The new mechanism for intermediate- and short-range nucleon-nucleon interaction

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Abstract

Arguments against the traditional Yukawa-type approach to \(NN\) intermediate- and short-range interaction due to scalar-isoscalar meson exchange are presented. Instead of the Yukawa mechanism for intermediate-range attraction some new approach based on formation of the symmetric six-quark bag in the state \(|(0s)^6[6]_X, L = 0\rangle\) dressed due to strong coupling to \(\pi\), \(\sigma\) and \(\rho\) fields are suggested. These new mechanism offers a strong intermediate-range attraction which replaces the effective \(\sigma\)-exchange (or excitation of two isobars in the intermediate state) in traditional force models. A similar mechanism with vector \(\rho\)-meson production in the intermediate six-quark state is expected to lead to a strong short-range spin-orbital nonlocal interaction in the \(NN\) system, which may resolve the long-standing puzzle of the spin-orbit force in baryons and in two-baryon systems. Illustrative examples are developed which demonstrate clearly how well the suggested new model can reproduce \(NN\) data. Strong interrelations have been shown to exist between the proposed microscopic model and the one-component Moscow \(NN\) potential developed by the authors previously and also with some hybrid models and the one-term separable Tabakin potential. The new implications
of the proposed model for nuclear physics are discussed.

I. INTRODUCTION

Since the middle of thirties when Yukawa proposed [1] his classic theory of the nuclear force, based on meson exchange between nucleons this concept, although improved and partially also modified during the last half century (see e.g. [2–7] and some review of works done until 1978 in the book of Brown and Jackson [8]), is basically still the same: the nuclear force is assumed to originate from the exchange of one or a few mesons between isolated nucleons. Though in the last two decades one added to the nucleons also other channels with one or two $\Delta$-isobars in the intermediate state [9], with isobars again interacting via meson exchange.

Based on this concept a large variety of potential models have been suggested in recent years to describe the $NN$ interaction, which fitted very accurately the experimental data for $NN$ scattering until the energy 300 MeV in the laboratory system.

However with the accumulation of many new data in the field of hadronic physics it became more and more evident that the traditional $NN$ interaction models (i.e. based on the meson exchange concept) suffers from numerous inner inconsistencies and discrepancies when e.g. the same meson-nucleon form factors have to have a different short-range behavior while describing very similar processes. In particularly, the same functional form of the $\pi NN$ form factor $F_{\pi NN}(q^2)$ has to have very different cut-off parameters $\Lambda_{\pi NN}$ to describe elastic and inelastic $NN$ scattering, or in description of two-body $2N$ and three-body $3N$ forces etc. (Some other numerous examples of such inconsistencies are discussed in Sect.II).

At the same time, due to radical improvements of the accuracy and the reliability of dynamical few-nucleon calculations, one begins to find also some numerous disagreements between the new experimental data and the results of the most accurate Faddeev calculations (for a list, although far from complete, of such disagreements in few-nucleon calculations see e.g. in [9]). It is very instructive that many of such disagreements cannot be removed by introducing phenomenological $3N$ forces into the calculations [10,11].
Some recent works in the field based on chiral perturbation theory (χPT) may serve as a very clear indicator for the degree of our understanding (or misunderstanding) of the fundamental NN interaction. This is especially true for the works [13,14]. There the authors have shown that within chiral perturbation theory without introduction of any cut-offs it was impossible to describe all the lowest partial waves even if one incorporates the excitation into intermediate ∆-isobars and the vector (viz. ρ− and ω-) meson exchanges. Thus, the quantitative description of lowest partial waves with $L = 0 - 2$ up to $E_{\text{lab}} = 300$ MeV requires already to go beyond the framework of the χPT. This problem becomes more urgent in passing to the intermediate energy region around $E_{\text{c.m.}} \approx 1$ GeV where a strong coupling to the meson production channels will require that the application of χPT gets even more complicated.

On the other hand, we consider critically in the Section III the problem of the existence and the role played in the fundamental NN force by a scalar-isoscalar light meson usually referred to as $σ$-meson. The $σ$-meson exchange is considered in the traditional OBE models as a main contribution responsible for the strong intermediate-range attraction between nucleons and eventually as the main component of nuclear binding (e.g. in the Walecka model). Despite of the very numerous attempts to find a well developed resonance in the $\pi\pi$ s-wave system undertaken in recent years, no definite evidence for such a well defined resonance has been found (see e.g. the recent review [15]). It is likely there is no such light scalar meson in free space.

Moreover, very recent studies of different groups have demonstrated [13,14,16] that the exchange of a correlated $\pi\pi$ pair in an S-state between nucleons leads to a repulsive rather than attractive contribution in the NN interaction. Thus, we should attribute the NN intermediate-range attraction to a generation of the two intermediate ∆-isobars (or at least to an $N\Delta$ intermediate state) [13,14]. But as it will be argued in Sect. IV, this intermediate $\Delta\Delta$ state has a strong overlap to the symmetric six-quark state $|(0s)^6[6]_X, L = 0, 2 >$ and thus the above $\Delta\Delta$ state can be replaced by an intermediate symmetric six-quark state strongly coupled to the 2π-channel.

Thus we tried to circumvent the problem in the treatment of lower partial waves by
refraining from the basic Yukawa idea of the meson exchange between (isolated) nucleons and we develop some new interaction mechanism on the basis of a quark model where quarks are strongly coupled to chiral fields.

Our treatment is based essentially on the group-theoretical considerations of the symmetries and on the algebraic recouplings in the six-quark system and the specific role played by fully the symmetric six-quark state \((0s)^6[6]_{x}[f]_{cs}\) in the \(NN\) interaction in lower partial waves. In particularly, one could even expect that such a fully symmetric 6\(q\) state, due to the maximal overlap of all six quarks (which implies some enhancement of \(q\bar{q}\) fluctuations inside such a state), may lead in the direction of a phase transition of the chiral symmetry (or partial chiral symmetry) restoration. This Goldstone limit, or even only approaching this limit, means, in accordance with the variational principle, the appearance of a strong additional attraction between quarks and thus also between two nucleons at intermediate range (i.e. at distances \(r_{NN} \sim 0.7 \div 1.2\) fm where such a dressed six-quark bag is localized).

The physical assumption about such a possible phase transition in a fully symmetric multiquark state or, to be more correct, about the approach to such a limit with an increasing density of quarks is the basis of the new model for the \(NN\) interaction presented here. Its rigorous justification can be found only by careful studies of chiral dynamics of the multiquark system with a detailed treatment of the \(qq\) correlations and strong coupling of such symmetric multiquark states with the Dirac sea of antiquarks. This problem which combines the chiral field theory and relativistic multiquark dynamics is a too complicated many-body problem to be handled today. However we do believe that it is already now possible to construct a chiral quark model with a minimal number of adjustable parameters, which will be able to describe the \(NN\) interaction in both lower and higher partial waves. To describe the latter our model should be combined with the presently already well developed \(\chi PT\) approach with \(\pi\)- and \(\pi\pi\)-exchanges between two nucleons.

How well the proposed model may work is illustrated by a simple model suggested here on the basis of the proposed new mechanism in Sect.V. In particular, the above simple model can describe perfectly all the lower \(NN\) phase shifts in a rather large energy range \(0 \div 600\) MeV. Hence, though we are unable presently to justify strictly the suggested new
model, its general framework looks completely natural and is in accordance with the general
concepts of quark models and chiral field dynamics.

The organization of this paper is following. In Sec. II we offer a critical look to the
OBE models and discuss the difficulties of traditional meson-exchange models with anomalously high cut-off parameters $\Lambda$ and also with respect to their application in few-nucleon
problems. Section III includes a critical discussion of the scalar meson puzzle in the light
of some new results. In particular, we argue here that the exchange by a $S$-wave $\pi\pi$
correlated pair with no $\Delta$-isobar in the intermediate state does not lead to any significant
intermediate-range attraction (which one ascribes usually to the $\sigma$-meson exchange) but
rather results in a repulsion between two nucleons. In Sec. IV we describe in detail the
new model for intermediate and short-range interaction and compare it with the traditional
Yukawa mechanism of $\sigma$- and $\rho$-exchange. Section V is devoted to interrelations between the
$NN$ interaction model suggested in this work and the potential models proposed previously.
In particular, we compare the new approach with the Moscow $NN$ potential developed in
previous years, with the hybrid quark compound bag (QCB) model and with the one-term
separable Tabakin potential. We elucidate the microscopic basis for the above models. In
Conclusion we summarize the main results of the work. Some algebraic details required
for the derivation of the basic formulas and some tables of the group-theoretical algebraic
coefficients are presented in the Appendix.

II. CRITIQUE OF THE BASIC ASSUMPTIONS OF OBE MODELS

Despite of the relative success in the description of the low-energy $NN$ scattering data up
to $E_{\text{lab}} = 350$ MeV, the traditional OBE models based on the initial Yukawa meson-exchange
mechanism for the nucleon-nucleon force are suffering from many inner contradictions and
inconsistencies. These contradictions concern not only the description of the $NN$ data
themselves but also e.g. the description for few-body data. All these contradictions and
inner inconsistencies form a large array of discrepancies either with accurate experiments or
other existing theories and these drawbacks seem to be almost unremovable today because
an improvement in one point leads very often to an appearance of discrepancies in other places. To avoid repetitions we arrange the discussion according to the following points.

A. The range of the \( NN \) force due to heavy meson exchange and the quark radius of the nucleon.

While the range of the \( \pi \)-exchange force (OPE) \( \lambda_\pi \simeq 1.45 \) fm is much larger than the quark radius of the nucleon \( < r_N > \simeq 0.6 \) fm so that the Yukawa \( \pi \)-exchange may be considered to occur mainly between two separated nucleons, the heavy-meson exchange with masses \( m \geq 600 \) MeV occurs mainly on the distances \( r_m \simeq 0.2 \div 0.8 \) fm where the two nucleons are strongly overlapping. Thus this heavy meson exchange happens mainly in the field of all six quarks of the participating nucleons. Hence in OBE models using such a heavy-meson mechanism it is first necessary to justify the employment of "free-space" meson-nucleon coupling constants and cut-off form factors. As a result of this, all existing OBE models have severe problems with the short-range cut-off parameters \( \Lambda \) \([6,7,9,17–19]\) (see especially the severe critique in \([17]\) and also in the next Subsection). Thus all the short-range parts of OBE potentials are treated purely phenomenologically \([5,8]\) but using at the same time the Yukawa framework which looks rather inadequate for such short ranges. Very recently an attempt \([13,14]\) undertaken to refrain from this short-range phenomenology but staying still within the framework of a meson-exchange model (with a perturbative chiral field-theory treatment of two-pion exchange) has demonstrated very clearly that the OBE+TPE model even taken consistently (and without phenomenological cut-off form factors) is able to describe only the higher \( NN \) partial waves. Hence the description of lower partial waves demands a non-perturbative dynamical treatment.

B. The difficulties with short-range cut-off form factors

It is well known \([3,4,17]\) that in all the OBE models the values of the cut-off parameter \( \Lambda_{mNN} \) \( (m = \pi, \sigma, \rho, \omega...) \) are strongly increased as compared with any microscopic model for meson-baryon coupling and also as compared with fits to the data of meson-nucleon...
scattering experiments \cite{20,21,18,19}. This disagreement is especially evident in the values of \( \Lambda_{\pi NN} \) which can be derived from the theory of \( \pi NN \) form factors \cite{17,20,22} and even from direct experiments \( N(e,e'\pi)N' \) in which a pion is knocked out from the pion cloud of the nucleon by fast electrons \cite{23}.

In any case the values of \( \Lambda_{\pi NN} \) taken in all OBE models to fit the \( NN \) data lie in the interval \[ 6,17 \]:

\[
\Lambda_{\pi NN}^{\text{OBE}} \simeq 1.3 \div 2.0 \text{ GeV}
\]

while all above mentioned direct estimates and experiments result in the values:

\[
\Lambda_{\pi NN}^{\text{N+theor}} \simeq 0.4 \div 0.8 \text{ GeV},
\]

i.e. a discrepancy of a factor 1/3 to 1/4 or even less.

Moreover, the choice of the strongly increased values \( \Lambda_{\pi NN} \simeq 1.3 \div 2.0 \text{ GeV} \) in microscopic nuclear models results in a strong enhancement of the pion field inside nuclei \cite{24} which is in drastic disagreement with many observations (see the numerous examples in the review \cite{24}). Thus, if even one assumes these strongly enhanced values of \( \Lambda_{\pi NN}^{\text{OBE}} \) required for the \( NN \) interaction in OBE models as some effective description of an unknown short-range part of the \( NN \)-interaction, this assumption turns out to be unacceptable for the description of the pion dynamics in nuclei with such models. But this is not the end of the story.

Even if one forgets for the moment the nuclear pion dynamics, the large value of \( \Lambda_{\pi NN} \simeq 1.3 \div 2.0 \text{ GeV} \) seems to be fully incompatible with the description of pion production in collisions \( pp \rightarrow pn\pi^+ \) \cite{25} and also with elastic backward \( p + d \) scattering \cite{18,19}. Let us to add to this collection still another example: a small value of \( \Lambda_{\pi NN} = 0.528 \text{ GeV} \) has been found by T. Cohen \cite{26} in his analysis for \( \pi NN \) form factor within the Skyrme model. A very similar value \( \Lambda_{\pi NN} = 0.63 \text{ GeV} \) (and \( \Lambda_{\rho NN} = 0.7 \text{ GeV} \)) has been extracted \cite{27} from an analysis of exclusive experiments \( NN \rightarrow N\Delta \) at incident proton energies a few GeV. There are also many other evidences which point very unambiguously to the necessity for soft cut-off parameters \( \Lambda_{\pi NN} \) and \( \Lambda_{\rho NN} \) for both the \( \pi NN \) and \( \rho NN \) form factors. Last not least the 3\( N \)-force models (via pion-exchanges) which describe accurately the 3\( N \)- and 4\( N \)-systems \cite{28,30} need still a soft cut-off parameter \( \Lambda_{\pi NN} \).
Quite a similar situation is observed also for other mesons, like $\sigma$, $\rho$ and $\omega$ for which one needs also large cut-off parameters $\Lambda$ in OBE models as compared to values given e.g. by the vector-dominance model (in case of $\rho$-mesons). In total, the problem with artificially enhanced values of the cut-off parameters seems to be almost unavoidable in the OBE models. For example, in the attempts to solve this problem Ueda [31] suggested to add the three-pion exchange contributions in the form of $\pi\rho$ and $\pi\sigma$ terms and also some ”heavy” pion $\Pi$ exchange. He found again that the cut-off parameter $\Lambda_{\Pi NN}$ for the $\Pi$-meson should be about 3 GeV (!) to fit the $NN$ scattering data.

Therefore in all these cases, i.e. really for the description of whole short-range part of the $NN$ interaction the Yukawa model shows an inner inconsistency even if the form factors are considered as an effective description of the interaction. A very similar critique of the short-range part of the $NN$ interaction in the current OBE models has been presented also by the Bochum group [17]. In the next Section we will demonstrate that there are also serious problems in a consistent interpretation of OBE models at intermediate ranges.

C. Few-body puzzles originating from the application of the conventional $NN$ interaction models to precise few-nucleon calculations

In recent years in high precision few-nucleon calculations which use the most realistic conventional $NN$ potentials for low ($< 200$ MeV) and intermediate energies ($200 – 300$ MeV), numerous marked disagreements with accurate modern experimental data have been found [12,29,32–35]. Because the full list of these disagreements and puzzles is rather long we will present here only the newest or best known ones.

(i) The best known disagreements have been found since the middle of seventies in $3N$- and $4N$-binding energies. The strong underbinding found in the $3N$ and $4N$ ground-state energies have been explained long ago with a significant contribution of a meson-exchange $3N$ force [28,29]. However this $3N$ force did not help really to understand quantitatively the remaining $3N$ puzzles, e.g. those pointed out in (ii) – (vi) below. Moreover, it was demonstrated very recently [30,37] that the conventional $3N$ forces
used fail quite evidently in the treatment of new high-precision experiments of $n\bar{d}$ and $\bar{p}d$ elastic scattering at energies $E_N \simeq 150 \div 300$ MeV in the backward hemisphere. Thus, the explanation given with the above 3\$ forces for binding energy puzzles must be also considered as an \textit{ad hoc} fit to specific data (see especially [10]).

(ii) The well known puzzle of the analyzing power $A_y$ for low-energy $\bar{n}d$ and $\bar{p}d$ scattering [38]. In recent years the situation with this puzzle has not improved but even got worse. The traditional three-nucleon force contribution does not help to remove the $A_y$ discrepancy. Moreover, very recently we made [39] a new high precision calculation for $A_y$ with our new one-component NN potential (the so called generalized orthogonality condition model [40]), which fits excellently the NN phase shifts in all low partial waves. We found for the analyzing power in this calculation almost the same disagreement with experiment as for the conventional NN models like AV18 etc. Because the off-shell behavior of the above NN model potential is strongly different from those for conventional NN models one can conclude from these results rather reliably that the explanation of $A_y$ puzzle is not related to different off-shell behaviors of the various NN potentials but demands some new type of spin dependent 3\$ forces.

(iii) A recent analysis by Scholten et al. [41] of new data from Osaka for $pp \rightarrow pp\gamma$ bremsstrahlung at $E_p = 390$ MeV discovered a large disagreement with predictions of the existing NN potential models. The data still could be explained by artificial enhancement of $\Delta$-isobar current contribution by a factor 1.7. Thus the situation here is similar as in the treatment of short-range NN interaction with conventional force models.

(iv) Recently it was found [36,37,42] that the so-called Sagara puzzle (disagreement for the backward $Nd$ elastic scattering near the minimum of the cross section) increases with growing energy. At $E_N = 200$ MeV in the lab system the disagreement is as large as 30\%. However if the conventional 3\$ force is taken into account the disagreement is considerably reduced but instead, there appears some larger disagreement for the
vector $A_y$ and the tensor $A_{xx}$ analyzing powers at the same backward angles.

(v) Very significant disagreements were discovered recently in $^3\text{He}(e, e'p)$ and $^3\text{He}(e, e'pp')$ reactions at moderate to high transferred momenta and energies $\omega$ \cite{43,44}. In particular, at energy transfer $\omega \approx 200$ MeV one observes in the first reaction a very big ($\sim 150 \div 200\%$) mismatch between the complete Faddeev 3N calculations and experimental data. The traditional MEC contribution does not help. It modifies the theoretical results only slightly.

(vi) Numerous disagreements with the data have also been found in recent four-nucleon calculations of the Lisbon \cite{34,45} and Grenoble groups \cite{12}. While the theoretical results of both are in very good agreement with each other.

This list may be continued much further (see e.g. recent reviews \cite{9}). So, the above few-body puzzles and disagreements found very recently together with long-standing puzzles are clearly signalling that the existing $NN$-force models (based on the meson-exchange mechanism) do not include at intermediate and short ranges some important nontrivial contribution. A candidate for such a nontrivial contribution which has been fully missed in previous $NN$ models is suggested in the present work.

III. THE SCALAR MESON PUZZLE AND THE PROBLEM OF THE INTERMEDIATE-RANGE $NN$ FORCE

The problem with scalar mesons and their role in the hadron-hadron interaction is attracting increased interest today (see e.g. \cite{15,46}). This interest concentrates on the experimental identification of the the scalar mesons and on their contribution to the description of hadron collisions, in particular to the $NN$ interaction.

According to the traditional point of view advocated for a long time by many ”constructors” of $NN$ potentials (see e.g. \cite{3,38}) an exchange by the ($\pi\pi$) correlated pair in relative $s$-wave between two pions in a combination with the excitation of intermediate $\Delta$-isobars is responsible for the strong intermediate-range attraction between nucleons \cite{3,38,47}. Further,
in the conventional picture, this strong attraction at short distances is fully compensated by a strong repulsion due to $\omega$-exchange [8,18].

Very recently, however, it was found by two groups independently [14,16] that the $(\pi\pi)$ $s$-wave correlation being treated consistently and taken itself, is unable to give any intermediate-range attraction but it results even in a rather strong short- and intermediate-range repulsion between nucleons. Thus, in the conventional meson-exchange mechanism, the main intermediate-range attraction should be associated only with the excitation of the intermediate-state $\Delta$-isobars. Some independent arguments for favour of this conclusion follow from the obvious failure to get this strong attraction from various microscopic models like the Skyrme soliton interaction model, the RGM $6q$ treatment with $qq$ interaction based on the Goldstone boson exchange [49] in which the $\Delta\Delta$ (or $\Delta N$) state excitation has been neglected.

A second important argument comes from the experimental search for the low-mass scalar-isoscalar meson [15,46]. While the highly excited scalars $f_0(1370)$ and $f_0(1500)$ have been identified more or less reliably in experiments, the identification of low-mass scalar meson resonances (which one refers often to as $\sigma$-meson and which one relates with the $\pi\pi$ $s$-wave resonance) is in no way well accepted. The spread in the mass and width estimates for these states are extremely large [13]. The estimates accepted today are as follows [50]:

$$m_\sigma = 400 \div 1200 \text{ MeV},$$

$$\Gamma_\sigma = 300 \div 500 \text{ MeV},$$

i.e. they are rather uncertain. Thus also from the experimental side the situation looks rather unsatisfactory. Therefore the attempt to interpret the basic internucleon attraction as originating from a Yukawa-type exchange of a scalar meson (the existence of which as a free particle is doubtful) seems to us not the best way to understand the intermediate-range interaction.

Nevertheless there is no doubt that some scalar meson contribution (of the $\sigma$-exchange type) is necessary for understanding numerous processes in hadron physics, e.g. for $\pi N$ and $NN$ interactions. Hence the above deep contradiction should be somehow resolved.
We propose here a new approach to solve this puzzle. This approach is in part based on the assumption that the scalar-isoscalar excitation of the QCD vacuum which is conventionally referred to as $\sigma$-meson is in essence not a real particle in free space (like e.g. the $\rho$-meson) but some sort of quasiparticle excitation inside hadrons, in particular inside a multiquark bag. This quasiparticle can thus exist inside the six-quark bag but not in free space. It can be understood very naturally from this assumption why it was impossible to observe this particle to date in the $\pi\pi$ final state interaction. Therefore one can conclude that such an exchange of a scalar-isoscalar quasiparticle may occur very naturally in the field of six valence quarks but that such a quasiparticle cannot couple with isolated nucleons in free space.

These ideas lead very naturally to the new basic mechanism of the intermediate-range $NN$-interaction presented in following Section.

**IV. THE DRESSED BAG MECHANISM FOR THE INTERMEDIATE AND SHORT RANGE $NN$ FORCE**

In order to give to the reader some clue to the suggested mechanism we display the respective graphs in FIG. 1.

![FIG.1](image-url). The traditional t-channel meson-exchange mechanism (a) compared to the new s-channel "dressed" bag mechanism (b) for the $NN$ interaction.
The Yukawa one-meson exchange mechanism displayed in FIG. 1(a) is confronted the new $s$-channel mechanism of the dressed bag intermediate state in FIG. 1(b). The two pion state produced in the lower vertex in FIG. 1(b) is modified in the high density six-quark bag in which chiral symmetry may be at least partially restored. The "$\sigma$" or a similar "scalar-isoscalar meson" is assumed to exist only in a high density environment and not in the vacuum, contrary to the $\pi$ and $\rho$ mesons. This mechanism can describe the short-range repulsion and the medium range attraction and replaces the $t$-channel exchange of $\sigma$- and an $\omega$-meson in conventional Yukawa-type $NN$ forces. The short-range repulsion arises here due to an additional requirement for mutual orthogonality of the $NN$- and 6q-channels (see Subsection 4.A).

Instead of the "$\sigma$"-meson in FIG. 1(b), other mesons like $\pi$ and $\rho$ can also be considered within this mechanism. The contributions of $\pi$, $\sigma$ and $\rho$ mesons will depend on the total angular momentum, spin-isospin and permutation symmetry of the respective six-quark state. Thus we adopt the $s$-channel quark-meson intermediate states, the transition amplitude being determined by $s$-channel singularities in sharp contrast to the Yukawa mechanism driven by $t$-channel meson exchange (see FIG.1(a)). Surely together with this specific six-quark mechanism we take into consideration also the traditional Yukawa mechanism for $\pi$-, $2\pi$- and $\rho$- (but not $\sigma$-) meson exchanges between isolated nucleons. However these meson-exchange contributions are essential only at the separations beyond the intermediate six-quark bag or in high partial waves ($L > 3$). In the lowest partial waves, the intermediate dressed six-quark bag gives a dominating contribution for the total $NN$ interaction. It is appropriate to refer henceforth to the present microscopic force model as a Moscow-Tübingen dressed bag model.
A. Short-range repulsion and orthogonality of the nucleon-nucleon and six-quark (bag) components

In our symmetry considerations we start from the well known results of previous works in this field \[51\text{-}61\]. If one assumes for the nucleon a wave function of three constituent quarks with a fully symmetric spatial part \([f_X] = [3]\) then the space (permutational) symmetry of the six-quark system can be presented as follows:

\[ [f_X]_{\text{even}} = [6] + [42], \quad \text{for even-parity partial waves} \]
\[ [f_X]_{\text{odd}} = [51] + [33], \quad \text{for odd-parity partial waves} \]

Further we adopt the nucleon wave function in the form

\[
N(123) = |(0s)^3[3]_X[21]_{CS,S,T} \rangle = \frac{1}{2}|(0s)^3[3]_X[21]_{CS,S,T} \rangle, \quad (1)
\]

where the coordinate part is the translationally-invariant harmonic oscillator (h.o.) state

\[
|(0s)^3[3]_X \rangle = |0s(\rho_1)\rangle|0s(\rho_2)\rangle, \quad |0s(\rho_1)\rangle \sim e^{-\rho_1^2/4b^2}, \quad |0s(\rho_2)\rangle \sim e^{-\rho_2^2/3b^2},
\]

\[
\rho_1 = \mathbf{r}_1 - \mathbf{r}_2, \quad \rho_2 = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2) - \mathbf{r}_3
\]

and \(\{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3\}\) are quark coordinates. Then the translationally-invariant shell-model (TISM) configuration for six-quark states \(\Psi_{6q}\) in the \(NN\) overlap region can be written as follows (with restriction to configurations with only minimal numbers of h.o. quanta):

\[
\Psi_{6q} \rightarrow |(0s)^6[6]_X, [f_{CS}], L = 0; ST \rangle + \sum_{f'} C_{f'}|(0s)^4(1p)^2[42]_X, [f'_{CS}], L = 0(2); ST \rangle,
\]

for even waves (with \([f_{CS}]=[2^3]\) for \(ST=10\) and \([2^21^2]\) for \(ST=01\) \(3)\)

and

\[
\Psi_{6q} \rightarrow |(0s)^5(1p)[51]_X, [f_{CS}], L = 1; ST \rangle + \sum_{f'} C_{f'}|(0s)^3(1p)^3[3]_X, [f'_{CS}], L = 1(3); ST \rangle,
\]

for odd waves (with \([f_{CS}]=[2^21^2]\) for \(ST=00\) and \([321]\) for \(ST=11\), \(4)\)

where \([f'_{CS}]=[42], [321], [2^3], [31^3], [21^4]\) are all possible color-spin (CS) Young schemes for the inner product \([2^3]_C \circ [42]_S\) for \(S=1\) and \([f'_{CS}]=[2^3]_C \circ [3^2]_S = [3^2], [41^2], [2^21^2], [1^6]\) for \(S=0\).
Let us consider first the triplet $S$-wave $NN$ scattering, e.g. in the channel $L = 0, ST = 10$. In this case both allowed six-quark configurations

\[ d_0 = |(0s)^6[6]_X, [2^4]_{CS}, L = 0; ST = 10 \rangle \text{ and } \]
\[ d_{L=0}^{f'} = |(0s)^4(1p)^2[42]_X, [f'_{CS}], L = 0; ST = 10 \rangle, \tag{5} \]

correspond to state vectors of very different nature: while the unexcited six-quark states $d_0$ include the states with a maximal overlap all six quarks, the states with mixed symmetry $d_{L=0}^{f'}$, $L=0$, are those with two excited $p$-shell quarks projected onto the $NN$ channel (with unexcited nucleons) correspond to cluster-like nodal $NN$ relative-motion wave functions $|2s(r)\rangle$ (see e.g. [56]):

\[ \langle N(123)N(456)|d_0 \rangle = \Gamma d_0 U_{f_0}^{NN} |0s(r)\rangle, \]
\[ \langle N(123)N(456)|d_{L=0}^{f'} \rangle = \Gamma d_{f'} U_{f'}^{NN} |2s(r)\rangle, \tag{6} \]

where $\mathbf{r} = \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) - \frac{1}{3}(\mathbf{r}_4 + \mathbf{r}_5 + \mathbf{r}_6)$ is the relative distance between two nucleons. In the $L = 2$ case the projection onto the $NN$ channel

\[ \langle N(123)N(456)|d_{L=2}^{f'} \rangle = \Gamma d_{f'} U_{f'}^{NN} |2d(r)\rangle \tag{7} \]

leads to the cluster-like $2d$ $NN$ state. (We denote by the symbols $f_0$, $f'$ the following Young schemes: $f_0 = \{[1^6]_{CST}, [2^3]_{CS}\}$ and $f' = \{[f'_{CS}], [f'_{CS}]\}$). In the Eqs. (3) and (7) $U_{f_0}^{NN}$ and $U_{f'}^{NN}$ are overlaps in the CST space (see Table V in Appendix), while $\Gamma d_0 \equiv \Gamma(s^6[6]_X) = 1$ and $\Gamma d_{f'} \equiv \Gamma(s^4p^2[42]_X L) = -\sqrt{\frac{4}{45}}(L=0,2)$ are trivial fraction parentage coefficients (f.p.c.) of the TISM.

It is well known that both types of configurations are mixed if one assumes the basic $qq$ interactions to be only the OGE (or other effective color-dependent interactions) [3, 57, 63]. (The states $d_{L=0}^{f'}$ and $d_{L=2}^{f'}$ are also mixed by effective $qq$ interactions and that gives rise to a new type of tensor force of quark origin, see Subsection 4.C.4). However, in a more microscopic treatment, the structure of fully symmetric states $d_0$ ($ST=10$ or 01) should be distinguished strongly from the mixed symmetry states $d_{L=0}^{f'}$ due to the enhanced quark density in non-excited symmetric states.
In fact, we are aware from previous studies (see e.g. [64–66]) based on chiral restoration effects in multiquark systems either in strong color-electric fields or in high density nuclear matter within the framework of the Nambu-Jona-Lasinio model, that some phase transition may happen in increasing the quark density or the temperature of the system. This phase transition leads to a restoration of the broken chiral symmetry. Thus it is very plausible to assume that in fully symmetric $6q$-states there occurs a transition to (at least) a partial restoration of chiral symmetry which leads, in turn, to some decrease in the mass difference of pions and scalar mesons (i.e. to some decrease of both the mass $m_\sigma$ and decay width $\Gamma_{\sigma \rightarrow 2\pi}$ of the $\sigma$ meson [64–66]). While the mutual overlap of six quarks in the mixed-symmetry states like $|s^4p^2[42]_X LST\rangle$ should be much smaller due to cluster-like structure of these states (see below). Hence the structure of the multiquark states with maximal spatial symmetry should be noticeably different from states of mixed symmetry and the mixing of both types of states should be damped.

The very important role which the $s^6$ bag surrounded by chiral fields plays for the intermediate-range attraction in $NN$ interaction is supported also from a very different point of view. In fact, very recently it was established within chiral perturbation theory [13,14,16] that in the $2\pi$-exchange diagrams conventionally associated to "\(\sigma\)-meson" exchange [2] the $\pi\pi$ s-wave correlation plays a minor role while the excitation of intermediate $\Delta$-isobars, in particular, the $\Delta\Delta$ channel in $S$-wave $NN$ interaction should be of prime importance in the $NN$ interaction. But the $\Delta\Delta$ channel has a very high threshold ($\sim 600$ MeV in c.m. system) and in low-energy $NN$ scattering (say, until $E_{\text{lab}} \simeq 300$ MeV) this channel is strongly suppressed and thus the respective intermediate $\Delta\Delta$ state should be well localized at rather short ranges $r_{\Delta\Delta} \sim 0.5 \div 0.8$ fm, i.e. well inside the overlap region of the two nucleons.

From the other side, the six-quark wavefunction $|(0s)^6[6]_X, L = 0\rangle$, being expanded into $N^*N^*$-components via fractional parentage coefficients (f.p.c.) [56,57,67], has a very significant $\Delta\Delta$-component. Thus, in the language of the quark model the fully symmetric six-quark configuration $|(0s)^6[6]_X, L = 0\rangle$ (surrounded by $\pi + \sigma + \rho$ fields) may replace very naturally the $\Delta\Delta$ intermediate-state channel in the traditional picture of $NN$ attraction.
We should also stress here that in the above $\Delta\Delta$ channel the $\Delta$-isobars – due to their strong mutual overlap – should be considered rather as ”structural” deltas and thus this conclusion is in a very good agreement with our suggestion to replace the strongly closed $\Delta\Delta$ channel with our ”dressed” $6q$ bag in a configuration $\left| (0s)^6[6]_X \right\rangle + \left| (\pi\pi) \right\rangle$ (see FIG. 1(b) and below).

Here we stress once again that we consider not the $t$-channel meson-exchange diagrams as in the Yukawa approach (FIG. 1(a)) but the $s$-channel eigen-energy diagrams where the energy and momenta of intermediate $\pi$, $\sigma$- and $\rho$-mesons are taken into consideration in an explicit form (see FIG. 1(b)).

Another basic fact for our approach is the microscopic calculations for the $NN$ interaction based on a model six-quark Hamiltonian, especially the results $^{51,54,56,51,61,63}$, with the important conclusion:

– the coherent superposition of the mixed symmetry states $\sum f' C_{f'} d_{f'}$ obtained from the $6q$-Hamiltonian being projected onto the $NN$ channel yields a large reduced width for the channel with two unexcited nucleons and a small reduced width for the channel with two $\Delta$’s, while the fully symmetric eigenstate $d_0$ is projected onto both $NN$ and $\Delta\Delta$ channels approximately with the same reduced width (or probability). This result makes it possible to identify the above mixed symmetry $6q$ states as related mainly to the proper $NN$ channel whereas the fully symmetric states $d_0$ in both channels $ST=01$ and 10 (see Eq. (5)), which are orthogonal to the former states, represent non-nucleonic bag-like states $^{58,58,60,6}$.

Thus the total six-quark wave function can be divided according to the above consideration into two mutually orthogonal components: cluster-like configurations with a node and bag-like configurations without a node:

$$\Psi_{6q} = \Psi_{bag} + \Psi_{cluster},$$  \hspace{1cm} (8)

$$\langle \Psi_{bag} | \Psi_{cluster} \rangle = 0.$$  \hspace{1cm} (9)

$^{1}$This statement in the form of very plausible conjecture has been suggested first as early as in 1986 $^{58,59}$.
This separation of $\Psi_{6q}$ into two mutually orthogonal components have been employed extensively in developing the so-called Moscow $NN$ potential \cite{58,59,70} which is rather successful in describing the $NN$ interaction on the level of a semi-phenomenological potential model \cite{10,11,73,69} and offered many interesting predictions and explanations for some long-standing puzzles. In particular, the orthogonality condition \cite{10} leads in low partial waves to almost stationary nodes in $NN$ scattering wave functions \cite{72,73,69} exactly at the place of the repulsive hard core in old phenomenological $NN$ potentials, like RHC (Reid Hard Core) etc. And thus this natural orthogonality condition plays the role of the repulsive core (or of its main part) at low partial waves in our approach. Due to the effect of orthogonality the strength of the repulsive core caused by $\omega$-meson exchange can be noticeably reduced \cite{73} and the $\omega NN$ coupling constant can be taken around the values $g_{\omega NN}^2/4\pi \simeq 5$ in nice agreement with $SU(3)$ predictions\cite{2}.

B. "Dressed" six-quark bags at short and intermediate $NN$ ranges

In the present work we make a next step in this direction and develop a microscopic model which can replace the intermediate and short range part of existing OBE potentials and which can simultaneously explain the enhanced meson-exchange vertices (large cutoff parameters) at short ranges in the traditional OBE approach to the $NN$ interaction.

The model uses an analogy between an excited atom or molecule and a multiquark bag with excited $p$-shell quarks like $s^4p^2$. Similarly to e.m. transitions in atomic physics we assume that each $p$-shell quark emits a pion – a "quantum" of the chiral field – in the process of its transition from the $p$- to $s$-shell (see FIG. 2).

\footnote{The authors are much obliged to Prof. Gerry Brown who attracted their attention just to this aspect of the problem related to the $\omega NN$ coupling constant puzzle.}
FIG. 2. Schematic representation of the two-pion emission in the transition of two $p$-shell quarks to an $s$ orbit.

The transition $NN^{L=0,2}(s^4p^2) \rightarrow 6q(s^6)+\sigma$ involving deexcitation of two quarks from the $p$-orbits in configurations $s^4p^2$ (and also the similar transition for $P$-waves $NN^{L=1}(s^3p^3) \rightarrow 6q(s^5p)+\sigma$) can be described as subsequent emission of two pions, each of them deexciting one $p$-orbit, followed by the formation of a $\pi\pi$ resonance inside the $6q$ bag giving the $\sigma$- or $\rho$-mesons. The mechanism of elastic $S$-wave $NN$ scattering via an intermediate symmetric $6q$ state $|(0s)^6[6]_X, L = 0\rangle + (2\pi)$ can be displayed by the graph in FIG. 3, where we assume the emission of intermediate $\sigma$- and $\rho$-mesons and their subsequent absorption which take place on a diquark (correlated quark pair) inside the $6q$ bag.

FIG. 3 The graph illustrating the $\sigma$- (or $\rho$-) meson emission and subsequent absorption by diquark pairs in the intermediate six-quark bag-like state.

Another variant of this process is displayed in FIG. 4 where the $\sigma$- and $\rho$-mesons are
formed from two independent pions in the process of their interaction inside the bag.

FIG. 4. The graph illustrates two sequential π-meson emissions and absorptions via an intermediate σ- (or ρ-) meson and the generation of a six-quark bag.

In the graph (FIG. 4) the pions are created in s-waves due to conservation of parity and angular momentum. The intermediate six-quark configuration $s^5p[51]_X$ (denoted by vertical dashed line in the graph) for even waves $L = 0, 2$ in the $NN$ channel has fixed quantum numbers which are determined by those in the initial ($NN$) and final ("dressed" bag $6q + \sigma$) states. The intermediate (after the first pion emission) state in the channel $ST = 01$, $J^P = 0^+$ has quantum numbers of the so-called $d'$-dibaryon (see, e.g. [32,74,75]):

$$|d'| = |(0s)^5(1p)[51]_X[321]_{CS}, LST = 110, J^P = 0^-\rangle. \quad (10)$$

The transition into the channel $ST = 10$, $J^P = 1^-$ goes via an intermediate state $d''$, which is a partner of the $d'$ with $S \to T$ interchanged:

$$|d''\rangle = |(0s)^5(1p)[51]_X[321]_{CT}, LST = 101, J^P = 1^-\rangle. \quad = |(0s)^5(1p)[51]_X[2^21^2]_{CS}, LST = 101, J^P = 1^+\rangle. \quad (11)$$

It should be noted that both states $d'$ and $d''$ have no direct coupling with the $NN$ channel due to the requirement of antisymmetrization and due to this feature they have been considered in some previous works as candidates for narrow dibaryon resonances (see
e.g. the discussion on $d'$ in Refs. [62,74,76]). Both transitions $NN^{L=0}(ST = 01, J^P = 0^+)$ → $d' + \pi$ and $NN^{L=0.2}(ST = 10, J^P = 1^-)$ → $d'' + \pi$ proceed with a spin and isospin flip of the quark emitting the S-wave $\pi$-meson. The further decays $d' \rightarrow d_0(ST = 01) + \pi$ and $d'' \rightarrow d_0(ST = 10) + \pi$ proceed with the spin-isospin-flip mechanism as well (for details see Refs. [62,76]). However in case of the coupling mechanism shown in FIG. 4, the existence of such narrow dibaryon resonances in the transition $s^4p^2 \rightarrow s^6 + 2\pi$ can destroy the coherence between the emissions of two pions and thus should be incompatible with such a mechanism.

C. Quark-model calculation of the transition operator from the $NN$ cluster channel to the ”dressed”-bag state

From a general point of view the calculation of the driving term corresponding to the transition displayed in FIG. 3 which includes a direct coupling of $\sigma$- and $\rho$-mesons with diquarks in the multiquark bag looks much more preferable and straightforward than the two-step mechanism of subsequent $\pi$-emissions and absorptions shown in FIG. 4. Unfortunately now we have no reliable estimates neither for the dynamics and probabilities of diquarks in the multiquark bag nor for the strength of coupling ($qq \rightarrow \sigma + (qq)$ (or ($qq \rightarrow \rho + (qq)$)). Therefore we postpone this estimation for the future work and present here the calculation only for the two-step mechanism of the coupling of a $\sigma$ to a six-quark bag.

The total amplitude for the transition $NN^{L=0.2}(s^4p^2) \rightarrow 6q(s^6) + \sigma$ to the ”dressed” bag is calculated as a contribution of the triangle graph shown in FIG. 5. Each of the two lower vertices are calculated in the framework of the phenomenologically successful $^3P_0$ quark-pair-creation model (QPCM) [77] which appears to be very useful [78-80] in the flux-tube picture of hadrons [81].
FIG. 5. Kinematical variables in the triangle diagram corresponding to the $\sigma$- (or $\rho$-) meson creation by two $\pi$-mesons formed in the transition of two $p$-shell quarks to the $s$-orbit (see also FIG. 4).

1. Elementary vertex operators

In the QPCM the transition operator for the emission of the pion $\pi^\lambda$ ($\lambda = 0, \pm$) by the sixth quark in a six-quark system can be written as (see Ref. [62] for details)

$$H^{(6)}_{\pi qq}(k_6; \rho_5, \rho'_5) = v \tau^{(6)}_{-\lambda} e^{i\frac{2\pi}{3} k_6} \hat{O}^{(6)}(\rho_5, \rho'_5) \sigma^{(6)} \left[ \frac{\omega_\pi}{2 m_q} \left( \frac{2}{i} \nabla_{\rho_5} + \frac{5}{6} k_6 \right) + \left( 1 + \frac{\omega_\pi}{12 m_q} \right) k_6 \right],$$

where the non-local factor is proportional to the pion wave function

$$\hat{O}^{(6)}(\rho_5, \rho'_5) = e^{-\frac{i}{2} k_6 \cdot (\rho_5 - \rho'_5)} \Psi_\pi(\rho_5 - \rho'_5).$$

Here $\rho_5$ and $\rho'_5$ are the relative coordinates of the 6th quark in the initial and final states respectively, viz.

$$\rho_5 = \frac{1}{5}(r_1 + r_2 + \ldots + r_5) - r_6, \quad \rho'_5 = \frac{1}{5}(r_1 + r_2 + \ldots + r_5) - r'_6.$$

We use shell-model quark configurations for the pion and the $\sigma$ meson

$$\pi^\lambda = |ss[2]_X LST = 001 J^P = 0^- \rangle \quad \sigma = |s^2s^2[4]_X, LST = 000, J^P = 0^+ \rangle,$$

that imply Gaussian wave functions:
\[
\Psi_\pi(\rho_\pi) \sim \exp(-\rho_\pi^2/4b_\pi^2),
\]
\[
\Psi_\sigma(\rho_\sigma; \rho_\pi(12), \rho_\pi(34)) \sim \exp\left[-\rho_\sigma^2/2b_\sigma^2 - \rho_\pi^2(12)/4b_\pi^2 - \rho_\pi^2(34)/4b_\pi^2\right],
\]
where \( \rho_\sigma = \frac{1}{2}(r_1 + r_2) - \frac{1}{2}(r_3 + r_4) \) and \( \rho_\pi(\sigma) = r_\pi - r_j \) (15)

In the limit of a point-like pion \( (b_\pi \rightarrow 0) \) the operator (12) goes to the standard pseudo-vector (PV) quark-pion coupling and the phenomenological constant \( v \) in Eq. (12) becomes the PV coupling constant:

\[
v = -i \frac{f_{\pi qq}}{m_\pi} \frac{1}{(2\pi)^{3/2}(2\omega_\pi)^{1/2}},
\]
where \( f_{\pi qq} \) should be normalized to the well known pion-nucleon PV coupling constant \( f_{\pi NN} = \frac{2}{3}f_{\pi NN} \).

The \( \pi + \pi \rightarrow \sigma \) transition amplitude is determined (see details in Ref. [82]) to be proportional to the overlap of the two wave functions of the \( \pi \)-meson with the \( \sigma \)-meson wave function:

\[
\langle \pi(12)\pi(34)|H_{\pi\pi\sigma}(\mathbf{k},\mathbf{k}')|\sigma \rangle = g_{\pi\pi\sigma}F_{\pi\pi\sigma}((\mathbf{k} - \mathbf{k}')^2), \quad F(q^2) = \exp(-\frac{1}{2}q^2b_\sigma^2),
\]
where the transition operator \( H_{\pi\pi\sigma} \) contains a phenomenological constant \( G \):

\[
H_{\pi\pi\sigma}(\mathbf{k},\mathbf{k}') = G \exp[i(\frac{\mathbf{k} - \mathbf{k}'}{2}) \cdot \rho_\sigma],
\]
and the effective coupling constant \( g_{\pi\pi\sigma} \) in Eq. (17) is proportional to the f.p.c. \( \Gamma_{\pi\pi\sigma} \) for decomposing the two pion \( q\bar{q} \) states in the configuration of Eq. (14): \( g_{\pi\pi\sigma} = G\Gamma_{\pi\pi\sigma} \). The coefficient \( \Gamma_{\pi\pi\sigma} \) includes contributions from both CST (color, spin, isospin) and coordinate overlaps.

2. Transition operator: Decomposition in basis of six-quark configurations

The total transition amplitude \( NN^{L=0}(s^4p^2) \rightarrow d''(d'') + \pi \rightarrow 6q(s^6) + \sigma \) can be written as

\[
A_{NN \rightarrow d0+\sigma}^{L=0(2)}(E; \mathbf{k}) = \int d^3r \, \Psi_E^{L=0(2)}(\mathbf{r}) \Omega_{NN \rightarrow d0+\sigma}(E; \mathbf{r}, \mathbf{k}),
\]
where $\Psi_E^{L=0(2)}(r)$ is the cluster part of the $NN$ wave function in the sense of Eqs. (8) - (9) and $E = 2m_N + \frac{p_N^2}{m_N}$. The transition operator $\Omega_{NN \to d_0 + \sigma}(E; r, k)$ in Eq. (18) is the contribution of the triangle diagram of FIG. 5. Note that the standard momentum representation of the amplitude could be obtained by inserting the plane-wave decomposition of the unit operator $I = \int d^3p_N |p_N\rangle\langle p_N|$ into Eq. (18):

$$T_{NN \to d_0 + \sigma}(E; p_N, k) = \int \frac{e^{i p_N \cdot r}}{\sqrt{(2\pi)^3}} \Omega_{NN \to d_0 + \sigma}(E; r, k) d^3r$$

(19)

We start from the quark-meson diagram of FIG. 4 and project the six-quark state in the left part of the diagram onto the two-nucleon clusters of the initial state. The full expression for the transition operator $\Omega_{NN \to d_0 + \sigma}$ can be written as an integral of the elementary six-quark transition amplitudes over both inner coordinates of quark clusters (viz. N(123), N(456), and also $\pi$ and $\sigma$) and the pion momenta $k_5 = \frac{k+q}{2}$ and $k_6 = \frac{k-q}{2}$ in the triangle diagram:

$$\Omega_{NN \to d_0 + \sigma}(E; r, k) = 15 \int d^3k_5 \int d^3k_6 \delta(k_5 + k_6 - k)$$

$$\times \sqrt{10} \langle N(123)N(456)|H^{(6)}_{\pi qq}(k_6)|d'(d'')\rangle \langle \pi(d')|G^{(0)}_{\pi d'}(E; k_6)\rangle \langle \pi(d'')|H^{(5)}_{\pi qq}(k_5)|d_0\rangle$$

$$\times G^{(0)}_{2\pi d_0}(E; k_5, k_6) \langle 2\pi|H_{\pi\pi\sigma}(k_5, k_6)\rangle \rangle \rangle$$

(20)

where $H^{(6)}_{\pi qq}$ ($H^{(5)}_{\pi qq}$) is the vertex operator (12) of the effective quark-pion coupling for the 6-th (5-th) quark in the diagram of FIG. 4 and $\sqrt{10}$ is a combinatorial factor ($\sqrt{\frac{6!}{3!2!2!}}$) in projection of the six-quark amplitude onto the $3q - 3q$ cluster channel. In Eq. (20), $G^{(0)}_{\pi d'}$ and $G^{(0)}_{2\pi d_0}$ are free Green functions for $\pi + d'$ and $2\pi + d_0$ systems:

$$G^{(0)}_{\pi d'}(E; k_6) = \left[E - m_{d'} - \frac{k_{d'}^2}{2m_{d'}} - \omega_\pi(k_6)\right]^{-1}, \quad \omega_\pi(k_6) = \sqrt{m_{\pi} + k_6^2},$$

$$G^{(0)}_{2\pi d_0}(E; k_5, k_6) = \left[E - m_{d_0} - \frac{(k_5 + k_6)^2}{2m_{d_0}} - \omega_\pi(k_5) - \omega_\pi(k_6)\right]^{-1}$$

(21)

Due to large masses of the intermediate six-quark states $d_0$ and $d'(d'')$ and also the $\sigma$- and $\rho$-mesons we use for them the nonrelativistic kinematics while for intermediate $\pi$-mesons we use the relativistic kinematics.
and the numerical factor in front of the integral in the r.h.s. of Eq. (20) is the number of $qq$ pair in the six-quark system.

In calculation of the amplitude Eq. (18) it is reasonable first to project the initial $NN$ state onto the basis of six-quark configurations by inserting the unit operator

$$I = \sum_{n,f} |n, f\rangle \langle n, f|$$

into the first vertex matrix element in the integrand of Eq. (20). Here we employ the full shell-model basis of six-quark configurations with quantum numbers of the initial $NN$ state $LST, J^P$, e.g.

$$|n, f\rangle = |s^{n_s}p^{n_p}[f_X] [f_{CS}]LST, J^P\rangle,$$

where symbols $n$ and $f$ are defined as $n=\{n_s, n_p\}, f = \{[f_X], [f_{CS}]\}$. In case of emission of $s$-wave pions, the excited six-quark configurations $d_{L=0}^{j=0(2)}$ in the sum (22) are only important (while the bag-like configuration $d_0$ does not contribute to the amplitude (18) because of the orthogonality condition (3) for the wave function $\Psi_\nu(r)$). Thus one can write the following decomposition of the vertex matrix element $N + N \rightarrow d'(d'')$ in the integrand of Eq. (20):

$$\sqrt{10} \langle N(123)N(456)|H_{\pi qq}^{(6)}(k_6)|d'(d'')\rangle$$

$$= \sum_{j} \sqrt{10} \langle N(123)N(456)|d_{f=0}^{L=0(2)}|d_{f=0}^{L=0(2)}|H_{\pi qq}^{(6)}(k_6)|d'(d'')\rangle$$

and use the overlap factors from Eqs. (3) and (7). As a result, the shell-model matrix elements of the vertex operator (12) will only contribute to the transition amplitudes of Eqs. (20) and (18).

All the matrix elements of interest are calculated based on the f.p.c. technique [53, 54, 55, 62] (see the details of calculation in Appendix) and are reduced to the standard form of the vertex matrix element as a product of a vertex constant $v f_{\pi AB}$, of a form factor $F_{\pi AB}(k_{\pi})$ and of a kinematical factor $\omega_\pi(k_{\pi})/m_q b$ (see Ref. [62] for details):

$$\langle d_{f=0}^{L=0(2)}|H_{\pi qq}^{(6)}(k_6)|d'\rangle = v \frac{\omega_\pi(k_6)}{m_q b} f_{\pi d'} f_{\pi d'} (k_6^2),$$

$$\langle d'|H_{\pi qq}^{(5)}(k_5)|d_0\rangle = v \frac{\omega_\pi(k_5)}{m_q b} f_{\pi d'} f_{\pi d'} (k_5^2).$$

(25)
Form factors $F_{\pi d_f d'}(k_b^2)$ and $F_{\pi d_0 d'}(k_b^2)$ in Eq. (25) do not depend on the index $f$ of configurations $d_f$ and have in shell-model representation the form:

$$F_{\pi d_f d'}^L(k_b^2) = (1 + a_L \frac{5k_b^2b^2}{24}) e^{-5k_b^2b^2/24}, \quad F_{\pi d_0 d'}(k_b^2) = e^{-5k_b^2b^2/24}$$  (26)

where $a_{L=0} = \frac{1}{3}$ and $a_{L=2} = -\frac{2}{3}$.

3. Calculation of the transition operator

Substituting the vertex amplitudes (17) and (25) into Eqs. (20) and (24) one obtains the following simple expression for the transition operator (20) in case of S and D partial waves in the initial $NN$-states:

$$\Omega_{NN \rightarrow d_0 + \sigma}^{L=0}(E; r, k) = g_0 e^{-5k_b^2b^2/48} D^L(E, k) |2s(r)\rangle,$$

$$\Omega_{NN \rightarrow d_0 + \sigma}^{L=2}(E; r, k) = g_2 e^{-5k_b^2b^2/48} D^L(E, k) |2d(r)\rangle,$$

$$\Omega_{NN \rightarrow d_0 + \sigma} = \sum_{L=0,2} \Omega_{NN \rightarrow d_0 + \sigma}^L$$  (27)

where the values $g_L$ are effective strength constants of transitions from (cluster) $NN$ states to the "dressed" bag configuration $N + N \rightarrow d_0 + \sigma$. The above calculation gives within the quark model the values $g_L$:

$$g_L = 15 \frac{f_{\pi q}^2}{m_\pi^2 m_q^2} \frac{1}{2} f_{\pi d_0 d'} g_{\pi \pi \sigma} \sum_f f_{\pi d_f d'}^L \Gamma_{d_f} U_{f}^{NN}, \quad L = 0, 2,$$  (28)

where the coefficients $\Gamma_{d_f}$ and $U_{f}^{NN}$ are defined according to Eqs. (6) and (7) while the vertex constants $f_{\pi d_0 d'}$ and $g_{\pi \pi \sigma}$ are taken from Eqs. (17) and (25).

Function $D^L(E, k)$ is obtained by an integration in Eq. (20) over inner momenta (the contribution of the triangular diagram of FIG. 5). By denoting the integration variable

$$q = k_5 - k_6$$  (29)

one can rewrite the integral in the form

$$D^L(E, k) = \int d^3 q \omega_{\pi}^{1/2}(\frac{k - q}{2}) \omega_{\pi}^{1/2}(\frac{k + q}{2}) \left[1 + \frac{5}{24} a_L (\frac{k - q}{2})^2 b^2 \right] e^{-q^2 B^2}$$

$$\times \left[ m_{d_0} + \frac{k^2}{2m_{d_0}} + \omega_{\pi}(\frac{k - q}{2}) + \omega_{\pi}(\frac{k + q}{2}) - E \right]^{-1} \left[ m_{d'} + \frac{1}{2m_{d'}} (\frac{k - q}{2})^2 + \omega_{\pi}(\frac{k - q}{2}) - E \right]^{-1},$$  (30)
where \( k = k_5 + k_6 \) (the \( \sigma \) meson momentum) is fixed. The result of integration depends on the absolute value of the vector \( k \) but not on its direction, i.e. \( D^L(E,k) = D^L(E,k) \). Moreover, one can substitute useful equalities:

\[
\omega_\pi((k-q)/2) + \omega_\pi((k+q)/2) = \omega_\pi((k-q)/2) + \omega_\pi((k+q)/2)
\]

\[
\omega_\pi((k-q)/2)\omega_\pi((k+q)/2) = \omega_\pi((k-q)/2)\omega_\pi((k+q)/2)
\]

(31)
to reduce the angular dependence of the integrand. Note that because of the Gaussian factor \( e^{-q^2 B^2} \) in the integrand with

\[
B^2 = 5b^2/48 + b_\pi^2/2
\]

the integral in Eq. (30) is convergent. Hence, at small values of \( kb \), one can use the following decomposition for \( D^L(E,k) \):

\[
D^L(E,k) = D_0(E,k) + \sum_{n=1} (k^2 b^2)^n D^L_{2n}(E)
\]

where \( D_0(E,k) \) is obtained by neglecting \( a_L \) in the numerator of the integrand. Then by making use a nonrelativistic approximation \( \omega_\pi(p) \approx \omega_{\text{nrel}} = m_\pi + p^2/2m_\pi \) in the last factor of the integrand one can carry out the integration over angular variables in \( D_0(E,k) \):

\[
D_0(E,k) = - \frac{8\pi m_{d\pi}}{k} \int_0^\infty dq \omega_{\pi}^{1/2}(k-q/2)\omega_{\pi}^{1/2}(k+q/2) e^{-q^2 B^2} m_{d_0} + \frac{k^2}{2m_{d_0}} + \omega_{\pi}(k-q/2) + \omega_{\pi}(k+q/2) - E \ln \left| 1 + \frac{(k-q)^2}{8m_{d\pi}^2 \Delta E} \right| 1 + \frac{(k+q)^2}{8m_{d\pi}^2 \Delta E} (33)
\]

In the integrand of Eq. (33) we use the definitions \( \Delta E = m_{d\pi} + m_\pi - E \) and \( m_{d\pi} = m_{d_\pi}/(m_{d_\pi} + m_\pi) \). It should be stressed that the approximations \( a_{L=0} \) and \( \omega_\pi \approx \omega_{\text{nrel}} \) do not play any significant role in integration over \( \cos \theta = k \cdot q/kq \) in Eq. (30) but without these approximations the explicit form of the integrand in Eq. (33) becomes very complicated, and thus a qualitative consideration of the behavior of the amplitude based on the decomposition (32) would be impossible. However, starting from Eq. (30) one can obtain easily an important result: in the limit \( k \to 0 \) the integral in Eq. (30) goes to a non-vanishing value

\[
D^L(E,0) = 4\pi \int_0^\infty \frac{(1 + \frac{5}{96} a_L q^2 b^2)\omega_{\pi}(q/2)e^{-q^2 B^2}}{(m_{d\pi} + \omega_{\pi}(q/2) + q^2/8m_{d\pi} - E)(m_{d_0} + 2\omega_{\pi}(q/2) - E)} q^2 dq,
\]

i.e. the proposed mechanism of \( \sigma - 6q \) coupling does not vanish at low \( \sigma \)-meson momenta. This should be very important for \( \sigma \)-meson effects in the NN system and in nuclei.
4. Separable form of the effective $NN$ interaction

In accordance with the diagram in FIG. 4 the contribution of virtual $d_0 + \sigma$ states to the $NN$ interaction in the $S$ and $D$ partial waves is defined by the matrix elements:

$$A_{NN \rightarrow d_0 + \sigma \rightarrow NN}^{L'L'} = \int d^3r'd^3r\Psi_{E}^{L''}(r')V_{E}^{L'L}(r',r)\Psi_{E}^{L}(r),$$

(35)

where $V_{E}^{L'L}(r',r)$ is the separable amplitude of the virtual transition:

$$V_{E}^{L'L}(r',r) = \int d^3k\Omega_{NN \rightarrow d_0 + \sigma}^{L''}(E;r',k)G_{\sigma}(0)d_{0\sigma}(E;k)\Omega_{NN \rightarrow d_0 + \sigma}^{L}(E;r,k).$$

(36)

Using the simple form of Eq. (27) for transition amplitudes $\Omega_{NN \rightarrow d_0 + \sigma}^{L''}(E;\cdot,k)$ and the free Green function in the form:

$$G_{\sigma}(0) = \frac{-1}{E - m_{\sigma} - m_{d_0} - \frac{k^2}{2m_{\sigma}} - \frac{k^2}{2m_{d_0}}},$$

one can obtain eventually a matrix separable potential of special form

$$V_{NqN}(E) = V_{E}^{L'L}(r',r) = D_{NqN}(E)\begin{pmatrix}
g_0^2|2s(r')\rangle\langle 2s(r)| & g_0|2d(r')\rangle\langle 2d(r)| 
g_2g_0|2d(r')\rangle\langle 2s(r)| & g_3^2|2d(r')\rangle\langle 2d(r)|
\end{pmatrix},$$

(37)

where the common energy-dependent factor $D_{NqN}$ has a complicated energy dependence

$$D_{NqN}(E) = \int k^2dk\exp\left(-\frac{5}{24}k^2\right)|D(E,k)|^2 \frac{1}{E - m_{\sigma} - m_{d_0} - \frac{k^2}{2m_{\sigma}} - \frac{k^2}{2m_{d_0}}}.$$

(38)

(we denote by $m_{\sigma d_0}$ the factor $m_{\sigma}m_{d_0}/(m_{\sigma} + m_{d_0})$ as usually) where the $D(E,k)$ is taken from Eq. (32). Note that $D_{NqN}$ implies some $L$-dependence as follows from Eq. (32). Starting from Eq. (32) one can write down for $D_{NqN}$ a representation

$$D_{NqN}(E) = \tilde{D}_{NqN}(E) + a_La_{L'}D_{NqN}'(E),$$

(39)

where $a_L$ is defined by Eq. (32). The contribution of the last term of Eq. (39) to the matrix elements of Eq. (37) can be taken into account through a renormalization of values $g_Lg_L'$ defined in Eq. (28). Thus, it is only a technical problem.

From the inspection of Eq. (37) we find a very important result that the separable potential in Eq. (37) gives rise to a new type of short-range tensor force of the non-Yukawa
type, which originates basically from the coupling between the input and output $L = 0$ and $L = 2$ form factors $|2s(r')\rangle$ and $\langle 2d(r)|$ through the intermediate $(0s)^6 + 2\pi$ states. In other words, this new tensor force comes from coupling of the cluster-like initial $NN$ channel with the mixed symmetry configuration $|s^4p^2[42]L = 0, 2\ ST = 10J^P = 1^-\rangle$ and the dressed bag intermediate states $(0s)^6 + 2\pi$. The above specific tensor term has a well expressed short-range character and this new interaction is very crucial to give a correct description of the mixing parameter $\varepsilon_1$ and the deuteron tensor structure with very soft cut-off factor $\Lambda_{\pi NN}$ in the OPE sector of our model. (We want to remind to the reader that a correct description of $\varepsilon_1$ at higher energies and deuteron $D$-state admixture requires the artificially enhanced $\Lambda_{\pi NN}$ values in traditional OBE models, see Section II.)

V. A SIMPLE MODEL

In this section we demonstrate that the mechanism of $NN$ interaction developed in preceding section is really able to describe $NN$ scattering in a wide energy region. For this aim we parameterize the basic potential components entering this model via a simple phenomenological form which includes the main features of the above mechanism.

Thus, the model interaction consists of three terms: the orthogonalizing potential $V_{\text{orth}}$ providing the condition of orthogonality between the proper $NN$ channel and the six-quark intermediate bag in $S$- and $P$-waves, the one-pion-exchange potential $V_{\text{OPE}}$ with soft dipole truncation, and the separable term $V_{NqN}$ with an energy dependence described by a pole (which is the simplest approximation to a quark-induced interaction corresponding to the separable amplitude $V_{\text{OPE}}^{LL}$) of the virtual transition $NN \rightarrow (6q + 2\pi) \rightarrow NN$) as illustrated by the graphs in FIG. 3 or FIG. 4:

$$V_{NN} = V_{\text{orth}} + V_{NqN} + V_{\text{OPE}},$$

(40)

$$V_{\text{orth}} = \lambda_0|\varphi_0\rangle\langle \varphi_0|, \quad (\lambda_0 \rightarrow \infty)$$

(41)

$$V_{\text{OPE}}(k) = \frac{f_\pi^2}{m^2} \frac{1}{k^2 + m^2} \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + k^2}\right)^2 \frac{1}{2} \sigma_1(k)\sigma_2(k)\frac{\tau_1\tau_2}{3}$$

(42)
The interaction $V_{NqN}$ for single channel case takes the form:

$$V_{NqN} = \frac{E_0^2}{E - E_0} \lambda |\varphi\rangle \langle \varphi|,$$

(43)

while the term $V_{NqN}$ for coupled channels is a (2×2)-matrix (compare with Eq.(37)):

$$V_{NqN} = \frac{E_0^2}{E - E_0} \begin{pmatrix} \lambda_{11} |\varphi_1\rangle \langle \varphi_1| & \lambda_{12} |\varphi_1\rangle \langle \varphi_2| \\ \lambda_{21} |\varphi_2\rangle \langle \varphi_1| & \lambda_{22} |\varphi_2\rangle \langle \varphi_2| \end{pmatrix},$$

(44)

where one assumes $\lambda_{12} = \lambda_{21}$. For all form factors in eqs.(41),(43),(44) we use the simple Gaussian form with one scale parameter $r_0$:

$$\varphi_i(r) = N_i r^{L_i+1} \exp \left( -\frac{1}{2} \left( \frac{r}{r_0} \right)^2 \right) \quad (45)$$

In the calculations the averaged pion mass $m = (m_{\pi_0} + 2m_{\pi_\pm})/3$, the averaged value of pion-nucleon coupling constant $f^2_\pi/(4\pi) = 0.075$, and a soft cut-off parameter with values $\Lambda = \Lambda_{\text{dipole}} = 0.50 \div 0.75$ GeV have been used.

The results of the fits of the model parameters $\lambda_k$ (or $\lambda_{jk}$), $r_0$ and $E_0$ to the $NN$ phase shift analysis data are displayed on FIGS. 6-8. It is quite evident this simple model describes the $NN$ low partial waves up to $E_{\text{lab}} = 600$ MeV very well. The model phase shifts and the mixing parameter $\varepsilon_1$ are compared in Figs. 6-8 with data of a recent phase shift analysis (SAID, solution SP99 [83]). There are three fitting parameters for each partial wave: $\lambda$ ($\lambda_k$ or $\lambda_{jk}$ for coupled channels), $r_0$ and $E_0$. The parameters of the projection operators (41) ($r_0$ for $V_{\text{orth}}$) are taken from our preceding work [40] where the deep local attractive potential (Moscow potential) have been constructed as an effective $NN$ one-component potential. The parameter $E_0$ corresponds to the sum of the six-quark bag energy and the effective $\sigma$-meson mass inside the six-quark bag. Its value is taken here in the range $600 \div 1000$ MeV. In accordance to our suggestions, it should be the same for all partial waves with definite parity. We found that the results depend on $E_0$ only weakly. All parameters found for $S$-, $P$- and $D$-waves are given in Table [I].

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4 It is still in accordance with our general algebraic multiquark formalism due to an appearance of the additional orthogonality condition (see the respective orthogonalizing potential (41)).
It is highly instructive to compare the present simple model based on the suggested new mechanism for $NN$ interaction with the well known phenomenological separable potential \[ [54] \] (so called Graz potential) which fits the same phase shifts until $E_{\text{lab}} = 500$ MeV. The reader can find from the comparison that the number of free parameters in the Graz potential exceeds very much those for our simple model \[ [44] \] whereas the energy range is smaller and the quality of the fit is worse for the Graz model. Thus our simple model describes $NN$ data more adequately than the Yamaguchi-type phenomenological model.

Moreover, it was very surprising to find out that such a simple model gives a very good description for $^1S_0$ phase shifts even up to $E_{\text{lab}} = 1200$ MeV (see FIG. 8)(at higher energies the $np$ phase shift analysis (PSA) is absent).

We want to discuss here especially the description of phase shifts in triplet coupled channels $^3S_1 - ^3D_1$. The crucial point is the behavior of the mixing parameter $\varepsilon_1$ with increasing energy. Without the separable (quark-bag induced) mixing potential (i.e. at $\lambda_{12} = 0$) the behavior of $\varepsilon_1$ is correct only at very low energies, but is in strong contradiction with the PSA at energies higher than 50 MeV (see the dashed line in FIG. 7.) The increase of the truncation parameter $\Lambda$ up to values 0.8 GeV does not help to get a better agreement with the data, but on the contrary, destroys the good description at low energies (the dotted line in the Figure). Introducing the quark-bag induced mixing ($\lambda_{12} \neq 0$ in Eq.(44)) allows us to reproduce the behavior of $\varepsilon_1$ (and $^3S_1 - ^3D_1$ phase shifts as well) with a reasonable accuracy until the energy as high as $E_{\text{lab}} \sim 600$ MeV, but only for sufficiently small values of $\Lambda_{\pi NN}$. The best fit for the $\varepsilon_1$ mixing parameter is shown on FIG. 7 (by solid line) with the potential parameter values given in Table 1 where the value of $\Lambda_{\pi NN} = 0.5936$ GeV.

In this case the condition:

$$\lambda_{12}^2 = \lambda_{11} \lambda_{22}$$ \hspace{1cm} (46)

is satisfied with high accuracy. Just this condition follows from our assumption that the quark-bag induced $S - D$ mixing arises due to coupling of the $NN$ channel with $L = 0, 2$ to a single $S$-wave six-quark state $|s^6 + 2\pi\rangle$ (see Eq.(37)). The increase of $\Lambda_{\pi NN}$ up to a value 0.8 GeV results in the violation of condition \[ (46) \] and in a significant deterioration of the
description of $\varepsilon_1$ (see the dot-dashed line in the FIG. 7). The other phase shifts ($^3S_1$ and $^3D_1$) are reproduced for all four variants with the same good accuracy, so that we present in FIG. 6 the results for one variant only.

Thus we can deduce from the results of our simple model presented in this Section, that the model is able to describe all phase shifts in low partial waves $L = 0 \div 2$ in a rather large energy interval $0 \div 600$ MeV. This good description and the comparison with the phenomenological Graz model seems support strongly the new dressed bag mechanism for the intermediate-range interaction suggested in the work. Moreover in the next Section we will show a tight interrelation existing between the current microscopic model and various phenomenological models of the $NN$ interaction proposed in earlier years.

VI. RELATIONSHIPS OF THE SUGGESTED MECHANISM TO OTHER INTERACTION MODELS

In this section we will discuss the interrelations of the new $NN$ mechanism suggested in this work with other models proposed in previous years and elucidate the microscopic grounds for some of them.

A. Relationship to the Moscow potential model

While the symmetry background of the Moscow potential models \cite{40,58–60,58–60,71,72} is rather similar to the present model the underlying mechanism and the particular realization are very different. In the above potential models one starts with a subdivision of the possible spatial (permutational) six quark symmetries of the total wavefunction into two types of different physical nature: $\Psi_{bag} ((0s)^6[6]) + \Psi_{NN} ((0s)^4(1p)^2[42])$ which are mutually orthogonal to each other. Then by excluding the bag-like components from the proper $NN$ channel one arrives at an effective interaction Hamiltonian in the $NN$ channel \cite{58,73} with an additional orthogonality condition constraint:

$$\left( T_R + V_{ME} + 10 \frac{|f\rangle \langle f|}{E - E_0} \right) \tilde{\chi} = E \tilde{\chi}$$ (47a)
\langle g|\tilde{\chi}\rangle = 0. \quad (47b)

\tilde{\chi}(R) is the wavefunction of the relative motion in the $NN$ channel which is renormalized through the overlap kernel $N(R, R')$ to have a probabilistic meaning \cite{3}. Here $V_{ME}$ is the sum of conventional meson-exchange potentials truncated at the proper (i.e. soft) values of $\Lambda_{mNN}$, while the form factor $f(R)$ in the separable term of Eq. (47a):

$$
\langle R|f\rangle \equiv f(R) = \langle \psi_{6q}|H|\psi_N\psi_N\rangle
$$

is the matrix element which couples the six-quark and $NN$ channels, and the function $g(R)$ in the orthogonality condition (47b) is taken as:

$$
\langle R|g\rangle \equiv g(R) = \langle \psi_{6q}|\psi_N\psi_N\rangle.
$$

Then in the initial version of the one-channel Moscow $NN$ potential \cite{58,72,39} one replaces both the separable term in (47a) and the orthogonality condition (47b) by one deep local potential where the deep-lying bound states (which are considered as "forbidden" states in the model) serve to provide the orthogonality condition (47b) due to the hermicity of the underlying potential.

Thus, the previous Moscow $NN$ potential model is in essence a phase-shift equivalent local effective potential to a highly non-local and energy-dependent model (47). Our next step was the generalized orthogonality-condition model \cite{40} where we still retained the deep local potential but disconnected really the bound state wavefunction in the potential from the orthogonality condition. Thus from this point of view the above $NN$ model can be considered as a generalized orthogonality condition model initially proposed by Saito in nuclear cluster physics \cite{85} as early as 1969. And very similar as in the cluster model, the deep attractive well of the one-channel Moscow $NN$ potential represents a local phase-shift equivalent potential for a nonlocal and energy-dependent interaction term in Eq. (47a) together with the orthogonality condition constraint (47b). As a result of the constraint, the $NN$ phase shifts in low partial waves ($S$ and $P$) display a behavior similar to phase shifts for a local repulsive core potentials \cite{86}. In other words, the orthogonality condition is equivalent in some sense to a repulsive core, but only on the energy shell. The orthogonality
condition results in a stationary (with respect to the energy variation) short-range node in the \( NN \) wavefunction of the relative motion rather than to a strong damping of the latter near the origin. Moreover the node position \( (r_n \simeq 0.6 \text{ fm}) \) coincides very nearly to the radius of repulsive core in the traditional force models like RSC \[86\] etc.

It is also instructive to know that the node position is very near to the size of the repulsive core only if the quark radius of the nucleon \( r_q \) is about 0.6 fm. If one assumes a smaller quark radius of the nucleon (around \( \bar{r}_q \simeq 0.35 \text{ fm} \)), as in some modern models for baryons \[87\], the node position is shifted inward and one needs extra repulsive terms \[73\] (in addition to the node) to describe adequately the \( NN \) phase shifts. In this way the short-range stationary node in wavefunction of the relative motion replaces a big portion of the repulsive core and thus the coupling constant for \( \omega \)-meson exchange may be reduced safely to the moderate values \( g_{\omega NN}^2/4\pi \simeq 5 \) dictated by \( SU(3) \) symmetry. Thus the new dressed bag model presented in the work gives a microscopic quark-meson realization for the previous Moscow-type \( NN \) models.

B. Interrelation with the Simonov’s QCB and other hybrid models

The total wavefunction of \( NN \) system, according to the Simonov’s quark-compound bag (QCB) model, is composed, similarly to our basic assumption \( (1) \) from two components of different nature: the quark compound bag part at small distances \( r \leq R_0 \) and the proper cluster-like \( NN \) component in peripheral region \( r > R_0 \) with \( R_0 \) being the matching radius between the two components. Similar arguments can be presented also for other hybrid models (e.g. for Kisslinger et al. model \[88\]). Then, analogously to our formal derivation \[73\], the bag-like component is eliminated in the QCB approach and one arrives at an effective one-channel Schrödinger equation for the \( NN \) component analogous to \((47a)\) where the transition form factor \( f(R) \) in the QCB model is chosen as a \( \delta \)-function centered at the transition radius \( R_0 \) plus energy dependent terms \[89\]. However, in the QCB approach in contrast to our model, two basic components, i.e. \( \Psi_{\bar{q}q} \) and \( \Psi_{NN} \) are taken to be nonorthog-
onal to each other. Thus, the very important constraint (47b) is absent in the QCB-model\[^5\]. However, when the two channels are orthogonalized in the QCB approach the scattering wavefunctions in $NN$ channel acquire a short-range node rather similar to our case but with a violation of the continuity at the matching radius $R_0$. Another important feature which distinguishes our two-component approach from the QCB and other hybrid models is the fact that the both components in our approach are treated in Hilbert space while in the hybrid models they are taken in configurational space.

But the main difference of the QCB approach to our current model is the fact that the QCB is, in essence, a phenomenological model (based on the $P$-matrix formalism) which does not consider any microscopical or field-theoretical aspects. However the fact that, starting with absolutely independent arguments (in fact we started more than two decades ago with the old phenomenological Moscow type $NN$ potential \[^70\]), we arrived at a model which, in its formal aspects, has many similarities with QCB, shows that both models reflect the underlying true physical picture rather adequately.

C. Interrelation with the separable Tabakin potential

There are also very interesting connections between our approach and the Tabakin potential. More than 30 years ago Tabakin, to facilitate drastically the Faddeev few-nucleon calculations, proposed \[^90\] the phenomenological one-term separable potential "with repulsion and attraction". The characteristic feature of the Tabakin potential which distinguishes it from many separable models proposed at that time is an oscillating behavior of the potential form factor $g(p)$ in $S$-waves:

$$V_T(p, p') = \lambda g_T(p) g_T(p')$$

(48a)

with

\[^5\]This nonorthogonality of two basic components in QCB leads to an appearance of some ghost state in infinity which can be considered as an analog of deeply bound "forbidden" states in our approach.
\[ g_T(p) = \frac{p^2 - p_0^2}{(p^2 + \beta^2)^2} \]  \hfill (48b)

The change in sign of \( g_T(p) \) at \( p = p_0 \) was able to produce a respective change in the sign of \( S \)-wave phase shift at \( E_{\text{lab}} \approx 300 \text{ MeV} \) due to appearance of the so called continuum bound state (CBS). In other words, the one-term Tabakin potential was able to describe both low-energy attraction and high-energy repulsion in \( S \)-wave of the \( NN \) interaction.

At that time the success of the Tabakin potential was considered as somewhat "accidental" and puzzling. However about a decade ago Nakaishi-Maeda has demonstrated \([92]\) that the Tabakin potential can be considered in a very good approximation as a first term in the unitary-pole expansion of \( t \)-matrix for the deep local Moscow \( NN \) potential while the scattering wavefunctions for both models display the short-distance stationary nodes (at \( r_n \approx 0.6 \text{ fm} \)) in very similar ways. Moreover, it has been shown \([92]\) that the continuum bound state in the Tabakin potential has the energy \( E \approx 300 \text{MeV} \) (in the lab. system) and is very similar in its structure to the "forbidden" bound state in the initial version of the Moscow potential.

Here we want to demonstrate that the analogy between the new quark-meson mechanism suggested in the present paper and the old Tabakin potential goes much further. In fact, the overlap factors \([6] \) between three-quark nucleon clusters and six-quark configurations \( |s^4p^2[42], L = 0, ST \rangle \) and \( |s^6[6], L = 0, ST \rangle \) lead inevitably to the nodal \( 2S \)-type relative motion form factors in our separable potential term \([41] \). In the momentum representation this form factor behaves like:

\[ g_{2S}(p) = N_{2S}(p^2 - p_0^2) \exp\left(-\frac{3p^2}{4p_0^2}\right) \]  \hfill (49)

which has the same nodal character with the same node position \( p_0^2 \) as the Tabakin's form factor \([48]\). The only difference is the power-like truncation factor \((p^2 + \beta^2)^{-2}\) in \([48]\) is

\[ \text{Almost simultaneously with the Tabakin work we suggested} \ [91] \text{ very similar separable potentials to describe cluster-cluster interaction for the systems such as} \ ^4\text{He}-^4\text{He}, \ ^4\text{He}-d \text{ etc. where all lowest partial phase shifts also change their sign (from positive to negative) at rather low energies.} \]
replaced in our case with the Gaussian form in [49] due to the use of six-quark shell-model basis in our calculations. Making use of the 2s-type form factor [49] will project out all the admixture of nodeless 0s-components in NN scattering wavefunctions giving by this way a stable short-range node in the S-wave at \( r_0 \approx 0.6 \) fm. Thus, the use of the oscillating 2s-type form factors replaces, in a good approximation, our orthogonality condition constraints (3) and (47b), resulting, in essence, in almost the same scattering wavefunctions. Therefore one can conclude that the Tabakin one-term potential agrees qualitatively with our new mechanism which dictates just a 2s-type oscillating character of the potential form factors or alternatively the necessity for the additional orthogonality constraint. This gives a quark microscopic interpretation for the success of the old phenomenological Tabakin potential. From here one can conclude that there are many completely independent arguments in favor of our new mechanism of interaction suggested in this work.

**VII. CONCLUSION**

In this paper we presented a critique of the conventional meson-exchange models of nuclear forces at intermediate and short ranges. We provided many arguments demonstrating clearly the inner inconsistencies and contradictions in modern OBE models for the short-range part of the interaction. There are also several observations in few-nucleon systems showing clearly that one cannot explain quantitatively and consistently many 3N and 4N experimental data with the existing \( NN \) models.

To find an alternative picture of the \( NN \) interaction we exploited the successful quark motivated semi-phenomenological models, viz. the Moscow [40,72,73] and Tübingen microscopic quark approaches [51,55,93], to develop them further. In this way we suggested in the present paper some new mechanisms for the intermediate- and short-range \( NN \) interaction. These mechanisms are distinguished from the traditional Yukawa concept for the meson exchange in the \( t \)-channel. We introduce a concept of the dressed symmetric six-quark bag in the intermediate state with \( s \)-channel propagation. In a tight connection to this mechanism we proposed also a new interpretation of the scalar-isoscalar \( \sigma \)-meson as a quasiparticle, i.e.
the particle can mainly only exist inside hadrons rather than as a real resonance in free space. In this respect the $\sigma$-mode should be treated differently as compared to other mesons like $\rho$, $\omega$ etc. The new interaction mechanism proposed here has been shown to lead to separable energy-dependent $s$-channel resonance-like interaction terms plus a term with a projection operator (in lowest partial waves) resulting from a constraint for a orthogonality condition.

In its final form the proposed interaction depends only on a few fundamental constants (quark-meson or diquark-meson coupling constants and the intermediate meson masses) so that eventually the total $NN$ force can be parametrized by a few free parameters only. However, at the present stage we prefer to employ the derived form of the interaction to build a simple model whose main goal is to illustrate how well the suggested mechanism can work. We found that by adjusting only three parameters of the model in each partial wave it is possible to describe excellently all lowest $NN$ phase shifts in a large energy interval $0 \div 600$ MeV, and $S$-waves even until 1200 MeV in the lab. frame. This gives some strong evidence that the suggested new microscopic mechanism of $s$-channel ”dressed” symmetric bag should work adequately.

The proposed interaction model has been demonstrated to give a natural microscopic background for previous phenomenological interaction models like the Moscow $NN$ potential, the Tabakin separable potential ”with attraction and repulsion” and also the QCB model by Simonov and other hybrid models. Thus it gives also very important bridges between at first glance absolutely disconnected models developed previously. In fact, without the present model it would be extremely hard or impossible to establish any correlations between, say, the Simonov QCB model and one-term Tabakin potential.

Another important result of the present model could be a possible resolution of a long-standing puzzle about the weak vector-meson contribution in baryon spectra and a strong spin-orbital splitting (due to the vector meson contribution) in the $NN$ interaction. If one assumes a significant quark-quark force due to vector-meson (or one gluon) exchange the vector coupling will result immediately also in strong spin-orbit splitting in baryon spectra. Very recently Glozman and Riska [87] described the absence of spin-orbital splitting in
negative parity excited baryon states in a model for the $qq$ interaction mediated essentially by Goldstone-boson exchange. However this model fails to explain the strong spin-orbit splitting in the $NN$ sector. Our explanation of the puzzle is based on the fact that there is no significant vector-meson contribution into $qq$ forces (in $t$-channel) but there is an important contribution of vector mesons in the dressing of the symmetric six-quark bag leading thereby to strong spin-orbital effects in the $NN$ interaction mediated by the "dressed" bag.

Moreover, the proposed model will lead to the appearance of strong $3N$ and $4N$ forces mediated by $2\pi$ and $\rho$ exchanges (see e.g. $3N$-forces graphs in FIG. 9).

![FIG. 9. Some graphs illustrating the new-type of 3N forces.](image)

It is easy to see that the new $3N$-forces include both central and spin-orbit components. Such a spin-orbit $3N$ force is extremely desirable to explain the low energy puzzle of the analyzing power $A_y$ in $Nd$ scattering and also the behavior of $A_y$ in the $3N$ system at higher energies $E_N \simeq 250 \div 350$ MeV at backward angles. The central components of the $3N$ and $4N$ forces are expected to be strongly attractive and thus they must contribute to $3N$- and $4N$-binding energies possibly resolving hereby the very old puzzle with the binding energies of the lightest nuclei. Moreover these strong contributions (as one can expect) of the above $3N$- and $4N$-forces mediated by the "$\sigma$-type" $2\pi$-exchange to the nuclear binding in a combination with strong relativistic effects predicted by our model can lead very
naturally to the relativistic hadrodynamics (i.e. the Walecka model) where the sigma-field constitutes the main agent for nuclear binding. The suggested new mechanism leads to a large number of new contributions for many nuclear physics observables like enhanced Coulomb displacements energies for isobar-analog states [71,73], enhanced spin-orbit splitting in the nuclear shell model, more significant relativistic effects, a serious renormalization of meson-exchange current contributions etc., etc. Future studies must show to what degree such expectations can be justified.

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APPENDIX A: DETAILS OF QUARK-MODEL CALCULATIONS

Here we consider some details for the quark-model calculations of the two-pion emission amplitude for the transition from the \(^3S_1(3D_1)\) NN state to the ”dressed” six-quark bag \(d_0 + \pi + \pi\). We will demonstrate here how to use the known fractional-parentage coefficient (f.p.c.) technique \([52,53,56,57,62]\) in calculations of the two-step processes \(d_f \to d'' + \pi \to d_0 + \pi + \pi\) in the channel \(ST = 10\ J^P = 1^-\). First we consider the two-pion emission in the two-quark subsystem ”56” (where ”5” and ”6” are quark numbers in the six-quark system ”123456”). We start from the 2s(2d) h.o. state of the 6-th quark in the \(d_f\)-state (see Sects. IV, Eqs. (3) and (10)-(11)) which, after S-wave pion emission, goes to the 1p h.o state in the 56-subsystem of the intermediate \(d''\) configuration. At the next step, the 5-th quark of the 56-subsystem emits another S-wave pion and the intermediate \(d''\) configuration goes to the final \(d_0\) configuration in which the 56-subsystem is in the 0s h.o. state. Therefore, we must take into consideration the following five non-vanishing elementary \(q \to q + \pi\) transition amplitudes in the h.o. quark basis:
(i) the two amplitudes

\[ T^{(6)}_{2s\rightarrow 1p}(j_{56} = 0) \equiv \langle 1p, s_{56} = 0 | j_{56} = 0, t_{56} = 1 | H^{(6)}_{\pi qq} | 2s, s_{56} = 1, t_{56} = 0 \rangle \]

\[ = i \sqrt{\frac{2}{3}} \frac{\omega_{\pi}(k_6)}{2m_q \alpha'} (1/2 || \sigma || 1/2) \sqrt{\frac{1}{6}} (1/2 || \tau || 1/2) F^{L=0}_L(k_6^2), \]

\[ T^{(5)}_{1p\rightarrow 0s}(j_{56} = 0) \equiv \langle 0s, s_{56} = 0, t_{56} = 0 | \pi d^{(5)}_{\pi qq} | 1p, s_{56} = 1, j_{56} = 0, t_{56} = 0 \rangle \]

\[ = i \sqrt{\frac{2}{3}} \frac{\omega_{\pi}(k_6)}{2m_q \alpha'} (1/2 || \sigma || 1/2) \sqrt{\frac{1}{6}} (1/2 || \tau || 1/2) F^{L=0}_L(k_6^2) \]  

(A1)

should be taken if the total angular momentum of the 56-subsystem \( j_{56} = 0 \) and

(ii) the three amplitudes

\[ T^{(6)}_{2s\rightarrow 1p}(j_{56} = 1) \equiv \langle 1p, s_{56} = 0 | j_{56} = 1, t_{56} = 1 | H^{(6)}_{\pi qq} | 2s, s_{56} = 1, t_{56} = 0 \rangle \]

\[ = -i \sqrt{\frac{2}{3}} \frac{\omega_{\pi}(k_6)}{2m_q \alpha'} (1/2 || \sigma || 1/2) \sqrt{\frac{1}{6}} (1/2 || \tau || 1/2) F^{L=2}_L(k_6^2), \]

\[ T^{(6)}_{2d\rightarrow 1p}(j_{56} = 1) \equiv \langle 1p, s_{56} = 0 | j_{56} = 1, t_{56} = 1 | H^{(6)}_{\pi qq} | 2d, s_{56} = 1, t_{56} = 0 \rangle \]

\[ = -i \sqrt{\frac{2}{3}} \frac{\omega_{\pi}(k_6)}{2m_q \alpha'} (1/2 || \sigma || 1/2) \sqrt{\frac{1}{6}} (1/2 || \tau || 1/2) F^{L=2}_L(k_6^2), \]

\[ T^{(5)}_{1p\rightarrow 0s}(j_{56} = 1) \equiv \langle 0s, s_{56} = 1, j_{56} = 0, t_{56} = 0 | \pi d^{(5)}_{\pi qq} | 1p, s_{56} = 0, t_{56} = 1 \rangle \]

\[ = -i \sqrt{\frac{2}{3}} \frac{\omega_{\pi}(k_6)}{2m_q \alpha'} (1/2 || \sigma || 1/2) \sqrt{\frac{1}{6}} (1/2 || \tau || 1/2) F^{L=2}_L(k_6^2) \]  

(A2)

should be taken in the case of \( j_{56} = 1 \).

To simplify matters we use shorthand notations \( T^{(6)}_{2s\rightarrow 1p}(j_{56} = 1) \), \( T^{(5)}_{1p\rightarrow 0s}(j_{56} = 0) \), etc. for the elementary amplitudes and omit spin, isospin and angular momentum projections (omitting the summation over these quantum numbers in the following expressions). The other shorthand notations are \( \alpha = \sqrt{\frac{5}{2}} b \), \( \alpha' = -\sqrt{\frac{5}{2}} \alpha \), where \( b \) is the scale parameter (r.m.s. radius) of the h.o basis functions, and

\[ F_0(k_6^2) = \exp(-\frac{5}{24} k_6^2 b^2), \quad F^L_2(k_6^2) = (1 + \frac{5}{24} a_L k_6^2 b^2) \exp(-\frac{5}{24} k_6^2 b^2), \]  

(A3)

where \( a_L = \frac{1}{3} \) if \( L=0 \) and \( a_L = -\frac{2}{3} \) if \( L=2 \). The functions in (A3) provide the \( k^2 \)-dependence of the form factors in the \( \pi d''d_f \) and \( \pi d_0 d'' \)-vertices (see Eq. (28) in Sect. IV)

\[ F^{(6)}_{\pi d_0 d''}(k_6^2) = F_0(k_6^2), \quad F^{L}_{\pi d''d_f}(k_6^2) = F^L_2(k_6^2) \]  

(A4)
The reduced matrix elements \((1/2||\sigma||1/2)\) and \((1/2||\tau||1/2)\) of the spin(isospin)-flip operators (i.e. \(\sigma\)- and \(\tau\)-matrices in the vertex operators \(H^{(6)}_{\pi qq}\) and \(H^{(5)}_{\pi qq}\)) are defined here in accordance with the Wigner-Ekkart theorem. Standard calculations give:

\[
(1/2||\sigma||1/2) = (1/2||\tau||1/2) = -\sqrt{6}
\]  

(A5)

Recall that for the desired amplitude we use the parametrization of Eq. (25) of Sect. 4

\[
15 \langle d_0|H^{(5)}_{\pi qq}(k_5)|d''\rangle \langle d''|H^{(6)}_{\pi qq}(k_6)|d_f\rangle = v^2 \frac{\omega_\pi(k_5)\omega_\pi(k_6)}{m_\pi^2 b^2} f_{\pi d_0 d_f} L_{\pi d' d_f} F_{d_0 d''} L_{k_5} F_{k_6}^L (k_5^2).
\]  

(A6)

Now one can calculate "the coupling constants" \(f_{\pi d_0 d_f}\) and \(f_{\pi d_0 d''}\) of this parametrization starting from the elementary amplitudes of Eqs. (A1)-(A2). For this purpose one can apply f.p.c.'s to separate the two-quark subsystem "56" from the six-quark configurations \(d_f, d''\) and \(d_0\) for all possible color-, spin-, isospin- and coordinate states of the quark pair ([\(f_{56}\)\(c= [2],[l^2], s_{56} =0,1, t_{56} =0,1, j_{56} =0,1\) for 2s-, 2d-, 1p-, and 0s-radial and orbital states). Recall that the f.p.c. technique implies summation over all possible states of the separated two-quark subsystem instead of summation over all numbers of quarks in the interaction operator. This scheme is particularly handy for application of the group-theoretical algebraic methods.

We use the invariants (i.e. Young schemes \([f_c]\), \([f_s]\), \([f_{cs}]\), \([f_t]\), \([f_{cst}]\) and \([f_x]\)) of the chain of symmetry groups (see, e.g. [57,62])

\[
SU(12)_{CST} \supset SU(6)_{CS} \times SU(2)_T \supset SU(3)_C \times SU(2)_S \times SU(2)_T,
\]

\[
SU(24)_{X_{CST}} \supset SU(12)_{CST} \times SU(2)_X
\]  

(A7)

for classification of six-quark, four-quark and two-quark states in the systems "123456", "1234" and "56" respectively. The f.p.c. for separation out of the pair "56" in the total \(X_{CST}\) space \(\Gamma_{X_{CST}}(q^6 \rightarrow q^4 \times q^2)\) is a product of "scalar factors" of the Clebsch-Gordan coefficients of groups \(SU(6)_{CS} \supset SU(3)_C \times SU(2)_S\), \(SU(12)_{CST} \supset SU(6)_{CS} \times SU(2)_T\) and \(SU(24)_{X_{CST}} \supset SU(2)_X \times SU(12)_{CST}\) taken from the reduction chain of Eq. (A7) (\(\Gamma_{CS}, \Gamma_{CST}\) and \(\Gamma_{X_{CST}}\)) and "orbital" f.p.c.'s \(\Gamma_X\) of translationally-invariant shell model (TISM)

\[
\Gamma_{X_{CST}}(q^6 \rightarrow q^4 \times q^2) = \Gamma_{CS} \Gamma_{CST} \Gamma_{X_{CST}} \Gamma_X
\]  

(A8)
The following extended notations for non-trivial scalar factors $\Gamma_{C,S}$ and $\Gamma_{CS,T}$ are used here (see e.g. [57]):

$$
\Gamma_{CS}^{S=1,2}([f_{CS}],[2^2]_{CS}],[2]_{CS}, s_{56} = 1) \equiv \left( \begin{array}{c|c|c}
[3]_c & [42]_s & [f_{CS}] \\
[2^2]_c \times [2]_c & [2^2]_s \times [2]_s & ([2^2]_CS \times [2]_CS)
\end{array} \right),
$$

$$
\Gamma_{CS}^{S=1,2}([f_{CS}],[2^2]_{CS}],[2]_{CS}, s_{56} = 0) \equiv \left( \begin{array}{c|c|c}
[2^3]_c & [42]_s & [f_{CS}] \\
[2^1]_c \times [2]_c & [2^2]_s \times [2]_s & ([2^2]_CS \times [2]_CS)
\end{array} \right),
$$

$$
\Gamma_{CS}^{S=1,2}([f_{CS}],[2^1]_{CS}],[2]_{CS}, s_{56} = 1) \equiv \left( \begin{array}{c|c|c}
[3]_c & [42]_s & [f_{CS}] \\
[2^1]_c \times [2]_c & [2^2]_s \times [2]_s & ([2^2]_CS \times [2]_CS)
\end{array} \right),
$$

$$
\Gamma_{CS}^{S=1,2}([f_{CS}],[2^1]_{CS}],[2]_{CS}, s_{56} = 0) \equiv \left( \begin{array}{c|c|c}
[3]_c & [42]_s & [f_{CS}] \\
[2^1]_c \times [2]_c & [2^2]_s \times [2]_s & ([2^2]_CS \times [2]_CS)
\end{array} \right), \quad (A9)
$$

Here the $[f_{CS}]$ are all the $CS$-Young schemes from the inner product

$$
[f_{CS}] = [2^3]_c \circ [42]_s = [42], [321], [2^3], [31^3], [21^4]
$$

Values of all the necessary scalar factors (A9) are shown in Tables II and III.

Only the Young schemes $[f_{CS}] = [2^21^2], [2^14], [1^6]$ are important for configurations $d_f$, $d''$ and $d_0$ ($[f_{CS}] = [\tilde{f}_X]$, where $[\tilde{f}_X]$ is the Young scheme conjugated to $[f_X]$). All the necessary scalar factors

$$
\Gamma_{CS,T}^{T=0}([f_{CS}],[2^2]_{CS}],[2]_{CS}, t_{56} = 0) \equiv \left( \begin{array}{c|c|c}
[f_{CS}] & [3^2]_T & [f_{CS}] \\
([2^2]_CS \times [2]_CS) & ([2^2]_T \times [2]_T) & ([1^4]_CS \times [1^4]_CS)
\end{array} \right),
$$

$$
\Gamma_{CS,T}^{T=0}([f_{CS}],[2^1]_{CS}],[2]_{CS}, t_{56} = 1) \equiv \left( \begin{array}{c|c|c}
[f_{CS}] & [3^2]_T & [f_{CS}] \\
([2^1]_CS \times [2]_CS) & ([3^2]_T \times [2]_T) & ([1^4]_CS \times [1^4]_CS)
\end{array} \right) \quad \text{(A11)}
$$

are shown in Table IV. The coefficients $\Gamma_{X,CS}$ are trivial weight factors $\Gamma_{X,CS}([6]_X([4] \times [2])) = 1$, $\Gamma_{X,CS}([51]_X([4] \times [2])) = \frac{1}{\sqrt{5}}$, and $\Gamma_{X,CS}([42]_X([4] \times [2])) = \frac{1}{\sqrt{5}}$ dependent only on the dimensions of irreducible representations of the symmetrical group for given Young schemes: $n_{[6]} = 1$, $n_{[51]} = 5$ and $n_{[42]} = 9$. The last factor in the right-hand side of Eq. (A8), the orbital f.p.c. of TISM $\Gamma_X$, takes the value dependent on the configuration, i.e. only five different values of $\Gamma_X$ are necessary:
\[ \Gamma_x(s^6[6](s^4[4] \times s^2[2])) = 1, \]
\[ \Gamma_x(s^4p^2 - s^52s[6](s^4[4] \times s^2s[2])) = \sqrt{\frac{1}{5}}, \]
\[ \Gamma_x(s^4p^2 - s^52d[6](s^4[4] \times s^2d[2])) = \sqrt{\frac{3}{5}}, \]
\[ \Gamma_x(s^5p[51](s^4[4] \times sp[2])) = -\sqrt{\frac{3}{5}}, \]
\[ \Gamma_x(s^4p^2[42](s^4[4] \times p^2[2])L = 0, 2) = -\sqrt{\frac{3}{10}}. \]  

(A12)

Thus the total transition amplitude \([A6]\) are expressed through the product of factors \([A1], [A2], [A9], [A12]\) summed over states of the pair "56" (the summation should be extended over all the possible two-quark states, but the fixed quantum numbers of the initial, intermediate and final states impose the restriction that only the summation over \(j_{56} = 0, 1\) and \([f_{56}]_{CS} = [2], [1^2]\) is allowed):

\[ 15 \langle d_0|H^{(5)}_{\pi qq}(k_5)|d'' \rangle \langle d''|H^{(6)}_{\pi qq}(k_6)|d_f \rangle = 15 \sum_{j_{56}=0,1} \sum_{[f_{56}]_{CS}=[2],[1^2]} \Gamma_x CS \left( [6],x([4] \times [2]) \right) \]
\[ \times \left[ \Gamma_x CS \left( [51],x([4] \times [2]) \right) \Gamma_x CS \left( [42],x([4] \times [2]) \right) \Gamma_x \left( s^6[6](s^4[4] \times s^2[2]) \right) \right] \]
\[ \times \left[ \Gamma_x \left( s^5p[51](s^4[4] \times sp[2]) \right) \Gamma_x \left( s^4p^2[42](s^4[4] \times p^2[2]) \right) L = 0, 2 \right] \]
\[ \times \Gamma_{CS}^{[2][3]} \left( [f_{1234}]_{CS} \times [f_{56}]_{CS} \right) \left( s_{56} \right) \left( \Gamma_{CS}^{[2][1^2]} \left( t_{56} \right) \Gamma_{CS}^{[2][1]} \left( t_{56} \right) \right) \]
\[ \times \Gamma_{CS}^{[2][1]} \left( [f_{1234}]_{CS} \times [f_{56}]_{CS} \right) \left( t_{56} \right) T^{(5)}_{1p \rightarrow 0s}(j_{56}) T^{(6)}_{2s(2d) \rightarrow 1p}(j_{56}) \]  

(A13)

Spin and isospin of the quark pair \(s_{56}(s'_{56}), t_{56}(t'_{56})\) in Eq. \((A13)\) depend on color quatum numbers of the pair. For example, \(t_{56} = 1(t'_{56} = 0)\) for \([f_{56}]_{CS} = [1^2]\) and \(t_{56} = 0(t'_{56} = 1)\) for \([f_{56}]_{CS} = [2]\). A general rule for \(s_{56}(s'_{56})\) is easy to understand from the right-hand side of Eqs. \((A9)\). One can remark that in the case of \(L=2\) (the \(3D_1\) initial state) the value \(j_{56} = 0\) does not contribute to the transition \(3D_1 \rightarrow 3S_1\) and only the term with \(j_{56} = 1\) should be taken in the right-hand side of Eq. \((A13)\). This leads to a difference in the value of the coupling constant \(f_{\pi d''d_f}\) for \(L=0\) and \(2\) that is marked by an additional superscript \(L: f_{\pi d''d_f}^L\).

Calculated values of the product \(f_{\pi d''d_f}^L f_{\pi d_0d''}\) are shown in Table [V]. Substituting these
values into Eq. (28) one obtains the following expression for factors $g_L$ in the transition operator (27)

$$g_L = g_{\pi\pi\sigma} \frac{f_{\pi q q}^2}{m_\pi^2} \frac{1}{m_q^2 b^2} \frac{1}{180} \times \begin{cases} \frac{-617}{1620 \sqrt{5}}, & L = 0 \\ \frac{55}{324}, & L = 2 \end{cases}$$

(A14)
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TABLE I. The model parameters for the different partial waves.

| $^2s^+L_J$ | $^1S_0(<600\text{ MeV})$ | $^1S_0(<1.2\text{ GeV})$ | $^1D_2$ | $^3S_1-^3D$ | $^3D_2$ | $^3D_3-^3G_3$ | $^1P_1$ | $^3P_0$ | $^3P_1$ | $^3P_2-^3F_2$ |
|------------|-----------------|-----------------|---------|-------------|---------|-------------|---------|---------|---------|-------------|
| $\Lambda$ (GeV) | 0.65             | 0.65             | 0.65    | 0.5936     | 0.5527  | 0.5936      | 0.7324  | 0.65    | 0.65    | 0.65       |
| $r_0^\text{orth}$ (fm) | 0.3943           | 0.3943           | 0.02463 | 0.3737     | 0.01038 | 0.002927    | 0.46572 | 0.3445  | 0.4491  | 0.03124    |
| $\lambda_{11}$ | 2.055            | 4.565            | 0.02463 | 7.201      | 0.01038 | 0.002927    | 28.74   | 0.02841 | 3.195   | -0.006486  |
| $\lambda_{12}$ | -                  |                  | 0.02463 | 0.007928   | 0.01038 | 0.002927    | 0.1753  | 0.02624 | 0.02841 | 0.000765   |
| $r_{01}$ (fm) | 0.59686          | 0.5106           | 0.79403 | 0.2294     | 0.86037 | 0.02624     | 0.455   | 0.44311 | 0.51749 | -0.006486  |
| $r_{02}$ (fm) |                  |                  |         | 0.45469    |         | 0.89971     | 0.44311 | 0.455   | 0.44311 | 0.70995    |
| $E_0$ (MeV) | 356              | 550              | 330     | 681        | 800     | 800         | 600     | 400     | 600     | 360        |
| $\chi^2$  | 1.09             | 3.9              | 0.028   | 1.7        | 0.062   | 0.11        | 0.167   | 0.14    | 0.13    | 0.71        |
TABLE II. Scalar factors $\Gamma^S_{c,s}([f_{cs}],[f'_{cs}],[f''_{cs}],s_{56})$ of the Clebsch-Gordon coefficients for the group $SU(6)_{cs} \subset SU(3)_c \times SU(2)_s$ (see Eq. (A9)).

$$S = 1$$

|           | $[2^2]_{cs} \times [2]_{cs}$ | $[21^2]_{cs} \times [1^2]_{cs}$ |
|-----------|-------------------------------|-------------------------------|
| $[2]_{c} \times [2]_{c}$ | $[21]_{c} \times [1^2]_{c}$ | $[2]_{c} \times [2]_{c}$ | $[21^2]_{c} \times [1^2]_{c}$ |
| $[2]_{s} \times [2]_{s}$ | $[31]_{s} \times [1^2]_{s}$ | $[31]_{s} \times [1^2]_{s}$ | $[2]_{s} \times [2]_{s}$ |
| $[42]_{cs}$ : | $\sqrt{\frac{1}{20}}$ | $-\sqrt{\frac{9}{20}}$ | 0 | 0 |
| $[321]_{cs}$ : | $\sqrt{\frac{8}{15}}$ | $-\sqrt{\frac{7}{15}}$ | $\sqrt{\frac{2}{9}}$ | $\sqrt{\frac{8}{27}}$ |
| $[2^3]_{cs}$ : | $\sqrt{\frac{5}{12}}$ | $\sqrt{\frac{7}{12}}$ | $\sqrt{\frac{1}{18}}$ | $-\sqrt{\frac{5}{44}}$ |
| $[31^3]_{cs}$ : | 0 | 0 | $-\sqrt{\frac{1}{18}}$ | $-\sqrt{\frac{25}{54}}$ |
| $[21^4]_{cs}$ : | 0 | 0 | $\sqrt{\frac{4}{9}}$ | $-\sqrt{\frac{4}{27}}$ |

TABLE III. Scalar factors $\Gamma^S_{c,s}([f_{cs}],[f'_{cs}],[f''_{cs}],s_{56})$ of the Clebsch-Gordon coefficients for the group $SU(6)_{cs} \subset SU(3)_c \times SU(2)_s$ (continued).

$$S = 0$$

|           | $[2^2]_{cs} \times [1^2]_{cs}$ | $[21^2]_{cs} \times [2]_{cs}$ |
|-----------|-------------------------------|-------------------------------|
| $[2]_{c} \times [2]_{c}$ | $[21^2]_{c} \times [1^2]_{c}$ | $[2]_{c} \times [2]_{c}$ | $[21^2]_{c} \times [1^2]_{c}$ |
| $[2]_{s} \times [1^2]_{s}$ | $[31]_{s} \times [2]_{s}$ | $[31]_{s} \times [2]_{s}$ | $[2]_{s} \times [1^2]_{s}$ |
| $[2^21^2]_{cs}$ : | $-\sqrt{\frac{7}{4}}$ | $\sqrt{\frac{7}{4}}$ | $-\sqrt{\frac{7}{2}}$ | $\sqrt{\frac{7}{2}}$ |
TABLE IV. Scalar factors $\Gamma^{T=0(1)}_{CS,T}(J_{CS}^f|J_{CS}^{f'},t_{56})$ of the Clebsch-Gordon coefficients for the group $SU(12)_{CST} \subset SU(6)_{CS} \times SU(2)_T$ (see Eq. (A11)).

|               | $T = 0$ |          |          |          |          | $T = 1$ |
|---------------|---------|----------|----------|----------|----------|---------|
|               | $[42]_{CS}$ | $[321]_{CS}$ | $[2^3]_{CS}$ | $[31^3]_{CS}$ | $[21^4]_{CS}$ | $[2^21^2]_{CS}$ |
| $|2^21^2\rangle_{CST}(T = 0)$ : |         |          |          |          |          |         |
| $(|2^2\rangle_{CS} \times |2\rangle_{CS}) \circ (|2\rangle_T \times |1^2\rangle_{T})$ | 1       | $-\sqrt{\frac{3}{8}}$ | $-\sqrt{\frac{3}{5}}$ | 0   | 0   | – |
| $(|2^1\rangle_{CS} \times |1^2\rangle_{CS}) \circ (|31\rangle_T \times |2\rangle_T)$ | 0       | $\sqrt{\frac{5}{8}}$ | $-\sqrt{\frac{3}{5}}$ | 1   | 1   | – |
| $(|1^6\rangle_{CST}(T = 0)$ : |         |          |          |          |          |         |
| $(|2^2\rangle_{CS} \times |2\rangle_{CS}) \circ (|2\rangle_T \times |1^2\rangle_{T})$ | 0       | 0       | $-\sqrt{\frac{7}{5}}$ | 0   | 0   | – |
| $(|2^1\rangle_{CS} \times |1^2\rangle_{CS}) \circ (|31\rangle_T \times |2\rangle_T)$ | 0       | 0       | $\sqrt{\frac{3}{5}}$ | 0   | 0   | – |
| $|2^21^2\rangle_{CST}(T = 1)$ : |         |          |          |          |          |         |
| $(|2^2\rangle_{CS} \times |2\rangle_{CS}) \circ (|2\rangle_T \times |1^2\rangle_{T})$ | –       | –       | –       | –       | –       | $-\sqrt{\frac{4}{9}}$ |
| $(|2^1\rangle_{CS} \times |1^2\rangle_{CS}) \circ (|31\rangle_T \times |2\rangle_T)$ | –       | –       | –       | –       | –       | $\sqrt{\frac{5}{6}}$ |

TABLE V. The products of coupling constants $f_{\pi d_0^m} f_{\pi d'' d_f}^{L=0(2)}$ for the two-step transition $d_f \rightarrow d'' + \pi \rightarrow d_0 + \pi + \pi$ with creation of the scalar-isoscalar $\pi + \pi$ pair ("$\sigma$-meson") and the overlap factor between the $NN$ and $d_f$ states $U^{NN}_f$.

Quantum numbers of $d_f$

$$s^4p^2 - s^52s(2d)[6]_{x}$$

|               | $[2^2]_{CS}$ | $[42]_{CS}$ | $[321]_{CS}$ | $[2^3]_{CS}$ | $[31^3]_{CS}$ | $[21^4]_{CS}$ |
|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $180 \times f_{\pi d_0^m} f_{\pi d'' d_f}^{L}$ : |               |               |               |               |               |               |
| $L = 0$       | $\frac{17}{18}$ | $-\sqrt{\frac{7}{9}}$ | $-\frac{72}{5} \sqrt{\frac{3}{5}}$ | $-\frac{13}{18}$ | $\frac{1}{9\sqrt{2}}$ | $-\frac{5}{9}$ |
| $L = 2$       | $\frac{13\sqrt{5}}{36}$ | $\frac{1}{2}$ | $-\frac{31}{36}$ | $-\frac{17\sqrt{5}}{36}$ | $-\frac{5\sqrt{5}}{36\sqrt{2}}$ | $-\sqrt{\frac{5}{18}}$ |
| $U^{NN}_f$ :  | $\sqrt{\frac{1}{9}}$ | $-\sqrt{\frac{9}{20}}$ | $\sqrt{\frac{16}{45}}$ | $\sqrt{\frac{1}{36}}$ | $-\sqrt{\frac{1}{18}}$ | 0 |
**Figure captions**

**FIG.1.** The traditional t-channel meson-exchange mechanism (a) compared to the new
s-channel ”dressed” bag mechanism (b) for $NN$ interaction.

**FIG.2.** Schematic representation of the two-pion emission in the transition of two $p$-shell
quarks to an $s$ orbit.

**FIG.3.** The graph illustrating the $\sigma$- (or $\rho$-) meson emission and subsequent absorption
by diquark pairs in the intermediate six-quark bag-like state.

**FIG.4.** The graph illustrates two sequential $\pi$-meson emissions and absorptions via an
intermediate $\sigma$- (or $\rho$-) meson and the generation of a six-quark bag.

**FIG.5.** The kinematic variables in the triangle diagram corresponding to the $\sigma$- (or $\rho$-) meson
generation from two $\pi$-mesons emerging in the transition of two $p$-shell quarks to the
$s$-orbit (see also FIG. 4).

**FIG.6.** The $NN$ phase shifts (in deg.) in our model in comparison with PSA data
(SAID, solution SP99).

**FIG.6.** (Continued.)

**FIG.7.** The mixing parameter $\varepsilon_1$ for different values of cut-off parameter $\Lambda_{\pi NN}$ (see
text).

**FIG.8.** The $^{1}S_0$ phase shifts fitted by means of our model ([11]) until $E_{\text{lab}} = 1200$ MeV.

**FIG.9.** Some graphs illustrating the new-type of $3N$ forces.
$^{1}S_0$