CONTRIBUTORS TO THE EVOLUTION OF THE PRIMORDIAL MAGNETIC FIELD FROM THE SMALL-SCALE COSMIC MICROWAVE BACKGROUND ANGULAR ANISOTROPY

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ABSTRACT

Recent observations of the cosmic microwave background (CMB) have extended the measured power spectrum to higher multipoles \( l \geq 1000 \), and there appears to be possible evidence for excess power on small angular scales. The primordial magnetic field (PMF) can strongly affect the CMB power spectrum and the formation of large-scale structure. In this paper, we calculate the CMB temperature anisotropies generated by including a power-law magnetic field at the photon last-scattering surface (PLSS). We then deduce an upper limit on the PMF based on our theoretical analysis of the power excess on small angular scales. We have taken into account several important effects such as the modified matter sound speed in the presence of a magnetic field. An upper limit to the field strength of \( |B_{\perp}| \leq 4.7 \, \text{nG} \) at the present scale of 1 Mpc is deduced. This is obtained by comparing the calculated theoretical result including the Sunyaev-Zeldovich (SZ) effect with recent observed data on the small-scale CMB anisotropies from the Wilkinson Microwave Anisotropy Probe (WMAP), the Cosmic Background Imager (CBI), and the Arcminute Cosmology Bolometer Array Receiver (ACBAR). We discuss several possible mechanisms for the generation and evolution of the PMF.

Subject headings: cosmic microwave background — magnetic fields — methods: numerical

1. INTRODUCTION

The possible existence of a PMF is an important question in modern cosmology. The PMF could influence a variety of phenomena in the early universe (Tsagas & Martens 2000; Grasso & Rubinstein 2001; Bertschart et al. 2004; Bruni et al. 2003; Clarkson et al. 2003; Giovannini 2004; Vallée 2004; Challinor 2005; Kaino & Ratra 2005; Dolgov 2005; Kosowsky et al. 2005; Gopal & Sethi 2005; Tashiro et al. 2006; Tashiro & Sugiyama 2006). However, there is still no firm understanding of the origin and evolution of the PMF, especially the observable magnetic field of 0.1–1.0 \( \mu \text{G} \) in galaxy clusters (Clarke et al. 2001; Xu et al. 2006). Recently, the origin of the PMF on such scales has been studied by a number of authors (Quashnock et al. 1989; Boyanovsky et al. 2003; Bamba & Yokoyama 2004; Berezhiani & Dolgov 2004; Hanayama et al. 2005; Takahashi et al. 2005; Ichiki et al. 2006). There is, however, a possible discrepancy between theory and observation on the scale of galactic clusters. Temperature and polarization anisotropies in the CMB provide very precise information on the physical processes in operation during the early universe. However, the CMB data from WMAP (Bennett et al. 2003), ACBAR (Kuo et al. 2004), and CBI (Mason et al. 2003) have indicated a potential discrepancy between the best-fit cosmological model to the WMAP data and observations at higher multipoles \( l \geq 1000 \). A straightforward extension of the fit to the WMAP data predicts a rapidly declining power spectrum in the large multipole range due to the finite thickness of the PLSS. However, the ACBAR and CBI experiments indicate continued power up to \( l \sim 4000 \). This discrepancy is difficult to account for by a simple retuning of cosmological parameters.

One possible interpretation of such excess power at high multipoles is a manifestation of the rescattering of CMB photons by hot electrons in clusters known as the thermal SZ effect (Sunyaev & Zeldovich 1980). Although there can be no doubt that some contribution from the SZ effect exists in the observed angular power spectrum, it has not yet been established conclusively that this is the only possible interpretation (Aghanim et al. 2001) of the small-scale power. Indeed, the best value of the matter fluctuation amplitude, \( \sigma_8 \), to fit the excess power at high multipoles is near the upper end of the range of the values deduced by other independent methods (Bond et al. 2005; Komatsu & Seljak 2002). Toward a solution of this problem, it has recently been proposed (Mathews et al. 2004) that a bump feature in the primordial spectrum may provide a better explanation for both the CMB and matter power spectra at small scales.

In this paper we consider another possibility. An inhomogeneous cosmological magnetic field generated before the CMB last-scattering epoch is also a plausible mechanism for producing excess power at high multipoles. Such a field excites an Alfvén wave mode in the baryon-photon plasma in the early universe and induces small rotational velocity perturbations. Since this mode can survive on scales below those at which Silk damping occurs during recombination (Jedamzik et al. 1998; Subramanian & Barrow 1998), it could be a new source of the CMB anisotropies on small angular scales. Analytic expressions for the temperature and polarization angular power spectra on rather larger angular scales (\( l \leq 500 \)) by Mack et al. (2002) based on the thin PLSS approximation have been derived for both vector and tensor modes. Subramanian & Barrow (2002) considered the vector perturbations in the opposite limit of smaller angular scales. Non-Gaussianity in fluctuations from the PMF has also been considered (Brown & Crittenden 2005). A strong magnetic field in the early universe, however, may conflict with the cosmological observations currently available. The combination of those studies and current observations places a bound on the strength of the PMF of \( B < 1.0–10 \, \text{nG} \).

In order to compare the CMB anisotropy induced by the PMF with observations more precisely, we need to perform numerical calculations of the fully linearized magnetohydrodynamic (MHD) equations along with a realistic recombination history of
the universe. In particular, we first need to develop a numerical method to predict the theoretical spectrum for intermediate angular scales, in which the analytic approximation becomes inappropriate. Recently, efforts along this line have been done (Lewis 2004) to obtain numerical estimates of the effects of the PMF on the CMB. In addition, Yamazaki et al. (2005b) have proven numerically that the PMF is an important physical process for higher multipoles $l$. They have constrained an upper limit to the PMF of $B < 3.9$ nG, based on a likelihood analysis of the data (Yamazaki et al. 2005a).

The WMAP data are very precise but do not extend to $l > 900$. The PMF, however, affects the CMB power spectrum for higher multipoles $l$ (Yamazaki et al. 2005a; Lewis 2004). Thus, one purpose of this paper is to place a new limit on the strength of the PMF by comparing our calculated result with the WMAP, CBI, and ACBAR data, the latter two of which have measured CMB anisotropies for higher multipoles $l$ than WMAP. We also discuss the consistency between our CMB constraints and the magnetic field in galactic clusters.

2. PRIMORDIAL STOCHASTIC MAGNETIC FIELD

Before recombination, Thomson scattering between photons and electrons along with Coulomb interactions between electrons and baryons were sufficiently rapid that the photon-baryon system behaves as a single tightly coupled fluid. Since the trajectory of plasma particles is bent by Lorentz forces in a magnetic field, photons are indirectly influenced by the magnetic field through Thomson scattering. Let us consider the PMF created at some moment during the radiation-dominated epoch. The energy density of the magnetic field can be treated as a first-order perturbation on a flat Friedmann-Robertson-Walker (FRW) background metric. In the linear approximation, the magnetic field evolves as a stiff source. Therefore, we can discard all back-reactions from the MHD fluid onto the field itself.

The electromagnetic tensor has the usual form,

$$F_{\alpha \beta} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix},$$  \tag{1}

where $E_i$ and $B_i$ are the electric and magnetic fields. Here we use natural units $c = \hbar = 1$. The energy momentum tensor for electromagnetism is

$$T^\alpha_{\beta \gamma} = \frac{1}{4\pi} \left( F^{\alpha \gamma} F^\beta_{\phantom{\beta} \gamma} - \frac{1}{4} g^{\alpha \beta} F_{\gamma \delta} F^{\gamma \delta} \right).$$  \tag{2}

The Maxwell stress tensor, $\sigma^{ik}$, is derived from the space-space components of the electromagnetic energy momentum tensor,

$$-T^i_{\phantom{i}j \phantom{j}k} = \sigma^{ik} = \frac{1}{a^2 4\pi} \left( E_i E^k + B^i B^k - \frac{1}{2} \delta^{ik} (E^2 + B^2) \right).$$  \tag{3}

Within the linear approximation (Durrer et al. 2000) we can discard the MHD back-reaction onto the field itself. The conductivity of the primordial plasma is very large, and it is “frozen-in” (Mack et al. 2002). This is a very good approximation during the epochs of interest here. Furthermore, we can neglect the electric field, i.e., $E \sim 0$, and can decouple the time evolution of the magnetic field from its spatial dependence, i.e., $B(\tau, x) = B(x)/a^2$ for very large scales. In this way we obtain the following equations,

$$T_{\phantom{i}j \phantom{j}k}^{00}_{\phantom{00} \text{EM}} = \frac{B^2}{8\pi a^2},$$  \tag{4}

$$T_{\phantom{i}j \phantom{j}k}^{0\text{-EM}} = T^{i\text{-EM}}_{\phantom{i}j \phantom{j}k} = 0,$$  \tag{5}

$$-T^i_{\phantom{i}j \phantom{j}k}^{\text{EM}} = \sigma^{ik} = \frac{1}{8\pi a^2} \left( 2B^i B^k - \delta^{ik} B^2 \right).$$  \tag{6}

2.1. Two-Point Correlation Function

We assume that the PMF, $B_\delta$, deviates from a homogeneous and isotropic distribution in a statistically random way. The power spectrum of fluctuations from a homogeneous and isotropic distribution can then be taken as a power law $P(k) \propto k^{n_B}$ (Mack et al. 2002), where $k$ is the wavenumber and the spectral index $n_B$ can be either positive or negative. Evaluating the two-point correlation function of the electromagnetic stress-energy tensor, we can obtain a good approximation to the vector isotropic spectrum for $k < k_C$ (Mack et al. 2002),

$$|\Pi^{(1)}(k)|^2 \approx \frac{1}{4(2n_B + 3)} \left( \frac{(2\pi)^{n_B + 3} B_x^2}{2\Gamma[(n_B + 3)/2] k_B^{n_B + 3}} \right)^2 \times \left( \frac{k_C^{2n_B + 3} + n_B}{n_B + 3} k^{2n_B + 3} \right)^2,$$  \tag{7}

where the vector-mode Lorentz force, $L(k)$, is given by $L(k) = k\Pi^{(1)}(k)$; $B_x$ is the comoving mean magnetic field amplitude obtained by smoothing over a Gaussian sphere of comoving radius $\ell$; and $k_B = 2\pi/\ell$ (where $\ell = 1$ Mpc in this paper). In this equation $k_C$ is the cutoff wavenumber in the power spectrum defined by

$$k_C \approx \left( 5.08 \times 10^{-2} \frac{B_x}{1 \text{ nG}} \right)^{2/n_B + 5} \times \left( \frac{k_B}{1 \text{ Mpc}^{-1}} \right)^{(n_B + 3)/(n_B + 5)} \left( \Omega_B h^2 \right)^{1/n_B + 5},$$  \tag{8}

where $h$ is the Hubble parameter in units of $100$ km s$^{-1}$ Mpc$^{-1}$ (Mack et al. 2002; Jedamzik et al. 1998; Subramanian & Barrow 1998). Since the magnetic field source term $\Pi^{(1)}(k)$ depends quadratically on the magnetic field strength, the explicit time dependence of the magnetic stress is given by $\Pi(\tau, k) = \Pi(k)/a^4$. The scalar mode of the magnetic field stress tensor $\Pi^{(0)}(k)$ is obtained via a similar calculation (Koh & Lee 2000),

$$|\Pi^{(0)}(k)|^2 \approx \frac{1}{2(2n_B + 3)} \left( \frac{(2\pi)^{n_B + 3} B_x^2}{2\Gamma[(n_B + 3)/2] k_B^{n_B + 3}} \right)^2 \times \left( \frac{k_C^{2n_B + 3} + n_B}{n_B + 3} k^{2n_B + 3} \right)^2.$$  \tag{9}

4 This has a form similar to that used in Mack et al. (2002) who treated two kinds of Fourier transforms of different normalizations. Here, we systematically adopt the same normalization for the Fourier transform. This removes the some of the uncertainty in the numerical calculations (Lewis 2004).
3. PERTURBATION EVOLUTION EQUATIONS

To obtain the scalar and vector perturbation evolution equations, we write the perturbed Einstein equation as

\[(\dddot{G}_{\mu\nu} + \delta G_{\mu\nu}) = 8\pi G (\bar{T}_{\mu\nu[FL]} + \delta T_{\mu\nu[FL]}),\]  \hspace{1cm} (10)

where \(T_{\mu\nu[FL]}\) is the energy-momentum tensor of a perfect fluid. The Einstein equation can then be separated into background and perturbation equations. The background Einstein tensor is given by

\[\ddot{G}_{\mu\nu} = \ddot{R}^\mu_\nu - \frac{1}{2} \delta^\mu_\nu \ddot{R},\]  \hspace{1cm} (11)

while the perturbation tensor is

\[\delta G_{\mu\nu} = \delta R^\mu_\nu - \frac{1}{2} \delta^\mu_\nu \delta R.\]  \hspace{1cm} (12)

The perturbed Einstein equation is then

\[\delta R^\mu_\nu - \frac{1}{2} \delta^\mu_\nu \delta R = 8\pi G \delta T^\mu_\nu[FL].\]  \hspace{1cm} (13)

The perfect fluid energy-momentum tensor has the form

\[T^\mu_\nu[FL] = p[FL] \delta^\mu_\nu + (\rho[FL] + p[FL]) U^\mu U_\nu,\]  \hspace{1cm} (14)

where the subscript [FL] denotes the fluid, while \(U^\mu = dx^\mu/(dt)^2\) is the four-velocity of the fluid relative to an observer in the frame in which the Einstein equation (13) is solved. The pressure \(p\) and energy density \(\rho\) of a perfect fluid at a given point are defined to be those measured by a comoving observer instantaneously at rest with respect to the fluid. For a fluid moving with a small coordinate velocity \(v^i = dx^i/dt\), \(v^i\) can be treated as a perturbation of the same order as \(\delta p[FL] = \rho[FL] - \bar{\rho}[FL]\), \(\delta p[FL] = p[FL] - \bar{p}[FL]\); where \(\bar{\rho}[FL]\) and \(\bar{p}[FL]\) are the background unperturbed state variables and \(t\) is the proper time. Then, to linear order in the perturbations, the energy-momentum tensor is given by

\[T^0_0[FL] = - (\bar{\rho}[FL] + \delta \rho[FL]),\]  \hspace{1cm} (15)

\[T^0_j[FL] = (\bar{\rho}[FL] + \delta \rho[FL]) v^j + T^0_j[0][FL],\]  \hspace{1cm} (16)

\[T^j_j[FL] = (\bar{p}[FL] + \delta p[FL]) \delta^j_j + \Sigma^i_j,\]  \hspace{1cm} (17)

Since we wish to consider the effect of the PMF, we combine the electromagnetic and perfect-fluid energy-momentum tensors,

\[T_{\alpha\beta} = T_{\alpha\beta}[FL] + T_{\alpha\beta}[EM].\]  \hspace{1cm} (18)

3.1. Boltzmann Equations

To evolve perturbations with the PMF, we start with the weak-field Boltzmann equation:

\[\frac{d}{d\tau} T(\tau, x, \dot{n}) = \frac{\partial}{\partial \tau} T + \dot{n} \nabla T = C[T] + G(h_{\omega}),\]  \hspace{1cm} (19)

which describes the evolution of the vector \(T\) under the Thomson collisional term \(C[T]\) and gravitational redshift in a perturbed metric \(G(h_{\omega})\) (Hu & White 1997).

As the radiation free streams, gradients in the distribution produce anisotropies. For example, as photons from different temperature regions intersect on their trajectories, the temperature difference is reflected in the angular distribution. This effect is represented in the Boltzmann equation (19) as a gradient term,

\[\hat{n} \cdot \nabla \rightarrow \hat{n} \cdot \frac{4\pi k}{3} \hat{y}^0,\]  \hspace{1cm} (20)

which includes the intrinsic angular dependence of the temperature and polarization distributions, \(T^0_0\) and \(\pm \hat{s} T^0_1\), respectively. We use the following two Clebsch-Gordan relations:

\[\left( s_1 T^m_1(s_2 T^m_2) \right) = \frac{s_{k_{l-1}}^m}{\sqrt{(2l+1)(2l-1) / 2}} \times \sum_{l,m,s} [(l_1, l_2; m_1, m_2 | l_1, l_2; l, m) \times (l_1, l_2; -s_1, s_2 | l_1, l_2; l, -s) \frac{4\pi}{2l+1} (s Y^m_l)],\]  \hspace{1cm} (21)

and

\[\frac{4\pi}{3} Y^0_l(s Y^m_l) = \frac{s_{k_{l+1}}^m}{\sqrt{(2l+1)(2l+3) / 2}} \times \sum_{l,m,s} [(l_1, l_2; m_1, m_2 | l_1, l_2; l, m) \times (l_1, l_2; -s_1, s_2 | l_1, l_2; l, -s) \frac{4\pi}{2l+3} (s Y^m_l)].\]  \hspace{1cm} (22)

Equation (22) couples multipoles \(l\) to the \(l \pm 1\) moments of the distribution. Here, the coupling coefficient is

\[s_{k_{l+1}}^m = \frac{1}{l} \frac{4l^2 - l^2 - s^2}{l^2}.\]  \hspace{1cm} (23)

The explicit form of the Boltzmann equations for the temperature and polarization follows directly from the Clebsch-Gordan relations. For the moments of the temperature fluctuations \(\Theta_i(m)\), we have (eq. [22]) with \(s = 0\)

\[\dot{\Theta}_i(m) = k \left[ \frac{\Theta_i(m+1)}{(2l+3)} - \frac{\Theta_i(m-1)}{(2l+1)} - \dot{T}_c \Theta_i + S_i^m (l \geq m),\]  \hspace{1cm} (24)

where the source term \(S_i^m\) accounts for the gravitational and residual scattering effects:

\[S_i^m = \left( \begin{array}{c} \dot{c} \Theta_i^0 - \phi \dot{c} v_{i h} + k \psi \dot{c} P^0 \\ 0 \quad \dot{c} s_{i h}^{(1)} + \dot{V} \end{array} \right).\]  \hspace{1cm} (25)

The presence of \(\Theta_i^0\) in equation (25) represents the fact that the isotropic temperature fluctuations are not destroyed by scattering. The Doppler effect enters the dipole \((l = 1)\) equation through the baryon velocity \(v_{i h}^0\) term.

In equations (24) and (25) we have introduced the optical depth \(\tau_c\) between epoch \(\tau\) and the present epoch \(\tau_0\):

\[\tau_c(\tau) \equiv \int_{\tau_0}^{\tau} \dot{c} (\tau') d\tau',\]  \hspace{1cm} (26)
where the combination $\tilde{\tau} e^{-\tau}$ is the visibility function. It expresses the probability that a photon last scattered between $\tau$ and $\tau + d\tau$. Hence, this quantity is sharply peaked at the last scattering epoch.

The term in brackets in equation (24) accounts for a free streaming effect that couples the $l$-modes to $l \pm 1$. This term implies that, in the absence of scattering, power is transferred down the hierarchy when $k \tau \geq 1$. This transfer merely represents a geometrical projection of fluctuations on the scale corresponding to $k$ at a distance $\tau$ that subtends an angle given by $l \sim k \tau$.

The main effect of scattering comes through the $\tilde{\tau}, \Theta_{l}^{(m)}$ term in equation (25). This term implies an exponential suppression of anisotropies with optical depth in the absence of sources.

Finally, the anisotropic nature of Compton scattering is expressed in equation (25) through

$$p^{(m)} = \frac{1}{10} \left[ \Theta_{2}^{(m)} - \sqrt{6} E_{2}^{(m)} \right].$$

These terms only involve the quadrupole moments of the temperature and $E$-polarization distributions.

The polarization evolution follows a similar derivation. Inserting $l \geq 2, m \geq 0$ into the Clebsch-Gordan relations of equation (22) with $s = \pm 2$ gives

$$\tilde{E}_{l}^{(m)} = k \left[ \frac{2\kappa_{l}^{m}}{2l-1} E_{l-1}^{(m)} - \frac{2m}{l(l+1)} B_{l}^{(m)} - \frac{2\kappa_{l+1}^{m}}{2l+3} E_{l+1}^{(m)} \right] - \tilde{\tau}_{l} E_{l}^{(m)} + \sqrt{6} P^{(m)} \delta_{l, 2} (l \geq m)$$

and

$$\tilde{B}_{l}^{(m)} = k \left[ \frac{2\kappa_{l}^{m}}{2l-1} B_{l-1}^{(m)} + \frac{2m}{l(l+1)} E_{l}^{(m)} - \frac{2\kappa_{l+1}^{m}}{2l+3} B_{l+1}^{(m)} \right] - \tilde{\tau}_{l} B_{l}^{(m)} (l \geq m).$$

### 3.2. Scalar Mode

We assume that the matter is representable as a perfect fluid and neglect the anisotropic pressure perturbations. We consider only adiabatic perturbations and neglect the entropy perturbations. The linearized perturbation equations that are obtained (Ma & Bertschinger 1995; Hu & White 1997; Koh & Lee 2000) from the Einstein equations up to first order are

$$3H^{2} \dot{\psi} + 3H \ddot{\phi} + k^{2} \ddot{\phi} = 4\pi G a^{2} \delta T_{0}^{0},$$

$$H \dot{\psi} + \ddot{\phi} = 4\pi G a^{2} (\rho + p) \psi^{(0)},$$

where

$$\rho = \rho_{PF} + \rho_{EM},$$

$$p = p_{PF} + p_{EM},$$

and

$$\dot{\psi} = -\frac{4}{3} \ddot{\phi} - 4 \dot{\phi},$$

$$\dot{\psi} = k^{2} \left( \frac{1}{4} \ddot{\phi} - \dot{\psi} \right) + k^{2} \ddot{\phi} + \frac{k^{2}}{3} (\dot{\phi} - \psi) = 4\pi G a^{2} \delta T_{0}^{0},$$

while $\delta^{(0)}$ and $\sigma$ are defined as

$$(\rho + p) \dot{\psi}^{(0)} = ik \delta T_{0}^{0},$$

$$(\rho + p) \sigma = \left( \frac{k b_{k}}{k^{2}} - \frac{\delta_{0}}{3} \right) \Sigma^{i}_{j},$$

and

$$\Sigma^{i}_{j} \equiv T^{i}_{j} - \frac{\delta^{i}_{k} T^{k}_{j}}{3}$$

denotes the traceless component of $T^{i}_{j}$. The conservation of energy-momentum is a consequence of the Bianchi identity:

$$T^{\mu\nu} = \partial_{\mu} T^{\nu}_{\mu} + \Gamma^{\nu}_{\alpha\beta} T^{\alpha\beta} + \Gamma^{\alpha}_{\alpha\beta} T^{\nu}_{\beta} = 0.$$  (39)

Including the energy-momentum tensor for the PMF and writing the magnetic energy density as

$$\rho_{EM} = \frac{B_{z}^{2}}{2\pi},$$

equation (39) leads to the following equations in $k$-space:

$$\dot{\delta} = -(1 + w) [e^{(0)} - 3 \psi] - 3H \left( \frac{\delta p}{\delta p} - w \right) \delta,$$

$$i^{(0)} = H(1 - 3w) \psi^{(0)} - \frac{\dot{w}}{1 + w} e^{(0)} + \frac{\delta p}{\delta p} \frac{k^{2} \delta}{1 + w} - k^{2} \psi + k^{2} \psi,$$

where $w \equiv p/\rho$. Here, we can cancel $\sigma_{EM}$ in equation (42) in the linear perturbation limit. Thus, in the motion and continuity equations for the scalar mode, we can just add the energy density and pressure of the PMF to the general energy density and pressure, respectively. From equations (41) and (42) we obtain the same form for the evolution equations of photons and baryons as that deduced in previous works (Ma & Bertschinger 1995; Hu & White 1997):

$$\dot{\delta} = -\frac{4}{3} \ddot{\phi} - 4 \dot{\phi},$$

$$i^{(0)} = k^{2} \left( \frac{1}{4} \ddot{\phi} - \dot{\psi} \right) + k^{2} \ddot{\phi} + \frac{k^{2}}{3} (\dot{\phi} - \psi) = 4\pi G a^{2} \delta T_{0}^{0},$$

$$\dot{\psi} = -\frac{\dot{a}}{a} \psi^{(0)} + c_{s}^{2} k^{2} \psi + \frac{4\dot{\rho}_{b}}{3 \rho_{b}} n_{e} \sigma_{T} \left[ \psi^{(0)} - \psi^{(0)} \right] + k^{2} \psi,$$

where $n_{e}$ is the free electron density, $\sigma_{T}$ is the Thomson scattering cross section, and $\sigma_{T}$, in the second term on the right-hand side of equation (44) is the shear stress of the photon with the PMF. In the presence of the PMF, the magnetic pressure should be included in the acoustic term. Therefore, the sound speed in the second term on the right-hand side of equation (46) should be

$$c_{s}^{2} = c_{0s}^{2} + \frac{\Pi^{(0)}}{4\pi a^{4} \rho_{b}},$$

where $c_{0s}$ is the baryon sound speed without the PMF (see Adams et al. 1996).
3.3. Vector Mode

The evolution of the vector potential $V(\tau, k)$ under the influence of the stochastic PMF can be written (Hu & White 1997; Mack et al. 2002) as

$$\dot{V}(\tau, k) + 2 \frac{a}{a} V(\tau, k) = - \frac{16\pi GL^{(1)}(\tau, k)}{a^2 k^2} - \frac{8\pi G a^2 p_\gamma \pi_\gamma + p_b \pi_b}{k},$$

(48)

where the dot denotes a conformal time derivative, and $p_i$ and $\pi_i$ are the pressure and the anisotropic stress, respectively, of the photons ($i = \gamma$) and neutrinos ($i = \nu$). Here, we have omitted the vector anisotropic stress of the plasma, which is negligible in general. In the absence of a magnetic source term, the homogeneous solution of equation (48) behaves $V \propto 1/a^2$. We take $a \propto \tau$ during the radiation-dominated epoch. The magnetic field therefore causes the vector perturbations to decay less rapidly ($\propto 1/a$ instead of $1/a^2$) with the universal expansion. Since the vector perturbations cannot generate density perturbations, we have $\delta^{(1)} = \delta^{(1)} = 0$, where $\delta^{(1)}$ and $\delta^{(1)}$ are the perturbations of the photon and baryon energy densities, respectively.

The magnetic field affects the photon-baryon fluid dynamics via a Lorentz force term in the baryon Euler equations. Following Hu & White (1997), the Euler equations for the neutrino, photon, and baryon velocities, $\nu_\gamma$, $\nu_\nu$, and $\nu_b$, respectively, are written as

$$i^{(1)}_\gamma - \dot{V} = - k \frac{\sqrt{3}}{5} \Theta^{(1)}_\gamma,$$

(49)

$$i^{(1)}_\nu - \dot{V} = - k \frac{\sqrt{3}}{5} \Theta^{(1)}_\nu,$$

(50)

$$i^{(1)}_b - \dot{V} = - \frac{\dot{a}}{a} \left[ i^{(1)}_b - V \right] - \frac{R}{a^4} i^{(1)}_b = \frac{L^{(1)}(\tau, k)}{a^4 (p_b + p_\gamma)},$$

(51)

where $R = (\rho_\gamma + p_\gamma)/a^4 (p_b + p_\gamma) \simeq (4/3)(\rho_\gamma/p_b)$ is the inertial density ratio between baryons and photons, while $\Theta^{(1)}_\gamma$ and $\Theta^{(1)}_\nu$ are quadrupole moments of the neutrino and photon angular distributions, respectively. These quantities are proportional to the anisotropic stress tensors. Equations (49)–(51) denote the vector equations of motion for the cosmic fluid, which arise from the conservation of energy-momentum.

4. TEMPERATURE POWER SPECTRA

We follow the total angular momentum representation introduced by Hu & White (1997) to compute the CMB power spectra induced by the stochastic PMF. A transparent description of the CMB anisotropy formation can be obtained by combining the intrinsic angular structure with that of the plane-wave spatial dependence. Each moment corresponds directly to an observable intrinsic angular structure with that of the plane-wave spatial dependence. Each moment corresponds directly to an observable

$$\Theta^{(1)}_{l}(\tau_0, k) = \int_0^{\tau_0} d\tau \left[ \frac{\dot{V} - i^{(1)}_b}{a^4 (p_b + p_\gamma)} \right],$$

(52)

This power spectrum of temperature anisotropies is to be added to that induced by the scalar density perturbations and compared with observational data.

5. RESULT AND DISCUSSIONS

We discuss the effects of scalar and vector modes of the PMF and also the SZ effect on the CMB in this section. Note that the current data for higher $l$ are known to be insensitive to the tensor mode, which we ignored in the present calculations. Figures 1 and 2 show the CMB power spectra generated by the PMF. The effects of the PMF become progressively more important at higher multipoles and eventually dominates over the scalar fluctuations for $l > 1000$. As a result, the potential discrepancy between the observed and theoretical CMB temperature anisotropies at higher multipoles $l$ is remarkably relaxed by the effects of the presence of a PMF (Yamazaki et al. 2005a, 2005b). In §§5.1–5.3 we discuss details of the behavior of the effects of the PMF on the CMB temperature anisotropies.

5.1. Scalar and Vector Modes

The left-hand side of Figure 1 shows the temperature anisotropies of the CMB power spectra that are generated by the PMF, and the right-hand side of Figure 1 shows the primary temperature anisotropies calculated in the $\Lambda$ cold dark matter (LCDM) model with and without the effects of the PMF. The scalar part of the primary temperature anisotropies consists of several different sources of metric perturbations including photon and neutrino energy density perturbations, as well as the perturbation due to the PMF. Therefore, the scalar mode generated by the PMF, as displayed by the green dash-dotted curves on the left-hand side of Figure 1, is evaluated by subtracting the two calculated CMB power spectra with and without the PMF in the $\Lambda$CDM model. Note that there is no such complication for the vector part of the primary temperature anisotropies because it arises uniquely from the vector mode of the PMF.

Both scalar and vector modes become larger with increasing field strength $|B_0|$ of the PMF, but the variation of the vector mode is much stronger than the scalar mode (see the right-hand side of Fig. 1). The scalar mode for the PMF gets complicated, as is discussed below in this subsection. However, the net effect is basically caused by two contributions: first, the increase of the sound speed for additional magnetic pressure to reduce the baryonic matter fluctuation amplitude and second, the changes of the metric and the total energy density that offset the first effect. Since the effect of the PMF for the vector mode is just an increase of the
Fig. 1.—Effects of the PMF strength $|B_{\lambda}|$ on the CMB. The five figures on the left-hand side show the CMB temperature anisotropies from the PMF. The red bold curves are the temperature anisotropies from vector and scalar modes of the PMF. The green dash-dotted curves are for the scalar mode, the blue dotted curves are for the vector mode, and the purple thin curves show the SZ effect. The upper parts of the figures show the combined effects on the CMB. The lower parts show the effects of the scalar mode obtained by subtracting the $\Lambda$CDM model without the PMF from the $\Lambda$CDM model with the PMF. The five figures on the right-hand side show the primary temperature anisotropies of the CMB with and without the PMF. The black short-dashed curves show the $\Lambda$CDM model without the PMF. The red bold curves are the $\Lambda$CDM model with the PMF. The purple thin curves are the SZ effect. The dots with green, gray, and blue error bars are the WMAP, ACBAR, and CBI data, respectively. On all figures, the power spectral index of the PMF is set to $n_B = -1.5$. In addition, the strengths of the PMF on both sides of this figure are set to be $|B_{\lambda}| = 8.0, 6.0, 4.0, 2.0$, and $1.0$ nG from top to bottom as labeled.
perturbation by the Lorentz force, the vector-to-scalar mode ratio increases as the strength of the PMF increases. The PMF affects both the energy density and pressure in the scalar mode. The influence on the energy density is the same as that on the baryon density. The magnetic pressure, however, increases the sound speed of the fluid, as discussed below equations (45) and (46). This produces interesting effects on the CMB (Adams et al. 1996; Tsagas & Maartens 2000; Yamazaki et al. 2006). In order to understand the effects of the change of the sound speed, $c_s$, boosted by the PMF, let us solve equations (45) and (46) in the WKB approximation,

$$
\delta \propto \frac{1}{\sqrt{c_s(\tau) a(\tau)}} \exp \left( -i \int c_s k d\tau \right) .
$$

Fig. 2.—Same as Fig. 1, but for effects of power spectral index $n_B$ of the PMF on the CMB. On all figures, the strength of the PMF is set to be $|B_k| = 4.0$ nG. In addition, the power spectral index of the PMF on both sides of this figure are set to $n_B = 0, -0.5, -1.0, -1.5,$ and $-2.0$ from top to bottom as labeled.
This solution clearly shows two different influences of \( c_i \) on the baryonic matter fluctuation: First, the increase of the sound speed due to the magnetic pressure results in a decrease in the amplitude of the evolution of \( \delta_b \) because of the prefactor of equation (56). This means that in the presence of a magnetic field, the plasma pressure increases by the repulsion of lines of magnetic force. Thus, gravitational collapse of the matter is delayed. Second, increasing the sound velocity makes the frequency of the baryonic fluctuation \( \delta_b \) larger because of the exponent of equation (56). This effect of the PMF on the density field makes different kinds of amplitude boosts that depend on \( k \) (Yamazaki et al. 2006).

The PMF shifts the acoustic oscillation for the baryon fluid either up or down depending on the combination of the sound speed \( c_i \) and wavenumber \( k \). Note, however, that since the amplitude of the PMF is so small, the difference between the shifted and nonshifted period of the acoustic oscillations essentially only depends on the wavenumber \( k \). This effect is strongly constrained by the observed data as displayed on the right-hand side of Figure 1.

When one compares the scalar and vector modes, the scalar mode makes only a minor contribution to the CMB except at lower amplitudes. However, since the magnetic field pressure delays the gravitational collapse of matter, its effect on the matter power spectrum for large-scale structure (LSS) is very interesting. Since in the present article we are only interested in the small-scale structure associated with larger multipoles \( l \), further details on this point will be discussed elsewhere (Yamazaki et al. 2006).

### 5.2. Dependence on \( B_z \)

The CMB power spectrum generated by a cosmological magnetic field is generally characterized by a broad peak at \( l \sim 1000 \text{--} 3000 \) for a reasonable range of \(-2 < n_B < 0 \), as shown in Figure 2. We display five cases of the CMB power spectra for \( B_z = 1, 2, 4, 8, \) and 10 nG in Figure 1. As mentioned above, for lower \( l \) and \( B_z \), the scalar mode dominates (bottom three cases for \( B_z = 1, 2, \) and 4 nG of the left-hand side of Fig. 1). For higher \( l \) and \( B_z \), the vector mode dominates. The amplitude is shown to increase with increasing magnetic field strength \( B_z \) for \( l > 1000 \), where the vector mode dominates. Some lengthy mathematical derivation of the power spectrum of fluctuations leads to \( C_l \propto B_z^4 \) for \( n_B < -3/2 \), and \( C_l \propto B_z^{4n_B} \) for \( n_B > -3/2 \) for higher multipoles \( l \) (Fig. 2).

### 5.3. Dependence on \( n_B \)

Figure 2 shows the \( n_B \) dependence of the CMB. For \( n_B < -3/2 \), the second term in equation (7) dominates. This \( k \)-dependence causes a strong angular dependence for the vector mode perturbation. This results in changing the peak position for different multipoles \( l \), depending on the power-law spectral index \( n_B \). On the other hand, peaks remain at nearly the same multipole \( l \sim 2000 \) independently of \( n_B \) for \( n_B > -3/2 \). This is because the vector mode perturbation from the magnetic field tends to be that of white noise when the first term in equation (7) dominates.

### 6. CONSTRAINT ON THE PMF IN A MCMC ANALYSIS

To set limits on the PMF, we have evaluated likelihood functions for fits to the WMAP, ACBAR, and CBI data over a wide range of the parameters, \( B_z \) and \( n_B \), for a stochastic PMF along with the usual six cosmological parameters, \( h, \Omega_b h^2, \Omega_c h^2, n_s, A_s, \) and \( \tau_c \). The quantity \( h \) is the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). The quantities \( \Omega_b h^2 \) and \( \Omega_c h^2 \) are the usual present baryon and cold dark matter closure parameters. The quantities \( n_s \) and \( A_s \) are the spectral index and the amplitude of the primordial scalar fluctuation, respectively. The parameter \( \tau_c \) is the optical depth. We adopt a flat cosmology. To explore this parameter space, we have employed the Markov chain technique (Lewis & Bridle 2002). We also take account of the SZ effect in our analysis. For that, we adopt a fixed prior of \( \sigma_8 = 0.9 \) (Spergel et al. 2003; Komatsu & Seljak 2002).

#### 6.1. Degeneracy

In our likelihood analysis of the magnetic field parameters, we continued the Markov chain Monte Carlo (MCMC) algorithm until the cosmological parameters converged to the values listed in Table 1. The inferred parameter values are consistent with those deduced in other analyses (Spergel et al. 2003; Mason et al. 2003; Kuo et al. 2004). We did not find obvious degeneracies of the magnetic field parameters with other cosmological parameters. We understand this for the following reasons. The PMF is constrained (Jedamzik et al. 1998; Mack et al. 2002) by the cutoff scale for the damping of Alfven waves as indicated by equation (8). Since this cutoff scale is small compared with the multipoles for the currently available data at \( l \ll 3000 \), the manifestation of this effect is remarkably seen as a monotonicly increasing contribution from both scalar and vector perturbations in the multipole region higher than \( l \sim 500 \) (Yamazaki et al. 2005b). This sensitivity of the CMB power spectrum to the PMF differs completely from those to the other cosmological parameters, which helps resolve the degeneracies among them. There is, however, a strong degeneracy between the magnetic field strength \( B_z \) and the power spectral index \( n_B \) because those parameters affect the amplitude simultaneously (see eqs. [7] and [9]).

#### 6.2. Limits on the PMF

In Figure 3 we show results of our MCMC analysis using the WMAP, ACBAR, and CBI data in the two-parameter plane, \( \{B_z, n_B\} \) versus \( n_B \). The 1 \( \sigma \) (68%) and 2 \( \sigma \) (95.4%) confidence level (CL)–excluded regions are bounded above by the thick curves as shown. We could not find a lower boundary of the allowed region at the 1 or 2 \( \sigma \) confidence level. Note, however, that we find a very shallow minimum with a reduced \( \chi^2 \approx 1.08 \). From Figure 3 we obtain an upper limit to the strength of the PMF at the 1 \( \sigma \) (95.4%) CL of

\[
|B_z| < 7.7 \text{ nG at 1 Mpc.}
\]  

This upper limit is particularly robust, as we have considered all effects on the CMB anisotropies, i.e., the effect of the ionization ratio, the SZ effect, and the effects from both the scalar and vector modes on the magnetic field, in the present estimate of \( |B_z| \) and \( n_B \).

### 7. PMF GENERATION AND EVOLUTION

In this section we discuss a multiple-generation and evolution scenario of the cosmological PMF that is motivated by the results of the present study.

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**Table 1**

| Parameter | Mean and 68% CL Errors | 95% CL Errors |
|-----------|-------------------------|---------------|
| \( \Omega_b h^2 \) | \( 0.0231 \pm 0.0011 \) | \( +0.0021 \) |
| \( \Omega_{\text{CDM}} h^2 \) | \( 0.120 \pm 0.011 \) | \( +0.024 \) |
| \( n_s \) | \( 0.976 \pm 0.027 \) | \( +0.034 \) |
| \( A_s \) | \( 0.903 \pm 0.129 \) | \( +0.194 \) |
| \( \tau_c \) | \( 0.126 \pm 0.039 \) | \( +0.11 \) |
| \( h \) | \( 0.727 \pm 0.037 \) | \( +0.070 \) |

Note.—Calculated corresponding cosmic expansion ages are \( t_0/Gyr = 13.35 \pm 0.21 \) (68% CL) and \( 13.35 \pm 0.41 \) (95% CL).
To begin with we adopt the following three constraint conditions:

1. A PMF strength $|B_1| \leq 7.7 \text{ nG (1 $\sigma$)}$ at 1 Mpc, as deduced above by applying the MCMC method to the WMAP, ACBAR, and CBI data.

2. The magnetic field strength in galaxy clusters is $0.1 \mu G < |B_{CG}| < 1 \mu G$ (Clarke et al. 2001; Xu et al. 2006). Hence, if the isotropic collapse is the only process that amplifies the magnetic field strength, the lower limit to the PMF is $\sim 1$–10 nG for the PLSS.

3. The gravity wave constraint on the PMF from Caprini & Durrer (2002). The big bang nucleosynthesis of the light elements depends on a balance between the particle production rates and the expansion rate of the universe, since the energy density of the gravity waves $\rho_{GW}$ contributes to the total energy density. However, $\rho_{GW}$ is constrained so that the expansion rate of the universe does not spoil the agreement between the theoretical and observed light-element abundance constraints for deuterium, $^3$He, $^4$He, and $^7$Li (Maggiore 2000; Yahiro et al. 2002).

Figures 3 and 4 summarize the various constraints on the PMF by these three conditions. The region bounded by the upper red solid curve is constrained by condition (1) as indicated, the green dash-dotted horizontal line in Figure 3 corresponds to the lower limit to the PMF from condition (2), and the Black solid, blue dotted, and pink dashed lines, respectively, are the upper limits of the produced PMF from big bang nucleosynthesis, the electroweak transition, and the inflation epoch. Figure 4 shows the allowed or excluded regions according to these multiple constraints depending on when the PMF was generated.

A. Nucleosynthesis: I+II+III region

$$1.0 \text{ nG} \leq |B_1| \leq 4.7 \text{ nG},$$
$$-3 \leq n_B \leq -2.40.$$  

B. Electroweak transition: II+III region

$$1.0 \text{ nG} \leq |B_1| \leq 4.7 \text{ nG},$$
$$-3 \leq n_B \leq -2.43.$$  

C. Inflation: III region

$$1.0 \text{ nG} \leq |B_1| \leq 4.7 \text{ nG},$$
$$-3 \leq n_B \leq -2.76.$$  

Obviously, the upper limits on both $|B_1|$ and $n_B$ become more stringent if the PMF is produced during an earlier epoch. Moreover, the limits we deduce are the strongest constraints on the PMF that have yet been determined. However, we caution that the evolution of the generation of the PMF during the LSS epoch is not well understood. Thus, if there are other effective physical processes for the generation and evolution of the PMF during the formation of LSS, our lower limit of the PMF parameters may decrease. In order to constrain the PMF parameters accurately, we should study the PMF not before but after the PLSS.

8. CONCLUSION

For the first time we have studied scalar mode effects of the PMF on the CMB. We have confirmed numerically without approximation that the excess power in the CMB at higher $l$ can be explained by the existence of a PMF. For the first time a likelihood analysis using the WMAP, ACBAR, and CBI data with an MCMC method has been applied to constrain the upper limit on the strength of the PMF to be

$$|B_1| < 7.7 \text{ nG}.$$  

We have also considered three conditions on the generation and evolution of the cosmological PMF: (1) our result, (2) the lower
limit on the PMF from the magnetic field of galaxy clusters, and
(3) the constraint on the PMF from gravity waves. Combining
these, we find the following concordance region for the PMF
parameters:

\[ 1 \text{ nG} < |B_j| < 4.7 \text{ nG}, \quad -3.0 < n_B < -2.4. \]

The PMF also affects the formation of LSS. For example,
magnetic pressure delays the gravitational collapse. It is thus
very important to constrain the PMF as precisely as possible.
If we combine our study and future plans to observe the CMB
anisotropies and polarizations for higher multipoles \( l \), e.g., via the
Planck Surveyor, we will be able to constrain the PMF more
accurately and explain the evolution and generation of the mag-
netic field on galaxy cluster scales along with the formation of
the LSS.

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APPENDIX
TIGHT-COUPLING APPROXIMATION

Since the Thomson opacity is larger before recombination, photons and baryons are tightly coupled. If the Thomson drag terms in
equations (44), (46), (50), and (51) are too large, it is difficult to solve these equations. Therefore, we here derive approximation
equations in this limit. The tight-coupling approximation for the scalar mode was introduced by Ma & Bertschinger (1995). Here,
therefore, we only need to introduce the vector mode in this appendix.

Using equations (50) and (51), we obtain the following equations for the photons and baryons in the tight-coupling approximation:

\[ v_b^{(1)} = \frac{1}{1 + R} \left\{ -\frac{\dot{a}}{a} v_b^{(1)} - R \left[ v_b^{(1)} - v_f^{(1)} \right] - \frac{3}{5} kR\Theta_{\gamma}^{(1)} + \frac{L^{(1)}}{a^4(\rho_b + p_b)} \right\}, \quad (A1) \]

\[ v_f^{(1)} - v_b^{(1)} = \frac{1}{\dot{\tau}_c} \left[ v_f^{(1)} + \left[ v_f^{(1)} - v_b^{(1)} \right] + \frac{3}{5} k\Theta_{\gamma}^{(1)} \right]. \quad (A2) \]

Writing \( \dot{v}_f^{(1)} = \dot{v}_b^{(1)} + \left[ \dot{v}_f^{(1)} - \dot{v}_b^{(1)} \right] \), we can rewrite (A2) as

\[ v_f^{(1)} - v_b^{(1)} = \frac{1}{\dot{\tau}_c} \left\{ v_f^{(1)} + \left[ v_f^{(1)} - v_b^{(1)} \right] + \frac{3}{5} k\Theta_{\gamma}^{(1)} \right\}, \quad (A3) \]

and using equations (A1) and (A2), we get

\[ v_f^{(1)} - v_b^{(1)} = \frac{1}{(1 + R)\dot{\tau}_c} \left\{ -\frac{\dot{a}}{a} v_f^{(1)} + \left[ v_f^{(1)} - v_b^{(1)} \right] + \frac{3}{5} k\Theta_{\gamma}^{(1)} + \frac{L^{(1)}}{a^4(\rho_b + p_b)} \right\}. \]

Differentiating equation (A4) and ignoring terms of more than second order in \( \dot{\tau}_c \), we get

\[ \dot{v}_f^{(1)} - \dot{v}_b^{(1)} = \frac{2R}{a(1 + R)} \left[ v_f^{(1)} - v_b^{(1)} \right] + \frac{1}{(1 + R)\dot{\tau}_c} \left\{ -\frac{\dot{a}}{a} v_f^{(1)} \right\} + O(\dot{\tau}_c^{-2}). \quad (A4) \]

Substituting equation (A4) into equation (A1) yields

\[ \dot{v}_f^{(1)} = \frac{1}{R} \left\{ \dot{v}_b^{(1)} - \frac{\dot{a}}{a} v_b^{(1)} - \frac{3}{5} kR\Theta_{\gamma}^{(1)} + \frac{L^{(1)}}{a^4(\rho_b + p_b)} \right\}. \quad (A5) \]

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Note added in proof.—After this paper was submitted for publication, the third-year WMAP data were released. Most of the results discussed here are unaffected by these new data. We note, however, that the magnitude of the SZ effect consistent with the WMAP analysis is now even smaller ($\sigma_8 = 0.75^{+0.03}_{-0.04}$), suggesting that the need for the effects of the PMF as described here is even greater.