Numerical 3D flow simulation of attached cavitation structures at ultrasonic horn tips and statistical evaluation of flow aggressiveness via load collectives

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Abstract. A compressible inviscid flow solver with barotropic cavitation model is applied to two different ultrasonic horn set-ups and compared to hydrophone, shadowgraphy as well as erosion test data. The statistical analysis of single collapse events in wall-adjacent flow regions allows the determination of the flow aggressiveness via load collectives (cumulative event rate vs collapse pressure), which show an exponential decrease in agreement to studies on hydrodynamic cavitation [1]. A post-processing projection of event rate and collapse pressure on a reference grid reduces the grid dependency significantly. In order to evaluate the erosion-sensitive areas a statistical analysis of transient wall loads is utilised. Predicted erosion sensitive areas as well as temporal pressure and vapour volume evolution are in good agreement to the experimental data.

1. Introduction
Ultrasonic horn transducers are widely used as a standard test case for the investigation of the material resistance to cavitation erosion (mass loss curves) [2]. If the horn tip diameter is sufficiently small and driven at high amplitude with ultrasonic frequency, large cavitation clouds are generated, which cover the complete horn tip surface. These clouds oscillate at their own frequency falling into a subharmonic range referred to as the acoustic excitation frequency [3]. Znidarcic et al. [3] denominate this cavitation type as ‘acoustic supercavitation’ as it exhibits acoustic as well as hydrodynamic characteristics.

Different authors (Dular et al. [4] and Znidarcic et al. [5]) have figured out that standard hydrodynamic cavitation models (mass transfer models based on the Rayleigh equation) - due to the simplifications of the Rayleigh equation - cannot correctly predict the vapour volume and subharmonic oscillation frequency compared to experiments. Therefore, Znidarcic et al. [5] have developed a new mass transfer model especially for the simulation of ‘acoustic supercavitation’, which takes into account the second derivative term of the Rayleigh-Plesset equation. Applying this model for the simulation of acoustic supercavitation, accurate results have been achieved in terms of attached cavity oscillation frequency, vapour volume and pressure pulsations.

In contrast to Znidarcic et al. [5] we apply a compressible density-based CFD flow algorithm for the simulation of the ultrasonic horn flow. This method does accurately capture compressible wave dynamics, which is directly associated with the occurrence of cavitation erosion [6]. For the investigation of the erosion sensitive areas a statistical analysis of transient wall loads is used, which has been successfully applied in the past to hydrodynamic cavitation in the case of micro channels [7] and hydrofoils [8]. Furthermore, a statistical analysis of single collapse events in wall-near flow regions is utilised to evaluate flow aggressiveness, which has been developed by Mihatsch et al. [1] and applied to axis-symmetric and planar nozzles [1, 9] (hydrodynamic cavitation).
In the present study the simulation results of the density-based flow solver and the erosion analysis procedure are compared to measurement data (vapour amount, pressure pulsations, cavity oscillation frequency and eroded sample surfaces) in order to:

- validate our in-house solver hydRUB for ‘acoustic supercavitation’;
- verify the erosion analysis procedure for application in acoustically induced cavitation.

2. Test cases

Two horn tip set-ups are investigated:

- **case A**: tip diameter of 16 mm with counter sample in 500 µm distance; driving frequency: 19.82 kHz; peak-to-peak amplitude: 35.8 µm.
- **case B**: tip diameter of 3 mm; driving frequency: 20 kHz; peak-to-peak amplitude: 164 µm.

Case A corresponds to the standard erosion test case where a counter sample is added below the horn tip sample [2]. The tip amplitude and the driving frequency are defined from in-house far-field microscopy imaging. Erosion sensitive areas are determined from surface pictures and confocal microscopy images of the horn tip sample [10]. For the assessment of erosion sensitive areas from our simulation results, we assume that the region of highest erosion probability corresponds to the location with the highest material removal. From previous numerical studies [10] it can be deduced that the total gap is filled with a large vapour cloud, which pulsates on subharmonic time scales. Accordingly, we assume case A - despite the fact that it has a larger tip diameter and is driven at medium amplitude - to induce ‘acoustic supercavitation’.

Case B is based on the set-up of Znidaric et al. [5] who have investigated the vapour volume structure and vapour dynamic with high-speed shadowgraphy images as well as the acoustic pressure propagation with a hydrophone (7 mm distance to the horn tip).

3. Method

For a detailed description of the numerical and physical model as well as the erosion analysis procedure we refer to previous publications [10, 11].

3.1. Numerical and physical model

Our in-house solver hydRUB [7, 8] is applied which has been extended by a compressible inviscid low Mach-number consistent Godunov-type flux formulation [12] and an optimised explicit Runge-Kutta scheme [12]. Velocity and density are reconstructed by the SMART [13] and MINMOD [14] scheme, respectively. A homogeneous mixture of liquid and vapour in equilibrium (undissolvable gas is neglected) is assumed. A barotropic equation of state with an isentropic phase change [15] is utilised.

3.2. Erosion analysis procedure

For the evaluation of erosion-sensitive areas we use a statistical analysis of transient wall loads for two combined erosion indicators (wall pressure and condensation rate) referred to as erosion probability [10].

Furthermore, the ‘Collapse Detector’ by Mihatsch et al. [9] is applied, which is based on a set of physical criteria to detect collapses of isolated vapour clouds and characterises the strength of the collapse pressure. In order to reduce the grid dependency of the collapse pressure and the event rate, scaling laws are utilised, which are based on the linear decay law of spherical waves [1]. Collapse pressure and event rate are scaled by the ratio of characteristic grid spacing and a uniform reference length scale \( x_{\text{ref}} \), and the event rate is further corrected by a power law with exponent \( \kappa = 2/3 \). In [1], \( \kappa = 2/3 \) is proposed based on a hydrodynamic cavitation test case.

3.3. Numerical set-up and verification

Slip wall boundary conditions are applied, and the horn tip surface is moving with an oscillating grid kinematics. The free surface is modelled as symmetry (a small part is modelled as Dirichlet pressure boundary and therefore fixes the pressure level to 1 bar) and accounts for pressure wave reflection. Water at room temperature is used. The flow region around the ultrasonic horn
is meshed with a double O-grid topology. For test case A a grid sensitivity study on three successively refined grids (factor 2 in each direction) is performed. The total number of cells per gap are: 21600 (coarse), 172800 (middle), 1382000 (fine). We find that the erosion probability on all grids features the same erosion sensitive areas on the tip: minimum erosion probability at the edge, maximal erosion probability at a ring surface and nearly continuous erosion probability in the middle of the sample, which is in excellent agreement with the material removal at the surface pictures (cf. figure 1). Furthermore, the same vapour dynamic - subharmonic oscillation of the vapour cloud inside the gap at every 10th driving cycle - is predicted on all grids (not shown).

![Figure 1](image1.png)

**Figure 1.** Erosion probability at the horn tip for different grid resolutions compared to an erosion surface picture.

![Figure 2](image2.png)

**Figure 2.** Temporal evolution of the virtual hydrophone pressure $p_{\text{hydroph}}$ and vapour volume $V_{\text{vap}}$ from the CFD simulation of test case B.

In order to evaluate temporal convergence behaviour the maximal deviation as well as the standard deviation of the erosion probability $P_{Er}$ (perimeter averaged) are compared between different analysis time intervals (not shown here). We conclude, that erosion sensitive areas can be reliably predicted after very short simulation time intervals (horn tip: $T_{\text{analyse}} = 2 \text{ms} \approx 40$ driving cycles) and that a temporally converged solution is reached at the horn tip sample six times faster than at the counter sample. Reason is the more frequent loading at the horn tip sample, as it is stressed at every driving cycle (partial collapses at the edge of the vapour cloud), whereas the counter sample is mainly stressed after each subharmonic cycle (every 10th cycle).

4. Results

4.1. Flow aggressiveness (case A)

The ‘Collapse Detector’ is evaluated for wall-adjacent (distance ≤ 100 µm) flow regions at the entire horn tip and counter sample (cf. figure 3). The load collectives show an exponential decrease. For a scaling exponent of $\kappa = 2/3$ grid dependence of the load collective at the horn tip can essentially be eliminated (figure 3a), whereas at the counter sample, a considerable grid dependence remains, in particular for high collapse pressure (not shown). A significant reduction of grid dependence at the counter sample for high loads is obtained by a $\kappa = 3/2$ (figure 3b). However, although not significant for erosion prediction, the results for low loads remain grid-dependent for $\kappa = 3/2$. Furthermore, a comparison of loads between tip and counter sample is no longer possible due to the different values of $\kappa$. In particular, the extrapolation to even higher loads (trend lines in figure 3) suggest a higher load and therefore higher erosion sensitivity at the counter sample than on the tip which is a contradiction to experimental findings. We conclude that there is no unique value of $\kappa$, and it may even vary within the counter sample area.

4.2. Cavitating flow data (case B)

In agreement to the flow measurements from Znidarcic et al. [3], we observe a large attached vapour cloud oscillating on subharmonic time scales and changing its shape from mushroom
Figure 3. Load collectives (cumulative event rate vs collapse pressure) for the near wall region (100 µm) of the horn tip (a) and the counter sample (b). Un-corrected and corrected (according to [1]) load collectives as well as high-load trend lines are included.

to cone shape until collapse. The measured subharmonic frequency of 5050 Hz (1/4 of driving frequency) can be well reproduced (cf. figure 2). The peak pressure at the virtual hydrophone is predicted in the same range as in the experiment (figure 2, 2 bar ≤ \( p_{\text{hydroph}} \) ≤ 4 bar [3]). However, the produced vapour volume is predicted too low (ca. 30 %) compared with experimental data. We suppose the discrepancy appears due to the experimental procedure of determining vapour volume using shadowgraphy images (fixed brightness threshold; assume rotational symmetric vapour volume; neglect of inside liquid phase). This assumption is confirmed, as the analogue numerical procedure via silhouettes of iso-surfaces (vapour volume fraction threshold: 10 %) predicts the vapour volume in better agreement to the experiment (not shown here).

5. Conclusions

We conclude that the compressible flow solver with barotropic cavitation model can well reproduce the main features of ‘acoustic supercavitation’ (attached cavity oscillation frequency, vapour amount and structure as well as pressure pulsations). Erosion sensitive areas can reliably be predicted. An exponential decrease of the load collective is also found for ‘acoustic supercavitation’. For grid independent load collectives, different scaling exponents \( \kappa \) have been figured out for the horn tip and counter sample, so that the choice of \( \kappa \) seems to be case dependent and needs a deeper analysis which is on-going.

References

[1] Mihatsch M S, Schmidt S J, Thalhammer M and Adams N A 2012 Proc. 8th Int. Symp. on Cavitation (Singapore)
[2] Pohl M and Stella J 2002 Wear 252 pp 501-11
[3] Znidarcic A, Mettin R, Cairo C and Dular M 2014 Physics of Fluids 26 023304
[4] Dular M, Mettin R, Znidarcic A and Truong V A 2012 Proc. 8th Int. Symp. on Cavitation (Singapore)
[5] Znidarcic A, Mettin R and Dular M 2015 Ultrasound Sonochemistry 22 pp 482-92
[6] Schmidt S J, Sezal I H, Schnerr G H and Thalhammer M 2007 Proc. 8th Int. Symp. Experimental and Computational Aerothermodynamics of Internal Flows (Lyon)
[7] Skoda R, Iben U, Morozov A, Mihatsch M S, Schmidt S J and Adams N A 2011 WIMRC Proc. 3rd Int. Cavitation Forum (Warwick)
[8] Skoda R, Iben U, Guenther M and Schilling R 2012 Proc. 8th Int. Symp. on Cavitation (Singapore)
[9] Mihatsch M S, Schmidt S J, Thalhammer M, Adams N A, Skoda R and Iben U 2011 WIMRC Proc. 3rd Int. Cavitation Forum (Warwick)
[10] Mottyll S, Mueller S, Niederhauser P, Hussong J, Huth S and Skoda R 2014 EPJ Web of Conferences 67 02078
[11] Deimel C, Mottyll S, Skoda R, Kiermeir J, Guenther M and Schilling R 2014 Proc. 8th Int. Conf. Computational Fluid Dynamics (Chengdu)
[12] Schnerr G H, Sezal I H and Schmidt S J 2008 Physics of Fluids 20 040703
[13] Gaskell P H and Lau A K C 1988 Int. J. Numerical Methods in Fluids 8 pp 617-41
[14] Harten A 1983 J. Computational Physics 49 pp 357-93
[15] Iben U 2002 Systems Analysis Modelling Simulation 42 pp 1283-307