Analysis of Water Supply and Demand based on Logistic Model

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Abstract. With the development of economy and the increasing growth rate of population, people’s increasing demand for water resources and the unreasonable exploitation and usage of water resource, water scarcity exists in many countries and regions. Accordingly, the research on how to solve the water scarcity is of great value. Firstly, considering the dynamic influence of time on water resource supply and demand and the two factors including the degree of water resource exploitation and per capita availability of water, the water resource supply and demand model is built to measure the capacity of the region to offer clean water. This paper selected the lower per capita water use in north China as the object. This water scarcity in this area is severe due to its large population, less natural water distribution, large proportion of agriculture and low usage of industrial and agricultural water. Secondly, building up the Logistic model and grey prediction model to predict the following 15 years’ population situation and the total water resource supply in north China. Use the intervention plan which combined waste water recycling and water transfer project to intervene water resources. Lastly, combined with intervention plan and introducing validity coefficients to improve the supply and demand model of water resources, it can be estimated that by the year of 2020, water scarcity in the certain area will be alleviated.

Key words: Water scarcity; Logistic Model; Grey Prediction Model; Supply-Demand Model; Cost-Effectiveness; Intervention Plan.

1. Background introduction
With the development of economy and the increase of population, human demand for water continues to increase. Water resource is an important resource which cannot be replaced. At present, many countries and regions have various degrees of water scarcity. According to the United Nations, the world has a population of a quarter in a state of water scarcity. Water use has been growing at twice the rate of population over the last century. There are two primary causes for water scarcity: physical scarcity and economic scarcity. Physical scarcity is where there is inadequate water in a region to meet demand. Economic scarcity is where water exists but poor management and lack of infrastructure limits the availability of clean water. Many scientists see this water scarcity problem becoming exacerbated with climate change and population increase. Due to the water scarcity there may be other reasons: (1) The growth of personal consumption;(2) The increase of industrial consumption; (3) Increasing pollution which depletes the supply of fresh water.
The current focus is to provide clean water for all of us. The supply of water must take into account the physical availability of water. Of course, social factors influence availability and distribution of clean water. For example, lack of adequate sanitation can cause a decrease in water quality. When analyzing issues of water scarcity, the following types of questions must be considered. (1) What are the geological, topographical, and ecological reasons for water scarcity; (2) The water scarcity problem is how to aggravate; (3) What is the potential for new or alternate sources of water (4) What are the demographic and health related problems tied to water scarcity? So, it has great significance to study the problem of water scarcity.

2. The Models

2.1. Develop the model of water resources supply and demand

First of all, this paper is going to discuss the issue from the perspectives of water resources demand and supply. The main components of water supply are natural water resources and technical water resources. Natural water resources mainly consist of surface water $x_1(t)$ and groundwater $x_2(t)$. Technical water resources consist of seawater desalting and recycling and purifying $x_3(t)$. Besides, environmental pollution will also decrease water resources supply, marked as $x_4(t)$. Water resources demand mainly consist of industrial water $y_1(t)$, agricultural water $y_2(t)$ and domestic water $y_3(t)$.

Total amount of water resources supply $S(t)$:

$$S(t) = x_1(t) + x_2(t) + x_3(t) - x_4(t)$$  \hspace{1cm} (1)

Total amount of water resources demand $Q(t)$:

$$Q(t) = y_1(t) + y_2(t) + y_3(t) - y_4(t)$$  \hspace{1cm} (2)

Here we define the demand-supply ratio of water resources as the degree of development and utilization of water resources, marked as $\eta(t)$. The situation of water resources developing will be determined according to the value of $\eta(t)$, and then decide whether there is water scarcity in the certain area.

Degree of Water Resources Development and Utilization:

$$\eta(t) = \frac{Q(t)}{S(t)}$$  \hspace{1cm} (3)

| development and utilization of water resources | division criterion          |
|-----------------------------------------------|-----------------------------|
| 0.3-0.5                                       | Slightly explode            |
| 0.5-0.7                                       | Moderate explode            |
| 0.7-1                                         | Heavily explode             |
| >1                                            | over-Explode                |
Considering the population growth in the certain area, suppose the population in the certain area at the certain moment is \( N(t) \), then the per capita water demand at Moment \( t \) should be \( R(t) = \frac{S(t)}{N(t)} \).

This paper uses this factor to assess the capability of water supply for its population in the certain area.

2.2. Water scarcity area

Using the UN water scarcity map, pick one country or region where water is either heavily or moderately overloaded. Explain why and how water is scarce in that region. Make sure to explain both the social and environmental drivers by addressing physical and/or economic scarcity.

2.2.1. Water scarcity area selection and reasons. First of all, this paper pick China where water is between heavily and moderately overloaded from the UN water scarcity map. Water supply and demand in China of 2015 disclosed by China Statistical Yearbook, when \( t = 2015 \), Degree of Water Resources Development and Utilization:

\[
\eta = \frac{Q}{S} = \frac{18590}{28600} = 0.65 \tag{4}
\]

Per capita amount of available water:

\[
R = \frac{S}{N} = \frac{28600}{13} = 1200(\text{m}^3) \tag{5}
\]

Refer the result to the internationally accepted standard of per capita water consumption, the extent of China’s water scarcity can be assessed.

2.3. Model-based Prediction of water resources scarcity situation and its impacts in 15 years

2.3.1. Prediction of Water Resources Situation. The following part is the prediction of north China’s water resources situation in the next 15 years, according to the 2010-2015 Population and Water Supply in north China disclosed by China Statistical Yearbook.

| Year | Total population (Million) |
|------|---------------------------|
| 2010 | 497.05                    |
| 2011 | 513.92                    |
| 2012 | 512.06                    |
| 2013 | 508.42                    |
| 2014 | 507.78                    |
| 2015 | 508.9                     |

First of all, developing Logistic Model to predict the population of north China in the next 15 years. Logistic model curve is mainly used to describe the growth patterns of biological population under the condition of limited environmental resources.

\( x(t) \): population at the Moment \( t \), \( x_m \): Maximum population (10 thousands), then
\[
\frac{dx}{dt} = r(x(1 - \frac{x}{x_m}))
\]  

(6)

Using the method of separation of variables in differential equations, we can solve the above Formula as

\[
x(t) = \frac{x_m}{1 + (\frac{x_m}{x_0} - 1)e^{-rt}}
\]  

(7)

Secondly, using data of Table 4 to establish a grey forecasting model in order to predict the total water supply in north China in the next 15 years. \(x^{(0)}\): Original variable sequence; \(x^{(1)}\): Original data sequence first order; \(a\): Development parameters of model; \(b\): Coordination coefficient of model; \(x^{(1)}(k+1)\): Estimated value of the first k+1 term of the first order of the original data sequence; \(x^{(0)}(k+1)\): Estimated value of item k+1 of the original data sequence.

Given observation data \(x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(N)\}\)

After accumulated \(x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(N)\}\)

Of which \(x^{(1)}(k) = \sum_{i=1}^{k} x^{(1)}(i), (k = 1, 2, \ldots, n)\)

Satisfies the first order linear differential equation model: \(\frac{dx^{(1)}}{dt} + ax^{(1)} = b\)

Finishing

\(x^{(0)}(k+1) = x^{(1)}(k+1) - x^{(1)}(k), (k = 0, 1, 2, \ldots)\)

(8)

2.3.2. The influence of water resources on residents. According to 2.3.1 forecast of water supply and demand in North China in the next fifteen years, see Figure 4. With the increase of time, the imbalance between supply and demand of water resources is increasing, the total demand of water resources is greater than the total supply, and the water resources are seriously insufficient. The specific supply and demand situation in the next 15 years in North China The total amount of surface water, groundwater and other water in the future 15 years in North China are predicted by the model, as shown in Figure 1:

![Figure 1. Change of water resources supply](image)

The results of agricultural water, industrial water and domestic water consumption in North China in the next 15 years are shown in Figure 2:
According to the results of the model prediction and the actual situation in North China, we illustrate the impact of water scarcity on the residential life from three aspects of agriculture, industry and life.

The direct impact of water shortage and environmental damage on industry performance lies in the constraint on industrial agglomeration. North China industry accounts for 24% of China’s total industry, but the water resources only take up 6% of the total water resources. With a large number of industrial enterprises gathering in the local area, it is bound to expand the demand for water resources, thus increasing water shortages and the production costs of enterprises. The constraint of industrial clustering of resource is further strengthened. At the same time, it will lead to the increase of the total discharge of industrial pollutants in the region, thus increasing the degree of environmental damage, and the environmental constraints on industrial agglomeration in this region are so strengthened.

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References
[1] LIU Yingqiu, Economic and social development of water security in north China, 2004.
[2] ZHU daoyuan, Mathematical modeling boutique cases [M]. Jiangsu: southeast university press, 2002.
[3] Nelson, guo days, a list of the party group. The grey system theory and application. Beijing: science press, 2000.
[4] Yuan Jiazu. Grey system theory and application. Beijing: science and technology press, 1991.
[5] Cuiping Ren, Qian Cheng. Research on the current situation of water resources based on the mathematical model. Earth and Environmental Science, 2018.
[6] Jiang Qiuyuan. Mathematical model [M]. Beijing: higher education press, 1987.
[7] An Overview of the State of the World’s Fresh and Marine Waters. 2nd Edition, 2008. (http://www.unep.org/dewa/vitalwater/index.html).
[8] The World’s Water: Information on the World’s Freshwater Resources. (http://worldwater.org).
[9] AQUASTAT. Food and Agriculture Organization of the United Nations. FAO Water Resources. (http://www.fao.org/nr/water/aquastat/water_res/index.stm).