WEAKLY NONLINEAR CLUSTERING FOR ARBITRARY EXPANSION HISTORIES

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ABSTRACT

Bouchet et al. showed that in an open or closed universe with only pressureless matter, gravitational instability from Gaussian initial conditions induces a normalized skewness, \( S_3 \equiv \langle \delta^3 \rangle / \langle \delta^2 \rangle^{3/2} \), which has only a very weak dependence on the nonrelativistic matter density. Here we generalize this result to a plethora of models with various contributions to the total energy density, including nonrelativistic matter, a cosmological constant, and other forms of missing energy. Our numerical results show that the skewness (and bispectrum) depend only very weakly (\( \lesssim 2\% \)) on the expansion history. Thus the skewness and bispectrum provide a robust test of gravitational instability from Gaussian initial conditions, independent of the underlying cosmological model.

Subject headings: cosmology: theory — galaxies: clusters: general — galaxies: statistics — large-scale structure of universe

1. INTRODUCTION

One of the predictions of the simplest and most popular inflationary models that distinguishes them from numerous alternatives such as topological-defect or isocurvature models is that large-scale structure grew from a Gaussian distribution of primordial density perturbations. However, even if the primordial distribution was Gaussian, subsequent gravitational evolution would introduce deviations from Gaussianity. A Gaussian distribution is symmetric with respect to overdense and underdense regions, but as gravitational amplification of density perturbations proceeds, overdensities can become arbitrarily large. On the other hand, any given region can become only so underdense (since densities must be positive). Thus the resulting distribution is necessarily skewed.

Peebles (1980) perturbatively calculated the normalized skewness induced by gravitational instability of initially Gaussian perturbations in an Einstein–de Sitter universe. More precisely, if \( \delta(x) \equiv \langle \rho(x) - \rho \rangle / \rho \) is the fractional density perturbation and \( \rho(x) \) is the density at position \( x \), then in an Einstein–de Sitter universe the statistic is \( S_3 \equiv \langle \delta^3 \rangle / \langle \delta^2 \rangle^{3/2} = 34/7 \). Bouchet et al. (1992) generalized this result to an open or closed universe containing only pressureless matter and found that the prediction for \( S_3 \) remains the same to less than 2\% for any reasonable value of \( \Omega_0 > 0.1 \). Thus they showed that measurements of \( S_3 \) should provide a robust test of Gaussian initial conditions, independent of \( \Omega_0 \), but only if the universe just consists of pressureless matter.

However, slow-roll inflation predicts that the universe is flat, while observations seem to point to a matter density \( \Omega_0 \approx (0.2–0.5) < 1 \). Therefore, if inflation is correct, then there must be some other missing energy density. The most widely considered form for this density is a cosmological constant (with pressure given by \( p = -\rho \)), but theorists have also realized that this missing energy might be characterized by any of a number of equations of state. Among these is some energy density that scales as \( a^{-2} \) (K matter), where \( a(t) \) is the scale factor of the universe (Kolb 1989; Kamionkowski & Toumbas 1996; Pen & Spergel 1997), something akin to an evolving cosmological constant, possibly generated by the dynamics of a scalar field (Ratra & Peebles 1988; Frieman et al. 1995; Coble, Dodelson, & Frieman 1997; Silveira & Waga 1997; Turner & White 1997; Caldwell, Dave, & Steinhardt 1998) or some combination of several forms of missing energy.

To test for primordial Gaussianity of density perturbations in this cornucopia of missing-energy models, we numerically calculate the induced skewness in flat models with a cosmological constant and/or other form(s) of energy. For completeness we also consider open and closed universes with a variety of equations of state. For all cosmological models investigated, the numerical results for \( S_3 \) never deviate from 34/7 by more than 2\% for models with \( \Omega_0 \gtrsim 0.1 \), and no more than about 1\% for \( \Omega_0 \gtrsim 0.3 \). Thus we conclude that the normalized skewness does indeed provide a robust test for primordial Gaussianity independent of the nature of the species that constitute the total energy density.

The bispectrum (Fourier transform of the induced three-point correlation function) has also been calculated, as a by-product of the calculation of the skewness, for Gaussian initial conditions (Peebles 1980; Fry 1984; Goroff et al. 1986) in an Einstein–de Sitter universe and in a universe with pressureless matter (Bouchet et al. 1992). Here we provide results for the bispectrum for the models we investigate.

The details of our calculation are presented in the next section. We write down the differential equation for an arbitrary expansion history, which must be solved to obtain the growth of the skewness. In § 3 we provide numerical results for the skewness and bispectrum for several forms of missing energy and for some models with various combinations of missing energy. We also provide an analytic approximation to these quantities as a function of \( \Omega_0 \) for flat cosmological-constant models. We then make some concluding remarks.

2. CALCULATION

The calculation of the bispectrum and skewness is outlined in detail in Peebles (1980) and Fry (1984), and we simply highlight the relevant steps below; Bouchet et al.
(1992) employ an alternative approach based on Lagrangian perturbation theory. The Friedmann equation for the expansion rate as a function of redshift $z$ is given by

$$H(z) = \frac{\dot{a}}{a} = H_0 E(z) ; \quad E(z) = \sqrt{\sum \Omega_i (1 + z)^{m_i + w_i}}, \quad (1)$$

where $H_0$ is the present-day value of the Hubble parameter, the dot denotes a derivative with respect to time $t$, and $\sum \Omega_i = 1$, where $\Omega_i$ represents the contribution to the overall energy density from species $i$ having equation of state $p = w_i \rho$. In this formulation, the curvature of the universe contributes an amount $\Omega_k$ to the total energy density and yields the term $\Omega_k (1 + z)^2$ in the sum of equation (1). Thus in a universe with, for example, nonrelativistic-matter density $\Omega_\Lambda$ and a cosmological-constant contribution to closure density of $\Omega_\Lambda$, $E(z) = [\Omega_\Lambda (1 + z)^3 + \Omega_\Lambda + (1 - \Omega - \Omega_\Lambda) (1 + z)^2]^{1/2}$. The deceleration can be written as

$$\ddot{a}/a = H_0^2 F(z), \quad (2)$$

and $F(z)$ can be obtained by differentiating equation (1). For example, for the $E(z)$ given above, $F(z) = \Omega_\Lambda - \Omega_\Lambda(1 + z)^3/2$.

If the fractional density perturbation is small ($\delta \ll 1$), the equations of motion for $\delta(x, t)$, the perturbation to the nonrelativistic-matter component, can be solved perturbatively using the expansion $\delta(x, t) = \delta^{(1)}(x, t) + \delta^{(2)}(x, t) + \cdots$, with $\delta^{(n+1)} \ll \delta^{(1)}$. We assume here that any other exotic dark matter components are to be unperturbed; we discuss below the validity of this approximation. The perturbative equations for the lowest order term, $\delta^{(1)}$, turn out to be separable, $\delta^{(1)}(x, t) = \delta_1(x) D_1(t)$, and the familiar linear-theory growth factor satisfies

$$\ddot{D}_1 + 2 \frac{\dot{a}}{a} \dot{D}_1 - \frac{3}{2} \Omega_\Lambda H_0^2 \left( \frac{a_0}{a} \right)^3 D_1 = 0 . \quad (3)$$

The next order equation of motion is

$$\ddot{\delta}^{(2)} + 2 \frac{\dot{a}}{a} \dot{\delta}^{(2)} - \frac{3}{2} \Omega_\Lambda H_0^2 \left( \frac{a_0}{a} \right)^3 \delta^{(2)} = \left[ \frac{3}{2} \Omega_\Lambda H_0^2 \left( \frac{a_0}{a} \right)^3 + \left( \frac{\dot{D}_1}{D_1} \right)^2 \right] D_1^2 \delta^{(1)}_1$$

$$+ \left[ 3 \Omega_\Lambda H_0^2 \left( \frac{a_0}{a} \right)^3 + 2 \left( \frac{\dot{D}_1}{D_1} \right)^2 \right] D_1^2 \delta^{(1)}_{1,i} \delta^{(1)}_{1,i},$$

$$\quad + \left( \frac{\dot{D}_1}{D_1} \right)^2 D_1^2 (\delta^{(1)}_{1,ij} \delta^{(1)}_{1,ij}), \quad (4)$$

where

$$\Delta_1(x) = -\frac{1}{4\pi} \int d^3x' \frac{\delta_1(x')}{|x - x'|}. \quad (5)$$

The homogeneous part of equation (4) is the same as that for equation (3), and it turns out that the solution for $\delta^{(2)}$ is dominated by the inhomogeneous terms on the right-hand side. Since the inhomogeneous part is the sum of three terms with two different time dependences, the solution for $\delta^{(2)}$ is not obviously separable. However, the complete solution is simply the sum of the solutions to the equations given by the homogeneous equation driven by each of these sources separately. We may thus write the solution as

$$\delta^{(2)} = (D_{2,a} + D_{2,b}) \delta^{(1)}_2 + (D_{2,a} + 2D_{2,b}) \delta^{(1)}_{1,ij} (\Delta_1, ij)$$

$$+ D_{2,b} \delta^{(1)}_{1,i} \delta^{(1)}_{1,ij}, \quad (6)$$

where the second-order growth factors satisfy

$$\dot{D}_{2,a} + 2 \frac{\dot{a}}{a} \dot{D}_{2,a} - \frac{3}{2} \Omega_\Lambda H_0^2 \left( \frac{a_0}{a} \right)^3 D_{2,a} = \frac{3}{2} \Omega_\Lambda H_0^2 \left( \frac{a_0}{a} \right)^3 D_1^2 ,$$

$$\dot{D}_{2,b} + 2 \frac{\dot{a}}{a} \dot{D}_{2,b} - \frac{3}{2} \Omega_\Lambda H_0^2 \left( \frac{a_0}{a} \right)^3 D_{2,b} = D_1^2 , \quad (7)$$

with the initial conditions that the growth factors and their first-time derivatives are zero at $t = 0$. Although it appears that there are two differential equations (for $D_{2,a}$ and $D_{2,b}$) to be solved for the second-order solution, it can be verified that $D_{2,b} = (D_1^2 - D_{2,a})/2$. Thus there is really only one independent second-order differential equation. The first of these equations (eq. [7]) can be solved analytically for the Einstein–de Sitter model with the result (for the growing mode) that $D_1 \propto T^{1/3}$ and $D_{2,a} = (3/7)T^2$. For more general models, equations (3) and (7) can be integrated numerically.

It is then straightforward to follow the arguments given in § 18 of Peebles (1980) to find the normalized skewness. In terms of the quantity $\mu = D_{2,a}/D_1^2$ (this is twice the parameter $\kappa$ in Bouchet et al. 1992), it is given by

$$S_3 = 4 + 2\mu = \frac{34}{7} + \frac{6}{7} \left( \frac{7}{3} \mu - 1 \right). \quad (9)$$

We have thus enumerated a simple prescription for calculating $S_3$ in a model with Gaussian initial conditions and an arbitrary expansion history.

In terms of the parameter $\mu$, the second-order correction to the density contrast is (Bouchet et al. 1992)

$$\delta^{(2)} = \frac{D_1^2}{2} \left[ (1 + \mu) \delta^{(1)}_2 + 2 \delta^{(1)}_{1,ij} \delta^{(1)}_{1,ij} + (1 - \mu) \delta^{(1)}_{1,ij} \delta^{(1)}_{1,ij} \right]. \quad (10)$$

If we compare equation (11) of Fry (1984) with equation (A2) of Goroff et al. (1986), it is clear that the symmetrized kernel of the second-order density solution for arbitrary expansion history can be written (cf. eq. [A1] of Goroff et al. 1986; eq. [57] of Catelan et al. 1995)

$$P_2^{E}(k_1, k_2) = \frac{1}{2} \left[ (1 + \mu) + \frac{k_1 \cdot k_2}{k_1 k_2} \left( \frac{k_1 + k_2}{k_1} \right) \right]$$

$$+ \left( 1 - \mu \right) \left( \frac{k_1 \cdot k_2}{k_1 k_2} \right)^2. \quad (11)$$

The bispectrum can be obtained from this kernel.

3. RESULTS

The solid line in Figure 1 shows the normalized skewness $S_3$ as a function of the nonrelativistic-matter density $\Omega_\Lambda$ in an open, flat, or closed model containing only pressureless matter; i.e., in a model with $E^2(z) = \Omega_\Lambda (1 + z)^3 + (1 - \Omega_\Lambda)(1 + z)^2$. [To clarify, the results do not really depend on the geometry of the universe, but rather on the expansion history $E(z).$] This reproduces the result of Bouchet et al. (1992), who found that the quantity $\mu$ is well approximated
for the cosmological-constant universe.

A flat model with nonrelativistic matter and some K-matter density that scales as $(1+z)^{-2}$, such as nonintersecting cosmic strings (Kolb 1989; Kamionkowski & Toumbas 1996; Pen & Spergel 1997), has the same $E(z)$ as the standard open model with the same nonrelativistic-matter density (since $w_k = -\frac{1}{3}$). Thus the skewness will be given by the solid curve in Figure 1.

Of course, the cases shown in the figure do not exhaust the full range of $E(z)$—and Martel (1995) has considered yet others—and we can think of no way to systematically study all possible cases. However, we have tried numerous strange combinations of the types of matter considered above, and in no case do we find any deviation from $34/7$ significantly larger than that in the open model. Several examples are shown in Table 1. In addition to flat $\Omega_0 < 1$ models with various combinations of a cosmological constant, $K$ matter, and $\Omega_1$ matter (including some with a negative cosmological constant), we also consider a closed cosmological-constant model that is just consistent with quasar-lensing statistics (White & Scott 1996) and an $\Omega_0 = 10$ closed model (Harrison 1993). We include these models to demonstrate the robustness of $S_3$ to dramatic variations in the expansion history. We do not suggest that all of these models are observationally tenable. These numerical experiments lead us to believe with good confidence that $S_3$ must differ from $34/7$ by no more than $2\%$ in any cosmological model with $\Omega_0 > 0.1$, and to no more than $\pm1\%$ for any model with $\Omega_0 \geq 0.3$ if structure grew via gravitational instability from a Gaussian distribution of primordial density perturbations.

4. DISCUSSION

We have generalized the result of Bouchet et al. (1992)—that the normalized skewness and bispectrum are only very weakly sensitive to $\Omega_0$ in models with only pressureless matter—to general Friedmann cosmologies with arbitrary contributions to the energy density, including the simplest cosmological-constant model. Prior work has shown that the normalized skewness depends only on a logarithmic derivative of the linear-theory growth factor $D_k$ (Scoccimarro et al. 1998; Catelan et al. 1995), and this suggested that the corrections to the Einstein–de Sitter value should be small. Our numerical results confirm that these corrections to the scaled skewness are indeed always small; they are never more than $2\%$ for $\Omega_0 \geq 0.1$ and $1\%$ for $\Omega \geq 0.3$. The robustness of these measures allows for excellent model-independent tests of structure formation from gravitational instability of a Gaussian distribution of primordial perturbations.

We have only considered the effect of the expansion on the skewness and bispectrum. One might wonder whether perturbations in the scalar field in models with a dynamical scalar field might affect the growth of structure. If so, then it is conceivable that the skewness would deviate from the gravitational-instability prediction. However, structure formation in this case would not be due simply to gravitational

\[
\mu \approx \frac{1}{3} \Omega_0^{2/63}.
\]

(12)

The short-dashed curve in Figure 1 shows our results for the normalized skewness $S_3$ as a function of the nonrelativistic-matter density $\Omega_0$ in a flat cosmological-constant model;\(^1\) that is, $E^2(z) = \Omega_0(1+z)^3 + (1 - \Omega_0)$. Note that $\Lambda < 0$ for models with $\Omega_0 > 1$. We see that the difference between $S_3$ and $34/7$ is even smaller in a cosmological-constant model than in an open model with the same $\Omega_0$. We find that $\mu$ is well approximated in these models by

\[
\mu \approx \frac{1}{3} \Omega_0^{-1/140}.
\]

(13)

To generalize even further, the long-dashed curve in Figure 1 shows the normalized skewness $S_3$ in a flat model with a nonrelativistic-matter density $\Omega_0$ plus some hypothetical matter density with $p = -2\rho/3$, scaling as $(1+z)^{-1}$; that is, $E^2(z) = \Omega_0(1+z)^3 + (1 - \Omega_0)(1+z)$. We will refer to this as "$\Omega_1$ matter." Again, the correction to $S_3$ is very small for any reasonable value of $\Omega_0$. The normalized skewness is well approximated in these models by

\[
\mu \approx \frac{1}{7} \Omega_0^{1/77}.
\]

(14)

Fig. 1.—Normalized skewness $S_3$ as a function of the nonrelativistic-matter density $\Omega_0$ in a universe with only nonrelativistic matter (solid line), a flat cosmological-constant model (short-dashed line), and a flat model with an energy density that scales as $(1+z)^{-1}$ (long-dashed line).

\begin{table}[h]
\centering
\caption{Predicted Skewnesses and $\mu$}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
$\Omega_0$ & $\Omega_1$ & $\Omega_2$ & $S_3$ & Deviation\(^a\) & $\mu$ \\
\hline
1 & 0 & 0 & 0 & 4.857 & 0 & 0.429 \\
0.3 & 0 & 7 & 0 & 0 & 4.865 & 0.15 & 0.432 \\
0.3 & 0 & 0 & 0 & 0 & 4.892 & 0.73 & 0.476 \\
0.3 & 0 & 0 & 0 & 0 & 4.844 & 0.26 & 0.422 \\
0.3 & 0.35 & 0.35 & 0 & 4.880 & 0.48 & 0.480 \\
0.3 & 0.35 & 0.35 & 0 & 4.854 & 0.07 & 0.427 \\
0.3 & 0 & 0.35 & 0.35 & 4.872 & 0.31 & 0.436 \\
0.3 & 0.35 & 0.35 & 0 & 4.872 & 0.31 & 0.436 \\
0.3 & 0.35 & 0.35 & 0 & 4.872 & 0.31 & 0.436 \\
0.3 & 0.35 & 0.35 & 0 & 4.872 & 0.31 & 0.436 \\
0.3 & 0.35 & 0.35 & 0 & 4.872 & 0.31 & 0.436 \\
10 & 0 & -9 & 0 & 4.809 & -0.99 & 0.405 \\
2 & 2 & -3 & 0 & 4.796 & -1.25 & 0.398 \\
\hline
\end{tabular}
\end{table}

\(^a\) Percentage deviation of $S_3$ from $34/7$.

\(^1\) Bouchet et al. (1995) have also integrated these equations numerically for the cosmological-constant universe.
instability, as we have assumed here. Moreover, calculations of the galaxy power spectrum in such scalar field models suggest that the scalar field is essentially smooth on these scales, so that scalar field perturbations do not impact structure on these scales (Coble et al. 1997; Caldwell, Dave, & Steinhardt 1998). Analytic approximations suggest that on small scales (≤ 100 h⁻¹ Mpc), scalar-field ordering should not significantly affect the skewness (Jaffe 1994). Thus we expect that in (nontopological) scalar field models, the prediction S₃ ≈ 3/7 hold for Gaussian initial conditions. In models with both hot and cold dark matter (see, e.g., Primack et al. 1995), the dark matter content affects the evolution of perturbations at early times during the linear regime. However, the nonlinear growth of perturbations after decoupling is simply due to gravitational infall. Therefore, S₃ should provide a robust probe of gravitational infall from Gaussian initial conditions independent of the nature of the dark matter.

In practice, other factors must be considered when comparing measurements of the observed skewness and bispectrum with the predictions above. For example, the skewness calculated above is that for an unsmoothed density field, while the observed distribution is intrinsically discrete. Juszkiewicz, Bouchet, & Colombi (1993) and Bernardeau (1994a, 1994b) considered the effects of smoothing, and it is straightforward to apply the discussion therein to generalize our results for S₃. In particular, for the case of smoothing with a spherical top-hat filter with radius R,

$$S_3(R) = S_3 - \frac{d \ln \sigma^2(R)}{d \ln R},$$

where S₃(R) is the result for the smoothed distribution, and σ² is the variance of the smoothed density field. In addition, the skewness and bispectrum presented here are those for the underlying mass distribution. Galaxy surveys probe the luminous-matter distribution, which, if biased relative to the matter, will be characterized by a different skewness and bispectrum. If the galaxy fractional density perturbation is written as

$$\delta_g(x, t) = b_1 \delta(x, t) + \frac{b_2}{2} \delta^2(x, t) + \cdots,$$

where b₁ is the linear bias term, b₂ is the first nonlinear term, and so on, then the skewness of the observed galaxy distribution is given by (Fry & Gaztañaga 1993; Juszkiewicz et al. 1995)

$$S_{3,g}(R) = \frac{1}{b_1} S_3(R) + \frac{3b_2}{b_1^2},$$

and similar corrections have been obtained for the bispectrum (Fry 1994; Matarrese, Verde, & Heavens 1997; Scoccimarro et al. 1998). More realistic calculations involving, for example, true scale-dependent power spectra or bias evolution have been obtained using numerical techniques (Juszkiewicz et al. 1993; Fry 1996; Jing & Börner 1997; Gaztañaga & Bernardeau 1998; Buchalter & Kamionkowski 1999), and nonlocal biasing schemes have also been considered (Catelan, Matarrese, & Porciani 1998; Catelan et al. 1998). On smaller scales, effects that arise in higher order in perturbation theory must be taken into account (Scoccimarro & Frieman 1996a, 1996b; Scoccimarro 1997).

There are also a number of effects introduced in realistic measurements that might affect comparison of the measured and predicted skewness. For example, redshift-space distortions in galaxy surveys must be considered (Hivon et al. 1995; Scoccimarro, Couchman, & Frieman 1998; Verde et al. 1998; Heavens, Matarrese, & Verde 1999), and projection effects must be treated properly in angular catalogs (Gaztañaga & Bernardeau 1998; Buchalter, Kamionkowski, & Jaffe 1999). Although there are a number of effects that complicate the comparison of theory with experiment, they are becoming increasingly well understood. Thus the model-independence of the predicted skewness and bispectrum for the matter distribution makes determinations of the bias from measurements of these quantities (Matarrese et al. 1997; Buchalter & Kamionkowski 1999) that much more robust.

The robustness of the scaled skewness to the expansion history supports the notion that the geometry of gravitationally collapsing objects is determined almost exclusively by the initial conditions. This leads us to speculate that the predictions for the higher order moments and correlation functions (e.g., kurtosis, four-point correlation function, trispectrum, and so forth) will be similarly independent of the expansion history. It would be interesting and straightforward to check this hypothesis numerically.

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