THEORETICAL INVESTIGATION OF TWO-FLUID ELECTROOSMOTIC FLOW IN MICROCHANNELS

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Abstract: This paper presents theoretical investigations of the pressure-driven two-liquid flow in microchannels with electroosmosis effect. For a fully developed, steady state, laminar flow of two liquids combined the pressure gradient and the electroosmotic effects, we have derived analytical solutions that relate the velocity profiles and flow rates to the liquid holdup, the aspect ratio of the microchannel, the viscosity ratio of the two liquids and the externally applied electric field.

1. Introduction

Electroosmosis refers to a liquid flow induced by an external electric field. When contacted with a polar solution, most surfaces acquire a finite surface charge density, which induces the electrically neutral solution to have a distribution of electrical charges. This high capacitance charged region of ions at the liquid/solid interface is known as the electrical double layer (EDL) [1]. EDL is primarily a surface phenomenon; its effect tends to appear when the typical channel dimension is of the order as the EDL thickness (Debye length). The thickness of EDL is dependent on the bulk ionic concentration and electrical properties of liquid, usually ranging from several nanometers to hundred. With the development of the biochip technology, electroosmosis has been widely used as a suitable pumping mechanism in the MEMS devices for chemical and biological analysis and medical diagnoses [2].

Many research works have been published for single conducting fluid involved electroosmotic flow and electroosmotic pump. Burgreen and Nakache[3] studied the effect of the surface potential on liquid transport through ultra-fine capillary slits under an imposed electrical field. Rice and Whitehead [4] studied the same problem in narrow cylindrical capillaries. Levine et al.[5] extended the Rice and Whitehead model to high zeta-potential for the electrokinetic flow. Mala et al.[6] reported microchannel flow and heat transfer on two parallel-plate and Yang et al.[7,8] studied electroviscous effect in rectangular microchannels. Maynes and Webb[9] analyzed the electroosmotic flow in a parallel plate microchannel and circular microtube under imposed constant wall heat flux and constant temperature boundary conditions.

In application, Zeng et al[10] fabricated an EOF pump by packing the 3.5μm diameter non-porous silica into 500-700μm diameter fused-silica capillaries. The proposed pump can generate maximum pressures in excess of 20atm and flow rates of 3.6μl/min for a 2kV applied potential. Chen et al [11] fabricated an EOF pump on soda-lime glass substrate using standard microlithography and chemical wet etching techniques. The pump provided a maximum pressure and flow rate performance of 0.33 atm and 1.5 μl/min at 1 kV applied potential.

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The aim of this work is to provide a theoretical analysis of the two-fluid flow combined pressure gradient and electroosmotic effect. In practice, the cross-section of microchannel made by modern micromachining technology is close to a rectangular shape. Analytical solutions of EDL in the conducting fluid and velocities of two fluids are obtained in the fully-developed section in a rectangular channel.

Consider a two-liquid system, in which one is a conducting liquid containing ionic charges with a charged smooth solid wall being contacted and another is a non-conducting fluid as shown in Figure 1. When an external electric field is applied across the conducting liquid, the external electric field interacts with these net charges within the double layers and creates electroosmotic body forces on the bulk conducting liquid [12]. If the external electric field varies, such applied electroosmotic body forces will be changed correspondingly. The electric potential in the conducting liquid due to the charged wall and velocity profiles of two liquids in rectangular channel are solved analytically.

\[ \nabla^2 \varphi = K \lambda \nabla^2 \varphi \]  
(1)

Where \( K = L \kappa \) is the ratio of the length scale \( L \) to the characteristic double layer thickness \( 1/\kappa \). Here \( \kappa \) is the Debye-Hückel parameter. The electric potential is equal to the zeta potential, \( \zeta \), at bottom channel wall (\( y=-h_1 \)) and \( \zeta \) at the side wall (\( x=w \)), but vanished at the interface because of two miscible liquids are considered. Normally, because that the channel walls are made of different materials. The solution of Eq. (1) has been obtain in the paper by Gao et al. [12]

We assume that the two liquids are Newtonian and incompressible and that the Reynolds number (\( Re = \rho_1 LV/\mu_1 \), where \( \mu \) is the liquid viscosity) of the flow is much smaller than unity so that inertia effects on the liquid flow may be neglected. For a fully developed flow, the momentum equation for the conducting liquid becomes

\[ \nabla^2 \rho = \text{Re} \frac{dp}{dx} - S \rho \]  
(2)

where \( \frac{dp}{dx} \) is a constant dimensionless pressure gradient. \( S \) is a parameter, which is defined as

\[ S = \frac{2z_e n_e E L}{\mu V} \]

For the nonconducting liquid, the momentum equation gives

![Figure 1. Two-fluid electroosmotic flow system and schematic of coordinate](image-url)
\[ \beta = \mu_2 / \mu_1 \]

where \( \beta = \mu_2 / \mu_1 \) is the dynamic viscosity ratio. At the interface \( \gamma = 0 \), matching conditions have to be obeyed. They are the continuities of velocity and hydrodynamic shear stress, which are represented as

\[ \begin{align*}
\vec{u} = \vec{u}_c & \\
\frac{\partial \vec{u}}{\partial y} = \beta \frac{\partial \vec{u}_c}{\partial y} & - M \vec{\rho}_c
\end{align*} \]  

(4)

where \( M = (\varepsilon k T E_z) / (\zeta_e U_\text{ref}^{\text{rel}}) \) and the dynamic viscosity ratio is defined as \( \beta = \mu_2 / \mu_1 \). Because of linearity, the velocity of the conducting liquid in Eq. (6) can be decomposed into two parts,

\[ \vec{u}_c = \vec{u}_c^e + \vec{u}_c^p \]  

(5)

where \( \vec{u}_c^e \) corresponds to the velocity component driven by the electroosmotic forces and \( \vec{u}_c^p \) corresponds to the velocity component by the pressure gradient. Consequently, the velocity of the nonconducting liquid is decomposed as

\[ \vec{u}_c = \vec{u}_c^e + \vec{u}_c^p \]  

(6)

where \( \vec{u}_c^e \) corresponds to the velocity component influenced by the electroosmotic flow in the conducting liquid and \( \vec{u}_c^p \) corresponds to the velocity component by the pressure gradient.

Using the separation of variables method, the analytical velocity components corresponding to the electroosmotic force are obtained as

\[ \vec{u}_c^e (\gamma, \zeta) = \sum_{j=1}^{\infty} \left[ \sinh(\lambda, \zeta) - \tanh(\lambda, \zeta) \cosh(\lambda, \zeta) \right] b \int \cos(\lambda, \zeta) \]  

(7a)

and

\[ \vec{u}_c^e (\gamma, \zeta) = \sum_{j=1}^{\infty} \left[ \sinh(\lambda, \zeta) - \tanh(\lambda, \zeta) \cosh(\lambda, \zeta) \right] b \int \cos(\lambda, \zeta) \]  

(7b)

The velocity components corresponding to the pressure gradient are

\[ \vec{u}_c^p (\gamma, \zeta) = \sum_{j=1}^{\infty} 4 \text{Re}(-1)^{j-1} \frac{\partial \Phi_j(\gamma)}{\partial \gamma} \cosh(\lambda, \zeta) \int \sinh(\lambda, \zeta) \cos(\lambda, \zeta) \]  

(7c)

and

\[ \vec{u}_c^p (\gamma, \zeta) = \sum_{j=1}^{\infty} 4 \text{Re}(-1)^{j-1} \frac{\partial \Phi_j(\gamma)}{\partial \gamma} \cosh(\lambda, \zeta) \int \sinh(\lambda, \zeta) \cos(\lambda, \zeta) \]  

(7d)

Where \( \Phi_j \) is parameter function \([12]\).

### 3. Results and discussion

#### 3.1. Effect of viscosity ratio

The flow characteristics depend on the coupling effect between the two fluids which involve the electrokinetic driving force in the conducting liquid and the interfacial phenomenon. The interfacial phenomenon is the balance of the modified stress force as shown in Eq. (4), which involves the opposite electrostatic force exerted on the interface and the hydrodynamic shear stress between the two liquid. The velocity at the liquid-liquid interface must match, i.e. the conducting liquid and the
non-conducting velocities must be the same and the forces must be balanced. To investigate the effect of viscosity ratio between the two fluids, the values of are chosen as different values. Fig. 2 shows the dimensionless velocity profiles at the symmetric line when the viscosity ratio is equal to 1, 10 and 100.

![Dimensionless velocity profiles](image)

**Figure 2.** Dimensionless velocity distribution at the symmetric line for different viscosity ratio

### 3.2. Effect of aspect ratio

The aspect ratio is defined as $\chi = h / w$ of the height of the microchannel to its width. The aspect ratios are large for most microchannels [2]. Fig. 3 shows the effect of aspect ratio with constant externally applied electric field. It is shown that its effects are less significant. However, for pressure driven flow, the effect of the aspect ratio cannot be neglected. This point is clearer from the results of pure pressure driven flow as shown in Fig. 4. At small aspect ratio, it is well known that the velocity represents as parabolic shape for a single liquid flow or two-liquid flow with same viscosity. At large aspect ratio, where the effect of side wall becomes significant, the flow presents ‘plug’ shapes even with electroosmotic effects. This is also why a two dimensional model is required.

![Dimensionless velocity distribution](image)

**Figure 3.** Dimensionless velocity distributions at the symmetric line for different aspect ratio of pure electroosmotic flow
4. Conclusions

An analysis of the two-liquid electroosmotic flow combined pressure gradient is presented in this work. The solution involves analytically solving the linear Poisson-Boltzmann equation, together with the modified Navier-Stokes equation considering the electroosmotic forces and pressure gradient as the body forces. Due to the presence of the surface free charge density at the liquid-liquid interface, the model accounts for both the force induced in the diffuse layer and the force exerted at the interface.

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