Neutrino phenomenology, muon and electron (g-2) under $U(1)$ gauged symmetries in an extended inverse seesaw model

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Abstract: The proposed work is an extension of the Standard Model, where we have introduced two anomaly free gauge symmetries i.e. $U(1)_{B-L}$ and $U(1)_{L_e-L_\mu}$ in an inverse seesaw framework. For this purpose, we have included three right handed neutrinos $N_{iR}$, three neutral fermions $S_{iL}(i = 1, 2, 3)$ and two scalar singlet bosons ($\chi_1, \chi_2$). We get a definite structure for neutrino mass matrix due to the aforementioned gauge symmetries. Thus, our model is able to predict the neutrino oscillation results which are in accordance with the experimental data and mostly supports normal ordering. The outcome comprises of active neutrino mass, mixing angles, mass square differences, CP violating phase etc. We also discuss neutrinoless double beta decay effective mass parameter $\langle m_{ee}\rangle$ which gives a strong evidence on the Majorana nature of neutrinos. Its predicted value is found to be well below the current experimental bounds of KamLAND-Zen, CUORE etc. Furthermore, as the extended gauge symmetries are local, hence, are associated with the corresponding gauge bosons i.e. $(Z_1, Z_2)$, which make our model feasible to explain current results of electron and muon $(g - 2)$ through neutral current interactions.

Keywords: Neutrino Physics, Seesaw mechanism, Anomalous magnetic moment of charged leptons, U(1) gauge symmetry
1 Introduction

Standard model (SM) is inordinately successful in explaining most of the properties of the hadrons, charged leptons, and gauge bosons with high precision. In spite of exceptional triumph, it lacks in explaining the tininess \([1]\) of neutrino mass, nature of the neutrinos i.e. (Dirac or Majorana), matter-antimatter asymmetry, muon anomalous magnetic moment \((g-2)_\mu\), dark matter content of the universe etc., contradicting the experimental evidences. To work within the domain of SM and generate small neutrino mass, Weinberg operator [2] becomes an unavoidable requisite. However, to sketch a model feasible for explaining other puzzling phenomena along with neutrino phenomenology seems difficult via dimension-5 operator. Therefore, to leap beyond the standard model (BSM) becomes more of an obligation and to do so we bring in right handed (RH) neutrinos into picture which lays the foundation for seesaw mechanism. Type-I or canonical [3-5] being the simplest seesaw mechanism which utilizes only three additional SM singlet RH neutrinos to showcase the
smallness of neutrino mass. The neutrino mass formula in this framework takes the form as shown below

\[ m_\nu = -\mathcal{M}_D\mathcal{M}_R^{-1}\mathcal{M}_D^T, \]  

(1.1)

where, \( \mathcal{M}_D, \mathcal{M}_R \) being the Dirac and heavy RH neutrino mass matrices respectively. In order to bring the scale of active neutrinos in the range of 0.1 eV constrained by cosmological bound [6], one needs heavy RH neutrinos to be as massive as \( \mathcal{O}(10^{14}) \) GeV. To prove the existence of such massive neutrino is impractical for current experiments, hence, its usage is very regular as seen in myriad literatures. On the other hand, additional variants of seesaw are type-II [7–10] seesaw with scalar triplets, type-III [11–14] seesaw with fermion triplets, linear seesaw [15–19], inverse seesaw [20–31] etc. Here, we explore the case of inverse seesaw using the extended symmetries, where we involve three right handed neutrinos \( N_{iR} \) and three neutral fermions \( S_{iL} \) (\( i = 1, 2, 3 \)). The inclusion of these six heavy neutrinos allow us to retain the inverse seesaw mass matrix eqn.(2.4), provided suitable charge assignements are made under the extended symmetries i.e. \( U(1)_{B-L} \) and \( U(1)_{L_e-L_\mu} \). The alluring feature of these symmetries is that they are anomaly free, hence, help us to achieve the motive of explaining neutrino phenomenology i.e. mixing angles (\( \sin^2 \theta_{13}, \sin^2 \theta_{12}, \sin^2 \theta_{23} \)), mass squared differences (\( \Delta m^2_{21}, \Delta m^2_{31} \)), CP phase (\( \delta_{CP} \)), Jarlskog invariant (\( J_{CP} \)), neutrinoless double beta decay (NDBD) mass parameter \( \langle m_{ee} \rangle \) etc. and discuss the current results of electron and muon (\( g - 2 \)).

As discussed above, the gauge symmetries being local, i.e. cancellation of gauge anomalies become a precondition. It is achieved by providing suitable charges to the \( N_{iR} \) and zero charge to \( S_{iL} \) under the above stated extended symmetries. Also, we have introduced two singlet scalar bosons (\( \chi_1, \chi_2 \)) contributing towards symmetry breaking. This leads to production of associated gauge bosons (\( Z_1, Z_2 \)) in MeV range consistent with most of the current experiments [32–37]. Whereas, experiments [38–41] designed to study magnetic moment of any particle helps us to know more about their characteristics meticulously. In this regard, deviation of (\( g - 2 \))\(_{\mu,e} \) values from their corresponding SM predictions, highlights a flaw of SM and also opens doors to new ideas, discussed, hitherto. The neutral gauge bosons coming from our model, contribute through the neutral current interaction Lagrangian towards these magnetic moment anomalies.

The outline of this paper comprises of the model framework which includes particles content, Lagrangian, mass matrices depicted in Sec. 2. Further, in Sec. 3, we present the numerical calculations providing the values of allowed parameter space helpful in describing the neutrino oscillation data. A detailed analysis of scalar sector has been provided in Sec. 4 and a comment on non-unitarity is imparted in Sec. 5. Additionally, in Sec. 6, a brief discussion on MeV range gauge bosons (\( Z_1, Z_2 \)) are incorporated with relevant plots. Sec. 7 depicts electron (\( g - 2 \)) and muon (\( g - 2 \)) explanations. Finally, our results are summarized in Sec. 8.
2 Model description

| Particle | \(SU(3)_C\) | \(SU(2)_L\) | \(U(1)_Y\) | \(U(1)_{B-L}\) | \(U(1)_{L_{\mu}-L_{\mu}}\) |
|----------|--------------|--------------|------------|--------------|----------------|
| \(\ell_L\) | 1            | 2            | -1         | -1           | 1, -1, 0       |
| \(\ell_R\) | 1            | 1            | -2         | -1           | 1, -1, 0       |
| \(N_{iR}\) | 1            | 1            | 0          | -1           | 1, -1, 0       |
| \(S_{iL}\) | 1            | 1            | 0          | 0            | 0              |
| \(H\)   | 1            | 2            | 1          | 0            | 0              |
| \(\chi_1\) | 1            | 1            | 0          | -1           | 1              |
| \(\chi_2\) | 1            | 1            | 0          | 1            | 0              |

Table 1: Particle and their corresponding charge assignment under \(SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{L_{\mu}-L_{\mu}} \times U(1)_{B-L}\) model.

In extension of SM, we have added three right-handed (RH) neutrinos \(N_{iR}\) and three neutral fermions \(S_{iL}, (i = 1, 2, 3)\) to achieve inverse seesaw mechanism for giving mass to active neutrinos in the sub-eV scale. The charges assigned to the RH and neutral fermions under \(U(1)_{B-L}\) is -1 and 0, while, \{1, -1, 0\} and 0 under \(U(1)_{L_{\mu}-L_{\mu}}\) respectively. In addition to above, we have included two extra singlet scalars \(\chi_1\) and \(\chi_2\) with assigned charges under \(U(1)_{B-L}\) as -1, 1 and under \(U(1)_{L_{\mu}-L_{\mu}}\) 1, 0 respectively. The Lagrangian for the leptonic sector of the proposed model is:

\[
\mathcal{L}_{\text{lepton}} \supset \mathcal{L}_{SM} + \left[ y_D^\alpha \bar{\ell}_e H N_{1R} + y_D^\mu \bar{\ell}_\mu H N_{2R} + y_D^\tau \bar{\ell}_\tau H N_{3R} \right] + \left[ y_{NS}^{11} \bar{N}_{1R} S_{1L} \chi_1 + y_{NS}^{12} \bar{N}_{1R} S_{2L} \chi_1 + y_{NS}^{13} \bar{N}_{1R} S_{3L} \chi_1 + y_{NS}^{33} \bar{N}_{3R} S_{3L} \chi_2 \right] + \left[ \mu_1 \bar{S}_{1L} S_{1L} + \mu_2 \bar{S}_{2L} S_{2L} + \mu_3 \bar{S}_{3L} S_{3L} \right] + \text{h.c.},
\]

(2.1)

where, \(\mathcal{L}_{SM}\) is the standard Lagrangian for leptonic sector, \(y_D^\alpha (\alpha = e, \mu, \tau)\) are the Yukawa couplings for Dirac interactions, \(y_{NS}^{i1}\) \((i = 1, 2, 3)\), \(y_{NS}^{33}\) are the couplings for heavy fermions and \(\mu_i, (i = 1, 2, 3)\) are the Majorana mass term of extra neutral fermion \(S_{iL}\). Thus, one can obtain the Dirac mass term \(\mathcal{M}_D\), the heavy fermion mass term \(\mathcal{M}_{NS}\) and the Majorana mass matrix \(\mathcal{M}_\mu\) as given below in Eqs. (2.2) and (2.3)

\[
\mathcal{M}_D = \frac{v_H}{\sqrt{2}} \begin{pmatrix}
    y_D^e & 0 & 0 \\
    0 & y_D^\mu & 0 \\
    0 & 0 & y_D^\tau
\end{pmatrix}, \quad (2.2)
\]

\[
\mathcal{M}_{NS} = \frac{1}{\sqrt{2}} \begin{pmatrix}
    y_{NS}^{11} v_1 & |y_{NS}^{12}| e^{i\phi} v_1 & y_{NS}^{13} v_1 \\
    |y_{NS}^{12}| e^{i\phi} v_1 & 0 & 0 \\
    y_{NS}^{13} v_1 & 0 & y_{NS}^{33} v_2
\end{pmatrix}, \quad \mathcal{M}_\mu = \begin{pmatrix}
    \mu_1 & 0 & 0 \\
    0 & \mu_2 & 0 \\
    0 & 0 & \mu_3
\end{pmatrix}. \quad (2.3)
\]
Table 2: Best-fit values of Yukawa couplings and vacuum expectation values.

| Parameters | Best fit values | Parameters | Best fit values |
|------------|----------------|------------|----------------|
| $y_D^{12}$ | $3.837 \times 10^{-6}$ | $y_{NS}^{12}$ | $9.588 \times 10^{-5}$ |
| $y_D^{13}$ | $9.984 \times 10^{-6}$ | $y_{NS}^{33}$ | $8.950 \times 10^{-5}$ |
| $y_D^{33}$ | $6.427 \times 10^{-6}$ | $v_1$ | $3.247 \times 10^{13}$ eV |
| $y_{NS}^{11}$ | $9.969 \times 10^{-5}$ | $v_2$ | $9.789 \times 10^{12}$ eV |
| $y_{NS}^{12}$ | $6.039 \times 10^{-5}$ | $\mu_i (i = 1, 2, 3)$ | $10$ eV |

In the expression of $M_{NS}$, we have taken $y_{NS}^{12}$ coupling as complex associated with phase $\phi$ and other Yukawa couplings are real by redefinition. The neutrino mass matrix thus can have the form in the basis $(\nu_L, N_R, S_L)$ as

$$M = \begin{pmatrix} \nu_L & N_R & S_L \\ \nu_L & 0 & M_D \\ N_R & M_D^T & 0 & M_{NS} \\ S_L & 0 & M_{NS}^T & M_\mu \end{pmatrix}. \quad (2.4)$$

Inverse seesaw is executed successfully by considering the assumption $M_\mu \ll M_D < M_{NS}$. As discussed in numerical calculation section (sec. 3), it is clear that $M_\mu$ is in the keV range, while, $M_D$ is in MeV range and $M_{NS}$ is considered in GeV range. The active neutrino mass matrix $m_\nu$ can be found from the expression

$$m_\nu = M_D^T (M_{NS}^{-1})^T M_\mu M_{NS}^{-1} M_D. \quad (2.5)$$

3 Numerical Calculations

The expressions for oscillation parameters related to the standard PMNS matrix $U_{PMNS}$ are as,

$$\sin^2 \theta_{13} = |U_{e3}|^2, \quad \sin^2 \theta_{23} = \frac{|U_{\mu3}|^2}{1 - |U_{e3}|^2}, \quad \sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}, \quad (3.1)$$

where, $U_{oi}$ ($\alpha = e, \mu, \tau, \ i = 1, 2, 3$) are the elements of $U_{PMNS}$ matrix. The expression for Jarlskog CP invariant given in terms of the PMNS matrix elements as

$$J_{CP} = \text{Im}[U_{e1} U_{\mu2} U_{e2}^* U_{\mu1}^*]. \quad (3.2)$$

In order to get neutrino phenomenology correlation plots, we make use of atmospheric and solar mass squared differences i.e. ($\Delta m^2_{31}$, $\Delta m^2_{21}$) in their $3\sigma$ range and other observables of neutrino oscillation ($\theta_{12}$, $\theta_{23}$, $\theta_{13}$) from Global-fit data [42] as represented below,

$$\sin^2 \theta_{12} = (0.02034 - 0.02430), \quad \sin^2 \theta_{23} = (0.407 - 0.618), \quad \sin^2 \theta_{13} = (0.269 - 0.343)$$

$$\Delta m^2_{31} = (2.43 - 2.598) \times 10^{-3} \text{ eV}^2, \quad \Delta m^2_{21} = (6.82 - 8.04) \times 10^{-5} \text{ eV}^2. \quad (3.3)$$
The parameter space utilized to get the results of neutrino sector with high precision is given below:

\[ y^\alpha \in [10^{-7}, 10^{-5}] , \quad y^1_{NS}(i = 1, 2, 3) , \quad y^{33}_{NS} \in [10^{-7}, 10^{-4}] , \]

\[ v_1 \in [10^{12}, 5 \times 10^{14}] \text{ eV}, \quad v_2 \in [10^{12}, 5 \times 10^{13}] \text{ eV}, \]

\[ \mu_i(i = 1, 2, 3) \in [10^{-2}, 9 \times 10^2] , \quad \phi \in [0, 2\pi]. \] (3.4)

It is evident that there are quite a few free parameters involved in obtaining the desired correlation plots between various oscillation parameters. Hence, when all of them are varied randomly they yield very weakly correlated scattered plots. In order to improve the correlation, we utilize \(\chi^2\) analysis, with required formula:

\[ \chi^2 = \sum_i \left( \frac{(P^\text{theo}_i - P^\text{exp}_i)^2}{(\Delta P^\text{exp}_i)^2} \right) \] (3.5)

where, \(P^\text{theo}\) is the value of any oscillation parameter coming from the theory, and \(P^\text{exp}\) is the experimentally measured value of that parameter with its 1\(\sigma\) uncertainty is denoted as \(\Delta P^\text{exp}_i\). There are five well-measured oscillation parameters \((\theta_{12}, \theta_{13}, \theta_{23}, \Delta m^2_{21}, \Delta m^2_{31})\) whose experimentally measured values are presented in [43]. We calculate the minimum \(\chi^2\) and the associated best-fit values of the free parameters as presented in Table 2, are used in generating the Figs.(1−4), while randomly varying the VEV’s \((v_1, v_2)\). In Fig.1, left (right) panel shows the correlation of sum of active neutrino masses \((\sum m_i)\) with \(\sin^2 \theta_{13}\) \((\sin^2 \theta_{12})\). Moving further, left panel of Fig.2 represents the correlation of \(\sum m_i\) with \(\sin^2 \theta_{23}\), while, right panel shows the correlation of the mixing angles i.e. \(\sin^2 \theta_{13}\) w.r.t \(\sin^2 \theta_{12}\). Left panel of Fig.3 shows the variation \(\sin^2 \theta_{23}\) with \(\sin^2 \theta_{13}\) and its right panel shows the dependency of \(\sum m_i\) with Jarlskog invariant \(J_{CP}\). In the left panel of Fig. 4, we project the correlation of Majorana phases \(\alpha_{21}\) and \(\alpha_{31}\), whereas right panel shows the variation of \(J_{CP}\) with \(\sin^2 \theta_{13}\).

**Figure 1**: Left (right) panel shows the variation of active neutrino mass value \((\sum m_i)\) with mixing angle \(\sin^2 \theta_{13}\) \((\sin^2 \theta_{12})\).
Comment on $0\nu\beta\beta$ decay

One of the long-standing open question in particle physics is: what is the nature of neutrino. To find its answer, many experiments are done and going to be done. Search for neutrinoless double beta decay (NDBD) is one of such experiments where the general expression is given...
\[(A, Z) \rightarrow (A, Z + 2) + 2e^+, \quad (3.6)\]

without any outgoing neutrinos. If the reaction is seen in experiment then it will be verified that neutrinos have Majorana nature as eqn. 3.6 is possible only if neutrino has its own antiparticle. Many experiments [44–55] have searched for the upper limit of the effective mass of NDBD and some future experiments are also going to be performed on the same. Table 3 shows some of the very recent experiments with their measurements of \(\langle m_{ee} \rangle\) (having 90% C.L.). And the NDBD effective mass \(\langle m_{ee} \rangle\) is very interestingly has the expression with oscillation parameters as

\[
\langle m_{ee} \rangle = |m_1 \cos^2 \theta_{12} \cos^2 \theta_{13} + m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} e^{i\alpha_{21}} + m_3 \sin^2 \theta_{13} e^{i(\alpha_{31} - 2\delta_{CP})}|, \quad (3.7)
\]

where, \(\delta_{CP}\) is CP violating phase. Fig. 5 shows the variation of average value of effective neutrinoless double beta decay mass with respect to sum of active neutrino mass. The bounds coming from the respective experiments are also indicated in shaded regions. The plot 5 gives a positive result towards the model acceptance.

### 4 Scalar sector

In this model, we have two singlet scalar bosons \(\chi_1, \chi_2\) to break the extra gauge symmetries beyond the SM. The full potential of the theory is as follows,

\[
V = m_H^2 (H^\dagger H) + \lambda_1 (H^\dagger H)^2 + m_1^2 (\chi_1^* \chi_1) + m_2^2 (\chi_2^* \chi_2) + \lambda_2 (\chi_1^* \chi_1)^2 + \lambda_3 (\chi_2^* \chi_2)^2 + \lambda_4 (H^\dagger H) (\chi_1^* \chi_1) + \lambda_5 (H^\dagger H) (\chi_2^* \chi_2) + \lambda_6 (\chi_1^* \chi_1) (\chi_2^* \chi_2), \quad (4.1)
\]

where \(\lambda_i, (i = 1, 2, 3, 4, 5, 6)\) are the coupling constants for the scalar bosons, and \(m_H^2, m_1^2, m_2^2\) are the mass terms for three bosons \(H, \chi_1, \chi_2\) respectively.

![Figure 5: Variation of average value of effective neutrinoless double beta decay mass is plotted against the sum of active neutrino mass \(\sum m_i\) where some experimental bounds have given from table 3.](image-url)
Table 3: Current experiments for searching neutrinoless double beta decay with 90% C.L.

4.1 CP-even sector:

Scalar fields $H, \chi_1, \chi_2$ can be parametrized in the basis of real and pseudo scalars as

$$H = \frac{v_H + h + i\eta}{\sqrt{2}}, \quad \chi_1 = \frac{v_1 + \chi'_1 + i\sigma_1}{\sqrt{2}}, \quad \chi_2 = \frac{v_2 + \chi'_2 + i\sigma_2}{\sqrt{2}},$$

(4.2)

where $v_H, v_1, v_2$ are vacuum expectation values (vev’s) of corresponding scalar bosons. $h, \chi'_1, \chi'_2$ are the CP-even sector of the scalar bosons and $\eta, \sigma_1, \sigma_2$ are CP-odd sector of the scalar bosons, and $\langle H \rangle = \frac{v_H}{\sqrt{2}}, \langle \chi_1 \rangle = \frac{v_1}{\sqrt{2}}, \langle \chi_2 \rangle = \frac{v_2}{\sqrt{2}}$. Thus, the CP-even mass matrix has the form in the basis of $(h, \chi'_1, \chi'_2)$ as

$$M^2_E = \begin{pmatrix}
    h & \chi'_1 & \chi'_2 \\
    \chi'_1 & 2\lambda_1 v_H^2 & \lambda_4 v_H v_1 & \lambda_5 v_H v_2 \\
    \chi'_2 & \lambda_4 v_H v_1 & 2\lambda_2 v_1^2 & \lambda_6 v_1 v_2 \\
    \end{pmatrix}.$$

(4.3)

With the constraints of LHC, we can safely take some assumptions to form $M^2_E$ as a three parameter mass matrix. Assuming that

$$\lambda_1 > \lambda_4, \lambda_5, \quad v_H < v_1, v_2, \quad \lambda_4 = \lambda_5,$$

(4.4)

the simplified CP even mass matrix has the form,

$$M^2_E = \begin{pmatrix}
    a & a & a \\
    a & b & c \\
    a & c & b \\
    \end{pmatrix},$$

(4.5)
where, \( a = 2 \lambda_1 v_H^2 \), \( b = 2 \lambda_2 v_1^2 \), \( c = \lambda_6 v_1 v_2 \). Required unitary matrix that diagonalizes CP even mass matrix can be written in the following form\(^1\)

\[
U_E = \begin{pmatrix}
1 & \theta \left( \cos \phi - \sin \phi \right) & \theta \left( \cos \phi + \sin \phi \right) \\
-\theta & \cos \phi & \sin \phi \\
\theta & -\sin \phi & \cos \phi \\
\end{pmatrix}.
\]  \( (4.6) \)

Here, \( \phi \) denotes the mixing between \( \chi'_{1} \) and \( \chi'_{2} \) and \( \theta \) is the mixing of Higgs sector with two scalar sectors. Now the mass eigenstate of these three scalar bosons is having the relation

\[
U^T_E M_E^2 U_E = \begin{pmatrix}
m_h^2 & 0 & 0 \\
0 & m_{\chi'_1}^2 & 0 \\
0 & 0 & m_{\chi'_2}^2 \\
\end{pmatrix} .
\]  \( (4.7) \)

After simplifying, we get \( \phi = \frac{5\pi}{4} \) and \( \theta = \frac{a}{c+b-a} \). For going to mass eigenbasis \((h, \chi'_1, \chi'_2)\) with simplified mass eigenvalues, we have the following relations:

\[
m_h^2 = a - 4a\theta + 2 (a + b) \theta^2 ,
\]
\[
m_{\chi'_1}^2 = -c + b ,
\]
\[
m_{\chi'_2}^2 = c + b + 2a\theta (2 + \theta) .
\]  \( (4.8) \)

There is another relation coming from solution of the parameters \( a, b \) and \( c \), given as

\[
\theta = \frac{-m_h^2 + m_{\chi'_2}^2 - \sqrt{-15m_h^4 - 10m_h^2 m_{\chi'_1}^2 + m_{\chi'_2}^2}}{4(2m_h^2 + m_{\chi'_2}^2)} ,
\]  \( (4.9) \)

which provides the constraints on the masses of the new scalars. For an example, by taking \( m_h = 125 \) GeV, \( \theta = 0.01 \), we have \( m_{\chi'_2} = 1.269 \) TeV. The expression \( \theta = \frac{a}{c+b-a} \) shows, \( b + c = 101a \) thus, from eqn. 4.8 we have \( a = 1.59 \times 10^4 \) GeV\(^2\). If we take \( c \to 0 \), i.e. \( \lambda_6 \to 0 \), the coupling between \( \chi'_1, \chi'_2 \) is negligible, then, \( b = 1.61 \times 10^6 \) GeV\(^2\), and \( m_{\chi'_1} \) becomes 1.268 TeV. The variation of two scalar bosons with respect to \( \theta \) are given in Fig. 6 where we have varied \( \theta \in [0.0, 0.1] \) to hold LHC bound. Fig.6a gives the dependency of \( \chi'_1 \) mass in variation of \( \theta \) and Fig.6b shows the same for second scalar boson \( (\chi'_2) \). From Fig. 6, we can see that the value of \( \chi'_1 \) mass is in the range \([448 \text{ GeV} - 1.28 \text{ TeV}]\) where second scalar mass is in the range \([469 \text{ GeV} - 1.28 \text{ TeV}]\).

5 Comments on non-unitarity

Active neutrino mixing matrix becomes non-unitary in nature due to the presence of extra sterile neutrinos. In inverse seesaw model, the existence of \( N_R \) and \( S_L \), and their mixing with active neutrinos lead non-unitarity in the mixing matrix \( N \) \([56]\). Change of mixing

\(^1\)detail calculation of \( U_E \) is given in appendix A
The matrix property (from unitary to non-unitary) also affect the neutrino oscillation results, so it is very important to calculate the limiting values of non-unitarity parameters coming from the theory.

The general formula for a unitary mixing matrix $\mathcal{U}$ in a model with $m$ number of sterile neutrinos (GeV or TeV mass scale) can be written as a compact form as:

$$
\mathcal{U} = \begin{pmatrix} 
N_{3\times3} & \Theta_{3\times m} \\
R_{m\times 3} & S_{m\times m} 
\end{pmatrix},
$$

(5.1)

where, $N_{3\times3}$ is the $(3 \times 3)$ active neutrino mixing matrix which is no longer unitary in nature and $\Theta_{3\times m}$, $R_{m\times 3}$ are the active-sterile neutrino mixing matrix, with, $S_{m\times m}$ the sterile-sterile neutrino mixing matrix. So, non-unitary active neutrino mixing matrix $N_{3\times3}$ can be decomposed by the expression:

$$
N = (1 - \eta) \ U_{PMNS},
$$

(5.2)

with the Hermitian parametrization matrix $\eta$:

$$
\eta = \frac{\Theta^\dagger \Theta}{2}.
$$

(5.3)

The matrix representation of $\eta$ coming from eqn.(5.2) explicitly written like [57]

$$
\eta = \begin{pmatrix}
\eta_{ee} & \eta_{e\mu} & \eta_{e\tau} \\
\eta_{e\mu}^* & \eta_{\mu\mu} & \eta_{\mu\tau} \\
\eta_{e\tau}^* & \eta_{\mu\tau}^* & \eta_{\tau\tau}
\end{pmatrix},
$$

(5.4)

with the constraint $\eta_{\alpha\beta}(\alpha, \beta = e, \mu, \tau) \ll 1$. From our model, the expression of $\Theta$ can be written as:

$$
\Theta = (\mathcal{M}_{NS}^T)^{-1} \mathcal{M}_D,
$$

(5.5)
and we have calculated the matrix element’s values of Θ as well as η with the help of table 2 which gives the matrix \((1 - η)\) as,

\[
1 - η = \begin{pmatrix}
0.9999 & 3.12837 \times 10^{-6} & 2.78463 \times 10^{-6} \\
3.12837 \times 10^{-6} & 0.9999 & 1.37 \times 10^{-5} \\
2.7846 \times 10^{-6} & 1.37 \times 10^{-5} & 0.9999
\end{pmatrix}, \tag{5.6}
\]

which lie on the satisfied lower bound of experimental data [58, 59].

6 Masses of gauge bosons

![Figure 7](image)

**Figure 7:** Plot (a) gives the allowed region for gauge coupling \(g_{11}\) with the variation of VEV \(v_2\) where plot (b) is the variation of gauge coupling \(g_{21}\) with VEV \(v_2\), plot (c) variation of gauge coupling \(g_{21}\) with respect to VEV \(v_1\) for three different fixed gauge boson masses \(m_{Z_2} = 20, 30, 40\) MeV, plot (d) gives the allowed region for gauge coupling \(g_{21}\) with the variation of gauge boson mass \(m_{Z_2}\). For all the plots, we have used the range value of \(v_1, v_2\) mentioned in eqn. 3.4.

Our model induces two massive gauge bosons, as the extended gauge symmetries are local in nature. The charge assigned \(χ_1\) under \(U(1)_{B-L}\) and \(U(1)_{L_μ - L_ν}\) as \(-1\) and \(+1\)
respectively. So the mass of the associated $Z_1$ is given as

$$m_{Z_1} = \sqrt{g_{11}^2 v_1^2 + g_{12}^2 v_2^2}, \quad (6.1)$$

where, $g_{11}$ and $g_{12}$ are the gauge couplings corresponding to $U(1)_{B-L}$ and $U(1)_{L_\mu-L_\tau}$ respectively. Similarly, we can have the mass of other gauge boson $Z_2$ as

$$m_{Z_2} = \sqrt{g_{21}^2 v_1^2}, \quad (6.2)$$

where, $g_{21}$ is the gauge coupling for the $Z_2$ boson corresponding to $U(1)_{B-L}$. From the consideration of MeV range gauge boson, we can see the dependency of gauge couplings on the vacuum expectation values $v_1, v_2$. The variation of gauge couplings with the vev’s of the extended symmetries are shown in Fig.7. From the plots, it can be said that the value of gauge couplings are in the range $\sim 10^{-6}$.

7 Electron and muon (g-2)

7.1 Electron (g-2)

![Feynman diagram](image)

Figure 8: (a) Feynman diagram for electron (muon) (where $l = e, \mu$) $(g - 2)$ where the mediator is $Z_1$ gauge boson, (b) same but through second gauge boson $Z_2$.

The anomalous magnetic moment for electron remains an open-question for recent particle physics experiments. The value of $(g - 2)_e$ is still not accurately measured unlike $(g - 2)_\mu$ with confusion related to its sign. Rubidium atom measurement gives us +ve value of $(g - 2)_e$ with 1.6σ discrepancy over SM [40],

$$(\Delta a_e)_{\text{Rb}} = (48 \pm 30) \times 10^{-14}, \quad (7.1)$$

whereas, for Cesium atom, [41] $\Delta a_e$ has 2.4σ discrepancy over SM as

$$(\Delta a_e)_{\text{Cs}} = (-87 \pm 36) \times 10^{-14}. \quad (7.2)$$
As there is an uncertainty in experimental value of \((g-2)_e\), we will go with more acceptable value of Rubidium atom measurement where \((g-2)_e\) value is +ve in sign. The Lagrangian which gives extra contribution to \((g-2)_e\) calculation, is through two gauge bosons

\[
\mathcal{L} = g_1 \bar{e} Z_1 \gamma^\mu e + g_2 \bar{e} Z_2 \gamma^\mu e,
\]  

(7.3)

with \(g_1, g_2\) being the corresponding gauge couplings. The Feynman diagrams for these interactions are depicted in Fig. 8 (with \(l = e\)), where the left diagram corresponds to the interaction occurring through \(Z_1\) while the right one is through \(Z_2\). The required expression coming from Feynman diagrams (i.e. Fig. 8a and Fig. 8b) can be written as \([60, 61]\):

\[
\Delta a_e = \int_0^1 \left( \frac{g_1^2}{4\pi^2} \frac{x^2(1-x)}{x^2 + \frac{m_{Z_1}^2}{m_e^2} (1-x)} + \frac{g_2^2}{4\pi^2} \frac{x^2(1-x)}{x^2 + \frac{m_{Z_2}^2}{m_e^2} (1-x)} \right) dx,
\]  

(7.4)

with \(x\) as Feynman parameter. Fig. 9a shows the dependence of electron anomalous magnetic moment with respect to gauge boson coupling \(g_1\), whereas, Fig. 9b is the variation of gauge coupling \(g_1\) with mass of gauge boson \(m_{Z_1}\). In Fig. 9c, the correlation between the gauge couplings has been shown, whereas, Fig. 9d is the variation of gauge coupling \(g_2\) with respect to \(Z_2\) mass.

### 7.2 Muon (g-2):

In the literature, there are considerable interest on the understanding of muon anomalous magnetic moment \([62–65]\). The recent measurement from Muon (\(g - 2\)) collaboration of Fermilab shows 4.2\(\sigma\) discrepancy in muon anomalous magnetic moment with respect to its SM result \([38]\):

\[
\Delta a_{\mu}^{\text{FNAL}} = a_{\mu}^{\exp} - a_{\mu}^{\text{SM}} = (25.1 \pm 59) \times 10^{-10}. \tag{7.5}
\]

Previously, Brookhaven National Laboratory shows the 3.3\(\sigma\) discrepancy from SM \([39]\):

\[
\Delta a_{\mu}^{\text{BNL}} = a_{\mu}^{\exp} - a_{\mu}^{\text{SM}} = (26.1 \pm 79) \times 10^{-10}. \tag{7.6}
\]

To explain the deviation, we should go beyond SM where a new type of interaction will be present in muon anomalous magnetic moment calculation. In our model, new gauge bosons \(Z_1, Z_2\) both are in MeV range so that they both can participate in the extra contribution on muon (\(g-2\)). The required Feynman diagrams are shown in Fig. 8 (\(l = \mu\)) where muon will interact with muon through gauge bosons \(Z_1\) and \(Z_2\). These two contributions will be added with the muon (\(g-2\)) calculation to give satisfactory value of \(\Delta a_{\mu}\). The neutral current interaction Lagrangian is:

\[
\mathcal{L} = -g_1 \bar{\mu} Z_1 \gamma^\mu \mu + g_2 \bar{\mu} Z_2 \gamma^\mu \mu,
\]  

(7.7)
Figure 9: Plot (a) shows the variation of electron anomalous magnetic moment $\Delta a_e$ with
gauge coupling $g_1$, plot (b) exhibits the change of gauge coupling $g_1$ with mass of gauge
boson $m_{Z_1}$, plot (c) shows the variation of one gauge coupling in respect to other one, plot
(d) depicts the variation of gauge coupling $g_2$ with the mass of gauge boson $m_{Z_2}$.

with $g_1$ and $g_2$ being two associated gauge couplings respectively. The expression for $\Delta a_\mu$
is as follows [60, 61]:

$$\Delta a_\mu = \int_0^1 \left( \frac{g_1^2}{4\pi^2} \frac{x^2(1-x)}{x^2 + \frac{m_{Z_1}^2}{m_\mu^2}(1-x)} + \frac{g_2^2}{4\pi^2} \frac{x^2(1-x)}{x^2 + \frac{m_{Z_2}^2}{m_\mu^2}(1-x)} \right) dx, \quad (7.8)$$

Fig.10a gives the variation of muon anomalous magnetic moment with respect to gauge
coupling $g_1$, whereas, Fig.10b represents the variation of gauge coupling $g_1$ with mass of
gauge boson $Z_1$ where both the anomalous magnetic moments are present along with CCFR
experimental bound. The Fig.10c plot shows the variation of the gauge couplings $g_1$ and
$g_2$ projecting a common region for $(g - 2)_{\mu,e}$. 
Figure 10: Plot (a) $\Delta a_\mu$ shows the dependency with respect to gauge coupling $g_1$. Plot (b) shows the variation of gauge coupling with respect to mass of gauge boson $m_{Z_1}$ where CCFR bound and electron and muon (g-2) results are given, plot (c) shows the change of gauge coupling $g_2$ with another gauge coupling $g_1$ and it also shows the common region for muon and electron.

8 Conclusion

In this article, we have described a complete study of neutrino phenomenology, non-unitarity and electron and muon ($g - 2$) in an extended SM with inverse seesaw mechanism. The model has additional three right-handed neutrinos and three neutral fermions with required quantum numbers so that we can cancel the gauge anomaly with two local gauge symmetries $U(1)_{L_e - L_\mu}$ and $U(1)_{B - L}$ which after symmetry breaking, give the mass to scalar sector as well as gauge bosons. As many works show that MeV scale gauge boson can explain most of the unsolved questions in particle physics, we have also taken two MeV range gauge bosons to explain muon, electron ($g - 2$) anomalies simultaneously. All the plots related with the sum of active neutrino masses ($\sum m_i$) show the acceptance of normal ordering as they ranges from 0.058 eV to 0.1 eV. With the help of minimization
of $\chi^2$, we have extracted best-fit values and strongly constrained the model free parameters for generating the plots. As a result, we have the value of Jarlskog invariant $J_{CP}$ coming from the model within $[-0.003, 0.003]$. Majorana phases $\alpha_{21}$ and $\alpha_{31}$ have values ranging from $[0, 260^\circ]$ and $[0, 360^\circ]$ respectively. The effective mass parameter's the upper bound of NDBD range $\langle m_{ee} \rangle \approx [0.005, 0.02]$ eV which certainly lies within the experimental bounds of KamLAND-Zen and CUORE. The calculated mass values of scalar singlets (TeV range) give a positive possibility of finding them in the future LHC experiments. Non-unitarity parameter $\eta$ also satisfies the experimental upper bounds. Apart from neutrino phenomenology, muon and electron $(g - 2)$ have been studied with the results of $(g - 2)_e$, $(g - 2)_\mu$ abiding the CCFR bound.

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**A Calculation of $U_E$**

For calculating the mixing matrix between the CP even scalar sectors, we have taken the mixing angles between three scalar bosons as:

$$
\theta_1 = \text{mixing angle between } h \text{ and } \chi'_{1} ,
$$

$$
\theta_2 = \text{mixing angle between } h \text{ and } \chi'_{2} ,
$$

$$
\phi = \text{mixing angle between } \chi'_{1} \text{ and } \chi'_{2}.
$$

The mixing matrix $U_E$ will be the product of three matrices, matrix having rotation along $Z$-axis, rotation along $Y$-axis and rotation along $X$-axis respectively. Thus the form of mixing matrix is:

$$
U_E = \begin{pmatrix}
-c_1 & s_1 & 0 \\
-s_1 & c_1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
c_2 & 0 & s_2 \\
0 & 1 & 0 \\
-s_2 & 0 & c_2
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
c_\phi & s_\phi & 0 \\
0 & -s_\phi & c_\phi
\end{pmatrix}
, \tag{A.1}
$$

$$
= \begin{pmatrix}
c_1c_2 & s_1c_\phi - c_1s_2s_\phi & s_1s_\phi + s_2c_1c_\phi \\
-s_1c_2 & c_1c_\phi + s_1s_2s_\phi & c_1s_\phi - s_1c_\phi s_2 \\
-s_2 & -s_\phi c_2 & c_2c_\phi,
\end{pmatrix}
, \tag{A.1}
$$
where, $c_i = \cos \theta_i$, $s_i = \sin \theta_i$ with $(i = 1, 2, \phi)$. The mixing angle between Higgs and other two scalar bosons should not be large to satisfy LHC bounds. So we have taken $\theta_1$ and $\theta_2$ to be small and for convenience $\theta_1 \sim \theta_2 \sim \theta$. After taking these assumptions,

$$U_E = \begin{pmatrix}
1 & \theta (\cos \phi - \sin \phi) & \theta (\cos \phi + \sin \phi) \\
\theta & \cos \phi & \sin \phi \\
-\theta & -\sin \phi & \cos \phi
\end{pmatrix},$$

(A.2)

which is the required mixing matrix $U_E$ for CP even scalar bosons.

References

[1] E. Ma, *Pathways to naturally small neutrino masses*, Phys. Rev. Lett. 81 (1998) 1171–1174, [hep-ph/9805219].

[2] S. Weinberg, *Baryon and Lepton Nonconserving Processes*, Phys. Rev. Lett. 43 (1979) 1566–1570.

[3] E. Ma, *Connection between the neutrino seesaw mechanism and properties of the Majorana neutrino mass matrix*, Phys. Rev. D 71 (2005) 111301, [hep-ph/0501056].

[4] A. Palcu, *Canonical Seesaw Mechanism in Electro-Weak SU(4)(L) x U(1)(Y) Models*, Mod. Phys. Lett. A 24 (2009) 2589–2600, [arXiv:0908.1636].

[5] H. Zhang, *Light Sterile Neutrino in the Minimal Extended Seesaw*, Phys. Lett. B 714 (2012) 262–266, [arXiv:1110.6838].

[6] E. Di Valentino, A. Melchiorri, and J. Silk, *Cosmological constraints in extended parameter space from the Planck 2018 Legacy release*, JCAP 01 (2020) 013, [arXiv:1908.01391].

[7] W. Rodejohann, *Type II seesaw mechanism, deviations from bimaximal neutrino mixing and leptogenesis*, Phys. Rev. D 70 (2004) 073010, [hep-ph/0403236].

[8] R. Ding, Z.-L. Han, L. Feng, and B. Zhu, *Confronting the DAMPE Excess with the Scotogenic Type-II Seesaw Model*, Chin. Phys. C 42 (2018), no. 8 083104, [arXiv:1712.02021].

[9] C. A. de Sousa Pires, F. Ferreira De Freitas, J. Shu, L. Huang, and P. Wagner Vasconcelos Olegário, *Implementing the inverse type-II seesaw mechanism into the 3-3-1 model*, Phys. Lett. B 797 (2019) 134827, [arXiv:1812.10570].

[10] P. V. Dong, L. T. Hue, H. N. Long, and D. V. Soa, *The 3-3-1 model with A4 flavor symmetry*, Phys. Rev. D 81 (2010) 053004, [arXiv:1001.4625].

[11] R. Franceschini, T. Hambye, and A. Strumia, *Type-III see-saw at LHC*, Phys. Rev. D 78 (2008) 033002, [arXiv:0805.1613].

[12] S. Goswami, K. N. Vishnudath, and N. Khan, *Constraining the minimal type-III seesaw model with naturalness, lepton flavor violation, and electroweak vacuum stability*, Phys. Rev. D 99 (2019), no. 7 075012, [arXiv:1810.11687].

[13] A. Biswas, D. Borah, and D. Nanda, *Type III seesaw for neutrino masses in U(1)_{B-L} model with multi-component dark matter*, JHEP 12 (2019) 109, [arXiv:1908.04308].
[14] P. Fileviez Perez, *Type III Seesaw and Left-Right Symmetry*, JHEP 03 (2009) 142, [arXiv:0809.1202].

[15] C. Arbeláez, C. Dib, K. Monsálvez-Pozo, and I. Schmidt, *Quasi-Dirac neutrinos in the linear seesaw model*, JHEP 07 (2021) 154, [arXiv:2104.08023].

[16] M. K. Behera, S. Mishra, S. Singirala, and R. Mohanta, *Implications of $A_4$ modular symmetry on Neutrino mass, Mixing and Leptogenesis with Linear Seesaw*, arXiv:2007.00545.

[17] M. K. Behera and R. Mohanta, *Linear seesaw in $A'_4$ modular symmetry with Leptogenesis*, arXiv:2201.10429.

[18] M. Hirsch, S. Morisi, and J. W. F. Valle, *A$_4$-based tri-bimaximal mixing within inverse and linear seesaw schemes*, Phys. Lett. B 679 (2009) 454–459, [arXiv:0905.3056].

[19] F. F. Deppisch, L. Graf, S. Kulkarni, S. Patra, W. Rodejohann, N. Sahu, and U. Sarkar, *Reconciling the 2 TeV excesses at the LHC in a linear seesaw left-right model*, Phys. Rev. D 93 (2016), no. 1 013011, [arXiv:1508.05940].

[20] E. Ma, *Radiative inverse seesaw mechanism for nonzero neutrino mass*, Phys. Rev. D 80 (2009) 013013, [arXiv:0904.4450].

[21] M. K. Behera, S. Singirala, S. Mishra, and R. Mohanta, *A modular $A_4$ symmetric scotogenic model for neutrino mass and dark matter*, J. Phys. G 49 (2022), no. 3 035002, [arXiv:2009.01806].

[22] M. K. Behera and R. Mohanta, *Inverse seesaw in $A'_4$ modular symmetry*, J. Phys. G 49 (2022), no. 4 045001, [arXiv:2108.01059].

[23] M. K. Parida and A. Raychaudhuri, *Inverse see-saw, leptogenesis, observable proton decay and $\Delta_{R}^{\pm}$ in SUSY SO(10) with heavy $W_R$, Phys. Rev. D 82* (2011) 053017, [arXiv:1007.5085].

[24] A. G. Dias, C. A. de S. Pires, P. S. Rodrigues da Silva, and A. Sampieri, *A Simple Realization of the Inverse Seesaw Mechanism*, Phys. Rev. D 86 (2012) 035007, [arXiv:1206.2590].

[25] A. G. Dias, C. A. de S. Pires, and P. S. R. da Silva, *How the Inverse See-Saw Mechanism Can Reveal Itself Natural, Canonical and Independent of the Right-Handed Neutrino Mass*, Phys. Rev. D 84 (2011) 053011, [arXiv:1107.0739].

[26] S. Centelles Chuliá, R. Srivastava, and A. Vicente, *The inverse seesaw family: Dirac and Majorana*, JHEP 03 (2021) 248, [arXiv:2011.06609].

[27] J. a. P. Pinheiro, C. A. de S. Pires, F. S. Queiroz, and Y. S. Villamizar, *Confronting the inverse seesaw mechanism with the recent muon g-2 result*, Phys. Lett. B 823 (2021) 136764, [arXiv:2107.01315].

[28] C. Pongkitivanichkul, N. Thongyoi, and P. Uttayarat, *Inverse seesaw mechanism and portal dark matter*, Phys. Rev. D 100 (2019), no. 3 035034, [arXiv:1905.13224].

[29] A. Abada, N. Bernal, A. E. C. Hernández, X. Marcano, and G. Piazza, *Gauged inverse seesaw from dark matter*, Eur. Phys. J. C 81 (2021), no. 8 758, [arXiv:2107.02803].
[30] T. Nomura, H. Okada, and S. Patra, *An inverse seesaw model with $A_4$-modular symmetry*, Nucl. Phys. B 967 (2021) 115395, [arXiv:1912.00379].

[31] H. An, P. S. B. Dev, Y. Cai, and R. N. Mohapatra, *Sneutrino Dark Matter in Gauged Inverse Seesaw Models for Neutrinos*, Phys. Rev. Lett. 108 (2012) 081806, [arXiv:1110.1366].

[32] H. K. Dreiner, J.-F. Fortin, J. Isern, and L. Ubaldi, *White Dwarfs constrain Dark Forces*, Phys. Rev. D 88 (2013) 043517, [arXiv:1303.7232].

[33] BABAR Collaboration, J. P. Lees et al., *Search for Darkonium in $e^+e^-$ Collisions*, Phys. Rev. Lett. 128 (2022), no. 2 021802, [arXiv:2106.08529].

[34] A. Anastasi et al., *Limit on the production of a low-mass vector boson in $e^+e^- \to U\gamma$, $U \to e^+e^-$ with the KLOE experiment*, Phys. Lett. B 750 (2015) 633–637, [arXiv:1509.00740].

[35] D. Croon, G. Elor, R. K. Leane, and S. D. McDermott, *Supernova Muons: New Constraints on $Z'$ Bosons, Axions and ALPs*, JHEP 01 (2021) 107, [arXiv:2006.13942].

[36] T. Araki, F. Kaneko, T. Ota, J. Sato, and T. Shimomura, *MeV scale leptonic force for cosmic neutrino spectrum and muon anomalous magnetic moment*, Phys. Rev. D 93 (2016), no. 1 013014, [arXiv:1508.07471].

[37] A. Hammad, A. Rashed, and S. Moretti, *The dark $Z'$ and sterile neutrinos behind current anomalies*, Phys. Lett. B 827 (2022) 136945, [arXiv:2110.08651].

[38] Muon $g-2$ Collaboration, B. Abi et al., *Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm*, Phys. Rev. Lett. 126 (2021), no. 14 141801, [arXiv:2104.03281].

[39] S. Charity, *Beam profile measurements using the straw tracking detectors at the Fermilab muon $g-2$ experiment, and a study of their sensitivity to a muon electric dipole moment*. PhD thesis, U. Liverpool (main), 2018.

[40] L. Morel, Z. Yao, P. Cladé, and S. Guellati-Khélifa, *Determination of the fine-structure constant with an accuracy of 81 parts per trillion*, Nature 588 (2020), no. 7836 61–65.

[41] H. Davoudiasl and W. J. Marciano, *Tale of two anomalies*, Phys. Rev. D 98 (2018), no. 7 075011, [arXiv:1806.10252].

[42] M. C. Gonzalez-Garcia, M. Maltoni, and J. Salvado, *Updated global fit to three neutrino mixing: status of the hints of theta13 > 0*, JHEP 04 (2010) 056, [arXiv:1001.4524].

[43] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, *Global fit to three neutrino mixing: critical look at present precision*, JHEP 12 (2012) 123, [arXiv:1209.3023].

[44] GERDA Collaboration, M. Agostini et al., *Improved Limit on Neutrinoless Double-$\beta$ Decay of $^{76}$Ge from GERDA Phase II*, Phys. Rev. Lett. 120 (2018), no. 13 132503, [arXiv:1803.11100].

[45] Majorana Collaboration, C. E. Aalseth et al., *Search for Neutrinoless Double-$\beta$ Decay in $^{76}$Ge with the Majorana Demonstrator*, Phys. Rev. Lett. 120 (2018), no. 13 132502, [arXiv:1710.11608].
A. S. Barabash et al., *Final results of the Aurora experiment to study $2\beta$ decay of $^{116}$Cd with enriched $^{116}$CdWO$_4$ crystal scintillators*, Phys. Rev. D 98 (2018), no. 9 092007, [arXiv:1811.06398].

NEMO-3 Collaboration, R. Arnold et al., *Results of the search for neutrinoless double-$\beta$ decay in $^{100}$Mo with the NEMO-3 experiment*, Phys. Rev. D 92 (2015), no. 7 072011, [arXiv:1506.05825].

NEMO-3 Collaboration, R. Arnold et al., *Measurement of the $2\nu\beta\beta$ decay half-life of $^{150}$Nd and a search for $0\nu\beta\beta$ decay processes with the full exposure from the NEMO-3 detector*, Phys. Rev. D 94 (2016), no. 7 072003, [arXiv:1606.08494].

CUORE Collaboration, A. Giachero et al., *New results from the CUORE experiment*, PoS ICHEP2020 (2021) 133, [arXiv:2011.09295].

KamLAND-Zen Collaboration, A. Gando et al., *Search for Majorana Neutrinos near the Inverted Mass Hierarchy Region with KamLAND-Zen*, Phys. Rev. Lett. 117 (2016), no. 8 082503, [arXiv:1605.02889]. [Addendum: Phys.Rev.Lett. 117, 109903 (2016)].

EXO Collaboration, J. B. Albert et al., *Search for Neutrinoless Double-Beta Decay with the Upgraded EXO-200 Detector*, Phys. Rev. Lett. 120 (2018), no. 7 072701, [arXiv:1707.08707].

NEMO Collaboration, A. S. Barabash and V. B. Brudanin, *Investigation of double beta decay with the NEMO-3 detector*, Phys. Atom. Nucl. 74 (2011) 312–317, [arXiv:1002.2862].

NEMO-3 Collaboration, J. Argyriades et al., *Measurement of the two neutrino double beta decay half-life of Zr-96 with the NEMO-3 detector*, Nucl. Phys. A 847 (2010) 168–179, [arXiv:0906.2694].

C. Arnaboldi et al., *A Calorimetric search on double beta decay of Te-130*, Phys. Lett. B 557 (2003) 167–175, [hep-ex/0211071].

M. J. Dolinski, A. W. P. Poon, and W. Rodejohann, *Neutrinoless Double-Beta Decay: Status and Prospects*, Ann. Rev. Nucl. Part. Sci. 69 (2019) 219–251, [arXiv:1902.04097].

C. Soumya, *Probing nonunitary neutrino mixing via long-baseline neutrino oscillation experiments based at J-PARC*, Phys. Rev. D 105 (2022), no. 1 015012, [arXiv:2104.04315].

M. Blennow, P. Coloma, E. Fernandez-Martinez, J. Hernandez-Garcia, and J. Lopez-Pavon, *Non-Unitarity, sterile neutrinos, and Non-Standard neutrino Interactions*, JHEP 04 (2017) 153, [arXiv:1609.08637].

E. Fernandez-Martinez, M. B. Gavela, J. Lopez-Pavon, and O. Yasuda, *CP-violation from non-unitary leptonic mixing*, Phys. Lett. B 649 (2007) 427–435, [hep-ph/0703098].

E. Fernandez-Martinez, J. Hernandez-Garcia, and J. Lopez-Pavon, *Global constraints on heavy neutrino mixing*, JHEP 08 (2016) 033, [arXiv:1605.08774].

S. R. Moore, K. Whisnant, and B.-L. Young, *Second Order Corrections to the Muon Anomalous Magnetic Moment in Alternative Electroweak Models*, Phys. Rev. D 31 (1985) 105.
[61] W. A. Bardeen, R. Gastmans, and B. E. Lautrup, Static quantities in Weinberg's model of weak and electromagnetic interactions, Nucl. Phys. B 46 (1972) 319–331.

[62] T. Aoyama et al., The anomalous magnetic moment of the muon in the Standard Model, Phys. Rept. 887 (2020) 1–166, [arXiv:2006.04822].

[63] A. Bodas, R. Coy, and S. J. D. King, Solving the electron and muon $g - 2$ anomalies in $Z'$ models, Eur. Phys. J. C 81 (2021), no. 12 1065, [arXiv:2102.07781].

[64] A. Kamada, K. Kaneta, K. Yanagi, and H.-B. Yu, Self-interacting dark matter and muon $g - 2$ in a gauged $U(1)_{\mu - \tau}$ model, JHEP 06 (2018) 117, [arXiv:1805.00651].

[65] T. Araki, F. Kaneko, Y. Konishi, T. Ota, J. Sato, and T. Shimomura, Cosmic neutrino spectrum and the muon anomalous magnetic moment in the gauged $L_{\mu} - L_{\tau}$ model, Phys. Rev. D 91 (2015), no. 3 037301, [arXiv:1409.4180].