Connexive Principles After a ‘Classical’ Turn in Medieval Logic

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The aim of this paper is to look at the arguments advanced by three Parisian arts masters about how to understand Prior Analytics II 4 and the more general discussion that medieval authors situate this in, revolving around the validity of various rules of inference. In particular, this paper argues that all three arts masters read Aristotle’s remarks as committing him to a weak form of connexive principles where the antecedent needs to be either true or at least formally possible; this is known in the contemporary literature as a weak connexive thesis.

1. Introduction

In chapter four of Book II of Aristotle’s Prior Analytics, one finds the following passage (Aristotle 1989, 64):

But it is impossible that the same thing should be necessitated by the being and by the not-being of the same thing. I mean, for example, that it is impossible that B should necessarily be great if A is white and that B should necessarily be great if A is not white. For whenever if this, A, is white it is necessary that that, B, should be great, and if B is great that C should not be white, then it is necessary if A is white that C should not be white. And whenever it is necessary, if one of two things is, that the other should be, it is necessary, if the latter is not, that the former should not be. If then B is not great A cannot be white. But if, if A is not white, it is necessary that B should be great, it necessarily results that if B is not great, B itself is great. But this is impossible. For if B is not great, A will necessarily not be white. If then if this is not white B must be great, it results that if B is not great, it is great, just as if it were proved through three terms.

This passage is interesting for a number of reasons. In the eyes of many Latin medieval authors, Aristotle was committing himself to the thesis that B does not follow from both A and its negation. Further in the passage, it also appeared to these authors that Aristotle committed himself to the view that it is impossible that ‘If not B then B’ holds (in modern notation \(\neg(\neg B \rightarrow B)\)). In modern discussions, the former principle is sometimes referred to as Abelard’s Thesis, while the latter principle is referred to as Aristotle’s First Thesis. Along with Boethius’s Thesis, \((A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)\), these two principles are often taken as characteristic theses of modern connexive logic. Connexive logic is an interesting and somewhat unusual non-classical logic. Nearly all of the systems of non-classical logic studied in contemporary philosophy (e.g. intuitionistic or relevance logic) are sub-logics of classical logic in the sense that all of the theorems of the non-classical logic in question are also theorems of classical logic. This is not the case for connexive logic. As can be easily checked, neither Aristotle’s First Thesis, Boethius’s Thesis, nor Abelard’s Thesis
are classically valid. This has led some to describe connexive logic as a contra-classical logic (Humberstone 2000).

Interestingly, connexive principles like Aristotle’s First Thesis and Boethius’s Thesis play an important role in discussions of hypotheticals by medieval authors from the eleventh century onward. In early medieval discussions of such hypothetical inferences, these inferences are accepted. For example, in twelfth and some thirteenth centuries works, one finds such inferences accepted by authors like Peter Abelard (Abelard 1970, 290 ll.25–27) and, in a restricted form, by Robert Kilwardby (Kilwardby 2015, 1141–1145). This is not an exhaustive list of authors from this period who accepted connexive principles; and indeed, one also finds authors in this period who accepted the classical thesis ‘from the impossible anything follows’ (ex impossible quodlibet, or hereafter EIQ). For example, Kilwardby accepted EIQ for accidental consequences, while accepting connexive principles for natural consequences. However, as has been discussed by Martin (Martin 1986, 2004), in the mid thirteenth century a number of objections were developed to connexive theses, which rendered the validity of these inferences questionable at best. An argument, apparently developed by William of Soissons and his followers, was used to show that connexive principles like those cited above are incompatible with EIQ (Martin 1986).1 The argument, which is parallel to C. I. Lewis’s several hundred years later, is the following:

\[
\begin{align*}
(1) & \quad P \& \neg P & \text{(Assumption)} \\
(2) & \quad P & \text{(Conjunction Elimination, 1)} \\
(3) & \quad \neg P & \text{(Conjunction Elimination, 1)} \\
(4) & \quad P \lor Q & \text{(Disjunction Introduction, 2)} \\
(5) & \quad Q & \text{(Disjunctive Syllogism, 3,4)}
\end{align*}
\]

This argument from William of Soissons et al. raised a number of questions for medieval authors who wanted to defend the validity of connexive logical principles. Another related issue concerns the acceptance of the rules ‘from A infer “A or B”’, and ‘from B infer “A or B”’, since it is straightforward to see that ‘A or not A’ follows from A and that ‘A or not A’ also follows from not A, which violates Boethius’s Thesis. Interestingly, Peter of Spain directly rejects the validity of Soissons’s argument, although he does not appear to explicitly endorse a connexive reading of the hypothetical (Peter of Spain 1992, 237), while Kilwardby argues that connexive principles need to be restricted to a particular notion of logical consequence, known as natural consequence (Kilwardby 2015, 1143). As logic develops into the late thirteenth and early fourteenth centuries, the notion of consequence described in the standard texts on logic becomes one that we would now describe as classical, and these texts no longer discuss connexive principles. This is the historical transition that I refer to as the ‘classical’ turn. For example, Walter Burley (Burley 1955, 61, 146) and John Buridan (Read 2015, 75) both explicitly endorse EIQ, and indeed Buridan states it as one of the characteristic results of a theory of consequence.

However, while all of these new logical ideas were being developed, the passage quoted above in Aristotle was still part of the corpus of logical texts discussed by medieval authors. So, a natural question is, how did later medieval authors interpret Aristotle and understand his reference to these connexive principles? In this paper, our focus will be on three arts masters writing at the University of Paris in the late thirteenth and early fourteenth centuries. Those masters are Rudulphus Brito, John Buridan, and the author of the Prior

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1 Hereafter referred to as ex impossible quodlibet or EIQ.
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Analytics commentary attributed to John Duns Scotus. We will argue that the arts masters at the University of Paris proposed two different ways of thinking about and defending connexive principles. One way was to argue that what Aristotle was doing was thinking about these kinds of inferences as closely connected to demonstrations ‘propter quid’. In other words, these inferences were seen as valid when applied to a particular subclass of arguments, but not as generally valid. As we will explain in more detail, these demonstrations are a class of valid knowledge producing argument, starting from true premises, and providing an answer to the question ‘why does the conclusion follow?’ (hence the name ‘propter quid’). We find this solution in the discussion in Rudulphus Brito’s Questions on the Prior Analytics (Brito 2016), and it is also alluded to by John Buridan in his Questions on the Prior Analytics (Buridan 1986). The second way is to think of these inferences as being restricted to a particular class of propositions. In particular, connexive inferences are valid, on this reading, if the propositions in question are ‘simple categoricals’ and these simple categoricals do not entail a contradiction. These propositions are described as being formally possible. In both cases, what these authors are doing is reading the connexive principles in a ‘weak’ way: Instead of applying to all inferences, these principles apply only to a subclass of arguments. In both cases, these arts masters endorse a weakened form of the various connexive principles when applied to the relevant class of argument. In the case of Brito, the premises need to be known and hence true (among other things), while for Buridan and Pseudo-Scotus the premises need to be formally possible. As a corollary, both Pseudo-Scotus and Buridan observe that such a solution, while rendering the connexive principles valid, does not keep them formally valid, unless the restrictions are explicitly acknowledged. As we shall see, this discussion also provides helpful insight into how these authors are conceiving of these issues, how these authors situate connexive and classical logical rules, and the development of these kinds of consequences from Brito to Buridan.

2. Background

Comparatively little is known about Radulphus Brito’s life.² Brito was born in the village of Ploudiry some time before 1273, with 1270 being a likely date. He became a master of arts at the University of Paris in 1290 and began his theological study in 1299, lecturing on the Sentences in 1309 and obtaining his theological doctorate in 1313/1314. He died some 6–7 years later in 1319/1320. Our focus in this paper is on Brito’s philosophical writings, most of which only exist in manuscript form. Fortunately for our purposes, his Questions on the Prior Analytics has been edited and will be the text we are working from (Brito 2016).

Buridan belongs to the generation writing after Brito. Born sometime before 1300, John Buridan was educated at the University of Paris. Buridan became an arts master in the mid 1320s and remained in the arts faculty for his entire career. His writings on logic, philosophy, and natural philosophy have attracted significant interest from modern scholars, with Buridan’s logical writings generally seen as possessing a high degree of both originality and subtlety. Our focus in this text will be on Buridan’s Questions on the Prior Analytics, where he discusses this issue. These issues do not emerge in either Buridan’s Summulae de Dialectica or in his Treatise on Consequences, his primary works on logic. However, in this paper we will be working from an unpublished edition of Buridan’s Questions on the Prior Analytics edited by Hubien (Buridan 1986).³

² See Courtenay 2005 and Deuffic 2002 for recent biographical information.
³ Unfortunately, the critical edition of John Buridan’s Questions on the Prior Analytics was left unfinished by Hubert Hubien upon his death. However, a typescript of the partial critical edition has been made available online at
Pseudo-Scotus is by far the most difficult of the authors to date, primarily because the *Questions on the Prior Analytics* we are working from was not written by John Duns Scotus, but internal evidence suggests that this work was written in the 1330s or early 1340s (*Read 2015*, 5–6). Without further information, it is not possible to further describe this author. For Pseudo-Scotus, we will be working from the 1891 Opera Omnia. The text that we are working from is known to have a number of textual issues. For our purposes in this paper, nothing of substance will hang on these issues, and we will be following the printed text.

The relationship between these three authors is not entirely clear, although the case of Buridan appears straight-forward. Buridan shows awareness of Pseudo-Scotus’s views in his other logical writings (*Read 2015*, 5–6) and, as should become clear below, the phrasing and referencing suggests one of the views he discusses in the context of these questions is Pseudo-Scotus’s. The textual evidence also suggests Buridan is reading Brito. As far as I can discern, there is nothing in the passages under discussion to suggest that Pseudo-Scotus is reading Brito (or vice versa, which would be impossible, given the likely dates).

As such, the paper will proceed as follows. Since the structures of the *Questions* texts written by these three authors are similar, and all three are written in the same style, we will discuss similar arguments put forward in three works in parallel. We will start by looking at the arguments that the three masters put forward in favour of the view that ‘the same thing follows from the same, affirmed and denied’, focusing here on the arguments put forward, and the overlap between them. With that in place, we will briefly comment on the sed contra (which is itself very brief), and then discuss the solutions discussed by each author. Finally, we will summarise the overall analysis provided by these arts masters and compare their approaches to modern approaches to connexive principles.

### 3. The question

Our focus in this paper is on one question found in each of these three authors. For Brito, the question is Question 7 of Book II.⁴ In the case of Pseudo-Scotus, it is question 3 of Book II, which addresses issues about connexive rules.⁵ Buridan raises this issue in question 7 of Book II.⁶

From a brief inspection of the questions, one can see some small but important differences between the three formulations of the question. The difference between Brito’s phrasing of the question (whether the rule is valid (lit. true)) and Buridan’s phrasing (whether it is possible) appears to be stylistic, with Brito asking, in effect, if this rule is valid and Buridan asking if it is possible for the same thing to follow from the same thing being and not being. Pseudo-Scotus’s question is more specific, focusing on whether from each part of a contradiction the same thing follows.

### 4. Initial arguments

In their discussions, Brito and Buridan both put forward four arguments in favour of the question being answered positively, while Pseudo-Scotus puts forward three. One argument, concerning disjunction introduction, is common to all three authors.

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⁴ ‘UTRUM HOC SIT VERUM: AD IDEM AFFIRMATUM ET NEGATUM NON SEQUITUR IDEM’ (*Brito 2016*, 469).
⁵ ‘Utrum ad utrumque contradictoriorum posit sequi idem’ (*Duns Scotus 1891*, II:183).
⁶ ‘UTRUM POSSIBLE SIT AD IDEM ESSE ET AD IPSUM NON ESSE SEQUI IDEM’ (*Burdian 1986*, L.II, Q.7, 1).
4.1. Disjunction introduction

The basic thrust of the argument, which is also found in the works of earlier authors, including Kilwardby (Kilwardby 2015, 1141), is that one can deduce the same disjunction from both parts of a disjunction if the two parts are contradictory. The rule is a straightforward application of what is now called or-introduction. That this is a problem for a connexive principle is straightforward to see, since if we let ‘A’ be ‘You are sitting’ and ‘not A’ be ‘You are not sitting’, and ‘B’ be ‘You are sitting or not sitting’, then clearly B follows from A, and it also follows from not A. As such, this appears to be a counterexample to both Boethius’s Thesis and Abelard’s Thesis. As an aside, it is unsurprising that we find this argument in Brito,7 as he often uses Kilwardby as a source. Indeed, it is likely that Kilwardby’s solution to this issue is explicitly discussed by Brito under the label of ‘the ancient’s opinion’. This is supported by a further reference to ‘the ancient’s opinion’ in Brito’s solution, which refers to Kilwardby’s distinction between accidental and natural consequences, but in an altered form.8 In addition, Brito’s solution to the disjunction-introduction problem appears to be a straightforward adaptation of Kilwardby’s point that the two parts of a disjunction follow by virtue of different things (Brito 2016, 474).9 It is also interesting to note that, while Brito and Buridan use exactly the same counter-example, Buridan’s reasoning is very close in phrasing to that of Pseudo-Scotus.10 Pseudo-Scotus’s text works out the example in more detail, but the thrust of the argument is nearly identical.11 The change from second person (‘You are seated’) to third person (‘Socrates runs’) does not appear to be a substantial change in the general point that is being made. It certainly does not change the overall logic of the argument and no comment about this is made in the discussion of Pseudo-Scotus or in Buridan. As such, the point at issue is that the rules for ‘or’ appear to create a problem for connexive principles.

4.2. Syllogism

Brito’s first argument and Buridan’s fourth argument both employ a syllogistic structure with changes in the premises (Table 1).

While the reading given in the critical edition of Brito is unclear as to why one would accept an invalid argument, the revised text is clearly valid. The argument is an instance of Cesare in Figure II. It is unclear whether this example is being used to reject Aristotle’s Thesis or Boethius’s Thesis, or, indeed, it may ‘work’ for both. Let the premises, taken together, be ‘B’ and let the conclusion be ‘C’. Then, if the argument is successful, it would establish that both ‘B entails C’ and ‘not B entails C’, and thus Boethius’s Thesis fails. The key question this argument should raise, is how exactly one should think about the pair of premises in the syllogism being given here. It appears that the notion ‘affirmed and denied’

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7 ‘Item, sequitur “si tu sedes, tu sedes vel non sedes” quia ad partem disiuncti sequitur totum disiunctum. Item sequitur “si tu non sedes, tu sedes vel non sedes;” et sic ad idem affirmatum et negatum, ut appareat, sequitur idem’ (Brito 2016, 469–470).
8 ‘Quaedam enim est quae tenet immediate de virtute sermonis, et sic non est verum quod “ad impossibile sequatur quodlibet,” et talis consequentia dicebatur ab ANTIQUIS “essentialis,” quia in antecedente includitur consequens. Alia est consequentia accidentalis, scilicet quae non tenet gratia formae nec de virtute sermonis, sed per quondam concomitantiam aliqvorum quae simul habent esse in veritate vel simul non esse’ (Brito 2016, 474). I would like to thank the anonymous referee for pointing out this passage.
9 Kilwardby’s solution is found in (Kilwardby 2015, 1143).
10 ‘Item, ista disiunctiua “tu sedes uel tu non sedes” sequitur ad istam “tu sedes” et “tu non sedes”; ergo eadem sequitur ad utrumque oppositum, quoniam istae duae opponuntur, scilicet “tu sedes” et “tu non sedes”’ (Buridan 1986, L.II, Q.7, 5).
11 ‘Tertio, quia ista disjunctiva, Socrates currit, vel Socrates non currit, sequitur ad istam, Socrates currit, et cum hoc sequitur ad istam, Socrates non currit ; igitur ad ambo contradictoria sequitur idem. Consequentia tenet, quia istae contradictunt. Antecedens probatur, quia quaelibet pro positio infert seipsam cum quacunque alia in una disjunctiva, et hoc gratia formae’ (Duns Scotus 1891, II:183–184).
Table 1. Syllogisms used in Brito’s first argument and Buridan’s fourth argument.

| Brito (Brito 2016, 469) | Buridan (Buridan 1986, L.II, Q.7, 6) |
|-------------------------|-------------------------------------|
| Every stone is an animal | No stone is an animal                |
| [No] human is an animala| Every human is an animal             |
| Therefore               | Therefore                            |
| No human is a stone     | No human is a stone                  |

*The critical edition text reads ‘omnes lapus est animal, omnes homo est animal’ (Brito 2016, 469). As was pointed out by one of the anonymous reviewers, this reading is obviously defective. Further, the critical apparatus lists ‘nullus’ as a reading present in some manuscripts. Here we amend the text to follow the reading that makes better logical sense.

is only being applied to the major premise, and not to the two premises taken together. As we shall see, Brito will put pressure on exactly this point.

Buridan’s version of the argument is slightly more complicated. In particular, Buridan assumes that the predicates of the premises, A and B, are contrary. As such, one is allowed to infer that ‘no B is A’, and so obtain an instance of the second syllogism. As before, the second of Buridan’s syllogisms is an instance of Cesare in Figure II, and so is valid.

What links both of these arguments is the presence of a false major premise. In the case of Brito, this is clear from the example, while in the case of Buridan it follows from the assumption that the predicates are contraries.

4.3. Necessarium sequitur ad quodlibet

Pseudo-Scotus and Buridan both put forward an argument against connexive principles that appeals to the principle ‘the necessary follows from anything’ (*necessarium sequitur ad quodlibet*).

The example that Pseudo-Scotus uses is ‘God is a substance, therefore God is’. The argument is compressed, but the main point is that both:

- God is a substance, therefore God exists
- And
- God is not a substance, therefore God exists

Both follow, because of the principle ‘the necessary follows from anything’. In this case, the ‘necessary’ is the proposition expression ‘God exists’, which was a standard medieval example of a necessarily true proposition. It is important to note here that the argument Pseudo-Scotus gives is a counter-example to both Aristotle’s Thesis and Boethius’s Thesis. Buridan’s discussion of this argument is extremely compressed, simply stating the rule. However, in his discussion of the question, he goes on to offer an argument that the principle ‘the necessary follows from anything’ follows from *ex impossibile quodlibet*, and that the inferences are both formally valid. The argument Buridan gives is straightforward but worth briefly discussing. Buridan argues as follows:

1. From the impossible anything follows.
2. Every necessary proposition is the contradictory of some impossible proposition.
3. If B follows from A, then the contradictory of A follows from the contradictory of B.

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12 ‘Item, fiat syllogismus sic “omne B est A et nullum C est A”; ergo nullum C est B; et si sumas praemissas praedictis contrarias, adhuc cadem conclusio conclutur, ut “nullum B est A et omne C est A; ergo nullum C est B”; ergo eadem conclusio sequitur ad aequas praemissas et ad oppositas earum’ (Buridan 1986, L.II, Q.7, 6).

13 ‘quia ad alterum contradictoriorum sequitur aliquod necessarium, ut verbi gratia, sequitur : Deus est substantia; igitur Deus est : igitur illud sequitur ad reliquum, per illam regulam, Necessarium sequitur ad quodlibet’ (Duns Scotus 1891, II:183).

14 ‘Et arguitur quod sic: quia sicut ad impossibile sequitur quodlibet, ita necessarium sequitur ad quodlibet; igitur sequitur ad utrumque oppositoris’ (Buridan 1986, L.II, Q.7, 3).
From (1) and (3), we have: from the ‘contradictory of anything’ follows the contradictory of the impossible. This with (2) gives us from the ‘contradictory of anything’ follows from the necessary. At this point, Buridan simply completes the proof. However, he is implicitly appealing to principle like ‘the contradictory of anything’ can itself be any proposition.

While Brito does not appeal to necessarium sequitur ad quodlibet, he does appeal to ex impossibile quodlibet. However, his discussion is rather different from Buridan, and will be treated as its own argument below.

4.4. Syllogisms from opposites

A syllogism from opposites is a special kind of syllogistic argument where the premise pair are either contrary or contradictory. Unlike in the previous example, which considers a particular syllogism, this argument focuses on a class of syllogism. Pseudo-Scotus and Buridan both put forward an argument using this kind of syllogism:

The idea here appears to be that since from a pair of opposed premises the same thing follows, one is allowed to infer that the conclusion follows from either part of the contradiction. The response of both Buridan\(^{15}\) and Pseudo-Scotus\(^{16}\) is that this argument is not good. The response is a natural one. For example, Buridan observes that just because something follows from the conjunction of two premises, it does not mean that an inference follows from one part of the conjunction alone,\(^{17}\) i.e. just because C follows from ‘A and B’, it does not entail that C follows from A alone, or that C follows from B alone.

4.5. Stand-alone arguments

The two arguments of Brito’s that we have not discussed do not have argumentative parallels with either Buridan or Pseudo-Scotus, although the ex impossibile quodlibet argument comes up in different ways in all three discussions.

The third argument that Brito gives is based on establishing that the same thing can follow from premises affirmed and denied using the principle ‘from the impossible anything follows’. Brito starts by observing that if ex impossibile quodlibet is granted, then this raises a problem for the principle under discussion.\(^{18}\) The argument Brito gives rests on Aristotle’s argument for the principle of non-contradiction in Metaphysics IV sections 3–6. This is interesting, because there is no discussion of the Metaphysics text relating to this question in either Pseudo-Scotus or Buridan. As we have already seen, both of those authors offer a ‘logical’ argument in support of the principle. The basic form of Brito’s argument is to observe that if one posits a contradiction as being true, then on the basis of what Aristotle said, it follows that everything is one, and so, if it follows that everything is one, you can infer whatever you like from what you have inferred.\(^{19}\) The key move in the second part appears to be based on the idea that if ‘everything is one’ is true, then things cannot be much more absurd than that. As such, anything else is also true.

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\(^{15}\) ‘Item, syllogismus ex oppositis constituitur ex praemissis suis tamquam ex utroque oppositorum, et tamen in eo sequitur una et eadem conclusio; igitur [...] et caetera’ (Buridan 1986, L.II, Q.7, 4).

\(^{16}\) ‘Secundo, quia in Syllogismo ex oppositis ad praemissas sequitur una conclusio; igitur ad utrumque contradictoriorum sequitur idem. Antecedens patet in isto secundo; et consequentia probatur, quia praemissae Syllogismi ex oppositis (?-there is a mark in the text) sunt ad invicem contradictoriae’ (Duns Scotus 1891, II:183).

\(^{17}\) ‘Ad aliam, quae arguabet de syllogismo ex oppositis, dico quod conclusio non sequitur ad utrumque praemissarum; immo ad neutram eorum sequitur, sed sequitur ex copulatia ex ambabus praemissis constituta’ (Buridan 1986, L.II, Q.7, 18).

\(^{18}\) ‘Sed idem affirmatum esse et negatum est impossibile, ergo ad idem affirmatum et negatum sequitur quodlibet, et sic sequatur idem’ (Brito 2016, 470).

\(^{19}\) ‘Probatio quod ex impossibili sequetur quodlibet, quia impossibile includit contradictoria. Sed qui ponit contradictoria simul esse vera, ponit omnia esse unum, ut probatur IV Metaphysicae, ergo qui ponit omnia esse unum, ponit quodlibet sequi ad quodlibet. Et sic qui ponit impossibile, ponit quodlibet, et sic ad impossibile sequitur quodlibet’ (Brito 2016, 470).
The final argument Brito gives in this question is based on a reading of the hypothetical where the antecedent and the consequent are linked by a cause. The argument is the follows:

1. If you are a man then God exists
2. If you are a donkey then God exists
3. If you are a donkey, then you are not a man
4. So if you are not a man, God exists. (from 2 & 3)

The first and last propositions provide a counter-example to the inference. However, as stated the argument does not look particularly plausible. In particular, just because ‘A entails B’ and ‘A entails C’, it need not follow that ‘B entails C’, nor indeed, that ‘C entails B’. Brito makes exactly this point in responding to the argument.

5. Sed contra

All three authors offer a very short sed contra (i.e. an argument in opposition to the initial objections) in defence of the opposite view. They all cite the authority of Aristotle, with Pseudo-Scotus explicitly quoting Aristotle Latinus. It is clear both because of the structure and what they go on to say, that despite the ‘classical’ arguments against connexive principles, of which Brito, Buridan, and Pseudo-Scotus are all clearly aware, they intend to find ways of defending connexive principles. It is to their solutions that we now turn. We will start by looking at Brito, who has a unique solution, and then focus on Pseudo-Scotus and Buridan, who take the same approach.

6. Solutions and responses

6.1. Brito’s solution

Brito’s solution to this question is to draw a distinction between two ways one can think about the same thing following from the same thing being affirmed and denied. One way to think about this is to think of this as following ‘formally and propter quid’ (formaliter et propter quid). Interestingly, Brito goes on to align this with a notion of formal consequence. The other way Brito simply refers to as ‘another mode’, and in this way of thinking about following, he says, the same thing does follow.20 According to Brito, Aristotle’s remarks are discussing following in the first sense, not the second.

The notion of ‘formally and propter quid’ is terminology that is taken from the theory of demonstration, which is grounded in the medieval reading of the Posterior Analytics. The discussion here is based on material that is common to many authors and will be illustrated with references to Buridan, as there is currently no critical edition of Brito’s Questions on the Posterior Analytics. It was standard among medieval authors to distinguish four possible questions that pertain to knowledge:21

1. Whether a thing is?
2. What a thing is?
3. How the thing is?
4. For what reason a thing is?

The basic idea is that to answer the first question, ‘whether a thing is’, one needs to provide a proof that the thing in question exists. To answer the second question, one need to prove what the thing in question is. To answer the third question, one needs to demonstrate the

20 ‘Si tamen intelligatur utrum alio modo ad idem affirmatum et negatum absolute sumptum sequatur idem, dicendum quod bene potest sequi idem’ (Brito 2016, 471).
21 In Latin these four questions correspond to: si est/utrum est?, quid est?, quia est?, and propter quid est?
way in which the thing exists. In the final case, one needs to explain the reason for what the thing is. As a precondition for the fourth kind of demonstration, it does need to be the case that the things being described by the demonstration exist.

Now, how does Brito’s requirement that these arguments be demonstrations propter quid help? The key observation is that the premises for this kind of demonstration need to be known to be true. One is not able to carry out this kind of demonstration using false propositions. As such, an argument from contradictory premises may be, in Brito’s words, materially valid, but it is not formally valid.22 What exactly Brito means by formally valid here is not spelt out, but considering how closely Brito is following Kilwardby in this analysis, it is likely to be a sense of formal validity closely related to that of Kilwardby, rather than the sense of formal validity that one finds in Buridan or in Pseudo-Scotus.

6.2. Pseudo-Scotus

Pseudo-Scotus begins his response by distinguishing three ways that a consequent can follow from the antecedent, namely formally, simply or as of now.23 According to Pseudo-Scotus, a formal consequence (i.e. a consequence where the consequent follows formally from the antecedent) is one where the consequence holds in all instances of an argument retaining the same form, but the terms given being changed.24 So, for example, ‘Some A is a B, therefore some B is an A’ is formally valid on this account, because regardless of what terms are used for A and B, the argument remains valid. Pseudo-Scotus is also clear about what he means by the terms and by the form. According to Pseudo-Scotus, the terms are the subject and predicate of the proposition while the form is essentially everything else (Duns Scotus 1891, II:184). In particular, Pseudo-Scotus mentions the copula, any syncategorematic expressions, the order, as well as both the number of propositions and the number of terms that occur in the argument. This is similar to the definition of formality that Buridan gives both in his discussion of the Prior Analytics text here and in his Treatise on Consequences.

A simple consequence for Pseudo-Scotus has a few parts:

1. It is a consequence where the consequent follows necessarily from the premises.
2. The consequence may not be valid in all other terms.
3. One can add a ‘necessary middle’ which, when added, makes the inference evident and formally valid.

Pseudo-Scotus’s illustration is the argument ‘A human runs, therefore an animal runs’.25 This inference, Pseudo-Scotus tells us, is perfected through the middle ‘Every human is an animal’. This definition requires some comments. The definition appears to allow for a formally valid argument to count as a simply valid one as well, but it is much broader, as it includes ones that are not formally valid. What separates these inferences from formal ones is that the argument can be made formally valid with the addition of a necessary middle. Note that the modal ‘necessary’ or its cognates do not need to be present in the proposition.

22 ‘Si; tu ergo quaeras utrum idem sequatur ad idem affirmatum et negatum gratia formae, dico quod non, quia ad unitatem causae sequitur unitas effectus formaliter. Sed praemissae sunt causae conclusionis, ergo ad easdem praemissas non sequitur nisi una conclusio formaliter’ (Brito 2016, 473).
23 ‘Quantum ad primum notandum, quod consequens potest sequi ad antecedens, vel formaliter, vel simpliciter, vel ut nunc solum’ (Duns Scotus 1891, II:184).
24 ‘Tunc dicitur sequi formaliter, quando in omnibus terminis est bona consequentia, retenta consimili forma arguendi, quantumcumque termini mutarentur’ (Duns Scotus 1891, II:184).
25 ‘Sed tunc dicitur aliquid sequi simpliciter, quando est necessaria consequentia antecedentis ad consequens in aliquis terminis: licet non valeat in omnibus terminis, retenta consimili forma arguendi, et quando per medium necessarium hujusmodi consequentia potest fieri evidens, et formalis, et sic ista consequentia est bona, Homo currit; igitur animal currit, quia fit formalis per istud medium, Omnis homo est animal, arguendo in Disamis’ (Duns Scotus 1891, II:184).
Thus ‘Every human is an animal’ is a necessary proposition, even though the modal expression ‘necessary’ is not present or implicit in any of the terms in the propositions. Second, what exactly does Pseudo-Scotus mean by a ‘necessary middle’? The expression ‘middle’ here could refer to a middle term, either one that is common, or it can refer to a proposition that serves as the link between the antecedent and the consequent. The example here is clearly of the second kind. Finally, Pseudo-Scotus says that a consequence follows as of now (ut nunc), if the middle that makes the consequence evident is not necessary, but only contingently true.

So, how is this distinction supposed to help? According to Pseudo-Scotus, if we are speaking about formal inferences, then anything follows from a contradiction. He argues for this in the following way, starting from contradictory assumption ‘Socrates runs and Socrates does not run’. He starts from the observation that from a conjunction both of its parts follow. So one may infer ‘Socrates runs’. From this, one can then infer ‘Socrates runs or you are in Rome’ by what we would now call disjunction introduction. This rule, Pseudo-Scotus notes, is also formally valid. He then applies disjunction elimination to conclude ‘You are in Rome’. Note that Pseudo-Scotus’s argument here implicitly contains a second instance of conjunction elimination. As such, the argument here rests on three inferences: conjunction elimination, disjunction introduction, and disjunctive syllogism and is an instance of the argument developed by William of Soissons in the twelfth century.

As Pseudo-Scotus goes on, he gives a number of properties that each type of inference generally has, and observes, in each case, that for none of these does the rule Aristotle gives hold. Instead, Pseudo-Scotus’s solution is what we might think of as a ‘restrictivist solution’. The second part of Pseudo-Scotus’s fourth conclusion states that, if neither part of a contradiction implies a contradiction, and both parts are simple categoricals, then the rule is universally true. What Pseudo-Scotus is doing here amounts to an endorsement of ‘weak connexivist’ principles. These principles are weak in the sense that they are not unrestrictedly valid, but are valid with various restrictions. What Pseudo-Scotus appears to have observed is that one can remove the ‘offending’ cases by requiring that the premises be consistent. This is based on two observations. The first is that if both parts of a contradictory pair (e.g. ‘A’ and ‘not A’) entail the same thing, then one of the pair entails the other. The second observation is that this does not hold for simple categorical statements that do not entail a contradiction. What Pseudo-Scotus is observing here is that if one removes this ‘offending’ case, then connexive principles like Aristotle’s First Thesis remain valid. But why does Pseudo-Scotus suggest that this is what Aristotle had in mind?

From his subsequent discussion, it appears that Pseudo-Scotus is observing that in Aristotle’s argument for connexive principles, he uses expressions that are of the form e.g. ‘B is large’ or ‘A is not white’. What Pseudo-Scotus appears to be thinking here is that the

26 ‘Prima est, quod ad quamlibet propositionem, quae manifeste implicat contrad Herrnionem, sequitur formaliter quaelibet alia, sicut sequitur, Socrates currit, et Socrates non currit; igitur tu es Romae’ (Duns Scotus 1891, II:184).
27 ‘Probatur, quia ad dictam copulativam sequitur quaelibet ejus pars gratia formae’ (Duns Scotus 1891, II:184).
28 ‘Socrates currit; igitur Socrates currit, vel tu es Romae, quia quaelibet propositio infert seipsam formaliter cum qualibet alia, in una disjunctiva’ (Duns Scotus 1891, II:184).
29 ‘et ultra sequitur, Socrates currit, vel tu es Romae, sed Socrates non currit, ut reservatum fuit; igitur tu es Romae, quod fuit probatum per illam regulam, Ex disjunctiva cum contradictoria unius partis ad reliquam partem est bona consequential’ (Duns Scotus 1891, II:184).
30 ‘Secunda, quod neutra pars contradicitoriarum implicet contrad Herrnionem, et ideo in simplicibus Categoricis regula est universaliter vera’ (Duns Scotus 1891, II:185).
31 ‘Secunda, quod neutra pars contradicitoriarum implicet contrad Herrnionem, et ideo in simplicibus Categoricis regula est universaliter vera. Probatur, si ad ambo contradictoria sequitur idem, tunc ad unum contradictiorum sequeretur relinquum, consequens est falsum in simplicibus Categoricis non implicantibus contradictionem’ (Duns Scotus 1891, II:185).
examples that illustrate the principle are restricted in a particular way, namely to statements where one has a single term and a single simple predicate affirmed of it. If we limit our attention to expressions of this form, then inferences like ‘God exists therefore God does not exist’, or ‘The anti-christ does not exist therefore the anti-christ exists’ do not follow. What Pseudo-Scotus is doing here is, in essence, imposing a syntactic requirement and consistency requirement on the expressions that occur in the argument. First, the expressions need to be simple categoricals. This rules out the possibility of assuming as the antecedent explicit contradictions, where each of the conjuncts is possible on its own, e.g. ‘A human is running and a human is not running’. It also blocks the argument from disjunctive introduction by fiat, as it were. For these kinds of arguments we are not allowed to infer disjunctive (or indeed molecular) conclusions. The second clause ensures that the simple terms are not themselves inconsistent. This is to block cases where the term may be tacitly impossible. While Pseudo-Scotus gives us no examples, it is not hard to think of potential candidates, e.g. ‘A chimera exists’ or liar sentences.

By way of conclusion, Pseudo-Scotus makes an interesting observation. He observes that the rule Aristotle has articulated here only holds for simple categoricals, but that it is not one that is formally valid. This observation points to an important corollary for the restrictivist strategy employed by Pseudo-Scotus. By requiring that the terms be consistent and syntactically restricted, it follows that these arguments cannot be formally valid.

Importantly, from a modern perspective, it can be shown that this sort of a strategy works. What Pseudo-Scotus has noticed is that the counter-examples to these connexive principles rest on being able to assume that the antecedent of a consequence is impossible. Pseudo-Scotus attributes to Aristotle the view that the rule only applies in a fairly restrictive set of circumstances. This strategy has some interesting parallels to one modern approach to connexive implication discussed by Graham Priest (Priest 1999).

6.3. Buridan

Buridan’s solution is to distinguish between a proposition that is formally contradictory and one that is formally possible. Buridan then goes on to observe that if a proposition is formally impossible, then anything you like follows from it formally. By a formal contradiction, Buridan is thinking of a proposition that includes a contradiction as part of its form. Two of the examples he uses are of a conjunction of contradictories (‘Every human runs and some human does not run’) and of a categorical proposition where the predicate is the infinite negation of the subject (‘Some human is a non-human’) before going on to give other possible cases. What this suggests is that the notion of ‘formal’ here refers to how formality works in a formal consequence, i.e. a formal contradiction is one that is contradictory, regardless of the particular terms under discussion. Buridan goes on to say that a formal necessity is the contradiction of a formal contradiction. So, for example, Buridan says that ‘Every human runs or some human does not run’ and ‘No human is a non-human’ are both formal necessities.

32 ‘ut dictum est, quia non sequitur, Deus est; igitur Deus non est, nec ad negativam sequitur affirmativa, quia non sequitur, Antichristus non est; igitur Antichristus est, quia utrobique antecedens est verum, et consequentia falsum’ (Duns Scotus 1891, II:185).
33 ‘Sic patet, quod illa regula, scilicet, Ad idem esse, et non esse, non sequitur idem, intelligitur solum in simplicibus Categoricis, et de consequentia formalis’ (Duns Scotus 1891, II:185–186).
34 It may be helpful to remember that on Buridan’s analysis, there is a difference between the proposition ‘Some A is not B’ and ‘Some A is non-B’. The first, Buridan argues, is true if there is no A, and this is the contradictory of ‘Every A is B’. ‘Some A is non-B’ is an affirmative statement, as the negation is modifying the predicate term, not the copula, and so has existential import. ‘Some A is not A’ can be true on Buridan’s analysis, e.g. when A is ‘Chimera’, while ‘Some A is non-A’ is false, and indeed appears to be impossible.
It is clear that Buridan’s solution to the question is essentially the same solution as that proposed by Pseudo-Scotus. Like Pseudo-Scotus, Buridan says that it appears to him that Aristotle intends this conclusion only for arguments with simple categorical propositions. According to Buridan, to be a simple categorical requires that the proposition not entail a contradiction on account of its form. Again, as in the case of Pseudo-Scotus this blocks the problematic molecular cases by fiat.

With this view having been developed, Buridan also notes that others say that in this passage Aristotle intended to only speak about demonstrative consequences propter quid. Buridan observes that, on this opinion, if the conclusion is false then it cannot have been demonstrated propter quid, because the argument cannot have been a demonstration. Likewise, if the conclusion is true, then it cannot have been demonstrated from a false premise because that is not how such demonstrations work. As such, the premises in such a demonstration also have to be true. While this is not the option that Buridan accepts, he says that this solution to the question also works. As we have seen above, this solution is clearly the one that Brito adopts. It would be interesting to see if there are any other concrete parallels between Brito and Buridan. In addition, it would be worthwhile to systematically trace the discussions of demonstrations propter quid in relation to connexive principle.

7. Analysis and modern comparison

What we have seen in these three authors are two broad strategies of thinking about the validity of contra-classical principles in a classical setting. Both strategies rest on a core insight, namely that the problem for connexive principles arises in cases where the premises of an argument are allowed to be false (of particular interest is the case where the premises are contradictory). What both of these solutions do is, in essence, restrict these arguments to require that the propositions involved be true, in the case of Brito, or at least non-contradictory, in the case of Scotus and Buridan. Interestingly, Brito’s strategy strengthens the notion of consequence so that connexive principles hold when the argument is a successful demonstration, and not ‘merely’ a logical validity. In particular, connexive rules are good when one is answering the question ‘why is such and such this way’. This approach, as Buridan points out in his discussion of the view, ends up requiring that the premises of the argument are jointly true, and the argument also needs to be valid. What Brito’s view does is more closely align this discussion in Aristotle with the theory of demonstration developed in the Posterior Analytics, which is a natural view given that many medieval authors tended to read the two books as being closely related. Buridan’s later discussion of the approach developed by Brito makes an interesting observation: if one restricts one’s attention to arguments that are sound, and not merely valid, one obtains a connexive notion of implication. This is because the instances that create problems for connexive principles are all cases where one or more of the premises are false.

In contrast, the approach taken by Pseudo-Scotus and Buridan is more ‘logical’ in nature. They both observe that if one restricts the premises in the argument to ones that are simple categoricals (for Buridan, categorical statements that do not imply a contradiction), then the rule holds. What is interesting is that both Pseudo-Scotus and Buridan note that such a rule is not formally valid. Indeed, for Pseudo-Scotus its goodness does not fall under any of the notions of consequence he considers. For Buridan, it is likely to be a kind of material consequence, which is only true because of the specific terms chosen.

The approach taken by Pseudo-Scotus and Buridan has some strong parallels with a modern analysis of connexive implication developed by Priest (Priest 1999). In this paper, Priest observes that it is possible to obtain a connexive notion of implication if one defines $A \Rightarrow B$ as $A \rightarrow B$ and $\Diamond A$, and that this notion of consequence is not closed under substitution of terms. The approach taken by Pseudo-Scotus and Buridan also makes this
observation. What they instead do is introduce the notion of formally possible and formally contradictory in an attempt to be explicit about the logical structure of these inferences. When we require that the premises be simple categoricals that do not imply a contradiction on their own, we require that the premises be jointly possible. They then observe that one obtains an acceptable reading of connexive rules. Thus, Priest’s observation was anticipated by Pseudo-Scotus and Buridan nearly 600 years ago.

8. Conclusion

The aim of this paper has been twofold. On the one hand, this paper looks at how three closely temporally and geographically linked medieval authors in the late thirteenth and early fourteenth centuries, Radulphus Brito, Pseudo-Scotus, and John Buridan, thought about connexive principles and their relationship to the classical rule ‘from the impossible anything follows’. This provides a basis for looking at how these authors thought about the connexive principles that had been discussed by twelfth century authors such as Peter Abelard, Peter of Spain, and Robert Kilwardby. Our aim has been to show that Brito, Pseudo-Scotus, and Buridan think about this problem in a closely related way, and find similar ways to defend connexive principles within a classical framework. All three authors advocate a solution that makes connexive inferences valid by imposing conditions on the premises allowed in the consequence. In the case of Brito, the argument needs to be a demonstration propter quid. Buridan, commenting on this, observes that this requires the premises to be true, and that this solution works. However, both Buridan and Pseudo-Scotus favour an analysis in which the premises need to be formally non-contradictory.

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