Solid-fluid coupling in a fully Lagrangian framework

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The simulation of multiphase flows consisting of granular and fluid phases is of great interest in a wide variety of industrial applications, such as chemical process engineering, design of conveyor systems and abrasion modeling. In this context, the granular phase is often described by the discrete element method, which calculates the trajectory of each individual solid particle in a Lagrangian manner while resolving inter-particle and geometry collisions. While the models behind individual interactions are generally not very complex, the necessary data structures and neighbor search algorithms often have a major impact on performance. Here, we present the coupling of this approach to a Lagrangian generalized finite difference method for the fluid phase which has been successfully used in a wide variety of practical applications. Coupling these two approaches enables us to treat all phases in a common framework and to use efficient and scalable data structures and algorithms. At the same time we retain the advantages of meshfree fluid solvers in free surface problems or rapidly changing flow geometries.

1 Model equations

We consider the unresolved coupling of a fluid and solid phase, which is based on the application of weighted averaging to the point-wise quantities and conservation equations of the continuous phase. For a fluid phase with constant density \( \rho_f \) and body force \( \mathbf{g} \), this results in the volume averaged Navier-Stokes (VANS) equations

\[
\frac{D(\varepsilon_f \rho_f)}{Dt} = -\varepsilon_f \rho_f \left( \nabla \cdot \langle \mathbf{u}_f \rangle \right) \\
\varepsilon_f \frac{D\langle \mathbf{u}_f \rangle}{Dt} = -\nabla \langle p \rangle + \nabla \cdot (\langle \tau_f \rangle) - \mathbf{F}_{fs} + \varepsilon_f \rho_f \mathbf{g} 
\]

with \( \varepsilon_f \), \( \langle \cdot \rangle \), \( \mathbf{F}_{fs} \) denoting the fluid phase volume fraction or porosity, the fluid phase intrinsic average and the fluid-solid interaction force density, respectively. For the averaged viscous stress we use \( \langle \tau_f \rangle = \tau_f \langle \mathbf{u}_f \rangle \) where \( \tau_f \) is the viscous stress in a Newtonian fluid under Stokes hypothesis. We further note that Reynolds type terms, resulting from small scale fluctuations about the averages, have been omitted. The momentum coupling term \( \mathbf{F}_{fs} \) is calculated through solid phase averaging of the coupling forces \( f_{fs,j} \) experienced by solid particles \( x_{s,j} \). We will restrict ourselves to contributions through pressure gradient, divergence of viscous stress tensor and drag force \( \mathbf{f}_{d,j} \), such that the weighted average with kernel \( w : \mathbb{R} \to \mathbb{R}_{\geq 0} \) reads

\[
\mathbf{F}_{fs}(\mathbf{x}) = \sum_j w(\|\mathbf{x} - x_{s,j}\|) f_{fs,j}, \quad \mathbf{f}_{fs,j} = -\nabla \langle p \rangle|_{x_{s,j}} + (\nabla \cdot \langle \tau_f \rangle)|_{x_{s,j}} + \mathbf{f}_{d,j} 
\]

2 Numerical scheme

We use a Lagrangian generalized finite difference method (GFDM)\textsuperscript{11} to solve the VANS equations and solid particle motion within a unified framework. The fluid phase is discretized by a point cloud and in our numerical scheme every time step begins with the movement of fluid points according to their attached velocities. Then, all necessary quantities are interpolated onto the solid particles so that coupling forces can be evaluated. For inter-particle and particle-wall collisions, a simple spring-damper model is employed. When the solid phase equations of motion have been integrated, the volume fraction \( \varepsilon_{s,i} \) and interaction force density \( F_{fs,i} \) at fluid point positions \( \mathbf{x}_{r,i} \) are calculated. We apply a discrete kernel re-normalization

\[
\varepsilon_{s,i} = \sum_j w_{ij} V_{s,j}, \quad F_{fs,i} = \sum_j w_{ij} f_{fs,j}, \quad w_{ij} = \frac{w(\|\mathbf{x}_{r,i} - x_{s,j}\|)}{\sum_k \Omega_k w(\|\mathbf{x}_{r,i} - x_{s,j}\|)} 
\]

which uses volumes \( |\Omega_k| \) from local triangulations calculated only on neighborhoods in the point cloud. Note that these local triangulations do not conflict with the meshfree nature of our approach since connectivity information is not stored and it is not required that they combine to a global domain discretization. In the calculation of these solid phase averages we can further take advantage of the neighborhood search intrinsic to the GFDM approach. When the solid phase averages are calculated we employ a solution procedure for the VANS equations that is based on a projection approach.

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3 Validation simulations

As a first test case we consider a single particle falling in a column filled with water under the effect of gravity and Stokes drag. We use a cylinder with quadratic base of side length $L = 40d_p$ and height $H = 60d_p$ so that the no-slip walls do not significantly influence the solid phase averaging or flow field at the particle position and choose a particle which has 2.5 times the density of the fluid. We compare the sedimentation velocity obtained from our simulations to the analytical results $v_0$ that can be obtained for the motion in a quiescent fluid. The considered time frame is chosen relative to the relaxation time $t_r = \rho_d d_p^2 / 18\mu$ and we evaluate the relative error $\epsilon_v$ in relative velocity $v(t) - v_0(x, t)$ when compared to $v_0$.

The results of this comparison are shown in Fig.1. We observe negligible errors for two different point cloud resolutions $h = 3d_{pc}$, $h = 6d_p$ corresponding to approximately 83000 and 13500 points, as well as both the one-way (solve single phase Navier-Stokes instead of VANS) and two-way coupling (full coupling with VANS). This example serves as a validation for the calculation of coupling forces at the particle position and to show that cases with little fluid disturbance can be reproduced.

As a next step in validating our scheme we want to consider a more sophisticated drag force and a case where the particles have significant influence on the fluid phase. We consider the flow of water through a channel with no-slip walls and without the effect of gravity and place a static block of particles randomly positioned across the whole cross section and half the channel length. From the bounding box of this block a volume fraction of $\epsilon_v = 0.3312$ is obtained. We prescribe different inflow velocities and compare the resulting pressure drop due to the particles with empirical correlations. As drag force we use the extension of single particle Schiller-Naumann drag through an exponential voidage function that has been fitted to extensive lattice Boltzmann simulations by Rong et al. [4].

The results of this comparison are shown in Fig.2. We observe that the predictions from Ergun's equation [2] were matched by our simulations quite well, however, especially at lower Reynolds number our results deviate slightly from the second correlation by Eisfeld and Schnitzlein [3]. Since neither the drag force modifications nor the pressure drop equations are always derived on the same basis, modeling errors are also present in such comparisons. Taking this into account we can conclude, that our simulations showed satisfactory results, which confirms the correct solid phase averaging and solution of volume averaged equations in the fluid phase.

4 Outlook

The next logical extension of the examples presented above would be a case where we combine the particle movement, solid phase averaging and solution of volume averaged equations to verify all components in tandem. Such a validation case could be the process of fluidization which provides a large pool of experimental and numerical results and is a typical field of application for unresolved fluid-solid coupling. With the last validation step completed, it will then be interesting to investigate the performance of our coupled solver in scenarios like free surface flows where the fully Lagrangian nature of our approach can show its advantages. In such contexts we might be faced with challenges in the averaging procedure due to the calculation of representative volumes in the fluid point cloud and particle movement across continuous phase interfaces.

References

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