Frequency Comb Generation at 800 nm in Waveguide Array Quantum Well Diode Lasers

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Abstract—A traveling wave model for a semiconductor diode laser based on quantum wells is presented as well as a comprehensive theoretical model of the lasing dynamics produced by the intensity discrimination and controllable loss of mode-coupling in a waveguide array. By leveraging a recently developed model for the detailed semiconductor gain dynamics, the temporal shape effects and nonlinear mode-coupling induced by the waveguide arrays can be characterized. Specifically, the enhanced pulse shaping and loss provided by the waveguides is capable of generating stable frequency combs at a wavelength of 800 nm in a GaAs device, a parameter regime for which it is difficult to obtain stable, broad comb generation using a single waveguide. Extensive numerical simulations showed that stable waveform generation could be achieved and optimized by an appropriate choice of the linear waveguide coupling coefficient, quantum well depth, and the input currents to the first and second waveguides. The model provides a demonstration that a compact, efficient and robust on-chip comb source can be produced in GaAs.

Index Terms—Diode lasers, optical frequency combs, waveguide arrays, quantum wells.

I. INTRODUCTION

O PTICAL frequency comb generation remains an important technological challenge, especially on chip scale devices which are important for practical engineering design and commercial applications. Chip scale devices are critical for reduction to practice implementations in applications ranging from frequency metrology and optical spectroscopy [1], [2], multi-heterodyne spectroscopy [3], optical atomic clocks [4], and arbitrary waveform synthesis [5]. However, practical physics considerations make reduction to chip scale diode lasers challenging due to the nonlinear phase shifts that occur from the fast carrier dynamics [6], and limit the pulse width within the cavity. A recent theoretical study developed a detailed traveling wave model of a semiconductor diode laser based on quantum wells [7]. The resulting frequency modulated comb showed potential for a compact, chip-scale comb source without additional external components at wavelength of 1550 nm. This work leveraged well known results that single-section diode lasers without saturable absorbers can also generate frequency-modulated (FM) mode locking [8]. Despite the success at 1550 nm, the generation of a broad, stable comb was difficult at the modified parameter settings of 800 nm where many engineering applications are relevant. In this work, we modify the detailed physics model introduced by Dong et al. [7] to include coupled waveguide arrays (WGAs) in order to generate stable waveforms at 800nm. WGAs have been demonstrated to enhance the nonlinear pulse shaping necessary to promote mode-locking [9], thus they are used in the present model to promote stable waveform generation at 800nm. Indeed, we show that with the addition of the WGAs, stable waveform generation can be achieved in a diode laser configuration, allowing for the possibility of chip scale frequency combs.

Optical frequency comb generation can be accomplished in a diverse number of laser settings. For instance, they can be generated in now standard mode-locked lasers such as Ti:Sapphire [10] and fiber lasers [11]. Generation in passive microresonators due to Kerr nonlinearity is also reported [12] as well as others based on microcavities, gain-switching lasers and phase modulators [13]–[15]. These standard and successful methods require a number of optical and fiber components that must be carefully aligned. They further often require bulky pump lasers and amplifiers for operation. So although they are ideally suited for laboratory experiments, they are difficult to implement outside of such laboratory settings. Diode lasers, in contrast, provide direct frequency comb generation, are portable and efficient from a chip-scale device and have been reduced to practice in numerous technologies [16], [17]. Two competing dynamics are at work in a typical laser of this type: a gain dynamic and a reverse-biased saturable absorber dynamic. A periodic train of short pulses is formed as a consequence of competition between these two effects, and its Fourier transform results in an optical frequency comb.
generation. It turns out that the stabilization of such pulses is limited by the nonlinear phase shifts that occur due to fast carrier dynamics [6]. However, as noted earlier, the FM mode-locked state in a single-section diode laser results in a continuous wave in time but a fixed, non-zero phase difference between comb lines because of the frequency modulation.

Quantum dot (QD) [18], [19] and quantum lasers (QDash) [20] have been an ideal source for studying FM mode-locking operation, with quantum well (QW) [21], [22] and bulk semiconductor lasers [8] also exhibiting this phenomenon. Given their practical importance, computational and theoretical models of semiconductor quantum well lasers have been extensively developed. Models can be fairly simple, including only a single rate equation and photon density variable [23], [24]. More complex models include multiple rate equations and material polarization [25]–[33] with various phenomenological parameters inserted. Existing models have failed to explain consistently the origin of the FM comb generation. This is, in part, due to the diverse and numerous nonlinear effects in the laser cavity. Following on previous QD single-section laser models [18], [34], Dong et al. [7] developed a more comprehensive detailed model for the QW diode laser dynamics. At 1550 nm, this model was able to produce stable waveforms and excellent performance characteristics. But in the application important 800 nm parameter regime, the model did not produce broad, stable combs.

The difficulty of generating combs at the shorter wavelength has been ascribed to the absence of spatial hole burning on the scale of a half-wavelength owing to carrier diffusion [35]. In the absence of the “fast” spatial holes due to standing waves, the “slow” spatial holes on the length scale of the cavity length yield multiple longitudinal modes that exchange energy in a periodic or chaotic manner, resulting in an unstable spectrum [36]. The difficulty in producing FM combs in such a detailed QW diode laser model at 800 nm motivates our exploration of using WGAs to enhance nonlinear pulse shaping for mode-locking and stabilize wave generation. Nonlinear mode-coupling in WGAs has been shown to produce the intensity discrimination necessary to produce mode-locking [37]–[39]. Indeed, semiconductor WGAs studied for CW lasing [40], [41] motivated a recent phenomenological model for mode-locking on a chip [9]. Here, we integrate the detailed semiconductor physics model of Dong et al. [7] into a WGA in order to achieve stable waveform generation at 800 nm. Indeed, we show that the extra pulse shaping and controllable loss induced by the WGAs is capable of producing frequency combs in this important parameter regime. As for the integrated device structure, a dispersion-compensating fiber is not necessary. Specifically, an OFC generator does not have to produce a train of short pulses. The integrated device described here operates in a frequency-modulated (FM) mode and produces coherent comb lines that have a deterministic quadratic phase relationship. A dispersion-compensating fiber is used here only to demonstrate the coherent nature of the comb. It is not required for any applications of frequency combs.

The paper is outlined as follows: Sec. II details the theoretical models necessary for constructing a detailed description of the laser cavity dynamics. The interplay of the various physics allow for the generation of stable waveforms.

II. THEORETICAL MODELS

The following subsections outline the key modeling components necessary for constructing a detailed description of the laser cavity dynamics. The interplay of the various physics allow for the generation of stable waveforms.

A. Waveguide Arrays

A traveling wave model for the generation of stable frequency combs of a semiconductor diode laser in a single waveguide has been studied previously [7]. Here we extend this model to GaAs, which has a higher central transition energy. GaAs lasers typically operate around 800 nm compared to 1550 nm in InGaAs [7]. However, we show that the single waveguide model does not yield stable waveform generation in this new parameter regime. The WGA architecture is a pulse shaping strategy that can help promote stable waveform generation. This is achieved by using the intensity discrimination of the WGAs.

This concept of a waveguide array mode-locked laser is shown in Figure 1. By preferentially coupling out low-intensity light to the neighboring waveguides, the electric field propagating in the first waveguide is shaped according to the intensity, dispersion and gain dynamics. This intensity discrimination is necessary for the generation of stable mode-locked pulses in a laser cavity [37]–[39]. Another effect of the coupled waveguides is to provide a controllable loss mechanism that can tailor the output spectrum by filtering out very high order longitudinal modes.

The leading-order equations governing the electric field dynamics and the linear, evanescent electric field coupling to the neighboring waveguides are given by

$$i \frac{d E_n}{d \xi} + C(E_{n-1} + E_{n+1}) + \beta |E_n|^2 E_n = 0,$$

where $E_n$ represents the electric field envelope in the $n$th waveguide in the array, $C$ represents the linear coupling coefficient, $\xi$ represents the normalized distance in the

![Fig. 1. Schematic of a waveguide array diode laser. The input current on the first waveguide provides with the satura...or experience net losses.](image-url)
waveguide array and $\beta$ the nonlinear self-phase modulation parameter [42]–[44].

Though the waveguide array can be formed by a different numbers of waveguides, numerical studies and stability analysis shows that using three or more waveguides can produce robust pulse shaping and intensity discrimination [44]. Indeed, a three waveguide structure has almost identical properties to the 41 waveguides considered in early WGA experiments [43], whereas two waveguides do not offer stable and robust dynamics [44]. Therefore, we consider a WGA architecture with three waveguides. The first waveguide is forward biased and gets a net gain from the pump. The second waveguide has zero net gain so that it is close to transparency. Waveguide three is reversed biased to increase the linear loss and shorten the recovery lifetime by sweeping out optically generated carriers, thus engineered to attenuate the electromagnetic energy that enters it. It should be noted that there are many other mechanisms that can produce saturable absorption in a laser cavity, with a diverse set of quantitative and qualitative models for characterizing its physical effects [45]. Here, we have chosen a quantitative model that can be engineered within the laser chip structure, i.e. we forego as much as possible any empirical or generic models.

B. Gain Model and Governing Equations

Here we briefly describe the non-trivial gain model [7], and extend it to the three waveguide array. Relaxation of the electrons from N-side to the separate confinement heterostructure (SCH) layer traps carriers inside the quantum well, as shown in Figure 2. In addition to the discretization and propagation of the wave equations, the carrier equations in the energy space are discretized as well in the gain model [7]. Despite the increased calculation expenses of the carrier equations, this model successfully captures the detailed cavity dynamics [7].

For the carriers trapped in the quantum well, light emitted is centered around different central frequencies because of the varied transverse energy $E_i$. As opposed to the confined $z$ direction, carriers in the quantum well have transverse momenta in the unconfined directions of $x$ and $y$. Specifically, the transverse energy $E_i$ is derived from the joint density of states as an integrated combination of carrier transverse energies, and contributes to light generation and recombination. Experimentally by changing the depth of the quantum well, we can constrain the carrier energy otherwise the carriers would easily escape. That is, the maximum transverse energy max($E_i$) can be equivalently adjusted by varying the quantum well depth. In this non-trivial gain model, all carrier Lorentzians are integrated in energy space [7].

The electric field, including forward and backward propagation is given by

$$E(\mathbf{z}, t) = E_+(\mathbf{z}, t)e^{i\mathbf{h}_{\mathbf{g}}\cdot \mathbf{z}} + E_-(\mathbf{z}, t)e^{-i\mathbf{h}_{\mathbf{g}}\cdot \mathbf{z}}. \quad (2)$$

Deriving from the slowly-varying envelope equation of $E_{\pm}(\mathbf{z}, t)$, combined with semiconductor Bloch equations of the total material polarization, and the carrier grating effects, the model [7] obtains

$$\pm \frac{\partial E_{\pm}}{\partial z} + \frac{1}{v_g} \frac{\partial E_{\pm}}{\partial t} = \frac{k''}{2} \frac{\partial^2 E_{\pm}}{\partial z^2}$$

$$- \alpha E_{\pm} - \left( \frac{\alpha_S}{2} + i\beta_S \right) (|E_{\pm}|^2 + 2|E_{\mp}|^2) E_{\pm} + S_{sp}$$

$$+ n_{qw} g_0 \frac{1}{2} \int \frac{dE_t}{h\omega_0} \left( \rho_{e,sch}^{E_t} + \rho_{p}^{E_t} - 1 \right) F_{\pm}(E_t, \mathbf{z}, t)$$

$$+ n_{qw} g_0 \frac{1}{2} \int \frac{dE_t}{h\omega_0} \rho_{g,sch}^{(a)}(\mathbf{E}_{\pm}(\mathbf{E}_t, \mathbf{z}, t)) \quad (3)$$

where $k''$ is the dispersion coefficient, $\alpha$ is the linear waveguide loss, $\alpha_S$ the two-photon absorption and $\beta_S$ the Kerr nonlinear coefficients. Electron (e) or hole (h) occupation probabilities are notated as $\rho_{e,sch}^{E_t}$, $\rho_{p}^{E_t}$ is the grating term and $S_{sp}$ and $F_{\pm}(E_t, \mathbf{z}, t)$ are respectively the spontaneous emission term and the filtered field derived in ref. [7].

The coupled carrier rate equations are given [7]

$$\frac{\partial \rho_{e,sch}^{E_t}}{\partial t} = \frac{\eta J_{in}}{q N_{c,v,sch}\hbar} \left( 1 - \rho_{e,sch}^{E_t} \right) / \tau_{sp}$$

$$+ \frac{n_{qw}}{E_t} \left( \rho_{e,sch}^{E_t} - \rho_{p}^{E_t} \right) / \tau_{e,h}$$

$$+ \rho_{e,sch}^{E_t} \frac{(1 - \rho_{p}^{E_t})}{\tau_{e,h}} - \rho_{e,sch}^{E_t} \frac{(1 - \rho_{p}^{E_t})}{\tau_{e,h}}$$

$$+ \frac{n_{qw}}{E_t} \left( \rho_{e,sch}^{E_t} - \rho_{p}^{E_t} \right) / \tau_{e,h}$$

$$- \rho_{e,sch}^{E_t} \frac{(1 - \rho_{p}^{E_t})}{\tau_{e,h}} - \rho_{e,sch}^{E_t} \bar{R}_{sl} - R_g. \quad (4a)$$

$$\frac{\partial \rho_{q,sch}^{E_t}}{\partial t} = \frac{\hbar}{n_{qw} h_{q,w} N_{r,w}} \left( \rho_{e,sch}^{E_t} - \rho_{q,sch}^{E_t} \right) / \tau_{sp}$$

$$- \rho_{q,sch}^{E_t} \frac{(1 - \rho_{p}^{E_t})}{\tau_{e,h}} - \rho_{q,sch}^{E_t} \bar{R}_{sl} - R_g. \quad (4b)$$

$$\frac{\partial \rho_{g,sch}^{E_t}}{\partial t} = \frac{\rho_{g,sch}^{E_t}}{\tau_{sp}} - 4k_D^2 D_{q,s} \rho_{g,sch}^{E_t} - 2g_0 \frac{\Delta E_t}{(h\omega_0)^2 h_{q,w} N_{r,w}}$$

$$\times \left( \frac{1}{2} (E_t^* F_e + F_e^* E_t) (\rho_{q,sch}^{E_t} + \rho_{p}^{E_t} - 1)$$

$$+ 2 \text{Re}(E_t^* F_e + F_e^* E_t) \rho_{g,sch}^{E_t}) \right) \quad (4c)$$

where $R_{sl}$ and $R_g$ are the recombination rates of population decay for stimulated emission and carrier grating, $N_{c,v,sch}$ and $N_r$ are the effective 3-D and 2-D density of states.
The parameter $g_0$ is the gain coefficient defined as
\[
g_0 = \frac{\Gamma_1 n_{q, w} q^2 m_r |\mathbf{e} \cdot \mathbf{p}|^2}{12\pi \hbar c e q \mu_0 \hbar \nu_{q, w}},
\]  
where $m_r = 1/(1/m_{e, w} + 1/m_{h, w})$ is the reduced effective mass. Other parameters are listed in Table I in details.

For simplicity, we rewrite the complicated term as a function of $G_{\pm}(E_{\pm})$ so that Eq. (3) is more compactly represented
\[
\pm \frac{\partial E_{\pm}}{\partial z} + \frac{1}{v_g} \frac{\partial E_{\pm}}{\partial t} + \frac{k''}{2} \frac{\partial^2 E_{\pm}}{\partial t^2} = -\frac{\alpha}{2} E_{\pm} - \left(\frac{\alpha_s}{2} + i\beta_s\right) |E_{\pm}|^2 + \frac{1}{2} |E_{\pm}^1|^2 E_{\pm}^1 + S_{sp} + G_{\pm} + iC(E_{\pm}^1 + E_{\pm}^2).
\]

These field equations are coupled with the carrier rate equations for the SCH and QW sections, of which the complete forms are shown in Dong et al. [7].

We extend this gain model to the waveguide array structure with three waveguides. By coupling out low-intensity components of the electric field to the neighboring waveguides, we can effectively shape the electric field propagating in the first waveguide through controllable loss and intensity discrimination, thus achieving highly robust stable waveform generation in the laser cavity.

The resulting equations describing the approximate dynamics of waveguide array mode-locking are thus given by
\[
\pm \frac{\partial E_{\pm}^1}{\partial z} + \frac{1}{v_g} \frac{\partial E_{\pm}^1}{\partial t} + \frac{k''}{2} \frac{\partial^2 E_{\pm}^1}{\partial t^2} = -\frac{\alpha}{2} E_{\pm}^1 - \left(\frac{\alpha_s}{2} + i\beta_s\right) |E_{\pm}^1|^2 + \frac{1}{2} |E_{\pm}^1|^2 E_{\pm}^1 + S_{sp} + G_{\pm} + iC(E_{\pm}^1 + E_{\pm}^2).
\]

For simplicity, the dimensionless coupling factor $C$ is maintained in the same way in our simulation between different waveguides. $C$ is determined by the designed parameters of the WGA, such as the waveguide separation along with its width and depth [46]. Specifically, with the linear coupling coefficient $c$ (m$^{-1}$) and waveguide length $L$ (m), $C = cL$.

Thus it can be adjusted via designing the waveguide array to realize optimal mode-locking of the output.

The carrier rate equations are also modified to simulate the weakly forward biased state and reverse biased state in the second and third waveguides. We simply set $J_{th} = 0$ in the rate equations for the second waveguide to get zero net gain. For the third waveguide, we simulate the voltage driven bias with an equivalent carrier drift with lifetime $\tau_d^{e,h}$ and replace the gain term as
\[
\frac{\partial \rho_{e,h}}{\partial t} = n_{q, w} \sum_{E_1} \left[ \rho_{q, w, E_1} \left( 1 - \frac{\rho_{e, h}^{e, h}}{\tau_{e, h}^{e, h}} + \frac{\rho_{e, h}^{e, h}}{\tau_{e, h}^{e, h}} \right) - \rho_{e, h}^{e, h} \left( 1 - \frac{\rho_{e, h}^{e, h}}{\tau_{e, h}^{e, h}} + \frac{\rho_{e, h}^{e, h}}{\tau_{e, h}^{e, h}} \right) \right] - \frac{\rho_{e, h}^{e, h}}{\tau_d^{e, h}} - \frac{\rho_{e, h}^{e, h}}{\tau_{sp}}.
\]

III. NUMERICAL RESULTS FOR STABLE WAVEFORM GENERATION

We solve the forward and backward wave equations (Eqs. (7)-(9)), coupled with the carrier rate equations [7] numerically using a robust predictor-corrector scheme which generically improves stability properties compared to the Euler algorithm [47]. We simulate 100 ns of operation of the waveguide array starting from noise with a time step of $\Delta t = 30$ fs. Our full model takes approximately 4 hours to run parallelized on x2 6-Core Intel Xeon CPU. The full simulation parameters are listed in Table I. Note that the major difference between GaAs and the previous material considered, InGaAsP, is that GaAs has a larger central transition frequency difference between GaAs and the previous material considered, InGaAsP, is that GaAs has a larger central transition frequency difference. For simplicity, we replace $\alpha$ by $\alpha_s$ with a significant decrease. These changes of parameters affect the formation of stable combs in the single waveguide model. Given the large parameter space to be explored using WAGAs for optimal design, we focus on (i) the linear waveguide coupling coefficient $C$ between the three waveguides, (ii) the input pump to the waveguide array, and (iii) the depth of the quantum well. The third waveguide is set to be reversed biased and maintains the same setup in the following experiments, while the first two waveguides have varied input pumps in different experiments. The other parameters are chosen to be the feasible parameters for an experimental design [7].
Fig. 3. Evolution of output power $P$ (mW) and power spectral density $|\hat{E}|^2$ (dBm/Hz) in the first waveguide with $I_{in} = 100$ mA and the coupling factor $C = 0$. (a) The temporal output and (b) the power spectral density of the temporal output in log scale. (c) (d) The zoomed power spectral density and temporal output of the first waveguide. The output electric field quasi-periodic and takes more than 200 ns to reach steady state.

Fig. 4. Evolution of output power $P$ (mW) and power spectral density $|\hat{E}|^2$ (dBm/Hz) in the first waveguide with $I_{1in} = 100$ mA, $I_{2in} = 0$ and the coupling factor $C = 1$. (a) The temporal output inside waveguide 1. A stable state is achieved for $t > 30$ ns. (b) A broad comb is shown in the power spectral density of the temporal output in log scale. (c) (d) The zoomed power spectral density and temporal output of the first waveguide. The output includes a periodic short burst on top of a continuous wave. Although it narrows down in the power spectral density, the spectrum is of higher intensity and a flatter plateau.

Similar to the single waveguide model, we specify the limits of $E_t$, that is, $\max(E_t)$, and the number of energy bins. As discussed previously, $\max(E_t)$ can be adjusted by varying the quantum well depth, and thus is experimentally controllable with appropriate fabrication design. The size of the energy bin ($dE_t$) is the energy discretization for numerical computation. It is chosen to be small relative to the homogeneous linewidth for numerical convergence of the gain integral. Smaller size of energy bins will greatly increase computation time. Note that our simulations also suggested that higher energy carriers can contribute to the total gain and thus affect the generation of the lasing dynamics. We choose $\max(E_t) = 50$ meV, with 25 energy bins to guarantee a small energy step for an accurate gain integral.

We first solved a single waveguide model by setting the coupling factor of $C = 0$ so there is no coupling between waveguide. The input currents to the first two waveguides are set to be $I_{1in} = 100$ mA and $I_{2in} = 0$ for the following experiments studying the impact of varying the coupling factor $C$. The quantum well depth $\max(E_t)$ is set to be 50 meV. As shown in Fig. 3, it took nearly 200 ns for the output to reach a quasi-periodic steady state with very high frequency oscillations and a spectrum indicating the presence of mode competition. Turning on the coupling factor with a small value, e.g. $C = 0.5$, does not stabilize the chaotic
Fig. 7. Evolution of output power $P$ (mW) and power spectral density $|E|^2$ (dBm/Hz) in the first waveguide with $I_{1\text{in}} = 100$ mA, $I_{2\text{in}} = 0$ and the coupling factor $C=3$. (a) The temporal output and (b) power spectral density of the temporal output in log scale of the first waveguide. The output power quickly dies to a low value. (c) (d) The zoomed power spectral density and temporal output of the first waveguide. The gain is insufficient to pump the wave compared to the energy loss and we are left with the low-power white noises in the cavity.

Fig. 8. Evolution of output power $P$ (mW) and power spectral density $|E|^2$ (dBm/Hz) in the first waveguide with $I_{1\text{in}} = 350$ mA, $I_{2\text{in}} = 0$ and the coupling factor $C = 3$. (a) The temporal output and (b) power spectral density of the temporal output in log scale of the first waveguide. (c) (d) The zoomed power spectral density and temporal output of the first waveguide. With increased input current to the first waveguide, the gain is sufficient to pump the wave compared to the energy loss. Interestingly, a second peak in the spectrum is generated, suggesting that the second waveguide is lasing as well.

electric field in the cavity. When the coupling factor $C$ is increased to unity, the electric field inside the cavity rapidly stabilizes to a mode-locked periodic state, see Fig. 4 in less than 50 ns and the spectrum exhibits a frequency comb with lines spaced by the round-trip frequency of 85.7 GHz. To show the periodic and coherent output is mode-locked, the spectrum and spectral phase are plotted on a linear scale, as shown in Fig. 5(a). The quadratic spectral phase shows that the frequency comb from this chip-scale device is coherent and can thus be used in applications such as spectroscopy and metrology. If one desires short pulses the quadratic phase can be compensated by propagating through another waveguide or fiber with anomalous dispersion, resulting in the output shown in Fig. 5(b). The required group delay dispersion is calculated to be 0.52 ps$^2$. If we increase the value of the coupling factor $C$ from 1 to 2 (Fig. 6), the mode-locked state still holds in the cavity, and the distance between comb lines is not changed. However, the output power of the electric field in the first waveguide is decreased. This is reasonable since more energy gets coupled to the neighboring waveguides and dissipated in the 3rd waveguide. If the coupling factor gets too large, for example, $C = 3$ in this case (Fig. 7), a large amount of energy gets coupled to the neighboring waveguides and we are no longer able to get to an equilibrium, or balance, of the gain and the loss dynamics, thus the mode locked state disappears. Interestingly, if the input current to the first waveguide is increased to $I_{1\text{in}} = 350$ mA with $C = 3$, the output power of the first waveguide is prevented from dying. Instead, we are left with a two-peak spectrum, as shown in Fig. 8. This suggests that the second waveguide is also lasing. Considering the second waveguide has no input current, the lasing is likely pumped by increased energy coupled out from the first waveguide with a larger coupling factor $C = 3$, and it equivalently balances the energy loss of cavity.

In the simulations above, the input current to the first waveguide is set to be $I_{1\text{in}} = 100$ mA. There is no energy pumping (gain from current injection) applied to the second and third waveguides in the array. Waveguide two experiences a net intrinsic loss with $\alpha = 5$ cm$^{-1}$, whereas an extra electron and hole sweep out with lifetime 0.1 ps and 0.3 ps exists in waveguide three as it is reverse biased.

With the energy step remaining as 2 meV but the quantum well depth $\max(E_t)$ increased to 100 meV, the mode-locked state does not hold with the coupling factor $C = 1$ and the electric field inside waveguide one falls back to the chaotic state, as shown in Fig. 9. Since the limit of $E_t$ equivalently contributes to the gain, we test another case when $\max(E_t)$ remains 50 meV but the input pump on the first waveguide is increased to 200 mA. Neither of these parameter regimes are capable of producing a repeatable waveform.

An extra gain to the second waveguide is added to make it nearly neutral from a gain-loss perspective in order to better shape the frequency combs. The results are shown in Fig. 10-13, where the input current to the first waveguide is maintained as 100 mA and the coupling factor $C$ is set
Fig. 10. Evolution of output power $P$ (mW) and power spectral density $|\hat{E}|^2$ (dBm/Hz) in the first waveguide with $I_{1in} = 100$ mA, $I_{2in} = 30$ mA and the coupling factor $C = 1$. (a) The temporal output and (b) the power spectral density of the temporal output in log scale of the first waveguide. (c) (d) The zoomed power spectral density and temporal output of the first waveguide. Compared to Fig. 6, the electric output still remains in a similar shape with a higher output power in the time domain.

Fig. 11. Evolution of output power $P$ (mW) and power spectral density $|\hat{E}|^2$ (dBm/Hz) in the first waveguide with $I_{1in} = 100$ mA, $I_{2in} = 50$ mA and the coupling factor $C = 1$. (a) The temporal output and (b) the power spectral density of the temporal output in log scale of the first waveguide. (c) (d) The zoomed power spectral density and temporal output of the first waveguide. The increased input pump to the second waveguide has a significant impact on the output of the waveguide array. The period of the electric output in the time domain is largely decreased while the power spectral density has a wider separation between each comb line.

Fig. 12. Evolution of output power $P$ (mW) and power spectral density $|\hat{E}|^2$ (dBm/Hz) in the first waveguide with $I_{1in} = 100$ mA, $I_{2in} = 80$ mA and the coupling factor $C = 1$. (a) The temporal output and (b) the power spectral density of the temporal output in log scale of the first waveguide. (c) (d) The zoomed power spectral density and temporal output of the first waveguide. Compared to Fig. 11, extra comb lines arise around the central lines due to a higher input pump.

the input current is increased to 50 mA, the distance between each comb line is increased to about 1THz and the electric field oscillates in a shorter period in the time domain, perhaps indicating a harmonically modelocked state. When the input current is increased to 80 mA, secondary comb lines appear around the central ones in the power spectral density but the electric field remains in the periodic state in the time domain. Increasing the input current to 100 mA destroys the balance between the loss and gain and the system falls into the chaotic state.

IV. CONCLUSION

In conclusion, we have presented computational evidence that a traveling wave model for a quantum well and the mode-coupling in a waveguide array can generate frequency combs at 800 nm. The mode coupling of the waveguide array provides the necessary intensity discrimination and controllable loss for pulse shaping stabilizing the generation of a repeatable waveform and frequency comb in the cavity. To experimentally realize stable, robust combs, the coupling factor between waveguides in the array must be optimized.

We explored the parameter space of WGA coupling factor $C$, input currents to the waveguides, waveguide biases, energy steps and energy limits, to understand the different performance characteristics of the waveguide array model and its dependency on the WGA parameters. The mode-locking behavior is sensitive to certain directions of the parameter space. In particular, if the coupling factor $C$ is too large, the excess coupling loss can lead to quenching of laser action. For properly chosen parameters the numerical results demonstrate the generation of frequency combs at 800 nm with the coupled waveguide array for a quantum well. This combined model of diode lasers can serve as an excellent candidate for compact, efficient and robust comb sources experimentally.

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