Analytical Expressions for the Hard-Scattering Production of Massive Partons

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Abstract. We obtain explicit expressions for the two-particle differential cross section $E_aE_cd\sigma(AB\rightarrow c\kappa X)/dcd\kappa$ and the two-particle angular correlation function $d\sigma(AB\rightarrow c\kappa X)/d\Delta\phi d\Delta y$ in the hard-scattering production of massive partons in order to exhibit the “ridge” structure on the away side in the hard-scattering process. The single-particle production cross section $d\sigma(AB\rightarrow cX)/dy_c dc_T$ is also obtained and compared with the ALICE experimental data for charm production in $pp$ collisions at 7 TeV at LHC.

1. Introduction
Knowledge of massive quark production processes in $pp$ collisions provides useful insight to guide our intuition in heavy quark production in nucleus-nucleus collisions. Analytical expressions for these processes summarize important features and essential dependencies so as to facilitate the uncovering of dynamical effects wherever they may occur. Similar analyses in massless quark production have led to new insights in the dominance of the hard-scattering process over a large $p_T$ domain and have paved the way for locating the boundary between the hard-scattering process and the flux-tube fragmentation process in high-energy $pp$ collisions [1, 2].

Accordingly, we would like to obtain $E_aE_cd\sigma(AB\rightarrow c\kappa X)/dcd\kappa$ for the two-particle differential cross section in the production of massive partons $c$ and $\kappa$. From such a general result, we integrate out the transverse momenta and obtain the two-particle angular correlation function $d\sigma/d\Delta\phi dy$ where $\Delta\phi=\phi_a-\phi_c$ and $\Delta y=y_b-y_c$, exhibiting analytically the “ridge” structure on the away side at $\Delta\phi\sim\pm\pi$ in the hard-scattering process. We subsequently examine $d\sigma(AB\rightarrow cX)/dy_c dc_T$ for the single-particle spectrum and compare with ALICE experimental data for charm production in $pp$ collisions at 7 TeV at LHC [3].

2. Hard Scattering Integral for $E_aE_cd\sigma(AB\rightarrow c\kappa X)/dcd\kappa$
In the parton model, the hard-scattering cross section for $AB\rightarrow c\kappa X$ is given by [4]

$$d\sigma(AB\rightarrow c\kappa X) = \sum_{ab} \int K_{ab} dx_a d\alpha_T dx_b d\beta_T G_{a/A}(x_a, \alpha_T)G_{b/B}(x_b, \beta_T)d\sigma(ab\rightarrow c\kappa), \quad (1)$$

where $(x_a, \alpha_T)$ and $(x_b, \beta_T)$ represent the momenta and $G_{a/A}$ and $G_{b/B}$ the structure functions of the incident partons $a$ and $b$ respectively, and $K_{ab}$ is the correction factor which can be obtained perturbatively [5] or it can also be approximated nonperturbatively [6]. The quantity $d\sigma(ab\rightarrow c\kappa)$ is the cross section element for the process $ab\rightarrow c\kappa$,

$$d\sigma(ab\rightarrow c\kappa) = \frac{1}{4[(a\cdot b)^2 - m_a^2 m_b^2]^{1/2}} |T_{fi}|^2 \frac{d^3c}{(2\pi)^3}2E_c \frac{d^3\kappa}{(2\pi)^3} \frac{d^4}{(2\pi)^4}(\delta^4(a + b - c - \kappa)). \quad (2)$$

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Here, we normalize the Dirac fields by \( \bar{u}u = 2m \). The quantity \(|T_{ji}|^2\) is related to \( d\sigma/dt \) by
\[
|T_{ji}|^2 = 16\pi [\hat{s} - (m_a + m_b)^2][\hat{s} - (m_a - m_b)^2] \frac{d\sigma(ab \rightarrow c\kappa)}{dt}.
\]
We consider the simplified case with \( m_a = m_b = 0 \) and treat \( aT, bT \) as small perturbations. The cross section element is then
\[
d\sigma(ab \rightarrow c\kappa) = \frac{s_{ab} d\sigma(ab \rightarrow c\kappa)}{2\pi} \frac{d^3c d^3\kappa}{E_c E_{\kappa}} \delta^4(a + b - c - \kappa),
\]
where \( \hat{s} = s_{ab} = (a + b)^2 \) that is different from \( s = s_{AB} = (A + B)^2 \). We get
\[
E_c E_{\kappa} d\sigma(AB \rightarrow c\kappa X) = \frac{1}{d^3c d^3\kappa} \sum_{ab} \int K_{ab} dx_a d^2x_b \frac{G_{a/A}(x_a, aT) G_{b/B}(x_b, bT)}{2\pi} \frac{\hat{s} d\sigma(ab \rightarrow c\kappa)}{dt} \delta^4(a + b - c - \kappa).
\]
We consider a factorizable structure function with a Gaussian intrinsic transverse momentum distribution,
\[
G_{a/A}(x_a, aT) = G_{a/A}(x_a) \frac{1}{2\pi\sigma^2} e^{-a^2/2\sigma^2}.
\]
Upon integrating over the transverse momenta \( aT \) and \( bT \), we obtain
\[
\frac{d\sigma(AB \rightarrow c\kappa X)}{dy_c dT_c d\phi_c dy_{\kappa} dT_{\kappa} d\phi_{\kappa}} = \sum_{ab} \int K_{ab} dx_a d^2x_b G_{a/A}(x_a) G_{b/B}(x_b) e^{(C_T + i\kappa_T)^2} \frac{\hat{s}}{2(2\pi\sigma^2)} \frac{d\sigma(ab \rightarrow c\kappa)}{dt} \delta(a_0 + b_0 - (c_0 + \kappa_0)) \delta(a_z + b_z - (c_z + \kappa_z)).
\]
To carry out the integration over \( x_a \) and \( x_b \), we write out the momenta in the infinite momentum frame,
\[
a = \left( x_a \frac{\sqrt{s}}{2} + \frac{a^2 + a_T^2}{2x_a \sqrt{s}} \right), \quad aT, \quad x_a \frac{\sqrt{s}}{2} - \frac{a^2 + a_T^2}{2x_a \sqrt{s}},
\]
\[
b = \left( x_b \frac{\sqrt{s}}{2} + \frac{b^2 + b_T^2}{2x_b \sqrt{s}} \right), \quad bT, \quad -x_b \frac{\sqrt{s}}{2} + \frac{b^2 + b_T^2}{2x_b \sqrt{s}},
\]
\[
c = \left( x_c \frac{\sqrt{s}}{2} + \frac{c^2 + c_T^2}{2x_c \sqrt{s}} \right), \quad cT, \quad x_c \frac{\sqrt{s}}{2} - \frac{c^2 + c_T^2}{2x_c \sqrt{s}},
\]
\[
\kappa = \left( x_\kappa \frac{\sqrt{s}}{2} + \frac{\kappa^2 + \kappa_T^2}{2x_\kappa \sqrt{s}} \right), \quad \kappa_T, \quad -x_\kappa \frac{\sqrt{s}}{2} + \frac{\kappa^2 + \kappa_T^2}{2x_\kappa \sqrt{s}},
\]
where \( x_c \) and \( x_\kappa \) can be represented by \( y_c \) and \( y_\kappa \),
\[
x_c = \frac{m_{cT} e^{y_c}}{\sqrt{s}}, \quad x_\kappa = \frac{m_{\kappa T} e^{y_\kappa}}{\sqrt{s}}.
\]
The two delta functions in Eq. (7) can be integrated to yield
\[
\frac{d\sigma(AB \rightarrow c\kappa X)}{dy_c dT_c d\phi_c dy_{\kappa} dT_{\kappa} d\phi_{\kappa}} = \sum_{ab} \int K_{ab} dx_a G_{a/A}(x_a) x_b G_{b/B}(x_b) e^{(C_T + i\kappa_T)^2} \frac{d\sigma(ab \rightarrow c\kappa)}{dt},
\]
where
\[
x_a = x_c + \kappa_T \frac{\sqrt{s}}{x_c s} - \frac{b^2 + b_T^2}{x_b s} = \frac{m_{cT} e^{y_c}}{\sqrt{s}} + \frac{m_{\kappa T} e^{-y_\kappa}}{\sqrt{s}} - \frac{b^2 + b_T^2}{x_b s},
\]
\[
x_b = x_\kappa + \kappa_T \frac{\sqrt{s}}{x_\kappa s} - \frac{a^2 + a_T^2}{x_a s} = \frac{m_{cT} e^{-y_c}}{\sqrt{s}} + \frac{m_{\kappa T} e^{y_\kappa}}{\sqrt{s}} - \frac{a^2 + a_T^2}{x_a s}.
\]
3. $c\kappa$ angular correlation $d\sigma(AB \to c\kappa X)/d\Delta\phi d\Delta y$

We can represent $c$ and $\kappa$ by $(y_c, \phi_c)$ and $(y_c + \Delta y, \phi_c + \Delta \phi)$, respectively. After averaging over $y_c$ and $\phi_c$, and integrating over $c_T, \kappa_T$, the correlation function (13) from the process $ab \to c\kappa$ is

$$\frac{d\sigma(AB \to c\kappa X)}{d\Delta\phi d\Delta y} = K_{ab} x_a G_{a/A}(x_a) x_b G_{b/B}(x_b) \delta_\sigma(\Delta\phi),$$

(15)

where $\delta_\sigma(\Delta\phi) = \frac{1}{4\pi\sigma^2} \int_0^\infty c_T dc_T \int_0^\infty \kappa_T dk_T \exp\left\{-\frac{c_T^2 + 2c_T\kappa_T \cos \Delta\phi + \kappa^2}{4\sigma^2}\right\} \frac{d\sigma(ab \to c\kappa)}{dt}$.

The above analytical expression assumes a simple form for $a=b$ and $c=\kappa$. The structure function can be represented in the form $x_a G_{a/A}(x_a) \propto (1 - x_a)^{\alpha_a}$ for which the two-particle angular correlation function becomes

$$\frac{d\sigma(AB \to c\kappa X)}{d\Delta\phi d\Delta y} \sim A \left[1 - \frac{2m_{cT}}{\sqrt{s}} \left[\cosh y_c + \cosh(y_c + \Delta y)\right] + 2 \left(\frac{m_{cT}}{\sqrt{s}}\right)^2 \left[1 + \cosh(2y_c + \Delta y)\right]\right]^{\alpha_a} \delta_\sigma(\Delta\phi).$$

(17)

If we consider $d\sigma(ab \to c\kappa)/dt$ to be approximately of the form

$$\frac{d\sigma(ab \to c\kappa)}{dt} = \frac{A}{(1 + c_{T}^2/m_{cT}^2)^{n/2}},$$

(18)

where $n=4$ from pQCD, then the integration over $c_T$ and $\kappa_T$ in Eq. (16) gives $\delta_\sigma(\Delta\phi)/A$ as shown in Fig. 1. The correlation function has maxima at $\Delta\phi \sim \pm \pi$ and a minimum at $\Delta\phi=0$. It is relatively flat in $\Delta y$ because $m_{cT}/\sqrt{s} \ll 1$. This gives the distribution in the form of a ridge structure on the away side at $\Delta\phi \sim \pm \pi$.

4. Production of massive quarks by gluons

We consider the process $gg \to c\bar{c}$, where analytical expressions $d\sigma/dt$ have been obtained earlier by Cambridge [7] and Glück, Owen, and Reya [8]. In the notation of [8], the cross section is

$$\frac{d\sigma(gg \to c\bar{c})}{dt} = \frac{\alpha_s^2}{64\pi^2} \left[12M_{ss} + \frac{16}{3}M_{tt} + \frac{16}{3}M_{uu} + 6M_{st} + 6M_{su} - \frac{2}{3}M_{tt}\right].$$

(19)

Upon writing out the above quantity as a function of $c_T, \bar{c}_T, \bar{y} = (y_c + y_c)/2$ and $\Delta y = y_c - y_c$, we get the heavy-quark pair production cross section

$$\frac{d\sigma(AB \to c\bar{c}X)}{dy_c c_T dc_T d\phi_c dy_c \bar{c}_T d\bar{c}_T d\bar{\phi}_c} \sim AK_{ab}(1 - x_a)^{\alpha_a}(1 - x_b)^{\alpha_b} \frac{\epsilon_{c_T + \bar{c}_T}}{2\pi(4\pi\sigma^2)} \frac{d\sigma(gg \to c\bar{c})}{dt},$$

(20)

where

$$\frac{d\sigma(gg \to c\bar{c})}{dt} = \frac{\alpha_s^2}{4^5m_{cT}^2\cosh^4\bar{y}} \left\{\frac{12}{\cosh^2\bar{y}} + \frac{64}{3}\cosh 2\bar{y} - 24\right\} + \left(\frac{m_{cT}}{3}\right)^2 \left[\frac{64}{3} + \frac{24\sinh^2\bar{y}}{\cosh^2\bar{y}} - \frac{8}{3}\right] + \left(\frac{m_{\bar{c}_T}}{3}\right)^2 \left[-\frac{64\cosh 2\bar{y}}{3} + \frac{8}{3}\left(\frac{1}{3}\cosh^2\bar{y}\right)\right],$$

$x_a = 2m_{cT} \cosh \bar{y} e^{\Delta y/2}$, and $x_b = 2m_{\bar{c}_T} \cosh \bar{y} e^{-\Delta y/2}$. This shows the back-to-back correlation of $c_T$ and $\bar{c}_T$ and the relatively flat distribution as a function of $\Delta y$.
5. Single-particle charm production

We need to integrate over $\kappa$ in Eq. (5) to get the single-particle distribution of $c$. Neglecting intrinsic $p_T$ and integrating over $\kappa$, we get

$$E_c d\sigma(AB \rightarrow cX) \approx \sum_{ab} \int dx_a dx_b G_{a/4}(x_a) G_{b/B}(x_b) \frac{d\sigma^{gg \rightarrow cc}}{dt} \delta(s + \hat{t} + \mu - m_c^2 - m_{\kappa}^2).$$

The integral over $x_a$ and $x_b$ can be evaluated by the saddle point method [1], and we get for $\bar{y} = y_c \approx 0$,

$$E_c d\sigma(AB \rightarrow cX) \approx K_{ab} (1 - x_a)^{y_a + 1/2} (1 - x_b)^{y_b + 1/2} \frac{1}{\sqrt{x_c}} \frac{d\sigma^{gg \rightarrow cc}}{dt}$$

which contains $1/m_T^4$, $m_c^2/m_T^6$, and $m_a^4/m_T^8$.

We examine the ALICE charm production cross section data in $pp$ collisions at 7 TeV [3] shown in Fig. 2. If we parametrize the data at mid-rapidity as $d\sigma/dy_{cT} dc_T \sim a/(1 + p_T^2/m_0^2)^{n/2}$, we find that the ALICE data can be fitted by a set of parameters given by

$$d\sigma/dy_{cT} dp_T|_{y=0} = a/(1 + p_T^2/m_0^2)^{n/2}$$

| Data | $a \ [\mu b/(GeV/c)^2]$ | $n$ | $m_0 \ [GeV]$ |
|------|----------------|-----|-------------|
| $D^0$ | 1600            | 5.8 | 3.5         |
| $D^+$ | 780             | 5.9 | 3.5         |
| $D^{++}$ | 808          | 5.7 | 3.5         |

![ALICE Data](image)

Fig. 2. ALICE data for charm production.

The extracted values of $n$ and $m_0$ are greater than those from the expected lowest-order results of $n = 4.5$ and $m_c \sim 1.5$ GeV in Eq. (22). This may arise from the final-state interactions correction factor $K_{ab}$ [6] for production of the $c\bar{c}$ pair in the color singlet state. The attractive color-singlet interaction between $c$ and $\bar{c}$ enhances the production of the pair at lower $p_T$ and increases the value of the effective mass of the produced charm meson.

6. Conclusion

We present analytical expressions for the hard-scattering production of massive quarks in order to guide our intuition, point out essential dependencies, and summarize important features. They will facilitate future comparisons with experimental data and pave the way for a better understanding of particle production processes.

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