Formation of terajet produced by artificial dielectric periodical structures on substrate

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Abstract. This paper presents an investigation of terajets formation by dielectric periodic structure at terahertz frequencies in effective medium regime (photonic metamaterial). The dispersions of effective permittivity for periodic structures formed by plastics (ABS, black) were analytically obtained for both regimes. The study of the interaction of a plane wave and artificial cubic dielectric on a substrate parallelepiped to form a subwavelength terahertz beams (terajet) was performed. The results of numerical simulation of S-parameters of structure are discussed. The subwavelength beam waist of terajets about 0.39λ – 0.41λ were obtained. The quality factor of terajets are discussed. The results may be applied for creation of methods to obtain subwavelength beams in the terahertz frequency range, and devices based on them. These investigations are opening new research lines such as mesoscale focusing devices for different sensing applications.

1. Introduction

Abundant literature is devoted to localization electromagnetic fields using photonic jets. 3D dielectric mesoscale particles have emerged as an important working tool of dielectric photonics over the past few years. For example, they are able upon plane-wave illumination (both normal and oblique) to produce sub-diffractive high intensity beams waist, called photonic terajets [1-2]. Terajets have been observed in the shadow region of a number of dielectric mesoscale objects such as spheres, cylinders, cubes, pyramids, axicons, hexahedrons and the like.

Photonic metamaterials (PM) have been a major research field for almost two decades with the main application as a superlens. Generally PM modelled as uniaxial anisotropic media.

One of the key concepts of the physics of artificial composites (or metamaterials) for electromagnetic waves is the possibility to describe their properties by effective parameters derived under the assumption that the structural elements of such a metamaterial are much smaller than the radiation wavelength. Anisotropic metamaterials is one outstanding class of metamaterials. It is
important to notice that its material parameters are not scalars but tensors, with different values of the principle components: dielectric function of uniaxial crystals is described by tensor with two different components corresponding to the directions along ($\varepsilon_\parallel$) and across ($\varepsilon_\perp$) the optical axe. Depending on the signs of these components, crystal represents a dielectric media ($\varepsilon_\parallel>0$, $\varepsilon_\perp>0$), metal ($\varepsilon_\parallel<0$, $\varepsilon_\perp<0$) or hyperbolic metamaterial ($\varepsilon_\parallel<0$, $\varepsilon_\perp<0$). So depending on the signs of the real parts of dielectric permittivities, the uniaxial metamaterials can behave as anisotropic dielectrics, hyperbolic materials, or anisotropic metals [3].

One important property of metamaterials is capability to scale their sizes from several millimetres to nanometers. Such property allows to use the well-known effects in any frequency range where the dielectric material properties are the same.

Artificial materials have been of interest in microwave/millimeter-wave and THz areas in recent years due to the distinctive features. The first attempt of artificial materials may date back to the late nineteenth century [4-7].

Recent development of artificially designed (structured) materials, which have shown unique electromagnetic properties, has provided a new horizons to a possibility of the diffraction limit overcoming.

For example, the authors [8] described an optical cylindrical metalens assembled by hexagonally arranged close-contact nanofibers. This lens forms a photonic jet with a full-width at half-maximum (FWHM) waist, 26% smaller than the Abby diffraction limit.

In this paper the dielectric periodic structure is represented by a cuboid (parallelepiped). Choosing this shape has some advantages over the traditional spherical particles with dimensions of a wavelength, for instance, for the spherical structure the maximum intensity of radiation is located inside the sphere, while for cuboid it is focused outside at the same refractive index contrast. The value of amplification (the ratio of maximum field intensity in the terajet to the intensity of incident radiation) for cuboid is greater than for sphere. In other words, dielectric cuboid acts as a lens with the focus in the form of terajet. Finally, manufacturing of cuboids is much easier than spherical particles [9].

2. Motivation

The idea is to take a dielectric cuboid particle, for example as in [1-2], and then realize it using a photonic crystal operating as an effective medium, that is, in the so-called metamaterial regime. This can be done using two dimensional photonic crystal consisting of parallel rods in air host medium, or air cylindrical holes inside dielectric matrix. Both of them can be homogenized and then it is possible to realize the specified cuboid. Such a device would operate as long as the effective medium approximation holds, that is, within the lowest photonic band gap.

It is well known that one method to produce tailored electro magnetic properties is by perforating a homogeneous slab [10-11]. However, precise machining and tight tolerances are required in this approach, particularly for high frequency applications. Sometimes the maximum number of holes is limited to prevent the material from physically cracking.

As it was mentioned above the artificial cuboid may be made of closely spaced dielectric or metallic cylinders or as perforated structure (photonic crystals). The comparison of such artificial cuboid was briefly considered in [12].

In our investigation the nondispersive black ABS plastic cuboid, placed on dielectric substrate, is perforated, with special distribution of cylindrical holes. The required refractive index is realized by drilling holes of appropriate sizes in a planar slab of same dielectric material. We consider all radii of the drill holes being identical and much smaller than the operating wavelength. The period of holes is small enough compared with the wavelength of light. Such subwavelength structures are not resolved by the light (in the sense of the far-field diffraction) and the heterogeneous structure behaves as a homogeneous material [13] with an effective refractive index.

In terms of Bloch waves, this means that all the Bloch modes supported by this periodic structure except the fundamental one are evanescent along the cylinder axis [14].
It has to be noted similar structures have been studied extensively in recent years for various purposes, especially in sub-diffraction imaging for both the near field and the far zone [15-18].

To manufacturing of perforated cuboid we have use three-dimensional (3D) printing technology, which constructs objects as successive layers [19]. It is one-step process and able to generate complex internal structures. It significantly simplifies the manufacture process and reduces material waste. Furthermore, 3D Printing can create perforated structures that are difficult to be realized by machining due to the mechanical strength of material. Compared with perforating a solid material, the design can be easily modified and rapidly prototyped in-house by using low-cost 3D-printing materials, and this is particularly useful for building laboratory prototypes. However, capabilities of dielectric materials are limited by resolution of 3D-printing devices about thirty of micrometers in our case.

Homogenization of periodic structures using analytical formula has been done long ago [20]. In this paper we use the Maxwell-Garnett effective medium theory (EMT), which [21-23] allows obtaining the information on the effective properties of composites [24-25].

Taking into account the depolarization factor is equal to 0.5 in case of electric field perpendicular to the optical axis and 0 in the case of polarization parallel to the axis of perforations, the effective permittivity of uniaxial structure in common case in the quasi-static approximation are given by [24-25].

\[ \varepsilon_{||} = \varepsilon_i + (1 - f)\varepsilon_a, \]
\[ \varepsilon_{\perp} = \frac{\varepsilon_a \varepsilon_i}{(\varepsilon_a + (1 - f)\varepsilon_i)}, \]  

where: \( f \) and \( 1-f \) denote the volume fractions of components 1 and 2, and the subscripts \( || \) and \( \perp \) indicate the cases with electric field polarized parallel and perpendicular to the interfaces of the structure, respectively. It has to be noted that the two extrema in equation (1) are called the Wiener bounds to permittivity [26], which set the absolute bound on all possible values of the effective permittivity of a two-phase composite [24].

3. Results

Let’s consider two different materials (photopolymer and air, respectively) having permittivity \( \varepsilon_a \) and \( \varepsilon_b \), that forming a composite of alternating cylinders with diameter \( d_a \) and distance between them \( d_b \). We consider TE and TM mode which propagate in the XY- and XZ- plane. The effective permittivity of the composite defines as \( \varepsilon_{TE, TM} \).

In approximation when the radii of the drill holes and period much smaller than the operating wavelength this EMT leads to the equations (1) for the TE and TM modes [24, 25], which can be written in the form:

\[ \varepsilon_{TE} = f_a\varepsilon_a + f_b\varepsilon_b, \]
\[ \varepsilon_{TM}^{-1} = f_a\varepsilon_a^{-1} + f_b\varepsilon_b^{-1}, \]  

where \( f_{a,b} = d_{a,b}/d_a + d_b \) is the volume fraction of the phase \( a,b \). It is followed from (2), that the refractive index components of the structure are less than the initial dielectric material refractive index.

It could be noted that although the TE waves have only electric field component perpendicular to the axis of perforations, the TM waves have the \( E \)-field components perpendicular as well as parallel to the axis of perforations. The detailed derivation of dispersion relations formulas can be found in [27].

The dispersions of effective refractive index of perforated structure for TE/TM modes (for photopolymer black ABS) normalized to design frequency are shown in Figure 1.
Figure 1. Normalized dispersion of effective refractive index for TE and TM wave.

From the Figure 1 it is followed that a stronger change in the refractive index is observed for the TM wave. Nevertheless, the changes in effective refractive index for TM wave are: 1.443 – 1.478, and for TE wave: 1.451 – 1.459. This photonic terajet structure will be very weakly dependent on polarization of incident wavefront because of symmetry structure of perforation at normal illumination. Moreover, at the intersection point of the dispersion curves for TM and TE waves, this artificial material will be isotropic.

Then, let us consider the scattering of a wave by an artificial cubic particle as a whole. In practice, the most commonly quoted parameters are $S_{11}$ which represents the reflection coefficient or return loss and $S_{21}$ the meaning of transmission coefficient.

It has to be noted that these parameters are used also for determining the effective refractive index of a composite structure [28-29] - so-called the Nicholsson-Ross-Weir method. This approach allows us to determine the material parameters of the medium from known amplitude reflection ($S_{11}$) and transmission ($S_{21}$) coefficients of S-matrix. Therefore, if the values of $S_{11}$ and $S_{21}$ are found, for example, by numerical simulations, it becomes possible to determine the effective refractive index $n_{\text{eff}}$. For a nonabsorbing medium [30], the expression for the effective refractive index is simplified (only component $S_{21}$ is used).

In this work, we use the numerical calculation based on Finite Integration Technique (FIT) implemented in the software environment of CST Microwave Studio® to determine S-parameters of the structure under consideration and terajet formation. In time domain the numerical effort of FIT increases more slowly with the problem size than other commonly employed methods [31]. In Figure 2 the results of numerical simulation of S-parameters are shown.

Figure 2. The transmission coefficient ($S_{12}$) and the scattering coefficient ($S_{11}$) of the transition in structure under the study.
From the analysis of Figure 2 it is followed that since this is a non-resonant metamaterial and has no conducting components, the flat cuboid based artificial lens have low loss, and can potentially operate over a relatively wide bandwidth.

The changes of the photonic terajet demonstrate its maximum field intensity and quality criterion of the jet \(Q\). The complex characteristic of photonic jet can be given with the aid of the modified so-called “quality criterion \(Q\)”, which combines all relevant jet parameters [32]. Instead of [32] we define \(Q\) as [9]: 
\[ Q = \frac{L_{\text{jet}} I_{\text{max}}}{\min(FWHM_{x,y})} \]
- the PNJ beam length \(L_{\text{jet}}\) is FWHM along z-axis, \(I\) – maximal value of field intensity along the photonic jet. Thus the photonic terajet’s length was calculated as the FWHM of intensity outside the particle (i.e., if the maximum intensity was found inside the particle, the half-maximum was counted from the surface).

It has been shown [33] that for optimal terajet beam parameters attainment is necessary to structure dimensions has to be smaller than the wavelength in a vacuum, the real part of the permittivity of the structure must be close to the permittivity of the background medium, and the imaginary part should be minimal.

![Figure 3. The FWHM (in wavelength) and normalized quality parameter Q for designed terajet.](image)

The results of simulating the main characteristics of the terajets are summarized in the form of complex characteristics and are presented in Figure 3. The minimal beam waist of terajet is 0.39 of wavelength, but this value corresponds to low intensity of a jet and thus \(Q\) factor is not high. At the point of maximal \(Q\) factor (due to maximal field intensity) the beam waist of terajet is about 0.41 of wavelength.

4. Conclusion
Sub-wavelength THz terajet by cuboid dielectric periodic structure at terahertz frequencies were investigated numerically using Finite Integration Technique. The resultant lens (in the form of cuboid) is highly compact, and has a flat profile, hence it may integrate well into larger systems of optical processing of terahertz radiation. The dispersions of effective permittivity for periodic structures formed by plastics (ABS) were analytically obtained using Maxwell-Garnett effective medium theory.

The dispersion of effective refractive index was used to define the effective refractive index of artificial cube which varying from 1.443 – 1.478 for TM wave, and 1.451 – 1.459 for TE wave.

The dependence of the FWHM of the terajet produced by the artificial cubic dielectric of the photopolymer type, wave polarization and location concerning the structure on the dielectric substrate was observed in this investigation. Thus, the FWHM has been demonstrated for TM-polarized wave in photopolymer ABS (black) which is equal to 0.41\(\lambda\).

The minimal beam waist of terajet is 0.39\(\lambda\), but this value corresponds to low intensity of a jet and thus \(Q\) factor is not high. At the point of maximal \(Q\) factor (due to maximal field intensity) the beam waist of terajet is about 0.41\(\lambda\). The choice of a compromise between the quality factor \(Q\) of the terajet
and the minimum resolution (in term of FWHM) is determined by the requirements for the practical use of the terajet.

A numerous potential applications of artificial material aided terajet are in far-field subwavelength imaging, terahertz microscopy, spectroscopy, and terahertz, enhanced nonlinearities, nanoscale wave guiding and strong light confinement, etc.

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