Temperature dependence of Quark and Gluon condensate In The Dyson-Schwinger Equations At Finite Temperature

Zhou Li-Juan¹, Zheng Bo¹, Zhong Hong-wei¹, Ma Wei-xing²

¹ School of Science, Guangxi University of Science and Technology, Liu Zhou, 545006, China
² Institute of High Energy Physics, Chinese Academy of Sciences, Beijing, 100049, China

Abstract

Based on the Dyson-Schwinger Equations (DSEs) with zero- and finite temperature, the two quark condensate, the four quark condensate and quark gluon mixed condensate in non-perturbative QCD state are investigated by solving the DSEs respectively at zero and finite temperature. These condensates are important input parameters in QCD sum rule with zero and finite temperature and properties of hadronic study. The calculated results manifest that the three condensates are almost independent of the temperature below the critical point temperature \( T_c \). The results also show that the chiral symmetry restoration is obtained above \( T_c \). At the same time, we also calculate the ratio of the quark gluon mixed condensate to the two quark condensate which could be quark virtuality. The calculations show that the ratio \( m_0^2(T) \) is almost flat in the region of temperature from 0 to \( T_c \), although there are drastic changes of the quark condensate and the quark gluon mixed condensate at this region of \( T_c \). The predicted ratio comes out to be \( m_0^2(T) = 2.41 GeV^2 \) for vacuum state at the Chiral limit, which suggests the significance that the quark gluon mixed condensate has played in OPE.

Key words: Dyson-Schwinger Equations at zero and finite temperature, Dynamical chiral symmetry breaking, Quark and gluon condensate.

PACS Number(s): 12.38.Lg, 12.38.Mh, 24.85.+p

*) The work was supported in part by National Natural Science Foundation of China (11365002), Guangxi Natural Science Foundation for Young Researchers (2013GXNSFBB053007), Guangxi Education Department (2013ZD049), Guangxi Grand for Excellent Researchers (2011-54), the Funds of Guangxi University of Science and Technology for Doctors (11Z16).
1 Introduction

With the development of heavy-ion collision experiments, more attentions have been turning to exploring the hot and dense QCD matter. The hot and dense matter can be studied via various approaches, such as: lattice QCD, QCD sum rules, Chiral perturbation theory as well as the Dyson-Schwinger equations (DSEs) and so on. Due to the asymptotic freedom feature of QCD, the QCD matter will take place a phase transition from hadronic phase, with quarks and gluons being bound states inside hadron, to the quark gluon plasma phase where the bound clusters of quarks and gluons have been deconfined at sufficient high temperature and/or density. Studying the Chiral condensates at zero- and finite temperature is a crucial importance of nuclear and hadronic physics research, even if for astrophysics and cosmology study.

According to QCD sum rules, quarks exist in the vacuum of non-perturbative QCD, which is densely populated by long-wave fluctuations of gluon fields. The order parameters of this complicated state are described by various vacuum condensates $\langle 0 | : \bar{q} q : | 0 \rangle$, $\langle 0 | : G_{\mu\nu}^a G_{\mu\nu}^a : | 0 \rangle$, $\langle 0 | : i g_s G_{\mu\nu}^a \sigma_{\mu\nu}^{\lambda} q : | 0 \rangle$, ..., which are the vacuum matrix elements of various singlet combinations of quark and gluon fields. In QCD sum rules, various condensates are input parameters so that they play an important role to reproduce various hadronic properties phenomenologically in the operator product expansion calculations (OPE)[1, 2, 3]. Contrary to the important quark condensate $\langle 0 | : \bar{q} q : | 0 \rangle$ and gluon condensate $\langle 0 | : G_{\mu\nu}^a G_{\mu\nu}^a : | 0 \rangle$, the quark gluon mixed condensate $\langle 0 | : i g_s G_{\mu\nu}^a \sigma_{\mu\nu}^{\lambda} q : | 0 \rangle$ characterizes the direct correlation between quarks and gluons, and together with the nonzero two quark condensate $\langle 0 | : \bar{q} q : | 0 \rangle$, it is responsible for the spontaneous breakdown of chiral symmetry. In our previous works, we have studied the various non-perturbative quantities at zero temperature by use of the DSEs in the ”rainbow” truncation, i.e. the quark condensate, the quark gluon mixed condensate, susceptibility, and so on[4]. Comparing our theoretical results with others, such as, QCD sum rules[5], Lattice QCD[6], we find our calculations are in a good agreement with them. Now, we want to extend the calculations of DSEs to nonzero temperature. As is known to all, solve DSEs at finite temperature is quite difficult, but with separable model interactions greatly simplifies the calculations[7, 8]. In the present work, we study the DSEs at finite temperature by use of the separable model interactions. The main interesting of this work lies on the consideration of nonzero temperature, which allow to study the QCD phase diagram along the axis of zero chemical potential, including deconfinement and the chiral symmetry restoration.

2 Dyson-Schwinger Equations at zero and finite temperature

2.1 Dyson-Schwinger Equations at zero temperature

To study quark and gluon condensates, we need to know the quark propagators, which determine various quark condensates and the quark gluon mixed condensates under the OPE constraints. The quark propagator in
configuration space is defined by

\[ S_f(x) = \langle 0| T q(x) \bar{q}(0) |0 \rangle. \]  

For the physical vacuum, the quark propagator can be divided a perturbative
\[ S_f^{PT}(x) \] and a non-perturbative part \[ S_f^{NPT}(x) \], one can write[9][10]

\[ S_f(x) = S_f^{PT}(x) + S_f^{NPT}(x). \]  

In momentum space, \( S_f^{NPT}(p) \) is related to the quark self-energy, so the quark propagator of DSEs can be written

\[ S_f^{-1}(p) = i\gamma \cdot p + m_f + \frac{4}{3} g_s^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S_f(k) \gamma^\nu(k,p) G_{\mu\nu}(p-k). \]  

In Eq.(3), \( g_s \) is the strong coupling constant of QCD with the usual \( \alpha_s(Q) \) by the relationship of \( \alpha_s = g_s^2(Q)/4\pi \). The \( G_{\mu\nu}(p-k) \) denotes fully dressed gluon propagator, and \( m_f \) is the current quark mass with the subscript \( f \) to stand for quark flavor. In Feynman gauge, the simplest separable Ansatz has following form[7][8]

\[ g_s^2 G_{\mu\nu}(p-k) \rightarrow \delta_{\mu\nu} G(p^2, k^2, p \cdot k), \]  

\[ G(p^2, k^2, p \cdot k) = D_0 F_0(p^2) F_0(k^2) + D_1 F_1(p^2) F_1(k^2)(p \cdot k), \]  

where \( D_0 \) and \( D_1 \) are two strength parameters, and \( F_0 \) and \( F_1 \) are corresponding form factors.

As it is impossible to solve the complete set of DSE’s, one has to find a physically acceptable way to truncate this infinite tower and make it soluble. To do it, we use a bare vertex \( \gamma^\nu \) to replace the full one \( \Gamma^\nu(k, p) \) in Eq.(3). This procedure is called as "Rainbow" approximation of DSEs. Thus, Eq.(3) then becomes to

\[ S_f^{-1}(p) = i\gamma \cdot p + m_f + \frac{4}{3} g_s^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S_f(k) \gamma^\nu(k,p) G_{\mu\nu}(p-k). \]  

An important observation is that the general form of the inverse quark propagator \( S_f^{-1}(p) \) can be rewritten in Euclidean space[11] as

\[ S_f^{-1}(p) = i\gamma \cdot p A_f(p^2) + B_f(p^2), \]  

with \( A_f \) and \( B_f \) are scalar functions of the \( p^2 \).

With "Rainbow" truncation, we can obtain the coupling integral equations for quark amplitudes \( A_f(p^2) \) and \( B_f(p^2) \) and these coupling equations take the form in the Feynman gauge

\[ [A_f(p^2) - 1]p^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} G(p-q) \frac{A_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} p \cdot q, \]  

\[ B_f(p^2) - m_f = \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} G(p-q) \frac{B_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)}. \]
2.2 Extension to Finite temperature

So far, we only consider the quark propagator at zero temperature. An extension to temperature dependence of the DSEs from the zero temperature to the finite temperature is systematically accomplished by a transcription of the Euclidean quark four momentum via $p \rightarrow p_n = (\omega_n, \vec{p})$, where $\omega_n = (2n+1)\pi T$ are the discrete Matsubara frequencies. Therefore, a sum over the Matsubara frequencies replace the integral over the energy.

The fully dressed quark propagator of DSEs at finite temperature can be written

$$S_f^{-1}(p_n, T) = i\vec{\gamma} \cdot \vec{p} A_f(p_n^2, T) + i\gamma_4 \omega_n C_f(p_n^2, T) + B_f(p_n^2, T),$$  \hspace{1cm} (10)

where $p_n^2 = \omega_n^2 + \vec{p}^2$. Due to the breaking of $O(4)$ symmetry in the four momentum space, we have three quark amplitudes $A_f$, $B_f$ and $C_f$. The solutions have the form $A_f(p_n^2, T) = 1 + a_f(T) F_1(p_n^2)$, $B_f(p_n^2, T) = m_f + b_f(T) F_0(p_n^2)$ and $C_f(p_n^2, T) = 1 + c_f(T) F_1(p_n^2)$, are defined by the temperature dependent coefficients $a_f(T)$, $b_f(T)$ and $c_f(T)$. The explicit form for $a_f(T)$, $b_f(T)$ and $c_f(T)$ is given by

$$a_f(T) = \frac{8D_1}{9} T \sum_n \int \frac{d^3p}{(2\pi)^3} F_1(p_n^2, \vec{p}^2) [1 + a_f(T) F_1(p_n^2)] d_f^{-1}(p_n^2, T),$$  \hspace{1cm} (11)

$$c_f(T) = \frac{8D_1}{3} T \sum_n \int \frac{d^3p}{(2\pi)^3} F_1(p_n^2, \omega_n^2) [1 + c_f(T) F_1(p_n^2)] d_f^{-1}(p_n^2, T),$$  \hspace{1cm} (12)

$$b_f(T) = \frac{16D_0}{3} T \sum_n \int \frac{d^3p}{(2\pi)^3} F_0(p_n^2) [m_f + b_f(T) F_0(p_n^2)] d_f^{-1}(p_n^2, T),$$  \hspace{1cm} (13)

where the denominator of the quark propagator $S_f(p_n, T)$, $d_f(p_n^2, T)$, is given by

$$d_f(p_n^2, T) = \vec{p}^2 A_f(p_n^2, T) + \omega_n^2 C_f(p_n^2, T) + B_f^2(p_n^2, T).$$  \hspace{1cm} (14)

As we know, solve DSEs at finite temperature is quite difficult, but with separable model interactions greatly simplifies the calculations. For simplicity, we choose the following form for the separable interaction form factor:

$$F_0(p^2) = \exp(-p^2/\Lambda_0^2),$$  \hspace{1cm} (15)

$$F_1(p^2) = \frac{1 + \exp(-p_0^2/\Lambda_0^2)}{1 + \exp((p^2 - p_0^2)/\Lambda_0^2)},$$  \hspace{1cm} (16)

which is successful used to describe the phenomenology of the light pseudoscalar mesons. Substituting Eqs.(15,16) into Eqs.(11,13) one can solving
gap equations for a given temperature $T$, and get the quark amplitudes $A_f$, $B_f$ and $C_f$, but there is need to control the appropriate number of Matsubara modes in calculation.

At the lowest dimension, quark and gluon condensates play essential role in describing properties of nuclear matter and hadron structure. The nonlocal quark condensate in calculation.

\[ B_x = 0 \] the local quark vacuum condensate is given by

\[ -3 \int_0^\infty \frac{p^2 dp^2}{p^2 A_f^2(p^2) + B_f^2(p^2)} \frac{B_f(p^2)}{B_f(p^2)} \frac{2J_1(\sqrt{p^2 x^2})}{\sqrt{p^2 x^2}}, \tag{17} \]

where $N_c = 3$ is number of colors. $J_1$ in Eq. (17) is Bessel function. When $x = 0$, the local quark vacuum condensate is given by

\[ \langle 0 | : \bar{q}(0)q(0) : 0 \rangle = -4N_c \int \frac{d^4p}{(2\pi)^4} \frac{B_f(p^2)}{p^2 A_f^2(p^2) + B_f^2(p^2)} \]

Another important physical quantity is the four quark condensate. The factorization hypothesis for the four quark condensate is well-known from the works of M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov[1], and has been extensively used in QCD sum rules through the operator product expansion approach. For the nonlocal four quark condensate $\langle 0 | : \bar{q}(0) \gamma_\mu \frac{\lambda_5}{2} q(0) \bar{q}(0) \gamma_\mu \frac{\lambda_5}{2} q(0) : 0 \rangle$, according to Ref. [14, 16, 17], we have

\[ \langle 0 | : \bar{q}(0) \gamma_\mu \frac{\lambda_5}{2} q(0) \bar{q}(0) \gamma_\mu \frac{\lambda_5}{2} q(0) : 0 \rangle \]

Similarly, the local ($x = 0$) four quark vacuum condensate is given by

\[ \langle 0 | : \bar{q}(0) \gamma_\mu \frac{\lambda_5}{2} q(0) \bar{q}(0) \gamma_\mu \frac{\lambda_5}{2} q(0) : 0 \rangle \]

which is consistent with the vacuum saturation assumption of Ref. [1].
Besides the quark condensate, the quark gluon mixed condensate is another important chiral order parameter, which plays an important role in QCD sum rules. In the frame work of the global color symmetry model (GCM), the quark gluon mixed condensate are given by

\[ \langle 0 | : \bar{q}(0)g\sigma \cdot G(0)q(0) : | 0 \rangle \]

\[ = - \frac{N_c}{16\pi^2} \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \frac{B_f(p^2)}{p^2 A_f^2(p^2) + B_f^2(p^2)} \left\{ 2A_f(p^2)(A_f(p^2) - 1) \right\} \]  

It is common belief that the quark condensate, which determines light quark mass, depends on temperature \( T \). In the case of finite temperature, one usually takes the same expression to study the temperature dependence of the quark condensate\[8\]. For simplicity, we takes the form

\[ \langle 0 | : \bar{q}(0)q(0) : | 0 \rangle_T = -4N_c T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \frac{B_f(p_n^2, T)}{p^2 A_f^2(p_n^2, T) + \omega_n^2 C_f^2(p_n^2, T) + B_f^2(p_n^2, T)} \]  

It is still a matter of debate for the four quark condensate when \( T \neq 0 \). It was shown in Ref.\[19\], that factorization hypothesis implies that the four quark condensate becomes dependent on the QCD renormalization scale. In addition, theoretical arguments from the chiral perturbation theory also do not support this approximation at next to leading order, except in the chiral limit\[20\]. For simplicity, we takes the form

\[ \langle 0 | : \bar{q}(0)\gamma_\mu \frac{\lambda^a_c}{2} q(0)\gamma_\mu \frac{\lambda^b}{2} q(0) : | 0 \rangle_T = -\frac{4}{9} \langle 0 | : \bar{q}(0)q(0) : | 0 \rangle_T^2, \]  

As to the quark gluon mixed condensate in \( T \neq 0 \) region, we have

\[ \langle 0 | : \bar{q}(0)g\sigma \cdot G(0)q(0) : | 0 \rangle_T \]

\[ = -36T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{B_f(p^2)}{p^2 A_f^2(p^2) + C_f(p^2) \omega_n^2 + B_f^2(p^2)} \left\{ (2 - A_f(p^2)) p^2 + (2 - C_f(p^2)) \omega_n^2 \right\} \right\} \]  

The DSEs provide a valuable non-perturbation, renormalisable, continuum tool for studying temperature dependent field theories. A number of physical phenomena such as confinement, dynamical chiral symmetry breaking, and temperature dependent of quark mass, which cannot be explained by perturbation treatments, can be understood in terms of its solution of the DSEs at zero and finite temperature. Eq.\[11\].
3 Calculations and results

We first solve the quark’s DSEs at zero temperature with parameters $m_u = 5.5 MeV$, $m_s = 117 MeV$, $\Lambda_0 = 758 MeV$, $\Lambda_1 = 961 MeV$, $p_0 = 600 MeV$, $D_0 \Lambda_0^2 = 219$, $D_1 \Lambda_0^2 = 40$, which are completely fixed by meson phenomenology calculated from the model as given in [8]. The obtained results of DSEs at zero temperature are displayed in Fig.1 and Fig.2. At the same time, we obtain the quark condensate $\langle \bar{q}q \rangle = (0.202 GeV)^3$ and the quark gluon mixed condensate $\langle \bar{q}q\sigma G|q \rangle = (0.455 GeV)^5$ at T=0.

In order to demonstrate the temperature dependence of quark propagators, we use Matsubara formula, and we then solve quark’s DSEs at nonzero temperature with the same gluon propagator and parameters. The results are given in Fig.3 and Fig.4. From Fig.3, we can find that, for low temperature, the vector parts of the quark propagator $A_f(0, T)$ and $C_f(0, T)$ coincide with each other, they are almost the same. However, for the temperature higher than about $T = 131 MeV$, they become distinctly different. That means the $O(4)$ symmetry has been broke.

Using the individual solutions of the quark’s DSEs at zero- and finite temperature, $A_f$, $B_f$ and $C_f$ we respectively obtain the properties of the QCD vacuum at zero- and nonzero temperature in the chiral limit case. The quark condensate $\langle \bar{q}q \rangle$, the four quark condensate $\langle \bar{q}\Gamma q\bar{q}\Gamma q \rangle$ and the quark gluon mixed condensate $\langle \bar{q}q\sigma G|q \rangle$ are important condensates of the lowest dimension, which reflect the non-perturbative structure of QCD vacuum state, and can be the chiral order parameter of QCD. In Fig.5, the temperature dependence of the two quark-, the four quark- condensate and the quark gluon mixed condensate in chiral limit in the separable model are plotted respectively. The critical temperature for the chiral symmetry restoration comes out to be $T_c = 131 MeV$. These order parameters give a same critical temperature and the same critical behavior. From Fig.5 we find these three condensates are almost independent of the temperature below $T_c$, while a clear signal of chiral symmetry restoration is shown up at $T_c$. We also calculate the ratio of the quark gluon mixed condensate to the two quark condensate. The results are shown in Fig.6. From the figure, we can see, although there are drastic changes of the quark condensate and the quark gluon mixed condensate near $T_c$, the ratio $m_0^2(T)$ is almost flat when temperature at the region from 0 to $T_c$. For vacuum state at the chiral limit, the ratio $m_0^2(0) = 2.41 GeV^2$, which is larger in comparison with the results from lattice QCD which is about $1 GeV^2$[22], and suggests the great significance that the quark gluon mixed condensate plays in OPE calculations.

In summary, we study fully dressed quark propagator $S_f(p^2, T)$ in QCD by using of the DSEs with zero and finite temperature under the ”rainbow” truncation, $\Gamma^{\nu} = \gamma^{\nu}$. We solve the DSEs numerically and get quark propagator functions, $A_f(p^2, T)$, $B_f(p^2, T)$ and $C_f(p^2, T)$ in Eq.(10) at two cases of $T = 0$ and $p^2 = 0$, and then we obtained the quark propagator $S_f(p^2, T)$. The resulting quark propagator has no Lehmann representation and hence there are no quark production thresholds in any calculations of observable. The absence of such thresholds admits the interpretation that $S_f(p^2, T)$ describes the propagator of a confined quark. With the solutions of the quark’s DSEs $A_f$, $B_f$ and $C_f$, the temperature dependence of the two quark condensate, the four quark condensate and the quark gluon mixed condensate in the chiral limit are obtained. We find these condensates have same critical tem-
perature for the chiral symmetry restoration and same critical behavior for QCD phase transition, though which characterize different aspects of QCD vacuum. At the same time, we study the ratio between quark gluon mixed condensate and the two quark condensate, and obtain the nontrivial result that the ratio is insensitive to temperature below the critical point $T_c$.

References

[1] M. A. Shifman, A.I. Vainshtein, V. Zakharov, Nucl. Phys. B 147, 385 (1979).

[2] L.J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. 127, 1 (1985).

[3] S. Narison, QCD Special Sum Rules, World Scientific, Singapore, 1989, and references therein.

[4] L. -J. Zhou, L. S. Kisslinger and W. -X. Ma, “Nonzero Mean Squared Momentum of Quarks in the Non-Perturbative QCD Vacuum,” Phys. Rev. D 82, 034037 (2010) [arXiv:hep-ph/0904.3558]; S. -M. Qin, L. -J. Zhou, Y. T. Gu and W. -X. Ma, “Vacuum condensate of QCD,” Chinese Physics C 32, 521 (2008).

[5] L. S. Kisslinger and O. Linsuain, arXiv: hep-ph /0110111; L. S. Kisslinger and M. A. Harly, arXiv: hep-ph/9906457; V. A. Novikov, M. A. Shifman, V. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Nucl. Phys. B 237, 525(1984).

[6] D. Takumi, I. Noriyoshi, O. Makoto and Hido Suganuma, Nucl. Phys. A 721, 934C (2003); V. M. Belyaev and B. L. Ioffe, Sov. Phys. JETP 56, 493 (1982); C. D. Roberts, R.T. Cahill, M. E. Sevior and N. Iannelle, Phys. Rev. D 49, 125 (1994); M. V. Polykov and C Weiss, Phys. Lett. B 387, 841(1996).

[7] C. J. Burden, L. Qian, C. D. Roberts, P. C. Tandy and M. J. Thomson, “Ground state spectrum of light quark mesons,” Phys. Rev. C 55, 2649 (1997) [arXiv:nucl-th/9605027].

[8] D. Blaschke, G. Burau, Y. .L. Kalinovsky, P. Maris and P. C. Tandy, “Finite T meson correlations and quark deconfinement,” Int. J. Mod. Phys. A 16, 2267 (2001) [arXiv:nucl-th/0002024].

[9] Zhou Li-juan, Ping Rong-gang, Ma Wei-xing, Commun. Theor. Phys., 42, 875 (2004).

[10] L. S. Kisslinger, T. Meissner, Phys. Rev. C 57, 1528 (1998); M. R. Frank and T. Meissner, Phys. Rev. C 53, 2410 (1996) [arXiv: hep-ph/9511016]; L. S. Kisslinger, M. Aw, A. Harey, and O. Linsuain, Phys. Rev. C 60, 065204 (1999).

[11] C. Roberts and S. Schmidt: Dyson-Schwinger equations: Density, temperature and continuum strong QCD, Progr. Part. Nucl. Phys. 45, Supplement 1, S1 (2000).
[12] T. Matsubara, Prog. Theor. Phys. 14, 351 (1955).

[13] D. Horvatic, D. Blaschke, D. Klabucar and A. E. Radzhabov, “Pseudoscalar Meson Nonet at Zero and Finite Temperature,” Phys. Part. Nucl. 39, 1033 (2008) [arXiv:hep-ph/0703115].

[14] L. S. Kisslinger and T. Meissner, “Structure of vacuum condensates,” Phys. Rev. C 57, 1528 (1998) [arXiv:hep-ph/9706423].

[15] T. Meissner, Phys. Lett. B 405, 8 (1997).

[16] H. -s. Zong, X. -f. Lu, J. -z. Gu, C. -H. Chang and E. -g. Zhao, “Vacuum Condensates in the Global Color Symmetry Model” Phys. Rev. C 60, 055208 (1999) [arXiv:nucl-th/9906078].

[17] H. -s. Zong, X. -f. Lu, E. -g. Zhao and F. Wang “Nonlocal Four-quark Condensate and Vacuum Susceptibilities,” Commun. Theor. Phys. 33, 687 (2000).

[18] Z. Zhang and W. -Q. Zhao, “Mixed quark-gluon condensate at finite temperature and density in the global color symmetry model,” Phys. Lett. B 610, 235 (2005) [arXiv:hep-ph/0406210].

[19] S. Narison and R. Tarrach, Phys. Lett. B 125, 217 (1983).

[20] A. Gomez Nicola, J. R. Pelaez and J. Ruiz de Elvira, “Non-factorization of four-quark condensates at low energies within Chiral Perturbation Theory,” Phys. Rev. D 82, 074012 (2010) [arXiv:hep-ph/1005.4370].

[21] A. Gomez Nicola, J. R. Pelaez and J. Ruiz de Elvira, “Scalar susceptibilities and four-quark condensates in the meson gas within Chiral Perturbation Theory,” Phys. Rev. D 87, 016001 (2013) [arXiv:hep-ph/1210.7977].

[22] T. Doi, N. Ishii, M. Oka and H. Suganuma, “Thermal effects on quark gluon mixed condensate from lattice QCD,” Phys. Rev. D 70, 034510 (2004) [arXiv:hep-lat/0402005].
Figure 1: $p^2$-dependence of quark self-energy amplitudes $A_f(p^2)$, subscript $f$ for the ud quark, the s quark and the chiral limit cases.
Figure 2: $p^2$-dependence of quark self-energy amplitudes $B_f(p^2)$, subscript $f$ for the ud quark, the s quark and the chiral limit cases.
Figure 3: T-dependence of quark self-energy amplitudes $A_f(0,T)$, and $C_f(0,T)$, subscript $f$ for the ud quark, the s quark and the chiral limit cases.
Figure 4: T-dependence of quark self-energy amplitudes $B_f(0, T)$, subscript $f$ for the ud quark, the s quark and the chiral limit cases.
Figure 5: T-dependence of the quark condensate, the four quark condensate and the quark gluon mixed condensate in the chiral limit cases.
Figure 6: T-dependence of the ratio of the quark gluon mixed condensate to the quark condensate, $m_0^2 = \frac{\langle 0|\bar{q}(0)\gamma_\sigma G(0)q(0)|0\rangle_T}{\langle qq \rangle_T}$, in the chiral limit cases.