Multi-scale asynchronous belief percolation model on multiplex networks

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Abstract
The studies of multiplex networks are increasingly popular in recent years. Modeling multiple complex systems as a multiplex network has refreshed our understanding about the structure and dynamics of various real-world systems. As an important variant of the voter models, belief formation dynamics such as the asynchronous belief percolation (ABP) model has attracted much attention from statistical physics and network science communities. Existing studies of the ABP model mainly focus on the applications to single networks, whereas how the structure of multiplex networks affects its dynamical behavior is still not well understood. To close this gap, we propose a multi-scale ABP model that takes into account the differential velocities of belief propagation at different subnetworks within the multiplex network. Using extensive computer simulations, we find that (i) increasing the degree correlation between subnetworks can promote nodes with minority belief to form stable clusters and (ii) minority nodes require less initial supports to survive in multiplex networks with respect to single networks. Our conclusion is robust against the detailed topology of the subnetworks that constitute the multiplex network.

1. Introduction

The studies of complex networks have provided many novel ideas and insights for understanding the structure and dynamics of complex systems, such as social ties that link different people and transportation networks that connect different cities [1–5]. In recent years, research efforts have shifted from studying each complex system as a single and isolated network to synthesizing multiple correlated systems as interdependent, multiplex or multilayer networks [6–11]. This evolving trend indeed echoes modern technical advancements, such as the Internet of Things, Smart Grid, Ubiquitous Computing, Mobile Web, Big Data and Artificial Intelligence, which contribute a novel perspective that different complex systems often have strong correlations and they can interact with each other through nontrivial means. For example, in our modern society, the same group of people (e.g. graduate students of the same college, close friends of the same workplace) can interact with each other through multiple communication routes including the face-to-face contact, instant message, cell-phone call, online chat and social media. This substantially increases the complexity in characterizing the dynamical processes (e.g. information spreading and cooperation behavior), because neglecting any layer of communication routes can result in biased or false-positive conclusions (e.g. source of rumors, fake news) [12–14].

Increasing research interests have focused on modeling complex systems as multiplex networks in which the same set of nodes can be connected through multiple layers of subnetworks [3–5]. For example, in social networks [15, 16], a group of people can have mutual interactions through direct contacts, mobile-phone calls, and social media; in transportation networks, cities can be connected through flight, railway and road networks. Various research topics emerge, ranging from the characterization of structural multiplexity [17–20] to the quantification of correlation properties [18, 21] to the exploration of dynamical processes happened on

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multiplex networks (e.g. diffusion processes [22–24], epidemic spreading [25–33], percolation [34–37], and evolutionary game [38–42]).

As an important variant of voter models, the dynamics of belief (or opinion) formation have attracted a great attention [43]. A seminal example is the asynchronous belief percolation (ABP) model proposed by Shao et al [44], in which minority and majority beliefs can coexist at the equilibrium state. Despite being introduced for more than a decade, existing studies of belief percolation dynamics mainly focus on the applications to single networks, whereas how the structure of multiplex networks affects the dynamical behavior of belief models is largely unclear. To close this knowledge gap, we propose a multi-scale modeling framework that extends the original ABP model [44] to multiplex networks. Our multi-scale ABP model takes into account the differential velocities of belief propagating at different layers of the multiplex network. Our model also addresses the effect of degree correlations between different layers on the belief dynamics. Using extensive computer simulations, we find that (i) the increase of degree correlations can promote the nodes with minority belief to form stable clusters, and (i) minority nodes require less initial supports to survive in multiplex networks with respect to single networks. Our conclusion is robust against the detailed topological structure of the subnetworks (e.g. Erdos–Renyi random networks or Barabasi–Albert scale-free (SF) networks) that constitute the multiplex network.

2. Model descriptions

2.1. ABP model

Inspired by the original belief percolation model [44], we introduce a nonlocal discount based on a resolution parameter $\gamma$ and propose the ABP model. Given a node $x$, let $\sigma_1, \sigma_2, \ldots, \sigma_k$ be the belief labels currently owned by the neighbors of $x$, $n_i$ be the number of neighbors of $x$ that have label $\sigma_i$, and $N$ be the overall number of nodes in the graph with label $\sigma_i$. As the belief updating of $x$, instead of choosing the label $\sigma_i$ that maximizes $n_i$ (as in the conventional belief propagation), we maximize the following function (see algorithm 1)

$$n_i = \gamma (v_i - n_i).$$

When $\gamma = 0$ the algorithm returns to label propagation; the reason for introducing the discount term is that when we decide to join a given community, we are increasing its density because $n_i$ new edges connect $x$ to existing members of the community, but we are at the same time decreasing it because of $v_i - n_i$ non-existing edges. Indeed, it can be shown that the density of the sparsest community at the end of the algorithm is never below $\gamma/1 - \gamma$.

Algorithm 1. The belief percolation model

Require: $\gamma$ - a random permutation of $G$’s nodes
1: $\pi \leftarrow (V, E)$, the resolution parameter $\gamma$
2: for all $x$: $\sigma_i \leftarrow x$, $v(x) \leftarrow 1$
3: while satisfy the stopping criterion, do
4: for $i = 0, 1, \ldots, N$ do
5: for every belief $\zeta$, $\zeta_i \leftarrow [\sigma_i \cap \zeta]$ and $n_i \leftarrow \arg \max (n_i - \gamma (v_i - n_i))$
6: $\zeta_i \leftarrow \arg \max (n_i - \gamma (v_i - n_i))$
7: decrement $v(\sigma_0)$
8: $\sigma_{0} \leftarrow \zeta_i$
9: increment $v(\sigma_0)$
10: end for
11: end while

In this paper, we consider a simple case that contains only two different types of beliefs. In a single network, each node is first assigned with a belief (or attitude) that is either positive $\sigma_+$ with probability $\delta$ or negative $\sigma_-$ with probability $1 - \delta$, respectively. And then each node updates its belief according to the algorithm 1. At each time step, every node simultaneously updates her belief according to the beliefs of all nodes obtained at the end of the last time step [44–47]. Note that the belief of a node remains unchanged if it only has one neighbor. All nodes will continue to update their statuses until they converge to an equilibrium state in which the fractions of positive and negative nodes remain unchanged.

2.2. ABP model on a multiplex network

A multiplex network comprises multiple layers of subnetworks, each of which characterizes a particular type of connections for the same set of nodes. For example, social network is a typical multiplex network because people
can interact with each other through multiple networking routes such as face-to-face contacts, cell-phone calls, online chats and social media. The commercial air-travel network can be refined as a multiplex network with different layers denoting the flight routes of different airplane carriers [48].

We generalize the original ABP model for a multiplex network with two layers as follows. Given a multiplex network, the belief of each node is initialized to be positive with probability $\delta$ and negative with probability $1 - \delta$, respectively. During subsequent $m$ ($m \geq 1$) time steps, we assume that only edges of the first layer of the multiplex network are active for belief propagation, which means that each node updates her belief in each of these $m$ time steps according to algorithm 1 among herself and her neighbors that belong to the first layer of the multiplex network. At the end of these $m$ time steps, the edges of the first layer are refrained for propagating beliefs, while the belief status of each node is inherited for her further updating. And then, the edges of the second multiplex-network layer become active. In each of the subsequent $n$ ($n \geq 1$) time steps, each node updates her belief using algorithm 1 for her neighbors that are directly connected in the second layer of the multiplex network. Following these $n$ time steps, the two layers of the multiplex network will be in turn active for $m$ and $n$ time steps, respectively. Their shifts will continue until the beliefs of all nodes converge to an equilibrium state during the activation of either the first or the second layer of the multiplex network. For this reason, at every iteration of belief percolation, the beliefs of each layer sequentially update in a fix order and no longer change all together. Thus the belief percolation algorithms on multiplex networks are implemented in an ‘asynchronous’ mode. In this study, we focus the discussion on a multiplex network with two layers, whereas the proposed multiplex ABP model can be straightforwardly applied to multiplex networks with three or more layers of subnetworks.

### 2.3. Degree correlation of a multiplex network

Aside from the effect of shifting active layers on the consensus of ABP models, we also investigate the effect of varying the degree correlation of a given multiplex network. For a multiplex network with two layers ($\alpha = 1, 2$), the degree of each node $i$ in the first and second layer is denoted by $k_i^{[1]}$ and $k_i^{[2]}$ that are summarized as a simple vector $K_i = \{k_i^{[1]}, k_i^{[2]}\}$. Denote $N$ as the number of nodes in the multiplex network. The correlation of degrees for nodes in the two layers can be defined using their linear correlation coefficient [49]

$$
\omega = \frac{\sum_{i=1}^{N} [(k_i^{[1]} - \langle k^{[1]} \rangle)(k_i^{[2]} - \langle k^{[2]} \rangle)]}{\sqrt{\sum_{i=1}^{N} (k_i^{[1]} - \langle k^{[1]} \rangle)^2 \sum_{i=1}^{N} (k_i^{[2]} - \langle k^{[2]} \rangle)^2}},
$$

where $\langle k^{[\alpha]} \rangle$ counts the average degree of the layer $\alpha$. With equation (2), a positive coefficient ($\omega > 0$) corresponds to an assortative multiplex network in which a node with a high degree in one layer is more likely to connect to nodes with high degrees in another layer, whereas a negative coefficient ($\omega < 0$) corresponds to a disassortative multiplex network in which a node with a high degree in one layer is more likely to connect to nodes with low degrees in another layer.

For each layer of the multiplex network, we consider its structure as an Erdős–Rényi (ER) network [1] or a SF network [2]. The degrees of nodes in an ER network is distributed as a Poisson distribution $P(k) = e^{-\langle k \rangle}\langle k \rangle^k/k!$, while the degrees of nodes in a SF network is assumed to distributed as a power law distribution, $P(k) \sim k^{-2.44}$ with minimum degree as $k_{\min} = 1$. As such, both of these two types of networks have an average degree $\langle k \rangle = 2$.

#### 2.3.1. The highest disassortative and assortative multiplex networks

We develop an algorithm to identify the two-layer multiplex network with minimal degree correlation. We first generate a sequence with nodes of the first layer sorted in ascending order according to their node degree. We then generate another sequence with nodes of the second layer sorted in descending order according to their node degree. Finally two nodes from different layers are paired together if they have the same rank in the two sequences. This treatment naturally reduces the degree correlation of a multiplex network to the minimum, because the higher the degree of a node is in one layer the lower the degree of the same node is in another layer. This heuristic claim can also be validated mathematically. Assume that we randomly pick up two nodes $X_i^{[1]}$, $Y_i^{[1]}$ from the first layer $L_1$, and the two nodes of their counterparts $X_i^{[2]}$ and $Y_i^{[2]}$ from the second layer $L_2$, with the following relationship:

$$
k_i^{[1]} > k_i^{[2]}, \quad k_i^{[2]} < k_i^{[1]}.
$$

If we exchange the correspondence of these two pairs of nodes by linking $X_i^{[1]}$ to $Y_i^{[2]}$ and $Y_i^{[1]}$ to $X_i^{[2]}$, the change in the degree correlation can be calculated by:
Let us further introduce two rewiring algorithms for constructing multiplex networks with varying degree correlations.

2.3.2. Multiplex networks with varying degree correlations

We perform simulations of the multi-scale ABP model on multiplex network where the two layers are either ER networks or SF networks. Each of the two models are studied in two parts to understand the influence of the two types of parameters on the belief dynamics: (1) the degree correlation between the two layers of a multiplex network; (2) the model parameter such as the time steps. In the following we first show the simulation results of the multi-scale ABP model on multiplex networks with different degree correlations when \( m \) and \( n \) are both 1. Then the multi-scale ABP model is studied on multiplex networks for different \( m \) and \( n \). Here, \( \rho_1 \) is the normalized size of the largest cluster of the positive beliefs, and \( \rho_2 \) is the normalized size of the second largest cluster of the positive opinion.

\[
\omega - \omega' = \sqrt{\sum_{i=1}^{N}(k_i^{[1]} - \langle k_i^{[1]} \rangle)^2} - \sqrt{\sum_{i=1}^{N}(k_i^{[2]} - \langle k_i^{[2]} \rangle)^2}
\]

\[
A = (k_X^{[1]} - \langle k_i^{[1]} \rangle)(k_X^{[2]} - \langle k_i^{[2]} \rangle) + (k_i^{[1]} - \langle k_i^{[1]} \rangle)(k_i^{[2]} - \langle k_i^{[2]} \rangle)
\]

\[
B = (k_X^{[1]} - \langle k_i^{[1]} \rangle)(k_X^{[2]} - \langle k_i^{[2]} \rangle) + (k_i^{[1]} - \langle k_i^{[1]} \rangle)(k_i^{[2]} - \langle k_i^{[2]} \rangle),
\]

where \( \omega \) and \( \omega' \) denote the degree correlations for the original and revised multiplex networks, respectively. Because the average degree of each layer is assumed equal to 2, we have

\[
A - B = (k_X^{[1]} - 2k_i^{[1]})(k_X^{[2]} - k_i^{[2]}) + (k_i^{[1]} - 2k_i^{[1]})(k_i^{[2]} - k_i^{[2]}),
\]

The relationship of equation (3) indicates that \( A - B < 0 \), and hence \( \omega < \omega' \). As such, the above algorithm ensures that the resulting multiplex network has the highest disassortative level. Similarly, we can generate a multiplex network with highest assortative correlation by building the two sequences with nodes of each layer sorted in ascending order according to their degree in that layer. With the one by one matches, the two networks are disassortatively connected due to the fact that the more friends one node has in \( L_1 \), the less neighbors it will have in \( L_2 \).

2.3.2. Multiplex networks with varying degree correlations

Let us further introduce two rewiring algorithms for constructing multiplex networks with varying degree correlations [41–50]. We first randomly match nodes from the two layers, which leads to a totally random multiplex network with degree correlation equaling 0. The assortative multiplex networks with positive degree correlation (\( \omega > 0 \)) can be generated by using the rewiring algorithm A. Specifically, we first randomly select two nodes from the first layer \( i^{[1]} \) and \( j^{[1]} \) as well as their counterpart nodes in the second layer \( i^{[2]} \) and \( j^{[2]} \). If these two pairs of nodes satisfy either of the following two conditions

\[
k_i^{[1]} > k_i^{[1]} \quad \text{and} \quad k_i^{[2]} < k_i^{[2]}
\]

or

\[
k_i^{[1]} < k_i^{[1]} \quad \text{and} \quad k_i^{[2]} > k_i^{[2]},
\]

we revise the correspondence between these two pairs by rewiring the connection from \( i^{[1]} \) to \( j^{[2]} \) and from \( j^{[1]} \) to \( i^{[2]} \). We continue this rewiring process until the degree correlation of the two layers fulfills our criterion.

The disassortative multiplex networks with negative degree correlation (\( \omega < 0 \)) can be obtained by using the rewiring algorithm B. Specifically, we first randomly pick up two nodes from the first layer \( i^{[1]} \) and \( j^{[1]} \) and their corresponding counterparts \( i^{[2]} \) and \( j^{[2]} \) from the second layer. We revise the correspondence of these two pairs by rewiring their interconnections if they fulfills either of the following conditions:

\[
k_i^{[1]} > k_i^{[1]} \quad \text{and} \quad k_i^{[2]} > k_i^{[2]}
\]

or

\[
k_i^{[1]} < k_i^{[1]} \quad \text{and} \quad k_i^{[2]} < k_i^{[2]}. \]

The rewiring process continues to proceed until the degree correlation reach our criterion.

3. Simulation results

We perform simulations of the multi-scale ABP model on multiplex network where the two layers are either ER networks or SF networks. Each of the two models are studied in two parts to understand the influence of the two types of parameters on the belief dynamics: (1) the degree correlation between the two layers of a multiplex network; (2) the model parameter such as the time steps. In the following we first show the simulation results of the multi-scale ABP model on multiplex networks with different degree correlations when \( m \) and \( n \) are both 1. Then the multi-scale ABP model is studied on multiplex networks for different \( m \) and \( n \). Here, \( \rho_1 \) is the normalized size of the largest cluster of the positive beliefs, and \( \rho_2 \) is the normalized size of the second largest cluster of the positive opinion.

3.1. The steady state of ABP model on multiplex networks

The steady state is reached when nodes keep in stable status at the end of \( m \) (or \( n \)) time steps of the belief dynamics in layer 1 (or 2). In order to better understand the steady state, we plot the ratio \( F(\delta) \), which is the fraction of the nodes that hold the same belief in the two layers, for different values of \( \omega \) in figure 1. Here, we fix the parameter \( \gamma = 0.1 \). We find that \( F(\delta) \approx 1 \) at most values of \( \delta \), which means most of the nodes hold the same
belief at each layer in the steady state regardless of the initial fraction $\delta$. These curves reach their bottoms at $\delta = 0.5$ with a value around 0.99. This can be understood as follows. Every node has a higher chance to be with the majority belief than with the minority belief, due to the formation rule that a node follows the majority belief of its local community. However when $\delta = 0.5$, the two beliefs (positive and negative) have the same probability to be the majority. Thus every node has the highest chance to be with different beliefs in the two layers when $\delta = 0.5$.

### 3.2. Multi-scale ABP model on multiplex networks with different degree correlations

With different correlation $\omega$ between each layer of ER–ER networks, $\rho_1$ and $\rho_2$ of the first network layer is shown in figures 2(a) and (b). Here, we fix the parameter $\gamma = 0.1$. It is clear to see that when the degree correlation increases from $-0.9$ to 0.9, the critical threshold $\delta_c$ also increases from 0.40 to 0.47. That is to say when the two layers of the multiplex network are more correlated in degree, (with the interactions between two layers) it is more difficult for the minority belief to survive. This is because in the ABP model, a node with more neighbors would be more likely to follow the majority, and such nodes can certainly influence many others. In the

![Figure 1](image1.png)

**Figure 1.** $F(\delta)$ is the fraction of nodes hold the same belief in the two layers of ER–ER multiplex networks. Here, we fix the parameter $\gamma = 0.1$.

![Figure 2](image2.png)

**Figure 2.** $\rho_1$ and $\rho_2$ as a function of $\delta$ in ER–ER networks of the multi-scale ABP model when $m$ and $n$ are both 1.
multiplex network with a small $\omega$, a node with a small number of neighbors in the first layer may have a large number of neighbors in the second layer. Such nodes may keep the minority belief in the first layer and spread it in the second layer. Obviously, there is less chance for the minorities to have such advantage in the multiplex network if the degree correlation is larger. Hence, it is more difficult for the minorities to survive in multiplex networks with a larger degree correlation.

Figures 3(a) and (b) show plots of $\rho_1$ and $\rho_2$ of the first layer as a function of the initial fraction $\delta$ in ER–SF networks. We can see that, the peak of $\rho_2$ shifts to the right with the increase of $\omega$, which is the same as the ER–ER network does. The higher the degree correlation is, the larger $\delta_c$ will be. The same phenomenon can also be observed in SF–ER and SF–SF networks that decreasing the degree correlation between the two layers makes the minority belief more stable.

Furthermore, we find that the value of the critical threshold $\delta_c$ of multiplex networks is larger than that of single networks which are characterized by the same degree distribution as the first layer. For example, for ER–ER multiplex networks with the lowest correlation $\omega = -0.1$, $\delta_c = 0.27$, while for single ER networks, $\delta_c = 0.26$. That is to say, minorities in the multiplex networks need more initial support to form the clusters at the steady state than minorities in single networks [51]. This might be understood by the fact that on a single network, one belief (the majority or minority) always tends to form clusters to keep the belief step by step. Though the fraction of the nodes with the minority belief decreases in the propagation, the nodes can always form a stable giant cluster in the steady state if the initial fraction is large enough. However on a multiplex network, after one time step in the first layer, though each belief forms clusters, such clusters will be broken up in the second layer due to the random interconnections between the two layers. That is to say, the beliefs seem to be randomly assigned to the nodes again in the second layer, so both beliefs have to form the clusters from the beginning while propagating in the second layer. Because of the majority belief formation rule, it does help the majority belief to propagate on the multiplex networks. Hence, the minorities always need a larger initial fraction (thus larger critical threshold) to form a giant cluster in the steady state.

### 3.3. Multi-scale ABP model on multiplex networks with different timescales

In order to study the different belief propagation speeds in the two layers, we perform simulations of the multi-scale ABP model on multiplex networks with the same degree correlation for different $m$ and $n$. Here, we fix the parameter $\gamma = 0.1$. Figures 3(a) and (b) show $\rho_1$ and $\rho_2$ of the first layer of ER–ER networks when $n = 1$ for
Figure 4. $\rho_1$ and $\rho_2$ as a function of $\delta$ in ER–ER networks with $n = 1$ of the multi-scale ABP model for different $m$.

Figure 5. $\rho_1$ and $\rho_2$ as a function of $\delta$ of the multi-scale ABP model on ER–ER networks with $m = 1$ for different $n$. 

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different $m$, $m$ and $n$ represent the time steps of the ABP model updates on the first layer and the second layer respectively. With the same $n$, the larger the $m$ is, the more rapidly the belief propagates in the first layer. We find that with a fixed $n$, the different values of $m$ influence the critical threshold $\delta_c$ of the multiplex networks very little. This is probably because after one time step of the ABP dynamics in one network, the size of the minority belief cluster is almost the same as that at the steady state, i.e. whatever $m$ is, the fraction of the minority belief is almost the same, thus the initial fraction of the second layer is the same.

However figures 5(a) and (b) exhibit a different phenomenon that with an increasing $n$, the critical threshold shifts to the left, which means that the more steps of the ABP dynamics on the second layer, the more easily the minorities could form a giant component at steady state in the first layer. The different influences of the time steps $m$ and $n$ on the belief dynamics may be understood by the fact that the largest cluster $\rho_1$ we observe is in the first layer of a multiplex networks. That is, the time steps of the ABP dynamics on the first layer and on the second layer show different impact on the critical threshold of the first layer.

3.4. Multi-scale ABP model on multiplex networks with different resolution parameter $\lambda$

It is observed that the label propagation algorithm just described tends to produce one or two giant clusters containing the majority of nodes. The presence of this giant component (largest cluster of opinions) is due to the very topology of social networks. To try to overcome this problem, we introduce the resolution parameter $\lambda$ in algorithm 1. The motivation behind is that it tends to produce clusters of opinions or beliefs with sizes that follow a heavy-tailed decreasing distribution, yielding both a huge number of small clusters with most number of nodes.

Figure 6 show $\rho_1$ and $\rho_2$, i.e. the size of the largest and second largest cluster of the positive beliefs, as a function of $\delta$ in the first and second layer of ER–ER multiplex networks, where the degree correlation between the two layers $\omega = 0.3$. Specially, in figures 6(a) and (c), one can found that $\rho_1$ in the first and second layer is smaller when $\lambda$ is large, which agree with the motivation behind the algorithm 1. In figures 6(a) and (c), we observe that the critical threshold $\delta_c$ of the multiplex networks is larger when $\lambda$ is smaller. Actually, we can consider $\lambda$ as a probability that introduces a random reassignment of the beliefs in the multiplex network, rather
than put them together in a cluster. However, the reassignment more likely leads the minorities, which may be in the minority belief cluster already, to a local majority neighborhood. Hence, the introduction of resolution parameter $\lambda$, in effect, helps the majority to devour the minority.

4. Conclusions

Because of the ubiquity of the asynchronous steady state in real-world belief competitions and the demand of the multiplex networks to characterize real-world systems, we propose a new multi-scale ABP model to study the dynamics of the ABP models on the multiplex networks. We consider a multiplex network with two layers, in which each layer is described by either a SF or an ER network. We investigate the influence of changing the degree correlation between the two layers of subnetworks on the belief propagation dynamics. In each full iteration of our multi-scale model, the ABP dynamics is first updated at the first layer for $m$ steps after which it is further updated at the second layer for $n$ steps. Extensive computer simulations show that enhancing the degree correlation between the two layers of the multiplex network can increase the critical threshold $\delta_c$. As such, which means when the multiplex network becomes more correlated it is harder for the minority belief to form a cluster. We also find that the time step $m$ slightly influences the critical threshold $\delta_c$ of the first layer, while $n$ has a stronger influence on that.

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