Research Article

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Synergistic effects of halloysite nanotubes with metal and phosphorus additives on the optimal design of eco-friendly sandwich panels with maximum flame resistance and minimum weight

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Abstract: The present work addresses the optimal design of sandwich panels made of flax fabric (FF)/vinyl ester (VE) composite face sheets and honeycomb VE core. The sandwich structures are first optimized in terms of flammability by obtaining the best combination of ammonium polyphosphate (APP), halloysite nanotube (HNT), and magnesium hydroxide (MH) as three flame retardants (FRs). Using the Taguchi method and horizontal burning test, it is shown that [6, 3, and 3%] and [1, 0.5, and 0%] are the optimal combinations of APP, HNT, and MH for the face sheets and core, respectively. Cone calorimeter test results indicate that the optimal FR combinations significantly decrease the mass loss rate (MLR), heat rate release (HRR), total smoke release (TSR), and maximum average release heat emission (MARHE). The FR sandwich structures are then geometrically optimized under compressive loads based on their weight. Different failure modes are considered as the design constraints of the optimization problem. Imperialist competitive algorithm (ICA), as a powerful meta-heuristic algorithm, is implemented to considerably reduce the computational cost of the optimization process. The results of this study show that proper combinations of FR additives can increase the flame retardancy while decreasing the weight of sandwich panels.

Keywords: honeycomb sandwich structures, flax fabric/vinyl ester composites, flame retardant additives, weight minimization, imperialist competitive algorithm

1 Introduction

With increasing global public awareness and concerns about environmental pollutants, as well as the overproduction of nonrenewable petroleum-based products, the use of natural fiber-reinforced composites has gained significant attention in recent years [1–4]. Flax is one of these natural materials that can be used in different configurations such as mats, rovings, fabrics, monofilament fibers, and yarns [5]. Owing to their low environmental impact, lightweight, low energy consumption, cost effectiveness, recyclability, and biodegradability, flax-reinforced composites have been successfully used in real applications such as automotive parts, construction, consumer goods, furniture, pipes, tanks, and rotor blades [6–10]. In particular, these natural fibers are a good alternative for conventional synthetic fibers used in sandwich structures [11–15].

Despite the excellent properties of flax fibers, their use in composite structures can be challenging. One of the most important issues is their low flame resistance [16–20]. Concerning the flax-reinforced sandwich structures, Kandare et al. [21] studied the fire retardancy of eco-friendly sandwich components composed of balsa core and flax-reinforced epoxy face sheets. They established the idea of using glass fiber/ammonium polyphosphate (APP) coating and presented the results of their research using a cone calorimeter. It was found that the flammable properties of the biodegradable sandwich structures can be significantly improved in the presence of the fire protective coating. Prabhakaran et al. [22] comprehensively explored the thermal conductivity, thermal expansion, flammability, and thermal stability of the environmental-friendly cork/flax/epoxy sandwich structures. Manobala et al. [23] compared the thermomechanical behavior of four types of sandwich panels obtained from a combination of flax-reinforced face sheets, glass-reinforced face sheets, coir cores, and
polyurethane cores. They measured and compared the thermal conductivity, flammability, and tensile strength of the structures. The experimental results indicated that the renewable sandwich structures consisting of flax/epoxy in face sheets and coir as core possess lower thermal conductivity and higher limiting oxygen index.

Honeycomb sandwich structures are lightweight engineering structures consisting of a thick and light honeycomb core covered by two relatively thin but stiff layers. Owing to their high efficiency, these sandwich structures have found extensive applications in diverse industries, including aerospace, automotive, railway, marine, and civil [24–26]. In this regard, numerous studies have addressed the physical, mechanical, and thermal properties of honeycomb structures [27–29] to optimize them for different purposes such as weight, deflection, impact, and sound transmission loss [30–34].

The present study is aimed to investigate the synergistic effects of APP, halloysite nanotube (HNT), and magnesium hydroxide (MH) additives on the optimal design of eco-friendly sandwich panels with flax fabric (FF)/vinyl ester (VE) composite face sheets and VE honeycomb core based on flame retardancy and weight. The design of the structures consists of two main parts: the design or selection of proper constituent materials and the geometrical design. Therefore, the optimal flame retardant (FR) materials are first determined for the face sheets and the core in Section 2. The main goal of this section is to optimize the amounts of three FR additives, namely APP, HNT, and MH, to maximize the flame retardancy of the structure regardless of its geometry and weight. The results of this section are obtained based on the horizontal test, cone calorimeter test, and compression test. Then, in Section 3, weight minimization of FR honeycomb sandwich panels subjected to compressive loads is investigated by considering various possible failure modes. The effects of optimal FR combinations (obtained from Section 2) on the minimum weight of sandwich panels are studied. Analytical approaches are used to predict the failure modes as the design constraints, and the optimization process is performed by imperialist competitive algorithm (ICA) that has been successfully implemented in many engineering optimization problems [35–38].

2 Flame retardancy optimization of sandwich panels

As mentioned earlier, the focus of the present work is on the design of sandwich panels, whose face sheets and the honeycomb core are made of FF/VE and VE, respectively.

It should be mentioned that the FF was purchased from Sungchang Industries (Korea) and the VE was supplied by CCP composites (Korea). To remove the greasy material on the fabrics, 2 wt% sodium hydroxide (NaOH) solution was used. The NaOH was obtained from Samchun Chemical Co., Korea. Since both the core and face sheets are flammable, FR additives were used to reduce the flammability of the structures. Among the various available FR additives, APP, HNT, and MH (from Samchun Pure Chemical, Korea) were selected. APP is an inorganic salt of polyphosphoric acid and ammonia whose flame resistance can be assigned to its high phosphorus and nitrogen contents. APP is an effective FR for thermoset resin such as epoxy and VE due to the involvement of both APP and resin in the charring during the combustion process. HNT is an efficient and popular FR among inorganic nanofillers due to tubular nanostructure, high aspect ratio, and natural availability [39]. The incorporation of HNT has a positive effect on the thermal degradation of VE-based intumescent FR systems. MH is an environmentally friendly and nontoxic FR mineral additive. Upon exposure to high temperatures, it experiences an endothermic decomposition, absorbs combustion heat, releases water vapor and nonflammable gas, and suppresses smoke. MgO is its main decomposition product with few effects in promoting polymer charring and low ability to form an effective barrier.

The following subsections briefly describe the optimization steps of FR sandwich panels.

2.1 Optimal FR face sheets

Taguchi design of experiment (DOE) is one the most popular and well-known statistical techniques that can significantly reduce the time and cost of experiments by decreasing the number of required tests. To this end, the factors (APP, HNT, and MH) and their levels (0, 3, and 6%) were defined. An L9 orthogonal array was selected to design the experiments for different combinations of the FRs, as shown in Table 1.

The FR composites were fabricated using the vacuum-assisted resin transfer molding (VARTM) technique considering different concentrations of APP, HNT, and MH. The FRs were spread between the layers of FFs before resin infusion. After preparing the nine composites proposed by Taguchi, horizontal flammability test was carried out according to the U94 standard test. This test determines the burning time and consequently the burning rate for the specimens. The digital pictures of one of the composite
specimens (H) before, during, and after the horizontal burning test are presented in Figure 1. Furthermore, Table 2 depicts the burning time (the time required for the flame to reach from the initial reference point to the terminal reference point) and burning rate of the nine composites. As observed, all combinations of APP, HNT, and MH prolong the burning time or decrease the burning rate of the pure composite (sample A). Note that specimen H is the composite with the highest flame retardancy (highest burning time) among the nine specimens.

The S/N ratios of the experimental data were then evaluated. Based on this analysis, the effect of each FR additive on the flammability of the composites can be evaluated, as shown in Figure 2. Accordingly, the incorporation of APP additive promotes the flame retardancy of the VE/FF composites whose effect increases at higher loadings. It is speculated that APP, which is liable for the formation of dense char via intumescent mechanism by producing phosphoric acid during combustion, can react with carbon agents (VE/FF), thereby, delay the burning time and resist further flame propagation. Moreover, the ammonium functional group present in the APP can serve as a blowing agent to enhance intercomponent space or dilute the oxygen content of the air and promote the flame retardancy. In the case of HNT, the addition of 3% HNT into the VE/FF composites delays the burning time by increasing flame retardancy. HNT can produce stable cohesive char,

![Figure 1: Schematic representation of specimen H for the face sheets: (a) before, (b) during, and (c) after burning in the horizontal burning test.](image-url)
which is responsible for trapping the flammable volatiles and forming solid barriers on the composites’ surface, reducing the decomposition rate while increasing the decomposition temperature. However, homogeneous dispersion of particles in the composites plays a vital role in the maintenance of the char layer. Hence, the higher loadings of HNT may deteriorate the fire resistance due to nonuniform distribution and agglomeration of the nanofillers.

Similarly, MH contents up to 3% lead to a significant flame retardancy. During decomposition of MH, the released magnesium oxide moiety and water serve as an insulator on the surface of the polymer composites and weakens the flame intensity, respectively. High MH loadings also decline the fire resistance due to the non synergistic effect with other additives as well as the agglomeration phenomenon.

S/N ratio analysis can also predict the best combination of the FRs for synergistic maximization of the flame retardancy of the VE/FF face sheets. Based on Figures 2, 6, 3, and 3% are the optimal values for APP, HNT, and MH to yield the maximum flame resistance in VE/FF face sheets, respectively. Therefore, further research was done by manufacturing a new composite coded as the J sample, with the optimal combination of FR additives. The horizontal burning test was carried out on the J sample to confirm its optimal flame retardancy behavior in terms of the burning time. The experiment showed that the burning time of the J sample reached a delay time of 462 s, almost 37% higher than the A sample and even 8% more than the H samples, indicating its superior flame retardancy behavior, as expected.

### 2.2 Optimal FR core

Here, similar to the previous subsection, the optimal combination of FR additives was obtained for the VE honeycomb core. Noteworthy, the face sheets are much more important than the core in the design of a flame-resistant sandwich composite structure due to their higher chance of exposure to fire. Therefore, in the present study, smaller amounts of FR additives (at three levels of 0, 0.5, and 1%) were used for the core compared to the face sheets to be cost effective. Similarly, nine different combinations of APP, HNT, and MH were suggested based on the Taguchi technique (Table 1). The proposed specimens

| Sample | Burning time (s) | Burning rate (mm/s) |
|--------|-----------------|---------------------|
| A      | 338             | 0.370               |
| B      | 362             | 0.345               |
| C      | 368             | 0.340               |
| D      | 407             | 0.307               |
| E      | 408             | 0.306               |
| F      | 344             | 0.363               |
| G      | 397             | 0.335               |
| H      | 428             | 0.292               |
| I      | 417             | 0.300               |

Table 2: L9 Horizontal burning test results for FF/VE composites used in the face sheets

![Figure 2: Main effects plot for S/N ratios for burning time.](image-url)
were prepared and subjected to the horizontal burn test, based on the UL94 standard. Figure 3 shows the digital snapshot of the specimens after the test. All composite specimens burned completely except composite H, in which the flame did not reach the terminal reference point. In this case, S/N analysis cannot be performed accurately. Therefore, the specimen H was selected as the optimal specimen.

2.3 Optimal FR adhesive layer

One of the main factors with significant influence on the thermal and mechanical performances of sandwich structures is the adhesive layer between face sheets and core. Despite the existence of different types of adhesives, in the present study, VE with the optimal FR combination for the core (obtained in Section 2.2) was used to bond the face sheets and core due to its compatibility with core materials and flame retardancy.

2.4 Optimal FR sandwich panels

In the previous three subsections, the optimal FR materials for the face sheets, core, and adhesive layer were selected. In this subsection, the effects of FRs on the flammability of sandwich panels are addressed. To this end, two types of sandwich panels were prepared. The first panel consisted of FF/VE composite face sheets, VE honeycomb core, and VE adhesive layer without any FR additives, whereas the second one was composed of the optimal FR flax-reinforced face sheets, optimal FR honeycomb core, and optimal FR adhesive layer. The process of making FR sandwich panels is briefly illustrated in Figure 4. First, two very thin J combination-based face sheets were fabricated using VARTM. Then, the honeycomb cores were made through four main steps: (1) designing a mold in design software, (2) making a plastic mold composed of PLA using a 3-D printer, (3) making a negative silicone mold, and (4) preparing the H combination-based VE composite and pouring it in the silicone honeycomb mold. Finally, the prepared face sheets were attached to the core by H combination-based adhesive layer and subjected to the 5 MPa pressure for 1 day.

After preparing the FR and non-FR sandwich panels, cone calorimeter test was carried out to provide an accurate and detailed comparison between their fire properties. Mass lost rate (MLR) is one of the most important flammability properties of the sandwich structures, especially in the early stages of burning. This is because in these structures, the thin face sheets bear almost all applied loads; moreover, they are first exposed to fire. Thus, structures possessing face sheets with high MLR will soon fail upon exposure to fire. The variations of this parameter during the first 6 min of the burning process are listed in Table 3. As observed, FR additives can considerably (18–32%) decrease the MLR for the sandwich panel. Furthermore, during the first 4 min from the ignition, the MLR of the non-FR sandwich panel gradually decreased; but then it began to increase. The reason was that after this time, the structure core was exposed to the flame, and due to its higher flammability relative to the face sheets, the MLR increased. Similar

![Figure 3: Schematic representation of nine different specimens for the core after burning in the horizontal burning test.](image-url)
behavior was observed for the burning of the FR-containing sandwich panels. The difference was in the longer time duration for core exposure to the flame, that is, the MLR began to increase 5 min after ignition. Heat rate release (HRR) is another prominent parameter in the flammability of composite structures. The cone calorimeter test data revealed that a proper FR combination can significantly decrement the HRR of sandwich structures (34–39%) in the first 6 min of the burning process. Again, a decrease–increase pattern was observed for both FR-containing and non-FR sandwich specimens due to the differences in flammability of composite face sheets and core. One more crucial parameter to evaluate the flammability of structures is the maximum average release heat emission (MARHE). The cone calorimeter machine showed MARHE values of 357.97 and 221.46 kW/m² for the non-FR and FR-containing sandwich panels, respectively. It implies the FR additives could decrease the MARHE of the sandwich composite by ~38%. Finally, the effects of FR additives on the amount of smoke production were examined. Total smoke release (TSR) of the non-FR specimen was 5495.6 m²/m², which decremented to 4629.0 m²/m² in the FR-containing sandwich composite. Overall, the cone calorimeter test showed the successful use of optimal combinations of APP, HNT, and MH to promote flame retardancy in the sandwich panels.

### 2.5 Compressive properties of the FR materials in the face sheets and core

So far, proper flame-resistant compositions were found. In the next step, an investigation was made to examine the effects of the optimal FR combinations on the compressive properties of materials used in the face sheets and core. This is because in Section 3, geometric design of FR and non-FR sandwich panels under compressive loads based on their weight is investigated, for which the values of compressive Young’s modulus and strength of the face sheets’ materials as well as the compressive Young’s

| Property | Unit          | Sandwich type | 1 min | 2 min | 3 min | 4 min | 5 min | 6 min |
|----------|---------------|---------------|-------|-------|-------|-------|-------|-------|
| MLR      | g/(s m²)      | Non-FR        | 19.18 | 15.74 | 12.85 | 12.25 | 12.74 | 13.36 |
|          |               | FR            | 15.78 | 12.40 | 9.93  | 8.83  | 8.71  | 9.32  |
| HRR      | kw/m²         | Non-FR        | 421.15| 326.53| 270.96| 265.29| 276.48| 289.35|
|          |               | FR            | 271.18| 216.58| 179.23| 163.94| 168.21| 182.52|
modulus of the core materials are required. To this end, compression test was conducted on four different types of specimens: (1) FF/VE composites without FRs, (2) FF/VE with [6, 3, and 3%] FRs, (3) pure VE, and (4) pure VE with [1, 0.5, and 0%] FRs. The experimental data obtained from the tests are presented in Table 4. As this table shows, the optimal FR combinations led to 6.35, 5.64 and 7.28% increase in the face sheets’ modulus, face sheets’ strength, and core modulus. In addition, the Poisson’s ratio was measured, the value of which was approximately 0.26 for FF/VE composite specimens and 0.31 for VE.

3 Weight minimization of sandwich panels

In this section, the effect of FR additives on the minimum required weight of sandwich panels is explored. A beam-shaped sandwich structure composed of FF/VE composite face sheets and VE honeycomb core is considered (see Figure 5). In this figure, \( L \), \( h_f \), \( h_c \), and \( H \) denote the length, face sheet thickness, core thickness, and total thickness, respectively. The honeycomb core consists of regular hexagons whose cell wall thickness and cell size are shown by \( t_{cell} \) and \( d_{cell} \). The sandwich beam is assumed to be subjected to compressive loads at its ends. Depending on the load amplitude and geometric parameters, different failure modes such as global buckling, face sheet wrinkling, face sheet dimpling, core shear instability, and face sheet failure may occur. The main goal here is to optimize the geometric parameters of the sandwich beam to minimize the structure weight while maintaining its resistance against the applied load with no failure modes. The influence of the optimal combinations of FRs on the optimal geometric parameters and consequently on the weight of the structure is investigated. The following equation expresses the weight per unit area of the honeycomb sandwich beam:

\[
W = 2h_f \rho_f + h_c \rho_c' + W_{\text{adhesive}},
\]

where \( \rho_f \) denotes the density of the face sheets, and \( \rho_c' \) shows the density of honeycomb core that can be calculated by having the density of VE \( \rho_{VE} \) using the approach developed in an earlier study [28]. It is necessary to mention that the density of the composite face sheets and VE is 1,220 and 1,030 kg/m³, respectively. Furthermore, the effects of FRs as well as the adhesive layer on the density of structures are ignored. Moreover, the mechanical properties obtained from the compressive test in Section 2.5 (see Table 4) are used in the optimization problem. Note that the best solution is found by ICA in order to speed up the optimization process. The constrained optimization problem can be expressed in the following form:

\[
\begin{align*}
\text{Minimize} \quad & W = W(h_f, h_c, t_{cell}, d_{cell}), \\
\text{Subject to} \quad & \sigma_{\text{applied}} < \sigma_{\text{GlobalBuckling}}, \\
& \sigma_{\text{applied}} < \sigma_{\text{Dimpling}}, \\
& \sigma_{\text{applied}} < \sigma_{\text{Wrinkling}}, \\
& \sigma_{\text{applied}} < \sigma_{\text{CoreShearStability}}, \\
& \sigma_{\text{applied}} < \sigma_{\text{FacesheetFailure}}.
\end{align*}
\]

In the following subsections, the analytical models of the failure modes are first briefly described. Then, ICA and its main steps are summarized. Finally, the optimization results are presented for different load amplitudes.

3.1 Failure modes

3.1.1 Global buckling equations

The first possible failure mode in sandwich beams under compressive loads is global buckling that shows the global instability of the structure. Consider a sandwich beam with honeycomb core and laminated composite face sheets, as shown in Figure 5. The sandwich beam is assumed to be under compressive in-plane loads at its ends. Based on piecewise low-order shear deformation theory [40], the displacement fields for the sandwich beam in the face sheets and core are expressed in the following forms:

**Table 4: Compression test results for the FR and non-FR materials used in the face sheets and core**

| Property                | Materials          | Type                         | Value   |
|-------------------------|--------------------|-------------------------------|---------|
| Compressive young modulus | FF/VE (face sheets) | Without FRs                  | 7.87 GPa |
|                         | VE (honeycomb core) | With optimal FR combination* | 8.37 GPa |
|                         |                    | Without FRs                  | 5.77 GPa |
|                         |                    | With optimal FR combination**| 6.19 GPa |
| Compressive strength    | FF/VE (face sheets) | Without FRs                  | 159.5 MPa|
|                         |                    | With optimal FR combination* | 168.5 MPa|

* 6% App, 3% HNT, and 3% MH.
** 1% App, 0.5% HNT, and 0% MH.
where subscripts $t$, $b$, and $c$ represent the top face sheet, bottom face sheet, and core, respectively. In addition, $\psi$ denotes the rotation angle of the straight line connecting the midpoint of the face sheet in the $xoz$ plane.

The von Karman’s strain–displacement relations for the face sheets and core are as follows:

$$w(x, z) = w(x),$$
$$u_t(x, z) = u_0(x) - \frac{h_c + h_f}{2} \psi - \left(z - \frac{h_c + h_f}{2}\right) \frac{\partial w}{\partial x},$$
$$u_b(x, z) = u_0(x) + \frac{h_c + h_f}{2} \psi - \left(z - \frac{h_c + h_f}{2}\right) \frac{\partial w}{\partial x},$$
$$u_c(x, z) = u_0(x) - z \left(\frac{h_c + h_f}{h_c} \psi - \frac{h_f}{h_c} \frac{\partial w}{\partial x}\right).$$

The stress components in the $k$th layer of the top composite face sheet ($kt$), $k$th layer of the bottom face sheet ($kb$), and the honeycomb core can be expressed by the following equation:

$$\sigma_{xi}^{kt} = \sigma^{kt}_{11} \varepsilon_{xi}^{kt}; \quad \sigma_{xs}^{kb} = \sigma^{kb}_{11} \varepsilon_{xs}^{kb}; \quad \sigma_{xi}^{c} = E_{c}^{*} \varepsilon_{xi}^{c};$$

where $Q_{11}$ denotes the equivalent stiffness coefficients of the face sheets [41]. Moreover, $E_{c}^{*}$ and $G_{c33}$ show the equivalent mechanical properties of the honeycomb core, which can be found in a study [28]. The stress resultants for the face sheets and core can be obtained using the following equation:

$$N_{xi} = A_{11}^{(0)} \varepsilon_{xi}^{(0)}; \quad N_{xs} = A_{11}^{(0)} \varepsilon_{xs}^{(0)}; \quad M_{xi} = D_{11}^{(1)} \varepsilon_{xi}^{(1)}; \quad M_{xs} = D_{11}^{(1)} \varepsilon_{xs}^{(1)};$$

where

$$A_{11}^{(0)} = \frac{h_f}{2}, \quad h_f = D_{11}^{(1)} \frac{1}{h_f}; \quad h_f = Q_{11} \int_{h_f/2}^{h_f/2}\left[z - \frac{h_c + h_f}{2}\right]^2 dz,$$

$$A_{c11} \varepsilon_{c11}^{(1)} = D_{c11}^{(1)} \varepsilon_{c11}^{(1)} = \frac{h_c}{2} G_{c33} dz.$$

Therefore, the strain energy ($U$) of the structure can be calculated.
\[
U = U_l + U_b + U_c = \frac{1}{2} \int_0^L \int_{-h/2}^{h/2} \sigma_{xExk} dx \times dz
+ \frac{1}{2} \int_0^L \int_{-h/2}^{h/2} \sigma_{xExk} dx \times dz
+ \frac{1}{2} \int_0^L \int_{-h/2}^{h/2} \sigma_{xExk} dx \times dz
g \times \left( \frac{n \pi}{L} \right)^2 G(z) - P_c = 0 \]

In the next step, the external work done by the compressive load with amplitude \( P_c \) is calculated as follows:

\[
V = -\frac{1}{2} \int_0^L P_c \frac{\partial w}{\partial x} dx.
\]

According to the principle of virtual work [42], we have:

\[
\delta U + V = 0.
\]

Two out-of-plane buckling equations can be obtained by combining equations (8–10) for the sandwich beam with immovable simply supported boundaries:

\[
\begin{align*}
g_x^{(3)} \frac{\partial^4 w}{\partial x^4} - g_x^{(2)} \frac{\partial^3 \psi}{\partial x^3} = S_{czz} \left( \frac{\partial^2 w}{\partial x^2} - \frac{\partial \psi}{\partial x} \right) + P_c \frac{\partial^2 w}{\partial x^2} = 0, \\
g_x^{(2)} \frac{\partial^3 w}{\partial x^3} - g_x^{(1)} \frac{\partial^2 \psi}{\partial x^2} = S_{czz} \left( \frac{\partial w}{\partial x} - \psi \right) = 0,
\end{align*}
\]

in which

\[
\begin{align*}
g_x^{(1)} &= \frac{(h_c + h_f)^2}{2} A_{f1} + \left( \frac{h_c + h_f}{h_c} \right)^2 D_{c11}; \\
g_x^{(2)} &= \frac{h_f}{h_c} \left( \frac{h_c + h_f}{h_c} \right) D_{c11}; \\
g_x^{(3)} &= 2 D_{f11} + \left( \frac{h_f}{h_c} \right)^2 D_{c11}; \quad S_{czz} = \left( \frac{h_c + h_f}{h_c} \right)^2 A_{c55}.
\end{align*}
\]

Moreover, the related boundary conditions can be derived as follows:

\[
\begin{align*}
x = 0, L: \begin{cases} \\
\delta w = 0 \quad & \text{or} \quad - \frac{\partial^3 w}{\partial x^3} + g_x^{(2)} \frac{\partial^2 \psi}{\partial x^2} + S_{czz} \left( \frac{\partial w}{\partial x} - \psi \right) = 0, \\
\frac{\partial w}{\partial x} = 0 \quad & \text{or} \quad g_x^{(3)} \frac{\partial^4 w}{\partial x^4} - g_x^{(2)} \frac{\partial^3 \psi}{\partial x^3} = 0, \\
\delta \psi = 0 \quad & \text{or} \quad - g_x^{(2)} \frac{\partial^3 w}{\partial x^3} + g_x^{(1)} \frac{\partial^2 \psi}{\partial x^2} = 0.
\end{cases}
\end{align*}
\]

The following solution is proposed for the buckling equations of simply supported sandwich beams:

\[
W(x) = \sum_{n} A_n \frac{n \pi}{L} x; \quad \varphi(x) = \sum_{n} B_n \cos \frac{n \pi}{L} x, \quad (14)
\]

where \( A_n \) and \( B_n \) are the unknown mode amplitudes and \( n \) represents the mode number. By substituting equation (14) into equation (11), we obtain the following equation:

\[
\begin{bmatrix}
\delta w = 0 \\
\frac{\partial w}{\partial x} = 0 \\
\delta \psi = 0
\end{bmatrix}^{T} \begin{bmatrix}
\frac{n \pi}{L}^4 G^{(3)} + \left( \frac{n \pi}{L} \right)^2 \left( S_{cz} - P_c \right) - \frac{n \pi}{L}^3 S^{(2)}_{cz} + \left( \frac{n \pi}{L} \right) S^{(1)}_{cz} - S_{czz}
\end{bmatrix}^{T} = 0.
\]

By setting the determinant of the above matrix of coefficients to zero, the following solution is obtained to predict the critical buckling loads of sandwich beams.

\[
P_{ct} = S_{czz} - \frac{1}{\left( \frac{n \pi}{L} \right)^4 G^{(3)} + \left( \frac{n \pi}{L} \right)^2 \left( S_{cz} - P_c \right) - \frac{n \pi}{L}^3 S^{(2)}_{cz} + \left( \frac{n \pi}{L} \right) S^{(1)}_{cz} - S_{czz}}.
\]

### 3.1.2 Face sheet dimpling (or monocell buckling)

Face sheet dimpling is a type of instability of honeycomb sandwich panels in which a face sheet over one cell buckles like a small plate supported by the cell walls. The critical stress for face sheet dimpling can be calculated by the following equation [43]:

\[
\sigma_{Dimpling} = 2 \frac{E_1 E_2}{(1 - \nu_{12}\nu_{21})} \frac{h_f}{d_{cell}},
\]

where \( E_f \) and \( E_2 \) denote the compressive moduli of the face sheets in the direction and perpendicular to the composite fibers. Furthermore, \( \nu_{12} \) and \( \nu_{21} \) show Poisson’s ratios for the face sheets.
3.1.3 Face sheet wrinkling

Wrinkling is a type of instability that may appear across many cells of the honeycomb core of sandwich panels in the direction of the applied load. In the present work, the following criterion is considered for this instability [43].

\[
\sigma_{\text{Wrinkling}} = \sqrt{\frac{2}{3} \frac{E_f E_z}{E_c} \frac{h_f}{h_c}}.
\]  

(18)

3.1.4 Core shear instability

The fourth constraint for designing honeycomb sandwich panels under in-plane compressive load is core shear instability. The critical stress for this instability can be calculated by the following equation [44]:

\[
\sigma_{\text{CoreShearStability}} = \frac{E_f U_{z2}}{H_c},
\]  

(19)

where

\[ U_{z2} = h_c G_{c13}^{*}, \quad H_c = 2h_f E_f, \]

in which, \( G_{c13}^{*} \) is calculated according to an earlier study [28].

3.1.5 Face sheet failure

When the stress created by the applied external force exceeds the equivalent strength of the face sheets, a failure may occur in them. Here, the Tsai–Wu criterion is used for determining the equivalent strength for the face sheets, the details can be found in an earlier study [45].

3.2 ICA

As a global search heuristic algorithm, ICA uses imperialism and imperialistic competition processes for optimization steps. Similar to other evolutionary algorithms, ICA begins with an initial population known as countries classified into two major groups: imperialists (i.e., stronger countries) and colonies (the rest of countries). Stronger imperialists possess more colonies. The major steps of the ICA can be listed as follows [46]:

Step 1: Selecting some random points of the function and initializing the empires.

Step 2: Moving the colonies to their most relevant imperialist (assimilating).

Step 3: If an empire possesses a colony stronger than the imperialist, the colony will substitute the imperialist (revolution).

Step 4: Computing the total power of empires (sum of the powers of imperialist and its colonies).

Step 5: Selecting the weakest colony (colonies) from the weakest empire and transferring it (them) to the empire with the highest likelihood of its (their) possession (imperialistic competition).

Step 6: Elimination of the weak empires.

Step 7: Stopping the process if only one empire remains, otherwise starting from Step 2.

For a detailed description of the proposed algorithm, see literature [46].

3.3 Verification study

3.3.1 Verification of global buckling results

Among the mentioned failure modes, the formulation proposed in Section 3.1.1 for global buckling of sandwich beams is new and has not been developed before, to the best of the authors’ knowledge. Therefore, a comparison study is required to verify the accuracy of the present analytical method. A comparison example is presented to evaluate the mechanical buckling loads of a sandwich beam made of aluminum face sheets and an orthotropic core. In this example, the material properties of the face sheets and core are as follows [47,48]:

- Aluminum Face Sheets: \( E = 70 \) GPa, \( v = 0.3 \)
- Orthotropic Core: \( E_1 = 1 \times 10^5 \) MPa, \( E_2 = 109 \) MPa, \( G_{12} = 26.6 \) MPa, \( v = 1 \times 10^{-5} \).

The dimensionless buckling loads of the structure are listed in Table 5 for different values of \( \frac{h_c}{h_f} \) and \( \frac{L}{h_f} \), considering various theories including mixed layer-wise theory (MLWT), zig–zag theory (ZZT), and global-local higher-order theory (GLHT). No significant difference is detected between the buckling loads estimated by the present study and those obtained from other theories [47,48]. Therefore, the developed solution of equation (16) is accurate enough to estimate the buckling load of sandwich beams.

3.3.2 Verification of ICA optimization results

As the second part of the verification study, the performance of the applied ICA is evaluated in weight
minimization of non-FR honeycomb sandwich beam under 200 kN/m compressive loads when the length of beam is 0.3 m. The values of design variables are assumed to vary in the following ranges:

\[ 0.3 \text{ mm} < h_f < 3 \text{ mm}, \quad 2 \text{ mm} < h_c < 20 \text{ mm}, \]
\[ 1 \text{ mm} < d_{\text{cell}} < 10 \text{ mm}, \quad 0.1 \text{ mm} < t_{\text{cell}} < 2 \text{ mm}. \]

The increments of the design variable are considered to be 0.3 mm for the face sheet thickness, 0.2 mm for the core thickness, and 0.1 mm for the other variables. First, all possible solutions (around \(1.65 \times 10^6\) solutions) are separately investigated. It is found that the exact optimum answer can be obtained at \( h_f = 1.2 \text{ mm}, \quad h_c = 18.4 \text{ mm}, \quad d_{\text{cell}} = 7.4 \text{ mm}, \) and \( t_{\text{cell}} = 0.5 \text{ mm}, \) which results in a weight of 5.2511 kg/m² for the structure.

The main goal of the optimization problem here is to obtain the minimum possible weight of the structure a little more than 5.51 kg/m². However, the minimum required weight can change as a result of processes such as revolution and assimilating in the ICA (see Section 3.2). As suggested by Figure 6, a sharp decrease in the predicted optimum value occurs during the next three iterations such that at the fourth iteration, the minimum possible weight is found to be around 5.27 kg/m². The results illustrated in Figure 6 show that the optimization algorithm still proposes smaller values as the weight of the structure at iteration 5, which is around 5.26 kg/m². It should be noted that the values shown in each iteration represent a local minimum. Therefore, the successful performance of the ICA requires reaching the global minimum. Based on Figure 6, the ICA can successfully suggest the optimal geometric parameters of \( h_f = 1.2 \text{ mm}, \quad h_c = 18.4 \text{ mm}, \quad d_{\text{cell}} = 7.4 \text{ mm}, \) and \( t_{\text{cell}} = 0.5 \text{ mm} \) at the seventh iteration, resulting in a weight of 5.2511 kg/m². This answer is the global minimum, and, as expected, there is no change in the optimal response with increasing iterations. It is worth noting that it only takes 1 min for the algorithm to reach this exact optimal response after seven iterations.

From this comparison, two conclusions can be drawn: (1) ICA is sufficiently powerful to handle the optimization problems, especially for the weight minimization of honeycomb sandwich beams and (2) ICA significantly reduces the computational costs. This confirms the reliability of the ICA results of the present study.

### 3.4 Optimization results

The main goal of the optimization problem here is to investigate the effects of FRs on the geometric design of honeycomb sandwich beams. For this purpose, the sandwich beam is presumed to be subjected to compressive

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### Table 5: The non-dimensional critical buckling loads

| \( \frac{h_f}{h_c} \) | \( \frac{l}{d} \) | Method | Present | MLWT [47] | ZYT [48] | GLHT [48] |
|-----------------|---------|--------|--------|----------|--------|--------|
| 5               | 5       |        | 0.014876 | 0.01432 | 0.01486 | 0.01484 |
| 10              | 5       |        | 0.041844 | 0.041084 | 0.04182 | 0.04182 |
| 50              | 5       |        | 0.364936 | 0.34319 | 0.3648 | 0.3648 |
| 25              | 5       |        | 0.009144 | 0.009031 | 0.009143 | 0.009142 |
| 10              | 5       |        | 0.031691 | 0.031096 | 0.03168 | 0.03168 |
| 50              | 5       |        | 0.155846 | 0.14385 | 0.1558 | 0.1558 |
| 50              | 5       |        | 0.008694 | 0.008555 | 0.008692 | 0.008692 |
| 10              | 5       |        | 0.027580 | 0.026762 | 0.02756 | 0.02756 |
| 50              | 5       |        | 0.090749 | 0.083230 | 0.09072 | 0.09072 |

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### Table 6: Parameters of ICA for the optimization process

| Parameters | Value |
|------------|-------|
| Number of initial countries | 100 |
| Number of initial imperialists | 6 |
| Number of decades | 40 |
| Revolution rate | 0.3 |
| \( \beta \) | 2 |
| \( \gamma \) | 0.5 |
| \( \zeta \) | 0.1 |

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![Figure 6: Convergence of the ICA to the optimal solution for a sandwich beam without FR additives (\( P_c = 200 \text{ kN/m} \)).](image-url)
loads with different amplitudes \( (P = 50, 100, \text{and} \ 200 \ \text{kN/m}) \). Again, the values of design variables are assumed to vary in the following ranges:

- \( 0.3 \ \text{mm} < h_f < 3 \ \text{mm}, \ 2 \ \text{mm} < h_c < 20 \ \text{mm}, \)
- \( 1 \ \text{mm} < d_{\text{cell}} < 10 \ \text{mm}, \ 0.1 \ \text{mm} < t_{\text{cell}} < 2 \ \text{mm}, \)

The increment is assumed to be 0.3 mm for the thickness of the face sheets while 0.2 mm for the core thickness and 0.1 mm for the cell wall thickness and cell size. ICA is implemented 10 times for each sandwich beam, and the best solution is selected as the optimal answer. Table 7 provides the optimization results for FR-containing and non-FR sandwich beams subjected to different compressive loads when \( L = 0.3 \ \text{m} \). It is observed that since the face sheets are mainly involved in withstanding the external compressive forces, the required thickness of face sheets may increase with enhancing the compressive mechanical loads. For example, the optimum value of the face sheet thickness is 0.6, 0.9, and 1.2 mm for the non-FR structure when the applied force is 50, 100, and 200 kN/m, respectively. Table 7 also reveals that the minimum required weight of the sandwich structures is somewhat less than that of non-FR structures. This is because, as discussed in Section 2.5, the FR combinations enhanced the compressive modulus and strength of materials. Therefore, they can withstand a higher compressive load. In other words, a smaller amount of materials is required to bear a given compressive load. However, the effect of FRs in reducing the minimum weight of the structure depends on the amount of the applied.

### 4 Conclusion

The present study addressed the synergistic effects of APP, HNT, and MH additives on the optimal design of honeycomb sandwich panels made of FF and VE in terms of flammability and weight. The flammability of the structure was first optimized by applying the best combinations of FR additives in the face sheets, core, and adhesive layer, using the Taguchi method and performing experimental tests. In the second step, the weight of sandwich structures was minimized by optimizing its geometric parameters including the thickness of the face sheet, core and cell wall and the size of the honeycomb cell. The following conclusions can be drawn:

- [6, 3, and 3%] and [1, 0.5, and 0%] were the best FR combinations of APP, HNT, and MH in the face sheets and core, respectively.
- Cone calorimeter test showed that the optimum amount of FRs in the sandwich structures led to an 18–32% decrease in MLR and a 34–39% decline in HRR during the first 6 min of the burning process.
- The experimental data obtained from the cone calorimeter test revealed that optimal FR combinations can decrement the MARHE and TSR in the burning process of the sandwich structure by 38 and 16%, respectively.
- Optimal FR combinations can also enhance the compressive modulus and strength of both FF/VE composite face sheets and VE honeycomb core.
- ICA is a powerful meta-heuristic optimization algorithm capable of predicting the optimum answers for engineering optimization problems, especially for weight minimization of honeycomb sandwich beams, with high accuracy. It can also significantly reduce the CPU time and consequently the computational costs of the optimization process.
- Using optimum amounts of APP, HNT, and MH, it is possible to design a sandwich panel with higher flame resistance and lighter weight.

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