Investigation of characteristics of waveguide channels system formed in PDLC with inhomogeneity of the amplitude-phase distribution of forming field

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Abstract. This work presents the results of modeling the influence of amplitude, phase and amplitude-phase heterogeneities of the forming field on the spatial distribution of the refractive index profile and, accordingly, the mode composition of radiation capable of propagating in waveguides formed by the holographic method in the photopolymer-liquid crystalline compositions. The spatial distribution of the refractive index is found by solving the system of kinetic equations. The mode composition of the radiation is determined by solving the dispersion equation for TE-modes.

1. Introduction
Currently, production of components for integrated optical devices is of great interest in the information sphere. One of the main elements of such devices is guiding system. The main task is to create components that do not require complicated technologies and have an affordable price. Currently, formation of waveguide channels is carried out by different methods, such as photolithography [1, 2], diffusion of titanium in a LiNbO3 crystal [3,4], holographic recording in photorefractive crystals [5], irradiation by a femtosecond laser [6], interference lithography in photoresist [7, 8] and others [9-11]. These methods make it possible to create both single waveguides and systems of waveguide channels. The main limitation of these methods is the fact that almost every one of them is either difficult to implement or requires expensive material.
One of effective methods for creating components for such devices may be holographic one [12, 13]. This method is simple to implement and also does not need material processing and the creation of additional components.
From the point of the possibility of forming guiding system by holographic method, much attention is paid to photorefractive crystals, photopolymer materials (PPM), photopolymer materials with a liquid crystal component (polymer-dispersed liquid crystals – PDLC) [14-17].
The use of PDLC allows to create a system of waveguide channels [18] controlled by an external electric field [19, 20] by holography methods. This material is a mixture of organic substances and is sensitive to radiation in the optical frequency range.
The purpose of this work is to investigate the holographic formation of waveguide channels in PDLC by a field with a non-uniform amplitude-phase distribution.

2. Formation geometry
A PDLC sample is a glass plate on which a mixture of monomer solution, liquid crystal and additional components are applied, with another glass plate or a transparent film superimposed [21].
PDLC has several advantages [22, 23], such as: high resolution, high diffraction efficiency, ability to control selective and diffraction properties, affordable material cost, no need for additional chemical processing of the finished waveguide system formed by holographic method.

For the formation of waveguide channels in the PDLC, a scheme for holographic diffraction structure (HDS) is used (Figure 1) [19, 20]. HDS is formed by illuminating the PDLC sample by two beams of laser radiation. During the structure formation following processes undergo in the sample: photopolymerization and diffusion of the components of the composition. The main characteristic parameter is ratio of rates of described processes. This parameter determines the character of optical properties changings [21].

In this paper, we consider formation by coherent monochromatic beams, the thickness of the PDLC layer is much smaller than the transverse size of forming beams [19].

3. Theoretical model

A three-dimensional model of holographic formation of inhomogeneous diffraction PDLC structures was developed in [19, 20]. This model takes into account the amplitude and phase inhomogeneities of the structure’s profile, as well as an arbitrary degree of nonlinearity of the recording process.

To find the dielectric constant tensor of PDLC during the formation process, the following expression is used:

\[
\hat{e}(\mathbf{r}, t) = (1 - \rho) \hat{\varepsilon}_p \cdot \hat{\mathbf{I}} + \sum_{m=0, \xi} \Delta \hat{\varepsilon}_p^m(\mathbf{r}, t) + \rho \left( \hat{\varepsilon}_o + \sum_{m=0, \xi} \Delta \hat{\varepsilon}_o^m(\mathbf{r}, t) \right)
\]

where \( \rho \) – volume fraction of LC; \( \hat{\mathbf{I}} \) – unit tensor; \( \hat{\varepsilon}_p \) – dielectric permittivity of photopolymer; 
\( \hat{\varepsilon}_o \) = \( \hat{\varepsilon}_o \cdot \hat{\mathbf{I}} + (\hat{\varepsilon}_o' - \hat{\varepsilon}_o') \cdot \mathbf{C} \mathbf{C} \) – dielectric tensor of LC; \( \hat{\varepsilon}_o', \hat{\varepsilon}_o' \) – tensor components measured with longitudinal and transverse orientations of an LC director \( \mathbf{C} \) respectively; \( m = o, e \) corresponds to the diffraction structures formed in the sample by ordinary and extraordinary waves, respectively; \( \mathbf{C} \mathbf{C} \) – means the tensor product of vectors (dyad); \( \Delta \hat{\varepsilon}_p^m(\mathbf{r}, t) \), \( \Delta \hat{\varepsilon}_o^m(\mathbf{r}, t) \) – perturbations of the dielectric constant tensor due to photopolymerization and diffusion processes, respectively.

The perturbation of dielectric tensor in general case is characterized by attenuation of optical radiation and inhomogeneity of spatial amplitude and phase distributions of forming field. The non-stationarity of tensor occurs during the formation of HDS and due to photo-induced change in the absorption coefficient of the material.

Formation of HDS is carried out by light field, which has a periodic spatial distribution of intensity, therefore, changes in the optical properties of the sample will also be characterized by a periodicity. For simplicity, in this paper, absorption of light radiation will not be taken into account.
Then perturbations of tensors $\Delta \hat{\varepsilon}_m^p(\mathbf{r}, \tau)$ and $\Delta \hat{\varepsilon}_m(\mathbf{r}, \tau)$ can be represented as a sum of spatial harmonics:

$$\Delta \hat{\varepsilon}_m^p(\mathbf{r}, \tau) = \sum_{i=0}^{H} \Delta \hat{\varepsilon}_{m^i}^p(\mathbf{r}, \tau) \cos(i \cdot \mathbf{K}^m \cdot \mathbf{r})$$

$$\Delta \hat{\varepsilon}_m(\mathbf{r}, \tau) = \sum_{i=0}^{H} \Delta \hat{\varepsilon}_{m^i}(\mathbf{r}, \tau) \cos(i \cdot \mathbf{K}^m \cdot \mathbf{r})$$

where $\tau = t/T_m$ – relative time; $T_m$ – characteristic diffusion time, $\mathbf{K}^m$ – gratings vectors. Then amplitudes of dielectric constant’s harmonics are connected with harmonics of refraction index in the following way:

$$\Delta \hat{\varepsilon}_m^p(\mathbf{r}, \tau) = 2n_p \Delta n_m^p(\mathbf{r}, \tau) \cdot \mathbf{I}$$

$$\Delta \hat{\varepsilon}_m(\mathbf{r}, \tau) = 2n_p \Delta n_m(\mathbf{r}, \tau) \cdot \mathbf{I} - \mathbf{CC}$$

where $n_p = \sqrt{\varepsilon_p}$ – unperturbed value of refractive index of polymer component, $n_m^0 = \sqrt{\varepsilon_m}$; $\Delta n_m^p(\mathbf{r}, \tau), \Delta n_m(\mathbf{r}, \tau)$ – amplitudes of refractive index harmonics for LC and polymer.

The amplitudes of refractive index harmonics ($i = 0 \ldots H$) are determined by solving the systems of $H+1$ kinetic equations for concentration of monomer and refractive index:

$$\frac{\partial M_m^p(\mathbf{r}, \tau)}{\partial \tau} = -i^2 M_m^p(\mathbf{r}, \tau) + \sum_{i=0}^{H} a_{ij}^m(\mathbf{r}, \tau) M_{m^i}(\mathbf{r}, \tau)$$

$$\frac{\partial n_m^p(\mathbf{r}, \tau)}{\partial \tau} = -\delta n_p \cdot \sum_{i=0}^{H} a_{ij}^m(\mathbf{r}, \tau) M_{m^i}(\mathbf{r}, \tau) - \mathbf{CC}$$

where $M_m^p(\mathbf{r}, \tau)$ – initial monomer concentration; $M_{m^i}(\mathbf{r}, \tau)$ – monomer concentration harmonics’ amplitudes; $\delta n_p, \delta n_d$ – coefficients describing the contributions of photopolymerization and diffusion in the HDS recording process. Also the following matrix of coefficients is introduced:

$$a_{ij}^m(\mathbf{r}, \tau) = -\begin{pmatrix}
  e_1^m & e_2^m & e_3^m & 0 & 0 & 0 & 0 & 0 \\
  2e_2^m & e_1^m & e_3^m & 0 & 0 & 0 & 0 & 0 \\
  2e_3^m & e_1^m & e_2^m & 0 & 0 & 0 & 0 & 0 \\
  0 & e_3^m & e_2^m & e_1^m & 0 & 0 & 0 & 0 \\
  0 & 0 & e_3^m & e_2^m & e_1^m & 0 & 0 & 0 \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  0 & 0 & 0 & 0 & 0 & e_3^m & e_2^m & e_1^m \\
  0 & 0 & 0 & 0 & 0 & e_2^m & e_3^m & e_1^m
\end{pmatrix}$$

where parameter $b_m^p(\mathbf{r}, \tau) = T_p^m(\mathbf{r}, \tau)/T_m(\mathbf{r})$ characterizes the ratio of characteristic times of photopolymerization $T_p^m(\mathbf{r}, \tau)$ and diffusion $T_m(\mathbf{r})$ processes.

The ratio of photopolymerization and diffusion processes rates determines the change in optical properties of PDLC. The characteristic times of the processes are determined by:

$$T_p^m(\mathbf{r}, \tau) = h \cdot \frac{1}{\sqrt{I_m(\mathbf{r}, \tau)}},$$

$$T_m(\mathbf{r}) = \frac{1}{T_m(\mathbf{r})},$$
\[ T_m(r) = \frac{1}{D_M |K^m(r)|^2}, \quad (8) \]

where \( h \) – a parameter that takes into account properties and composition of photopolymerizable composition; \( D_M \) – monomer diffusion coefficient; \( I^m(r, \tau) \) – intensity distribution of formation field taking into account the photoinduced change in the absorption coefficient.

According to (7) the spatial amplitude distribution of recording field is taken into account in \( T_p^m(r, \tau) \) through the expression for interference pattern \( I^m(r, \tau) \), and the phase distribution, according to (8), is taken into account in \( T_m(r) \) through the expression for the gratings vectors \( K^m(r) \).

Reference [24] contains a technique by which one can find a solution for the spatial harmonics of the refractive index:

\[ n^m_{pi}(r, \tau) = \delta n_p \cdot \sum_{i=0}^{H} \sum_{p=0}^{H} A^m_{i,p}(r, \tau) \frac{\exp\left(\lambda^m_{i,p}(r) \cdot \tau\right) - 1}{\lambda^m_{i,p}(r)}, \quad (9) \]

where notation and method of calculating the coefficients \( A^m_{i,p}(r, \tau), \lambda^m_{i,p}(r) \) are introduced in [25].

Kinetics of spatial profile of refractive index is defined as the sum of spatial harmonics:

\[ n^m(r, \tau) = n_0 + \sum_{i=0}^{H} \sum_{p=0}^{H} \left[ n^m_{pi}(r, \tau) + n^m_{pi}(r, \tau) \right] \cos(i \cdot K^m \cdot r), \quad (10) \]

where \( n_0 \) – the value of refractive index of the material prior to the recording process.

Equations (2) - (5), (9) and (10) are a general solution of the nonlinear process of HDS formation in PDLC, taking into account amplitude-phase heterogeneity of formation field.

The phase distribution of recording field along the \( x \) axis is represented as an expansion in a Taylor series:

\[ \phi^m(x) = \phi^m_0 + \phi^m \cdot x + 0.5 \phi^m \cdot x^2, \quad (11) \]

then \( T_m(r) = T_m \cdot (1 + \phi^m \cdot x / \phi^m)^2 \), where \( T_m \) corresponds to a uniform phase front (plane wave).

Parameter \( b^m(r, \tau) \) describes the influence of inhomogeneities of forming field on the formation kinetics; this parameter also takes into account the degree of nonlinearity of recording process:

\[ b^m(r, \tau) = b^m \cdot f(r, \tau), \quad (12) \]

where \( b^m \) – value in the center of sample and \( f^m(r, \tau) \) takes into account the amplitude and phase heterogeneity.

Figure 2 (a) shows the normalized amplitude profile of the interference pattern in the transverse coordinate \( A^m_{\text{norm}}(x) = I^m(x, \tau) / I^m_0(\tau) \), where \( I^m_0(\tau) \) – intensity of formation field in the center of the formed HDS.

Figure 2 (b) shows the normalized phase distribution of forming field \( \phi^m(x) = \phi^m(x) / \phi^m \).
Figure 2. Characteristics of recording field: a) normalized amplitude distribution; b) normalized phase distribution.

4. Numerical simulations
To determine the effect of inhomogeneity of amplitude-phase distributions of formation field on the change in refractive index of material, numerical simulation of the kinetics of HDS formation was carried out using two beams of laser radiation, excluding attenuation. The polarization of recording beams coincides with extraordinary waves in the sample. Amplitude profiles of harmonics in the form \( n_i(e^e_i)(r,\tau)=n_{ps}(r,\tau)+n_{ph}(r,\tau) \) are calculated by expression (10), taking into account separately the amplitude and phase, as well as the amplitude-phase inhomogeneities of formation field (Figure 2). Wavelength \( \lambda = 0.633 \) \( \mu \)m, angles of incidence \( \theta_1 = \theta_2 = 10^\circ \). Parameter \( b^e \) in the center of the sample is taken equal to 0.3, since when the value is less than one the refractive index profile is close to rectangular. And also when \( 0.1 < b^e < 0.4 \) the contribution of higher spatial harmonics exceeds the contribution of first harmonic [24].

Due to non-uniformity of the formation field at different points of system being formed, the spatial distribution of interference pattern will be different. And, therefore, the radiation will have different effects on the formation process. It follows from this that the parameter \( b^e \) at different points of will be different too. And therefore the amplitudes of harmonics and type of refractive index profile will be different.

Figure 3 shows spatial profiles of refractive index for amplitude-phase (a), amplitude (b) and phase (c) inhomogeneity at three points of the recording field. On this picture the distance from the center of the formed structure \( (n\Lambda / 2) \) to the center of the formation field is indicated in mm. \( \Lambda \) – structure period, \( n \) – structure number. \( \Lambda \approx 1.8 \) \( \mu \)m

5. Calculation of mode composition of radiation in waveguides
Further, the formed HDS was investigated as a system of periodically located waveguide channels. Similar to [20], on the basis of dispersion equation solution, the mode composition of radiation, which can propagate in waveguide channels formed by a non-uniform recording radiation field, was determined.

First, form of the refractive index profile of formed structure was approximated by function (13).
Figure 3. The spatial profile of refractive index with amplitude-phase (a), amplitude (b) and phase (c) inhomogeneity of the recording field.

Figure 4. The mode composition of the waveguide channels, taking into account the amplitude-phase, amplitude and phase inhomogeneities of the recording field.
Figure 4 shows the mode composition for waveguide channels, taking into account the recording fields inhomogeneity at various points. Figures 3 and 4 show that when structure is formed by an inhomogeneous field, refractive index profile varies in different areas of the material. And guided mode composition of waveguides formed at different points changes according to this profile.

\[ n(x) = n_2 \sqrt{1 - 2\Delta \left( \frac{x}{a} \right)^a}, \]  

(13)

where \( n_2 \) – formed channel’s refractive index; \( \Delta \) – relative refractive index difference; \( a \) – waveguide profile parameter.

Maximum number of guided modes in a waveguide is determined from expression (14):

\[ M = \frac{a}{a + 2} a^2 k^2 n_2^2 \Delta. \]  

(14)

6. Conclusion

Thus, this paper presents modeling results of the influence of formation field’s inhomogeneity on the characteristics of PDLC waveguide system. Model used is taking into account the anisotropy of optical properties of material and the inharmonic character of structure being formed. Refractive index profiles, as well as guided mode compositions for fields with amplitude, phase and amplitude-phase inhomogeneities are determined. The results obtained in conjunction with results of [19, 20] can be used to develop integrated optical devices based on photopolymer-liquid crystal compositions.

7. References

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