Numerical modelling of mangrove merged with seawall for investigating wave mitigation over flat topography

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Abstract. In this study, numerical modelling to see the effect of mangrove merged with seawall on wave mitigation over flat topography is developed. The model is based on Navier-Stokes equations. Resistance of the mangrove is described using Manning friction, whereas the seawall is created from particles and added directly to the topography. Numerical solution of the model is solved using Smoothed Particle Hydrodynamics (SPH). Two types of seawalls; vertical and incline seawall are used. Results of the numerical simulations show that existence of the mangrove merged with vertical seawall can reduce 10.52191% in wave height and prevent the wave to pass the seawall. Whereas the existence of the mangrove merged with incline seawall can reduce 15.39512% in wave height but the wave still passes the seawall.

1. Introduction

In this study, we do numerical modelling to see the effect of mangrove merged with seawall on wave mitigation especially over flat topography. Researches focused on wave mitigation due to mangrove (coastal vegetation) or only seawall can be found in many literatures for instances see [1], [2], [3], [4], and [5] for mangrove (coastal vegetation) modelling and [6], [7], [8], [9], and [10] for modelling of overtopping of seawall. But, according to our knowledge, the numerical modelling of combination of them is still rare.

In the literatures, to model wave mitigation or overtopping can be based on shallow water equations, Boussinesq type equations, or Navier-Stokes type equations. Further, numerical solution of the models can be grid based method (e.g. staggered finite volume method) or meshless method (e.g. Smoothed Particle Hydrodynamics (SPH)). Formulation of resistance of Mangrove (coastal vegetation) can be governed as porous medium or as friction. Whereas the seawall is governed directly in the topography. Further, in SPH modelling, the seawalls are created from particles. Detail explanation of the porous medium modelling can be seen in [3], elaboration of the friction consideration of mangrove (coastal vegetation) can be found in detail in [1], and detail description of the seawall modelling can be read in [8].

In this article, we aim to develop numerical model of the wave mitigation due to mangrove (coastal vegetation) merged with vertical or incline seawall over flat bottom. The resistance of the mangrove is formulated by Manning friction and the seawall is added directly to the topography. Here we also consider pressure gradient \( \nabla P \), gravitational force \( g \) and stress tensor \( \tau \) as the forces acting on the fluid.
The model will be solved numerically using SPH method. Formulations of the model are given as follows:

\[ \frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u}, \]
\[ \frac{D\mathbf{u}}{Dt} = - \frac{1}{\rho} \nabla P + \mathbf{g} - \frac{c_f |\mathbf{u}|}{H^{1/3}} + \frac{1}{\rho} \nabla \cdot \mathbf{\tau}, \]  
\[ \frac{D\mathbf{x}}{Dt} = \mathbf{u}, \]  
\[ \frac{D\mathbf{p}}{Dt} = \mathbf{0}, \]

where \( \rho \) represents fluid density, \( \mathbf{u} \) is flow velocity, \( c_f = 0.25 \) is the Manning friction coefficient, \( H \) is water depth, \( \mathbf{x} \) denotes position of particles and \( \mathbf{\tau} \) is defined by \( \tau_{ij} = 2\eta \zeta_{ij}, \) \( \zeta_{ij} = \frac{1}{2} (\partial_i u_j + \partial_j u_i), \) and \( \eta = 10^{-3} \text{ Pa.s} \) is the molecular dynamics viscosity.

In this article, brief explanation of the SPH method will be given in Section 2. Results of numerical experiments and discussions will be carried out in Section 3. At the end, some conclusions are presented in Section 4.

2. SPH formulations

In the SPH method, finite numbers of particle are used to discretize the fluid. Each particle has some physical properties and moves according to the governing equations. Formulations of the governing equations in SPH term are based on particle approximation. To get the particle approximation of any function, an integral interpolation using a specific kernel function is needed. The kernel function has some properties. Further, there are some kernel functions that can be chosen, in this study the common cubic spline kernel is used. Explanation of the SPH method can be read in detail in some references for instances see [11], [12], and [13]. All information about the SPH method written here exists in the references.

Let \( \phi(x) \) denotes the function that will be interpolated using the kernel function \( W \) with smoothing length \( h \) and \( m \) is notation of mass of the fluid particle

\[ \phi(x) = \int \phi(x') W(x - x', h) dx. \]  

In the discrete form, the integral interpolation can be written as

\[ \phi(x_i) = \sum_j m_j \frac{\phi_j}{\rho_j} W(x_i - x_j, h). \]  

Where the summation is calculated for all particles \((j)\) in the compact support area of the kernel function with radius \(2h\).

Using the method, SPH formulations of the governing equations are given as follows

\[ \frac{D\rho_i}{Dt} = \sum_{j=1}^{N} m_j \left( \mathbf{u}_j - \mathbf{u}_i \right) \cdot \nabla W_{ij}, \]  

\[ \frac{D\mathbf{u}_i}{Dt} = - \sum_{j=1}^{N} m_j \left( \rho_i + \rho_j \right) \nabla_i W_{ij} + \mathbf{g} - \frac{c_f |\mathbf{u}_j|}{H^{1/3}} + \frac{1}{\rho} \left( \nabla \cdot \mathbf{\tau} \right)_i. \]  

Notation \( \Pi_{ij} \) denotes artificial viscosity added to ensure numerical stability. Here the artificial viscosity is formulated as in [1] with coefficient \( \alpha_1 = 0.025 \).

Formulation of the stress tensor is adopted from Shaimy in [14]. Note that, in the reference, the stress tensor is written in power-law model with coefficient \( n \), whereas in this article the coefficient is equal to one, \( n = 1 \). Therefore the SPH formulation of the stress tensor is written as follows. Notation \( a \) and \( b \) denote 2D coordinate (\( x \) and \( y \) direction) and \( \delta^{ba} \) is delta Dirac function.

\[ \frac{1}{\rho} \left( \nabla \cdot \mathbf{\tau} \right)_i^a = \sum_{j=1}^{N} m_j \left( \frac{\mu_c b^a}{\rho_i \rho_j} + \frac{\mu_s b^a}{\rho_i \rho_j} \right) \frac{\partial W_{ij}}{\partial x_i^a}, \]  

\[ \zeta_i^{ba} = \sum_{j=1}^{N} m_j \frac{\mathbf{u}_j b^a}{\rho_j} \frac{\partial W_{ij}}{\partial x_i^a} + \sum_{j=1}^{N} m_j \frac{\mathbf{u}_j b^a}{\rho_j} \frac{\partial W_{ij}}{\partial x_i^a} - \frac{2}{3} \sum_{j=1}^{N} m_j \frac{\mathbf{u}_j b^a}{\rho_j} \cdot \nabla_i W_{ij} \delta^{ba}. \]
Basically, pressure of each particle can be updated using equation of state or by solving pressure Poisson equation. In current article, equation of state described in [13] is used with constant $\gamma = 7$. The position of the particles is updated using Leapfrog time integration.

3. Numerical experiments and discussions

Two numerical experiments are conducted to see influence of the mangrove merged with seawall. Two type of seawalls are used in the experiment namely vertical seawall and incline seawall. Note that the surface of particles in this article is taken using algorithm provided in [15].

3.1. Wave mitigation due mangrove merged with vertical seawall

The numerical experiment of vertical seawall is conducted in computational domain $\Omega = [0, 14]$. The seawall and mangrove are located at position $10 \text{ m} \leq x \leq 11.5 \text{ m}$ with height $0.3 \text{ m}$ over initial water depth $1 \text{ m}$ and $8 \text{ m} \leq x \leq 9.5 \text{ m}$, respectively. At the beginning, 3679 particles are created consecutively with spacing 0.05 m. To generate solitary wave, wavemaker as described in [16] is used. Note that both wavemaker and seawall are created from particles. Further, position of the seawall is fixed and remain unchanged. All particles move according to the governing equations with smoothing length $h = 0.046 \text{ m}$ and time step $\Delta t = 5 \times 10^{-4} \text{ s}$. As an illustration see Figure 1. Results of the simulation are presented in Figure 2.

Figure 1. Computational domain and set up of simulation in case mangrove merged with vertical seawall (right) and only seawall (left).

Figure 2 shows water surface of simulation in case mangrove merged with vertical seawall (red color) and in case only vertical seawall (green color). As shown in Figure 2(a), at the beginning all particles are set at rest so the water surface for both cases is zero. Figure 2(b) describes profile of water surface due to movement of wavemaker. The wavemaker moves horizontally, the movement will push the water particles and create solitary wave as shown in the figure. The wave propagates to the vertical seawall. In the figure, the influence of the mangrove placed at position $8 \text{ m} \leq x \leq 9.5 \text{ m}$ is not seen clearly yet. In Figure 2(c), the crest of the wave passes through the mangrove area therefore the reduction on the wave amplitude is seen clearly. Our recorded data of peak of the wave as shown in the figure are $0.493266 \text{ m}$ (in case only vertical seawall) and $0.441365 \text{ m}$ (in case mangrove merged with vertical seawall). In other words, the existence of the mangrove can reduced the wave amplitude especially for result given in the figure the reduction is 10.52191%.

Figure 2(d) – 2(f) show wave overtopping phenomena. In Figure 2(d), the wave start to propagate over the seawall. The outer particles in case of only vertical seawall and mangrove merged with vertical seawall reach position $x = 10.506839$ and $x = 10.156806$, respectively. In Figure 2(e) and 2(f), the wave flows over the seawall. In the only seawall case, the wave reaches position $x = 11.838261$ (Figure 2(e)) and $x = 12.208062$ (Figure 2(f)). Whereas in the other case the wave moves back and goes down from the seawall as shown in Figure 2(e) and 2(f). The Manning friction is added in the momentum equation (see the governing equations) which means existence of the mangrove reduces velocity of the fluid. Therefore the wave in case mangrove merged with seawall is behind the wave of only seawall case and do not pass the seawall.
Figure 2. Water surface of simulation in case mangrove merged with vertical seawall (sm, red) and only seawall (s, green) at time $t = 0\ s$ (a), $2.05\ s$ (b), $3.25\ s$ (c), $3.6\ s$ (d), $4.5\ s$ (e), and $4.75\ s$ (f).

3.2. Wave mitigation due to mangrove merged with incline seawall

The numerical set up in this simulation is the same as the previous simulation. The different is shape of the seawall, note that the width and height of the seawall are the same as the vertical seawall. In this case, the seawall has inclination as shown in Figure 3. Note that, total particles of this simulation is 4720 particles. Results of the simulation are given in Figure 4.

Figure 3. Computational domain and set up of simulation in case mangrove merged with incline seawall (right) and only seawall (left).
Figure 4 presents comparison of the surface of water in numerical experiment in case only incline seawall and in case mangrove merged with incline seawall. Figure 4(a) describes water surface at the beginning of simulation. The same as the vertical seawall case, at the beginning all particles are set at rest. Figure 4(b) shows the wave profile due to movement of the wavemaker. In Figure 4(c), the reduction of the wave is occurred. Further the reduction is 17.61248% respect to the only seawall case. In Figure 4(d) the wave reach the end of the seawall. Here we record the outer particle of each case and the different of the height is 15.39512% respect to the particle height of only incline seawall case. Figure 4(e) and 4(f) show the overtopping phenomena. In the only seawall case, more particles pass the seawall compare with the mangrove merged with seawall. The furthest position of the particles presented in the Figure 4(f) are $x = 12.07104$ for the merged case and $x = 12.75081$ for the only seawall case.

From the numerical experiments, we get that the mangrove can reduce both wave amplitude and wave speed with percentage of reduction as mention before. Whereas the structure of the seawall influence of the wave overtopping phenomena. It is clearly shown in Figure 2 and Figure 4 that in case incline seawall has more particle that pass the seawall than the vertical seawall case at the same time. In the incline seawall case, the wave propagates and flows over the inclination, whereas in the vertical seawall case, the wave propagates and hit the seawall directly.
4. Conclusions

Numerical model on wave mitigation due to mangrove merged with seawall has been developed. The model based on Navier-Stokes type equations. The resistance of the mangrove is formulated using Manning friction and the seawall is modeled directly in the topography. The seawall is created from particles. The mangrove can reduce both wave amplitude and wave speed, whereas the structure of the seawall influence of the wave overtopping phenomena. The incline seawall case has more particles that pass the seawall than the vertical seawall case at the same time. Results of the numerical simulations show that existence of the mangrove merged with vertical seawall can reduce 10.52191\% in wave height and prevent the wave to pass the seawall. Whereas the existence of the mangrove merged with incline seawall can reduce 15.39512\% in wave height but the wave still passes the seawall.

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References

[1] Iryanto and P H Gunawan 2016 J. Phys.: Conf. Ser. 693 012013
[2] Magdalena I, Pudjaprasetya S and Wiryanto L 2014 Advances in Applied Mathematics and Mechanics 6 680-692
[3] Shao S 2010 Coastal Engineering 57 304-316
[4] Khng X Y, Teh S Y and Koh H L 2017 AIP Conference Proceedings 1870 040009
[5] Yao Y, Tang Z, Jiang C, He W and Liu Z 2018 Journal of hydro-environment research 19 78-87
[6] Li T, Troch P and De Rouck J 2004 Journal of Computational Physics 198 686-726
[7] Orszaghova J, Borthwick A G L and Taylor P H 2011 Coastal Engineering Proceedings 1 15
[8] Shao S, Ji C, Graham D I, Reeve D E, James P W and Chadwick A J 2006 Coastal engineering 53 723-735
[9] Jiang C, Liu X, Yao Y, Deng B and Chen J 2017 Journal of Earthquake and Tsunami 11 1740006
[10] Luo M, Reeve D E, Shao S, Karunarathna H, Lin P and Cai H 2019 Engineering Analysis with Boundary Elements 103 160-171
[11] Liu G R and Liu M B 2003 Smoothed particle hydrodynamics: a meshfree particle method (World scientific)
[12] Monaghan J J 1992 Annual review of astronomy and astrophysics 30 543-574
[13] Monaghan J J 1994 Journal of computational physics 110 399-406
[14] Shaimy R A F 2013 Simulation of SPH method for Coutte-Poiseuille flow of power law fluid within a gap filled with obstacles Master's thesis (Institut Teknologi Bandung, Bandung)
[15] Barecasco A, Terissa H and Naa C F 2013 arXiv preprint arXiv:1309.4290
[16] Iryanto and Pudjaprasetya S R 2017 East Asian Journal on Applied Mathematics 7 728-740