Generalized space–time noncommutative inflation

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Abstract. We study the noncommutative inflation with a time-dependent noncommutativity between space and time. From the numerical analysis of power law inflation, there are clues that the CMB spectrum indicates a nonconstant noncommutative inflation. Then we extend our treatment to the inflation models with more general noncommutativity and find that the scalar perturbation power spectrum depends sensitively on the time varying of the spacetime noncommutativity. This stringy effect may be probed in the future cosmological observations.

Keywords: string theory and cosmology, inflation, physics of the early universe

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1. Introduction

As a successful theory describing the very early universe, inflation [1]–[3] can solve many problems in standard Big Bang cosmology, such as the large-scale smoothness, horizon and unexpected relic problems. More importantly, inflation also predicts a scale-invariant spectrum on CMB which has been confirmed quite precisely in cosmological experiments, such as the WMAP three year result [4]. Although inflation has achieved many spectacular successes, it is not perfect and has some conceptual problems [5]. One of them is the trans-Planckian problem [6]. During inflation, the physical wavelength of each comoving mode expands exponentially and escapes the horizon. The inflationary modes which have just reentered the horizon at the present time correspond to a perturbation with a very small physical wavelength, perhaps smaller than the string scale or even the Planck scale. So the usual assumption that these inflationary modes originate from the local Minkowski vacuum is not robust. More seriously, near the Planck scale, the quantum effects of gravity should be large and the usual semiclassical treatment in the inflationary models, in which the weakly self-coupled scalar field theory is coupled to classical gravity, are known to break down. An effective way to take the trans-Planckian effect into account is to modify the dispersion relation of the perturbations [6]. With the modified dispersion relation, one can calculate the spectrum of the fluctuations and compare with experiments. Therefore the scale-invariant spectrum in the inflationary cosmology gives a window to study the trans-Planckian physics and hopefully has important implications for a quantum gravity theory.

In [7], Brandenberger and Ho suggested that the stringy spacetime uncertainty principle possibly affects the primordial density fluctuations and leads to trans-Planckian effects during inflation. Equivalently, one may realize the uncertainty relation from the noncommutative relation

$$[t, x_p] = i l_s^2,$$

where \( t \) and \( x_p \) are, respectively, the physical time and physical radial coordinate. Here, choosing the radial coordinate makes sure that the noncommutativity does not spoil the...
global symmetries of the whole classical background. In practice, it is more convenient to introduce another noncommutative relation

\[ [\tau, x] = i\alpha^2, \] (2)

where \( \tau \) is another time coordinate such that the Friedmann–Robertson–Walker (FRW) metric for a spatially flat universe could be written as

\[ ds^2 = dt^2 - a^2(t) dx^2 = a^{-2}(\tau) d\tau^2 - a^2(\tau) dx^2. \] (3)

Note that \( \tau \) is not the conformal time and \( x = a(t)x \). The scalar inflation models have been studied by considering the scalar field coupled to the classical background (3) in terms of Moyal star product induced by the noncommutative relation (2). Due to the noncommutativity, the coupling of the scalar field fluctuations with the background is nonlocal in time. This leads to a different spectral index from the usual commutative inflation in the infrared region. Also the model predicts sufficient running of the spectral index \([7],[10]–[13]\). In recent research, noncommutativity inflationary cosmology has been generalized and extended in various ways: see \([14]\) for noncommutative tachyon inflation; \([15]\) for curvature fluctuations of noncommutative inflation; \([16,17]\) for noncommutative brane inflation; \([18]\) for noncommutative eternal inflation; other related works could be found in \([19]–[23]\).

It is believed that the trans-Planckian physics is closely related to the underlying fundamental theory of quantum gravity. In string theory, which is the best quantum gravity theory we have, noncommutative geometry can be realized quite naturally. One simple way is to consider the Dp-brane in the presence of a constant Neveu-Schwarz–Neveu-Schwarz (NSNS) \( B \) field in the flat spacetime background. The low energy effective theory of Dp-brane is a noncommutative field theory \([24]\). In general, due to the presence of nonconstant NSNS or Ramond–Ramond (RR) background and the curved background, more general noncommutativity may appear on the Dp-brane \([25]–[27]\). Though the quantum field theory with the general noncommutativities has not been studied as much as the usual case, they are important to the study of the string theory. On the other hand, in some cosmology scenarios based on the \( D \)-brane, such as the brane world scenario, the possible noncommutativity on the brane could not be the one in (1) due to the involved bulk flux and background. It is very possible that the more precise cosmological observational data on the CMB spectrum in the future may shed more light on the trans-Planckian physics, and even decide the details of the noncommutativity. Therefore, it is interesting to investigate the physical implications of the general noncommutativity in the inflationary cosmology. In this paper we generalize the usual noncommutativity (1) to the one with time dependence:

\[ [t, x_p] = i\theta(t), \] (4)

and consider its cosmological implications.

This paper is organized as follows. In section 2 we first discuss the time-dependent commutative relationship and introduce a new star product with constant commutative relation in redefined space and time coordinates. Then we analyse the power law inflation with a time-dependent noncommutativity. Power law inflation exhibits clearer effects of noncommutativity and our numerical analysis shows that a time-dependent noncommutativity is quite reliably encoded in the WMAP data. In section 3, we
extend the discussion to the inflation models with general scalar potential and general noncommutativity. We develop a slightly new method to treat the slow rolling noncommutative inflation and calculate the spectrum and the spectral index. We find that the spectral index relies on the time-varying of the noncommutative parameter very sensitively. We conclude in section 4.

2. Power law noncommutative inflation

In the (quasi-)de Sitter spacetime, we assume that noncommutativity of proper time and physical distance is \([t, x_p] = i\theta(t)\). We restrict that \(\theta\) is only a function of time. This requirement can preserve both the spatial translational and rotational symmetry of the FRW metric. To describe the scalar field theory coupled with the classical background, it is more convenient to define another commutative relation

\[ [\tau, x]_\ast = i\frac{a}{b}l_s^2 \]

such that we can use the standard Moyal product

\[ (f * g)(x, \tau) = e^{-(i/2)l_s^2 \left( \partial_x \partial_{\tau'} - \partial_{\tau} \partial_x \right)} f(x, \tau) g(y, \tau') \bigg|_{y=x, \tau'=\tau}, \]

where \(x\) is the conformal coordinate, \(x_p = a(t)x\), and \(\tau\) is a redefinition of the time variable, through the equation \(d\tau = b(t)dt\). This leads to the commutative relation (4)

\[ [t, x_p]_\ast = i\frac{a}{b}l_s^2 = i\theta(t). \]

Note that \(l_s\) is not the same as the string length, since \(\sqrt{\theta}\) is the effective string scale. When \(\theta\) is constant, the noncommutativity returns to the one in [7].

In the power law inflation we have \(a(t) = a_0 t^n\) and for simplicity we assume \(b = a^{m/2}\), so the time-dependent noncommutative parameter is of the power law form,

\[ \theta(t) = a_0^{1-(m/2)} l_s^2. \]

Obviously the variable \(m\) characterizes the time-varying of the noncommutativity. The right-hand side of the equation may leak a constant coefficient, but it cannot affect the final result by the redefinition of \(a_0\). In this case, \(a(t) = a_0 t^n = a_0 \tau^{n/(mn/2+1)} = (\tau/l)^{n/(mn/2+1)}\), where \(l\) is only the constant coefficient. When we consider the more general inflation potentials in the next section, all the power law assumptions, including the assumption of \(a(t)\) and \(\theta(t)\), will disappear.

In the 1+1 dimensions, the noncommutative action reveals the fact that gravitational background and scalar field interact with each other nonlocally in time. The classical background is the flat FRW metric

\[ ds^2 = -dt^2 + a(t)^2 dx^2 = -b(\tau)^2 d\tau^2 + a(\tau)^2 dx^2. \]

The action is

\[ S = \int d\tau dx \sqrt{-\text{det} g} \left( \partial_\tau \phi \ast b^2 \ast \partial_\tau \phi - (\partial_x \phi)^\ast a^{-2} \ast (\partial_x \phi) \right). \]

Through Fourier transformation, the action in the momentum space is

\[ S = V \int d\tau d\mathbf{k} \frac{a}{2b} (\beta_+ \partial_\tau \phi + \partial_\tau \phi - \beta_- \phi \partial_\tau \phi), \]
where
\[ \beta_k^+(\tau) = \frac{1}{2}(b^2(\tau - t_s^2 k) + b^2(\tau + t_s^2 k)), \] (12)
\[ \beta_k^-(\tau) = \frac{1}{2}(a^{-2}(\tau - t_s^2 k) + a^{-2}(\tau + t_s^2 k)). \] (13)

One essential point is that there exists a cutoff on the wavenumber \( k \) in spatial slices, due to the uncertainty principle of space and time
\[ \Delta \tau \Delta x \geq t_s^2, \] (14)
which is derived from the commutative relation
\[ [\tau, x] = t_s^2. \] (15)

According to the spacetime and energy–time uncertainty relationship, we have an upper bound
\[ k \leq k_0(\tau) \equiv \left( \frac{\beta_k^+}{\beta_k^-} \right)^{1/4} t_s^{-1}. \] (16)

It means that there is a time-dependent minimal wavelength when we detect spacetime. Note that the above bound is different from the one in [7] due to the time dependence of the noncommutative parameter. This will lead to different behavior in the power spectrum, as we will show soon.

In an inflationary scenario, the fluctuations in the UV and IR regions have different spectral indexes. When the equal sign holds in (16), we say that it gives the creating time or the initial condition of \( k \) modes. The UV region is the region that the modes are created in the Hubble radius, while the IR region is the region that the initial condition of modes are already outside the Hubble radius. There is a turning point in time that the IR region converts to the UV region. Modes, coinciding with the initial condition, are created. When their wavelength is the Hubble radius \( k_0(\tau) = aH \), it is the time transforming between UV and IR. The turning point requests \( k_0/a = H \):
\[ k_0 = \left( \frac{\beta_k^+}{\beta_k^-} \right)^{1/4} t_s^{-1} = \{ a_0^{m+2}[(\tau + k t_s^2)^{mn/2}(\tau - k t_s^2)]^{2n/(mn/2+1)} \}^{1/4} t_s^{-1} = aH = a_0 t^{m+1}. \] (17)

The larger \( m \) we choose, the larger \( k_0(\tau) \). If \( m \) is becoming larger, the condition emerges later during inflation. It also means that we can see the IR region in the sky earlier.

To calculate the power spectrum, we can write the action in the effective conformal time coordinate
\[ S \simeq V \int_{k < k_0} d\tilde{\eta} dk \frac{1}{2} y_k^2(\tilde{\eta})(\phi'_{+k} \phi'_{+k} - k^2 \phi_{+k} \phi_{+k}), \] (18)
where
\[ d\tilde{\eta} = \left( \frac{\beta_k^-}{\beta_k^+} \right)^{1/2} d\tau = b \left( \frac{\beta_k^-}{\beta_k^+} \right) dt = \frac{1}{\lambda a} dt, \] (19)
\[ y_k = \frac{a}{b} (\beta_k^- \beta_k^+)^{1/4}. \] (20)
The primes denote derivatives with respect to \( \tilde{\eta} \).
Considering the scalar perturbation in the $1 + 3$ dimensions, the action is

$$S = V \int_{k < k_0} d\tilde{\eta} d^3k \frac{1}{2}\tilde{z}_k^2(\tilde{\phi}_k - k^2\phi_k),$$

(21)

where $z_k$ is a smeared version of $z$:

$$z_k = z y_k, \quad z = \frac{a\dot{\phi}}{H}.$$  

(22)

The scalar power spectrum can be calculated following [28] and be expressed as

$$P_R = \frac{k^2}{4\pi^2 z_k^2(\tilde{\eta}_k)}.$$  

(23)

The choice of $\tilde{\eta}_k$ is from

$$k^2 = \frac{z''_k}{z_k}.$$  

(24)

First, consider the UV region. In the power law inflation, $z \propto a$, up to a constant. According to the relation between $d\tau$ and $d\eta$ (19), we can derive the relation between $d/d\eta$ and $d/d\tau$ in the second-order approximation. With (24) we can get the time when the modes exceed the horizon, then using (23), the result of the power spectrum is figured out. In the UV region,

$$P_R = \left( \frac{n(2n-1)}{(mn/2 + 1)} \right)^{n/(n-1)} n \frac{(lp)}{8\pi^2} l^{-2/(n-1)} k^{-2/(n-1)} \left( 1 + A \left( \frac{k_c}{k} \right) \right)^{(mn-2n-4)/(n-1)},$$

(25)

the spectral index is

$$n_s = 1 + \frac{d\ln P_R}{d\ln k} = 1 - \frac{2}{n - 1} - \frac{mn/2 - n + 2}{n - 1} A \left( \frac{k_c}{k} \right)^{(mn-2n-4)/(n-1)},$$

(26)

and the running of the spectral index is

$$\alpha_s \equiv \frac{dn_s}{d\ln k} = \left( \frac{mn/2 - n + 2}{n - 1} \right)^2 A \left( \frac{k_c}{k} \right)^{(mn-2n-4)/(n-1)},$$

(27)

where

$$A = \frac{n}{8(n - 1)(2n - 1)(mn/2 + 1)^2} \left( \frac{m - 4}{2} (m^3 + m^2 + 14m + 24)n^3 + (\frac{3}{2}m^3 + 8m^2 + 22m + 8)n^2 + (-3m^2 + 22m + 32)n + 4(2 - m) \right),$$

(28)

$$k_c = l_p^{(2n-2)/n} l^{-(n/2)-1},$$

(29)

and $l_p$ is the reduced Planck length. When $m = 2$, the results return back to the one in [10, 12]. From the result, we have to require $(m - 2)n \lesssim O(1)$ in order to have the scale-invariant spectrum. If only considering the first-order approximation, the result is the same as the common inflation without noncommutativity.
In the IR region, the effect of the noncommutativity is important. We can also take the second-order approximation as the UV region, but it is much more complicated and is actually not necessary in the following discussions. So we only give out the power spectrum in the first-order approximation, which turns out to be enough to illustrate the physical picture and see the variation between different noncommutativities.

From the initial condition (16), we obtain

\[ k \equiv \left( \frac{\beta_k}{\beta_l} \right)^{1/4} l_s^{-1} = \left\{ \alpha_0^{m+2} \left[ (\tau + kl^2_s)^{m/2} (\tau - kl^2_s)^{2n/(mn/2+1)} \right] \right\}^{1/4} l_s^{-1}. \]  

(30)

The equation can be rewritten in the form

\[ \left( \frac{\tau}{kl^2_s} + 1 \right)^{m/2} \left( \frac{\tau}{kl^2_s} - 1 \right) = (\alpha_0^{m/2+1} l_s^{2})^{-(mn/2+1)/n} (kl^2_s)^{(2/n)+(m/2)-1}. \]  

(31)

In the IR region, \( k \) is so small that we can use the condition to simplify the equation. Assume that \( \tau \propto k^x \). Because the value of the right hand side is not negative, \( x \leq 1 \). If \( (2/n) + (m/2) - 1 > 0 \), the right hand side is a small quantity; and if \( (2/n) + (m/2) - 1 < 0 \), the right hand side is a large quantity. When \( x = 1 \), we can solve the equation in the approximation of \( \tau \simeq kl^2_s \), which requires \( n(1 - (m/2)) < 2 \). We obtain

\[ \tau = kl^2_s + \left( \frac{k^2 l^2_s}{\alpha_0^{m/2+1} l_s^{2}} \right)^{(mn/2+1)/n} (2kl^2_s)^{-m/2}. \]  

(32)

Then,

\[ \mathcal{P}_R \sim k^2 z_k^{-2} (\tilde{\eta}_k) \sim k^{((3/2)mn-3n+4)/(1/2mn+1)}. \]  

(33)

If \( x < 1 \), we can obtain

\[ x = \frac{mn + 2}{(m/2) + 1)n}. \]  

(34)

and \( \tau \propto k^x \). From \( x < 1 \), we get \( n(1 - (m/2)) > 2 \). Roughly speaking, this condition requires \( m < 2 \), i.e. the effective string length is decreasing during inflation. Then

\[ \mathcal{P}_R \sim k^2 z_k^{-2} (\tilde{\eta}_k) \sim k^0. \]  

(35)

The future cosmology evolution may reveal the effect of the IR region relating to the earlier period of inflation. If \( n(1 - (m/2)) < 2 \), the spectrum looks bluer than the one in [7]. If \( n(1 - (m/2)) > 2 \), the spectrum looks redder than [7]. Furthermore the condition of \( n(1 - (m/2)) > 2 \) may reveal new phenomena that the spectrum is scale-invariant in the first approximation.

It would be illuminating to use our model to fix the experimental data. There are four parameters, \( l_s \), \( l_s \), \( m \), \( n \), in our model. From the WMAP data, we know

\[ \mathcal{P}_R = 2.95 \times 10^{-9} \quad \text{at} \quad k = 0.002 \text{ Mpc}^{-1}, \]
\[ n_s = 0.93 \pm 0.03, \quad \frac{dn_s}{d \ln k} = -0.031^{+0.016}_{-0.017} \quad \text{at} \quad k = 0.05 \text{ Mpc}^{-1}, \]  

(36)
\[ n_s = 1.16 \pm 0.10, \quad \frac{dn_s}{d \ln k} = -0.085 \pm 0.043 \quad \text{at} \quad k = 0.002 \text{ Mpc}^{-1}. \]
Figure 1. Here $n_s$ is the spectral index and $k$ is the comoving mode. As our observable universe gets larger in the future, noncommutative inflation with decreasing noncommutativity predicts that the spectrum will turn from red to blue clearly.

Figure 2. Here $\alpha$ is the running of the spectral index and $k$ is the comoving mode. The running becomes faster in the later period. It is due to the stronger effects of noncommutativity in the earlier period of inflation.

Using (25)–(27), we may determine the variation of noncommutativity, i.e. the variable $m$. When we require the $|A(k_c/k)mn/(n-1)| < 0.5$, the numerical analysis shows that the available data of $m$ is 2.2–2.3. Since $m > 2$, it illustrates the fact that the noncommutativity is decreasing during the inflation period.

In figures 1 and 2, we choose $n = 14.9$, $m = 2.2$, $l_s = 6.07 \times 10^{-32}$ cm, $l = 1.575 \times 10^{-23}$ cm, which are in the allowable scope of the WMAP data. From the numerical analysis above, we show that the WMAP data really permits the existence of a time-dependent noncommutativity in the power law inflation. Furthermore, our model predicts that the spectrum will turn bluer and the running of the index will be faster.

3. More general noncommutative inflation models

In this section, we extend the above discussions on power law inflation to more general cases, where a general inflaton potential other than the exponential potential is considered. Moreover, we also consider the more general spacetime noncommutativity without the assumption $\theta(t) \sim a^{1-(m/2)}$. To simplify our discussion, here we assume a new slow roll condition on the time-dependent spacetime noncommutativity, namely we require that

$$
\sigma \equiv \frac{\dot{\theta}}{H\theta}, \quad |\sigma| \ll 1.
$$

(37)
And, as usual, we impose the slow roll conditions for the Hubble constant and the background inflaton field:

\[ \epsilon = -\frac{\dot{H}}{H^2}, \quad \delta = -\frac{\ddot{\varphi}}{H\dot{\varphi}}, \quad |\epsilon| \ll 1, \quad |\delta| \ll 1. \quad (38) \]

The equation of motion for the perturbations can be written as [28]

\[ u''_k + \left( k^2 - \frac{z_k''}{z_k} \right) u_k = 0, \quad (39) \]

where \( u_k \) is the canonically normalized perturbation variable \( u_k \equiv -z_k R_k \), and \( R_k \) is the comoving curvature perturbation.

In the noncommutative case, \( z_k''/z_k \) can be expanded as

\[ \frac{z_k''}{z_k} = \left( \lambda^2 a^2 H^2 \right) \left\{ \frac{\dot{z}_k}{H^2 z_k} + \left( 1 + \frac{\lambda}{H \lambda} \right) \frac{\ddot{z}_k}{H z_k} \right\}, \quad (40) \]

where, as defined in (19), \( \lambda \) denotes the difference between the usual conformal time and the modified conformal time \( \tilde{\eta} \).

With the slow roll condition on \( \theta \) in mind, we can expand the corresponding quantities up to the leading order of the \( l_s \) corrections as

\[ y_k = 1 + \mu, \quad \lambda = 1 - \mu, \quad \mu \equiv H^2 \theta^2 \frac{k^2}{a^2}. \quad (41) \]

The validity of this expansion requires \( \mu < 1 \). This is the same requirement as the perturbation is generated in the UV region.

Compared with the \( \mu \) parameter defined in the work [13], \( \mu \equiv H^2 l_s^4 k^2 / a^2 \), we see that the time varying noncommutativity can be thought of as a redefinition of \( \mu \).

Using the slow roll approximation, we can integrate out the modified conformal time as\(^5\)

\[ \tilde{\eta} = \int \frac{dt}{a} (1 + \mu) = -\frac{1}{aH} \left( 1 + \epsilon + \frac{1}{3} \mu \right). \quad (42) \]

Inserting (42) and (41) into (40), the equation (39) becomes

\[ u''_k + \left( 1 + \frac{8}{3} H^2 \theta^2 \right) k^2 u_k - \frac{2}{\eta^2} \left( 1 + 3\epsilon - \frac{3}{2} \delta \right) u_k = 0. \quad (43) \]

We see that, in spite of using the modified conformal time, the difference between this classical equation of motion and the usual commutative one is only a rescale of \( k \). As is shown in [6, 29], this rescale property also holds in the quantum level for the initial conditions. So the comoving curvature perturbation can be written as a function of \( (1 + \frac{4}{3} H^4 \theta^2) k \):

\[ R_k = R((1 + \frac{4}{3} H^4 \theta^2) k). \quad (44) \]

\(^5\) In [13], \( \mu \) has been considered as nearly a constant. It is because the horizon crossing formula is used in [13] for calculating the power spectrum. So the time variable is set to the horizon crossing point. However, in dealing with the equation of motion of a single comoving mode \( k \), \( \mu \) should be varying very fast with time because \( a \) expands exponentially.
The perturbation spectrum takes the form

\[ P_R = \frac{k^3}{2\pi^2} \left| R \left( 1 + \frac{4}{3} H^4 \theta^2 \right) k \right|^2 = \left( 1 - 4H^4 \theta^2 \right) \frac{k^3 |R(k)|^2}{2\pi^2} \]  

(45)

where \( R(k) \) is the usual comoving curvature perturbation calculated in the commutative inflation models. We have used the fact that \( P_R \) should produce a nearly scale-invariant spectrum.

Then we have the spectral index \( n_s \) and the running of the spectral index \( \alpha_s \):

\[ n_s - 1 \equiv \frac{\text{d} \ln P_R}{\text{d} \ln k} \bigg|_{k=aH} = 16\mu - 4\epsilon + 2\delta - 8\sigma, \quad \alpha_s \equiv \frac{\text{d} n_s}{\text{d} \ln k}, \]  

(46)

where \( \mu \) is calculated at \( k = aH \).

Note that it is the slow roll parameter \( \sigma \) on \( \theta \) rather than \( \theta \) itself that appears in the spectral index. Therefore even the noncommutativity is very tiny, its variation could affect the spectral index strongly. From (46), it is easy to find that, if the noncommutativity \( \theta(t) \) decreases with time, the spectrum turns out to be bluer. While if \( \theta(t) \) increases with time, the spectrum turns out to be redder.

In particular, if \( \theta \) decreases rapidly at around 60 e-folds before the end of inflation, then the power spectrum can be very blue in the small multipole moment \( l \) region. In this case, the power spectrum can be greatly suppressed. This provides a possible solution to the problem of the lack of the CMB anisotropy power on the largest angular scales. Note that to make a very blue spectrum with \( n_s - 1 \sim O(1) \), it is still possible that \( \sigma \) and \( \mu \) are both smaller than 1, so both the \( \mu \) expansion and the slow roll approximation are still valid.

It is remarkable that, in the discussion above, to treat the slow rolling noncommutative inflation, we developed a slightly new method to get the power spectrum and spectral index. To check its validity, let us consider two special cases.

As the first example, let us consider the constant commutator limit, where \( \theta = t_s^2 \). This has been considered in \([7,13]\). In this case, the spectrum and the spectral index agree completely with the ones in \([13]\).

For another example, consider the power law inflation with a monomial commutator. We have discussed this case in section 2, using the method developed in \([7]\). Once again, the results of this section match with the ones in section 2 correctly.

The result (46) opens up a window for studying the noncommutative inflation models. It shows that the spectral index depends strongly on how \( \theta \) evolves with time. Even a slow rolling change of the noncommutativity could result in a very different spectral index. This issue is interesting because, on the one hand, the question how the string theory gives the trans-Planckian effect in the inflationary cosmology should be taken into consideration seriously. For example, in some brane world scenarios, the noncommutativity on the brane is induced by the bulk background flux. In a curved time-dependent background, how the flux evolves and give rise to the time-dependent noncommutativity is an interesting question to ask. There have already been some efforts to study string evolution in the inflationary background \([30]\). However, a lot more needs to be done to get a complete inflationary string theory picture and determine the background field evolution during inflation. On the other hand, it is exciting that precise measurements on the CMB spectrum may provide a probe for the underlying spacetime geometry emergent from string theory in a dynamical way.
4. Conclusion

In this paper we discussed the noncommutative inflation models with a time-dependent noncommutativity. In the power law inflation case, we found that the WMAP data could encode the time dependence of the noncommutativity. Our discussion on general inflationary models showed that the power spectral index sensitively relies on the time varying of the noncommutativity. This suggests that more precise observational data in the future may tell us the details of the noncommutative inflationary scenario and probe the underlying physics of quantum gravity.

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