$B$ DECAYS TO CHARMLESS $VP$ FINAL STATES

Amol S. Dighe
Enrico Fermi Institute and Department of Physics, University of Chicago
5640 S. Ellis Avenue, Chicago IL 60615

and

Michael Gronau
Theoretical Physics Division, CERN
CH-1211 Geneva 23, Switzerland

and

Jonathan L. Rosner
Enrico Fermi Institute and Department of Physics, University of Chicago
5640 S. Ellis Avenue, Chicago IL 60615

ABSTRACT

The CLEO Collaboration has now observed the decays $B^+ \to \omega \pi^+$ and $B \to \omega K^+$ with branching ratios of $(1.1^{+0.6}_{-0.5} \pm 0.2) \times 10^{-5}$ and $(1.5^{+0.7}_{-0.6} \pm 0.3) \times 10^{-5}$, respectively. These are the first reported decays to charmless final states involving a vector ($V$) and a pseudoscalar ($P$) meson. The implications of these decays for others of $B$ mesons to charmless $VP$ final states are explored. In a model-independent approach, using only flavor SU(3) symmetry, several tests are proposed for an anticipated hierarchy among different contributions to decay amplitudes.

PACS numbers: 13.25.Hw, 14.40.Nd, 11.30.Er, 12.15.Ji
I. INTRODUCTION

The CLEO Collaboration [1] has now observed the decays $B^+ \to \omega \pi^+$ and $B \to \omega K^+$ with branching ratios of $(1.1^{+0.6}_{-0.5} \pm 0.2) \times 10^{-5}$ $(2.9\sigma)$ and $(1.5^{+0.7}_{-0.6} \pm 0.3) \times 10^{-5}$ $(4.3\sigma)$, respectively. These are the first reported decays of $B$ mesons to charmless final states involving a vector ($V$) and a pseudoscalar ($P$) meson. $VP$ final states may be crucial in studies of CP violation in $B$ decays [2].

We have previously applied flavor SU(3) symmetry [3, 4, 5] to decays of the form $B \to PP$ [6, 7, 8, 9, 10, 11, 12, 13], and made some preliminary remarks about $VP$ decays in Refs. [8] and [13]. In the latter paper relations are defined between SU(3) amplitudes and quark diagrams for $VP$ decays. The observation of the $\omega \pi^+$ and $\omega K^+$ modes, and the existence of limits on other $VP$ modes at levels close to those expected [14], make an updated analysis relevant at this time. The $2.9\sigma$ level of the $\omega \pi^+$ signal requires that we regard it as preliminary.

We decompose amplitudes for $B \to VP$ decays into linear combinations of reduced matrix elements in Section II. Applications of the relations implied by these decompositions, suggesting a variety of tests for an anticipated hierarchy among different contributions, are discussed in Section III. Our results are compared with attempts to calculate decay modes \textit{a priori} with the help of specific models in Section IV. We conclude (with a brief summary of experimental prospects) in Section V.

II. SU(3) DECOMPOSITION

In Tables 1 and 2 we list the $VP$ modes of nonstrange $B$ mesons for strangeness-preserving and strangeness-changing decays, respectively. Our notation is as follows:

1. As a language equivalent to flavor SU(3), we employ an overcomplete set of quark diagrams [5], which we denote by $T$ (tree), $C$ (color-suppressed), $P$ (QCD-penguin), $S$ (additional penguin contribution involving flavor-SU(3)-singlet mesons, called $P_1$ in Ref. [14]), $E$ (exchange), $A$ (annihilation) and $PA$ (penguin annihilation). The last three amplitudes, in which the spectator quark enters into the decay Hamiltonian, are expected to be suppressed by $f_B/m_B$ ($f_B \approx 180$ MeV) and may be neglected to a good approximation. The presence of higher-order electroweak penguin contributions [15] introduces no new SU(3) amplitudes, and in terms of quark graphs merely leads to a substitution [8, 10]

$$T \to t \equiv T + P_{EW}^C, \quad C \to c \equiv C + P_{EW},$$

$$P \to p \equiv P - \frac{1}{3}P_{EW}^C, \quad S \to s \equiv S - \frac{1}{3}P_{EW},$$

where $P_{EW}$ and $P_{EW}^C$ are color-favored and color-suppressed electroweak penguin amplitudes.

2. We use the phase conventions of Ref. [3] for pseudoscalar mesons, the mixing assumption $\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$ and $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$, and the corresponding phase conventions for vector mesons with $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\phi = s\bar{s}$.
Table 1: $\Delta S = 0$ $B \to VP$ decays. Coefficients of amplitudes are to be divided by denominator factor.

| Amplitude | Denom. | $t_P$ | $t_V$ | $c_P$ | $c_V$ | $p_P$ | $p_V$ | $s_P$ | $s_V$ |
|-----------|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| $B^+ \to \rho^+ \pi^0$ | $-\sqrt{2}$ | 1 | 1 | 1 | 1 | -1 | |
| $\rho^0\pi^0$ | $-\sqrt{2}$ | 1 | 1 | -1 | 1 | |
| $\omega\pi^+$ | $\sqrt{2}$ | 1 | 1 | 1 | 1 | 2 | |
| $\phi\pi^+$ | 1 | 1 | 1 | 1 | 1 | 1 | |
| $\rho^+\eta$ | $-\sqrt{3}$ | 1 | 1 | 1 | 1 | 1 | |
| $\rho^0\eta'$ | $\sqrt{6}$ | 1 | 1 | 1 | 1 | 4 | |
| $K^{*+}\bar{K^0}$ | 1 | 1 | 1 | |
| $\bar{K}^{*0}K^+$ | 1 | |
| $B^0 \to \rho^- \pi^0$ | -1 | 1 | 1 | |
| $\rho^+\pi^-$ | -1 | 1 | 1 | |
| $\rho^0\pi^0$ | 2 | -1 | -1 | 1 | 1 | 2 | |
| $\omega\pi^0$ | 2 | 1 | -1 | 1 | 1 | 2 | |
| $\phi\pi^0$ | $\sqrt{2}$ | 1 | 1 | 1 | 1 | 1 | |
| $\rho^0\eta$ | $-\sqrt{6}$ | -1 | 1 | 1 | 1 | 1 | |
| $\rho^0\eta'$ | $2\sqrt{3}$ | -1 | 1 | 1 | 1 | 4 | |
| $\omega\eta$ | $-\sqrt{6}$ | 1 | 1 | 1 | 1 | 2 | 1 | |
| $\omega\eta'$ | $2\sqrt{3}$ | 1 | 1 | 1 | 1 | 2 | 4 | |
| $\phi\eta$ | $-\sqrt{3}$ | 1 | 1 | 1 | 1 | 1 | 1 | |
| $\phi\eta'$ | $\sqrt{6}$ | 1 | 1 | 1 | 1 | 1 | 1 | |
| $K^{*0}\bar{K^0}$ | 1 | 1 | |
| $\bar{K}^{*0}K^+$ | 1 | |

3. We denote strangeness-preserving ($\Delta S = 0$) amplitudes by unprimed letters and strangeness-changing ($|\Delta S| = 1$) amplitudes by primed letters.

4. The suffix on each amplitude denotes whether the spectator quark is included in a pseudoscalar ($P$) or vector ($V$) meson.

5. Each decay amplitude involves positive or negative integer coefficients multiplying the indicated reduced amplitudes and divided by a common denominator factor.

III. APPLICATIONS

A. Hierarchies of amplitudes

One can immediately identify certain amplitudes likely to be most important in $B \to VP$ decays. In the corresponding $PP$ decays (which may be denoted by similar amplitudes without the subscripts), the amplitudes are expected to obey an
 approximate hierarchy \[6, 7, 8, 12, 16\]. The process \(B^0 \rightarrow K^+\pi^-\) is observed with a branching ratio somewhat in excess of \(10^{-5}\), while \(B^0 \rightarrow \pi^+\pi^-\) is expected to have a branching ratio not vastly different from this. Thus we deduced in previous work that \(|t| \simeq |p'|\), while \(|p/t| \simeq |t'/p'| \simeq \lambda\), where \(\lambda \equiv V_{us} \simeq 0.22\). We do not have an estimate for \(|c/t|\) or \(|c'/t'|\). We expect \(|c/t|\) to be small on the basis of color-suppression arguments. However, \(|c'/t'|\) may be larger due to the electroweak penguin term in \(c'\) [see Eq. (1)]. The large branching ratios for \(B^+ \rightarrow K^+\eta'\) (about \(7 \times 10^{-5}\)) and \(B^0 \rightarrow K^0\eta'\) (about \(5 \times 10^{-5}\)) \[17\] indicate the importance of the \(s'\) amplitude at a level comparable to that of \(p'\) \[12, 18\].

A similar hierarchy appears to apply to the \(VP\) decays. The fact that the \(B^+ \rightarrow \omega\pi^+\) and \(B^+ \rightarrow \omega K^+\) branching ratios are comparable to one another and each of order \(10^{-5}\) indicates that the dominant contribution to \(\omega\pi^+\) is most likely \(t_V\), while the dominant contribution to \(\omega K^+\) is most likely \(p'\). We expect the \(s'\) contribution to be relatively unimportant; this contribution would involve a coupling of the \(\omega\) and \(\phi\) which violated the Okubo-Iizuka-Zweig (OZI) rule favoring connected quark diagrams. Such couplings are probably much more important for \(\eta\) and \(\eta'\) than for vector mesons. Specifically, the penguin amplitude \(s'_V\), coupling to the flavor SU(3) singlet component of the \(\eta\) and and \(\eta'\), can be as large as or even larger than \(p'\). A similar situation seems to hold in decays to two light pseudoscalar mesons \[12\].
B. Tests for smallness of amplitudes

How can one learn more about which $VP$ amplitudes are important? One way of using the tables is to compare charged $B$ decays and neutral $B$ decays to each other. This can teach us something about the magnitudes of some of the amplitudes. Consider, for instance, the eight pairs of $|\Delta S| = 1$ processes listed in Table 2. The following approximate amplitude equalities test the smallness of certain contributions. The relations between $B^+$ and $B^0$ amplitudes are independent of SU(3) breaking. In each case we list only the final state.

1. Smallness of $c'_{PV}$:

$$
\rho^+ K^0 \approx \sqrt{2}(\rho^0 K^0), \quad \sqrt{2}(\rho^0 K^+) \approx \rho^- K^+,
K^{*0} \pi^+ \approx \sqrt{2}(K^{*0} \pi^0), \quad \sqrt{2}(K^{*+} \pi^0) \approx K^{*+} \pi^- .
$$

The $c'_{PV}$ amplitudes contain color-favored electroweak penguin terms which may not be negligible (see Eq. (1) and Ref. [15]), and indeed provide important contributions in a priori calculations to be discussed in Sec. IV.

2. Smallness of $t'_{PV}$:

$$
\omega K^+ \approx \omega K^0 , \quad K^{*+} \eta \approx K^{*0} \eta , \quad K^{*+} \eta' \approx K^{*0} \eta' .
$$

3. Smallness of $s'_{P}$:

$$
-\rho^0 K^+ \approx \omega K^+ , \quad K^{*0} \pi^+ \approx \phi K^+ .
$$

The first relation is sensitive to any breakdown of nonet symmetry (unequal decay constants for $\rho$ and $\omega$ mesons). The second relation is sensitive to SU(3)-breaking effects since it involves comparing an amplitude with nonstrange quark pair production to one with strange quark pair production; the form factors are also likely to differ [13].

In addition, a number of approximate triangle relations hold, such as

$$
\sqrt{2}(\rho^+ K^0) \approx \rho^0 K^0 + \omega K^0 ,
$$

all of whose sides have a $p'_{V}$ contribution, so the decay rates may be significant. The shape of the amplitude triangle may tell us about the relative magnitudes and phases of $c'_{P}$ and $p'_{V}$. Since we expect $c'_{P}$ to be smaller than $p'_{V}$, this triangle will be a “squashed” one.

4. The last relation among the eight pairs of amplitudes, $\phi K^+ = \phi K^0$, is exact and follows from isospin.
Assuming that both \( s' \) and \( c' \) are small, one also finds \( \rho^0 K^0 \approx \omega K^0 \). Lipkin [18] has pointed out that if \( s' \) is small [as checked, for example, by relations such as Eq. (4)], but if the \( B^0 \to \rho^0 K^0 \) and \( B^0 \to \omega K^0 \) rates are unequal, then both \( p' \) and \( c' \) amplitudes must be present, and they are close enough in amplitude for interference to be observed. Whether this enhances the possibility of observing direct CP violation [18] remains an open question. The most likely source of a contribution to \( c' \) is an electroweak penguin amplitude with the same weak phase as \( p' \), so a direct CP asymmetry is unlikely in the neutral decays. In the more readily observed charged decays \( B^+ \to (\rho^0, \omega) K^+ \), one would need interference between \( t' \) and \( t'' \) to see a CP asymmetry. In our approach one cannot infer anything about \( \Delta S = 0 \) amplitudes do not exhibit simple isospin relations which test the smallness of some amplitudes. Still, one can make the following observations about large and small amplitudes:

1. The largest amplitudes are expected to be \( t_P \), \( t_V \). Therefore, the following 7 processes are expected to have the largest rates: \( \rho^+ \pi^0 \), \( \rho^0 \pi^+ \), \( \omega \pi^+ \), \( \rho^+ \eta \), \( \rho^+ \eta' \), \( \rho^- \pi^+ \), and \( \rho^+ \pi^- \).

2. Smaller decay rates (equal in \( B^+ \) and \( B^0 \) decays) measure different kinds of penguin amplitudes:
   - \( p_P \) is measured in \( \bar{K}^* K^+ \approx \bar{K}^* K^0 \);
   - \( p_V \) is measured in \( K^{*+} \bar{K}^0 \approx K^{*0} \bar{K}^0 \);
   - \( s_P \) is measured in \( \phi \pi^+ = \sqrt{2}(\phi \pi^0) = -\sqrt{3}(\phi \eta) = \sqrt{6}(\phi \eta') \);
   - \( s_V \) is measured by the combinations \( \sqrt{6}(\rho^+ \eta') + \sqrt{3}(\rho^0 \eta) \), \( 2\sqrt{3}(\rho^0 \eta') + \sqrt{6}(\rho^0 \eta) \), and \( 2\sqrt{3}(\omega \eta') + \sqrt{6}(\omega \eta) \), implying rate relations if \( s_V \) is small.

C. Relations testing for presence or absence of \( s'_V \)

Whereas the amplitude \( p'_V \) can be measured directly in \( B^+ \to \rho^+ K^0 \), it is much more difficult to determine the amplitude \( s'_V \) contributing only to decays involving the \( \eta \) and \( \eta' \). We recall that the presence of the corresponding sizable amplitude \( s \) in \( B \to PP \) decays was manifested by the particularly large \( B \to K \eta' \) decay rate [12].

Several tests for the presence of the singlet amplitude \( s'_V \) can be constructed. \( s'_V \) is less likely to be small than \( s'_P \), since the axial anomaly can affect \( \eta \) and \( \eta' \) flavor-singlet couplings [19]. A large corresponding singlet amplitude in \( |\Delta S| = 1 \) \( B \to PP \) decays was found to be needed to explain the observed \( B \to K \eta' \) rate [12].

In the event that \( s'_V \) is negligible, several triangle relations hold among the amplitudes for \( B^+ \to \rho^0 K^0 \) and \( B^+ \to K^{*+}(\pi^0, \eta, \eta') \), and among the amplitudes for \( B^0 \to K^{*0}(\eta, \eta') \) and \( B^0 \to \rho^+ K^0 \). If we are willing, moreover, to neglect \( c'_V \) in comparison with other amplitudes, a set of triangle relations analogous to the above, but with the decays \( B^0 \to K^{*0}(\eta, \eta') \) replacing \( B^+ \to K^{*+}(\eta, \eta') \) and with \( K^{*0} \pi^+ \) replacing \( -\sqrt{2}(K^{*+} \pi^0) \), should hold.
Under some circumstances, interference terms between contributions of different amplitudes to rates will cancel when suitable sums of rates are constructed. Thus, one finds that if \( s'_V \) can be neglected, the triangle relations mentioned above imply

\[
\mathcal{B}(B^+ \to K^{*+}\pi^0) + \mathcal{B}(B^+ \to \rho^+K^0) = \mathcal{B}(B^+ \to K^{*+}\eta) + \mathcal{B}(B^+ \to K^{*+}\eta') \quad . \quad (6)
\]

Similarly, with \( c'_V \) small in addition, one finds

\[
\mathcal{B}(B^+ \to K^{*0}\pi^+) / 2 + \mathcal{B}(B^+ \to \rho^+K^0) = \mathcal{B}(B^0 \to K^{*0}\eta) + \mathcal{B}(B^0 \to K^{*0}\eta') \quad . \quad (7)
\]

(Here and later we neglect phase space effects.) Both sides of these two relations contain contributions from the \( p'_V \) amplitude, which we expect to be significant. The failure of either of the above two equations to hold would indicate a significant \( s'_V \) contribution. In that case, we may proceed to determine \( s'_V \) as follows.

When the amplitude \( s'_V \) is not neglected, the triangle relations for \( B^+ \) decays discussed above are replaced by

\[
\begin{aligned}
-\sqrt{2}(K^{*+}\pi^0) - (\rho^+K^0) + s'_V &= -\sqrt{3}(K^{*+}\eta) \quad , \\
-\sqrt{2}(K^{*+}\pi^0) + 2(\rho^+K^0) + 4s'_V &= \sqrt{6}(K^{*+}\eta') \quad .
\end{aligned}
\]

Moreover, if \( c'_V \) can be neglected, one can write

\[
\begin{aligned}
(K^{*0}\pi^+ - (\rho^+K^0) + s'_V &= -\sqrt{3}(K^{*0}\eta) \quad , \\
(K^{*0}\pi^+) + 2(\rho^+K^0) + 4s'_V &= \sqrt{6}(K^{*0}\eta') \quad .
\end{aligned}
\]

In the complex plane, let \( (\rho^+K^0) = (1,0) \) [so that all the amplitudes and phases in these relations are measured in units of \( (\rho^+K^0) \)]. Since only \( p'_V \) contributes to \( (\rho^+K^0) \), all the amplitudes will henceforth be given in terms of \( p'_V \) as the unit.

Let \(-\sqrt{2}(K^{*+}\pi^0) = (a, b), (K^{*0}\pi^+) = (c, d), \) and \( s'_V = (e, f) \). Then, squaring the above four equations to obtain four rate relations, and constraining \( a^2 + b^2 \) and \( c^2 + d^2 \) using the rates for the decays \( B^+ \to K^{*+}\pi^0 \) and \( B^+ \to K^{*0}\pi^+ \), we have six equations in the six unknowns \( a, b, c, d, e, f \). One is required to measure seven different decay rates, but all of them involve \( p'_V \), so none of them should be very small.

**D. Where does the spectator quark end up?**

The distinction between amplitudes \( a_P \) or \( a'_{P'} \) (\( a \equiv t, c, p, s \)) (in which the spectator quark is incorporated into a pseudoscalar meson) and \( a_V \) or \( a'_{V'} \) (in which the spectator ends up in a vector meson) is responsible for the large number of reduced amplitudes in the \( VP \) case, as compared to the simpler \( PP \) decays. Some hint that these amplitudes may not have equal magnitudes is provided by the upper bound \[ \mathcal{B}(B^+ \to \phi K^+) < 0.53 \times 10^{-5}, \] as compared with \( \mathcal{B}(B^+ \to \omega K^+) = (1.5^{+0.7}_{-0.6} \pm 0.3) \times 10^{-5}, \) implying \( \mathcal{B}(B^+ \to \phi K^+)/\mathcal{B}(B^+ \to \omega K^+) < 1. \) The \( \phi K^+ \) amplitude is dominantly \( p'_P \), while the \( \omega K^+ \) amplitude is mainly \( p'_V/\sqrt{2} \). If the \( p'_P \) and \( p'_V \) amplitudes were equal, we should have expected \( \mathcal{B}(B^+ \to \phi K^+) = 2\mathcal{B}(B^+ \to \omega K^+) \). The amplitude \( p'_V \) can be measured directly in \( B^+ \to \rho^+K^0 \) and, neglecting \( c'_P \), in
Table 3: Some $B$ decay modes capable of distinguishing between $a_P$-type and $a_V$-type amplitudes.

| Decay mode            | Dominant amplitude | Signal events | Expected background | $\mathcal{B}$ upper limit $\times 10^{-6}$ |
|-----------------------|--------------------|---------------|---------------------|------------------------------------------|
| $B^+ \to \rho^+\pi^0$ | $t_P$              | 8             | $5.5 \pm 1.2$       | 77                                       |
| $\rho^0\pi^+$         | $t_V$              | 4             | $2.3 \pm 0.3$       | 43                                       |
| $\omega\pi^+$         | $t_V$              | 8.8           |                     | (a)                                      |
| $\phi\pi^+$           | $s_P$              | 0             |                     | 5.6                                      |
| $\rho^+K^0$           | $p'_V$             | 0             |                     | 48                                       |
| $\rho^0K^+$           | $p'_V$             | 1             | $3.8 \pm 0.2$       | 19                                       |
| $\omega K^+$          | $p'_V$             | 12            |                     | (a)                                      |
| $\phi K^+$            | $p'_P$             | 0             |                     | 5.3                                      |
| $K^{*0}\pi^+$         | $p'_P$             | 2             | $1.0 \pm 0.6$       | 41                                       |
| $K^{*+}\pi^0$         | $p'_P$             | 4             | $1.9 \pm 0.7$       | 99                                       |

| $B^0 \to \rho^-\pi^+$ | $t_V$              | (b)           |                     | (c)                                      |
| $\rho^+\pi^-$         | $t_P$              | (b)           |                     | (c)                                      |
| $\phi\pi^0$           | $s_P$              | 0             |                     | 6.5                                      |
| $\rho^-K^+$           | $p'_V$             | 2             | $2.0 \pm 0.4$       | 35                                       |
| $\rho^0K^0$           | $p'_V$             | 0             |                     | 39                                       |
| $\phi K^0$            | $p'_P$             | 2             |                     | 42                                       |
| $K^{*+}\pi^-$         | $p'_P$             | 3             | $0.7 \pm 0.2$       | 72                                       |
| $K^{*0}\pi^0$         | $p'_P$             | 0             | $1.1 \pm 0.3$       | 28                                       |

(a) Signal observed (see text).
(b) Sum of channels has 7 events above expected background of $2.9 \pm 0.7$.
(c) Upper limit on average branching ratio is $88 \times 10^{-6}$.

$B^0 \to \rho^0K^0$. To confirm that indeed $|p'_V| > |p'_P|$, it would be useful to compare $p'_V$ measured in this way with $p'_P$ measured in $B^+ \to K^{*0}\pi^+$.

The assumption of equal and opposite $p'_P$ and $p'_V$ amplitudes lies behind a prediction by Lipkin [18] that one expects constructive interference between the nonstrange and strange components of the $\eta$, and destructive interference in the $\eta'$, for the decays $B^+ \to K^{*+}(\eta, \eta')$. This is valid if the penguin transition $\bar{b} \to \bar{s}$ leads to an intermediate $\bar{s}u$ final state accompanied by any number of gluons, as long as there is not some fundamental asymmetry in the wave function between the $\bar{s}$ and the $u$. If the final light $q\bar{q}$ pair is then produced in a flavor-SU(3) invariant manner, the $p'_P$ and $p'_V$ amplitudes will be equal and opposite. (Gluons must be present; otherwise one could rotate away any $\bar{b} \to \bar{s}$ transition by a redefinition of quark fields [20].)

The full generality of Lipkin’s argument is less obvious. The transition $\bar{b} \to \bar{s}+$ (meson) has different Lorentz structure when the meson is a pseudoscalar (our $p'_V$ amplitude) than when it is a vector (our $p'_P$ amplitude). The $p'_V$ and $p'_P$ amplitudes indeed fail to be equal and opposite in several explicit calculations to be discussed in Sec. IV.
One might ask whether there is any evidence so far for amplitudes of the $a_P^{(\prime)}$ type. In Table 3 we collect a number of decay modes which can shed light on this question. We list the mode, the amplitude expected to dominate, and the number of signal events, expected backgrounds, and upper limits on the branching ratios in units of $10^{-6}$ reported by CLEO II [14]. We do not list the coefficients of the dominant amplitudes, which may be found in Tables 1 and 2.

None of the upper limits shown in Table 3 conflicts with the expectation that the $t_V$ amplitude has already been seen in $B^+ \to \omega \pi^+$ and the $p_V^{(')}$ amplitude has already been seen in $B^+ \to \omega K^+$. If these are the dominant amplitudes one expects

$$B(B^0 \to \rho^- \pi^+) = 2B(B^+ \to \rho^0 \pi^+) = 2B(B^+ \to \omega \pi^+)$$  \hspace{1cm} (12)$$

and

$$B(B^+ \to \rho^+ K^0) = B(B^0 \to \rho^- K^+)$$

$$= 2B(B^+ \to \rho^0 K^+) = 2B(B^0 \to \rho^0 K^0) = 2B(B^+ \to \omega K^+) \hspace{1cm} . \hspace{1cm} (13)$$

The case of $B^0 \to \rho^\pm \pi^\mp$ is particularly interesting since the excess of signal over expected background is the largest of any in Table 3, but the process requires flavor tagging in order to separate the $t_V$-dominated decay $B^0 \to \rho^- \pi^+$ from the $t_P$-dominated decay $B^0 \to \rho^+ \pi^-$. Interesting statements about the relative magnitudes of amplitudes will require about a factor of 3 more data than those on which Table 3 was based.

### E. Relations between $\Delta S = 0$ and $|\Delta S| = 1$ decays

Comparison of decay rates between $\Delta S = 0$ and $|\Delta S| = 1$ processes can help in determining ratios of magnitudes of CKM elements. For example, one expects

$$\frac{B(B^+ \to K^{*-0} \pi^+)}{B(B^+ \to K^{*+0} K^+)} = \frac{B(B^+ \to \rho^+ K^0)}{B(B^+ \to K^+ \pi^0)} = \frac{|V_{ts}|}{|V_{td}|}^2 , \hspace{1cm} (14)$$

assuming the top quark dominates the penguin amplitudes in these sets of processes. In certain ratios SU(3) breaking form factor effects cancel out. (See Ref. [13] for a more complete discussion).

Once the dominant amplitudes (such as $t_V$ and $p_V^{(')}$, for which we already have evidence) have been mapped out, one can use flavor SU(3) to anticipate the smaller amplitudes (like $t_V^{(')}$ and $p_V$). Taking account of SU(3) breaking, we have

$$t_V^{(')}/t_V = p_P^{(')} = \lambda (f_K/f_{\pi}) . \hspace{1cm} \hspace{1cm} (15)$$

For penguin amplitudes in the flavor-SU(3) limit,

$$p_V/p_V^{(')} = s_P/s_P^{(')} = s_V/s_V^{(')} = s_P/s_P^{(')} = V_{td}/V_{ts} \hspace{1cm} . \hspace{1cm} (16)$$

(assuming top-quark dominance of the penguin amplitudes). One can then, in the manner of Ref. [12], search for processes in which two amplitudes with two different weak phases contribute with comparable strength. If these amplitudes have different
strong phases as well, there is a possibility of a large CP asymmetry in comparing
the rate for a process with its charge conjugate.

IV. PREDICTIONS OF SPECIFIC MODELS

Calculations of $B \rightarrow (PP, VP, VV)$ decay rates over the past ten years, involving
assumptions about factorization and using specific $B$-to-light-meson form factors,
include those of Refs. [1, 21, 22, 23, 24, 25, 26, 27]. For the most part, these works have successfully anticipated those $B \rightarrow PP$ decays observed at a branching ratio
level of $10^{-5}$ or greater, such as $B^0 \rightarrow K^+\pi^-$. The decay $B^0 \rightarrow \pi^+\pi^-$ is expected to
correspond to a branching ratio not much below $10^{-5}$.

A common feature of all the calculations is their expectation that $B(B^+ \rightarrow \omega K^+)/B(B^+ \rightarrow \phi K^+) \ll 1$, in disagreement with the CLEO results [1]. In our
language, these calculations predict $|p'_\omega/p'_\phi| \ll 1$; in the work of Ref. [22], the contribution of $p'_\omega$ is vanishingly small in certain decays. This is a result of the specifically
chosen form factors used to calculate hadronic matrix elements of penguin operators.

The authors of Ref. [27] propose a new source of penguin terms, associated with
charmed quarks in the loop rather than top quarks as is conventionally assumed (see
also Ref. [28]). Their prediction of the $B^+ \rightarrow \omega K^+ / \phi K^+$ ratio of rates nonetheless
remains too small, with $B(B^+ \rightarrow \omega K^+)/B(B^+ \rightarrow \phi K^+) \approx 1/6$ in contradiction
to experiment. This assumption implies weak phases of $\text{Arg} (V_{cb}^* V_{cd}) = \pi$ and $\text{Arg}
(V_{ts}^* V_{cs}) = 0$ for $\Delta S = 0$ and $|\Delta S| = 1$ penguin amplitudes, respectively. If the top
quark were dominant instead, one would have weak phases of $\text{Arg} (V_{cb}^* V_{cd}) = -\beta$ and
$\text{Arg} (V_{ts}^* V_{cs}) = \pi$. One cannot distinguish between a weak phase of $\pi$ and one of zero
in the $|\Delta S| = 1$ transitions, but it should be possible to tell the difference between
the weak phase of zero and $-\beta$ for the $\Delta S = 0$ amplitudes. That distinction lies
beyond the scope of the present work.

The relative signs of contributions from $p'_p$ and $p'_\omega$ in the model of Ref. [1] are
such that one expects $B(B \rightarrow K^*\eta) > B(B \rightarrow K^*\eta')$ (see also Ref. [15]). For the
corresponding decays with $K$ replacing $K^*$, the prediction [1, 18] is $B(B \rightarrow K\eta) <
B(B \rightarrow K\eta')$, as observed for charged $B$’s [17].

The authors of Ref. [1] do not include the $s'_t$ terms, which may be important.
For the corresponding $PP$ decays, numerous authors [12, 13, 19] have noted that the
singlet amplitude $s'$ is required to understand the large $B \rightarrow K\eta'$ rate. There is some
question as to the origin of this singlet amplitude. It most likely originates as a result of
the gluonic anomaly in the axial U(1) current, but may be manifested in various
ways, e.g., through an admixture of $c\bar{c}$ pairs or gluon pairs in the $\eta'$ (and, to a lesser extent, $\eta$) wave function. The absence of a meaningful constraint on the strange-
quark content of the $\eta'$ (achievable by measuring the rate for $\phi \rightarrow \eta'\gamma$) [29]
leaves some room for such admixtures. A direct coupling of singlet pseudoscalar mesons
through penguin-type diagrams introduced in Ref. [1] is another possibility.

The models do seem to predict roughly the right magnitude for $t_\omega$, which we expect
to dominate the observed decay $B^+ \rightarrow \omega\pi^+$. An expectation of Ref. [1] common to
other models is that $|t_p| > |t_\omega|$. In that case, one should expect $B(B^+ \rightarrow \rho^+\pi^0) >
B(B^+ \rightarrow \rho^0\pi^+)$ and $B(B^0 \rightarrow \rho^+\pi^-) > B(B^0 \rightarrow \rho^-\pi^+)$. As one sees in Table 3, there
Table 4: Branching ratios for penguin-dominated $B$ decays in two models, in units of $10^{-6}$.

| Decay                  | Chau et al. | Ciuchini et al. |
|------------------------|-------------|-----------------|
| $B^+ \to \rho^0 K^+$   | 0.6         | 7               |
| $B^+ \to \omega K^+$   | 1.4         | 5.4             |
| $B^+ \to K^*\pi^+$     | 8.8         | 52              |
| $B^+ \to \phi K^+$     | 14          | 33              |

is no evidence yet for or against this hierarchy.

We find that many of our Eqs. (2)–(4) are not satisfied by the models of Chau et al. (Ref. [5]) and Ciuchini et al. (Ref. [27]). The sources of the violations are of some interest.

In Ref. [5], the annihilation amplitude seems to have a large effect. We have noted that the $\phi K^+$ and $\phi K^0$ amplitudes should be equal by isospin when we neglect annihilation. Chau et al. find (including annihilation) that the branching ratios of these processes are $14 \times 10^{-6}$ and $9 \times 10^{-6}$, respectively. This means that the ratio of the annihilation amplitude to $p'_P$ must be at least 0.25 (which is surprisingly large) if $p'_P$ and annihilation interfere constructively in $\phi K^+$. Otherwise, annihilation is even larger. Note that in the calculation of Ref. [5] (as in the others) $p'_P$ (rather than $p'_V$) is the dominant amplitude in $|\Delta S| = 1$ decays. Thus, the effects of annihilation in other processes (dominated by the smaller $p'_V$) are even larger. For instance, these authors find $\mathcal{B}(B^+ \to \rho^0 K^0) = 0.34 \times 10^{-6}$ and $2\mathcal{B}(B^0 \to \rho^0 K^0) = 0.70 \times 10^{-6}$, while these numbers should be approximately equal by our Eq. (2) if $c'_P$ is negligible.

In the calculation by Ciuchini et al. the last two rates come out to be quite different as a result of a significant electroweak penguin contribution to the $c'_P$ amplitude. The authors find $\mathcal{B}(B^+ \to \rho^+ K^0) = 30 \times 10^{-6}$ and $2\mathcal{B}(B^0 \to \rho^0 K^0) = 10 \times 10^{-6}$. The one-to-two orders of magnitude enhancement relative to the results of Ref. [5] comes from the “charming penguin” terms. This illustrates the large spread of model-predictions.

In Table 4 we compare some other results from Ref. [5] (see also Ref. [24]) and Ref. [27], where again rates are enhanced by “charming penguins”. The first authors neglected $s'_P$. In our treatment, the branching ratios for the processes in the first and second rows should be approximately equal; so should those for the processes in the third and fourth rows. The differences in Ciuchini et al. show the effect of $s'_P$ and involve some nonet-symmetry and SU(3) breaking effects. The combined effect, resulting in amplitude differences at a level of 20%, is not unexpected. On the other hand, the rate difference between the first two processes in Chau et al., arising from nonet-symmetry breaking alone, seems quite large for such effects.

V. CONCLUSIONS

The decays $B^+ \to \omega \pi^+$ (still requiring confirmation) and $\omega K^+$ seen at branching ratio levels of about $10^{-5}$ by the CLEO Collaboration [1] can be used, with the help of flavor SU(3), to anticipate the observability of other charmless $B \to VP$ decays in the
near future. We have indicated which amplitudes in the flavor-SU(3) decomposition are likely to be large as a result of present evidence. These consist of a strangeness-preserving “tree” amplitude $t_V$ and a strangeness-changing penguin amplitude $p'_V$. In both cases the subscript indicates that the spectator quark is incorporated into a vector $(V)$ meson.

Other decays depending on the amplitude $t_V$ are $B^+ \rightarrow \rho^0 \pi^+$ and $B^0 \rightarrow \rho^- \pi^+$. If $t_V$ is the dominant amplitude in these processes, we expect $\Gamma(B^+ \rightarrow \rho^0 \pi^+) = \Gamma(B^+ \rightarrow \omega \pi^+)$ and $\Gamma(B^0 \rightarrow \rho^- \pi^+) = 2\Gamma(B^+ \rightarrow \omega \pi^+)$. Furthermore, model calculations predicting $|t_P| > |t_V|$ imply that decays expected to be dominated by $t_P$, such as $B^+ \rightarrow \rho^+ \pi^0$ and $B^0 \rightarrow \rho^+ \pi^-$, will also have branching ratios in excess of $10^{-5}$.

An appreciable value for the amplitude $p'_V$, somewhat of a surprise in conventional models, implies that $B \rightarrow \rho K$ decays should be observable at branching ratio levels in excess of $10^{-5}$. The smallness of the ratio $B(B^+ \rightarrow \phi K^+)/B(B^+ \rightarrow \omega K^+)$ indicates that $|p'_P| < |p'_V|$. The amplitude $p'_P$ should dominate not only $B \rightarrow \phi K$ but also $B \rightarrow K^+ \pi^-$ decays. Evidence for any of these would then tell us the magnitude of $p'_P$. The relative phase of $p'_P$ and $p'_V$ is probed by $B \rightarrow K^*(\eta, \eta')$ decays.

We have argued that singlet amplitudes $s, s'$, corresponding to disconnected quark diagrams, are more likely to be appreciable when pseudoscalar mesons $\eta, \eta'$ are disconnected from the rest of the diagram than when vector mesons $\omega, \phi$ are disconnected. Thus, we expect $|s'_V| > |s'_P|$ and $|s_V| > |s_P|$. (Recall that the subscript refers to the meson in which the spectator quark is incorporated.) We have suggested several tests for non-zero singlet amplitudes, including a number of triangle and rate relations, and have outlined a program for determining the magnitude and phase of $s'_V$.

Once the dominant $t, p'$, and $s'_V$ amplitudes have been determined, flavor SU(3) predicts the amplitudes $t'_t, t'_p$, and $s'_V$. One can then (cf. Ref. [12]) determine which processes are likely to exhibit noticeable interferences between two or more amplitudes, thereby having the potential for displaying direct CP-violating asymmetries.

The CLEO Collaboration [1] has also reported the observation of the decay $B \rightarrow \phi K^*$, with a branching ratio of $(1.3_{-0.2}^{+0.7}) \times 10^{-5}$ when charged and neutral modes are combined. (Isospin invariance implies equal rates for the two.) Decays of the form $B \rightarrow VV$ are more complicated than $B \rightarrow PP$ or $B \rightarrow VP$ decays because of the three possible partial waves in the final state. Once these are separated out, for example using decay angular distributions [30], an analysis similar to the one presented here becomes possible for $B \rightarrow VV$ decays as well.

ACKNOWLEDGMENTS

We are grateful to M. Ciuchini, H. J. Lipkin, M. Neubert, J. G. Smith, and S. Stone for helpful discussions. This work was performed in part at the Aspen Center for Physics, and supported in part by the United States Department of Energy under Grant No. DE FG02 90ER40560 and by the United States – Israel Binational Science Foundation under Research Grant Agreement 94-00253/2.

References
[1] CLEO Collaboration, M. S. Alam et al., CLEO CONF-23, August 6, 1997, presented at Lepton-Photon Symposium, Hamburg, 1997.

[2] For a review see, e.g., Y. Nir and H. Quinn, Ann. Rev. Nucl. Part. Sci. 42, 211 (1992).

[3] D. Zeppenfeld, Zeit. Phys. C 8, 77 (1981).

[4] M. Savage and M. Wise, Phys. Rev. D 39, 3346 (1989); 40, 3127(E) (1989).

[5] L. L. Chau et al., Phys. Rev. D 43, 2176 (1991).

[6] M. Gronau, O. Hernández, D. London, and J. L. Rosner, Phys. Rev. D 50, 4529 (1994).

[7] M. Gronau, O. Hernández, D. London, and J. L. Rosner, Phys. Rev. D 52, 6356 (1995).

[8] M. Gronau, O. Hernández, D. London, and J. L. Rosner, Phys. Rev. D 52, 6374 (1995).

[9] M. Gronau and J. L. Rosner, Phys. Rev. D 53, 2516 (1996).

[10] A. S. Dighe, M. Gronau, and J. L. Rosner, Phys. Lett. B 367, 357 (1996); 377, 325(E) (1996);

[11] A. S. Dighe, Phys. Rev. D 54, 2067 (1996).

[12] A. S. Dighe, M. Gronau, and J. L. Rosner, CERN report CERN-TH/97-185, Enrico Fermi Institute Report No. EFI 97-34, [hep-ph/9707521], submitted to Phys. Rev. Letters.

[13] M. Gronau and J. L. Rosner, Phys. Lett. B 376, 205 (1996).

[14] CLEO Collaboration, presented by J. G. Smith at the 1997 Aspen Winter Physics Conference on Particle Physics, Aspen, CO, January, 1997. See also D. M. Asner et al., Phys. Rev. D 53, 1039 (1996); J. G. Smith, to be published in Ann. Rev. Nucl. Part. Sci.

[15] R. Fleischer, Zeit. Phys. C 62, 81 (1994); Phys. Lett. B 321, 259 (1994); 332, 419 (1994); N. G. Deshpande and X.-G. He, Phys. Lett. B 336, 471 (1994); Phys. Rev. Lett. 74, 26 (1995); N. G. Deshpande, X.-G. He, and J. Trampetic, Phys. Lett. B 345, 547 (1995).

[16] J. Silva and L. Wolfenstein, Phys. Rev. D 49, R1151 (1994).

[17] CLEO Collaboration, Cornell University report CLEO CONF 97-22a, EPS97-333, presented to European Physical Society Meeting, Jerusalem, August 1997.
[18] H. J. Lipkin, Argonne National Laboratory report ANL-HEP-CP-97-45, presented at The Second International Conference on B Physics and CP Violation, Honolulu, HI, March 24–27, 1997; Phys. Rev. Lett. 46, 1307 (1981); Phys. Lett. B 254, 247 (1991).

[19] D. Atwood and A. Soni, hep-ph/9704357; W.-S. Hou and B. Tseng, hep-ph/9705304; A. Datta, X.-G. He, and S. Pakvasa, hep-ph/9707256; A. L. Kagan and A. A. Petrov, UCHEP-27/UMHEP-443, hep-ph/9707354; H. Fritzsch, CERN-TH/97-200, hep-ph/9708348.

[20] S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964).

[21] M. Bauer, B. Stech, and M. Wirbel, Zeit. Phys. C 34, 103 (1987).

[22] N. G. Deshpande and J. Trampetić, Phys. Rev. D 41, 895 (1990); see also N. G. Deshpande in B Decays (Revised 2nd Edition), edited by S. Stone World Scientific, Singapore, 1994), p. 587.

[23] A. DeAndrea, N. Di Bartolomeo, R. Gatto, and G. Nardulli, Phys. Lett. B 318, 549 (1993); A. DeAndrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Lett. B 320, 170 (1994).

[24] G. Kramer and W. F. Palmer, Phys. Rev. D 52, 6411 (1995); Zeit. Phys. C 66, 429 (1995).

[25] R. Aleksan et al., Phys. Lett. B 356, 95 (1995).

[26] H.-Y. Cheng and B. Tseng, IP-ASTP-03-97/NTU-TH-97-08, hep-ph/9707310; IP-ASTP-04-97/NTU-TH-97-09, hep-ph/9708211.

[27] M. Ciuchini, E. Franco, G. Martinelli, and L. Silvestrini, CERN report CERN-TH/97-30, hep-ph/9703353; M. Ciuchini, R. Contino, E. Franco, G. Martinelli, and L. Silvestrini, CERN report CERN-TH/97-188, hep-ph/9708222. This last work contains an extensive list of further references.

[28] A. J. Buras and R. Fleischer, Phys. Lett. B 341, 379 (1995).

[29] J. L. Rosner, Phys. Rev. D 27, 1101 (1983); in Proceedings of the International Symposium on Lepton and Photon Interactions at High Energy, Kyoto, Aug. 19-24, 1985, edited by M. Komura and K. Takahashi (Kyoto Univ., Kyoto, 1985), p. 448.

[30] A. S. Dighe, I. Dunietz, H. J. Lipkin, and J. L. Rosner, Phys. Lett. B 369, 144 (1996).