Particle-fluid interaction forces as the source of acceleration PDF invariance in particle size

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Abstract

The conditions allowing particle suspension in turbulent flow are of interest in many applications, but understanding them is complicated both by the nature of turbulence and by the interaction of flow with particles. Observations on small particles indicate an invariance of acceleration PDFs of small particles independent of size. We show to be true the postulated role of particle/fluid interaction forces in maintaining suspension. The 3D-PTV method, applied for two particle phases (tracers and inertial particles) simultaneously, was used to obtain velocity and acceleration data, and through the use of the particle's equation of motion the magnitude of forces representing either the flow or the particle interaction were derived and compared. The invariance of PDFs is shown to extend to the component forces, and lift forces are shown to be significant.

1 Introduction

Some of the most long-standing open questions in fluid dynamics pertain to the modeling of the movement of particles in turbulence. Mixing, deposition and resuspension of particles from a wall are examples of processes which are topics of current research interest in a variety of scientific fields. These and similar processes are key mechanisms in a variety of applications. Examples include transport of
lead contamination [1], channel dredging [2], indoor distribution of allergens [3], and design of stirred chemical reactors [4], to mention just a few.

Modeling of particle motions in turbulent flow is difficult because it involves both the modeling of the surrounding flow field, and resulting pressure gradients; and modeling of the particle-flow interaction, which involves the local flow around the particle and the forces resulting from stress applied on the particle by the flow [5]. In the equation of motion of a particle moving in a fluid pressure gradient appears as a term depending only on the absolute Lagrangian acceleration of the fluid around the particle, in contrast to other terms that depend on the relative velocity between the particle and the flow. In very dense suspensions of particles, the two-way or four-way coupling which means also modeling of feedback of particles on the flow and particle-particle interactions, are needed [5], but these are not in the scope of the present work.

In the research fields dealing with the suspension of particles from the walls into a flow (hereinafter called resuspension), different approaches were developed according to the distinction between the importance of the pressure gradient forces (i.e. only flow) versus the interaction forces, strongly dependent on the relative particle-fluid velocity. For example, studies on “dust devils”, atmospheric vortices that entrain dust, focus on static pressure gradients as a key resuspension mechanism (entitled “$\Delta P$ effect”), and separately from resuspension through aerodynamic drag effect [6, 7]. A contrary example is from the fields of aerosol and powder resuspensions where the central approach focuses on the particle-flow interaction, and requires detailed models of the forces applied to the surface of the particle by the flow. Several types of models exist, some focus particularly on particle resuspension into the flow [8, 9, 10, 11] and some developed to model the general motion of a particle in flow [12, 13].

The present study focuses on the relative importance of the pressure gradient and the particle-flow interaction forces on inertial particles in turbulent flows. The interaction forces are several terms that arise from the interaction of the particle with the flow due to different mechanisms and on different time and length scales. Such terms (reviewed in section 2) may include the well known Stokes form drag, the Basset history term, Faxen corrections, lift terms (for which a possible
candidate is the Saffman lift term which arises from asymmetric stresses around the particle) and other, yet unknown terms. Although most investigations of particle motions assume lift terms negligible for small particles, a question whether this result holds for turbulent flows and particles with diameter of the order of the Kolmogorov length scale, is still a matter of scientific debate.

Some recent numerical simulation results (e.g. [14]) have found a significant lift force, as well as recent measurements of vorticity experienced by large particles in turbulence provide an argument supporting the significance of lift effects [15]. Although the models applied to the motion of small (point-like) particles are different from those for large (e.g. compared to the Kolmogorov length scale of turbulent flow) particles, recent studies of particle motion in turbulence show that the probability density function (PDF) of particle acceleration, when normalized to its standard deviation (hereinafter called standardized distribution), is independent of the particle size. This result stands for particles of size ranging from point-like particles of the size much smaller than the Kolmogorov length scale to the particles of diameter several times larger than the Kolmogorov scale.

Calzavarini et al. [16] proposed that acceleration distribution of small particles that are expected to represent the flow field represent the acceleration (and therefore the pressure gradient forces) experienced by the flow. However, another key mechanism is required to explain the self-similarity of acceleration distribution of the large particles. The authors [16] pointed out that the “drag forces” acting on the particles require more study in order to explain the self-similarity of standardized acceleration PDF. The present study is presenting such explanation, linked to the particle-fluid interaction forces, including drag and lift terms at once.

The fact that the normalized acceleration distribution functions are self-similar independently of the particle size and density, at least for the particles which are much smaller or at the order of Kolmogorov scale, is important for modeling. The acceleration PDF obtained for a particular turbulent flow using numerical simulations or experiments can be translated into the distribution function for the particles, allowing a quantitative predictions of sediment transport, dispersion and mixing.

This study uses a unique capability to experimentally obtain both the turbulent
velocity field in the proximity of the particles and the trajectories of particles themselves, in order to shed some light on the underlying distributions of the forces of inertia, pressure, drag and lift. We show that in turbulent flow, the contribution of lift to particle acceleration is of the same order as that of drag, so that the common practice of neglecting lift is not applicable for particles that are not much smaller than the Kolmogorov length scale.

Our method involves the extraction of fluid and particle velocities and acceleration from data obtained by three-dimensional Particle Tracking Velocimetry (3D-PTV), as described in section 2.1; this data, in conjunction with the equation of motion of a particle in flow, yields the forces from pressure gradients and from other interactions, using a processing technique described in detail in the following section. We follow with concluding remarks about the relative velocity as a mechanism characterizing the flow’s suspension capacity.

2 Materials and methods

2.1 Experimental setup

The study is based on unique experimental data obtained by two three-dimensional particle tracking velocimetry (3D-PTV) systems recording simultaneously the turbulent flow and motion of inertial particles in the same observation volume. The quasi-homogeneous and quasi-isotropic turbulent flow in the observation volume is maintained by eight counter-rotating disks on both sides of the volume. Detailed description of the experimental set-up and the methods of analysis are given by Guala et al. [17] and reproduced here for brevity.

The experiment has been carried out in a glass tank of $120 \times 120 \times 140$ mm$^3$ in which the flow is forced mechanically from two sides by two sets of four rotating disks with artificial roughness elements. The observation volume of approximately $30 \times 30 \times 30$ mm$^3$ was centered with respect to the forced flow domain, mid-way between the disks. More detailed information about the flow that can be produced by this forcing device, depicted in Figure 2.1, has been presented by Liberzon et al. [18], where smooth and baffled disks were compared.
Utilization of the 3D-PTV technique is of special importance since the second phase (i.e. the solid particles) essentially represents Lagrangian objects, and this experimental method allows measuring their motion, distribution (e.g. clustering) and dynamics, along with their interaction with the carrier fluid, measured in the same frame of reference.

A sketch of the experimental apparatus is shown in Figure 1 as reproduced from ref. [17]. Two 3D-PTV systems were synchronized at a frame rate of 500 fps. One system, consisting of a Photron-Ultima high-speed camera (1024 x 1024 pixels) with a 4-view image splitter, was set to record images of the Rhodamine labeled silica gel particles, as the second phase. A dichroic red filter and a low aperture were employed to filter out the light scattered by the 40–60 µm neutrally buoyant polystyrene flow tracer particles (Microparticles GmbH, density 1.03 g/cm³). The second 3D-PTV system, consisting of four MC1310 cameras (Mikrotron GmbH, 1280 x 1024 pixels), linked to a real-time digital video recording system (IO Industries) was used for measuring the fluid phase. These cameras were equipped by dichroic green filters. Typical images of the tracers and the solid particles are shown in Figure 3, where we can see that there is no contamination by the silica particles of the images in the system devoted to the fluid tracers, and vice versa. The observation volume of 30 x 30 x 30 mm³ was illuminated by a continuous 20 W Ar-Ion laser (Spectra-Physics).

Ref. [17] had measured different combinations of tracers and inertial particles, from which we present results of one data set using spherical porous glass particles with mean diameter of 500 micrometers and density of 1.45 g/cm³. For the details of the 3D-PTV method and data processing the readers are referred to Refs. [17, 19].

Raw data obtained by 3D-PTV may contain some measurement noise. Similarly to the previous works (e.g. [19, 20]) the position noise is filtered using a Savitsky-Golay filter [21] applied to each Cartesian component of the position along particle trajectories.

It has been shown [20] that a fit time of the order of the Kolmogorov time scale is sufficient to filter out noise without losing data. Therefore it is sufficient in our case to use a cubic polynomial over a 5-point time window in the Savitsky-Golay filter.
Figure 1: (Left) tank and rotating disks schematic, (Right) Experimental setup schematic.

2.2 Equation of motion of a particle

A particle equation of motion, i.e. an equation that balances the acceleration of the particle times its mass, $m_p a_p$, with the sum of forces acting on it, can be obtained from first principles by integrating the Navier-Stokes equation over a control volume around a particle.

The Navier-Stokes equations are notoriously nonlinear and hard to handle in all but a small class of simple cases. When applied to turbulent flow, they require a combination of assisting mathematical concepts, and a judicious use of scale relations. It is therefore inherent to the source of this equation, that several derivations are possible, depending on what can be assumed about the problem at hand and which terms can be neglected. In many derivations, the common concept that arises from the mathematical development is that of undisturbed fluid velocity, i.e. the velocity that the fluid would have had at the absence of the particle. In this way the flow is separated into the flow field as it would have been without particles, and the disturbance field.

This has allowed for an important application of the particle equation of motion: reconstruction of the undisturbed flow field from measurement of tracer particles.
The disturbance field created by the tracers is considered negligible compared to the undisturbed field, in a quantifiable way [22, 23].

Another application of the particle motion equation is for incorporating particle motion in numerical simulations (e.g. [24] and many others). When simulating particles in flow, a common approach is to separate the modeling of the undisturbed flow field from the modeling of particle motion, and use coupling of Eulerian flow simulation with Lagrangian particle tracking, whose underlying equation is the particle equation of motion.

The often-quoted review work of Maxey and Riley [25] provided two derivations: the state-of-the-art one of Corrsin and Lumley [26], and the corrected original derivation. The Corrsin-Lumley form is

$$
m_p \frac{dV_i}{dt} = m_i \left( \frac{Du_i}{Dt} - \nu \nabla^2 u_i \right) - \frac{1}{2} m_i \frac{d}{dt} \left( V_i - u_i \right) - \frac{6\pi a \mu}{\sqrt{\pi \nu}} \left( V_i - u_i \right) - \frac{6\pi a^2 \mu}{\nu} \int_{-\infty}^{t} \frac{d}{dt} \left( V_i - u_i \right) \sqrt{\frac{t - \tau}{\pi \nu}} d\tau + g_i \left( m_p - m_i \right) ; \quad i = 1, 2, 3$$

using the following variables: for the particle, its velocity $V_i$, radius $a$ and particle mass $m_p$; for the carrier fluid, the undisturbed fluid velocity at the particle centre $u_i$, the mass of fluid displaced by the particle, $m_i$, the fluid viscosity $\mu$ and its kinematic viscosity $\nu$.

An important notation convention is that the derivatives $d/dt$ and $D/Dt$ represent Lagrangian derivatives, following the particle and the containing fluid element respectively, so that (bold face symbols denote the vector quantities):

$$
\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla
$$

$$
\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla
$$

The terms on the right side of Eq. 1 are the different force terms acting on the particle: the pressure-gradient force $F_p$, added-mass force $F_a$, Stokes drag $F_S$, Basset history term $F_B$, and buoyancy $F_g$, respectively.

Maxey and Riley [25] kept the buoyancy term, but replaced the other terms with
expressions different by the undisturbed velocity Laplacian (also known as the Faxen corrections). Recent simulations have shown [27] that the Faxen correction becomes significant only for particles with diameter of several times the dissipative (Kolmogorov) length scale of the flow; in our experiments, the inertial particle size is of the order of the dissipative length scale. Therefore we use here the form of the terms that does not contain the Faxen corrections, implying that the pressure term can be estimated from the Lagrangian acceleration of the fluid:

\[ F'_p \equiv m_I \frac{Du}{Dt} \]  

(4)

An important term that is not included in these equations of motion which were also derived in parallel by Gatignol [28] for the point-like particles is the lift force. This term can, for instance, be formulated as shear-induced lift, suggested by Saffman [29]. If we add the definition of the relative velocity \( W_i = V_i - u_i \) of the particle, then the lift force is proportional to the cross-product of the relative rotation with the relative velocity:

\[ F_{Sa} \propto (\nabla W) \times W \]  

(5)

This is not the only possible formulation of the lift force(s) that can arise from asymmetric stresses on the surface of the particle, and the asymmetry is expressed using the relative velocity gradient or alternatively using the local undisturbed flow vorticity. The Saffman lift term has been found negligible in laminar flows but may be important in turbulent flows, where fluctuating velocity and vorticity are likely to induce asymmetric stresses.

For the purpose of the present study, the pressure-gradient force on the right-hand side and the total force on the particle on the left-hand side are the only terms that are of interest individually, hence the terms relating to particle-fluid interaction will be grouped together into a so-called interaction force term \( F_r \equiv F_S + F_B + F_{Sa} \). Hence the equation of motion may be rewritten in the short form used here throughout,

\[ m_p \frac{dV}{dt} - F'_p - F_a - F_g - F_r = 0 \]  

(6)
A similar method of analysis has been employed previously by Sridhar and Katz \cite{30} in their study of bubble motion in a laminar vortex. The authors \cite{30} have decomposed $F_r$ into two orthogonal components: the drag force, parallel to the relative velocity, $F_D \parallel v$, and the lift force perpendicular to the relative velocity, $F_L \perp w$. The decomposed terms provided the drag and lift coefficients that were expressed as functions of the local particle Reynolds number.

Although the method of Ref. \cite{30} appears promising and it has been expanded from spherical bubbles to ellipsoidal ones \cite{31}, it has not been further developed for turbulent flows. The original work of Ref. \cite{30} uses constructions suitable for laminar flow, and makes use of an interpolation method to obtain undisturbed velocity without examining its accuracy. The merit of the method was demonstrated using a comparison of measured trajectories to trajectories predicted from theory, but this approach is not feasible in turbulent flows. For this reason we will present a different method to compare interpolation methods, explained in section 2.3.

An additional simplification introduced in the work of \cite{30} was to include the added mass force term in the left hand side term and create the so-called inertia force,

$$F_I = m_p \frac{dv}{dt} - F_a$$

(7)

For the purpose of this work we can also treat the particle and added mass together as a particle of increased mass which is exposed to the same pressure gradient, and the particle-fluid interaction forces are derived from the velocity gradients around the particle. Following this notation, the form of the equation of motion used henceforth is the following:

$$F_I - F_p' - F_g - F_r = 0$$

(8)

and the interaction force is decomposed into two orthogonal components, $F_r = F_D + F_L$. 
2.3 Undisturbed fluid velocity interpolation

The undisturbed fluid velocity field $\mathbf{u}$ is a central concept in all the aforementioned derivations. It is assumed that the flow tracers (i.e. small, neutrally buoyant and inertialess particles) velocity $\mathbf{V}$ represents the fluid flow velocity at their locations. However, in most two phase flows, the second phase (bubbles, droplets, inertial particles, etc.) cannot be approximated as flow tracers and the undisturbed fluid flow velocity at their position must be obtained through interpolation of the fluid velocity of the flow tracers at positions close to the particle.

In Ref. [30] neighboring bubbles were used to estimate the fluid velocity in the vicinity of any other bubble, serving both as flow tracers and as a second phase. The measurements were performed in two-dimensional settings, and a convenient bilinear interpolation method was used to obtain the undisturbed fluid velocity field. For a three dimensional case, a different interpolation method is needed. Moreover, we need to evaluate possible sources of error – if the “flow tracers” (for example, the small bubbles) used in Ref. [30], are not ideal tracers in all flow conditions (e.g., in high strain regions), then the undisturbed flow velocity at the position of the particle from the second phase, $u_i(X(t), t)$ would contain both the interpolation error and the error due to the imperfection of the “flow tracers”.

In order to develop the method of Sridhar and Katz [30] for turbulent flows, we performed a careful assessment of the three dimensional interpolation technique. There are many interpolation methods that could conceivably be used. Some interpolation methods are generally applicable to any interpolation problem, and some are tailored for preserving certain properties of the flow (e.g. limited acceleration variance, or zero velocity divergence); the later, however, often come at the expense of other properties of the flow. A comprehensive survey of the suitability of all interpolation methods to our task is a very broad undertaking. Out of all the methods we tested, we chose the Inverse Distance Weighting interpolation [32], which is computationally simple and achieves good results compared to other methods. The interpolated value under this method is

\[
\mathbf{u}_i = \frac{\sum_j u_j r_{ij}^{-p}}{\sum_j r_{ij}^{-p}}
\]
where the exponent \( p \) is an arbitrary free parameter. In this study we are able to do a careful analysis by comparing tracers velocity to interpolated values using the same set of flow tracers. Since flow tracers presumably have the same velocity as that of the carrying fluid, the optimal power \( p \) would minimize the least-squares difference between the particle velocity and the interpolated velocity at the same position. Ideally, in error-free measurements one could obtain null difference for ideal tracers, or estimate the correct relative velocity for real tracer particles. For a full set of \( n \) flow tracers velocity measurements for all the frames, we minimize

\[
S = \frac{1}{n} \sum_{j=1}^{n} \sqrt{W_i(j) W_i(j)}
\]

(9)

where \( W_i(j) \) is the fluid-relative velocity vector for particle \( j \). The value minimizing \( S \) in our data is \( p = 1.5 \).

### 3 Results and discussion

#### 3.1 Relative velocity

Figure 2a shows the relative velocity between particles and the fluid surrounding them. For the tracers, the same set of particles is used both for the fluid velocity interpolation and for the tracer velocity measurement. Although the tracer self-test, given in Eq. 9 minimized the average relative velocity to 0.07 m/s, the distribution of tracers relative velocity has tails going out to ±0.2 m/s, for very few events. This represents the addition of tracer nonideality to the interpolation error. The tracers are found throughout the flow and encounter different flow conditions, some of them expose a relatively slow dynamic response even for the tracers. The inertial particles have a wider relative velocity distribution, showing that the heavier particles are also slower to respond to the flow, as one might expect.

In figure 2b, the velocity PDFs are normalized by the standard deviation of each distribution, providing a normalized, or standardized, distribution. Although the PDFs are then close, they do not exactly “collapse” as is the case of the force PDFs
Especially in the tails of the distribution, tracers’ normalized relative velocities are somewhat higher. It can be seen from the differences between the two shapes of the standardized distributions that flow tracers experience some peculiar flow conditions or flow regions that inertial particles avoid or filter out. The fact that the tracers have a non-negligible relative velocity, although significantly smaller than that of inertial particles, indicates that there would be some error in undisturbed velocity estimate that stems from the possibility of tracers not following the flow faithfully. Apparently these extreme cases (turbulent events that correspond to the tails of the distribution) will contribute to the noise in the PDFs of force terms acting on inertial particles, as shown in section 3.2. However, only a negligibly small fraction of samples shows a significant relative velocity, and therefore the noise appears only toward the tails of the force distributions and does not affect the outcome of the study.

In addition, our experimental data of synchronously measured flow velocity and the motion of the inertial particles, provides the option to directly probe the commonly applied assumption on the relative velocity of the inertial particles, $W$, to be of the same order of magnitude as their absolute Lagrangian velocity, $V$, e.g. Ref. [15]. The PDFs of relative and absolute velocities of inertial particles are shown in figure 3 together with the Gaussian fit for each case. The standard deviation of the relative velocity is $\sigma_W = 0.105 \text{ m/s}$ and it is slightly higher than that of
the absolute velocity, $\sigma_V = 0.076$ m/s. The velocities are of the same order of magnitude, yet the distributions show that the relative velocity has broader tails, which is a sign of intermittent turbulent flow in its proximity. The differences between relative and absolute velocity of inertial particles is a sort of measure of its low-pass filtering effect. Except these strong turbulence regions (or time events) the relative velocity and the absolute velocity of the inertial particles are correlated statistically. Figure 3 shows this in a joint PDF of the velocity and the relative velocity. The correlation is close to 1, although not perfect as we can find events with low absolute and high relative velocity, and vice versa.

### 3.2 Forces on the particles in turbulent flow

The major focus of this study is on the estimates of the force terms acting on inertial particles along their trajectories in a turbulent flow. The key method is the analysis of the forces acting on inertial particles as compared to those acting on flow tracers, while both estimates obtained using the same dataset and the same interpolation. Comparing forces acting on tracers and inertial particles effectively, we need to compensate for the different size and density of tracers as compared to inertial particles. Since the absolute values are of different orders of magnitude, we will normalize each probability distribution function (PDF) to its standard deviation, emphasizing the shape of the distribution.
The standard deviation values used to normalize the PDFs in this section are noted in Table 1. The pressure force scaling ratio is of interest in that it serves as an indication of the way tracers see the flow as opposed to inertial particles. Since the pressure force is defined as \( m \frac{Du}{Dt} \) (eq. 4), it should scale by the mass ratio if the distribution of \( \frac{Du}{Dt} \) seen by tracers is the same as that seen by inertial particles. Our measurement indicates that the scaling is close to the mass ratio but somewhat smaller. Some of the difference may be a result of a certain size distribution of the particles around the nominal value. The rest of the difference is linked with the low-pass filtering effect discussed in section 3.1.

The scaling of inertia force, conversely, is larger than the mass ratio. This shows that as a particle grows larger, the role of surface forces grows relative to the role of the pressure force. The particle follows the flow less closely, and hence is affected more by forces related to the relative velocity and its gradient, i.e. drag and lift. This is also seen by the scaling of the drag and lift forces, which are also larger than the mass ratio.

First we consider in Figure 4 the PDFs of the inertia force, \( F_I \) (fig. 4a) and the pressure gradient force, \( F'_p \) (fig. 4b) for tracers and inertial particles.

The standardized distribution of inertia force \( F_I \), which here stands for the particle acceleration, is similar for both tracers and inertial particles, independent of their size and density, as reported previously [16]. This result is counter-intuitive, because tracers and inertial particles are exposed to different accelerations. Recalling that the left hand side of the equation of motion balances the right hand side which is a sum of other forces, we emphasize that the observed similarity of
the two distributions does not come from the same source for the inertial particles and tracers. Each type of particles experiences different interaction forces both by amplitude and by the combination of different terms. The similarity of the standardized PDFs implies that the particle-fluid interaction forces are in some sense responsible to coordinate the particle motion in the flow, keeping the particle from developing a high relative velocity.

The standardized distributions of the pressure force term of the inertial particles and the tracers also behave similarly. In this case, however, the result is expected. This is because for both data sets, the distribution is obtained by interpolation from the tracers series, and only scaled by the particle mass, which is not seen in the standardized plot. The only possible difference between tracers and inertial particles in generating the pressure force PDFs is that the points of interpolation are different, gathered from the different paths taken by tracers and inertial particles. However, both sets of interpolation points are large enough to map the entire flow, so that the standardized PDFs show self-similarity.

The similarity of both inertia (particle acceleration) and pressure (fluid acceleration) force terms provides another peculiar result. Despite the fact that the order of magnitude of forces are different, and despite the different combination of particle-interaction forces (more lift for large particles, different drag components,
etc.) because both the pressure force and the inertia force show similar PDFs, also the drag and lift force terms, have to show similarity in the distributions. As we rigorously prove in appendix A the self-similarity of the total interaction force in these conditions is a mathematical necessity if additionally the interaction forces are statistically independent of the undisturbed fluid velocity. This can happen if the interaction force is a function of not only the relative velocity, but also its gradient, and possibly other factors. Our data shows that the independence of pressure gradient force and total interaction forces is real, as shown by the self-similarity of the total interaction force PDF, figure 5a.

What we find more surprising is that the similarity of standardized distributions is apparent not only in the sum of interaction forces, but also in the orthogonal components of drag (parallel to relative velocity), fig. 5b, and lift (perpendicular to it), fig. 5c. This is a combined result of the interaction force PDFs similarity and isotropy of the flow and low buoyancy, such that the relative velocity and its gradient have no preferred direction. From figure 6 it becomes clear that the drag and lift components have the same magnitude, contrary to the common assumption that lift is small compared to drag forces.

All of the force distributions fit a stretched exponential form, except for the pressure force distribution, which is skewed, and each side fits separately to a stretched exponential with different parameters. In isotropic turbulence, the pressure force distribution should be symmetric, as shown by analytic considerations as well as simulations, therefore the pressure force distribution indicates that for this experiment a certain anisotropy is created by a secondary flow, perhaps related to the system’s geometry. In contrast, the inertia force PDFs are symmetric, as would be expected from particles in isotropic turbulence. The anisotropy of the pressure force is masked by the other forces acting on the particle. These seem to be affected more by the small scales of turbulence, which are independent of the secondary flow. Such result lends support for the mechanism of lift suggested by Kim and Balachander, which focus on fluctuations of a size scale smaller than the particle as the main mechanism responsible for generation of interaction forces.

Analyzing the distributions of the different force components as defined in Eq. 8...
(a) Standardized interaction force ($F_r$ in eq. 8)

(b) Drag force ($F_D$) PDFs

(c) Lift force ($F_L$) PDFs

Figure 5: Dependent force terms, standardized
we can provide here for the first time the plot that emphasizes the relative contributions of each one of the terms. The relative importance of the pressure gradient (fluid-driven) versus the components of fluid-particle interaction forces (relative velocity dependent) is given in figure 6, where not normalized distributions for both the inertial particles and the tracers are shown. In both cases, the distribution of the interaction forces is wider than that of the pressure force and closer to the inertial force. The ability of the particle-fluid interaction forces to mask out the skewness of the pressure gradient force is a result of their relative strength.

In figure 7, we present the distributions of the inertial particles in real values and distributions of tracers scaled by $0.14 \times 10^4$ which is the mass ratio of the inertial particles and tracers. The collapse shows that the distributions are very similar in both cases, despite the three orders of magnitude differences with few exceptions: the drag and lift that tracers experience are shifted more towards the lower values, as compared to the inertial particles. However, in general, we found the similarities between the distributions to be the central (and unexpected) result of this study.
4 Conclusions

The similarity of acceleration PDFs of particles with different size and density is a relatively recent observation that helps to understand better particle motion in turbulent flows. It shows that, at least for particles less than few times the Kolmogorov scales of the flow, the behavior of particles can be estimated using some properties of the turbulent flow, regardless of the particle size. Before this study the similarity was observed only for the distributions of accelerations of the particles.

We analysed a unique data set of directly and simultaneously measured Lagrangian trajectories of flow tracers along with the (almost) neutrally buoyant inertial particles which are ten times larger than the tracers and of the order of magnitude of the Kolmogorov length scale [17]. The particles and the flow were measured in a quasi-isotropic, quasi-homogeneous turbulent flow at $Re_\lambda \sim 250$ between counter-rotating baffled disks. The analysis allowed a deeper look into the distributions of the forces using a suitable formulation of particles’ equation of motion. The central point is our ability to accurately estimate the undisturbed fluid velocity at the position of each inertial particle and along its Lagrangian trajectory, obtained by interpolation from neighbor tracers. After examining several interpolation methods, we chose Inverse Distance Weighting with a parameter that minimizes the error due to interpolation by testing the tracer “relative velocity”.

Figure 7: PDFs of the forces for particles (solid curves) and tracers (dashed curves) in logarithmic and linear scales.
The distributions of forces acting on inertial particles and flow tracers in this turbulent flow indicate that both the pressure gradient force and the particle-fluid interaction forces play a significant role, while the interaction forces exhibit larger values. This means that in a flow of this kind, an accurate model of the interaction forces would be essential in order to faithfully predict particle movement. Furthermore, the role of lift is found to be of the same magnitude as that of drag. Thus the models of particles in turbulence, that are not much smaller than the Kolmogorov scale, must include a lift term.

Standardized PDFs of acceleration, expressed here as inertia force, $F_i$, collapse onto a single curve for both the flow tracers and the inertial particles. The unexpected result is the similar collapse of the distributions of the interaction forces. We derive a mathematical expressions of the PDFs demonstrating this as a consequence of the orthogonal decomposition and the fact that the sum is self-similar.

The relative velocity distribution of inertial particles is broader than that of smaller particles, but much closer to a Gaussian process. This distribution also helps to understand the similarity of the drag force PDFs. We can show using our directly measured values that the relative velocity is well correlated with the particle velocity, emphasizing the known effect of inertia: as the flow gets faster, the slow particle response leads to high relative velocities. The correlation is, nevertheless, not perfect and there are also significant relative velocity values associated with other regions of the flow.

The particle acceleration PDFs are symmetric, in contrast to the asymmetric pressure gradient distribution. This indicates that the mechanism generating particle-fluid interaction forces depends more on the small-scale turbulent fluctuations, as has been recently proposed [35].

The present findings shall be taken with caution - one cannot expect that the PDFs of the force terms will keep its shape similar for any arbitrary large particle size, even for neutrally buoyant particles such as those studied in this work. We note that although similar, as size grows, the relative velocity distribution widens and particles effectively filter out strong turbulent events (in terms of fluid velocity). We believe that the self-similarity of the PDF shape is maintained as long as the relative velocity growth can compensate for the increased size.
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A Proof of self-similarity propagation

In this section we provide a proof to the claim above, that if the inertia force curves of two different particles in the same statistical flow have the same standardized shape, then so will the standardized PDF of the total particle-fluid interaction force.

The following definitions are used for a continuous random variable $X$:

- Its standard distribution is $\sigma_X$.
- The standard-deviation normalized variable is $X^\sigma \equiv X/\sigma_X$.
- Its probability density function is $P_X(x)$ where $x$ is any value that can be taken by $X$.

Using this terminology, the formal theorem is thus: Let $G, D_1, D_2$ be continuous independent random variables, and $m_1, m_2$ scalars. Let

\begin{align}
I_1 & = m_1 G + D_1 \\
I_2 & = m_2 G + D_2
\end{align}

Theorem: If $P_{I_1^\sigma}(x) = P_{I_2^\sigma}(x)$, then $P_{D_1^\sigma}(x) = P_{D_2^\sigma}(x)$ and the distributions hold the relation

$$\frac{m_1 \sigma_{D_1}}{\sigma_{I_1}^2} = \frac{m_2 \sigma_{D_2}}{\sigma_{I_2}^2}$$

The specific fluid-dynamic meaning of each term is explained in section A.2.
A.1 Proof

Divide eq. [10] by \( \sigma_{I_1} \) and multiply each term \( X \) on the right hand by \( \frac{\sigma_X}{\sigma_X} \) to get

\[
I_1\sigma_{I_1} = m_1 \frac{\sigma_G \sigma_{D_1}}{\sigma_{I_1}} + \frac{\sigma_{D_1}}{\sigma_{I_1}} D_1\sigma_{I_1} \equiv \sigma \tag{12}
\]

The PDF of a sum of independent random variables is the convolution of the PDFs of the summed terms, hence

\[
P_{I_1} (x) = m_1 \frac{\sigma_G \sigma_{D_1}}{\sigma_{I_1}^2} \left( P_{G^*} * P_{D_1} \right) (x) \tag{13}
\]

Apply the same procedure, mutatis mutandis, to \( I_2 \). Now substitute into the given

\[
P_{I_1} (x) = P_{I_2} (x)
\]

To remove the common distribution \( G \) we take the Fourier transform of both sides,

\[
m_1 \frac{\sigma_G \sigma_{D_1}}{\sigma_{I_1}^2} \hat{P}_{G^*} (\omega) \hat{P}_{D_1} (\omega) = m_2 \frac{\sigma_G \sigma_{D_2}}{\sigma_{I_2}^2} \hat{P}_{G^*} (\omega) \hat{P}_{D_2} (\omega) \tag{15}
\]

so that \( P_{G^*} \) cancels out, then after the inverse transform is applied,

\[
m_1 \frac{\sigma_G \sigma_{D_1}}{\sigma_{I_1}^2} P_{D_1} (x) = m_2 \frac{\sigma_G \sigma_{D_2}}{\sigma_{I_2}^2} P_{D_2} (x) \tag{16}
\]

Rearranging,

\[
\frac{P_{D_1} (x)}{P_{D_2} (x)} = \frac{m_2 \sigma_{D_2}}{m_1 \sigma_{D_1}} \left( \frac{\sigma_{I_1}}{\sigma_{I_2}} \right)^2 \equiv \alpha \tag{17}
\]

where \( \alpha \) is a constant.

Now, each probability density function must integrate to 1 at infinity,

\[
\int_{-\infty}^{\infty} P_{D_1} (x) \, dx = 1 \tag{18}
\]

\[
\int_{-\infty}^{\infty} P_{D_2} (x) \, dx = 1 \tag{19}
\]
and from eq. [17]

\[ \int_{-\infty}^{\infty} P_{D_1^x} (x) \, dx = \int_{-\infty}^{\infty} \alpha P_{D_2^x} (x) \, dx = \alpha \int_{-\infty}^{\infty} P_{D_3^x} (x) \, dx = \alpha \]  

(20)

So that with eq. [18] we get \( \alpha = 1 \). This allows us to rearrange eq. [17] in two ways:

- The lefthand side expands to \( P_{D_1^x} (x) = P_{D_2^x} (x) \),
- the righthand side expands to

\[ \frac{m_1 \sigma_{D_1}}{\sigma_{I_1}^2} = \frac{m_2 \sigma_{D_2}}{\sigma_{I_2}^2} \]

QED.

As an aside we note that the converse claim, \( P_{D_1^x} (x) = P_{D_2^x} (x) \to P_{I_1^x} (x) = P_{I_2^x} (x) \), is also trivially true, as immediately seen from eq. [13].

A.2 Application of the proof to distributions of particles in turbulent flow

The standardized inertia force is the sum of several force terms acting on the particle. In its simplest form, the particle’s equation of motion may be written as

\[ m_p \frac{dV}{dt} = m_F \frac{dU}{dt} + D \]  

(21)

where \( D \) is the sum of forces emanating from the interaction of the particle with the fluid, collecting surface forces such as Stokes form drag, Saffman lift, Basset history term etc.

In terms of the theorem above, we define

\[ m_p \frac{dV}{dt} \equiv I \]

\[ \frac{dU}{dt} \equiv G \]
When the same flow contains two types of particles, e.g. small, neutrally buoyant tracers together with larger inertial particles, we have $I_{1,2}, m_{F1,2}, D_{1,2}$. But $G$ is common to both particle sizes, as they both share the same flow. Substituting these terms into eq. 21 we get eqns. 10.

The observation that acceleration PDFs are invariant translates into $P_{I_1\sigma}(x) = P_{I_2\sigma}(x)$, which is the condition for the propagation of invariance theorem, yielding $P_{D_1\sigma}(x) = P_{D_2\sigma}(x)$.

The conclusion is that the PDF of total particle-fluid interaction forces must show standardized invariance, independent of the pressure gradient force, if standardized acceleration PDFs are invariant.

References

[1] Rebecca L Lankey, Cliff I Davidson, and Francis C McMichael. Mass balance for lead in the california south coast air basin: An update. Environmental Research, 78(2):86 – 93, 1998.

[2] Edward Peltier, Pavan Ilipilla, and David Fowle. Structure and reactivity of zinc sulfide precipitates formed in the presence of sulfate-reducing bacteria. Applied Geochemistry, 26(9-10):1673 – 1680, 2011.

[3] Yoojeong Kim, Ashok Gidwani, Barbara E. Wyslouzil, and Chang W. Sohn. Source term models for fine particle resuspension from indoor surfaces. Building and Environment, 45(8):1854 – 1865, 2010.

[4] Inci Ayranci, Márcio B. Machado, Adam M. Madej, Jos J. Derksen, David S. Nobes, and Suzanne M. Kresta. Effect of geometry on the mechanisms for off-bottom solids suspension in a stirred tank. Chemical Engineering Science, 79(0):163 – 176, 2012.

[5] C.E. Brennen. Fundamentals of Multiphase Flow. Cambridge University Press, 2005.
[6] R Greeley, MR Balme, JD Iversen, S Metzger, R Mickelson, J Phoreman, and B White. Martian dust devils: Laboratory simulations of particle threshold. *Journal of Geophysical Research-Planets*, 108(E5), MAY 17 2003.

[7] M. Balme and A. Hagermann. Particle lifting at the soil-air interface by atmospheric pressure excursions in dust devils. *Geophysical Research Letters*, 33(19), OCT 14 2006.

[8] G. Ziskind. Particle resuspension from surfaces: Revisited and re-evaluated. *Reviews in Chemical Engineering*, 22(1-2):1–123, 2006.

[9] E. Rabinovich and H. Kalman. Incipient motion of individual particles in horizontal particle-fluid systems: B. Theoretical analysis. *Powder Technology*, 192(3):326–338, JUN 25 2009.

[10] M.W. Reeks and D. Hall. Kinetic models for particle resuspension in turbulent flows: theory and measurement. *Journal of Aerosol Science*, 32(1):1 – 31, 2001.

[11] F. Zhang, M. Reeks, and M. Kissane. Particle resuspension in turbulent boundary layers and the influence of non-gaussian removal forces. *Journal of Aerosol Science*, 58(0):103 – 128, 2013.

[12] W. Cheng, Y. Murai, M.-A. Ishikawa, and F. Yamamoto. An algorithm for estimating liquid flow field from PTV measurement data of bubble motion. *Transactions of Visualization Society of Japan*, 23(11):107–114, 2003.

[13] M.A. Rizk and S.E. Elghobashi. The motion of a spherical-particle suspended in a turbulent-flow near a plane wall. *Physics of Fluids*, 28(3):806–817, 1985.

[14] J. Yao and M. Fairweather. Particle deposition in turbulent duct flows. *Chemical Engineering Science*, 84:781–800, DEC 24 2012.

[15] Robert Zimmermann, Yoann Gasteuil, Mickael Bourgoin, Romain Volk, Alain Pumir, and Jean-François Pinton. Rotational intermittency and turbulence induced lift experienced by large particles in a turbulent flow. *Phys. Rev. Lett.*, 106:154501, Apr 2011.

25
[16] E. Calzavarini, R. Volk, M. Bourgoin, E. Leveque, J. F. Pinton, and F. Toschi. Acceleration statistics of finite-sized particles in turbulent flow: the role of Faxen forces. *Journal of Fluid Mechanics*, 630:179–189, JUL 10 2009.

[17] Michele Guala, Alexander Liberzon, Klaus Hoyer, Arkady Tsinober, and Wolfgang Kinzelbach. Experimental study on clustering of large particles in homogeneous turbulent flow. *Journal of Turbulence*, 9(34):1–20, 2008.

[18] A. Liberzon, M. Guala, B. Lüthi, W. Kinzelbach, and A. Tsinober. Turbulence in dilute polymer solutions. *Physics of Fluids (1994-present)*, 17(3):–, 2005.

[19] B Lüthi, A Tsinober, and W Kinzelbach. Lagrangian measurement of vorticity dynamics in turbulent flow. *Journal of Fluid Mechanics*, 528:87–118, APR 10 2005.

[20] G. A. Voth. *Lagrangian Acceleration Measurements in Turbulence at Large Reynolds Numbers*. PhD thesis, Cornell University, 2000.

[21] William H. Press et al. *Numerical Recipes: The Art of Scientific Computing, Third Edition*. Cambridge University Press, 2007.

[22] R. Mei. Velocity fidelity of flow tracer particles. *Experiments in Fluids*, 22(1):1–13, 1996.

[23] A Melling. Tracer particles and seeding for particle image velocimetry. *Measurement Science & Technology*, 8(12):1406–1416, DEC 1997.

[24] M. Mattson and K. Mahesh. Simulation of bubble migration in a turbulent boundary layer. *Physics of Fluids*, 23(4):045107, 2011.

[25] M. R. Maxey and J. J. Riley. Equation of motion for a small rigid sphere in a nonuniform flow. *Physics of Fluids*, 26:883–889, April 1983.

[26] S. Corrsin and J. Lumley. On the equation of motion for a particle in turbulent fluid. *Applied Scientific Research, Section A*, 6(2-3):114–116, 1956.

[27] Enrico Calzavarini, Romain Volk, Emmanuel Lévêque, Jean-François Pinton, and Federico Toschi. Impact of trailing wake drag on the statistical
properties and dynamics of finite-sized particle in turbulence. *Physica D: Nonlinear Phenomena*, 241(3):237 – 244, 2012. Special Issue on Small Scale Turbulence.

[28] R Gatignol. The Faxèn formulas for a rigid particle in an unsteady non-uniform stokes-flow. *Journal de Mecanique Theoretique et Appliquee*, 2(2):143–160, 1983.

[29] PG Saffman. Lift on a small sphere in a slow shear flow. *JOURNAL OF FLUID MECHANICS*, 22(2):385–&, 1965.

[30] G. Sridhar and J. Katz. Drag And Lift Forces On Microscopic Bubbles Entrained By A Vortex. *Physics Of Fluids*, 7(2):389–399, FEB 1995.

[31] Barry Ford and Eric Loth. Forces on ellipsoidal bubbles in a turbulent shear layer. *Physics of Fluids*, 10(1):178–188, 1998.

[32] Donald Shepard. A two-dimensional interpolation function for irregularly-spaced data. In *Proceedings of the 1968 23rd ACM national conference*, ACM ’68, pages 517–524, New York, NY, USA, 1968. ACM.

[33] M Holzer and E Siggia. Skewed, Exponential pressure distributions from Gaussian velocities. *Physics of Fluids A-Fluid Dynamics*, 5(10):2525–2532, OCT 1993.

[34] Prakash Vedula and P. K. Yeung. Similarity scaling of acceleration and pressure statistics in numerical simulations of isotropic turbulence. *Physics of Fluids (1994-present)*, 11(5):1208–1220, 1999.

[35] Jungwoo Kim and S. Balachandar. Mean and fluctuating components of drag and lift forces on an isolated finite-sized particle in turbulence. *Theoretical and Computational Fluid Dynamics*, 26(1-4):185–204, JAN 2012.