The formation of higher-order hierarchical systems in star clusters

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ABSTRACT

We simulate open clusters containing up to 182 stars initially in the form of singles, binaries and triples. Due to the high interaction rate a large number of stable quadruples, quintuples, sextuples, and higher-order hierarchies form during the course of the simulations. For our choice of initial conditions, the formation rate of quadruple systems after about 2 Myr is roughly constant with time at $\sim 0.008$ per cluster per Myr. The formation rate of quintuple and sextuple systems are about half and one quarter, respectively, of the quadruple formation rate, and both rates are also approximately constant with time. We present reaction channels and relative probabilities for the formation of persistent systems containing up to six stars. The reaction networks for the formation and destruction of quintuple and sextuple systems can become quite complicated, although the branching ratios remain largely unchanged during the course of the cluster evolution. The total number of quadruples is about a factor of three smaller than observed in the solar neighbourhood.

Key words: methods: N-body simulations – binaries: general – stars: statistics – open clusters and associations: general.

1 INTRODUCTION

A sizable fraction of (and possibly all) stars are formed in stellar clusters (Lada & Lada 2003, de Grijs et al. 2003, Fall et al. 2005). Investigations of starburst regions in other galaxies reveal that star clusters are predominantly of low mass. For the small Magellanic cloud and the galaxies M33 and M51, Lamers et al. 2003 find that the initial cluster mass function follows a power law distribution with a slope of about -2. The minimum mass in their sample ($M_{cl} \approx 10^4 M_\odot$) is mainly a result of observational bias. The mean mass of star clusters in the solar neighbourhood is $\langle M_{cl} \rangle \approx 1000 M_\odot$ (Kavchynko et al. 2005), suggesting that the mass function found by de Grijs et al. (2003) should be extended to even lower masses. The vast majority (96 $\pm$ 2%) of stars of spectral type O do not belong to a known association, and may have formed in small star clusters (de Wit et al. 2003) and subsequently been ejected by dynamical slingshots (Gualandris et al. 2004; Aarseth 2005). From this it has been argued that the majority of stars are born in small clusters (Kroupa 1995a), many of which may disperse within a few tens of Myr of their formation (Bastian et al. 2005; Fall et al. 2005). In this case, the population of Galactic field stars originates mainly from low-mass clusters and, as a consequence, the majority of stellar multiples in the Galactic field were also born in relatively small star clusters.

The population of binary stars in the solar neighbourhood has been studied extensively for the field (Duquennoy & Mayor 1991; Couteau 1993, 1995) and for some nearby star clusters (Kouwenhoven et al. 2005). Higher-order multiplicities are much more difficult to find than binaries. Still, the Multiple Star Catalogue (MSC) contains 728 systems comprising 3–7 stars. The catalogue is claimed to be complete to a distance of 10 pc from the Sun (Tokovinin 1997, 1999). The majority of the listed systems are triples, and orbital parameters are provided where they are available (Sterzik & Tokovinin 2002). The MSC contains 558 triples, 138 quadruples, 25 quintuples, and 7 sextuples.

We define the multiplicity fraction as the number of objects with a given multiplicity divided by the total number of single stars and multiples:

$$f_i = n_i/N,$$

Here $n_i$ is the number of objects with multiplicity $i$ and $N$ is the sum of the number of single stars ($N_S$), binaries ($N_B$), triples ($N_T$), quadruples ($N_Q$), quintuples ($N_{Qn}$) and sextuples ($N_{Sx}$). Assuming that 15–25% of all systems have
multiplicity larger than two (Tokovinin 2004), we arrive at the following multiplicity fractions: \( f_2 = 0.11-0.19 \), \( f_{3d} = 0.03-0.05 \), \( f_{4d} = 0.005-0.009 \) and \( f_{5c} = 0.001-0.002 \). (A summary of the multiplicities found in the MSC and their configuration is presented in Table 2).

If most stars are born in relatively small star clusters, then the majority of the field population must originate in small clusters and the proportions in which single stars, binaries, triples and higher order multiples occur in the field should be reproducible by computer simulations of such star clusters. Once these clusters dissolve, the numbers of higher-order multiples are no longer affected by cluster dynamical evolution, although internal stellar evolution may still affect their relative numbers. For example, a triple can evolve into a binary when two of its components merge in an unstable phase of Roche-lobe overflow. Thus, the relative multiplicity fraction is not frozen in when the cluster dissolves. Stellar evolution tends to reduce the multiplicity, although these effects become less important for older populations.

Several recent theoretical studies have discussed the formation of multiple systems. These studies approach the subject from two distinct perspectives: (1) stellar dynamical models, in which gravitational interactions are computed between point-mass stars, and (2) hydrodynamical simulations of protocluster evolution.

Purely dynamical interactions between single stars are clearly ineffective in producing a sufficiently high fraction of binaries (Aarseth 1974, Aarseth 2004). In addition, binary orbital periods tend to be too short (Clarke 1993). Inclusion of hydrodynamical effects can boost the formation rate of binaries, but at the cost of reducing further their orbital periods (Delgado-Donate et al. 2003, Goodwin & Kroupa 2003). Increasing the number of stars in the simulations worsens the problem, in the sense that typically only a few binaries are formed per star cluster, and higher-order multiples are very rare (Heggie & Aarseth 1992). Simulations which start with a large proportion of binaries can reproduce the observed binary frequency (Kroupa 1992, Aarseth 2004), but studies of the formation of higher-order multiples in star clusters containing primordial binaries but no primordial triples fail to reproduce either the fraction or the orbital characteristics of triples observed in the field (Portegies Zwart et al. 2004).

Protocluster evolution, in which gas coagulates to form stars or subclusters in the form of hierarchical stellar systems, may be able to account for a high proportion of binary and possibly triple systems in young star clusters (Bate et al. 2003). The hydrodynamical breakup of protostellar cores could be a dominant mechanism for the formation of binaries, and possibly triples, in this environment. These methods, however, fail to produce higher-order multiplicities (White & Ghez 2001). From these studies it would seem that star clusters can form from their parent molecular cloud with a rich population of binaries and triples, but without significant numbers of quadruples, quintuples and sextuples relative to one another are roughly consistent with observations.

2 INITIAL CONDITIONS

The clusters simulated in this study are initialised by selecting positions and velocities for 100 objects distributed according to a King (1963, 1966) model with \( W_0 = 6 \). For each object we randomly draw a mass between \( m_{\text{min}} = 0.1 M_\odot \) and \( m_{\text{max}} = 30 M_\odot \) from a Kroupa mass function (Kroupa 1998), resulting in a total cluster mass of about \( 45 M_\odot \). After adding binary and triple components, the masses of the simulated clusters average about \( 67 M_\odot \). Finally, we scale the velocities of all stars and multiple centers of mass in the cluster in such a way that the entire system is in virial equilibrium, in standard \( N \)-body units (Heggie & Mathieu 1986), and scale the cluster to a virial radius \( r_{\text{vir}} = 0.1 \) pc, corresponding to a half-mass radius of about \( 0.08 \) pc. Given the rapid expansion of the clusters in the first few Myr we consider this a reasonable choice. We call this model S (see Tab. 1). For a \( \sim 67 M_\odot \) star cluster in the solar neighbourhood, the Jacobi radius in the Galactic tidal field is \( r_J \approx 5.5 \) pc, which is an order of magnitude larger than the initial tidal radius of the adopted King model. For clarity we therefore neglect the tidal field in our simulations. Note that later we describe additional simulations with larger initial cluster radii, in which case this assumption may break down.

Starting from model S, we generate model B by randomly selecting 50 stars, which are converted into binaries by adding a companion (secondary) star and orbital parameters. Again, before starting the simulation, the velocities of all single stars and binary centres of mass are rescaled so that the entire system is again in virial equilibrium. The mass of the secondary is randomly chosen from a uniform distribution between \( m_{\text{min}} \) and the mass of the primary. The orbital parameters are chosen from empirical distributions describing approximately the inner orbits of hierarchical triple systems in the MSC (Tokovinin 1997, 1999). The orbital period \( P_{\text{bin}} \) of the binaries is selected by generating a random variable \( X \) between 0 and 1 and setting

\[
P_{\text{bin}}(X) = 0.09687 X^{-1} - 0.09677 \quad \text{years.} \tag{1}
\]

The eccentricity for each binary orbit is generated by selecting another random number \( Y \) uniformly in \([0,1]\), and defining

\[
e_{\text{bin}}(Y) = 1.16 \max(0, Y - 0.4). \tag{2}
\]

If for any binary, the separation at periastron is smaller than five times the maximum stellar radius (from Eggleton et al. 1980), new orbital parameters are selected randomly.
These distributions (Eqs. 1 & 2) are presented in Figures 1 and 2 and compared with the MSC data. For clarity, and due to our lack of understanding of selection effects in the discovery of multiple systems, we decided deliberately to stay close to the observed distributions for the orbital period and eccentricity. An additional argument for this choice is that our simple prescription for the generation of initial conditions allows our initial conditions to be easily reproduced. The fact that the binary period distribution does not exactly reproduce the observed distribution is therefore not a major concern. The other binary parameters (inclination, orbital phase and ascending node) are chosen randomly (Hut & Bahcall 1983).

Model T is generated by adding a third (outer) star to 32 randomly selected binaries in model B. The fraction of triples thus obtained is based on the observed fraction of higher order systems in the solar neighborhood. No simulations were performed with primordial quadruples or higher-order hierarchies. The mass of the tertiary star is randomly chosen between $m_{\text{min}}$ and the mass of the inner binary. The orbital period $P_{\text{trip}}$ for the outer orbit of the triple is selected from a distribution approximating the observed MSC distribution of outer orbital periods in triples:

$$P_{\text{out}}(Z) = 46.8Z^{-1} - 45.8 \text{ years},$$ \hspace{1cm} (3)

and the eccentricity for the outer orbit is generated with

$$e_{\text{out}}(W) = 0.80W,$$ \hspace{1cm} (4)

where $Z$ and $W$ are uniformly distributed between 0 and 1. The other parameters for the outer orbit are chosen randomly, as for the binary orbits. Finally, we check the stability of the triple using Eq. 90 from Mardling & Aarseth 2001, and new parameters for the outer orbit are selected randomly if the triple turns out to be dynamically unstable, or if the separation at pericentre between the outer star and the binary is less than five times the maximum stellar radius. As in model B, the entire system is then restored to virial equilibrium by rescaling the velocities of single stars and the centre of mass velocities of binaries and triples.

Figure 1 compares the observed distributions for the inner and outer orbital period and eccentricity for 68 observed triple systems (solid curves; Tokovinin 1997, 1999), and the distributions (dashed lines) generated from Eqs. 1 and 4.

Table 1. Initial parameters and final fraction of higher-order multiples for the various simulations. Each simulation consists of 100 ‘particles’ which can be single stars, binaries, or triple systems. With the adopted number of binaries ($n_B$) and triples ($n_T$) the total number of stars $n_s$ then ranges between 100 to 182. Each simulation was initialised from a King model with $W_0 = 6$ and with a virial radius $r_{\text{vir}} = 0.1 \text{ pc}$. Some additional simulations were performed with larger virial radii (1 pc and 3 pc). When the simulations are terminated at $t = 55 \text{ Myr}$, each cluster has (on average) produced $n_{Q4}$ quadruples, $n_{Q5}$ quintuples and $n_{Sx}$ sextuple systems. The final row indicates the total number of simulations performed for each set of initial conditions, each varying only in the original random seed used.

![Figure 1. Cumulative distribution of the inner and outer orbital periods of 68 observed triple systems (solid curves; Tokovinin 1997, 1999), and the distributions (dashed lines) generated from Eqs. 1 and 4.](image1)

![Figure 2. Cumulative distribution of the inner and outer orbital eccentricity for 68 observed triple systems (solid curves; Tokovinin 1997, 1999), and the distributions (dashed lines) generated from Eqs. 2 and 4.](image2)
3 METHODS

Once the initial realizations are generated, our star clusters are evolved using the kira integrator in the Starlab software package [Portegies Zwart et al. 2000]. The equations of motion are integrated using a fourth-order predictor-corrector Hermite scheme (Makino & Aarseth 1992), using block time steps [McMillan 1986a,b; Makino, 1991; see also Aarseth 2003 for an overview]. Stellar and binary evolution are modeled with using SeBa [Portegies Zwart & Verbunt 1996; Portegies Zwart et al. 1997]. For stable hierarchical triples, we follow the internal evolution of the inner binary, including Roche-lobe overflow, the effect of stellar-wind mass loss, supernovae and the emission of gravitational waves. In our prescription within SeBa, the outer star in a stable triple cannot initiate a phase of mass transfer to the inner binary by Roche-lobe overflow, but the inner binary is evolved according to the prescriptions for isolated binaries. Binaries in higher-order multiples are evolved, so it is in principle possible to have a system consisting of six stars, in which three binaries each in states of mass transfer, orbit one other. Such a situation, however, is quite rare. All simulations were performed on 2.8 GHz workstations Intel Pentium 4 processor, and took about 1 hour each.

In order to clarify some of the results on multiple hierarchies we first provide a brief description of our treatment and identification procedure for multiple systems. The kira integrator employs a dynamic tree structure that represents an N–body system as a mainly 'flat' tree having individual stars and the centres of mass of multiple systems as leaves. Binary, triple, and more complex multiples are represented as binary subtrees below their top-level centre of mass nodes. The tree structure determines both how node dynamics is implemented and how the long-range gravitational force is computed, and also provides a convenient means of identifying transient structures.

The tree evolves dynamically according to simple heuristic rules: particles that approach 'too close' to one another are combined into a centre of mass and binary node; and when a node becomes 'too large' it is split into its binary components. These rules apply at all levels of the tree, allowing arbitrarily complex systems to be followed. In practice, the term 'too close' is taken to mean that two objects approach within roughly the 90° deflection distance for typical stellar masses and speeds (see Figure 7.2 on page 421 of Binney & Tremaine 1987, and also Heggie & Hut 2003, where we adopt \( \theta = \pi \) to compute the 90° deflection distance). ‘Too large’ means that the node’s diameter exceeds \( \sim 2.5 \) times this distance. The simulation software prints out summary information each time the tree structure changes; these data are the basis for the data analysis in this paper.

Reactions between higher order multiple systems are generally chains of events that cause a particular hierarchy to change, whether or not the multiple systems are stable. The start of a reaction is signalled by a change in the hierarchical structure of a multiple. In order to enable a clear identification of specific reactions, we introduce a time window \( \Delta t \), within which a reaction must be completed in order to be counted. We set \( \Delta t = 0.01 N\)–body time units, which is about the orbital period of a 1kT binary and which corresponds to \( \sim 550 \) years in model T.

A reaction can often be decomposed into a series of events in which only a subset of the total number of stars participates, each occurring within a smaller time window. These events are counted as separate reactions. Reactions that last for longer than \( \Delta t \) are counted multiple times, each time the reaction is counted according to the dominant hierarchy, which sometimes changes continuously. Reactions that last for more than \( \Delta t \) but that do not experience abrupt changes in hierarchy are counted only once. Thus, short-lived resonances are only counted once, but those which last for many \( N\)–body time units and change configuration may be counted many times. Such long-lived resonances, however, are rare.

In our methodology, we distinguish between multiples in terms of the time they remain intact (as just described). The survival time of a multiple is taken to be the interval between the instant when the multiple forms and the time at which components are lost or new ones acquired. If the survival time is more than one \( N\)–body time unit (\( 1/2\sqrt{2} \) standard crossing times), we call the multiple ‘persistent.’ Should the survival time be less than this, the multiple is called ‘transient.’ An overview of number of transients and persistent multiples created in our simulation model T is presented in tables [2] and [3] in §3.3.

3.1 Evolution of structural parameters

We concentrate here on the three models S, B and T with an initial virial radius \( r_{\text{vir}} = 0.1 \) pc, but additional simulations have been performed with \( r_{\text{vir}} = 1 \) pc and \( r_{\text{vir}} = 3 \) pc. The latter two models with primordial triples are identified as T1 and T3 for \( r_{\text{vir}} = 1 \) pc and \( r_{\text{vir}} = 3 \) pc, respectively. Figure 3 compares the evolution of the bound mass of models T, T1 and T3. The means of the T1 and T3 runs are plotted as dashed and dotted curves, respectively. The gray shaded area in Figure 3 shows the results (mean \( \pm 1\sigma \)) for the 67 simulations of model T. Table 1 summarises the results of the various simulations.

The time at which model T loses half its mass is \( t_{\text{hm}} = 15 \pm 15 \) Myr, much shorter than for the other two models. A larger initial cluster radius results in a much lower rate of mass loss. Mass loss for model T3 is governed by stellar evolution. In model T, stellar mass loss is less important, and most mass loss is a result of dynamical ejection. The sudden divergence between the curves for models T1 and T3 around 20 Myr is associated with the onset of significant dynamical activity in T1 at that time (roughly 2–3 initial relaxation times into the evolution). Models S, B, and T had comparable overall mass evolution, indicating that an initially high proportion of triple systems has little influence on the mass loss rate for these clusters; rather, it is mostly the relaxation of the cluster that drives dynamical mass loss. Simulations that generated more long lived high-order multiples (\( n > 4 \)) did, however, tend to have considerably higher mass loss rates.

The evolution of the cluster half-mass radii for model T are presented in Figure 4. Our initial choice of 0.1 pc as the initial cluster virial radius seems somewhat small compared with the observations in the open cluster catalogue (Dias et al. 2002). However, this is quickly compensated by...
the rapid expansion of the cluster, driven by a combination
of stellar mass loss, dynamical heating by multiple systems,
and mass stratification (Merritt et al. 2004). After about
25 Myr the simulated cluster has expanded beyond about
6 pc, which, according to our earlier estimate, would exceed
the cluster's Jacobi radius in the Galactic tidal field. Al-
though we ignore the tidal field in our simulations, we argue
that a cluster which expands beyond this radius should be
considered dissolved. Some of the smaller observed clusters
younger than \( \sim 20 \) Myr may be satisfactorily reproduced
by model T. At later times, model T tends to expand too
rapidly compared to the observed cluster population. How-
ever, since our models are unlikely to survive this long if the
Galactic tidal field was taken properly into account the com-
parison may not be appropriate. We note, however, that the
expected effect of the Galactic field will be to cause the mass,
and hence the half–mass radius, of the cluster to begin to
decline after 25 Myr, possibly improving the agreement be-
tween the model and the observations, but we do not pursue
that possibility further here. The initially larger models (T₁
and T₃) may provide somewhat better descriptions of the
observed clusters at later times.

3.2 Evolution of multiplicity

Our main simulations (model T) started with only single
stars, binaries and triples, but in time a relatively rich popu-
lation of higher-order multiples formed. During the evolution
of model T, the binary and triple fractions hardly change,
while the numbers of stable hierarchical systems consisting
of 4 stars (\( n_{Qd} \)), 5 stars (\( n_{Qs} \)) and 6 stars (\( n_{Sx} \)) increase
gradually with time. This is illustrated in Figures 3 and 6,
which show the numbers of higher-order multiples (quadru-
uples, quintuplets and sextuples) per cluster as functions of
time for models T and T₁.

During the first few million years of model T, the num-
ber of quadruples increases sharply from zero at birth to
about 0.35 per cluster at \( t \simeq 2 \) Myr. After initial rapid
increase, the formation rate drops by about a factor of 20,
and subsequently remains constant until the end of the simu-
lation. The numbers of quintuples and sextuples show a sim-
ilar trend of rapid increase at early times and significantly
slower growth for the rest of the evolution of the cluster, but
their rates start to increase only after delay times of about
4 and 10 Myr for the quintuples and sextuples, respectively.
The numbers of quadruples, quintuples and sextuples at late
times in model T are quite well approximated by a simple
least-squares linear fit:

\[
\begin{align*}
    n_{Qd}(t) &= 8.0 \times 10^{-3} t + 0.34, \\
    n_{Qs}(t) &= 3.4 \times 10^{-3} t + 0.11, \\
    n_{Sx}(t) &= 1.4 \times 10^{-3} t + 0.08. 
\end{align*}
\]

The formal error on these fits is 4–7%. The formation rate
drops by about a factor of two for each higher hierarchy
from quadruples to sextuples. In figure 6, similar trends are
observable for model T₁.

It is tempting to interpret the delay in the formation
of quintuplets and sextuples as the time required to estab-
lish the channels through which these higher-order systems
form—we first create quadruples from triples, then quintu-
uples from quadruples, and finally sextuples from quintu-
iques.
Figure 5. Numbers of quadruple (solid curve), quintuple (dashes) and sextuple (dots) systems per cluster as functions of time. The numbers plotted are valid for the number of systems present at the end of one $N$–body time unit. These data are averaged over the 67 runs of model T. The dashed lines are fits through the rightmost portion of the data (see Eq. 5).

Figure 6. Numbers of quadruple (solid curve), quintuple (dashes) and sextuple (dots) systems per cluster as functions of time. The numbers plotted are valid for the number of systems present at the end of one $N$–body time unit. These data are averaged over the 84 runs of model T$_1$.

Thus, while it is admittedly difficult to discern the relevant dynamical details in a single set of runs, we interpret Figure 5 as follows. The initial half-mass relaxation time of model T is $\sim$ 250 kyr. The relatively rapid initial rise in the number of multiples represents the early period of strong dynamical activity when binaries and triples sink to the cluster centre and interact, releasing energy and, as a side effect, producing some higher-order systems. By 3 Myr the (average) cluster has expanded fivefold in radius and its density has dropped, reducing the quadruple formation rate mainly...
by depleting the numbers of single stars. Subsequently, the remnant of the cluster is rich in binaries and triples and, as the density declines, conditions become more suitable for forming and retaining quintuple and sextuple systems. (A typical triple in these runs is hard, in the stellar dynamical sense, but barely so, making it unlikely that a hierarchical system of two such triples could form and survive at the initial density of model T.)

By the end of the simulations (at \( \sim 55 \) Myr) a total of 22% (15 out of 67) of the runs have not produced any persistent hierarchy consisting of 4 or more stars that survived to the end of the calculation. A total of 37% (25) of the simulations have 1 multiple present after 55 Myr, 30% (20) have 2 multiples, and only 10% (7) of the runs have 3 stable higher-order multiple systems present after this time. None of the simulations had more than three multiples containing 4 or more stars upon termination. Short-lived higher-order systems, however, are common in each of the simulations, and a total of 7921 higher-order multiples were formed. However, most were destroyed before the end of the simulation (see §3.3 for details, and in particular Table (2)).

Once the cluster disperses, the surviving multiples become part of the Galactic field, and we may usefully compare their numbers with the MSC (§1). Assuming that the field was assembled from the remnants of clusters similar to simulation T (of which, on average, \( \sim 100 \) single stars, binaries, and stable multiples remain by the end of the calculation), the resultant frequencies of triples, quadruples, quintuples, and sextuples are, respectively, 0.25, 0.008, 0.003, and 0.002. Thus our particular choice of initial binary and triple parameters under produces quadruples and higher-order multiples by a factor of \( \sim 2-3 \) compared to the MSC, although the relative frequencies of these systems are somewhat encouraging. We now discuss in more detail the breakdown of these frequencies within each class.

Figure [b] gives the the equivalent to Figure [a] but for simulation T1. Also in this case the number of multiples increases quite dramatically in the first few Myr to level off at later time, although the overall formation rate of higher order multiples is lower than in the models T. Several higher order multiples were formed in simulation T3, but their number was rather small and they are omitted from the figure.

### 3.3 The hierarchy of multiple systems

During the simulations, the configurations of the multiple systems continually change, as do the numbers of components. In this section we discuss the various hierarchical structures, and the frequencies with which they appear in the simulations. In §3.4 we further explore the channels through which higher-order multiples are formed and destroyed. For both this subsection and the next we concentrate on model T.

In tables (2) and (3) we present an overview of the transients and persistent multiples created in simulation model T. The multiples which escape the cluster are included in this table, as well as the multiples which remain bound. A single star in these tables is identified by the letter S. We denote a pair of bound objects by putting parentheses around them, with the more massive of the two always to the left. For brevity we introduce separate notation for a binary (a pair of stars), which we write as B \( \equiv \) (S,S). A triple can have

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**Table 2. Overview of the stellar multiplicities observed in the MSC and in our 67 simulations of model T. The first column gives this configuration of the multiple. A single star is designated by an S, a binary by a B. A binary consists of two single stars and could be written as \( (S, S) \equiv B \). The entries are ordered by mass, with the more massive component always positioned on the left. The parentheses indicate the hierarchy of the multiple system.**

| Quadruple conf. | MSC | Total | pers. | pers. |
|----------------|-----|-------|-------|-------|
| (B,B)          | 69  | 1171  | 5.080 | 290   |
| (B,(B,B))      | 61  | 1093  | 2.43  | 209   |
| (B,(B,B))      | 4   | 945   | 0.97  | 165   |
| (B,(B,B))      | 4   | 358   | 4.45  | 41    |
| (B,(B,B))      | 0   | 297   | 0.62  | 37    |
| Total          | 138 | 3864  | 742   |
| Quintuple conf. | MSC | Total | pers. | pers. |
| (B,(B,B))      | 9   | 949   | 3.08  | 202   |
| (B,(B,B))      | 7   | 854   | 2.16  | 145   |
| (B,(B,B))      | 5   | 142   | 2.21  | 20    |
| (B,(B,B))      | 4   | 14    | 0.33  | 2     |
| (B,(B,B))      | 0   | 205   | 0.83  | 32    |
| (B,(B,B))      | 0   | 162   | 1.83  | 29    |
| (B,(B,B))      | 0   | 112   | 0.70  | 23    |
| Total          | 25  | 2501  | 460   |
| Sextuple conf. | MSC | Total | pers. | pers. |
| (B,(B,B))      | 3   | 88    | 6.0   | 28    |
| (B,(B,B))      | 2   | 1     | 0.01  | 0     |
| (B,(B,B))      | 1   | 699   | 1.74  | 132   |
| (B,(B,B))      | 1   | 121   | 0.74  | 11    |
| (B,(B,B))      | 0   | 381   | 0.36  | 15    |
| (B,(B,B))      | 0   | 67    | 1.11  | 66    |
| (B,(B,B))      | 0   | 46    | 0.33  | 4     |
| (B,(B,B))      | 0   | 21    | 1.2   | 1     |
| (B,(B,B))      | 0   | 17    | 1.3   | 7     |
| (B,(B,B))      | 0   | 14    | 0.57  | 2     |
| (B,(B,B))      | 0   | 13    | 3.7   | 0     |
| (B,(B,B))      | 0   | 9     | 0.6   | 2     |
| (B,(B,B))      | 0   | 8     | 2     | 2     |
| (B,(B,B))      | 0   | 4     | 0.1   | 2     |
| (B,(B,B))      | 0   | 5     | 0.5   | 2     |
| (B,(B,B))      | 0   | 5     | 0.3   | 0     |
| (B,(B,B))      | 0   | 4     | 0.2   | 0     |
| (B,(B,B))      | 0   | 4     | 0.08  | 0     |
| (B,(B,B))      | 0   | 2     | 0.01  | 0     |
| (B,(B,B))      | 0   | 3     | 0.8   | 2     |
| (B,(B,B))      | 0   | 1     | 0.05  | 0     |
| (B,(B,B))      | 0   | 2     | 0.01  | 0     |
| (B,(B,B))      | 0   | 3     | 0.1   | 0     |
| (B,(B,B))      | 0   | 2     | 0.09  | 0     |
| (B,(B,B))      | 0   | 2     | 0.01  | 0     |
| (B,(B,B))      | 0   | 1     | 0.8   | 0     |
| Total          | 1   | 1556  | 227   |
two configurations: (B,S) if the binary is more massive than the star orbiting it, or (S,B) if the outer star is more massive. Table 2 draws a distinction between the total number of configurations and those which are persistent. Table 3 compares the simulated hierarchies directly with those observed.

The most long-lived multiple configuration, on average, consists of 4 stars arranged in the classical quartet (B,B), in which two binaries orbit one another. A total of 1171 reactions led to such a configuration. Of these, 290 resulted in persistent systems, with an average lifetime of about 20 Myr. Aarseth (2004) has commented on the formation of such binary–binary systems, and on their longer lifetimes compared to other, less compact, configurations. Due to their smaller cross sections, they are less likely to have a fatal encounter with another object than are strictly hierarchical systems. The ratios of the numbers of the various quadruple configurations to the total number of quadruples in the MSC are: 0.50 for (B,B), 0.44 for ((B,S),S), 0.05 for ((B,B),S) and (S,(B,S)). The configuration (S,(S,B)), in which the inner binary is less massive than the two single stars orbiting it, has not been observed.

To compare these numbers with the results of our simulations of model T, we multiply the frequencies at which these various configurations are formed by their average lifetimes. The probability of observing any of the above quadruple configurations during a simulation is then 0.54 for (B,B), 0.23 for ((B,S),S), 0.14 for (S,(B,S)), 0.07 for (S,(B,B)), and 0.01 for the unobserved (S,(S,B)). These relative ratios are comparable to those actually observed.

In the observations it is often hard to determine the masses of the component stars, so we present in Table 3 a reduced version of Table 2 in which we list the observed numbers of hierarchies, but group all systems with the same physical hierarchy together. Thus, quadruple systems are reduced to just two types (B,B) and ((B,S),S), the latter category then contains all possible permutations in which a binary is orbited by two single stars: ((B,S),S), (S,(B,S)) and (S,(S,B)). Among the quintuple configurations, the strictly hierarchical systems ((B,S),S) are vastly outnumbered by the triple-binary pairs ((B,S),B) in the simulations as well as in the observations. In the observed sample, however, permutations of ((B,S),B) are much more abundant than in our simulations. The differences between the occurrences of observed and simulated multiplicities is quite striking. The origin of these discrepancies is not trivial to understand; it could stem from observational selection effects or be a result of our choice of initial binary and triple parameters.

### 3.4 Multiplicity formation and destruction reactions

A gravitational dynamical interaction in which the multiplicity of one object changes can be described in terms of creation and destruction reactions. Upon the dissociation of a binary, two single stars appear, and when two single stars form a bound pair a binary is created. A cascade of such reactions then becomes a multiplicity reaction network. It is straightforward to extend this description to reactions in which more than two stars participate, although the reactions and the reaction network can become quite complicated. We now use this perspective to describe the formation channels for systems consisting up to six stars.

The configurations listed in Table 2 and Table 3 are often formed in complex interactions involving single stars and higher-order systems. In this section we concentrate on the individual formation and destruction reactions for multiplicities of up to 6 stars.

Figure 7 presents the most common reactions leading to the ‘creation’ (liberation) of single stars due to the gravitational decay of multiple systems (left), and the corresponding ‘destruction’ (consumption) reactions resulting in the loss of single stars (right). Each ‘ladder’ represents a reaction channel; the symbols S, B and T on the rungs refer to single, binary and triple stars, respectively. An arrow is drawn from the originating level to the final state of the system. Dashed arrows indicate the reduction of a higher-order system to one of lower order—a binary being disrupted or a triple being reduced to a single star and a binary. Such a reduction could be caused by an encounter with another cluster member, by stellar evolution (e.g. a supernova event).
Figure 7. Dynamical reactions for the creation (left) and destruction (right) of single stars. In 67 simulations of model T, a total of 4330 unique creation reactions were identified, versus 3245 unique destruction reactions. Unique meaning that if a particular reaction involving a certain set of single star components occurred more than once during a simulation it is counted only once. A reaction results in the creation or destruction of at least one single star. In the leftmost creation reaction, for example, two single stars are created from the destruction of a single binary via stellar dynamics such as a supernova event of due to the background potential of the star cluster, whereas in the second creation reaction one single star and one binary form from a triple. The numbers along the top line show the relative frequency in relation to all reactions encountered of this particular reaction. Note that only the most common reactions are given, hence these numbers do not add up to unity. The text to the right indicates the state of the system. The following terminology is used: S for single stars, B for binaries, T for triples, Qd, Qn and Sx for quadruples, quintuples and sextuples, respectively.

Figure 8. Principal reactions leading to the creation and destruction of binaries: 2639 creation and 2628 destruction reactions.

or, in the case of triple reduction, by the internal dynamics of a dynamically unstable system. These different scenarios are not represented separately in these diagrams because the effect on the multiplicity of the components is the same. Solid arrows indicate capture, as when a single star or small system combines (typically in the presence of another cluster member) with another small system to form a higher order multiple. Note that, although this presentation might suggest that interactions occur in isolation, this is usually not the case. Many interactions take place near the core of the cluster, and there may be many other stars nearby which can carry away small amounts of energy and angular momentum. It is sometimes hard to identify the star(s) responsible for the binding energy and angular momentum of a post-encounter bound pair. Our simulations include stellar and binary evolution and allow for stellar collisions to take place, but in order not to unnecessarily complicate matters, these processes are not displayed separately here.

The number at the top of each reaction channel indicates its relative frequency in our simulations. Thus the left panel in Figure 7 presents reactions in which single stars are created by the ionisation of a binary (40%) or by destruction of a hierarchical triple (20%), whereas the right panel describes the ‘destruction’ of single stars by absorption into a higher-order system binary (39%) or triple (22%). The sums of the creation and destruction frequencies should each be unity, but for clarity only the most important reactions are shown. The remaining interactions comprise a wealth of low-probability, sometimes rather arcane reactions contributing to the overall cluster evolution.

Figures 8 and 12 show analogous reaction channels for the formation and destruction of binary through sextuple systems. The symbols Qd, Qn and Sx refer to quadruples, quintuples and sextuples. Note the important role played by triples in the formation of higher-order multiples. Quadruples and quintuples are most commonly formed from a triple that absorbs a single star or a binary, whereas sextuples tend to form from the combination of two triples. (See also Table 2 where sextuples consisting of two triples orbiting one another are relatively common.) We find similar trends in the destruction of the higher-order multiples. In particular, the decay of sextuples to form two triples is quite striking.

The reaction frequencies presented in these figures are computed for the entire simulation of model T, even though one might argue that the early phase of rapid multiple creation ($t < 2 \text{ Myr}$) could produce reactions different from those found the later, slower phase of the evolution (Fig. 5). Interestingly, there is no significant difference between the types of reactions seen during the early and late phases.

3.5 The orbital parameters of the hierarchies

It is not trivial to compare orbital parameters of the observed multiples with those in the simulations, in part due to the large number of parameters and due to severe selection effects. Complete orbital solutions are not available for any of the multiples listed in the MSC, but for four quadruples the MSC lists the orbital periods and eccentricities for each of the three orbits and also the masses of the four stars; those systems are HD 08065+1757, HD 05569+0939, HD 11128+3205 and HD 11171-2414.

Nevertheless we attempt to compare the orbits of the multiples in the MSC with those obtained in our simulations. For clarity we limit ourselves here to the population of quadruple systems. Fig. 13 (top panel) shows the orbital separations and eccentricities for all quadruples in the MSC. The bottom panel shows the same information for persistent quadruples at an age of 55 Myr for model T. Small symbols
Figure 9. Reactions leading to the creation and destruction of triples: 2700 creation and 2969 destruction reactions.

Figure 10. Reactions leading to the creation and destruction of quadruples: 1247 creation and 1229 destruction reactions.

Figure 11. Reactions leading to the creation and destruction of quintuples: 724 creation and 760 destruction reactions.

Figure 12. Reactions leading to the creation and destruction of sextuples: 511 creation and 530 destruction reactions.
indicate hierarchical systems \((B, S), S\), whereas large symbols show data for the \((B, B)\) configuration. For hierarchical quadruples, the outermost, intermediate, and inner orbits are represented by small bullets, triangles, and circles, respectively. For binary–binary systems, the large bullets represent the outer orbit, while the large triangles and circles indicate, respectively, the inner orbits containing the most massive and least massive stars.

Many of the ‘outer’ orbits in the observed quadruples are missing from the sample in the MSC, which reflects the few bullets in Fig. 13. The outer orbits seem to be cut off at around \(10^3\) AU. The simulated sample of outer orbits, however, extends all the way to about \(10^5\) AU. The few known outer orbits in the observed sample of quadruple systems strongly suggests that these orbits are simply not observed, and that the observational selection effect becomes important at around a few hundred AU.

There does not seem to be a specific selection effect regarding the orbital parameters of the inner binary orbits in the 69 \((B, B)\) quadruples listed in the MSC, as for the primary as well as the secondary binary a total of 46 are observed. The inter-binary orbit, however, is quite heavily affected by selection effects, as the MSC lists only 10 binaries with known period and eccentricity (the large bullets in the top panel of Fig. 13).

These different selection effects are best seen in the large number of missing bullets in the top panel of Fig. 13 which suggests that the outer orbits in hierarchical systems, as well as the inter-binary orbits in \((B, B)\) systems, are severely affected by observational selection effects. The other seem to be rather well represented by the observations, with the possible exception of the larger number of circularized binaries in the observation and some excess of short period eccentric binaries in the simulations. This suggests that tidal circularisation in our simulations is less effective that in nature. This should not come as a surprise, as tidal effects are not taken into account in higher order systems in which an inner binary is perturbed by an outer object, as is often the case in these quadruple systems.

We have performed similar analyses comparing the orbital parameters for the quintuples and sextuple systems in the MSC with simulation T, and the results of these are quite similar, in the sense that the observational selection effects for determining orbital parameters gradually increase for larger periods. In these cases we also find that the simulations tend to overproduce short-period systems with high eccentricities relative to the observations, at the expense of short-period circular orbits. We decided not to show the plots for these higher order multiples, as the quality of the observational data declines with increasing multiplicity.

4 DISCUSSION AND CONCLUSIONS

We have performed simulations of star clusters which initially consisted of single stars, binaries and triples. The initial conditions were selected to match the young (< 50 Myr) star clusters in the solar neighbourhood, consisting of at most a few hundred stars. The initial conditions for the binaries and triples were selected based on the observed populations, which we transform directly to input parameters. For simplicity, observational selection effects are neglected, both in determining the distribution functions from which we generate our initial conditions, and in our final comparison with observations.

After initialisation we run the simulations for about 55 Myr (corresponding to about 1000 N-body time units, or about 350 dynamical crossing times of the system) and compare our results with the observed star clusters and multiples in the field. Our models with initial half-mass radii of about 0.1 pc are consistent with several observed young (< 20 Myr) star clusters. The rapid expansion of our simulated clusters is attributed partly to stellar mass loss and partly to dynamical effects. Our simulations fail to explain the small (< 2 pc) star clusters at ages > 20 Myr, although
we argue that proper inclusion of tidal effects would mitigate this discrepancy.

The stars in the simulations are initially distributed as single stars, binaries and triples. During the evolution, multiple systems containing more than 4 stars form as a result of strong dynamical interactions among primordial single stars, binaries and triples. After the first few million years the fraction of higher-order systems remains roughly constant.

Our simulation models form significant numbers of hierarchical systems consisting of four or more stars when compared to similar simulations without primordial triples. We measure the fractions of systems containing up to 6 stars, and find a steady increase in their number. After the start of the simulation the number of higher-order multiples rapidly increases up to an age of about 2 Myr, after which their net formation rate drops by about a factor of 20. The number of multiples consisting of 4 stars increases at about twice the rate as those containing 5 stars, which again increase at about twice the rate of system with 6 stars.

Among the higher-order multiples that form in our simulations, the strictly hierarchical systems are the least likely to survive. This is most easily seen in the quadruples, among which the configuration consisting of two binaries orbiting one another are by far the most common. Among the quintuples the most likely stable configuration is a triple star orbiting by a binary. Most sextuple systems consist of two triples orbiting one another.

The relative frequency of stable hierarchies in our simulations is generally comparable to those observed in MSC, but with some notable exceptions. In four observed systems, a massive binary is hierarchically orbited by three single stars. In our simulations such sextuple systems are extremely rare (relative frequency < 1%). On the other hand, a sextuple system consisting of three binaries orbiting one another as a hierarchical triple is quite common in our simulations, but none are observed in the population of multiple systems in the solar neighbourhood. These interesting differences may originate from observational selection effects or from specific choices in our initial conditions.

We also present the principal interactions in which multiple stars are created and destroyed. Reactions in which single stars (~ 50%) or triples (~ 30%) participate are most common, simply because such systems are most abundant. Most sextuple systems are formed from an interaction between two triple systems, and most quadruples form from a triple and a single star. The actual relative importance of various reactions in the network may be quite different for a different choice of initial conditions. The initial fractions of stars, binaries and triples may play a crucial role here, and a shallower initial density profile may boost the number of high order multiples and may even allow for systems containing larger numbers of stars. We expect, however, that multiplicities of 4 or higher will remain relatively rare compared with systems consisting of fewer stars.

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