Holonomic quantum computation in subsystems

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We introduce a generalized method of holonomic quantum computation (HQC) based on encoding in subsystems. As an application, we propose a scheme for applying holonomic gates to unencoded qubits by the use of a noisy ancillary qubit. This scheme does not require initialization in a subspace since all dynamical effects factor out as a transformation on the ancilla. We use this approach to show how fault-tolerant HQC can be realized via 2-local Hamiltonians with perturbative gadgets.

Introduction.—A basic requirement for the construction of a reliable quantum computer is the ability for high-fidelity storage and manipulation of quantum information. Storage is most generally achieved through encoding in suitable physical degrees of freedom that may be protected from unwanted interactions through symmetries or by active operations. At the same time, manipulating quantum information requires addressing the relevant degrees of freedom in a precise and robust way.

Holonomic quantum computation (HQC) is one method that promises a resilient way of information processing through all-geometric, adiabatic control [1]. In this approach, logical states are encoded in a degenerate eigenspace of the Hamiltonian, and gates are realized by adiabatically varying the Hamiltonian along suitable paths in the space of control parameters. This gives rise to geometric transformations inside the eigenspace [2], which depend on certain global properties of the path and are thus robust against local fluctuations that preserve those properties. [3]. This method employs encoding in a subspace, which is only a special case of the most general form of encoding possible [4]—encoding in virtual subsystems [5]. The latter type of encoding has numerous applications in the area of decoherence control, ranging from noiseless subsystems that offer protection through more efficient encoding or in cases where no subsystem protection exists [6], to operator error-correcting codes [7] that allow for simplified recovery methods leading to improved fault-tolerance thresholds [8]. Given the operational robustness of the holonomic approach, a natural question is whether a generalized method for HQC compatible with encoding in subsystems is possible.

In this paper, we answer the above question affirmatively. The paper reports three main results. The first one is a general framework for HQC in subsystems. A distinctive feature of this framework is that it involves performing simultaneous computations in the different eigenspaces of the Hamiltonian. We show that given sufficient control over the parameters of a Hamiltonian, it is possible to generate any combination of geometric transformations in its eigenspaces. This possibility has been suggested in Refs. [9, 10], but a proof has been lacking. A remarkable consequence of this result is the possibility to apply purely geometric operations on a given system by the use of a noisy ancilla. This approach does not require preparation of the system in any subspace since all dynamical effects are absorbed by the ancilla. The latter is particularly appealing as it avoids the problem of imperfect initialization [10]. The second main result is a scheme for universal quantum computation on qubits based on this principle. The scheme offers a robust way of gate implementation which in comparison to standard HQC is less demanding in the preparation stage and insensitive to certain transitions between energy levels.

The third main result is a scheme for fault-tolerant (FT) HQC based on 2-local interactions. The ultimate scalability of any method of computations requires the ability for FT implementation that guarantees the existence of an accuracy threshold [11]. The first such HQC scheme [12] requires 3-local interactions that cannot be reduced to 2-local using standard techniques without losing fault tolerance. As 3-local interactions can be very difficult to engineer, the question of whether FTHQC can be implemented using 2-local Hamiltonians is especially important in view of possible experimental implementations. We show that our scheme for HQC with noisy ancillas is readily compatible with the FT techniques on stabilizer codes [11] and can be reduced to 2-local by perturbative gadgets [13] in a FT way.

Quantum holonomies.—Consider an isodegenerate family of Hamiltonians \( \{H_\lambda\} \) on an \( N \)-dimensional Hilbert space, continuously parameterized by a parameter \( \lambda \) in a control-parameter manifold \( \mathcal{M} \): \( H_\lambda = \sum_{\nu=1}^{R} \varepsilon_n(\lambda) \Pi_n(\lambda) \), where \( \{\varepsilon_n(\lambda)\}_{\nu=1}^{R} \) are the \( R \) different \( d_n \)-fold degenerate eigenvalues of \( H_\lambda \), \( \{\Pi_n(\lambda)\} \) are the projectors on the corresponding eigenspaces. Let \( \lambda^\mu \) be local coordinates on \( \mathcal{M} \) \( (1 \leq \mu \leq \dim \mathcal{M}) \) and \( \{|n;\alpha\lambda;\gamma\rangle\}_{\alpha=1}^{d_n(\lambda)} \) be an orthonormal basis of the \( n^{th} \) eigenspace of the Hamiltonian at the point \( \lambda \). Consider a time-dependent Hamiltonian \( H(t) := H_\lambda(t) \) obtained by varying the control parameters along a curve \( \lambda(t) \) in \( \mathcal{M} \). If the change of \( \lambda \) is adiabatic, the transformation generated by \( H(t) \) is \( U(t) = U_{A_{\lambda}}(t) = \exp(i \int_0^t d\tau H(\tau)) = \sum_{\nu=1}^{R} e^{i\omega_n(t)} U_{A_{\nu}}(t) \), where \( \omega_n(t) = -\int_0^t d\tau \varepsilon_n(\lambda(\tau)) \) are dynamical phases, and \( U_{A_{\nu}}(t) = \mathcal{P} \exp(i \sum_{\alpha} \frac{\partial}{\partial A_{\alpha}} A_{\nu}(t)) \), where \( \mathcal{P} \) denotes path-ordering. The adiabatic connections are \( A_\nu = \sum_{\mu} A_{\nu\mu} d\lambda^\mu \), where \( A_{\nu\mu} \) has matrix elements \( (A_{\nu\mu})_{\alpha\beta} = \langle n;\alpha\lambda;\gamma|\frac{\partial}{\partial A_{\beta}}|n;\beta\lambda;\gamma \rangle \) [2]. When the path \( \lambda(t) \) forms a loop \( \gamma(t) \), \( \gamma(0) = \gamma(T) = \lambda_0 \), the unitary ma-
the form $\varepsilon H \equiv U_\varepsilon V H V^\dagger U_\varepsilon$ is called the holonomy associated with the loop. The set $\text{Hol}(A_\varepsilon) = \{ U_\varepsilon \gamma \in L_{A_\varepsilon}(\mathcal{M}) \}$, where $L_{A_\varepsilon}(\mathcal{M}) = \{ \gamma : [0, T] \to \mathcal{M} | \gamma(0) = \gamma(T) = \lambda_0 \}$ is the space of all loops based on $\lambda_0$, is a subgroup of $U(d_{\text{M}})$ called the holonomy group. In Ref. [1], it was observed that in the generic case, the adiabatic connection corresponding to the $n^{th}$ energy level of the Hamiltonian family is irreducible, i.e., the holonomy group $\text{Hol}(A_\varepsilon)$ is equal to $U(d_{\text{M}})$, and therefore adiabatic holonomies can be used for universal quantum computation in the $n^{th}$ eigenspace of the Hamiltonian.

**HQC in subsystems.**—A (virtual) subsystem $[\mathcal{B}]$ is any tensor factor of a subspace of a system's Hilbert space $\mathcal{H}^S$. We will consider decompositions of the form

$$\mathcal{H}^S = \bigoplus_{i=1}^m \mathcal{H}^A_i \otimes \mathcal{H}^B_i,$$

where logical states are encoded in the subsystems $\mathcal{H}^A_i$, and will be interested in universal HQC in $\{ \mathcal{H}^A_i \}$. Eq. (1) describes the most general form of encoding of information and plays a fundamental role in the theory of quantum error correction [4] where it provides the structure of preserved information under open-system dynamics [4]. (Note that most generally all subsystems $\mathcal{H}^A_i$ can be used for encoding and computation simultaneously [13].)

**Lemma.** Let $H(0)$ be a Hamiltonian with at least two different eigenvalues. It is possible to implement any combination of holonomies in the eigenspaces of $H(0)$ through a suitable adiabatic cyclic change of $H(0)$.

**Comment.** It is known that given sufficient control over the parameters of a Hamiltonian we can generate holonomically any unitary operation in a given eigenspace [1]. The question we address here is whether it is possible to generate an arbitrary combination of holonomies in the different eigenspaces. This property may not be obvious. For example, in the case of a two-level Hamiltonian, the evolution of one eigenspace completely determines the evolution of the other one. Since the holonomy in a given eigenspace depends entirely on the evolution of that eigenspace, this raises the question if the two eigenspaces can undergo arbitrary independent holonomies. We now show, by construction, that this is possible.

**Proof.** It is sufficient to show that it is possible to generate a universal set of gates in any given eigenspace while at the same time generating the identity operation in the rest of the eigenspaces. Without loss of generality, we will assume that there are only two eigenspaces; if there are more, we can always operate within the subspace spanned by two of them by varying only the restriction of the Hamiltonian on that subspace. Then the initial Hamiltonian can be written $H(0) = \varepsilon_1 \Pi_1 + \varepsilon_2 \Pi_2$, where $\Pi_1, 2$ are the projectors on the ground and excited eigenstates, and $\varepsilon_1 < \varepsilon_2$ are their corresponding eigenvalues. Observe that $H(0)$ is invariant under unitaries of the form $V = V_1 \oplus V_2$, where $V_{1,2}$ are unitaries on the subspace with projectors $\Pi_{1,2}$, respectively. Let the Hamiltonian vary along a loop $H(t)$, $H(0) = H(T)$, which satisfies the adiabatic requirement to some satisfactory precision. To this precision, the resulting unitary transformation can be written $U(T) = T \exp(-i \int_0^T dt H(t)) = e^{-i \omega_1 U_1 + e^{-i \omega_2 U_2}$, where $U_1$ and $U_2$ are the holonomies resulting in the two eigenspaces, and $\omega_{1,2} = \int_0^T d\gamma_{1,2}(t)$ are dynamical phases. Observe that the Hamiltonian $V H(t) V^\dagger$, where $V = U_1 \oplus U_2$, gives rise to the holonomies $V_1 U_1 V_1^\dagger$ and $V_2 U_2 V_2^\dagger$, respectively. This follows from the fact that the overall unitary transformation generated by $V H(t) V^\dagger$ is equal to $V U(t) V^\dagger$ where $U(t)$ is the unitary generated by $H(t)$. Note that $V H(t) V^\dagger$ is a valid loop based on $H(0)$ with the same spectrum as that of $H(t)$.

Imagine that we want to generate holonomically a gate $W_1$ in the ground space of the Hamiltonian while at the same time realizing the identity holonomy $I_2$ in the excited space. Choose any loop $H(t)$ which gives rise to the holonomy $W_1^{d_2}$ in the ground space, where $d_2$ is the dimension of the excited space (we know that such a loop can be found). Let this loop result in the holonomy $W_2$ in the excited space. The latter can be written $W_2 = \sum_{j=1}^{d_2} e^{i \alpha_j |j\rangle \langle j|}$, where $\{ |j\rangle \}$ is an eigenbasis of $W_2$ and $e^{i \alpha_j}, \alpha_j \in R$, are the corresponding eigenvalues. Consider the unitary $C_2$ which cyclically permutes the eigenvectors $\{ |j\rangle \}$: $C_2 |j\rangle = |j + 1\rangle$, where we define $|d_2 + 1\rangle \equiv |1\rangle$. We can implement the desired combination of holonomies in the two eigenspaces as follows. First apply $H(t)$. This results in the holonomies $W_1^{d_2}$ and $W_2$ in the ground and excited spaces, respectively. Next, apply $(I_1 \oplus C_2) H(t) (I_1 \oplus C_2)^\dagger$. This generates the holonomies $W_1^{d_2}$ and $C_2 W_2 C_2^\dagger = \sum_{j=1}^{d_2} e^{i \alpha_{j-1} |j\rangle \langle j|}$ (we have defined $\alpha_{-1} \equiv \alpha_{d_2}$). The combined effect of these two operations is $W_1^{2d_2}$ and $\sum_{j=1}^{d_2} e^{i (\alpha_j + \alpha_{j-1})} |j\rangle \langle j|$. We next apply $(I_1 \oplus C_2) H(t) (I_1 \oplus C_2)^\dagger$, which generates the holonomies $W_1^{2d_2}$ and $C_2 W_2 C_2^\dagger = \sum_{j=1}^{d_2} e^{i \alpha_{j-2} |j\rangle \langle j|}$. The net result becomes $W_1^{2d_2}$ and $\sum_{j=1}^{d_2} e^{i (\alpha_j + \alpha_{j-1} + \alpha_{j-2})} |j\rangle \langle j|$. We continue this for a total of $2d_2$ rounds, which results in the net holonomic transformations $W_1^{2d_2} = W_1$ and $e^{i (\alpha_1 + \alpha_2 + \ldots + \alpha_{2d_2})} \sum_{j=1}^{d_2} |j\rangle \langle j| \propto I_2$. This completes the proof.

The proof uses sequences of loops. In the next section, we will see that depending on the task it may be possible to find constructions based on a single loop.

**Theorem 1.** Consider a non-trivial subsystem decomposition of the form [11]. Choose an initial Hamiltonian in the form $H(0) = \bigoplus_{i=1}^m I^A_i \otimes H^B_i$, where $H^B_i$ are operators on $H^B_i$ such that all eigenvalues of $H^B_i$ are different.
from the eigenvalues of $H^B$ for $i \neq j$. In the case when $m = 1$, we impose the additional requirement that $H^B$ has at least two different eigenvalues. By varying adiabatically this Hamiltonian along suitable loops in a sufficiently large control manifold, it is possible to generate a unitary of the form $U = \bigoplus_i W_i^A \otimes V_i^B$, where \{ $W_i^A$ \} is an arbitrary set of geometric transformations on \{ $H^A_i$ \}.

Proof. Denote the eigenvalues of $H^B$ by $\omega_{\alpha_i}, \alpha_i = 1, \ldots, d_i$, and the projectors on their corresponding eigenspaces $H^B_{\alpha_i}$ by $\Pi^B_{\alpha_i}$. Then $H(0)$ has the spectral decomposition $H(0) = \sum_{i=1}^{m} \sum_{\alpha_i=1}^{d_i} \omega_{\alpha_i} \Pi^A_i \otimes \Pi^B_{\alpha_i}$, where $\Pi^A_i$ is the projector on $H^A_i$. According to Lemma 1, we can implement holonomically any combination of unitary transformations in the different eigenspaces of $H(0)$ up to an overall phase. Thus by applying the holonomy $W_i^A \otimes W_i^B$ in each of the eigenspaces $H^A_i \otimes H^B$, for $\alpha_i = 1, \ldots, d_i$, where $W_i^B$ are arbitrary unitaries on $H^B_{\alpha_i}$, we obtain the net unitary $U = \bigoplus_i W_i^A \otimes \bigoplus_i e^{i\phi_{\alpha_i}} W_i^B \equiv \bigoplus_i W_i^A \otimes V_i^B$, where $e^{i\phi_{\alpha_i}}$ are dynamical phases.

HQC without initialization.—From Theorem 1 one can see that in the case when the Hilbert space factors as $\mathcal{H} = H^A \otimes H^B$, it is possible to apply holonomic computation in subsystem $H^A$ without initializing the state of the system in any subspace. In particular, if we are given a system $H^A$ in an unknown state, we can append to it another ancillary system $H^B$, also in an unknown state, and apply any desired transformation holonomically on the first system. Since this approach does not require the preparation of pure ancillary states, it can be advantageous in implementations where the latter is difficult, such as nuclear magnetic resonance (NMR) [10].

We now present an explicit scheme for universal computation on qubits based on this principle. The scheme uses a single ancillary gauge qubit. We will show how to implement a universal set of one- and two-qubit gates. We will build the necessary loops by interpolations between points in the space of Hamiltonians of the form $H(t) = f(t)H(0) + g(t)H(T)$, where $f(0) = g(T) = 1, f(T) = g(0) = 0$. This Hamiltonian interpolates between $H(0)$ and $H(T)$ during a time interval $T$. The interpolating Hamiltonians that we will be using (to be described below) have two energy levels of equal degeneracy, and their energy gaps are non-zero unless the entire Hamiltonian vanishes, i.e., the interpolations can be realized adiabatically for sufficiently long time $T$ and smooth choice of $f(t)$ and $g(t)$. For an adiabatic interpolation of the above type we will use the short notation $H(0) \rightarrow H(T)$.

Let us label the two qubits on which we will be applying the gates by 1 and 2, and the gauge qubit by 3. In order to apply a single-qubit gate, say, on qubit 2, we will use the starting Hamiltonian $H(0) = I_2 \otimes X_3$. (Here by $X_i, Y_i, Z_i$ and $I_i$ we denote the Pauli matrices and the Identity acting on the $i^{th}$ qubit). We first apply

\[ I_2 \otimes X_3 \rightarrow Z_2 \otimes Z_3. \quad (2) \]

This results in a geometric transformation and a dynamical phase in each eigenspace. Let us denote the purely geometric part of the resulting unitary by $U_{2,3}$ (the exact form of $U_{2,3}$ is not important since we will undo it later). At this point we can apply a unitary whose geometric part is equal to an arbitrary gate on qubit 2 according to a method described in Ref. [17]. For example, the interpolation $Z_2 \otimes Z_3 \rightarrow X_2 \otimes Z_3$ gives rise to the geometric operation $R_{2Z_2}$, where $R_2$ is the Hadamard gate on qubit 2. The interpolation $Z_2 \otimes Z_3 \rightarrow -e^{i\frac{\pi}{2}}X_2 \otimes Z_3 \rightarrow -Z_2 \otimes Z_3$ results in the geometric operation $T_2Z_2X_2$, where $T_2$ denotes the $\pi/8$ gate on qubit 2. These two gates are a universal set of single-qubit gates. Let $G_2$ be the gate from the above set which we want to implement. After the corresponding interpolation, the net geometric transformation becomes $G_2U_{2,3}$ and the Hamiltonian is transformed to $G_2Z_2G_2^\dagger \otimes Z_3$. We can now “undo” the unitary $U_{2,3}$ by applying the interpolation $G_2Z_2G_2^\dagger \otimes Z_3 \rightarrow I_2 \otimes X_3$. The latter is the inverse of Eq. (2) up to the single-qubit unitary transformation $G_2$, i.e., it results in the transformation $G_2U_{2,3}G_2$. Thus the net result is $G_2U_{2,3}G_2^\dagger G_2U_{2,3} = G_2$, which is the desired unitary on qubit 2. Note that the relative dynamic phase between the ground and excited spaces, which accumulates during the procedure, at the end is equivalent to a transformation on qubit 3.

For universal computation, we also need a nontrivial two-qubit gate. We can start again by the interpolation (2) which results in the geometric transformation $U_{2,3}$. At this point we can apply, for example, the interpolation $I_1 \otimes Z_2 \otimes Z_3 \rightarrow I_1 \otimes Y_2 \otimes Z_3 \rightarrow Z_1 \otimes Z_2 \otimes Z_3$, which results in the gate $S_1^1N_1^2$, where $S_1 = T_2^2$ and $N_1^2$ is the “controlled not” gate with qubit 1 the control, and qubit 2 the target [17]. To “undo” the operation $U_{2,3}$, we apply the transformation $Z_1 \otimes Z_2 \otimes Z_3 \rightarrow I_1 \otimes Z_2 \otimes X_3$, which is the inverse of (2) up to the transformation $S_1^1N_1^2$. The net result is $S_1^1N_1^2U_{2,3}^\dagger N_1^2S_1^1N_1^2U_{2,3} = S_1^1N_1^2$.

We note that unlike the standard holonomic approach, here each eigenspace of the Hamiltonian undergoes the same geometric operation, which supplies the scheme with additional robustness. The scheme is insensitive to those transitions between the two energy levels that are equivalent to local operations on the transformed gauge qubit. The scheme uses 2- and 3-local Hamiltonians.

FTHQC with 2-local Hamiltonians.—The theory of fault tolerance [11] guarantees that, if errors during the implementation of a given gate are sufficiently uncorrelated and improbable, an arbitrarily long computation can be implemented reliably with a modest resource overhead. The first proposal for FTHQC [12] uses the encoding present in a stabilizer code and Hamiltonians that are elements of the instantaneous stabilizer or gauge group of the code. These Hamiltonians couple qubits in the same block, but errors do not propagate as each eigenspace is subject to the same transversal operation. That scheme...
requires 3-local Hamiltonians. Even though every Hamiltonian can be simulated by a 2-local one via the so called perturbative gadgets\cite{13}, the locality of that scheme cannot be reduced by a direct application of these techniques. Since the simulated Hamiltonian couples qubits in the same code block, an error on one of the gadget ancillas\cite{13} can spread to multiple qubits within a block. The scheme from the previous section suggests an alternative approach to FTHQC. Any FT protocol on qubit stabilizer codes can be decomposed into transversal one- and two-qubit gates (these are gates that couple only the corresponding qubits from different blocks). In addition, one requires the preparation of a special ancillary state such as $$|00...0\rangle + |11...1\rangle)/\sqrt{2}$$, which is done non-transversally. Transversality guarantees that a single error during an encoded operation results in at most one error per block of the code. Our scheme for holonomic one- and two-qubit gates is readily compatible with this approach: we can apply the same operations as in a standard FT protocol\cite{11} by coupling every qubit or pair of qubits in the code to an “external” gauge qubit as described in the previous section. Obviously, a single error during the implementation of a transversal operation cannot propagate to multiple qubits in a block because the latter do not interact. In contrast to the previous approach which can be understood as performing HQC inside the subsystem containing the protected information, this scheme performs HQC in the entire system and does not require Hamiltonians that depend on the code. Furthermore, here each 3-local Hamiltonian can be reduced to 2-local via perturbative gadgets as the gadget ancillas would couple to at most one qubit inside a block. A complete fault-tolerance analysis is beyond the scope of this paper, but we note that the use of extra qubits increases the chance for an error during a single gate. In addition, the 3-qubit gadget decreases the minimum gap of the original Hamiltonian by a factor $$\sim \varepsilon^{-3}$$ where $$\varepsilon$$ is the perturbation parameter in which the approximation to order $$O(\varepsilon^4)$$ is carried out\cite{13}. Thus for maintaining a given precision, the time for implementing a two-qubit gate would have to increase by a similar factor which decreases the allowed rate for environment noise. A way around this could be to look for non-perturbative implementations. Since the no-initialization property is not crucial for fault tolerance per se, implementations with an ancilla in a known state can also be considered.

**Conclusion.**—In summary, we have introduced a general framework for HQC in subsystems, showing that it is possible to realize simultaneously independent HQC in the subsystems $$\mathcal{H}_1^i$$ in any nontrivial decomposition of the form \cite{11}. As an application, we proposed a robust scheme for applying purely geometric gates to unencoded qubits by the use of a noisy ancillary qubit. We used this approach to show that 2-qubit Hamiltonians are universal for FTHQC. We hope that our results will open new avenues for quantum information processing implementations that combine the robustness of the holonomic control with the most general form of encoding. An interesting future direction would be to extend the present results to the theory of geometric phases based on dynamical invariants\cite{13}, which encompasses non-adiabatic, mixed-state and open-path holonomies. Since geometric phases have wide applications, associating holonomies with subsystems could find use beyond the field of quantum computing as well.

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**Note.**—Recently, an independent scheme for (open-path) FTHQC with 2-local Hamiltonians via perturbative gadgets was proposed by D. Bacon and S. T. Flammia in Ref.\cite{19}.

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