Influence of Bearing’s Flexibility on the Working of Cycloid Drive

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Abstract. In this article are reviewed calculate method of force distribution with accounting bearing flexibility on cycloid reducer elements of the types KHV and 2KV. There are analyzed schemas such as gear without machining errors, with constant errors on the pins and with random errors from machining tolerance. Schemas of force distribution on the pins of the types KHV and 2KV are presented. Relation between bearings flexibility and force distribution on the pins are determined, and also relation between machining errors and force distribution on the pins. Advantages and disadvantages of various methods of load estimation in the reducer are elucidated, appliance of this methods for calculation of various types cycloid reducers are analyzed.

1. Introduction

Modern engineering requires that gears possess such properties as compactness, rigidity and accuracy. These characteristics correspond to the cycloid speed reducers, which is widely used as rotary axes of machines, manipulators [1,2] and other technological equipment. Achievement of such requirements is obtained due to the multiple-tooth contact, however, this leads to the fact that the cycloid reducer forms a statically uncertain system and, as a result, special requirements to the processing accuracy are applied and the force calculation becomes more complicated. The bearing rigidity makes a significant contribution to the accuracy of the reducer. Thus, the development of cycloid reducer new models, their optimization, production and implementation are quite expensive processes.

Let's consider the most promising cycloid reducers such as KHV and 2KV according to Kudravtsev [3]. The KHV gear (fig. 1.a) consist of input shaft with an eccentric $h$, which confers the cycloidal gear rotation movement $g[4]$. This movement is limited by the mechanism of parallel cranks $w$ relative to the output shaft $v$. 
Figure 1. Kinematic schemas

The gear 2KV (fig. 1.b) consists of a center gear with involute teeth a, it engages with several spur gears g, which are positioned on eccentric shafts h, and cycloidal gear f, which engages with the pin wheel b, and carrier v. Given that are several eccentric shafts, they work like parallel cranks.

2. Calculation methods
The existing method of calculation assumes that all the details of the cycloidal reducers are considered absolutely rigid, except for the points of contact of the cycloidal gear with the pins [5]. The analytical model of this method is presented in fig. 2.a. The pin wheel turns on angle $\beta$, when subjected to external torque $T$. Contact friction is neglected, the reducer is produced without errors, relation between the normal force and the contact displacement is linear.

Given these assumptions, the strength in the $i$-th pin will be equal to:

$$F_i = \tau_i \delta_i = \tau_i \beta (P_i \times N_i),$$

where $\delta_i$ – contact displacement of the cycloidal gear with the $i$-th pin [6], $\tau_i$ – rigidity of contact between cycloidal gear and $i$-th pins, $P_i$ – radius vector of the trochoid profile at the point of contact with the $i$-th pin from the origin of the cycloidal gear, $N_i$ – normal to the trochoid profile at the point of contact with the $i$-th pin (fig. 3).
Then sum of the moments will be equal to the moment applied to the pin wheel:

$$\sum (P_i \times N_j)^2 \beta = T \quad (2)$$

Following that we substitute [7] the values then the force in the pin will be equal to:

$$F_i = \frac{T}{4(ez_i)^2} \frac{\sin(zc_i)}{\sqrt{1+\lambda^2-2\lambda\cos(zc_i)}} \quad (3)$$

The maximum force will act on the pin, which is opposite the pole of the cycloidal gear. This method calculation is valid for the gear of produced with high accuracy, and the distribution of the load among simultaneously operating pins is satisfied.

In order for accounting machining tolerances, the following method was proposed (fig.2.b). The cycloidal gear, the pin wheel, the carrier, the disk of the mechanism of parallel cranks and other parts of the reducer are considered absolutely rigid. The points of contact the cycloidal gear with the i-th pin and the points of contact the cycloidal gear with the j-th finger are considered flexible. Their flexibility is communicated with the relative movement of part's points. Contact friction is neglected. The reducer is produced with errors, which are characterized by the total error vector $\Delta_i$.

Consequently, under the action of external forces, the cycloidal gear moves by the value of $u$ along the x-axis and by the value of $v$ along the y-axis, together this the cycloidal gear is rotated by the angle $\alpha$ and the pin wheel by the angle $\beta$. We write these four unknown parameters in the form of a vector:

$$\chi = \{u \ v \ \alpha \ \beta\}^T \quad (4)$$

$B_i$ is denoted the matrix that converts the cycloidal gear and the pin wheel position vector $\chi$ into the local displacement vector $\delta_i$:

$$\delta_i = B_i \chi + \Delta_i \quad (5)$$

$F_i$ is denoted a vector characterizing the loads with which the bearings and contact points act on the cycloidal gear in the direction of local displacements $\delta_i$. We introduce the matrix $D_i$ to convert the vector of local displacements $\delta_i$ into the vector $F_i$,

$$F_i = D_i \delta_i = D_i (B_i \chi + \Delta_i) \quad (6)$$

Denote by $R$ - the vector of external loads acting on the reducer:

$$R = \begin{pmatrix} F_{cx} \\ F_{cy} \\ 0 \\ T \end{pmatrix} \quad (7)$$

where $F_{cx}, F_{cy}$ – projection of centrifugal force, which acts on the cycloidal gear along the axes.

After some calculation we produce:

$$[\sum_{i=1}^{n} B_i^T D_i B_i] \chi = R - \sum_{i=1}^{n} B_i^T D_i \Delta_i. \quad (8)$$

Solving this system of equations, we obtain the values of global displacements, then we can determine the reactions in the points of contact.
Consider the different types of contact points and the corresponding matrices $B$ and $D$.

For contact points of the cycloidal gear with the $p$-th pin, the matrix $B$ has the form:

$$
B_p = \begin{bmatrix}
N_p x \\
N_p y \\
N_p x - P_p y N_p x \\
e(N_p y \cos \varphi - N_p x \sin \varphi) + P_p x N_p y - P_p y N_p x
\end{bmatrix}.
$$

(9)

The matrix $D_p$ for converting the vector of local displacements $\delta_p$ into the vector $F_p$ (if $\delta_p > 0$):

$$
D_p = \begin{cases}
C, & \text{if } \delta \geq 0; \\
0, & \text{if } \delta < 0,
\end{cases}
$$

(10)

where $C$ - the rigidity of the contact of the cycloidal gear with the $p$-th pin.

For contact points of the cycloidal gear with the $f$-th finger of parallel cranks mechanism, the matrix $B$ has the form:

$$
B_f = \begin{bmatrix}
N_f x \\
N_f y \\
N_f x - P_f y N_f x \\
0
\end{bmatrix}.
$$

(11)

The matrix $D_f$ will be equivalent to $D_p$. For contact points of the cycloidal gear with the $b$-th bearing, the matrix $B$ has the form:

$$
B_b = \begin{bmatrix}
1 & 0 & -P_b y & 0 \\
0 & 1 & P_b x & 0
\end{bmatrix}.
$$

(12)

and matrix $D_b$

$$
D_b = \begin{bmatrix}
C_b & 0 \\
0 & C_b
\end{bmatrix},
$$

(13)

where $C_b$ - the rigidity of $b$-th bearing [8].

The task is solved iteratively, because of the number of working pins is unknown in advance.

### 3. Case study

The results of the calculation are presented in Figure 4, which shows the strength of constraints effect on the cycloidal gear. The red color indicates the strength of the pins, the green color indicates the fingers, and the blue color indicates the bearings.
c. forces distribution in KHV with constant error (-0.005 mm) of diameter pins
d. forces distribution in 2KV with constant error (-0.005 mm) of diameter pins
e. forces distribution field in KHV with random errors from tolerance (-0.005 mm, 0) of diameter pins
f. forces distribution field in 2KV with random errors from tolerance (-0.005 mm, 0) of diameter pins

Figure 4. The results of the calculation

It will be seen from Fig. 4.a and 4.b, when calculating a gear produced without any machining errors [9], the distribution of forces of pins corresponds to the traditional method of calculation. If we (Fig. 4.c, 4.d) add constant machining error (-0.005 mm) of diameter pins, then less number of pins are loaded and the maximum force are increase. In both cases, the KHV scheme is less desired, because of bearing flexibility on the eccentric and the reaction of the mechanism of parallel cranks make a great contribution to forces distribution and the most loaded pin is shifted along of the external moment. However, usually the pins are produced with different accuracy. To calculate, we took random errors from the machining tolerance (Fig. 4.e, 4.f). Forces distribution fields turned out to be more uniform for reducers KHV and 2KV.

4. Summary
The traditional calculation method gives a good result for 2KV reducer. If we take into account bearings flexibility and the machining tolerances of all parts, then we can obtain a more reliable forces distribution. Accounting for bearings flexibility increases the maximum force in pins by 30%, and accounting for machining errors increases the maximum force in pins twice. In case of cycloidal gear from polyamide the number of working pins increase on 2-3 pieces [10].
5. References
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