On the Compression of Cryptographic Keys

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Abstract

Any secured system can be modeled as a capability-based access control system in which each user is given a set of secret keys of the resources he is granted access to. In some large systems with resource-constrained devices, such as sensor networks and RFID systems, the design is sensitive to memory or key storage cost. With a goal to minimize the maximum users’ key storage, key compression based on key linking, that is, deriving one key from another without compromising security, is studied. A lower bound on key storage needed for a general access structure with key derivation is derived. This bound demonstrates the theoretic limit of any systems which do not trade off security and can be treated as a negative result to provide ground for designs with security tradeoff. A concrete, provably secure key linking scheme based on pseudorandom functions is given. Using the key linking framework, a number of key pre-distribution schemes in the literature are analyzed.

I. INTRODUCTION

In any computer system offering security-related services, it is a basic necessity that its users have access to some private information to give them leverage over an adversary. These secret pieces of information are commonly known as (cryptographic) keys. The key is usually used as input to protocols or algorithms for identification, secrecy and authentication purposes. Nearly all such systems can be modeled as a capability-based access system in which each resource is assigned a secret key and a user granted with access right to the resource would be given its key. For example, in secure group communication [3], each conference group is assigned a conference key which is given to all users belonging to the group so that the communication of the group could be kept secret and message authentication can be achieved within the group.

Ideally, the security requirement of a typical system (not limited to secure group communication) is that all users outside a particular group or not granted access to a resource should not be able to obtain or compute the key for it even by collusion. For instance, in secure group communication, it is necessary to ensure that all users outside a certain conference group (whose key is treated as a resource key) should not be able to derive the group key from their keys.

In most cases, the storage needed at each user could be too large to be practical. For example, in a typical access control system, if a user has a high level of privilege, his device may need to store a considerable number of keys. Since the cost of the tamper-resistant storage for the keys increases linearly with the size of the key storage, it is thus worthwhile to study techniques to generate all these keys from a smaller seed or compress the key materials. There is a similar problem facing emerging applications like sensor networks and RFID tags. Despite the dropping cost of secure storage, key storage is still a big concern in these applications, involving low cost embedded devices which have to store a considerable amount of secret keys. Compressing key materials is essential to the scalability of such designs.

To ensure correctness of the operation of all cryptographic algorithms, the key compression needs to be lossless. Besides, to protect a resource from unauthorized access by collusion of compromised users, the key compression should not leak information that can ease unauthorized access to any resource key not given to the compromised users. This paper studies techniques to create dependency between resource keys (to derive one key from another) so as to reduce the
storage requirement on each user device. In other words, we exploit the redundancy in privileged group memberships for key compression. The goal is to minimize the maximum of user key storage over all users. To link keys together, we need to consider the access memberships of all the resources in the system to avoid compromising the security of some resources. We investigate the limit of this key derivation approach by deriving a bound on maximum compression achievable without compromising the security of any resource key.

Due to their simplicity, existing work in the literature such as [10], [20], [7] only considers monotonic access structures. Whereas, this paper considers a much more general access structure without posing any restrictions on what properties it must have. The results of this paper are general enough to cover most practical application scenarios. Note also that the applicability of the model we use is not limited to symmetric or shared key systems. For asymmetric key systems, the model depicts the possession of private keys and the resources represent all algorithms requiring a private key input. For instance, a resource could represent the decryption algorithm of a certain public key cryptosystem and its keys represent the required private keys to achieve a successful decryption of a certain ciphertext encrypted using the corresponding public key. The access control model we consider in this paper could cover a wide range of actual systems, including those not designed for access control purposes.

The contribution of this paper is three-fold. First, we derive the lower bound on key storage needed for a general access structure if key dependency is created between keys held by a user. This lower bound corresponds to the theoretical limit on maximum key compression achievable in an ideal access structure without key compromise. We also show that this bound is tight by giving some concrete examples in sensor network key pre-distribution, which are either bound-achieving or close to this bound. Second, we give a practical, provably secure key linking scheme (for a general access structure) based on pseudorandom functions (PRF). We also provide a reduction proof of security for this construction. Third, we demonstrate how to apply the key linking framework to reduce key storage in pairwise key pre-distribution schemes for sensor networks. We have to emphasize that, unlike the existing schemes with key re-use such as [14], [6], the resulting key storage reduction does not come with a price of lowering the resilience or security against compromised nodes. The only trade-off is lowering the security guarantee from the information-theoretic sense to the computational-complexity-theoretic sense (due to the use of pseudorandom functions), which in essence makes no difference in practice.

In the next section, we present the definitions of access structures. In Sections III and V, we present the key storage lower bound and the key linking construction based on pseudorandom functions respectively. Then we consider applying the key linking framework to key pre-distribution for sensor networks in Section VI. Finally, we have some discussions in Section VII and conclude in Section VIII.

II. ACCESS STRUCTURE

We use an access control system to model a security system; a fairly wide range of applications can be covered by this model. The access structure of a typical system depicts the relations between users and keys/resources. A graphical presentation is shown in Figure 1.

Suppose \( U = \{u_1, u_2, \ldots, u_n\} \) is the set of users and \( R = \{r_1, r_2, \ldots, r_m\} \) the set of resources in a system. Let \( 2^U \) be the set of all subsets of \( U \) and denote the set of all possible secret keys

1The access membership of a resource is the subset of legitimate users granted access right to it.

2In nearly all of the existing key pre-distribution schemes for sensor networks, in order to lower the key storage requirement, the same key is used for links between several pairs of nodes. So when a key is exposed to an adversary due to a compromised node, all these links will be compromised instead of one, thus lowering the resilience of the network against compromised nodes.
Fig. 1. A Typical Access Structure Graph

by $K$. Each resource $r_j \in \mathcal{R}$ is associated with a key $k_j \in K$ and an ordered pair $(P_j, F_j)$ with $P_j \subseteq \mathcal{U}$ and $F_j \subset 2^\mathcal{U}$; $P_j$ is the subset of privileged users granted access to $r_j$ whereas each element in $F_j$ corresponds to a forbidden subset of users which should not be able to access $r_j$ even if all of them collude. Then the access structure of a system has the following definition.

**Definition 1.** The access structure $\Gamma$ of a security system $(\mathcal{U}, \mathcal{R}, K)$ is the following set of 4-tuples: $\{(r_j, k_j, P_j, F_j) : r_j \in \mathcal{R}, k_j \in K, P_j \subseteq \mathcal{U}, F_j \subseteq 2^\mathcal{U}\}$.

In the definition of an access structure, a system is not required to guard against all illegitimate users outside the privileged group of a resource from accessing it. In practical scenarios, usually, only a bounded number of illegitimate users in collusion could be excluded; there is a tradeoff of security for storage. However, this paper considers an ideal access structure which is the most desired setting as raised by Naor et. al. [18] in the context of broadcast encryption. An access structure is ideal if all the illegitimate users to any resource in the system are excluded from accessing it.

**Definition 2.** An access structure $\Gamma = \{(r_j, k_j, P_j, F_j)\}$ for a security system $(\mathcal{U}, \mathcal{R}, K)$ is ideal if $\mathcal{U} \setminus P_j \subseteq F_j$, $\forall r_j \in \mathcal{R}$.

In a security system, the access structure is associated with a key assignment scheme. The set of keys held by a user may not be exactly the same as that of the resources he could access, but should allow him to compute all the resource keys he needs. An access structure graph, whose definition is given below, incorporates a key assignment to an access structure.

**Definition 3.** Given a set of users $\mathcal{U} = \{u_1, u_2, \ldots, u_n\}$, a set of resources $\mathcal{R} = \{r_1, r_2, \ldots, r_m\}$ and a set of keys $K$, an access structure graph $G$ for the system is a bipartite graph with vertex set $V(G) = \mathcal{U} \cup \mathcal{R}$ and edge set $E(G) \subseteq \mathcal{U} \times \mathcal{R}$, and the following properties hold:

- $(u_i, r_j) \in E(G)$ if and only if $u_i$ can access $r_j$.
- Each resource vertex $r_j$ is associated with a privileged user subset $P_j \subseteq \mathcal{U}$ such that $(u_i, r_j) \in E(G)$ if and only if $u_i \in P_j$.
- Each resource vertex $r_j$ is associated with a key $k_j \in K$.

The associated key assignment of an access structure graph is said to be secure and sound if the following holds: a user $u_i$ can compute the key $k_j$ if and only if $(u_i, r_j) \in E(G)$ for all $1 \leq j \leq m$. Note that existing key pre-distribution schemes for sensor networks with key re-use [14], [6] do not satisfy this requirement of security and soundness.

**III. A KEY STORAGE LOWER BOUND WITH KEY DEPENDENCY**

This section uses the access structure graph defined in Section II to derive a lower bound on the key storage requirement if dependency is created between keys.
In an access structure graph, the degree of each user vertex $u_i$ is the key storage requirement at $u_i$ assuming the users store the resource keys directly and each key has the same length, whereas, the degree of each resource vertex $r_j$ is the number of privileged users who can access it, which is the same as $|P_j|$. Let the key storage at user $u_i$ be $d_i$, the goal is to minimize $\max_{u_i \in U} \{d_i\}$.

Usually the resource keys should be picked independently at random to ensure security. However, for some users, storing multiple keys may be redundant. For instance, if a privileged group $P_1$ is the subset of another say $P_2$, that is, $P_1 \subset P_2$, then it is redundant for a user in $P_1$ to store $k_2$ (the key for $P_2$) in addition to $k_1$ (the key for $P_1$). If $k_2$ can be derived from $k_1$, then the storage at each $u_i \in P_1$ would be reduced by one key, equivalent to removing the edge $(u_i, r_2)$ from $G$ and adding a new edge between $r_1$ and $r_2$ (the key dependency). Note that the resulting graph is no longer bipartite.

Given two keys $k_j$ and $k'_j$ for privileged subsets $P_j$ and $P'_j$, if $k'_j$ is derived from $k_j$, all users in $P_j$ would have access to $k'_j$. As a result, to ensure that the key linking does not compromise security, it is necessary to make sure that $P_j \setminus P'_j = \emptyset$ (the empty set). In other words, $P_j \subset P'_j$ if $P'_j \not\subset P_j$. Otherwise, a user not in $P'_j$ (but in $P_j$) would have access to $k'_j$. Subject to this constraint, the best achievable key storage reduction is given by the following theorem.

**Theorem 1:** If dependency is created between keys while maintaining the ideal access structure and security of a system, depending on the access structure, the best achievable maximum storage at each user is at least $\lceil \frac{m}{n} \rceil$ where $n$ is the total number of users and $m$ is the total number of resources with distinct access membership.

**Proof.** To maintain the security and access structure, a key $k'_j$ can be derived from another key $k_j$ only if $P_j \subset P'_j$ and the users in $P'_j \setminus P_j$ need to store $k'_j$ while users in $P_j$ can generate $k'_j$ from $k_j$.

If a key $k'_j$ can be generated from $k_j$, then $|P'_j \setminus P_j| \geq 1$ since $P_j \subset P'_j$ (Note that the $m$ resources have distinct access membership). That is, at least one user in $P'_j$ needs to store $k'_j$. In other words, after key linking, each resource vertex in the access structure graph should have at least one edge coming from the set of user vertices. If we denote the number of edges coming from a user vertex to $r_j$ by $y_j \geq 1$ and the degree of a user $u_i$ by $x_i$, then

$$\sum_{i=1}^{n} x_i = \sum_{j=1}^{m} y_j \geq \sum_{j=1}^{m} 1 = m.$$  

In the best case, the degrees of any two users $u_i$ and $u'_i$ should not differ by more than 1. Hence, the maximum user degree $\max_{u_i \in U} \deg(u_i) \geq \lceil \frac{m}{n} \rceil$. $\blacksquare$

The result of Theorem 1 does not assume any concrete construction for creating the key dependency. It is rather general, discussing whether a particular key could be derived from another while maintaining security and what the best achievable key storage reduction would be. In the best scenario, a $\frac{1}{n}$ reduction factor could be achieved by eliminating all redundancy in the privileged group memberships of a system. The lower bound in Theorem 1 is also tight as can be seen from the example below.

Shown in Figure 2 is an example for the complete secure group communication with 4 users. Originally, each user has to store $2^{11} - 1 = 7$ keys. Note that $m = 11$ and $n = 4$, and hence $\lceil \frac{m}{n} \rceil = 3$. After key linking, the maximum number of keys of a user is $4 > 3$.

\(^3\)Such a derivation is possible if $k_2$ would not leak out information about $k_1$ practically. We show in the next section how such a derivation can be instantiated by a pseudorandom function.

\(^4\)Note that two fictitious nodes are added to achieve a lower storage; the lower bound stated in Theorem 1 holds here because the only effect of adding these fictitious nodes is that two resource nodes are added to the original access structure graph, which in essence increases the value of $m$. 

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Clearly, as long as there exist resource nodes (in the access structure graph) sharing a non-empty intersection between their access membership sets, key linking could always be possible between them, resulting in storage reduction at some users. However, it is not necessarily true that \( \max_{u_i \in \mathcal{U}} \{ d_i \} \), the maximum of the key storage per user (over all users), can always be reduced. We may achieve storage reduction at all users but the one with maximum storage particularly when the access structure graph is very irregular. For instance, shown in Figure 3 is a case where key linking cannot lead to a reduction on the maximum key storage per user (which is originally 3); no matter how key dependency is created, the maximum key storage per user is still 3 while the lower bound should be \( \lceil \frac{5}{8} \rceil = 1 \). Whether the lower bound on the maximum key...
storage per user (as stated in Theorem 1) can be achieved and whether key linking can reduce the maximum key storage per user depends on the access structure. Loosely speaking, if the access structure graph is dense, it is likely that a reduction on the maximum key storage per user can be achieved through key linking; if the degrees of user nodes are regular (that is, each user has access to roughly the same number of resources), and the sequence of the degrees of the resource nodes (in ascending order) does not have a sharp difference between two consecutive elements (that is, there does not exist a resource node having a considerably larger access membership set than others), key linking is most effective with the resulting maximum key storage per user closest to the lower bound.

A general key linking algorithm is designed (given in the next section) to run experiments on different access structure graphs. The results actually agree with the above observations.

IV. AN ALGORITHM TO FIND A KEY LINKING PATTERN

Depicted below is a general algorithm for finding a key linking pattern for any given access structure graph. This algorithm converts an access structure graph into one with key linking. It runs as follows:

**An algorithm to find a key linking pattern.**

**Input:** an access structure graph  
**Output:** an access structure graph with key linking

1) Sort the resource nodes according to the size of their privileged groups and assign an index (0,1,2, ..... ) to each node accordingly.
2) Pick the node with largest index to start with and set it as the current node.
3) From current node, pick a node with the next smaller index and set its index as the find-pointer.
4) Check if the node at find-pointer is a subset of the current node.
   a) If yes, mark a link at the find-pointer node to the current node and go to step 5.
   b) If not, decrease the find-pointer by 1 and repeat step 4 if the find-pointer is greater than zero, otherwise, go to step 5.
5) Decrease the current node index by 1.
6) Repeat 3-5 until the current node index is 0.

V. A KEY DERIVATION SCHEME BASED ON PSEUDORANDOM FUNCTIONS

In order to generate a key $k'$ from another key $k$, we could consider $k$ as a seed to some pseudorandom generator $g$ which outputs $k'$\[^5\] The requirement of a suitable generator is that, without the knowledge of the seed $k$, to any computationally efficient algorithm (i.e. polynomial-time), the output of $g$ is indistinguishable from any random number picked uniformly from the key space. This would ensure that the view to anyone (computationally bounded and without the knowledge of the seed) is almost identical to that without key linking, thus guaranteeing that nobody could learn any information about the seed key from the generated keys. This computational indistinguishability requirement is essential to ensuring the security of the whole system. The

\[^5\]The output space of $g$ should be the same as the key space.
explanation is as follows: Note that the resulting keys from the generator is to be used as an input key to a certain cryptographic algorithm or protocol whose security guarantee is usually based on the assumption that the input key is uniformly picked from the key space. In fact, it can be shown that, if the distribution of the key generator output is computationally indistinguishable from a uniform distribution over the key space, the security guarantee of cryptographic primitives like encryption and message authentication codes holds.

Although one-way functions or pre-image resistant hash functions have a long history of being used for linking messages in message authentication [5], [16], it should be noted that the direct application of a one-way function as the key generator is not sufficient to achieve the goal of key secrecy here. A more careful composition of one-way functions is needed for linking keys together, namely, a pseudorandom function (PRF) whose definition is as follows.

**Definition 4.** Let \( f : \{0,1\}^{l_x} \times \{0,1\}^{l_i} \rightarrow \{0,1\}^{l_o} \) be a function which takes a seed key \( s \in \{0,1\}^{l_s} \) and an input string \( x \in \{0,1\}^{l_i} \) and outputs another string \( y \in \{0,1\}^{l_o} \) (i.e. \( y = f_s(x) \)). \( f_s(\cdot) \) is is said to be taken from a pseudorandom function ensemble with index \( s \) if it satisfies that, with \( s \) uniformly picked from \( \{0,1\}^{l_s} \) and kept secret, all computationally efficient algorithms \( A \) given a set \( Z = \{(x',y') : y' = f_s(x')\} \) of evaluations of \( f_s \) at \( x' \in X \) of his choice could tell whether a given \( y \) is the output of \( f_s(\cdot) \) on input \( x \not\in X \) or randomly picked form \( \{0,1\}^{l_o} \) with a negligible advantage in \( l_s \) for all \( x \), where the advantage of an algorithm \( A \) for a given \( x \) is defined as follows:

\[
\left| Pr[s \leftarrow \{0,1\}^{l_s}; y = f_s(x) : A(Z,x,y) = 1] - Pr[y \leftarrow \{0,1\}^{l_o} : A(Z,x,y) = 1]\right|.
\]

Suppose \( f \) is a PRF. To generate a key \( k' \) for a resource (or privileged group) with label \( r' \) from another key \( k \) for a resource with label \( r \), we could consider the concatenation of the labels \( r || r' \) as an input string to \( f \) and generate \( k' \) as \( k' = f_k(r || r') \). In the next section, privileged group identities in a sensor network are used as resource labels. The property of \( f \) ensures that nobody (computationally bounded), without the knowledge of \( k \), would be able to distinguish \( k' \) from a key directly picked from the key space with a non-negligible advantage. This also guarantees that nobody could extract \( k \) from \( k' \). If there is a PPT algorithm \( A \) which can extract \( k \) from \( k' \), then it could be used to tell whether a given \( k' \) is generated form \( f \) or randomly picked as follows: run \( A \) on \( k' \) to extract \( k \) and check if \( k' \neq f_k(r || r') \); \( k' \) is a generated from \( f \) if and only if \( k' = f_k(r || r') \); otherwise, flip a coin to make a random/wild guess. Hence, the key extraction problem is at least as difficult as solving the decisional problem non-negligibly better than a wild guess. Conjectured pseudorandom functions which are efficient for the purpose here include AES-OMAC [2] and SHA-HMAC [15]. For example, if \( h(\cdot) \) denotes the HMAC function, \( f_k(x) \) can simply be implemented as \( h(k || x) \) where \( k || x \) denotes the concatenation of the secret key \( k \) and the public input \( x \).

It is natural to worry about whether such indistinguishability preserves if \( f \) is used to generate a series of keys, that is, whether \( k_t \) is still computationally indistinguishable from a random

\(^6\)Two probability distributions are said to be computationally indistinguishable when no polynomial-time distinguishing procedure can tell them apart. In other words, given a sample which could be picked from either of the two distributions, no sufficiently efficient algorithm can tell whether the sample is from the first distribution or the second.

\(^7\)Recall that a one-way function is one which is easy to evaluate in one direction but hard in the reverse. In some implementation, the output of a one-way function may leak a significant fraction of the input bits. For example, suppose \( f : \{0,1\}^2 \rightarrow \{0,1\}^2 \) is a one-way function leaking no input bit, we could construct another one-way function \( f' : \{0,1\}^{2l} \rightarrow \{0,1\}^{2l} \) in the following way: \( f'(x_1||x_2) = x_1||f(x_2) \) where \( x_1,x_2 \in \{0,1\}^l \). This is still a one-way function but leaks half of the input bits. Consequently, one might be able to distinguish between its output and a uniformly picked random number.

\(^8\)We will call \( f_z \) a pseudorandom function for the sake of simplicity despite the loss of rigor.
Theorem 2: Suppose \( f : \{0,1\}^k \times \{0,1\}^* \rightarrow \{0,1\}^k \) is a PRF, \( k \) is uniformly picked from \( \{0,1\}^k \), and \( k_1 = f_k(r||r_1) \); \( k_2 = f_{k_1}(r_1||r_2) \); \ldots; \( k_t = f_{k_{t-1}}(r_{t-1}||r_t) \). If \( t \) is polynomially many in \( l_k \), then \( \{k_t\} \approx U_{l_k} \) (denoting the two distributions are computationally indistinguishable\(^9\)) where \( \{k_t\} \) is the distribution of \( k_t \) and \( U_{l_k} \) is the uniform distribution over \( \{0,1\}^{l_k} \).

Proof. Suppose we look at the generation of \( k_t \) from \( k_{t-1} \) and assume that \( \{k_{t-1}\} \approx U_{l_k} \) with an indistinguishability coefficient \( \epsilon_{t-1} \) (defined as the maximum indistinguishability advantage achievable by any poly-time algorithm); that is, \( \epsilon_{t-1} \) is negligible. We know that \( k_t = f_{k_{t-1}}(r_{t-1}||r_t) \) and wish to show that \( \{k_t\} \approx U_{l_k} \). We use the standard hybrid argument with the hybrid distribution \( K_t’ = \{k_t’ : s_i \leftarrow \{0,1\}^k; k_t’ = f_{s_i}(r_{t-1}||r_t)\} \). From the property of the PRF, \( K_t’ \approx U_{l_k} \) with an indistinguishability coefficient \( \epsilon_f \) (negligible). We argue that \( K_t’ \approx \{k_t\} \) by contradiction. Suppose there is a PPT algorithm \( \mathcal{A} \) which can distinguish between \( K_t’ \) and \( \{k_t\} \) with a distinguishability advantage \( \epsilon’_t \), then it can be used to distinguish between \( \{k_{t-1}\} \) and \( U_{l_k} \).

The construction is as follows: for a given \( s \in \{0,1\}^k \), compute \( k = f_s(r_{t-1}||r_t) \) and run \( \mathcal{A} \) on \( k \). If \( s \in \{k_{t-1}\} \), then \( k \in \{k_t\} \), whereas, if \( s \in U_{l_k} \), then \( k \in K_t’ \). Thus this perfectly simulates the challenge of \( \mathcal{A} \) in a real attack and could be used to distinguish between \( \{k_{t-1}\} \) and \( U_{l_k} \) (a contradiction to our assumption). Hence, \( \epsilon’_t \leq \epsilon_{t-1} \).

Overall, \( \{k_t\} \approx U_{l_k} \) with an indistinguishability coefficient \( \epsilon_t \leq \epsilon’_t + \epsilon_f = \epsilon_{t-1} + \epsilon_f \). Note that when \( i = 1 \), \( \epsilon_0 = 0 \) since \( k_0 = k \in U_{l_k} \). Summing over \( i \), we have \( \epsilon_t \leq t \epsilon_f \). Since \( \epsilon_f \) is negligible in \( l_k \), if \( t \) is polynomially many, then \( \epsilon_t \) remains negligible in \( l_k \). This concludes the proof.

Since the security guarantee of a pseudorandom function is computationally complexity based, key linking based on a pseudorandom function is computationally secure.

VI. KEY LINKING FOR PAIRWISE KEY PRE-DISTRIBUTION IN SENSOR NETWORKS

In this section, we look at three examples of applying key linking to sensor network key pre-distribution (KPS). In a sensor network, each node is preloaded with a set of keys in its key ring in such a way that it can establish a pairwise key with another node in its physical neighborhood with reasonably high probability (mainly for mutual entity authentication); the model considered here is the same as that in [14], [6], [4], [12], [11]. In pairwise KPS, each privileged group consists of two users or nodes. We will ignore the repeated usage of keys, which trades off security for reduced key storage; but the discussion below should also apply to that case.

A. A Graph-theoretic Representation of KPS for Sensor Networks

When two sensor nodes share a common key, they can mutually authenticate each other. We can easily represent this keying or trust relationship in a graph; that is, the sensor nodes are represented as vertices and an edge exists between two vertices if the corresponding sensor nodes share a common key. This graph is called a keying relationship graph in the following discussion. Note that the keying relationship graph is a logical graph and does not reflect the actual network topology of the sensor network during deployment. We assume there are \( n \) sensors.

\(^9\)That is, no polynomial time algorithm can distinguish whether a given sample is from the former or latter distributions.
B. Key Linking for KPS in Sensor Networks

a) Example 1 — KPS for sensor networks with one or multiple base stations [19]: In [19], the base station of a sensor network (with \( n \) nodes) has a master key which is used (with PRF) to derive different keys, with each one being shared between the base station and a different sensor node. That is, each sensor node and the base station only needs to store a single key. The keying relationship graph is simply a star with the base station at the centre. This is indeed a special instance of the access structure graph discussed in Section III here, the number of users is \((n + 1)\) (including the base station) and the number of resources is \( n \). Applying Theorem [1], the maximum key storage in the best case is \( \left\lceil \frac{n}{n+1} \right\rceil = 1 \). Hence, the design in [19] is indeed optimal in its context. By a similar token, we could apply Theorem [1] to cases with multiple base stations.

b) Example 2 — KPS with perfect connectivity in the key relationship graph: Ideally, to ensure any pair of physical neighboring nodes in the deployed network to be able to find a shared key, each node needs to store \((n - 1)\) keys (without key linking) if there are \( n \) nodes labeled from 0 to \((n - 1)\). That is, the keying relationship graph \( G \) is a complete graph. This storage requirement is trivially impractical. Since there are \( \binom{n}{2} \) possible groups, if key linking is applied, the maximum key storage in the best case is \( \left\lceil \frac{n(n-1)}{2n} \right\rceil = \left\lceil \frac{n-1}{2} \right\rceil \) (Theorem 1); the maximum possible reduction factor is \( \frac{1}{2} \) (still not good enough).

The implementation of the linking could be done as follows. Without loss of generality, assume \( n \) is odd. A user \( i \) needs to store one seed key \( k_i \) and \( \frac{n-1}{2} \) other derived keys \( \{ k_{ji} = f_k(i||j) : j = (i-d) \bmod n, d \in [1, \frac{n-1}{2}] \} \) where \( k_{ji} \) is the pairwise key between node \( j \) (where \( j = (i-d) \bmod n, d \in [1, \frac{n-1}{2}] \)) and node \( i \). For the pairwise key between node \( i \) and node \( j' \) (where \( j' = (i+d) \bmod n, d \in [1, \frac{n-1}{2}] \)), \( k_{ij'} = f_k(i||j') \). That is, for the \( \frac{n-1}{2} \) nodes in front of node \( i \), node \( i \) has to store the derived keys, whereas, for the \( \frac{n-1}{2} \) nodes behind it, it can derive the pairwise key from \( k_i \). As a result, the overall key storage per node is \( \frac{n-1}{2} + 1 \).

c) Example 3 — KPS with bounded connectivity in the key relationship graph: In many cases, due to storage constraint, each node can only share a common key with another \( c \) nodes with \( c < n \). That is, each vertex in the keying relationship graph has a bounded degree. Without loss of generality, assume \( c \) is even.

The total number of edges of the resulting keying relationship graph \( G' \) is \( \frac{n}{2} \) which is the total number of possible groups. If key linking is applied, the maximum key storage per node in the best case is \( \left\lceil \frac{n}{2c} \right\rceil = \left\lceil \frac{n}{2} \right\rceil \); the maximum reduction factor is again \( \frac{1}{2} \). In the best possible case, a node \( i \) would only need to store one key \( k_i \) and \( \frac{c}{2} \) other derived keys \( k_{ji} \). The problem of determining which half of the \( c \) pairwise keys are derived from \( k_i \) and which half are obtained from other \( \frac{c}{2} \) nodes could be solved by finding an Eulerian tour over \( G' \). An Eulerian tour over a graph \( G \) is a tour along the edges of \( G \) so that each edge is passed exactly once. Such a tour exists in a graph \( G \) if \( G \) has at most two vertices with an odd degree; this is fulfilled for \( G' \) in question. The Fleury’s algorithm (shown in Appendix) with running time \( O(|E(G)|) \) (where \(|E(G)|\) is the total number of edges in \( G \)) can be used for finding an Eulerian tour [8] and the set of edges of each vertex would be partitioned by the tour into two halves, one marked as incoming edges and the other as outgoing edges. Now we could derive all pairwise keys on an outgoing edge of node \( i \) using \( k_i \) and set the keys of the incoming edges as \( k_{ji} \) derived from \( k_j \) of another node \( j \).

Regarding the case with an odd number of edges in a connectivity graph, edges could be added to make the graph Eulerian. If there are vertices in a connectivity graph \( G \) with an odd number of edges, the total number of such vertices should be even\(^{10} \). We could simply partition such a

\(^{10}\) The sum of the degrees of all vertices of a graph is even. If a vertex has an odd degree, then there must exist another vertex with an odd degree to make the total sum even. That is, vertices with an odd degree come in pairs.
subset of vertices into pairs and assign an edge to each pair, then the resulting graph is Eulerian.

While a regular keying relationship graph is considered in this example, the result and technique apply to a more general keying relationship graph as long as one can partition the edges connecting each vertex into two parts. Reduction on maximum key storage could always be achieved.

Recall that given any key of a node on a particular outgoing edge of the key relationship graph, it is computationally infeasible to find the keys on other outgoing edges of the node, thus guaranteeing the resilience of compromised nodes. Any collusion of compromised nodes would not threat the security of the remaining nodes since we have considered an ideal access structure and it is computationally hard for the collusion to find any key not originally held by them if a pseudorandom function is used.

VII. DISCUSSIONS

While there is always reduction in the average storage whenever there is redundant membership, key linking may not lead to reduction on the maximum key storage per user in some access structure. Under the framework of constraints considered in this paper, it could be difficult to achieve reduction on the maximum storage in those cases. An analogy to this situation is when Huffman encoding for the equiprobable case. This sets the limits of any scheme if trading off security is not considered. If further reduction on the maximum key storage is necessary, trading off the ideal access structure is one possible solution and combinatorial techniques could apply as in [3], [1], [17], [13]. Alternatively, we could consider a set of keys as a long bit string (instead of a set of individual keys) and create the linking on a bit-by-bit basis using the technique of correlated pseudorandomness [9]; however, the gain also comes as a result of trading off security; now a non-privileged user may learn some of the bits of a resource key he is not supposed to whereas the key linking technique considered in this paper would not leak out information that can be efficiently extracted by a non-privileged user.

VIII. CONCLUSION

As applications involving low cost devices like sensor networks and RFID emerge, memory cost (for secure key storage) which is usually not a concern has become an essential constraint to designing security systems. To alleviate this, the key storage requirement could possibly reduced by creating dependency among secret keys stored in a user device, that is, key linking. Key linking exploits redundancy in privileged group memberships for key compression.

We derive an upper bound for maximum achievable key compression in a system with ideal, general access structure. This bound is tight and can somehow be treated as a negative result, which demonstrates that without trading off security, considerable key storage reduction may not be achievable. We also show a provably secure instantiation of key linking scheme using pesudorandom functions. We show how to apply the key linking technique to reduce key storage in pairwise key pre-distribution in wireless sensor networks; the storage reduction is still not sufficient to give efficient schemes which again demonstrate the cost in efficiency loss we have to pay if no security tradeoff is considered. The results actually provide ground for proposals which trade off security for efficiency.

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APPENDIX

Fleury’s Algorithm

Fleury’s algorithm constructs an Euler circuit in a graph (if it’s possible). The algorithm runs as follows:

1. Pick any vertex to start.
2. From that vertex pick an edge to traverse, considering following rule: never cross a bridge of the reduced graph unless there is no other choice.
3. Darken that edge, as a reminder that you can’t traverse it again.
4. Travel that edge, coming to the next vertex.
5. Repeat Steps 2-4 until all edges have been traversed, and you are back at the starting vertex.

By “reduced graph” we mean the original graph minus the darkened (already used) edge.