Navigating four dimensions – upper secondary students’ understanding of movement in spacetime

Magdalena KERSTING
University of Oslo, Department of Physics, Postboks 1048, 0316 Oslo, Norway

Abstract. In contrast to classical physics, general relativity (GR) interprets gravity as geometry. Objects follow geodesic curves in four-dimensional spacetime and curvature creates the illusion of a gravitational force. Despite the scientific relevance of this theory, educational research in the domain of GR is scarce. This study contributes to a growing body of knowledge concerning secondary students’ understanding of GR. Based on a thematic analysis of audio records of student discussions of 97 Norwegian physics students (18–19 years), we present results on students’ understanding of geodesic movement in spacetime. Our findings can give guidance to improve learning and instruction of a key concept of GR at the upper secondary school level.

1. Introduction
In the last years, physics educators have made first attempts of introducing Albert Einstein’s general theory of relativity to secondary school curricula [1]. Initial efforts have focused on developing appropriate teaching approaches that rely on qualitative understanding [2], geometrical approaches [3], or simplified mathematical treatments [4]. However, to make teaching and learning successful, there remains the need to study students’ knowledge and conceptual understanding of key features of GR.

1.1. Movement in spacetime
One central feature of GR relates to the motion of objects in four-dimensional spacetime. Spacetime provides the dynamic setting in which GR takes place and in which the phenomenon of gravity arises. Famously, John Archibald Wheeler summarized the relativistic model of gravity by saying that “spacetime tells matter how to move; matter tells spacetime how to curve” [5]. Einstein explained this dynamic interplay between the geometry of spacetime and the movement of objects by drawing on the concept of geodesic curves. Geodesic curves generalize the notion of straight lines to the realms of curved spaces. According to Einstein, free objects that are not subject to any external force will follow geodesic curves through spacetime. It is the curvature of spacetime that manifests itself as gravitational phenomena: There is no force of gravity acting on a falling object – the object follows a geodesic path in a curved universe.

To illustrate Einstein’s model of movement in spacetime, physics educators recently introduced “Einstein’s first law” in analogy to Newton’s first law [4]. While Newton’s first law states that objects move along straight lines if no external forces act on them, Einstein’s first law translates this statement to a four-dimensional setting. The law takes into account the reality of curved space and warped time: Objects move along geodesic curves in spacetime if no external forces act on them (Figure 1).

There are two crucial differences between Newton and Einstein’s model of gravity. First, Newton models space and time as static entities against which the laws of physics unfold. Einstein merges...
space and time into a dynamic fabric that takes an active role in the unfolding of these laws. Second, in classical physics, objects fall down to Earth because the force of gravity acts on them; objects resting on the ground experience no net force. In GR, this model is turned upside down. There exists no force of gravity: Objects at rest are subject to an upward-pointing normal force and it is only in free fall that objects are free of any force acting on them. Thus, according to Einstein, a ball falling to the ground is not subject to a downward-facing force, but it follows a geodesic curve which is the straightest possible path through curved space and warped time [4,6]. The force of gravity is an illusion created by the geometry of our universe.

![“Einstein’s law”](image)

**Figure 1.** Einstein’s law serves as a generalisation of Newton’s first law in the setting of curved space and warped time. Screenshot is taken from www.viten.no/relativity.

1.2. Literature review: student understanding of movement in spacetime

Since topics of GR are relatively new in teaching and instruction at the upper secondary school level, there is not much empirical evidence on how secondary school students conceptualize four-dimensional spacetime or geodesic motion. Yet, research with college and undergraduate students can give helpful guidance because upper secondary school students often are of comparable age to freshmen students at the college or university level. A recent investigation into undergraduate students ideas about the curvature of the universe concluded that the majority of students drew on their previous knowledge of geometry: Students understood curvature similar to how scientists used the term by referring to properties such as the dimensions, shape, or amount of bend [7].

Yet, whereas the curved geometry of the universe does not seem to pose significant problems, other findings suggest that undergraduate learners struggle to explain motion in spacetime when shifting between classical and modern perspectives of gravity. To describe the motion of freely-falling objects in gravitational fields according to Newton and Einstein, students have to “crosswalk” between two
conceptual frameworks [8]: Undergraduate students struggle with this transition because of the altered view of gravity and the altered view of freely falling systems [9,10]. The few available studies on secondary school students understanding of GR, mainly conducted in Norway [2] and Australia [11], conclude that learners find the concept of spacetime and topics of “Einsteinian physics” engaging and motivating. However, these findings allow not much insight into the detailed learning processes of students, their conceptual struggles, and successful ways of teaching relativistic concepts.

In an attempt to give a more comprehensive account of secondary school students’ understanding of spacetime, a recent study looked at the metaphorical language of final year physics students that worked with concepts of gravity and spacetime [12]. Even though many students reasoned about spacetime similar to experts, Newtonian views proved to remain strong: Several students confused cause and effect in the domain of GR by drawing on the force of gravity to explain curvature of the universe. Moreover, an exploratory case study with a pair of upper secondary physics students in Norway found that movement was a key issue when students conceptualized gravity and curvature. While the students could draw on the Einsteinian view of gravity to explain planetary movement, they struggled to grasp the gravitational influence on objects at rest [8]. Thus, while researchers have started to investigate upper secondary students’ conceptual understanding of movement in spacetime, there is clearly the need to gain a more comprehensive understanding to make teaching and instruction of GR successful.

1.3. Research questions

The starting point of this study was the scarcity of comprehensive accounts of secondary school students’ conceptual understanding in GR. The aim was to gain deeper insight into upper secondary students’ understanding of movement in spacetime related to gravity and curvature. Two research questions guided the investigations:

1) What characterizes upper secondary school students’ understanding of movement in four-dimensional spacetime?

2) What are difficulties and challenges that upper secondary school students face when conceptualizing movement along geodesic curves?

2. Project background and methods

This study is part of the larger design-based research project ReleQuant that develops digital learning environments and studies students’ learning processes in modern physics [2,13]. The project takes a sociocultural stance towards learning [14] and emphasizes the use of language in physics education. In particular, the learning environments invite students to discuss key topics repeatedly through structured interactions with peers and teachers.

This study reports from the second of three consecutive classroom trials of the learning environment. The trial was implemented in five upper secondary physics classes with in total 97 students (18-19 years) in three Norwegian schools in spring 2017. To collect data, ReleQuant researchers employed a novel approach of fostering collaborative learning: In several built-in activities, students were asked to discuss in pairs or small groups, record their conversations with mobile phones, and send the records to the teacher afterwards. In line with design-based research principles [15] and based on results from the first trial that showed that students struggled to relate gravity to movement in spacetime [8], the second version of the learning environment presented students with the discussion task shown in figure 2.
Figure 2. Students were asked to discuss movement in spacetime through structured interactions within a collaborative learning environment. Screenshot is taken from www.viten.no/relativity.

The data comes from 21 audio-recorded discussions of small groups of 2-5 students. After transcribing the audio files, methods of thematic analysis [16] were used to unpack students’ understanding of movement in spacetime. The codes emerged inductively and were related to spatial and temporal movement as well as Newtonian and Einsteinian conceptions of spacetime. In a second step and in line with the methodology of thematic analysis, the codes were then reviewed and grouped into three themes. Table 1 shows a summary of the final themes and codes. In addition to the thematic analysis, each group discussion was tagged by whether or not the group came to the correct conclusion: Students in the classroom did not follow a geodesic curve because they were not in free fall – the external normal force acted on them.

Table 1. Overview of the themes and codes of the thematic analysis of student discussions.

| Theme                                      | Codes                                      | Explanation                                                                 |
|--------------------------------------------|--------------------------------------------|-----------------------------------------------------------------------------|
| Physical Conception                        | • Newtonian conception                     | This theme comprises the two different physical frameworks students drew on to explain gravity. The codes tag instances of Newtonian and Einsteinian explanations. |
|                                            | • Einsteinian conception                   |                                                                             |
| Nature of Movement                         | • spacetime movement                       | This theme categorizes the ways students talked about their movement in spacetime. The theme encompasses three codes - movement in space, movement in time, and movement in spacetime. |
|                                            | • temporal movement                        |                                                                             |
|                                            | • spatial movement                         |                                                                             |
| Movement along geodesic curves             | • shortest path                            | This theme categorizes the ways students talked about geodesic movement. Students conceptualized geodesic curves in various ways and the codes correspond to the different classifications of geodesic curves. |
|                                            | • fastest path                             |                                                                             |
|                                            | • straight path                            |                                                                             |
|                                            | • geodesic force                           |                                                                             |
|                                            | • free fall                                |                                                                             |
|                                            | • geodesic Einstein                        |                                                                             |
3. Results
The presentation of the results follows the three main themes of the thematic analysis: The presentation moves from students’ general understanding of gravity to their understanding of movement in spacetime to their understanding of movement along geodesic curves.

Generally, most groups were able to state the difference between Newton and Einstein’s theories of gravity. Students displayed a sound understanding of Newton’s force model and related the strength of the gravitational force to the mass of objects. Einstein’s model invited the groups to address a broader range of topics: First, students were able to state that gravity, in the Einsteinian framework, is not considered to be a force. Second, students discussed the geometry and curvature of spacetime to explain gravity. Moreover, the groups talked about concepts of gravity and spacetime in visual ways, often acknowledging their wish or need to visualize these concepts further, and often referred to the popular rubber sheet analogy:

\textbf{Student 1:} According to Newton, why do you experience a pull towards the ground? And according to Einstein? So, according to Newton, there is

\textbf{Student 2:} force of gravity

\textbf{Student 1:} the force of gravity that is from the mass of the Earth that pulls on us.

\textbf{Student 2:} Yes.

\textbf{Student 1:} And we, but why do you experience a pull towards the ground according to Einstein? And then there is something with... no, that’s actually a good question. Because, even though if the rubber sheet analogy, right?

\textbf{Student 2:} Mhhm.

\textbf{Student 1:} It stood actually further up [in the learning environment] that we are pulled down by warped time actually.

\textbf{Student 2:} Mhhm.

\textbf{Student 1:} We don’t stand, in a way, in the curved spacetime, but if we think that spacetime is like continuous with a lot of layers on top of each other or many of those layers. That each layer curves, that if you have a layer right on top of the North Pole, which curves as well, then it makes maybe sense. [...] 

Students broadly conceptualized their own movement in spacetime according to movement in space and movement in time. The codes for both classifications appeared equally often even though students seemed to be more certain to explain their movement in space than their movement in time. Space was mostly understood in terms of the universe and spatial movement was often exemplified via planetary movement such as the elliptical movement of the Earth around the Sun or via the movement of galaxies in an expanding universe.

\textbf{Student:} [...] we go around the Sun, and we go around our own solar system, and other solar systems, and we go around our own axis.

The groups acknowledged temporal movement by the observation that they did not stand still in time. Yet, students admitted that it was difficult to visualize movement in time.

\textbf{Student 1:} Yes, we move along the time dimension all the time.

\textbf{Student 2:} Yes.

\textbf{Student 1:} Also when we stand still.

\textbf{Student 2:} Yes, and...

\textbf{Student 1:} This is nothing we see, so it is difficult to... maybe difficult to visualize, but it is a form of movement.

While many groups seemed to be comfortable with the idea of generic movement in spacetime, very few understood the subtleties of movement along geodesic curves. Indeed, only four out of 21
groups came to the right conclusion that they did not follow a geodesic curve. And only one of those four groups gave a correct explanation drawing on the concept of free fall:

**Student 1:** We move in spacetime right now.
**Student 2:** We do this all the time.
**Student 1:** We accelerate upwards in spacetime.
**Student 2:** We do that. We resist the curvature of the Earth.
**Student 1:** Yes.
**Student 2:** And we move forward in time all the time.
**Student 1:** Yes.
**Student 2:** Yes.
**Student 1:** And then we follow also a geodesic. No, we don’t do that. We don’t do that because if we had been in free fall then we would have followed a geodesic curve. Now we are influenced by the normal force as well.
**Student 1:** Yes, yes, we accelerate upwards.

Other groups tried to reason with forces and tried to connect geodesic movement to an absence of forces but did not succeed in finding the right answer.

**Student 1:** Do you follow a geodesic curve in spacetime right now?
**Student 2:** Hmm.
**Student 1:** [...] objects that are not influenced by forces follow a geodesic curve in spacetime.
**Student 2:** Are we influenced by forces? We are influenced by forces, aren’t we? If gravity is not a force.
**Student 1:** Is it not a force?
**Student 2:** That’s what we started with.
**Student 1:** The force of gravity is not a force, but then we don’t move along a curve. No, we move along a curve because we are not influenced by forces.
**Student 2:** Gravity is just a geometric phenomenon after all.
**Student 1:** So we move along a curve now.
**Student 2:** We move along a geodesic curve now.
**Student 1:** Curve in spacetime, that’s what we do you know.
**Student 2:** That’s what we do you know.
**Student 1:** Yes, but then we say this.

Interestingly, students often stated that they were following a geodesic curve because Einstein said so. There seemed to be a common (mis)conception that movement in spacetime is the same as movement along geodesic curves:

**Student 1:** Are we moving in spacetime right now? And do we follow a geodesic curve?
**Student 2:** Yes, one does that.
**Student 1:** Yes, because all objects with mass do this.
**Student 2:** Anyway, according to Einstein.
**Student 1:** But not according to Newton. So according to Newton, why do we experience a pull towards the Earth?
**Student 2:** That’s because of the force of gravity after all.

Even though students struggled to answer the question whether or not they moved along geodesic curves, they were able to draw on various different ways of describing a geodesic. The two most common ways of characterizing geodesic curves were through the concept of the straightest and shortest path through spacetime. Many students discussed whether they were literally fowling the straightest or shortest path through spacetime:
Student 1: Do we move along a geodesic curve? I don’t know.
Student 2: What do you think?
Student 3: No, we surely don’t do that.
Student 2: We don’t necessarily move along the straightest path. We get bent a bit here and bent a bit there.
Student 1: As far as I know, we don’t get that. We have to have a goal to follow the shortest path.

Indeed, this geometric understanding of a geodesic curve might have prevented the groups from drawing a connecting to the physical state of being in free fall.

4. Discussion and Conclusion
With the aim of gaining deeper insight into upper secondary school students’ understanding of movement in four-dimensional spacetime, this study set out to characterize difficulties and challenges that students faced when conceptualizing movement along geodesic curves in relation to gravity and curvature.

The findings show that students were able to explain their movement in spacetime by drawing on movement in space and movement in time. Common examples of students’ spatial movement included the Earth circling around the Sun or the movement of our galaxy in an expanding universe. Even though students realized that they move in time because they grow older, they admitted that temporal movement was hard to visualize. While generic movement in spacetime did not pose significant challenges to students, the concept of movement along geodesic curves did. Students displayed a broad variety of different ways of characterizing geodesic curves such as the shortest, straightest, or fastest path between two points or as a curve on which no forces act. This variety in responses shows that students were able to use different characterizations of geodesics, which, in turn, suggests that they had a good understanding of the concept of a geodesic curve as such. Yet, movement along geodesic curves in spacetime challenged almost all groups. Only few groups were able to connect the geometric description of a geodesic curve as the straightest path in a curved space to the physical state of being in free fall or alternatively, to the state of not being affected by external forces. Thus, there seems to be a gap in students’ understanding between the geometric framework of GR and the physics of gravity. This finding challenges an observation by Bandyopadhyay and Kumar [9] who stated that the separation between the conceptual and technical aspects of GR is possible in a meaningful way. While it seems that one can separate the technical from the conceptual features in a learning sequence on GR, students eventually struggle to stitch the pieces together again.

It is important to address some obvious limitations of this study. First, the data of this study comes from five upper secondary physics classes in three Norwegian schools with in total 97 students. While the analysis of the audio records of the group discussions allows insight into students’ spontaneous ideas and reasoning with concepts of gravity and spacetime, the findings are of course not generalizable to other contexts. Moreover, letting students record their own discussions could have led to a bias in the responses that were collected. It is likely that not all groups of students submitted their records and the data set might consist of those discussions that were done by high-achieving or motivated groups of students. However, the great variation in the responses suggests that data came from a variety of achievement levels.

Identifying student challenges in relation to concepts of gravity and geodesic movement can not only give insight into learning processes of upper secondary physics students. The findings allow improving teaching and instruction of GR as well. The results of this study influenced the design of the ReleQuant learning environment that addresses free fall and geodesic movement in spacetime explicitly in its final unit on spacetime.¹

¹The Norwegian Center for Science Education launched the final version of the learning environment General Relativity in January 2018: www.viten.no/relativity
When presenting Einstein’s model of gravity it is important to be explicit about Einstein’s and Newton’s description of free fall. This study supports a conclusion of Bandyopadhyay and Kumar [10] that the “most important cognitive transition that needs to be affected […] is the altered view of an inertial frame wherein a freely falling frame in uniform gravity (which is an accelerating non-inertial frame in the Newtonian view) is inertial”. Indeed, the findings of this study corroborate the importance of a sound instructional emphasis on the concept of free fall. However, contrasting Newton’s and Einstein’s views of gravity should not lead to oversimplified presentations of the topic. In this study, students displayed a common misconception that every object follows a geodesic curve through spacetime. Instructional units should thus help students connect the idea of free fall and geodesic movement while taking into account these common misconceptions.

5. References

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