A model of strongly coupled heavy vector resonances for fermion masses and mixings.

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We build a viable and predictive extension of the SM with a heavy vector in the fundamental $SU(2)_L$ representation and nine SM singlet scalar fields, consistent with the current pattern of SM fermion masses and mixings. The masses of the light active neutrinos are generated from radiative seesaw mechanism at one loop level mediated by the neutral components of the heavy vector as well as by the left handed Majorana neutrinos. We carry out an analysis of the predictions in the lepton sector, where the model is only viable for inverted neutrino mass hierarchy.

I. INTRODUCTION

Despite its great consistency with the experimental data, the Standard Model (SM) is unable to explain several issues such as, for example, the number of fermion generations, the observed pattern of fermion masses and mixings, etc. Whereas in the quark sector, the mixing angles are small, in the lepton sector two of the mixing angles are large, and one mixing angle is small. Neutrino experiments have brought clear evidence of neutrino oscillations from the measured neutrino mass squared splittings. The three neutrino flavors mix and at least two of the neutrinos have non vanishing masses, which according to neutrino oscillation experimental data must be smaller than the SM charged fermion masses by many orders of magnitude.

That SM “flavor puzzle” motivates to build models with additional scalars and fermions in their particle spectrum and with an extended gauge group, supplemented by discrete flavour symmetries, which are usually spontaneously broken, in order to generate the observed pattern of SM fermion masses and mixing angles. Recent reviews of discrete flavor groups can be found in Refs. [1–5]. Several discrete groups such as $S_3$ [6–31], $A_4$ [32–72], $S_4$ [73–90], $D_4$ [91–99], $Q_6$ [100–110], $T_7$ [111–120], $T_{13}$ [121–124], $T'$ [125–132], $\Delta(27)$ [133–154], $\Delta(54)$ [155], $\Delta(96)$ [156–158], $\Delta(6N^2)$ [159–161], and $A_5$ [162–173] have been implemented in extensions of the SM, to provide a nice description of the observed pattern of fermion masses and mixing angles.

On the other hand, given the current lack of experimental evidence in favor of the traditional big paradigms of Physics beyond the Standard Model, it seems prudent to explore more exotic paths. In recent years, for instance, some groups have pay attention to spin-1 fields transforming in the fundamental representation of $SU(2)_L$. This kind of field may naturally appears, for instance, in models such as: Higgs-Gauge Unification[175] and Composite Higgs[176].

In this work we build an extension of the Standard Model (SM) where the SM gauge symmetry is supplemented by the $S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$ discrete group and the particle content is extended to include nine SM scalars singlets, two left handed neutrinos $N_{nL}(n = 1, 2)$, singlets under the SM gauge group and a $SU(2)_L$ doublet of heavy vectors. Our model is consistent with the SM fermion masses and mixings. The effective neutrino mass matrix arises through radiative seesaw and the physical observables of the lepton sector agree with their experimental values only for the scenario of inverted neutrino mass hierarchy.

The paper is organized as follows. In section II we explain our model. In Sec. III we focus on the discussion of quark masses and mixing and give our corresponding results. In Sec. IV we discuss the implications of our model on lepton masses and mixings. We conclude in section V. Appendix A provides a concise description of the $S_3$ discrete group.

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II. THE MODEL

We propose an extension of the Standard Model (SM) where the SM gauge symmetry is supplemented by the $S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$ discrete group and the particle content is extended to include the SM scalars singlets $\varphi, \chi, \xi, \eta, \sigma, \rho_k (k = 1, 2)$, two left handed neutrinos $N_{nL} (n = 1, 2)$, singlets under the SM gauge group and a $SU(2)_L$ doublet of heavy vectors. The full symmetry $\mathcal{G}$ of our model experiences the following two step spontaneous breaking:

$$\mathcal{G} = SU(3)_C \times SU(2)_L \times U(1)_Y \times S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$$

$$\downarrow \Lambda_{int}$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\downarrow v$$

$$SU(3)_C \times U(1)_Q$$

where the symmetry breaking scales satisfy the hierarchy $\Lambda_{int} > v$ and $v = 246$ GeV is the electroweak symmetry breaking scale.

In the present model the fermion sector is extended by introducing two heavy left handed Majorana neutrinos, which are singlets under the SM group. The quark, lepton and scalar assignments under the $S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$ discrete group are shown in Tables I, II and III, respectively.

| $q_{1L}$ | $q_{2L}$ | $q_{3L}$ | $u_{1R}$ | $u_{2R}$ | $u_{3R}$ | $d_{1R}$ | $d_{2R}$ | $d_{3R}$ |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $S_3$   | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       |
| $Z_2$   | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
| $Z_6$   | 0       | 0       | 0       | 0       | 0       | 0       | 3       | 3       |
| $Z_8$   | -2      | -1      | 0       | 2       | 1       | 0       | 2       | 1       |
| $Z_{12}$| 0       | 0       | 6       | 6       | 0       | 0       | 0       | 0       |

Table I: Quark assignments under $S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$.

| $l_{1L}$ | $l_{2L}$ | $l_{3L}$ | $l_{1R}$ | $l_{2R}$ | $l_{3R}$ | $N_{1L}$ | $N_{2L}$ |
|----------|----------|----------|----------|----------|----------|----------|----------|
| $S_3$    | 1        | 1        | 2        | 2        | 1        | 1        | 1        |
| $Z_2$    | 0        | 0        | 0        | 0        | 0        | 1        | 1        |
| $Z_6$    | 0        | 2        | 0        | 0        | 0        | 3        | 3        |
| $Z_8$    | -2       | 4        | 0        | 0        | 4        | 0        | 0        |
| $Z_{12}$ | 0        | 6        | 0        | 0        | 6        | 0        | 0        |

Table II: Lepton assignments under $S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$.

| $\phi$ | $\varphi$ | $\chi$ | $\xi$ | $\eta$ | $\sigma$ | $\rho_1$ | $\rho_2$ |
|--------|------------|--------|-------|-------|----------|----------|----------|
| $S_3$  | 1          | 1      | 2     | 2     | 1        | 1        | 1        |
| $Z_2$  | 0          | 0      | 0     | 0     | 0        | 0        | 0        |
| $Z_6$  | 0          | -2     | 0     | -3    | 0        | -3       | 0        |
| $Z_8$  | 0          | -1     | 0     | -1    | -2       | 0        | 0        |
| $Z_{12}$| 0         | 0      | 0     | 0     | 0        | -3       | -2       |

Table III: Scalar assignments under $S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$.

Here the dimensions of the $S_3$ irreducible representations are specified by the numbers in boldface and the different $Z_2 \times Z_6 \times Z_8 \times Z_{12}$ charges are written in additive notation.

The $S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$ assignment of the heavy vector in the fundamental $SU(2)_L$ representation is:

$$V_\mu \sim (1, 1, 0, 0, 0).$$
With the above particle content, the following Yukawa terms for the quark and lepton sectors as well as the interaction terms of the heavy vector with the heavy left handed Majorana neutrinos arise:

\[
\mathcal{L}_Y^{(q)} = y_{33}^{(u)} q_{3L} \bar{u}_{3R} + y_{22}^{(u)} q_{2L} \bar{u}_{2R} \frac{\eta}{\Lambda} + y_{11}^{(u)} q_{1L} \bar{u}_{1R} \frac{\eta^2}{\Lambda^2} + y_{12}^{(u)} q_{2L} \bar{u}_{1R} \frac{\eta^2 \rho_1^2}{\Lambda^4} + y_{13}^{(u)} q_{3L} \bar{u}_{1R} \frac{\eta^2 \rho_2^2}{\Lambda^5} + y_{12}^{(d)} q_{1L} \bar{d}_{2R} \frac{\eta^2 \rho_1^2}{\Lambda^4} + y_{13}^{(d)} q_{1L} \bar{d}_{3R} \frac{\eta^2 \rho_2^2}{\Lambda^5} + y_{11}^{(d)} q_{1L} \bar{d}_{1R} \frac{\eta^2 \rho_1^2}{\Lambda^4} + y_{12}^{(d)} q_{2L} \bar{d}_{1R} \frac{\eta^2 \rho_2^2}{\Lambda^5} + y_{13}^{(d)} q_{3L} \bar{d}_{1R} \frac{\eta^2 \rho_2^2}{\Lambda^5} + y_{12}^{(d)} q_{1L} \bar{d}_{2R} \frac{\eta^6}{\Lambda^{10} - \eta^6 - \eta^4 - \eta^2 - 1} + h.c.,
\]

\[
\mathcal{L}_Y^{(l)} = y_{11}^{(l)} l_{1L} \phi_{1R} \frac{\eta^2 \rho_2^2 \rho_3^2}{\Lambda^2} + y_{22}^{(l)} (l_{1L} \phi \chi)_l + y_{13}^{(l)} l_{1L} \phi_{3R} \frac{\eta^2 \rho_1^2}{\Lambda^4} + y_{23}^{(l)} (l_{1L} \phi \chi)_l + y_{33}^{(l)} (l_{1L} \phi \chi)_l + l_{3R} \rho_3^2 + \frac{1}{2} \sum_{n=1}^{2} m_{N_n} N_{nL} N_{nL} + h.c.,
\]

\[\mathcal{L}_{V1N} = y_{1n}^{(V)} l_{1L} \gamma^\mu V_\mu N_{nL} \frac{\sigma}{\Lambda} + \sum_{n=1}^{2} y_{2n}^{(V)} l_{1L} \gamma^\mu V_\mu N_{nL} \frac{\xi}{\Lambda},\]

where \(y_{ij}^{(u,d)} (i, j = 1, 2, 3), y_{11}^{(l)}, y_{22}^{(l)}, y_{13}^{(l)}, y_{23}^{(l)}, y_{33}^{(l)}, y_{11}^{(V)} \) and \(y_{2n}^{(V)} \) are dimensionless couplings.

From the interactions terms of the heavy vector with the heavy left handed Majorana neutrinos we find:

\[\mathcal{L}_{V1N} \supset y_{1n}^{(V)} l_{1L} \gamma^\mu V_\mu N_{nL} \frac{\sigma}{\Lambda} + y_{2n}^{(V)} \left( l_{2L} + \sqrt{2} l_{3L} \right) \gamma^\mu V_\mu N_{nL} \frac{\xi}{\Lambda}.\]

Besides that, as the hierarchy among charged fermion masses and quark mixing angles mass emerges from the breaking of the \(S_3 \times Z_6 \times Z_8 \times Z_{12} \) discrete group, we set the VEVs of the SM singlet scalar fields with respect to the Wolfenstein parameter \(\lambda = 0.225\), and the model cutoff \(\Lambda\), as follows:

\[v_\chi \sim v_\rho_1 \sim v_\rho_2 \sim v_\phi \sim v_\chi \sim v_\xi = v_\sigma = \Lambda_{int} = \lambda \Lambda.\]

The role of the different discrete group factors of the model is explained in the following. The \(S_3, Z_6, Z_8\) and \(Z_{12}\) discrete groups allow to reduce the number of model parameters and set the SM charged lepton mass hierarchy, which is crucial to get viable textures for the lepton sector consistent with the current pattern of lepton masses and mixings, as we will show in Section [IV]. We use the \(S_3\) discrete group because since it is the smallest non-Abelian group that has been considerably studied in the literature. The \(S_3, Z_6, Z_8\) and \(Z_{12}\) symmetries determine the allowed entries of the charged lepton mass matrix. In addition, the \(Z_6\) symmetry separates the \(S_3\) scalar doublet \(\chi\) participating in the charged lepton Yukawa interactions from the \(S_3\) doublet \(\xi\) that appear in the neutrino Yukawa terms. Furthermore, the \(Z_8\) and \(Z_{12}\) symmetries are crucial for explaining the tau and muon lepton masses and for providing the Cabibbo sized value for the reactor mixing angle \(\theta_{13}\) as well as the Cabibbo sized corrections to the atmospheric mixing angle \(\theta_{23}\), without tuning the charged lepton Yukawa couplings. The smallness of the electron masses is explained by the \(S_3, Z_6, Z_8\) and \(Z_{12}\) discrete symmetries. We use the \(Z_{12}\) discrete symmetry since it is the smallest cyclic symmetry that glue \(\rho_1^2\) and \(\rho_2^2\) with \(l_{3R}\), considering \(l_{3R}\) charged under this symmetry. The \(Z_2\) symmetry, under which only the Majorana neutrino and the heavy vector are charged, is introduced in order to avoid a tree level type I seesaw mechanism for the generation of the light active neutrino masses.

We assume the following VEV pattern for the \(S_3\) doublet SM singlet scalars \(\chi\) and \(\xi\):

\[\langle \chi \rangle = v_\chi (1, 0), \quad \langle \xi \rangle = v_\xi \left( 1, \sqrt{2} \right),\]

which are natural solutions of the scalar potential minimization equations for the whole region of parameter space as shown in detail in Ref. [21].
III. QUARK MASSES AND MIXINGS

From the quark Yukawa terms of Eq. (3), it follows that the SM quark mass matrices are given by:

$$M_U = \begin{pmatrix} a_{11}^{(u)} \lambda^6 & a_{12}^{(u)} \lambda^5 & a_{13}^{(u)} \lambda \\ a_{21}^{(u)} \lambda^5 & a_{22}^{(u)} \lambda^4 & a_{23}^{(u)} \lambda \\ a_{31}^{(u)} \lambda^4 & a_{32}^{(u)} \lambda^3 & a_{33}^{(u)} \lambda \\ \end{pmatrix} \frac{v}{\sqrt{2}},$$

$$M_D = \begin{pmatrix} a_{11}^{(d)} \lambda^7 & a_{12}^{(d)} \lambda^6 & a_{13}^{(d)} \lambda^5 \\ a_{21}^{(d)} \lambda^6 & a_{22}^{(d)} \lambda^5 & a_{23}^{(d)} \lambda^4 \\ a_{31}^{(d)} \lambda^5 & a_{32}^{(d)} \lambda^4 & a_{33}^{(d)} \lambda^3 \\ \end{pmatrix} \frac{v}{\sqrt{2}},$$

where $\lambda = 0.225$, $v = 246$ GeV and $a_{ij}^{(u,d)}$ ($i, j = 1, 2, 3$) are $O(1)$ parameters. Since the charged fermion mass and quark mixing pattern is caused by the spontaneous breaking of the $S_3 \times Z_6 \times Z_8 \times Z_{12}$ discrete group and in order to simplify the analysis, we adopt the following benchmark scenario:

$$a_{12}^{(u)} = a_{21}^{(u)}, \quad a_{31}^{(u)} = y_{13}^{(u)}, \quad a_{32}^{(u)} = y_{23}^{(u)},$$

$$a_{12}^{(d)} = |a_{12}^{(d)}| e^{-i\tau_1}, \quad a_{21}^{(d)} = |a_{12}^{(d)}| e^{i\tau_1},$$

$$a_{13}^{(d)} = |a_{13}^{(d)}| e^{-i\tau_2}, \quad a_{31}^{(d)} = |a_{13}^{(d)}| e^{i\tau_2}, \quad a_{23}^{(d)} = |a_{23}^{(d)}|,$$

Furthermore we set $a_{33}^{(u)} = 1$, which is suggested by naturalness arguments.

For the quark mass matrices given above and considering the benchmark scenario previously described, we look for the eigenvalue problem solutions reproducing the experimental values of the quark masses $177, 178$, quark mixing parameters and CP violating phase $179$, under the condition that the effective parameters $a_{ij}^{(u,d)}$ ($i, j = 1, 2, 3$) be most close to $O(1)$. Applying the standard procedure we find the following solution:

$$a_{11}^{(u)} \simeq 0.58, \quad a_{22}^{(u)} \simeq 2.19, \quad a_{12}^{(u)} \simeq 0.67,$$

$$a_{13}^{(u)} \simeq 0.80, \quad a_{23}^{(u)} \simeq 0.83, \quad a_{11}^{(d)} \simeq 1.96,$$

$$a_{12}^{(d)} \simeq 0.53, \quad a_{13}^{(d)} \simeq 1.07, \quad a_{22}^{(d)} \simeq 1.93,$$

$$a_{23}^{(d)} \simeq 1.36, \quad a_{33}^{(d)} \simeq 1.35, \quad \tau_1 \simeq 9.56^\circ, \quad \tau_2 \simeq 4.64^\circ.$$  

As shown in Table IV, the obtained quark masses $177, 178$, quark mixing angles and CP violating phase $179$ are

| Observable | Model value | Experimental value |
|------------|-------------|--------------------|
| $m_u(MeV)$ | 1.44        | $1.45^{+0.56}_{-0.45}$ |
| $m_c(MeV)$ | 656         | $635 \pm 86$        |
| $m_t(GeV)$ | 177.1       | $172.1 \pm 0.6 \pm 0.9$ |
| $m_d(MeV)$ | 2.9         | $2.9^{+0.5}_{-0.4}$  |
| $m_s(MeV)$ | 57.7        | $57.7^{+16.8}_{-15.7}$ |
| $m_b(GeV)$ | 2.82        | $2.82^{+0.09}_{-0.01}$ |
| $\sin \theta_{12}$ | 0.225 | 0.225 |
| $\sin \theta_{23}$ | 0.0412 | 0.0412 |
| $\sin \theta_{13}$ | 0.00351 | 0.00351 |
| $\delta$ | $64^\circ$ | $68^\circ$ |

Table IV: Model and experimental values of the quark masses and CKM parameters.

in very good agreement with the experimental data.

IV. LEPTON MASSES AND MIXINGS

From the interactions terms of the heavy vector with the heavy left handed Majorana neutrinos of Eq. (6), it follows that light active neutrino masses are generated from a one loop level radiative seesaw mechanism mediated by the
active neutrino mass matrices are respectively given by:

From the terms given above and the relations given by Eqs. (7) and (12), we find that the charged lepton and light left handed Majorana neutrinos and by the real and imaginary parts of the neutral components of the heavy vector in the fundamental $SU(2)_L$ representation, as illustrated in Fig. 1. Thus, the elements of the light active neutrino mass matrix take the form:

\[
(M_{\nu})_{11} = \sum_{n=1}^{2} (y_{1n}^{(V)})^2 f(m_{Re} V^o, m_{Im} V^o, m_{N_n}) \left(\frac{\nu_{\lambda}}{\Lambda}\right)^2 m_{N_n} ,
\]

\[
(M_{\nu})_{22} = \sum_{n=1}^{2} (y_{2n}^{(V)})^2 f(m_{Re} V^o, m_{Im} V^o, m_{N_n}) \left(\frac{\nu_{\lambda}}{\Lambda}\right)^2 m_{N_n} = 2 (M_{\nu})_{11} ,
\]

\[
(M_{\nu})_{12} = (M_{\nu})_{21} = \sum_{n=1}^{2} (y_{1n}^{(V)})(y_{2n}^{(V)}) f(m_{Re} V^o, m_{Im} V^o, m_{N_n}) \left(\frac{\nu_{\lambda}}{\Lambda}\right)^2 m_{N_n} = (M_{\nu})_{11} ,
\]

\[
(M_{\nu})_{13} = (M_{\nu})_{31} = \sqrt{2} \sum_{n=1}^{2} (y_{1n}^{(V)})(y_{2n}^{(V)}) f(m_{Re} V^o, m_{Im} V^o, m_{N_n}) \frac{\nu_{\lambda} \nu_{\sigma}}{\Lambda^2} m_{N_n} ,
\]

\[
(M_{\nu})_{23} = (M_{\nu})_{32} = \sqrt{2} \sum_{n=1}^{2} (y_{2n}^{(V)})^2 f(m_{Re} V^o, m_{Im} V^o, m_{N_n}) \frac{\nu_{\lambda} \nu_{\sigma}}{\Lambda^2} m_{N_n} ,
\]

\[
(M_{\nu})_{33} = 2 \sum_{n=1}^{2} (y_{2n}^{(V)})^2 f(m_{Re} V^o, m_{Im} V^o, m_{N_n}) \left(\frac{\nu_{\lambda}}{\Lambda}\right)^2 m_{N_n} .
\]

From the terms given above and the relations given by Eqs. (7) and (12), we find that the charged lepton and light active neutrino mass matrices are respectively given by:

\[
M_l = \frac{v}{\sqrt{2}} \begin{pmatrix}
0 & a_1 \lambda^9 & 0 & a_4 \lambda^4 \\
0 & a_2 \lambda^5 & a_5 \lambda^4 & 0 \\
a_1 \lambda^9 & a_2 \lambda^5 & 0 & a_3 \lambda^3 \\
a_4 \lambda^4 & a_5 \lambda^4 & a_3 \lambda^3 & 0
\end{pmatrix},
\]

\[
M_{\nu} = \sum_{n=1}^{2} \begin{pmatrix}
(y_{1n}^{(V)})^2 & (y_{1n}^{(V)})(y_{2n}^{(V)}) & \sqrt{2} y_{1n}^{(V)} y_{2n}^{(V)} \\
y_{1n}^{(V)} y_{2n}^{(V)} & (y_{2n}^{(V)})^2 & \sqrt{2} y_{2n}^{(V)} \\
\sqrt{2} y_{1n}^{(V)} y_{2n}^{(V)} & \sqrt{2} y_{2n}^{(V)} & (y_{2n}^{(V)})^2
\end{pmatrix} \lambda^2 f(m_{Re} V^o, m_{Im} V^o, m_{N_n}) m_{N_n}
\]

\[
\left(\begin{array}{c}
Z \\
Y \\
X
\end{array}\right) \begin{pmatrix}
\sqrt{2} Y \\
\sqrt{2} X \\
2 X
\end{pmatrix}
\]

where $a_1, a_2, b_1, c_1, y_{1n}^{(V)}$ and $y_{2n}^{(V)}$ are $O(1)$ dimensionless parameters, whereas $X, Y, Z$ are dimensionful parameters.
possibilities for the neutrino–heavy-vector coupling constant: 10

it is possible to extract, using the light active neutrino mass matrix elements, the mass of the left-handed sterile

X which corresponds to the eigenvalue problem solutions reproducing the experimental values of the neutrino mass

scale in this kind of models with heavy resonances [181–188].

an active neutrino and sterile neutrino. In our analysis we have set Λ = 3 TeV, which is a typical value for the cutoff

values of the charged lepton masses are taken from Ref. [177], whereas the range for experimental values of neutrino mass

squared splittings and leptonic mixing parameters, are taken from Ref. [180].

which are given by the following relations:

\[ X \simeq \sum_{n=1}^{2} \left( y_{2n}^{(V)} \right)^2 \lambda^2 f (m_{Re V^0}, m_{Im V^0}, m_{N_n}) m_{N_n}, \]  

(14)

\[ Y \simeq \sum_{n=1}^{2} \left( y_{1n}^{(V)} \right) \left( y_{2n}^{(V)} \right) \lambda^2 f (m_{Re V^0}, m_{Im V^0}, m_{N_n}) m_{N_n}, \]  

(15)

\[ Z \simeq \sum_{n=1}^{2} \left( y_{1n}^{(V)} \right)^2 \lambda^2 f (m_{Re V^0}, m_{Im V^0}, m_{N_n}) m_{N_n}, \]  

(16)

being \( f (m_{Re V^0}, m_{Im V^0}, m_{N_n}) \) a one loop function, which is given by:

\[
\begin{align*}
    f (m_{Re V^0}, m_{Im V^0}, m_{N_n}) &= \frac{\Lambda^2}{m_{Re V^0}^2} - \frac{\Lambda^2}{m_{Im V^0}^2} + \frac{m_{Re V^0}^2}{m_{Re V^0}^2 - m_{N_n}^2} \ln \left( \frac{m_{Re V^0}^2}{m_{N_n}^2} \right) \\
    &\quad - \frac{m_{Im V^0}^2}{m_{Im V^0}^2 - m_{N_n}^2} \ln \left( \frac{m_{Im V^0}^2}{m_{N_n}^2} \right) \\
    &\quad + \left( \frac{m_{N_n}^4}{m_{Re V^0}^2 (m_{Re V^0}^2 - m_{N_n}^2)} - \frac{m_{N_n}^4}{m_{Im V^0}^2 (m_{Im V^0}^2 - m_{N_n}^2)} \right) \ln \left( \frac{\Lambda^2 + m_{N_n}^2}{m_{N_n}^2} \right).
\end{align*}
\]  

(17)

The charged lepton masses, the neutrino mass squared splittings and the leptonic mixing parameters can be very

well reproduced for the scenario of inverted neutrino mass hierarchy in terms of natural parameters of order one,

as shown in Table V starting from the following benchmark point:

\[ a_1 \simeq 1.96168, \quad a_2 \simeq 1.03698, \quad a_3 \simeq 0.84294, \quad |a_4| \simeq 1.00752, \quad \arg (a_4) \simeq 218^\circ, \]

\[ a_5 \simeq -0.597641, \quad X \simeq 16.5289 \text{ meV}, \quad Y \simeq -0.219701 \text{ meV}, \quad Z \simeq 49.5616 \text{ meV}, \]  

(18)

which corresponds to the eigenvalue problem solutions reproducing the experimental values of the neutrino mass

squared splittings and leptonic mixing parameters. From the values of the X, Y and Z parameters given above,

it is possible to extract, using the light active neutrino mass matrix elements, the mass of the left-handed sterile

neutrinos \( (m_N) \) provided the values of the neutrino–heavy-vector coupling and the mass of the Heavy neutrino. As

an illustration, we fix the mass of the heavy vectors to \( m_{Im V^0} \approx m_{Re V^0} = 0.5 \) TeV and we consider two different

possibilities for the neutrino–heavy-vector coupling constant: \( 10^{-6} \) and \( 10^{-3} \). For these cases we found \( m_N = 0.5 \)

TeV and 600 GeV, respectively. Thus, for the later case of \( y_{1n}^{(V)} \approx 10^{-3} (n,n = 1, 2) \), the heavy vector will decay into

an active neutrino and sterile neutrino. In our analysis we have set \( \Lambda = 3 \) TeV, which is a typical value for the cutoff

scale in this kind of models with heavy resonances [181] [182].

| Observable | Model value | Experimental value |
|------------|-------------|-------------------|
| \( m_e \) [MeV] | 0.487 | 0.487 | 0.487 |
| \( m_\mu \) [MeV] | 102.8 | 102.8 ± 0.0003 | 102.8 ± 0.0006 | 102.8 ± 0.0009 |
| \( m_\tau \) [GeV] | 1.75 | 1.75 ± 0.0003 | 1.75 ± 0.0006 | 1.75 ± 0.0009 |
| \( \Delta m_{21}^2 \) [10^{-5}eV^2] (IH) | 7.55 | 7.55±0.20_{-0.16} | 7.20−7.94 | 7.05−8.14 |
| \( \Delta m_{13}^2 \) [10^{-3}eV^2] (IH) | 2.42 | 2.42±0.03_{-0.04} | 2.34−2.47 | 2.31−2.51 |
| \( \sin^2 \theta_{12} \) (IH) | 3.20 | 3.20±0.20_{-0.16} | 2.89−3.59 | 2.73−3.79 |
| \( \sin^2 \theta_{13} \) (IH) | 5.33 | 5.51±0.18_{-0.30} | 4.91−5.84 | 4.53−5.98 |
| \( \sin^2 \theta_{13} \) (IH) | 2.248 | 2.220±0.074_{-0.076} | 2.07−2.36 | 1.99−2.44 |

Table V: Model and experimental values of the charged lepton masses, neutrino mass squared splittings and leptonic
mixing parameters for the inverted (IH) mass hierarchy. The model values for CP violating phase are also shown.
The experimental values of the charged lepton masses are taken from Ref. [177], whereas the range for experimental values of neutrino mass
squared splittings and leptonic mixing parameters, are taken from Ref. [180].
Furthermore, we found a leptonic Dirac CP violating phase of $-50.28^\circ$ and a Jarlskog invariant close to about $-2.25 \times 10^{-2}$ for the inverted neutrino mass hierarchy. Now, let us consider the effective Majorana neutrino mass parameter:

$$m_{ee} = \left| \sum_j U_{ej}^2 m_{\nu j} \right|,$$

where $U_{ej}$ and $m_{\nu j}$ are the PMNS leptonic mixing matrix elements and the neutrino Majorana masses, respectively. The neutrinoless double beta $(0\nu\beta\beta)$ decay amplitude is proportional to $m_{ee}$. With the model best fit values in Table V we find

$$m_{ee} \simeq 15.3434 \text{ meV}.$$  

This is within the declared reach of the next-generation bolometric CUORE experiment [189] or, more realistically, of the next-to-next-generation ton-scale $0\nu\beta\beta$-decay experiments. The current most stringent experimental upper limit $m_{ee} \leq 160 \text{ meV}$ is set by $T_{1/2}^{0\nu\beta\beta}(^{136}\text{Xe}) \geq 1.1 \times 10^{26} \text{ yr}$ at 90\% C.L. from the KamLAND-Zen experiment [190].

V. CONCLUSIONS

We have built a viable and predictive extension of the SM with a heavy vector in the fundamental $SU(2)_L$ representation and nine SM singlet scalar fields that successfully explains the current pattern of SM fermion masses and mixing angles. The model incorporates the $S_3$ family symmetry, which is supplemented by the $S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$ discrete group. The observed hierarchy of SM charged fermion masses and mixing angles comes from the breaking of the $S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$ discrete group. The light active neutrino masses arise from a radiative seesaw mechanism at one loop level. The obtained values of the physical observables for the quark sector are compatible with the experimental data, whereas the ones for the lepton sector also do but only for the inverted neutrino mass spectrum. In what concerns the inverted neutrino mass hierarchy, the obtained leptonic mixing parameters are in excellent agreement with the experimental data. Our model predicts an effective Majorana neutrino mass parameter of $m_{ee} \simeq 15.3 \text{ meV}$ and a Jarlskog invariant of the order of $10^{-2}$ for the inverted neutrino mass spectrum.

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Appendix A: The product rules for $S_3$.

The $S_3$ discrete group contains 3 irreducible representations: $1$, $1'$ and $2$. Considering $(x_1, x_2)^T$ and $(y_1, y_2)^T$ as the basis vectors for two $S_3$ doublets and $y'$ an $S_3$ non trivial singlet, the multiplication rules of the $S_3$ group for the case of real representations take the form [1]:

\begin{equation}
\begin{pmatrix}
x_1 \\
x_2 \\
y_1 \\
y_2
\end{pmatrix}_2 \otimes \begin{pmatrix}
y_1 \\
y_2
\end{pmatrix}_2 = (x_1 y_1 + x_2 y_2)_1 + (x_1 y_2 - x_2 y_1)_1' + \begin{pmatrix}
x_2 y_2 - x_1 y_1 \\
x_1 y_2 + x_2 y_1
\end{pmatrix}_2,
\end{equation}

\begin{equation}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix}_2 \otimes \begin{pmatrix}
y'
\end{pmatrix}_1' = \begin{pmatrix}
-x_2 y' \\
x_1 y'
\end{pmatrix}_2,
\end{equation}

\begin{equation}
\begin{pmatrix}
x
\end{pmatrix}_1' \otimes \begin{pmatrix}
y'
\end{pmatrix}_1' = \begin{pmatrix}
x y'
\end{pmatrix}_1.
\end{equation}

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