An Extended Framework for Marginalized Domain Adaptation

Gabriela Csurka, Boris Chidlovski, Stéphane Clinchant and Sophia Michel
Xerox Research Center Europe (XRCE),
6 chemin Maupertuis, 38240 Meylan, France,
Firstname.Lastname@xrce.xerox.com

Abstract
We propose an extended framework for marginalized domain adaptation, aimed at addressing unsupervised, supervised and semi-supervised scenarios. We argue that the denoising principle should be extended to explicitly promote domain-invariant features as well as help the classification task. Therefore we propose to jointly learn the data auto-encoders and the target classifiers. First, in order to make the denoised features domain-invariant, we propose a domain regularization that may be either a domain prediction loss or a maximum mean discrepancy between the source and target data. The noise marginalization in this case is reduced to solving the linear matrix system $AX = B$ which has a closed-form solution. Second, in order to help the classification, we include a class regularization term. Adding this component reduces the learning problem to solving a Sylvester linear matrix equation $AX + BX = C$, for which an efficient iterative procedure exists as well. We did an extensive study to assess how these regularization terms improve the baseline performance in the three domain adaptation scenarios. We present experimental results on two image and one text benchmark datasets, conventionally used for validating domain adaptation methods. We report our findings and comparison with state-of-the-art methods.

1 Introduction

While huge volumes of unlabeled data are generated and made available in many domains, the cost of acquiring data labels remains high. Domain Adaptation (DA) problems arise each time we need to leverage labeled data in one or more related source domains, to learn a classifier for unseen or unlabeled data in a target domain. The domains are assumed to be related, but not identical and this domain shift occurs in multiple real-world applications, such as named entity recognition or opinion extraction across different text corpora, etc.

In this paper, we build on the domain adaptation work based on noise marginalization [9]. In deep learning, a denoising autoencoder (DA) learns a robust feature representation from training examples. In the case of domain adaptation, it takes unlabeled instances of both source and target data and learns a new feature representation by reconstructing the original features from their noised counterparts. A marginalized denoising autoencoder (MDA) marginalizes the noise at training time and thus does not require an optimization procedure using explicit data corruptions to learn the model parameters but computes the model in closed form. This makes MDAs scalable and computationally faster than the regular denoising autoencoders. The principle of noise marginalization has been successfully extended to learning with corrupted features [35], link prediction and multi-label learning [10], relational learning [12], collaborative filtering [32] and heterogeneous cross-domain learning [33, 46].

In this paper we extend the previous efforts and propose a larger framework for the marginalized domain adaptation. The marginalized domain adaptation refers to a denoising of source and target instances that explicitly makes their features domain invariant and eases the target prediction. We propose two extensions to the MDA. The first extension is a domain regularization, aimed at generating domain invariant features. Two families of such regularization are considered; one is based on the domain prediction principle, inspired by the adversarial learning of neural networks [25]; the second uses the maximum mean discrepancy (MMD) measure [31].

The second extension to the MDA is a class regularization; it allows to generate a classifier for target instances which can be learned jointly with the domain invariant representation.

Our framework works in supervised, unsupervised and semi-supervised settings, where the source data is completed with a few labeled target data, massive unlabeled target data or both, respectively. In all cases, the noise marginalization is maintained, thus ensuring the scalability and computational efficiency. We show how to jointly optimize the data denoising and the domain regularization, and how to marginalize the noise, which guarantees the closed-form solution and thus the computational efficiency. When the joint optimization is extended to the target prediction, the solution does not have a closed form, but is the solution of a Sylvester linear matrix equation $AX + XB = C$, for which efficient iterative methods can be used.

The remainder of the paper is organized as follows. In Section 2 we revise the prior art. Section 3 presents the components of the marginalized domain adaptation, including in-
stance denoising, domain and class regularizations and target classifier learning. The joint loss minimization is detailed in Section 4. In Section 5 we describe two image and one text datasets we used, the experimental settings. We report the evaluation results which are grouped and analyzed by the three settings, namely, unsupervised, supervised and semi-supervised ones. Section 6 discusses the open questions and concludes the paper.

2 State of the art

Domain adaptation for text data has been studied for more than a decade, with applications in statistical machine translation, opinion mining, and document ranking [18, 47]. Most effective techniques include feature replication [17], pivot stance denoising, domain and class regularizations and target collections [11]. Domain adaptation has equally received a lot of attention in computer vision[1]. A considerable effort to systematize different shallow domain adaptation and transfer learning techniques has been undertaken in [29, 38, 16]. These studies distinguished three main categories of domain adaptation methods. The first category aims at correcting sampling bias [44]. The second category is in line with multi-task learning where a common predictor is learned for all domains, which makes it robust to domain shift [8]. The third family seeks to find a common representation for both source and target examples so that the classification task becomes easier [57]. Finally, an important research direction deals with the theory of domain adaptation, namely when adaptation can be effective and guaranteed with generalization bounds [8].

More recently, deep learning has been proposed as a generic solution to domain adaptation and transfer learning problems [13, 26, 34]. One successful method which aims to find common features between source and target collection relies on denoising autoencoders. In deep learning, a denoising autoencoder is a one-layer neural network trained to reconstruct input data from partial random corruption [43]. The denoisers can be stacked into multi-layered architectures where the weights are fine-tuned with costly back-propagation. Alternatively, outputs of intermediate layers can be used as input features to other learning algorithms. This learned feature representation was applied to domain adaptation [26], where stacked denoising autoencoders (SDA) achieved top performance in sentiment analysis tasks. The main drawback of SDAs is the long training time, and Chen et al. [9] proposed a variant of SDA where the random corruption is marginalized out. This crucial step yields a unique optimal solution which is computed in closed form and eliminates therefore the need for back-propagation. In addition, features learned with this approach lead to a classification accuracy comparable with SDAs, with a remarkable reduction of the training time [9].

More recently, deep learning architectures have demonstrated their ability to learn robust features and that good transfer performances could be obtained by just fine-tuning the neural network on the target task [13]. While such solutions perform relatively well on some tasks, the refinement may require a significant amount of new labeled data. More recent works proposed better strategies than fine-tuning, by designing deep architecture for the domain adaptation task. For example, Ganin et al. [25] has shown that adding a domain prediction task while learning the deep neural network leads to better domain-invariant feature representation. Long et al. [34] proposed to add a multi-layer adaptation regularizer, based on a multi-kernel maximum mean discrepancy (MMD). These approaches obtained a significant performance gain which shows that transfer learning is not completely solved by fine-tuning and that transfer tasks should be addressed by appropriate deep learning representations.

3 Domain adaptation by feature denoising

We define a domain \( D \) as the composition of a feature space \( \mathcal{X} \subset \mathbb{R}^d \) and a label space \( \mathcal{Y} \). A given task in the domain \( D \) (classification, regression, ranking, etc.) is defined by a function \( h : \mathcal{X} \rightarrow \mathcal{Y} \). In the domain adaptation setting, we assume working with a source domain \( D^s \) represented by the feature matrix \( X^s \) and the corresponding labels \( Y^s \), and a target domain \( D^{t} \) with the features \( X^{t} \).

We distinguish among three scenarios of domain adaptation, depending on what is available in the target domain:

- **Unsupervised (US)** setting, where all available target instances are unlabeled. In this case, \( X^{t} \) is empty and the labeled data, denoted by \( X_{l} \) contain only the labeled source examples, \( X_{l} = X^s \).
- **Supervised (SUP)** setting, where few labeled target instances \( X^{l}_{l} \) are available at training time. In this case, we have \( X_{l} = [X^s, X^{l}_{l}] \).
- **Semi-supervised (SS)** setting, where massively unlabeled \( (X^{u}_{l}) \) target data are available together with few labeled \( (X^{l}_{l}) \) data at the training time.

In what follows we propose a framework to address all three scenarios in one uniform way. It aims at finding such a transformation of source and target data that minimizes the following loss function:

\[
L = L_1 + \lambda L_2 + \gamma L_3, 
\]

where

- \( L_1 \) is the data denoising loss on all data \( X = [X^s, X^{l}] \),
- \( L_2 \) is the cross-domain classification loss on labeled data \( X_{l} = [X^s, X^{l}_{l}] \) with labels \( Y_{l} = [Y^s, Y^{l}_{l}] \),
- \( L_3 \) is the alignment loss on unlabeled data \( X^{u}_{l} \),
• $L_3$ is the domain regularization loss on source and target data $X$.

Parameters $\lambda$ and $\gamma$ capture the trade-off between the three terms. All losses and parameters are described in the following subsections. Intuitively, minimizing the total loss (3.1) can help exploring the implicit dependencies between the data denoising, the domain regularization and the cross-domain classification.

In this paper we study the case when all three terms in (3.1) belong to the class of squared loss functions. More precisely:

• $L_1 \equiv L_1(X, W)$ is the instance denoising loss under the dropout law; we minimize the square loss $\|X - XW\|^2$ between the corrupted data $X$ and the original data $X$ denoised with the linear transformation $W$. This term is the core element of the marginalized denoising autoencoder (MDA) [9].

• $L_2 \equiv L_2(X_t, Y_t, W, Z_t)$ is the class regularization loss, aimed at learning a (multi-class) ridge classifier $Z_t$ from the available corrupted and denoised instances $X_t, W$. The term is defined as $\|Y_t - X_tWZ_t\|^2$. It can be seen as a generalization of the Marginalized Corrupted Features (MCF) framework [36] with a square loss (the MCF corresponds to the case when $W = I_d$).

• $L_3 \equiv L_3(X^*, X^t, W)$ is the domain regularization loss that expresses the discrepancy between the source and target domains. We explore two options for this term. One is based on the empirical maximum mean discrepancy (MMD), taking into account the class labels when available; the other uses a pre-trained domain classifier to regularize the total loss.

We follow the marginalized framework for optimizing the loss on corrupted data [9, 36], and minimize the loss expectation $E[L]$. To simplify the reading, we denote the expected loss values $E[L]_i$ also with $L_i$.

By minimizing the marginalized expected loss (3.1), $\arg\min_{W, Z_t} L$, we obtain optimal solutions for the transformation matrix $W$ and classifier $Z_t$. This can be achieved in two different ways, namely:

• $W$ and $Z_t$ are learned sequentially. In this case we first set $\lambda = 0$ and learn $W$ by minimizing $L_1 + \gamma L_3$. Then for the fixed $W$ we learn $Z_t$ from $L_2$. Except the supervised MMD in $L_3$, the learning of $W$ remains unsupervised, including the supervised and semi-supervised settings. The target labels in these cases are used at the second step, when the classifier $Z_t$ is learned.

• $W$ and $Z_t$ are learned jointly. In this case we iteratively optimize the joint loss with respect to $W$ and $Z_t$. To initialize the iterative process, we set $W = I_d$ and minimize $L$ to compute $Z_t$, then we fix $Z_t$ and optimize $L$ with respect to $W$, and so on. The process is repeated until convergence. In practice we observed that the convergence is achieved after several iterations.

In the following subsections we describe in details and discuss each of the three loss terms, in Section 4 we address their different combinations.

### 3.1 Domain Instance Denoising

The first term we consider is the loss used by the Marginalized Denoising Autoencoder (MDA) [9]. Its basic idea is to reconstruct the input data from a partial random corruption [36] with a marginalization that yields optimal reconstruction weights in a closed form. The MDA loss can be written as

$$L_1 \equiv \frac{1}{M} \sum_{m=1}^{M} \|X - \tilde{X}_m W\|^2 + \omega \|W\|^2,$$

where $\tilde{X}_m \in \mathbb{R}^N \times \mathbb{R}^d$ is the $m$-th corrupted version of $X$ by random feature dropout with a probability $p$ and $\omega \|W\|^2$ is a regularization term. In order to avoid the explicit feature corruption and an iterative optimization, Chen et al. [9] showed that by considering the limit case $M \to \infty$, the weak law of large numbers allows to rewrite the loss $L_1$ as its expectation and the optimal $W$ can be written as (see Appendix for details):

$$W = (Q + \omega I_d)^{-1} P,$$

where $P$ and $Q$ depend only on the covariance matrix $X^\top X$ and the noise level $p$.

One main advantage of the MDA is that it requires no label and therefore can be applied in all three settings US, SUP and SS. Note in the supervised case $X_t = [X^*, X^t]$ includes only few target examples to learn $W$.

### 3.2 Learning with marginalized corrupted features

Inspired by the Marginalized Corrupted Features (MCF) approach [36], we propose to marginalize the following loss:

$$L_2 \equiv \frac{1}{M} \sum_{m=1}^{M} \|Y_t - \tilde{X}_t m WZ_t\|^2 + \delta \|Z_t\|^2,$$

where $Z_t \in \mathbb{R}^d \times \mathbb{R}^C$ is a multi-class classifier (each column corresponds to one of the $C$ classes), $Y_t \in \mathbb{R}^N \times \mathbb{R}^C$ is a label matrix, where $y_{nc} = 1$ if $x_n$ belongs to class $c = 1, \ldots, C$, and -1 otherwise, and $\delta \|Z_t\|^2$ is a regularization term. When $W = I_d$, we obtain the MCF baseline where the classifier is learned directly with the corrupted features.
Moreover, when \( p = 0 \), we obtain the ridge classifier learned with the original features.

Given \( \mathbf{W} \), the multi-class classifier \( \mathbf{Z}_t \) can be computed in closed form using the expected loss of (3.4) (see derivations in the Appendix):

\[
(3.5) \quad \mathbf{Z}_t = (1 - p)(\mathbf{W}^T \mathbf{Q}_t \mathbf{W} + \delta \mathbf{I}_d)^{-1}\mathbf{W}^T \mathbf{X}_t^* \mathbf{Y}_t.
\]

The computation of \( \mathbf{Z}_t^* \) requires the labeled data \( \mathbf{X}_t \), that contain the source (US) or possibly target data (SUP, SS).

### 3.3 Reducing the discrepancy between domains

The domain regularization term \( \mathcal{L}_3 \) in (3.1) is aimed at bringing the target domain closer to the source domain, by minimizing the discrepancy between the domains. In the following, we explore three options for the term \( \mathcal{L}_3 \), namely (1) a classical empirical MMD using the linear kernel, (2) its supervised version where the discrepancy is minimized between the class means and (3) a domain classifier \( \mathbf{Z}_D \) trained on the uncorrupted data to distinguish between the source and target data.

#### 3.3.1 Reducing the MMD between domains

The minimization of maximum mean discrepancy (MMD) \( \mathcal{L}_3 \) between the source and target domains is the state of art approach widely used in the literature. It is often integrated in feature transformation learning \([2, 3, 7]\) or used as a regularizer for the cross-domain classifier learning \([20, 34, 42]\). The MMD is defined as a distance in the reproducing kernel Hilbert space (RKHS). In practice, its empirical version is used, as it can be written as \( Tr(\mathbf{K} \mathbf{N}) \), where

\[
\mathbf{K} = \begin{bmatrix} \mathbf{K}^{s,s} & \mathbf{K}^{s,t} \\ \mathbf{K}^{t,s} & \mathbf{K}^{t,t} \end{bmatrix} \quad \text{and} \quad \mathbf{N} = \begin{bmatrix} \frac{1}{N_s N_s} & \frac{1}{N_s N_t} \\ \frac{1}{N_t N_s} & \frac{1}{N_t N_t} \end{bmatrix},
\]

where \( \mathbf{K}^{a,b} \) is the kernel distance matrix between all elements of \( \mathbf{X}^a \) and \( \mathbf{X}^b \), \( 1^{a,b} \) is a constant matrix of size \( N_a \times N_b \) with all elements being equal to 1, and \( N_s, N_t \) are the number of source and target examples.

We integrate this loss in the total one \( \mathcal{L} \) by considering the MMD between the source and target data after the denoising. To be able to marginalize out the loss and to keep our solution linearly solvable, we use the MMD with the linear kernel. Intuitively, this corresponds to minimizing the distance between the two centroids of the source and target data after denoising. The corresponding loss can be expressed as follows

\[
(3.6) \quad \mathcal{L}_m = \frac{1}{M} \sum_{m=1}^{M} Tr(\mathbf{W}^T \mathbf{X}_m^* \mathbf{N} \mathbf{X}_m \mathbf{W}).
\]

After marginalizing the expected loss, we obtain \( \mathbb{E}\mathcal{L}_3 = Tr(\mathbf{W}^T \mathbf{M} \mathbf{W}) \), where \( \mathbf{M} = \mathbb{E}[\mathbf{X}^T \mathbf{N} \mathbf{X}] \) (see the derivations in the Appendix).

#### 3.3.2 Reducing the MMD between the domain class means

The MMD requires no labels and can be computed between all available source all target instances. If we have labeled source and labeled target examples we can go one step further and modify the MMD to measure the distance between the means (centroids) of corresponding classes in the source and target domains \([33]\). The corresponding loss is the following

\[
(3.7) \quad \mathcal{L}_c = \frac{1}{M} \sum_{m=1}^{M} Tr(\mathbf{W}^T \mathbf{X}_m^* \mathbf{C} \mathbf{X}_m \mathbf{W}),
\]

where:

\[
\mathbf{C}_{ij} = \begin{cases} 
\frac{1}{N_s N_s} & \text{if } x_i, x_j \in D^s \text{ & } y_i = y_j = c \\
\frac{1}{N_t N_t} & \text{if } x_i, x_j \in D^t \text{ & } y_i = y_j = c \\
\frac{1}{N_s N_t} & \text{if } x_i \in D^s, x_j \in D^t \text{ & } y_i = y_j = c \\
0 & \text{if } x_i \in D^t, x_j \in D^s \text{ & } y_i = y_j = c \\
\end{cases}
\]

\( N^c_s \) is the number of source instances from the class \( c \) and \( N^c_t \) is the number of target instances from the class \( c \). Note that the "otherwise" item above includes all cases where \( y_i \neq y_j \) and where either \( x_i \) or \( x_j \) is unlabeled.

Similarly to \( \mathcal{L}_m \) in (3.6), we can marginalize out the expected loss \( \mathcal{L}_c \) and obtain \( \mathbb{E}\mathcal{L}_c = Tr(\mathbf{W}^T \mathbf{M} \mathbf{W}) \), where \( \mathbf{M} = \mathbb{E}[\mathbf{X}^T \mathbf{C} \mathbf{X}] \).

#### 3.3.3 Learning a domain classifier

As the last option of the domain regularization \( \mathcal{L}_3 \) in (3.1), we explore a loss based on the domain classifier \([14]\). Inspired by [23] who proposed to regularize intermediate layers in a deep learning model with a domain prediction task, [14] combines the domain prediction regularization with the MDA. We develop a similar regularization term for our extended framework, and use it jointly with the feature denoising term \( \mathcal{L}_1 \) and the class regularization \( \mathcal{L}_2 \).

The idea of this domain regularization is to denoise data in such a way that pushes source data towards the target and hence allows the cross-domain classifier to perform better on the target. This is done by first learning a domain classifier \( \mathbf{Z}_D \in \mathbb{R}^N \) using a regularized ridge classifier learned on the uncorrupted data. The regularized loss is defined as \( \|\mathbf{Y}_D - \mathbf{XZ}_D\|^2 + \alpha \|\mathbf{Z}_D\|^2 \), where \( \mathbf{Y}_D \in \mathbb{R}^N \) are the domain labels (-1 for source and +1 for target). The closed form solution is the following

\[
(3.8) \quad \mathbf{Z}_D = (\mathbf{X}^T \mathbf{X} + \alpha \mathbf{I}_d)^{-1}(\mathbf{X}^T \mathbf{Y}_D).
\]

Then the loss we consider in our unified framework is:

\[
(3.9) \quad \mathcal{L}_d = \frac{1}{M} \sum_{m=1}^{M} \|\mathbf{Y}_T - \mathbf{X}_m \mathbf{WZ}_D\|^2,
\]

where \( \mathbf{Y}_T = 1_N \) is a vector containing only ones (all denoised instances should be predicted as target).
Table 1: All models of the extended framework, with the corresponding notations, losses and solutions.

| Method | Loss | Closed-form solution for W |
|--------|------|---------------------------|
| S1     | $\mathcal{L}_1$ | $(Q + \omega I_d)^{-1}P$ |
| S1M    | $\mathcal{L}_1 + \gamma \mathcal{L}_m$ | $(Q + \omega I_d + \gamma M_d)^{-1}P$ |
| S1C    | $\mathcal{L}_1 + \gamma \mathcal{L}_c$ | $(Q + \omega I_d + \gamma M_c)^{-1}P$ |
| S1D    | $\mathcal{L}_1 + \gamma \mathcal{L}_d$ | $Q^{-1}(P + \gamma (1 - p)X^\top Y^\top Z_d^T)(I_d + \gamma Z_d Z_d^\top)^{-1}$ |
| J12    | $\mathcal{L}_1 + \lambda \mathcal{L}_2$ | $A = A_{12} = Q_{12}^{-1}(Q + \omega I_d), B = B_{12} = \lambda Z_l Z_l^\top, \quad C = C_{12} = Q_{12}^{-1}(P + \lambda (1 - p) X_l^\top Y_l Z_l^\top)$ |
| J12M   | $\mathcal{L}_1 + \lambda \mathcal{L}_2 + \gamma \mathcal{L}_m$ | $A = Q_{12}^{-1}(Q + \omega I_d + \gamma M_l), B = B_{12} = C = C_{12}$ |
| J12C   | $\mathcal{L}_1 + \lambda \mathcal{L}_2 + \gamma \mathcal{L}_c$ | $A = Q_{12}^{-1}(Q + \omega I_d + \gamma M_c), B = B_{12} = C = C_{12}$ |
| J12D   | $\mathcal{L}_1 + \lambda \mathcal{L}_2 + \gamma \mathcal{L}_d$ | $A = Q_{12}^{-1} Q, B = A_{12}(I_d + \gamma Z_d Z_d^\top)^{-1}, \quad C = (C_{12} + Q_{12}^{-1} \gamma (1 - p) X_l^\top Y_l Z_d^T)(I_d + \gamma Z_d Z_d^\top)^{-1}$ |

4 Minimizing the total loss

In the previous section we described three terms of the loss function $\mathcal{L}$. Now, we discuss two main cases of minimizing the total loss. First, we discuss the sequential case, where we first learn $W$ using only the data without labels ($\mathcal{L}_1$ or $\mathcal{L}_1 + \gamma \mathcal{L}_3$), and then we learn the classifier $Z_l$ or any other classifier. Second, we describe the joint case where $W$ and $Z_l$ are learned jointly, by iteratively minimizing the total loss $\mathcal{L}_1 + \lambda \mathcal{L}_2 + \gamma \mathcal{L}_d$. In both cases we consider three options for the domain regularization $\mathcal{L}_d$ and discuss three domain adaptation scenarios, $US$, $SUP$ and $SS$.

All mentioned combinations of the losses form different models; we denote them as follows. The sequential methods are prefixed by a character $S$ followed by the indexes of the losses used. For example, when we learn $W$ with $\mathcal{L}_1 + \gamma \mathcal{L}_d$, the method is denoted $S1D$. When we learn $W$ and $Z_l$ jointly, we prefix the method by a character $J$ followed by the loss indexes. For example, the method $J12C$ means that we optimize $\mathcal{L}_1 + \lambda \mathcal{L}_2 + \gamma \mathcal{L}_c$. All the combinations are summarized in Table 1.

4.1 Sequential framework

In this first case, we obtain $W$ in an unsupervised manner and then learn a classifier $Z_l$ using the denoised features. To get $W$, we set $\lambda = 0$ and minimize $\mathcal{L} = \mathcal{L}_1 + \gamma \mathcal{L}_3$. For each option of loss $\mathcal{L}_3$, we get closed-form solutions for $W$, denoted $S1$, $S1M$, and $S1D$. All the solutions are presented in Table 1. Any model can be deployed in three domain adaptation scenarios. In the $SUP$ and $SS$ cases, we additionally exploit the class labels using $\mathcal{L}_3 = \mathcal{L}_c$ (see S1C in Table 1).

Once $W$ is computed, we learn $Z_l$ using (35) or use any other classifier by feeding it with the denoised features $X_l W$. In the $US$ case, the classifiers are learned with the denoised source features, while in the $SUP$ and $SS$ cases the classifier exploits additionally the labeled target data.

4.2 Joint framework

In this case, $W$ and $Z_l$ are learned jointly, by alternatively optimizing the total loss $\mathcal{L}$ in variables $W$ and $Z_l$. We start by initializing $W$ with $I_d$ and minimize the loss in $Z_l$, then we fix $Z_l$ and compute $W$, and so on. The process is repeated until convergence for a certain threshold.

The partial derivatives of $\mathcal{L}$ with respect to $Z_l$ depend on $\mathcal{L}_2$ only, this makes solution (35) always valid. The partial derivatives with respect to $W$ can be written as a Sylvester linear matrix equation $AW + WB = C$, that we solve using the Bartels-Stewart algorithm (40). Depending on which loss is used as $\mathcal{L}_3$, we obtain three versions of the Sylvester equation, denoted $J12$, $J12M$ and $J12D$ and detailed in Table 1.

Note that for $J12D$ we do not use the regularizer term $\omega \|W\|^2$ in the loss, in order to be able to reduce the partial derivatives to solving a Sylvester equation. Furthermore, in the $SUP$ case, as $Q = Q_l$, and $P = P_l$, if we remove $\omega \|W\|^2$ from $J12$ we obtain a closed form solution $W = Q_l^{-1}(P_l + \lambda (1 - p) X_l^\top Y_l Z_l^T)(I_d + \lambda Z_l Z_l^\top)^{-1}$. Note that in our experiments we found that the results with the term (by solving a Sylvester equation) and without (a closed form solution) are similar, but the latter case is much faster.

5 Experimental Results

In the experimental section, we pursue a number of important goals. First, we want to assess all models proposed in the previous sections in three domain adaptation scenarios. Second, we evaluate the impact of the domain regularization $\mathcal{L}_d$ and the class regularization $\mathcal{L}_c$ on the denoising matrix $W$ and target classifier $Z_l$. Finally, we report the performance of the sequential and joint learning cases, we analyze our results and compare them to the state-of-the-art.

---

The code for all models is available at [http://github.com/sclincha/xrce_msda_da_regularization](http://github.com/sclincha/xrce_msda_da_regularization)
This section is organized as follows. In Section 5.1, we briefly describe three datasets used in the experiments, then Section 5.2 describes the experimental setting, including the methods and parameters used. In Section 5.3, we compare the sequential and joint models with different loss combinations in the US, SUP and SS settings, for all datasets. Finally, in Section 5.4, we compare our best performing models with the state-of-the-art.

5.1 Datasets All experiments are conducted on three domain adaptation datasets, well known in image processing and sentiment analysis communities.

OFF31 and OC10. Two most popular datasets used to compare visual domain adaptation methods are the Office31 dataset [39] (OFF31) and the Office+Caltech10 [28] (OC10). The former consists of three domains: Amazon (A), dslr (D) and Webcam (W) with images of 31 products (classes). The latter contains 10 of the 31 classes for the same domains and includes an extra domain from the Caltech collection. For all images, we use the Decaf TF6 features [19] with the full training protocol [27] where all source data is used for training.

AMT. A standard dataset for textual domain adaptation is the Amazon dataset of text products reviews; it includes four domains: Books (b), DVD (d), Kitchen (k) and Electronics (e) preprocessed by Blitz et al. [5]. Reviews are considered as positive if they have more than 3 stars, and negative otherwise. We adopt the experimental setting of [25] where documents are represented by a bag of unigrams and bi-grams with the 5000 most frequent common words selected and a tf-idf weighting scheme.

5.2 Methods and settings Most models proposed in the previous sections produce the denoising matrix \( W \) and the target classifier \( Z_t \), which can be inferred sequentially or jointly. The sequential approach learns first the matrix \( W \) and then a target classifier \( Z_t \) for a fixed \( W \). The joint approach learns \( W \) and \( Z_t \) jointly by alternating the updates of \( W \) and \( Z_t \) using the same loss \( L \).

For each dataset, we consider all domains and take all possible source-target pairs as domain adaptation tasks. For example, for the OFF31 dataset, we consider six following adaptation tasks: \( D \rightarrow A \), \( A \rightarrow D \), \( A \rightarrow W \), \( W \rightarrow D \), \( A \rightarrow W \), and \( D \rightarrow W \). Similarly, for the OC10 and AMT datasets, we consider all possible source-target pairs as adaptation tasks. To compare the different models, we report averaged accuracies over all adaptation tasks for a given dataset. In the supervised (SUP) and semi-supervised (SS) scenarios, we randomly select 3 target instances per class to form the target training set, and use the rest for the test.

In addition to the sequential and joint model learning, we include in our framework two standard classifiers, the nearest neighbor (NN) classifier and the Domain Specific Class Means (DSCM) as they represent a valuable alternative to the ridge target \( Z_t \) classifier [15]. In DSCM, a target test example is assigned to a class by using a soft-max distance to the domain-specific class means. The main reason for choosing these classifiers is that NN is related to retrieval (equivalent to precision at 1) and NCM with clustering, so the impact of \( W \) on these two extra tasks is indirectly assessed.

Fed with the denoised instances, obtained with matrix \( W \), these classifiers help assess the value of our framework for domain adaptation tasks.

To ensure the fair comparison of all methods, we run all experiments with a unique parameter setting. All selected parameter values are explained below: Besides, cross-validation on the source is not the best way to set model parameters for transfer learning and domain adaptation [45].

- we set \( \lambda = \gamma = 1 \) as term weights, this corresponds to the equal weighting in the global loss (3.1);
- we set \( \omega = 10^{-2} \) as \( W \) norm regularization in (3.3) (as in [9]);
- we set the dropping noise level for \( P \) and \( Q \) in (3.3): \( p = 0.5 \) for image datasets and \( p = 0.9 \) for AMT, as text representations are initially very sparse and a higher noise level is required;
- we set \( \alpha = \delta = 1 \) for the classifier regularization terms in (3.4) and (3.8);
- we consider a single layer MDA only, to enable a fair comparison of different loss combinations and learning methods.

To reveal all strong and weak points of our framework, we compare all models and classifiers with two baselines. The first baseline is denoted BL and provided by the classifier learned on the original features, without denoising. The classifier \( Z_t \) is learned using (3.5) with \( p = 0 \) and hence \( Q = S \). The second baseline refers to the original MDA method [19] and corresponds to S1 method in our framework. It uses the loss \( L_1 \) to build \( W \) in an unsupervised manner and learns a classifier on the denoised features.

In the following subsections, we compare the methods of our framework to the baselines, for all domain adaptation settings and all datasets.

---

*In this case, while we do not have the guarantee a global minimum, we observed in general to quick convergence of the loss (only a few iterations)."
5.3 Comparing domain adaptation methods

5.3.1 Unsupervised Setting In this case, labeled data are available from the source domain only, $X_t = X^s$. We compare the different models described in Section 4 (see the summary in Table 1). For each method and each domain adaptation task, we learn the $Z_t$ classifier and the NN and DSCM classifiers applied to the denoised features. The accuracy results are averaged per dataset and reported in Table 2.

The first observation is that all BL baselines get improved by the MDA (S1). Second, on the text data (AMT), the $Z_t$ classifier performs the best. Third, on the image collections the picture is more complex, with the NN showing the globally highest accuracy. If we compare the baseline S1 and extended methods, we can conclude the following. In sequential framework, the domain regularization S1D often improves over S1 for the linear classifier $Z_t$ and the DSCM, but not of the NN. In the joint framework, the class regularization $L_2$ degrades the linear classifier $Z_t$ results but improves the DSCM classifier.

To conclude, in the unsupervised domain adaptation, the best strategy may depend on the data type, regularization and classification method. If we compare methods by averaging their results over the rows in Table 2, S1D with the average 75.7% appears as the best strategy over all classifiers and datasets, followed by J12. Both outperform the baseline S1 with the average 74.5%. This suggests that the best strategy is to learn the denoising $W$ with the domain regularization and then to learn any classifier from denoised features.

5.3.2 Supervised Setting In this case we have $X = X_t = [X^s, X^t]$, $Q_t = Q$ and unlabeled target data is unavailable. In Table 3 we report the evaluation results for the different models using $Z_t$, NN and DSCM classifiers. It is easy to note that all models in the supervised case behave quite different from the unsupervised one.

5.3.3 Semi-supervised Setting In this case, a large set of unlabeled target data is available together with a small set of labeled target data. We explore if the proposed methods are able to take advantage of the two. Table 4 shows the results of different models on the three datasets, and them to the baselines.

Sequential framework. Compared to the supervised case, having more target examples makes the domain regularization $L_3$ to either have no effect or slightly improve the results.

Joint framework. Adding class regularization $L_2$ either improves or does not change the results, except learning $Z_t$ for AMT, where a significant drop is observed. Adding $L_d$ often decreases the performance, while adding $L_c$ is less harmful. In general, in the semi-supervised case J12 is the best strategy for the image datasets. For the text dataset, like in the unsupervised case, using S1D with $W$, followed by learning a classifier on the denoised features seems to be a better option.

5.4 Comparison with the state of the art We complete the experimental section by comparing our results to the state of art results, in the unsupervised and semi-supervised
Table 4: Semi-supervised domain adaptation on OC10, OFF31 and AMT. Bold indicates the best result per dataset, underline indicates the improvement over L1.

|       | OC10   | OFF31  | AMT   |
|-------|--------|--------|-------|
|       | nn dscm | Zi     | nn dscm | Zi     |
| BL    | 90.8   | 91.6   | 88.1   | 77.6   | 76.6   | 70.3   | 73.3   |
| S1    | 91.1   | 91.9   | 90.4   | 78.4   | 76.9   | 74.6   | 82.4   |
| S1D   | 91.2   | 91.9   | 90.7   | 78.1   | 77.3   | 74.5   | 82.2   |
| S1C   | 91.3   | 92.1   | 90.4   | 78.5   | 77.6   | 74.5   | 82.4   |
| J12   | 92.3   | 91.8   | 89.1   | 80.8   | 80.5   | 74.7   | 76.6   |
| J12D  | 87.9   | 89.8   | 89.1   | 80.8   | 80.4   | 74.7   | 76.6   |
| J12C  | 92.1   | 91.6   | 89.1   | 80.8   | 80.2   | 74.9   | 76.6   |

Concerning the semi-supervised scenario, it is much less used and most papers report results with SURF BOV features and the sampling protocol [39, 28]. We therefore tested our methods on OC10 with L12C+DSCM and BOV features averaged over the 20 random samples; and we get an accuracy of 55.8% that is above most state of art results, including GFK [23] (48.6%), SA [23] (53.6%), MMDT [30] (52.5%).

6 Conclusion

We proposed an extended framework for domain adaptation, where the state-of-the-art marginalized denoising autoencoder is extended with domain and class regularization terms, aimed at addressing unsupervised, supervised and semi-supervised scenarios. The domain regularization drives the denoising of both source and target data toward domain invariant features. Two families of domain regularization, based on domain prediction and the maximum mean discrepancy, are proposed. The class regularization learns a cross-domain classifier jointly with the common representation learning. In all cases, the models can be reduced to solving a linear matrix equation or its Sylvester version, for which efficient algorithms exist.

We presented the results of an extensive set of experiments on two image and one text benchmark datasets, where the proposed framework is tested in different settings. We showed that adding the new regularization terms allow to outperform the baselines and help design best performing strategies for each adaptation scenarios and data types. Compared to the state of art we showed that despite of their speed and relatively low cost, our models yield comparable or better results than existing feature transformation methods but below highly expensive non-linear methods with additional data processing such as the landmark selection or those using deep architectures requiring costly operations both at training and at test time. Furthermore, similarly to the stacked MDA framework, we can easily stack several layers together with only forward learning, where the denoised features of the previous layer become the input of a new layer and non-linear functions can be applied between the layers.

References

[1] R. Aljundi, R. Emonet, D. Muselet, and M. Sebban. Landmarks-based kernelized subspace alignment for unsupervised domain adaptation. In Proc. of CVPR, (IEEE), 2015.
[2] M. Baktashmotlagh, M. Harandi, B. Lovell, and M. Salzmann. Unsupervised domain adaptation by domain invariant projection. In Proc. of ICCV, (IEEE), 2013.
[3] S. Ben-David, J. Blitzer, K. Crammer, and F. Pereira. Analysis of representations for domain adaptation. In NIPS, 2007.
[4] J. Blitzer, R. McDonald, and F. Pereira. Domain adaptation with structural correspondence learning. In EMNLP, 2006.
[5] J. Blitzer, S. Kakade, and D. P. Foster. Domain Adaptation with Coupled Subspaces. In AISTATS, 2011.
[6] K. M. Borgwardt, A. Gretton, M. J. Rasch, et al. Integrating structured biological data by kernel maximum mean discrepancy. Bioinformatics, 2006.
[7] L. Bruzzone and M. Marconcini. Domain adaptation problems: A dasvm classification technique and a circular validation strategy. (PAMI), 2010.
[8] M. Chen, K. Q. Weinberger, and J. Blitzer. Co-training for domain adaptation. In NIPS, 2011.

\(^7\)We exclude the supervised scenario as rarely addressed in the literature.

\(^8\)We report results from [22].
Table 5: Unsupervised adaptation on the OFF3L: (top) feature transformation methods with FC6 features; (middle) our results with using FC6 features; (bottom) the deep domain adaptation methods. Best results per task are in bold. Underline indicates best results on FC6 features.

| Method          | A→W | D→W | W→D | D→A | W→A | A→D | Mean |
|-----------------|-----|-----|-----|-----|-----|-----|------|
| GFK+SVM [41]    | 37.8| 81.0| 86.9| 34.8| 31.4| 44.8| 49.1 |
| SA+SVM [41]     | 35  | 74.5| 81.5| 32.3| 30.1| 41.3| 49.1 |
| TCA+SVM [41]    | 36.8| 82.3| 84.1| 32.9| 28.9| 40.6| 50.9 |
| CORAL+SVM [41]  | 48.4| 96.5| 99.2| 44.4| 41.9| 53.7| 64.0 |
| L12 + NN        | 50.5| 94.3| 98.0| 46.4| 42.8| 53.8| 64.3 |
| L1D + RDG       | 53.9| 91.2| 95.1| 47.3| 49.0| 55.5| 65.3 |
| DDC [42]        | 59.4| 92.5| 91.7| 52.1| 52.2| 64.4| 68.7 |
| RGrad [25]      | 67.3| 94.0| 93.7| -   | -   | -   | -    |
| DAN [24]        | 68.5| 96.0| 99  | 54  | 53.1| 67.0| 72.9 |

[9] M. Chen, Z. Xu, K. Q. Weinberger, and F. Sha. Marginalized denoising autoencoders for domain adaptation. In ICML, 2012.
[10] Z. Chen, M. Chen, K. Q. Weinberger, and W. Zhang. Marginalized denoising for link prediction and multi-label learning. In AAAI, 2015.
[11] Z. Chen and B. Liu. Topic modeling using topics from many domains, lifelong learning and big data. In ICML, 2014.
[12] Z. Chen and W. Zhang. A marginalized denoising method for link prediction in relational data. In ICDM, 2014.
[13] S. Chopra, S. Balakrishnan, and R. Gopalan. DLID: Deep learning for domain adaptation by interpolating between domains. In ICML Workshop (WREPL), 2013.
[14] S. Clinchant, G. Csurka, and B. Chidlovskii. A domain adaptation regularization for denoising autoencoders. In Proc. of ACL, 2016.
[15] G. Csurka, B. Chidlovskii, and F. Perronnin. Domain adaptation with a domain specific class means classifier. In TASK-CV, ECCV workshop, 2014.
[16] G. Csurka. Domain Adaptation for Visual Applications: A Comprehensive Survey. CoRR, arXiv:1702.05374, 2017.
[17] H. Daumé. Frustratingly easy domain adaptation. CoRR, arXiv:0907.1815, 2009.
[18] H. Daumé III and D. Marcu. Domain adaptation for statistical classifiers. Journal of Artificial Intelligence Research, 2006.
[19] J. Donahue, et al. Decaf: A deep convolutional activation feature for generic visual recognition. CoRR, arXiv:1310.1531, 2013.
[20] L. Duan, I. W. Tsang, and D. Xu. Domain transfer multiple kernel learning. Transactions of Pattern Recognition and Machine Analyses (PAMI), 34(3):465–479, 2012.
[21] L. Duan, I. W. Tsang, D. Xu, and T.-S. Chua. Domain adaptation from multiple sources via auxiliary classifiers. In Proc. of ICML, pages 289–296, 2009.
[22] N. Farajidavar, T. deCampos, and J. Kittler. Transductive transfer machines. In Proc. of ACCV, pages 623–639, 2014.
[23] B. Fernando, A. Habrard, M. Sebban, and T. Tuytelaars. Unsupervised visual domain adaptation using subspace alignment. In Proc. of ICCV, (IEEE), pages 2960–2967, 2013.
[24] Y. Ganin and V. Lempitsky. Unsupervised domain adaptation by backpropagation. In Proc. of ICML, 2015.
[25] Y. Ganin et al. Domain-adversarial training of neural networks. CoRR, arXiv:1505.07818, 2015.
[26] X. Glorot, A. Bordes, and Y. Bengio. Domain adaptation for large-scale sentiment classification: A deep learning approach. In Proc. of ICML 2011.
[27] B. Gong, K. Grauman, and F. Sha. Connecting the dots with landmarks: Discriminatively learning domain invariant features for unsupervised domain adaptation. In ICML, 2013.
[28] B. Gong et al. Geodesic flow kernel for unsupervised domain adaptation. In CVPR, (IEEE), 2012.
[29] R. Gopalan, R. Li, V. M. Patel, and R. Chellappa. Domain adaptation for visual recognition. Foundations and Trends in Computer Graphics and Vision, 8(4), 2015.
[30] J. Hoffman, E. Rodner, J. Donahue, T. Darrell, and K. Saenko. Efficient learning of domain-invariant image representations. In Proc. of ICLR, 2013.
[31] J. Huang, A. Smola, A. Gretton, K. Borgwardt, and B. Schölkopf. Correcting sample selection bias by unlabeled data. In Proc. of NIPS, 2007.
[32] S. Li, J. Kawale, and Y. Fu. Deep collaborative filtering via marginalized denoising auto-encode. In CIKM, 2015.
[33] Y. Li, M. Yang, Z. Xu, and Z. Zhang. Learning with marginalized corrupted features and labels together. In Proc. of AAAI, volume arXiv:1602:07332, 2016.
[34] M. Long, Y. Cao, J. Wang, and M. I. Jordan. Learning transferable features with deep adaptation networks. In Proc. of ICML, 2015.
[35] M. Long, J. Wang, G. Ding, J. Sun, and P. S. Yu. Transfer feature learning with joint distribution adaptation. In Proc. of ICCV, (IEEE), 2013.
[36] L. v. d. Maaten, M. Chen, S. Tyree, et al. Learning with marginalized corrupted features. In Proc. of ICML, 2013.
[37] S. J. Pan, J. T. Tsang, Ivor W and Kwok, and Q. Yang. Domain adaptation via transfer component analysis. Transactions on Neural Networks, 2011.
[38] S. J. Pan and Q. Yang. A survey on transfer learning. Transactions on Knowledge and Data Engineering, 2010.
[39] K. Saenko, B. Kulis, M. Fritz, and T. Darrell. Adapting visual category models to new domains. In Proc. of ECCV, 2010.
[40] D. C. Sorensen, and Y. Zhou. Direct methods for matrix Sylvester and Lyapunov equations. In Journal of Applied Mathematics, 2003(6), 277-303.
[41] B. Sun, J. Feng, and K. Saenko. Return of frustratingly easy
domain adaptation. In Proc. of AAAI, 2016.

[42] E. Tzeng, J. Hoffman, N. Zhang, K. Saenko, and T. Darrell. Deep domain confusion: Maximizing for domain invariance. CoRR, arXiv:1412.3474, 2014.

[43] P. Vincent, H. Larochelle, Y. Bengio, and P.-A. Manzagol. Extracting and composing robust features with denoising autoencoders. In Proc. of ICML, 2008.

[44] Z. Xu and S. Sun. Multi-source transfer learning with multi-view adaboost. In Proc. of NIPS, pages 332–339, 2012.

[45] E. Zhong, W. Fan, Q. Yang, O. Verscheure, and J. Ren. Cross validation framework to choose amongst models and datasets for transfer learning. In Proc. PKDD (ECML), 2010.

[46] J. T. Zhou, S. J. Pan, I. W. Tsang, and Y. Yan. Hybrid heterogeneous transfer learning through deep learning. In Proc. of AAAI, 2014.

[47] M. Zhou and K. C. Chang. Unifying learning to rank and domain adaptation: Enabling cross-task document scoring. In Proc. of SIGKDD, 2014.

Appendix

In this section we derive and show the partial derivatives of each expected loss terms according to $W$ and when relevant according to $Z_l$. In our derivations we used the fact that the trace is linear and it commutes with the expectations and used the derivative formulas of the trace from [7]:

$$L_1 = \mathbb{E}[Tr((X - \bar{X}X)^\top (X - \bar{X}X))] + \omega\|W\|^2$$
$$= Tr(X^\top X) - 2Tr(\mathbb{E}[X^\top X]W) + Tr(W^\top \mathbb{E}[X^\top X]W) + \omega\|W\|^2$$

where $\bar{X}$ is the random variable representing the corrupted $X^{\muu}$ features, $\mathbb{E}[X] = (1-p)X$, $P = \mathbb{E}[X^\top X]$ and $Q = \mathbb{E}[X^\top X]$. If we denote by $S$ the covariance matrix $X^\top X$ of the uncorrupted data, we have $P = (1-p)S$ and:

$$Q_{lj} = \begin{cases} S_{lj}(1-p)^2, & \text{if } i \neq j, \\ S_{lj}(1-p), & \text{if } i = j. \end{cases}$$

The partial derivatives of $L_1$ can be written as:

$$\frac{\partial L_1}{\partial W} = -2P + 2(Q + \omega I_d)W$$

Note that $L_2$ (3.4) and $L_d$ (3.9) are similar (we have $Z_D$ instead of $Z_l$, $Y_{\tau}$ instead of $Y_l$ and $\delta = 0$). Therefore we derive here the expected loss and its derivatives derivatives only for $L_2$:

$$L_2 = \mathbb{E}[Tr((Y_l - \bar{X}_l W Z_l)^\top (Y_l - \bar{X}_l W Z_l))] + \delta\|Z_l\|^2$$
$$= Tr(Y_l^\top Y_l) - 2Tr(Y_l^\top \mathbb{E}[\bar{X}_l]W Z_l) + Tr(Z_l^\top W^\top \mathbb{E}[X_l^\top X_l]W Z_l) + \delta\|Z_l\|^2$$
$$= \|Y\|^2 - 2(1-p)Y_l^\top X_l W Z_l + \delta\|Z_l\|^2$$

where $X_l$ is the labeled part of the data $[X^l, X_l^\muu]$, where $X_l^\muu$ is empty in the unsupervised scenario and $Q_l$ is computed as $Q$ but with $S_l = X^\top X$. In the case of $\ell_2$ (but not $\ell_d$) we derive the partial derivatives also according to $Z_l$:

$$\frac{\partial L_2}{\partial W} = -2(1-p)Y_l^\top X_l Z_l + 2Q_l W Z_l Z_l^\top$$
$$\frac{\partial L_2}{\partial Z_l} = -2(1-p)W^\top X_l^\top Y_l + 2(W^\top Q_l W + \delta I_d)Z_l$$

Finally, in the case of MMD the marginalized loss becomes:

$$L_m = Tr(W^\top \mathbb{E}[X^\top X] W) = Tr(W^\top MW)$$

yielding to the partial derivatives $\frac{\partial L_m}{\partial W} = 2MW$, where $M = \mathbb{E}[X^\top X]$ can be computed as $Q$ in (??) using $S_m = X^\top X$ instead of $S$. For $L_c$ we have the loss equal to $2M_c W$, where $M_c$ is computed with $S_c = X^\top_c C X_c^\top$.