Fluctuations of Particle Yield Ratios in Heavy-Ion Collisions

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Abstract

We study the dynamical fluctuations of various particle yield ratios at different incident energies. Assuming that the particle production yields in the hydronic final state are due to equilibrium chemical processes ($\gamma = 1$), the experimental results available so far are compared with the hadron resonance gas model (HRG) taking into account the limited momentum acceptance in heavy-ion collisions experiments. Degenerated light and conserved strange quarks are presumed at all incident energies. At the SPS energies, the HRG with $\gamma = 1$ provides a good description for the measured dynamical fluctuations in $(K^+ + K^-)/(\pi^+ + \pi^-)$. To reproduce the RHIC results, $\gamma$ should be larger than one. We also studied the dynamical fluctuations of $(p + \bar{p})/(\pi^+ + \pi^-)$. It is obvious that the energy-dependence of these dynamical fluctuations is non-monotonic.

1 Introduction

Understanding of the dynamical properties of hot and dense matter is a key question in the heavy-ion collisions experiments. The phase structure and event-by-event fluctuations [1, 2, 3] have been suggested to provide comprehensive characteristics of the particle production yields. They are essential observations to examine the hypothesis about the equilibrium of the chemical processes in the hadron final state [4]. The event-by-event fluctuations of certain particle yields have been studied at SPS and RHIC energies [5, 6, 7]. Therefore, it is natural to study the energy-dependence of the particle yield ratios and the event-by-event fluctuations using the hadron resonance gas model (HRG), as it provided a good description for the thermodynamical evolution of the hadronic system below the critical temperature [8, 9, 10] and has been used to characterize the conditions deriving the chemical freeze-out [11, 12].

The hypothetical chiral symmetry breaking restoration and deconfined phase transition to quark-gluon plasma (QGP) are to be characterized by remarkable fluctuations in the particle production yields [11, 2, 3], which are accompanied by dynamical and volume fluctuations as well. The latter can simply be eliminated, when taking into consideration the dimensionless particle yield ratios [3].

In this letter, we try to answer the questions, whether the strangeness quarks $q_s$ should enhance the dynamical fluctuations and whether the critical endpoint could be localized by means of event-by-event fluctuations in the hydronic final state. We make predictions for the dynamical and statistical fluctuations of the ratios of different particle yields in dependence on incident energy. Apparently, the dynamical fluctuations strongly depend on sort of the particle yields. In some particle yield ratios, the dynamical fluctuations are smaller than the statistical ones. In others, the dynamical fluctuations are slightly greater than the statistical ones. The energy dependence is non-monotonic.

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All these predictions are phenomenologically of great interest and hope to encourage experimental attempts to measure their event-by-event fluctuations in a wide range of incident energy.

2 Model

The hadron resonances treated as a free gas \[8, 9, 10, 13, 14\] are conjectured to add to the thermodynamic pressure in the hadronic phase. This statement is valid for free as well as strong interactions between the hadron resonances themselves. It has been shown that the thermodynamics of strongly interacting system can be approximated to an ideal gas composed of hadron resonances \[13, 15\].

At finite temperature \(T\), strangeness \(\mu_S\) and baryo-chemical potential \(\mu_B\), the pressure of one sort of hadron resonance reads

\[
p(T, \mu_B, \mu_S) = \frac{g}{2\pi^2} T \int_0^\infty k^2 dk \ln \left[1 \pm \gamma \lambda_B \lambda_S e^{-\varepsilon(k)/T} \right],
\]

where \(\varepsilon(k) = (k^2 + m^2)^{1/2}\) is single-particle energy and \(\pm\) stands for bosons and fermions, respectively. \(g\) in the front of the integration is the spin-isospin degeneracy factor and \(\gamma \equiv \gamma_n \gamma_q \gamma_m\) stand for the quark phase space occupancy parameters, where \(n\) and \(m\) being number of light and strange quarks, respectively. \(\lambda = \exp(\mu/T)\) is the fugacity, where \(\mu\) is the chemical potential multiplied by corresponding charge. Summing over all hadron resonances results in the final thermodynamic pressure in the hadronic phase, as no phase transition is conjectured in HRG.

The quark chemistry is given by relating the hadronic chemical potentials to each of the quark constituents; \(\mu_B = 3\mu_q\) and \(\mu_S = \mu_q - \mu_s\), where \(q\) and \(s\) being the light and strange quark quantum number, respectively. The baryo-chemical potential for the light quarks is averaged as \(\mu_q = (\mu_u + \mu_d)/2\) and the strangeness chemical potential \(\mu_S\) is calculated as a function of \(T\) and \(\mu_B\) under the assumption of strange quarks conservation \[13\].

In grand canonical ensemble, the particle density is no longer constant.

\[
\langle N \rangle = \sum_i g_i \frac{2\pi^2}{2} \int dk k^2 \frac{\gamma e^{(\mu_i - \varepsilon_i)/T}}{1 \pm \gamma e^{(\mu_i - \varepsilon_i)/T}},
\]

\[
\langle (\Delta N)^2 \rangle = \sum_i g_i \frac{2\pi^2}{2} \int dk k^2 \frac{\langle N_i \rangle}{1 \pm \gamma e^{(\mu_i - \varepsilon_i)/T}}.
\]

To count the number density \(N\) and fluctuations \((\Delta N)^2\) in the hadronic final state, the chemical freeze-out processes have to be taken into account, i.e. the hadron resonances should finally decay to stable particles or resonances.

\[
\langle N_i^{final} \rangle = \langle N_i^{direct} \rangle + \sum_{j \neq i} b_{j\rightarrow i} \langle N_j \rangle,
\]

\[
\langle (\Delta N_{j\rightarrow i})^2 \rangle = b_{j\rightarrow i} (1 - b_{j\rightarrow i}) \langle N_j \rangle + b_{j\rightarrow i}^2 \langle (\Delta N_j)^2 \rangle
\]

where \(b_{j\rightarrow i}\) is the branching ratio for the decay of \(j\)-th to \(i\)-th particle. In this work, the chemical freeze-out is characterized by \(s/T^3\) \[14\], where \(s\) is the entropy density.
The fluctuations of particle yield ratios of particle 1 and particle 2 read [3]

\[ \sigma_{N_1/N_2}^2 = \frac{\langle (\Delta N_1)^2 \rangle}{\langle N_1 \rangle^2} + \frac{\langle (\Delta N_2)^2 \rangle}{\langle N_2 \rangle^2} - 2 \frac{\langle \Delta N_1 \Delta N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} \] (6)

In this expression, we include all possible fluctuations, i.e., dynamical and statistical as well. The third term of Eq. (6) counts for the fluctuations from hadron resonances that decay into particle 1 and particle 2, simultaneously. In such a mixing channel, all correlations including the quantum statistical ones are taken into account. Obviously, this decay channel results in strong correlated particles and correspondingly fluctuations.

To extract the statistical fluctuations, we apply Poisson scaling to the mixed decay channels[1].

\[ \langle \sigma_{N_1/N_2}^2 \rangle_{stat} = \frac{1}{\langle N_1 \rangle} + \frac{1}{\langle N_2 \rangle} \] (7)

Detector acceptance factor and resolutions are main sources for statistical fluctuations. Subtracting Eq. (7) from Eq. (6) we get the dynamical fluctuations of the particle yield ratio \( N_1/N_2 \).

\[ \langle \sigma_{N_1/N_2}^2 \rangle_{dyn} = \frac{\langle N_1^2 \rangle}{\langle N_1 \rangle^2} + \frac{\langle N_2^2 \rangle}{\langle N_2 \rangle^2} - \frac{\langle N_1 + N_2 \rangle + 2\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} \] (8)

3 Results

The experimentally measured dynamical fluctuations of particle yield ratios \((K^+ + K^-)/(\pi^+ + \pi^-)\) are systematically confronted with the theoretical predictions in Fig. 1. An earlier attempt to compare with preliminary results has been reported in [3, 5]. It was found that the theoretical and experimental ratios of dynamical to statistical fluctuations are compatible with each other. Individual fluctuations themselves are not.

Depending on \( \gamma \), HRG is apparently able to predict various particle yield ratios at a wide range of incident energy. At SPS energies, HRG with \( \gamma = 1 \) provides a good description for the experimentally measured dynamical fluctuations [5, 7]. To reproduce the RHIC results, \( \gamma \) should be larger than one. The dynamical fluctuations of \((p + \bar{p})/(\pi^+ + \pi^-)\) are depicted in Fig. 2. Few comments are in order at this moment.

- The dependence on \( \sqrt{s} \) is non-monotonic. The fluctuations can be suppressed and/or enhanced at different \( \sqrt{s} \)
- Strangeness fluctuations are positive and enhanced with \( \sqrt{s} \). There are remarkable minima at the top SPS energies
- At high energies, the fluctuations smoothly increase with \( \sqrt{s} \)

In Fig. 1 we compare measured dynamical fluctuations of \((K^+ + K^-)/(\pi^+ + \pi^-)\) yield ratios with HRG model. At \( \gamma = 1.0 \), we find an excellent agreement at SPS energies. At RHIC energy, the measured fluctuations are above the theoretical ones. One has to allow \( \gamma \) to have values larger than one, in order to reproduce the data. The explanation for this disagreement would be two-fold. First, the RHIC measurements are still preliminary [7]. The final measurements might modify the \( \gamma \)-value reported here. Second, we refer to our

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1Experimentally, there are various methods to eliminate the statistical fluctuations [7]. The frequently used one is the counting of particle yield ratios from mixing events.
previous study of the particle yield ratios in heavy-ion collisions \[16\], where we concluded that the statistical models, like HRG, with \(\gamma = 1\) slightly overestimate the particle yield ratios at RHIC energy. Therefore we should assign to \(\gamma\) values larger than one. What we observe here apparently supports such a conclusion that the quark phase space occupancy factor, characterized by \(\gamma\), is obviously modified at RHIC energies. A connection between \(\gamma\) and hypothetical phase transition is discussed in \[16\]. Therefore, the modification of \(\gamma\)-value might be understood, if phase transition to QGP takes place.

Additionally, we see in Eq. 8 that \(\langle N_i \rangle\), where \(i\) equals 1 or 2, inversely proportional to \(\sigma\), assuming that the contribution from the two-particle-channel is negligible \(\langle N_1 N_2 \rangle\). Should HRG model overestimate the particle yield ratios at RHIC energy, as the case studied in \[16\], straightforwardly, we expect that the dynamical fluctuations \(\sigma\) should be underestimated at \(\gamma = 1.0\).

So far we can conclude that the quark phase space occupancy factor obviously depends on the incident energy. Up to the top SPS energy, the quark phase space occupancy factor is most probably saturated \((\gamma = 1.0)\). At RHIC energies, a phase transition to new state of matter has been reported \[17\]. This can be seen in our analysis, if the quark phase space occupancy factor is modified. Allowing \(\gamma\) to have values larger than one, HRG bests reproduce the particle yield ratios as reported in \[11, 12, 16\] and their dynamical fluctuations, the subject of this work.

The statistical model analysis of \((K^+ + K^-)/((\pi^+ + \pi^-))\) dynamical fluctuations reported in \[18\] let to the conclusion that the non-equilibrium model, equivalent to \(\gamma \neq 1\), would provide an acceptable description at top SPS and RHIC energies. The definition of the chemical freeze-out and the mechanism assuring conserved strangeness in the hadronic final state would be responsible for this contradiction with HRG.
At low incident energy, the energy density or temperature and the degrees of freedom are not high enough to cause non-equilibrium phase transition to QGP \([13, 14]\). As the hadronic matter goes into a new phase of nearly deconfined quarks and gluons, the quark phase space occupancy factor should correspondingly change. The fitted values of \(\gamma\) used to draw the figure 1 in \([18]\) are not explicitly given to discuss them here. Taking into account the momentum acceptance, the HRG results show that \(\gamma = 1\) best reproduces the dynamical fluctuations at low SPS energies.

As the HRG calculations are performed in grand canonical formulism, the so-called conservation laws \([19]\) would be taken into account. With the conservation laws we mean - among others - the difference between the canonical (CE) and grand canonical ensembles (GCE). At high energies, equivalent to large volume \(V\), - in principle - there is almost no difference \([19]\) between CE and GCE. The conservation laws are discussed at vanishing \([19]\) and finite chemical potential in the final particle multiplicity \([20]\). In GCE, the control variables are \(\mu\), \(V\) and \(T\). The total number density \(N\) is therefore allowed to fluctuate. Therefore \(N\) is related to CE by the Legendre transformation.

In HRG, we studied this problem from another point of view \([13]\) and found that the ideal quantum Boltzmann gas formulism applied in HRG takes into account the interactions and the quantum statistics in the system. Furthermore, the ideal quantum Boltzmann gas overcomes the limitations in classical Boltzmann ideal gas, that the entropy for instance is only specified within an undetermined additive constant in the way that it determines such an additive constant in the high temperature limits of quantum Fermi- and Bose-gas. Quantitative estimation of the conservation laws at finite chemical potential, i.e. different incident energies, is included in the parameter \(q\) \([19]\), where \(q\) is probability to detect \(n\) particle density out of total \(N\) particle density produced in the whole momentum space. The averaged particle density detected (accepted) read

\[
\langle n \rangle = q \langle N \rangle  \\
\langle(\Delta n)^2 \rangle = q^2 \langle(\Delta N)^2 \rangle + q(1-q) \langle N \rangle
\]

where \(\langle N \rangle\) and \(\langle(\Delta N)^2 \rangle\) are given in Eq. 2 - Eq. 5.

In Fig. 2 the dynamical fluctuations of non-strangeness \((p + \bar{p})/(\pi^+ + \pi^-)\) yield ratios are depicted as a function of \(\sqrt{s}\). At \(q = 1\), HRG obviously overestimate the dynamical fluctuations. It is worth noticing that \(\sigma_{dyr}\) excursively increase at \(\sqrt{s} \approx 15\) GeV. The negative values are to be understood as dominant statistical fluctuations, especially the ones not explicitly included in Eq. 7. The negative values might also refer to dominant fluctuations in the proton-pion channel.

Taking into account an acceptance factor \(q < 1\), the experimental data can be reproduced. As noticed in previous figure (Fig. 2), the quark phase space occupancy factor is expected to be modified at incident energies higher than SPS. Taking this fact into account, the dynamical fluctuations of \((p + \bar{p})/(\pi^+ + \pi^-)\) explosively switch to positive values. If this expectation turns to be correct, we expect that the dynamical fluctuations of \((p + \bar{p})/(\pi^+ + \pi^-)\) at RHIC energies might not be as smooth as the SPS ones.

4 Discussions and conclusions

At high temperatures, the hadronic matter is conjectured to go through chiral symmetry breaking restoration and deconfined phase transition(s) to partonic phase at almost same critical temperature \([21]\). Depending on order of phase transition, which in turn depends
Fig. 2: The dynamical fluctuations of non-strangeness yield ratio \((p + \bar{p})/(\pi^+ + \pi^-)\). The experimental results (open circles) are taken from [5, 6]. The negative values point to dominant statistical fluctuations. The factor \(q\) relates canonical with grand canonical ensembles (see the text).

on quark flavors and masses, fluctuations in particle yields are likely expected, especially if the transition causes out-of-thermal equilibrium. The three pions, \(\pi^0, \pm\), are the lightest Goldstone bosons resulting from chiral symmetry breaking. Therefore, strong fluctuations in the pion fields are likely expected during the transition in chiral symmetry.

The chiral transition associated with massless up and down quarks is of second order. If the quark masses are accounted with, the transition is either a slightly first order or just a cross over. The transition is explicitly a first order, if strange quark with very light mass is included. Also the deconfinement phase transition depends on the quark flavors and their masses. For three quark flavors, the transition is likely of first order. In this case the transition occurs via nucleation of hadronic bubbles in the background of QGP. For two flavors, the transition is a smooth cross-over.

This discussion might illustrate how strong are the pion dynamical fluctuations connected with the chiral phase transition at high temperature and out-of-thermal equilibrium. These fluctuations are given the generic name of disoriented chiral condensates (DCC) [22].

The dynamical fluctuations associated with strong first order of phase transition are likely very large. The continues second order or cross over phase transition might wish out large part of dynamical fluctuations in the final state. On the other hand, the dynamical fluctuations are conjectured to slow down near the second order phase transition. This has been confirmed in classical systems, solid state physics. In quantum field theory, the long-wavelength (spinodal) modes will be quenched through the second order phase transition [23].

The Fluctuations of quark number \(n_q\) have been studied in lattice QCD [24]. It has been found that the \(T\)-dependence of \(n_q\) fluctuations is dominated by the analytic part of the partition function. Across the critical temperature, there is a smooth increase in
the fluctuations. Almost same results are reported in this work. We therefore can use the fluctuations to characterize the phase transition.

In our analysis, the critical temperature is not exactly specified, as HRG does implement any phase transition. The most economic way is to study the dynamical fluctuations in the hadronic final state, such a way we can also compare with the experimental results. We picked up two particle yield ratios, for them we have extensive experimental estimations at a wide range of incident energies. We find that the dynamical fluctuations depend on the particle yields and incident energy.

Tab.1 gives the relation between dynamical and statistical fluctuations $\frac{\sigma_{\text{dyn}}}{\sigma_{\text{stat}}}$ at SPS and RHIC energies. For the strangeness particle yield ratios, the statistical fluctuation increases against the dynamical ones. For non-strangeness particle yield ratios, we get just the opposite.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\sigma_{\text{dyn}}/\sigma_{\text{stat}} & 12.3 & 17.3 & 62.4 & 200 \\
\hline
(K + K^-)/ (\pi^+ + \pi^-) & 0.0143 & 0.0140 & 0.0115 & 0.0088 \\
(p + \bar{p})/(\pi^+ + \pi^-) & -0.0143 & 0.0139 & 0.0309 & 0.0317 \\
\hline
\end{array}
\]

Tab. 1: $\sigma_{\text{dyn}}/\sigma_{\text{stat}}$ of the particle yield ratios at $\gamma = 1$ and $q = 0.75$ and various $\sqrt{s}$. Using this quantity, we can estimate how large are the dynamical fluctuations compared to the statistical ones when the produced particles are chemically frozen. The validity of these predictions apparently depend on the conditions controlling the chemical equilibrium in the final state.

The energy dependence of dynamical fluctuations is non-monotonic. This can also be seen in Tab.1. For $(K + K^-)/ (\pi^+ + \pi^-)$, the fluctuations decrease up to top SPS energy, afterwards very slowly increase with $\sqrt{s}$. $(p + \bar{p})/(\pi^+ + \pi^-)$ have negative dynamical fluctuations at SPS energies. At higher energies, their dynamical fluctuations might jump to positive values. The SPS energy might not high enough to lead to strongly out-of-equilibrium phase transition. The situation is different at RHIC energies. On the other hand, the lattice QCD simulations show that the transition in the region of the phase diagram corresponding to the RHIC energy - and very early universe - is a cross over. This continuous phase transition is not strong enough to secure out-of-equilibrium. Furthermore, any anomalous phenomenon, like the dynamical fluctuations, should be washed out in the final state, as the transition is a smooth and continuous cross-over or second order. This might be the reason why although new state of matter should be created at RHIC energies [17], there is no abrupt change in the dynamical fluctuations with increasing $\sqrt{s}$. That the dynamical fluctuations smoothly increase with $\sqrt{s}$ in agreement with the lattice simulations for $n_q$ fluctuations [24], might support the conclusions that any anomalous phenomenon associated with non-equilibrium phase change likely would be washed out, if the phase change is smooth and does not cause out-of-equilibrium.

In our analysis, we assume that the particle production is due to chemical equilibrium processes controlling the final state, i.e., $\gamma = 1$. That our models can very well reproduce the experimental measurements means that the equilibrium freeze out scenario is apparently correct, especially up to SPS energies. Nevertheless, energy scan down to $\sqrt{s} = 10$ GeV turns to be a crucial step to verify the worthwhile behavior of particle production [14, 16] and now dynamical fluctuations.

It is also worth extracting information about the role of different decay channels in the energy-dependence of dynamical fluctuations. It will be a further propose to study the effect of chemical non-equilibrium processes on event-by-event dynamical fluctuations.
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