Magnetic Field Dependence of the Level Spacing of a Small Electron Droplet

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Abstract

The temperature dependence of conductance resonances is used to measure the evolution with the magnetic field of the average level spacing $\Delta \epsilon$ of a droplet containing $\sim 30$ electrons created by lateral confinement of a two-dimensional electron gas in GaAs. $\Delta \epsilon$ becomes very small ($< 30 \mu eV$) near two critical magnetic fields at which the symmetry of the droplet changes and these decreases of $\Delta \epsilon$ are predicted by Hartree-Fock (HF) for charge excitations. Between the two critical fields, however, the largest measured $\Delta \epsilon = 100 \mu eV$ is an order of magnitude smaller than predicted by HF but comparable to the Zeeman splitting at this field, which suggests that the spin degrees of freedom are important.

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In a recent Letter, Klein et al. [1] showed that a droplet of electrons created by lateral confinement of a two-dimensional electron gas undergoes changes in its symmetry at two critical magnetic fields. On one hand, the exchange and the confinement energies favor a compact electron distribution, on the other hand, the Coulomb repulsion favors a diffuse occupation. The magnetic field $B$ alters the balance between these two effects, causing the ground state (GS) to change [2]. Correlations are also expected to play an important role [3]. Such symmetry breaking is expected to be accompanied by low energy excitations. In particular, Hartree-Fock (HF) [2] calculations, which predict these changes in symmetry, also predict that these are accompanied by a decrease of the average excitation energy of the droplet. We report, here, measurements of the average level spacing $\Delta \epsilon(B)$ which confirm these qualitative predictions. Between the two critical magnetic fields, however, the largest measured $\Delta \epsilon$ is much smaller than predicted by current theories for charge excitations. The maximum $\Delta \epsilon(B)$ is close to the Zeeman energy, which suggests that spin excitations are important.

The device we use consists of a heavily $n$-doped GaAs substrate covered by a first layer of AlGaAs and a top layer of GaAs [4], both layers grown by molecular-beam epitaxy. The strong electric field created by the the band offset between AlGaAs and GaAs, as well as the positive voltage $V_b$ applied to the substrate on the bottom, creates a two-dimensional electron gas (2DEG) at the AlGaAs/GaAs interface. Ti-Au electrodes are deposited on the top surface 100nm above the 2DEG and fashioned by electron beam lithography. A negative voltage maintained constant during the experiment, is applied to the top electrodes. This confines the electrons laterally in a potential that is approximately a two-dimensional harmonic oscillator characterised by the energy $\hbar \omega_0 [4]$. In addition, the electrodes define two potential barriers, through which electrons must tunnel to enter and exit the droplet. The 2DEG regions outside the left and right barriers form the leads. The current through the droplet is measured as a function of the voltage $V_b$, as well as that between the left and right leads $V_{lr}$.

Figure 1(a) illustrates the conductance $G$ through the droplet for $V_{lr} < k_B T/e$, where $G$
is independent of $V_{lr}$. Sharp peaks arise from resonant tunneling: the $N$th peak occurs at a voltage $V_N$ such that $eV_N$ is proportional to $(E_N - E_{N-1})$, the energy difference between a $(N)$-particle and $(N-1)$-particle GS. At high temperatures, this difference is dominated by the Coulomb charging energy of the droplet. At very low temperatures, however, there is a quantum mechanical contribution to $(E_N - E_{N-1})$ arising from the confinement of the droplet to a small region of space. When the GS of the droplet changes, $(E_N - E_{N-1})$ changes as well and thus $V_N$ provides a spectroscopic probe of the GS energy of the droplet.

Figure 1(b) illustrates how this addition spectroscopy \cite{1,5,6} is done by following the $B$-dependence of the peak position at the base temperature of our dilution refrigerator corresponding to an electron temperature of 50mK (see caption of Fig.2). At high $B$, the kinetic energy of the electrons is quantized in Landau levels (LLs) of index $n = 0, 1, ...$. The large structure in $V_N(B)$ below 1.5T corresponds to the transfer of electrons from higher to lower $n$ LLs. The change in behavior at 1.5T indicates a different phenomenon. Above this field all the electrons are in the lowest LL (i.e. all electrons have the same kinetic energy) and the jumps in $V_N$ are caused by the flips of electron spins. The spins flip, not because of their magnetic moment, which is small in GaAs ($g = -0.4$), but because of the Coulomb interaction \cite{5,1}. The latter scales as $e^2/\epsilon \ell_B$ where $\ell_B = \sqrt{\hbar c/eB}$ is the magnetic length and $\epsilon$ is the dielectric constant. Increasing $B$ decreases $\ell_B$, which increases the Coulomb repulsion, causing the electrons to spread out \cite{5,1}.

Klein et al. \cite{1} have shown that just below $B_c$ (indicated in Fig.1(b)) the GS of the droplet is a singlet. Their experiment shows that for a droplet with a small number of electrons $N \sim 30$ and a large $\hbar \omega_0 \sim 2meV$, the GS has a compact charge distribution at low fields. The compact occupation of the lowest LL is achieved when the charge density corresponds to filling fraction (the ratio of electron to flux-quantum density) equal to 2 throughout the droplet, i.e. when both spin states are equally occupied. Above $B_c$, the growth in the Coulomb repulsion causes the charge to spread out, causing the electrons to flip their spins in order to minimize the loss of exchange energy. Thus the transition at $B_c$ results from a change in symmetry from zero to finite total spin.
A different kind of symmetry breaking occurs at high fields. Above $1.9B_c$ all spins are polarized and the droplet must find a new way to spread out as $B$ increases. In HF, the GS between $1.9B_c$ and $B_r = 2.5B_c$ (Fig.1(c)) is the so-called maximum density droplet (MDD) \cite{7,2}, the compact charge distribution for a spin polarized droplet with filling fraction 1 throughout the droplet. In HF, the symmetry change at $B_r$ is analogous to a liquid-gas phase transition and the order parameter is the charge density. Klein et al. observe experimentally that $B_r$ is lower than predicted by HF, as seen by comparing Fig.1(b) and (c), suggesting that correlations play an important role in this higher field symmetry breaking \cite{8,3}.

HF predicts a dramatic decrease in the average excitation energy $\Delta \epsilon$ of the droplet near $B_c$ and $B_r$. One way we determine $\Delta \epsilon$ is from the cross-over from single to multiple level transport as $T$ is increased \cite{9}. We carefully select conductance peaks that have an exponential tail at base temperature (Fig.1(a)). This indicates that the peak shape is dominated by the thermal broadening of the energy distribution of the electrons in the reservoir \cite{9} and that quantum fluctuation effects \cite{10} are comparatively small. In contrast, a Lorentzian tail is the characteristic signature of the regime where the latter effects are important. Also, we measure the temperature dependence at $B$ fields well separated from the steps in Fig.1(b), for at the cusps associated with these we expect degenerate GS’s.

When $k_BT < \Delta \epsilon$, the current is limited by a single quantum level and the conductance peak profile is given by the derivative of the Fermi-Dirac distribution function. The data in Fig.1(a) are well fit by the formula \cite{11}

$$G(V_b) = \frac{e^2}{h} \sum_{N=1}^{\infty} \frac{\Gamma_N}{4k_BT} \cosh^{-2} \left( \frac{\alpha e V_b - V_N}{2k_BT} \right).$$

(1)

$\Gamma_N$ is the tunneling matrix element, and the factor $\alpha$ converts a change in $V_b$ to a shift in the electrostatic potential of the droplet \cite{12}. Thus, the amplitude of the peak $G_{\text{max}}$ decreases as $1/T$ with increasing $T$. However, when $k_BT$ becomes larger than $\Delta \epsilon$, excited states as well as the GS participate in the conductance. The number of levels participating then grows as $T/\Delta \epsilon$, but each channel still contributes a weight that varies as $1/T$, so the total conductance becomes temperature independent. The cross-over of $G_{\text{max}}$ from $1/T$
to constant provides a measure of $\Delta \epsilon$. Beenaker $^{[11]}$ has calculated the line shape in the multi-level regime assuming a constant $\Delta \epsilon$ and a constant $\Gamma_N$ for all levels in the excitation spectrum of the $(N)$-electron droplet. He finds that the following is a good approximation:

$$G(V_b) \approx \frac{e^2}{\hbar} \sum_{N=1}^{\infty} \frac{\Gamma_N}{2\Delta \epsilon} \cosh^{-2} \left( \frac{\alpha e V_b - V_N}{2.5k_BT} \right).$$

(2)

The cross-over temperature of $G_{max}$ from $1/T$ to constant is at $\Delta \epsilon/2$.

Concomitant with this cross-over of the peak amplitude, the peak profile also changes subtly, and this shows up in measurements of the full width at half maximum (FWHM) $^{[9]}$. The FWHM is proportional to $T$ for both single- and multiple-level transport, but the FWHM is larger in the multiple-level regime. The increase of the FWHM also provides an indication of when $k_BT$ is comparable to $\Delta \epsilon$.

Figure 2(a) shows the $T$-dependence of the conductance peaks at $B/B_c = 1.6$. The inverse of $G_{max}$ as a function $T$ is plotted in Fig.2(b). The cross-over from $1/T$ to constant is quite clear. The FWHM as a function of $T$ for one of the peaks in Fig.2(a) is plotted in Fig.2(c). At $T \approx 0.5K$ where the amplitude cross-over takes place, the FWHM also deviates from its low $T$-linear behavior.

Small differences from the model of Eqs.(1) and (2) are observed in Fig.2(b). We observe that $1/G_{max}$ decreases slightly with increasing $T$ above 600mK. This can be explained by taking into account the variation of $\Gamma$ and $\Delta \epsilon$ with excitation energy for fixed $N$. One expects that levels of increasing excitation energy have smaller $\Delta \epsilon$ and larger $\Gamma$ because of deviations from parabolicity of the confining potential and because of narrowing of the two tunnel barriers at higher energy.

Figures 2(d-f) show analogous measurements at $B/B_c = 2.1$ close to $B_r$. Clearly $\Delta \epsilon$ is smaller near the field at which the symmetry breaking occurs. The $T$-dependence of the FWHM is also consistent with a small $\Delta \epsilon$. The FWHM is linear in $T$ down to the lowest temperature measured, with a slope larger than the one measured in Fig.2(c), suggesting that the droplet is already in the multiple-level regime at the lowest temperatures.

Figure 3(a) shows the $B$-field dependence of $\Delta \epsilon$ extracted from data like those in Fig.2.
for devices with 30 and 50 electrons in the droplet. Although the values of $B_c$ differ by 0.2T between the two cases, the experimental results for the two droplets are consistent when plotted as a function of $B/B_c$. The level spacing nearly vanishes at $B/B_c = 1$ and 2.2 where the two symmetry changes occur. Between these two critical fields, $\Delta \epsilon$ reaches a maximum value of 100$\mu$eV.

The inset of Fig.3(b) illustrates the HF prediction for the level spacing. We have calculated the gap between the GS and the lowest excited state as a function of $B$. The latter energy vanishes at level crossings, which occur near steps in Fig.1(c), and rises to a maximum between steps. We have plotted, in the inset of Fig.3(b), the lines that join all these local maxima. In the experiment, we have also measured $\Delta \epsilon$ between steps. The overall shape of Fig.3(a) is reproduced by HF: $\Delta \epsilon$ vanishes near $B_c$, rises to a maximum, and then falls again precipitously near $B_r$. However, the largest values of $\Delta \epsilon$ predicted by HF are ten times larger than observed experimentally.

Another way to measure $\Delta \epsilon$ is to use tunneling excitation spectroscopy (TES) [13,14]. In this scheme, the differential conductance is measured as a function of $V_{ir}$ and the quantum levels are seen as peaks. Two spectra are shown in Fig.4. At $B/B_c = 1.6$, where $\Delta \epsilon$ is largest, the level spacing measured this way agrees well with that measured from the $T$-dependence. However at $B/B_c = 2.1$, close to $B_r$, TES gives a value somewhat larger than does the temperature dependence, although the value is much smaller than at $B/B_c = 1.6$. TES measures the single-particle excitations [15] while the $T$-dependence also includes many-body excitations. This may be the reason for this discrepancy.

The observation of very small $\Delta \epsilon$ near $B_c$ and $B_r$ confirms the interpretation of Klein et al. [1] that symmetry breaking occurs at these critical fields. However the observation that the largest $\Delta \epsilon$ is much smaller than the value predicted by HF calls into question the nature of the low-lying excitations of the MDD.

The existing models that describe the low-lying excitations near the MDD can be separated in two categories depending on whether they consider charge or spin excitations. Consider, first, the former, which assumes that both GS and excited states are spin polar-
ized. This class includes the HF calculation of Chamon and Wen [2], which uses as a basis the single-particle states of the symmetric gauge, without level mixing. The single-particle states are labeled by an angular momentum index $m > 0$, and they represent circular orbits of radius $\ell_B \sqrt{2(m+1)}$. In the MDD GS, all the innermost orbitals are occupied ($m = 0, 1, \ldots, N - 1$) and the charge density has the spatial distribution corresponding to filling fraction 1 uniformly over the droplet. This compact charge distribution is incompressible. As $B$ increases, however, the HF charge distribution approaches the classical dome-like shape corresponding to a state which is compressible throughout the droplet. The first step in this transformation is the edge reconstruction at $B_r$ where holes are first introduced in the interior of the droplet. This transition, from an incompressible state to a compressible one, gives rise to the abrupt decrease of the level spacing at $B_r$. The single-particle level spacing in the compressible state can be evaluated from the expression $2\hbar^2/m^*r^2g_s \sim 40\mu eV$ where $m^*$ is the effective mass, $g_s = 1$ is the spin-degeneracy and $r = 200nm$ is the radius of the MDD. A better approximation than HF is the model of Oaknin et al. [16] in which charge magnetoexcitons are the low-lying excitations. These are single electron-hole pair excitations of the MDD which correspond to moving an electron from the interior to the exterior of the droplet. While such excitons give rise to excited states in the MDD, they become stable in the GS above $B_r$. Above $B_r$, as in HF the electron occupancy is reduced within a few $\ell_B$ inside the edge of the MDD. Other models that incorporate correlations include the work of Kamilla and Jain [17,3] that study excitations of non-interacting composite fermions. In all the models we have discussed, the level spacing in the MDD state is of order $e^2/\ell_B$, which is approximately 7meV near 3T, the $B$-field at which the MDD is formed for the data in Fig.3(a), and is therefore much larger than the $\Delta \epsilon$ we observe.

In contrast, the Zeeman energy is 75$\mu eV$ at 3T (using $g = -0.4$), a value closer to our measured $\Delta \epsilon$. Several calculations have appeared recently on spin-wave like excitations [18] in a droplet [19]. By canting the spins of the electrons gradually over the MDD, the droplet reduces the cost in exchange energy of having two neighbouring electrons with opposite spins. Such states may also be a better description of the GS near $B_r$. Even if the MDD is a good
description of the GS, excited states that involve spin excitations may have lower energy
than those involving charge excitations, which cost both confinement and exchange energies.
In particular, a uniform rotation of all spins costs no exchange or confinement energy at all.
The total spin quantum numbers of the spin-polarized MDD are \( S^2 = (N/2 + 1)N/2 \) and
\( S_z = N/2 \). The first excited state has the same \( S^2 \) as the MDD but \( S_z = N/2 - 1 \) and the
energy gap is then given by the Zeeman energy. This excited state can also be obtained
by including vertex corrections in the Coulomb interactions \([20]\). While, this gives \( \Delta \epsilon \) of
the right order of magnitude, the actual excited states may be more complex, involving an
admixture of both spin and charge excitations depending on \( N \) and the Zeeman energy.

We have plotted in Fig.3(b) the lowest \( \Delta \epsilon(B) \) predicted for either spin excitations or
charge excitations. The upper branch of the trapezoidal shape in Fig.3(b) is the Zeeman
energy. The two abrupt decreases at \( B_c \) and \( B_r \) are the low-lying charge excitations predicted
by HF near the fields at which symmetry changes.

Our measurements confirm the predictions of HF that \( \Delta \epsilon \) vanishes near two critical fields
at which changes in the symmetry of the droplet occur. Between these two critical fields,
however, the spacing is of the order of the Zeeman energy suggesting that the low-lying
excitations are those of the total spin of the droplet rather than charge excitations included
in HF and some other models.

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FIGURES

FIG. 1. (a) Linear conductance through the droplet as a function of the voltage on the bottom gate $V_b$ at $B = 2.7T$. (b) Variation with $B$ of the position of the peak near 0.160V. (The $B$-sweep rate is 0.025 G/sec.) We count 13 steps between $B_c$ and $2B_c$ which means that $N = 26$ or 27. (c) HF calculation of the resonant energy as a function of $B$ for a droplet containing $N = 27$ electrons and $\hbar\omega_0 = 2.1\text{meV}$. The model has the same number of steps and the same $B_c$ as the experiment. Note that the height of the steps is larger in HF (c) than in the experiment (b) suggesting that the excitation gap is also larger in HF.

FIG. 2. Conductance vs. $V_b$ for $T$ ranging from 100mK to 600mK in increments of 100mK, (a) measured at $B/B_c = 1.6$ and (d) at $B/B_c = 2.1$. (b) and (e) Inverse of the conductance peak vs. $T$ for the peaks in (a) and (d) respectively. The cross-over from $1/T$ to constant determines $\Delta\epsilon/2$. (c) and (f) Full width at half maximum vs. $T$ for one peak in (a) and (d) respectively. The error bars are determined by comparing the behavior of other peaks. In (c), the deviation from the low $T$ straight line gives a similar value of $\Delta\epsilon/2$ as in (b). The width and amplitude at base temperature indicate that the 2DEG is at 50mK, whereas the base temperature of the dilution refrigerator is 25mK, measured by nuclear orientation thermometry.

FIG. 3. (a) $\Delta\epsilon$ obtained using the analysis described in the text as a function of $B/B_c$. The open circles are for a droplet containing $N \sim 30$ electrons and the solid circles are for one containing $N \sim 50$ electrons. (b) Inset: HF calculation of the $B$-dependence of the gap between the GS and the first excited state. The gap vanishes near each of the steps in Fig.1(c) and rises to a maximum between. The solid line joins all the local maxima between steps. The dashed line is the Zeeman energy. Main: $B$-dependence of the smallest of these two curves shown in the inset. The result illustrates $\Delta\epsilon(B)$ for both charge or spin excitations.

FIG. 4. Tunneling excitation spectra, $dI/dV_{tr}$ as a function of $V_{tr}$ at (a) $B/B_c = 1.6$ and (b) $B/B_c=2.1$. $U \sim 0.5\text{meV}$ is the Coulomb gap and $\Delta\epsilon$ is the level spacing. A factor $\beta = 0.7$ converts the $x$-axis scale to meV units [12].
(a)

(b)

(c)

(d)

(e)

(f)

$G^{-1}_{\text{max}} \times 10^3 (h/e^2)$

$\Delta \epsilon / 2$

$T$ (K)

FWHM ($10^{-6}$ V)

$\Delta \epsilon / 2$

$T$ (K)
