Diamagnetic Persistent Currents and Spontaneous Time-Reversal Symmetry Breaking in Mesoscopic Structures

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(Received January 16, 2022)

Recently, new strongly interacting phases have been uncovered in mesoscopic systems with chaotic scattering at the boundaries by two of the present authors and R. Shankar. This analysis is reliable when the dimensionless conductance of the system is large, and is nonperturbative in both disorder and interactions. The new phases are the mesoscopic analogue of spontaneous distortions of the Fermi surface induced by interactions in bulk systems and can occur in any Fermi liquid channel with angular momentum \( m \). Here we show that the phase with \( m \) even has a diamagnetic persistent current (seen experimentally but mysterious theoretically), while that with \( m \) odd can be driven through a transition which spontaneously breaks time-reversal symmetry by increasing the coupling to dissipative leads.

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The interplay of disorder and interactions is a rich source of unexplained phenomena in the bulk, especially in two dimensions [1], despite three decades of theoretical effort [2–4]. In mesoscopic systems one is confronted with phenomena not seen in the bulk, such as Coulomb Blockade oscillations [5,6] of the zero-bias conductance, or persistent currents in mesoscopic rings in a small external magnetic field [7–14] (for reviews see [15]). It has been realized in the last several years that one needs to take interactions seriously in order to understand the experimental Coulomb Blockade peak spacing statistics [6]. This understanding has led to Universal Hamiltonian treatments for weak-coupling (electron gas [6] and strong-coupling regime (the dimensionless conductance \( g \) plays the role of \( N \)). Details of this treatment will appear soon [24].

In this Letter, treating Coulomb Blockade and persistent currents within the same approach, we show that the mesoscopic Pomeranchuk phases display unexpected signatures in the persistent current, including a diamagnetic persistent current (seen experimentally [12–14] but so far unexplained) in a model without superconductivity for \( m \) even, and spontaneous time-reversal symmetry breaking for \( m \) odd.

We want the effective Hamiltonian in an energy window of width the Thouless energy \( E_T \) around the Fermi energy. This is the regime of validity [15] of Random Matrix Theory (RMT) [25]. For ballistic structures \( E_T = h v_F / L \), where \( v_F \) is the Fermi velocity and \( L \) is the linear system size. The dimensionless conductance is defined as the number of single-particle energy states (of mean spacing \( \Delta \)) in this window \( g = E_T / \Delta \). Our effective Hamiltonian [18,23] has a noninteracting part representing the chaotic scattering at the walls, and a Fermi-liquid-like interacting part which conserves momentum. The order parameter [23] \( \sigma \) of the Pomeranchuk phase is a two-dimensional vector whose magnitude \( \sigma \) and direction \( \chi \) represent the size and direction of the maximum Fermi surface distortion. The shape of the deformed Fermi surface is given by \( \sigma \cos(\theta - \chi) \), where \( \theta \) is the angular momentum channel in which the instability occurs. To determine the behaviour of \( \sigma \) the fermions are integrated out and an effective action is obtained [23], the dominant part of which is self-averaging for large \( g \).

The “kinetic” term \( f(\omega) \) behaves like \( \omega^2 \) for \( \omega \ll \Delta \).
The averaged energy landscape of to the self-averaging contributions determined earlier specific to the particular disorder realization in addition energy automatically contains the contributions which are the effective potential at its global minimum. This en-
troductory regime is simply the value of the strong-coupling regime is [23] simply the value of the effective potential at its global minimum. This effective potential can be obtained as the ground state energy \( \mathcal{E}(\sigma) \) of a noninteracting fermion Hamiltonian where \( \sigma = \sigma(\cos \chi + \sin \chi) \) appears as a parameter:

\[
(H_\sigma(\chi))_{\alpha\beta} = \varepsilon_\alpha \delta_{\alpha\beta} - g\sigma M_{\alpha\beta}(\chi)
\]

where the first term encodes the chaotic scattering (with the eigenvalues \( \varepsilon_\alpha \) controlled by RMT), and \( M_{\alpha\beta}(\chi) \) represents the coupling between the collective mode \( \sigma \) and particle-hole excitations of the fermions [23]. In the large-\( g \) limit, the ground state energy of the system in the strong-coupling regime is [23] simply the value of the effective potential at its global minimum. This energy automatically contains the contributions which are specific to the particular disorder realization in addition to the self-averaging contributions determined earlier [23]. The averaged energy landscape of \( \sigma \) in the strong-coupling phase would be a “Mexican Hat”, with rotational symmetry [23]. The sample-specific contributions break this symmetry completely for \( m \) even, leading to a single minimum, and to a two-fold symmetry for \( m \) odd, as can be seen in Figure 1.

To see the relation of the above to the persistent current we note that [15]

\[
I(\phi) = -\frac{\partial F(\phi)}{\partial \phi},
\]

where \( \phi \) is the external flux piercing the sample. At zero temperature the Free energy \( F \) is just the ground state energy \( \mathcal{E} \), so we desire to obtain \( \mathcal{E} \) as a function of \( \phi \). We obtain this by taking the noninteracting part of the Hamiltonian from the RMT ensemble of crossover Hamiltonians [25] parametrized as

\[
H_{\text{cross}} = \sqrt{\frac{1}{1+C^2\phi^2}} \left( H_S + C\phi H_A \right)
\]

where \( H_{S,A} \) represent symmetric (\( T \) preserving) and antisymmetric (\( T \) breaking) random matrices drawn from their respective normalized ensembles [25], and \( C \) is a factor of order unity which depends on the shape of the sample and the precise nature of the chaotic scattering at the boundary [17].

Figure 2 shows the dependence of \( \mathcal{E} \) on the crossover parameter \( C\phi \) for \( m \) even and odd, and clearly shows the diamagnetic behaviour for \( m \) even. To see why this is special, consider what is known about persistent currents [15]. In a mesoscopic ring penetrated by a flux, the ground state energy has to be periodic in the flux, since an integer number of flux quanta can be gauged away.

\[
I_{\text{pers}}(\phi) = -\frac{\partial F}{\partial \phi} = I_1 \sin(2\pi\phi/\phi_0) + I_2 \sin(4\pi\phi/\phi_0) + \cdots
\]

where \( \phi_0 = h/e \) is the flux quantum. Only the even moments \( I_{2n} \) survive disorder-averaging [15].

In the noninteracting case the typical, fluctuating values of the Fourier coefficients are (for small \( n \) ) \( I_{n,\text{typ}} \approx E_T/\phi_0 \) while the average is \( \langle I_{2n} \rangle \approx \Delta/\phi_0 \). Experiments typically measure \( \langle I_{2n} \rangle \) and a few other low-order harmonics [9–14].

Interactions, when included in renormalized first-order perturbation theory [8], produce \( \langle I_2 \rangle \approx \mu^* E_T/\infty \) where \( \mu^* \) (of order 1) is the dimensionless Cooper-channel interaction at low energies. Thus, interactions enhance the average persistent current, but if \( \mu^* > 0 \) (\( I_2 \)) should be paramagnetic, while if \( \mu^* < 0 \) it should be diamagnetic. This prediction [8], while of the same order-of-magnitude as the experiments [9–14], has the wrong sign. Materials that show no sign of superconductivity (implying that \( \mu^* > 0 \)) show [11–14] a diamagnetic \( (I_2) \). Many explanations have been proposed to account for this puzzle (a recent one being Ref. [26]), but the question remains open, as summarized in Ref. [27]. In this context a diamagnetic persistent current of order \( E_T/\phi_0 \) in a model
without superconductivity is striking. The fact that our treatment is nonperturbative in the interactions [23] enables us to evade the usual sign [8]. Our approach, while suggestive, is not directly applicable to the experiments on Au and Ag rings [12,14] since the samples are not likely to be in the strong-coupling regime, and are not fully in the ballistic limit (the elastic mean free path is of the same order as the system size). On the other hand, our theory would apply directly to ensembles of ballistic GaAs rings of the type used in Refs. [11,13], but at stronger coupling.

Let us now turn to \( m \) odd. The exact degeneracy of the two global minima separated by \( \pi \) in the angle \( \chi \) can be seen from Figure 1, and can be proved analytically using the relation \( H_\sigma^g(\chi) = H_\sigma(\chi + \pi) \). The two degenerate minima are related by the time-reversal transformation \( T \). A particular value of \( \chi \) leads to a distortion of the Fermi surface along the direction specified by \( \chi \). Under \( T \), \( k \to -k \) and a distortion of the Fermi surface for odd \( m \) maps to an inequivalent state at \( \chi + \pi \) with the same energy, since the underlying Hamiltonian is \( T \)-invariant. The ground state of a Hamiltonian quantum system with a two-fold degenerate potential is the symmetric combination of the two minima. This applies to the isolated mesoscopic structure, whose dynamics is Hamiltonian at energy scales smaller than \( \Delta \) (for energy scales in the range \( \Delta \ll \omega \ll E_T \) the dynamics is dissipative with ohmic dissipation, see Eq. (2)). The splitting between the symmetric and antisymmetric combinations is the tunneling amplitude between the two minima, here \( \Delta e^{-g} \). The two minima correspond to states carrying opposite persistent currents, and are macroscopically distinguishable.

The coupling of the mesoscopic structure to the leads produces ohmic dissipation at arbitrarily low energies. This is precisely the case of the Caldeira-Leggett model [28] considered and solved by Chakravarty [29], and Bray and Moore [29]. The effective action of our model at low energies (\( \omega \ll \Delta \)) is

\[
g \int dt (V(\chi(t)) + \frac{1}{2} (dx/dt)^2) + \frac{2g \ell^2(\sigma)^2}{\pi \Delta^2} \int \frac{dt dt'}{(t-t')^2} \sin^2 \left( \frac{\chi - \chi'}{2} \right)
\]

where \( \chi \) is the angle of \( \sigma \) in the Mexican Hat, \( V(\chi) \) is the doubly-degenerate realization-specific potential of Figure 1, and \( \Gamma \) is the level width induced by coupling to the leads. The long-range interaction in imaginary time comes from a \( |\omega|^n \) "kinetic" term, which in turn arises from the Landau damping of \( \sigma \) due to decay into particle-hole pairs at arbitrarily low energies, possible because each formerly sharp level \( \alpha \) is broadened by coupling to the leads.

The model has a weak-dissipation phase in which the ground state is still the symmetric superposition of the two minima, and a strong-dissipation phase in which the particle is localized in one minimum. The transition between the two phases [29] occurs for \( g(\Gamma/\Delta)^2(\sigma^2) \approx 1 \). For large enough \( g \) even a weak coupling to the leads \( (\Gamma \approx \Delta/\sqrt{\ell}, \text{since } (\sigma^2) \approx 1) \) is sufficient to meet this criterion, and leads to localization in one minimum of the twofold degenerate effective potential even at zero temperature, corresponding to a spontaneous breaking of \( T \). The \( T \)-breaking transition could be monitored by measuring the peak-height statistics [30].

If one turns on an external flux, one minimum moves up in energy as the flux increases while the other moves down. The ground state (which moves down) displays a paramagnetic persistent current of order \( E_T/\phi_0 \). In the isolated dot, or in the case with weak dissipation, one starts with a symmetric superposition of the two minima as the zero-flux ground state. As \( \phi \) increases the system crosses over to fully \( T \)-broken dynamics when the energy difference of the two minima is greater than their splitting in zero field, which is \( \Delta e^{-g} \). Thus, the crossover will occur for an external flux \( \phi_\chi \approx \phi_0 e^{-g} \), as compared to the noninteracting crossover flux \( \phi \approx \phi_0/\sqrt{\ell} \). For strong enough dissipation, the ground state already breaks \( T \), and the variation of \( \mathcal{E} \) contains a term first-order in \( \phi \), which implies a spontaneous persistent current at zero flux.

Figure 3 shows the ensemble-averaged second Fourier coefficient \( \langle I_2 \rangle \) for \( m \) even and odd. Ideally one would find a periodic behaviour of \( \mathcal{E} \) with \( \phi \). This cannot be captured by the crossover Hamiltonian, but must be put in by hand. Since we do not know the number \( C \) connecting the crossover parameter to the flux \( \phi \), there is an inherent ambiguity in this procedure. We show \( \langle I_2 \rangle \) for three different choices for \( C \). As can be seen, the qualitative results are unaffected.

Finite temperature produces additional interesting effects for \( m \) odd. For temperatures exceeding the tunnel splitting \( \Delta e^{-g} \) the superposition states are irrelevant, and one can think of the two minima as separately thermally occupied. Because of their (near) degeneracy, there will be a Curie-like susceptibility of \( 1/T \) of the persis-

**FIG. 3.** The ensemble-averaged second Fourier coefficient of the persistent current in units of \( \Delta/\phi_0 \) as a function of coupling \( |u_m| \geq |u_m^*| = 1/\ln 2 \) for \( m = 1 \) and \( m = 2 \) and \( g = 20 \).
tent current (and therefore the magnetization) to external flux.

In summary, we have shown that some surprising signatures of the mesoscopic Pomeranchuk regimes show up in the persistent current. There is a diamagnetic persistent current (for $m$ even) without any superconductivity. The $m$ odd case undergoes a spontaneous time-reversal symmetry-breaking transition as the coupling to the leads is increased, and displays a spontaneous persistent current (at zero flux) in the $T$-broken phase. It would be very interesting to explore the behaviour of persistent currents in the quantum critical regime [31], where large fluctuations of the order parameter $\sigma$ and finite quasiparticle lifetime [32] at low energies are expected [23].

We thank R. Shankar for illuminating discussions and E. Mucciolo for pointing out Ref. [21], and the National Science Foundation for partial support under grants DMR 98-04983 (DH and HM) and DMR 0071611 (GM).

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