Persisting Neurons

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Most algorithms used in neural networks (NN)-based learning tasks are strongly affected by the choices of initialization. Good initialization can avoid sub-optimal solutions and alleviate saturation during training. However, designing improved initialization strategies is a difficult task and our understanding of good initialization is still very primitive. Here, we propose persistent neurons, a strategy that optimizes the learning trajectory using information from previous converged solutions. More precisely, we let the parameters explore new landscapes by penalizing the model from converging to the previous solutions under the same initialization. Specifically, we show that persistent neurons, under certain data distribution, is able to converge to more optimal solutions while initializations under popular framework find bad local minima. We further demonstrate that persistent neurons helps improve the model’s performance under both good and poor initializations. Moreover, we evaluate full and partial persistent model and show it can be used to boost the performance on a range of NN structures, such as AlexNet and residual neural network. Saturation of activation functions during persistent training is also studied.

1 Introduction

Neural networks (NN)-based architectures become the dominant learning approach for many tasks including image classification, speech recognition. These methods have been applied to many other domains like potential molecules discovery [1][2][3][4]. For achieving state of art performance, deeper neural network architectures are used, starting LeNet-5 to AlexNet and residual neural networks (ResNet) [2][5][6]. Most NN-based algorithms rely on backward propagation [7] to update the parameters in the network. Back-propagating the gradient optimizes a training criterion with respect to a set of parameters $w$. Iterating the training processing aims at finding the function $f(w)$ that minimizes some expected loss. Starting for initial parameter set $w_0$, the position of $w$ is updated every training iteration. The evolution history of $w$ corresponds to a trajectory in the parameter space.

Neural networks have highly non-convex loss surface and the number of local optima and saddle points can grow exponentially as the number of parameters increases [8]. At the same time, as the structure goes deeper, gradient vanishing/exploding exacerbates the learning. These barriers hinder the trajectory converging to the optimal points for the expected loss. A wide array of methods have been developed for improving the trainability of neural networks. Among all these methods, a good initialization is critical to achieve a desired functionality. Good weight initialization constitutes a favourable starting point in parameter space. It also helps overcome the saturation problem and render learning trajectory more effective [9][10]. In most learning tasks, finding a global minimum for small network sizes can be NP-hard [11], and a proper initialization of the weights in a neural network is critical to the final convergence [9][12][13].

Motivation In previous research [9][12], the authors show good initialization leads to better performance. By monitoring the number of dead (or nearly dead) neurons, the authors also show that the model is less saturated with a better initialization. Instead of converging to local minima and stop updating, the parameters are more likely to reach the global optima or lower loss function values. However, recent studies show these initialization methods are not as solid as we think, and the definition of a good initialization is still ambiguous. Although the widely used initialization [9][12] methods show advantages on a range of tasks, there are also results suggest that the method proposed in [12] strikingly fails on a shallow network [14]. A potential explanation is that people designed and compared different...
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Figure 1: Loss surface with two minima.

parameterizations by solely looking at the distribution of activations at the initialization stage instead of analyzing the trajectories of gradient descent (GD) [12][13]. These initialization methods also do not consider different optimization strategies, but in practice, different gradient-based methods can lead to completely different results.

This suggests that we should utilize more information from the trajectories. Suppose the parameters are not initialized at favourable locations, which is very common as our understanding of a proper initialization is very primitive. Can we keep optimizing the trajectory and achieve the same advantages as better initialization by modelling the learning tasks using information from the trajectories? To be specific: instead of engineering at the start of the trajectories, by learning from the end of the trajectories, can we improve the trainability and achieve advantages as follows?

- If the network is born with poor initialization, can we design a method to optimize the trajectory to let the model have the same accuracy as a good initialized situation?
- Is the new trajectory capable of alleviating the gradient vanishing problem and not converging to sub-optimal minima or saddle points as a good initialized situation?

The statement using information from the end of trajectories seems counterintuitive since we are not able to know anything about the destination at the beginning. As most learning tasks boil down to using deterministic descent method to reach particular minima, the results will keep the same if the initialization points remain unchanged. The parameters won’t generate different solutions unless we use different initializations.

Here, we propose persistent neurons to help optimize the learning trajectory. Persistent neurons is an approach for regularizing neural network using past optimization information and thus changes the gradient update during the training. As mentioned earlier, the model’s parameters may converge to saddle points or local minima during gradient-based optimization. In persistent training, the weights start from the same initial point \( w_{ini} \) every time. We conjecture that the converged minima after the first training is not the global optimal point in the non-convex loss surface. To prevent the model converging to the same region, the loss term in the model includes additional penalties on the previous converged parameters recursively. The updated loss function is \( f_n = f_{n-1} + g_{reg}(w_{n-1}) \) where \( f_{n-1} \) is the loss function of last persistent iteration and \( w_{n-1} \) is the converged parameters in the last training, respectively.

2 Persistent training on low dimension

Before empirically verifying the viability in deep neural networks, we start from a two-dimensional model for better visualizing. We use gradient descent to update the parameters. We first define a loss function consists of two parameters: the function is written:

\[
f(w) = f(x, y) = -\exp\left(-\frac{1}{5}((x - 2)^2 + (y + 2)^2)\right) - \frac{3}{2}\exp(-(x + 2)^2 + (y + 1)^2)
\]

(1)

\( f \) has two minima: \( (x_a, y_a) = (2, -2) \) and \( (x_b, y_b) = (-2, -1) \) respectively. The loss surface is shown in Figure[1]. There are two basins of attraction and the initialization of the weights is critical to its convergence. The initial parameter \( w_{ini} \) is set as \((-0.335, 1.4)\), which locates in the middle between two basins of attraction. The gradient descent is:

\[
x_{t+1} = x_t - \eta \nabla f(w)_x \\
y_{t+1} = y_t - \eta \nabla f(w)_y
\]

where \( t \) is the time step, \( \eta \) is the learning rate and \( \nabla f(w)_x, y \) is the gradient along \( x \) and \( y \) directions. Figure[2] shows the path and loss contour in the parameter space: after 50,000 steps gradient descent steps, \( w \) converges to the sub-optimal
minima \((x_a, y_a)\). Now we apply persistent training to let \(w\) discover another basin. We add the previous converged parameters \(w_0 = (x_a, y_a)\) into the iterative loss function. This eventually leads the new parameters getting rid of the attraction from the last converged basin. The updated loss function can be expressed as:

\[
f_1 = f + g_{\text{reg}}(w_0)
\]  

where \(g_{\text{reg}}(w_0, w) = \lambda \times \frac{|w_0^T w|}{||w_0||} \). \(\lambda = 0.1\) is hyperparameter that controls the penalty term on the previous converged weights. Smaller \(\lambda\) requires more persistent training iterations.

Figure 3 shows the loss surface and the optimizing trajectory of \(f_1\). The weights descend from the same initial point \(w_{\text{ini}}\) but converge to different basin (the global optima in this case). This iterative training method can be extended to more complex loss surfaces with more sub-optimal minima. The loss function during the \(n_{th}\) iteration contains the regularization of all previous converged parameters \(w_0, 1, 2, 3, \ldots n-1\). By monitoring the performance on the validating data, the championship parameters can thus be chosen.

### 3 Persistent training on fully connected network

In the last section, we showed that the parameters are able to get rid of the bad minima using persistent training. In the two-dimensional example, the initialization plays a deciding role in determining the final converged parameters. If we re-initialize the weights, for instance, assign the parameters to the left region in Figure 2, then GD is able to find the optima. In this section, we compare the re-initialization and persistent training. We show that the re-initialization under popular
framework (He initialization for ReLU) fails in a simple shallow network and always produces similar results (degenerate functions in the parameter space) while persistent training is able to find different classes of function [12].

**Problem Setting** Consider a three layer network with \( m \in \mathbb{N} \) hidden units each layer. \( x, y \in \mathbb{R} \) and the data is shown in Figure 4. We are interested in finding a function that fits the \( y \) value. The three layer neural network defines a function \((\mathbb{R} \rightarrow \mathbb{R})\):

\[
f_W(x) = W3\sigma(W2\sigma(W1x + b1) + b2) + b3
\]

(3)

Where \( W3 \in \mathbb{R}^{(1,m)} \), \( W2 \in \mathbb{R}^{(m,m)} \) and \( W1 \in \mathbb{R}^{(m,1)} \), \( b1, b2, b3 \) are the bias terms and \( \sigma \) is the ReLU activation function. We train parameters using GD with momentum = 0.9 with mean square loss between the ground truth \( y \) and the predicted value \( f_W(x) \). We first initialize the weight using the method proposed in [12] and train the networks with 50000 steps. The result is shown in Figure 4. We note that the model fails to fit the nonlinear trend and generate a kink (non-differential point).

For generating more reasonable predictors with different converged parameters, we first try re-initialization. During 100 times re-initializations, the frequency of observing the 'kink' in predicted functions is very high. Figure 5 shows the loss history during 100 times re-initializations, which corresponds to the lowest training loss of 0.33.

The failure of most random seeds implies that the initialization itself does not circumvent the parameters converging to sub-optimal solutions. Unlike the two dimensional cases in the previous section, re-initialization fails to improve the model’s capability or to solve the 'kink' problem. These empirical results show that the commonly used initialization

![Figure 5: Loss history during 100 times re-initializations using different random seeds, seed = 14 has the lowest loss.](image)

![Figure 6: NN predictor’s championship performance during re-initializations (corresponds to the lowest loss value model).](image)

**Algorithm 1** Training NN predictor with \( N \) persistent iterations

1: **Input**: Initialization: \( W_{1ini}, W_{2ini}, W_{3ini}, \lambda = 0.01, m = 32 \), persistent iterations: \( N \)
2: **Output**: NN predictor \( f_W(x) \)
3: Train the three layer model \( \text{// learning rate=0.001, 50,000 iterations} \)
4: Save converged flattened parameters \( W1, W2, W3 \) as \( W_{10}, W_{20}, W_{30} \). \( \text{// plain model training} \)
5: \( \mathcal{P} = \mathcal{L}(W_{1ini}, W_{2ini}, W_{3ini}) - \mathcal{L}(W_{10}, W_{20}, W_{30}) \) \( \text{// } \mathcal{L} \text{ is the loss function} \)
6: for \( \text{iteration} = 1, 2, \ldots, N \) do \( \text{// persistent training starts} \)
7: \( \text{persistent-loss} = \sum_{i=0}^{\text{iteration}-1} \left( \frac{||W1_i||^2}{||W1||^2} + \frac{||W2_i||^2}{||W2||^2} + \frac{||W3_i||^2}{||W3||^2} \right) \) \( \text{// persistent penalty} \)
8: \( \mathcal{L}_{\text{persistent}} = \lambda \times \mathcal{P} \times \text{persistent-loss} + \mathcal{L} \)
9: Train \( \mathcal{L}_{\text{persistent}} \) \( \text{// learning rate=0.001, 50,000 iterations} \)
10: Save converged flattened parameters as \( W_{1\text{iteration}}, W_{2\text{iteration}}, W_{3\text{iteration}} \)
11: end for
12: return \( f_W(x) = f(W_{1N}, W_{2N}, W_{3N})(x) \)
strategy does not help training under certain data distribution. Here, we use persistent training as an alternative strategy to solve this problem by utilizing information from previous converged trajectories. Algorithm 1 shows the persistent training pseudocode for this fully connected network. We extract the previous converged parameters and add them as additional penalties to change the trajectories in the parameter space. Figure 7 shows the corresponding loss using persistent training with $\lambda = 0.01$. We train the model for 100 iterations (1 plain model and 99 persistent iterations). Figure 8 shows the championship predictor during persistent training, the championship model has loss value of $0.10$ while the 100 times random initializations’ lowest loss is $0.33$. During persistent training, unlike re-initialization where the ‘kink’ arises repeatedly, we observe that the non-differentiable point no longer exists after few training iterations ($\text{loss} \ll 4.2$). The championship persistent predictor, though parameterized under unfavourable initialization, exhibits nonlinearity and no longer performs linear regressions. Figure 7 shows that after several persistent training iterations, the loss values maintain at a reasonable range instead of further increasing to $\sim 4.2$. This suggests that most persistent NN predictors no longer generate ‘kink’, and parameters in the shallow network avoid converging to bad minima. The results on shallow networks further strengthen the claim that persistent training is intrinsically different from re-initialization, which solely changes the start point of trajectory.

## 4 Full and partial persistent model

In the previous section, we showed that persistent training helps improve the model’s capability while re-initialization fails. This suggests that the method proposed in [12], though widely used, exhibit instability on certain data distribution. The definition of universal initialization is still very unclear to us. In the previous fully connected layer example, the initialization is a poor method. However, in many learning tasks, the method in [12] has proven to be a satisfying approach for initialization. In well-initialized models, can persistent training still improve the models’ performance? In the following, we investigate persistent training with different models, including LeNet-5 to AlexNet and ResNet, under standard initialization.

### Full persistent model

The loss for $n_{th}$ persistent training iteration is:

$$L_n(w) = L(w) + \lambda \cdot |L(w_{ini}) - L(w_0)| \cdot \sum_{k=0}^{n-1} \sum_l |w^T_k w^l| / ||w^l_k||^2$$

(4)

Where $L(w)$ is the network’s loss function and $w \in \mathbb{R}^{N_1 \times 1}$ denotes the weights, $N_1$ is the number of parameters in layer $l$. $w^l$ corresponds to the flattened weights at layer $l$. $w^l_k$ corresponds to the flattened converged weights at layer $l$ after $k_{th}$ persistent training. $|L(w_{ini}) - L(w_0)|$ is an estimation of the loss surface’s depth from it’s start point $w_{ini}$ to the plain model’s final loss and $\lambda > 0$ is hyperparameter that controls the strength of persistent penalties, $w_0$ corresponds to plain model’s converged parameters.

We sample the $w_{ini}$ from two different distributions: initialization 1, 2 are sampled from normal distribution with zero mean and standard deviation 0.05 and 0.2, respectively, standing for a good initialization and bad one. We denote them as $w_{ini1}$ and $w_{ini2}$. We apply persistent training on LeNet-5[5] with ReLU activation for classifying CIFAR-10 dataset. We use Adam optimizer[16] with a batch size of 256, the deterministic option is turned on, and data shuffling is
turned off. The learning rate is 0.001 and the persistent penalty term $\lambda = 0.01$. We iteratively train the network for 20 times (1 plain model and 19 persistent models). There are 50000 training images and 10000 testing images in CIFAR-10, in our experiment, the original test data is randomly split into two datasets (5000 each) for validating and testing.

![Figure 9: Validation and test error of persistent iterations on CIFAR-10(Initialization 1).](image)

![Figure 10: Validation and test error of persistent iterations on CIFAR-10(Initialization 2).](image)

Figure 9 and 10 show the persistent training results. The black and blue solid horizontal lines show the validation and test error for the plain LeNet-5, the dashed lines denote the errors of persistent training. The plain model with initialization 1($w_{\text{ini},1}$) has a validation accuracy of 62.40% with test accuracy 62.84%, while for initialization 2($w_{\text{ini},2}$) the validation accuracy is 57.30% with test accuracy 60.58%. After persistent training on $w_{\text{ini},2}$ for 19 iterations (shown in Figure 10), the championship validation accuracy is boosted from 57.30% to 63.94% with test accuracy 63.82%, surpassing the well-initialized $w_{\text{ini},1}$ scenario. This suggests that persistent training can make up the gap between poorly-born and well-born neurons.

Furthermore, we apply persistent training on well-born neurons. As shown in Figure 9, persistent training also helps improve the model with $w_{\text{ini},2}$ (validation accuracy = 65.00% and test accuracy = 64.76%).

Figure 11 shows the training curves corresponding to the championship persistent model with $w_{\text{ini},1}$ and $w_{\text{ini},2}$. In the plain model regime, the well-initialized model has both lower errors in training and validating (transparent ones). However, as we applied persistent model on $w_{\text{ini},2}$, we can see the championship model (blue ones) has lower validating error than the plain model with $w_{\text{ini},1}$ (red transparent) even though there exists a large gap between their training accuracy. Our observations on persistent training suggest that smaller training error does not always indicate higher population accuracy.

**Partial persistent model** The full weight persistent model leverages all layer’s previous converged weight during persistent training. This section introduces the partial persistent model and applies it to a variety of architectures. The partial persistent model only takes a random layer’s parameters into persistent training. Let $l^*$ be a random layer. The loss for $n$th iteration is:

$$\mathcal{L}_n(w) = \mathcal{L}(w) + \lambda \cdot |\mathcal{L}(w_{\text{ini}}) - \mathcal{L}(w_0)| \cdot \sum_{k=0}^{n-1} \sum_{l} \frac{|w_k^T w_l^s|}{||w_k^s||^2} \delta(l - l^*)$$  \hspace{1cm} (5)$$

We apply partial persistent training on LeNet-5 (batch size=256), AlexNet and ResNet-18 (batch size=128) for 40, 100 and 350 epochs using Adam optimizer [16] with learning rate 0.001. The persistent hyperparameter $\lambda$ is set as 0.001. Default weights initialization scheme in PyTorch [17] is used for all the layers. Figure 13 and 14 show the persistent training results on the three networks. Championship models’ training curves are shown in Figure 15 and 16. Our observations suggest that persistent training helps improve the models’ accuracy. For LeNet-5, the championship persistent model achieves validation accuracy 63.58% and test accuracy 63.40% while for plain model the accuracies are 62.54% and 61.86%; persistent training boosts the validation accuracy from 67.98% to 68.82% and test accuracy from 67.28% to 68.68% on AlexNet; the validation accuracy increases from 91.16% to 91.96% and test accuracy from 90.86% to 91.48% on ResNet-18, the persistent neurons’ achievement on ResNet-18 is a large improvement as ResNet-50, a significantly deeper model, has a test accuracy of 91.14%. Our results also suggest that ResNet, though considered with smooth loss landscape geometry where non-convexities should not be problematic [18] [19], can still converge to sub-optimal regions [19]. The persistent training drives the weights leaving the sub-optimal solutions and
arriving at new locations with improved validation accuracy. Our empirical observations suggest that ResNet's loss landscape can still have different optima.
5 Neuron dynamics in persistent training

To further evaluate the neurons’ behaviour during the training process and understand the persistent mechanism, we study the saturation condition during the training. Previous studies suggest that standardized initialization methods will generate less saturated neurons during training, thus beneficial for the learning tasks\[9]\[12]. Saturation, which corresponds to zero gradients, is an unfavourable phenomenon during updating the parameters\[20]. The saturation of the network is an important feature that can be used as a descriptor of the training process, as well as understand the behaviour of the network itself\[21].

Previous studies suggest that a less saturated model will have better performance. Inspired by this insight, a variety of methods have been developed to alleviate the saturation\[22]\[23]\[24]. For example,\[9]\[12] engineers on the initialization stage and uses careful initialization, BatchNorm shifts the distribution of activations, so the parameters are less likely to get stuck in the saturated regime\[23], different activation functions, such as leaky rectified linear unit (Leaky ReLU), parametric rectified linear unit (PReLU) and randomized leaky rectified linear units (RReLU) are developed in order to maintain the gradient in a stable scale\[25]. As early mentioned, persistent neurons achieves pronounced results on different models by learning from the previous trajectories. To further study whether persistent neurons is comparable to a good initialization which efficiently tackles this saturation problem. A natural way of investigating this is looking at the neurons dynamics during training. Here we employ a 7 layer NN with Tanh activation for studying the saturation behaviour during partial persistent training. The CIFAR dataset uses the same random split for test and validation as previous models. Figure 18 shows the persistent training history where the 5th persistent training corresponds to the championship model.

Figure 19 shows the saturation behaviour during persistent training with hyperbolic tangent function as activation for CIFAR-10 classification with Xavier initialization proposed in \[9]. The neurons become less saturated as the persistent training iteration increases. During our persistent training, the 5th iteration corresponds to the best performance. The performance does not show monotonic behaviour with respect to persistent iterations. Our experiments suggest that a less saturated model does not always imply better accuracy.

Figure 20 shows the activation values normalized histogram of the plain model and persistent championship model. The saturated plain model’s activations distribute mostly at the extremes (asymptotes -1 and 1). The persistent training mitigates the saturation and re-distributes more weights on the linear or near-linear regions where the gradients can flow well. As the number of persistent iteration increases, the parameters are repulsed from all previous converged minima/saddle points. These repulsion forces the model to explore different landscapes, resulting in a different path and less saturated behaviour. In persistent training, the championship model evolves within a certain saturation level, solely eliminating the saturation (increasing the persistent training iteration) does not boost the performance and can even weaken the model.

6 Conclusion

In this paper, we propose persistent neurons. Persistent neurons uses information from the previous training trajectories and regularizes the model to converge to new parameters under the same initialization. We show that the standardized initialization methods, which solely utilize and analyze the start of the optimization trajectories, can fail on certain
data distribution and persistent training helps overcome the problem and converges to more optimal solutions. This is achieved by incorporating additional information from the end of the trajectories, which is typically not leveraged in previous research. We also show that persistent training achieves gains in performance in both well-initialized and poor-initialized condition. Furthermore, we show by utilizing the previous converged parameters’ locations, the partial persistent training boosts the performance on a range of models. We also show that persistent neurons alleviates the saturation problem. Persistent neurons presents a new approach to address some of the main concerns and limitations of persistent training and be easily generalized to more learning tasks.

References

[1] Geoffrey Hinton, Li Deng, Dong Yu, George E Dahl, Abdel-rahman Mohamed, Navdeep Jaitly, Andrew Senior, Vincent Vanhoucke, Patrick Nguyen, Tara N Sainath, et al. Deep neural networks for acoustic modeling in speech recognition: The shared views of four research groups. *IEEE Signal processing magazine*, 29(6):82–97, 2012.
[2] Alex Krizhevsky, Ilya Sutskever, and Geoffrey E Hinton. Imagenet classification with deep convolutional neural networks. In *Advances in neural information processing systems*, pages 1097–1105, 2012.

[3] Junshui Ma, Robert P Sheridan, Andy Liaw, George E Dahl, and Vladimir Svetnik. Deep neural nets as a method for quantitative structure–activity relationships. *Journal of chemical information and modeling*, 55(2):263–274, 2015.

[4] Yann LeCun, Yoshua Bengio, and Geoffrey Hinton. Deep learning. *nature*, 521(7553):436–444, 2015.

[5] Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, 86(11):2278–2324, 1998.

[6] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 770–778, 2016.

[7] David E Rumelhart, Geoffrey E Hinton, and Ronald J Williams. Learning internal representations by error propagation. Technical report, California Univ San Diego La Jolla Inst for Cognitive Science, 1985.

[8] Peter Auer, Mark Herbster, and Manfred KK Warmuth. Exponentially many local minima for single neurons. In *Advances in neural information processing systems*, pages 316–322, 1996.

[9] Xavier Glorot and Yoshua Bengio. Understanding the difficulty of training deep feedforward neural networks. In *Proceedings of the thirteenth international conference on artificial intelligence and statistics*, pages 249–256, 2010.

[10] Dumitru Erhan, Yoshua Bengio, Aaron Courville, Pierre-Antoine Manzagol, Pascal Vincent, and Samy Bengio. Why does unsupervised pre-training help deep learning? *Journal of Machine Learning Research*, 11(Feb):625–660, 2010.

[11] Avrim Blum and Ronald L Rivest. Training a 3-node neural network is np-complete. In *Advances in neural information processing systems*, pages 494–501, 1989.

[12] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Delving deep into rectifiers: Surpassing human-level performance on imagenet classification. In *Proceedings of the IEEE international conference on computer vision*, pages 1026–1034, 2015.

[13] Dmytro Mishkin and Jiri Matas. All you need is a good init. *arXiv preprint arXiv:1511.06422*, 2015.

[14] David Holzmüller and Ingo Steinwart. Training two-layer relu networks with gradient descent is inconsistent. *arXiv preprint arXiv:2002.04861*, 2020.

[15] Ingo Steinwart. A sober look at neural network initializations. *arXiv preprint arXiv:1903.11482*, 2019.

[16] Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*, 2014.

[17] Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, et al. Pytorch: An imperative style, high-performance deep learning library. In *Advances in Neural Information Processing Systems*, pages 8024–8035, 2019.

[18] Felix Draxler, Kambis Veschgini, Manfred Salmhofer, and Fred A Hamprecht. Essentially no barriers in neural network energy landscape. *arXiv preprint arXiv:1803.00885*, 2018.

[19] Hao Li, Zheng Xu, Gavin Taylor, Christoph Studer, and Tom Goldstein. Visualizing the loss landscape of neural nets. In *Advances in Neural Information Processing Systems*, pages 6389–6399, 2018.

[20] Anna Rakitianskaia and Andries Engelbrecht. Measuring saturation in neural networks. In *2015 IEEE Symposium Series on Computational Intelligence*, pages 1423–1430. IEEE, 2015.

[21] Janusz Kolbusz, Pawel Rozycki, Oleksandr Lysenko, and Bogdan M Wilamowski. Neural networks saturation reduction. In *International Conference on Artificial Intelligence and Soft Computing*, pages 108–117. Springer, 2018.

[22] Bing Xu, Ruitong Huang, and Mu Li. Revise saturated activation functions. *arXiv preprint arXiv:1602.05980*, 2016.

[23] Sergey Ioffe and Christian Szegedy. Batch normalization: Accelerating deep network training by reducing internal covariate shift. *arXiv preprint arXiv:1502.03167*, 2015.

[24] Ian Goodfellow, Yoshua Bengio, and Aaron Courville. *Deep learning*. MIT press, 2016.

[25] Bing Xu, Naiyan Wang, Tianqi Chen, and Mu Li. Empirical evaluation of rectified activations in convolutional network. *arXiv preprint arXiv:1505.00853*, 2015.