Production Ratios of Strange Baryons from QGP with Diquarks

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Abstract

Assuming that axial-vector and scalar diquarks exist in the Quark-Gluon Plasma near the critical temperature $T_c$, baryons can be produced through the processes of quarks and diquarks forming $\frac{1}{2}^+$ baryon states. Ratios of different baryons can be estimated through this method, if such kind of QGP with diquarks can exists.

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1. Introduction

The Quark-Gluon Plasma is generally concerned to consist of quarks and gluons. While diquarks may exist in QGP too. They have the color $\{3^*\}$ representation, and the flavor $\{6\}$ or $\{3\}$ representation[1]. That means an SU(3) sextet of axial-vector diquarks and an SU(3) triplet of scalar diquarks. If axial-vector and scalar diquarks exist near the critical temperature $T_c$ (not too high to melt diquarks[2][3]) and approximate thermal equilibrium could form, baryon production can be described as the process of quark and diquark forming a $\frac{1}{2}^+$ baryon state. Ratios of different baryons can be estimated through this method. Since strange baryon production is widely discussed[4][5][6][7] and has a upper limit in the hadronic gas model[8], the ratio beyond that limit can predict that the QGP has formed or not in the relativistic heavy ion collisions.

2. Quark-Diquark Model

In the SU(6) quark-diquark model, baryon wave functions can be written as these forms[1][9][10][11],

$$|p\rangle^\dagger = \frac{1}{\sqrt{18}} [\sqrt{2}(\sqrt{2}V_+(uu)d^\dagger - V_0(uu)d^\dagger) - (\sqrt{2}V_+(ud)u^\dagger - V_0(ud)u^\dagger) + 3S(ud)u^\dagger],$$

$$|n\rangle^\dagger = \frac{1}{\sqrt{18}} [-\sqrt{2}(\sqrt{2}V_+(dd)u^\dagger - V_0(dd)u^\dagger) + (\sqrt{2}V_+(ud)d^\dagger - V_0(ud)d^\dagger) + 3S(ud)d^\dagger],$$

$$|\Lambda\rangle^\dagger = \frac{1}{\sqrt{12}} [(\sqrt{2}V_+(us)d^\dagger - V_0(us)d^\dagger) - (\sqrt{2}V_+(ds)u^\dagger - V_0(ds)u^\dagger) + S(us)d^\dagger - S(ds)u^\dagger + 2S(ud)s^\dagger],$$

$$|\Sigma^0\rangle^\dagger = \frac{1}{\sqrt{36}} [2(\sqrt{2}V_+(ud)s^\dagger - V_0(us)s^\dagger) - (\sqrt{2}V_+(us)d^\dagger - V_0(us)d^\dagger) - (\sqrt{2}V_+(ds)u^\dagger - V_0(ds)u^\dagger) + 3S(us)d^\dagger + 3S(ds)u^\dagger],$$
And they can be rewritten as

\[
\begin{align*}
| p \rangle^\dagger &= \frac{1}{\sqrt{3}}[B(V_{uu}, d) - \sqrt{\frac{1}{2}}B(V_{ud}, u) + \sqrt{\frac{3}{2}}B(S_{us}, u)], \\
| n \rangle^\dagger &= \frac{1}{\sqrt{3}}[-B(V_{dd}, u) + \sqrt{\frac{1}{2}}B(V_{ud}, d) + \sqrt{\frac{3}{2}}B(S_{ud}, d)], \\
| \Lambda \rangle^\dagger &= \frac{1}{\sqrt{3}}[\sqrt{\frac{3}{4}}B(V_{us}, d) - \sqrt{\frac{3}{4}}B(V_{ds}, u) + \sqrt{\frac{1}{4}}B(S_{us}, d) - \sqrt{\frac{1}{4}}B(S_{ds}, u) + B(S_{ud}, s)], \\
| \Sigma^0 \rangle^\dagger &= \frac{1}{\sqrt{3}}[B(V_{ud}, s) - \sqrt{\frac{1}{4}}B(V_{us}, d) - \sqrt{\frac{1}{4}}B(V_{ds}, u) + \sqrt{\frac{3}{4}}B(S_{us}, d) + \sqrt{\frac{3}{4}}B(S_{ds}, u)],
\end{align*}
\]

where B represent a \((\frac{1}{2})^+\) baryon state.

So, for \(p, n, \Lambda\) and \(\Sigma^0\),

\[
\begin{align*}
\frac{dp}{dt} &= \frac{1}{3} \cdot \left[ \Gamma(V_{uu}, d, p) + \frac{1}{2} \Gamma(V_{ud}, u, p) + \frac{3}{2} \Gamma(S_{us}, u, p) \right], \\
\frac{dn}{dt} &= \frac{1}{3} \cdot \left[ \Gamma(V_{dd}, u, n) + \frac{1}{2} \Gamma(V_{ud}, d, n) + \frac{3}{2} \Gamma(S_{ud}, d, n) \right], \\
\frac{d\Lambda}{dt} &= \frac{1}{3} \cdot \left[ \frac{3}{4} \Gamma(V_{us}, d, \Lambda^0) + \frac{3}{4} \Gamma(V_{ds}, u, \Lambda^0) + \frac{1}{4} \Gamma(S_{us}, d, \Lambda^0) + \frac{1}{4} \Gamma(S_{ds}, u, \Lambda^0) + \Gamma(S_{ud}, s, \Lambda^0) \right], \\
\frac{d\Sigma^0}{dt} &= \frac{1}{3} \cdot \left[ \Gamma(V_{ud}, s, \Sigma^0) + \frac{1}{4} \Gamma(V_{us}, d, \Sigma^0) + \frac{1}{4} \Gamma(V_{ds}, u, \Sigma^0) + \frac{3}{4} \Gamma(S_{us}, d, \Sigma^0) + \frac{3}{4} \Gamma(S_{ds}, u, \Sigma^0) \right].
\end{align*}
\]

Now, baryon productions can be calculated by combining different processes of quarks and diquarks forming \((\frac{1}{2})^+\) medium baryon states. \(\Xi^0\), \(\Xi^-\) and other \((\frac{1}{2})^+\) baryons can be calculated through similar methods.

### 3. Baryon Production with Diquarks

If diquarks could exist in the Quark-Gluon Plasma near the critical temperature, the contribution of direct 3-quark interaction can be neglected, for the 3-body interaction has much larger initial phase space and the interacting probability is much smaller compared to the quark-diquark interaction. Thus, baryon production can be described as a combination of different processes of quarks and diquarks forming \((\frac{1}{2})^+\) baryons, as

\[
\frac{dB}{dt} = \sum C^2_{\text{CG}}(D_{q_1q_2}, q_3, B) \Gamma(D_{q_1q_2}, q_3, B),
\]

and \(C^2_{\text{CG}}(D_{q_1q_2}, q_3, B)\) is the Clebsch-Gordan coefficient to represent the state of quark-diquark coupling shown in equations \((2)\), and

\[
\Gamma(D_{q_1q_2}, q_3, B) = |v_{D_{q_1q_2}} - v_{q_3}| \rho_D \rho_{q_3} \sigma(D_{q_1q_2} + q_3 \rightarrow B),
\]

where quarks and diquarks are under Fermi and Bose distributions,

\[
\begin{align*}
\rho_D &= \frac{\omega_D}{\pi^2} \int_{m_D}^{\infty} \frac{ppE_DdE_D}{e^{\frac{pp + m_D}{\omega_D}} - 1}, \\
\rho_q &= \frac{\omega_q}{\pi^2} \int_{m_q}^{\infty} \frac{ppE_qdE_q}{e^{\frac{pp + m_q}{\omega_q}} + 1}
\end{align*}
\]
\[ \sigma(D + q \rightarrow B) = (2\pi)^4 \int \frac{\omega_D \omega_q |M|^2}{4E_D E_q |v_D - v_q|} \delta^4(p_D + p_q - p_B) \delta(p_B - m_B^2) \theta(E_B) \frac{d^4p_B}{(2\pi)^3}, \]  

(7)

The result is \[12\]

\[ \frac{dB}{dt} = \sum C^2_{c_{ij}}(D_{q_1 q_2}, q_3, B) \frac{3\omega_D \omega_q |M|^2}{32\pi^2} T^2 F_{FB}(q_3, D_{q_1 q_2}, B, T), \]  

(8)

where

\[ T^2 F_{FB}(q, D, B, T) = \int \int \frac{dE_q dE_D}{(e^{\frac{E_q - p_q}{T}} + 1)(e^{\frac{E_D - p_D}{T}} - 1)}, \]  

(9)

and the integrating ranges are

\[ m_q \leq E_q \leq \infty \]
\[ m_D \leq E_D \leq \infty \]

and

\[ E_q E_D \geq \frac{1}{4m_B^2} \{ 4(E_q + E_D)(m_q^2 E_D + m_D^2 E_q) + [m_B^2 - (m_q + m_D)^2][m_B^2 - (m_q - m_D)^2] \} \]

For axial-vector diquarks, one has

\[ L_{intV} = ig\bar{B}\gamma_{\mu}\gamma_5 q V_{\mu}, \]  

(10)

\[ |M_V^2| = \frac{g^2}{3} \left[ \frac{(m_B^2 - m_q^2)^2}{m_V^2} + m_B^2 + m_q^2 - 2m_V^2 + 6m_B m_q \right], \]  

(11)

For scalar diquarks,

\[ L_{intS} = ig\bar{B}q S, \]  

(12)

\[ |M_S^2| = g^2([m_B + m_q]^2 - m_S^2], \]  

(13)

Where the diquark mass is assumed as \( m_{D}(q_1 q_2) = m_{D0} + m_{q1} + m_{q2} \) and the difference of axial-vector diquark mass and scalar diquark mass is neglected as a simple assumption. \( m_{D0} \) here is about 500-700 MeV. That means the contribution of gluons to build a baryon is mainly subjected to the diquark and the current mass \( m_q \) can be used in the calculations. \( m_{D0} \) should not be smaller than the masses of constituent quarks. Additionally, it is assumed that g is same in these reactions.

4. Results

As the quark masses have wide ranges, mean masses are used in the calculations. (Note, the results greatly depend on the quark masses, defferences may reach about 10 percentages if extreme value of quark masses are used, and the lighter the quark masses are, the higher the ratios grow). The critical temperature is set as \( T_c = 170 MeV \[13\][14] \) for \( \mu_B \) near 0 MeV. The ratios of strange baryons to proton (or neutron) grow as the base mass of diquark \( m_{D0} \) grows, and reduce as the differences of the quark masses of different flavors grow. Some results are listed in table 1 below (calculation errors are less than 1%), for \( \mu_B = 0 MeV, \mu_S = 0 MeV. \)
| $m_{D0}$ (MeV) | 500  | 550  | 600  | 650  | 700  |
|---------------|------|------|------|------|------|
| $\Lambda/p$, $\Lambda/\bar{p}$ (%) | 49.0 | 51.4 | 54.4 | 58.2 | 63.8 |
| $\Lambda/n$, $\Lambda/\bar{n}$ (%) | 50.0 | 52.6 | 55.6 | 59.6 | 65.4 |
| $\Sigma^0/p$, $\Sigma^0/\bar{p}$ (%) | 58.1 | 61.4 | 66.3 | 73.8 | 86.1 |
| $\Sigma^0/n$, $\Sigma^0/\bar{n}$ (%) | 59.2 | 62.7 | 67.7 | 75.6 | 88.3 |
| $\Lambda/(p+n)$, $\Lambda/\bar{n}$ (%) | 24.8 | 26.0 | 27.5 | 29.4 | 32.3 |

Table 1  Relative yields of baryons from QGP with diquarks at $\mu_B = 0 MeV$

When setting $\mu_B = 45 MeV$ (RHIC [15, 16]), $\mu_S = \mu_q - \mu_s$ in the plasma can be calculated by the strangeness conservation and some results including anti-baryon/baryon ratios are listed in table 2 below. Where $\mu_S$ is the strangeness chemical potential, $\mu_q = \frac{1}{3}\mu_B$ is the chemical potential of $u, d$ quark and $\mu_s$ is the chemical potential of $s$ quark.

| $m_{D0}$ (MeV) | 500  | 550  | 600  | 650  | 700  |
|---------------|------|------|------|------|------|
| $\mu_S$ (MeV) | 20.0 | 19.6 | 19.2 | 18.8 | 18.4 |
| $\Lambda/p$ (%) | 43.6 | 45.9 | 48.6 | 52.3 | 57.6 |
| $\Lambda/n$ (%) | 44.4 | 46.9 | 49.8 | 53.5 | 59.0 |
| $\Sigma^0/p$ (%) | 51.9 | 55.1 | 59.7 | 66.8 | 78.3 |
| $\Sigma^0/n$ (%) | 52.9 | 56.3 | 61.0 | 68.4 | 80.3 |
| $\Lambda/(p+n)$ (%) | 22.0 | 23.2 | 24.6 | 26.4 | 29.1 |
| $\Lambda/\bar{p}$ (%) | 55.1 | 57.6 | 60.8 | 64.7 | 70.7 |
| $\Lambda/\bar{n}$ (%) | 56.2 | 58.9 | 62.1 | 66.3 | 72.6 |
| $\Sigma^0/\bar{p}$ (%) | 64.9 | 68.4 | 73.5 | 82.6 | 94.7 |
| $\Sigma^0/\bar{n}$ (%) | 66.2 | 69.9 | 75.1 | 83.5 | 97.1 |
| $\Lambda/(p+n)$ (%) | 27.8 | 29.1 | 30.1 | 32.8 | 35.8 |
| $\bar{p}/p$ (%) | 59.5 | 59.8 | 60.0 | 60.4 | 60.7 |
| $\Lambda/\Lambda$ (%) | 75.3 | 75.1 | 74.9 | 74.8 | 74.7 |
| $\Sigma^0/\Sigma^0$ (%) | 74.4 | 74.2 | 73.9 | 73.7 | 73.4 |
| $\Sigma^0/\Sigma^0$ (%) | 94.6 | 93.8 | 93.0 | 92.2 | 91.5 |

Table 2  Relative yields of baryons from QGP with diquarks at $\mu_B = 45 MeV$

From these calculations, one can find that the production of $\Sigma^0$ is higher than the production of $\Lambda$. That is because $|M|^2$ is larger than $|M|^2$ here and the $\Sigma^0$ production has the contribution of the big term $\Gamma(V_{ud,s},\Sigma^0)$ while the $\Lambda$ production has not.

5. Hadronic Gas Model

For the hadronic gas of thermal and chemical equilibrium, the relative yields of hadrons can be calculated [8, 12, 17], based on the free gas model.

The temperature of hadronic matter should be lower than the critical temperature $T_c$ of phase transition, so the relative yields of strange hadrons from hadronic matter of temperature $T_c$ can give the upper limits of the relative yields of strange hadrons [8]. And the production of $\Sigma^0$ is smaller than the production of $\Lambda$ due to the free gas model.

The upper limits of some strange baryons for $T_c = 170 MeV$, $\mu_B = 0 MeV$ are
$$\frac{\Lambda}{p} < 43.6\%, \quad \frac{\Lambda}{n} < 43.9\%.$$
Figure 1: Ratios of $\Sigma^0$ and $\Lambda$ over proton via diquark base mass (for $\mu_B = 0 \text{MeV}$ and $\mu_B = 45 \text{MeV}$) and the upper limits of hadronic gas model.

$$\frac{\Sigma^0}{p} < 30.2\%, \quad \frac{\Sigma^0}{n} < 30.4\%,$$

$$\frac{\Lambda}{p + n} < 21.9\%,$$

For $\mu_B = 45 \text{MeV}$, then $\mu_S = 9.8 \text{MeV}$, the difference is small,

$$\frac{\Lambda}{p} < 41.2\%, \quad \frac{\Lambda}{n} < 41.5\%, \quad \frac{\Sigma^0}{p} < 28.5\%, \quad \frac{\Sigma^0}{n} < 28.7\%,$$

$$\frac{\Lambda}{p + n} < 32.0\%, \quad \frac{\Sigma^0}{p + n} < 32.2\%,$$

$$\frac{\Lambda}{p + n} < 66.1\%$$

6. Discussions

The ratios of strange baryons over proton or neutron produced from QGP with diquarks are higher than the hadronic gas limits (shown in fig.1). This may become a criterion to judge if QGP (with diquarks) has been formed in the collisions and quark masses are not too heavy. Especially, the production of $\Sigma^0$ is higher than the production of $\Lambda$ here. Additionally, anti-baryon/baryon ratios from experimental data can be fit well by tuning $\mu_B$ and $m_{D_0}$, for example, $\mu_B = 45 \text{MeV}$ and $m_{D_0} = 650 \text{MeV}$, except $\Xi/\Xi$ is a bit larger (exp. $0.82 \pm 0.08$).

Moreover, for the RHIC experiments, the baryonic potential is quite small. The results given here are based on this condition. But for large baryonic potentials, Bose condensate may occur, and the products of some baryons may form a peak there, if QGP could
exist at those conditions.

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