Ilten, Nathan Owen; Lewis, Jacob; Przyjalkowski, Victor
Toric degenerations of Fano threefolds giving weak Landau-Ginzburg models. (English)
J. Algebra 374, 104-121 (2013).

Fano/Landau-Ginzburg mirror symmetry predicts that the mirror of a smooth projective Fano variety \( X \) should be a Landau-Ginzburg model (LG model for short) \((Y, f)\), that is, a non-compact Kähler manifold endowed with a Morse function \( f \), so that, roughly speaking, the algebraic geometry of \( X \) is related to the symplectic geometry of \((Y, f)\). In the paper under review, the authors consider toric LG models, that is, models of the form \((T, f)\), where \( T \) is an algebraic torus and \( f \) a Laurent polynomial on \( T \). Such a pair is a very weak LG model if the constant term of the regularized I-series of \( X \) (also known as the \( J \) function, it is the generating series of \( 1 \)-pointed Gromov-Witten invariants) is equal to the constant term series of \( f \) (that is, the generating series of the constant terms of powers of \( f \)). It is moreover weak if \((T, f)\) can be compactified to an open Calabi-Yau variety.

Toric LG models are simpler because \( f \) is then a Laurent polynomial, hence its analytical properties are easier to study than those of general Morse functions. Another motivation, which is central to the paper under review, is the idea that the LG model of any toric degeneration of \( X \) should be the same as the LG model of \( X \). In fact, this approach was used in \([V. V. Batyrev, et al., Nucl. Phys., B 514, No.3, 640–666 (1998; Zbl 0896.14025)]\) to compute the LG model of Grassmannians. However, for the sake of completeness, it should be mentioned that in some cases, such as for the quadric threefold, the toric LG model does not contain all critical points of \( f \), which leads to difficulties explained in Appendix B.2 of \([T. Eguchi, K. Hori and C-S. Xiong, Int. J. Mod. Phys. A 12, No. 9, 1743–1782 (1997; Zbl 1072.32500)]\).

The main result of the paper under review is the fact that every Fano threefold of Picard rank one has a weak Landau-Ginzburg model associated to a toric degeneration, and that this model has a compactification as a family of \( K3 \) surfaces of Picard rank 19. So the LG models of these varieties, previously computed in \([V. Przyjalkowski, “ Weak Landau-Ginzburg models for smooth Fano threefolds”, arXiv:0902.4668]\), are indeed equal to those of their toric degenerations. The two main steps in the proof are a construction of toric degeneration for each case, and the compactification of the LG model as a Calabi-Yau family. This last point offers the most difficulties, and is a significant obstacle to generalisations in higher dimension.

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MSC:

14J33 Mirror symmetry (algebro-geometric aspects)
14J45 Fano varieties
14M25 Toric varieties, Newton polyhedra, Okounkov bodies
14J28 \( K3 \) surfaces and Enriques surfaces
32G20 Period matrices, variation of Hodge structure; degenerations

Keywords:
toric varieties; Landau-Ginzburg models; mirror symmetry; degenerations

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