Six-Dimensional Fixed Points from Branes

Ilka Brunner, Andreas Karch
Institut für Physik, Humboldt-Universität,
Invalidenstr. 110, D-10115 Berlin

Abstract: We review the construction of six dimensional $N = 1$ fixed points in a brane picture involving D6 branes stretching between NS 5 branes.

Introduction: The classical Hanany-Witten brane setup. Hanany and Witten [2] invented a brane configuration in IIB theory to study 3 dimensional $N = 4$ supersymmetric gauge theories. The basic ingredients they used are IIB NS 5 branes, D5 branes and D3 branes in flat space. The worldvolumes of the branes are in the following directions

\[
\begin{array}{c|cccccccc}
\text{NS 5} & x^0 & x^1 & x^2 & x^3 & x^4 & x^5 & x^6 & x^7 & x^8 & x^9 \\
\text{D 5} & 0 & 1 & 2 & - & - & - & 7 & 8 & 9 & - \\
\text{D 3} & 0 & 1 & 2 & - & - & - & 6 & - & - & - \\
\end{array}
\]

It can be checked that this configuration preserves 1/4 of the supersymmetries, that is 8 supercharges, as required for $N = 4$ in $d = 3$. The 3 branes are suspended between the 5 branes in the $x_6$ direction. The 3d, $N = 4$ theory is realized in the directions which are common to all branes. The point of view we take is that the 5 branes are much heavier than the 3 branes because they have two extra dimensions. The low energy dynamics is determined by the lowest dimensional brane in the setup. Moving around the 3 brane corresponds to changing the moduli of our theory. If we move around 5 branes this corresponds to changing parameters like masses, coupling constants, FI-terms.

What is the field theory on a 3 brane suspended between two NS 5 branes? On an infinite 3 brane there is a theory of a vector multiplet $V_{N=8}$ with 16 supercharges.

\footnote{Based on a talk given at the “31st International Symposium Ahrenshoop on the Theory of Elementary Particles” Buckow, September 2-6, 1997.}
The multiplet contains a vector and scalars corresponding to the fluctuations of the 5 brane in the transversal directions. In our setup, the $x_6$ direction is finite, therefore we are left with a $2 + 1$ dimensional field theory. The effect of the NS 5 branes is that SUSY is broken to $N = 4$. The 3 brane motion in the 7, 8, 9–direction is locked because the 3 brane positions in these directions have to agree with the 5 brane positions. Also, the scalar $A_6$ coming from the dimensional reduction of the vector is projected out by the boundary conditions on the NS 5 brane. Altogether, we are left with an $N = 4$ vector multiplet. To enhance the gauge group to $U(N_c)$ we can put $N_c$ 3 branes on top of each other. The gauge coupling of the theory is related to the distance between the NS 5branes:

$$\frac{1}{g_{YM}^2} = \frac{\Delta x_6}{g_s}$$

Here, $g_s$ is the string coupling and $1/g_s$ would be the gauge coupling on an infinite D3. We can include matter multiplets in the fundamental representation by putting D5 branes in between the NS branes. Strings can then stretch between the D5 and D3 branes yielding matter in the fundamental representation. The mass of these multiplets corresponds to the distance (in the 345 direction) between the D5 and D3 branes. An alternative way to include matter is to add semi-infinite 3 branes to the left and right of the NS branes. Matter arises from strings stretching between a semi-infinite and finite piece of the 3 brane. The two descriptions of matter multiplets are related by the Hanany Witten effect: We move the D5 branes off to infinity. When they cross an NS brane a new D3 brane is created, which ends on the NS brane.

The Hanany Witten setup can be T-dualized along the directions 3,4,5. The dimensions of the D brane stretched between the two NS branes increases in each step. This enables us to study theories with 8 supercharges in various dimensions.

**Bending and RR charge conservation.**

In higher dimensions it becomes very important to take into account the disturbance caused by the D branes ending on the NS branes [12]. The end of a Dd brane looks like a magnetic monopole in the worldvolume of the NS 5 brane or as a charged particle on the $6 - d$ dimensional subspace transverse to the Dd brane. Dd branes ending from different sides on the NS brane contribute with opposite charge. The consequence is that the NS branes do not have a definite $x_6$ position, but the $x_6$ coordinate obeys a Laplace equation:

$$\Delta x_6(y) = 0,$$

where $y$ parametrizes the transversal space. The “true” $x_6$ coordinate of the NS brane is the $x_6$ value far away from the disturbance. We can analyze the behaviour in various dimensions by looking at the solutions to the Laplace equation in various dimensions. In three dimensions, the case analyzed in the previous paragraph, the solution behaves as

$$x_6 = \frac{1}{|y|} + \text{constant},$$
such that for $|y| \to \infty$ we get a definite value, which we can call the $x_6$ position of the 5 brane.

In $d = 4$ we obtain a logarithmic behaviour. The distance in $x_6$ between two NS branes is proportional to the 4 dimensional gauge coupling, which is known to diverge logarithmically in 4 dimensions. This is reproduced in the brane picture.

In 5 dimensions, the transversal space is one dimensional and we obtain a linear bending of the NS brane. This can also be seen from the fact that RR charge has to be conserved at the vertices where different 5 branes come together. If we characterize a 5 brane by its charge under the NS and RR 2 forms, then a D brane has charge $(0,1)$ and an NS brane $(1,0)$. If they end on each other, a $(1,1)$ brane emerges from the vertex.

**The basic 6d brane setup.**

We now want to study what happens in 6 dimensions in some more detail [5, 10, 11]. We consider D6 branes stretching between NS branes.

![Figure 1: The brane configuration under consideration, giving rise to a 6 dimensional field theory. Horizontal lines represent D6 branes, the crosses represent NS 5 branes.](image)

The configuration is shown in figure 1. The worldvolume of the NS brane lies completely inside the worldvolume of the D6 which ends on it. We include matter by semi-infinite D6 branes extending to both sides of the NS branes and will discuss D8 branes later on. Because there are no transversal directions of the NS 5 brane left, there is no room for bending. The RR charge has to cancel exactly at each vertex. The net charge is given by the number of D6 branes ending from one side minus the number of D6 branes ending from the other side. Thus, we only get a consistent picture if:

$$N_c = n_l = n_r,$$

where $n_l$ ($n_r$) denotes the number of D6 ending from the left (right). The total number of flavor giving semi infinite D6 is therefore

$$N_f = n_l + n_r = 2N_c.$$  

**The low energy field theory.**

What is the low energy field theory interpretation of this brane setup? According to the above philosophy, we have to look for the lowest dimensional brane in the setup. In our setup, the NS branes are as light as the finite D6 brane pieces. Only
the semi infinite D6 branes are heavy. This is different from brane configurations leading to lower dimensional field theories, where the NS branes could always be considered as heavy and their motions determined parameters in the theory. The theory on a IIA NS 5 brane is the theory of a (0,2)– tensor multiplet. This multiplet consists of a tensor and 5 scalars (and fermions). Because of the presence of the D6 branes, one half of the SUSY is broken and we are left with a (0,1) theory. The tensor multiplet decomposes into a (0,1) tensor, which only contains one scalar, and a hypermultiplet, which contains 4 scalars. The hypermultiplet is projected out from the massless spectrum because the position of the semi-infinite D6 branes fixes the position of the NS branes, so that fluctuations in the transversal directions are suppressed. The scalar in the tensor multiplet corresponds to motions of the 5 branes in the $x_6$ direction. We have two NS 5 branes and therefore two tensor multiplets, but effectively we keep only one of them because one of the scalars can be taken to describe the center of mass motion of the system. The vev of the other scalar gives us the distance between the NS branes. It is therefore related to the coupling of the theory. If the two 5 branes come together we arrive at a strong coupling fixed point. This theory contains tensionless strings coming from virtual membranes stretching between the 5 branes.

Altogether, the branes describe an SU($N_c$) theory with a tensor and $N_f$ hypers. The brane analysis gives the result that for a consistent theory the number of fundamentals has to be $N_f = 2N_c$. It predicts a strong coupling fixed point with this matter content.

**Inclusion of D8 branes.**

So far, we included fundamental matter multiplets by semi-infinite D6 branes. It should also be possible to describe the matter content by higher dimensional D branes between the NS branes [6]. In our case, these are D8 branes. D8 branes are charged under a RR nine form potential. The dual field strength of it is a constant. This constant is related to the cosmological constant appearing in massive IIA supergravity. The D8 branes divide space-time into different regions with different cosmological constant. Whenever we cross a D8 brane, the cosmological constant jumps by one unit. This is important for our brane configuration because there is a term in the action of massive IIA supergravity which is proportional to the IIA mass parameter and which couples the NS two form field $B$ to the field strength of the 7 form potential under which the D6 is charged. The coupling reads:

$$-m \int d^{10}x B \wedge *(F^{(2)})$$

($F^{(2)}$ is the field strength of the dual one form potential.) This modifies the equations of motion for the 7 form potential and therefore the RR charge cancellation condition. If a D6 brane ends on an NS brane, the equations of motion for the 7 form potential, or equivalently the Bianchi identity for the dual two form field strength is

$$dF^{(2)} = d \ast F^{(8)} = \theta(x_7)\delta^{(456)} - mH,$$
where $H = dB$. The $\delta$ term is the source term coming from the D6 brane ending on the NS brane. In the presence of $m$ D8 branes the number of D6 branes ending from left and right on the NS branes should differ by $m$. In this way, a D8 placed in between the two NS branes has the same effect as a semi infinite D6 ending on the NS brane.

**The field theoretical point of view.**

We have seen that the brane construction leads via RR charge conservation to a restriction on the number of vectors and hypers in the theory. This restriction can be reproduced from a field theory point of view by an analysis of the gauge anomaly. In six dimensions the anomaly arising from vectors and hypers is

$$I = \text{tr}_{adj} F^4 - \sum_R n_R \text{tr} F^4$$

$R$ denotes the representation of the matter multiplets. In our case we only have $N_f$ fundamental matter multiplets. We can convert the trace in the adjoint to a trace in the fundamental representation and obtain:

$$I = (a - N_f) \text{tr}_f F^4 + c(\text{tr}_f F^2)^2$$

where $a$ and $c$ are group dependent factors. If the theory is consistent, the anomaly has to be cancelled. There are various options: In the simplest case, both the prefactor of the $F^4$ term and $c$ vanish. In this case the theory is certainly consistent. If only the prefactor of the $F^4$ term vanishes but $c$ does not vanish, the anomaly can be cancelled by introducing a Green Schwarz tree-level counterterm. This counterterm involves an antisymmetric tensor field. In the case $c > 0$ the anomaly can be cancelled without introducing gravity, whereas in the case $c < 0$ we can only get a consistent theory if we introduce gravity. Note that in 6d a tensor can be divided into a self dual and an anti self dual part. One piece is contained in the gravity multiplet and the other in a tensor multiplet. The tensor multiplet furthermore contains a scalar $\phi$, whereas the gravity multiplet does not contain scalars. In the remaining case that the prefactor of the $F^4$ term does not vanish, the anomaly cannot be cancelled. Our gauge group is $SU(N_c)$. Here, $a = 2N_c$ and $c = 6$, so that we are in the situation, where the anomaly can be cancelled by introducing a tensor, if $N_f = 2N_c$. This is precisely the result obtained from the brane picture. For $N_c \leq 3$, $SU(N_c)$ does not have an independent fourth order Casimir. In this case, the condition $c > 0$ only imposes an upper bound on the number of flavors. Global anomalies restrict the possible matter content further in these cases [3]. The only additional possibilities for $SU(2)$ is 10 flavors, which can be realized by introducing an orientifold and using $SU(2) \sim Sp(2)$. The additional possibilities in the $SU(3)$ case are 0 and 12 flavors and can not be seen in the brane picture.

From the brane picture we have predicted a strong coupling fixed point, when the expectation value of the scalar in the tensor multiplet vanishes. To verify this from a field theory point of view, we look at the following part of the action:

$$\frac{1}{g^2} \text{tr} F_{\mu\nu}^2 + \sqrt{c} \phi \text{tr} F_{\mu\nu}^2$$
We see, that one can absorb the bare gauge coupling into the expectation value of the tensor to get an effective coupling

\[
\frac{1}{g_{eff}^2} = \sqrt{c}\phi
\]

This is the effective coupling we see in the brane picture. At \( \phi = 0 \) there is a strong coupling fixed point [1].

**Modifications of the basic setup.**

We can introduce further building blocks into our basic brane setups to obtain other 6 dimensional fixed points. Of course, we can build a chain of \( k \) NS branes connected by D6 branes, leading to a product of SU gauge groups. This leads to a gauge group \( SU(N_c)^k \) with bifundamentals and \( 2N_c \) fundamentals. We can also introduce orientifold 6 planes parallel to the D6 branes, leading to Sp or SO gauge groups, depending on the sign of the orientifold projection. The charge of the orientifold is twice or minus twice the charge of a D6 brane. Furthermore, it changes sign when it passes the NS branes. Taking this into account, RR charge conservation leads for one group factor to the condition \( N_f = N_c - 8 \) for SO and \( N_f = N_c + 8 \) for Sp. This is in agreement with anomaly cancellation. Combining these two options we get gauge groups \( \{Sp(2N_c) \times SO(2N_c + 8)\}^{k+1/2} \) Furthermore, we can introduce O8 planes parallel to flavor giving D8 branes. If an NS 5 is stuck to the D8, we get SU gauge groups with an antisymmetric or symmetric tensor (depending on the charge of the O8).

Many such possibilities have been studied in [10, 11]. The resulting theories can also be obtained from 5 branes at ALE singularities [1].

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