Challenging Fundamental Limitations in Electromagnetics with Time-Varying Systems

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Abstract: Time-varying components provide an exceptional opportunity to explore novel means for building efficient electromagnetic devices. While such explorations date back to more than half a century ago, recent years have experienced a renewed and increased interest into the design of dynamic electromagnetic systems. This resurgence has been partly fueled by the desire to surpass the performance of conventional devices, and to enable systems that can challenge various well-established physical bounds, such as the Bode-Fano limit, the Chu limit, etc. Here, we overview this emerging research area and provide a concise and systematic summary of the most relevant applications for which the relevant physical bounds can be overcome through time-varying elements. Besides leading to devices with superior performances, such research endeavors open new possibilities and might offer insight towards future all-electromagnetic/optical technologies.

Index Terms: Time-varying systems, physical limits, temporal modulation, impedance matching, absorption, antennas, delay lines, electromagnetic camouflage.

1. Introduction

The study of electromagnetic wave interactions with time-varying media dates back to more than half a century ago, when several pioneering works [1], [2], [3], [4], [5], [6], [7], [8], [9], [10] revealed the rich physics arising due to such interactions. Although in the subsequent decades several other works have further explored the implications of wave propagation in dynamic media [11], [12], [13], [14], it was not until recently when the field gained considerable momentum [15] partly owing to the improved fabrication and characterization techniques at various frequency regimes [16], [17], [18]. Consequently, it was soon discovered that time-dependent materials can enable many interesting and novel phenomena, such as interband photonic transitions [19], [20], nonreciprocal systems [21], [22], efficient wavelength conversion [23], [24], [25], [26], temporal aiming [27], Fresnel drag [28], negative extinction [29], entangled photon generation [30], [31], parametric oscillations [32], synthetic dimensions [33], topological phase transition [34], nonreciprocal gain [35], among others [36]. Several recent publications also further investigated the fundamental aspects of wave propagation in time-varying media, such as Refs. [37], [38], [39], [40], [41], [42], thus providing a better understanding of the physical implications of time-varying media.

Apart from enabling novel effects, time-varying materials and components may be also used to improve a given device to provide better functionality in terms of operational bandwidth, size, efficiency of the relevant process, etc. [36]. In some cases, such improvements may even exceed what is normally allowed by well-established physical bounds, as conventional physical bounds are typically derived under the assumption of linearity and time-invariance (LTI systems). Indeed, as mentioned above, time-variance in the system properties may enable various functionalities not present in a conventional setting, such as parametric gain [35] or spectral changes [19], [23], [24], [25], [26], [43], which are generally not accounted for when deriving conventional bounds for LTI systems, and can therefore be exploited to bypass such physical limits. Consequently, time-varying materials and components provide an exceptional opportunity to realize superior devices not restricted by various conventional limits.
2. Applications

In this section we provide a concise overview of the most relevant applications of time-varying electromagnetic systems in the context of overcoming various physical bounds of conventional LTI systems. In this regard, this section is divided into various subsections, each of which focuses on a specific application or research topic, with the goal to systematically guide the reader through this emerging research area. We will not discuss some major research directions that also seek to overcome various issues present in conventional systems and broaden their functionality, such as nonreciprocity [21], [22] or synthetic dimensions [33] based on time modulation, since these topics are already covered in detail in the literature. Instead, we will focus on topics of particular interest to the IEEE Antennas and Propagation community, including how time-varying systems impact the fundamental limitations of antennas [44], broadband impedance matching networks [45], and radar absorbers [46].

2.1. Impedance Matching

Efficient power transfer from a source to a load with minimal reflections is arguably one of the most important tasks in electromagnetic engineering. Thus, when designing impedance matching networks to minimize reflections, it becomes essential to identify the limitations and the optimal configurations of the system. In this regard, a matching-bandwidth limit was introduced for linear, time-invariant and passive systems by Bode in 1945 [47], which was later generalized by Fano for arbitrary load impedances [45]. The resulting Bode-Fano limit provides a theoretical upper bound on the bandwidth over which a certain reflection reduction can be attained, independent of the complexity of the employed matching network (only assumed to be reactive), and it is therefore a very powerful tool to assess the maximum bandwidth of various electromagnetic systems, such as antennas [48] or optical invisibility cloaks [49]. To overcome this limitation one strategy that has been explored in the literature is to break the underlying assumption of passivity. Indeed, several proposals utilizing active “non-Foster” elements have shown that the bandwidth of small antennas [50] and cloaking devices [51], [52], [53], [54] can be extended beyond what is possible with a passive system. However, active elements typically increase the noise level of the system [55] and can lead to potential instabilities (unbounded oscillations in time) [54], [56], [57], thereby disrupting the desired functionality.

An alternative option to go beyond the Bode-Fano limit is to break the assumption of time-invariance, i.e., allow temporal variation in the system properties. In this context, Ref. [58] employed time-varying “switched” matching networks to achieve a better bandwidth performance compared to time-invariant networks. More recently, Ref. [59] provided another insightful method, which involves switching the system parameters while a broadband signal (a pulse) is within the matching network and before it reaches the load, as shown in Figs. 1(a) and 1(b). Typically, the temporal switching would lead to reflected waves due to the temporal discontinuity [60], however it was shown that the transmission efficiency can still be maximized by optimizing several parameters, including the impedance of the matching transmission line and the phase velocity of the pulse within the matching line before and after the switching event [59]. Since temporal switching increases the number of free parameters that can be tuned compared to the time-invariant case, one may achieve optimal performances that are otherwise out of reach in a conventional time-invariant setting, especially for large load-source impedance contrast values (see Fig. 1(c)) [59]. This proposal was further extended from hard (abrupt) to “soft” temporal switching in Ref. [61], which could be more feasible to implement in practice. An adverse byproduct of the temporal switching operation is the spectral compression or broadening of the pulse, which necessitates the use of additional analog or digital devices to restore the original pulse spectrum and, thus, complicates the overall matching system. Moreover, this method is feasible only for short-time signals (as quasi-monochromatic signals would require impractically long matching lines), and requires knowledge that the pulse is present within the matching line (which implies the presence of some fast detection mechanism able to trigger the switching event). Even with these limitations, this approach show a realistic path toward ultra-broadband matching networks.
Fig. 1. (a) A temporally switched transmission line can be employed to match a source impedance $R_G$ to a load impedance $R_L$ beyond the Bode-Fano limit. (b) Such a switched matching line can be realized with parallel varactor diodes and banks of series inductors, whose properties are changed abruptly in time at $t = t_s$. (c) The maximum transmission efficiency for various $R_L/R_G$ values shows that the temporally switched case (solid blue) provides a better performance compared to the non-switched time-invariant case (dashed red line) especially for large source-load contrast values. (d),(e) A transmission line terminated with a time-varying inductance $L(t)$. (d) With a suitable time modulation, the inductor can accumulate energy without any reflections. (e) By time-reversing the temporal modulation in (d) the energy trapped by the inductor can be released. (f) Time modulation profile of the inductance comprising both the trapping (before $t = 2.5$ s) and releasing regimes (after $t = 2.5$ s). (g) Temporal variation of the incident signal and the reflected signal (obtained theoretically and through simulations) at the load position for a load inductance temporally modulated as in panel(f). Panels (a-c) are reproduced with permission from Ref. [59], American Physical Society. Panels (d-g) are reproduced with permission from Ref. [62], American Physical Society.

beyond the Bode-Fano constraints.

For the quasi-monochromatic regime, on the other hand, a transmission line where the load reactance itself is varied in time has been used to impedance match a time-harmonic source a purely reactive load and to accumulate and release energy on demand (see Fig. 1(d-g)) [62]. In this context, it was noted that a time-modulated reactance can emulate a resistance and, can, therefore act as a “virtual” absorber [62]. An intuitive picture explaining this result was also provided, showing that temporally modulating the reactance with such a tangent function is equivalent to a short-circuited transmission line with the shorted end moving away from the source with the same velocity as the phase velocity of the incident signal [62]. Therefore, the incident wave does not reflect back since it never “reaches” the reflecting termination. Consequently, energy from a time-harmonic source can be trapped with no reflections and released on demand (see Figs. 1(f) and 1(g)).

Lastly, another interesting application of temporal modulation in the context of impedance matching is related to the problem of capturing and trapping an incident electromagnetic wave into a lossless cavity, with no reflections. This can be achieved by temporally modulating the radiation leakage from the cavity, such that it perfectly cancels the direct reflection from the cavity at all time instants [63]. In this regard, we have also shown that by temporally modulating the shell of a compact core-shell scatterer (acting as an open cavity supporting a nonradiating mode), an incident broadband pulse can be captured within the cavity with no reflection, effectively forcing a broadband signal into a narrowband resonator with ideal efficiency (this may also be interpreted as spectrum compression through temporal modulation with the additional constraint of zero reflections). [64] See Supplementary Material for further details.

2.2. Antenna Performance Enhancement

Antenna miniaturization is another major research area within the applied electromagnetism community with many potential benefits for various areas, such as communication systems, sensor networks, implanted devices, and navigation systems. While it is generally possible to reduce the size of an antenna down to subwavelength scales for a given design frequency, this typically comes at the price of reduced bandwidth and radiation efficiency. In this regard, the trade-off
between size and performance for electrically small antennas was investigated more than half a century ago in several seminal papers by Wheeler [65], Chu [66], and later by Harrington [67], resulting in, as it is known today, the Chu-Harrington limit [68]. The limit as derived by Chu is based on an equivalent circuit model for the modal impedance of the lowest-order spherical wave radiated by an antenna enclosed within a spherical volume. Regardless of the antenna structure and LTI material, this equivalent circuit for spherical waves becomes more reactive as the size of the spherical volume is reduced, leading to a higher Q factor and a smaller bandwidth (consistent with the fact that, for example, a short wire dipole has a strongly reactive input impedance). These analyses were later refined by other researchers (e.g. [44], [69], [70]), further underlying the fact that for a linear time-invariant passive antenna one cannot simultaneously optimize size, bandwidth and efficiency.

Following Chu’s work, antenna engineers realized very early the need for active (i.e., non-passive) designs, nonlinear materials, and time-varying components to increase the communication bandwidth of wireless systems based on small antennas. In this context, for example, nonlinear ferrite materials were exploited to realize time-varying inductors, which were used to overcome the limitations of small antennas in VLF transmitters by shifting the carrier frequency [71]. Subsequent works (see, for instance, Refs. [72], [73], [74], [75], [76]) further demonstrated the unique opportunities of employing time-varying “switched” elements (which affect the transient response, and therefore the bandwidth, of radiation) to design antennas with improved characteristics.

More recently, with the renewed surge of interest in temporal modulation in electromagnetic systems, research into time-varying antenna designs have been reinvigorated. For example, Ref. [77] confirmed that the antenna bandwidth can be increased significantly through impedance modulation. Another more recent work employed “Floquet” impedance matching [78] (see Fig. 2) to overcome the Chu-Harrington limit by noting that this limitation is fundamentally rooted within the more general Bode-Fano limit (see Section 2.1) [79]. Indeed, as recognized by Chu, a more accurate definition of the fractional bandwidth of the antenna is in terms of the tolerable reflection coefficient over the band (and not simply the inverse of the Q factor), and therefore the antenna bandwidth is limited by the Bode-Fano limit applied to Chu’s equivalent circuit for spherical waves. As discussed in [78], this limitation can be surpassed by periodically modulating the matching network components at twice the frequency of the antenna resonance frequency to impart parametric gain [32] into the system. A realistic simulation of the proposed concept was also demonstrated based on a small loop antenna (see Figs. 2(b)), which includes periodically modulated variable capacitances (varactors). Through numerical calculations (see Fig. 2(c)), it was shown that the bandwidth of the transducer gain can be significantly enhanced owing to the temporal modulation compared to the time-invariant case, and can even surpass the Chu-Harrington limit, which is a rather remarkable result given the fact that parametric phenomena are typically narrowband. A possible drawback of this Floquet scheme is that the parametric gain may potentially lead to instabilities. However, unlike antenna systems based on active non-Foster elements [80], [81], [82], the dispersion of the parametric gain can be easily controlled through the modulation properties to bring the system into the stable regime. Moreover, it was also pointed out that deliberately inducing an impedance mismatch by modifying the source impedance can act as a negative feedback to help ensure stability and also increase the operational bandwidth [78]. Indeed, a stable parametric enhancement effect was later experimentally demonstrated in Ref. [83] in a fabricated time-varying loop antenna operating at RF frequencies (see Fig. 2(d)). The parametric enhancement due to the time-modulated varactor was verified through reflection $|S_{11}|$ measurements (where $|S_{11}| > 1$ was observed over a relatively broad bandwidth), which showed good agreement with effective circuit model calculations, as shown in Fig. 2(e). The measured data was then used to evaluate the radiated power from the antenna, which was shown to surpass the limitations of linear time-invariant passive antennas [83]. In this context, other recent studies also investigated the use of time-varying parametric amplifiers with the aim of broadening the antenna bandwidth [84], [85]. This was achieved by first trading efficiency with bandwidth (in line with the Bode-Fano limit), and subsequently using parametric up-converters based on time-varying reactive elements to increase the antenna gain. It was also noted that such parametric amplifiers...
have various advantages, including inherent stability and low-noise operation [84], [85].

### 2.3. Electromagnetic Absorption

Wave absorption plays a crucial role in various electromagnetic systems, ranging from solar energy harvesting to scattering minimization for radar or anechoic chamber applications. Such systems are designed to have highly absorbing properties within the bandwidth of interest. A key limitation of conventional electromagnetic absorbers originates from the trade-off between the absorption bandwidth and the thickness of the absorbing material. Within this context, using the analytical properties of the reflection coefficient, Rozanov [46] showed that there exists a ultimate thickness to bandwidth ratio $\Delta \lambda / d$ for passive time-invariant absorbers backed by a perfect conductor. Specifically, for the case of a nonmagnetic absorber, the upper limit for this ratio becomes $\Delta \lambda / d < 16/|\ln \rho_0|$, where $\rho_0$ is the lowest reflectance within the operational bandwidth. Clearly, the absorption bandwidth can therefore be enlarged by increasing the thickness of the absorber or the tolerable reflection coefficient. Prominent examples of broadband absorbers include cascaded [87], [88] or adiabatically tapered [89], [90], [91] structures which typically have thicknesses on the order of several wavelengths. However, for many applications, such as radar absorbing or solar energy harvesting, a thin absorber with a large bandwidth is highly desirable due to practical and economic reasons, which motivates the large interest in overcoming the “Rozanov bound”. In this direction, it was shown in Ref. [86] that temporally switching the relative permittivity of the absorbing material can be used to extend the bandwidth of thin absorbers beyond what is possible with time-invariant structures (see Fig. 3[a]), similar to the use of temporally switched transmission lines for broadband impedance matching as discussed in Section 2.1. An important drawback of these approaches is that, while reflections are reduced within the bandwidth of the incident wave, significant reflections may be induced outside this frequency range as the temporal switching can lead to frequency conversion. While this may
not pose a problem for certain narrowband systems, it would become an issue in the context of, for example, broadband radar and stealth technology, as the target may become even more easily detectable due to the increased reflections outside the original signal bandwidth, which could then be picked up by sufficiently broadband or multi-band detectors. To overcome this problem, a possible strategy is to modulate both the real and imaginary part of the permittivity in such a way that the modulation spectrum becomes “unilateral” (or “spectrally causal”), as we theoretically demonstrated in a recent publication [43]. Such an approach ensures that back-reflections due to the temporal modulations are prohibited over a very wide frequency range and, therefore, the incident wave is perfectly absorbed with no reflections over wide bandwidths (see Fig. 3(b),(c)). Along similar lines, a recent work theoretically demonstrated, through Green’s function analysis and numerical optimizations, that a suitable switching of both the conductivity and permittivity leads to a reduction of reflection over the whole frequency spectrum [92]. As an alternative strategy, a recent experimental work [93] also demonstrated broadband absorption by creating an energy-trap through switched electronic components triggered by the pulse entering the absorbing region. While the strategies described above work for broadband pulses and are based on non-periodic temporal modulations (e.g., “switching”), perfect absorption for quasi-monochromatic signals has also been realized by employing periodic temporal modulations [94]. Specifically, it was shown that perfect absorption can be achieved even with infinitesimal losses owing to coherent multichannel illumination with Floquet time-periodic driving schemes. A single-channel coherent perfect absorber based on a time-Floquet engineered reflector without any physical mirror (ground plane) was also demonstrated, which might be useful in certain application scenarios where metallic or Bragg mirrors are not available.

2.4. Other Applications

Our list of applications which can benefit from temporal modulation is certainly not exhaustive. In the Supplementary Material, we provide several other application examples that might be of interest to readers also from different backgrounds, such as optics and photonics.

3. Outlook

Although the study of time-varying electromagnetic systems dates back more than half a century, recent years have seen a surge of renewed interest into such dynamic schemes, largely moti-
vated by the need to realize novel electromagnetic systems that are superior to conventional LTI ones for various applications, from high-speed full-duplex communication systems, to broadband stealth technology, to high-efficiency energy harvesting and energy transfer. However, despite the promising recent progress in this field, we believe several challenges should be taken into consideration and carefully addressed before such systems can be adopted into practical use.

Since a time-varying system requires external power to operate, such active systems might become susceptible to instabilities (unbounded oscillations) that can disrupt the desired functionality. Therefore, especially for systems involving parametric modulations, a careful analysis is essential to assess and ensure stability [78]. Moreover, the additional energy pumped into the system [95] should be carefully assessed to determine the figure of merits (such as power efficiency) of the design in a proper manner [96] and also to assess whether the power required by the temporal modulation is beyond practical reach and/or may damage the system.

Frequency dispersion is another fundamental property of materials arising due to causality. Hence, any realistic system involves, to some degree, frequency-variations in the material constitutive parameters. However, especially for numerical studies involving time-varying structures, such dispersion effects are often ignored and the time-variation is assumed to be independent of frequency for simplicity [20]. Therefore, any realistic implementation of such proposals will necessarily experience performance deterioration due to frequency dispersion of the material properties. We note that several recent works [97], [98], [99] have studied material dispersion effects in the context of temporal modulations. We believe further investigations in this area are important to enhance our understanding of time-varying materials and may lead to improved designs with more realistic properties.

For schemes requiring synchronization between the incident wave and the temporal variation, another challenge is that fast detectors and switching mechanisms are essential to trigger the modulation in the presence of the incident wave. Such additional components might complicate the design and reduce its practical applicability. Hence, more research is required to integrate such components into time-varying systems in a practically useful manner.

Dissipation, unless desired to realize absorbers, is another major challenge that needs to be overcome to realize certain practical applications with time-varying components. For instance, epsilon-near-zero materials have recently become the subject of interest in the context of time modulation at optical frequencies, as they can be efficiently controlled through external optical pumps [18]. However, such materials still suffer from high losses especially around their epsilon-near-zero wavelengths. Hence, the search for better epsilon-near-zero materials with lower losses is an important area of research (note that, theoretically, nothing prevents the existence of an epsilon-near-zero material that is dissipationless, albeit frequency dispersive, within a certain frequency window). An alternative strategy could be to exploit (rather than to eliminate or compensate) the losses in such materials to realize time-dependent loss profiles that accompany the time-varying refractive index profiles. In this context, we recently provided a general strategy to exploit the synergy between these two time-modulation profiles [43]. Investigations along these directions may also enhance our understanding of wave dynamics in complex (“non-Hermitian”) time-varying media.

Despite these challenges, we believe that the use of time-varying materials to overcome the limitations of conventional LTI electromagnetic systems is an exciting and promising research area that might lead to novel and superior devices in the future as more research is conducted. Spatially-varying engineered structures, such as frequency selective surfaces, diffraction gratings, photonic crystals, metamaterials, etc., have revolutionized the field of electromagnetics in recent decades. Along the same vein, it would be safe to say that the new insight and the new tools provided by the field of time-varying electromagnetic systems will enable further advances and possible revolutions in the forthcoming decades for electromagnetic research.
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