COLOR TRANSPARENCY AND THE VANISHING
DEUTERIUM SHADOW

L. L. Frankfurt\textsuperscript{a,e}, W. R. Greenberg\textsuperscript{b}, G. A. Miller\textsuperscript{c}, M. M. Sargsyan\textsuperscript{a,f}, M. I. Strikman\textsuperscript{d,e}

\textsuperscript{(a)} School of Physics and Astronomy, Tel Aviv University, 69978, Israel,
\textsuperscript{(b)} Department of Physics, University of Pennsylvania, Philadelphia, PA 19104
\textsuperscript{(c)} Department of Physics, University of Washington, Seattle, WA 98195
\textsuperscript{(d)} Department of Physics, Pennsylvania State University, University Park, PA 16802
\textsuperscript{(e)} Institute for Nuclear Physics, St. Petersburg, Russia
\textsuperscript{(f)} Yerevan Physics Institute, 375036, Armenia

Abstract

We find that the final state interactions in the $d(e,e'np)$ amplitude depend strongly on the final momentum of the spectator nucleon. This means that color transparency effects can be studied at rather low four-momentum transfer $Q^2 \geq 4$ (GeV/c)$^2$ using un-polarized and polarized deuteron targets.
The Continuous Electron Beam Accelerator Facility CEBAF and HERMES (DESY) are about to begin running experiments. These new installations offer unprecedented luminosity and a continuous beam \[1,2\]. Several electron-deuteron scattering experiments (including some with polarized targets) are planned to study \(ed\) reactions at CEBAF and HERMES. The new features of these experiments is the ability to detect also the final state nucleons (hadrons) in coincidence with scattered electrons covering transferred energy range \(1\, (\text{GeV}/\text{c})^2 \leq Q^2 \leq 20\, (\text{GeV}/\text{c})^2\).

We point out that these very same experiments can be used to gain insight into how quantum chromodynamics influences nuclear interactions at fairly low values of \(Q^2\). To understand this, recall some well known properties of the deuteron. First, the deuteron is the best understood nuclear system, with a wave function determined experimentally in a wide momentum range \[3\]. Second, it has long been known that the total cross section for projectile-deuterium scattering deviates from the sum of the neutron and proton cross sections because one of its particles sometimes lies in the shadow cast by the other \[4\]. The earliest estimates are that

\[
\sigma_d = \sigma_n + \sigma_p - \frac{\sigma_n\sigma_p}{4\pi} < d \left| \frac{1}{r^2} \right| d >, \tag{1}
\]

where \(r\) is the operator representing the internucleon separation and \(|d\rangle\) is the deuteron wave function. The \(\frac{1}{r^2}\) behavior causes the second scattering to occur for small distances \(r\) despite the typically large \(\approx 4\, \text{fm}\) separation between the \(n\) and the \(p\). This is exploited below.

In QCD, the absorption of a high momentum virtual photon by a nucleon leads to the production of a small-sized color singlet state for sufficiently high values of \(Q^2\). We optimistically term the small-sized wave packet a point like configuration PLC. Such a state would not interact with the spectator nucleon, and the deuteron would lose its shadow. This vanishing of a final state interaction (FSI) has been termed color transparency (CT).

Despite intense experimental and theoretical investigation no unambiguous evidence for this novel effect has been observed. It is our view that a PLC is actually produced, but that it expands as it propagates through the nucleus \[3\]. The expanded system interacts strongly and obscures the physics of the initial PLC. The time or distance required for the expansion is of the order of \(l_h \sim 0.4(p/\text{GeV})\, \text{fm}\), where \(p\) is the momentum of the PLC. If \(l_h\)
is greater than the nuclear radius the expansion effects are minimal. This condition, which can be achieved for very high values of the momentum transfer, has not yet been met in an experiment.

The precise d(e,e’pn) experiments we discuss could provide the long-sought signature of CT. The deuteron wave function peaks for $r \approx 1.8 \text{ fm}$, and the relevant distances for rescattering are determined by operators which include a $\frac{1}{r^2}$ dependence. Thus for the deuteron (and other light nuclei) the PLC needs only to remain of small size for short propagation distances.

Suppose an incident virtual photon of four momentum ($\nu, \vec{q}$) leads to the detection of an outgoing nucleon with the large momentum $\vec{p}_f = \vec{q} - \vec{p}_s$, and the other “spectator” nucleon of momentum $\vec{p}_s(p^z_s, p^t_s)$, in which the subscript $z$ and $t$ denotes a direction parallel and perpendicular to that of $\vec{q}$. The importance of FSI is maximized by using so-called “perpendicular kinematics” \cite{6}, in which the light cone fraction of the deuteron momentum carried by a spectator nucleon (mass $m$) $\alpha \equiv \sqrt{m^2 + p^2_s - p^z_s} \approx 1$, but $p_t$ is not negligible. Then the spectator momentum $\vec{p}_s$ is approximately perpendicular to $\vec{q}$.

The scattering amplitude $\mathcal{M}$, including the $np$ final state interaction, can be written using the eikonal approximation as

$$\mathcal{M} = \langle p^z_s, \vec{p}_t | d > - \frac{1}{4i} \int \frac{d^2 k_t}{(2\pi)^2} \langle p^z_s, \vec{p}_t - \vec{k}_t | d > \times$$

$$f^{np}(\vec{k}_t) [1 - i\beta],$$

where $\vec{p}_s = p^z_s - (E_s - m) \frac{M_d + \nu}{|q|}$, $E_s = \sqrt{p^2_s + m^2}$ and $M_d$ is the mass of the deuteron. Spin indices and factors arising from the electron-nucleon interaction are suppressed to simplify the notation. The factor $\beta$ accounts for theta function of the eikonal Green’s function. However when $\alpha \to 1 \beta \approx 0$ \cite{6}. The function $f^{np}$ represents the FSI between the outgoing nucleons. We use a parametrization

$$f^{pn} = \sigma^{pn}_{tot} (i + a_n) e^{-b_n k^2_t/2},$$

for the $np$ scattering amplitude. The quantities $\sigma^{pn}_{tot}$, $a_n$ and $b_n$, at $Q^2 > 3$ (GeV/c)$^2$ depend weakly on the momentum of the knocked-out nucleon with $\sigma^{pn}_{tot} \approx 40$ mb, $a_n \approx -0.2$ and $b_n \approx 6 - 8$ GeV$^{-2}$ for the kinematics of our interest.
The sensitivity to CT effects that we observe rests on the very different $\vec{p}_t$ dependence of the two terms of Eq. (2) at $\alpha \approx 1$. What can one expect? For $\vec{p}_t \approx 0$, the ratio of the second term to the first is of the order of $-\sigma_{tot}^{m}/16\pi R_d^2$ and is small and negative. Thus, at low $p_t$, final state interactions reduce the value of the computed cross section. This is the shadowing effect mentioned above. But the first term falls more rapidly than the second as the magnitude of $\vec{p}_t$ increases. This is because the fall off is controlled by the large deuteron size in the first term and by the small range of the $np$ interaction in the second term. (In the limit of zero range ($b_n \to 0$), the second term is proportional to $\int d^3r \frac{1}{r} < \vec{r}|d> \cdot$) As $p_t$ increases from zero, the relative importance of the shadowing grows. However, if $p_t$ is further increased, the value of $|\frac{M}{<\vec{p}_t|d>}|^2$ actually increases!

We define the transparency $T$ as the ratio of the measured cross section (or calculated cross section with FSI) to the one calculated in the plane wave impulse approximation (PWIA):

$$T(Q^2, \vec{p}_t, \alpha) \equiv \frac{\sigma_{d(e,e'pm)}^{FSI}(Q^2, \vec{p}_t, \alpha)}{\sigma_{d(e,e'pm)}^{PWIA}(Q^2, \vec{p}_t, \alpha)}.$$  \hspace{1cm} (4)

Fig. 1 shows the dramatic dependence of $T$ on the magnitude of $\vec{p}_t$ as a function of $Q^2$ for an unpolarized target.

At $p_t \leq 200$ MeV/c the FSI lead to shadowing, with $T(p_t = 0) \approx 0.97$ and a much smaller $T(p_n^t = 0.2$ GeV/c) $\approx 0.5$. But for $p_t > 300$ MeV/c one finds $T > 1$. These features are apparent for all $Q^2 \geq 2$ (GeV/c)$^2$, but are more significant for the larger values of $Q^2$.

Other kinematics are examined in Ref. [7].

Including the effects of CT would change the results of Fig. 1. For sufficiently large $Q^2$ the final state interactions would be eliminated entirely. But for values of $Q^2 \leq 10$ GeV$^2$, the deuteron is not completely transparent. Our calculations must include the effects of PLC expansion. We use two models which account for the formation of the PLC and their evolution to the normal hadronic state: the quantum diffusion model [8] and the three state model of Ref. [9]. For both models, the parameters we use are in the range consistent with the $(p,2p)$ [10] and SLAC (e,e'p) [11] data.

The reduced interaction between the PLC and the spectator nucleon can be described in terms of its transverse size and distance $z$ from the photon absorption point. In the quantum diffusion model the PLC-N scattering amplitude takes the form [12]:

\hspace{1cm}
\[ f^{PLC,N}(z, k_t, Q^2) = i\sigma_{tot}(z, Q^2) \cdot e^{\frac{b_2 t}{2}} \times \frac{G_N(t \cdot \sigma_{tot}(z, Q^2)/\sigma_{tot})}{G_N(t)}, \]

where \( t = -k_t^2 \), and \( G_N(t) \) is the Sachs form factor. In Eq. (5) \( \sigma_{tot}(z, Q^2) \) is the effective total cross section of the interaction of the PLC at the distance \( z \) from the interaction point. This is [8]:

\[
\sigma_{tot}(z, Q^2) = \sigma_{tot}^{pn} \left\{ \left( \frac{z}{l_h} + \frac{\langle r_t(Q^2)^2 \rangle}{\langle r_t^2 \rangle} (1 - \frac{z}{l_h}) \right) \Theta(l_h - z) + \Theta(z - l_h) \right\},
\]

where \( l_h = 2p_n/\Delta M^2 \), with \( \Delta M^2 = 0.7 \text{ GeV}^2 \). Here \( \langle r_t(Q^2)^2 \rangle \) represents the transverse size of the initially produced configuration. Several realistic models indicate [13] that this is negligibly small for \( Q^2 \geq 1.5 \text{ GeV}^2 \).

The three state models, which allows also the computation of resonance production cross sections, is based on the assumption that the hard scattering operator acts on a nucleon to produce a non-interacting \(|PLC\rangle\) which is a superposition of three baryonic states:

\[ |PLC\rangle = \sum_{m=N,N^*,N^{**}} F_{m,N}(Q^2) |m\rangle, \]

where \( F_{m,N}(Q^2) \) are elastic \((m = N)\) and inelastic transition form factors. We assume that all form factors have the same \( Q^2 \)-dependence and also neglect possible spin effects in the form factors. CT is introduced by the statement that the initially produced PLC undergoes no FSI [9],

\[ T_S |PLC\rangle = 0, \]

where \( T_S \) is the matrix representing the soft final state interactions. \( T_S \) is represented by the most general \( 3 \times 3 \) Hermitian matrix consistent with Eq. 8. We use \( T_S \) of Ref. [4], with the parameters \( M_{N^*}^*=1.4 \text{ GeV}, M_{N^{**}}^*=1.8 \text{ GeV}, \epsilon = 0.17, F_{N,N}/F_{N,N^*} = 1.0, F_{N^*,N}/F_{N,N^{**}} = 3.1 \).

We compare the predictions of these two models of CT in Fig. 2. The ratios of quantities \( T \) of Eq. (3) computed with FSI according to CT - \( T^{CT} \) or according to the usual Glauber approximation - \( T^{GA} \) are shown. We find \( T^{CT}/T^{GA} > 1 \) for \( p_t \leq 200 \text{ MeV}/c \), and \( T^{CT}/T^{GA} < 1 \) for \( p_t > 300 \text{ MeV}/c \).
One may also compute and measure ratios of cross sections for different values of $p_t$. This quantity represents the ratio of directly measured experimental quantities, and does not require additional normalization to the corresponding PWIA calculation (as in Eq (3)). A study of Fig. 2 shows that the effects of color transparency can modify such ratios by as much as 30% for $Q^2$ as low as 6-10 (GeV/c)$^2$.

We next discuss the possibility of using a polarized deuteron target to investigate color coherent effects. Using different target polarizations emphasizes the role of the deuteron $d$-state causing smaller space-time intervals to be probed. For numerical estimates we consider the asymmetry $A_d$ measurable in electrodisintegration of the polarized deuteron:

$$A_d(Q^2, \vec{p}_s) = \frac{\sigma(1) + \sigma(-1) - 2\sigma(0)}{\sigma(1) + \sigma(0) + \sigma(-1)}$$

(9)

where $\sigma(s) \equiv \frac{d\sigma^{s,z}}{dE_{e'}d\Omega_{e'}dp}$, $s$ and $s_z$ are the spin and its $z$ component of the deuteron.

It is useful to recall some properties of the $s$- $u(k)$ and $d$- $w(k)$ state wave functions in momentum space. The quantity $u(k)$ decreases as $k$ increased from 0, and changes sign at $k \approx 400$ MeV/c, while $w(k)$ grows with $k$ from a negative minimum at $k \approx 100$ MeV/c. Thus in some range of momenta, the $w(k)$ is comparable to (or larger) than $u(k)$ (for details see [3]). These well-established features cause the tensor polarization (the numerator of Eq. (3)) to be comparable to the unpolarized cross section (denominator of Eq. (3)). In particular, at $p_t \approx 300$ MeV/c and in the PWIA the asymmetry calculated according to Eq. (9) is close to unity [4]. Deviations from unity originate predominantly from the effects of FSI.

We present the $Q^2$ dependence of the asymmetry $A_d$ for “perpendicular” kinematics, at $p_t = 300$ MeV/c. This figure clearly demonstrates the computed importance of CT effects.

The reliability of our interpretation of an experimental measurement depends on the dominance of FSI in causing deviations of $T$ and $A_d$ away from the plane wave results. We claim that competing effects are restricted to small values by the chosen conditions: (a) only small nucleon momenta in the deuteron $\leq 300 - 350$ MeV/c are relevant here, (b) perpendicular kinematics, $\alpha \approx 1$, (c) the observables are the ratios of experimental quantities in (nearly) similar kinematical conditions, (d) the FSI amplitude of Eq. (2) is dominated by small values of nucleon Fermi momenta.
The conditions (a) and (b) are sufficient to suppress relativistic effects of nucleon motion in the initial state. One measure of such effects is the difference between the nucleon Fermi momenta defined in the light cone \[15\] and nonrelativistic theories of deuteron: 
\[
\sqrt{m^2 + p_t^2} - m^2 - \sqrt{k_z^2 + p_t^2} \mid_{\alpha \to 1} \approx p_t^3/8m^2.
\]
This is small if (a) and (b) are satisfied. The deuteron wave functions are well-known if condition (a) holds \[3\]. Relativistic effects and nucleon binding effects were examined closely in Ref. \[7\] and the overall effects are no more than a few percent of \(T\) and \(A_d\).

The influence of meson exchange currents (MEC) and \(\Delta\)-isobar contributions (IC) \[10\] are mechanisms which are potentially competitive with the influence of FSI. But, for our kinematics MEC are suppressed by the structure of the \(\gamma^*N \to N\pi\) transition matrix. Such transitions are related to the nucleon sea quarks. These are not very important for \(Q^2 \geq 1\) GeV\(^2\) and \(x_{Bj} \sim \alpha \sim 1\) where valence quarks dominate.

The role of IC is also expected to be small, because the \(\gamma^*N \to \Delta\) transition form factor decreases more rapidly with \(Q^2\) than elastic nucleon form factors \[17\]. The \(\Delta\) contributions are further suppressed because the \(\Delta N \to NN\) amplitude is predominantly real and decreases rapidly with energy (since it is dominated by pion exchange) whereas the FSI effects we study are determined by the imaginary part of the soft rescattering amplitude. Another FSI channel involves the final state pion charge-exchange (CHE) interaction. In our kinematics \(-t \geq 0.05\) (GeV/c)\(^2\). The charge-exchange amplitude drops more strongly with \(-t\) than the elastic amplitude, causing the correction to be small (see e.g. \[18\]).

To estimate MEC, IC, and CHE contributions numerically the predictions of Glauber approximations were compared in Ref. \[7\] with the predictions of the model of Arenhövel et al. \[19\], which includes all of those effects. For perpendicular kinematics, the combined influence of MEC, IC and CHE cause only \(\sim 10\%\) changes in computed values of \(T\) for \(Q^2 = 1\) (GeV/c)\(^2\) and \(p_t = 400\) MeV/c. These contributions even smaller for larger values of \(Q^2\) and \(p_t\).

For perpendicular kinematics, the combined influence of the various correction terms discussed above are typically no more than \(\sim 5\%\) and \(\sim 10\%\), at \(Q^2 > 2\) (GeV/c)\(^2\), for unpolarized \(T\) and polarized measurements \(A_d\). This is significantly smaller than the effects of CT that we predict.
The use of “perpendicular” kinematics significantly increases the sensitivity to FSI and allows smaller than average internucleon distances to be probed. Depending on whether the Born or rescattering term dominates the Glauber approximation to the scattering amplitude, CT effects cause either an increase or decrease of the $d(e, e'pn)$ cross section. Based on this, we suggest that the ratios of $d(e, e'np)$ cross sections at $\alpha \approx 1$, $p_t \sim 200$ MeV/c and at $\alpha \approx 1$, $p_t \sim 300$ MeV/c be measured. Our calculations of this ratio predict 20-40% effects for $Q^2 \sim 4 - 10$ (GeV/c)$^2$. We also suggest that the tensor asymmetry in the $d(e, e'pn)$ reaction be measured. Here the conventional Glauber approximation predicts a huge FSI and the CT models predict a 50 - 100% change in the asymmetry at $Q^2 \sim 4 - 10$ (GeV/c)$^2$. The measurements we suggest could provide definitive evidence for or against color transparency.

This work is supported in part by the U.S. Department of Energy and BSF.
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FIGURES

FIG. 1. $T$ of Eqs. (4) and (2), $\alpha = 1$, $\vec{p}_s \approx \vec{p}_t$.

FIG. 2. $p_t$ and $Q^2$ dependence of ratios of $T^{GA}/T^{CT}$ of Eq. (4), $\alpha = 1$. a) quantum diffusion, b) three state model.

FIG. 3. $A_d(Q^2, \vec{p}_s, \vec{p}_t)$ of Eq. (9). Solid line - GA, dashed line quantum diffusion model, dash-dotted - three state model, dotted line PWIA. $\alpha = 1$, $p_t = 300$ MeV/c.
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