Remarks Concerning the Study of Four-Jet Events from Hadronic Decays of the $Z^0$.

H. Jeremie, P. Leblanc, E. Lefebvre, D. Davignon, G. Karapetian; Université de Montréal.

Abstract: The angular correlations of four-jet events from hadronic decays of the $Z^0$ have been studied in the past mainly to extract from them the fundamental constants of quantum chromodynamics called colour factors. Previous studies have used all the available phase-space in order to maximize statistics. In this note we want to point out the possibility that significant differences between experiment and theory in restricted regions of phase-space might have escaped detection. Such differences could be a harbinger of the existence of new particles. Some preliminary results are presented.

1 Introduction

In second-order QCD perturbation theory four-jet events arise from the production of four hard partons and their subsequent fragmentation. The cross-section for four-parton production by electron-positron annihilation can then be written as:

$$\frac{d\sigma (x)_{\text{quatre-partons}}}{dx} = f_q D_{qqq}(x) + (1-f_q) D_{qgq}(x)$$  (1)

where $D_{qqq}(x)$ and $D_{qgq}(x)$ are differential, normalized, theoretical distributions of variables $x$ for events whose final state consists of four quarks, and events consisting of two quarks and two gluons respectively, while $d\sigma (x)_{\text{four-parton}}/dx$ is the normalized distribution of the data. The value for the fraction $f_q$ is a fundamental prediction of QCD, it is approximately proportional to $T_R$, one of the QCD colour factors. It can be extracted from a sample of four-jet events by varying $f_q$ to obtain a best fit to a measured distribution, if $D_{qqq}(x)$ and $D_{qgq}(x)$ are sufficiently different from each other (see f.ex. figure 3d). Such differences arise because in the case of $qqq$ events the primary quark-antiquark pair radiates a polarized gluon which splits into two quarks of spin 1/2, while in the $qgqg$ case this intermediate gluon splits into two gluons of spin one. Angular momentum conservation then requires the distribution of the final quark-antiquark pair with respect to the initial pair to be different from that where the final partons are gluons. For practical reasons we preferred to work directly with $f_q$ than with $T_R$, the two approaches being equivalent.

If the intermediary gluon would have the possibility to split into fermions other than those with the canonical QCD flavours, the value of $f_q$ (or $T_R$) would show an increase, constituting thus a possible signal of some new physics [2].

---

1 e-mail: jeremie@lps.umontreal.ca
2 this is the English version of an article which appeared in French in ref. [1]
3 $D_{qgq}(x)$ contains events where both quarks radiate a gluon as well as events where one radiated gluon splits into two gluons.
The main theoretical distributions we used here were obtained from the second-order \(O(\alpha_s^2)\) QCD matrix element calculation by Ellis, Ross and Terrano (called ERT model hereafter) [3], as implemented in the JETSET simulation package [4], which includes hadronization of the four partons. The parameters of the model were adjusted so as to reproduce energy-energy correlations and event-shape distributions measured by OPAL [5], and the scaled invariant mass cut-off of the ERT four-parton generator was \(y_{\text{min}}=0.01\).

Higher order QCD processes will also contribute to the production of four-jet events, in which case the value of \(f_q\) derived from eq.(1) will no longer yield a correct measurement of this quantity. However, \(f_q\) still provides a useful method of comparison between data and models. Since this method relies only on comparisons of shapes of angular correlations and no actual four-quark events are ever identified, we call the measured quantity the “apparent” fraction of four quark events.

To gauge the influence of higher orders, a hybrid model combining matrix elements and parton showers, and a matrix element model generating pure five-parton events were also investigated (see refs.[6] and [7]). Such a hybrid model is expected to simulate a number of higher order effects, while the 5-parton events are one of the important contributions of next-to-leading order (NLO) calculations.

Using all events produced in the entire region of phase-space (except for a threshold), previous investigations (refs. [8] to [15]) found good agreement with QCD predictions for the overall global value of \(T_R\), equivalent to \(f_q\).

As previously mentioned, in this work we will try to obtain more information by examining the evolution of \(f_q\) as a function of variables which permit a subdivision of the phase-space. We chose the variable \(m_3 + m_4\), the sum of the masses of the two least energetic jets.

2 Experimental Method

2.1 Event Selection

The events were recorded with the OPAL detector [16] at LEP\(^5\), details of the trigger selection and on-line filtering system can be found in refs. [17] and [18]. A standard selection of multihadronic events was applied [19]. For the present analysis charged particles, measured in the central tracking system of the OPAL detector, and clusters formed by showers in the electromagnetic and hadronic calorimeter were used. Additional corrections were applied to minimize double counting of charged particles in the central detector and the calorimeters [20].

\(^4\)The values of the fragmentation parameters we used were: PARJ(21)=0.49, PARJ(41)=1.8, PARJ(42)=0.6.

\(^5\)Although data taken with the OPAL detector were used, the analysis itself was done independently from the OPAL collaboration. Only the authors cited above assume responsibility for this analysis.
2.2 Reconstruction of Four Jets

We assigned the observed particles to jets, whose directions reflect approximately the directions of the hard partons presumed to be emitted before hadronization\(^6\). In this analysis we employ the DURHAM jet-finder \([21]\), which uses a jet resolution parameter between a pair of jets defined as

\[
y_{ij} = \frac{2}{E_{\text{vis}}^2} \cdot \min(E_{i}^2, E_{j}^2)(1 - \cos \theta_{ij})
\]

where \(E_i, E_j\) correspond to the energies of jets \(i\) and \(j\), while \(\theta_{ij}\) is the angle between them, and \(E_{\text{vis}}\) is the visible energy of the whole event.

The jet-finding algorithm combines iteratively each pair of particles \(k\) and \(l\) with the smallest \(y_{kl}\) into a new pseudo-particle by adding the individual four-momenta, until exactly four pseudo-particles are reconstructed, which are defined to be four jets. Then the minimum value of \(y_{ij}\) of all the possible combinations of the four jets is determined. This value is called \(y_{34}\) and is used to preclassify the events\(^7\). If one stops the iteration of the jet-finding algorithm at five jets, then the corresponding value of \(y_{ij}\) is called \(y_{45}\).

The usual way to choose a sample of four jet events is to specify a fixed value of a parameter called \(y_{\text{cut}}\) and select events such that \(y_{45} < y_{\text{cut}} < y_{34}\). The quantities \(y_{45}\) and \(y_{34}\) represent those values of \(y_{\text{cut}}\) where the event makes a transition from a five-jet classification to a four-jet classification, and four-jet to three-jet, respectively. This method of selecting four-jet events is not sufficient to exclude five- and more parton events reconstructed as four-jet events. We therefore accepted an event if \(y_{34} > 0.012\), but demanded \(y_{45} < 0.006\). This eliminates a large fraction of unresolved five-parton events in the sample, without reducing appreciably the number of four-jet events (22% for data but only 8% for ERT, at the detector level).

From an analysis of \(3.3 \times 10^6\) accepted hadronic events by OPAL at the \(Z^0\) peak we extracted \(8.4 \times 10^4\) four-jet events selected as described above.

A sample of \(14.3 \times 10^4\) simulated four-jet events was used to correct for detector and acceptance effects. They were generated with the ERT matrix element calculation we want to test and subjected to the same cuts as the data.

2.3 Measurement of the Bengtsson-Zerwas correlation

We used as angular variable mostly the Bengtsson-Zerwas \([22]\) correlation, other correlations are also possible \([23]\). It measures the angle \(\chi_{\text{BZ}}\) between the two vectors \(\vec{p}_1 \times \vec{p}_2\) and \(\vec{p}_3 \times \vec{p}_4\). Here \(\vec{p}_1\) to \(\vec{p}_4\) are the momentum vectors of jets 1 to 4, where the numbering refers to jets ordered with respect to their measured energy (\(E_1 > E_2 > E_3 > E_4\)).

\(^6\)The Monte Carlo generator model generates first partons, then fragments them into particles and finally tracks these particles through the detector. These three stages will be referred to in the text as parton level, particle level and detector level respectively.

\(^7\)This definition of \(y_{34}\) is not to be confused with a jet resolution parameter \(y_{ij}\) limited to two individual jets numbered three and four!
Ordering increases the probability that jets 1 and 2 do indeed originate from the primary quarks as required. To minimize the sensitivity to exchanges between jets 1 and 2 or 3 and 4, we used the symmetrized version of the B-Z correlation

$$\chi_{BZ} = \angle(\vec{p}_1 \times \vec{p}_2), (\vec{p}_3 \times \vec{p}_4); \text{ if } \chi_{BZ} > \frac{\pi}{2}, \chi_{BZ} = \pi - \chi_{BZ}$$

so that effectively $\chi_{BZ}$ runs from zero to $\frac{\pi}{2}$. To obtain well defined planes, the angles between jets 1 and 2, and between jets 3 and 4, were required to be smaller than 160°. The number of accepted events quoted above includes this cut. The Bengtsson-Zerwas correlation discriminates well between $q\bar{q}q\bar{q}$ and $q\bar{q}gg$ events, since the polarization of the intermediate gluon [24] influences directly the distribution of its decay products, either two quarks or two gluons. It measures only angles and it relies on jet energies only in so far as energy ordering is required. The angular resolution, as obtained from comparing the ERT Monte Carlo at the particle level with that at the detector level, was 14 degrees RMS.

### 3 Results

![Figure 1: Comparison of the distributions of $m_3 + m_4$ between real data and ERT simulation (normalised with respect to each other). An overall correction of 1 GeV has been applied to the scale of the ERT distribution in order to make the two distributions coincide better. The hashed histogram corresponds to the distribution of four-quark events ($q\bar{q}qq$), while the dotted curve represents the $q\bar{q}gg$ events.](image)

The variable which we will use to subdivide the phase-space is $m_3 + m_4$, the sum of the intrinsic masses of the two least energetic jets. Before proceeding to calculate the correlations and fit them to the data, we have to assure ourselves that the raw distributions of the events as a function of $m_3 + m_4$ are satisfactorily reproduced by the ERT simulation. Figure 1 shows that this is the case.

In figure 2a one can find the results for $f_q$ (full circles), obtained by adjusting $f_q$ (i.e. the relative proportions of the theoretical $q\bar{q}q\bar{q}$ and $q\bar{q}gg$ correlations) to obtain a best fit to
the experimental data\(^8\). One sees that the data follow approximately the ERT predictions (histogram) until 10 GeV, for values above 10 GeV there are substantial differences.

Figure 2: a.) Comparison of the distribution of \(f_q\) as a function of \(m_3 + m_4\) for the data (full circles) with the ERT prediction (histogram).

b.) Comparison of the distribution of \(f_q\) as a function of \(m_3 + m_4\) for a mixture of 5 parton events (15\%) and 4 parton events (85\%) (full circles) with the ERT prediction (histogram). The proportions of the mixture have been adjusted so as to reproduce the global (integrated) value of \(R_{4q} = 1.32\).

c.) Comparison of the distribution of \(f_q\) as a function of \(m_3 + m_4\) for the hybrid model (full circles) with the ERT prediction (histogram). Here also the proportion of the hybrid events has been adjusted so as to reproduce the global (integrated) value of \(R_{4q} = 1.32\). All errors are statistical only.

If we integrate over all events, the global fit yields an apparent fraction of

\[ f_q^{exp} = 0.108 \pm 0.009 \] compared to a theoretical value of \(f_q^{th} = 0.082 \pm 0.001\), (errors statistical).

A convenient way of comparing experiment to theory is to form the ratio

\[ R_{4q} = \frac{f_q^{exp}}{f_q^{th}}, \] which yields

\[ R_{4q} = 1.32 \pm 0.10 \ (\chi^2/dg=0.9) \] for all accepted events. The cited error being purely statistical, such a value of \(R_{4q}\) represents an approximate agreement between data and theory.

But if we divide the available phase-space into two regions, one with \(m_3 + m_4 < 10 \text{ GeV}\) and the other with \(m_3 + m_4 > 10 \text{ GeV}\), we find

for \(m_3 + m_4 < 10 \text{ GeV}: \ R_{4q} = 0.88 \pm 0.11 \ (\chi^2/dg=0.24), \) while

for \(m_3 + m_4 > 10 \text{ GeV}: \ R_{4q} = 2.98 \pm 0.35 \ (\chi^2/dg=1.3), \) errors statistical only.

These rather large differences between the values of \(R_{4q}\) for the two regions of phase-space constitute the most important result of this analysis.

\(^8\)Some bins in the region of high \(m_3 + m_4\) in figure 2a of ref.[1] have been combined, yielding the results displayed here.
Let us now form the double quotient

\[
Q = \frac{R_{4q}(m_3^2 + m_4^2 > 10)}{R_{4q}(m_3^2 + m_4^2 < 10)} = 3.39 \pm 0.57,
\]

which is yet another convenient way to represent our results.

| cut | \(\Delta Q\) |
|-----|-------------|
| a.) \(y_{34} > 0.015\) | \(\pm 0.03\) |
| b.) \(|\cos \theta_{jet}| \leq 0.95\) | \(-0.19\) |
| c.) number of tracks and clusters in each jet > 5 | \(+0.52\) |
| d.) \(\chi_{BZ} \geq 9\) degrees | \(-0.34\) |
| e.) \(\chi_{BZ} \leq 81\) degrees | \(+0.18\) |
| f.) \(y_{45} < 0.009\) | \(\pm 0.42\) |
| g.) \(\theta_{34} \leq 140\) | \(\pm 0.15\) |
| h.) without correction of the ERT scale | \(-0.05\) |
| i.) configuration CH+EM, without hadronic clusters | \(+0.33\) |
| k.) JADE algorithm [25] instead of DURHAM | \(-1.13\) |
| l.) N-R correlation [23] instead of B-Z | \(-0.07\) |
| m.) \(\theta_{34}\) distribution instead of B-Z | \(+0.10\) |
| n.) constrained fit: \(\sum E_i = E_{tot}; \sum \vec{p}_i = 0\) | \(-0.21\) |
| o.) statistical error | \(\pm 0.57\) |
| total systematic error | \(-1.30\) |
| total error | \(\pm 1.2\) |

Table 1: Systematical errors of the quotient \(Q\)

An estimation of the sytematical errors of this quantity can be found in table 1. In this table we consider the variations of the following quantities:

a.) increase of the jet resolution criterium, \(y_{34}\), to 0.015;
b.) elimination of jets which point approximately in the direction of the beam pipe;
c.) elimination of jets with few particles;
d.) and e.) elimination of small and large angles for \(\chi_{BZ}\);
f.) admission of a larger number of 5-jet events;
g.) limitation on \(\theta_{34}\), the angle between jet 3 et jet 4, to 140 degrees instead of 160;
h.) without the correction of 1 GeV for the Monte Carlo scale;
i.) jets calculated without using hadronic clusters and without correction for double counting of energy;
j) using the old JADE algorithm (see ref. [25]) with less jet finding resolution;
k.) replace \(\chi_{BZ}\) by the Nachtmann-Reiter correlation (see ref. [23]);
l.) replace \(\chi_{BZ}\) by \(\theta_{34}\);
m) replace \(\chi_{BZ}\) by \(\theta_{34}\);
n.) fit with constraint by imposing conservation of total energy and momentum.

One finds:

\[Q = 3.39^{+0.97}_{-1.42},\]

so that \(Q\) becomes approximately
\[ Q = 3.4 \pm 1.2. \]

The difference between experiment and theory, expressed by the value of \( Q > 1 \), is approximately an effect of two standard deviations.

Figure 3: a.) Comparison of the distribution of \( \chi_{BZ} \) between data (full circles) and ERT prediction (histogram) for all accepted events (before fitting for the best value of \( f_q \)).

b.) Comparison of the distribution of \( \chi_{BZ} \) between data (full circles) and ERT prediction (histogram) for events with \( m_3 + m_4 < 10 \text{ GeV} \).

c.) Comparison of the distribution of \( \chi_{BZ} \) between data (full circles) and ERT prediction (histogram) for events with \( m_3 + m_4 > 10 \text{ GeV} \).

d.) Comparison of the \( \chi_{BZ} \) distribution for the four-quark events (full circles) with the one for two quark and two gluon events (histogram), as calculated with the ERT prediction for all accepted events. Note the difference in scale for this last graph compared to the three previous ones.

All the distributions are normalised with respect to each other, they are not corrected for detector effects and all errors are only statistical.

In figure 3 we display directly the \( \chi_{BZ} \) distributions which yielded the above-mentioned results. In figure 3a the experimental and theoretical (leading order) distributions are shown for all accepted events, while in figures 3b and c these distributions are displayed for events with \( m_3 + m_4 > 10 \text{ GeV} \) and \( m_3 + m_4 > 10 \text{ GeV} \) respectively. In these three cases the difference between experiment and theory can be expressed by the values for the chi-squares per degree of freedom, which are 1.8, 0.24, and 4.9 respectively. Visual inspection confirms that the difference between experiment and theory is largest for case “c”, \( m_3 + m_4 > 10 \text{ GeV} \). After adjustment by fit (the corresponding distributions are not shown here), the chi-squares become 0.9, 0.1 and 1.3 respectively. Figure 3d is an illustration of the theoretical shape differences between \( q\bar{q}q\bar{q} \) and \( q\bar{q}g\bar{g} \) events (for all accepted events according to ERT), which make our type of analysis possible.
4 Discussion

Figure 2a shows that there are deviations of the experimental distributions from the theoretical ones which concentrate in the region of high values of the variable $m_3 + m_4$. Such deviations could also be caused by the absence of higher order effects in the ERT simulation. To get an idea of the influence such higher order effects might have, we show in figure 2b the results one obtains if one replaces the experimental distributions by distributions explicitly containing five-parton events in addition to the 4-parton events furnished by ERT. The mixture is adjusted so as to reproduce the observed value of 1.32 for $R_{4q}$. In figure 2c one finds the results for a hybrid calculation, where each of the four partons from ERT is followed by a parton shower, before being fragmented into particles. These events are also mixed with regular four-parton events so as to reproduce the observed value of $R_{4q}$. In both cases one expects such calculations to show trends associated with higher order effects. In figures 2b and 2c these calculations are represented as fictitious data (full circles) to be compared with calculations where such effects are not present (histogram). One notices that indeed such higher order effects have a tendency to increase with the value of $m_3 + m_4$, but more gradually than the actually observed experimental results of figure 2a. Another possibility to explain such results could be the appearance of new fermions in the region of 10 to 15 GeV for $m_3 + m_4$, i.e. 5 to 7 GeV for each of the emitted particles. (see f.ex. refs. [26, 27]). At this stage of the analysis it is not possible to discriminate between these possibilities, but it is hoped that this research note motivates other groups to verify these results with more precision.

5 Summary

The Bengtsson-Zerwas [22] angular correlation, calculated with the second order ERT [3] simulation, has been compared with the correlation measured for hadronic four-jet events from the disintegration of the $Z^0$. Shape differences between experiment and theory have been observed. The differences concentrate in the region of $m_3 + m_4 > 10$ GeV, where $m_3 + m_4$ is the sum of the intrinsic masses of the two least energetic jets

References

[1] H.Jeremie, P.Leblanc and E.Lefebvre, Can. Jour. Phys. 84 (2006) 411.
[2] G.Farrar, Phys.Lett. B 265 (1991) 395.
[3] R.K.Ellis, D.A. Ross and A.E. Terrano, Nucl. Phys. B 178 (1981) 421.
[4] T.Sjöstrand, Comp. Phys. Comm. 39 (1986) 347; T.Sjöstrand and M.Bengtsson, Computer Phys. Comm. 43 (1987) 367.
[5] OPAL collaboration, P.D.Acton et al., Phys. Lett. B 276 (1992) 547.

[6] J.André and T.Sjöstrand, Phys. Rev. D 57 (1998) 5767.

[7] K.Hagiwara and D.Zeppenfeld, Nucl. Phys. B 313 (1989) 560; F.Wäckerle, Diplomarbeit Karlsruhe 1994, IEKP-KA/93-19.

[8] ALEPH collaboration, D.Decamp et al., Phys. Lett. B 284 (1992) 151.

[9] DELPHI collaboration, P.Abreu et al., Phys. Lett. B 255 (1991) 466; DELPHI collaboration, P.Abreu et al., Zeit. f. Phys. C 59 (1993) 357.

[10] DELPHI collaboration, P.Abreu et al., Phys. Lett. B 414 (1997) 401.

[11] OPAL collaboration, R. Akers et al., Zeit. f. Phys. C 68 (1995) 519.

[12] OPAL collaboration, R. Akers et al., Zeit. f. Phys. C 65 (1995) 367.

[13] ALEPH collaboration, R.Barate et al., Zeit. f. Phys. C 76 (1997) 1.

[14] OPAL collaboration, G.Abbiendi et al., Eur. Phys. J. C 20 (2001) 601.

[15] ALEPH collaboration, R.Barate et al., Eur. Phys. J. C 27 (2003) 1.

[16] OPAL collaboration, K.Ahmet et al., Nucl. Instr. and Meth. A 305 (1991) 275.

[17] M. Arignon et al., Nucl. Instr. and Meth. A 313 (1992) 103.

[18] D. Charlton et al., Nucl. Instr. and Meth. A 325 (1993) 129.

[19] OPAL collaboration, G.Alexander et al., Zeit. f. Phys. C 52 (1991) 175.

[20] OPAL collaboration, K.Ackerstaff et al., Eur. Phys. J. C 2 (1998) 213.

[21] S. Catani, Y. L. Dokshitzer, M.Olsson, G.Turnock and B.R.Webber, Phys. Lett. B 269 (1991) 432.

[22] M.Bengtsson and P.Zerwas, Phys. Lett. B 208 (1988) 306.

[23] O.Nachtmann and A.Reiter, Zeit. f. Phys. C 16 (1982) 45.

[24] H.A. Olson, P. Osland, I. Overbo, Phys. Lett. B 89 (1980) 221.

[25] JADE collaboration, S.Bethke et al., Phys. Lett. B 213 (1988) 235.

[26] E.L.Berger al., Phys. Rev. Lett. 86 (2001) 4231.

[27] E.L.Berger et al., Phys. Rev. D71 (2005) 14007.