Incomplete fusion studies near Coulomb barrier: a modified sum rule model

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Abstract. The excitation functions of the evaporation residues, produced via complete fusion and incomplete fusion reactions of $^{11}$B + $^{122}$Sn, were measured for the projectile energy of around 6 MeV/A by the off-line gamma spectrometry. The cross sections have been compared with the statistical model code Projected Angular Momentum Coupled Evaporation (PACE4). The original sum rule model underestimated the ICF cross sections. We therefore made modification in the model mainly to incorporate the energy dependence in the definition of critical angular momentum. Using this modified sum rule model, we found a significant improvement in the results.

1. Introduction
Heavy ion reactions with beam energies above the Coulomb barrier may be classified as complete fusion reactions (CF) and incomplete fusion reactions (ICF). While at lower beam energies CF is the dominating process, ICF emerges as a competing process, generally at high energies. Early experiments to study ICF reaction mechanism were confined to high beam energies of $\sim$ 10 MeV/A\textsuperscript{[1]} but recently there have been reports for ICF competing with CF even at beam energies as low as 5 MeV/A\textsuperscript{[2-4]}. Many models have been proposed to explain the underlying reaction mechanism of ICF reactions. Breakup fusion model by Udagawa and Tamura \textsuperscript{[5]}, pre-equilibrium emission particle model by Bondorf \textit{et al.} \textsuperscript{[6]}, multistep direct reaction theory by Zegrebaev \textit{et al.} \textsuperscript{[7]}, and sum rule model by Wilczyński \textit{et al.} \textsuperscript{[1]} are some of them. But none of them could satisfactorily explain all the observations associated with the ICF reactions, in particular, at low beam energy \textsuperscript{[8]}.

The aim of the present work was to understand the ICF reaction mechanism using the sum rule model \textsuperscript{[1]}, just above the coulomb barrier within the beam energy range of 5-7 MeV/A. The experimentally determined values of the cross section for CF as well as ICF were compared with the model predictions which were found to be rather low. We revised the concept of the critical angular momentum ($l_{cr}$) as the angular momentum corresponding to the distance of closest approach \textsuperscript{[9]}. We added an extra term in the definition of $l_{cr}$ used in the sum rule model \textsuperscript{[1]}. With this modification we
achieved an improvement in the theoretically calculated values when compared with the experimental results.

2. Experimental details
The experiment consisted of irradiating the target foils by the projectile, followed by off-line data collection. The projectile beam of $^{11}$B was delivered by the 14UD Pelletron accelerator at the Tata Institute of Fundamental Research, Mumbai, India. The self-supporting targets of enriched $^{122}$Sn of thickness 1.5-2.0 mg/cm$^2$ were prepared by the rolling technique. During each run of experiments, a stack of two targets were used for three beam energies of 67, 73 and 78 MeV. The first foil acted as an energy degrader for the second foil. This enabled us to obtain the irradiation at six different beam energies of 64, 67, 70, 73, 75 and 78 MeV. The beam current was 20 – 40 nA. Immediately after the irradiation, the data collection was carried out off-line by a coaxial high purity germanium detector (HPGe). The spectra consisted of the characteristic $\gamma$-peaks corresponding to the daughter nuclei of the evaporation residues of the dominant reaction channels, as listed in table 1 [10]. The evaporation residues formed but not detected were either short lived ($T_{1/2} < 3$ min) or stable.

| Reaction Channel | Evaporation residues (Parent nucleus) | Daughter nucleus | $E_{\gamma}$ (keV) | $T_{1/2}$ | $a_{\gamma}$ (%) |
|------------------|---------------------------------------|-----------------|------------------|---------|----------------|
| 4n               | $^{129}$Cs                           | $^{129}$Xe      | 371.9            | 32.06 hr | 30.6           |
| 5n               | $^{129}$Cs                           | $^{128}$Xe      | 442.9            | 3.62 min | 26.8           |
| 6n               | $^{127}$Cs                           | $^{127}$Xe      | 124.7            | 6.25 hr  | 11.4           |
| p5n              | $^{127}$Xe                           | $^{127}$I       | 202.8            | 36.34 day| 68.7           |
| $\alpha$9n      | $^{128}$I                            | $^{128}$Xe      | 442.9            | 24.99 min| 12.6           |
| $\alpha$3n      | $^{126}$I                            | $^{126}$Te      | 388.6            | 12.93 day| 35.6           |
| $\alpha$5n      | $^{124}$I                            | $^{124}$Te      | 602.7            | 4.17 day | 62.9           |
| $\alpha$6n      | $^{123}$I                            | $^{123}$Te      | 158.9            | 13.22 hr | 83.3           |
| 2$\alpha$3n     | $^{122}$Sb                           | $^{122}$Te      | 564.2            | 2.72 day | 70.7           |

3. Data analysis and experimental results
The energy calibration of the data was done using the standard radioactive source $^{152}$Eu. Figure 1 shows a typical gamma spectrum observed at the beam energy of 70 MeV. The characteristic gamma rays of the daughter nuclei of the decaying evaporation residues have been labeled. Absolute efficiency of the detector was obtained by placing the $^{152}$Eu source at the target position. Peak fitting program gf3 [11] was used to determine the areas of the peaks. For the overlapping peaks, multiple peak fitting was required. It was essential to know the peak-width accurately to determine the correct area of the peaks. To achieve this, a relation of the type $W^2 = A + BE_{\gamma}$ ($A$ and $B$ are positive constants) between peak width ($W$) and gamma energy ($E_{\gamma}$), was fitted to the spectrum of the radioactive source $^{152}$Eu.

3.1. Determination of the cross section
The cross section, $\sigma$, of an evaporation residue was obtained from the measured peak areas (PA) using the following equation,

$$PA(t_2 - t_1) = \sum_{\gamma} N_{\gamma} a_{\gamma} \sigma I_0 [(1 - \exp(-\lambda T)) \exp(-\lambda t_2) - \exp(-\lambda t_1)],$$

(1)
where \( N_T \) is the number of target nuclei per unit area, \( \lambda \) is the decay constant of the evaporation residue, \( a_\gamma \) is the branching intensity of the characteristic gamma ray, \( \varepsilon_\gamma \) is the absolute efficiency of detection of the gamma ray, \( T \) is the irradiation time, \( I_b \) is the beam intensity, \( t_1 \) is the start time and \( t_2 \) is the stop time of the data acquisition.

There was a case in which two residues \(^{128}\text{Cs} \) and \(^{128}\text{I} \) decayed to the same daughter \(^{128}\text{Xe} \). Such a decay mode was referred as the double decay mode, the analysis of which we discuss now. The gamma transition of energy 443 keV \((2^+ \rightarrow 0^+)\) belonging to \(^{128}\text{Xe} \), was obtained through the \( \beta^- \) decay \( (T_{1/2} = 24.99 \text{ min}) \) of \(^{128}\text{I} \) and EC decay \( (T_{1/2} = 3.62 \text{ min}) \) of \(^{128}\text{Cs} \), shown in the inset of figure 2. The total area of the 443 keV gamma peak, called Area, consisted of two parts: \( P_1 \) and \( P_2 \) corresponding to the decay via \(^{128}\text{I} \) and \(^{128}\text{Cs} \), respectively. If \( \sigma_1 \) and \( \sigma_2 \) denote the values of the cross section for \(^{128}\text{Cs} \) and \(^{128}\text{I} \), respectively, the Area is given as,

\[
\text{Area} = P_1 \times \sigma_1 + P_2 \times \sigma_2 .
\]

\[
P_1 = \lambda_1 N_T \varepsilon_\gamma a_\gamma I_b \left[ 1 - \exp(-\lambda_1 T) \right] \left[ \exp(-\lambda_1 t_1) - \exp(-\lambda_1 t_2) \right]
\]

\[
P_2 = \lambda_2 N_T \varepsilon_\gamma a_\gamma I_b \left[ 1 - \exp(-\lambda_2 T) \right] \left[ \exp(-\lambda_2 t_1) - \exp(-\lambda_2 t_2) \right].
\]

The symbols in equation (3) have the same meaning as in equation (1) except that subscript 1 refers to \(^{128}\text{Cs} \) while subscript 2 refers to \(^{128}\text{I} \). A straight line was expected from equation (2) when we plotted the ratio of \( P_1 \) and \( P_2 \) on the x-axis and the ratio of Area and \( P_2 \) on the y-axis. Such a plot is shown in figure 2, as an example for the beam energy of 70 MeV. The data points correspond to different values of \( t_1 \) and \( t_2 \). From the straight line fit to the data points we obtained the cross section values \( \sigma_1 = 546.1 \pm 12.1 \text{ mb} \) for \(^{128}\text{Cs} \) and \( \sigma_2 = 3.8 \pm 0.6 \text{ mb} \) for \(^{128}\text{I} \).

![Figure 1. Gamma spectrum observed off-line in the reaction \(^{11}\text{B} + ^{122}\text{Sn} \) at the beam energy of 70 MeV.](image-url)
3.2 Results and discussion

The cross sections of the residues formed by the neutron evaporation from the compound nucleus were found to be very large. These nuclei were $^{129}$Cs, $^{129}$Cs, $^{127}$Cs and $^{127}$Xe formed after the emission of 4n, 5n, 6n and p5n, respectively. We used the statistical model code PACE4 [12], employing the Bass model, to calculate the values of the cross section. The level density parameter $a = A/8$, where $A$ is the mass number of the compound nucleus, was used in the code for the best agreement with the experimental results. All the other optical model parameters were set to their default values. Figure 3 gives a comparison of the experimentally determined cross sections with the calculations. The general behavior of the experimental curves followed the trend obtained by the PACE4 calculation. However, there was large disagreement between the experiment and the calculation for certain residues, e.g. $^{127}$Xe. Moreover, the PACE4 predicted significant cross sections for the residues like $^{125}$I, $^{128}$Xe formed from decay of the compound nucleus. They were not observed in our off-line experiment because they are either stable or have very long half lives.

Figure 4 presents the cross sections of $^{123,124,126,128}$I and $^{122}$Sb. For all these residues, the PACE4 results were found to be much lower than the experimental results. From the experimental finding of the enhancement in the cross section of the iodine and antimony isotopes, we inferred that their production was largely due to ICF reactions, i.e. the pre-equilibrium emission of $\alpha$ and $2\alpha$ (or $^8$Be) ejectiles, respectively. Since, the experimental cross sections of iodine and antimony isotopes have contribution from both ICF and CF reactions, the ICF contribution was extracted by subtracting the CF cross section obtained by the PACE4 from the experimental values.
Figure 3. Cross sections of residues formed via CF reactions. Solid symbols are experimental values and the corresponding hollow symbols are PACE4 calculated values.

Figure 4. Cross sections of residues formed via emission of $\alpha$ and $2\alpha$ (or $^8$Be). Solid symbols are experimental values and the corresponding hollow symbols are PACE4 results.
3.3 Cross section of complete fusion reaction

The total absolute cross sections of CF and ICF reaction were obtained by summing the cross sections of the corresponding residues which were found to have appreciable yield.

\[ \sigma_{\text{CF}}(^{11}\text{B}) = \sum_{x,y} (^{11}\text{B}, x\text{yp}) \] (4)

In the present case,

\[ \sigma_{\text{CF}}(^{11}\text{B}) = \sum_{x} \sigma(^{133-x}\text{Cs}) + \sum_{x,y} \sigma(^{133-x-y}\text{Xe}) = \sigma(^{129}\text{Cs}) + \sigma(^{128}\text{Cs}) + \sigma(^{127}\text{Cs}) + \sigma(^{127}\text{Xe}) , \] (5)

where \( x \) and \( y \) are the number of neutrons and protons evaporated from the compound nucleus, respectively.

3.4 Cross section of incomplete fusion reactions

As mentioned earlier, the contribution due to ICF reaction for each of the \( \alpha \) channel was obtained by subtracting PACE4 value from the experimental result. Let \( x \) denote the number of evaporated neutrons. The total cross section of the ICF reaction through pre-equilibrium emission of \( \alpha \) particle was calculated as

\[ \sigma_{\text{ICF}}(^{11}\text{B}, \alpha) = \sum_{x} (^{11}\text{B}, \alpha x) \] (6)

\[ \sigma_{\text{ICF}}(^{11}\text{B}, \alpha) = \sigma(^{129}\text{I}) + \sigma(^{127}\text{I}) + \sigma(^{126}\text{I}) + \sigma(^{125}\text{I}) + \sigma(^{124}\text{I}) + \sigma(^{123}\text{I}) \]

Of these, \( ^{129}\text{I} \) and \( ^{127}\text{I} \) are stable isotopes, and were not observed experimentally.

Similarly, total cross section of ICF reaction via pre-equilibrium emission of \( 2\alpha \) (or \(^{8}\text{Be}\)) was obtained as,

\[ \sigma_{\text{ICF}}(^{11}\text{B}, 2\alpha) = \sum_{x} (^{11}\text{B}, 2\alpha x) = \sigma(^{122}\text{Sb}) . \] (8)

The other Sb isotopes were not observed in the data.

4. Sum rule model

The original sum rule model (OSRM) was proposed by Wilczyński et al. [1] to unify CF and ICF into a single framework. The model has been successfully utilized in explaining the experimentally observed cross section in the beam energy of \( \sim 10 \text{ MeV/nucleon} \).

This model enabled us to calculate absolute cross section of \( i \)th reaction channel, which is given as

\[ \sigma(i) = \pi \lambda^2 \sum_{\ell=0}^{\ell_{\text{max}}} (2\ell + 1)N_{\ell}(i)p(i) , \] (9)

where \( \lambda \) is the reduced de Broglie wavelength, \( \ell_{\text{max}} \) is the maximum angular momentum calculated from the prescription given in [9,13,14]. The reaction probability \( p(i) \), for the \( i \)th reaction channel (including CF), is proportional to the ground state Q-value, \( Q_{\text{gs}}(i) \),

\[ p(i) \propto \exp \left[ \frac{Q_{\text{gs}}(i) - Q_{\ell}(i)}{T} \right] , \] (10)

where \( T \) is a parameter. The change in the Coulomb interaction energy, \( Q_{\ell}(i) \), due to the transfer of charge, is given as
where \(Z_1^{\text{in}}, Z_2^{\text{in}}, Z_1^{\text{f}}, Z_2^{\text{f}}\) are the atomic numbers of the constituents of the dinuclear system before and after the transfer of charge, and \(q_c\) is a parameter. Each of the CF and ICF processes are viewed as forming dinuclear systems. This dinuclear system consists of the projectile and the target before the reaction takes place. After the fusion, the dinuclear system becomes an ejectile and the compound nucleus (target + captured fragment). The transmission coefficient \(T_e(i)\) is given as

\[
T_e(i) = \left[1 + \exp\left(\frac{\ell - \ell_{\text{lim}}(i)}{\Delta_e}\right)\right]^{-1},
\]

where \(\Delta_e\) is the diffuseness of the \(T_e(i)\) - distribution. The normalization of the probability is done as

\[
N_e = \left[\sum_i T_e(i)p(i)\right]^{-1}.
\]

The limiting angular momentum \(\ell_{\text{lim}}(i)\) is defined as [15]

\[
\ell_{\text{lim}}(i) = \frac{\text{mass of projectile}}{\text{mass of captured fragment}} \ell_{\text{cr}}(\text{target + captured fragment}).
\]

The main feature of this model is the generalized concept of angular momentum. According to this concept the ICF reactions are localized in the successive \(\ell\)-windows above the critical angular momentum for the CF reaction. This window is automatically created by the prescription given in the model, once the critical angular momentum \(\ell_{\text{cr}}\) is defined for all the ICF as well as CF reaction channels. Thus, the quantity \(\ell_{\text{cr}}\) is an important ingredient and is calculated by the balance of nuclear and Coulomb forces when the target and captured fragment are at the sum of their half-density radii. Such a definition of critical angular momentum in OSRM does not involve the energy dependence of the projectile. Our calculation did not give the satisfactory results when the OSRM was exactly followed. The most important discrepancy was the low value of the calculated cross section for the ICF reaction as compared to the experimental result (see figure 6).

We have tried to revise the concept of the critical angular momentum by introducing the energy dependence in its definition. Our idea was similar to the work of Glass and Mosel [9] where they indeed found experimental results consistent with the variation of critical angular momentum with energy of the projectile. Here we propose a model called the modified sum rule model (MSRM), in which we added an extra term in the definition of \(\ell_{\text{cr}}\) in the following manner,

\[
\left(\ell_{\text{cr}} + \frac{1}{2}\right)^2 = \frac{\mu(C_1 + C_2)^2}{\hbar^2} \left[4\pi\gamma \frac{C_1C_2}{C_1 + C_2} - \frac{Z_1Z_2e^2}{(C_1 + C_2)^2} + \frac{Kb^2}{(C_1 + C_2)^3}\right].
\]

Where \(C_1\) and \(C_2\) are the half-density radii [13] of the target and the captured fragment, \(\gamma\) is the surface tension coefficient. The third term in the square bracket is a new term, where \(K\) is the average beam energy in centre of mass frame and \(b\) is the classical impact parameter which was parameterized as \(0.3(C_1 + C_2)\) in our calculation. Except for the definition of \(\ell_{\text{cr}}\) in MSRM, the other formalism was kept identical to the OSRM. The parameters utilized in our calculation with reference to those used in OSRM [1] have been listed in table 2.
Figure 5 gives the probability distribution of various reaction channels as a function of the angular momentum $\ell$. The localization of various reaction channels in different $\ell$ windows with some overlaps has emerged as expected in OSRM.

Figure 6 represents a comparison of the experimental results with the OSRM and the MSRM. We observed only those residues which were decaying during the off-line data collection of gamma-ray spectra. Nevertheless, we were able to obtain the cross sections of most of the dominating reaction channels of CF as well as ICF reactions. Since there were definitely some missing channels in our data, as mentioned earlier, our experimental cross section values should be taken as the lower limits of the actual values. The calculation with MSRM gives a reasonable agreement with the experimental results and the agreement seems to improve with the beam energy.

Figure 5. Probability distribution of ICF reactions of $^{11}$B + $^{122}$Sn based on the modified sum rule model.

Table 2. The parameter values used in the sum rule model calculations

| Parameter | OSRM | MSRM |
|-----------|------|------|
| $T = 3$ MeV | $T = 3$ MeV |
| $q_c = 0.06$ fm$^{-1}$ | $q_c = 0.08$ fm$^{-1}$ |
| $\Delta_\ell = 2\hbar$ | $\Delta_\ell = 1\hbar$ |
| $b = 0.3$ (C$_1$+C$_2$) | |
| $K = 60$ MeV | |
| $\ell_{\text{max}} = 15\%$ increment above OSRM | |
Figure 6. Cross sections for the CF and ICF (α and 8Be) reactions in 11B + 122Sn. Solid points are the experimental values and corresponding hollow and half-filled points are the results of the calculation based on MSRM and OSRM, respectively.

5. Conclusion
We have experimentally measured the lower limits of the cross section for the CF and ICF (α and 8Be) reactions in 11B + 124Sn. The range of beam energy was 5-7 MeV/nucleon. The original sum rule model which works well in the regime of high energy was not successful in explaining our experimental results. We modified the sum rule model in the definition of the critical angular momentum to fit our experimental results along with small changes in the values of other parameters. The calculations produced results close to the experimental values. Further investigation is currently underway to check the applicability of MSRM with the same parameter values in other reactions 10B + 124Sn and 11B + 124Sn.

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