Hadronic Light-by-Light Contribution to Muon g-2

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Abstract

I present the main results obtained in a recent work together with Eduardo de Rafael and Arkady Vainshtein on the hadronic light-by-light contribution to muon g-2. We came to the estimate \( a_{\mu}^{HLbL} = (10.5 \pm 2.6) \times 10^{-10} \). Here, some emphasis is put in pointing out where the future KLOE2 two-photon experimental program can help to reduce the present model dependence of \( a_{\mu}^{HLbL} \).

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1 Introduction

One of the six possible photon momenta configuration to the hadronic light-by-light (HLbL) contribution to the muon anomalous magnetic moment $a = (g_\mu - 2)/2$ is shown in Fig. 1 and described by the vertex function

$$\Gamma^\mu(p_2, p_1) = -e^6 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{\Pi^{\mu\nu\rho\sigma}(q, k_1, k_3, k_2)}{k_1^2 k_2^2 k_3^2} \times \gamma_\nu(p_2 + k_2 - m)^{-1} \gamma_\rho(p_1 - k_1 - m)^{-1} \gamma_\sigma$$

where $q \to 0$ is the momentum of the photon that couples to the external magnetic source, $q = p_2 - p_1 = -k_1 - k_2 - k_3$ and $m$ is the muon mass.

![Figure 1: Hadronic light-by-light scattering contribution.](image)

The dominant contribution to the hadronic four-point function

$$\Pi^{\mu\nu\alpha\beta}(q, k_1, k_2, k_3) = i^3 \int d^4x \int d^4y \int d^4z e^{i(-k_1 \cdot x + k_2 \cdot y + k_3 \cdot z)} \langle 0 | T [V^\mu(0)V^\nu(x)V^\rho(y)V^\sigma(z)] | 0 \rangle$$

comes from the three light quark ($q = u, d, s$) components in the electromagnetic current $V^\mu(x) = [\bar{q}\hat{Q}\gamma^\mu q](x)$ where $\hat{Q}$ denotes the quark electric charge matrix. We are interested in the limit $q \to 0$ where current conservation implies

$$\Gamma^\mu(p_2, p_1) = -\frac{a_{\text{HLbL}}^{\text{HLbL}}}{4m} [\gamma^\mu, \gamma^\nu] q_\nu.$$  

Here I would like to describe the main results of [1]. For recent reviews of previous work [2–8], see [9–11].
2 Numerical Conclusions and Prospects

The discussion in [1] lead the authors to give the following numerical conclusions according to an $1/N_c$ expansion [12] –such expansion works reasonably well:

- **Contribution from $\pi^0, \eta$ and $\eta'$ exchanges**: Implementing a new OPE constraint into a neutral pion exchange model [8], the authors of Ref. [8] obtained $(11.4 \pm 1.0) \times 10^{-10}$ for this contribution. Within the ENJL model the momenta higher than a certain cutoff is accounted separately via quark loops [4, 5] while in the OPE based model these momenta are already included into the result. Assuming that the bulk of high energy quark loops are associated with pseudo-scalar exchange Ref. [4, 5] obtains $(10.7 \pm 1.3) \times 10^{-10}$ after adding these to the neutral pion exchange within the ENJL model. Taking into account this discussion, the authors of [1] quote as central value the one in [8] with the largest error quoted in [4, 5]:

$$a_{\text{HLbL}}^{\text{π, η, η'}} = (11.4 \pm 1.3) \times 10^{-10}. \quad (4)$$

- **Contribution from pseudo-vector exchanges**: The analysis done in [1] suggests that the errors quoted within the large $N_c$ ENJL model are underestimated. Taking the average within both estimates and raising the present uncertainty to cover both, Ref. [1] quote

$$a_{\text{HLbL}}^{\text{pseudo-vectors}} = (1.5 \pm 1.0) \times 10^{-10}. \quad (5)$$

- **Contribution from scalar exchanges**: The ENJL model should give a good estimate of these large $N_c$ contributions, the authors of [1] therefore keep the result from [4] but with a larger conservative error to cover for other unaccounted higher resonances that give negative contributions:

$$a_{\text{HLbL}}^{\text{scalars}} = -(0.7 \pm 0.7) \times 10^{-10}. \quad (6)$$

- **Contribution from dressed pion and kaon loops**: The next-to-leading in $1/N_c$ contributions are the most complicated to calculate at present. In particular, the charged pion loop shows a large instability due to model dependence. This and the contribution of higher resonances loops was taken into account in [1] by taking the central value as the full VMD result quoted [4] with again a large conservative error:

$$a_{\text{HLbL}}^{\text{π+, dressed loop}} = -(1.9 \pm 1.9) \times 10^{-10}. \quad (7)$$

Adding the contributions above and errors in quadrature, as well as the small charm quark contribution $a_{\text{HLbL}}^{\text{charm}} = 0.23 \times 10^{-10}$, one gets our best estimate [1]

$$a_{\text{HLbL}} = (10.5 \pm 2.6) \times 10^{-10}. \quad (8)$$
The proposed new $g_\mu - 2$ experiment accuracy goal of $1.4 \times 10^{-10}$ calls for a considerable improvement in the present calculations. The use of further theoretical and experimental constraints could result in reaching such accuracy soon enough. In particular, imposing as many as possible short-distance QCD constraints [2–6,8] has result in a better understanding of the numerically dominant $\pi^0$ exchange. At present, none of the light-by-light hadronic parametrization satisfy fully all short distance QCD constraints. In particular, this requires the inclusion of infinite number of narrow states for other than two-point functions and two-point functions with soft insertions [13]. A numerical dominance of certain momenta configuration can help to minimize the effects of short distance QCD constraints not satisfied, as in the model in [8].

Recently, an off-shell form factor for the $\pi^0$ neutral exchange has been discussed in [14] to get $a^{\text{HLbL}}_\mu$ – the numerical values for the $\pi^0$ exchange obtained are very similar to the ones quoted above. How to take off-shellness effects consistently in the full four-point function (2) remains however an open question [14].

More experimental information on the decays $\pi^0 \to \gamma\gamma^*$, $\pi^0 \to \gamma^+\gamma^*$ and $\pi^0 \to e^+e^-$ (with radiative corrections included [15–17]) can also help to confirm some of the neutral pion exchange results.

A better understanding of other smaller contributions but with comparable uncertainties needs both more theoretical work and experimental information. This refers in particular to pseudo-vector exchanges. Experimental data on radiative decays and two-photon production of these and other C-even resonances can be useful in that respect. Experimental information on processes $\pi^0\pi^0 \to \gamma\gamma^*$ and $\pi^+\pi^- \to \gamma^+\gamma^*$ would be very welcome for that. For instance, these processes are related to the two-photon coupling of the lightest QCD resonance –the $\sigma$ [18–21].

New approaches to the pion dressed loop contribution, together with experimental information on the vertex $\pi^+\pi^-\gamma^+\gamma^*$ would also be very welcome. Measurements of two-photon processes like $e^+e^- \to e^+e^-\pi^+\pi^-$ can be useful to give information on that vertex and again could reduce the model dependence.

The two-gamma physics program at KLOE2 will be very useful and well suited in the processes mentioned above which information can help to decrease the present model dependence of $a^{\text{HLbL}}_\mu$.

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