Research on WT-SVD Bilayer Filter Denoising for Downhole Signal

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Abstract. Aiming at the problem of singularity and mutation points after wavelet denoising with improved layered threshold when denoising the downlink signal received in the downhole in rotary steering drilling, a bilayer filtering denoising method based on singular value decomposition is proposed. This method combines wavelet denoising and singular value decomposition denoising, and adopts the WT-SVD bilayer filtering denoising method to construct a matrix of the wavelet improved layered threshold denoising signal, and then performs singular value decomposition and recovery on the matrix signal. The number of selected singular values and the structure of the reconstruction matrix were determined through experiments, and the signal waveforms, signal-to-noise ratio, root mean square error and correlation coefficients of three different denoising methods including singular value decomposition, wavelet improved layered threshold and WT-SVD bilayer filtering are compared and studied. The final experimental results show that the WT-SVD bilayer filtering denoising method is effective.

1. Introduction
In the rotary steering drilling system, the surface monitoring device uses a variable drilling fluid displacement method to send a control instruction to the downhole steering tool via a downward channel. The downhole signals constitute the control instructions in the form of "three drops and three rises" pulse width coding [1]. The downlink communication receiving device in the downhole steering tool obtains the corresponding change in the drilling fluid displacement by detecting and collecting the output voltage of the downhole mud motor. After identifying and interpreting the generation time and sequence of the pulse edges of the downlink signal and the pulse widths of the five instruction codes, an accurate downlink control command word is obtained [2]. Noise is generated due to the environmental interference during the transmission of the transmitted signal. Therefore, denoising the received downhole signal has become a key technology for obtaining control instructions.
The wavelet threshold denoising method proposed by Donoho proved that there is an optimal global threshold in the wavelet transform process, but the signal after denoising has delay and oscillation [3-4]. In addition, many research on the reasonable selection of thresholds and threshold functions shows that different thresholds and threshold functions have different effects on the denoising process [5-6]. The improved layered threshold denoising method is a more effective and flexible denoising method after improving the threshold function. Although this method basically removes the noise, it still fails to solve the problem of singularity and mutation points in the downhole received and transmitted signals, and these singular points and abrupt points will seriously affect the recognition of the pulse width signal [7].

2. WT-SVD bilayer filtering denoising method

WT-SVD (Wavelet Transform-Singular Value Decomposition) bilayer filtering denoising is a wavelet denoising method based on the singular value decomposition, which combines wavelet transform (WT) denoising and singular value decomposition (SVD) denoising to achieve the signal’s denoising requirements. The principle of denoising is as follows: wavelet denoising is performed on the downhole signal, and then the singular value decomposition method is used to further process the denoised signal.

2.1. Wavelet layered threshold denoising

Compared with the global threshold, the layered threshold denoising method is based on the correlation of the wavelet coefficients to determine the appropriate threshold denoising of each layer. It will not seriously "kill" the wavelet coefficients and the high-frequency part of useful signal, and can ensure the integrity of the signal [8]. Therefore, in view of the shortcomings of the soft threshold, hard threshold, and global threshold methods, and the differences in noise coefficients at different scales after wavelet decomposition, this paper uses an improved layered threshold denoising method in the first process of filtering and denoising to ensure smoothness after the threshold is applied transition [9]. The expression of the improved threshold function used in this method is shown in equation (1).

\[
\hat{w}_{(j,k)} = \begin{cases} 
\text{sgn}(w_{(j,k)}) \left\{ \frac{\lambda^2}{|w_{(j,k)}| + \lambda e^{\lambda^2/|w_{(j,k)}|^2}} \right\} & |w_{(j,k)}| \geq \lambda \\
0 & |w_{(j,k)}| < \lambda 
\end{cases}
\]

(1)

When the absolute value of the wavelet coefficient is less than the critical threshold, the wavelet coefficient is directly set to 0 as a conventional threshold function. When the absolute value of the wavelet coefficient is greater than or equal to the critical threshold, a new expression is used to shrink the wavelet coefficient. Figure 1 shows a comparison of soft threshold, hard threshold, and improved threshold functions.

![Figure 1. Comparison of soft, hard and improved threshold functions.](image)

The improved threshold function is between soft and hard threshold functions, and has continuity and flexibility in the wavelet domain, overcomes the discontinuity and reconstruction-induced
oscillations of the hard threshold function, and the constant deviation of the soft threshold function [10-11].

2.2. Singular value decomposition denoising

The singular value decomposition denoising method mainly reflects the correspondence between the information components of the signal and the singular values of the reconstruction matrix. Smaller singular values reflect components with less information in the signal, while larger singular values reflect components with larger amount of information [12]. Therefore, it is necessary to reasonably remove signal components that reflect small singular values and use large singular values to recover the original signal. It can not only filter out the noise interference, but also keep the characteristic information of the original signal from being destroyed, and finally obtain the denoised signal more accurately. The basic steps of singular value decomposition and denoising are:

(1) Constructing a matrix by using the downlink signal \( S = (s(1), s(2), s(3), ..., s(N)) \);

(2) Perform singular value decomposition on the matrix \( E \) to obtain \( U \in \mathbb{R}^{m \times m}, \Sigma \in \mathbb{R}^{m \times m} \) and \( V \in \mathbb{R}^{n \times n} \), then \( E = U \cdot \Sigma \cdot V^T \);

(3) Select a larger singular value for the singular value matrix, and obtain a processed matrix \( \Sigma^* \);

(4) Use the matrices \( U \in \mathbb{R}^{m \times m}, V \in \mathbb{R}^{n \times n} \) and \( \Sigma^* \) to reconstruct the matrix to obtain \( E^* = U \cdot \Sigma^* \cdot V^T \);

(5) A new data sequence \( S^* \) is constructed from all the elements in the first row of the matrix \( E^* \) and the elements in the second row from the \( n \)-th column to the \( m \)-th row and the \( n \)-th column, that is the signal after denoising.

The wavelet denoising method using only the improved layered threshold can basically remove the noise, but still cannot solve the problem of singularity and mutation points in the downhole received and transmitted signal. Therefore, this paper combines improved layered threshold denoising and singular value decomposition denoising, and performs bilayer filtering denoising to remove the remaining noise in the signal to achieve better denoising effect.

3. Key points of bilayer filtering denoising

The key steps in applying bilayer filtering denoising to signal processing to reduce signal noise include two aspects: selecting the number of non-zero singular values and determining the structure of the reconstruction matrix.

3.1. Selection of the number of singular values

Although the singular value decomposition method has certain denoising advantages, there are still some difficulties in selecting the number of singular values. If the number of selected singular values is large, there will still be a lot of noise in the denoised signal; on the contrary, when the number is small, the original signal will be destroyed, and cause signal distortion [13]. To this end, different schemes are used to select the number of singular values, such as: ① the characteristic mean method; ② the differential spectrum method; ③ the singular median method. Through a large number of experiments, it is concluded that when \( L \leq n \), \( p = L \); when \( L > n \), there is always \( p = 2n \). Among them, \( L \) represents the number of rows selected by the reconstruction matrix, \( n \) represents the number of dominant frequencies after one-dimensional signal fast Fourier transform, and \( p \) represents the number of singular values selected by the reconstruction matrix after singular value decomposition.

3.2. Determination of the reconstruction matrix structure

When selecting the number of matrix rows, the general method is to try it out based on specific signals, observe and analyze the characteristics of each set of singular value matrices, and expect to obtain a
satisfactory set of singular value matrix components [14]. This trial-and-error method requires a lot of calculations, and it is demanding on the signal analysis experience of researchers.

For the matrix \( E = U \cdot \Sigma \cdot V^T \) constructed by the downhole signals, the amount of information \( \delta_i \) contained in each component \( \Sigma_i \) signal obtained by the separation is determined by the corresponding singular value \( \sigma_i \). The smaller \( \sigma_i \), the smaller the corresponding information amount \( \Sigma_i \), where the information amount \( \Sigma_i \) contained in \( \delta_i \) can be calculated by equation (2).

\[
\delta_i = \frac{\sigma_i}{\sigma_1 + \sigma_2 + \cdots + \sigma_r}, \quad i = 1, 2, \ldots, r
\]

Signal components with relatively small amounts of information have little effect in practice, so the number of rows and columns of a matrix can be appropriately selected according to this measure.

The specific method of selecting the number of rows and columns of the matrix is as follows: Select different rows in turn to construct the matrix, use the singular value of the corresponding matrix to obtain the components of each information amount, and then analyze the connections and changes between the components. If it starts with a certain information component \( \delta_i \), and the subsequent information components gradually approach zero in order, the remaining components after the \( i \)-th information component have little effect, so the number of rows \( m = i \) of the matrix can be determined. The advantage of this method is that it does not need to calculate and analyze the amount of information of each component, and only uses singular values to judge. It greatly reduces the amount of calculation based on a comprehensive inspection of the information contained in all singular values. Finally, for the calculation of the number of columns, the continuous truncation method is \( n = \text{int}(N/m) \), and the Hankel matrix method is \( n = N - m + 1 \).

4. Experiment and result analysis

4.1. Determination of the number of singular values

Figure 2 is a group of signals received downhole, including a signal sequence of 512 sampling points, that is, \( N = 512 \). Perform fast Fourier transform on the signal sequence to obtain the number and distribution of the main frequencies. The spectrum of the signal is shown in Figure 3. The number of main frequencies of the signal can be obtained. The main frequency is the number of points indicated by the arrows in the figure, that is, the number of points with large amplitude.

![Figure 2. Downlink signal received downhole.](image-url)
The biggest advantage of singular value decomposition is not only the ability to remove noise points, but the more important point is to simplify the data. Based on the singular value matrix data results obtained by performing singular value decomposition on the transmitted signal, combining the characteristics of singular value decomposition, it is concluded that the singular values decay rapidly from large to small, and 256 singular values are obtained after decomposition. According to formula (2), it can be obtained that the sum of the singular values of the first 10% and even the first 1% accounts for more than 95% of the sum of the singular values. These singular values basically include the information contained in the signal.

Figure 4 is the signal-to-noise ratio (SNR) curve when the WT-SVD reconstruction matrix takes different singular values. It shows the trend of the SNR of the two noise reduction methods of singular value decomposition (SVD) and wavelet transform-singular value decomposition (WT-SVD). It can be seen that the signal-to-noise ratio curve is increasing with the number of singular values. When the number of singular values reaches 18, the upward trend is gradually slower. However, we cannot judge the denoising effect based on this criterion alone. Therefore, by selecting four representative singular values of 10, 18, 25, and 40, a waveform comparison experiment is performed on the denoised signal.

Figure 5 shows the comparison of signal waveforms after denoising with different singular values. It can be seen that the signal waveform is smoothest when the number of singular values is 18, and the singular points are basically removed. Combining Figures 4 and 5, it can be concluded that the denoising effect is best when the number of singular values is 18.
4.2. Determination of the reconstruction matrix structure

In the rotary steering drilling system, the number of sampling points \( N = 512 \) of the downstream signal is sampled. When using the Hankel matrix method to construct the matrix, the optimal number of rows should be 256 and the number of columns should be determined as 257 when considering each sampling point. The reason is that when the number of lines is \( L = 2,3,\ldots,255 \) in sequence, the subsequent sampling points in the signal will not be obtained, and its number is \( 512 - 2L \), which will eventually cause some useful information to be removed. When the number of lines is \( L = 257,258,\ldots,511 \), the number of signal sampling points will be insufficient. There are a total of \( 2L - 512 \) sampling points missing, and repeated acquisition of the signal sampling point values will result in information redundancy. Therefore, the denoising effect is best when the number of rows of the reconstruction matrix is half of the number of sampling points, and it is finally determined that the number of rows of the reconstruction matrix is \( L = 256 \).

4.3. Comparative experiment of multiple denoising methods

In order to verify the denoising effect of the WT-SVD bilayer filtering denoising method, the singular value decomposition (SVD) denoising method, the Sym6 wavelet improved layered threshold (WT) denoising method, and the wavelet transform-singular value decomposition (WT-SVD) denoising method have been compared with simulation experiments. Figure 6 shows the signal waveforms of the three methods after denoising.

From the signal waveform in Figure 6, it can be seen that the original signal is greatly affected by noise. Pure singular value decomposition and denoising cannot effectively remove noise. After Sym6 wavelet improved layered threshold denoising, most noise has been filtered out. However, there are still some singularities, indicating that the noise has not been completely eliminated. The singular value decomposition denoising method is used to denoise the signal after the improved layered threshold denoising, which basically eliminates all the noise contained in the signal, and the signal waveform is smoother and the distortion is smaller.

| Signal-to-noise ratio (SNR) | Root mean square error (RMSE) | Correlation coefficient \( \rho \) |
|-----------------------------|------------------------------|-------------------------------|
| SVD                        | 33.7227                      | 0.2601                        | 0.9910                        |

Table 1. SNR, RMSE and correlation coefficient of different noise reduction methods.
From the data in Table 1, it can be seen that the signal-to-noise ratio of the signal is 37.7109, the root mean square error is 0.1643, and the correlation coefficient is 0.9964 after WT-SVD bilayer filtering denoising. These three indicators have been improved compared to the previous two denoising methods, indicating that the WT-SVD bilayer filtering denoising method is suitable for the analysis and processing of the received signal downhole, and can achieve better denoising effects. The signals denoised by bilayer filtering can be more conducive to accurate identification of the three-drop and three-liter pulse widths.

5. Conclusions
This paper proposes a bilayer filtering denoising method based on singular value decomposition. By combining improved layered threshold denoising and singular value decomposition denoising, a WT-SVD bilayer filtering denoising method is obtained. After applying it to the actual downhole received signal, the improved layered threshold denoising is applied to the downlink signal, and then the matrix is constructed using Hankel matrix method. After determining the number of singular values and the structure of reconstruction matrix, the matrix is subjected to singular value decomposition and reconstruction to recover the signal. The experiment compares the WT-SVD bilayer filtering denoising method with other methods, and verifies that the method can effectively remove singularity and mutation points in noise. It has a better denoising effect and provides a guarantee for the coding identification of the downhole received signal.

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