A defense of Hellwig-Kraus reductions

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Abstract
Aharonov and Albert analyze a thought experiment which they believe shows that quantum mechanical state reductions occur along temporal hypersurfaces in Minkowski space. They conclude that the covariant state reduction theory of Hellwig and Kraus does not apply. In Part I of this paper we disagree with this interpretation of the A-A experiment, and show the adequacy of the H-K theory. In Part II we examine the belief that H-K reductions produce self contradicting causal loops, and/or give rise to absurd boundary conditions. These objections to the theory are shown to be unfounded.

PACS 03.65 - Quantum mechanics
PACS 03.65.Bz - Foundations, theory of measurement

1 Introduction

When the position of a quantum mechanical particle is measured, its state undergoes a collapse that is instantaneous and universal. If that were not so, there would be a finite probability of finding the particle in two different places at once. The collapse must furthermore be such that the particle cannot be simultaneously found at two different places relative to any Lorentz observer. It follows that state reduction for a position measurement must be effective throughout the entire space-like region surrounding the measurement. This illustrates the Hellwig-Kraus reduction thesis which claims more generally that any measurement reduces the state in all of the surrounding space-like region [1]. Accordingly, state reduction is felt all along the surface of the backward time cone, and at all events forward of that surface.

Several authors believe that there exist special measurements that do not result in a Hellwig-Kraus type of reduction [2],[3],[4]. The reductions associated
with these measurements are said to occur across hypersurfaces of time, in violation of the covariant H-K reduction scheme. The prototype of this kind of measurement is given in a thought experiment proposed by Aharonov and Albert in Ref. 2.

Part I

2 The Aharonov & Albert experiment

Two spatially separated 1/2 spin particles are initially prepared in the state

$$|0,0\rangle = |J_z = 0, J^2 = 0\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2\} \quad (1)$$

The first particle is made to interact with a detector at point $x_1$, and the second particle is made to interact with another detector at point $x_2 \neq x_1$, where the interactions are simultaneous in some Lorentz Frame. Prior to these interactions, the detectors are brought together for the purpose of correlating their internal variables in a specific way. After the interactions, Aharonov & Albert show that neither one of the detectors has measured a single spin component of either particle by itself. Instead, the detectors have together recorded the fact that the particles continue to be in the state $|0,0\rangle$. On this basis, A&A claim to have designed “...a system of purely local experiments which measures a nonlocal property of the system at a well defined time...”(Ref. 2, p.363). That is true so long as “measures” refers only to a measurement interaction, and does not include the observation that the author believes is necessary to produce the state reduction associated with measurement. A&A do not make this distinction. They focus entirely on the interaction as though it is sufficient to bring about a state reduction. They conclude that this particular reduction takes place across a (flat) hypersurface of time, which means that the covariant (cone-shaped) reduction of Hellwig and Kraus cannot apply.

The author believes that a proper analysis of any state reduction must include an account of all detector observations as well as their interactions. Any observation is intrinsically local, and is said by von Neumann to trigger a measurement process that he calls Process I (see Sect. 5). It is claimed here that any such measurement will sharply locate the vertex of a Hellwig-Kraus (cone shaped) reduction at the observation site; and therefore, that the A&A experiment does not result in (flat) temporal hypersurface reductions.
3 Nonlocal nondemolition

Consider the case shown in Fig. 1 in which one of the detector-particle interactions (event 1) takes place before the other (event 2) in some Lorentz frame. The world lines of particles #1 and #2 are parallel to each other and to the \( t \)-axis, where the particles together occupy the state \(|0,0\rangle\). The two detectors (the square boxes) are initially brought together at event \( I \) in order to prepare their variables as specified in A&A (Ref. 2, Eqs. 13, 18). The detectors then separate for the purpose of interacting with the particles at the space-like events 1 and 2. After these interactions, the detectors turn back to re-unite with one another at event \( M \) so their variables can be jointly compared. The last step is not discussed by A&A, yet joint comparison is essential. Prior to this reunion the separate detector variables are indefinite, which is why neither detector by itself can measure the spin state of either particle. However, the detectors are correlated by the initial preparation in such a way that their combined values are definite (Ref. 2, p. 362-3). This combined definiteness applies before event 1, and again after event 2 in the Lorentz frame of Fig. 1. It is this that allows an observer at event \( M \) to make a definite measurement on the combined apparatus, confirming that the particles remain in the state \(|0,0\rangle\). Aharonov and Albert call this a nonlocal nondemolition experiment because it measures a nonlocal state without destroying it.

Of course, it is not necessary to bring the detectors together if the information can be communicated to event \( M \) by some other means. It is only necessary to insure that such information is correctly combined prior to an observation.

The detector’s combined variables will be correlated any time before event 1 and after event 2 as has been said. But A&A show that this correlation is destroyed at any time between events 1 and 2. Between these events they say, ”. . . the full state will not be separable into a state of the two-particle system and a state of the measuring apparatus . . .” (Ref. 2, p.364). The interaction at event 1 therefore disturbs the system in such a way as to entangle the particle and detector states in Hilbert space, and this disturbance is rectified by the interaction at event 2. In a Lorentz frame in which events 1 and 2 are simultaneous, this disturbance does not occur.

4 Sequential cycles

We now subject each detector to two interactions (see Fig. 2), where each detector moves along with, and remains close to, the particle that it monitors. The
first detector on the left makes contact with particle #1 (not shown) at events 1 and 3, and the second detector on the right makes contact with particle #2 (not shown) at events 2 and 4. Both events on the right are assumed to have a space-like relationship to both events on the left.

These detectors have been initially prepared at an event such as $I$ in Fig. 1, but in this case they are not reunited for the purpose of measurement. Instead, a measurement is affected by communicating information about the detector’s variables to a common event like $M$ by other means. This information can be taken from any pair of events along the world lines of the detectors, such as events $a$ and $v$, or $b$ and $u$. We will say that the detectors have been compared at events $m$ and $n$ when information from these two events has been brought together at the common event for comparison. This comparison will not by itself produce a state reduction. We will say that the detectors have been observed at events $m$ and $n$ when the comparison at those two events has been externally
“observed” at the common event. This observation will result in a reduction that is reflected back along the surface of the backward time cone of the common event (through nonlocal correlations) to the detectors and the particles. These are reduced when their world lines penetrate the surface of the backward time cone of the common event, as prescribed by Hellwig and Kraus.

As previously stated, when the detectors are compared at events $a$ and $u$ or $b$ and $v$, their combined variables will be definite and they will register the fact that the particles occupy the state $|0, 0\rangle$. If, additionally, the detectors are observed at events $a$ and $u$ or $b$ and $v$, then the particle state $|0, 0\rangle$ will be empirically verified and will be identically reduced.

We also learned in the previous section that if the detectors are compared at events $b$ and $u$, their combined variables will be indefinite. This reflects the fact that the system is disrupted after event 1 and before event 2. Therefore, an observation of detectors at events $b$ and $u$ will be accompanied by a state reduction that decouples the particles. Such an observation is irreversible. It will disrupt the system in a way that event 2 cannot restore.

If the system survives events 1 and 2 without a disruptive observation, we will have completed a nondemolition cycle. Events 3 and 4, complete another nondemolition cycle. This cycle can be repeated many times so long as there is no permanent disruption of the system due to an observation of the detectors at events such as $b$ and $u$, or $d$ and $w$.

Since nothing happens to alter the states between event $w$ and $v$ (projecting backward in time), the particle and detector states at events $d$ and $v$ are
also non-separable in Hilbert space. This means that when the detectors are compared at events \(d\) and \(v\), their combined variables will be indefinite. And if the detectors are observed at events \(d\) and \(v\), the resulting irreversible reduction will permanently disrupt the system, destroying the state \(|0,0\rangle\) by decoupling the particles from one another. This, we say, happens because the pair of events 3 and 2 is demolitional.

If Fig. 2 is Lorentz transformed in such a way as to make event 2 occur before event 1, then by a similar reasoning the combined variables at events \(b\) and \(z\) will also be indefinite. Therefore, events 1 and 4 are demolitional as well. It appears that observations of either of the cross pairs 1 and 4, or 3 and 2, will irreversibly disrupt the particle state, whereas parallel pair observations will not. This conclusion is Lorentz invariant.

All of the above assumes that the detectors are correctly prepared at event \(I\). With a different initial preparation of the detectors, Aharonov and Albert show that events 3 and 2 can be made to be nondemolitional, and event 1 and 2 would then be demolitional. In this case, the change in initial preparation is one that would cause detector \#1 to emerge from event 1 in such a way that a comparison of the combined variables (after 1 and before 2) would be definite.

Aharonov and Albert draw a different conclusion from sequential experiments like those in Fig. 2. For them, the parallel event pairs 1-2, and 3-4 constitute complete nondemolition cycles that automatically include state reduction, and this is supposedly achieved without having to introduce an external observer (Ref. 2, p. 365). They also say that the cross event pairs 3-2, and 1-4, are complete cycles (again, including state reduction without the benefit of an observer), but this time the result is demolitional. As a result, we have the odd situation that the four unobserved events in Fig. 2 are said to leave the system in the original state \(|0,0\rangle\), and at the same time, they are said to leave it irreversibly reduced in a disrupted state. A&A accept this result at face value, allowing all such conflicting state reductions to be realized. They formalize the apparent contradiction by representing reduced states as surface functionals, thereby allowing the different reductions to apply along different temporal hypersurfaces. This can be done in such a way as to preserve Lorentz invariance, which otherwise appears to be lost [5].

In this paper we say that competing reductions are not equally realized. A realized reduction will be either demolitional or nondemolitional, and the one that actually occurs depends on which events are witnessed by an outside observer.
5 Two processes

John von Neumann’s theory of measurement introduces a special non-Schrödinger process, which he calls Process I, that describes the collapse of the state function under measurement. It changes a pure quantum mechanical state into a classical mixture. The standard Schrödinger evolution, which he calls Process II, can only change pure states into pure states [6].

If the detectors in Fig. 2 are not observed, they will evolve under Schrödinger (von Neumann’s Process II) causing their joint variables to vary back and forth between definite and indefinite values. This would produce no paradox, no irreversible reduction, and no difficulty with the ordinary meaning of Lorentz invariance.

We have seen in the previous section that detector information must be brought to a common location for external examination. It is here that we make the observation that initiates von Neumann’s Process I, and locates the vertex of a Hellwig-Kraus reduction at the observation site. Since we decide when and where we are going to make an observation, we are the ones who decide whether the associated reduction will preserve, or permanently destroy the particle state. We can choose to observe events 1 and 2, or 3 and 4, and preserve the state $|0,0\rangle$; or we can choose to observe events 3 and 2, or 1 and 4, and irreversibly destroy the state.

Choosing when to measure a quantum mechanical system is identical with deciding when to impose additional boundary conditions. It opens a closed system to further conditionals. It is difficult for many physicists to believe that there exists an independent process that competes with Schrödinger at such a fundamental level, but this is the implication of von Neumann’s theory of measurement. Von Neumann showed that the boundary between a quantum mechanical system and a measuring device is arbitrary. However, there must always be a boundary, and there is always something on the observer side of that boundary that cannot be included in the system. If everything were includable, then the wider system would be entirely Schrödinger driven. It would then be unable to undergo a state reduction. Therefore, something that is intrinsically outside the system gives rise to state reduction. Von Neumann himself believed this “something” to be related to the existence of consciousness, as have several others [8][9][10] including the author [11][12]. But however consciousness may or

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1. This boundary has nothing to do with the microscopic/macroscopic distinction. According to von Neumann, a macroscopic measuring device can always be included inside of a quantum mechanical system.

2. This statement precludes solutions of the type proposed by Ghiradi et al. [7].
may not enter into the picture, reduction represents a boundary making ability of the quantum mechanical observer that cannot be set aside. These boundaries are an essential part of quantum epistemology inasmuch as a quantum mechanical universe without definite boundaries is alien to human experience. We humans are constrained to deal with classical reality at the boundary of a quantum reality, and are therefore limited to a theory that deals with finite quantum mechanical systems that exist outside of ourselves. Heisenberg writes, “...it is important to remember that in natural science we are not interested in the universe as a whole, including ourselves, but we direct our attention to some part of the universe and make that the object of our studies” [13]. There is something about ourselves, as quantum mechanical observers, that places us firmly outside of the Schrödinger-driven world of quantum mechanics. When this distinctive role of the observer is taken into account, a la von Neumann, the Aharonov and Albert experiment is found to be fully consistent with the covariant reduction theory of Hellwig and Kraus.

**Part II**

6 **Causal loops**

Some find a Hellwig-Kraus reduction disturbing in the way that it projects its influence backward in time (relative to a given Lorentz frame), thereby raising the prospect of a self contradictory causal loop (Ref. 3, p.1696).

Imagine that the spin of the two particles in the state $|0,0\rangle$ are observed by two different (and separated) observers, where there is now no attempt to preserve the state as in Part I. Suppose the first particle is observed at event $A$ in Fig. 3, and is found to have a positive spin. Because of correlations, the second particle at event $B$ will then be found to have a negative spin.

Neither particle can be found in the twice reduced region III, for otherwise there would be a Lorentz frame in which one of the particles would be found in two different places at once. This means that the reduced state of event $A$ is confined to the space-like region around event $A$ that overlaps the backward time cone of event $B$ (labeled region II in Fig. 3). Similarly, the reduced state of event $B$ is confined to region I in Fig. 3. We ignore what happens in the forward time cones of events $A$ and $B$.

We can find the unrenormalized reduced state of event $A$ by operating on $|0,0\rangle$ in Eq. 1 with $\langle \uparrow |$, the measured spin of the first particle at $A$. 


This reduced single particle ket occupies the immediate backward time cone of event $B$ as shown in region II of Fig. 3. It tells us that the observation at event $A$ leaves only one spin possibility at event $B$. When the second particle is subsequently observed at $B$, the result is

$$\langle \uparrow | 0, 0 \rangle = \frac{1}{\sqrt{2}} | \downarrow \rangle_2$$

whose square magnitude is 0.5, the joint probability of finding the first particle with spin up, and the second particle with spin down. The reduced state of event $B$ (shown in region I) can be found in a similar way.

What may be disturbing about this theory is that the reduced state in the immediate backward time cone of event $B$ in Fig. 3 is a function of what happens at event $A$. If the observer at $A$ decides to look at his apparatus, that decision clearly influences the form of the reduced state leading into event $B$. One might therefore suppose that observer $A$ could send a superluminal message to observer $B$ by deciding not to look. But that will not work. When $B$ records a negative spin, he does not know if that happens because he is looking at the original state $|0, 0 \rangle$ and just happens to measure ‘spin down’, or because he is looking at the reduced state of event $A$ which makes ‘spin down’ the only possibility at

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\[^4\text{von Neumann's non-relativistic Process I for this particular renormalized reduction gives the state } | \uparrow \rangle_1 | \downarrow \rangle_2 \text{ that exists any time after event } A \text{ and before event } B \text{ in the Lorentz frame of Fig. 3. This is compatible with the relativistic H-K result if we let } | \uparrow \rangle_1 \text{ be limited to the future time cone of event } A, \text{ and let } | \downarrow \rangle_2 \text{ be limited to the backward time cone of event } B.\]
event $B$. Therefore, A’s decision to observe, or not, cannot send a decipherable message to observer B.

Although observer A can choose to make an observation, he cannot choose the outcome; and so, he cannot choose B’s outcome either. He can no more send a message to B than he can send one to himself by this means. The fact that B’s outcome depends on A’s outcome is called outcome dependence, and the fact that B’s result is independent of A’s decision to make an observation is called parameter independence (Ref. 4, Sect. 8.2).

An ensemble of experiments of this kind is also unable to communicate superluminally. The observer at event A can only record the probability of finding ‘spin up’, and he will get 0.5 no matter what the observer at event $B$ does or experiences. In order to confirm that the joint probability is 0.5 for the $(\uparrow, \downarrow)$ spin combination, it will be necessary for observers A and B to get together (at some later time) to compare notes - to verify correlations. Superluminal communication is then no longer a consideration.

7 Non-local Boundaries

There is still another objection that is raised in connection with the reduction scheme in Fig. 3. The two observations at events $A$ and $B$ in that figure are a pair of boundary conditions that are placed on the original state $|0, 0\rangle$. However, neither one is anywhere near the boundary of that function. They are both some distance away from the shaded area in Fig. 3 over which they have joint jurisdiction. Cohen & Hiley comment on this consequence of the Hellwig-Kraus theory saying, “. . . the two-particle wave function is reduced before either particle has been subjected to a measurement, which seems absurd” (Ref. 3, p. 1695). The author disagrees.

We know that the correlations found in this two particle spin system act nonlocally through distance. There is nothing absurd about that, or at least, there is nothing new about that since the discovery of Bell’s inequalities. Certainly these correlations can be nonlocal through time as well as distance in any Lorentz frame - so long as they do not extend into the backward time cone of a given measurement. There is therefore no reason why the joint events A and B in Fig. 3 should not be the terminal boundary conditions of the shaded area in that figure, as well as the initial conditions of the reductions in regions I and II.
References

[1] K. E. Hellwig, K. Kraus, *Phys. Rev. D*, 1, 566 (1970)

[2] Y. Aharonov, D. Z. Albert, *Phys. Rev. D*, 24, No. 2, 359 (1981)

[3] O. Cohen, B. J. Hiley, *Found. Phys.*, 25, No. 12, 1669 (1995)

[4] B. d’Espagnat, *Veiled Reality*, Addison-Wesley, Reading, Mass., p.210 (1995)

[5] Y. Aharonov, D. Z. Albert, *Phys. Rev. D*, 29, No. 2, 228 (1984)

[6] J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Princeton University press, Princeton, NJ, p. 351

[7] G. C. Ghirardi, A. Rimini, T. Weber, *Phys. Rev. D*, 34, No. 2, 470 (1986)

[8] E. Wigner, ”Remarks on the Mind-Body Question”, In *Scientist Speculates*, I.J. Good (ed), Basic Books, New York (1962); Reprint In: E. Wigner, *Symmetries and Reflections*, Indiana University press, Bloomington (1967)

[9] F. London, E. Bauer, *La theorie de l’observation en mecanique quantique*, Hermann & Cie, Paris, 1939.

[10] O. Costa de Beauregard, *Found. Phys.*, 6, 539 (1976)

[11] R. Mould, *Found. Phys.*, 28, No. 11, 1703 (1998)

[12] R. Mould, to appear in *Found. Phys.*, 29, No. 12, (1999). Also, see the author’s home page [http://nuclear.physics.sunysb.edu/~mould](http://nuclear.physics.sunysb.edu/~mould), or the Archives [http://xxx.lanl.gov/abs/quant-ph/9908077](http://xxx.lanl.gov/abs/quant-ph/9908077)

[13] W. Heisenberg, *Physics and Philosophy*, Harpers & Row, New York (1958)

[14] J. S. Bell, *Physics* 1, 195 (1964)