The inclusive radiative $B$-decay is a sensitive probe of new physics, especially if related to the virtual exchange of a charged Higgs boson. Supersymmetric models provide a particularly interesting example.

In the limit of exact supersymmetry, $\text{BR}(b \to s\gamma) = 0$, due to the vanishing of any magnetic-moment transition operator. We illustrate the impact of this constraint for realistic values of the supersymmetry-breaking parameters.
Recently it has been pointed out [1, 2] that the experimental upper bound on the branching fraction for the inclusive decay mode \( b \rightarrow s\gamma \), \( \text{BR}(b \rightarrow s\gamma) < 8.4 \times 10^{-4} \) at 90\% C.L. [3], imposes a severe constraint on the existence of a charged Higgs boson. Indeed the absence of large hadronic uncertainties in the theoretical prediction for the inclusive radiative \( B \)-decay allows one to extract a reliable limit on the charged Higgs mass, which depends only on \( \Lambda_{QCD} \) and the top-quark mass \( (m_t) \), and which is more stringent than the limits from \( B^0 - \bar{B}^0 \) mixing, \( K \) physics, or direct collider experiments.

It has also been argued [1] that the experimental bound on \( \text{BR}(b \rightarrow s\gamma) \) excludes large portions of the parameter space of the supersymmetric Higgs sector and, in particular, it pre-empts LEP-II searches. This conclusion has been reached by considering only the \( W \) boson and charged Higgs contribution to \( \text{BR}(b \rightarrow s\gamma) \), but neglecting the effects of the supersymmetric partners. In this letter, we want to show that the inclusion of these effects strongly modifies the result. This is due to the fact that, in the limit of exact supersymmetry, any magnetic moment-transition operator vanishes [4] and therefore \( \text{BR}(b \rightarrow s\gamma) = 0 \). As supersymmetry-breaking terms are turned on, the rate for the decay \( b \rightarrow s\gamma \) no longer vanishes, but it can still be considerably suppressed by the approximate cancellation. The possibility of having, in the supersymmetric case, a rate for \( b \rightarrow s\gamma \) lower than the standard model one was already pointed out in ref. [5], performing a general exploration of the parameter space. Here we show why and where such a suppression should particularly be expected.

The branching ratio for \( b \rightarrow s\gamma \), in units of the branching ratio for the semileptonic \( b \) decay, is:

\[
\frac{\text{BR}(b \rightarrow s\gamma)}{\text{BR}(b \rightarrow c\ell\bar{\nu})} = \frac{6\alpha}{\pi I(m_c/m_b)} \left[ \frac{\eta_{14} A_{\gamma} + \frac{8}{3}(\eta_{14}^2 - \eta_{16}^2) A_g + C}{1 - \frac{2}{3\pi} \alpha_s(m_b) f(m_c/m_b)} \right]^2,
\]

where \( \eta = \alpha_s(m_Z)/\alpha_s(m_b) \). In eq. (1) \( I \) is the phase-space factor, \( I(x) = 1 - 8x^2 + 8x^6 - 24x^4 \log x \), and \( f \) is the QCD correction factor, \( f(m_c/m_b) = 2.41 \) [3], for the semileptonic process, and \( C \) is a coefficient coming from operator mixing in the leading logarithmic QCD corrections computed in ref. [7] and given in the appendix. Finally \( A_{\gamma} \) and \( A_g \) are the coefficients of the effective operators for \( bs \)-photon and \( bs \)-gluon interactions:

\[
L_{\text{eff}} = G_F \sqrt{\frac{\alpha}{8\pi^3}} V_{tb} V_{ts}^* m_b \left[ A_{\gamma} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu} + A_g \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a \right] + \text{h.c.}
\]
The contributions to $A_\gamma$ and $A_g$ from the exchange of $W$ bosons \[8\], charged Higgs bosons \[9\], and supersymmetric particles \[5\] have all been computed already, and we present them in the appendix. Here we want to single out the effect of the supersymmetric contribution, but avoid the dependance on many unessential unknown parameters. We therefore consider the limit in which: (i) the diagonal gaugino and higgsino mass terms are neglected; (ii) the two Higgs vacuum expectation values are equal ($\tan \beta = 1$); (iii) all squarks, other than the scalar partners of the top quark, have the same mass $\tilde{m}$; (iv) the supersymmetry-breaking trilinear coupling $A$ is zero, and therefore the two scalar partners of the top quark have mass $m_t = \tilde{m}^2 + m_W^2$.

Assumptions (i), (ii), and (iv) do not sizeably affect our results, as discussed in the appendix, but allow a great simplification of their description. In particular the value of $\tan \beta$ is irrelevant for the Higgs contribution \[1, 2\] and for the chargino contributions (see appendix), as long as $\tan \beta \geq 1$, which is always the case in the interesting supersymmetric models. With this choice of parameters, the two charginos are exactly degenerate with the $W$ boson. Assumption (iii) corresponds to a hypothesis of minimal flavor violation, which means that the Yukawa couplings are the only source of flavor non-conservation. The authors of ref. \[5\] also studied the possibility of flavor violation caused by a misalignment of squarks and quarks, after renormalization effects are taken into account, which gives rise to gluino-mediated flavor-changing neutral currents. We ignore here these effects, which are strongly model-dependent and usually rather small, if one considers flavor-symmetric boundary conditions for the squark mass matrices at the unified scale.

Under the above-stated hypothesis, the coefficients $A_\gamma$ and $A_g$ become:

$$A_{\gamma,g} = \frac{3}{2} x f_{\gamma,g}^{(1)} + \frac{y}{2} \left[ f_{\gamma,g}^{(1)}(y) + f_{\gamma,g}^{(2)}(y) \right] + z \left[ f_{\gamma,g}^{(1)}(z) + \frac{1}{2} f_{\gamma,g}^{(2)}(z) \right]$$

$$- (2x + z) f_{\gamma,g}^{(1)}(x + z) - \frac{(x + z)}{2} f_{\gamma,g}^{(2)}(x + z), \quad (3)$$

$$x \equiv \frac{m_t^2}{m_W^2}, \quad y \equiv \frac{m_t^2}{m_H^2}, \quad z \equiv \frac{\tilde{m}^2}{m_W^2}, \quad (4)$$

where $m_H$ is the charged-Higgs mass and the functions $f_{\gamma,g}^{(1,2)}$ are defined in the appendix. Equation (3) exhibits exact cancellation as we approach the supersymmetric limit $z \to 0, y \to x$. 

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Figure 1 shows the numerical evaluation of eq. (1), with $A_{\gamma,g}$ as in eq. (3) and $\alpha_s(m_Z) = 0.118$, $\text{BR}(b \to c e \bar{\nu}) = 10.7\%$, $m_b = 4.8$ GeV, $m_c/m_b = 0.3$.

The chargino contributions to this rate die out as $\tilde{m}$ becomes large. Because of negative interference of these same contributions, the reduction of the rate, for any given charged-Higgs mass, is apparent from the figures. Notice that there also is a large portion of parameter space where $\text{BR}(b \to s \gamma)$ is actually smaller than the standard model prediction. The vanishing of the rate in the supersymmetric limit would occur in these contour plots for $\tilde{m} = 0$ and $m_H = m_W$. As discussed in the appendix, the results shown in fig. 1 for special values of the supersymmetric parameters are representative of a significantly larger portion of the parameter space, whenever $\tilde{m} \gtrsim 200$ GeV.

We infer from the existing literature that the uncertainty in the prediction of $\text{BR}(b \to s \gamma)$ coming from the uncertainties in the values of $\alpha_s(m_Z)$ [7] and in higher-order QCD corrections [10] is less than about 25\%. This point is important enough to deserve further studies.

The expected improvement in the experimental bound on $\text{BR}(b \to s \gamma)$ will undoubtedly provide very useful information on the allowed supersymmetric parameter space. It is very important to realize that a significant deviation, in either direction, from the standard model prediction may indeed occur [5].

We thank A. Masiero for useful comments.

Appendix

In this appendix we present the coefficients $A_{\gamma}, A_{g},$ and $C$, which appear in eq. (1).

We assume all squarks, other than the partners of the top quark, to be degenerate with mass $\tilde{m}$. The $2 \times 2$ top squark mass matrix is diagonalized by an orthogonal matrix $T$ such that:

$$
T \begin{pmatrix}
\tilde{m}_1^2 + m_t^2 & A \tilde{m} m_t \\
A \tilde{m} m_t & \tilde{m}_2^2 + m_t^2
\end{pmatrix} T^{-1} = \begin{pmatrix}
\tilde{m}_{t1}^2 & 0 \\
0 & \tilde{m}_{t2}^2
\end{pmatrix},
$$

where $A$ is the supersymmetry-breaking trilinear coupling. Defining $M$ to be the weak gaugino mass and $\mu$ the Higgs mixing parameter, the chargino mass matrix is diagonalized by two unitary $2 \times 2$ matrices $U$ and $V$, according to:

$$
U^* \begin{pmatrix}
M & m_W \sqrt{2} \sin \beta \\
m_W \sqrt{2} \cos \beta & \mu
\end{pmatrix} V^{-1} = \begin{pmatrix}
\tilde{m}_{\chi1}^2 & 0 \\
0 & \tilde{m}_{\chi2}^2
\end{pmatrix}.
$$
The contributions to $A_{\gamma,g}$ from $W$, charged Higgs, and charginos are respectively:

\[ W : \quad A_{\gamma,g} = \frac{3}{2} \frac{m_t^2}{m_W^2} \left( \frac{m_t^2}{m_W^2} \right) \]
\[ (7) \]

\[ H : \quad A_{\gamma,g} = \frac{1}{2} \frac{m_t^2}{m_H^2} \left[ \frac{1}{\tan^2 \beta} f_{\gamma,g}^{(1)} \left( \frac{m_t^2}{m_H^2} \right) + f_{\gamma,g}^{(2)} \left( \frac{m_t^2}{m_H^2} \right) \right] \]
\[ (8) \]

\[ \tilde{\chi} : \quad A_{\gamma,g} = \sum_{j=1}^{2} \left( \frac{m_W^2}{\tilde{m}_{\chi_j}^2} \right) \left| V_{j1} \right|^2 \left( \frac{\tilde{m}_{t_k}^2}{\tilde{m}_{\chi_j}^2} \right) \left[ f_{\gamma,g}^{(1)} \left( \frac{\tilde{m}_{t_k}^2}{\tilde{m}_{\chi_j}^2} \right) \right] - \sum_{k=1}^{2} \left( V_{j1} T_{k1} - V_{j2} T_{k2} m_t \sqrt{2 m_W \sin \beta} \right)^2 \left( \frac{\tilde{m}_{t_k}^2}{\tilde{m}_{\chi_j}^2} \right) \]
\[ - \frac{U_{j2}}{\sqrt{2 \cos \beta \tilde{m}_{\chi_j}}} \left( V_{j1} f_{\gamma,g}^{(3)} \left( \frac{\tilde{m}_{t_k}^2}{\tilde{m}_{\chi_j}^2} \right) - V_{j2} \right) T_{k1} f_{\gamma,g}^{(3)} \left( \frac{\tilde{m}_{t_k}^2}{\tilde{m}_{\chi_j}^2} \right) \], \]
\[ (9) \]

where:

\[ f_{\gamma}^{(1)}(x) = \frac{7 - 5x - 8x^2}{36(x - 1)^3} + \frac{x(3x - 2)}{6(x - 1)^4} \log x \]
\[ (10) \]

\[ f_{\gamma}^{(2)}(x) = \frac{3 - 5x}{6(x - 1)^2} + \frac{(3x - 2)}{3(x - 1)^3} \log x \]
\[ (11) \]

\[ f_{\gamma}^{(3)}(x) = (1 - x) f_{\gamma}^{(1)}(x) - x \left( \frac{3}{2} f_{\gamma}^{(2)}(x) - \frac{23}{36} \right) \]
\[ (12) \]

\[ f_{g}^{(1)}(x) = \frac{2 + 5x - x^2}{12(x - 1)^3} - \frac{x}{2(x - 1)^4} \log x \]
\[ (13) \]

\[ f_{g}^{(2)}(x) = \frac{3 - x}{2(x - 1)^2} - \frac{1}{(x - 1)^3} \log x \]
\[ (14) \]

\[ f_{g}^{(3)}(x) = (1 - x) f_{g}^{(1)}(x) - x \left( \frac{3}{2} f_{g}^{(2)}(x) - \frac{1}{3} \right) \]
\[ (15) \]

In fig. 1 we have considered the limit in which $M, \mu = 0$, $\tan \beta = 1$, and $A = 0$. This implies:

\[ U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad V = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \tilde{m}_{\chi_1,2} = m_W, \]
\[ (16) \]
\[
T = \begin{pmatrix}
1 & 0 \\
0 & 1 
\end{pmatrix}, \quad \tilde{m}_{t_{1,2}}^2 = \tilde{m}^2 + m_t^2,
\]
and eqs. (7)–(9) reduce to eq. (3).

One can then consider the case \( A \neq 0 \), holding \( M = \mu = 0 \). This corresponds to
\[
T = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 \\
-1 & 1 
\end{pmatrix}, \quad \tilde{m}_{t_{1,2}}^2 = \tilde{m}^2 + m_t^2 \pm A\tilde{m}m_t,
\]
and the chargino contribution to \( A_{\gamma,g} \), eq. (9), reduces to:
\[
\bar{\chi} : \quad A_{\gamma,g} = z f_{\gamma,g}^{(1)}(z) + \frac{z}{2} f_{\gamma,g}^{(2)}(z) - \sum_{k=1}^{2} \left[ \frac{(x + w_k)}{2} f_{\gamma,g}^{(1)}(w_k) \frac{(w_k)}{4} f_{\gamma,g}^{(2)}(w_k) \right].
\]

We have verified that the inclusion of \( A \neq 0 \), as in eq. (19), does not sizeably modify the results presented in fig. 1, at least for \(|A| < 2\). For \(|A| > 2\), small values of \( \tilde{m} \) are forbidden by the condition \( \tilde{m}_{t_{1,2}}^2 > 0 \).

Next we consider the case \( M = \mu = A = 0 \), and arbitrary \( \tan \beta \). The chargino contribution, eq. (9), becomes:
\[
\bar{\chi} : \quad A_{\gamma,g} = -\frac{x + z}{4 \cos^4 \beta} \left[ f_{\gamma,g}^{(1)} \left( \frac{x + z}{2 \cos^2 \beta} \right) + \frac{1}{2} f_{\gamma,g}^{(2)} \left( \frac{x + z}{2 \cos^2 \beta} \right) \right] \quad (20)
+ \frac{z}{4 \cos^4 \beta} \left[ f_{\gamma,g}^{(1)} \left( \frac{z}{2 \cos^2 \beta} \right) + \frac{1}{2} f_{\gamma,g}^{(2)} \left( \frac{z}{2 \cos^2 \beta} \right) \right] - \frac{x}{4 \sin^4 \beta} f_{\gamma,g}^{(1)} \left( \frac{x + z}{2 \sin^2 \beta} \right).
\]
It is easy to verify that eq. (20) has only a very weak (logarithmic) dependence on \( \tan \beta \) in the limit of large \( \tan \beta \). We have also numerically checked that the results shown in fig. 1 are not greatly affected by values of \( \tan \beta > 1 \), as long as \( \tilde{m} \gtrsim 200 \) GeV.

Finally one can also consider the case for which \( M \) and \( \mu \) are different from zero. From the explicit form of eq. (9), it is easy to see that the corrections to this amplitude with respect to eq. (3) are of relative order \( M/\tilde{m} \) or \( \mu/\tilde{m} \), with the dominant contribution coming from the helicity-flipped term in the gaugino-higgsino line.

We conclude that the particularly simple case illustrated in fig. 1 is representative of a significantly larger portion of parameter space.

Finally the coefficient \( C \) can be derived from ref. [4], which contains a complete calculation of the leading logarithmic QCD corrections to the
effective $bs\gamma$ interactions. Including the effect of four-quark operators for on-shell photons, we obtain:

$$C = \sum_{i=1}^{8} b_i \eta^{d_i}$$  \hspace{1cm} (21)

$$b = (-0.69, 1.55, -0.20, -0.04, -0.10, -0.02, -0.43, -0.07)$$
$$d = (0.696, 0.609, 0.409, -0.423, 0.146, -0.899, 0.261, -0.522),$$

where $\eta = \alpha_s(m_Z)/\alpha_s(m_b) = 0.548$. 
Figure caption

Fig. 1: Contour plots of BR($b \rightarrow s\gamma$) in the plane ($m_H, \tilde{m}$) of the charged Higgs mass $m_H$ and the common squark mass $\tilde{m}$, for $m_t = 120$ (1a), 150 (1b), and 180 GeV (1c), under the assumptions stated in the text ($M = \mu = A = 0$, tan $\beta = 1$, see appendix). The present 90% C.L. bound, BR($b \rightarrow s\gamma$) $< 8.4 \times 10^{-4}$ [3], and the standard model prediction, for any given $m_t$, are shown by solid lines.

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