Research Article
M-Polynomials and Degree-Based Topological Indices of the Molecule Copper(I) Oxide

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1. Introduction

1.1. Application Background. A graph that represents the construction of a molecule and also their connectivity is known as a molecular graph, and such a representation is generally known as topological representations of molecule. Molecular graphs are normally characterized by means of exclusive topological basis for parallel of chemicals shape of a molecule with organic, chemical, or bodily homes. Study of graph has some programs of various topological indices in quantitative structure-activity relationship (QSAR) and quantitative structure-property relationship (QSPR), digital screenings, and computational drug designing citations as shown in [1, 2]. Thus far, several exclusive topological indices have been established, and maximum of them are most effective graph descriptors in [3, 4]; apart, some indices have proven their parallel with organic, chemical, or physical residences of secure molecules in [5–17].

In the field of mathematics, any graph has vertices and edges that are represented by the atoms and chemical bonds. Graph that represents the construction of molecules and their connectivity is known as a molecular graph, and such representation is usually referred as topological representation of molecules. There are some significant topological indices like distance-based topological indices, degree-based topological indices, and primarily based topological indices. Among these works, distance primarily based topological indices unit works out a crucial task in a chemical graph started, specifically in chemistry [18,19]. Many fields have many features that can be solved with the help of graphs. In the physiochemical compounds or network systems, we have a tendency to abstractly outline exclusive ideas in modeling of mathematics. We have a tendency to refer to as the distinctive names, such as Randić index and national capital index.

A topological index is a numerical parameter of a graph and describes its topology. It describes the molecular shape numerically and is applied within the advancement of qualitative structure-activity relationships (QSARs). The following are the 3 types of topological indices:
(1) Degree-based.
(2) Distance-based.
(3) Spectral-based.

Degree-based topological indices were studied extensively and may be correlated with many residences of the under study molecular compounds. There is a strong relationship among distance-based and degree-based topological indices in [20]. Most commonly known invariants of such kinds are degree-based topological indices. These are actually the numerical values that correlate the structure with various physical properties, chemical reactivities, and biological activities. Topological indices are sincerely the numerical values that relate the shape to one of a kind of physical residences, artificial reactivity, and natural biological activities [21, 22].

Loads of research has been executed inside the course of M-polynomial, as in the case of Munir et al., processed M-polynomial and related lists of triangular boron nanotubes in [6], polyhex nanotubes in [23], nanostar dendrimers in [4], and titania nanotubes in [5]. M-Polynomials and topological lists of V-phenylenic nanotubes and nanotori. In this paper, the objective is to process the M-polynomial of the crystallographic realistic structure of the atom copper(I) oxide (Cu2O) [8, 24].

1.2. Crystallographic Structure of Cu2O(m; n). Copper oxide is a p-type semiconductor and inorganic compound. Copper oxide is a chemical element with formula Cu2O(m; n). Cu2O(m; n) is a certainly happening reddish coral that is particularly used in chemical sensors and solar orientated cells in [8, 24]. It has many advantages such as photochemical effects, stability, pigment, a fungicide, nontoxicity, and low cost. It has potential applications in new energy, sensing, sterilization, and other fields. It has narrow band gap and is easily excited by visible light.

Cu2O(m; n) is additionally responsible for the pink shading in Benedict’s test and is the essential cause to select Cu2O (see Figures 1 and 2). The promising projects of Cu2O(m; n) are mainly on chemical sensors, sunlight-based cells, photocatalysis, lithium particle batteries, and catalysis. Here, we have taken into consideration a monolayer of Cu2O(m; n) for satisfaction. To ultimate the basis for Cu2O(m; n), we pick out the setting of this graph as Cu2O(m; n) be the chemical graph of copper(I) oxide with (m; n) unit cells within the aircraft.

2. Definitions and Literature Review

2.1. M-Polynomial. M-Polynomial is defined by S. Klavžar or E. Deutsch in 2015 [3, 8]. Within the factors of degree-based topological indices, we compete necessary role of M-polynomial. Readers can refer to [9–17, 27–35]. It is the foremost general progressive polynomial and an additionally closed formula alongside 10 distance-based topological indices is given by M-polynomial. It is explained as

\[ M(G, a, b) = \sum_{d_i \leq j \leq \Delta} m_{ij}(G) a^i b^j, \]

and we have \( \delta = \text{Min}\{|r\mid r \in V(G)\} \) and \( \Delta = \text{Max}\{|r\mid r \in V(G)\} \), where \( m_{ij}(G) \) is the edge \( E(G) \), where \( i \leq j \).

2.2. Degree-Based Topological Indices. Any purpose on a graph which does not build upon numbering of its vertices is molecular descriptor. This is also called as topological index. Topological indices are most useful in the field of isomeric discrimination, chemical validation, QSAR, QSPR, and a pharmaceutical drug form. Topological indices are accessed from the system of molecule.

There are some important degree-based topological indices defined, and the first Zagreb index was introduced by Gutman and Trinajstić as follows:

\[ M_1(G) = \sum_{r,s \in E(G)} (d_r + d_s). \]

The second modified Zagreb index is defined as

\[ M_2(G) = \sum_{r,s \in E(G)} \frac{1}{d(r)d(s)}. \]

General 1st and 2nd multiplicative Zagreb indices are introduced by Kulli, Stone, Wang, and Wei and are stated as

\[ MZ_{1II}^a(G) = \prod_{r,s \in E(G)} (d_r + d_s)^a, \]

\[ MZ_{2II}^a(G) = \prod_{r,s \in E(G)} (d_r + d_s)^a. \]

The general 1st and 2nd Zagreb indices proposed by Kulli, Stone, Wang, and Wei are stated as

\[ Z_{1}^a(G) = \sum_{r,s \in E(G)} (d_r + d_s)^a, \]

\[ Z_{2}^a(G) = \sum_{r,s \in E(G)} (d_r d_s)^a. \]

In 1987, Fajtlowicz in [36] proposed the harmonic index and stated

\[ H(G) = \sum_{r,s \in E(G)} \frac{2}{d_r + d_s}. \]

The inverse sum index is defined:

\[ I(G) = \sum_{r,s \in E(G)} \frac{d_r d_s}{d_r + d_s}. \]

Symmetric division index is described as

\[ SS D(G) = \sum_{r,s \in E(G)} \frac{\min(d_r, d_s)}{\max(d_r, d_s) + \min(d_r, d_s)}. \]

SU and XU recognized general Randić index or general multiplicative Randić index stated as follows (Table 1):
Theorem 1. Crystallographic structure of the graph of copper(I) oxide $G \approx Cu_2O[\{m; n\}]$, where $n; m \geq 1$. We have

\[
M(G; a; b) = f(a; b) = (4m + 4n - 4)ab^2 + (4mn - 4m - 4n + 4)a^2b^2 + 4mnab^4.
\]

(11)

Proof. Suppose $G$ be the crystallographic structure of $Cu_2O[\{i; m; n\}]$. The edge set of $Cu_2O[\{i; m; n\}]$ has the following three partitions by Figures 1 and 2:

\[
E_1 = E_{(1,2)} = \{ e = rs \in E(G) | d_r = 1; d_s = 2 \}, \\
E_2 = E_{(2,2)} = \{ e = rs \in E(G) | d_r = 2; d_s = 2 \}, \\
E_3 = E_{(2,4)} = \{ e = rs \in E(G) | d_r = 2; d_s = 4 \},
\]

such that

\[
|E_1(G)| = 4mn + 4n - 4, \\
|E_2(G)| = 4mn - 4m - 4n + 4, \\
|E_3(G)| = 4mn.
\]

(13)

Thus, the M-polynomial of $Cu_2O[\{i; m; n\}]$ is

\[
M(G; a; b) = \sum_{i \leq j} m_{ij}(G)a^ib^j,
\]

\[
M(G; a; b) = \sum_{i \leq 1} m_{12}(G)ab^2 + \sum_{2 \leq 2} m_{22}(G)a^2b^2 + \sum_{2 \leq 4} m_{24}(G)a^2b^4,
\]

\[
M(G; a; b) = \sum_{rs \in E_1} m_{12}(G)ab^2 + \sum_{uv \in E_2} m_{22}(G)a^2b^2 + \sum_{uv \in E_3} m_{24}(G)a^2b^4,
\]

\[
M(G; a; b) = |E_1(G)|ab^2 + |E_2(G)|a^2b^2 + |E_3(G)|a^2y^4,
\]

\[
M(G; a; b) = (4m + 4n - 4)ab^2 + (4mn - 4m - 4n + 4)a^2b^2 + 4mnab^4.
\]

(14)

Theorem 2. Crystallographic structure of the graph of copper(I) oxide $G \approx Cu_2O[\{m; n\}]$, where $n; m \geq 1$. We have

\[
M_1(G) = 40mn - 4m - 4n + 4.
\]

Proof. Suppose $G$ be the crystallographic structure of $Cu_2O[\{i; m; n\}]$. We have to find

\[
M(G; a; b) = f(a; b) = (4m + 4n - 4)ab^2 + (4mn - 4m - 4n + 4)a^2b^2 + 4mn \times a^2b^4.
\]

(15)
Table 1: Formulas of degree-based topological indices from M-polynomial.

| Topological Indices | \( f(t, s) \) | \( M(G; t, s) \) |
|---------------------|----------------|-----------------|
| First Zagreb index  | \( t + s \) | \( M_1(G; t, s) = (D_t + D_s)M(G; t, s) \) |
| Second Zagreb index | \( ts \)    | \( M_2(G; t, s) = (D_tD_s)M(G; t, s) \) |
| Second modified Zagreb index | \( \frac{1}{ts} \) | \( m \cdot M_2(G; t, s) = \left( \frac{\delta_1(\delta)}{\delta} \right) M(G; t, s) \) |
| General Randić index, \( a \neq 0 \) | \( \left( ts \right)^a \) | \( R_a(G) = \left( D_t^aD_s^a \right)M(G; t, s) \) |
| Inverse general Randić index, \( a \neq 0 \) | \( \frac{1}{(ts)^a} \) | \( RR_a(G) = \left( D_t^{-a}D_s^{-a} \right)M(G; t, s) \) |
| Symmetric division index | \( \left( t^2 + s^2 \right)/ts \) | \( SS D(G) = \left| D_t - \delta D_s \right| \) |
| Harmonic index       | \( \frac{2}{t + s} \) | \( H(G) = 2\delta \delta \cdot M(G; t, s) \) |
| Inverse sum index    | \( \frac{ts}{t + s} \) | \( I(G) = \delta_1\delta_2 \cdot M(G; t, s) \) |

\[ D_t = s(\partial/\partial s)M(G; t, s)|_{t = s = 1}, \quad D_s = t(\partial/\partial t)M(G; t, s)|_{t = s = 1}, \delta_1 = \int_0^1 \frac{M(G; y, s)}{ydt}, \delta_2 = \int_0^1 \frac{M(G; t, y)}{yds}, \quad f = M(G; t, t), Q_a = x^aM(G; t, s), a \neq 0. \]

The 3D plot of first Zagreb index is given in Figure 3 (for \( u = 1 \) left, \( v = 1 \) middle, and \( w = 1 \) right), and we see the dependent variables of the first Zagreb index on the involved parameters.

**Theorem 3.** Crystallographic structure of the graph of copper(I) oxide \( G \approx CuO[0m; n] \), where \( n; m \geq 1 \). We have \( M_2(G) = 48mn - 8m - 8n + 8 \).

**Proof.** Suppose \( M(G; a; b) = (4m + 4n - 4)ab^2 + (4mn - 4n - 4m + 4) \times a^2b^2 + 4mn \times a^2b^4. \)

\[ (21) \]

We have to find \( D_bD_a; \) first, we take \( D_a; \)

\[ D_a = (4m + 4n - 4)ab^2 + (4mn - 4n - 4m + 4)^3a \times a \times b^2 + 4mn \times a^2b^4; \]

\[ (22) \]

Now, take \( D_b; \)

\[ D_bD_a(f; a; b) = (4m + 4n - 4)ab \times b^2 + 4mn \times a^2b^4; \]

\[ (23) \]

The second Zagreb index is

\[ M_2(G) = D_bD_a(f; a; b) |_{a = b = 1}; \]

\[ M_2(G) = 2(4m + 4n - 4) + 4(4mn - 4m - 4n + 4) + 32mn; \]

\[ (24) \]

After solving, the result is

\[ M_2(G) = 48mn - 8m - 8n + 8. \]

The 3D plot of second Zagreb index is given in Figure 4 (for \( u = 1 \) left, \( v = 1 \) middle, and \( w = 1 \) right), and we see the dependent variables of the second Zagreb index on the involved parameters.

**Theorem 4.** Crystallographic structure of the graph of copper(I) oxide \( G \approx CuO[0m; n] \), where \( n; m \geq 1 \), and we have

\[ M_3^m(G) = \frac{3}{2}mn + m + n - 1.3. \]

\[ (26) \]
Proof. suppose
\[ M(G; a; b) = (4m + 4n - 4)ab^2 + (4mn - 4m - 4n + 4)a^2b^2 + 4mnab^4. \]

Now, we have to find \( S_aS_b \); first, we find \( S_a \):
\[ S_a = \int_0^{a} \frac{f(x, b)}{x} dx, \]
\[ f(x, b) = (4n + 4m - 4)xb^2 + (4mn - 4m - 4n + 4)x^2b^2 + 4mnxb^4, \]
\[ \frac{f(x, b)}{x} = (4m + 4n - 4)b^2 + (4mn - 4n - 4m + 4)xb^2 + 4mnxb^4. \]

Taking integration on both sides,
\[ \int_0^{a} \frac{f(x, b)}{x} dx = \int_0^{a} \left( (4m + 4n - 4)ab^2 + \right. \]
\[ \left. + \int_0^{a} (4mn - 4m - 4n + 4)xb^2 dx + \right. \]
\[ \left. + 4mn \int_0^{a} xdxb^4, \right. \]
\[ S_a = (4m + 4n - 4)ab^2 + \]
\[ + \frac{1}{2} (4mn - 4n - 4m + 4)a^2b^2 + 2mnab^4. \]

Now, take \( S_b \) and then...
\[
S_a S_b f(a, b) = (4m + 4n - 4)ax^2 \\
+ \frac{1}{2}(4mn - 4m - 4n + 4)a^2x^3 + 2mna^2x^4,
\]
\[
S_a S_b f(a, b) = \frac{1}{2}(4m + 4n - 4)ab^2 \\
+ \frac{1}{4}(4mn - 4m - 4n + 4)a^2b^2 + \frac{1}{2}mna^2b^4.
\] (30)

\[
^m M_2 (G) = S_a S_b f(a, b)|_{a=b=1} = \frac{1}{2}(4m + 4n - 4) + \frac{1}{4}(4mn - 4m - 4n + 4) + \frac{1}{2}mn
\]
\[
= (2m + 2n - 2) + (mn - m - n + 1) + \frac{1}{2}mn
\] (31)
\[
= 2m - m + 2n - n - 2 + 1 + mn\left(1 + \frac{1}{2}\right).
\]

After solving, the result is
\[
^m M_2 (G) = \frac{3}{2}mn + m + n - 1. \tag{32}\]

The 3D plot of modified second Zagreb index is given in Figure 5 (f or u = 1 left, v = 1 middle, and w = 1 right), and we see the dependent variables of the modified second Zagreb index on the involved parameters. □

**Theorem 5.** Crystallographic structure of the graph of copper(I) oxide \(G \approx Cu_20[m; n]\), where \(n; m \geq 1\), and we have
\[
R_a(G) = \left(2^{a^2} - 2^{2a^2}\right)(m + n - 1) + \left(2^{a^2} + 2^{3a^2}\right)mn. \tag{33}\]

**Proof.** suppose
\[
M(G; a; b) = (4m + 4n - 4) \times ab^2 + (4mn - 4m - 4n + 4) \\
\times a^2b^2 + (4mn) \times a^2b^4. \tag{34}\]

We have to find \(D_a D_b\) first, and we find \(D_a:\)
\[
D_a = (4m + 4n - 4) \times ab^2 + 2(4mn - 4m - 4n + 4) \\
\times a^2b^2 + 8mn \times a^2b^4. \tag{35}\]

Now, take \(D_b:\)
\[
D_a D_b = (4m + 4n - 4)a \times 2b \times b + 2(4mn - 4m - 4n + 4)a^2 \\
\times 2b \times b + 2(4mn)a^2 \times 4b^3 \times b. \tag{36}\]

Take \(a\) on the above equation:

\[\text{Now, the second modified Zagreb index is}\]

\[D_a^D_b^D = 2^{a^2} (4m + 4n - 4)ab^2 + 4^{a^2} (4mn - 4m - 4n + 4)a^2b^2 \\
+ 8^{a^2} (4mn)a^2b^4, \]
\[
D_a^D_b^D = 2^{a^2} m + n - 1ab^2 + 2^{2a^2} (mn - m - n + 1)a^2b^2 \\
+ 2^{4a^2} mn a^2b^4. \tag{37}\]

Now, the general Randić index is
\[
R_a(G) = D_a^D_b^D (f(a, b))|_{a=b=1}, \]
\[
R_a(G) = 2^{a^2} (m + n - 1) + 2^{2a^2} (mn - m - n + 1) + 2^{3a^2} mn, \]
\[
R_a(G) = 2^{a^2} m + 2^{a^2} n - 2^{2a^2} + 2^{2a^2} mn - 2^{3a^2} m \\
- 2^{2a^2} m + 2^{2a^2} + 2^{3a^2} mn. \tag{38}\]

The result is
\[
R_a(G) = \left(2^{a^2} - 2^{2a^2}\right)(m + n - 1) + \left(2^{2a^2} + 2^{3a^2}\right)mn. \tag{39}\]

The 3D plot of Randić index is given in Figure 6 (f or u = 1 left, v = 1 middle, and w = 1 right), and we see the dependent variables of the Randić index on the involved parameters. □

**Theorem 6.** Crystallographic structure of the graph of copper(I) oxide \(G \approx Cu_20[m; n]\), where \(n; m \geq 1\), and we have
\[
RR_a(G) = \left[\frac{1}{2^{a^2 - 2}} - \frac{1}{2^{3a^2 - 2}}\right](m + n) + \left[\frac{1}{2^{a^2 - 2}} + \frac{2}{2^{3a^2 - 2}}\right](mn) \\
+ \left[\frac{1}{2^{a^2 - 2}} + \frac{1}{2^{2a^2 - 2}}\right]. \tag{40}\]
Proof. Suppose
\[ M(G; a; b) = (4m + 4n - 4) \times ab^2 + (4mn - 4m - 4n + 4) \]
\[ \times a^2b^2 + (4mn) \times a^3b^4. \]

(41)

Now, we have to find \( S_a S_b \), and first, we find \( S_a \):
\[ S_a = (4m + 4n - 4) \int_0^a dx \cdot b^2 + (4mn - 4m - 4n + 4) \]
\[ \cdot \int_0^a xdx \cdot b^2 + 8mn \int_0^a xdx \cdot b^4 \]
\[ S_a = (4m + 4n - 4)ab^2 + 2(mn - m - n + 1)a^2b^2 \]
\[ + 4mna^2b^4. \]

(42)

Similarly, take \( S_b \):
\[ S_a S_b = 4(m + n - 1)a \int_0^b xdx + 2(mn - n - m + 1)a^2 \]
\[ \cdot \int_0^b xdx + 4mna^2 \int_0^b x^3 dx, \]
\[ S_a S_b = 2(m + n - 1)ab^2 + (mn - m - n + 1)a^2b^2 \]
\[ + mna^2b^4. \]

Take \( \alpha \) on the above equation:
\[ S_a S_b = \frac{1}{2^{\alpha+2}} (m + n - 1)ab^2 + \frac{1}{2^{2\alpha+2}} (mn - m - n + 1)a^2b^2 \]
\[ + \frac{1}{2^{3\alpha+2}} mna^2b^4. \]

(43)

The inverse Randić is
\[ RR_a(G) = (f(a, b))_{a=0} = \frac{1}{2} \left( m + n - 1 \right) \]
\[ + \frac{1}{2^{m-2}} (mn - m - n + 1) \]
\[ + \frac{2}{2^{m-2}} mn = \left[ \frac{1}{2} + \frac{2}{2^{m-2}} \right] (mn) + \left[ \frac{1}{2} + \frac{1}{2^{m-2}} \right] \]
\[ (45) \]

The 3D plot of inverse Randić index is represented in Figure 7 (for \( u = 1 \) left, \( v = 1 \) middle, and \( w = 1 \) right), and we see the dependent variables of the inverse Randić index on the involved parameters. □

**Theorem 7.** Crystallographic structure of the graph of copper(I) oxide \( G = Cu_2[0m; n] \), where \( n; m \geq 1 \), and we have \( SS \ D(G) = 18mn + 2m + 2n - 2 \).

**Proof.** Suppose
\[ M(G; a; b) = (4m + 4n - 4) \times ab^2 + (4mn - 4m - 4n + 4) \]
\[ \times a^3b^2 + (4mn) \times a^2b^4. \]
\[ (46) \]

First, we have to find \( S_b \):

\[ SS \ D(G) = \left[ \frac{1}{2} (4m + 4n - 4) + (4mn - 4m - 4n + 4) + 2mn \right] + \left[ 2(4m + 4n - 4) + (4mn - 4m - 4n + 4) + 8mn \right], \]
\[ SS \ D(G) = (2m + 2n - 2) + (4mn - 4m - 4n + 4) + (2mn + 8m + 8n - 8) + (4mn - 4m - 4n + 4) + 8mn, \]
\[ SS \ D(G) = (2m - 4m + 8m - 4m) + (2n - 4n + 8n - 4n) - (2 - 4 + 8 - 8) + (4mn + 2mn + 4mn + 8mn). \]
\[ (52) \]

After the calculation, the result is
\[ SS \ D(G) = 18mn + 2m + 2n - 2. \]
\[ (53) \]

The 3D plot of symmetric division index is given in Figure 8 (for \( u = 1 \) left, \( v = 1 \) middle, and \( w = 1 \) right), and we see the dependent variables of the symmetric division index on the involved parameters. □

**Theorem 8.** Crystallographic structure of the graph of copper(I) oxide \( G = Cu_2[0m; n] \), where \( n; m \geq 1 \), and we have
\[ H(G) = \frac{5}{3} (m + n - 1) + \frac{7}{3} mn. \]
\[ (54) \]
\[ S_a f(x, b) = 4(m + n - 1) \int_0^a x^2 \, dx + 4(mn - m - n + 1) \cdot \int_0^a x^3 \, dx, \]
\[ S_a f(a, b) = \frac{4}{3}(m + n - 1)a^3 + \frac{1}{2}(mn - m - n + 1)a^4 + 2mn a^6 . \] (57)

The harmonic index is
\[ H(G) = 2S_a f(a, b)|_{a=1} \]
\[ = 2 \left[ \frac{4}{3}(m + n - 1) + \frac{1}{2}(mn - m - n + 1) + \frac{2}{3}mn \right] , \]
\[ H(G) = 2 \left[ \left( \frac{4}{3} - \frac{1}{2} \right)m + \left( \frac{4}{3} - \frac{1}{2} \right)n + \left( \frac{1}{3} - \frac{4}{3} \right) + \left( \frac{1}{2} + \frac{1}{2} \right) \right]mn , \]
\[ H(G) = 2 \left[ \frac{5}{6}m + \frac{5}{6}n + \frac{7}{6}mn - \frac{5}{6} \right] . \] (58)

Now, the result is
\[ H(G) = \frac{5}{3}(m + n - 1) + \frac{7}{3}mn . \] (59)

The 3D plot of harmonic index is given in Figure 9 (for \( u = 1 \) left, \( v = 1 \) middle, and \( w = 1 \) right), and we see the dependent variables of the harmonic index on the involved parameters. \( \Box \)

**Theorem 9.** Crystallographic structure of the graph of copper(I) oxide \( G \approx Cu20[m; n] \), where \( n; m \geq 1 \), and we have
\[ S_aJD_aD_b f(a, b)) = \frac{44}{3}mn - \frac{4}{3}(m + n - 1) . \] (60)

**Proof.** Suppose
\[ M(G; a; b) = (4m + 4n - 4) \times ab^2 + (4mn - 4m - 4n + 4) \times a^3b^4 . \] (61)

First, we have to find \( D_b \).
\[
D_b f(a, b) = 8(m + n - 1)ab^2 + 8(mn + m + n - 1)a^2b^2 \\
+ 32mnab^4.
\]

Take \( D_a \):
\[
D_a D_b f(a, b) = 8(m + n - 1)ab^2 + 16(mn - n - m + 1)a^3b^2 \\
+ 64mnab^4.
\]

Take \( J_f(a, b) \):
\[
J D_a D_b f(a, b) = 8(m + n - 1)x^3 + 16(mn - n - m + 1)x^4 \\
+ 64mnx^6.
\]

Take \( S(a) \):
\[
S_a J D_a D_b f(a, b) = \frac{8}{3}(m + n - 1)a^3 \\
+ 4(mn - n - m + 1)a^4 + \frac{32}{3}mnab^6.
\]

The inverse sum index is
\[
S_a J D_a D_b (f(a, b))|_{a=1} = \frac{8}{3}(m + n - 1) \\
+ 4(mn - n - m + 1) + \frac{32}{3}mn \\
= \left(\frac{8}{3} - 4\right)m + \left(\frac{8}{3} - 4\right)n \\
+ \left(\frac{32}{3} + 4\right)mn + \left(4 - \frac{8}{3}\right).
\]
After the calculation, the result is
\[
S_{a,b}(D_a D_b (f(a,b))) = \frac{44}{3} mn - \frac{4}{3} (m + n - 1).
\] (67)

The 3D plot of inverse sum index is given in Figure 10 (\(f\) or \(u = 1\) left, \(v = 1\) middle, and \(w = 1\) right), and we see the dependent variables of the inverse sum index on the involved parameters.

**Data Availability**

No data were used in this study.

**Disclosure**

All authors have not any fund, grant, and sponsor for supporting publication charges.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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