Creutz Ratios From Color-Truncated Lattice Configurations

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Abstract

We investigate whether information about Creutz ratios is encoded, separately, in each gluon color component of numerically generated lattice configurations. Working in SU(2) lattice gauge theory in Landau gauge, we set two of the three gluon color components to zero, and compensate for the loss of two-thirds of the fluctuation by simply rescaling the remaining component by a factor of $\sqrt{3}$. Creutz ratios are then computed with this "abelianized" configuration. We find that the Creutz ratios of loops constructed from abelianized links converge to the usual Creutz ratios in the scaling regime.

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It has been shown in a number of studies [1] that if the link configurations of SU(2) lattice gauge theory are transformed to the maximal abelian gauge (in which links are maximally diagonal [2]), and then the link variables are "abelianized" by truncation to a set of diagonal components, that the Creutz ratios computed from these "abelian" links match the Creutz ratios computed from the full link variables quite closely. The motivation of this procedure was to test the monopole confinement mechanism proposed by 't Hooft [3] (for a critical discussion, see ref. [4]).

In this note we address the question of whether the reproduction of Creutz ratios by color-truncated configurations is unique to the maximal abelian gauge, or whether this is a property of other gauges as well. If the latter, it would suggest that information about the QCD string tension is encoded, separately, in each color component of the gluon field, at least in certain gauges.

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We consider Landau gauge fixing in D=4 SU(2) lattice gauge theory; the gauge-fixing condition is that

\[ \sum_{x} \sum_{\mu=1}^{4} \text{Tr}[U_\mu(x) + U_\mu^+(x - e_\mu)] \text{ is maximized} \] (1)

with SU(2) link variables,

\[ U_\mu(x) = a_0 I + i a_k \sigma_k \quad \sum_{k=0}^{3} a_k^2 = 1 \] (2)

and \( \sigma_k \) the Pauli matrices. Having numerically generated and (Landau) gauge-fixed a lattice configuration, by the standard Monte Carlo methods, we might try to "abelianize" the configuration by setting

\[ a_1 = a_2 = 0 \] (3)

and then rescaling \( U \) to restore unitarity. However, such a procedure could hardly be expected to reproduce Creutz ratios, since it basically just throws away most of the link fluctuation. In maximal abelian gauge this is not such a difficulty, because in that gauge it is normally the case that

\[ a_3^2 >> a_1^2 + a_2^2 \] (4)

i.e. most of the fluctuation is in the color 3-direction. In Landau gauge, however,

\[ < a_1^2 >= < a_2^2 >= < a_3^2 > \] (5)

3By a "color component" we are referring to gluon color degrees of freedom in the adjoint representation, i.e. 3 components for SU(2), 8 components for SU(3), etc.
so the truncation (3) is much more severe. To mimic the effect of the truncated degrees of freedom using the remaining degree of freedom, let us just rescale $a_3$ by a constant

$$a'_3 = \sqrt{3}a_3$$

so that

$$< (a'_3)^2 > = < a_1^2 > + < a_2^2 > + < a_3^2 >$$

and then rescale each link variable to restore unitarity. The proposal is therefore to associate, with each link variable $U$, an "abelianized" link variable $U'$ according to the rule

$$U = a_0I + ia_k\sigma^k \implies U' = \frac{a_0I + i\sqrt{3}a_3\sigma^3}{a_0^2 + 3a_3^2}$$

and then calculate Wilson loops constructed from the abelianized variables.

With this prescription, we have calculated Creutz ratios $\chi[I, I]$ for loops built from the abelianized $U'$ links, up to $I = 4$ and $\beta = 2.7$, on a $12^4$ lattice. The lattice was thermalized for 5000 iterations, data for loops up to $3 \times 3$ was taken every every 5th iteration of the next 5000 iterations; data for $\chi(4, 4)$ was taken every 10th iteration of the next 40000 iterations following thermalization. The results, for the Creutz ratios of the abelianized loops (open triangles) compared to the usual Creutz ratios (solid triangles) are shown in Figure 1. The convergence of the abelianized ratios to the usual ratios, at larger $\beta \geq 2.3$, is quite clear.

Since there is nothing special about the 3-color in Landau gauge, we conclude that somehow each gluon color in this gauge has encoded, separately, information about the Creutz ratios. We do not, at this point, interpret this result as either supporting or discrediting any particular confinement mechanism, although it suggests that the maximal abelian gauge is not unique in reproducing Creutz ratios from color truncated configurations. Our data indicates only that, in Landau gauge, the magnetic disorder which must account for an area law falloff of Wilson loops appears to be present in each color separately, and that the full disorder is obtained from the disorder of any single component by a naive rescaling of that component by a factor of $\sqrt{3}$.

It is not hard to understand, for small loops at weak couplings, why the $\sqrt{3}$ rescaling would reproduce the standard Wilson loop values. At weak couplings, in Landau gauge, Wilson loops

$$W(C) = \text{Tr} < P \exp \left[ ig \oint A_\mu \sigma^a A^a_\mu dx^\mu \right] >$$

and then calculate Wilson loops constructed from the abelianized variables.
can be approximated by

\[ W(C) \approx \text{Tr} \exp \left[ -\frac{g^2}{8} \oint dx^\mu \oint dy^\nu < A^a_\mu(x) A^a_\nu(y) > I \right] \]  \hspace{1cm} (10)

and in this approximation

\[ W(C) = \text{Tr} \exp \left[ -\frac{g^2}{8} \oint dx^\mu \oint dy^\nu 3 < A^3_\mu(x) A^3_\nu(y) > I \right] \]

\[ = \text{Tr} \exp \left[ -\frac{g^2}{8} \oint dx^\mu \oint dy^\nu < A^{a_\mu}_a(x) A^{a_\nu}_a(y) > I \right] \]  \hspace{1cm} (11)

where the abelianized field configuration

\[ A^{a_\mu}_a(x) = \begin{cases} \sqrt{3} A^3_\mu(x) & a = 3 \\ 0 & a \neq 3 \end{cases} \]  \hspace{1cm} (12)

corresponds, at weak couplings, to the color truncation rule in eq. (8). Therefore, it would not be surprising to find that, e.g., one-plaquette loops in the scaling region constructed from abelianized links agree with the usual one-plaquette values. The agreement for small loops is simply a consequence of the fact that the gluons are weakly interacting at small scales. The surprise is that the abelianized links, constructed with a naive \( \sqrt{3} \) rescaling, produce \( \chi \)-ratios which are fairly close to the usual values up to the largest (4 \( \times \) 4) loops studied here. At these loop sizes, and, e.g., \( \beta = 2.4 \), the effect of confinement should be manifest, and the gluons are expected to be highly interactive. Even if magnetic disorder is encoded in a single gluon color component, it is not obvious why the naive rescaling produces \( \chi \)-ratios (and therefore string tensions) which are so nearly correct.

Possibly the approximation (10) may have some validity outside perturbation theory (at least in Landau gauge). This would explain why a simple \( \sqrt{3} \) rescaling gives nearly the right answer. It remains, however, to explain why eq. (10), which is reminiscent of one-gluon exchange, has any validity at all in a strongly coupled regime. All we can say, at this point, is that our data is consistent with that approximation.

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References

[1] T. Suzuki and I. Yotsuyanagi, Phys. Rev. D42 (1990) 4257,  
S. Hioki et. al., Phys. Lett. B272 (1991) 326.

[2] A. Kronfeld, M. Laursen, G. Schierholz, and U.-J. Wiese, Nucl. Phys. B 293 (1987) 461.

[3] G. ’t Hooft, Nucl. Phys. B138 (1978) 1.

[4] L. DelDebbio, M. Faber, and J. Greensite, Nucl. Phys. B414 (1994) 594.

Figure Caption

**Fig. 1** Creutz ratios $\chi[I, I]$ vs. $\beta$, for $I = 1 - 4$. Solid triangles are the usual ratios, open triangles are Creutz ratios obtained from color-truncated (“abelianized”) configurations.
Figure 1

Data for various indices:

- $\chi_{1,1}$
- $\chi_{2,2}$
- $\chi_{3,3}$
- $\chi_{4,4}$

Plots showing the relationship between $\beta$ and $\chi$ for different indices, with markers for Normal Lattice and Abelianized Lattice, and a line indicating $\log(4/\beta)$. The plots are on a logarithmic scale for the y-axis.