Manifestly Gauge Invariant Models of Chiral Lattice Fermions

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Abstract
A manifestly gauge invariant lattice action for nonanomalous chiral models is proposed which leads in the continuum limit to the theory free of doublers.

1 Introduction
In this paper I present a gauge invariant lattice formulation of the anomaly free chiral models like the $SO(10)$ or the standard model. It will be proven in the framework of perturbation theory that all the fermion doublers decouple in the continuum limit and the physical content of the the theory is the usual one of the standard model.

The present model is a gauge invariant extension of the model proposed in our paper [1], in which the gauge invariance was broken for a finite lattice spacing and restored in the continuum limit.

To provide the doublers with big masses we use the Yukawa–Wilson interaction of fermions with Higgs mesons. This possibility was widely discussed last years mainly in the framework of the Smit-Swift model [2], [3], [4], [5]. (For detailed references and review of other approaches see [6], [7], [8]).

However the models used so far did not solve the problem. To give the doublers big masses one needs a large Yukawa coupling. It prevents using the perturbation theory as a big Yukawa coupling produces strong effects on the Higgs mesons interaction. On the other hand the strong coupling analysis leads to the conclusion that the light fermions in these models are noninteracting [9], [10], [11].

In this paper I shall show that if one introduces the additional gauge invariant interaction with the Pauli-Villars (PV) fields, and higher covariant derivative kinetic terms for the Higgs and Yang-Mills fields, the effective Yukawa-Wilson coupling becomes weak and the the model can be treated perturbatively.

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2 The Model

We consider the SO(10) model described by the Lagrangian:

\[ L = -\frac{1}{4}(F_{\mu\nu}^{ij})^2 + i \sum_l \bar{\psi}^i_l(\hat{\partial} - igA^{ij} \sigma_{ij})\psi^i_l + L_H \]  

(1)

Here \( \sigma_{ij} \) are the SO(10) generators: \( \sigma_{ij} = 1/2[\Gamma_i, \Gamma_j] \), where \( \Gamma_i \) are Hermitian \( 32 \times 32 \) matrices which satisfy the Clifford algebra: \( [\Gamma_i, \Gamma_j] = 2\delta_{ij} \). The chiral SO(10) spinors \( \psi_{\pm} = 1/2(1 \pm \Gamma_{11})\psi \), where \( \Gamma_{11} = \Gamma_1 \Gamma_2 \ldots \Gamma_{10} \), describe the 16-dimensional irreducible representation of SO(10) including quark and lepton fields. We assume also that the spinors \( \psi_+ \) are left-handed \( \psi_+ = 1/2(1 - \gamma_5)\psi_+ \). Index \( l \) numerates generations. \( L_H \) includes interactions of Higgs mesons and will not be written explicitly.

We start with the analysis of one loop fermion diagrams. Let us take as the gauge invariant regularization of the Lagrangian (1) the following lattice action

\[ I = I_{YM} + I_\psi + I_{YW} + I_H \]  

(2)

Here \( I_{YM} \) is the standard Wilson lattice action for the SO(10) gauge fields, \( I_\psi \) describes the gauge invariant interaction of the gauge fields with the original fermions \( \psi_l \), and also with the fermionic, \( \psi_r \), and bosonic \( \chi_r, \bar{\chi}_r \), PV fields

\[ I_\psi = \sum_{x,\mu,l} \left[ -\frac{1}{2ia} \bar{\psi}_l^i(x)\gamma^\mu U_{\mu}(x)\psi_l^i(x + a_\mu) \right] \]  

(3)

\[ + \sum_{x,\mu,r} \left[ -\frac{1}{2ia} \bar{\psi}_r(x)\gamma^\mu U_{\mu}(x)\psi_r(x + a_\mu) - \frac{M_r}{2} \bar{\psi}_r(x)C_DCT_{11} \bar{\psi}_r(x) \right] \]

\[ + \sum_{x,\mu,r} \left[ -\frac{1}{2ia} \bar{\chi}_r(x)\gamma_{\mu11} U_{\mu}(x)\chi_r(x + a_\mu) - \frac{1}{2ia} \bar{\chi}_r(x)\Gamma_{11} U_{\mu}(x)\bar{\chi}_r(x + a_\mu) \right] \]

\[ + \frac{M_r}{2} \left[ \bar{\chi}_r(x)C_D C \bar{\chi}_r(x) + \bar{\chi}_r(x)C_D C \bar{\chi}_r(x) \right] + h.c. \]

Here \( U_{\mu}(x) \) is the usual lattice gauge field \( U_{\mu} = \exp \{ ig\sigma_{ij}A_{\mu}^{ij} \} \). The matrix \( C \) is a conjugation matrix for the SO(10) group: \( C\sigma = -\sigma^T C \), the matrix \( C_D \) is the usual charge conjugation matrix. The terms proportional to \( M_r \) are the gauge invariant Majorana mass terms for the PV fields. The number of PV fields with the mass \( M_r \) will be denoted by \( C_r \). Contributions of bosonic and fermionic PV fields to spinorial loops differ by sign.

\( I_{YW} \) is the Yukawa–Wilson action which produces via spontaneous symmetry breaking the Wilson like mass terms lifting the values of the doublers masses:

\[ I_{YW} = -\frac{N}{2} \sum_{l,i,x,\mu} \left[ \tilde{\psi}_l^i(x)\varphi^i(x)\Gamma^i U_{\mu}(x)C_D C \tilde{\psi}_l^{+T}(x + a_\mu) \right] \]  

(4)

\[ + \tilde{\psi}_l^i(x + a_\mu)\Gamma^i C_D C \varphi^i(x + a_\mu)U_{\mu}^{+T}(x)\tilde{\psi}_l^{+T}(x) - 2\tilde{\psi}_l^i(x)\Gamma^i C_D C \varphi^i(x)\tilde{\psi}_l^{+T}(x) \]
\[-N \sum_{r,x,\mu} \left[ \bar{\psi}_r(x) \varphi^i(x) \Gamma^i U_\mu(x) C D \bar{\psi}_r^T(x + a_\mu) \\
+ \bar{\psi}_r(x + a_\mu) \Gamma^i C D \varphi^i(x + a_\mu) U_\mu^T(x) \bar{\psi}_r^T(x) - 2 \bar{\psi}_r(x) \Gamma^i C D \varphi^i(x) \bar{\psi}_r^T(x) \right] \]

\[-N \sum_{r,x,\mu} \left[ \bar{\tilde{\chi}}_r(x) \varphi^i(x) \Gamma^i U_\mu(x) C D \bar{\tilde{\chi}}_r^T(x + a_\mu) + \bar{\tilde{\chi}}_r(x + a_\mu) \varphi^i(x + a_\mu) \Gamma^i C D U_\mu^T(x) \bar{\tilde{\chi}}_r^T(x) \\
- 2 \bar{\tilde{\chi}}_r(x) \Gamma^i C D \varphi^i(x) \bar{\tilde{\chi}}_r^T(x) \right] + h.c. \]

Here \( \varphi^i \) is the Higgs field realising the 10-dimensional representation of \( SO(10) \). \( N \) is a large dimensionless parameter which becomes infinite in the continuum limit. We shall put \( a^{-1} = \lambda N \), where \( \lambda \) is a fixed mass scale.

Finally \( I_H \) is the Higgs action including the gauge invariant kinetic term and the Higgs potential which we do not write explicitly. We also did not introduce the scalar–fermion interaction producing finite masses for physical leptons and quarks. It can be done in a standard way and will not be discussed here.

A nonzero expectation value of the Higgs field \( < \varphi_{10} > = v \) produces the Wilson terms giving all the doublers the big masses \( \sim N v \). The price we pay for that is the presence of the large Yukawa coupling \( N \sim \lambda a^{-1} \). Due to this coupling the heavy particles may not decouple in the framework of perturbation theory and in the limit \( a \to 0 \) new vertices including scalar and vector fields may arise leading to breakdown of the weak coupling expansion.

We shall show that in our model due to the presence of the PV fields there is a compensation between the contributions of bosonic and fermionic fields leading to decoupling of the heavy particles and the absence of additional vertices.

Let us consider the Higgs meson scattering amplitudes generated by the Yukawa interaction (\(^4\)). For the diagram with \( L \) external lines we have:

\[ \Pi_L \sim N^L \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left\{ \frac{1 - \gamma_5}{2} \left[ \frac{(1 + \Gamma_{11})}{2} \ldots V^{i_L} S(p + Q_{L-1}) \ldots V^{i_1} S(p) \right] \right\} \]

\[ + \sum_{r,\pm} \frac{(1 + \Gamma_{11})}{2} V^{i_L} S_r(p + Q_{L-1}) \ldots V^{i_1} S_r(p) \]

\[ Q_i = \sum_{n=1}^i k_n \text{ where } k_n \text{ are the external momenta. Here the factor } l \text{ in the first term is due to the presence of } l \text{ generations of original fields.} \]

\( V^i \) is the interaction vertex

\[ V^{i_L} = \Gamma^{i_L} C D \sum_{\mu} (\cos(p + Q_{L-1})_\mu a - 1) \]

and \( S_r \) are the propagators

\[ S_{\psi_+ \psi_+} = \frac{\hat{s}}{s^2 + m^2}, \]
\[
S_{\psi^+ \psi^+} = S_{\bar{\psi} \bar{\psi}} = S_{\chi^+ \chi^+} = S_{\bar{\chi} \bar{\chi}} = -S_{\chi^- \chi^-} = -S_{\bar{\chi} \bar{\chi}} = \frac{s}{s^2 + m^2 + M_r^2}, \quad (8)
\]

\[
S_{\psi_1^+ \psi_1^+} = S_{\psi_1^+ \psi_1^+} = \frac{\Gamma_{10} C_{Dm}}{s^2 + m^2 + M_r^2}, \quad (9)
\]

\[
S_{\psi_1^+ \psi_1^+} = S_{\psi_1^+ \psi_1^+} = S_{\bar{\psi} \bar{\psi}} = S_{\chi^+ \chi^+} = S_{\bar{\chi} \bar{\chi}} = \frac{\Gamma_{10} C_{Dm}}{s^2 + (m^2 + M_r^2)}, \quad (10)
\]

\[
S_{\psi_1^- \psi_1^-} = S_{\psi_1^- \psi_1^-} = S_{\chi^- \chi^-} = S_{\bar{\chi} \bar{\chi}} = S_{\bar{\chi} \bar{\chi}} = \frac{M_r C_D C_{T11}}{s^2 + (m^2 + M_r^2)}, \quad (11)
\]

Here
\[
s_\mu = a^{-1} \sin(p_\mu a),
\]

\[
m = N v \sum_\mu (1 - \cos(p_\mu a)). \quad (12)
\]

Note that the contribution of the original fields includes only positive chirality projections \(1/2(1 + \Gamma_{11})\) whereas the PV fields have both positive and negative chirality projections.

Let us show that in the limit \(a \to 0\) the heavy particles decouple. It is convenient to separate the integration domain in (3) into two parts

\[
V_{in} : |p| < \lambda N^\gamma; \quad V_{out} : |p| > \lambda N^\gamma; \quad \gamma < \frac{1}{3} \quad (14)
\]

In the domain \(V_{in} | ap | \ll 1\) and one can use the expansion over \((ap)\). For example the diagrams including the propagators (4), (8) look as follows

\[
\Pi_{(in)}^L \sim N^{-L} \int_{V_{in}} \frac{\text{Tr} \left[ \left( p + Q_{L-1} \right)^2 \left( \hat{p} + \hat{Q}_{L-1} \right) \ldots p^2 \hat{p} \left( \frac{1 + \Gamma_{11}}{2} \right) \right] d^4p}{\left[ \left( p + Q_{L-1} \right)^2 + M_r^2 a^2 \right] \ldots \left[ p^2 + M_r^2 a^2 \right]} \quad (15)
\]

One sees that \(\Pi_{in}^L < N^{(4+L)\gamma - L}\) and \(\Pi_{in}^L \to 0\) when \(a \to 0\). The same estimate is valid also for diagrams including the propagators (4), (11) provided \(M_r < \lambda N^\gamma\).

In the domain \(V_{out}\) the corresponding expression for \(\Pi_{out}^L\) after rescaling the integration variables \(ap = u\) may be written in the form

\[
\Pi_{out}^L \sim N^{L+4} \int_{V_{out}} du \text{Tr} \left\{ \left( 1 - \gamma_5 \right) \left[ \frac{(1 + \Gamma_{11})}{2} V^{i_2} S(u + aQ_{L-1}) \ldots V^{i_1} S(u) \right] \right\} \quad (16)
\]

\[
+ \sum_{r_1 \pm} \left( \frac{1 \pm \Gamma_{11}}{2} \right) V^{i_1} S_r(u + aQ_{L-1}) \ldots V^{i_1} S_r(u)
\]

Let us consider the first term in eq. (16).
Near the edges of Brillouin zones \( n = (\pi, 0, 0, 0) \) e.t.c. the vertex function \( V_i \sim O(1) \) whereas the propagators may be written as follows

\[
S(u) \sim a \left( \frac{\hat{u}}{u^2 + \mu^2} \right) \tag{17}
\]

where \( \mu^2 \sim O(1) \). One sees that the diagrams with arbitrary number of external Higgs lines are divergent in the limit \( a \to 0 \), leading to nonrenormalizability of the theory. Higher derivative terms also arise. In particular \( \Pi^2 \) contains a finite term \( \sim k^4 \). It illustrates a nondecoupling of heavy particles in the model with the Yukawa coupling of the order of the cut-off.

However the presence of the PV fields changes the situation drastically. The PV fields produce the analogous contributions to eq.\((14)\) which compensate the contribution of the original fields.

For definiteness we consider now the case of even number of generations, e.g. \( l = 2 \). As we have already mentioned the Higgs meson scattering amplitudes eq.\((16)\) contain the contribution of the positive chirality physical fields \( \psi^1_{\pm} \), and PV fields \( \psi^l_{\chi}, \chi^r \) of both chiralities. Now we shall show that the contributions of the positive and negative chirality fields are equal in the limit \( a \to 0 \).

Indeed the trace over the \( SO(10) \) indices in the eq.\((13)\) is proportional to

\[
\begin{align*}
\text{Tr} \left[ (1 \pm \Gamma_{11}) \Gamma_{i_1} \ldots \Gamma_{i_L} \right] \\
\Gamma_{11} \sim \varepsilon^{i_1 \ldots i_{10}} \Gamma_{i_1} \ldots \Gamma_{i_{10}}
\end{align*}
\]

where \( \varepsilon^{i_1 \ldots i_{10}} \) is antisymmetric with respect to the permutations of any two indices. The term proportional to \( \Gamma_{11} \) is different from zero only if the product of \( \Gamma_i \) can be reduced to the sum of the terms proportional to the product \( \Gamma_{1} \ldots \Gamma_{10} \).

In the limit \( a \to 0 \) the contribution of this term to the scattering amplitudes to the leading order is proportional to

\[
a^{-4} \int dk_1 \ldots dk_{10} \varphi^{i_1}(k_1) \ldots \varphi^{i_{10}}(k_{10}) \varepsilon^{i_{12} \ldots i_{10}} \delta(k_1 + \ldots + k_{10}) \tag{20}
\]

This term is zero due to antisymmetry of \( \varepsilon^{i_1 \ldots i_{10}} \). Next to the leading term is proportional to \( a^{-2} \) and contains two additional momenta \( k_{i_j}, k_{i_m} \), and there is also a finite term with four momenta \( k_{i_1} k_{i_{10}} k_{i_r} k_{i_s} \). All other terms vanish in the limit \( a \to 0 \). The terms with two and four momenta are zero due to antisymmetry of \( \varepsilon^{i_1 \ldots i_{10}} \).

Therefore one can drop the terms proportional to \( \Gamma_{11} \) in the eq.\((10)\) and instead of summing over \( \pm \) to multiply the last term by the factor 2. Now the first and the second terms in the eq.\((14)\) have the same structure, and expanding this equation in terms of \( M_r^2 \) one gets

\[
\Pi_L \sim 2 \left\{ \frac{1}{a^4} \left( \sum_r C_r + 1 \right) f_1(k) + \frac{1}{a^2} \left( \sum_r C_r M_r^2 \right) f_2(k) \right. \\
+ \left( \sum_r C_r M_r^4 \right) f_3(k) \right\} + O(a) \tag{21}
\]
Assuming the PV conditions

\[ \sum C_r + 1 = 0; \quad \sum C_r M_r^2 = 0; \quad \sum C_r M_r^4 = 0; \quad (22) \]

we see that \( \Pi^L \rightarrow 0 \) when \( a \rightarrow 0 \).

No new problems arise for the mixed diagrams including scalar and vector external lines. Their contribution also vanish in the continuum limit (in fact it follows from the gauge invariance of the theory and vanishing of the diagrams with only scalar external lines).

Therefore all spinorial loops generated by the Yukawa–Wilson interaction do not contribute in the continuum limit and the heavy particles decouple.

In the case of an odd number of generations the same conclusion holds if one introduces the infinite number of PV fields following the procedure described in [12], [1]. One should take in the eq.s (2,3) the summation over \( r \) from \(-\infty \) to \(+\infty \) (\( r > 0 \) corresponding to the positive chirality PV fields, \( r < 0 \) to the negative chirality PV fields and \( r = 0 \) to the original field \( \psi^+ \)) and put \( M_r = M|_r | \). The analytic expression for the spinor loop with \( L \) external lines can be written in the same way as in ref.[1] and after summation over \( r \) looks as follows

\[ \Pi_L \sim \int_{\frac{\pi}{a}}^{\frac{\pi}{a}} \sum_{l=0}^{n-1} \frac{A_l(p, Q_l)}{\sqrt{M^2 (s^2 + m^2) \sinh \left( \frac{\pi \sqrt{s^2 + m^2}}{M} \right)}} d^4 p \quad (23) \]

where \( Q_l = k_1 + \ldots + k_l \), \( k_i \) are the external momenta, and \( A_l \) are some polynomials over \( p, Q_l \). This expression decreases exponentially when \( a \rightarrow 0 \) providing decoupling of heavy particles in the odd number generation case as well. (As was indicated in the ref.[1] in the lattice model it is not necessary to take an infinite number of the PV fields. It is sufficient to take \( |r| \leq \tilde{N}(a) \) where \( \tilde{N}(a) \rightarrow \infty \) when \( a \rightarrow 0 \).

Up to now we considered only spinorial loops. However the diagrams with internal lines of Higgs and Yang-Mills fields generated by the Yukawa-Wilson coupling are also proprtional to the large parameter \( N \) and for these diagrams the compensation mechanism described above does not work. If one tries to develop a perturbation expansion for the action (2) one finds an infinite series of divergent diagrams indicating the failure of weak coupling expansion. This decease may be cured by the method analogous to the one used in ref[1]. The Yang-Mills and Higgs actions should be modified by introducing higher order covariant derivatives of sufficiently high order. By choosing appropriately the parameter \( \Lambda \) multiplying the higher derivative term in the modified action one can make the contribution of these diagrams vanishing in the continuum limit.

This mechanism may be illustrated by the analysis of the fermion selfenergy diagram generated by the Yukawa-Wilson coupling. The corresponding integral looks as follows

\[ \Sigma \sim N^2 \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \left( 1 - \gamma_5 \right) \left( 1 + \Gamma_{11} \right) V^i S(p + Q) D(p) V^k d^4 p \quad (24) \]
Here $D(p)$ is a modified Higgs field propagator corresponding to the sixth order derivatives in the regularised action

$$D = \left[ a^{-2} \sum_{\mu} \left( \cos(p_{\mu}a) - 1 \right) + \Lambda^2 \left( \sum_{\mu} \cos(p_{\mu}a) - 1 \right)^3 \right]$$

(25)

$S(p)$ is the fermion propagator (8) and $V^i$ is the interaction vertex (6).

To analyse this integral we separate as before the integration domain into $V_{\text{in}}$ and $V_{\text{out}}$ according to the eq.(14). In the domain $V_{\text{in}}$ one can use the expansion over $(pa)$ and the integral looks as follows (omitting irrelevant matrix factors)

$$\Sigma_{\text{in}} \sim N^2 \int_{V_{\text{in}}} \frac{(q+p)^4 a^4(q+p)}{(q+p)^2 + (v\lambda^{-1})^2 a^2(q+p)^4 \left[ p^2 + \Lambda^2 a^6 p^6 \right]}$$

(26)

In the limit $a \to 0$, $\Sigma_{\text{in}} < \lambda N^2^{-5\gamma} \to 0$. In the domain $V_{\text{out}}$ after rescaling the integration variables $ap_{\mu} = u_{\mu}$ one has

$$\Sigma_{\text{out}} \sim a^{-2} N^2 \int_{V_{\text{out}}} \frac{s(u + Qa) V(u + Qa) V(u + Qa)}{s^2(u + Qa) + m^2(u + Qa) \left[ (\cos(u) - 1) + \Lambda^2 a^2(\cos(u) - 1)^3 \right]}$$

(27)

For small $a$, $\Sigma_{\text{out}} < \lambda^3 N^{-5} \Lambda^{-2}$. Choosing $\Lambda^2 = \lambda^2 N^5$, $5 \ll \beta \ll 6$ we can make this term vanishing in the continuum limit. Exactly the same arguments are applied to the selfenergy diagram with the fermion propagator (8) and all other diagrams with internal Higgs or Yang-Mills lines. We note that the condition $\Lambda^2 \ll \lambda^2 N^5$ means that the masses $M_{\Lambda}$ of additional excitations which appear due to the presence of higher derivative regulators are less than $\lambda N^{1.5}$. In particular one can take $M_{\Lambda} \sim M_r$ where $M_r$ are the PV masses. Both $M_{\Lambda}$ and $M_r$ correspond to the poles in the domain $V_{\text{in}}$.

3 Discussion

The construction discussed above is obviously applicable also to the standard model as the gauge group of the standard model is a subgroup of $SO(10)$, and to get its lattice version it is sufficient to put in our action all vector fields except for the gluons and electroweak bosons equal to zero.

In the scaling limit our model coincides essentially with the model of ref [1] if one introduces to the latter a gauge invariant interaction of Higgs fields. However it has an advantage of being manifestly gauge invariant for a finite lattice spacing, which makes it more suitable for nonperturbative calculations. In particular there is no problem of nonperturbative gauge fixing.

The gauge invariant action we propose is given by the eq. (2) written in terms of shifted Higgs fields $\varphi \to \varphi + v$ and modified by introducing higher derivative kinetic term for for the Higgs and Yang-Mills fields We proved that the weak coupling expansion near the ground state corresponding to the quadratic piece of this action reproduces in the continuum limit a manifestly gauge invariant perturbation theory
for the $SO(10)$ model. As was discussed above the perturbation theory for the standard model is obtained by reducing the $SO(10)$ group to the subgroup $SU(3) \otimes SU(2) \otimes U(1)$.

In this paper we made no attempts to study a real phase structure of the lattice model which requires essentially nonperturbative treatment. It remains to be seen if our conclusions are valid beyond the perturbation theory. We note that there is an important difference between our construction and the original Smit-Swift model. In the original model the physical reason for the absence of interacting chiral fermions was the formation of scalar-fermion bound states due to the strong Yukawa-Wilson coupling. In our case, as has been shown in the previous section, the effective Yukawa-Wilson interaction is weak (vanishing in the scaling limit). The effective interaction between Higgs mesons induced by the fermion loops is suppressed by the PV fields, and the interaction induced by the exchange of Higgs fields is suppressed by the higher derivative regulators. In fact the only relevant domain in the momentum space in our model is the domain near $p = 0$ ($V_n$ as defined by the eq. (14)). In this domain the doublers are absent and the Yukawa-Wilson interaction is negligible.

We repeat once again that the most important problem is to develop for the analysis of this model a reliable nonperturbative approach which allows to study the phase structure of the theory and to check if the conclusions about the doublers decoupling remain valid and if the light fermions interact nontrivially in the continuum limit. Our analysis has been done in the framework of the weak coupling expansion. We showed that although the Yukawa–Wilson coupling in our model is large, perturbation theory can be applied. In comparison with the model of ref. [1] the present construction has an advantage of being manifestly gauge invariant for a finite lattice spacing which makes it more suitable for nonperturbative calculations. In particular there is no problem of nonperturbative gauge fixing. However the most important problem is to develop for the analysis of this model a reliable nonperturbative approach and to check if the conclusion about the doublers decoupling remains valid outside of perturbation theory and if the light fermions interact nontrivially in the continuum limit.

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