Effect of structural defects on anomalous ultrasound propagation in solids during second-order phase transitions

Pavel V. Prudnikov, Vladimir V. Prudnikov, Evgenii A. Nosikhin
Omsk state university, Department of theoretical physics, pr.Mira 55A, Omsk 644077, Russia

(Dated: September 10, 2008)

The effect of structural defects on the critical ultrasound attenuation and ultrasound velocity dispersion in Ising-like three-dimensional systems is studied. A field-theoretical description of the dynamic effects of acoustic-wave propagation in solids during phase transitions is performed with allowance for both fluctuation and relaxation attenuation mechanisms. The temperature and frequency dependences of the scaling functions of the attenuation coefficient and the ultrasound velocity dispersion are calculated in a two-loop approximation for pure and structurally disordered systems, and their asymptotic behavior in hydrodynamic and critical regions is separated. As compared to a pure system, the presence of structural defects in it is shown to cause a stronger increase in the sound attenuation coefficient and the sound velocity dispersion even in the hydrodynamic region as the critical temperature is reached. As compared to pure analogs, structurally disordered systems should exhibit stronger temperature and frequency dependences of the acoustic characteristics in the critical region.

PACS numbers: 64.60.Ak, 64.60.Fr, 64.60.Cn, 71.23.-k, 43.35.+d

I. INTRODUCTION

The progress in understanding the nature of critical phenomena is mainly related to the theoretical and experimental studies of critical dynamics in condensed matter. However, the descriptions of the nonequilibrium behavior of systems during phase transitions still contain a number of unsolved problems. This is due to the fact that studying the dynamic properties of critical fluctuations, which have anomalously high amplitudes and slow damping, encounters problems that are more complex than the problems that arise when equilibrium properties are described. Qualitatively, this is caused by the necessity of taking into account the interactions between order-parameter fluctuations and other longlived excitations.

The dynamics of phase transitions contains a number of physically important processes that are determined by the behavior of a multispin correlation function and, thus, are particularly complex for a theoretical description. These are, for example, thermal processes near the critical point in a liquidgas system, the attenuation of electromagnetic-field energy that accompanies magnetic resonance phenomena, and the anomalous attenuation and scattering of acoustic waves in various media during phase transitions. The latter processes are important, since they underlie resonance and ultrasonic methods of studying critical dynamics.

The unique feature of ultrasonic methods is the fact that, at temperatures that are close to a second-order phase transition temperature in magnetic systems and systems with structural phase transitions, researchers detect both anomalously strong ultrasound attenuation and an anomalous change in the ultrasound velocity, which can easily be observed in experiment (Figs. 1, 2). These phenomena are caused by the interaction of low-frequency acoustic oscillations with longlived order-parameter fluctuations, which produce a random force that disturbs normal acoustic regimes by means of magnetostrictive spinphonon interaction. In this process, relaxation and fluctuation attenuation mechanisms can be distinguished. The relaxation mechanism, which is due to a linear dynamic relationship between sound waves and an order parameter, manifests itself only in an ordered phase, where the statistical average of the order parameter is nonzero. Since the relaxation of the order parameter near a phase-transition point proceeds slowly, this mechanism plays an important role in the dissipation of low-frequency acoustic oscillations. The fluctuation attenuation mechanism, which is determined by a quadratic relation between the deformation variables in the Hamiltonian of a system with order-parameter fluctuations, manifests itself over the entire critical-temperature range. To date, there exist a considerable number of works that deal with a theoretical description of the ultrasonic anomalies that appear in condensed matter during phase transitions and give an adequate explanation of experimental results.

One of the most interesting and important problems from both experimental and theoretical viewpoints is the study of the influence of structural defects on the ultrasound propagation characteristics in materials undergoing phase transformations. The structural disorder induced by impurities or other structural defects plays a key role in the behavior of real materials and physical systems. The matter of particular interest is the effect of frozen structural defects, whose presence can manifest itself in a random perturbation of a local transition temperature, as it occurs, for instance, in ferro- and antiferromagnetic systems in the absence of an external mag-
The magnetic field. The statistical features inherent in systems with a frozen disorder create considerable difficulties for both the analytical description of the behavior of such systems and the experimental methods of their investigation. According to the heuristic Harris criterion \[10\], the effect of frozen point defects becomes noticeable and induces a new type of critical behavior if the critical exponent for the heat capacity of a pure system is positive. As was demonstrated in previous studies, this criterion is only met for Ising-like systems. Therefore, the effect of point structural defects on the critical behavior is negligible in systems with a multicomponent order parameter, such as the XY model and the Heisenberg model. Therefore, one of the most challenging problems from a physical point of view is the study of the effect of structural defects on the critical behavior of systems with a single-component order parameter, in which the presence of a structural disorder leads to a substantial change in the critical-behavior characteristics.

However, the problem of the effect of structural defects on the characteristics of ultrasound propagation in materials that undergo phase transformations still remains unsolved because of the complexity of the theoretical description of the four-spin correlations of order-parameter fluctuations, which determine acoustic characteristics. Pawlak and Fechner \[11\] attempted to describe the effect of point defects on the ultrasound propagation parameters near a critical temperature using the first-order \(\varepsilon\)-expansion. However, as was shown in our work \[12\], some mistakes crept in the description of this phenomenon in \[11\]; in particular, they used wrong diagrams for taking into account the dynamic effects of the interaction of order-parameter fluctuations via a defect-induced field and they did not use diagrams that give a noticeable contribution to the attenuation coefficient. Moreover, earlier investigations \[13, 14\] based on the field-theoretical description of pure and disordered systems in two-loop and higher approximations with the use of the method of summation of asymptotic series demonstrated that the results obtained in the lowest-order \(\varepsilon\) expansion can only be considered as a crude estimate, especially for disordered systems. Thus, the results obtained in \[11\] require reevaluation using a more precise approach. For this purpose, in this work we performed a correct field-theoretical description of the effect of structural defects on the anomalous critical ultrasound attenuation and the anomalous change in the ultrasound velocity in three-dimensional Ising-like compressible systems with allowance for both the fluctuation \[12\] and relaxation attenuation mechanisms without using the \(\varepsilon\)-expansion method.

II. THE MODEL

For phase transitions in compressible systems, the relation between an order parameter and elastic deformations is an important factor. As was first shown in \[15\], the critical behavior of compressible systems with a quadratic striction is unstable in regard to the relation between an order parameter and acoustic modes, and a first-order phase transition that is close to a second-order phase transition is realized. However, as was clarified in \[16\], the conclusions made in \[15\] are valid only at low pressures and, beginning with a certain threshold pressure, the deformation effects induced by an external pressure...
change the order of the phase transition. 

The Hamiltonian of a disordered compressible Ising model can be written as

$$H = H_{el} + H_{op} + H_{int} + H_{imp}. \quad (1)$$

The contribution of the deformation degrees of freedom is determined as

$$H_{el} = \frac{1}{2} \int \text{d}^dx \left( C_{11}^0 \sum_\alpha u_{\alpha \alpha}^2 + 2 C_{12}^0 \sum_{\alpha \beta} u_{\alpha \beta} u_{\beta \alpha} + 4 C_{44}^0 \sum_{\alpha < \beta} u_{\alpha \beta}^2 \right), \quad (2)$$

where $u_{\alpha \beta}(x)$ are the components of the deformation tensor and $C_{ij}^k$ are the elastic constants. The use of an isotropy approximation for $H_{el}$ is caused by the fact that, in the critical region, the system behavior parameters are determined by an isotropic fixed point of renormalization-group transformations, while the anisotropy effects are negligible. The magnetic component $H_{op}$ is represented in the form of the Ginzburg-Landau-Wilson Hamiltonian

$$H_{op} = \int \text{d}^dx \left[ \frac{1}{2} \tau_0 S_\alpha^2 + \frac{1}{2} \left( \nabla S \right)^2 + \frac{1}{4} u_0 S^4 \right], \quad (3)$$

where $S(x)$ is the spin order parameter, $u_0$ is a positive interaction constant, and $\tau_0 = (T - T_{0c})/T_{0c}$ is the reduced phase-transition temperature. The $H_{int}$ component determines spin-phonon interaction,

$$H_{int} = \int \text{d}^dx \left[ g_0 \sum_\alpha u_{\alpha \alpha} S^2 \right], \quad (4)$$

where $g_0$ is the quadratic-striction parameter. The effect of defects is taken into account by the term

$$H_{imp} = \int \text{d}^dx \left[ \Delta \tau(x) S^2 \right] + \int \text{d}^dx \left[ h(x) \sum_\alpha u_{\alpha \alpha} \right], \quad (5)$$

where random and Gaussian-distributed variables $\Delta \tau(x)$ and $h(x)$ determine local phase-transition temperature fluctuations and random stress fields, respectively.

To perform calculations, it is convenient to use the Fourier components of the deformation variables in the form

$$u_{\alpha \beta} = \tilde{u}_{\alpha \beta}^{(0)} + V^{-1/2} \sum_{q \neq 0} \tilde{u}_{\alpha \beta}(q) \exp(irq), \quad (6)$$

where $q$ is the wavevector, $V$ is the volume, $\tilde{u}_{\alpha \beta}^{(0)}$ is the uniform deformation tensor, and $\tilde{u}_{\alpha \beta}(q) = 1/2 [q_\alpha u_{\alpha \beta} + q_\beta u_{\alpha \beta}]$. We introduce an expansion in terms of the normal coordinates,

$$\bar{u}(q) = \sum_\lambda \bar{e}_\lambda(q) Q_{\lambda},$$

where $\bar{e}_\lambda(q)$ is the polarization vector.

We then perform integration with respect to the off-diagonal components of the uniform part of the deformation tensor $u_{\alpha \beta}^{(0)}$, in the statistical sum (they are not essential for the critical behavior of the system in an elastically isotropic medium) and obtain a Hamiltonian for the system in the form of a functional for the spin order parameter $S(q)$ and the normal coordinates of the deformation variables $Q_\lambda(q)$

$$\tilde{H} = \frac{1}{2} \int \text{d}^dq \left( \tau_0 + q^2 \right) S_q S_{-q} + \int \text{d}^dq \left( h_q Q_{\lambda} + \frac{1}{2} \sum \int \text{d}^dq \Delta \tau_q S_{q1} S_{q1 - q} + \frac{1}{4} \sum \int \text{d}^dq \left( S_q S_{-q} \right)^2 \right) \quad (7)$$

where

$$w_0 = \frac{3g_0^2}{2V (4C_{12}^0 - C_{11}^0)}, \quad q = C_{11}^0 + 4C_{12}^0 - 4C_{44}^0.$$ 

The relaxation critical dynamics of compressible systems is described by dynamic equations of the type of generalized Langevin equations,

$$\dot{S}_q = -\Gamma_0 \frac{\partial \tilde{H}^*}{\partial S_{-q}} + \xi_q + \Gamma_0 h_q,$$

$$\dot{Q}_{\lambda} = -\frac{\partial \tilde{H}^*}{\partial Q_{-\lambda}} - q^2 D_0 Q_{\lambda} + \eta_q + h_Q,$$ 

where $\Gamma_0$ and $D_0$ are bare kinetic coefficients; $\xi_q(x,t)$ and $\eta_q(x,t)$ are Gaussian-distributed quantities that have the character of a random force and $h_q$ and $h_Q$ are the fields thermodynamically conjugated to the spin and deformation variables, respectively.

When solving the set of nonlinear equations with Hamiltonian $\tilde{H}(S, Q)$ iteratively, we can single out the elastic-variable response function $D(q,\omega)$, which is determined as

$$D(q,\omega) = \frac{\delta \langle [Q_{\omega,\lambda}] \rangle}{\delta h_Q} = \langle [Q_{\omega,\lambda}Q_{-\omega,-\lambda}] \rangle,$$ 

and the spin-variable response function $G(q,\omega)$

$$G(q,\omega) = \frac{\delta \langle [S_{\omega,\lambda}] \rangle}{\delta h_S} = \langle [S_{\omega,\lambda}S_{-\omega,-\lambda}] \rangle,$$ 

where $\langle \ldots \rangle$ stands for statistic averaging over random Langevin forces, $[\ldots]$ stands for the averaging over the fluctuations of random fields $\Delta \tau_{-q}$ and $h_q$ that are specified by structural defects, and $\omega$ is the characteristic ultrasonic vibration frequency.
Using the Dyson representation, we present the $G(q, \omega)$ and $D(q, \omega)$ response functions in the form

$$G^{-1}(q, \omega) = G_0^{-1}(q, \omega) + \Pi(q, \omega), \quad (11)$$
$$D^{-1}(q, \omega) = D_0^{-1}(q, \omega) + \Sigma(q, \omega). \quad (12)$$

The bare $G_0(q, \omega)$ and $D_0(q, \omega)$ response functions are determined as:

$$D_0(q, \omega) = \left(\omega^2 - a_0 q^2 - i\omega D_0 q^2\right)^{-1},$$
$$G_0(q, \omega) = \left(i\omega / \Gamma_0 + (\tau_0 + q^2)^{-1}\right)^{-1}.$$

In the low-temperature phase, the response function contains an additional relaxation contribution

$$S_q = M \delta_{q,0} + \varphi_q, \quad (13)$$

with the magnetization

$$M = \begin{cases} 0, & T > T_c, \\ B |T - T_c|^\beta, & T < T_c, \end{cases} \quad (14)$$

where $B$ is the phenomenological relaxation parameter and $\varphi_q$ is the fluctuation part of the order parameter.

The self-energy part $\Sigma(q, \omega)$ of the $D(q, \omega)$ response function is directly related to the dynamic characteristics of ultrasound propagation [18].

As a result, the ultrasonic attenuation coefficient can be expressed through the imaginary part of $\Sigma(q, \omega)$

$$\alpha(\omega, \tau) \sim \omega \text{Im}\Sigma(0, \omega), \quad (15)$$

and the sound velocity dispersion is expressed through its real part,

$$c^2(\omega, \tau) \sim c^2(0, \tau) \sim \text{Re}\left(\Sigma(0, \omega) - \Sigma(0, 0)\right). \quad (16)$$

We calculated $\Sigma(q, \omega)$ in a two-loop approximation. The diagrammatic representation of $\Sigma(q, \omega)$ is shown in Fig. 2. These Feynman diagrams contain $d$-dimensional integration (in our case, $d = 3$).

When approaching the critical point, the correlation length $\xi$ tends to infinity, and, when $\xi^{-1} \ll \Lambda$ (where $\Lambda$ is the cutoff parameter of the integration over wavevectors), the system characteristics demonstrate their asymptotic scaling behavior for wavevectors $q \ll \Lambda$. Thus, the calculation of these quantities can be carried out in the limit $\Lambda \to \infty$. The application of a renormalization-group procedure eliminates the divergences that appear in the thermodynamic variables and kinetic coefficients at $\Lambda \to \infty$.

To calculate attenuation coefficient $\alpha(q, \omega)$ and ultrasound velocity dispersion $c(\omega, \tau)$ and to eliminate the divergences in $\Sigma(q, \omega)$ at $q \to 0$ we used the matching method in [19], which was then generalized for the description of the dynamic behavior of a system in [20].

Thus, using a scaling relationship for the dynamic response function

$$D(q, \omega, \tau) = e^{-(2-\eta)l} D\left(q e^l, (\omega / \Gamma_0) e^{2l}, e^{l/\nu}\right), \quad (17)$$

we can calculate the right-hand side of the equation for some constant value $l = l^*$, at which not all of the arguments in the response function disappear simultaneously. The choice of $l^*$ is determined by the condition

$$\left[\left(\omega / \Gamma_0\right) e^{4l^*}\right]^{4/\nu} + \left[\left(\tau e^{l^*} / \nu\right)^{2\nu} + q^2 e^{2l^*}\right]^2 = 1, \quad (18)$$

renormalization-group transformation, namely, to find a relation between the behavior of the system in the precritical regime at a low value of reduced temperature $\tau$ and the behavior of the system in a regime far from the critical mode, i.e., without divergences in $\Sigma(q, \omega)$. As was demonstrated in [20], matching condition [18] provides an infrared cutoff for all diverging quantities. Based on Eq. (18), we find the solution for $l^*$ in the form of a functional dependence on $\omega$ and $\tau$, which is specified by the static critical exponent $\nu$ of the correlation length and by the dynamic critical exponent $z$,

$$e^{l^*} = \tau^{-\nu} \left[1 + (y/2)^{4/\nu}\right]^{-1/4} \equiv \tau^{-\nu} F(y), \quad (19)$$

where $y = \omega \tau^{-z \nu} / \Gamma_0$ is the argument of the $F(y)$ function.

As is known from the theory of ultrasound scattering in solids near a phase-transition temperature [8, 9], the expression for the imaginary part of $\Sigma(\omega, \tau)$ in the asymptotic limit ($\tau \to 0$, $\omega \to 0$) can be defined by a scaling function $\phi(y)$

$$\text{Im}\Sigma(\omega, \tau) / \omega \sim \tau^{-\alpha - z \nu} \phi(y), \quad (20)$$

which depends on the single generalized variable $y$. At the same time, for the imaginary component of the self-energy part, the following scaling relationship is valid [18]

$$\frac{\text{Im}\Sigma(\omega)}{\omega} = e^{(\alpha + z \nu) \nu} \frac{\text{Im}\Sigma(\omega e^{z \nu})}{\omega e^{z \nu}}.$$
The substitution of $e^{\tau}$ from Eq. (19) into the right-hand side of this expression allows calculating the $\phi(y)$ scaling function.

In the asymptotic limit $\tau \to 0, \omega \to 0$, the expression for the real part of $\Sigma(\omega, \tau)$ can be determined using another scaling function $f(y)$,

$$\text{Re} \left( \Sigma(0, \omega) - \Sigma(0, 0) \right) = \tau^{-\alpha} \left( f(y) - f(0) \right). \quad (21)$$

The real component of the self-energy part satisfies the scaling relation

$$\text{Re} \left( \Sigma(0, \omega) - \Sigma(0, 0) \right) = e^{\nu/\nu} \text{Re} \left( \Sigma(0, \omega e^{z}) - \Sigma(0, 0) \right). \quad (22)$$

The dynamic scaling functions calculated in the two-loop approximation have the form

$$\phi(y) = \frac{g^2 \Gamma_0}{\pi} F_{\alpha/\nu+1/2} - \frac{1}{\sqrt{2}} \left[ 1 - \frac{(\Delta + 1)^{1/2}}{\sqrt{\Delta}} \right] -$$

$$- M^2 \frac{3g^2 \Gamma_0}{2\pi^2} \frac{F_{\alpha/\nu-1/2} - \frac{1}{\sqrt{\Delta}}}{y^2} \left[ 1 - \frac{(\Delta + 1)^{1/2}}{\sqrt{\Delta}} \right] -$$

$$- \frac{3g^2 \Gamma_0}{y^3} F_{\alpha/\nu+1/2} - \frac{1}{\sqrt{2}} \frac{1 - (\Delta + 1)^{1/2}}{\sqrt{\Delta}} \left( \Delta - 1 \right)^{1/2} -$$

$$\times \frac{g^2 v^* \Gamma_0}{12 \pi^3} \frac{F_{\alpha/\nu-\Delta} - \frac{1}{\sqrt{2}}}{y^2} \ln \Delta,$$

$$f(y) = \frac{g^2 \Gamma_0}{\pi} F_{\alpha/\nu+1/2} - \frac{1}{\sqrt{2}} \left[ 1 - \frac{(\Delta - 1)^{1/2}}{\sqrt{\Delta}} \right] -$$

$$- M^2 \frac{3g^2 \Gamma_0}{2\pi^2} \frac{F_{\alpha/\nu-1/2}}{y^2} \left[ 1 - \frac{(\Delta - 1)^{1/2}}{\sqrt{\Delta}} \right] -$$

$$- \frac{3g^2 \Gamma_0}{y^3} F_{\alpha/\nu+1/2} - \frac{1}{\sqrt{2}} \frac{1 - (\Delta + 1)^{1/2}}{\sqrt{\Delta}} \left( \Delta - 1 \right)^{1/2} -$$

$$\times \frac{g^2 \Gamma_0}{12 \pi^3} \frac{F_{\alpha/\nu-\Delta}}{y^2} \arctan(\Delta^2 - 1)^{1/2},$$

where $g^*$, $u^*$ and $v^*$ are the magnitudes of the interaction vertices at the fixed point of the renormalization-group transformations that corresponds to the critical behavior of the disordered compressible Ising model [21]. The terms in Eqs. (23) and (24) that are proportional to $M^2$ describe the relaxation contribution for the scaling functions of the attenuation coefficient and the sound velocity dispersion. In our subsequent numerical calculations of the scaling functions, we used the value $\nu = 0.70$ from [21] for the corresponding fixed point. The value of the dynamic exponent $z = 2.1653$ was taken from [22], where the critical dynamics of a disordered Ising model was analyzed within the framework of a relaxation model. The use of this value of exponent $z$ is valid in the case of disordered Ising-like systems with a negative heat-capacity exponent, since the relation between an order parameter and elastic deformations in the critical dynamics of a compressible system exerts no substantial influence on the relaxation properties of the order parameter.

### III. ANALYSIS OF RESULTS AND CONCLUSIONS

Perturbation-theory series are known to be asymptotic, and the vertices of the interaction of order-parameter fluctuations in the fluctuation range $\tau \to 0$ are too high to provide the direct application of Eqs. (23) and (24). Therefore, to extract the necessary physical information from the derived expressions, we apply the PadéBorel method, which is used to sum up asymptotic series, that was generalized to a three-parameter case. Then, the forward and inverse Borel transformations have the form

$$\phi(w, u, v) = \sum_{i,j,k} c_{ijk} w^i u^j v^k = \int_0^\infty e^{-w} B(wt, ut, vt) dt,$$

$$B(w, u, v) = \sum_{i,j,k} \frac{c_{ijk}}{(i + j + k)!} w^i u^j v^k,$$

where $w = g^2$.

To analytically continue the Borel transform of the function, we introduce a series in an auxiliary variable $\lambda$

$$\tilde{B}(w, u, v, \lambda) = \sum_{k=0}^\infty \lambda^k \sum_{i=0}^k \sum_{j=0}^{k-i} \frac{c_{ij}}{k!} w^i u^j v^{k-i-j}, \quad (26)$$

and substitute it to the Padé [L/M] approximation at the point $\lambda = 1$. This procedure was proposed and approved in [23] to describe the critical behavior of a number of systems containing several vertices of the interaction of order-parameter fluctuations. The fact [23] that the system retains its symmetry during the application of the Padé approximants in variable $\lambda$ becomes substantial for the description of multivertex models. In this work, we calculated the scaling functions in the two-loop approximation using approximant [1/1]. The behavior of the dynamic $\phi(y)$ and $f(y)$ scaling functions calculated with summation methods for pure and disordered systems is shown in Figs. 3(a) and 3(b) on a loglog scale. Depending on the interval of changing variable $y$, the following asymptotic regions can be distinguished in the behavior of $\phi(y)$ and $f(y)$: a hydrodynamic region, where $y \sim \omega \xi^\nu \sim (g^2)^\nu < 1$, and a critical region $y \sim \omega \xi^\nu \gg 1$, which determines the behavior of the system near the phase transition temperature ($\tau = (T - T_c)/T_c \ll 1$). As is seen from these curves at $y \ll 1$, the presence of a structural disorder does not affect the behavior of the $\phi(y)$ and $f(y)$, scaling functions and, consequently, the behavior of this system.
FIG. 3: Scaling functions (a) $\phi(y)$ and (b) $f(y)$ for (1) pure and (2) disordered systems at $T > T_c$ and (1') and (2') at $T < T_c$ ($\phi_0 = \phi(0), f_0 = f(0)$), respectively.

However, it begins to manifest itself in the crossover region $10^{-1} < y < 10$ and exerts an essential effect in the critical region $y > 10$.

As follows from Eqs. (15) and (20), the attenuation coefficient can be expressed as

$$\alpha(\omega, \tau) \sim \omega^2 \tau^{-\nu z} \phi(y),$$  

and, using Eqs. (16) and (21), we can write the relation for the sound velocity dispersion in the form

$$c^2(\omega, \tau) - c^2(0, \tau) \sim \tau^{-\alpha} (f(y) - f(0)).$$  

The results of the calculations of the asymptotic dependences of the attenuation coefficient and the sound velocity dispersion for the critical and hydrodynamic regions are given in the table. The characteristics of their frequency and temperature dependences were determined in the range $10^{-3} \leq y \leq 10^{-1}$, for the hydrodynamic regime and in the range $10 \leq y \leq 10^3$, for the critical regime. Note that, according to [20], the real temperature range $10^{-3} \leq \tau \leq 10^{-1}$ in ultrasonic studies of phase transitions corresponds to the range $1 \leq y \leq 10^2$, i.e., it covers the crossover region and the beginning of the critical region (precritical regime).

It follows from the table that anomalously strong ultrasound attenuation should be observed in both pure and disordered systems. For the disordered model, the increase in the attenuation coefficient as the phase-transition temperature is approached is expected to be stronger than that in the pure model even in the hydrodynamic region, whereas in the critical region, the disordered system should exhibit stronger frequency and temperature dependences of the attenuation coefficient as compared to the pure system.

These conclusions are supported by the model representation of the results of the numerical calculations of the critical temperature behavior of the attenuation coefficient for both the pure and disordered systems performed at $B = 0.3$ and $\omega/\Gamma_0 = 0.0015$. (Fig. 4). These values were determined when we compared the calculated temperature dependence of the attenuation coefficient and the results of experimental studies of pure FeF$_2$ samples (Fig. 4 (dots) [1], which demonstrate Ising-like behavior in the critical region.

An analysis of the data related to the sound velocity dispersion (see table) demonstrates that, as compared to pure analogs, a structural disorder in Ising-like systems leads to a stronger temperature dependence of the sound velocity dispersion in both the hydrodynamic and critical regions and is characterized by an increase in the exponent of the temperature dependence (a decrease in the absolute value) when going from the hydrodynamic region.
to the critical region. However, the effect of a structural disorder has the converse character for the exponent of the frequency dependence of the sound velocity dispersion: in the hydrodynamic region, the exponents of these two types of systems coincide, whereas, in the critical region, the sound velocity dispersion of a disordered system has a stronger frequency dependence compared to the pure system. The exponent decreases strongly when going from the hydrodynamic to the critical region.

A particularly important result of our investigation consists in the predicted manifestation of the dynamic effects of structural defects on anomalous sound attenuation and sound velocity dispersion over a wider temperature range near the critical temperature (already in the hydrodynamic region) in comparison with other experimental methods [24], which require a narrow temperature range (of about $\tau \approx 10^{-4}$) to be studied for revealing these effects. Thus, the results obtained can serve as a reference for purposeful experimental investigations of the dynamic effects of structural defects on the critical behavior of solids using acoustic methods via the detection of the influence of structural defects on the frequency and temperature dependences of the ultrasound attenuation coefficient and ultrasound velocity dispersion.

### Table I: Asymptotic behavior of the sound attenuation coefficient and the sound velocity dispersion in the critical, precritical, and hydrodynamic regimes for pure and disordered systems

| Regim          | Pure               | Disordered          |
|----------------|--------------------|---------------------|
|                | $T < T_c$          | $T > T_c$           |
|                | $T < T_c$          | $T > T_c$           |
| Critical       | $y = 10^4 \div 10^4$ | $\omega^{0.98 \pm 0.08}$ | $\omega^{1.05 \pm 0.17}$ | $\omega^{1.12 \pm 0.10}$ | $\omega^{1.21 \pm 0.24}$ |
| Precritical    | $y = 10^3 \div 10^2$ | $\omega^{1.08 \pm 0.21}$ | $\omega^{1.20 \pm 0.37}$ | $\omega^{1.23 \pm 0.25}$ | $\omega^{1.37 \pm 0.48}$ |
| Hydrodynamic   | $y = 10^{-3} \div 10^{-1}$ | $\omega^{2.38 \pm 1.38}$ | $\omega^{2.38 \pm 1.38}$ | $\omega^{2.14 \pm 1.44}$ | $\omega^{2.14 \pm 1.44}$ |

| Regim          | Pure               | Disordered          |
|----------------|--------------------|---------------------|
|                | $T < T_c$          | $T > T_c$           |
|                | $T < T_c$          | $T > T_c$           |
| Critical       | $y = 10^4 \div 10^3$ | $\omega^{0.11 \pm 0.25}$ | $\omega^{0.34 \pm 0.54}$ | $\omega^{0.36 \pm 0.31}$ | $\omega^{0.49 \pm 0.66}$ |
| Precritical    | $y = 10^3 \div 10^2$ | $\omega^{0.30 \pm 0.49}$ | $\omega^{1.08 \pm 1.48}$ | $\omega^{0.41 \pm 0.54}$ | $\omega^{1.01 \pm 1.45}$ |
| Hydrodynamic   | $y = 10^{-3} \div 10^{-1}$ | $\omega^{2.65 \pm 2.65}$ | $\omega^{2.65 \pm 2.65}$ | $\omega^{2.95 \pm 2.95}$ | $\omega^{2.95 \pm 2.95}$ |

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