Exact Results for the SU(∞) Principal Chiral Model

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Abstract. Exact correlation functions of $N \times N$ matrix-valued quantized fields are not generally known, even as $N$ approaches infinity (the planar limit). This is in stark contrast to isovector field theories, which have straightforward $1/N$-expansions. I review the method by which exact correlation functions of the large-$N$ limit (1 + 1)-dimensional sigma model with $\text{SU}(N) \times \text{SU}(N)$ symmetry. This field theory is asymptotically free, with a dynamically generated mass gap. The technique is a combination of the $1/N$-expansion of the $S$ matrix and Smirnov's form-factor axioms. I briefly discuss how to extract the short-distance behavior of the exact correlation function, which can be compared with perturbation theory.

Feynman diagrams in the large-$N$ limit of matrix theories are planar [1]. Planar diagrams are a small subclass of all the diagrams generated at finite $N$. The cardinality of planar diagrams grows only as a power of the number of loops, in contrast to the cardinality of all diagrams, which grows as the factorial of the number of loops. Furthermore, the expansion in $1/N$ is analogous to the perturbative expansion in a string theory, where the diagrams themselves play the role of worldsheets. This analogy has been nearly completely realized in the AdS/CFT correspondence. For field theories which are not conformal, little progress has been made, unless an ultraviolet cut-off is present, with a large bare coupling (in a more realistic approach, the bare coupling is infinitesimal). In contrast, the $1/N$-expansion an effective way to calculate properties of $N$-component isovector theories (such as the $O(N)$ sigma model); the reason is that a saddle point in a Lagrange multiplier appears. Unfortunately, such saddle-point methods do not work for matrix field theories.

Here I review some recent work on the correlation functions of the $\text{SU}(N) \times \text{SU}(N)$ principal chiral sigma model (PCSM) in one space and one time dimension. This model is integrable, and its particle spectrum and $S$ matrix have been known for some time [2]. Karowski and Weisz found an exact form factor for the current operator of the $N = 2$ PCSM [3]. For $2 < N < \infty$, the PCSM has bound states, which complicates the determination of exact form factors. By combining integrability and the $N \to \infty$ limit, exact form factors and exact Green’s functions can be found [4], [5]. A key ingredient is Smirnov’s axioms for form factors [6].

The action of the $\text{SU}(N) \times \text{SU}(N)$-symmetric PCSM in Minkowski spacetime is

$$S = \frac{N}{2g_0^2} \int d^2x \, \eta^{\mu\nu} \, \text{Tr} \, \partial_\mu U(x) \, \partial_\nu U(x),$$

where $U(x) \in \text{SU}(N)$ is the bare field, $\mu, \nu = 0,1$, $U(x) \in \text{SU}(N)$ (that is, $U(x)$ is an $N \times N$ unitary matrix of determinant one), and the metric is that of flat Minkowski space, $\eta^{00} = 1$, $\eta^{ij} = -1$.
\(\eta^{11} = -1, \eta^{01} = \eta^{10} = 0\). An implicit ultraviolet cut-off \(\Lambda\) is present in (1). The action has the invariance \(U(x) \rightarrow V_L U(x) V_R\), for two constant matrices \(V_L, V_R \in SU(N)\). The PCSM is asymptotically ultraviolet free. Analytic and numerical evidence strongly indicates that the spectrum has a mass gap \(m\). Particle masses are given by the sine formula:

\[
m_r = m \frac{\sin \frac{\pi r}{N}}{\sin \frac{\pi}{N}}, \quad r = 1, \ldots, N - 1.
\]

If \(m\) is fixed, as \(N \rightarrow \infty\), the only surviving bound state of the elementary particles corresponds to \(r = N - 1\). This is the antiparticle. The binding energy, for \(1 < r < N - 1\), vanishes. Furthermore, the residues of the \(1 < r < N - 1\)-bound-state poles of the S matrix also vanish. Thus, in the large-\(N\) limit, only the elementary particle and its antiparticle remain in the spectrum.

The S matrix of two elementary particles of the PCSM, with incoming rapidities \(\theta_1\) and \(\theta_2\) (the components of the momenta are \((p_j)_0 = m \cosh \theta_j, (p_j)_1 = m \sinh \theta_j\), outgoing rapidities \(\theta'_1\) and \(\theta'_2\) and rapidity difference \(\theta = \theta_{12} = \theta_1 - \theta_2\) is

\[
S_{PP} = S_{PP}(\theta) \ 4\pi \delta(\theta'_1 - \theta_1) \ 4\pi \delta(\theta'_2 - \theta_2),
\]

where \(S_{PP}(\theta)\) is a function which acts on the quantum numbers of the particles. The general convention is to refer to \(S_{PP}(\theta)\) by the term “S matrix”, rather than \(S_{PP}\). The \(1/N\)-expansion of the S matrix is explicitly

\[
S_{PP}(\theta) = \left[1 + O(1/N^2)\right] \left[1 - \frac{2\pi i}{N\theta} (P_L \otimes 1_R + 1_L \otimes P_R) - \frac{4\pi^2}{N^2\theta^2} P_L \otimes P_R\right],
\]

where \(P_L\) and \(P_R\) exchange left and right color indices, respectively. The S matrix elements of one particle and one antiparticle \(S_{AP}(\theta)\) can be found from (3), using crossing.

The bare field \(U(x)\) has vanishing correlations, as \(g_0\) is taken to zero and the ultraviolet cut-off is removed. Thus, \(U(x)\) is not the physical field of the renormalized quantum field theory. There is, however, such a physical field, \(\Phi(x)\), which is a complex \(N \times N\) matrix. This scaling field is not directly proportional to the unitary matrix \(U(x)\), but there is an equivalence

\[
\Phi(x) \sim Z(g_0, \Lambda)^{-1/2} U(x),
\]

in the sense that

\[
\frac{1}{N} \langle 0| \text{Tr} \ \Phi(x) \Phi(0) \dagger |0\rangle = Z(g_0, \Lambda)^{-1} \frac{1}{N} \langle 0| \text{Tr} \ U(x) U(0) \dagger |0\rangle.
\]

The renormalization factor \(Z(g_0(\Lambda), \Lambda)\) goes to zero as \(\Lambda \rightarrow \infty\). The coupling is a function of the cut-off \(g_0(\Lambda)\), with the property that the mass gap \(m(g_0(\Lambda), \Lambda)\) is independent of \(\Lambda\). The field \(\Phi\) is normalized by

\[
\langle 0| \Phi(0)_{b_0 a_0} |P, \theta, a_1, b_1\rangle_{in} = N^{-1/2} \delta_{b_0 a_1} \delta_{b_0 b_1},
\]

where the ket on the right is a one particle \((r = 1)\) state, with rapidity \(\theta\) and we implicitly sum over left and right colors \(a_1\) and \(b_1\), respectively. There is no contribution from the one-antiparticle state (with \(r = N - 1\)):

\[
\langle 0| \Phi(0)_{b_0 a_0} |A, \theta, b_1, a_1\rangle_{in} = 0.
\]

In much of what follows, vacuum expectation values, such as those in (4), are Wightman functions, rather than Schwinger functions. This means that there is no time-ordering of local
operators. Towards the end of this article, I will discuss the short-distance behavior of Euclidean Schwinger functions.

Knowledge of form factors, that is matrix elements of local operators, may be used to express Wightman functions through completeness:

$$\mathcal{W}(x) = \frac{1}{N} \langle 0 | \text{Tr} \Phi(x) \Phi(0)^\dagger | 0 \rangle = \sum_X \frac{1}{N} \langle 0 | \Phi(x) | X \rangle_{\text{in}} \langle X | \Phi(0)^\dagger | 0 \rangle,$$

where $X$ denotes an arbitrary asymptotic state. A basic form factor is a matrix element of an operator $\mathcal{B}(x)$ between multi-particle states, that is

$$f^\mathcal{B} (\theta_1, \ldots, \theta_M)_{j_1 \ldots j_M} = \langle 0 | \mathcal{B}(0) | \theta_M, j_M, \ldots, \theta_1, j_1 \rangle_{\text{in}},$$

where $j_1, \ldots, j_M$ are the quantum numbers of the excitation (including whether it is a particle or antiparticle). Other matrix elements may be found by crossing and the Lorentz-transformation properties of the operator $\mathcal{B}(x)$. Smirnov’s axioms [6] are:

1. **Scattering Axiom** or Watson’s theorem. This is

$$f^\mathcal{B} (\theta_1, \ldots, \theta_i, \theta_{i+1}, \theta, \theta_{i+2}, \ldots, \theta_M)_{j_1 \ldots j_i \ldots j_{i+1} \ldots j_{i+2} \ldots j_M} = S^{k_i k_{i+1}}_{j_{i+1} j_{i+2}} (\theta_1 - \theta_{i+1}) f^\mathcal{B} (\theta_1, \ldots, \theta_{i-1}, \theta, \theta_{i+1}, \theta_{i+2}, \ldots, \theta_M)_{j_1 \ldots j_{i-1} k_i k_{i+1} j_{i+2} \ldots j_M}. \quad (8)$$

2. **Periodicity Axiom**, or generalized crossing, which states

$$f^\mathcal{B} (\theta_1, \ldots, \theta_M)_{j_1 \ldots j_M} = f^\mathcal{B} (\theta_M - 2\pi i, \theta_1, \ldots, \theta_{M-1})_{j_M j_1 \ldots j_{M-1}}. \quad (9)$$

3. **Annihilation Pole Axiom**. A justification for this axiom in the context of the SU($\infty$) PCSM is given in the first of Refs. [4]. The axiom states:

$$i \operatorname{Res} f^\mathcal{B} (\theta_1, \ldots, \theta_M)_{j_1 \ldots j_M} \big|_{\theta_M = \theta_M - 2\pi i} = f^\mathcal{B} (\theta_1, \ldots, \theta_{M-2})_{j_1 \ldots j_{M-2}} C_{j_{M-1} j_M}$$

$$- S^{k_{M-1} k_1}_{\theta_1 \theta_{M-1}} (\theta_1 - \theta_{M-1}) S^{k_2 k_3}_{\theta_2 \theta_{M-2}} (\theta_2 - \theta_{M-2}) \cdots S^{k_{M-3} k_{M-2}}_{\theta_{M-3} \theta_{M-2}} (\theta_{M-3} - \theta_{M-2})$$

$$\times S^{k_{M-2} k_{M-1}}_{j_M j_{M-2}} (\theta_{M-2} - \theta_{M-1}) f^\mathcal{B} (\theta_1, \ldots, \theta_{M-2})_{k_1 \ldots k_{M-2}} C_{k_{M-1} j_M}, \quad (10)$$

where $C$ is the charge-conjugation matrix.

4. **Lorentz-Invariance Axiom**. If an operator $\mathcal{B}(x)$ carries Lorentz spin $s$, the form factors must transform under a boost $\theta_j \rightarrow \theta_j + \alpha$ for all $j = 1, \ldots, M$ as

$$f^\mathcal{B} (\theta_1 + \alpha, \ldots, \theta_M + \alpha) = e^{s \alpha} f^\mathcal{B} (\theta_1, \ldots, \theta_M), \quad (11)$$

The spin for the scaling field is zero.

5. **Minimality Axiom**. This is the assumption that the form factors consistent with the other axioms have as much analyticity as possible.

Paradoxically, Axioms 1. and 3. appear trivial at large-$N$, because interactions (see (3)) are of order $1/N$. The resolution of the paradox is that the color indices of the in-state are contracted in such a way as to produce extra factors of $N$ [4], [5]. These contractions produce an effective $S$ matrix between particles and and antiparticles. The rules for constructing exact form factors are described below.

The general matrix element of $\Phi(0)$ between the vacuum and an $(M - 1)$-antiparticle, $M$-particle state has many terms. By comparing it to the $S$-matrix element describing the scattering
of $M$-particles, we can determine the leading part of the form factors in the $1/N$-expansion. This part is proportional to $N^{-M+1/2}$. Left and right permutations (in the permutation group may be written $S_M$) by $\sigma$ and $\tau$, respectively and $\sigma$ and $\tau$ take the set of numbers $0$, $1$, $2$, $\ldots$, $M-1$ to $\sigma(0)$, $\sigma(1)$, $\ldots$, $\sigma(M-1)$ and $\tau(0)$, $\tau(1)$, $\ldots$, $\tau(M-1)$, respectively. The most general form factor of the renormalized field is

$$\langle 0 \mid \Phi(0)_{b_0 a_0} \mid I_1, \theta_1, C_1; I_2, \theta_2, C_2; \ldots; I_{2M-1}, \theta_{2M-1}, C_{2M-1} \rangle = \frac{\sqrt{N}}{N^M} \sum_{\sigma, \tau \in S_M} F_{\sigma\tau}(\theta_1, \theta_2, \ldots, \theta_{2M-1}) \prod_{j=0}^{M-1} \delta_{a_j a_{\sigma(j)+M}} \delta_{b_j b_{\tau(j)+M}},$$

(12)

where $I_j$ is $A$ or $P$ and $C_j$ consists of the color pair $a_j, b_j$. The order $(1/N)^0$ parts of the coefficients of the tensors

$$N^{-M+1/2} \prod_{j=0}^{M-1} \delta_{a_j a_{\sigma(j)+M}} \delta_{b_j b_{\tau(j)+M}},$$

(13)

that is $F_{\sigma\tau}^0(\theta_1, \theta_2, \ldots, \theta_{2M-1})$, are the same, no matter the order of the particles in the in-state ket of (12), except for a phase, as we explain below.

The function $F_{\sigma\tau}$ can be expanded in powers of $1/N$, i.e.

$$F_{\sigma\tau}(\theta_1, \theta_2, \ldots, \theta_{2M-1}) = F_{\sigma\tau}^0(\theta_1, \theta_2, \ldots, \theta_{2M-1}) + \frac{1}{N} F_{\sigma\tau}^1(\theta_1, \theta_2, \ldots, \theta_{2M-1}) + \cdots.$$

(14)

Only the leading term on the right-hand side of (14) is known.

Suppose we interchange two adjacent excitations in the left-hand side of (12). Axiom 1 implies that as $N \to \infty$:

(i) If both excitations are antiparticles or both excitations are particles, the result is the interchange of the rapidities of these two excitations, in the function $F_{\sigma\tau}^0$.

(ii) If one excitation is an antiparticle with rapidity $\theta_j$ and colors $a_j, b_j$ and the other excitation is a particle with rapidity $\theta_k$ and colors $a_k, b_k$, and $\sigma(j) + M \neq k$, $\tau(j) + M \neq k$, there is no effect on the function $F_{\sigma\tau}^0$.

(iii) If one excitation is an antiparticle with rapidity $\theta_j$ and colors $a_j, b_j$ and the other excitation is a particle with rapidity $\theta_k$ and colors $a_k, b_k$, and $\sigma(j) + M = k$, $\tau(j) + M \neq k$, then $F_{\sigma\tau}^0$ is multiplied by the phase $\frac{\theta_{j+k+\pi i}}{\theta_{j-k-\pi i}}$.

(iv) If one excitation is an antiparticle with rapidity $\theta_j$ and colors $a_j, b_j$ and the other excitation is a particle with rapidity $\theta_k$ and colors $a_k, b_k$, and $\sigma(j) + M \neq k$, $\tau(j) + M = k$, then $F_{\sigma\tau}^0$ is multiplied by the phase $\frac{\theta_{j+k+\pi i}}{\theta_{j-k-\pi i}}$.

(v) If one excitation is an antiparticle with rapidity $\theta_j$ and colors $a_j, b_j$ and the other excitation is a particle with rapidity $\theta_k$ and colors $a_k, b_k$, and $\sigma(j) + M = k$, $\tau(j) + M = k$, then $F_{\sigma\tau}^0$ is multiplied by the phase $\left(\frac{\theta_{j+k+\pi i}}{\theta_{j-k-\pi i}}\right)^2$.

These conditions yield a choice of $F_{\sigma\tau}^0(\theta_1, \theta_2, \ldots, \theta_{2M-1})$, satisfying the annihilation-pole axiom and having as much analyticity as possible:

$$F_{\sigma\tau}^0(\theta_1, \theta_2, \ldots, \theta_{2M-1}) = \frac{(-4\pi)^{M-1} K_{\sigma\tau}}{\prod_{j=1}^{M-1} (\theta_j - \theta_{\sigma(j)+M + \pi i})(\theta_j - \theta_{\tau(j)+M + \pi i})},$$

(15)
where

$$K_{\sigma\tau} = \begin{cases} 1, & \sigma(j) \neq \tau(j), \text{ for all } j \\ 0, & \text{otherwise} \end{cases}$$

(16)

Notice that the expression for $K_{\sigma\tau}$ insures the absence of double poles. Thus $K_{\sigma\tau}$ is unity if and only if the composite permutation $\sigma \circ \tau^{-1}$ has no fixed points, i.e. has the smallest possible fundamental character in $S_M$. The number of pairs $\sigma$ and $\tau$ in $S_M$ satisfying this condition is $(M-1)!M!$. Equations (15) and (16) completely determine the form factors in the large-$N$ limit.

Now that the form factors are determined, the Wightman function can be found. It is convenient to write (6) as

$$W(x) = \frac{1}{N} \sum_{a_0,b_0} \sum_{X} \langle 0|\Phi(0)_{b_0a_0}|X\rangle\ln \ln \langle X|\Phi(0)_{b_0a_0}|0\rangle^*|0\rangle e^{ipx \cdot x}$$

$$= \frac{1}{N} \sum_{a_0,b_0} \sum_{X} \langle 0|\Phi(0)_{b_0a_0}|X\rangle^2 e^{ipx \cdot x},$$

where $p_X$ is the momentum eigenvalue of the state $|X\rangle$. After some work, the result is the remarkably simple expression:

$$W(x) = \int \frac{d\theta}{4\pi} e^{ixp} + \frac{1}{4\pi} \sum_{l=1}^{\infty} \int d\theta_1 \cdots d\theta_{2l+1} e^{i \sum_{j=1}^{l} x_j p_j} \prod_{j=1}^{2l} \frac{1}{(\theta_j - \theta_{j+1})^2 + \pi^2}. \tag{17}$$

For $m|x| \gg 1$, the expression (17) decays exponentially, as expected. On the other hand, if (17) is exact for all choices of $x$, then for $m|x| \ll 1$, the Wick-rotated time-ordered product of two scaling field operators should behave as

$$\frac{1}{N} \langle 0|\mathcal{T} \text{ Tr } \Phi(x)\Phi(0)^\dagger|0\rangle = C_2 (\ln m|x|)^2 + C_1 \ln m|x| + C_0 + O(1/\ln m|x|), \tag{18}$$

for some constants $C_2, C_1, \text{etc.}$

Here is the standard argument for (18) (see for example, Ref. [7]). After a Wick rotation $x^0 \to ix^0$, the regularized Euclidean correlation function is $G(|x|, \Lambda) = N^{-1} \langle 0|\mathcal{T} \text{ Tr } \Phi(x)\Phi(0)^\dagger|0\rangle$. This function and the coupling $g_0(\Lambda)$ satisfy the renormalization group equations

$$\frac{\partial \ln G(R, \Lambda)}{\partial \ln \Lambda} = \gamma(g_0) = \gamma_1 g_0^2 + \cdots, \quad \frac{\partial g_0(\Lambda)}{\partial \ln \Lambda} = \beta(g_0) = -\beta_1 g_0^3 + \cdots, \tag{19}$$

respectively. The coefficients of the anomalous dimension $\gamma_1(g_0)$ and the beta function $\beta_1(g_0)$ are $\gamma_1 = (N^2 - 1)/2N^2$ and $\beta_1 = 1/(4\pi)$. For large $\Lambda$, $G(R, \Lambda)$ becomes a function of the product of the two variables $G(R\Lambda)$. Integrating (19) yields the leading behavior

$$G(R, \Lambda) \sim C[\ln(R\Lambda)]^{\gamma_1/\beta_1}. \tag{20}$$

As $N \to \infty$, the power $\gamma_1/\beta_1$ becomes 2.

Performing the Wick-rotation on (17) is done by setting $x^4 = 0$ and replacing $x^0$ by $iR$, with $R > 0$. This changes the phases in (17) by $\exp ip_j, x \to \exp -mR \cosh \theta_j$. It is convenient to introduce $L = \ln \frac{1}{mR}$. As $mR$ becomes small, $\exp -mR \cosh \theta_j$ becomes approximately the characteristic function of $(-L, L)$, equal to unity for $-L < \theta < L$ and zero everywhere else (the same argument was used to explain how walls form in the Feynman-Wilson gas [8]). This basic
idea was used to find the scaling behavior of Ising-model correlation functions [9] from the exact form factors [10]. The short-distance approximation to the Euclidean two-point function is

\[ G(mR) = \frac{L}{2\pi} + \frac{1}{4\pi} \sum_{l=1}^{\infty} \int_{-L}^{L} d\theta_1 \cdots \int_{-L}^{L} d\theta_{2l+1} \frac{1}{\prod_{j=1}^{2l} (\theta_j - \theta_{j+1})^2 + \pi^2} . \]  

(21)

This series (21) is the partition function of a polymer in a box of size 2L. The \( j \)th atom in the polymer chain is located at \( \theta_j \). The long-range potential energy is \( \ln[(\theta_j - \theta_{j+1})^2 + \pi^2] \), between connected pairs of atoms in the chain.

Redefining the variables by \( \theta_j = Lu_j \), (21) becomes

\[ G(mR) = \frac{L}{2\pi} + \frac{L}{4\pi} \sum_{l=1}^{\infty} \int_{-1}^{1} du_1 \cdots \int_{-1}^{1} du_{2l+1} \frac{1}{\prod_{j=1}^{2l} (u_j - u_{j+1})^2 + (\pi/L)^2} . \]

(22)

The factors in the series (22) are closely related to the generators of the Poisson semigroup. Consider the transfer operators \( P(a) \), whose matrix elements are defined by \( \langle u'|P(a)|u \rangle = a((u' - u)^2 + a^2)^{-1/2} \), where \( u' \) and \( u \) are arbitrary real numbers. These operators form the Poisson semigroup, with the composition law \( P(a)P(b) = P(a + b) \). Specifically, \( P(a) = \exp -a\Delta^{1/2} \), where \( \Delta^{1/2} = \sqrt{-d^2/dx^2} \) is the fractional Laplacian in one dimension.

As I will discuss in a future publication, Eq. (18) follows from the existence of a self-adjoint extension of \( \Delta^{1/2} \) on the Hilbert space of functions which vanish outside the open interval \( u \in (-1, 1) \). This operator has an infinite set of discrete eigenvalues \( \lambda_n \), of the eigenfunctions \( \varphi_n(u) \), \( n = 1, 2, \ldots \), with \( 0 < \lambda_1 < \lambda_2 < \cdots \), with \( \varphi_n(\pm 1) = 0 \). Discussion of the spectrum of \( \Delta^{\alpha/2} \), with real \( \alpha \in (0, 2) \), may be found in [11]. For \( u, u' \in (-1, 1) \),

\[
\frac{1}{L((u - u')^2 + (\pi/L)^2)} = \langle u'|e^{-\frac{1}{2}H(L)}|u \rangle ,
\]

where the operator in the exponent is

\[ H(L) = \Delta^{1/2} + O(L^{-1}) . \]

(23)

Summing over \( l \) in Eq. (22) yields

\[ G(mR) = \frac{L}{4\pi} \sum_{n=1}^{\infty} \left[ \int_{-1}^{1} du \varphi_n(u) \right]^2 \frac{1}{1 - \exp[-2\pi \lambda_n/L + O(L^{-2})]} . \]

(24)

The expression (24) yields the first two coefficients in (18):

\[ C_2 = \frac{1}{8\pi^2} \sum_{n=1}^{\infty} \left[ \int_{-1}^{1} du \varphi_n(u) \right]^2 \lambda_n^{-1} , \quad C_1 = \frac{1}{4\pi} = 0.0796 . \]

(25)

An upper bound on the leading coefficient \( C_2 \) is obtained by replacing \( \lambda_n \) in (25) by \( \lambda_1 \). This yields \( C_2 < \frac{1}{4\pi^2 \lambda_1} = 0.0219 \), from the numerical result \( \lambda_1 = 1.1577 \), found in the second of Ref. [11].

To summarize, the short-distance behavior of the exact correlation function agrees with that predicted by perturbation theory. This, in my opinion, is convincing support of the validity of the large-\( N \) form factors, found in Refs. [4], [5].
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