Radiative transitions of doubly charmed baryons in lattice QCD

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We evaluate the spin-3/2 → spin-1/2 electromagnetic transitions of the doubly charmed baryons on 2+1 flavor, 323 × 64 PACS-CS lattices with a pion mass of 156(9) MeV. A relativistic heavy quark action is employed to minimize the systematic errors on charm-quark observables. We extract the magnetic dipole, M1, and the electric quadrupole, E2, transition form factors. We find that the M1 transition is dominant and light degrees of freedom (u/d- or s-quark) play the leading role. E2 form factors, on the other hand, are found to be negligibly small, which in turn, has minimal effect on the helicity and transition amplitudes. We predict the decay widths and lifetimes of Ξcc∗+/+ and Ωcc∗ transitions based on our results. Differences in kinematical and dynamical factors with respect to the Nγ → Δ transition is discussed. A comparison to Ωcc∗ → Ω0 transition and a discussion on systematic errors related to the choice of heavy quark action are also given.

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I. INTRODUCTION

Recently there has been a profound interest in the spectroscopy and the structure of charmed baryons. Even though there are many states yet to be confirmed and discovered by experiments, charmed baryon sector holds its theoretical appeal. Binding of two heavy quarks and a light quark provides a unique view for confinement dynamics. All of the singly charmed ground-state baryons, which were predicted by the quark model, have been experimentally observed [1–5]. Observation of the doubly-charmed baryons, on the other hand, have been challenging for experiments. First observation of the doubly charmed baryon was reported by SELEX collaboration in 2002 [6]. Mass of the Ξcc++ baryon was reported as 3519±1 MeV/ c2. However, none of the following experiments could confirm this result [7–10], until very recently LHCb Collaboration discovered the isospin partner of Ξcc++, namely Ξcc∗++, [11], containing two c quarks and one u quark. Mass of Ξcc∗++ reported by LHCb is 3621.40 ± 0.72 ± 0.27 ± 0.14 MeV/ c2, approximately 100 MeV larger than the SELEX finding and in agreement with lattice QCD predictions. This mass difference between the two isospin partners has been discussed with various theoretical approaches [12–15].

Spin-1/2 doubly-charmed baryons sit at the top layer of the flavor-mixed symmetric 20-plet of the SU(4) multiplet. In this layer, Ξcc++ and Ξcc∗++ are the isospin doublets, I = 1/2, and Ωcc is the isospin singlet, I = 0. Spin-3/2 doubly-charmed baryons Ξcc∗∗++, Ξcc∗∗ and Ωcc∗ sit at the third layer of the flavor-symmetric 20-plet with the same isospin assignments.

Electromagnetic properties of the baryon transitions give information about their internal structures and shape deformations. Examining the radiative transitions of doubly charmed baryons is a crucial element of understanding the heavy-quark dynamics. In our previous works, we have studied the Ωc→ Ω0 and Ξc→ Ξ0 transitions in lattice QCD [16, 17]. Being motivated by the recent experimental discovery of the Ξcc∗+ baryon, we extend our investigations to the spin-3/2 → spin-1/2 electromagnetic transitions of the doubly charmed baryons. Such transitions are of particular interest for experimental facilities such as LHCb, PANDA, Belle II and BESIII to search for further states.

Spin-3/2 → spin-1/2 transitions are governed by three transition form factors, namely, the magnetic dipole (M1), the electric quadrupole (E2) and the electric charge quadrupole (C2). We study the Sachs form factors and the helicity amplitudes of these transitions and extract the decay width and the lifetime. Electromagnetic transitions of the doubly charmed baryons have also been studied within the heavy hadron chiral perturbation theory [18–20], in the context of quark models [21–26] and by QCD sum rules [27, 28].

This paper is organized as follows: In Section II, we give the formulation of the transition kinematics. Section III presents the details of our lattice setup. We present and discuss our results in Section IV, and summarize the work in Section V.
II. LATTICE FORMULATION

Electromagnetic transition form factors for a $B\gamma \to B^*$ process are encoded into baryon matrix elements written in the following form:

$$\langle B^*(p', s') | J_\mu | B(p, s) \rangle = i \sqrt{\frac{2}{3}} \left( \frac{m_{B^*} - m_B}{E_{B^*}(p') E_B(p)} \right) \bar{u}_\tau(p', s') \sigma^\mu u(p, s),$$

(1)

where $B$ and $B^*$ denote spin-1/2 and spin-3/2 baryons, respectively. $p$ and $p'$ denote the initial and final four momenta and, $s$ and $s'$ denote the spins. $u(p, s)$ is the Dirac spinor and $u_\tau(p, s)$ is the Rarita-Schwinger spin vector. Operator $\sigma^\mu$ can be parameterized in terms of Sachs form factors $G_{M1}$, $G_{E2}$ and $G_{C2}$, denoting the magnetic dipole, the electric quadrupole and the electric charge quadrupole transition form factors, respectively. The kinematical factors are defined as

$$K_{M1}^{\tau} = -3 \left( (m_{B^*} + m)^2 - q^2 \right)^{-1} \frac{i \epsilon^{\tau \mu \nu} P^\nu q^\tau (m_{B^*} + m_B)}{2m_{B^*}},$$

(3)

$$K_{E2}^{\tau} = -K_{M1}^{\tau} - 6 \Omega^{-1}(q^2) i \epsilon^{\tau \beta \alpha \nu} P^\nu q^\alpha q^\beta \gamma_5 (m_{B^*} + m_B)/m_{B^*}.$$  

(4)

$$K_{C2}^{\tau} = -3 \Omega^{-1}(q^2) q^\tau (q^2 P^\mu - q \cdot P q^\mu) i \gamma_5 (m_{B^*} + m_B)/m_{B^*}.$$  

(5)

Here $q = p' - p$ is the transferred four-momentum, $P = (p' + p)/2$ and

$$\Omega(q^2) = \left( (m_{B^*} + m_B)^2 - q^2 \right) \left( (m_{B^*} + m_B)^2 - q^2 \right).$$

(6)

The Rarita-Schwinger spin sum for the spin-3/2 field in Euclidean space is given by

$$\sum_s u_\sigma(p, s) \bar{u}_\tau(p, s) = -i \frac{\gamma \cdot p + m_{B^*}}{2m_{B^*}} \left[ g_{\sigma \tau} - \frac{1}{3} \gamma_\sigma \gamma_\tau + \frac{2 p \cdot p}{3 m_{B^*}^2} - \frac{i}{3 m_{B^*}^2} \right],$$

(7)

and the Dirac spinor sum by

$$\sum_s u(p, s) \bar{u}(p, s) = -i \frac{\gamma \cdot p + m_{B^*}}{2m_{B^*}}.$$  

(8)

To extract the form factors we use the following two- and three-point correlation functions,

$$\langle G_{\sigma, B^*}^B(t; p; \Gamma_4) \rangle = \sum_x e^{-i p \cdot x} \frac{1}{4} \times \langle \text{vac} | T \bar{\eta}_\sigma(x) \eta\alpha_\beta'(0) \rangle \langle \text{vac} \rangle,$$

(9)

$$\langle G_{\sigma, B^*}^B(t; p; \Gamma_4) \rangle = \sum_x e^{-i p \cdot x} \frac{1}{4} \times \langle \text{vac} | T \bar{\eta}(x) \eta\alpha'(0) \rangle \langle \text{vac} \rangle,$$

(10)

$$\langle G_{\sigma, B^*}^B(t_2, t_1; p', p; \Gamma) \rangle = -i \sum_{x_2, x_1} e^{-i p \cdot x_2} e^{i q \cdot x_1} \Gamma_{\alpha \alpha'} \langle \text{vac} | T \bar{\eta}\alpha_\sigma(x_2) j_{\mu}(x_1) \eta\alpha'(0) \rangle \langle \text{vac} \rangle,$$

(11)

where the spin projection matrices are given as

$$\Gamma_i = \frac{1}{2} \begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix}, \quad \Gamma_4 = \frac{1}{2} \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}.$$  

(12)

Here, $\alpha, \alpha'$ are the Dirac indices, $\sigma$ and $\tau$ are the Lorentz indices of the spin-3/2 interpolating field and $\sigma_i$ are the Pauli spin indices. Spin-1/2 state is created at $t = 0$ and it interacts with the external electromagnetic field at time $t_1$ while it propagates to fixed-time $t_2$ where the final spin-3/2 state is annihilated.

We choose the interpolating fields similarly to those of $\Delta$ and $N$ as

$$\eta_\mu(x) = \frac{1}{\sqrt{3}} \epsilon^{ijk} \{ 2 [c^{T}(x) C \gamma_\mu \ell^j(x)] c^k(x) + [c^{T}(x) C \gamma_\mu c^j(x)] \ell^k(x) \},$$

(13)

$$\eta(x) = \epsilon^{ijk} [c^{T}(x) C \gamma_5 \ell(x)] c^k(x).$$

(14)
where $c$ denotes charm quark and $i$, $j$, $k$ are the color indices. Since we study the $\Xi_{cc}^+, \Xi_{cc}^+$ and $\Omega_{cc}$ baryons, $\ell$ is selected as $u$, $d$ and $s$ quark, respectively. Charge conjugation matrix is defined as $C = \gamma_4 \gamma_2$. Interpolating field in Equation (13) has been shown to have minimal overlap with spin-1/2 states and therefore does not need any spin-3/2 projection [30].

To extract the form factors, we calculate the following ratio of the two- and three-point functions:

$$R_{\sigma}(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma; \mu) = \frac{\langle G^{B^*}_\sigma J^{B^*}(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma) \rangle}{\langle \delta_{ij} G^{B^*}_\delta(t_2; \mathbf{p}; \Gamma_4) \rangle} \left[ \frac{\delta_{ij} G^{B^*}_\delta(2t_1; \mathbf{p}; \Gamma_4)}{G^{B^*}(2t_1; \mathbf{p}; \Gamma_4)} \right]^{1/2}. \quad (15)$$

In the large Euclidean time limit, $t_2 - t_1 \gg a$ and $t_1 \gg a$, time dependence of the correlators are eliminated so that the ratio in Equation (15) reduces to the desired form

$$R_{\sigma}(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma; \mu) \propto \frac{t_1 \gg a}{t_2 - t_1 \gg a} \Pi_{\sigma}(\mathbf{p}', \mathbf{p}; \Gamma; \mu). \quad (16)$$

We choose the ratio in Equation (15) from among several other alternatives [31–34] as it leads to a good plateau region and signal quality [16].

Sachs form factors can be singled out choosing appropriate combinations of Lorentz direction $\mu$ and projection matrices $\Gamma$. Similar to our work in Ref. [16], we fix the kinematics for $B \gamma \rightarrow B^*$ (spin-3/2 at rest) as

$$G_{C2}(q^2) = C(q^2) \frac{2m_B}{q^2} \Pi_k(q_k, 0; l \Gamma_k; 4); \quad (17)$$

$$G_{M1}(q^2) = C(q^2) \frac{1}{q^2} \left[ \Pi_l(q_k, 0; \Gamma_k; l) - \frac{m_B}{E_B} \Pi_k(q_k, 0; \Gamma_l; l) \right]; \quad (18)$$

$$G_{E2}(q^2) = C(q^2) \frac{1}{q^2} \left[ \Pi_l(q_k, 0; \Gamma_k; l) + \frac{m_B}{E_B} \Pi_k(q_k, 0; \Gamma_l; l) \right]; \quad (19)$$

where

$$C(q^2) = 2\sqrt{6} \frac{E_B m_B}{m_B + m} \left( 1 + \frac{m_B}{E_B} \right)^{1/2} \left( 1 + \frac{q^2}{3m_B^2} \right)^{1/2}. \quad (20)$$

Here, $k$ and $l$ are two distinct indices running from 1 to 3. For real photons, only $G_{M1}$ and $G_{E2}$ contribute. $G_{C2}$ does not play any role since it is proportional to the longitudinal helicity amplitude. In this work, we focus on the $M1$ and $E2$ transition form factors only due to poor signal-to-noise ratio of the $C2$ form factor with a limited number of gauge configurations.

### III. LATTICE SETUP

#### A. Gauge Configurations

We have run our simulations on gauge configurations generated by the PACS-CS collaboration [35] with the non-perturbatively $O(a)$-improved Wilson quark action and the Iwasaki gauge action. Details of the gauge configurations are given in Table I. Simulations are carried out with near physical $u, d$ sea quarks of hopping parameter $\kappa_{ud}^{sea} = 0.13781$. This corresponds to a pion mass of approximately 156 MeV [35]. Hopping parameter for the sea $s$ quark is fixed to $\kappa_s^{sea} = 0.13640$. It has been shown that it is feasible to carry-out simulations involving charm quarks on

| $N_s \times N_t$ | $N_f$ | $a$ [fm] | $a^{-1}$ [GeV] | $L$ [fm] | $\beta$ | $c_{sw}$ | $\kappa_{ud}^{sea}$ | $\kappa_s^{sea}$ | $m_\pi$ [MeV] |
|------------------|------|--------|-------------|--------|--------|--------|--------------|-------------|-----------|
| $32^3 \times 64$ | $2+1$ | 0.0907(13) | 2.176(31) | 2.90 | 1.90 | 1.715 | 0.13781 | 0.13640 | 156(7)(2) |

ensembles with physical light dynamical quarks [36]. Since the ensemble we employ has almost-physical quark masses,
we omit an extrapolation to the physical light quark mass point. A comparison of our previous \( m_{\Omega} \) results from Ref. [37] (extrapolated value: 2.740(24) GeV) and Ref. [16] (this ensemble: 2.750(15) GeV) along with a more recent \( \chiPT \) form extrapolation on \( m_{\Sigma} \) (extrapolated: 2.487(31) GeV vs. this ensemble: 2.486(47) GeV) from Ref. [38] indicates that almost-physical ensemble values agree with extrapolated results. Therefore, we consider the extracted values on this ensemble as final, which eliminates one source of systematic error.

**B. Strange quark mass re-tuning**

We have been unable to reproduce the experimental \( \Omega \) mass in our previous studies with \( \kappa_s = 0.13640 \) as tuned by the PACS-CS Collaboration to physical strange quark mass with respect to the mass of \( \Omega \) baryon. Our determination of the mass of \( \Omega \) on the \( \kappa_{\text{sea}} = 0.13781 \) ensemble with \( \kappa_{\text{val}} = 0.13640 \) is \( m_{\Omega} = 1.790(17) \) GeV, which overestimates the experimental value by \( \sim 6\% \) [39]. It is, however, in agreement with the PACS-CS value reported from the same ensemble, \( m_{\Omega} = 1.772(7) \) GeV [35]. A crude analysis of the \( m_{\Omega} \) values reported by PACS-CS is shown in Figure 1. We employ a linear and a \( \chiPT \) form [40] for extrapolation, both of which overestimate the experimental value. This issue with the tuning of \( \kappa_s \) has been raised in some works in the literature as well [41, 42]. Therefore we opt-in to use a partially quenched strange quark \( m_{\text{val}} \neq m_{\text{sea}} \) and adopt the value \( \kappa_{\text{val}} = 0.13665 \) reported in Ref. [41] while keeping \( a^{-1} = 2.176(31) \) GeV. We find \( m_{\Omega} = 1.674(30) \) GeV with the re-tuned \( \kappa_s \) value.

**C. Heavy quark action and quark mass tuning**

It is well known that the Clover action has \( \mathcal{O}(m_Q a) \) discretization errors that might become significant for charm quarks. Although we have successfully utilized the Clover action for charm quarks in our previous works while accounting for the associated errors, in this work we improve our simulations with a relativistic heavy quark action. We employ the so-called \textit{Tsukuba action}, proposed by Aoki et al. [43], which is designed to remove the leading cutoff effects of order \( (m_Q a)^n \) and reduce it to \( \mathcal{O}(f(m_Q a)(a\Lambda_{QCD})^2) \) where \( f(m_Q a) \) is an analytic function around the \( m_Q a = 0 \) point and can be removed further by tuning the parameters of the action non-perturbatively. As a result,
only $O((a\Lambda_{QCD})^2)$ discretization errors remain. The action is

\[ S_\Psi = \sum_{x,y} \bar{\Psi}_x D_{x,y} \Psi_y, \]

(21)

where $\Psi$s are the heavy quark spinors and the fermion matrix is given as

\[ D_{x,y} = \delta_{xy} - \kappa_Q \sum_{\mu=1}^{3} \left[ (r_s - \nu \gamma_\mu)U_{x,\mu} \delta_{x+\hat{\mu},y} + (r_s + \nu \gamma_\mu)U^\dagger_{x,\mu} \delta_{x+\hat{\mu},y} \right] - \kappa_Q \left[ (1 - \gamma_4)U_{x,4} \delta_{x+\hat{4},y} + (1 + \gamma_4)U^\dagger_{x,4} \delta_{x+\hat{4},y} \right] - \kappa_Q \left[ c_B \sum_{\mu,\nu} F_{\mu\nu}(x) \sigma_{\mu\nu} + c_E \sum_{\mu} F_\mu(x) \sigma_{\mu4} \right] \delta_{xy}. \]

(22)

Here, the parameters $r_s$, $\nu$, $c_B$ and $c_E$ should be tuned in order to remove the discretization errors appropriately. We adopt the perturbative estimates for $r_s$, $c_B$ and $c_E$ [44] and non-perturbatively tuned $\nu$ value [45]. We re-tune $\kappa_Q$ non-perturbatively so as to reproduce the relativistic dispersion relation,

\[ E_{1S}^2(p) = E_{1S}^2(0) + c_{\text{eff}}^2 |p|^2, \]

(23)

for 1S spin-averaged charmonium state. We extract the energies of the pseudoscalar and vector charmonium states from the two-point correlation functions of the interpolating fields \[ \chi(x) = \bar{c} \gamma_5 c, \quad \chi_\mu(x) = \bar{c} \gamma_\mu c. \]

(24)

The values of the parameters and extracted charmonium masses are given in Table II. Masses of the charmonium states are in very good agreement with the experimental results. We give the extracted static masses, $E_{1S}^2(0)$, and effective speed of light, $c_{\text{eff}}^2$, in Table III and Figure 2 shows the dispersion relation. Hyperfine splitting is a simple prediction one can get from this exercise and is also a good indicator for the severeness of the discretization errors. Experimental $V - PS$ hyperfine splitting is $\Delta E_{V - PS} = 113$ MeV where our results yield $\Delta E_{V - PS} = 116(4)$ MeV. We do not include disconnected diagrams in this calculation hence the effects of the possible annihilation of the $\eta_c$ and $J/\psi$ into light hadrons are neglected. This mechanism would mainly affect the $\eta_c$ meson and lead to a mass shift of $\Delta M_{\eta_c} = -3$ MeV [46]. Considering this correction, our hyperfine splitting estimate increases by 3 MeV in good agreement with the experimental result.

| $\kappa_Q$ | $r_s$ | $\nu$ | $c_B$ | $c_E$ | $m_{\eta_c}$ [GeV] | $m_{J/\psi}$ [GeV] | $m_{1S}$ [GeV] | $\Delta E_{V - PS}$ [MeV] |
|-----------|-------|------|------|------|----------------|----------------|-------------|-----------------|
| 0.10954007 | 1.1881607 | 1.1450511 | 1.9849139 | 1.7819512 | 2.984(2) | 3.099(4) | 3.071(4) | 116(4) |

TABLE II. Parameter values of the relativistic heavy quark action and masses of pseudoscalar, vector and 1S charmonium states as well as the $V - PS$ hyperfine splitting.

| $|p|^2$ | $E_{1S}(0)$ [GeV] | $E_{1S}(0)$ [GeV] | $c_{\text{eff}}^2$ |
|--------|----------------|----------------|---------------|
| 2      | 1.41111 ± 0.00150591 | 3.07058 ± 0.00327686 | 1.00818 ± 0.0159342 |
| 3      | 1.41113 ± 0.00150235 | 3.07063 ± 0.00326911 | 1.00538 ± 0.0169947 |
| 4      | 1.41117 ± 0.00149903 | 3.07071 ± 0.00326189 | 1.00186 ± 0.0175885 |
| 5      | 1.41122 ± 0.00149308 | 3.07082 ± 0.00324894 | 0.998545 ± 0.0185763 |
| 6      | 1.41127 ± 0.00148551 | 3.07092 ± 0.00323247 | 0.995832 ± 0.0197037 |

TABLE III. Extracted static masses, $E_{1S}(0)$, in lattice and physical units and effective speed of light, $c_{\text{eff}}^2$ from the dispersion relation analysis with different momenta. $|p|^2$ column indicates the number of momentum units used for the analysis.
D. Simulation Details

We make our simulations at the lowest allowed lattice momentum transfer \( q = 2\pi/L \), corresponding to three-momentum squared value of \( q^2 = 0.183 \) GeV\(^2\), where \( L = N_s a \) is the spatial extent of the lattice. We use a simple scaling method as in Ref. [31] in order to estimate the values of the form factors at zero momentum. We assume that the momentum-transfer dependence of the transition form factors is the same as the momentum dependence of the \( \Omega^{\ast}_{cc} \) and \( \Xi^{\ast}_{cc} \) baryon’s charge form factors. Such a scaling was used in previous analyses [31] and also suggested by the experimental analysis of the proton form factors. The scaling method provides more precise determination of the form factor values at zero momentum since extrapolations in finite momentum have to build on a functional form that suffer from large statistical errors. With the aid of this simple scaling, \( G_{M1}(0) \) is estimated by

\[
G^{s,c}_{M1}(0) = G^{s,c}_{M1}(q^2) \frac{G^{E0}(0)}{G^{E0}(q^2)}.
\]  

We consider quark contributions separately due to the fact that their charge form factor contributions scale differently. We have observed that [37, 47] the light-quark contribution produces a soft form factor while that of the heavy quark is harder, which falls off more slowly with increasing momentum-transfer squared. Since we found out similar results for different kinematics in our previous works [16], we fix the kinematics to where the spin-3/2 baryon is produced at rest and the spin-1/2 has momentum \(-q\).

In order to increase statistics, we insert positive and negative momenta in all spatial directions and make a simultaneous fit over all data. We consider current insertion along all spatial directions. The source-sink time separation is fixed to 12 lattice units (1.09 fm), which has been shown to be enough to avoid excited-state contaminations for electromagnetic form factors of charmed baryons [37]. We have computed various source-sink pairs by shifting them by 12\( a \). All statistical errors are estimated by a single-elimination jackknife analysis. The vector current we utilize in our simulations is the point-split lattice vector current

\[
j_\mu = \frac{1}{2} [\bar{q}(x + \mu)U_\mu(1 + \gamma_\mu)q(x) - \bar{q}(x)U_\mu(1 - \gamma_\mu)q(x + \mu)] ,
\]

which is conserved by Wilson fermions, thus eliminates the need for renormalization.
In order to improve the ground-state coupling, non-wall smeared source and sink are smeared in a gauge-invariant manner using a Gaussian form. In the case of light and strange quarks, we choose the smearing parameters so as to give a rms radius of \( r_{\text{rms}}^{l,s} \sim 0.5 \) fm. We have measured the size of the charm quark charge radius to be small compared to the light and strange quarks, both in mesons [48] and baryons [37]. Therefore, we adjust the smearing parameters to obtain \( r_{\text{rms}}^c = (r_{\text{rms}}^{l,s})/3 \). We use wall-source/sink method [48] which provides a simultaneous extraction of all spin, momentum and projection components of the correlators. The wall source/sink is a gauge-dependent object that requires fixing the gauge. We fix the gauge to Coulomb, which gives a somewhat better coupling to the ground state.

The effects of disconnected diagrams are neglected in this work since they are noisy and costly to compute. Furthermore contributions of disconnected diagrams to isovector electromagnetic form factors are usually suppressed [49]. We also expect the sea-quark effects to be suppressed in our results.

**IV. RESULTS AND DISCUSSION**

**A. Baryon masses**

We extract the masses of spin-1/2 and spin-3/2 \( \Omega_{cc} \) and \( \Xi_{cc} \) baryons using their respective two-point correlation functions defined in Equations (9) and (10). In case of spin-3/2 baryons, an average over spatial Lorentz indices is taken. Two-point correlation functions reduce to

\[
\langle G^{BB}(t; p; \Gamma_4) \rangle \simeq Z_B(p) \bar{Z}_B(p) e^{-E_B(p)t} (1 + O(e^{-\Delta E t}) + \ldots),
\]

where the mass of a baryon is encoded into the leading order exponential behavior and can be identified for the \( p = (0, 0, 0) \) case when the excited states are properly suppressed. We perform an effective mass analysis,

\[
m_{\text{eff}}(t + \frac{1}{2}) = \ln \frac{G^{BB}(t; 0; \Gamma_4)}{G^{BB}(t + 1; 0; \Gamma_4)},
\]

in order to estimate a suitable fit window, \([t_i, t_f]\), for the correlation functions and extract the masses by performing a non-linear regression analysis via Equation (27). It is possible to take the contributions of first excited states into account as correction terms to Equation (27) to enhance the analysis, however, we find it to be an excessive treatment considering the precision and agreement of our results. Initial time slice \( t_i \) is chosen by intuition where the data starts to form a plateau while the fit window is extended to the time slice until when the signal is deemed to be lost. Effective mass plots are shown in Figure 3. Fit regions are determined to be \([t_i, t_f]\) = [17, 23], [17, 23], [14, 30] and [18, 30] for \( \Xi_{cc} \), \( \Xi^*_{cc} \), \( \Omega_{cc} \) and \( \Omega^*_{cc} \) baryons respectively. Our results are given in Table IV and shown in Figure 4 in comparison to other determinations by various lattice collaborations and the experimental values where available. Note that our results are obtained at a pion mass of \( m_\pi \approx 156 \) MeV and compare well to those from other lattice collaborations which are either on physical-quark mass point or extrapolated to physical quark mass and considers the continuum limit.

**TABLE IV.** Extracted \( \Xi_{cc} \), \( \Xi^*_{cc} \), \( \Omega_{cc} \), and \( \Omega^*_{cc} \) masses as well as those of other lattice collaborations and experimental values. Errors quoted by the other collaborations correspond to statistical and various systematical errors.

|                   | This work | PACS-CS [45] | ETMC [30] | Briceno et al. [50] | Brown et al. [46] | RQCD [51] | Experiment [11] |
|-------------------|-----------|-------------|-----------|---------------------|--------------------|-----------|-----------------|
| \( m_{\Xi_{cc}} \) [GeV] | 3.626(30) | 3.603(22)   | 3.568(14)(19)(11) | 3.595(39)(20)(6) | 3.610(23)(22) | 3.610(21) | 3.62140(72)(27)(14) |
| \( m_{\Xi^*_{cc}} \) [GeV] | 3.693(48) | 3.706(28)   | 3.652(17)(27)(3) | 3.648(42)(18)(7) | 3.692(28)(21) | 3.694(18) | —               |
| \( m_{\Omega_{cc}} \) [GeV] | 3.719(10) | 3.704(17)   | 3.658(11)(16)(50) | 3.679(40)(17)(5) | 3.738(20)(20) | 3.713(16) | —               |
| \( m_{\Omega^*_{cc}} \) [GeV] | 3.788(11) | 3.779(18)   | 3.735(13)(18)(43) | 3.765(43)(17)(5) | 3.822(20)(22) | 3.785(16) | —               |
FIG. 3. Effective mass plots for the doubly charmed baryons. Shaded bands show the fit regions. Empty symbols are slightly shifted to the right for a clearer view.

FIG. 4. Comparison of our masses. See Table IV for references.
B. Form factors

Since we have all possible Lorentz, momentum, polarization and current indices, we define an average over correlation function ratios,

\[ \Pi_1 = \frac{C(q^2)}{|q|} \frac{1}{6} \sum_{k,l} \Pi_l(q_k, 0; \Gamma_k; l), \quad \Pi_2 = \frac{C(q^2)}{|q|} \frac{1}{6} \sum_{k,l} \Pi_k(q_k, 0; \Gamma_l; l), \]  

and rewrite Equations (18) and (19) as,

\[ G_{M1}(q^2) = \Pi_1 - \frac{m_{B^*}}{E_{B^*}} \Pi_2, \]  

\[ G_{E2}(q^2) = \Pi_1 + \frac{m_{B^*}}{E_{B^*}} \Pi_2. \]

Note that the factor in front of the \( \Pi_2 \) term simplifies to \( m_{B^*}/E_{B^*} = 1 \) since we only calculate the kinematical case where the spin-3/2 particle is at rest. Also let us remind the reader that we omit the \( C_2 \) form factor due to its poor signal-to-noise ratio. \( \Pi_1, \Pi_2 \) terms and the \( G_{M1}(q^2) \) for \( \Omega_{cc}^+ \gamma \rightarrow \Omega_{cc}^{++} \) and \( \Xi_{cc}^- \gamma \rightarrow \Xi_{cc}^{--} \) are illustrated in Figure 5 and Figure 6 as functions of the current insertion time, \( t_1 \), for both quark sector contributions. Fit regions are chosen as \( t_1 = [7, 9] \) and \( t_1 = [9, 11] \) for \( \Omega_{cc}^+ \), \( c \) and \( s \) quarks respectively. For the \( \Xi_{cc}^- \) transitions fit regions are chosen as \( t_1 = [5, 8] \) and \( t_1 = [5, 7] \) for \( c \) and \( u/d \) quarks respectively.

Once we have the \( \Pi_1 \) and \( \Pi_2 \) terms, it is straightforward to compute the \( G_{M1} \) and \( G_{E2} \). \( \Pi_1 \) and \( \Pi_2 \) contributions have similar magnitudes with opposite signs hence they combine destructively for \( G_{E2} \) resulting in a vanishing value. Since the value of \( G_{E2} \) is consistent with zero, it might be more sensitive to systematic errors. For one, we extract \( G_{E2} \) by two numerically differing but analytically identical procedures. First, we compute it by performing fits to the \( \Pi_1 \) and \( \Pi_2 \) terms separately and then combining the fit results and secondly, by combining the \( \Pi_1 \) and \( \Pi_2 \) terms and then performing a fit to the sum. These two procedures are identical and should result in same values except the numerical fluctuations. We find that these two approaches are consistent with each other. Another source of the systematic error might be our negligence of the disconnected diagrams. Although their contribution is suppressed with respect to that of connected diagrams, they might become significant since the disconnected contributions vanish in this case. We expect that the electric quadrupole form factor to be consistent with zero, the reason for the high error for \( G_{E2} \) is due to the data fluctuates between negative and positive axis. We observed that the mean values and the standard deviation are slightly changed in the calculations made without using the \( G_{E2} \).

Total form factors can easily be obtained using the individual quark contributions according to the formula,

\[ G_{M1,E2}(Q^2) = \frac{2}{3} G_{M1,E2}(Q^2) + c_l G_{M1,E2}(Q^2), \]

where \( c_l = -1/3 \) for the \( d \) and \( s \) quarks and \( c_l = 2/3 \) for the \( u \) quark corresponding to \( \Xi_{cc}^{++}, \Omega_{cc}^{++} \) and \( \Xi_{cc}^{++} \) baryons, respectively. We use the scaling assumption in Equation (25) to extract the values of the form factors at \( Q^2 = 0 \).

Our results for the \( M1 \) and \( E2 \) form factors are compiled in Table V. Note that the charm quark contributions include a factor of 2 accounting for the number of valence charm quarks. A close inspection of the quark sector contributions shows that the \( M1 \) form factors are dominantly determined by the light quarks, in agreement with our expectations based on our previous conclusions [16, 17, 37]. The \( \ell \)-quark contribution is visibly larger than the \( c \)-quark contribution. This pattern is also consistent with the hyperon transition form factors [31]: Heavier quark contribution is systematically smaller than that of the light quarks. Contributions of \( s \)- and \( \ell \)-quark sector is similar when switching from a \( \Omega_{cc} \) baryon to a \( \Xi_{cc} \). The charm quark contribution is also similar and suppressed as well which is in agreement with our previous conclusions [37, 47]. Note that, for the \( G_{M1} \) form factors, the mean value of the \( s \)-quark contribution is larger compared to that of \( \ell \)-quark, while the errors for \( \ell \)-quark is considerably larger than those of the \( s \)-quark. For this reason, the hierarchy of the contributions (excluding the charm quark) remains inconclusive.

Previously, we have calculated magnetic moments and charge radii of charmed baryons on a wide range of pion masses changing from \( m_\pi \sim 156 \text{ MeV} \) to \( m_\pi \sim 700 \text{ MeV} \) [37, 39, 47]. We argue in Ref. [39] that the finite size effects that might be arising to due \( m_\pi L < 4 \), are not severe, which we expect to be the case in this calculation also. Moreover, magnetic moments and the charge radii of the \( \Xi_{cc} \) and \( \Omega_{cc} \) baryons found to be similar. Interestingly, magnetic moments of the individual \( s \)- and \( \ell \)-quark sectors for \( \Xi_{cc} \) and \( \Omega_{cc} \) baryons as well were found to be similar within their error bars. Both observations are consistent with the pattern that we see in our current results of \( G_{M1} \) form factors of \( \Xi_{cc}^{++} \gamma \rightarrow \Xi_{cc}^{++} \) and \( \Omega_{cc}^{++} \gamma \rightarrow \Omega_{cc}^{++} \) transitions.
Sachs form factors can be related to phenomenological observables such as the helicity amplitudes and the decay width of a particle. Relation between the Sachs form factors of a $B^*$ at rest and the standard definitions of electromagnetic transition amplitudes $f_{M1}$ and $f_{E2}$ are given as \([52, 53]\)

\[
\begin{align*}
    f_{M1}(q^2) &= \frac{\sqrt{4\pi\alpha}}{2m_B} \left( \frac{|q|}{m_B} \right)^{1/2} \frac{G_{M1}(q^2)}{1 - q^2/(m_B + m_{B^*})^2}^{1/2}, \\
    f_{E2}(q^2) &= \frac{\sqrt{4\pi\alpha}}{2m_B} \left( \frac{|q|}{m_B} \right)^{1/2} \frac{G_{E2}(q^2)}{1 - q^2/(m_B + m_{B^*})^2}^{1/2},
\end{align*}
\]

where $\alpha$ is the fine structure constant. Helicity amplitudes $A_{1/2}$ and $A_{3/2}$ are defined as linear combinations of the
transition amplitudes as

\[ A_{1/2}(q^2) = -\frac{1}{2} [f_{M1}(q^2) + 3f_{E2}(q^2)], \]
\[ A_{3/2}(q^2) = -\frac{\sqrt{3}}{2} [f_{M1}(q^2) - f_{E2}(q^2)]. \]

The decay width is defined as [54]

\[ \Gamma = \frac{m_B \cdot m_{B'}}{8\pi} \left( 1 - \frac{m_{B'}^2}{m_B^2} \right)^2 \left\{ |A_{1/2}(0)|^2 + |A_{3/2}(0)|^2 \right\}, \]

in terms of the helicity amplitudes where we have used the constraint \( q = (m_B^2 - m_{B'}^2)/2m_B \) at \( q^2 = 0 \). An alternative definition of the decay width in terms of the Sachs form factors can be written as

\[ \Gamma = \frac{\alpha}{16} \frac{(m_B^2 - m_{B'}^2)^3}{m_B^2 m_{B'}^2} \left\{ 3|G_{E2}(0)|^2 + |G_{M1}(0)|^2 \right\}. \]

We give our estimates for the helicity amplitudes, decay widths and lifetimes in Table VI. Both definitions of the decay width return consistent results. Since mass splittings between these baryons kinematically forbid an on-shell strong decay channel, the total decay rates are almost entirely determined in terms of the electromagnetic mode. In comparison to \( N\gamma \to \Delta \) transition [54], we observe roughly one order of magnitude suppression in the helicity amplitudes. Considering that the form factors are directly related to the transition matrix elements and thus to the interesting internal dynamics, it is desirable to compare the form factors as well. One can derive the dominant \( M1 \) form factor of the \( N\gamma \to \Delta \) transition by inputting the PDG quoted \( A_{1/2} \) and \( A_{3/2} \) helicity amplitudes into Equation (35) and following the steps backwards. This calculation returns \( G_{N\gamma\to\Delta}^{M1}(0) = 3.063^{+0.102}_{-0.096} \), which is at the same order as the \( M1 \) form factors of the \( \Omega_{cc}^* \) and \( \Xi_{cc}^* \) transitions. This suggests that the suppression in the helicity amplitudes and decay widths is dominantly due to the kinematic factors, i.e. the heavier baryon masses and the smaller mass splitting.
Assuming the \( u \) - and \( d \)-quark has the same contribution within the \( \Delta^+ \) baryon, individual quark contributions (without electric charge and quark number factors) can be deduced as \( G_{M1,\gamma}^{N\gamma \rightarrow \Delta}(0) = G_{N\gamma \rightarrow \Delta}(0) = G_{N\gamma \rightarrow \Delta}(0) \) with the help of Equation (32). In contrast to the charm quark contributions, this reveals a suppression of around one order of magnitude in \( G_{M1}^{cc}(0) \). Decay widths are smaller by three orders of magnitude which is directly correlated with the similar decrease in the kinematical factor of Equation (38). \( \Omega_{cc}^{++} \) turns out to have the narrowest width, hence is the longest living amongst the doubly charmed baryons while the \( \Xi_{cc}^{*+} \) and \( \Xi_{cc}^{++} \) have similar widths and lifetimes.

### TABLE VI. Results for the helicity amplitudes, decay widths and lifetimes. Zero-momentum values are obtained using the simple scaling assumption given in Equation (25).

| \( Q^2 \) [GeV\(^2\)] | \( f_{M1} \) \( 10^{-2}[\text{GeV}^{-1/2}] \) | \( f_{E2} \) \( 10^{-2}[\text{GeV}^{-1/2}] \) | \( A_{1/2} \) \( 10^{-2}[\text{GeV}^{-1/2}] \) | \( A_{3/2} \) \( 10^{-2}[\text{GeV}^{-1/2}] \) | \( \Gamma \) [keV] | \( \tau \) [10\(^{-18}\) s] |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \Omega_{cc}^{++} \rightarrow \Omega_{cc}^{++} \) | 0.181 | 1.108(35) | 0.013(15) | -0.574(28) | -0.947(32) | — | — |
| \( \Xi_{cc}^{++} \rightarrow \Xi_{cc}^{++} \) | 0.180 | 1.115(141) | -0.027(118) | -0.516(192) | -0.989(157) | — | — |
| \( \Xi_{cc}^{*++} \rightarrow \Xi_{cc}^{*++} \) | 0.180 | 1.205(248) | 0.048(229) | 0.531(368) | 1.084(289) | — | — |

Electromagnetic transitions of the doubly charmed baryons have also been studied within the heavy hadron chiral perturbation theory \([18–20]\), in the context of quark models \([21–26]\) and by QCD sum rules \([27]\). Electromagnetic decays of doubly charmed baryons are found to be suppressed, which is qualitatively in agreement with our results. Bag model predictions \([55, 56]\) for decay widths are one order of magnitude larger than our results. Quark model predictions are even larger \([13, 25, 57]\) as well as the chiral perturbation theory’s \([19]\) and QCD sum rules’ \([28]\). It is worthy to note that considering the kinematic suppression, \( G_{M1}^{cc} \) values have to be larger by at least one order of magnitude or \( E2 \) transition should play a significant role for such a large width. This seems highly unlikely as we have found that the \( E2 \) transition is almost non-existent while the heavy-quark contribution to \( M1 \) transition is heavily suppressed and the light quark contribution is not enhanced enough to compensate the change.

#### C. Systematic errors

Since we switch to a relativistic heavy quark action in this analysis, while keeping the rest of the setup the same, we use this opportunity to quantify the systematic errors on charm observables in comparison to using a Clover action prescription \([16]\). To this end, we re-calculate the \( \Omega_{c}\gamma \rightarrow \Omega_{c}^{*} \) transition form factors, which follows the same procedures described in previous sections. A comparison of our results are given in Table VII. Note that the \( \kappa_{s}^{val} \) value we use in this and the previous work differs, therefore the change in \( \Omega_{c} \) and \( \Omega_{c}^{*} \) masses cannot solely be attributed to the change of the charm quark action. Strange quark observables also differ due to the same reason. \( G_{E2}^{c}(Q^2) \) is not a reliable observable either since its charmed-sector results are consistent with zero in both cases. A clear comparison can be made using the \( G_{M1}^{c}(Q^2) \) form factor for which we see a \( \sim 20\% \) deviation. We provide the full results of the analysis from 730 measurements in Tables VIII and IX for completeness. The updated decay width is \( \Gamma = 0.096(14) \) keV, approximately 20\% larger than but still in agreement within errors with the previous estimation of \( \Gamma = 0.074(8) \) keV \([16]\), leaving the conclusions unchanged.

### TABLE VII. Mass of \( \Omega_{c} \) and \( \Omega_{c}^{*} \) as well as the charm-sector of the \( \Omega_{c}\gamma \rightarrow \Omega_{c}^{*} \) transition form factors at \( Q^2 = 0.180 \text{ GeV}^2 \).

| \( m_{\Omega_{c}} \) [GeV] | \( m_{\Omega_{c}^{*}} \) [GeV] | \( G_{M1}^{c}(Q^2) \) | \( G_{E2}^{c}(Q^2) \) |
|-----------------|-----------------|-----------------|-----------------|
| Bahtiyar et al. \([16]\) | 2.750(15) | 2.828(15) | -0.167(33) | -0.008(26) |
| This work | 2.707(11) | 2.798(24) | -0.209(30) | -0.010(23) |
| Exp. | 2.695(2) | 2.766(2) | — | — |
TABLE VIII. Results for $G_{M1}$ and $G_{E2}$ form factors of the $\Omega_c \gamma \rightarrow \Omega_c^*$ transition at the lowest allowed four-momentum transfer and at zero momentum transfer. Quark sector contributions to each form factor are given separately.

| $Q^2$ [GeV$^2$] | $G_{M1}(Q^2)$ | $G_{M1}'(Q^2)$ | $G_{M1}(Q^2)$ | $G_{E2}(Q^2)$ | $G_{E2}'(Q^2)$ | $G_{E2}(Q^2)$ |
|---------------|--------------|----------------|---------------|---------------|---------------|---------------|
| 0.180         | 1.456(102)   | −0.209(30)     | −0.625(43)    | −0.195(11)    | 0.010(23)     | 0.059(43)     |
| 0             | 1.748(122)   | −0.215(31)     | −0.725(50)    | −0.234(134)   | 0.010(24)     | 0.071(52)     |

TABLE IX. Results for the helicity amplitudes and the decay width of the $\Omega_c \gamma \rightarrow \Omega_c^*$ transition. Helicity amplitudes are given at finite and zero momentum transfer. Zero-momentum values are obtained using the scaling assumption in Equation (25).

| $Q^2$ [GeV$^2$] | $f_{M1}$ | $f_{E2}$ | $A_{1/2}$ | $A_{3/2}$ | $\Gamma$ [keV] | $\tau$ [10$^{-18}$ s] |
|---------------|---------|---------|----------|----------|---------------|----------------|
| 10$^{-2}$[GeV$^{-1/2}$] | 10$^{-2}$[GeV$^{-1/2}$] | 10$^{-2}$[GeV$^{-1/2}$] | 10$^{-2}$[GeV$^{-1/2}$] | [keV] | [10$^{-18}$ s] |
| 0.180         | −0.951(66) | −0.090(65) | 0.341(99) | 0.901(85) | —             | —             |
| 0             | −1.104(76) | 0.109(79) | 0.389(119) | 1.050(101) | 0.096(14)     | 6.889(997)    |

V. SUMMARY AND CONCLUSIONS

We have evaluated the radiative transitions of doubly charmed baryons in 2+1-flavor lattice QCD and extracted the magnetic dipole ($M1$) and electric quadrupole ($E2$) form factors as well as the helicity amplitudes and the decay widths. We have extracted the individual quark contributions to the $M1$ and $E2$ form factors and found that $M1$ form factors are dominantly determined by the light quarks. $E2$ form factor contributions are found to be negligibly small and its absence has a minimal effect on the observables. The helicity amplitudes are observed to be suppressed roughly by one order of magnitude in comparison to the $N\gamma \rightarrow \Delta$ transition’s. $M1$ form factors are found to be at the same order with that of the $N\gamma \rightarrow \Delta$, suggesting that the kinematical factors play a more important role in suppressing the helicity amplitudes and the decay widths in the heavy quark systems. $\Omega_c^*$ has the smallest decay width hence is the longest living amongst the lowest-lying spin-3/2 doubly charmed baryons. Our results qualitatively agree with the predictions of other approaches however there is a quantitative disagreement of around one order of magnitude. We have also provided updated results for the $\Omega_c \gamma \rightarrow \Omega_c^*$ transition computed with a relativistic heavy quark action and estimated the systematic error due to using a Clover action. Our results are particularly suggestive for experimental facilities such as LHCb, PANDA, Belle II and BESIII to search for further states.

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