The decay $K_L \to \pi^0 \nu \bar{\nu}$ is an excellent probe of the nature of CP violation. It is almost entirely CP-violating, and hadronic uncertainties are negligible. Experiments which hope to detect the decay are currently being planned. We calculate the decay rate in several extensions of the standard model Higgs sector, including the Liu-Wolfenstein two-doublet model of spontaneous CP-violation and the Weinberg three-doublet model. In a model with an extra doublet, with CP-violation arising from the CKM sector, the rate can increase by up to 50%. However, in models in which the CP violation arises either entirely or predominantly from the Higgs sector, we find that the decay rate is much smaller than that of the standard model, unless parameters of the model are fine-tuned.
1 Introduction

One of the deepest mysteries in theoretical physics concerns the nature and origin of the CP violation observed in the kaon system. Although it can be accommodated within the three generation standard model, most extensions of the standard model contain additional sources of CP violation\cite{1}. A primary motivation for the construction of B-factories is to explore CP violation in a regime in which it is expected to be considerably larger.

Within the standard model, much of the effort in understanding CP violation has focused on finding the values of the CKM matrix, which are parameterized by the Wolfenstein parameterization\cite{2, 3}, in which one has

\begin{align}
|V_{us}| &= \lambda; \quad |V_{cb}| = A\lambda^2; \quad V_{ub} = A\lambda^3(\rho - i\eta); \quad V_{td} = A\lambda^3(1 - \bar{\rho} - i\bar{\eta})
\end{align}

where \(\bar{\rho} \equiv \rho(1 - \lambda^2/2)\) and \(\bar{\eta} \equiv \eta(1 - \lambda^2/2)\). From K- and B- decays, these parameters can be determined. Unfortunately, the interpretations of many current measurements of CP violation in the kaon system, as well as of many future measurements in the B system, are plagued by theoretical uncertainties. These result from the absence of precise non-perturbative calculations of hadronic matrix elements. For example, determination of \(V_{cb}\) and \(V_{ub}\) to an accuracy of better than 5% and 10% respectively may not be possible without a significant improvement in the determination of hadronic matrix elements. In addition, loop-induced decays also contain significant theoretical uncertainties, which affect the predictions (and interpretations) for \(\epsilon'/\epsilon\), \(B^0 - \bar{B}^0\) mixing, etc. As emphasized by Buras et al.\cite{4}, even with optimistic assumptions about the theoretical and experiment errors, it will be difficult to achieve an accuracy better than \(\pm 0.15\) in \(\rho\) and \(\pm 0.05\) in \(\eta\).

As also emphasized by Buras and others\cite{5}, there are two processes in which the hadronic uncertainties are significantly reduced, and two processes in which they are virtually absent. The former two are \(K^+ \rightarrow \pi^+ \nu\bar{\nu}\) and the ratio of \(B^0_d - \bar{B}^0_d\) mixing to
$B^0_s - \overline{B}^0_s$ mixing. The gold-plated decays, in which theoretical uncertainties are extremely small, are the CP asymmetry in $B^0_d \rightarrow \psi K_s$ and the decay $K_L \rightarrow \pi^0\nu\overline{\nu}$. Buras\cite{4} has noted that measurement of the CP asymmetry in $B^0_d \rightarrow \psi K_s$ plus a measurement of the branching ratio for $K_L \rightarrow \pi^0\nu\overline{\nu}$ would allow a determination of all of the elements of the CKM matrix without any significant hadronic uncertainties, assuming that the CP violation is entirely in the CKM matrix. In this paper, we will be concentrating on the mode $K_L \rightarrow \pi^0\nu\overline{\nu}$, which (up to $O(\epsilon)$ corrections) is entirely CP-violating and free of substantial hadronic uncertainties.

The expected branching ratio for $K_L \rightarrow \pi^0\nu\overline{\nu}$ in the standard model is approximately $3 \times 10^{-11}$. This is many orders of magnitude smaller than the current upper bound of $5.8 \times 10^{-5}$\cite{3}. However, upcoming experiments are expected to improve the bound to $10^{-8}$, and preliminary studies at CEBAF and BNL\cite{7} claim that, not only could $K_L \rightarrow \pi^0\nu\overline{\nu}$ be detected, but as many as 100 events could be seen. Although these studies are only in a very preliminary stage, a 10% measurement of the branching ratio does not appear to be impossible within the next decade.

As discussed above, virtually all extensions of the standard model contain additional sources of CP violation. One might expect the branching ratio for $K_L \rightarrow \pi^0\nu\overline{\nu}$ to be different in these models. Although the branching ratio has been calculated in the standard model, including QCD corrections\cite{8}, we know of no calculations of the branching ratio in models in which CP violation arises from a source other than the CKM matrix.

Given the potential precision of a measurement of $K_L \rightarrow \pi^0\nu\overline{\nu}$, and the likelihood of additional sources of CP violation in extensions of the standard model, it is important to calculate the branching ratio in these extensions. Even if it is some time before the necessary precision is reached, one should still look at the branching ratio in extensions of the standard model in the hope that some models might have a significantly higher rate–this might motivate “intermediate” experiments which might not reach the standard
model rate. For example, the electric dipole moment of the neutron is very small in the standard model, and is not in reach of experiments, but extensions of the model can have a much larger rate, and this has provided strong motivation for experiments which have lowered the bound substantially (ruling out several models in the process).

In this paper, we will calculate the rate for $K_L \to \pi^0 \nu \bar{\nu}$ in models with an extended Higgs sector. Such models are the simplest extensions of the standard model, and have additional sources of CP violation. Models with additional gauge groups, such as left-right models, and supersymmetric models, are currently under investigation.

In the next section, we will review the standard model result for $K_L \to \pi^0 \nu \bar{\nu}$, and then consider the simplest extension of the standard model, in which a single Higgs doublet is added to the standard model, and yet all of the CP violation still arises from the CKM matrix. In Section 3, the most general two-doublet model in which CP is violated spontaneously will be considered, along with the Weinberg three-doublet model; in one subsection, the effects of neutral Higgs bosons will be considered and in the next subsection, the effects of charged Higgs bosons will be included. Finally, in Section IV, we present our conclusions.

2 The standard model and simplest extension

The calculation of $K_L \to \pi^0 \nu \bar{\nu}$ amounts to determining the coefficient of the effective Lagrangian for $d \bar{s} \to \nu \bar{\nu}$, and evaluating the hadronic matrix element. The matrix element will be the same as that in semileptonic $K_L$ decay, and thus in the ratio of the rate for $K_L \to \pi^0 \nu \bar{\nu}$ to that of the semileptonic decay, the matrix element will cancel. There are two types of diagrams which contribute to this effective Lagrangian. The first are $Z$-penguins, generated by an induced $d \bar{s} Z$ coupling, and are shown in Figure 1. The second consist of box diagrams, shown in Figure 2.
Inami and Lim\textsuperscript{9} have calculated these contributions, in the limit that external masses and momenta are much smaller than the internal masses. The amplitudes are then described by an effective four-fermion interaction: The effective Lagrangian is given by the form

\[ L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi}\sin^2\theta_W V_{ts}^* V_{td} \left( 4 D \pi \gamma_\mu d_L \sum_{i=1}^{3} \nu_{L_i} \gamma_\mu \nu_{L_i} \right) \]  

(2)

where the sum is over the three neutrino flavors. Using unitarity, Inami and Lim show that one can calculate the contribution due to the top quark, and then (ignoring the up and charm quark masses) subtract the mass independent part, so that only the CKM matrix elements involving the top quark enter. In the CP-violating decay, \( K_L \to \pi^0 \nu \bar{\nu} \), the imaginary part of \( L_{\text{eff}} \) will enter.

Including the contribution of the box diagrams (in the limit that lepton masses are ignored compared to the top),

\[ D(x_t) = \frac{x_t}{4} \left[ \frac{3x_t - 6}{(1 - x_t)^2} \ln \frac{x_t + 2}{x_t - 1} \right] \]  

(3)

where \( x_t \equiv m_t^2/M_W^2 \). The ratio of branching ratios is then

\[ \frac{B(K_L \to \pi^0 \nu \bar{\nu})}{B(K^+ \to \pi^0 e^+ \nu)} = \frac{3}{4\pi^4} \frac{\tau_{K_L}}{\tau_{K^+}} \left( \frac{G_F^2 m_W^4}{A^4} \right)^2 \lambda^8 \eta^2 D(x_t)^2 \]  

(4)

which gives a branching ratio of \( \sim 3 \times 10^{-11} \) in the standard model (using \( \eta = 0.35 \) (see ref. \textsuperscript{4})).

We begin our consideration of extensions beyond the standard model by looking at the simplest extension: the two-doublet model, in which CP violation occurs through the CKM sector. In this case, the rate will also depend on the imaginary part of \( V_{ts}^* V_{td} \), and the only change will be the addition of charged Higgs loops in Figure 1.

In this simplest extension, one Higgs doublet couples to one quark charge, and the other couples to the other quark charge. The detailed vertices and Lagrangian are well-known\textsuperscript{10} (and can be obtained from the \( \xi = 0 \) limit of the model discussed in the next
section). The neutral Higgs boson interactions are flavor-conserving, and thus will not contribute to the diagrams of Fig. 1. The only difference is that we now have physical charged Higgs bosons in the loop instead of just W and Goldstone bosons. The charged Higgs bosons appear in diagrams (a), (b), (d) and (h) (note that there is no ZWH vertex in the model). The divergences in these diagrams cancel, and we find that the ratio of the contribution of charged Higgs boson loops to the amplitude relative to the standard model result, $R$ is

$$R = -\frac{1}{4} \cot^{2} \beta \frac{(1 - x_{t})^{2}}{(1 - x)^{2}} \left( \frac{(x(4 - x) - 2x^{2} \cos 2\theta_{W}) \ln x + x(1 - x)(3 - 2 \cos 2\theta_{W})}{(3x_{t} - 6) \ln x_{t} - (1 - x_{t})(2 + x_{t})} \right)$$

where $x \equiv (m_{t}/m_{H^{+}})^{2}$ and $\tan \beta$ is the ratio of the vacuum expectation values of the two Higgs doublets (in most unified models it is greater than unity, and must be greater than 0.5 for perturbation theory to be valid). For a charged Higgs mass of 150 (250, 400) GeV, the ratio is $R = .32 (.20, .12)$ times $\cot^{2} \beta$. Thus, for $\tan \beta$ near unity, this can increase the branching ratio by a factor of 1.74 for a charged Higgs mass of 150 GeV. It should be noted that this model has a lower bound on the charged Higgs mass arising from $b \to s\gamma$ of 200 GeV[11], which gives an increase in the branching ratio of approximately 50% (for $\tan \beta \sim 1$).

Belanger et al.[12] have also considered the rate for $K_{L} \to \pi^{0}\nu\overline{\nu}$ in this model. Their results are consistent with ours. They note that the ratio of the rates is given by $(1 + R)^{2}Q$, where $Q$ is the ratio of the CKM parameters $(A^{4} \eta^{2})$ as determined from experiments including the effects of the charged Higgs to the values of these parameters as determined from experiments in the standard model (without the charged Higgs). The value of $Q$ is consistent with unity, since no discrepancy with the standard model is seen. However, by scanning parameter-space, and requiring all experimental results to be within the 90% confidence level, they show that there is a region of parameter-space in
which the value of $Q$ can be somewhat larger, leading to a larger rate. Our philosophy is that this involves charged Higgs effects in experiments other than $K_L \to \pi^0 \nu \bar{\nu}$, and that by the time the experiment is done, the uncertainties in $(A^4 \eta^2)$ will be much smaller, in the range of 10 percent\[4\]. Nonetheless, one should be aware that the extraction of the CKM angles in this model may give results different from those in the standard model.

3 Spontaneous CP-violation

Another attractive mechanism for CP-violation is spontaneous CP-violation\[13\]. This cannot occur in the single Higgs model, and thus requires extension of the Higgs sector. If one adds one more Higgs doublet, then one can violate CP spontaneously, but at the cost of tree level flavor-changing neutral currents (FCNC). The discrete symmetry that is usually implemented to eliminate such currents will also eliminate the spontaneous CP violation\[14\]. One has two choices: break the discrete symmetry by parameters which are sufficiently small that FCNC are not phenomenologically problematic, or keep the discrete symmetry and enlarge the Higgs sector by adding the third doublet. The former option was analyzed in detail by Liu and Wolfenstein\[15\], the latter is the model of Weinberg\[16\]. We first consider the Liu-Wolfenstein model.

The model contains two Higgs doublets, and the most general CP-invariant Yukawa coupling and Higgs potential is

$$
-L_Y = \overline{\Psi}_L^i (F_{ij} \tilde{\Phi}_2 + \xi F_{ij}^\prime \tilde{\Phi}_1) U^a_{Rj} + \overline{\Psi}_L^i (G_{ij} \Phi_1 + \xi G_{ij}^\prime \Phi_2) D^a_{Rj} + \text{h.c.},
$$

$$
V = -\mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2
+ \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 \left[ (\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right]
+ \frac{1}{2} \xi' (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) (\lambda_6 \Phi_1^\dagger \Phi_1 + \lambda_7 \Phi_2^\dagger \Phi_2)\]
$$

Here, $\xi$ and $\xi'$ are small parameters which determine the amount by which the discrete
symmetry \((\Phi_2 \leftrightarrow -\Phi_2, \ D^o_R \leftrightarrow -D^o_R)\) which eliminates FCNC is broken. The fact that both Higgs doublets couple to all of the fermions ensures the existence of FCNC, since diagonalizing the quark mass matrix will not automatically diagonalize the Yukawa coupling matrices. Minimizing the potential yields

\[
\langle \Phi_1 \rangle = \sqrt{\frac{1}{2}} \left( \begin{array}{c} 0 \\ v_1 \end{array} \right), \quad \langle \Phi_2 \rangle = \sqrt{\frac{1}{2}} \left( \begin{array}{c} 0 \\ v_2 e^{i\alpha} \end{array} \right).
\] (7)

The CP-violating phase \(\alpha\) is given by

\[
\cos \alpha = -\xi' \frac{\lambda_6 v_1^2 + \lambda_7 v_2^2}{4\lambda_5 v_1 v_2}.
\] (8)

Liu and Wolfenstein discuss two limiting cases. If \(\xi = 0, \xi' \neq 0\), then the model becomes an earlier model of Branco and Rebelo[17]. Here, CP-violation occurs in the Higgs sector, however, there are no FCNC at tree level, and thus in order to obtain a \(\Delta S = 2\) CP-violation one must go to two loops. As a result, the value of \(\epsilon\) is too small. The second case is if \(\xi' = 0, \xi \neq 0\), then the CP-violating phase is \(\pi/2\). As Liu and Wolfenstein discuss, spontaneous CP violation in this limit is the same as introducing a purely imaginary Yukawa coupling \(i\xi\) which breaks the discrete symmetry. Although this model is certainly viable, there is no natural mechanism for ensuring \(\xi' = 0\), although they use this limit in their numerical examples, as will we.

In this model, there will be contributions to the \(K_L \to \pi^0\nu\bar{\nu}\) rate from charged Higgs loops (as in the simple model in the last section, albeit with very different couplings), as well as from neutral Higgs loops. Since CP violation has a different origin in this model, one might hope to avoid the \(V_{ts}^* V_{td}\) suppression factor present in the standard model result.

3.1 Neutral Higgs bosons

We will first consider effects of neutral Higgs bosons. Since the neutrinos are very light, their interactions with Higgs bosons will be negligible, and thus box diagrams will not
contribute. We have only corrections to the $\bar{s}dZ$ vertex, and the internal fermion line will be a $b$-quark, rather than a top quark. It is clear that we will need two flavor-changing neutral current couplings, so the result will be proportional to $\xi^2$.

The flavor-changing Yukawa couplings can be found from the Yukawa terms in Eq. 6. The couplings of the neutral complex fields, $\phi_1$ and $\phi_2$, to down-type quarks are given by

$$-\mathcal{L}_Y = \bar{D}'_{Li}(G_{ij}\phi_1 + \xi G'_{ij}\phi_2)D'_{Rj} + \text{h.c.}$$

where the primes indicate the weak eigenstate basis. Plugging in $v_1/\sqrt{2}$ and $v_2 e^{i\alpha}/\sqrt{2}$ for the vacuum expectation values, and defining $D'_R \to e^{-i\alpha}D'_R$ yields the mass matrix

$$M_d = \frac{1}{\sqrt{2}}(G + \frac{e^{-i\alpha}G'}{v_1})v_1 \equiv M_d + e^{-i\alpha} \frac{v_2}{v_1} M'_d$$

where flavor indices have been suppressed. There are three neutral physical Higgs fields and one neutral Goldstone boson, which we denote by $H_j$, with $j = 1 - 4$ where $H_4$ is the Goldstone boson (the calculation is done in the Feynman gauge, so the Goldstone boson mass is the $Z$-boson mass). To rotate to the fermion mass eigenstate basis, we need to define

$$N \equiv V_L M'_d V_R^\dagger$$

where $V_{L,R}$ rotate $D'_L$ and $D'_R$ into their mass eigenstates $D_L$ and $D_R$. We then find that the general flavor-changing Yukawa coupling of $\bar{D}'_{Li}D'_{Rj}H_k$ is given by

$$i \frac{(\sqrt{2}G_F)^{\frac{3}{2}}}{\cos^2 \beta} \xi D_i \left[ e^{i\alpha} N_{ij} (S_{2k} + iS_{4k}) R + e^{-i\alpha} N_{ji}^* (S_{2k} - iS_{4k}) L \right] D_j H_k$$

Here, $L$ and $R$ are $\frac{1}{2}(1 \mp \gamma_5)$, $\tan \beta \equiv v_2/v_1$ and $S_{ij}$ is the matrix which diagonalizes the $4 \times 4$ Higgs mass matrix. $S_{ij}$ depends on parameters in the Higgs potential and is essentially undetermined. Note that if $\xi' = 0$, then the $4 \times 4$ matrix divides into two $2 \times 2$ matrices (the scalar and pseudoscalar matrices, respectively), and then either $S_{2k}$ or $S_{4k}$ will vanish, greatly simplifying the vertex.

9
Due to the proliferation of parameters, we will now greatly simplify the calculation by taking the special case $\xi' = 0$, as was done by Liu and Wolfenstein. There is a potential delicacy with that limit. If the Lagrangian is CP-invariant (i.e. all of the CP-violation arises spontaneously), then only $\xi$ and $\xi'$ can violate CP. Any effect proportional to $\xi^2$ only will then not violate CP (as discussed above, $\xi' = 0$ is equivalent to multiplying $\xi$ by $i$). However, one can certainly have a model in which there is both explicit and spontaneous CP violation, thus the $N$ matrices need not be real. In that case, our results will not be significantly affected by this assumption. Even if one assumes that the Lagrangian is CP-invariant, and relaxes the $\xi' = 0$ assumption, then there will be terms of $O(\xi'^2)$, as well as $O(\xi^3)$, which do violate CP; these terms will be $O(\xi')$ or $O(\xi)$ times terms that we will calculate. In that particular case, under the assumption that the Lagrangian is CP-invariant, our numerical results would be somewhat larger than the actual result (note that there are no real bounds on the size of $\xi'$ other than it is “small”). We will discuss the implications of $\xi' \neq 0$ later.

The neutral Higgs loops contribute to diagrams (a), (b) and (d) in Fig. 1., in which the $G^-$ is replaced by a neutral Higgs (and the $u_i$ fermion is replaced by a $d_i$; the leading contribution will come from internal $b$-quarks. Note that under the assumption $\xi' = 0$, the scalars and pseudoscalars decouple, and the $Z$ boson only couples to a scalar plus a pseudoscalar. As a result, diagram (h) doesn’t contribute to the vector $\bar{\tau} \gamma_\mu d$ effective Lagrangian. In addition, the need for two flavor-changing neutral current vertices implies that both fermion vertices must involve a Higgs boson, and thus diagrams (f) and (g) will not contribute. A further simplification, for the sake of illustration, can be made by taking all of the neutral scalars to have the same mass as the Goldstone boson, i.e. $M_Z$—we will discuss the results of relaxing this assumption shortly. In that case, the resulting sum over the four Higgs boson contributions just becomes $\sum_k (S^2_{2k} + S^2_{4k})$, which
is 2. The effective Lagrangian from these loops is found to be
\[ \mathcal{L} = \frac{G_F}{4\sqrt{2}\cos^2\beta} 4\pi r^2 T_w \bar{\sigma}_\mu \gamma^\mu \] (13)

where
\[ T_1 = \xi^2 \frac{N_{sb} N_{db}^* - N_{bs}^* N_{bd}}{m_W^2} \left( x_b (4 - x_b) \ln(x_b) + \frac{3x_b}{(1 - x_b)^2} \right) \] (14)
and \( x_b \equiv m_b^2 / m_H^2 \). Note that if we relax the assumption that the Higgs masses will be the Z mass, but still assume that they are degenerate, then this result will hold except for a slightly different contribution from the Goldstone boson. One expects, of course, the lightest of the Higgs bosons to give the biggest contribution.

Using this result, we can find the ratio of amplitudes, \( A \) for \( K_L \rightarrow \pi^0 \nu \bar{\nu} \) in this model to that in the standard model. This gives
\[ \frac{A_{new}}{A_{SM}} = \frac{\xi^2}{\cos^4\beta} \frac{\text{Im}(N_{sb} N_{db}^* - N_{bs}^* N_{bd})}{m_b^2} T_2 \] (15)

where
\[ T_2 \equiv \frac{m_b^2}{2m_W^2} \frac{1}{\text{Im}(V^*_{ts}V_{td}) x_t} \left[ \frac{(4 - x_t) \ln(x_t)}{(1 - x_t)^2} + \frac{3}{1 - x_t} \right] \] (16)

Using the Wolfenstein parametrization, the \( \text{Im}V^*_{ts}V_{td} \) term is \( A\eta \lambda^5 \), which is (for \( \eta \approx .35 \)) \( 1.8 \times 10^{-4} \). Using neutral Higgs masses of \( m_Z \), as discussed earlier, we find
\[ \frac{A_{new}}{A_{SM}} = .06 \frac{\xi^2}{\cos^4\beta} \frac{\text{Im}(N_{sb} N_{db}^* - N_{bs}^* N_{bd})}{m_b^2} \] (17)

At first sight, it appears that this ratio could be quite large. In virtually all models, the value of \( \tan \beta \) ranges from unity to \( m_t / m_b \sim 35 \). At the upper end of the range, \( \cos^4\beta \) can be as small as \( 10^{-6} \). If \( \xi \sim 0.1 \), and the \( N \) matrix elements are the size of the largest mass scale expected (\( m_b \)), then the ratio could be several hundred, leading to a rate as much as five orders of magnitude greater than the standard model rate.
However, the value of $\xi N$ is not arbitrary. It contributes to $\epsilon$ and thus is constrained. Liu and Wolfenstein have calculated the neutral Higgs contribution to $\epsilon$. In the two-generation case, they find, taking the Higgs scalar masses to be $100$ GeV,

$$\frac{\xi^2}{\cos^4 \beta} = \frac{2 \times 10^{-3}}{\cos^{2/3} \beta \sin^{2/3} \beta} \left( \frac{1}{(\sigma + \sigma')^{2/3} (N_{12} - N_{21})^{4/3} (N_{12} + N_{21})^{2/3}} \right)$$

(18)

where one writes $N_{ij}$ in terms of its real and imaginary parts: $N_{ij} = N_{ij}' + i \xi \tan \beta n_{ij}$ and defines

$$\sigma \equiv -\frac{n_{12} + n_{21}}{N_{12}' + N_{21}'} \quad \sigma' \equiv \frac{n_{21} - n_{12}}{N_{12}' + N_{21}'}.$$  

(19)

Since physical quantities can only depend on the product $\xi N$, the expressions for $\sigma$ and $\sigma'$ depend on a particular convention. Liu and Wolfenstein scale $\xi$ by assuming that $N_{12} - N_{21} = m_s \sin \theta_c \simeq \sqrt{m_dm_s}$. With this convention, they argue that the natural values of $\sigma$ and $\sigma'$ are of $O(1)$, and that if one assumes that the $N$ matrices have the same structure as the quark mass matrices, then all of the terms in parentheses in Eq. (18) should be of $O(1)$. Writing the terms in parentheses as $A'$, we can then write (with this convention)

$$\frac{A_{new}}{A_{SM}} = 1.2 \times 10^{-4} A' \frac{1}{\cos^{2/3} \beta \sin^{2/3} \beta} \frac{\Im(N_{sb} N_{db}^* - N_{bd}^* N_{bs})}{m_b^2}$$

(20)

Of course, the expression in Eq. (18) is only valid in the two-generation case. In the general case, the expression in parentheses will be much more complicated. Nonetheless, the result in Eq. (20) will be unaltered, and one still also expect the value of $A'$ to be $O(1)$.

Even if $\tan \beta \sim m_t/m_b$, this ratio will be no greater than one percent, and thus unmeasurable. The only way to get a large rate would be to assume that either $A'$ is much greater than unity (which requires extensive fine-tuning) or that the off-diagonal terms in the $N$ matrix are much larger than the largest mass scale in the down-quark sector. Neither of these seems likely. In addition, the requirement that Higgs mediated
$B - \overline{B}$ mixing not be too large gives strong constraints\textsuperscript{[18]} on $N_{bd}$, which we find to be approximately $\xi N_{bd}/m_b \leq 0.007$, which further constrains the ratio.

### 3.2 Charged Higgs Bosons

What about the contribution of charged Higgs bosons in the Liu-Wolfenstein model? In this model, the coupling of the charged Higgs bosons to fermions is given by

$$\mathcal{L} = -i(2\sqrt{2}G_F)^{1/2} \left( H^+ U (\Gamma_1 L + \Gamma_2 R)D + H^{-} \overline{D} (\Gamma_1^\dagger R + \Gamma_2^\dagger L) U \right)$$

(21)

where

$$\Gamma_1 = V^\dagger_L [\cot \beta M_u - \xi e^{i\alpha} N_u / \sin^2 \beta]$$

$$\Gamma_2 = [\tan \beta M_d - \xi e^{-i\alpha} N_d / \cos^2 \beta] V^\dagger_L$$

(22)

Here, the matrices $M_u$ and $M_d$ are diagonal, and the matrices $N_u$ and $N_d$ are defined as in the neutral Higgs case. $V_L$ is the CKM matrix. Note that if $\xi = 0$, the couplings reduce to the usual two-Higgs model.

The diagrams are the same as in the two-Higgs case, and only internal top quarks are considered. The effective Lagrangian arising from diagrams (a), (b) and (d) is found to be

$$\mathcal{L}_1 = \frac{G_F}{8\sqrt{2}} \frac{\alpha}{4\pi \sin^2 \theta_W} T_1 \overline{\nu}_L \gamma^\mu \nu_L$$

(23)

where

$$T_1 = \frac{(\Gamma_1^\dagger)^{st} (\Gamma_1)^{td} - (\Gamma_2^\dagger)^{st} (\Gamma_2)^{td}}{m_W^2} \left( \frac{x(4-x)\ln x}{(1-x)^2} + \frac{3x}{1-x} \right)$$

(24)

and $x \equiv m_t^2 / m_H^2$. In this case, diagram (h) also contributes, and the effective Lagrangian is

$$\mathcal{L}_2 = \frac{G_F}{4\sqrt{2}} \frac{\alpha}{4\pi \sin^2 \theta_W} T_2 \overline{\nu}_L \gamma^\mu \nu_L$$

(25)
and

$$T_2 = -\cos 2\theta_W \frac{(\Gamma_1)^\dagger_{st}(\Gamma_1)_{td} + (\Gamma_2)^\dagger_{st}(\Gamma_2)_{td}}{m_W^2} \left( \frac{x^2 \ln x}{(1 - x)^2} + \frac{x}{1 - x} \right)$$  \quad (26)

The $\xi = 0$ part of the effective Lagrangian is identical to the simplest extension considered in the last section, in which there is no spontaneous CP-violation and the CKM matrix is real. What about the $\xi^2$ terms? The $\Gamma$ factors become

$$(\Gamma_1)^\dagger_{st}(\Gamma_1)_{td} \pm (\Gamma_2)^\dagger_{st}(\Gamma_2)_{td} = \xi^2 \left( \frac{(V_L N_u^\dagger)^{st}(N_u V_L^\dagger)_{td}}{\sin^4 \beta} \pm \frac{(N_d^\dagger V_L)_{st}(V_L N_d)_{td}}{\cos^4 \beta} \right)$$  \quad (27)

Once again, we don’t know the values of the $N_u$ and $N_d$ matrix elements, but can assume that they are not much larger than the top and bottom masses, respectively. Consider the contribution of the $N_d$ terms. They give an expression which is identical to that of the neutral case except for some extra $V_L$ matrices and replacing $x_b = m_b^2/m_{H^+}^2$ with $x = m_t^2/m_{H^+}^2$. This latter change will reduce the size of the final result (due to the absence of the large logarithm), and it is unlikely that including the CKM matrices will increase the result, and thus the contribution of the $N_d$ terms will also be very small.

The ratio of the contribution of the $N_u$ terms to the standard model result is (choosing $m_{H^+} = 150$ GeV and using Eq. (18))

$$\left| \frac{A_{new}}{A_{SM}} \right| \approx 10^{-5} A' \cos^{10/3} \beta \frac{\sin^{14/3} \beta}{m_t^2} Im[(V_L N_U^\dagger)_{st}(N_u V_L^\dagger)_{td}]$$  \quad (28)

Even if one chose to ignore the CKM factors, and assume that $N_u$ is of order $m_t$, then, since $\tan \beta \geq 1$, this is no more than $0.02A'$, and thus will also not be large (unless, as discussed earlier, one fine-tunes to make $A'$ large. We conclude that the $\xi^2$ effects are not significant.

There is a cross-term which is linearly dependent on $\xi$ We find that

$$\left| \frac{A_{new}}{A_{SM}} \right| \approx 10^{-2} \sqrt{A'} \cos^{8/3} \beta \frac{m_t [V_{st}(N_u V_L)_{td} + V_{dt}(N_u V_L)_{ts}]}{m_b^2}$$  \quad (29)

Again, if one assumes that $(N_u V_L)_{td}$ is approximately $m_t V_{td}$, this is approximately $3 \times 10^{-3} \sqrt{A'}$, which is not measurable [19]
If one assumes that the CP violation is entirely spontaneous, i.e. that there is no CKM CP-violation, then this model has the ability, as shown by Liu and Wolfenstein, to explain all observed CP-violating phenomena. However, as we have seen, it will generally give a much smaller rate for $K_L \to \pi^0 \nu \bar{\nu}$ than the standard model. Note that, as discussed earlier, if one does not assume $\xi' = 0$, then the result will be $O(\xi)$ or $O(\xi')$ times smaller than the terms that we have calculated.

Perhaps the most well-known model of spontaneous CP violation is the Weinberg model[16]. Although bounds from the neutron electric dipole moment and $b \to s \gamma$ seem to rule out the model[20], it might survive with some fine-tuning and other similar models might still be viable. This model assumes that there are no tree-level flavor changing neutral currents, and as a result three Higgs doublets are needed in order to violate CP spontaneously. All CP violation is to come from the Higgs sector, and thus the CKM matrix is real. Since there are no tree-level flavor changing neutral currents, neutral Higgs bosons will not contribute to the $K_L \to \pi^0 \nu \bar{\nu}$ decay at one-loop. There are two charged Higgs bosons (in addition to the charged Goldstone boson), whose couplings to fermions are given by

$$\mathcal{L}_Y = (2\sqrt{2} G_F)^{1/2} \sum_{i=1}^2 \left( \alpha_i U_L M_D D_R + \beta_i U_R M_U V_L D_L \right) H_i^+ + \text{h.c.} \quad (30)$$

where $V_L$ is the real CKM matrix. The CP violation occurs in the (complex) parameters $\alpha_i$ and $\beta_i$. The observed CP violation parameter $\epsilon$ is proportional to $\sum_i \text{Im} \left( \alpha_i \beta_i^* \right)/m_{H_i^+}^2$. Since the neutron electric dipole moment is proportional to the same parameter, it is predicted in the model (modulo long-distance effects), and, as discussed above, tends to give too large a value[20].

In the calculation of the contribution of the charged Higgs bosons to the diagrams in figure 1, we find that all terms are proportional to $\alpha_i^* \alpha_i$ or to $\beta_i^* \beta_i$, and thus have no imaginary part; the one-loop penguin contributions vanishes. This is not surprising,
since the value of $\epsilon$ and of the neutron electric dipole moment involve the operator $d\sigma_{\mu\nu}s$ whereas we are here interested in $d\gamma_{\mu}s$, and the extra $\gamma$ matrix is needed to give the $\alpha_i\beta_i^*$ structure instead of $\alpha_i\alpha_i^*$. There will be a one-loop box contribution, but this will be suppressed by two powers of the tau-lepton mass divided by $M_W$. Thus the rate for $K_L \rightarrow \pi^0\nu\bar{\nu}$ in the Weinberg model will be much lower than that of the standard model.

4 Conclusions

The process $K_L \rightarrow \pi^0\nu\bar{\nu}$ is an extremely promising probe of the nature of CP violation. It is almost entirely CP-violating and is free of significant hadronic uncertainties. The branching ratio, which is calculated quite precisely in the standard model, is small, but within reach of currently planned experiments, and its measurement to 10% accuracy may be possible. In this paper, we have calculated the branching ratio in models in which the CP violation arises either completely or partially from an extended Higgs sector. We have concentrated on the Liu-Wolfenstein and Weinberg models, although the results should be fairly general. In spite of potentially large contributions, it has been shown that when the constraints caused by fitting the value of $\epsilon$ are included, the contribution of both neutral and charged Higgs bosons to the branching ratio become very small. Thus, in a model in which most or all of the CP violation arises from the Higgs sector, the branching ratio for $K_L \rightarrow \pi^0\nu\bar{\nu}$ will be much smaller than the standard model result, and thus unmeasurable.

We thank David Atwood for several useful discussions. This work was supported by the National Science Foundation grant No. NSF-PHY-9306141.

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Figure 1: Corrections to the $\pi dZ$ vertex in the standard model. $G$ refers to the charged Goldstone boson.
Figure 2: Box diagrams contributing to $K_L \rightarrow \pi^0 \nu \bar{\nu}$.
