Modelling of DC motor and educational application in Cyber-physical systems

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Abstract. Present paper describes the development and implementation of the DC motor simulation model in the framework of Cyber-physical systems and Internet of Things for the educational purposes. In order to provide the testbed for students, the control system and simulation model should be developed in unified environment. This has been achieved by using web technologies such as nodejs and cloud9 IDE. Simulation model is derived in the form of the differential equations. The MATLAB/Simulink block diagram is provided. The model has been tested and successfully validated for the pulse input function.

1. Introduction
Cyber-physical systems (CPS) and Internet of Things (IoT) are research fields with significant interest. However, in order to study such systems, the appropriate testbed should be developed [1, 2]. Present paper describes our effort to provide a simple DC motor simulation model and implement it in the CPS&IoT framework. In order to provide the testbed, one should have a proper environment, which would enable easy development of control algorithms, simulation model as well as application on mobile devices with web access.

In order to provide the students with the proper testbed, the following technologies will be applied: javaScript, nodejs, Arduino, Firmata, Ubuntu Linux and cloud9 IDE. Here, the idea is, that the control algorithms for the DC motor position control problem are coded by students from scratch. Further, the model should be derived and implemented in javaScript. This would provide the knowledge of controlling the real world system as well as develop simulation model. By development of the model, the algorithms could be then programmatically developed in order to perform efficient control.

Although the DC motor position control is well worked out, the incorporation of the modelling principles within the CPS&IoT is new. The hardware is readily available and affordable, however, the whole procedure of constructing the CPS&IoT system and development of simulation model (digital twin) is challenging with no proper materials available. Besides, the whole system should work also on mobile devices and internet clients. Another important aspect is, that such systems should be easily in integrated in the cloud/internet environment for example, for using cloud services, such as speech recognition [3] or machine vision. Such integration is only possible by applying up to date web technologies which provides important advantage.
2. Methodology
Relation between the angular velocity and torque produced by the motor can be expressed as [4, 5, 6]:

\[ T = I_L \dot{\omega} \]  

(1)

where \( T \) is torque, \( I_L \) moment of inertia and \( \dot{\omega} \) is the rate at which the angular velocity is changing with the units \( \text{rad/s}^2 \). This could be understood as the analogy of the Newton's second law for rotational bodies. \( I_L \) is expressed as

\[ I_L = m r^2 \]  

(2)

where \( m \) is the mass of the load and \( r^2 \) is squared distance from the centre of the rotation. Electrical setup of the motor could be described by the following equation:

\[ V_S = R i(t) + L \frac{d i(t)}{dt} + V_B \]  

(3)

where \( V_S \) is supplied voltage to the motor terminals, \( R \) resistance of the coils, \( i(t) \) current, \( L \) inductance and \( V_B \) back Electro Motive Force (EMF) voltage which is proportional to the angular speed:

\[ V_B = K_B \omega \]  

(4)

where \( \omega \) is angular speed and \( K_B \) back EMF constant.

Equation that relates the torque applied to the load in terms of the current going to the motor is the following:

\[ T = K_T i(t) \]  

(5)

where \( K_T \) is the torque constant.

If we neglect the inductive part of the motor, the following differential equation should be considered:

\[ \frac{d \omega(t)}{dt} = \frac{K_T}{I_L} \frac{V_S - K_B \omega(t)}{R} - b \omega(t) \]  

(6)

where \( b \) is the damping ratio which diminishes the effective voltage applied to the motor in the proportion of the angular velocity. DC motor output shaft position could be expressed as:

\[ \int \rho \omega(t) dt \]  

(7)

where \( \rho \) is the gear ratio of the motor. Example of the model implementation in MATLAB/Simulink (Release 2014) is shown in figure 1. The model has two integrators for transforming angular acceleration, \( \dot{\omega} \), speed \( \omega \) and position. One can observe the feedback from damping as well as back EMF. At the input the pulse function is used providing us with +1V and -1V as the input signal. This is converted to the Pulse Width Modulation (PWM) signal from the microcontroller which is pushed to the H-bridge (DRV8838 Single Brushed DC Motor Driver Carrier Item #2990). At the output shaft the gear ratio \( \rho \) is taken into account. Several other constants have been experimentally determined as will be explained in the next section of the paper.
Figure 1. Simulink model of a small geared DC motor.

In order to determine certain parameters of the model the following measurements were performed: a) relation between rotor rotation and back EMF voltage, b) relation between PWM from microcontroller and output voltage and c) relation between PWM and rotor rotation frequency. The measurements are shown in figure 2. One could observe linear relations. The linear interpolation is provided which could be used for parameters estimation.

Besides the performed measurements other parameters should also be considered. Since the microcontroller is at hand, with the current and voltage sensors, several other parameters could be determined, for example relation between the current and the torque. Here the acceleration on the axle should be measured.

3. Results

Table 1 shows the example parameters of the DC motor, which were used at simulation. Parameters’ names as well as the units are also provided.
Table 1. Example parameters of the DC motor.

| Parameter | Parameter name        | Value   | Unit           |
|-----------|-----------------------|---------|----------------|
| b         | damping coefficient   | 0.01    | N*m/(rad/s)    |
| KT        | torque constant       | 0.025   | N*m/A          |
| Rho       | gear ratio            | 100     | No unit        |
| IL        | moment of inertia     | 0.000001| kg*m^2         |
| R         | resistance of the motor| 2   | Ohm            |
| Kb        | back EMF constant     | 0.0019  | V/(rad/s)      |

Test input to the DC motor was a pulse function which value was in range from -1V to +1V. This transfers to the PWM value range of +36 and -36. The x-axis represents time steps where 200 timesteps is equal to 4 seconds.

Figure 3. Test input function which was put to the DC motor. On the left side, the PWM signal is shown while on the right the input voltage on the motor terminals is shown.

Figure 4 shows the results of DC motor position for four parallel simulation and real runs. Again, on the x-axis is time, where value 200 represents 4 seconds. On y-axis the position is shown where 0 represents 0 degrees and 1023 represents 300 degrees. This is the range of output potentiometer which was used as the feedback encoder at position control. With the green line, the actual response of the motor to the test signal in figure 3 is shown and with the purple line the results of the simulation are shown. The four runs were completed in order to test reputability of the system.

Figure 4. The position of DC motor for four simulation runs.
4. Validation
The validation was performed on \( n \) points according to the following equations [7, 8]. The coefficient of determination is defined as:

\[
r^2 = \left[ \frac{1}{n} \sum_{i=1}^{n} (S_i - \overline{S}) \overline{A} \right] / S_{\overline{S}A_i}^2
\]

(8)

Mean Absolute Percent Error (MAPE):

\[
MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{S_i - \overline{A}}{A_i} \right|
\]

(9)

Bias component of MSE (Mean Squared Error):

\[
U^m = \left( \overline{S} - \overline{A} \right)^2 / MSE
\]

(10)

Variation component of MSE:

\[
U^s = \left( S_{\overline{S}} - A_i \right)^2 / MSE
\]

(11)

Covariation component of MSE:

\[
U^c = 2(1-r)S_{\overline{S}A_i} / MSE
\]

(12)

where \( A_t \) are actual values gained from the DC motor by the microcontroller input and \( S_t \) are simulated values as computed according to the input parameters.

According to the provided measures the values of the validation criterions were calculated and gathered in table 2 which shows validation statistics [7, 8]. Mean Absolute Percent Error (MAPE) is below 5% for the case of four runs. Both bias components of MSE \( U^m \) are less than 20%. Unequal variation, \( U^s \) accounts for less than 2%. The largest is the Covariation component of MSE \( U^c \), more than 85% indicating that simulated dynamics tracks the real process reasonably well but diverges point-by-point which is due to the random component of the DC motor response. One should also consider, that the data set is provided for only four runs so repetition of the experiment would provide more accuracy.

| Statistics                              | Run1  | Run2  | Run3  | Run4  | AVG   |
|----------------------------------------|-------|-------|-------|-------|-------|
| Number of data points (n)              | 201   | 201   | 201   | 201   | 201   |
| Coefficient of Determination (R^2)     | 0.92  | 0.94  | 0.92  | 0.94  | 0.93  |
| Mean Absolute Percent Error (MAPE)     | 0.05  | 0.04  | 0.04  | 0.04  | 0.04  |
| Mean Square Error (MSE)                | 645.25| 550.51| 651.61| 507.09| 588.62|
| Root Mean Square Error (RMSE)          | 25.4  | 23.46 | 25.53 | 22.52 | 24.23 |
| Bias component of MSE (Um)             | 0.03  | 0.17  | 0.16  | 0.17  | 0.13  |
| Variation component of MSE (Us)        | 0.05  | 0.03  | 0.0   | 0     | 0.02  |
| Covariation component of MSE (Uc)      | 0.92  | 0.84  | 0.84  | 0.83  | 0.85  |

These are the basic criterions for the continuous model validation. The most informative is MAPE, which determines the average percentage of model deviation. Being in range of 4% one could expect, that the model is reasonably fitted to the real DC motor position output.
5. Discussion
The modelling of the DC motor in the framework of the CPS&IoT is a challenging task. One has to consider programming of entire control system as well as the simulation model from the scratch.

Important advantage at dealing with such a task is application of the javaScript/ECMAScript for entire development. This reduces the complexity of the task since only one programming language is used from the server side, microcontroller as well as the GUI on mobile devices. Such an approach has been proven as useful in several real world and experimental setups [3, 9, 10, 11].

Validation of the model showed, that it adequately describes the basic relations in the electro-motive system. For further improvement of the model additional measurements should be conducted.

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References
[1] Škraba A, Stanovov V, Semenkin E and Kofjač D 2016 Putting Cloud 9 IDE on the Wheels for Programming Cyber-Physical Internet of Things Platforms Providing Educational Prototypes. 13th International Conference on Informatics in Control, Automation and Robotics
[2] Škraba A et al 2018 Development of educational cyber-physical Internet of Things platform study of the PID controller 7th Mediterranean Conference on Embedded Computing (MECO) Budva 1-4
[3] Škraba A, Stojanović R, Zupan A, Koložvari A and Kofjač D 2015 Speech-Controlled Cloud-Based Wheelchair Platform for Disabled Persons Microprocessors and Microsystems Elsevier
[4] Hale F 2000 Introduction to control system analysis and design (Raleigh, NC: Francis J. Hale)
[5] Nise N 2015 Control Systems Engineering (John Wiley)
[6] Lyshevski S 2003 Control systems theory with engineering applications (Mumbai: Jaico Pub., published in arrangement with Birkhauser Boston)
[7] Oliva R 1995 A Vensim ® Module to Calculate Summary Statistics for Historical Fit. D-4584 System Dynamics Group, Massachusetts Institute of Technology, Cambridge MA. Available at http://metasd.com/wp-content/uploads/2010/03/D4584theil.pdf
[8] Sterman J D 1984 Appropriate Summary Statistics for Evaluating the Historical Fit of System Dynamics Models Dynamica 10(2) 51-66
[9] Škraba A, Koložvari A, Kofjač D, Stojanović R, Stanovov V and Semenkin E 2016 Streaming pulse data to the cloud with bluetooth LE or NODEMCU ESP8266 5th Mediterranean Conference on Embedded Computing (MECO) Bar 428-31
[10] Škraba A, Koložvari A, Kofjač D, Stojanović R, Stanovov V and Semenkin E 2017 Prototype of group heart rate monitoring with NODEMCU ESP8266 6th Mediterranean Conference on Embedded Computing (MECO) Bar 1-4
[11] Kofjač D, Stojanović R, Koložvari A and Škraba A 2018 Designing a low-cost real-time group heart rate monitoring system Microprocessors and Microsystems 63 75-84