Game Theory in Military Decision: An Anecdote

T. Karthy¹, S. Vaishnavi² and A. Barath³
Department of Mathematics, Faculty of Engineering & Technology, SRM institute of science & Technology, Kattankulathur – 603 203, Chennai, Tamilnadu, India.

E-mail: karthyt@srmist.edu.in, vaishnavisridhar26@gmail.com, barathkesavan99@gmail.com

Abstract. The goal of this paper is to explore possibilities of applying game theory in making strategic decisions in military. The plot revolves around the victory of the U.S Army over Japanese Army in regaining the island New Guinea. The decisions leading to victory and failure are solved using Two-person zero sum game.

1. Introduction
Game theory is the analysis of strategic interaction among rational decision-makers using mathematical models. This is applied in various fields of research to explain why an person makes a specific decision and how one individual's decisions influence others. In 2003, in determining courses of action, LTC Cantwell introduced a technique using zero-sum games to enhance military decision-making. He suggested using ordinal values to fill in the USA and the opponent's zero-sum payoff matrix, and then solve the game[1]. Proposed a method and illustrated Cantwell’s example by replacing the ordinal values with cardinal values using multi-attribute decision making techniques. The result was more accurate preferences and utilities. The analysis was by transforming the game from a zero-sum to a non-zero sum game and examined the solutions[2]. mentioned the use of AHP & TOPSIS, a hybrid multi-attribute decision-making methodology, to rate the nine phases of a terrorist attack based on data from twenty-one separate terrorist incidents. We use these frequencies to gain insight into the terrorist activity by using statistical methods to analyze frequencies[3]. proposed a new approach for sensitivity analysis in multi-attribute decision-making problems where if one attribute’s weights change, then we can assess changes in the problem's outcomes.

In this paper, we have used different strategies for U.S Army Vs Japanese army where winning army is assumed to be U.S. Army and Japanese Army Vs. U.S. Army where Japanese army is considered to win the battle. Based on the strategies, different methodologies are adopted to show the how the respective win the title in the battle. This paper is structured as follows: Preliminaries in section 2, Methodologies in section 3, Plot in section 4, in section 5 Main results are discussed and finally in section 6 Conclusion is given.
2. Preliminaries

Definition 2.1: “The outcome of the situation is controlled the decision of all parties involved. Such a situation is termed as competitive situation.”

Definition 2.2: “Game is any set of circumstances that has a result dependent on the actions of two or more decision-makers.”

Definition 2.3: “A game involving ‘n’ players is called n person game.”

Definition 2.4: “If the algebraic sum of gains and losses of all the players is zero in a game, then it is called two-person zero sum game. Otherwise it is called non zero sum game.”

Definition 2.5: “The gain resulting from a two-person zero sum game is represented in the matrix form is called payoff matrix.”

Definition 2.6: “Saddle point of a payoff matrix is that position in the payoff matrix where the maximum of row minima coincides with the minimum of column maxima, the value payoff at the saddle point is called value of the game or saddle point.”

Definition 2.7: If a player knows exactly what the other player would do in a deterministic situation and the goal function is to maximize success, pure strategy is the rule of judgment to choose the correct action.

Definition 2.8: “Zero sum game with two players are called rectangular game.”

Definition 2.9: “When max-min is not equal to min-max, then pure strategy fails and this type of strategy is called mixed strategy.”

3. Methodologies

3.1. Dominance property

Sometimes, it is observed that one of the pure strategies of either player is inferior to atleast one of the remaining ones. The superior strategies are said to dominate the inferior ones. In such cases of dominance, we can reduce the size of pay-off matrix by deleting those strategies which are dominated by others.

3.1.1 General rule.
- If all the element of $x^{th}$ row, $\leq$ corresponding elements of $y^{th}$ row, then $x^{th}$ row is dominated by $y^{th}$ row, therefore, $x^{th}$ row is eliminated.
- If all the elements of $x^{th}$ column are $\geq$ corresponding elements of $y^{th}$ column, $x^{th}$ column is dominating $y^{th}$ column. Therefore, $x^{th}$ column is eliminated.
3.2 Mixed strategy

For any 2×2 two-person zero sum game without saddle point having the pay off matrix for player A is

\[
\begin{pmatrix}
M & A \\
A & \lambda
\end{pmatrix}
\]

The optimum mixed strategies:

Strategy for U.S. Army: \( S_f = \begin{pmatrix} M \\ A \end{pmatrix} \) and Strategy for Japanese army: \( S_j : \begin{pmatrix} M \\ A \end{pmatrix} \)

Where \( p_1 = \frac{a_{21} - a_{11}}{\lambda} \) and \( p_2 = 1 - p_1 \)

\( q_1 = \frac{a_{21} - a_{11}}{\lambda} \) and \( q_2 = 1 - q_1 \)

And Value of the game = \( \frac{a_{21}a_{12} - a_{11}a_{22}}{\lambda} \)

3.2.1 Graphical method. Graphical method is used only in games with no saddle point, and having a pay-off matrix of type 2×n or n×2. General rule:

- Draw two parallel axis say axis1 and axis2.
- Mark the points in the graph and plot the graph form axis2 to axis1.
- For 2xn, the shading region is lower envelope, for nx2 the shading region is upper envelope.
- The intersection of two lines is called the max-min point say H.
- The lines through max-min point gives the pay-off matrix.

4. Plot

The plot revolves around the battle between U.S Army (under the guidance of General Kenney) and Japanese Army. Where General Kenney of allied air force (US) struggled to regain island of new guinea from the hands of Japanese Army. Some information leaked after Japanese action that is Japanese Army will assemble at Rabaul and then sailing over to Lae. So General Kenney and his troops estimated that Japanese Army take 3 days to sail from Rabaul to Lae. General wants to maximize the no. of sailing days by air attack. They had two routes to sail through, Southern direction (clear weather) and the northern direction (worst climatic conditions). So General had a clear idea that one of the routes will be used. So, he proceeded with the following strategies:

i. Mission(place)
ii. Situation and course of action
iii. Analysis of opponent course of action
iv. Comparing of equipments
v. Decision

The northern area was covered by aircrafts and few aircrafts were roaming around southern area just to inform the arrival of the enemy. General’s plan was sending the few attacking aircrafts towards southern direction and other equipped attacking aircrafts towards northern direction. If the Japanese Army chooses the northern direction to sail it guarantees two more days of extension for the Japanese army to reach the shore by attacking them with aircrafts. Else, they happen/choose to sail in southern direction it might take 1-3 more days of extension due to attacking by aircrafts.

By comparing the probability of arriving and with some strategies, the General chose the northern direction because it guarantees two days of extension by attack of aircraft.

Japanese Army, had two choices:

i. Northern direction (poor weather and so less visibility)
ii. Southern direction

In favour of General’s prediction Japanese Army followed northern route and suffered in the attack for two days. And there was a struggle in reaching the shore due to explosion which extended their arrival to shore by two more days. The main motive of this paper is to analyse the usage of game theory in real-time application in various fields in particularly military forces.
4.1 schematic representation of the plot

The representation given by Haywood [4] describes the northern and southern route between Lae and Rabaul.

![Schematic representation of the plot](image)

**Figure 1.** Representation of the plot.

4.2 Assumptions

i. All defence forces act rationally.

ii. Each soldier attempts to maximize success or to minimize casualty.

iii. Complete relevant information is known to each soldier.

iv. Each soldier, makes individual decision without direct communication.

v. Pay-off matrix is constructed with respect to winning/losing probability, situations of the soldiers with respect to days, equipment and so on.

5. Main results

5.1 U.S Army vs Japanese Army

Strategies are applied based on General Kenney’s decision shows the winning of U.S Army

5.1.1 Strategy – I. Matrix is constructed with respect to days where N represents Northern Direction and S represents Southern Direction, the entries in the matrix represent the number of extension days due to air attack.

|       | N  | S  |
|-------|----|----|
| U.S Army | 2 | 2 |

Min(row)- 2,1
Max(column)- 2,3
Max(min)- 2
Min(max)- 2
Max(min) = Min(max) which is called the value of the game = 2
Here the value of the game represents the minimum assured days of air attack.

5.1.2. Strategy-II. Matrix is constructed with respect to equipment, where M represents man power, A represents Aircraft, the entries in the matrix represent the probability values for U.S. Army to win the Japanese army, the entries in the matrix are probability values for winning of U.S. Army

\[
\begin{bmatrix}
M & A \\
0.6 & 0.4 \\
0.4 & 0.6
\end{bmatrix}
\]

Japanese Army

Min(row) - 0.4, 0.4
Max(column) - 0.6, 0.6
Max(min) - 0.4
Min (max) - 0.6
Max(min) ≠ Min(max)
The saddle point is not equal so, pure strategy fails. Games without saddle point is called mixed strategy and the matrix is solved for mixed strategy,

Let the given pay off matrix be

\[
\begin{bmatrix}
M & A \\
\alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22}
\end{bmatrix}
\]

The optimum mixed strategies are

Strategy for U.S. Army: \[S_U = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}\]

and Strategy for Japanese army: \[S_J = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}\]

Now \(\lambda = \alpha_{11} p_1 + \alpha_{22} q_2 - \alpha_{12} p_1 - \alpha_{21} q_2\)

\(= (0.6+0.6) - (0.4+0.4)\)

\(= 1.2 - 0.8\)

\(= 0.4\)

\(\alpha_1 = \frac{0.6 - 0.4}{0.4} \approx 1 \text{ and } \alpha_2 = 2\)

\(\lambda = \frac{0.6 - 0.4}{0.4} \approx 1\)

\(q_1 = \frac{0.6 - 0.4}{0.4} \approx 1 \text{ and } q_2 = 2\)

Value of the game = \(\frac{0.6(0.6) - 0.4(0.4)}{0.4} = \frac{1}{2}\)

Strategies for U. S Army and Japanese Army

\[S_U = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}, \quad S_J = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}\]

Here the value of the game represents the minimum assured probability for winning of U.S. army using equipment.

5.1.3. Strategy – III. Matrix is constructed with respect to equipment (crisp values), where M represents Man Power, S represents Ship, A represents Aircraft, the entries in the matrix are taken as 1 for winning, 0 for losing.

Japanese Army

\[
\begin{bmatrix}
M & A & S \\
1 & 1 & 1
\end{bmatrix}
\]
This pure strategy can be solved using dominance property. From the matrix we see that \( C_1 \) is dominating \( C_3 \). Therefore, omit column \( C_1 \). The resultant reduced matrix is,

\[
\begin{bmatrix}
1 & 0 \\
1 & 1 \\
0 & 0 \\
\end{bmatrix}
\]

Now, \( R_1 \) is dominated by \( R_2 \), therefore, omit \( R_1 \). We get, \( 2 \times 2 \) matrix

\[
\begin{bmatrix}
A & S \\
1 & 1 \\
0 & 0 \\
\end{bmatrix}
\]

Min(row)- 1, 0
Max(column)-1, 1
Max(min)- 1
Min (max)- 1
Max(min) = Min(max) which is called the value of the game = 1, Here the value of the game represents the minimum assured days of air attack.

5.2 Japanese Army vs U.S Army

Strategies are applied based on Japanese army decision shows the winning of Japanese army

5.2.1. Strategy-I. Matrix is constructed with respect to days where \( N \) represents Northern Direction, \( S \) represents Southern Direction, the entries in the matrix represent the number of extension days due to air attack.

\[
\begin{bmatrix}
U.S Army & S & N \\
Japanese Army & 1 & 3 \\
\end{bmatrix}
\]

Min(row)- 1,2
Max(column)- 2,3
Max(min)- 2
Min(max)- 2
Max(min) = Min(max) which is called the value of the game = 2.Here the value of the game represents the minimum assured days of air attack.

5.2.2. Strategy –II. Matrix is constructed with respect to equipment, where, \( M \) represents Man Power, \( S \) represents Ship, \( A \) represents Aircraft, the entries in the matrix are probability values for winning of Japanese army.

\[
\begin{bmatrix}
U.S Army & M & A & S \\
Japanese Army & 0.1 & 0.4 & 0.5 \\
& 0 & 0 & 0 \\
& 0.3 & 0.2 & 0.5 \\
\end{bmatrix}
\]
Applying dominance property for the matrix, $R_2$ is dominated by $R_3$. So, omitting $R_2$ we get a $3 \times 2$ matrix,

\[
\begin{pmatrix}
M & A & S \\
0.1 & 0.4 & 0.5 \\
0.3 & 0.2 & 0.5
\end{pmatrix}
\]

Now let us construct a graph using this matrix.

![Graph showing the max-min point and pay-off matrix](image)

**Figure 2.** construct a graph using this matrix.

The max-min point is H. The lines passing through max-min point gives the pay-off matrix

\[
\begin{pmatrix}
M & A \\
0.1 & 0.4 \\
0.3 & 0.2
\end{pmatrix}
\]

Min(row)- 0.1, 0.2
Max(column)-0.3, 0.4
Max(min)- 0.2
Min (max)- 0.3
Max(min)≠Min(max) . Here pure strategy fails.
Therefore, each player plays with certain probabilistic fixation. This type of strategies called mixed strategy. Let the given pay off matrix be

\[
\begin{pmatrix}
M & A \\
0.1 & 0.4 \\
0.3 & 0.2
\end{pmatrix}
\]

The optimum mixed strategies

Where strategy for Japanese army: $S_J = \begin{pmatrix} q_1 & q_2 \end{pmatrix}$ and strategy for U.S. army: $S_U = \begin{pmatrix} q_1 & q_2 \end{pmatrix}$
Now \( \lambda = (\bar{a}_{11} + \bar{a}_{22}) - (\bar{a}_{12} + \bar{a}_{21}) \)
\[= (0.1 + 0.2) - (0.4 + 0.3) \]
\[= 0.3 - 0.7 = -0.4 \]

\[p_1 = \frac{\bar{a}_{22} - \bar{a}_{21}}{\lambda} = \frac{0.2 - 0.3}{-0.4} = \frac{1}{2} \quad \text{and} \quad p_2 = \frac{1}{2} \]

\[q_1 = \frac{\bar{a}_{21} - \bar{a}_{22}}{\lambda} = \frac{0.2 - 0.4}{-0.4} = \frac{1}{2} \quad \text{and} \quad q_2 = \frac{1}{2} \]

Value of the game
\[\frac{\bar{a}_{11} \bar{a}_{22} - \bar{a}_{12} \bar{a}_{21}}{\lambda} = \frac{0.1(0.2) - 0.4(0.3)}{-0.4} = -0.1 \]

Value of the game = 0.1

Here the value of the game represents the minimum assured probability for winning of Japanese army using equipment.

5.2.3. Strategy-III. Matrix is constructed with respect to equipment (crisp values), where, M represents Man Power, S represents Ship, A represents Aircraft, the entries in the matrix are probability values for winning of Japanese army.

| U.S Army | \( M \) | \( A \) | \( S \) |
|----------|--------|--------|--------|
| \( M \)  | 0.4    | 0.1    | 0.5    |
| \( A \)  | 0      | 0      | 0      |
| \( S \)  | 0.5    | 0      | 0.5    |

Japanese Army

This pure strategy can be solved using dominance property.

From the above matrix we can say that \( R_2 \) is dominated by \( R_3 \)

Therefore, omitting \( R_2 \). We get a 3×2 matrix,

\[
\begin{bmatrix}
M & A \\
0.4 & 0.1 & 0.5 \\
S & 0.5 & 0 & 0.5
\end{bmatrix}
\]

Further \( C_3 \) is dominating \( C_2 \). So, we omit \( C_3 \) and hence we obtain 2×2 matrix,

\[
\begin{bmatrix}
M & A \\
0.4 & 0.1 \\
S & 0.5 & 0
\end{bmatrix}
\]

Min(row)- 0.1, 0
Max(column)- 0.5, 0.1
Max(min)- 0.1
Min (max)- 0.1
Max(min) = Min(max) which is called the value of the game = 0.1

Here the value of the game represents the minimum assured probability for winning of Japanese army using equipment.

6. Conclusion
Usage of different strategies based on decision making reflects the result of any battle. We can conclude that decision making plays an important role in determining success. However, in our plot
General Kenney’s strategy was the turning point for winning of the battle. And these strategies will probably yield success and thus we can continue to employ these strategies in a battle to assist the decision makers. We conclude that game theory provides insights to military planning and strategy.

7. References
[1] William P Fox 2016 Applied Game Theory to Improve Strategic and Tactical Military Decisions Journal of Defense Management 6 1-7
[2] William P Fox 2014 TOPSIS in business analytics. InEncyclopedia of business analytics and optimization Journal of Defense Management 4 1-6
[3] Alireza, Alinezhada and Abbas Amini Sensitivity analysis of TOPSIS technique: the results of change in the weight of one attribute on the final ranking of alternatives 2011 Journal of Optimization in Industrial Engineering 7 23-28
[4] Haywood O G, 1954 Journal of The Operations Research Society of America 365-384.
[5] Ravid I1990 Military decision, game theory and intelligence: An anecdote. Operations Research 38 260-4
[6] Kenny G C 1949 General Kenney Reports, Little, Brown, Boston, Mass.
[7] Morgenstern O and Von Neumann J 1953 Theory of games and economic behaviour Princeton university press
[8] McDonald J 1996 Strategy in poker business & war WW Norton & Company
[9] Smith Jr NM Walters SS Brooks FC and Blackwell DH 1953 The Theory of Value and the Science of Decision a Summary J .OPER RES SOC. 1103-13
[10] Williams JD 1986 The Compleat Strategyst Being a primer on the theory of games of strategy Courier Corporation