Method and device for dynamic modelling of rubbery materials applied to human soft tissues. Part II: device and experimental results

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Abstract. The paper presents the experimental results obtained on an experimental device where a horizontal rubber wire is stretched by a transversal oscillating force - that is a body with an acceleration sensor attached, placed at the middle of it that oscillates freely. A nonlinear model was proposed for the experimental test rig, the differential equation of motion was offered and a series of curves were traced and compared to the experimental ones. One can conclude that the theoretical model certifies very well the behaviour of the real model. An open problem remains the manner of adopting the parameters characteristic to the dissipative element of the system.

1. Introduction
Analyzing the shape of traction force-deformation curves corresponding to human tissues, [1], [2], it was noticed a good agreement between the experimental curve stress-strain of a human tendon and the same curve traced for a rubber cord during a traction test. This fact suggested the idea of modelling soft human tissues as a fascicle of wires made of rubber. Besides elastic characteristics, the rubber also presents rheological properties that together with inertial characteristics will state the dynamical behaviour of the systems where the rubber elements will operate.

The second section of the paper presents the experimental results obtained on a test rig where rubber cords are used. The device is designed by authors and constructed in the laboratory.

In the third section, a dynamic model is proposed and the differential equation, its integration and preliminary conclusions are deduced.

Conclusions are presented in the last section and it is emphasised that the theoretical model proposed describes accurately the actual behaviour of the tissue.

2. Device and experimental results
Based on the observations from the first part of the paper, a device was built, with two rubber cords (1) were pre-stressed with the same traction force. On the midst of these cords was attached a mass (2) which can vibrate in vertical plane, figure 1. On the body of the mass is fixed an acceleration sensor (3). The body is (2) is moved downwards with controlled amplitude, with the aid of limiting part (4). When the body is set free, it oscillates and the data offered by the acceleration sensor [3], are saved in digital form and processed afterwards.
During the tests it was noticed that it is practically impossible to ensure identical behaviours to both wires. The main causes are the related to ensuring the same pre-stretching force for the two cords and mounting the suspended body precisely at the middle of the wire. As a consequence, after recording the signal generated by the acceleration sensor one could perceive the “beat” phenomenon produced by the different values of periods of oscillation resultant from the two wires. Thus, it was abandoned the two wires variant and a single one was used, without pre-stretching.

In figure 2 there are presented the results of two successive, identical launchings of the mobile body, delayed by a period of several tens of minutes, in order to complete an image upon system’s repeatability behaviour. As it can be observed from figure 2, except for a restrain on horizontal, the signals are quasi-identical. Also form figure 2 it can be noticed an evident signal damping and the presence of a noise overlapped on the body’s vibration. To eliminate the noise, the numerical filtering was performed using computer software. In figure 3 there are presented the initial signal and the filtered one, [4], [5].

From figure 3, besides the important damping, other two conclusions are drawn: a) the motion is a quasi-periodical one since the time intervals between two instants when the acceleration passes through zero in the same direction are equal; b) the asymmetry of the signal with respect to equilibrium position, fact that directs to the conclusion that the dynamical system proposed should be a non-linear one. In figure 4 there are presented the experimental data and the results of interpolation.
of these data with a curve of type $y(t) = A \exp(\mu t) \cos(\omega t + \phi)$, characteristic to the model of a dynamic system having the behaviour described by a second order homogeneous differential equation with constant coefficients. The parameters of interpolation curve were established using the subroutine `genfit` of MATHCAD14, [6]. The asymmetrical aspect of acceleration plot is noticeable in figure 4.

After numerical integration of filtered signal of acceleration, the velocity and vertical displacement of the middle point of rubber wire, where the weight is fixed. Using these data, the curve from phase diagram and the hysteresis curve were traced and the graphs are presented in figure 5 and figure 6, respectively.

![Figure 5. Phase diagram](image1)

![Figure 6. Hysteresis loop](image2)

3. Theoretical model proposition

In the subsequent section is presented the theoretical model proposed to describe the behaviour of the suspended body motion.

![Figure 7. Model for the rubber cord](image3)

![Figure 8. Forces acting upon the body](image4)

The rubber wire is modelled as an elastic element linked in parallel with a damper. The damper has a variable characteristic. The idea of using such a type of damper is due to Hunt and Crossley, [7]. Their challenge to model the damped contact of two elastic balls using a linear model conducted to a hysteresis curve that didn’t close in origin as a loop but remained opened. This fact contradicts the problem stating that at the instants of contact initiation and contact detachment the interaction forces between the balls should be zero. To overcome this drawback, Hunt and Crossley showed that the damping force is required to be proportional both to deformation velocity and to deformation raised at power $3/2$ (power equal to the exponent of deformation from the force expression in the case of Hertzian contact, [8]).

The expression of elastic force $F_e$ from the stretched rubber wire was deduced by interpolation of experimental data with a curve having the form, [9], in the first part of the paper, namely:
\[ f(x) = -k \left[ \frac{1}{2} - \tanh(x/a) \right] + bx^3 \]  \hspace{1cm} (1)

where \( k \), \( a \) and \( \beta \) are constants and \( x \) is the strain (specific elongation). Denoting by \( y \) the vertical displacement of the mass, then the elongation of the wire is:

\[ \Delta l = \sqrt{l_0^2 + y^2} - l_0 \]  \hspace{1cm} (2)

where \( l_0 \) is the initial, unstretched length of the wire and \( l \) is the current length. Therefore, the strain is:

\[ x = \frac{\Delta l}{l_0} = \frac{\sqrt{l_0^2 + y^2}}{l_0} - 1 \]  \hspace{1cm} (3)

Considering that in relation (2) the displacement \( y \) is a function of time \( t \), the derivation with respect to time of relation (2) leads to the expression of velocity of deformation of the rubber cord:

\[ v(t) = \frac{y(t) \dot{y}(t)}{\sqrt{l_0^2 + y(t)^2}} . \]  \hspace{1cm} (4)

For the damping force, the general form is proposed:

\[ F_d(t) = \mu \left[ v(t) \dot{v}(t) \right]^{\alpha} \]  \hspace{1cm} (5)

where \( \mu \), \( \alpha \) and \( \beta \) are constants. The vector equation of motion of the body is (figure 8):

\[ Ma = G + F_1 + F_2 \]  \hspace{1cm} (6)

where \( M \) and \( G \) are the mass and the weight of the body, respectively \( F_1 \), \( F_2 \) are the tractions from the two wire branches, each one equal in module with the sum between an elastic force and damping force (according to the model from figure 7):

\[ F = F_e + F_a = \phi \left( \frac{\sqrt{l_0^2 + y(t)^2} - l_0}{l_0} \right) + \mu \left[ \frac{y(t) \dot{y}(t)}{\sqrt{l_0^2 + y(t)^2}} \right]^\alpha \left( \frac{\sqrt{l_0^2 + y(t)^2} - l_0}{l_0} \right)^\beta \]  \hspace{1cm} (7)

The projection of relation (6) on vertical direction gives the scalar equation:

\[ Ma(t) = -Mg - 2(F_e + F_a) \sin \theta , \]  \hspace{1cm} (8)

where:

\[ \sin \theta = \frac{y}{\sqrt{l_0^2 + y^2}} \]  \hspace{1cm} (9)
The motion equation takes the final form:

\[
M \ddot{y}(t) = -g \left\{ f \left( \frac{l_0^2 + y(t)^2 - l_o}{l_0} \right) + \mu \left[ \frac{y(t) \dot{y}(t)}{\sqrt{l_0^2 + y(t)^2}} \right] \right\} \left[ \frac{\sqrt{l_0^2 + y(t)^2} - l_o}{\sqrt{l_0^2 + y(t)^2}} \right] \frac{y(t)}{\sqrt{l_0^2 + y(t)^2}}
\]  

(10)

The equation (10) is a second order nonlinear differential equation, [10]. To integrate this equation, applying a numerical procedure is necessary. The authors used the Runge-Kutta, [11], method of fourth order. It is obvious that for obtaining a tangible solution, knowing the parameters \( \mu \), \( \alpha \) and \( \beta \) that characterize the damping force, is required. In a first stage, the characteristic parameters were chosen: \( \mu = 0.7 \), \( \alpha = 1 \) and \( \beta = 0 \), corresponding to a linear viscous damper.

Concerning the characteristics of rubber wire, the parameters from equation (1) are the ones found experimentally in the first part of the paper: \( \kappa = 13.11 \); \( \alpha = 0.328 \); \( b = 0.174 \); \( l_0 = 0.25 \). The mass of the body attaché to the wire is \( M = 0.22 \, \text{Kg} \). In figure 9 and figure 10 there are presented the variations of displacement and velocity, respectively, for the theoretical model. With these variations, using the relation (10), the acceleration can be found directly, the numerical derivation being unnecessary.

\[ \gamma_e \]

\[ \dot{\gamma}_e \]

\[ \ddot{\gamma}_e \]

Figure 9. Displacement of theoretical model

Figure 10. Velocity of theoretical model

In figure 10, the displacement \( \gamma_e \) represents the value around which the oscillatory motion performs. This value is coincident to the equilibrium elongation value, when the body is acted by its own weight and by the traction forces from cords. To find this value, it is considered the equilibrium case when the damping force is not present in the structure of the traction force, and therefore it is necessary:

\[
2 f \left( \frac{l_0^2 + y(t)^2 - l_o}{l_0} \right) \frac{y(t)}{\sqrt{l_0^2 + y(t)^2}} = -Mg
\]  

(11)

Equation (11) is transcendent and was solved using the Newton-Raphson algorithm and gives the elongation value for the equilibrium state of the body. In figure 11 and figure 12 there are presented the phase diagram and the hysteresis loops. The variation of force from the elastic wire was also represented in figure 12.
Comparing the curves from the phase space and the hysteresis loops for the theoretical model, with the ones obtained experimentally, it can be affirmed that the proposed model can describe very well the behaviour of the actual system. The manner of determining the values $\mu$, $\alpha$ and $\beta$ that describe as accurate as possible the dissipative component of the system is the aim of future research.

4. Conclusions

The paper proposes a test rig and methodology for characterizing mechanical properties of rubber when used as substitute for soft human tissues. Initially, different tests applied to rubber bodies of diverse shapes and tracing the force-deformation diagram characteristic to each type of loading are considered. A comparison with the results from literature shows a good agreement between the experimental curve and the theoretical one for the traction of a filiform rubber body. For such type of body, the experimental points from the traction curve were established and afterwards, the data were interpolated with a function having a form proposed in technical literature.

Next, the paper presents a test rig, consisting of a vibratory system, made of an horizontal rubber cable, having attached at its midst a mass of small dimensions. An acceleration transducer is attached to the mobile body, used for acquiring experimental data. The repeatability of the results was tested in a first stage. Then, the experimental data were filtered and integrated in order to obtain, for body oscillatory motion, the time dependency of velocity and elongation. These data were used in plotting the experimental curves for phase space and hysteresis loops that attested the nonlinear character of the dynamical system considered and the occurrence of a significant damping.

Finally, a theoretical model was proposed for simulation of dynamic experimental system, and the equation of motion was deduced that proved to have a strong non-linear character. The equation of motion was integrated numerically, and following the integrating process, the time dependencies of velocity and elongation were plotted, together with the phase space and hysteresis loops. A qualitative comparison showed that the proposed theoretical describes very well the dynamical behaviour of the experimental dynamic system. For future research directions, the manner to deduce from experimental data the optimum values of the parameters characterizing the dissipative component of the system is aimed.

5. References

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