Prediction of the point of impact in an anisotropic plate

M Koabaz 1, T Hajzargarbashi 2, T Kundu 2 and M Deschamps 1

1 Université de Bordeaux, CNRS, UMR 5469, Laboratoire de Mécanique Physique, 351 cours de la libération, Talence, F-33405, France.

2 Department of Civil Engineering and Engineering Mechanics, University of Arizona, Tucson, AZ 85721, United States.

Co-authors’ E-mail addresses:

m.koabaz@lmp.u-bordeaux1.fr

talieh@email.arizona.edu

tkundu@email.arizona.edu

m.deschamps@lmp.u-bordeaux1.fr

Abstract

Locating the point of impact of a foreign object in a plate is important for continuous health monitoring of structures. A new method based on an optimization scheme has been recently proposed to locate the point of impact in anisotropic plates by analyzing the times of arrival of the ultrasonic signals at the passive sensors attached to the plate. Following this optimization based technique, in this paper the impact point on an anisotropic plate is predicted from the acoustic emission data. Experiments are carried out with a carbon-epoxy plate where the impact point is modeled by an acoustic source. A Parallel Pre-stressed Actuator (PPA) is used as the acoustic source and the acoustic signals at different locations are received by adhesively bonded acoustic sensors. The source point is then predicted and compared with its actual location. Related theory is also presented in the paper.

1. Introduction

The point of impact for an isotropic plate can be located after detecting the acoustic emission signal by at least three sensors and applying the triangulation technique. However, if the plate is anisotropic then the triangulation technique does not work, because this technique assumes that the wave velocity is the same in all directions, which is not true for anisotropic plates.

An alternative method based on an optimization scheme was proposed by Kundu et al. [1-4] to locate the point of impact in anisotropic plates by analyzing the time of arrival of the ultrasonic signals received by the passive sensors attached to the plate. Recently Hajzargarbashi et al. [5] improved the impact point prediction technique. Following their modification, in this paper the impact point on an anisotropic plate is predicted from the acoustic emission data. The method originally proposed in Kundu et al. [1] is based on the optimization technique – minimization of a non-linear objective function or error function. However, the proposed objective function in that reference had the inherent problem of multiple singularities which was overcome in Kundu et al. [2] by modifying the objective function. The optimization technique was further improved by Hajzargarbashi et al. [5]. Using this latest optimization technique the impact point in a Carbon-epoxy plate is predicted and compared with the exact point of impact.
The method proposed in this paper for locating the point of impact works well for any type of anisotropy because it uses the experimentally obtained direction dependent Lamb wave velocity profile.

2. Formulation

The formulation presented in Kundu et al. [1-2] and Hajzargarbashi et al. [3] is repeated here. Let the time of detection of the acoustic signal at the i-th station be $t_{id}$. If the time of impact be $t_c$ then the travel time for the signal from the impact point to the station location is:

$$t_i = t_{id} - t_c \tag{1}$$

Note that in (1) both $t_{id}$ and $t_c$ are defined with respect to the same time of reference. If the coordinates of a receiving sensor $S_i$ is $(x_i, y_i)$ with $i = 1, 2, 3\ldots n$ and the impact point coordinate is $(x_0, y_0)$ then the distance of the sensor from the impact point is given by:

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} \tag{2}$$

The time of travel of the wave to the sensor locations is denoted by $t_i$ and the velocity $c(\theta)$ of the wave in the plate is a function of the wave propagation direction $\theta$, therefore, one can write:

$$d_i = c(\theta_i)t_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} \tag{3}$$

From (3) it is possible to define an error function $E(x_0, y_0)$, also known as the objective function in the optimization literature. Values of $(x_0, y_0)$ can be obtained by minimizing the objective function. The objective function should be expressed in terms of relative times of arrival as defined below, instead of the actual times of arrival that are difficult to obtain:

$$t_{ij} = t_{id} - t_{jd} = (t_i + t_c) - (t_j + t_c) = t_i - t_j \tag{4}$$

It removes the need for explicit knowledge of the time of impact, $t_c$. If the wave velocity is $c(\theta)$ then the relative times of arrival are defined as:

$$t_{12} = t_1 - t_2 = \frac{d_1}{c(\theta_1)} - \frac{d_2}{c(\theta_2)} = \frac{c(\theta_2)d_1 - d_2c(\theta_1)}{c(\theta_1)c(\theta_2)} \tag{5}$$

$$= \frac{c(\theta_2)\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} - \sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2}c(\theta_1)}{c(\theta_1)c(\theta_2)}$$

The error function $E(x_0, y_0)$ can be defined as:

$$E(x_0, y_0) = (t_1 - t_2)c(\theta_1)c(\theta_2) - \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}c(\theta_2) + \sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2}c(\theta_1) \tag{6}$$

Ideally, for the correct values of $(x_0, y_0)$ the error function should give a zero value while for wrong values of $(x_0, y_0)$ it should give a positive value. Therefore, we need to minimize the value of this error function or objective function. To give equal importance or bias to three measurements of arrival times at the three sensor locations...
the error function can be defined in a different manner as shown below. With this definition of the error function, the impact point location prediction should not be strongly influenced by the experimental error in any one time of arrival measurement [5]:

\[
E(x_0, y_0) = (t_1 - t_2)e(\theta_1)c(\theta_2) - \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \cdot c(\theta_1) + \sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2} \cdot c(\theta_2) \\
+ (t_1 - t_3)e(\theta_3)c(\theta_3) - \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \cdot c(\theta_3) + \sqrt{(x_3 - x_0)^2 + (y_3 - y_0)^2} \cdot c(\theta_1) \\
+ (t_2 - t_3)e(\theta_2)c(\theta_3) - \sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2} \cdot c(\theta_2) + \sqrt{(x_3 - x_0)^2 + (y_3 - y_0)^2} \cdot c(\theta_2) (7)
\]

By some optimization scheme the impact point \((x_0, y_0)\) can be obtained by minimizing the error function given in (7).

3. Experimental investigation

Experiments were carried out on a 910 mm x 795 mm carbon-epoxy plate where the impact point is modeled as an acoustic source. The plate is 3 mm thick with fibers going in the 0° direction. A Parallel Pre-stressed Actuator (PPA) is used as the acoustic source as shown in figure 1, and the acoustic signals at different locations are received by adhesively bonded acoustic sensors. Figure 2 shows the experimental setup.

Wave velocity in different directions of the plate are obtained by placing 10 sensors at 10° angular intervals varying from 0° to 90°. Velocity measurements are shown in figure 3 at 150 KHz. The signal received from the sensor was visualized on an oscilloscope like the signal shown in figure 4. Signals were then averaged before being recorded on a computer. Angles were measured from the direction of the fibers (0° direction) to its perpendicular direction (90° direction). For every direction, the envelopes were calculated to extract the Energy velocities of the exited modes. Knowing the distance \(d\), separating the transmitter and the receiver, and dividing this distance by the arrival times, one can obtain the velocity of the guided wave mode.
The experimental signal at 150 kHz is quite simple in the fiber direction, but it becomes much more complex in the cusp range, see figure 5.

To avoid the complexity that comes from the cusp of the SH mode, the experiment was carried out at 45 kHz frequency that excites only $A_0$ and $S_0$ modes. The experimental measurement of the velocity of $A_0$ mode is shown in figure 6.
After obtaining the wave velocity profile in the carbon-epoxy plate a second experiment is carried out to predict the impact point from the acoustic emission data. For this experiment four passive sensors are adhesively bonded to the plate as shown in figure 7. Sensors 1, 2, 3 and 4 (S1, S2, S3 and S4) are placed away from the point of impact. The location of the acoustic source is denoted as S0; it corresponds to the impact point. From the time history of the acoustic source (denoted as S0 in figure 7) the exact time of impact can be obtained.

It is not easy to measure the exact time of arrival of the signals from the time history plots (envelope of the signals) because there is some ambiguity in these plots about the exact time of arrival or the starting point of the signals. Arrival of the weak extensional mode or the $S_0$ mode is hidden in the low level noise present in the time history plots. The arrival time of the stronger $A_0$ mode is obtained from the plots by the threshold technique. The arrival time of the first noticeable peak that is greater than the noise level is recorded as the arrival time of the ultrasonic energy.

4. Optimization of the objective function

Since the objective function (see Eq. (7)) uses the differences in the recorded arrival times the small errors in the arrival time recording, mentioned in the previous section should not affect the final results significantly. If there is redundancy in the number of sensors then the effects of these small errors on the impact location estimate also would tend to be minimized. In saying so it should also be kept in mind that guided waves propagating in a composite plate are dispersive and attenuative. Therefore, if the propagation distances from the impact point to different receivers vary significantly then, due to dispersion and attenuation, the shapes of the signal recorded by
different receivers look significantly different and recording the arrival time by the threshold technique may not give accurate results. In that situation time-domain correlation technique [9, 10] applied to two time-domain signals recorded by two sensors can be followed to accurately obtain the difference between the arrival times at the two sensors than what is considered here.

The \( x \) and \( y \) values corresponding to the minimum of the objective function (see Eq. (7)) should give the impact point or the optimized coordinate values \((x_0, y_0)\). Because of multiple local minima of the objective function the standard optimization schemes such as the Simplex algorithm [11] do not always converge to the absolute minimum. However, successful convergence is possible by finding the \( y \)-coordinate corresponding to the minimum values of the function for different \( x \)-values, then among these minimum values the absolute minimum is identified; \( x \) and \( y \) values corresponding to this absolute minimum are the coordinates of the impact point.

Hajzargarbashi et al. [5] have shown that if the number of receiving acoustic sensors is increased from 3 to 4 then the accuracy of the impact point prediction is significantly improved. In the objective function it is not necessary to consider all 4 receiving sensors simultaneously, 4 combinations of 3 sensor sets can be used to predict the impact point. To construct a set of 3 sensors from a group of 4, one sensor is left out. In this manner 4 different sets of three sensors can be constructed from a group of 4 sensors. Impact points predicted using 4 different sets of 3 sensors and 1 set of 4 sensors are shown in figures 8 to 11. Different points in these figures are generated by randomly giving a small variation (less than 5 percent) in the recorded arrival time and plotting the point where the optimization scheme converges, figures 8 and 9 correspond to two slightly different objective functions – one given by Kundu et al. [2] generates figure 8 and the second objective function proposed by Hajzargarbashi et al. [5] produces figure 9. These two objective functions are denoted as “old” [2] and “new” [5] objective functions. Both figures 8 and 9 are obtained from 4 sets of 3 sensors. When the times of arrival of all four sensors are considered simultaneously in the objective function instead of considering 4 sets of 3 sensors separately then figures 10 and 11 are obtained from these two objective functions. The velocity profile shown in figure 6 has been used to predict the impact point. Note that the accuracy of the final prediction is about the same for the two objective functions but the scattering in the predicted data points due to the measurement error in the arrival times decreases with the new objective function.

![Figure 8: Impact points exact and predicted using the old objective function for 4 sets of 3 sensors.](image8)

![Figure 9: Impact points exact and predicted using the new objective function for 4 sets of 3 sensors.](image9)
5. Conclusion

Optimization schemes proposed by Kundu et al. [1-4] and Hajzargarbashi et al. [5] to locate the acoustic source in an anisotropic plate by analyzing the times of arrival of the ultrasonic signals are used in this paper. Following this optimization based technique, the acoustic source point on a Carbon-epoxy plate is predicted from the acoustic emission data. Experiments are carried out with an acoustic source (PPA) placed on the plate modeling a point of impact. It is shown that the source point can be correctly predicted by this technique when the anisotropic nature of the velocity profile in the plate is considered.

Acknowledgement:

Third author T. Kundu would like to acknowledge the financial support he received from the University of Bordeaux where he spent one month as an invited professor during this collaborative research.

References

[1] Kundu T, Das S and Jata K V 2007 Point of Impact Prediction in Isotropic and Anisotropic Plates from the Acoustic Emission Data J. Acoust. Soc. Am. 122 (4) 2057-2066.

[2] Kundu T, Das S, Martin S A and Jata K V 2008 Locating Point of Impact in Anisotropic Fiber Reinforced Composite Plates Ultrasonics 48(3) 193-201.

[3] Kundu T, Das S and Jata K V 2009 Health Monitoring of a Thermal Protection System using Lamb Waves Structural Health Monitoring: An International Journal 8(1) 29-45.

[4] Kundu T, Das S and Jata K V 2009 Impact Point Detection in Stiffened Plates by Acoustic Emission Technique Smart Materials & Structures 18 Article #035006.

[5] HajZargarbashi T, Kundu T and Bland S 2010 A new algorithm for detecting impact points in anisotropic plates by the acoustic emission technique Health Monitoring of Structural and Biological Systems IV, Ed. T. Kundu, SPIE's 17th Annual International Symposium on Smart Structures and Materials & Nondestructive Evaluation and Health Monitoring, San Diego, California, March 7-11, 2010.

[6] Kessler S S, Spearing S and Soutis C 2002 Damage detection in composite materials using lamb wave methods Smt. Mater. Str. 12 795–803.

[7] Kundu T, Das S and Jata K V 2007 An improved technique for locating the point of impact from the acoustic emission data Proc. SPIE 6532.
[8] Sachse W and Sancar S 1986 Acoustic emission source location on plate-like structures using a small array of transducers *U.S. Patent* No. 4,592,034.

[9] Kosel T, Grabec I and Muzic P 2000 Location of acoustic emission sources generated by air flow *Ultrasonics* 38 824–826.

[10] Ziola S M and Gorman M R 1991 Source location in thin plates using cross-correlation *J. Acoust. Soc. Am.* 90 2551–2556.

[11] Nelder J A and Mead R 1965 A simplex method for function minimization *Comp. J.* 7 308–315.