Born-Infeld cosmology with scalar Born-Infeld matter

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Abstract
Cosmology in Eddington-inspired Born-Infeld gravity is investigated using a scalar Born-Infeld field (e.g. tachyon condensate) as matter. In this way, both in the gravity and matter sectors we have Born-Infeld-like structures characterized by their actions and via two separate constants, $\kappa$ and $\alpha^2$, respectively. With a particular choice of the form of $\dot{\phi}$ (the time derivative of the Born-Infeld scalar), analytical cosmological solutions are found. Thereafter, we explore some of the unique features of the corresponding cosmological spacetimes. For $\kappa > 0$, our solution has a de Sitter-like expansion both at early and late times, with an intermediate deceleration sandwiched between the accelerating phases. On the other hand, when $\kappa < 0$, the initial de Sitter phase is replaced by a bounce. Our solutions, at late time, fit well with available supernova data—a fact we demonstrate explicitly. The estimated properties of the Universe obtained from the fitting of the $\kappa > 0$ solution, are as good as in $\Lambda$CDM cosmology. However, the $\kappa < 0$ solution has to be discarded due to the occurrence of a bounce at an unacceptably low redshift.

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I. INTRODUCTION

General relativity (GR) is largely successful as a classical theory of gravity, especially with the recent detection of gravitational waves. However, there do exist many unsolved puzzles. Among them is the problem of singularities in GR which is expected to be resolved by quantum gravity. Dark matter and dark energy also do not seem to be well understood within the framework of GR. Thus, in order to address some of the above-mentioned problems, it is not unusual to construct theories which deviate from GR within the classical framework, inside matter distributions, or in the strong-field regime. This expectation has led many researchers to actively pursue modified theories of gravity in the classical domain and also in quantum theory. One such modification is inspired by the well-known Born-Infeld electrodynamics where we are able to regularize the infinity in the electric field at the location of a point charge [1]. With a similar determinantal structure \[ \sqrt{-\det(g_{\mu\nu} + \kappa R_{\mu\nu})} \] as in the action of Born-Infeld electrodynamics, a gravity theory in the metric formulation was first suggested by Deser and Gibbons [2]. In fact, the determinantal form of the gravitational action existed earlier in Eddington’s re-formulation of GR in de Sitter spacetime [3]. This formulation is affine and the connection is the basic variable instead of the metric. However, the coupling of matter remained a problem in Eddington’s approach.

Later, Vollick [4] introduced the Palatini formulation of Born-Infeld gravity and worked on various related aspects. He also introduced a nontrivial and somewhat artificial way of coupling matter in such a theory [5, 6]. More recently, Banados and Ferreira [7] have come up with a formulation where the matter coupling is different and simpler compared to Vollick’s proposal. We will focus here on the theory proposed in Ref. [7] and refer to it as Eddington-inspired Born–Infeld (EiBI) gravity, for obvious reasons. Note that the EiBI theory has the feature that it reduces to GR in vacuum.

Interestingly, EiBI theory also falls within the class of bimetric theories of gravity (also called bi-gravity). The current bimetric theories have their origin in the seminal work of Isham, Salam and Strathdee [8]. Several articles have appeared in the last few years on various aspects of such bi-gravity theories. In Ref. [9], the authors pointed out that the EiBI field equations can also be derived from an equivalent bi-gravity action. This action is closely related to a recently discovered family of unitary massive gravity theories which are built as bi-gravity theories. Several others have contributed in this direction, in various
Let us now briefly recall Eddington–inspired Born–Infeld gravity. The central feature here is the existence of a physical metric which couples to matter and another auxiliary metric which is not used for matter couplings. One needs to solve for both metrics through the field equations. The action for the theory developed in Ref. [7] is given as

$$S_{BI}(g, \Gamma, \Psi) = \frac{c^3}{8\pi G \kappa} \int d^4x \left[ \sqrt{-g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)} - \lambda \sqrt{-g} \right] + S_M(g, \Psi)$$

(1)

where $\Lambda = \frac{\lambda - 1}{\kappa}$. Variation with respect to $\Gamma$, done using the auxiliary metric $q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}(q)$, gives

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}(q)$$

(2)

Variation with respect to $g_{\mu\nu}$ gives

$$\sqrt{-q} q^{\mu\nu} = \lambda \sqrt{-g} g^{\mu\nu} - \frac{8\pi G}{c^4} \kappa \sqrt{-g} T^{\mu\nu}$$

(3)

where the $T^{\mu\nu}$ components are in the coordinate frame. In order to obtain solutions, we need to assume a $g_{\mu\nu}$ and a $q_{\mu\nu}$ with unknown functions, as well as a matter stress-energy ($T^{\mu\nu}$).

Thereafter, we write down the field equations and obtain solutions using some additional assumptions about the metric functions and the stress-energy.

Work on various fronts has been carried out on various aspects of this theory in the last few years. Astrophysical scenarios have been discussed in Refs. [12–20]. Spherically symmetric solutions were obtained in Refs. [7, 21–25]. A domain wall brane in a higher-dimensional generalization was analyzed in Ref. [26]. Generic features of the paradigm of matter-gravity couplings were analyzed in Ref. [27]. In Ref. [28], authors showed that EiBI theory admits a nongravitating matter distribution, which is not allowed in GR. Some interesting cosmological and circularly symmetric solutions in 2 + 1 dimensions were shown in Ref. [29]. Constraints on the EiBI parameter $\kappa$ have been obtained from studies on compact stars [12, 16], tests in the Solar System [30], astrophysical and cosmological observations [14], and nuclear physics studies [31]. In Ref. [32], a major problem related to surface singularities was noticed in the context of stellar physics. However, gravitational backreaction was suggested as a cure to this problem in Ref. [33]. In Ref. [34], the authors proposed a modification to EiBI theory by taking its functional extension in a way similar to $f(R)$ theory. Recently, the authors of Ref. [35] used a different way of matter coupling by taking the Kaluza ansatz for the five-dimensional EiBI action in a purely metric formulation, and then compactify it.
using Kaluza’s procedure to get a four-dimensional gravity coupled in a nonlinear way to electromagnetic theory.

Much work in EiBI gravity is devoted to cosmology. In Refs. [7, 9, 36], the authors showed the nonsingularity of the Universe filled by any ordinary matter. Linear perturbations have been studied in the background of the homogeneous and isotropic spacetimes in the Eddington regime [37, 38]. Bouncing cosmology in EiBI gravity was emphasized as an alternative to inflation in Ref. [39]. The authors of Ref. [40] studied a model described by a scalar field with a quadratic potential, which results in a nonsingular initial state of the Universe leading naturally to inflation. They also investigated the stability of the tensor perturbations in this inflationary model [41] while the scalar perturbation was studied in Ref. [42]. Other relevant work has been reported in Refs. [43–45]. Large-scale structure formation in the Universe and the integrated Sachs-Wolfe effect are discussed in Ref. [46]. Further efforts in this line were reported in Refs. [47–50].

In Ref. [25], we considered the Born-Infeld structure in both the gravity and matter sectors and obtained new spherically symmetric static spacetimes when EiBI gravity is coupled to Born-Infeld electrodynamics. In this article, we investigate a similar problem in EiBI cosmology where we consider a Born-Infeld scalar field in the matter part. In particular, we use the tachyon condensate scalar because of its Born-Infeld-like structure in the action. The tachyon condensate scalar field arises in the context of theories of unification such as superstring theory [51]. There have been several articles in the literature where cosmology with the tachyon scalar field was discussed [52–64]. The energy-momentum tensor of the BI scalar tachyon condensate can be split into two parts; one with zero pressure (dark matter) and another with $p = -\rho$ (dark energy) [65]. This facilitates the description of dark energy and dark matter using a single scalar field, a fact we shall use for our model in this article.

We organize our article as follows. In Sec. II we discuss the basic setup of the BI scalar field in the cosmological background. Next, in Sec. III we obtain a constant negative (effective) pressure solution as the special case of the general assumption on the form of the time derivative of the scalar field. Then, we fit the solutions with the supernova data and test its viability in Sec. IV. Finally, in Sec. V we summarize our results.
II. COSMOLOGY WITH A BORN-INFELD SCALAR

The action for the BI scalar (e.g. tachyon condensate) is

\[ S_M = -\frac{1}{c} \int \sqrt{-g} \alpha_T^2 \mathcal{V}(\phi) \sqrt{1 + \alpha_T^{-2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} \, d^4x \]  

(4)

where, \( \mathcal{V}(\phi) \) is the potential for the scalar field and \( \alpha_T \) is the constant parameter. The resulting stress-energy tensor components have the following general expression:

\[ T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} \]

\[ = \mathcal{V}(\phi) \left[ \frac{g^{\mu\alpha} g^{\nu\beta} \partial_\alpha \phi \partial_\beta \phi - g^{\mu\nu} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - g^{\mu\nu} \alpha_T^2}{\sqrt{1 + \alpha_T^{-2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi}} \right] \]

(5)

By varying the action with respect to (w.r.t.) \( \phi \), we get the equation of motion of the scalar field

\[ \partial_\nu \left[ \frac{\mathcal{V}(\phi) \sqrt{-g} g^{\mu\nu} \partial_\mu \phi}{\sqrt{1 + \alpha_T^{-2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi}} \right] = \alpha_T^2 \sqrt{-g} \mathcal{V}'(\phi) \sqrt{1 + \alpha_T^{-2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi} \]

(6)

where \( \mathcal{V}'(\phi) \) is derivative of the potential w.r.t. the scalar field. The equation (6) also ensures the conservation of the stress-energy tensor (\( \nabla_\mu T^{\mu\nu} = 0 \)). For a homogeneous and isotropic, spatially flat Universe, we assume the following ansatz for the physical spacetime metric:

\[ ds^2 = -U(t) c^2 dt^2 + a^2(t) \left[ dx^2 + dy^2 + dz^2 \right] \]

(7)

The equation (6) leads to the following equation of motion of the scalar field (\( \phi \)):

\[ \frac{\ddot{\phi}}{c^2 \alpha_T^2 U - \dot{\phi}^2} + \frac{3\dot{\phi}}{c^2 \alpha_T^2 U} \left( \frac{\dot{a}}{a} \right) + \frac{\mathcal{V}'(\phi)}{\mathcal{V}(\phi)} - \frac{\dot{\phi} \dot{U}}{2U(c^2 \alpha_T^2 U - \dot{\phi}^2)} = 0 \]

(8)

where dots denote the derivatives w.r.t. \( t \) and primes denote the derivatives w.r.t. \( \phi \). Also, the stress-energy tensor for the BI scalar field can be re-written as that of an analogous perfect fluid, i.e. \( T_{\mu\nu} = (p_\phi + \rho_\phi c^2) u_\mu u_\nu + p_\phi g_{\mu\nu} \), where \( p_\phi \) and \( \rho_\phi \) are the equivalent pressure and energy density, respectively, in the comoving frame. The \( \rho_\phi \) and \( p_\phi \) are expressed in terms of the scalar field as

\[ \rho_\phi = \frac{\alpha_T^2 \mathcal{V}(\phi)}{c^2 \sqrt{1 - \dot{\phi}^2 U^{-1} \alpha_T^{-2} c^{-2}}} \]

(9)

\[ p_\phi = -\alpha_T^2 \mathcal{V}(\phi) \sqrt{1 - \dot{\phi}^2 U^{-1} \alpha_T^{-2} c^{-2}} \]

(10)
We can re-express the scalar potential in terms of $\rho_\phi$ and $p_\phi$ in the following way,

$$V(\phi) = \sqrt{-p_\phi \rho_\phi c^2 / \alpha_T^2}$$  \hspace{1cm} (11)

Using Eqs. (9) and (10), Eq. (8) can be rewritten as

$$\frac{\dot{\rho}_\phi}{\rho_\phi} = -3 \left( \frac{\dot{a}}{a} \right) \left( \frac{\dot{\phi}^2}{U c^2 \alpha_T^2} \right)$$  \hspace{1cm} (12)

Generally, in a scalar field cosmology, we choose the form of the potential for the scalar field. This is because we do have an extra degree of freedom in the field equations. Here, we exploit this and choose the form of $\dot{\phi}$ instead of $V(\phi)$. We assume the form of $\dot{\phi}$ as

$$\dot{\phi}^2 = \frac{U c^2 \alpha_T^2}{1 + C_1 a^n}$$  \hspace{1cm} (13)

where $C_1 > 0$ and $n > 0$. Thus, using the Eq. (13) we solve Eq. (12) and obtain

$$\rho_\phi = C_2 (a^{-n} + C_1)^{3/n}$$  \hspace{1cm} (14)

$$p_\phi = -C_1 C_2 c^2 (a^{-n} + C_1)^{3/n-1}$$  \hspace{1cm} (15)

where $C_2$ is an integration constant. At this point, $\alpha_T$ is just a scaling parameter for the scalar field and the potential. But this is changed if we relate $C_1$ with $\alpha_T$. We choose $C_1^{3/n} = \alpha_T^2$. Then the potential becomes [from Eq. (11)]

$$V = C_2 c^2 \left( a^{-n} / \alpha_T^{2n/3} + 1 \right)^{3/n-1/2}$$  \hspace{1cm} (16)

where the potential is now expressed in terms of the scale factor in parametrized form. For this choice of $C_1$, $V \rightarrow C_2 c^2$ for $a \rightarrow \infty$. This becomes a universal character (i.e. for arbitrary $n > 0$). For $a \rightarrow \infty$, $\rho_\phi \rightarrow \alpha_T^2 C_2$ and $p_\phi \rightarrow -\alpha_T^2 C_2 c^2$. On the other hand, for the given $n$ and $C_2$, $\alpha_T$ becomes the control parameter for modifying the behaviour of the potential and all other quantities in the early Universe.

### III. A CONSTANT NEGATIVE PRESSURE SOLUTION

In GR, the cosmological solution with a constant negative pressure can be found. It may be a bouncing solution where the scale factor is given by

$$a(t) = a_0 \left( \cosh \frac{t}{t_0} \right)^{\frac{1}{n}}$$  \hspace{1cm} (17)
The corresponding energy density profile is
\[ \rho = \frac{1}{6\pi G t_0^2} \left( 1 - \frac{a_0^3}{a^3} \right) \] (18)

The other possibility is of a singular solution where the scale factor and the energy density are given by
\[ a(t) = a_0 \left( \frac{\sinh \frac{t}{t_0}}{1 + \frac{a_0^3}{a^3}} \right) \] (19)
\[ \rho = \frac{1}{6\pi G t_0^2} \left( 1 - \frac{a_0^3}{a^3} \right) \] (20)

In EiBI gravity coupled to a BI scalar, with the choice of \( n = 3 \) in Eq. (15), \( p_\phi = -\alpha^2_\Lambda C_2 c^2 \) (constant negative pressure). We would now like to construct the solution for the physical line element. Let us assume an ansatz for the auxiliary metric
\[ ds^2_q = -V(t)c^2 dt^2 + b^2(t) \left[ dx^2 + dy^2 + dz^2 \right] \] (21)

From the Eq. (3) we get two independent equations which we rewrite in the following way,
\[ a = \frac{b}{C_0} \sqrt{V/U}, \] (22)
\[ \rho_\phi = \frac{c^2}{8\pi G \kappa} \left( \frac{C_0^3 U^2}{V^2} - 1 \right) \] (23)

where \( C_0 = 1 + 8\pi G \kappa \alpha^2_\Lambda C_2 c^{-2} \) is a new constant parameter. Note that \( C_0 \) has both \( \kappa \) and \( \alpha^2_\Lambda \) in its definition. We will see that \( C_0 \) is one of the parameters which we will use while fitting our final solution with supernova observations. We also assume \( \lambda = 1 \) (\( \Lambda = 0 \)). From the \( \Gamma \) variation, we further have two independent equations [Eq. (2)]. After a simple algebraic manipulation these two equations become
\[ \frac{\dot{b}^2}{b^2} = \frac{c^2}{6\kappa} \left( 2V + U - \frac{3V^2}{C_0^2 U} \right) \] (24)
\[ \frac{d}{dt} \left( \frac{\dot{b}}{b} \right) - \frac{1}{2} \frac{\dot{b}V}{bV} = \frac{c^2}{2\kappa} \left( -U + \frac{V^2}{C_0^2 U} \right) \] (25)

To obtain the solution, we need to solve the four equations (22), (23), (24), and (25). However, we have five unknowns: \( a, b, U, V, \) and \( \rho_\phi \). Therefore, we have the freedom to choose the functional form of any one of the unknowns. Since we are free to choose the auxiliary metric functions we assume \( b(t) = b_0 \exp(\bar{H}_b t) \), where \( b_0 \) and \( \bar{H}_b \) are two arbitrary nonzero constants. Thus, from Eq. (24), we arrive at
\[ U^2 + 2 \left( V - \frac{3\kappa \bar{H}_b^2}{c^2} \right) U - \frac{3V^2}{C_0^2} = 0 \] (26)
The above quadratic equation (26) has two roots. We choose the one for which \( U > 0 \). Therefore, we have

\[
U = \sqrt{ \left( V - \frac{3\kappa \bar{H}_b^2}{c^2} \right)^2 + \frac{3V^2}{C_0^2} - \left( V - \frac{3\kappa \bar{H}_b^2}{c^2} \right) } \tag{27}
\]

Further, from Eq. (25), we have

\[
\frac{\dot{V}}{V} = \frac{c^2}{H_b \kappa} \left( U - \frac{V^2}{C_0^2 U} \right) \tag{28}
\]

At this juncture, we introduce a new variable \( X = C_0 \left( 1 - 3\kappa \bar{H}_b^2 e^{-2V-1} \right) \) and rewrite the Eq. (28) as

\[
\frac{\dot{X}}{2X - \sqrt{X^2 + 3}} = -2\bar{H}_b \tag{29}
\]

Integrating the Eq. (29), we get

\[
\frac{(2X - \sqrt{X^2 + 3})^2}{\sqrt{X^2 + 3} - X} = \frac{C_3}{b^5} \tag{30}
\]

where \( C_3 \) is the constant of integration. Using Eqs. (22), (27), and (30), we compare the two expressions of \( \rho_\phi \) - one from field equations (i.e. Eq. 23) and the other from the conservation equation (i.e. Eq. 14). Since the field equations in EiBI theory satisfy the conservation equation, the constant \( C_3 \) is fixed and has the expression: \( C_3 = 144\pi^2 G^2 \kappa^2 C^2_2 C_0 e^{-4} \). The new variable \( X \) is related to \( V \). Since \( V \) appears in the auxiliary metric as a coefficient of \( dt^2 \), \( X \) is, in a sense related to a redefinition of the time variable. In what follows, we will see this connection more explicitly.

A. \( \kappa > 0 \)

Using Eqs. (22), (27), and (30) we rewrite the expression of \( a \) as a function of \( X \):

\[
a^3 = \frac{4\pi G \kappa C_2}{C_0 c^2} \left[ \frac{\sqrt{X^2 + 3} + X}{\sqrt{X^2 + 3} - 2X} \right] \tag{31}
\]

From Eq. (14), we note that \( \rho_\phi = \alpha_n^2 C_2(a^{-3}a_T^{-2} + 1) \) for \( n = 3 \) and \( C_1 = \alpha_T^2 \). So, \( \rho_\phi > 0 \) necessitates \( a^3 > 0 \). Then, for \( \kappa > 0 \), \( X \leq 1 \) and \( a \in (0, \infty) \) maps onto \( X \in (-\infty, 1) \). Similarly, we express the other metric function \( U \) and the coordinate \( t \) as functions of \( X \)

\[
U = \frac{3\kappa \bar{H}_b^2}{c^2} \left[ \frac{\sqrt{X^2 + 3} - X}{C_0 - X} \right] \tag{32}
\]

\[
t = t_0 + \frac{1}{6\bar{H}_b} ln \left[ \frac{C_3 (\sqrt{X^2 + 3} - X)}{(2X - \sqrt{X^2 + 3})^2} \right] \tag{33}
\]
where \( t_0 = -\ln(b_0)/\dot{H}_0 \) is an arbitrary constant. The relation between \( t \) and \( X \) in Eq. (33) is invertible and we may write \( X \) as a function of \( t \) though the expression takes a complicated form. Therefore, it is better to express the metric functions \( a(t) \) and \( U(t) \) of the physical metric [Eq. (7)] in parametric form [Eqs. (31), (32), and (33)], where \( X \) is the parameter.

We define the cosmological time \( \tau \)

\[
\tau = \int \sqrt{U} dt
\]

\[
= \frac{\sqrt{3\kappa}}{2c} \int \left[ \frac{\sqrt{X^2 + 3} - X}{C_0 - X} \right]^{1/2} \frac{dX}{(\sqrt{X^2 + 3} - 2X)} + \text{const.}
\]

In order to understand the evolution of \( a \), we compute the deceleration parameter \( (q) \)

\[
q = -a \frac{d^2 a}{d\tau^2} \left/ \left( \frac{da}{d\tau} \right)^2 \right.
\]

\[
= -1 + \left( \frac{\sqrt{X^2 + 3} - 2X}{\sqrt{X^2 + 3} - X} \right) \left[ 1 + \frac{X}{\sqrt{X^2 + 3}} + \frac{\sqrt{X^2 + 3} + X - C_0}{2(C_0 - X)} \right]
\]

Interestingly, we note that, for \( X = 1 \) (i.e. \( a \rightarrow \infty \)), \( q = -1 \). Also, for \( X \rightarrow -\infty \) (i.e. \( a \rightarrow 0 \)), \( q \rightarrow -1 \). Therefore, both at early and late times, the Universe undergoes a de Sitter phase in our model. Also, for some values of \( C_0 \), the Universe may undergo a decelerated expansion state \( (q > 0) \) [see Fig. 1].

Using Eqs. (31) and (34), we plot the scale factor \( a \) as a function of the cosmological time \( (\tau) \) [Fig. 2(a)]. In the plot, we choose a value of \( C_0 \) such that we are able to note an initial loitering phase (with an acceleration) followed by a decelerated expansion phase and then an accelerated expansion at late times. From Fig. 2(d) we note that the effective energy density \( (\rho_\phi) \) decreases as \( a \) increases and approaches a nonzero minimum value at large \( a \). Though \( \rho_\phi \) diverges at \( a \rightarrow 0 \), it takes an infinite time \( (\tau \rightarrow -\infty) \) to reach that point. Thus, the Universe is nonsingular. Such a nonsingular de Sitter inflationary phase is a common feature of the Universe with constant pressure, in 3 + 1 EiBI cosmology. The accelerated expansion of the Universe at late times is due to the fact that at late times the effective pressure \( (p_\phi) \) is related to the energy density \( (\rho_\phi) \) as \( p_\phi \approx -\rho_\phi c^2 \). The decelerated expansion in between the two accelerated phases is due to the fact that the repulsive nature of gravitating matter is less dominant than its attractive character, during this phase.

In Fig. 3 we show the variation of the scalar field as a function of \( \tau \), which results in such a cosmological solution. We also plot the associated potential \( V(\phi) \) as a function of \( \phi \).
FIG. 1. The deceleration parameter \((q)\) is plotted as function of \(X\). In the plot, \(X\) ranges from \(-\infty\) to 1 for \(a \in (0, \infty)\). \(C_0\) [see Eq. (35)] takes different values for different curves in the plot. For \(C_0 = 1.01, 1.2, 1.5\), \(q\) has a transition from negative to positive values and then again to a negative value.

An analytical expression for \(V\) as a function of \(\phi\) is, unfortunately, not available due to the non-invertible nature of \(\phi(a)\). We use Eqs. (13), (16), and (34) for the plotting. Here, the scalar field could be either decaying or growing in time for the same solution [see Eq. (13)]. These features are shown in the top and bottom panels respectively, with the associated potentials. However, in both situations, the potentials approach a nonzero minimum value and that occurs at late times of the Universe.

During the loitering phase, the scale factor grows exponentially \((a \sim a_0 \exp(2\sqrt{2c\tau}/\sqrt{3\kappa}))\). On the other hand, it can be shown that during the period of inflation, the Universe expanded by a factor of \(e^{60}\) in \(10^{-32}\) seconds. Using this as input, the bound on \(\kappa\) becomes \(\kappa \lesssim 0.67 \times 10^{-50} \text{ m}^2\) which is greater than the Planck length square \((l_p = 1.6 \times 10^{-35} \text{ m})\). However, the de Sitter expansion of the Universe at very late times is different from it and depends on the other BI parameter \(\alpha_T^2\) \((a \propto \exp(\sqrt{8\pi GC_2\alpha_T^2}/3\tau))\). Thus, in two different extreme regimes, the evolution of the scale factor depends on the two BI parameters independently. However, the intermediate phase depends on the product of these two parameters \((\kappa\alpha_T^2)\).
FIG. 2. The scale factor $a(\tau)$ is plotted as a function of cosmological time. For all the plots, we set $8\pi G = 1$, $c = 1$ and choose $\kappa = 0.5$, $\alpha_T^2 = 5.0$, $C_2 = 0.001$, and $C_0 = 1.0025$. We choose an initial value $a = 0.06$ at $\tau = 0.06$. (a) The initial loitering state followed by a deceleration and late time acceleration are highlighted. (b) A zoomed version of (a) is shown to illustrate the loitering phase and the following transition to the deceleration phase. Note that the loitering phase also includes an acceleration, where the scale factor has an exponential growth ($a \sim a_0 \exp(2\sqrt{2c\tau}/\sqrt{3}\kappa)$). (c) The loitering phase is shown again by changing the linear scaling to a logarithmic scaling in the vertical axis. (d) The energy density ($\rho_\phi$) for the scalar field is plotted as a function of $\tau$.

B. $\kappa < 0$

For $\kappa < 0$, $a^3 > 0$ implies that $X \geq 1$ [see Eq. (31)]. For $X = 1$, $a \to \infty$, but for $X \to \infty$, $a^3 \to 8\pi G|\kappa|C_2/C_0c^2$. Also, we note that $da/d\tau = 0$ at $X \to \infty$. So, for $\kappa < 0$, there is a nonzero minimum scale factor $a_B = (8\pi G|\kappa|C_2/C_0c^2)^{1/3}$ at which the Universe undergoes a bounce. In Fig. 4, the deceleration parameter ($q$) is plotted as a function $X$ for different
FIG. 3. (a) The scalar field ($\phi$) is plotted as a function of $\tau$. We assume $\dot{\phi} < 0$ (decaying). (b) $V(\phi)$ is plotted for the case $\dot{\phi} < 0$. (c) $\phi(\tau)$ assuming $\dot{\phi} > 0$ (growing). (d) $V(\phi)$ for the $\dot{\phi} > 0$ case.

In all plots here, we choose an initial condition $\phi = 1.0$ at $\tau = 0.06$.

values of $C_0$. Since $\kappa < 0$, so $C_0 < 1$; we also assume $C_0 > 0$. In Fig. 4, we note that $q \to -\infty$ as $a \to a_B$ (near the bounce). This is because at $a_B$, $da/d\tau = 0$, but $d^2a/d\tau^2$ has a nonzero finite value. However, in this case too, the Universe undergoes a de Sitter stage at late times ($q \to -1$ for $X \to 1$).

In Fig. 5(a), we show the variation of the scale factor ($a$) as a function of the cosmological time ($\tau$). We choose the parameter values in such a way that we see the bounce followed by decelerated expansion and finally the de Sitter stage at late times. The plot of the effective energy density ($\rho_\phi$) in the Fig. 5(b) shows a regular profile throughout. Here we mention that the nonsingular bouncing early Universe is a common feature of EiBI cosmology. The BI scalar field provides the additional late time acceleration.
The deceleration parameter \( q \) is plotted as a function of \( X \) for \( \kappa < 0 \). In the plot, \( X \) ranges from 1 to \( \infty \) for \( a \in (a_B, \infty) \). \( C_0 \) [see Eq. (35)] takes different values for different curves in the plot. For \( C_0 = 0.9999, 0.99, 0.97, q \) has a transition from negative to positive to negative values. For all values of \( C_0 \), \( q \to -\infty \) for \( X \to \infty \) or, \( a \to a_B \).

In Fig. 6 we plot the scalar field \( \phi(\tau) \) and the associated potential \( V(\phi) \). We note that, for both decaying and growing nature of the scalar field, the potential remains invariant.

The scale factor \( a(\tau) \) is plotted as the function of the cosmological time \( \tau \) for \( \kappa < 0 \). We set \( 8\pi G = 1, c = 1 \) and choose \( \kappa = -0.1, \alpha_T^2 = 5.0, C_2 = 0.001 \), and \( C_0 = 0.9995 \). We choose an initial value \( a = 0.06 \) at \( \tau = 0.06 \). (b) The corresponding energy density \( \rho_\phi \) for the scalar field is also plotted as a function of \( \tau \).
FIG. 6. (a) The scalar field \( \phi \) is plotted as a function of \( \tau \). We assume \( \dot{\phi} < 0 \) (decaying). (b) \( V(\phi) \) is plotted for the case \( \dot{\phi} < 0 \). (c) \( \phi(\tau) \) assuming \( \dot{\phi} > 0 \). (d) \( V(\phi) \) for the case \( \dot{\phi} > 0 \). In all plots here, we choose an initial condition \( \phi = 0.0 \) at \( \tau = 0.06 \).

IV. OBSERVATIONAL TEST OF THE SOLUTIONS FROM FITTING THE SUPERNova DATA

So far we have found that the resulting cosmological solutions exhibit an early-time accelerated expansion as well as a late-time acceleration. There can also exist a phase of decelerated expansion lying in between the two accelerated phases. For \( \kappa > 0 \), the early Universe grows exponentially (similar to an inflationary scenario) and for \( \kappa < 0 \), the early Universe undergoes a bounce. In this section, we fit the aforementioned solutions with the supernova data and test their viability in describing the evolution of the Universe at late times. However, we do not consider any other field as a part of the matter sector apart from
the BI scalar field. Here, we follow the approach adopted in [65], where the BI scalar field is treated as a single candidate having the capacity of exhibiting different equations of state at different scales and making a transition from \( p = 0 \) (cold dark matter) at small scales to \( p = -\rho c^2 \) (dark energy) at large scales. The total energy density is, likewise, split into two parts: (a) pressureless dust and (b) dark energy, \( \rho = \rho_{DM} + \rho_{DE} \)

\[
\rho_{DE} = -\rho_{DE} c^2
\]

Thus, from Eqs. (9) and (10), we get

\[
\rho_{DM} = \frac{\mathcal{V}(\phi) \dot{\phi}^2}{c^4 U \sqrt{1 - \frac{\dot{\phi}^2}{c^2 U \alpha^2}}}; \quad \rho_{DE} = \frac{\alpha^2 \mathcal{V}(\phi)}{c^2 U \sqrt{1 - \frac{\dot{\phi}^2}{c^2 U \alpha^2}}}
\]

In our case, we get \( \rho_{DM} = C_2/a^3 \) and \( \rho_{DE} = -p_{DE}/c^2 = \alpha^2_7 C_2 \) (a constant) [see Eqs. (14) and (15) with \( n = 3 \) and \( C_1 = \alpha^2_7 \)]. This is similar to ΛCDM cosmology. However, the cosmological constant \( \Lambda = \alpha^2_7 C_2 \) is not an ad-hoc quantity but generated from the BI scalar field with a specific choice of \( \dot{\phi}^2 \) (or, alternatively with an equivalent choice of the potential function).

**A. \( \kappa > 0 \)**

Using Eqs. (31) and (34), we rewrite the Hubble function as

\[
H(z) = H_0 \left[ \frac{(\sqrt{X^2(z) + 3} - X(z))(C_0 - X(z))(X_0^2 + 3)}{(\sqrt{X_0^2 + 3} - X_0)(C_0 - X_0)(X^2(z) + 3)} \right]^{1/2}
\]

where \( z \) is the redshift defined as \( a = 1/(1 + z) \). \( H_0 \) and \( X_0 \) are the present day values of the Hubble function and \( X(z) \). The expressions for \( X(z) \) and \( H_0 \) are given as

\[
X(z) = \frac{1 - \tilde{a}_0^3(1 + z)^3}{\sqrt{1 + 2\tilde{a}_0^3(1 + z)^3}}
\]

\[
H_0 = \frac{2c}{\sqrt{3\kappa}} \left[ \frac{(C_0 - X_0)(\sqrt{X_0^2 + 3} - X_0)}{X_0^2 + 3} \right]^{1/2}
\]

where, \( \tilde{a}_0^3 = 4\pi G\kappa C_2/C_0 c^2 = (\sqrt{X_0^2 + 3} - 2X_0)/(\sqrt{X_0^2 + 3} + X_0) \). Using the Eqs. (38) and (39), we can define the luminosity distance for the observed supernova at the redshift \( z \) as
\[ D_L(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')} = cd_L(z)/H_0, \]

where \( d_L(z) \) is the Hubble free luminosity distance. Therefore, the Hubble free luminosity distance becomes

\[
d_L(z; X_0, C_0) = (1 + z) \int_0^z H_0 \frac{dz'}{H(z')} = (1 + z) \int_0^z \left[ \frac{(\sqrt{X_0^2 + 3} - X_0)(X_0^2(z') + 3)}{(\sqrt{X_0^2(z') + 3} - X(z'))(C_0 - X(z'))(X_0^2 + 3)} \right]^{1/2} dz'
\]

Using Eq. (41) as the model, we fit the supernova data. There are two parameters \( C_0 \) and \( X_0 \), which are both dimensionless. From the best fit parameter values (of \( C_0 \) and \( X_0 \)), we can estimate the best fit values of \( q_0 \) (present day value of the deceleration parameter), \( \Omega_{DM0} \) (ratio of present day matter density to total energy density of the Universe), \( \Omega_{DE0} \) (for dark energy), \( \kappa \), and \( \alpha_T^2 \). To carry out all of this, we write down the useful relations next. We use Eq. (35) to evaluate \( q_0 \). We get the expression of \( \Omega_{DM}(z) \) in terms of \( X_0 \) and \( C_0 \), [using Eqs. (23), (31), and (40)]

\[
\Omega_{DM}(z) = \frac{\rho_{DM}(z)}{\rho_{\phi}(z)} = \frac{2C_0(\sqrt{X_0^2 + 3} - X_0)(\sqrt{X_0^2 + 3} - 2X_0)(1 + z)^3}{3 \left[ C_0 (\sqrt{X_0^2(z) + 3} - X(z))^2 - 1 \right]} \]

Further, \( \Omega_{DE}(z) \) can also be expressed as

\[
\Omega_{DE}(z) = \frac{\rho_{DE}}{\rho_{\phi}(z)} = 1 - \Omega_{DM}(z)
\]

where \( \Omega_{DM0} = \Omega_{DM}(0) \) and \( \Omega_{DE0} = \Omega_{DE}(0) \). We can evaluate \( \alpha_T^2 \) using

\[
\alpha_T^2 = \frac{\Omega_{DE0}}{\Omega_{DM0}}
\]

Finally, the expression of \( \kappa \) is

\[
\kappa = \frac{4c^2}{3H_0^2} \frac{(C_0 - X_0)(\sqrt{X_0^2 + 3} - X_0)}{(X_0^2 + 3)}
\]

where, we use a prior value of \( H_0 = 70 \text{ km/sec/Mpc} \).

To fit the Supernova data with our model, we follow the method used in [66], wherein the authors have studied the expansion history of the universe up to a redshift \( z = 1.75 \) using the 194 Type Ia supernovae (SNe Ia) data. However, in our case we use the more recent Union2.1 Compilation data [67]. The observational dataset consists of the values of the distance modulus \( (m_i(z_i) - M) \) and redshifts \( z_i \) with their corresponding errors. Each distance modulus is related to the corresponding luminosity distance \( D_L \) of the SNe Ia by

\[
m(z) = M + 5 \log_{10} \left[ \frac{D_L(z)}{Mpc} \right] + 25
\]
The observed distance modulus can be translated to $d_L^{obs}(z_i)$. For a given model $H(z; a_1, a_2, ..., a_n)$, one can also theoretically predict the $d_L^{th}(z)$ using the Eq. (41). The best fit values of the model parameters $(a_1, a_2, ..., a_n)$ are estimated by minimizing the $\chi^2(a_1, a_2, ..., a_n)$ which, in this case, is given by

$$\chi^2(a_1, a_2, ..., a_n) = \sum_{i=1}^{580} \left( \log_{10} d_L^{obs}(z_i) - \log_{10} d_L^{th}(z_i) \right)^2 + \left( \frac{\partial \log_{10} d_L(z_i)}{\partial z_i} \sigma_z \right)^2$$

(47)

where $\sigma_z$ is the 1σ redshift uncertainty of the data and $\sigma_{log_{10} d_L(z_i)}$ is the 1σ error of $log_{10} d_L^{obs}(z_i)$. The error in redshift $\sigma_z$ is estimated from the uncertainty due to peculiar velocities, $\Delta v = \Delta (cz) = 500 \text{ km/s}$, i.e. $\sigma_z = \Delta z = (500 \text{ km/s})/c$.

The resulting best fit parameter values are

$$X_0 = 0.912 \quad ; \quad C_0 = 1.372 \quad ; \quad \chi^2_{min} = 497.926.$$  

(48)

If $\chi^2_{min}/d.o.f = \chi^2_{min}/(N - n) \lesssim 1$ ($N$: number of data points, $n$: number of parameters), the fit is good and the data are consistent with the considered model $H(z; a_1, ..., a_n)$. Here, $\chi^2_{min}/d.o.f = 0.861$. In Fig. 7(a) the variation of the luminosity distance with respect to the redshift $z$ is shown for the best fit parameter values, along with the observed data points.

In Fig. 7(b) we show the 1σ and 2σ confidence levels in the parameter space $(X_0 - C_0)$. Using the best fit parameter values, the estimated values of $q_0$, $\Omega_{DM0}$, and $\Omega_{DE0}$ are

$$q_0 = -0.605^{+0.026}_{-0.054} \quad ; \quad \Omega_{DM0} = 0.255^{+0.051}_{-0.021} \quad ; \quad \Omega_{DE0} = 0.745^{+0.016}_{-0.051}.$$  

(49)

These are in reasonably good agreement with ΛCDM cosmology [67]. Further, we plot $\Omega_{DM}(z)$ and $\Omega_{DE}(z)$ in Fig. 8. This figure once again demonstrates the viability of our model with observations, at least at late times.

We mention an important point here. Using the critical energy density of the Universe as obtained in the framework of GR (i.e. $\rho_c = 3H_0^2/8\pi G$), one can write down the present-day values of the density parameters corresponding to dark matter and dark energy as $\Omega'_{DM0} = 8\pi GC_2/3H_0^2$ and $\Omega'_{DE0} = 8\pi GC_2\alpha_T^2/3H_0^2$. Here, $\Omega'_{DM0} + \Omega'_{DE0} \neq 1$, as it should be in a modified theory of gravity. In our case, with the best fit values of the parameters $C_0$ and $X_0$, we find that $\Omega'_{DM0} + \Omega'_{DE0}$ is very close (but not equal) to one ($\Omega'_{DM0} = 0.254$ and $\Omega'_{DE0} = 0.741$). Therefore, irrespective of our definition of the density parameters, our model for late-time evolution seems to work well.
FIG. 7. (a) Observed SNe Ia (Union2.1 Compilation data) Hubble free luminosity distance \((d_L)\) with the fitted curve (thick black solid line) is shown. In the plot we use \(c = 3 \times 10^5 \text{km/s}\). The best fit parameter values are \(X_0 = 0.912\) and \(C_0 = 1.372\); \(\chi^2_{\text{min}}/d.o.f = 0.861\). (b) 1σ and 2σ error plots are shown in two-dimensional parameter space \((X_0, C_0)\).

The estimated value of the theory parameter for the EiBI gravity and the value of the
FIG. 8. The plot shows the evolution of $\Omega_{DM}$ (solid curve) and $\Omega_{DE}$ (dashed curve) with redshift ($z$) in our best-fit model for $\kappa > 0$. The universe is completely dominated by matter beyond $z \sim 10$.

parameter for the scalar field (matter) sector are

$$\kappa = 3.1 \ (Gpc)^2 \quad ; \quad \alpha^2_T = 2.9. \quad (50)$$

Using $\kappa$, $G$, and $c$, we can define mass ($[M]_{BI}$), time ($[T]_{BI}$), and length ($[L]_{BI}$). $[M]_{BI} = \sqrt{\kappa} G^{-1} c^2 = 7.13 \times 10^{52} \text{ kg}$, $[T]_{BI} = \sqrt{\kappa}/c = 1.76 \times 10^{17} \text{ s}$, and $[L]_{BI} = \sqrt{\kappa} = 1.76 \text{ Gpc}$.

B. $\kappa < 0$

We also fit the bouncing (i.e. $\kappa < 0$) solution with the same supernovae data. In this case, the expression for the luminosity distance function becomes

$$d_L(z; X_0, C_0) = (1 + z) \int_0^z \left[ \frac{(\sqrt{X_0^2 + 3} - X_0)(X_0 - C_0)(X^2(z') + 3)}{(\sqrt{X^2(z') + 3} - X(z'))(X(z') - C_0)(X_0^2 + 3)} \right]^{1/2} dz' \quad (51)$$

where, $X(z) = \frac{2 + a_B^3(1+z)^3}{2\sqrt{1 - a_B^3(1+z)^3}} \quad (52)$

$$a_B^3 = \frac{4X_0 - 2\sqrt{X_0^2 + 3}}{\sqrt{X_0^2 + 3} + X_0} \quad (53)$$

and, $H_0 = \frac{2c}{\sqrt{3|\kappa|}} \left[ \frac{(X_0 - C_0)(\sqrt{X_0^2 + 3} - X_0)}{X_0^2 + 3} \right]^{1/2} \quad (54)$
Here, $X_0 \geq 1$ and $0 < C_0 < 1$. The minimum scale factor ($a_B$) corresponds to the maximum redshift $z_{\text{max}} = 1/a_B - 1$. The best fit parameter values are $X_0 = 1.011$, $C_0 = 0.973$ with $\chi^2_{\text{min}}/\text{d.o.f.} = 0.862$. Using the best fit value of $X_0$ in Eq. (33), the maximum redshift becomes $z_{\text{max}} = 3.51$. This is absurd as $z_{\text{max}}$ should be greater than the redshift corresponding to CMB radiation ($z \approx 1000$). Thus the bouncing Universe model is ruled out though it fits well with the data.

V. CONCLUSIONS

Let us now briefly summarize our findings.

We have looked at cosmologies in EiBI gravity with a Born-Infeld scalar (tachyon condensate) in the matter part of the total action. Thus, we have incorporated a Born-Infeld structure in both the gravity and the matter sectors of the theory. We have two control parameters; $\kappa$ of dimension $L^2$ for the gravity sector and the dimensionless $\alpha_2^2 T$ for the matter sector.

In our approach here, we have assumed a form of $\dot{\phi}^2$ instead of assuming a scalar potential. For a particular choice of a parameter $n$ (i.e. $n = 3$), the problem reduces to a situation where the Universe is driven by a perfect fluid of constant negative pressure. We obtain the analytical solution for such a set up. For $\kappa > 0$, the Universe undergoes a de Sitter expansion stage both at early and late times. In between, there could be a decelerated expansion depending on the value of $\kappa \alpha_2^2 T$. For $\kappa < 0$, there is a difference in the picture through the occurrence of a bounce instead of the de Sitter expansion at early times. However, at late times, the Universe still undergoes the de Sitter expansion even for $\kappa < 0$.

Qualitatively, similar behaviour can also be achieved for $n > 3$ in the form of $\dot{\phi}^2$ (Eq. 13). For such a different choice, the effective pressure $p_\phi$ becomes bounded. This is named the Maximal Pressure State (MPS) in [40], which provides a de Sitter stage in the expansion history of the Universe at the early times for $\kappa > 0$, in EiBI gravity. But for $\kappa < 0$ and $n > 3$, a similar bounce does occur supporting the generic bounce character in EiBI cosmology. We also note that, in our case, the equation of a state for the BI scalar (tachyon condensate) becomes $p_\phi \approx -\rho_\phi c^2$ at late times. Thus, at late times, the Universe undergoes a de Sitter expansion phase. In fact, it seems that we do not need any exotic matter source for producing late time acceleration in EiBI gravity, due to the above-mentioned property.

We have split the total energy-momentum tensor of the BI scalar (tachyon condensate) into
two parts: one for dark matter \((p = 0)\) and the other as dark energy \((p = -\rho c^2)\). In our special case \((i.e. n = 3 \text{ in the equation of } \dot{\phi}^2)\), the dark energy has constant negative pressure. Therefore, this is equivalent to \(\Lambda\)CDM cosmology though the effective cosmological constant is generated from the BI scalar. However, for \(n > 3\), we would have an evolving dark energy. It may be noted that though we have a viable background cosmological model, issues such as inflation and reheating will have to be addressed in greater detail. We hope future investigations will throw light on these topics.

We have shown that the supernova data fit well with both the late time solutions for \(\kappa > 0\) and \(\kappa < 0\). However, we discard the \(\kappa < 0\) solution because our fit predicts an unacceptable value of the redshift where the Universe may undergo a bounce. With a different choice of \(\dot{\phi}\) and additional matter, it may be possible to introduce new parameters and obtain a viable \(\kappa < 0\) solution. Remarkably, the cosmological properties estimated from the supernova data fit of the \(\kappa > 0\) solution is as good as in \(\Lambda\)CDM cosmology. It is possible that instead of using the special case \(n = 3\), one may use the general form of \(\dot{\phi}^2\) and keep \(n\) as an additional fitting parameter, in order to figure out how dark energy evolves.

Since we have analytical solutions which are not too complicated, it may be worthwhile to attempt a study of cosmological perturbations using this model as a background cosmology. Such a study with adequate observational tests and checks will surely help in establishing EiBI cosmology on a firmer footing, in future.

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