THEORY OF “JITTER” RADIATION FROM SMALL-SCALE RANDOM MAGNETIC FIELDS AND PROMPT EMISSION FROM GAMMA-RAY BURST SHOCKS

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Received 2000 January 18; accepted 2000 May 2

ABSTRACT

We demonstrate that the radiation emitted by ultrarelativistic electrons in highly nonuniform, small-scale magnetic fields is different from synchrotron radiation if the electron’s transverse deflections in these fields are much smaller than the beaming angle. A quantitative analytical theory of this radiation, which we refer to as jitter radiation, is developed. It is shown that the emergent spectrum is determined by statistical properties of the magnetic field. The jitter radiation theory is then applied to internal shocks of γ-ray bursts (GRBs). The model of a magnetic field in GRBs proposed by Medvedev & Loeb in 1999 is used. The spectral power distribution of radiation produced by the power-law–distributed electrons with a low-energy cutoff is well described by a sharply broken power law: \( P(\omega) \propto \omega^{-1} \) for \( \omega \leq \omega_{jm} \) and \( P(\omega) \propto \omega^{-(p-1)/2} \) for \( \omega \geq \omega_{jm} \), where \( p \) is the electron power-law index and \( \omega_{jm} \) is the jitter break frequency, which is independent of the field strength but depends on the electron density in the ejecta, \( \omega_{jm} \propto n_{e}^{-1/2} \), as well as on the shock energetics and kinematics. The total emitted power of jitter radiation is, however, equal to that of synchrotron radiation.

Since large-scale fields may also be present in the ejecta, we construct a two-component, jitter + synchrotron spectral model of the prompt γ-ray emission. Quite surprisingly, this model seems to be readily capable of explaining several properties of time-resolved spectra of some GRBs, such as (1) the violation of the constraint on the low-energy spectral index called the synchrotron “line of death,” (2) the sharp spectral break at the peak frequency, inconsistent with the broad synchrotron bump, (3) the evidence for two spectral subcomponents, and (4) possible existence of emission features called “GRB lines.” We believe these facts strongly support both the existence of small-scale magnetic fields and the proposed radiation mechanism from GRB shocks. As an example, we use the composite model to analyze GRB 910503, which has two spectral peaks. At last, we emphasize that accurate GRB spectra may allow precise determination of fireball properties as early as several minutes after the explosion.

Subject headings: gamma rays: bursts — magnetic fields — radiation mechanisms: nonthermal

1. INTRODUCTION

The conventional paradigm of the generation of radiation by relativistic electrons in magnetic fields is totally based on the theory of synchrotron radiation. We demonstrate that this theory is invalid if a magnetic field is tangled on very short spatial scales, and we develop a quantitative theory of radiation in this case. Apparently, the required short-scale fields may naturally be present in astrophysical shocks. Here we focus on radiation from γ-ray burst (GRB) shocks, for which a detailed theory of the formation of magnetic fields has recently been elaborated.

The relativistic blast-wave model of cosmological GRBs (see, e.g., a review by Piran 1999) explains fairly well many observational features of this phenomenon, such as the rapid variability of γ-ray flux, the prompt optical flash, the light curves and spectra of delayed afterglows, etc. This model interprets the prompt γ-ray flash as synchrotron radiation emitted by Fermi-accelerated electrons in internal shocks propagating in the ejecta (Rees & Mészáros 1994) and then Lorentz-boosted to the γ-ray band. The afterglows are explained in a similar way, as the emission from an external shock (Mészáros & Rees 1993) propagating into the interstellar medium. To achieve the observed very high luminosities, the magnetic field in the GRB shocks must be of nearly equipartition strength, \( \epsilon_B = B^2 / 8\pi e_r \sim 1 \), where \( e_r \) is the thermal energy density of the shocked material.

Until very recently, the equipartition assumption was completely unjustified. Medvedev & Loeb (1999) have shown that the relativistic two-stream instability is capable of producing magnetic fields with \( \epsilon_B \sim 10^{-1} \) to \( 10^{-5} \) in both internal and external shocks. Observations of afterglow spectra and light curves yield values of \( \epsilon_B \) from \( \sim 10^{-1} \) to \( 10^{-2} \) for GRB 970508 (Wijers & Galama 1998; Frail, Waxman, & Kulkarni 2000; Granot, Piran, & Sari 1999) and to \( \sim 10^{-5} \) for GRB 990123 and GRB 971214 (Galama et al. 1999). Recent detection of polarization of the optical afterglow of GRB 990510 (Covino et al. 1999; Wijers et al. 1999) indicates that the geometry of the magnetic field in the shock is consistent with the predictions of Medvedev & Loeb (1999) for collimated outflows (Ghisellini & Lazzati 1999; Gruzinov 1999; Sari 1999).

The magnetic field produced in GRB shocks randomly fluctuates on a very small scale of roughly the relativistic skin depth, which is \( \lambda_B \sim 10^2 \) cm in internal shocks, for instance. On the other hand, the emitting ultrarelativistic electrons have much larger Larmor radii. Therefore, the electron trajectories are not helical, as they would be in a homogeneous field. Thus, the theory of synchrotron radiation derived for homogeneous fields is not applicable, and the spectrum of the emergent radiation is different. Such a situation has never been considered in the astrophysical literature.

In this paper we investigate the effect of small-scale magnetic fields on the properties of radiation. For concreteness, we focus primarily on internal shocks. We show that there

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are two regimes, depending on the ratio of the particle’s deflection angle and the relativistic beaming angle. Which regime is realized depends on the magnetic field properties, $B$ and $\lambda_B$, but is independent of the particle’s energy. When deflections are large compared to beaming, synchrotron radiation is emitted. Otherwise, when the particle’s deflections are small, a new type of radiation—jitter radiation—is produced. A quantitative analytical theory of this radiation is developed in this paper. For the power-law–distributed electrons with a cutoff, $N(\gamma) \sim \gamma^{-p}$ for $\gamma \geq \gamma_{\text{min}}$, where $\gamma$ is the particle’s Lorentz factor and $p$ is the index, the emergent spectrum has the following properties. First, the spectral power peaks at the so-called jitter frequency, $\omega_{\text{jm}} = \omega(\gamma_{\text{min}})$, which, unlike the synchrotron frequency, is independent of the magnetic field strength but instead depends on the particle density in the shock, $\omega_{\text{jm}} \propto n^{1/2}$. Second, at low frequencies, $\omega \lessapprox \omega_{\text{jm}}$, the spectral power scales as $P(\omega) \propto \omega^1$, in contrast to the synchrotron spectrum, for which $P(\omega) \propto \omega^{1/3}$. The high-frequency portion is, however, determined by the electrons, $P(\omega) \propto \omega^{-(p-1)/2}$. Third, the total (i.e., frequency-integrated) powers emitted in the jitter and synchrotron regimes are identical.

Since large-scale fields (e.g., due to a magnetized progenitor) may also be present in the shocked material, we construct a composite, two-component jitter + synchrotron (JS) spectral model of the prompt $\gamma$-ray emission. We then compare the predictions of this model with presently available data collected (mostly) by the Burst and Transient Source Experiment (BATSE) on the Compton Gamma Ray Observatory (CGRO). It turns out that the JS model is able to explain naturally some properties of the GRB spectra that are inconsistent with a simple synchrotron shock model. First, almost half of all BATSE bursts violate the so-called synchrotron “line of death” prediction, i.e., their low-frequency spectral indices are greater than the maximal admissible value of $1/4$ (Preece et al. 1998). The spectra of these bursts are, however, consistent with the steeper $\omega^1$ law of jitter radiation. Second, the sharp change of the spectral index at a peak frequency seen in some bursts is also consistent with our model. Third, the JS model theoretically supports the fact that some GRBs have two spectral subcomponents (Pendleton et al. 1994). Fourth, there is an indication that spectra of some GRBs exhibit spectral features (“GRB lines,” see Briggs 2000 for a review and some BATSE candidates). It should be noted, however, that the results from Ginga and BATSE are somewhat controversial. We demonstrate that a linelike spectral feature may be associated with a weak jitter component in a synchrotron-dominated spectrum. If a future analysis confirms that “GRB lines” are real features and not instrumental (or other) artifacts, then they provide, together with the synchrotron component, precise information about properties of cosmological fireballs just a few hundred seconds after the explosion. We illustrate this on the example of GRB 910503, which has been observed by all four instruments on CGRO and which exhibits a second spectral peak at roughly twice the synchrotron peak frequency. A simple fit of a spectral shape readily yields the value of the magnetic field, $e_B \approx 4 \times 10^{-4}$ in the shock, which is in agreement with results of a completely different analysis by Chiang & Dermer (1999). It should be noted that a reliable identification/detection of the jitter spectral features will provide direct evidence that the magnetic fields in GRBs are generated by the two-stream instability, since this is the only presently known mechanism capable of producing small-scale, large-amplitude fields in shocks.

Finally, we emphasize that the advantage of our JS model is that it was specially designed not in order to explain peculiarities of the GRB spectra but solely to study the physical effect of small-scale magnetic fields. The phenomenon considered in this paper is quite general and clearly relevant not only to the emission from internal GRB shocks. A similar mechanism of emission is expected to operate in external GRB shocks and possibly in more conventional supernova shocks and blazar jets.

The rest of this paper is organized as follows. A qualitative consideration of the radiation mechanisms is presented in § 2. In § 3 the structure and properties of magnetic fields in GRB internal shocks are discussed. In § 4 we present a quantitative analytical theory of jitter radiation. A two-component, jitter + synchrotron spectral model of the prompt $\gamma$-ray emission is presented in § 5. We compare the predictions of the model with recent observational results in § 6. Finally, § 7 is the conclusion.

2. RADIATION FROM SMALL-SCALE FIELDS: GENERAL CONSIDERATION

Let us consider a plasma at rest threaded by a magnetic field. Let us now consider an ultrarelativistic electron with the Lorentz factor $\gamma \gg 1$ moving in a magnetic field. Because of beaming, the emerging radiation is concentrated in a narrow cone with the opening angle $\Delta \theta \sim 1/\gamma \ll 1$ in the direction of the particle’s velocity. In a uniform magnetic field the electron moves along a helical trajectory, so that the radiation seen by an observer consists of short pulses repeated every cyclotron period. The synchrotron spectrum therefore consists of a large number of cyclotron harmonics, the envelope of which is determined by the inverse duration of the pulses (Rybicki & Lightman 1979). The spectrum is peaked near the critical synchrotron frequency $\omega_c = (3/2)\gamma^2 eB/m_e c$, where $B_\bot = B \cos \chi$ and $\chi$ is the particle’s pitch angle.

If the magnetic field is randomly tangled and the correlation length is less than a Larmor radius of an emitting electron, then the electron experiences random deflections as it moves through the field. Its trajectory is, in general, stochastic. This is similar to the collisional motion of an electron in a medium. Bremsstrahlung quanta are emitted in every collision. Unlike the bremsstrahlung case, here “collisions” are due to small-scale inhomogeneities of the magnetic field rather than to electrostatic fields of other charged particles. Since the Lorentz force depends on the particle’s velocity, the emergent spectrum will be somewhat different from pure bremsstrahlung. There is also an alternative physical interpretation of the process. For an ultrarelativistic electron, the method of virtual quanta applies (Rybicki & Lightman 1979). In the rest frame of the electron, the magnetic field inhomogeneity with wavenumber $k$ is transformed into a transverse pulse of electromagnetic radiation with frequency $kc$. This radiation is then Compton-scattered by the electron to produce observed radiation with frequency $\sim \gamma^2 kc$ in the lab frame.

Keeping this general physical picture in mind, we now analyze the problem in more detail. Let us consider a non-uniform random magnetic field with a typical correlation scale $\lambda_B$; the Larmor radius of the electron, $\rho_e = \gamma m_e c^2/\epsilon B_\bot$, is less or comparable to $\lambda_B$. The emerging spectrum depends
on the relation between the particle’s deflection angle, \( \alpha \), and the beaming angle, \( \Delta \theta \) (Landau & Lifshitz 1975). For ultrarelativistic particles and small deflection angles, the latter is estimated as follows. The particle’s momentum is \( p \sim \gamma m_e c \). The change in the perpendicular momentum due to the Lorentz force acting on the particle during the transit time \( t \sim \lambda_B/c \) is \( p_{\perp} = F_{\perp} t \sim eB_{\perp}\lambda_B/c \). The angle \( \alpha \) is then
\[
\alpha \sim \frac{eB_{\perp}\lambda_B}{m_e c^2} \gamma \frac{\lambda_B}{\rho_e}.
\] (1)

It is interesting to note that this ratio is independent of the particle’s energy (i.e., \( \gamma \)) and is determined by the properties of the magnetic field only, i.e., by \( B \) and \( \lambda_B \). It is more convenient, however, to use the wavevector, \( k_B \), as a measure of the magnetic field scale, instead of \( \lambda_B \sim k_B^{-1} \). We now define the deflection-to-beaming angle ratio as follows:
\[
\delta \equiv \frac{\gamma}{k_B \rho_e} \sim \frac{\alpha}{\Delta \theta}.
\] (2)

There are two limiting cases.

First, \( \delta \sim \alpha/\Delta \theta \gg 1 \), i.e., the deflection angle is much larger than the beaming angle (see Fig. 1a). Then, an observer sees radiation coming from short segments (“patches”) of the electron’s trajectory that are nearly parallel to the line of sight (very much like the case of pure synchrotron radiation). The magnetic field in every patch is almost uniform, but it varies from patch to patch. The radiation is completely identical to synchrotron radiation from large-scale weakly inhomogeneous magnetic fields.

Second, \( \delta \sim \alpha/\Delta \theta \ll 1 \), i.e., the deflection angle is smaller than the beaming angle, so that the electron’s entire trajectory is seen by an observer, as shown in Figure 1b. The particle moves along the line of sight almost straight and experiences high-frequency jittering in the perpendicular direction as a result of the random Lorentz force. We therefore refer to the emerging radiation as “jitter” radiation. Its spectrum is determined by random accelerations of the particle. Let us imagine an electron moving ultrarelativistically along the line of sight with a constant velocity; the transverse accelerations of the electron are small. In the laboratory frame, the electron passes through the magnetic field inhomogeneities having a typical scale \( \lambda_B \sim k_B^{-1} \) with the velocity \( c \). In the particle’s frame (i.e., where its parallel velocity vanishes), the field correlation scale is \( \lambda_B' \sim \lambda_B/\gamma \sim (k_B \gamma)^{-1} \) as a result of the Lorentz transformation. The electron’s perpendicular acceleration changes significantly during \( \tau \sim \lambda_B'/c \sim (\gamma k_B c)^{-1} \), so that the characteristic frequency of the emitted radiation is \( \omega_{ej} \sim \gamma^{-1} \gamma k_B c \). In the laboratory frame, this frequency is boosted to \( \omega = \gamma \omega_{ej} \). Thus, the spectrum of the emergent radiation is peaked at the frequency \( \omega_{ej} \sim \gamma^2 k_B c \). This frequency is higher than the critical synchrotron frequency in the uniform magnetic field of the same strength, \( \omega_{\epsilon}\gamma \), namely,
\[
\frac{\omega_{\epsilon}}{\omega_{ej}} \approx \frac{3}{2} \delta \ll 1,
\] (3)
as follows from equation (2).

We should warn here that despite their apparent similarity, the jitter and free-electron laser emission mechanisms are quite different. The wiggler field in free-electron lasers is appropriately adjusted for the electron motion to be in phase with the produced radiation field to emit coherent radiation. Jitter radiation is, in general, incoherent.

3. THE STRUCTURE OF THE MAGNETIC FIELD IN GRB SHOCKS

To proceed further, a model for a magnetic field in GRB shocks is required. We use the only presently available quantitative theory of the magnetic field generation in shocks proposed by Medvedev & Loeb (1999). To be specific, we focus on internal shocks that produce \( \gamma \)-ray emission. External shocks that are responsible for the delayed afterglows may be treated similarly and will be considered in a future publication.

Shock fronts are shown to be natural sites of the magnetic field generation. Right before a shock, the inflowing (in the shock frame) bulk plasma particles meet the outflowing particles that were reflected (scattered) from the shock. Such a two-stream motion is kinetically unstable. The emergent magnetic field is random with zero mean and lies in the plane of the shock front, i.e., perpendicular to the shock velocity. In principle, all plasma species participate in the instability. We assume the protons and electrons to be the only species and discuss their contributions separately.

It is important to emphasize that the generated magnetic field fills the entire volume of a shock shell and is not located within a thin layer of order several skin depths near the front. There is a gas flow through a shock. Because of flux freezing, the generated magnetic field is transported with the shocked material downstream. This material is replenished with a fresh one where a new magnetic field is thus continuously produced. Since the magnetic field is long lived and does not decay in a dynamical time, as indicated by numerical simulations (see references in Medvedev & Loeb 1999), this field will be present in the entire ejecta.

3.1. Fields Produced by the Electrons and Protons

In this subsection we briefly summarize the main results of the theory of Medvedev & Loeb (1999) for future reference. Since electrons are light, the instability induced by them is rapid: the typical e-folding length (i.e., the e-folding time times the shock speed) is much smaller than the char-

Fig. 1.—Emission from various points along the particle’s trajectory. (a) \( \alpha \gg \Delta \theta \); emission from selected parts (bold portions) of the trajectory is seen by an observer. (b) \( \alpha \ll \Delta \theta \); emission from the entire trajectory is observed.
characteristic shock thickness determined by the Larmor radius of heavier protons. Therefore, the magnetic field energy grows rapidly and reaches the approximate equipartition with the electron kinetic energy,

$$\frac{B_e^2}{8\pi} = \eta_e \gamma_{\text{int}} m_e c^2 n = \frac{m_e}{m_p} e_t \eta_e,$$

i.e., \( \eta_{Be} = B_e^2/8\pi e_t = (m_e/m_p) \eta_e \), where \( \eta_e \) is the efficiency factor for the electrons that incorporates uncertainties due to the nonlinear phase of the instability; one infers from numerical simulations that generically \( \eta_e \approx 0.1-0.01 \), \( \gamma_{\text{int}} \) is the relative Lorentz factor of two colliding shells that produce an internal shock, \( \gamma_{\text{int}} \sim 2-4 \), and \( n \) is the number density of particles in the expanding shell, before the shock. The spatial correlation scale, \( k_{Be} \), of the field behind the shock is

$$k_{Be} = \frac{4\gamma_{\text{int}} \omega_{pe}}{2^{1/4} \gamma_e c},$$

where \( \omega_{pe} = 4\pi e^2 n/m_e \) is the electron plasma frequency squared, \( \gamma_e \) is the initial effective thermal Lorentz factor of the streaming electrons, and an extra factor of \( 4\gamma_{\text{int}} \) is due to the relativistic shock compression.

The protons may generate magnetic fields too. Since they are much heavier than electrons, the spatial coherence length and the \( e \)-folding length are comparable to the thickness of the collisionless shock. Therefore, the field does not have enough time to grow during the flow transit through the shock. It may, however, grow behind the shock if the two-stream motion of the protons persists there. If this is the case, then there are two possibilities. First, if there is no energy transfer from the protons to the electrons or if it is slow compared to the rate of the field growth, then the magnetic field energy may be as large as

$$\frac{B_e^2}{8\pi} = \eta_p \gamma_{\text{int}} m_p c^2 n = e_t \eta_p \sim 0.1 e_t,$$

provided \( \eta_p \sim 0.1 \). Second, if the energy transfer is fast, which may be the case in fields that are in equipartition with the electrons or stronger, then the protons may efficiently damp their energy into the emitting electrons, so that the resultant field will be \( B_e^2/8\pi \sim B_p^2/8\pi \). Alternatively, if no magnetic field is generated downstream of the shock, the strength of the field produced by the protons may be orders of magnitude lower. Which case becomes realized can be learned from numerical particle simulations or from observations. We keep \( \eta_{Be} = B_e^2/8\pi e_t \), as a parameter. The characteristic correlation scale of the generated magnetic field is

$$k_{BP} = \frac{\omega_{pe}}{2^{1/4} \gamma_e c},$$

where \( \omega_{pe} = 4\pi e^2 n/m_p \) and \( \gamma_e \sim 2 \) is the initial effective thermal Lorentz factor of the streaming protons. The compression factor, \( 4\gamma_{\text{int}} \), is absent because the field is likely produced after the compression occurs.

### 3.2. The Model

From equations (2), (4), and (5) we estimate the \( \delta_e \)-parameter for the electrons,

$$\delta_e = \frac{1}{2^{1/4}} \left( \frac{\gamma_e}{\gamma_{\text{int}}} \right)^{1/2} \sqrt{\eta_e} \equiv \phi \sqrt{\eta_e} \lesssim 1.$$

The exact value of \( \phi = 2^{-3/4}(\gamma_e/\gamma_{\text{int}})^{1/2} \) is somewhat uncertain: \( \gamma_e \) may evolve during the instability from its initial value \( \gamma_e \sim 2-3 \) to \( \gamma_e \sim \gamma_{\text{int}} \sim 3-4 \) as a result of nonlinear effects.\(^2\) The numerical prefactor may also be affected. Therefore, we assume that possible values of \( \delta_e \) are in the range \( 1 \lesssim \delta_e \lesssim 10^{-4} \) (given the uncertainty in \( \eta_e \) from 0.1 to 0.01) and generically \( \delta_e \sim 0.1 \). The \( \delta \)-parameter for the protons is

$$\delta_p = 2^{-1/4}(\gamma_p \gamma_{\text{int}})^{1/2} \frac{m_p}{m_e} \sqrt{\eta_p} \gtrsim 1,$$

unless \( \eta_p \) is too small: \( \eta_p \lesssim 10^{-7} \) (i.e., \( e_{Be} \lesssim 10^{-3} e_{Be} \)).

As will be shown below in § 4, a spatial spectrum of the magnetic field \( B_e \) with \( \delta_e < 1 \) is required to calculate the spectrum of jitter radiation. This distribution of \( B_e \) over scales is difficult to find from first principles because it is determined by fully nonlinear dynamics of the instability process. Some constraints may, however, be drawn. First, the two-stream instability produces magnetic fields in a finite range of scales, \( 0 \leq k \leq k_{\text{crit}, e} \), where \( k_{\text{crit}, e} \sim k_{Be} \) to within a numerical factor of order unity. Second, the field grows until it becomes strong enough to deflect the particles in the transverse direction by \( \sim 1 \) radian on a scale of the field coherence length, i.e., \( B_e^{-1} \sim \rho_p \sim k_{Be}^{-1} \). If we now assume that each Fourier harmonic, \( B_e \), is amplified independently of others, we obtain \( B_e \propto k \) for \( k \lesssim k_{\text{crit}, e} \). This is the lower limit: the spectrum of the magnetic field can only be steeper than linear.

In reality, all Fourier harmonics are coupled. Thus, when at least one harmonic reaches the subequipartition strength, the streaming electrons are isotropized by random Lorentz forces. This prevents the growth of other harmonics. Therefore, the spectrum of the field will be steeper than linear and will have a maximum near \( k_{Be} \). The simplest possible model is a power law,

$$B_k = \begin{cases} C_B k^\mu & \text{if } 0 \leq k \leq k_{Be}, \\ 0 & \text{otherwise}, \end{cases}$$

where \( C_B \) is a normalization constant and \( \mu \geq 1 \) is a power-law spectral index of the magnetic field, being a free parameter here. The constant \( C_B \) may be determined using Parseval’s theorem,\(^3\) \( \int k^0 \int |k|^2 B_k^2 dk = \pi \int_0^\infty B^2_L dk \), where \( L \) is the system size. Taking into account that \( \int B_k^2 dx = B_L^2 L = B^2 e cT \), where \( T \) is the total duration of the pulse, we write

$$C_B^2 = \pi(2\mu + 1)cT B^2_{Be} k_{Be}^{(2\mu + 1)}. \quad (11)$$

The magnitude of the field, \( B_k \), is given by equation (4).

Strictly speaking, this model may be used for not too small values of \( k : k > k_{\text{min}, e} \) where \( k_{\text{min}} \) is set by the condition \( \gamma_e/\gamma_{\text{crit}, e} = 1 \) (see eq. [2]). The field harmonics with \( k < k_{\text{min}} \) are large-scale ones, and they contribute to synchrotron radiation. Using equation (2), it is easy to obtain \( k_{\text{min}} = \delta_e k_{Be} \). It is also useful to introduce the small-scale field component as follows:

$$B^2_{SS} = \int_{k_{\text{min}}}^{k_{Be}} B^2_k dk \quad \frac{B^2_{SS}}{B^2_e} = 1 - \delta^{2\mu + 1}. \quad (12)$$

\(^2\) Note that the inflowing electrons are cold; they are not Fermi-accelerated yet. Note also that \( \gamma_e \) cannot be greater than \( \gamma_{\text{int}} \) for the instability to operate. Otherwise, no magnetic field is produced.

\(^3\) We use the following definition of the Fourier transform:

$$f(\omega) = \int f(t) e^{-i\omega t} dt.$$
Hereafter we omit the subscripts to $\delta$; this should not cause any confusion.

To summarize, we assume the following structure of the magnetic field sketched in Figure 2. First, there is a magnetic field produced by the electrons, $B_1$, the magnitude of which is given by equation (4). A large fraction of this field contributes to the small-scale component, in accordance with equation (12). Its distribution over spatial scales is described by a power law with index $\mu$ (see eq. [10]). The rest, $B^2_2 - B^2_{SS}$, contributes to the large-scale component. Second, there is a magnetic field produced by the protons, $B_p$. This field is a large-scale field. Both $B_1$ and $B_p$ are random with zero mean. Third, there could be an ordered magnetic field left from a magnetized progenitor, $B_\ast$. We define the total large-scale magnetic field as follows:

$$B^2_{LS} = B^2_0 + (B^2_1 - B^2_{SS}), \quad B^2_0 = B^2_\ast + B^2_p,$$

where $B_0$ is the fraction of the large-scale field that is not produced by the electrons.

4. RADIATION FROM SMALL-SCALE FIELDS: QUANTITATIVE THEORY

In § 2 we qualitatively demonstrated that there are two regimes of radiation. An ultrarelativistic electron propagating through small-scale fields generated only by the electrons ($\delta \ll 1$) emits jitter radiation. The electron moving through larger scale fields generated only by the protons ($\delta \gg 1$) emits synchrotron radiation. The radiation spectrum from a magnetic field with a broadband distribution over scales is neither of the above and must be calculated by appropriate scale averaging of a particle trajectory. However, for a bimodal distribution of § 3.2 such that the magnetic energy in $B_1$ harmonics for which $\delta \sim 1$ (i.e., where the transition from a jitter to a synchrotron regime occurs) is small, the separation of scales is possible. The resultant radiation will be approximately a composition of the jitter and synchrotron spectra.\footnote{Physically, a particle moves along a helical trajectory about field lines of a large-scale field. This trajectory is slightly perturbed by high-frequency jittering in the (instantaneous) transverse direction as a result of a small-scale field component. The intensities of the spectral subcomponents are determined by the magnetic field energy densities at large and small scales.}

For sufficiently small values of $\delta$ such that $\alpha \ll \Delta \theta$, the velocity $v$ of a particle is almost constant, whereas its acceleration $\omega \equiv \dot{\theta}$ varies with time. Calculating the Fourier component of the electric field using the Liénard-Wiechert (retarded) potentials, one arrives at the following expression for the total energy emitted per unit solid angle $d\Omega$ per unit frequency $d\omega$ (Landau & Lifshitz 1975, § 77; see also Rybicki & Lightman 1979, § 3.2):

$$dW = \frac{e^2}{2\pi c^3} \left(\frac{\omega}{\omega_0}\right)^4 \left[ n \times \left( n - \frac{v}{c} \right) \times w_{\omega} \right]^2 d\Omega \frac{d\omega}{2\pi},$$

where $w_{\omega} = \int w e^{i\omega t} dt$ is the Fourier component of the particle’s acceleration, $\omega = \omega_0(1 - n \cdot v/c)$, and $n$ is the unit vector pointing toward the observer.

First, we can simplify the vector expression in equation (14). Indeed, in the ultrarelativistic case the longitudinal component of the acceleration is small compared to the transverse component, $\omega / w_\omega \sim 1 / \gamma^2 \ll 1$. Therefore, $v$ and $w$ are approximately perpendicular to each other. Second, the dominant contribution to the integral over $d\Omega$ comes from small angles $\theta \sim 1 / \gamma$ with respect to the particle’s velocity. Therefore, we approximately write $\omega = \omega_0 (1 - v/c + \theta^2/2) \approx \omega_0 (1 - v^2/c^2 + \theta^2) = \omega_0 (\theta^2 + \gamma^{-2})$. We can now replace integration over the solid angle $d\Omega \approx \theta d\theta d\phi$ with integration over $d\phi d\omega / \omega$ and integrate equation (14) over the azimuthal angle, $\phi$, from 0 to $2\pi$. The spectral energy finally becomes

$$dW = \frac{e^2}{2\pi c^3} \int_{\omega_0/\gamma^2}^{\infty} \frac{d\omega'}{\omega'^{3.2}} \left[ 1 - \frac{\omega}{\omega_0 \gamma^{2}} + \frac{\omega^2}{2\omega_0^2 \gamma^{4}} \right] d\omega' = \frac{e^2}{2\pi c^3} B_k^2 \frac{dW_{\omega}}{\gamma m_e c}.$$

The leading term in the above equation is due to high-frequency “jittering” of the electron as it moves through the random magnetic field. The second and third terms in the parentheses are corrections due to the angular structure of the radiation field convolved with the relativistic beaming.

The acceleration $w$ is found from the equation of motion, $\dot{p} = (e/c)w \times B$. In general, $B$ may vary both in amplitude and in direction. In relativistic GRB shocks, radiation is beamed; only a conical section from0 to $2\pi$. The spectral energy finally becomes

$$dW = \frac{e^2}{2\pi c^3} \int_{\omega_0/\gamma^2}^{\infty} \frac{d\omega'}{\omega'^{3.2}} \left[ 1 - \frac{\omega}{\omega_0 \gamma^{2}} + \frac{\omega^2}{2\omega_0^2 \gamma^{4}} \right] d\omega' = \frac{e^2}{2\pi c^3} B_k^2 \frac{dW_{\omega}}{\gamma m_e c}.$$

For the magnetic field distribution, we use the model given by equation (10).

- Fig. 2.—Model for the magnetic field in GRB shocks.
where and is the classical electron radius. As is expected, the characteristic frequency of the emergent radiation is

\[ \omega_{j} = \gamma^{2} B_{0} c = 2^{7/4} \gamma^{2} \gamma_{\text{int}} \gamma_{e}^{-1/2} \omega_{pe} \]  

(18)

(see eqs. [5] and [8]). The function \( J(\xi) \) is defined as

\[ J(\xi) = (2 \mu + 1) e^{2 \mu \frac{t}{\delta}} \int_{\mu}^{\infty} \left( \min \left\{ 2, \frac{t}{\delta} \right\} - I(\xi) \right) d\xi, \]  

(19)

where \( I(\xi) = \int_{\mu}^{\infty} (1 - \xi + \frac{1}{2} \xi^{2}) \gamma_{\text{int}} d\gamma_{\text{int}} \) and \( \min(a, b) \) denotes the smallest of \( a \) and \( b \). From equations (2), (4), and (18) we have \( \omega_{j} = \gamma^{2} B_{0} / \delta m_{e} c \). Equation (17) may now be cast in the form

\[ P(\omega) = \frac{e^{2}}{2c} \delta^{2} \frac{\omega_{j}}{\gamma^{2} B_{0}} J \left( \frac{\omega}{\omega_{j}} \right). \]  

(20)

This spectrum is shown in Figure 3 for several values of \( \mu \). In general, the steeper the field distribution, \( B_{0} \propto k^{4} \), the more peaked is the radiation spectrum and the closer the peak frequency to \( 2 \omega_{j} \). Note that the discontinuity of the slope is artificial. This is due to the \( k_{\text{min}} \) cutoff because the model we use does not continuously interpolate between the jitter and synchrotron radiation limits (i.e., between the \( \delta \ll 1 \) and \( \delta \gg 1 \) cases). This discontinuity is less prominent for large \( \mu \) and small \( \delta \).

The total emitted power may be obtained by integrating equation (17) over frequencies. A simpler way is to express it in terms of the electron’s acceleration, i.e., \( dW/dt = (2e^{2} \gamma / 3c^{3})(w_{\|}^{2} + \gamma^{2} w_{\perp}^{2}) \) (Rybički & Lightman 1979) and substitute \( w_{\|} = 0 \) and \( w_{\perp} \) given by equation (16). The result is

\[ dW/dt = (2/3) r_{e}^{2} c \gamma^{2} B_{0}^{2}. \]

(21)

Note that this expression is identical to that for synchrotron radiation in which a uniform field, \( B_{0}^{2} \), is replaced with its average, \( B_{0}^{2} = \langle B_{0}^{2} \rangle \).

We are now able to calculate the jitter radiation spectrum from an ensemble of electrons. We assume that electrons are accelerated in the shock to a power-law distribution \( N(\gamma) = C_{N} \gamma^{-p} \) (where \( C_{N} \) is a normalization constant) with a minimum Lorentz factor, i.e., \( \gamma_{\text{min}} \leq \gamma < \infty \). The index \( p \) must be \( p \geq 2 \) so that energy does not diverge at large \( \gamma \). We assume the standard value, \( p = 2.5 \) (Sari, Narayan, & Piran 1996), unless stated otherwise. The spectrum is found from

\[ P_{\text{ens}}(\omega) = \int_{\gamma_{\text{min}}}^{\infty} N(\gamma) P(\omega) d\gamma, \]

(22)

noting that \( \omega_{j} = \omega_{j}(\gamma) \propto \gamma_{e}^{-1/2} \), i.e., in exact analogy with synchrotron radiation, \( \omega_{j} \propto \gamma_{e}^{-1/2} \). We obtain

\[ P_{\text{ens}}(\omega) = \frac{C_{N}}{2^{3/4} \gamma_{\text{int}} \gamma_{e}^{1/2} \omega_{pe}} \int_{0}^{\infty} \omega_{j}^{p} \left( \omega_{j}^{2} - (\omega_{j})^{2} \right)^{-p/2} P(\omega) d\omega, \]

(23)

where we introduced the characteristic frequency of radiation \( \omega_{j} \equiv \omega_{j} / \gamma_{\text{min}} \) (see eq. [18]).

\[ \omega_{j} = 2^{7/4} \gamma_{\text{int}} \gamma_{e}^{-1/2} \omega_{pe} \]  

(24)

At low and high frequencies, the spectra may be obtained analytically as follows:

\[ P_{\text{ens}}(\omega) \propto \begin{cases} \left( \omega / \omega_{j} \right)^{1} & \text{if } \omega \ll \omega_{j} \text{,} \\ \left( \omega / \omega_{j} \right)^{p-3/2} & \text{if } \omega \gg \omega_{j}. \end{cases} \]

(25)

At high frequencies the spectrum is analogous to the synchrotron case. The electron power-law index determines the slope. At low frequencies the spectrum is linear in frequency, in contrast to the synchrotron spectrum, which is softer and scales as \( P_{\text{syn}}(\omega) \propto \omega^{1/3} \). The spectrum of radiation from single-speed electrons is peaked at \( \omega \approx \omega_{pe} \) for large \( \mu \)’s, i.e., for a peaked magnetic field distribution, as discussed above (see Fig. 3). Therefore, the spectral break due to the \( \gamma_{\text{min}} \) cutoff occurs at

\[ \nu_{\text{break}} \approx 2 \omega_{pe} / \pi = \left( 2^{7/4} / p \right) \gamma_{\text{min}}^{3/2} \gamma_{e}^{1/2} \omega_{pe} \]

\[ \approx 6.0 \times 10^{6} \gamma_{\text{min}}^{3/2} \gamma_{e}^{1/2} n_{10}^{1/2} \text{ Hz}, \]

(26)

where \( n_{10} \equiv n / 10^{10} \text{ cm}^{-3} \). The above frequency is calculated in the frame of the relativistic expanding shell. This frequency is boosted in the observer’s frame by a factor of \( \gamma_{sh}^{-1} \).

The model of jitter radiation contains two extra parameters, compared to the model of synchrotron radiation. These parameters are the deflection-to-beaming ratio, \( \delta \), and the magnetic field index, \( \mu \), which determines the peakedness of the magnetic field over spatial scales. We now investigate how the spectrum of the emergent radiation depends on these parameters.

\[ \text{Small-scale magnetic fields present in the shock provide effective collisions in the otherwise collisionless plasma and make the Fermi acceleration operate (Medvedev & Loeb 1999).} \]

\[ \text{Recent studies indicate that the high-frequency spectral power index in the prompt GRBs is approximately } -1, \text{ which translates to } p \approx 3. \text{ We keep } p = 2.5 \text{ for illustrative purposes.} \]

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5 We keep this integral in a general form because it contains logarithmic terms for \( \mu = 0.5, 1, \) and 1.5.

6 Small-scale magnetic fields present in the shock provide effective collisions in the otherwise collisionless plasma and make the Fermi acceleration operate (Medvedev & Loeb 1999).

7 Recent studies indicate that the high-frequency spectral power index in the prompt GRBs is approximately -1, which translates to \( p \approx 3 \). We keep \( p = 2.5 \) for illustrative purposes.
Figure 4.—Spectral power $P(\omega)$ of radiation (log-log plot, arbitrary units) emitted by the power-law–distributed electrons vs. $\omega/\omega_{\text{cm}}$ for $\delta = 0.9, 0.7, 0.5, 0.3, \text{and } 0.1$ (from top to bottom). Here $\mu = 1$ and $p = 2.5$. The synchrotron spectrum with the same magnetic field strength is shown (dashed curve) for comparison. The value of $B_{\text{SS}}/B_{\text{e}}$ is kept fixed for all $\delta$.\n
Figure 4 represents the spectral power, $P(\omega)$, given by equations (20) and (23), as a function of $\omega/\omega_{\text{cm}}$, where $\omega_{\text{cm}} = \omega_{\text{cm}}(\nu_{\text{min}})$ for various values of $\delta$. The dashed curve represents the synchrotron spectrum that corresponds to the same magnetic field strength. Note that the frequency is normalized by the synchrotron break frequency, $\omega_{\text{cm}}$, (not the jitter break frequency, $\omega_{\text{jm}}$), to emphasize that, for the fixed field magnitude, $B_{\text{SS}}$, the jitter frequency increases with decreasing $\delta$, in accordance with equation (3). One can see that jitter radiation is well described by the broken power law given by equation (25). Notice also that there is little change in the spectrum shape near the break frequency and the overall decrease of $P(\omega)$ as $\delta$ decreases.

Figure 5 shows $P(\omega)$ versus $\omega/\omega_{\text{cm}}$ for a few values of the magnetic field index, $\mu$. In general, the ratio $B_{\text{SS}}/B_{\text{e}}$ depends on $\mu$. To highlight the effect of the magnetic spectrum shape alone, we keep $B_{\text{SS}}/B_{\text{e}}$ fixed for all $\mu$. One can see from Figure 5 that $\mu$ has little effect on the overall shape of the radiation spectrum. However, as one goes from a flat ($\mu = 0$) to a peaked ($\mu = 10$) magnetic field distribution, the change in slope at the break frequency becomes more abrupt.

4.2. Synchrotron Radiation, $\delta \gg 1$

For completeness, we also consider the case of $\delta \gg 1$. In this case, $\omega/\Delta \theta \gg 1$, while still $\lambda_\mu/\rho_1 \ll 1$ for energetic electrons. The spectral power emitted by a single electron is given as (Landau & Lifshitz 1975, §§ 77 and 74)

$$\frac{dW}{dt d\omega} = \frac{e^2}{3\pi e c^5} \left( \bar{\omega} J_\epsilon \left( \frac{\omega}{\bar{\omega}} \right) \right),$$

(27)

where $\bar{\omega}_J = (3/2)\nu^2 eB_{\text{j}}/m_e c, \langle \ldots \rangle$ denotes the average over the electron's trajectory, and the tilde denotes the local (instantaneous) quantity. Here $F(\xi) = \bar{\omega} J_\epsilon \left( \frac{\omega}{\bar{\omega}} \right) K_{5/3}(\xi d_\epsilon)$ and $K_{5/3}(x)$ is the modified Bessel function of order $5/3$. As for the spectrum from a plasma, the observed radiation comes from regions with different field strength. Therefore, one has to replace $B_{\text{j}}$ in equation (27) with the ensemble average, $\bar{B} = \langle B_{\text{j}}^2 \rangle^{1/2}$, which rigorously yields the standard synchrotron spectrum.

5. JITTER + SYNCHROTRON MODEL OF GRB EMISSION

We now use the model of magnetic fields discussed in § 3.2 to construct a GRB spectral model. All calculations are performed in the frame of the expanding shell. The transition to the observer's frame is obvious. The magnetic field in the shock shell was shown to be subdivided into small-scale and large-scale components. The small-scale field, for which $\delta \leq 1$, yields jitter radiation. Its spectral power is given by equations (20) and (23). The large-scale field, for which $\delta > 1$, yields synchrotron radiation. The relevant theory may be found in Rybicki & Lightman (1979). The composite spectrum thus contains both jitter and synchrotron components and may be schematically represented as follows:

$$P_{J+S}(\omega) = P_J(\omega; B_{\text{SS}}, \delta, \mu) + P_S(\omega; B_{\text{LS}}).$$

(28)

This approximation is valid for the field distribution from § 3.2 unless $\delta \sim 1$, as discussed in the beginning of § 4. It is convenient to normalize frequencies onto the jitter frequency, $\omega_{\text{jm}}$, which is independent of the magnetic field strength (see eq. [26]). The synchrotron-to-jitter frequency ratio is then

$$\frac{\omega_{\text{cm}}}{\omega_{\text{jm}}} = \frac{\omega_{\text{e}}}{\omega_{\text{j}}} \approx \frac{3}{2} \frac{B_{\text{LS}}}{B_{\text{SS}}} \delta,$$

(29)

as follows from equations (3) and (13).

Figure 6 represents the spectrum emitted by single-speed electrons for the same values of $\mu$ as in Figure 3. One can clearly see a sharp feature on top of the broad synchrotron spectrum. Integrating this spectrum over the power-law distribution of $\gamma$'s, we obtain the composite JS model of GRBs. A typical example is shown in Figure 7. In general, there are two bumps: the sharp one is near the jitter frequency and the other, a broad bump, is associated with synchrotron emission. Depending on the ratio $B_{\text{LS}}/B_{\text{SS}}$, these bumps may overlap to produce either featureless broad or sharply
Fig. 6.— Arbitrarily normalized, composite power spectra $P_{J,S}(\omega)$ (log-log plot) emitted by a single electron for several values of the magnetic field spectral index, $\mu = 1$ (short-dashed curve), $\mu = 3$ (long-dashed curve), and $\mu = 10$ (solid curve) for $\delta = 0.6$ and $B_0^2/B_{SS}^2 = 3$.

peaked spectra as well. The high-frequency tail always scales as $P_{J,S}(\omega) \propto \omega^{-(p-1)/2}$. The dependence of the spectrum on $\delta$ is displayed in Figure 8. As an example, we take $B_0^2/8\pi = B_{SS}^2/8\pi$ and $\mu = 10$ (for such $\mu$'s $B_{SS}^2/8\pi \approx B_0^2/8\pi$, and therefore $B_{LS}^2/8\pi \approx B_0^2/8\pi$). A sharp jitter feature is easily seen in the spectrum. As $\delta$ decreases, the synchrotron bump moves toward lower frequencies and decreases in amplitude. The jitter peak decreases even faster, and the spectral feature becomes less prominent. The other parameter, $\mu$, determines the ratio $B_{LS}^2/B_{SS}^2$, but, besides this, its effect on the spectrum is weak (see Fig. 5) and is therefore not shown.

The spectrum of radiation depends on the relative magnitudes of the large- and small-scale field components. Figure 9 represents spectra for $\mu = 10$ and various values of the ratio $B_0^2/B_{SS}^2 \sim B_{LS}^2/B_{SS}^2$. Figure 9a shows the spectral power versus frequency. For strong large-scale fields, the synchrotron spectral component dominates. As $B_{LS}^2/B_{SS}^2$ decreases, the synchrotron peak moves toward lower frequencies. The amplitude of the synchrotron peak decreases too. The position and the amplitude of the jitter peak remain almost equal.

Fig. 7.— Typical composite power spectrum $P_{J,S}(\omega)$ (log-log plot, arbitrary units) for the power-law–distributed electrons with $p = 2.5$ for $\mu = 1$, $\delta = 0.3$, and $B_0^2/B_{SS}^2 = 0$ is shown (solid curve). The synchrotron (long-dashed curve) and jitter (short-dashed curve) subcomponents are also shown.

Fig. 8.— Composite power spectra $P_{J,S}(\omega)$ (log-log plot, arbitrary units) for the power-law–distributed electrons with $p = 2.5$ for $\delta = 0.9, 0.3, 0.1, 0.033, B_0^2/B_{SS}^2 = 1$, and $\mu = 10$.

Fig. 9.— Composite spectra (log-log plots, arbitrary units) for the power-law–distributed electrons with $p = 2.5$ for $B_0^2/B_{SS}^2 = 10^{-4} − 10^4$, $\delta = 0.2$, and $\mu = 10$. (a) Spectral power $P_{J,S}(\omega)$; (b) spectral flux $F_{J,S}(\omega) \equiv P_{J,S}(\omega)/\omega$. 

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unchanged. Thus, in the limit $B_{LS}^2/B_{SS}^2 \to 0$, the spectrum becomes purely jitter. It is illustrative to depict the spectral flux, $F(\omega) \equiv P(\omega)/\omega$ (observationally, this quantity is proportional to the photon spectral flux, $N(E)$, measured in units of photons s$^{-1}$ cm$^{-2}$ keV$^{-1}$), as presented in Figure 9b. It is seen that the slope of the flux below the jitter break continuously decreases as $B_{LS}^2/B_{SS}^2$ decreases. The synchrotron asymptotic slope $\propto \omega^{-2/3}$ is shown for comparison. It is interesting that for $B_{LS}^2/B_{SS}^2 \leq 10^{-1}$ (the actual value depends on $\delta$; the larger $\delta$, the larger the ratio), there is a portion in the spectrum that has the power-law index being less than $-\frac{2}{3}$ and approaching zero in the limit of the vanishing large-scale field. We discuss this property in § 6 in the context of the “line of death” for synchrotron radiation in GRBs.

Finally, it is worthwhile to compare the peak spectral fluxes of jitter, $F_{J,\text{max}} \equiv F_j(\omega_{jm})$, and synchrotron, $F_{S,\text{max}} \equiv F_S(\omega_{em})$, radiation,

$$F_{J,\text{max}} = f(p, \mu)\delta^2,$$

(30)

where $f(p, \mu)$ is a function of two power-law indices, $p$ and $\mu$. The above equation may be readily obtained from equations (23), (20), and, for instance, (27). For $\mu \gg 1$, the function $f$ depends on $\mu$ only weakly, while, given the spectrum, $p$ is found by fitting the large frequency slope. Therefore, for a given spectrum with two subcomponents, both significant parameters, namely, $B_{LS}^2/B_{SS}^2$ and $\delta$, are uniquely found from equations (29) and (30).

6. COMPARISON WITH OBSERVATIONS

In § 5 we constructed a composite JS model of radiation emitted from internal shocks of GRBs, assuming that the magnetic fields are produced in these shocks via the relativistic two-stream instability (Medvedev & Loeb 1999). Several observational predictions can now be made. We compare them with presently available observational data. For future reference, we define the photon spectral index, $s$, as follows:

$$F(\omega) \propto \omega^s.$$

(31)

Here we point out that $F(\omega) \equiv P(\omega)/\omega \propto N(E)$, where $N(E)$ is the number of observed photons per unit time per unit energy range per unit area.

We have shown that prompt GRB spectra consist of two spectral subcomponents, namely, synchrotron and jitter. The synchrotron component is, as usual, well approximated by the smoothly broken power law, also referred to as the Band function (Band et al. 1993) or the GRB function. The jitter component is better approximated by a sharply broken power law with the hard and soft photon indices being equal to $s = -(p + 1)/2$ and $s = 0$, respectively. The position of the jitter peak is independent of the magnetic field strength, in contrast to the synchrotron peak, but depends on the particle (electron) density in the relativistic expanding shell (see eq. [26]).

6.1. Low-Frequency Spectra and the “Line of Death”

The optically thin synchrotron model of GRBs makes a solid prediction. Namely, the soft photon spectral index must be in the range $-3/2 \leq s \leq -2/3$, depending on the strength of the magnetic field if strong synchrotron cooling of the emitting electrons in the fireball is taken into account (Katz 1994; Sari et al. 1996). Thus, the photon index in this model can never be greater than $-\frac{2}{3}$, creating a testable “line of death” for the synchrotron shock model (Katz 1994). There is, however, growing observational evidence that many bursts violate this prediction (Crider et al. 1997; Strohmayer et al. 1998; Preece et al. 1998; Frontera et al. 2000). For instance, Preece et al. (1998) have studied time-resolved spectra of the bursts collected by the Large Area Detector (LAD) from the BATSE instrument. They found that 23 out of 137 bursts violate the optically thin synchrotron model. Frontera et al. (2000) demonstrated that about 50% of time-resolved spectra of the bursts observed before 1998 May by the Wide Field Camera on board BeppoSAX also violate this model. The proposed explanations, which include Compton upscattering of low-energy photons (Liang et al. 1997), synchrotron self-absorption (Papathanassiou 1999), and the influence of the pair annihilation in the fireball photosphere (Eichler & Levinson 2000), though possible, seem rather ad hoc and suffer from drawbacks. The Compton upscattering model strictly requires a single upscattering event per photon that requires the column density to self-adjust to a few grams per square centimeter. The self-absorption model results in large optical depths and thus weak emitted flux and very low radiation efficiency. The photospheric model requires an extremely low baryonic load (for more discussion see Mészáros 2000).

The JS model provides a natural resolution of this puzzle. Figure 10 represents the photon index as a function of a logarithm of frequency for the synchrotron and jitter cases. When jitter radiation dominates, the low-energy photon spectrum in the synchrotron shock model can never be greater than creating a testable “line of death” for the synchrotron shock model (Katz 1994). There is, however, growing observational evidence that many bursts violate this prediction (Crider et al. 1997; Strohmayer et al. 1998; Preece et al. 1998; Frontera et al. 2000). For instance, Preece et al. (1998) have studied time-resolved spectra of the bursts collected by the Large Area Detector (LAD) from the BATSE instrument. They found that 23 out of 137 bursts violate the optically thin synchrotron model. Frontera et al. (2000) demonstrated that about 50% of time-resolved spectra of the bursts observed before 1998 May by the Wide Field Camera on board BeppoSAX also violate this model. The proposed explanations, which include Compton upscattering of low-energy photons (Liang et al. 1997), synchrotron self-absorption (Papathanassiou 1999), and the influence of the pair annihilation in the fireball photosphere (Eichler & Levinson 2000), though possible, seem rather ad hoc and suffer from drawbacks. The Compton upscattering model strictly requires a single upscattering event per photon that requires the column density to self-adjust to a few grams per square centimeter. The self-absorption model results in large optical depths and thus weak emitted flux and very low radiation efficiency. The photospheric model requires an extremely low baryonic load (for more discussion see Mészáros 2000).

The JS model provides a natural resolution of this puzzle. Figure 10 represents the photon index as a function of a logarithm of frequency for the synchrotron and jitter cases. When jitter radiation dominates, the low-energy photon index tends toward its asymptotic value of $s = 0$, thus shifting the “line of death” to harder spectra. In principle, indices as large as $s = 1/2$ are allowed by our model. Positive values of $s$, however, are quite unlikely. These predictions are in good agreement with observational data. Namely, only two bursts out of those studied by Preece et al. (1998) are above the $s = 0$ line by more than 1 $\sigma$, and all bursts studied by Frontera et al. (2000) except the peculiar one, GRB 970111, fall below this line. At last, these three
bursts are still consistent with \( s = \frac{1}{2} \) within statistical uncertainties.

### 6.2. Sharply Broken Power-Law Spectra of GRBs

Most of the observed bursts are well fitted by the GRB function (Band et al. 1993). However, there are bursts with a sharp curvature of the spectrum at the break energy. For such bursts a broken power-law model generates better \( \chi^2 \) fits than the GRB function (see, e.g., Preece et al. 1998). In terms of the JS model, there are two cases in which such spectra occur. They can be observationally distinguished by the value of the soft (i.e., low-energy) photon index. First, it is the case of purely jitter radiation: \( B_{1s}/B_{ss} \rightarrow 0 \), and there is no synchrotron peak (or this peak is too weak and is at low energies, outside the detector range; see Fig. 9). These spectra are flat, \( s \approx 0 \), at low energies. Second, for large \( \delta \approx 1 \) and the equipartition between the large- and small-scale fields, \( B_{1s}/B_{ss} \approx 1 \), similar spectral shapes are also obtained (see Fig. 8). In this case, the soft photon index is \( s \approx -2/3 \), or it is in the range \(-3/2 \leq s \leq -2/3 \) if synchrotron cooling of the electrons is taken into account (Sari et al. 1996). Thus, we expect fewer broken power-law bursts that have soft photon indices in the range \(-2/3 \leq s \leq 0 \). Figure 2 of Preece et al. (1998) indeed reveals a gap between the low-energy bursts with \( s \approx 0 \) and \( s \leq -2/3 \). However, a large number of such bursts is required to draw a statistically significant conclusion. We should also point out that if no ordered field is present in the ejecta, then \( B_{1s} \approx B_{ss} \) is equivalent to \( B_{1s}^2/8\pi \approx B_{ss}^2/8\pi \). This, in turn, naturally implies strong energy coupling and rough equipartition between the protons and the electrons, as discussed in § 3.1.

### 6.3. Two-Component Spectral Structure of GRB Emission

The analysis done by Pendleton et al. (1994) of the bursts from the first BATSE catalog collected by LAD demonstrates that there is a large number of bursts that have high-energy photon indices (50–300 keV range) that are much larger than the low-energy ones (20–100 keV range). No such behavior is observed at other energy ranges. Thus, this result may indicate the presence of more than one spectral component in the energy range 40–100 keV. (The absence of similar behavior at higher energies seems to rule out the inverse Compton origin of a second spectral component.) On the other hand, such spectra are completely consistent with the JS model. They generally correspond to the well-separated synchrotron and jitter peaks and are characterized by small \( \delta \)'s, \( \delta \approx 0.1 \), and weak large-scale fields, \( B_{1s}/B_{ss} \ll 0.01 \), as is clearly seen from Figure 9.

### 6.4. “Lines” in GRB Spectra

Whether emission and/or absorption features, often referred to as lines, in GRB spectra are real has remained an open question for a long time (see, e.g., Briggs 2000 and Ryde 2000 for discussion). These features have been observed, for instance, by the KONUS experiment, the Venera mission (Mazets et al. 1981), and Ginga (Murakami et al. 1988). At that time, it was widely believed that these features were cyclotron lines due to a highly magnetized progenitor. The BATSE Spectroscopy Detectors (SDs) refined these results. Palmer et al. (1994) studied almost 200 bursts detected by SDs. No convincing line features were found. In a more recent analysis by Briggs (2000) of more than 100 bursts observed by SDs before 1996 May 31, about 10 highly significant line features were found. These are low-energy (\( \sim 50 \) keV) emission features. Clearly, this controversy requires further studies. However, it seems likely that if lines exist, they are rare.

Not too surprisingly, the two-component JS model is able to produce spectra with emission-line-like features. These are not lines in a strict sense, because at energies higher than the “line” peak the spectrum has no curvature and falls down as a power law with \( s = -(p + 1)/2 \). Few such spectra are seen in Figure 8. These spectra are obtained (1) for rather low values of \( \delta (\delta \leq 0.3) \), otherwise the synchrotron and jitter peaks overlap, and (2) for a narrow range of \( B_{1s}/B_{ss} \) (0.1 \( \lesssim B_{1s}/B_{ss} \lesssim 1 \)), otherwise either the spectral components are too well separated if the value of this ratio is small, or the jitter peak is too weak and unobservable if the ratio is large. We thus conclude that the spectra with emission features are not very common. They require some parameter tuning and therefore must be rare. Moreover, inhomogeneities and highly variable conditions in the GRB shocks may smear out weak spectral features completely.

At last, we present an illustrative example. Figure 11 shows the spectrum of GRB 910503 obtained using all capability of CGRO’s four experiments (Schaefer et al. 1998). Here the energy flux \( \nu F_{\nu} \propto E^2 N(E) \propto \alpha P(\nu) \) in units of \( \text{photons cm}^{-2} \text{s}^{-1} \text{keV}^{-1} \times (E/100 \text{ keV})^2 \) is plotted versus frequency in units of keV. A sharp second peak at \( \sim 2 \text{ MeV} \) is clearly seen. The solid curve in the figure represents a visual (i.e., not \( \chi^2 \)) spectral shape fit using the JS model. Here we fit the separation of the two peaks and their relative amplitudes; we did not fit the absolute flux and position of the synchrotron spectral break, which depend on the energetics of the fireball. Note that the intensity of a second, high-frequency (jitter) component is low. The values inferred from this fit are the following: \( B_{1s}/B_{ss} = 7 \), \( \delta = 0.07 \), \( p = 3.9 \), and we took \( \mu = 10 \). We calculate the efficiency of the magnetic field generation in the shock from equation (2), assuming \( \phi = 1/4 \) as a typical value. We obtain \( \eta_s \approx 0.08 \). The total magnetic field energy is calculated as \( \epsilon_B = \epsilon_{Be}(1 + B_{1s}^2/B_{ss}^2) \), where \( \epsilon_{Be} \) is found from equation (4), and we used \( B_{ss} \approx B_s \) for large \( \mu \). We obtain \( \epsilon_B \approx 4 \times 10^{-4} \), which is in agreement with the conclusion drawn by Chiang & Dermer (1999) from a completely different analysis that the magnetic field in this burst is well below the equipartition, \( \epsilon_B \lesssim 10^{-2} \). Their argument arises from the fact that the low-energy spectrum of this burst is consistent with \( F(\nu) \propto \nu^{-2/3} \). Evidently, synchrotron losses are
small, and the emitting electrons do not form a cooled distribution, which otherwise would result in $F(\omega) \propto \omega^{-3/2}$. This is possible only for very small values of $\epsilon_B$.

7. CONCLUSION

In this paper we have shown that radiation produced by relativistic electrons in magnetic fields may differ quite substantially from synchrotron. This radiation, referred to here as jitter radiation, is produced in the magnetic fields that are highly inhomogeneous on very small spatial scales. Such fields are likely to be present in GRB shocks. We developed a quantitative theory of jitter radiation. Jitter radiation has a different spectrum, and its peak frequency is independent of the field strength. However, the total (i.e., frequency-integrated) emitted power of jitter radiation depends on the field strength and is exactly identical to that of synchrotron radiation. We also constructed a composite, two-component JS spectral model of the prompt GRB emission. Predictions of this model seem to be in excellent agreement with presently available data and likely resolve some puzzling spectral properties of the prompt \( \gamma \)-ray emission. All this, we think, strongly supports the hypothesis that (1) the proposed jitter radiation mechanism operates in astrophysical objects and (2) the magnetic field is generated in shocks by the two-stream instability. We emphasize that a reliable identification/detection of the jitter spectral features will provide direct evidence that the magnetic field in GRBs is due to the two-stream instability, since we are presently unaware of any other mechanism that is capable of producing the required small-scale, large-amplitude fields. In general, the detection of both spectral components in GRB spectra would be a powerful and precise tool for investigating the properties of cosmological fireballs.

It is important to emphasize that the phenomenon of jitter radiation is intrinsic not only to internal shocks. Similar conditions (i.e., strong, small-scale fields) are expected to occur in external shocks that produce delayed afterglows, as well as in more conventional supernova shocks and relativistic jets. More speculatively, the above mechanism could allow us to study the magnetic and electric fields in reconnection regions (remember that reconnection occurs on the electron skin depth scales) and the small-scale structure of magnetic turbulence and cascade (e.g., in the interstellar medium).

The theory presented in this paper is only a “first step.” It is incomplete in the sense that it does not include the effects of both ordered and large-scale random magnetic fields in a self-consistent way and hence does not smoothly interpolate between the two extremes of synchrotron and jitter mechanisms. The synchrotron self-absorption has been omitted. The model also does not consider some related effects. For instance, it is likely that random electric fields (i.e., strong Langmuir turbulence) are present in the cosmological collisionless shocks. These fields will definitely affect the radiation spectrum via a similar mechanism. It is also unclear now whether the particle’s motion is ergodic in random fields (i.e., whether a particle “samples” strong and weak fields statistically homogeneously) and what could be the effect of nonergodicity on the observed spectrum. All these issues will be addressed in future publications.

The author is grateful to Ramesh Narayan and Norm Murray for their interest in this work, various insightful comments, and useful discussions and to George Rybicki and Avi Loeb for discussions. This work has been supported by NSF grant PHY 9507695.

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