Phase disorder on laser beam combining in a Talbot resonator

Yeojin Chung, Alejandro B. Aceves
Department of Mathematics, Southern Methodist University, Dallas, TX 75275, USA.

We study the role that phase disorder plays in the mode discrimination characteristics of a Talbot resonator. Two cases considered here correspond to a six-core and a twelve-core photonic crystal fiber.

I. INTRODUCTION

Fiber laser arrays are currently considered as one of the most viable options for state of the art high power lasers [1]. Key to their effectiveness is to coherently combine the output of as large a number of amplifiers as possible. Coherent beam combining can be implemented either passively [2-8], or actively [9], with the first one being preferred given that in principle it would be easier to implement. Passive schemes are based on linear mode coupling of optical elements and with the advancement of technology in multicore fiber arrays, many of the recent schemes make use of such arrays. Regardless of the beam-combining architecture in all instances, we define $E_{\text{output}} = \sum_i \sum_j S_{ij} E_j$ as the output field that results from combining $N$ electric fields $E_j$ emerging from individual and ideally identical amplifiers. Here $S_{ij}$ defines the passive coupling scheme. The objective of any combining scheme is effectively one of efficient mode selectivity or mode discrimination. This is the case if for example there is an eigenmode of $S_{ij}$ whose (largest) eigenvalue is much bigger than the rest. Furthermore maximum coherence is achieved if the dominant mode is such that the phases of $E_i^{(\text{out})} = \sum_j S_{ij} E_j$ are identical. Having said this, it is typical that in most if not all instances, as the number of elements $N$ in the arrays increase, the coherence decreases [10]. This is due to the fact that most of perturbations appearing during the propagation of the fields in the (ideally identical) individual amplifiers affect the path length, thus in turn the phase. This combined with the fact that as $N$ increases mode discrimination decreases, one expects coherence to diminish. For each passive linear scheme, it is important to characterize its robustness to phase disorder; in particular it would be useful to determine how the coherence changes when there is disorder in the input phases and how it scales with $N$.

Given the fast technological advances in multicore fibers, they present perhaps the most suitable architecture for passive beam combining and in this paper we discuss such multicore arrays in a Talbot resonator [11] as proposed in [12]. The natural modes in this resonator are due to self-imaging, which according to the Talbot effect happens to images formed by periodic objects, such as a periodic diffraction grating. The combination of periodicity and Fresnel diffraction is such that the image (mode) is reproduced every multiple of the Talbot length $z_T = \frac{a^2}{\lambda}$, where $a$ is the period of the grating and $\lambda$ is the light wavelength.

The next section presents the main results highlighting the role of phase disorder on mode selectivity. While our work deals with one particular beam combining scheme, we believe this behavior is generic of all passive, linear schemes. This is why in section III we briefly discuss nonlinear schemes and their suitability to achieve high coherence for large arrays.
II. TALBOT RESONATOR

In this section, we present the results of numerical simulation about the role of phase disorder on beam combining. We limit our study to the scheme based on a Talbot resonator for a photonic crystal fiber geometry \[12\]. We will consider two periodic like profiles: a hexagonal one consisting of six cores, equally spaced, one at the center of the hexagon. The second case is a rectangular array of 12 cores. In either case the generated modes will take the form

\[ E_m(x, y, 0) = \sum_{j=1}^{n} \exp(-a[(x-x_j)^2 + (y-y_j)^2])e^{i\phi_j^{(m)}} \]  

with \( n = 6 \) or \( n = 12 \).

The figure of merit to be studied is the correlation function,

\[ \gamma(z) = \frac{\int E_n^*(x, y, z = 0)E_m(x, y, z)dx\,dy}{\int |E_m(x, y, z = 0)|^2dx\,dy}, \]  

where ideally you would like to work in a regime for which \( \gamma \) is not only high for the in-phase mode, but separates well from other modes. These two properties are, for the Talbot resonator what determines if mode selectivity is achieved.

Figures 1, 2 show the dependence of correlation function \( \gamma(z) \) on distance from the output of waveguide in a 6-core and 12-core PCFs, respectively. The distance is in units of the lattice constant \( \Lambda \), where \( \Lambda/\lambda = 4.51 \).

For each supermode \( m \), we consider the deterministic phase factor \( \phi_j^{(m)} = 2m(j-1)\pi/6 \) associated with core \( j \), where \( j = 1, \ldots, 6 \) for 6-core PCF and \( j = 1, \ldots, 12 \) for 12-core PCF. As shown in both figures 1, 2 it is the case that in the absence of disorder both the \( N = 6 \) cores in a hexagonal lattice and the \( N = 12 \) cores in a rectangular lattice, mode selectivity is achieved. We should point out that figure 1 reproduces the figure 9 in \[12\].

The rest of the figures represent averaged values \( \langle \gamma \rangle \) computed over many realizations of different strengths of the random phase perturbations. We assume in all instances the phase inputs are \( iid \). Figures 3, 4, 5 correspond to different realizations of the random phase factor in a 6-core PCF with same parameters as figure 1 for increasing disorder strengths. That is, in our simulations the phase includes a random variable \( \xi \) uniformly distributed so that \( \phi_j^{(m)} = 2m(j-1)\pi/6 + \xi \) and with strengths \( 0 \leq \xi \leq \pi/2 \) (figure 3), \( 0 \leq \xi \leq \pi \) (figure 4), and \( 0 \leq \xi \leq 2\pi \) (figure 5). We clearly observe that while the contrast (separation of different curves) between the in-phase \( (m = 0) \) mode and the others does not diminish by much for noise levels \( 0 \leq \xi \leq \pi/2 \) (figure 3) it does so in a significant way once the noise strength increases (figure 4). That is, even if one were able to control path lengths so that the noise is only half a period \( 0 \leq \xi \leq \pi \) this scheme becomes ineffective. Notice that if \( 0 \leq \xi \leq 2\pi \) (figure 5) there is no mode discrimination at all. We should notice that for the operational wavelengths in fiber lasers, small perturbations (e.g., nonlinear phase, fiber lengths, thermal effects, gain induced perturbations to name some) are likely to modify the phase in multiples of \( 2\pi \).

Figure 6 shows the average of \( \gamma(z) \) over different realizations of random phase factor in a 12-core PCF, where \( \phi_j^{(m)} = 2m(j-1)\pi/3 + \xi \) for \( j = 1, \ldots, 12, \, m = 0, 1, 2, \) and \( 0 \leq \xi \leq 2\pi \). The similarity of the outcome with the 6-core case suggests that noise will “erase” the contrast irrespective of the number of fibers or the geometry.
FIG. 1: Dependence of correlation function $\gamma(z)$ on distance from the output of waveguide in a 6-core PCF. The phase factor $\phi_{j}^{(m)}$ is considered deterministic. The distance $z$ is in units of lattice constant $\Gamma$, where $\Lambda/\lambda = 4.51$.

FIG. 2: Dependence of correlation function $\gamma(z)$ on distance from the output of waveguide in a 12-core PCF. The phase factor $\phi_{j}^{(m)}$ is considered deterministic. The distance $z$ is in units of lattice constant $\Gamma$, where $\Lambda/\lambda = 4.51$.

III. NONLINEAR COUPLING SCHEMES

The previous section highlights the sensitivity to phase disorder and the loss of coherent beam combining efficiency. Furthermore, it is generally the case (with or without disorder) that as the number of elements in a fiber laser array increases, the coherence diminishes for linearly based coupling schemes \[10\]. The role of phase disorder in linearly based coupling schemes we believe will be similar to what we just showed for the Talbot resonator discussed above. Altogether, new paradigms need to be sought. Here we suggest that having a nonlinear coupling scheme should be considered as a viable alternative. As we know from many studies not only in laser dynamics but in other areas of science, synchronization of weakly coupled oscillators
is universal. Whether one describes laser modes, social networks, fireflies or chemical reactions to name some, a phase transition where incoherent oscillations lock into an in-phase state is present at a critical coupling strength. For this to happen, the system requires to be nonlinear, but the details need not be identical. This behavior is best represented in the work by Kuramoto on a “globally” weakly coupled nonlinear chain of oscillators,

\[
\frac{d\theta_i}{dt} = \tilde{\omega}_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\theta_i - \theta_j), \quad i = 1, 2, \ldots, N, \tag{3}
\]

where \(\theta_i(t)\) is the phase of the \(i\)-th oscillator and \(\tilde{\omega}_i\) is its natural frequency. In this model, disorder comes from the fact that the frequencies are random with a Lorenzian probability density.
FIG. 5: Average of $\gamma(z)$ for $N = 6$. Random phase factor is considered as $\phi_j^{(m)} = 2m(j - 1)\pi/6 + \xi$ for $0 \leq \xi \leq 2\pi$.

FIG. 6: Average of $\gamma(z)$ for $N = 12$. Random phase factor is considered as $\phi_j^{(m)} = 2m(j - 1)\pi/3 + \xi$ for $0 \leq \xi \leq 2\pi$.

$$g(\omega) = \frac{\delta}{\pi[\delta^2 + (\omega - \omega_0)^2]}.$$ \hfill (4)

Kuramoto found a phase transition in that the order parameter

$$r(t) = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j(t)} \right|$$ \hfill (5)

takes the asymptotic ($N \to \infty$ and $t \to \infty$) value.
\[ r = 0, \quad K < 2\delta \]
\[ = \sqrt{1 - \frac{2\delta}{K}}, \quad K \geq 2\delta \]

which translates into enhanced locking as the coupling strength goes past a critical value.

Perhaps more relevant case to coherent beam combining which can still be viewed as some form of phase transition is that of spatial and spatio-temporal localization in fiber arrays. Here the nonlinear index of refraction together with linear nearest neighbor coupling is modeled by the discrete nonlinear Schrödinger equation (DNLSE),

\[ i \frac{\partial a_{nm}}{\partial z} = (\Delta a)_{nm} + n_2 |a_n|^2 a_{nm}, \]

where the term \( \Delta a \) represents a general linear coupling scheme. Specific to cw lasers, the possibility of reaching an output that is spatially localized into a few cores would enhance coherence if it is measured by

\[ r = \frac{|\sum_{nm} a_{nm}|^2}{\sum_{nm} |a_{nm}|^2}. \]

As theory \cite{14} and experiment \cite{15} have shown, at sufficiently high powers (the equivalent of having high enough \( K \) in the oscillator model), an initially broad distribution of power reaches in propagation a localized mode. We are currently exploring the role that the disorder of incident phases plays in the dynamics of spatial and spatio-temporal localization. It is also the case that non-Hamiltonian perturbations to coupled systems like the DNLSE may produce an attractor in the form of a highly coherent state. As a proof of principle, recent work \cite{16} on large systems of nonlinearly coupled oscillators shows one can find asymptotic dynamics towards an attracting highly coherent state. Figure \( 4 \) shows a dynamics for an array of 17 oscillators reaching a state of maximum coherence.

Finally, nonlinear coupling schemes can perform in a very robust manner even in the presence of losses. In fact, small losses (that is departing from a conservative system where stable equilibrium states are centers) should facilitate synchronization \cite{17}.

\[ r = \frac{|\sum_{nm} a_{nm}|^2}{\sum_{nm} |a_{nm}|^2}. \]

IV. CONCLUSIONS

Phase disorder is always present in the amplification and propagation of light in optical fibers. This work presents the deterrent effect it has on the efficient beam combining in a Talbot resonator. Two cases of 6 and 12 elements are considered as a way to explore the role if any, \( N \) the number of elements plays on the efficiency of coherent coupling. We found for these two cases that mode selectivity characterized by the separation of the parameter \( \gamma \) for different modes lost if the noise in the phases is of strength equal to \( 2\pi \). While the results are limited to one particular linear scheme, we believe all linear schemes will show an eventual decrease in efficiency not only by increasing the number of elements as previously stated \cite{10}, but also in terms of expected phase disorder generated during propagation in the individual fiber amplifiers. The last section provides a brief discussion of a new paradigm whereby considering nonlinear coupling mechanisms, phase transitions towards high coherence are at least theoretically feasible in the important \( N \to \infty \) case.
FIG. 7: Time evolution of $E = \frac{1}{\Sigma u_{n}} \sum |u_{n}|$ for an initial condition with $E = 0.528646241$ for an array of 17 nonlinear coupled oscillators model [10].

V. ACKNOWLEDGEMENTS

A. Aceves acknowledges the collaboration and fruitful discussions with Dr. Erik Bochove from the Air Force Research Laboratory.

[1] D. J. Richardson, J. Nilsson and W. A. Clarkson, “High power lasers: current status and future perspectives [Invited]”, J. Opt. Soc. Am. B 27, B63-B92 (2010).
[2] Y. Huo, P. Cheo and G. King, “Fundamental mode operation of a 19-core phased locked Yb-doped fiber amplifier”, Opt. Exp. 25, 6230-6239 (2004).
[3] H. Bruesselbach, D. C. Jones, M. S. Mangir, M. Minden and J. L. Rogers, “Self-organized coherence in fiber laser arrays”, Opt. lett. 30, 1339-1341 (2005).
[4] Y. Huo and P. K. Cheo, “Analysis of transverse mode competition and selection in multicore fiber lasers”, JOSA B 22, 2345-2349 (2005).
[5] E. J. Bochove, P. K. Cheo and G.G. King, “Self-organization in a multicore fiber laser array”, Opt. Lett. 28, 1200-1202 (2003).
[6] L. Liu, Y. Zhou, F. Kong, Y. C. Chen and K. K. Lee, “Phase locking in a fiber laser array with varying path lengths”, Appl. Phys. Lett. 85, 4837-4839 (2004).
[7] A. Shirakawa, T. Saitou, T. Sekiguchi, and K. Ueda, “Coherent addition of fiber lasers by use of a fiber coupler”, Opt. Exp. 10, 1167-1172 (2002).
[8] T. Wu, W. Chang, A. Galvanusias and H. Winful, “Model for passive coherent beam combining in fiber laser arrays”, Opt. Exp. 17, 19509-19518 (2009).
[9] T. M. Shay, V. Benham, J. T. Baker, A. D. Sanchez, D. Pilkington and C. A. Lu, “Self-Synchronous and Self-Referenced Coherent Beam Combination for Large Optical Arrays”, IEEE Journ. of Selected Topics in Quantum Elec., 13, 480-486 (2007).
[10] D. Kouznetsov, J-F Bisson, A. Shirakawa and K. Ueda, “Limits of Coherent Addition of Lasers: Simple Estimate”, Optical Review 12, 445-447 (2005).
[11] H. F. Talbot, “Facts relating to optical science”, Philos. Mag. 9, 401-407 (1836); L. Rayleigh, “On copying diffraction gratings and on some phenomenon connected therewith”, Philos. Mag. 11, 196-205 (1881).
[12] A. Mafi and J. V. Moloney, “Phase locking in a passive multicore photonic crystal laser”, JOSA B 21, 897-902 (2004).
[13] Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence*, Dover Publications (1984).

[14] A. B. Aceves, C. De Angelis, G. G. Luther, A. M. Rubenchik and S. K. Turitsyn, “Energy Localization in nonlinear fiber arrays: Collapse effect compressor”. Phys. Rev. Lett. 75, 73-76 (1995).

[15] S. Minardi, F. Eilenberger, Y. V. Kartashov, A. Szameit, U. Ropke, J. Kobelke, K. Schuster, H. Bartelt, S. Nölte, L. Torner, F. Lederer, A. Tunnermann, and T. Pertsch, “Three-Dimensional Light Bullets in Arrays of Waveguides”, Phys. Rev. Lett. 105, 263901 (2010).

[16] P. Panayotaros and A. B. Aceves, “Stabilization of coherent breathers in perturbed Hamiltonian coupled oscillators”, Submitted for publication (2011).

[17] V. Eckhouse, M. Fridman and A. Friesem “Loss Enhanced Phase Locking in Coupled Oscillators”, Phys. Rev. Lett. 100, 024102 (2008).