HUNTING FOR GIANT CELLS IN DEEP STELLAR CONVECTIVE ZONES USING WAVELET ANALYSIS

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ABSTRACT
We study the influence of stratification on stellar turbulent convection near the stellar surface and at various depths by carrying out three-dimensional, high-resolution hydrodynamic simulations with the Anelastic Spherical Harmonic code. Four simulations with different radial-density contrasts corresponding to different aspect ratios for the same underlying 4 Myr, 0.7 M⊙ pre-main-sequence star model are performed. We highlight the existence of giant cells that are embedded in the complex surface convective patterns using a wavelet and time-correlation analysis. Next, we study their properties, such as lifetime, aspect ratio, and spatial extension, in the different models according to the density contrast. We find that these giant cells have a lifetime larger than the stellar period, with a typical longitudinal width of 490 Mm and a latitudinal extension increasing with the radial-density contrast, surpassing 50° in the thickest convective zone. Their rotation rate is much larger than the local differential rotation rate, also increasing with radial-density contrast. However, their spatial coherence as a function of depth decreases with density contrast due to the stronger shear present in these more stratified cases.

Key words: convection – hydrodynamics – stars: low-mass – stars: pre-main sequence – stars: rotation – turbulence

Online-only material: color figures

1. INTRODUCTION

Most low-mass stars have a deep convective envelope. Understanding its dynamical properties is of fundamental interest to explain the evolution and redistribution of heat, energy, and angular-momentum transport in stars. It is also a key step toward global comprehension of the generation and organization of magnetic fields within these stars. Little is known about the spectral energy distribution (SED) as a function of depth within stellar convective zones. For instance, is there a clear separation between different spatial scales in convective patterns, or can they genuinely manifest themselves at the stellar surface? Unfortunately, the Sun is the only star where we can observe with enough spatial resolution the multi-scale properties of surface convection. Convective scales are considered to range from small-scale granulation patterns with 1–2 Mm diameter cells with a typical lifetime of 5 minutes to intermediate scales such as supergranulation cells with a size of 20–50 Mm and a lifetime of one day to possibly giant cells of a few 100 Mm with a lifetime of several weeks (Nordlund et al. 2009; Rieutord & Rincon 2010).

The origin of such a large range of spatio-temporal scales and the physical processes behind these distinctive scales, possibly including mesogranulation, have been debated for decades. Simon & Weiss (1968) proposed the location of H, He, and metal recombination layers to explain the existence of granules, supergranules, and giant cells. Rast (2003) proposed an advection/fragmentation process in the upper layers of the convective zone to explain mesogranulation and supergranulation from an initial random distribution of downflow plumes. However, a turbulent origin for these convective patterns is currently convincing. Whereas granulation and supergranulation have clear evidence of existence via, respectively, G-band or white-light-intensity observations and time-averaged dopplergrams over 30 minutes or magnetograms, intermediate mesogranulation is still controversial. Rieutord et al. (2000) stressed the spatial and time-filtering biases in correlation-tracking techniques (see Meunier 2006 for a detailed discussion). As far as the giant cells are concerned, they initially were predicted by Simon & Weiss (1968) to be an efficient way of transporting heat over several density scale heights, thanks to the lower superadiabatic temperature gradient necessary for convection over such large scales. But they are very difficult to measure because they are merged within stronger signals such as granulation or supergranulation, which have to be removed correctly. Moreover, looking for such a large-scale signal implies subtracting both the global differential rotation of the Sun and the limb effect and disentangling possible line shifts due to the solar magnetic field in active regions. Evidence for an underlying organization of the convective flows was advocated by Lisle et al. (2004), who found a north–south alignment of supergranulation patterns possibly due to the presence of giant cells. Spectroscopic studies (LaBonte et al. 1981; Scherrer et al. 1986) found cells with a longitudinal extension around 45° and a typical velocity lower than 10 m s⁻¹, but the use of magnetically sensitive Fe i lines could alter these conclusions. Giant cells might also have been detected indirectly in velocity spectra from SOHO/Michelson Doppler Imager (MDI) time-averaged data in Hathaway et al. (2000), with significant power at low spherical-harmonics degrees (l < 30) and a proper rotation rate similar to that of the Sun. Another approach consists of developing full-sphere simulations, resolving only the largest scales of turbulent convection down to supergranulation with mean flows. They can be used as a tool to probe these giant cells and are useful to develop specific proxies to help with observational detection. Most local Cartesian simulations, computed at a higher turbulent degree with detailed radiation processes included, are able to probe the properties of granulation (Stein & Nordlund 1998; Vogler 2005). From the simulations and the observational data, there is a recent trend to bring together the different scales of convection, contrary to the previous paradigm, which identified specific discrete scales for granulation, mesogranulation, and supergranulation as different features of convection. This is well highlighted by looking at the velocity spectrum \( \sqrt{\langle \text{FFT}(V)^2 \rangle} \), which increases linearly with the wave number \( k \) over the broad range of spatial scales presented in Figure 22 of the Nordlund et al. (2009) review.

In preparation for future work dedicated to low-mass young stars presenting much deeper convective zones than the Sun, and
thus possibly more extended giant cells, we try to quantify their identity in our simulations and determine if their properties depend on the size of the convective zone and how they might modify the dynamics of convection by adverting smaller-scale motions. Actually, because the local density scale height decreases much less toward the surface in low-mass, pre-main-sequence (PMS) stars than within the Sun, we can expect a lower horizontal expansion rate of giant cells near the surface. This could imply a potentially larger lifetime due to the weaker distortions by the mean flows. Indeed, these giant cells should also be important in the global stellar-activity cycle, particularly downstream of the convective zone to the stellar surface. Finally, it is also important to note that the equatorial modes associated with the linear growth of convection instability, discussed by Busse & Cuong (1977) and Gilman (1975), could be candidates for these giant cells if they survive in the nonlinear regime when turbulence is fully developed.

In this paper, we present a set of high-resolution, three-dimensional (3D) simulations of a subpart of the whole convective zone for a low-mass young star, assuming various radial-density contrasts. The initial 3D stellar models implemented in our simulations are presented in Section 2 with the numerical methods and boundary conditions used to compute them. Next, the wavelet analysis pipeline developed specifically for this study is detailed in Section 3, with the main methods used to probe the existence of stellar giant cells and their characteristics as the main goal of this paper. Then, we present our results showing the properties of convection in the different models in Section 4, hunt for giant cells and their global properties in Section 5, and conclude in Section 6.

2. THE 3D STELLAR-MODEL SETUP

Four different models with an e-folding radial-density contrast from 13 to 272 are computed. The different main characteristics of the four models computed are presented in Table 1 and discussed in Section 4. A typical run for these simulations lasts 3000 days, i.e., several convective turnover times. The outer boundary condition is common to all models and is located at 0.98 $R_*$ below the stellar surface to keep the anelastic treatment of the Anelastic Spherical Harmonic (ASH) code correct. The inner boundary location for each model is determined to have the chosen density contrast. We restrict the model to very slow rotators equal to the mean solar-rotation rate. However, common low-mass young stellar objects (YSOs) such as CTTS have mean stellar angular velocity as high as 5 $\Omega_*$ and have been studied already by Ballot et al. (2007). Other spectral types of YSOs have been studied by Brown et al. (2008, 2010).

2.1. Equations and Boundary Conditions

The simulations described here were performed with the ASH code. ASH solves the 3D anelastic equations of motion in a rotating spherical geometry using a pseudospectral semi-implicit approach (e.g., Clune et al. 1999; Brun et al. 2004). These equations are fully nonlinear in velocity variables and linearized in thermodynamic variables with respect to a spherically symmetric mean state. This mean state is taken to have density $\bar{\rho}$, pressure $P$, temperature $T$, and specific entropy $S$; perturbations about this mean state are written as $\rho$, $P$, $T$, and $S$. Conservation of mass, momentum, and energy in the rotating reference frame are therefore written as

$$\nabla \cdot (\bar{\rho} \mathbf{v}) = 0, \quad (1)$$

$$\bar{\rho} \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + 2\Omega_\odot \times \mathbf{v} \right) = -\nabla P + \rho g - [\nabla \bar{P} - \bar{\rho} g] - \nabla \cdot \mathbf{D}, \quad (2)$$

$$\bar{\rho} \tilde{T} \frac{\partial S}{\partial t} = \nabla \cdot [\kappa \bar{\rho} c_p \nabla (\bar{S} + T) + \kappa \bar{\rho} \tilde{T} \nabla (\bar{S} + S)] - \bar{\rho} \tilde{T} \mathbf{v} \cdot \nabla (\bar{S} + S) + 2\bar{\rho} \mathbf{v} e_{ij} e_{ij} - 1/3 (\nabla \cdot \mathbf{v})^2 + \bar{\rho} \epsilon, \quad (3)$$

where $c_p$ is the specific heat at constant pressure, $\mathbf{v} = (v_r, v_\theta, v_\phi)$ is the local velocity in spherical geometry in the rotating frame of constant angular velocity $\Omega_\odot = \Omega_\odot \hat{\mathbf{e}}_\phi$, $g$ is the gravitational acceleration, $\kappa$ is the radiative diffusivity, $\epsilon$ is the heating rate per unit mass due to gravitational contraction, and $\mathbf{D}$ is the viscous stress tensor, with components

$$D_{ij} = -2\bar{\rho} \mathbf{v} e_{ij} - 1/3 (\nabla \cdot \mathbf{v}) \delta_{ij}, \quad (4)$$

where $e_{ij}$ is the strain-rate tensor. Here, $\mathbf{v}$ and $\kappa$ are effective eddy diffusivities for vorticity and entropy. To close the set of equations, linearized relations for the thermodynamic fluctuations are taken as

$$\frac{\rho}{\bar{\rho}} = \frac{P}{\bar{P}} - \frac{T}{\bar{T}} = \frac{P}{\gamma P} - \frac{S}{c_p}, \quad (5)$$

assuming the ideal gas law

$$\bar{P} = R \bar{\rho} \tilde{T}, \quad (6)$$

where $R$ is the gas constant. The effects of compressibility on convection are taken into account by means of the anelastic approximation, which filters sound waves that would otherwise severely limit the time steps allowed by the simulation.
For boundary conditions at the top and bottom of the domain, we impose

1. impenetrable walls:
   \[ v_r = 0 | r = r_{bot}, r_{top}, \]

2. stress-free conditions:
   \[ \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) = \frac{\partial}{\partial r} \left( \frac{v_\phi}{r} \right) = 0 | r = r_{bot}, r_{top}, \]

3. and a constant entropy gradient:
   \[ \frac{\partial S}{\partial r} = \text{cst} | r = r_{bot}, r_{top}, \]

Convection in stellar environments occurs over a large range of scales. Numerical simulations cannot, with present computing technology, consider all these scales simultaneously. We therefore seek to resolve the largest scales of nonlinear flows because we are interested in the global transport of heat, energy, and angular momentum in convective envelopes, and particularly the existence of giant cells. We do so within a large-eddy-simulation (LES) formulation, which explicitly follows largescale flows while employing subgrid-scale (SGS) descriptions for the effects of unresolved motions. Here, those unresolved motions are treated as enhancements to viscosity and thermal diffusivity (\( \nu \) and \( \kappa \)), which are thus effective eddy viscosities and diffusivities. For simplicity, we have taken these to be functions of radius alone and have scaled them as the inverse of the square root of the mean density. However, the deepest model, \( \Delta \rho = 272 \), only depends on the inverse of the cubic root of the mean density to avoid overly increasing the horizontal resolution in order to fully resolve the convective patterns near the base of the convective zone. We emphasize that currently tractable simulations are still many orders of magnitude away, in parameter space, from the highly turbulent conditions likely to be found in real stellar convective zones. These large-eddy simulations should therefore be viewed only as indicators of the properties of real flows.

### 2.2. ASH Numerical Methods

To numerically solve the anelastic equations of motion with ASH, we use a pseudospectral method (Glatzmaier 1984; Canuto 1999; Fornberg 1998). The velocity and thermodynamic variables within ASH are expanded in spherical harmonics \( Y_m^\ell(\theta, \phi) \) in the horizontal directions and in Chebyshev polynomials \( T_n(r) \) in the radial direction. Spatial resolution is thus uniform everywhere on a sphere when a complete set of spherical harmonics of degree \( \ell \) is used, retaining all azimuthal orders \( m \). We truncate our expansion at degree \( \ell = \ell_{max} \), which is related to the number of latitudinal mesh points \( N_\theta \) (here, \( \ell_{max} = (2N_\theta - 1)/3 \)), take \( N_\theta = 2N_0 \) azimuthal mesh points, and utilize \( N_r \) collocation points for projection onto the Chebyshev polynomials. A semi-implicit, second-order Crank–Nicholson scheme is used to determine the time evolution of linear terms, whereas an explicit, second-order Adams–Bashforth scheme is employed for the advective and Coriolis terms. The ASH code has been optimized to run efficiently on massively parallel supercomputers, such as the IBM SP-6, SGI Altix, or IBM Blue Gene/P, and has demonstrated excellent scalability on such machines.
HEALPix maps. This corresponds to the parameter Nside in the ring format for HEALPix, where data are organized for shell slices, conversion into the FITS format is done using the ang2pix routine, which allows direct representation of our data in the ring format for HEALPIX, where data are organized for each iso-latitude circle from the north pole to the south pole. From our binary output containing different resolutions, with a dyadic description of the different levels, is useful for developing efficient wavelet-decomposition algorithms and fast statistical analysis for high-resolution maps.

The undecimated, isotropic wavelet transform is used for its efficiency because the sum of all wavelet scales and the coarsest-resolution image provides exactly the original image. The analyzing wavelet $\Psi$ for each spatial scale $n$ is defined by the difference between the two-order, three-box-spline $\Phi$ (whose shape is very similar to a Gaussian), with consecutive cutoff frequencies having a dyadic distribution and starting from the Nyquist frequency defined by $l_c = \frac{\Delta}{2} N_\theta$ for the first scale: $\Psi = \Phi_{\frac{\Delta}{\pi}} - \Phi_{\frac{\Delta}{2\pi}}$. The interested reader can find all the details of such a wavelet transformation in Starck et al. (2006). Wavelet coefficients for each scale highlight different features in the convective patterns; the finest scale details the complex downflow structures, whereas the coarsest one allows the detection of large-scale flows.

For each simulation output file and each chosen depth in the convective zone, the wavelet analysis is performed for seven scales and for each variable studied. An illustration of this wavelet decomposition is presented in Figure 3, which shows the analysis of radial velocity for the shell slice near the stellar surface at $r = 0.97 R_\star$ for the model $\Delta \rho = 100$. At the top left-hand side, the complex convective patterns of such a high-resolution simulation are highlighted by asymmetries between fine downflows and broader upflows. The first two scales correspond to fine structures in the downflow plumes. Scale 3 focuses on the downflow network, whereas large-scale structures are stressed in scales 6 and 7. Scales 4 and 5 correspond to intermediate-scale structures linked to broader upflows of convective cells. Finally, the last scale shows the large-scale structures that could correspond to giant convective shells.

3. THE WAVELET ANALYSIS PIPELINE

3.1. The HEALPIX and MRS Package Applied to Stellar Convection

Because the simulations that we perform correspond to spherical shells, and large-scale structures are embedded among complex patches of convection, especially near the surface, the use of a wavelet decomposition adapted to this spherical geometry is a powerful tool for detecting giant cells. The HEALPIX1 pavement of the sphere into equal-area blocks at different resolutions, with a dyadic description of the different levels, is useful for developing efficient wavelet-decomposition algorithms and fast statistical analysis for high-resolution maps on the full sphere. From our binary output containing different shell slices, conversion into the FITS format is done using the ang2pix routine, which allows direct representation of our data in the ring format for HEALPIX, where data are organized for each iso-latitude circle from the north pole to the south pole. Before applying this transformation, overbinning by a factor of two is applied to fit to the nine levels of resolution of the HEALPIX maps. This corresponds to the parameter $N_{\text{side}} = 2^9 = 512$ and does not modify the initial horizontal resolution of our simulations, which is typically 1024 × 2048 pixels.

Next, the Multi-ReSolution (MRS) package developed by Starck et al. (2006) is used to perform wavelet analysis. The undecimated, isotropic wavelet transform is used for its

![Figure 2. Total (solid curve), convective (dashed curve), and radiative (dotted line) luminosities from the CESAM calculations of our 0.7 $M_\odot$ YSO model at 4 Myr old. These luminosities are normalized with respect to the stellar luminosity. The different inner boundaries for the convective zone of our set of simulations correspond to the vertical dashed lines.](image)

Various methods are used to detect the signal corresponding to the largest scales of convection in our attempt to hunt for giant cells.

1. First, we analyze the amplitude of the large-scale flows by calculating the ratio between the rms radial velocity at scale 3 with that at scale 7. We also distinguish between the signal in upflows and downflows.

2. Next, we perform a time-correlation analysis both on the complete image and on scale 7 to quantify the lifetime of the detected large-scale structures. To do this, we analyze a 60-day time series of longitude--latitude maps of the radial velocity, i.e., about two stellar periods for each model. Then, we calculate the autocorrelation function (acf) as defined below and already used in Miesch et al. (2008) for each lag time $\tau$ with respect to the initial snapshot:

$$\text{acf}(r, \Omega, \tau) = \frac{\int_{0}^{2\pi} \int_{0}^{\pi} \int_{0}^{\Omega} v_i(\tau = 0, \theta, \phi) v_i(\tau, \theta, \phi') \sin \theta d\theta d\phi}{\int_{0}^{2\pi} \int_{0}^{\pi} v_i^2(\tau = 0, \phi) \sin \theta d\theta d\phi}, \quad (7)$$

where the convective patterns are reprojected on the new reference frame $\phi' = \phi - \Omega \tau$, corotating with the chosen tracking velocity $\Omega_t$. The optimal tracking velocity $\Omega_{\text{opt}}$, partly due to the stellar differential rotation, is defined for each latitudinal range $[\theta_1, \theta_2]$ among a large range of values $\Omega_t$ at depth $r$ to maximize the acf.

Then, we define the typical lifetime of large-scale structures as the time $\tau_c$ where the acf is bigger than a defined threshold (typically 0.5 for this study) for each band of latitude.

3. Finally, spatial cross-correlations with depth are performed to probe the radial extension of the largest scales.

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1 Developed by NASA, initially for the analysis of CMB data.
4. CONVECTION PROPERTIES

4.1. General Characteristics of Our Simulations

The main characteristics of our convective models are presented in Table 1. These simulations are done with a Prandtl number equal to one, and the top dissipative coefficients are chosen to have the same value, equal to \(2 \times 10^{12} \text{ cm}^2 \text{ s}^{-1}\). For all four models to ensure the same level of turbulence near the stellar surface. Indeed, all other parameters, such as the density scale height or the radial-velocity profiles (see Figure 6), are the same in the upper layers of our simulations for all models, as we model the same underlying star with different depths for the convective zone. The decrease in the convective Rossby number as the convective zone becomes deeper indicates that the influence of rotation on convection becomes more important. Indeed, thicker convective zones lead to an increase in time for convective flows to move up and down; hence, the Coriolis force has more time to act. The main parameter among these models that makes the most difference is the aspect ratio \(R_\ast/L\), where \(L = r_{\text{top}} - r_{\text{bottom}}\). Its large variation is due to the smooth slope of the density profile of this low-mass young star. This entails a great variation in the Rayleigh and Taylor numbers in our four simulations by more than three orders of magnitude. Finally, we obtain Reynolds numbers calculated from \(v_{\text{rms}}\) speeds in the mid-layer between 100 and 720. But Reynolds numbers based on peaked velocities are around 1500–2000 and are similar for each model, thus yielding one of the most turbulent simulations obtained with ASH until now.

4.2. Turbulent Convective Structures According to the Density Contrast

Even though the models have significantly different \(R_\ast/L\) ratios, this has in appearance little influence on the convective patterns close to the stellar surface, as shown in Figure 4. We clearly see the complexity of the convective patterns, with a network of narrow downflow lanes surrounded by a mosaic of convective upflows. Such a small-scale tiling of our models near the surface makes it difficult to extract information about the potential underlying giant cells from the surface. We also note that case \(\Delta \rho = 13\) possesses slightly smaller convective cells at high latitudes than the three other cases. This is due to the thinness of the convective envelope.
Indeed, provided that the level of turbulence is sufficient (i.e., a fully nonlinear state has been achieved), the small-scale convective patterns are linked to the local density scale height $H_\rho$, which is common to all models and equal to 10 Mm. To be more precise, from the conservation of mass in the anelastic approximation and for typical $H_\rho \ll R_\ast$, we can deduce that the horizontal size $H_L$ of the convective patterns is close to $H_L = \alpha H_\rho$, where $\alpha \approx v_{\text{horiz}}/v_r$ characterizes the anisotropy of the flow. We find that for the $\Delta \rho = 13$ model, $\alpha = 7.5$, which gives $H_L = 75$ Mm at $r = 0.97 R_\ast$. The SED computed in the polar regions is peaked at a spherical-harmonic degree $l = 70$ corresponding to the same $H_L$ as deduced above. This value is slightly larger than the depth of the convective zone (around 60 Mm) for the case $\Delta \rho = 13$. This relationship holds for the other models. We find that the anisotropy factor increases to 10.3 for the $\Delta \rho = 272$ model corresponding to $H_L = 103$ Mm. This difference explains the slightly smaller size of the convective cells at high latitudes for the $\Delta \rho = 13$ model.

By looking at the shape of the convective patterns, it is also important to stress the complex deformation of convective cells by differential rotation everywhere, and particularly in the strong shearing layer at mid-latitude. There is a clear difference between the large convective cells near the equator, always stretched longitudinally, and the more horizontally isotropic convective cells at larger latitudes. The fluctuating temperature structure is also common to all models, with a weak range of variations up to 1.4 K. We also clearly note the good correlation between velocity flows and temperature fluctuations (i.e., cold structures sink). In particular, the stronger convective plumes at the downflow interstices are correlated with intense cold spots in the temperature map. This is typical of high-resolution turbulent convection.

To appreciate the 3D structure of the flow, a 3D rendering of the radial velocity for each model (done with SDVision; see Pomarede et al. 2008) is presented on the last row of Figure 4. These figures clearly show the turbulent nature of the flows in our realized convective models. The convective patterns are similar in all latitudes for the $\Delta \rho = 13$ model, whereas the $\Delta \rho = 100$ and 272 models show convective cells extended through the whole convective zone and parallel to the rotation axis at high latitudes. Convective structures modify their orientation to accommodate for the Coriolis force at the equator. They tend to drift at low latitudes from being purely radial (as imposed by gravity) to being aligned with the rotation axis. This underlines the influence of rotation in these latter models, where $R_o < 1$. The drawing of some stream lines of
the velocity field in these aligned convective cells shows flows that have a spiraling motion within both downflows and upflows parallel to the rotation axis at this high-latitude position, which is vastly different from the simplistic cartoon of convective rolls. We also see the effect of the differential rotation that tilts the convective cells in depth. Convective structures thus form an angle with respect to a pure radial direction both in the \( r \) and \( \theta \) planes. This is at the origin of Reynolds stresses and associated angular momentum.

One can also look at how the energy is transferred radially in the different models after they reach a relaxed state (see Figure 5). The various expressions for each transport process (namely, radiative, kinetic, viscous, or unresolved fluxes) are defined, for instance, in Brun et al. (2004). We focus on the two extreme cases, the others having intermediate properties. First, only 70% of stellar luminosity is reached at the inner edge of the \( \Delta \rho = 272 \) model, going much deeper in the convective zone than the \( \Delta \rho = 13 \) model. This highlights again that the main source of energy in this PMS star is provided by gravitational contraction, as already discussed in Section 2.3, and is a consequence of a radially extended heating source. We note the shape of the unresolved flux \( L_{\text{ed}} \), which is defined such that it increases quickly from \( r = 0.96 R_* \) to unity at the outer-boundary condition localized at \( r = 0.98 R_* \). The transport of energy by viscosity (dashed line) is always negligible in these models. The overluminosity of the outward enthalpy flux (between 130% and 140% of the stellar surface luminosity) characterizing convective flows is due to the asymmetry between the narrow-cooling downflows and the broad, warm upflows in compressible convection, as already illustrated in Figure 4. This yields a strong inward kinetic energy flux and forces the convection to carry up to 40% more than the stellar surface-radiated flux. This radial convective luminosity resulting from our 3D simulations is very different from the initial 1D model presented in Figure 2 that are based on the mixing-length theory (Cox & Giuli 1968).

In our models, turbulent convection is complex, asymmetric, and carries an overluminous flux. As explained in Section 2.3, the radiative luminosity is enhances at the inner boundary to mimic the flux emitted by the unresolved portion of the star.

Further, studying the azimuthally and longitudinally average-rms radial-velocity distribution \( \langle v_{\text{rms}} \rangle \) also provides a clue to the global radial-convective structure and is a means to check that the inner-boundary condition does not disturb and introduce artifacts in our study in the bulk of the convective zone. Figure 6 shows such a radial distribution of \( v_{\text{rms}} \) (also indicating the location of the last shell slices for each density-contrast case). First, the velocity profile close to the stellar surface is common to all models for the shell slices at \( r = 0.97 R_* \), and \( r = 0.95 R_* \) with a maximum averaged radial velocity of 68 m s\(^{-1}\). Second, there is a clear difference between models near the inner-boundary location with respect to the highest density-contrast case (\( \Delta \rho = 272 \)) because the impenetrable boundary condition demands a cancellation of radial velocity. We see that near the bottom, the flow is around 20–30 m s\(^{-1}\) in most cases, except in the \( \Delta \rho = 13 \) model, where it reaches \( \approx 45 \) m s\(^{-1}\) before dropping swiftly to zero.

After having pointed out the global convective organization and the importance of rotation, we now study how the interplay between these two ingredients gives rise to differential rotation (see Figure 7). Differential rotation is dominated by a band of slow/fast jets. They are mostly cylindrical. Differential rotation is retrograde at low latitudes in the low-density-contrast cases and becomes prograde in the stronger-density cases. This change in behavior is linked to the fact that the influence of rotation in the nonlinear regime becomes important for these latter models with \( R_c < 1 \), as already seen in Gilman (1979), Glatzmaier & Gilman (1982), and Browning et al. (2004). Actually, as the convective eddies rise, they are more and more influenced by rotation,
being tilted away from purely radial motions (see also Pedlosky 1987). This leads to a gradual change of the redistribution of angular momentum by convection via Reynolds stresses (see, for instance, Brun & Toomre 2002 and Miesch et al. 2008). The rotation contrast is similar for all models, although a little smaller for high-density-contrast cases. No particular effort has been made to introduce the influence of a tachocline via thermal forcing (Miesch et al. 2006) because we are interested mostly in convective structure and not mean-flow profiles (see Ballot et al. 2007 for a discussion of that effect in young stars).

5. HUNTING FOR GIANT CELLS

After having stressed the existence of large-scale structures in our simulations, we study their spatiotemporal coherence. This should allow us to identify them as giant cells and finally to characterize their 3D structure and properties according to the density contrast, as well as their capacity to transport heat, energy, and angular momentum.

5.1. Extracting the Giant Cell Signal

Figure 3 shows that the large-scale structures are localized mainly at low latitudes (mainly lower than 30°) and that their typical width is around 40°, i.e., 490 Mm. The “potato-like” shape found here for these giant structures differs from the analysis of solar data by Beck et al. (1998), which, on the contrary, found longitudinally extended cells focused at low latitudes.

The amplitude of the large-scale structures increases with contrast density (see Figure 8), especially for the shell slices corresponding to the middle depth of the computed domain of each case, showing the strengthening of the potential underlying convective cells with depth. Near the surface, the signal in the large scale from both upflows and inflows stays quite small (typically 5% of the speed flows at scale 3, as already visible in Figure 3) because the giant cells are disturbed by smaller convective cells there. We also note in Figure 8 the effect of the impenetrable inner-boundary condition, except for the case Δρ = 272, where the deepest shell slice is still far from the
inner boundary. By looking at SED (Figure 9), we also remark that the spectral kinetic-energy distribution at each depth in the low spherical-harmonic-degree (l) range increases with the density contrast by a factor of five between the $\Delta \rho = 13$ and $\Delta \rho = 272$ cases. This is due not only to faster flow in the more stratified case but also to a change of slope in the spectra for the range of $l < 10$. Particularly, there is a maximum at $l = 7$–10 for the $\Delta \rho = 13$ model, which is less pronounced in the stronger density-contrast case. In the analyses of convective instability done by Chandrasekhar (1961), Roberts (1972), and Dormy et al. (2004), the most unstable mode for the full sphere is $l = 1$. As the shell becomes thinner, the most unstable mode drifts toward higher $l$. One can crudely understand this by the fact that convective instability triggers mainly convective rolls with a characteristic scale on the order of the aspect ratio of the shell that corresponds to the mode $l = 14$ in the $\Delta \rho = 13$ model, whereas the characteristic scale in the $\Delta \rho = 272$ model is instead $l = 2$. But we also see that the turbulent nature of our simulations makes it almost impossible to pick up a single $l$ mode, and that is why our wavelet analysis is better suited to finding giant cells that are made of a complex combination of $l$ modes.

Now that we have identified a clear large-scale signal, we wish to analyze its properties. Particularly, we can characterize the temporal coherence of these large-scale structures by trying to track them over long timescales, at least on the order of the stellar-rotation period.

5.2. Time Correlation Analysis

We analyze here the time correlation series for all models near the stellar surface at $r = 0.97 R_\star$. The result is presented in Figure 10. The threshold for the autocorrelation function is fixed at 0.5, corresponding mainly to the transition between the red and green colors on the different plots. For the full images composed of the whole range of scales, the lifetime is very similar for our models and equal to 1.5–2 days. This indicates that the overall density contrast of the models has little influence on the lifetime of convective cells near the surface. This result confirms that surface convection is well controlled by the local density scale height, which is common to all models here with $H_\rho = 10$ Mm. By looking at the correlation only for the largest scale (i.e., scale 7), we emphasize its much greater lifetime, about five-fold longer than that of the small-scale convective patterns. We also remark that the other bands of the autocorrelation function, visible only in the analysis of this large scale, correspond to a “stroboscopic” effect that appears when the chosen tracking velocity differs from the proper angular velocity of these giant cells and is also another means to estimate mean longitudinal width.

We also find that the lifetime of the large-scale structures seems to decrease with the density contrast from $\tau = 60$ days to $\tau = 12$ days. However, on closer inspection, especially for the cases $\Delta \rho = 37$ and $\Delta \rho = 100$, there is a modulation of the autocorrelation function with respect to the lag time that translates to a lifetime of $\tau = 50$ days if we consider a slightly lower threshold of the autocorrelation function.

This conclusion is confirmed by looking at the time–longitude plots (Figure 11), where we can track these large-scale structures in all models, at least for the 60 days studied here, while remarking that they also undergo many distortions in the high-density-contrast cases, which can entail modulations in the amplitude of the acf observed in Figure 10. These modulations can be understood by realizing that the stronger differential rotation achieved in the high-density-contrast models can shear more easily these large-scale structures, thus diminishing their temporal correlation. We can also see that these large-scale convective patterns have a slightly stronger amplitude when the density contrast increases, even near the stellar surface, as already found with the previous $v_{\text{rms}}$ global method as a function of depth (see Section 5.1).

Finally, we can have a look at the evolution of the lifetime of these large structures with respect to depth. Figure 12 shows how the flow is structured both at small and large

Figure 8. Wavelet analysis of the rms radial-velocity amplitude for upflows. Similar results are obtained for downflows.

Figure 9. Spectral kinetic-energy distribution focused on low $l$ modes at different depths for the $\Delta \rho = 13$ (left) and $\Delta \rho = 272$ (right) models. The upper solid line corresponds to $r = 0.97 R_\star$, the dashed one to $r = 0.95 R_\star$, the triple dot-dashed one to $r = 0.92 R_\star$, the dot-dashed one to $r = 0.85 R_\star$, the long dashed one to $r = 0.75 R_\star$, and the bottom solid one to $r = 0.6 R_\star$. 

Figure 10. The Astrophysical Journal, 728:115 (15pp), 2011 February 20 Bessolaz & Brun
Figure 10. Autocorrelation function (acf, see Equation (7)) computed close to the surface ($r = 0.97 R_*$) and for the latitudinal band [$0^\circ$–$20^\circ$], assuming the proper tracking rate $\Omega_t$ for each model. From left to right by increasing density contrast, the acf is computed both for the full (all scales) radial-velocity maps (upper figures) and only the largest scale (lower figures). Note the much larger temporal range for the large-scale analysis.

(A color version of this figure is available in the online journal.)

Figure 11. Time–longitude diagrams at the equator and close to the stellar surface ($r = 0.97 R_*$) for all models with respect to the increasing density contrast downward, without (left) and with (right) the optimal tracking rate taken from the time-correlation analysis.

(A color version of this figure is available in the online journal.)
Figure 12. Radial-velocity map both for the full image and only the largest scale (7) deeper in the convective zone at $r = 0.95 R_\ast$, $r = 0.85 R_\ast$, and $r = 0.75 R_\ast$ for the model $\Delta \rho = 100$, and the linked acf for the latitudinal band [0$^\circ$–20$^\circ$] corresponding to the largest scale.

(A color version of this figure is available in the online journal.)

scales for three different depths in the $\Delta \rho = 100$ model, based on the autocorrelation analysis of the largest scale. The convective patterns in these deeper shells are mainly dominated by the complex distribution of the downflow lanes that have a north–south orientation, and we observe fine structures in the downflow plumes that penetrate throughout the entire convective zone. We can see that the signal at large scales is stronger than that near the stellar surface (see, for comparison, Figure 3) by a factor of five. We can see that the lifetime of these large-scale patterns increases at $r = 0.95 R_\ast$, showing that the flow is less disturbed by the smaller scales at this depth with respect to the stellar surface. Otherwise, the optimal tracking-rotation rate obtained for these large-scale patterns is the same at each depth, except at the deepest one. This confirms that their spatio-temporal coherence is maximal from $r = 0.98 R_\ast$ to $r = 0.85 R_\ast$ in this case. Thus, these giant cells seem to propagate at their own rate. The spatial correlation in depth becomes weaker below $r = 0.85 R_\ast$, although the amplitude of the large-scale signal is maximal at this depth, as shown in Figure 8. Actually, we remark in Figure 12 that at deeper depth, the rotation rate is different, indicating that giant cells are not extended over the entire depth of the convective zone for the deepest model. One can understand this behavior from the fact that the depth $r = 0.85 R_\ast$ corresponds to the transition between two consecutive giant cells in the radial direction. The same behavior in depth is found for the $\Delta \rho = 272$ model, whereas the giant cells fill in radially across the entire depth of the convective zone in the models with lower density contrast. We thus deduce from this analysis that giant cells have a radial extension of 0.13 $R_\ast$.

5.3. Influence of Mean Flows and Aspect Ratios on Resulting Convective Patterns

It is interesting to note that the stronger the density contrast is, the quicker these giant cells move with respect to the mean stellar differential rotation of each model. This is illustrated in Table 2. The small-scale structure of convection corresponds grossly to the supergranulation scale in our simulations, and also has a slightly greater rotation rate with respect to the local differential rotation. This is a well-known characteristic and has already been observed in simulations by Miesch et al. (2008) and in solar data (see Meunier 2006). However, giant cells have a very different rate. In most cases, they are prograde with respect to both the global rate and local differential rotation. For the models $\Delta \rho = 13$ and 37, it is striking to note that they move in a counterstreaming direction.

The latitudinal extension of the large-scale structures can also be compared between all models discussed in this study, to see the influence of the domain aspect ratio, which varies between 2 and 14 as reported in Table 1 and provides an insight into the 3D shape of these cells. An important latitudinal angle is also reported in Table 1, corresponding to the tangent
cylinder for all models, i.e., the imaginary cylinder tangent to the inner boundary that intersects the outer-boundary sphere at a given colatitude. First, the graphical analysis of the large-scale structures visible from scale 7 of our wavelet analysis shows that the longitudinal width of the giant cells is nearly constant around 40° for all models, as the multi-branch of the acf seen in Figures 10 and 12 reveals. It is also important to note that the longitudinal extension is much larger than the wavelength deduced from the number of downflow lanes visible at the equator in Figure 4 (more than 20 lanes at the equator). Their latitudinal extent increases with increasing density contrast. Indeed, these cells are well focused near the equator and do not reach latitudes beyond 30° for the Δρ = 13 case, whereas these cells are extended to high latitudes (>60°) for the Δρ = 272 case. We thus see that giant cells have different radial and horizontal aspect ratios depending on the depth of the convective zone in which they are embedded.

The evolution of the large-scale-structure lifetime, normalized to the maximal lifetime found for each model versus latitude, is shown in Figure 13. For each model, there is a general trend for a steep decrease in the timescale toward high latitudes, which corresponds to the position of the proper tangent cylinder. However, we note that this decrease is less pronounced when density contrast increases. Indeed, these cells are well focused near the equator and do not reach latitudes beyond 30° for the Δρ = 13 case, whereas these cells are extended to high latitudes (>60°) for the Δρ = 272 case. We thus see that giant cells have different radial and horizontal aspect ratios depending on the depth of the convective zone in which they are embedded.

The spatial coherence of these giant cells at different depths is also studied by cross-correlating the different shell slices of each model. One finds that the Δρ = 100 and Δρ = 272 models show a decrease in this coherence for r < 0.85 R*, contrary to the lower-density cases, as noted in Figure 12 for the Δρ = 100 model. The meridional-circulation profiles of these models (see Figure 14) give us a hint to understand this property. Whereas the low-density-contrast cases (panels (a) and (b)) present circulations extended over the entire depth of the modelized convective zone, larger-density cases (panels (c) and (d)) present several circulation cells in the radial direction at low latitudes. However, in these cases, one finds thicker circulation cells filling most of the depths that are aligned with the rotation axis at higher latitudes, corresponding to polar convective cells. It is important to stress that the meridional-circulation profile gives only an indication of the radial extension, because the giant cells are obviously non-axisymmetric.

We summarize our findings in Table 3, giving the estimated mean aspect ratio of the giant cells for each model by distinguishing their properties according to the latitudinal range of the high-density-contrast models. We find that they are quite extended horizontally at low latitudes. However, their depth is limited to ≈110 Mm, except at high latitudes for the thicker cases.
Using our wavelet-analysis pipeline, we can see how these giant cells transport enthalpy, kinetic energy, and angular momentum with respect to smaller scales. Figure 15 shows such an analysis for the specific radial enthalpy, radial kinetic energy, and angular-momentum fluxes transported both by the entire flows and only the largest scales in the $\Delta \rho = 272$ model. We focus here on one depth corresponding to $r = 0.95 R_*$ to obtain better contrast because, for instance, the radial-specific enthalpy flux tends toward zero near the surface due to the chosen impenetrable boundary conditions. The different quantities are computed by taking into account only the fluctuating components as defined in Brun & Palacios (2009).

First, we find that the enthalpy transport by giant cells is always directed outward at scale 7. This represents about 3% of the total enthalpy flux, whereas strong negative enthalpy fluxes are found at the edges of convective cells at smaller scales.

Second, strong negative kinetic-energy fluxes are localized in the fine structures of the downflows between convective cells, whereas positive kinetic-energy fluxes correspond to upflows in the full image. At scale 7, there is a good correlation between the giant cell signal at low latitudes detected via the radial-velocity field and kinetic-energy fluxes, i.e., negative kinetic-energy fluxes are located in the downflows of giant cells. The amplitude of the signal at scale 7 corresponds to 2% of the signal in the full image.

Third, the specific angular-momentum flux due to the $\theta-\phi$ component of the Reynolds stress is involved in the latitudinal
transport of angular momentum (see Brun & Toomre 2002 for details). Whereas this transport is complex at small scales, scale 7 shows a dominant transport equatorward at low latitudes, but also an important transport poleward particularly in the southern hemisphere at mid-latitudes. The transport by scale 7 represents 5% of the total signal.

Finally, the third column of Figure 15 shows the reconstruction of the variables without the giant cells’ signal (i.e., we have subtracted the signal corresponding to scale 7). This indicates the importance of large-scale structures in transporting enthalpy outward, particularly because the snapshot without this signal leads to a clear lack of transport of enthalpy outward. The difference is less obvious for the other quantities, but, as we have already seen, these large scales contribute a few percent to the overall transport.

Hence, we find that these large-scale convective structures contribute to the global distribution of heat, energy, and angular momentum in a systematic way, imposing a large-scale trend on the smaller-scale flow.

6. DISCUSSION

In this paper, we have studied the general properties of multi-scale convection within deep stellar convective zones. First, the convective spatial structure near the stellar surface is well constrained by the local density scale height, which is a consequence of the conservation of mass in stratified media. However, this convective organization at small scales also depends on the thickness of the shell to some extent, as shown in the polar regions of the Mρ = 13 case that possess the thinnest shell. Still, global stratification has negligible influence on surface convective patterns whenever the thickness of the convective zone is sufficient (>60 Mm in our study done with \( H_ρ = 10 \) Mm at the surface).

By using a wavelet analysis, we have stressed that there is a clear signal at large scales (>200 Mm), with a typical radial velocity around 2 m s\(^{-1}\), which is much more elongated in latitude when the density contrast increases (see Figure 13). This “latitudinal extent” estimate of the giant cells can be a means to determine the depth of the convective zone via correlation tracking. This could also be an interesting observational method to try to detect preferentially these giant cells at high latitudes. These large structures have a proper tracking rate greater than the local differential rotation, and the discrepancy increases linearly with density contrast. It is important to understand that the giant cells detected here, even though they have several characteristics similar to those of linear convective equatorial modes, are clearly different in nature. Gilman (1975) showed that these equatorial modes are the most unstable in the linear regime for high \( m \) modes (>20) when the Taylor number increases \( (Ta > 10^3) \) for a quite thin convective zone \( (R/L = 5) \). In a later work, Glatzmaier & Gillman (1982) found a decrease in the most unstable \( m \) mode when thicker convective zones are considered. However, the longitudinal extent of our giant cells is very similar for all models, despite the great range of Taylor numbers covered in these simulations. Gilman (1975) also showed that these equatorial modes are well confined at low latitudes (<35°), which is not the case for our high-density-contrast models. These giant cells are also different from the basic convective rolls because they are not extended over the full depth of the convective zone, particularly up to mid-latitudes for the realistic highly turbulent high-density-contrast cases, as shown in Figure 4.

We thus believe that the signal seen at large scales is genuine and instructive in revealing observational, stellar giant cells. As in Lisle et al. (2004), we find that giant cells are predominantly oriented along the north–south direction. More recently, Hanasoge et al. (2010) looked for the existence of giant cells using a time–distance analysis of numerical simulations of solar convection with the ASH code. After a calibration of MDI solar-surface observations from these simulations, they obtained an upper limit for the longitudinal velocities around 10 m s\(^{-1}\) for \( l < 10 \) corresponding to the range of spatial scales identified in our simulations. In Figure 12, we have found a maximal radial velocity for the giant cell signal of 12.9 m s\(^{-1}\). A wavelet analysis of \( v_ρ \) and \( v_θ \) gives, respectively, maximal velocities of 11 m s\(^{-1}\) and 7 m s\(^{-1}\), consistent with the error bars of Hanasoge et al. (2010) as reported in their Figure 5. However, the direct comparison of their results to ours is not so straightforward, because we model a young star with a density stratification and velocity profiles that differ from the Sun. On the other hand, it is likely that simulations done with the ASH code possess different energy spectra in large scales with respect to reality because the range of scales available spans only about three decades. This could lead to an overestimation of the amplitude of the giant cell signal even if our simulations clearly show evidence of its existence.

The lifetime for these giant cells is much greater than the stellar-rotation rate. These giant cells take part in a systematic way in the transport of heat, energy, and angular momentum, although their contribution does not exceed a few percentage points of the transport carried out by the whole range of flows. There are actually two types of giant cells in the deepest convective zones, corresponding to \( Δρ = 100 \) and 272 and separated roughly by the tangent cylinder. The giant cells localized at low latitudes do not span the entire convective zone due to the strong low-latitude radial shear in these models, and we find mainly two of them in the radial direction (i.e., around 110 Mm in thickness). In contrast, the giant cells at higher latitudes are aligned with the rotation axis and coherent through the entire thickness of the convective zone. These two different modes of convection, with very different aspect ratios of convective cells, can be understood by the different orientation of gravity and Coriolis force with respect to the flow combined with the increased influence of rotation on these deep convective zones. We have thus found that giant cells are likely to be present in deep convective zones and that they are more easily detected at mid-latitudes.

The presence of giant cell flows in our purely hydrodynamic simulations suggests that some large-scale organization of turbulent stellar convection is possible. In the presence of magnetic fields, this large-scale organizing of the flow may contribute to the global scale dynamo observed in most solar-type stars. We intend to study in the near future (N. Bessolaz & A. S. Brun 2011, in preparation) the subtle interplay between multi-scale convection, rotation, and magnetic fields in young solar-type stars that possess deep convective zones and possibly giant cells.

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REFERENCES

Ballot, J., Brun, A. S., & Turck-Chièze, S. 2007, ApJ, 669, 1190
Beck, J. G., Duvall, T. L., Jr., Scherrer, P. H., & Hoeksema, J. T. 1998, in Proc.
SOHO 6/GONG 98 Workshop, Structure and Dynamics of the Interior of
the Sun and Sun-like Stars, ed. S. G. Korzennik & A. Wilson (Noordwijk:
ESA), 725
Brown, B. P., Browning, M. K., Brun, A. S., Miesch, M. S., & Toomre, J.
2008, ApJ, 689, 1354
Brown, B. P., Browning, M. K., Brun, A. S., Miesch, M. S., & Toomre, J.
2010, ApJ, 711, 424
Browning, M. K., Brun, A. S., & Toomre, J. 2004, ApJ, 601, 512
Brun, A. S., Miesch, M. S., & Toomre, J. 2004, ApJ, 614, 1073
Brun, A. S., & Palacios, A. 2009, ApJ, 702, 1078
Brun, A. S., & Toomre, J. 2002, ApJ, 570, 865
Busse, F. H., & Cuong, P. G. 1977, Geophys. Astrophys. Fluid Dyn., 8, 17
Canuto, V. M. 1999, ApJ, 524, 311
Chandrasekhar, S. 1961, Hydrodynamic and Hydromagnetic Stability (Oxford:
Clarendon)
Chune, T. L., Elliott, J. R., Glatzmaier, G. A., Miesch, M. S., & Toomre, J.
1999, Parallel Comput., 25, 361
Cox, J. P., & Giuli, R. T. 1968, Principles of Stellar Structure (New York: Gordon
and Breach)
Dormy, E., Soward, A. M., Jones, C. A., Jault, D., & Cardin, P. 2004, J. Fluid
Mech., 501, 43
Fornberg, B. 1998, A Practical Guide to Pseudospectral Methods, Cambridge
Monographs on Applied and Computational Mathematics (Cambridge: Cambridge
Univ. Press)
Gilman, P. A. 1975, J. Atmos. Sci., 32, 1331
Gilman, P. A. 1979, ApJ, 231, 284
Glatzmaier, G. A. 1984, Journal of Computational Physics, 55, 461
Glatzmaier, G. A., & Gilman, P. A. 1982, ApJ, 256, 316
Hanasoge, S. M., Duvall, T. L., DeRosa, M. L. 2010, ApJ, 712, L98
Hathaway, D. H., Beck, J. G., Bogart, R. S., Bachmann, K. T., Khatri, G., Petitto,
J. M., Han, S., & Raymond, J. 2000, Sol. Phys., 193, 299
Iben, I., Jr. 1965, ApJ, 141, 993
Labonte, B. J., Howard, R., & Gilman, P. A. 1981, ApJ, 250, 796
Lisle, J. P., Rast, M. P., & Toomre, J. 2004, ApJ, 608, 1167
Meunier, N. 2006, Observations of the Solar Surface Dynamics, Vol. 21 (Tarbes
Cedex: EAS Publications Series)
Miesch, M. S., Brun, A. S., DeRosa, M. L., & Toomre, J. 2008, ApJ, 673,
557
Miesch, M. S., Brun, A. S., & Toomre, J. 2006, ApJ, 641, 618
Morel, P. 1997, A&A, 124, 597
Nordlund, Å., Stein, R. F., & Asplund, M. 2009, Living Rev. Sol. Phys., 6, 2
Pedlosky, J. 1987, Geophysical Fluid Dynamics (New York: Springer)
Pomarede, D., Fidaali, Y., Audit, E., Brun, A. S., Massef, F., & Teyssier, R.
2008, in ASP Conf. Ser. 386, Numerical Modeling of Space Plasma Flows:
Astronom 2007, ed. N. V. Pogorelov, E. Audit, & G. P. Zank (San Francisco,
CA: ASP), 327
Porter, D. H., & Woodward, P. R. 2000, ApJS, 127, 159
Rast, M. P. 2003, ApJ, 597, 1200
Rieutord, M., & Rincon, F. 2010, Liv. Rev. Sol. Phys., 7, 2
Rieutord, M., Roudier, T., Malherbe, J. M., & Rincon, F. 2000, A&A, 357, 1063
Roberts, P. H. 1972, Phil. Trans. R. Soc. London, 272, 663
Scherrer, P. H., Bogart, R. S., & Hoeksema, J. T. 1986, BAAS, 18, 702
Simon, G. W., & Weiss, N. O. 1968, Z. Astrophys., 69, 435
Starck, J.-L., Moudden, Y., Abrial, P., & Nguyen, M. 2006, A&A, 446, 1191
Stein, R. F., & Nordlund, A. 1998, ApJ, 499, 914
Vögler, A. 2005, Mem. Soc. Astron. Ital., 76, 842