Information transfer model of natural processes: from the ideal gas law to the distance dependent redshift

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Information theory provides shortcuts which allow to deal with complex systems. The basic idea one uses for this purpose is the maximum entropy principle developed by Jaynes. However, an extensions of this maximum entropy principle to systems far from thermal equilibrium or even to non-physical systems is problematic because it requires an adequate choice of constraints. In this paper we apply the information theory in an even more abstract way and propose an information transfer model of natural processes which requires no choice of adequate constraints. It is, therefore, directly applicable to systems far from thermal equilibrium and to non-physical systems. We demonstrate that the information transfer model yields well known laws, which, as yet, have not been directly related to information theory, such as the radioactive decay law, Fick’s first law and Hubble’s law.

I. INTRODUCTION

It is generally accepted that, from a unifying point of view, information theory provides shortcuts which allow to deal with complex systems [1]. The basic idea one uses for this purpose is the maximum entropy principle developed by Jaynes [2]. We repeat here the summary of the basic ideas as formulated by Haken ([1] p. 33): We start from macro-observables which may fluctuate and whose mean values are known. We distinguish the macro-variables by an index $k$ and denote their mean values by $f_k$. We wish then to make a guess at the probability distribution $p_j$ of the system over states labelled by the index $j$. This is achieved under the maximization of the information $i$

$$i = - \sum_j p_j \ln p_j$$

under the constraint that

$$\sum_j p_j f_j^{(k)} = f_k$$

(2)

Evidently, $f_j^{(k)}$ is the contribution of state $j$ to the macro-variable labelled by $k$. Furthermore we require

$$\sum_j p_j = 1$$

(3)

i.e. that the probability distribution $p_j$ is normalized to unity. As was shown by Jaynes [2], this principle allows us to derive the basic formulae of thermodynamics in a very short and elegant way. For this derivation the constraints refer to the conserved quantities of a closed system, i.e. energy, particle numbers etc. The crux of the problem of extending this maximum entropy principle to systems far from thermal equilibrium or even to non-physical systems lies in the adequate choice of constraints...

This summary shows that an extension of the maximum entropy principle to systems far from thermal equilibrium or to non-physical systems is problematic because it requires the choice of adequate constraints. In this paper we will apply the information theory in an even more abstract way which requires no choice of adequate constraints. For that it is convenient to apply the definition of information which was originally given by Hartley [3]. He described the information $I$ by the number of selections, $n$,

$$I = K_s n$$

(4)

where $K_s = K_0 \ln s$ is a constant which depends on the number of symbols, $s$, available at each selection and $K_0$ defines the unit of information. If $K_0 = 1/\ln 2$ the unit of information is given in bits. Taking into account the definition $i = I/(nK_0)$ and setting $p_j = 1/s = constant$ with $j = 1 \ldots s$ we get equation (4) also from equation (1). That is, Hartley’s equation (4) describes the information if the probability to use a symbol is equal for all symbols.

II. INFORMATION TRANSFER MODEL

It is well known from information theory [4] that any technical process can transfer at best the complete information $I_q$ from an information source $q$ to an information destination $u$. That is, we have for the information $I_u$, which the information destination $u$ receives from the information source $q$, the inequality

$$I_u \leq I_q$$

(5)

Fig. 1 shows a simplified version of Shannon’s diagram of a general communication system (see Fig. 1 in Shannon’s work [2]) where the transfer system comprises a transmitter, a channel, a receiver and a noise source. We
are familiar with information transfer during a technical process: The information source \( q \) could be for example a computer, labelled \( q \), which communicates via a transfer system (e.g. via the world wide web) with a computer, labelled \( u \). One would say in this case that there is information transfer from the computer \( q \) to the computer \( u \).

The simplified diagram in Fig. 1 allows us to include formally also natural processes in the transfer system. To distinguish this approach clearly from mere technical communication systems we will call it more generally information transfer model (of natural processes). We make no detailed assumptions concerning the nature of the process but will restrict the considered processes by condition 1:

**Condition 1** The considered natural process can be sufficiently described by only two independent generic process variables \((q, u)\) and is able to transfer information.

The values of the generic process variables are in the general case not restricted so that we have the condition

\[
-\infty \leq q \leq +\infty \quad \text{and} \quad -\infty \leq u \leq +\infty
\]

Because the two variables are independent we can formally assign them to the information source \( q \) and the information destination \( u \) of the information transfer model (Fig. 1). That is, in the context of the proposed information transfer model we now introduce a new point of view: We say that there is information transfer from the variable \( q \) to the variable \( u \). It is, therefore, convenient to call variable \( q \) the source variable and variable \( u \) the destination variable.

We must now find a definition of information which is directly related to the generic variables of the considered process. Take for example a rod of length \( |\Delta q| \) from which we cut (select) \( n = |\Delta q| / |\delta q| \) pieces (number of length selections, \( n \)) of smaller length \(|\delta q|\). It is obvious that we have the condition \( n \geq 1 \). It is also a well known fact that the length \(|\delta q|\) will become quantized for microscopic values of \(|\delta q|\) so that we have the further condition \(|\delta q| > 0\). That is, we cannot select an infinite number of pieces, \( n \), from a rod of limited length \(|\Delta q| < \infty\). The only possible way to select an infinite number of pieces \((n \to \infty)\) is given if we select pieces of limited length \(|\delta q| < \infty\) from a rod of infinite length \(|\Delta q| \to \infty\). In this special example we considered length selections. Generalizing we now define process variable selections

\[
n_q = |\Delta q| / |\delta q| \geq 1 \quad \text{and} \quad n_u = |\Delta u| / |\delta u| \geq 1
\]

where \( n_q \) is the number of source variable selections and \( n_u \) is the number of destination variable selections. \(|\Delta q|\) and \(|\Delta u|\) are absolute values of the process variables and \(|\delta q|\) and \(|\delta u|\) are absolute values of the process signals. A graphical illustration of symbols and naming conventions to define process variable selections is given in Fig. 2.

The notation “process signal” is motivated by the circumstance that we need a detector to detect these absolute values so that we define the term

\[
\left\langle \frac{|\delta q|}{|\delta u|} \right\rangle_{\text{detector}} \quad \text{or} \quad \left\langle \frac{|\delta u|}{|\delta q|} \right\rangle_{\text{detector}}
\]

and call it detector. As an example consider the detector

\[
p = \left( |\delta W| / |\delta V| \right)_{\text{detector}}
\]

where the detector value is the pressure \( p \) of gas particles in a box of volume \( V \). That is, this detector relates an absolute value of the process source signal, \(|\delta W|\), “emitted” from the process work \( W \) to an absolute value of the process destination signal, \(|\delta V|\), “detected” at the process volume \( V \), and delivers as output value the pressure \( p \). From this example it becomes obvious that a pressure of \( p = 0 \) is contradictory to the considered process, i.e. gas particles in a box at a given temperature, because a pressure of \( p = 0 \) means that there is not any gas particle in the box. To avoid this contradiction we have to exclude \(|\delta W| = 0\) and \(|\delta V| = \infty\) in this special example. Generalising we get the following condition for the absolute values of the process signals

**Condition 3** \( 0 < |\delta q| < \infty \) and \( 0 < |\delta u| < \infty \)
According to Hartley’s definition of information we can now define an information \( I \) which is directly related to the two independent process variables \( q \) and \( u \)

\[
I_q = K^q_s n_q = K^q_s \frac{\Delta q}{|\delta q|}
\]

\[
I_u = K^u_s n_u = K^u_s \frac{\Delta u}{|\delta u|}
\] (8)

Applying inequality (5) we have the general inequality

\[
I_u = K^u_s \frac{|\Delta u|}{|\delta u|} \leq I_q = K^q_s \frac{|\Delta q|}{|\delta q|}
\] (9)

where the constants \( K^q_s \) and \( K^u_s \) are related to the given natural process. Because we make no detailed assumptions about the nature of the process we must consider \( K^q_s \) and \( K^u_s \) as unknown constants in our model. For that we define the dimensionless positive process dependent constant \( \kappa \)

\[
0 < \kappa = \frac{K^u_s}{K^q_s} < \infty
\] (10)

and call it the information transfer index of a process. Using the constant \( \kappa \), inequality (9) becomes

\[
\kappa n_u \leq n_q \text{ or } \kappa \frac{|\Delta u|}{|\delta u|} \leq \frac{|\Delta q|}{|\delta q|}
\] (11)

A. The ideal information transfer process

In the case of ideal information transfer the complete information \( I_q \) from the information source \( q \) is sent to the information destination \( u \) so that we have the condition

**Condition 4** \( I_u = I_q \)

We refer to this condition as ideal information transfer condition. It is now useful to define the corresponding information transfer process.

**Definition 1** An Ideal Information Transfer (IIT) process is a natural process which fulfils condition 4 and the ideal information transfer condition. That is, we have a natural process which enables ideal information transfer \( (I_u = I_q) \) from the source variable \( q \) to the destination variable \( u \).

Because it is forbidden that any process (technical or natural) transfers more information to the destination than the source sends (that is, \( I_u > I_q \) or \( \kappa n_u > n_q \) is forbidden) one can formulate a sufficient condition for a global IIT process.

**Condition 5** If any natural process can be described globally by two independent absolute values of the global process variables \( q_{\text{global}} = |\Delta q| \) and \( u_{\text{global}} = |\Delta u| \), it must become an IIT process according to definition 1 for a sufficiently large absolute value of the global destination variable \( (u_{\text{global}} \to \infty) \) if the absolute value of the global source variable is limited \( (q_{\text{global}} < \infty) \). If a natural process is a global IIT process for a given large absolute value of the global destination variable, \( u_{\text{global}} = |\Delta u|^* \), it stays a global IIT process for \( u_{\text{global}} > u_{\text{global}}^* \).

We will call this condition the global IIT process condition. In an analogous way one can formulate a sufficient condition for a local IIT process.

**Condition 6** We consider any natural process which is described locally by two independent absolute values of the process variables \( (|\Delta q|, |\Delta u|) \). If there is locally a sufficiently large number of destination variable selections \( n_u \) given so that we have \( \kappa n_u >> 1 \), the natural process must become an IIT process according to definition 1 if the absolute value of the source variable becomes locally sufficiently small \( (|\Delta q| \to 0) \). If a natural process is a local IIT process for a given small absolute value of the source variable, \( |\Delta q|^* \), it stays a local IIT process for \( |\Delta u| < |\Delta q|^{*} \).

We will call this condition the local IIT process condition. The justification of conditions 4 and 5 is given in the appendix.

For an IIT process according to definition 1 we get from inequality (11) the following equation

\[
\frac{|\delta q|}{|\delta u|} = \frac{1}{\kappa} \frac{|\Delta q|}{|\Delta u|}
\] (12)

where \( |\Delta q| \) is the absolute value of the source variable, \( |\Delta u| \) is the absolute value of the destination variable and \( \kappa \) the information transfer index. It is useful to call this equation the ideal detector equation (of an IIT process). The notation “detector” is a synonym for any (real or virtual) measurement device which delivers the output value \( (|\delta q| / |\delta u|)_{\text{detector}} \) in an experiment. Because of equation (12) we have to measure the absolute value of only one process variable and can calculate the absolute value of the conjugate process variable by using the output value of the detector device (if the natural process is an IIT process and if we know the information transfer index \( \kappa \)).

B. Ideal transfer laws of the IIT process

We now ask which absolute value of the destination variable we can expect for a given absolute value of the source variable if we consider an IIT process. That is we are looking for a functional relation like

\[
\langle |\Delta u| \rangle = F (|\Delta q|)
\] (13)

where \( \langle |\Delta u| \rangle \) is the expected absolute value of the destination variable and \( |\Delta q| \) is a given absolute value of the information source variable. We will call such a functional relation between the generic process variables...
an *ideal transfer law* if it results from ideal information transfer and is hence related to an IIT process. To derive the function $F$ in equation (13) we postulate that the absolute values of the process signals ($|\delta q|$, $|\delta u|$) be small enough to be considered as infinitesimal increments ($|d(\Delta q)|$, $|d(\Delta u)|$) of the absolute values of the process variables ($|\Delta q|$, $|\Delta u|$). This means that the process variable selections must be sufficiently large.

**Condition 7** $n_q = \frac{|\Delta q|}{|\delta q|} \gg 1$ and $n_u = \frac{|\Delta u|}{|\delta u|} \gg 1$

We will refer to this condition as the *ideal transfer law condition*. The ideal detector equation (12) now becomes

$$\left(\frac{d(\Delta q)}{|d(\Delta u)|}\right)_{\text{detector}} = \frac{|d(\Delta q)|}{|d(\Delta u)|} = \frac{|\delta u|}{|\delta q|} = \frac{1}{\kappa} |\Delta q|$$ (14)

If we integrate equation (14) we get the ideal transfer law we are looking for. Before we do this, it is important to distinguish strictly between so called constant and floating information sources so that we define:

**Definition 2** A constant information source is given if the absolute value of the information source variable is constant, $|\Delta q| = |\Delta q_0|$, in the ideal detector equation (13). If this is not the case a floating information source is given.

For an IIT process with a constant information source according to definition 2 equation (14) becomes

$$\frac{d |\Delta q|}{d |\Delta u|} = \pm \frac{1}{\kappa} \frac{|\Delta q_0|}{|\Delta u|} \quad \text{with} \quad |\Delta q_0| = \text{constant}$$ (15)

If we integrate equation (15) we have

$$\pm \frac{\kappa}{|\Delta q_0|} \int_{|\Delta q_{\text{ref}}|}^{(|\Delta q|)} d |\Delta q'| = \int_{|\Delta u_{\text{ref}}|}^{(|\Delta u|)} \frac{1}{|\Delta u|} d |\Delta u|$$ (16)

where $\langle |\Delta u| \rangle$ is the expected absolute value of the information destination variable and $|\Delta q|$ is a given absolute value of the information source variable. The subscript $\text{ref}$ indicates absolute reference values of the IIT process which are unknown in the context of the information transfer model. The general solution of equation (16) is

$$\langle |\Delta u| \rangle = |\Delta u_{\text{ref}}| \exp \left( \pm \kappa \frac{|\Delta q| - |\Delta q_{\text{ref}}|}{|\Delta q_0|} \right)$$ (17)

We will refer to this equation as the *ideal transfer law of a constant information source*. One can rearrange equation (17) also as follows

$$\langle |\Delta u| \rangle = |\Delta u_0| \exp \left( \pm \kappa \frac{|\Delta q|}{|\Delta q_0|} \right)$$

with $|\Delta u_0| = |\Delta u_{\text{ref}}| \exp \left( \mp \kappa \frac{|\Delta q_{\text{ref}}|}{|\Delta q_0|} \right)$ (18)

where $|\Delta u_0|$ corresponds to $|\Delta q| = 0$.

For an IIT process with a floating information source according to definition 2 equation (14) becomes

$$\frac{d |\Delta q|}{d |\Delta u|} = \pm \frac{1}{\kappa} \frac{|\Delta q|}{|\Delta u|}$$ (19)

If we integrate this equation we have

$$\int_{|\Delta q_{\text{ref}}|}^{(|\Delta q|)} \frac{1}{|\Delta q|} d |\Delta q'| = \int_{|\Delta u_{\text{ref}}|}^{(|\Delta u|)} \frac{1}{|\Delta u|} d |\Delta u|$$ (20)

where $\langle |\Delta u| \rangle$ is the expected absolute value of the information destination variable and $|\Delta q|$ is a given absolute value of the information source variable. Solving the integrals gives the general solution

$$\langle |\Delta u| \rangle^{\pm 1/k} = \frac{1}{R_{\kappa}} \frac{R_{\kappa}}{|\Delta q|}$$

with $R_{\kappa} = \frac{\kappa}{|\Delta u_{\text{ref}}|} \frac{|\Delta q_{\text{ref}}|}{|\Delta q_0|}$ (21)

We will refer to this equation as the *ideal transfer law of a floating information source*. It is convenient to call $R_{\kappa}$ the reference constant (of an IIT process) because it is directly related to the absolute reference values ($|\Delta q_{\text{ref}}|$, $|\Delta u_{\text{ref}}|$) which are unknown in the context of the information transfer model.

### C. Ideal point particles

The defining feature of a point particle is being zero-dimensional. It is an appropriate representation of any physical object whose size, shape, and structure is irrelevant in the given context. Consider for example ideal gas particles in a box of volume $V$. Because ideal gas particles have no volume the expected volume, $\langle V \rangle$, which is related to the ideal gas particles is equal to the box volume, $\langle V \rangle = V$. Generalising we now say that in the context of the information transfer model, ideal point particles obey the condition

**Condition 8** $\langle |\Delta u| \rangle = |\Delta u|$ so that we will refer to it as *ideal point particle condition*. Because of condition 8 one can combine the ideal detector equation (12) and the ideal transfer law (equation (17) or equation (21)) if one considers ideal point particles and if the ideal transfer law condition 7 is fulfilled.

Furthermore, we say that in the context of the information transfer model ideal point particles enable an ideal information transfer and are hence related to an IIT process according to definition 1. The available number of process variable selections $N$, which are the central quantities to express the transferred information, can then directly be related to the available number of ideal point particles, $N_{\text{ideal}}$, so that we have in the case of ideal information transfer, $I_0 = I_q$, the relation

$$\frac{|\Delta u|}{|\delta u|} = \frac{|\Delta q|}{\kappa |\delta q|} = N_{\text{ideal}}$$ (22)
We will refer to this equation as ideal point particle equation and we will in the following discussion always consider a huge number of ideal point particles, \( N_{\text{ideal}} \gg 1 \), so that the ideal transfer law condition is always fulfilled.

### III. GAS PARTICLES IN A BOX

We now demonstrate that one can derive the ideal gas law if one applies the global IIT process condition. For that we consider \( N \) real gas particles in a box. An important quantity to describe the \( N \) real gas particles is the gas pressure, \( p \), which is measured by a pressure detector

\[
p = \left( \frac{|\delta W|}{\delta V} \right)_{\text{detector}}
\]

where \( W \) is the work which contributes to the change of the gas box volume \( V \). This detector defines two independent process variables \((W, V)\). To proceed the discussion we must assume that our considered process, moving real gas particles in a box, fulfills condition. Any real gas can deliver only a limited absolute value of work so that we can consider this work as the global information source of the process \((W = q_{\text{global}} = |\Delta q| < \infty)\). Consequently, we can consider the volume as the global information destination of the process. According to the global IIT process condition the process, moving real gas particles in a box, must become a global IIT process if the absolute value of the global information destination variable, \( V \), is sufficiently large \((V = u_{\text{global}} = |\Delta u| \to \infty)\). For an IIT process our pressure detector will deliver an output value according to the ideal detector equation (setting \( q_{\text{global}} = |\Delta q| = W \) and \( u_{\text{global}} = |\Delta u| = V \))

\[
p = \left( \frac{|\delta W|}{\delta V} \right)_{\text{detector}} = \frac{1}{\kappa} \frac{W}{V}
\]

where the term \( W/\kappa \) can be determined by this equation if we measure \( p \) and \( V \). Condition is a sufficient condition to justify ideal information transfer and, therefore, equation (24). A more descriptive explanation for the ideal information transfer is that, if the volume \( V \) becomes sufficiently large, the average spacing between the \( N \) real gas particles becomes large enough so that the real gas particles can be considered as an ensemble of point particles with no form of interaction (ideal gas particles). Its feature is then to transfer information in an ideal way (condition so that the moving real gas particles in a box, becomes an IIT process according to definition.

For an IIT process we can also apply the ideal point particle equation and get in this case

\[
\frac{V}{|\delta V|} = \frac{1}{\kappa} \frac{W}{|\delta W|} = N \gg 1
\]

where \( N \) is the number of ideal gas particles in a box of volume \( V \). Considering the ideal detector equation of the IIT process we have the relation

\[
pV = \frac{W}{\kappa} = N |\delta W|
\]

and from thermodynamics we know that for an ideal gas

\[
pV = N k_B T
\]

where \( k_B \) is the Boltzmann constant and \( T \) the temperature. Comparing equations (27) and (28) we see that the unknown absolute value of the process (energy) signal, \(|\delta W|\), is given by

\[
|\delta W| = k_B T
\]

and is hence proportional to the temperature \( T \). From equation (26) we can calculate the average absolute work value per gas particle

\[
\bar{W} = \frac{W}{N} = \kappa |\delta W| = \kappa k_B T
\]

and know from thermodynamics that \( \kappa = 3/2 \) for an ideal monatomic gas.

#### A. The isothermal volume change

We consider an isothermal volume change of the ideal gas. That is, our gas box is thermally in good contact to the surroundings so that the (initial) energy \( W \) remains constant \((W = W_0 = \text{constant})\) if we start to change the initial volume \( V \). According to the global IIT process condition the process, moving real gas particles in a box, remains a global IIT process if the absolute value of the global destination variable, \( V \), stays sufficiently large during the isothermal volume change. For an IIT process we can apply the ideal transfer law of a constant information source (setting \(|\Delta q| = W_0, |\Delta u| = V\).

\[
\langle V \rangle = V_{\text{ref}} \exp \left( \frac{\Delta W}{W_0} \right)
\]

with \( \Delta W = \pm (W - W_{\text{ref}}) \)

where \( \langle V \rangle \) is the expected value of the gas volume for a given work, \( \Delta W \), and \( V_{\text{ref}} \) is the reference volume. The ideal transfer law condition is valid because of equation (24), and the ideal gas particles obey the ideal point particle condition so that we have

\[
\langle V \rangle = V
\]

Combining equations (30) and (31) we get

\[
\Delta W = \frac{1}{\kappa} W_0 \ln \left( \frac{V}{V_{\text{ref}}} \right)
\]

The work \( \Delta W \) is zero if there is no change of the volume. However, if the gas (pressure) increases the box volume \((V > V_{\text{ref}})\), we gain mechanical work \( \Delta W > 0 \). And if the box volume decreases \((V < V_{\text{ref}})\), the gas consumes mechanical work \( \Delta W < 0 \).
B. An adiabatic volume change

We now consider an adiabatic volume change of the ideal gas. That is, our gas box is thermally isolated so that the (initial) absolute value of energy, $W$, begins to change (to “float”) if we start to change the initial volume $V$. If we consider only sufficiently large volumes, $V$, the process, moving real gas particles in a box, remains an IIT process during our adiabatic volume change according to condition 5. For an IIT process we can apply the ideal transfer law of a floating information source (using equation (21) and setting $q_{\text{global}} = |\Delta q| = W$ and $u_{\text{global}} = |\Delta u| = V$).

$$\langle V \rangle^{-1/\kappa} = \frac{1}{\kappa} R_\kappa W$$  \hspace{1cm} (33)

where the negative sign indicates that $W$ decreases if $V$ increases. $\langle V \rangle$ is the expected gas volume for a given energy, $W$, which contributes to the volume change and $R_\kappa$ is the reference constant of the IIT process. The ideal transfer law condition 7 is valid because of equation (25) and we can again apply the ideal point particle condition (31) which allows us to combine equations (24) and (33) so that we get a relation between the experimentally accessible quantities $p$ and $V$

$$p \frac{V^{\gamma+1}}{V^{\gamma}} = \frac{1}{R_\kappa} = \text{constant}$$  \hspace{1cm} (34)

This is the well known relation between $p$ and $V$ if an ideal gas undergoes a reversible adiabatic change, where we know from thermodynamics that $\gamma = 5/3$, and thus $\kappa = 3/2$, for an ideal monatomic gas. Notice that we would derive an equivalent relation (34) with the same arguments for polytropic processes.

IV. CONSTANT INFORMATION SOURCES

The example of real gas particles in a box has shown that real gas particles will “transform” to ideal point particles if the gas box volume becomes sufficiently large. It is plausible and descriptive to assume that the reason for this “transformation” is a sufficiently large distance between the real gas particles so that interactions between the gas particles become negligible. However, in the context of the proposed information transfer model such descriptive arguments are not necessary because the global IIT process condition 5 is sufficient to explain this “transformation”. We see that the argument of ideal information transfer is more general than the descriptive argument that the interactions between the gas particles become negligible. In this subsection we will discuss two examples of constant information sources, where we can generally expect an exponential law like equation (17) if the information transfer is ideal. Especially the second example will show that the descriptive argument of negligible interactions between point particles is not necessary to explain ideal information transfer. In such a case the only argument is the global IIT process condition 5 to understand why ideal information transfer occurs.

A. The radioactive decay process of unstable atomic nuclei

We will now discuss the radioactive decay of $N$ unstable atomic nuclei where we know that this decay process can be described by an exponential radioactive decay law

$$\langle N \rangle = N_0 \exp \left(-\frac{|\Delta t|}{\tau}\right)$$  \hspace{1cm} (35)

where $\tau$ is a time constant, $\langle N \rangle$ is the expected number of unstable atomic nuclei at the end of the decay time interval, $|\Delta t|$, and $N_0$ is the number of unstable atomic nuclei at $|\Delta t| = 0$. The unstable atoms can be embedded in a solid medium so that there are interactions with the stable matrix atoms and/or interactions between the unstable atoms. However, it is plausible to assume that physical processes in the nuclei of the unstable atoms can be considered as independent from such interactions. That is, we can idealise the $N$ unstable atomic nuclei as ideal point particles if we consider their radioactive decay only. An important feature of ideal point particles is ideal information transfer so that we can conclude that equation (35) reflects an ideal transfer law of a constant information source. Comparing equation (35) and equation (18) we can obviously define in the context of the information transfer model a constant absolute value of time as the information source, $|\Delta t| = 0 = \kappa \tau = \text{constant}$. Consequently the absolute value of the information destination variable will be the number $N$ of available unstable atomic nuclei ($|\Delta u| = N$).

These were descriptive arguments to see why the radioactive decay law (35) reflects an ideal transfer law (18) of a constant information source. We will now give a more rigorous derivation of equation (35) by means of the information transfer model. Like for the example of the real gas particles we start again with a technical detector device

$$\text{decay rate} = \frac{\langle |\delta N| \rangle_{\text{detector}}}{|\delta t|}$$  \hspace{1cm} (36)

which measures the decay rate of unstable atomic nuclei in this example. Analogously to the pressure detector (24) this detector defines two independent process variables ($t, N$). To proceed the discussion we must assume that our considered process, the radioactive decay of unstable atomic nuclei, fulfils condition 11. According to the global IIT process condition 5 the radioactive decay process must become a global IIT process if the number $N$ of available unstable atoms is sufficiently large. For an IIT process we can apply the ideal detector equation (12) which becomes in this case (setting $|\Delta q| = t_0$ and $|\Delta u| = N$)
\[ \left( \frac{|\delta N|}{|\delta t|} \right)_{\text{detector}} = \frac{\kappa N}{t_0} \quad \text{and} \quad t_0 = \text{constant} \quad (37) \]

and we can apply the ideal point particle equation (22) which becomes in this case

\[ \frac{N}{|\delta N|} = \frac{t_0}{\kappa |\delta t|} = N >> 1 \quad \text{and} \quad t_0 = \text{constant} \quad (38) \]

We see that the process variable selections, \(N/|\delta N|\) or \(t_0/|\delta t|\), are equal or, respectively, proportional to the number \(N\) of available unstable atomic nuclei. The absolute values of process signals become, therefore,

\[ |\delta N| = 1 \quad \text{and} \quad |\delta t| = \frac{t_0}{\kappa N} \quad (39) \]

That is, our detector \[ \delta N \] will detect one process (decay) signal during the absolute value of the process value of the signal \( |\delta t| = t_0/(\kappa N) \). The ideal transfer law condition (7) is valid because of equation (38) so that the ideal transfer law (18) becomes in this case

\[ \langle N \rangle = N_0 \exp \left( -\frac{|\Delta t|}{\tau} \right) \quad \text{with} \quad \tau = \frac{t_0}{\kappa} \quad (40) \]

which is the decay law (35) of unstable atomic nuclei. The constant absolute value of the information source, \( t_0 \), and the information transfer index \( \kappa \) are unknown constants for the decay process but are related to the measurable time constant \( \tau \). For the measurement of the time constant \( \tau \) we can apply the ideal point particle condition (8) \( \langle N \rangle = N \), so that we can combine the ideal detector equation (37) and the ideal transfer law (40)

\[ \text{decay rate} = \left( \frac{|\delta N|}{|\delta t|} \right)_{\text{detector}} = \frac{N_0}{\tau} \exp \left( -\frac{|\Delta t|}{\tau} \right) \quad (41) \]

This relation between detector and transfer law allows us to determine experimentally the time constant \( \tau \) of unstable atomic nuclei by a technical realisation of the detector and by our laboratory clock which measures \( |\Delta t| \).

### B. The isothermal vacancy formation process in a crystalline solid

Considering the Gibbs free energy \( G \) of a crystal with \( N_o \) occupied lattice sites and \( N_v \) vacant lattice sites, the number of vacancies \( N_v^{eq} \) at equilibrium is given by

\[ \frac{\partial G}{\partial N_v} \bigg|_{T,p} = 0 \quad (42) \]

which yields (e.g. [8] p. 27)

\[ n_v^{eq} = \exp \left( -\frac{g_f}{k_B T} \right) \quad \text{with} \quad n_v^{eq} = \frac{N_v^{eq}}{N_o + N_v^{eq}} \quad (43) \]

where \( n_v^{eq} \) is the fraction of vacant lattice sites at equilibrium, \( g_f \) is the Gibbs free energy for the formation of a vacancy, \( k_B \) the Boltzmann constant and \( T \) the temperature.

In contrast to the radioactive decay process there are no descriptive arguments to see that equation (13) reflects an ideal transfer law of a constant information source. Contrary to statistical thermodynamics the simple but universal approach of the information transfer model will principally not allow us to understand why vacancies come into existence in a crystal. However, the information transfer process and especially its global IIT process condition (5) will enable us to see that the isothermal vacancy formation process is an IIT process according to definition (1). The most convenient way to apply the information transfer model is to start the discussion with a technical detector device which corresponds to definition (7) because such a detector defines immediately two independent process variables. If there is no technical detector available we can define a plausible virtual detector. In this case we define

\[ \text{virtual vacancy detector} = \left( \frac{|\delta W|}{|\delta N|} \right)_{\text{detector}} \quad (44) \]

where \( W \) is the work which contributes to the change of the number of vacancies, \( N \), in the crystal. This detector is highly plausible because a change of the number of vacancies will require work. To proceed the discussion we must assume that the vacancy formation process fulfills condition (4). Like for the ideal gas we consider \( W \) as the information source variable so that \( N \) becomes the information destination variable. Our crystal is thermally in good contact to the surroundings so that the energy, \( W \), which contributes to the change of the number of vacancies remains always constant (\( W = W_0 = \text{constant} \)). That is, we have a constant information source in the context of the information transfer model. According to the global IIT process condition (5) the formation process of real vacancies in a crystal must becomes a global IIT process if the absolute value of the global destination variable, \( N \), becomes sufficiently large. For an IIT process we can apply the ideal transfer law (17) of a constant information source (setting \( |\Delta u_0| = W_0 \), \( |\Delta u| = N \))

\[ \langle N \rangle = N_{\text{ref}} \exp \left( -\kappa \frac{|\Delta W|}{W_0} \right) \quad \text{with} \quad \Delta W = (W - W_{\text{ref}}) \quad (45) \]

where \( \langle N \rangle \) is the expected number of vacancies for a given energy \( |\Delta W| \) and \( N_{\text{ref}} \) is a reference number. \(|\Delta W| \) is simply a “certain amount of energy” in the context of the information transfer model. A physical interpretation of \(|\Delta W| \) requires the application of statistical thermodynamics. The setting of the right sign in equation (17) requires also some knowledge concerning the considered process. In this case we know that the expected number of vacancies decreases if \(|\Delta W| \) increases. If the formation of vacancies would require no energy, \(|\Delta W| = 0 \), all
lattice sites would be empty, \( \langle N \rangle = N_{\text{ref}} = N_{\text{latt}} \). That is, as reference number of the considered process we can set the available number of lattice sites of the crystals.

In the case of ideal information transfer we could consider real gas particles as ideal point particles. In an analogous way we can now consider in a crystal the real particles located at the lattice sites as ideal point particles. If we define a vacancy as a real “particle” located at a lattice site it can be considered in the case of ideal information transfer, too, as an ideal point particle. That is, the sum of all ideal particles is equal to the number of all available lattice sites in the crystal, \( N_{\text{ideal}} = N_{\text{latt}} \), so that the ideal point particle equation \( (22) \) becomes

\[
\frac{N}{\partial N} = \frac{W_0}{\kappa |\delta W|} = N_{\text{latt}} \gg 1
\]

and \( W_0 = \text{constant} \) \( (46) \).

The ideal transfer law condition \( \frac{7}{7} \) is valid because of equation \( (16) \). Considering that we have \( W_0 = \kappa N_{\text{latt}} |\delta W| \) and \( N_{\text{ref}} = N_{\text{latt}} \), equation \( (15) \) becomes

\[
\langle n \rangle = \exp \left( -\frac{|\Delta w|}{|\delta W|} \right) \quad \text{with}
\]

\[
\langle n \rangle = \frac{\langle N \rangle}{N_{\text{latt}}} \quad \text{and} \quad |\Delta w| = \frac{|\Delta W|}{N_{\text{latt}}} \quad (47)
\]

where \( \langle n \rangle \) is the expected fraction of vacant lattice sites for a given energy \( |\Delta w| \). Comparing equations \( (17) \) and \( (43) \) we now can physically assign the energy \( |\Delta w| \) to the Gibbs free energy for the formation of a vacancy, \( |\Delta w| = g_f \). That is, the absolute value of the process (energy) signal, \( |\delta W| \), for the vacancy formation process is then given by

\[
|\delta W| = k_B T \quad (48)
\]

This is a remarkable result because we have seen that the absolute value of the process (energy) signal for the process of ideal gas particles in a box is also given by \( k_B T \) (see equation \( (28) \)). Because of condition \( \frac{8}{8} \) we have \( 0 < |\delta W| < \infty \) so that in the context of the information transfer model temperatures of \( T = 0 \) and \( T = \infty \) are not allowed.

V. THE RANDOM WALK PROCESS

We now demonstrate that one can derive Fick’s first law (e.g. \( \frac{6}{6} \) p. 1) if one applies the local IIT process condition \( \frac{6}{6} \) which states that we will have ideal information transfer if the absolute value of the source variable is sufficiently small. For that we consider the random walk process of point particles. If a certain number \( N \) of point (tracer) particles are, in a thought experiment, suddenly introduced on the \( yz \) plane at \( x = x_0 \), then, in the time interval \( |\Delta t| \), some particles would progress a distance, \( |\Delta x| \); still others, \( |\Delta x_2| \); etc. The Einstein-Smoluchowski equation says that (e.g. \( \frac{7}{7} \) p. 380)

\[
\overline{\Delta x^2} = \frac{(\Delta x_1)^2 + (\Delta x_2)^2 + \ldots + (\Delta x_n)^2}{N}
\]

\[
= 2D |\Delta t| \quad (49)
\]

where \( \overline{\Delta x^2} \) is the mean square distance and \( D \) is the diffusion coefficient. This equation cannot tell us how many particles travel a distance, \( |\Delta x_1| \); how many, \( |\Delta x_2| \); etc. It is a simple but fundamental law which is related to any randomly walking point particles.

Comparing equation \( (19) \) with the ideal transfer law \( (21) \) of a floating information source we get a similar result if we set \( |\Delta u| = |\Delta x| \), \( |\Delta q| = |\Delta t| \), \( k = 1/2 \) and \( R_{n=1/2} = D \)

\[
\langle |\Delta x| \rangle^2 = 2D |\Delta t| \quad (50)
\]

That is, in the context of the information transfer model we consider the square of an expected travelling distance \( \langle |\Delta x| \rangle \) of the tracer point particles (this is not the mean travelling distance, because \( \overline{\Delta x} = 0 \)) but rather the mean square distance \( \overline{\Delta x^2} \). Furthermore, the diffusion coefficient \( D \) is considered in the context of the information transfer model more generally as a reference constant of an IIT process (in this case an IIT process with an information transfer index \( k = 1/2 \)). According to the local IIT process condition \( \frac{7}{7} \) the random walk process must become a local IIT process if the absolute value of the source variable becomes locally sufficiently small \( (|\Delta q| = |\Delta t| \rightarrow 0) \). For an IIT process we can apply the ideal point particle equation \( (22) \) which now becomes (setting \( |\Delta q| = |\Delta t| \), \( |\Delta u| = |\Delta x| \) and \( k = 1/2 \))

\[
\overline{\Delta x} = 2|\Delta t| = N \gg 1 \quad (51)
\]

We see that the process variable selections, \( |\Delta x| / |\delta x| \) or \( |\Delta t| / |\delta t| \), are equal or, respectively, proportional to the considered huge number \( N \) of randomly walking local tracer particles. This guarantees that the ideal transfer law condition \( \frac{7}{7} \) is also valid if \( |\Delta x| \) is in the range of the average jump length of the considered tracer particles.

The experimental study of a random walk process implies the collective observation of a huge number of single tracer particles, so that one can define a local tracer particle flux

\[
j_x = \frac{1}{A_0} \frac{\partial N}{\partial t} \quad (52)
\]

where \( \partial N \) is the infinitesimal number of tracer particles which pass through the unit area \( A_0 \) in \( x \) direction during the infinitesimal time interval \( \delta t \). The local concentration \( C \) of the tracer particles is given by definition as

\[
C = \frac{1}{A_0} \frac{\partial N}{\partial x} \quad (53)
\]
We can now consider the local ratio between flux and concentration

$$\frac{|\dot{j}_x|}{C} = \left(\frac{\delta x}{\delta t}\right)_{\text{detector}}$$  \hfill (54)

which yields a virtual detector $(|\delta x|/|\delta t|)_{\text{detector}}$ in the context of the information transfer model. Because the random walk process is locally an IIT process we can apply the ideal detector equation (12) so that equation (54) becomes

$$\frac{|\dot{j}_x|}{C} = \left(\frac{\delta x}{\delta t}\right)_{\text{detector}} = \frac{1}{2}\frac{|\Delta x|}{|\Delta t|}$$  \hfill (55)

The tracer particles obey locally the ideal point particle condition $\langle|\Delta x|\rangle = |\Delta x|$, so that we can combine the ideal transfer law (50) and the ideal detector equation (55)

$$DC = |\dot{j}_x||\Delta x|$$  \hfill (56)

We can now eliminate the explicit absolute value of the destination variable, $|\Delta x|$, if we apply a partial differentiation of equation (56) for a given local flux $j_x$

$$\left|\frac{\partial(DC)}{\partial x}\right| = \left|D\frac{\partial C}{\partial x}\right| = |\dot{j}_x|$$  \hfill (57)

which yields Fick’s first law (e.g. [6] p. 1)

$$\dot{j}_x = -D\frac{\partial C}{\partial x}$$  \hfill (58)

if we omit the absolute value bars. This discussion shows that Fick’s familiar first law is, in the context of the information transfer model, the result of a local IIT process.

VI. IDEAL GLOBAL POINT PARTICLE PROPAGATION PROCESSES

We consider a source of propagating point particles which is located at $r = 0$ and assume that we have a technical measurement device at the detector point in Fig. 3 which detects the propagating particles located originally in the source point and which delivers a measurement value with the dimension of a velocity

$$\text{technical detector} = \left(\frac{\delta r}{\delta t}\right)_{\text{detector}}$$  \hfill (59)

This value measured by the detector is always a superposition of the contribution which results from the relative movement of the detector relative to the particle source and the contribution which results from the considered particle propagation process

$$\left(\frac{\delta r}{\delta t}\right)_{\text{detector}} = \left(\frac{\delta r}{\delta t}\right)_{\text{rel. mov.}} + \left(\frac{\delta r}{\delta t}\right)_{\text{propag.}}$$  \hfill (60)

To apply the information transfer model we must consider a detector without any movement relative to the particle source $(\langle\delta r/\delta t\rangle_{\text{rel. mov.}} = 0)$.

The particle source is embedded in a (mathematically) infinite volume so that we can consider the distance $r$ as the global information destination of the particle propagation process. The age $t$ (in the sense of a duration or, quite generally, of a limited absolute value of time) is limited and can be considered as the global information source of the particle propagation process. According to the global IIT process condition $\eta$ the propagation process of particles must become a global IIT process if the distance from the particle source is sufficiently large ($r = u_{\text{global}} \to \infty$). For an IIT process we can apply the ideal detector equation (12) (setting $|\Delta u| = r$ and $|\Delta q| = t$)

$$\left(\frac{\delta r}{\delta t}\right)_{\text{detector}} = \frac{r}{t}$$  \hfill with $r > 0$, $t > 0$ (61)

where the detector value $(\delta r/\delta t)_{\text{detector}}$ is always positive for propagating particles so that we can omit the absolute value bars. The only constant which is to be determined experimentally is the information transfer index $\kappa$. If we have no appropriate experimental detector device or no information concerning $r$ or $t$, the information transfer index $\kappa$ can also be derived from the ideal transfer law (21) of a floating information source (setting $|\Delta u| = r$ and $|\Delta q| = t$)

$$\langle r \rangle^{1/\kappa} = \frac{1}{\kappa}R_\kappa t$$  \hfill (62)

where $\langle r \rangle$ is the expected distance of the propagating particles from the particle source at the age $t$ and $R_\kappa$ is...
the reference constant of an IIT process (a particle propagation process in this case). Notice that \( r \) in the ideal detector equation (61) and \( \langle r \rangle \) in the transfer law (62) represent usually two different distances from the particle source. The detector position \( r \) is controlled by the user of the detector device, whereas the expected distance \( \langle r \rangle \) is a floating distance value which floats according to the transfer law (62) with the age \( t \) of the particle propagation process. An experimentalist can control \( \langle r \rangle \), at best, indirectly by controlling the age \( t \) of the process (i.e., the duration of the experiment).

We will now apply equations (61) and (62) to two generic processes: First, the propagation of tracer particles during the diffusion in solids, and second, the propagation of photons in vacuum over large distances.

### A. The tracer particle diffusion process

We consider the diffusion process of tracer particles in an infinite homogeneous isotropic medium without the action of any external forces. The tracer particle source is located in the centre of a spherical coordinate system (the source point in Fig. 3). The ideal detector equation at the detector point in Fig. 3 now becomes (using equation (61) and setting \( \kappa = 1/2 \))

\[
\frac{\langle j_r \rangle}{C} = \left( \frac{\delta r}{\delta t} \right)_{\text{detector}} = \frac{r}{2t} \tag{63}
\]

where \( \langle j_r \rangle \) is the radial tracer particle flux and \( C \) is the concentration of tracer particles. In subsection \[ \text{V} \] it was shown that \( \kappa = 1/2 \) for a tracer diffusion process. The validity of the ideal detector equation (61) requires a sufficiently large distance of the detector from the particle source.

One can directly prove that the ideal detector equation (63) is valid for infinitesimally small tracer particle sources. For that we consider the solution for the tracer particle concentration distribution for a point diffusion source in an infinite volume (\[ \text{V} \] p. 29)

\[
C = \frac{M}{8(\pi D t)^{3/2}} \exp \left( -\frac{r^2}{4Dt} \right) \tag{64}
\]

where \( M \) is the total amount of diffusing tracer particles and \( D \) is the diffusion coefficient. The radial tracer particle flux, \( j_r \), is given by (e.g. \[ \text{V} \])

\[
j_r = -D \frac{\partial C}{\partial r} \tag{65}
\]

Using solution (64) we get equation (63). It is, however, well known from diffusion theory (e.g. \[ \text{V} \]) that a realistic tracer diffusion source with a given radius \( R > 0 \) can be approximated as an infinitesimally small tracer diffusion source if we consider the diffusion process at a distance \( r \) sufficiently far away from the tracer diffusion source \( (r >> R) \). That is, equation (63) is also valid for realistic tracer particle sources if we restrict our discussion to sufficiently large distances \( (r >> R) \) of the detector from the particle source.

Let us now assume in a thought experiment that we have a small freely movable virtual tracer particle detector device which is able to measure separately \( j_r(r, t) \) and \( C(r, t) \) at any distance \( r \) in the medium and which yields the measured ratio \( j_r/C \) as output value. If we could move such a virtual detector device in the medium at low temperatures (a frozen tracer particle distribution) the output result would be equal to the relative velocity between our tracer particle source and our virtual detector. Our virtual detector device at rest would measure \( j_r/C = 0 \) at very low temperatures because of \( j_r = 0 \). At sufficiently high temperatures the tracer particle flux \( j_r \) becomes detectable and we can measure a detector output value, \( j_r/C \). According to the global IIT process condition \[ \text{V} \] the tracer diffusion process must become a global IIT process if the distance \( r \) from the tracer particle source is sufficiently large. That is, our virtual detector device will then deliver an output value, \( j_r/C \), according to equation (63). A special feature of the ideal detector equation (63) is that we can calculate the age \( t \) of the tracer diffusion process by the output value, \( j_r/C \), of our virtual detector device if we know the distance \( r \) of the virtual detector device from the tracer particle source.

### B. The photon propagation process in vacuum

Let us now apply the obtained relations to a cosmological propagation process. Redshift occurs when the electromagnetic radiation that is emitted from an object is shifted toward the red end (less energetic end or longer wavelength) of the electromagnetic spectrum. The redshift parameter \( z > 0 \) is used to describe the change in wavelength. It is defined as

\[
z = \frac{\lambda_o - \lambda_e}{\lambda_e} \tag{66}
\]

where \( \lambda_e \) is the emitted and \( \lambda_o \) is the observed wavelength. Conversely, a decrease in wavelength is called blueshift \( (z < 0) \).

Redshift measurements are an important method in astronomy to get information about the radial velocity, \( v_r \), of a radiation source (star or galaxy) relative to the observer (e.g. \[ \text{V} \] p. 100)

\[
z(v_r) = \frac{v_r}{c} = \frac{1}{c} \frac{dr}{dt} \tag{67}
\]

where \( dr \) is the infinitesimal radial displacement of the radiation source during the infinitesimal time interval \( dt \) and \( c \) is the vacuum light velocity (we restrict this discussion to \( v_r << c \)). This velocity related redshift in equation (67) reflects our experimental experience for photon emission sources which are not very far away from the observer (on the cosmological length scale).
The photons detected during a redshift measurement have their origin in a star or a galaxy which can always be considered as the source point in Fig. 3 at sufficiently large distance \( r \). The detector point in Fig. 3 is the location of our laboratory in three dimensional space where we measure the redshift at process age \( t > 0 \). At process age \( t = 0 \) the radiation source started to emit the first photons. We now consider the term \( z \times c \) as the output value of a redshift measurement device

\[
zc = \left( \frac{\delta r}{\delta t} \right)_{\text{detector}}
\]

which yields a detector in the context of the proposed information transfer model. If we consider the case where the redshift measurement device has no radial velocity relative to the photon source equation (67) would yield the result \( z = z(v_r) = 0 \). Applying, however, the ideal detector equation (61) for propagating particles we have in this case

\[
z(r, t) c = \left( \frac{\delta r}{\delta t} \right)_{\text{detector}} = \frac{r}{t} \quad \text{if} \quad v_r = 0
\]

which yields a distance and time dependent red shift value \( z = z(r, t) \). According to the global IIT process condition [5] the process of propagating photons must become a global IIT process if the distance \( r \) from the photon source is sufficiently large. That is, our detector (a redshift measurement device) will then deliver an output value according to equation (69). To determine the unknown information transfer index \( \kappa \) we consider the ideal transfer law of the process. In subsection V we had concluded by means of the ideal transfer law that \( \kappa = 1/2 \) for a random walk process (see equation (69)). For a photon propagation process the ideal transfer law (62) becomes

\[
\langle r \rangle = R_{\kappa=1} t
\]

because it is experimentally well confirmed that electromagnetic radiation propagates with a constant velocity, where \( \langle r \rangle \) is the expected distance of the photons from a photon source of age \( t \). The reference constant \( R_{\kappa=1} \) (with the dimension of a velocity) is a medium dependent constant which is equal to \( c \) in vacuum. We can now conclude that \( \kappa = 1 \) for a photon propagation process so that the ideal detector equation (69) finally becomes

\[
z(r, t) c = \frac{r}{t} \quad \text{if} \quad v_r = 0
\]

where \( r \) is the distance of the detector (a redshift measurement device) from the observed photon source and \( t \) is the age of the observed photon source (star or galaxy). That is, if we know the distance \( r \) from a star or galaxy and are not moving relative to the star or galaxy \( (v_r = 0) \) we can determine its age \( t \) from redshift measurements (if the distance \( r \) is sufficiently large). Or vice versa, if we know the age of a star or galaxy we can determine its distance if \( v_r = 0 \).

FIG. 4: Hubble’s published radial velocity-distance relation among extra-galactic nebulae (see [10] for the meaning of the different points and lines).

If we can only measure distances \( r \) and redshifts \( z \) from the observable galaxies, a reasonable assumption is that the observable galaxies have comparable ages \( t_0 \). Plotting such measurements in a diagram enables us to estimate the average age \( t_0 \) of the observed galaxies. Hubble [10] published distance dependent redshift measurements in 1929. Fig. 4 shows Hubble’s published velocity of each galaxy in his sample relative to the Milky Way. The proportionality constant, \( H_0 = \langle t_0 \rangle^{-1} \), is called the Hubble constant. Using this Hubble constant equation (71) becomes

\[
c z(r) = H_0 r \quad \text{if} \quad v_r = 0
\]

and \( \tau_0 = \frac{1}{H_0} = \text{constant} \) (72).

The discussion in our paper shows that Hubble’s constant, \( H_0 \), can correspond only to the average age of all observed galaxies because every single galaxy shows a distance dependent redshift according to equation (71), where \( t \) is the age of the observed galaxy. Interestingly enough the proposed information transfer model yields results which are in agreement with cosmological observations concerning the redshifts of stars and galaxies: Narlikar and Arp [11] state “the Hubble constant is simply determined by the age of the galaxies which comprise the relation” (see our equation (72)).

VII. SUMMARY

Inequality [3] is a well known fundamental law from information theory [3] which states that any technical process can transfer at best the whole information from an information source to an information destination. To discuss the consequences of this law for natural processes we
propose an information transfer model where the transfer system explicitly includes any natural processes. To develop this model we first simplified Shannon’s diagram of a general communication system [3], where the transfer system consists of transmitter, channel, receiver and noise source, by our diagram in Fig. 1. The basic idea is that we can then formally replace the technical process by any natural process which can be described sufficiently by two independent generic process variables \( q \) and \( u \) (condition 1). Because of the independence of the generic process variables we can now formally consider them as information source \( q \) and information destination \( u \) of the information transfer model (Fig. 1).

Starting from this point of view it was necessary to find a definition of information which is directly related to process variables. For that we defined process variable selections, applied the definition of information which was originally given by Hartley [3] and obtained finally a working expression for the information based on the two independent process variables (equation (8)). In the next step we defined an IIT process (definition 1) which enables ideal information transfer from the source variable to the destination variable (condition 1).

Because it is forbidden that any process (technical or natural) transfers more information to the information destination than the information source sends we derived sufficient conditions for a global IIT process and a local IIT process. We demonstrated that one can derive the ideal gas law, the radioactive decay law, the formation of vacancies in crystals and Hubble’s law if one applies the global IIT process condition (A4). Applying the local IIT process condition (A5) allowed us to derive Fick’s first law.

There is no doubt that one cannot violate a fundamental law (inequality (5)) which is valid for technical processes if one replaces the technical process by any natural process which fulfills condition 1. Our abstract definition of information concerning generic process variables, which is based on Hartley’s [3] approach, yields, interestingly enough, several well known laws from the simple model presented here.

**APPENDIX A: JUSTIFICATION OF IIT PROCESS CONDITIONS**

1. **Justification of condition 5**

According to condition 5 we have \( |\delta q| > 0 \). That is, if we consider the case where the absolute value of a global source variable is limited \( (q_{\text{global}} = |\Delta q| < \infty) \) we have a limited number of source variable selections

\[
n_q = \frac{q_{\text{global}}}{|\delta q|} < \infty \quad \text{if} \quad q_{\text{global}} < \infty \quad \text{and} \quad |\delta q| > 0 \quad (A1)
\]

Because of inequality (11) we have now

\[
\kappa n_u \leq n_q < \infty \quad \text{with} \quad 0 < \kappa < \infty \quad (A2)
\]

Considering, furthermore, the case that the absolute value of the global destination variable is allowed to become unlimited \( (u_{\text{global}} = |\Delta u| \to \infty) \) we would expect an unlimited number of destination variable selections

\[
n_u = \frac{u_{\text{global}}}{|\delta u|} \to \infty \quad \text{if} \quad u_{\text{global}} \to \infty
\]

and \( |\delta u| < \infty \) \( (A3) \)

if we take only into account that \( |\delta u| \) is limited \( (|\delta u| < \infty) \) according to condition 3. However, according to inequality (A2) an unlimited \( n_u \) is forbidden for the considered case \( (q_{\text{global}} < \infty) \). To solve this conflict we must conclude that \( n_u \) can never become infinity but remains limited according to the relation

\[
n_u = \frac{n_u}{\kappa} < \infty \quad \text{if} \quad u_{\text{global}} \to \infty \quad \text{and} \quad q_{\text{global}} < \infty \quad (A4)
\]

no matter how large \( u_{\text{global}} \) becomes. That is, if equation (A4) is correct for a given large \( u^*_{\text{global}} \) it remains correct for \( u_{\text{global}} > u^*_{\text{global}} \). Equation (A4) represents condition 5.

2. **Justification of condition 6**

We now consider the case that there is a huge number of destination variable selections, \( n_q \), given so that we have \( \kappa n_q > 1 \). Because of inequality (11) we now have

\[
1 < \kappa n_u \leq n_q \quad \text{with} \quad 0 < \kappa < \infty \quad (A5)
\]

We would expect that the number of source variable selections becomes small \( (\approx 1) \)

\[
n_q = \frac{|\Delta q|}{|\delta q|} \to 1 \quad \text{if} \quad |\Delta q| \to |\delta q| > 0 \quad (A6)
\]

if \( |\Delta q| \) becomes sufficiently small. However, according to inequality (A5) \( n_q \to 1 \) is now forbidden for the considered case \( (\kappa n_u > 1) \). To solve this conflict we must conclude that \( n_q \) can never become a small number \( (\approx 1) \) but remains a huge number \( (>> 1) \) according to the relation

\[
n_q = \kappa n_u >> 1 \quad \text{if} \quad |\Delta q| \to 0 \quad (A7)
\]

no matter how small \( |\Delta q| \) becomes. That is, if equation (A7) is correct for a given small \( |\Delta q| \) it remains correct for \( |\Delta q| < |\Delta q|^* \). Equation (A7) represents condition 6.
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