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Deterministic Generation of Large-Scale Entangled Photonic Cluster State from Interacting Solid State Emitters

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The ability to create large highly entangled “cluster” states is crucial for measurement-based quantum computing. We show that deterministic multiphoton entanglement can be created from coupled solid state quantum emitters without the need for any two-qubit gates and regardless of whether the emitters are identical. In particular, we present a general method for controlled entanglement creation by making direct use of the always-on exchange interaction, in combination with single-qubit operations. This is used to provide a recipe for the generation of two-dimensional, cluster-state entangled photons that can be carried out with existing experimental capabilities in quantum dots.

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The cluster state quantum computing paradigm is believed to be the most feasible approach for photonic quantum computing. In this approach, the difficulty of entanglement creation between photons is shifted to the upfront creation of a highly entangled multiqubit cluster state [1]. To date, photonic cluster states have been created by passing parametrically down-converted pairs of entangled photons through linear optic elements and subsequent measurement of a photon. This process is inherently probabilistic, and as a result creating a cluster state larger than a few photons is a formidable task [2]. In previous theoretical work it was shown [3] that a periodically pumped quantum dot (QD) can produce a cluster state string of photons. Very recently, there has been an experimental breakthrough materializing this deterministic approach and generating a one-dimensional cluster state [4]. However for applications, larger dimensional graph states are needed. To that end, a proposal [5] generalized the scheme of Ref. [3] to a pair of QDs, introducing the idea that entangled emitters can emit entangled photons. The main challenges with that approach are that it requires the application of experimentally demanding two-qubit entangling gates between the emitters, and that it assumes the two QDs do not interact in the absence of optical pulses. Although there are ongoing efforts to demonstrate this idea, these issues make the experiments challenging. The recent experimental progress of Ref. [4] makes a practical protocol to generate a higher dimensional cluster state with existing resources a particularly timely topic. In addition to quantum computing, the deterministic creation of large-scale cluster states would also impact quantum communications [6,7].

Here we present a deterministic protocol for generating two-dimensional photonic cluster states which requires no externally driven two-qubit gates and which allows for, and in fact makes crucial use of, an always-on coupling between emitters. The necessary entanglement is built up by free evolution and the photons are generated in an entangled state through optical pumping of the emitters. For QDs it is the always-on exchange interaction between the spins that provides entanglement. Remarkably, we show that with carefully chosen pulse sequences this entanglement is sufficient to generate a cluster state when combined with single-qubit gates already demonstrated in experiments. We provide the pulse sequences that implement the required evolution for two distinct cases, emitters with (i) equal and (ii) unequal Zeeman splittings.

While the scheme we describe is applicable to any pair of emitters coupled with Heisenberg type interaction, we focus on QDs, because they are very efficient emitters, and the coupled-QD system has been studied and understood very well experimentally [8–11]. We consider a pair of stacked epitaxial QDs with a thin enough barrier between such that they are tunnel coupled. A bias voltage controllably loads single carriers (electrons or holes) into each QD. We consider the bias regime where each QD contains a single electron. The electrons can virtually tunnel into the opposite QD, and thus there is an effective exchange interaction between them (Fig. 1). Recent experimental advances based on this system demonstrated ultrafast coherent control, including single spin rotations and entanglement control [11].

In previous work [5], we showed that to generate a photonic cluster state the required evolution of the two emitters should have the form $U_{\text{target}} = cz(A \otimes A)$, where $cz$ is the conditional-Z gate between the two spins, given by the matrix diag$(1, i, 1, -1)$ and $A$ is either the
which for simplicity we fix perpendicular to the axis. As a result, the symmetry of the problem is different in each case, resulting in evolution operators of distinct symmetries and thus different pulse sequences. In each case we decompose the evolution operator using the Cartan decomposition [15] into a product of single-spin operations and purely two-spin operations. This allows us to identify the parameter regime that maximizes the entanglement generated between the two spins and to isolate the purely single-spin part of the evolution, which may be used or may need to be compensated with single-qubit gates.

**Equal Zeeman frequencies.**—In this case the evolution operator can be decomposed into the form

$$U_{eq} = (e^{-i\omega_1 t_s} \otimes e^{-i\omega_1 t_s}) e^{-iJt_s^1 s_2^1 s_2^2}.$$  

(1)

The purely two-spin operator in this equation comes from the exchange interaction. It is well known that Heisenberg exchange interaction yields an entangling gate, the so-called square root of swap, $U_{ss}$. After evolution time $t = \pi/(2J) \equiv \tau_1$, $U_{eq}$ is equivalent to $U_{ss}$ up to single-qubit rotations and is thus maximally entangling:

$$U_{eq}^{max} = (e^{-i(\pi/2)s_j} \otimes e^{-i(\pi/2)s_j}) e^{-i(\pi/2)s_1^1 s_2^1} \equiv (e^{-i(\pi/2)s_j} \otimes e^{-i(\pi/2)s_j}) U_{ss}. $$  

(2)

This evolution operator can be used to generate the target gate $U_{target} = cz(A \otimes A)$ in the case where $A$ is a $\pi/2$ rotation about $x$. To see this, first express $cz$ in terms of $U_{ss}$ (up to a global phase):

$$cz = (e^{-i(\pi/2)s_j} \otimes e^{i(\pi/2)s_j}) U_{ss}(1 \otimes e^{-i\pi x s_j}) U_{ss}.$$  

(3)

The target gate is

$$U_{target} = cz(A \otimes A).$$
Using Eqs. (2)–(4) we have
\[
U_{\text{target}} = cz(e^{-i(\pi/2)s_z} \otimes e^{-i(\pi/2)s_z})\,.
\]

Thus, for equal Zeeman frequencies, we may generate the target gate using two sets of single-qubit gates interspersed with periods of free evolution.

Unequal Zeeman frequencies.— The unequal Zeeman splitting case is somewhat more involved, as the symmetry of the system is lower. As a result, the evolution operator does not have a simple decomposition as in the equal Zeeman case above. The evolution operator in the product spin basis is symmetric, with \( u_{24} = u_{13} \), \( u_{13} = u_{22} \), and \( u_{44} = u_{11} \). The expressions for the matrix elements are in [16].

A key difference from the equal Zeeman case is that the condition for maximal entanglement, in addition to \( J \), also depends on \( \omega_1 \), \( \omega_2 \). When
\[
t = \frac{2n\pi}{\sqrt{J^2 + (\omega_1 - \omega_2)^2}} = \frac{(m + 1)\pi}{J} \equiv \tau_2
\]
with \( n, m \) positive integers, the free evolution amounts to an Ising gate up to single qubit operations:
\[
U_{\text{uneq}}^{me} = (e^{i\phi_3} \otimes e^{i\phi_3})e^{-ix_3}s_zs_z,
\]
with
\[
\phi = \pm \pi \sqrt{4n^2 - (2m + 1)^2} \frac{(\omega_1 + \omega_2)}{2(\omega_1 - \omega_2)} \pm k\pi,
\]
with \( k \) integer from which we obtain the constraint
\[
n > |1 + 2m|/2.
\]
Condition (5) requires choosing the integers \( n \) and \( m \) such that the values of \( \omega_1 \), \( \omega_2 \), and \( J \) fall in a physical range.

To construct the sequence that will give us the target evolution we make an ansatz using single spin rotations about the \( z \) and the \( x \) axes in addition to the maximally entangling free evolution \( U_{\text{uneq}}^{me} \). In fact we notice that the square of \( U_{\text{uneq}}^{me} \) gives a separable evolution of the two spins amounting to rotations about the \( x \) axis for both spins. This is a resource we also use as it will provide additional \( x \) rotations without the need of external pulses. We equate our ansatz sequence to the target gate, \( cz(H \otimes H) \), and examine whether this equation has a general solution and in that case determine the angles \( \phi_1, \phi_2, \phi_3 \). Multiplying both sides on the right by \((H \otimes H)\) and inserting the identity before the entangling part of \( U_{\text{uneq}}^{me} \) in the form \((H \otimes H)^2\) we notice that we can introduce \( cz \) on the left-hand side, since \((H \otimes H)e^{ix_3} \otimes \phi_3((H \otimes H)\) is proportional to \( cz \) up to single-qubit rotations. We thus reduce the problem to a single spin equation:
\[
e^{ix_3}\phi_2e^{i\alpha}(2\phi - \pi)e^{ix_3}\phi_1e^{ix_3}\phi_3e^{ix_3}\phi_3He^{ix_3}(-\pi/2) = 1.
\]

In the middle of the expression in Eq. (9) the rotations about the \( x \) axis are left separate to emphasize that one originates from free evolution while the other one is implemented by external fields. The solution of Eq. (9) is
\[
\phi_1 = \pi/2, \quad \phi_2 = \pi - \phi, \quad \phi_3 = 3\pi/2 - 2\phi, \quad \alpha = 5\pi/4.
\]

We thus find that the overhead for generating each pair of entangled photons is two single-qubit \( z \) rotations and a single-qubit \( x \) rotation. The protocols for generating the cluster states for equal and unequal Zeeman frequencies are shown in Fig. 2.

Implementing the single-qubit \( x \) rotations.—For the \( x \) rotations we have several options: using longer pulses, i.e., spectral selectivity; combining free evolution with \( z \) rotations; specially engineering or selecting QD parameters to simplify the evolution. Because our primary goal is to find protocols based on experimentally demonstrated

FIG. 2. (a) Circuit generating entangled photons from QDs, with the spin-spin gate \( G \) shown for (b) equal Zeeman frequencies and (c) unequal Zeeman frequencies.

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capabilities, we will focus on creating the x rotation by combining z rotations and free evolution. Below we discuss this approach in detail both for the equal and the unequal Zeeman frequency cases as applicable.

Free evolution for appropriate time interval $t_x$ (for the equal Zeeman case, $t_x = 2\pi J$ and for the unequal $t_x = 4\pi \sqrt{\omega^2 + (\omega_1 - \omega_2)^2}$) yields an x rotation by angle $\chi$ for each qubit [for the equal Zeeman case, $\chi = -2\pi \omega/J$ and for the unequal $\chi = 2\phi$, with $\phi$ in Eq. (7)]. Combining this with the ansatz

$$e^{i\gamma z_1} = e^{-i\frac{\gamma}{2} z_1} e^{i\frac{\gamma}{2} z_1} e^{-i\frac{\gamma}{2} z_1} e^{i\frac{\gamma}{2} z_1} e^{-i\frac{\gamma}{2} z_1},$$

which is satisfied for certain ranges of $\chi$, limiting the ratio of the physical parameters to certain values, allows us to tune the x-rotation angle $\phi$ by adjusting the z-rotation angles $\xi$ and $\psi$. Out of the physically viable ranges for the system parameters ($\omega$ and $J$), in the equal-Zeeman case we select the large range $\omega \in [0.58 J, 0.87 J]$, a condition that can be achieved by tuning the magnetic field. In the unequal Zeeman case, Eq. (10) gives a condition on the ratio of the $g$ factors of the two emitters. The relevant range here is much narrower, so an alternative approach may be more desirable. By selecting parameters appropriately, we can avoid the x rotation altogether in the pulse sequence forming $U_{\text{ans}}$. By combining the two middle x rotations in Eq. (9) we see that in the special case when $\phi = 3\pi/4$ the externally induced x rotation is not needed ($\phi_x = 0$). This condition gives us a relation between the Zeeman frequencies of the two QDs through the constraint

$$\omega_1 = \frac{\gamma + 1}{\gamma - 1} \omega_2,$$

$$\gamma = \frac{\pm 3/2 \pm 2k}{2\sqrt{4n^2 - (2m + 1)^2}},$$

with $k$ a positive integer. A large number of solutions can be obtained by varying $n$, $m$, $k$. More importantly, a reasonably large number of solutions persists for physically relevant parameter regimes, $\omega_1/\omega_2 \in (1, 1.2)$. For example, $n = 14$, $m = 13$, $k = 45$ gives $\gamma = 183/(2\sqrt{55})$ so that $\omega_1/\omega_2 \approx 1.18$, which should be achievable experimentally, e.g., by polarizing the nuclear spins [17–19] in one QD to obtain an effective local magnetic field through the Overhauser term of the hyperfine interaction, $I_S S_0$ or via the use of micromagnets [20,21], which would provide a more deterministic approach.

Error analysis.—We now address how well the sequences presented above perform in the presence of errors, such as uncertainty in the system parameters. In the equal Zeeman case, we consider an error, $\omega \rightarrow \omega(1 + \eta)$ and plot the fidelity of the pulse sequence of Eq. (10), defined as $\mathcal{F} = |\text{Tr}(U U_{\text{idem}}^\dagger)|/|\mathcal{U}|^2$, as a function of the ratio $\omega/J$ for the allowed regime of interest and as a function of $\eta$, Fig. 3(a). We find that the fidelity is robust, even for high percentage of error in $\omega$. The remaining operations do not depend on $\omega$, so its fluctuations will not affect other parts of the full sequence.

In the unequal Zeeman case, we consider an error in the Zeeman ratio discussed above, $\omega_1/\omega_2 = 1.18 + \eta$. The fidelity is shown in Fig. 3(b) and it depends more sensitively on the error. Such an error in $\eta$ could occur from fluctuations in the Overhauser field, which has typical widths on the order of 10s of MHz [22,23]. Narrowing the nuclear spin distribution [22,23] to 1 MHz (corresponding to $\eta \sim 0.001$) would guarantee high fidelity. Despite the higher sensitivity to error, this regime has the advantage of not requiring a rotation about $x$ at all.

The $z$ gates are assumed to be instantaneous compared to the other timescales in the system. This is an excellent approximation, as faster pulses lead to a polarization selection rule for circularly polarized light, which in turn leads to higher fidelities [24].

Additional sources of error are the finite trion lifetime and the finite spin coherence time. The implicit assumption has been that the QDs emit photons immediately after excitation, which requires that the spontaneous emission time be much faster than the Larmor precession periods and the timescale of the exchange interaction. In addition, the total pulse sequence should be much shorter than the spin coherence time so that a large enough cluster can be generated before the spin decoheres. The spontaneous emission time in free space is $\sim 1$ ns, and it can be made faster through the Purcell effect by embedding the QD into a cavity [25–27]. Then an emission timescale on the order of 100 ps can be achieved [25,26]. Importantly, coupling to a cavity still allows for optical spin rotations by using off-resonant pulses [28]. Reference [3] showed that the ratio of the Zeeman frequency over the spontaneous emission rate can be as high as 10%–20% with reasonably low errors. This constrains the Larmor period and exchange interaction timescale to be on the order of 10 ns (1 ns) or longer for free space (cavity-mediated) emission. The sequences of free evolution and pulses have a duration roughly given by several times $\pi/J$. Taking $J \sim \omega_1 \sim 2\pi \times 1$ GHz, each period should be $\sim 20$ ns. The coherence time of the electron spin $T_2$ is several $\mu$s in free-induction decay.
and can be extended using decoupling sequences. Based on these values we estimate we can obtain a cluster state of size at least $2 \times 100$, an order of magnitude larger than the state of the art.

In conclusion, we developed a method to generate a large 2D entangled photonic cluster state using coupled emitters. We showed in detail how this would work in a QD molecule with current experimental capabilities. Our approach can be adapted to other systems, including point defects, trapped ions, etc. Adding emitters to the system would increase the cluster state beyond two photons in the vertical direction. This could be done with a chain of emitters and decoupling to select at any one time coupling between only two neighboring emitters.

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[1] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. 86, 5188 (2001).
[2] X.-L. Wang, L.-K. Chen, W. Li, H.-L. Huang, C. Liu, C. Chen, Y.-H. Luo, Z.-E. Su, D. Wu, Z.-D. Li, H. Lu, Y. Hu, X. Jiang, C.-Z. Peng, L. Li, N.-L. Liu, Y.-A. Chen, C.-Y. Lu, and J.-W. Pan, Phys. Rev. Lett. 117, 210502 (2016).
[3] N. H. Lindner and T. Rudolph, Phys. Rev. Lett. 103, 113602 (2009).
[4] I. Schwartz, D. Cogan, E. R. Schmidgall, Y. Don, L. Gantz, O. Kenneth, N. H. Lindner, and D. Gershoni, Science 354, 434 (2016).
[5] S. E. Economou, N. Lindner, and T. Rudolph, Phys. Rev. Lett. 105, 093601 (2010).
[6] D. Buterakos, E. Barnes, and S. E. Economou, Phys. Rev. X 7, 041023 (2017).
[7] A. Russo, E. Barnes, and S. E. Economou, Phys. Rev. B 98, 085303 (2018).
[8] H. J. Krenner, M. Sabathil, E. C. Clark, A. Kress, D. Schuh, M. Bichler, G. Abstreiter, and J. J. Finley, Phys. Rev. Lett. 94, 057402 (2005).
[9] E. A. Stöffer, M. Scheibner, A. S. Bracker, I. V. Ponomarev, V. L. Korenev, M. E. Ware, M. F. Doty, T. L. Reinecke, and D. Gammon, Science 311, 636 (2006).
[10] M. F. Doty, M. Scheibner, A. S. Bracker, I. V. Ponomarev, T. L. Reinecke, and D. Gammon, Phys. Rev. B 78, 115316 (2008).
[11] D. Kim, S. G. Carter, A. Greilich, A. Bracker, and D. Gammon, Nat. Phys. 7, 223 (2011).
[12] S. E. Economou and T. L. Reinecke, Phys. Rev. B 78, 115306 (2008).
[13] D. Press, T. D. Ladd, B. Zhang, and Y. Yamamoto, Nature (London) 456, 218 (2008).
[14] A. Greilich, S. E. Economou, S. Spatzek, D. R. Yakovlev, A. D. Reuter, A. D. Wieck, T. L. Reinecke, and M. Bayer, Nat. Phys. 5, 262 (2009).
[15] B. Kraus and J. I. Cirac, Phys. Rev. A 63, 062309 (2001).
[16] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.123.070501 for the full expressions of the matrix elements in the unequal Zeeman frequencies case and a derivation of the expressions in the manuscript.
[17] B. Eble, O. Krebs, A. Lemaître, K. Kowalik, A. Kudelski, P. Voisin, B. Urbaszek, X. Marie, and T. Amand, Phys. Rev. B 74, 081306(R) (2006).
[18] A. Hogele, M. Kroner, C. Latta, M. Claassen, I. Carusotto, C. Bulutay, and A. Imamoglu, Phys. Rev. Lett. 108, 197403 (2012).
[19] M. Gullans, J. J. Krich, J. M. Taylor, H. Bluhm, B. I. Halperin, C. M. Marcus, M. Stopa, A. Yacoby, and M. D. Lukin, Phys. Rev. Lett. 104, 226807 (2010).
[20] Y.-S. Shin, T. Obata, Y. Tokura, M. Pioro-Ladriere, R. Brunner, T. Kubo, K. Yoshida, and S. Tarucha, Phys. Rev. Lett. 104, 046802 (2010).
[21] A. J. Sigillito, J. C. Loy, D. M. Zajac, M. J. Gullans, L. F. Edge, and J. R. Petta, Phys. Rev. Applied 11, 061006 (2019).
[22] H. Bluhm, S. Foletti, D. Mahalu, V. Umansky, and A. Yacoby, Phys. Rev. Lett. 105, 216803 (2010).
[23] G. Ethier-Majcher, D. Gangloff, R. Stockill, E. Clarke, M. Hugues, C. Le Gall, and M. Atatüre, Phys. Rev. Lett. 119, 130503 (2017).
[24] S. E. Economou, L. J. Sham, Y. Wu, and D. G. Steel, Phys. Rev. B 74, 205415 (2006).
[25] M. D. Birowosuto, H. Sumikura, S. Matsuo, H. Taniyama, P. J. van Veldhoven, R. Notzel, and M. Notomi, Sci. Rep. 2, 321 (2012).
[26] Y. A. Kelaita, K. A. Fischer, T. M. Babinec, K. G. Lagoudakis, T. Sarmiento, A. Rundquist, A. Majumdar, and J. Vučković, Opt. Mater. Express 7, 231 (2017).
[27] P. M. Vora, A. S. Bracker, S. G. Carter, T. M. Sweeney, M. Kim, C. S. Kim, L. Yang, P. G. Breton, S. E. Economou, and D. Gammon, Nat. Commun. 6, 7665 (2015).
[28] S. G. Carter, T. M. Sweeney, M. Kim, C. S. Kim, D. Solenov, S. E. Economou, T. L. Reinecke, L. Yang, A. Bracker, and D. Gammon, Nat. Photonics 7, 329 (2013).