Direct CP violation in $D^+ \to K^0(\bar{K}^0)\pi^+$ decays as a probe for new physics

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Abstract

In this paper we investigate CP violation in charged decays of $D$ meson. Particularly, we study the direct CP asymmetry of the Cabibbo favored non-leptonic $D^+ \to \bar{K}^0\pi^+$ and the doubly Cabibbo-suppressed decay mode $D^+ \to K^0\pi^+$ within standard model, two Higgs doublet model with generic Yukawa structure and left right symmetric models. In the standard model, we first derive the contributions from box and di-penguin diagrams contributing to their amplitudes which are relevant to the generation of the weak phases essential for non-vanishing direct CP violation. Then, we show that these phases are so tiny leading to a direct CP asymmetry of order $10^{-11}$ in both decay modes. Regarding the two Higgs doublet model with generic Yukawa structure and after taking into account all constraints on the parameter space of the model, we show that the enhanced direct CP asymmetries can be 6 and 7 orders of magnitudes larger than the standard model prediction for $D^+ \to \bar{K}^0\pi^+$ and $D^+ \to K^0\pi^+$ respectively. Finally, within left right symmetric models, we find that sizable direct CP asymmetry of $\mathcal{O}(10^{-3})$ can be obtained for the decay mode $D^+ \to \bar{K}^0\pi^+$ after respecting all relevant constraints.
I. INTRODUCTION

Heavy meson decays can serve as a probe for New Physics (NP) beyond the Standard Model (SM). Of particular interest, CP violation in heavy mesons decays can discriminate between many extensions of beyond SM physics that have new complex couplings of the new particles to quarks or leptons. These couplings provide the sources of the so called weak phases which are essential for having non vanishing CP violation. In the SM, complex couplings can arise only in the Cabibbo-Kobayashi-Maskawa (CKM) matrix describing the quark mixing [1, 2]. The couplings of the interactions of the charged quarks to $W^\pm$ gauge bosons are proportional to the CKM matrix elements. Thus, with the presence of such interactions, CP violation can be generated in the SM. However, the CP violation in the SM is too small to account for the observed baryon asymmetry which play an important role in the domination of matter in our local regions in the universe.

In the mesons sector, CP violation has been observed in the kaon and B mesons [3–6]. Regarding D mesons, the $D^0 - \bar{D}^0$ mixing was discovered in 2007 after combining the results from BABAR [7], Belle [8] and CDF [9]. Later, the mixing has been observed at LHCb [10] and at Belle [11]. Concerning direct CP violation in D meson decays, search in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ has been carried at LHCb [12, 13], Fermilab [14] and Belle [15]. Recent search at LHCb with sensitivities that have reached a level of $10^{-3}$ has shown that $A_{CP}(D^0 \rightarrow K^+K^-) = (0.04 \pm 0.12 \pm 0.10)\%$ and $A_{CP}(D^0 \rightarrow \pi^+\pi^-) = (0.07 \pm 0.14 \pm 0.11)\%$. [16]. Here, $A_{CP}$ stands for the time-integrated CP asymmetry and clearly, the results show that no sign of direct CP violation in these decay modes.

Two body non-leptonic D decays can be sorted into Cabibbo-Favored (CF), singly Cabibbo-suppressed (SCS) and Double Cabibbo Suppressed (DCS) according to the suppression factor $\lambda \simeq |V_{us}| \simeq |V_{cd}|$ appears in their amplitudes. In the SM, previous studies showed that direct CP-asymmetry of order $10^{-3}$ can be obtained for some SCS decay modes [17, 18]. For examples, the CP asymmetries of the decays $D^0 \rightarrow K_sK^*$ and $D^0 \rightarrow K_s\bar{K}^*$ have been estimated to be as large as $3 \times 10^{-3}$ [18]. With more investigations in SCS decay modes, within SM also, a large CP-asymmetry of order $10^{-2}$ has been predicted for the mode $D^0 \rightarrow K_sK_s$ [19]. Turning to the CF and DCS two body D decays, the asymmetries, within SM, are expected to be so tiny and of order $\lesssim 10^{-9}$ for $D^0 \rightarrow K^-\pi^+$ and $D^0 \rightarrow K^+\pi^-$ as estimated in our earlier studies in Refs. [20, 21]. The result motivated us to explore, also in
the same studies, NP effects in these decay processes where we have shown that in some extensions of the SM sizable CP asymmetry of order $10^{-2}$ can be obtained. In this work, we extend our studies in Refs. [20, 21] to explore direct CP violation in charged $D$ decays to CF and DCS $K\pi$ final states. In particular, we consider the CF mode $D^+ \to \bar{K}^0\pi^+$ and the DCS $D^+ \to K^0\pi^+$ decay mode. The direct CP asymmetries of $D^+ \to K^0(\bar{K}^0)\pi^+$ are expected to be different than those of $D^0 \to K^+\pi^-$ as the strong CP violating phases contributing to these processes have different origins. In our work in Ref. [20] we found that sizeable direct CP asymmetry of $D^0 \to K^-\pi^+$ can be generated in a specific new physics model namely, in no-manifest Left-Right Symmetric (LRS) model. In this study, we inspect if the model can still lead to sizeable CP asymmetries after taking into account the up to date constraints from collider physics, flavor physics, and low-energy precision measurements.

This paper is organized as follows: in section II we derive the amplitudes of the CF and DCS non-leptonic $D^+ \to K^0(\bar{K}^0)\pi^+$ decays in the framework of the SM and give our estimations of their direct CP asymmetries. Motivated by the almost null values of the asymmetries, we extend the analysis to include two possible candidates of NP models. These NP candidates are based on the presence of new charged scalars as in general two Higgs models in section III and new charged bosons as in no-manifest LRS in section IV. Finally, we conclude in sect. V.

II. DIRECT CP ASYMMETRY OF CF AND DCS NON LEPTONIC $D^+ \to K\pi^+$ DECAYS IN THE STANDARD MODEL

In general the effective Lagrangian describing CF and DCS $D^+ \to K\pi^+$ decays can be expressed as

$$\mathcal{L}_{\text{eff.}} = \frac{G_F}{\sqrt{2}} V_{cq}^* V_{uq} \left[ \sum_{i, a} c_{1ab}^i (\bar{q} \Gamma^i c_a) (\bar{u} \Gamma_i q_b) + \sum_{i, a} c_{2ab}^i (\bar{u} \Gamma^i c_a) (\bar{q} \Gamma_i q_b) \right] \quad (1)$$

Here $i = S, V$ and $T$ stands for scalar (S), vectorial (V) and tensorial (T) operators respectively. The Latin indexes $a, b = L, R$ and $q_{L, R} = (1 \mp \gamma_5)q$. In Eq. (1) we have $q \neq q'$ where $q$ and $q'$ can be $d$ or $s$ down-type quark. For CF decays $q = s$ and $q' = d$ while for DCS decays we have $q = d$ and $q' = s$.  

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In the SM the contributions from tree-level and loop-level diagrams, shown in Fig., lead to the effective Hamiltonian that can be expressed as

$$\mathcal{H}_{\text{eff.}}^{SM} = \frac{G_F}{\sqrt{2}} V_{cq}^* V_{uq} \left[ c_1 \left( \bar{q} \gamma_\mu c_L \right) \left( \bar{u} \gamma_\mu q'_L \right) + \left( c_2 \bar{u} \gamma_\mu c_L \right) \left( \bar{q} \gamma_\mu q'_L \right) \right] + \text{h.c.}$$

$$= \frac{G_F}{\sqrt{2}} V_{cq}^* V_{uq} \left( c_1 Q_1 + c_2 Q_2 \right) + \text{h.c.} \quad (2)$$

In the framework of naive factorization approximation (NFA), the amplitude of a given decay process under concern can be obtained using $\mathcal{H}_{\text{eff.}}^{SM}$ via

$$A_{D^+ \to K^+ \pi^+}^{SM} = \langle K^+ \pi^+ | \mathcal{H}_{\text{eff.}}^{SM} | D^+ \rangle \quad (3)$$

Upon evaluating the matrix elements of the operators in Eq.(2), we obtain the amplitude of the CF decay mode $D^+ \to \bar{K}^0 \pi^+$ and DCS decay mode $D^+ \to K^0 \pi^+$ as

$$A_{D^+ \to \bar{K}^0 \pi^+}^{SM} = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[ (a_1 + \Delta a_1^{sd}) X_{D^+ \to \bar{K}^0}^{\pi^+} + (a_2 + \Delta a_2^{sd} + \Delta a_2^{sd \bar{K}^0}) X_{D^+ \to \bar{K}^0}^{\pi^+} \right],$$

$$A_{D^+ \to K^0 \pi^+}^{SM} = i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{us} \left[ (a_1 + \Delta a_1^{ds}) X_{D^+ \to \bar{K}^0}^{\pi^+} + (a_2 + \Delta a_2^{ds} - \Delta a_2^{ds \bar{K}^0}) X_{D^+ \to \bar{K}^0}^{\pi^+} \right] \quad (4)$$

with $X_{P_2 P_3}^{P_1}$ is given by

$$X_{P_2 P_3}^{P_1} = i f_{P_1} \Delta_{P_2 P_3}^2 F_0^{P_2 P_3}(m_{P_1}^2), \quad \Delta_{P_2 P_3}^2 = m_{P_2}^2 - m_{P_3}^2 \quad (5)$$

here $f_P$ is the $P$ meson decay constant and $F_0^{P_2 P_3}$ is the form factor. In Eq.(4) the coefficients $a_1 = c_1 + c_2 / N_C$ and $a_2 = -(c_2 + c_1 / N_C)$, where $N_C$ is the color number, account for the tree-level contributions to the amplitudes. These contributions originate from integrating out the $W^\pm$ boson mediating the tree-level diagrams. On the other hand, and in the same equations, $\Delta a_{1,2}^{sd,ds}$ and $\Delta a_{2}^{sd \bar{K}^0,ds \bar{K}^0}$ account for the contributions to the amplitudes originating from integrating out the $W^\pm$ boson mediating the box and di-penguin diagrams in Fig.1. Their expressions can be obtained from the following expressions upon setting $q = s$ and $q' = d$ for CF decay mode $D^+ \to \bar{K}^0 \pi^+$ and $q = d$ and $q' = s$ for DCS decay mode $D^+ \to K^0 \pi^+$.
FIG. 1. Feynman diagrams for DCS processes: left (right) di-penguins contribution (box) contribution. For CF processes we make the replacements \( d \leftrightarrow s \) in each diagram.

\[
\Delta a_{qq'} \simeq -\frac{G_F m_W^2}{\sqrt{2} \pi^2 V_{cq}^* V_{uq'}} B_{x}^{qq'} - \frac{G_F \alpha_S}{4 \sqrt{2} \pi^3 V_{cq}^* V_{uq'}} \left[ \frac{\kappa}{2} \left( 1 - \frac{1}{N^2} \right) \right] P_g^{qq'}
\]

\[
\Delta a_{qD}^{qq'} \simeq -\frac{G_F m_W^2}{\sqrt{2} \pi^2 V_{cq}^* V_{uq'}} B_{x}^{qq'}
\]

\[
\Delta a_{qK}^{qq'} \simeq -\frac{G_F \alpha_S}{4 \sqrt{2} \pi^3 V_{cq}^* V_{uq'}} \frac{3 m_q m_c}{8 N} \chi^{K_0} P_g^{qq'}
\]

(6)

where \( \kappa = \frac{(m_D^2 + m_K^2)}{2} + 3 m_{\pi}^2/4 \) and \( \chi^{K_0} = \frac{m_{K_0}^2}{[(m_c - m_u)(m_d + m_s)]} \simeq 2. \) The quantities \( B_{x}^{qq'} \) and \( P_g^{qq'} \) originate from the box and di-penguin diagrams respectively and they are given as

\[
B_{x}^{qq'} = V_{cD}^* V_{uD} V_{uq'}^* V_{Uq'} f(x_U, x_D)
\]

\[
P_g^{qq'} = [V_{cD}^* V_{uD} E_0(x_D)] [V_{Uq}^* V_{Uq'} E_0(x_U)]
\]

(7)

with \( U = u, c, t \) and \( D = d, s, b, x_q = (m_q/m_W)^2 \) and \( f_{UD} \equiv f(x_U, x_D) \) where \[22\]

\[
f(x, y) = \frac{7 x y - 4}{4(1-x)(1-y)} + \frac{1}{x-y} \left[ \frac{y^2 \log y}{(1-y)^2} \left( 1 - 2x + \frac{xy}{4} \right) - \frac{x^2 \log x}{(1-x)^2} \left( 1 - 2y + \frac{xy}{4} \right) \right]
\]

and the Inami function \( E_0(x) \) is given as

\[
E_0(x) = \frac{1}{12(1-x)^4} \left[ x(1-x)(18 - 11x - x^2) - 2(4 - 16x + 9x^2) \log(x) \right]
\]

(8)

In NFA, there is no source for the strong CP conserving phases required for having non vanishing direct CP asymmetries. Consequently this factorization approximation is irrelevant to the study of CP violation. On the other hand the mass of the charm quark is not heavy
enough to allow for a sensible heavy quark expansion, such as in QCD factorization and soft
collinear effective theory, and it is not light enough for the application of chiral perturbation
theory. A possible approach to study charm decays in a model-independent way is
the so called the diagrammatic approach. Within this approach, the amplitude is
decomposed into parts corresponding to generic quark diagrams according to the topologies
of weak interactions. For each one of these topological diagrams, the related magnitude and
relative strong phase can be extracted from the data without making further assumptions,
apart from flavor SU(3) symmetry.

In the diagrammatic approach the amplitudes of the the CF decay mode \( D^+ \to \bar{K}^0 \pi^+ \)
and DCS decay mode \( D^+ \to K^0 \pi^+ \) can be written as

\[
A_{D^+ \to \bar{K}^0 \pi^+} = V_{cs}^* V_{ud} (T + C)
\]

\[
A_{D^+ \to K^0 \pi^+} = V_{cs}^* V_{ud} (C'' + A'')
\]

where \( T \) represents the tree level color-allowed external W-emission quark diagram, \( C \) and
\( C'' \) denote the color-suppressed internal W-emission diagram and \( A'' \) is the W-annihilation
diagram. Comparing Eq.(9) and Eq.(11) we find that

\[
T = \frac{G_F}{\sqrt{2}} (a_1 + \Delta a_{1d}^s) f_\pi (m_D^2 - m_K^2) F_0^{DK}(m_\pi^2)
\]

\[
C = \frac{G_F}{\sqrt{2}} (a_2 + \Delta a_{2d}^s + \Delta a_{2dK}^s) f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2)
\]

\[
C'' = \frac{G_F}{\sqrt{2}} (a_1 + \Delta a_{1d}^s) f_D (m_K^2 - m_\pi^2) F_0^{K\pi}(m_D^2)
\]

\[
E'' = \frac{G_F}{\sqrt{2}} (a_2 + \Delta a_{2d}^s - \Delta a_{2dK}^s) f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2)
\]

The direct CP asymmetry of the CF decay mode \( D^+ \to \bar{K}^0 \pi^+ \) can be expressed as

\[
A^{SM}_{CP}(D^+ \to \bar{K}^0 \pi^+) = \frac{|A^{SM}_{D^+ \to \bar{K}^0 \pi^+}|^2 - |\bar{A}^{SM}_{D^+ \to K^0 \pi^+}|^2}{|A^{SM}_{D^+ \to \bar{K}^0 \pi^+}|^2 + |\bar{A}^{SM}_{D^+ \to K^0 \pi^+}|^2} = \kappa \sin(\phi_2 - \phi_1)
\]

with

\[
\kappa = \frac{2r \sin(\alpha)}{|1 + r|^2}
\]

here \( r = |C/T| \) and \( \alpha = \alpha_C - \alpha_T \). The phases \( \alpha_C \) and \( \alpha_T \) are the strong phase of the
amplitudes $C$ and $T$ respectively. The weak phases $\phi_1$ and $\phi_2$ are defined through

$$\phi_1 = \tan^{-1}\left(\frac{|\Delta a_{1d}^s| \sin \Delta \phi_1}{a_1 + |\Delta a_{1d}^s| \cos \Delta \phi_1}\right)$$

$$\phi_2 = \tan^{-1}\left(\frac{|\Delta a_{2d}^s + \Delta a_{2d}^{sK^0}| \sin \Delta \phi_2}{a_2 + |\Delta a_{2d}^s + \Delta a_{2d}^{sK^0}| \cos \Delta \phi_2}\right)$$

(14)

where the phases $\Delta \phi_1$ and $\Delta \phi_2$ are the phase of $\Delta a_{1d}^s$ and $\Delta a_{2d}^s + \Delta a_{2d}^{sK^0}$ respectively.

Regarding the DCS decay mode $D^+ \rightarrow K^0 \pi^+$, the direct CP asymmetry can be expressed as

$$A_{CP}^{SM}(D^+ \rightarrow K^0 \pi^+) = \frac{|A_{D^+ \rightarrow K^0 \pi^+}^{SM}|^2 - |\bar{A}_{D^+ \rightarrow K^0 \pi^+}^{SM}|^2}{|A_{D^+ \rightarrow K^0 \pi^+}^{SM}|^2 + |\bar{A}_{D^+ \rightarrow K^0 \pi^+}^{SM}|^2} = \kappa' \sin(\phi_2' - \phi_1')$$

(15)

where

$$\kappa' = \frac{2r' \sin(\alpha')}{|1 + r'|^2} \quad (16)$$

with $r' = |A''/C''|$ and $\alpha' = \alpha_{A''} - \alpha_{C''}$. The phases $\alpha_{A''}$ and $\alpha_{C''}$ are the strong phase of the amplitudes $A''$ and $C''$ respectively. The weak phases $\phi_1'$ and $\phi_2'$ are defined through

$$\phi_1' = \tan^{-1}\left(\frac{|\Delta a_{1d}^s| \sin \Delta \phi_1'}{a_1 + |\Delta a_{1d}^s| \cos \Delta \phi_1'}\right)$$

$$\phi_2' = \tan^{-1}\left(\frac{|\Delta a_{2d}^s - \Delta a_{2d}^{sK^0}| \sin \Delta \phi_2'}{a_2 + |\Delta a_{2d}^s - \Delta a_{2d}^{sK^0}| \cos \Delta \phi_2'}\right)$$

(17)

here the phases $\Delta \phi_1'$ and $\Delta \phi_2'$ are the phase of $\Delta a_{1d}^s$ and $\Delta a_{2d}^s + \Delta a_{2d}^{sK^0}$ respectively.

Using he fitted values $T = (3.14 \pm 0.06) \times 10^{-6}$ GeV, $C = C'' = (2.61 \pm 0.08) \times 10^{-6}$ GeV, $e^{-i(152 \pm 1)^0}$ and $A'' = (0.39^{+0.13}_{-0.09}) \times 10^{-6} e^{i(31 \pm 3)^0}$ we find that $\kappa \simeq -0.23$ and $\kappa' \simeq -0.01$. On the other hand using $a_1 = 1.2 \pm 0.1$ and $a_2 = -0.5 \pm 0.1$ and the expressions in Eqs. (14, 17) we find that $\sin(\phi_2' - \phi_1') \simeq -2.0 \times 10^{-10}$ and $\sin(\phi_2' - \phi_1') \simeq 4.8 \times 10^{-9}$. Thus, from Eqs. (12, 13) we find that $A_{CP}^{SM}(D^+ \rightarrow \bar{K}^0 \pi^+) \simeq 4.6 \times 10^{-11}$ and $A_{CP}^{SM}(D^+ \rightarrow K^0 \pi^+) \simeq -5.7 \times 10^{-11}$. Clearly the predicted direct CP asymmetries within SM are so tiny in both decay modes due to the highly suppressed generated weak phases originating leaving a room for a possible enhancement from new physics beyond SM that have new weak phases.

III. MODELS WITH CHARGED HIGGS CONTRIBUTIONS

Possible extensions of the SM include the two Higgs doublet models (2HDM) [29, 30]. Based on their couplings to quarks and leptons, these models can be classified to several
types such as: type I, II or III (for a review see ref. [31]). Among these types 2HDM type III (2HDM III) is of a particular interest to our study. This can be attributed to the presence of complex couplings of Higgs to quarks. These couplings are relevant for generating the desired CP violating weak phases. In the literature, 2HDM III has gain interest as it can explain $B \to D\tau\nu$, $B \to D^*\tau\nu$ and $B \to \tau\nu$ simultaneously while other types such as 2HDM I and 2HDM II cannot [32].

In 2HDM III the physical mass eigenstates are $H_0$ (heavy CP-even Higgs), $h_0$ (light CP-even Higgs) and $A_0$ (CP-odd Higgs) and $H^\pm$. In this model both Higgs doublets can couple to up-type and down-type quarks. As a consequence the couplings of the neutral Higgs mass eigenstates can induce flavor violation in Neutral Currents at tree-level. In the down sector these flavor violating couplings are stringently constrained from flavor changing neutral current processes [32, 33]. Thus in the following we consider only charged Higgs couplings to quarks that can be expressed as [32, 34]:

$$L_{H^\pm}^{\text{eff}} = \bar{u}_f \Gamma_{u_f d_i}^{H^\pm LR \text{ eff}} P_R d_i + \bar{u}_f \Gamma_{u_f d_i}^{H^\pm RL \text{ eff}} P_L d_i,$$

where

$$\Gamma_{u_f d_i}^{H^\pm LR \text{ eff}} = \sum_{j=1}^{3} \sin \beta V_{fj} \left( \frac{m_{d_i}}{v_d} \delta_{ji} - \epsilon_{ji}^d \tan \beta \right),$$

$$\Gamma_{u_f d_i}^{H^\pm RL \text{ eff}} = \sum_{j=1}^{3} \cos \beta \left( \frac{m_{u_f}}{v_u} \delta_{jji} - \epsilon_{jji}^u \tan \beta \right) V_{ji}$$

Here $v_u$ and $v_d$ denote the vacuum expectations values of the neutral component of the Higgs doublets, $\tan \beta = v_u/v_d$ and $V$ is the CKM matrix. Applying the Feynman-rules given in Eq. (18) allows us to calculate the contributions to the total amplitude originating from tree-level Charged Higgs mediation.

The contribution of charged Higgs to the effective Hamiltonian can be written as

$$Q_1^{H^\pm} = (\bar{q} P_R c)(\bar{u} P_L q'),$$

$$Q_2^{H^\pm} = (\bar{q} P_L c)(\bar{u} P_R q'),$$

$$Q_3^{H^\pm} = (\bar{q} P_R c)(\bar{u} P_L q'),$$

$$Q_4^{H^\pm} = (\bar{q} P_R c)(\bar{u} P_R q').$$

(20)
where as before for CF decays $q = s$ and $q' = d$ while for DCS decays we have $q = d$ and $q' = s$. The Wilson coefficients $C_i^H$, at the electroweak scale, can be expressed as

\[
C_i^{H^\pm} = \frac{\sqrt{2}}{G_F V_{cq}^* V_{uq}^* m_H^2} \left( \sum_{j=1}^{3} \cos \beta V_{jq} \left( \frac{m_u}{v_u} \delta_{j1} - \epsilon_{j1}^u \tan \beta \right) \right) \left( \sum_{k=1}^{3} \cos \beta V_{kj}^* \left( \frac{m_c}{v_u} \delta_{k2} - \epsilon_{k2}^u \tan \beta \right) \right),
\]

\[
C_2^{H^\pm} = \frac{\sqrt{2}}{G_F V_{cq}^* V_{uq}^* m_H^2} \left( \sum_{j=1}^{3} \sin \beta V_{1j} \left( \frac{m_{q'}}{v_d} \delta_{jq'} - \epsilon_{jq'}^d \tan \beta \right) \right) \left( \sum_{k=1}^{3} \sin \beta V_{2k}^* \left( \frac{m_q}{v_d} \delta_{kq} - \epsilon_{kq}^d \tan \beta \right) \right),
\]

\[
C_3^{H^\pm} = \frac{\sqrt{2}}{G_F V_{cq}^* V_{uq}^* m_H^2} \left( \sum_{j=1}^{3} \cos \beta V_{jq} \left( \frac{m_u}{v_u} \delta_{j1} - \epsilon_{j1}^u \tan \beta \right) \right) \left( \sum_{k=1}^{3} \sin \beta V_{2k}^* \left( \frac{m_q}{v_d} \delta_{kq} - \epsilon_{kq}^d \tan \beta \right) \right),
\]

\[
C_4^{H^\pm} = \frac{\sqrt{2}}{G_F V_{cq}^* V_{uq}^* m_H^2} \left( \sum_{k=1}^{3} \cos \beta V_{kq} \left( \frac{m_c}{v_u} \delta_{k2} - \epsilon_{k2}^u \tan \beta \right) \right) \left( \sum_{j=1}^{3} \sin \beta V_{1j} \left( \frac{m_{q'}}{v_d} \delta_{jq'} - \epsilon_{jq'}^d \tan \beta \right) \right).
\]

(21)

In order to evaluate the contributions of the charged Higgs to the amplitudes of the decay modes under consideration we need to discuss the restraints imposed on the flavor-changing parameters $\epsilon_{ij}^{u,d}$ appear in the expressions of $C_i^{H^\pm}$ above. We consider first the down sector and discuss the possible constraints that can be imposed on $\epsilon_{ij}^d$. For the case $i \neq j$, stringent bounds on $\epsilon_{ij}^d$ from considering flavor changing neutral current (FCNC) processes due to the tree level neutral Higgs exchange \[32, 33\]. As a result, they cannot contribute significantly to the decay modes under investigation. Thus we are left with $\epsilon_{11}^d$, $\epsilon_{22}^d$ and $\epsilon_{33}^d$. The couplings $\epsilon_{11}^d$ and $\epsilon_{22}^d$ can be severely constrained by applying the naturalness criterion of ’t Hooft to the quark masses. In view of the criterion, the smallness of a quantity is only natural if a symmetry is gained in the limit in which this quantity is zero \[32\]. Consequently, it is unnatural to have large accidental cancellations without a symmetry forcing these cancellations. Applying the naturalness criterion to the quark masses leads to the bounds given as \[33\]

\[
|v_u(d)\epsilon_{ij}^{d(u)}| \leq |V_{ij}^{CKM}| \max \left[ m_{d_i(u_i)}, m_{d_j(u_j)} \right] \quad \text{for } i < j
\]

\[
|v_u(d)\epsilon_{ij}^{d(u)}| \leq \max \left[ m_{d_i(u_i)}, m_{d_j(u_j)} \right] \quad \text{for } i \geq j.
\]

(22)

Clearly, due to the smallness of the $d$ and $s$ quark masses, the constraints on $\epsilon_{11}^d$ and $\epsilon_{22}^d$ are so strong. Thus we are left with $\epsilon_{33}^d$ which is irrelevant to the decay modes we are
interested in. Putting all together, we can safely neglect terms proportional to the couplings \( \epsilon_{ij} \) in \( C^H_{i} \).

We turn now to discuss the constraints that can be set on the couplings \( \epsilon_{ij}^u \). Again, applying the naturalness criterion of ’t Hooft to the \( u \) quark mass we find that, using second line of Eq.(22), the constraint on \( \epsilon_{11}^u \) is so severe. As a result we can drop terms proportional to \( \epsilon_{11}^u \) in \( C^H_i \). Thus, to a good approximation, we can finally write

\[
\begin{align*}
C_{1}^{H^\pm} &\simeq - \frac{\sin 2\beta V_{3q}^* \epsilon_{31}^u}{\sqrt{2} G_F V^*_{cq} V_{wq} m_{H}^2} \left( \frac{m_c}{v_u} V_{2q}^* - \epsilon_{22}^u \tan \beta V_{2q}^* \epsilon_{32}^u \tan \beta V_{3q}^* \right), \\
C_{2}^{H^\pm} &\simeq C_{3}^{H^\pm} \simeq 0 \quad (23)
\end{align*}
\]

where we have neglected the terms that are proportional to \( \epsilon_{12}^u \epsilon_{21}^u \) due to the strong constraint \( |\epsilon_{12}^u \epsilon_{21}^u| < 2 \times 10^{-8} \) from \( D - \bar{D} \) mixing \([33]\). Moreover, the bound also implies that \( |\epsilon_{12,21}| < \sqrt{2} \times 10^{-4} \) in the absence of a symmetry that protect one of these parameters from being much smaller than the other one. As a consequence, we neglected terms proportional to \( \epsilon_{12}^u \) in the above Wilson coefficients. We also neglected terms suppressed by the up quark mass.

We proceed now to calculate the amplitudes of the decay processes of interest. For CF decay modes \( D^+ \rightarrow \bar{K}^0 \pi^+ \), the total amplitude, including Higgs contribution, can be written as

\[
A_{D^+ \rightarrow \bar{K}^0 \pi^+}^{SM+H^\pm} \simeq -i G_F V^*_{cs} V_{ud} \left[ (a_1 + \Delta a_1^H) X_{D^+ \bar{K}^0}^{\pi^+} + (a_2 + \Delta a_2^H \bar{K}^0) X_{D^+ \pi^+}^{\bar{K}^0} \right],
\]

with

\[
\Delta a_1^H = \chi_{1}^{\pi^+} (C_1^H - C_4^H), \quad \Delta a_2^H \bar{K}^0 = \frac{1}{2N} (C_1^H - \chi_{1}^{\bar{K}^0} C_4^H) \quad (25)
\]

The quantities \( C_{1,4}^H \) can be obtained from \( C_{1,4}^H \) by setting \( q = s \) and \( q' = d \) and

\[
\chi_{1}^{\pi^+} = \frac{m_s^2}{(m_c - m_s)(m_u + m_d)} \quad (26)
\]

In the case of DCS decay modes \( D^+ \rightarrow K^0 \pi^+ \), the total amplitude can be expressed as

\[
A_{D^+ \rightarrow K^0 \pi^+}^{SM+H^\pm} = i G_F V^*_{cd} V_{us} \left[ (a_1 + \Delta a_1^H D^+) X_{K^0 \pi^+}^{D^+} + (a_2 + \Delta a_2^H K^0) X_{D^+ \pi^+}^{K^0} \right],
\]

(27)
where

\[ \Delta a_{1}^{H D^+} = \chi^{D^+} (C_{1}^{H} + C_{4}^{H}), \quad \Delta a_{2}^{H K^0} = \frac{1}{2N} (C_{1}^{H} - \chi^{K^0} C_{4}^{H}) \] (28)

The quantities \( C_{1,4}^{H} \) can be obtained from \( C_{1,4}^{H} \) by setting \( q = d \) and \( q' = s \) and

\[ \chi^{D^+} = \frac{m_{D^+}^2}{(m_c + m_d)(m_u - m_s)}. \] (29)

In a recent study a lower bound on the charged Higgs mass in 2HDM of Type II has been set after taking into account all relevant results from direct charged and neutral Higgs boson searches at LEP and the LHC, as well as the most recent constraints from flavour physics \cite{35}. The bound reads \( m_{H^\pm} \gtrsim 600 \text{ GeV} \) independent of \( \tan \beta \). This bound should be also respected in 2HDM III \cite{32}.

For \( \tan \beta = 50 \), \( m_H = 600 \text{ GeV} \) and keeping only dominant terms, after considering constraints imposed on the \( \epsilon_{ij}^u \) studied in details in Ref.\cite{33}, we find that

\[ \Delta a_{1}^{H \pm} \simeq 0.001 \epsilon_{22}^{u} \]
\[ \Delta a_{2}^{H K^0} \simeq 0.0001 \epsilon_{22}^{u} \]
\[ \Delta a_{1}^{H D^+} \simeq 0.278 \epsilon_{22}^{u} \]
\[ \Delta a_{2}^{H K^0} \simeq 0.003 \epsilon_{22}^{u} \] (30)

We proceed now to discuss the constraints imposed on the coupling \( \epsilon_{22}^{u} \). The processes \( D_{(s)} \to \tau \nu, D_{(s)} \to \mu \nu \) can constraint the real part of \( \epsilon_{22}^{u} \) while the constraints on the imaginary part of \( \epsilon_{22}^{u} \) are weak \cite{33}. Regarding the imaginary part of \( \epsilon_{22}^{u} \) which is relevant for generating direct CP violation, and for \( m_{H^\pm} = 600 \text{ GeV} \), \( \tan \beta = 50 \), the constraints from the electric dipole moment of the neutron reads \(-0.16 \lesssim \text{Im}(\epsilon_{22}^{u}) \lesssim 0.16 \) \cite{33}. Other processes such as \( D - \bar{D} \) mixing and \( K - \bar{K} \) mixing can be used to set bounds on \( \epsilon_{22}^{u} \). However these bounds are weaker than the bounds obtained from \( D_{(s)} \to \tau \nu, D_{(s)} \to \mu \nu \) and the electric dipole moment of the neutron \cite{20,33}.

The real parts of \( \Delta a_{1}^{H} \) and \( \Delta a_{2}^{H} \) are expected to be much smaller than the SM contributions, \( a_1 \) and \( a_2 \), and hence we can be safely neglect them and keep only the imaginary parts required for generating the weak phases.

The direct CP asymmetry of the CF decay mode \( D^+ \to \bar{K}^0 \pi^+ \), including Higgs contributions, can be expressed as
\[ A_{CP}^{SM+H}(D^+ \rightarrow \bar{K}^0\pi^+) = \frac{|A_{D^+\rightarrow \bar{K}^0\pi^+}^{SM+H}|^2 - |\bar{A}_{D^+\rightarrow \bar{K}^0\pi^+}^{SM+H}|^2}{|A_{D^+\rightarrow \bar{K}^0\pi^+}^{SM+H}|^2 + |\bar{A}_{D^+\rightarrow \bar{K}^0\pi^+}^{SM+H}|^2} = \kappa \sin(\phi_2^H - \phi_1^H) \]  

(31)

where \( \kappa \) is given as before and the weak phases \( \phi_1^H \) and \( \phi_2^H \) are defined through

\[
\phi_1^H = \tan^{-1} \left( \frac{|\Delta a_1^H| \sin \Delta \phi_1^H}{a_1} \right)
\]

\[
\phi_2^H = \tan^{-1} \left( \frac{|\Delta a_2^H| \sin \Delta \phi_2^H}{a_2} \right)
\]

(32)

where \( \Delta \phi_1^H \) and \( \Delta \phi_2^H \) are the phases of \( \Delta a_1^H \) and \( \Delta a_2^H \) respectively. Regarding the DCS decay mode \( D^+ \rightarrow K^0\pi^+ \), the direct CP asymmetry can be expressed as

\[ A_{CP}^{SM+H}(D^+ \rightarrow K^0\pi^+) = \frac{|A_{D^+\rightarrow K^0\pi^+}^{SM+H}|^2 - |\bar{A}_{D^+\rightarrow K^0\pi^+}^{SM+H}|^2}{|A_{D^+\rightarrow K^0\pi^+}^{SM+H}|^2 + |\bar{A}_{D^+\rightarrow K^0\pi^+}^{SM+H}|^2} = \kappa' \sin(\phi_2^H - \phi_1^H) \]  

(33)

where \( \kappa' \) is given as before and the weak phases \( \phi_1'^H \) and \( \phi_2'^H \) are defined through

\[
\phi_1'^H = \tan^{-1} \left( \frac{|\Delta a_1'^H| \sin \Delta \phi_1'^H}{a_1} \right)
\]

\[
\phi_2'^H = \tan^{-1} \left( \frac{|\Delta a_2'^H| \sin \Delta \phi_2'^H}{a_2} \right)
\]

(34)

where \( \Delta \phi_1'^H \) and \( \Delta \phi_2'^H \) are the phases of \( \Delta a_1'^H \) and \( \Delta a_2'^H \) respectively. Assuming maximum value of \( \text{Im}(c_{22}') \), we obtain CP asymmetry \( A_{CP}^{SM+H}(D^+ \rightarrow \bar{K}^0\pi^+) \approx 3.8 \times 10^{-5} \) and \( A_{CP}^{SM+H}(D^+ \rightarrow K^0\pi^+) \approx 4.5 \times 10^{-4} \). This result show that charged Higgs contributions to the amplitudes of these decay modes can enhance the direct CP asymmetry 6 and 7 orders of magnitudes for the CF decay mode \( D^+ \rightarrow \bar{K}^0\pi^+ \) and the DCS decay mode \( D^+ \rightarrow K^0\pi^+ \) respectively.

**IV. A NEW CHARGED GAUGE BOSON AS LEFT RIGHT MODELS**

In this section, we consider a new physics model based on the gauge group \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) [36–45]. Assuming no mixing between \( W_L \) and \( W_R \) gauge bosons, the contributions from new diagrams, similar to the SM tree-level diagrams with \( W_L \) is replaced by a \( W_R \), to the effective Hamiltonian governs \( D \rightarrow K\pi \) decays can be expressed as:

\[
\mathcal{H}_{LR} = \frac{G_F}{\sqrt{2}} \left( \frac{g_{R\mu W}}{g_{L\mu W_R}} \right)^2 V_{Rq}^* V_{Rq'} \left[ c_1' (\bar{q} \gamma_\mu c_R) (\bar{u} \gamma_\mu q_R') + c_2' (\bar{u} \gamma_\mu c_R q_R') \right] + \text{h.c.} \]  

(35)
here $g_L$ and $g_R$ denote the gauge $SU(2)_L$ and $SU(2)_R$ couplings respectively and $q$ and $q'$ can be different light down-type quarks. The masses $m_W$ and $m_{W_R}$ represent the $SU(2)_L$ and $SU(2)_R$ charged gauge boson masses respectively and $V_R$ is the quark mixing matrix in the right sector in analogy to the CKM quark mixing matrix, $V_{CKM} \equiv V$, in the left sector of the charged quark currents. ATLAS and CMS have set stringent limits on $m_{W_R}$, in the $3.5 - 4.4$ TeV region based on their latest analyses with 37 fb$^{-1}$ and 35.9 fb$^{-1}$ luminosities, respectively, at $\sqrt{s} = 13$ TeV [40–51]. These analyses rely on the assumptions that the model is manifestly left-right symmetric i.e. $g_L = g_R$ and that $V_R$ is either diagonal, or $V_R = V_R$. Clearly, due to the stringent limits on $m_{W_R}$ and the assumptions of $V_R$, one expects that no sizeable CP asymmetry can be obtained in this class of left right symmetric models for both CF and DCS decay modes of $D \to K\pi$ decays.

Previous studies showed that sizable CP asymmetries can be obtained in the Charm and muon sectors in a general left right symmetric model [20, 52, 53]. In this model, the mixing between the left and the right gauge bosons is allowed and the left-right symmetry is not manifest at unification scale. In order to estimate the CP asymmetries of the CF and DCS decay modes of $D \to K\pi$ decays, in the framework of this model, we start by parameterizing the charged current mixing matrix as [43, 53, 54]

$$
(W_L^\pm) = \begin{pmatrix} \cos \xi & -\sin \xi \\ e^{i\omega} \sin \xi & e^{i\omega} \cos \xi \end{pmatrix} \begin{pmatrix} W_1^\pm \\ W_2^\pm \end{pmatrix} \simeq \begin{pmatrix} 1 & -\xi \\ e^{i\omega} \xi & e^{i\omega} \end{pmatrix} \begin{pmatrix} W_1^\pm \\ W_2^\pm \end{pmatrix}
$$

(36)

where $\xi$ is a mixing angle, $W_1^\pm$ and $W_2^\pm$ are the mass eigenstates and $\omega$ is a CP violating phase. Hence, the charged currents interaction in the quark sector can be expressed as

$$
\mathcal{L} \simeq -\frac{1}{\sqrt{2}} \bar{Q} \gamma_\mu (g_L V P_L + g_R \xi \bar{V} \bar{P} R) D W^\dagger_1 - \frac{1}{\sqrt{2}} \bar{Q} \gamma_\mu (-g_L \xi \bar{V} P_L + g_R \bar{V} R P_R) D W^\dagger_2
$$

(37)

where $\bar{V} R = e^{i\omega} V R$. Upon integrating out $W_1$ in the usual way and neglecting $W_2$ contributions, given its mass is much higher, we obtain the effective Hamiltonian relevant to the CF and DCS $D \to K\pi$ decays as:

$$
\mathcal{H}_{\text{eff.}}^{q \bar{q}'} = \frac{4G_F}{\sqrt{2}} \left\{ c_1 \left[ \bar{q} \gamma_\mu (V^* P_L + \frac{g_R}{g_L} \bar{V} R P_R)_{c q} \right] \left[ \bar{u} \gamma^\mu (V P_L + \frac{g_R}{g_L} \bar{V} P_R)_{u q'} \right] \right\} + c_2 \left[ \bar{q}_\alpha \gamma_\mu (V^* P_L + \frac{g_R}{g_L} \bar{V} R P_R)_{c q} \right] \left[ \bar{u}_\beta \gamma^\mu (V P_L + \frac{g_R}{g_L} \bar{V} P_R)_{u q'} \right] + \text{h. c.}
$$

(38)
where \( \alpha, \beta \) are color indices and \( q, q' \) are different light down-type quarks. The terms of the effective Hamiltonian proportional to \( \xi \) are:

\[
\Delta H_{\text{eff}}^{q'q} \simeq \frac{G_F g_R}{\sqrt{2} g_L} \xi \left[ c_1 \bar{q}_\alpha \gamma_\mu V^*_c q \sigma_{\mu
u} \bar{q}_R \gamma_\nu + c_1 \bar{q}_\alpha \gamma_\mu \tilde{V}_c \sigma_{\mu
u} \bar{q}_R + c_2 \bar{q}_\alpha \gamma_\mu V^*_c q \sigma_{\mu
u} \bar{q}_R \gamma_\nu \right] + \text{h. c.} \quad (39)
\]

Upon evaluating the matrix elements of the operators in Eq. (39), we obtain the new contribution to the amplitude of the CF decay mode \( D^+ \to \bar{K}^0 \pi^+ \) by setting \( q = s \) and \( q' = d \)

\[
A_{D^+ \to \bar{K}^0 \pi^+}^{LR} = -\frac{i G_F g_R}{\sqrt{2} g_L} \xi \left[ c_1 V^*_c V_R \left( X_{D^+ \to \bar{K}^0}^{\pi^+} - \frac{2}{N} \lambda^{\pi^+} X_{D^+ \to \bar{K}^0}^{\pi^+} \right) + c_1 \bar{V}_c V_{ud} \left( X_{D^+ \to \bar{K}^0}^{\pi^+} - \frac{2}{N} \lambda^{\pi^+} X_{D^+ \to \bar{K}^0}^{\pi^+} \right) \right]
\]

\[
= \frac{i G_F g_R}{\sqrt{2} g_L} \xi \left( V^*_c V_{ud} - \bar{V}_c V_R \right) \left( a_1 X_{D^+ \to \bar{K}^0}^{\pi^+} - 2 \lambda^{\pi^+} a_2 X_{D^+ \to \bar{K}^0}^{\pi^+} \right) \quad (40)
\]

and thus, the total amplitude, including SM contribution, can be written as

\[
A^{SM+LR}_{D^+ \to \bar{K}^0 \pi^+} \simeq -\frac{i G_F g_R}{\sqrt{2} g_L} V^*_c V_{ud} \left[ (a_1 + \Delta a_1^{LR}) X_{D^+ \to \bar{K}^0}^{\pi^+} + (a_2 + \Delta a_2^{LR \bar{K}^0}) X_{D^+ \to \bar{K}^0}^{\pi^+} \right], \quad (41)
\]

with

\[
\Delta a_1^{LR} \simeq -\frac{g_R}{g_L} \xi \left( V^*_R - \bar{V}_c \right) a_1, \quad \Delta a_2^{LR \bar{K}^0} \simeq \frac{2 g_R}{g_L} \xi \left( V^*_R - \bar{V}_c \right) \lambda^{\bar{K}^0} a_2 \quad (42)
\]

The direct CP asymmetry of the CF decay mode \( D^+ \to \bar{K}^0 \pi^+ \), including the new contributions, can be expressed as

\[
A_{CP}^{SM+LR}(D^+ \to \bar{K}^0 \pi^+) = \frac{\left| A^{SM+LR}_{D^+ \to \bar{K}^0 \pi^+} \right|^2 - \left| A^{SM+LR}_{D^+ \to \bar{K}^0 \pi^+} \right|^2}{\left| A^{SM+LR}_{D^+ \to \bar{K}^0 \pi^+} \right|^2 + \left| A^{SM+LR}_{D^+ \to \bar{K}^0 \pi^+} \right|^2} = \kappa \sin(\phi_2^{LR} - \phi_1^{LR}) \quad (43)
\]

where \( \kappa \) is given as before and the weak phases \( \phi_1^{LR} \) and \( \phi_2^{LR} \) are defined through

\[
\phi_1^{LR} = \tan^{-1} \left( \frac{\Delta a_1^{LR}}{a_1} \right), \quad \phi_2^{LR} = \tan^{-1} \left( \frac{\Delta a_2^{LR \bar{K}^0}}{a_2} \right) \quad (44)
\]

where \( \Delta \phi_1^{LR} \) and \( \Delta \phi_2^{LR} \) are the phases of \( \Delta a_1^{LR} \) and \( \Delta a_2^{LR \bar{K}^0} \) respectively. We turn now to the DCS decay mode \( D^+ \to K^0 \pi^+ \). proceeding in a similar way as before, upon evaluating the matrix elements of the operators in Eq. (39) and setting \( q = d \) and \( q' = s \) we find that the new contribution to the amplitude can be given as
Thus, the total amplitude after including SM contribution can be expressed as

\[ A_{D^+ \rightarrow K^{0\pi^+}}^{SM+LR} = -\frac{iG_F g_R}{\sqrt{2} g_L} \xi \left[ -c_1 V_{cd}^{*} V_{us}^{R} \left( X_{K^{0\pi^+}}^{D^+} - \frac{2}{N} \lambda \right) + c_1 V_{cd}^{*} V_{us}^{R} \left( X_{K^{0\pi^+}}^{D^+} - \frac{2}{N} \lambda \right) \right] \]

\[ -c_2 V_{cd}^{*} V_{us}^{R} \left( -2 \lambda X_{K^{0\pi^+}}^{D^+} + \frac{1}{N} \lambda \right) + c_2 V_{cd}^{*} V_{us}^{R} \left( -2 \lambda X_{K^{0\pi^+}}^{D^+} + \frac{1}{N} \lambda \right) \]

\[ = \frac{iG_F g_R}{\sqrt{2} g_L} \xi \left( V_{us}^{R} V_{us}^{*} - V_{cd}^{*} V_{cd}^{R} \right) \left( a_1 X_{K^{0\pi^+}}^{D^+} - 2 \lambda \right) \]

(45)

Thus, the total amplitude after including SM contribution can be expressed as

\[ A_{D^+ \rightarrow K^{0\pi^+}}^{SM+LR} = i \frac{G_F}{\sqrt{2}} V_{cd}^{*} V_{us} \left[ (a_1 + \Delta a_1^{LR}) X_{K^{0\pi^+}}^{D^+} + (a_2 + \Delta a_2^{LR K^0}) X_{K^0}^{D^+} \right], \]

(46)

where

\[ \Delta a_1^{LR} \simeq \frac{g_R}{g_L \lambda} \xi \left( V_{us}^{R} + V_{cd}^{R} \right) a_1, \quad \Delta a_2^{LR K^0} \simeq -\frac{2g_R}{g_L \lambda} \xi \left( V_{us}^{R} + V_{cd}^{R} \right) \]

(47)

with \( \lambda = V_{us} \). The direct CP asymmetry in this case can be then expressed as

\[ A_{CP}^{SM+LR}(D^+ \rightarrow K^{0\pi^+}) = \frac{|A_{D^+ \rightarrow K^{0\pi^+}}^{SM+LR}|^2 - |A_{D^+ \rightarrow K^{0\pi^+}}^{SM+LR}|^2}{|A_{D^+ \rightarrow K^{0\pi^+}}^{SM+LR}|^2 + |A_{D^+ \rightarrow K^{0\pi^+}}^{SM+LR}|^2} = \kappa' \sin(\phi_2^{LR} - \phi_1^{LR}) \]

(48)

where \( \kappa' \) is given as before and the weak phases \( \phi_1^{LR} \) and \( \phi_2^{LR} \) are defined through

\[ \phi_1^{LR} = \tan^{-1} \left( \frac{|\Delta a_1^{LR}| \sin \Delta \phi_1^{H}}{a_1} \right) \]

\[ \phi_2^{LR} = \tan^{-1} \left( \frac{|\Delta a_2^{LR K^0}| \sin \Delta \phi_2^{LR}}{a_2} \right) \]

(49)

where \( \Delta \phi_1^{LR} \) and \( \Delta \phi_2^{LR} \) are the phases of \( \Delta a_1^{LR} \) and \( \Delta a_2^{LR K^0} \) respectively.

In order to give an estimation of the direct CP asymmetries in Eqs. (43, 48) we need to determine the allowed values of the left right mixing angle \( \xi \) and the elements of the matrix \( \bar{V} \) relevant to the decay processes under consideration. Information about the allowed values of the left right mixing angle \( \xi \) can be inferred from the measurement of the muon decay parameter \( \rho \), which governs the shape of the overall momentum spectrum, performed by the TWIST collaboration [55, 56]. This parameter is related to \( \xi \) via [55]:

\[ \rho \simeq \frac{3}{4} \left[ 1 - 2 \left( \frac{g_R}{g_L} \xi \right)^2 \right] \]

(50)

Defining \( \xi = \frac{g_R}{g_L} \xi \) and for the TWIST value, from their latest global fit given in Table VII in Ref. [56], \( \rho = 0.74960 \pm 0.00019 \) we obtain the allowed 2σ range of \( \xi \)
We turn now to discuss the allowed values of the elements of the matrix $\bar{V}^R$ appear in Eqs. (42, 47). These elements are $\bar{V}_{ud}^R$, $\bar{V}_{cs}^R$, $\bar{V}_{us}^R$ and $\bar{V}_{cd}^R$. The real parts of these elements will not produce any weak CP violating phase required for generating direct CP asymmetry. In addition, their contributions to the amplitudes will be always suppressed by a factor $\zeta$ and thus, to a good approximation, can be neglected compared to the SM contributions. As a result, we only need to determine the allowed values of the imaginary parts of $\bar{V}_{ud}^R$, $\bar{V}_{cs}^R$, $\bar{V}_{us}^R$ and $\bar{V}_{cd}^R$.

In a recent study, the authors of Ref. [57] have listed the bounds from collider physics, flavor physics, and low-energy precision measurements on the complex couplings of the $W^\pm$ boson to right-handed quarks. Particularly, these bounds are applied to the couplings in the left-right symmetric models that are generated from the mixing between the charged gauge bosons of the $SU(2)_R$ and $SU(2)_L$. As shown in Ref. [57], the experimental value of $(\epsilon'/\epsilon)_K$ and the stringent bounds on the electric dipole moment of the neutron can lead to strong bounds on $Im(\bar{V}_{ud}^R)$ and $Im(\bar{V}_{us}^R)$. From that study we find that $Im(\bar{V}_{ud}^R)$ and $Im(\bar{V}_{us}^R)$ can be as large as $9 \times 10^{-4}$ and $2 \times 10^{-4}$ respectively. Moreover, $Im(\bar{V}_{cd}^R)$ can be as large as $2 \times 10^{-3}$ without violating the strongest bounds on the electric dipole moment of the neutron. The result of the study in Ref. [57] showed also that the dominant constraint on $\zeta Im(\bar{V}_{cs}^R)$ results from the process $K_L \rightarrow \pi^0 e^+ e^-$ and $\zeta Im(\bar{V}_{cs}^R)$ can have a maximum allowed value $7 \times 10^{-3}$. This result shows that we can set $Im(\bar{V}_{cs}^R) \simeq O(1)$ without violating the imposed constraints. Taking these values into account, we obtain $|A^{SM+LR}_{CP}(D^+ \rightarrow \bar{K}^0 \pi^+)| \simeq O(10^{-3})$ which is 8 orders of magnitude larger than the SM prediction. For the other decay mode we find that $|A^{SM+LR}_{CP}(D^+ \rightarrow K^0 \pi^+)| \simeq O(10^{-7})$ which is only 4 orders of magnitude larger than the SM prediction.

V. CONCLUSION

In this work we have studied CP violation in charged decays of $D$ meson. In particular, we have investigated the direct CP asymmetry of the Cabibbo favored non-leptonic $D^+ \rightarrow \bar{K}^0 \pi^+$ and the doubly Cabibbo-suppressed decay mode $D^+ \rightarrow K^0 \pi^+$ within standard model, two Higgs doublet model with generic Yukawa structure and left right symmetric models.
In the standard model, we have shown that the generated weak phases at loop-level are so tiny resulting in direct CP asymmetries at the order $10^{-11}$ in both decay modes.

Regarding the two Higgs doublet model with generic Yukawa structure, after taking into account all relevant constraints on the parameter space of the model, we have found that charged Higgs contributions to the amplitudes can enhance the direct CP asymmetries by 6 and 7 orders of magnitudes with respect to their standard model predictions for $D^+ \to \bar{K}^0\pi^+$ and $D^+ \to K^0\pi^+$ respectively. Finally, we have shown that due to the strong constraints on the parameter space of the LRS models no sizable direct CP asymmetries can be achieved for the doubly Cabibbo-suppressed decay mode $D^+ \to \bar{K}^0\pi^+$. However, this is not the case for the Cabibbo favored non-leptonic $D^+ \to \bar{K}^0\pi^+$ decay mode where sizable direct CP asymmetry of $\mathcal{O}(10^{-3})$ still can be obtained after respecting all relevant constraints on the parameter space of the model. This result should motivates search for direct CP violation in $D^+ \to \bar{K}^0\pi^+$ decay mode at colliders.

ACKNOWLEDGMENTS

This work was partially support by CONACYT projects CB-259228 and CB-286651 and Conacyt-SNI.

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