Bifurcation analysis of hydro-turbine regulating system with saturation nonlinearity for hydropower station with upstream and downstream surge chambers

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Abstract. A nonlinear mathematical model of hydraulic turbine regulating system is applied to describe hydropower stations with upstream and downstream surge chambers. This model features saturation nonlinearity including pipeline system and turbine regulating system used in stability analysis. First, the existence conditions and direction of Hopf bifurcation are obtained. Second, based on the algebraic criteria for the occurrence of Hopf bifurcation, the stability domain is drawn in a coordinate system, where the proportional gain \( K_p \) is the abscissa and the integral gain \( K_i \) is the ordinate. Third, the nonlinear dynamic behaviour of a regulating system with different state parameters are analyzed, and the variations of the system stability around the two sides of the bifurcation point are numerically calculated. Based on this work we conclude that the Hopf bifurcation of system is supercritical. The bifurcation parameters that are far from the bifurcation point would be advantageous to the rapid system regulation needed to sustain equilibrium. Furthermore, it is established that using a PID controller is more conducive to stability than a PI controller. The unit stability regulation gets worse by taking into account the saturation nonlinearity.

1. Introduction

Currently, upstream and downstream surge chambers are being increasingly applied to the hydropower stations with complex arrangements in water delivery power generation and pumped storage power stations. For this type of power station, how to ensure the stability of the hydro-turbine regulating system and the regulating performance are big problems that need to be addressed. Compared to a power station without a surge chamber or only one surge chamber, this arrangement can lead to a deterioration of the system’s dynamic performances and even to resonance, due to the water-level fluctuations in the upstream and downstream surge chambers on account of the inappropriate design of the surge chamber or incorrect selection of the speed controller’s parameters. These could cause great harm to the power station and prevent its safe and stable operation.

There has been a great deal of research into the stability of the hydro-turbine regulating system in hydropower stations. Guo et al. overcame the Thoma Assumption, established a complete mathematic models for the pressure pipeline system and hydro-turbine system under the rigid hammer assumption \([1-4]\). Moreover, they conducted a linear processing on the model of a hydro-mechanical speed governing system, derived the critical sectional area for the stability of the upstream surge chamber \([5]\).
As to the power stations with upstream and downstream surge chambers, Yang and Chen et al. conducted many studies on the system’s stability based on linear models [6-7].

However, the hydro-turbine regulating system is a nonlinear system. Whether under large or small fluctuations, the nonlinear effects induced by the system’s nonlinearity and the changes in parameters cannot be neglected. By taking the nonlinear factors in the hydraulic system into account, Bao et al. analyzed the nonlinear stability of the hydropower station with a surge chamber and the major influence factors based on the Hopf bifurcation theory [8]. Ling et al. considered the saturated nonlinear elements in a hydro-turbine regulating system and took the nonlinearity of the flow equation into account when studying a system with surge chambers. In addition, they conducted a Hopf bifurcation analysis on the systems [9-11]. In the Institute of Electrical and Electronic Engineers’ (IEEE) PES Working Group on Prime Mover 1992, Yuan et al. proposed a nonlinear model for a hydro-turbine in combination with the first-order model of a generator, they constructed a model of a single-pipe single-turbine nonlinear hydro-turbine regulating system by taking the output amplitude limiting elements of the hydro-turbine into account [12]. Guo et al. uses the Hopf bifurcation theory to study the stability of hydro-turbine governing system of hydropower station with sloping ceiling tailrace tunnel [13]. According to the previous research results, both the nonlinearity of the hydraulic system and the nonlinearity of the regulating system can impose adverse effects on the system’s stability to a certain extent. However, the above-described studies focused on hydropower stations with only upstream surge chamber, and thereby they lacked the effects of the nonlinear factors in upstream and downstream double surge chambers.

In this article, for a hydropower station with upstream and downstream surge chambers, the effects of the saturation nonlinearity of the speed regulator on the regulating system stability were investigated under small fluctuations based on the Hopf bifurcation theory. Firstly, the nonlinear mathematic models of the pipeline system and the hydro-turbine regulating system were constructed. Accordingly, the conditions and directions of the Hopf bifurcation in this nonlinear dynamic system were analyzed and the algebraic criterion for bifurcation was determined. Then, based on the algebraic criterion, the system’s stable domain was plotted using the proportional gain $K_p$ and the integral gain $K_i$ as the horizontal and vertical coordinates, respectively. Finally, the system’s stability given different state parameters were analyzed using the stable domain, and the variations in the system’s stability on both sides of the bifurcation point were simulated. Then suitable conclusions were drawn from the results.

2. Mathematical model

The diversion power system of the hydropower station with upstream and downstream double surge chambers is shown in figure 1. According to the rigid water hammer model, the fundamental equations are listed below.

![Diversion power system of hydropower station with upstream and downstream surge chambers.](image)

(1) Pipe network equations [14]

Momentum equation of diversion tunnel:

upstream: \[ z_1 - 2q_1h_1 + H_0 = T_w \frac{dq_1}{dt} \]  (1)

downstream: \[ z_2 - 2q_2h_2 + H_0 = T_w \frac{dq_2}{dt} \]  (2)
Continuity equation of surge chamber:
upstream: \( q_1 = q_t - T_{f1} \frac{dz_1}{dt} \) \hspace{1cm} (3)
downstream: \( q_2 = q_t - T_{f2} \frac{dz_2}{dt} \) \hspace{1cm} (4)

Momentum equation of penstock:
\[ h = -z_1 - z_2 - T_{wt} \frac{dq_t}{dt} - 2q_t h_0 / H_0 \] \hspace{1cm} (5)

(2) Hydraulic turbine regulating system equations [15]

Moment equation and flow equation of hydraulic turbine:
\[ m_i = e_i h + e_x x + e_y y \] \hspace{1cm} (6)
\[ q_i = e_q h + e_q x + e_q y \] \hspace{1cm} (7)

First derivative differential equation of generator:
\( T_a \frac{dx}{dt} = m_i - (m_e + e_e x) \) \hspace{1cm} (8)

Dynamic equation of hydraulic servo:
\[ T_y \cdot \frac{dy}{dt} + y = u \] \hspace{1cm} (9)

Figure 2. Structure of the PID governing system with nonlinearity.

Figure 3. Nonlinear link of saturation.

Figure 4. Hyperbolic nonlinear function.

According to the transfer function for the parallel PID-type speed controller:
\[ G_{PID}(s) = K_p + K_i / s + K_d s \] \hspace{1cm} (11)

The control output can be rewritten as:
\[ z = K_p (r - x) + K_i x_8 - K_d x  \] \hspace{1cm} (12)
\[ x_8 = r - x \] \hspace{1cm} (13)

Where \( x_8 \) denotes the intermediate state variable.

Set \( u_m = 1 \) and \( z_0 = 1 \). The parameters of \( a \) and \( b \) were appropriately selected so that the hyperbolic function could approach the amplitude limiting nonlinear element. In this article, \( a \) and \( b \) were set as 1 and 0.75, respectively.

According to equations (1-13), the eight-dimensional differential equations of the dynamic model of the hydro-turbine with saturated nonlinear element in the water diversion power system with upstream and downstream surge chambers can be expressed as:
3. Bifurcation existence
The nonlinear dynamic system as described in equation (14) can be written in the following form: 
\[ \dot{x} = f(x, \nu), \] in which \( \nu \) denotes the system’s bifurcation parameter. Since \( \dot{x} = 0 \), the only equilibrium point of the system, \( x_0 = (q_{10}, q_{20}, q_{10}, q_{20}, z_{10}, z_{20}, y_0, x_0, y_0, z_0, x_0) \), can be calculated:

\[
\begin{align*}
q_{10} &= q_{20} = q_0 = (e_0, y_0, e_0, y_0) / (1 + 2 e_0 h_0 / H_0) \\
z_{10} &= 2 q_0 h_0 / h_0 \\
z_{20} &= 2 q_0 h_20 / H_0 \\
x_0 &= r \\
y_0 &= \frac{m_s e_0 (1 + 2 e_0 h_0 / H_0) - [(e_0 - e_0) (1 + 2 e_0 h_0 / H_0) - 2 e_0 e_0 h_0 / H_0] e_0 r}{e_0 e_0 + (e_0 e_0 - e_0 e_0) (1 + 2 e_0 h_0 / H_0)} \\
x_0 &= b K_r^{-1} \tan \frac{y_0}{a}
\end{align*}
\]

where \( h_0 = h_{10} + h_{10} + h_{20} \).

The Jacobian matrix of the system \( \dot{x} = f(x, \nu) \) at the equilibrium point \( x_0 \) can be described as:

\[
J(\nu) = Df(x_0, \nu) = \begin{bmatrix}
\frac{\partial q_1}{\partial q_1} & \cdots & \frac{\partial q_1}{\partial x_k} \\
\frac{\partial q_1}{\partial q_1} & \cdots & \frac{\partial q_1}{\partial x_k} \\
\vdots & \ddots & \vdots \\
\frac{\partial x_k}{\partial q_1} & \cdots & \frac{\partial x_k}{\partial x_k}
\end{bmatrix}
\]

The characteristic equation of the Jacobian matrix, \( \det(J(\nu) - \chi I) = 0 \), can be expanded as:

\[
\chi^8 + a_1 \chi^7 + a_2 \chi^6 + a_3 \chi^5 + a_4 \chi^4 + a_5 \chi^3 + a_6 \chi^2 + a_7 \chi + a_8 = 0
\]

According to the Hopf bifurcation theorem [8], when \( \nu = \nu_c \), the following expression can be obtained: \( a_i > 0 \) \( (i = 1, 2, 3, 4, 5, 6, 7, 8) \), and

\[
\Delta_s = \begin{pmatrix}
a_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_1 & a_2 & a_1 & 0 & 0 & 0 & 0 & 0 \\
a_1 & a_1 & a_1 & 0 & 0 & 0 & 0 & 0 \\
a_2 & a_1 & a_1 & a_1 & a_1 & 1 & 0 \\
a_2 & a_1 & a_1 & a_1 & a_1 & a_1 & 1 & 0 \\
0 & a_1 & a_1 & a_1 & a_1 & a_1 & 1 & 0 \\
0 & 0 & a_1 & a_1 & a_1 & a_1 & 0 & a_1 \\
0 & 0 & 0 & a_1 & a_1 & a_1 & 0 & 0
\end{pmatrix} = 0
\]
This means that, when $\nu = \nu_c$, equation (17) has a pair of pure imaginary characteristic roots: $\chi_{1,2} = \pm ia_0$.

Then take the derivative of equation (17) with respect to the parameter $\nu$:

$$\frac{d\chi}{d\nu} = \frac{a_1\chi^2 + a_2\chi^3 + a_3\chi^4 + a_4\chi^5 + a_5\chi^6 + a_6\chi^7 + a_7\chi + a_8}{8\chi^2 + 7a_2\chi + 6a_3\chi^2 + 5a_4\chi^3 + 4a_5\chi^4 + 3a_6\chi^5 + 2a_7\chi^6 + a_8)$ (19)

in which $a_i = \frac{da_i}{d\nu}$ ($i = 1, 2, 3, 4, 5, 6, 7, 8$).

If $\nu = K_i$ was selected as the bifurcation function and the power station A was taken as the example, at the bifurcation point ($K_d = 2$, $K_i = 0.5$, $K_p = 0.8744547$), $a_0 = 0.243$:

$$\text{Re}\frac{d\chi}{d\nu}\bigg|_{\nu = \nu_c} = 0.197 > 0$$ (20)

Therefore, based on the Hopf bifurcation theorem [8], the calculated Hopf bifurcation is supercritical. According to the bifurcation direction, we can deduce that when $\nu < \nu_c$, the equilibrium point of the system is a stable focus, and when $\nu < \nu_c$, the periodic motion is bifurcated from the system’s equilibrium position. Therefore, a stable limit cycle is generated that leads to a continuous vibration in the system.

4. Bifurcation analysis

Take a real hydropower station A with upstream and downstream double surge chambers as an example, the bifurcation analysis was conducted on the saturation nonlinearity of the hydro-turbine regulating system. The basic information of this power station is as follows: rated head $H_0 = 419$ m, rated flow $Q_0 = 81.56$m$^3$/s and $T_r = 9.864$s. The working condition was calculated using the system’s nominal output as the normal operating condition. The other parameters are as follows: $T_{a1} = 1.718$s, $T_{a2} = 2.586$s, $T_{a3} = 1.154$s, $h_{10} = 2.329$m, $h_{10} = 6.236$m and $h_{20} = 1.683$m. The ideal transfer coefficients of the hydro-turbine were $e_i = 1.5$, $e_x = -1$, $e_y = 1$, $e_{q1} = 0.5$, $e_{q2} = 0$ and $e_{aq} = 1$; the load self-regulation coefficients were set as: $e_x = 0$ and $g = 9.81$(m/s$^2$); and the reaction time constant of the servomotor was set to $T_e = 0.2$s. The rotating speed disturbance and load disturbance were set as 0 and -0.1, respectively, i.e., $\nu = 0$ and $m_y = -0.1$.

4.1 Stable domain

The bifurcation points of the Hopf bifurcation are the critical points for the system stability. The curve consisting of the bifurcation points in different states is referred to as the bifurcation line. The complete parameter plane can be divided into the stable and unstable domains by the bifurcation line. The position of the bifurcation line determines the stable states and stability margin of the system in the parameter plane under different conditions, i.e., it can measure the system’s dynamic properties.

In this article, the system’s bifurcation line was plotted using $K_d$ and $K_i$ as the horizontal and vertical ordinates, respectively. Since $a_i > 0$ in the region of the existence of bifurcation, $K_{i_c} < 4.36$s. Figure 5 displays the bifurcation line when $K_{i_c} = 2$s.

Since the system’s Hopf bifurcation is supercritical, for the same $K_d$ and $K_p$, when $\nu < \nu_c$, i.e., $K_i < K_{i_c}$, the system is stable, otherwise, the system is unstable. Set $K_i = 0.5$s$^{-1}$, there are two bifurcation points for $K_p$, denoted as $S_i$ and $S_c$, and the corresponding theoretical values of $K_{i_c}$ are 0.874 and 3.349, respectively. According to the foregoing analysis, when $0.84<K_{i_c}<3.349$, the system is stable and the state variable tends to decay and converge to a stable equilibrium point; when $K_{i_c} = 0.874$ or $K_{i_c} = 3.349$, the system is unstable, and the state variable will appear as an equal amplitude oscillation and converge to a stable limit cycle. Then the system’s bifurcation characteristics were analyzed using the system’s rotating speed $x$ as the state variable. The program was compiled with MATLAB, and the calculation was conducted using the local maximum algorithm. The bifurcation diagram was plotted as
shown in figure 6. One can observe that the bifurcation points of $K_p$ are $T_1$ and $T_2$, and the corresponding values of $K_p^*$ are consistent with the previous theoretical values (0.874, 3.349).

**Figure 5.** Bifurcation line.

**Figure 6.** Bifurcation diagram when rotating speed $x$ varies with $K_p$ ($K_i=0.5s^{-1}$).

### 4.2 Dynamic response characteristic

This section aims to verify the foregoing results of the theoretical analysis and obtain an in-depth insight into the dynamic response characteristics of the hydraulic turbine regulating system with saturable nonlinearity. Six points from figure 5 were selected for numerical simulations. The values of the state parameters at different points are listed in Table 1.

| State point/state parameter | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ |
|-----------------------------|-------|-------|-------|-------|-------|-------|
| $K_d$ (s)                   | 2     | 2     | 2     | 2     | 2     | 2     |
| $K_i$ (s$^{-1}$)            | 0.3   | 0.4   | 0.5   | 0.5   | 0.6   | 0.7   |
| $K_p$                       | 0.874 | 0.874 | 0.874 | 3.349 | 3.349 | 3.349 |

Theoretical states of phase space trajectory corresponding to different variables.

| Theoretical states of phase space trajectory | Equilibrium point | Equilibrium point | Bifurcation point | Bifurcation point | Limit cycle | Limit cycle |
|---------------------------------------------|-------------------|-------------------|-------------------|-------------------|--------------|--------------|

By means of numerical simulations, the dynamic response processes and the phase space trajectories of the characteristic variables ($q_t$, $x$, $y$) corresponding to the six state points in Table 1 were calculated as shown in figure 7.

We can observe from figure 7 that:

1. The phase space trajectories of the variable response via numerical simulations fits well with the results of the theoretical analysis. For the bifurcation points $S_1$ and $S_4$, after the disturbance was applied to the system, the variables $q_t$, $x$ and $y$ exhibited the equal amplitude oscillations, which corresponded to the stable limit cycle in the phase space. For the points $S_2$ and $S_3$ in the stable domain, the variables converged to the equilibrium points after several cycles of damped oscillation, and the corresponding phase space trajectories were reached at the stable equilibrium points after several cycles of motion. For the points $S_5$ and $S_6$ in the unstable domain, the variables $q_t$, $x$ and $y$ firstly diverged after the disturbance, then tended to approach a stable equal amplitude oscillation and the corresponding phase space trajectories were the limit cycles that firstly diverged and were then stable.
(a) Dynamic response process of $S_1$.

(b) Dynamic response process of $S_4$.

(c) Phase space trajectory of $S_1$.

(d) Phase space trajectory of $S_4$.

(e) Dynamic response process of $S_2$.

(f) Dynamic response process of $S_3$.

(g) Phase space trajectory of $S_2$.

(h) Phase space trajectory of $S_3$. 
By comparing the results from the points $S_2$ and $S_3$, the smaller the value of $K_i$ (i.e., the state parameter is further away from the bifurcation points) the faster the system will return to the equilibrium point, which indicates that a state parameter in the stable domain far away from the bifurcation point increases the stability of the system. Similarly, when comparing the points $S_5$ and $S_6$, the larger the value of $K_i$ (i.e., the state parameter is further away from the bifurcation points) the faster the system diverges and the larger the steady-state amplitude of the characteristic variable will be, which indicates that a state parameter in the stable domain far away from the bifurcation point decreases the stability of the system.

4.3 Comparison between PID-type and PI-type controller

If the PID-type speed controller was replaced by a PI-type controller in the regulating system, The transfer function of the PI-type controller can be written as [15-16]:

$$G_p(s) = K_p + K_i/s$$

(21)

Which corresponds to the transfer function of the PID-type speed controller when $K_d=0$. As stated above, figure 8 displays the bifurcation set of $K_d$.

As shown in figure 8, when the PID controller was used, the larger the value of $K_d (0<K_d<K_d^*)$, the larger the stability of the domain will become. In contrast, when the PI controller was used, i.e., when $K_d=0$, the system’s stable domain was significantly smaller than that of the PID controller, which suggests that the adoption of a PID controller is more favourable for the system stability than the adoption of a PI controller.

4.4 Comparison between nonlinearity and linearity
If the nonlinearity of the speed regulating system is not taken into account, the only difference between the linear dynamic model and the nonlinear dynamic model (i.e. equation (14)) of the hydro-turbine regulating system of the hydropower station with upstream and downstream double surge chambers is the expression of $y'$. For the linear dynamic model, we have

$$y = \frac{1}{T_y} \left[ K_p (r - x) + K_d x - K_i x - y \right] \quad (22)$$

The stability of the above linear system was analyzed to obtain the stable domain of the PID parameter space. Figure 9 displays the bifurcation line with the same $K_d$, from which we can observe that the stable domain of a nonlinear system is smaller than that of a linear system with the same $K_d$ value.

5. Conclusions
Using the Hopf bifurcation theory, this article conducted a bifurcation analysis of a hydro-turbine regulating system with upstream and downstream double surge chambers, in which the saturable nonlinearity of the speed controller was taken into account. The following conclusions can be drawn due to the study of the system’s stability from the results of the bifurcation analysis.

(1) The Hopf bifurcation of the hydro-turbine regulating system with upstream and downstream double surge chambers in which the saturable nonlinearity of the speed controller is taken into account is supercritical. In the $K_p-K_i$ coordinate system, the bifurcation points constitute the bifurcation line that divides the system’s stable and unstable domains. In the stable domain, the further the system’s parameter is away from the bifurcation point, the faster the phase space trajectory of the characteristic variable will return to the stable equilibrium point. In contrast, along the bifurcation line or in the unstable domain, the phase space trajectory of the characteristic variable of the system parameter forms a stable limit cycle.

(2) The use of a parallel PID-type controller is more beneficial to the system’s stability than the use of a PI-type controller.

(3) The consideration of saturable nonlinear elements is adverse to the system’s stability.

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Appendix
The subscript 0 represents the value of initial moment. $i=1, 2, t$, where $l$ and 2 represent diversion tunnel and tailrace tunnel, respectively, $t$ represents penstock.

| $Q_i$, $L_i$, $f_i$ | flow, length, sectional area of pipeline | $H$ | working head |
|-----------------|---------------------------------|-----|--------------|
| $F$             | area of surge chamber           | $h_i$ | head loss of pipeline |
| $\Delta Z$     | change of surge chamber water level | $n$ | rotational speed |
| $T_y$           | servomotor response time constant | $Y$ | guide vane opening |
| $T_{ui}$        | time constant of fluid inertia   | $F_i$ | area of surge chamber |
| $M_{eg}$, $M_i$ | resistance torque, dynamic torque | $T_u$ | time constant of unit inertia |
| $K_p$, $K_d$, $K_i$ | proportional, derivative, integral gain | $e_g$ | load self-regulation coefficient |
| $e_{dx}$, $e_x$, $e_y$ | transfer coefficients of turbine torque | $r$ | rotating speed disturbance |
| $e_{dth}$, $e_{qth}$, $e_{qiv}$ | transfer coefficients of turbine flow | $z$, $u$ | control output |
| $m_{d} = (M_d - M_{d0})/M_{d0}$ | relative dynamic torque | $x = (n - n_0)/n$ | relative rotational speed |
| $m_{g} = (M_g - M_{g0})/M_{g0}$ | relative resistance torque | $y = (Y - Y_0)/Y_0$ | relative guide vane opening |
\[ z_i = \Delta z_i / H_0 \] relative change of surge chamber water level

\[ q_i = (Q_i - Q_0) / Q_0 \] relative change of the flow of pipeline

\[ h = (H - H_0) / H_0 \] relative working head of the turbine

\[ T_{ri} = F_i H_0 / Q_0 \] time constant of surge chamber

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