Heat Transmission Reinforcers Induced by MHD Hybrid Nanoparticles for Water/Water-EG Flowing over a Cylinder

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Abstract: The assumptions that form our focus in this study are water or water-ethylene glycol flowing around a horizontal cylinder, containing hybrid nanoparticles, affected by a magnetic force, and under a constant wall temperature, in addition to considering free convection. The Tiwari–Das model is employed to highlight the influence of the nanoparticles volume fraction on the flow characteristics. A numerical approximate technique called the Keller box method is implemented to obtain a solution to the physical model. The effects of some critical parameters related to heat transmission are also graphically examined and analyzed. The increase in the nanoparticle volume fraction increases the heat transfer rate and liquid velocity; the strength of the magnetic field has an adverse effect on liquid velocity, heat transfer, and skin friction. We find that cobalt nanoparticles provide more efficient support for the heat transfer rate of aluminum oxide than aluminum nanoparticles.

Keywords: circular cylinder; free convection; hybrid nanofluid; MHD; Tiwari–Das model

1. Introduction

Flowing fluids are an important medium for transmitting energy, which cannot be neglected in several industrial, pharmaceutical, engineering, and food applications. This has led researchers to focusing on either searching for the best fluids that provide good heat transferability or trying to improve the efficiency of these media. Water was initially the best choice due to its abundance and ease of obtaining, and it possesses many critical thermophysical properties such as high heat capacity and thermal conductivity, so it met the needs of most industrial and engineering heat transfer applications. The mixture of water and ethylene glycol (EG) has also been used extensively in many energy systems, as ethylene glycol possesses distinct characteristics, such as its low freezing and high boiling points, high thermal conductivity, stability over a wide temperature range, and low viscosity, so it is suitable in operations requiring pumping. Accordingly, the combination of water and ethylene glycol is almost freeze-resistant, making it useful for charging operations and as an anti-freeze material in vehicles.

The physicist Feynman [1] was the first to address the nanoscale idea, as he suggested that the physical properties of materials differ according to the size of their particles. In the early 1990s, the scanning tunneling microscope was invented, which enabled researchers to study and monitor materials at the atomic level, which contributed to the wide spread of nanotechnology in several industrial, medicine, and engineering application [2–7]. Further experimental research and numerical simulation revealed that the lower the viscosity of the fluid and the greater its specific temperature, density, and thermal conductivity, the more
efficient and able it is to transfer energy. In the field of heat transfer, Choi and Eastman [8] were amongst the first to improve the thermal features of flowing liquids through the adoption of nanoparticles. Eastman et al. [9] confirmed that combining copper nanoparticles with ethylene glycol promotes its thermal conductivity. Chon et al. [10] analyzed the impact of temperature and the size of ultrafine particles on the thermal conductivity of a nanoliquid. Xuan and Li [11] determined the most important factors affecting the rate of energy transmission, such as volume fraction, shapes, and dimensions of ultrafine particles; see also Refs. [12–14]. In 2007, Tiwari and Das [15] presented a mathematical model highlighting the effective role of the volume fraction of ultrafine particles in energy transmission. Since then, many mathematical models have been constructed that relied on this model. Nazar et al. [16] used Tiwari–Das’s nanofluid model to examine the combined convection flow of a nanofluid past a cylinder in a porous medium. Dinarvand et al. [17] constructed a mathematical model for investigating the mixed convection flow of a nanoliquid that relied on Tiwari–Das’s model. Swalme et al. [18] simulated the boundary layer flow of a micropolar nanoliquid over a sphere using the Tiwari–Das model. Hamarsheh et al. [19] modeled the behavior of Casson nanofluid flow under the effect of a magnetic field with the aid of the Tiwari–Das model.

As an extension and development of the proposal presented by Choi and Eastman [8], hybrid nanoparticles have been introduced to the field of heat transmission through fluids. The idea of hybrid nanoparticles is based on the synthesis of two nanomaterials to produce a nanocomposite that possesses upgraded thermal characteristics. Suresh et al. [20] found that fabrication of Cu-Al₂O₃ nanocomposite including a small amount of copper, which has a high thermal conductivity with aluminum and has multiple features such as excellent stability and chemical inertness, shows properties integrated with those of alumina nanoparticles. Baghbanzadeh et al. [21] discussed a method of incorporating silica nanospheres (SiO₂) and multiwall carbon nanotubes (MWCNTs) to form hybrid nanoparticles; they also reported the optimal ratio of SiO₂ to MWCNTs in the nanocomposite that produces the highest thermal conductivity. Zhou et al. [22] produced a polymer nanocomposite with high efficiency in terms of thermal conductivity based on the hybrid nanoparticles idea. The reader is also referred to Refs. [23–28]. These improvements in thermal properties exhibited by the hybrid nanoparticles led them to being considered in several numerical studies related to heat transfer. Labib et al. [29] employed the mixture model to demonstrate the improvements in the heat transmission using CNTs–Al₂O₃ hybrid nanoparticles. Moghadassi et al. [30], in their numerical studies, found that adding small quantities of copper particles to a mono-nanofluid, which consisted of water and aluminum oxide, boosted the rate of heat transfer by 5%. Rostami et al. [31] numerically investigated the combined convection flow of SiO₂–Al₂O₃–water hybrid nanofluid past a vertical plate. Abdel-Nour et al. [32] employed the Galerkin finite element technique to address entropy generation in the flow of water hosting Al₂O₃–Cu hybrid nanocomposite in the presence of magneto-free convection. More numerical studies were conducted on hybrid nanofluids in Refs. [33–38].

Magneto-hydrodynamics or magneto-fluid dynamics (MHD) plays a vital role in many physical and engineering applications that depend on heat transmission mainly through electrically conductive fluids. Its applications are most evident with plasma propulsion, MHD molten pump jets or thrusters, flow control around hypersonic and re-entry vehicles, MHD power generation, biomedical applications, and others [39–41]. MHD is the science interested in studying the interaction between a magnetic field and electrically conducting moving fluids. The exposure of an electric-conducting fluid to an orthogonal magnetic field results in the creation of a type of force called Lorentz force, which affects all physical quantities related to heat transfer. Many research studies have examined the many aspects of this physical phenomenon in the heat transfer field. Sheikholeslami and Rokni [42] presented an article reviewing most of the previous studies that simulated the flow of nanofluids under a magnetic field. Jamaludin et al. [43] examined the boundary layer flow of nanofluids over a permeable sheet in the case of convection combined with slip.
conditions and the MHD effect. Dogonchi et al. [44] numerically studied the impacts of a magnetic field and joule heating on a nanofluid over a stretching plate considering Brownian motion. Alwawi et al. [45,46] modeled the magneto-free/mixed convection flow of Casson nanofluid over a solid sphere. The reader may also refer to Refs. [47–49].

The aforementioned literature produced our motivation to investigate hybrid particles promoting the heat transfer of water and ethylene glycol-water (EGW) flowing around a horizontal cylinder, which is widely used in heat transfer processes, especially in cooling. A variable orthogonal magnetic field was considered. The numerical approach of the solution to the mathematical model was computed using the Keller box method. In addition, we conducted comparisons between the physical quantities related to heat transfer for both host fluids.

2. Mathematical Model Formulation

As explained in Figure 1, we assume that the boundary layer flow of water/water-ethylene glycol, supported by hybrid nanocomposite over a circular cylinder, is affected by a transverse magnetic field. We also consider free convection and constant wall temperature. \( x \) and \( y \) indicate the axis that is measured on the circumference of the cylinder starting from the stagnation point and the axis that measures the vertical distance to its surface, respectively. \( a, T_w, T_\infty, g, \) and \( B_0 \) refer to the radius of circular cylinder, constant wall temperature, ambient temperature, gravity vector, and magnetic field strength, respectively.

![Figure 1. Physical model configuration.](image)

Relying on the above consideration and under the usual Boussinesq incompressible hybrid nanofluid model, and applying the boundary layer approximations, the governing model of continuity, momentum, and energy equations can be described as [50–53]:

\[
\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} = 0
\]

subject to:

\[
\hat{u} = \hat{v} = 0, \quad T = T_w \text{ at } \hat{y} = 0, \\
\hat{u} \to 0, \quad T \to T_w \text{ at } \hat{y} \to \infty,
\]
The non-dimensional variables are expressed as:
\[ y = Gr^{1/4} \left( \hat{q} \right), \quad u = \left( \frac{\hat{q}}{\hat{f}} \right) Gr^{-1/2} \hat{q}, \quad v = \left( \frac{\hat{q}}{\hat{f}} \right) Gr^{-1/4} \hat{q} \]
\[ \theta = \frac{T - T_w}{T_w - T_\infty}, \quad \hat{p} = \frac{p - p_0}{\hat{f} (\hat{f}/\hat{f})^2} \]

where \( Gr = g \beta f (T_w - T_\infty) \hat{f}^{2} \) is the Grashof number.

Table 1 displays the thermophysical properties of both the mono and hybrid nanofluids.

| Properties of the Mono Nanofluid | Properties of the Hybrid Nanofluid |
|----------------------------------|-----------------------------------|
| \( \rho_{nf} = (1 - \chi) \rho_f + \lambda \rho_s \) | \( \rho_{hnf} = (1 - \chi_2) [(1 - \chi_1) \rho_f + \chi_1 \rho_{p_1}] + \chi_2 \rho_{p_2} \) |
| \( (\rho c_p)_{nf} = (1 - \chi) (\rho c_p)_f + \chi (\rho c_p)_s \) | \( (\rho c_p)_{hnf} = (1 - \chi_2) [(1 - \chi_1) (\rho c_p)_f + \chi_1 (\rho c_p)_{p_1}] + \chi_2 (\rho c_p)_{p_2} \) |
| \( \beta_{nf} = (1 - \chi) \beta_f + \beta_s \) | \( \beta_{hnf} = (1 - \chi_2) [(1 - \chi_1) \beta_f + \chi_1 \beta_{p_1}] + \chi_2 \beta_{p_2} \) |
| \( k_{nf} = \frac{(k_f + 2k_s)}{(k_f + k_s)} \) | \( k_{hnf} = \frac{(k_2 + 2k_1 - 2k_0) (k_2 - k_1)}{k_2 + 2k_1 + \chi k_2 (k_1 - k_2)} \) |
| \( \sigma_{nf} = \frac{(\sigma c_v)}{\sigma f} \) | \( \sigma_{hnf} = \frac{(\sigma c_v + 2\sigma c_f - 2\sigma_1 (\sigma c_f - \sigma c_0))}{\sigma c_v + 2\sigma c_f + \chi (\sigma c_f - \sigma c_2)} \) |

Table 1. Thermo-physical properties. Data from [53].

Using the non-dimensional variable in Equation (6), the properties listed in Table 1, and the boundary layer approximations (as \( Gr \to \infty \)), we have \( (\partial p/\partial y) = 0 \) and \( (\partial p/\partial x) = 0 \) [52]. Consequently, we ignore the associated terms in the previous equations as follows:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \( (7) \)

\[ u \frac{\partial u}{\partial y} + v \frac{\partial u}{\partial y} = \frac{\rho_f}{\rho_{nf}} \left( \frac{1}{(1 - \chi)^2 (1 - \chi_2)^2} \right) \hat{f} y \sin x - \frac{\rho_f}{\rho_{nf}} \frac{\sigma_{nf}}{\sigma f} M u, \]

\[ = \frac{1}{Pr} \left[ \frac{k_{nf}}{k_f} \right] (1 - \chi_2) [(1 - \chi_1) \rho_f + \chi_1 (\rho c_p)_f] + \chi_2 (\rho c_p)_{p_2} / (\rho c p)_f \right] \hat{f} y \sin x \]

where \( M = \left( \frac{\hat{f} \hat{f}^2}{\hat{f}} \right) \) is the magnetic parameter, \( Pr = v_f / \alpha f \) is the Prandtl number, and the boundary conditions are:

\[ u = v = 0, \quad \theta = 1, \quad \text{at} \ y = 0, \]
\[ u \to 0, \quad \theta \to 0, \quad \text{as} \ y \to \infty. \]  \( (10) \)

The reduction process of Equations (7)–(10) requires using the following transformation [51]:

\[ \psi = xf (x, y), \quad \theta = \psi (x, y), \]  \( (11) \)

where \( \psi \) is the stream function given by:

\[ u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \]  \( (12) \)
Consequently, we obtain the following reductive equations:

\[
\frac{\rho f}{\mu_{nf}} \left( \frac{1}{(1-\chi_1)^{x}} \right) \frac{\partial^3 f}{\partial x^3} + f \frac{\partial^2 f}{\partial x^2} - \left( \frac{\partial f}{\partial y} \right)^2 - \frac{\rho f}{\mu_{nf}} \frac{\partial w_{nf}}{\partial y} M \frac{\partial f}{\partial y} + \frac{1}{\mu_{nf}} (1-\chi_2) [(1-\chi_1) f + \chi_1 \frac{\partial^2 w_{nf}}{\partial y^2}] + \chi_2 \frac{\partial^2 w_{nf}}{\partial y^2} \right] \frac{\partial^2 M}{\partial y^2} = (13)
\]

subject to:

\[
f = \frac{\partial f}{\partial y} = 0, \quad \theta = 1 \text{ at } y = 0, \quad \frac{\partial f}{\partial y} \rightarrow 0, \quad \theta \rightarrow 0, \text{ as } y \rightarrow \infty.
\]

As \(x\) approaches zero (known as the stagnation point), Equations (13)–(15) transform to the following ODEs:

\[
\frac{\rho f}{\mu_{nf}} \left( \frac{1}{(1-\chi_1)^{x}} \right) \frac{\partial f}{\partial \theta^3} + f \frac{\partial^2 f}{\partial \theta^2} - \left( \frac{\partial f}{\partial \theta} \right)^2 + \frac{1}{\mu_{nf}} (1-\chi_2) [(1-\chi_1) f + \chi_1 \frac{\partial^2 w_{nf}}{\partial \theta^2}] + \chi_2 \frac{\partial^2 w_{nf}}{\partial \theta^2} \theta - \frac{\rho f}{\mu_{nf}} \frac{\partial w_{nf}}{\partial \theta} M \frac{\partial f}{\partial \theta} = 0,
\]

and the boundary conditions are:

\[
f(0) = f'(0) = 0, \quad \theta(0) = 1 \text{ as } y = 0, \quad f' \rightarrow 0, \quad \theta \rightarrow 0 \text{ as } y \rightarrow \infty,
\]

where the primes refer to differentiation with respect to \(y\).

To examine and analyze the flow characteristics, two physical quantities are highlighted: the local skin friction coefficient \(C_f\) and the local Nusselt number \(Nu\), which were described by Alwawi et al. [50] as follows:

\[
C_f = \left( \frac{\tau_w}{\rho f U^2_{\infty}} \right), \quad Nu = \left( \frac{a q_w}{k_f (T_{\infty} - T_w)} \right),
\]

where

\[
\tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right) \hat{y} = 0, \quad a q_w = -k_{nf} \left( \frac{\partial T}{\partial y} \right) \hat{y} = 0
\]

By applying the non-dimensional variables in Equations (6) and (15), \(C_f\) and \(Nu\) transform to:

\[
C_f = Gr^{-1/4} \frac{1}{(1-\chi_1)^{2.5} (1-\chi_2)^{2.5}} \frac{\partial^2 f}{\partial y^2} (x, 0), Nu = -Gr^{1/4} \frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial y} (x, 0),
\]

3. Numerical Approach

The model that governs our context is treated numerically based on the Keller box method (KBM). Generally, the PDEs of the boundary layer flow problems of a hybrid fluid are solved numerically using this method. The Keller box is an implicit finite difference scheme method used for finding precise numerical results for the PDFs, which was constructed by Keller and Bramble [54]. Recently, it was used by Swalmeh et al. [55] and
Hamarsheh et al. [19] to find the numerical results of the influences of convection boundary layer flow in a micropolar nanofluid.

Briefly, KBM has four steps: (1) converting the governing equations of nonlinear PDEs into a first-order system; (2) using the central differences method to obtain the finite difference equations; (3) linearizing these finite difference equations via Newton’s method, and then systemizing them to matrix-vector form; (4) applying the block tri-diagonal elimination method to solve the matrix-vector linear system. The numerical results of the physical quantities for the obtained linear system were tabulated and plotted using MATLAB version 7.0.0 (MathWorks, Natick, MA, USA).

The values of the parameters that were considered were $\chi = 0.1, 0.15, 0.2$ and $M > 0$. The thermo-physical properties of the nanoparticles and the host fluid used in the mathematical calculation are reported in Table 2.

Table 2. Different values of thermo-physical properties of nanoparticles and host fluid. Data from [56–58].

| Physical Properties | Host Fluid | Nanoparticles |
|---------------------|------------|---------------|
|                     | H2O + EG 50% | Water | Co (Cobalt) | Al | Al2O3 |
| $k$ (W/mK)          | 0.425      | 0.613        | 100        | 237 | 40   |
| $\rho$ (kg/m$^3$)   | 1056       | 997.1        | 8900       | 2701 | 3970 |
| $\rho c_p$ (J/kgK)  | 3288       | 4179         | 420        | 902  | 765  |
| $B \times 10^{-5}$ (K$^{-1}$) | 0.000341  | 21           | $1.3 \times 10^{-5}$ | 2.31  | 0.85  |
| $\sigma_s$ m$^{-1}$ | 0.00509    | $5.5 \times 10^{-6}$ | $1.602 \times 10^7$ | $3.5 \times 10^7$ | $1 \times 10^{-10}$ |
| Pr                  | 29.86      | 6.2          | -          | -    | -    |

The verification of the accuracy of the current numerical results for $C_f$ and Nu using the prior results published by Merkin [59] and Alwawi et al. [50] is demonstrated in Tables 3 and 4.

Table 3. Comparison of $C_f$ with previous numerical results ($\chi_1 = \chi_2 = 0$, and $M = 0$) at $Pr = 1$.

| x       | Merkin [59] | Alwawi et al. [50] | Current Study |
|---------|-------------|--------------------|---------------|
| 0       | 0.0000      | 0.0000             | 0.0000        |
| $\pi/6$ | 0.4151      | 0.4158             | 0.4148        |
| $\pi/3$ | 0.7558      | 0.7538             | 0.7544        |
| $\pi/2$ | 0.9579      | 0.9563             | 0.9573        |
| $2\pi/3$| 0.9756      | 0.9735             | 0.9751        |
| $5\pi/6$| 0.7822      | 0.7480             | 0.7469        |
| $\pi$   | 0.3391      | 0.3311             | 0.3330        |

Table 4. Comparison of Nu with previous numerical results ($\chi_1 = \chi_2 = 0$, and $M = 0$) at $Pr = 1$.

| x       | Merkin [59] | Alwawi et al. [50] | Current Study |
|---------|-------------|--------------------|---------------|
| 0       | 0.4214      | 0.4214             | 0.4219        |
| $\pi/6$ | 0.4161      | 0.4162             | 0.4166        |
| $\pi/3$ | 0.4007      | 0.4008             | 0.4010        |
| $\pi/2$ | 0.3745      | 0.3744             | 0.3744        |
| $2\pi/3$| 0.3364      | 0.3360             | 0.3355        |
| $5\pi/6$| 0.2825      | 0.2754             | 0.2714        |
| $\pi$   | 0.1945      | 0.1926             | 0.1939        |

4. Results and Discussion

Figures 2 and 3 illustrate the impact of growth magnetic parameter M on the Nusselt number and the skin friction of water and water-EG hybrid nanofluids at fixed nanoparticles volume fractions ($\chi_1 = 0.1$, $\chi_2 = 0.05$). This magnetic current suppresses both the rate of energy transmission and the skin friction of both fluids, which occurs because the passage of a magnetic current across a flowing fluid formulates a type of force that
curbs the energy of the particles of the fluid. The figures show the basic role of cobalt and aluminum in supporting the heat transfer of aluminum oxide, produced due to the efficiency of aluminum in raising the rate of heat transfer.

Figure 2. Effect of magnetic field on skin friction. (a) H$_2$O vs. H$_2$O + EG 50%; (b) Co-Al$_2$O$_3$ vs. Al$_2$O$_3$.

Figure 3. Effect of magnetic field on Nusselt number. (a) H$_2$O vs. H$_2$O + EG 50%; (b) Co-Al$_2$O$_3$ vs. Al$_2$O$_3$.

Figures 4 and 5 show the opposite effect of the imposed magnetic field on the fluid velocity and the direct effect on the temperature, which were expected due to the creation of the Lorentz force, which works to slow the movement of the fluid and thus reduce its velocity, resulting in an increase in its temperature. Interestingly, the hybrid nanofluid, composed of water-ethylene glycol and Co-Al$_2$O$_3$ had a lower temperature compared with the hybrid nanofluid consisting of water and Co-Al$_2$O$_3$ with an increased magnetic parameter, which means less energy dissipation and increased stability. However, when we compared the hybrid nanoparticles fabricated from the synthesis of Co and Al$_2$O$_3$ with the hybrid nanoparticles formed from the synthesis of Al and Al$_2$O$_3$, the latter was found to be better as it showed the lowest temperature.
2.5 6 40 3.5 10 8 20 60 0.5 60 2.5 0.5 140 100 2 100 20 180 4 180 160 140 80 180 4 60 4 1 80 160 20 120

Figure 4. Effect of magnetic field on velocity. (a) H₂O vs. H₂O + EG 50%; (b) Co/Al₂O₃ vs. Al₂O₃.

Figure 5. Effect of magnetic field on temperature. (a) H₂O vs. H₂O + EG 50%; (b) Co/Al₂O₃ vs. Al₂O₃.

Figures 6 and 7 highlight the behavior of the Nusselt number and skin friction with increasing volume fraction coefficient of cobalt (Co) or aluminum (Al); the volume fraction coefficient of aluminum oxide (χ₁ = 0.1) and magnetic parameter M = 0.1 remained fixed. We found that the increase in the volume fraction coefficient promoted the rate of heat transfer and skin friction. The increase in χ₂, which helps promote the density and thermal conductivity of the mono nanoliquid, resulted in increases in both the skin friction coefficient and the Nusselt number. Notably, the combination of Al₂O₃/water-EG was more efficient in terms of energy transfer rate, and its skin friction coefficient was the highest.

The effect of the growth in the nanoparticle volume fraction (χ₂) for cobalt and aluminum on the velocity and temperature profiles with constant aluminum oxide (χ₁ = 0.1) and magnetic parameter (M = 0.1) is demonstrated in Figures 8 and 9. As expected, the growth in the nanoparticles volume fraction, whether for cobalt or aluminum, contributed significantly to the acceleration of heat transfer from outside the surface of the cylinder to the mono nanoliquid, which aided in the expansion of the thermal boundary layers. As a result, the temperature of the mono nanoliquid rose. Conversely, the increase in nanoparticles volume fraction, whether for cobalt or aluminum, inhibited mono nanoliquid velocity.
Figure 6. Effect of nanoparticles volume fraction on skin friction.

Figure 7. Effect of nanoparticles volume fraction on Nusselt number.

Figure 8. Effect of nanoparticles volume fraction on velocity.
1. The heat transmission rate of the hybrid-nanofluid is higher compared with that of the mono nanofluid.
2. The heat transmission rate of ethylene glycol-water is higher than that of water, regardless of the incorporated hybrid nanoparticles.
3. The magnetic parameter tends to raise the temperature profile, whereas it has a reverse effect on skin friction, velocity, and Nusselt number.
4. Aluminum and cobalt nanoparticles are essential to support the thermal properties of aluminum oxide, which promote energy transfer.
5. $C_f$, $Nu$, and the temperature of Al$_2$O$_3$/ethylene glycol-water nanofluid increase and by the addition of nanoparticles (Co and Al); they consequently formed a hybrid nanofluid with different values of $M$ and $\chi_2$.
6. MHD hybrid nanofluid flow is clearly affected by the nanoparticle volume fraction ($\chi_2$) and magnetic ($M$) parameters. So, the findings of this study provide additional information to the field of fluid mechanics.

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Abbreviations

- \( a \) Radius of cylinder
- \( B_0 \) Strength of magnetic field
- \( C_f \) Local skin friction coefficient
- \( Gr \) Grashof number
- \( g \) Gravity vector
- \( k \) Thermal conductivity
- \( M \) Magnetic parameter
- \( Pr \) Prandtl number
- \( q_w \) Surface heat flux
- \( T \) Temperature of the fluid
- \( T_\infty \) Surrounded temperature
- \( \hat{u} \) Dimensional x-component of velocity
- \( \hat{v} \) Dimensional y-component of velocity
- \( u \) Nondimensional x-component of velocity
- \( v \) Nondimensional y-component of velocity
- \( \hat{p} \) Dimensional fluid pressure
- \( p \) Nondimensional fluid pressure
- \( \nu_f \) Kinematic viscosity of base fluid
- \( \alpha \) Thermal diffusivity
- \( \beta \) Thermal expansion
- \( \sigma \) Electrical conductivity
- \( \theta \) Temperature of nanofluid
- \( \mu_f \) Dynamic viscosity
- \( \rho \) Density
- \( \rho c_p \) Heat capacity
- \( \tau_w \) Wall shear stress
- \( \chi_1 \) Volume fraction of Al\(_2\)O\(_3\)
- \( \chi_2 \) Volume fraction of Al/Co
- \( \psi \) Stream function

Subscript

- \( hn_f \) Hybrid-nanofluid
- \( s \) Nanoparticles
- \( nf \) Nanofluid
- \( f \) Host fluid
- \( s1 \) Al\(_2\)O\(_3\) Nanoparticles
- \( s2 \) Al or Co Nanoparticles

References

1. Feynman, R.P. There’s plenty of room at the bottom. In Engineering and Science Magazine; California Institute of Technology: Pasadena, CA, USA, 1960.
2. Silva, G.A. Introduction to nanotechnology and its applications to medicine. Surg. Neurol. 2004, 61, 216–220. [CrossRef]
3. Sozer, N.; Kokini, J.L. Nanotechnology and its applications in the food sector. Trends Biotechnol. 2009, 27, 82–89. [CrossRef]
4. Contreras, J.; Rodriguez, E.; Taha-Tijerina, J. Nanotechnology applications for electrical transformers—A review. Electr. Power Syst. Res. 2017, 143, 573–584. [CrossRef]
5. Razvarz, S.; Chavez, L.F.G.; Vargas-Jarillo, C. Nanotechnology applications in industry and heat transfer. In Proceedings of the Latin American Symposium on Industrial and Robotic Systems, Tampico, Mexico, 30 October–1 November 2019; pp. 1–8.
6. Noman, M.T.; Petru, M.; Amor, N.; Yang, T.; Mansoor, T. Thermophysiological comfort of sonochemically synthesized nano TiO\(_2\) coated woven fabrics. Sci. Rep. 2020, 10, 17204. [CrossRef] [PubMed]
7. Noman, M.T.; Petru, M.; Amor, N.; Louda, P. Thermophysiological comfort of zinc oxide nanoparticles coated woven fabrics. Sci. Rep. 2020, 10, 21080. [CrossRef]
8. Choi, S.U.; Eastman, J.A. Enhancing Thermal Conductivity of Fluids with Nanoparticles; Argonne National Lab.: DuPage County, IL, USA, 1995.
9. Eastman, J.A.; Choi, S.; Li, S.; Yu, W.; Thompson, L. Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. Appl. Phys. Lett. 2001, 78, 718–720. [CrossRef]
10. Chon, C.H.; Kihm, K.D.; Lee, S.P.; Choi, S.U.S. Empirical correlation finding the role of temperature and particle size for nanofluid (Al\(_2\)O\(_3\)) thermal conductivity enhancement. Appl. Phys. Lett. 2005, 87, 153107. [CrossRef]
11. Xuan, Y.; Li, Q. Heat transfer enhancement of nanofluids. *Int. J. Heat Fluid Flow* **2000**, *21*, 58–64. [CrossRef]
12. Wang, X.-Q.; Mujumdar, A.S. Heat transfer characteristics of nanofluids: A review. *Int. J. Therm. Sci.* **2007**, *46*, 1–19. [CrossRef]
13. Triasiksri, V.; Wongwises, S. Critical review of heat transfer characteristics of nanofluids. *Renew. Sustain. Energy Rev.* **2007**, *11*, 512–523. [CrossRef]
14. Putra, N.; Roetzel, W.; Das, S.K. Natural convection of nano-fluids. *Heat Mass Transf.* **2003**, *39*, 775–784. [CrossRef]
15. Tiwari, R.K.; Das, M.K. Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *Int. J. Heat Mass Transf.* **2007**, *50*, 2002–2018. [CrossRef]
16. Nazar, R.; Tham, L.; Pop, I.; Ingham, D. Mixed convection boundary layer flow from a horizontal circular cylinder embedded in a porous medium filled with a nanofluid. *Transp. Porous Med.* **2011**, *86*, 517–536. [CrossRef]
17. Dinarvand, S.; Abbassi, A.; Hosseini, R.; Pop, I. Homotopy analysis method for mixed convective boundary layer flow of a nanofluid over a vertical circular cylinder. *Therm. Sci.* **2015**, *19*, 549–561. [CrossRef]
18. Swalmeh, M.; Alkasasbeh, H.T.; Hussanan, A.; Mamat, M. Heat transfer flow of Cu-water and Al2O3-water micropolar nanofluids about a solid sphere in the presence of natural convection using Keller-box method. *Results Phys.* **2018**, *9*, 717–724. [CrossRef]
19. Hamarsheh, A.S.; Alwawi, F.A.; Alkasasbeh, H.T.; Rashad, A.M.; Idris, R. Heat transfer improvement in MHD natural convection flow of graphite oxide/carbon nanotubes-methanol based casson nanofluids past a horizontal circular cylinder. *Processes* **2020**, *8*, 1444. [CrossRef]
20. Suresh, S.; Venkitaraj, K.; Selvakumar, P.; Chandrasekar, M. Synthesis of Al2O3–Cu/water hybrid nanofluids using two step method and its thermal physical properties. *Colloids Surfaces A Physicochem. Eng. Asp.* **2011**, *388*, 41–48. [CrossRef]
21. Baghbanzadeh, M.; Rashidi, A.; Rashtchian, D.; Lotfi, R.; Amrollahi, A. Synthesis of spherical silica/multiwall carbon nanotubes hybrid nanostructures and investigation of thermal conductivity of related nanofluids. *Thermochem. Acta* **2012**, *549*, 87–94. [CrossRef]
22. Zhou, W.; Chen, Q.; Sui, X.; Dong, L.; Wang, Z. Enhanced thermal conductivity and dielectric properties of Al/β-SiCw/PVDF composites. *Compos. Part A Appl. Sci. Manuf.* **2015**, *71*, 184–191. [CrossRef]
23. Chen, L.F.; Cheng, M.; Yang, D.J.; Yang, L. Enhanced thermal conductivity of nanofluid by synergistic effect of multi-walled carbon nanotubes and Fe3O4 nanoparticles. *Appl. Mech. Mater.* **2014**, *548–549*, 118–123. [CrossRef]
24. Esfe, M.H.; Saedodin, S.; Biglari, M.; Rostamian, H. Experimental investigation of thermal conductivity of CNTs-Al2O3/water: A statistical approach. *Int. Commun. Heat Mass Transf.* **2015**, *69*, 29–33. [CrossRef]
25. Yarmand, H.; Gharehkhani, S.; Shirazi, S.F.S.; Goodarzi, M.; Amiri, A.; Sarsam, W.S.; Alehashem, M.; Dahari, M.; Kazi, S. Study of synthesis, stability and thermo-physical properties of graphene nanoplatelet/platinum hybrid nanofluid. *Int. Commun. Heat Mass Transf.* **2016**, *77*, 15–21. [CrossRef]
26. Leong, K.; Ahmad, K.K.; Ong, H.C.; Ghazali, M.; Baharum, A. Synthesis and thermal conductivity characteristic of hybrid nanofluids—A review. *Renew. Sustain. Energy Rev.* **2017**, *75*, 868–878. [CrossRef]
27. Sajid, M.U.; Ali, H.M. Thermal conductivity of hybrid nanofluids: A critical review. *Int. J. Heat Mass Transf.* **2018**, *126*, 211–234. [CrossRef]
28. Huang, T.; Zhang, G.; Gao, Y. A novel silver nanoparticle-deposited aluminum oxide hybrids for epoxy composites with enhanced thermal conductivity and energy density. *Compos. Interfaces*. **2019**, *26*, 1001–1011. [CrossRef]
29. Labib, M.N.; Nine, J.; Afrianto, H.; Chung, H.; Jeong, H. Numerical investigation on effect of base fluids and hybrid nanofluid in forced convective heat transfer. *Int. J. Therm. Sci.* **2013**, *71*, 163–171. [CrossRef]
30. Moghadassi, A.; Ghomi, E.; Parvizian, F. A numerical study of water based Al2O3 and Al2O3–Cu hybrid nanofluid effect on forced convective heat transfer. *Int. J. Therm. Sci.* **2015**, *92*, 50–57. [CrossRef]
31. Rostami, M.N.; Dinarvand, S.; Pop, I. Dual solutions for mixed convective stagnation-point flow of an aqueous silica–alumina hybrid nanofluid. *Chin. J. Phys.* **2018**, *56*, 2465–2478. [CrossRef]
32. Abdel-Nour, Z.; Aissa, A.; Mebarek-Oudina, F.; Rashad, A.M.; Ali, H.M.; Sahnoun, M.; El Ganaoui, M. Magneto-hydrodynamic natural convection of hybrid nanofluid in a porous enclosure: Numerical analysis of the entropy generation. *J. Therm. Anal. Calorim.* **2020**, *141*, 1981–1992. [CrossRef]
33. Khan, U.; Shafiaq, A.; Zaib, A.; Baleanu, D. Hybrid nanofluid on mixed convective radiative flow from an irregular variably thick moving surface with convex and concave effects. *Case Stud. Therm. Eng.* **2020**, *21*, 100660. [CrossRef]
34. Aly, E.H.; Pop, I. MHD flow and heat transfer over a permeable stretching/shrinking sheet in a hybrid nanofluid with a convective boundary condition. *Int. J. Numer. Methods Fluid Flow* **2019**, *29*, 3012–3038. [CrossRef]
35. Zainal, N.A.; Nazar, R.; Naganthan, K.; Pop, I. MHD mixed convection stagnation point flow of a hybrid nanofluid past a vertical flat plate with convective boundary condition. *Chin. J. Phys.* **2020**, *66*, 630–644. [CrossRef]
36. Hanif, H.; Khan, I.; Shafie, S. Heat transfer exaggeration and entropy analysis in magneto-hybrid nanofluid flow over a ver-tical cone: A numerical study. *J. Therm. Anal. Calorim.* **2020**, *141*, 2001–2017. [CrossRef]
37. Gul, T.; Khan, A.; Bilal, M.; Alreshidi, N.A.; Mukhtar, S.; Shah, Z.; Kumam, P. Magnetic dipole impact on the hybrid nanofluid flow over an extending surface. *Sci. Rep.* **2020**, *10*, 8474. [CrossRef] [PubMed]
38. El-Zahar, E.; Rashad, A.; Saad, W.; Seddeek, L. Magneto-hybrid nanofluids flow via mixed convection past a radiative circular cylinder. *Sci. Rep.* **2020**, *10*, 10494. [CrossRef]
39. Al-Hababbeh, O.; Al-Saqqa, M.; Safi, M.; Khater, T.A. Review of magnetohydrodynamic pump applications. *Alex. Eng. J.* **2016**, *55*, 1347–1358. [CrossRef]
40. Farrokhi, H.; Otuya, D.O.; Khimchenko, A.; Dong, J. Magnetohydrodynamics in biomedical applications. In *Nanofluid Flow in Porous Media*; IntechOpen: London, UK, 2020.

41. Messerle, H. *Magnetohydrodynamic Electrical Power Generation*; Wiley: London, UK, 1995.

42. Sheikholeslami, M.; Rokni, H.B. Simulation of nanofluid heat transfer in presence of magnetic field: A review. *Int. J. Heat Mass Transf.* 2017, 115, 1203–1233. [CrossRef]

43. Jamaludin, A.; Naganithran, K.; Nazar, R.; Pop, I. Thermal radiation and MHD effects in the mixed convection flow of Fe$_3$O$_4$–water ferrofluid towards a non-linearly moving surface. *Processes* 2020, 8, 95. [CrossRef]

44. Dogonchi, A.S.; Chamkha, A.J.; Hashemi-Tilehnoee, M.; Seyyedi, S.M.; Haq, R.-U.; Ganji, D.D. Effects of homogeneous-heterogeneous reactions and thermal radiation on magneto-hydrodynamic Cu-water nanofluid flow over an expanding flat plate with non-uniform heat source. *J. Cent. South Univ.* 2019, 26, 1161–1171. [CrossRef]

45. Alwawi, F.A.; Alkasasbeh, H.T.; Rashad, A.; Idris, R. MHD natural convection of sodium alginate Casson nanofluid over a solid sphere. *Results Phys.* 2020, 16, 102818. [CrossRef]

46. Alwawi, F.A.; Alkasasbeh, H.T.; Rashad, A.M.; Idris, R. A numerical approach for the heat transfer flow of carboxymethyl cellulose-water based casson nanofluid from a solid sphere generated by mixed convection under the influence of Lorentz force. *Mathematics* 2020, 8, 1094. [CrossRef]

47. Upadhya, S.M.; Devi, R.L.V.R.; Ragu, C.S.K.; Ali, H.M. Magnetohydrodynamic nonlinear thermal convection nanofluid flow over a radiated porous rotating disk with internal heating. *J. Therm. Anal. Calorim.* 2021, 143, 1973–1984. [CrossRef]

48. Tlili, I.; Samrat, S.; Sandeep, N. A computational frame work on magnetohydrodynamic dissipative flow over a stretched region with cross diffusion: Simultaneous solutions. *Alex. Eng. J.* 2021, 60, 3143–3152. [CrossRef]

49. Khan, I.; Hussain, A.; Malik, M.; Mukhtar, S. On magnetohydrodynamics Prandtl fluid flow in the presence of stratification and heat generation. *Phys. A Stat. Mech. Appl.* 2020, 540, 123008. [CrossRef]

50. Alwawi, F.A.; Alkasasbeh, H.T.; Rashad, A.; Idris, R. Heat transfer analysis of ethylene glycol-based Casson nanofluid around a horizontal circular cylinder with MHD effect. *Proc. Inst. Mech. Eng. Part C* 2020, 234, 2569–2580. [CrossRef]

51. Ali, M.; Hussain, A.; Gorla, R.S.R. Natural convection flow from an isothermal horizontal circular cylinder with temperature dependent viscosity. *Heat Mass Transf.* 2005, 41, 594–598. [CrossRef]

52. Aghamajidi, M.; Yazdi, M.; Dinarvand, S.; Pop, I. Tiwari–Das nanofluid model for magnetohydrodynamics (MHD) natural-convective flow of a nanofluid adjacent to a spinning down-pointing vertical cone. *Propuls. Power Res.* 2018, 7, 78–90. [CrossRef]

53. Khashi‘iel, N.S.; Arifin, N.M.; Nazar, R.; Hafidzuddin, E.H.; Wahi, N.; Pop, I. Magnetohydrodynamics (MHD) axisymmetric flow and heat transfer of a hybrid nanofluid past a radially permeable stretching/shrinking sheet with Joule heating. *Chin. J. Phys.* 2020, 64, 251–263. [CrossRef]

54. Keller, H.B. A new difference scheme for parabolic problems. In *Numerical Solution of Partial Differential Equations–II*; Elsevier: Amsterdam, The Netherlands, 1971; pp. 327–350.

55. Swalmeh, M.; Alkasasbeh, H.T.; Hussanan, A.; Thoi, T.N.; Mamat, M. Microstructure and inertial effects on natural convection micropolar nanofluid flow about a solid sphere. *Int. J. Ambient. Energy* 2019, 1–12. [CrossRef]

56. Nabwey, H.A.; Khan, W.A.; Rashad, A.M. Lie group analysis of unsteady flow of kerosene/coalt ferrofluid past a radiated stretching surface with Navier slip and convective heating. *Mathematics* 2020, 8, 826. [CrossRef]

57. Kumar, K.A.; Sandeep, N.; Sugunamma, V.; Animasaun, I.L. Effect of irregular heat source/sink on the radiative thin film flow of MHD hybrid ferrofluid. *J. Therm. Anal. Calorim.* 2020, 139, 2145–2153. [CrossRef]

58. Sheikholeslami, M.; Hayat, T.; Alsaedi, A. MHD free convection of Al$_2$O$_3$–water nanofluid considering thermal radiation: A numerical study. *Int. J. Heat Mass Transf.* 2016, 96, 513–524. [CrossRef]

59. Merkin, J. Free convection boundary layer on an isothermal horizontal cylinder. In *Proceedings of the American Society of Mechanical Engineers and American Institute of Chemical Engineers, Heat Transfer Conference*, St. Louis, MO, USA, 9–11 August 1976.