Creation of physical modes from unphysical fields

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September 1, 2014

Abstract

We present a basic idea and a toy model that physical modes originate from unobservable fields. The model is defined on a higher-dimensional space-time and has fermionic symmetries that make fields unphysical, and observable modes can appear through a dimensional reduction.

1 Introduction

The existence of the standard model fields leads to the following basic questions, concerning the structure of the model. Why is the gauge group $SU(3) \times SU(2) \times U(1)$? Why exist three families of quarks and leptons? What is the origin of the weak scale? It is hard to answer them completely without some powerful guiding principles and/or a more fundamental theory. What we can do at present is to simplify the questions and to find possible solutions based on some conjectures.

We adopt the fantastic idea that our world comes into existence from “nothing”, as a conjecture [1]. Nothing here means an empty world whose constituents are only unphysical objects. Based on it, we speculate that local fields emerge from unobservable ones by unknown mechanisms. Our question is how physical fields come from the world with only a vacuum state as the physical state.

In this paper, we study a mechanism to create physical fields from unobservable ones, in the expectation that a useful hint on the origin of our world is provided. We present a basic idea and a toy model defined on a higher-dimensional space-time. The model has fermionic symmetries, and higher-dimensional fields form non-singlets under their transformations and become unphysical. We show that some singlets can appear after the dimensional reduction and become physical.

The outline of this paper is as follows. In the next section, we give our basic idea. In section 3, we present a toy model that physical modes come from unobservable ones

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through a dimensional reduction. In the last section, we give conclusions and discussions.

2 Creation of physical modes

Our basic idea is summarized as follows. We assume that the world just after the birth of space-time is effectively described by a theory with unphysical particles on a higher dimensional space-time.

Let the fermionic conserved charges $Q^A_f$ satisfy the algebraic relations,

$$\{Q^A_f, Q^B_f\} = \sum_i f^{ABi} N^i, \quad [N^i, Q^A_f] = i \sum_B f^{iAB} Q^B_f, \quad [N^i, N^j] = i \sum_k f^{ijk} N^k,$$  

(1)

where $\{O_1, O_2\} \equiv O_1 O_2 + O_2 O_1$, $[O_1, O_2] = O_1 O_2 - O_2 O_1$ and $N^i$ are some bosonic conserved charges. $f^{ABi}$, $f^{iAB}$ and $f^{ijk}$ are the structure constants that satisfy the relations,

$$\sum_i \left( f^{ABi} f^{iCD} + f^{BCi} f^{iAD} + f^{CAi} f^{iBD} \right) = 0,$$  

(2)

$$\sum_j f^{ABj} f^{jik} + \sum_C \left( f^{iBC} f^{CAk} + f^{iAC} f^{CBk} \right) = 0,$$  

(3)

$$\sum_k f^{ijk} f^{kAC} - \sum_B \left( f^{jAB} f^{iBC} + f^{iAB} f^{jBC} \right) = 0,$$  

(4)

$$\sum_l \left( f^{ijl} f^{lkm} + f^{jkl} f^{lim} + f^{kil} f^{ljm} \right) = 0,$$  

(5)

from the Jacobi identities,

$$[[Q^A_f, Q^B_f], Q^C_f] + [[Q^B_f, Q^C_f], Q^A_f] + [[Q^C_f, Q^A_f], Q^B_f] = 0,$$  

(6)

$$[[Q^A_f, Q^B_f], N^i] - [[Q^B_f, N^i], Q^A_f] + [[N^i, Q^A_f], Q^B_f] = 0,$$  

(7)

$$[[N^i, Q^A_f], Q^B_f] + [[Q^B_f, N^i], N^j] + [[Q^A_f, N^j], N^i] = 0,$$  

(8)

$$[[N^i, N^j], Q^A_f] + [[N^j, Q^A_f], N^i] + [[Q^A_f, N^i], N^j] = 0.$$  

(9)

The system is supposed to be schematically expressed by the Lagrangian density $\mathcal{L}_{(4+n)D}$ or the Hamiltonian density $\mathcal{H}_{(4+n)D}$ such that

$$\mathcal{L}_{(4+n)D} = \delta^{(1)} \mathcal{R}_{(4+n)D}^{(1)} = \delta^{(2)} \mathcal{R}_{(4+n)D}^{(2)} = \cdots = \delta^{(s)} \mathcal{R}_{(4+n)D}^{(s)},$$  

(10)

$$\mathcal{H}_{(4+n)D} = i \{Q_f^{(1)}, \mathcal{R}_{(4+n)D}^{(1)}\} = i \{Q_f^{(2)}, \mathcal{R}_{(4+n)D}^{(2)}\} = \cdots = i \{Q_f^{(s)}, \mathcal{R}_{(4+n)D}^{(s)}\},$$  

(11)

where $\delta_f^{(r)}$ ($r = 1, 2, \cdots, s$) are defined by $\xi_f^{(r)} \mathcal{D}_{(4+n)D} = i \{\xi_f^{(r)}. Q_f^{(r)}, \mathcal{R}_{(4+n)D}\}$ with Grassmann parameters $\xi_f^{(r)}$, and $Q_f^{(r)}$ are linear combinations of $Q^A_f$, i.e., $Q_f^{(r)} \equiv \sum_A a_A^{(r)} Q^A_f$ with some constants $a_A^{(r)}$. $\mathcal{D}_{(4+n)D}$ ($= \mathcal{D}_{(4+n)D}(x, y)$) is an operator on the higher-dimensional space-time, where $x$ and $y$ stand for the coordinates of a 4-dimensional (4D) space-time and an extra space, respectively.
Physical states denoted by $|\text{phys}\rangle$ can be selected by imposing the following conditions on states,

$$Q_f^{(r)}|\text{phys}\rangle = 0 \quad (r = 1, 2, \cdots, s).$$

(12)

The conditions (12) are interpreted as counterparts of the Kugo-Ojima subsidiary condition in the BRST quantization [2, 3, 4].

The system does not evolve because of the relation $\langle \text{phys}|{\mathcal{H}}_{(4+n)D}|\text{phys}\rangle = 0$ derived from (11) and (12). Every field belongs to a member of non-singlets under the fermionic charges $Q_f^{(r)}$, i.e., every particle pairs with its ghost partner that is related to by $Q_f^{(r)}$, and it is unphysical. Then, only the vacuum $|0\rangle$ survives as the physical state.

For the emergence of a physical mode, it is necessary to disappear its ghost partner. Based on the orbifold breaking mechanism that some modes are eliminated by orbifolding the extra space [5], we assume that the structure of space-time changes, the configuration of fields are altered, and then the boundary conditions of fields on the extra space are determined dynamically. After the dimensional reduction to the 4D space-time, the system is schematically expressed by the Lagrangian density $L_{4D}$ or the Hamiltonian density $H_{4D}$ such that, in $s'$ different ways,

$$L_{(4+n)D} \rightarrow L_{4D} = L + \delta_f^{(r')} \mathcal{R}_{4D}^{(r')},$$

(13)

$$H_{(4+n)D} \rightarrow H_{4D} = H + i\{Q_f^{(r')}, \tilde{\mathcal{R}}_{4D}^{(r')}\},$$

(14)

where $\delta_f^{(r')}$ ($r' = 1, 2, \cdots, s'$) are defined by $\zeta^{(r')} \delta_f^{(r')} \mathcal{O}_{4D} = i\{\zeta^{(r')} Q_f^{(r')}, \mathcal{O}_{4D}\}$ with Grassmann parameters $\zeta^{(r')}$ and an operator $\mathcal{O}_{4D}(= \mathcal{O}_{4D}(x))$ defined on the 4D space-time, and $Q_f^{(r')}$ are fermionic charges.

Physical states can be selected by imposing the following conditions on states,

$$Q_f^{(r')}|\text{phys}\rangle = 0 \quad (r' = 1, 2, \cdots, s').$$

(15)

Unless $\mathcal{H}$ is written by an exact form for $Q_f^{(r')}$, $\mathcal{H}$ can contain 4D $Q_{f(4D)}^{(r')}$ singlet fields, i.e., ghost partnerless particles. Then, physical states including them appear in the system.

3 An example

3.1 5-dimensional toy model

We give a toy model defined on a 5-dimensional (5D) flat space-time. Let us begin with the Lagrangian density,

$$L_{5D} = \partial_M \varphi^\dagger \partial^M \varphi - m^2 \varphi^\dagger \varphi + \partial_M c_\varphi^\dagger \partial^M c_\varphi - m^2 c_\varphi^\dagger c_\varphi$$

$$= \sum_{a=1}^{N} \left( \partial_M \varphi^{a\dagger} \partial^M \varphi^a - m^2 \varphi^{a\dagger} \varphi^a + \partial_M c_\varphi^{a\dagger} \partial^M c_\varphi^a - m^2 c_\varphi^{a\dagger} c_\varphi^a \right),$$

(16)
where $M = 0, 1, 2, 3, 5$, $\varphi^a = \varphi^a(x, y)$ are ordinary 5D complex scalar fields yielding the commutation relations, and $c^a_\varphi = c^a_\varphi(x, y)$ are 5D complex scalar fields yielding the anti-commutation relations. Here $x^\mu (\mu = 0, 1, 2, 3)$ and $x^5$ are denoted by $x$ and $y$, respectively. Both $\varphi^a$ and $c^a_\varphi$ form N-plets of $U(N)$.

Based on the formulation with the property that the hermitian conjugate of canonical momentum for a variable is just the canonical momentum for the hermitian conjugate of the variable [6], we define the conjugate momenta of $\varphi$, $\varphi^\dagger$, $c_\varphi$ and $c^\dagger_\varphi$ as

$$
\pi \equiv \left( \frac{\partial \mathcal{L}_{5D}}{\partial \dot{\varphi}} \right)_R = \varphi^\dagger, \quad \pi^\dagger \equiv \left( \frac{\partial \mathcal{L}_{5D}}{\partial \dot{\varphi}^\dagger} \right)_L = \varphi,
$$

$$
\pi_{c_\varphi} \equiv \left( \frac{\partial \mathcal{L}_{5D}}{\partial \dot{c}_\varphi} \right)_R = \dot{c}_\varphi^\dagger, \quad \pi^\dagger_{c_\varphi} \equiv \left( \frac{\partial \mathcal{L}_{5D}}{\partial \dot{c}^\dagger_\varphi} \right)_L = \dot{c}_\varphi,
$$

where $R$ and $L$ stand for the right-differentiation and the left-differentiation, respectively.

By solving the Klein-Gordon equations $\left( \Box + m^2 \right) \varphi = 0$ and $\left( \Box + m^2 \right) c_\varphi = 0$, we obtain the solutions

$$
\varphi(x) = \int \frac{d^4k}{\sqrt{(2\pi)^42k_0}} \left( a(k)e^{-ikx} + b^\dagger(k)e^{ikx} \right),
$$

$$
\varphi^\dagger(x) = \int \frac{d^4k}{\sqrt{(2\pi)^42k_0}} \left( a^\dagger(k)e^{ikx} + b(k)e^{-ikx} \right),
$$

$$
\pi(x) = i \int d^4k \sqrt{\frac{k_0}{2(2\pi)^4}} \left( a^\dagger(k)e^{ikx} - b(k)e^{-ikx} \right),
$$

$$
\pi^\dagger(x) = -i \int d^4k \sqrt{\frac{k_0}{2(2\pi)^4}} \left( a(k)e^{-ikx} - b^\dagger(k)e^{ikx} \right),
$$

$$
c_{\varphi}(x) = \int \frac{d^4k}{\sqrt{(2\pi)^42k_0}} \left( c(k)e^{-ikx} + d^\dagger(k)e^{ikx} \right),
$$

$$
c_{\varphi}^\dagger(x) = \int \frac{d^4k}{\sqrt{(2\pi)^42k_0}} \left( c^\dagger(k)e^{ikx} + d(k)e^{-ikx} \right),
$$

$$
\pi_{c_\varphi}(x) = i \int d^4k \sqrt{\frac{k_0}{2(2\pi)^4}} \left( c(k)e^{ikx} - d^\dagger(k)e^{-ikx} \right),
$$

$$
\pi^\dagger_{c_\varphi}(x) = -i \int d^4k \sqrt{\frac{k_0}{2(2\pi)^4}} \left( c(k)e^{-ikx} - d^\dagger(k)e^{ikx} \right),
$$

where $k_0 = \sqrt{k^2 + m^2}$ and $kx = k^M x_M$.

The system is quantized by regarding variables as operators and imposing the following commutation and anti-commutation relations on the canonical pairs,

$$
\{\varphi^a(x, t), \pi_{a^\dagger}(y, t)\} = i\delta^{a\dagger} \delta^4(x - y), \quad \{\varphi^\dagger(a)(x, t), \pi^a(y, t)\} = i\delta^{aa} \delta^4(x - y),
$$

$$
\{c^a_\varphi(x, t), \pi_{c^\dagger_\varphi}(y, t)\} = i\delta^{a\dagger} \delta^4(x - y), \quad \{c^\dagger_\varphi(x, t), \pi^c_\varphi(y, t)\} = -i\delta^{aa} \delta^4(x - y),
$$

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and otherwise are zero. Or equivalently, for operators \( a(k), a^\dagger(k), b(k), b^\dagger(k), c(k), c^\dagger(k), d(k) \) and \( d^\dagger(k) \), the following commutation relations are imposed on,

\[
\begin{align*}
[a^a(k), a^{a\dagger}(l)] &= \delta^{aa}\delta^4(k-l), & [b^a(k), b^{a\dagger}(l)] &= \delta^{aa}\delta^4(k-l), \\
[c^a(k), c^{a\dagger}(l)] &= \delta^{aa}\delta^4(k-l), & [d^a(k), d^{a\dagger}(l)] &= -\delta^{aa}\delta^4(k-l),
\end{align*}
\]

(29) and otherwise are zero.

The state vectors in the Fock space are constructed by operating creation operators \( a^\dagger(k), b^\dagger(k), c^\dagger(k) \) and \( d^\dagger(k) \) from the vacuum state \( |0\rangle \) that satisfy \( a^a(k)|0\rangle = 0 \), \( b^a(k)|0\rangle = 0 \), \( c^a(k)|0\rangle = 0 \) and \( d^a(k)|0\rangle = 0 \). Note that the system contains negative norm states as seen from the relation \( [d^a(k), d^{a\dagger}(l)] = -\delta^{aa}\delta^4(k-l) \). For instance, from the relation,

\[
\int d^4k |f^a(k)|^2 = -\int d^4k \int d^4l f^{a\ast}(k)f^a(l)|0\rangle\langle d^a(k), d^{a\dagger}(l)|0\rangle = -\int d^4k f^a(k)d^{a\dagger}(k)|0\rangle^2,
\]

(31)

we find that the state \( \int d^4k f^a(k)d^{a\dagger}(k)|0\rangle \) has a negative norm. Here, \( f^a(k) \) are some square integrable functions.

The \( \mathcal{L}_{5D} \) is invariant under the \( U(N) \rangle \) \( SU(N) \times U(1) \) transformation,

\[
\begin{align*}
\delta^a \varphi &= ie^aT^a \varphi, & \delta^a \varphi^\dagger &= -ie^a \varphi^\dagger T^a, & \delta^a \varphi = ie^a T^a c_\varphi, & \delta^a \varphi^\dagger = -ie^a c_\varphi^\dagger T^a, \\
\delta \varphi &= ie \varphi, & \delta \varphi^\dagger &= -ie \varphi^\dagger, & \delta c_\varphi = ie c_\varphi, & \delta c_\varphi^\dagger = -ie c_\varphi^\dagger
\end{align*}
\]

(32) and the fermionic transformations,

\[
\begin{align*}
\delta_F \varphi &= -\zeta^a T^a c_\varphi, & \delta_F \varphi^\dagger &= 0, & \delta_F c_\varphi = 0, & \delta_F c_\varphi^\dagger &= \zeta^a \varphi^\dagger T^a, \\
\delta_F \varphi &= -\zeta c_\varphi, & \delta_F \varphi^\dagger &= 0, & \delta_F c_\varphi = 0, & \delta_F c_\varphi^\dagger &= \zeta \varphi^\dagger, \\
\delta_F^\dagger \varphi &= 0, & \delta_F^\dagger \varphi^\dagger &= \zeta^\dagger c_\varphi^\dagger T^a, & \delta_F^\dagger c_\varphi = \zeta^\dagger T^a \varphi, & \delta_F^\dagger c_\varphi^\dagger = 0, \\
\delta_F^\dagger \varphi &= 0, & \delta_F^\dagger \varphi^\dagger &= \zeta^\dagger c_\varphi, & \delta_F^\dagger c_\varphi = \zeta^\dagger \varphi, & \delta_F^\dagger c_\varphi^\dagger = 0,
\end{align*}
\]

(33) where \( \epsilon^a (a = 1,2,\cdots,N^2-1) \) and \( \epsilon \) are infinitesimal real parameters, and \( \zeta^a \) and \( \zeta \) are Grassmann parameters.

The above transformations are generated by the conserved charges as follows,

\[
\begin{align*}
\delta^a \Theta_{5D} &= i[\epsilon^a N^a, \Theta_{5D}], & \delta \Theta_{5D} &= i[\epsilon N^0, \Theta_{5D}], \\
\delta_F^a \Theta_{5D} &= i[\zeta^a Q_F^a, \Theta_{5D}], & \delta_F \Theta_{5D} &= i[\zeta Q_F, \Theta_{5D}], \\
\delta_F^{a\dagger} \Theta_{5D} &= i[\zeta^{a\dagger} Q_F^{a\dagger}, \Theta_{5D}], & \delta_F^{a\dagger} \Theta_{5D} &= i[\zeta^{a\dagger} Q_F, \Theta_{5D}],
\end{align*}
\]

(38)

where \( N^a \) and \( N^0 \) are the \( SU(N) \) and \( U(1) \) conserved hermitian charges, and \( Q_F^a, Q_F, Q_F^{a\dagger} \) are the fermionic conserved charges. Note that \( \delta_F \) is not generated by a hermitian operator, different from the generator of the BRST transformation in systems with first class constraints \[41\] and that of the topological symmetry \[7, 8\].
From \( Q_F^a, Q_F, Q_F^{\dagger} \) and \( Q_F^\dagger \), we can construct the fermionic conserved hermitian charges \( Q_1^a, Q_2^a, Q_1 \) and \( Q_2 \) such that

\[
Q_1^a = Q_F^a + Q_F^{\dagger a}, \quad Q_2^a = i(Q_F^a - Q_F^{\dagger a}), \quad Q_1 = Q_F + Q_F^{\dagger}, \quad Q_2 = i(Q_F - Q_F^{\dagger}).
\]

(41)

The conserved charges satisfy the algebraic relations,

\[
[N^\alpha, N^\beta] = i \sum_{\gamma=1}^{N^2-1} f^{\alpha\beta\gamma} N^\gamma, \quad [N^\alpha, Q_1^\beta] = i \sum_{\gamma=1}^{N^2-1} f^{\alpha\beta\gamma} Q_1^\gamma, \quad [N^\alpha, Q_2^\beta] = i \sum_{\gamma=1}^{N^2-1} f^{\alpha\beta\gamma} Q_2^\gamma,
\]

\[
\{Q_1^a, Q_1^\dagger^a\} = \sum_{\gamma=1}^{N^2-1} f^{ab\gamma} N^\gamma, \quad \{Q_2^a, Q_2^\dagger^a\} = \sum_{\gamma=1}^{N^2-1} f^{ab\gamma} N^\gamma,
\]

\[
\{Q_1^a, Q_2^\dagger^a\} = \frac{1}{N} \delta^{ab} N^0 + \sum_{\gamma=1}^{N^2-1} d^{ab\gamma} N^\gamma, \quad (Q_1)^2 = N^0, \quad (Q_2)^2 = N^0, \quad \{Q_1, Q_2\} = 0,
\]

(42)

where \( f^{ab\gamma} \) and \( d^{ab\gamma} \) are structure constants of the Lie algebra \( su(N) \) that satisfy the relations \( [T^a, T^\beta] = i \sum_{\gamma=1}^{N^2-1} f^{ab\gamma} T^\gamma \) and \( [T^a, T^\beta] = \frac{1}{N} \delta^{ab} I + i \sum_{\gamma=1}^{N^2-1} d^{ab\gamma} T^\gamma \) \((I \text{ is the } N \times N \text{ unit matrix})\), and \( N^0 \) commutes to every charge.

The \( \mathcal{L}_{5D} \) is rewritten as

\[
\mathcal{L}_{5D} = \delta_F \mathcal{R}_{5D} = \delta_F^{\dagger} \mathcal{R}_{5D}^{\dagger} = \delta_F^{\dagger} \mathcal{L}_{5D}^{\dagger} = -\delta_F^{\dagger} \delta_F \mathcal{L}_{5D},
\]

(43)

where \( \delta_F \) and \( \delta_F^{\dagger} \) are defined by \( \delta_F = \zeta \delta_F \) and \( \delta_F^{\dagger} = \zeta^\dagger \delta_F^{\dagger} \), and \( \mathcal{R}_{5D}, \mathcal{R}_{5D}^{\dagger}, \mathcal{L}_{5D} \), and \( \mathcal{L}_{5D}^{\dagger} \) are given by

\[
\mathcal{R}_{5D} = \partial_M c^\dagger_{\phi} \partial^M \phi - m^2 c^\dagger_{\phi} \phi, \quad \mathcal{R}_{5D}^{\dagger} = \partial_M \phi^\dagger \partial^M c^\phi - m^2 \phi^\dagger c^\phi.
\]

(44)

\[
\mathcal{L}_{5D} = \partial_M \phi^\dagger \partial^M \phi - m^2 \phi^\dagger \phi,
\]

(45)

respectively.

The Hamiltonian density \( \mathcal{H}_{5D} \) is written in the \( Q_F \) and \( Q_F^{\dagger} \) exact forms such that

\[
\mathcal{H}_{5D} = i\{Q_F, \mathcal{R}_{5D}\} = -i\{Q_F^{\dagger}, \mathcal{R}_{5D}^{\dagger}\} = \{Q_F, \{Q_F^{\dagger}, \mathcal{H}_{5D}\}\} = -\{Q_F^{\dagger}, \{Q_F, \mathcal{H}_{5D}\}\},
\]

(46)

where \( Q_F, Q_F^{\dagger}, \mathcal{R}_{5D}, \mathcal{R}_{5D}^{\dagger} \), and \( \mathcal{H}_{5D}^{\dagger} \) are given by

\[
Q_F = \int \left(-\pi c^\phi + \phi^\dagger c^\phi \right) d^4 x, \quad Q_F^{\dagger} = \int \left(-c^\phi \pi^\dagger + \pi c^\phi \right) d^4 x,
\]

\[
\mathcal{R}_{5D} = \pi c^\phi \pi^\dagger + \nabla c^\phi \nabla \phi + m^2 c^\phi \phi, \quad \mathcal{R}_{5D}^{\dagger} = \pi^{\dagger} c^\phi + \nabla^\dagger \nabla c^\phi + m^2 \phi^\dagger c^\phi,
\]

\[
\mathcal{H}_{5D}^{\dagger} = \pi^{\dagger} \phi^\dagger + \nabla^\dagger \phi^\dagger \nabla \phi + m^2 \phi^\dagger \phi.
\]

(47)

To formulate our model in a consistent manner, we use a feature that a conserved charge can, in general, be set to be zero as an auxiliary condition. We impose the following subsidiary conditions on states to select physical states,

\[
Q_F|\text{phys}\rangle = 0 \quad \text{and} \quad Q_F^{\dagger}|\text{phys}\rangle = 0.
\]

(50)
Note that $Q_{1}^{|\text{phys}\rangle} = 0$ means $\langle \text{phys}|Q_{F} = 0$, and $N_{0}|\text{phys}\rangle = 0$ is also imposed on from the relation $|Q_{F}, Q_{c}^\dagger \rangle = N_{0}$. We find that all states, except for the vacuum state $|0\rangle$, are unphysical because they do not satisfy (50). This feature is understood as the quartet mechanism [2, 3]. The projection operator $P^{(n)}$ on the states with $n$ particles is given by

$$P^{(n)} = \frac{1}{n} \left( a^{\dagger} P^{(n-1)} a + b^{\dagger} P^{(n-1)} b + c^{\dagger} P^{(n-1)} c - d^{\dagger} P^{(n-1)} d \right) \quad (n \geq 1),$$

and is written by

$$P^{(n)} = i \{ Q_{F}, R^{(n)} \},$$

where $R^{(n)}$ is given by

$$R^{(n)} = \frac{1}{n} \left( c^{\dagger} P^{(n-1)} a + b^{\dagger} P^{(n-1)} b \right) \quad (n \geq 1).$$

We find that any state with $n \geq 1$ is unphysical from the relation $\langle \text{phys}|P^{(n)}|\text{phys}\rangle = 0$ for $n \geq 1$. Then, we understand that both $\varphi$ and $c_{\varphi}$ become unphysical, and only $|0\rangle$ is the physical one. This can be regarded as a field theoretical version of the Parisi-Sourlas mechanism [9].

Using $Q_{1} (\equiv Q_{F} + Q_{c}^\dagger)$ and $Q_{2} (\equiv i(Q_{F} - Q_{c}^\dagger))$, $\mathcal{H}_{5D}$ is rewritten as

$$\mathcal{H}_{5D} = i \{ Q_{1}, \mathcal{R}_{1} \} = i \{ Q_{2}, \mathcal{R}_{2} \},$$

where $\mathcal{R}_{1}$ and $\mathcal{R}_{2}$ are given by

$$\mathcal{R}_{1} = \frac{1}{2} \left( \mathcal{R}_{5D} + \mathcal{R}_{5D}^\dagger \right), \quad \mathcal{R}_{2} = \frac{1}{2i} \left( \mathcal{R}_{5D} - \mathcal{R}_{5D}^\dagger \right),$$

respectively. We can select only the vacuum state as the physical state by imposing the following subsidiary conditions on states, in place of (50),

$$Q_{1}|\text{phys}\rangle = 0 \quad \text{and} \quad Q_{2}|\text{phys}\rangle = 0.$$  

### 3.2 Dimensional reduction

We show that singlets under fermionic transformations can appear through a dimensional reduction and become physical modes.

We assume that the structure of space-time changes into $M^{4} \times S^{1}/Z_{2}$. Here, $M^{4}$ is the 4D Minkowski space-time and $S^{1}/Z_{2}$ is the 1-dimensional (1D) orbifold obtained by dividing a circle $S^{1}$ with the $Z_{2}$ reflection of 5-th coordinate such as $y \rightarrow -y$ or $y \rightarrow 2\pi R - y$ ($R$ is the radius of $S^{1}$).

If we require that the Lagrangian density should be single-valued on $M^{4} \times S^{1}/Z_{2}$, the following boundary conditions are allowed,

$$\varphi(x, -y) = \eta^{0} P_{0} \varphi(x, y), \quad \varphi(x, 2\pi R - y) = \eta^{1} P_{1} \varphi(x, y),$$

$$c_{\varphi}(x, -y) = \eta^{0} P_{0} c_{\varphi}(x, y), \quad c_{\varphi}(x, 2\pi R - y) = \eta^{1} P_{1} c_{\varphi}(x, y),$$

where

$$\eta^{0} = \begin{cases} 1 & \text{for} \quad x \in [0, R), \\ -1 & \text{for} \quad x \in [R, 2R). \end{cases}$$

$$\eta^{1} = \begin{cases} 1 & \text{for} \quad x \in [0, \pi), \\ -1 & \text{for} \quad x \in [\pi, 2\pi). \end{cases}$$

...
where $\eta^0$ and $\eta^1$ are intrinsic $Z_2$ parities of $\varphi$, $\eta^0_c$ and $\eta^1_c$ are intrinsic $Z_2$ parities of $c_\varphi$, and $P_0$ and $P_1$ are $N \times N$ matrices that satisfy $(P_0)^2 = I$ and $(P_1)^2 = I$, respectively.

We assume that the boundary conditions are determined by an unknown mechanism, and take $(\eta^0, \eta^1) = (1, 1)$, $(\eta^0_c, \eta^1_c) = (-1, -1)$, and

\[
P_0 = \text{diag}(1, \cdots, 1), \quad P_1 = \text{diag}(1, \cdots, -1, \cdots, -1).
\]

Then, $\varphi$ and $c_\varphi$ are given by the Fourier expansions,

\[
\varphi^{a_+}(x, y) = \frac{1}{\sqrt{\pi R}} \varphi_0^{a_+}(x) + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \varphi_n^{a_+}(x) \cos \left( \frac{n}{R} y \right),
\]

\[
\varphi^{a_-}(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \varphi_n^{a_-}(x) \cos \left( \frac{n}{R} y \right),
\]

\[
c^{a_+}_\varphi(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} c_n^{a_+}(x) \sin \left( \frac{n}{R} y \right),
\]

\[
c^{a_-}_\varphi(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} c_n^{a_-}(x) \sin \left( \frac{n}{R} y \right),
\]

where $\varphi_0^{a_+}(x), \varphi_n^{a_+}(x)$ and $c_n^{a_+}(x)$ ($a_+ = 1, \cdots, k, a_- = k + 1, \cdots, N, n = 1, 2, \cdots$) are 4D fields.

After inserting the expansions (60 – 63) into (16) and integrating the 5-th coordinate, we obtain the 4D Lagrangian density,

\[
\mathcal{L}_{4\text{D}} = \sum_{a_+ = 1}^{k} \left( \partial_\mu \varphi_0^{a_+ T} \partial^\mu \varphi_0^{a_+} - m^2 \varphi_0^{a_+ T} \varphi_0^{a_+} \right) + \sum_{a_+ = 1}^{k} \sum_{n=1}^{\infty} \left[ \partial_\mu \varphi_n^{a_+ T} \partial^\mu \varphi_n^{a_+} - \left( m^2 + \frac{n^2}{R} \right) \varphi_n^{a_+ T} \varphi_n^{a_+} \right] + \partial_\mu c_n^{a_+ T} \partial^\mu c_n^{a_+} - \left( m^2 + \frac{n^2}{R} \right) c_n^{a_+ T} c_n^{a_+} \right] + \sum_{a_- = k+1}^{N} \sum_{n=1}^{\infty} \left[ \partial_\mu \varphi_n^{a_+ T} \partial^\mu \varphi_n^{a_-} - \left( m^2 + \frac{n^2}{R} \right) \varphi_n^{a_+ T} \varphi_n^{a_-} \right] + \partial_\mu c_n^{a_- T} \partial^\mu c_n^{a_-} - \left( m^2 + \frac{n^2}{R} \right) c_n^{a_- T} c_n^{a_-} \right].
\]

The $\mathcal{L}_{4\text{D}}$ is invariant under the $SU(k) \times SU(N-k) \times U(1)$ transformation,

\[
\delta^{\hat{a}} \varphi_0 = i \epsilon^{\hat{a}} T^\hat{a} \varphi_0, \quad \delta^{\hat{a}} \varphi_0^\dagger = -i \epsilon^{\hat{a}} \varphi_0^\dagger T^\hat{a}, \quad \delta \varphi_0 = i \epsilon \varphi_0, \quad \delta \varphi_0^\dagger = -i \epsilon \varphi_0^\dagger,
\]

\[
\delta^{\hat{a}} \varphi_n = i \epsilon^{\hat{a}} T^\hat{a} \varphi_n, \quad \delta^{\hat{a}} \varphi_n^\dagger = -i \epsilon^{\hat{a}} \varphi_n^\dagger T^\hat{a}, \quad \delta \varphi_n = i \epsilon \varphi_n, \quad \delta \varphi_n^\dagger = -i \epsilon \varphi_n^\dagger, \quad \delta \varphi_n = i \epsilon \varphi_n, \quad \delta \varphi_n^\dagger = -i \epsilon \varphi_n^\dagger, \quad \delta \varphi_n = i \epsilon \varphi_n, \quad \delta \varphi_n^\dagger = -i \epsilon \varphi_n^\dagger,
\]

where $\epsilon^{\hat{a}}$ ($\hat{a} = 1, 2, \cdots, k^2 + (N-k)^2 - 2$) and $\epsilon$ are infinitesimal real parameters, $T^\hat{a}$ are the elements of Lie algebra concerning $SU(k) \times SU(N-k)$, and $\varphi_0, \varphi_n$ and $c_n$ stand for multiplets $\varphi_0^{a_+}, \varphi_n^{a_+}$ and $c_n^a$ ($a = 1, 2, \cdots, n$), respectively.
The $\mathcal{L}_{4D}$ is also invariant under the fermionic transformations,

$$
\delta^\dagger_{\hat{c}} \varphi_n = -\zeta^\dagger c_n, \quad \delta_{\hat{F}}^\dagger \varphi_n = 0, \quad \delta^\dagger_{\hat{c}} c_n = 0, \quad \delta_{\hat{F}}^\dagger c_n = \zeta^\dagger \varphi_n, \quad (68)
$$

$$
\delta^\dagger_{\hat{c}} \varphi_n = -\zeta c_n, \quad \delta_{\hat{F}}^\dagger \varphi_n = 0, \quad \delta^\dagger_{\hat{c}} c_n = 0, \quad \delta_{\hat{F}}^\dagger c_n = \zeta \varphi_n, \quad (69)
$$

$$
\delta^\dagger_{\hat{c}} \varphi_n = 0, \quad \delta_{\hat{F}}^\dagger \varphi_n = \zeta^\dagger c_n T^\dagger, \quad \delta^\dagger_{\hat{c}} c_n = \zeta^\dagger T^\dagger \varphi_n, \quad \delta_{\hat{F}}^\dagger c_n = 0, \quad (70)
$$

$$
\delta^\dagger_{\hat{c}} \varphi_n = 0, \quad \delta_{\hat{F}}^\dagger \varphi_n = \zeta^\dagger c_n, \quad \delta^\dagger_{\hat{c}} c_n = \zeta^\dagger \varphi_n, \quad \delta_{\hat{F}}^\dagger c_n = 0, \quad (71)
$$

where $\zeta^\dagger$ and $\zeta$ are Grassmann parameters.

The $\mathcal{L}_{4D}$ is rewritten by

$$
\mathcal{L}_{4D} = \sum_{a_s=1}^{k} \sum_{n=1}^{\infty} \left[ \partial_{\mu} c_n^a \partial^{\mu} \varphi_n^a - \left( m^2 + \left( \frac{n}{R} \right)^2 \right) c_n^a \varphi_n^a \right] + \mathcal{L}_{KK},
$$

$$
\mathcal{L}_{KK} = \delta_{\hat{F}}^\dagger \mathcal{R}_{4D} = \delta_{\hat{F}}^\dagger \mathcal{R}^\dagger_{4D} = \delta_{\hat{F}}^\dagger \delta_{\hat{F}}^\dagger \mathcal{L}^\varphi_{4D} = -\delta_{\hat{F}}^\dagger \delta_{\hat{F}}^\dagger \mathcal{L}^\varphi_{4D},
$$

(72)

where $\delta_{\hat{F}}$ and $\delta_{\hat{F}}^\dagger$ are defined by $\delta_{\hat{F}} = \zeta \delta_{\hat{F}}$ and $\delta_{\hat{F}}^\dagger = \zeta^\dagger \delta_{\hat{F}}$, and $\mathcal{R}_{4D}$ and $\mathcal{L}^\varphi_{4D}$ are given by

$$
\mathcal{R}_{4D} = \sum_{a_s=1}^{k} \sum_{n=1}^{\infty} \left[ \partial_{\mu} c_n^a \partial^{\mu} \varphi_n^a - \left( m^2 + \left( \frac{n}{R} \right)^2 \right) c_n^a \varphi_n^a \right] + \mathcal{R}^\dagger_{4D},
$$

$$
\mathcal{L}^\varphi_{4D} = \sum_{a_s=1}^{k} \sum_{n=1}^{\infty} \left[ \partial_{\mu} \varphi_n^a \partial^{\mu} \varphi_n^a - \left( m^2 + \left( \frac{n}{R} \right)^2 \right) \varphi_n^a \varphi_n^a \right] + \mathcal{L}^\dagger_{4D},
$$

(73)

In the similar way, the Hamiltonian density $\mathcal{H}_{4D}$ is written by

$$
\mathcal{H}_{4D} = \sum_{a_s=1}^{k} \left[ \pi_0^a \pi_0^a + \nabla \varphi_0^a \nabla \varphi_0^a + \varphi_0^a \varphi_0^a \right] + \mathcal{H}_{KK},
$$

$$
\mathcal{H}_{KK} = i\{Q_{F(4D)}, \tilde{\mathcal{R}}_{4D}\} = -i\{Q_{F(4D)}^\dagger, \tilde{\mathcal{R}}_{4D}^\dagger\} = \{Q_{F(4D)}^\dagger, \{Q_{F(4D)}, \mathcal{H}^\varphi_{4D}\}\},
$$

(75)

where $Q_{F(4D)}$, $\tilde{\mathcal{R}}_{4D}$ and $\mathcal{H}^\varphi_{4D}$ are given by

$$
Q_{F(4D)} = \int \sum_{a_s=1}^{k} \sum_{n=1}^{\infty} \left[ -\pi_n^a c_n^a + \varphi_n^a \pi_n^a \right] d^3x,
$$

$$
\tilde{\mathcal{R}}_{4D} = \sum_{a_s=1}^{k} \sum_{n=1}^{\infty} \left[ \pi_n^a \pi_n^a + \nabla c_n^a \nabla \varphi_n^a + \left( m^2 + \left( \frac{n}{R} \right)^2 \right) c_n^a \varphi_n^a \right]
$$

$$
+ \sum_{a_s=1}^{k} \sum_{n=1}^{\infty} \left[ \pi_n^a \pi_n^a + \nabla c_n^a \nabla \varphi_n^a + \left( m^2 + \left( \frac{n}{R} \right)^2 \right) c_n^a \varphi_n^a \right],
$$

(77)
Here, \( \pi^a_0 = \dot{\phi}_0^a \), \( \pi^a_0 = \dot{\phi}_0^a \), \( \pi_n = \dot{\phi}_n \), \( \pi^a_n = \dot{\phi}_n^a \), \( \pi_n^c = \dot{c}_n^a \), and \( \pi^a_n = \dot{c}_n^a \).

We impose the following subsidiary conditions on states,

\[
Q^\dagger_{\text{F(4D)|phys}} = 0 \quad \text{and} \quad Q_{\text{F(4D)|phys}} = 0 ,
\]

and select physical states. From (68) – (71), Kaluza-Klein (KK) modes \( \phi^a_n(x) \) and \( c^a_n(x) \) (\( n = 1, 2, \cdots \)) form non-singlets of fermionic symmetries and become unphysical. In contrast, \( \phi_0^a(x) \) are singlets of fermionic symmetries and become physical fields. In this way, it is shown that the physical modes \( \phi_0^a(x) \) appear from the unphysical fields \( \phi^a(x, y) \), after the dimensional reduction.

### 4 Conclusions and discussions

We have studied a mechanism to create physical fields from unobservable ones, based on the conjecture that our world comes into existence from nothing. We have proposed the idea that physical modes can appear through the dimensional reduction from unobservable particles on a higher-dimensional space-time, and presented the toy model with fermionic symmetries that make 5D fields unphysical forming non-singlets of those symmetries, and 4D singlet modes come from through the compactification with the orbifold breaking mechanism\(^1\).

There are many subjects left behind to examine the origin of our world.

First one is to explore how the topology change occurs and how boundary conditions are determined. It is interesting to investigate a dynamical determination of the structure of space-time and the pattern of boundary conditions.

Second one is to construct a realistic theory including the standard model as a low-energy theory and a fundamental theory at a higher-energy scale, based on our conjecture and idea. Larger fermionic symmetries would be needed to formulate unphysical theories including gauge bosons and gravitons. It is challenging to construct an interacting model containing our coexisting system as a subsystem, after the example of the gauge fixing term and the Faddeev-Popov ghost term\(^{12}\) in gauge theories\(^{2,3,4}\) and non-gauge model with BRST scalar doublets\(^{13,14}\).

These studies would shed new light on the origin of our space-time and the standard model, and provide us a hint on the structure of ultimate theory.

\(^{1}\) As an attempt different from the standard lore, toy models of fermions have been presented with the feature that a finite number of Kaluza-Klein modes survive without inducing masses after the integration over extra coordinates by imposing the invariance under space-time reflections and a shift relating extra space on the Lagrangian density\(^{10,11}\).
Acknowledgments

The author thanks Prof. T. Kugo for valuable discussions and useful comments. This work was supported in part by scientific grants from the Ministry of Education, Culture, Sports, Science and Technology under Grant Nos. 22540272.

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