Neutrino Masses from Loop-Induced Dirac Yukawa Couplings

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Abstract

We consider a possibility to naturally explain tiny neutrino masses without the lepton number violation. We study a simple model with $SU(2)_L$ singlet charged scalars ($s_{1}^{\pm}, s_{2}^{\pm}$) as well as singlet right-handed neutrino ($\nu_{R}$). Yukawa interactions for Dirac neutrinos, which are forbidden at the tree level by a softly-broken $Z_2$ symmetry, are induced at the one-loop level via the soft-breaking term in the scalar potential. Consequently neutrinos obtain small Dirac masses after the electroweak symmetry breaking. It is found that constrains from neutrino oscillation measurements and lepton flavor violation search results (especially for $\mu \rightarrow e\gamma$) can be satisfied. We study the decay pattern of the singlet charged scalars, which could be tested at the LHC and the ILC. We discuss possible extensions also, e.g. to introduce dark matter candidate.

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I. INTRODUCTION

The neutrino oscillation data provide evidence that neutrinos have tiny masses [1–5], which can be understood as a clear signature for physics beyond the standard model (SM). The simplest way to obtain neutrino masses may be to introduce $SU(2)_L$-singlet right-handed neutrinos, $\nu^i_R \ (i = 1-3)$, which have Yukawa interaction with the SM Higgs boson. Then Dirac masses for neutrinos are generated after the electroweak symmetry breaking. However, in this naive mechanism, the Yukawa coupling constants for neutrinos have to be unnaturally small ($\lesssim 10^{-11}$) in comparison with those for the other fermions. The most familiar idea to solve the problem would be the seesaw mechanism [6] by introducing Majorana mass terms for $\nu^i_R$. Taking the Majorana masses much larger than vacuum expectation value of the SM Higgs boson, very light Majorana neutrinos are obtained without excessively small Yukawa coupling constants.

Another interesting possibility to avoid tiny Yukawa coupling constants is the radiative generation of Majorana masses for $\nu_L$ without Dirac mass terms. The original model was proposed by Zee [7], in which Majorana neutrino masses are obtained at the one-loop level by the dynamics of the extended Higgs sector\(^1\). There have been several variant models in this direction \([8, 11, 13]\)\(^2\). Some of them \([9, 11, 13]\) include dark matter candidates by imposing an unbroken $Z_2$ symmetry which forbids the Dirac masses for neutrinos. In those models (seesaw and radiative ones), neutrinos are regarded as Majorana particles whose mass terms cause lepton number violating phenomena such as the neutrinoless double beta decay. Lepton number violation (LNV), however, has not yet been confirmed by experiments. Thus it is valuable to investigate a possibility that the tiny neutrino masses are generated in theories where the lepton number is conserved and Yukawa coupling constants are not extremely small.

Radiative generation of masses for Dirac neutrinos would be an interesting possibility. There were several studies in past for such a scenario in various frameworks such as the left-right symmetry [14, 15], supersymmetry (SUSY) [16], and extended models within the SM

\(^1\) If leptons couple with only one of two doublet scalar fields in the Zee model in order to eliminate the flavor changing neutral current interaction, the model cannot satisfy neutrino oscillation data [8].

\(^2\) In ref. [10], the second singlet fermion is added to the model in ref. [9] in order to satisfy neutrino oscillation data.
TABLE I: Particle contents of the 1LDNM. Here \( L_\ell, \ell_R, \) and \( \Phi \) are the \( SU(2)_L \)-doublet fields of left-handed leptons, the right-handed charged leptons, and the \( SU(2)_L \)-doublet scalar field in the SM, respectively. Three column from the right show particles added to the SM.

The simplest model seems to be the one in ref. [17], where Dirac neutrino masses are generated at the one-loop level by introducing two \( SU(2)_L \)-singlet charged scalar fields \( s_1^\pm, s_2^\pm \) as well as \( \nu_R^i \). In this letter, we show the one-loop Dirac neutrino model (1LDNM) is compatible with neutrino oscillation data although this was overlooked in [17]. We can find parameter sets which satisfy also constraints from searches for lepton flavor violation. We discuss the collider phenomenology of charged scalars under these parameter sets. Their decay pattern into leptons can be a characteristic feature of the 1LDNM, by which the model could be tested at the LHC and the International Linear Collider (ILC).

We also discuss some extensions of the model briefly; accommodating dark matter candidates, case with Majorana masses for \( \nu_R \), and so on.

II. THE MODEL

Particle contents of the 1LDNM are listed in Table I. Three \( SU(2)_L \)-singlet neutral fermions \( \nu_R^i \) \( (i = 1-3) \) are introduced such that Dirac masses for neutrinos exist. A softly-broken \( Z_2 \) symmetry is imposed in the model so that Dirac masses can be forbidden at the tree level, where \( \nu_R^i \) are assigned to be \( Z_2 \)-odd. Dirac neutrino masses are generated at the one-loop level by utilizing \( SU(2)_L \)-singlet charged scalars, \( s_1^+ \) and \( s_2^+ \), where \( s_2^+ \) is taken as a \( Z_2 \)-odd particle which can couple with \( \nu_R^i \). The Yukawa interactions, the Higgs potential, and the Dirac neutrino mass generation in this model are presented below in order.
The Yukawa interactions for leptons are given by

$$\mathcal{L}_{\text{Yukawa}} = y_\ell \overline{L}_\ell \Phi L_{\ell R} + f_{\ell\ell'} \overline{L}'_\ell i\sigma_2 L_{\ell' R'} s_1^+ + h_{\ell\ell'} (\overline{L}_{\ell R})^c \nu_{\ell R}^c s_2^+ + \text{h.c.},$$

(1)

where $\Phi$ is the SM Higgs doublet field. The superscript $c$ denotes the charge conjugation and $\sigma_i$ ($i = 1-3$) are the Pauli matrices. We take the basis where the Yukawa coupling matrix for charged leptons has been diagonalized as $y_\ell$. Notice that the matrix $f$ is antisymmetric, while the matrix $h$ takes somehow an arbitrary form. Although $h_{\ell\ell'}$ and $f_{\ell\ell'}$ are basically complex numbers, $f_{\ell\ell'}$ can be taken to be real numbers by using rephasing of three $L_{\ell}$ (and $\ell R$ to keep $y_\ell$ real) without loss of generality. Furthermore we can take the basis where $\nu_{\ell R}^c$ are mass eigenstates (of real positive mass eigenvalues). Then neutrino oscillation data give relations between the elements of $f_{\ell\ell'}$ and $h_{\ell\ell'}$ as shown later.

The Higgs potential is written as

$$V = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 + \mu_1^2 |s_1^+|^2 + \mu_2^2 |s_2^+|^2 + \left\{ \mu_3^2 s_1^+ s_2^- + \text{h.c.} \right\}$$

$$+ \lambda_1 |s_1^+|^4 + \lambda_2 |s_2^+|^4 + \left\{ \lambda_3 (s_1^+ s_2^-)^2 + \text{h.c.} \right\} + \lambda_4 |s_1^+|^2 |s_2^+|^2$$

$$+ \lambda_5 (\Phi^\dagger \Phi)|s_1^+|^2 + \lambda_6 (\Phi^\dagger \Phi)|s_2^+|^2,$$

(2)

where $\mu^2 > 0$. Although $\mu_3^2$ and $\lambda_3$ can be complex parameters, they become real by rephasing $s_1^+$ and $s_2^+$. Thus there is no complex parameter in the Higgs potential. Notice that $\mu_3^2$ is the soft-breaking parameter for the $Z_2$ symmetry we imposed. The quartic coupling constants should satisfy the following conditions in order to avoid that the potential is unbounded from below:

$$\lambda > 0, \quad \lambda_1 > 0, \quad \lambda_2 > 0,$$

(3)

$$\omega_1 \equiv 2\lambda_1 + \lambda_5 \sqrt{\frac{\lambda_1}{\lambda}} > 0, \quad 2\sqrt{\omega_1 \lambda_2} + 2\lambda_3 + \lambda_4 + \lambda_6 \sqrt{\frac{\lambda_1}{\lambda}} > 0,$$

(4)

$$\omega_2 \equiv 2\lambda_2 + \lambda_6 \sqrt{\frac{\lambda_2}{\lambda}} > 0, \quad 2\sqrt{\omega_2 \lambda_1} + 2\lambda_3 + \lambda_4 + \lambda_5 \sqrt{\frac{\lambda_2}{\lambda}} > 0,$$

(5)

$$\omega_{12} \equiv 2\lambda_1 + (2\lambda_3 + \lambda_4) \sqrt{\frac{\lambda_1}{\lambda_2}} > 0, \quad 2\sqrt{\omega_{12} \lambda_2} + \lambda_5 + \lambda_6 \sqrt{\frac{\lambda_1}{\lambda_2}} > 0.$$

(6)

Mass eigenstates of two charged scalar fields are given by a mixing angle $\theta_\pm$ as

$$\begin{pmatrix} H_1^+ \\ H_2^+ \end{pmatrix} \equiv \begin{pmatrix} \cos \theta_\pm & -\sin \theta_\pm \\ \sin \theta_\pm & \cos \theta_\pm \end{pmatrix} \begin{pmatrix} s_1^+ \\ s_2^+ \end{pmatrix}, \quad \tan 2\theta_\pm = \frac{2\mu_3^2}{m_{s_1^+}^2 - m_{s_2^+}^2},$$

(7)
where we used $m^2_{s_1^\pm} \equiv \mu_1^2 + \lambda_5 v^2/2$ and $m^2_{s_2^\pm} \equiv \mu_2^2 + \lambda_6 v^2/2$ with $v \equiv \sqrt{2\langle \phi^0 \rangle} = 246$ GeV. Clearly, $m^2_{s_1^\pm} > 0$ and $m^2_{s_2^\pm} > 0$ are necessary for $\langle s_1^+ \rangle = \langle s_2^+ \rangle = 0$. Masses of $H_1^\pm$ and $H_2^\pm$ are expressed as

$$m^2_{H_1^\pm} = \frac{1}{2} \left\{ m^2_{s_2^\pm} + m^2_{s_1^\pm} - \sqrt{(m^2_{s_2^\pm} - m^2_{s_1^\pm})^2 + 4\mu_3^4} \right\},$$

$$m^2_{H_2^\pm} = \frac{1}{2} \left\{ m^2_{s_2^\pm} + m^2_{s_1^\pm} + \sqrt{(m^2_{s_2^\pm} - m^2_{s_1^\pm})^2 + 4\mu_3^4} \right\},$$

where $H_1^\pm$ is defined as the lighter one. It is required to satisfy $m^2_{s_1^\pm} m^2_{s_2^\pm} - \mu_3^4 > 0$ so that $m^2_{H_1^\pm} > 0$ at $\langle s_1^+ \rangle = \langle s_2^+ \rangle = 0$. The LEP experiment constrains masses of charged scalar fields to be greater than 73-107 GeV at the 95% confidence level (see mass limits for $H^\pm$ from doublet fields and charged sleptons in ref. [20]).

In Fig. 1 the one-loop diagram for the Dirac neutrino mass is shown. The Dirac neutrino mass matrix $M_{\nu D}$ for $(M_{\nu D})_{\ell i} \bar{\nu}_L^i \nu_R^i$ is obtained as follows:

$$(M_{\nu D})_{\ell i} = C \left(f^T_{\ell\ell'} m_{\ell\ell'} \right) h_{\ell i}, \quad C \equiv \frac{\sin 2\theta^\pm}{16\pi^2} \ln \frac{m^2_{H_1^\pm}}{m^2_{H_2^\pm}}.$$  

Needless to say, $\theta^\pm = 0$ and $\pi/2$ are not acceptable to obtain nonzero neutrino masses. Since we are taking the basis where $\ell$ and $\nu_L^i$ are mass eigenstates, the Dirac mass matrix can be

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3 The same diagram was used in ref. [14] for the left-right symmetric model, where Yukawa coupling constant $h_{\ell i}$ in the 1LDNM is replaced by $f_{\ell\ell'}$. Current neutrino oscillation data cannot be satisfied in the case.
where \( m_i \) (\( i = 1-3 \)) are neutrino mass eigenvalues (\( m_i \geq 0 \)). The matrix \( U_{\text{MNS}} \) is the Maki-Nakagawa-Sakata (MNS) matrix \([21]\) which is expressed in the standard parametrisation as

\[
U_{\text{MNS}} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix} \begin{pmatrix}
c_{13} & 0 & s_{13} e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13} e^{i\delta} & 0 & c_{13}
\end{pmatrix} \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

where \( c_{ij} \) and \( s_{ij} \) stand for \( \cos \theta_{ij} \) and \( \sin \theta_{ij} \), respectively. One of the \( m_i \)'s is zero in this model because \( \text{Det}(M_{\nu D}) \times \text{Det}(f^T) = 0 \). Current neutrino oscillation data \([1-5]\) allow two choices; either \( m_1 = 0 \) or \( m_3 = 0 \). Two squared mass differences \( \Delta m^2_{ij} \equiv m_i^2 - m_j^2 \) are taken as \( \Delta m^2_{21} = 7.5 \times 10^{-5} \text{eV}^2 \) and \( |\Delta m^2_{31}| = 2.3 \times 10^{-3} \text{eV}^2 \).

For \( m_1 = 0 \), we have the following relations which were not found in ref. \([17]\):

\[
\begin{align*}
h_{\mu 1} &= -\frac{m_e (U_{\text{MNS}})_{\mu 1}^*}{m_\mu (U_{\text{MNS}})_{e1}} h_{e1}, \\
h_{\mu 2} &= -\frac{m_e (U_{\text{MNS}})_{\mu 1}^*}{m_\mu (U_{\text{MNS}})_{e1}} h_{e2} + \frac{(U_{\text{MNS}})_{e2} m_2}{C m_\mu f_{\mu \tau}}, \\
h_{\mu 3} &= -\frac{m_e (U_{\text{MNS}})_{\mu 1}^*}{m_\mu (U_{\text{MNS}})_{e1}} h_{e3} + \frac{(U_{\text{MNS}})_{e3} m_3}{C m_\mu f_{\mu \tau}}, \\
h_{\tau 1} &= \frac{m_e (U_{\text{MNS}})_{\tau 1}^*}{m_\tau (U_{\text{MNS}})_{e1}} h_{e1}, \\
h_{\tau 2} &= \frac{m_\mu (U_{\text{MNS}})_{\mu 2}}{m_\tau (U_{\text{MNS}})_{\tau 2}} h_{e2}, \\
h_{\tau 3} &= \frac{m_\mu (U_{\text{MNS}})_{\mu 3}}{m_\tau (U_{\text{MNS}})_{\tau 3}} h_{e3}, \\

f_{e\mu} &= \frac{(U_{\text{MNS}})_{\tau 1}^*}{(U_{\text{MNS}})_{e1}} f_{\mu \tau}, \\
f_{e\tau} &= \frac{(U_{\text{MNS}})_{\mu 1}^*}{(U_{\text{MNS}})_{e1}} f_{\mu \tau}.
\end{align*}
\]

When we fix the MNS matrix and neutrino masses, six elements of \( h_{\ell i} \) and two elements of \( f_{\ell' \ell} \) in the left-hand side of these equations are given by five variables (\( h_{e1}, h_{e2}, h_{e3}, f_{\mu \tau}, \) and \( C \)). Rephasing of the massless \( \nu^0_{\ell 2} \) makes \( h_{e1} \) real. Two phase degrees of freedom remain in \( h_{\ell i} \). However, eqs. \([19]\) and \([20]\) show that the CP violating phase \( \delta \) in the MNS matrix vanishes because \( f_{\ell' \ell} \) are real.
We assume for simplicity the so-called tribimaximal mixing \(^22\) \((s_{23}^2 = 1/2, s_{12}^2 = 1/3, s_{13}^2 = 0)\) which agrees with neutrino oscillation data well. A simple example of the parameter set (the set A) which satisfy eqs. (13)-(20) is

\[
h = 1.1 \times 10^{-2}
\begin{pmatrix}
1 & 1.8 \times 10^{-1} & 1 \\
-2.4 \times 10^{-3} & -9.4 \times 10^{-4} & 1.0 \times 10^{-3} \\
1.4 \times 10^{-4} & -4.0 \times 10^{-6} & 5.9 \times 10^{-5}
\end{pmatrix},
\]

(21)

\[
f = 1.1 \times 10^{-2}
\begin{pmatrix}
0 & 0.5 & 0.5 \\
-0.5 & 0 & 1 \\
-0.5 & -1 & 0
\end{pmatrix},
\]

(22)

\[
m_{H_1^\pm} = 150 \text{ GeV}, \quad m_{H_2^\pm} = 200 \text{ GeV}, \quad \theta_\pm = 0.1 \text{ rad.}
\]

(23)

Notice that some elements of \(h_{\ell i}\) (especially, \(h_{\tau i}\)) tend to be small because of ratios of charged lepton masses in eqs. (13)-(18) while all elements of \(f_{\ell\ell'}\) are in the same order of magnitude.

For \(m_3 = 0\), we obtain

\[
h_{\ell 1} = \frac{m_\mu (U_{\text{MNS}})_{\ell 3}}{m_e (U_{\text{MNS}})_{\mu 3}} h_{\mu 1} + \frac{(U_{\text{MNS}})_{\tau 1} m_1}{C m_e f_{\tau e}},
\]

(24)

\[
h_{\ell 2} = \frac{m_\mu (U_{\text{MNS}})_{\ell 3}}{m_e (U_{\text{MNS}})_{\mu 3}} h_{\mu 2} + \frac{(U_{\text{MNS}})_{\tau 2} m_2}{C m_e f_{\tau e}},
\]

(25)

\[
h_{\ell 3} = \frac{m_\mu (U_{\text{MNS}})_{\ell 3}}{m_e (U_{\text{MNS}})_{\mu 3}} h_{\mu 3},
\]

(26)

\[
h_{\tau 1} + \frac{m_e (U_{\text{MNS}})_{\ell 1}}{m_\tau (U_{\text{MNS}})_{\tau 1}} h_{\ell 1} = -\frac{m_\mu (U_{\text{MNS}})_{\mu 1}}{m_\tau (U_{\text{MNS}})_{\tau 1}} h_{\mu 1},
\]

(27)

\[
h_{\tau 2} + \frac{m_e (U_{\text{MNS}})_{\ell 2}}{m_\tau (U_{\text{MNS}})_{\tau 2}} h_{\ell 2} = -\frac{m_\mu (U_{\text{MNS}})_{\mu 2}}{m_\tau (U_{\text{MNS}})_{\tau 2}} h_{\mu 2},
\]

(28)

\[
h_{\tau 3} = \frac{m_\mu (U_{\text{MNS}})_{\tau 3}}{m_\tau (U_{\text{MNS}})_{\mu 3}} h_{\mu 3},
\]

(29)

\[
f_{\ell\mu} = -\frac{(U_{\text{MNS}})_{\ell 3}}{(U_{\text{MNS}})_{\mu 3}} f_{\tau e},
\]

(30)

\[
f_{\mu\tau} = -\frac{(U_{\text{MNS}})_{\mu 3}}{(U_{\text{MNS}})_{\mu 3}} f_{\tau e}.
\]

(31)

Notice that eq. (28) is the same as eq. (17). The phase of \(h_{\mu 3}\) is absorbed by the massless \(\nu_R^3\) while two phase degrees of freedom remain in \(h_{\mu 1}\) and \(h_{\mu 2}\). Equation (31) means \(\delta = 0\) similarly to the case of \(m_1 = 0\). It is worth to mention that we obtain \(s_{23}^2 = 1/2\) and \(s_{13}^2 = 0\) independently of \(h_{\ell i}\) if \(f_{\ell\mu} = -f_{\tau e}\) and \(f_{\mu\tau} = 0\), respectively. Such conditions on \(f_{\ell\ell'}\) might be given by some discrete symmetry. Equations (24)-(31) for the tribimaximal mixing are
satisfied by the following example (the set B) with values in eq. (23):

\[
h = 8.7 \times 10^{-3} \begin{pmatrix}
-7.0 \times 10^{-1} & 1 & 0 \\
1 & 1 & 1 \\
6.0 \times 10^{-2} & 6.0 \times 10^{-2} & 6.0 \times 10^{-2}
\end{pmatrix},
\]

(32)

\[
f = 8.7 \times 10^{-3} \begin{pmatrix}
0 & 1 & -1 \\
-1 & 0 & 0 \\
1 & 0 & 0
\end{pmatrix}.
\]

(33)

III. PHENOMENOLOGY

In this section, we consider the constraint from the lepton flavor violating (LFV) decays of charged leptons and the prospect for the LHC physics.

A. Lepton flavor violation

The most stringent constraint on this model from the LFV decays of charged leptons is given by the experimental bound on the branching ratio (BR) of \( \mu \to e\gamma \), \( \text{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11} \) [23]. The branching ratio in this model is calculated as

\[
\text{BR}(\mu \to e\gamma) \simeq \frac{\alpha}{768\pi G_F^2} \left\{ 16 f_{\mu e}^2 f_{\mu \tau}^2 \left( \frac{c_+^2}{m_{H_1^+}^2} + \frac{s_+^2}{m_{H_2^+}^2} \right)^2 + |(hh)_{\mu e}|^2 \left( \frac{s_+^2}{m_{H_1^+}^2} + \frac{c_+^2}{m_{H_2^+}^2} \right)^2 \right\},
\]

(34)

where \( c_+ \equiv \cos \theta_+ \) and \( s_+ \equiv \sin \theta_+ \). We ignore fermion masses in the loop integration and the electron mass in the final state. The parameter set A (eqs. (21)-(23)) results in \( \text{BR}(\mu \to e\gamma) = 2.9 \times 10^{-12} \). This means not only that the 1LDNM can satisfy the current bound on \( \text{BR}(\mu \to e\gamma) \) but also that the BR can be in the expected sensitivity of experiments in the future. On the other hand, the set B (eqs. (23), (32) and (33)) satisfies the experimental bound with a much smaller value \( \text{BR}(\mu \to e\gamma) = 7.5 \times 10^{-15} \). This is because \( f_{\mu \tau} \) and \( h_{e3} \) for \( m_3 = 0 \) are proportional to small \( s_{13} \). Although \( \text{BR}(\tau \to \mu\gamma) \sim 10^{-13} \) for the set B is much larger than \( \text{BR}(\mu \to e\gamma) \), it is also far from experimental sensitivity.

In the 1LDNM the coupling constants \( f_{\ell\ell'} \) and \( h_{\ell 3} \) can be \( \mathcal{O}(10^{-2}) \). Then the box diagram contributions to \( \mu \to eee \), which are proportional to the eighth power of these coupling constants, can be smaller than the current experimental upper bound although it becomes crucial in models where some of coupling constants are \( \mathcal{O}(1) \) [24].
B. Prospects at the LHC

The charged scalar boson $H_1^\pm$ is expected to be produced at the LHC if it is light. The production cross section via $q\bar{q} \rightarrow \gamma^* Z^* \rightarrow H_1^+ H_1^-$ with $\sqrt{s} = 14$ TeV is 23 fb for $m_{H_1^\pm} = 150$ GeV for example. The partial decay widths of $H_1^- \rightarrow \ell\nu$ ($\ell = e, \mu, \tau$) are given by

$$\Gamma(H_1^- \rightarrow \ell\nu) \equiv \sum_{\ell'} \Gamma(H_1^- \rightarrow \ell'\nu) \simeq \frac{m_{H_1^\pm}}{16\pi} \left( 4c_\pm^2 \sum_{\ell'} |f_{\ell\ell'}|^2 + s_\pm^2 \sum_i |h_{\ell i}|^2 \right), \quad (35)$$

where fermion masses are neglected.

If $H_1^\pm$ is made dominantly from $s_\pm^\pm$, its decay is determined by $f_{\ell\ell'}$. By using eqs. (19) and (20) for $m_1 = 0$ and the tribimaximal mixing, we obtain

$$\text{BR}(H_1^- \rightarrow e\nu) : \text{BR}(H_1^- \rightarrow \mu\nu) : \text{BR}(H_1^- \rightarrow \tau\nu) = 2 : 5 : 5. \quad (36)$$

The set A (eqs. (21)-(23)) gives approximately the same result. Equations (30) and (31) for $m_3 = 0$ and the tribimaximal mixing give

$$\text{BR}(H_1^- \rightarrow e\nu) : \text{BR}(H_1^- \rightarrow \mu\nu) : \text{BR}(H_1^- \rightarrow \tau\nu) = 2 : 1 : 1. \quad (37)$$

The set B (eqs. (32), (33), and (23)) gives the same result in a good approximation. These results are robust because the matrix structure of $f_{\ell\ell'}$ is restricted very well. Therefore, if $H_1^\pm \simeq s_1^\pm$, this model predicts $\text{BR}(H_1^- \rightarrow \tau\nu)/\text{BR}(H_1^- \rightarrow \mu\nu) \simeq 1$. On the other hand, if $H_1^\pm$ is made dominantly from $s_2^\pm$, its partial decay widths are controlled by $h_{\ell i}$. For $m_1 = 0$, eqs. (13) and (16) show $h_{\tau 1} \sim h_{\mu m_\mu/m_\tau}$. Furthermore, we have $h_{\tau 3} \sim h_{\mu m_\mu/m_\tau}$ with eq. (18) for $\theta_{13} = 0$. For $m_3 = 0$, eq. (29) also means $h_{\tau 3} \sim h_{\mu m_\mu/m_\tau}$. Thus, it seems reasonable to expect $h_{\tau i} \sim h_{\mu m_\mu/m_\tau}$ ($i = 1-3$). Then we have

$$\frac{\text{BR}(H_1^- \rightarrow \tau\nu)}{\text{BR}(H_1^- \rightarrow \mu\nu)} \sim \frac{m_\mu^2}{m_\tau^2} \sim 10^{-2}. \quad (38)$$

As the result, this model is likely to give $\text{BR}(H_1^- \rightarrow \tau\nu)/\text{BR}(H_1^- \rightarrow \mu\nu) \lesssim 1$ due to the discussion above.

If $H_2^\pm$ is also light and the production cross section is significant, the decays into $\ell\nu$ ($\ell = e, \mu, \tau$) smear the relation discussed above to some extent. Otherwise we can test the model at the LHC as well as the ILC by measuring the above characteristic pattern of the decay branching ratios.
The partial decay width for $h^0 \rightarrow \gamma\gamma$ of the SM Higgs boson $h^0$, which is caused at the one-loop level in the SM, is affected by additional one-loop diagrams with $H_i^+$ and $H_2^+$. The contributions to the SM prediction depend on $\lambda_5$ and $\lambda_6$ as well as Higgs masses. When coupling constant for $h^0H_i^+H_i^-$ ($i = 1$ or $2$) is positive (negative), the additional loop effect from $H_i^+$ gives a destructive (constructive) contribution to the SM prediction. These contributions can amount to $\mathcal{O}(10)$% deviations [25]. If such the effect is detected at the LHC when the light SM Higgs boson is discovered, it can be an important indirect signature of the charged singlet scalar bosons.

IV. DISCUSSIONS

Possible extensions of the 1LDNM are discussed in this section. First, we try to introduce dark matter candidates which do not exist in the model. Next, we consider the case with lepton number violation. Even if lepton number is not conserved and Majorana mass terms for $\nu_R^i$ are allowed, the mechanism to suppress the Dirac mass term is fruitful.

A. Introducing dark matter candidates

A possibility to accommodate dark matter candidates would be to impose an unbroken $Z_2$ symmetry (we call it $Z'_2$) to this model in addition to the softly-broken $Z_2$ symmetry such that all particles in the loop are $Z'_2$-odd. Since the SM charged leptons in the loop in Fig. 1 cannot be $Z'_2$-odd, they must be replaced by newly introduced $Z'_2$-odd fermions which can be understood as the fourth generation leptons. The $Z'_2$-odd Dirac neutrino could be the lightest $Z'_2$-odd particle which is stable. However, it cannot be identified as the dark matter because the spin-independent scattering cross section on a nucleon is too large to satisfy current data of direct searches [26] due to the diagram mediated by the $Z$ boson. Therefore, such a minimal extended model is excluded.

We may consider the other model by taking different scalar particle contents, where the dark matter candidate enters and Dirac neutrino masses are induced radiatively. Such a

4 Instead of the $Z'_2$ symmetry, lepton number can be used when it is conserved. For example, a fermion (boson) with a lepton number 2 (1) could be stable. From this point of view, lepton number conservation seems fit well for introducing dark matter candidates.
model can be found in ref. [18] where the following exact $Z'_2$-odd particles are introduced: an $SU(2)_L$-doublet scalar field ($\Phi_2 = (\phi^+_2, \phi^0_2)^T$) and a real neutral singlet scalar ($s^0$) as well as a neutral singlet Dirac fermion ($N$). In ref. [18], a real scalar dark matter (Re($\phi^0_2$) or $s^0$) and the so-called Dirac leptogenesis [27] via $N$ decay are considered. If the Dirac fermion $N$ is the lightest $Z'_2$-odd particle, the dark matter is different from its anti-particle in contrast with the dark matter of the Majorana particle. It could be compatible with the asymmetric dark matter scenario [28]. The Dirac leptogenesis would be also achieved by the decay of $\phi^0_2$.

Another simple possibility is the $R$-parity-conserving SUSY extension where the lightest supersymmetric particle becomes a candidate for dark matter. The detailed study will be presented elsewhere.

B. Baryogenesis

There seem to be two possible extensions of this model in order to realise baryogenesis, although the detailed analysis on them is beyond the scope of this letter. One is the electroweak baryogenesis [29]. The scalar sector should be extended in order to have CP-violating phases and also to achieve strong first order phase transition at the electroweak symmetry breaking. The other is an application of the Dirac leptogenesis [27]. It is known that the leptogenesis is possible without LNV. The number to be converted to the baryon asymmetry is free from the $\nu^i_R$ number because the sphaleron does not act on the gauge singlet fields $\nu^i_R$.

C. Lepton number violation

The mechanism to induce the Dirac mass terms for neutrinos can be applied also to the lepton number violating case, in which $\nu^i_R$ have Majorana masses as $M_i(\nu^i_R)^c\nu^i_R$. Then, the type-I seesaw mechanism is realized at the two-loop level via the one-loop induced Dirac masses. This model was studied in refs. [17, 30]. By this loop suppression mechanism, $M_i$ are much lighter than those in the tree-level seesaw mechanism. Consequently, $M_i$ can

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A one-particle-irreducible two-loop diagram also exists for the Majorana masses of $\nu_L$, which seems to be overlooked in ref. [17].
be at the TeV scale without excessive fine tuning on coupling constants. Such TeV scale Majorana neutrinos could be tested at the LHC as well as the ILC.

In the two-loop seesaw model, we can remove the soft-breaking term of the $Z_2$ symmetry. Then the Majorana masses for $\nu_L$ are generated at the three-loop level and the lightest $\nu_R$ becomes a dark matter candidate. This model coincides with the model proposed by Krauss, Nasri and Trodden [9].

V. CONCLUSIONS

We have investigated a simple model (1LDNM) with the mechanism for radiative generation of Dirac neutrino masses without introducing lepton number violation. In the 1LDNM, the Yukawa interaction $\bar{L}\Phi R\nu_R$ is absent at the tree level because of the softly-broken $Z_2$ symmetry, so that it is induced at the one-loop level by the soft-breaking in the mixing between $s_1^\pm$ and $s_2^\pm$. Tiny neutrino masses are generated from the TeV scale dynamics. We have found the model can be compatible with the current neutrino oscillation data as well as LFV search results (especially for $\mu \rightarrow e \gamma$). There is no CP-violating phase in the MNS matrix in this model. It is possible that BR($\mu \rightarrow e \gamma$) becomes large enough to be discovered by experiments in near future. The 1LDNM is likely to give $\text{BR}(H_1^- \rightarrow \tau \nu) / \text{BR}(H_1^- \rightarrow \mu \nu) \lesssim 1$. Characteristic features of $H_1^\pm$ are expected to be tested at the LHC and the ILC. We also have discussed several possible extensions of the model, which implement dark matter candidates, mechanism for baryogenesis, and the radiative type-I seesawlike scenario by using one-loop suppressed Dirac masses.

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