Measurement of CP Asymmetries in $B^0 \rightarrow \phi K^0$ and $B^0 \rightarrow K^+ K^- K^0_s$ Decays

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(The BABAR Collaboration)
We measure the time-dependent CP asymmetry parameters in $B^0 \to K^+K^-K^0$ based on a data sample of approximately 227 million $B$-meson pairs recorded at the $\Upsilon(4S)$ resonance with the BABAR detector at the PEP-II $B$-meson Factory at SLAC. We reconstruct two-body $B^0$ decays to $\phi(1020)K_s^0$ and $\phi(1020)K_l^0$, and the three-body decay $K^+K^-K^0_S$ with $\phi(1020)K_l^0$ excluded. For the $B^0 \to \phi K^0_S$ decays, we measure $\sin 2\beta_{\text{eff}}(\phi K^0) = +0.50 \pm 0.25$ (stat)$^{+0.05}_{-0.06}$ (syst). The $B^0 \to K^+K^-K^0_S$ decays are dominated by $K^+K^-$ S-wave, as determined from an angular analysis; we measure $\sin 2\beta_{\text{eff}}(K^+K^-K^0_S) = +0.55 \pm 0.22$ (stat) $\pm 0.04$ (syst) $\pm 0.11$ (CP), where the last error is due to the uncertainty in the fraction of CP-even contributions to the decay amplitude. We find no evidence for direct CP violation.

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In the Standard Model (SM) of particle physics, the decays $B^0 \to K^+K^-K^0$ are dominated by $b \to s\bar{s}s$ gluonic penguin amplitudes, but can also be affected by amplitudes that are suppressed by elements of the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix. These CKM-suppressed amplitudes cannot be precisely known in a model-independent way, but are in general expected to be small. Let $2\beta_{\text{eff}}$ be the CP-violating phase difference between decays with and without mixing, and $\beta = \arg (V_{td}V_{td}^*)$ where $V_{td}$ are elements of the CKM quark mixing matrix. The difference $|\beta - \beta_{\text{eff}}|$ is expected to be nearly zero, with theoretical uncertainties of a few degrees for $B^0 \to \phi K^0$. Larger uncertainties exist for $B^0 \to K^+K^-K^0_S$ with $B^0 \to \phi K^0_S$ decays excluded, due in part to an extra CKM-suppressed tree amplitude contribution.

Since additional decay diagrams with non-SM particles and interactions introducing new CP-violating phases may contribute to $\beta_{\text{eff}}$, measurements of $\sin 2\beta_{\text{eff}}$ in these channels and their comparisons with the SM expectation are sensitive probes for physics beyond the SM. The value of $\sin 2\beta$ has been measured in $B^0 \to J/\psi K^0$ with an average of 0.742$\pm$0.037. The BABAR and Belle collaborations have measured $\sin 2\beta_{\text{eff}}$ in $\phi K^0$ $(+0.47 \pm 0.34^{+0.08}_{-0.06}$ with 114 million $B\bar{B}$ pairs) and $-0.96 \pm 0.50^{+0.09}_{-0.11}$ with 152 million $B\bar{B}$ pairs (without $\phi K^0_S$ only), respectively, and in $K^+K^-K^0_S$ excluding $\phi K^0$ $(+0.57 \pm 0.26 \pm 0.04^{+0.17}_{-0.06}$ with 122 million $B\bar{B}$ pairs) and $+0.51 \pm 0.26 \pm 0.05^{+0.18}_{-0.00}$ with 152 million $B\bar{B}$ pairs, respectively.

At $B$ factories, the neutral $B$ mesons are exclusively produced in pairs. We select events for which one $B$ ($B_{\text{rec}}$) is reconstructed as $B^0 \to K^+K^-K^0$ and the other ($B_{\text{tag}}$) is partially reconstructed as either $B^0$ or $\bar{B}^0$. We define $\Delta t = t_{\text{rec}} - t_{\text{tag}}$ to be the difference between the proper decay times of the $B$ mesons. The decay rate $f_+(f_-)$ for the final state $J$ when the $B_{\text{tag}}$ decays as a $B^0$ ($\bar{B}^0$) is given by

$$f_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} [1 \pm S_J \sin (\Delta m \Delta t) \mp C_J \cos (\Delta m \Delta t)],$$

where $\tau_{B^0}$ is the $B^0$ lifetime and $\Delta m$ is the $B^0-\bar{B}^0$ mixing frequency. The parameter $S_J$ is non-zero if there is CP violation in the interference between decays with and without mixing, while a non-zero value for $C_J$ would entail direct CP violation. In the limit where the CKM-suppressed amplitudes do not contribute, the SM predicts no direct CP violation ($C_J = 0$) since the dominant decay amplitudes have the same CP-violating phase, and that $S_J = -\eta_f \times \sin 2\beta_{\text{eff}}$. For $B^0 \to \phi K^0_S$ decays, the effective eigenvalue $\eta_f = -1$, for $B^0 \to \phi K^0_L$ $\eta_f = +1$. For $B^0 \to K^+K^-K^0_S$ decays, $\eta_f = 2f_{\text{even}} - 1$, where $f_{\text{even}}$ is the fraction of CP-even contributions to the $B^0 \to K^+K^-K^0_S$ amplitude. Then the value of $\eta_f$ depends on the angular momentum of the $K^+K^-$ system: it is -1 for relative $P$-wave and +1 for $S$-wave.

In this paper, we present a measurement of $\sin 2\beta_{\text{eff}}$ with almost twice the number of events as for the previous BABAR results. We reconstruct $B^0$ candidates in two independent modes, $\phi K^0$ (with the $K^0$ either a $K^0_s$ or a $K^0_L$) and $K^+K^-K^0_s$ (with the $\phi$ mass region excluded). $K^0_s$’s are detected via their $\pi^+\pi^-$ decay only. We extract the CP asymmetry parameters using extended maximum-likelihood fits. Using an angular moment analysis, we extract the $K^+K^-$ $P$-wave fractions in the data. These fractions are used to check the assumption that $\eta_f = -1$ for $\phi K^0_s$ and $+1$ for $\phi K^0_L$ by bounding the $S$-wave contamination in the $\phi$ mass region, and to measure $\eta_f$ for $K^+K^-K^0_S$. This analysis is based on 227 million $B\bar{B}$ pairs collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ storage rings at SLAC, operating at the $\Upsilon(4S)$ resonance (center-of-mass (c.m.) energy $\sqrt{s} = 10.58$ GeV). In Ref. we describe the silicon vertex tracker (SVT) and drift chamber (DCH) used for track and vertex reconstruction, the electromagnetic calorime-
ter (EMC) and instrumented flux return (IFR) used for \( K^0_s \) reconstruction, and the detector of internally reflected Cherenkov light (DIRC), which, together with the EMC, the IFR, and the ionization \( dE/dx \) from the SVT and DCH, is used for particle identification.

The \( B^0 \)-candidate reconstruction and selection is similar to that described in Refs.\(^5\) and \(^11\). We consider a \( K^+K^- \) pair to be a \( \phi \) candidate if its invariant mass is within 15 MeV/\( c^2 \) (about three times the apparent width in the \( K^+K^- \) invariant mass spectrum) of the central \( \phi \) mass value \(^13\). For a given \( B \rightarrow \phi \) meson pair, we obtain \( \Delta t \) from the measured distance between the fully reconstructed \( B_{\text{rec}} \) meson decay point and the \( B_{\text{tag}} \) decay point along the beam direction, and the known boost of the \( \Upsilon(4S) \) system (\( \beta^2 = 0.56 \)). A multivariate tagging al-
gerument {\[ \phi \]} algorithm determines the flavor of the \( B_{\text{tag}} \) meson \(^8\) and classifies it in one of seven mutually exclusive tagging categories.

We use two kinematic variables to discriminate between signal \( B \) decays and combinatorial background. The energy difference between the measured \( e^+e^- \) c.m.
energy of the \( B \) candidate and \( \sqrt{s}/2 \) is \( \Delta E \). Its distribution peaks at zero for signal, with a width of about 20 MeV for \( \phi K^0_S \) and \( K^+K^-K^0_S \). The width is only about 3 MeV for \( \phi K^0_S \), because for this mode we constrain the \( B^0 \) candidate’s mass to the nominal value \(^13\). The beam-energy-substituted mass, \( m_{\text{ES}} \), is used for candidates without a \( K^0_S \). It is defined as \( m_{\text{ES}} = \sqrt{(s/2 + p_B \cdot p_B - E_i^2 - p_B^2)} \), where the \( B \) momen-
tum \( p_B \) and the four-momentum of the initial state \( (E_i, p_i) \) are defined in the laboratory frame. It peaks at the \( B^0 \) mass for signal, with a width of about 3 MeV.

For \( \phi K^0_S \) candidates, we require \( \Delta E < 100 \) MeV and \( m_{\text{ES}} > 5.21 \) GeV/\( c^2 \); for \( K^+K^-K^0_S \) candidates, we require \( \Delta E < 80 \) MeV; and for \( K^+K^-K^0_S \) candidates, we require \( \Delta E < 200 \) MeV and \( m_{\text{ES}} > 5.2 \) GeV/\( c^2 \).

The dominant background is continuum \( e^+e^- \rightarrow q\bar{q} \) events; these tend to be jet-like in the \( e^+e^- \) c.m. frame, while \( B \) decays tend to be spherical. To enhance discrimination between signal and continuum, we use Fisher discriminants \( (\mathcal{F}) \) to combine four event-shape-related variables \(^3\) \(^11\). The other background originates from \( B \) decays. For the \( \phi K^0 \) final state, opposite-\( CP \) contributions from the \( K^+K^-K^0 \) fin-

nal state \( (K^+K^-S \text{-wave}) \) are estimated from data with a moment analysis \(^11\) (see below) to be less than 6.6% at 95% confidence level. The mode \( \phi K^0 \) has additional background. Its dominant \( CP \) contamination is from the mode \( \phi K^0 \rightarrow \phi K^0\pi^0 \), for which we expect approximately eight events in the region \( \Delta E < 10 \) MeV. In the final likelihood fit we explicitly parameterize backgrounds from \( B \) decays both with and without charm.

For the \( K^+K^-K^0_S \) mode, we apply invariant mass cuts to suppress background from \( B \) decays that proceed through a \( b \rightarrow c \) transition, namely those containing \( D^0, J/\psi, \chi_{c0}, \) or \( \psi(2S) \) decaying into \( K^+K^- \), or \( D^+ \) or \( D_S^+ \) decaying into \( K^+K^0_S \). Finally, to suppress \( B \) decays into final states with pions, we require the rate for a charged pion to be misidentified as a kaon to be less than 2%.

A total of 4,300, 8,238, and 27,368 events have a candidate that passes the \( \phi K^0_S, \phi K^0_{L}, \) or \( K^+K^-K^0_S \) selection criteria, respectively. From simulation, we find the final selection efficiencies for signal to be 40%, 20% and 26%, respectively.

We extract the \( K^+K^-K^0_S \), \( \phi K^0_S \), and \( \phi K^0_{L} \) event yields and \( CP \) parameters with two extended maximum-likelihood fits. One is to the \( K^+K^-K^0_S \) candidates; the other is to both the \( \phi K^0_S \) and \( \phi K^0_{L} \) candidates, with the assumption that \( C_{\phi K^0_S} = C_{\phi K^0_{L}} \) and \( S_{\phi K^0_S} = -S_{\phi K^0_{L}} \). We verified the fit procedure for the \( \phi K^0 \) mode with samples of \( \phi K^+ \) and \( J/\psi K^0 \) events. We found for the former a null asymmetry as expected, and for the latter results that are consistent with previous measurements \(^9\). We verified the fit procedure for the \( K^+K^-K^0_S \) mode with a sample of \( K^0_SK^0_S \) \( K^+ \) events, for which we found a null asymmetry as expected.

The likelihood function used in each extended maximum-likelihood fit to its \( N_k \) candidates tagged in category \( k \) is

\[
\mathcal{L}_k = e^{-N_k} N_k \sum_{i=1}^{n_B} \left( \epsilon_{i,k} P_{i,k}^S + N_{C,k} P_{i,k}^C + \sum_{j=1}^{n_B} N_{B,j} \epsilon_{j,k} P_{j,k}^B \right)
\]

where \( N_k \) is the sum of the signal, continuum, and \( n_B \) \( B \)-background yields tagged in category \( k \); \( N_S \) is the number of \( \phi K^0_S \), \( \phi K^0_{L} \), or \( K^+K^-K^0_S \) signal events; \( \epsilon_k \) is the fraction of signal events tagged in category \( k \); \( N_{C,k} \) is the number of continuum background events tagged in category \( k \); \( N_{B,j} \) is the number of \( B \)-background events of class \( j \); and \( \epsilon_{j,k} \) is the fraction of \( B \)-background events of class \( j \) tagged in category \( k \). Each \( B \)-background class comprises similar \( B \) decays. The \( B \)-background event yields are fixed parameters and are zero for the \( \phi K^0_S \) sample. The total likelihood \( \mathcal{L} \) is the product of the likelihoods for each tagging category.

The probability density functions (PDFs) \( P^S_{i,k}, P^C_{i,k}, \) and \( P^B_{j,k} \), for signal, continuum background, and \( B \)-background class \( j \), respectively, are the products of the PDFs of the discriminating variables. The signal PDF is thus given for the \( K^+K^-K^0_S \) sample by \( \mathcal{P}(m_{\text{ES}}) \cdot \mathcal{P}(\Delta E) \cdot \mathcal{P}(\mathcal{F}) \cdot \mathcal{P}(\Delta t; \sigma_{\Delta t}) \), for the \( \phi K^0_S \) sample by \( \mathcal{P}(m_{\text{ES}}) \cdot \mathcal{P}(\Delta E) \cdot \mathcal{P}(\mathcal{F}) \cdot \mathcal{P}(m_{K^0}) \cdot \mathcal{P}(\cos \theta_H) \cdot \mathcal{P}(\Delta t; \sigma_{\Delta t}) \), and for the \( \phi K^0_{L} \) sample by \( \mathcal{P}(\Delta E) \cdot \mathcal{P}(\mathcal{F}) \cdot \mathcal{P}(m_{K^0}) \cdot \mathcal{P}(\cos \theta_H) \cdot \mathcal{P}(\Delta t; \sigma_{\Delta t}) \), where \( \theta_H \) is the angle between the \( K^+ \) candidate and the parent \( B_{\text{rec}} \) flight direction in the \( K^+K^- \) rest frame. The quantity \( \sigma_{\Delta t} \) is the uncertainty in the measurement of \( \Delta t \) for a given event. The time-dependent \( CP \) parameters defined in Eq. \(^3\), diluted by the effects of mistagging and the \( \Delta t \) resolution, are contained in \( P^C_{\Delta t}(\Delta t; \sigma_{\Delta t}) \). As in our \( J/\psi K^0 \) analysis \(^2\), the \( \Delta t \)-resolution function for signal and \( B \)-background
events is a sum of three Gaussian distributions, which have two distinct means as well as three distinct widths. The widths are the error of the measured $\Delta t$ scaled by three independent factors.

In the fits to data, we leave unconstrained the parameters describing the CP asymmetry, the $\Delta t$-resolution functions, the tagging characteristics, and the event yields. We also leave unconstrained the means of the signal $m_{EE}$ and $\Delta E$ Gaussian PDFs, the widths of the signal $\Delta E$ PDFs, the mean of the signal $m_{KK}$ PDF (which is parameterized by a relativistic $P$-wave Breit-Wigner function), and all parameters of the $K^+K^-K^0$ candidates’ signal PDF for $F$. We take from simulation any other parameters of the $m_{EE}$, $\Delta E$, $F$, $\cos \theta_H$, and $m_{KK}$ PDFs for signal and $B$ background. The parameters describing the signal and $B$-background $\Delta t$-resolution function are determined by a simultaneous fit to an independent sample of reconstructed $\bar{B}^0$ decays to flavor eigenstates, with more than 100,000 events \cite{ref}. We use the world-averaged values for $\tau_{\bar{B}^0}$ and $\Delta m_d$ \cite{ref2}. The fits to the $\phi K^0$ and $K^+K^-K^0$ candidates have a total of 35 and 34 free parameters, respectively.

We use an angular moment analysis based on the $\cos \theta_H$ distribution to extract the $K^+K^-K^0$ CP content, and also to bound the $S$-wave contamination in the $\phi$ mass region. In this approach, we expand the decay distribution for a given $K^+K^-$ invariant mass in terms of moments $\langle P_\ell \rangle$ of conveniently normalized Legendre polynomials $P_\ell(\cos \theta_H)$:

$$|A(m_{KK})|^2 = \sum_\ell \langle P_\ell \rangle \cdot P_\ell(\cos \theta_H),$$  \hspace{1cm} (3)

where $A(m_{KK})$ is the mass-dependent decay amplitude. We normalize $P_\ell(\cos \theta_H)$ such that the integral of $P_\ell(\cos \theta_H)^2$ over $\cos \theta_H$ from $-1$ to $1$ equals unity. We extract the moments by summing over all events:

$$\langle P_\ell \rangle = \sum_j P_\ell(\cos \theta_{H,j}) \, W_j / \varepsilon_j,$$  \hspace{1cm} (4)

where $W_j$ is the weight for event $j$ to belong to the signal decay and is calculated by the sPlot technique of Ref.\cite{ref3}. The efficiency $\varepsilon_j$ is evaluated from a large MC sample in bins of $m_{KK}$ and $\cos \theta_H$. Limiting ourselves to the two lowest partial waves, we can write the total decay amplitude in terms of the $S$-wave ($CP$-even) and the $P$-wave ($CP$-odd) amplitudes,

$$A(m_{KK}) \approx A_S(m_{KK})P_0(\cos \theta_H) + e^{i\phi_p}A_P(m_{KK})P_1(\cos \theta_H),$$ \hspace{1cm} (5)

where $\phi_p$ is the relative phase between the real partial-wave amplitudes $A_S(m_{KK})$ and $A_P(m_{KK})$. If we compare Eq. \cite{ref4} to Eq. \cite{ref5}, we can relate the moments (of order $\ell \leq 2$) to the wave intensities and thus to the total fraction of $CP$-even events, $f_{\text{even}}$, as

$$f_{\text{even}} = \frac{A_S(m_{KK})^2}{A_S(m_{KK})^2 + A_P(m_{KK})^2} + 1 - \sqrt{\frac{5}{4}}\langle P_2 \rangle,$$ \hspace{1cm} (6)

where $A_S(m_{KK})^2$ and $A_P(m_{KK})^2$ are the $S$- and $P$-wave intensities, respectively. In the normalization, the total number of signal events is $\sqrt{2}\langle P_0 \rangle$.

**TABLE I:** Systematic uncertainties on the CP parameters.

| Source          | $S_{0K}$ | $C_{0K}$ | $S_{KKK}$ | $C_{KKK}$ |
|-----------------|---------|---------|----------|----------|
| Detector effects| ±0.02   | ±0.02   | ±0.02    | ±0.01    |
| DCSD            | ±0.01   | ±0.03   | ±0.00    | ±0.03    |
| Fit bias        | ±0.01   | ±0.01   | ±0.02    | ±0.01    |
| $B^0$-$\bar{B}^0$ tagging | ±0.01   | ±0.02   | ±0.00    | ±0.01    |
| $S$-wave contamination | +0.06   | -0.02   | -        | -        |
| Other           | ±0.03   | ±0.02   | ±0.01    | ±0.01    |
| Total           | +1.04   | ±0.05   | ±0.03    | ±0.04    |

Systematic errors on the $CP$-asymmetry parameters are listed in Table I. We account for uncertainties in the $\Delta t$ resolution, the beam-spot position, and the detector alignment. We also estimate errors due to the effect of doubly CKM-suppressed decays (DCSD) of the $B_{\text{tag}}$ \cite{ref7}. The uncertainty due to possible biases in the fit procedure is conservative and includes effects on the $CP$ parameters of correlations among the fit variables, which have been determined with full-detector MC simulations. Uncertainties in the $B^0$-$\bar{B}^0$ tagging efficiency in both signal and background are also included. Finally, we account for errors due to the $CP$ content of the background, uncertainties in the PDF parameterization, and the uncertainties of $\tau_{\bar{B}^0}$ and $\Delta m_d$ \cite{ref8}. For each mode we add the individual contributions in quadrature to obtain the total systematic uncertainty.

We also consider the systematic error due to the $CP$-even fraction of the $K^+K^-K^0$ mode. We do not find evidence for the existence of higher moments $\langle P_\ell \rangle$, $\ell = 3 \ldots 6$, which could arise from intermediate $D$-wave decays into $K^+K^-$ or decays proceeding through an isospin-1 resonance into $K^+K^0$. Nevertheless, we estimate a systematic error from the $D$-wave by examining the $\langle P_2 \rangle$ moment in the $K^+K^-$ mass region (1.1–1.7 GeV/$c^2$, corresponding to the $f_2(1270)$, $a_2(1320)^0$, and $f_2'(1525)$ resonances, and assuming that $\langle P_2 \rangle$ arises only from $D$-wave and $S$-$D$ interference. Since the moment itself is consistent with zero, we assign a systematic error of 4% based on the $\langle P_2 \rangle$ error. We account for the possible presence of $a_0(980)^+$, $a_0(1450)^+$, and $a_2(1320)^+$ in the $K^+K^0$ subsystem (4.6%). We also estimate a bias due to the modeling of the efficiency from MC events (2.5%). We find the total systematic error on $f_{\text{even}}$ to be ±0.06. This leads to a systematic error on $\sin 2\beta_{\text{eff}}$ of ±0.11.
Table I shows the measured CP parameters and yields from the final extended maximum-likelihood fits. Note that when fitting $\sin 2\beta_{\text{eff}}$ for $K^+K^-K^0_{S}$, we constrain $C_{KKK}$ to zero. All yields are consistent with our previously measured branching fractions. Figure 1 shows the signal-enhanced distributions of $m_{ES}$ for $\phi K_S^0$ and $K^+K^-K^0_{S}$ events and of $\Delta E$ for $\phi K_L^0$ events, together with the result from the final extended maximum-likelihood fits after applying a requirement on the ratio of signal likelihood to signal-plus-background likelihood (computed without the displayed variable) to reduce the background. The requirement is chosen to roughly maximize $N_S^2/(N_S + N_C)$ where $N_C$ is the total number of continuum events, and is applied only for the purpose of making these plots. The curves are projections from the likelihood fits for total yield (solid lines), continuum background (short dashed lines), and total background (long dashes in (b) only). The efficiency of the likelihood-ratio cut is (a) 79% for signal and 5% for background, (b) 35% for signal, 16% for background, and 3% for continuum background, and (c) 77% for signal and 5% for background.

For the $K^+K^-K^0_{S}$ final state including the $\phi$ mass region, the distributions of the $S$- and $P$-wave intensities, and the CP-even fraction, as a function of $K^+K^-$ invariant mass, are shown in Fig. 2. The total fraction of CP-even events with the $\phi$ mass region excluded is given in Table II. We successfully verified our value of $f_{\text{even}}$ with a different method that uses the event rates in $B^+ \to K^+K^0_S K^0_S$ and the isospin-related channel $B^0 \to K^+K^-K^0_S$.

To summarize, in a sample of 227 million $B\bar{B}$ meson pairs, we measure the CP content and CP parameters in $B^0$-meson decays into $\phi K^0$, and into $K^+K^-K^0_S$ with the $\phi$ mass region excluded. We determine the fraction of CP-even and CP-odd contributions with an angular analysis. In $B^0 \to K^0$, our values for $\sin 2\beta_{\text{eff}}$ and $C_{\phi K}$ are in good agreement with our previously published values.
FIG. 3: Distributions of $S$- and $P$-wave intensities and CP even fraction as a function of $K^+ K^−$ invariant mass. Notice that the first bin integrates a wider mass range than the φ resonance occupies. Insets show $S$- and $P$-wave intensities in the φ mass region.

ues [8], and the small $S$-wave contamination is treated as a systematic uncertainty. In $B^0 \rightarrow K^+ K^- K^0$, the $K^+ K^-$ system is observed to be dominated by $S$-wave; this, along with the measured value of $\sin 2\beta_{\text{eff}}$, is consistent with previous measurements based on isospin symmetry [9, 10]. Both of our $\sin 2\beta_{\text{eff}}$ values are consistent to within one standard deviation with the value of $\sin 2\beta$ measured in $B^0 \rightarrow c\bar{c}s$ decays [6].

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