Hierarchical Quark Mixing and Bimaximal Lepton Mixing on the Same Footing

C. S. Kim and J. D. Kim

Physics Department, Yonsei University, Seoul 120-749, Korea

(March 22, 2018)

Abstract

We show that not only the hierarchical quark CKM mixing matrix but also the “bimaximal” lepton flavor mixing matrix can be derived from the same mass matrix ansatz based on the broken permutation symmetry, by assuming the hierarchy of neutrino masses to be $m_1 \simeq m_2 << m_3$. We also reproduce the recently measured angle of unitary triangle, $\sin 2\beta$, as well as all the observed experimental values of $V_{\text{CKM}}$ of the quark CKM matrix. And we predict Jarlskog rephasing invariant quantity, $J_{\text{CP}} \simeq 0.18 \times 10^{-4}$, and the upper bound of the same quantity in the lepton sector, $J_{\text{CP}}^l \leq 0.012$, which may be indeed large enough to generate the lepton number violation of the universe.

PACS number(s): 11.30Er, 12.15Ff, 14.60Pq
1 Introduction

The flavor mixing and fermion masses and their hierarchical patterns remain to be one of the basic problems in particle physics. Within the Standard Model, all masses and flavor mixing angles are free parameters and no relations among them are provided. As an attempt to derive relationship between the quark masses and quark mixing hierarchies, a quark mass-matrix Ansatz was suggested about two decades ago [1]. This in fact reflects the calculability [2] of the flavor mixing angles in terms of the quark masses. Of several Ansatz proposed, the canonical mass matrices of the Fritzsch type [1, 3] and its variations [4] had been generally assumed to predict the hierarchical Cabibbo-Kobayashi-Maskawa (CKM) matrix [5] or the Wolfenstein mixing matrix [6] except the unexpectedly heavy top-quark mass, $m_t$.

For the lepton sector, we now have some evidence for lepton flavor mixing. The observations of the solar neutrino deficit [7, 8] and the atmospheric neutrino deficit [9] can be explained by neutrino oscillations, and these in turn indicate nonzero neutrino masses and mixing. The elements of the lepton flavor mixing matrix are determined from the neutrino oscillation experiments. The results of the atmospheric neutrino experiments can be explained by maximal mixing between the muon neutrino and tau neutrino states, and those of the solar neutrino experiments can be explained by vacuum oscillation with maximal mixing between the electron neutrino and muon neutrino, although they may also be explained through matter enhanced neutrino oscillation. These results imply the “bimaximal” mixing pattern between three flavor neutrinos [10].

Thus, it is likely that the “bimaximal” mixing pattern of the lepton sector is quite different from the hierarchical mixing of the quark sector. At the first glance, the origin of the lepton flavor mixing seems to be quite different from that of the quark sector. However, in this Letter, we will show that the lepton flavor mixing can be obtained via diagonalization of the mass matrix based on the broken permutation symmetry, exactly as in the quark sector. In order to do that, we will first derive the quark mixing matrix from the mass matrix Ansatz based on the broken permutation symmetry. Then, we will extend it to the lepton sector. We will, in particular, show that the “bimaximal” mixing of the lepton flavor can be obtained in such a scheme.

Recently a new general class of mass matrix Ansatz, that respects the quark mass hierarchy of the quark flavor mixing matrix, has been studied. That is a generalization of various specific forms of mass matrix by successive breaking of the maximal permutation symmetry.
The resulting mass matrix in the hierarchical basis is of the form \[1\]:

\[
M_H = \begin{pmatrix}
0 & A & 0 \\
A & D & B \\
0 & B & C \\
\end{pmatrix}.
\] (1)

The matrix \(M_H\) contains four independent parameters even in the case of real parameters so that the genuine calculability is lost. In order to maintain the calculability, one has to make additional Ansatz to provide any relationship between two of the four independent parameters, as shown in Ref. \[12\]. In this Letter, however, we do not make additional Ansatz to keep the calculability but rather introduce additional parameter besides three fermion masses. This additional parameter will be determined from the best fit to the measured quark CKM mixing matrix and the observed neutrino mass hierarchy.

The parameters \(A, B, C\) and \(D\) can be expressed in terms of the fermion mass eigenvalues. In view of the hierarchical pattern of the quark masses, it is natural to expect that \(A < D << C\), and then one can take the mass eigenvalues to be \(-m_1, m_2\) and \(m_3\). The trace and the determinant of \(M_H\) should be given, respectively,

\[
Tr(M_H) = -m_1 + m_2 + m_3, \quad \text{and} \quad Det(M_H) = -m_1 m_2 m_3.
\]

From those relations, we obtain the following form of fermion mass matrix:

\[
M = \begin{pmatrix}
0 & \frac{m_1 m_2 m_3}{m_3 - \epsilon} & 0 \\
\frac{m_1 m_2 m_3}{m_3 - \epsilon} & m_2 - m_1 + \epsilon & \omega(m_2 - m_1 + \epsilon) \\
0 & \omega(m_2 - m_1 + \epsilon) & m_3 - \epsilon \\
\end{pmatrix},
\] (2)

in which the analytic relation between two parameters \(\epsilon\) and \(\omega\) is given by

\[
\omega^2 = \frac{\epsilon(m_3 - m_2 + m_1 - \epsilon)(m_3 - \epsilon) - \epsilon m_1 m_2}{(m_3 - \epsilon)(m_2 - m_1 + \epsilon)^2}.
\] (3)

For given three fermion masses, \(\omega^2\) is determined by fixing the parameter \(\epsilon\). Therefore, the mixing matrix can be expressed in terms of three fermion mass eigenvalues and additional parameter \(\epsilon\). Moreover we have to restrict the parameter range, \(0 < \epsilon < (m_3 - m_2)\), so that the mass matrix is to be real symmetric. Although there are four independent parameters in the mass matrix, Eq. (2), it will be shown in Sections 2 and 3 that by varying the additional parameter \(\epsilon\) we can reproduce not only the hierarchical quark CKM mixing matrix but also the “bimaximal” lepton mixing matrix from the same mass matrix Ansatz of Eq. (2).

The real symmetric mass matrix \(M\) can be diagonalized by an orthogonal matrix \(U\) as follows:

\[
UMU^\dagger = \text{diag}(-m_1, m_2, m_3).
\] (4)
The analytic formula for the orthogonal matrix is obtained, after simple algebra,

\[
U = \left( \begin{array}{ccc}
\sqrt{\frac{m_2 m_3}{Y_1}} (m_3 + m_1 - \epsilon) & -\sqrt{\frac{m_1 (m_3 - \epsilon)}{Y_1}} (m_3 + m_1 - \epsilon) & \sqrt{\frac{m_1 (m_3 - \epsilon)}{Y_1}} \omega (m_2 - m_1 + \epsilon) \\
\sqrt{\frac{m_1 m_3}{Y_2}} (m_3 - m_2 - \epsilon) & \sqrt{\frac{m_2 (m_3 - \epsilon)}{Y_2}} (m_3 - m_2 - \epsilon) & -\sqrt{\frac{m_2 (m_3 - \epsilon)}{Y_2}} \omega (m_2 - m_1 + \epsilon) \\
\sqrt{\frac{m_1 m_2}{Y_3}} \epsilon & \sqrt{\frac{m_1 (m_3 - \epsilon)}{Y_3}} \epsilon & \sqrt{\frac{m_1 (m_3 - \epsilon)}{Y_3}} \omega (m_2 - m_1 + \epsilon)
\end{array} \right),
\]

where the normalization factors are

\[
Y_1 = (m_2 m_3 + m_1 (m_3 - \epsilon))(m_3 + m_1 - \epsilon)^2 + m_1 (m_3 - \epsilon) \omega^2 (m_2 - m_1 + \epsilon)^2,
\]

\[
Y_2 = (m_1 m_3 + m_2 (m_3 - \epsilon))(m_3 - m_2 - \epsilon)^2 + m_2 (m_3 - \epsilon) \omega^2 (m_2 - m_1 + \epsilon)^2,
\]

\[
Y_3 = (m_1 m_2 + m_3 (m_3 - \epsilon)) \epsilon^2 + m_3 (m_3 - \epsilon) \omega^2 (m_2 - m_1 + \epsilon)^2.
\]

Using the above formulae, one can calculate the quark and lepton flavor mixing matrices. Since the flavor mixing matrix for the quark sector is CKM matrix, we will determine the parameter \( \epsilon \) from the best fit to the measured CKM matrix elements by using the running quark masses at 1 GeV scale. As shown in Section 2, the CP-violating phase \( \delta \) will also be determined from the best fit analysis.

In the lepton sector, the mixing matrix can be expressed in terms of the lepton masses. Thanks to the evidence for the neutrino oscillations and nonzero neutrino masses, one can phenomenologically construct the lepton mixing matrix in such a way as to be consistent with the present neutrino experiments. Preferring the vacuum oscillation solutions for the solar and atmospheric neutrino deficits, we may take into account the “bimaximal” mixing scenario \[10\]. In Section 3, we will show how the near “bimaximal” mixing matrix can be achieved from our same mass matrix Ansatz by assuming the hierarchy of neutrino masses to be \( m_1 \simeq m_2 << m_3 \) and taking parameter \( \epsilon \) appropriately.

## 2 Quark Flavor CKM Mixing Matrix

Now let us consider the quark sector with the form of mass matrix, Eq. (2). We will take the up-type and down-type quark mass matrix as follows:

\[
M_u = M,
\]

\[
M_d = PMP^{-1},
\]

where the CP-violating phase \( \delta \) is from \( P = \text{diag}(\exp(i \delta), 1, 1) \), and for \( M_{u,d} \) the corresponding mass matrices are with eigenvalues

\[
(-m_u, m_c, m_t) \quad \text{for up type quarks},
\]

\[
(-m_d, m_s, m_b) \quad \text{for down type quarks}.
\]
Note that we have used the same parameter $\epsilon$ for up- and down-type quark mass matrices.

The CKM matrix can be obtained by

$$V_{\text{CKM}} = U_u P U_d^\dagger P^{-1}, \quad (11)$$

from the orthogonal matrices of $U_u, U_d$ of Eq. (4). Explicit formulae of the CKM matrix elements are as follows:

$$V_{ud} = \left\{\sqrt{m_u m_t m_s m_b} + e^{-i\delta} \sqrt{m_u (m_t - \epsilon) m_d (m_b - \epsilon)} (m_t + m_u - \epsilon)(m_b + m_d - \epsilon) + e^{-i\delta} \sqrt{m_u (m_t - \epsilon) m_d (m_b - \epsilon)} \omega^u \omega^d (m_c - m_u + \epsilon)(m_s - m_d + \epsilon)\right\}/\sqrt{Y_1^u Y_1^d}, \quad (12)$$

$$V_{us} = \left\{e^{i\delta} \sqrt{m_u m_t m_s m_b} - \sqrt{m_u (m_t - \epsilon) m_s (m_b - \epsilon)} (m_t + m_u - \epsilon)(m_b - m_s - \epsilon) - \sqrt{m_u (m_t - \epsilon) m_s (m_b - \epsilon)} \omega^u \omega^d (m_c - m_u + \epsilon)(m_s - m_d + \epsilon)\right\}/\sqrt{Y_2^u Y_2^d}, \quad (13)$$

$$V_{ub} = \left\{e^{i\delta} \sqrt{m_u m_t m_s m_b} - \sqrt{m_u (m_t - \epsilon) m_b (m_b - \epsilon)} (m_t + m_u - \epsilon)\right\}/\sqrt{Y_3^u Y_3^d}, \quad (14)$$

$$V_{cd} = \left\{\sqrt{m_u m_t m_s m_b} - e^{-i\delta} \sqrt{m_c (m_t - \epsilon) m_d (m_b - \epsilon)} (m_t - m_c - \epsilon)(m_b + m_d - \epsilon) - e^{-i\delta} \sqrt{m_c (m_t - \epsilon) m_d (m_b - \epsilon)} \omega^u \omega^d (m_c - m_u + \epsilon)(m_s - m_d + \epsilon)\right\}/\sqrt{Y_1^c Y_1^d}, \quad (15)$$

$$V_{cs} = \left\{e^{i\delta} \sqrt{m_u m_t m_s m_b} + \sqrt{m_c (m_t - \epsilon) m_s (m_b - \epsilon)} (m_t - m_c - \epsilon)(m_b - m_s - \epsilon) + \sqrt{m_c (m_t - \epsilon) m_s (m_b - \epsilon)} \omega^u \omega^d (m_c - m_u + \epsilon)(m_s - m_d + \epsilon)\right\}/\sqrt{Y_2^c Y_2^d}, \quad (16)$$

$$V_{cb} = \left\{e^{i\delta} \sqrt{m_u m_t m_s m_b} + \sqrt{m_c (m_t - \epsilon) m_b (m_b - \epsilon)} (m_t - m_c - \epsilon)\right\}/\sqrt{Y_3^c Y_3^d}, \quad (17)$$

$$V_{td} = \left\{\sqrt{m_u m_c m_s m_b} - e^{-i\delta} \sqrt{m_t (m_t - \epsilon) m_d (m_b - \epsilon)} \epsilon (m_b + m_d - \epsilon) + e^{-i\delta} \sqrt{m_t (m_t - \epsilon) m_d (m_b - \epsilon)} \omega^u \omega^d (m_c - m_u + \epsilon)(m_s - m_d + \epsilon)\right\}/\sqrt{Y_3^t Y_3^d}, \quad (18)$$

$$V_{ts} = \left\{e^{i\delta} \sqrt{m_u m_c m_s m_b} + \sqrt{m_t (m_t - \epsilon) m_s (m_b - \epsilon)} \epsilon (m_b - m_s - \epsilon) - \sqrt{m_t (m_t - \epsilon) m_s (m_b - \epsilon)} \omega^u \omega^d (m_c - m_u + \epsilon)(m_s - m_d + \epsilon)\right\}/\sqrt{Y_3^t Y_3^d}, \quad (19)$$

$$V_{tb} = \left\{e^{i\delta} \sqrt{m_u m_c m_s m_b} + \sqrt{m_t (m_t - \epsilon) m_b (m_b - \epsilon)} \epsilon^2 + \sqrt{m_t (m_t - \epsilon) m_b (m_b - \epsilon)} \omega^u \omega^d (m_c - m_u + \epsilon)(m_s - m_d + \epsilon)\right\}/\sqrt{Y_3^t Y_3^d}, \quad (20)$$

where $Y_{1}^{u(d)}, Y_{2}^{u(d)}, Y_{3}^{u(d)}$, and $\omega^{u(d)}$ are given in Eqs. (8),(7),(8),(8), with fermion masses replaced by up-type (down-type) quark masses. Because the up-type quarks have mass hierarchy, $m_u \ll m_c \ll m_t$, we can approximate the quantities $Y_{1}^{u} \simeq Y_{2}^{u} \simeq m_{t}^{4} (m_{c} / m_{t})$ and $Y_{3}^{u} \simeq m_{t}^{4} (\epsilon / m_{t})$. The orthogonal matrix $U_u$, which diagonalizes the up-type quark mass
matrix, may be approximated
\[ U_u \simeq \left( \begin{array}{ccc}
1 & -\sqrt{\frac{m_u}{m_c}} & \sqrt{\frac{m_u}{m_c}}
\sqrt{\frac{m_u}{m_c}} & 1 & -\sqrt{\frac{m_u}{m_c}}
0 & \sqrt{\frac{m_u}{m_c}} & 1
\end{array} \right). \tag{21} \]

Similar approximations are applied to the down-type quarks as well because of the observed similar mass hierarchy, \( m_d \ll m_s \ll m_b \). Here we have presumed that \( \epsilon \ll m_t, m_b \).

Due to the mass hierarchy in the quark sector, we can express the CKM matrix elements in the leading approximation,
\[ V_{us} \simeq e^{i\delta} \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}, \tag{22} \]
\[ V_{ub} \simeq -\sqrt{\frac{m_u}{m_c}} \left( \frac{\epsilon}{m_b} - \sqrt{\frac{\epsilon}{m_t}} \right), \tag{23} \]
\[ V_{cb} \simeq \sqrt{\frac{\epsilon}{m_b}} - \sqrt{\frac{\epsilon}{m_t}}. \tag{24} \]

Note that our result for \( |V_{us}| \) in Eq. (22) is the same as that of the Fritzsch model [1], and the ratio of \( |V_{ub}| \) to \( |V_{cb}| \) is \( |V_{ub}| / |V_{cb}| \simeq \sqrt{m_u/m_c} \). The predictions for \( V_{ub} \) and \( V_{cb} \) are different from those of the Fritzsch Ansatz. The free parameter \( \epsilon \) gives us flexibility to reproduce measured values of \( |V_{ub}| \) and \( |V_{cb}| \) at the low energy scale, even if top quark mass is rather large. We can also obtain the value of Jarlskog factor from the relation
\[ J_{CP} \equiv \text{Im}(V_{ub} V_{td} V_{ud}^* V_{tb}^*) \simeq \sqrt{\frac{m_u}{m_c}} \sqrt{\frac{m_d}{m_s}} \frac{\epsilon}{m_b} \sin \delta. \tag{25} \]

For the numerical work, we have used the running quark masses at 1 GeV scale [13]:
\[ m_u = 4.88 \pm 0.57 \text{ MeV}, \quad m_d = 9.81 \pm 0.65 \text{ MeV}, \]
\[ m_c = 1.51 \pm 0.04 \text{ GeV}, \quad m_s = 195.4 \pm 12.5 \text{ MeV}, \]
and \[ m_t = 475 \pm 80 \text{ GeV}, \quad m_b = 7.2 \pm 0.6 \text{ GeV}. \]

The experimental values of CKM mixing matrix elements [17, 18] are
\[ |V_{us}| = 0.2196 \pm 0.0023, \quad |V_{cb}| = 0.0395 \pm 0.0017, \quad |V_{ub}/V_{cb}| = 0.08 \pm 0.02. \]

We have made \( \chi^2 \) analysis to obtain two parameters, mass matrix parameter \( \epsilon \) and the phase \( \delta \), which induce the three observed experimental values of CKM matrix. From numerical computation, we find that the best fit to the measured CKM matrix elements occurs at
\[ \epsilon = 14.2 \text{ MeV} \quad \text{and} \quad \delta = 1.47 \text{ radians} \quad \text{with} \quad \chi^2_{\text{min}} = 1.36. \]
For 90% significance level, the two parameters are in the range of $12.8 \text{MeV} < \epsilon < 15.5 \text{MeV}$ and $1.42 < \delta < 1.52$. The resulting CKM mixing matrix is then given, as in central values,

$$V_{\text{CKM}} = \begin{pmatrix}
0.9755 - 0.0124i & -0.0334 + 0.2172i & -0.0022 + 0.0003i \\
0.0335 + 0.2170i & 0.9747 + 0.0124i & 0.0395 \\
0.0008 - 0.0083i & -0.0387 & 0.9992
\end{pmatrix}. \quad (26)$$

Note that the predicted ratio $|V_{ub}/V_{cb}|$ prefers lower bound on the experimental measurement, $\sim 0.06$. The Jarlskog factor is estimated as

$$J_{\text{CP}} = (0.18 \pm 0.02) \times 10^{-4}. \quad (27)$$

The moduli of the quark mixing matrix elements and the quantity $J_{\text{CP}}$ do not depend on the parametrization chosen. We can rewrite the CKM mixing matrix (26) with the standard parametrization as used in the particle data book [18]:

$$V_{\text{CKM}} = \begin{pmatrix}
0.9756 & 0.2197 & 0.0006 - 0.0022i \\
-0.2196 & 0.9748 & 0.0395 \\
0.0081 - 0.0021i & -0.0387 - 0.0005i & 0.9992
\end{pmatrix}. \quad (28)$$

Within the $1\sigma$ range, the mixing angles and the phase are corresponding to

$$\theta_{12} = (12.7 \pm 0.2)^\circ, \quad \theta_{23} = (2.3 \pm 0.2)^\circ, \quad \theta_{13} = (0.13 \pm 0.01)^\circ \quad \text{and} \quad \delta_{13} = (75 \pm 3)^\circ,$$

$$\iff \quad V_{us} \simeq \sin \theta_{12} = 0.2197 \pm 0.0030, \quad V_{cb} \simeq \sin \theta_{23} = 0.0395 \pm 0.0020 \quad \text{and} \quad V_{ub} = \sin \theta_{13} \exp(-i\delta_{13}) = (0.0006 \pm 0.0002) - (0.0022 \pm 0.0002)i.$$

We often use the unitary triangle for the study of CP violation. Our model predicts that one of the angles of the unitary triangle is

$$\beta = \arg(-V_{cd}V_{cb}^* / V_{td}V_{tb}^*) = (15 \pm 2)^\circ, \quad \text{and} \quad \sin 2\beta = 0.50 \pm 0.06,$$

which is consistent with the CDF results [19], $\sin 2\beta = 0.79^{+0.41}_{-0.44}$.

### 3 Lepton Mass Matrix and Neutrino Oscillation

Now let us consider the lepton sector with the same mass matrix Ansatz of Eq. (2). We take the CP phase $\delta = 0$, for a while, so that CP is conserved in the lepton sector. At present, the elements of the lepton flavor mixing matrix are determined from the neutrino oscillation experiments. Recent atmospheric neutrino experiments from Super-Kamiokande [3] show evidence for neutrino oscillation and hence for a nonzero neutrino mass. The results indicate the maximal mixing between the muon neutrino and tau neutrino states with a mass squared difference

$$\Delta m^2_{32} \sim 2 \times 10^{-3} \text{ eV}^2.$$
On the other hand, the solar neutrino deficit \(^7, \text{8}\) may be explained through matter enhanced neutrino oscillation \([i.e., \text{the Mikheyev-Smirnov-Wolfenstein (MSW) solution}]^{[13]}\) if

\[
\delta m_{\text{solar}}^2 \simeq 6 \times 10^{-6} \text{ eV}^2 \quad \text{and} \quad \sin^2 2\theta_{\text{solar}} \simeq 7 \times 10^{-3} \quad \text{(small angle case)},
\]

or

\[
\delta m_{\text{solar}}^2 \simeq 9 \times 10^{-6} \text{ eV}^2 \quad \text{and} \quad \sin^2 2\theta_{\text{solar}} \simeq 0.6 \quad \text{(large angle case)},
\]

and through the long-distance vacuum neutrino oscillation called “just-so” oscillation \([14]\) if

\[
\delta m_{\text{solar}}^2 \simeq 10^{-10} \text{ eV}^2 \quad \text{and} \quad \sin^2 2\theta_{\text{solar}} \simeq 1.0.
\]

However, the recent data on the electron neutrino spectrum reported by Super-Kamiokande \([8]\) seem to favor the “just-so” vacuum oscillation, even though the small angle MSW oscillation and the maximal mixing between the atmospheric \(\nu_\mu\) and \(\nu_\tau\) have been taken as a natural solution for the neutrino problems \([13]\). Then, these results from neutrino experiments imply that three flavor neutrinos are oscillating along \textit{bimaximal} mixing pattern with the observed mass hierarchy, \(\Delta m_{21}^2 \ll \Delta m_{32}^2\) or \(m_1 \simeq m_2 \ll m_3\). Bimaximal neutrino mixing has been studied in the recent literature \([10]\).

Now we show how the \textit{nearly} “bimaximal” mixing in neutrino oscillation can be achieved from our mass matrix \textit{Ansatz} of Eq. (2). The flavor (or weak) neutrino states are superposition of mass states and we may write

\[
|\nu_\alpha> = \sum_{i=1}^3 V_{\alpha i}^l |\nu_i>, \tag{29}
\]

where \(\alpha = e, \mu, \tau\), and index \(i\) represents mass eigenstate of neutrino. Let us denote the unitary matrices, which make the neutrino mass matrix \(M_\nu\) and charged lepton mass matrix \(M_l\) diagonal, as \(U_\nu\) and \(U_l\),

\[
U_\nu M_\nu U_\nu^T = \text{diag}(-m_1, m_2, m_3), \quad U_l M_l U_l^T = \text{diag}(-m_e, m_\mu, m_\tau),
\]

where \(m_1, m_2,\) and \(m_3\) are neutrino masses hereafter, and \(M_\nu\) and \(M_l\) have the matrix form of Eq. (2).

The mixing matrix \(V_{\text{CKM}}^l\) in neutrino oscillations is related to \(U_\nu\) and \(U_l\) as follows:

\[
V_{\text{CKM}}^l = U_l^* U_\nu^T, \tag{30}
\]

where \(T\) means the transpose. Since the charged lepton family has mass hierarchy \(m_e \ll m_\mu \ll m_\tau\), the approximate form of the orthogonal matrix \(U_l\) can be obtained, like in the
quark case,
\begin{equation}
U_l \simeq \begin{pmatrix}
1 & -\frac{m_e}{m_\mu} & 0 \\
\frac{m_e}{m_\mu} & 1 & -\frac{e^l}{m_\tau} \\
0 & \frac{e^l}{m_\tau} & 1
\end{pmatrix},
\end{equation}
where we assumed that $e^l \ll m_\tau$. Now by choosing the parameter $e^l \simeq m_3/2$, we take the neutrino mass matrix Ansatz such that its masses satisfy the observed hierarchy of $m_1 \simeq m_2 \ll m_3$. And the neutrino mass matrix has form of Eq. (2) with $e^l \simeq m_3/2$,
\begin{equation}
M_\nu \simeq \begin{pmatrix}
0 & \sqrt{2m_1m_2} & 0 \\
\sqrt{2m_1m_2} & m_3/2 & m_3/2 \\
0 & m_3/2 & m_3/2
\end{pmatrix}.
\end{equation}
In this case we can also write $U_\nu$ approximately as follows,
\begin{equation}
U_\nu \simeq \begin{pmatrix}
\sqrt{\frac{m_2}{m_1+m_2}} & -\sqrt{\frac{m_1}{m_1+m_2}} & \frac{1 - e^l}{m_3} \\
\sqrt{\frac{m_1}{m_1+m_2}} & \sqrt{\frac{m_2}{m_1+m_2}} & 1 - \frac{e^l}{m_3} \\
0 & \frac{e^l}{m_3} & \sqrt{1 - \frac{e^l}{m_3}}
\end{pmatrix}.
\end{equation}
Then, using Eq. (30), we can obtain expressions for the mixing matrix elements such as
\begin{align*}
V_{e1}^l & \simeq \sqrt{\frac{m_2}{m_1+m_2}} + \frac{m_e}{m_\mu} \sqrt{\frac{m_1}{m_1+m_2}} \sqrt{1 - \frac{e^l}{m_3}}, \\
V_{e2}^l & \simeq \sqrt{\frac{m_1}{m_1+m_2}} - \frac{m_e}{m_\mu} \sqrt{\frac{m_2}{m_1+m_2}} \sqrt{1 - \frac{e^l}{m_3}}, \\
V_{e3}^l & \simeq -\frac{m_e}{m_\mu} \sqrt{\frac{e^l}{m_3}},
\end{align*}
\begin{align*}
V_{\mu_1}^l & \simeq -\sqrt{\frac{m_1}{m_1+m_2}} \sqrt{1 - \frac{e^l}{m_3}} + \frac{m_e}{m_\mu} \sqrt{\frac{m_2}{m_1+m_2}}, \\
V_{\mu_2}^l & \simeq \sqrt{\frac{m_2}{m_1+m_2}} \sqrt{1 - \frac{e^l}{m_3}} + \frac{m_e}{m_\mu} \sqrt{\frac{m_1}{m_1+m_2}}, \\
V_{\mu_3}^l & \simeq \frac{e^l}{m_3},
\end{align*}
\begin{align*}
V_{\tau_1}^l & \simeq \sqrt{\frac{m_1}{m_1+m_2}} \sqrt{\frac{e^l}{m_3}}, \\
V_{\tau_2}^l & \simeq -\sqrt{\frac{m_2}{m_1+m_2}} \sqrt{\frac{e^l}{m_3}}, \\
V_{\tau_3}^l & \simeq \sqrt{1 - \frac{e^l}{m_3}}.
\end{align*}
As one can easily see, the "bimaximal" mixing is nearly achieved when we set $e^l \simeq m_3/2$ in
the lepton sector. Explicit lepton flavor mixing matrix $V_{\text{CKM}}^l$ may be written as

$$V_{\text{CKM}}^l \approx \begin{pmatrix}
\sqrt{\frac{1}{2}} + \frac{1}{2} \sqrt{\frac{m_e}{m_\mu}} & \sqrt{\frac{1}{2}} - \frac{1}{2} \sqrt{\frac{m_e}{m_\mu}} & -\sqrt{\frac{1}{2}} \sqrt{\frac{m_e}{m_\mu}} \\
-\frac{1}{2} + \frac{1}{2} \sqrt{\frac{m_e}{m_\mu}} & \frac{1}{2} + \frac{1}{2} \sqrt{\frac{m_e}{m_\mu}} & \sqrt{\frac{1}{2}} \\
\frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}}
\end{pmatrix}.$$ (43)

Notice that the value of $V_{e3}^l$ is not exactly zero but small,

$$|V_{e3}^l| \approx \frac{1}{\sqrt{2}} \sqrt{\frac{m_e}{m_\mu}} \approx 0.05,$$ (44)

which is consistent with the bound obtained from CHOOZ experiment [20] $|V_{e3}^l| \leq 0.22$ if $(m_3^2 - m_1^2) > 10^{-3}$ eV$^2$. Although the exact “bimaximal” neutrino mixing matrix predicts zero for the $V_{e3}^l$ element, a nonvanishing small $V_{e3}^l$ element is not completely excluded. Since the $\nu_\mu \rightarrow \nu_e$ appearance channel is sensitive to the product $|V_{\mu3}^l V_{e3}^l|^2$ and $\nu_\mu \rightarrow \nu_\tau$ appearance channel is sensitive only to $|V_{\mu3}^l|^2$, we can determine the element $V_{e3}^l$ by combining the regions to be probed in both channels. We expect that this will be performed in the future experiments such as K2K and MINOS.

Finally after including the CP-violating leptonic phase $\delta^l$, we can also calculate the theoretical upper bound of rephasing-invariant quantity analogous to the Jarlskog invariant in the quark sector,

$$J_{\text{CP}}^l \approx \frac{\sqrt{m_1 m_2}}{m_1 + m_2 - m_3} \sqrt{1 - \frac{\epsilon^l}{m_3}} \sqrt{\frac{m_e}{m_\mu}} \sin \delta^l \leq 0.012.$$ (45)

Compared to the quark sector result, $J_{\text{CP}}^l$ of Eq. (27), we find that $J_{\text{CP}}^l$ can be surprisingly large, and the maximum amount of CP violation in the generation of the lepton number violation of the universe may be indeed large [21].

**Acknowledgments**

We thank G. Cvetic and S.K. Kang for careful reading of the manuscript and their valuable comments. C.S.K. wishes to acknowledge the financial support of 1997-sughak program of Korean Research Foundation, Project No. 1997-011-D00015. The work of J.D.K. was supported in part by a postdoctoral grant from the Natural Science Research Institute, Yonsei University in 1999.
References

[1] H. Fritzsch, Phys. Lett. B73, 317 (1978); Nucl. Phys. B155, 189 (1979).

[2] A.C. Rothman and K.S. Kang, Phys. Rev. Lett. 43, 1548 (1979).

[3] K.S. Kang and S. Hadjitheodoridis, Phys. Lett. B193, 504 (1987); H. Harari and Y. Nir, Phys. Lett. B195, 586 (1987).

[4] F. Cuypers and C.S. Kim, Phys. Lett. B254, 462 (1991); and the references therein.

[5] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 625 (1973).

[6] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).

[7] B.T. Cleveland et al., Nucl. Phys. B (Proc. Suppl.) 38, 47 (1995); Kamiokande Collaboration: Y. Fukuda et al., Phys. Rev. Lett. 77, 1683 (1996); GALLEX Collaboration: W. Hampel et al., Phys. Lett. B388, 384 (1996); SAGE Collaboration: J.N. Abdurashitov et al., Phys. Rev. Lett. 77, 4708 (1996).

[8] Super-Kamiokande Collaboration: talk by Y. Suzuki at Neutrino-98, Takayama, Japan (June, 1998).

[9] Super-Kamiokande Collaboration: Y. Fukuda, et al., Phys. Rev. Lett. 81, 1562 (1998).

[10] V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, Phys. Lett. B437, 107 (1998); H. Georgi and S.L. Glashow, hep-ph/9808293; S.K. Kang and C.S. Kim, Phys. Rev. D59, 091302 (1999); M. Jezabek and Y. Sumino, Phys. Lett. B457, 139 (1999); Y. Wu, hep-ph/9905222; C.H. Albright and S.M. Barr, hep-ph/9906292; and the references therein.

[11] H. Fritzsch and Z. Xing, Phys. Lett. B353, 114 (1995); P. Kauss and S. Meshkov, Phys. Rev. D42, 1863 (1990).

[12] K.S. Kang and S.K. Kang, Phys. Rev. D56, 1511 (1997); K.S. Kang, S.K. Kang, C.S. Kim and S.M. Kim, hep-ph/9808419.

[13] L. Wolfenstein, Phys. Rev. D17, 2369 (1978); S. P. Mikheyev and A. Smirnov, Yad. Fiz. 42, 1441 (1985); Nuovo Cimento 9C, 17 (1986).
[14] V. Barger, R.J. Phillips and K. Whisnant, Phys. Rev. D24, 538 (1981); S.L. Glashow and L.M. Krauss, Phys. Lett. B190, 199 (1987); A.S. Joshipura and M. Nowakowski, Phys. Rev. D51, 2421 (1995).

[15] T. Yanagida, talk at Neutrino-98, Japan (June, 1998); P. Ramond, talk at Neutrino-98, Japan (June, 1998).

[16] H. Fusaoka and Y. Koide, Phys. Rev. D57, 3986 (1998).

[17] OPAL Collaboration: K. Ackerstaff et al., Phys. Lett. B395, 128 (1997); ALEPH Collaboration: D. Buskulic et al., Phys. Lett. B395, 373 (1997).

[18] Particle Data Group, Eur. Phys. J. C3, 103 (1998).

[19] CDF Collaboration: CDF/PUB/BOTTOM/CDF/4855.

[20] The CHOOZ Collaboration: M. Apollonio et al., Phys. Lett. B420, 397 (1998); S.M. Bilenky and C. Giunti, Phys. Lett. B444, 379 (1998).

[21] Y. Liu and U. Sarkar, hep-ph/9906307.