Non-paraxial Airy beams

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We report the propagation dynamics of Airy light beams under non-paraxial conditions. It is studied using the general approach which deals with Fourier expansion of the beam. We show the transformation of the beam from the Airy light beam as the paraxiality parameter decreases. The role of evanescent waves in non-paraxial regime is discussed. © 2009 Optical Society of America

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During some last years it was reported [1] about exciting experimental realization of the Airy light beams. There were indeed confirmed the important nondiffracting and accelerating properties of these beams. From the theoretical point of view the Airy beams are described by the diffraction equation [2]

\[
\frac{\partial u}{\partial z} + \frac{1}{2k} \frac{\partial^2 u}{\partial z^2} = 0, \tag{1}
\]

which is valid only for paraxial waves.

In the current Letter we study the properties of non-paraxial Airy beams, which are the exact solutions of the Maxwell equations. We will perform the theoretical analysis of propagation of non-paraxial beams and this can offer the new problems for experimentalists in the field.

Let us consider the light beam propagating along z-direction in isotropic medium with dielectric permittivity \(\varepsilon\) and magnetic permeability \(\mu\). We suppose that in the initial plane \(z = 0\) the field is distributed according to the Airy function as \(\text{Ai}(x/x_0)\exp(ax/x_0)\), where \(a\) is a decay factor limiting the beam energy at \(x < 0\), \(x_0\) is an arbitrary transverse scale. Then this field changes during propagation according to the Maxwell equations.

If \(a = 0\), we get to the ordinary Airy light beam, which is non-diffractive as it was discovered in the pioneering work [3]. However, such a beam should possess the infinite energy to be created and, therefore, cannot be realized experimentally. Parameter \(a\) confines the beam to be generated, so that it is not more non-diffractive, but can be experimentally generated. Note that there are other possibilities to confine the beam [4].

It is well-known that an arbitrary beam distribution in the plane \(z = 0\) can be constructed using the plane waves. Each plane wave propagates as it follows from the Maxwell equations:

\[
\mathbf{H}(x, z, k_x) = \left(c_1(k_x)\mathbf{e}_y - c_2(k_x)\frac{\mathbf{e}_y \times \mathbf{k}}{k_0\mu}\right)e^{ik_xx + ik_xz},
\]

\[
\mathbf{E}(x, z, k_x) = \left(c_2(k_x)\mathbf{e}_y + c_1(k_x)\frac{\mathbf{e}_y \times \mathbf{k}}{k_0\varepsilon}\right)e^{ik_xx + ik_xz}, \tag{2}
\]

where the amplitudes \(c_1\) and \(c_2\) (in general, complex numbers) describe polarization of partial plane waves. The dependencies on the transverse wavenumber \(k_x\) in the amplitudes \(c_1\) and \(c_2\) describe the profile of the beam. If \(c_1 = 0\) (\(c_2 = 0\)) then wave is TE (TM) polarized. Transverse \(k_x\) and longitudinal \(k_z\) wavenumbers are connected with the dispersion equation, so that

\[k_z(k_z) = \sqrt{k_0^2\varepsilon - k_x^2},\]

where \(k_0 = \omega/c\) is the wavenumber in vacuum. Also, the wavevector is of the form \(\mathbf{k} = k_0\mathbf{e}_x + k_z\mathbf{e}_z\).

In further the dimensionless parameters are introduced: transverse wavenumber \(q = k_zx_0\), wavenumber in vacuum \(\chi = k_0x_0\), transverse \(\tilde{x} = x/x_0\) and longitudinal \(\tilde{z} = z/k_0x_0^2\) spatial ranges.

To obtain the Airy light beam in initial plane \(z = 0\) we assume that the amplitudes are equal to [2]

\[c_1,2(q) = \frac{1}{2\pi}A_{1,2} \text{e}^{-aq^2} e^{i(q^3 - 3a^2q - ia^3)/3}, \tag{3}\]

where \(A_1\) and \(A_2\) are complex constants to provide an arbitrary beam polarization.

Now the Fourier transform (the superposition of the plane waves incident at different angles) is for the propagation of a non-paraxial Airy beam. For example, for the electric field of the beam one reads

\[
\mathbf{E}(\tilde{x}, \tilde{z}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(A_2\mathbf{e}_y + \frac{A_1}{\varepsilon\chi}(-q\mathbf{e}_z + \zeta(q)\mathbf{e}_x)\right)\]

\[\times e^{iq^2 + i\zeta(q)z}e^{-aq^2} e^{i(q^3 - 3a^2q - ia^3)/3} dq, \tag{4}\]

where \(\zeta(q) = \sqrt{\chi^2\varepsilon\mu - q^2}\).

This beam is the superposition of the TE and TM Airy beams. Both of them are described in the same manner. Therefore, we can investigate only TE Airy beam. TM beam is characterized by another amplitude coefficient \(A_2\). TE Airy beam has only \(y\) electric field component and follows from Eq. (3), when \(A_1 = 0\).

Paraxial Airy beams, which are usual for experiments [2,5–7], can be obtained, when the spectrum of propagating waves is wide. It corresponds to the high-frequency radiation, which is described by the paraxiality param-
Fig. 1. The distribution of normalized intensity of the Airy beam under paraxial condition ($\chi = 100$) calculated using Eq. (1). The decay parameter $a = 0.1$. Refractive index is $n = 1$.

The parameter $\chi \gg 1$. Then one can use the approximate expression $\sqrt{\chi^2 \varepsilon \mu - q^2} \approx \chi \sqrt{\varepsilon \mu} - \frac{q^2}{2 \chi \sqrt{\varepsilon \mu}}$, and the exact solution transforms to the paraxial one

$$E_y(x, z) = \frac{e^{i n x^2 z}}{2\pi} \int_{-\infty}^{\infty} A_2 e^{i q x - i q^2 z / \chi} x e^{-a q^2 e^{i (q^2 - 3a^2 q - ia^3)/3} dq},$$

where $n = \sqrt{\varepsilon \mu}$ is the refractive index. The integral can be expressed using the closed-form expressions [2] as

$$E_y(x, z) = A_2 e^{i n x^2 z / \chi} \mathrm{Ai} \left[ x - \left(\frac{z'}{2}\right)^2 + ia z' \right] \times \exp \left( a x - a^2 z' / 2 - i (z'^3 / 12 - a^2 z' / 2 - i a z' / 2) \right),$$

where $z' = z / n$. The equivalence of Airy beam descriptions given by Eq. (1) for large parameter $\chi$ and paraxial formulae [1] and [4] can be directly verified. Fig. 1 shows the distribution of normalized beam intensity $|E_y(x, z)|^2 / |A_2|^2$ calculated from Eq. (1) in paraxial case (for great number $\chi$). One can see the typical structure of Airy beam and its main property – acceleration – and compare them with previously reported results obtained on the basis of Eq. [3] [2, 5].

As the paraxiality parameter $\chi$ decreases, the beam structure becomes more and more different that in Fig. 1. When coordinate $z$ is small, the non-paraxiality weakly influences (see Fig. 2(a)). The acceleration of the non-paraxial Airy beam is the same as that of the paraxial one, because it is determined by the amplitude spectrum [4]. Only after $z = 2$ the beam starts to diverge rapidly. At first, the side lobes disappear, while the main lobe is still strong. After $z \sim 4$ the non-paraxial Airy beam dissipates. The discussed Airy beam has the non-paraxiality parameter $\chi$ comparable with the unity. The properties of the paraxial beams quickly restore, if $\chi$ increases. For example, for $\chi = 10$ the spatial range, where the acceleration is excellently seen, significantly raises (approximately by two times). When $\chi$ further decreases, the beam’s acceleration is getting less and less pronounced (see Fig. 2(b), (c), and (d)). Now the beams propagate almost strongly ahead, while the side lobes rapidly attenuate.

The explanation of the behavior of the non-paraxial Airy beams can be provided with dividing the spectrum of the plane waves forming the beam into two parts. The first one describes propagating waves and corresponds to...
Fig. 4. The distribution of normalized intensity of the non-paraxial Airy beam at \( \chi = 0.1 \) corresponding to (a) propagating waves, (b) evanescent waves, (c) total beam. Parameters: \( a = 0.1 \) and \( n = 1 \).

the waves with real longitudinal wavenumber \( k_z \), therefore, \( -\chi \sqrt{\varepsilon \mu} < q < \chi \sqrt{\varepsilon \mu} \). The other qs (\( q < -\chi \sqrt{\varepsilon \mu} \) and \( q > -\chi \sqrt{\varepsilon \mu} \)) are for the evanescent waves, which decay during propagation because of imaginary wavenumber \( k_z \). When parameter \( \chi \) is great, the spectrum of the propagating waves is wide, while evanescent waves can be neglected. That is why one uses the paraxial approximation for the field of the Airy beam (3). When \( \chi \) is comparable with the unity, both propagating and evanescent waves should be taken into account. In Fig. 4 we show the field distributions of propagating and evanescent beams, when the total beam energy corresponds to the Airy beam. For \( \chi = 10 \) (not shown) the profile of the propagating beam coincides with that of the total (both propagating and evanescent) intensity distribution. The maximal value in the profile of the evanescent waves is less than \( 10^{-10} \), therefore, the evanescent waves can be certainly neglected. For \( \chi = 1 \) the contributions of the evanescent and propagating waves are comparable. The energy of the propagating waves is concentrated in the main lobe, that is why it propagates at long distance (see Fig. 2 (c)). The energy of the evanescent waves is localized in the side lobes. This is the reason, why the side lobes rapidly decay. In Fig. 3(b) the beam with very narrow spectrum of the propagating waves is shown. We can observe very low energy of the propagating waves. This beam can be approximately considered as the evanescent Airy beam. Note that the sum of the energies of the propagating and evanescent waves is not the total energy of the beam, because the last includes also the interference term (it arises due to the interference of the propagating and evanescent waves).

In Fig. 1 the propagation of the almost evanescent Airy beam is shown. The low intense propagating beam propagates without spreading at the shown distance. However, it yields very low input to the total intensity. In spite of the suppressed intensity of the evanescent beam its main lobe propagates at long distance without significant change of the intensity. It should be noted that the realistic distance of beam’s propagation decreases with diminution of paraxiality parameter \( \chi \), because \( z = \chi x_0 \).

As for the energy flux of the TE and TM Airy beams, it has \( x \) and \( z \) non-zero components. More interesting situation arises for a superposition of these partial beams, which realizes a specially polarized Airy beam. Then the Poynting vector possesses also \( y \) component. However, this case require separate investigation. Similar study for the Bessel beams was earlier performed in Ref. [8].

In conclusion, we have proposed the non-paraxial Airy beam as the exact solution of the Maxwell equations. We have theoretically analyzed the propagation of this beam and interpreted its properties using those of propagating and evanescent waves. We have proved the non-paraxiality parameter \( \chi \) to be large enough (at least \( \chi > 10 \)) to provide the paraxial beam and its properties (acceleration). If \( \chi \) is small, the evanescent waves prevail and the beam rapidly decays. The beam with small \( \chi \) can be regarded as the evanescent Airy beam.

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