Empirical Photometric Redshifts of Luminous Red Galaxies and Clusters in SDSS

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ABSTRACT
In this work I discuss the necessary steps for deriving photometric redshifts for luminous red galaxies (LRGs) and galaxy clusters through simple empirical methods. The data used is from the Sloan Digital Sky Survey (SDSS). I show that with three bands only (\textit{gri}) it is possible to achieve results as accurate as the ones obtained by other techniques, generally based on more filters. In particular, the use of the \textit{(g-i)} color helps improving the final redshifts (especially for clusters), as this color monotonically increases up to \textit{z} \sim 0.8. For the LRGs I generate a catalog of \sim 1.5 million objects at \textit{z} < 0.70. The accuracy of this catalog is \sigma = 0.027 for \textit{z} \leq 0.55 and \sigma = 0.049 for 0.55 < \textit{z} \leq 0.70. The photometric redshift technique employed for clusters is independent of a cluster selection algorithm. Thus, it can be applied to systems selected by any method or wavelength, as long as the proper optical photometry is available. When comparing the redshift listed in literature to the photometric estimate, the accuracy achieved for clusters is \sigma = 0.024 for \textit{z} \leq 0.30 and \sigma = 0.037 for 0.30 < \textit{z} \leq 0.55. However, when considering the spectroscopic redshift as the mean value of SDSS galaxies on each cluster region, the accuracy is at the same level as found by other authors: \sigma = 0.011 for \textit{z} \leq 0.30 and \sigma = 0.016 for 0.30 < \textit{z} \leq 0.55. The photometric redshift relation derived here is applied to thousands of cluster candidates selected elsewhere. I have also used galaxy photometric redshifts available in SDSS to identify groups in redshift space and then compare the redshift peak of the nearest group to each cluster redshift. This procedure provides an alternative approach for cluster selection, especially at high redshifts, as the cluster red sequence may be poorly defined.

Key words: surveys – galaxies: distances and redshifts – galaxies: clusters.

1 INTRODUCTION
Recent galaxy redshift surveys (2dFGRS, \textsuperscript{[2001]}\textsuperscript{Colless et al.}; SDSS, \textsuperscript{[2000]}\textsuperscript{York et al.}) have provided the astronomical community a unique view of the local universe (\textit{z} \sim 0.1). Such surveys are based on spectrographs that simultaneously observe hundreds of objects. Although the improvement respective to a decade ago is enormous, larger and mainly deeper general spectroscopic surveys are not yet possible with current instrumentation. Note that surveys targeting specific populations, such as Luminous Red Galaxies (SDSS, \textsuperscript{[2001]}\textsuperscript{Eisenstein et al.}) or star-forming galaxies (Wiggle-z, \textsuperscript{[2007]}\textsuperscript{Glazebrook et al.}), can sample much larger volumes. For the mean time, photometric redshifts provide a valuable alternative to probe faint sources within large areas.

Photometric redshift techniques are essentially a mechanism to convert photometric properties of galaxies (such as colors) into redshift and physical properties (e.g., luminosity and type). Thus, with the proper choice of passbands and the use of an accurate photometric redshift algorithm, it is possible to map the distant universe in three dimensions. These surveys represent a powerful tool for studying the statistical properties of galaxies and their evolution.

There are several photometric redshift estimators developed to date. These can be generally classified either as empirical or template-based methods. In the first case a direct relation is obtained through the comparison of the photometric properties (colors) and spectroscopic redshifts. Such empirical relations can be derived, for instance, through polynomial fitting (\textsuperscript{[1993]}\textsuperscript{Connolly et al.}) or neural networks (\textsuperscript{2004}Collister & Lahav). The template-based algorithms rely on the availability of a set of galaxy templates. These should accurately represent the distribution of galaxy SEDs...
and their evolution with look back time \cite{Csabai2003}. Hybrid photometric redshift techniques have also been proposed in the last few years. They combine the advantages of empirical and template fitting methods by iteratively improving the concordance between photometric data and the spectral energy distributions. In other words, the template spectra is reconstructed to best match the observed photometric measurements of each galaxy \cite{Budavari2004, Csabai2003}.

Luminous red galaxies are marked by uniform spectral energy distributions, characterized by a strong break at 4000 Å due to the accumulation of a number of metal lines. The shift of this feature through different filters is strong correlated with redshift. These galaxies are also known to be some of the most luminous objects in the universe and are preferentially found at high density environments, rendering these objects an interesting tool for selecting and studying clusters. All that said, it is clear that LRGs comprise an optimal population to derive accurate photometric redshifts to very large distances.

This paper describes the construction of a large photometric redshift catalog of LRGs at $z < 0.70$. This catalog is based on simple polynomial fitting of the relations between galaxy colors and spectroscopic redshifts. I explore the use of different colors from SDSS, showing that with three bands only it is possible to achieve results comparable to more elaborated empirical techniques, such as ANNz \cite{Collister2004}, kd-trees or the nearest-neighbour method \cite{Csabai2003}. In addition, I employ similar relations to derive photometric redshifts of galaxy clusters.

When estimating redshifts of clusters the main drawback is the need to apply a background correction when selecting probable cluster galaxies seen in two dimensions. I discuss different possibilities when minimizing the background effects, showing that the most precise results can be achieved when selecting the reddest galaxies (the selection is based on the $u-r$ color). Photometric redshift estimates of clusters rely on precise values of their median color. Elliptical or S0 galaxies comprise the main population in the central regions of galaxy clusters. Thus, one would like to use these galaxy types when estimating the typical colors of clusters. As it is shown in §4.2, at low redshifts a simple statistical background correction is enough to minimize the influence of galaxies that do not belong to the clusters, and to accurately estimate cluster colors. However, at higher redshifts this simple correction leads to an underestimation of cluster colors compared to the expected values for ellipticals. That is also due to the increase with redshift in the fraction of blue galaxies in clusters \cite{Butcher1984}. To circumvent this problem, the use of the $u-r$ color plays a key role to help the selection of early type systems and thus reduce the scatter of the observed colors of clusters. More details are found in §4.2.

This paper is divided as follows. In the next section I describe the SDSS survey, which is used as the basis for obtaining the empirical relations and evaluate the results. In section 3 I describe the selection of LRGs and the photometric redshift technique employed for these objects. The same is done for clusters in §4, where I also make considerations about redshift accuracy. I also use galaxy photometric redshifts from SDSS for the identification of groups in redshift space. I summarize the results in §5. Through this work I assumed a cosmology with $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$ and $H_0 = 100$ $h$ km s$^{-1}$ Mpc$^{-1}$, with $h$ set to 0.7.

## 2 DATA

The photometric and spectroscopic data for this paper were taken from the fifth release of the Sloan Digital Sky Survey \cite{York2000}. The SDSS consists of an imaging survey of $\pi$ steradians of the northern sky in five optical passbands ($ugriz$), from 3,500–8,900 Å. This will provide photometry for of order $5 \times 10^7$ galaxies. Spectroscopic will provide redshifts and spectra for $\sim 10^6$ of these. The survey is carried out using a 2.5 m telescope, an imaging mosaic camera with 30 CCDs, two fiber-fed spectrographs and a 0.5 m telescope for the photometric calibration. The imaging survey is taken in drift-scan mode and the data are processed with a photometric pipeline (PHOTO) specially written for the SDSS data.

Targets for spectroscopy are selected by the targeting pipeline from the imaging. Spectroscopic fibers are assigned to the targets by a tiling algorithm \cite{Blanton2003}. The minimum distance of 55 arcsec between the fibers leads to a loss of $\sim 6\%$ of galaxies, which is the main source of incompleteness.

The spectroscopic survey is originally divided in three samples. The 'Main', flux-limited sample, has a median redshift of 0.104 and a limiting magnitude of $r_{\text{petro}} \sim 17.77$ \cite{Strauss2002}. As this limit is much brighter than that for the imaging, the redshift completeness is nearly 100\%. The second sample is the luminous red galaxy sample (LRG), which is approximately volume-limited to $z \approx 0.38$ \cite{Eisenstein2001}, extending to $z \approx 0.55$. Finally, the quasar sample is defined by objects with colors distinct from those of ordinary stars. The completeness of this sample depends somewhat on redshift. In particular, the completeness is low for $2.4 < z < 2.9$, where the quasar and stellar loci cross; it is similarly low at redshifts around 3.5 and 4.5.

In addition to these data, the 2dF-SDSS LRG and Quasar Survey (2SLAQ; \cite{Cannon2006}) has been recently completed. This survey exploits the high-quality SDSS imaging combined with the extraordinary spectroscopic capabilities of the Two-degree Field (2dF) instrument on the 3.9-m Anglo-Australian Telescope. It then results in a spectroscopic redshift catalog of $\sim 13,000$ LRGs at $0.4 < z < 0.7$. These data can be found in the data release five (DR5) of SDSS or directly from the 2SLAQ website\footnote{http://www.2slaq.info/}.

In this paper I use photometric data from SDSS and spectroscopic redshifts from SDSS and 2SLAQ surveys (available within SDSS).

All the data selected from SDSS is from the DR5. I have selected only objects from the “Galaxy” view (so that only PRIMARY objects are allowed) in order to avoid duplicate observations. Standard flags for clean photometry are also enforced. When selecting spectra and imaging, a joined query of the Galaxy and SpecObj (objects with clean spectra) views is performed. All the magnitudes retrieved from SDSS are de-reddened (corrected for extinction) model magnitudes.
3 SELECTION OF LUMINOUS RED GALAXIES

The selection criteria adopted for constructing a photometric sample of LRGs is analogous to the description given in Padmanabhan et al. (2003), which is aimed at selecting a uniform sample of LRGs at 0.2 < z < 0.7. Two different criteria are applied for selecting a low redshift sample (Cut I, z < 0.4) and a high redshift sample (Cut II, z > 0.4). Initially, two color tracks are defined:

\[
\begin{align*}
    c\perp & \equiv (r - i) - (g - r)/4 - 0.18, \\
    d\perp & \equiv (r - i) - (g - r)/8 \approx r - i .
\end{align*}
\]

(1) (2)

Then, the following color cuts are applied

Cut I: \( |c\perp| < 0.2 \); 

Cut II: \( d\perp > 0.55 \), \( g - r > 1.4 \),

(3) (4) (5)

The final cut, \( g - r > 1.4 \), is effective on isolating the sample from the stellar locus. In addition to these selection criteria, all galaxies with \( g - r > 3 \) and \( r - i > 1.5 \) are eliminated. These last constraints are helpful on removing stars with unusual colors, without discarding real galaxies (Padmanabhan et al. 2003). However, it is important to keep in mind that a 5% stellar contamination may still be present, as pointed out by Collister et al. (2007) (see section 4 of their paper), who applied similar selection criteria. Nonetheless, their criteria leads to the selection of more objects than the one adopted here.

These color cuts are still not enough to select LRGs from SDSS (see discussion in Eisenstein et al. 2001). Therefore, additional cuts in magnitude are applied. First, a color track which is approximately parallel to the low-redshift locus is defined for Cut I

\[
c\parallel = 0.7(g - r) + 1.2(r - i - 0.18) .
\]

(6)

Then, the following cuts are implemented

Cut I: \( \tau_{\text{Peto}} < 13.6 + c\parallel/0.3 \), \( r_{\text{Peto}} < 19.7 \)

(7)

Cut II: \( i < 18.3 + 2d\perp \), \( i < 20 \)

(8)

\( r_{\text{Peto}} \) is used for consistency with the original SDSS LRG target selection. Except for the numerical values of the magnitude cuts in equation 7, Cut I is identical to the SDSS LRG Cut I. The numerical values for Cut II are chosen to derive a population consistent with the first cut. At the redshift range sampled by Cut II the 4000 Å break is moving through the \( r \) band. As a consequence, the \( r \) band K-corrections are very sensitive to redshift. Thus, using the \( i \) band for Cut II leads to a more robust selection.

When applying these criteria to select LRGs from the DR5 of SDSS a total of 578,160 galaxies are selected using Cut I and 896,988 through Cut II. The combined sample, after excluding overlapping galaxies, adds to 1,459,536 luminous red galaxies. This sample is from now on called the photometric sample. Note that the high redshift sample has approximately 74% of the MegaZ-LRG catalog (Collister et al. 2007). This last catalog was selected from the data release 4 of SDSS (DR4). If the same criteria adopted by Collister et al. (2007) is applied to DR5 the number of LRGs retrieved is \( \sim 1.4 \) million. So, the high redshift sample in the current work (Cut II) actually represents \( \sim 64% \) of the MegaZ-LRG catalog. That is due to the different criteria employed here. I allow only objects at \( 1.4 < g - r < 3 \) and \( r - i < 1.5 \), while Collister et al. (2007) uses \( 0.5 < g - r < 3 \) and \( r - i < 2 \). Besides that, they select galaxies at \( d\perp > 0.5 \), while here the adopted cut is \( d\perp > 0.55 \). When imposing that galaxies should have spectroscopic measured redshifts and applying the same criteria as above, there are 197,956 luminous red galaxies in SDSS. Out of these, 186,572 are at low redshifts (Cut I) and 11,384 at high redshifts (Cut II). The small number of galaxies with spectra available at high redshift is due to the fact that 2SLAQ was restricted to a small number of fields located in the equatorial stripe of the SDSS survey area. This set is called the spectroscopic sample. Note that this sample does not include stars, as those were removed according to their spectroscopic identification.

3.1 Photometric redshifts of LRGs

The empirical photometric redshift estimators rely on the existence of a training set of objects with spectroscopic redshifts. This set should be representative, in terms of photometry and redshift, of the target sample which will be used later on. The training set used here is the spectroscopic sample (with 197,956 objects) mentioned above. Actually, this sample is divided into a “training” and “evaluation” samples. I randomly selected 10,000 objects out of the 197,956 LRGs to be the “training” set. The remaining 187,956 galaxies are kept to play the role of an “evaluation” sample. The training set is then used to derive an empirical relation between galaxy colors and redshift, which is then applied to the the evaluation set. I found that increasing the training sample to 20,000 objects does not represent a meaningful gain in accuracy. Forcing the “random” selection to have a fixed percentage at \( z > 0.40 \) (say 60%) also leads to similar results.

On what follows I discuss which colors are best suited for deriving an empirical relation used for photometric redshift estimates. In Figure 1 I show the variation with redshift of the apparent magnitude \( r \) and of five SDSS colors, namely \( u-g \), \( g-r \), \( (g-i) \), \( r-i \), \( (i-z) \). The data points represent the 10,000 galaxies randomly selected for the training sample. On each bandpass K-corrections are obtained through the convolution of a SED characteristic of early type galaxies (taken from Coleman, Wu, & Weedman 1980; CWW from now on) with the SDSS filters. The expected colors in different redshifts are the result of adding the color of an elliptical galaxy at zero redshift and the difference in K-corrections between two bands. The zero redshift colors are taken from table 3 of Fukugita, Shimasaku, & Ichikawa (1993). These color tracks are shown by the solid lines (red in the electronic edition) of panels (b)-(f) of Figure 1. A small offset was noticeable between the color tracks and the data points. I estimated these offsets (a factor < 0.15) and took them into account for the figure. In panel (d) the dotted line (green in the electronic edition) indicates a second order polynomial fit to the relation between \( (g-i) \) color and redshift. In this...
Figure 1. The variation of apparent magnitude \( r \) and five SDSS colors with redshift. The six panels show the following parameters versus redshift: (a) magnitude; (b) \((u-g)\); (c) \((g-r)\); (d) \((g-i)\); (e) \((r-i)\); (f) \((i-z)\). The solid line (red in the electronic version) on each panel indicates the expected color variation of early type galaxies. In panel (d) the dotted line (green in the electronic edition) indicates a second order polynomial fit to the relation between \((g-i)\) color and redshift. In the same panel the dashed line (blue in the electronic figure) shows the result of a fourth degree polynomial.

Panel the dashed line (blue in the electronic figure) shows the result of a fourth degree polynomial.

A few features are readily noticed from the inspection of this figure. First, \((u-g)\) shows a large scatter and does not follow the color track expected for an elliptical galaxy (due to the lower sensitivity of the \( u \) filter, specially for LRGs). Second, the \((i-z)\) color shows little variation with redshift. Thus, we do not expect these two colors to contribute in a meaningful way for a photometric redshift estimator. Third, the \((g-r)\) shows the expected large variation at low redshifts \((z < 0.35)\), becoming nearly flat afterwards. The \((r-i)\) color shows the opposite trend, being nearly constant at \( z < 0.35 \) and increasing fast for higher-\( z \). The \((g-i)\) combines the results of the two previous colors, showing a large variation at low-\( z \) and a less steep dependence at higher-\( z \). These results are mainly associated to the shifting of the 4000 Å break between the \( g \) and \( r \) filters at \( z \sim 0.35 \). Last, it is worth to note that the \((r-i)\) color has a much smaller scatter in comparison to the ones based on the \( g \)-band \((u-g, g-r, g-i)\). That happens cause the reddest filters are better suited for sampling these types of galaxies.

I then used the training data to estimate different empirical relations between colors and redshift. These relations are based on a variety of combinations of colors, with the use of the \( r \) band magnitude in a few cases. These relations are then applied to the 187,956 galaxies of the evaluation sample. The redshift accuracy is characterized by the residual between the spectroscopic and photometric redshifts \((\delta z = z_{\text{spec}} - z_{\text{phot}})\) and the standard deviation

\[
\sigma = \sqrt{\frac{1}{N-1} \sum (\delta z_i - \mu)^2},
\]

where \( \delta z_i \) is the residual for the i-th galaxy and \( \mu = \langle \delta z \rangle \) is the mean residual. The sum is performed over all \( N \) data points. I also computed the mean and standard deviation
The mean, standard deviation, and fraction of LRGs with a valid photometric redshift ($z_{\text{phot}} > 0$ and $|\delta z| < 0.10$). All rows show the results of polynomial fits obtained using different parameters. The first row lists the results when applying a polynomial of fourth order to the relation between the color ($g-i$) and redshift. In the second row I list the results when using the colors ($g-i$) and ($r-i$). Those based on the $r$ magnitude and colors ($g-r$) and ($r-i$) are shown in the third row. In the fourth row the results represent the use of the $r$ magnitude and colors ($g-r$) and ($r-i$). The fifth row has the results when using the colors ($g-r$), ($g-i$) and ($r-i$). In the sixth row I show the results obtained when adding the color ($r-z$) to the previous set. The seventh has the result of a second order polynomial to the colors ($g-r$), ($g-i$), ($r-i$) and ($i-z$). Details about the fits are given in the text. In the last row I show the results achieved when using the color track based on the elliptical template from Coleman, Wu, & Weedman (1980) for the ($g-i$) color.

(Now called $\mu_0$ and $\sigma_0$) for the case where the residual is weighted by the factor $1 + z_{\text{spec}}$ ($|\delta z| = (z_{\text{spec}} - z_{\text{phot}})/(1 + z_{\text{spec}})$). When computing $\mu$ and $\sigma$ (or $\mu_0$ and $\sigma_0$) I only use galaxies with $|\delta z| > 0.10$. This gives equivalent results to reject outliers at the 5-s level. Including the gross outliers increases the standard deviation, as $\sigma$ is very sensitive to the presence of outliers. For instance, in the fifth row of Table 1 (results based in the colors $g-r$, $g-i$ and $r-i$) the values of $\sigma$ and $\sigma_0$ are raised from 0.027 and 0.023 to 0.032 and 0.026, respectively.

The mean and standard deviation, as well the fraction of galaxies with a valid $z_{\text{phot}}$ (the number of gross outliers is 100 minus this fraction), obtained for different empirical relations are summarized in Table 1. The last row of this table has the results obtained when considering the color track based on the elliptical template from Coleman, Wu, & Weedman (1980) for the ($g-i$) color. In other words, I simply use the track exhibited as a solid line in panel (d) of Figure 1 to compute redshifts from the observed ($g-i$) color. The remaining rows (1-7) in Table 1 show the results for the following polynomial fits:

$$z_{\text{phot}} = A + B(g-i) + C(g-i)^2 + D(g-i)^3 + E(g-i)^4,$$

$$z_{\text{phot}} = A + B(g-i) + C(r-i),$$

$$z_{\text{phot}} = A + Br + C(g-r) + D(r-i),$$

$$z_{\text{phot}} = A + Br + C(g-i) + D(r-i),$$

$$z_{\text{phot}} = A + B(g-r) + C(g-i) + D(r-i),$$

$$z_{\text{phot}} = A + B(g-r) + C(g-i) + D(r-i) + E(i-z) + F(g-r)^2 + G(g-i)^2 + H(r-i)^2 + I(i-z)^2.$$

From the inspection of Table 1 we see no large differences among the different relations. It is interesting to note that the results based on the ($g-i$) color track (using the SED from CWW) show the highest fraction of galaxies with a valid photometric redshift. However, the scatter determined in this way is a little larger when compared to other solutions. Besides that fact, the inspection of the relation $z_{\text{phot}} \times z_{\text{spec}}$ shows a poor correlation for $z_{\text{spec}} > 0.30$ (for the TEMP-CWW results). We also note that the results based on the $r$ band magnitude and colors have more outliers than the solutions based on two or more colors (but no magnitude). The result based on the $r$ magnitude and colors ($g-r$) and ($r-i$) is at the same level of the one that uses ($g-i$), instead of ($g-r$). So, for LRGs, the ($g-i$) does not seem to be superior to the ($g-r$) for redshift estimation. That is not true for galaxy clusters though (§4). I have also tried second and fourth order polynomial fits to the ($g-i$) color only. However, the overall results were not better. In Table 1 we can see that the fraction of outliers increases for this ($g-i$) fourth degree solution. Finally, it is interesting to see that the use of the $z$ filter trough the $r-z$ or $i-z$ colors does not help improving the results. The scatter is at the same level when adding the $r-z$ color, increasing a little if the $i-z$ is employed with a second order fit.

I also estimated the uncertainty in the photometric redshift estimates, of each galaxy, through propagation of errors. In this process I consider the error in the coefficients of the empirical fits, as well as the photometric errors for each magnitude or color. These uncertainties are estimated as shown in the equation below, where I use the fractional uncertainties in the coefficients (such as $A$, $B$, ...) and in the color and magnitudes used on each fit ($r, g-r, g-i,...$)

$$\Delta z_{\text{phot}} = z_{\text{phot}} \sqrt{(\Delta A/A)^2 + \cdots + (\Delta (g-r)/(g-r))^2 + \cdots}.$$ (10)

After inspecting Table 1 and plots of the $z_{\text{phot}} \times z_{\text{spec}}$ relation I decided to consider as the final catalog of photometric redshifts the one based on the ($g-r$), ($g-i$) and ($r-i$) colors. The coefficients for the empirical relation derived for this case are

$$A = -0.3068 \pm 0.0006$$

$$B = \quad 6.2005 \pm 0.1333$$

$$C = -5.9933 \pm 0.1331$$

$$D = \quad 6.4932 \pm 0.1324$$

This decision was mainly motivated by the fact that this relation is the one to produce fewer outliers and to show small individual galaxy redshift errors. The results based on relations that involve the magnitude $r$ have large galaxy error estimates, due to the large fractional uncertainties in the magnitude coefficient ($\Delta A/A$). That is the main reason for not adopting one of these relations. A plot with the comparison between $z_{\text{phot}}$ and $z_{\text{spec}}$ obtained with the ($g-r$), ($g-i$) and ($r-i$) colors is shown on Figure 2. Here I plot 15,000 randomly selected points from the evaluation sample (comprising 187,956 galaxies). It is important to mention that these results are in good agreement to what has been found by other authors [Padmanabhan et al. 2003; Collister et al. 2007] and the residuals show no systematic trends with $z_{\text{spec}}$. When considering only galaxies...
4 GALAXY CLUSTERS

The knowledge of clusters redshifts is essential for estimating other physical parameters of these systems (such as luminosity and richness). Accurate redshifts are also crucial for large scale structure studies. Perhaps the oldest and simplest way to get photometric redshifts is through the use of single band galaxy magnitudes within the cluster region. However, the use of colors turns the photometric redshifts of clusters much more accurate in the last years. Color based techniques explore the fact that clusters have a large population of early type galaxies which are characterized by a strong break at 4000 Å. The observation of this feature results from Goto et al. (2002) use the matched filter (MF) technique were intentionally truncated at z = 0.5 (as they ran the MF up to this redshift). The redshift estimates from Kim et al. (2002), based on a matched filter (MF) technique were intentionally truncated at z = 0.5 (as they ran the MF up to this redshift). The results from Goto et al. (2002) use the g − r color track for redshift estimation, but as seen from their figure 14, their photo-zs are truncated at z ∼ 0.44, while the spectroscopic sample used for comparison goes to z = 0.5 and they probably detect higher redshift systems (considering the magnitude limit adopted). In other words, although accurate they underestimate the redshifts (at least in the high-z regime).

4.1 Selection of a Training Sample of Galaxy Clusters

For the determination of photometric redshifts of galaxy clusters the first step that should be taken is the compilation of a list of objects with measured spectroscopic redshifts. Unfortunately, there are not so many clusters with spectra taken at z > 0.3, which biases our sample to low-z clusters. Our calibration sample consists of 512 clusters over the area covered by SDSS (DR5). These come from Struble & Roed (1999); Holden et al. (1999); Vikhlinin et al. (1998); Carlberg et al. (1996) and Mullis et al. (2003). The combined sample of these references contains 1805 clusters in the whole sky. After selecting all clusters with redshifts at 0.02 ≤ z ≤ 0.55 and outside 70.0 < α < 110.0 or 270.0 < α < 300.0 we are left with a list of 1055 clusters. The right ascension limits are meant to avoid most systems outside the SDSS region. However, many systems that do not overlap with SDSS are still allowed in this list. Then I select data from SDSS for all these clusters. Those falling off the SDSS limits will obviously contain no galaxies. For the remaining I generate finding charts, which are inspected to see if the regions around each cluster (8.0 Mpc x 8.0 Mpc) are well sampled (I exclude clusters with excised regions near their centers). The final list comprises 512 systems. The sample size may not be large enough

\[
\begin{align*}
\text{Comparison between} & \quad \text{Figure 2.} \\
\text{the estimator led to a negative photometric redshift I set} & \quad \text{z} <= 0.55, \quad \sigma = 0.027 \quad \text{and} \quad \sigma_0 = 0.023, \quad \text{while for} \quad \text{galaxies at} \quad 0.55 < z <= 0.70, \quad \sigma = 0.049 \quad \text{and} \quad \sigma_0 = 0.040.
\end{align*}
\]

\[
\begin{align*}
\text{Table 2 presents the luminous red galaxy catalog} & \quad \text{derived from the relation based on} \quad (g - r), \quad (g - i) \quad \text{and} \quad (r - i) \quad \text{colors. The parameters listed are} \quad \text{ra, dec, u, g, r, i, z, u, err, g, err, r, err, i, err, z, err, z, phot, err, z, phot} \quad \text{and} \quad \text{objID (the object ID within SDSS). The magnitudes are the de-reddened model magnitudes. When the estimator led to a negative photometric redshift I set} \quad z, \text{phot and err, z, phot} \quad \text{to} \quad -9.99. \quad \text{That happens for 8,509 galaxies (only 0.6% of the total catalog).}
\end{align*}
\]
to properly include evolutionary effects. However, the dominant population of red galaxies in clusters is believed to evolve passively with redshift. So, for clusters, the average color estimated from these galaxies should provide a clear correlation with redshift. In the future it will be interesting to use larger training samples of clusters for empirical photometric redshift estimators.

The next step is to investigate which colors provide the best connection to spectroscopic redshifts. When doing that I noticed that the relations between magnitude or colors to redshift are very well established. However, caution should be taken for the background correction (§3.2) and also to the selection of clusters which will be used as the evaluation sample. I noticed that most of the 512 clusters can be well represented by the color tracks of elliptical galaxies. However, there are a few outliers, specially at low redshifts, which should definitely be avoided when training a photo-z estimator. These are mainly associated to wrong redshifts (clusters with few galaxies with available spectra) or projection effects (if there is one or more clusters aligned to a low redshift system the color inferred will probably be wrong).

I then decided to gather further information from NED to exclude clusters with a small number of galaxies with redshift available. When doing that I select from NED all galaxies within 3 arcmins of each cluster center. After inspecting the information retrieved for each cluster I kept only those with at least 3 galaxies with a concordant redshift. Besides that, NED also provides a “special note” for some clusters, meaning that there is some peculiarity with the object (most of times it is a double system or there are different redshifts listed for it). Most of the 512 clusters are from Struble & Rood (1999) who also gives the number of galaxies used for measuring the redshift (may be different from above, as the aperture is not 3'). I then impose that the clusters should not have the “special note” in NED and have at least 3 galaxies in Struble & Rood (1999). The final training set comprises 132 systems. Nearly all high redshift clusters are kept (z > 0.4).

| ra  | dec  | u     | g     | r     | i     | z     | u-err | g-err | r-err | i-err | z-err | z-phot | err-zp | objID |
|-----|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|--------|-------|
| 0.001571 | 14.982869 | 26.399 | 22.074 | 20.319 | 19.488 | 19.079 | 0.067 | 0.159 | 0.050 | 0.039 | 0.126 | 0.47247 | 0.06714 | 58772077442560743 |
| 0.002045 | -0.793661 | 22.227 | 21.282 | 19.469 | 18.829 | 18.490 | 0.069 | 0.083 | 0.025 | 0.023 | 0.073 | 0.38900 | 0.03420 | 587724179523227851 |

**Table 2.** Example table of the LRG catalog containing 1,459,536 objects. The magnitudes listed are the de-reddened model magnitudes. The full table is available on-line or upon request to the author.
used to compute the mean magnitude and median colors of the clusters.

I tested three different possibilities for obtaining these corrected cluster counts. The procedure is executed exactly as described above, but in the first case I consider all galaxies with the r band magnitude less than 21 (m_r ≤ 21, which is approximately the star/galaxy separation limit of SDSS) and in the second I count only galaxies with m_r ≤ 21, but with m^* − 3 ≤ m_r ≤ m^* + 3, where m^* is the apparent characteristic magnitude of clusters. I consider the bright end values of the double-Schechter cluster luminosity function (LF) obtained by Popesso et al. (2006). They found α = −1.09 and M^* = −20.94 within R_200. This value of M^* is converted to the same cosmology used here and to the proper value at z = 0 (taking the mean redshift of their sample as z = 0.1). Then, for different redshifts I adopted an evolutionary correction to the value of M^*, given by M^*(z) = M^*(0) + Qz, with Q = −1.4 (Yee & López-Cruz 1994). The absolute characteristic magnitude (M^*) is converted to m^* through the application of the distance modulus formula to each cluster redshift (the variation of m^* with redshift is shown in the upper panels of Figure 4). The third test done is to compare the values obtained when considering all galaxies to the results found when imposing a selection according to the (u − r) color.

In Figure 4 I show the dependence of mean apparent magnitude r_{mean} (two upper panels) and two SDSS colors (median values; four lower panels) with redshift. The colors exhibited are g-r and r-i. In the top panels the solid line represents the expected variation of the apparent characteristic magnitude of clusters, while in the four lower panels the solid line indicates the expected color variation of E galaxies. In all panels, every black dot is a cluster (the 132 of the training sample), while the triangles with error bars (red in the electronic edition) indicate the mean values in redshift bins of 0.05. The left panels (a, b and c) show the results for the case where counts are taken at m^* − 3 ≤ m_r ≤ m^* + 3 and a color cut (in u-r) is applied for selecting galaxies. The exact values for this cut are chosen after comparing the u-r color tracks of Es and Sbcs in figure 3. Initially, I selected only galaxies with u − r ≥ 2.40, which already led to very good results. However, I found that these could still be improved if a variable cut (with redshift) was applied. The choice of this cut affects most the higher redshift clusters (z > 0.20). I finally decided to select only the galaxies with u − r ≥ 2.00 for clusters at z ≤ 0.20, u − r ≥ 2.30 at 0.20 < z ≤ 0.40 and u − r ≥ 2.45 at z > 0.40. I found these cuts to give the most accurate photometric redshifts. For comparison Strateva et al. (2001) find that early and late types can be well separated by a simple color cut at u − r = 2.22.

In the right panels of figure 4 I show the results of not applying one of the constraints mentioned above (the fixed luminosity range m^* − 3 ≤ m_r ≤ m^* + 3; or the color cut in u-r). Panel (d) shows the results for r_{mean} when using all galaxies at r ≤ 21 (counts are not restricted to a fixed luminosity range, but the u-r color cut is still enforced). Finally, panels (e) and (f) have the median color variations when I do not impose a color cut in u-r (but the fixed luminosity range is still applied).

It is worth mentioning that the further constraints applied above (in luminosity and color) are intended to improve the background correction, which is done by the subtraction of every cluster histogram (in magnitude and colors) by the background distribution. From panel (a) we can see that the mean cluster magnitudes show a strong variation up to redshift ~ 0.4. After that the relation tends to become flatter. That is due to the magnitude limit considered for the survey (r = 21.0), which renders the cluster counts at m^* − 3 ≤ m_r ≤ m^* + 3 truncated for high redshift systems (as m^* + 3 extrapolates r = 21.0). Applying a correction to the r_{mean} values, to take in account this truncation, results in no meaningful improvement in the accuracy of the photometric redshifts. When comparing panels (a) and (d) there is a remarkable difference between computing r_{mean} within a fixed luminosity range (m^* − 3 ≤ m_r ≤ m^* + 3, for instance) or using the full survey limits (I considered all galaxies with r ≤ 21 for panel d). The main effect is the overestimation of the counts at low redshifts (z < 0.15), which leads to the flattening of the r_{mean} × z relation in this regime. That happens because when using all galaxies at r ≤ 21 for low-z clusters, we sample magnitudes that are too faint in comparison to the relevant regime of a cluster LF (such as m^* − 3 ≤ m_r ≤ m^* + 3). Then, the r_{mean} values become biased towards higher values, with also a noticeable increase in the scatter. For clusters at z > 0.15 there is no visible difference because m^* + 3 is always close to r = 21 (or the survey limit is even extrapolated for high-z systems). A similar discussion, but for richness computation (instead of r_{mean}) is done in Lopes et al. (2006). On what regards colors, the use of all galaxies at r ≤ 21 has no large impact. We only see very few clusters that have their median colors offset from the color tracks, increasing a little the scatter.
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Figure 4. The variation of mean apparent magnitude \( r_{\text{mean}} \) and two SDSS colors (median values) with redshift. The top two panels show the \( r_{\text{mean}} \) variation, while the lower four panels exhibit the median color variations (\( g-i \) in the two middle panels and \( r-i \) in the lower two). The left panels (a, b and c) have the results when considering counts at \( m^* - 3 \leq m_r \leq m^* + 3 \) and with a color cut (\( u-r \)) applied for galaxy selection (this cut is explained in the text). Panel (d) shows the results for the mean magnitude when using all galaxies at \( r \leq 21 \) (counts are not restricted to a fixed luminosity range, but the \( u-r \) color cut is still applied). Finally, panels (e) and (f) have the median color variations when I do not impose a color cut in \( u-r \) (but the fixed luminosity range, \( m^* - 3 \leq m_r \leq m^* + 3 \), is still enforced). The solid line on each panel indicates the expected color variation of early type galaxies (except for panels (a) and (d) where they indicate the expected variation of the apparent characteristic magnitude of clusters). Each black dot represents one of the 132 clusters of the training sample, while the triangles (red in the electronic edition) with error bars indicate the mean values in redshift bins of 0.05.

From the comparison of panels (b) and (e) we notice a clear trend for underestimation of the \( g-i \) color at \( z > 0.15 \). The effect is more pronounced at \( z > 0.40 \). The same effect is noticed for the \( g-r \) color (not shown in the plot). For \( r-i \) the effect is not too drastic and we only see a mild underestimation at \( z > 0.40 \). Nonetheless, these results are very useful to show the relevance of imposing a color cut (according to \( u-r \)) for measuring cluster colors. By doing so, we can restrict the analysis to early type galaxies, rendering the derived colors in good agreement to the expectations of elliptical galaxies.

4.3 Photometric redshifts of galaxy clusters

I then proceed to derive empirical relations to estimate photometric redshifts of clusters. That is done in a similar way to what is shown in section 3.1 for LRGs, but here I use the mean magnitudes and median colors of clusters, estimated as in the left panels of figure 4. In other words, on top of the background corrections I require galaxies to have \( m^* - 3 \leq m_r \leq m^* + 3 \) and also impose a color cut (in \( u-r \)), as described above. For the 132 clusters of the training sample I derived the values of \( r_{\text{mean}}, (g-i)_{\text{median}} \) and \( (r-i)_{\text{median}} \), which are shown in figure 4. An empirical relation between these three parameters and redshift is then derived. Other colors are also obtained and will later be used for comparison to the results based on the two above. This relation can be expressed by

\[
\begin{align*}
\zeta_{\text{phot}} = A + Br_{\text{mean}} + C(g-i)_{\text{median}} + D(r-i)_{\text{median}},
\end{align*}
\]
analogous to the ones employed for LRGs. The derived coefficients are

\[ A = -0.4424 \pm 0.0084 \]
\[ B = 0.0076 \pm 0.0006 \]
\[ C = 0.2382 \pm 0.0024 \]
\[ D = 0.2126 \pm 0.0042 \]

To assess the accuracy of the photo-z estimator this relation is applied to all 512 clusters with known spectroscopic redshifts (section 4.1). However, as we would do with clusters with unknown redshifts, we have to start the procedure with a guess redshift and iterate it until the photometric redshift difference between two iterations is less than 0.01. A maximum of 10 iterations is allowed. Convergence is not found for only 2 of the 512 clusters. This iterative procedure is necessary as we sample 0.5 \(h^{-1}\) Mpc and \(m^\ast - 3 \leq m_r \leq m^\ast + 3\) for each cluster, but we do not know what the redshift of the cluster is (used to determine this radius and luminosity range for galaxy selection). So, we start with a guess value (\(z_{\text{guess}} = 0.15\)), compute the mean magnitude and median colors and apply the empirical relation obtained above. Then we use the new redshift estimate to repeat the procedure until convergence is achieved. The \(z_{\text{phot}} \times z_{\text{spec}}\) comparison is shown on the top panel of Figure 5, while the weighted residuals is exhibited in the lower panel. Clusters with \(|\delta_m| < 0.10\) (see definition in §3) are shown as filled circles (489 or 96% of the 512). The remaining have open circles. If I had used more than 132 clusters (for instance, all the 512) to derive the photometric redshift empirical relation the final results would have a slightly increased scatter. That is due to the fact that some clusters have spectroscopic redshifts derived from a small number of galaxies. That is why it is important to use a clean sample to train the calibration (§4.1).

In Table 3 I summarize the results obtained for relations based on different combinations of mean \(\tau\) magnitude and median colors. Analogously to what was done for LRGs, I only use clusters with \(|\delta_m| < 0.10\) when computing \(\mu\) and \(\sigma\) (or \(\mu_0\) and \(\sigma_0\)). Had I included the gross outliers, the standard deviation for the \(r_{\text{giri}}\) relation (third row of Table 3) is raised from 0.026 and 0.023 to 0.045 and 0.039 (values of \(\sigma\) and \(\sigma_0\), respectively). Note that the fraction of outliers is only \(\sim 5\%\). See also the next section for some considerations regarding accuracy.

The results in this table are first shown for all clusters and then for those at \(z \leq 0.30\) and \(z > 0.30\). On each case, the first four rows list the results considering galaxies at \(m^\ast - 3 \leq m_r \leq m^\ast + 3\) and with the \(u-r\) color cut applied. The fifth row exhibits the values when not imposing a fixed luminosity range, while the last row has the results without a color cut. When considering all clusters (but only the first four rows), we see no large differences, except for the values achieved for the \(r_{\text{grgiri}}\) relation (\(r_{\text{mean}}, (g-r)_{\text{median}}\) and \((r-i)_{\text{median}}\)) which have \(\sigma\) (or \(\sigma_0\)) a little higher than the rest. The same is also true for low redshift clusters (which dominate the sample) and the high redshift ones. Actually, the fraction of clusters at high-\(z\) with a valid redshift decreases a little for the \(g_{\text{giri}}\) and \(r_{\text{grgiri}}\) cases. Considering that, I decided to adopt the redshifts obtained with \(r_{\text{mean}}, (g-i)_{\text{median}}\) and \((r-i)_{\text{median}}\), as they have fewer outliers and low dispersions at all redshifts. The coefficients for equation 11 are listed above. These results show the importance of using \(r_{\text{mean}}\) and \((g-i)_{\text{median}}\) for photometric redshift estimation at \(z < 0.55\) in SDSS. The use of \((g-r)_{\text{median}}\) leads to an increased scatter (mainly at high-\(z\)), while using only colors (without \(r_{\text{mean}}\)) gives lower completeness at high-\(z\).

On what regards the results without the luminosity or color constraints (rows 5 and 6), they are noticeably worst than when these constraints are enforced. When not applying the luminosity restriction the results are at a similar level than what is obtained in the \(r_{\text{grgiri}}\) case (except for the number of outliers at high-\(z\)). Similar values are also derived for the case where no color cut is applied, but only when considering low-\(z\) clusters. At \(z > 0.30\) the standard deviation and fraction of outliers achieved without the color cut is the highest among all. That tells us how important is the selection of early type galaxies for cluster photo-z estimation at higher redshifts. Without this pre-selection the cluster regions will likely be contaminated by lower redshift sources or, more importantly, by blue galaxies (which have an increased fraction at higher redshifts). Thus increasing

| Relation     | \(\mu\)   | \(\sigma\) | \(\mu_0\) | \(\sigma_0\) | Fraction(%) |
|--------------|-----------|------------|-----------|-------------|-------------|
| \(g_{\text{giri}}\) | -0.0021   | 0.026      | -0.0015   | 0.023       | 95.1        |
| \(r_{\text{grgiri}}\) | -0.0056   | 0.028      | -0.0052   | 0.025       | 95.3        |
| \(rgi_{\text{giri}}\) | -0.0046   | 0.026      | -0.0040   | 0.023       | 94.9        |
| \(gr_{\text{giri}}\) | -0.0023   | 0.026      | -0.0021   | 0.023       | 95.1        |
| \(rgi_{\text{giri}} (r \leq 21)\) | -0.0021   | 0.028      | -0.0021   | 0.025       | 95.1        |
| \(rgi_{\text{giri}} (\text{no u-r cut})\) | -0.0021   | 0.029      | -0.0018   | 0.026       | 93.9        |

Table 3. The mean, standard deviation and fraction of clusters with a valid photometric redshift (\(z_{\text{phot}} > 0\) and \(|\delta_m| < 0.10\)). All rows show the results of polynomial fits obtained using different parameters. In the first row I list the results when using the colors \((g-r)\) and \((r-i)\). Those based on the \(r\) magnitude and colors \((g-r)\) and \((r-i)\) are shown in the second row. In the third row the results represent the use of the \(r\) magnitude and colors \((g-i)\) and \((r-i)\). The fourth row has the results when using the colors \((g-r)\), \((g-i)\) and \((r-i)\). Fifth row exhibits the results obtained with colors \(r_{\text{mean}}, (g-i)_{\text{median}}\) and \((r-i)_{\text{median}}\), but not being restricted to galaxies at \(m^\ast - 3 \leq m_r \leq m^\ast + 3\). All galaxies at \(r \leq 21\) are used in this case. In the sixth row the results are again for \(r\) magnitude and colors \((g-i)\) and \((r-i)\), but the galaxies selected have no \(u-r\) cut applied. In the continuation of the table, the same type of information is also given for low \((z \leq 0.30)\) and high redshift clusters \((z > 0.30)\).
the final error and biasing the results to low values. In other words, a simple background correction is not enough, even considering that we are working with a small aperture (0.50 h\(^{-1}\) Mpc).

4.4 Considerations about photometric redshift accuracy

It is important to mention that the redshift accuracy achieved in this work is lower than what other authors found with SDSS data. I estimated \( \sigma = 0.024 \) for \( z \leq 0.30 \), while Goto et al. (2002) had estimated uncertainties of \( \sigma = 0.015 \) and Koester et al. (2007) of \( \sigma = 0.01 \) at \( 0.10 < z < 0.30 \). Note that Goto et al. (2002) show residuals for clusters at \( z > 0.08 \). The inclusion of lower redshift clusters in our sample \( (z > 0.05) \) helps increasing the scatter, but by no means can explain the difference to other results.

Koester et al. (2007) estimate redshifts as part of the selection of clusters. Each cluster has assigned to it the redshift of a galaxy (brightest cluster galaxy, BCG) that maximizes the likelihood of representing a cluster center. Goto et al. (2002) have the redshift estimates done after cluster detection (with the cut and enhance method). Their estimates are based on an early version of the maxBCG technique, and are not identical to the ones from Koester et al. (2007). For a given redshift, they start counting the number of galaxies within the cluster detection radius brighter than \( M_r = -20.25 \) and within \( \pm 1 \) mag in \( g - r \) around the color prediction for elliptical galaxies (Fukugita, Shimasaku, & Ichikawa 1995). The procedure is repeated for several redshifts in steps of \( \delta z = 0.01 \). After the background is taken in account on each bin, the redshift of the bin with the largest number of galaxies is considered the cluster estimated redshift.

In this work redshifts are obtained through the application of an empirical relation to \( r_{\text{mean}}, (g-i)_{\text{median}} \) and \( (r-i)_{\text{median}} \). The process is started with a guess redshift \( (z_{\text{guess}} = 0.15) \) and is iterated until convergence is achieved. For the 512 clusters used here the photometric redshift accuracy is simply given by the comparison of the measured spectroscopic redshift and the photometric estimate. So, it is clear that this process is not guided whatsoever. For comparison, Koester et al. (2007) estimate the accuracy of their estimates by selecting all clusters from their catalog that have spectroscopic redshifts for their BCGs. For those, they compare \( z_{\text{phot}} \) and \( z_{\text{spec}} \), where \( z_{\text{phot}} \) is the cluster photo-z estimated from the maxBCG algorithm and \( z_{\text{spec}} \) is the spectroscopic redshift measured for the BCG galaxy in question. That seems a fair comparison, but the authors also recognize that \(~16\%\) of their clusters suffer from projection effects, which could affect cluster redshift estimates based on several galaxies (and not only the BCG). On the other hand, Goto et al. (2002) seem to guide the comparison of \( z_{\text{phot}} \) and \( z_{\text{spec}} \) in their words: “the redshift of the SDSS spectroscopic galaxy within the detected radius and with nearest spectroscopic redshift to the estimation is adopted as the real redshift”. Such procedure obviously biases the comparison to small redshift offsets, based on a single galaxy.

I decide to estimate the residuals in a similar way to what was done by Koester et al. (2007) and Goto et al. (2002). For that purpose I did not consider the ’Main’ flux-limited sample of SDSS. Instead, I used only the 197, 956 LRGs, with spectra available, selected in §3. In the first case I selected the nearest galaxy to the cluster center (within a maximum aperture of \( 60'' \)). If the LRG selected is close to the center it might be the BCG of the cluster. However, it is important to note that I do not make any magnitude or color requirement for that selection. This simple approach results in \( \sigma = 0.021 \) (or \( \sigma_0 = 0.017 \)) for the full redshift range of the clusters used here. When I restrict the sample to clusters at \( 0.10 \leq z \leq 0.30 \) I find \( \sigma = 0.014 \) (or \( \sigma_0 = 0.011 \)). The last results are closer to Koester et al. (2007) for the same redshift interval. For the full sample there are 111 (~23\%) clusters (out of 489 with a valid photo-z), while there are 71 at \( 0.10 \leq z \leq 0.30 \).

To perform a comparison to Goto et al. (2002) I did something similar to what they did. The only difference is to use a radius of \( 0.50 \) h\(^{-1}\) Mpc, instead of the “detection” radius available within their catalog. That should not result in meaningful differences as you do not want to select a galaxy that is too far from the cluster center. So, within \( 0.50 \) h\(^{-1}\) Mpc I select the LRG with the closest spectroscopic redshift to the value of \( z_{\text{phot}} \) for each cluster. For 301 clusters (~62\%) there is at least one LRG inside \( 0.50 \) h\(^{-1}\) Mpc. Out of those, 206 are at \( 0.10 \leq z \leq 0.30 \). I found \( \sigma = 0.018 \) (or \( \sigma_0 = 0.017 \)) for the full sample and \( \sigma = 0.017 \) (or \( \sigma_0 = 0.015 \)) at \( 0.10 \leq z \leq 0.30 \).

One problem with these tests is the fact that correlating the value of \( z_{\text{phot}} \) for a cluster with \( z_{\text{spec}} \) for a single galaxy may lead to wrong matches due to projection effects. As these clusters represent a combination of objects from different catalogs in the literature they are not supposed to have their centroid perfectly matched with a BCG (substructure can affect the centroid determination). That
is not the case of the catalog from [Koester et al. (2007)]. So, if their BCG has an spectroscopic observation and their code works properly, they will have a good correlation between $z_{\text{phot}}$ and $z_{\text{spec}}$ (except, perhaps, for clusters with strong projection effects). Nonetheless, it is encouraging that the two tests above result in an improved accuracy.

To minimize the influence of projection effects I also estimated the accuracy in a third way. For each cluster I select two tests above result in an improved accuracy. For each cluster I select all LRGs within 360″ from the cluster center. From those, I check if the galaxy closest to the centroid is at a maximum distance of 120″. If it is, I assume the redshift of this LRG as a reference ($z_{\text{ref}}$). Then, from all the other galaxies (LRGs) selected within 360″ I take those that have a maximum redshift difference $|z_{\text{rg}} - z_{\text{ref}}|$ of 0.030. If I end up with at least three galaxies I take the mean of these redshifts to be the value of $z_{\text{spec}}$. When comparing these to $z_{\text{phot}}$ I find $\sigma = 0.016$ (or $\sigma = 0.018$) for the full redshift range and $\sigma = 0.011$ (or $\sigma = 0.010$) at $0.10 \leq z \leq 0.30$. There are 125 clusters (∼26%) in the full sample and 81 in the restricted redshift interval. Note that this process is not guided whatsoever. I use an aperture that does not scale with redshift (360″) and check if there are at least three LRGs at the same redshift, taking as reference the redshift of the LRG closest to the cluster center (within 120″). As I impose a minimum number of three galaxies, projection effects are minimized and we can see that the accuracy is greatly improved, reaching the 0.01 level found by [Koester et al. 2007].

4.5 Application of the photometric redshift estimator

As an application of the empirical photometric redshift relation obtained in §4.3 I used SDSS data to derive new redshift estimates for the supplemental version of the Northern Sky Optical Cluster Survey (NoSOCS, Lopes et al. 2004). This cluster catalog contains candidates to $z \sim 0.50$, but the redshift estimates were based in a simple magnitude-redshift relation. For that purpose, I use the gap-technique described in [Katgert et al. 1996] and Olsen et al. 2005, which identifies gaps in the redshift distribution that are larger than a given value to separate groups. The gap size adopted is $\Delta z = 0.005$ ($1+z$) [Olsen et al. 2005], which is approximately 1,500 km/s in the restframe. I considered the photometric redshifts available in the training sample to select all galaxies within 0.50 $h^{-1}$ Mpc of the cluster center. When performing galaxy selection, the only requirement I make is that the photo-z of the galaxy should be greater than 0.01 (to avoid very nearby structures and failures within the SDSS estimates). After applying the gap-technique, a number of groups in z-space is identified for each cluster.

The next step is to assess the significance of each of these groups. For that I consider the area of 400 square degrees described above. For each group I draw 1000 sets of galaxies from the 400 square degrees catalog. These sets have the same number of galaxies as in the cluster region where the group was identified. The gap-technique is applied exactly as before and then I check the probability of finding groups with at least the same number of galaxies at the redshift of the original group. A field group is considered if its redshift is within ±0.005 of the group identified in the cluster region. The significance is given by the difference between one and the achieved probability. I only consider groups that are significant at the 99% level.

From all the significant groups I select the one that has the smallest redshift difference to the spectroscopic value of the cluster in question. This group should also have at least three member galaxies (most have many more) and be found within 3′ of the cluster center. I have also ran this group identification procedure with one slight modification, which is the gap size. That was modified to $\Delta z = 500(1 + \exp(-(N - 6)/33))/c$, where $N$ is the number of galaxies found in the redshift survey of a cluster [Adami et al. 1998], and $c$ is the speed of light in km/s.

In Figure 6 I summarize the results obtained for the two gap sizes adopted. In the bottom panel (b) the gap size considered is $\Delta z = 0.005(1 + z)$, being $\Delta z = 500(1 + \exp(-(N - 6)/33))/c$ in the upper panel (a). The total number of clus-
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5 CONCLUSIONS

In this work I described the construction of a large catalog of luminous red galaxies with photometric redshifts at \( z < 0.70 \) within SDSS\(^2\). This catalog is based on an empirical relation to derive the photo-\( z \)s. Such relation uses three bands only for achieving results as accurate as those obtained by other methods (sometimes based in more bands; Padmanabhan et al. 2003; Collister et al. 2007). The comparison of photometric and spectroscopic redshifts show no large systematic effects in the redshift range probed, which suggests that this sample is suitable for large-scale structure studies.

I have also investigated what the main systematics are in the estimation of photometric redshifts of galaxy clusters at \( z < 0.55 \). That represents an improvement respective to some deep cluster catalogs derived from SDSS. When these catalogs have accurate photometric redshift estimates, these are truncated at \( z < 0.44 \) (Goto et al. 2002). When they go a little further \((z = 0.5)\) the estimates are not as accurate (Kim et al. 2002). I also showed the relevance of using the \( g-i \) color, and to a lesser extent the mean \( r \) magnitude, to improve the photometric redshift accuracy, especially at high-\( z \) \((z > 0.4)\). Besides that, I show that on top of a “traditional” background correction it is very important to select galaxies from a fixed luminosity range and perform a careful selection of early type galaxies. In this work, this pre-selection of red galaxies is made through a variable cut (in redshift) in the \( u-r \) color. The main advantage of this type of selection is to only require that clusters should exhibit a population of early type galaxies towards their cores. There is no need for these galaxies to exhibit a narrow red sequence. This type of requirement is very important for the cases where the photometric errors are large or the red-sequence is still being formed, which could be the case at high redshifts.

The results obtained for clusters are independent of the way these are selected. So, the methodology described in this work should be valid for any type of clusters, selected by different techniques and wavelengths. The only requirement is to have the proper filters for separating early and late type galaxies and to track the 4000 \( \AA \) break. For SDSS data this method works for clusters at \( z < 0.55 \). Other considerations are made regarding accuracy. I show that the results shown here are in good agreement to previous works.

The empirical relation derived for clusters is applied to 7409 clusters from the NoSOSCs supplemental catalog (Lopes et al. 2004) which are found within SDSS. For these clusters I was able to update the photometric redshift estimates, deriving more accurate values than before (when using only magnitudes within the DPOSS data). This cluster catalog, with the new redshifts, will be updated in a future work, where substructure and superposition effects will be investigated. This catalog is also being used to derive velocity dispersions and mass estimates for the lower redshift systems \((z < 0.1)\) and to investigate scaling relations in clusters.

Finally, I tried to identify groups in redshift space using photometric redshifts of galaxies available in SDSS. I found that for \( \sim 60\% \) of clusters (mostly at \( z < 0.4 \)) it is possible to clearly identify a group using only photometric redshifts of galaxies. When comparing \( \Delta z_{\text{spec}} \) of clusters to the photometric redshift of the nearest group, \( \sigma = 0.011 \). If a different gap size is employed when searching for the groups the rate of identified systems increases to \( \sim 80\% \), but the accuracy is a little worst (\( \sigma = 0.014 \)). This procedure represents an al-

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\(^2\) The LRG catalog can be retrieved from the electronic edition of this journal or by request to the author.
ternative approach for cluster detection, based on galaxy photometric redshifts.

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