Multiresolution Subspace-Based Optimization Method for the Retrieval of 2-D Perfect Electric Conductors

Xiuzhu Ye®, Senior Member, IEEE, Francesco Zardi®, Marco Salucci®, and Andrea Massa®, Fellow, IEEE

Abstract—Perfect electric conductors (PECs) are reconstructed integrating the subspace-based optimization method (SOM) within the iterative multiscaling approach (IMSA). Without a priori information on the number of the scatterers and modeling their electromagnetic (EM) scattering interactions with a (known) probing source in terms of surface electric field integral equations, a segment-based representation of PECs is retrieved from the scattered field samples. The proposed IMSA-SOM inversion method is validated against both synthetic and experimental data by assessing the reconstruction accuracy, the robustness to the noise, and the computational efficiency with some comparisons, as well.

Index Terms—Inverse problems, inverse scattering, iterative multiscaling approach (IMSA), metallic scatterers, microwave imaging, multiresolution, perfect electric conductor (PEC), subspace-based optimization method (SOM).

NOMENCLATURE

BCS Bayesian compressive sensing.
CG Conjugate gradient.
EM Electromagnetic.
GAs Genetic algorithms.
IMSA Iterative multiscaling approach.
ISP Inverse scattering problem.
LS Level set.
LSF Local shape function.
MoM Method of moments.
PEC Perfect electric conductor.
SNR Signal-to-noise ratio.
SOM Subspace-based optimization method.
SVD Singular value decomposition.
TE Transverse electric.
TM Transverse magnetic.

I. INTRODUCTION

Detecting metallic objects and retrieving the scatterer shape with microwave imaging techniques are common tasks in many practical applications. For instance, the detection of concealed weapons is of paramount importance [1], [2], [3], [4] in critical environments, such as airports and living spaces. In the industrial context, EM imaging has been used to localize metallic contaminants in food products [5] as well as cracks in metallic pipelines [6], [7], [8]. As for the medical framework, online sensors have been designed to guide surgeons in the removal of metallic shrapnels [9]. Moreover, subsurface imaging of metallic objects is of great importance for the detection of landmines [10], [11] and for the inspection of underground infrastructures [12], [13], [14], [15].

Generally speaking, metallic objects are, in practice, often modeled as PECs, and their EM interactions with the probing source have been modeled with reliable techniques in the state-of-the-art literature. Depending on the mathematical modeling, which is a consequence of the representation chosen for the unknown PEC objects, the PEC imaging methods can be roughly divided in volume-based and surface-based approaches.

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In volume-based representations, the investigation domain is discretized in subunits, namely, pixels, of unknown dielectric properties to be retrieved by often modeling the field scattered by each PEC pixel as a multipole expansion in cylindrical harmonics. For instance, the LSF is an example of a volume-based formulation where a Boolean variable is associated with each pixel for denoting whether it is PEC or empty [16]. Accordingly, the PEC ISP is rephrased as a binary optimization one, and then solved with GAs [17], [18]. Otherwise, the derivative-based optimization algorithms, such as CGs [16], [19], [20], are applied when the binary descriptors are relaxed to continuous ones.

In [21], a different volume-based representation is proposed to exploit a fast BCS technique for retrieving the zeroth order coefficients of the multipole expansion at each pixel location, which are then postprocessed to classify each pixel as PEC or empty. Such an approach has been further developed in [22] by considering a multipole expansion of higher order. A higher order expansion has been also adopted in [23] to solve the ISP involving both PEC and dielectric scatterers. Another volume-based representation for mixed targets, which relies on integral scattering equations instead of a multipole expansion, has been presented in [24], [25], and [26]. More specifically, a complex permittivity value has been assigned to each pixel, and metallic objects have been modeled as dielectrics with a large, but finite, imaginary part.

Differently from volumetric models, surface-based representations of the PEC scatterers only consider the outer contour, since it is enough to model the scattering behavior of objects impenetrable to EM waves. Subject to the (also approximate) knowledge of the position and number of disconnected targets, arbitrary PEC shapes have been described with either the coefficients of Fourier series [27], [28], [29], [30] or the control points of spline curves [29], [30], [31], [32], [33], [34]. In both cases, the surface descriptors have been retrieved by applying the Newton–Kantorovich method [27], the differential evolution optimizer [31], [33], and the GAs [28], [29], [30], [32], [34].

Without a priori information on the scattering scenario (i.e., the number and the positions of the objects), the LS method has been successfully used in [35] to reconstruct the PEC contour. Under the same hypothesis (i.e., without a priori knowledge), another surface-based method, which is based on the discretization of the investigation domain in a grid of segments, has been introduced in [36] and further refined in [37]. Each segment has been then classified as PEC or non-PEC with an inversion procedure that relies on the SOM. Such an approach has been successfully applied to both TM and TE illuminations [37], [39], as well. Alternatively, considering a qualitative method as the first step can help in detecting the number of unknowns, thus enabling a more successful application of surface-based approaches [51].

Unlike PEC inversion methods that require a priori information, which might be inaccurate or even unavailable, the reconstruction approaches that use pixel- or segment-based representations of the investigation domain can effectively model arbitrary connected and non-connected PEC geometries, but they need a large number of unknowns to achieve a high-resolution imaging. This causes a heavier ill-posedness of the ISP at hand, which implies the use of effective regularization countermeasures, as well as an increase in the computational cost that might result unacceptable (especially) if computationally intensive optimization methods are adopted for the scattering-data inversion.

To address these issues, a novel strategy, which is based on the IMSA as applied to a segment-based representation of the investigation domain of the ISP, is proposed hereinafter for the retrieval of PEC objects. As a matter of fact, the IMSA has been extensively applied to mitigate the nonlinearity and to alleviate the nonuniqueness of the ISP when dealing with dielectric scatterers [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50], [51]. The underlying idea is that the data inversion is iteratively performed for the reconstruction of only a portion of the whole investigation domain, called region of interest (RoI), which is iteratively reduced to implement a synthetic zoom on the scatterers. In this way, the number of unknowns is kept close to the limited amount of information collectable from the measurements [52] for both mitigating the ill-posedness of the ISP [40] and reducing the computational burden, but the spatial resolution of the reconstruction is increased only in the RoIs where the scatterers have been detected.

Because of its flexibility and being a meta-level strategy, the IMSA has been integrated with a variety of solution strategies (e.g., the BCS [47], the particle swarm optimization [46], and the SOM [43], [44]) and different formulations (e.g., Lippmann–Schwinger integral equations or contraction integral equations [48]). Moreover, it has been used to deal with both 2-D [49] and 3-D [45] inversion problems.

In this article, the IMSA has been combined with the SOM to tackle the PEC imaging problem, since this latter method independently reduces the ill-posedness of the ISP [53] by constraining the unknown currents, induced on the investigation domain, to a low-dimensional subspace. Moreover, such an integration has been already successfully exploited for the retrieval of dielectric scatterers [43], [44] even though with a pixel-based representation of the investigation domain. Compared with other state-of-the-art techniques, the IMSA-SOM tool for the PEC reconstruction does not require a priori information on the location or the number of the scatterers, and it has limited computational needs because of the profitable integration of the IMSA with an efficient optimization strategy based on CG.

The outline of this article is as follows. The mathematical formulation of the ISP arising in PEC imaging is described in Section II, while the proposed IMSA-SOM inversion method is detailed in Section III. Section IV is devoted to the numerical and experimental assessment with some comparisons, as well. Finally, concluding remarks are drawn (Section V).

II. Mathematical Formulation

Let $D$ be a 2-D square investigation domain of side $L_D$ laying on the $xy$ plane within an unbounded free-space background with permittivity and permeability $\varepsilon_0$ and $\mu_0$, respectively (Fig. 1), where unknown PEC scatterers have to
be retrieved. Indicating with $f$ the working frequency and assuming a time-dependency factor $e^{-j2\pi f t}$ hereinafter, such a scenario is probed by a set of $V$ TM $z$-polarized monochromatic plane waves impinging from the angular directions \{\phi_i; \nu = 1, \ldots, V\} and having (known) electric field equal to $E^{(\nu)}(r) = E_{\text{inc}}^{(\nu)}(r) \hat{z}$ [$r = (x, y)$] ($\nu = 1, \ldots, V$).\footnote{Throughout this article, a bold notation is used to indicate a physical vector ($\mathbf{a}$), a single bar indicates an algebraic vector or 1-D tensor ($\bar{a}$), and double bars indicate a matrix or 2-D tensor ($\overline{a}$).} For each $\nu$th ($\nu = 1, \ldots, V$) incidence, the electric field $E^{(\nu)}(r) = E_{\text{tot}}^{(\nu)}(r) \hat{z}$ is measured at $M$ positions, \{\mathbf{r}^{(\nu)}_m; m = 1, \ldots, M\}, of the observation domain $D_{\text{obs}}$ external to $D$.

In order to numerically deal with the problem at hand, the investigation domain is partitioned into $N$ square pixels identified by $Q$ line edges (Fig. 1). A binary variable $P_q$ is assigned to each $q$th ($q = 1, \ldots, Q$) segment to denote the membership to the PEC when $P_q = 1$, while $P_q = 0$ stands for the background, so that the dielectric distribution in $D$ is univocally identified by the algebraic binary vector $\overline{P}$ ($\overline{P} \triangleq \{P_q; q = 1, \ldots, Q\}$). From an EM viewpoint, each $q$th ($q = 1, \ldots, Q$) edge is modeled in terms of an associated piecewise constant equivalent current, $J^{(q)}(\mathbf{r}_q) = J_z^{(q)}(\mathbf{r}_q) \hat{z}$, centered at the barycenter of the $q$th segment, $\mathbf{r}_q$, subject to the condition that for each $\nu$th ($\nu = 1, \ldots, V$) view

$$\overline{J} = \overline{P} \cdot \overline{J}^{(q)} = \overline{0},$$

where $\cdot$ denotes the dot product, which forces the equivalent current to be zero on non-PEC segments. In (1), $\overline{I}$ is the identity matrix, $\overline{P}$ is the diagonal matrix whose $q$th ($q = 1, \ldots, Q$) diagonal element is equal to $P_q$, $\overline{J}^{(q)}$ is the $Q$-size complex-valued algebraic vector whose $q$th ($q = 1, \ldots, Q$) entry is equal to $J_z^{(q)}(\mathbf{r}_q)$, and $\overline{0}$ is the $Q$-size null vector.

Moreover, the $\nu$th ($\nu = 1, \ldots, V$) scattered field, which is defined as $E^{\text{scat}}_{\text{sc}}(\mathbf{r}) \triangleq E^{(\nu)}(\mathbf{r}) - E_{\text{inc}}^{(\nu)}(\mathbf{r})$, is related to the corresponding equivalent current, $J^{(\nu)}(\mathbf{r})$, through the following relationship:

$$E^{\text{scat}}_{\text{sc}} = \overline{G} \cdot \overline{J},$$

where $\overline{G}$ is the Green’s matrix, mapping the equivalent current in $\mathbf{r}_q$ ($q = 1, \ldots, Q$) ($\mathbf{r}_q \in D$) to the scattered field at $\mathbf{r}_r$, whose $(\nu, t)$th entry is equal to [38], [55]

$$\overline{G}_{r q} = \begin{cases} -k \eta W H_{0}^{(t)}(k |\mathbf{r}_r - \mathbf{r}_q|), & \text{if } t \neq q \\ -k \eta W \frac{4}{4} + j \frac{2}{\pi} \left( \ln \left( \frac{2kW}{4} \right) - 1 \right), & \text{if } t = q \end{cases}$$

$k (k \triangleq (2\pi/\lambda))$ and $\eta (\eta \triangleq \mu_0/\varepsilon_0)$ being the wavenumber and the background impedance, respectively, $\lambda$ is the wavelength at $f$, $W$ is the length of one edge, $H_{0}^{(t)}$ is the Hankel function of the first kind and zeroth order, $\gamma \approx 1.781$ is a constant [38], [54], and $\| \cdot \|$ stands for the $L_2$-norm.

Depending on the location of the probing point $\mathbf{r}_r$, (2) can be further customized. When $\mathbf{r}_r \in D_{\text{obs}}$ (i.e., $\mathbf{r}_r = \mathbf{r}_{\text{inc}}^{(\nu)}$), (2) is rewritten as follows:

$$\overline{E}^{\text{scat}}_{\text{sc}} = \overline{G}_{\text{ext}} \cdot \overline{J},$$

where $\overline{G}_{\text{ext}}$ is now an $M \times Q$ matrix with the $(m, q)$th $(m = 1, \ldots, M; q = 1, \ldots, Q)$ entry given by $\overline{G}_{\text{ext}}_{m q} = \overline{G}_{r q} (t \neq q, m = t)$, while $\overline{E}^{\text{scat}}_{\text{sc}}$ is the complex algebraic vector of size $M$ of the scattered field samples (i.e., $\overline{E}^{\text{scat}}_{\text{sc}} = \{\mathbf{E}^{\text{scat}}_{\text{sc}}(\mathbf{r}_m); m = 1, \ldots, M\}$). Otherwise (i.e., $\mathbf{r}_r \in \overline{D}$), it turns out that $\overline{E}^{\text{scat}}_{\text{sc}} = \overline{G}_{\text{int}} \cdot \overline{J}$

$$\overline{E}^{\text{scat}}_{\text{sc}} = \overline{G}_{\text{int}} \cdot \overline{J},$$

where $\overline{G}_{\text{int}}$ is a $Q \times Q$ matrix with the $(p, q)$th $(p, q = 1, \ldots, Q)$ entry given by $\overline{G}_{\text{int}}_{p q} = \overline{G}_{r q} (t = p)$, while $\overline{E}^{\text{scat}}_{\text{sc}} = \{\mathbf{E}^{\text{scat}}_{\text{sc}}(\mathbf{r}_q); q = 1, \ldots, Q\}$.

Dealing with PECs, the boundary condition (i.e., the tangential total field vanishes in correspondence with the PEC segments) can be exploited to derive the following:

$$\overline{P} \cdot \overline{E}^{\text{inc}}_{\text{tot}} = \overline{0}$$

$$\overline{E}^{\text{inc}}_{\text{tot}} = \overline{E}^{\text{inc}} + \overline{E}^{\text{scat}},$$

Therefore:

$$\overline{P} \cdot \overline{E}^{\text{inc}}_{\text{tot}} + \overline{G}_{\text{int}} \cdot \overline{J}^{(q)} = \overline{0}$$

$$I_{\text{inc}}^{\text{tot}} = \{I^{\text{inc}}_{\text{tot}}(\mathbf{r}_q); q = 1, \ldots, Q\}$$.

According to such a formulation and the segment-based representation of the PEC scatterers, the inverse problem at hand is then rephrased as that of determining $\overline{P}$ and $\overline{E}^{(q)}_{\text{inc}}; \nu = 1, \ldots, V$ that fulfill (1), (4), and (7).

III. INVERSION METHOD

The solution of the inverse problem formulated in Section II is addressed with an approach based on the integration of the SOM within the IMSA scheme. More specifically, the scattering-data inversion is carried out by means of an iterative strategy that performs $S$ successive “zooming” steps. At each
s-th (s = 1, . . . , S; s being the step index) step, the PEC profile of the RoI, which is the portion of the investigation domain D where the scatterer has been estimated to lie, is retrieved by means of a deterministic algorithm based on the SOM as applied to (1), (4), and (7). Such a reconstruction is then exploited to improve the RoI estimate by also enhancing the spatial resolution of the retrieved guess. The process is repeated until a data-matching convergence criterion holds true.

The implementation of such a multilevel process needs the following: 1) to redefine the problem unknowns, {Ψ; ΨC}; so that the SOM, which is a continuous-variable optimization method, can be fruitfully applied (Section III-A); 2) to customize the SOM [38] to the problem at hand (Section III-B); 3) to define a suitable cost function that faithfully links the ISP at hand to its mathematical representation, so that the solution is the global minimum of the cost function itself (Section III-C); and 4) to customize the meta-level IMSA strategy to both such a formulation (i.e., problem unknowns and cost function) and the integration with the optimization level (i.e., the SOM; Section III-D). These items will be detailed or briefly recalled in the following.

A. Unknowns Coding

Since the unknown vector Ψ is binary, it is rewritten as a function of the set of auxiliary continuous variables, {ζv: q = 1, . . . , Q}, as follows:

\[ P_q = \frac{1}{1 + \exp(-b_\xi Competence)} \]  

(q = 1, . . . , Q), the coefficient b being defined as in [37]. Consequently, since \( \xi_q > 0 \) (\( \xi_q < 0 \)) implies that \( P_q = 1 \) (\( P_q = 0 \)); that is, the q-th (q = 1, . . . , Q) segment is PEC (empty), and the same information coded in the Q-size binary vector \( \Psi \) can be now drawn from the same size continuous vector \( \Xi = \{\xi_q: q = 1, . . . , Q\} \).

B. SOM Implementation

According to the SOM guidelines [53], the q-th (q = 1, . . . , V) equivalent current \( \Psi_q \) is decomposed in two terms

\[ \Psi_q = \Psi^D_q + \Psi^A_q \]  
The deterministic part and the ambiguous one, respectively. The former, \( \Psi_q \) (q = 1, . . . , V), is computed from (4) by applying the SVD to the external Green matrix

\[ \Psi^D = \Psi^D \Psi^D \Psi^D \]  

where \( H \) stands for conjugate transposition, \( \Psi^D \) = \{\( \Psi_q \); q = 1, . . . , Q\} is the M × Q matrix whose q-th (q = 1, . . . , Q) column is the M-size left-singular vector (i.e., \( \Psi^D_q \)), \( \Psi^D \) is the Q × Q diagonal matrix of the Q singular values of \( \Psi^D \) (i.e., \( \Psi^D \) = \{\( \sigma_q \); q = 1, . . . , Q\}), and \( \Psi^D \) = \{\( \Psi^D_q \); q = 1, . . . , Q\} is the Q × Q matrix whose q-th (q = 1, . . . , Q) column is the Q-size right-singular vector (i.e., \( \Psi^D_q \)). More specifically, \( \Psi^D_q \) (q = 1, . . . , V) is given by

\[ \Psi^D_q = \sum_{q=1}^Q \frac{\sum_{q=1}^Q \sigma_q^D \Psi^D_q}{\sigma_q^2} \Psi^D_q \]  

where \( \sigma_q \) is the SVD truncation threshold, which is adaptively set as follows [43]:

\[ \sigma_q = \arg \min \left\{ \frac{\sum_{q=1}^Q \sigma_q^D \Psi^D_q}{\sigma_q^2} - \alpha \right\} \]  

(12)

\( \alpha \) (0 < \( \alpha \) < 1) being a real user-defined calibration parameter (see Section IV-B).

The ambiguous part of the current, \( \Psi^A_q \), is yielded as the linear combination of the remaining \( (Q - \sigma_q) \) right-singular vectors

\[ \Psi^A_q = \sum_{q=Q-a+1}^Q \Psi^A_q \Psi^A_q \]  

where \( \Psi^A_q \) = \{\( \Psi_q^A \); q = 1, . . . , (Q - \( \sigma_q \))\} is the unknown complex algebraic vector of the weights of the q-th (q = 1, . . . , V) ambiguous current, while \( \Psi^A_q \) is the corresponding \( (Q - \sigma_q) \) × V size matrix \( \Psi^A_q = \{\Psi_q^A; q = 1, . . . , V\} \).

C. Cost Function Definition

The cost function \( F \) is defined to quantify the mismatch between actual and estimated values in both the scattered field and the induced equivalent currents

\[ F(\Psi^A_q, \Psi^C_q) = \sum_{q=1}^Q \left[ F^C_q(\Psi^C_q, \Psi^C_q) + F^C_q(\Psi^A_q, \Psi^A_q) \right] \]  

(14)

(15)

while \( F^C_q \) is given by

\[ F^C_q(\Psi^A_q, \Psi^A_q) = \frac{\left\| \Psi^A_q - \Psi^A_q \right\|^2}{\left\| \Psi^A_q \right\|^2} \]  

(16)

being the normalized error in fulfilling conditions (1) and (7) on the \( q \)-th (q = 1, . . . , V)-induced equivalent current \( \Psi^D_q \), which is equal to \( \Psi^D_q = \sum_{q=1}^Q \frac{\Psi^D_q}{\sigma_q^D} \Psi^D_q + \sum_{q=Q-a+1}^Q \Psi^A_q \Psi^A_q \) according to (11) and (13).
D. IMSA Implementation

The algorithmic customization of the IMSA meta-level to integrate the SOM-based optimization for the retrieval of PECs can be described through the following multistep iterative (i being the iteration index) process.

1) Initialization: Set the step index to \( s = 1 \) and the RoI to the whole investigation domain \( D = (D^{(1)} = D) \).

2) IMSA Loop:
   a) Unknowns Setup (\( i = 0 \)): If \( s = 1 \), then set \( \overline{w}_{i}^{(s)} = \overline{z}_{i}^{(s)} = \overline{1} \). Otherwise (i.e., \( s > 1 \)), map the trial solution from the previous zooming step into the current \( s \)-th discretization grid of the RoI \( D^{(s)} \) (i.e., \( \overline{w}_{i}^{(s)} = \Phi_{1}(\overline{w}_{i}^{(s-1)}); D^{(s)} \) and \( \overline{z}_{i}^{(s)} = \Phi_{1}(\overline{z}_{i}^{(s-1)}); D^{(s)} \), \( \Phi \) being the mapping operator from the grid of \( D^{(s-1)} \) to the finer one of \( D^{(s)} \).

   b) Scattering-Data Inversion: Compute the \( s \)-th step trial solution \( (\overline{z}_{i}^{(s)}; \overline{w}_{i}^{(s)}) \) within the RoI \( D^{(s)} \) by solving the following optimization problem:

\[
(\overline{z}_{i}^{(s)}, \overline{w}_{i}^{(s)}) = \arg \min_{\overline{z}, \overline{w}} \{ F(\overline{z}, \overline{w}) \} \tag{17}
\]

with \( I \) iterations of the deterministic two-step CG algorithm in [37] (i.e., \( \overline{z}_{q}^{(s)} = (\overline{r}_{q}^{(s)})_{1=0;i=1}^{Q} \) and \( \overline{w}_{i}^{(s)} = (\overline{r}_{i}^{(s)})_{1=0;i=1}^{Q} \) from starting \( \overline{z}_{i}^{(s)} \) and \( \overline{w}_{i}^{(s)} \), respectively.)

   c) Step Check: Stop the IMSA loop if the maximum number of zooming steps is reached (i.e., \( s = S \)) and output the estimated solution by setting \( \overline{z}_{\text{opt}}^{(s)} = \overline{z}_{i}^{(s)} \) and \( \overline{w}_{\text{opt}}^{(s)} = \overline{w}_{i}^{(s)} \).

   d) Roll Update: Compute the \( s \)-th estimate of the PEC indicator vector \( \overline{F}^{(s)} \) through (8) with \( \zeta_{q} \leftarrow \rho_{q}^{(s)} \) and apply the “filtering and clustering” operations [40] to determine the new RoI, \( D^{(s+1)} \), by defining its center, \( \mathbf{r}_{D}^{(s+1)} = (x_{D}^{(s+1)}, y_{D}^{(s+1)}) \), and side, \( L_{D}^{(s+1)} \), as follows:

\[
\zeta_{D}^{(s+1)} = \sum_{q=1}^{Q} \sum_{i=1}^{Q} \frac{z_{q}^{(s)} P_{q}^{(s)}}{\sum_{q=1}^{Q} P_{q}^{(s)}} \tag{18}
\]

(\( \zeta \in \{x; y\} \))

\[
L_{D}^{(s+1)} = 2 \times \sqrt{\sum_{q=1}^{Q} \sum_{i=1}^{Q} \frac{P_{q}^{(s)}}{\sum_{q=1}^{Q} P_{q}^{(s)}}} \tag{19}
\]

   e) Roll Check: Terminate the IMSA loop if the zooming factor \( \eta^{(s)} \), which is defined as follows:

\[
\eta^{(s)} = \frac{L_{D}^{(s+1)} - L_{D}^{(s)}}{L_{D}^{(s+1)}} \tag{20}
\]

is below a user-defined threshold \( \eta_{\text{min}} \) (i.e., \( \eta^{(s)} \leq \eta_{\text{min}} \)) and set the problem solution to the current trial one (i.e., \( \zeta_{\text{opt}} = \zeta_{D}^{(s)} \) and \( \overline{w}_{\text{opt}} = \overline{w}_{D}^{(s)} \)). Otherwise, update the IMSA loop index (i.e., \( s \leftarrow (s + 1) \)) and restart the “IMSA Loop.”

IV. Numerical and Experimental Validation

This section is devoted to illustrate the results of the validation of the proposed inversion method and to give some indications on its performance in different scenarios and under various conditions. Toward this end, representative test cases, concerned with both synthetic and experimental scattering data, will be discussed.

To quantitatively assess the effectiveness of the data inversion/reconstructions, suitable error functions, customized to the segment-based representation of PEC scatterers, are defined and used. Namely, they are the total reconstruction error

\[
\Xi_{\text{tot}} \equiv \frac{1}{Q} \sum_{q=1}^{Q} [(1 - \rho_{q}^{\text{true}}) \rho_{q}^{\text{opt}} + \rho_{q}^{\text{true}} (1 - \rho_{q}^{\text{opt}})] \tag{21}
\]

(\( \Xi_{\text{tot}} = 0 \) if \( \rho_{q}^{\text{opt}} = \rho_{q}^{\text{true}} \) and \( \Xi_{\text{tot}} = 1 \) if \( \rho_{q}^{\text{opt}} = \bar{T} - \rho_{q}^{\text{true}} \)), the internal reconstruction error

\[
\Xi_{\text{int}} \equiv \frac{1}{Q} \sum_{q=1}^{Q} \rho_{q}^{\text{true}} (1 - \rho_{q}^{\text{opt}}) \tag{22}
\]

(\( \Xi_{\text{int}} = 0 \) if \( \rho_{q}^{\text{opt}} = 1 \) and \( \Xi_{\text{int}} = 1 \) if \( \rho_{q}^{\text{opt}} = 0 \), \( 0 \leq q \leq Q_{\text{int}} \)), and the external reconstruction error

\[
\Xi_{\text{ext}} \equiv \frac{1}{Q} \sum_{q=1}^{Q} (1 - \rho_{q}^{\text{true}}) \rho_{q}^{\text{opt}} \tag{23}
\]

(\( \Xi_{\text{ext}} = 0 \) if \( \rho_{q}^{\text{opt}} = 0 \) and \( \Xi_{\text{ext}} = 1 \) if \( \rho_{q}^{\text{opt}} = 1 \), \( 0 \leq q \leq Q_{\text{ext}} \)), \( \Xi_{\text{true}} \) and \( \Xi_{\text{opt}} \) being the true/actual and the reconstructed PEC indicator vectors, respectively, while \( Q_{\text{int}} = Q - Q_{\text{ext}} \), \( Q_{\text{ext}} \) being the segments of the investigation domain external to the support of the PEC scatterer.

Unless stated otherwise, the following reference scenario has been considered throughout the numerical assessment. A measurement setup at a frequency of \( f = 300 \) [MHz] where a plane wave probes a square investigation domain, \( D \), of side \( L_{D} = 3 \lambda \) by impinging from \( V = 27 \) different angular directions \( \{\phi_{n} = 2 \pi ((n - 1)/V); n = 1, \ldots, V\} \). The electric field coming from the interaction between the probing source and the scatterers laying in \( D \) has been measured by \( M = 27 \) ideal probes uniformly spaced on a circular observation domain, \( D_{\text{obs}} \), external to the investigation domain, with radius \( r_{\text{obs}} = 2.2 \lambda \). The \( M \times V \) synthetic scattering data have been numerically generated with an MoM solver by densely discretizing \( D \) with square cells \( \lambda/50 \)-sided. To emulate real data, the scattered field samples have been then blurred with an additive white Gaussian noise characterized by different SNRs. According to the guidelines in [52], the inverse problem at hand has been solved by uniformly partitioning \( D \) in \( N = 18 \times 18 \) subdomains.\(^2\) Moreover, the maximum number of zooming steps of the IMSA-SOM has been set to \( S \leq 6 \), while the value of the zooming threshold has been fixed to \( \eta_{\text{min}} = 0.2 \) according to [49].

\(^2\)The number of subdomains is set according to the degrees of freedom of the scattered field to \( N \approx ((2ka)^2/2) \), where \( a = (L_{D}/2)^{1/2} \) [52].
A. Illustrative Example

In this section, a detailed step-by-step description of the IMSA-SOM method as applied to an illustrative example is provided. Toward this end, a square PEC object of side $0.6\lambda$ has been chosen [see the red contour in Fig. 2(a)] and reconstructed by processing noisy scattered data with $SNR = 40$ [dB].

According to the multistep iterative procedure described in Section III-D, the RoI has been first initialized to the whole investigation domain ($D^{(1)} = D$). At the first IMSA step ($s = 1$), an SOM-based reconstruction of $D^{(1)}$ [green pattern—Fig. 2(a)] has been performed. Fig. 2(a) shows the reconstructed PEC profile with the cyan segments of the PEC indicator vector $P^{(s)}_j\forall j = 1$. Starting from such an estimate of the target position and shape, the RoI is updated by defining the green patterned region $D^{(2)}$ in Fig. 2(b). The second ($s = 2$) IMSA step has been then carried out by applying the SOM inversion to image $D^{(2)}$. As expected, owing to the improved RoI estimate, the reconstructed PEC map is significantly closer to the actual one [Fig. 2(b)]. In turn, such an improvement enables a further shrinking of the RoI, $D^{(3)}$, toward the actual support of the PEC [Fig. 2(c)]. At the successive IMSA step ($s = 3$), the SOM inversion is repeated by yielding the PEC indicator vector $P^{(s)}_j\forall j = 1$ as mapped in Fig. 2(c). The IMSA loop has been then stopped, since the zooming factor $(\eta)_{s=4} = 5.32 \times 10^{-2}$ was below the threshold ($\eta_{min} = 0.2$) as pointed out in Fig. 2(c) where the RoI matches very closely the shape of the actual target, and no further reconstruction enhancements were expected further zooming.

For the sake of completeness, the behaviors of both the cost function, $F$, and the reconstruction error, $\Xi_{tot}$, throughout the iterative ($i = 1, \ldots, I$) multistep ($s = 1, \ldots, S$) IMSA-SOM process, are shown in Fig. 3.

B. Control-Parameters Calibration

The IMSA-SOM inversion method depends on its control parameters (Section III), namely, the $\alpha$ threshold, which regulates the adaptive SVD truncation process, and the maximum number of iterations of the deterministic two-step CG algorithm [37], $I$, performed at each $s$th ($s = 1, \ldots, S$) IMSA step.

To give the interested readers some insights on the sensitivity of IMSA-SOM to these control parameters by also motivating the choice of the control setup used throughout the whole validation, the results of a study on a circular PEC object $\lambda/2$ in radius are reported in the following. More in detail, the inversion process has been repeated for each choice of the values of $\alpha$ and $I$ by processing different noisy data, so that the inferred indications are independent on the noise level.

The outcomes of such a calibration phase are summarized in Fig. 4 in terms of the total reconstruction error. In particular, Fig. 4(a) shows the behavior of $\Xi_{tot}$ versus $\alpha$ when fixing $I = 1000$, while the dependence on $I$ ($\alpha = 0.6$) is analyzed.
in Fig. 4(b). It turns out [Fig. 4(a)] that the α value impacts significantly on the inversion accuracy, the fluctuations of $\Xi_{\text{tot}}$ being quite large, since, for instance, $\Xi_{\text{tot}} = 0.1$ ($\Xi_{\text{tot}} = 0.01$) corresponds to 10(1)% of wrongly reconstructed segments on the total number. To better illustrate such an outcome, let us observe the reconstructions when $\alpha = 0.0$ [Fig. 5(b)] and $\alpha = 1.0$ [Fig. 5(c)]. In the first case, the SVD truncation is too selective, and only a limited portion of the PEC is correctly retrieved [Fig. 5(b)], while there is no truncation when $\alpha = 1.0$, and the inversion generates many artifacts outside the PEC domain [Fig. 5(c)]. For completeness, the total, internal, and external error indexes corresponding to each test case in Fig. 5 are reported in Table I.

As for the sensitivity of the IMSA-SOM performance on $I$, Fig. 4(b) indicates that, as expected, the higher the $I$ value, the smaller the reconstruction error is, but only until a threshold around $\approx 1000$ iterations. Indeed, if the deterministic minimization of $\mathcal{F}$ is stopped too early (e.g., $I = 30$), the optimum of (14) has not yet been reached, and the reconstruction is suboptimal [e.g., Fig. 5(d)]. On the other hand, increasing $I$ beyond a threshold value yields negligible improvements as confirmed by the comparison between the inversion results when setting $I = 1000$ [Fig. 5(a)] or $I = 1500$ [Fig. 5(e)].

The optimal setup for $\alpha$ and $I$ has been then chosen by considering $C = 4$ different SNR levels, $\Upsilon = \{\Upsilon_c; c = 1, \ldots, C\} = \{10, 20, 40, 60\}$ [dB], and applying the following rule:

$$\varsigma_{\text{opt}} = \min_c \sum_{c=1}^C \arg \min_{\varsigma} \{\Xi_{\text{tot}}^\varsigma \}_{\text{SNR}=\Upsilon_c}$$

(24)

The result has been $\alpha_{\text{opt}} = 0.6$ and $I_{\text{opt}} = 1000$. 

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### Table I

| Fig. | $\alpha$ | $I$ | $\Xi_{\text{tot}}$ | $\Xi_{\text{int}}$ | $\Xi_{\text{ext}}$ |
|------|---------|-----|-------------------|-------------------|-------------------|
| 3(a) | 0.6     | 1000| 1.77             | 6.48              | 1.26              |
| 3(b) | 0.0     | 1000| 9.68             | 93.5              | 0.57              |
| 3(c) | 1.0     | 1000| 6.97             | 5.86              | 7.09              |
| 3(d) | 0.6     | 30   | 3.90             | 3.40              | 3.96              |
| 3(e) | 0.6     | 1500| 1.77             | 6.48              | 1.26              |
A proof of the effectiveness of such a choice is shown in Fig. 5(a) where the IMSA-SOM reconstruction when SNR = 10 [dB] is reported, the total error being $\|\Xi_{\text{tot}}\|_{\text{SNR}=10} \approx 1.77 \times 10^{-2}$.

C. Numerical Assessment

Once calibrated, the performance of the IMSA-SOM has been assessed in comparison with a competitive state-of-the-art inversion approach. For a fair comparison, the single-resolution SOM in [37] has been considered as a reference.\(^3\)

The first test case is concerned with a “T”-shaped PEC object whose larger edge is 0.6λ. Such a shape presents corners and cavities making the reconstruction process even harder. Fig. 6 shows the behavior of the error indexes versus the SNR for both the IMSA-SOM and the SOM. As it can be noticed, the reconstruction accuracy of the IMSA-based method is significantly better than that of the single-resolution SOM, the improvement of the total reconstruction error ranging from around 30\% (SNR = 5 [dB]) being $\Delta \Xi_{\text{tot}} \approx (\Xi_{\text{tot}}^{\text{SOM}} - \Xi_{\text{tot}}^{\text{IMSA-SOM}})/\Xi_{\text{tot}}^{\text{SOM}}$. Moreover, it turns out that the multizooming approach reduces mainly the external error, while the internal one is almost equivalent for both methods. Such an outcome is pictorially pointed out by the map of the retrieved PEC profiles in Fig. 7 where the representative examples of inversion when processing data at SNR = 20 [dB] [Fig. 7(a) and (b)], SNR = 10 [dB] [Fig. 7(c) and (d)], and SNR = 5 [dB] [Fig. 7(e) and (f)] are reported. More in detail, Fig. 7(a) and (b) shows that both techniques are able to correctly localize the PEC object within the investigation domain, but the IMSA-SOM better shapes it. The differences between the two inversion approaches become more and more evident increasing the noise level to SNR = 10 [dB] [Fig. 7(c) versus (d)] and up to SNR = 5 [dB] [Fig. 7(e) versus (f)] when the SOM image reveals some disconnected artifacts in the surrounding of the bottom-left corner of the “T” boundary, as well, so that $\Delta \Xi_{\text{tot}} \approx 55\%$.

The second experiment is aimed at assessing the reliability of the IMSA-SOM in reconstructing PEC shapes that are not exactly mapped into the gridding of $\mathcal{D}$. Toward this purpose, a “Diamond” object whose larger edge is $0.5\lambda$ and edges tilted with respect to the discretization grid has been considered as a benchmark. The results in Fig. 8 indicate that the IMSA-SOM reduces the value of the total error of the SOM up to $\Delta \Xi_{\text{tot}} \approx 61\%$ (SNR = 5 [dB]), the minimum improvement being equal to $\Delta \Xi_{\text{tot}} \approx 47\%$ (SNR = 40 [dB]).

\(^3\)According to the guidelines in [37], a $\sqrt{10}$-sided uniform grid has been chosen to discretize the investigation domain $\mathcal{D}$ for the single-resolution inversion, so that $(N^{\text{SOM}} = 30 \times 30)$.\(^4\)
mainly derives from the ability of the IMSA to model more accurately the tilted edges of the diamond contour because of the use of a denser discretization in the RoI.

Still referring to the “Diamond” PEC, the dependence of the IMSA-SOM inversion on the size of the scatterer has been evaluated next by varying the diagonal, $d$, between $d = 0.5 \lambda$ and $d = 1.5 \lambda$. Fig. 10 gives the values of the total reconstruction error, $\Xi_{\text{tot}}$, versus $d$ for different SNRs. It turns out that, whatever the combination of the target size and the noise level, the IMSA-SOM performs better than the SOM. However, it is worth noticing that, while the error values tend to be quite close for larger dimensions of the PEC

Table II—Numerical Assessment (“Diamond” PEC Object, $M = V = 27$, SNR = 5 [dB])—Reconstruction Error Indexes

| $d / \lambda$ | $\Xi_{\text{tot}} \times 10^{-1}$ | $\Xi_{\text{tot}} \times 10^{-1}$ | $\Xi_{\text{tot}} \times 10^{-1}$ |
|--------------|-----------------|-----------------|-----------------|
| SNR = 40 [dB]| IMSA-SOM        | SOM             | IMSA-SOM        | SOM             |
| 1.1          | 2.58            | 5.45            | 7.77            | 9.38            |
| 0.7           | 1.29            | 3.27            | 0.0             | 0.0             |
| 0.3           | 0.37            | 1.42            | 0.0             | 0.0             |

$((d / \lambda) \to 1.5)$ and high SNRs, the advantage of using the IMSA strategy becomes greater as the size is lowered, and the noise level is getting heavier.

To further confirm these conclusions, Fig. 11(a)–(f) shows the PEC profiles retrieved when processing data with SNR = 5 [dB]. When $d = 1.1 \lambda$ [Fig. 11(a) and (b)], the SOM image presents both spurious artifacts outside the actual contour and a wrong empty internal region, which are properly avoided by the IMSA-SOM. Moving to the case $d = 0.7 \lambda$, the IMSA-SOM confirms to be more accurate in shaping the actual scatterer as well as in estimating its support [Fig. 11(c) versus (d)]. This is even more evident when $d = 0.3 \lambda$ [Fig. 11(e) versus (f)]. The computed errors reported in Table II further quantitatively corroborate such observations.

The successive experiment has been devoted to infer the highest spatial resolution, $R$ (i.e., the minimum distance at which two disconnected objects can be distinguished), achievable by the IMSA-SOM. More specifically, two PEC circles of radius 0.1 $\lambda$ have been considered, and the minimum distance
Fig. 12. Numerical assessment (“Two Circles” PEC objects, \( M = V = 27 \), SNR = 20 dB)—plots of the total reconstruction error, \( \Xi_{\text{tot}} \), as a function of the interscatter distance, \( D \).

Fig. 13. Numerical assessment (“Two Circles” PEC objects, \( M = V = 27 \), SNR = 20 dB)—maps of the PEC profile retrieved by (a), (c), and (e) IMSA-SOM and (b), (d), and (f) SOM when processing the data scattered by the actual PEC objects spaced by (a) and (b) \( D = 0.5 \lambda \), (c) and (d) \( D = 0.35 \lambda \), and (e) and (f) \( D = 0.3 \lambda \).

Table III

| \( D \) [\( \lambda \)] | \( \Xi_{\text{tot}} \) [\( \times 10^{-2} \)] | \( \Xi_{\text{true}} \) [\( \times 10^{-2} \)] | \( \Xi_{\text{est}} \) [\( \times 10^{-2} \)] |
|----------------|-----------------|-----------------|-----------------|
| 0.50           | 0.81            | 3.05            | 0.00            | 0.71            | 3.71            |
| 0.35           | 0.93            | 2.94            | 0.56            | 0.90            | 3.04            |
| 0.30           | 0.89            | 2.22            | 1.98            | 1.13            | 0.89            | 2.23            |

Fig. 14. Experimental assessment (Dataset “rectTM_cent,” \( V = 36 \), \( M = 49 \)—maps of the PEC profile retrieved by (a) IMSA-SOM and (b) SOM.

between their boundaries, \( D \), has been varied within the range \( 0.3 \lambda \leq D \leq 0.7 \). Fig. 12 shows the total reconstruction error, \( \Xi_{\text{tot}} \), as a function of the object distance, \( D \), when SNR = 20 [dB], the complete set of error indexes being reported in Table III for completeness. Unlike the IMSA-SOM, where \( \Xi_{\text{tot}} \) is almost flat in all the range of variation of \( D \) and equal to \( \Xi_{\text{tot}} \approx 0.01 \), the plot of the SOM error shows a hill-like behavior with \( \Delta \Xi_{\text{tot}} \geq 55\% \) when \( D/\lambda \leq 0.57 \). Fig. 13(a)–(f) illustrates these deductions by showing the profiles reconstructed in correspondence with three representative values of \( D \), namely, \( D = 0.5 \lambda \) [Fig. 13(a) and (b)], \( D = 0.35 \lambda \) [Fig. 13(c) and (d)], and \( D = 0.3 \lambda \) [Fig. 13(e) and (f)]. When the interobjects distance is \( D = 0.5 \lambda \), both single and multiresolution SOM techniques distinguish two separated scatterers [Fig. 13(a) and (b)], even though the “bare” SOM overestimates the size of the circles, and it also retrieves spurious artifacts in the proximity of the actual PECs [Fig. 13(b)]. Decreasing the distance below \( D \approx 0.5 \lambda \) [e.g., \( D = 0.35 \lambda \)—Fig. 13(d)], the single resolution is no more able to recognize two objects as correctly done by the IMSA-based inversion [Fig. 13(c) versus (d)]. If \( D \) is further shortened [e.g., \( D = 0.3 \lambda \)—Fig. 13(e) and (f)], neither of the two methods can resolve the two disconnected supports. However, while the IMSA-SOM map in Fig. 13(e) shows some “ghost” shadows in between the PEC circles, the SOM also detects wrong artifacts far from the actual scatterers [Fig. 13(f)].

Similar results have been obtained in other test cases during an exhaustive numerical assessment; thus, we are quite confident to state that the spatial resolution of the IMSA-SOM is \( R_{\text{IMSA-SOM}} = 0.35 \lambda \), that is a 30% improvement over the single-resolution SOM being \( R_{\text{SOM}} = 0.5 \lambda \).

D. Experimental Assessment

After the numerical validation with synthetic scattering data, this section is devoted to the assessment of the IMSA-SOM inversion strategy against experimental data measured in a real environment. Toward this end, the “rectTM_cent” and “rectTM_dece” datasets, provided by the Institut Fresnel, have been considered [56]. These two datasets have been generated by measuring the EM interactions between a metallic cylinder of section \( 1.27 \) cm \( \times \) \( 2.45 \) cm and the EM field radiated by a horn antenna located \( 72 \) cm away and working at \( f = 8 \) [GHz]. More in detail, the scatterer under test has been illuminated from \( V = 36 \) different angular directions, while the scattered electric field has been collected by \( M = 49 \) measurement probes uniformly located on a circular observation...
domain $\mathcal{D}_{\text{obs}}$ of radius $\rho_{\text{obs}} = 76$ [cm]. The two datasets differ for the position of the object with respect to the center of the measurements systems (i.e., $(x_0, y_0) = (-0.5, -0.75)$ [cm]—Dataset “rectTM_cent”; $(x_0, y_0) = (0, 4)$ [cm]—Dataset “rectTM_dece”).

The inversions of the “rectTM_cent” dataset are shown in Fig. 14(a) and (b). One can observe that both methods correctly localize the unknown object, but the SOM gets worse, since (once again) it overestimates the size of the cylinder as confirmed by the values of the reconstruction errors in Table IV (i.e., $\Delta \Xi_{\text{tot}} \approx 55\%$).

Concerning the computational issues, it turns out that the IMSA-SOM allows a computational saving$^4$ with respect to the bare SOM of about $\Delta t \approx 77\%$ ($\Delta t \triangleq (T^{\text{SOM}} - T^{\text{IMSA-SOM}})/T^{\text{SOM}}$). Indeed, even though the IMSA-SOM repeats up to $S = 6$ times the inversion process, while the SOM does it only once, the dimension of the inversion problem at hand is much smaller (i.e., $N_{\text{IMSA-SOM}} \ll N_{\text{SOM}}$) despite the higher spatial resolution yielded at the convergence.

Similar outcomes can be drawn for the “rectTM_dece” dataset, the IMSA-SOM improvements in both reconstruction accuracy and computational costs being $\Delta \Xi_{\text{tot}} \approx 50\%$ and $\Delta t \approx 72\%$ (Table IV). For completeness, the PEC profiles retrieved by the multiresolution/steps and the single-step SOM implementations are shown in Fig. 15.

### V. Conclusion

A novel inversion strategy, named IMSA-SOM, has been developed to address the ISP for 2-D PECs. The proposed strategy combines the IMSA and the SOM, and it has proved to be reliable and highly effective in a wide range of scenarios and under different conditions. Indeed, the developed inversion approach has been tested against both numerical and experimental scattering data by considering complex shapes and closely located scatterers, as well.

$^4$On a standard laptop computer with i5-8265U processor and 8-[GB] RAM.

TABLE IV

|                      | $\Xi_{\text{tot}} [\times 10^{-2}]$ | $\Xi_{\text{int}} [\times 10^{-2}]$ | $\Xi_{\text{ext}} [\times 10^{-2}]$ | $T$ [min] |
|----------------------|-------------------------------------|-------------------------------------|-------------------------------------|-----------|
| **IMSA-SOM**         | 1.67                                | 3.71                                | 0.0                                 | 1.71      |
| **SOM**              | 0.0                                 | 0.0                                 | 3.80                                | 124       |
| **IMSA-SOM**         | 1.39                                | 2.80                                | 0.0                                 | 1.42      |
| **SOM**              | 0.0                                 | 0.0                                 | 2.87                                | 120       |

Compared with the state-of-the-art literature on the subject, to the best of the authors’ knowledge, the main outcome of this article and of the related research work is that the developed innovative method for the PEC reconstruction is a reliable, flexible, and computationally efficient inversion tool, robust to the noise on the scattering data, as well, that effectively mitigates the nonlinearity and the ill-posedness of the imaging problem at hand.

Future works, beyond the scope of the current manuscript, will be aimed at extending the formulation to 3-D geometries as well as at customizing the proposed implementation to buried objects scenarios of great applicative interest. Moreover, the extension of the proposed IMSA-SOM to deal with multiple targets located in different portions of the reconstruction domain (as done in [51] and [57]) will be addressed. Toward this end, the joint zooming on multiple disconnected RoIs to take into account the nonlinearity related to the multiple scattering phenomena among the objects as well as the exploitation of a preliminary qualitative initialization step to detect the number of unknowns [51] will be investigated.

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XiuZhu Ye (Senior Member, IEEE) received the bachelor’s degree in communication engineering from the Harbin Institute of Technology, Harbin, China, in 2008, and the Ph.D. degree in electrical engineering from the National University of Singapore (NUS), Singapore, in 2012.

She was an Assistant Professor with Beihang University, Beijing, China. Since 2012, she has been a Post-Doctoral Researcher with the Department of Electronics and Communication Engineering, NUS. She joined the Department of Information and Electronics, Beijing Institute of Technology, Beijing, in 2019, as an Associate Professor and the Ph.D. Director. She is an expert in microwave imaging system, inverse problems, and electromagnetic theory. She has authored or coauthored more than 80 scientific articles in the related areas, including more than 30 papers in top journals. Her current research interests include biomedical imaging system, through wall imaging and related machine learning algorithms. She is also familiar with all types of microwave imaging systems.

Dr. Ye serves as the Youth Deputy Director at EMC branch and member of council at the Young Women Scientists Club in China Electronics Society.

Francesco Zardi received the B.Sc. degree in telecommunications and electronic engineering and the M.Sc. degree in information and communications engineering from the University of Trento, Trento, Italy, in 2017 and 2019, respectively, where he is currently pursuing the Ph.D. degree at the International Doctoral School in Information and Communication Technology of Trento.

He is a Senior Researcher with the ELEDIA Research Center, University of Trento. His research interests include advanced radar architectures and electromagnetic diagnostic techniques.

Marco Salucci (Senior Member, IEEE) received the M.S. degree in telecommunication engineering from the University of Trento, Trento, Italy, in 2011, and the Ph.D. degree from the International Doctoral School in Information and Communication Technology of Trento, University of Trento, in 2014.

He was a Post-Doctoral Researcher with CentraleSupélec, Paris, France, and the Commissariat à l’Énergie Atomique et aux Énergies Alternatives (CEA), Paris. He is currently an Assistant Professor at the Department of Civil, Environmental, and Mechanical Engineering (DICAM), University of Trento, and a Research Fellow of the ELEDIA Research Center, University of Trento. His research interests include inverse scattering, biomedical and ground penetrating radar (GPR) microwave imaging techniques, antenna synthesis, and computational electromagnetics with a focus on system-by-design methodologies integrating optimization techniques and artificial intelligence for real-world applications.

Dr. Salucci was a member of the COST Action TU1208 “Civil Engineering Applications of Ground Penetrating Radar.” He is a member of the IEEE Antennas and Propagation Society. He is an Associate Editor for Communications and Memberships of the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION. He serves as an Associate Editor for the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION and the IEEE OPEN JOURNAL OF ANTENNAS AND PROPAGATION. He serves as a Reviewer for different international journals, including the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, the IEEE JOURNAL ON MULTISCALE AND MULTIPHYSICS COMPUTATIONAL TECHNIQUES, and the IET Microwaves, Antennas and Propagation.

Andrea Massa (Fellow, IEEE) received the Laurea (M.S.) degree in electronic engineering and the Ph.D. degree in electrical engineering and computer science (EECS) from the University of Genoa, Genoa, Italy, in 1992 and 1996, respectively.

He is currently a Full Professor of electromagnetic fields at the University of Trento, Trento, Italy, where he is involved in teaching electromagnetic fields, inverse scattering techniques, antennas and wireless communications, wireless services and devices, and optimization techniques. He is the Director of the network of federated laboratories “ELEDIA Research Center” (www.eledia.org) located in Brunei, China, Czech, France, Greece, Italy, Japan, Peru, and Tunisia with more than 150 researchers. He is a Chang-Jiang Chair Professor at the University of Electronic Science and Technology of China (UESTC), Chengdu, China, a Professor at CentraleSupélec, Paris, France, and a Visiting Professor at Tsinghua University, Beijing, China.

He is also an Adjunct Professor at Penn State University, State College, PA, USA, a Guest Professor at UESTC, and a Visiting Professor at the Missouri University of Science and Technology, Rolla, MO, USA, Nagasaki University, Nagasaki, Japan, the University of Paris Sud, Bures-sur-Yvette, France, Kumamoto University, Kumamoto, Japan, and the National University of Singapore. He has authored or coauthored more than 900 scientific publications among which more than 350 on international journals (more than 13,500 citations—H-index = 60 [Scopus]; more than 11,000 citations—H-index = 58 [ISI-Web of Science]); and more than 22,000 citations—H-index = 87 [Google Scholar]) and more than 550 in international conferences where he presented more than 200 invited contributions (more than 40 invited keynote speaker) (www.eledia.org/publications). He has organized more than 100 scientific sessions in international conferences and has participated to several technological projects in the national and international framework with both national agencies and companies (18 international projects, more than 5M; eight national projects, more than 5M; ten local projects, more than 2M; 63 industrial projects, more than 10M; six university projects, more than 300k). His research interests include inverse problems, analysis/synthesis of antenna systems and large arrays, radar systems synthesis and signal processing, cross-layer optimization and planning of wireless/RF systems, semantic wireless technologies, system-by-design and material-by-design (metamaterials and reconfigurable materials), and theory/applications of optimization techniques to engineering problems (telecommunications, medicine, and biology).

Prof. Massa is a Fellow of the Institution of Engineering and Technology (IET) and the Electromagnetic Academy. He is a member of the Editorial Board of the Journal of Electromagnetic Waves and Applications, a permanent member of the the Photonics and Electromagnetics Research Symposium (PIERS) Technical Committee and the European Microwave Week (EuMW) Technical Committee, and an European School of Antennas (ESoA) member. He has been appointed in the Scientific Board of the “Società Italiana di Elettromagnetismo (SIEmi)” and elected in the Scientific Board of the Interuniversity National Center for Telecommunications (CNIT). In 2011, he has been appointed by the National Agency for the Evaluation of the University System and National Research (ANVUR) as a member of the Recognized Expert Evaluation Group (Area 09, “Industrial and Information Engineering”) for the evaluation of the researches at the Italian University and Research Center for the period 2004–2010. Furthermore, he has been elected as the Italian Member of the Management Committee of the COST Action TU1208 “Civil Engineering Applications of Ground Penetrating Radar.” He is the Senior DIGITEO Chair at L2S-CentraleSupélec and CEA LIST, Saclay, France, and the University Carlos III of Madrid (UC3M)-Santander Chair of Excellence at UC3M, Spain. He was an IEEE Antennas and Propagation Society (AP-S) Distinguished Lecturer from 2016 to 2018. He served as an Associate Editor for the IEEE TRANSACTION ON ANTENNAS AND PROPAGATION from 2011 to 2014. He serves as an Associate Editor for the International Journal of Microwave and Wireless Technologies.

H-index = 54 [ISI-WoS]; and more than 22.000 citations—h-index = 87 [Google Scholar]