Whitening of the Quark-Gluon Plasma

Stanislaw Mrówczyński
Soltan Institute for Nuclear Studies
ul. Hoża 69, PL - 00-681 Warsaw, Poland
and Institute of Physics, Świętokrzyska Academy
ul. Świętokrzyska 15, PL - 25-406 Kielce, Poland
E-mail: mrow@fuw.edu.pl

Abstract.
Parton-parton collisions do not neutralize local color charges in the plasma but they only redistribute them among momentum modes. Color diffusion and conductivity are discussed as processes responsible for the plasma whitening. The conductivity and diffusion coefficients are computed and the time evolution of the color density is studied. The conductivity is shown to be more effective than the diffusion, it whitens the plasma even before the momentum degrees of freedom are thermalized.

1. Introduction
This talk is based on two works done in collaboration with Cristina Manuel [1, 2]. In the first paper [1], we have analyzed the local equilibrium of the quark-gluon plasma which is defined as a state of maximal local entropy. Using the kinetic equations with the collision terms of the Waldmann-Snider form, we have proved that such a state is generically colorful, i.e. the color four-current is non-vanishing. Thus, the collisions, which are responsible for equilibration of the parton momenta, do not neutralize the local color density. Since the color current is (covariantly) conserved in every collision process, the inter-parton collisions redistribute the color charges among various momentum modes but they do not change a local macroscopic color charge. Consequently, if the color charges are not homogeneously distributed in the process of the plasma production due to, say, statistical fluctuations, the inter-parton collisions will not neutralize the system. On the other hand, the global equilibrium of the quark-gluon plasma is locally colorless because of the maximum entropy principle. We assume here that the system does not carry a global color charge and that it does not experience an external chromodynamic field. Once the inter-parton collisions are not responsible for the neutralization of the local charges, one has to invoke other collective mechanism to whiten the plasma. This is the subject of the second article [2].

Local charges are neutralized due to the currents that flow in the system. We consider the diffusive currents generated by the charge density gradient (Fick’s law) and the ohmic currents caused by the chromoelectric field (Ohm’s law) which is, in turn, induced by the charge density. The color conductivity of the quark-gluon plasma has been studied for long time [3, 4, 5, 6, 7, 8, 9, 10], but only recently the problem has been well understood [11, 12, 13, 14, 15, 16, 17, 18]. As far as we know, the color diffusion was only briefly discussed by Heiselberg [10]. In all these papers, the plasma near the colorless global equilibrium was
studied. We are, however, interested in the plasma that is locally colorful. Thus, in Sec. 2 we derive the diffusion and conductivity coefficients in such a plasma, and then, in Sec. 3 the temporal evolution of the color charge density is considered.

2. Color Diffusion and conductivity coefficients

Using the transport theory, we derive here the diffusive and ohmic currents in a plasma that is locally colorful. The transport equations of quarks, antiquarks and gluons read

\[(D^0 + v \cdot D)Q(p, x) - \frac{g}{2} \{E + v \times B, \nabla_p Q(p, x)\} = C[Q, \tilde{Q}, G],\]

\[(D^0 + v \cdot D)\tilde{Q}(p, x) + \frac{g}{2} \{E + v \times B, \nabla_p \tilde{Q}(p, x)\} = \tilde{C}[Q, \tilde{Q}, G],\]

\[(D^0 + v \cdot D)G(p, x) - \frac{g}{2} \{E + v \times B, \nabla_p G(p, x)\} = C_g[Q, \tilde{Q}, G].\]

The (anti-)quark on-mass-shell distribution functions \(Q(p, x)\) and \(\tilde{Q}(p, x)\), which are \(N_c \times N_c\) hermitean matrices, belong to the fundamental representation of the SU\((N_c)\) group, while the gluon distribution function \(G(p, x)\), which is a \((N_c^2 - 1) \times (N_c^2 - 1)\) matrix, belongs to the adjoint representation. The covariant derivative \(D^\mu \equiv \partial^\mu - ig[A^\mu(x), \cdots]\), the chromoelectric and chromomagnetic fields, \(E\) and \(B\), which enter the transport equations also belong to either the fundamental or adjoint representation, correspondingly. To simplify the notation, we use the same symbols \(D^0, D, E\) and \(B\) to denote a given quantity in the fundamental or adjoint representation. \(x \equiv (t, x)\) denotes the four-position while \(p\) is the three-momentum. Because the partons are assumed to be massless, the velocity \(v\) equals \(p/|p|\). The collision terms \(C, \tilde{C}\) and \(C_g\) will be discussed later on.

We are interested in a state close to the colorful local equilibrium. When the effects of quantum statistics are neglected, the (on-mass-shell) local equilibrium distribution functions are [1]

\[Q^{eq}(p, x) = \exp\left[-\beta(x)(u_\nu(x)p^\nu - \mu_b(x) - \tilde{\mu}(x))\right],\]

\[\tilde{Q}^{eq}(p, x) = \exp\left[-\beta(x)(u_\nu(x)p^\nu + \mu_b(x) + \tilde{\mu}(x))\right],\]

\[G^{eq}(p, x) = \exp\left[-\beta(x)(u_\nu(x)p^\nu - \tilde{\mu}_g(x))\right],\]

where \(p^\mu = (E_p, p)\), and \(E_p = |p|\) for massless quarks and antiquarks, and for gluons; \(\beta = 1/T\), \(u^\mu\), \(\mu_b\) denote, respectively, the inverse temperature, hydrodynamic velocity and baryon chemical potential; the colored chemical potentials of quarks (\(\tilde{\mu}\)) and of gluons (\(\tilde{\mu}_g\)) obey the relationship

\[\tilde{\mu}_g(x) = 2T^a \text{Tr}[\tau^a \tilde{\mu}(x)] = \mu_a(x)T^a ,\]

where \(\tau^a, T^a\) with \(a = 1, \ldots, N_c^2 - 1\) are the SU\((N_c)\) group generators in the fundamental and adjoint representations, normalized as \(\text{Tr}[\tau^a \tau^b] = \frac{1}{2} \delta^{ab}\) and \(\text{Tr}[T^a T^b] = N_c \delta^{ab}\). In the local equilibrium state there is a non-vanishing color charge density and current. However, the current vanishes in the local rest frame because the distribution functions are locally isotropic.

We further consider small deviations from local equilibrium, and we write down the quark distribution function as \(Q(p, x) = Q^{eq}(p, x) + \delta Q(p, x)\). Assuming that

\[|Q^{eq}| \gg |\delta Q| , \quad |D^\mu Q^{eq}| \gg |D^\mu \delta Q| , \quad |\nabla_p Q^{eq}| \gg |\nabla_p \delta Q| ,\]

where

and taking into account the local isotropy of the equilibrium state, the transport equation (1) can be approximated as

$$\left( D^0 + v \cdot D \right) Q^{eq} + \frac{g}{2} \{ E, \nabla p Q^{eq} \} \equiv L[\delta Q] ,$$

where $L[\delta Q]$ is the collision term linearized around the local equilibrium distribution function. Analogous equations hold for antiquarks and gluons. Let us recall here that the collision terms evaluated for the local equilibrium distribution functions (2) vanish [1].

To get the transport coefficients of color diffusion and conductivity we assume that the linearized collision terms $L, \bar{L}$ and $L_g$ satisfy the relationship

$$\int \frac{d^3p}{(2\pi)^3} v L[\delta Q] = -\gamma \int \frac{d^3p}{(2\pi)^3} v \delta Q ,$$

where $1/\gamma$ is the characteristic relaxation time which, for simplicity, is assumed to be the same for quarks, antiquarks and gluons. The relation is obeyed [15] by the Waldmann-Snider collision term linearized around global (colorless) equilibrium. The relationship (10) is also trivially satisfied by the collision term in the relaxation time approximation, which, unfortunately, violates the covariant current conservation. However, the collision term can be easily improved. In analogy to the so-called BGK collision term [19], one constructs the expression [2] which obeys both the covariant current conservation and the relation (10).

Using Eq. (10), the color current (in the adjoint representation), which is generated by deviations from equilibrium, gets the form

$$j^a(x) = -d D_{ab} \rho^b(x) + \sigma_{ab} E^b(x) ,$$

where $D_{ab} = \delta_{ab} \nabla + g f^{acb} A^c$, the diffusion constant equals $d = 1/3\gamma$ and the color conductivity tensor reads

$$\sigma_{ab} = \frac{g^2}{3\gamma} \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} \left( \text{Tr}[\{\tau^a, \tau^b\}(Q^{eq} + \bar{Q}^{eq})] + \text{Tr}[\{T^a, T^b\} G^{eq}] \right) .$$

When the equilibrium is colorless, the conductivity $\sigma$ is proportional to the unit matrix in color space.

3. Temporal evolution of the color density

The color current generated by both the gradient of color density and the chromoelectric field is given by Eq. (11). The covariant current conservation $D^0 \rho + D j = 0$ combined with the Gauss law $D E(x) = \rho(x)$, provides the equation

$$[D^0 - d D^2 + \sigma] \rho(x) = 0 ,$$

where the term $[D, \sigma] E$ has been neglected. It can be shown [2] that $[D, \sigma] E \ll \sigma D E$ in the small coupling limit. Since the commutator $[D^0, \sigma]$ can be also shown to be small, Eq. (13) is solved by

$$\rho(x) = e^{-\sigma t} n(x)$$

with $n$ satisfying the diffusion equation

$$[D^0 - d D^2] n(x) = 0 .$$

$\rho(x)$ and $n(x)$ obey the initial condition $\rho(0, x) = n(0, x) = \rho_0(x)$. One shows [2] that the conductivity dominates over the diffusion for the long wavelength charge density modes with the wave vector $k$ obeying $k^2 < \sigma/d$. 

424
4. Discussion

The question arises what is the characteristic time of plasma whitening. The only reliable estimates of the time scales of interest have been found for the perturbative quark-gluon plasma close to the global (colorless) equilibrium. The characteristic time scale of the color conductivity and of the parton-parton collisions at momentum transfer of order \( g^2 T \) are \([8, 15]\)

\[
\frac{1}{\sigma} \sim \frac{\ln(1/g)}{T}, \quad t_{\text{soft}} \sim \frac{1}{g^2 \ln(1/g) T}.
\]

As seen, \( 1/\sigma \ll t_{\text{soft}} \). Since the evolution of color degrees of freedom is dominated by the soft collisions, the relaxation time \( 1/\gamma \) is of order \( t_{\text{soft}} \). Our derivation of the color conductivity and the further analysis apply for \( t \geq 1/\gamma \) as \( 1/\gamma \) is the minimal time scale which enters the linearized transport equation (9) with the collision term (10). Taking into account the above estimate of color conductivity, the charge density is found to vanish at \( t \geq 1/\gamma \). Because the characteristic time scale of color dissipation cannot be shorter than \( t_{\text{soft}} \), the whitening of the quark-gluon plasma occurs at \( t \sim t_{\text{soft}} \). For shorter times the color density is expected to oscillate.

To equilibrate the system’s momentum degrees of freedom, there are needed parton-parton collisions at the momentum transfer of order \( T \). Such collisions occur at the time scale \([15]\)

\[
t_{\text{hard}} \sim \frac{1}{g^2 \ln(1/g) T},
\]

which in the perturbative limit is much longer than \( t_{\text{soft}} \). Thus, we conclude that the plasma becomes white first and then the momentum degrees of freedom are thermalized.

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