ALOCK-PACZYŃSKI TEST WITH MODEL-INDEPENDENT BAO DATA

FULVIO MELIA\(^1\),‡ AND MARTIN LÓPEZ-CORREDOIRA\(^2,3\)

ABSTRACT

Cosmological tests based on the statistical analysis of galaxy distributions are usually dependent on the evolution of the sources. An exception is the Alcock-Paczyński (AP) test, which is based on the changing ratio of angular to spatial/redshift size of (presumed) spherically-symmetric source distributions with distance. Intrinsic redshift distortions due to gravitational effects may also have an influence, but there is now a way to overcome them: with the inclusion in the AP test of an observational signature with a sharp feature, such as the Baryonic Acoustic Oscillation (BAO) peak. Redshift distortions affect only the amplitude of the peak, not its position. As we will show here, the use of this diagnostic, with newly acquired data on the anisotropic distribution of the BAO peaks from SDSS-III/BOSS-DR11 at average redshifts \(z = 0.57\) and \(z = 2.34\), disfavors the current concordance (ΛCDM) model at 2.7σ. A statistically acceptable fit to the AP data with wCDM (the version of ΛCDM with a dark-energy equation of state \(\omega_{de} \equiv p_{de}/\rho_{de}\) rather than \(\omega_{de} = \omega_\Lambda = -1\)) is possible only with \(\omega_{de} = -0.24^{+0.60}_{-0.42}\) and \(\Omega_m = 0.74^{+0.22}_{-0.33}\). Within the context of expanding Friedmann-Robertson-Walker (FRW) cosmologies, these data strongly favor the zero ‘active mass’ equation-of-state, the basis for the \(R_h = ct\) Universe, in which \(\rho + 3p = 0\), where \(\rho\) and \(p\) are, respectively, the total density and pressure of the cosmic fluid. In \(R_h = ct\), however, the currently inferred CMB and BAO scales would have to be different, while a strong point in favor of ΛCDM is that in this model the same acoustic length would be responsible for both.

Keywords: cosmology: cosmological parameters – cosmology: distance scale – cosmology: observations – cosmology: theory – baryon acoustic oscillations

1. INTRODUCTION

A highly desirable goal in cosmology is the acquisition of model-independent data that can be used to test theoretical models and to optimize their parameters. There are two main types of cosmological observations that shed light on the geometry of the Universe: a measurement of the fluctuations in the Cosmic Microwave Background Radiation (CMB) and the analysis of large-scale structure via the inferred distribution of galaxies (Schade et al. 1997; Podsiadlowski et al. 2008; Kapahi 1987; López-Corredoira 2010; Lubin & Sandage 2001; Lerner et al. 2014). The quality and quantity of both kinds of data have progressed significantly in recent decades. CMB anisotropies provide the most evident support for the concordance (ΛCDM) model, but one should find an independent confirmation of this theory and its parameters because CMB anisotropies may be generated/modified by mechanisms other than those in the standard picture (Narlikar et al. 2007; Angus & Diaferio 2011; López-Corredoira 2013; Melia 2014), and may also be contaminated by other effects (López-Corredoira 2007). Cosmological tests using surveys of galaxies have also been developed to provide information on the geometry of the Universe: for instance, the Hubble diagram or the angular-size test. However, both of these need some assumptions in order to provide cosmological information.

As of today, Hubble diagrams constructed from the apparent magnitude (taking into account K-corrections) vs. distance or redshift of first-rank elliptical galaxies in clusters require a strong evolution of galaxy luminosities to fit FRW cosmologies (Schade et al. 1997). For Type Ia Supernovae (Kowalski et al. 2008) (SNIa) embedded in these galaxies, or for gamma-ray bursts (Wei 2010), one assumes zero evolution, and the standard model fits the data, though some systematic effects may be affecting the result, including a possible evolution in the metallicity of SNIa progenitors (Podsiadlowski et al. 2008), possible internal extinctions (Balázs et al. 2006), time variations of the grey-dust absorption of light from these supernovae in various types of host galaxies (Bogomazov & Tutukov 2011), and observational selection effects. The angular-size vs. redshift test has been developed by several authors (Kapahi 1987), using sources seen at radio, near-infrared and visible wavelengths. All these applications produce an angular size \(\theta \sim z^{-1}\) up to \(z \sim 3\), suggesting a strong evolution in galactic radii. The fact that galaxies with the same luminosity apparently were six times smaller at \(z = 3.2\) than at \(z = 0\) (López-Corredoira 2010) makes it difficult to compare different models. The surface brightness (known as the ‘Tolman’ test) also depends strongly on the assumption of galaxy evolution, so the results of this test (Lubin & Sandage 2001; Lerner et al. 2014) may vary hugely depending on one’s interpretation.

An alternative cosmological test based on the Alcock & Paczyński (AP) (Alcock & Paczyński 1979; López-Corredoira 2014) approach evaluates the ratio of observed angular size to radial/redshift size. The main advantage of this test is that, regardless of whether or
not galaxies may have evolved with redshift, it depends only on the geometry of the Universe. However, redshift distortions induced by peculiar velocities (Kaiser 1987; Matsubara & Suto 1996; Hamilton 1998), described in linear perturbation theory by the parameter $\beta$, can also have an influence. There is now a way to overcome this possible contamination—via the inclusion in the AP test of an effect with a sharp feature, such as the Baryon Acoustic Oscillation (BAO) peak, for which the degeneracy between redshift and geometric and gravitational distortions is almost completely broken (Font-Ribera et al. 2014). The reason is that the value of $\beta$ affects the amplitude of the peak, not its position. In this letter, we carry out the AP test using two Baryon Acoustic Oscillation (BAO) peak positions that significantly increase its precision over what has been achieved thus far. In so doing, we demonstrate that the concordance (ACDM) model is strongly disfavored by these new data. Instead, the AP test favors a model with zero ‘active mass’, i.e., with the equation of state $\rho + 3p = 0$, where $\rho$ and $p$ are, respectively, the total density and pressure of the cosmic fluid (Melia 2007; Melia & Shevchuk 2012).

2. THE ALCOCK-PACZYŃSKI TEST

Given a spherically symmetric structure, or distribution of objects, with radius

$$s_{||} = \Delta z \frac{d}{dz} d_{\text{com}}(z)$$  \hspace{1cm} (1)

along the line of sight and a radius

$$s_{\perp} = \Delta \theta (1 + z)^m d_A(z)$$  \hspace{1cm} (2)

(where $m = 1$ with expansion, while $m = 0$ for a static universe) perpendicular to the line of sight, the ratio

$$y \equiv \frac{\Delta z}{z} \frac{s_{\perp}}{s_{||}}$$  \hspace{1cm} (3)

depends only on the cosmological comoving distance, $d_{\text{com}}(z)$, and the angular-diameter distance, $d_A(z)$, and is independent of any source evolution.

Previous applications of the galaxy two-point correlation function to measure a redshift-dependent scale for the determination of $y(z)$ were limited by the difficulty in disentangling the acoustic length in redshift space from redshift distortions due to internal gravitational effects (López-Corredoira 2014). A serious drawback with this process is that inevitably one had to either pre-assume a particular model, or adopt prior parameter values, in order to estimate the level of contamination. And the wide range of possible distortions (i.e., values of $\beta$; see, e.g., Eqs. 4 and 5 in López-Corredoira 2014) for the same correlation-function shape resulted in seriously large errors.

But this situation has changed dramatically in the past few years with the use of reconstruction techniques to enhance the quality of the galaxy two-point correlation function and with the much more precise determination of the Ly-$\alpha$ and quasar auto- and cross-correlation functions, resulting in the measurement of BAO peak positions to better than ~4% accuracy. In this letter, we determine $y(z)$ using two BAO peak positions: 1) the measurement of the BAO peak position in the anisotropic distribution of SDSS-III/BOSS DR11 galaxies at $z = 0.57$ (Anderson et al. 2014), in which a technique of reconstruction to improve the signal/noise ratio was applied. This technique should not affect the position of the BAO peak, so the measured parameters are independent of any cosmological model. And 2) the self-correlation of the BAO peak in the Ly-$\alpha$ forest in the SDSS-III/BOSS DR11 data at $z = 2.34$ (Delubac et al. 2015), plus the cross-correlation of the BAO peak of QSOs and the Ly-$\alpha$ forest in the same survey (Font-Ribera et al. 2014).

In both of these measurements, the template used for the correlation function was drawn from the concordance model. However, the actual shape of the BAO peak does not affect the calculation of its centroid position, both along the line-of-sight and in the direction perpendicular to it, when its FWHM is very narrow. Any shape could be used, and the results would be the same. The peak’s narrowness mitigates the impact of redshift distortions, which affect the peak’s amplitude, but not its location. Any modifications to this amplitude as a function of distance may produce a second-order shift to the centroid, but always very negligibly. This conclusion is reflected, for instance, in the fact that, although the errors for the parameter $\beta$ quoted in Table 2 of Delubac et al. (2015) are very large, the relative error bars for $d_A(z)$ and $H(z)$ are much smaller. If the BAO peak measurements were sensitive to $\beta$, their error bars would be much bigger.

The angular-diameter distance $d_A(z)$ and Hubble constant $H(z)$ are related to the variable $y(z)$ via the expression

$$y(z) = \left(1 + \frac{1}{z}\right) \frac{d_A(z)^* H(z)}{c}$$ \hspace{1cm} (4)

where $d_A(z)^*$ is the measured angular-diameter distance assuming an expanding Universe, that is, $d_A(z)^* = d_A(z)$ if there is expansion and $d_A(z)^* = \frac{d_A(z)}{1+z}$ for a static universe. Very importantly, the quantity $y(z)$ is independent of the uncertain (co-moving) acoustic scale $r_s$, since $d_A(z)^* \propto r_s$, while $H(z) \propto 1/r_s$, so the dependence on $r_s$ cancels out in the product. (Sometimes an alternative definition of this ratio, $F(z) \equiv z y(z)$, has been used in the literature, but these properties are the same for both working definitions.)

The values of $d_A(z)$ and $H(z)$ in terms of the fiducial and unknown acoustic scales are: $d_A(z) = (0.57)(r_s^{\text{fiducial}}/r_s) = 1421\pm20$ Mpc, $H(z) = (0.57)(r_s/r_s^{\text{fiducial}}) = 96.8\pm3.4$ km s$^{-1}$ Mpc$^{-1}$, with a correlation coefficient between both parameters of 0.534 (Anderson et al. 2014), which lead, through Equation [4], to $y(z = 0.57) = 1.264\pm0.056$; $c/H(r_s) = 9.15^{+0.20}_{-0.21}$ and $d_A(z = 2.34)/r_s = 10.93^{+0.35}_{-0.34}$ (see Eqs. 22 and 23 in Delubac et al. 2015), plus the adoption of a correlation coefficient of -0.6 between $c/H$ and $d_A$ (Delubac et al. 2015), which lead to $y(z = 2.34) = 1.706\pm0.083$. The uncertainties in $y(z)$ are found by propagating the errors in $H(z)$ and $d_A(z)$ through Equation (4) including the covariance terms derived from the given correlation coefficients.

These are the only two BAO measurements currently available that measure both $d_A(z)$ and $H(z)$ with small statistical errors. We do not include measurements of $d_A(z)$ and $H(z)$ based on the anisotropic two-point correlation function at shorter scales (Chuang & Wang 2012;
Blake et al. 2012; Reid et al. 2012), since these are biased by the pre-assumption of a cosmological model, or the adoption of priors, used to characterize the internal redshift distortions (i.e., the parameter $\beta$).

In figure 1, we compare various model predictions with these two BAO measurements of $y(z)$, following the conventional methodology of the AP cosmological test (Alcock & Paczyński 1979; López-Corredoira 2014). We here plot the function for the wCDM model (the version of $\Lambda$CDM with a dark-energy equation of state $\omega_{de} \equiv p_{de}/\rho_{de}$ different from $\omega_{\Lambda} = -1$), in addition to the predictions of the standard $\Lambda$CDM model itself with fiducial parameter values $\Omega_m = 0.3$ and $\omega_{de} = -1$. The (heavy) dashed curve represents the (expanding) $R_h = ct$ (Melia 2007; Melia & Shevchuk 2012) and (static) STL (López-Corredoira 2014) cosmologies, whose $y(z)$ functions are (coincidentally) identical (see text).

$$d_{\text{com}}(z) = (1+z)d_A(z) = \frac{c}{H_0}\ln(1+z), \quad (5)$$

and has a $y(z)$ coincident with that of the Static/Tired Light (STL) model (López-Corredoira 2014), though its representation of the Universe is based on the FRW metric with expansion, while STL is static. This coincidence arises because for STL $d_{\text{com}}(z) = d_A(z) = \frac{ct}{H_0}\ln(1+z)$, which corresponds to a factor $(1+z)$ difference with the angular diameter distance $d_A(z)$ in the $R_h = ct$ Universe. The latter also has a Hubble parameter $H(z) = H_0(1+z)$, while $H$ is constant in STL. So the various factors of $(1+z)$ all cancel in the formulation of $y(z)$ when calculated with $d'_A(z)$ according to Equation 1. The $R_h = ct$ model represents a cosmology with zero ‘active mass,’ i.e., with the equation of state $\rho + 3p = 0$, where $\rho$ and $p$ are, respectively, the total density and pressure of the cosmic fluid. This model has successfully passed all other cosmological tests applied to it thus far (Melia 2013a; Wei et al. 2013; Melia 2013b; Melia 2014b; Wei et al. 2014b; Melia 2015a; Melia 2015b; Wei et al. 2015).

3. Discussion

3.1. Model Comparison with the Alcock-Paczyński Test

The $R_h = ct$ Universe (and, coincidentally, also the static STL cosmology) fits the data very well without any ad hoc optimization of free parameters. For the $\Lambda$CDM/wCDM cosmology, we see that the fiducial parameter values ($\Omega_m = 0.3, \omega_{\Lambda} = -1$) are excluded at a 99.34% C.L. (equivalent to 2.7$\sigma$), while even allowing $\Omega_m$ to be free with a fixed $\omega_{\Lambda} = -1$ (best fit for $\Omega_m = 0.200_{-0.041}^{+0.051}$) is excluded at 2.1$\sigma$. The only way to reconcile the data with the wCDM cosmology at an exclusion C.L. < 68% is to set $\omega_{de} = -0.24_{-0.42}^{+0.60}$ much higher than the fiducial value $\omega_{de} = -1$, and $\Omega_m = 0.74_{-0.33}^{+0.22}$, also not consistent with its concordance value. The confidence level contours are shown in figure 2.

Our results reinforce the conclusions drawn by Delubac et al. (2015), Abdalla et al. (2014) and Aubourg et al. (2014), who also remarked on the tension between the measurement at $z = 2.34$ and predictions of the standard model, based on various methods of analysis. Abdalla et al. (2014) also pointed out that values of $\omega_{de} > -1$ were necessary to alleviate the tension for the datum $H(z =
2.34), while Aubourg et al. (2014) realized this tension, but did not explore such possible solutions because they fitted the cosmological parameters using a combination of BAO+-(Supernovae data and/or the CMB). However, our results are based on the simultaneous analysis of both model-independent BAO data (at $z = 0.57$ and 2.34), which more emphatically excludes the standard model, while strongly favoring the $R_0 = ct$ Universe.

Our results are stronger than the tension reported by Delubac et al. (2015), Abdalla et al. (2014) and Aubourg et al. (2014), for the principal reason that the Alcock-Paczyński test amplifies the deviation of the measurements relative to the ΛCDM prediction. These authors reported a tension at the $\gtrsim 2\sigma$ level. The AP test, however, compares the observed product $d_A(z)H(z)$ with that of the model, and since both BAO measurements of $d_A(z)$ and $H(z)$ are higher than those expected in the standard model, the AP discrepancy is larger than simply that of the quadrature sum of the two individual discrepancies. And while Aubourg et al. (2014) did not find any models that substantially improved the agreement between theory and the Ly-α forest BAO measurement without worsening the corresponding fit to other data, they did not consider the $R_0 = ct$ cosmology. Our results here show that, insofar as the AP test by itself is concerned, this model (and coincidentally the STL model) does in fact yield a much better fit to the BOSS measurements than does the standard model.

A comparison between these results and those reported in López-Corredoira (2014) highlights the dramatic improvement in the measurement of the BAO peak position that has led to the compelling conclusions drawn in this letter. Though the measurements of $y(z)$ based on the galaxy two-point correlation function used in that earlier work were good enough to rule out all but the concordance and STL models (the $R_0 = ct$ Universe was not included in that comparison, but its $y(z)$ function is identical to that of STL), the errors arising from the contamination due to internal redshift distortions were still too large for us to discriminate between these two cosmologies. As we can see from figure 1, however, the significant improvement in the precision with which the BAO peak position is measured now (Anderson et al. 2014; Delubac et al. 2015) strongly disfavors the standard model.

3.2. The Acoustic Scale

The surprising and emphatic result summarized in figure 1 now compels us to contrast the inability of ΛCDM to pass the AP test against one of its most impressive successes—the interpretation of the CMB and BAO lengths as arising from a single, consistent acoustic scale. This ‘standard ruler’ is believed to be responsible for the multi-peak structure in the CMB power spectrum, leaving also its indelible imprint on the large-scale structure we see today. In many ways, the optimization of the parameters in ΛCDM relies critically on the correct interpretation of this ‘sound horizon.’

One may question some of the assumptions made in calculating the acoustic scale in ΛCDM, including its estimation from the comoving distance $r_s$ traveled by a sound wave rather than from an actual solution to the geodesic equation in an expanding medium, but the fact that $r_s^\text{CMB}/r_s^\text{BAO} \sim 1$ within the measurement error strongly favors the standard model. By comparison, the measured value of $r_s$ at $z = 2.34$ is $\sim 148$ Mpc in $R_0 = ct$ (comparable to the value in ΛCDM), though increasing significantly to $r_s \sim 600$ Mpc for the CMB. The actual value depends on the redshift at last scattering, ranging from $\sim 567$ Mpc at $z = 700$ to $\sim 740$ Mpc at $z = 2000$.

If the results of the AP test are reliable, and $R_0 = ct$ is indeed the correct cosmology, it is therefore unlikely that the CMB and BAO length scales have a common origin. In addition, the fact that the same scale fits both sets of data in the standard model would then be an unlikely coincidence, providing some observational support for ΛCDM. Needless to say, a satisfactory resolution of this conflict will benefit considerably from additional high-precision measurements of the BAO scale using a broader redshift coverage than is currently available. A quick inspection of figure 1 suggests that to distinguish between the various models, the AP test is best suited to redshifts $z \gtrsim 2$—the higher the better.

4. Conclusions

The results of our analysis in this paper show that, if the measurements of $d_A(z)$ and $H(z)$ derived from the BAO peak anisotropic distributions are correct, the concordance ΛCDM model, optimized to fit SNIa and CMB data, does not pass the AP test. This contrasts with the model’s success in accounting for the CMB and BAO scales with a single acoustic horizon. However, in light of the AP results, one may begin to question its status as a true ‘concordance’ model. Instead, the AP test using these model-independent data strongly favors the $R_0 = ct$ Universe, which has thus far also been strongly favored by model selection tools in other one-on-one comparative tests with ΛCDM (Melia 2013a; Wei et al. 2013; Melia 2013b; Melia 2014b; Wei et al. 2014a; Wei et al. 2014b; Melia 2015a; Melia 2015b; Wei et al. 2015). The consequences of this important result are being explored elsewhere, including the growing possibility that inflation may have been unnecessary to resolve any perceived ‘horizon problem’ and therefore may have simply never happened (Melia 2013).

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Table 1
Results of the χ²-test, using the N = 2 points of the BAO peak data: the minimum reduced χ²_red ≡ χ²/(N − ν), where ν is the number of free parameters; best-fit free parameters (if any); and associated probability of the models.

| Model                      | χ²_red, min | Free parameters       | Probability     |
|----------------------------|-------------|-----------------------|-----------------|
| ΛCDM; Ω_m = 0.3, ω_de = -1 | 5.01        | —                     | 6.6 × 10⁻³       |
| ΛCDM; Ω_m free; ω_de = -1  | 6.72        | Ω_m = 0.193⁺0.044⁻0.038 | 9.5 × 10⁻³       |
| wCDM; Ω_m, ω_de free      | —           | Ω_m = 0.74⁺0.22⁻0.33; ω_de = -0.24⁺0.60⁻0.42 | —               |
| R_h = ct (or STL)          | 0.09        | —                     | 0.956           |