Production of $\eta_c(1S, 2S)$ in $e^+e^-$ and $pp$ collisions

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We derive the light-front wave function (LFWF) representation of the $\gamma^*\gamma^* \rightarrow \eta_c(1S), \eta_c(2S)$ transition form factor $F(Q_1^2, Q_2^2)$ for two virtual photons in the initial state. For the LFWF, we use different models obtained from the solution of the Schrödinger equation for a variety of $c\bar{c}$ potentials. We compare our results to the BaBar experimental data for the $\eta_c(1S)$ transition form factor, for one real and one virtual photon. We observe that the onset of the asymptotic behaviour is strongly delayed and discuss applicability of the collinear and/or massless limit.

In addition, we present a thorough analysis of $\eta_c(1S, 2S)$ quarkonia hadroproduction in $k_\perp$-factorisation in the framework of the light-front potential approach for the quarkonium wave function. The off-shell matrix elements for the $g^*g^* \rightarrow \eta_c(1S, 2S)$ vertices are derived. We discuss the importance of taking into account the gluon virtualities. We present the transverse momentum distributions of $\eta_c$ for several models of the unintegrated gluon distributions. Our calculations are performed for four distinct parameterisations for the $c\bar{c}$ interaction potential consistent with the meson spectra. We compare our results for $\eta_c(1S)$ to measurements by the LHCb collaboration and present predictions for $\eta_c(2S)$ production.
1. Introduction: Description of the mechanism $\gamma^*\gamma^* \rightarrow \eta_c(1S, 2S)$

Complementary information for meson structure in quantum chromodynamics can be provided by the study of electromagnetic form factors as well as meson-photon transition form factors. Over a span of several years the attention has been paid mostly on the case of light pseudoscalar meson-photon transition form factors e.g. $\eta, \eta'$, $\eta^0$. Similar analysis have been performed for $\eta_c$ production. The mass of the $\eta_c$ assure a hard scale and validate the perturbative approach even for zero virtuality. Here we will focus on calculating transition form factor for both virtual photon in the light-front frame. For illustration in Fig. 1 we present the generic diagram for the process and the particle momenta involved into corresponding calculation.

![Figure 1: Generic diagram for production of $\eta_c(1S)$ or the first excited state $\eta_c(2S)$](image)

The general form of the photon-photon fusion amplitude reads:

$$M_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \rightarrow \eta_c) = 4\pi\alpha_{em}(-i)\varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F(Q_1^2, Q_2^2),$$

where $Q_1^2, Q_2^2$ are photon virtualities and light-front representation of the transition form factor:

$$F(Q_1^2, Q_2^2) = e_e^2\sqrt{4m_c} \int \frac{dz d^2k}{(k - (1 - z)q_2)^2 + z(1 - z)q_1^2 + m_c^2} \psi(z,k).$$

In order to construct the form factor $F(Q_1^2, Q_2^2)$, we have used light-front wave functions $\psi(z, k)$ (see Fig. 2). These wave functions are obtained in a few steps, the first step is to solve the Schrödinger equation for several model of $c\bar{c}$ potential and then the obtained non-relativistic radial space wave function $u(r)$ are transformed to momentum space $u(p)$ (for more details see [1, 2]). Due to Terentev prescription: $p = k$, $p_z = (z - 1/2)M_{c\bar{c}}$ valid for weakly bound system we can rewrite:

$$\Psi_{\lambda}\lambda(z,k) = \bar{u}_\lambda(zP_+, k)\gamma_5 \gamma_\lambda((1 - z)P_+ - k) \psi(z, k), \quad \psi(z, k) = \frac{\pi}{\sqrt{2M_{c\bar{c}}}} \frac{u(p)}{p}.$$

Here $M_{c\bar{c}} = \sqrt{(k^2 + m_c^2)/(1 - z)}$ is the invariant mass of the $c\bar{c}$ state which depends on momenta of quarks. It is worth to notice that we treat $\eta_c$ meson as a bound state of $c\bar{c}$ and therefore we assume that dominant component in the Fock-state expansion comes from $c\bar{c}$:
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Figure 2: Radial light-front wave function obtained for the Buchmüller-Tye potential.

$$|\eta_c; P_+, P\rangle = \sum_{i, j, \Lambda, \lambda} \frac{\delta^i_j}{\sqrt{N_c}} \int \frac{dz d^2k}{z(1 - z) 16\pi^2} \Psi_{i\Lambda}(z, k_c) c_{i\lambda}(zP_+, p_c) \rho^{\lambda}(1 - z)P_+, p_c) \rangle + \ldots$$ (4)

We vary our numerical results by analysing several models of interaction potential from the literature. In Fig. 3 we present normalized form factor $F(Q^2, 0)/F(0, 0)$ for one real and one virtual photon compared to BaBar data [3]. The description of the experimental data appears to be related not only to applied potential model, but also to lower value of $c$-quark mass $m_c$.

Figure 3: Normalized transition form factor $F(Q^2, 0)/F(0, 0)$ as a function of photon virtuality $Q^2$. The BaBar data are shown for comparison [3].

2. Production of $\eta_c(1S, 2S)$ in $pp$ collisions

We applied [4] LF form-factor in $pp$ collision by adjusting quantum numbers to gluon-gluon process. In the $k_T$-factorization approach, gluons are off-shell, $q_i^2 = -q_i^2$ and their four momenta:

$$q_1 = (q_{1+}, 0, q_1), \ q_2 = (0, q_{2-}, q_2), \ q_{1+} = x_1 \sqrt{\frac{s}{2}}, \ q_{2-} = x_2 \sqrt{\frac{s}{2}}.$$ (5)
We can write the cross section for inclusive $\eta_c(1S)$ or $\eta_c(2S)$ production in the form:

$$d\sigma = \int \frac{dx_1}{x_1} \int \frac{d^2q_1}{\pi q_1^2} F(x_1, q_1^2, \mu_F^2) \int \frac{dx_2}{x_2} \int \frac{d^2q_2}{\pi q_2^2} F(x_2, q_2^2, \mu_F^2) \frac{1}{2x_1x_2s} |M|^2 d\Phi(2 \to 1), \quad (6)$$

where the phase space element: $d\Phi(2 \to 1) = (2\pi)^4 \delta^{(4)}(q_1 + q_2 - p) \frac{d^3p}{(2\pi)^3} \delta(p^2 - M_{\eta_c}^2)$.

$$M^{ab} = \frac{q_1^\mu q_2^\nu}{|q_1||q_2|} M_{\mu\nu}^{ab} = \frac{q_1 q_2^\nu}{|q_1||q_2|} n^\mu n^\nu M_{\mu\nu}^{ab} = \frac{x_1 x_2 s}{2|q_1||q_2|} n^\mu n^\nu M_{\mu\nu}^{ab}. \quad (7)$$

Therefore, the matrix element reads: $n^\mu n^\nu M_{\mu\nu}^{ab} = 4\pi\alpha_s (-i) [q_1, q_2] \text{Tr}[t^a t^b] / \sqrt{N_c} I(q_1^2, q_2^2)$ and averaging over colors, we obtain the final result:

$$\frac{d\sigma}{d\gamma d^2p} = \int \frac{d^2q_1}{\pi q_1^4} F(x_1, q_1^2) \int \frac{d^2q_2}{\pi q_2^4} F(x_2, q_2^2) \delta^{(2)}(q_1 + q_2 - p) \times \frac{\pi^3 a_s^2}{N_c(N_c^2 - 1)} |q_1||q_2| \sin(\phi_1 - \phi_2) I(q_1^2, q_2^2)^2. \quad (8)$$

The $I(q_1^2, q_2^2)$ above is related to the form factor for $\gamma^* \gamma^* \to \eta_c$: $F(Q_1^2, Q_2^2) = \frac{e^2 \sqrt{N_c}}{s^{1/2}} I(q_1^2, q_2^2)$, and going to NRQCD limit the transition form factor takes the form:

$$F_{\text{NRQCD}}(Q_1^2, Q_2^2) = \frac{4e^2 \sqrt{N_c}}{\sqrt{\pi M_{\eta_c}} M_{\eta_c}^3} \frac{1}{Q_1^2 + Q_2^2} R(0), \quad (9)$$

where $R(0)$ is radial wave function of the potential-model at the spatial origin.

The normalization of the cross section crucially depends on the value of the form factor at on-shell point, thus we extract $F(0, 0)$ from experimental value of the radiative decay width (see Table 1) in leading order:

$$\Gamma_{\text{LO}}(\eta_c \to \gamma\gamma) = \frac{\pi}{4} a_s^2 M_{\eta_c}^3 |F(0, 0)|^2. \quad (10)$$

and the expression for the width at Next Leading Order, according to Ref. [5], is:

$$\Gamma_{\text{NLO}}(\eta_c \to \gamma\gamma) = \Gamma_{\text{LO}}(\eta_c \to \gamma\gamma) \left(1 - \frac{20 - \pi^2 a_s}{3} \right). \quad (11)$$

In Figs. 4 and 5 we present differential cross section as a function of transverse momentum for prompt $\eta_c(1S)$ and $\eta_c(2S)$ production compared with the LHCb data for $\sqrt{s} = 7, 8$ TeV [7] and preliminary experimental data for $\sqrt{s} = 13$ TeV[8] for the interval in rapidity $2.0 < y < 4.5$. In the numerical calculation we used several unintegrated gluon distributions and we applied form factor calculated from the power-law potential as explained in the section 1. In Fig. 6 we show the transverse momentum distribution of the $\eta_c(1S)$ (left panel) and $\eta_c(2S)$ (middle panel) with form
we present the results with different normalization of the form
factor, normalized to experimental value, exact value from light front wave functions and point
like form factor. We wish to point out that it is important to take into account gluon virtualities in
the \( \eta_c \) prompt hadroproduction. This kind processes are also a good probe of Unintegrated Gluon
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Figure 6: Distribution in transverse momentum of the $\eta_c(1S)$ (left panel) and the $\eta_c(2S)$ (middle panel) for form factor calculated, from different potential models, with the same normalization at the on shell point. On the right panel comparison of the results with different normalization of the form factor for the Power-law potential.

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