Convection and convective overshooting in stars more massive than 10 $M_\odot$

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Abstract

We examine how the mixing length parameter $\alpha_P$ and the overshooting parameter $\delta_{ov}$ affect the properties of convective cores and convective envelopes in stars more massive than 10 $M_\odot$. First, we show that a larger value of $\alpha_P$ leads to a stronger mixing, a smaller chemical gradient, a higher effective temperature, and a smaller stellar radius. We then find that if a star develops convective core overshooting during the main sequence phase, the star will enter the red (super)giant phase earlier than a star without core overshooting. Finally, we find that a convective envelope leads to a discontinuity of the hydrogen profile above the hydrogen burning shell. Convective envelope overshooting can facilitate the occurrence of blue loop in the Hertzsprung–Russell diagram.

Key words: stars: evolution — stars: interiors — supergiants

1 Introduction

Massive stars have convective cores and radiative envelopes. Convection can mix different chemical elements in the convection zone, and bring heavy elements generated by the nuclear reaction region to the stellar surface. It is also an important energy transport mechanism. Local mixing length theory (MLT) developed by Böhm-Vitense (1958) is the most widely used to describe convection in stellar models. The mixing length $l$ in convective regions is specified as a multiple of the pressure scale height $l = \alpha_P H_p$, where $\alpha_P$ is a free parameter and $H_p$ is the pressure scale height. Pasetto et al. (2014) and Magic, Weiss, and Asplund (2013) showed that the mixing length parameter $\alpha_P$ was of great importance for determining the convective energy transport, and it directly affected the stellar interior temperature structure. However, from the theoretical point of view a knowledge of $\alpha_P$ is still a long-standing problem.

According to the local MLT, the boundary of a convection core is fixed where the acceleration of the fluid element is zero. However, a convective cell may not have a velocity of zero when it arrives at the expected convective core boundary, and will continue to move some distance before coming to rest. Such a phenomenon is referred to as the convective overshooting. Classical treatment of convective overshooting is based on the nonlocal MLT (Zahn 1991). The predicted properties of stellar evolution are highly sensitive to the degree of convective core overshooting in the main sequence and core helium burning phases (Maeder 1975, 1976; Stothers & Chin 1981; Bertelli et al. 1985; Chen & Han 2002, 2003; Noels et al. 2010; Montalbán et al. 2013). Moreover, Bressan et al. (2015) and Ding and Li (2014) showed that convective overshooting and semiconvection greatly affected the extension of mixing and the rate of chemical mixing in the convective envelopes.

A red supergiant (RSG) is an evolved, massive (10–30 $M_\odot$) core helium burning star (e.g., Meynet & Maeder 2000; Eldridge et al. 2008; Brott et al. 2011; Davies et al. 2013). It represents a key phase in the evolution of stars more massive than 10 $M_\odot$. However, convection overshooting in the evolution of stellar models from blue to red
in the post-main sequence phase and from red to blue in the blue loop phase has not been studied systematically.

In this paper, we attempt to investigate the properties of the convective cores and the convective envelopes of stars more massive than 10 \( M_\odot \) under different convection and convective overshooting assumptions. Special attention is given to the convective overshooting at the bottom of the convective envelope during the central helium burning phase. Section 2 describes the evolution code, and gives the input parameters for the models. Section 3 gives the main results and discussions in details. In section 4 we summarize our conclusions.

### 2 Models

We employ a stellar evolution code (Eggleton 1971, 1972, 1973), which has been updated by Han, Podsiadlowski, and Eggleton (1994) and Pols et al. (1995, 1998). The code uses a self-adaptive non-Lagrangian mesh. Both convective and semiconvective mixings are treated as diffusion processes. The stellar structure equations, the mesh equations, and the diffusion equations are solved simultaneously. A stellar model is well represented by only 199 mesh points at any stage of its evolution.

#### Table 1. Four sets of evolutionary models.

| Case   | \( \alpha_p \) | \( \delta_{ov} \) |
|--------|-----------------|-------------------|
| Case 1 | 1.5             | 0.0               |
| Case 2 | 1.5             | 0.12              |
| Case 3 | 2.0             | 0.0               |
| Case 4 | 2.0             | 0.12              |

![Fig. 1. Evolutionary tracks of 10, 15, 20, 30, 40, and 60 \( M_\odot \) stars on the HR diagram.](image-url)
Fig. 2. Internal structures of 20 and 30 $M_\odot$ stars, from the MS to the exhaustion of central helium. Filled regions represent the fully convective zones.

up to central carbon burning (Eggleton 2008, 2009). In this paper, we use 499 mesh points. The stellar structure will probably be more accurate than if using only 199 mesh points.

We follow the work of Schröder, Pols, and Eggleton (1997) and Pols et al. (1998). Standard MLT is used by our code to describe the convection. Convective overshooting is based on the stability criterion itself, the $\nabla$ prescription, by incorporating a condition that the mixing occurs in a region with $\nabla_{\text{rad}} > \nabla_{\text{ad}} - \delta$, where $\nabla_{\text{rad}}$ and $\nabla_{\text{ad}}$ are the radiative and adiabatic temperature gradients, respectively. Parameter $\delta$ is defined as the product of a free parameter $\delta_{\text{ov}}$, the overshooting parameter, and a conveniently chosen factor which depends only on the ratio $\zeta$ of radiation pressure to gas pressure:

$$\delta = \frac{\delta_{\text{ov}}}{(2.5 + 20\zeta + 16\zeta^2)}. \quad (1)$$

The $\nabla$ prescription with $\delta_{\text{ov}} = 0.12$ leads to the values of overshooting length $l_{\text{ov}}$ that vary between 0.22 $H_\odot$ and 0.4 $H_\odot$ for the low-mass and high-mass stars.

Evolutionary models are computed for 10, 15, 20, 30, 40, and 60 $M_\odot$ stars. For each mass, four sets of evolutionary models from the main sequence (MS) to the end of central helium (or carbon) burning phase are computed (see table 1). The first set with a ratio of $\alpha_p = 1.5$ and the third with 2.0 are computed without a certain amount of overshooting. The second set with $\alpha_p = 1.5$ and the fourth with 2.0 are computed with convective overshooting $\delta_{\text{ov}} = 0.12$.

Mass loss comes from Reimers (1975) and de Jager, Nieuwenhuijzen, and van der Hucht (1988). We do not consider here the effect of rotation or magnetic fields. The effect of rotation on the structure and evolution of stars is a complicated process and has been studied by Heger, Langer, and Woosley (2000) and Käpylä et al. (2007). The effect of magnetic fields on stellar convection has also been investigated by Cantiello and Braithwaite (2011).

3 Results
3.1 Hertzsprung–Russell diagram

In figure 1 we show four cases of evolutionary tracks for each mass in the Hertzsprung–Russell (HR) diagram. We can see that stars evolve toward a lower effective temperature. First we compare case 1 with case 2 for each mass. It can be noted that convective core overshooting produces an extended track during the MS phase. Case 2 has a larger luminosity than case 1 for most of the time. Maeder and Meynet (1987) concluded that convective core overshooting models with an MS termination in the range of spectral type B0 or later ($\log T_{\text{eff}} < 4.47$) produced an MS widening. On the one hand, convective core overshooting makes the nuclear reaction region larger and increases the
MS lifetime. The chemical discontinuity outside the convective core is removed by overshooting. On the other hand, core overshooting increases the amount of fuel available to the nuclear burning region, and enhances the energy production. The obtained conclusions are also applicable to cases 3 and 4.

Secondly, we check cases 2 and 4 for each mass. We find that case 4 has a higher effective temperature than case 2 during the central helium burning phase. It indicates that the conducting heat of convection is affected deeply by $\alpha_p$. A larger value of $\alpha_p$ means that buoyancy can do more work on the convective cells. Thus, the degree of convective mixing is enhanced, and the efficiency of convective heat transfer is increased.

3.2 Internal structure

3.2.1 Lifetime

Four cases of the internal structure of 20 and 30 $M_\odot$ stars are shown in figure 2. It is evident that convective core overshooting significantly delays the occurrence of the central helium burning phase and increases the MS lifetime. The lifetime depends on the ratio $q_{cc} M/L$, where $q_{cc}$ is the initial core mass fraction and $M$ and $L$ are the stellar mass and luminosity, respectively (Maeder & Meynet 1987). In cases 2 and 4, there are more massive convective cores at the MS phase, and the thermonuclear reactions increase the MS lifetime $t_{H}$. However, during the helium burning phase the lifetime $t_{He}$ is decreased by convective core overshooting. This is due to the fact that the evolutionary tracks of models remain at a higher luminosity. As a result, convective core overshooting reduces the ratio $t_{He}/t_{H}$.

3.2.2 Convective envelope

In figure 2, we can see the radial extent and location of the subsurface convection zones in our 20 and 30 $M_\odot$ stars from the zero age MS to roughly the end of core hydrogen burning. Cantiello et al. (2009) studied the convection zones in the outer envelope of hot massive stars during the MS phase. They found that these near-surface convection zones were caused by opacity peaks associated with iron and helium ionization. Furthermore, our models...
Fig. 4. Stellar radius in our 15, 20, and 30 $M_\odot$ stars during the core helium burning phase.

show that during the helium burning phase the stars possess deep convective envelopes, which are located above the hydrogen burning shell. Frick and Strittmatter (1972) found that the hydrogen burning shell was located just outside the helium core, and the stellar envelope would contract in the early core helium burning stage. In the late stage of central helium burning, helium abundance was fairly low and the stellar core would contract. Then the stellar envelope expanded correspondingly and entered the red (super)giant phase. Lai and Li (2011) indicated that convective envelope overshooting would lead to the increase of opacity and radiative temperature gradient $\nabla_{\text{rad}}$ during the red giant branch phase. As a result, a convective stable region below the base of the convective envelope will be converted to a convective unstable region.

However, we notice that case 1 has a larger convective envelope than case 3, and similarly case 2 has a larger convective envelope than case 4 during the central helium burning phase. This indicates that a larger $\alpha_p$ produces more efficient mixing in the chemical gradient region, leading to a negligible chemical gradient and suppressing the semi-convection. Stothers and Chin (1985) and Bressan, Chiosi, and Bertelli (1981) found that core overshooting reduced convective instability in the envelope.

It is striking to notice that both cases 1 and 3 have a small intermediate convective zone, just above the convective core, in the early stage of core helium burning. The reason is that the model without core overshooting leads to a higher hydrogen abundance outside the convective core. As a result, opacity and $\nabla_{\text{rad}}$ will be larger. So semiconvection as well as full-convection can be observed in the early core helium burning phase.

3.2.3 Stellar radius

In figure 3 we show the behaviour of radius $R$ (in solar units) as a function of time in our 15, 20, and 30 $M_\odot$ stars. Overshooting from the core evidently provides a larger reservoir of available nuclear fuel during the core hydrogen burning phase and consequently it produces an increase in the lifetime of the core hydrogen burning phase. In figure 4,
during the core helium burning phase, the most notable feature is that case 1 has a larger radius than case 3, and case 2 has a larger radius than case 4. As we know well, the radius of a star is related to temperature and luminosity by

$$\frac{R}{R_\odot} = \left(\frac{L}{L_\odot}\right)^{1/2} \left(\frac{T}{T_\odot}\right)^{-2}.$$  \hspace{1cm} (2)

In cases 3 and 4 the value $\alpha_p$ equals 2.0, which is larger than in cases 1 and 2 ($\alpha_p = 1.5$). It means that a stronger mixing ($\alpha_p = 2.0$) will increase the effective surface temperature of a star. Hence the radius of a star decreases at a given luminosity. Lai and Li (2011) showed that a larger value of $\alpha_p$ would normally increase the efficiency of energy transport, and lead to a higher effective temperature and a smaller stellar radius.

3.2.4 Core mass
In figure 5, for each mass the evolution of the core mass as a function of time is presented. The boundary of the convective core is chosen on the Schwarzschild criterion. We can see that cases 2 and 4 including overshooting have more massive convective cores than cases 1 and 3 during the MS phase. Stars leave the MS phase when the hydrogen in their cores is exhausted, at the same time that convection will stop in stellar cores and the masses of the convective cores approximate to zero. The helium core contracts on the thermal timescale. Eventually, the core temperature is high enough for stars to ignite their central helium supply, and convection then begins again. It should be noticed that cases 1 and 3 overlap, and cases 2 and 4 overlap. The values $\alpha_p$ in cases 1 and 3 equal 1.5 and 2.0, respectively. It
indicates that the difference in value on $\alpha_p$ has a negligible influence on the core mass.

3.3 Core potential

When does a star enter the red (super)giant phase? Lauterborn, Refsdal, and Weigert (1971) and Mowlavi and Forestini (1994) showed that core potential $\phi_c = M_c/R_c$ described sufficiently the evolution of stellar models from blue to red in the post-MS phase. $M_c$ and $R_c$ are the mass and radius (in solar units) of the core, respectively, which extends from the centre to the base of the hydrogen burning shell. Lauterborn, Refsdal, and Weigert (1971) showed that a star lay closer to the Hayashi line in the HR diagram for higher $\phi_c$. A critical potential $\phi_{1cc} = M_{1c}/R_{1c}$ can be found when the effective surface temperature of a star reaches log ($T_{eff}$) = 3.6. When the core potential exceeds the critical value $\phi_{1cc}$, the star becomes an RSG. In cases 2 and 4, $\phi_c$ rapidly exceeds $\phi_{1cc}$ (or $\phi_{2cc}$). This result can be understood by considering during the MS phase convective core overshooting makes the nuclear reaction region larger, and that the central temperature rises, and further that helium burning happens earlier. The star becomes an RSG just at the onset of core helium burning.

3.3.1 Blue loop

Blue loops that emerge from the region of red (super)giants on the HR diagram during the core helium burning phase are still poorly known, especially in stellar models of intermediate and high mass (Stothers & Chin 1981, 1991; Alongi et al. 1991). Our models develop huge convection envelopes during the RSG phase. Downward convective overshooting at the bottom of the convective envelope will drive the discontinuity of hydrogen into the interior deeper. When the hydrogen burning shell moves outward and encounters the discontinuity, overshooting from the
envelope provides the amount of available fuel. Thus the energy production is enhanced, and the stellar luminosity as well as the effective temperature is increased. The star then becomes a blue supergiant. Lai and Li (2011) and Lauterborn, Refsdal, and Weigert (1971) showed that the occurrence of blue loop critically depended on the hydrogen profile just above the hydrogen burning shell. However, Huang and Weigert (1983) and Bertelli, Bressan, and Chiosi (1985) found that convective core overshooting during the MS phase could significantly suppress the subsequent development of the blue loop. Xu and Li (2004) showed that convective core overshooting took away the chemical gradient profile outside the stellar core, as well as the abundance discontinuity formed later by downward penetration of the convection envelope.

4 Conclusions

In this paper, we investigate the properties of convective cores and convective envelopes in stars more massive than 10 M⊙. Different values of the mixing length parameter αp and the overshooting parameter δov result in different theoretical predictions. The main conclusions are summarized as follows.

(i) Convective core overshooting makes a larger convective core, increases the MS lifetime, and extends the MS band in the HR diagram.

(ii) Increasing the mixing length parameter αp leads to an enhancement of the effective mixing and an increase of the convective heat transfer efficiency in the convective region. Thus, the star has a smaller chemical gradient, a higher effective temperature, and a smaller stellar radius.

(iii) If a star develops convective core overshooting during the MS phase, the star will enter the red (super)giant phase earlier than a star without core overshooting.

(iv) Convective envelope leads to a discontinuity of hydrogen profile above the hydrogen burning shell. Convective envelope overshooting can facilitate the formation of blue loop in the HR diagram.

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