The structure of hybrid neutron star in Einstein-Λ gravity

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In this paper, we investigate the structure of neutron stars by considering both the effects of the cosmological constant and the existence of quark matter for neutron stars in Einstein’s gravity. For this purpose, we use a suitable equation of state (EoS) which includes a layer of hadronic matter, a mixed phase of quarks and hadrons, and a quark matter in the core. To investigate the effect of the cosmological constant on the structure of hybrid neutron stars, we utilize the modified TOV equation in Einstein-Λ gravity. Then we drive the mass-radius relation for different values of the cosmological constant. Our results show that for small values of the cosmological constant (Λ), especially for the cosmological constant from the cosmological perspective (Λ = 10^{−52} m^{−2}), Λ has no significant effect on the structure of hybrid neutron stars. But for higher values, for example, by considering Λ > 10^{−14} m^{−2}, this quantity affects the maximum mass and radius of these stars. We find an upper limit for the cosmological constant as Λ < 9 × 10^{−13} m^{−2}, based on the fact that the gravitational redshift cannot be more than 1 for stars. The maximum mass and radius of these stars decrease by increasing the cosmological constant Λ. Also, by determining and analyzing radius, the compactness, Kretschmann scalar, and gravitational red shift of the hybrid neutron stars with M = 1.4M⊙ in the presence of the cosmological constant, we find that by increasing Λ, they are contracted. Also, our results for dynamical stability show that these stars satisfy this condition.

Keywords: Hybrid neutron star, Structure, Einstein-Λ gravity, Cosmological constant

I. INTRODUCTION

Einstein gravity is a successful theory of gravity, especially for explaining the motion of planets and stars at the macroscopic scale, the solar system phenomena, and light bending to precision measurements of the orbits of binary pulsars [1], so far since it is strongly valid under the weak gravitational field approximation. In addition, the most recent cosmological observations are consistent with the standard cosmological models built on Einstein’s gravity. Another powerful prediction of Einstein gravity is related to the presence of gravitational waves, which was detected by the LIGO detections in 2016 [2]. However, there is a mysterious late-time acceleration phase in our Universe. Indeed, the observations of Supernova type Ia (SN Ia) showed that the expansion of our Universe is currently undergoing a period of acceleration [3, 4]. The Einstein gravity cannot describe this acceleration. In order to explain this acceleration there are some modified theories of gravity (see refs. [5–11], for more details). It is notable that the realization of gravitational wave astronomy provides us with the possibility of discriminating among Einstein gravity and other gravity theories [12]. In other words, by improving the results of the LIGO detections, we can determine the validation of Einstein gravity extensions. Also, the future of gravitational theories in the framework of gravitational wave astronomy after the recent GW detections is discussed in ref. [13]. Among these modified gravities, adding a (cosmological) constant (Λ) to Lagrangian of Einstein gravity is a simplified theory to explain this acceleration [14, 15] (Historically, Einstein presented this constant to save the universe from expanding, but then he rejected it after discoveries by Hubble).

Because of the high density in the core of neutron stars, which is one of the densest objects in the universe, a quark matter is expected in their interior [16–19]. This system with such high density is the interest and favorite case for physicists and astrophysicists [20, 21]. There are a lot of uncertainties in the calculations and compositions of the neutron star. Thus people use different models and various equations of state to compare their results with the observational data of neutron stars. In this work, we intend to obtain the structure properties of a neutron star with a quark core which is named hybrid star. For this purpose, we assume a hybrid star composed of three parts: a layer

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of hadrons in the surface, a hadron-quark mixed matter in the middle, and a pure quark matter in the center of the star.

To determine the properties of stars, we need a hydrostatic equilibrium equation (HEE) that satisfies static gravitational equilibrium. The TOV equation was the first HEE equation used to calculate the structure of stars. It was derived in Einstein gravity by Tolman, Volkoff and Oppenhimer [39–41]. Many authors have been studied compact stars by using TOV equation [42–49]. It is worth mentioning that, for studying compact objects such as neutron stars in modified theories of gravity, we must extract the modified TOV equation. For example, the modified TOV equation in dilaton gravity [50], vector-tensor-Horndeski theory of gravity [51], gravity’s rainbow [52, 53], F(R) and F(G) gravities [54–56], massive gravity [57, 58], mimetic gravity [59, 60], and F(R, T) gravity [61], have been evaluated (see [62–75], for more details). Therefore, we have to modify the TOV equation to study the structure of neutron stars with a quark core in Einstein-Λ gravity.

In this paper, we use a suitable EoS which includes three parts, a hadronic matter layer (LOCV Method [76]), a mixed part of quarks and hadrons (with Gibss conditions), and a quark matter in core (MIT bag model) where studied in some literature [77, 78]. The mathematical form of this EoS presented as a polynomial function,

\[ P = \sum_{i=1}^{7} a_i E^{7-i}, \]

in which \( a_i \) are as

\[ a_1 = 1.194 \times 10^{-57}, \quad & \quad a_2 = -0.246 \times 10^{-40}, \]
\[ a_3 = 2.011 \times 10^{-25}, \quad & \quad a_4 = -8.123 \times 10^{-10}, \]
\[ a_5 = 1.656 \times 10^{6}, \quad & \quad a_6 = -1.201 \times 10^{21}, \]
\[ a_7 = 2.915 \times 10^{35}. \]

II. MODIFIED TOV EQUATION IN EINSTEIN-Λ GRAVITY

The action of the Einstein gravity with the cosmological constant in 4-dimensions is given by

\[ I_G = \frac{1}{2\kappa} \int d^4x \sqrt{-g} (R - 2\Lambda) + I_{Matt}, \]

where \( R \) is the Ricci scalar and \( I_{Matt} \) is the action of matter field. \( \kappa = \frac{8\pi G}{c^4} \), \( c \) is the velocity of light and \( G \) is the Newtonian gravitational constant. Varying the action \( (3) \) with respect to the metric tensor \( g_{\mu\nu} \), the equation of motion can be written as

\[ R^\nu_\mu + \frac{1}{2} R g^\nu_\mu + \Lambda g^\nu_\mu = \frac{8\pi G}{c^4} T^\nu_\mu, \]

where \( R^\nu_\mu \) is the symmetric Ricci tensor. \( T^\nu_\mu \) is the energy-momentum tensor of perfect fluid as \( T^\mu_\nu = (c^2 \epsilon + P) U^\mu U^\nu - P g^\mu_\nu \) (where \( \epsilon \) and \( P \) are density and pressure of the fluid which are measured by local observer, respectively, and \( U^\mu \) is the fluid 4-velocity). Considering the field equation \( (4) \) and the mentioned energy-momentum tensor with a spherical symmetric spacetime in the following form

\[ ds^2 = c^2 f(r) dt^2 - \frac{dr^6}{g(r)} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

the HEE in Einstein-Λ gravity is given \[80\]

\[ \frac{dP}{dr} = \frac{3c^2 GM + r^3 (\Lambda c^4 + 12\pi GP)}{c^2 r^3 [6GM - c^2 r (\Lambda r^2 + 3)]} (c^2 \epsilon + P), \]

where for \( \Lambda = 0 \) this equation reduces to TOV equation (see \[80\], for more details).
III. STRUCTURE OF HYBRID NEUTRON STARS IN THE PRESENCE OF THE COSMOLOGICAL CONSTANT

To determine structures of hybrid neutron stars in the presence of the cosmological constant, we use the HEE in the Einstein-Λ gravity and the mentioned EoS in Eq. (1). By numerical integration of this equation, we determine the structure of hybrid stars. The results of maximum gravitational mass and the radius-mass diagram for the hybrid neutron stars are presented in Figs. 1 and 2.

![FIG. 1: Maximum gravitational mass for the hybrid neutron stars at different Λ.](image1)

![FIG. 2: Mass-radius diagram for the hybrid neutron stars at different Λ.](image2)

Our obtained results for the maximum gravitational mass and the corresponding radius, and also other structure properties of the hybrid neutron stars have been presented in Table 1. In this table (Table 1), it is seen that for $\Lambda = 10^{-52} m^{-2}$ (from the cosmological perspective, the amount of cosmological constant is about $10^{-52} m^{-2}$), there is
TABLE I: Structure properties of hybrid neutron stars for different Λ.

| Λ(m$^{-2}$) | $M_{\text{max}}$ ($M_\odot$) | $R$ (km) | $R_{\text{Sch}}$ (km) | $z$ | $M_{BB}$ ($M_\odot$) |
|------------|-----------------|--------|-----------------|---|-----------------|
| 0          | 1.80            | 10.002 | 5.31            | 0.46 | 3.01            |
| $1.0 \times 10^{-52}$ | 1.80 | 10.002 | 5.31 | 0.46 | 3.01 |
| $1.0 \times 10^{-50}$ | 1.80 | 10.002 | 5.31 | 0.46 | 3.01 |
| $1.0 \times 10^{-15}$ | 1.80 | 10.002 | 5.31 | 0.46 | 3.01 |
| $1.0 \times 10^{-14}$ | 1.76 | 9.82 | 5.15 | 0.51 | 2.94 |
| $2.0 \times 10^{-13}$ | 1.73 | 9.66 | 5.02 | 0.56 | 2.87 |
| $5.0 \times 10^{-13}$ | 1.64 | 9.40 | 4.87 | 0.72 | 2.64 |
| $7.0 \times 10^{-13}$ | 1.59 | 9.25 | 4.72 | 0.85 | 2.47 |
| $7.5 \times 10^{-13}$ | 1.58 | 9.17 | 4.64 | 0.88 | 2.42 |
| $8.0 \times 10^{-13}$ | 1.57 | 9.14 | 4.60 | 0.92 | 2.38 |
| $8.5 \times 10^{-13}$ | 1.56 | 9.06 | 4.57 | 0.96 | 2.34 |
| $9.0 \times 10^{-13}$ | 1.55 | 8.98 | 4.54 | 1.00 | 2.29 |

no effect on the structure of hybrid neutron star. In addition, for the cosmological constant in the range $10^{-14} m^{-2} \geq \Lambda \geq 10^{-52} m^{-2}$, this parameter does not have significant effects on the structure of hybrid neutron stars. But for $\Lambda > 10^{-14} m^{-2}$, the maximum mass and radius of hybrid neutron star decrease when $\Lambda$ increases. Here, we can ask this question: "why do the maximum mass of hybrid neutron stars decrease when the value of cosmological constant increases?". We will answer this question after evaluating the effects of $\Lambda$ on the other properties of hybrid neutron stars.

Now, we want to study the other properties of hybrid neutron stars such as Schwarzschild radius, average density, gravitational redshift, and Buchdahl-Bondi bound in the presence of $\Lambda$. We calculate these properties as follows.

The modified Schwarzschild radius in Einstein-Λ gravity is obtained as \[ (7) \]

\[
R_{\text{Sch}} = \frac{(3GM + \sqrt{\frac{c^4}{\Lambda} + 9G^2M^2})\Lambda^2c^4}{\Lambda^2c^4 - c^2}, \]

for $\Lambda = 0$, the above equation reduces to the Schwarzschild radius in Einstein gravity. $M$ is the mass of a hybrid neutron star. The modified Schwarzschild radius depends on the maximum mass and cosmological constant (see Table I). We see that the modified Schwarzschild radius decreases by increasing the cosmological constant.

Another quantity that we have brought in Table I is the gravitational redshift in the presence of the cosmological constant from the following formula \[ (8) \]

\[
z = \frac{1}{\sqrt{1 - \frac{2GM}{c^2R} - \frac{\Lambda}{3}R^2}} - 1.\]

Our calculations are presented in Table I. We find that the gravitational redshift increases with increasing $\Lambda$. According to the fact that the gravitational redshift cannot be more than 1 for compact objects, we find an upper limit on $\Lambda$. Our results indicate that there is the upper limit on the cosmological constant for $\Lambda < 9 \times 10^{-13} m^{-2}$.

The Buchdahl-Bondi bound is another quantity that we have to respect. The modified Buchdahl-Bondi bound in Einstein-Λ gravity is given by $M \leq M_{BB}$ \[ (9) \]

\[
M_{BB} = \frac{2c^2}{9G}R - \frac{\Lambda c^2}{3G}R^3 + \frac{2c^2}{9G}R\sqrt{1 + 3\Lambda R^2}.\]

This relation reduces to $M \leq \frac{4c^2}{9G}R$ for $\Lambda = 0$. The calculated results of $M_{BB}$ are presented in Table I. These results indicate that the obtained stars respect to this bound.
IV. DYNAMICAL STABILITY

To consider dynamically stability condition the adiabatic index \((\gamma)\) is plotted in radius parameter in Fig. 3. This condition (which is \(\gamma > \frac{4}{3}\)) implies that stars are stable against the radial adiabatic infinitesimal perturbations. The \(\gamma\) can be obtained by the following relation,

\[
\gamma = \frac{\rho c^2 + P}{\rho c^2} dP \frac{dP}{dp}.
\]

(10)

\[
\text{FIG. 3: Adiabatic index versus radius for different } \Lambda.
\]

In addition to the above approaches, we investigate the dynamical stability by using Shapiro and Teukolsky’s point of view. They believe that for dynamical stability, the pressure-averaged value of adiabatic index \(\bar{\gamma} = \frac{\int_{r_0}^{r} \gamma p^2 dr}{\int_{r_0}^{r} p^2 dr}\), must be greater than \(\frac{4}{3}\). The determined results of the average value of \(\bar{\gamma}\) for different cosmological constants are as follows

\[
\begin{align*}
\bar{\gamma}_{\Lambda=1 \times 10^{-14}} & = 2.480, \\
\bar{\gamma}_{\Lambda=1 \times 10^{-13}} & = 2.700, \\
\bar{\gamma}_{\Lambda=2 \times 10^{-13}} & = 2.695, \\
\bar{\gamma}_{\Lambda=5 \times 10^{-13}} & = 2.694, \\
\bar{\gamma}_{\Lambda=7 \times 10^{-13}} & = 2.686.
\end{align*}
\]

(11)

The obtained results in Fig. 3 and value of \(\bar{\gamma}\), show that by applying the effect of the cosmological constant to hybrid neutron stars, these objects are stable against the radial adiabatic infinitesimal perturbations.

V. OTHER PROPERTIES: CONTRACTION

According to our results in the Table I and Fig. 2 we find that, when the structure of the hybrid neutron stars are calculated with HEE in Einstein-\(\Lambda\) gravity theory, the radius of these stars reduces with increasing the cosmological...
TABLE II: Radius and other properties of the hybrid neutron stars with $M = 1.4M_\odot$ for different $\Lambda$.

| $\Lambda (m^{-2})$ | $R (km)$ | $\sigma$ | $K (10^{-8} m^{-2})$ | $z$ |
|-------------------|--------|--------|-----------------|-----|
| 0                 | 10.76  | 0.38   | 1.15            | 0.270 |
| $1.00 \times 10^{-52}$ | 10.76  | 0.38   | 1.15            | 0.274 |
| $1.00 \times 10^{-14}$ | 10.76  | 0.38   | 1.16            | 0.278 |
| $1.00 \times 10^{-13}$ | 10.42  | 0.39   | 1.43            | 0.327 |
| $1.46 \times 10^{-13}$ | 10.34  | 0.39   | 1.53            | 0.350 |
| $2.00 \times 10^{-13}$ | 10.20  | 0.40   | 1.67            | 0.379 |
| $5.00 \times 10^{-13}$ | 9.76   | 0.41   | 2.36            | 0.547 |

constant, $\Lambda$, and therefore the stars are contracted. To see better contraction of these stars due to the effect of cosmological constant, we have brought the result of the radius of the hybrid neutron stars with $M = 1.4M_\odot$ in Table II. Just as this table shows, the radius of hybrid stars with gravitational mass equals $1.4M_\odot$, decrease with increasing $\Lambda$. For more details see Fig. 2. From this figure, we see that the hybrid neutron stars with $M = 1.4M_\odot$ have different radii. In other words, by increasing the value of the cosmological constant, the radius of these stars decreases. At the same time, their mass is constant ($M = 1.4M_\odot$), which confirms the existence of a contraction.

A. Strength of Gravity

To evaluate the strength of gravity, we investigate the compactness and the Kretschmann scalar of these compact objects in the presence of the cosmological constant in the following parts.

**Compactness:** For a spherical object, the compactness of the star may be defined as $\sigma = \frac{R_{\text{Schw}}}{R}$. Considering this definition of compactness, we calculate it for different values of the cosmological constant in Table II. Our results show that the strength of gravity increases when the value of the cosmological constant increases.

**Kretschmann scalar:** Notably, we can study the Kretschmann scalar to measure curvature in a vacuum.

$$K = \sqrt{R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta}} = 2\Lambda \sqrt{\frac{2}{3} + \frac{4\sqrt{3}GM}{c^2R^3}}. \quad (12)$$

As we can see in Table II, the Kretschmann scalar increases by increasing the cosmological constant.

Here our analysis of compactness, Kretschmann scalar and redshift of hybrid neutron stars in the presence of the cosmological constant show that by increasing $\Lambda$, these quantities increase, whereas the radius of these stars decreases. In other words, these stars become very compact when $\Lambda$ increases.

According to the above results, now we can answer the above question "why do the maximum mass of hybrid neutron stars decrease when the value of the cosmological constant increases?" **Answer:** It is known that from the cosmological point of view, by adding the cosmological constant to Einstein’s gravity, this theory can explain the accelerated expansion of our Universe. In other words, this constant creates a pressure or repulsive force inside of universe (the large scale), which leads to this acceleration. According to our results, by increasing $\Lambda$, the hybrid neutron stars became very compact. Indeed, the cosmological constant inside of hybrid neutron stars or compact objects (the small scale) may act as an attractive force (opposite of effect it on universe). As we know, for having a balance between attractive force (gravitational force due to mass of compact object + the cosmological constant) and repulsive force (internal pressure), the maximum mass of compact objects decreases by increasing the strength of gravity (or by increasing the value of the cosmological constant).

VI. SUMMARY AND CONCLUSION

In this work, we checked the effect of the cosmological constant on the structure of neutron stars with quark matter (hybrid neutron stars). Because of high densities in the interior of neutron stars, we believe that these compact stars contain a deconfined quark phase. Therefore, we consider a crust of hadronic matter, a mixed phase of quark and hadronic matters, and a quark core for a neutron star. To calculate the structure of the hybrid neutron stars, we used the HEE in the Einstein-\(\Lambda\) gravity that is a modified TOV equation. By numerically integrating this HEE,
we determined these stars’ gravitational mass and radius. The results showed that for Λ smaller than $10^{-14} \, m^{-2}$ (especially $\Lambda = 10^{-52} \, m^{-2}$, the cosmological constant from the cosmological perspective), Λ does not affect the structure of hybrid neutron star. But for $\Lambda > 10^{-14} \, m^{-2}$, the structure of this star is affected by the cosmological constant. Our results showed that hybrid neutron stars’ maximum mass and radius decrease by increasing the cosmological constant, Λ. The compactness, the Kretschmann scalar, and gravitational redshift increase with Λ. Therefore we can conclude that a hybrid neutron star is contracted when the cosmological constant increases.

Another exciting result was related to dynamical stability. By applying the effect of the cosmological constant to these stars, the dynamical stability is kept inside them. In addition, we found an upper limit on the cosmological constant which was for $\Lambda < 9 \times 10^{-13}m^{-2}$.

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