Capability analysis of Puck damage model in predicting the damage behavior of unidirectional composite laminates under different scenarios

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Abstract. Composite materials have been broadly used in many engineering fields because of their distinctive properties. They have advantages over metals as their properties offer significant advantages such as very good strength to weight ratio. However, failure in composite structures due to interlaminar and intralaminar raises a maintenance concern because it can lead to invisible damage. Therefore, this paper presents the capability of three-dimensional Puck failure criteria with gradual degradation rule to predict the structural responses, as well as the onset and propagation of failure due to variation of the situation in a composite laminate. The proposed damage model is achieved via the implementation of user-subroutines both in statics and explicit analysis using Abaqus software. Three different scenarios are investigated which were open-hole tension, low-velocity impact, and multi-bolt double-lap joint. The force-displacement and force-time curves are recorded and compared with experimental data taken from literature to measure the performance of such damage model. It is found out that the model adopted here responses well with test curves and demonstrates the high capability of predicting the damage in the direction of in-plane and out-of-plane in a composite laminate.

1. Introduction

A composite material is produced using at least two constituent materials with essentially different physical or chemical properties that, when combined, produce a material with characteristics different from the individual components. The common mixture of composite material consists of fiber reinforcement combined with matrix resin. The fiber reinforcement has a good character such as brittleness, high strength, and stiffness, while the matrix resin has high shear properties, low density and easy to fabricate where resulting in producing a composite which has very good strength to weight and stiffness to weight ratios. Furthermore, composite structures are fabricated with tailoring properties in a required or desired specification. They have been increasingly used in load-bearing structures such as aircraft and automobile industries as they offer significant advantages like strength to weight capacity over metals [1].

During the manufacture, maintenance, and service life, composites may suffer a variety of damages in tension, compression and out-of-plane direction. In composite laminates, damage phenomena can be divided into two main groups: (1) interlaminar failure (delamination), and (2) intralaminar failure (fiber
and matrix breakage including fiber-matrix decohesion) [2]. The majority of research in composite damage is focusing on open-hole tension (OHT) [3,4], open-hole compression (OHC)[5–7], composite joints [8–10], and low-velocity impact [11–14]. From the previous researches published elsewhere, the most complicated works are emphasized on the development and validation of failure theory for the laminates. Hashin [15] started his ideas in developing the criteria than can identify different modes of failure which is a versatile formulation as compared to the polynomial formulation [16,17]. Puck and Schürmann [18] improvised the work of Hashin and introduced the Inter-fiber failure concept (IFF) where the prediction of failure is based on fracture plane resulted from stresses acting on the fracture plane. The enhancement of Puck damage criteria was done by Davila et al. [19] where they incorporated with a fracture mechanics approach to evaluate the in-situ strength. Further development of failure theories was concentrated on damage mechanics approach where the idea conceptually originates from research of Kachanov [20]. His idea later is adopted for in-plane failure damage criteria developed by Ladeveze and Le Dantec [21]. Despite having all those theories, the total failure of composite laminates cannot be achieved when the progression law is neglected. The most common technique to degrade the structural stiffness matrix is by using the ply-discount approach. The laminates become weaker whenever the reduction of elastic stiffness parameters or the stiffness matrix itself took place.

This paper presents a detailed Finite Element Model (FEM) of open-hole tension (OHT), low-velocity impact (LVI) and multi-bolt double-lap joint of composite laminates. All numerical analyses are carried out using Puck failure criteria together with gradual stiffness decay, whereas the efficiency of the proposed damage model is evaluated by comparing with experimental data. The FE-model is developed using Abaqus software and numerical calculation is realized by using user-subroutines UMAT (tension) and VUMAT (impact). The structural responses such as force-displacement and contact force-time histories are recorded and analyzed to measure the performance of the damage model.

2. Composite damage model
To simulate the onset and progression of failure in composite laminate, a versatile and efficient progressive damage model (PDM) consists of failure theory to predict the initiation of damage, and damage evolution law to monitor the damage progression since the damage is accumulated throughout the layers.

2.1. Puck failure criteria and its constitutive law
In this work, the onset of failure is detected using three-dimensional (3D) Puck failure criteria, while the degradation of stiffness is achieved via the ply-discount method using a gradual degradation scheme. Table 1 shows the formulation used in predicting fiber failure in both tension and compression, respectively.

| Fiber failure (FF) | FF in tension | FF in compression |
|-------------------|--------------|------------------|
|                   | $f^T_{FF} = \frac{1}{R^T} \left[ \sigma_{11} - \left( v_{11} - v_{1\|f} m_{\sigma} \frac{E_t}{E_{\|f}} \right) \left( \sigma_{22} + \sigma_{33} \right) \right]$ for $[\ldots] \geq 0$ | $f^C_{FF} = \frac{1}{R^C} \left[ \sigma_{11} - \left( v_{1\|} - v_{1\|f} m_{\sigma} \frac{E_t}{E_{\|f}} \right) \left( \sigma_{22} + \sigma_{33} \right) \right]$ for $[\ldots] < 0$ |

Based on Puck and Schürmann [18], $R^T$ and $R^C$ are the tensile and compressive longitudinal strengths, while $v_{1\|}$ and $v_{1\|f}$ are the major in-plane Poisson’s ratio, and fiber Poisson’s ratio, respectively. The elastic properties of lamina in fiber directions and longitudinal directions are denoted as $E_{\|f}$ and $E_t$, respectively, and $m_{\sigma}$ is referred to magnification factor where the value of carbon fiber reinforced polymer (CFRP) and glass fiber reinforced polymer (GFRP) are taken as 1.1 and 1.3 [22], respectively.
Derivation for inter-fiber failure (IFF) has been made based on the assumption that stresses acting on the fracture plane \( \langle \rho \rangle \) causes the damage in the matrix compound. The action-plane fracture criteria have been formulated using stresses \( \sigma_n, \tau_{n\parallel} \) and \( \tau_{n\perp} \) which act on a plane parallel to the fibers and at an angle \( \theta \). These stresses are calculated with the aid of the following transformation:

\[
\begin{pmatrix}
\sigma_n(
\theta) \\
\tau_{n\parallel}(
\theta) \\
\tau_{n\perp}(
\theta)
\end{pmatrix}
= 
\begin{pmatrix}
c^2 & s^2 & 2sc \\
-sc & sc & (c^2 - s^2) \\
0 & 0 & s
c
\end{pmatrix}
\begin{pmatrix}
\sigma_{{22}} \\
\sigma_{{33}} \\
\tau_{{23}} \\
\tau_{{31}} \\
\tau_{{21}}
\end{pmatrix};
\quad c = \cos \theta, \quad s = \sin \theta
\]  

Table 2 summarises the functions used in forecasting the onset of matrix failure. Equation (3) was used to identify the matrix cracking in tension or compression.

### Table 2. Inter-fiber failure (FF) according to Puck failure criteria

| Inter-fiber failure (IFF) | IFF in tension for \( \sigma_n \geq 0 \) | \( f_{\text{IFF}}(\theta) = \left( \frac{1}{(R_\perp)} \cdot R_\parallel \psi \right)^{\frac{1}{2}} \left( \sigma_n(\theta) \right) \] |
|--------------------------|------------------------------------------|------------------------------------------------------------------|
| Inter-fiber failure (IFF)| IFF in compression for \( \sigma_n < 0 \) | \( f_{\text{IFF}}(\theta) = \left( \frac{1}{R_\parallel} \psi \right)^{\frac{1}{2}} \left( \sigma_n(\theta) \right) \] |

Other parameters such as Puck’s inclination parameters \( p_{l\psi}^t \) and \( p_{l\psi}^c \) used in IFF formulation have been extracted from Puck et al. [22] for a detailed explanation on additional parameters used here. The definition of the inclination parameters \( p_{l\psi}^t \) and \( p_{l\psi}^c \) at angle \( \psi \) can be written as:

\[
p_{l\psi}^i = \frac{p_{l\psi}^t}{R_\parallel} \quad \cos^2 \psi + \frac{p_{l\psi}^c}{R_\parallel} \sin^2 \psi; \quad i = t, c
\]

\[
\cos^2 \psi = \frac{\tau_{n\parallel}^2}{\tau_{n\perp}^2 + \tau_{n\parallel}^2}; \quad \sin^2 \psi = \frac{\tau_{n\parallel}^2}{\tau_{n\perp}^2 + \tau_{n\parallel}^2}
\]

\[
R_\perp = R_\perp^t; \quad R_\parallel = S_{21}; \quad R_\parallel = \frac{R_\parallel^t}{2(1 + p_{l\psi}^c)}
\]

The parameter \( \psi \) denotes the shear angle in action plane, \( R_\perp \) is failure resistance normal to fibers direction, while \( R_{l\psi}, R_{l\perp} \) and \( R_{l\parallel} \) are the fracture resistances of the action plane due to the shear stressing. In equation (8), \( R_{l\perp}^t \) and \( R_{l\perp}^c \) are the transverse tensile and compressive strength, respectively, while \( S_{21} \) is the in-plane shear strength of composite.

Whenever the fiber and inter-fiber failure functions reach unity (=1), then the structural stiffness matrix is updated using a reduced stiffness matrix, \( C^d \):

\[
C^d = \begin{bmatrix}
\beta C_{11} & kC_{12} & kC_{13} & 0 & 0 & 0 \\
kC_{21} & kC_{22} & kC_{23} & 0 & 0 & 0 \\
kC_{31} & kC_{32} & kC_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & \beta \omega G_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & \beta \omega G_{13} & 0 \\
0 & 0 & 0 & 0 & 0 & \beta \omega G_{23}
\end{bmatrix}
\]

\[
\beta = 1 - d_f, \kappa = (1 - d_f)(1 - d_m) \quad \omega = (1 - S_m d_m)(1 - S_m d_m)
\]
In the constitutive law, $d_f$ and $d_m$ are used as the global damage variables corresponding to FF and IFF, respectively. Individual damage variables based on failure mode are represented by $d_f^f$, $d_f^c$, $d_m^t$, and $d_m^c$ for fibre failure in tension and compression and inter-fiber failure in tension and compression, respectively. The relationship between global and local variables is defined as $d_f = 1 - (1 - d_f^f)(1 - d_f^c)$ and $d_m = 1 - (1 - d_m^t)(1 - d_m^c)$. The value for control parameters ($S_m^t = 0.9$, and $S_m^c = 0.5$) were used for controlling the matrix cracking as recommended in Abaqus manual.

2.2. Cohesive law
One of the dominant types of failure in composite laminate is delamination or debonding. It occurs in the thickness direction and caused from out-of-plane type of loading. Since Puck formulation does not consider delamination effect, cohesive contact formulation was used in this numerical exercise, where the complete implementation was carried out using the embedded equations in Abaqus software itself. The prediction of delamination was achieved through the functions given in table 3 and equation (10).

| Table 3. Failure criteria for delamination used in this article |
|---------------------------------------------------------------|
| **Approach**                      | **Cohesive contact interface**                  |
| Damage initiation               | Quadratic nominal stress criterion              |
| $\left\{ \frac{(t_n)}{t_n^0} \right\}^2 + \left\{ \frac{(t_s)}{t_s^0} \right\}^2 + \left\{ \frac{(t_t)}{t_t^0} \right\}^2 = 1$ |
| Damage evolution                | Power law fracture criterion                   |
| $\left\{ \frac{(G_n)}{(G_n)^0} \right\}^\alpha + \left\{ \frac{(G_s)}{(G_s)^0} \right\}^\alpha + \left\{ \frac{(G_t)}{(G_t)^0} \right\}^\alpha = 1$ |

The traction vector can be defined as:

$$t = \begin{bmatrix} t_n \\ t_s \\ t_t \end{bmatrix} = \begin{bmatrix} K_{nn} & K_{ns} & K_{nt} \\ K_{sn} & K_{ss} & K_{st} \\ K_{tn} & K_{ts} & K_{tt} \end{bmatrix} \begin{bmatrix} \varepsilon_n \\ \varepsilon_s \\ \varepsilon_t \end{bmatrix}$$

(10)

From the formula, $t$ is the nominal traction stress vector which consists of three components $t_n$, $t_s$, and $t_t$ while $K_{ii}(i = n, s, t)$ is the stiffness coefficient corresponding to the three stresses in cohesive element and $\varepsilon_i (i = n, s, t)$ is the three strains at material orientation.

3. Model validation

3.1. Material properties
In this paper, three different case studies were performed to test the capability of proposed progressive damage model to predict the initiation and final failure of composite laminates: (a) Open-hole tension (b) low-velocity impact, and (c) bolted joint. These three cases used different material properties since the references are taken from different publications. The properties were listed in table 4 and 5 for material and cohesive properties, respectively.

| Mechanical properties | T300/1034C | CCF300/Epoxy | T300/5228A |
|-----------------------|------------|--------------|------------|
| Longitudinal modulus, $E_1$ (GPa) | 146.8 | 123.91 | 144.0 |
| Transverse modulus, $E_2 = E_3$ (GPa) | 11.4 | 9.72 | 9.31 |
| The in-plane shear modulus, $G_{12} = G_{13}$ (GPa) | 6.1 | 4.53 | 4.68 |
Transverse shear modulus, $G_{23}$ (GPa) 3.8 2.56 3.0
The in-plane Poisson’s ratio, $v_{12} = v_{13}$ 0.3 0.288 0.31
Transverse Poisson’s ratio, $v_{23}$ 0.5 0.347 0.31
Longitudinal tensile strength, $R_{\parallel}^t$ (MPa) 1730 1762.3 1633
Longitudinal compressive strength, $R_{\parallel}^c$ (MPa) 1379 1362.2 1021
Transverse tensile strength, $R_{\perp}^t$ (MPa) 66.5 71.1 53.8
Transverse compressive strength, $R_{\perp}^c$ (MPa) 268.2 218.3 212
The in-plane shear strength, $R_{\perp\parallel}$ (MPa) 58.2 83.5 80.4

Table 5. Cohesive properties used in LVI simulation [24].

| $K_{mn}$ (MPa) | $K_{ss}$ (MPa) | $K_{tt}$ (MPa) | $t_n^0$ (MPa) | $t_s^0$ (MPa) | $t_t^0$ (MPa) | $G_n^c$ (J/m$^2$) | $G_s^c$ (J/m$^2$) | $G_t^c$ (J/m$^2$) |
|----------------|----------------|----------------|---------------|---------------|---------------|-----------------|-----------------|-----------------|
| 1 x 10$^5$     | 1 x 10$^5$     | 1 x 10$^5$     | 80            | 80            | 80            | 0.556           | 1.497           | 1.497           |

3.2. Finite element model

All finite element model (FEM) was developed using Abaqus 6-13 using solid reduced integration brick element (C3D8R) with enhanced hourglass control. This modeling part aims to mimic the test setup as close as possible.

3.2.1. Scenario I: Open-hole tension

The plies were modeled in stacked condition, which had one element in each ply in direction of thickness. Mesh size is quite fine at the vicinity of the hole, whilst coarser size further away from the notch as illustrated in Figure 1. No end tabs required for this virtual test because of the existence of notch. The dominant failure was expected to occur in the area of the hole. The dimension of central notch composite laminates were length, $L = 203.2$ mm, width, $W = 25.4$ mm, thickness, $t = 2.616$ mm, and diameter of the notch, $D = 6.35$ mm. The laminate was tested in tension, and composed of plies made of T300/1034C with ply orientation of [0/(±45)$\times$90]$^3$.

The simulation was performed using Abaqus/Standard solver and achieved via user-material subroutine (UMAT). One end of the specimen was clamped, and another end was displaced until the total failure.

![Figure 1. Dimension and boundary conditions of OHT coupon.](image1.png)

![Figure 2. Complete boundary conditions and relevant dimensions in LVI specimen.](image2.png)
3.2.2. Scenario II: Low-velocity impact

A 3D-FE model of an LVI specimen was modeled based on Abaqus/Explicit requirement. The set-up used here mimicked the test set-up according to ASTM D7136/D7136M-15 standard. The laminate was meshed with enhanced solid reduced integration brick element (C3D8R), while the impactor, rubber-clamps and base support were meshed using rigid element (R3D4) since the stiffness of the items are very high when compared with the laminate. Boundary conditions were simulated by fixing all DOEs (degree of freedom) of the fixture base and clamps. Besides, the initial velocity of LVI was 2577 mm/s in the vertical direction was prescribed to the impactor in this 3D FE model. The impactor was locked in all directions except in the direction of loading as can be seen in figure 2.

The diameter of the spherical punch was 16 mm, and the lumped mass 5.36 kg was specified in reference point of the rigid, while the moment of inertia was set to be a large value to prevent rotation of the punch. The nominal thickness of CCF300/epoxy carbon fiber composite laminates was 4mm. The original stacking sequence of the actual composite layer [45/0/−45/90]_4s was simplified to [45/0/−45/90_4s/−45/0/45] for the sake of computational cost. This approach is a so-called global-local approach and successfully implemented in Han et al. [26] works. The general contact was defined between the punch and composite laminate, base-support and bottom ply, rubber-clamps and top ply and between each ply, ignoring the effect of tangential friction force. The contact force between the punch and the laminate and the reaction force and displacement of compressive reference points were recorded and later was used for the failure analysis. Fiber breakage and matrix damage were predicted through the form of a user-defined variable (SDV).

3.2.3. Scenario III: Multi-bolt double-lap joint

The joint was modeled using the eight-node brick element (C3D8R) with hourglass controlled. The laminates were integrated into the model using a composite layup editor in Abaqus/CAE, where six-sub laminates were grouped with each layup contain four layers in the thickness direction. The composite specimens were made from T300/5228A carbon/epoxy composite material, having a stacking sequence of [0/45/90/−45]_3s. The material properties of this material are listed in table 3. Three bolts made of aerospace-grade Titanium alloy with a diameter of 6 mm each were arranged according to figure 3, where modulus and Poisson’s ratio were 110 GPa and 0.3, respectively.

For the sake of simplification, washers between laminates and bolts were not modeled and the bolts were assembled with one body (bolt and nut are modelled as one part). No bolt torques were applied to the system due to the finger tight consideration. To simulate the contact interaction, the surface to surface contact was constructed, where “hard” contact was applied in the normal direction to transfer enough pressure between contact surfaces. The penalty formulation was utilized in the model with a friction coefficient of 0.2 between all contact surfaces.
The simulation was performed under static analysis and achieved by Abaqus/Standard platform using the user-material subroutine (UMAT) to predict damage behavior in such type of joint. The details of composite failure criteria and evolution law are shown in table 1.

4. Result and discussion

4.1. Analysis of OHT of T300/1034C composite laminate

The numerical model with progressive damage model is validated with experimental results taken from the literature [27]. The load-displacement history between simulation and experiment is shown in figure 4(a).

Figure 4. (a) Comparison of force-displacement curve of test data and simulation of OHT coupon, and (b) Damage pattern recorded at the outer layer for fiber breakage and matrix cracking in both direction.

It can be said that the result of the simulation is highly acceptable since it followed the trend produced from the experiment. However, the proposed damage model is unable to precisely tracked the final pattern due to the non-linear functions in Puck formulation. Based on the experiment, the laminate lost its carrying capacity at 15.9 kN, while the FE model predicted the total failure at 14.64 kN which is 7.89% under-prediction from the benchmark.

Figure 4(b) demonstrates the failure patterns in fiber and inter-fiber(matrix) due to the tension load for the T300/1034C composite system. The red color represents total failure, whereby others represent partially fail and no failure conditions. The failure is concentrated in the direction of tension either in fiber or inter-fiber direction. It is observed clearly that both fiber and matrix cracking contributed to final failures to the structure as highlighted in red color. The onset of failure originated from the notch due to high concentration stress. The averaged crack paths are similar in both failure modes. The result also revealed that the net-tension failure obtained from the simulation was similar to the reference [27].

4.2. Structural response and damage analysis of LVI of CCF300/epoxy composite laminate

In this section, the CCF300/epoxy composite laminate is used as panel for low-velocity impact simulation under the velocity of 2.577 m/s. Based on figure 5(a), the contact force duration calculated from simulation is shorter than the test curve, which probably due to different stiffness and boundary condition applied to the model [26]. The result exhibited the performance of proposed damage model is sufficiently predict the peak force and the structural response where the difference was only 9.74%.
Figure 5. (a) Comparison of contact force-time curve of test data and simulation of LVI plate, and (b) Average damage patterns represent the fiber breakage and matrix cracking in both directions.

Under the impact energy of 4.45 J/mm (2.577 m/s), No significant contribution of fiber failure as can be depicted in figure 5(b). This is because the energy applied to the structure is low to break the fiber either in tension or compression. The matrix failure dominates the monolayers having the orientation of 0°, 45°, and -45°, while almost no effect on 90° layers. Another type of failure occurs in this case is delamination due to high inter-laminar tension and shear stresses in the impact zone [13] as illustrated in figure 6. There were six interface layers developed in the model, which all layer shows the existence of delamination (red-colour). Delamination spreads away from the location under the impact point and propagates to the outward regions (top to bottom). The interface between 45° and 0° layers was the most critical zone where the damage is highly affected from the delamination. It is pointed out that, delamination area become greater when increasing the impact loading.

Figure 6. Delamination in-between the cohesive interfaces.

4.3. Structural response and damage analysis of T300/5228A carbon/epoxy composite under multibolt double-lap joint

A comparison of the force-displacement curve between experiment and simulation is shown in figure 7. The damage model agreed well in predicting the average maximum load of 25762 N. The structure exhibits linear deformation and non-linear pattern once it started to recognize the damage onset and continue with similar behavior until final failure. The FE model can predict well the linear stage and satisfactorily estimates the non-linear path.
Figure 7. Comparison of reaction force-displacement curve of multi-bolt double lap joint between test data and simulation of T300/5228A composite system.

The numerical result shows in figure 8 concluded that the initial damage, as well as the final failure type, is bearing failure. Bolt-3 (right) experiences larger notch size as compared with other two bolts, due to its location closer to load application area. Both fiber breakage and matrix cracking occur in compression direction, while failure in loading direction shows less significant in the double lap joint scenario.

Figure 8. Damage patterns at the final stage (total failure).

5. Conclusion
In the present study, the numerical validations with three types of structural tests are implemented using 3D Puck failure theory incorporated with gradual degradation law. In the FE model, 3D solid elements are widely used to precisely predict the onset of failure as well as the damage accumulation in a composite laminate. The proposed failure model was able to predict the damage behavior of different
loading scenarios and capture well the damage initiation and progression on composite laminates. It is hoped that the failure formulation can be a great tool for future analysis in another form of loadings.

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