Differentiating Neutrino Models on the Basis of $\theta_{13}$ and Lepton Flavor Violation

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Abstract.
We show how models of neutrino masses and mixings can be differentiated on the basis of their predictions for $\theta_{13}$ and lepton flavor violation in radiative charged lepton decays and $\mu - e$ conversion. We illustrate the lepton flavor violation results for five predictive SO(10) SUSY GUT models and point out the relative importance of their heavy right-handed neutrino mass spectra and $\theta_{13}$ predictions.

Keywords: Neutrino models, lepton flavor violation

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1. INTRODUCTION

Many models exist in the literature which attempt to explain the observed neutrino masses and mixings. The viable models agree with the presently known neutrino oscillation parameters falling within the $2\sigma$ ranges [1]:

\[
\begin{align*}
\sin^2 \theta_{12} &= 0.28 - 0.37, \\
\sin^2 \theta_{13} &= 0.38 - 0.63, \\
\sin^2 \theta_{13} &\leq 0.033.
\end{align*}
\]

The data suggests the approximate tri-bimaximal mixing texture of Harrison, Perkins, and Scott [2]:

\[
U_{PMNS} = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix},
\]

with $\sin^2 \theta_{23} = 0.5$, $\sin^2 \theta_{12} = 0.33$, and $\sin^2 \theta_{13} = 0$.

The reason for the plethora of models still in agreement with experiment of course can be traced to the inaccuracy of the present data. In addition, there are a number of unknowns that must be still determined: the hierarchy and absolute mass scales of the light neutrinos; the Dirac or Majorana nature of the neutrinos; the CP-violating phases of the mixing matrix; any departure of the reactor neutrino angle, $\theta_{13}$ from 0°; any departure of the atmospheric neutrino mixing angle, $\theta_{23}$, from 45°; whether the approximate tri-bimaximal mixing is a softly-broken or accidental symmetry; and the magnitude of the charged lepton flavor violation? In this presentation we survey some of the models to determine what they predict for $\theta_{13}$, the hierarchy, and lepton flavor violation.

2. THEORETICAL FRAMEWORK AND MODELS

The observation of neutrino oscillations implies that neutrinos have mass, with the mass squared differences given in Eq.(1). Information concerning the absolute neutrino mass scale has been determined by the combined WMAP, SDSS, and Lyman alpha data [3] which place an upper limit on the sum of the masses, $\sum m_i \leq 0.17$ eV. An extension of the SM is then required, and possible suggestions include one or more of the following:

- the introduction of dim-5 effective non-renormalizable operators;

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• the addition of right-handed neutrinos with their Yukawa couplings to the left-handed neutrinos;
• the addition of direct mass terms with right-handed Majorana couplings;
• the addition of a Higgs triplet with left-handed Majorana couplings.

The general $6 \times 6$ neutrino mass matrix in the $B(v_{\alpha L}, N'^\alpha_{\alpha L})$ flavor basis of the six left-handed fields then has the following structure in terms of $3 \times 3$ submatrices:

$$
\mathcal{M} = \begin{pmatrix}
M_L & M_N^T \\
M_N & M_R
\end{pmatrix},
$$

(3)

where $M_N$ is the Dirac neutrino mass matrix, $M_L$ the left-handed and $M_R$ the right-handed Majorana neutrino mass matrices. With $M_L = 0$ and $M_N << M_R$, the type I seesaw formula is obtained [3]

$$
M_\nu = -M_L^T M_R^{-1} M_N,
$$

(4)

for the light Majorana neutrinos, while if $M_L \neq 0$ and $M_N << M_R$, one obtains the type II seesaw formula [5],

$$
M_\nu = M_L - M_L^T M_R^{-1} M_N.
$$

(5)

The effective light Majorana mass matrix is complex symmetric and can be diagonalized by a unitary transformation, $U_{\nu L}$, to give

$$
M_\nu^{\text{diag}} = U_{\nu L}^T M_\nu U_{\nu L} = \text{diag}(m_1, m_2, m_3),
$$

(6)

with real, positive masses down the diagonal. On the other hand, the Dirac charged lepton mass matrix is diagonalized by a bi-unitary transformation according to

$$
M_E^{\text{diag}} = U_{E L}^T M_E U_{EL} = \text{diag}(m_e, m_\mu, m_\tau).
$$

(7)

The neutrino mixing matrix, $V_{PMNS}$ [6], is then given by

$$
V_{PMNS} \equiv U_{EL}^T U_{\nu L} = U_{PMNS} \Phi,
$$

$$
\Phi = \text{diag}(e^{i\chi_1}, e^{i\chi_2}, 1),
$$

(8)

in terms of the approximately tri-bimaximal mixing matrix, $U_{PMNS}$ and the phase matrix, $\Phi$, since an arbitrary phase rotation of $U_{\nu L}$ is not possible in the above.

Models which have been introduced to explain the neutrino oscillation phenomena fall into two categories. There are those based on some lepton flavor symmetry such as $\mu = \tau$ interchange symmetry, the more restrictive $S_3$ or $A_4$ flavor symmetry, and $SO(3)$ or $SU(3)$ flavor symmetries. Many attempts have been made to explore the location of texture zeros for the lepton mass matrices, with the hope that some flavor symmetry might be identified that way. A more ambitious class of models is based on $SU(5)$ or $SO(10)$ grand unification, where one attempts to explain the masses and mixings in the quark sector as well as the lepton sector. These models are said to have a “minimal” Higgs structure, if the Higgs bosons responsible for electroweak symmetry breaking transform as the $10$, $126$ dimensional representations of $SO(10)$, and possibly the $120$, $45$ or $54$ representations. They lead to symmetric or antisymmetric entries in the mass matrices. Other $SO(10)$ models have Higgs bosons which transform as $10$, $16$, $\overline{16}$, and $45$ representations and are referred to as “lopsided,” since lopsided contributions to the down quark and charged lepton mass matrices can occur due to the $SU(5)$ structure of the $16$’s. For recent reviews, cf. Ref. [7].

3. SURVEY OF MODEL PREDICTIONS FOR $\theta_{13}$

In a previous publication [8] we have made a survey of 63 models in the literature which give the large mixing angle (LMA) solution for the solar neutrino oscillations and firm and reasonably restrictive predictions for the reactor neutrino angle. The cutoff date for the selection of models was chosen to be May 2006. In this study we found most of the models predict a value in the range of $10^{-4} < \sin^2 \theta_{13} < 0.04$ with a normal hierarchy preferred by 3:1. The results are displayed in Fig. 1 in the form of a $\sin^2 \theta_{13}$ histogram, where the models are distinguished according to their type, and each is assigned the same area on the histogram though their imprecise predictions may cover several bins.
FIGURE 1. Histogram of $\sin^2 \theta_{13}$ predictions for all 63 models.

Since the planned Double CHOOZ and Daya Bay reactor experiments \cite{9} will reach $\sin^2 2\theta_{13} \sim 0.01$, roughly half of the models will be eliminated, either by the positive or negative signal of disappearance of $\bar{\nu}_e$’s from their beams. But even if a positive signal for the disappearance is seen, Fig. 1. indicates that of the order of 5 - 10 models will still survive, depending upon the accuracy of the measurement and the resilience of the models.

Meanwhile the MEG experiment \cite{10} at PSI is beginning to look for the $\mu \rightarrow e\gamma$ decay mode. With plans to lower the present branching ratio limit \cite{11} of $1.2 \times 10^{-11}$ down to the $10^{-13}$ range, this experiment may serve as an even more immediate selector of models. With this in mind, we turn to the subject of charged lepton flavor violation as a further distinguishing feature of the models proposed.

FIGURE 2. Example of a Feynman diagram for $\mu \rightarrow e\gamma$ with a neutrino mass insertion in the SM.
4. LEPTON FLAVOR VIOLATION IN RADIATIVE DECAYS

It is of interest to look first at charged lepton flavor violation in the SM with the addition of three massive right-handed neutrinos. In this case, individual $L_e$, $L_\mu$ and $L_\tau$ lepton numbers are not conserved since Majorana mass terms are not forbidden. The lepton flavor violation then arises in 1-loop diagrams, where the neutrino insertion involves lepton flavor-changing Yukawa couplings. Examples of such diagrams are given in Fig. 2 for the process $\mu \rightarrow e\gamma$. The $U_{ik}$ are elements of the PMNS mixing matrix, while the $Y_{ik}$ are Yukawa couplings for $i, k = 1, 2, 3,$ and $N^c_i$ is one of the heavy left-handed conjugate neutrinos. The branching ratio is given by \[ BR_{21} \equiv \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow \nu\mu\nu\bar{\nu})} \]
\[ = \frac{3\alpha}{32\pi} \sum_k |U_{\mu k}^* m^2_{N^c_k} U_{e k}|^2 \]
\[ \simeq 3\alpha 128\pi \left( \frac{\Delta m^2_{21}}{M^2_{G}} \right)^2 \sin^2 2\theta_{12} \sim 10^{-54}. \] (9)

In this extended SM, the branching ratio is immeasurably small, due to the approximate GIM cancellation and the extremely small mass ratios of the left-handed neutrinos to the massive right-handed neutrinos. For such models, the MEG experiment would be expected to give a null result.

In SUSY GUT models, slepton - neutralino and sneutrino - chargino loops contribute to the radiative decays as shown in Fig. 3.

In the constrained MSSM (CMSSM) with universal soft masses and trilinear couplings \[13\], charged lepton flavor violation arises from evolution of the Yukawa couplings and soft SUSY-breaking parameters. Thus with more comparable heavy masses in the loops and no GIM mechanism anticipated, the LFV branching ratios can be much larger \[14\].

In the leading log approximation the largest contribution comes from the left-handed slepton mass matrix yielding a branching ratio \[ BR_{21} = \frac{\alpha^3}{G_F m^8_s} |(m^2_{LL})_{ji}|^2 \tan^2 \beta, \] (10)
where \[ (m^2_{LL})_{ji} = -\frac{1}{8\pi^2} m^2_0 (3 + A^2_0/m^2_0) Y^+_{jk} \log \left( \frac{M_G}{M_k} \right) Y_{ki}. \] (11)
with the Yukawa couplings specified in the lepton flavor basis and the right-handed Majorana matrix diagonal, so $M_k$ is just the kth heavy right-handed neutrino mass, while $M_G$ is the GUT scale typically equal to $2 \times 10^{16}$ GeV and $m_s$ is some typical SUSY scalar mass. Petcov and collaborators \[15\] have shown that the full evolution effects as first calculated by Hisano, Moroi, Tohe, and Yamaguchi \[16\] can be extremely well approximated by Eq. (10), if one sets \[ m^8_s \simeq 0.5 m^2_0 M^2_{1/2} (m^2_s + 0.6 M^2_{1/2})^2. \] (12)

We shall see that for the SUSY GUT models to be considered, the MEG experiment will be able to observe the predicted $\mu \rightarrow e\gamma$ branching ratios or place further restrictions on those models.
5. GENERIC APPROACH TO LEPTON FLAVOR VIOLATION IN SUSY GUTS

Many papers have studied predictions for the lepton flavor violating processes in SUSY GUT models by adopting a generic approach [17]. Following the procedure of Casas and Ibarra [18], with the charged lepton and right-handed Majorana neutrino mass matrix diagonal, the seesaw formula can be inverted to yield the Yukawa neutrino coupling matrix

\[ Y_V = \frac{1}{\sqrt{2} \sin \beta} D_N \sqrt{M_R} D_V \sqrt{M_D} U_{PMNS}^T \]  

in terms of a complex orthogonal \( R \) matrix which allows for various unknown right-handed neutrino mixings. By using soft SUSY-breaking benchmarks, adopting various heavy right-handed neutrino masses and various \( R \) parametric angles and phases, one can "predict" the radiative LFV branching ratios. The results are typically presented in the form of Monte Carlo scatter plots. One finds the results strongly depend on \( \tan \beta \), \( M_3 \), \( \theta_{13} \), and the \( R \) parameters. The branching ratios are found to increase by powers of 10 as \( \theta_{13} \) varies from \( 0^\circ \rightarrow 10^\circ \), while the present branching ratio bound for \( \mu \rightarrow e\gamma \) restricts \( M_3 \) to lie lower than \( 10^{12} \sim 10^{13} \) GeV.

6. EXAMPLES OF PREDICTIVE SUSY GUT MODELS

Instead of adopting a generic approach as described above, we are interested in determining whether one can differentiate between various SUSY GUT models on the basis of their LFV predictions, even if they have similar predictions for \( \sin^2 \theta_{13} \). For this purpose we have selected five \( SO(10) \) SUSY GUT models which are highly predictive, i.e., their model parameters are precisely specified by the authors.

The models differ in the flavor symmetry chosen and the Higgs representations used to break the \( SO(10) \) symmetry at the GUT scale and the electroweak symmetry at the weak scale. We list the models, references, flavor symmetry and Higgs representations.

- AB (Albright-Barr) \([19] \): \( U(1) \times Z_2 \times Z_2 \) with \( 10, 16, 45 \)
- CM (Chen-Mahanthappa) \([20] \): \( SU(2) \times (Z_2)^3 \) with \( 10, 126 \)
- CY (Cai-Yu) \([21] \): \( S_4 \) with \( 10, 126 \)
- DR (Dermisek-Raby) \([22] \): \( D_3 \) with \( 10, 45 \)
- GK (Grimus-Kuhblok) \([23] \): \( Z_2 \) with \( 10, 120, 126 \)

Salient features of each model are listed in Table I. The CM model has a relatively large prediction for \( \sin^2 \theta_{13} \) which should be easily accessible to the future Double CHOOZ and Daya Bay reactor experiments. The AB, CY and DR model predictions for \( \sin^2 \theta_{13} \sim 0.0025 \) are barely within reach of those experiments. The GK model prediction would most likely require a neutrino factory to reach that level. Note that relatively low values of \( \tan \beta \) are preferred for four of the models, while the DR model favors a high value. Relatively mild hierarchies for the massive right-handed neutrinos are predicted for the CM, DR and GK models, but the heaviest one occurs in increasing order for these three models, ranging from \( 7 \times 10^{12} \) to \( 2 \times 10^{13} \) GeV. The CY model predicts a degenerate right-handed spectrum with a lower value of \( 2.4 \times 10^{12} \). The AB model, on the other hand, puts the heaviest one at \( 2.4 \times 10^{14} \) GeV and the lower two nearly degenerate at \( 4.5 \times 10^8 \) GeV. Resonant leptogenesis \([24] \) is possibly an interesting feature of that model. The five predictive models studied in this work thus cover a wide range of possibilities and suggest that charged lepton flavor violation can play an important role in further narrowing the list of viable candidates.

7. RADIATIVE LEPTON FLAVOR VIOLATION PREDICTIONS

We now turn to the predictions for the branching ratios for \( \mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma \) and \( \tau \rightarrow e\gamma \) in the five predictive models under consideration. We use the shorthand convention \( BR_{ij}, BR_{32}, \) and \( BR_{31} \) for these respective branching ratios. Working in the CMSSM scenario with universal soft parameters \( m_0, M_1/2 \) and \( A_0 \) for a given \( \tan \beta \) and \( \text{sgn}(\mu) \), we find a variety of plots are possible:

- \( BR_{ij} \) vs. \( M_{1/2} \) for fixed \( A_0 = 0 \) and different choices of \( m_0 \).
- \( A_0/m_0 \) vs. \( M_{1/2} \) scatterplot with a color scheme to indicate the branching ratio ranges.
TABLE 1. Higgs representations, flavor symmetries, and other noteworthy features of the five $SO(10)$ SUSY GUT models considered in this work.

| Models | Higgs Content | Flavor Symmetry | $M_R$ (GeV) | $\tan \beta$ | $\sin^2 \theta_{13}$ | Interesting Features |
|--------|---------------|----------------|------------|-------------|----------------------|---------------------|
| AB     | 10, 16, 126   | $U(1) \times Z_2 \times Z_2$ | $2.4 \times 10^{14}$ | 5 | 0.0020 | (2.6$^7$) | Large $M_R$ hierarchy with lightest two nearly degenerate leads to resonant leptogenesis. |
| CM     | 10, 126       | $SU(2) \times (Z_2)^3$ | $7.0 \times 10^{12}$ | 10 | 0.013 | (6.5$^5$) | Large $M_R$ hierarchy with heaviest more than 3 orders of magnitude below GUT scale; large $\sin^2 \theta_{13}$. |
| CY     | 10, 126       | $S_4$ | $2.4 \times 10^{12}$ | 10 | 0.0029 | (3.1$^5$) | Degenerate $M_R$ spectrum 4 orders of magnitude below GUT scale. |
| DR     | 10, 45        | $D_3$ | $5.8 \times 10^{13}$ | 50 | 0.0024 | (2.8$^5$) | Mild $M_R$ hierarchy almost 3 orders of magnitude below GUT scale. |
| GK     | 10, 120, 126  | $Z_2$ | $2.0 \times 10^{15}$ | 10 | 0.00031 | (1.0$^5$) | Mild $M_R$ hierarchy just 1 order of magnitude below GUT scale; rather small $\sin^2 \theta_{13}$. |

- Ratio of the branching ratios, BR32/BR21 and BR31/BR21 on log-log plots, where for example,

$$\log BR32 = \log BR21 + \log \left(\frac{(Y^2_{ν_L} Y_{ν_R})_{32}}{(Y^2_{ν_L} Y_{ν_R})_{21}}\right)^2$$

with $L \equiv \log(M_3^2/M_2^2)$. Due to the factorization of the soft-breaking parameters and the GUT model parameters in the approximation of Eqs. (11) and (12), the above yields a straight line with unit slope and intercept given by the second term on the right. The length of the straight line depends on the soft parameter constraints applied.

For the sake of brevity, we shall present only the third type of plots. For a more complete description we refer the reader to a longer paper submitted for publication $[25]$. We have imposed the following soft parameter constraints $[26]$:

For $\tan \beta = 5, 10$:

- $m_0 :$ 50 $\rightarrow$ 400 GeV
- $M_{1/2} :$ 200 $\rightarrow$ 1000 GeV
- $A_0 :$ $-4000$ $\rightarrow$ 4000 GeV

For $\tan \beta = 50$:

- $m_0 :$ 500 $\rightarrow$ 4000 GeV
- $M_{1/2} :$ 200 $\rightarrow$ 1500 GeV
- $A_0 :$ $-50$ $\rightarrow$ 50 TeV

(15)

In addition it is desirable to impose WMAP dark matter constraints $[3]$ in the neutralino, stau or stop coannihilation regions $[27]$, where the lightest neutralino is the LSP. These more restrictive constraints are well described by the quadratic polynomial for the soft scalar mass in terms of the soft gaugino mass $[28]$:

$$m_0 = c_0 + c_1 M_{1/2} + c_2 M_{1/2}^2,$$

where $c_i = c_i (A_0, \tan \beta, \text{sgn} (\mu)).$

(16)

where $m_0$ is bounded since $M_{1/2}$ is bounded. If $M_{1/2}$ is too small, the present experimental bound on the Higgs mass of $m_H \geq 114$ GeV may be violated or the neutralino relic density in the early universe will be too small, while if $M_{1/2}$ is too large the neutralino relic density will be too large.
FIGURE 4. Branching ratio predictions for $\tau \to \mu + \gamma$ vs. branching ratio predictions for $\mu \to e + \gamma$ in the five models considered. The soft SUSY breaking constraints imposed apply for the thin line segments, while the more restrictive WMAP dark matter constraints apply for the thick line segments. The present experimental constraints are indicated by the dashed lines.

FIGURE 5. Branching ratio predictions for $\tau \to e + \gamma$ vs. branching ratio predictions for $\mu \to e + \gamma$ in the five models considered. The same additional conventions apply as in Fig. 4,
In Figs. 4 and 5 we have plotted BR32 and BR31 vs. BR21 on log-log graphs. The thin line segments for each model observe the soft parameters constraints imposed, while the heavier line segments observe the more restrictive WMAP dark matter constraints. The vertical dashed line reflects the present BR21 bound \cite{11}, while the horizontal dashed line refers to the present BR32 or BR31 experimental limit, respectively \cite{29}. It is clear from these two plots that the ongoing MEG experiment stands the best chance of confirming the predictions for or eliminating the GK and AB models. Even with a super-B factory, the present experimental bounds on the BR32 and BR31 branching ratios can only be lowered by one or two orders of magnitude at most \cite{30}.

The above figures apply in the case where $A_0 = 0$ is selected for the common trilinear scalar coupling. We show in \cite{25} that as $|A_0|$ is allowed to depart from zero, the predicted branching ratios increase. Hence the line segments in Figs. 4 and 5 represent lower limits and extend upward somewhat at $45^\circ$ as $|A_0|$ is increased.

8. LEPTON FLAVOR VIOLATION IN $\mu - e$ CONVERSION

Lepton flavor violation can also occur in the $\mu - e$ conversion process in $^{81}Ti$, where $\mu + Ti \rightarrow e + Ti$. The $\mu - e$ conversion branching ratio is the conversion rate scaled by the capture rate for the process, $\mu + Ti \rightarrow \nu_\mu + Sc$. The one-loop diagrams involving $\gamma$, $Z$ and Higgs penguins all contribute along with box diagrams, but in the CMSSM scenario the $\gamma$ penguin has been shown to dominate \cite{31}. We show two such diagrams involving slepton-neutralino and sneutrino-chargino loops in Fig. 6, where the effects of the virtual $N_L^c$ and $\tilde{N}_L^c$ with their Yukawa couplings appear in slepton loops.

The $\mu - e$ conversion branching ratio on Ti vs. BR21 is plotted in Fig. 7 for the five GUT models, where the tighter WMAP dark matter constraints have been imposed, again for the case of $A_0 = 0$. The present conversion branching ratio limit for the Ti experiment \cite{32} is shown at $4 \times 10^{-12}$. It is projected that such an experiment will be able to reach down to $10^{-17}$ in a first round and down to $10^{-18}$ in a second generation experiment. While the expectation is that the MEG experiment may be able to reach a limit of $10^{-13}$ branching ratio for $\mu \rightarrow e\gamma$, it is apparent that a $\mu - e$ conversion experiment which eventually reaches a limit of $10^{-18}$ will be considerably more powerful. Potentially such an experiment would be able to eliminate all five models, and probably all such SUSY GUT models in the CMSSM scenario, if no positive signal is found.

![Figure 6](image-url)  
**FIGURE 6.** Examples of Feynman diagrams for slepton - neutralino and sneutrino - chargino contributions to $\mu - e$ conversion in SUSY models with slepton mass insertions.
FIGURE 7. Branching ratio predictions for $\mu \rightarrow e$ conversion vs. branching ratio predictions for $\mu \rightarrow e + \gamma$ in the five models considered. The more restrictive WMAP dark matter constraints apply for the thick line segments shown. Note that the predictions for the CM and CY models nearly overlap.

9. CONCLUSIONS

We have tried to differentiate models based on $\sin^2 \theta_{13}$ and charged lepton flavor violation predictions. Our study was initially based on 63 models available in the literature prior to June 2006. There we found that a normal neutrino mass hierarchy is preferred 3:1. Moreover, future Double CHOOZ and Daya Bay reactor experiments will be able to eliminate roughly half of the 63 neutrino models surveyed, if their sensitivity reaches $\sin^2 2\theta_{13} \simeq 0.01$ as planned. Still of the order of five models have similar values for $\sin^2 \theta_{13}$ in the interval 0.001 - 0.08. We have suggested that charged lepton flavor violation experiments may be able to further distinguish them. If the now-running MEG experiment sees positive signals for $\mu \rightarrow e \gamma$, all non-SUSY models or models which do not involve new physics will be ruled out.

We then narrowed our study to five predictive $SO(10)$ SUSY GUT models in the literature. All five models have type I seesaw mechanisms implying normal hierarchy. Their predictions for $\sin^2 2\theta_{13}$ lie in the ranges of 0.05 for the Chen-Mahanthappa model, 0.01 for the Albright-Barr, Cai-Yu, and Dermisek-Raby models, and 0.001 for the Grimus-Kubock model. Previous studies of generic $SO(10)$ models have concluded that the LFV branching ratios depend critically on $\theta_{13}$ and the heaviest right-handed neutrino mass, $M_3$. Here we find that $M_3$ appears to be more important. If the MEG experiment can reach an upper bound of $10^{-13}$ for the $\mu \rightarrow e \gamma$ branching ratio, it will rule out the Grimus-Kubock and Albright-Barr models if no positive signal is seen. If a $\mu \rightarrow e$ conversion experiment can be performed and reach a branching ratio limit of $10^{-18}$ as projected, it can potentially rule out all five models considered.

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