Steric repulsion and van der Waals attraction between flux lines in disordered high $T_c$ superconductors

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We show that in anisotropic or layered superconductors impurities induce a van der Waals attraction between flux lines. This attraction together with the disorder induced repulsion may change the low $B$ - low $T$ phase diagram significantly from that of the pure thermal case considered recently by Blatter and Geshkenbein [Phys. Rev. Lett. 77, 4968 (1996)].

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Conventional type-II superconductors show in addition to the flux repulsing Meissner state a second superconducting (Abrikosov) phase in which the magnetic induction $B$ enters the material in the form of quantized flux lines (FLs) which form a triangular lattice. Each FL carries a unit flux quantum $\Phi_0 = h c / 2e$. The Abrikosov lattice is characterized by a non-zero shear modulus $c_{66}$, which vanishes at the upper and lower critical field, $H_{c2}$ and $H_{c1}$, where continuous transitions to the normal and the Meissner state, respectively, occur. In his mean-field solution Abrikosov treats FLs as stiff rods. Close to the lower critical field $H_{c1}$, their interaction becomes exponentially weak and hence the FL density $l^{-2} = B / \Phi_0$ vanishes as $|\ln h|^{-2}$ where $h = (H - H_{c1}) / H_{c1}$ denotes the reduced field strength $\mathbb{1}$.

Thermal fluctuations roughen the FLs resulting in a possible melting of the Abrikosov lattice close to $H_{c1}$ and $H_{c2}$, respectively, because of the softening of $c_{66}$. This applies in particular to high-$T_c$ materials with their elevated transition temperatures and their pronounced layer structures $\mathbb{2}$. At present, it is not clear whether the transition to the normal phase at high field happens in these materials via one or two transitions. However, melting of the FL lattice has clearly been observed experimentally $\mathbb{3}$.

At low fields a first order melting transition to a liquid phase and a change in the critical behavior of $B$ has been predicted some time ago by Nelson $\mathbb{4}$. Quantitatively the influence of thermal fluctuations is described by a thermal length scale $L_T = \Phi_0^2 / (16 \pi^2 T) \approx 2 cm K / T \mathbb{5}$. $L_T$ has a simple physical meaning: an isolated flux line of length $L_T$ shows a thermal mean square displacement of the order of the London penetration length $\lambda$. Besides a shift of $H_{c1}$, large scale thermal fluctuations lead close to $H_{c1}$ to an entropic repulsion $\sim (\lambda^2 / L_T l)^2$ between FLs which dominates over the bare interaction for small $h$ and hence $B \sim h \mathbb{6}$. (Here we measure all FL interactions in units of $\xi_0 = (\Phi_0 / 4 \pi \lambda)^2 = L_T / \lambda^2$).

More recently, Blatter and Geshkenbein $\mathbb{7}$ found that in anisotropic or layered superconductors short scale fluctuations give rise also to an attractive van der Waals (vdW) interaction $\mathbb{8}$. For FLs separated by a distance $R$ the strength of this interaction is of the order of $\lambda^2 L_T (d + \varepsilon R) R^4$. $\varepsilon^2 = m / M \ll 1$ denotes the anisotropy of the material with $m$ and $M$ the effective masses parallel and perpendicular to the $ab$ plane, and $d$ the interlayer spacing. $\lambda$ and $\lambda / \varepsilon$ are then the screening lengths parallel and perpendicular to the layers, respectively.

The competition among the bare, the entropic and the vdW interactions leads to an interesting phase diagram at low $B$ values. In particular, Blatter and Geshkenbein $\mathbb{9}$ find at low $T$ a first order transition between the Meissner and the Abrikosov phase.

So far fluctuation effects have been discussed for a clean superconductor. It is well known, however, that in type–II superconductors FLs have to be pinned in order to prevent dissipation from their motion under the influence of an external current. Therefore, besides the thermal fluctuations one has to take into account the effect of disorder. Randomly distributed pinning centers lead indeed to a destruction of the Abrikosov lattice $\mathbb{10}$, but as has been recently shown, for not too strong disorder FLs may form a (Bragg–) glass phase which is characterized by quasi long–range order of the FL lattice $\mathbb{11}$. For low $B$ this phase undergoes a melting transition to a pinned liquid state. Inside this phase, disorder induced effects are expected to dominate over those of thermal fluctuations for sufficiently low $T$. It is therefore the aim of the present paper to consider the influence of the disorder fluctuation induced forces between the FLs, and to study the low $B$ phase diagram. In particular we find, that the latter deviates substantially from that found in Ref. $\mathbb{12}$ for pure systems. The rest of the paper is organized as follows. We first reconsider the disorder mediated steric repulsion between the FLs and then derive the disorder induced vdW interaction. Finally we discuss the phase diagram at low $B$ and $T$.

Let us start with the steric repulsion, which results from the long wave length fluctuations of FLs. Using a simple scaling argument, it was found in Ref. $\mathbb{13}$, that the disorder dominated steric repulsion between FLs is of the order

$$\lambda^{2/\zeta} L_{\text{dis}}^{-2} l^{2-2/\zeta} = (T_{\text{dis}} / \varepsilon_0 \xi)^2 (\xi / l)^{2/\zeta - 2}$$

(1)

where the roughness exponent $\zeta$ of a single FL in a random potential is about $5/8$ in $D = 3$ dimensions $\mathbb{14}$. Here $\xi$ is the correlation length in the superconducting
planes and \( L_{\text{dis}} \) denotes a disorder–related length scale with a similar meaning as \( L_T \). At low temperatures, \( L_{\text{dis}} \approx L_T T_R^{1/\zeta-2}/T_{\text{dis}} \), where \( \kappa = \lambda/\xi \), denotes the Ginzburg-Landau parameter. \( T_{\text{dis}} \) is defined in 3, and it is identical with \( T_R^{\text{a,iso}} \) of ref. 3. The steric interaction results from the fact, that the FL is confined to a cylindrical cage, formed by its six nearest neighbors. It therefore cannot gain the full energy decrease a free FL can obtain from the disorder.

This argument has been later put into question [2], since the FL might overcome the averaged repulsive potential \( (\lambda/l)^{D-1} \) of the cage and hence leave it to follow its optimal path. Naively, this should be the case if \( (\lambda/l)^{D-1} < \lambda^{1/\zeta} L_{\text{dis}}^{\zeta-2}/L_T \) or \( D > D_\zeta = (2/\zeta(D)) - 1 \). Such an argument is indeed correct for thermal fluctuations, where \( \zeta = 1/2 \) in all dimensions: for \( D > 3 \) the steric repulsion becomes ineffective since the entropy gain from leaving the cage outweighs the repulsion 13.

In the case of disorder, the situation is however different. At low \( T \) a FL which leaves its cage can only increase its energy gain if it follows the optimal path. Let us neglect for the moment the repulsive interaction between the lines. Since the optimal path for a line with a given initial point has a river delta like shape, two lines separated initially by a distance \( nl \) will merge after a transient region of length \((nl)^{1/\zeta} \ll L \). Let us now switch on again the repulsion. Since the lines follow the same path, repulsion leads to an energy increase of the order \( L \). This has to be compared with the free energy gain by following the optimal path instead of the path inside the cage, which is of the order \( L^{2\zeta-1} \) and hence smaller than the repulsion since \( \zeta < 1 \). Thus, a line leaving its cage would increase its energy instead of decreasing it unlike the thermal case, where lines can avoid each other and still gain entropy. Consequently, the simple argument of Ref. 10 applies which results in 3.

The above argument can indeed be derived in a somewhat more formal way by using a renormalization group (RG) analysis of interacting FLs 3. In the large wavelength limit, the FLs of length \( L \) are described by the Hamiltonian

\[
\mathcal{H} = \int_0^L dz \sum_{i=1}^{N} \left\{ \frac{\xi}{2} \mathbf{R}_i^2 + \varepsilon_{\text{pin}}(\mathbf{R}_i, z) + \sum_{j \neq i} v_0 \delta_\lambda(\mathbf{R}_{ij}) \right\},
\]

where \( \mathbf{R}_i = \partial \mathbf{R}_i(z)/\partial z \) and \( \mathbf{R}_{ij} = \mathbf{R}_i - \mathbf{R}_j \). \( \mathbf{R}_i(z), \mathbf{R}_{ij} \) denotes the position vector of the \( i \)-th FL, \( \xi \) its stiffness constant which is of the order \( \varepsilon_0 \) in the long wavelength limit. The random pinning potential fulfills \( \varepsilon_{\text{pin}}(\mathbf{R}, z) \equiv 0 \) and 3

\[
\varepsilon_{\text{pin}}(\mathbf{R}, z) \varepsilon_{\text{pin}}(\mathbf{R}, 0) = \frac{1}{\mathcal{N}} \frac{\xi_0^2}{\delta_\lambda(\mathbf{R})} \delta(z) k(\mathbf{R})
\]

with \( k(x) = 1 \) for \( x \ll 1 \) and \( k(x) \approx (1/x^2) \ln x \) for \( x \gg 1 \), respectively. \( T_{\text{dis}}^3 \) is proportional to the impurity concentration. \( \delta_\lambda(\mathbf{R}) \) represents a \( \delta \)-function smeared out over a scale \( \lambda \) and \( v_0 \approx 4\pi \lambda^2 \varepsilon_0 \). Since we are interested in the diluted limit \( l \gg \lambda \), the RG flow of \( \Delta \) and \( v \) can be obtained by considering only two interacting lines as in Ref. [14].

Close to \( D = 2 \) dimensions, we find \((\partial v/\partial \varepsilon_0) = 2(1/\zeta - 1)\varepsilon \) and hence \( v \) remains a relevant perturbation even above two dimensions, in contrast to the results obtained in Ref. [13]. We, therefore, conclude that the result (1) is valid for all \( D \). It is worth remarking here, that in contrast to the present case, a single FL in a cylindrical cage does indeed show a depinning transition from this potential in \( D > 2 \) dimension [15].

Next we consider the attractive van der Waals interaction. We start with the case of extreme anisotropy \( \varepsilon = 0 \), in which the Josephson coupling between the layers can be neglected. The repulsive interaction between two pancake vortices 1 and 2 separated by a distance \( R \) in the same layer is then larger than their attractive interaction in different layers by a factor \( \lambda/d \) \( \gg 1 \). In considering the interaction of different FLs consisting of pancake vortices we therefore restrict ourselves to pancakes in the same layer. Since their interaction is \( 2d\ln(\xi/R) \), the interaction of two pancake dipoles resulting from the displacements \( \mathbf{u}_1, \mathbf{u}_2 \) of pancakes 1 and 2 in the same layer is \( U_{12} = -(2d/R^2)(2(\mathbf{u}_1 \cdot \mathbf{n}))(\mathbf{u}_2 \cdot \mathbf{n}) - \mathbf{u}_1 \cdot \mathbf{u}_2 \), where \( \mathbf{n} \) is the unit vector in the direction connecting the two FLs. A displacement \( \mathbf{u} \) will indeed induce a displacement \( \mathbf{u}_2 \) because of the force \( f_{12} = -\partial U_{12}/\partial \mathbf{u}_2 \). The actual response of the pancake 2 is limited by the elastic force \( f_{22} \sim -(\partial u_2/\partial x)^2 \ln(\pi \lambda^2) \) for \( k \approx \pi/d \). With \( f_{12} + f_{22} = 0 \) and averaging over different configurations of \( \mathbf{u}_1 \), the vdW–interaction per unit length is

\[
V_{\text{vdW}} \sim -\frac{\lambda^2}{R^4} \frac{1}{\ln \pi \lambda/d} \mathbf{u}_1 \cdot \mathbf{u}_2
\]

Inserting the result for the short wavelength thermal fluctuations \( \langle \mathbf{u}_1 \rangle = (2T/\varepsilon_0 d) \mathbf{u}^2 \ln(\pi \lambda/d) \), we obtain the vdW attraction given in Ref. [3], apart from a numerical factor.

In the case of an impure superconductor \( \langle \mathbf{u}_1 \rangle \) is given by the mean square displacement of a pancake in a potential which is a superposition of a parabolic elastic potential \( \varepsilon_0 d(\mathbf{u}/2\pi \lambda)^2 \ln(\lambda/d) \), resulting from the dispersive elastic constant for \( k \approx \pi/d \), and a random potential \( \int_0^L dz \varepsilon_{\text{pin}}(\mathbf{u}, z) \). Rewriting 3, for \( L \approx d \) in dimensionless quantities \( r = R/\xi \) and \( t = d/z \), it is easy to see, that the ground state displacement \( \mathbf{u}_0/\xi \) is only a functional of \( (\lambda \pi d)^{1/2} \varepsilon_{\text{pin}}(r, t) \). \( \varepsilon_{\text{pin}}(r, t) \) is the dimensionless random potential of mean zero and variance unity and

\[
\Delta(x) = \frac{x^2}{\ln^2(1 + x^2)}, \quad \Delta_0 = \left( T_{\text{dis}}^3 \xi^2 \varepsilon_0 \right)^3
\]

Thus at \( T \ll T_{\text{dis}} \), \( \langle \mathbf{u}_0^2 \rangle \approx \mathbf{u}_0^2 = \xi^2 f(\Delta(\pi d)/d) \). Perturbation theory applies for \( \Delta \ll 1 \) and gives \( f(\Delta) \approx \Delta^\eta \) with \( \eta = 1 \). For larger \( \Delta \) no exact result is known. Imry–Ma

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arguments and a variational treatment give \( \eta = 1/2 \) with a logarithmic correction \([4]\), whereas we find \( \eta \approx 4/9 \) from simulations. For BSCCO with \( \varepsilon = 1/300, \lambda \approx 2000 A, \xi \approx 20 A, \quad d = 15 A, \) and \( T_{\text{dis}} = 45 K \) \([4]\), \( \Delta_0 \approx 91 \) and \( \Delta \approx 261, i.e. \) we are in the non-perturbative regime. In Ref. \([2]\) a much larger value of \( T_{\text{dis}} = 210 K \) was reported which results \( \Delta_0 \approx 9.261 \times 10^3 \) and \( \Delta \approx 2.6 \times 10^4 \). The final form for the disorder induced vdw interaction in the decoupled limit is therefore

\[
V_{\text{vdW}}^{(\text{dis})} \approx -\frac{\kappa^{-2} \varepsilon_0}{\ln \pi \lambda / d} \left( \frac{\lambda}{R} \right)^4 \left[ \Delta_0 \left( \frac{\pi \lambda / d}{\ln^2 (\pi \lambda / d)} \right) \right]^{\eta}.
\]

(6)

From \([3]\) and \([2]\), we conclude that, in the decoupled limit, the vdw interaction will be dominated by disorder fluctuations for \( T \ll T_{\text{dis}}. \) For \( \Delta \gg 1 \) (and \( \eta = 1/2 \)), \( T_{\text{dis}} = \kappa (T_{\text{dis}} d / \varepsilon_0 \lambda^2)^{1/2} \), whereas for \( \Delta \ll 1 \), \( T_{\text{dis}} = \kappa^4 (T_{\text{dis}} d / \varepsilon_0 \lambda^2)^{1/2} \left( \frac{\varepsilon_0 \lambda}{d \ln (\pi \lambda / d)} \right) \). With the above values this results in \( T_{\text{dis}} = 8K \) for \( T_{\text{dis}} = 45K \) \([4]\), and \( T_{\text{dis}} = 83K \) for \( T_{\text{dis}} = 210K \) \([4]\).

Next we consider the continuous anisotropic case \( \varepsilon > 0 \). Inspection of the perturbation theory \([4]\) shows, that if the disorder induced interaction between FLs is neglected the vdw interaction can be written in the following form

\[
V_{\text{vdW}} = -\frac{1}{4\pi} \left( \frac{\Phi_0^2}{4\pi} \right)^2 \int_0^{\pi/d} \frac{dk}{2\pi} \left[ V_{\text{int}}^{(\text{dis})}(k, R) \right]^2 k^4 \times (C_T^2(k) + 2C_T(k)C_{\text{dis}}(k)),
\]

(7)

where the interaction between two vortex segments is

\[
V_{\text{int}}^{(\text{dis})}(k, R) = -\frac{1}{2\pi R} \left( \frac{\varepsilon}{\lambda R^{\gamma}} \right)^{1/2} \left( \frac{\varepsilon_0}{\lambda R} \right)^{1/2} K_1(\varepsilon R) \sqrt{1 + \lambda^2 k^2 / \lambda^2},
\]

with \( K_\nu \) the \( \nu \)th order modified Bessel function. The two different correlations \( C_T \) and \( C_{\text{dis}} \) are defined as \( C_T(k) = \langle \mathbf{u}_k \mathbf{u}_{-k} \rangle - \langle \mathbf{u}_k \rangle \langle \mathbf{u}_{-k} \rangle = T / (\varepsilon_0(k) k^2) \) \([4]\) and \( C_{\text{dis}}(k) = \langle \mathbf{u}_k \rangle \langle \mathbf{u}_{-k} \rangle \). The first contribution in \([4]\) resulting from \( C_T(k) \), yields the thermal vdw interaction which was considered in Ref. \([4]\). The second contribution comes from the disorder and vanishes as the impurity concentration vanishes. To find \( C_{\text{dis}}(k) \) we have to distinguish the cases \( \lambda k \ll 1 \) and \( \lambda k \gg 1 \). For \( \lambda k \ll 1 \) we may neglect the dispersion of \( \varepsilon(k) \) and use the known result for \( u_k^2 \) on large scales \([3]\). This gives \( C_{\text{dis}}(k) \sim \xi^2 k^{-1} (\Delta(\lambda k))^2 \xi^2/3. \) On the contrary, for \( \lambda k \gg 1 \), the strong dispersion of \( \varepsilon(k) \) essentially decouples pancake vortices in different layers. Hence \( C_{\text{dis}} \approx \frac{u_k^2 d}{d} \), apart from logarithmic corrections. Using the asymptotic behavior of the Bessel function \( K_1(z \to 0) \sim z^{-1} \) and \( K_1(z \to \infty) \sim e^{-z} \), it is easy to see, that the main contribution to the second integral in \([4]\) comes from large \( k, k \lesssim \pi / (d + \varepsilon R) \). In a convenient interpolation form the disorder induced van der Waals interaction for \( \lambda < R < \lambda / \varepsilon \) is then given by

\[
V_{\text{vdW}}^{(\text{dis})}(R) \approx -\frac{\varepsilon}{4\pi} \left( \Delta_0 \frac{\lambda}{d} \right)^{1/2} \left( \frac{d}{d + \varepsilon R} \right) \left( \frac{\varepsilon_0 \lambda}{d + \varepsilon R} \right)^{1/2} \left( \frac{\Delta_0 \lambda}{d + \varepsilon R} \right)^{1/2} \left( \frac{\varepsilon_0 \lambda}{d + \varepsilon R} \right)^{1/2} \left( \frac{\varepsilon_0 \lambda}{d + \varepsilon R} \right)^{1/2}
\]

(8)

Eqn. \([4]\) is the main result of this paper. For \( \varepsilon R \ll d \) it changes over to \([4]\). Thus similar to the thermal case, the vdw attraction decays as \( R^{-4} \) or \( R^{-5} \) for \( \varepsilon R < d \) or \( \varepsilon R > d \), respectively. Eqn. \([4]\) can indeed be written in the form of the thermal vdw interaction \([4]\) if we replace there \( T \) by \( T_{\text{dis}}. \)

In the last part we analyze the phase diagram at low \( T \) and \( H \geq H_{c1}. \) The bare repulsion between the flux lines \( 2K_0(R/\lambda) \) and their vdw attraction result in a minimum of their interaction energy at a distance \( R_{\text{min}} \approx \alpha \lambda \) (\( \ll l \)). For not too low temperatures \( \alpha \) is about 20 and only weakly \( T \) dependent. The same applies to the width of the minimum which is of the order of \( \beta \lambda \) with \( \beta \approx 10. \) Since the vdw attraction is strongly distance dependent, its main contribution comes from those configurations where the line pair is at a distance \( R_{\text{min}}. \) To lowest order in \( \Delta_0 \), we can estimate the average vdw interaction by considering the configurations of a single line in the absence of any FL interaction. With \( u \approx \lambda (L / T_{\text{dis}})^{1/2} \) for the displacement of a single FL, we find from \( u \approx l \) for the mean distance \( L \) between two line segments reaching a minimum \( L_b \approx L_{T_{\text{dis}}} (1/\lambda)^{1/4}. \) The length \( L_b \) of the segment over which the line stays in the minimum follows from the same argument as \( L_b \approx L_{T_{\text{dis}}} \beta^{1/4}. \) Thus, the contribution of the vdw attraction to the Gibbs free energy density is of the order

\[
\frac{1}{T^2} V_{\text{vdW}}(R_{\text{min}}) \frac{L_b}{L_b} \approx V_{\text{vdW}}(R_{\text{min}}) \left( \frac{\lambda}{l} \right)^{2+1/4} \beta^{1/4} \frac{1}{\lambda^2},
\]

(9)

which is much larger than the vdw interaction at the mean distance \( l. \) For thermal fluctuations (\( \zeta = 1/2 \)) the mean vdw attraction has therefore the same \( l \) dependence as the entropic repulsion. In this case, the result can also be obtained by mapping the problem onto 2d-Bosons \([5]\). For disorder induced fluctuations, however, the vdw attraction decays faster than the steric repulsion. The total Gibbs free energy density for \( \varepsilon R_{\text{min}} \ll d \) can be written in the following form

\[
G(x; H, T, T_{\text{dis}}) \approx \frac{\varepsilon_0}{\lambda^2 x^2} \left( x K_0(x) + \frac{\gamma T - \delta T}{x^2} \right) + \frac{\gamma_{\text{dis}}}{x^{3/5}} - \frac{\delta_{\text{dis}}}{x^{3/5}} - \hbar \ln \kappa \}
\]

(10)

which has to be minimized with respect to \( x = l / \lambda. \) Here \( z = 6 \) is the number of nearest neighbors for a triangular lattice. \( \gamma_{\text{dis}} \approx 0.08 (T / \varepsilon_0 \lambda)^2 \) \([4]\) and \( \gamma_{\text{dis}} \approx c_{\text{dis}}(T_{\text{dis}} / \varepsilon_0 \lambda)^2 \kappa^{1/2} \) denote the strength of the entropic and disorder dominated steric repulsion respectively. Expression \([10]\) has to be considered as an interpolation between the regimes dominated by the bare interaction at high \( B \) and the different fluctuation induced interactions at low \( B, \) respectively.
prefactors of the terms following from the vdW interactions are 
\[ \delta_T \approx c_T \frac{\beta}{\alpha^3} \left( \frac{1}{\xi} \right) \left( T / (\xi_0 \Lambda \ln^2 (\pi a / \xi_0)) \right) \] and 
\[ \delta_{\text{dis}} \approx \tilde{c}_{\text{dis}} \frac{\beta^{5/2}}{(1/\xi)} \left( T_{\text{dis}} / \xi_0 \right)^{3 \eta \kappa^{-2+6 \eta}} \ln \frac{\pi}{\xi} \left( 1+2 \eta \right) (\lambda / d)^{-1+\eta}. \]

The precise determination of the coefficients \( c_T \), \( c_{\text{dis}} \), \( \tilde{c}_{\text{dis}} \) as well as of \( \alpha \) and \( \beta \) is beyond the scope of this paper. To find them would require an RG treatment similar to that performed in [13] for short range repulsive FLs in the thermal case. In this case, instead of singling out a typical distance \( R_{\text{min}} \), the contributions of the vdW interaction from all distances will be taken into account according to their statistical weight. We postpone this study to a future publication [20] and discuss here the phase diagram (Fig. 1) only qualitatively.

The coexistence regime is well below the transition to the Bragg glass phase where the quasi long range order of the lattice persists. This transition line is beyond the reach of this analysis and corresponds to a much higher value of \( B_{\text{BG}} \).

In conclusion, we have obtained a disorder induced van der Waals attraction and a steric repulsion between the flux lines in anisotropic or layered superconductors. A qualitative analysis of the low field phase diagram for such impure systems predicts a rich phase diagram with a first order transition between two pinned gas/liquid phases for not too strong disorder. For larger disorder the first order transition disappears and \( B \sim (H - H_{c1})^{5/3} \) decreases continuously as \( H \to H_{c1} \).

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FIG. 1. The low- \( B \) phase diagram of an impure anisotropic superconductor. Thick lines correspond to continuous transitions. White regions indicate the coexistence regimes of the adjacent phases. \( B = 0 \) corresponds to the Meissner phase. For sufficiently weak disorder and low \( T \) a low density pinned gas phase is separated from a high density pinned liquid phase by a first order transition where \( B \) jumps from \( B_1 \) to \( B_2 \). With increasing disorder, the \( B_1 \) and \( B_2 \) lines merge as indicated by the dashed line, leading to a critical point at \( T_{\text{cl}} \). For large disorder \( T_{\text{cl}} \) moves towards \( T_{\text{cu}} \) and the first order transition disappears completely.

For \( T = 0 \), to begin with, we find from \([10]\) that for increasing \( H \) the transition from the Meissner phase to the pinned liquid phase is continuous since the steric repulsion dominates over the vdW interaction for large \( x = l / \lambda \) and hence \( B \sim l^{5/3} \). Increasing \( H \) further, for weak enough disorder (but \( \Delta > 1 \)) the vdW attraction will dominate over the steric repulsion in an intermediate range of \( x \) resulting in a discontinuous transition from a dilute to a dense pinned liquid phase. At even higher \( B \) a second discontinuous transition to the Bragg-glass phase takes place. At finite but low \( T \) this picture is essentially unchanged and will then cross over smoothly to the thermal phase diagram studied in \([5]\). For stronger disorder the steric repulsion dominates at \( T = 0 \) for all values of \( B \) and the discontinuous transition disappears. The first order transition between the low (or zero) \( B \) phase and the dense pinned liquid phase is now shifted to higher \( T \) and will disappear for sufficiently strong disorder completely. The latter is most likely the situation in BSSCO with impurity concentration corresponding to \( T_{\text{dis}} = 45K \) or higher.

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