Transmitter Localization using Quantum Sensor Networks

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Abstract—Quantum sensors (QSs) are able to measure various physical phenomena with extreme sensitivity. QSs have been used in several applications such as atomic interferometers, but very few applications of quantum sensor networks (QSNs) have been proposed or developed. We look at a natural application of QSNs—localization of an event (in particular, of an RF transmission). In this paper, we develop a viable technique for the localization of a radio-frequency (RF) transmitter using QSNs. Our approach poses the localization problem as a well-studied quantum state discrimination problem, and addresses the challenges in its application to the localization problem. In particular, a quantum state discrimination solution can suffer from high probability of error, especially when the number of states (i.e., number of potential transmitter locations, in our case) can be high. We address this challenge by developing a two-level localization approach, which localizes the transmitter in a coarser and finer way in the respective levels. We evaluate our approaches on a custom-built QSN simulator, and our evaluation results show that our proposed techniques achieve high accuracy in simulated settings.

I. Introduction

Quantum sensors, being strongly sensitive to external disturbances, are able to measure various physical phenomena with extreme sensitivity. These quantum sensors interact with the environment and have the environment phenomenon or parameters encoded in their state [6]. A group of distributed quantum sensors, if prepared in an appropriate entangled state, can further enhance the estimation of a single continuous parameter, improving the standard deviation of measurement by a factor of $1/\sqrt{m}$ for $m$ sensors (Heisenberg limit) [8]. Recently, experimental physicists successfully demonstrated a reconfigurable entangled radio-frequency photonic sensor network [18], [17]. The experiments establish a connection between the entanglement structure and the achievable quantum advantage in different distributed sensing problems.

Recently, many protocols have been developed for estimation of a single parameter or multiple independent parameters [8], [12] using one or multiple (possibly, entangled) sensors. But, the use of a distributed set of QSs working collaboratively to estimate more complex physical/environmental phenomena has not been explored much. In this paper, we develop a scheme for the accurate localization of events using a quantum sensor network (QSN); in particular, we consider the localization of an RF transmitter and develop a viable technique that demonstrates a novel use of QSNs in a practical application.

RF localization is a key technology for location-based services. An improvement in RF localization will be very beneficial to an array of mobile applications and thus generate huge economic effects. For example, accurate localization in supermarkets, libraries, museums, and augmented reality applications can be very useful. In addition, in shared spectrum systems [4], we may want to guard the spectrum by detecting and localizing unauthorized transmitters that jam the spectrum illegally. Classical techniques for RF transmitter localization involve localization via triangulation [19] or fingerprinting techniques [2] (see [5] for a survey).

Potential of Quantum Sensors for Precise Localization. Quantum sensing is an emerging field that leverages the quantum properties of light and matter at atomic/subatomic scales and has the potential to sense signals at an unprecedented level of precision. For example, physicists in the year 2016 used squeezed quantum states to improve the sensitivity of the Laser Interferometer Gravitational-wave Observatory (LIGO) detector and successfully detected gravitational waves. In [18], researchers use some distributed entangled RF-photonic quantum sensors to estimate the amplitude and phase of a radio signal, and the estimation variance beats the standard quantum limit by over 3 dB. In RF-photonic sensors, RF signals are carried over to the optical domain by electro-optic transducers.

Motivated by the above, in this paper, we develop techniques for localization using a distributed quantum sensor network.

Transmitter Localization using QSNs. Our approach to transmitter localization using QSNs essentially involves posing the localization problem as a quantum state discrimination problem [3] which is to identify the specific state a given quantum state is in (from a given set of states in which the system can be) by performing quantum measurements on the given quantum system. The overall architecture is illustrated in Fig. 1. First, a probe state is generated and distributed to the QSN. Then, once the quantum sensors have been impacted (i.e., the overall quantum state changed) due to the transmission from the transmitter’s signal, an appropriate quantum global measurement is made on the set of quantum sensors.

The outcome of the measurement determines the quantum state, and thus, the location of the transmitter. However, the above process can be erroneous, as the state determined may be incorrect. This paper’s goal is to develop an approach with a minimal probability of error.
Contributions. To the best of our knowledge, ours is the first work to investigate an event-localization technique using quantum sensor networks. In this context, we make the following key contributions.

1) We frame the event-localization problem as a well-studied quantum state discrimination (QSD) problem, which allows us to develop viable event-localization schemes using quantum sensors.

2) For the specific case of RF transmitter localization, we design various localization schemes, viz., OneLevel, TwoLevel and TwoLevel-Pro, to accurately localize a RF transmitter in a given area deployed with quantum sensors.

3) We model how a quantum sensor’s state evolves due to received power from a transmitter at a certain distance. Using this model, we evaluate our localization schemes and demonstrate their effectiveness.

II. Sensor Model, Problem, and Related Work

In this section, we start with modeling the impact of a received signal on a quantum sensor state. We then formulate the quantum localization problem, and discuss related work.

Quantum Sensor Model. Impact on a quantum sensor due to a physical phenomenon is typically modeled by an appropriate unitary operator. In our context, a quantum sensor evolves due to the received power over a time window. Let the initial pure state of the sensor be \( |\psi_0\rangle \), and let the impact on the sensor due to received power be represented by the unitary operator \( U(\alpha) \) where \( \alpha \) depends on the received power, fixed time window over which the sensor’s state evolves, etc. The evolved state of the sensor is given by: \( |\psi_\alpha\rangle = U(\alpha) |\psi_0\rangle \). The unitary operator \( U(\alpha) \) describing the interaction between a qubit and the environment is modeled as \[ U(\alpha) = e^{-i\alpha G} \tag{1} \]

where \( G \) is the generator, which is a Hermitian operator. \( G \) depends on the type of \( \alpha \) parameter \[ \alpha \] which includes quadrature displacement [18], phase shift [10], qubit phase rotation, etc. In our context, \( \alpha \) is assumed to be a phase shift, and thus, our generator follows the expression in [10]: \( \hat{G} = \sigma_z / 2 \), where \( \sigma_z \) is the Pauli-Z matrix. The phase shift \( \alpha \) is fundamentally a function of the received power \( P_R \), and we assume \( \alpha \) to be directly proportional to \( P_R \).

\[ \alpha = c P_R \tag{2} \]

where \( c \) is an appropriate constant. From the above, we get

\[ \hat{U}(\alpha) = \begin{bmatrix} e^{-icP_R} & 0 \\ 0 & e^{icP_R} \end{bmatrix} \tag{3} \]

Eqn. [3] models how a quantum sensor evolves due to a received signal from a transmitter; in essence, the transmitter’s signal triggers a phase shift in the quantum sensor.

Finally, we assume a simple signal propagation model, viz., the log-distance model which gives \( P_R \) (in dBm) as:

\[ P_0 - 10\beta \log_{10}(d) + \chi \tag{4} \]

where \( P_0 \) is the transmit power (in particular, the power received at 1 m away from the transmitter), \( d \) is the Euclidean distance between the transmitter and the sensor, \( \beta \) is the path-loss exponent, and \( \chi \) is the shadowing effect that can be represented by a zero-mean Gaussian distribution.

Impact on Multiple Quantum Sensors. Consider a set of \( m \) quantum sensors distributed over an area, with a combined \( m \)-dimensional quantum state of \( |\psi_0\rangle \). Consider an intruder at a certain specific location in the area. Let \( U(\alpha_i) \) be the impact on the \( i \)th sensor due to the transmitter. Then, the overall impact on the combined quantum state is represented by a tensor product of \( m \) individual unitary operators, i.e., \( \hat{U}(\alpha) = \bigotimes_{i=1}^{m} \hat{U}(\alpha_i) \), and the evolved state is \( \hat{U}(\alpha) |\psi_0\rangle \).

Problem Definition. Consider a network of quantum sensors distributed in a geographic area and a potential transmitter/intruder in the area. Let the initial state of the system of quantum sensors network be \( |\psi_0\rangle \). As described above, due to the transmission from the intruder, the quantum state evolves to \( |\psi_\alpha\rangle = U(\alpha) |\psi_0\rangle \). The transmitter localization problem is to determine the location of the transmitter based on the evolved quantum state \( |\psi_\alpha\rangle \).

Related Work. Radio transmitter localization using a set of sensors/receivers has been widely studied [21], [5]. Localization methods can be roughly classified into two types: geometry-based and fingerprinting-based. The geometry-based method includes multilateration (by measuring time-of-flight between the transmitter and multiple sensors) or triangulation (by measuring angle-of-arrival of the transmitter at multiple sensors) [19]. The fingerprinting-based method [2] has a training phase which records the signal fingerprint for certain locations; localization is then achieved by matching the real-time signal to the recorded fingerprint(s). Here,
Parameter Estimation using Quantum Sensors. Prior works on parameter estimation using quantum sensors include: estimation of single [8] or multiple independent parameters [12], estimation of a single linear function over parameters [11], and estimation of multiple linear functions [13]. Our transmitter localization problem can be looked upon as a novel single parameter (TX location) estimation problem based on sensor measurements that are functions (based on signal propagation model and distance) of the parameter being estimated.

III. Methodology and Proposed Schemes

Quantum State Discrimination (QSD). Given a quantum state $|\phi\rangle$ that is known to be equal to one of the states (known as target states) in the set $\{|\phi_1\rangle, |\phi_2\rangle, \ldots, |\phi_n\rangle\}$, the quantum state discrimination (QSD) problem is to determine which state $|\phi\rangle$ really is. In general, each target state $|\phi_i\rangle$ may be associated with a prior probability $q_i$; in this paper, we assume uniform prior. The QSD problem is typically solved using a series of measurements or a single measurement—as defined below. It is known that if the target states $\{|\phi_i\rangle\}$ are not mutually orthogonal, then there is no quantum measurement capable of perfectly (without error) distinguishing the states. Thus, a QSD solution may give an erroneous answer—i.e., guess the state to be in $|\phi_j\rangle$ when the state is really in $|\phi_j\rangle$ for some $i \neq j$. Thus, a QSD solution is associated with an overall probability of error (PoE), and the optimization goal of the QSD problem is to determine the measurement (or a sequence of measurements) that minimizes the PoE.

General Measurements. A general measurement [11] is defined by matrices $M_1, M_2, \ldots, M_n$ such that $\sum_i M_i^\dagger M_i = I$ where $M_i^\dagger$ is the conjugate transpose of $M_i$. If this general measurement is carried out on a pure state, we see the outcome “i” with probability $p(i) = \langle \phi | M_i^\dagger M_i | \phi \rangle$. Thus, if we associate the outcome “i” with the given state $|\phi\rangle$ being in the target state $|\phi_i\rangle$, the probability of error (PoE) for the given measurement $\{M_i\}$ is given by $\sum_i \sum_{j \neq i} \langle \phi_i | M_i^\dagger M_j | \phi_i \rangle$.

If we are only interested in the probability of outcomes (as in our context), the above general measurement can also be represented by the set of positive semi-definite matrices (PSD) $\{E_i = M_i^\dagger M_i\}$ where $\sum_i E_i = I$. This representation is called positive-operator valued measure (POVM); in this paper, we use this representation of measurement for simplicity.

Core Idea: TX Localization as QSD. Consider a geographic area where a transmitter can be at a set of potential locations $\{l_1, l_2, \ldots, l_n\}$. For simplicity, let us assume that the transmission power is constant. Let the initial state of the quantum system, composed of say $n$ distributed quantum sensors, be $|\psi_0\rangle$. When the transmitter $T$ is at a location $l_i$, let the impact of the $T$’s transmission from location $l_i$ evolve the overall state of the quantum system to $|\psi_i\rangle$ based on the model described in the previous section. Now, consider the set of target states $\{|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_n\rangle\}$ corresponding to the set of potential locations of the transmitter. Then, localizing the transmitters, i.e., determining the location $l_i$ from where the transmission occurred, is tantamount to solving the QSD problem with the target states $\{|\psi_i\rangle\}$ and thus determining the state the quantum system has transformed to as a result of the transmission.

Selection of Initial State and Measurement. In the above context, our goal is to select an initial state $|\psi_0\rangle$ and the POVN measurement (i.e., PSD matrices $\{E_1, \ldots, E_n\}$, one for each possible outcome/location) such that the overall PoE is minimized — for a given setting of transmitter location, quantum sensors, and signal propagation model. The optimization problem of selecting an optimal combination of initial state and POVM in our context is beyond the scope of this work. Here, we use a non-entangled uniform superposition pure initial state $|\psi_0\rangle = \sum_{i=0}^{n-1} \frac{1}{\sqrt{n}} |i\rangle$. For a given initial state and target states, determining an optimal POVM can be shown to be a convex optimization problem and can be solved using an appropriate semi-definite program (SDP) [7]. However, due to scalability challenges in solving the SDP, whose size is exponential in the number of quantum sensors involved, in this paper, we use a well-known measurement known as pretty-good-measurement (PGM) [9] which is known to perform pretty well in general settings [9]. The PGM POVM is given by:

$$E_i = q_i \rho_i^{-1/2} \rho_i \rho_i^{-1/2}$$

where $q_i$ is the prior probability and $\rho_i = |\psi_i\rangle \langle \psi_i|$ is the density matrix of the $i^{th}$ target state $\psi_i$, and $\rho = \sum_i q_i \rho_i$.

Basic OneLevel Scheme: Key Challenges. The above-described methodology is essentially our basic OneLevel localization scheme, see Fig. 2. That is, the OneLevel scheme localizes the transmitter by first determining the set of target states $\{|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_n\rangle\}$ corresponding to the set of potential locations of the transmitter, and then, localizes the transmitter in real-time by performing QSD over the evolved quantum state using PGM measurement. We assume that the
only potential locations of the transmitter are the centers of the cells of the grid (we relax this assumption in Section IV).

The key challenges in the OneLevel scheme are twofold: (i) It is likely to incur a high probability of error due to a large number of target states (equal to the number of potential transmitter locations). (ii) A global POVM measurement over a large number of sensors can be difficult to implement in practice [20]; even ignoring the communication cost of teleporting the qubits to a central location, the main challenge arises due to the complexity of the circuit or hardware required to implement a POVM over a large number of qubit states. We address these challenges by designing two-level localization schemes as described below.

**TwoLevel Scheme.** TwoLevel solves the above-mentioned challenges by localizing the transmitter by using two levels of POVMs, with each POVM requiring a measurement over a much fewer number of sensors and with a much fewer number of possible target states. We discretize the given area into a grid; each unit of the grid is called a cell. A block is a group of neighboring cells that form a rectangle. In Fig. 3 (a), a grid has 4 × 4 cells and 2 × 2 blocks. The thick dotted lines depict the blocks while the non-thick dotted lines depict the cells. In general, for a $N \times N$ grid with $N^2$ cells, we construct blocks by dividing the entire grid into $\sqrt{N} \times \sqrt{N}$ blocks — yielding $N$ blocks in the whole area, with each block comprised of $\sqrt{N} \times \sqrt{N} = N$ cells. Without loss of generality, we assume $\sqrt{N}$ to be an integer in our discussion. The basic idea of the TwoLevel scheme is to localize the transmitter in two stages: first, localize the transmitter at a block level (Fig. 3 (a)); and then, within that block, localize the transmitter at the cell level (Fig. 3 (b)). The sensors, target states, and POVMs used for localization at these two stages are different. Such a two-stage localization scheme naturally addresses the above-mentioned challenges by reducing both the number of sensors as well as target states required at each stage. We describe the scheme in more detail below.

**Coarse-Level Localization.** The coarse level concerns localizing the transmitter at the block level, and is based on coarse-level sensors deployed over the entire given area. The target states for the coarse-level QSD/localization are the states corresponding to the location at the center of each block in the given area. As mentioned above, since the number of blocks is $N$, the number of target states for the Coarse-Level localization is $N$. The POVM measurement for the coarse-level localization is constructed using Eqn. 5 for the PGM measurement over the target states derived from the impact of the transmitter at coarse-level discrete locations (i.e., center of the blocks) on the coarse-level sensors. Note that in reality, the transmitter is never at the center of the blocks—but, we stipulate that a block’s center is a reasonable representative of the actual locations of the transmitter in that block. More formally, let $\{L_1, L_2, \ldots, L_N\}$ denote the centers of the blocks in the area, and $S$ be the coarse-level sensors. Let $\hat{U}_i$ denote the impact on $S$ when the transmitter is at location $L_i$. Then, the target states for the coarse-level localization are

$\{\hat{U}_i | \psi_0 \rangle \}$ where $| \psi_0 \rangle$ is the initial state of $S$. These target states are used to determine the POVM measurements as per Eqn. 5 and thus, determine the block.

**Fine-Level Localization.** Once the transmitter has been localized within a block $B$ via coarse-level localization, the transmitter is then localized at a cell level within $B$. For fine-level localization, each block $B$ has a set of fine-level sensors $S(B)$ deployed within $B$ (which need not be disjoint from the coarse-level sensors). The target states for fine-level localization within $B$ correspond to the potential locations of the transmitter within $B$ which are the centers of the cells within $B$, see Fig. 3 (b), and is derived from the impact of the transmitter’s signal at the fine-level sensors $S(B)$. Note that at the fine-level localization phase, only the sensors $S(B)$ where $B$ is the block selected in the previous coarse-level localization are involved. Note that $S(B_1)$ and $S(B_2)$ from two different blocks need not be disjoint. More formally, let $\{l_1, l_2, \ldots, l_N\}$ denote the centers of the cells in the block $B$ selected by the coarse-level localization phase, and $S(B)$ be the fine-level sensors. Let $\hat{U}_i$ denote the impact on $S(B)$ when the transmitter is at location $l_i$. Then, the target states for the fine level localization are $\{\hat{U}_i | \psi_0 \rangle \}$ where $| \psi_0 \rangle$ is the initial state of $S(B)$. These target states are used to determine the POVM measurement as per Eqn. 5 and thus, determine the cell within the block $B$, which is the TX location.

**TwoLevel-Pro Scheme.** The above described TwoLevel scheme addresses the scalability challenges sufficiently, and also works reasonably well in practice as observed in our evaluations. However, we also observed in our evaluations that the TwoLevel scheme has a high error rate when the transmitter is situated at the edge of the blocks; this is unsurprising as a transmitter on either side of a block’s borders (i.e., on different blocks, but at the block edge) is likely to have a similar impact on the sensors. See Fig. 4 (a). To circumvent
The quantum measurement is in-valuate our localization schemes. We use two performance metrics to Performance Metrics.

repeatedly as described in Section III, and in multiple levels to determine the target state or the TX location. This is done localize a transmitter, we first compute the evolved state using which are then used to construct the POVM as per Eqn. 5. To

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Simulator. Our custom-built simulator is written in Python and uses Numpy and Scipy libraries to perform matrix opera-

ions. In the simulator, we first determine the target states which are then used to construct the POVM as per Eqn. 5. To localize a transmitter, we first compute the evolved state using the model described in Section III, and then, use the POVM to determine the target state or the TX location. This is done repeatedly as described in Section III and in multiple levels (coarse, fine, border) depending on the localization scheme.

Performance Metrics. We use two performance metrics to evaluate our localization schemes.

1) Cell-Classification accuracy (CCacc), in percentage.
2) Localization error (Lerr), in meters.

IV. Evaluation

In this section, we evaluate our developed schemes using a custom-built simulator. We observe that as expected the performance of TwoLevel-Pro is superior to that of TwoLevel which is in turn superior to OneLevel. Simulator. Our custom-built simulator is written in Python and uses Numpy and Scipy libraries to perform matrix operations. In the simulator, we first determine the target states which are then used to construct the POVM as per Eqn. 5. To localize a transmitter, we first compute the evolved state using the model described in Section III, and then, use the POVM to determine the target state or the TX location. This is done repeatedly as described in Section III and in multiple levels (coarse, fine, border) depending on the localization scheme.

Performance Metrics. We use two performance metrics to evaluate our localization schemes.

1) Cell-Classification accuracy (CCacc), in percentage.
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We deploy the given sensors uniformly over the area; for the TwoLevel and TwoLevel-Pro schemes, we deploy the fine-level sensors along the block borders so that most of them can be used by two blocks. We don’t use more than 8 sensors for any single QSD instance—since the memory and computing requirements for storing and implementing a POVM becomes prohibitive beyond that. E.g., a POVM for 256 target-states over 12 sensors requires 69 GB of main memory storage.

Multi-shot Discrimination. The quantum measurement is intrinsically probabilistic and the single-shot discrimination can incur a high probability of error. One way to reduce this probability of error is to repeat the discrimination process many times and pick the most frequent measurement outcome. Such repeated measurements is commonly done in quantum sensing [6] and computing [16]. In our context, the repetitions must be done while the transmitter remains fixed.

Discrete and Continuous Settings. We consider two settings: discrete and continuous. In both settings, the training (i.e., determination of target states and POVMs) is done based on the transmitter being at the cells’ centers. During the actual evaluation/localization, in the discrete setting, the transmitter is still restricted to be at the cells’ centers, while in the continuous setting, the transmitter may be located anywhere in the area.

Discrete Setting: We start with evaluating the performance of OneLevel for varying grid size; see Fig. 5 (a). We use either 4 or 8 quantum sensors in the whole area. We see that the CCacc of OneLevel decreases with the increase in the area, i.e., grid size in terms of cell numbers. This is expected as a larger grid results in a larger number of target states for the QSD problem, and thus, a higher probability of error. As expected, we observe that localization-accuracy using 8 sensors is higher than for 4 sensors.

To evaluate TwoLevel and TwoLevel-Pro schemes, we use only the largest 16 x 16 grid. The grid is divided into 4 x 4 blocks each of 4 x 4 cells. In addition, there are 21 border blocks for TwoLevel-Pro. We use 8 coarse-level quantum sensors, and 40 total fine-level quantum sensors with each of the 16 blocks using 4 fine-level sensors each. Note that the set of fine-level sensors for the

We need 256 matrices each of size $2^{12} \times 2^{12}$, with each matrix element being a complex number requiring 16 bytes.
blocks need not be disjoint. Fig. 5 (b) plots the CC_{acc} of TwoLevel and TwoLevel-Pro for the discrete setting for varying standard deviation of the zero-mean noise. We see that the performance of TwoLevel and TwoLevel-Pro is significantly better than OneLevel. In particular, for noise SD of 1, the localization-accuracy of TwoLevel and TwoLevel-Pro is 92.5% and 99.2% respectively, compared to only 11.7% and 29.7% of OneLevel using 4 and 8 quantum sensors respectively. We note that although the total number of quantum sensors deployed for TwoLevel and TwoLevel-Pro is much higher than OneLevel, localizing a transmitter in TwoLevel or TwoLevel-Pro still uses only a small number of sensors. In particular, TwoLevel uses 12 sensors (8 coarse, 4 fine) to localize a transmitter, while TwoLevel-Pro uses a few more. Fig. 5 (b) also shows that TwoLevel and TwoLevel-Pro schemes are robust against noise, i.e., when the noise level increases, the performance hardly drops. This robustness is due to the repeated measurements.

**Continuous Setting.** Fig. 6 plots the performance of the OneLevel, TwoLevel, and TwoLevel-Pro schemes in the more general continuous setting, in terms of the cumulative distribution of L_{err} of the schemes. We see that TwoLevel and TwoLevel-Pro significantly outperform OneLevel by a wide margin at both 50th percentile (representing the median performance) and 95th percentile (representing the tail performance). TwoLevel-Pro performs better than TwoLevel mainly after the 80th percentile—these represent the instances where TwoLevel incurs errors due to the transmitter being at the edge of blocks (where TwoLevel-Pro is able to provide an improvement). We don’t use the cell-classification metric in the continuous setting, as it is not very meaningful here.

**V. CONCLUSION AND FUTURE WORK**

Our work presents a novel and promising application of quantum sensor networks. We have developed schemes for the localization of events (in particular, transmission by an RF transmitter) using a quantum sensor network. Our work has significant opportunities for improvement. In our future work, we plan to address the optimization problem of determining the initial state and whether an entangled initial state helps. We would like to explore schemes that use restricted forms of measurement (on a computational basis) and investigate quantum machine learning techniques to optimize the measurement.

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