Adaptive Output Consensus of Heterogeneous Nonlinear Multi-agent Systems: A Distributed Dynamic Compensator Approach

Guangqi Li, Long Wang

Abstract—Distributed dynamic compensators, also known as distributed observer, play a key role in the output consensus problem of heterogeneous nonlinear multi-agent systems. However, most existing distributed dynamic compensators require either the compensators’ information to be exchanged through communication networks, or that the controller for each subsystem satisfies a class of small gain conditions. In this note, we develop a novel distributed dynamic compensator to address the adaptive output consensus problem of heterogeneous nonlinear multi-agent systems with unknown parameters. The distributed dynamic compensator only requires the output information to be exchanged through communication networks. Thus, it reduces the communication burden and facilitates the implementation of the dynamic compensator. In addition, the distributed dynamic compensator converts the original adaptive output consensus problem into the global asymptotic tracking problem for a class of nonlinear systems with unknown parameters. Then, by using the adaptive backstepping approach, we develop an adaptive tracking controller for each subsystem, which does not require the small gain conditions as in previous studies. It is further proved that all signals in the closed-loop system are globally uniformly bounded, and the proposed scheme enables the outputs of all the subsystems to track the output of leader asymptotically. A simulation is presented to illustrate the effectiveness of the design methodology.

Index Terms—Distributed control, distributed dynamic compensator, heterogeneous nonlinear multi-agent systems, adaptive output consensus.

I. INTRODUCTION

The consensus problem of multi-agent systems has attracted many researchers, due to its widespread potential applications in various fields. Its objective is to design a distributed control law such that the states or the outputs of all agents achieve an agreement. The control law is distributed in the sense that each agent’s controller only uses information from the agent and its neighboring agents. During the past decades, the consensus problem for multi-agent systems has been extensively studied from various perspectives [1]-[14]. For more details, please refer to the surveys [15]-[17] and the references cited therein.

Recently, more attention has been paid to the heterogeneous nonlinear multi-agent systems [18]-[24]. Distributed dynamic compensators, also called distributed observer, are useful in dealing with the output consensus problem of heterogeneous nonlinear multi-agent systems. This problem can be addressed in two steps. First, a local dynamic compensator is designed for each agent, and the outputs or states of all compensators achieve consensus through a proper collaborative control strategy. Then, the output regulation theory is applied to constructing controller, forcing the output of each agent to track the output of local compensator. Based on this method, the output consensus problem has been addressed for different classes of heterogeneous nonlinear multi-agent systems [20]-[23]. For example, the output synchronization problem was investigated in [20] for heterogeneous nonlinear multi-agent systems. The cooperative output regulation problem was addressed for heterogeneous nonlinear multi-agent systems with unknown and non-identical control directions [22]. Unfortunately, all the distributed dynamic compensators in [20]-[24] require the compensator information to be exchanged through communication networks. The compensator information is not physical but artificial, hence exchanging such information must incur additional communication complexity and burden. In many physical circumstances, each agent can only observe or measure the output information of its neighboring agents. As a result, it is more desirable to design distributed controller under output communication. However, the output communication also brings new challenges in designing controller, and new design technique is required. In a recent paper [24], a general framework was proposed to address the output consensus problem of heterogeneous nonlinear multi-agent systems under output communication. Actually, the distributed dynamic compensator constructed under output communication and each subsystem can be viewed as a interconnection system. Then, the controller satisfying a class of small gain conditions is designed for each subsystem to address the tracking problem of the interconnection system. However, the small gain conditions result in sufficiently large control gains, and for some nonlinear systems with completely unknown parameters, it is unable or difficult to design controller satisfying small gain conditions.

In this note, a novel distributed dynamic compensator is developed to address the adaptive output consensus problem for heterogeneous nonlinear multi-agent systems with unknown parameters. The distributed dynamic compensator only requires the output information to be exchanged through communication networks. This considerably reduces the communication burden and facilitates the implementation of the dynamic compensator. In addition, the distributed dynamic compensator converts the adaptive output consensus problem of heterogeneous nonlinear multi-agent systems with unknown parameters into the problem of global asymptotic tracking for a class of nonlinear systems with unknown parameters. Then, by using adaptive backstepping approach, we develop an adaptive tracking controller for each subsystem, without requiring the small gain conditions [24]. It is further proved that all signals in the closed-loop system are globally uniformly bounded, and the proposed scheme enables the outputs of all the subsystems to track the output of leader asymptotically. A simulation is presented to illustrate the effectiveness of the design methodology.

The adaptive output consensus problem has also been addressed via adaptive backstepping approach [20] for nonlinear multi-agent systems with unknown parameters [27]-[29]. Compared with these results [27]-[29], our designed methodology has the following advantages:

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In [27]-[29], some restrictive conditions were imposed, e.g., each agent needs to know the state information and nonlinear functions of its neighbors [27]-[28], or the filter information of its neighbors [29], and the system orders of all agents need to be the same. However, in our work, the system orders are not the same for all agents, and only the output information is exchanged through communication network.

Our design methodology is more flexible than the methods in [27]-[29]. Actually, by means of the distributed dynamic compensator, we can use different control approaches to design tracking controller for each subsystem. Thus, our proposed methodology can be used to address the output consensus problems of heterogeneous nonlinear multi-agent systems with non-identical structure, and hence the output consensus problem of multi-agent systems with unknown and non-identical control directions. However, it is difficult to apply the methods in [27]-[29] to these problems, even for the case that the system orders of all agents are the same.

In [27]-[28], each agent required constructing additional local estimates to account for the unknown parameters of its neighbors’ dynamics. This inevitably results in a much complex controller. However, in our work, each agent does not need to construct the additional local estimates.

The rest of this note is organized as follows. In Section 2, we formulate our problem statement, and give some useful lemmas. In Section 3, we first develop a novel distributed dynamic compensator and the essential supremum norm, respectively. Moreover, a function of all agents are the same.

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III. MAIN RESULTS

A. A novel distributed dynamic compensator

In this section, a novel distributed dynamic compensator is developed to address the challenges caused by heterogenous dynamics. First, we consider the following dynamic compensator:

\[
\dot{\eta}_{i,l} = A\eta_{i,l} - K C \eta_i + \eta_i, i = 1, \ldots, r_i,
\]

\[
\dot{\eta}_{i,r_i+1} = A\eta_{i,r_i+1} - K C \sum_{j=0}^{N} a_{ij} (\eta_{i,r_i+1} - \eta_{i,l}) - K e_{vi},
\]

\[
\dot{\gamma}_i = C \eta_i, i = 1, \ldots, N,
\]

where \(K \in \mathbb{R}^{1 \times 1}\) is a constant matrix to be designed later. Let \(\tilde{e}_i = \eta_i - \tilde{y}_i, i = 1, \ldots, N\), and \(\tilde{e} = [\tilde{e}_1, \ldots, \tilde{e}_N]^T\). Moreover, let \(\tilde{\eta}_i = \eta_i - x_{0i}, i = 1, \ldots, N, l = 1, \ldots, r_i + 1\). Then, for this dynamic compensator, we have the following results.

**Theorem 1:** Consider the dynamic compensator with \(K\) being designed by (13). Under Assumption 2, there exist a KL-function \(\beta\) and a K-function \(\gamma\) such that for \(i = 1, \ldots, N, l = 1, \ldots, r_i + 1\),

\[
\|\tilde{\eta}_{i,l}\| \leq \beta(\|\eta_i(0)\|, t) + \gamma(\|\tilde{e}\|), t \geq 0.
\]

In particular, if \(\lim_{t \to \infty} \tilde{e}_{i,l} = 0, i = 1, \ldots, N\), then \(\lim_{t \to \infty} \tilde{\eta}_{i,l}(t) = 0, i = 1, \ldots, N, l = 1, \ldots, r_i + 1\).

**Proof.** From (2) and (3), we have

\[
\dot{\tilde{\eta}}_{i,l} = A\tilde{\eta}_{i,l} - K C (\tilde{\eta}_{i,l} - \tilde{\eta}_{i,l+1}), l = 1, \ldots, r_i,
\]

\[
\dot{\tilde{\eta}}_{i,r_i+1} = A\tilde{\eta}_{i,r_i+1} - K C \sum_{j=0}^{N} a_{ij} (C \tilde{\eta}_{i,r_i+1} - C \eta_i + \tilde{y}_i - y_j).
\]

Observe that

\[
\sum_{j=0}^{N} a_{ij} (C \tilde{\eta}_{i,r_i+1} - C \eta_i + \tilde{y}_i - y_j)
\]

\[
= \sum_{j=0}^{N} a_{ij} (y_i - \tilde{y}_i) + a_{0i} C \tilde{\eta}_{i,r_i+1} + \sum_{j=0}^{N} a_{ij} (C \tilde{\eta}_{i,r_i+1} - C \eta_i + \tilde{y}_j - y_j).
\]

Thus, submitting (2) into (3) yields

\[
\dot{\tilde{\eta}}_{i,l} = A\tilde{\eta}_{i,l} - K C (\tilde{\eta}_{i,l} - \tilde{\eta}_{i,l+1}), l = 1, \ldots, r_i,
\]

\[
\dot{\tilde{\eta}}_{i,r_i+1} = A\tilde{\eta}_{i,r_i+1} - a_{0i} KC \tilde{\eta}_{i,r_i+1} + \sum_{j=0}^{N} a_{ij} KC (\tilde{\eta}_{i,r_i+1} - \tilde{\eta}_{i,l-1}) - K \sum_{j=0}^{N} a_{ij} \tilde{e}_{i,l} + K \sum_{j=0}^{N} a_{ij} \tilde{\eta}_{i,j}.
\]

Let \(\tilde{\eta} = [\tilde{\eta}_{i,1}, \ldots, \tilde{\eta}_{i,r_i+1}]^T\) and \(\tilde{\eta} = [\tilde{\eta}_{i,1}, \ldots, \tilde{\eta}_{i,N}]^T\). Moreover, we define the block matrix \(\tilde{L} \in [\tilde{L}_{ij}]_{i,j=1}^{N}\) as

\[
\tilde{L}_{ii} = \begin{bmatrix}
1 & -1 & 0 & \cdots & 0 & 0 \\
0 & 1 & -1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 1 & -1 \\
0 & 0 & 0 & \cdots & 0 & \sum_{j=0}^{N} a_{ij}
\end{bmatrix} \in \mathbb{R}^{(r_i+1) \times (r_i+1)},
\]

\[
\tilde{L}_{ij} = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 0
\end{bmatrix} \in \mathbb{R}^{(r_i+1) \times (r_j+1)}, i \neq j.
\]

Then, the system can be expressed as

\[
\tilde{\eta} = [I_{(r_i+1) \times (r_i+1)} \otimes A - (\tilde{L} + \tilde{\Delta}) \otimes KC] \tilde{\eta} + M \tilde{e},
\]
where the derivative of $V_{i,k-1}$, by induction, satisfies

$$V_{i,k-1} = - \sum_{l=1}^{k-1} c_{i,l} \hat{e}_{i,l} + \hat{e}_{i,k-1} \hat{e}_{i,k} + \theta_i^T (\hat{\theta}_i - \tau_{i,k-1})$$

$$- \sum_{l=2}^{k-1} \hat{e}_{i,l} \frac{\partial \alpha_{i,l-1}}{\partial \theta_i} (\hat{\theta}_i - \tau_{i,k-1}).$$

(26)

A direct calculation leads to

$$\dot{V}_{i,k} = - \sum_{l=1}^{k-1} c_{i,l} \hat{e}_{i,l} + \hat{e}_{i,k-1} \hat{e}_{i,k} + \theta_i^T (\hat{\theta}_i - \tau_{i,k-1})$$

$$- \sum_{l=1}^{k-1} \hat{e}_{i,l} \frac{\partial \alpha_{i,l-1}}{\partial \theta_i} (\hat{\theta}_i - \tau_{i,k-1}) + \hat{e}_{i,k} \hat{e}_{i,k+1}$$

$$+ \hat{e}_{i,k} \left[ \alpha_{i,k} + \psi_i \theta_i \right]$$

$$- \sum_{l=1}^{k-1} \frac{\partial \alpha_{i,k-1}}{\partial \theta_i} (x_{i,l+1} + \psi_i \theta_i)$$

$$- \sum_{l=1}^{k-1} \frac{\partial \alpha_{i,k-1}}{\partial \psi_i} (A_{i,l} - KC(\eta_i - \eta_{i+1})).$$

Then, choosing the tuning function $\tau_k$ and the virtual control signal $\alpha_{i,k}$ as

$$\tau_k = \tau_{k-1} + \left( \psi_k - \sum_{l=1}^{k-1} \psi_l \frac{\partial \alpha_{i,l-1}}{\partial \theta_i} \right) \hat{e}_{i,k}$$

$$\alpha_{i,k} = -c_{i,k} \hat{e}_{i,k} - \hat{e}_{i,k-1} \hat{e}_{i,k+1} + \theta_i^T (\hat{\theta}_i - \tau_{i,k-1})$$

$$+ \sum_{l=2}^{k-1} \frac{\partial \alpha_{i,l-1}}{\partial \theta_i} \left( \psi^{T}_i \theta_i - \sum_{l=2}^{k-1} \frac{\partial \alpha_{i,k-1}}{\partial \theta_i} \frac{\partial \alpha_{i,l-1}}{\partial x_{i,l}} \right)$$

one can obtain

$$\dot{V}_{i,k} = - \sum_{l=1}^{k} c_{i,l} \hat{e}_{i,l} + \hat{e}_{i,k} \hat{e}_{i,k+1} + \theta_i^T (\hat{\theta}_i - \tau_{i,k})$$

$$- \sum_{l=2}^{k} \hat{e}_{i,l} \frac{\partial \alpha_{i,l-1}}{\partial \theta_i} (\hat{\theta}_i - \tau_{i,k}).$$

(28)

**Step $r_k$**. The derivative of $\hat{e}_{i,r_k}$ is computed as

$$\dot{e}_{i,r_k} = u_i + \psi_i \theta_i - \sum_{l=1}^{r_k-1} \frac{\partial \alpha_{i,r_k-1}}{\partial x_{i,l}} (x_{i,l+1} + \psi_i \theta_i)$$

$$- \sum_{l=1}^{r_k-1} \frac{\partial \alpha_{i,r_k-1}}{\partial \psi_i} (A_{i,l} - KC(\eta_i - \eta_{i+1})).$$

The Lyapunov function is constructed as

$$V_{i,r_k} = V_{i,r_k-1} + \frac{1}{2} \hat{e}_{i,r_k}^2,$$

(30)

and the adaptive control law $u_i$ is chosen as

$$\dot{\theta}_i = \tau_{i,r_k} := \tau_{i,r_k-1} + \left( \psi_{i,r_k} - \sum_{l=1}^{r_k-1} \psi_l \frac{\partial \alpha_{r_k-1}}{\partial x_{r_k}} \right) \hat{e}_{i,r_k},$$

$$u_i = \alpha_{i,r_k} := -c_{i,k} \hat{e}_{i,k} - \hat{e}_{i,k-1} \hat{e}_{i,k+1} - \psi_i \theta_i$$

$$+ \sum_{l=1}^{r_k-1} \frac{\partial \alpha_{r_k-1}}{\partial x_{r_k}} (x_{i,l+1} + \psi_i \theta_i) + \sum_{l=1}^{r_k-1} \frac{\partial \alpha_{r_k-1}}{\partial \psi_i} (A_{i,l} - KC(\eta_i - \eta_{i+1})).$$

(31)
Remark 3: In [27, 28], each agent required constructing additional local estimates to account for the unknown parameters of its neighbors’ dynamics. This inevitably results in a much complex controller. From [44], it is easy to know that our designed controller for each agent does not need the additional local estimates.

C. Stability analysis

Theorem 2: Consider the closed-loop system consisting of the N nonlinear subsystems [11], the leader [2], the dynamic compensator [1] and the adaptive controllers [31]. Under Assumptions 1 and 2, all signals in the closed-loop system are globally uniformly bounded, and asymptotic consensus tracking of all the subsystems’ output to \( y_0 \) is achieved, i.e., \( \lim_{t \to \infty} [y_i(t) - y_0(t)] = 0 \) for \( i = 1, \cdots, N \).

Proof. By using \( \ref{eq:28} \) with \( k = r_i - 1 \), we have

\[
\dot{V}_{i,r_i} = - \sum_{l=1}^{r_i-1} c_{i,l} \dot{e}_{i,l}^2 + \dot{e}_{i,r_i} \dot{e}_{i,r_i} + \ddot{\theta}_i (\dot{\theta}_i - \tau_{i,r_i-1} - 1) - \sum_{l=2}^{r_i-1} \dot{e}_{i,l} \frac{\partial \alpha_{i,l-1}}{\partial \theta_i} (\dot{\theta}_i - \tau_{i,r_i-1}). \tag{32}
\]

Then, from \( \ref{eq:29} \), the derivative of \( V_{i,r_i} \) can be computed as

\[
\dot{V}_{i,r_i} = - \sum_{l=1}^{r_i-1} c_{i,l} \dot{e}_{i,l}^2 + \dot{e}_{i,r_i} \dot{e}_{i,r_i} + \ddot{\theta}_i (\dot{\theta}_i - \tau_{i,r_i-1}) - \sum_{l=2}^{r_i-1} \dot{e}_{i,l} \frac{\partial \alpha_{i,l-1}}{\partial \theta_i} (\dot{\theta}_i - \tau_{i,r_i-1}) + \dot{e}_{i,r_i} \left[ u_i - \alpha_i \dot{e}_{i,r_i} + \alpha_{i,r_i} \right] + \psi_{i,r_i} \theta_i \left( - \sum_{l=1}^{r_i} \frac{\partial \alpha_{i,l-1}}{\partial \theta_i} (x_{i,l+1} + \psi_{i,l} \theta_i) + \frac{\partial \alpha_{i,r_i-1}}{\partial \theta_i} (x_{i,r_i+1}) - 1 \right) - \sum_{l=1}^{r_i} \frac{\partial \alpha_{i,l-1}}{\partial \theta_i} (A_{i,l} - KC (y_{i,l} - y_{i,l+1})). \tag{33}
\]

Submitting \( \ref{eq:31} \) into \( \ref{eq:33} \) results in

\[
\dot{V}_{i,r_i} = - \sum_{l=1}^{r_i} c_{i,l} \dot{e}_{i,l}^2 - \sum_{l=2}^{r_i} \dot{e}_{i,l} \frac{\partial \alpha_{i,l-1}}{\partial \theta_i} (\dot{\theta}_i - \tau_{i,r_i}) + \ddot{\theta}_i (\dot{\theta}_i - \tau_{i,r_i}) + \dot{e}_{i,r_i} (u_i - \alpha_{i,r_i}). \tag{34}
\]

Thus, according to the adaptive controller \( \ref{eq:34} \), one have

\[
\dot{V}_{i,r_i} = - \sum_{l=1}^{r_i} c_{i,l} \dot{e}_{i,l}^2. \tag{35}
\]

Form \( \ref{eq:30} \), it is clear that \( \dot{\theta}_i \in L_\infty \) and \( \dot{e}_{i,l} \in L_\infty \cap L_2, l = 1, \cdots, r_i \). Then, from Theorem 1 and Assumption 1, one have \( \dot{\eta}_i, \dot{\theta}_i \in L_\infty \) and \( \dot{\eta}_i, \dot{\theta}_i \in L_\infty \) for \( l = 1, \cdots, N \), \( i = 1, \cdots, r_i \). This implies \( \dot{e}_{i,l} \in L_\infty, l = 1, \cdots, r_i \). Therefore, all signals in the closed-loop system are globally uniformly bounded. Then, from \( \ref{eq:29} \), we have \( \dot{e}_{i,l} \in L_\infty \). In addition, according to Lemma 3 and the fact \( \dot{e}_{i,1} \in L_\infty \cap L_2, i = 1, \cdots, N \), one can verify that \( \lim_{t \to \infty} \dot{e}_{i,1} = 0, i = 1, \cdots, N \). This together with Theorem 1, implies that \( \lim_{t \to \infty} (y_i - y_0) = \lim_{t \to \infty} (y_i(t) - y_0(t)) + \lim_{t \to \infty} C (\dot{\eta}_i - x_0) = 0 \). The proof is completed.

Remark 4. It should be noted that in [24], the uncertain parameters must belong to a known compact set, and the controller for each subsystem needs to satisfy a class of small gain conditions. The small gain conditions may result in sufficiently large control gains, and for some nonlinear systems with completely unknown parameters, it is unable or difficult to design controller satisfying small gain conditions. However, in our work, by means of the novel distributed dynamic compensator [1], the controller [31] does not require the small gain conditions, and it hence can be adopted to the case that the uncertain parameters are completely unknown.

Remark 5. By means of the distributed dynamic compensator [1], we can use different control approaches to design tracking controller for each subsystem. For example, for some agents with the following dynamic:

\[
\dot{x}_{i,l} = x_{i,l+1} + \psi_{i,l} (x_{i,l}, \cdots, x_{i,l}), l = 1, \cdots, r_i - 1,
\]

\[
\dot{x}_{i,r_i} = u_i + \psi_{i,r_i} (x_{i,1}, \cdots, x_{i,r_i}),
\]

\[
y_i = x_{i,1}, \tag{36}
\]

where only the output \( y_i \) can be measured by agent \( i \), a linear-like output feedback controller can be designed for [49] via the dynamic gain scaling technique [39], which avoids the repeated derivatives of the nonlinearities depending on the observer states and the dynamic gain in backstepping approach. Thus, our proposed methodology can be applied to the leader-following output consensus problem of heterogeneous nonlinear multi-agent systems with unknown parameters and unknown non-identical control directions. Actually, by combination of adaptive backstepping technique and Nussbaum-type function, one can design an adaptive controller for each subsystem such that \( \lim_{t \to \infty} (y_i - y_i) = 0 \). Specifically, for the following heterogeneous nonlinear multi-agent systems with unknown and non-identical control directions:

\[
\dot{x}_{i,l} = x_{i,l+1} + \psi_{i,l} (x_{i,l}, \cdots, x_{i,l}) \dot{y}_i, l = 1, \cdots, r_i - 1,
\]

\[
\dot{x}_{i,r_i} = b_i u_i + \psi_{i,r_i} (x_{i,1}, \cdots, x_{i,r_i}) \dot{y}_i,
\]

\[
y_i = x_{i,1}, i = 1, \cdots, N, \tag{37}
\]

where \( b_i \) is a nonzero constant with unknown sign, the adaptive controller for each subsystem is designed as

\[
u_i = -N_i (k_i x_0) x_{i,l},
\]

\[
\dot{k}_i = -\dot{e}_{i,r_i} x_{i,r_i}, \tag{38}
\]

where \( N_i (k_i) \) is a Nussbaum function and \( \alpha_{i,r_i} \) is defined in [41]. Following the similar proof in Theorem 2 with a minor modification, one can prove that all signals in the closed-loop system are globally bounded, and \( \lim_{t \to \infty} (y_i - y_i) = 0 \).

IV. AN ILLUSTRATIVE EXAMPLE

In this section, we consider a heterogeneous nonlinear multi-agent system connected by a communication graph shown in Figure 1, where the weighted adjacency matrix \( A \) satisfies \( a_{ij} = 1 \) if and only

![Fig. 1: The communication graph.](image-url)
if \((v_i, c_i) \in \mathcal{E}\). The system is composed of agents with unknown parameters. In particular, agents \(i, i = 1, 2, 3\) are described by

\[
\begin{align*}
\dot{x}_{i,1} &= x_{i,2} + x_{i,1}^2 \theta_i, \\
\dot{x}_{i,2} &= u_i + \sin(x_{i,1}) \theta_i, \\
y_i &= x_{i,1},
\end{align*}
\]

while agents \(i, i = 3, 5\) are described by

\[
\begin{align*}
\dot{x}_{i,1} &= u_i + \cos(x_{i,1}) \theta_i, \\
y_i &= x_{i,1}.
\end{align*}
\]

The leader’s signal \(y_0\) is generated by the linear system (4) with

\[
A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad C = [1 \ 0].
\]

Obviously, it can be seen that \((A, C)\) is detectable, and Assumptions 1-2 are satisfied. Therefore, by Theorem 2, we can design a distributed adaptive controller of form (3) for all agents such that all signals in the closed-loop system are globally uniformly bounded and \(\lim_{t \to \infty} (y_i - y_0) = 0, i = 1, \ldots, N\). Following the design procedure in Section III, one can design the distributed adaptive controller as follows.

- **Step 1.** The distributed dynamic compensator is given in (4) with \(N = 5, r_1 = r_2 = r_3 = 2, r_4 = r_5 = 1\), and

\[
K = [17.3081 \ 5.3019]^T.
\]

- **Step 2.** For agents \(i = 1, 2, 3\), the first error \(\epsilon_{i,1} = x_{i,1} - \hat{y}_i\), the virtual control signal \(\alpha_{i,1}\) and the first tuning function \(\tau_{i,1}\) are given by (22). The second error \(\epsilon_{i,2}\), the update law \(\hat{\theta}_i\) and the controller law \(u_i\) are given, respectively, by

\[
\begin{align*}
\epsilon_{i,2} &= x_{i,2} - \alpha_{i,1}, \\
\hat{\theta}_i &= \tau_{i,1} \left(\sin(x_{i,2}) - x_{i,2}^2 \frac{\partial \alpha_{i,1}}{\partial x_{i,1}} \right) \epsilon_{i,2}, \\
u_i &= -c_i \epsilon_{i,2} - \epsilon_{i,1} - \sin(x_{i,2}) \hat{\theta}_i + \frac{\partial \alpha_{i,1}}{\partial x_{i,1}} (x_{i,2} + x_{i,1}^2 \hat{\theta}_i) \\
&\quad + \frac{\partial \alpha_{i,1}}{\partial x_{i,1}} [A \eta_{i,1} - KC(\eta_{i,1} - \eta_{i,2})] + \frac{\partial \alpha_{i,1}}{\partial \theta_i} \epsilon_{i,2} \\
&\quad + \frac{\partial \alpha_{i,1}}{\partial \theta_i} [A \eta_{i,2} - KC(\eta_{i,2} - \eta_{i,3})].
\end{align*}
\]

- **Step 3.** For agents \(i = 4, 5\), the error \(\epsilon_{i,1}\), the update law \(\hat{\theta}_i\) and the controller law \(u_i\) are given, respectively, by

\[
\begin{align*}
\epsilon_{i,1} &= x_{i,1} - \hat{y}_i, \\
\hat{\theta}_i &= \cos(x_{i,1}) \epsilon_{i,1}, \\
u_i &= -c_i \epsilon_{i,1} - \cos(x_{i,1}) \hat{\theta}_i + CA \eta_{i,1} - CKC(\eta_{i,1} - \eta_{i,2}).
\end{align*}
\]

Simulation is performed with \(c_{i,1} = c_{i,2} = c_{i,3} = 1, i = 1, 2, 3, j = 4, 5\), \(\theta_1 = 2.5, \theta_2 = 1.2, \theta_3 = -2, \theta_4 = -1, \theta_5 = 0.5\), and the following initial conditions:

\[
\begin{align*}
x_{1}(0) &= [0.1, -0.2]^T, \quad x_{2}(0) = [0.5, 1.2]^T, \quad x_{3}(0) = [-2, 1]^T, \\
x_{4}(0) &= -0.5, \quad x_{5}(0) = 0.25, \quad x_{10}(0) = [1, -1], \quad \hat{\theta}(0) = 1.2, \\
\hat{\theta}_2(0) &= -1, \quad \hat{\theta}_4(0) = 0.5, \quad \hat{\theta}_5(0) = 0.2, \quad \hat{\theta}_6(0) = -0.75, \\
\eta_{1,1}(0) &= [0.1, 0.2]^T, \quad \eta_{1,2}(0) = [1, -1.5]^T, \quad \eta_{1,3}(0) = [-1, -0.2]^T, \\
\eta_{2,1}(0) &= [0.5, -0.5]^T, \quad \eta_{2,2}(0) = [0.25, 0.3]^T, \quad \eta_{2,3}(0) = [0.5, 0.2]^T, \\
\eta_{3,1}(0) &= [0.5, -0.4]^T, \quad \eta_{3,2}(0) = [0.6, -1]^T, \quad \eta_{3,3}(0) = [3, -0.2]^T, \\
\eta_{4,1}(0) &= [2, -1.4]^T, \quad \eta_{4,2}(0) = [2, 1]^T, \\
\eta_{5,1}(0) &= [1, 2]^T, \quad \eta_{5,2}(0) = [0.5, -0.75]^T.
\end{align*}
\]

The simulation results are shown in Figure 2, which shows the effectiveness of the design methodology.

**V. CONCLUSION**

The adaptive output consensus problem has been investigated in this note for a class of heterogeneous nonlinear multi-agent systems with unknown parameters. A novel distributed dynamic compensator has been developed to address the challenges caused by heterogeneous dynamics. The distributed dynamic compensator only requires the output information to be exchanged through communication networks. In addition, it can convert the original adaptive consensus problem into the problem of global asymptotic tracking for a class of nonlinear systems with unknown parameters. By means of adaptive backstepping approach, we have developed an adaptive tracking controller for each subsystem, which does not require the small gain conditions as in [24]. It has been proved that all signals in the closed-loop system are globally uniformly bounded, and the proposed scheme enables the outputs of all subsystems to track the output of leader asymptotically.

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