The van Hiele geometry thinking levels: gender and school type differences

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Abstract

This research shows that it would be very appropriate to divide the teaching of geometry into different levels according to van Hiele theory. Secondary schools, with regard to their specialization, should determine what levels they want to achieve, and adapt teaching geometry to that goal. Our research also shows that Van Hiele levels are equally suitable for both genders.

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Selection and peer-review under responsibility of Cognitive-counselling, research and conference services (c-crcs).

Keywords: van Hiele levels; geometry; mathematics; math education

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1. Problem Statement

Van Hiele [1957] described the model of geometric thinking using three aspects: the existence of levels, properties of the levels and the movement from one level to the next level. According to van Hiele, there are five levels of geometric thinking which are labeled: Level 1 recognition (visualization), Level 2 analysis, Level 3 order (abstraction), Level 4 deduction and Level 5 rigor. Usiskin [1982] tested the ability of the van Hiele model to describe and predict the performance of students in secondary school geometry. His test clearly distributed students according to the level they achieved, except Level 5. Currently there are two basic lines of research based on the van Hiele theory in the world: one transferring the van Hiele theory to other areas of mathematics (Boolean Algebra, Function – Analysis – Calculus), and another one using dynamic geometry SW to achieve higher van Hiele levels – see De Villiers.

1.1. Research questions

Two key questions were asked in this part of study – the existence of differences between males and females and among three types of secondary schools.

1.2. Purpose of Study

The main goal of the whole study was to verify the validity of the Usiskins - van Hiele geometry test in the Czech educational system, according to the achieved level. Obtained data can be further statistically processed from another point of view. The results of the study could be used to modify the method of teaching and evaluating geometry at the Czech secondary schools.

2. Research Methods

Basic method: Van Hiele geometry test adapted by Usiskin (except Level 5) and translated into the Czech language. 5 questions are used for each of the four levels (Level 1...Level 4) of geometric thinking. A level is achieved if and only if 3 or more than 3 out of 5 questions are answered correctly.

Methods of data analysis: descriptive statistics, comparing binomial distributions, effect-size value.

Research sample: N=215, gender 111M, 104F, age 15-17 years, schools: G - secondary general school (“grammar school / gymnasium”) 112, T - secondary technical school 55, B - secondary business school 48.

3. Findings

There are results of level N in binary variable - 0 means that a level was not achieved, 1 means that a level was achieved. In general, 96.7% of students achieved Level 1, 86.5% achieved Level 2, 39.1% achieved Level 3 and 8.8% achieved Level 4. The table shows percentage of students who achieved different levels and confidence interval for binomial distribution (left, right) at 0.05 significant level. Results:

| Level   | Percentage | Left | Right |
|---------|------------|------|-------|
| Level 1 | 0.97       | 0.94 | 0.99  |
| Level 2 | 0.87       | 0.82 | 0.91  |
| Level 3 | 0.39       | 0.33 | 0.46  |
| Level 4 | 0.09       | 0.05 | 0.13  |
There is no intersection of confidence intervals, so these levels differentiated population into 5 different groups (no level, Level 1, …, Level 4). This part of analysis is more precisely described at conference The Learner 2013, Rhodes (Haviger, Vojkůvková)

3.1. Gender point of view:

An analysis of gender differences was carried out by comparing two proportions (two binomial distributions) and effect size as an absolute value of difference between percentages. Results:

Table 2. Gender

| Level  | Perc F | Perc M | Sig | Effect size |
|--------|--------|--------|-----|-------------|
| Level 1 | 0.946  | 0.990  | 0.030 | 0.044       |
| Level 2 | 0.838  | 0.894  | 0.113 | 0.056       |
| Level 3 | 0.306  | 0.481  | 0.004 | 0.174       |
| Level 4 | 0.054  | 0.125  | 0.033 | 0.071       |

There are statistically significant differences between male and female in levels 1, 3 and 4, but effect size is bigger than 10% only on Level 3.

Graphical representation of the results:

![Graph](image)

Fig. 1. Comparison of genders

3.2. Type of school point of view:

An analysis of differences among school types was created between each pair in the same way as with gender.
Comparison of general school (“gymnasium”) and business school results:

Table 3. General school (G) vs business school (B)

| Level  | Perc G | Perc B | Sig.  | Effect size |
|--------|--------|--------|-------|-------------|
| Level 1| 0.982  | 0.896  | 0.007 | 0.086       |
| Level 2| 0.920  | 0.625  | 0.000 | 0.294       |
| Level 3| 0.536  | 0.167  | 0.000 | 0.369       |
| Level 4| 0.125  | 0.063  | 0.119 | 0.062       |

There are statistically significant differences between general school (“gymnasium”) and business school in levels 1, 2 and 3. Effect size is bigger than 10% only on levels 2 and 3.

Comparison of general school (“gymnasium”) and technical school results:

Table 4. General school (G) vs technical school (T)

| Level  | Perc G | Perc T | Sig.  | Effect size |
|--------|--------|--------|-------|-------------|
| Level 1| 0.982  | 1.000  | 0.159 | 0.017       |
| Level 2| 0.920  | 0.964  | 0.140 | 0.044       |
| Level 3| 0.536  | 0.291  | 0.001 | 0.244       |
| Level 4| 0.125  | 0.036  | 0.033 | 0.088       |

There are statistically significant differences between general school (“gymnasium”) and technical school in levels 3 and 4 only. Effect size is bigger than 10% only on level 3.

Comparison of business school and technical school:

Table 5. Business school (B) vs technical school (T)

| Level  | Perc B | Perc T | Sig.  | Effect size |
|--------|--------|--------|-------|-------------|
| Level 1| 0.896  | 1.000  | 0.007 | 0.104       |
| Level 2| 0.625  | 0.964  | 0.000 | 0.338       |
| Level 3| 0.167  | 0.291  | 0.068 | 0.124       |
| Level 4| 0.063  | 0.036  | 0.269 | 0.026       |

There are statistically significant differences between business school and technical school in 1 and 2 levels only. Effect size is bigger than 10% on levels 1, 2 and 3.

Graphical representation of the results:
4. Conclusions

There are statistically significant differences between genders, but effect sizes of differences are bigger than 10% for level 3 only.

Bigger differences can be seen among three different types of schools. Low results of business school students are given by the school’s focus – more algebra than geometry is taught there. Excellent results at Level 1 and Level 2 of technical school indicate that its students have mastered applied geometry very well but they are not interested in theoretical concepts. Best results in Levels 3 and 4 were obtained from general schools (“gymnasium”). This type of school in the Czech Republic traditionally prepares the students for university studies.

Summary: Approximately the same framework of teaching geometry is applied at all types of schools in the Czech Republic. This research shows that it would be very appropriate to divide the teaching of geometry into different levels according to van Hiele. Secondary schools, with regard to their specialization, should determine what levels they want to achieve, and adapt teaching geometry to that goal. Also, our research shows that Van Hiele levels may be used for both genders equally.

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