Redistribution of Internal Forces in RC Beams due to Fire Action

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Abstract. Due to the fire temperature action material properties (for RC structures - for concrete and reinforcing steel) are subjected to degradation. This results in the reduction of stiffness of cross-section of the elements and finally in redistribution of internal forces for statically indeterminate structural systems (for continuous beams and slabs, for frames). Thus it is important to take into account these effects while carrying out the fire design of such structures, because as a result of internal forces redistribution the ultimate limit state in fire situation may be achieved earlier than for analysis neglecting these effects. In the paper there is analyzed the question of stress redistribution for cross-sections of RC beam fixed at both ends subjected to the action of dead and live loads on its upper surface as well as the fire temperature acting from the other three sides of the beam. The temperature change with the time was assumed in accordance with standard fire curve (ISO 834). Analysis was carried out with the finite element method (FEM) with the application of Abaqus program. Results of analysis indicate the importance of redistribution effects for longer fire duration time (more than 90 min) and the redistribution includes the reduction in values obtained for span cross-section and increase in support one.

1. Introduction

The action of fire temperature onto RC structural elements results in the change in thermal/physical and mechanical material properties for concrete and reinforcing steel. One of the basic mechanical parameter influencing the behaviour of the RC elements in fire situations is concrete compressive strength. With the increase in the temperature this parameter is subjected to reduction – for the temperature level 400 ÷ 500°C concrete compressive strength reaches only the value of about 50% of the initial strength (determined at the temperature equal to 20°C), while for about 800 ÷ 900°C it becomes practically negligible [1, 2, 3]. The detailed effect of elevated temperatures onto thermal and mechanical parameters of concrete depends on numerous factors: concrete type, test regime, loading technique, load level, heat rate and maximum temperature level, moisture conditions and others [2, 3]. Behaviour of RC structural elements cross-section depends also significantly on reinforcing steel mechanical properties. The yielding stress reduces with the temperature, which is especially visible for the temperatures between 400 and 600°C – within this range loss in the initial value may reach 50%.

Important physical material parameter that influences the distribution of stresses in the RC element in fire situations is thermal elongation coefficient responsible for the relative change in elements dimensions while heating. Its value is constant only for small temperature changes, while for fire conditions becomes non-linear function of temperature. According to [4] assuming linear thermal elongation coefficient for the whole analyzed scope of fire temperatures while determining the stresses caused by high temperatures may lead to faulty results. Distribution of temperature field within the
cross-section of element is influenced by the following material properties: specific heat, thermal conductivity, density. All these parameters are non-linear temperature functions which results in non-linear heat transfer problem.

Similar situation is for reinforcing steel, where thermal properties depend on the temperature. But the influence of steel thermal properties onto distribution of temperature in element is almost negligible due to the mass of concrete constituting the element. For RC elements in statically indetermined structural systems the change in the material properties and characteristics of the cross-sections stiffness as a result of temperature increase due to the fire action leads to redistributions of stresses/internal forces along the elements length and in its cross-sections. The analysis of this problem with the application of FEM (Abaqus program) will be presented in the work. The redistribution of internal forces for continuous beams or slabs as well as for frames may lead to their pre-mature failure due to excessive bending moments or shear forces.

2. Description of the analyzed RC beam

The conducted analysis includes RC beam fixed at both ends with the span length equal to 6,0 m and cross-section dimensions: 0,3m (width) by 0,50m (height). The beam is made of concrete strength class C35/45 and reinforced with steel RB500W (bar diameter equal to 20mm). Concrete nominal cover is taken as 40mm. General view of the beam as well as the arrangement of reinforcement is presented in figure 1.

![Figure 1. Analyzed RC beam: a) general view of the element – longitudinal cross-section; b) arrangement of the major reinforcement for the span; c) arrangement for the major reinforcement for the support](image)
3. Material models

Analyzed beam was modelled within Abaqus program as solid type element, constructed from concrete cubic body with embedded reinforcement in the form of wire-type element with given diameter. Element was modelled in 3D stress state. Application of wire-type element made it impossible to take into account the change in steel properties with the temperature and such elements within the program do not allow to account for the heat transfer process. But as it was stated in introduction such assumption and modelling do not lead to significant differences in results.

3.1. Model for concrete

Basic material functions and relationships applied here in were taken from PN-EN 1992-1-2 [1]. Firstly, thermal/physical parameters for concrete (specific heat, thermal conductivity, thermal elongation coefficient, density) were applied which are responsible for temperature field distribution within each point of the RC element. Then, mechanical properties of concrete and steel (concrete compressive and tensile strengths, reinforcing steel yielding stress) were used which determines the mechanical response of the element.

Additionally, the change of concrete elasticity modulus as a function of temperature was taken according to [6] – by reduction factor defined as following:

\[ k_{E,T} = 1,0 \text{ for } T < 150^\circ C \]  
\[ k_{E,T} = \frac{700 - T}{550} \text{ for } T \geq 150^\circ C \]

In order to model the concrete behaviour in applied FEM program (Abaqus) three types of models may be used: smeared crack concrete model, brittle crack concrete model and concrete damaged plasticity model. For the purpose of conducted analysis Concrete Damaged Plasticity (CDP) model was used as it makes possible to model non-elastic concrete behaviour as a material with different properties at compression and tension. Such model assumes two major mechanism of concrete defects: cracking due to tension and crushing from compression. In cases of modelling the complex non-linear problems (unsteady and coupled ones) application of the exact material behaviour description may cause some problems for the stability of solution and may become non-effective due to the calculation duration time. For such situations the simple method for elastic-plastic material description as bi-linear one can be used – see: figure 2.

3.2. Model for reinforcing steel

Reinforcing steel for the analysis in FEM program was modeled as wire-type element. Thus, the influence of thermal properties of steel as a temperature function is not taken into account while determining the temperature field distribution in concrete cross-section. The properties of steel are then assumed as the initial values (as determined at temperature equal to 20°C), both for elastic and
plastic range. Mechanical properties of reinforcing steel at elevated temperatures were taken according to PN-EN 1992-1-2 [1]. Elasticity modulus for steel is assumed as constant value equal to 200 GPa, while the Poisson’s ratio is taken as 0.3. Similarly to concrete, due to the complexity of problem modelling, to illustrate the elastic-plastic behaviour of the material bi-linear model with horizontal line was assumed.

4. Model for accounting the temperature effect

Analysing un-identified heat flow it is necessary to assume initial temperature of the system – for the considered RC beam case this was assumed at the level 20°C. Heat exchange between the heated air (by the fire inside the compartment of building) and analyzed structural element is conducted by three ways: conduction inside the RC beam, convection on the contact zone between concrete and surrounding air, thermal radiation. The method of heat exchange between the heated air and the concrete element is determined by the boundary conditions – in the analyzed case there were assumed the conditions of the third type, i.e. the heat exchange between those two media will be realized by convection and thermal radiation through defining air temperature and heat transfer coefficient by convection (rate of absorption for radiation) for a given time. It was assumed that the beam is heated from three sides (side and bottom surfaces) and the change in air temperature is realized according to standard (ISO 834) temperature – time curve [8]. On the top surface of the beam there was assumed the constant temperature equal to 20°C. Service load (dead load and live load) was applied to the beam before the action of temperature and remained constant during the whole heating process.

Based on recommendations given in [1,8] heat transfer coefficient by convection was assumed as: 25 W/(m²K) – for heated surfaces and 4 W/(m²K) – for un-heated surface. Rate of absorption for radiation was taken as 0.70 according to [1,8]. The air temperature change during the fire was linear approximated with sections which length depends on the rate of temperature increase in time.

5. Results of analysis

As a result of conducted analysis of RC beam with the application of FEM program there were obtained distributions of temperatures as well as the substitute stress distributions for different cross-sections of the element for given fire duration time. What is more, it was possible to compare mentioned distributions for different thermal properties of concrete – as a constant initial value or as a variable being the function of temperature.

5.1. Temperature distribution

In figure 3 there is presented the comparison of temperature distribution within the beam cross-section for two cases: with assuming thermal properties of concrete as constant equal to values determined as for 20°C and with taking into account the influence of temperature onto the variation of these parameters in compliance with [1]. It may be concluded that ignoring the dependence between the thermal properties of concrete and temperature (decrease in the values of thermal conductivity and increase in specific heat) results in quite significant over-estimation of temperatures. This is mainly due to not taking into account the fact of decreasing concrete thermal conductivity with the temperature which is responsible for the slower rate of heat transfer with the increase in fire duration time.

In some of the practical engineering approaches to the question of fire design for RC structural elements it may be assumed the constant values of concrete thermal properties. Then the analysis for fire resistance of the element is based on the smaller cross-sections (as the bigger part of concrete is degraded as a consequence of higher temperatures inside the cross-sections) and as a result lower levels of load bearing capacities for the cross-sections are obtained. So this approach is on the safe side for the verification of fire resistance for the RC elements.
| Fire duration time | Constant thermal properties for concrete as defined for temperature 20°C | Variable thermal properties of concrete as a temperature function according to [1] |
|--------------------|------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| 30 min             | ![30 min temperature distribution](image)                             | ![30 min temperature distribution](image)                                       |
| 60 min             | ![60 min temperature distribution](image)                             | ![60 min temperature distribution](image)                                       |
| 90 min             | ![90 min temperature distribution](image)                             | ![90 min temperature distribution](image)                                       |
| 120 min            | ![120 min temperature distribution](image)                            | ![120 min temperature distribution](image)                                      |

**Figure 3.** Temperature distribution for the RC cross-section for two analyzed data sets of concrete thermal properties.
5.2. Analysis of results for Mises substitute stress distributions

In order to compare the obtained results of stresses in complex stress state with material strength parameters for uni-axial stress state effort hypothesis are applied. One of the most commonly used is amorphous strain energy hypothesis (Mises/H-M-H). It provides good results in comparison with the experiments for elastic-plastic material. According to amorphous strain energy hypothesis the effort of material at given point is conditioned by the density of amorphous strain energy despite the type of stress state.

For three-axial stress state the energy of amorphous strain is defined as:

$$\Phi_t = \frac{(1+\nu)}{6E} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]$$  \hspace{1cm} (2)

and for the uni-axial stress state it can be expresses as:

$$\Phi_t = \frac{(1+\nu)}{3E} \sigma^2$$  \hspace{1cm} (3)

As a result of comparison of presented equations there may be obtained the formula for substitute (reduced) stress (H-M-H or Mises):

$$\sigma_{H-M-H} = \frac{1}{\sqrt{2}} \sqrt{\left( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right)}$$  \hspace{1cm} (4a)

or

$$\sigma_{H-M-H} = \frac{1}{\sqrt{2}} \sqrt{\left[ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6 (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) \right]}$$  \hspace{1cm} (4b)

Figure 4. Haigh-Becker 3D space: $\sigma_1$, $\sigma_2$, $\sigma_3$ – principal stresses

In the applied FEM program (Abaqus) reduced stresses are signed with symbol „MISES”. Within the Haigh-Becker space for the case of three-axial stress state this condition defines the space inside the cylinder with infinite length which axis coincides with axiator axis (figure 4).

Substitute/equivalent plastic strain (PEEQ) in Abaqus program defines the scalar value of plastic strain at the considered point. PEEQ for strains are equivalent for MISES substitute stress for stresses.

In figure 5 there are shown Mises stresses maps for the analyzed RC beam together with the longitudinal cross-sections for stresses for selected cross-section height level (26,46cm and 38,22cm).
The results of analysis in the form of Mises stress maps determined for support and span cross-sections for fire duration time from 0 min (only mechanical/service load on the beams) to 120 minutes are presented in figure 6. It is worth mentioning that all results were obtained for concrete thermal properties being the functions of temperature. Analysing Mises stresses maps for the span cross-section of considered element there may be noticed bigger values of substitute stresses in the upper part of cross-section (4,74 MPa) than in the lower part (3,19 MPa) before applying of fire temperature load, which can be explained by different values of concrete compressive and tensile strengths. That is why the first plastic strain in cross-section is observed in the lower part (appearance of crack in the tensile zone) and as a consequence “re-grouping” of stresses takes place in the cross-section with the stress increase in compressive zone (upper part of cross-section).

After 30 minutes of fire duration time non-elastic strains may be observed for the edge of cross-section. Analysing the stress maps after 60 minutes one can observe the reduction in stresses for lower part of the cross-section due to its heating and degradation of concrete tensile strength. After yielding of upper and lower fibers of the cross-section, re-grouping of stresses take place with the increase in the middle part. Then, with longer fire duration time there may be noticed the reduction in stresses in the whole cross-section.
Conducting the similar analysis for the support cross-section, there are observed plastic strains for the upper part of the cross-section (cracks in the tensile zone). Due to the higher values of stresses in the support zone than in the span cross-section just after few minutes of fire duration non-elastic strains appeared in the whole cross-section. Maximum value of substitute stress in the support cross-section from mechanical (service) load is observed in the lower part (compressive zone) due the higher concrete compressive than tensile strength. First plastic strain appears even before the fire action in the upper part (tensile zone).

As a result of standard fire action after 30 minutes it may be observed the increase in stresses in the whole cross-section. After 60 minutes due to temperature increase and degradation of concrete mechanical parameters there is observed the reduction in stresses in the lower part and increase in the other parts of the cross-section. Starting from 90 minutes of fire duration time it may be observed the decrease in stresses in the upper and lower part of the cross-section, but the increase for the other part, which is the opposite effect than for the span cross-section.
Generally, it may be concluded that after 90 minutes of fire duration time, redistribution of stresses between the support and span cross-sections takes place – stresses in span are reduced, while in the support cross-section increases.

There was also carried out the comparison of results for stresses obtained for concrete thermal properties assumed as constant values and taken as a temperature function. Assuming constant values in analysis results not only in higher temperatures inside the cross-sections of the beam (see: figure 3), but also in significantly higher stresses in cross-sections, both for span and support sections of the analyzed beam.

6. Conclusions
From the conducted analysis the following conclusion may be drawn:

- While designing RC beams in statically in-determined systems with required fire resistance at least R90 it is necessary to take into account redistribution of internal stresses/forces in cross-section along the beam length. As a result of stress/forces „re-grouping” there is observed the increase in their values for support section and reduction – in span sections. Hence, it is necessary to provide appropriate amount of reinforcing steel in regions over supports.
- In order to design RC structures rationally for the fire situation it is necessary to use concrete thermal properties as a temperature functions while determining temperature distribution. Application of constant values of concrete thermal properties leads to overestimation of unfavourable temperature effects onto the structure and to underestimation of the load bearing capacity.
- While carrying out the advanced coupled thermal – displacement analysis for fire situation, thermal/physical and mechanical properties of materials (structural concrete, reinforcing steel) should be taken as temperature dependent functions. Using the constant values of these parameters leads to the overestimation of stresses.
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