Seesaw Extended MSSM and Anomaly Mediation without Tachyonic Sleptons

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Abstract

Superconformal anomalies provide an elegant and economical way to understand the soft breaking parameters in SUSY models; however, implementing them leads to the several undesirable features including: tachyonic sleptons and electroweak symmetry breaking problems in both the MSSM and the NMSSM. Since these two theories also have the additonal problem of massless neutrinos, we have reconsidered the AMSB problems in a class of models that extends the NMSSM to explain small neutrino masses via the seesaw mechanism. In a recent paper, we showed that for a class of minimal left-right extensions, a built-in mechanism exists which naturally solves the tachyonic slepton problem and provides new alternatives to the MSSM that also have automatic $R$-parity conservation. In this paper, we discuss how electroweak symmetry breaking arises in this model through an NMSSM-like low energy theory with a singlet VEV, induced by the structure of the left-right extension and of the right magnitude. We then study the phenomenological issues and find: the LSP is an Higgsino-wino mix, new phenomenology for chargino decays to the LSP, degenerate same generation sleptons and a potential for a mild squark-slepton degeneracy. We also discuss possible collider signatures and the feasibility of dark matter in this model.

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I. INTRODUCTION

One of the leading candidates for TeV scale physics is the supersymmetric extension of the Standard Model (SM) \[1\] since it resolves an outstanding SM conceptual issue: the gauge hierarchy problem (or why \( M_Z \ll M_{Pl} \) is stable under radiative corrections). It also leads to gauge coupling unification as well as a candidate for dark matter of the universe if two additional assumptions are made: a grand desert until \( M \sim 10^{16} \text{ GeV} \) for gauge unification, and exact \( R \)-parity for dark matter. In addition it has the potential to explain the origin of spontaneous breaking of electroweak symmetry. Of course, supersymmetry (SUSY) has to be a broken symmetry to conform with observations because no superpartner particles have been observed yet. Understanding the nature and origin of this SUSY breaking is a major challenge which has commanded a great deal of attention. An attractive and elegant mechanism is to use the superconformal anomaly \[2, 3\] to break supersymmetry in the manner that has been dubbed Anomaly Mediated Supersymmetry Breaking (AMSB). AMSB provides an ultra-violet insensitive way to determine the soft SUSY breaking parameters \[4, 5\] as they depend only on the TeV scale gauge Yukawa couplings of the low energy theory. Consequently, it considerably reduces the number of arbitrary parameters of the SUSY breaking sector. It also provides a heavy gravitino which has a number of cosmological advantages.

A major problem of AMSB is that when implemented in the Minimal Supersymmetric Standard Model (MSSM), it leads to negative slepton mass-squares—an unacceptable scenario since it leads to the breakdown of electric charge (sometimes called the tachyonic slepton problem). Another stumbling block to realistic AMSB model building is electro-weak symmetry breaking (EWSB): the explicit \( \mu \) term in the MSSM gives a \( B\mu \) that is too large, while extensions like the Next-to Minimal Supersymmetric Standard Model (NMSSM) fail to generate a \( \mu \) term that is large enough. A number of attempts have been made to extend the MSSM in order to cure these problems \[6, 7, 8, 9, 10, 11, 12, 13, 14, 15\], usually with a focus on the tachyonic slepton problem.

Since in AMSB models the SUSY breaking profile is crucially dependent on the low energy theory, an interesting question arises as to whether AMSB still has the same problems when the MSSM extended to accommodate neutrino masses. In a recent paper \[16\], we pointed out that when the MSSM is minimally extended to the supersymmetric left-right (SUSYLR)
model with $B - L = 2$ triplets to implement the seesaw mechanism, the low energy particle content and interaction profile changes just enough to cure the negative slepton mass square problem. A key feature responsible for this cure is the appearance of a naturally light $SU(2)_L$ triplet and a doubly-charged singlet which have leptonic Yukawa interactions. In Ref. [16], we explained how SUSYLR fixes the tachyonic slepton problem of AMSB and also noted some of the gross distinguishing features of the model—such as the appearance of $B - L = 2$ triplets, doubly-charged Higgs bosons, and a pair of additional heavy Higgs doublets all with masses around the mass scale of conformal SUSY breaking, $F_\phi$—typically in the tens of TeVs. Since then another paper has explored the relationship of neutrinos and AMSB in the context of deflected AMSB [17].

In this paper, which should be viewed as a sequel to ref. [16], we attempt to present a complete phenomenologically acceptable model addressing questions such as EWSB, and dark matter. A summary of our results is as follows:

- We show that the model below the $F_\phi$ scale is the NMSSM with a singlet superpotential mass term, $\mu_N$. This term is necessary for EWSB and can arise from the SUSYLR framework necessary for the solution to the tachyonic slepton problem.

- One implication of the similarity to the NMSSM below the TeV scale is that the magnitude of the $B\mu$-term is of the desired magnitude.

- We present the sparticle spectrum of the model for a generic choice of the parameters and in particular we display the lightest superparticle which can be the dark matter of the universe. We find that same generation sleptons are degenerate and that a possibility exists for degenerate sleptons and squarks.

- We find that the mass difference between the chargino and the lightest neutralino in our model is much larger than the Minimal Anomaly Mediated Supersymmetry Breaking (mAMSB) models where a universal scalar mass corrects the tachyonic slepton mass problem.

The paper is organized as follows: in Section II we review the basic ingredients of the SUSYLR model that is the framework of our discussion; in Section III we show the multi-TeV scale spectrum of the model and discuss how it solves the negative slepton mass square problem of the model; in Section IV we discuss the effective theory below the $F_\phi$ TeV
scale and show how electroweak symmetry breaking arises. In Section [V] we display the sparticle spectrum and compare it with that in some other benchmark SUSY models with different SUSY breaking mechanisms. For the allowed parameter space of our model, we find a Higgsino-wino mixture to be the Lightest Supersymmetric Particle (LSP) and mention its prospects as the dark matter of the universe. We finish with a brief discussion of the ultraviolet consequences of this model in Section [VI] and a conclusion.

II. MINIMAL SUSYLR MODEL CURES THE PROBLEMS OF AMSB: A BRIEF REVIEW

In generic AMSB models the soft SUSY breaking parameters associated with the superfield combination $\Phi_i \Phi_{j*}$ are determined by the anomalous dimensions $\gamma^{ij}_a(g_a, Y_{\ell mn})$ and the scaling functions $\beta^{ai}_g(g_a, Y_{ij}^{\ell mn}), \beta^{ijk}_Y(g_a, Y_{\ell mn})$ of the low energy theory:

$$
(m^2)^{ij}_a = -\frac{1}{4}|F_\phi|^2 \left[ \frac{1}{2} \frac{\partial \gamma^{ij}_a}{\partial g_a} \beta^{ai}_g + \frac{\partial \gamma_a^{ij}}{\partial Y_{\ell mn}} \beta^{\ell mn}_Y + \text{h.c.} \right] \tag{1}
$$

$$
a^{ijk} = \beta^{ijk}_Y F_\phi \tag{2}
$$

$$
M_{\lambda a} = \frac{\beta^{a}_g}{g_a} F_\phi \tag{3}
$$

Here $F_\phi$ is the SUSY breaking scale in the gauge where the conformal compensator $\phi$ has the form

$$
\phi = 1 + F_\phi \theta^2 \tag{4}
$$

with $F_\phi$ as an input parameter having a value in the 10s of TeV range. The remainder of our notational conventions can be found in Appendix [A].

It is clear from Eq. (1) that when this formula is applied to the MSSM, the slepton mass-squares are negative due to the positive (asymptotically non-free) $SU(2) \times U(1)_Y$ gauge couplings’ $\beta$ functions and the nearly zero lepton Yukawa couplings\footnote{While the Yukawa coupling of $\tau$ might be significant, the first and second generation leptons have negligible Yukawa couplings}. As pointed out in Ref. [16], this problem is cured by extending the MSSM to SUSYLR due to the following property: the effective theory below the seesaw scale $v_R$ contains a set of $SU(2)_L$ triplets and doubly-charged fields, both having Yukawa couplings to the left- and right-handed leptons.
respectively. Their masses are naturally in the multi-TeV range despite the high seesaw scale due to an accidental global symmetry of the theory [18, 19]. Furthermore, provided these new couplings are of order 1, the slepton masses squares can be made positive. Thus, SUSYLR not only explains the small neutrino masses by means of the seesaw mechanism, but its marriage with AMSB cures the negative slepton mass-square problem. The resulting theory combines the predictive power of AMSB, explains neutrino masses, and retains a natural dark matter candidate due to the theory’s automatic conservation of $R$-Parity below the right-handed scale. It also contains a mechanism for generating an appropriate singlet vacuum expectation value (VEV) in the effective low energy NMSSM-like superpotential. In the following subsections, we fill in the details.

A. The Left-Right Model

The particle content of a SUSYLR model is shown in Table I. As the model is left-right symmetric, it contains both left- and right-handed higgs bosons—in this case $B − L = ±2$ triplets so that $R$-parity may be preserved (a task for which $B − L = 1$ doublets are not suitable). The presence of both $SU(2)_L$ and $SU(2)_R$ triplets means that parity is a good symmetry until $SU(2)_R$ breaks. While the seesaw mechanism may be achieved with only $SU(2)_R$ higgs fields, demanding parity forces the presence of left-handed triplets. The inclusion of both these fields then leads to positive left- and right-handed slepton masses.

To be explicit, the fields of Table I transform under parity as

$$Q \leftrightarrow -i \tau_2 Q^c, \quad L \leftrightarrow -i \tau_2 L^c, \quad \Phi_a \rightarrow \Phi^\dagger_a,$$

$$\Delta \leftrightarrow \Delta^c \dagger, \quad \bar{\Delta} \leftrightarrow \bar{\Delta}^c \dagger, \quad S, N \rightarrow S^*, N^*$$

so that the fully parity symmetric superpotential is

$$W_{\text{SUSYLR}} = W_Y + W_H + W_{\text{GSPNR}} + W_{\text{GSVNR}}$$

with

$$W_Y = i y^a_Q Q^T \tau_2 \Phi_a Q^c + i y^a_L L^T \tau_2 \Phi_a L^c + i f_c L^T \tau_2 \Delta^c L^c + i f L^T \tau_2 \Delta L$$

$$W_H = (M_\Delta \phi - \lambda_S S) \left[ \text{Tr}(\Delta^c \Delta^c) + \text{Tr}(\Delta \bar{\Delta}) \right] + M_S^2 \phi^2 S + \frac{1}{2} \mu_S \phi S^2 + \frac{1}{3} \kappa_S S^3$$

$$+ \lambda_N^{ab} S \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{3} \kappa_N N^3$$
\[
\begin{array}{|c|c|}
\hline
\text{Fields} & SU(3)^c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
\hline
Q & (3, 2, 1, +\frac{1}{2}) \\
Q^c & (\bar{3}, 1, 2, -\frac{1}{2}) \\
L & (1, 2, 1, -1) \\
L^c & (1, 1, 2, +1) \\
\Phi_a & (1, 2, 2, 0) \\
\Delta & (1, 3, 1, +2) \\
\bar{\Delta} & (1, 3, 1, -2) \\
\Delta^c & (1, 1, 3, -2) \\
\bar{\Delta}^c & (1, 1, 3, +2) \\
S, N & (1, 1, 1, 0) \\
\hline
\end{array}
\]

\text{TABLE I: Assignment of the matter and Higgs fields' representations of the left-right symmetry group (except for } U(1)_{B-L} \text{ where the charge under that group is given.)}

\[ W_{\text{GSPNR}} = \frac{\lambda_A}{M_X^2} \text{Tr}^2(\Delta \bar{\Delta}) + \frac{\lambda_A^c}{M_X^2} \text{Tr}^2(\Delta^c \bar{\Delta}^c) \]
\[ + \frac{\lambda_B}{M_X^2} \text{Tr}(\Delta \bar{\Delta}) \text{Tr}(\Delta \bar{\Delta}) + \frac{\lambda_B^c}{M_X^2} \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\bar{\Delta} \bar{\Delta}^c) \]
\[ + \frac{\lambda_C}{M_X^2} \text{Tr}(\bar{\Delta} \Delta) \text{Tr}(\bar{\Delta} \bar{\Delta}) \]
\[ + \frac{\lambda_S}{M_X^2} \text{Tr}(\bar{\Delta} \Delta) S^2 + \frac{\lambda_S^c}{M_X^2} \text{Tr}(\Delta^c \Delta^c) S^2 + \ldots \]  

\[ W_{\text{GSVNR}} = \frac{\lambda_D}{M_{\text{Pl}}^2} \text{Tr}(\Delta \Delta) \text{Tr}(\Delta^c \Delta^c) + \frac{\lambda_D}{M_{\text{Pl}}^2} \text{Tr}(\bar{\Delta} \bar{\Delta}) \text{Tr}(\bar{\Delta} \bar{\Delta}^c) \]
\[ + \frac{(\lambda_A)^{ab}}{M_{\text{Pl}}^2} \text{Tr}(\Delta \bar{\Delta}) \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{(\lambda_A^c)^{ab}}{M_{\text{Pl}}^2} \text{Tr}(\Delta^c \bar{\Delta}^c) \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) \]
\[ + \frac{2\lambda_A^{c ab}}{M_{\text{Pl}}^2} \text{Tr}(\bar{\Phi}_a \tau_2 \Phi_b^T \tau_2 \bar{\Delta}) + \frac{2\lambda_A^{c ab}}{M_{\text{Pl}}^2} \text{Tr}(\Delta^c \bar{\Delta} \Phi_a^T \tau_2 \Phi_b \bar{\Delta}^c) \]
\[ + \frac{\lambda_N}{M_{\text{Pl}}^2} \text{Tr}(\bar{\Delta} \Delta) N^2 + \frac{\lambda_N^c}{M_{\text{Pl}}^2} \text{Tr}(\Delta^c \Delta^c) N^2 \]
\[ + \frac{\lambda_S}{M_{\text{Pl}}^2} \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) S^2 + \frac{\lambda_M}{M_{\text{Pl}}^2} S^2 N^2 + \ldots \]
Furthermore, parity demands that the couplings be related as

\[ y^a_Q = (y^a_Q)^\dagger \quad y^a_L = (y^a_L)^\dagger \quad f = f^c \quad M_\Delta = M^* \]

\[ \lambda_S = \lambda_S^* \quad M^2_S = (M^2_S)^* \quad \mu_S = \mu_S^* \quad \kappa_S = \kappa_S^* \]

\[ \lambda_N = \lambda_N^\dagger \quad \kappa_N = \kappa_N^* \]

We have also imposed a discrete $\mathbb{Z}_3$ symmetry on Eq. (5) with

\[ (Q, Q^c, L, L^c, \Delta, \Delta^c, \Phi_a, N) \rightarrow e^{2\pi i/3}(Q, Q^c, L, L^c, \Delta, \Delta^c, \Phi_a, N), \]

\[ (\bar{\Delta}, \bar{\Delta}^c) \rightarrow e^{4\pi i/3}(\bar{\Delta}, \bar{\Delta}^c) \]

and $S$ invariant. This symmetry is necessary to keep one singlet light below the right-handed scale since it forbids terms such as

\[ W_{\mathbb{Z}_3} = \kappa_{12} SN^2 + \kappa_{21} S^2 N + \lambda_N^\dagger N \text{Tr}(\Delta^c \bar{\Delta}^c) \]

which would generate a large, $O(v_R)$, mass for $N$. Yet because it is a global symmetry, it will be violated by gravitational effects\(^2\) leading to Eq. (5) containing the non-renormalizable terms of Eq. (9) (which are accordingly suppressed by the planck scale $M_{\text{Pl}}$).

The superpotential Eq. (5) must also contain the additional non-renormalizable terms given by Eq. (8) if the theory is to preserve $R$-parity and be phenomenologically viable\(^2\),\(^18\),\(^19\). These terms preserve the $\mathbb{Z}_3$ symmetry and are therefore suppressed by the next new scale of physics, which we have chosen to call $M_X$. We will show that it is possible to fix $M_X$ in Section III, where we consider the $F_\phi$ scale theory.

Meanwhile, the Higgs potential given by Eq. (7) dictates that the VEV for the right-handed superfields are

\[ \langle S \rangle = \frac{M_\Delta}{\lambda_S} \phi \]

\[ \langle \Delta^c \rangle \langle \bar{\Delta}^c \rangle = \langle S \rangle \left( \frac{M_\Delta \kappa_S}{\lambda_S^2} + \frac{\mu_S}{\lambda_S} \right) \phi + \frac{M^2_S}{\lambda_S} \phi^2 \]

With $M_\Delta \sim \mu_S \sim v_R \sim 10^{11}$ GeV, where $v_R$ is the right-handed breaking scale. Eq. (12) should be evident from the form of the superpotential; Eq. (13) requires Eq. (7) to be recast

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\(^2\) For example, if a particle charged under this symmetry falls into a blackhole, there is no way to ascertain the amount of this charge the blackhole contains. This can be contrasted with a gauged symmetry where Gauss’s law may be utilized to determine the charge enclosed.
as
\[ W_H \supset -\lambda S \text{Tr}(\Delta \bar{\Delta}) + M_S^2 \phi^2 + \frac{1}{2} \mu S \phi S + \frac{1}{3} \kappa S^2 \] (14)

The non-renormalizable terms will shift the right-handed scale VEVs by at most \( \sim M_\Delta^2 / M_X \ll M_\Delta \) so they may be safely be ignored. The theory then remains ultra violet (UV) insensitive below \( v_R \) and hence respects the AMSB trajectory below this scale. Yet even though the particles remain on their AMSB trajectory, the negative slepton mass-squares problem is still solved due to the additional low-scale yukawa couplings \( f \) and \( f_c \).

To see why these yukawas survive, consider the Higgs sector of Eq. (5) before \( SU(2)_R \) breaks and setting the non-renormalizable terms to zero—essentially leaving just the terms in Eq. (7). This superpotential has a complexified \( U(6) \) symmetry\(^3\) involving the \( \Delta \)'s and the \( \Delta^c \)'s (similar symmetry arguments are discussed in [20], but because the authors used a parity odd singlet, there was only a complexified \( U(3) \) symmetry). When \( SU(2)_R \) breaks, the \( U(6) \) is reduced to a \( U(5) \) yielding 22 real degrees of freedom that are massless. The \( D \)-terms and the gauge fields consume 6 of these, leaving a total of 16 massless modes. The surviving 16 massless real degrees of freedom are the two doubly-charged \( SU(2)_L \) singlets and the two left-handed triplets.

Only the non-renormalizable terms of Eqs. (8) and (9) break the \( U(6) \) symmetry, and therefore the mass of the Higgsino must be
\[ \mu_{\Delta, \bar{\Delta}} \sim \mu_{DC} \sim \frac{v_R^2}{M_X} \] (15)

The SUSY breaking bilinear terms generated by AMSB will force these masses to be at least \( F_\phi \) giving
\[ M_X \lesssim \frac{v_R^2}{F_\phi}. \] (16)

Thus, the scale of new physics is determined by the right-handed scale and the SUSY breaking scale.

The mass matrix for the left-handed triplets and doubly-charged Higgses have a similar form, here we state the doubly-charged matrix:
\[ M_{DC} = \mu_{DC}^2 \begin{pmatrix} 1 & 1 - \epsilon_{\Delta} \\ 1 - \epsilon_{\Delta} & 1 \end{pmatrix} \] (17)

\(^3\) A complexified \( U(6) \) is a \( U(6) \) with its parameters taken to be complex. Its existence in Eq. (7) can be seen by defining two new fields \( \Delta \equiv (\Delta, \Delta^c) \) and \( \bar{\Delta} \equiv (\bar{\Delta}, \bar{\Delta}^c) \)—which are complex 6-vectors—and combining the trace over each separately to \( \text{Tr}(\Delta \bar{\Delta}) \).
where $\mu_{DC} \simeq F_\phi$ and $\epsilon_\Delta = 1 - \frac{B_\Delta}{\mu_{DC}}$. The eigenvalues of this mass matrix are $m^2_{DC} = \epsilon_\Delta \mu^2_{DC}$ and $M^2_{DC} = 2\mu^2_{DC}$. Since $\epsilon_\Delta$ depends on $\mu_{DC}$, and $\mu_{DC}$ can be adjusted through the coupling it contains, one doubly-charged Higgs can be made light. On the whole, we expect the two doubly charged scalar masses to be above 1 TeV (for the lighter one) and $F_\phi$ (for the heavier one). Note that there is no such splitting between the fermionic partners, which remain heavy with a mass of about $\mu_{DC}$. A similar argument applies to the left-handed triplets.

Finally, because the masses of the $SU(2)_L$ triplets and the doubly-charged particles will be around $F_\phi$, they are of the correct size to influence the low-scale theory: if the masses had been large, $F_\phi \ll \mu_{DC} \ll v_R$, then they would have merely introduced another trajectory preserving threshold that decoupled from the low scale theory. However, because these particles remain in the low-scale theory, the effect of their couplings is important. For the sleptons the relevant terms are

$$W \supset f_c \Delta^{c-}e^c e^c + i f L^T \tau_2 \Delta L$$

with the surviving yukawa couplings $f_c$ and $f$ providing positive mass-squares to the scalar leptons$^4$

To make this explicit we write down the slepton masses with the contributions of these additional interactions (taking the $SU(2)_L \times U(1)_Y$ gauge couplings to be $g_2$ and $g_1$ respectively):

$$m^2_{\tilde{e}c} = \frac{1}{2} \frac{|F_\phi|^2}{(16\pi^2)^2} \left[ 8 f^\dagger_c (Y^a_L)^T (Y^a_L)^* f_c + 12 (Y^a_L)^\dagger f f^\dagger Y^a_L 
+ 8 f^\dagger_c f_c \left[ (Y^a_L)^\dagger Y^a_L + 4 f^\dagger_c f_c + \text{Tr} (f^\dagger_c f_c) \right] + 4 (Y^a_L)^\dagger Y^a_L \left[ (Y^b_L)^\dagger Y^b_L + 2 f^\dagger_c f_c \right] 
+ 2 (Y^a_L)^\dagger Y^b_L \left[ 2 (Y^b_L)^\dagger Y^a_L + \text{Tr} \left( 3 (Y^b_Q)^\dagger Y^a_Q + (Y^b_L)^\dagger Y^a_L \right) \right] + 4 (\lambda^{cb}_N)^* \lambda^{ca}_N \right] 
- 2 g^2_1 \left[ 24 f^\dagger_c f_c + 3 (Y^a_L)^\dagger Y^a_L + 26 g^2_1 \right] - 6 g^2_2 (Y^a_L)^\dagger Y^a_L + h.c. \right] \right)$$

$^4$ Note that slepton mass squares can also be positive for theories with a right handed scale lower than $10^{11}$ GeV. We choose the high scale version since neutrino masses in this case do not require any fine tuning of Yukawa couplings.
\[
m_\ell^2 = \frac{1}{2} \left( \frac{|F_\phi|^2}{16\pi^2} \right)^2 \left[ 6f(Y_\ell^a)^T (Y_\ell^a)^* f^\dagger + 4Y_\ell^a f^\dagger f_c(Y_\ell^a)^\dagger \right. \\
+ 6 \left[ (Y_\ell^a)^\dagger (Y_\ell^a)^* + 12f f^\dagger + 2\text{Tr}(f^\dagger f) \right] f f^\dagger + 2 \left[ Y_L^b(Y_L^b)^\dagger + 3ff^\dagger \right] Y_L^a(Y_L^a)^\dagger \\
+ Y_L^b(Y_L^a)^\dagger \left[ 2Y_L^a(Y_L^b)^\dagger + \text{Tr} \left( 3(Y_Q^a)^\dagger Y_Q^a + (Y_L^b)^\dagger Y_L^a \right) + 4(\lambda_N^{cb})^* \lambda_N^{ca} \right] \\
- g_1^2 \left( 18f f^\dagger + 3Y_L^a(Y_L^a)^\dagger + 13g_1^2 \right) - 3g_2^2 \left( 14f f^\dagger + Y_L^a(Y_L^a)^\dagger + 3g_2^2 \right) + \text{h.c.} \right] \]  

(20)

Taking
\[
m_{an} = \frac{F_\phi}{16\pi^2},
\]  

(21)
assuming that \( f, f_c \) are diagonal in flavor space (an assumption required to satisfy constraints from lepton flavor violating experiments [21]), and neglecting the first and second generation yukawa couplings simplifies Eqs. (19) and (20) to
\[
m_e^2 = m_{an}^2 \left[ 40f_{c1}^4 + 8f_{c1}^2(f_{c2}^2 + f_{c3}^2) - 48f_{c1}^2g_1^2 - 52g_1^4 \right] 
\]  

(22)
\[
m_e^2 = m_{an}^2 \left[ 84f_1^4 + 12f_1^2(f_2^2 + f_3^2) - 6f_1^2(3g_1^2 + 7g_2^2) - 13g_1^4 - 9g_2^4 \right] 
\]  

(23)
for the first generation.\(^5\) We then only need
\[
f_1(F_\phi) \simeq f_2(F_\phi) \simeq f_{c1}(F_\phi) \simeq f_{c2}(F_\phi) \gtrsim 0.6
\]  

(24)
to make the sleptons positive (from the detailed analysis of Section[VB].

These couplings and the masses of the doubly-charged field and the left-handed triplets are experimentally constrained from muonium-antimuonium oscillations [22] which demands that
\[
\frac{f_{c1}f_{c2}}{4\sqrt{2m_{DC}^2}} \approx \frac{f_1f_2}{4\sqrt{2m_{\Delta\Delta}^2}} < 3 \times 10^{-3}G_F;
\]  

(25)
The minimum \( f \) values that satisfies Eq. (24) implies a lower bound on the masses of the doubly-charged and left-handed triplet Higgs field to be about \( m_{DC}, m_\Delta \geq 2 \text{ TeV} \). The lighter end of this range is clearly accessible at the Large Hadron Collider (LHC).

It is worth noting that even though the \( f, f_c \) are diagonal, one may obtain large neutrino mixing. As already noted, the neutrino masses arise from the type I seesaw [23, 24, 25, 26, 27].

\(^5\) The expressions for the smuon may be gotten by taking \( f_1 \leftrightarrow f_2 \) and \( f_{c1} \leftrightarrow f_{c2} \).
formula given by:

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D$$

$$= \frac{v_{wk}^2 \sin^2 \beta}{v_R^2} y^T f^{-1} y_\nu$$  \hspace{1cm} (26)$$

Note that the Yukawa coupling matrix $y_\nu$ is arbitrary and can be easily arranged to give large mixings even though $f$ is diagonal and we can fit the neutrino data by appropriate choice of parameters.

**III. BETWEEN SCALES: $v_R$ TO $F_\phi$**

Once $SU(2)_R$ breaks around the seesaw scale of $10^{11}$ GeV, the effective theory contains the NMSSM, an extra set of higgs doublets, a pair of left-handed triplets, and the doubly-charged fields$^6$. The non-renormalizable terms of Eq. (25) also influence the form of the lower scale theory and produce some important effects that aid in construction of a realistic low-energy theory. One significant contribution comes from the higher dimensional operators: the generation of a mass term for $N$. Specifically the terms

$$\frac{\lambda_N^c}{M_{Pl}} \text{Tr}(\Delta^c \bar{\Delta}^c) N^2 + \frac{\lambda_M}{M_{Pl}} S^2 N^2$$

(27)

generate a superpotential term of $\mu_N \phi N^2$ when $\Delta_c, \bar{\Delta}_c,$ and $S$ get a VEV. The mass $\mu_N$ is given by$^7$

$$\mu_N \equiv \frac{\lambda_N^c}{M_{Pl}} \langle \Delta^c \rangle \langle \bar{\Delta}^c \rangle + \frac{\lambda_M}{M_{Pl}} \langle S \rangle^2 \simeq \frac{v_R^2}{M_{Pl}}$$  \hspace{1cm} (28)$$

Because the $v_R$ threshold preserves the AMSB trajectory, this explicit mass term produces a SUSY breaking bilinear term proportional to $F_\phi$

$$\int d^2 \phi \mu_N \phi N^2 \supset \mu_N F_\phi N^2 \equiv b_N N^2$$

(29)

with $b_N$ given as

$$b_N = \mu_N F_\phi \simeq \frac{v_R^2}{M_{Pl}} F_\phi.$$  \hspace{1cm} (30)$$

In Section [IV] this term will be shown to play an important role in EWSB; for now it suffices to note that if $b_N$ is to be of the expected order of $M_{SUSY}^2$, then the right-handed scale

$^6$ The resulting theory with the additional particle content might be aptly labeled the NMSSM++

$^7$ We choose to denote the scalar component of the superfield $X$ as $\mathbf{X}$ to avoid confusion between the superfield and its scalar component. This allows us to write more meaningful expressions such as $\langle X \rangle / \langle \mathbf{X} \rangle = \phi$
must be around $v_R \simeq 10^{11}$ GeV. Constraining $v_R$ automatically determines the scale of new physics $M_X$ from Eq. (16): $M_X \lesssim 10^{16} - 10^{18}$ GeV. The end result is that the order of magnitude of all the scales of the theory are fixed.

Furthermore, the non-renormalizable terms can also be used to simplify the low-energy theory, though this is not necessary. Consider the terms

$$\frac{\lambda_c^{a b}}{M_{P l \phi}} \operatorname{Tr} \left( \Delta_c \bar{\Delta}^c \right) \operatorname{Tr} \left( \Phi_a \tau_2 \Phi_b^{T T} \tau_2 \right) + \frac{2 \lambda_c^{a b}}{M_{P l \phi}} \operatorname{Tr} \left( \Delta_c \tau_2 \Phi_a^{T T} \Phi_b \bar{\Delta}^c \right)$$

which yield a low energy mass matrix for the $\Phi$’s that is not symmetric between $\Phi_1$ and $\Phi_2$ (due to the second term). The asymmetry generates an operator of the form:

$$W \supset i M H_{u2} \tau_2 H_{d1}$$

without the corresponding $H_{u1} H_{d2}$ term. This allows a large mass, say of order $F_\phi$, for $H_{u2}$ and $H_{d1}$ while leaving $H_{u1}$ and $H_{d2}$ light. The resulting VEVs for $H_{u2}$ and $H_{d1}$ will then be suppressed by $M$ and will not play a role in the theory below $F_\phi$.

Finally, as discussed in Section II, the non-renormalizable terms yield masses around $F_\phi$ for the left-handed triplets as well as the doubly-charged fields. These fields therefore decouple from the electroweak scale theory along with the extra bi-doublet due to the doublet-doublet splitting mechanism discussed above. This leaves the low energy theory as the NMSSM and we use this to explore electroweak symmetry breaking as well as the remaining consequences of the low-energy theory.

### IV. EWSB

Naively it would be expected that the resulting low-energy theory is merely the NMSSM (since the remaining particle content is precisely that theory), but if this were the case, the model would not be able to achieve a realistic mass spectrum—the singlet $N$ would get a very small VEV, and the Higgsino would be lighter than allowed by experiment [28]. The origin of this problem is best illustrated with a toy model:

#### A. Toy Exposition

Consider a superpotential given by

$$W_{\text{toy}} = \frac{1}{3} \kappa N^3$$

(33)
where $N$ is a singlet field with no gauge symmetries. The resulting scalar potential, including SUSY breaking, is

$$V_{\text{toy}} = \kappa^2 |N|^4 + \frac{1}{3}(a_\kappa N^3 + a_\kappa^* N^*^3) + m_N^2 |N|^2. \quad (34)$$

Assuming the parameters $\kappa$, $a_\kappa$, and $\langle N \rangle$ are real, the minimization condition for Eq. (34) is

$$2\kappa^2 \langle N \rangle^2 + a_\kappa \langle N \rangle + m_N^2 = 0 \quad (35)$$

and the solution is given as

$$\langle N \rangle = -\frac{a_\kappa \pm \sqrt{a_\kappa^2 - 8\kappa^2 m_N^2}}{2\kappa^2} \quad (36)$$

The soft couplings $a_\kappa$ and $m_N$ are determined by AMSB via Eqs. (1) and (2):

$$a_\kappa = \frac{F_\phi}{16\pi^2} \frac{6\kappa^3}{m_{\text{an}}} \quad (37)$$

Substituting these into Eq. (36) yields

$$\langle N \rangle = \frac{F_\phi}{16\pi^2} \frac{\kappa}{4} (-6 \pm \sqrt{-60}) \quad (38)$$

and the large negative under the radical demonstrates the inability to achieve a real, non-zero VEV in this model.

The source of the problem can be identified by examining the potential of $N$. To expose the difficulty, it is helpful to define

$$x \equiv \frac{\kappa \langle N \rangle}{m_{\text{an}}} \quad (39)$$

and re-write Eq. (34) as

$$\langle V_{\text{toy}} \rangle = \frac{1}{4m_{\text{an}}^4} x^4 + x^3 + 3\kappa^2 x^2 \quad (40)$$

where the AMSB expressions of Eq. (37) have been substituted. For the potential to have a non-trivial minimum, it is necessary that the cubic term dominate for some value of $x$ (since this term is the only one that provides a negative contribution to the potential); however, for large $\kappa$, the $x^2$ term will always be larger than the cubic term. Meanwhile, for small $\kappa$ the quartic term will dominate the expression. Therefore, if there is any chance for the $x^3$
term to create a minimum other than zero, it must be that \( \kappa \simeq 1 \). This leaves the potential as

\[
\frac{\langle V_{\text{toy}} \rangle}{4m_{\text{an}}^4} = \frac{1}{4} x^4 + x^3 + 3x^2
\]

where it now becomes clear that neither large \( x, x \sim 1 \), nor small \( x \) will have the cubic term dominate the expression—leaving the only minimum as the trivial one. Thus, the heart of the problem is that AMSB predicts the cubic term’s coefficient such that it will always be weaker than either the quartic or quadratic regardless of the parameter regime.

The same problem carries over to the full NMSSM, as pointed out in [28]. In this model, the additional coupling of \( N \) to \( H_u \) and \( H_d \) does not alter the relative strengths of \( N \)’s quartic, cubic, or quadratic terms, but it does add a linear term to the potential, \( a_\lambda v_u v_d N \). The induced linear term shifts the trivial minimum away from zero, but keeps it small. The minimization condition for \( N \) can then be approximated as

\[
\tilde{\mu}_N^2 \langle N \rangle - \frac{1}{2\sqrt{2}} a_\lambda v^2 \sin 2\beta = 0
\]

with \( \tilde{\mu}_N^2 \simeq m_{\text{an}}^2 \) being essentially the AMSB predicted soft SUSY breaking mass for \( N \). The maximum value occurs when \( \sin 2\beta = 1 \) so we have that

\[
\langle N \rangle \lesssim \frac{a_\lambda v^2}{2\tilde{\mu}_N^2 \sqrt{2}} \simeq \frac{1}{2\sqrt{2}} \frac{v^2}{m_{\text{an}}} \simeq 22 \text{ GeV}
\]

The small \( \langle N \rangle \) then results in a chargino mass which falls below the LEP II bound of about 94 GeV.

Given this limitation of the NMSSM, it is desirable to explore methods that either alter the relative strengths of the terms or yield a large tadpole term for \( N \). The former may be done by adding vector-like matter (as in [6]), while the latter was explored in [28] by introducing a linear term for \( N \). We propose here a different solution that alters the relative strengths and is already present in the model.

**B. Low Energy Theory**

The superpotential of Eq. (5) contains in its non-renormalizable terms the key to solving the small \( \langle N \rangle \) problem: as discussed in Section III, the terms of Eq. (9) generate a mass term for \( N \) given by Eq. (28). This mass term then yields a SUSY breaking bilinear term given by Eq. (30). The size of \( b_N \) is quite conveniently around the SUSY breaking scale.
and also provides a means of turning the net mass-square of $N$ negative. To establish this property we now turn to the effective $M_{\text{SUSY}}$-scale theory.

The effective superpotential responsible for EWSB (valid for $M_{\text{SUSY}} < Q \ll F_\phi$) is

$$W|_{M_{\text{SUSY}}} = i y_u Q^T \tau_2 H_u u^c + i y_d Q^T \tau_2 H_d d^c + i y_e L^T \tau_2 H_d e^c$$

$$+ i \lambda N H_u^T \tau_2 H_d + \frac{1}{2} \mu_N N^2 + \frac{1}{3} \kappa N^3$$

(44)

and the SUSY breaking potential is

$$V_{\text{SB}}|_{M_{\text{SUSY}}} = m_Q^2 Q^T Q + m_{u^c} u^c u^c + m_{d^c} d^c d^c + m_{L^c} L^c L^c + m_{e^c} e^c e^c$$

$$+ m_H^2 H_u^T H_u + m_{H_d^T} H_d^T H_d + m_N^2 N^* N$$

$$+ \left[ i a_u Q^T \tau_2 H_u u^c + i a_d Q^T \tau_2 H_d d^c + i a_e L^T \tau_2 H_d e^c + \text{h.c.} \right]$$

$$+ \left[ \frac{i}{2} a_N L H_u^T \tau_2 H_d - \frac{1}{2} b_N N^2 + \frac{1}{3} a_N N^3 + \text{h.c.} \right]$$

$$- \frac{1}{2} (M_3 \lambda_3 \lambda_3 + M_2 \lambda_2 \lambda_2 + M_1 \lambda_1 \lambda_1 + \text{h.c.})$$

(45)

The resulting Higgs sector potential is

$$V = V_F + V_D + V_{\text{SB}}$$

(46)

with $V_F$ and $V_D$ the typical SUSY contribution:

$$V_F = |\lambda|^2 |N|^2 (|H_u|^2 + |H_d|^2) + |i \lambda H_u^T \tau_2 H_d + \mu_N N + \kappa N^2|^2$$

(47)

$$V_D = \frac{1}{8} (g_1^2 + g_2^2) (|H_u|^2 - |H_d|^2)^2 + \frac{1}{2} g_2^2 |H_u^T H_d|^2$$

(48)

The potential of Eq. (46) can be made to spontaneously break electroweak symmetry giving

$$\langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix} \quad \langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix} \quad \langle N \rangle = \frac{n}{\sqrt{2}}$$

(49)

and we take the usual definitions: $v_u = v \sin \beta$ and $v_d = v \cos \beta$. The minimization conditions are

$$m_{H_u}^2 - \frac{1}{8} (g_2^2 + g_1^2) v^2 \cos 2\beta + \frac{1}{2} \lambda^2 (n^2 + v^2 \cos^2 \beta) - \frac{n}{\sqrt{2}} (\bar{a}_\lambda + \frac{\lambda \kappa}{\sqrt{2}}) \cot \beta = 0$$

(50)

$$m_{H_d}^2 + \frac{1}{8} (g_2^2 + g_1^2) v^2 \cos 2\beta + \frac{1}{2} \lambda^2 (n^2 + v^2 \sin^2 \beta) - \frac{n}{\sqrt{2}} (\bar{a}_\lambda + \frac{\lambda \kappa}{\sqrt{2}}) \tan \beta = 0$$

(51)

$$\bar{m}_N^2 + \kappa^2 n^2 + \frac{1}{2} \lambda^2 v^2 + \frac{n \bar{a}_\lambda}{\sqrt{2}} - \frac{1}{2} v^2 \left( \frac{\bar{a}_\lambda}{n \sqrt{2}} + \lambda \kappa \right) \sin 2\beta = 0$$

(52)
The tilded variables are introduced to display the deviations from the usual NMSSM due to the presence of the term $\mu_N$ in Eq. (44). These constructs are defined as

$$\tilde{a}_\lambda \equiv a_\lambda + \lambda \mu_N$$
$$\tilde{a}_\kappa \equiv a_\kappa + 3 \kappa \mu_N$$
$$\tilde{m}_N^2 \equiv m_N^2 + \mu_N^2 - b_N$$

Of particular interest is Eq. (55), which may be recast using Eq. (30) of Section III:

$$\tilde{m}_N^2 = m_N^2 + \mu_N^2 - \mu_N F_\phi$$
$$\approx m_N^2 - \mu_N F_\phi$$
$$\approx \left( \frac{\lambda^4}{(16 \pi^2)^2} F_\phi - \mu_N \right) F_\phi$$

The second line follows from the fact that $\mu_N \sim O\left( \frac{M_{\text{SUSY}}^2}{F_\phi} \right) \sim O\left( \frac{F_\phi}{(16 \pi^2)^2} \right)$ and therefore the $\mu_N^2$ term is negligible compared to the other terms. The last line uses the AMSB expression for the scalar mass-squared, assuming it is dominated by the $\lambda$ contribution. As can be seen, due to the $\lambda^4$ suppression, it is relatively easy to adjust $\mu_N$ to the appropriate value to make $\tilde{m}_N^2$ negative and therefore induce a singlet VEV of the correct size. Given that $\lambda(M_{\text{SUSY}}) \lesssim 0.5$ (from constraints of perturbativity to the right-handed scale) and that $\mu = \frac{\lambda n}{\sqrt{2}}$, it is only necessary for $n \gtrsim 300$ GeV to achieve chargino masses above the LEP II bound. Figure 1 shows that such values are easily attainable in this situation. In the figure, constant $n$ contours are plotted in the $\mu_N-\kappa(v_R)$ plane treating the VEVs of the Higgs doublets as constant background values with $\tan \beta = 3.25$, $F_\phi = 33$ TeV, and $\lambda(v_R) = 0.5$. The ample parameter space therefore demonstrates that this inherent property of our model easily provides a means to resolve the conflict between AMSB and the NMSSM.

The resulting mass spectrum for this $\tilde{\text{NMSSM}}$ is quite similar to the NMSSM (see [29])—
FIG. 1: Constant $n$ contours in the $\mu_N$-$\kappa(v_R)$ plane where the curves, from top to bottom, correspond to $n = -10000, -7500, -5000, -2500$ and $-1000$ GeV. A constant value of $\tan \beta = 3.25$ has been assumed with $F_\phi = 33$ TeV and $\lambda(v_R) = 0.5$.

particularly for the scalar and charged Higgses\(^8\) whose mass matrices are given by

$$M^2_S = $$

$$
\begin{pmatrix}
\frac{v^2}{4} (g_1^2 + g_2^2) + \frac{n v_u}{\sqrt{2} v_d} \tilde{A}_\Lambda & \frac{n v_d}{\sqrt{2} v_u} (4 \lambda^2 + g_1^2 + g_2^2) - \frac{n}{\sqrt{2}} \tilde{A}_\Lambda & \lambda^2 N_u - \frac{n}{\sqrt{2}} \tilde{a}_\Lambda - \lambda \kappa N_d \\
\frac{n v_d}{\sqrt{2} v_u} (4 \lambda^2 + g_1^2 + g_2^2) - \frac{n}{\sqrt{2}} \tilde{A}_\Lambda & \frac{v^2}{4} (g_1^2 + g_2^2) + \frac{n v_u}{\sqrt{2} v_d} \tilde{A}_\Lambda & \lambda^2 N_d - \frac{n}{\sqrt{2}} \tilde{a}_\Lambda - \lambda \kappa N_u \\
\lambda^2 N_u - \frac{n v_d}{\sqrt{2} \tilde{a}_\Lambda - \lambda \kappa N_d} & \lambda^2 N_d - \frac{n}{\sqrt{2} \tilde{a}_\Lambda - \lambda \kappa N_u} & 2 n^2 \kappa^2 + \frac{n}{\sqrt{2} \tilde{a}_\Lambda - \lambda \kappa N_u}
\end{pmatrix}
$$

(56)

and

$$M^2_C = \frac{v^2}{2 v_d v_u} \left[ \sqrt{2} n \tilde{A}_\Lambda + v_d v_u \left( \frac{1}{2} g_2^2 - \lambda^2 \right) \right] ;$$

(57)

defining $\tilde{A}_\Lambda \equiv \tilde{a}_\Lambda + \frac{\lambda \kappa N}{\sqrt{2}}$.

On the other hand, the pseudoscalar mass matrix gets a contribution from the $b_N$ term which is rather large and typically guarantees that the heavier pseudoscalar is mostly singlet.

\(^8\) Simply substitute the appropriate variables with their tilded form: $(a_\kappa, a_\lambda, m^2_N) \rightarrow (\tilde{a}_\kappa, \tilde{a}_\lambda, \tilde{m}^2_N)$ Typically, however, $\mu_N$ is rather small and so the untilded variables make a good approximation to the tilded ones.
Its mass matrix is given by:

\[
M^2_P = \begin{pmatrix}
\frac{1}{\sqrt{2}} \tilde{A}_\lambda \frac{v^2 n}{v_u v_d} & \frac{v}{\sqrt{2}} \left( a_\lambda - \lambda \mu_N - \sqrt{2} \lambda \kappa n \right) \\
\frac{v}{\sqrt{2}} \left( a_\lambda - \lambda \mu_N - \sqrt{2} \lambda \kappa n \right) & \frac{v_n v_d}{n \sqrt{2}} \left( \tilde{a}_\lambda + 2 \lambda \kappa n \sqrt{2} \right) - 3 \tilde{a}_\kappa \frac{n}{\sqrt{2}} + 2 b_N + 8 \kappa \mu_N \frac{n}{\sqrt{2}}
\end{pmatrix}
\] (58)

The neutralino and chargino mass matrices remain similar to the NMSSM, and in the bases \((\tilde{B}, \tilde{W}, \tilde{H}_u, \tilde{H}_d, \tilde{N})\), \((\tilde{W}^+, \tilde{H}_u^+, \tilde{W}^-, \tilde{H}_d^-)\) they are:

\[
M_{\chi^0} = \\
\begin{pmatrix}
M_1 & 0 & M_Z \sin \beta \sin \theta_W & -M_Z \cos \beta \sin \theta_W & 0 \\
0 & M_2 & -M_Z \sin \beta \cos \theta_W & M_Z \cos \beta \cos \theta_W & 0 \\
M_Z \sin \beta \sin \theta_W & -M_Z \sin \beta \cos \theta_W & 0 & -\frac{\alpha}{\sqrt{2}} n & -\frac{\alpha}{\sqrt{2}} v_d \\
-M_Z \cos \beta \sin \theta_W & M_Z \cos \beta \cos \theta_W & -\frac{\alpha}{\sqrt{2}} n & 0 & -\frac{\alpha}{\sqrt{2}} v_u \\
0 & 0 & -\frac{\alpha}{\sqrt{2}} v_d & -\frac{\alpha}{\sqrt{2}} v_u & \sqrt{2} \kappa n + \mu_N
\end{pmatrix}
\] (59)

\[
M_{\chi^\pm} = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}; \quad X = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & \mu \end{pmatrix}
\] (60)

respectively.

C. A Brief Summary of Scales

With EWSB achieved and the mass spectrum given, we now have a complete picture of the physics starting at the high scale \(v_R\) and coming down to the electroweak scale. The theory starts as a parity-conserving SUSYLR model with AMSB generating the SUSY breaking, breaks down to the NMSSM++ (but without introducing new SUSY breaking effects), and finally ends up at \(M_{\text{SUSY}}\) as the \(\tilde{\text{NMSSM}}\) (as elucidated in Figure[2]). We may now turn our attention to the rich phenomenological consequences of this theory.

V. PHENOMENOLOGY

The following numerical values are based on our parameter running scheme. We run the gauge coupling values from the electroweak scale to the right-handed scale, \(v_R = 2 \times 10^{11}\)
taking the $F_\phi$ threshold into account by decoupling the triplets and doubly charged fields. Yukawa couplings are then inputs at the right-handed scale: the third generation values for the SM couplings ($y_Q, y_L$) and all three generations of the seesaw couplings ($f, f_c$). These are evolved down to the SUSY scale \cite{30, 31}. Because of parity $f = f_c$ at the right-handed scale and all of the off diagonal terms are taken to be negligible due to lepton flavor violating constraints. We also assume that the first and second generation seesaw couplings are equal ($f_2 = f_1$) for simplicity. Soft terms follow their AMSB trajectory, given by Eqs. (1), (2) and (3) down to the $F_\phi$ scale, below which the soft terms are evolved to the SUSY scale using the usual Renormalization Group Equation (RGE)s of the NMSSM \cite{32}. Note that due to the mass splitting between the Higgsinos and Higgses of both the doubly charged and left-handed triplets described in Section \ref{sec:IIA} there will be some corrections to the SUSY RGEs. These corrections will depend on the mass splitting and will be fairly small.

Numerical results will be compared to popular SUSY breaking models: Minimal Supergravity (mSUGRA), Minimal Gauge Mediated Supersymmetry Breaking (mGMSB) and
mAMSB—an AMSB in which the slepton mass problem is fixed by adding a universal mass, $m_0$ to all sfermion soft masses [3]. Note that slepton phenomenological comparisons to mAMSB also apply to [11] since the additional $R$-parity violating lepton sector Yukawa coupling is analogous to adding a universal slepton mass.

A. The Spectrum

Before engaging in the full details of the various sectors of the model, it is helpful to take a step back and look at the overall spectrum. Figure 3 examines the bosinos and Figure 4 the sfermions in this model and compares their masses to similar points in parameter space for mSUGRA, mGMSB and mAMSB calculated from isajet [33] (matching between the different points were done based on the gluino mass). The columns of the bosino chart, Figure 3 from left to right are gluino, neutralino and chargino. The columns of the sfermion chart, Figure 4 from left to right are: left-handed first generation, right-handed first generation, lightest mass third generation (and third generation neutrinos), heaviest third generation and gluinos—for comparison with the bosino chart. The Higgses and the mostly singlino neutralino have not been included to keep from cluttering the plots, although their masses are reported in Table II.

The most striking general feature of these figures is the degeneracy of the spectrum between colored and electroweak particles in LR-AMSB. While this is very dependent on the seesaw couplings (Table II shows slepton masses that are lighter than the squarks due to smaller values of the seesaw couplings), it is a possibility that is difficult to achieve in other models. Table II also shows the Higgs masses. Here $H_3$ and $A_2$ are the mostly singlet scalar and pseudoscalar. Due to the large size of the singlet VEV, these fields decouple from the spectrum as does the mostly singlino $\tilde{\chi}_5^0$. The neutral scalar Higgs masses stated include the full radiative corrections due to top and stop loops [34]. These corrections need $m_t \gtrsim 600$ GeV which implies $F_\phi \gtrsim 33$ TeV to allow the Higgs to be above the LEP II bound. In the following subsections, we will continue to explore this spectrum, focusing on the sleptons, squarks and finally the neutralinos and charginos.
FIG. 3: From left to right, columns correspond to charginos, neutralinos and gluino masses at $\tan \beta = 3.25$ and $\text{sgn}\,\mu = +1$. The parameter points are: $F_\phi = 33$ TeV, $f_1(v_R) = f_3(v_R) = 3.5$ for LR-AMSB; $m_0 = 209$ GeV, $m_{\tilde{\chi}^0_1} = -300$ GeV and $A_0 = 265$ GeV for mSUGRA; $\Lambda = 99$ TeV, $M_{\text{mess}} = 792$ TeV and $N_5 = 1$ for mGMSB; $F_\phi = 33$ TeV and $m_0 = 645$ GeV for mAMSB (here we also matched to the lightest slepton).

B. Sleptons

We start this discussion by analyzing the seesaw yukawa couplings $f$ and $f_c$. In all the work that follows, we take their maximum value at $v_R$ to be $\sim \sqrt{4\pi} \sim 3.5$ based on perturbativity arguments. Of immediate note is the fixed point-like behavior of these
TABLE II: Mass spectrum for the LR-AMSB point given in Figure 3. Slepton masses are reported for \( f_1(v_R) = f_3(v_R) = 3.5/1.4 \). Higgs masses are also reported here as well as the mostly singlino neutralino.

couplings. This can be seen in Figure 5 which plots \( f_{c1} \) versus the log of the energy scale for initial values of (a) \( f_3(v_R) = 0 \) and (b) \( f_3(v_R) = 3.5 \); the curves, in ascending order, correspond to \( f_1(v_R) = 0.25, 0.5, 0.75, 1, 2.25, 3.5 \). Increasing the initial value of \( f_3 \) decreases the value of \( f_1 \) at the TeV scale as can be seen by comparing Figure 5a and Figure 5b.

Similar plots can be drawn for the other couplings: \( f_1, f_{c3} \) and \( f_{c1} \), but their qualitative behavior follows those in Figure 5. Table III illustrates the quantitative differences in the values of the fixed points. For initial values of \( f_1 \) and \( f_3 \) greater than 1.5, these values are correct up to 2%. The higher values for the right-handed sector \( (f_c) \) are due to the slower running caused by the broken \( SU(2)_R \) symmetry.

This fixed point like behavior translates into an upper bound for the slepton masses. This can be seen in Figure 6 which displays the dependence of the selectron masses on the initial
|      | $f_3$ | $f_1$ | $f_{c3}$ | $f_{c1}$ |
|------|-------|-------|----------|----------|
| Fixed Point Value | 0.64  | 0.64  | 0.67     | 0.67     |

**TABLE III:** Fixed point values at the DC scale for the seesaw couplings assuming initial values are above 1.5 for the data point used in Figure 5.

value of the seesaw coupling. For this plot $f_1(v_R) = f_3(v_R)$ has been assumed for simplicity. For $f \geq 0.5$ the yukawa coupling contribution is comparable in size to the gauge coupling contribution in the AMSB mass expression, *e.g.* Eq. (19). The mass’ quartic dependence on the seesaw couplings is reflected in its steep rise near 0.5 and its rapid surpassing of the LEP II bound. At a value of $f \sim 1$ this steep ascent slows down indicating the onset of the fixed point behavior, beyond which the low energy observable $f(F_\phi)$ values are approximately 0.6.

The masses of the other sleptons follow the behavior of Figure 6, a general feature of which is the mild degeneracy between the left and right-handed slepton masses. This seems a bit contrary to Eqs. (22) and (23), which show that the factor for $f_1^4$ term for the left-handed sleptons is twice as large as that of the right-handed sleptons. However, this term is capped by the fixed-point of $f_1$ and the negative SU(2)$_L$ contribution happens to be a little less than half of this value (an accidental cancelation) yielding the degeneracy.

This is an interesting situation phenomenologically since it numerically falls in between mSUGRA/mGMSB and mAMSB. In mSUGRA, left-handed slepton masses get larger positive contributions from $M_2$ as they run from the ultraviolet. In mGMSB boundary conditions dictate that the left-handed to right-handed mass ratio is about 2 : 1. Meanwhile, in mAMSB, both sectors get the same contribution from $m_0$, the universal masses needed to make the sleptons non-tachyonic, which drops out in the mass splitting at tree level. Furthermore, there are accidental cancellations in the anomaly induced splittings related to the gauge contributions and in the $D$-term contributions [7, 35]. The upshot of this is that the mass splitting is usually dominated by loop-level effects and is quite small [7].

As a concrete example for mAMSB, including the first loop leading log, the difference between the masses squared with $\tan \beta = 3.25$ and $F_\phi = 33$ TeV is given by $\Delta_e = m_{\tilde{e}_L}^2 - m_{\tilde{e}_R}^2 \sim 751$ GeV$^2$ [7, 35]. The corresponding percent difference, defined as $\frac{\Delta_e}{(m_{\tilde{e}_L} + m_{\tilde{e}_R})^2}$, is then highly dependent on the masses of the selectrons. For selectron masses above the mass
FIG. 4: From left to right, columns correspond to first generation left-handed, first generation right-handed, lightest third generation and heaviest third generation sfermions. The final column consists of gluino masses for comparison with Figure 3. Input parameters are as given in Figure 3.
FIG. 5: Plots of $f_{c1}$ versus the log of the energy scale. The lines correspond, in ascending order, to $f_1(v_R)$ values of 0.25, 0.5, 0.75, 1, 2.25 and 3.5 for (a) $f_3(v_R) = 0$ and (b) $f_3(v_R) = 3.5$.

FIG. 6: Plot of $m_{\tilde{\chi}_1}^e$ (dashed) and $m_{\tilde{e}}$ as a function of $f_1(v_R) = f_3(v_R)$ for $F_\phi = 33$ TeV. The greyed-out region has been excluded by LEP II and the line at around 417 GeV is the mass of the neutralino, the LSP in this case.

of the LSP given in Figure 6, $\sim 450$ GeV, the percent difference is less than 1%. However, in LR-AMSB, the percent difference can rise as high as 5% as demonstrated in Figure 7 which gives contours for constant mass percent differences. Resolution of slepton masses from end-point lepton distribution of the selectron decays at lepton collider is roughly 2% [36], making the measurement of such mass differences feasible. Therefore, measurements of mild mass differences of about $3 - 5\%$ will single this model out from the large mass differences of
mSUGRA and mGMSB while potentially discriminating it from the small mass differences of mAMSB (although this will highly dependent on the values of the seesaw couplings).

![Graph](image)

**FIG. 7:** Constant contours of $\frac{m_{\tilde{e}} - m_{\tilde{e}c}}{m_{\tilde{e}} + m_{\tilde{e}c}} \times 100\%$ in the $f_3(v_R) - f_1(v_R)$ plane. The unlabeled contours on the left side of the plot, from left to right, correspond to 2%, 3%, 4% and 5%. The dashed vertical (horizontal) contour corresponds to a $\tilde{\tau}_1$ ($\tilde{e}^c$) constant contour of mass equal to that of the LSP (417 GeV). The shaded region is excluded by LEP II bounds of 81.9 GeV (94 GeV) on the mass of $\tilde{\tau}_1$ ($\tilde{e}^c$).

Constant mass contours for the right-handed selectron are plotted in Figure 8 in the $f_3(v_R) - f_1(v_R)$ plane. This plot allows a study of how the masses change with respect to both seesaw couplings. The horizontal and vertical grayed-out contours are ruled out due to LEP II bounds on the lightest stau and selectron masses of 81.9 GeV and 94 GeV respectively. Mass contours increase from left to right and correspond to $m_{\tilde{e}} = 200, 300, 417$ (the mass of the lightest neutralino, indicated with a dashed contour) 500, 550, 600, 610, 615, 620, 625, 630 GeV. The horizontal dashed curve represents a constant $m_{\tilde{\tau}_1}$ contour at the mass of the lightest neutralino. Since the selectron is a first generation slepton its mass is mainly governed by $f_1$, Eq. (20), explaining the small dependence on $f_3$ for smaller values of $f_1$. Two things are clear from this plot: the fixed point like behavior—reflected in the fact that for large $f_1$ an equal change in mass requires a larger change in $f_1$—and the decrease of the $f_1$ fixed point with the increase of $f_3$. This latter point is responsible for the curving to the right of the contours at high $f_1$ values and was mentioned earlier with regards to Figure 5.

As a final remark on the sleptons, notice that the contours in Figure 8 corresponding
FIG. 8: Mass contours for the right-handed selectron mass, $m_{\tilde{e}}$ in the $f_3(v_R)$–$f_1(v_R)$ plane.

The horizontal and vertical grayed-out contours are ruled out due to LEP II bounds on the lightest stau and selectron masses of 81.9 GeV and 94 GeV respectively. Constant mass contours for the selectron mass $m_{\tilde{e}} = 94$ (the LEP lower bound), 200, 300, 417 (the mass of the lightest neutralino, indicated with a dashed contour) 500, 550, 600, 610, 615, 620, 625, 630 GeV, for $F_\phi = 33$ TeV. The dashed horizontal curve corresponds to a $m_{\tilde{\tau}_1}$ constant contour equal to the mass of the lightest neutralino. The influence of $f_3(v_R)$ is apparent at large values of $f_1(v_R)$ and $f_3(v_R)$. Larger increases in $f_1(v_R)$ are needed for as the mass increases because of the fixed point like behavior of $f_1$.

to the LSP suggest more stringent lower bounds on the seesaw couplings than the LEP II bounds. These are necessary so that the lightest neutralino will be the LSP and therefore a possible dark matter candidate. The values indicated in the plot correspond to low energy values of the seesaw couplings that are only about 10% off from their fixed-point value, $f_{c_1}, f_{c_3}, f_1, f_3 \sim 0.6$. Therefore, for succesful dark matter, the seesaw couplings can be expected to be larger than about 0.5. This can be checked by a quick calculation, since the lightest neutralino mass is approximately the wino mass (see Section V D) and depends only on $F_\phi$. Meanwhile, the selectron mass depends on $f_1 \sim f_3 \equiv f$, which we can set equal to each other as an approximation, and $F_\phi$. Given that the selectrons must be heavier then the LSP, for a viable dark matter candidate, yields

$$f(F_\phi) \gtrsim 0.58.$$  \hspace{1cm} (61)
C. Squarks

Squark masses in mAMSB decrease with energy due to the increase of $SU(2)_L$ and $U(1)_Y$ gauge couplings, which contribute negatively to their masses. At a certain energy scale, the negative contributions take over and the AMSB expressions for the squark soft masses become negative. In our case this happens at an earlier scale due to the increase size of the $SU(2)_L$ and $U(1)_Y$ beta functions from the extra triplet and the doubly charged fields. Normally we would have expected this to show up at high temperatures and lead to breakdown of color gauge symmetry. However, at high temperatures the vacuum of the theory is also affected by temperature corrections. Consequently the mass-square term of the squarks will have the form $\mu^2(T)\tilde{q} \simeq (-M^2_{\text{AMSB}} + \lambda T^2)$. The first term only grows logarithmically with temperature whereas the second term grows quadratically. The coefficient $\lambda$ is positive so that the net effect is that $\mu^2(T)\tilde{q}$ remains positive at high temperature and leaving color gauge symmetry intact in the early universe.

It is also worth noting that because non-asymptotically free gauge couplings contribute negatively to masses, the right-handed squarks are slightly heavier than left-handed squarks. This is different than mSUGRA and mGMSB where all gauge couplings yield positive contributions making left-handed squarks heavier, see Figure 4. Furthermore, the squarks in this model can be degenerate with the sleptons.

D. Bosinos and the LSP

Because all superpartners eventually decay into the LSP, its makeup is an important part of SUSY collider phenomenology and dark matter prospects. Therefore understanding that makeup is an important task. Cosmological constraints rule out a charged or colored LSP, hence limiting the choices to the sneutrino or the lightest neutralino. The former, in typical models, makes a poor dark matter candidate (relic abundances are too light; much of its mass range ruled out by direct detection). It is therefore more interesting to consider the lightest neutralino as the LSP, the candidate in common SUSY scenarios (except in mGMSB where it is the next to lightest SUSY particle but has the same collider significance).

The lightest neutralino will be some mixture of the wino, bino and Higgsino. Its gaugino
composition follows from the gaugino mass ratio which is easily calculated and relatively independent of the point in parameter space. In AMSB this ratio depends on both the gauge couplings and the gauge coupling beta functions, $b$. The latter is important since this is where the effects of the light triplets and doubly-charged Higgs are felt (see Table IV for $b$ for values in LR-AMSB compare to AMSB based on MSSM particle content). It is calculated to be: $M_3 : M_2 : M_1 \sim 1.3 : 1 : 1.3$. The striking characteristic of this ratio is its degeneracy when compared to mSUGRA/mGMSB, $M_3 : M_2 : M_1 \sim 3 : 1 : 0.3$ or even mAMSB $M_3 : M_2 : M_1 \sim 8 : 1 : 3.5$. 

Specifically then, the LSP will have a large wino component where in mAMSB it is all wino, and there will also be some non-negligible mixing with the bino. Note that in mSUGRA (mGMSB) the sole contribution to this ratio is from the gauge couplings and therefore the LSP (Next-to Lightest Supersymmetric Particle (NLSP)) is always mostly bino. The Higgsino contribution is not independent of other parameters and therefore is not as predictable, but numerical results show that it is typically a little bit lighter than the wino (its value decreases compared to the wino as $F_\phi$ is increased). Therefore, the LSP will be some combination mostly Higgsino with significant wino content and and a little bit of bino. The mixed Higgsino state will correspond to $\chi^0_2$, $\chi^0_3$ and $\chi^2_3$ will be mostly wino with some Higgsino (percent values will be complementary to those of $\chi_1$), and $\chi^0_4$ will be mostly bino.

|        | $b_1$ | $b_2$ | $b_3$ |
|--------|-------|-------|-------|
| MSSM   | $\frac{33}{5}$ | 1     | -3    |
| LR-AMSB| $\frac{78}{5}$  | 6     | -3    |

**TABLE IV:** Values for the $b$ parameter in the MSSM and LR-AMSB. Note the larger values in LR-AMSB for $SU(2)_L$ and $U(1)_Y$.

An immediate consequence of the degeneracy of the gauginos is a more natural heavy LSP, closer in mass to both the gluinos and squarks. Naturalness suggests that squark masses are not much larger than 1 TeV, to minimize fine tuning in the Higgs mass, therefore:

$$F_\phi \lesssim 63 \text{ TeV}$$
yielding:

\[ M_1 \lesssim 1350 \text{ GeV} \]
\[ M_2 \lesssim 980 \text{ GeV} \] \hspace{1cm} (62)

This is a much larger value than the upper bound in mAMSB \( M_2 \lesssim 200 \text{ GeV} \) \(^{35}\) and therefore has less of its parameter ruled out by experimental data.

Another point to consider is that the Higgsino and wino form isospin doublets and triplets with the appropriate charginos. Therefore when they play the role of the lightest neutralino, there is potential for a very small mass difference between the lightest neutralino and the lightest chargino. This is very pronounced in mAMSB where the mass difference of the mostly wino neutralino and chargino is on the order of 100s of MeVs including leading radiative corrections. Analytical approximations for this quantity for large \( \mu \) have been given in \(^{7, 35, 42}\). Such approximations are not as useful in LR-AMSB since the relevant mass scales: \( \mu, M_1, \) and \( M_2 \) are relatively of the same order (the singlino contribution is much larger than these); however, an analytic expression for the minimum of the mass difference is attainable.

First note that a Higgsino mixing exists in the neutralino matrix, absent in the chargino sector. This mixing goes to zero as \( \tan \beta \to 1 \) hence indicating that for \( \tan \beta = 1 \) the mass difference is minimal (when \( \tan \beta \to 1 \) and \( \tan \theta_W \to 0 \) the global custodial \( SU(2) \) becomes an exact symmetry making the mass difference zero). The eigenvalues of the two matrices can than be expanded for \( \tan \beta = 1 \) using the approximation \( M_1 \sim M_2 > \mu \gg M_Z \), this yields, to first order:

\[ \Delta \tilde{\chi}_1 \equiv m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 2 \sin^2 \theta_W \frac{M_Z^2}{M_1} \] \hspace{1cm} (63)

The second order term is positive definite so that \( \Delta \tilde{\chi}_1 \) can in fact be used as a minimal value for the mass splitting. Notice that the \( \Delta \tilde{\chi}_1 \to 0 \) as \( \tan \theta_W \to 0 \) as argued above (and that \( \Delta \tilde{\chi}_1 \to 0 \) as \( M_1 \to \infty \) since this also restores the custodial symmetry when \( \tan \beta = 1 \).

The form of Eq. (63) is convenient since the only free parameter it depends on is \( F_\phi \) (through \( M_1 \)), which also controls the squark masses. Applying the natural upper bound for \( M_1 \) from Eq. (62) yields:

\[ \Delta \tilde{\chi}_1 > 1.4 \text{ GeV} \] \hspace{1cm} (64)
This is larger than the mAMSB value of a few 100s of MeV. Exact values for the mass difference are given in Figure 9 as a function of \( \mu \equiv \frac{1}{\sqrt{2}} \lambda n \) with: \( \lambda = 0.26 \), \( \tan \beta = 3.25 \) and the singlino mass term \( 2\left( \mu_N + \frac{1}{\sqrt{2}} n \kappa \right) = 2M_1 \). The line at 165 GeV represents the asymptotic value for large \( M_2 \) in mAMSB at the one loop level\(^7\) and below the dotted line the squark masses are above a TeV and hence the Higgs mass becomes finetuned.

FIG. 9: Mass difference of the lightest chargino and neutralino as a function of \( \mu \) for \( \lambda = 0.26 \), \( \tan \beta = 3.25 \) and the singlino mass term \( 2\left( \mu_N + \frac{1}{\sqrt{2}} n \kappa \right) = 2M_1 \). From top to bottom, \( M_2 = 1.1\mu \), \( 1.5\mu \), \( 2\mu \) and \( 3\mu \). The line at 0.165 GeV is the asymptotic value for large \( M_2 \) in mAMSB, while the dotted curve is represents where squark masses are at about a TeV. Below this curve, the Higgs mass is somewhat fine-tuned.

E. Collider Signatures

The small size of \( \Delta \tilde{\chi}_1 \) can potentially be problematic at a collider because the soft decay products, \( X \), in the process \( \chi_1^+ \rightarrow X \chi_1^0 \), will not be visible. This is a feature shared by both LR-AMSB and mAMSB. The difference is that the larger value of \( \Delta \tilde{\chi}_1 \) for LR-AMSB might produce prospects of detection if \( X = \tau \) or a hard \( \mu \); however, this advantage is counterbalanced by a faster chargino decay eliminating chances of long-lived charged tracks with no muon chamber activity. Regardless, similar situations have been analyzed and found to be manageable for both lepton colliders\(^{43}\) and the Tevatron\(^{42,44}\).

On the other hand, LHC studies of mAMSB have focused on mSUGRA like signals
Such signals are heavily dependent on lepton final states and are based on left-handed squark decays to mostly wino charginos and neutralinos. These in turn can decay leptonically producing trilepton signals or same sign dilepton signals, both of which have potentially manageable backgrounds. For in mAMSB, though, the the wino states are the lightest and will not decay leptonically. Hence the the right-handed squarks take the place of the left-handed ones decaying into the mostly bino neutralino (which can decay leptonically). Yet, since there is no corresponding chargino to the bino, signals such as the trilepton and the same sign dilepton signal may not be possible.

The situation in LR-AMSB is more analogous to mSUGRA: right-handed squarks will decay to the LSP which has some bino content. Meanwhile, the left-handed squarks may decay either to the lightest chargino/neutralino, or, more likely (because of their higher wino content), to $\chi_3^0$ and $\chi_2^+$. These may then decay leptonically depending on the slepton masses ($e.g$, $f(\nu_R) = 1.4$ in Table II) giving the familiar signals: trilepton and same sign dilepton. Note that it is also possible that decay of $\chi_1^+$ will produce leptonic signals since $\Delta \tilde{\chi}_1$ is larger. These considerations would help differentiate this model from mAMSB, while the degeneracies in the gaugino sector and same generation slepton will differentiate it from mSUGRA and mGMSB. These differences between the various scenarios are summarized in Table V.

|                     | mSUGRA and mGMSB | mAMSB | LR-AMSB |
|---------------------|-------------------|-------|---------|
| $M_3 : M_2 : M_1$   | 3 : 1 : 0.3       | 8 : 1 : 3.5 | 1.3 : 1 : 1.3 |
| $|M_1|, |M_2|$ (GeV) Naturalness upperbound | 130, 260$^a$ | 640, 200 | 1350, 980 |
| Same generation slepton mass percent difference | $\sim 150\%$ | $\sim 2\%$ | $\sim 4\%$ |
| Possibility of slepton-squark degeneracy | No | No | Yes |

$mGMSB$ only, in mSUGRA sfermion and gaugino mass are determined by two separate parameters.

**TABLE V:** A list of phenomenological characteristics of interest in mSUGRA, mGMSB, mAMSB and LR-AMSB.
F. Triplets and Doubly-Charged Higgses

The interplay between AMSB and the left-handed and doubly-charged Higgses leads to interesting phenomenology and is worth summarizing here. Because they play the central role of saving the slepton masses from a tachyonic fate, their masses must be around the $F_\phi$ scale. This puts a bound on the right-handed scale scale, $v_R \lesssim 10^{12}$ GeV, which is not the case when these particles appear in mSUGRA and mGMSB [20, 47]. It is also possible, through mixing due to bilinear $b$-terms Eq. (17), that one triplet and one doubly charged Higgs will be light, $\mathcal{O}(1$ TeV) and therefore accessible at the LHC. Their presence would also be felt indirectly in upcoming muonium-antimuonium oscillation experiments since their couplings to first and second generation leptons must be large. For sleptons above the LEP II bound:

$$f_1(F_\phi) \sim f_2(F_\phi) \sim f_{c1}(F_\phi) \sim f_{c2}(F_\phi) \sim 0.5$$

and for sleptons above the lightest neutralino (for a good dark matter candidate):

$$f_1(F_\phi) \sim f_2(F_\phi) \sim f_{c1}(F_\phi) \sim f_{c2}(F_\phi) \sim 0.6$$

Based on Figure 5a and Figure 8. On the other hand, all the triplet and doubly-charged Higgsinos will remain heavy, $\mathcal{O}(F_\phi)$, and undetectable at the LHC or low energy experiments.

G. Dark matter

As noted in the previous section, the LSP in our model is a predominantly Higgsino wino mix with very little bino (about 1%). Since the annihilation rate for such an LSP is large, its relic density from conventional annihilation arguments is not enough to explain the observed $\Omega_m$ of the universe of 20%. This issue has been discussed earlier in ref. [48], according to which the decay of the gravitino in the late stage of the universe to non-thermal winos will generate enough density to make it a viable dark matter. A similar mechanism would work in this mostly Higgsino case since the crucial ingredients are similar: the LSP mass (this a similar), its interactions with the gravitino (again, this is similar between the two cases because of the similar masses) and its annihilation rate (these are also the same since wino and Higgsino annihilation takes place through a t-channel chargino exchange proportional with $\alpha_2$ strength). Like [48], we have scanned over the parameters and found

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that such dark matter does evade current bounds on direct detection set by CDMS Soudan and EDELWEISS but will be detectable by future experiments.

VI. BEYOND $v_R$

In this section, we comment on the ultraviolet behaviour of the theory. As we see from Figure 10 below, despite the new contributions to $SU(2)_L$ and $U(1)_Y$ beta functions, all couplings remain perturbative until about $10^{11}-10^{12}$ GeV. Our effective field theory approach below this scale should hold without any problem. Once we are above this scale, the couplings could maintain perturbativity if there are extra dimensions due to negative contributions from vector gauge KK modes of the theory if the inverse radius of the extra dimensions are around $10^{11}$ GeV or so. Such extra dimensions could also be the origin of the Planck suppressed operators that we have used in our discussion.

![Figure 10: Inverse gauge couplings as a function of the log of the energy scale. The $v_R$ scale is at about $10^{12}$ GeV at which point $\alpha_R^{-1}$ begins to run with its curve being indistinguishable from $\alpha_2^{-1}$ due to parity.](image)

VII. CONCLUSION

In summary, we have elaborated on our suggestion that minimally extending MSSM to account for neutrino masses in a way that $R$-parity remains an automatic symmetry of the
theory allows for a solution to the tachyonic slepton problem of anomaly mediated supersymmetry breaking. Interestingly, the solution requires that parity symmetry remain exact above the $v_R$ scale. Among the new results, we show how to obtain radiative electroweak symmetry breaking and a reasonable $B\mu$ term in this class of models. We also discuss the sparticle spectrum of the model in detail and show how it differs from that of mAMSB as well as other widely discussed supersymmetry scenarios. A new feature of this model is the presence of new TeV scale $SU(2)_L$ triplets and doubly charged $SU(2)_L$ singlet fields, whose phenomenology has been the subject of many papers [50, 51, 52, 53, 54, 55]. We believe that the model discussed here is a serious alternative to the mAMSB whose further phenomenological implications need to be explored in detail.

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APPENDIX A: NOTATION CONVENTIONS

In this appendix we summarize our notational conventions. Given a superpotential defined as

$$W = L^i\Phi_i + \frac{1}{2!}\mu^{ij}\Phi_i\Phi_j + \frac{1}{3!}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{4!}\frac{\lambda^{ijkl}}{M}\Phi^i\Phi^j\Phi^k\Phi^l + \cdots$$  \hspace{1cm} (A1)

with a corresponding lagrangian of

$$\mathcal{L} = \int d^4\theta \left( Z^i_j\Phi_i\Phi^j + \cdots \right) + \left[ \int d^2\theta \left( W + \mathcal{W} + W_\alpha \right) + \text{h.c.} \right]$$  \hspace{1cm} (A2)

the anomalous dimensions, $\gamma^i_j$, and $\beta$-functions, $\beta^i_L$, $\beta^{ij}_\mu$, $\beta^{ijk}_Y$, at a given energy scale $Q$ are defined by

$$\gamma^i_j = \frac{d \ln Z^i_j}{d \ln Q} = 4C_a(\Phi_i) g^2_a\delta^i_j - Y_{jpq}Y^{jpq}$$  \hspace{1cm} (A3)

$$\beta^i_L = \frac{d L^i}{d \ln Q} = -\frac{1}{2}L^j\gamma^i_j$$  \hspace{1cm} (A4)

$$\beta^{ij}_\mu = \frac{d \mu^{ij}}{d \ln Q} = -\frac{1}{2}\mu^{ip}\gamma^j_p \hspace{1cm} (j \leftrightarrow i)$$  \hspace{1cm} (A5)

$$\beta^{ijk}_Y = \frac{d Y^{ijk}}{d \ln Q} = -\frac{1}{2}Y^{ijp}\gamma^j_p \hspace{1cm} (k \leftrightarrow i) \hspace{1cm} (k \leftrightarrow j)$$  \hspace{1cm} (A6)
Furthermore, we choose the sign of the soft SUSY breaking terms by specifying that

\[ V_{SB} = \frac{1}{2} \left( m^2 \right)_{ij} \Phi_i \Phi_j^* + \ell^i \Phi_i + \frac{1}{2!} b_{ij} \Phi_i \Phi_j + \frac{1}{3!} a_{ijk} \Phi_i \Phi_j \Phi_k + \text{h.c.} \]  

(A7)

**APPENDIX B: BETWEEN SCALES: \( v_R \) TO \( F_\phi \)**

The superpotential between the \( v_R \) and \( F_\phi \) scale is:

\[
W_{\text{NMSSM++}} = \frac{1}{2 \lambda} \left( y_t^a \Phi_t^a + y_b^a \Phi_b^a \right) Q^T \tau_2 \Phi_t^a + \frac{1}{2} |f_{3i}|^2 + \frac{1}{3} \kappa |N|^3 
\]

(B1)

Where \( a = 1, 2 \), the MSSM Yukawa matrices have been approximated by the third generation diagonal term, the seesaw Yukawa couplings are diagonal and the subscript \( i \) represents lepton generation and is summed. The gamma functions for the theory between the \( v_R \) and \( F_\phi \) are:

\[
\gamma_Q = \frac{1}{8 \pi^2} \left( y_t^a y_t^a + y_b^a y_b^a - \frac{5}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{1}{30} g_1^2 \right) 
\]

(B2)

\[
\gamma_{Q_1} = \frac{1}{8 \pi^2} \left( -8 g_3^2 - 8 g_2^2 - \frac{1}{30} g_1^2 \right) 
\]

(B3)

\[
\gamma_{Q_3} = \frac{1}{8 \pi^2} \left( 2 y_t^a y_t^a - \frac{8}{3} g_3^2 - \frac{8}{15} g_1^2 \right) 
\]

(B4)

\[
\gamma_{Q_4} = \frac{1}{8 \pi^2} \left( -8 g_3^2 - \frac{8}{15} g_1^2 \right) 
\]

(B5)

\[
\gamma_{Q_5} = \frac{1}{8 \pi^2} \left( 2 y_b^a y_b^a - \frac{8}{3} g_3^2 - \frac{2}{15} g_1^2 \right) 
\]

(B6)

\[
\gamma_{Q_6} = \frac{1}{8 \pi^2} \left( -8 g_3^2 - \frac{2}{15} g_1^2 \right) 
\]

(B7)

\[
\gamma_{L_3} = \frac{1}{8 \pi^2} \left( y_t^a y_t^a + 6 |f_{3i}|^2 - \frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 \right) 
\]

(B8)

\[
\gamma_{L_1} = \frac{1}{8 \pi^2} \left( 6 |f_{1i}|^2 - \frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 \right) 
\]

(B9)

\[
\gamma_{e^c} = \frac{1}{8 \pi^2} \left( 2 y_t^a y_t^a + 4 |f_{3i}|^2 - \frac{6}{5} g_1^2 \right) 
\]

(B10)

\[
\gamma_{e^c} = \frac{1}{8 \pi^2} \left( 4 |f_{1i}|^2 - \frac{6}{5} g_1^2 \right) 
\]

(B11)
\[ \gamma_N = -\frac{1}{8\pi^2} (2|\kappa|^2 + 2\lambda^{ab}\lambda^{ab}) \]  
\[ \gamma_{H_{ua}}^{H_{ub}} = -\frac{1}{8\pi^2} \left( 3y^a_t g^b_t + \lambda^{ac}\lambda^{bc} - \delta^{ab} \left( \frac{3}{2} g^2_2 + \frac{3}{10} g^2_1 \right) \right) \]  
\[ \gamma_{H_{da}}^{H_{db}} = -\frac{1}{8\pi^2} \left( 3y^a_b g^b_b + y^a_t g^b_t + \lambda^{ac}\lambda^{cb} - \delta^{ab} \left( \frac{3}{2} g^2_2 + \frac{3}{10} g^2_1 \right) \right) \]  
\[ \gamma_{\Delta} = -\frac{1}{8\pi^2} \left( 2|f_3|^2 + 2|f_2|^2 + 2|f_1|^2 - 4g^2_2 - \frac{6}{5} g^2_1 \right) \]  
\[ \gamma_{\Delta^{c--}} = -\frac{1}{8\pi^2} \left( 2|f_3|^2 + 2|f_2|^2 + 2|f_1|^2 - \frac{24}{5} g^2_1 \right) \]  
\[ \gamma_{\Delta^{--}} = -\frac{1}{8\pi^2} \left( \frac{24}{5} g^2_1 \right) \]  

These expressions were used for the slepton masses in Eqs. (22) and (23). The third generation squark masses can also be written down (here we assume real yukawa couplings for simplicity):

\[ m^2_{Q_3} = \frac{1}{4} F^2_\phi \left\{ \frac{b_1}{7\pi^2} \left[ \frac{2b_2}{5\pi^2} - \frac{2b_3}{3\pi^2} \right] 
\begin{align*} 
&+ \frac{4g^2_1}{9\pi^2} \left[ \frac{2}{10} \left( 3(y^c_t)^2 + (y^c_b)^2 - \frac{13}{9} g^2_2 - 3g^2_2 - \frac{8}{3} g^2_3 \right) + \frac{y^c_t}{16\pi^2} \left( 3y^a_t y^c_t + \lambda^{ad}\lambda^{cd} \right) \right] 
\end{align*} \right\} \]  
\[ m^2_{\tilde{e}c} = \frac{1}{4} F^2_\phi \left\{ \frac{b_1}{9\pi^2} \left[ \frac{2b_2}{3\pi^2} \right] 
\begin{align*} 
&+ \frac{8g^2_1}{16\pi^2} \left[ \frac{2}{10} \left( 3(y^c_t)^2 + (y^c_b)^2 - \frac{13}{9} g^2_2 - 3g^2_2 - \frac{8}{3} g^2_3 \right) + \frac{y^c_t}{16\pi^2} \left( 3y^a_t y^c_t + \lambda^{ad}\lambda^{cd} \right) \right] \right\} \]  
\[ m^2_{\tilde{e}c} = \frac{1}{4} F^2_\phi \left\{ \frac{b_1}{18\pi^2} \left[ \frac{2b_2}{3\pi^2} \right] 
\begin{align*} 
&+ \frac{8g^2_1}{16\pi^2} \left[ \frac{2}{10} \left( 3(y^c_t)^2 + (y^c_b)^2 - \frac{13}{9} g^2_2 - 3g^2_2 - \frac{16}{3} g^2_3 \right) 
\end{align*} \right\} \]  

Were the first generation squark masses can be found by using the third generation mass expressions with yukawa couplings set to zero and \( b_A = \left( \frac{28}{5}, 6, -3 \right) \) for \( A = (1, 2, 3) \).
Typically, the largest contribution to these are given by:

\[ m^2_q \sim F^2_\phi \frac{\alpha^2_3(F_\phi)}{2\pi^2}. \]  

(B22)

The present LEP bound on the Higgs mass of 114 GeV can then roughly be translated to give a lower bound of about 600 GeV on the top squark mass. Using \( \alpha_3(F_\phi) \simeq 0.08 \) in the above expressions, we can translate this squark mass bound to a lower limit on \( F_\phi \) of about 30 TeV. We have used this in all our calculations in the text.

**ADDENDUM**

The purpose of this addendum is to clarify certain aspects of the detailed model implementing the idea described in the main body of the paper. We first show that the model defined in Eqs. (5)–(9) has new diagrams at the \( F_\phi \) scale that dominate the contributions noted in the text, making the sleptons tachyonic below \( F_\phi \). It is then noted that the model permits an additional term in the kähler potential that is crucial to restoring the low-energy phenomenology and leaves the presented results unaltered.

To elucidate the issues at the \( F_\phi \) scale, it is useful to first consider a simplified model with a superpotential of

\[
W_{\text{simp}} = (\lambda_S S - M_\Delta \phi) (\Delta^c \bar{\Delta}^c - M_{\Delta}^2 \phi^2) + \frac{\lambda_A^c}{M_X \phi} \text{Tr}^2(\Delta^c \bar{\Delta}^c) + \frac{\lambda_B}{M_X \phi} \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c)
\]

(C1)

and fields as defined in the text. The mass scales \( M_\Delta, M_S \) are assumed to be of the same order as \( v_R \), the right-handed scale.

The superfields of Eq. (C1) acquire a VEV given by

\[
\langle S \rangle = \frac{M_\Delta}{\lambda_S} \phi \\
\langle \Delta^c \rangle = \langle \bar{\Delta}^c \rangle = M_S \phi
\]

(C2) (C3)

and, as expected, the VEVs are proportional to \( \phi \) indicating this is an AMSB preserving threshold. It is worth noting that preserving AMSB is a direct result of the superconformal invariance of the VEV structure which is itself a result of the VEVs being induced by terms that preserve the superconformal symmetry.
FIG. 11: One loop yukawa mediated contributions to the selectron from integrating out the doubly-charged particles at $\mu_{\text{DC}}$. The fields $d^{++}$, $D^{++}$ represent the mass eigenstates of the scalars $\Delta^{c-}$ and $\bar{\Delta}^{c+}$.

Now once the superfields are shifted by their VEVs, the non-renormalizable terms give rise to an effective mass term for the (otherwise massless) doubly-charged fields:

$$W_{\text{simp}} \supset M_{\phi}^2 \Delta^{c-} \bar{\Delta}^{c+} = \mu_{\text{DC}} \phi \Delta^{c-} \bar{\Delta}^{c+}, \quad (C4)$$

where $\mu_{\text{DC}} \equiv \frac{M_{\phi}^2}{M_X \phi}$.

As discussed in the text, $\mu_{\text{DC}} \geq F_\phi$ to avoid tachyonic doubly-charged particles; however, given the form of Eq. (C4), it is evident the threshold associated with the doubly-charged particles also preserves AMSB, which is true even if it is at $F_\phi$.

But $\mu_{\text{DC}} \sim F_\phi$ has additional threshold corrections to the remaining low-scale particles that are important [8, 56]. These effects are governed by the ratio

$$\delta \equiv \frac{b_{\text{DC}}}{\mu_{\text{DC}}} = \frac{F_\phi}{\mu_{\text{DC}}} \quad (C5)$$

which measures the splitting of the messenger scalar fields’ masses due to SUSY breaking\(^9\). The usual AMSB expressions for the low-scale particles are zero order in $\delta$, and are dominant for $\mu_{\text{DC}} \gg F_\phi$; however, for $\mu_{\text{DC}} \sim F_\phi$ the one-loop yukawa-mediated contributions also become important. For the selectron, all such diagrams are shown in Figure\(^{11}\) The sum

\(^9\) If the scalar mass matrix of Eq. (17) has the eigenvalues $m_{\pm}^2$, then $(m_+^2 - m_-^2)/\mu_{\text{DC}}^2 = 2\delta$
of the graphs in Figure 10 yield a scalar mass-squared correction of

\[
\Delta m^2_{\phi} = -\frac{2}{3} \frac{f_1^2}{16\pi^2} \mu_{DC}^2 \delta^4 \sim -\frac{2}{3} \frac{F_\phi^2}{16\pi^2} f_1^2, \tag{C6}
\]

where the second expression takes \( \mu_{DC} \) around \( F_\phi \). This expression is always negative and larger in magnitude than the AMSB expressions, which are suppressed by an additional factor of \( 1/16\pi^2 \).

At this stage it would appear that combining the seesaw mechanism with AMSB has actually made the problem worse, since the sleptons are now ‘more negative' by a factor of \( 16/\pi^2 \). This is not, however, the situation because the model itself permits additional terms that are not expressed in the superpotential. In fact, the full model of Eqs. (5)–(9) allow the kähler potential term

\[
K \supset k \frac{\phi^+}{\phi} \text{Tr}(\Delta \Delta + \Delta^c \bar{\Delta}^c) \tag{C7}
\]

with \( k \) an order one constant.

A term such as Eq. (C7) has been studied before\[57, 58\], and it was pointed out that it yields an effective superpotential term of

\[
\int d^4 \theta \ K \supset \int d^4 \theta \ k \frac{\phi^+}{\phi} \text{Tr}(\Delta \Delta + \Delta^c \bar{\Delta}^c) \quad \rightarrow \quad W \supset k \frac{\phi^+}{\phi} \text{Tr}(\Delta^c \bar{\Delta}^c). \tag{C8}
\]

The presence of this effective SUSY mass term then alters the mass matrix for the doubly-charged particles given in Eq. (17) to

\[
\mathcal{M}_{DC} = \begin{pmatrix}
|\mu_{DC} + k F_\phi^+|^2 & \mu_{DC} F_\phi - |k F_\phi|^2 \\
\mu_{DC}^* F_\phi^+ - |k F_\phi|^2 & |\mu_{DC} + k F_\phi^+|^2
\end{pmatrix} \tag{C9}
\]

with \( \mu_{DC} \sim v_R^2/M_X \) as before. Since \( k \) and \( \mu_{DC} \) are free parameters (\( k \) is an arbitrary \( \mathcal{O}(1) \) constant while \( \mu_{DC} \) depends on the non-renormalizable couplings), Eq. (C9) may be tuned so that all the fields are at \( M_{SUSY} \):

\[
|\mu_{DC} + k F_\phi^+|^2 \sim \frac{|F_\phi|}{16\pi^2} \\
\mu_{DC} F_\phi - |k F_\phi|^2 \sim \left( \frac{|F_\phi|}{16\pi^2} \right)^2 \tag{C10}
\]

The tunings Eq. (C10) permit both the doubly-charged fermions and the doubly-charged scalars to remain in the theory to the TeV scale and retain the AMSB trajectory for all the particles. A similar argument allows the left-handed triplets to persist until \( M_{SUSY} \).
While both the doubly-charged scalars and fermions survive to the TeV scale, the muonium-antimuonium constraints given in Eq. (25) still force these particles’ masses to be at or above 2 TeV. If they reside right near this lower bound, the LHC may produce both doubly-charged scalars and fermions (as opposed to just the scalars as presented in the paper).

Because this new particle content survives to the TeV scale, the AMSB expression may be utilized at that scale to determine the soft masses. The presence of the new yukawa couplings $f$ and $f_c$ for the sleptons will then cause them to be positive. In the analysis of Section V these AMSB expressions were evaluated at $F_\phi$ for both squarks and sleptons, then used as boundary conditions to evolve the masses down to $M_{\text{SUSY}}$. As the parameters do not run significantly from $F_\phi$ to $M_{\text{SUSY}}$ (it is only two orders of magnitude), the numerical results presented in the paper remain valid within the expected uncertainty.

**NOTE**

After this paper was published, the authors were informed of [59] which discusses an alternative scenario to avoiding tachyonic sleptons. The authors regret this omission and their oversight which prevented it appearing in the printed paper.

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