Dynamical Effects beyond Mean Field in Drip Line Nuclei

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New structure problems are raised in nuclei near drip lines by recent RIB experiments. We pointed out the importance of dynamical effect beyond mean field theories to the new structure problems.

§1. Introduction

Recently, nuclear structure in neutron-rich nuclei has attracted much attention because of its exotic nature, such as halo and skin\(^{1)\text{-}^3}\). Recent advances of secondary beam techniques allow us to measure useful structure information on masses, interaction cross sections and excited states. The shell structure is one of essential issues in nuclear structure. Recently, the magic numbers in the neutron-rich region have been extensively studied both experimentally and theoretically. For example, the melting of the N=8 closed shell was pointed out due to the large mixing of 1p\(_{1/2}\) and 2s\(_{1/2}\) orbitals in \(^{11}\)Be\(^4\), \(^{11}\)Li\(^5\) and \(^{12}\)Be\(^6\). The disappearance of the N=20 closed shell was also shown in \(^{32}\)Mg experimentally\(^7\). Very recently a possible new magic number is pointed out at N=16 by the analysis of the neutron separation energies and the interaction cross sections\(^8\).

So far the nuclear structure problems in light nuclei have been studied mostly by using mean field theory\(^9\) and shell model calculations\(^10\). It is known that the correlations beyond the mean field have substantial effects on the single-particle energies and electromagnetic transitions. In particular, the effect of collective excitations has been studied by the particle-vibration coupling and found to be important\(^11,12\). In this paper, we present the study of dynamical correlations on single-particle energies in light neutron-rich nuclei based on the \(^{10}\)Be and \(^{24}\)O core by using the particle-vibration coupling model\(^13\). The polarization charges for electric transitions in drip line nuclei have been discussed also by using the particle-vibration coupling model elsewhere\(^14,15\). In the present version of the model, firstly Hartree-Fock (HF) calculations with Skyrme interactions are performed to obtain the zeroth-order single-particle energies and the wave functions. Secondly, we couple to them the natural parity vibrational states with \(J^\pi=1^-, 2^+\) and \(3^-\) by using the particle-vibration coupling model.\(^{*)\) E-mail address:sagawa@u-aizu.ac.jp

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coupling Hamiltonian derived from the same Skyrme interaction, and the HF wave functions. In Section 2, we describe our model. Numerical results are given in Section 3. Summary is given in Section 4.

§2. Particle-vibration coupling model

The particle-vibration coupling Hamiltonian has the form

\[ H_{PV} = \sum_{\alpha \beta} \sum_{\lambda n \mu} \langle \alpha | \delta g_n(r) v(r) Y_{\lambda \mu}(\hat{r}) | \beta \rangle a_\alpha^\dagger a_\beta, \]

(2.1)

where \( \alpha, \beta \) label the single-particle states, \( v(r) \) is the form factor associated to the particle-hole interaction \( V_{ph} \) derived from the Skyrme force \( (V_{ph}(\vec{r}_1, \vec{r}_2) = v(r_1) \delta(\vec{r}_1 - \vec{r}_2)) \), and the radial transition density \( \delta g_n(r) \) can be derived from the RPA phonon.

The second-order correction due to the RPA phonons to the single-particle energies coming from the perturbing Hamiltonian (2.1) is obtained by the real part of the four diagrams depicted in Fig. 1. The imaginary part, which gives rise to the width of the single-particle states, is not discussed in the present work. The corresponding energy shifts for the diagrams can be found, e.g., in Ref.\textsuperscript{17}, and are given to be,

\[
\Delta \varepsilon_\alpha = Re \sum_{\lambda, n} \frac{1}{2j_\alpha + 1} \left[ \sum_{\beta > F} \frac{|\int dr u_\alpha(r) u_\beta(r) v(r) \delta g_n(r)|^2}{\varepsilon_\alpha - (\varepsilon_\beta + \omega_n) + i\eta} \langle \alpha | Y_\lambda | \beta \rangle^2 \right] + \sum_{\beta < F} \frac{|\int dr u_\alpha(r) u_\beta(r) v(r) \delta g_n(r)|^2}{\varepsilon_\alpha - (\varepsilon_\beta - \omega_n) - i\eta} \langle \alpha | Y_\lambda | \beta \rangle^2, (2.2)
\]

where \( \varepsilon_\alpha \) and \( \varepsilon_\beta \) are HF single-particle energies, \( u_\alpha \) and \( u_\beta \) are the radial HF wave functions and \( \omega_n \) is the RPA phonon energy. The sum over \( \beta \) spans unoccupied particle states (occupied hole states) when the notation \( \beta > F \) (\( \beta < F \)) is employed. If the state \( \alpha \) is a particle state, the first and second terms in the above equation correspond to the upper and lower left-hand side diagrams (a) and (b) in Fig. 1. If \( \alpha \) is a hole state, the first and second terms correspond, respectively, to the lower and upper right-hand side diagrams (d) and (c) in Fig. 1. On quite general grounds, it is expected that the real parts of Eq. (2.2) are larger for the “leading” diagrams (a) and (c) shown in Fig. 1 than the diagrams (b) and (d) in Fig. 1. This is because the energy denominators of the upper row diagrams (a) and (c) are expected to be smaller.
than those of the diagrams (b) and (d). Furthermore, the energy denominator of the contributions (b) and (d) have opposite sign, so their contributions should cancel partially those of the leading terms. In $^{24}\text{O}$ we will find exceptions to these rules due to the possibility that the leading contributions are not possible because of phase space limitations (hole intermediate states may not be available in these light nuclei if large angular momenta are required by selection rules). This will be one of the important qualitative messages of the present work.

§3. Results

We present numerical results for the single-particle energies in the $^{10}\text{Be}$ and $^{24}\text{O}$ core. The shell structure of $^{10}\text{Be}$ core in the present study is compared with the results of Ref.\textsuperscript{18} (see also the coupled-channel calculation of Ref.\textsuperscript{19}). Since it is somewhat questionable to apply the self-consistent HF+RPA approach to obtain realistic single-particle energies and collective excited states in very light nuclei, we adopt a phenomenological approach for $^{10}\text{Be}$. Namely, we start from a Woods-Saxon potential as the zeroth-order potential and calculate the particle-vibrational coupling with empirical phonon energies and transition strengths.

In $^{10}\text{Be}$, the existing Skyrme forces give a HF single-particle spectrum with a large $1p_{1/2}$-$1p_{3/2}$ gap. The interaction SIII gives, for instance, $\varepsilon(1p_{1/2})=-3.80$ MeV and $\varepsilon(1p_{3/2})=-11.13$ MeV. Using a standard Woods-Saxon parameterization\textsuperscript{20},

\begin{equation}
U = (-V_0 + V_r \frac{N - Z}{A} f(r) + V_{ls}(\vec{l} \cdot \vec{s}) R_0^2 \frac{f'(r)}{r})
\end{equation}

(with $V_0 = 55$ MeV, $V_r = 33$ MeV, $V_{ls} = \alpha_{ls} V$ and $\alpha_{ls} = 0.44$, and where $f(r)$ is the usual Fermi function with $r_0 = 1.27$ fm and $a = 0.67$ fm), the energy gap is reduced but the Fermi surface lies in a similar position as in the Skyrme-HF case. In fact, the Woods-Saxon results are $\varepsilon(1p_{1/2})=-3.93$ MeV and $\varepsilon(1p_{3/2})=-9.36$ MeV. This $1p_{1/2}$ single-particle state, together with the $2s_{1/2}$ unoccupied state, are
shown in the first column of Fig. 2.

The vibrational states included in our model space are limited to those for which experimental information is available. The energy and reduced transition probability \( B(E2, 0 \rightarrow 2^+_1) \) of the lowest \( 2^+_1 \) state in \(^{10}\text{Be} \) has been measured to be 3.37 MeV and 52 e\( ^2 \)fm\(^4 \), respectively\(^{22}\). The effect of coupling between the single-particle states and this phonon can be seen in the second column of Fig. 2. The coupling reduces the \( 2s_{1/2}-1p_{1/2} \) gap from 5 to 2 MeV, melting the \( N=8 \) shell closure. Interpreting those as the \( 1^+_2 \) and \( 1^-_2 \) levels of \(^{11}\text{Be} \), the shifts induced by phonon coupling are not enough to give an inversion of the two levels. This is at variance with Ref.\(^{18}\), in which the inversion of the two levels was obtained by a particle-vibration coupling model with a non-standard Woods-Saxon parameterization for the mean field: a large diffuseness \( a=0.9 \) fm has been employed and the spin-orbit parameter \( \alpha_{ls} \) has been changed from 0.44 to 0.8 just for p-states. Thus, in such a modified Woods-Saxon potential, the \( 1p_{1/2} \) and \( 2s_{1/2} \) states lie close enough so that the shift induced by phonons can reverse their order.

We checked the above perturbative results for the \( 1^+_2 \) state by diagonalizing the Hamiltonian (2.1) in the five-dimensional model space with \( 2s_{1/2}, 1d_{5/2}\otimes 2^+, 2d_{5/2}\otimes 2^+, 1d_{3/2}\otimes 2^+, 2d_{3/2}\otimes 2^+ \) configurations and found consistent results with the above calculations. The wave function of the ground state is essentially given by

\[
|1^+_2 \rangle = 0.921 |2s_{1/2} \rangle + 0.356 |1d_{5/2} \otimes 2^+ \rangle.
\]

In the case of \(^{24}\text{O} \) core, the HF spectrum with SIII interaction is shown in the first column of Fig. 3. We have then calculated \( 0^+, 1^+, 2^+, 3^+ \) and \( 4^+ \) vibrational states by the self-consistent RPA model. We first check the numerical accuracy of the model. The energy and proton (neutron) moments \( M_p \) (\( M_n \)) of the collective lowest \( 2^+ \) state are obtained by three different methods for the RPA calculations. The energy and proton (neutron) moments are \( E=3.90 \) MeV and \( M_p \) (\( M_n \))=1.8 (10) fm\(^2 \), respectively, with unoccupied levels calculated using the harmonic oscillator basis.

In the case of the coupling with the lowest \( 2^+_1 \) state, the \( 1d_{3/2} \) and \( 2s_{1/2} \) states
have moderate shifts. Although the phonon is collective, there is substantial cancellation, in the case of the 2s$_{1/2}$ hole, between the diagrams (c) and (d) in Fig. 1 with the 1d$_{5/2}$ and 1d$_{3/2}$ intermediate state, respectively. In the case of the 1d$_{3/2}$ particle the cancellation occurs between the two diagrams (a) and (b) with the 1d$_{3/2}$ and 2s$_{1/2}$ intermediate states, respectively. There is not such cancellation for the 1d$_{5/2}$ state because the diagram (c) of Fig. 1, with intermediate 2s$_{1/2}$ state, gives a large effect and has negative sign, contrary to the general rule. The remaining quadrupole states have non-negligible effect although they are lying at higher energies. The 3$^-$ states play a crucial role as far as the single-particle energy gap is concerned and deserves a special discussion. The large amount of low-energy (i.e., below 10 MeV) 3$^-$ strength explains the large absolute values of the energy shifts. In particular, two states at 6.07 MeV and at 7.33 MeV absorb 22% and 19% of the energy weighted sum rule strength, respectively. The fact that the 2s$_{1/2}$ hole state is pushed down in energy is due to the absence of contributions from the diagram (c) of Fig. 1 because of angular momentum selection rules, and also due to the strong contribution from the diagram (d) with the resonant-like 1f$_{7/2}$ intermediate state. These unusual effects are possible only in light nuclei where occupied states span only low values of the angular momentum, and were never found in systematic studies of heavier nuclei, for example, near $^{208}$Pb. It is also interesting to observe experimentally the predicted collective 3$^-$ states in $^{24}$O to test the validity of the present theoretical study of the shell gaps.

§4. Summary

Effects of the collective modes on the shell structure have been investigated in the nuclei near the $^{10}$Be and $^{24}$O core, at the neutron drip line. Energy shifts of the single-particle states are calculated by the particle-vibration coupling model taking into account the coupling to low-lying collective vibrational states. In the case of $^{10}$Be core, the coupling to the lowest 2$^+$ state is found to be the most important effect to lower and raise the single-particle energies of 2s$_{1/2}$ and 1p$_{1/2}$ states, respectively, and leads to a much narrower gap between the two states than that of the Woods-Saxon potential. In the case of $^{24}$O core, the single-particle energies below the Fermi level behaves unusually in the calculations with the particle(hole)-vibration coupling compared with heavy nuclei near the $^{208}$Pb core. The single-particle energy of the 1d$_{5/2}$ state is lowered by the coupling to the collective 2$^+$ states which gives an upward shift in heavy nuclei. Moreover the energy of 2s$_{1/2}$ state is also lowered by the coupling to 3$^-$ states due to very specific effects found in the case of $^{24}$O core, i.e., the blocking of the available configuration space for the particle(hole)-vibration couplings.

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