Investigation of the State Vectors and Prediction of the Orbital Elements for Spot-6 Satellite during 1300 periods with Perturbations

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Abstract
Computer simulations were carried out to investigate the dependence of the main perturbation parameters (Sun and Moon attractions, solar radiation pressure, atmosphere drag, and geopotential of Earth) on the orbital behavior of satellite. In this simulation, the Cowell method for accelerations technique was adopted, the equation of motion with perturbation was solved by 4th order Runge-Kutta method with step (1/50000) of period to obtain the state vectors for position and velocity. The results of this simulation have been compared with data that available on TLEs (NORD data in two line elements). The results of state vectors for satellites (Cartosat-2B, Gsat-14 and Spot-6) shows excellent correlation and this is leading us to extend our study for (spot-6) satellite to include the orbital behavior during 13000 periods under the effect of one type of perturbation or all types. The results indicate that all perturbation have clear effect on spot-6 orbit, reduced the perigee and apogee about 3 Km. during 89 days, also the time of period reduced 4.7 sec. Other conclusions present that the perigee angle increases 28.01 degree with any perturbation accept SRP. Furthermore, the geopotential have a big periodic effect but the atmospheric drag have accumulated effect on most orbital elements.

Keywords: satellite orbit, orbital prediction, orbital perturbations, orbital elements.

1. Introduction:
The orbital elements are the parameters that required defining the elliptical orbit. These elements are considering in classical of two body systems, which used under Kepler's law. The six parameters of orbital elements were used in astronomy and orbital mechanics, are the same elements that define the size and shape of the ellipse orbit are eccentricity (e) and semi major axis (a), the inclination (i) is vertical tilt of the ellipse with respect to the equatorial plane, the longitude of the ascending node (Ω) is horizontally orients the node of the ellipse. Argument of perigee (ω) is the distance angle of orbital plane measured from the ascending node to the perigee point, the mean anomaly at epoch defines the position of the orbiting along the ellipse at a specific time (t). In this work we used the data of state vector for (spot-6) satellite that execute from NORD date to show that the variation of orbital elements individual under effect the perturbations of $J_2$, atmospheric drag, solar radiation pressure and third body attraction during 1300 revolutions.
2. Methods:

Calculation the state vectors of elliptical satellite orbit from orbital elements

To calculate the state vectors of the satellite at specified time (t) we should know the time of perigee passage to calculate the mean anomaly (M), after that calculated the eccentric anomaly (E) (3).

The perigee radius is:

\[ r_p = h_p + R_e \]  \hspace{1cm} (1)

Where \( h_p \) is height of satellite at perigee, \( R_e \) is mean radius of Earth = 6378.165 Km.

The semi major axis (a) calculated by the equation below:

\[ r_p = a(1 - e) \]  \hspace{1cm} (2)

The mean motion (n) can be written as following:

\[ n = \frac{2\pi}{T} = \sqrt{\frac{\mu}{a^3}} \]  \hspace{1cm} (3)

The unperturbed orbital period (T) is given by Kepler's 3rd law.

The mean anomaly in any time, which used to describing the location of the satellite in an orbit, is:

\[ M = n(t - t_p) \]  \hspace{1cm} (4)

Where \( t_p \) : the perigee time.

The eccentric anomaly (E), which use using Kepler equation of the satellite in an orbit as calculated:

\[ E = M + e_o \sin E \]  \hspace{1cm} (5)

This equation cannot be solved directly. There are some methods for finding (E), and finally a formula, which gives an approximate result, is Newton-Raphson method for finding successively better approximations to the roots of a real valued function (4).

To find the Cartesian coordinate \((x_w, y_w)\) and \(z_w\) to the satellite in its orbit as the following (5,6):

\[
\begin{align*}
x_w &= a(\cos E - e) \\
y_w &= a\sqrt{1 - e^2} \sin E \\
z_w &= 0
\end{align*}
\]

The displacement radius (r) will be:

\[ r = a(1 - e \cos E) \]  \hspace{1cm} (6)

The velocity and its components were computed as (6):

\[
\begin{align*}
\dot{x}_w &= \sqrt{\mu} \sin f \\
\dot{y}_w &= \mu \left( e + \cos f \right) \\
\dot{z}_w &= 0 \\
\dot{r} &= \sqrt{\mu a e \sin E}
\end{align*}
\]

Convert the position and velocity to components from the orbital plane to the Earth equatorial plane by Gaussian matrix \( R_{ij} \), which contains Euler angle \((i, \Omega, \omega)\) [4]. Multiply \( R_{ij} \).
With the position and velocity components:
Where $R^{-1}$ is the inverse of Gauss matrix:

$$R^{-1} = \begin{bmatrix}
R_{11} & R_{21} & R_{31} \\
R_{12} & R_{22} & R_{32} \\
R_{13} & R_{23} & R_{33}
\end{bmatrix}$$

Where the matrix elements are (5):
$R_{11} = \cos \omega \cos \Omega - \sin \omega \sin \Omega \cos i$, $R_{21} = -\sin \omega \cos \Omega - \cos \omega \sin \Omega \cos i$, $R_{31} = \sin \Omega \sin i$  
$R_{12} = \cos \omega \sin \Omega + \sin \omega \cos \Omega \cos i$,  
$R_{22} = -\sin \omega \sin \Omega + \cos \omega \cos \Omega \cos i$, $R_{32} = -\cos \Omega \sin i$  
$R_{13} = \sin \omega \sin i$, $R_{23} = \cos \omega \sin i$, $R_{33} = \cos i$

Then the equatorial Cartesian coordinate of the satellite and its distance from the center of Earth

\[
x = R_{11}x_w + R_{21}y_w + R_{31}z_w \\
y = R_{12}x_w + R_{22}y_w + R_{32}z_w \\
z = R_{13}x_w + R_{23}y_w + R_{33}z_w \\
r = (x^2 + y^2 + z^2)^{1/2}
\]

\[
\dot{x} = R_{11}\dot{x}_w + R_{21}\dot{y}_w + R_{31}\dot{z}_w \\
\dot{y} = R_{12}\dot{x}_w + R_{22}\dot{y}_w + R_{32}\dot{z}_w \\
\dot{z} = R_{13}\dot{x}_w + R_{23}\dot{y}_w + R_{33}\dot{z}_w \\
\dot{r} = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2}
\]

Converting state vectors to orbital elements

The elliptical orbital elements can be calculated at all steps of revolution at any time $t$

where:

\[
M = M + n (T_p/\text{no. of step})
\]

From the equatorial component of position, velocity and angular momentum can be calculated as follows (6, 7):

\[
\vec{h} = \vec{r} \times \dot{\vec{r}}
\]

or

\[
\begin{bmatrix}
h_x \\
h_y \\
h_z
\end{bmatrix} =
\begin{bmatrix}
y\dot{z} - z\dot{y} \\
z\dot{x} - x\dot{z} \\
x\dot{y} - y\dot{x}
\end{bmatrix}
\]

\[
h = \sqrt{h_x^2 + h_y^2 + h_z^2}
\]

The inclination ($i$) is represent by:

\[
\tan i = \frac{\sqrt{h_x^2 + h_y^2}}{h_z}
\]

The longitude of ascending node ($\Omega$) is representing by:

\[
\tan \Omega = \frac{h_x}{-h_y}
\]
The semi-major axis is representing by:

\[ a = \left( \frac{2}{r} - \frac{\mu}{v^2} \right)^{-1} \]  

(16)

The eccentricity (e) is represents as:

\[ e = \sqrt{(1 - \frac{r}{a})^2 + \frac{xx + yy + zz}{\sqrt{\mu a}}} \]  

(17)

The eccentric anomaly (E) is represents as:

\[ \tan E = \left( 1 - \frac{r}{a} \right) \left( xx + yy + zz \right) \sqrt{\mu} \]  

(18)

The true anomaly (f) is represents as:

\[ \tan \frac{f}{2} = \frac{1 + e}{1 - e} \tan \frac{E}{2} \]  

(19)

Numerical Integration Method

The equation of motion with perturbation is (8,9):

\[ \ddot{r} + \frac{\mu}{r^3} r = a_p \]  

(22)

Orbital Perturbations:

The perturbations in this project are perturbing accelerations on satellite’s motion [10]:

\[ a_p = a_g + a_{\text{drag}} + a_{\text{solar}} + a_{3\text{rd b}} \]

Where

- \( a_g \): geopotential(non-spherical of Earth) acceleration.
- \( a_{\text{drag}} \): atmospheric drag acceleration.
- \( a_{\text{solar}} \): solar radiation pressure acceleration.
- \( a_{3\text{rd b}} \): 3rd body gravity acceleration.

Geopotential acceleration

The perturbing acceleration effects on satellite due to the Earth’s gravity potential represent as function of J2 parameter (10, 11):

\[ X_g = (\mu \text{sun}/r^3) * \left[ \left( \frac{2}{r} \right) J2 \left( \frac{\mu}{r} \right)^2 \right\]  

\[ \times \left( 5 \left( \frac{Z}{r} \right)^2 - 1 \right) \]  

(23)

\[ \text{where: } J2 = 0.0010826283 \quad \mu \text{sun} = 1.327124 \times 10^{11}. \]

Atmospheric Drag Acceleration

The velocity of the satellite relative to atmospheric as shown in the following (10, 12).

\[ \vec{V}_r = \vec{V}_{in} - \vec{V}_{\text{Earth}} \times \vec{r} \]  

(24)

Where

- \( \vec{V}_r \): is the satellite velocity vector relative to atmospheric.
\( \vec{V}_i \): is the inertial velocity for satellite.
\( \vec{v}_E \): is the rotational velocity for Earth.
\( \vec{r} \): is the inertial satellite position vector.

The perturbing acceleration of the satellite according to atmospheric drag can be stated as [12]:

\[
a_{\text{Drag}} = -\frac{1}{2} C_D \frac{A}{m} \rho \frac{V_r^3}{v_r} \tag{25}
\]

\( V_r \): is the relative speed between satellite and the atmosphere.
\( C_D \): is the drag coefficient, we using the value is 2.2.
\( A \): is the satellite cross-section area, we using the value is 5.2 m².
\( m \): is represent the mass of satellite, we using the value is 900 kg.
\( \rho \): is the air density at satellite altitude is get as in (11) or calculated using NLRMSISE-00.

**Solar Radiation Pressure (SRP)**

Solar radiation pressure is the force that effect on the satellite, due to the momentum flux from sun; the solar radiation pressure is show below as (10, 11):

\[
P_\odot = \frac{\Phi_\odot}{c} = \frac{L_\odot}{4\pi r^2 c} \tag{26}
\]

Where
- \( \Phi_\odot \): is the intensity of the solar flux4.56×10⁻⁶.
- \( c \): is the speed of light.
- \( L_\odot \): is the solar luminosity.
- \( r \): is the sun distance from the Earth depending on Julian date (JD).

The used function is JD=Julian date (2014, 5, 28, 18, 13, 0), with step tp/(3600*24).

The perturbing acceleration can be stated as (2):

\[
a_\odot = \frac{L_\odot}{4\pi c \text{ mass}} C_R \frac{r_{\text{sat}}-r_\odot}{\|r_{\text{sat}}-r_\odot\|^2} f_s \frac{1}{C_R} \tag{31}
\]

Where
- Area: is representing of the satellite area.
- mass: is represent of the satellite mass.
- \( C_R \): is representing of the radiation pressure coefficient (the used value is 1.3).
- \( \epsilon \): is representing of the body reflectivity.
- \( f_s \): is representing the shadow function \( f_s = 1 \) when the satellite out the shadow.

\( r_\odot \): is the position of the sun, the drag coefficient equal to CD = 2.2, the reflectivity equal to \( \epsilon = 0.31 \), for the shadow function \( f_s \) it is calculated comparing the satellite position with the Earth and the Sun (12, 13).

**Third body perturbation**

The term third body refers to any other body in space besides the Earth, which could have a gravitational affect on the satellite orbit. In fact, the significant influences on the orbit it come from the sun and the moon those main sources of perturbations. Since the gravitational pull, force exerted on satellite orbit causes its orbit variation in all orbital elements. The equation that describes the perturbing acceleration according to the third body can be seen as follows equation (13, 14, 15):

\[
a_{3rdB} = GM_{3rdB} \left( \frac{S}{S^3} - \frac{r_{3rdB}}{r_{3rdB}^3} \right) \tag{27}
\]

\[
S = r_{3rdB} - r_{\text{sat}} \tag{28}
\]

Where
- \( S \): relative position for satellite
r_{3rdB} \text{: position vector for the 3rd body.}

r_{sat} \text{: Position vector for the satellite.}

These positions were computed at the date and time by a function in Mat-Lab as following: JD=Julian date (2014,5,28,18,13,0), with step $t_p/(3600 \times 24)$.

3. Results and Discussion:

We used Matlab program to solve the equation of orbital motion that has been developed with perturbations, the behavior of orbital elements under perturbation shown in the following figures, also the program was applied on more satellite data from TLEs that taken from the Web repository http://celestrak.com. To calculate the state vector (S.V.) for three satellites and calculate the orbital elements for one satellite (spot-6) with single perturbation and with all perturbations as show table 2 and figs (1-7).

The input data for (spot-6) satellite are:

- $h_a=698.89$; $\text{mean}=279.7434$; $\Omega=215.8134$; $e=0.0001368$; $\omega=80.3963$;
- mean motion $=14.58528066 \times (2\pi)/(24 \times 3600)$.
- $Re=6378.137$; $\pi=3.141592653589793$; $\mu=398600.4$; $\text{error} = 10^{-10}$

1. predicate the state vector for satellites

Table (1): Compare the state vectors between the TLEs and our program results at the same mean anomaly.

| State Vectors | Cartosat-2B (TLEs) | Cartosat-2B (our results) | Spot 6 (TLEs) | Spot 6 (our results) | Gsat-14 (TLEs) | Gsat-14 (our results) |
|---------------|--------------------|--------------------------|---------------|---------------------|----------------|----------------------|
| X (km)        | -6231.7560         | -6234.8471               | -5736.9414    | -5740.1912          | 36095.3223600 | 36103.0071          |
| Y (km)        | -3189.4018         | -3180.6681               | -4136.9553    | -4130.5381          | -21779.412    | -21764.8162         |
| Z (km)        | 14.8069            | 13.9406                  | 15.1814       | 12.2793             | 3.4839025     | 6.86152945          |
| $r_{mag}$(km) | 7000.5204          | 7000.6716                | 7072.9857     | 7072.9215           | 42157.029002  | 42156.0713          |
| $V_x$ (km/s)  | 0.4537396          | -0.33406481              | -0.6124123    | -0.5190191          | 1.588695300   | 1.58772211          |
| $V_y$ (km/s)  | 0.9400898          | 1.0001334                | 0.8782634     | 0.9451343           | 2.6330807     | 2.63373232          |
| $V_z$ (km/s)  | 7.4776527          | 7.4760139                | 7.4303591     | 7.4292566           | -0.06768204   | -0.01061538         |
| $V_{mag}$(km/s)| 7.5501615          | 7.5501169                | 7.5071055     | 7.5071469           | 3.07523442    | 3.075289821         |

The state vectors for the satellites (Cartosat-2B, Gsat-14 and Spot 6) that calculated by using the program and the state vectors for same satellites that given by TLEs that shown in table (1). The results show a good agreement to predicate the state vectors for satellites. The estimated values of state vector components, for the low satellites less than 120 meter in position and the velocity less than 0.01 m/s. For high satellite, orbit the difference in position less than 950 meter while in velocity less than 0.04 m/s.
2. The perturbations effects during 1300 periods for spot 6 satellite:

Note: in all figures: The argument of perigee (ω) written as AOP, LOAN(deg.)= Ω -180 (deg.), period written as Tp, the time of 1300 revolution for spot-6 equal about 89 days.

2.1 Results of Solar Radiation Pressure perturbation (SRP) for 1300 periods:

Figure (1): The variation of orbital elements during 1300 periods due to SRP effect.
2.2 Results of geopotential perturbation (J2) for 1300 periods:

Figure (2): The variation of orbital elements during 1300 periods due to geopotential perturbation.
2.3 Results of Sun attraction during 1300 periods:

Figure (3): The variation of orbital elements during 1300 periods due to Sun attraction.

2.4. Results of Lunar attraction perturbation for 1300 periods:

Figure (4): The variation of orbital elements during 1300 periods due to lunar attraction.
2.5 Results of atmospheric drag perturbation during 1300 periods:

![Graphs showing orbital element changes over 1300 periods due to atmospheric drag perturbation.]

Figure (5): The variation of orbital elements during 1300 periods due to atmospheric drag perturbation.
2.6 Results of all perturbations during 1300 periods for spot 6 satellite:

Figure (6-a): The variation of orbital elements during 1300 periods due to all perturbation.

Figure (6-b): The variation of orbital elements during 1 period due to all perturbation.
Figure (7): The variation of r and v during 1300 periods due to all perturbation.

Table (2): The final orbital elements of spot 6 with all perturbation during 1300 periods. Tp initial=98.7295050518214, Tp final (with all pert.)=98.647063132365929

| State | a (km)       | e             | i (deg)         | Ω (deg)         | ω (deg)         | f (deg) after 1300 rev. |
|-------|--------------|---------------|-----------------|-----------------|-----------------|-------------------------|
| without | 7076.0594220 | 9906          | 0.0001368000    | 1981            | 35.813399996    | 479                     |
| Solar  | 7078.0680356 | 0.0001368388  | 98.198699811    | 35.813399782    | 98.198699811    | 279.3679288             |
| Lunar  | 7078.1355298 | 0.0001368401  | 98.198587150    | 35.813321440    | 108.47201371    | 279.3679286             |
| Drag   | 7078.1243194 | 0.0001368999  | 98.199135771    | 35.813426803    | 108.47165645    | 279.3679286             |
| SRP    | 7078.0658963 | 0.0001368387  | 98.198694963    | 35.813387118    | 108.47169437    | 279.3679288             |
| J4     | 7071.2582724 | 98.202675516  | 35.815604931    | 108.49102365    | 279.367943       |
| All    | 7072.1197343 | 0685          | 98.203185174    | 335.815217269   | 1411            | 279.3679418             |
4. Discussions the results of orbital elements under perturbations during 1300 periods (or 82 days) for spot 6 satellite.

Figure (1) shows the orbital elements during 1300 periods under the effect of the effect of SRP the semi major axis is linear increase about 2 km. The eccentricity and RAAN are secular decrease respectively 0.0000067 and 0.0001 degree. The inclination and argument of perigee are secular increase respectively 0.0001 and 3.44 degree solar attraction. From the evaluation of the semi major axis, it is clear that the amplitude of the variation is of order of is 0.0005 km. The eccentricity drops up 0.00000013. The inclination is linear trend with slight decline about 0.0027. The RAAN is increase linear about 0.0021 degree. The argument of perigee is periodic variation drop about 0.06 degree and the true anomaly is shape as saw tooth according to the mean motion of satellite.

Figure (2) depicts orbital elements during 30 days under effect the lunar attraction. The semi major axis is osculating about 0.0016 km. The eccentricity periodic with secular variation drops up 0.00000095. The inclination is periodic with secular increase about 0.0017 degree. The RAAN is secular with low amplitude changing about 0.003 degree. The argument of perigee is periodic change about 0.6 degree.

Figure (3) describes orbital elements during 30 days under effect the atmospheric drag. The semi major axis is decrease linearly about 0.041 km. The eccentricity drops up 0.00000002 in secular decrease. The inclination is linear trend with slight decline. The RAAN is periodic with low amplitude. The argument of perigee is periodic variation drop about 0.0024 degree and the true anomaly is shape as saw tooth according to the mean motion of satellite.

The Sun and the Moon attraction have a similar effect on the a,e,AOP,Tp where all them increase linearly with little ripple in the Moo's influence as show in fig(3) and fig(4) otherwise the inclination(i) and LOAN with Sun attraction have a turbulent curved down word as in fig(3), Figure (4) show that in the case of the Moon attraction the variation i and LOAN have cycles like sin function with amplitude increase of node angle, that depend on the satellite-Moon distance as will as the Moon perigee months.

In the case of atmospheric drag as show in fig(5) the behavior of a,e,AOP,Tp like them behavior in the case of the moon attraction. LOAN and i are increase linearly with same turbulent. That mean the tangent velocity of satellite of this high is more effect than the radial velocity because the low density of atmosphere.

Figure (6a) shows orbital elements variation during 1300 cycles under the effect of all perturbations, in this case, the semi-major axis (a) has secular curved decrease about 2km. per 1300 cycles also the behavior of eccentricity. a,e have similar change with J2 effect, that mean J2 has a dominate effect on the orbit size and shape. The behavior of I,LOAN and AOP have a mix to geopotential and drag effects that shows that geopotential predominant and the atmospheric drag has a cumulative effect on them.

Figure (6b) show the behavior of the orbital elements with all perturbations through one period; a,e,i and LOAN have two cycles like sine or cosine functions with small variation at the end of period. AOP and true anomaly have one period with small increase in value at the end of period.

Figure(7) show the variations of satellite distance, velocity and period with all perturbations the distance (r) from the Earth center change through any period between perigee 7075 km. and apogee 7077 km. which depend on the eccentricity. The distance (r) has a secular curved decrease with time at the same time. The velocity (v) has a secular curved increased. The period decreases like the behavior of semi-major axis because they are related to Kepler's third law.

Table (2) is show variations of the orbital elements after 1300 periods with all perturbations individually and collectively. The table data agree with fig(6a) and fig(7).
5. Conclusions:
1- All perturbations have clear effect on spot-6 orbit, reduced the perigee and apogee about 3 Km. during 89 days.
2- The time of period reduced 4.7 sec. through 89 days with all perturbations.
3- The perigee angle increases 28 degree with any perturbation accepts SRP.
4- The geopotential have a big periodic effect but the atmospheric drag have accumulated effect on most orbital elements.
5- through one period accept true anomaly (0-360)degree linearly, all other orbital elements have periodic variation like sine or cosine function with change in amplitude because the perturbation effects.

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