Analysis of the relationship between the distance barriers GaAs and GaAs with the transmission coefficient and the reflection coefficient

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Abstract. The tunneling effect is a phenomenon of the behavior of particles with small energy having a probability to penetrate the barrier potential. The transmission coefficient is the ratio of breakthrough particles to incoming particles. The reflection coefficient is the ratio of reflected particles to incoming particles. The tunneling effect is analyzed using the Schrodinger equation with the matrix propagation method. The purpose of this study is to analyze the relationship between the distance barriers with the transmission coefficient and the reflection coefficient. The barriers used are GaAs and GaAs with potential energy of 1.424 eV with a bandgap of 0.565 nm. The researcher used 1 eV with a distance between barriers 0.25-2 nm with a change in the distance between barriers is 0.25 nm. Changes in the distance between barriers GaAs and GaAs affect the value of the transmission coefficient and the reflection coefficient. Changes in the distance between the barrier produce a transmission coefficient and a reflection coefficient that is periodic because the distance between the barriers has the form of complex exponential functions. This research application is used to design and develop new structures in improving the quality of semiconductor devices such as diodes, transistors, and IC.

1. Introduction
The development of physics in the 20th century give birth to a new field of physics namely quantum mechanics that focus on the phenomenon of the interaction of radiation with matter. Quantum mechanics is born because classical mechanics cannot explain the relationship between radiation and matter. Classical mechanics explains the dynamic motion of macroscopically. Physicists Maxwell, Einstein and Louise de Broglie made important contributions in the creation of quantum mechanics. Maxwell believes that light can behave as a wave according to the phenomenon of interference and diffraction. Einstein argued that light can behave as particles according to the photoelectric effect, Einstein’s opinion was supported by Compton according to the Compton effect phenomenon which shows that light behaves as particles. Louise de Broglie concludes these studies by adhering to the law of natural symmetricity, therefore Louise de Broglie argues that light has two behaviors namely behavior as waves and behavior as particles, where those behaviors do not appear simultaneously [1]. The dualism effect of light can explain the phenomenon of a tunneling effect which at that time could not be explained by classical mechanics.
The tunneling effect is a particle phenomenon that can penetrate a barrier even though the energy possessed by particles is smaller than the barrier energy. The barrier is an area of potential energy that blocks electrons. The types of barrier potential are stair potential, boundary square potential, infinite square potential, finite square potential and square potential with walls. The phenomenon of tunneling effect is not all particles that are reflected this is known as the transmission coefficient and the reflection coefficient.

The transmission coefficient is the ratio between breakthrough particles with incoming particles and the reflection coefficient is the ratio of reflected particle to incoming particles, which can be expressed by the following formula \( T = |t|^2 = \left| \frac{E}{A} \right|^2 \) and \( R = |r|^2 = \left| \frac{B}{A} \right|^2 \). The relation of the transmission coefficient and the reflection coefficient is \( T + R = 1 \) [2]. To find out the value of the transmission coefficient and reflection coefficient using the Schrodinger equation. The Schrodinger equation is of two types namely steady and non-steady. A steady Schrodinger equation means that a time dependent Schrodinger equation is formulated with the equation \((-i\hbar)^2 \frac{\partial^2}{\partial x^2} \psi(x, t) + V(x, t)\psi(x, t) = i\hbar \frac{d}{dt} \psi(x, t)\). Schrodinger’s equation is non-steady, meaning that the time-dependent Schrodinger equation is formulated with the \( \left[ \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x)\psi(x) \right] = E\psi(x) \) [3]. All of these Schrodinger equations obey the law of conservation of energy, de Broglie’s hypothesis and behave mathematically that is single and linear [4]. Some research results related to the tunneling effect are the maximum transmission coefficient on energy 0.75 eV with Graphene material [5], the maximum transmission coefficient on energy 0.9 eV with GaN, SiC dan GaAs materials [6], the maximum transmission coefficient on energy 0.5133 eV with GaAs dan Pbs materials [7]. Based on this, the transmission coefficient and reflection coefficient are influenced by changes in energy and the semiconductor material used.

Semiconductor material is material under certain circumstances behaves as an insulator and conductor. Low semiconductor conductivity at normal temperatures means the material is an insulator and if the temperature is raised the conductivity increases so that the semiconductor material is conductor [8]. There are two types of semiconductors namely intrinsic and extrinsic. Intrinsic semiconductors are said to be pure semiconductors because they have not been interfered with by other atoms. An extrinsic semiconductor is a semiconductor material that has been mixed with other atoms, this is done to improve the electrical properties of semiconductor materials. Mixed atoms are called dopants and the process of mixing other atoms into pure semiconductors is called doping [9]. This research uses GaAs semiconductor material which is included in the extrinsic semiconductor material. Gallium Arsenide (GaAs) has a bandgap 0.565 nm and a potential height of 1.424 eV. Gallium Arsenide is one of the semiconductor materials used in the manufacture of high-speed electronic devices such as laser diodes and transistors [7].

2. Method
This research uses a matrix propagation method to solve the tunneling effect problem. The steps in the matrix propagation method are (1) finding the wave function solution in each state (2) calculating \( p_{\text{step up}} \) (3) calculating \( p_{\text{free}} \) (4) calculating \( p_{\text{step down}} \) (5) calculating total propagation by multiplying \( p_{\text{step up}} \cdot p_{\text{free}} \cdot p_{\text{step down}} \) (6) calculating the transmission coefficient and the reflection coefficient [10]. An overview of the stages of matrix propagation can be seen in Figure 1 below:
Figure 1. The potential of a finite square barrier case E < V

Regions I, II, and III have the following wave functions respectively:

\[ \psi_1 = \frac{A}{\sqrt{k_1}} e^{ik_1x} + \frac{B}{\sqrt{k_1}} e^{-ik_1x} \]  
\[ \psi_2 = \frac{C}{\sqrt{k_2}} e^{ik_2x} + \frac{D}{\sqrt{k_2}} e^{-ik_2x} \]  
\[ \psi_3 = \frac{F}{\sqrt{k_1}} e^{ik_2x} + \frac{G}{\sqrt{k_1}} e^{-ik_2x} \]

with \( k_1 = \frac{\sqrt{2mE}}{\hbar} \) and \( k_2 = \frac{\sqrt{2m(V-E)}}{\hbar} \) in the case of the tunneling effect the electron energy is less than the potential barrier energy. After knowing the wave function solution in each state, the next process is to calculate the \( p_{step up} \) by meeting the continuity requirements at the limit \( x=0 \).

\[ \left. \frac{\psi_1}{\sqrt{k_1}} \right|_{step} = \left. \frac{\psi_2}{\sqrt{k_1}} \right|_{step} \]
\[ \left. \frac{d\psi_1}{dx} \right|_{step} = \left. \frac{d\psi_2}{dx} \right|_{step} \]

The equation is changed in the form of a matrix:

\[ \frac{1}{\sqrt{k_1}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{\sqrt{k_2}} \begin{bmatrix} k_2 \\ ik_2 \end{bmatrix} \begin{bmatrix} C \\ -ik_1 \end{bmatrix} \]

By using the inverse matrix help obtained \( p_{step up} \) as follows:

\[ p_{step up} = \frac{1}{2\sqrt{k_1k_2}} \begin{bmatrix} k_1 - ik_2 & k_1 + ik_2 \\ k_1 + ik_2 & k_1 - ik_2 \end{bmatrix} \]

After \( p_{step up} \) is known, the next process is to calculate \( p_{free} \) by way of:

\[ Ce^{k_2x_1} = F \]
\[ De^{-k_2x_1} = G \]
The equation is changed in the form of a matrix and using the inverse matrix help obtained \( p_{\text{free}} \) as follows:

\[
\begin{bmatrix}
e^{k_2 L_1} & 0 \\
0 & e^{-k_2 L_1}
\end{bmatrix}
\begin{bmatrix}
C \\
D
\end{bmatrix}
= 
\begin{bmatrix}
F \\
G
\end{bmatrix}
\]

(10)

\[
p_{\text{free}} = 
\begin{bmatrix}
e^{k_2 L_1} & 0 \\
0 & e^{-k_2 L_1}
\end{bmatrix}
\]

(11)

To find the \( p_{\text{step down}} \) of a wave function solution using equations (1) and (2), using the continuity conditions obtained:

\[
p_{\text{step down}} = \frac{1}{2\sqrt{k_1 k_2}} \begin{bmatrix}
k_2 + i k_1 & k_2 - i k_1 \\
k_2 - i k_1 & k_2 + i k_1
\end{bmatrix}
\]

(12)

The next process is to find total propagation by multiplying \( p_{\text{step up}}, p_{\text{free}}, \) dan \( p_{\text{step down}} \) as in the equation (12).

\[
P = \frac{1}{2\sqrt{k_1 k_2}} \begin{bmatrix}
k_1 - ik_2 & k_1 + ik_2 \\
k_1 + ik_2 & k_1 - ik_2
\end{bmatrix}
\begin{bmatrix}
e^{k_2 L_1} & 0 \\
0 & e^{-k_2 L_1}
\end{bmatrix}
\frac{1}{2\sqrt{k_1 k_2}} \begin{bmatrix}
k_2 + i k_1 & k_2 - i k_1 \\
k_2 - i k_1 & k_2 + i k_1
\end{bmatrix}
\]

(13)

Equation (12) will produce total propagation with a matrix:

\[
P = \begin{bmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{bmatrix}
\]

(14)

Equation (14) will produce a transmission coefficient with the equation:

\[
T = \left( \frac{1}{p_{11}} \right)^2
\]

(15)

\[
T = \left( 1 + \left( \frac{k_1^2 + k_2^2}{2k_1 k_2} \right)^2 \sinh^2(k_2 L_1) \right)^{-1}
\]

(16)

The stages of the matrix propagation method also apply to the double barrier potential. Coefficient transmission on the double barrier tunneling effect is:

\[V(\chi)\]

```
......... I ........ II ........ III ........ IV ........ V
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\[0 \quad L_1 \quad x_1 \quad L_2 \quad x_2 \quad L_3 \quad x_3 \]

**Figure 2. Double Potential barrier case E <V**

The state of the particles in regions I, II, and III is represented by the wave function as in equations (1), (2), and (3). The wave function in regions IV and V is:

\[
\Psi_4 = \frac{H}{\sqrt{k_3}} e^{k_3 x} + \frac{i}{\sqrt{k_3}} e^{-k_3 x}
\]

(17)

\[
\Psi_5 = \frac{f}{\sqrt{k_1}} e^{ik_1 x} + \frac{M}{\sqrt{k_1}} e^{-ik_1 x}
\]

(18)
After knowing the wave function in each state, the second step is to calculate $p_{\text{step up}}$, $p_{\text{free}}$, and $p_{\text{step down}}$. For $p_{\text{step up}}$, $p_{\text{free}}$, and $p_{\text{step down}}$, the first barrier is the same as equation (7), (11) and (12). The third step before going to the second barrier is $p_{\text{free}}$ between the barriers:

$$F e^{k_1 L_2} = H$$
$$G e^{-k_1 L_2} = I$$

The equation is in the form of a matrix and using the inverse matrix help is obtained $p_{\text{free}}$ is:

$$\begin{bmatrix} e^{k_1 L_2} & 0 \\ 0 & e^{-k_1 L_2} \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix} = \begin{bmatrix} H \\ I \end{bmatrix}$$

(21)

The fourth step by fulfilling the continuity requirements is obtained $p_{\text{step up}}$, $p_{\text{free}}$, and $p_{\text{step down}}$ on the second barrier is:

$$p_{\text{step up 2}} = \frac{1}{2\sqrt{k_1 k_3}} \begin{bmatrix} k_1 - ik_3 & k_1 + ik_3 \\ k_1 + ik_3 & k_1 - ik_3 \end{bmatrix}$$
$$p_{\text{free 2}} = \begin{bmatrix} e^{-k_3 L_3} & 0 \\ 0 & e^{k_3 L_3} \end{bmatrix}$$
$$p_{\text{step down 2}} = \frac{1}{2\sqrt{k_1 k_3}} \begin{bmatrix} k_3 + ik_1 & k_3 - ik_1 \\ k_3 - ik_1 & k_3 + ik_1 \end{bmatrix}$$

(23)

(24)

(25)

The fifth step is to calculate the total propagation so that the following equation is obtained:

$$\rho = \frac{1}{2\sqrt{k_1 k_2}} \begin{bmatrix} k_1 - ik_2 & k_1 + ik_2 \\ k_1 + ik_2 & k_1 - ik_2 \end{bmatrix} e^{-k_2 L_2} \begin{bmatrix} e^{-k_2 L_1} & 0 \\ 0 & e^{k_2 L_1} \end{bmatrix} \frac{1}{2\sqrt{k_1 k_2}} \begin{bmatrix} k_2 + ik_1 & k_2 - ik_1 \\ k_2 - ik_1 & k_2 + ik_1 \end{bmatrix} e^{-k_3 L_3} \begin{bmatrix} e^{-k_3 L_3} & 0 \\ 0 & e^{k_3 L_3} \end{bmatrix} \frac{1}{2\sqrt{k_1 k_3}} \begin{bmatrix} k_3 + ik_1 & k_3 - ik_1 \\ k_3 - ik_1 & k_3 + ik_1 \end{bmatrix}$$

(26)

With the help of equation (14) a transmission coefficient is obtained:

$$T = \left| \frac{t_1 t_2}{1 - (r_1 r_2) e^{-2ik_1 L_2}} \right|^2$$

(27)

Where $t_1$ is the transmissivity of the first barrier and $t_2$ is the transmissivity of the second barrier, $r_1$ is a reflection on the first barrier and $r_2$ is a reflection on the second barrier.

### 3. Results and Discussion

This research uses particles namely electrons because electrons are particles that can move freely. Electrons have a wave number $k_1$ when the electron is outside the potential barrier and electrons that penetrate the barrier so that it is in the barrier has a wave number $k_2$. Particles that make it out of the barrier have the same wave number as electrons before connecting the barrier namely $k_1$. The wave number value $k_1 > k_2$ because the electron energy is not affected by the potential energy barrier when the electron is in the area outside the potential barrier, so that it has momentum $p = \sqrt{2mE}$, but the electron will be affected by the potential energy barrier when the electron enters the energy barrier, so
the momentum possessed is \( P = \sqrt{2m(V - E)} \). The formula \((V - E)\) is obtained because in this case the potential barrier energy is greater than the electron energy.

The wave function in each state has a difference, the wave function in a state outside the barrier is in the form of a complex exponential function resulting in a sinusoidal function. The wave function in the state inside the barrier is in the form of an exponential function with a graph decreasing exponentially because the electron energy has been affected by the potential energy of the barrier. Electrons outside the barrier are said to behave like matter and electrons when they penetrate the analysis are electrons acting as waves because waves have characteristics that are transmitted and reflected when viewed as matter electrons cannot penetrate a barrier because matter does not have the characteristics to penetrate something.

The relationship between the distance barrier with the transmission coefficient and the reflection coefficient analyzed by a computer program that is Matlab. The Matlab program helps in processing data and produces a graph of the relationship between the barriers with the transmission coefficient and the graph of the relationship between the barriers with the reflection coefficient. The Matlab program used is 1 eV electron energy with a barrier distance of 0.25-2 nm with a change in barrier distance of 0.25 nm, where the general semiconductor layer has a size of ~ 20 nm. The graph of relationship the distance between barriers with the transmission coefficient and the graph of relationship the distance between barriers with the reflection coefficient can be seen in Figure 3.

![Figure 3](image)

**Figure 3.** Results of computer simulations (left side of the graph of relationship the distance between barrier with the transmission coefficient and the right side of the graph of relationship the distance between barrier with the reflection coefficient).

The simulation results of Figure 3 show that in the case of the pure tunneling effect changes in the distance between barriers have contributed to the value of the transmission coefficient and the reflection coefficient. Electrons that have 1 eV energy have a chance to penetrate the distance between the barriers that is getting bigger. The numerical results of a tunneling effect are shown in Table 1 with nine data.

| Distance Between Barrier (nm) | Transmission Coefficient | Reflection Coefficient |
|-------------------------------|--------------------------|------------------------|
| 0,25                          | 0,4432                   | 0,5568                 |
The first data with the distance between barrier 0.25 nm produces a particle penetration probability is 0.4432 and the reflected particle probability is 0.5568. The first data until the fourth data increases with the maximum transmission coefficient value obtained at a distance between 1 nm barrier, but after the fourth data transmission coefficient decreases this causes the reflection coefficient to increase because of the sum of the transmission coefficients and the reflection coefficient equal to 1. The first to eighth data can be said to be periodic because the greater the distance between barrier, the transmission coefficient value is not smaller but obtains data whose values go up and down. This is because the distance between a barrier is in the form of a complex exponential function for either the transmission coefficient or the reflection coefficient.

4. Conclusion
Changes in the distance between barriers GaAs and GaAs affect the value of the transmission coefficient and the reflection coefficient. The transmission coefficient value is inversely proportional to the distance between the barriers so that the data obtained starts from the minimum value and rises exponentially. The reflection coefficient value is linear with the distance between the barriers so that the data obtained starts from the maximum value and decreases exponentially. The data obtained are periodic because the distance between the barriers is in the form of complex exponential functions. This research application is used to design and develop new structures in improving the quality of semiconductor devices such as diodes, transistors, and IC. Further research can be carried out using changes in the distance between the barriers which are smaller than 0.25 nm.

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