Coherent states in quantum cosmology

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In the realm of a quantum cosmological model for dark energy in which we have been able to construct a well-defined Hilbert space, a consistent coherent state representation has been formulated that may describe the quantum state of the universe and has a well-behaved semiclassical limit.

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I. INTRODUCTION

In a previous paper \cite{1} a general, simple quantum description was constructed for a model of an homogeneous and isotropic universe filled with a fluid described by an equation of state, \( p = w \rho \), being \( p \) and \( \rho \) the pressure and the energy density of the fluid, respectively, and \( w \) is a constant parameter. That model can be regarded to be an approximate idealization in which only a particular kind of energy dominates the universe along its time evolution, from the beginning to the end. Among these particular dominating energies, most emphasis was paid to the case of dark energy. The great advantage of this mode is that it is analytically solvable and, therefore, able to neatly show the analogy between quantum mechanical open systems and quantum cosmology, and take it quite far at least formally, even though quantum cosmology adds some exceptional features to the formalism. Thus, an analytically solvable model for which the complete quantum development can be tracked can be considered.

On the other hand, coherent states have been always considered as mathematical objects with applications in quantum physics. In fact, the large number of their applications lead to the introduction for new definitions of particular quantum systems for which coherent states are involved. Coherent states can be constructed from the algebras which are behind their definition. More precisely, in the literature we usually deal with Heisenberg algebras to obtain them. Nevertheless, in some works \cite{2,3} coherent states for some quantum systems are constructed from the so-called Generalized Heisenberg Algebras (GHA).

In this paper we shall take advantage from the above property of the model to investigate the role that coherent states, obtained from a GHA, may play in a cosmological model. We in fact obtain general expressions for the cosmic wavefunctions that describe coherent states, which can be taken as a basis for further developments of this subject.

We outline the paper as follows. In sec. II we give a brief summary of the cosmological model, reviewing the basics aspects of its Hilbert space. Coherent states are formulated and described in sec. III and, in section IV, we give some conclusions and further comments.

II. A COSMOLOGICAL MODEL

The model considered in ref. \cite{1} consists of a Friedman-Lemaître-Robertson-Walker (FLRW) universe filled with a fluid described by the equation of state, \( p = w \rho \), where \( w \) is a constant parameter. For a gauge \( \mathcal{N} = a^3 \), where \( \mathcal{N} \) is the lapse function and \( a \equiv a(t) \) is the cosmic scale factor, the Hamiltonian of the system is given by,

\[
H = -\frac{2\pi G}{3} a^2 p_n^2 + \rho_0 a^{3(1-w)},
\]

where \( p_n \) is the conjugate momenta to the scale factor, \( G \) is the gravitational constant, and \( \rho_0 \) is the energy density of the fluid at the coincidence time \cite{1}. Then, a set of Hamiltonian eigenfunctionals can be obtained. In the configuration space, they are given as

\[
\phi_n(a) = N_n a^n \mathcal{J}_n(\lambda a^n),
\]

in which \( N_n \) is a normalization constant, \( \alpha \) is a parameter depicting the factor ordering ambiguity, \( \mathcal{J}_n \) is a Bessel function of the first kind and order \( n \), and,

\[
q = \frac{3}{2}(1-w), \quad \lambda = \frac{1}{\hbar q} \sqrt{\frac{3}{2\pi G \rho_0}}.
\]

The functions given by Eq. (2) correspond to the following eigenvalue problem,

\[
\hat{H}\phi_n(a) = \mu_n \phi_n(a),
\]

with

\[
\mu_n = q^2 n^2 - \epsilon_0^2,
\]

where \( \epsilon_0^2 \) is a factor which depends on the factor ordering choice. Now, we have to impose some boundary conditions in order to construct wavefunctionals which can describe the state of the universe. In particular, when
the fluid is dark energy (\( w < -\frac{1}{3} \)), the boundary conditions are: i) the wavefunctionals have to be regular everywhere, even when the metric degenerates, \( a \to 0 \), and ii) they should vanish at the big rip singularity when \( a \to \infty \). The boundary conditions are satisfied by the Hamiltonian eigenfunctionals when we impose the following restrictions on the parameter \( \alpha \),
\[
-qn \leq \alpha < \frac{q}{2} \sim \frac{3}{2}.
\]
Now, in order to develop the usual machinery of quantum mechanics, we need to construct a well-defined Hilbert space. It is usually an impossible task, in general, when gravitational fields are taken into account, since they appear non-renormalizable infinities in the formalism. Just in the case of very simplified minisuperspaces, a regularization process can be made and, then, a Hilbert space can be however defined.
We can start by defining the Hamiltonian eigenstates, \( |n> \), to be those states represented in the configuration space by the wavefunctionals given in Eq. (2), i.e.,
\[
<n|a>=<a|n>=\phi_n(a),
\]
as the wave functionals considered so far are real functions. Then, let us define the scalar product to be,
\[
<f|g>=\int_0^\infty da W(a) f(a)g(a),
\]
where \( W(a)=a^k \) is a weight function. Thus, the orthogonality relations between the Hamiltonian eigenstates turn out to be,
\[
<n|m>=\frac{N_n N_m}{q}\int_0^\infty du u^{k+2\alpha+1} \frac{1}{q^2} \Gamma(n+1)\Gamma(m+1)
\]
with \( u = a^2 \), and using the standard bibliography [4], this integral can be performed for the following values,
\[
n+m+1 > 1 - \frac{k+2\alpha+1}{q} > 0.
\]
For instance, for a weight function, \( W(a)=a^{2\alpha+1} \), the orthogonality relations are,
\[
<n|m> = \sqrt{\frac{\pi}{2q^2 \Gamma(2n+1)\Gamma(2m+1)}}
\]
where, \( \sqrt{q}=\lambda q=\frac{1}{\sqrt{2\pi}} \sqrt{\frac{3}{\pi e^2} \rho_0} \). In particular, they do not show any problem with the normalization of the zero mode because,
\[
\langle 0|0> = N_0^2 \sqrt{\frac{\pi}{2q^2 \Gamma(2)}},
\]
which can be normalized by choosing an appropriate value of the normalization constant, \( N_0 \). However, by Eq. (11), these relations do not form an orthogonal basis for the representation of the quantum state of the universe. Nevertheless, using the scalar product [3], we can define an orthonormal basis, in terms of Laguerre polynomials. For the value \( k = \frac{q}{2} - (2\alpha+1) \) in the integration measure, we can use the following set of functionals,
\[
\psi_n(a) = N_n a^{\frac{4\alpha+q}{2}} e^{-\frac{4\alpha}{q}} \Lambda_n(\lambda a^q),
\]
where,
\[
\Lambda_n(x) = \sum_{m=0}^n \left( \frac{n}{m} \right) \frac{(-x)^m}{m!},
\]
is the Laguerre polynomial of order \( n \). They form an orthonormal basis with appropriate values of the normalization constants, \( N_n \). Then, the Hamiltonian eigenstates can be decomposed into the basis defined by the set \( \{ \psi_n \} \) as,
\[
\phi_n(a) = \sum_{m=0}^\infty C_{nm}\psi_m(a),
\]
where the coefficients, \( C_{nm} \), are given by,
\[
C_{nm} = \langle \psi_m|\phi_n> = \int_0^\infty da a^{2\alpha+1} \psi_m(a)\psi_n(a).
\]
We can then develop the usual formalism of quantum mechanics in this orthonormal basis.
The standard procedure of constructing coherent states is clearer when we work with an orthonormal basis of Hamiltonian eigenstates. Let us therefore consider the scalar product [3], for \( k = -(2\alpha+1) \). In that case, the orthogonality relations for the Hamiltonian eigenstates can be written as [1],
\[
\langle n|n> = 1, \forall n,
\]
\[
\langle n|m> = 0, |n-m| \text{ even},
\]
\[
\langle n|m> = \frac{4}{\pi} (-1)^{\frac{n(n-m)}{2}} \sqrt{n+m} , |n-m| \text{ odd}.
\]
The price that we have to pay when using this scalar product is that then we need a regularization procedure for the zero mode [1]. Its advantage is that the set of Hamiltonian eigenstates can be split in two sets, for even and odds modes, which are orthogonal to each other, separately. Then, we can use an analogous formalism to that developed in [3]. Let us split the Hilbert space as
\[
\mathcal{H} = \mathcal{H}_+ \oplus \mathcal{H}_-,
\]
where the subspaces, \( \mathcal{H}_+ \) and \( \mathcal{H}_- \), are chosen to be,
\[
f_+(u) \in \mathcal{H}_+ \Rightarrow f_+(u) = \sum_{n=0}^\infty C_{2n}\phi_{2n}(u) \otimes \chi_+
\]
\[
f_-(u) \in \mathcal{H}_- \Rightarrow f_-(u) = \sum_{n=0}^\infty C_{2n+1}\phi_{2n+1}(u) \otimes \chi_-(20)
\]
with some constants $C_k$, and,
\[
\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\]
(21)

The subspaces, $\mathcal{H}_+$ and $\mathcal{H}_-$, turn out to be the subspaces of even and odd functionals, and any function belonging

\[
\langle f | g \rangle = \lim_{l_p \to 0} \int_{l_p}^{\infty} du W_1(u) f^+(u) g(u) = \lim_{l_p \to 0} \int_{l_p}^{\infty} du W_1(u) (f_+(u) g_+(u) + f_-(u) g_-(u)),
\]
(23)

weighted by the function, $W_1(u) = u^{-(2\alpha+1)}$, with the limit being introduced to regularize the zero mode. With this scalar product the basis $\{\phi_n\}_{n \in \mathbb{N}}$ becomes orthonormal,

\[
\langle n | m \rangle = \langle \phi_n | \phi_m \rangle = \xi \lim_{l_p \to 0} \int_{l_p}^{\infty} \frac{1}{u} J_n(u) J_m(u),
\]
(24)

where $\xi$ is given by the usual scalar product between the vectors, $\chi_{\pm}$, i.e.,

\[
\begin{align*}
n, m \text{ even} & \Rightarrow \xi = \chi_+^\dagger \chi_+ = 1 \\
n, m \text{ odd} & \Rightarrow \xi = \chi_-^\dagger \chi_- = 1 \\
n \text{ even}, m \text{ odd} & \Rightarrow \xi = \chi_+^\dagger \chi_- = 0 \\
n \text{ odd}, m \text{ even} & \Rightarrow \xi = \chi_-^\dagger \chi_+ = 0.
\end{align*}
\]
(25)

Therefore, we have obtained an orthogonal basis of Hamiltonian eigenfunctional for a universe filled with dark energy. Now, we can apply the formalism described in ref. [7] to construct coherent states for the model being considered.

III. COHERENT STATES

The interest of formulating coherent states in cosmology is two fold. On the one hand, this would prepare the mechanics of the universe to further, potentially, generalizable new developments, and, on the other hand, to enhance the analogy between usual quantum mechanics and cosmology.

In what follows we shall use the formalism to construct coherent states for a system described by a generalized algebra $\hat{\mathfrak{g}}$, given by

\[

\begin{align*}
H_0 A^\dagger &= A^\dagger f(H_0) \\
AH_0 &= f(H_0) A \\
[A^\dagger, A] &= H_0 - f(H_0),
\end{align*}
\]
(26-28)

where $A$, $A^\dagger$ and $H_0$ are the generators of the algebra, and $f(x)$ is called the characteristic function of the system. $H_0$ is the Hamiltonian of the physical system under
to the space $\mathcal{H}$ can be decomposed as,

\[
f(u) = f_+(u) \oplus f_-(u).
\]
(22)

Let us define now, in this space, the following scalar product for any two functions, $f(u), g(u) \in \mathcal{H},$

\[
H_0 |m\rangle = \epsilon_m |m\rangle,
\]
(29)

and $A^\dagger$ and $A$ are the generalized creation and annihilation operators,

\[
A^\dagger |m\rangle = N_m |m+1\rangle, \quad A|m\rangle = N_{m-1} |m-1\rangle,
\]
(30-31)

where, $N_m^2 = \epsilon_{m+1} - \epsilon_m$. The use of a generalized algebra $\hat{\mathfrak{g}}$ would add a parametrization through the characteristic function, $f(H_0)$, that allows us to have a systematic covering of distinct potentials for the given system. The customary Heisenberg algebra is recovered in the limit value $f(x) = 1 + x$ [6].

Then, the coherent states are defined to be the eigenstates of the generalized annihilation operator,

\[
A|z\rangle = z|z\rangle,
\]
(32)

where $z$ is a generally complex number.

Since we have a Hamiltonian spectrum for the model of a dark energy dominated universe, $H|n\rangle = \epsilon_n |n\rangle$, we can now find the characteristic function, $f(x)$, and the quantum excitation levels can be written as $\epsilon_{n+1} = f(\epsilon_{n})$. Choosing a factor ordering $\alpha = \beta$ so that $\epsilon_0^2 = 0$, we have,

\[
\epsilon_{n+1} = q^2(n+1)^2 = \epsilon_n + 2q\sqrt{\epsilon_n} + q^2 = (\sqrt{\epsilon_n} + q)^2 = f(\epsilon_n).
\]
(33)

The spectrum of the case being considered is formally similar to the spectrum for a free particle in a square well potential $\hat{\mathfrak{g}}$, and the computation to follow can be done in a parallel way.

Thus, the coherent states are given by,

\[
|z\rangle = N(z) \sum_{n=0}^{\infty} \frac{z^n}{N_{n-1}} |n\rangle,
\]
(34)

where,

\[
N_n = N_0 N_1 \cdots N_n,
\]
(35)
with, for consistency, $N_{-1}! = 1$. Therefore, since $N_{n-1}^2 = \epsilon_n - \epsilon_0$, it can be checked that in our case,

$$N_{n-1}! = q^n n!,$$

and the coherent states can be written then as,

$$|z\rangle = N(z) \sum_{n=0}^{\infty} \frac{z^n}{q^n n!} |n\rangle. \tag{37}$$

This expression can be formally simplified, in terms of the creation operator, since the state $|n\rangle$ can be written as,

$$|n\rangle = \frac{1}{N_{n-1}!} (A^\dagger)^n |0\rangle = \frac{1}{q^n n!} (A^\dagger)^n |0\rangle. \tag{38}$$

The coherent states can be expressed then with a formal expression,

$$|z\rangle = N(z) \sum_{n=0}^{\infty} \left( \frac{z A^\dagger}{q^2} \right)^n \frac{1}{(n!)^2} |0\rangle = N(z) I_0 \left( 2 \sqrt{\frac{z A^\dagger}{q^2}} \right) |0\rangle, \tag{39}$$

where, $I_0(x)$, is the modified Bessel function of the first kind of order zero.

Now, we have to impose the following conditions in order to get the so-called Klauder’s coherent states [6] (KCS):

i/ Normalization:

$$\langle z|z\rangle = 1, \tag{40}$$

ii/ Continuity in the label, $z$:

$$|z - z'| \to 0 \Rightarrow \| |z\rangle - |z'\rangle \| \to 0, \tag{41}$$

iii/ Completeness:

$$\int d^2 z \ W(z)|z\rangle \langle z| = 1. \tag{42}$$

The normalization condition can be found by choosing an appropriate normalization function, $N(z)$. In terms of the Hamiltonian eigenvectors, the norm of the coherent states can be expressed as,

$$1 = \langle z|z\rangle = N^2(z) \sum_{n,m=0}^{\infty} \frac{z^n (z^*)^m}{q^n q^m n! m!} \langle n|m \rangle \tag{43}$$

Therefore, the normalization function, $N(z)$, ought to be chosen as,

$$N^{-2}(z) = \sum_{n=0}^{\infty} \left( \frac{|z|}{q} \right)^{2n} \frac{1}{(n!)^2} = I_0 \left( \frac{2|z|}{q} \right). \tag{44}$$

Then, the normalized coherent states can be written as,

$$|z\rangle = \left( I_0 \left( \frac{2|z|}{q} \right) \right)^{-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{|z|^n}{q^n n!} |n\rangle, \tag{45}$$

or rescaling the variable $|z|$ as $|z| \to q|z|$, it can be re-expressed as,

$$|z\rangle = \left( I_0 \left( 2|z| \right) \right)^{-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{|z|^n}{n!} |n\rangle. \tag{46}$$

In terms of the creation operator, using Eq. (39), the coherent states can then be also written as,

$$|z\rangle = [I_0 \left( 2|z| \right)]^{-\frac{1}{2}} I_0 \left( 2 \sqrt{\frac{|z| A^\dagger}{q}} \right) |0\rangle. \tag{47}$$

In the configuration space, the wave functionals corresponding to the coherent states (46) can be expressed in terms of the scale factor, $a$, and the variable $z$, in the form,

$$\langle a|z\rangle = \varphi_z (a) \equiv \varphi (a, z) = [I_0 (2|z|)]^{-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{|z|^n}{n!} \phi_n (a), \tag{48}$$

where the function $\varphi (a, z)$ has to be interpreted as a functional of paths for the scale factor, $a(t)$, and the variable $z$. These coherent wave functionals satisfy the boundary conditions imposed in the previous section as they are satisfied by the Hamiltonian eigenfunctionals. When the scale factor degenerates, in the limit $a \to 0$, using the asymptotic expansions for Bessel’s functions, the coherent wavefunctionals can be written as,

$$\varphi (z, a) \approx \frac{a^\alpha}{\sqrt{I_0 (2|z|)}} \sum_{n=0}^{\infty} \frac{|z|^n (\lambda a^q)^n}{n! 2^n n!} = a^\alpha I_0 \left( \frac{2\lambda |z| a^q}{\sqrt{I_0 (2|z|)}} \right), \tag{49}$$

which may express the known boundary condition of the universe. If $\alpha$ would vanish, then Eq. (49) expressed the Vilenkin’s tunneling condition [8] as it took on a constant value. If $\alpha > 0$, then Eq. (49) would vanish in accordance with the Hartle-Hawking no boundary proposal [9].

In the opposite limit, for large values of the scale factor, the introduced boundary condition is also obeyed. The limit of large values of the scale factor is equivalent to the semiclassical limit, where $\hbar \to 0$. In both cases, the asymptotic expansions of Bessel’s functions are the same, and the Hamiltonian eigenfunctionals go as,

$$\phi_n (a) \approx \sqrt{2 \pi \lambda a^q} \cos \left( \lambda a^q - \frac{\pi}{2} n - \frac{\pi}{4} \right). \tag{50}$$

Then, the coherent states can be written as,
for large values of the scale factor. Since in this model the classical action is \( S_c = \lambda a^q \), it turns out that the functional \( \varphi(z, a) \) can be also expressed as,

\[
\varphi(z, a) \approx \frac{\cos(|z| - S_c(a)) - \sin(|z| - S_c(a))}{\sqrt{\pi S_c(a) I_0(2|z|)}} \to 0 \quad (a \to \infty).
\]

The coherent states, in the semiclassical limit are those represented in Fig. 1 and Fig. 2. There is a set of maxima for the coherent wave functionals, for the values \( z_k \) given by,

\[
|z_k| = S_c(a) - \arctan \left( \frac{2S_c(a) - 1}{2S_c(a) + 1} \right) + 2k\pi.
\]

Therefore, we have obtained expressions for normalized coherent states, in the configuration space. They satisfy the imposed boundary conditions, both, in the limit of large values of the scale factor and when it degenerates. The same limit for large values of the scale factor runs for the semiclassical limit, in which the coherent states should represent, by the Hartle criterion, valid semiclassical approximations. That is the case because Eq. (51) is, for any value of the parameter \(|z|\), an oscillatory function of the classical action with a prefactor which goes to zero as the scale factor grows up.

The second condition for coherent states to be a set of KCS amounts to the continuity in the label \( z \). It is easy to check that this condition is satisfied. For a given pair of complex numbers, \( z = re^{i\theta} \) and \( z' = r'e^{i\theta'} \), which are very close to one another, \( r \approx r' \) and \( \theta - \theta' \approx 0 \), the scalar product between them is given by,

\[
\langle z|z' \rangle \approx \frac{1}{\sqrt{I_0 \left( \frac{2\pi}{q} \right) I_0 \left( \frac{2\pi'}{q} \right)}} \sum_{n=0}^{\infty} \frac{(r'r'^*)^n}{q^{2n/(1!^2)}} \approx 1,
\]

when \( r' \to r \), so the norm of the difference between two coherent states goes to zero as they approach,

\[
|| |z\rangle - |z'\rangle ||^2 = 2(1 - \langle z|z' \rangle) \approx 0.
\]

The third condition on the completeness of coherent states to be a set of KCS, can be fulfilled by including an appropriate weight function for the integration in the variable \( z \); i.e., it should be satisfied that

\[
\int d^2z W_2(z)|z\rangle\langle z| = 1,
\]

which in our case implies,

\[
2\pi \sum_{n=0}^{\infty} \frac{|n\rangle\langle n|}{(n!)^2} \int_0^{\infty} d|z| W_2(z) |z|^{2n} \frac{I_0(2|z|)}{I_0(2|z|)} = 1.
\]
This corresponds to choosing a weight function
\[ W_2(|z|) = \frac{2|z|}{\pi} J_0(2|z|) K_0(2|z|), \quad (58) \]
in the formalism of ref. [6], and also amounts to the fulfillment of the completeness condition. The latter condition comes from the equalities,
\[
\int d^2 z W_2(z) |z\rangle\langle z| = 4 \sum_{n=0}^{\infty} \frac{|n\rangle\langle n|}{(n!)^2} \int_0^{\infty} d|z| K_0(2|z|) |z|^{2n+1}
= \sum_{n=0}^{\infty} |n\rangle\langle n| = 1, \quad (59)
\]
where we have used [4],
\[
\int_0^{\infty} dx K_0(2x)x^{2n+1} = \frac{(n!)^2}{4}. \quad (60)
\]

IV. CONCLUSIONS AND FURTHER COMMENTS

We have obtained a set of Klauder coherent states for a dark energy dominated universe. They satisfy the boundary conditions and may lead to valid semiclassical approximations. Coherent states might represent a continuous set of states ascribable to classical universes, which are in this way interpretable as a multiverse. The different universes differ from one another in a smooth way by the value taken on by the parameter \( z \).

The distinction between on-shell (\( \hat{H}\phi_n = 0 \)) and off-shell (\( \hat{H}\phi_n \neq 0 \)) contributions depends on the choice of the factor ordering. This ultimately implies that, if the factor ordering choice becomes eventually related to the particular choice of a time variable, the different Hamiltonian eigenstates may represent the so-called ground state wave functional for particular choices of the time variable, in the configuration space. In that case, coherent states can be interpreted as the ground state for a given time variable, i.e., for particular reference system.

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