Massive neutrinos, Lorentz invariance dominated standard model and the phenomenological approach to neutrino oscillations

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Abstract

For the electroweak interactions, the massive neutrino perturbative kinematical procedure is developed in the massive neutrino Fock space. The perturbation expansion parameter is the ratio of neutrino mass to its energy. This procedure, within the Pontecorvo–Maki–Nakagawa–Sakata (PMNS)-modified electroweak Lagrangian, calculates the cross-sections with the new neutrino energy projection operators in the massive neutrino Fock space, resulting in the dominant Lorentz invariant standard model massless flavor neutrino cross-sections. As a consequence of the kinematical relations between the massive and massless neutrinos, some of the neutrino oscillation cross-sections are Lorentz invariance violating. But all these oscillating cross-sections, some of which violate the flavor conservation, being proportional to the squares of neutrino masses are practically unobservable in the laboratory. However, these neutrino oscillating cross-sections are consistent with the original Pontecorvo neutrino oscillating transition probability expression at short time (baseline), as presented by Dvornikov. From these comparisons, by mimicking the time dependence of the original Pontecorvo neutrino oscillating transition probability, one can formulate the dimensionless neutrino intensity-probability, by phenomenologically extrapolating the time $t$, or, equivalently the baseline distance $L$ away from the collision point for the oscillating differential cross-section. For the incoming neutrino of 10 MeV in energy and neutrino masses from Fritzsch analysis with the neutrino mixing matrix of Harrison, Perkins and Scott, the baseline distances at the first two maxima of the neutrino intensity are $L \simeq 281$ and 9279 km. The intensity $I$ at the first maximum conserves the flavor, while at the second maximum, the intensities violate the flavor, respectively, in the final and initial state. At the end some details are given as to how one should be able to verify experimentally these neutrino oscillations away from the collision point. In the material presented here one reinforces the notion that the massless flavor neutrino can be considered as the superposition of three massive neutrinos.

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1. Introduction

The flavor changing neutrino oscillation experiments, such as The Super-Kamiokande [1], SNO [2], KAMLAND [3] as well as Homestake Collaboration [4], clearly require massive neutrinos as have been exhibited, for example, by Bilenky et al [5], Giunti and Laveder [6] and Kayser [7]. In discussing the neutrino oscillations, one assumes that the left-handed flavor massless neutrino fields $\nu_{\alpha L}$, with $\alpha = e, \mu$ and $\tau$, are unitary linear combinations of the massive neutrino fields $\nu_{iL}$.
and analogously for the states (see [5–10] and references therein),
\[ v_{\alpha L} = U_{\alpha i} v_{i L} \quad (i = 1, 2, 3; \alpha = e, \mu, \tau), \]  

\[ |v_{i L}| = U_{\alpha i}^* |v_{i L}| \quad (i = 1, 2, 3; \alpha = e, \mu, \tau), \]  

where \( U \) is the unitary Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix and \( v_{i L} \) is a left-handed neutrino field associated with mass \( m_i \) (see for example [5–11]). In the study of flavor neutrino oscillations, the flavor state from (2) is used, via the Schrödinger equation, to calculate the oscillation probability \( P_{\alpha\beta} (L) \) for the flavor neutrino oscillation transition \( v_\alpha \to v_\beta \) at a very large distance \( L \) (~ thousands of km) (see for example [5–11]).

Not long after the suggestion by Pontecorvo [12] that massive neutrinos oscillate, which appears to be the easiest to treat with the Schrödinger equation (consult [5–11]), people started looking at how the neutrino masses influence the field theoretic treatment of the standard model (SM). One of the earliest paper that addressed that problem was by Schrock [13]. He studied mostly the decays where the neutrinos and antineutrinos appear in final states, such as the \( \pi, K \) and the nuclear \( \beta \) decays. Such decays are friendly to the application of the PMNS unitary transformations to the established SM. Here, the kinematics is then of the massive neutrinos augmented with the PMNS unitary matrix dependence. He proposed tests that showed promise for determination of neutrino masses and lepton mixing. Specifically, the proposed tests of the \( \pi \) and \( K \) decays could detect the neutrino masses in the 1–400 MeV range and those for the nuclear \( \beta \) decays in the 0.1 keV–5 MeV range. Today it is known that all these ranges are too high as it appears that every neutrino mass is below 1 eV.

More recently, Li and Liu [14] studied the connection of massive neutrinos to the SM by studying the inequivalent vacua model (see [15] and references therein); here, the transformation between the Fock space of neutrino mass states and the unitary inequivalent flavor states is a Bogoliubov transformation [14, 15]. This transformation, for instance, yields the corrections to the Pontecorvo neutrino oscillation probability that are of \( O(m^2) \) (\( m \) denoting generically any of the three neutrino masses). However, the problem is that it also yields that the branching ratio of \( W^+ \to e^+ + v_\mu \) to \( W^+ \to e^+ + v_\tau \) is \( O(m^2) \), which contradicts the Hamiltonian that one started from. In other words, in the inequivalent vacua model, there is a flavor changing current such as \( W^+ \to e^+ + v_\mu \) and the branching ratio is different from that of the SM with zero neutrino masses. In the appendix, Li and Liu [14] show that the neutrino oscillation effects are large enough to neglect the inequivalent vacua model effects; that is, the sum of all three decay widths of \( W^+ \to e^+ + v_\mu, \tau \) equals the width of \( W^+ \to e^+ + v_\nu, \mu, \tau \) in the SM.

It is well known that all by itself, the SM with massless flavor neutrinos has been remarkably successful in describing the laboratory experimental data, such as the neutrino scattering, at low and medium energies (see, for instance, [8, 9]). So one can then ask whether the SM cross-sections can be derived when starting, instead of with the massless flavor neutrino fields \( v_{\alpha L} \), with the massive neutrino fields \( v_{\alpha L} \). In this paper, the answer to this question is in the affirmative.

To proceed in this direction, as suggested by (1), the application of the PMNS substitution rule (3) transforms the SM Lagrangian density with the massless neutrino fields into the one with the massive neutrino fields (4):

The PMNS substitution : \( v_{\alpha L} \to U_{\alpha i} v_{i L} \)  

\[ \alpha, \beta, \ldots, \epsilon = e, \mu, \tau; i, j, a, \ldots, b = 1, 2, 3; \]

\[ l_{\alpha L} = \left( \frac{U_{\alpha i} v_{i L}}{a_{\alpha L}} \right); \]

\[ \epsilon_{L,R} = P_{L,R} \epsilon, \]

\[ P_{L,R} = \frac{1}{2} (1 \mp \gamma^5), \]

\[ L^\text{Lepton}_{W,\text{int}} = \frac{g}{\sqrt{2}} \sum_{\epsilon=\pm} \left[ \bar{v}_{L}(x) U_{\alpha i}^* \gamma^\mu \epsilon_{L}(x) W^\mu_{\alpha}(x,+) \right. \]

\[ + \bar{v}_{L}(x) \gamma^\mu U_{\alpha i} v_{i L}(x) W^\mu_{\alpha}(x,+) \right], \]

\[ W^\mu(x, \pm) = \frac{1}{\sqrt{2}} \left[ W^\mu(x, 1) \mp i W^\mu(x, 2) \right], \]

\[ L^\text{Lepton}_{Z,\text{int}} = \frac{g}{c_W} Z_{\mu}(x) \sum_{\epsilon=\pm} \left[ \bar{v}_{L}(x) T^3 \gamma^\mu i L_{\mu}(x) \right. \]

\[ \left. - s_W^2 \bar{\epsilon}(x) \gamma^\mu \epsilon(x) \right] \]

\[ = \frac{g}{4 c_W} Z_{\mu}(x) \sum_{\epsilon=\pm} \left[ \bar{\tau}_{\alpha}(x) U_{\alpha i}^* \gamma^\mu (1 - \gamma^5) \right. \]

\[ \times U_{\alpha b} v_{b L}(x) + \bar{\tau}(x) \gamma^\mu \left[ (4 s_W^2 - 1) + \gamma^5 \right] \epsilon(x) \right], \]

\[ s_W = \sin \theta_W, \]

\[ c_W = \cos \theta_W. \]

Since the Lagrangian densities (4) contain the massive neutrino fields, all the calculations are now done formally in the massive neutrino Fock space. The massless neutrinos will be the mass state neutrinos in the limit of negligible masses as a result of the perturbative neutrino kinematical procedure.

2. Perturbative kinematical procedure for calculating the neutrino differential cross-sections

A free neutrino spinor field with the mass \( m_i \), \( i = 1, 2, 3 \) is written generally with the creation and annihilation operators as

\[ v_i(x) = \frac{1}{(2\pi)^{3/2}} \int d^4q q^0 \sum_s e^{iqx} u(q, s) a(q, s) \]

\[ + e^{-iqx} v(q, s) b^\dagger(q, s), \]

\[ q^0 = (\bar{q}^2 + m_i^2)^{1/2}. \]

The perturbative kinematics is based on the fact that the neutrino mass \( m_i \) \( (m_i < 1 \text{ eV}) \) is generally much smaller than its absolute momentum value \( |q| \). Therefore it is convenient to start with the ‘massless’ four-component neutrino momentum \( q^\mu_{(\gamma)} \) with fixed flavor parameter \( \gamma \)

\[ q^\mu_{(\gamma)} = (\bar{q}^2, q^0_{(\gamma)}, q^0_{(\gamma)}), \quad q^2_{(\gamma)} = 0, \quad \gamma = e, \mu, \tau. \]
Next, one assumes that under this flavor parameter $\gamma$ are grouped together three massive neutrinos, say, $\nu_i$ with masses $m_i$; $i = 1, 2, 3$, then the difference among their energies $|\Delta q^0_{ij}(\nu_i, \nu_j)| \equiv |\Delta m^2_{ij}|/q^0_{ij} = [(m^2_i - m^2_j)/q^0_{ij}]$ is much smaller than the quantum-mechanical uncertainty of the energy [16]. As a consequence, in this case with fixed $\gamma$ it is impossible to distinguish the emission of neutrinos with different masses in the neutrino processes [16]. Hence, the three massive neutrinos, satisfying these quantum mechanical conditions, can be viewed as superposing themselves to form the flavor neutrino $\nu_\gamma$ as depicted by relations (1) and (2). With this in mind, with $q^\mu_{i(y)} = (q^x_{i(y)}, q^2_{i(y)}, q^3_{i(y)})$ as the four momentum of the massive neutrino with mass $m_i$ the perturbative kinematics can be presented as

$$q^\mu_{i(y)} \simeq q^{\mu 0}_{i(y)} - q^{\mu 0}_{i(y)} m_i^2 / 2q^0_{i(y)} + O(m_i^4), \quad i = 1, 2, 3; \gamma(\text{fixed}) = (e, \mu, \tau);$$

$$q^{0}_{i(y)} \simeq \tilde{q}^{0}_{i(y)} + m_i^2 / 2q^0_{i(y)} + O(m_i^4),$$

$$q_i^{0}_{(y)} \simeq -m_i^2 + O(m_i^4). \quad (7)$$

In (7) the terms with $O(m_i^6)$ will be neglected and the fixed parameter $\gamma$, as already established, is the neutrino flavor. Thus with this perturbative kinematics $q^\mu_{i(y)}$ ceases to be a true Lorentz four-momentum, while $q^{\mu 0}_{i(y)}$ remains so. However, since $q^{\mu 0}_{i(y)}$, as shown in (7), is the main part of $q^\mu_{i(y)}$, the main portions of cross-sections are expected to be Lorentz invariant (LI), while the Lorentz invariance violation (LIV) will be generated by $g^{\mu 0}m_i^2/2q^0_{i(y)}$ from (7). The LIV terms are expected to be very small due to the smallness of neutrino masses. Taking these relations into account, within the massive neutrino Fock space the differential cross-sections with flavor neutrinos are calculated. The question, of course, is: is the result consistent with the SM?

To continue, in analogy to $q^\mu_{i(y)}$, one now introduces $\tilde{s}_{i(y)}$ and $\tilde{s}_{i(y)}$ to denote, respectively, the helicity operators and eigenvalues for $i = 1, 2, 3$ massive neutrinos comprising the massless flavor neutrino $\nu_\gamma$. The helicity operator and eigenvalue of the massless flavor neutrino $\nu_\gamma$ are denoted, respectively, as $\tilde{s}_{i(y)}$ and $\tilde{s}_{i(y)}$. And, the effects of the massive- to massless-neutrino kinematical relation (7) on these helicity eigenvalues are simply what one can call the ordinary massive- to massless-neutrino helicity relation:

$$\tilde{s}_{i(y)} = \tilde{s}_{i(y)} \cdot \sigma / |\tilde{s}_{i(y)}| \perp \tilde{s}_{i(y)},$$

$$\tilde{s}_{(y)} = \tilde{s}_{(y)} \rightarrow \tilde{s}_{(y)} \rightarrow s_{(y)} = s_{(y)}, \quad i = 1, 2, 3; \gamma(\text{fixed}) = (e, \mu, \tau). \quad (8)$$

As a consequence of (7) and (8), with spinor indices suppressed, the contractions of massive neutrino free-field operators with the massive neutrino and antineutrino states are, respectively

$$\langle 0 | v(x, I) | \gamma_{i(y),} s_{i(y)} = \{ / (2\pi)^3 \}^{\gamma_{i(y)} \delta(\mathrm{k} - q_{i(y)})} \delta(s_{i(y)}, s_{i(y)}), \quad (9)$$

$$|\tilde{q}_{i(y),} s_{i(y)} = |v(x, k) |0\rangle = \{ / (2\pi)^3 \}^{\gamma_{i(y)} \delta(\mathrm{k} - q_{i(y)})} \delta(s_{i(y)}, s_{i(y)}),$$

where $s_{i(y)}$ and $\delta$ have the same kind of interrelationship as $s_{i(y)}$ and $\gamma$ in (8), etc.

Since, as shown in (7)–(9), the superposed three massive neutrinos contain the single flavor designation, either in the initial or final state, say, $\gamma$ and $\delta$, the process can be denoted as $\nu(\gamma) + \alpha(P_1) \rightarrow \nu(\delta) + \beta(P_2)$. From the Lagrangian densities (4) the amplitude and its Hermitian conjugate for the process containing massive neutrinos are built around these respective flavor designations, $\gamma$ and $\delta$, so that the generic amplitudes are given, respectively, as

$$S^\mathrm{ampl} = \sum_{i, j, \ldots} \delta(s_{i(y)} + P_{i(1)} - q_{(y)} - P_{(2)}) i M_{i, j, \ldots},$$

$$S^\mathrm{ampl} = \sum_{k, \ldots} \delta(s_{i(y)} + P_{i(1)} - q_{(y)} - P_{(2)}) (-i) M^*_{i, j, \ldots}. \quad (10)$$

Here, the momenta indicate the actual massive neutrino–lepton scattering, and different Latin indices indicate possibilities of summation with the $U$ matrices, which, however, here is not necessary to be explicit. To derive the cross-section, with the help of equations (7)–(9), one needs

$$S^\mathrm{ampl} = \sum_{i, j, \ldots} \left\{ \delta^2(s_{i(y)} + P_{i(1)} - q_{(y)} - P_{(2)}) + \delta^2(s_{i(y)} + P_{i(2)} - q_{(y)} - P_{(1)}) \right\} \right\} \left\{ M_{i, j, \ldots}(M^*_{i, j, \ldots}) \right\}$$

$$+ O(m^4). \quad (11)$$

The final result in (11) is the consequence of general delta function property $\delta(x, y) = 0$. The terms with $O(m^4)$, denoting the fourth power of products of variety of $m_1, m_2$, etc., are neglected. It follows that while the Fock space contains the massive neutrino states, the cross-section will utilize the kinematics of massless flavor neutrinos.

Next, one needs the spinor expressions, appearing in (9), to reflect, respectively, the kinematical and helicity relations in order to facilitate the cross-section calculations:

$$u(\tilde{q}_{i(y)}, s_{i(y)}) = \sqrt{2 (m_i + q^{\mu 0}_{i(y)})} u(m_i, \tilde{0}, s_{i(y)}),$$

$$u(m_i, \tilde{0}, s_{i(y)}) = (\pm 1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (12)$$
\[ q_{i,\alpha} = \gamma_\mu q_{i,\alpha}; \]
\[ \overline{u}(q_{i,\alpha}, s_{i,\alpha}) = \overline{u}(m_i, 0, s_{i,\alpha}) \frac{m_i - q_{i,\alpha}}{\sqrt{2(m_i + q_{i,\alpha}^0)}}. \]  \hspace{1cm} (12)

For a process with \( \gamma \) and \( \delta \) flavor designations, \( \nu(\gamma') + \alpha(P_1) \rightarrow \nu(\delta) + \beta(P_2) \), in cross-section evaluations, one will deal with the neutrino energy projection operator over the positive energy states. Furthermore, rather than averaging over, one simply sums over the massive neutrino helicity degrees of freedom. Consistent with the ordinary neutrino helicity relation (8), the sum is carried over only the equal helicity eigenvalues:
\[ s_{i,\alpha} = s_{k,\alpha} = s_{(\alpha)} = \sum_{s_{p,\alpha} = k_{i,\alpha}} u(q_{i,\alpha}, s_{i,\alpha}) \otimes \overline{u}(q_{k,\alpha}, s_{k,\alpha}) \]
\[ = \sum_{s_{\alpha}} u(q_{i,\alpha}, s_{\alpha}) \otimes \overline{u}(q_{k,\alpha}, s_{\alpha}) \]
\[ = \frac{1}{2} \left[ q_{(i,\alpha)}^{+} q_{(k,\alpha)}^{+}; +, c \right] , \]
\[ [q_{(i,\alpha)}^{+}, q_{(k,\alpha)}^{+}; +, c] = \left[ \frac{m_i - q_{(i,\alpha)}^{+}}{2}, \frac{m_k - q_{(k,\alpha)}^{+}}{2} \right] \frac{1 + \gamma^0}{2^{1/2}} \]
\[ \delta_{\alpha}\delta_{\alpha = 1, 2, 3; \gamma = e, \mu \text{ or } \tau}, \]  \hspace{1cm} (13)

where \( + \) sign refers to the positive energy states and \( c \) refers to the fact that the equal helicity eigenvalues in the sum yield the coherent result. (The incoherent projection operators \( [q_{(i,\alpha)}^{+}, q_{(k,\alpha)}^{+}; +, i] \) with unequal helicity eigenvalues \( s_{i,\alpha} \neq s_{k,\alpha} \) are not dealt with here.) The relation (13) defines the spinorial massive neutrino to massless neutrino helicity relation and it is consistent with the ordinary helicity relation (8). Carrying out the indicated operations in relation (13) as a power series over the neutrino masses/energy, one obtains for the neutrino energy projection operator over the positive energy states the following:
\[ \begin{align*}
[q_{(i,\alpha)}^{+}, q_{(k,\alpha)}^{+}; +, c] &= \frac{1}{2} \left[ q_{(i,\alpha)}^{+}, q_{(k,\alpha)}^{+}; +, c \right] \frac{1}{n} , \\
[q_{(i,\alpha)}^{+}, q_{(k,\alpha)}^{+}; +, c] &= -q_{(k,\alpha)}^{+} , \\
[q_{(i,\alpha)}^{+}, q_{(k,\alpha)}^{+}; +, c] &= m_k + \left( m_k - m_i \right) \frac{q_{(j,\alpha)}^{0}}{2q_{(i,\alpha)}^{0}} , \\
[q_{(i,\alpha)}^{+}, q_{(k,\alpha)}^{+}; +, c] &= -\frac{m_i - m_j}{2q_{(i,\alpha)}^{0}} q_{(i,\alpha)}^{0} + m_i m_j \gamma^0 \frac{1}{2q_{(i,\alpha)}^{0}} .
\end{align*} \]  \hspace{1cm} (14)

The coherent energy operator \( [q_{(i,\alpha)^+}, q_{(k,\alpha)^+}; +, c] \) generates the electroweak interactions that are the same as the SM interactions plus the LIV neutrino oscillation processes that are negligible since their cross-sections are proportional to the squares of neutrino masses and, as such, are essentially zero. Relation (14) is in essence the procedure for calculating the cross-sections for the processes requiring only the neutrino energy projection operators over the positive energy states.

3. Applications to the differential cross-section calculations

As established earlier and consistent with (11), the quasi-elastic electroweak process with massive neutrinos present, to \( O(m^2) \), can be denoted with the kinematics that uses just the massless flavor neutrinos:
\[ v(q_{(\gamma)}) + \alpha(P_1) \rightarrow v(q_{(\delta)}) + \beta(P_2); \]
\[ y = \frac{q_{(\gamma)}^0 - q_{(\delta)}^0}{q_{(\gamma)}^0} = \frac{p_{(2)}^0 - p_{(1)}^0}{q_{(\gamma)}^0}, \]  \hspace{1cm} (15)

where \( y \) is the normalized energy transfer. Now, although working in the massive neutrino Fock space, relation (11) says that the kinematics for the cross-sections for the quasi-elastic scattering of the massless flavor neutrinos is determined with flavor neutrino momenta according to \( \delta_{\alpha}(q_{(\gamma)}^0 + P_{(1)}^0 - q_{(\delta)}^0 - P_{(2)}^0) \). Furthermore, since (see also [8])
\[ \int dy = \frac{1}{2\pi} \int d\sigma(q_{(\delta)}) d\sigma(P_{(2)}) \times \delta_{\alpha}(q_{(\gamma)}^0 + P_{(1)}^0 - q_{(\delta)}^0 - P_{(2)}^0), \]  \hspace{1cm} (16)

the normalized neutrino (or charged lepton) energy transfer \( y = (q_{(\gamma)}^0 - q_{(\delta)}^0)/q_{(\gamma)}^0 \) cannot affect Lorentz invariance of any of the differential cross-sections.

Also, in view of (11), the cross-section normalization factor is defined with respect to the massless flavor neutrino momenta:
\[ B = \frac{1}{(2\pi)^3} \left| \left(P_{(1)} \cdot q_{(\gamma)}^0 \right)^2 - P_{(1)}^2 q_{(\gamma)}^0 \right|^{1/2} = \frac{1}{(2\pi)^3} \left| \left(P_{(1)} \cdot q_{(\gamma)}^0 \right) \right|. \]

In explicit evaluations, one uses the following short-hand notation:
\[ m_{ab} = \sum_i U_{ai} m_i U_{ij}^*, \quad m'_{ab} = \sum_i U_{ai} m_i^2 U_{ij}^*. \]  \hspace{1cm} (17)

4. Deriving the differential cross-sections with new energy projection operators for the flavor neutrino processes within the massive neutrino Fock space

\[ d\sigma_{\gamma} \] — from the Lagrangian density in (4), the free neutrino field (5), the kinematical relation (7), the relations (8) and (9), one derives in the usual way the \( W \)-exchange \( S_W \) and \( S_W' \) matrix elements for the process in (15). Specifically, with the Fierz rearrangement and repeated indices summing up, one has,
\[ S_W = \sum_{i,j} \delta_{\alpha\beta} \left( q_{(i,\gamma)} + P_{(1)} - q_{(j,\delta)} - P_{(2)} \right) \frac{\gamma^\mu U_{ai}^* U_{ij} U_{ji}^* U_{ai}^*}{(2\pi)^3} \times \frac{i q_{(i,\gamma)}^2}{8 M_W^2} \]  \hspace{1cm} (18)
and $S_L$ is obtained from (18) as shown in (10). The contribution to the process (15) due to the $W$-exchange from (18), after taking into account (11), (17), $\sqrt{S}g^2 = 8M^2_0G$, and the fact that $s_{i(y)}, s_{j(\delta)}, \ldots$, obey, respectively, the ordinary and spinorial helicity relation, (8) and (13), the standard procedure gives,

$$\frac{d\sigma_W(m)}{dy} = \frac{d\sigma_W(c^2)}{dy} \frac{G^2}{4\pi} \left\{ \begin{array}{c}
\delta_{y(\gamma)} \\
\delta_{y(\delta)}
\end{array} \right\} \times \sum_{i,j,k,h} \left( U^\dagger_{il} U_{ry} U_{ph} U^\dagger_{ga} \right) \left( U_{lj} U^\dagger_{ry} U^\dagger_{j(h) \delta} \right)
\times \left[ \text{Tr} \left( M_1 - P_{(1)} \right) \gamma_\mu \left( 1 - \gamma^5 \right) \right]
\times \left[ \text{Tr} \left( M_2 - P_{(2)} \right) \gamma_\mu \left( 1 - \gamma^5 \right) \right]
\times \left[ q_{(y),y(\gamma)} + \gamma^\nu \left( 1 - \gamma^5 \right) \right]
\times \left[ q_{(\delta),y(\delta)} + \gamma^\nu \left( 1 - \gamma^5 \right) \right],
$$

where $m$ symbolically denotes dependence on $m_{1,2,3}$. Next, with gamma matrices traces carried out, yields,

$$\frac{d\sigma_W(m)}{dy} = \frac{d\sigma_W(SM)}{dy} \left[ 1 + m^2_{\mu\nu} \left( \frac{1}{q^2_{(y)}} + \frac{1}{q^2_{(\delta)}} \right) \right]
- \frac{2G^2\delta_{y(\gamma)}}{\pi \left\{ \left( P_{(1)} \cdot q_{(y)} \right) \right\}} \left\{ \begin{array}{c}
\delta_{y(\gamma)} m_{\mu\nu} m_{\delta\lambda} \\
\delta_{y(\delta)} m_{\mu\nu} m_{\delta\lambda}
\end{array} \right\} \times \left[ \text{Tr} \left( M_1 - P_{(1)} \right) \gamma_\mu \left( 1 - \gamma^5 \right) \right]
\times \left[ \text{Tr} \left( M_2 - P_{(2)} \right) \gamma_\mu \left( 1 - \gamma^5 \right) \right]
\times \left[ q_{(y),y(\gamma)} + \gamma^\nu \left( 1 - \gamma^5 \right) \right]
\times \left[ q_{(\delta),y(\delta)} + \gamma^\nu \left( 1 - \gamma^5 \right) \right].
$$

One can notice that, while the negligible LIV is associated with the neutrino mass, the LI Standard Model result is formally identified with zero neutrino mass limits

$$\frac{d\sigma_W(m)}{dy} = \frac{d\sigma_W(SM)}{dy} \left[ 1 + m^2_{\mu\nu} \left( \frac{1}{q^2_{(y)}} + \frac{1}{q^2_{(\delta)}} \right) \right] + O(m^4; LIV),
$$

one can summarize the neutrino flavor transitions for the $W$-exchange neutrino processes. Flavor conserving are LI to $O(m = 0)$ terms and negligible LIV to $O(m^2)$ terms. Flavor violating are negligible LIV to $O(m^3)$ terms. The $d\sigma_S/dy$—as in the previous case, from the Lagrangian density in (4), the free neutrino field (5), the kinematical relation (7), the contractions (9), one derives in the usual way the $Z$-exchange $S_Z$ and $S'_Z$ matrix elements for the process in (15). Specifically, one has

$$S_Z = \sum_{i,j} \frac{G^2}{4\pi} \left\{ \begin{array}{c}
\delta_{y(\gamma)} \\
\delta_{y(\delta)}
\end{array} \right\} \times \frac{1}{Z^2} \left( P_{(1)} - q_{(y)} \right) \delta_{ij} \delta_{y(\gamma)U_{y(\delta)}}
\times \left[ \text{Tr} \left( M_1 - P_{(1)} \right) \gamma_\mu \left( 1 - \gamma^5 \right) \right]
\times \left[ \text{Tr} \left( M_2 - P_{(2)} \right) \gamma_\mu \left( 4q^2_{(\delta)} - 1 + \gamma^5 \right) \right]
\times \left[ \text{Tr} \left( M_1 - P_{(1)} \right) \gamma_\mu \left( 1 - \gamma^5 \right) \right]
\times \left[ \text{Tr} \left( M_2 - P_{(2)} \right) \gamma_\mu \left( 1 - \gamma^5 \right) \right],
$$

(22)

while $S'_Z$ is obtained from the $S_Z$ through the Hermitian conjugation. In what follows, one will find the following shorthand notation very useful:

$$u_0 = s^0_{(y)},
\quad u_1 = 2s^0_{(y)} - 1,
\quad z_1 = s^0_{(y)} (2s^0_{(y)} - 1) + \frac{1}{2},
\quad z_2 = s^0_{(y)} (2s^0_{(y)} - 1),
\quad z_3 = s^0_{(y)} - \frac{1}{2},
\quad z_4 = s^0_{(y)} (s^0_{(y)} - 1) + \frac{1}{2},
\quad z_1 + z_3 = 2s^0_{(y)}.
$$

After taking into account that $c^2_{\mu\nu} M^2 Z = M^2_0$, and the fact that the helicities, $s_{i(y)}, s_{j(\delta)}, \ldots$ obey both the ordinary and the spinorial helicity relations (8) and (13), the standard procedure yields the general expressions:

$$\frac{d\sigma_W(m)}{dy} = \frac{d\sigma_W(SM)}{dy} \left[ 1 + m^2_{\mu\nu} \left( \frac{1}{q^2_{(y)}} + \frac{1}{q^2_{(\delta)}} \right) \right]
\times \left[ \text{Tr} \left( M_1 - P_{(1)} \right) \gamma_\mu \left( 2z_3 + \frac{1}{2} \gamma^5 \right) \right]
\times \left[ \text{Tr} \left( M_2 - P_{(2)} \right) \gamma_\mu \left( 2z_3 + \frac{1}{2} \gamma^5 \right) \right]
\times \left[ \text{Tr} \left( q_{(y),y(\gamma)} + \gamma^\nu \left( 1 - \gamma^5 \right) \right) \right].
$$

(24)

The coherent energy operator expansion according to (14), with evaluating the traces of gamma matrices, yields

$$\frac{d\sigma_Z(m)}{dy} = \frac{d\sigma_Z(SM)}{dy} \left[ 1 + m^2_{\mu\nu} \left( \frac{1}{q^2_{(y)}} + \frac{1}{q^2_{(\delta)}} \right) \right]
- \frac{G^2 m_{\gamma\mu} m_{\gamma\nu} \delta_{y(\gamma)}}{8\pi \left\{ \left( P_{(1)} \cdot q_{(y)} \right) \right\}} \left[ \begin{array}{c}
\frac{1}{q^2_{(y)}} \\
\frac{1}{q^2_{(\delta)}}
\end{array} \right] \left[ \begin{array}{c}
M_1 M_2 z_2 (q_{(y)} \cdot q_{(\delta)}) \\
M_1 M_2 z_2 (q_{(y)} \cdot q_{(\delta)})
\end{array} \right]
+ \left( P_{(1)} \cdot q_{(y)} \right) \left( P_{(2)} \cdot q_{(y)} \right) (z_1 + z_3)
+ \left( P_{(1)} \cdot q_{(y)} \right) \left( P_{(2)} \cdot q_{(y)} \right) (z_1 - z_3)
+ \left( P_{(2)} \cdot q_{(y)} \right) \left( P_{(2)} \cdot q_{(y)} \right) (z_1 + z_3)
+ \left( P_{(2)} \cdot q_{(y)} \right) \left( P_{(2)} \cdot q_{(y)} \right) (z_1 - z_3)
+ \left( P_{(1)} \cdot q_{(y)} \right) \left( P_{(1)} \cdot q_{(y)} \right) (z_1 + z_3)
.$$
formally zero neutrino mass limits:

\[ \frac{d\sigma_{Z}(SM)}{dy} = \frac{G^2 \delta_{gg} \delta_{\alpha\beta}}{\pi \left| (P_{11})_{(q_{(y)})}\right|^2} \left[ M_{1} M_{2} z_{2} (q_{(y)}) \right] + O(m^4), \] (25)

Here also, the negligible LIV is associated with the neutrino mass while the LI Standard Model result is identified with formally zero neutrino mass limits:

\[ \frac{d\sigma_{Z}(m)}{dy} = \frac{d\sigma_{Z}(SM)}{dy} + O(m^2; LIV). \] (26)

While the terms of \( O(m = 0) \) are LI and flavor conserving, the negligible LIV terms of \( O(m^2) \) are either flavor violating or flavor conserving.

\[ \frac{d\sigma_{W,Z}/dy—here, the differential cross-section for the quasi-elastic neutrino scattering (15) due to the overlapping \( S \)-matrix elements from the \( W \) and \( Z \) is given as a sum of its components after taking into account relations (11), (18) and (22). Importantly, again taking into account the fact that helicities, \( s_{(i,j)}, s_{(j,k)}, \ldots \) obey both the ordinary and spinorial helicity relations, (8) and (13), the standard procedure yields the general expression:

\[ s_{(e,y)} = s_{(e,y)} = s_{(i,j)} = s_{(i,j)} \]

\[ s_{(h,\delta)} = s_{(h,\delta)} = s_{(i,j)} = s_{(i,j)} \]

\[ \frac{d\sigma_{W,Z}(m)}{dy} = \frac{d\sigma_{W,Z}(m)}{dy} + O(m^2; LIV). \] (27)

Of course, one cannot avoid the coherent energy operator expansion according to (14), and evaluating the traces of gamma matrices one obtains

\[ \frac{d\sigma_{W,Z}(m)}{dy} = \frac{d\sigma_{W,Z}(SM)}{dy} - \frac{G^2 m_{\nu y} m_{\delta y} \delta_{\alpha\beta}}{2\pi \left| (P_{11})_{(q_{(y)})}\right|^2} \left[ M_{1} M_{2} w_{0} (q_{(y)}) \right] + O(m^4), \]

\[ \frac{d\sigma_{W,Z}(m)}{dy} = \frac{d\sigma_{W,Z}(SM)}{dy} - \frac{G^2 m_{\nu y} m_{\delta y} \delta_{\alpha\beta}}{2\pi \left| (P_{11})_{(q_{(y)})}\right|^2} \left[ M_{1} M_{2} w_{0} (q_{(y)}) \right] + O(m^4), \]

Again, the negligible LIV is associated with the neutrino mass, while the LI Standard Model result is identified with formally zero neutrino mass limits:

\[ \frac{d\sigma_{W,Z}(m)}{dy} = \frac{d\sigma_{W,Z}(SM)}{dy} + O(m^2; LIV). \] (28)

The overwring \( W \)- and \( Z \)- exchanges cross-section terms of \( O(m = 0) \) are LI and flavor conserving, while the negligible terms of \( O(m^2) \) carry the LIV terms with both the conserved and violated flavor.

5. Phenomenological neutrino cross-section intensity from time extrapolation of the neutrino oscillation scattering

Dvornikov in the classical field theoretical model [17] quotes the neutrino oscillation-transition probability with the characteristic Pontecorvo dimensionless argument \( \Delta m^2 t/4E \).

\[ P(t) = \sin^2(2\theta_{\nu e}) \sin \left( \frac{\Delta m^2 t}{4E} \right), \] (30)

where \( \theta_{\nu e} \) is the vacuum mixing angle, \( \Delta m^2 = m_1^2 - m_2^2 \) is the mass squared difference and \( E \) is the energy of the system (detailed description of these and other parameters is given in [17]). The interest in [17] comes from the fact that at \( t = 0 \), one can expand (3) for small Pontecorvo argument as \( \sin \left( \frac{\Delta m^2 t}{4E} \right) = \frac{\Delta m^2 t}{4E} \cdots \). Or, turning the logic around, one could think of the scattering theory presented here as an approximation of a more general description, yet to be derived, where \( \sin \left( \frac{\Delta m^2 t}{4E} \right) \) occurred but for the laboratory measurements was approximated with \( \frac{\Delta m^2 t}{4E} \). So, with this reversed logic, one can assume that the calculated neutrino intensity, defined appropriately from differential cross-sections, can be analyzed at the practical baseline region if in it every dimensionless Pontecorvo argument of the
\[ \Delta m^2 \] is replaced with \( \frac{\Delta m^2}{4\pi^2} \). Other terms remain unaffected and, if proportional to the squares of neutrino masses, may be even neglected. However, the recoil, which Dvornikov [17] in the classical approach did not have, here will have to be taken into account and, if possible, averaged out in the calculated neutrino intensity in the short baseline region.

To pursue the idea from [17] on the phenomenological level, say, on the neutrino scattering process as described by the differential cross-section (28), it is necessary to work in the laboratory frame, \( P_1 = (0, M_1) \). Next thing is to concentrate on the incoming neutrino energy in (28) with the help of the neutrino energy transfer

\[
y = \frac{q_{(\gamma)} - q_{(\delta)}}{q_{(\gamma)}},
\]

(31)

\( q_{(\delta)} = (1 - y) q_{(\gamma)} \).

So that after substituting (31) into (28) one obtains for the differential cross-section

\[
\frac{d\sigma_{(W,Z)}(m)}{dy} = \frac{d\bar{\sigma}_{(W,Z)}(SM)}{dy} \delta_{\alpha\beta} \left\{ \frac{1}{1 + \left(1 - y\right)^2} - \frac{m_y m_0}{4 q_{(\gamma)}^2} \left[ \delta_{\alpha\gamma} + \delta_{\alpha\beta} \right] \left( \frac{1}{1 - y} \right)^2 \right\}
\]

\[
- \frac{G^2 m_y m_0}{\pi} \left[ \frac{u_0 M_1 M_2}{w_0} \left( \frac{\delta_{\alpha\gamma}}{1 - y} \right) + \delta_{\alpha\beta} \left( \frac{P_{(\gamma)} \cdot q_{(\delta)}}{q_{(\gamma)}} \right) \right],
\]

(32)

\[
\frac{d\bar{\sigma}_{(W,Z)}(SM)}{dy} = \frac{2G^2}{\pi} \left[ M_1 M_2 u_0 \left( \frac{q_{(\gamma)} \cdot q_{(\delta)}}{q_{(\gamma)}} \right) - u_1 M_1 q_{(\gamma)} \left( P_{(\gamma)} \cdot q_{(\delta)} \right) \right].
\]

(33)

In order to bring out the new features, one ‘removes’ the SM cross-section from the newly derived cross-section, through the definition of the neutrino cross-section intensity, which is given by (30), will allow the introduction of the dimensionless Pontecorvo argument (30).

\[ I_{(W,Z)}(m, \theta) = I_1 P_{(\gamma)} + I_2 P_{(2\beta)} + I_3 P_{(\delta)} \]

\[ = \frac{2G^2}{\pi} \left[ M_1 M_2 u_0 \left( \frac{q_{(\gamma)} \cdot q_{(\delta)}}{q_{(\gamma)}} \right) - u_1 M_1 q_{(\gamma)} \left( P_{(\gamma)} \cdot q_{(\delta)} \right) \right].
\]

(34)

where the dimensionless intensity \( I_{(W,Z)}(m, \theta) \) from (34) will serve also as unnormalized probability. As shown in (31) and (35), the angle \( \theta \) dependence comes through the kinematics. The expression for the normalized neutrino energy transfer \( y \) follows from scattering kinematics and the energy–momentum conservation (more details can be found in [9]). As seen in (35), for fixed \( q_{(\gamma)} \), \( y \) depends only on \( \cos^2 \theta \), which would be averaged over straightforwardly.

Relation (34), of course, is an instantaneous intensity at the time of interaction. This instant can be defined as \( t_0 = 1/q_{(\gamma)} \). For \( q_{(\gamma)}^0 = 10 \text{MeV}, t_0 = 6.58 \times 10^{-21} \text{s} \), one can also define a ‘distance’ with \( L_0 = 1/q_{(\gamma)}^0 \) to give for the same neutrino energy \( L_0 = 1.97 \times 10^{-12} \text{cm} \). Of course, what counts are relative times and distances. Here, \( t_0 \) and \( L_0 \) are given for the sake of convenience. Now, as described at the beginning of this section, one can perform the time extrapolation with sinus functions on the arguments that are in the Pontecorvo forms, while others, with neutrino mass square dependences, can be dropped. The result of this procedure is, where for the sake of simplicity, the processes are denoted with just the flavor quantum numbers:

\[ I_{(W,Z)}(m, \theta) = I_1 P_{(\gamma)} + I_2 P_{(2\beta)} + I_3 P_{(\delta)} \]

\[ = \frac{2G^2}{\pi} \left[ M_1 M_2 u_0 \left( \frac{q_{(\gamma)} \cdot q_{(\delta)}}{q_{(\gamma)}} \right) - u_1 M_1 q_{(\gamma)} \left( P_{(\gamma)} \cdot q_{(\delta)} \right) \right].
\]

(36)

\[ I_1 (P_{(\gamma)} + P_{(\delta)}) = -\delta_{\alpha\beta} \delta_{\alpha\gamma} \left[ 1 + \left(1 - y\right)^2 \right] \sin \frac{m_y^2 t}{4 q_{(\gamma)}},
\]

\[ I_2 (P_{(\gamma)} + P_{(\delta)}) = -\delta_{\alpha\beta} \delta_{\alpha\gamma} \frac{1}{\left(1 - y\right)^2} \sin \frac{m_y m_0 t}{4 q_{(\gamma)}},
\]

\[ I_3 (P_{(\gamma)} + P_{(\delta)}) = -\delta_{\alpha\beta} \delta_{\alpha\gamma} \frac{1}{\left(1 - y\right)^2} \sin \frac{m_y m_0 t}{4 q_{(\gamma)}},
\]

(37)

One notices that, since \( q_{(\gamma)}^0 \) is the incoming neutrino energy, the intensities in (36) depend on the scattering angle \( \theta \) only through \( y \) (compare with (35)). So in (37), it is shown how to get averaged, over the scattering angle, the total intensity \( I_{(W,Z)}(m) \) from the sum of individual averaged intensities. Since the dimensionless intensities serve also as unnormalized probabilities, one has to look just at zero to positive values.
when connecting to observations; this has to be done for the individual as well as for the total intensities, unaveraged and averaged:

\[
\text{Observ.: } I_i \geq 0, \langle I_i \rangle \geq 0, \tag{38}
\]

\[
\text{Unobserv.: } I_i \leq 0, \langle I_i \rangle \leq 0.
\]

\[
\text{Observ.: } I_{[W,Z]}(m, \theta) \geq 0, I_i \geq 0; \left\{ I_{[W,Z]}(m, \theta) \right\} \geq 0, \langle I_i \rangle \geq 0, \tag{39}
\]

\[
\text{Unobserv.: } I_{[W,Z]}(m, \theta) \leq 0, I_i \geq 0; \left\{ I_{[W,Z]}(m, \theta) \right\} \leq 0, \langle I_i \rangle \geq 0.
\]

One notices that (36) has three independent parts as the flavor numbers \( \gamma \) and \( \delta \) are generally independent of \( \alpha \) and \( \beta \). Hence (36) is describing, respectively, the flavor conserving \( \alpha = \beta = \gamma = \delta \) transition, and the flavor violating \( \alpha = \beta = \gamma \neq \delta \) and \( \alpha = \beta = \delta \neq \gamma \) transitions. Already at this point, one can see that one has here the physics beyond the SM. The interesting thing is that if one forces \( \gamma \) and \( \delta \) to be identical with \( \alpha \), one obtains that (36), (37) \( \approx 0 \) for any value of \( t \) or \( L \); this is another indication that physics went beyond the SM.

At this point, it is the easiest to continue if one has the explicit values for neutrino masses and the neutrino mixing matrix. The masses accepted here are from the analysis by Fritzsch [18] with values,

\[
m_1 = 0.004 \text{ eV}, \quad m_2 = 0.01 \text{ eV}, \quad m_3 = 0.05 \text{ eV}, \quad (40)
\]

while the neutrino mixing matrix is due to Harrison et al [19]

\[
(U_{\alpha \nu}) = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\
-\dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{3}} & -\dfrac{1}{\sqrt{2}} \\
-\dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{2}} \\
\end{pmatrix} . \quad (41)
\]

Now, taking that the target at rest is an electron, one has \( M_1 = M_2 \approx 0.5 \text{ MeV} \), while one retains the incoming energy of the neutrino at \( q^0_{(\nu)} = 10 \text{ MeV} \) and looks for \( L_+ = L_\nu \) and \( L_- < L_\nu \), at which the values of \( +\sin \alpha \) and \( -\sin \alpha \) functions in (36) become +1, yielding first large intensities. The result is

\[
\sin \frac{m_\nu^2}{4q^0_{(\nu)}} t = 1 \rightarrow L_+ = \frac{2\pi q^0_{(\nu)}}{m_\nu^2}, \tag{42}
\]

\[
-\sin \frac{m_\nu m_{\nu \tau}}{4q^0_{(\nu)}} t = 1 \rightarrow L_- = \frac{6\pi q^0_{(\nu)}}{m_\nu m_{\nu \tau}}.
\]

Next, one realizes that, to a good approximation, these distances away from the source, where the first large intensities occur, are given as \( L_\pm = L_\pm \approx L_\nu \), that is, with just \( L_\pm \). As to the specific place where one notices these maxima, one has to involve the kinematics from (31) and (35).

Relation (42) is a general relation; as already mentioned, from (36), one actually has three possibilities for transitions: one flavor conserving and two flavor violating. Using the values for the neutrino masses in (40) and the neutrino matrix in (41), one deduces the distances from (42) to be

(43)

Flav. conserv.) \( I_1 \left( P_{(1a)\nu} \rightarrow P_{2(\beta)\nu} + P_{(\delta)\nu} \right) \delta_{\beta\gamma} \delta_{\beta\delta} \); \( \alpha, \beta = e; \gamma, \delta = e : L_+ (v_{(e)} \rightarrow v_{(e)}) \approx 281 \text{ km.} \)

Flav. viol.) \( I_2 \left( P_{(1a)\nu} + v_{(\gamma)} \rightarrow P_{2(\beta)\nu} + v_{(\delta)} \right) \delta_{\beta\gamma} \); \( \alpha, \beta, \gamma = e; \delta = \mu : L_- (v_{(\mu)} \rightarrow v_{(\mu)}) \approx 9279 \text{ km.} \)

Flav. viol.) \( I_3 \left( P_{(1a)\nu} + v_{(\gamma)} \rightarrow P_{2(\beta)\nu} + v_{(\delta)} \right) \delta_{\beta\gamma} \); \( \alpha, \beta, \gamma = e, \delta = \mu : L_- (v_{(\mu)} \rightarrow v_{(e)}) \approx 9279 \text{ km.} \)

The distances are rather large, particularly for the flavor violating oscillations, \( v_{(e)} \rightarrow v_{(\mu)} \) and \( v_{(\mu)} \rightarrow v_{(e)} \), which one can attribute to the small values of neutrino masses. One notices that among these three distances in (43) two are the same, \( L_- (v_{(e)} \rightarrow v_{(e)}) = L_- (v_{(\mu)} \rightarrow v_{(e)}) \).

For \( I_1 \) and \( I_3 \) intensities, it helps to look at \( \theta \) dependence of \( (1 - y)^{-2} \) in order to see at which angles they are best detected. Here are some of the most significant values:

\[
(1 - y)^{-2} \begin{cases}
\theta = 0; 0.1; 0.4; 0.5; \frac{\pi}{2} \\
= 1681; 60.34; 2.26; 1.7; 1.
\end{cases}
\]

Indeed, \( I_1 \) and \( I_3 \) have rather large maxima in the forward, \( \theta = 0 \), direction, the direction that should be favorable for detecting neutrino oscillation.

One may as well look at the average values of intensities over the scattering angle \( \theta \). For this one needs the scattering angle average quantity \( \langle (1 - y)^{-2} \rangle \) in (36). For the electron target, which is a very light target, one notices very large values in the forward, \( \theta = 0 \), direction and rather smooth thereafter up to \( \theta = \pi/2 \). The large value in question of 1681 is at \( \theta = 0 \). This comes basically from the smallness of the electron mass. Another meaning of the large value at \( \theta = 0 \) is that the recoil activities are strong at these angles when the target is light, and weak in the perpendicular directions. The straightforward numerical integration from more detailed evaluation of \( (1 - y)^{-2} \) yields for the average values

\[
\begin{align*}
\langle (1 - y)^{-2} \rangle &= \frac{2}{\pi} \int_0^{\pi/2} d\theta (1 - y)^{-2} \approx 59, \\
\langle I_1 \rangle &\left( P_{(1a)\nu} + v_{(\gamma)} \rightarrow P_{2(\beta)\nu} + v_{(\delta)} \right) \delta_{\beta\gamma} \delta_{\beta\delta} 60 \sin \frac{m_\nu^2 t}{4q^0_{(\nu)}}, \\
\langle I_2 \rangle &\left( P_{(1a)\nu} + v_{(\gamma)} \rightarrow P_{2(\beta)\nu} + v_{(\delta)} \right) = -\delta_{\beta\gamma} \sin \frac{m_\nu m_{\nu \tau} t}{4q^0_{(\nu)}}, \\
\langle I_3 \rangle &\left( P_{(1a)\nu} + v_{(\gamma)} \rightarrow P_{2(\beta)\nu} + v_{(\delta)} \right) = -\delta_{\beta\gamma} \delta_{\beta\delta} 59 \sin \frac{m_\nu m_{\nu \tau} t}{4q^0_{(\nu)}}.
\end{align*}
\]

The numbers 60, 1 and 59 have the meanings of relative strengths with respect to each other of these dimensionless intensities. Then according to definitions in (38) and (39), for flavor conserving part, \( \langle I_{[W,Z]}(m, \theta) \rangle \approx 100\% \quad \{I_1(\alpha + \alpha \rightarrow \alpha + \alpha)\} \), where one identified 60 = 100\%. One may say that the flavor violating parts \( \langle I_2 \rangle \) and \( \langle I_3 \rangle \) split 100\% as \( \langle I_{[W,Z]}(m, \theta) \rangle \approx 2\% \quad \{I_2(\alpha + \alpha \rightarrow \alpha + \delta)\} \) and \( 98\% \quad \{I_3(\alpha + \gamma \rightarrow \alpha + \alpha)\} \).

(45)
Now, unlike in the classical field theoretical model of Dvornikov [17], here one has quantum field theory with the particle kinematics so that the scattering angle $\theta$ between the incoming neutrino and the scattered electron comes into play. So far, one was dealing with the incoming neutrino and derived the intensities. However, at distances of 281 and 9279 km, one can hope to detect the outgoing neutrino energy $q^0_{(\gamma)}$, while, at the same time, measure the scattering angle $\theta$ of the recoiled electron. From relations (31) and (35), one now correlates $q^0_{(\delta)}$, $q^0_{(\gamma)}$ and angle $\theta$ through the relation

$$P_{(\delta)} = \left(0, M_1 \right), \vec{q}_{(\delta)} = q^0_{(\gamma)} \frac{\cos \phi}{\sin \theta} \left[ q^0_{(\gamma)} - q^0_{(\delta)} + 2M_1 \right]$$

$$+ q^0_{(\gamma)} M_1 (M_1 - 2q^0_{(\delta)}) - q^0_{(\delta)} M_1^2 = 0; \quad (46)$$

$$q^0_{(\gamma)} = \left| \vec{P}_{(\gamma)} \right| \cos \theta + q^0_{(\delta)} \cos \phi,$$  

$$\left| \vec{P}_{(\gamma)} \right| \sin \theta = q^0_{(\delta)} \sin \phi. \quad (48)$$

Relation (46), for example, associates the neutrino scattering with a measured outgoing neutrino energy. Relations (47) and (48) correlate the electron recoil with incoming and outgoing neutrino energies. Specifically, with the knowledge of $\theta$ and $q^0_{(\gamma)}$, one can calculate the value for $q^0_{(\delta)}$; for instance for $\theta \approx 0$, (46) gives $q^0_{(\delta)} \approx 0.24 \text{ MeV}$. From (47), one has that $|\cos \phi| = |q^0_{(\gamma)} - |\vec{P}_{(\gamma)}| \cos \theta| (q^0_{(\delta)})^{-1} \leq 1$. From (47) and/or (48), one can solve for two quantities, assuming that others are known. The presumed oscillation of the $\delta$-neutrino is occurring along the line with the angle $\phi$ with respect to $\vec{P}_{(\gamma)}$ and the corresponding intensity is determined by relations (45). Although, by and large, one expects $q^0_{(\delta)} < |\vec{P}_{(\gamma)}|$, which implies $0 < \sin \phi$, nevertheless, smaller angle $\theta$ would imply also smaller angle $\phi$. Of course in addition to relations (46) and (47) one can find other relations that can serve the purpose of studying the neutrino oscillations from scattering experiments.

6. Discussion

One thing that one notices right away is the fact that while the LIV is very real, because it is associated with the $O(m^2)$ terms, it is negligible in the scattering-like experiments. Therefore, the ‘massless’ SM is basically LI because the neutrinos have masses that are $\leq 1 \text{ eV}$. Because the LIV terms are proportional to $O(m^2)$, they play no role in the laboratory experiments where the SM dominates. As shown by relations (30)–(34), the extrapolation of the negligible portion of the laboratory neutrino oscillation scattering cross-sections into the practical baseline oscillation differential cross-sections should make them now observable at reasonable distances from the interaction region, particularly through the help of the neutrino cross-section intensity (43). The most interesting thing is that this extrapolated differential cross-section contains the flavor conserving and two kinds of flavor violating parts.

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