Trapped-ion antennae for the transmission of quantum information

M. Harlander,1 R. Lechner,1 M. Brownnutt,1 R. Blatt,1,2 and W. Hänsel1,2

1Institut für Experimentalphysik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
2Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Otto-Hilitmair-Platz 1/Technikerstraße 21a, A-6020 Innsbruck, Austria

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More than one hundred years ago Heinrich Hertz succeeded in transmitting signals over a few meters to a receiving antenna using an electromagnetic oscillator and thus proving the electromagnetic theory developed by James C. Maxwell\textsuperscript{1}. Since then, technology has developed, and today a variety of oscillators is available at the quantum mechanical level. For quantized electromagnetic oscillations atoms in cavities can be used to couple electric fields\textsuperscript{2, 3}. For mechanical oscillators realized, for example, with cantilevers\textsuperscript{4, 5} or vibrational modes of trapped atoms\textsuperscript{6} or ions\textsuperscript{7, 8}, a quantum mechanical link between two such oscillators has, to date, been demonstrated in very few cases and has only been achieved in indirect ways. Examples of this include the mechanical transport of atoms carrying the quantum information\textsuperscript{9} or the use of spontaneously emitted photons\textsuperscript{10}. In this work, direct coupling between the motional dipoles of separately trapped ions is achieved over a distance of 54 $\mu m$, using the dipole-dipole interaction as a quantum-mechanical transmission line\textsuperscript{11}. This interaction is small between single trapped ions, but the coupling is amplified by using additional trapped ions as antennae. With three ions in each well the interaction is increased by a factor of seven as compared to the single-ion case. This enhancement facilitates bridging of larger distances and relaxes the constraints on the miniaturization of trap electrodes. This represents a new building block for quantum computation and also offers new opportunities to couple quantum systems of different natures.

The exchange of quantum information between qubits at remote sites is a key feature required to render quantum computation truly scalable\textsuperscript{12}. The dipole-dipole interaction offers a link between separate quantum systems without the need to shuttle particles between sites. The interaction strength depends on the orientation and distance of the dipoles and is in general given by

$$U_{dd} = \frac{1}{4\pi\epsilon_0} \frac{\vec{d}_1 \cdot \vec{d}_2 - 3(\hat{\vec{d}}_1 \cdot \vec{c}_r)(\hat{\vec{d}}_2 \cdot \vec{c}_r)}{r^3},$$

where $d_i$ are the interacting dipoles, $r$ and $\vec{c}_r$ denote the magnitude and direction of their separation.

Here this interaction is explored using ions or ion strings held in two separate potential wells of a linear segmented ion trap (see Fig.1), where the interacting dipoles are produced by the oscillating charges. As the dipole-dipole interaction decreases rapidly with trap separation, it is advantageous to bring the trapping wells as close together as possible. However, the generation of small inter-well distances requires similarly small distances, $d$, between the ions and the trap electrodes. This requirement runs counter to the effort to keep a larger ion-electrode separation in order to reduce “anomalous heating”\textsuperscript{13, 14} with its $d^{-4}$ scaling, and also the effects of technical noise of the applied voltages\textsuperscript{15}. Various routes may be taken to balance these competing requirements. In one approach, the heating rate may be reduced through the use of cryogenic temperatures\textsuperscript{16}. Here, another approach is taken that uses more ions in the individual traps and enables interaction over larger distances. The additional ions work as “antennae” that increase the motional dipole moment at the respective trapping site.

Given longitudinal alignment of the traps with one par-
ticle in each well, the dipole-dipole interaction (Eq. [1]) is:

\[ U_{dd} = -\frac{q_1 q_2 \Delta z_1 \Delta z_2}{2\pi\epsilon_0 r^3} \]

\[ = -\frac{\hbar}{2} \Omega_c \left( a_1 + a_1^\dagger \right) \left( a_2 + a_2^\dagger \right) \]

\[ \approx -\frac{\hbar}{2} \Omega_c \left( a_1 a_2^\dagger + a_1^\dagger a_2 \right) \]

with

\[ \Omega_c = \frac{q_1 q_2}{2\pi\epsilon_0 \sqrt{m_1 m_2 \omega_1 \omega_2}} \]

Here \( q_i \) and \( m_i \) refer to the charge and mass of the particles, \( \Delta z_i = \sqrt{\frac{\hbar}{2m_i \omega_i^2}}(a_i + a_i^\dagger) \) denotes the vibrational amplitude of the motion of ion \( i \). The quantum-mechanical creation and annihilation operators, \( a_i \) and \( a_i^\dagger \) act on the individual harmonic oscillators with frequencies \( \omega_i \). Rapidly oscillating terms have been neglected in Eq. 3.

At resonance, i.e., for \( \omega_1 = \omega_2 \), the coupling described by Eq. [3] leads to a complete exchange of motional states between the two ions after time \( T_{swap} = \pi/\Omega_c \). This is analogous to two coupled pendula connected by a (massless) spring. If one pendulum initially oscillates while the other is at rest, the motion is periodically exchanged between them. The first pendulum comes to a complete stop after a characteristic time, \( T_{swap} \), proportional to the associated spring constant. A quantum-mechanical description may be given using the vibrational quanta, \( n_i \), often labelled “phonons”, in the individual wells. Under the dipole-dipole interaction an initial motional state \( |n_1, n_2\rangle \) becomes entangled with all other motional states \( |n_1', n_2'\rangle \) respecting \( n_1' + n_2' = n_1 + n_2 \). Only at odd (even) multiples of \( T_{swap} \) is the swapped (original) basis state recovered. Notably, the initial state \( |0, 1\rangle \) evolves into the Bell-state \( (|0, 1\rangle + |1, 0\rangle)/\sqrt{2} \) after time \( T_{swap}/2 \), yielding a maximally entangled state of motion. This motional entanglement can be mapped onto the internal electronic state of the ions[17].

In the experiment presented the coherent energy exchange is demonstrated between singly charged \( ^{40}\text{Ca}^+ \) ions. They are held in an ion trap with gold-on-alumina electrodes arranged in a two-layer geometry (cf. Fig. 1) similar to the one described in Ref. 18. Applying DC voltages of up to 110 V to seven adjacent electrode pairs, a double-well potential with a trap separation of 54 \( \mu \)m and trap frequencies of 537 kHz is created (see Methods). The ions are Doppler-cooled on the \( S_{1/2}-P_{1/2} \) transition at 397 nm using a single, elliptically shaped laser beam, and detected by collecting the fluorescence light on an EM-CCD camera and on a photo-multiplier tube. Two 729 nm laser beams, individually focused on the two trapping sites, are used to perform sideband cooling on the \( S_{1/2}-D_{5/2} \) transition and to map out the sideband spectrum[19]. During the 4 ms period of sideband cooling the 729 nm beam is alternated between the two trapping sites at the approximate rate of the energy exchange, leading to an imbalance of phonon population between the traps. Immediately following the cooling cycle a mean phonon number \( \langle n_1 \rangle = 3.9(4) \) is observed in the first well, by comparing Rabi-flops on the red- and blue-sideband[19]. Fig. 2 shows the oscillatory behaviour of the phonon number in this well as a function of waiting time. The theoretical fit to the data indicates an exchange within \( T_{swap} = 222(10) \) \( \mu \)s and an initial phonon population of \( \langle n_2 \rangle = 9(1) \) in the second well. Currently, the ions experience a heating rate of 1.3(7) quanta per millisecond, which is comparable to other room-temperature traps[20].

Ideally, the exchange rate would be significantly larger than the average heating rate. To enhance the exchange rate strings of several ions can be used in the trapping wells. Approximating the individual ion strings as point-like objects with appropriately increased charge and mass, the resulting dipoles scale with the square root of the number of ions (see Eq. [1]), leading to an overall increase of the interaction rate proportional to the total number of ions. Using lighter ion species for antennae would enhance the coupling even more.

This increase is demonstrated by mapping out the ions’ excitation spectrum as the trapping frequencies of the individual sites are scanned through the resonance con-
The dipole-dipole coupling manifests as an avoided crossing, separating the mode frequencies by $\Omega_c$. Close to resonance, the motion of the ion strings is strongly coupled, and the oscillation can be excited with 729 nm light on either of the two trapping sites [19]. Fig. 3 (a) shows an example of this avoided crossing measured with two ions in each well while Fig. 3 (b) represents an individual sideband spectrum. The motional spectra of five ion configurations have been analyzed, using up to three ions in each well. The configurations and corresponding mode frequencies are displayed in Fig. 4. The curves present calculations using a common set of fit parameters for the potential and for the action of the control voltage $U_{ax}$ (see Methods). For the configuration with one ion per well, the observed splitting of 1.9 kHz agrees within one standard deviation with the energy-exchange rate from Fig. 2 which was measured with the same trap parameters. The data further show that the splitting is increased from 1.9(3) kHz to 14(1) kHz by using up to three ions in each well, without any modification of the external potential. The seven-fold increase of the coupling is beyond the factor three that is expected from a simple point-charge model and is due to the anharmonicity of the individual potential wells: as the outer potential walls are steeper than the inner ones, the ion-strings’ centers of mass get closer as more ions are added. At the same time, the average oscillation frequency is reduced. Both of these effects lead to an increase in the coupling strength (see Eq. 4). The extent of the ion strings provides an additional increase when it becomes non-negligible with respect to the inter-well distance.

The demonstrated coupling [21] can be used in diverse schemes to create entanglement or to perform gates. The creation of Bell-states is discussed above and requires

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**Fig. 3.** (a) Oscillation frequencies of four trapped ions (two in each well) as a function of the axial control voltage $U_{ax}$, yielding a mode splitting of 5.5(3) kHz. The data points correspond to peaks in individual sideband spectra taken on the $S_{1/2}-D_{5/2}$ transition. The error bars are smaller than the dot size. (b) Example of an individual sideband spectrum.

**Fig. 4.** Experimentally observed dipole-dipole coupling for various ion configurations in a double-well potential. The graphs display the oscillation frequencies of the two lowest vibrational modes as a function of control voltage $U_{ax}$ and reveal the mode splitting at resonance. (a) Unmodified trap frequencies, as measured with a single ion, in the first or second well, respectively. (b) Spectra with up to three ions in each well as depicted in the insets. The lines represent predictions from numerical calculations and fit all data simultaneously. A small drift in the control voltage of 7 mV over one hour has been taken into account, aligning the spectra on top of one another. By the use of three ions in each potential well the coupling is increased from 1.9(3) kHz to 14(1) kHz (see text). The vertical elongation of the ion pictures is due to aberrations, arising from an off-axis position of the ions relative to the imaging axis.
creation of linear cluster states within two steps. A two-dimensional cluster state would only require two further steps by pairwise coupling of columns \[24\]. The amplification of the dipole-dipole interaction is not limited to ions. The presented technique may be directly transferred to the coupling of trapped Rydberg atoms \[25\]. Furthermore, atomic and ionic systems may be combined. Neutral atoms have already been brought close to ions \[26\], \[27\]. When the ions’ oscillation frequency is tuned into resonance with adjacent levels of a Rydberg atom, coupling between ions and Rydberg atoms may become achievable.

\section*{METHODS}

Calculation of eigenfrequencies in the double-well potential.

A symmetric double-well potential is created along the trap axis by applying 2.8 V, 110.4 V, -16.8 V, 13.7 V, -16.8 V, 110.4 V, and -33.0 V to the seven adjacent electrodes pairs marked in Fig.1(b). The outermost electrode pairs are used to compensate for the asymmetry induced by the taper zone. The control voltage, \(U_{\text{ax}}\), is additionally applied to the leftmost electrode pair, creating a nearly-homogeneous electric field in the center region. This shifts the trap frequencies of the two wells in opposite directions and is used to tune the wells into and out of resonance. In the region of interest the double-well potential is well described by a fourth-order polynomial, where the 0th order term does not influence the dynamics and the third-order coefficient has been eliminated by appropriate choice of origin:

\[ U_{\text{pot}}(z) = \alpha_1 \cdot z + \alpha_2 \cdot z^2 + \alpha_4 \cdot z^4. \]  

The polynomial coefficients \(\alpha_2\) and \(\alpha_4\) determine both the uncoupled trap frequency, \(\omega_0\), and the inter-well separation, \(r\), at the symmetry point \[28\] (\(\alpha_1 = 0\)). For the theoretical calculations, the action of the tuning voltage is modelled as a linear contribution to the coefficients \(\alpha_1 = 4.61 \times 10^{-24}\) m\(^{-1}\) and \(\alpha_2 = 0.9 \times 10^{-20}\) m\(^{-2}\). The equilibrium positions for \(N\) ions, \(z_0^N = (z_{0,1}, ..., z_{0,N})\), are those minimizing the total energy, \(U_{\text{tot}}^N\), including the Coulomb interaction:

\[ U_{\text{tot}}^N(z^N) = \sum_{i=1}^{N} U_{\text{pot}}(z_i) + \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{q_i^2}{4\pi \epsilon_0} \frac{1}{|z_i - z_j|}. \]  

The theoretical data for the vibrational mode frequencies, \(\omega_i\), in Fig.2 are computed from the two lowest eigenvalues of the Jacobian matrix of the total energy, \(U_{\text{tot}}^N(z^N)\), at positions \(z_0^N\). These correspond to the modes where the individual ion strings move as a whole, approximating the motion of single particles.

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Correspondence should be addressed to R.B. (email: rainer.blatt@uibk.ac.at).
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