Unification of electromagnetic noise and Luttinger liquid via a quantum dot

Karyn Le Hur and Mei-Rong Li

Département de Physique, Université de Sherbrooke, Sherbrooke, Québec, Canada J1K 2R1

(Dated: March 23, 2022)

We investigate the effect of dissipation on a small quantum dot (resonant level) tunnel-coupled to a chiral Luttinger liquid (LL) with the LL parameter $K$. The dissipation stems from the coupling of the dot to an electric environment, being characterized by the resistance $R$, via Coulomb interactions. We show that this problem can be mapped onto a Caldeira-Leggett model where the (ohmic) bath of harmonic oscillators is governed by the effective dissipation strength $\alpha = (2\tilde{K})^{-1}$ with $\tilde{K}^{-1} = K^{-1} + 2R/R_K$ and $R_K = h/e^2$ the quantum of resistance. Experimental consequences are discussed and the limit $K = 1/2^+$ is thoroughly studied at small $R/R_K$ through the spin-boson-fermion model.

PACS numbers: 73.23.Hk, 71.10.Pm, 72.70.+m

A quantum dot can be viewed as a simple artificial atom exhibiting charge quantization, and its charge can now be measured with a very high accuracy with the aid of an electrometer based on a single-electron transistor. The coupling of the quantum dot to a macroscopic reservoir of electrons inevitably produces quantum charge fluctuations on the dot. This generic phenomenon has been vividly investigated theoretically both in the case of a large metallic box with a very dense spectrum and in the opposite limit of a two-level system. The reservoir of electrons may be a two-dimensional (2D) Fermi-liquid lead or a 1D structure [e.g., a fractional quantum Hall edge state (FQHES) or a quantum wire] where interacting electrons form Tomonaga-Luttinger liquid (LL). Some recent endeavors have been accomplished by Cedraschi et al. and by one of us to understand the role of dissipation—coming from the capacitive coupling of the dot to an electric environment with an ohmic resistance—on the charge quantization of a quantum dot, but with the limitation of free electrons in the reservoir lead. In this Letter, we explore dissipation effects on the small quantum dot (resonant level) coupled to a chiral LL (CLL) which has been previously introduced by Furusaki and Matveev. We seek to provide a unified picture of the role of interactions in the one-channel conductor and of the zero-point fluctuations of the electric environment generalizing the case of a single junction. Of interest to us is to understand the nature of the quantum phases emerging through the dissipative mesoscopic structure shown in Fig. 1, and hence to discuss physical implications for the occupation probability on the quantum dot. We highlight that the physics explored here is sufficiently appealing to experimentalists to carry out activities similar to those already existing on superconducting qubits capacitively coupled to lossy transmission lines in GaAs/AlGaAs heterostructures.

The quantum dot of interest in Fig. 1 is small enough such that we can only restrict ourselves to the highest-occupied level. This leads to a two-level system. The gate voltage $V_g$ is fixed such that the two states in which the level is occupied ($|1\rangle$) or not ($|0\rangle$) are almost degenerate. We can thus resort to an orbital spin-1/2 operator, $S$, to describe these two states: $S_z = 1/2$ corresponds to the state $|1\rangle$ and $S_z = -1/2$ to $|0\rangle$; $S^+$ flips the state from $|0\rangle$ to $|1\rangle$, and $S^-$ vice versa. In the presence of electromagnetic noise, the charging Hamiltonian takes the form

$$H_c = \epsilon S_z + (e\tilde{Q}_0/C_g)S_z. \quad (1)$$

The detuning $\epsilon$ depending on the gate voltage $V_g$ is the energy difference that an electron must overcome if it wants to tunnel between the dot and the lead; here we concentrate on the region close to $\epsilon = 0$ (resonant level). The second term in Eq. (1) arises from the extra capacitive coupling between the dot and the gate voltage fluctuations (the quantum noise) $\delta V_g(t) = \tilde{Q}_0/C_g$, with $\tilde{Q}_0$ denoting the charge fluctuation operator on the gate capacitor $C_g$ emerging from the finite impedance $Z(\omega)$ of this term has not been previously considered in Ref. 3. Akin to Refs. 4 and 11, we find appropriate to model the impedance $Z(\omega)$ in a microscopic fashion through a long dissipative transmission line composed of an infinite collection of $LC_0$ oscillators (Fig. 1); assuming $C_t = C_g$, the latter being governed by the Hamilto-
where the charge (fluctuation) operator $\hat{Q}(x)$ and the phase operator $\phi(x)$ obey the commutation relation $[\phi(x), \hat{Q}(y)]/\epsilon = i\delta(x - y)$. According to Ref. [11], $\hat{Q} = \sqrt{2} \int_0^1 dx \cos(\pi x/2) \hat{Q}(x)$. At low frequency $\omega \ll \omega_c = 1/(RC)$ where the resistance $R = \sqrt{L/C}$, the transmission line gives an impedance $Z(\omega) = R/(1 + i\omega/\omega_c) \approx R$.

Now, we allow an electron to tunnel between the resonant level and the reservoir lead, i.e., the CLL. Even though the CLL model is the most natural description of the FQHES [12] the charge of the edge excitations depend sensitively on how contacting is done [13]. Therefore, semi-infinite quantum wires — where the wires are coupled to the level only at the edge $x = 0$ [6, 12] as shown in Fig. 1 — represent a more judicious realization of the CLL for our proposal. We only consider the case of spinless electrons which implies that all the electrons have been completely spin-polarized by applying an external magnetic field. The kinetic part of the CLL reads

$$H_{\text{chiral}} = \frac{v}{4\pi} \int_{-\infty}^{+\infty} \left( \frac{d\phi}{dx} \right)^2 dx,$$

where $v$ is the Fermi velocity and the chiral boson field $\varphi(x)$ obey the commutation relations $\{\varphi(x), \varphi(y)\} = i\pi \delta(x - y)$. The tunneling processes between the CLL and the level can be described by the Hamiltonian

$$H_{\Delta} = (\Delta/\sqrt{2\pi a})(e^{i\varphi(0)/\sqrt{K}}S^+ + \text{H.c.}),$$

where $a$ is a short-distance cutoff, $\Delta$ the tunneling amplitude, $K < 1$ the LL parameter, and we have exploited the bosonized form $\hat{\Psi}(0) = (1/\sqrt{2\pi a}) \exp(i\varphi(0)\sqrt{K})$ of an electron operator $\hat{\Psi}(0)$ at $x = 0$. In the case of the FQHES, $K$ must be clearly identified as the Landau level filling factor [12]. The total Hamiltonian takes the form $H_{\text{tot}} = H_{\text{c}} + H_{\text{noise}} + H_{\text{chiral}} + H_{\Delta}$. We make the unitary transformation $U_t = \exp(iS_z/\epsilon_0)$, where $\hat{\phi}_0 = \sqrt{2} \int_0^1 dx \cos(\pi x/2) \hat{\phi}(x)$ is the conjugate operator to $\hat{Q}(0)/\epsilon$, such that the noise contribution in $H_{\text{c}}$ is completely absorbed in the tunneling part as

$$\hat{H}_{\Delta} = U_t^\dagger H_{\Delta} U_t = (\Delta/\sqrt{2\pi a})(e^{i\varphi_0 e^{i\varphi(0)/\sqrt{K}}S^+ + \text{H.c.}}).
$$

Apparently, the tunneling of an electron between the dot and the CLL must be mediated by excitations in the environmental bosonic modes. It is crucial to bear in mind the large time behavior [16]

$$K(t) = \langle \hat{\phi}_0(t)\hat{\phi}_0(0) \rangle - \langle \hat{\phi}_0^2 \rangle \simeq -2\pi \ln(|\omega_c|t),$$

where $r = R/R_K$ with $R_K = h/e^2 \simeq 25.8k\Omega$ being the quantum of resistance. Let us first establish the renormalization group (RG) equation for the dimensionless tunneling amplitude $\Delta = \Delta/\sqrt{\Lambda}$; $\Lambda = \min(\omega_c, \delta\epsilon)$ is the high-energy cutoff in our model, $\delta\epsilon$ the level spacing on the dot, and we must equate the frequency cutoff of the CLL to $v/a = \Lambda$ (we set $\hbar = k_B = 1$ and $v$ is a dimensionless parameter). Expanding the partition function to second order in $\Delta$ and using $K(t)$ in Eq. 11 give

$$d\Delta/dl = [1 - (2K)^{-1} - r] \Delta,$$

where the RG variable is $l = \ln(\Lambda/T)$ with $T \ll \Lambda$ denoting the temperature. This already allows us to distinguish two different regimes according to the parameter $K$ defined as $1/K = 1/K + 2\pi$. For $K \ll 1/2$, $\Delta$ is an irrelevant perturbation which means that the physics is dominated by a level being weakly coupled to the CLL, and a perturbation theory in $\Delta$ is appropriate. This stands for the “localized” phase where the level is occupied for $\epsilon < 0$ and unoccupied for $\epsilon > 0$, resulting in a jump in the occupation probability $\langle S_z \rangle_\epsilon$ of the level ($\epsilon$ is the electron energy relative to the Fermi energy) at $\epsilon = 0$. When $K \gg 1/2$, the level coupling to the CLL is a relevant perturbation that will lift the degeneracy of the ground state at $\epsilon = 0$ and lead to a continuous function of $\epsilon$ for $\langle S_z \rangle_\epsilon$ [12]. This is the “delocalized” realm where an electron is resonating back and forth between the CLL and the dot. To get a better description of the strong-coupling fixed point for $K \gg 1/2$ as well as to investigate $\langle S_z \rangle_\epsilon$ on a more quantitative level, we derive an effective Caldeira-Leggett (or ohmic spin-boson) theory [12].

Noting that the level orbital spin only couples to the local CLL mode $\varphi(0)$ and to the “local” noise mode $\phi_0$ (along the lines of Ref. 13):

$$S_{\text{loc}} = (T/2\pi) \sum_{\omega_n} |\omega_n| \varphi(\omega_n) \varphi(-\omega_n),$$
From Eq. (9), it becomes transparent that the suppressed noise spectrum in the electric environment at low energy 

\[ \Gamma = \Lambda \tilde{\Delta} \frac{2}{3K^{2/3}} \]

Note that the original Bethe-Ansatz results [22]. The emerging Kondo scale \( \Gamma \) corresponds explicitly to the energy scale at which the coupling \( \Delta \) gets strongly renormalized in Eq. (7): \( \Gamma = \Lambda \Delta^{2/3(3K-1)} \). The Kondo ground state for energies smaller than \( \Gamma \) can be viewed as the complete screening of the orbital spin \( S_z \) in the absence of magnetic field (\( \epsilon = 0 \)) implying \( \langle S_z \rangle_{\epsilon=0} = 0 \); equivalently, the prominent tunneling process smears the quantization of charge on the dot, i.e., makes the occupation probability \( \langle S_z \rangle_{\epsilon=0} \) continuous around the Fermi level, and thus \( \mathcal{J} = 0 \) [shown as the lower stair region in Fig. 2(b)]. This is a clear distinction between the localized and delocalized phases which in principle should be accessible experimentally [2].

The case of \( K = 1/2^{+} \) deserves a special treatment. We still resort to the original Hamiltonian \( H_{\text{tot}} \). For \( r \ll 1 \), the tunneling process in Eq. (4) becomes a marginal operator such that higher order terms in \( \Delta \) will play a role in the RG flow of Eq. (7). This will slightly modify the value of \( R_c \). We resort to the spin-fermion model (SBFM) or equivalently the Bose-Fermi Kondo model [24] to make a zoom into this area denoted by SBFM in Fig. 2(a). We referonize \( H_{\Delta} \) in Eq. (4):

\[ H_{\Delta} = J_{\perp} \left( \psi_{\uparrow}^{\dagger}(0) \psi_{\uparrow}(0) S_{z}^{+} + \text{H.c.} \right), \]

with the dimensionless Kondo coupling \( J_{\perp} = \Delta \frac{2}{3 \pi v / \Lambda} \) and \( \psi_{\uparrow}^{\dagger}(x) \psi_{\uparrow}(x) = \left( \Lambda / (2 \pi v) \right)^{1/2} \exp(i \sqrt{2} \varphi(x)) \). Now, let us rewrite \( H_{\epsilon} \) in Eq. (11) exactly like in Ref. [5]:

\[ H_{\epsilon} = (\epsilon + \sqrt{\Phi}) S_{z} + J_{\perp} S_{z} \left( \psi_{\uparrow}^{\dagger}(0) \psi_{\uparrow}(0) - \psi_{\downarrow}^{\dagger}(0) \psi_{\downarrow}(0) \right), \]

where \( \Phi = e \delta V_{\perp} / \sqrt{r} \) represents the bosonic noise coupled to the level, and to be fully consistent we have included the Ising part \( J_z \) of the Kondo coupling which emerges in the renormalization procedure or which might also be induced by a small Coulomb interaction \( u S_{z} d \varphi(0) / d x \) between electrons in the CLL and the quantum dot. Thus far we have assumed \( u = 0 \), so at the bare level \( J_z = 0 \). According to Refs. [5, 11], we can then predict that the delocalized-localized transition will occur at
a finite but small $R_c$; in the limit of very small $J_\perp$, we rigorously obtain $R_c = R_K J_\perp$. We finally expect a small jump in the value of $R_c$ at $K = 1/2$ as depicted in Fig. 2 (a). A quantitative discussion for larger values of $\Delta$ including a finite Coulomb repulsion $\Delta$ would be addressed elsewhere through a numerical RG approach [22].

The above analysis can also be generalized to the situation where the small dot in Fig. 4 is replaced by a large metallic grain with $\delta e \to 0$. We can assume that such a metallic dot is similar to a small quantum wire governed by a Fermi liquid theory, i.e., with a LL parameter $K_g = 1$; charging effects are again taken into account through Eq. (1) but now $S_1$ must be viewed rigorously as a projective operator acting on the two charge states $Q = 0$ and $Q = 1$. This is indeed a valid description when focusing on local physics around $\phi = 0$. A quantitative discussion for larger values of $\Delta$ is obtained from a numerical RG approach [24].

References [4, 6, 8] allows us to predict a KT phase transition in the Caldeira-Leggett theory extending the previous works of K.L.H. [5].

To summarize succinctly, we have investigated dissipative interactions in the CLL and of the zero-point fluctuations in the electronic environment through the Kondo physics; the derived models for the transmission lines. For a generic case of $C_i \neq C_g$, there is an extra term in the noise Hamiltonian ($1/2C_i - 1/2C_g \delta C_0^2$, and one would thus not reach the Johnson-Nyquist noise level $S_{\text{noise}}$ shown in Eq. 3. On the other hand, the resistance $R$ is the only relevant physical parameter and the transmission lines are purely a phenomenological model for it. Therefore, $R$ leads to the Johnson-Nyquist noise which can be produced from the transmission lines only for $C_i = C_g$.

This assumption is required for the exact diagonalization of the model for the transmission lines. For a generic case of $C_i \neq C_g$, there is an extra term in the noise Hamiltonian ($1/2C_i - 1/2C_g \delta C_0^2$, and one would thus not reach the Johnson-Nyquist noise level $S_{\text{noise}}$ shown in Eq. 3. On the other hand, the resistance $R$ is the only relevant physical parameter and the transmission lines are purely a phenomenological model for it. Therefore, $R$ leads to the Johnson-Nyquist noise which can be produced from the transmission lines only for $C_i = C_g$.

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17. The correction to the partition function to second or-
\[
\delta Z \approx -\Delta^2 \Lambda/(2\pi)^2a \int d\tau_1 \int d\tau_2 K(\tau_1 - \tau_2)(a/(v|\tau_1 - \tau_2|))^{1/K}
\]
where we must equate \(\omega_c = \Lambda = v/a\) and \(\tau_i = it_i \gg 1/\Lambda\). \(\delta Z\) must be independent from the energy cutoff \(\Lambda\), which results in Eq. (7).