Effects of linear modification on the performance of finite length journal bearings

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Abstract Journal bearings are widely used in industry due to their favourable characteristics such as relatively low cost of manufacturing and maintenance, high load carrying capacity, and useful dynamic characteristics for rotor systems. Journal bearing misalignment, however, may have negative consequences for the general performance of this type of bearing. Misalignment commonly occurs due to errors in manufacturing tolerances, installation errors, wear, shaft deflection, and asymmetrical loading of systems. This paper presents a solution for hydrodynamic problems in journal bearings of finite length based on a 3D misalignment model and linear modification of the bearing profile to reduce the effect of misalignment on the characteristics of the journal bearing. The results showed that linear modification of misaligned journal bearings reduces the maximum pressure and improves the level of film thickness; additionally, the dynamic characteristics of the journal bearing are also improved by such linear profile modification.

Keywords: Journal bearing, Linear modification, 3D Misalignment, Finite length

1. Introduction

Journal bearings are the most common type of bearings. These consist of two main parts, the outer cylinder (bearing) and the shaft (journal), which rotates at an angular speed. A small clearance separates the two parts; this is filled with lubricant to reduce friction, wear, and dynamic imbalance in the system. Journal bearings are used in most high-speed rotating machines, including gas turbines, steam turbines, hydro-electric turbines, electric generators, and compressors. A fully aligned case rarely exists in the practical use of this type of bearing, as misalignment occurs due to many reasons, including inaccurate manufacturing tolerances, shaft deformation, and asymmetric bearing loads. The general performance of any journal bearing is significantly affected by misalignment, and in extreme cases of misalignment, the pressure increases while the film thickness drops, which may cause contact between the surfaces of the bearing and the journal. Axial profile modification (linear chamfer) has thus been proposed as a solution, to improve the level of film thickness and reduce maximum pressure.

The problem aligning of journal bearings has drawn the attention of several researchers due to the wide use of these types of bearing. A numerical method was presented in [1] for calculating the dynamic and static characteristics of journal bearings in which the Reynolds equation was solved using finite difference method. The results were thus based on the Reynolds boundary condition method for L/D = 0.5 and 1. The performance of journal bearings was examined in [2], which solved the problem of
misalignment for different values of bearing parameters for L/D=1. These results showed that the thermal effects in misaligned case were more significant than in aligned cases. The study in [3] presented a method for reducing side flow leakage in the case of lightly loaded bearings by chamfering the edges of the inner surface of the bearing, with results that showed that when chamfer length increased, the load carrying capacity and the temperature of the bearing surface decreased.

A numerical study of a thermo-hydrodynamic journal bearing was done in [4] to consider misalignment effects. That study examined the effects of local or a global defects on bearing geometry to determine ways to improve the performance of misaligned journal bearing, while [5] provided a detailed analysis of journal bearings to assess their dynamic characteristics with regard to the deformation of the bearing structure and journal misalignment. An analytical study was presented in [6] of hydrodynamic journal bearing characteristics with reference to a misalignment model caused by shaft deformation. The results showed that the pressure distribution and film thickness changed when misalignment was taken into consideration. The researchers in [7] analysed the problem of journal bearing misalignment caused by shaft deformation, identifying that such misalignment reduced the minimum film thickness and increased the maximum pressure, while [8] presented a numerical solution for calculating the dynamic and static characteristics of journal bearings based on solving the Reynolds equation using a finite element method, with various values of length to diameter ratio used in that work.

An approach to diagnosing bearing defects (wear and misalignment) using an artificial neural network was developed in [9], while [10] proposed a new analytical method to solve the Reynolds’ equation in order to determine the static and dynamic characteristics of finite length journal bearings. Researchers in [11] then used journal bearing with variable geometry to reduce the vibration amplitude, with results that showed the amplitude of vibration decreased by 70% in comparison with that of a classical journal bearing. Journal bearings with two partial arcs, variable geometry, and adjustable dynamic coefficients were then used in [12], which showed that the rotor vibrations were decreased when this type of arc journal bearing was used. An analytical solution to the dynamic coefficients of finite length journal bearings was given by [13], which produced a numerical solution under the assumption of π-film boundary conditions. A more general means of considering the effects of misalignment on journal bearings in numerical terms was offered in [14], which outlined a procedure for applying boundary conditions to the numerical solution to calculate the pressure distribution. Researchers in [15] investigated the effect of bushing profiles on the performance of elasto-hydrodynamic journal bearing, with the effects of misalignment and asperity contact between the surfaces studied using numerical solutions (FDM in conjunction with the over relaxation method, while [16] used a relatively simple experimental method to obtain the dynamic coefficients and minimum film thickness of journal bearings depending on the journal orbit.

This paper presents a solution for misalignment issues in journal bearings, a common problem in industry. Axial bearing profile modification (linear chamfer) was thus used to investigate the effects of changing the bearing geometry on film thickness, pressure distribution, and the dynamic coefficients of journal bearings.

2. Governing equations

The Reynolds equation is the basic equation for hydrodynamic pressure distribution, while the film thickness equation is derived from journal bearing geometry. The Reynolds equation is [10]

$$\frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial p}{\partial z} \right) = 6\eta \rho \omega \frac{\partial h}{\partial x}$$

(1)

while the equation for film thickness is [14]:

$$h = c(1 + \epsilon_r \cos(\theta))$$

(2)

These equations can be written in non-dimensional form as
\[
\frac{\partial}{\partial \theta} \left( H^3 \frac{\partial P}{\partial \theta} \right) + \frac{R_L^2}{L} \frac{\partial}{\partial Z} \left( H^3 \frac{\partial P}{\partial Z} \right) = \frac{\partial H}{\partial \theta} \tag{3}
\]

and

\[ H = 1 + \varepsilon_r \cos(\theta) \tag{4} \]

where

\[ x = R\theta \ ; \ H = h/c \ ; \ Z = z/L \ ; \ \text{and} \ \ P = \frac{P - P_o}{\eta \omega} \left( \frac{c^2}{R^2} \right) \]

\(P\): Pressure (dimensionless);
\(P_o\): Atmospheric pressure
\(\varepsilon_r\): Eccentricity ratio

\(H\): Film thickness in a dimensionless form.

3. The misalignment effect

Misalignment decreases \(H_{m_{\text{min}}}\) and increases \(P_{m_{\text{ax}}}\). Deviation between the bearing axis and journal axis causes misalignment, and shifts may occur in the vertical plane, horizontal plane, or both planes. Figure 1 shows an example of deviation due to the misalignment (3D), where \(\delta h\) represents the horizontal misalignment and \(\delta v\) is the vertical misalignment; these are functions of the \(z\) position. \(\delta v_{m_{\text{ax}}}\) and \(\delta h_{m_{\text{ax}}}\) represent the deviations at the bearing edges, which are used as input parameters to study the effect of misalignment.

\[ \begin{align*}
\delta v(z) &= \delta v_{m_{\text{ax}}} (1 - 2z/L) \ \text{for} \ z \leq L/2 \\
\delta v(z) &= \delta v_{m_{\text{ax}}} (2z/L - 1) \ \text{for} \ z > L/2 \\
\delta h(z) &= \delta h_{m_{\text{ax}}} (1 - 2z/L) \ \text{for} \ z \leq L/2 \\
\delta h(z) &= \delta h_{m_{\text{ax}}} (2z/L - 1) \ \text{for} \ z > L/2 \tag{5}
\end{align*} \]

In dimensionless form, these are,

\[ \begin{align*}
\Delta v(z) &= \Delta v_{m_{\text{ax}}} (1 - 2Z) \ \text{for} \ Z \leq 1/2 \\
\Delta v(z) &= \Delta v_{m_{\text{ax}}} (2Z - 1) \ \text{for} \ Z > 1/2 \\
\Delta h(z) &= \Delta h_{m_{\text{ax}}} (1 - 2Z) \ \text{for} \ Z \leq 1/2 \tag{6}
\end{align*} \]
\[ \Delta h(z) = \Delta h_{\text{max}} (2Z - 1) \text{ for } Z > 1/2 \]

where \( \Delta = \delta / c \) is the misalignment in dimensionless form, and \( Z = z / L \).

**Figure 2.** Deviation of journal bearing centre caused by misalignment for (a) \( z \leq L/2 \); (b) \( z > L/2 \) [14]

### 4. Linear modification

The axial profile modification of a bearing can be used to overcome the negative consequences of misalignment. In this work, linear modification as shown in figure 3 was used, achieved from each side of the bearing at a height \( c_r \) along a distance \( z_0 \).

**Figure 3.** Linear chamfer location: (a) Longitudinal section; (b) 3D representation.

The equation of linear modification (see figure 4) is

\[ f(z) = Az + B \]  \hspace{1cm} (7)

**Figure 4.** Linear modification of the bearing.
To find the constants A and B, the following boundary conditions were used:

For the Left side \((z \leq z_m)\)

- At \(z = 0\) \(\to f(z) = c_m\)
- At \(z = z_m\) \(\to f(z) = 0\)

These conditions are applied in equation (7) to give:

\[ c_m = B \quad , A = \frac{c_m}{z_m} \]

Therefore, when \(z_m \leq L/2\), the general equation of modification is

\[ f(z) = c_m \left( 1 - z \frac{1}{z_m} \right) \]  \hspace{1cm} (8)

Similarly, for the Right side \((z \geq L - z_m)\)

- At \(z = 0\) \(\to f(z) = c_m\)
- At \(z = L - z_m\) \(\to f(z) = 0\)

and the constants in equation (7) become

\[ B = c_m - \frac{c_m}{z_m} L \quad , A = \frac{c_m}{z_m} \]

For the right side, the general equation of modification thus becomes

\[ f(z) = c_m \left( 1 + \frac{1}{z_m} (z - L) \right) \]  \hspace{1cm} (9)

Equations (8) and (9) can be written in a dimensionless form as

\[ F(z) = \frac{f(z)}{c} = c_r \left( 1 - Z \frac{1}{z_o} \right) \]  \hspace{1cm} (10)

and

\[ F(z) = \frac{f(z)}{c} = c_r \left( 1 + \frac{1}{z_o} (Z - 1) \right) \]  \hspace{1cm} (11)

The parameters \(z_o\) and \(c_r\) in equations (10 and 11) are dimensionless where, \(z_o = z_m / L\) and \(c_r = c_m / c\). The two parameters \((z_o, c_r)\) can then be used to calculate the effectiveness of the bearing modification.

5. **Numerical solution**

The governing equations require a numerical solution to give the pressure distribution and the film thickness based on the Reynolds boundary condition method. Figure 5 illustrates the discretisation of the pressure gradients:
Figure 5. Discretisation of finite difference.

where

\[
\frac{\partial}{\partial \theta} \left( H^3 \frac{\partial P}{\partial \theta} \right) = \frac{H^3 \frac{\partial P}{\partial \theta} b - H^3 \frac{\partial P}{\partial \theta} a}{\Delta \theta} \tag{12}
\]

\[
\frac{\partial P}{\partial \theta} |_b = \frac{P(i+1,j) - P(i,j)}{\Delta \theta}
\]

\[
\frac{\partial P}{\partial \theta} |_a = \frac{P(i,j) - P(i-1,j)}{\Delta \theta} \tag{12a}
\]

\[
H^3 |_b = \left[ \frac{H_{(i+1,j)} + H(i,j)}{2} \right]^3
\]

\[
H^3 |_a = \left[ \frac{H(i,j) + H(i-1,j)}{2} \right]^3 \tag{12b}
\]

\[
\frac{\partial H}{\partial \theta} = \frac{H_{(i+1,j)} - H_{(i-1,j)}}{2 \Delta \theta}
\]

A similar procedure can be used for the pressure gradient in the z direction. Substituting the outcome for these into equation (3) gives

\[
P_{(i,j)} = \frac{1}{\gamma} \left[ H_b^3 P(i+1,j) + H_a^3 P(i-1,j) + B_1 B_2 H_c^3 P(i+1) + B_1 B_2 H_d^3 P(i-1) - C_1 H_{(i+1,j)} + C_1 H_{(i-1,j)} \right]
\]

\[
B_1 = \frac{R^2}{L^2}, B_2 = \frac{d \theta^2}{dz^2}, C_1 = \frac{d \theta}{2}
\]

\[
\gamma = H_b^3 + H_a^3 + B_1 B_2 H_c^3 + B_1 B_2 H_d^3
\]
The load components in dimensionless form are thus

\[ W_t = \int_0^1 \int_0^{\theta_{cav}} P \sin \theta R L d\theta \, dz \]

\[ W_r = \int_0^1 \int_0^{\theta_{cav}} P \cos \theta R L d\theta \, dz \]

\[ W = \sqrt{W_t^2 + W_r^2} \] (15)

and the angle between the line of centres and the load, \( W \) (attitude angle) is

\[ \beta = \tan^{-1} \frac{W_t}{W_r} \] (16)

The dynamic coefficients can thus be represented by the following equations where the stiffness coefficients are

\[ K_{xx} = \int_0^1 \int_0^{2\pi} \left( \frac{\partial P}{\partial X} \right) \cos \theta \, d\theta \, dz \]

\[ K_{xy} = \int_0^1 \int_0^{2\pi} \left( \frac{\partial P}{\partial Y} \right) \cos \theta \, d\theta \, dz \]

\[ K_{yx} = \int_0^1 \int_0^{2\pi} \left( \frac{\partial P}{\partial X} \right) \sin \theta \, d\theta \, dz \]

\[ K_{yy} = \int_0^1 \int_0^{2\pi} \left( \frac{\partial P}{\partial Y} \right) \sin \theta \, d\theta \, dz \] (17)

and the damping coefficients are

\[ C_{xx} = \int_0^1 \int_0^{2\pi} \left( \frac{\partial P}{\partial X} \right) \cos \theta \, d\theta \, dz \]

\[ C_{xy} = \int_0^1 \int_0^{2\pi} \left( \frac{\partial P}{\partial Y} \right) \cos \theta \, d\theta \, dz \] (18)

\[ C_{yx} = \int_0^1 \int_0^{2\pi} \left( \frac{\partial P}{\partial X} \right) \sin \theta \, d\theta \, dz \]

\[ C_{yy} = \int_0^1 \int_0^{2\pi} \left( \frac{\partial P}{\partial Y} \right) \sin \theta \, d\theta \, dz \]

The derivatives \( \frac{\partial P}{\partial X}, \frac{\partial P}{\partial Y}, \frac{\partial P}{\partial X} \) and \( \frac{\partial P}{\partial Y} \) are also required to calculate the dynamic coefficients. These derivatives are evaluated as follows:
\[
\frac{\partial H}{\partial t} = \Delta \dot{X} \cos \theta + \Delta \dot{Y} \sin \theta
\]

\[
\frac{\partial H}{\partial X} = \cos \theta
\]

\[
\frac{\partial H}{\partial Y} = \sin \theta
\]

\[
\frac{\partial H}{\partial \theta} = -\Delta X \sin \theta + \Delta Y \cos \theta
\]

Differentiation of the Reynolds equation with respect to \(X\) gives

\[
\frac{\partial}{\partial \theta} \left( H^3 \frac{\partial P_x}{\partial \theta} \right) + \alpha \frac{\partial}{\partial Z} \left( H^3 \frac{\partial P_x}{\partial Z} \right) = -\frac{\partial}{\partial \theta} \left( 3H^2 \cos \theta \frac{\partial P}{\partial \theta} \right) - \alpha \frac{\partial}{\partial Z} \left( 3H^2 \cos \theta \frac{\partial P}{\partial Z} \right) - \sin \theta
\]

(19)

while differentiation with respect to \(Y\) gives

\[
\frac{\partial}{\partial \theta} \left( H^3 \frac{\partial P_y}{\partial \theta} \right) + \alpha \frac{\partial}{\partial Z} \left( H^3 \frac{\partial P_y}{\partial Z} \right) = -\frac{\partial}{\partial \theta} \left( 3H^2 \sin \theta \frac{\partial P}{\partial \theta} \right) - \alpha \frac{\partial}{\partial Z} \left( 3H^2 \sin \theta \frac{\partial P}{\partial Z} \right) - \cos \theta
\]

(20)

Further differentiation with respect to \( \dot{X} \) and \( \dot{Y} \) respectively, gives

\[
\frac{\partial}{\partial \theta} \left( H^3 \frac{\partial P_x}{\partial \theta} \right) + \alpha \frac{\partial}{\partial Z} \left( H^3 \frac{\partial P_x}{\partial Z} \right) = \cos \theta
\]

(21)

\[
\frac{\partial}{\partial \theta} \left( H^3 \frac{\partial P_y}{\partial \theta} \right) + \alpha \frac{\partial}{\partial Z} \left( H^3 \frac{\partial P_y}{\partial Z} \right) = \sin \theta
\]

(22)

Equations (19), (20), (21), and (22) can then be solved numerically using the following general form:

\[
\vec{P}_{(i,j)} = \frac{1}{\psi} [(\Delta \theta)^2 \text{RHS} - H_a^3 \vec{P}_{(i+1,j)} - H_b^3 \vec{P}_{(i-1,j)} - \alpha C_2 H_d^3 \vec{P}_{(i,j+1)} - \alpha C_2 H_c^3 \vec{P}_{(i,j-1)} + C_1 H_{(i+1,j)} - C_1 H_{(i-1,j)}] \]

(23)

Where:  \( \alpha = \frac{R^2}{L^2}, C_1 = \frac{\Delta \theta}{2}, C_2 = \frac{(\Delta \theta)^2}{(\Delta Z)^2} \)

\( \psi = -H_b^3 - H_a^3 - \alpha C_2 H_c^3 - \alpha C_2 H_d^3 \).

and the dynamic coefficients can be determined numerically as

\[
K_{xx} = \sum_{j=1}^{M} \sum_{i=1}^{N} \left( \frac{\partial P_{(i,j)}}{\partial X} \right) \cos \theta \Delta \theta \Delta Z
\]

\[
K_{xy} = \sum_{j=1}^{M} \sum_{i=1}^{N} \left( \frac{\partial P_{(i,j)}}{\partial Y} \right) \cos \theta \Delta \theta \Delta Z
\]
\[ K_{yx} = \sum_{j=1}^{M} \sum_{i=1}^{N} \left( \frac{\partial P(i,j)}{\partial x} \right) \sin \theta \Delta \theta \Delta Z \]  
(24)

\[ K_{yy} = \sum_{j=1}^{M} \sum_{i=1}^{N} \left( \frac{\partial P(i,j)}{\partial y} \right) \sin \theta \Delta \theta \Delta Z \]

\[ C_{xx} = \sum_{j=1}^{M} \sum_{i=1}^{N} \left( \frac{\partial P(i,j)}{\partial x} \right) \cos \theta \Delta \theta \Delta Z \]

\[ C_{xy} = \sum_{j=1}^{M} \sum_{i=1}^{N} \left( \frac{\partial P(i,j)}{\partial y} \right) \cos \theta \Delta \theta \Delta Z \]  
(25)

\[ C_{yx} = \sum_{j=1}^{M} \sum_{i=1}^{N} \left( \frac{\partial P(i,j)}{\partial x} \right) \sin \theta \Delta \theta \Delta Z \]

\[ C_{yy} = \sum_{j=1}^{M} \sum_{i=1}^{N} \left( \frac{\partial P(i,j)}{\partial y} \right) \sin \theta \Delta \theta \Delta Z \]

6. Results and Discussion

Mesh density (number of points investigated) significantly affects the accuracy of results. The effect of mesh density on \( P_{\text{max}} \) and \( H_{\text{min}} \) is thus examined in this section. Figure 6 illustrates the effect using a range of total numbers of mesh points \( (k) \) between 55 and 64,800; this clarifies that, when the value of \( k > 300 \), \( P_{\text{max}} \) and \( H_{\text{min}} \) are not significantly affected by the number of mesh points. However, \( k = 16,471 \) was used in the current work to ensure the independence of the results from the number of mesh points.

![Figure 6](image_url)

**Figure 6.** Effect of number of points on \( H_{\text{min}} \) and \( P_{\text{max}} \).

The results of the 3D misalignment model applied in this work were compared with the results presented by Xu et al. (2015) [17]. That study used the following equation to calculate film thickness:

\[ h = c + e_o \cos(\theta - \varphi_o) + e'\left( \frac{z}{D} - \frac{1}{2} \right) \cos(\theta - \alpha - \varphi_o) \]

The comparison was performed for various values of \( \Delta h_{\text{max}} \) and \( \Delta v_{\text{max}} \) at three positions along the bearing length, as shown in Table 1. The maximum observed difference was 0.138%.
Table 1. Verification of the 3D misalignment model.

| $\Delta h_{\text{max}}$ | $\Delta v_{\text{max}}$ | $Z$ | $H_{\text{min}}$ current work | $H_{\text{min}}$ [17] | % Difference |
|------------------------|------------------------|-----|-------------------------------|------------------------|--------------|
| 0.2                    | 0.4                    | 0.256 | 0.1188423                      | 0.1188691              | 0.0225       |
|                        |                        | 1    | 0.2587435                      | 0.2587611              | 0.0068       |
|                        |                        | 0    | 0.2145261                      | 0.2145728              | 0.0218       |
| 0.3                    | 0.3                    | 0.256 | 0.3060354                      | 0.3060359              | 0.0002       |
|                        |                        | 1    | 0.2105620                      | 0.2105656              | 0.0017       |
|                        |                        | 0    | 0.0739046                      | 0.0739235              | 0.0256       |
| 0.52                   | 0.52                   | 0.256 | 0.3253831                      | 0.3253831              | 0             |
|                        |                        | 1    | 0.0550872                      | 0.0551634              | 0.1381       |

The pressure distribution (3D) for the aligned and misaligned cases is illustrated in figure 7. $P_{\text{max}}$ is 1.1842 for the aligned case, as shown in figure 7a, while for the misaligned case, $P_{\text{max}}$ is 1.8138, as shown in figure 7b, where the maximum pressure is also seen to increase by 53%. Two pressure spikes can be seen, close to the boundaries of the solution space; these local spikes can be attributed to the misalignment effect, as the film thickness takes on lower values at these locations. The relatively high level of pressure at these locations may result in a significant negative effect on the life of the bearing.

![Figure 7](image1.png)

(a) (b)

Figure 7. Pressure distribution (3D) (a) aligned case and (b) misaligned case. (L/D =1.5 and $\varepsilon_r$=0.7).

Figure 8 shows the 3D pressure distribution for modified bearings using different values for chamfer parameters. In the misaligned case shown in figure 7, $P_{\text{max}}$ is 1.8138. When the chamfer parameters are $c_r = 0.2$ and $z_o = 0.2$, the max pressure is 1.39 (23.4 % reduction), as shown in figure 8a. In figure 8b, however, $P_{\text{max}}$ is 1.497 when the chamfer parameters are $c_r = 0.3$ and $z_o = 0.5$. Linear modification is thus seen to decrease $P_{\text{max}}$ when $c_r \leq 0.3$ and $z_o \leq 0.5$. Figure 8c shows that the maximum pressure increases when the chamfer parameters increase; the max pressure in this figure is 2.49 when the chamfer parameters are $c_r = 0.5$ and $z_o = 0.5$. High values of $c_r$ should thus be avoided. The corresponding results for 2D film thickness for this case are shown in figure 9. The $H_{\text{min}}$ values at these chamfer parameters are 0.195, 0.182, and 0.104 respectively. In the misaligned case the value of minimum film thickness is 0.055, suggesting that the linear modification based on the chamfer parameters used in this figure increases $H_{\text{min}}$ by 254.5%, 230.9%, and 89% respectively in comparison with the misaligned case.
Figure 8. Pressure distribution (3D) in the case of misalignment with linear modification at chamfer parameters (a) $c_r = 0.2$, $z_o = 0.2$, (b) $c_r = 0.3$, $z_o = 0.5$, and (c) $c_r = 0.5$, $z_o = 0.5$. (L/D =1.5).
Figure 9. Film thickness distribution (2D) in the case of misalignment with linear modification at chamfer parameters (a) $c_r = 0.2$, $z_o = 0.2$, (b) $c_r = 0.3$, $z_o = 0.5$, and (c) $c_r = 0.5$, $z_o = 0.5$. (L/D = 1.5).

Figure 10 shows the effect of the chamfer parameters ($z_o, c_r$) on $H_{min}$ and $P_{max}$. The results for the aligned case without modification are represented by the horizontal green line in this figure. The misalignment parameters used were $\Delta h_{max} = \Delta v_{max} = 0.52$, which cause a significant increase in the
maximum pressure. The range of $z_o$ is 0 to 0.5, while the values of $c_r$ are in the range 0 to 2. The value of $P_{max}$ is 1.8138 in the case of a misaligned bearing, and using the linear modification at $z_o \leq 0.5$ and $c_r < 0.5$ causes $P_{max}$ to decrease. However, when $c_r \geq 0.5$ and $z_o > 0.3$, $P_{max}$ increases, as shown in figure 10a. This outcome again emphasises that high values of $c_r$ should be avoided. The value of $H_{min}$ in the case of misalignment is 0.055. In general, linear modification increases $H_{min}$ in comparison with the misalignment case, as shown in figure 10b.

![Figure 10](image_url)

Figure 10. Effect of chamfer parameters on (a) maximum pressure and (b) minimum film thickness, at L/D=1.5

$$\Delta h_{max} = \Delta v_{max} = 0.52, z_o = 0 \text{ to } 0.5, \text{ and } \varepsilon_r = 0.7.$$  

Figure 11 illustrates the effect of the linear chamfer on the stiffness coefficients $K_{xx}$, $K_{xy}$, $K_{yx}$, and $K_{yy}$. A wide range of chamfer parameters were used such that $z_o$ was between 0 and 0.5, while the values of $c_r$ were held in a range of 0 to 2. Figure 11a shows that $K_{xx}$ was 2.864 in the aligned case, and this value was increased to 3.588 due to misalignment, an increase of 25.28% in $K_{xx}$. The linear chamfer reduced the level of $K_{xx}$: when $c_r = 0.3$ and $z_o = 0.2$, the reduction in $K_{xx}$ was 4.5% as compared to the misaligned case. Using higher values for the chamfer parameters led to an increase in $K_{xx}$, however. Figure 11b shows that the $K_{xy}$ value for the aligned case was 3.01, while in the misaligned case, this decreased to 2.44. Thus, when $c_r = 0.5$ and $z_o = 0.25$, the level of $K_{xy}$ is very close to that in the aligned case. Figure 11c illustrates the influence of the chamfer on $K_{yx}$. In the aligned case, $K_{yx}$ is -0.109, which decreases to -1.391 due to the misalignment. Using the chamfer parameters $c_r = 0.3$ and $z_o = 0.3$ increased $K_{yx}$ by 69.9%. Figure 11d shows that in the aligned case, $K_{yy}$ is 1.741, while in the misaligned case this value increases to 2.773. In general, the linear modification reduces the level of $K_{yy}$, which is reduced by 33.1% when the chamfer parameters are $c_r = 0.3$ and $z_o = 0.3$. 

![Figure 11](image_url)
Figure 11. Effect of chamfer parameters on stiffness coefficients: (a) $K_{xx}$, (b) $K_{xy}$, (c) $K_{yx}$ and (d) $K_{yy}$ at $L/D=1.5$ and $\Delta h_{max} = \Delta v_{max} = 0.52$ and $z_o = 0$ to 0.5 and $\epsilon_r = 0.7$.

Figure 12 illustrates the results on the damping coefficients ($C_{xx}$, $C_{xy}$, $C_{yx}$ and $C_{yy}$) in dimensionless form. In Figure 12a, the $C_{xx}$ in the aligned case is seen to be 5.838; it is 6.212 in the case of misalignment.
In the linear chamfer, this level decreases, coming very close to the aligned case when the chamfer parameters are $c_r=0.3$ and $z_o=0.35$. The values of $C_{xy}$ are illustrated in Figure 12b. In the aligned case, $C_{xy}$ is 1.899; this value is decreased to 0.903 in the misaligned case, while the linear chamfer shows a general increase at this level of 50% when chamfer parameters are $c_r=0.3$, $z_o=0.3$. Figure 12c shows the effect of the linear chamfer on $C_{yy}$. In the aligned case $C_{yy}$ is 1.951, while in the case of misalignment, $C_{yy}$ is 3.249, which represents an increase of 66.5%. The chamfer causes a reduction in the levels of $C_{yy}$, and when the chamfer parameters are $c_r=0.3$ and $z_o=0.3$, a reduction of 33.5% is obtained in comparison with the misaligned case. These results represent an important outcome: linear modification using a relatively low range of $c_r$ and $z_o$ improves the dynamic characteristics of the bearings as well as reducing $P_{max}$ and offering significant increases in $H_{min}$.

![Figure 12](image_url)

**Figure 12.** Effect of chamfer parameters on damping coefficients (a) $C_{xx}$, (b) $C_{xy}$, and (c) $C_{yy}$, at $L/D=1.5$ and $\Delta h_{max} = \Delta v_{max} = 0.52$ and $z_o = 0$ to 0.5 and $\varepsilon_r = 0.7$. 
7. Conclusions

This paper presents a solution to the hydrodynamic problem of journal bearings of finite length based on a 3D misalignment model. A linear modification of the bearing profile was used to reduce the effect of misalignment on the characteristics of journal bearings. The finite difference method was used to solve the governing equations numerically, utilising the Reynolds boundary condition method. The results showed that misalignment causes noticeable pressure spikes close to the edges of the solution space. Using a first order profile for modification increases the levels of film thickness by 277%, reduces pressure spikes from misalignment by 18.5%, and generally improves the dynamic characteristics of journal bearings. However, higher values of chamfer parameters should be avoided.

References

[1] Lund J W and Thomsen, K K 1978 A Calculation Method and Data for the Dynamic Coefficients of Oil-Lubricated Journal Bearings ASME New York,

[2] Mokhtar M O A , Safar S.and Abd-El-Rahman M , A M, 1985 An Adiabatic Solution of Misaligned Journal Bearings. Journal of Tribology ASME, Vol.107, pp. 263-267

[3] Nacy S M 1997 Effect of Chamfering on Side-Leakage Flow Rate of Journal Bearings wear Vol. 212(1), pp. 95–102

[4] Bouyer J and Fillon, M 2003 Improvement of the THD Performance of a Misaligned Plain Journal Bearing Journal of Tribology ASME, Vol. 125(2), pp. 334-342

[5] Ebrat O, Mourelatos Z P Vlahopoulos N and Vaidyanathan K 2004 Calculation of Journal Bearing Dynamic Characteristics Including Journal Misalignment and Bearing Structural Deformation Tribology Transactions, Vol.47, pp. 94-102.

[6] Sun J and Changlin G 2004 Hydrodynamic Lubrication Analysis of Journal Bearing Considering Misalignment Caused by Shaft Deformation Tribology International ELSEVIER, Vol.37, pp. 841–848

[7] Sun J Gui C, Li Z and Li Z 2005 Influence of Journal Misalignment Caused by Shaft Deformation Under Rotational Load on Performance of Journal Bearing Part J: Journal of Engineering Tribology, Vol.219 (4), pp. 275–283.

[8] Boukhelef D Bounif A and Bouzid DA 2011 Dynamic characterization and stability analysis of hydrodynamic journal bearing using the FEM, Dynamics of mechanical systems Vol.17(5), pp. 503-509.

[9] Saridakis K M, Nikolakopoulos P G, Papadopoulos C A and Dentsoras A.J 2012 Identification of wear and misalignment on journal bearings using artificial neural networks Part J: Journal of Engineering Tribology. Vol.226(1), pp. 46-56.

[10] Chasalevris A and Sfyris D 2013 Evaluation of the Finite Journal Bearing Characteristics, Using the Exact Analytical Solution of the Reynolds Equation. Tribology International Vol.57, pp. 216–234/

[11] Chasalevris A and Dohnal F 2015 A journal bearing with variable geometry for the suppression of vibrations in rotating shafts: Simulation, design, construction and experiment Mechanical Systems and Signal Processing Vol.52-53, pp. 506–528.

[12] Chasalevris A and Dohnal F 2016 Enhancing Stability of Industrial Turbines Using Adjustable Partial Arc Bearings. Journal of Physics : Conference Series Vol.744
[13] Dyk S, Rend J, Byrtus M and Smolik L 2018 Dynamic Coefficients and Stability Analysis of Finite-Length Journal Bearings Considering Approximate Analytical Solutions of the Reynolds Equation. Tribology International, Elsevier Ltd, 130, pp. 229–244.

[14] Jamali H U and Al-Hamood A 2018 A New Method for the Analysis of Misaligned Journal bearing Tribology in Industry, Vol.40, No.(2), pp.213–224.

[15] Liu C, Zhao B, Li W and Lu X 2019 Effects of bushing profiles on the elastohydrodynamic lubrication performance of the journal bearing under steady operating conditions. Mechanics and Industry, Vol.20, No 2

[16] Zhou Y, Shao L, Zhang C, Ji F, Liu J, Li G, Ding S, Zhang Q and Du F 2020, Numerical and experimental investigation on dynamic performance of bump foil journal bearing based on journal orbit Chinese Journal of Aeronautics. https://doi.org/10.1016/j.cja.2019.12.001

[17] Xu G, Zhou J, Geng H, Lu M, Yang L, Yu L 2015 Research on the Static and Dynamic Characteristics of Misaligned Journal Bearing Considering the Turbulent and Thermohydrodynamic Effects. Journal of Tribology ASME, Vol. 137(2).