Analysis of Localized Plasticity Pattern and Ultrasound Parameters

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Abstract: The quantitative regularities of phase macro-wave generation are generalized for the stage of linear work hardening observed for a range of metals and alloys. Elastic-plastic strain invariant is introduced and its nature is discussed. The existence of elastic-plastic strain invariant is due to the entropy production rate in the system by localized plasticity wave generation.

Keywords: ultrasound velocity, plastic deformation, metals.

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1. INTRODUCTION

A series of studies of plastic deformation in solids was carried on; the research results were summarized in the monograph [1]. It has been established that the plastic deformation tends to localize on the macro-scale level in any solid from the yield limit to the failure. The localization phenomenon is manifested as spontaneous layering of the material bulk into non-deformed and actively deforming volumes, which are arranged over the test sample, making up a clearly defined pattern [1]. Each pattern is in close correspondence with the law of deformation hardening acting at the given flow stage. The order of appearance of patterns is as follows: (i) switching auto-waves; (ii) phase auto-waves; (iii) stationary dissipative structures. At the pre-fracture stage collapse of auto-waves would occur [1 - 3].

The emergence of patterns in the deforming medium is evidently due to the self-organization of its defect structure [4]. It is well known that the processes involved in the self-organization of an open medium would result in a decrease in the rate of entropy production [5], which suggests that a detailed description of plastic deformation localization processes can be discussed in terms of entropy production rate [6-8]. This will be discussed and exemplified where necessary in what follows. A more convenient and therefore preferred is the stage of linear deformation hardening where phase auto-waves of localized plasticity propagate at the rate \( V_{aw} \approx 10^5 \ldots 10^{-4} \) m/s [1]; the auto-wave length, \( \lambda \), is of order \( 10^{-2} \) m, no matter what kind of material.

2. ELASTIC-PLASTIC STRAIN INVARIANT

The quantitative analysis was made of the research results [9-12]. It was found that the characteristics of phase auto-waves propagating at the stage of linear deformation hardening, i.e. auto-wave length, \( \lambda \), and auto-wave velocity, \( V_{aw} \), as well as the deforming medium’s characteristics, i.e. interplanar distance, \( \chi \) and transverse wave rate, \( V_t \), enter the following
dimensionless relation:

\[ \frac{\lambda \cdot V_{aw}}{\chi \cdot V_t} \equiv \hat{Z} \approx \frac{2}{3} \pm \frac{1}{4}, \quad (1) \]

Relation (1) has been called elastic-plastic strain invariant. It is based on the original data obtained for a range of metals and alloys listed in the Table (note that the values \( \lambda \) and \( V_{aw} \) are quantitative characteristics of localized plastic deformation, while \( \chi \) and \( V_t \) play the same role for elastic deformation).

Hence, relation (1) indicates that elastic and plastic deformation processes occurring in the medium are interrelated, which provides additional insights into the kinetics of form changing. Its usefulness in application has to be verified by exploring the nature of invariant (1). This problem can be addressed by testing the hypothesis for the normal distribution of the quantity \( \hat{Z} \) for different materials. This hypothesis can be accepted on the condition that linear correlation exists between the values of the variable \( \hat{Z} \) on the one hand and the quantiles of normal distribution on the other hand. To establish a correlation between the above two sets, we shall apply a graphical method [11] for plotting a dependence of values of the variable \( \hat{Z} \) on quantiles of the normal distribution \( Q \), which were tabulated for the probabilities \( i/(n+1) \) (here \( n=16 \) is the number of studied metals and alloys; \( i = 1, 2, 3 \ldots 16 \)). Calculations have been made which show that the correlation coefficient for the quantities \( Q \) and \( \hat{Z} \) is \( \sim 0.6 \), which suggests that the distribution of the invariant \( \hat{Z} \) is close to normal. The spread of values \( \hat{Z} \) is accountable to inaccurate experimental determination of the values \( \lambda \) and \( V_{aw} \) for the phase auto-waves of localized plastic flow.

The following are theoretical considerations of the physical nature of the invariant, which are based on thermodynamic estimates. The plastic deformation is described in terms of transformation of elastic and plastic strain fields, which occurs in a concerted manner in the deforming solid. The process kinetics is determined by the values which enter invariant (1). The distribution of elastic and plastic strain fields occurs at rates \( \sim V_t \) and \( \sim V_{aw} \), respectively; the characteristic spatial scales of the processes are determined by the values \( \chi \) and \( \lambda \). Thus, invariant (1) reflects the intricate reciprocal dependence of processes involved in the redistribution of strains and stresses by the plastic flow: due to stress relaxation, deformation would occur, while a change in the deformation level would initiate rearrangement of the elastic stress field.

3. THE NATURE OF THE ELASTIC-PLASTIC STRAIN INVARIANT

It is demonstrated in [1] that the auto-wave phenomenon appearing in the deforming solid is described by a set of parabolic differential equations of rate, which have been derived for variations in the strains and stresses, i.e. \( \dot{\varepsilon} = f(\varepsilon, \sigma) + D_\varepsilon \varepsilon^\prime \) and \( \dot{\sigma} = g(\varepsilon, \sigma) + D_\sigma \sigma^\prime \), respectively (here \( D_\varepsilon \varepsilon^\prime \) and \( D_\sigma \sigma^\prime \) are ‘diffusion’ type members responsible for the spatial distribution of strains and stresses; \( f(\varepsilon, \sigma) \) and \( g(\varepsilon, \sigma) \) describe local relaxation acts [2]). The coefficients \( D_\varepsilon \) and \( D_\sigma \) have the same dimension as the diffusion coefficient \( L^2 \cdot T^{-1} \). In view of the diffusivity of strain and stress redistributions, there is reason to believe [11] that the flows of
strains and stresses, i.e. \( j_e = \dot{\varepsilon}/A \) and \( j_\sigma = \dot{\sigma}/A \) (here \( A = 1 \) is sample cross-section area) are proportional to both the strain and stress gradients, i.e. \( \nabla \varepsilon \) and \( \nabla \sigma \), respectively; hence, the interdependence of the values \( \sigma \) and \( \varepsilon \) is taken into account, considering that \( \sigma(\varepsilon) \). In view of the fact that the deformation, \( \varepsilon \) is the ‘key’ variable employed for plastic flow description, the following set of equation is obtained:

\[
\begin{align*}
\dot{j}_e & \sim \dot{\varepsilon} = D_{\varepsilon\varepsilon} \nabla \varepsilon + D_{\varepsilon\sigma} \nabla \sigma \\
\dot{j}_\sigma & \sim \dot{\sigma} = D_{\sigma\varepsilon} \nabla \varepsilon + D_{\sigma\sigma} \nabla \sigma.
\end{align*}
\]  

The flows \( j_e \) and \( j_\sigma \) are interdependent. Onsager’s reciprocity principle \([12,13]\) asserts symmetries for the coefficients from the set of (2); hence, the equality \( D_{\varepsilon\varepsilon} = D_{\sigma\sigma} \) holds good.

By the plastic deformation, structural changes will occur; the variation in the production of entropy \( (S) \) in the system changes in concert with the variables \( \varepsilon \) and \( \sigma \). The pace of variation is formally determined by the derivatives \( \partial S/\partial \varepsilon \) and \( \partial S/\partial \sigma \) \([7, 8]\). By analogy with \([10]\), the flows \( j_e \) and \( j_\sigma \) in (2) are proportional to \( \partial S/\partial \varepsilon \) and \( \partial S/\partial \sigma \); hence,

\[
\dot{\varepsilon} = \Gamma_{\varepsilon\varepsilon} \partial S/\partial \varepsilon \quad \text{(3)}
\]

\[
\dot{\sigma} = \Gamma_{\sigma\varepsilon} \partial S/\partial \varepsilon \quad \text{(4)}
\]

respectively. According to Onsager’s reciprocity principle \([13]\), the following equality also holds good for the coefficients of (3) and (4)

\[
\Gamma_{\varepsilon\varepsilon} = \Gamma_{\sigma\varepsilon}. \quad \text{(5)}
\]

From dimensional considerations, it is convenient to write

\[
\Gamma_{\varepsilon\varepsilon} = \hat{T}\ddot{u}/(\lambda V_{av})^2 \quad \text{(6)}
\]

\[
\Gamma_{\sigma\varepsilon} = \hat{T}\ddot{u}/(\chi V)^2, \quad \text{(7)}
\]

where \( \hat{T} \) is the rate of temperature variation and \( \ddot{u} \) is acceleration of the rate. The latter two values may be regarded as local characteristics of the deformed material behavior within the plastic deformation nucleus \([13,14]\). In this case, \( \dot{\varepsilon} = \text{const} \) and \( \ddot{u} = \text{const} \); hence,

\[
\lambda V_{av} \approx \chi V, \quad \text{(8)}
\]

which determines the elastic-plastic strain invariant with precision to the constant \( \hat{Z} \). It is thus believed that the elastic-plastic strain invariant is the consequence of the non-linear connection between strains \( \varepsilon \) and stresses \( \sigma \), which determines the plastic flow kinetics of the deformed medium.

To estimate the magnitude of the value \( \hat{Z} \), equation (1) is written as the product of two dimensionless ratios, i.e.
\[ \frac{\hat{\lambda} \cdot V_{\text{aw}}}{\hat{x} \cdot V_t} = \frac{\hat{\lambda} \cdot V_{\text{aw}}}{\hat{x} \cdot V_t} = \tilde{Z} = \frac{2 \pm 1}{3} < 1. \] (9)

The latter ratios might be assigned a statistical meaning [5]. Thus the quantity \( \frac{\hat{\lambda}}{\hat{x}} > 1 \) is taken to be the ratio of auto-wave scale to the minimal possible (lattice) scale or else to the possible number of sites in the system where localized plasticity waves are generated, while the quantity \( V_{\text{aw}} / V_t < 1 \) is taken to be a measure for choosing auto-wave rate from the range of possible values, i.e. \( 0 \leq V_{\text{aw}} \leq V_t \). In view of the above, the concept of the nature of invariant (1) is formulated as follows. The general variation in the entropy production can be formulated for the deformed system in which auto-waves are generated as the sum of a static (scale) contribution and a kinematic (rate) one, i.e.

\[ \Delta S = \Delta S_{\text{stat}} + \Delta S_{\text{kin}} = \hat{S} < 0, \] (10)

where \( \Delta S < 0 \) expresses a general decrease in the entropy production for the deformed system where localized plasticity waves are generated [5, 15].

Let us evaluate the entropy gain signs (10). In view of the above, the static contribution is estimated using the Boltzmann formula for entropy as follows:

\[ \Delta S_{\text{stat}} = k_B \ln \frac{\hat{\lambda}}{\hat{x}} > 0, \] (11)

since \( \frac{\hat{\lambda}}{\hat{x}} > 1 \) (here \( k_B \) is the Boltzmann constant). The contribution of rate is estimated as

\[ \Delta S_{\text{kin}} = k_B \ln \frac{V_{\text{aw}}}{V_t} < 0, \] (12)

since \( V_{\text{aw}} / V_t < 1 \).

Thus from (10-12) follows that

\[ \ln \frac{\hat{\lambda}}{\hat{x}} + \ln \frac{V_{\text{aw}}}{V_t} = \hat{S} / k_B < 0 \] (13)

and, correspondingly,

\[ \frac{\hat{\lambda}}{\hat{x}} \cdot \frac{V_{\text{aw}}}{V_t} = \exp \left( \frac{\hat{S}}{k_B} \right) < 1. \] (14)

Apparently,

\[ \hat{Z} = \exp \left( \frac{\hat{S}}{k_B} \right) = \frac{2}{3}, \] (15)

hence, \( \hat{S} = k_B \cdot \ln 2/3 \approx -0.4k_B \) as calculated per elementary relaxation act [16].

4 CONCLUSION

The elastic-plastic strain invariant has been established experimentally. The analysis of its
nature suggests that the development of plastic deformation occurs via auto-wave processes of localized plasticity, involving elastic deformation processes. Hence, the both types of deformation are interdependent, which is suggested by the plastic flow diagram $\sigma(\varepsilon)$.

The nature of the elastic-plastic strain invariant is discussed herein. In the given approach, the plastic deformation is regarded as a self-organization process, which occurs in the defect structure of the deformed system and involves a decrease in the production of entropy.

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