Diquarks, Pentaquarks and Dibaryons

Shi-Lin Zhu

Department of Physics, Peking University, BEIJING 100871, CHINA
(Dated: October 28, 2018)

We explore the connection between pentaquarks and dibaryons composed of three diquarks in the framework of the diquark model. With the available experimental data on H dibaryon, we estimate the Pauli blocking and annihilation effects and constrain the \( P = + \) pentaquark SU(3)_F singlet mass. Using the \( \Theta^+ \) pentaquark mass, we estimate \( P = - \) dibaryon mass.

PACS numbers: 12.39.Mk, 12.39.-x
Keywords: Pentaquark, Dibaryon, Diquark

I. INTRODUCTION

Baryons in the conventional quark model are color singlets composed of three quarks. So their color wave function is anti-symmetric. Pauli principle requires the total wave function of three quarks be anti-symmetric. For the \( L = 0 \) ground state baryons, their orbital wave function is symmetric. Therefore their spin-flavor wave function is totally symmetric, corresponding to the nucleon octet and Delta decuplet with positive parity. The mass splitting between the members of the SU(6) multiplet is caused by either the color-spin interaction from the gluon exchange or the flavor-spin interaction from the pseudoscalar meson exchange.

Quark model has been very successful in the classification of baryons [1]. However, quantum chromodynamics (QCD) as the underlying theory of strong interaction, allows a much richer baryon spectrum. Especially there may exist hybrid baryons (qqqG) and multiquark baryons such as pentaquarks (qqqqq), dibaryons (qqqqq) etc. Since Jaffe proposed the H dibaryon in 1977 [2], there has been extensive experimental search of this state. There also exist discussions of other possible dibaryons in literature [3, 4]. Up to now, none of these non-conventional baryon states has been established experimentally except pentaquarks.

The surprising discovery of \( \Theta^+ \) pentaquark [2, 4] last year is one of the most important events in hadron physics for the past decades. There have appeared more than two hundred pentaquark papers in literature within one year. Its quantum number, internal structure, decay mechanisms and underlying dynamics are all open [5, 6].

Jaffe and Wilczek proposed the diquark picture for pentaquarks [5, 6]. The diquark is very similar to an antiquark in many aspects. This feature leads to deep connection between pentaquarks and dibaryons which are composed of three diquarks. In this short note, we will explore this connection.

II. \( P = + \) PENTAQUARKS VS \( P = - \) DIBARYONS

Within the framework of the diquark model, we discuss the connection between \( P = + \) pentaquarks and those \( P = - \) dibaryons which are composed of three diquarks and contain one orbital excitation between diquarks.

Jaffe and Wilczek proposed that the \( \Theta^+ \) pentaquark is composed of two diquarks and one strange anti-quark \( \bar{s} \). They argued that the light quarks are strongly correlated. Two light quarks tend to form a scalar diquark in the \( 3_c, \bar{3}_F \) representation whenever possible. The lighter the quark mass, the stronger the correlation. The one-gluon-exchange interaction and the instanton induced interaction seem to support such an idea.

Since the pentaquark is a color singlet, the color wave function of the two diquarks within the pentaquark must be antisymmetric \( 3_C \). In order to get an exotic anti-decuplet, the two scalar diquarks combine into the symmetric SU(3)_F \( 6_F : [ud]^2, [ud][ds], [su]^2, [su][ds], [ds]^2, \) and \( [ds][ud] \). Bose statistics demands symmetric total wave function of the diquark-diquark system, which leads to the antisymmetric spatial wave function with one orbital excitation. The resulting anti-decuplet and octet pentaquarks have \( J^P = \frac{1}{2}^+, \frac{3}{2}^+ \). The resulting flavor wave functions are collected in Table [6].

Throughout our discussion we assume exact isospin symmetry. We denote the up and strange quark mass by \( m_u, m_s \) and the \([ud], [us]\) diquark mass by \( m_{[ud]}, m_{[us]} \). Since the same quark exists in the two diquarks, Pauli blocking effect may raise the spectrum by \( E_{pb} \). However the centrifugal barrier from the orbital excitation makes two diquarks far apart. One expects that the Pauli blocking effect is less significant for \( P = + \) pentaquarks than for \( P = - \) pentaquarks.

In contrast, the quark and anti-quark annihilation effect tends to lower the spectrum. There are two kinds of possible annihilation mechanism. For example, the \( \bar{u} \) may annihilate with the up quark in either \([ud]\) or \([us]\) diquark. Such a mechanism lowers the pentaquark mass by \( E_{ann} \). The second possibility is that the \( \bar{u} \) and down quark in the \([ud]\) diquark annihilates into a virtual K or \( K^* \), which may also lower the pentaquark mass by \( E_{ann} \). \( E_{ann} \) is probably greater than \( E_{pb} \). After taking into
We list lying dibaryons with excitation. One may wonder whether one can get a low lying dibaryon with diquarks and one anti-quark by Jaffe and Wilczek [8]. The quark content of the proton-type dibaryon is \( \bar{u} \bar{d} \) to \( \bar{u} \bar{P} \), and \( \bar{d} \bar{q} \) to \( \bar{P} \bar{q} \). Its mass is as low as 1530 MeV even with one orbital excitation. One may wonder whether one can get a low lying dibaryon with \( L = 1 \) after replacing the anti-quark in \( \Theta^+ \) by a diquark. 

Now let’s discuss \( P = - \) dibaryons composed of three scalar diquarks with \( L = 1 \). Its color wave function is anti-symmetric. Its spin wave function is symmetric since diquarks are scalars. Bose statistics requires the total wave function to be symmetric. Hence the product of the flavor and orbital wave function is anti-symmetric. Diquarks are scalars. Their total wave function is symmetric. Hence their flavor wave function is antisymmetric. Diquarks combine into a color singlet so their color wave function is antisymmetric. Diquarks are scalars. Their total wave function is symmetric. Hence their flavor wave function is symmetric. 

Let’s move on to those dibaryons which are composed of three diquarks and have no orbital excitation. Three \( \Xi_c \) diquarks combine into a color singlet so their color wave function is antisymmetric. Diquarks are scalars. They obey Bose statistics. Their total wave function should be symmetric. Since there is no orbital excitation between scalar diquarks, their spin and spatial wave functions are symmetric. Hence their flavor wave function must be totally anti-symmetric. I.e., the resulting dibaryon is a \( SU(3)_F \) singlet with positive parity, which is nothing but the \( H \) dibaryon proposed by Jaffe long time ago [2]. Another \( P = + \) dibaryon with two P-waves between the diquarks and \( L = 0 \) could also be low lying [20]. 

Within the diquark framework, it was pointed out that lighter pentaquarks can be formed if the two scalar diquarks are in the antisymmetric \( SU(3)_F \) representation. We follow Ref. [8] and use the lower index “6” to denote the \( 6 \) representation. The only \( \Sigma^0 \) diquark with the quark content \([ud][us][ds]\) with the quark content \([ud][us][ds]\), its mass can be estimated as 

\[
M_{SU^0} = 2m_{[us]} + m_{[ud]} + E_L + 2E_{L=0}^\text{p} + E_{L=1}^\text{p} \tag{3}
\]

For the \( \Xi^0 \) dibaryon with the quark content \([ud][ud][us]\), 

\[
M_{\Xi^0} = 2m_{[ud]} + m_{[us]} + E_L + 2E_{L=0}^\text{p} + 2E_{L=1}^\text{p} \tag{4}
\]

For the nucleon-type dibaryon \( N_0 \) with the quark content \([us][us][ds]\), 

\[
M_{N_0} = 3m_{[us]} + E_L + 2E_{L=0}^\text{p} + 2E_{L=1}^\text{p} \tag{5}
\]

For the \( \Sigma^0 \) dibaryon with the quark content \([us][us][ds]\), its mass can be estimated as 

\[
M_{\Sigma^0} = 2m_{[us]} + m_{[ud]} + E_L + 2E_{L=0}^\text{p} + 2E_{L=1}^\text{p} \tag{6}
\]

\[
\frac{1}{\sqrt{3}} \left( [ud][su] - \bar{u} + [ds][ud] - \bar{d} + [su][ds] - \bar{s} \right) \tag{7}
\]

Since the same quark exists within two dibaryons, Pauli blocking effect may raise the spectrum by \( E_{L=0}^\text{p} \). In contrast, the quark and anti-quark annihilation effect tends to lower the spectrum by \( E_{L=0}^\text{ann} \). Since there is no orbital excitation, the diquarks are in \( \Xi \)-wave. \( E_{L=0}^\text{p} \) can be quite significant and \( E_{L=0}^\text{p} \gg E_{L=1}^\text{p} \). 

The \( P = - \) pentaquark singlet mass may be estimated as 

\[
M_1 = \frac{1}{3} \left( 2m_u + m_s + 2m_{[ud]} + 4m_{[us]} \right) + E_{L=0}^\text{p} - 2E_{L=0}^\text{ann} - 2E_{L=0}^\text{ann} \tag{8}
\]

Replacing the antiquark in Eq. (7) by the corresponding diquark we arrive at the \( H \) dibaryon with the diquark content \([ud][us][ds]\). Its mass reads 

\[
M_H = m_{[ud]} + m_{[us]} + 3E_{L=0}^\text{p} \tag{9}
\]

IV. DISCUSSION

We follow Ref. [8] and use \( m_u = 360 \text{ MeV}, m_s = 460 \text{ MeV}, m_{[ud]} = 420 \text{ MeV}, m_{[us]} = 580 \text{ MeV} \). If we
naively ignore the Pauli blocking and annihilation effects, we get

\[ M_{\Lambda_0} = M_{\Theta^+} + 2m_{[us]} - m_{[ud]} - m_s = 1710\text{MeV} \] (10)

where we have used \( M_{\Theta^+} = 1530\text{MeV} \). Such a low lying dibaryon with negative parity is clearly in conflict with the experimental data. In other words, the Pauli blocking and annihilation effects are important.

We may make a rough estimate of \( E_{L=0}^{pb} \) from available experimental information on \( H \) dibaryon. If \( H \) really exists, it must be a very loosely bound state which is typically around 240 MeV.

Clearly this \( P=- \) isoscalar dibaryon state is probably (40 ~ 80) MeV above \( \Lambda\Lambda \) threshold. So it’s unstable against \( P\)-wave \( \Lambda\Lambda \) and \( \Xi N \) strong decays. Its width is expected to be not very broad. This state could be searched at RHIC.

The author thanks F. E. Close and Q. Zhao for helpful communications. This project was supported by the National Natural Science Foundation of China under Grant 10375003, Ministry of Education of China, FANEDD and SRF for ROCS, SEM.

\[ M_{\Lambda_0} = (2270 \sim 2310)\text{MeV} \] (14)

This \( P=- \) isoscalar dibaryon state is probably (40 ~ 80) MeV above \( \Lambda\Lambda \), \( \Xi N \) threshold. So it’s unstable against \( P\)-wave \( \Lambda\Lambda \) and \( \Xi N \) strong decays. But it’s possibly stable against \( \Xi N \pi \) or \( \Sigma \pi \) S-wave strong decays. Its width is expected to be not very broad. This state could be searched at RHIC.

[1] Particle Dada Group, Phys. Rev. D 66, 010001 (2002).
[2] R. Jaffe, Phys. Rev. Lett. 38, 195 (1977).
[3] J. L. Ping et al. Phys. Rev. C 62, 054007 (2000).
[4] Z. Y. Zhang et al., Phys. Rev. C 61, 065204 (2000); Q. B. Li et al., Nucl. Phys. A 683, 487 (2001); V. B. Kopeliovich, Nucl. Phys. A 639, 75c (1998).
[5] T. Nakano et al., Phys. Rev. Lett. 91, 012002 (2003).
[6] V. V. Barmin et al., hep-ex/0304040; S. Stepanyan et al., hep-ex/0307018; J. Barth et al., hep-ph/0307083; A. E. Arataynet al., hep-ex/0309042; V. Kubarovsky et al., hep-ex/0311046; A. Aasarpatian et al., hep-ex/0312044; A. Aliev et al., hep-ex/0401024; COSY-TOF Collaboration, hep-ex/0403011.
[7] D. Diakonov et al., Z. Phys. A 359, 305 (1997).
[8] R. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003), hep-ph/0312360.
[9] Shi-Lin Zhu, Phys. Rev. Lett. 91, 232002 (2003).
[10] F. Stancu and D. O. Riska, hep-ph/0307010; A. Hosaka, hep-ph/0307222; C. E. Carlson et al., hep-ph/0312325.
[11] M. Karliner and H. J. Lipkin, hep-ph/0307243.
[12] F. Csikor et al., hep-lat/0309050; S. Sasaki, hep-lat/0310014; T.-W. Chiu, T.-H. Hsieh, hep-ph/0403020.
[13] R. Bijker et al., hep-ph/0310281.
[14] J. Ellis et al., hep-ph/0410127.
[15] T. D. Cohen, hep-ph/0309111, hep-ph/0312191.
[16] X.-C. Song and Shi-Lin Zhu, hep-ph/0403093; Y.-X. Liu, J.-S. Li, C.-G. Bao, hep-ph/0411197.
[17] F. Huang et al., hep-ph/0310040; B. Wu and B.-Q. Ma, hep-ph/0311331.
[18] X.-G. He and X.-Q. Li, hep-ph/0403191; H. Y. Cheng et al., hep-ph/0403232.
[19] P.-Z. Huang et al., hep-ph/0311108, hep-ph/0401199; Y.R. Liu et al., hep-ph/0312074, hep-ph/0404123; W. Li et al., hep-ph/0312362.
[20] A. Zhang et al., hep-ph/0403210.
[21] A.W. Thomas, K. Hicks and A. Hosaka, hep-ph/0312083; Y. Oh, H. Kim, S. H. Lee, hep-ph/0312229; S. T. Nam, A. Hosaka and H.-C. Kim, hep-ph/0401074; Q. Zhao, J. S. Al-Khalili, hep-ph/0312348.
[22] F. E. Close and J. J. Dudek, Phys.Lett. B586, 75 (2004).
[23] F. E. Close and J. J. Dudek, Phys.Lett. B583, 278 (2004); F. E. Close, private communication.
[24] T. D. Cohen, Talk at the conference of "Pentaquark States: Structure and Properties", Trento, Italy, Feb. 2004.
[25] P. Bicudo, hep-ph/0405086.
[26] J. R. Ahn et al., Phys. Rev. Lett. 87, 132504 (2001).
[27] H. Takahashi et al., Phys. Rev. Lett. 87, 212502 (2001).
TABLE I: Flavor wave functions in Jaffe and Wilczek’s model [8]. $\langle q_1 q_2 | q_3 q_4 \rangle = \sqrt{2} \langle q_1 q_2 | q_3 q_4 \rangle + \langle q_3 q_4 | q_1 q_2 \rangle$ or $|q_1 q_2|^2 = |q_1 q_2| |q_1 q_2|$ is the diquark-diquark part.

\[
\begin{array}{|c|c|}
\hline
(Y, I, J_3) & \text{10} & (Y, I, J_3) & \text{8} \\
\hline
(2,0,0) & \sqrt{\frac{2}{3}}[ud][us]_+ s + \sqrt{\frac{2}{3}}[ud]_+ d & - & - \\
\frac{1}{2}, \pm \frac{1}{2} & & \frac{1}{2}, \pm \frac{1}{2} & & \frac{1}{2}, \pm \frac{1}{2} & \\
(1, \frac{3}{2}, \pm \frac{1}{2}) & \sqrt{\frac{2}{3}}[ud][us]_+ \tilde{s} + \sqrt{\frac{2}{3}}[ud]_+ \tilde{d} & - & - \\
(1, \frac{3}{2}, \mp \frac{1}{2}) & \sqrt{\frac{2}{3}}[ud][us]_+ \bar{s} + \sqrt{\frac{2}{3}}[ud]_+ \bar{d} & - & - \\
(0,1,1) & \sqrt{\frac{3}{2}}[ud][us]_+ \tilde{d} + \sqrt{\frac{1}{2}}[us]_+ \tilde{s} & - & - \\
(0,1,0) & \sqrt{\frac{3}{2}}[ud][ds]_+ \bar{d} + [ud][us]_+ \bar{s} & - & - \\
(0,1,-1) & \sqrt{\frac{3}{2}}[ud][ds]_+ \tilde{u} + \sqrt{\frac{1}{2}}[ds]_+ \tilde{s} & - & - \\
(-1, \frac{3}{2}, \pm \frac{1}{2}) & \sqrt{\frac{2}{3}}[us][ds]_+ \bar{d} + \sqrt{\frac{1}{3}}[us]_+ \bar{s} & - & - \\
(-1, \frac{3}{2}, \mp \frac{1}{2}) & \sqrt{\frac{2}{3}}[ds][us]_+ \bar{d} + \sqrt{\frac{1}{3}}[ds]_+ \bar{s} & - & - \\
(-1, \frac{3}{2}, -\frac{1}{2}) & [ds]_+ \bar{u} & - & - \\
\hline
\end{array}
\]

TABLE II: $P = +$ pentaquark masses with the correction from Pauli blocking and the annihilation effects. $E_L$ is the orbital excitation energy.