First order dissipative hydrodynamics from an effective covariant kinetic theory

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The first order hydrodynamic evolution equations for the shear stress tensor, the bulk viscous pressure and the charge current have been studied for a system of quarks and gluons, with a non-vanishing quark chemical potential and finite quark mass. The first order transport coefficients have been obtained by solving an effective Boltzmann equation for the grand-canonical ensemble of quarks and quasigluons. We adopted temperature dependent effective fugacity for the quasiparticles to encode the hot QCD medium effects. The non-trivial energy dispersion of the quasiparticles induce mean field contributions to the transport coefficients whose origin could be directly related to the realization of conservation laws from the effective kinetic theory. Further, the relative significance of dissipative quantities has been investigated through their respective ratios. Both the QCD equation of state and chemical potential are seen to have a significant impact on the QGP evolution. The first order viscous corrections to the time evolution of temperature along with the description of pressure anisotropy of the system have also been explored.

Keywords: Effective kinetic theory, Quark-gluon plasma, Dissipative evolution, Quark chemical potential, Pressure anisotropy.

I. INTRODUCTION

High energy heavy-ion collision (HIC) experiments in Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) have realized the existence of a new state of matter-quark gluon plasma (QGP), as a near-perfect fluid [1,2]. Relativistic hydrodynamics has been successfully employed to describe the space-time evolution of the created deconfined nuclear matter; see Refs. [3–9] for recent reviews. On the other hand, the input parameters such as equation of state and transport coefficients, have been estimated from the microscopic theories. The inclusion of dissipative effects in the QGP evolution is significant for explaining the quantitative behavior of experimental observables in the HIC, i.e., collective flow, transverse momentum spectra etc. [10,11]. The theoretical explanation of the hadron elliptic flow at RHIC with dissipative hydrodynamic evolution provide the evidence of transport processes in the QCD medium [15]. The relevance of the transport process in the HIC is reconfirmed in [16–19].

There have been various approaches/​attempts for the estimation of the transport parameters of the hot QCD medium [20–29]. To explore the relative significance of transport parameters, their ratios have been studied in recent literatures [30,31]. The quantitative estimation of shear viscosity from experiments has been widely investigated in several works [32,40]. In parallel, there have been some attempts to study the effect of bulk viscosity in the evolution of the QGP [41–45]. Notably, the effect of dissipative charge current has received less attention compared to the viscous coefficients in the framework of dissipative hydrodynamics. This can be attributed to the fact that the net baryon number and chemical potential are insignificant in the very high energetic collisions. However, for the lower collision energies probed in the RHIC beam energy scan and for upcoming experiments at Facility for Antiproton and Ion Research (FAIR), the baryon chemical potential can no longer be neglected. In addition to the effects of the chemical potential, the finite quark mass corrections are also significant in the evolution of the QGP in this context. This sets the motivation to investigate the hydrodynamic evolution of the QGP with a non-vanishing baryon chemical potential and finite quark mass.

The description of the QCD medium evolution requires the knowledge of microscopic description of thermodynamic quantities of the medium along with the appropriate momentum distribution functions of its effective degrees of freedom (quasi-quarks/antiquarks and quasigluons). To that end, encoding the thermal medium effects in the hot QCD equations of state (computed within lattice QCD or Hard Thermal Loop theory) in terms of quasiparton degrees of freedom with nontrivial dispersion relations has turned out to be a viable approach. The quasiparticle description of thermodynamic and transport properties of the hot QCD/QGP medium have been investigated in several works [27,31,46–53]. In the current analysis, we utilize the effective fugacity quasiparticle model (EQPM) [54,55] for the effective microscopic description of the QGP. The microscopic framework for the estimation of transport coefficients in the current analysis is done within the covariant kinetic approach using relativistic Boltzmann equation. We employ the Chapman-Enskog like iterative method to solve the relativistic transport equation with relaxation time approximation (RTA) for the collision kernel, along with a mean field term arising from the quasi-particle description of QGP.
The mean field term in the effective covariant kinetic theory with the EQPM can be realized from the conservation laws as described in the Ref. [56]. The goal of the current analysis is to investigate these mean field corrections to the dissipative quantities with non-vanishing baryon chemical potential and quark mass, within EQPM. The relative behavior of different dissipative processes can be estimated with the respective ratios of their transport coefficients in the light of the mean field contributions and finite quark chemical potential. We study the viscous corrections to the time evolution of temperature and pressure anisotropy by analyzing the boost invariant longitudinal expansion. These aspects are crucial in the investigation of the hydrodynamic evolution of the QGP from the covariant effective kinetic theory.

The manuscript is organized as follows. The mathematical formulation of the first order dissipative hydrodynamic evolution equations from the EQPM covariant kinetic theory along with the description of longitudinal Bjorken flow is presented in section II. Section III deals with the discussions on the mean field contributions and the relative significance of transport coefficients. Finally, in section IV, the conclusion and outlook have been presented.

II. FORMALISM

The formalism for the estimation of dissipative hydrodynamic evolution of the QGP consists of the quasiparticle modeling followed by the setting up of the effective covariant kinetic theory of the system away from equilibrium. The current analysis is based on a covariant kinetic theory for hot QCD medium recently developed by Chandra and Mitra [54] employing the effective fugacity quasiparticle model [51, 55]. Here, we have extended the approach to investigate the transport properties of the hot QCD medium with finite quark chemical potential and quark/antiquark masses. There are several other quasiparticle models present in the literature to describe hot QCD medium which include models with effective masses for the quasiparticles [57, 62], a self-consistent, single parameter quasiparticle models with temperature dependent effective mass [63, 64], NJL and PNJL based quasiparticle models [65, 67], and recently proposed quasiparticle model based on the Gribov-Zwanziger quantization [68, 70].

Notations and Conventions: In this article, we have used the following notations and conventions. The quantity \( u_μ \) is the fluid velocity (normalized to unity) and in the fluid rest frame \( u^μ = (1, 0, 0, 0) \). The metric tensor is taken to be \( g^{μν} = \text{diag}(1, -1, -1, -1) \). The subscript \( k \) used in the manuscript implies the particle species, \( k = (g, q, q) \), where \( g, q \) and \( \bar{q} \) denotes gluons, quarks and antiquarks, respectively. The quantity \( g_k \) represents the degeneracy factor of the \( k \)-th species. We choose the appropriate gluon and quark/antiquark degeneracy factors respectively as \( g_g = N_s \times (N_c^2 - 1) \) and \( g_q = N_s \times N_c \times N_f \), where \( N_f = 3 \) is the number of flavors, \( N_s = 2 \) is the spin degrees of freedom and \( N_c = 3 \) is the number of colors.

A. QCD thermodynamics and the effective covariant kinetic theory with finite chemical potential

Realizing the hot QCD medium as a Grand-canonical ensemble, the EQPM interprets the hot QCD equation of states (EoS) with quasigluon and quasiquark/antiquark effective fugacities. Here, we have considered the \( (2 + 1) \)-flavor lattice QCD EoS for the effective description of QGP [71, 72]. The EQPM energy-momentum tensor can be defined in terms of dressed momenta \( \tilde{p}_k \) of \( k \)-th particle species and takes the following form [56].

\[
T^μν(x) = \sum_k g_k \int d\tilde{P}_k \tilde{p}_k^{μ} \tilde{p}_k^{ν} f_k(x, \tilde{p}_k) + \sum_k δω_k g_k \int d\tilde{P}_k \frac{\langle \tilde{p}_k^{μ} \tilde{p}_k^{ν} \rangle}{E_k} f_k(x, \tilde{p}_k),
\]

where \( k = (g, q, q) \) represents the particle species, \( f_k(x, \tilde{p}_k) \) is the quasiparton distribution function, \( \langle \tilde{p}_k^{μ} \tilde{p}_k^{ν} \rangle \equiv \frac{1}{Z} \frac{1}{Z} \left( \sum_α \frac{1}{z_α} \exp \left[ -\beta \left( E_α - \mu_α \right) \right] \right) \frac{1}{Z} \frac{1}{Z} \left( \sum_β \frac{1}{z_β} \exp \left[ -\beta \left( E_β - \mu_β \right) \right] \right) \tilde{p}_k^{μ} \tilde{p}_k^{ν} \) and \( d\tilde{P}_k \equiv \frac{d^4 \tilde{P}_k}{(2\pi)^4} \) is the momentum integral measure. In the above equation, \( \tilde{p}_k^μ \) is the “dressed” four-momentum of particles of species \( k \) defined later in terms of bare particle four momentum \( p_k^μ \) and effective fugacity. We consider nonzero quark mass \( m_q \) of different flavor (with \( m_u = 3 \text{ MeV}, m_d = 5 \text{ MeV} \) and \( m_s = 100 \text{ MeV} \) for up, down and strange quarks, respectively) and energy \( E_k = \sqrt{\tilde{p}_k^2 + m_q^2} \) for quarks/antiquarks whereas for gluons \( E_k = |\tilde{p}_k| \).

The covariant form of EQPM parton distribution functions in equilibrium, with a non-zero baryon chemical potential \( \mu_g \) can be defined as

\[
f_q^{\prime} = \frac{z_q \exp \left[ -\beta(u_μ p_μ - \mu_q) \right]}{1 + z_q \exp \left[ -\beta(u_μ p_μ - \mu_q) \right]},
\]

\[
f_q = \frac{z_q \exp \left[ -\beta(\mu_q - p_μ) \right]}{1 + z_q \exp \left[ -\beta(\mu_q - p_μ) \right]},
\]

\[
f_q^{\prime} = \frac{z_q \exp \left[ -\beta(\mu_q - p_μ) \right]}{1 - z_q \exp \left[ -\beta(\mu_q - p_μ) \right]},
\]

where we define the scalar product \( u_μ p^μ \equiv u_μ p^μ \) and the inverse temperature \( \beta \equiv 1/T \). The quantities \( z_q, z_q^{\prime} \) and \( z_q \) denote the temperature dependent effective fugacity parameter of the quarks, anti-quarks and gluons, respectively, that encode the hot QCD medium effects in the quasiparticle description of the QGP. The effective fugacities are not related with any conserved number current in the hot QCD medium and thus retain the same form in the case of a small finite baryon chemical potential. Therefore the fugacity parameter for quark and
antiquark is same, i.e., \( z_q = z_{\bar{q}} \) in the EQPM description of the QGP \cite{31,33,34}. Therefore, in the rest of this article, we denote the fugacity parameter for both quasiquark and antiquark by \( z_q \). Since the effective fugacities are not related with any conserved number current in the QGP medium, the temperature dependence of the effective fugacity parameter remain unaltered with the finite chemical potential, as discussed in the Ref. \cite{31}. As expected, in the limit of vanishing quark chemical potential, i.e., \( \mu_q = 0 \), the equilibrium distribution functions for quasiquark and anti-quark becomes identical \( f_q^0 \equiv f_{\bar{q}}^0 \).

The dispersion relation encodes the collective excitation of quasipartons and relates the quasiparticle (dressed) four-momenta \( \tilde{p}_k^\mu \) and the bare particle four-momenta \( p_k^\mu \) as follows,

\[
\tilde{p}_k^\mu = p_k^\mu + \delta \omega_k u^\mu, \quad \delta \omega_k = T^2 \partial_T \ln(z_k).
\]

The zeroth component of the four-momenta is modified as,

\[
\tilde{p}_k^0 \equiv \omega_k = E_k + \delta \omega_k.
\]

Next, we focus on the net baryon four-current \( N^\mu \) which is defined as the difference of baryon and anti-baryon four-current \cite{31,33}. The quasiparticle description of the flow in terms of dressed momenta has the following form \cite{33}

\[
N^\mu(x) = g_q \int d\tilde{p}_q \tilde{p}_q^\mu \left[ f_q(x, \tilde{p}_q) - f_{\bar{q}}(x, \tilde{p}_q) \right] + \delta \omega_q \frac{d\tilde{p}_q^\mu}{E_q} \left[ f_q(x, \tilde{p}_q) - f_{\bar{q}}(x, \tilde{p}_q) \right],
\]

where \( \langle \tilde{p}_q^\mu \rangle \equiv \Delta_\mu^{\tilde{p}_q} \tilde{p}_q^\mu \) is the irreducible tensor of rank one. Note that the quark and antiquark four-momentum is same i.e. \( \tilde{p}_q^\mu = \tilde{p}_{\bar{q}}^\mu \) since both sectors have identical mass \( m_q \). The relevant thermodynamic quantities such as energy density, pressure, the speed of sound and number density can be obtained from their basic thermodynamic definitions.

From Eq. (1), we obtain the expression of the energy density \( \varepsilon \) and the pressure \( P \), respectively, within the EQPM by using the following definitions

\[
\varepsilon \equiv u_{\mu} u_{\nu} T^{\mu \nu}, \quad P \equiv -\frac{1}{3} \Delta_{\mu \nu} T^{\mu \nu},
\]

along with the matching condition \( \varepsilon = \varepsilon_0 \) and \( n = n_0 \) where the subscript ‘0’ represents equilibrium quantities.

In the case of finite quark mass, \( m_q \), and non-vanishing baryon chemical potential, \( \mu_q \), the total energy density and pressure can be expressed in terms of modified Bessel function of second kind, \( K_n(y) \), and Polylog functions as

\[
\varepsilon = \sum_{l=1}^{\infty} g_q \frac{y^2 T^4 (-1)^{l-1} z_q^l \cosh(\alpha l)}{\pi^2 l^2} \left[ K_1(ly) - K_0(ly) \right] + \frac{2 \delta \omega_q}{yT} \left( K_3(ly) - K_1(ly) \right) + g_q \frac{T^3}{\pi^2} \times \left[ 3T \text{PolyLog} [4, z_q] + \delta \omega_q \text{PolyLog} [3, z_q] \right],
\]

and

\[
P = \sum_{l=1}^{\infty} g_q \frac{y^2 T^4 (-1)^{l-1} z_q^l \cosh(\alpha l)}{\pi^2 l^2} K_2(ly) + g_q \frac{T^4}{\pi^2} \text{PolyLog} [4, z_q],
\]

where \( y \equiv \beta m_q \) and \( \alpha \equiv \beta \mu_q \). In the \( z_q \to 1 \), i.e., \( \delta \omega_q \to 0 \) limit, the expressions for energy density and pressure given in Eqs. (9) and (10), respectively, reduce to that obtained in Ref. \cite{29}. Similarly, one can obtain the quasiparticle net baryon density using the definition \( n \equiv u_q N^\mu \) along with the matching conditions. The net baryon density is then given by

\[
n = \sum_{l=1}^{\infty} g_q \frac{y^2 T^3 (-1)^{l-1} z_q^l \sinh(\alpha l)}{\pi^2 l} K_2(ly).
\]

Note that in the limit of vanishing chemical potential, \( \alpha \to 0 \), net baryon density disappears.

From Eqs. (9) and (10), one can obtain the results for massless case by setting \( m_q = 0 \). In this case, we obtain the EQPM energy density and pressure of the hot QGP with a non-zero \( \mu_q \) in terms of Polylog functions and have the following form,

\[
\varepsilon = \frac{3 T^4}{\pi^2} \left[ g_q \text{PolyLog} [4, z_q] - g_q \left( \text{PolyLog} [4, -z_q e^{-\alpha}] + \text{PolyLog} [4, -z_q e^{\alpha}] \right) \right] + \delta \omega_q \frac{g_q T^3}{\pi^2} \left( \text{PolyLog} [3, -z_q e^{\alpha}] - \text{PolyLog} [3, -z_q e^{-\alpha}] \right),
\]

and

\[
P = \frac{T^4}{\pi^2} \left[ g_q \text{PolyLog} [4, z_q] - g_q \left( \text{PolyLog} [4, -z_q e^{\alpha}] + \text{PolyLog} [4, -z_q e^{-\alpha}] \right) \right].
\]

Similarly, the net baryon density in the massless limit takes the following form,

\[
n = \frac{g_q T^3}{\pi^2} \left( \text{PolyLog} [3, -z_q e^{-\alpha}] - \text{PolyLog} [3, -z_q e^{\alpha}] \right).
\]

Note that the results of Eqs. \cite{13,14} matches with that obtained in Ref. \cite{27}. We also observe that in the limit \( z_q \to 1 \), Eqs. \cite{13,14} reduces to that obtained in Ref. \cite{24}.

The macroscopic definition of viscous tensor and the baryon diffusion current requires the non-equilibrium part of the distribution function of the particles. For the system close to local thermodynamic equilibrium,
the non-equilibrium quasiparton phase space distribution function takes the form \( f_k = f_k^0 + \delta f_k \), where \( \delta f_k / f_k^0 \ll 1 \) and the equilibrium distribution \( f_k^0 \) are given in Eqs. (17)-(19) for \( k = (q, q, g) \). Macroscopically, the energy-momentum tensor in the non-equilibrium case can be decomposed as,

\[
T^{\mu\nu} = \varepsilon u^\mu u^\nu - (P + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu},
\]

where \( \Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu \) is the projection operator orthogonal to the fluid velocity, \( \pi^{\mu\nu} \) is the shear stress tensor and \( \Pi \) is the bulk viscous pressure. Similarly, the baryon four-current can be macroscopically described as,

\[
N^\mu = nu^\mu + n^\mu.
\]

Note that the above expressions for energy-momentum tensor and particle four-current are written for fluid four-velocity defined in Landau frame.

The projection of \( T^{\mu\nu} \) and \( N^\mu \) conservation equations along and orthogonal to \( u^\mu \) gives,

\[
\dot{\varepsilon} + (\varepsilon + P + \Pi) \theta - \pi^{\mu\nu} \sigma_{\mu\nu} = 0, \quad (\varepsilon + P + \Pi) u^\mu \nabla^\alpha (P + \Pi) + \Delta^{\mu\nu} \partial_\alpha \pi^{\mu\nu} = 0, \quad \dot{n} + n \theta + \partial_\mu n^\mu = 0,
\]

where \( \theta \equiv \partial_\mu u^\mu \) is the expansion scalar, \( \dot{A} \equiv u^\mu \partial_\mu A \) represents the comoving derivative, \( \nabla^\alpha \equiv \Delta^{\alpha\beta} \partial_\beta \) is a space-like derivative operator which is orthogonal to \( u^\mu \) and \( \sigma^{\mu\nu} \equiv \Delta^{\alpha\beta} \partial_\alpha u^\beta \). Here we define a four-index tensor \( \Delta^{\mu\nu}_{\alpha\beta} \equiv \frac{1}{2} (\Delta^{\mu\alpha} \Delta^{\nu\beta} + \Delta^{\nu\alpha} \Delta^{\mu\beta}) \frac{1}{2} \Delta^{\mu\nu} \Delta_{\alpha\beta} \) which is a traceless symmetric projection operator orthogonal to the fluid velocity.

The expressions for the derivatives of \( \alpha \) and \( \beta \) can be obtained from Eqs. (17)-(19) and taking the following form,

\[
\dot{\beta} = \chi_\beta \theta + O(\delta^2), \quad \dot{\alpha} = \chi_\alpha \theta + O(\delta^2),
\]

\[
\nabla^\mu \beta = -\beta \dot{u}^\mu + \frac{n}{\varepsilon + P} \nabla^\mu \alpha + O(\delta^2),
\]

with \( \chi_\beta \) and \( \chi_\alpha \) taking the following form,

\[
\chi_\beta = \left[ J_{q_0}^{(0)+} (\varepsilon + P) - J_{q_0}^{(0)-} n - J_{q_0}^{(0)+} n / J_{q_0}^{(0)+} + J_{q_0}^{(0)-} n - J_{q_0}^{(0)+} n / J_{q_0}^{(0)+} + J_{q_0}^{(0)-} n - J_{q_0}^{(0)+} n \right] / J_{q_0}^{(0)+} + J_{q_0}^{(0)-} n - J_{q_0}^{(0)+} n,
\]

and,

\[
\chi_\alpha = \left[ J_{g_{0q}}^{(0)+} (\varepsilon + P) - J_{g_{0q}}^{(0)+} n - J_{g_{0q}}^{(0)-} n / J_{g_{0q}}^{(0)+} + J_{g_{0q}}^{(0)-} n - J_{g_{0q}}^{(0)+} n / J_{g_{0q}}^{(0)+} + J_{g_{0q}}^{(0)-} n - J_{g_{0q}}^{(0)+} n \right] / J_{g_{0q}}^{(0)+} + J_{g_{0q}}^{(0)-} n - J_{g_{0q}}^{(0)+} n.
\]

Here \( J_{q_0}^{(r)+} \) are the thermodynamic integrals defined as

\[
J_{q_{nm}}^{(r)+} = \frac{g_y}{2\pi^2 (2m + 1)!} \int_0^{\infty} d \tilde{p}_q \left| (u, \tilde{p}_q) \right|^{-2m-r-1} \times (| \tilde{p}_q |)^{2m+2} f_q \tilde{f}_q,
\]

\[
J_{g_{nm}}^{(r)} = \frac{g_y}{2\pi^2 (2m + 1)!} \int_0^{\infty} d \tilde{p}_g \left| (u, \tilde{p}_g) \right|^{-2m-r-1} \times (| \tilde{p}_g |)^{2m+2} f_g \tilde{f}_g,
\]

where \( f_q^\pm = f_q \tilde{f}_q \pm f_q \tilde{f}_q \) and \( \tilde{f}_q \equiv (1 - a f_q) \) with \( a = -1 \) and \( +1 \) for Bose-Einstein and Fermi-Dirac statistics, respectively. The expressions of these integral coefficients appearing in Eqs. (22) and (23), in terms of temperature and chemical potential, are given in Appendix A.

The shear stress tensor \( \pi^{\mu\nu} \) can be expressed in terms of \( \delta f_k \) within EQPM as follows [56],

\[
\pi^{\mu\nu} = \sum_k g_k \Delta^{\mu\nu}_{\alpha\beta} \int d\tilde{p}_k \tilde{p}_k^\alpha \tilde{p}_k^\beta \frac{\delta f_k}{E_k},
\]

where \( k = (g, q, \bar{q}) \) represents the particle species. Similarly, the bulk viscous pressure \( \Pi \) and the particle diffusion current \( n^\mu \) can also be defined as,

\[
\Pi = -\frac{1}{3} \sum_k g_k \Delta_{\alpha\beta} \int d\tilde{p}_k \tilde{p}_k^\alpha \frac{\delta f_k}{E_k},
\]

\[
n^\mu = g_q \Delta_{\alpha} \int d\tilde{p}_q \tilde{p}_q^\alpha \frac{\delta f_q - \delta f_{\bar{q}}}{E_q - \delta f_{\bar{q}}}. \]

We will use the above equations for dissipative quantities to obtain their first-order expressions and corresponding transport coefficients.

The relativistic transport equation quantifies the rate of change of quasiparton phase space distribution function in terms of collision integral \( C[f_k] \) and has the following form

\[
P_{\mu}^e \partial_\mu f_k(x, \tilde{p}_k) + F_{\mu}^e (u \cdot \tilde{p}_k) \frac{\partial \pi}{\partial h} f_k = C[f_k],
\]

where \( P_{\mu}^e = -\partial_\mu (\delta f_k u^\mu) \) is the force term defined from the conservation of energy momentum and particle flow. In the current EQPM framework, the collision integral is defined in the relaxation time approximation (RTA), where the thermal relaxation \( \tau_R \) linearizes the collision term as [73]

\[
C[f_k] = -\frac{\delta f_k}{\tau_R}.
\]

To obtain \( \delta f_k \), we solve the relativistic Boltzmann equation with RTA using the Chapman-Enskog like iterative expansion.

B. First order dissipative evolution equation

The first order correction to distribution functions for quarks, anti-quarks and gluons can be obtained from the
Boltzmann equation, Eq. \[(29)\], by considering an iterative Chapman-Enskog like solution \([76, 77]\). For the current effective kinetic theory, we obtain the following form,

\[
\delta f_q = \tau_R \left[ \frac{\bar{p}_q}{u_p} \partial_q - \beta \delta \omega_q \right] f_q \tilde{f}_q,
\]

(31)

\[
\delta f_\pi = \tau_R \left[ \frac{\bar{p}_\pi}{u_p} \partial_q - \beta \delta \omega_\pi \right] f_\pi \tilde{f}_\pi,
\]

(32)

\[
\delta f_g = \tau_R \left[ \frac{\bar{p}_g}{u_p} \partial_q - \beta \delta \omega_g \right] f_g \tilde{f}_g,
\]

(33)

With the expressions for \(\delta f_q\) obtained in Eqs. \[(31)-(33)\], we can obtain the first order evolution equation for dissipative quantities from Eqs. \[(29)-(30)\].

Assuming the thermal relaxation time \(\tau_R\) to be independent of particle four-momenta and keeping terms up to first-order in gradients, we obtain

\[
\pi^{\mu\nu} = 2 \tau_R \beta_\pi \sigma^{\mu\nu},
\]

\[
\Pi = -\gamma \epsilon, \quad \Pi^{\mu} = \kappa_n \nabla^\mu \epsilon,
\]

(34)-(35)

where the coefficients have the following form,

\[
\beta_\pi = \beta \left[ \tilde{J}_{q 42}^{(1)+} + \tilde{J}_{q 42}^{(1)} + (\delta \omega_q) \tilde{L}_{q 42}^{(1)+} + (\delta \omega_q) \tilde{L}_{q 42}^{(1)} \right],
\]

(37)

\[
\beta_{\Pi} = \beta \left[ \frac{\lambda \beta}{\beta} \left( \tilde{J}_{q 31}^{(0)+} + \tilde{J}_{q 31}^{(0)} + (\delta \omega_q) \tilde{L}_{q 31}^{(0)+} + (\delta \omega_q) \tilde{L}_{q 31}^{(0)} \right) + \frac{\lambda}{\beta} \left( \tilde{J}_{q 31}^{(0)+} + \tilde{J}_{q 31}^{(0)} + (\delta \omega_q) \tilde{L}_{q 31}^{(0)+} + (\delta \omega_q) \tilde{L}_{q 31}^{(0)} \right) \right.

\]

\[
+ \frac{\lambda}{3} \left[ \tilde{J}_{q 42}^{(1)+} + \tilde{J}_{q 42}^{(1)} + (\delta \omega_q) \tilde{L}_{q 42}^{(1)+} + (\delta \omega_q) \tilde{L}_{q 42}^{(1)} \right] - (\delta \omega_q) \tilde{J}_{q 21}^{(1)+} - (\delta \omega_q) \tilde{J}_{q 21}^{(1)} \right],
\]

(38)

\[
\beta_\Pi = \left[ \frac{n}{(\epsilon + P)} \right] \left[ \tilde{J}_{q 21}^{(0)+} + (\delta \omega_q) \tilde{L}_{q 21}^{(0)+} \right] - \tilde{J}_{q 21}^{(1)+}
\]

\[
- (\delta \omega_q) \tilde{L}_{q 21}^{(1)}. \]

(39)

The thermodynamic integrals labeled by \(\tilde{L}_{k n m}^{(r)\pm}\) appearing in the above expressions are defined as,

\[
\tilde{L}_{q n m}^{(r)\pm} = \frac{g_q}{2 \pi^2 (2m + 1)!} \int_0^\infty d \|\tilde{p}_q\| \frac{(u_q \tilde{p}_q)^{n-2m-r-1}}{E_q} \times (\|\tilde{p}_q\|)^{2m+2} f_q^{(r)\pm},
\]

(40)

\[
\tilde{L}_{q n m}^{(r)} = \frac{g_q}{2 \pi^2 (2m + 1)!} \int_0^\infty d \|\tilde{p}_q\| \frac{(u_q \tilde{p}_q)^{n-2m-r-1}}{\|\tilde{p}_q\|} \times (\|\tilde{p}_q\|)^{2m+2} f_q \tilde{f}_q,
\]

(41)

The expressions for the integral coefficients \(\tilde{J}_{k n m}^{(r)\pm}\) appearing in Eqs. \[(37)-(39)\] are given in Appendix \(A\) in terms of temperature and chemical potential.

By comparing the Eqs. \[(34)-(36)\] with the relativistic Navier-Stokes equations \([78]\), we can obtain the coefficients of bulk viscosity, shear viscosity and charge conductivity as \(\beta_\pi \tau_R = \eta, \beta_\Pi \tau_R = \zeta\) and \(\beta_{n} \tau_R = \kappa_n\), respectively. Note that we consider a special case where the relaxation times for all particle species are same. The general case with different thermal relaxation time is left for future analysis. The form of above integrals for the massive and massless case, are presented in the Appendix \(A\).

C. Longitudinal boost-invariant expansion

To model the dissipative hydrodynamical evolution of the QGP formed in the heavy-ion collision experiments, we employ the Bjorken’s prescription \([79]\) for one-dimensional boost invariant expansion. Here, we consider the case of vanishing baryon chemical potential. The evolution equation of the energy density for purely longitudinal boost-invariant expansion can be expressed in terms of Milne coordinates \((\tau, x, y, \eta)\), where \(\tau = \sqrt{t^2 - z^2}\) and \(\eta = \tanh^{-1}(z/t)\) resulting in \(u^\mu = (1, 0, 0, 0)\) with the metric tensor given by \(\sigma^{\mu\nu} = (1, -1, -1, -1/t^2)\) \([77]\).

Employing the Milne coordinate system, the energy evolution equation Eq. \[(17)\] gets simplified to,

\[
\frac{d \epsilon}{d \tau} = - \left( \frac{\epsilon + P}{\tau} \right) + \left( \frac{\zeta + 4 \eta/3}{\tau^2} \right),
\]

(43)

where we have used \(\theta = 1/\tau, \Pi = -\zeta/\tau, \Pi^{\mu\nu} = \Phi/\tau\) and \(\Phi = 4 \eta/3 \tau\). We numerically solve Eq. \[(43)\] to study the evolution of viscous nuclear matter with the values of dissipative quantities given in Eq. \[(37)\] and Eq. \[(38)\], imposing the LEOs. The initial condition in RHIC (for Pb-Pb collision) is \(T = 0.36 \text{ GeV at } \tau_0 = 0.6 \text{ fm} \) and in LHC (for Au-Au collision) is \(T = 0.5 \text{ GeV at } \tau_0 = 0.4 \text{ fm} \). We estimated the temperature evolution by assuming relaxation time to be same for both bulk and shear parts \((\tau_R = \tau_0 = \tau_1 = 0.25 \text{ fm})\). With these conditions, we can investigate the proper time evolution of longitudinal pressure \((P_L)\) to transverse pressure \((P_T)\), \(P_L/P_T \equiv (P + \Phi) / (P + \Pi + \Phi/2)\), where \(P\) is the equilibrium thermodynamic pressure.

III. RESULTS AND DISCUSSIONS

We initiate the discussion with the temperature dependence of mean field corrections to the shear tensor, bulk viscous pressure and the particle diffusion, respectively, of the hot QGP with finite quark chemical potential. The
The relative significance of charge conductivity $\kappa_n$ and shear viscosity $\eta$ could be understood in terms of the ratio $\kappa_n T/\eta$. Within RTA, the quantity $\beta_\pi T/\beta_\Pi = \kappa_n T/\eta$ is plotted in Fig. 3 (left panel) for different quark chemical potential $\mu_q$. The ratio becomes almost constant at high temperature regimes, whereas it drops for the low temperature regime, indicating that the conductivity of the medium is relatively small to the shear viscosity in the regime $T \lesssim 2.5 T_c$ as compared to very high temperature regimes. We observe a similar trend in the temperature behavior of the mean field force term from the effective theory appears as the mean field corrections to the transport coefficients of the system. As mentioned earlier, the force term consists of the modified part of the EQPM dispersion relation $\delta\omega_k \equiv T^2 \partial_T \ln z_k$. At high temperature region $T/T_c \sim 2.5$, where $T_c = 0.17$ GeV is the transition temperature, the fugacity parameters $z_k$ are slowly varying functions of temperature. Since $\delta\omega_k$ is the temperature gradient of the $z_k$, the mean field effects are not significant at higher temperature region. The mean field contributions to first order coefficients of the shear tensor and bulk viscous pressure at quark chemical potential $\mu_q = 0.1$ GeV are depicted in Fig. 1 (left panel).

Since the mean field corrections at high temperature regimes are negligible, the ratio asymptotically tends to unity. However, the mean field contributions due to the quasiparticle excitation are significant in the lower temperature regime. The effects of mean field contributions to the dissipative quantities are shown in Fig. 1. We observe the quantitative difference in the $\beta_\pi$ and $\beta_\Pi$ with and without the mean field corrections at low temperature regimes. In Fig. 1 (right panel), the mean field effects to the first order coefficient of particle diffusion is shown for different quark chemical potential $\mu_q$. The dependence of finite quark mass and baryon chemical potential to the mean field contributions are separately shown in the left and right panel of Fig. 1 respectively. The mean field correction to the transport parameters with binary, elastic collisions at $m_q = 0$ and $\mu_q = 0$ is described in Ref. [56]. The effects of quark mass and chemical potential are visible in the low temperature regimes whereas in the higher temperature regimes the mean field contributions are almost independent on $m_q$ and $\mu_q$.

In Fig. 2, we show the temperature dependence of the ratio of the coefficient of the bulk viscous tensor to that of the shear tensor $\frac{\beta_n}{\beta_\pi}$ at $\mu_q = 0.1$ GeV. In the RTA, the ratio becomes $\frac{\beta_n}{\beta_\pi} = \frac{\zeta}{\eta}$, where $\zeta$ is the bulk viscosity of the hot QGP medium. Within the EQPM, the squared speed of sound tends to the Boltzmann limit $\frac{1}{3}$ at very high temperature. Since the bulk viscosity is proportional to this term $\frac{1}{\zeta}$, the effective description of $\zeta$ with EQPM tends to zero at high temperature regime. We observe that the temperature behavior of the ratio $\frac{\beta_n}{\beta_\pi}$ has a decreasing trend with the increase in temperature. We observe that the quark mass correction and mean field corrections are more visible in the low temperature regime near to the transition temperature $T_c$. Further, we compared the results with other parallel work and the lattice results. We found that our observations are consistent with the results of [22][24].
dimensionless ratio $\frac{\kappa_q T^2}{\eta \pi^2 T}$, in which $\kappa_q$ is the coefficient of thermal conductivity. The quantity $\frac{\kappa_q}{\eta}$ is defined as $\frac{\kappa_q}{\eta} = (\beta_n/\beta_\pi) \frac{(\varepsilon + P)}{nT}$ within RTA [74]. In the high temperature limit, the ratio reduces to $\frac{\kappa_q}{\eta} = C \frac{\pi^2 T}{\mu_q^2}$ as shown in the Fig. 3 (right panel).

For the non-interacting QGP $z_k = 1$, the value of the constant becomes $C = 95/81$ [74]. However, it should be noted that for the realistic EoS, the value of $C$ is equal to 5/4. The EoS dependence of the viscous coefficients and conductivity are captured by the effective quasiparton fugacities. Also, the mean field effects on the ratio are more visible in lower temperature regime whereas the effects are negligible at higher temperature limit. This is due to the fact that in the high temperature regime, the value of $z_k$ approaches unity. In the ideal limit $z_k = 1$, the EQPM results can be reduced to the first order dissipative hydrodynamic evolution equation with appropriate coefficients as described in [74].

In Fig. 3 (left panel), we depicted the proper time evolution of temperature and pressure anisotropy in ideal and first order hydrodynamics with initial temperature $T_0 = 500$ MeV at proper time $\tau_0 = 0.4$ fm. We assumed the Navier-Stokes initial condition for shear and bulk viscous part respectively as $\Phi = 4\eta/3\tau_0$ and $\Pi = -\zeta/\tau_0$, with the thermal relaxation time $\tau_R = 0.25$ fm. The temperature evolution based on the first order dissipative hydrodynamics shows slower temperature drop with proper time compared to the ideal evolution. Following the temperature evolution, the proper time dependence of the pressure anisotropy $P_L/P_T$ is shown in Fig. 3 (right panel). We see that compared to the first order evolution results for $P_L/P_T$ in case of non-interacting Boltzmann particles [74], there is a slightly faster approach to isotropization in the present EQPM model.

**IV. CONCLUSION AND OUTLOOK**

In this paper, we have derived the first order dissipative hydrodynamic evolution equations within an effec-
tive covariant kinetic theory by realizing the system as a grand canonical ensemble of gluons and quarks, with a finite baryon chemical potential $\mu$, and non-zero quark mass $m_q$. The covariant effective kinetic theory is employed for the hot QCD matter within the EQPM. The thermal medium effects have been encoded through the EQPM by introducing the lattice equation of state in phase space momentum distribution through the effective fugacity parameter. We observed that the mean field contributions that emerge from the covariant kinetic theory induce sizable modification to the first order coefficients of the shear stress tensor, bulk viscous pressure and the particle diffusion of the hot QGP medium in the temperature regime near to $T_c$. However, the modifications to the first order coefficients are negligible at higher temperatures ($T \geq 2.5 T_c$). In the massless limit, our estimations at $\mu = 0$ agree with results of Ref. [56], for the binary elastic collisions.

We further studied the ratio of viscous coefficients $\frac{\beta}{\kappa}$ and $\frac{\kappa_{\theta^2}}{\beta}$ respectively within RTA for different quark chemical potential. We found that at the lower temperature the charge conductivity is relatively smaller compared to the results at higher temperatures. Also, the effect of the baryon chemical potential is more visible in the temperature regime near to $T_c$. The proper time evolution of temperature and pressure anisotropy are seen to be sensitive to the viscous effects and the equation of state. Finally, various predictions of the current work, turned out to be consistent with the other parallel results.

The analysis presented in the manuscript is the first step towards the higher order (second and third) dissipative hydrodynamic evolution equation from the effective covariant kinetic theory within the EQPM. The investigation of the hydrodynamic evolution equations for the hot magnetized QGP medium (magnetohydrodynamics) would be another interesting direction to pursue. In addition, deriving the transport coefficients using a more realistic collision term is another problem worth investigating. We leave these problems for future work.

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**Appendix A: Thermodynamic integrals**

1. Massive case

For the case of massive quasipartons, the scalar thermodynamic integrals $\tilde{j}^{(r)\pm}_{k\,nm}$ and $\tilde{j}^{(r)\pm}_{k\,nm}$ can be expressed in terms of the modified Bessel function of second kind as shown in the following:

\[
\tilde{j}^{(1)+}_{q\,42} = \frac{g_q T^3 y^3}{240 \pi^2} \sum_{l=1}^{\infty} (-1)^{l-1} z_q^l \cosh(l\alpha) \left[ K_0(l\gamma) - 7K_3(l\gamma) + 22K_1(l\gamma) + 16K_{1,1}(l\gamma) \right] \\
- \delta \omega_q \frac{g_q T^3 y^3}{60 \pi^2} \sum_{l=1}^{\infty} (-1)^{l-1} z_q^l \cosh(l\alpha) \left[ K_4(l\gamma) - 8K_2(l\gamma) + 15K_0(l\gamma) - 8K_{1,2}(l\gamma) \right],
\]

\[
\tilde{j}^{(0)-}_{q\,21} = -\frac{g_q T^3 y^3}{12 \pi^2} \sum_{l=1}^{\infty} (-1)^{l-1} z_q^l \sinh(l\alpha) \left[ K_4(l\gamma) - 4K_2(l\gamma) + 3K_0(l\gamma) \right] \\
+ \delta \omega_q \frac{g_q T^3 y^3}{12 \pi^2} \sum_{l=1}^{\infty} (-1)^{l-1} z_q^l \sinh(l\alpha) \left[ K_3(l\gamma) - 5K_1(l\gamma) + 4K_{1,1}(l\gamma) \right],
\]
\[
\tilde{J}^{(0)}_{q\ 21} = - \frac{g_4 T^4 y^4}{12\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_4(l_\alpha) - 4K_2(l_\alpha) + 3K_0(l_\alpha) \right] \right.
\]
\[
+ \delta_\omega q_4 \frac{T^3 y^3}{12\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_3(l_\alpha) - 5K_1(l_\alpha) + 4K_{i,1}(l_\alpha) \right] \right],
\]  
(A3)

\[
\tilde{J}^{(1)}_{q\ 21} = - \frac{g_4 T^3 y^3}{12\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_3(l_\alpha) - 5K_1(l_\alpha) + 4K_{i,1}(l_\alpha) \right] \right.
\]
\[
+ \delta_\omega q_4 \frac{T^2 y^2}{3\pi^2} \sum_{l=1}^{\infty} (-1)^{l-1} q^4 \cosh(\alpha) \left[ K_2(l_\alpha) - 3K_0(l_\alpha) + 2K_{i,2}(l_\alpha) \right],
\]  
(A4)

\[
\tilde{J}^{(0)}_{q\ 31} = - \frac{g_4 T^5 y^5}{16\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_5(l_\alpha) - 3K_3(l_\alpha) + 2K_1(l_\alpha) \right] \right],
\]  
(A5)

\[
\tilde{J}^{(0)}_{q\ 30} = \frac{g_4 T^5 y^5}{16\pi^2} \sum_{l=1}^{\infty} l(-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_5(l_\alpha) + K_3(l_\alpha) - 2K_1(l_\alpha) \right]
\]
\[
+ \delta_\omega q_4 \frac{T^4 y^4}{4\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_4(l_\alpha) - K_0(l_\alpha) \right] \right],
\]  
(A6)

\[
\tilde{J}^{(0)}_{q\ 20} = \frac{g_4 T^4 y^4}{8\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \sinh(\alpha) \left[ K_4(l_\alpha) - K_0(l_\alpha) \right] \right.
\]
\[
+ \delta_\omega q_4 \frac{T^3 y^3}{4\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \sinh(\alpha) \left[ K_3(l_\alpha) - K_1(l_\alpha) \right] \right],
\]  
(A7)

\[
\tilde{J}^{(0)}_{q\ 10} = \frac{g_4 T^3 y^3}{4\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \sinh(\alpha) \left[ K_3(l_\alpha) - K_1(l_\alpha) \right] \right],
\]  
(A8)

\[
\tilde{J}^{(0)}_{q\ 30} = \frac{g_4 T^5 y^5}{16\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_5(l_\alpha) + K_3(l_\alpha) - 2K_1(l_\alpha) \right] \right]
\]
\[
+ \delta_\omega q_4 \frac{T^4 y^4}{4\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_4(l_\alpha) - K_0(l_\alpha) \right] \right],
\]  
(A9)

\[
\tilde{J}^{(1)}_{q\ 42} = \frac{g_4 T^4 y^4}{120\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_4(l_\alpha) - 8K_2(l_\alpha) + 15K_0(l_\alpha) - 8K_{i,2}(l_\alpha) \right] \right],
\]  
(A10)

\[
\tilde{L}^{(0)}_{q\ 31} = -\frac{2 g_4 T^4 y^4}{24\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_4(l_\alpha) - 4K_2(l_\alpha) + 3K_0(l_\alpha) \right] \right],
\]  
(A11)

\[
\tilde{L}^{(0)}_{q\ 21} = -\frac{g_4 T^3 y^3}{12\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \sinh(\alpha) \left[ K_3(l_\alpha) - 5K_1(l_\alpha) + 4K_{i,1}(l_\alpha) \right] \right],
\]  
(A12)

\[
\tilde{L}^{(1)}_{q\ 21} = -\frac{g_4 T^2 y^2}{6\pi^2} \sum_{l=1}^{\infty} \left[ (-1)^{l-1} \frac{1}{4} q^4 \cosh(\alpha) \left[ K_2(l_\alpha) - 3K_0(l_\alpha) + 2K_{i,2}(l_\alpha) \right] \right],
\]  
(A13)

where the function \(K_{i,n}(l_\alpha)\) is defined as,
\[
K_{i,n}(l_\alpha) = \int_0^\infty \frac{d\theta}{(\cosh \theta)^n} \exp (-l_\alpha \cosh \theta).
\]  
(A14)
2. Massless case

For the massless case and non-vanishing baryon chemical potential, the thermodynamic integrals given in Eqs. \[(37)\] \[(39)\] take the following form,

\[
\dot{J}_{q 42}^{(1)+} = \frac{2g_q T^5}{5\pi^2} \left[ -2 \left\{ \text{PolyLog} [4, -e^\alpha z_q] + \text{PolyLog} [4, -e^{-\alpha} z_q] \right\} + \frac{\delta \omega_q}{T} \left\{ \text{PolyLog} [3, -e^{-\alpha} z_q] + \text{PolyLog} [3, -e^\alpha z_q] \right\} \right],
\]

\[
\dot{J}_{q 21}^{(0)-} = \frac{g_q T^4}{\pi^2} \left\{ \text{PolyLog} [3, -e^\alpha z_q] - \text{PolyLog} [3, -e^{-\alpha} z_q] \right\} - \frac{\delta \omega_q}{3T} \left\{ \text{PolyLog} [2, -e^{-\alpha} z_q] - \text{PolyLog} [2, -e^\alpha z_q] \right\},
\]

\[
\dot{J}_{q 21}^{(1)+} = \frac{g_q T^4}{3\pi^2} \left\{ \text{PolyLog} [2, -e^{-\alpha} z_q] + \text{PolyLog} [2, -e^\alpha z_q] \right\} + \frac{\delta \omega_q}{T} \left\{ \log [1 + e^{-\alpha} z_q] + \log [1 + e^\alpha z_q] \right\},
\]

\[
\dot{J}_{q 31}^{(0)+} = -\frac{4g_q T^5}{\pi^2} \left[ \text{PolyLog} [4, -e^{-\alpha} z_q] + \text{PolyLog} [4, -e^\alpha z_q] \right],
\]

\[
\dot{J}_{q 30}^{(0)+} = \frac{6g_q T^5}{\pi^2} \left[ 2 \left\{ \text{PolyLog} [4, -e^\alpha z_q] - \text{PolyLog} [4, -e^{-\alpha} z_q] \right\} - \frac{\delta \omega_q}{3T} \left\{ \text{PolyLog} [3, -e^{-\alpha} z_q] + \text{PolyLog} [3, -e^\alpha z_q] \right\} \right],
\]

\[
\dot{J}_{q 20}^{(0)-} = \frac{g_q T^4}{\pi^2} \left\{ 3 \left\{ \text{PolyLog} [3, -e^{-\alpha} z_q] - \text{PolyLog} [3, -e^\alpha z_q] \right\} + \frac{\delta \omega_q}{T} \left\{ \text{PolyLog} [2, -e^{-\alpha} z_q] + \text{PolyLog} [2, -e^\alpha z_q] \right\} \right\},
\]

\[
\dot{J}_{q 10}^{(0)+} = -\frac{g_q T^3}{\pi^2} \left[ \text{PolyLog} [2, -e^{-\alpha} z_q] + \text{PolyLog} [2, -e^\alpha z_q] \right],
\]

\[
\dot{L}_{q 42}^{(1)+} = -\frac{g_q T^4}{5\pi^2} \left[ \text{PolyLog} [3, -e^{-\alpha} z_q] + \text{PolyLog} [3, -e^\alpha z_q] \right],
\]

\[
\dot{L}_{q 31}^{(0)+} = \frac{g_q T^4}{2\pi^2} \left[ \text{PolyLog} [3, -e^{-\alpha} z_q] + \text{PolyLog} [3, -e^\alpha z_q] \right],
\]

\[
\dot{L}_{q 21}^{(0)-} = \frac{g_q T^3}{3\pi^2} \left[ \text{PolyLog} [2, -e^{-\alpha} z_q] - \text{PolyLog} [2, -e^\alpha z_q] \right],
\]

\[
\dot{L}_{q 21}^{(1)+} = -\frac{g_q T^2}{6\pi^2} \left[ \log [1 + e^\alpha z_q] + \log [1 + e^{-\alpha} z_q] \right],
\]

For the gluonic case we obtain,

\[
\dot{J}_{q 42}^{(1)} = \frac{6g_q T^5}{15\pi^2} \left[ 2 \text{PolyLog} [4, z_q] - \frac{\delta \omega_q}{T} \text{PolyLog} [3, z_q] \right],
\]
\[ \tilde{j}^{(0)}_{g,31} = -\frac{4g_s T^5}{\pi^2} \text{PolyLog} \{4, z_g\}, \]  
(A28)

\[ \tilde{j}^{(0)}_{g,21} = -\frac{9g_s T^4}{\pi^2} \text{PolyLog} \{3, z_g\}, \]  
(A29)

\[ \tilde{j}^{(0)}_{g,30} = \frac{6g_s T^5}{\pi^2} \left[ 2 \text{PolyLog} \{4, z_g\} + \frac{\delta \omega_2}{T} \text{PolyLog} \{3, z_g\} \right], \]  
(A30)

\[ \tilde{L}^{(1)}_{g,42} = \frac{g_s T^4}{5\pi^2} \text{PolyLog} \{3, z_g\}, \]  
(A31)

\[ \tilde{j}^{(0)}_{g,31} = -\frac{9g_s T^4}{\pi^2} \text{PolyLog} \{3, z_g\}. \]  
(A32)

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