Electromagnetic spin-1 form-factor free of zero-modes *

Abstract. The electromagnetic current $J^+$ for spin-1, is used here to extract the electromagnetic form-factors of a light-front constituent quark model. The charge ($G_0$), magnetic ($G_1$) and quadrupole $G_2$ form factors are calculated using different prescriptions known in the literature, for the combinations of the four independent matrix elements of the current between the polarisation states in the Drell-Yan frame. However, the results for some prescriptions relying only on the valence contribution breaks the rotational symmetry as they violate the angular condition. In the present work, we use some relations between the matrix elements of the electromagnetic current in order to eliminate the breaking of the rotational symmetry, by computing the zero-mode contributions to matrix elements resorting only to the valence ones.

Keywords Spin-1 Particles • Electromagnetic Current • Electromagnetic Form Factors • Light-Front Field Theory.

1 Introduction

Light-front models are useful to describe hadronic bound states, like mesons or baryons due to its particular boost properties [1; 2; 3]. However, the light-front description in a truncated Fock-space breaks the rotational symmetry because the associated transformation is a dynamical boost [4; 5; 6; 7; 8; 9]. Therefore, an analysis with covariant and analytical models, can be useful to pin down the main missing features in a truncated light-front Fock-space description of the composite system. In this respect, the rotational symmetry breaking of the plus component of the electromagnetic current, evaluated with the Drell-Yan condition (momentum transfer $q^+ = q^0 + q^3 = 0$), was recently analysed relying on a model of a composite spin 1 two-fermion bound state with an analytical form of the vertex [4; 5]. It was shown [4; 5; 7; 10; 11; 12] that if pair terms are ignored in the evaluation of the matrix elements of the electromagnetic current, the covariance of the electromagnetic form factors is broken. The scalar nature of the form factors is restored only when pair terms or zero modes contributions are included in the computation of the current matrix elements [5; 7; 11; 13].

However, the extraction of the electromagnetic form factors of a spin-1 composite particle from the microscopic matrix elements of the plus component of the current ($J^+ = J^0 + J^3$) in the Drell-

---

* Presented by J. P. B. C. de Melo at LIGHT-CONE 2014, May, NCSU-USA

J. P. B. C. de Melo and Anacé N. Silva
Laboratorio de Física Teórica e Computação Científica, Universidade Cruzeiro do Sul, 01506-000, São Paulo, Brazil.

T. Frederico and Clayton S. Mello
Departamento de Física, Instituto Tecnológico de Aeronáutica, DCTA, 12228-900, São José dos Campos, Brazil.
Yan frame, based only on the valence component of the wave function, is plagued by ambiguities due to an expected rotational invariance breaking with a truncated state [14; 15]. In the Breit frame, with the particular choice of the momentum transfer along the transverse direction, where the Drell-Yan-condition is valid, the plus component of the electromagnetic current has four independent matrix elements, although only three form factors, $G_0, G_1,$ and $G_2$ are sufficient to parametrize the current of a vector particle. Then, one is in principle free to choose different combinations of these matrix elements to compute form factors. But, these matrix elements satisfy an identity, the angular condition 

\[ m \text{ regularization parameter} \]

\[ \Lambda \text{ for the final state.} \]

It was demonstrated in [72] where \( \epsilon \) choice of matrix element of the current eliminated by assuming the angular condition. 

results. It was demonstrated in [10] with a simplified model of the vertex and light-cone polarizations to compute matrix elements, that the prescription from [14] eliminates zero-modes contributions to the form factors. Here, we will generalize such results by deriving analytical relations for the matrix elements [19], where pair terms are implicitly included, even considering only the valence contribution, and after that the numerical calculations of the spin-1 form factors with all prescriptions found in the literature agree.

We should add that zero-modes or pair terms in the matrix elements of the electromagnetic current appear in different ways. For example, in the case of spin-0 particles, like for the pion or kaon, the plus component of the electromagnetic current calculated in the Breit frame with the Drell-Yan condition does not have such contributions in simple analytical and covariant models of the vertex [13; 20; 21; 22; 23].

2 Covariant Model for Composite Vector Particle

The electromagnetic current of a spin 1 particle has the following general form:

\[ J_{\mu}^\alpha = \left[ F_1(q^2)g_{\alpha\beta} - F_2(q^2)\frac{q_\alpha q_\beta}{2m^2} \right](p^\mu + p'^\mu) - F_3(q^2)(q_\alpha g^\mu_\beta - q_\beta g^\mu_\alpha) , \]

(1)

where \( m_v \) is the mass of the vector particle, \( q^\mu \) is the transfer momentum, \( p^\mu \) and \( p'^\mu \) are the on-shell initial and final momentum, respectively. Here, the spin-1 particle is identified with a \( \rho \) meson. Linear combinations of the covariant form factors \( F_1, F_2 \) and \( F_3 \) allow to get the charge \((G_0)\), magnetic \((G_1)\) and quadrupole \((G_2)\) form factors (see e.g. [4]).

In the impulse approximation, the matrix elements of the electromagnetic current \( J_{ji}^+ \), are obtained from the loop integral [4],

\[ J_{ji}^+ = i \int \frac{d^4k}{(2\pi)^4} \frac{Tr[\epsilon^\alpha_j \Gamma_\beta(k, k - p_f)(k - \not{p}_f + m)\gamma^\mu(k - p_i + m)^e_i(k, k - p_i)(k + m)]A(k, p_f)A(k, p_i)}{(k^2 - m^2 + ie)((k - p_i)^2 - m^2 + ie)((k - p_f)^2 - m^2 + ie)} \]

(2)

where \( \epsilon_\mu \) and \( \epsilon_i \) are the polarization four-vectors of the final and initial states, respectively and \( m \) is the quark mass. The electromagnetic form factors are calculated in the Breit frame with the Drell-Yan-West condition, \( q^+ = 0 \), which gives the momentum transfer \( q^\mu = (0, q_x, 0, 0) \), the particle initial momentum \( p^\mu = (p_0^v, -q_x/2, 0, 0) \) and the final one \( p'^\mu = (p_0^v, q_x/2, 0, 0) \). We use the definition \( \eta = -q^2/4m_v \), and then the energy of the vector particle is \( p_0 = m_v\sqrt{1 + \eta} \).

The polarization four-vectors in the instant-form basis are given by \( \epsilon_{\mu}^v = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \) \( \epsilon_{\mu}^x = (0, 0, 1, 0) \), \( \epsilon_{\mu}^y = (0, 0, 0, 1) \), for the initial state and by, \( \epsilon_{\mu}^v = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \) \( \epsilon_{\mu}^x = \epsilon_{\mu}^y, \) \( \epsilon_{\mu}^x = \epsilon_{\mu}^z \), for the final state.

In order to make finite the photon-absorption amplitude; we use following regularization function \( A(k, p_f) = N/((p - k)^2 - m_R^2 + i\epsilon)^2 \) associated with a model for the vector particle vertex with a regularization parameter \( m_R \). The vertex function of the spin-1 particle [4] in terms of the quark momentum is given by

\[ \Gamma^\mu(k, p) = \gamma^\mu - \frac{m_v}{2} \frac{2k^\mu - p^\mu}{p.k + m_v m - i\epsilon} \].

(3)
In the equation above, the vector particle is on mass shell; and \( m_v \) is the vector bound state mass.

In the next section, the extraction schemes of the electromagnetic form factor from the plus component of the electromagnetic current and the angular condition are discussed.

3 Angular Condition and Prescriptions to Compute Vector Particle Form Factors

The angular condition for plus component of the electromagnetic current, Eq. (2), in the Breit frame \( (q^+ = 0) \), is given by the equation below, with the light-front, \( (I^+_{m^m}) \) and instant form, \( (J^{\prime \prime}_m) \), spin-basis, \([4; 24]\):

\[
\Delta (q^2) = (1 + 2\eta)I_{11}^+ + I_{1-1}^+ = (1 + \eta)(J^{\prime \prime}_{yy} - J^{\prime \prime}_{zz}) = 0 .
\]

(4)

The prescription adopted by Grach and Kondratyuk \([14]\) corresponds to eliminate the matrix element \( I_{00}^+ \) in the light-front spin basis, from the calculation of the electromagnetic form factors. In references \([10; 11; 19; 26; 27]\), it was explored the fact that the zero-mode contribute only to the matrix element \( I_{00}^+ \) in the light-front spin basis. In Ref. \([15]\) the prescription from Grach and Kondratyuk was used to calculate deuteron form factors, where \( I_{00}^+ \) is eliminated by the angular condition, and in this case the results for the electromagnetic form factors are free of the zero mode contribution \([11; 19; 26]\). However, zero modes are present in the matrix elements of the current in the case of the other prescriptions \([16; 17; 18; 25]\), and consequently the electromagnetic form factors are not free of such contributions (see the discussion in \([19]\)).

In Ref. \([19]\), it was derived relations fulfilled by the zero-mode or \( Z \)-diagram contributions to matrix elements of the current in the instant-form basis:

\[
J^{+Z}_{xx} + \eta J^{+Z}_{zz} = 0 , \; J^{+Z}_{zz} + \sqrt{\eta} J^{+Z}_{yy} = 0 \; \text{and} \; J^{+Z}_{yy} = 0 .
\]

(5)

From the angular condition (4) and the fact that \( J^{+}_{yy} \) does not have zero-modes, i.e., \( J^{+}_{yy} = 0 \); we can write the non-vanishing matrix elements for the \( Z \)-diagrams in terms of the valence matrix elements, starting with \( J^{+Z}_{zz} = J^{+V}_{yy} - J^{+V}_{zz} \). The final form of the matrix elements of the current, where the zero-mode contributions are eliminated in favor of the valence ones through the use of (5) are:

\[
\begin{align*}
J^{+}_{xx} &= J^{+V}_{xx} - \eta (J^{+V}_{yy} - J^{+V}_{zz}) \\
J^{+}_{zz} &= J^{+V}_{yy} - \sqrt{\eta} (J^{+V}_{yy} - J^{+V}_{zz}) \\
J^{+}_{yy} &= J^{+V}_{yy} ,
\end{align*}
\]

(6)

and also for light-front spin basis, \( I^+_{m^m} \), is written below:

\[
I^+_{11} = 0 , \; I^+_{10} = 0 , \; I^+_{1-1} = 0 , \; I^+_{00} = (1 + \eta)J^{+Z}_{zz} .
\]

(7)

The equations above, make clear, in the case of the light-front spin basis, that only \( I^+_{00} \), has a zero mode contribution \([19; 27]\), similar results are obtained also in the ref. \([10; 11]\). The prescription of Grach and Kondratyuk \([14]\) is written in terms of instant form spin basis and also in the light-front spin basis:

\[
\begin{align*}
G^{GK}_0 &= \frac{1}{3}(3 - 2\eta)I_{11}^+ + 2\sqrt{2\eta}I_{10}^+ + I_{1-1}^+ = \frac{1}{3}(J^{+}_{xx} + (2 - \eta)J^{+}_{yy} + \eta J^{+}_{zz}) . \\
G^{GK}_1 &= 2[I_{11}^+ - \frac{1}{\sqrt{2\eta}}I_{10}^+] = J^{+}_{yy} - J^{+}_{zz} - \frac{J^{+}_{xx}}{\sqrt{\eta}} , \\
G^{GK}_2 &= \frac{2\sqrt{2}}{3}[-\eta I_{11}^+ + \sqrt{2\eta}I_{10}^+ - I_{1-1}^+] = \frac{\sqrt{2}}{3}(J^{+}_{xx} - (1 + \eta)J^{+}_{yy} + \eta J^{+}_{zz}) ,
\end{align*}
\]

(8)
and with the use of the relations given by the Eq.(6) and Eq.(7), zero-mode contributions are absent due to fact that the matrix elements $I_{00}^+$ is excluded [19]. Other prescriptions from the literature are given by [18],

$$G_0^{CCKP} = \frac{1}{3(1+\eta)} \left[ \frac{3}{2} - \eta \right] (I_{11}^+ + I_{00}^+) + 5\sqrt{2\eta} I_{10}^+ + (2\eta - \frac{1}{2}) I_{1-1}^+ = \frac{1}{6} [2J_{xx}^+ + J_{yy}^+ + 3J_{zz}^+]$$

$$G_1^{CCKP} = \frac{1}{(1+\eta)} \left[ J_{xx}^+ + 3(1+2\eta) + I_{00}^+ - I_{1-1}^+ - \frac{2(1-\eta)}{\sqrt{2\eta}} I_{10}^+ \right] = \frac{J_{xx}^+}{\sqrt{\eta}}$$

$$G_2^{CCKP} = \frac{\sqrt{\eta}}{3(1+\eta)} \left[ -\eta I_{11}^+ + 2\sqrt{2\eta} I_{10}^+ - \eta I_{00}^+ + (\eta + 2) I_{1-1}^+ \right] = \frac{\sqrt{\eta}}{3} [J_{xx}^+ - J_{yy}^+].$$

The prescription from reference [16] builds the form factors of the spin-1 particle as follows:

$$G_0^{BH} = \frac{1}{3(1+2\eta)} \left[ (3-2\eta)I_{00}^+ + 8\sqrt{2\eta} I_{10}^+ + 2(2\eta - 1) I_{1-1}^+ \right]$$

$$G_1^{BH} = \frac{2}{(1+2\eta)} \left[ I_{00}^+ - I_{1-1}^+ + \frac{(2\eta - 1)}{\sqrt{2\eta}} I_{10}^+ \right]$$

$$G_2^{BH} = \frac{2\sqrt{2}}{3(1+2\eta)} \left[ \sqrt{2\eta} I_{10}^+ - \eta I_{00}^+ - (\eta + 1) I_{1-1}^+ \right]$$

The prescription from reference [17] gives the following from factors:

$$G_0^{FFS} = \frac{2}{3(1+2\eta)} \left[ (2\eta + 3) I_{11}^+ + 2\sqrt{2\eta} I_{10}^+ - \eta I_{00}^+ + (2\eta + 1) I_{1-1}^+ \right] = \frac{1}{3} [J_{xx}^+ + 2J_{yy}^+]$$

$$G_1^{FFS} = G_1^{CCKP}$$

$$G_2^{FFS} = G_2^{CCKP}.$$  

In the expressions of the form factors given by prescriptions detailed above [16; 17; 18] $I_{00}^+$ is present, which implies zero-modes contributions to the electromagnetic form factors. In the next section, we present the calculations of the electromagnetic form factors with these prescriptions.

4 Results and Summary

As suggested in Refs. [14; 15], the elimination of the matrix elements $I_{00}^+$ through the angular condition, as discussed in [11; 19] cancels out the zero-modes contribution to the form factors. In this case, it is possible to calculate the electromagnetic form factor only from the valence terms of the matrix elements of the plus component of the current in the Breit-frame with $q^+ = 0$. The relations (6) and (7) provides a practical way to compute the electromagnetic form factors of the spin-1 particle free of zero-modes, which is also verified numerically in the following figures 1, 2 and 3.

The parameters for the model are the constituent quark mass, $m = 0.430$ GeV, $\rho$ meson mass, $m_\rho = 0.770$ GeV, and the regulator mass, $m_R = 3.0$ GeV. The regulator mass is chosen to reproduce the experimental value of decay constant for the $\rho$ meson, which is $f_\rho = 0.152 \pm 0.008$ GeV [28]. The present value of $m_R$ is different form the one used in [4], where the decay constant was not fitted.

Our new numerical results are shown in figures 1, 2 and 3. The calculations were performed for the charge, magnetic and quadrupole form factors, with the instant form and the light-front spins basis. The prescriptions to extract form factors from the matrix elements of the plus component of the
Fig. 1 The figures show the electromagnetic form factors, $G_0$ and $G_1$, calculated with the plus component of the current with the prescriptions labelled as GK\[14\] (8), BH\[16\] (10), CCKP\[18\] (9) and FFS\[17\] (11) with and without zero-mode contributions (Z-terms).

Fig. 2 The quadrupole form factor from calculations labelled as in Fig. 1.

current given by Eqs. (8-11) were compared in the figures with the full covariant result (see ref. [4]). The charge form factor presents a zero, as shown in Fig. 1, which is dominated by valence terms with the position weakly affected by the inclusion of the zero-mode contribution to the form factor. The zero modes present a tail relevant for large momentum transfers for $G_0$ and $G_2$, while for $G_1$ the effect is somewhat attenuated. These observations deserve further investigations considering also different models for the $\rho$ Bethe-Salpeter amplitude, as the symmetric model proposed in [19].

In summary, in this work we checked numerically that after the inclusion of the contribution from zero-modes, the covariance of the form factors is restored and all prescriptions adopted here produced exactly the same results for the $\rho$ meson model adopted here. We also present a practical formula to calculate the matrix elements of the current in any spin basis computing only valence contributions.

Acknowledgments. This work was supported in part by the Brazilian agencies FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo), CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) and CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior). Benefited from access to the computing facility of the Centro Nacional de Supercomputação at the Federal University of Rio Grande do Sul (CESUSP/UFRGS). J. P. B. C. de Melo thanks the organizer of the Light-Cone 2014 for the invitation.
References

1. Terentev, M. V.: *On the structure of wave functions of mesons as bound states of relativistic quarks*. Sov. J. Nucl. Phys. 24, 106 (1976).
2. Kondratyuk, L. A. and Terentev, M. V.: *The Scattering Problem For Relativistic Systems With Fixed Number Of Particles*. Sov. J. Nucl. Phys. 31, 561 (1980).
3. Brodsky, S. J., Pauli, H.-C. and Pinsky, S. S.: *Quantum chromodynamics and other field theories on the light cone*. Phys. Rep. 301, 299 (1998).
4. de Melo, J. P. B. C. and Frederico, T.: *Covariant and light front approaches to the rho meson electromagnetic form factors*. Phy. Rev. C 55, 2043 (1997).
5. de Melo, J. P. B. C., Sales, J. H. O., Frederico, T. and Sauer, P. U.: *Pairs in the light front and covariance*. Nucl. Phys. A 631, 574c (1998).
6. Naus, H. W. L., de Melo, J. P. B. C. and Frederico, T.: *Ward-Takahashi identity on the light front*. Few-Body Systems. 24, 99 (1998).
7. de Melo, J. P. B. C., Frederico, T., Naus H. W. L. and Sauer, P. U.: *Pion electromagnetic current in the light cone formalism*. Nucl. Phys. A 660, 219 (1999).
8. Grach, I. L. and Kondratyuk, L. A.: *Electromagnetic form factor Of Deuteron In Relativistic Dynamics. Two Nucleon And Six Quark Components*. Sov. J. Nucl. Phys. 39, 198 (1984).
9. de Melo, J. P. B. C., El-Bennich, Bruno, and Frederico, T.: *The photon-pion transition form factor: incompatible data or incompatible models? Few Body Syst. 55, 373 (2014).*
10. Bakker, B. L. G. and Ji, C. R.: *The Vector meson form factor analysis in light front dynamics*: Phy. Rev. D65, 110001 (2002).
11. Choi, Ho-Meoyng and Ji, C. R.: *Electromagnetic structure of the rho meson in the light front quark model*. Phy. Rev. D 70, 053015 (2004).
12. de Melo, J. P. B. C., Frederico, T., Pace, E. and Salmé, G.: Frame dependence of the pair contribution to the pion electromagnetic form factor in a light front approach: Braz. Jour. of Phys. 33, 301 (2003).
13. de Melo, J. P. B. C., Naus, H. W. L. and Frederico, T.: *Pion electromagnetic current in the light cone formalism*. Phy. Rev. C 59, 2278 (1999).
14. Grach, I. L. and Kondratyuk, L. A.: *Electromagnetic form factor Of Deuteron In Relativistic Dynamics. Two Nucleon And Six Quark Components*. Sov. J. Nucl. Phys. 39, 198 (1984).
15. Grach, I. L., Kondratyuk, L. A. and Strikman, M.: *Is the structure in the Deuteron magnetic form factor at $Q^2$ approximately $=2-G e v^2$ a new evidence for nuclear core?*. Phy. Rev. Lett. 62, 387 (1989).
16. Brodsky, S. J. and Hiller, J. R.: *Universal properties of the electromagnetic interactions of spin one systems*. Phys. Rev. D46, 2141 (1992).
17. Frankfurt, L. L., Frederico, T. and Strikman, M.: *Deuteron form factors in the light cone quantum mechanics 'good' component approach*. Phy. Rev. C48, 2182 (1993).
18. Chung, P. L., Polyzou, W. N., Coester, F. and Keister, B. D.: *Hamiltonian Light Front Dynamics of Elastic electron Deuteron Scattering*. Phys. Rev. C37, 2000 (1988).
19. de Melo, J. P. B. C. and Frederico, T.: *Light-Front projection of spin-1 electromagnetic current and zero-modes*. Phys.Lett. B708, 87 (2012).
20. de Melo, J. P. B. C., Frederico, T., Pace, E. and Salmé, G.: *Pair term in the electromagnetic current within the front form dynamics: Spin-0 case*. Nucl. Phys. A 07, 399 (2002).
21. Pereira, Fabiano P., de Melo, J. P. B. C., Frederico, T. and Tomio, L.: *Kaon Electromagnetic Form Factor in the Light-Front Formalism*. Physics of Elementary Particles and Atomic Nuclei, Vol. 36, 217 (2005).
22. da Silva, Edson O., de Melo, J. P. B. C., El-Bennich, Bruno and Filho, Vouto C.: *Pion and kaon elastic form factors in a refined light-front model*. Phy. Rev. C 86, 038202 (2012).
23. de Melo, J. P. B. C., Tsushima, K., El-Bennich, Bruno, Rojas, E., Frederico, T.: *Pion structure in the nuclear medium*. Phys. Rev. C90, 035201 (2014).
24. Keister, B. D. and Polizou, W. N.: *Relativistic Hamiltonian dynamics in nuclear and particle physics*. Adv. Nucl. Phy. 20, 225 (1991).
25. Cardarelli, F., Grach, I. L., Narodetsky, I. M., Salme, G. and Simula, S.: *Electromagnetic form factors of the rho meson in a light front constituent quark model*. Phy. Lett. B 369, 393 (1995).
26. de Melo, J. P. B. C. and Frederico, T.: *Electromagnetic form factor of a composed vector particle in the light-front*, hep-ph/0311079.
27. de Melo, J. P. B. C. and Frederico, T.: *Eletroweak Form Factors in the Light-Front for Spin-1 Particles*. Few-Body Syst. 52, 403 (2012).
28. The Review of Particle Physics K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014).
