Ludlum configuration for the measurement of mass attenuation coefficients at high energy photons by using Compton scattering method

M F Mohd Yusof¹,², N Mohd Isa², A B Abd Kadir², H Salleh²

¹School of Health Sciences, Universiti Sains Malaysia, 16150, Kelantan, Malaysia
²Medical Physics and Secondary Standard Dosimetry Laboratory, Malaysian Nuclear Agency, 43000 Selangor, Malaysia

Abstract. The measurements of mass attenuation coefficients by using Compton scattering technique was carried out by using the Ludlum configuration. A ¹³⁷Cs sealed source was used and attenuated at angles between 30 and 75° to provide scattered gamma energies between 337.72 and 564.09 keV. The mass attenuation coefficients of solid water and perspex were measured and compared to the calculated value of water by using XCOM. The results showed that the measured mass attenuation coefficients of solid water and Perspex® phantoms were in agreement to the values of water within 6.84 and 7.20% average percentage of discrepancies. The results indicated the suitability of the Ludlum configuration for the measurement of mass attenuation coefficients by using the Compton scattering method.

1. Introduction
The value mass attenuation coefficients become the most important parameter to determine the absorption and scattering characteristics collectively known as attenuation properties of a material towards ionizing radiations [1]. Different types of materials would have different values of mass attenuation coefficients as it is proportional to the mass density, effective atomic numbers and electron density of the materials. Therefore two materials are postulated to have similar attenuation properties towards ionizing radiations when the values of mass attenuation coefficients are similar.

The values of mass attenuation coefficients of materials can be theoretically calculated by using the photon cross section database known as XCOM [2]. This program enables the user to calculate the mass attenuation coefficients of known element and compounds at very wide range of photon energies and had been used by many researchers. The mass attenuation coefficients of a material at low energy photons on the other hand can be experimented based on the transmission of photons through the material based on the Beer-Lambert law [3-5]. A previous study suggested that the measurement of mass attenuation coefficients of materials at high energy photons can be measured by using the Compton scattering method by using a high energy gamma emitter such as ¹³⁷Cs [6]. This method provides better accuracy of measurement at experimented gamma energy ranges as the scattering of the incident scattering gamma to an attenuator is commonly inelastic at specific gamma energies.

The Ludlum configuration had been used for the measurement of lead equivalent of materials and several quality control measurements in the Malaysian Nuclear Agency (Nuklear Malaysia). A
previous study showed that the Ludlum configuration can be used for the measurement of mass attenuation coefficients at high energy photons using the transmission method [7]. Therefore, this study focused on the use of the Ludlum configuration for the measurement of mass attenuation coefficients at high energy photons by using the Compton scattering method.

2. Methodology

2.1. Experimental Set Up and System Calibration

The Ludlum configuration used in this study consists of the gas type Ludlum detector connected to a computer analysis and a $^{137}\text{Cs}$ sealed source that provided gamma peak energy of 662 keV. The $^{137}\text{Cs}$ sealed source was encapsulated in a lead container with collimation size of 0.1 cm to simulate the line source projection. An Al plate with approximate thickness of 0.1 cm was used as an attenuator to obtain the scattered photons. The Al plate was placed between the detector and the source at 20 cm distances from both the detector and the source as shown in Figure 1. The Ludlum Detector was placed at angles of $30^\circ$, $45^\circ$, $60^\circ$ and $75^\circ$ to measure the scattered gamma energies. The scattered gamma energies was calculated based on the previous work by Limkitjaroenporn et al. [6] by using the equation of,

$$E_{\gamma'} = \frac{E_\gamma}{1 + (1 - \cos \theta) E_\gamma/mc^2}$$  \hspace{1cm} (1)

where $E_\gamma$ and $E_{\gamma'}$ is the incident and scattered gamma energy respectively, $\theta$ is the angle of scattered gamma and $m$ is the electron rest mass [8][9]. This equation easily derived by assuming a relativistic collision between gamma ray and an electron initially at rest [6].

The energy calibration was performed on the Ludlum configuration to determine the linearity of signal in term of count per minute (CPM) of the detector at different photon energies. The calibration was performed by using $^{133}\text{Ba}$, $^{137}\text{Cs}$ and $^{60}\text{Co}$ sealed sources that provided gamma peak energies of 0.36, 0.662 and 1.3 MeV respectively.

![Figure 1](image_url)  \hspace{1cm} Figure 1. The experimental set up of the Ludlum configuration for the measurement of mass attenuation coefficients by using Compton scattering method.
Table 1. The scattered gamma energies at different angles of measurements.

| Scattering angle, $\theta$ | Incident gamma energy, $E_\gamma$ (keV) | Scattered gamma energy, $E'_\gamma$ (keV) |
|---------------------------|----------------------------------------|------------------------------------------|
| 0                         | 662                                    | 662.00                                   |
| 30                        |                                        | 564.09                                   |
| 45                        |                                        | 479.90                                   |
| 60                        |                                        | 401.76                                   |
| 75                        |                                        | 337.72                                   |

2.2 Experimented Materials

Two types of water equivalent phantom materials, the solid water and Perspex® phantoms were used in this study. The solid water phantoms is an epoxy resin-based materials commonly used for dosimetric and quality control measurements in radiotherapy. The Perspex® phantoms on the other hand is a methyl methacrylate-based materials commonly used as phantoms for diagnostic imaging. Both materials are made water equivalent with density close to water (1.0 g/cm$^3$). The elemental compositions, density and effective atomic number of solid water and Perspex® phantoms is presented in Table 2.

Table 2. The chemical compositions, density and effective atomic number of the phantom materials used in the study.

| Phantom      | Material              | Chemical composition          | Density (g/cm$^3$) | Effective atomic number, $Z_{\text{eff}}$ |
|--------------|-----------------------|------------------------------|--------------------|------------------------------------------|
| Solid water  | Epoxy resin           | C$_{21}$H$_{25}$ClO$_5$     | 1.00$^1$           | 7.35$^1$                                 |
| Perspex®     | Methyl Methacrylate   | (C$_5$O$_2$H$_8$)$_n$       | 1.18$^1$           | 6.48$^1$                                 |

$^1$Khan [8]

2.3 Measurement of Linear and Mass Attenuation Coefficients

The linear attenuation coefficient of the phantom samples was measured based on the transmission of photon through the samples based on the Beer-Lambert equation of,

$$I = I_0 e^{-\mu x}$$  \hspace{1cm} (2)

with $I_0$ and $I$ is the initial and transmitted photon respectively, $\mu$ is the linear attenuation coefficient of the sample medium and $x$ is the thickness of the sample medium. The linear attenuation coefficient can be calculated by rearranging Equation 2 into the equation

$$\mu = \frac{\ln I/I_0}{x}$$  \hspace{1cm} (3)

The mass attenuation coefficient, $\mu/\rho$ of the phantom materials can be calculated by dividing the value of linear attenuation coefficient with the density of the phantom material. The theoretical value of mass attenuation coefficients can be calculated by using the XCOM software. The molecular formula of the phantom materials and the range of photon energies were inserted into the calculation software and the tabulated data on the mass attenuation coefficients of the phantom materials were obtained. The theoretical values of mass attenuation coefficients of the phantom materials were compared to their measured values from the Ludlum configuration measurement.

3. Results and discussions

3.1. Calibration Curve of the System

The energy calibration curve of the Ludlum configuration is illustrated in Figure 2. The energy calibration graph showed an excellent linearity of signal detected by the Ludlum configuration shown
by the linear regression value \( R^2 = 0.999 \). The result indicated the suitability of the Ludlum set up for measurement of mass attenuation coefficients at high energy photons.

![Energy Calibration Curve](image)

**Figure 2.** The energy calibration curve of the Ludlum configuration at different photon energies

### 3.2. Linear and Mass Attenuation Coefficients of Materials using Compton Scattering Methods

The measured linear and mass attenuation coefficients of the solid water and Perspex® phantoms in comparison to their respective XCOM values at all experimented scattered gamma energies are presented in Table 3 and 4 respectively. The comparison of mass attenuation coefficients of solid water and Perspex® phantoms to their respective XCOM values are illustrated in Figure 3 and 4 respectively. The results showed that the linear and mass attenuation coefficients of both solid water and Perspex® phantoms decreased at increased photon [6][7]. The results also showed that the measured mass attenuation coefficients of the phantom materials were in agreement to their respective theoretical values measured using XCOM calculations within 2.72 and 4.35% percentage of differences all experimented scattered photon energies in solid water and Perspex® respectively. The measurement of mass attenuation coefficient at transmitted gamma energies (angle = 0°) also showed an agreement between the measured and theoretical values by XCOM within 4.47 and 6.18% percentage of differences by solid water and Perspex® respectively. The paired sample t-test was calculated between measured and theoretical values of mass attenuation coefficients and presented in Table 5. The results showed that there was no significant different between the theoretical values to the measured values of mass attenuation coefficients of the phantom samples. This indicated the consistency between the experimental values of the mass attenuation coefficients to the theoretical values.

| Energy | Solid water \( \mu/\rho \) (cm\(^2\)/g) | Solid water \( \mu \) (cm\(^{-1}\)) | \( \mu/\rho \) (cm\(^2\)/g) Percentage Difference (%) |
|--------|-----------------------------------|-----------------------------------|-----------------------------------------------|
| 662    | 0.067                             | 0.060                             | 0.070                                         | 4.47                                          |
| 564.09 | 0.086                             | 0.085                             | 0.085                                         | 1.16                                          |
| 479.90 | 0.094                             | 0.096                             | 0.096                                         | 2.13                                          |
| 401.76 | 0.103                             | 0.105                             | 0.105                                         | 1.94                                          |
| 337.72 | 0.107                             | 0.110                             | 0.110                                         | 2.72                                          |
Table 4. The mass attenuation coefficients of Perspex® phantoms measured using Ludlum configuration and XCOM.

| Energy (keV) | Perspex®<sub>XCOM</sub> (cm<sup>2</sup>/g) | Perspex®<sub>meas</sub> (cm<sup>2</sup>/g) | Percentage Difference (%) |
|-------------|---------------------------------|---------------------------------|--------------------------|
| 662         | 0.097                           | 0.107                           | 6.18                     |
| 564.09      | 0.103                           | 0.117                           | 3.88                     |
| 479.90      | 0.109                           | 0.124                           | 3.67                     |
| 401.76      | 0.115                           | 0.130                           | 4.35                     |
| 337.72      | 0.118                           | 0.135                           | 3.39                     |

Figure 3. The mass attenuation coefficients of the solid water phantoms by using the XCOM calculation and measurements.

Figure 4. The mass attenuation coefficients of Perspex® phantoms by using the XCOM calculation and measurements.
Table 5. The mass attenuation coefficients of solid water and Perspex® phantoms in comparison to the theoretical values by using XCOM.

| Paired samples       | Mean  | t    | df | Sig. (2-tailed) |
|----------------------|-------|------|----|-----------------|
| Solid water_{meas}-Solid Water_{XCOM} | 0.096 | 1.457 | 3  | 0.241           |
| Perspex_{meas}-Perspex_{XCOM}          | 0.111 | 17   | 3  | 0.040           |

4. Conclusion

The gamma energies reduced at increased scattered angles. The measured mass attenuation coefficients of the phantom materials decreased when measured at higher scattered gamma energies. The measured mass attenuation coefficients by using the Ludlum configuration by using the Compton scattering method showed good agreement to the theoretical values measured by using the XCOM software at all experimented gamma energies. The results indicated the suitability of the Ludlum configuration for the measurement of mass attenuation coefficients of materials by using the Compton scattering method.

5. References

[1] Cevik U and Baltas H 2007 Measurement of the mass attenuation coefficients and electron densities for BiPbSrCaCuO superconductor at different energies Nucl. Instr. Meth. Phys. Res. B 256 p 619-625.
[2] Berger M J and Hubbell J H. 1987 XCOM: Photon cross-sections on a personal computer, NBSIR, 1987, 87-3597 The Phys. of Radiation Therapy ed M F Khan (Philadelphia: Lippincott Williams & Wilkins, 4th ed 2010).
[3] Hill R F, Brown S, and Baldock C 2007 Evaluation of the water equivalence of solid phantoms using gamma ray transmission measurements. Radiat. Maes. 43 p 1258-1264.
[4] Shakhreet B Z, Bauk S, Tajuddin A A, and Shukri A 2009 Mass attenuation coefficients of natural Rhizophora spp. wood for X-rays of 15.77–25.27 keV range, Radiat. Prot. Dosim., 135 p 47-53.
[5] Marashdeh M W, Bauk S, Tajuddin A A, and Hashim R 2012 Measurement of mass attenuation coefficients of Rhizophora spp. binderless particleboards in the 16.59–25.26 keV photon energy range and their density profile using x-ray computed tomography. Appl. Radiat Isot. 70 p 656–662.
[6] Limkitjaroenporn P, Kaewkhao J, Chewpraditkul W, and Limsuwan P 2012 Mass attenuation coefficient and effective atomic number of Ag/Cu/Zn alloy at different energy by Compton scattering technique Procedia Eng. 32 p 847-854.
[7] Mohd Yusof M F, Tajuddin A A, Hashim R, Bauk S, Isa N M, and Md Isa M J 2017 Mass attenuation coefficients of tannin-added Rhizophora spp. particleboards at 16.59-25.26 keV photons, and 137Cs and 60Co gamma energies. Radiol. Phys. Technol. 10(3) p 331-339.
[8] Knoll G F 2000 Radiation Detection and Measurement (New York: John Wiley and Sons Inc. 3rd ed)
[9] Trousfanidis N 1983 Measurement and Detection of Radiation (New York: Hemisphere Publishing).

Acknowledgement

The author would like to thank the financial support by the Bridging Grant scheme no. 304.PPSK.6316145 by Universiti Sains Malaysia.