A Realistic Approach to the $\Xi NN$ Bound-State Problem based on Faddeev Equation

K. Miyagawa · M. Kohno

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Abstract The Faddeev equations for the $\Xi NN$ bound-state problem are solved where the three $S=−2$ baryon-baryon interactions of Jülich-Bonn-München chiral EFT, HAL QCD and Nijmegen ESC08c are used. The $T$-matrix $T_{\Xi NN}$ obtained within the original $\Lambda\Lambda-\Xi N-\Sigma\Sigma / \Xi N-\Lambda\Sigma-\Sigma\Sigma$ coupled-channel framework is employed as an input to the equations. We found no bound state for Jülich-Bonn-München chiral EFT and HAL QCD but ESC08c generates a bound state with the total isospin and spin-parity $(T,J^P) = (1/2,3/2^+)$ where the decays into $\Lambda\Lambda N$ are suppressed.

Keywords $S=−2$ hypernuclear system · Faddeev equation · coupled-channel interaction

1 Introduction

In the last decade, the description of $S=−2$ baryon-baryon interactions has been significantly developed; in addition to conventional meson-theoretical approaches, the ways established on chiral effective theory and lattice QCD simulation have made remarkable progress. Following this development, analyses of three-baryon systems with $S=−2$ have appeared [1, 2,3]. The interactions adopted there are, however, more or less simplified, and the results obtained appear to be still primitive. This paper presents an analysis of the $\Xi NN$ system as a bound state on the basis of Faddeev equation, which uses the three descriptions of the interaction for the $\Lambda\Lambda-\Xi N-\Sigma\Sigma$ and $\Xi N-\Lambda\Sigma-\Sigma\Sigma$ coupled systems: Jülich-Bonn-München chiral EFT (Jülich Ch-EFT) [4,5,6], HAL QCD [7,8], and Nijmegen ESC08c [9].

An advantage of the Faddeev approach is that the inputs to the equations are two-body $T$-matrices. Notice that the $\Lambda\Lambda$, $\Xi N$ and $\Sigma\Sigma$ systems are coupled in $^1S_0$ for $t$ (isospin)=0 states, while $\Xi N$ is not coupled to other channels in $^3S_1$,$^3D_1$. On the other hand, for $t =1,$
ΞN and ΛΣ are coupled in $^1S_0$, while ΞN, ΛΣ and ΣΣ are coupled in $^3S_1-^3D_1$. After precisely solving these two-body coupled-channel problems, we use the $T$-matrix $T_{ΞN,ΞN}$ as the input to $ΞN$ Faddeev equations. Although entire couplings in the three-body space are not included, this usage of the coupled-channel $T$-matrix is a significant step toward realistic analyses of $S=-2$ hypernuclear systems.

We are also interested in the fact that Jülich Ch-EFT and HAL QCD give quite similar $ΞN$ phase shifts throughout the $S$-wave spin- and isospin channels [4,8]. In particular, they both predict a structure close to the $ΞN$ threshold that is related to the coupling to the $ΛΛ$ state. Thus, we first describe in detail the $S=-2$ interactions employed in Sect. 2. The $ΞNN$ Faddeev equations and the results are presented in Sect. 3.

In Fig. 1, $ΞN S$-wave phase shifts generated by Jülich Ch-EFT (red lines) and HAL QCD (blue lines) are shown for isospin $t=0$ and $t=1$ states. Interestingly, these two interactions give quite similar phase shifts except at lower energies of the $^3S_1$, $t=1$ state. The $^1S_0$ phase shift for $t=0$ to which the $ΛΛ$ channel is coupled also shows noticeable behavior; it indicates a strongly attractive feature quickly rising up to 80 degrees from the threshold. In contrast to this, ESC08c gives repulsive phase shift for the $^1S_0$, $t=0$ state as shown in Fig. 2. A characteristic of ESC08c is that a bound state exists in $^3S_1-^3D_1$ for $t=1$ [9]. However, as realized from Fig. 2 the force used here for this partial wave is not so attractive as it generates a bound state. The numerical code used [10] is given by one of the authors of Ref. [9], and we examine rigorously the

![phase_shifts](image_url)
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Fig. 2 $\Xi N\, ^1S_0$ (left panel) and $^3S_1$ (right panel) phase shifts generated by ESC08c for isospin $t = 0$ (dashed lines) and $t = 1$ (solid lines).

phase shifts shown in Fig. 2. Thus, ESC08c employed in this paper is not identical to the original, but nevertheless in Sec. 3, the three-body results for ESC08c will be shown for reference.

Since the $\Xi N$ state can decay into $\Lambda\Lambda$ in $^1S_0$ for $t = 0$, we closely investigate the $T$-matrices for this channel. In Fig. 3, both of $|T_{\Xi N,\Xi N}|^2$ and $|T_{\Delta \Lambda,\Xi N}|^2$ by Jülich Ch-EFT and HAL QCD show visible cusps just at the $\Xi N$ threshold, which are caused by an inelastic virtual-state pole close to the threshold [4,8]. In more detail, the real and the imaginary parts of $T_{\Xi N,\Xi N}$ below the $\Xi N$ threshold are illustrated in Fig. 4. As indicated on the left panel, the magnitude of the imaginary part of Jülich Ch-EFT is negligibly small compared to that of the real part. This is the reason why we utilize only the real part of $T_{\Xi N,\Xi N}$ as an input to the $\Xi NN$ Faddeev calculation, and treat it as a bound state problem. By contrast, the imaginary part of ESC08c has a significant magnitude indicated by the red line on the right panel, which makes us unable to address the $\Xi NN$ system as a bound state.
3 ΞNN Faddeev equation and Results

The Faddeev equations for the system consist of two nucleons and a hyperon can be seen in many literatures. Let us now assign the number 1 to Ξ, 2 and 3 to two nucleons, and impose antisymmetry to the total wave function Ψ:

\[ P_{23} \Psi = -\Psi \]

where \( P_{23} \) is the transposition operator for two nucleons. Then the Faddeev components for the bound-state problem satisfy the coupled equations,

\[ \psi^{(23)} = G_{0}^{NN} (1 - P_{23}) \psi^{(12)} \]

\[ \psi^{(12)} = G_{0}^{\Xi NN} (\psi^{(23)} - P_{23} \psi^{(12)}) \]

(1)

where \( \psi = \psi^{(23)} + (1 - P_{23}) \psi^{(12)} \). To solve this coupled set, we follow the way used in [11, 12]: the set of integral equations is put into the form

\[ \eta(E) \tilde{\psi} = \tilde{K}(E) \tilde{\psi} \]

(2)

with \( \eta(E) \) added to the left, and this eigenvalue problem is solved at a fixed energy \( E \) below the \( \Xi NN \) threshold. If a bound state exists, an eigenvalue such as \( \eta(E_b) = 1 \) can be found at the bound-state energy \( E_b \).

We first analyze the state with the total isospin and spin-parity \( (T, J^{\pi}) = (1/2, 1/2^+) \), which is most likely bound owing to the contribution from the \( \Xi \)-deuteron configuration. As mentioned in Sect. 2, the calculations are performed only for Jülich Ch-EFT and HAL QCD, where the imaginary part of \( T_{\Xi NN} \) is negligibly small below the threshold and only the real part is incorporated. In Fig. 5, eigenvalue \( \eta(E) \) is shown as a function of \( E \) below the \( \Xi d \) threshold at \( -2.225 \text{ MeV} \). The energy \( E \) is set to zero at the \( \Xi NN \) threshold. Although the \( \Xi \) phase shifts in the \( t = 0, 1 \) state show a strongly attractive feature, the eigenvalues are far from \( \eta(E) = 1 \), thus the \( \Xi NN \) system is not bound. Through detailed investigations, we confirm that the \( \Xi NN, t = 1 \) force for \( 1S_0 \) which shows repulsive behavior in Fig. 1 prevents the state from binding.

We also study the \( (T, J^{\pi}) = (1/2, 3/2^+) \) state. In this case, the overlap of the \( \Xi NN \), \( 1S_0 \) state with the total spin \( J^{\pi} = 3/2^+ \) in angular-momentum coupling is negligible, and decays into \( \Lambda NN \) are suppressed. Hence, we perform bound-state calculations for ESC08c in
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Fig. 5 Eigenvalue $\eta(E)$ of the Faddeev kernel $\tilde{K}(E)$ as a function $E$ below the $\Xi d$ threshold. The energy $E$ is set to zero at the $\Xi NN$ threshold. The red and blue lines indicate the values generated by Jülich Ch-EFT and HAL QCD respectively.

addition to Jülich Ch-EFT and HAL QCD. No bound state is found also in this $(T,J^P) = (1/2, 3/2^+) \text{ state for Jülich Ch-EFT and HAL QCD, but a bound state exists at } E = -3.05 \text{ MeV for ESC08c. The attraction in the } \Xi^N_1S_1 \text{ state for } t=1 \text{ shown in Fig. 2 is the main contribution to this binding. In conclusion, we have performed the } \Xi NN \text{ bound-state calculations for the } (T,J^P) = (1/2, 1/2^+) \text{ and } (T,J^P) = (1/2, 3/2^+) \text{ states using the coupled-channel } T \text{-matrix } T_{\Xi^N_1S_1} \text{ with negligible imaginary parts below the threshold for Jülich Ch-EFT and HAL QCD. It turns out that no bound state exists. In spite of the } \Xi N \text{ strong attraction in the } ^1S_0, t=0 \text{ state, repulsive effects from the isosin partner, } ^1S_0, t=1 \text{ state prevent the binding. In contrast, ESC08c generates a bound state at } E = -3.05 \text{ MeV for } (T,J^P) = (1/2, 3/2^+) \text{ where the decays into } \Lambda \Lambda \text{ are suppressed owing to negligible angular-momentum coupling. This is brought about by an attractive feature of the } ^3S_1, t=1 \text{ state for ESC08c.}

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