Resonant photo-ionization of Yb$^+$ to Yb$^{2+}$

Simon Heugel$^{1,2}$, Martin Fischer$^{1,2}$, Vladimir Elman$^1$, Robert Maiwald$^{1,2}$, Markus Sondermann$^{1,2}$ and Gerd Leuchs$^{1,2,3}$

1 Max Planck Institute for the Science of Light, Guenther-Scharowsky-Str. 1/building 24, D-91058 Erlangen, Germany
2 Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU), Department of Physics, Staudtstr. 7/B2, D-91058 Erlangen, Germany
3 Department of Physics, University of Ottawa, Ottawa, Ont. K1N 6N5, Canada

E-mail: gerd.leuchs@mpl.mpg.de

Received 26 August 2015, revised 20 October 2015
Accepted for publication 6 November 2015
Published 4 December 2015

Abstract
We demonstrate the controlled creation of a $^{174}$Yb$^{2+}$ ion by photo-ionizing $^{174}$Yb$^+$ with weak continuous-wave lasers at ultraviolet wavelengths. The photo-ionization is performed by resonantly exciting transitions of the $^{174}$Yb$^+$ ion in three steps. Starting from an ion crystal of two laser-cooled $^{174}$Yb$^+$ ions localized in a radio-frequency trap, the verification of the ionization process is performed by characterizing the properties of the resulting mixed-species ion-crystal. The obtained results facilitate fundamental studies of physics involving Yb$^{2+}$ ions.

Keywords: ytterbium, light–matter interaction, photo-ionization

(Some figures may appear in colour only in the online journal)

1. Introduction

The ionic species Yb$^{2+}$ has been proposed for several studies in the field of fundamental physics, namely for tests on the temporal variation of the fine structure constant [1–3] and for investigations on the efficient interaction of a single photon with a single atom in free-space [4]. For the latter application the isotope $^{174}$Yb$^{2+}$ is of particular relevance, since its level scheme comprises a nearly isolated resonance transition which is close to a pure two-level system. Trapping such an ion in the focus of a parabolic mirror [5] and exciting it with linear-dipole radiation is expected to enable interaction of light and matter with almost unit coupling efficiency [4, 6, 7].

In this contribution we present the creation of $^{174}$Yb$^{2+}$ starting from a trapped $^{174}$Yb$^+$ ion. The choice of an unfavorable ionization method might complicate or even hinder the desired experiments due to the creation of charges in the trapping environment. In particular, due to the small, sub-wavelength extent of the focal spot of a deep parabolic mirror, any charge potential created during the ionization process will shift the ion out of the focus of the parabola. Therefore, ionization by electron-beam bombardment as performed in [3] might not be an option.

Previously, several production methods have been applied for the generation of multiply charged ions. Most widely used is the production of ions by laser ablation from solid state targets [8, 9]. An alternative is the production of ion-plasmas from vacuum arc sources [10], potentially followed by further beam ionization to higher charged states. Ions generated with these methods have also been trapped subsequently to their generation, e.g. in a Penning trap [10] or in a linear Paul trap [8]. Methods to generate multiply charged ions with a higher level of control have been demonstrated in [3, 11, 12]. There, singly charged ions have been generated and trapped inside a radio-frequency ion-trap. The subsequent ionization to the doubly charged state has been performed by electron-beam bombardment [3], field ionization using femtosecond laser pulses [12], or non-resonant direct photo-ionization with a vacuum-ultraviolet light source [11]. The work in [3] has to be highlighted here as the first report on the successful production and trapping of Yb$^{2+}$. However, all of the aforementioned methods suffer from drawbacks with respect to our particular experimental scenario. As already indicated above, methods inducing stray charges are not acceptable for experiments where light is to be focused tightly onto a quantum object in a well defined manner. Furthermore, the weak focusing of all auxiliary beams used in our set-up (see [5]) would require high absolute powers when, e.g. using ionization techniques based on pulsed lasers.

In contrast, here we demonstrate the resonant photo-ionization of a single trapped and laser cooled Yb$^+$ ion to Yb$^{2+}$, requiring only about $100 \mu W$ of laser power while...
simultaneously minimizing charging of the trap environment and obtaining high ionization efficiency. Section 2 introduces the pursued ionization scheme and gives an estimate for the expected ionization rate. In section 3 we report the experimental results, followed by a discussion and an outlook in the last section.

2. Resonant photo-ionization scheme

The photo-ionization scheme used here involves a set of three transitions as shown in figure 1. In the first step the \([\text{Xe}]^4f^{14}5d\), \(J = 3/2\) level (abbreviated as 5d\(_{1/2}\) in the following) is excited via the 6s\(_{1/2} \rightarrow 6p\(_{1/2}\) transition and a subsequent decay 6p\(_{1/2} \rightarrow 5d\(_{3/2}\). The latter occurs with a probability of 0.5\% [13]. The notation 6s\(_{1/2}\) is used as an abbreviation for the \([\text{Xe}]^4f^{14}6s\), \(J = 1/2\) level and 6p\(_{1/2}\) for \([\text{Xe}]^4f^{14}6p\), \(J = 1/2\). In the next step the \([\text{Xe}]^4f^{14}7p\), \(J = 1/2\) level (shortly 7p\(_{1/2}\)) is excited via the transition starting from 5d\(_{3/2}\) using light with a wavelength of 245 μm. Starting from the 7p\(_{1/2}\) level a transition towards an unbound state of the valence electron is realized by excitation with the same wavelength.

On the basis of the theoretical data in [14] the decay paths of the 7p\(_{1/2}\) level can be analyzed. The strongest decay with a probability of 69.1\% occurs via the \(7p_{1/2} \rightarrow 4f^{14}7s, J = 1/2\) transition from where finally either the cooling cycle (see below) or the \([\text{Xe}]^4f^{14}5d\), \(J = 5/2\) level (shortly 5d\(_{5/2}\)) is entered. This level is entered mainly via the intermediate \([\text{Xe}]^4f^{14}6p\), \(J = 3/2\) level. The total probability to enter the 5d\(_{5/2}\) level via the decay cascade from the 7p\(_{1/2}\) level is estimated to be about \(5 \times 10^{-3}\). The decay \(7p_{1/2} \rightarrow 6s_{1/2}\) occurs with a probability of 17.7\%. The probability for the decay \(7p_{1/2} \rightarrow 5d_{3/2}\) is found to be 12.4\%.

The 5d\(_{5/2}\) level is reported to have a lifetime of 7.2 ms [15]. Due to this relatively long lifetime the efficiency of the excitation of the 7p\(_{1/2}\) level in the above described scheme would be lowered significantly. The 5d\(_{5/2}\) level is therefore cleared directly via the 5d\(_{5/2}\) \(\rightarrow [3/2]_{1/2}\) transition with light at a wavelength of 976 nm. Here, \([3/2]_{1/2}\) is the abbreviation for the \([\text{Xe}]^4f^{14}5d_{6s}, J = 3/2\) level. In addition the 5d\(_{5/2}\) level can decay to the \([\text{Xe}]^4f^{14}6s^2\), \(J = 7/2\) level (shortly \(f_{7/2}\)) with a probability of 83\% [15]. A lifetime of 3700 d is reported for the latter level [16], which is cleared via the \(f_{7/2} \rightarrow [5/2]_{3/2}\) transition with the wavelength 638 nm [17], where \([5/2]_{3/2}\) designates the \([\text{Xe}]^4f^{14}5d_{6s}, J = 5/2\) level. The upper levels of the transitions driven at 638 nm and 976 nm decay back into ground state [14, 17].

In the experiment the four transitions mentioned above are driven by diode-laser based light-sources. The inclusion of the two repumping-lasers for the 5d\(_{5/2}\) and \(f_{7/2}\) levels allows for the efficient excitation of the 7p\(_{1/2}\) level which in turn maximizes the photoionization probability.

The performance of this photoionization scheme can be estimated from the steady-state of the atomic multi-level system excitation according to the action of the lasers driving the transitions shown in figure 1 and the cross-section of the photoionization from 7p\(_{1/2}\). The rate \(R\) for the ionization process can be calculated as:

\[
R = p_{7p} \sigma_{245} F_{245},
\]

with the excitation probability \(p_{7p}\) of the 7p\(_{1/2}\) level, the photoionization cross-section \(\sigma_{245}\) for the photoionization starting from that level and the flux \(F_{245}\) of photons at wavelength 245 nm, which is used for ionization.

The expected excitation probability for the 7p\(_{1/2}\) level has been computed using the theoretical transition-rate data found in [18]. It was found to be \(p_{7p,\text{max}} = 9.5 \times 10^{-5}\) if all driven transitions are strongly saturated.

The cross-section for the photoionization from 7p\(_{1/2}\) level has been calculated following a quantum-defect-theory based approach [19]. References [20, 21] provide two different parametrizations of the corresponding transition matrix elements. The quantum defect values are derived on the basis of the level energies listed in [22] and the \([\text{Xe}]^4f^{14}\)-series ionization-limit from [23]. For the scheme proposed here,
these calculations give $\sigma_{245} = 5.5\text{ Mb}$ according to [20] and $\sigma_{245} = 7.2\text{ Mb}$ according to [21].

If the $5d_{5/2} \rightarrow 7p_{1/2}$ transition is efficiently saturated the total ionization rate can be approximated according to equation (1) as

$$R \approx 4.1 \cdot 10^{-6}\frac{\text{m}^2}{\text{J}} \cdot \frac{P_{245}}{w_0},$$

with a total continuous-wave power of $P_{245}$ for the excitation laser at wavelength 245 nm and a gaussian beam-waist $w_0$. For $w_0 = 10\ \mu\text{m}$ and $P_{245} = 100\ \mu\text{W}$ one finds a total ionization rate of $R \approx 4.1\text{ s}^{-1}$.

We end this section in stressing the importance of the repumping lasers at 976 nm and 638 nm wavelength. Starting from the $7p_{1/2}$ level, the ionization to Yb$^{2+}$ is calculated to be equally probable as decaying to the $f_7/2$ level for a laser power at 245 nm that is more than three orders of magnitude larger than the one used in our experiments. For high ionization success probabilities, one would have to increase the incident power by at least another two orders of magnitude. As discussed below, at such high power levels detrimental effects occurred in our experiments. However, for the low 245 nm laser power leading to successful ionization the probability for ionization is considerably lower than for decaying to the branch ending up in the $f_7/2$ level. Therefore, the laser at 638 nm wavelength clearing this level is necessary for achieving practical ionization rates in the experiment. The same holds true for the 976 nm laser depopulating the $5d_{5/2}$ level which is the main path for entering the $f_7/2$ level.

3. Experimental results

A scheme of the experimental setup for the photoionization-experiments is shown in figure 2. The ions are confined within a stylus ion-trap [24] and the necessary laser-beams are superimposed and focused onto the ions. A set of auxiliary electrodes is employed to compensate stray electric DC-fields and to probe the motional dynamics of the ion crystals. For the latter purpose low-amplitude AC signals are supplied to one of these electrodes. All involved light-sources are commercially available diode-based laser-systems based on external-cavity diode lasers tuned by frequency selective optical feedback. In addition to the lasers needed for the ionization process, radiation at 935 nm is needed for laser cooling Yb$^+$ [25], where the main cooling transition is the one at 369 nm. Since light at 935 nm wavelength empties the $5d_{5/2}$ level its application is detrimental for the ionization process. Therefore, the 935 and 245 nm light paths are equipped with acousto-optic modulators serving as beam choppers. This facilitates switching between ionization and cooling cycles. The fluorescence light at wavelength 369 nm is filtered and imaged onto a single-photon sensitive camera (EMCCD). The Yb$^+$ ions themselves are produced by isotope selective photo-ionization from a stream of neutral Yb atoms. The latter are evaporated from a resistively heated oven containing Yb foil. The photo-ionization scheme from neutral Yb to Yb$^+$ is explained in [26, 27].

As a first step we investigated the $5d_{3/2} \rightarrow 7p_{1/2}$ transition by applying the laser at 245 nm wavelength and monitoring the fluorescence from the $6s_{1/2} \rightarrow 6p_{1/2}$ cooling transition. For this purpose all involved laser powers were chosen such that an effective saturation parameter $s_{245} \approx 0.02$ was reached when compared to the signal levels obtained from the theoretical model described above. The laser at 935 nm is switched off in these experiments, while the 245 nm beam now serves as a repumper closing the cooling cycle.

The result of a spectral scan of the 245 nm laser across the $5d_{3/2} \rightarrow 7p_{1/2}$ transition is displayed in figure 3. From the fluorescence counts as a function of the laser frequency we extract a center wavelength of 245.426 nm. In all subsequent ionization experiments the ionization laser was tuned to this

Figure 2. Scheme of the ion-trap setup and laser-configuration for the ionization experiments. All lasers except the laser at the wavelength 245 nm are applied at a strongly saturated power-level. The lasers at 245 and 935 nm are alternated using acousto-optic modulators acting as beam choppers. The ion motion is probed utilizing one of the auxiliary electrodes.

Figure 3. Scan of the $5d_{3/2} \rightarrow 7p_{1/2}$ transition at 245 nm with a saturation parameter of $s_{245} \approx 0.02$. The fluorescence from the decay $6p_{1/2} \rightarrow 6s_{1/2}$ at 369 nm is detected. The repumping lasers at 638 and 976 nm are each applied at a strongly saturating power-level.
line center. From the spectral width of the fluorescence curve a value of $\tau = 13.5 \pm 2.1$ ns is found for the lifetime of the 7$p_{1/2}$ level. No other experimentally obtained values for this lifetime and wavelength could be found. The theoretical data in [18] gives $\tau = 24.6$ ns, whereas a value of $\tau = 10$ ns can be derived from the data in [28].

The successful ionization in the experimental runs is verified utilizing a crystallized pair of ions, similar to what has been done in [11, 12]. Initially a pair of $^{174}\text{Yb}^+$ ions is loaded and laser-cooled into a crystallized state. Following to this step the photoionization process is started by applying the laser-radiation according to the scheme shown in figure 1. During this period the laser at 935 nm is switched off. Afterwards the outcome of this photoionization attempt is probed using the laser-cooling scheme with the repumping-laser at 935 nm switched on and the laser at 245 nm switched off. In the experiment this sequence has been automated using accousto-optical modulators with a switching rate of typically 50 Hz.

Once one of the two ions is turned dark the photoionization sequence is interrupted and the remaining crystal consisting of a ‘dark’ and a ‘bright’ ion is analyzed. At this point the dark ion remains sympathetically cooled by the laser cooled bright ion. A first indication for the successful ionization of the dark ion can be found in the bright ion’s position shift, see also [11, 12]. A doubly charged dark ion will be trapped closer to the trap-center pushing the singly charged bright ion further off. This situation is shown in figure 4.

The extent of this displacement can be calculated for an arbitrary trapping potential. A two-ion crystal aligns along the weakest axis of the radio-frequency (RF) ion-trap secular potential. The one-dimensional model potential for this situation is given by

$$ V = \sum_{l=1,2} \frac{1}{2} M \left(2\pi\nu_l\right)^2 x_l^2(t) + \frac{Q}{|x_1(t) - x_2(t)|}, $$

with the ion mass $M$, the secular-motion frequencies $\nu_l$ of ion $l$, the factor $Q = q_1 q_2 / 4 \pi \varepsilon_0$ and the charges $q_{1,2}$ of the ions. In the following the ion with index 1 will be assumed to be the singly charged bright ion.

We define

$$ \eta = \frac{\nu_2}{\nu_1}. $$

A value of $\eta = 2$ would be expected for a singly and a doubly charged ion if the trapping potential is only constituted of the RF field. This changes in the presence of static electric fields. Then, due to the difference in the scaling of the resulting forces onto a particular ion charge the value of $\eta$ will differ from 2.

The equilibrium positions $X_1, X_2$ for each of the two ions are found to be:

$$ X_1^3 = \frac{Q}{\left(1 + \eta^{-2}\right)^2 \omega_1^2}, $$

$$ X_2 = -\eta^{-2} X_1. $$

with $\omega_1 = 2\pi \nu_1$. The plot in figure 5 shows the normalized position of the remaining bright ion as a function of $\eta$.

In the first ionization attempts the laser at 245 nm was applied at power levels much higher than 100 $\mu$W. As suggested by equation (1) and also by intuition, larger powers should result in larger ionization rates. Contrary to these expectations we could not observe successful ionization but a spatial shift of the ions or even ion loss instead. These effects can be attributed to strong charging of non-conducting materials in the vacuum chamber induced by the deep-ultraviolet laser. For an investigation of charging effects on trapping performance we refer to [29]. Hence, the experiments
were continued with the ionization laser operating at low power levels.

Fluorescence images acquired before and after a presumably successful ionization attempt are displayed in figure 4. By averaging over a series of such images we obtain the ions’ positions as well as the center of the trapping potential. From these data we deduce a normalized displacement of $XX_{11} = (1.74 \pm 0.04)_h$, where $X_{11}$ and $X_{1}$ are the distances of the bright $Yb^+$-ion to the trap center after and before the ionization attempt, respectively. Within the error margins this displacement is compatible with the expectation for a $Yb^+$–$Yb^{2+}$ crystal, see figure 5. A first estimate for the parameter $\eta$ based on this result yields $\eta = 2.07$, while even larger deviations from $\eta = 2$ are possible within the error margins. As mentioned above this deviation from $\eta = 2$ hints towards the influence of static electric fields onto the trapping potential.

The results found above are limited in accuracy by the discretization of the camera images. For evaluating $\eta$ more accurately and validating the ionization success the motional eigenfrequencies of the crystal are investigated, as has been also done in [3, 11]. For the potential in equation (3) the two crystal-eigenfrequencies can be found to be

$$\nu_{\text{com}} = \nu_1 \cdot \frac{\eta^4 + 6\eta^2 - \sqrt{\eta^8 + 14\eta^4 + 1} + 1}{2\eta^2 + 2}$$

(7)

$$\nu_{\text{bre}} = \nu_1 \cdot \frac{\eta^4 + 6\eta^2 + \sqrt{\eta^8 + 14\eta^4 + 1} + 1}{2\eta^2 + 2}.$$  

(8)

The ‘com’-mode represents the eigenmode where both ions oscillate in phase, whereas in the ‘breathe’-mode both ions oscillate exactly out of phase. The plots in figure 6 show the dependence of these frequencies on the parameter $\eta$.

In the experiment these crystal-eigenfrequencies are probed by applying a corresponding RF-electric field using auxiliary electrodes [30]. Figure 7 shows images of the ions’ fluorescence during excitation of the two motional modes along the weak trap axis. Initially a value of $\nu_{\text{com}}(\eta = 1) = 474 \text{ kHz}$ had been found for the crystal with two singly charged ions. The excitation of the ‘com’-mode of the mixed-species crystal gives $\eta = 2.13$. From the excitation of the same crystal’s ‘breathe’-mode $\eta = 2.14$ is found. The appearance of $\eta \approx 2$ for both modes already hints towards a successful photoionization of one of the two $Yb^+$ ions.

As a last step, we combine the results of the measurements on the spatial shift of the remaining bright ion and the motional frequencies. Using equation (5), we determine $q_x$ for the dark ion from $(X_1(\eta)/X_1(1))^3$ and the average value of $\eta = 2.135$ as found from the motional dynamics of the ion crystal. This procedure yields $q_x = (1.96 \pm 0.14)e$ in good agreement with a doubly charged ion.

4. Discussion and outlook

The above analysis clarifies that the remaining dark ion is indeed a doubly charged ytterbium ion. Even the quantitative agreement of $\eta$ for both excited crystal modes alone leads to
the same conclusion, since obviously other combinations of \( \{\nu_s, \nu_f\} \) can be ruled out by the observation of the fluorescence from the remaining Yb\(^+\) ion. The slight deviation from \( \eta = 2 \) can be attributed to the influence of static electric fields present in the experiment. This kind of effect is expected to occur in a stylus-type ion-trap system where the trapped ions are widely exposed to their environment.

In the experiment as described here the photoionization process itself takes approximately one second. Using a laser power of approximately 100 \( \mu \)W for the ionization laser the process could be run with near unit efficiency. For this laser power, the time span of one second until successful ionization as stated above constitutes an upper limit, since the duty cycle for employing the ionization lasers is 50\%. Therefore the ionization rate is on the order of the one expected from equation (2). At larger powers two effects could be observed: The trapping of the ions is disturbed by the disturbance of the electric trapping potential from strong photo-electric charges and, occasionally, dark states of singly charged ytterbium are produced. The mechanism for the production of these dark states could not be identified yet.

Since the doubly charged species is now readily available, spectroscopy experiments on the Yb\(^{2+}\) resonance \( ^1S_0 \rightarrow ^3P_1 \) transition at a wavelength of 252 nm can be initiated. As discussed in [3] this is complicated by the branching of the \( ^3P_1 \) level towards possible dark-states. Hence, the next experimental steps include the spectroscopy on possible repumping transitions, with the most likely candidate being the \( ^3P_2 \rightarrow ^3P_1 \) transition at 1578 nm wavelength.

Furthermore, measurements of the life-time of the \( ^3P_1 \) state obtained for Yb\(^{2+}\) ions in plasma [9] indicate a maximum possible rate of fluorescence photons of about \( 2 \times 10^6 \) s\(^{-1}\) on full saturation. Therefore, trapping Yb\(^{2+}\) ions in a parabolic-mirror trap system providing high photon-collection efficiency [5] is expected to support this task.

Finally, we aim at using the Yb\(^{2+}\) ions in experiments on efficient light–matter interaction in free space [7], with the ultimate goal of absorbing a single photon with close to unit efficiency.

Acknowledgments

G L thanks J Bergquist for stimulating discussions. M S and G L are grateful to the Deutsche Forschungsgemeinschaft for financial support. G L acknowledges financial support from the European Research Council under the Advanced Grant ‘PACART’.

References

[1] Dzuba V A, Flambaum V V and Marchenko M V 2003 Phys. Rev. A 68 022506
[2] Dzuba V A and Flambaum V V 2008 Phys. Rev. A 77 012515
[3] Schauer M M, Danielson J R, Feldbaum D, Rahaman M S, Wang L B, Zhang J, Zhao X and Torgerson J R 2010 Phys. Rev. A 82 062518
[4] Sondermann M, Maiwald R, Konermann H, Lindlein N, Peschel U and Leuchs G 2007 Appl. Phys. B 89 489–92
[5] Maiwald R, Golla A, Fischer M, Bader M, Heugel S, Chalopin B, Sondermann M and Leuchs G 2012 Phys. Rev. A 86 043431
[6] Golla A, Chalopin B, Bader M, Harder I, Mantel K, Maiwald R, Lindlein N, Sondermann M and Leuchs G 2012 Eur. Phys. J. D 66 100
[7] Fischer M, Bader M, Maiwald R, Golla A, Sondermann M and Leuchs G 2014 Appl. Phys. B 117 797–801
[8] Campbell C, Steele A, Churchill L, DePalatis M, Nayler D, Matsukevich D, Kuzmich A and Chapman M 2009 Phys. Rev. Lett. 102 233004
[9] Zhang Z G, Li Z S, Svanberg S, Palmeri P, Quinet P and Biémont E 2001 Eur. Phys. J. D 15 301–5
[10] Gruber L, Holder J, Steiger J, Beck B, DeWitt H, Glassman J, McDonald J, Church D and Schneider D 2001 Phys. Rev. Lett. 86 636–9
[11] Feldker T, Pelzer L, Stappel M, Bachor P, Kolbe D, Walz J and Schmidt-Kaler F 2014 Appl. Phys. B 114 11
[12] Kwapien T, Eichmann U and Sandner W 2007 Phys. Rev. A 75 063418
[13] Olmschenk S, Youngc K C, Moehring D L, Matsukevich D N, Mauz P and Monroe C 2007 Phys. Rev. A 76 052314
[14] Biémont E, Palmeri P and Quinet P 2000 A new database of astrophysical interest Toward a New Millennium in Galaxy Morphology ed D Block et al (Dordrecht: Springer) pp 635–7 ISBN 978–94–010–5801–8
[15] Taylor P, Roberts M, Gateva-Kostova S V, Clarke R B M, Barwood G P, Rowley W R C and Gill P 1997 Phys. Rev. A 56 2699
[16] Roberts M, Taylor P, Barwood G P, Gill P, Klein H A and Rowley W R C 1997 Phys. Rev. Lett. 78 1876
[17] Gill P, Klein H A, Levick A P, Roberts M, Rowley W R C and Taylor P 1995 Phys. Rev. A 52 909–12
[18] Biémont E, Palmeri P and Quinet P 2012 D.r.e.a.m., database on rare earths at Mons University (http://w3.umons.ac.be/astro/dream.shtml)
[19] Seaton M 1958 Mon. Not. R. Astron. Soc. 118 504
[20] Burgess A and Seaton M 1958 Rev. Mod. Phys. 30 992–3
[21] Peach G 1967 Mem. R. Astron. Soc. 71 13
[22] Ralphenko Y, Kramida A and Reader J 2011 N A T 2012 NIST atomic spectra database (ver. 4.1.0) (online), (http://physics.nist.gov/asd)
[23] Huang W, Xu X, Xu C, Xue M and Chen D 1995 J. Opt. Soc. Am. B 12 961–3
[24] Maiwald R, Leibfried D, Britton J, Bergquist J C, Leuchs G and Wineland D J 2009 Nat. Phys. 5 551–4
[25] Bell A S, Gill P, Klein H A, Levick A P, Tam C and Schnier D 1991 Phys. Rev. A 44 20–3
[26] Hosaka K, Webster S A, Blythe P J, Stannard A, Beaton D, Margolis H S, Lea S N and Gill P 2005 IEEE Trans. Instr. Meas. 54 759–62
[27] Balzer C, Braun A, Hannemann T, Pape C, Etlinger M, Neuhauser W and Wunderlich C 2006 Phys. Rev. A 73 041407
[28] Fawcett B and Wilson M 1991 At. Data Nucl. Data Tables 47 241 – 317
[29] Harlander M, Brownmatt M, Hänsel W and Blatt R 2010 New J. Phys. 12 093035
[30] Naegeler H, Blatt R, Eschner J, Schmidt-Kaler F and Leibfried D 1998 Opt. Express 3 89–96