AN ANALYTIC STUDY OF THE GRAVITATIONAL WAVE PULSAR SIGNAL WITH SPIN DOWN EFFECTS

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Abstract

In this work, we present the analytic treatment of the Fourier Transform (FT) of the Gravitational Wave (GW) signal from a pulsar including spin down corrections in a parametrized model discussed by Brady et. al. The formalism lends itself to a development of the FT in terms of well known special functions and integrals defining the spin down moments.

1 Introduction

The detection of gravitational waves (GW) from astrophysical sources is one of the most outstanding problems in experimental gravitation today. Large laser interferometric gravitational wave detectors like the LIGO, VIRGO, LISA, TAMA 300, GEO 600 and AIGO are potentially opening a new window for the study of a vast and rich variety of nonlinear curvature phenomena. This network of gravitational wave detectors can confirm that GW exist and monitoring these wave forms give important information on their amplitudes, frequencies and other important physical parameters.

The detection of GW necessitates the substantial accumulation of Signal to Noise (S/N) over long observation periods. The data analysis for continuous GW like, for example, from rapidly spinning neutron stars is an important problem for ground based interferometric detectors that demands analytic, computational and experimental ingenuity.

In recent works [2, 3] we have implemented the Fourier transform (FT) of the Doppler shifted GW signal from a pulsar with the Plane Wave Expansion in Spherical Harmonics (PWESH). Spherical-harmonic multipole expansions are used throughout theoretical physics. Indeed, they arise wherever one deals with fields, be they electromagnetic, gravitational, hydrodynamical and solid body etc. The expansion of a plane wave in spherical harmonics has a variety of applications not only in quantum mechanics and electromagnetic theory [4], but also in many other fields. In linear theories, such as vacuum electromagnetic-wave theory, the problem is relatively simple: the field’s multipole components evolve independently of each other; there is no coupling. However, in nonlinear theories like general relativity, it is more difficult. A number of researchers have used spherical-harmonic expansions for a variety of problems in general relativity, including problems where nonlinearity shows up as Kip Thorne [5] has pointed out.
in detail. The relationships between scalar spherical harmonics incorporated in our present work and the various other harmonics like the pure spin vector harmonics related to the Regge-Wheeler harmonics have been discussed [5]. The basis states in the PWESH expansion form a complete set and facilitate such a study. It also turns out that the consequent analysis of the Fourier Transform (FT) of the GW signal from a pulsar has a very interesting and convenient development in terms of the resulting spherical Bessel, generalized hypergeometric function, the Gamma functions and the Legendre functions. Significant analytic and numerical studies have been carried out by many, for example, by Olver [6], R.C. Thorne [7], van der Laan and Temme [8] and Cherry [9]. Hypergeometric functions also arise in the analysis of gravitationally radiating binary stars [10]. The generalized hypergeometric functions in our analysis find their extensions to the H-functions, Meijer G-functions [11, 12] and the Heun [13, 14] functions.

Both rotational and orbital motions of the Earth and spin down of the pulsar can be considered in this analysis which happens to have a nice analytic representation for the GW signal in terms of the special functions above. The signal can then be studied as a function of a variety of different parameters associated with both the GW pulsar signal as well as the orbital and rotational parameters. Regardless of the immediate usefulness of such an analysis for GW data community which we do indeed look forward to the numerical analysis of this analytical expression for the signal offers a challenge for efficient and fast numerical and parallel computation. The plane wave expansion approach was also used by Bruce Allen and Adrian C. Ottewill [15] in their study of the correlation of GW signals from ground-based GW detectors. They use the correlation to search for anisotropies from stochastic background in terms of the \( l, m \) multipole moments. Our PWESH formalism enables a similar study. Recent studies of the Cosmic Microwave Background Explorer have raised the interesting question of the study of very large multipole moments with angular momentum \( l \) and its projection \( m \) going up to very large values of \( l \sim 1000 \). Such problems warrant an intensive analytic study supplemented by numerical and parallel computation.

However, the important spin down corrections were not included in our previous works. A considerable amount of work has been done on the search for the continuous GW signals. Livas, Jones and Niebauer [16] have investigated, in separate works, the time series of continuous GW signals that incorporated modulation of the motion of the detector over limited regions of parameter space. They have not considered pulsar spin down, and restricted their analysis to small areas of the sky. In this work, we present a parametrized model clearly discussed by Brady et al [1, 17] for the gravitational wave frequency and the phase measured at the ground based detector that includes the spindown parameters.

Spin down corrections are important in the analysis of GW signals. Brady et al [1, 17] indicate that most of the S/N is accumulated during the final stages of spin down of pulsars. There could exist a class of pulsars which spin down mainly due to gravitational radiation reaction [18]. The frequency scales as \( f \propto \tau^{-1/4} \) in these types of pulsars, where \( \tau \) is the age of the pulsar. Assuming
that the mean birth rate for such pulsars in our galaxy is $\tau_B^{-1}$, the nearest one should be a distance $r = \sqrt{\tau_B/\tau}$ from earth, where $R \simeq 10$ kpc is the radius of the galaxy. This estimate has been provided by Brady et.al \[1\]. We use the expression for $h_c$ provided by Thorne \[19\] and \[20, 21, 22\].

The amplitude, $h_c$ is given as,

$$h_c = \frac{16\pi^2G\epsilon f^2}{c^4}$$  \hspace{1cm} (1)

where $G$ is the gravitational constant, $f$ is the sum of the frequency of rotation of the star and the precession frequency, $I$ is the moment of inertia with respect to the rotation axis, $\epsilon$ is the poloidal ellipticity of the star and $r$ is the distance to the star. Hughes et. al, suggest that $h_c \leq 10^{-24}$.

The frequency domain characteristic of the GW signal consists of two components with carrier frequencies $f_0$ and $2f_0$ (Jaranowski et. al.\[23\]) that are both amplitude and phase modulated. The amplitude modulation in their analysis is determined by functions which split each of the two components into five lines that connect the GW and the earth rotation frequencies. They observe that the frequency modulation (FM) broadens the lines. They make estimates for a kilohertz GW frequency, the spin-down age $\tau = 40$ years, and an observation time of 120 days for the maximum frequency shifts due to the neutron star spin-down, Earth’s orbital motion and Earth’s diurnal motion which are, respectively, of the order $\sim 8$, $\sim 0.1$, and $\sim 10^{-3}$ Hz. In this paper, we have considered a single component GW frequency, $f_0$ and neglected the amplitude modulation.

The paper is organized as follows. In Section 2 we outline the FT of the GW signal from the pulsars which is relevant to the detection of continuous gravitational waves. We focus attention on spin down effects which indicate significant frequency evolution over periods of several weeks of observation. In this section, we use the expression for a parametrized model of the expected gravitational waveform, including modulating effects due to detector motion. We derive the analytic form of the FT with spin down corrections in the following section. The expressions for the spin down moments are computed using the familiar technique of differentiation with respect to a parameter. We present our conclusions in section 4.

2 Frequency and phase evolution with Spin Down

Manchester (1992) and Kulkarni (1992) have suggested that pulsars lose rotational energy by electromagnetic braking, particle emission and emission of GW \[23, 24\]. Therefore, the rotational frequency is not completely stable, and varies over a timescale $t$ which is of order the age of the pulsar. Younger pulsars with periods of tens of milliseconds have the largest spin down rates. Current observations suggest that spindown is primarily due to electromagnetic braking. Brady et. al. have suggested a sufficiently general model of the frequency evolution to cover all possibilities in the study of pulsar GW signal detection.
For observing times $t_{obs}$ much less than $t$, the frequency drift is small. We use a slightly modified form of the parametrization given by Brady and Creighton (2000)[17] in that it differs by an factor of $1/2$, and the summation index, $k$ starts from zero. Our expression is the Doppler modulated version of the earlier power series of the frequency evolution given by [1]. The parametrized model for frequency and phase evolution with spin down corrections is given in the following form.

$$f(t; \lambda) = \frac{f_0}{2} \left( 1 + \frac{\vec{v} \cdot \hat{n}}{c} \right) \left( 1 + \sum_{k=0}^{\infty} f_k \left[ t + \frac{\vec{v} \cdot \hat{n}}{c} \right]^k \right)$$ (2)

$$\phi(t; \lambda) = \pi f_0 \left( t + \frac{\vec{v} \cdot \hat{n}}{c} + \sum_{k=0}^{\infty} \frac{f_k}{k+1} \left[ t + \frac{\vec{v} \cdot \hat{n}}{c} \right]^{k+1} \right)$$ (3)

where $\vec{r}(t)$ denotes the detector position, $\hat{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ is a unit vector in the direction of the source. The angles $\theta, \phi$ are associated with the pulsar source; $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$ is the velocity of the detector and $f_k$ are the spin down parameters. Here $|f_k| \leq \tau_{min}^{-k}$ and $\tau_{min}$ is the spin down age in years. Here $\tau = \frac{f}{df/dt}$ is life in years of the pulsars. For the extreme case of the gravitational-wave frequency of $10^3$ Hz, the spin-down age is $\tau = 40$ years for pulsars in our galaxy and $\geq 10^7$ years for millisecond pulsars. Our analysis considers the equality situation, that is, $|f_k| = \tau_{min}^{-k}$. The phase $\phi$ depends on the frequency $f_0$, $k$ spin-down parameters, and on the angles $\phi, \theta$ and the co-latitude $\alpha$ of the detector.

In the earlier analysis of Valluri et.al. [3], the spin down corrections were neglected. In their analysis of the FM of a monochromatic plane wave, they characterize the motion of the Earth (and detector) by: (a) assuming the orbit of the Earth to be circular. (b) neglecting the effect of the Moon and the perturbation effects of Jupiter on the Earth’s orbit. In this work, in addition to incorporating spin down effects, we also include the approximate correction for the eccentricity of the orbital motion of the Earth.

With these assumptions, (a) and (b), and ignoring spin down corrections we arrive at the following expression of the FT for the GW pulsar signal [3], rewritten more concisely as,

$$\tilde{h}(f) = S_{nlm}(\omega_0, \omega_{orb}, T_{Er}, n, l, m, A, R, k, \alpha, \theta, \phi) = \sum_{n=\infty}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \psi_0 \psi_1 \psi_2 \psi_3 \psi_4$$

$$= \sum_{n=\infty}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \psi_0 \psi_1 \psi_2 \psi_3 \psi_4$$

$$\psi_0(n, l, m, \alpha, \theta, \phi) = 4\pi i^{l} Y_{lm}(\theta, \phi) N_{lm} P_i^m(\cos \alpha)$$ (4)
\[ \psi_1(n, \theta, \phi, T_{Er}, f_0, A) = T_{Er} \sqrt{\frac{\pi}{2}} e^{-i \frac{2\pi f_0 A}{c} \sin \theta \cos \phi} i^n e^{-in\phi} J_n \left( \frac{2\pi f_0 A \sin \theta}{c} \right) \]  
(6)

\[ \psi_2(l, \omega_{orb}, \omega_r, n, m, R) = \begin{cases} 1 - e^{i \pi (l-B_{orb})} \
 1 - e^{i2\pi f_0 A} \end{cases} \frac{1}{2^{2l+1}} \]  
(7)

\[ \psi_3(k, l, m, \omega_{orb}, \omega_r) = k^{l+\frac{1}{2}} \frac{\Gamma (l+1)}{\Gamma \left( l + \frac{3}{2} \right) \Gamma \left( \frac{l+B_{orb}+1}{2} \right) \Gamma \left( \frac{l-B_{orb}+1}{2} \right)} \]  
(8)

\[ \psi_4(k, l, m, \omega_{orb}, \omega_r) = F_3 \left( l+1; l + \frac{3}{2}; \frac{l+B_{orb}+2}{2}, \frac{l-B_{orb}+2}{2}; -\frac{k^2}{16} \right) \]  
(9)

The angle \( \alpha \) is the co-latitude detector angle and angles \( \theta, \phi \) are associated with the pulsar source. Here \( \omega_0 = 2\pi f_0, \omega_{orb} = \frac{2\pi}{T_{orb}} \) (\( T_{orb} = 365 \) days, \( T_{Er} = 1 \) day), \( B_{orb} = 2 \left( \frac{\omega_0 - \omega_r}{\omega_r} + \frac{n}{2} + \frac{n\omega_{orb}}{\omega_{orb}} \right) \), \( k = \frac{4\pi f_0 R_E \sin(\alpha)}{c} \) (\( R_E \) is the radius of Earth, \( c \) is the velocity of light) and \( A \) is the sun-earth distance.

### 3 Calculation of the FT including Spin Down Corrections

Considering the spin down corrections in Equation (2), we have for the summation term in \( \phi(t; \lambda) \) in Equation (3),

\[ \sum_{k=0}^{\infty} \frac{f_k}{k+1} \left[ t + \frac{\omega_{orb}}{c} \right]^{k+1} \]

Upon using the definition, \( |f_k| = \tau_{\text{min}}^{-k} \) we have for this summation,

\[ \sum_{k=0}^{\infty} \frac{\tau_{\text{min}}^{k+1}}{k+1} \left[ t + \frac{\omega_{orb}}{c} \right]^{k+1} \]

This power series represents a logarithmic function of the form \(-\ln(1-x)\). It is interesting to note that this is the \( n = 1 \) case of the multi-valued polylogarithm function \( L_n(x) \) and the phase evolution, apart from the factor \( i\pi f_0 \) is one of the versions of the multi-valued Lambert-W function [25]. Hence, we have,

\[ -\tau_{\text{min}} \ln \left( 1 - \frac{t + \frac{\omega_{orb}}{c}}{\tau_{\text{min}}} \right) \]

\[ \therefore \phi(t) = e^{i\pi f_0 \left( t + \frac{\omega_{orb}}{c} \right) - \tau_{\text{min}} \ln \left( 1 - \frac{t + \frac{\omega_{orb}}{c}}{\tau_{\text{min}}} \right)} \]

\[ = e^{i\pi f_0 \left( t + \frac{\omega_{orb}}{c} \right) - \tau_{\text{min}} \ln \left( 1 - \frac{t + \frac{\omega_{orb}}{c}}{\tau_{\text{min}}} \right)} \]

5
Using the binomial expansion on equation (10), we obtain,

\[ e^{i \pi f_0 (t + \frac{\vec{r} \cdot \hat{n}}{c})} \left[ 1 + \frac{t + \frac{\vec{r} \cdot \hat{n}}{c}}{\tau_{\min}} + \frac{(i \pi f_0 \tau_{\min})(i \pi f_0 \tau_{\min} + 1)}{2} \left( \frac{t + \frac{\vec{r} \cdot \hat{n}}{c}}{\tau_{\min}} \right)^2 + \ldots \right] \]

(11)

We define the Spindown Moment Integrals as follows:

\[ \int_0^T e^{i \pi f_0 (t + \frac{\vec{r} \cdot \hat{n}}{c})} \cdot e^{-i 2 \pi ft} \cdot \left( t + \frac{\vec{r} \cdot \hat{n}}{c} \right)^k \, dt \]

(12)

for \( k = 0, 1, 2, 3, \ldots \), where \( k = 0 \) denotes the absence of spin down corrections and recovers the formula for the FT given in Equations (4-9).

Thus, a generic spin down moment integral could be written from the derivative of a generic integral,

\[ I_{\text{generic}} = \int_0^T e^{i \pi f_0 (t + \frac{\vec{r} \cdot \hat{n}}{c}) - i 2 \pi ft} \, dt \]

(13)

Hence, we have the \( k \)-th derivative as,

\[ \frac{d^k I_{\text{generic}}}{df_0^k} = \int_0^T \left[ i \pi \left( t + \frac{\vec{r} \cdot \hat{n}}{c} \right) \right]^k e^{i \pi f_0 (t + \frac{\vec{r} \cdot \hat{n}}{c}) - i 2 \pi ft} \, dt \]

(14)

for \( k = 0, 1, 2, 3, \ldots \).

This \( k \) derivative gives the Corresponding Spin down moment integral. Thus, \( I_{\text{generic}} \) has been analytically evaluated. Its derivatives with respect to \( f_0 \) can be done by symbolic packages like Maple, Mathematica etc. It should be noted that the FT of the GW signal using the later parametrization given by Brady and Creighton (2000) can also be analytically incorporated in a similar manner of derivation given above.

It should be noted that previously these spin down corrections were incorporated by splitting up a time interval, say \( T_0 \) into \( M \) equal parts each of interval \( \Delta t \) (\( T_0 = M \Delta t \)) so that the signal is monochromatic in each interval or "window"[22]. This is the idea behind "stacking" where FTs are incoherently combined by adding their power spectra. "Tracking" as suggested by Brady et. al (2000), Papa and Schutz, (cited in review article gr-qc/9802020) is the attempt to track weak lines in individual FTs in successive data sets to identify persistent signals. We express our final result as a coherent sum over any given time interval as a closed form solution.

In our original FT, we have not included the effects of orbital eccentricity which arise due to the orbital motion of Earth. Denoting the orbital eccentricity
by \(e_\oplus (= 0.017)\), we find that by replacing the usual Sun-Earth distance, \(A\) by the following modified \(A\), is a reasonable estimate of the correction.

\[
A \rightarrow A\sqrt{1 - e_\oplus^2}
\]  

(15)

The correction in the phase of the GW signal due to the effects of orbital eccentricity can also be accurately estimated as has already been pointed out by Jaranowski et al. (1998).

\section{Conclusions}

Recently, new mechanisms e.g. r-mode instability of spinning neutron stars and temperature asymmetry in the interior of the neutron star with misaligned spin axis have been discussed in the literature [21, 22]. The continuous GW signal may consist of frequencies which are multiple of some basic frequencies. Brady and Creighton have given estimates of the characteristic strain of gravitational waves from an active r-mode instability in a newborn neutron star suggesting that these sources will be detectable by the enhanced interferometers in LIGO out to distances 8 Mpc; the rate of supernovae is 0.6 per year within this distance [20, 17].

We have presented in this paper the rudiments of a simple analysis for spin down for sources of continuous gravitational waves. For the more computationally-intensive search over all sky positions and spin down parameters, it is important to be able to calculate the smallest number of independent parameter values which must be sampled in order to cover the entire space of signals. The PWESH has the potential to improve the numerical accuracy and convergence of analytic FT’s and spin down corrections associated with the GW signal.

The study of templates in time-domain has been made by many research workers (Schutz 1991; Krolak 1997; Brady et al. 1998; Brady, Creighton 2000; Jaranowski et al. 1998; Jaranowski, Krolak 1999, 2000 [26]). However, the analysis in the frequency domain has the advantage of incorporating the spectral noise density of the interferometer. The data output at the interferometer is available in discrete form and the question arises if the analytical FT is in fact a very convenient tool. However, the FFT has a resolution limited to \(1/T_0\) as pointed by Srivastava and Sahay [22]. It is hoped that the analytical FT will provide useful insights.

We intend to further estimate from our spin down analysis the number of templates required for matched filtering analysis. We intend to make use of our approach developed for all frequencies to evaluate the Fitting Factor (FF) as given by Apostolatos [27] and considered for low frequencies by Srivastava and Sahay (2002). The time-delay type effects [28] due to general relativity and other corrections for the ephemeris have been treated in detail by Cutler in his Ligo Algorithmic Library (LAL) Barycenter Package (available at: http://www.lsc-group.phys.uwm.edu/) and will also be taken into account.

A new expansion was explored by MacPhie et.al. [4] both in the case of scalar and vector harmonics. This can be extended to tensor harmonics as well, and
work in that direction will be carried out. This would be of possible relevance to study the multipole formalisms for gravitational radiation [5].

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