On the neutrino vector and axial vector charge radius

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Abstract. A Majorana neutrino is characterized by just one flavor diagonal electromagnetic form factor: the anapole moment, that in the static limit corresponds to the axial vector charge radius \( \langle r_A^2 \rangle \). As is the case for the vector charge radius of a Dirac neutrino, proving that this quantity is a well defined physical quantity is non trivial. I will describe briefly the origin of the long standing controversy about the physical or non physical nature of the neutrino charge radius. Then I will argue that, in contrast to Dirac neutrino electromagnetic form factors, for Majorana neutrinos cosmological and astrophysical arguments do not provide useful informations on \( \langle r_A^2 \rangle \). Therefore this quantity has to be studied by means of terrestrial experiment. Finally, I will discuss the constraints that can be derived on \( \langle r_A^2 \rangle \) for the tau neutrino from a comprehensive analysis of the data on single photon production off Z-resonance, and I will conclude with a few comments on \( \nu_\mu \) scattering data from the NuTeV, E734, CCFR and CHARM-II collaborations and on the limits implied for \( \langle r_A^2 \rangle \) for the muon neutrino.

INTRODUCTION

Experimental evidences for neutrino oscillations \([1, 2]\) imply that neutrinos are the first elementary particles whose properties cannot be fully described within the Standard Model (SM). This hints to the possibility that other neutrino properties might substantially deviate from the SM predictions, and is presently motivating vigorous efforts, both on the theoretical and experimental sides, to understand more in depth the physics of neutrinos and of their interactions.

Neutrinos electromagnetic interactions can play an important role in a wide variety of domains, as for example in cosmology \([3]\) and in astrophysics \([4, 5]\). The electromagnetic properties of Dirac neutrinos are described in terms of four form factors. The matrix element of the electromagnetic current between an initial neutrino state \( \nu_i \) with momentum \( p_i \) and a final state \( \nu_j \) with momentum \( p_j \) reads \([6, 7]\)

\[
\langle \nu_j^\mu(p_j)|J_\mu^{\text{EM}}|\nu_i^\nu(p_i)\rangle = i\bar{u}_j\Gamma_\mu^{\nu\mu}(q^2)u_i;
\]

\[
\Gamma_\mu^{\nu\mu}(q^2) = (q^2\gamma_\mu - q_\mu q) [V^\nu(0) - A^0(0)] + i\sigma_{\mu\nu}q^\nu [M^\nu(0) + E^\nu(0)];
\]

where \( q = p_j - p_i \), and the \((ij)\) indexes denoting the relevant elements of the form factor matrices have been left implicit. In the \( j = i \) diagonal case, \( M^\nu \) and \( E^\nu \) are called the magnetic and electric form factors, that in the limit \( q^2 \to 0 \) define respectively the neutrino magnetic moment \( \mu = M^\nu(0) \) and the (CP violating) electric dipole moment \( \epsilon = E^\nu(0) \), while the form factors \( q^2 V^\nu(q^2) \) and \( q^2 A^\nu(q^2) \) correspond to non vanishing neutrino charge distributions of the form

\[
\rho_{\nu}(r) = \int \frac{d^3q}{(2\pi)^3} q^2 V(q^2) e^{i\vec{q}\cdot\vec{r}}
\]

that are induced by the virtual transitions \( \nu_i \rightleftharpoons l^- W^+ \) \((l = e,\mu,\tau)\). Intuitively, these distributions can be depicted as a positive core surrounded by a negative cloud:

In the static limit, the reduced Dirac form factor \( V^\nu(q^2) \) and the neutrino anapole form factor \( A^\nu(q^2) \) are related to the vector and axial vector charge radius \( \langle r_V^2 \rangle \) and \( \langle r_A^2 \rangle \) through:

\[
\langle r_V^2 \rangle = -6 V^\nu(0); \quad \langle r_A^2 \rangle = -6 A^\nu(0).
\]

In the following we will refer to these form factors as the vector and axial vector charge radius also when \( q^2 \neq 0 \).

A long standing controversy about the possibility of consistently defining a gauge invariant, physical, and
The neutrino form factors appearing in the definition of an improved and process independent neutrino charge radius \[3\] has been recently settled \[3, 10, 11, 12\]. The controversy was related to the general problem of defining improved one-loop Born amplitudes in \(SU(2) \times U(1)\) processes like e.g. \(e^+e^- \rightarrow f\bar{f}\). Let us consider schematically the amplitude for this process in QED. We can include the leading one-loop corrections while retaining a Born-like amplitude for this process in QED. We can include the like e.g. \(\Pi\) where \(\frac{1}{e^2} = \frac{1}{e_0^2} - \Pi_f(q^2)^2\),

\[
\mathcal{M}_{QED}^0 = e^2 \frac{J_0^f \times J_0^f}{q^2} \rightarrow \mathcal{M}_{QED}^1 = e^2 \frac{J_0^f \times J_0^f}{q^2} - \Pi_f(q^2),
\]

where \(\Pi_f(q^2)\) is the reduced photon self energy. In this approach it is consistent to neglect the additional corrections due to the photons box diagrams, since they are separately gauge invariant. The same is not possible in the SM. A factorized Born-like expression (with suitably renormalized couplings) for the one-loop neutral current amplitude of the form:

\[
\mathcal{M}_{NC} = e^2 \frac{J_0^f \times J_0^f}{q^2} + \left( \frac{e^2}{s_{\gamma^*}^2} \right) \frac{J_0^f \times J_0^f}{q^2} - M_Z^2 - i M_Z^2 \Gamma_Z
\]

(with \(f \neq e, \nu_e\)) in general is not gauge invariant!

A sketchy argument to explain the problem goes as follows: if one tries to define for example an improved effective \(e^+e^-Z\) coupling that includes the one-loop vertex corrections (fig. 1a) one finds that the ‘improper vertex’ corresponding to the mixed \(\gamma-Z\) self energy (fig. 1b) should also be included in order to obtain a finite expression. Still, the result is gauge dependent, and gauge invariance can be recovered only by including also the \(W\) box diagrams (fig. 1c). One can easily convince himself that things work in this way, by considering the cut of the diagrams sketched in fig. 1 that involve \(W\) bosons, as depicted in fig. 2. Each single diagram obtained by cutting the vertex, the mixed self-energy and the \(W\) box represents only a partial contribution to the full amplitude for the process \(e^+e^- \rightarrow W^+W^-\), and of course a single partial contribution has no well defined physical meaning: only the full amplitude is a physical quantity. On the other hand, the \(W\) box diagrams needed to recover gauge invariance connect initial state fermions to the final states, and therefore depend on the specific process. This indicates that at one-loop, the amplitude cannot be straightforwardly written in the factorized form \[3\], and that the definition of an improved and process independent \(e^+e^-Z\) vertex is indeed problematic. This problem

\[
\begin{align*}
\text{FIGURE 2.} & & \text{A subset of diagrams contributing at 1-loop to } e^+e^- \rightarrow f\bar{f}, \text{ cut to show the relation with } e^+e^- \rightarrow W^+W^- \text{.}
\end{align*}
\]

\[\text{is even more acute when one tries to define the charge radius of a neutrino as a physical, process independent property, intrinsic to this particle. This is due to the absence at tree level of a neutrino-photon coupling, so that there is no well defined quantity even at the lowest order.}

Consistent definitions of one-loop gauge invariant vertexes, self-energies and box diagrams in the electroweak sector of the SM were first obtained by Deagrassi and Sirlin \[14\] through the systematic application of a procedure previously developed in QCD, the so-called pinch technique \[15\]. More recently, the same technique was used to construct a neutrino charge radius at one-loop, which is independent of the gauge fixing parameter, of the gauge fixing scheme, and of the particular scattering process \[12\]. That such a construction is possible can be intuitively understood by observing that for neutrino scattering off right handed polarized fermions, the \(W\) box diagrams are absent to begin with, and thus no ambiguity can arise \[3\]. This suggested a way to derive a unique decomposition of loop contributions that separately respect gauge invariance, and from which a process independent charge radius could be defined as an intrinsic property of the neutrino. Furthermore, in \[10, 12\] it was argued that the so-defined charge radius is a true physical observable, in the sense that its value could be extracted, at least in principle, from experiments.

Coming back to the neutrino form factors appearing in

\[
\begin{align*}
\text{FIGURE 1.} & & \text{The vertex (a), mixed } A-Z \text{ self-energy (b) and } W \text{ box diagrams (c) that contribute to construct a gauge invariant 1-loop amplitude.}
\end{align*}
\]

\[
\text{\[2\] In the 2002 Review of Particle Physics} [13] \text{the limits on the neutrino charge radius are reported as limits on "nonstandard contributions to neutrino scattering". The corresponding section begins with the statement: "We report limits on the so-called neutrino charge radius squared in this section. This quantity is not an observable physical quantity, and this is reflected in the fact that it is gauge dependent."}
\]
not all of them survive when the neutrinos are Majorana particles \( (\nu^M) \). In the non-diagonal case \( (\nu^M_i \neq \nu^M_j) \) and in the limit of CP invariance the electromagnetic interaction is described by just two form factors \( [6] \). If the initial and final Majorana neutrinos involved in the process have the same CP parity, only \( E^M_{ji}(q^2) \) and \( A^M_{ji}(q^2) \) are non vanishing, while if the CP parity is opposite, the electromagnetic interaction is described by \( M^M_{ji}(q^2) \) and \( V^M_{ji}(q^2) \). Finally, in the Majorana diagonal case \( \nu^M_i = \nu^M_j \) the only surviving form factor is the anapole moment \( A^M(q^2) \). As discussed in \([16]\), this last result can be inferred from the requirement that the two identical fermions final state in \( \gamma \rightarrow \nu^M \nu^M \) be antisymmetric, and it holds regardless of the assumption of CP invariance.

In the SM the neutrino electromagnetic form factors have extremely small values \([17]\). Due to the left-handed nature of the weak interactions, the numerical value of the vector and axial vector charge radius coincide, and for the different \( \nu_e \), \( \nu_\mu \) and \( \nu_\tau \) flavors they fall within the range \( \langle r^2_{eA} \rangle \approx (1 - 4) \times 10^{-33} \text{cm}^2 \) \([18]\). However, it could well be that, due to new physics, the electromagnetic interactions of neutrinos are enhanced with respect to the SM expectations. In general, the strongest limits on anomalous form factors come from astrophysical and cosmological considerations. For example the neutrino magnetic moments can be constrained from consideration of stellar energy losses through plasma photon decay \( \gamma \rightarrow \nu \nu \) \([18]\), from the non-observation of anomalous energy loss in the Supernova 1987A neutrino burst as would have resulted from the rapid emission of superweakly interacting right handed neutrinos \([13]\), or from Big Bang nucleosynthesis arguments. In the last case constraints are obtained by requiring that spin flipping Dirac magnetic moment interactions should be weak enough not to populate the right handed neutrinos degrees of freedom at the time of the freeze out of the neutron-to-proton ratio \([3]\).

Since the charge radii do not couple neutrinos to on-shell photons, the corresponding interactions are not relevant for stellar evolution arguments. However, in the Dirac case, right handed neutrinos can still be produced through e.g. \( e^+e^- \rightarrow \nu_R \bar{\nu}_R \), and therefore constraints from the Supernova 1987A as well as from nucleosynthesis do apply. They yield respectively \( |\langle r^2 \rangle| \lesssim 2 \times 10^{-33}\text{cm}^2 \) \([19]\) and \( |\langle r^2 \rangle| \lesssim 7 \times 10^{-33}\text{cm}^2 \) \([20]\).

However, if neutrinos are Majorana particles, they do not have light right-handed partners, and the previous constraints do not apply. In this case, in particular for \( \nu_\tau \), a large anapole moment resulting in an interaction even stronger than electroweak could be allowed. In the early Universe such an interaction could keep \( \nu_\tau \) in thermal equilibrium long enough to experience a substantial reheating from the annihilation process \( e^+e^- \rightarrow \nu_\tau \bar{\nu}_\tau \), and in turn this could affect the Universe expansion and change the abundance of primordial \( ^7\text{H}e \). In section 2 we will argue that this is not so, and that even an interaction one order of magnitude stronger than electroweak would hardly affect Helium abundance at the observable level. We can conclude that constraints on the Majorana neutrino axial charge radius can be obtained only from terrestrial experiments.

The present laboratory limits for the electron neutrino are \(-5.5 \times 10^{-32} \leq \langle r^2_{eA}(\nu_e) \rangle \leq 9.8 \times 10^{-32}\text{cm}^2 \) \([21]\). In the Dirac case these limits apply to the sum \( \langle r^2_{eA}(\nu_e) \rangle + \langle r^2_{Ae}(\nu_e) \rangle \) as well. Limits for \( \nu_\mu \) have been derived from neutrino scattering experiments \([22, 23]\). They are about one order of magnitude stronger than for \( \nu_e \), and will be discussed in section 4. Due to the fact that intense \( \nu_\tau \) beams are not available in laboratories, to date no direct limits on \( \langle r^2_{Ae}(\nu_\tau) \rangle \) have been reported by experimental collaborations. However, under the assumption that a significant fraction of the solar neutrinos converts into \( \nu_\tau \), the limit \( \langle r^2_{eA}(\nu_\tau) \rangle \lesssim 2 \times 10^{-33}\text{cm}^2 \) was derived from the SNO and Super-Kamiokande data \([24]\). A limit on the \( \nu_\tau \) vector charge radius (Dirac case) was obtained from TRISTAN data on \( e^+e^- \rightarrow \nu\nu\gamma \) single photon production \([25]\). As is discussed in section 3, the same data can be used to set limits also on the anapole moment of a Majorana \( \nu_\tau \).

In the next section we will briefly analyze the possibility of deriving constraints on the axial charge radius of Majorana neutrinos from nucleosynthesis. In section 3 we will study the bounds on the tau neutrino charge radius implied by the TRISTAN and LEP experimental results. In section 4 we will discuss the constraints on the muon neutrino charge radius from the NuTeV, CHERMILL, CCFR and the BNL E734 experiments. They result in the following 90\% c.l. limits:

\[
-8.2 \times 10^{-32}\text{cm}^2 \leq \langle r^2_{Ae}(\nu_\tau) \rangle \leq 9.9 \times 10^{-32}\text{cm}^2, \tag{7}
\]

\[
-5.2 \times 10^{-33}\text{cm}^2 \leq \langle r^2_{Ae}(\nu_\mu) \rangle \leq 6.8 \times 10^{-33}\text{cm}^2. \tag{8}
\]

For \( \langle r^2_{Ae}(\nu_\tau) \rangle \) we could not find new experimental results that would imply better constraints than the existing ones \([21]\). We just mention that the Bugey nuclear reactor data from the detector module closest to the neutrino source (15 meters) \([26]\) should imply independent limits of the same order of magnitude than the existing ones.

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\(^3\) These values correspond to the \( q^2 = 0 \) limit, and decrease with increasing energies with a logarithmic behavior.

\(^4\) In the SM with right handed neutrinos the \( \nu_R \) cannot be produced through the charge radius couplings, since the vector and axial vector contributions exactly cancel. Therefore, the quoted limits implicitly assume that, because of new physics contributions, one of the two form factors dominates and no cancellations occur.

\(^5\) These limits are twice the values published in \([21]\) since we are using a convention for \( \langle r^2_{Ae}(\nu_\tau) \rangle \) that differs for a factor of 2.
NUCLEOSYNTHESIS

In this section we study the impact on the primordial Helium abundance $Y$ of an axial charge radius large enough to keep a Majorana $\nu_e$ in thermal contact with the plasma down to temperatures $T < 1$ MeV. In this case the neutrinos would get reheated by $e^+e^-$ annihilation, and this would affect the Universe expansion rate. To give an example, assuming that one neutrino species keeps in thermal equilibrium until $e^+e^-$ annihilation is completed ($T \ll m_e$) has the same effect on the expansion as $\Delta \nu = 1 - (4/11)^{4/3} \approx 0.74$ additional neutrinos.

The amount of Helium produced in the early Universe is determined by the value of the neutron to proton ratio $n/p$ at the time when the $ne^+ \leftrightarrow p\nu$ and $n\nu \leftrightarrow pe^-$ electroweak reactions freeze out. This occurs approximately at a temperature $T_{fo} \approx 0.7$ MeV [27, 28]. Apart for the effect of neutron decay, virtually all the surviving neutrons end up in $^3He$ nuclei. Assuming no anomalous contributions to the electron neutrino reactions, the freeze out temperature can only be affected by changes in the Universe expansion rate, which is controlled by the number of relativistic degrees of freedom and by their temperature. If $\nu_e$ have only standard interactions, at the time of the freeze out they are completely decoupled from the thermal plasma. However, an anomalous contribution to the process $e^+e^- \leftrightarrow \nu_e\bar{\nu}_e$ would allow them to share part of the entropy released in $e^+e^-$ annihilation.

The maximum effect is achieved assuming that the new interaction is able to keep the $\nu_e$ thermalized down to $T_{fo}$. The required strength of the new interaction can be estimated by equating the rate for an anomalously fast $e^+e^- \leftrightarrow \nu_e\bar{\nu}_e$ process $\Gamma_{\nu_e} \propto \langle \sigma v \rangle_{\nu_e}$ to the Universe expansion rate $\Gamma_U = (8\pi\rho/3m_p^2)^{1/2}$. In the primary formula (\langle \sigma v \rangle) the thermally averaged cross section times relative velocity, $\sigma_v \approx 0.365 T^3$ is the number density of electrons, $\rho \approx 1.66 g_\ast^{1/2} (T^2/m_p)$ is the Universe energy density with $g_\ast \approx 10.75$ the number of relativistic degrees of freedom, and $m_p$ is the Plank mass. The thermally averaged cross section can be written as $\langle \sigma v \rangle \equiv \kappa G_\nu^2 T^2$ where $G_\nu \equiv (2\pi^2 \alpha/3) \langle r_A^2 \rangle$ parametrizes the strength of the interaction that we assume sensibly larger than the Fermi constant $G_F$, and $\kappa \approx 0.2$ has been introduced to allow direct comparison with the SM rate $\langle \sigma v \rangle^{SM} \approx 0.2 G_F^2 T^2$ [27]. By setting $\Gamma_{\nu_e} = \Gamma_U$ at $T = T_{fo}$, we obtain $G_\nu \approx 13 \times 10^{-3}$ GeV$^{-2}$. Therefore, to keep the $\nu_e$ thermalized until the ratio $n/p$ freezes out, an interaction about ten times stronger than electroweak is needed. However, even in the presence of such a large interaction, Helium abundance would only be mildly affected. This is because at $T \approx 0.7$ MeV $e^+e^-$ annihilation is still not very efficient, and the photon temperature is only slightly above the temperature of the thermally decoupled neutrinos: $(T_\gamma - T_e)/T_e \approx 1.5%$ [27]. This induces a change in the primordial Helium abundance $\Delta Y \approx +0.04 (\Delta T_\nu/T_e)$ which is below one part in one thousand. This effect could possibly be at the level of the present theoretical precision [25], however, it is far below the present observational accuracy, for which the errors are of the order of one percent [30].

LIMITS ON $\nu_\tau$ VECTOR AND AXIAL VECTOR CHARGE RADIUS

Limits on $\langle r_V^2 \rangle$ and $\langle r_A^2 \rangle$ for $\nu_\tau$ can be set using experimental data on single photon production through the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. A large set of data from TRISTAN and from LEP, spanning the energy range from 58 GeV up to 207 GeV, was analyzed in [31]. Given that the form factors run with the energy, separate results were presented for data collected in different energy ranges: below $Z$ resonance (TRISTAN), between $Z$ resonance and the threshold for $W^+W^-$ production (LEP-1.5), and finally for the data above $W^+W^-$ production (LEP-2). Due to the much larger statistics collected at high energy, a combined fit of all the data does not give any sizable improvement with respect to the LEP-2 limits, that therefore represent the strongest bounds.

The SM cross section for the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ is given by [32]$$\frac{d\sigma_{\nu\bar{\nu}\gamma}}{dx dy} = \frac{2\alpha/\pi}{x(1-y^2)} \left[ \left(1 - \frac{x^2}{2}\right) + \frac{y^2}{4} \right] \times \left\{ N_\nu \sigma_v(s',g_\nu,g_A) + \sigma_a(s') + \sigma_v(s') \right\}$$ (9)

where $\sigma_v$ corresponds to the lowest order $s$ channel $Z$ boson exchange with $N_\nu = 3$ the number of neutrinos that couple to the $Z$ boson. For later convenience in $\sigma_v$, we have explicitly shown the dependence on the electron couplings $g_\nu = -1/2 + 2 sin^2 \theta_W$ and $g_A = -1/2$, where $\theta_W$ is the weak mixing angle. The additional two terms $\sigma_a$ and $\sigma_v$ in (9) correspond respectively to $Z-W$ interference and to $t$ channel $W$ boson exchange in $\nu_\tau$ production. The kinematic variables are the scaled photon momentum $x = E_\gamma/E_{beam}$ with $E_{beam} = \sqrt{s}/2$, the reduced center of mass energy $s' = s(1-x)$, and the cosine of the angle between the photon momentum and the incident beam direction $y = \cos \theta_p$. The expressions for the lowest order cross sections appearing in (9) read:

$$\sigma_v(s) = \frac{s G_F^2}{6\pi} \frac{4(1 + \frac{g_\nu^2 + g_A^2}{3}) M_Z^2}{(M_Z^2 - s)^2 + M_Z^2 \Gamma_Z^2}$$ (10)

$$\sigma_a(s) = \frac{s G_F^2}{6\pi} \frac{(g_\nu + g_A)(M_Z^2 - s) M_Z^2}{(M_Z^2 - s)^2 + M_Z^2 \Gamma_Z^2}$$ (11)

$$\sigma_t(s) = \frac{s G_F^2}{6\pi}$$ (12)
where \( G_F \) is the Fermi constant, \( \alpha \) the fine structure constant, \( M_Z \) and \( \Gamma_Z \) the mass and width of the Z boson. Few comments are in order. Eq. (9) was first derived in [32]. It holds at relatively low energies where \( W \) exchange in the \( t \) channel can be legitimately approximated as a contact interaction. This amounts to neglect the momentum transfer in the \( W \) propagator, and to drop the \( W - \gamma \) interaction, so that the photons are emitted only from the electron lines. While this approximation is sufficiently good at TRISTAN energies, to analyze the LEP data collected above \( Z \) resonance some improvements have to be introduced. We will use an improved approximation where finite distance effects are taken into account in the \( W \) propagator, however we will still work in the limit of vanishing \( W - \gamma \) interactions, since the corresponding effects are of higher order in a leading log approximation and for our scopes can be safely neglected. Finite distance \( W \) exchange effects can be taken into account in the previous expressions by replacing \( \sigma_{st}(s) \) and \( \sigma_t(s) \) by

\[
\sigma_{st}(s) \cdot F_{st} \left( \frac{s}{M_W^2} \right), \quad \text{and} \quad \sigma_t(s) \cdot F_t \left( \frac{s}{M_W^2} \right), \tag{13}
\]

respectively, where \( M_W \) is the \( W \) boson mass, and

\[
F_{st}(z) = \frac{3}{\bar{z}} \left[ (1+z)^2 \log(1+z) - z \left( 1 + \frac{3}{2} z \right) \right], \tag{14}
\]

\[
F_t(z) = \frac{3}{\bar{z}} \left[ -2(1+z) \log(1+z) + z(2+z) \right]. \tag{15}
\]

The contact interaction approximation is recovered in the limit \( z \to 0 \) for which \( F_{st,t}(z) \to 1 \).

An anomalous interaction due to non-vanishing \( \nu_t \) axial and axial vector charge radii can be directly included in (3) by redefining the \( Z \) boson exchange term in the following way:

\[
N_s \sigma_t(s', g_V, g_A) \longrightarrow \left( N_s - 1 \right) \sigma_t(s', g_V, g_A) + \sigma_t(s', g_V', g_A) \tag{16}
\]

where

\[
g_V'(s') = g_V - \left[ 1 - \frac{s'}{M_Z^2} \right] \delta, \tag{17}
\]

\[
\delta = \frac{\sqrt{\pi} \alpha}{3 G_F} \left[ \langle r_V^2 \rangle + \langle r_A^2 \rangle \right]. \tag{18}
\]

The substitution \( g_V \to g_V' \) in (16) takes into account the new photon exchange diagram for production of left-handed \( \nu_t \). In the Dirac case, \( s \)-channel production of right handed \( \nu_t \) through photon exchange must also be taken into account. This yields a new contribution that adds incoherently to the cross section, and that can be included by adding inside the brackets in (3) the term

\[
\sigma_R(s') = \frac{s' G_F^2}{6 \pi} (\delta')^2, \tag{19}
\]

\[
\delta' = \frac{\sqrt{\pi} \alpha}{3 G_F} \left[ \langle r_V^2 \rangle - \langle r_A^2 \rangle \right]. \tag{20}
\]

In the SM \( \langle r_V^2 \rangle = \langle r_A^2 \rangle \) and therefore there is no production of \( \nu_R \) through these couplings. For a Majorana neutrino \( \delta' = 0 \) and \( \langle r_V^2 \rangle = 0 \), and thus the limits on anomalous contributions to the process \( e^+ e^- \rightarrow \nu \nu \gamma \) translate into direct constraints on the axial charge radius \( \langle r_A^2 \rangle \).

**Limits from TRISTAN**

The three TRISTAN experiments AMY [34], TOPAZ [35] and VENUS [36] have searched for single photon production in \( e^+ e^- \) annihilation at a c.m. energy of approximately \( \sqrt{s} = 38 \) GeV. Anomalous contributions to the cross section for \( e^+ e^- \rightarrow \nu \nu \gamma \) would have been signaled by an excess of events. Limits on a Dirac \( \nu_t \) charge radius from the TRISTAN data were derived in [35]. The analysis was extended in [31] to the include the case of a Majorana neutrino with an anapole moment. The details of TRISTAN results can be found in table 1 of ref. [31]. For a Majorana \( \nu_t \) (\( \delta' = 0 \) and \( \langle r_V^2 \rangle = 0 \)) the data imply the following 90 \% c.l. limits:

\[-3.7 \times 10^{-31} \text{cm}^2 \leq \langle r_A^2 \rangle \nu_t \leq 3.1 \times 10^{-31} \text{cm}^2. \tag{21}\]

For the Dirac case, the associated production of right-handed states through \( \sigma_R \) in (19) allows us to constrain independently the vector and axial vector charge radius. The 90 \% c.l. limits on \( r^2_{V+A} \nu_t \equiv \langle r_V^2 \nu_t \rangle + \langle r_A^2 \nu_t \rangle \) are:

\[-2.1 \times 10^{-31} \text{cm}^2 \leq r^2_{V+A} \nu_t \leq 1.8 \times 10^{-31} \text{cm}^2. \tag{22}\]

As we have already mentioned, strictly speaking the constraints just derived cannot be directly compared with the LEP constraints analyzed below, since the two experiments are proving neutrino form factors at different energy scales. Of course, since our limits constrain essentially only physics beyond the SM, it is not possible to make a sound guess of the form of the scaling of the form factors with the energy, which is determined by the details of the underlying new physics. However, if we assume a logarithmic reduction of the form factors with increasing energy as is the case in the SM, then we would expect a moderate reduction of about \( \approx 0.65 \) when scaling from TRISTAN to LEP-1.5 energies, and an additional reduction of about \( \approx 0.75 \) from LEP-1.5 up to LEP-2 measurements at 200 GeV.

**Limits from LEP**

Limits on \( \langle r_V^2 \rangle \) and \( \langle r_A^2 \rangle \) were derived from the observation of single photon production at LEP in a com-
completely similar [31]. Contrary to magnetic moment interactions that get enhanced at low energies with respect to electroweak interactions, the interaction corresponding to a charge radius scale with energy roughly in the same way than the electroweak interactions, and therefore searches for possible effects at high energy are not in disadvantage with respect to low energy experiments. It is for this reason that LEP data above the Z resonance are able to set the best constraints on the vector and axial vector charge radius for the $\tau$ neutrino.

All LEP experiments have published high statistics data for the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ for c.m. energies close to the Z-pole; however, due to the dominance of resonant Z boson exchange, these data are not useful to constrain anomalous neutrino couplings to s-channel off-shell photons. Therefore, only off-resonance data collected above $Z$ resonance in the in the energy range 130 GeV – 207 GeV, were used in the analysis in [31]. The data were divided into two sets: LEP-1.5 data collected below $W^+W^-$ production threshold (table 2 of ref. [31]) and LEP-2 data collected above $W^+W^-$ threshold up to 207 GeV (table 3 of ref. [31]).

**LEP-1.5:** The ALEPH [57], DELPHI [58] and OPAL [34, 40, 41] collaborations have published data for single photon production at c.m. energies of 130 GeV and 136 GeV. During the fall 1995 runs ALEPH [42] and DELPHI [45] accumulated about 6 pb$^{-1}$ of data for each experiment, observing respectively 40 and 37 events. In the same runs OPAL [34, 40] collected a little less than 5 pb$^{-1}$ observing 53 events. In addition, OPAL published data also for the 1997 runs (at the same energies) [41] collecting an integrated luminosity of 5.7 pb$^{-1}$ and observing 60 events. With a total integrated luminosity of about 28 pb$^{-1}$ LEP-1.5 implies the following 90 % c.l. limits:

\[-5.9 \times 10^{-31} \text{cm}^2 \leq \langle r_A^2 (\nu_\tau) \rangle \leq 6.6 \times 10^{-31} \text{cm}^2 \quad (23)\]

for the axial vector charge radius of a Majorana $\nu_\tau$, and

\[-3.5 \times 10^{-31} \text{cm}^2 \leq r_{V+A}^2 (\nu_\tau) \leq 3.7 \times 10^{-31} \text{cm}^2 \quad (24)\]

for the Dirac case. In spite of the much larger statistics, the limits from LEP-1.5 are roughly a factor of two worse than the limits from TRISTAN in [21] and [22]. The main reason for this is that at LEP-1.5 energies initial state radiation tends to bring the effective c.m. energy $s'$ of the collision close to the Z resonance, thus enhancing $Z$ exchange with respect to the new photon exchange diagram.

**LEP-2:** Above the threshold for $W^+W^-$ production the four LEP experiments reported results for 24 different data points, corresponding altogether to about 1.6 nb$^{-1}$ of data (see table 3 of ref. [31]).

ALEPH [42, 43, 44] published data for ten different c.m. energies, ranging from 161 GeV up to 209 GeV. DELPHI [45] published data collected at 183 GeV and 189 GeV, and gave separate results for the three major electromagnetic calorimeters, the High density Projection Chamber (HPC) covering large polar angles, the Forward ElectroMagnetic Calorimeter (FEMC) covering small polar angles, and the Small angle THe Calorimeter (STIC) that covers the very forward regions. In three papers [44, 47, 48] L3 reported the results obtained at 161 GeV, 172 GeV, 183 GeV and 189 GeV. Finally, OPAL published data for four different c.m. energies [39, 40, 41, 45]. The 90 % c.l. limits implied by LEP-2 data read

\[-8.2 \times 10^{-32} \text{cm}^2 \leq \langle r_A^2 (\nu_\tau) \rangle \leq 9.9 \times 10^{-32} \text{cm}^2 \quad (25)\]

for the Majorana case, and

\[-5.6 \times 10^{-32} \text{cm}^2 \leq r_{V+A}^2 (\nu_\tau) \leq 6.2 \times 10^{-32} \text{cm}^2 \quad (26)\]

for a Dirac $\nu_\tau$.

These limits are about a factor of four stronger than the limits derived in [23] from the SNO and Super-Kamiokande observations and than the limits obtained in [25] from just the TRISTAN data.

It is worth mentioning that the limits on $\langle r_V^2 (\nu_\tau) \rangle$ and $\langle r_A^2 (\nu_\tau) \rangle$ are almost uncorrelated between them (see figure 3 of ref. [31]). The possibility of bounding simultaneously the vector and axial vector charge radii stems from the fact that in $e^+e^-$ annihilation also the right-handed neutrinos can be produced, and they couple to the photon through a combination of $\langle r_V^2 \rangle$ and $\langle r_A^2 \rangle$ which is linearly independent with respect to the one that couples the left-handed neutrinos. In contrast, neutrino scattering experiments do not involve the right handed neutrinos, and therefore can only bound the combination $\langle r_V^2 \rangle + \langle r_A^2 \rangle$.

Before concluding this section, we should mention that independent limits could also be derived from the DONUT experiment, through an analysis similar to the one presented in [51] that yielded limits on the $\nu_\tau$ magnetic moment. We have estimated that the constraints from DONUT would be at least one order of magnitude worse than the limits obtained from LEP; however, it should be remarked that these limits would be inferred directly from the absence of anomalous interactions for a neutrino beam with an identified $\nu_\tau$ component [31].

**LIMITS ON $\nu_\mu$ VECTOR AND AXIAL VECTOR CHARGE RADIUS**

The NuTeV collaboration has recently published a value of $\sin^2 \theta_W$ measured from the ratio of neutral current to charged current in deep inelastic $\nu_\mu$-nucleon scattering [53]. Their result reads

$$\sin^2 \theta_W^{(n)} = 0.2277 \pm 0.0013 \pm 0.0009 \quad (27)$$
where the first error is statistical and the second error is systematic. In order to derive limits on neutrino electromagnetic properties one should compare the results obtained in neutrino experiments to a value of $\sin^2 \theta_W$ determined from experiments that do not involve neutrinos. Currently, the most precise value of $\sin^2 \theta_W$ from non-neutrino experiments comes from measurements at the Z-pole and from direct measurements of the W-mass \cite{13}. In the numerical calculations in \cite{53} we have used the value for $\sin^2 \theta_W$ obtained from a global fit to electroweak measurements without neutrino-nucleon scattering data, as reported in \cite{52, 53}:

$$\sin^2 \theta_W = 0.2227 \pm 0.00037.$$  

The effect of a non-vanishing charge radius can be taken into account through the replacement $g_V \rightarrow g_V - \delta$ in the formulas for $\nu_e$-nucleon and $\nu_{\mu}$-electron scattering \cite{54}, where $\delta$ is given in \cite{13}. Since there are no right-handed neutrinos involved, there is no effect proportional to $\delta'$ and therefore only $\delta \propto \langle r_V^2 (\nu_\mu) \rangle + \langle r_A^2 (\nu_\mu) \rangle$ can be constrained. Upper and lower limits can be directly derived by comparing $\sin^2 \theta_W$ with the quoted value of $\sin^2 \theta_W$ from non-neutrino experiments. Since the results for neutrino experiments and the measurements at the Z-pole are not consistent at the $1 \sigma$ level, in the following equations (29)-(31) we will (conservatively) combine the errors by adding them linearly.

From the NuTeV result \cite{24}, we obtain the 90 % c.l. upper limit:

$$r_{V+A}^2 (\nu_\mu) \leq 7.1 \times 10^{-33} \text{ cm}^2,$$

where $r_{V+A}^2 (\nu_\mu) \equiv \langle r_V^2 (\nu_\mu) \rangle + \langle r_A^2 (\nu_\mu) \rangle$. However, since \cite{27} hints to a non-vanishing value of $\delta$, no lower limit is obtained from this measurement (see fig. 3). A reanalysis of the E734 data on $\nu_{\mu}$-$e$ and $\bar{\nu}_{\mu}$-$e$ scattering \cite{24} yields the 90 % c.l. limits:

$$-5.7 \times 10^{-32} \text{ cm}^2 \leq r_{V+A}^2 (\nu_\mu) \leq 1.1 \times 10^{-32} \text{ cm}^2.$$  

(30)

Note that in ref. \cite{23} the E734 collaboration is quoting a lower limit about 3.6 times and an upper limit about 7.5 times tighter than the ones given in (30). This is because of various reasons: first of all, as was pointed out in \cite{54}, in \cite{23} an inconsistent value for $G_F$ was used that resulted in bounds stronger by approximately a factor of $\sqrt{2}$. In addition, the errors were combined quadratically, which, due to the large negative trend in their data, resulted in a much stronger upper bound on $r_{V+A}^2 (\nu_\mu)$ than the one quoted here. Finally, our value of $\delta$ is defined through the shift $g_V \rightarrow g_V - \delta$ of the SM vector coupling, consistently for example with the notation of \cite{54}, while the convention used by the E734 Collaboration \cite{23}, as well as by CHARM II \cite{22}, define $\delta$ as a shift in $\sin^2 \theta_W$. This implies that our limits are larger for an additional factor of 2 with respect to the results published by these two collaborations.

From the CHARM II neutrino-electron scattering data \cite{23} we obtain at 90 % c.l.:

$$-5.2 \times 10^{-33} \text{ cm}^2 \leq r_{V+A}^2 (\nu_\mu) \leq 2.2 \times 10^{-32} \text{ cm}^2.$$  

(31)

These limits differ from the numbers published by the CHARM II collaboration \cite{23} not only because of the mentioned factor of 2 in the definition of $\delta$, but also because the present value of $\sin^2 \theta_W$ \cite{13} is smaller than the one used in 1995 in the CHARM II analysis.

From the data published by the CCFR collaboration \cite{55} one can deduce

$$-5.3 \times 10^{-33} \text{ cm}^2 \leq r_{V+A}^2 (\nu_\mu) \leq 6.8 \times 10^{-33} \text{ cm}^2.$$  

(32)

The four limits discussed above are represented in fig. 3, that makes apparent the level of precision of the NuTeV result. By combining the upper limit from CCFR \cite{22} and the lower limit from CHARM II \cite{23} we finally obtain:

$$-5.2 \times 10^{-33} \text{ cm}^2 \leq r_{V+A}^2 (\nu_\mu) \leq 6.8 \times 10^{-33} \text{ cm}^2.$$  

(33)

It is well known that the NuTeV result shows a sizable deviation from the SM predictions \cite{54}, and as a consequence it also appears to be inconsistent (at the 90 % c.l.) with $\delta = 0$. In fact, strictly speaking their result $\langle r_V^2 (\nu_\mu) \rangle + \langle r_A^2 (\nu_\mu) \rangle = (4.20 \pm 1.64) \times 10^{-33} \text{ cm}^2$ (1 $\sigma$ error) could be interpreted as a measurement of $\langle r_V^2 (\nu_\mu) \rangle + \langle r_A^2 (\nu_\mu) \rangle$, that becomes consistent with zero only at approximately 2.5 standard deviations. However, while the quoted value is not in conflict with other experimental limits, we believe that it would not be easy to construct a model that could generate a neutrino charge radius of the required size, without conflicting with other high precision electroweak measurements.

\textbf{FIGURE 3.} 90 % c.l. limits on $(\langle r_V^2 \rangle + \langle r_A^2 \rangle)$ for the muon neutrino derived from (a) E734 at BNL \cite{23}, (b) CHARM II \cite{22}, (c) CCFR experiment \cite{55}, and (d) from the NuTeV result \cite{24}. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{90 % c.l. limits on $(\langle r_V^2 \rangle + \langle r_A^2 \rangle)$ for the muon neutrino derived from (a) E734 at BNL \cite{23}, (b) CHARM II \cite{22}, (c) CCFR experiment \cite{55}, and (d) from the NuTeV result \cite{24}.}
\end{figure}
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