True thermal antenna with hyperbolic metamaterials

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A thermal antenna is an electromagnetic source which emits in its surrounding, a spatially coherent field in the infrared frequency range. Usually, its emission pattern changes with the wavelength so that the heat flux it radiates is weakly directive. Here, we show that a class of hyperbolic materials, possesses a Brewster angle which is weakly dependent on the wavelength, so that they can radiate like a true thermal antenna with a highly directional heat flux. The realization of these sources could open a new avenue in the field of thermal management in far-field regime.

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The thermal radiation \[ \text{a hot body emits in its background results from a well-known incoherent emission process. The local charges in the medium (electrons, ions or partial atomic charges) oscillate thanks to thermal fluctuations and as the corresponding oscillators are usually delta correlated they radiate incoherently in their surrounding. A direct consequence of this mechanism is the absence of privileged directions of emission. However, in 1986 and 1988, Hesketh et al. [3, 4] showed that a textured surface of a doped silicon sample could behave as a thermal antenna, that is as a spatially coherent source thanks to the presence of a surface plasmon polariton, a surface wave whose the electromagnetic field is spatially correlated. Since this pioneer work, numerous spatially coherent sources [3–15] have been proposed. However, sources with extremely directional emission patterns have been achieved so far at a given single frequency mainly. The emission angle of these sources generally changes significantly with respect to the wavelength throughout the Planck window. It follows that the heat flux they radiate, which results from the spectral integration of the directional monochromatic emissivity weighted by the Planck distribution function, is not notably directional. Today, the development of broadband angular selective sources in the infrared range remains a challenging problem. In this Letter we show that a class of hyperbolic materials (HM) can be used to achieve a ‘true thermal antenna’ which radiates a highly directional heat flux in its surrounding.

To start, let us consider an arbitrary semi-infinite planar anisotropic medium at temperature \( T \) surrounded by a bosonic field at zero temperature. According to the theory of fluctuational electrodynamics [10] the radiative heat flux lost in its surrounding by this medium in the direction \( \mathbf{u} = \frac{\mathbf{k}}{\kappa} (\kappa, \gamma_0) \) with \( \gamma_0 = \sqrt{\omega^2/c^2 - \kappa^2} \) can be formally written into a Landauer-like form [17–19]

\[
\Phi(\kappa) = 2 \int_0^\infty d\omega \frac{\Theta(\omega, T)}{2\pi} \mathcal{T}(\omega, \kappa) \frac{d^2 \kappa}{(2\pi)^2},
\]

where \( \kappa := (k_x, k_y)^\parallel \) is the parallel component of wavevector (with the constraint \( |\kappa| < \omega/c \) for the propagating waves), \( \Theta(\omega, T) := \frac{\hbar \omega}{e^\omega - 1} \) is the mean energy of a harmonic oscillator in thermal equilibrium at temperature \( T \) and \( \mathcal{T} \) denotes the transmission coefficient associated to each mode \((\omega, \kappa)\) which reads

\[
\mathcal{T}(\omega, \kappa, d) := \frac{1}{2} \text{Tr}[(1 - \mathcal{R}^\dagger \mathcal{R})].
\]

Here we have introduced the reflection operator for both polarization states \((s, p)\)

\[
\mathcal{R} := \begin{pmatrix} r^{ss} & r^{sp} \\ r^{ps} & r^{pp} \end{pmatrix}.
\]

When further introducing \( I_\omega^0(T) := \Theta(\omega, T) \frac{\omega^2}{4\pi c^2} \) the spectral intensity of a blackbody at the frequency \( \omega \) and using the generalized emissivity

\[
\epsilon(\omega, \mathbf{u}) := \mathcal{T}(\omega, \kappa)
= \frac{1}{2} \left( |r^{ss}|^2 - |r^{pp}|^2 \right)
- |r^{sp}|^2 - |r^{ps}|^2,
\]

the flux radiated by the source in the direction \( \mathbf{u} \) can be written as

\[
\Phi(\kappa) = \int_0^\infty d\omega \frac{\epsilon(\omega, \mathbf{u}) I_\omega^0(T) d^2 \kappa}{\pi}.
\]
Notice that expression \( \text{(4)} \) includes the emissivity in the two different polarization states \( s \) and \( p \) and the emissivity in the cross-polarized states \( sp \) and \( ps \) as well allowing so to deal with arbitrary anisotropic sources. When the source displays an azimuthal symmetry the directional heat flux simplifies to

\[
d\Phi(\theta) = 2 \cos \theta \sin \theta d\theta \left( \int_0^\infty d\omega \epsilon(\omega, \theta) I_\omega^0(T) \right).
\]  

(6)

Both expressions \( \text{(5)} \) and \( \text{(6)} \) are the classical expressions predicted by the Kirchoff’s theory \[2\]. An inspection of this expressions clearly shows that an angular drift in the emission pattern for the source with respect to the frequency leads to a heat flux which is less directive. Therefore, in order to get a highly directional heat flux, we need to have a source with a spatial degree of coherence which does not change significantly all over the Planck window.

Below, we show that a class of HM precisely behaves like that. To this end, we consider a uniaxial anisotropic medium with a dielectric permittivity tensor \( \epsilon = \epsilon_\parallel(x \otimes x + y \otimes y) + \epsilon_\perp z \otimes z \), \( \epsilon_\parallel \) being the permittivity parallel to the surface and \( \epsilon_\perp \) the permittivity along the normal to its surface. For this medium the components of the reflection operator are

\[
\begin{align*}
    r^{ss} &= \frac{\gamma_0 - \gamma_s}{\gamma_0 + \gamma_s}, \\
    r^{pp} &= \frac{\epsilon_\parallel \gamma_0 - \gamma_p}{\epsilon_\parallel \gamma_0 + \gamma_p}, \\
    r^{ps} &= r^{sp} = 0
\end{align*}
\]  

(7) \hspace{1cm} (8) \hspace{1cm} (9)

with \( \gamma_s = \sqrt{\epsilon_\parallel \omega^2/c^2 - \kappa^2} \) and \( \gamma_p = \sqrt{\epsilon_\parallel \omega^2/c^2 - \epsilon_\perp / \epsilon_\parallel \kappa^2} \). From these expressions it can be directly seen that by imposing the condition \( r^{pp} = 0 \) that the Brewster angle \( \theta_B \) is given, using the identity \( \kappa = \omega/c \sin \theta_B \), by

\[
\theta_B = \arcsin \sqrt{\frac{\epsilon_\perp (\epsilon_\parallel - 1)}{\epsilon_\parallel \epsilon_\perp - 1}}.
\]  

(10)

In Fig.1 this angle is plotted in the \((\epsilon_\parallel, \epsilon_\perp)\) plane that is for arbitrary uniaxial media when losses are negligible. Each quadrant corresponds to a specific class of anisotropic medium. When both parameters \( \epsilon_\parallel \) and \( \epsilon_\perp \) are positive the medium is a standard uniaxial crystal with an ellipsoidal iso-frequency surface. On the contrary, if both parameters are negative the medium is a metallic-like anisotropic medium. In this case, the iso-frequency surface is purely imaginary and the medium does not support propagating modes. In the two others quadrants, \( \epsilon_\parallel \) and \( \epsilon_\perp \) have opposite signs. In both cases, the iso-frequency relations \( \kappa_\parallel^2 + \kappa_\perp^2 = \omega^2/c^2 \) define hyperboloidal surfaces. When \( \epsilon_\parallel > 0 \) and \( \epsilon_\perp < 0 \) the HM is called type I HM while in the case where \( \epsilon_\parallel < 0 \) and \( \epsilon_\perp > 0 \) it is of type II. One associates to these two types of HM two different iso-frequency surfaces: a one two-sheeted hyperboloid for type I HM and a one-sheeted hyperboloid for type II HM. As shown in Fig. 1 both types of HM possesses a Brewster angle in some regions of \((\epsilon_\parallel, \epsilon_\perp)\) plane. For the type I HMs, this angle only exists when \( \epsilon_\parallel \leq 1 \) and we observe its drift towards grazing angles as the value of \( \epsilon_\parallel \) increases. On the contrary for type II HMs, a Brewster angle only exists when \( \epsilon_\perp \leq 1 \). But what is worthwhile to note is that the Brewster angle changes very few with the value of \( \epsilon_\parallel \) making these media potentially good candidates to design a coherent thermal antenna with a weak angular variation of emission angle with respect to the frequency.

Figure 2: Reflection in polarization \( s \) of HM for the type I \((\epsilon_\parallel > 0)\) and type II \((\epsilon_\parallel < 0)\). The light zone corresponds to the region where the reflection is close to 1.
To confirm this prediction, let us investigate the reflection coefficients of HMs. These reflection coefficients are plotted in Figs. 2 and 3. In s-polarization, we see that, while the reflection of type I HMs is relatively weak for any angle of incidence, the reflection of type II HMs is very close to 1 showing so that their thermal emission is almost entirely p-polarized. Moreover, the reflection coefficient in p-polarization of type I and II HMs, plotted in Fig. 3 for two different values of $\epsilon_\perp$, confirm the weak variation of Brewster angle of HMs of type II when $\epsilon_\perp$ is smaller than one. They also demonstrate, according to relation (11) that the thermal emission of these media is restricted to an angular sector beyond a critical angle. For $\epsilon_\perp = 0.5$ (see Fig.3-a) this angle is located around 45°. This critical angle increases (not shown in Fig. 3) with the value of $\epsilon_\perp$. It can be derived by the condition that $\gamma_p = 0$ which is equivalent to $\kappa = \epsilon_\perp \omega / c$ or $\theta = \arcsin(\epsilon_\perp)$ [20] showing again that the condition $0 \leq \epsilon_\perp \leq 1$ must be necessarily fulfilled.

Finding a natural HM of type II that displays the required properties over a broad spectral range in the Planck frequency window is a tricky task. However, a metamaterial can be designed for that purpose. To this end, we consider an artificial composite structure formed by alternating layers of materials of permittivity $\epsilon_1$ and $\epsilon_2$. In the long-wavelength limit the structure behaves like an homogeneous uniaxial crystal [21] with

$$\epsilon_\parallel = f \epsilon_1 + (1 - f) \epsilon_2, \quad (11)$$
$$\epsilon_\perp = \frac{\epsilon_1 \epsilon_2}{f \epsilon_2 + (1 - f) \epsilon_1}, \quad (12)$$

where $f$ denotes the filling factor with respect to medium 1. It is easy to show that to obtain a HM of type II with $\epsilon_\perp < 1$ the two dielectric constants must satisfy the following inequalities

$$f \epsilon_1 + (1 - f) \epsilon_2 < 0, \quad (13)$$
$$\epsilon_2 (\epsilon_1 - f) + (f - 1) \epsilon_1 < 0, \quad (14)$$
$$f \epsilon_2 + (1 - f) \epsilon_1 < 0, \quad (15)$$
$$\epsilon_1 \epsilon_2 < 0. \quad (16)$$

A graphical solution of this system (see Supplemental Material [22]) shows that the possible values of $\epsilon_1$ and $\epsilon_2$ are restricted to a special region which itself depends on the filling fraction. On the other hand, when fixing $\epsilon_1$ and $\epsilon_2$ this system of conditions leads to limits of the filling fraction for which we can expect to have a broad band Brewster angle. Guided by these conditions we consider as a realistic example of a hyperbolic thermal antenna a metamaterial composed by alternating Zirconium Nitride (ZrN) and gold (Au) layers. ZrN is a low index dielectric in the infrared range (Re($\epsilon_1$) varies between 0.59 and 0.95 in the wavelength range between 4 $\mu$m and 80 $\mu$m. [23]).

For the second material we take gold whose the dielectric constant is well described by a simple Drude model

$$\epsilon_2 = 1 - \frac{\omega_p^2}{\omega (\omega + i \gamma)}, \quad (17)$$

with the plasma frequency $\omega_p = 13.71 \times 10^{15}$ rad/s and the electron damping $\gamma = 4.05 \times 10^{13}$ rad/s.

In Fig. 4 we plot the angular heat flux at an equilibrium temperature of 300K emitted by ZrN-Au layered structures with different filling factors. For this temperature, the Wien’s wavelength is $\lambda_W = 9.66\mu m$ and about 95% of the radiative energy is emitted by the source between 0.5$\lambda_W = 4.83\mu m$ and 4.5$\lambda_W = 43.47 \mu m$. This spectral range corresponds precisely to the region where ZrN has a low dielectric constant. For small filling factors, the structure has a metallic behavior very similar to that one of a gold sample and it is therefore weakly emitting. However, as the filling factor of ZrN increases, a lobe of emission evolves at oblique incidence reaching a maximum value when $f = 0.99$ which means that only
1% of the structure is metallic. The directivity observed for \( f = 0.99 \) results from the invariance of Brewster angle. On the other hand, since the reflectivity for \( s \)-polarization is almost equal to 1, the thermal emission at the Brewster angle equals only one-half of the blackbody value. Note that, for other filling fractions like \( f = 0.1 \), we can also see an emission peak in Fig. 4. From the values of the permittivities for \( f = 0.1 \) we have (see \cite{22}) \( \epsilon_\perp \gg 1 \). Hence the emission peak for \( f = 0.1 \) is not due to a pure broadband Brewster angle effect. This conclusion is backed by the fact that the thermal emission is much lower than one-half of the blackbody value.

Although the thermal antenna designed above radiates most of its radiative power in a specific direction of space, its angle of thermal emission is very oblique. However, in principle, according to expression \cite{11} a thermal antenna can be engineered to emit at an arbitrary emission angle by properly tuning the value of \( \epsilon_\perp \). In particular, we note that when \( \epsilon_\perp \to 0 \) (i.e. for epsilon near zero (ENZ) HM of type II) the emission angle is orthogonal to the surface. Around this value, the emission angle varies as \( \theta_\text{B} \simeq \sqrt{\epsilon_\perp (|\epsilon_\parallel| + 1)} \) so that provided that \( \epsilon_\parallel \) is not too large, this angle remains close to zero.

In summary, we have predicted that a class of HM can emit most of their radiative power in privileged directions of space. This result paves the way for highly directive radiative heat sources. We believe that these true thermal antennas should find broad applications in the field of fundamental sciences and for a number of applications such as thermal management, heat-driven control of chemical reaction or thermal regulation in biology. However, further works are needed to achieve true thermal antenna in arbitrary emission angles. This will require to find the appropriate effective properties of HMs. Alternatively, another direction of research could be the development of broadband angular selective photonic hypercrystals \cite{24}, metamaterials with periodic spatial variation of both anisotropic and isotropic materials, which could combine the strong selectivity of photonic crystals with the singular properties of anisotropic media.

![Figure 4: Angular heat flux emitted by a ZrN-Au layered structure at \( T = 300 \) K for different filling factors in ZrN. The flux is normalized by the blackbody emission.](image-url)