Optimization of Vehicle Trajectories Considering Uncertainty in Actuated Traffic Signal Timings

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Abstract—This paper introduces a robust green light optimal speed advisory (GLOSA) system for fixed and actuated traffic signals considering a probability distribution. These distributions represent the domain of possible switching times from the signal phasing and timing (SPaT) messages. The system finds the least-cost (minimum fuel consumption) vehicle trajectory using a computationally efficient $A^*$ algorithm incorporated within a dynamic programming (DP) procedure to minimize the vehicle's total fuel consumption. Constraints are introduced to ensure that vehicles do not collide with other vehicles, run red indications, or exceed a maximum vehicular jerk for passenger comfort. Results of simulation scenarios are evaluated against empirical comparable trajectories of uninformed drivers to compute fuel consumption savings. The proposed approach produced significant fuel savings compared to an uninformed driver behavior amounting to 37% on average for deterministic SPaT and 30% for stochastic SPaT data. A sensitivity analysis was performed to understand how the degree of uncertainty in SPaT predictions affects the optimal trajectory's fuel consumption. The results present the required levels of confidence in these predictions to achieve savings in fuel consumption. Specifically, the study demonstrates that the proposed system can be within 85% of the maximum savings if the timing error is (±3.3 seconds) at a 95% confidence level. Results also emphasize the importance of more reliable SPaT predictions as the time to green decreases relative to the time the vehicle is expected to reach the intersection given its current speed.

Index Terms—Actuated traffic signals, optimizing vehicle trajectories, uncertain switching times, eco-driving, stochastic optimization.

I. INTRODUCTION

T

HE global energy system is severely affected by shortages in fuel supply. Excessive use of fossil fuels exacerbates climate change and results in more heat waves that impose global health threats [1]. The transportation sector is responsible for about 28% of the total energy consumed by the U.S. (based on 2021 statistics), of which 90% comes from petroleum products [2]. This sector also contributes 27% of the total greenhouse gas emissions, resulting in air quality degradation and climate change [3], [4], [5]. As a result, there is a global trend to reduce the consumption of fossil fuels in the transportation sector by raising awareness of the crisis, using energy-efficient systems, and using alternative energy sources.

Eco-driving is an efficient and cost-effective system that can significantly reduce vehicle fuel consumption and was proven to achieve fuel savings of up to 45% [6]. Eco-driving at signalized intersections is most commonly referred to as green light optimal speed advisory (GLOSA). Traffic signals introduce travel delays caused by vehicle stops during red indications. Traffic signals also impose additional fuel consumption due to idling and aggressive acceleration, which is estimated to be 2.8 million gallons in the U.S. [7], [8].

Researchers have conducted studies to improve signal control and eco-driving systems to reduce fuel losses at intersections using infrastructure-to-vehicle (I2V) communication technologies [9]. Q-learning, artificial neural networks, fuzzy logic, and game theory were all employed to optimize traffic signal controllers [10], [11], [12], [13]. In parallel, dynamic eco-driving models have been developed over the past decade [14]. These models attempt to reduce vehicle fuel consumption caused by unnecessary decelerations and accelerations upstream and downstream of signalized intersections, which allows vehicles to reduce idling at red lights using signal phasing and timing (SPaT) information.

In fixed-time signal controllers, SPaT messages indicate exact information about signal switching times, which enables vehicles to deterministically optimize their speed profile. This optimization significantly reduces fuel consumption levels. Kamalanathsharma and Rakha [15] developed an eco-cooperative adaptive cruise control (Eco-CACC) system that uses I2V communication to recommend fuel-efficient speeds for a single vehicle approaching a fixed-time traffic signal controller. The algorithm’s objective function was explicitly set to minimize fuel consumption. Using a modified $A^*$ algorithm, the problem was solved through dynamic programming (DP) to achieve average fuel savings of up to 30%.

A reliable switching time is not currently available in SPaT messages for actuated traffic signal controllers, which are widely deployed in the U.S., due to real-time signal control updates. Previous research used the available data in SPaT messages to predict a most likely switching time or exceed a maximum vehicular jerk for passenger comfort.
A. Eco-Driving at Actuated Traffic Signals

Given the uncertainty in switching time predictions, significant levels of fuel savings can still be achieved, as indicated by Mousa et al. [17], who presented an eco-driving application that optimizes the vehicle’s trajectory in terms of fuel consumption and emissions at semi-actuated traffic signals. The application’s framework uses detectors located 300 meters upstream of the minor approaches, which allows the prediction of red signal indications at major approaches. The system demonstrated a reduction in fuel consumption of up to 23.2%. However, it should be noted that this framework is limited to semi-actuated signals and does not take into account fully actuated controllers. The study also used a brute-force algorithm to find the optimal trajectory, which is time-consuming and may not be practical for real-world applications. Furthermore, the framework would require additional infrastructure equipment for real-world application, which would add to the cost of implementation in real-world scenarios.

Regarding fully actuated signals, Mahler and Vahidi [18] developed an algorithm to plan the vehicle’s optimal velocity while approaching actuated traffic signals. SPaT predictions in their algorithm are based on historical data and real-time phase information. This study used a simplified cost function representing fuel consumption levels rather than an actual fuel consumption model to avoid the associated computational complexity. Moreover, the control algorithm does not take into consideration the signal switching time prediction error, which leads to a risk of running a red light when the vehicle is close to the signal stop bar. Results showed fuel savings of up to 6% over the uninformed driver for actuated signals.

Hao et al. developed an eco-approach and departure (EAD) application to optimize vehicle trajectories for actuated signals [19], [20]. The framework takes into account the calculated maximum and minimum time to the next signal phase and the movement of the vehicle ahead detected by radar. The study found that fuel consumption can be reduced by up to 12% under optimal conditions. However, the study was limited by relying on a range of predicted signal timings from minimum to maximum specified values, rather than a prediction of the switching time probability distribution, which results in relatively low fuel consumption savings.

Another stochastic eco-driving control system was developed by Sun et al. [21]. This system deals with a vehicle proceeding through multiple actuated traffic signal controllers with uncertain SPaT information. The problem was formulated as a chance-constrained stochastic program, in which an effective red-light duration (ERD) concept was introduced to capture the randomness in the traffic signal switching times. Using DP, the study reported 40% savings in fuel consumption with a 5% sacrifice in arrival times. However, using distributionally robust optimization (DRO) may not be the most efficient option, as it considers the worst-case scenario among all possible distributions, which can lead to overly conservative trajectories that sacrifice potential gains in performance. Additionally, using DRO adds significantly to the computational complexity, with the fuel consumption model already having a high level of computational complexity due to using engine and transmission torque and the gear number in the calculations, making it inefficient for real-time applications.

Typaldos et al. [22] proposed a framework for a GLOSA system for both deterministic and stochastic settings of signal switching times. The framework combined analytical solutions and stochastic dynamic programming (SDP). The results of the study indicated that the proposed framework had a promising potential for fuel savings. However, it was also noted that computation for the SDP solution took several minutes when executed on a normal PC, making the developed system potentially unsuitable for real-time applications, as it would not provide real-time advisory speeds efficiently from a vehicle perspective. In addition, the developed system does not use an explicit form of fuel consumption minimization as an objective function for the optimization problem.

This research addresses the limitations of previous efforts by developing a GLOSA system that provides instantaneous advisory acceleration/deceleration policies to vehicles approaching fully actuated traffic signal controllers. The system utilizes a progressively updated probability distribution of the signal switching times at each time step while ensuring vehicle safety by not running a red indication or aggressively decelerating. Additionally, the system takes advantage of existing infrastructure, making it a low-cost option for field implementation and testing. The optimization system also explicitly minimizes vehicle fuel consumption and provides a quantification measure of acceptable levels of switching time prediction error from a practical GLOSA application standpoint, which will be further discussed in subsequent sections.

II. Problem Statement

Various researchers have devised methods for dealing with uncertainty in SPaT predictions and making eco-driving control decisions that achieve fuel savings despite that uncertainty. However, the literature lacks information on the extent of the effect of SPaT uncertainty on achievable fuel savings. This makes it difficult to assess when a SPaT prediction is good enough or when the error in SPaT prediction is within reasonable limits. Knowledge of the effect of a SPaT information error on the achievable fuel consumption can help assess the validity of the SPaT most likely time prediction from an application standpoint. This can also help with choosing a statistical or machine learning model. Better models can be chosen by looking at their prediction error distributions and having a reference for how this error is expected to affect the fuel consumption savings given optimal control during eco-driving. This study also investigates the effect of the degree of uncertainty in SPaT predictions on the achievable fuel savings given optimal vehicle control (Fig. 1).

A. Study Contribution

This study makes several contributions to the existing body of knowledge and has potential benefits to practitioners including infrastructure operators and automotive OEMs. This study also addresses the drawbacks in the literature mentioned above. The main contributions can be summarized as follows:
Extending the deterministic vehicle control algorithm proposed by Kamalanathsharma and Rakha [15] to a stochastic algorithm, which explicitly minimizes vehicle fuel consumption. In addition, the system also accounts for vehicle jerk constraints to ensure passenger comfort.

Comparing the fuel consumption savings resulting from the proposed system with actual field trajectories for the case of uninformed drivers.

Detailing a framework for the safe application of GLOSA given a stochastic prediction. The system’s safety is ensured through a risk assessment procedure to prevent red light violations and aggressive braking.

Identifying benchmarks for practitioners attempting to use statistical or machine learning models to predict SPaT most likely switching times regarding both model bias and variance required to achieve the benefits of GLOSA.

Providing OEMs with an objective quantification of the effect of level of confidence in SPaT predictions that they can use to reliably control their vehicles.

III. METHODOLOGY

In this paper, we extend the work of Kamalanathsharma and Rakha that introduced a deterministic vehicle control algorithm [15] by constraining the vehicle acceleration level with comfortable levels of jerk and by including the case of actuated signal controllers. The problem of optimizing vehicle trajectory is formulated as a robust optimal control problem, in which the real-time uncertain signal switching time is used to calculate and update the optimal vehicle control policy in real time. The updated policy is based on new information received through I2V communication, in which a switching time probabilistic distribution is transmitted to the vehicle in real time.

The optimization system explicitly minimizes vehicle fuel consumption as an objective function. DP and the A* algorithm are used to solve the problem numerically to mitigate the computational complexity. The robustness of the algorithm toward uncertain signal information is ensured through a risk assessment procedure to prevent red light violations. The eco-driving approach simulation results are compared to the case of an uninformed driver approaching a traffic signal without prior information about the switching time as a baseline. The uninformed driver data were retrieved from a field experiment conducted at the Virginia Smart Road test facility at the Virginia Tech Transportation Institute [23].

This study adopts a four-step research methodology:

1) Define the eco-driving problem at hand with all different analysis scenarios.

2) Develop a holistic eco-driving system that takes stochasticity and changing information into account without violating any of the constraints imposed by the vehicle and the traffic signal.

3) Evaluate the performance of the system under:
   a) The different pre-defined scenarios.
   b) Various levels of bias and variance introduced in the predictions.

4) Identify the effect of the vehicle’s initial speed on the overall system performance.

A. Base Analysis Scenarios

The base scenario is defined as follows: we have a vehicle traveling on a single-lane approach, with no other traffic, with a free-flow speed ($v_0$) of 17.8 m/s (40 mph), which is common for urban arterials. As shown in Fig. 2, $X_0$ is the beginning position, which is 250 meters upstream of the stop bar of an actuated traffic signal controller located at the position $X_M$. The choice of 250 meters is due to the reliable range of dynamic short-range communication (DSRC) and cellular vehicle-to-everything (C-V2X) existing technologies beyond which significant message losses and interference occur [24], [25]. The vehicle should accelerate to the desired speed at or before the position $X_N$, which is 180 meters downstream of the stop bar. The vertical grade (G) is chosen to be 3% either uphill or downhill. This is the same setup used in the field experiment [23], which allows us to use field data collected about uninformed drivers as a baseline.

The assumption is that the vehicle is within the communication range of an actuated traffic signal controller. The signal is assumed to be transmitting SPaT information to the vehicle in real time. Given the stochastic nature of actuated signals, SPaT information contains uncertain most likely signal switching times from red to green; this information is in the form of a probability distribution of the switching time or a confidence level in the most likely switching time. Assuming a normal distribution for the errors, a confidence level can be used to infer a standard deviation and a mean for the distribution.

B. Additional Scenarios

The same problem setup is applied across multiple different scenarios. As shown in Table I, we have the uninformed driver (Scenario I), in which the vehicle driver approaches a traffic
signal that is currently red. The driver is not aware of the traffic signal switching time, which randomly takes one value from the set of switching times (10, 15, 20, or 25 s). In this scenario, the vehicle trajectory data are retrieved from a previous field experiment conducted on the Smart Road test facility.

In simulated Scenarios II and III, the vehicle is provided with signal switching time information to optimize the vehicle trajectory. In Scenario II, a fixed time signal is simulated by providing the vehicle with the exact switching time. A deterministic vehicle trajectory optimization algorithm is used to plan the acceleration/deceleration policy. In Scenario III, an actuated traffic signal controller is simulated by providing the vehicle with stochastic switching times.

### C. Stochastic Data Generation

Real data are gathered from the intersection between Gallows Road and Prosperity Avenue/Park Tower Drive in Northern Virginia, and predictions are made replicating the same model proposed by Eteifa and Rakha [16]. The predictions and true values are aggregated to construct a lookup table to be used in the simulations. The lookup table is then binned according to the true value of switching time into 1-second bins. For each 1-second bin, field data (true values and predictions) are used to define the error distribution specific to that time-to-green true value. For example, the error distribution if only 3 seconds are remaining would be significantly narrower than if 20 seconds are remaining until the green phase. Note that these bins are used to replicate a dynamic state of a prediction model in the field where the SPaT information will contain a most likely time that is updated every 1 second and the vehicle is expected to respond to that new prediction.

Even though the data used in the simulations are actual field data and the predictions are made by an actual model, it should be noted that the methodology presented is not specific to the model used and can be extended to any model with any error distribution. The sensitivity analysis presented in Section D of the results and analysis shows the effect of varying the error of the model, represented by the bias and variance of a normal distribution, on the overall performance of the proposed stochastic GLOSA system.

### IV. STOCHASTIC PROBLEM STATEMENT AND FORMULATION

To develop an optimal trajectory planning tool for GLOSA, the problem is formulated as a stochastic optimal control problem. The exact signal switching time is assumed to be unknown by the system. Instead, the most likely switching time or a probabilistic distribution of switching times is used to plan the trajectory. We assume that the vehicle receives uncertain signal information in real time as it approaches the intersection.

The problem is identified as follows: let \( t \) be a discretized time variable that belongs to the set \( \{ t_0, \ldots, t_f \} \), where \( t_0 \) is the initial timestamp when the vehicle enters the system, and \( t_f \) is the final timestamp when the vehicle arrives at the destination. The period \( \Delta t \) is the difference between two timestamps, which is assumed to be a decisecond in this application.

Equation (1) shows the objective function, which minimizes the summation of the expected value of fuel consumption given the speed control policy represented by the required acceleration/deceleration level \( a_t \) at each timestamp \( t \), where \( FC_t \) is the instantaneous fuel consumption at time \( t \).

\[
\text{Min} \sum_{t=t_0}^{t_f} E(FC_t(a_t)) \cdot \Delta t
\]  

Further, let \( x_t \) be a discretized distance variable at time \( t \) that belongs to the space \( X \subset \mathbb{R}^n \), and \( X_t \) is the vehicle position at time \( t \). \( v_t \) is the speed variable at time \( t \) that depends on the acceleration/deceleration level \( a_t \), which is governed by either the throttle input \( f_{th,t} \) or the braking deceleration level.

\[
X_t = \sum_{t=t_0}^{t_f} x_t = \sum_{t=t_0}^{t_f} v_t \cdot \Delta t
\]

The admissible speed policy space is constrained by the vehicle dynamics, acceleration, jerk, and other system control constraints that ensure safety and comfort.

The problem constraints are illustrated as follows: (3) shows the upstream distance constraint, where the vehicle traveled upstream distance is equal to the traveled distance from the initial position \( X_0 \) at time \( t_0 \) until the time when the signal switches to green at position \( X_S \). Knowing that the switching time is uncertain in this case, the expected value of the switching time \( t_s \) is used. The traveled upstream distance is upper bounded by the position of the stop bar \( X_M \).

Note that using the expected value of the switching time denotes a risk-neutral attitude, where the upstream optimal trajectory is planned regardless of the risk of running a red light. However, this risk is eliminated by introducing a risk assessment procedure, where a critical stopping distance \( d_{sr,t} \) is calculated at every time step \( t \). When the remaining distance to the stop bar is less than or equal to \( d_{sr,t} \), the vehicle policy is set to decelerate at the maximum allowable rate \( \alpha \). This maximum deceleration rate is set to \(-6.0m/s^2\), which is considered a comfortable level of deceleration to stop. This is similar to an approach described in the literature that defined a last-resort parabola for the minimum stopping distance [26].

\[
X_s = \sum_{t=t_0}^{t_f} v_t \cdot \Delta t \leq X_M v_{t_f} + \Delta t
\]

\[
= v_t + \alpha \cdot \Delta t \quad \forall t \leq E(t_s), \quad \forall X_t \text{ where } X_M - X_t \leq d_{sr,t}
\]
Similarly, the downstream speed policy is constrained to cover the downstream distance in addition to the remaining distance to the stop bar in the case when the signal switches before the vehicle reaches the position $X_M$ (5). Note that this constraint is applied after the true switching time $t_s$ is revealed.

$$\sum_{t=t_s}^{t_f} v_t \Delta t = X_N + (X_M - X_s) \quad (5)$$

The speed policy is further constrained by the kinematic equation (6), where $a_t$ is the vehicle acceleration/deceleration level. Equation (7) shows the speed limit constraint. Finally, equation (6), where $a_t$ is the vehicle acceleration/deceleration level. Equation (7) shows the speed limit constraint. Finally, equation (6), where $a_t$ is the vehicle acceleration/deceleration level. Equation (7) shows the speed limit constraint.

$$v_t + a_t \Delta t \forall t \in \{t_0, \ldots, t_f\} \quad (6)$$

$$v_t \leq v_{lim} \forall t \in \{t_0, \ldots, t_f\} \quad (7)$$

$$a_t + \Delta t \leq a_t + 1.3 \cdot \Delta t \forall t \in \{t_0, \ldots, t_f\} , \forall f_{br,t} \quad (8)$$

V. UNDERLYING SYSTEMS

Vehicle dynamics models are utilized to define the spatiotemporal variables according to the current state variables and the acting forces on the vehicle including the tractive, aerodynamic, rolling, and grade resistance forces. As the solution space is discretized in time and space, vehicle dynamics and fuel consumption models are used to evaluate the optimal vehicle trajectory at each time step $\Delta t$.

A. Vehicle Dynamics Model

Based on the throttle level or braking level inputs, it is required to model the vehicle’s acceleration and deceleration resulting behavior considering all the acting forces and constraints on the vehicle. As the scope of this research is optimizing the trajectory of light-duty vehicles, the dynamic model for light-duty vehicles on varied terrain was used [28]. The vehicle acceleration, tractive, and resistance forces are computed in this model as follows:

- The vehicle acceleration is computed by dividing the net force acting on the vehicle, which is the tractive force minus the resisting force, by the vehicle mass $m$ (9), where $F_t$ and $R_t$ are the tractive and resistance forces at time $t$, respectively.

$$a_t = \frac{F_t - R_t}{m} \quad (9)$$

- The vehicle’s tractive force is computed as the acting force by the engine. This force is upper bounded by the maximum tractive force between the vehicle tires and the roadway pavement (10), where $f_{br,t}$ is the throttle input (from 0 to 1), $\eta_d$ is the driveline efficiency, and $\beta$ is the factor that accounts for gear shift impacts (set to 1.0 for light-duty vehicles) [28]. $P_t$ is the vehicle power at time $t$, $v(t)$ is the vehicle speed at time $t$, $M_{fa}$ is the vehicle mass on the tractive axle (kg), $\mu$ is the road friction or adhesion coefficient, and $g$ is the gravitational acceleration in ($\text{m/} \text{s}^2$).

$$F_t = \min \left[ \frac{3600 f_{br,t} \eta_d \beta P(t)}{v(t) M_{fa} g} \mu \right] \quad (10)$$

- The acting resistance force on the vehicle is the summation of the vehicle’s rolling, aerodynamic, and grade resistance forces (11), where $\rho$ is the air density at sea level and $25^\circ \text{C}$, $C_d$ and $C_h$ are the vehicle drag coefficient and the altitude correction factor, respectively, $A_f$ is the vehicle frontal area, and $c_\rho, c_\alpha, \text{and} c_r$ are the rolling resistance constants.

$$R_t = \frac{\rho}{25.91} C_d C_h A_f v_t^2 + m g \frac{C_R}{1000} (c_\rho + c_\alpha + c_r) + m g G_t \quad (11)$$

B. Fuel Consumption Model

It is required to estimate accurate rates of fuel consumption levels consistent with actual in-field measurements. To achieve that, the Virginia Tech Comprehensive Power-Based Fuel Consumption model (VT-CPFM-1) is used [29]. This model overcomes the limitations of existing fuel consumption models in the literature by eliminating the bang-bang control behavior. In addition, it is easily calibrated using publicly available vehicle data. It is also known for its simplicity and accuracy in calculating instantaneous fuel consumption from the instantaneous vehicle power. Further details about the model can be found in the literature [29]. The formulation of the used model is provided below (12):

$$FC_t = \begin{cases} \alpha_0 + \alpha_1 P_t + \alpha_2 P_t^2 & \forall P_t > 0 \\ \alpha_0 & \forall P_t \leq 0 \end{cases} \quad (12)$$

The parameters $\alpha_0, \alpha_1, \text{and} \alpha_2$ are the model constant calibrated for the specific vehicle in use. The instantaneous vehicle power $P_t$ is calculated according to (13).

$$P_t = \left( \frac{R_t + 1.04 m a_t}{3600 \eta_d} \right) v_t \quad (13)$$

Here $a_t$ and $v_t$ are the instantaneous acceleration and speed variables, respectively, $m$ is the vehicle mass, and $\eta_d$ is the driveline efficiency. As the vehicle used in the field test is a 2014 Cadillac SRX, we used this vehicle’s calibrated parameters so that we can compare the simulated results with the field experiment results as a baseline for our calculations. The vehicle parameters are provided in TABLE II.

VI. SOLUTION APPROACH

A. Stochastic Dynamic Programming

Given the nonlinear stochastic optimization setting in this particular problem, finding the optimal speed policy is a
computationally complex procedure. This complexity needs to be reduced to enhance the real-world applicability of this algorithm to find, in a feasible time frame, a heuristic solution close enough to the optimal one. DP is one of the most powerful methods to solve stochastic optimization problems in discretized time utilizing Bellman’s principle of optimality, and it is well known for its significant reduction of computation complexity [30].

In our problem, we have a single control variable \( (m = 1) \), which is either a throttle input or a deceleration level. The problem is solved over a control time horizon of length \( T = (t_f - t_0) / \Delta t \) (where \( t_0 \) is the simulation starting time, and \( t_f \) is the final time when the vehicle reaches position \( X_N \)). Thus, the solution space would be \( R^{m \times T} \). DP decomposes the problem to a sequence of \( T \) problems on \( R^m \), which is a significant reduction in computational complexity.

As depicted in Fig. 3, the solution approach is described as follows:

1) The vehicle enters the upstream link with an initial state \( S_0 \) at position \( X_0 \) while the signal is red.

2) Based on the current speed, the system calculates the critical distance \( d_{cr,t} \) required to decelerate at the maximum allowable deceleration level \( (\alpha = -6m/s^2) \). In addition, the system checks the remaining distance \( d_i \) to the signalized intersection stop line.

3) While the signal is red, the system evaluates the risk of running the red light. The risk is evaluated by comparing the remaining distance to the intersection with the critical stopping distance. In this case, there are two possibilities:

a) The red-light violation risk occurs if the remaining distance is less than or equal to the critical distance. In this case, a deceleration policy with the rate \( \alpha \) is adopted.

b) If there is no violation risk, the system will receive SPaT information, generate the next upstream state policy \( S_{t+1} \) for the vehicle using the A* algorithm, and repeat Steps 1 through 3.

4) If the signal turns green, the system will generate the next downstream policy states until reaching position \( X_N \) using the A* algorithm.

B. Safety Distance Buffer

Preventing the vehicle from running a red light is ensured through a risk assessment procedure by calculating the critical stopping distance \( d_{cr,t} \) based on the current speed and a desired deceleration level as \( v_f^2 / 2a_{des} \), where \( a_{des} \) is the desired deceleration level. When the vehicle’s distance to the intersection is less than the critical distance, the vehicle will start decelerating to a stop, as long as the signal indication is still red. This procedure is defined to prevent the vehicle from running a red light and to ensure that it decelerates at some acceptable level, which might happen due to the uncertainty in the switching time predictions. However, this procedure can also lead to unnecessary fuel losses when the vehicle starts decelerating, the signal switches to green, and then the vehicle accelerates back to the desired speed, losing some fuel.

To mitigate this issue, an enhancement is proposed by incorporating a real-time calculated safety time buffer \( (B) \). This buffer represents an additional delay imposed by the system to prevent the vehicle from reaching the critical distance to the intersection and activating the risk assessment procedure. As shown in Fig. 4, the safety buffer \( (B) \) is calculated in real time using the expected time to green \( (TTG) \) and the critical distance \( d_{cr,t} \), according to the analytical solution shown in (14). It is shown that the vehicle avoids reaching the critical distance to the intersection until the signal switches to green. It is also noted that in some cases, when the switching time is relatively long, the vehicle will have to stop anyway and idle at the intersection until the light turns green.

\[
B = \frac{TTG}{d_{cr,t}} - 1 \tag{14}
\]

C. The A* Algorithm

The A* algorithm is a pathfinding algorithm that uses a heuristic cost estimate to determine the next state policy.
leading to the shortest path. It was used previously in the literature to plan the vehicle trajectory in deterministic settings [15]. The $A^*$ algorithm is used to estimate the least-cost next-state acceleration/deceleration policy by assuming this policy would remain the same for the remainder of the time horizon.

As shown in Fig. 5, the $A^*$ cost estimate heuristic is calculated over two sections, upstream and downstream, and is described as follows:

1) In the upstream section, the $A^*$ algorithm iterates in an outer loop for each next-state upstream admissible policy; the algorithm assumes the policy would remain the same until the reaching position $X_s$. The policies that violate the expected red-light condition and other constraints are considered infeasible. The system computes the upstream section fuel consumption $U_i$ for each feasible policy $i$.

2) For each upstream feasible policy $i$, the system iterates in an inner loop generating the next downstream admissible policies based on the vehicle state at position $X_s$. Similarly, each admissible downstream policy is assumed to remain the same until reaching the destination at the position $X_N$. The system computes the downstream section fuel consumption $D_{ij}$ at each downstream policy $j$ for each upstream policy $i$.

3) Using the upstream and downstream heuristic cost estimates, the $A^*$ algorithm selects the upstream next-state policy based on the minimum total fuel consumption of the two sections $U_i + D_{ij}$ for each upstream policy $i$.

4) When the vehicle reaches the downstream section, the system will only iterate in the inner loop, and the policy with minimum fuel consumption $D_j$ is selected.

VII. RESULTS AND ANALYSIS

A. Optimal Policy for Deterministic SPaT Information

In the case of deterministic SPaT information, the trajectory optimization system is applied to our problem in the defined scenarios. As the system receives the signal switching time from the controller, the optimal trajectory is computed to achieve the best fuel economy. As shown in Fig. 6 and Fig. 7, at TTG=10 seconds, the optimal policy is to cruise at the current speed. That is because the travel time to the intersection at the current speed would be greater than the time to green. As such, the system applies the throttle level that overcomes the resistance forces to keep the vehicle cruising at the current speed and keeps the fuel losses at a minimum.

For the case of TTG=15 seconds, the system recognizes that an amount of delay is required to reach the stop line after the signal switches. Accordingly, the system applies a level of deceleration to achieve that delay. Similarly, at TTGs of 20 and 25 seconds, the system applies the optimal deceleration levels to achieve the required delay. Downstream of the traffic signal, the system determines the optimal acceleration policy to reach the destination position with the desired speed for all cases.

B. Optimal Policy for Stochastic SPaT Information

The stochastic GLOSA system is applied for the defined scenarios with different distributions, where a sampled value from the probability distribution of the switching time is transmitted to the system at each $\Delta t$. The system generates the policy that achieves the best fuel economy according to the perceived information at this point, which generates the policy behavior as shown in Fig. 8 for the downhill setting and Fig. 9 for the uphill setting. The system controls the fluctuation intensity based on the acceleration/deceleration policy smoothing and the jerk limit, such that the maximum allowable jerk limit is not violated to ensure proper passenger comfort levels.

In the case at TTG=10 seconds, the system’s optimal policy is to cruise with a slight decrease in the speed close to the stop.
bar, when the vehicle is about to activate the safety assessment strategy while the traffic light is red. When the traffic signal switches to green, the vehicle accelerates again to reach the destination position at the desired speed. In this case, the controlled vehicle avoided unnecessary deceleration behavior as in Scenario I. As such, the system produced a significant fuel-optimal trajectory.

Similarly, for the cases at TTG = 15, 20, and 25 seconds, the system generates the optimal deceleration policy, while continuously checking the risk region. If the risk region is reached and the light is still red, as in the case of TTG = 25 seconds, the vehicle will activate the deceleration policy to stop until the light turns green. That is when the vehicle will update its policy to accelerate again to reach the desired speed at the destination position.

C. Fuel Consumption Savings for Deterministic and Stochastic Settings

Compared to the uninformed driver, the optimization system showed significant savings in fuel consumption levels for both the deterministic and stochastic settings. The results compare the overall performance in terms of fuel savings in the case when the SPaT information is deterministic (Scenario II) to when SPaT information is stochastic (Scenario III) using the uninformed driver (Scenario I) as a baseline.

The stochastic SPaT results are obtained by averaging the results for all the different analysis scenarios. As depicted in Fig. 10, the overall average fuel savings are 37% and 30% for the deterministic and stochastic settings, respectively. The highest fuel savings is 63% for a deterministic TTG and 51% for a stochastic TTG when driving downhill with a TTG of 15 seconds. This is because minimal throttle is needed to overcome resistance and cruise in this scenario. With a TTG of 15 seconds, the most savings occur compared to uninformed drivers, as slowing down a little is needed to reach the stop bar at high speed by the time it turns green.

The case of TTG = 25 seconds is when the switching time is too long for the vehicle to arrive at the stop bar without stopping. In this case, it is implied that the fuel saving is mainly due to optimizing the acceleration profile in the downstream section. That is shown by the fact that the average fuel consumption savings for deterministic and stochastic SPaT information are relatively close (21% in Scenario II and 19% in Scenario III). However, as the switching time decreases, a notable difference between the two scenarios is achieved. This is when optimizing the trajectory in the upstream section plays a more effective role in decreasing the total fuel consumption.

It is shown that optimizing the upstream trajectory incorporates significant portions of additional fuel savings that can be achieved. These savings amount to up to 21% for deterministic SPaT and 19% for stochastic SPaT. This demonstrates the importance of having reliable predictions for traffic signal switching times.

Our robust GLOSA system shows superior fuel efficiency compared to systems proposed in other studies. Mahler and Vahidi reported a fuel consumption saving of 6% [18]. Hao et al. achieved savings of up to 12% [19], [20]. Sun et al. [21] reported fuel savings of 40%, which is higher than our reported savings but less than our maximum savings. The reason for this is that the Intelligent Driver Model (IDM)
was used as a baseline [21]. In this case, IDM demonstrated much more aggressive acceleration and stopping behavior compared to the uninformed drivers in our empirical data. This effect compounded at consecutive intersections, leading to a much higher baseline fuel consumption compared to the case of a single intersection.

D. Effect of Bias and Variance

According to the field data, the prediction uncertainty can be represented by a bias and variance term. The variance is represented by a standard deviation that reflects the random errors in any given SPaT prediction model. The bias can occur due to systematic errors or processes in the controller that the model can fail to take into consideration. This process is performed to mimic the actual noised predictions that the system will receive while approaching the intersection.

As the switching time prediction quality impacts the optimal trajectory, a sensitivity analysis is conducted to visualize the impact of the normal distribution parameters (the mean error bias and standard deviation) on the optimal fuel consumption levels. A total of 320 runs were performed using different values of the mean error bias and standard deviation (Fig. 11). The analysis showed that the change in standard deviation values has a significant effect on fuel consumption, compared to the effect of the bias value, which was not shown to be significant. Additional uncertainty in the prediction led to losses in fuel savings except in the case where TTG=15 seconds. Fig. 12 shows that in the case of TTG=15 seconds, the fuel consumption decreases as the stochasticity increases. This behavior is explained by considering that the vehicle’s travel time from the initial position to the intersection equals the switching time, according to the initial speed of 17.8 m/s (40 mph). The vehicle approaches the stop line at a time when the signal is about to switch to green. However, the system activates the risk assessment procedure to ensure that no violations of traffic signal timings are made. Therefore, the vehicle begins to decelerate until the signal switches to green at some point. By introducing additional stochasticity, the vehicle can be delayed enough so that the signal will switch before the vehicle enters the risk zone and activates the risk assessment procedure. That is the only case in which more stochastic information results in better fuel economy.

E. The Effect of Varying Initial Speed

To further evaluate the system response to the stochastic SPaT information, we vary the initial vehicle speed from 17.8 m/s (40 mph) to 13 m/s (29 mph) (Fig. 13). For the slower initial speed, the system generates similar speed profiles regardless of the switching time prediction uncertainty in the cases where TTG is 10, 15, and 20 seconds. The policy is to provide a minimal level of constant acceleration such that the vehicle reaches the stop bar within the green period and reaches the destination at the maximum possible speed (17.8 m/s). That is because the optimal policy given the lower speed will have the vehicle arrive at the intersection late enough to accommodate the switching time probability distribution. In cases where TTG is 25 seconds, the system will generate a decreasing speed policy until the vehicle has to stop due to activating the risk assessment procedure. Note that the slow deceleration is followed by a sharp deceleration as the vehicle is within the safe stopping distance from the stop bar. This highlights the previously mentioned point that the prediction accuracy becomes more important the closer TTG is to the time until reaching the intersection given the initial speed.

F. Impact of Confidence in SPaT Information on Fuel Consumption

This section discusses the level of confidence in the switching time prediction required to achieve significant fuel economy.
consumption savings in the stochastic setting of the optimal velocity planning problem. Given that the fuel consumption savings in the deterministic setting are the upper bound of the stochastic setting, we need to identify how the level of uncertainty decreases the savings. In other words, we need to identify how the probability distribution standard deviation and bias can affect the fuel savings compared to the upper bound. Fig. 14 shows the relationship between the parameters of the switching time probability distribution and the proportion of fuel savings, where the maximum savings occur in the case when there is no uncertainty in the switching time information (prediction standard deviation and mean bias equal zero). The savings proportion decreases as these parameters increase. It is shown that we can achieve fuel savings in the stochastic setting of more than 85% of the maximum possible savings when the switching time distribution standard deviation is less than 1.25 seconds and the mean bias is less than 0.8 seconds. In other words, 85% of the possible fuel savings can be achieved with a switching time prediction error of up to ±3.3 seconds for a 95% confidence level. This information gives us insights into the required confidence in SPaT signals, where the exact signal switching time is unknown.

VIII. CONCLUSION AND RECOMMENDATIONS FOR FURTHER RESEARCH

A GLOSA system was developed to compute the optimal vehicle trajectory in the vicinity of a traffic signal. The system deals with fixed time as well as actuated traffic signals, where the exact signal switching time is unknown with an expected time to green provided to the vehicle through I2V communication. Using fuel consumption minimization as the objective function, the problem is solved through a DP procedure utilizing the A* algorithm to find the minimum-cost path. A risk mitigation procedure is incorporated in the formulation to ensure safe deceleration levels and no red light violations. Additionally, the passengers’ comfort is achieved by controlling the vehicle jerk due to fluctuations caused by errors in the expected switching time at each time step.

The simulation results of deterministic and stochastic switching time settings were compared to empirical data for an uninformed driver based on experiments conducted at the Virginia Tech Transportation Institute. Results show that significant fuel savings can be achieved with an average of 37% and 30% for the deterministic and stochastic settings, respectively. Additionally, the sensitivity analysis showed that the system is robust to the errors included in the uncertain switching time predictions, where the system will adjust the vehicle trajectory in real time according to the updated predicted expected switching times. It also showed how the initial speed affects the vehicle acceleration and deceleration fluctuations, which is useful in planning trajectories in a corridor that has consecutive signal controllers.

In the case of uncertain switching times, the proposed system can achieve more than 85% of the possible savings (for fixed time signals) if the timing error is (±3.3 seconds) at a 95% confidence level. It was shown that the system is more sensitive to prediction errors when the time to the light turning green is close to the time required to reach the intersection given its current speed.

Future research directions include testing the stochastic optimizer for different levels of market penetration within surrounding uninformed drivers when queues form upstream of the traffic signal, and for different levels of traffic congestion, as was done for the case of deterministic signal timings [31], [32], [33]. Additionally, the algorithm adaptation to the case of consecutive signal controllers can be further explored.

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