Analysis and correction of geometrical error-induced pointing errors of a space laser communication APT system

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ABSTRACT

The geometrical error caused pointing error is an inevitable problem in space satellite laser communication terminals which can affect the pointing accuracy of the APT (acquisition pointing and tracking) system greatly, in order to facilitate the assembling of the APT system and improve the performance of the laser communication system, the geometrical error sensitivity about the APT pointing accuracy is analyzed based on multi-body kinematics method in this paper, the error transformation matrix is derived and the geometrical error is analyzed, the simulation results provide some pointing error distribution regulars which are conductive to assembling. Based on the above research, the geometrical error correction experiment is performed and the pointing accuracy of the APT system is tested, the expectation value of the pointing error can reach 29.9 $\mu$rad which is greatly improved. This research can provide technical references for the design and analysis of space laser communication terminals.

KEYWORDS

Pointing accuracy; geometrical error; multi-body kinematics; space laser communication

1. Introduction

Due to its advantages of higher-capacity communication rate, stronger anti-interference ability, cost-effectiveness, longer link distance at lower size, weight and power terminal, etc., satellite optical communication (SOC) has become a promising scheme that has been deeply studied by scholars all over the world.\textsuperscript{[1–3]} Since the satellite laser communication distance is very long, so the beam used in SOC system is very narrow, the beam angle can be 0.1\textendash 0.01\% of the microwave communication radiation which is approximately tens of microradians,\textsuperscript{[4]} therefore, high pointing accuracy is necessary to ensure the link performance such as acquisition probability, low bit error rate, stable link tracking, etc.

Generally, the APT (acquisition, pointing and tracking) system is utilized to control the beam pointing direction. Different APT have different advantages, the periscope-type APT is used in LCT (laser communication terminal) for the excellent astigmatism elimination performance.\textsuperscript{[5]} In Japan’s next-generation LEO (low earth orbit) system, the U-shape frame is utilized in SOTA (small optical transponder).\textsuperscript{[6]} A L-type APT terminal is adopted in the LLST (lunar laser space terminal) system for the advantages of small size and light weight.\textsuperscript{[7]} However, in these APT systems, the pointing error of the optical axis induced by geometrical error is usually ineluctable such as unperpendicularity error of azimuth and elevation axis, rotation error, lens installation error, etc. Many scholars pay attention to geometrical error-induced pointing error in previous
studies. Wu et al.\cite{8} analyzed pointing error vary regular of periscope-type APT system azimuth and elevation axes based on geometric optical method. Yan et al.\cite{9} analyzed the pointing error of the theodolite type APT. For the pointing error induced optical energy attenuation, Bai et al.\cite{10} proposed a spot prediction method. For the 45° folding mirror of the periscope-type APT system, Song et al.\cite{11} analyzed the thermal deformation-induced pointing error. Ding et al.\cite{12} studied gravity-induced deformation of the T-type APT system. During the assembling process we need to eliminate the geometrical errors as far as we can, but different errors have different effects on beam pointing accuracy in a different type of APT. In order to assemble the APT system effectively, it is necessary to correct the geometrical errors according to its pointing accuracy sensitivity; however, this question gains little attention in previous studies.

Mainly pointing accuracy analysis methods for the beam pointing equipment are geometric optical vector method, spherical function model method and multi-body method.\cite{13,14} The geometric optical method is used in the pointing analysis of LCT, this method is appropriate for small error system which needs to build complex vector transfer matrix.\cite{15} The spherical function model method is used to correct the static error of the horizon telescope which needs to build complex spherical function model.\cite{16} The multi-body method from the view point of the effect of body position errors on pointing error vary regular,\cite{17} by establishing the functionality structure of the beam pointing system, then according to the relative motion between the two topological bodies to deduce out the error transformation matrix.

In previous studies, most scholars focused on the influence regularity of the dynamic error and static error on the pointing error, or high sensitivity detector\cite{18} but there are few studies focus on the error sensitivity analysis. Motivated by improving pointing accuracy of the APT system of the space laser communication terminal, aiming at the requirements of real project, the error transformation matrix of the L-type APT is established and the geometrical error sensitivity about pointing accuracy is analyzed based on multi-body method in this paper, then the correction experiments is performed. The results showed that the geometrical error is effectively corrected by the established analytical model. This study can provide some guidance for design, manufacturing and assembling process of the APT system.

The paper is organized as follows. The system introduction is shown in Section 2. The multi-body modeling and geometrical error sensitivity analysis is derived in Section 3. The correction experiment is performed in Section 4. At last, the conclusion is given in Section 5.

2. Introduction of the APT system

2.1. Coordinate description

The APT system performs two basic functions: pointing to the counter terminal and redirecting light from the counter terminal into the surface of the receive detector, here we take the former case as research target. According to actual engineering problems the L-type APT is adopted as

| Nomenclature |
|--------------|
| APT          | Acquisition, pointing and tracking |
| SOC          | Satellite laser communication       |
| LCT          | Laser communication terminal        |
| LEO          | Low earth orbit                     |
| SOTA         | Small optical transponder            |
| LLST         | Lunar laser space terminal          |
| FORF         | fixed optical reference frame        |
| AZORF        | Azimuth optical reference frame      |
| ELORF        | Elevation optical reference frame    |
| STE          | Static error                         |
| DYE          | Dynamic error                        |
| DAQ          | Data acquisition                     |
| IFOV         | Instantaneous field of view          |
| CCD          | Charge coupled device                |
| LCE          | Laser communications equipment       |
| LUCE         | Laser utilizing communications equipment |
the research target in this paper. The coordinate system of the L-type APT used in our project is shown in Figure 1.

The L-type APT incorporates five parts which are as follows: Housing, azimuth axis, folding mirror, elevation axis, and the telescope. The Housing which performs the functions of compression, collimation or expand the laser beam is fixed on the gas floating platform, its coordinate $o$–$XYZ$ is fixed which is regarded as the fixed optical reference frame (FORF), the outgoing laser parallels to axis $o$–$Z$. The coordinate system of $a$–$X_AzY_AzZ_Az$ is the optical reference frame of azimuth (AZORF), point $a$ is the centroid of azimuth axis and $a$–$Z_Az$ parallels to the laser direction if there's no axis assemble error. The coordinate system of $p$–$X_My_Mz_M$ is the optical reference frame of the folding mirror, point $p$ is the centroid of the ellipse mirror, the folding mirror can be regarded as part of a multi-body system, the angle between its normal and $p$–$X_Mt$ is $45^\circ$. The fourth body is elevation axis, the coordinate $e$–$X_ElY_ElZ_El$ is the elevation optical reference frame (ELORF) and $e$–$X_El$ parallels to elevation axis. The last coordinate system $t$–$X_TeY_TeZ_Te$ is the optical reference frame of telescope and $t$–$Z_Te$ parallels to the send out laser direction if there is no geometrical error exists, point $t$ is the centroid of the telescope.
2.2. The pointing error illustrated

The pointing error is the angle difference between the location where the telescope is pointing and the actual direction it points to. The illustration of the pointing error is shown in Figure 2, the vector of pointing errors is given as \( \Delta A \) (Azimuth) and \( \Delta E \) (Elevation) which can be expressed as the following function

\[
\begin{align*}
\Delta A &= A' - A = f(A,E) + \varepsilon \\
\Delta E &= E' - E = g(A,E) + \eta
\end{align*}
\]

where \( \Delta A \) and \( \Delta E \) are the pointing errors in the azimuth and elevation axis, \( A' \) and \( E' \) are the ideal pointing direction in azimuth axis and elevation axis, \( A \) and \( E \) are the actual beam pointing direction, \( f(A,E) \) and \( g(A,E) \) are functional representations of the rotation model of the azimuth and elevation axis, \( \varepsilon \) and \( \eta \) are the errors associated with assembling.

3. Pointing error modeling and simulation

3.1. Modeling of the multi-body functional structure

In order to visualize the abstract structures of the terminal, we need to refine out its functionality structure, then we can describe the motion relationship between the bodies. Huston et al.\cite{19} utilize the low-order body array method to describe the relationship. According to the natural growth of the body order to assign its number and order, then this method builds the coordinate transformation model based on the relative motion of adjacent low-order bodies, after that we can obtain the motion relationship between any two low-order bodies.

The motion relationship of two adjacent bodies in the topological structure is shown in Figure 3. \( B_1 \) and \( B_2 \) are the adjacent moving bodies, \( O_1 \) and \( O_2 \) are the coordinates of the actual position of \( B_1 \) body and \( B_2 \) body, \( O_{m1} \) and \( O_{m2} \) are the ideal position and actual position of the moving reference coordinate respectively, \( O'_2 \) is the ideal position of the \( B_2 \) body coordinate. Then \( P'_2 \), \( P_2 \) and \( P_{2e} \) represent the ideal position vector, actual position vector and the position error vector of the moving reference coordinate of \( B_1 \) body respectively. \( S'_2 \), \( S_2 \) and \( S_{2e} \) denote the ideal moving vector, actual moving vector and moving error vector of \( B_2 \) body respectively.

The position/attitude matrix of \( B_2 \) to \( B_1 \) can be expressed as a 4 × 4 homogeneous matrix \( _1^2 T \) which is the product of four matrix: attitude transformation matrix, attitude error matrix, position transformation matrix and position error matrix. Assuming that \( _1^2 T_s \), \( _1^2 T_{se} \) denote the attitude transformation matrix and attitude error matrix, \( _1^2 T_p \) and \( _1^2 T_{pe} \) denote the position transformation matrix and position error matrix of \( B_2 \) to \( B_1 \). Then the transformation matrix \( _1^2 T \)
can be written as
\[ \mathbf{T}^{21} = \mathbf{T}^{p2} \mathbf{T}^{p0} \mathbf{T}^{e2} \mathbf{T}^{se} \] (2)

Assuming that there’s no error exist in the motion between the two adjacent bodies, \( \mathbf{T}^{21} \) can be expressed as
\[ \mathbf{T}^{21}(0) = \mathbf{T}(e = 0) = \mathbf{T}^{p2} \mathbf{T}^{e} \] (3)

According to the structure of the APT system in Figure 1, its topological structure incorporates four low-order bodies and a zero-order body. Because the Housing is fixed to the gas floating platform (which is fixed to the ground), so the Housing’s order is zero. Then arrange the remaining four bodies in order: azimuth axis corresponding body 1, 45° folding mirror corresponding body 2, elevation axis corresponding body 3, and the telescope corresponding body 4, the schematic diagram is shown in Figure 4.

Assuming the vector of the laser sent out from the laser diode from Housing is \( \mathbf{r} \), its vector is \([0,0,1,0]^T\) in homogeneous coordinates according to Figure 1, and the vector of the laser from the telescope axis is \( \mathbf{r}' \), then \( \mathbf{r}' \) can be expressed as the transformation of the position/attitude matrix of the five adjacent bodies on vector \( \mathbf{r} \), according to Equation (2), this process can be expressed as
\[ \mathbf{r}' = \mathbf{T}^{31} \mathbf{T}^{21} \mathbf{T}^{10} \mathbf{r} \] (4)

and according to Equation (3) the pointing vector \( \mathbf{r}' \) which without geometrical errors is
\[ \mathbf{r}' = \mathbf{T}^{0} \mathbf{r} \] (5)

then the pointing error \( \Delta \mathbf{r} \) can be written as
\[ \Delta \mathbf{r} = \mathbf{r}' - \mathbf{r} \] (6)

3.2. Geometrical error analysis

Verticality error and rotation error of the optical element or axis will affect the accuracy of the optical axis. For a turntable-type APT system the main errors are static error (verticality error) and dynamic error (rotation error), these errors will affect the pointing direction of the telescope, meanwhile, for the folding mirror the error will affect the optical axis pointing direction. Each of these errors can be characterized as several transformations of the coordinate system.
Shown in Figure 5 is an illustration of the errors. In Figure 5(a), the static errors of azimuth axis are \( \delta_{ax} \) for which coordinate system \( a\text{-}XAzYAzZAz \) turn \( \delta_{ax} \) degree around axis \( a\text{-}X \) to coordinate \( o\text{-}XYZ \) and \( \delta_{ay} \) for which coordinate system \( a\text{-}XAzYAzZAz \) turn around \( \delta_{ay} \) degree around axis \( a\text{-}Y \) to coordinate \( o\text{-}XYZ \). The dynamic errors of azimuth axis are \( \alpha_a \) for which coordinate system \( a\text{-}XAzYAzZAz \) turn \( \alpha_a \) degree around axis \( a\text{-}XAz \) and \( \beta_a \) for which coordinate system \( a\text{-}XAzYAzZAz \) turn \( \beta_a \) degree around axis \( a\text{-}YAz \) and rotational positioning error \( \gamma_a \) for which coordinate system \( a\text{-}XAzYAzZAz \) turn \( \gamma_a \) degree around axis \( a\text{-}ZAz \). Assuming that the turntable turns \( \theta_{Az} \) degree around azimuth axis, then, according to the above analysis the transformation matrix of the azimuth axis can be expressed as

\[
1\text{e}T(e) = \text{Rot}(X, \delta_{ax}) \cdot \text{Rot}(Y, \delta_{by}) \cdot \text{Rot}(Z, \theta_{Az}) \cdot \text{Rot}(X_{Az}, \alpha_a) \cdot \text{Rot}(Y_{Az}, \beta_a) \cdot \text{Rot}(Z_{Az}, \gamma_a)
\]  
(7)

where \( \text{Rot}(X_{Az}) \) represents the rotation transformation around axis \( X_{Az} \) which can be derived out through coordinate transformation matrix.\(^{[20]}\)

\[
\text{Rot}(x) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(x) & -\sin(x) & 0 \\
0 & \sin(x) & \cos(x) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\]  
(8)
Then, the static induced error transformation matrix of the telescope can be expressed as:

\[
Rot(y) = \begin{bmatrix}
\cos(y) & 0 & \sin(y) & 0 \\
0 & 1 & 0 & 0 \\
-\sin(y) & 0 & \cos(y) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\]

(9)

\[
Rot(z) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(z) & -\sin(z) & 0 \\
0 & \sin(z) & \cos(z) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

(10)

Common sense shows us that the translation of the axis does not introduce angular errors, so the displacement error is ignored in Equation (7). The main errors of 45° folding mirror are slope angle error \(\beta_{py}\) and wedge angle error \(\gamma_{px}\). The slope angle error can be described as the deviation of the 45° mirror turns \(\beta_{py}\) degree around \(Y_{M1}\) and the wedge angle error can be described as the mirror turns \(\gamma_{px}\) degree around its semi-major axis. According to the plane mirror reflection transmission matrix, the transformation matrix of the mirror performs on the optical axis can be expressed as:

\[
2^T_i(e) = \begin{bmatrix}
2\beta_{py} & -\sqrt{2}\gamma_{py} & 1 & 0 \\
-\sqrt{2}\gamma_{py} & 1 & \sqrt{2}\gamma_{py} & 0 \\
1 & \sqrt{2}\gamma_{py} & -2\beta_{py} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(11)

Seen in Figure 5(b) the static error of elevation axis is \(\delta_{cy}\) for which coordinate system \(e-X_{El}Y_{El}Z_{El}\) turn \(\delta_{ey}\) degree around axis \(p-Y_{M}\) (if there is no error exists in 45° mirror) to coordinate \(p-X_{M}Y_{M}Z_{M}\) and \(\delta_{ez}\) for which coordinate system \(e-X_{El}Y_{El}Z_{El}\) turn \(\delta_{ez}\) degree around axis \(p-Z_{M}\) to coordinate \(p-X_{M}Y_{M}Z_{M}\). The dynamic errors of the elevation axis are \(\alpha_e\) for which coordinate system \(e-X_{El}Y_{El}Z_{El}\) turn \(\alpha_e\) degree around axis \(e-X_{El}\) and \(\beta_{a}\) for which coordinate system \(e-X_{El}Y_{El}Z_{El}\) turn \(\beta_{a}\) degree around axis \(e-X_{El}\) and rotational positioning error \(\gamma_{a}\) for which coordinate system \(e-X_{El}Y_{El}Z_{El}\) turn around \(\gamma_{a}\) degree around axis \(e-X_{El}\). Assuming that the turntable turns \(\theta_{El}\) degree around elevation axis, then, based on above analysis the elevation axis transformation is:

\[
3^T_2(e) = Rot(Y, \delta_{cy}) \cdot Rot(Z, \delta_{cz}) \cdot Rot(X, \theta_{El}) \cdot Rot(X_{El}, \alpha_e) \cdot Rot(Y_{El}, \beta_a) \cdot Rot(Z_{El}, \gamma_a)
\]

(12)

The telescope static errors are induced by assembling errors, these errors can be described as the non-perpendicularity between telescope optical axis and the elevation shaft. Seen in Figure 5(c) is the illustration of the telescope static error. \(\delta_{tx}\) is the telescope static error angle which coordinate system \(e-X_{El}Y_{El}Z_{El}\) turn around the axis \(e-X_{El}\) and \(\delta_{ty}\) is the error angle which coordinate system \(e-X_{El}Y_{El}Z_{El}\) turn around the axis \(e-Y_{El}\) and the final coordinate system of the telescope is \(t-X_{Y_{t}}Z_{t}\). Then, the static induced error transformation matrix of the telescope can be expressed as:

\[
4^T_3(e) = Rot(Y_{El}, \delta_{ty}) Rot(X_{El}, \delta_{tx})
\]

(13)

Based on the above analysis, the sending out beam vector \(r'\) and \(r''\) can be derived out based on Equations (4)–(10). Then according to Figure 2 the pointing error angle of azimuth axis and elevation axis can be written as:

\[
\begin{align*}
\Delta A &= a \tan^{-1} \left( \frac{\Delta r_x}{\Delta r_z} \right) \\
\Delta E &= a \tan \left( \frac{\Delta r_y}{\Delta r_z} \right)
\end{align*}
\]

(14)
3.3. Sensitivity analysis

Due to the diversity of errors, it brings a lot of inconvenience to assembling process of the APT system, for the sake of research period and improving the accuracy, it is necessary to analyze the geometrical error sensitivity of the space pointing equipment, so we can assemble the APT effectively. The geometrical error sensitivity can be defined as the variation of the pointing error with the increase of the unit geometrical error. This can be written as

\[ S = \int \int \frac{\partial \Delta \theta}{\partial e_k} d\Omega/\Omega(k = 1, 2, 14), \tag{15} \]

where \( S \) denotes the geometrical error sensitivity, \( \Delta \theta \) denotes the variation of pointing error angle, \( e_k \) denotes the increase amount of each kind error and \( \Omega \) denotes the motion space of the APT system (in our system the rotate angle range of the azimuth axis is \( \pm 180^\circ \) and the rotate angle range of the elevation axis is \( 0^\circ - 90^\circ \)). The generation of geometrical error is random, generally each of the errors follows normal distribution, but Equation (15) can not reflect the ergodicity of the error sensitivity. As we know, the pointing error varies with the geometrical error increase, in order to analysis the error sensitivity in the whole pointing space of the APT system it is need to take different random error value into consideration, in this paper the Monte Carlo method is utilized. First, set the range of the dynamic error which need to be examined and take out one of them randomly, the static error is also given. Then establish the beam propagation model according to Equations (2)–(14), based on this model we can solve out the instantaneous beam pointing angle with and without geometrical error respectively, meanwhile, calculate out the beam pointing error. Repeating the above procedure 10,000 times at each pointing location of the APT, at last, we can calculate out the error sensitivity value according to Equation (15). The simulation results (normalized value) can be seen in Figure 6.

From the data in Figure 6, the geometrical errors of the 45° folding mirror of \( \gamma_{py} \) and \( \beta_{py} \) have the greatest impact on the pointing accuracy, then is the azimuth axis rotation error \( \beta_a \). The azimuth axis geometrical error \( \gamma_a \) and \( \alpha_a \) have the lowest sensitivity. So we are required to reduce \( \gamma_{py} \), \( \beta_{py} \), and \( \beta_a \) as far as possible during the assembling process. Under certain geometrical error conditions, the pointing error of the APT varies with azimuth angle \( \theta_{Az} \) and elevation angle \( \theta_{El} \) is

![Figure 6. The geometrical error sensitivity of the pointing error.](image)
simulated, and the results are shown in Figure 7. When the geometrical errors are not corrected and we assuming that the static error (STE) is 10⁻⁶ and the dynamic error (DYE) has little perturbation (1°) around 10⁻⁶, under this situation the maximum value of pointing error is 273 μrad (see in Figure 7(a)), there are two extreme position \( \theta_{Az} = 1\text{rad}, \quad \theta_{El} = 1.5\text{rad} \) and \( \theta_{Az} = -1.5\text{rad/} \theta_{El} = 0\text{rad} \), where the maximum and minimum pointing error occurred easily. In Figure 7(b) we assume that the static error reduced to 5⁻⁶ and the dynamic error remains unchanged, we can found that the maximum value of the pointing error is reduced which is 215 μrad and the pointing error has great variation with azimuth rotation angle. When we reduce dynamic error to 5⁻⁶ we can found that the overall level of pointing error reduced but there still exist a maximum pointing error which is 221 μrad in Figure 7(c). In Figure 7(d) we reduced both of static error and dynamic error to 5⁻⁶, the results shown that the pointing error distribution revert to the state of Figure 7(a) but the overall size has gone down and the maximum pointing error is 127 μrad.

4. Pointing error correction experiments

The mathematical model of the pointing error induced by geometrical errors have been established, the above research and analysis can provide some references for us during the assembling process. Because of the folding mirror is in the middle position of the APT system, it is hardly to assemble the APT if just take the beam pointing error from the telescope as a feedback. Owing to the complex structure of the APT system, the monolithic assembly method is inappropriate for the accumulative errors. Here, we adopt the modular assembling method. Firstly, assemble each of the modules separately, then assemble all the modules together. During the assembling process of each module, a benchmark coordinate is selected as reference, assuming that the pointing
The accuracy threshold of each module is \( \delta_i^* \), if the pointing error \( \delta_i \) meets the requirements (\( \delta_i < \delta_i^* \) in Figure 8), then the assembling process move on to the next step. After each module is assembled, then begin to adjust the interface accuracy of any two modules in order of Housing, azimuth axis, elevation axis, telescope axis.

According to the simulation results of section 3.3, the errors that have greatest effect on beam pointing accuracy are the geometrical errors \( \gamma_{py} \) and \( \beta_{py} \) of the folding mirror, then is the azimuth axis rotation error \( \beta_{ax} \). Therefore, these three kinds of errors need to be corrected carefully. The APT beam pointing error correction platform is built which is shown in Figure 9. The system consists of the beam power attenuation link, off-axis collimator (the focal length is 3 m), the APT, an inrun (the slope angle is 45°) and a turntable which can rotate 360 degree in one direction. The APT is fixed on the slope table and the table is fixed on the turntable (in this way, one-dimensional motion can be converted to two-dimensional motion) and data acquisition system (DAQ card and computer). The IFOV (Instantaneous field of view) of the CCD (Charge Coupled Device) used in the APT is 6.4 \( \mu \)rad/pixel.

The commonly used axis accuracy test methods are static test method and dynamic test method, the static test method needs to record the data at each stop position. So, here we choose dynamic test method, start rotating the turntable and the CCD start sampling (100 Hz) the center of the laser spot when the target is tracked steadily, so the miss distance between the laser spot center and the center of the CCD is used as the pointing error of the APT which is caused by geometrical errors.
The test results can be seen in Figure 10. The maximum pointing error of elevation axis is 51.2 μrad (8pixel × 6.4IFOV), and the expectation is 21.7 μrad, the maximum pointing error of azimuth axis is 54.4 μrad and the expectation is 20.5 μrad, so the composite pointing error is \((21.7^2 + 20.5^2)^{1/2} = 29.9\ \text{μrad}\) which is smaller than the simulation results. In fact, the pointing accuracy of the APT is affected by variety of factors, such as the platform vibration, the geometrical induced error, the detector’s noise or control error, etc. But it is mostly due to vibration and the geometrical induced errors during the dynamic test process. So, it is believed that the geometrical induced APT pointing error is smaller than 29.9 μrad. However, when the vibration intensity changes, the influence of geometrical induced error on the APT pointing accuracy is also different\([23]\), and this problem needs further analysis.

The typical value of tracking accuracy of a APT system is around 30 μrad such as LCE 32 μrad, LUCE 36 μrad\([24]\). This means that the geometrical error is corrected effectively in this paper. This is benefit from the analytical results of the error sensitivity of the APT system by using the multi-body kinematics method. Therefore, this method can provide some valuable references for the APT system assembling, which is also applicable for other types of APT such as periscope type and U-type frame APT systems.

5. Conclusion

Motivated by the requirements for high pointing accuracy of the APT system in space laser communication, the geometrical error induced APT pointing error is analyzed, discussed and some principles are discovered. The multi-body topological structure of the L-type theodolite APT is given, it is geometrical error is analyzed and the error transformation matrix is derived, then the geometrical error sensitivity about APT pointing error analysis is performed, the simulation results shown that the non-perpendicular error of the 45° folding mirror has a bigger contribution to APT pointing error than other geometrical error, the pointing error distribution is greatly affected by azimuth axis rotation angle, extremely the maximum pointing error occurs easily when \(\theta_{Az} = 1\ \text{rad}/\theta_{El} = 1.5\ \text{rad}\) or \(\theta_{Az} = -1.5\ \text{rad}/\theta_{El} = 0\ \text{rad}\). At last, the correction experiment is performed and the pointing accuracy is tested, the pointing accuracy is greatly improved based on the modular assembling method which reduced from 256 μrad to 29.9 μrad. This work is conductive to design of high performance space laser communication system which will provide some valuable guidance. Especially during the APT assembling process, this method can reduce unnecessary assembling steps and save time.
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