New type of lenses based upon left-handed materials

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Abstract

New type of lenses which are a slab of a left-handed material embedded into a regular material is proposed. These two materials should have equal refractive indices. Lenses with a focal length larger than the width of the slab can be constructed. These lenses should be easier to make than the well known Veselago lens, because the materials of the Veselago lens should obey an additional matching condition. Lenses of new type have multiple foci and might be useful for the 3D imaging.

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In his seminal work Veselago has introduced the concept of left-handed materials (LHM’s). In a simplest case the LHM’s are materials with simultaneously negative electric permittivity $\varepsilon$ and magnetic permeability $\mu$ in some frequency range. It is easy to show that in LHM the vectors $\mathbf{k}, \mathbf{E}, \mathbf{H}$ form a left-handed set, while in usual materials ($\varepsilon > 0$, $\mu > 0$) they form a right-handed set. If the imaginary parts of $\varepsilon$ and $\mu$ are small, the electromagnetic waves (EMW’s) propagate in the LHM but they have some unusual properties. All these properties originate from the fact that in the isotropic LHM the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ is anti-parallel to the wave vector $\mathbf{k}$.

Consider a propagation of the EMW from a point source located at the point $z = -a$ through an infinite slab of the LHM with the thickness $d$ and a usual right-handed material (RHM) at $z < 0$ and $z > d$ (Fig. 1). We assume below that refractive indices of the LHM and the RHM are the same. This assumption excludes the total internal reflection. Then transmission coefficient of the slab is non-zero and it is obvious that $S_z > 0$ everywhere at $z > -a$ because the energy propagates from its source. Directions of the vector $\mathbf{k}$ for different rays are shown by arrows. They should be chosen in such a way that at both interfaces tangential components of vector $\mathbf{k}$ for incident, reflected and refracted waves are the same. Another condition is that the component $k_z$ should be parallel to $S_z$ in the RHM and anti-parallel in the LHM. It follows that the Snell’s law for the RHM-LHM interfaces has an anomalous form: $\sin i / \sin r = -n'/n$, where $i$ and $r$ are the angles of incidence and refraction respectively, $n' = \sqrt{|\varepsilon'||\mu'|/\varepsilon_0\mu_0}$ and $n = \sqrt{\varepsilon\mu/\varepsilon_0\mu_0}$ are positive refractive indices for LHM and RHM respectively. The angles of reflection are equal to the angles of incidence. Refractive index in the LHM’s is often defined as negative. It has been shown recently that introduction of the negative refractive index for the LHM’s is unnecessary and even misleading.

Recently a method of fabricating of the left-handed metamaterials on the basis of metallic photonic crystals has been found and the San Diego group has reported the first observation of the anomalous transmission and even the anomalous Snell’s law. Both observations have been interpreted as the result of negative $\varepsilon$ and $\mu$. The speculations about the nature of
negative $\epsilon$ and $\mu$ in the proposed metamaterials are still controversial, but the very existence of the LHM seems to be demonstrated.

Pendry has proposed that the Veselago lens (VL) is a perfect lens in a sense that the width of its foci does not have usual wave length limitation. This is still a controversial point. Our paper is based upon geometrical optics only so that this unresolved question does not appear.

In this paper we propose and analyze another type of lenses, which has geometrical construction similar to the VL. In these lenses $\epsilon'\mu' = \epsilon\mu$ like in the VL, but $\epsilon' \neq -\epsilon$, $\mu' \neq -\mu$. Since $n' = n$ the absolute value of the angles of incidence, reflection and refraction is the same and this system may also work as a lens (see Fig. 2). The most important difference between the new type of lenses and the VL is the presence of reflected waves. As a consequence, the new lenses produce a periodic array of 3D images of different intensity. The lenses should be a lot easier to manufacture than the VL, because the materials for them should obey one condition only ($\epsilon\mu = \epsilon'\mu'$) instead of two in the case of the VL ($\epsilon = -\epsilon'$, $\mu = -\mu'$). Another advantage is that the new lenses can have a focal length greater than the width of the slab $d$ and produce images of objects located at distances larger than $d$ (Fig. 2(B)). In contrast, VL produces real images of the objects located at a distance closer than $d$ from the lens only.

One can see that both the VL and the new lenses are absolute instruments because they image stastically three-dimensional domains and the optical length of any curve in the object space is equal to the optical length of their images. Note, that the above properties of the absolute instrument should be valid in the limit of geometrical optics only. For the VL this 3D domain is limited by the condition $-d \leq z \leq 0$, while for the new lenses it is the whole half-space $z < 0$. Since the LHM’s have been already obtained we think that the new lenses might be extremely important for 3D imaging.

In the rest of this paper we calculate the positions of the multiple foci of the new lenses and the distribution of intensities among these foci. Since the angles of incidence, reflection and refraction are equal to each other, a simple geometric construction gives the following equations for the positions of the foci

\[ z = a - m_0 d \quad \text{and} \quad z = \pm (2dm - a), \quad m = m_0 + 1, m_0 + 2, ..., \quad m_0 \text{ - even (or zero)} \]

\[ z = \pm (2dm - a), \quad m = m_0, m_0 + 1, ..., \quad m_0 \text{ - odd} \]
where $m_0 = \text{Int}[a/d]$, $a$ is the distance from the point source to the slab.

To study the intensities in the foci one should know the reflection and transmission coefficients at the RHM-LHM and LHM-RHM interfaces. As has been shown by Veselago,[1] one can get them from the regular Fresnel expressions by substituting the absolute values of $\epsilon'$ and $\mu'$. Then the reflection and transmission coefficients at the RHM-LHM interface $r$, $t$ and LHM-RHM interface $r'$, $t'$ have a form

$$t = \frac{2\epsilon}{\epsilon + |\epsilon'|}, \quad r = \frac{|\epsilon'| - \epsilon}{|\epsilon'| + \epsilon}$$

$$t' = \frac{2|\epsilon'|}{\epsilon + |\epsilon'|}, \quad r' = \frac{\epsilon - |\epsilon'|}{\epsilon + |\epsilon'|}$$

These equations are valid for an arbitrary angle of incidence and polarization[2]. The multiple scattering approach gives the following expression for the intensities in the foci in the case when $a < d$

$$I(a) = (1 - r^2)I_0$$

$$I(2dm - a) = (1 - r^2)^2r^{4m-4}I_0, \quad m = 1, 2...$$

$$I(-[2dm - a]) = (1 - r^2)^2r^{4m-2}I_0, \quad m = 1, 2...$$

where $I_0$ is the intensity of the source. The sum of the intensities over all foci at the right of the slab is $I_R = I_0(1 - r^2)/(1 + r^2)$. The net intensity in the foci at the left of the slab is $I_L = I_0r^2(1 - r^2)/(1 + r^2)$. One can check that $I_R + I_L + I_{\text{lost}} = I_0$, where $I_{\text{lost}} = r^2I_0$ is the intensity of the light which did not contribute to any of the foci and which is the light reflected from the first interface. Another energy conservation statement is that the intensity in the focus inside the slab $z = a$ is equal to the sum of the intensities over all foci at the left and at the right of the slab: $I_L + I_R = I(a) = (1 - r^2)I_0$. At a given value of $n'$ lenses of the new type differ from each other by reflection coefficient $r$. For a specified refractive index $n'$ one might achieve different results by choosing proper values of $r$. To obtain maximum intensity in the closest to the lens focus one should make $r$ close to 0 (the limit $r = 0$ corresponds to the VL with one focus only outside of the slab). If $r$ close to 1, the intensities change slowly from one focus to another, thus one obtains an array of images with almost equal intensity.

Now let us study the three dimensional images produced by our lens and their spatial orientation relatively to the source (Fig. 2(A)). One can see that the images at the left side
of the slab are inverted, however the images at the right side have the same orientation as the source. The image inside the slab is also inverted.

When the distance from the source to the slab satisfies the relation \( d < a < 2d \), the intensities in the foci are

\[
I(2d - a) = (1 - r^2)r^2I_0
\]

\[
I(2dm - a) = (1 - r^2)^2r^{4m-4}I_0, \quad m = 2, 3...
\]

\[
I([-2dm - a]) = (1 - r^2)^2r^{4m-2}I_0, \quad m = 1, 2...
\]

The sum of the intensities over all foci at the right of the slab is \( I_R = I_0r^4(1 - r^2)/(1 + r^2) \).

The net intensity in the foci at the left of the slab \( I_L = I_0r^2(1 - r^2)/(1 + r^2) \). It is easy to show that \( I_R + I_L + I_{\text{lost}} = I_0 \), where in this case \( I_{\text{lost}} = [r^2 + (1 - r^2)^2]I_0 \) is the intensity of the light which did not contribute to any of the foci. As well as in the previously considered case of \( a < d \) we have that \( I_L + I_R \) is equal to the intensity in the focus inside the slab \( I(2d - a) = r^2(1 - r^2) \). At \( r \) close to 1 the intensities in the foci outside the slab change slowly from one focus to another. Figure 2(B) also shows the 3D images and their orientations. In this case the images at the left side of the slab are inverted, while the images at the right side and the image inside the slab are not. Note, that at \( a > d \) the VL does not have any real images.

Thus, we proposed a new type of lenses on the basis of the LHM. They are easier for manufacturing than the VL, they have multiple foci, and they can produce 3D images at the distances greater than the width of the LHM slab.

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1. V. G. Veselago, Sov. Phys.-Solid State 8, 2854 (1967).
2. N. Garcia and M. Nieto-Vesperinas, Phys. Rev. Lett. 88, 207403 (2002).
3. P. M. Valanju, R. M. Walser, and A. P. Valanju, Phys. Rev. Lett. 88, 187401 (2002).
4. J. B. Pendry, Phys. Rev. Lett. 85, 3966 (2000).
5. R. A. Shelby, D. R. Smith, and S. Schultz, Science 292, 77 (2001).
6. A. L. Pokrovsky and A. L. Efros, Solid State Comm. 124, 283 (2002).
7. D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. 84, 4184 (2000).
FIG. 1: Reflection and refraction of light outgoing from a point source at $z = -a$ and passing through the slab of the LHM at $0 < z < d$. Refraction of light is described by the anomalous Snell’s law. The arrows represent the direction of the wave vector. The reflected waves are shown by dashed lines near each interface only. The slab is surrounded by the usual RHM. (a) $n' > n$. (b) The Veselago lens ($n' = n$). The reflected waves are absent, all rays pass through two foci.

8 P. Markos and C. M. Soukoulis, Phys. Rev. B 65, 033401 (2001).
9 A. L. Pokrovsky and A. L. Efros, Phys. Rev. Lett. 89, 093901 (2002).
10 G. W. ’t Hooft, Phys. Rev. Lett. 87, 249701 (2001).
11 J. M. Williams, Phys. Rev. Lett. 87, 249703 (2001).
12 F. D. M. Haldane, cond-mat/0206420.
13 A. L. Pokrovsky and A. L. Efros, cond-mat/0202078.
14 M. Born and E. Wolf, Principles of Optics (Pergamon Press, Oxford, 1980), p. 143.
15 J. D. Jackson, Classical Electrodynamics (Willey & Sons, New York, 1998), p. 305.
FIG. 2: Multiple 3D images ("fishes") of the object marked by the bulb. The active element of the lens is a slab made of the LHM with $\epsilon'\mu' = \epsilon\mu$, but $\epsilon' \neq -\epsilon$ and $\mu' \neq -\mu$. The arrows show the directions of the wave vectors, which are opposite to the Poynting vector inside the slab. (A) $a < d$; (B) $d < a < 2d$. 