Analytical solution and finite element method simulation on the temperature characteristics of disk cutter during tunnel boring machine driving

Yihan Wu¹, Yimin Yao¹ and Lu Zheng¹,²*

¹College of Civil Engineering, Fuzhou University, Fuzhou, Fujian 350116, P.R. China
²Sichuan University – The Hong Kong Polytechnic University Institute for Disaster Management and Reconstruction, Sichuan University, Chengdu 610207, China

*Corresponding author: zhengluz@fzu.edu.cn, ORCID: 0000-0003-3674-2148

Abstract: With the rapid economic and social development in China, using the full-face rock tunnel boring machine (TBM) in construction is becoming increasingly common. As the principal tool for rock breaking, the efficiency and service life of disk cutters used by the TBM will affect project progress and cost. The failure mechanism of disk cutters is still unclear. The specific impact of the changing temperature during rock breaking on tool failure behavior remains unclear. Therefore, this paper studies the temperature characteristics of disk cutters using the theoretical solution and finite element method (FEM) simulation. The finite difference method discretized the heat conduction equation and boundary conditions. The temperature distribution and variation are solved using coding in MATLAB. FEM simulation using ABAQUS is established to characterize the heat transfer process and analyze the temperature field. Several experiments are conducted for verification. The simulated results correlate well with analytical ones, proving the correctness and accuracy of the proposed method.

Keywords: Disk cutter; Heat transfer; Transient temperature distribution; Finite difference method; ABAQUS

1. Introduction
Recently, as urbanization increases, constructing various tunnels and underground projects are rapidly growing, and using the tunnel boring machine (TBM) becomes more common and important. Modern TBM s use disk cutters as the principal tool to break the tunnel surface of a rock mass. The disk cutters are in direct contact with the rock mass and constant wear. After the cutters are worn, they cause an overload of surrounding cutters, abnormal tunneling parameters, and other unusual phenomena. Disk cutters should be replaced timeously¹,². However, the literature³ indicates that the time related to detecting, maintaining, and replacing disk cutters reaches 30%–40% of the total construction time. Furthermore, the cutter replacement space in the TBM is limited; it is challenging to replace them⁴. Thus, improving the performance and extending the service life of disk cutters has become a principal problem.

During excavation, cutting tools experience a temperature rise. Yang et al.⁵ established an analytical solution to describe the changing drill temperature during rock excavation, verified by laboratory
experiments. Their results show that high working temperatures cause the failure of drills. Wang et al.\cite{6} analyzed the temperature and thermo-stress field variations of the cutting blade during the rock breaking process using finite element method (FEM) simulation using ANSYS. They highlighted appropriate operating parameters that could control its working temperature, increasing the service life and improving cutting efficiency. Yang et al.\cite{7} also analyzed rapid cutting heat-gathering in a short time and the consequent high temperature and thermos-stress during high-speed dry cutting. They highlighted that the temperature control of cutting tools is an effective countermeasure to solve tool wear.

The above research shows that the temperature state of cutting tools is related to their working performances and service lives, significantly influencing the total excavation progress. Therefore, analyzing the temperature field and its following thermo-stress is critical to improving the life and reliability of cutting tools. However, a lack of analytical solutions exist for the transient temperature field of disk cutters, strangling quick analysis of the consequent thermo-stress field and obstructing setting up proper countermeasures. This paper presents an analytical solution to access the temperature characteristics of disk cutters by proposing an analytical model of transient thermal transfer using the heat transfer theory, discretized using the finite difference method. The temperature distribution and variation are solved through coding in MATLAB. Furthermore, FEM simulations based on ABAQUS are performed for verification.

2. Analytical solution for the transient thermal transfer model of disk cutters

2.1. Model assumptions
To establish a rational and feasible disk cutter model to solve temperature characteristics, the basic assumptions are given as follows.
(1) The disk cutter material is isotropic, and the thermal characteristic parameters are constants; they do not change with temperature.
(2) Heat flow density $q_1$ is stably applied to the edge of the disk cutter and propagated radially.
(3) The initial external boundary is the environmental temperature (40°C in this study).
(4) Except for the disk cutter’s edge, the other parts are insulating surfaces.
(5) The disk cutter itself does not generate heat inside.

2.2. One-dimensional transient thermal conductivity equation
The transient thermal conductivity equation of a one-dimensional free heat source can be defined as
\[
\rho C \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}.
\]  
(1)

The edge is short and in a radial direction. The disk cutter can be simplified (Figure 1), ignoring the different thicknesses of its edge. Figure 2 shows the thermal structure of the heat transfer problem domain. Because the top and bottom surfaces are insulating according to the assumptions above, the temperature does not change along the thickness direction. The model can be simplified as a plane-temperature problem.

![Figure 1. Half-space of disk cutter model.](image1)

![Figure 2. Thermal structure.](image2)

The mathematical expressions are
\[
\rho c \frac{dT}{dt} = k \frac{\partial^2 T}{\partial x^2}, \quad (0 \leq x \leq l),
\]  \hspace{1cm} \text{(2)}

BC1: \left. k \frac{dT}{dx} \right|_{x=0} = \alpha_1(T - T_0) - q_1, \quad \text{and} \hspace{1cm} \text{(3)}

BC2: \left. k \frac{dT}{dx} \right|_{x=l} = -\alpha_2(T - T_0), \hspace{1cm} \text{(4)}

where \( k \) is the thermal conductivity, \( q_1 \) is the density of thermal inflow, \( \alpha_1 \) is the heat transfer coefficient between the cutter edge and the water spray, \( T_0 \) is the environmental temperature, \( \alpha_2 \) is the heat transfer coefficient of the cutter and shaft, and \( l \) is the distance between the inner and outer diameters of the disk cutter.

2.3. Discrete equation of the internal node

Figure 3 shows the discrete diagram of the forward difference method for temperature field analysis.

![Figure 3. Diagram of forward difference method.](image)

From Figure 3, \( i \) represents the inner node and \( n \) the \( n\Delta t \) moment. Its node equation is written using the forward difference method for that node \((i,n)\). The difference equation of the internal node is

\[
sT_{i+1}^n + (1 - 2s)T_i^n + sT_{i-1}^n = T_{i+1}^{n+1}, \quad s = \frac{\beta \Delta t}{(\Delta x)^2}, \quad \beta = \frac{k}{\rho c},
\]  \hspace{1cm} \text{(5)}

where \( s \) is the stability parameter \((\leq 1/2)\), \( \rho \) is the density of the disk cutter, and \( c \) is the specific heat capacity.

2.4. Discrete equation of the boundary node

Figures 4 and 5 represent the outer and inner boundary nodes, respectively.

![Figure 4. The outer boundary node.](image)  \hspace{1cm}  \text{Figure 5. The inner boundary node.}
**Figure 4** shows the boundary point 0. An additional fictitious point – 1 is set outside the boundary point 0. The left node difference equation can be set using **Equation (5)**. The central difference format can be obtained according to the boundary condition (3).

\[ T_{0}^{n+1} = 2sT_{1}^{n} + s \frac{2Vx}{k} q_{1} + s \frac{2Vx}{k} \alpha_{1}T_{0} + (1 - 2s - s \frac{2Vx}{k} \alpha_{1})T_{0}^{n} \]  

(6)

To ensure the convergence and stability of the boundary point discrete equation, the following conditions must be met,

\[ 1 - 2s - s \frac{2Vx}{k} \alpha_{1} \geq 0. \]  

(7)

Suppose a node \( m + 1 \) is fictitious outside the inner boundary point \( m \) (**Figure 5**). Similarly, the central difference scheme can be obtained.

\[ T_{m}^{n+1} = 2sT_{m-1}^{n} + s \frac{2Vx}{k} \alpha_{2}T_{0} + (1 - 2s - s \frac{2Vx}{k} \alpha_{2})T_{m}^{n} \]  

(8)

\[ 1 - 2s - s \frac{2Vx}{k} \alpha_{2} \geq 0 \]  

(9)

Finally, the heat conduction matrix of a single medium disk cutter can be obtained.

\[
\begin{bmatrix}
1 - 2s - s \frac{2\Delta x}{k} \alpha_{1} & 2s \\
2s & 1 - 2s & s \\
& s & 1 - 2s & s \\
& & \ddots & \ddots & \ddots \\
& & & s & 1 - 2s & s \\
& & & & 2s & 1 - 2s - s \frac{2\Delta x}{k} \alpha_{2} \\
\end{bmatrix}
\begin{bmatrix}
T_{0}^{n} \\
T_{1}^{n} \\
\vdots \\
T_{m-1}^{n} \\
T_{m}^{n}
\end{bmatrix}
\]

(10)

**3. Case study**

MATLAB codes the heat conduction matrix formed by the difference equations of disk cutters and solves the temperature distribution to the final equilibrium state. The geometric parameters of the 17-inch Robbins disk cutter and the rock mass characteristic parameters of Colorado red granite are adopted. **Table 1** shows the specific parameters of the disk cutter. **Table 2** shows the specific rock mass parameters. The disk cutter penetration into the rock mass is 8 mm, and the cutter’s rotating speed is 6.65 rpm. The time step \( \Delta t \) is 1 s, and the space step \( \Delta x \) is 8.3 mm. The heat flow density \( q_{1} \) is determined using reference [8]. **Figure 6** shows the overall temperature change of the disk cutter. From **Figure 6**, the temperature is highest at the disk cutter’s edge and the lowest at its inner boundary. A rapid temperature increase occurs at the edge where the disk cutter contacts the rock mass, and heat generates. It also reaches a steady state in a shorter time. The temperature increase is not so obviously close to the inner boundary, causing a longer time to the steady state.

**Table 1. Parameters of the 17-inch Robbins disk cutter.**

| Type                | Cutter diameter | Shaft diameter | Blade angle | Blade width | Arc radius |
|---------------------|-----------------|----------------|-------------|-------------|------------|

4
Table 2. Parameters of the Colorado Red Granite.

| Parameters                        |       |
|-----------------------------------|-------|
| Single-axial compressive strength, MPa | 158   |
| Shear strength, MPa               | 22.8  |
| Tensile strength, MPa             | 6.78  |
| Elastic modulus, GPa              | 41.0  |
| Poisson ratio                     | 0.234 |
| Cohesion, MPa                     | 27.9  |
| Friction coefficient, °           | 0.7   |
| Internal friction angle, °        | 35    |
| Angle between shear plane and horizontal plane, ° | 20    |

4. Numerical verification

4.1. Numerical model
The FEM model was established based on the 17-inch Robbins disk cutter and numerically analyzed the temperature field variation during rock breaking. In modeling, the disk cutter is simplified, and the cutter shaft and other small, refined parts are omitted. The quadrilateral mesh is adopted (Figure 7). The linear element DC2D4(standard) in the implicit solver is used. DC2D4 is quadrilateral first-order linear, suitable for heat transfer analysis.

4.2. Boundary and operating conditions
The boundary and operating conditions are the same as those listed in the case study. The disk cutter’s initial temperature is also set to 40°C. The cutter rotating speed is 6.65 rpm, and the penetration is 8 mm.

4.3. Numerical analysis of disk cutter temperature field
The specific parameters of the disk cutter (Table 1) and the specific rock mass parameters (Table 2), respectively, are used. The disk cutter’s temperature field distribution is obtained through ABAQUS analysis. Figure 8 shows the distribution of a single medium disk cutter at different times. From Figure 8, the temperature values differ at different points of the disk cutter during rock breaking. The distribution trend is the same. The temperature at the disc cutter’s edge is the highest, and the cutter’s temperature at the shaft is the lowest. The temperature rises faster at the point closer to the edge. For example, from Figure 8 (d), the temperature at the cutter edge is close to 6 times the shaft’s temperature. The increase in temperature during 1500–2000 s is less than that during 500–1000 s.

4.4. Comparison
This section describes the temperature variation curves during rock breaking obtained by analytical solution and ABAQUS simulation. The penetrations are 6 mm and 10 mm, respectively (Figure 9). FEM is based on the minimum potential energy principle. Its elastic deformation energy is the lower bonder-bound solution; the element stiffness is slightly stiffer. The corresponding displacement field is slightly smaller than that in the analysis. Therefore, the FEM solution, which adopts temperature as a basic variable, will also be the lower bonder and smaller than the temperature field analysis. From Figure 9 and Table 3, the curve obtained using ABAQUS analysis is close to that obtained using the analytical solution.
Figure 8. The Overall Temperature Distribution at Each Time of the Plate Rob.

(a) 500 s  
(b) 1000 s  
(c) 1500 s  
(d) 2000 s

Figure 9. Comparison temperature curves through analytical and numerical solutions at the disc cutter’s edge.

(a) 6 mm penetration  
(b) 10 mm penetration

Table 3. Transient temperature distribution errors.

| Penetration | 6 mm | 10 mm |
|-------------|------|-------|
| Error       | 5.96%| 6.34% |
5. Conclusions

(1) Under the same operating conditions, the disc cutter temperature distributions demonstrate consistency between the analytical solution and numerical simulation, proving the correctness and accuracy of the proposed model.

(2) Combined with specific cases, the disc cutter’s transient temperature distribution is analyzed. A gradient of internal temperature occurs when the disc hob cuts rock. The closer to the disc cutter’s edge, the higher the temperature is, and the shorter the time to reach the steady state. The closer to the shaft, the lower the temperature is, and the longer the time to reach the steady state.

Acknowledgments

This study was funded by the National Key R & D Program of China (No. 2017YFC1501001-03), the National Natural Science Foundation of China under Grant No. 51878265 and 41977233.

References

[1] Yuan LB, Liu J, Zhao H, Yang ZY and Xu C 2019 Wear law and life prediction of disc cutter of shield in water-rich pebble Drift Ground. J. Tunnel construction. 039(010), pp 1712-1719.

[2] Li Q, Gan PL 2020 On cutter wearing control technology of the shield passing through mixed strata. J. Modern Tunnelling Technology. 57(01), pp 168-174.

[3] Zhang HM 2011 Mechanical analysis of TBM disc cutter damage mechanism and its application J. Modern Tunnelling Technology. 48(01), pp 61-65.

[4] Wu J, Yuan DJ, Li XG, Jin DL and Shen X 2017 Analysis on wear mechanism and prediction of shield cutter. J. China J. Highw. Transp. 30(08), pp 109-116.

[5] Yang XF, Li XH and Lu YY 2011 Temperature analysis of drill bit in rock drilling. J. Journal of Central South University (Science and Technology). 42(10), pp 3164-3169.

[6] Wang YY, Ting Q, Xia YM and Lv D 2014 Cutting thermo-mechanical coupling analysis of shield cutter. J. Machinery design & manufacture. (01), pp 100-103.

[7] Yang X, Cao HJ, Du YB, Xu L and Chen YP 2018 Regulation and control method for tool temperature in high-speed dry cutting processes based on specific cutting energy J. China mechanical engineering. 29(21), pp 2559-2564.

[8] Yao YM 2021 Temperature and thermal stress field analysis of disk cutter during rock breaking. Master thesis, Fuzhou University.