Relativistic Aharonov–Bohm effect in the presence of two-dimensional Coulomb potential

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We obtain exact solutions to the Dirac equation and the relevant binding energies in the combined Aharonov–Bohm–Coulomb potential in 2+1 dimensions. By means of solutions obtained the quantum Aharonov–Bohm (AB) effect is studied for free and bound electron states. We show that the total scattering amplitude in the the combined Aharonov–Bohm–Coulomb potential is a sum of the Aharonov–Bohm and the Coulomb scattering amplitudes. This modifies expression for the standard Aharonov–Bohm cross section due to the interference these two amplitudes with each other.

I. INTRODUCTION

The quantum Aharonov–Bohm (AB) effect, predicted by Aharonov and Bohm [1], was analyzed in many physical sides in numerous theoretical and experimental works (see, [2]). The AB effect occurs when electrons travel in a certain configuration of a vector potential \( A_\mu \) in which the corresponding magnetic flux is confined to a finite-radius tube topologically equivalent to a cylinder. In the case of cylindrical external field configuration, where a natural assumption is that the relevant quantum mechanical system is invariant along the symmetry \( \{ z \} \) axis, the system then becomes essentially two-dimensional in the \( xy \) plane [3].

When an electron travels in an Aharonov–Bohm potential the electron wave function acquires a (topological) phase which further influences interference pattern. The vector potential can produce observable effects because the relative (gauge invariant) phase of the electron wave function, correlated with a non-vanishing gauge vector potential in the domain where the magnetic field vanishes, depends on only the magnetic flux [3]. In definite sense one can say that the Aharonov–Bohm effect is due to the topological properties of a space of electron wave functions in a topologically nontrivial background.

In works [4, 5] the contribution in the AB amplitude, which can arise from the inclusion the spin-orbit interaction of the electron magnetic moment with the electric field oriented along the solenoid axis, was theoretically studied in the nonrelativistic approximation. This effect has been recently confirmed in experiment [6]. We note that this quantum system also has the axial symmetry.

Many physical phenomena occurring in quantum systems of electrically charged fermions, which have the axial symmetry, can be studied more effectively by means of the corresponding Dirac equation in 2+1 dimensions that enables one to consider the relativistic effects.

A permanent interest in this topic also is stimulated by the studies of (2+1)-dimensional models in both high-temperature superconductivity [7] and particle theory (including the quantum Hall effect [8] and the degenerate planar semiconductors with low-energy electron dynamics [9]) [7-14]).

In section II we study the electron states in an Aharonov–Bohm potential and discuss the topological properties of a space of electron wave functions in 2+1 dimensions. In section III we obtain exact solutions of the Dirac equation for bound electron states in 2+1 dimensions in the combined Aharonov–Bohm–Coulomb potential. In section IV solutions of the Dirac equation for the scattering problem in 2+1 dimensions in the combined Aharonov–Bohm–Coulomb potential are obtained. In section V the Aharonov–Bohm scattering in the combined Aharonov–Bohm–Coulomb potential are discussed.

We use the units where \( c = \hbar = 1 \).

II. ELECTRON STATES IN AN AHARONOV–BOHM POTENTIAL

The most wide configuration of external fields, in which exact solutions of the Dirac equation in 2+1 dimensions are managed to find in the form of special functions, seems to be the configuration of an

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Aharonov–Bohm potential

\[ A^0 = 0, \quad A_x = -\frac{By}{r^2}, \quad A_y = \frac{Bx}{r^2}; \quad A^0 = 0, \quad A_r = 0, \quad A_\varphi = \frac{B}{r}, \quad B = \frac{\Phi}{2\pi}, \]

\[ r = \sqrt{x^2 + y^2}, \quad \varphi = \arctan(y/x), \]  

and as well as a vector

\[ A^0(r) = \frac{a}{er^2}, \quad A_r = 0 \]  

(where \( e \) is the electrical charge of a fermion) and a scalar \( U(r) = -b/r \) Coulomb potentials.

In three-dimensional space such a configuration is the magnetic field of an infinitely thin solenoid creating a finite magnetic flux \( \Phi \) in the \( z \) direction (the magnetic field \( B_z = \Phi \delta(r) \)) and the electric field of a thin thread oriented along the solenoid axis perpendicular to the plane \( z = 0 \) and having both electric and scalar charges of constant linear densities \( a/2 \) and \( b/2 \), respectively. The interaction with a scalar field can be introduced in theory by means of replacement \( m \to m + U \), where \( m \) is a fermion mass. In our problem this scalar interaction is not actual and so below is not considered.

In 2+1 dimensions, the Dirac \( \gamma^\mu \)-matrix algebra is known to be represented in terms of the two-dimensional Pauli matrices, \( \sigma_j \) [12]. In addition, two kinds of fermions can be introduced in accordance with the signature of the two-dimensional Dirac matrices [12]

\[ \eta = \frac{i}{2} \text{Tr} (\gamma^0 \gamma^1 \gamma^2) = \pm 1, \]

where two signs of \( \eta \) correspond to two nonequivalent representations of the Dirac matrices. We choose

\[ \gamma^0 = \eta \sigma_3, \quad \gamma^1 = i \sigma_1, \quad \gamma^2 = i \sigma_2. \]

It will be noted that the model with charged fermions is invariant under the charge conjugation operation and the transformation \( m \to -m \), which is equivalent to the transformation \( \gamma^\mu \to -\gamma^\mu \) or \( \eta \to -\eta \). Hence, we can fix signs \( e \) and \( m \).

Let us consider an electron of mass \( m > 0 \) and charge \( e \) in the \( xy \) plane in potential [11]. The Dirac equation in 2+1 dimensions in potential \( A_\mu \) is

\[ (\gamma^\mu \hat{P}_\mu - m) \Psi = 0. \]  

(3)

Here \( \hat{P}_\mu = -i \partial_\mu - eA_\mu \) is the generalized electron momentum operator.

We seek solutions of Eq. (3) in [11] in the form

\[ \Psi(t, \mathbf{x}) = \frac{1}{\sqrt{2\pi}} \exp(-iEt + il\varphi)\psi(r, \varphi), \]  

(4)

where \( E \) is the electron energy, \( l \) is an integer, and

\[ \psi(r, \varphi) = \begin{pmatrix} f(r) \\ g(r)e^{il\varphi} \end{pmatrix}. \]  

(5)

Electron wave function in field [11] (limited at \( r = 0 \)) has the form

\[ \psi_{\nu}(r, \varphi) = e^{-iEt + il\varphi} \frac{\pi p}{2E} \left( \sqrt{\frac{E + \eta m}{E - \eta me^{2i\varphi}}} J_{\nu+1}(pr) \right). \]  

(6)

Here \( p = \sqrt{E^2 - m^2} \), and \( J_\nu(pr) \) is the Bessel function of the index

\[ \nu = |l + eB|, \]

It should be noted that the irregular solution can be eliminated only at the definite limitation on admissible values of \( |\nu| \). Indeed, in order that the irregular (the Neumann function \( N_\nu(pr) \)) solution can be eliminated we need to allot it on the “background” of the regular solution \( J_\nu(pr) \) at \( r \to 0 \) what leads to condition \( |l + eB| > 0 \).

Wave functions are normalized by condition

\[ \int \psi_{\nu}^* \psi_{\nu'} \delta^2 x = 2\pi \delta_{\nu,\nu'} \delta(p - p'), \]  

(7)

\[ \delta(p - p') = \begin{cases} 1 & \text{if } p = p' \\ 0 & \text{if } p \neq p' \end{cases} \]
At $B = 0$ one recover the free electron solutions in 2+1 dimensions from (9).

We note that solutions (10) are one-valued only when the index $\nu$ is an integer, for example $l + s$. In this case the magnetic field flux is quantized as

$$\Phi = 2\pi \hbar s/e \equiv \Phi_0 s,$$

where $\Phi_0$ is the elementary magnetic flux, and $eB = s$. If $eB$ is not an integer solutions (10) are many-valued.

One can define the scattering amplitude (SA) in a conventional manner. We assume that the incident electron wave is from the left and the wave function is normalized by the standard manner, i.e. the upper boundary conditions for solutions at limit, continuity, and uniqueness, in difference from the usual topological numbers which are by the topological quantities introduced here characterize such properties of solutions as the invariance under time translations of equations (8).

Here $f(\varphi)$ is the scattering amplitude.

Written $\psi(r, \varphi)$ in the form

$$\psi(r, \varphi) = \sum_{l=-\infty}^{\infty} A_l J_{|\nu|}(pr)e^{il\varphi},$$

it is easily to show that we must put

$$A_l = e^{-i(\pi/2)(l+eB)}.$$

The scattering amplitude is proportional to $S_l - 1 \equiv e^{2i\delta_l} - 1$, where $\delta_l = (\nu - l)\pi = eB\pi \equiv e\Phi/2\hbar$ are the partial phase shifts. They depend upon only total magnetic flux $\Phi$.

Coefficient before $e^{il\varphi}/\sqrt{l}$, is the standard Aharonov–Bohm amplitude

$$f_A(\varphi) = \frac{1}{\sqrt{2\pi l}} \frac{e^{-i\varphi(s-1/2)} \sin(e\Phi/2)}{\sin(\varphi/2)}.$$  

Here $e\Phi = 2\pi s + 2\pi \Delta$, $-1/2 \leq \Delta \leq 1/2$.

In order to give “topological” properties of solutions one can define the so-called “topological” current

$$J^\mu = \psi^* e^{\mu\nu}\partial_\nu \bar{\psi} + e\partial_\nu A_\nu \psi = -i\psi^* e^{\mu\nu}\bar{\psi} \partial_\nu \psi + j^\mu,$$

where $\partial_\mu \equiv \partial/\partial x_\mu$, and $\psi$ is free electron wave function (at $B = 0$). Currents $J^\mu$ and $j^\mu$ satisfy continuity equations $\partial_\mu J^\mu = \partial_\mu j^\mu = 0$ and, therefore,

$$Q = \frac{1}{2\pi} \int J^0 d\mathbf{r} = \int \psi^* \left( -i \frac{\partial}{\partial \varphi} + 2\pi eB \delta(\mathbf{r}) \right) \psi d\mathbf{r},$$

and

$$q = \frac{1}{2\pi} \int j^0 d\mathbf{r} = eB \int \psi^* \delta(\mathbf{r}) \psi d\mathbf{r} = \frac{eB}{\hbar c} = \left[ \frac{\Phi}{\Phi_0} \right] \left[ \frac{\Phi}{\Phi_0} \right]_d$$

(where $[\Phi/\Phi_0]$ is the integer part of $\Phi/\Phi_0$) are conserved. Quantity $[\Phi/\Phi_0]$ is the “topological number”, and $q - [\Phi/\Phi_0]$ can be called the “topological defect”.

Note that the topological quantities introduced here characterize such properties of solutions as the invariance under time translations and uniqueness, in difference from the usual topological numbers which are by the boundary conditions for solutions at $r \to \infty$. The latter is conserved due to the finiteness of energy. In our case the “topological defect” is of importance. It occurs due to the branching of solutions in the point $x, y = 0$. One can as well say that if $eB$ is not an integer than an electron wave function in potential (11) gains “topological” phase.
III. BOUND STATES

Now we consider the bound electron states and binding energies in the combined Coulomb–Aharonov–Bohm potentials. Looking for solutions in form \(4\) after simple standard rearrangement, we obtain for \(f(r)\) and \(g(r)\)

\[
\frac{df}{dr} - \frac{l + eB}{r} f + \left( E + m + \frac{a}{r} \right) g = 0, \\
\frac{dg}{dr} + \frac{1 + l + eB}{r} g - \left( E - m + \frac{a}{r} \right) f = 0.
\]

(15)

Here and below we put \(\eta = 1\). The functions \(f(r)\) and \(g(r)\) are normalized

\[
\int_0^\infty (|f|^2 + |g|^2) dr = 1.
\]

(16)

Further, following \(12\) for \(f(r)\) and \(g(r)\), we obtain

\[
f(r) = \sqrt{m + Ee^{-\rho/2}} \rho^{-1}(Q_1 + Q_2), \\
g(r) = -\sqrt{m - Ee^{-\rho/2}} \rho^{-1}(Q_1 - Q_2),
\]

(17)

where

\[
\rho = 2\lambda r, \quad \lambda = \sqrt{m^2 - E^2},
\]

(18)

\(\gamma\) is determined by

\[
\gamma = \frac{1}{2} \pm \sqrt{(l + eB + 1/2)^2 - a^2},
\]

(19)

and solutions (finite at \(\rho = 0\)) are expressed in terms of the confluent hypergeometric function \(F(b, c; z)\):

\[
Q_1 = AF \left( \gamma - \frac{1}{2} - \frac{aE}{\lambda}; 2\gamma; \rho \right), \\
Q_2 = CF \left( \gamma + \frac{1}{2} - \frac{aE}{\lambda}; 2\gamma; \rho \right).
\]

(20)

The constants \(A\) and \(C\) are related by

\[
C = \frac{\gamma - 1/2 - Ea/\lambda}{l + eB + 1/2 + ma/\lambda} A.
\]

(21)

At \((l + eB + 1/2)^2 > a^2\) \(\gamma\) is real, and must be chosen positive. If \(a^2 > (l + eB + 1/2)^2\) then both roots of \(\gamma\) are imaginary and corresponding wave functions oscillate at \(r \to 0\). So the pure Coulomb field in 2+1 dimensions can be considered only for \((l + eB + 1/2)^2 > a^2\).

In order to \(Q_1\) and \(Q_2\) were normalized they must reduce to polynomials. For \(F(b, c; z)\) it means that \(b\) must be equal to a negative integer or zero, therefore

\[
\gamma - \frac{1}{2} - \frac{Ea}{\lambda} = -n.
\]

(22)

It is easily to show that admitted values of the quantum number \(n\) are: 0, 1, 2, . . . for \(l + eB + 1/2 > 0\) and 1, 2, 3, . . . for \(l + eB + 1/2 < 0\), and the discrete electron energy levels are given by

\[
E_n = m \left[ 1 + \frac{a^2}{(n + \sqrt{(l + eB + 1/2)^2 - a^2)^2})} \right]^{-1/2}.
\]

(23)

Functions \(20\) are one-valued only for integers \(eB\). It is seen from \(23\) that the Aharonov–Bohm potential influences the radiation spectrum of electron. In addition, if \(eB\) are not integers, the phases of electron wave functions of bound states also depend upon flux parameter \(eB\) owing to factor \(e^{-iE_n(eB)t}\).
It should be remarked that solutions of Klein–Gordon equation contain parameter
\[
\gamma^s = \sqrt{(l + eB)^2 - a^2},
\]
therefore
\[
E_n^s = m \left[ 1 + \frac{a^2}{(n - 1/2 + \sqrt{(l + eB)^2 - a^2)^2}} \right]^{-1/2}.
\]
This expression makes sense only when \(|l + eB| > a^2\), a condition that forbids the existence of the \(l = 0\) energy level at \(B = 0\).

**IV. AHARONOV–BOHM SCATTERING IN THE PRESENCE OF A COULOMB POTENTIAL**

The wave functions of the continuous spectrum \((E > m)\) can be obtained from \([15]\) by means of replacements
\[
\sqrt{m - E} \rightarrow -i\sqrt{E - m}, \quad \lambda \rightarrow -ip, \quad -n \rightarrow \gamma - 1/2 - iaE/p.
\]
These functions should also be normalized anew. After replacements \([26]\), let us represent \(f\) and \(g\) in the form
\[
\left( \begin{array}{c} f \\ g \end{array} \right) = A' e^{ipr(2pr)} \gamma^{-1} [e^{i\xi} F(\gamma - 1/2 - i\mu, 2\gamma, -2ipr) + e^{-i\xi} F(\gamma + 1/2 - i\mu, 2\gamma, -2ipr)]
\]
where \(A'\) is the normalization constant and
\[
\mu = \frac{aE}{p}, \quad e^{-2i\xi} = \frac{\gamma - 1/2 + i\mu}{l + eB + 1/2 - i\mu}, \quad \mu' = \frac{ma}{p} \equiv \mu \frac{m}{E}.
\]
We note that quantity \(\xi\) is real.

After simple transformations given for three-dimensional case in \([15]\), we obtain
\[
\left( \begin{array}{c} f \\ g \end{array} \right) = \sqrt{E \pm m \over E - m} \left( \begin{array}{c} (2pr) \gamma^{-1} |\Gamma(\gamma + 1/2 + i\mu)|^{-1} \Gamma(2\gamma) \\ \Gamma(\gamma + 1/2 + i\mu) \end{array} \right) e^{i\pi/2} \\
e^{-i(\pi/2 - \gamma + 1/2 + \mu \ln 2pr - \arg \Gamma(\gamma + 1/2 + i\mu))} \left( \begin{array}{c} 3 \\ \Re \end{array} \right) [e^{i(pr + \xi)} F(\gamma - 1/2 - i\mu, 2\gamma, -2ipr)].
\]
Here \(\Gamma(z)\) is \(\Gamma\) function.

Asymptotically, the wave function has the form
\[
\left( \begin{array}{c} f \\ g \end{array} \right) = \sqrt{2(E \pm m) \over E r} \left( \begin{array}{c} \sin \cos \\ \cos \sin \end{array} \right) (pr + \delta_l + \mu \ln 2pr - \pi l/2),
\]
where
\[
\delta_l = \xi - \pi \gamma/2 - \arg \Gamma(\gamma + 1/2 + i\mu) + \pi/4 + \pi l/2,
\]
and
\[
e^{2i\delta_l} = \frac{l + eB + 1/2 - i\mu'}{\gamma - 1/2 + i\mu} \frac{\Gamma(\gamma + 1/2 - i\mu)}{\Gamma(\gamma + 1/2 + i\mu')} e^{i\pi(l - \gamma + 1/2)}.
\]
The expression for the analytical continuation of Eq. \([31]\) in the range \(E < m\)
\[
e^{2i\delta_l} = \frac{l + eB + 1/2 + (am)/\lambda \Gamma(\gamma + 1/2 - (aE)/\lambda)}{\gamma - 1/2 - (aE)/\lambda} \frac{\Gamma(\gamma + 1/2 + (aE)/\lambda)}{\Gamma(\gamma + 1/2 + (aE)/\lambda)} e^{i\pi(l - \gamma + 1/2)}
\]
has the poles at the points where $\gamma + 1/2 - (aE)/\lambda = 1 - n, \ n = 1, 2\ldots$, as well as at the point $\gamma - 1/2 - (aE)/\lambda = -n = 0$. In these points the energy levels are discrete. Near the poles with $n \neq 0$, it is easily to obtain

$$e^{2i\delta_l} \approx (-1)^{n+l} \frac{(l + eB + R + 1/2)\lambda^3}{\Gamma(n + 1)\Gamma(2\gamma + n)n^2a(E - E_0)} e^{-i\pi(\gamma - 1/2)}. \quad (33)$$

The residue of function $\exp(2i\delta_l)$ in its pole is related to the coefficient in the asymptotic expression of the wave function of the corresponding bound state as follows

$$f \approx A_0 e^{-\lambda r}, \quad g = \sqrt{\frac{m - E}{m + E}} f. \quad (34)$$

where

$$A_0 = \left[ \sqrt{\frac{m + E}{m - E}} \frac{(l + eB + ma/\lambda + 1/2)\lambda^3}{2m^2 a\Gamma(n + 1)\Gamma(2\gamma + n)} \right]^{1/2} (2\lambda r)^{\gamma + n - 1/2}. \quad (35)$$

Now we consider the scattering problem in 2+1 dimensions in the combined Coulomb–Aharonov–Bohm potentials. The total phase shifts according to (29) are

$$\delta_l = -\pi\gamma/2 + \pi/4 + \pi l/2 + \xi - \arg \Gamma(\gamma + 1/2 + i\mu) \equiv \delta_{AB} + \delta_l^a, \quad (36)$$

where

$$\delta_{AB} = -\pi\gamma/2 + \pi/4 + \pi l/2 \quad (37)$$

and

$$\delta_l^a = \xi - \arg \Gamma(\gamma + 1/2 + i\mu) \quad (38)$$

are the phase shifts due to the Aharonov–Bohm and the Coulomb potential, respectively. Note that $\delta_{AB}$ and $\delta_l^a$ weakly depend on $a$ and $eB$, respectively.

The total scattering amplitude is proportional to

$$f_{tot}(\varphi) \sim \sum_{l=0}^{\infty} \left[ \exp(2i\delta_{AB} + 2i\delta_l^a) - 1 \right]. \quad (39)$$

The difference in the square brackets is written in the form \[16\]

$$\exp(2i\delta_{AB} + 2i\delta_l^a) - 1 = [\exp(2i\delta_{AB}) - 1] + [\exp(2i\delta_{AB})(\exp(2i\delta_l^a) - 1)]. \quad (40)$$

The Coulomb phases mainly contribute in the scattering amplitude at large $l$, so their contribution can be calculated in the quasi-classical approximation. After simple calculations and taking into account Eq. (11), we obtain

$$f_{tot}(\varphi) = \frac{1}{\sqrt{2\pi p\sin(\varphi/2)}} \left[ \sin(\pi eB)e^{-i\varphi(s - 1/2)} + \frac{am}{p}\sin(\pi eB) \right] e^{i\pi eB} = f_{AB}(\varphi) + f_a(\varphi). \quad (41)$$

From Eq. (11) it follows that these two amplitudes interference with each other in the scattering cross section:

$$d\sigma = |f_{tot}(\varphi)|^2 d\varphi = \frac{d\varphi}{2\pi p\sin^2(\varphi/2)} \left[ \sin^2 \pi eB + \left( \frac{2am}{p} \right) \sin \pi eB \cos(s \varphi + \varphi/2 + \pi eB) \right]. \quad (42)$$

From Eq. (14) it is seen that the periodic dependence of the interference term in the cross section differs for toward ($\varphi = 0$) and backward ($\varphi = \pi$) scattering.
V. RESUME

It is shown that the gauge-invariant (observable) quantities are the quantum-mechanical phases of electron wave functions and the energies of bound states in both quantum and classical mechanics. The gauge-invariant phase is the phase, which acquires the electron wave function when the electron travels along a closed path which encircles a thin solenoid (oriented along the axis $z$) in the plane $z = 0$. It will be recalled that the quantum wave associated with each electron in the entrance splits into two wave packets that go around the solenoid with different sides. The ways of these wave packets intersect in the exit to result in a closed contour. So, though the Aharonov–Bohm potential satisfies equation $[\nabla \times \mathbf{A}] = 0$ everywhere in the plane except the point $x = y = 0$, the integral that gives the magnetic flux $\Phi$ through a closed contour $C$ enclosed by the wave packets

$$\oint C \mathbf{A} \cdot d\mathbf{s} = \Phi$$

is defined unambiguously.

Classical electron (with the energy $E$ and the moment of momentum $L_0$ with respect to axis $z$) trajectory in the Aharonov–Bohm potential is line

$$r = \frac{r_{\min}}{\cos(\varphi - \varphi_0)}$$

$$r_{\min} = \frac{L_0 c + eB}{\sqrt{E^2 - m^2 c^4}}$$

and the scattering angle is zero.

Electron energy expressed via the action variables $J_r$ and $J_\varphi$ is

$$E_n = m c^2 \left[ 1 + \frac{a^2}{(c J_r + \sqrt{(c J_\varphi + eB)^2 - a^2})^2} \right]^{-1/2}$$

After quantization ($J_r = \hbar n$, $J_\varphi = \hbar (l + 1/2)$) it follows from this formula Eq. (23).

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