Spin relaxation under identical Dresselhaus and Rashba coupling strengths in GaAs quantum wells

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Spin relaxation under identical Dresselhaus and Rashba coupling strengths in GaAs quantum wells is studied in both the traditional collinear statistics, where the energy spectra do not contain the spin-orbit coupling terms, and the helix statistics, where the spin-orbit couplings are included in the energy spectra. We show that there is only marginal difference between the spin relaxation times obtained from these two different statistics. We further show that with the cubic term of the Dresselhaus spin-orbit coupling included, the spin relaxation time along the (1,1,0) direction becomes finite, although it is still much longer than than that along the other two perpendicular directions. The properties of the spin relaxation along this special direction under varies conditions are studied in detail.

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I. INTRODUCTION

Semiconductor spintronics which aims at making use of electron spin degrees of freedom has attracted much attention both theoretically and experimentally owing to its potential applications such as spin transistors. The realization of such device requires a long spin relaxation time (SRT) which should be at least longer than the time of one expected operation. Therefore it is essential to understand the mechanism of the spin relaxation/dephasing (R/D) time which is due to the lack of the bulk inversion symmetry and the spin-orbit interaction in crystals without inversion center. This splitting is equivalent to an effective magnetic field (EMF) $\mathbf{h}(\mathbf{k})$ acting on the spin, with its magnitude and orientation depending on $\mathbf{k}$. In quantum wells (QW’s) this EMF is composed of the Rashba term due to the lack of the structure inversion symmetry and the Dresselhaus term due to the lack of the bulk inversion symmetry. Their contributions to the energy spectra as well as the spin R/D have been widely studied in the literature. Under some special conditions, such as when the growth direction of the QW is along (110) axis or when the growth direction is along (001) axis but the strengths of the Rashba and the Dresselhaus terms are comparable, the SRT shows strong anisotropy and along some special polarization direction one can get extremely long SRT. Schliemann et al. further proposed a new spin transistor based on this strong anisotropy of the SRT when the strengths of the Rashba and the Dresselhaus terms are identical. Very recently Jiang and Wu proposed to use strain to get extremely long spin dephasing time (also relaxation time) in GaAs (001) QW’s where there is no anisotropy along different directions.

Among these studies, Wu proposed a many-body approach by setting up the kinetic spin Bloch equations and solving it self-consistently with all the scattering, i.e., the electron-electron, electron-phonon and electron-nonmagnetic impurity scattering explicitly included. By this approach, one not only gets the spin R/D time due to the single-particle effective spin-flip scattering, but also obtains that due to the many-body effects which have shown to be dominant in some n-type semiconductor QW’s and spin transport. The many-body effect here is not only referred to be the effect of the Coulomb scattering, but also expanded to include the spin R/D due to the inhomogeneous broadening from the EMF combined with the spin conserving scattering. Moreover, this approach also includes the counter-effect from the scattering to the inhomogeneous broadening from the anisotropic EMF, which has been shown to be also very important in the spin kinetics. In this paper, we apply this approach to study the spin relaxations in GaAs (001) QW’s with the identical Rashba and Dresselhaus spin-orbit coupling strengths, especially at the case when the cubic term of the Dresselhaus term is included. The later has been investigated in the single-particle picture without counting the many-body effects.

Another issue we address in this paper is the influence of the two spin eigenstates on the spin kinetics. It is noted that in our previous works as well as many other works in the literature, the equilibrium state is taken as the Fermi distribution of electrons in conduction band without the spin-orbit coupling from the DP term. Therefore, the energy spec-
trum of electrons is \( \varepsilon_k = k^2/2m^* \), with \( m^* \) denoting the effective mass of electrons, and the eigenstates of spin are the eigenstates of \( \sigma_\alpha \), i.e., \( \chi_\uparrow = (1,0)^T \) and \( \chi_\downarrow = (0,1)^T \), which are collinear in the laboratory coordinates. In the following we refer to the Fermi distribution composed by these eigenstates (without the spin-orbit coupling) to be the collinear Fermi distribution and the statistics to be the collinear statistics. In the meantime, there is another equilibrium state used in the literature which is the Fermi distribution of electrons with the eigenstates being those of electrons in the conduction band with the DP spin-orbit coupling. Therefore the energy spectrum is now being

\[
\varepsilon_{k,\eta} = k^2/2m^* + \eta |\mathbf{h}(k)| \tag{1}
\]

with \( \eta = \pm 1 \) for the two spin branches. The eigenfunctions of spin are now

\[
|\eta\rangle = \frac{1}{\sqrt{2}}[\chi_\uparrow + \eta \chi_\downarrow] \tag{2}
\]

with \( \hat{h}(k) = h_x(k) + ih_y(k) \). The spin polarizations therefore strongly depend on the momentum \( k \). In the following we refer to the Fermi distribution composed by these eigenstates (with the spin-orbit coupling) to be the helix Fermi distribution and the statistics to be the helix statistics as the eigenstates shows helix spin structure when the DP EMF is composed only by the Rashba term.\(^{34} \) Some works are performed with the helix statistics especially those dealing with spin relaxation in quantum dots\(^{35,36} \) and spin Hall conductivity.\(^{37,38,39} \) Very recently Lechner and Rössler\(^{40} \) and Grimaldi\(^{41} \) gave some formal theory to discuss the spin R/D using the helix statistics. However, whether the collinear statistics widely used in the spin R/D calculation is adequate in two dimensional electron gas (2DEG) is unknown yet. In principal, when the DP spin-orbit coupling is weak, the collinear statistics is good enough for the spin dephasing calculation. However, if one comes to the strong coupling regime which can be obtained in 2DEG confined in narrow quantum wells,\(^{42,43,44,45} \) then two things happen: One is that the strong DP EMF causes strong interference effect and induces strong spin depolarization which is irrelevant to the spin dephasing effect; Another is that the statistical equilibrium state might have to be changed into that in the helix statistics. The later then changes all the scattering and might change the spin R/D when the DP EMF is strong. Nevertheless, it is not so obvious as when the EMF is very strong, the spin polarization decreases dramatically due to the interference effect long before the scattering plays its role. In this paper we address this issue by solving the kinetic spin Bloch equations with both collinear and helix statistics.

We organize the paper as follows: We present our model and kinetic spin Bloch equations with both the helix and the collinear statistics in Sec. II. Then we present our numerical results in Sec. III. We first compare the scattering with two spin statistics in Sec. III A and IIIB, but without the Coulomb scattering. Then we discuss the spin relaxation under the identical Dresselhaus and Rashba coupling strengths in GaAs QW’s with the collinear statistics by including all the scattering. We conclude in Sec. IV.

II. MODEL AND KINETIC SPIN BLOCH EQUATIONS

We construct the kinetic spin Bloch equations\(^{31,32} \) by using the nonequilibrium Green function method\(^{46} \) with these two different statistics and write them in the unified form as follows:

\[
\dot{\rho}_k + \dot{\rho}_k|_{\text{coh}} + \dot{\rho}_k|_{\text{scatt}} = 0 . \tag{3}
\]

Here \( \rho_k \) represents a single particle density matrix of electrons with wavevector \( k \). Often we project the density matrix in the laboratory spin space (the collinear spin space) which is constructed by basis with spin states oriented relative to a fixed direction, i.e., \( \chi_\uparrow \) and \( \chi_\downarrow \). In such spin space, the diagonal matrix elements \( \rho_{k,\sigma,\sigma'} = f_{k,\sigma} \) with \( \sigma = \uparrow \) or \( \downarrow \) describe the distribution functions of electrons with momentum \( k \) and spin \( \sigma \) orienting along the laboratory \( z \)-axis. The off-diagonal elements \( \rho_{\uparrow,\downarrow} \) describe the corrections (coherence) between the spin-up and spin-down states, with its real and imaginary parts standing for the spin polarizations along the \( x \)- and \( y \)-axes respectively.

The coherent part of the two sets of Bloch equations with different statistics are the same and are given in the collinear spin space as

\[
\dot{\rho}_k|_{\text{coh}} = i[H_R + H_D - \sum_q V_q\rho_k|_{q-\mathbf{k}}, \rho_k] , \tag{4}
\]

with \( H_R(D) = |\mathbf{h}_R(D)| \cdot \sigma \) representing the Rashba\(^6 \) (Dresselhaus\(^7 \)) Hamiltonian and \( \sigma \) standing for the Pauli matrices. It describes the spin precession along the direction of the EMF. It is noted that the Hartree-Fock term \( \sum_q V_q\rho_k|_{q-\mathbf{k}} \) in the coherent part cannot flip the total electron spin as

\[
\sum_k \left[ \sum_q V_q\rho_k|_{q-\mathbf{k}} , \rho_k \right] = 0 . \tag{5}
\]

Therefore the DP term is the only one that flips the total spin polarization. For GaAs (001) QW with well width \( a \) and under the infinite-well-depth assumption, \( \mathbf{h}_R(k) = \alpha(k_y, -k_z, 0) \) and \( \mathbf{h}_D(k) = \gamma(k_x k_y^2 - (\frac{\pi}{a})^2), k_y(\frac{\pi}{a})^2 - k_z^2, 0 \). Here \( \alpha \) is the strength of the Rashba term and \( \gamma \) is the material-specific strength of the Dresselhaus term. \( \alpha \) can be tuned by the external gate voltage.\(^6 \) By changing the external gate voltage, one may have \( \beta = \gamma(\frac{\pi}{a})^2 \).\(^{4,36} \)

If one further ignores the cubic term in \( \mathbf{h}_D(k) \), at the special polarization direction \( (1,1,0) \), Eq. (2) becomes |\( \eta\rangle = (1, \eta)^T \) which is independent on \( k \). \( V_q \) in Eq. (4) reads \( V_q = \sum_{\mathbf{q}} \frac{4\pi e^2}{\kappa_0 (\mathbf{q} + i\mathbf{q}_z + \kappa_0)} |I(i\mathbf{q}_z)|^2 \). Here \( \kappa_0 \) denotes
the static electric constant and \( \kappa^2 = 4\pi e^2 N_e/(ak_BT) \) stands for the Debye-Hücke screening constant. \( N_e \) represents the 2D electron density. The form factor \( |I(iq_j)|^2 = \pi^4 \sin^2 y/[y^2(\pi^2 - y^2)]^2 \) with \( y = q_a/2 \). The square bracket \([A,B] = AB - BA\) is the commutator. It is noted that \( \dot{\rho}_k^{\text{coh}} \) is independent on the specific spin statistics and can be expanded as

\[
\dot{f}_{k,\uparrow}^{\text{coh}} = -\dot{f}_{k,\downarrow}^{\text{coh}} = 2\text{Im}[P_k \rho_{k,\uparrow\downarrow}],
\]

\[
\rho_{k,\uparrow\downarrow}^{\text{coh}} = -iP_k(f_{k,\uparrow} - f_{k,\downarrow}) - i\Delta_k \rho_{k,\uparrow\downarrow},
\]

in which \( \{\ldots\}^\dagger \) is the Hermite conjugate of the same terms in the previous \( \{\ldots\} \). The subscript \( \xi \) denotes the spin branch which is \( \sigma = \uparrow, \downarrow \) in the collinear statistics and \( \eta = \pm \) in the helix statistics. \( N_i \) in Eq. (8) is the impurity density and \( |U_{q_i}|^2 = \sum_{q_i} \{4\pi Z_i e^2/|\kappa_0(q_i^2 + q_z^2)| \}^2 |I(iq_i)|^2 \) is the impurity potential with \( Z_i \) denoting the charge number of the impurity. \( |g_{q_{\xi \lambda}}|^2 \) and \( N_{q_{\xi \lambda}} = [\exp(\Omega_{q_{\xi \lambda}}/k_BT) - 1]^{-1} \) are the matrix element of the electron-phonon interaction and the Bose distribution function with phonon energy spectrum \( \Omega_{q_{\xi \lambda}} \) at phonon mode \( \lambda \) and wavevector \((q, q_z)\) respectively. It is noted that the scattering terms are different for different statistics. With the collinear statistics, the matrix \( T_{k,\xi} = T_{k,\sigma} \) is \( 1/2 \), a constant number, and the energy spectrum \( \varepsilon_{k,\xi} = \varepsilon_{k,\eta} \equiv \varepsilon_k \). Then the scattering terms are exactly the same as those in Refs. 18,19. However, with the helix statistics, the scattering terms become more complicated. \( T_{k,\xi} = T_{k,\eta} \) is a \( 2 \times 2 \) matrix and becomes \( k \)-dependent:

\[
T_{k,\eta} = \frac{1}{2}{\begin{bmatrix} 1 & \eta \cdot \mathbf{h}(k) \end{bmatrix} \cdot \sigma}. \tag{9}
\]

Moreover, the energy spectrum in the \( \delta \)-functions in Eq. (8) \( \varepsilon_{k,\eta} \) is anisotropic in \( k \) [Eq. (1)]. When the strength of the DP term tends to zero, the scattering terms in these two statistics become identical.

Besides the collinear spin space, there is another spin space (the helix spin space) used in the literature\(^34\,40\) which is constructed by spin states \( |+\rangle \) and \( |-\rangle \). The density matrix and the two sets of kinetic spin Bloch equations with different statistics can also be written in the helix spin space and the physics itself is the same as that in the collinear spin space. They are given in Appendix A.

It is noted that

\[
\sum_k \dot{\rho}_k^{\text{scatt}} = 0 \tag{10}
\]

for both statistics as all the scattering here is the spin-conserving scattering.

The scattering terms tend to drive the out-of-equilibrium system back to the equilibrium one \( \rho_0(k) = \{\exp[H_0(k) - \mu]/\kappa_0 k_BT + 1\}^{-1} \) with \( H_0(k) = \frac{k^2}{2m} + \mathbf{h}(k) \cdot \sigma \) for the collinear statistics and \( H_0(k) = \frac{k^2}{2m} + \mathbf{h}(k) \cdot \sigma \) for the helix statistics. This can be seen from the fact that \( \dot{\rho}_k^{\text{scatt}} = 0 \) when \( \dot{\rho}_k = \rho_0(k) \) for each corresponding statistics. The equilibrium state \( \rho_0 \) can be further writ-
\[\rho_0(k) = \left[\exp((H_0 - \mu)/k_BT) + 1\right]^{-1}\]
\[= \sum_\xi f(\varepsilon_{k,\xi} - \mu) T_{k,\xi}\]  \hspace{1cm} (11)

for both statistics, with \(f(x) = \left[\exp(x/k_BT) + 1\right]^{-1}\) the Fermi distribution and \(\mu\), the chemical potential. For the collinear statistics, Eq. (11) returns to the familiar case \(\rho_0(k) = f(\varepsilon_k - \mu)\).

It is further noted that besides the energy spectrum \(\varepsilon_{k,\xi}\) in the two statistics is different, the spin polarization in the equilibrium state \(S^0_{\text{collinear}} = Tr[\rho_0(k)]\sigma]/2\) for each \(k\) also differs in the two statistics. For the collinear states in the collinear statistics, there is not any polarization \(S^0_{\text{collinear}} = 0\) for each \(k\). However, in the helix statistics \(S^0_k = \sum_k S^0_k\) is still zero.

Although in different statistics, the way of the spin relaxation might be different due to the different equilibrium states. Here we first show analytically that the conclusion that when the strengths of the Rashba and the Dresselhaus spin-orbit couplings are identical and the cubic term of the Dresselhaus term is ignored, the SRT along the direction \(\hat{n}_1 = \frac{1}{\sqrt{2}}(1, 1, 0)\) is infinite, \(^4\) does not depend on the statistics. This is because with the identical strengths of the Rashba and the Dresselhaus spin-orbit couplings and in the absence of the cubic term of the Dresselhaus term, the EMF is now proportional to \(\hat{n}_1\): \(h(k) = |h(k)|\hat{n}_1\) and hence \(H_R + H_D = |h(k)|\hat{n}_1 \cdot \sigma\).

The total spin polarization along \(\hat{n}_1\) in the collinear space is \(S_{\hat{n}_1}(t) = S \cdot \hat{n}_1\), with \(S = \sum_k S_k\) and \(S_k = Tr[\rho_k \sigma]/2\). Then from Eq. (3) one has, in both statistics,

\[\frac{\partial}{\partial t} S_{\hat{n}_1} + i \sum_k Tr[(H_R + H_D, \rho_k |\hat{n}_1 \cdot \sigma)] = 0\]  \hspace{1cm} (12)

with the help of Eqs. (5) and (10). It is easy to verify that the second term of the equation is zero as \(Tr[(H_R + H_D, \rho_k |\hat{n}_1 \cdot \sigma)] = |h(k)|Tr[\hat{n}_1 \cdot \sigma \rho_k \hat{n}_1 \cdot \sigma - \rho_k |\hat{n}_1 \cdot \sigma] = 0\). Then the polarization \(S_{\hat{n}_1}(t) = S_{\hat{n}_1}(t = 0)\) does not relax in both statistics.

### III. NUMERICAL RESULTS

It is noted that when the cubic term of the Dresselhaus term is included, the electron spin polarization along any direction relaxes, even when the strengths of the Rashba and Dresselhaus terms are identical. Moreover, even when the spin lifetime along \(\hat{n}_1\) is infinity, the spin lifetimes along the two perpendicular directions \(\hat{n}_2 = \frac{1}{\sqrt{2}}(1, -1, 0)\) and \(\hat{z}\) are finite. It is not clear yet how the different statistics change these SRT’s. This is what we are going to address first in following.

It is seen that all the unknowns to be solved appear nonlinearly in the coherent and the scattering parts. Therefore the kinetic spin Bloch equations have to be solved self-consistently to obtain the temporal evolutions of the electron distribution functions \(f_{k,\sigma}(t)\) and the spin coherence \(\rho_{k,\uparrow\downarrow}(t)\). The SRT along any direction \(\hat{n}\) is obtained by fitting the envelope of the quantity \(S\) under a small initial polarization along \(\hat{n}\) direction

\[\rho_k(t = 0) = \frac{1}{2} \left[ \sum_{\xi=\pm 1} f(\varepsilon_k - \mu_{\xi}) + \sum_{\xi=\pm 1} \xi f(\varepsilon_k - \mu_{\xi}) \hat{n} \cdot \sigma \right]\].  \hspace{1cm} (13)

By choosing spin dependent chemical potentials \(\mu_{\pm 1}\), one can construct an initial state with the given electron density \(N_e = \sum_k Tr(\rho_k)\) at \(4 \times 10^{11}\) cm\(^{-2}\) and the initial spin polarization \(P_\hat{n} = S_k/N_e\) along the direction \(\hat{n}\) at 5 \%. In the numerical calculation, how to compute the scattering terms are the central problem. For the collinear statistics, this problem has been solved successfully with high accuracy and fast CPU speed. The details of the numerical scheme has been laid out in Ref. 19. For the helix statistics, however, things are much more complicated. This is because in the helix energy spectrum, it is impossible to divide the momentum space into a set of discrete grids in the way that after performing the energy conservation (\(\delta\)-functions) in the scattering terms, all the final states still belong to this set. Therefore we still divide the momentum space by the mesh as that in Ref. 19 but approximate the \(\delta\)-function by the Gaussian distribution \(\delta(x) = \lim_{\sigma \to 0} \exp(-x^2/\sigma^2)/(\sqrt{\pi}\sigma)\) and take the width to be \(\sigma = 0.5\Delta E\) with \(\Delta E\) representing the energy interval of the grids. This approximation requires much more grid points to converge the results. Moreover, it also costs one more integration as the \(\delta\)-function cannot be carried out analytically in the discrete space. Therefore the CPU time becomes much longer in the helix statistics.

In the following, we study the spin relaxation under identical strengths of the Dresselhaus and the Rashba couplings and with the cubic term of the Dresselhaus term included. We first show the temporal evolution of the spin polarization along all directions, \(i.e., \hat{n}_1, \hat{n}_2\) and \(\hat{z}\) in these two statistics. Then we investigate the SRT along the direction \(\hat{n}_1\) with the impurity scattering and the electron-LO phonon scattering included to compare the effect of different statics to the SRT. Finally we give the SRT with all the scattering including at different temperatures, electron and impurity densities and well widths with the collinear statistics.

In the numerical calculation, we only need to consider electron-longitudinal optical (LO) phonon scattering for the electron-phonon scattering as we focus on the high temperature regime (\(T > 120\) K). The matrix element of the electron-LO phonon interaction is \(|g_{qL,LO}|^2 = \{\alpha \Omega^{3/2}/[\sqrt{2\mu(q^2 + q_0^2)]}\}|I(iq_z)|^2\) with \(\alpha = \ldots\)
\[ e^2 \mu/(2\Omega_{\text{LO}}) \left( \kappa^{-1} - \kappa_0^{-1} \right). \] \( \kappa_\infty \) is the optical dielectric constant and \( \Omega_{\text{qfs}} = \Omega_{\text{LO}} \) is the LO-phonon frequency. The material parameters of GaAs are listed as following: \( \gamma = 0.067 m_0 \) with \( m_0 \) the free electron mass, \( \kappa_\infty = 10.8, \kappa_0 = 12.9 \) and \( \Omega_{\text{LO}} = 35.4 \) meV. The strengths of the Dresselhaus SOC and Rashba SOC are \( \gamma = 25 \) meV \( \text{Å}^3 \) and \( \alpha = \gamma(\pi)^2 \) with \( \alpha = 5 \) nm unless otherwise specified. \(^4\)

A. Temporal evolutions of spin polarization in collinear and helix statistics

In Fig. 1, we plot the temporal evolutions of the spin polarization along the three directions: \( \hat{z} \) [Fig. 1(a)], \( \hat{n}_2 \) [Fig. 1(b)] and \( \hat{n}_1 \) [Fig. 1(c)] with the electron-LO phonon and the electron-impurity scattering included at temperature \( T = 200 \) K in the system with identical strengths of the Rashba and Dresselhaus coupling. The impurity density \( N_i \) is \( 0.5N_e \). The evolutions within these two statistics are plotted in the same figures for comparison. It is seen from the figure that with the cubic Dresselhaus term included, the SRT along \( \hat{n}_1 \) becomes finite (several hundred picoseconds), but still much longer than the other two perpendicular directions \( \hat{z} \) and \( \hat{n}_2 \) (less than 1 ps). This is because of the strong anisotropy from the EMF \( \mathbf{h}(\mathbf{k}) = h_{1}(\mathbf{k})\hat{n}_1 + h_{2}(\mathbf{k})\hat{n}_2 \) with \( h_{1}(\mathbf{k}) = \sqrt{2} \gamma E \left( k_y - k_x \right) (\frac{\pi}{a})^2 + \frac{k_y k_x}{\alpha} \) and \( h_{2}(\mathbf{k}) = \gamma k_y k_p \frac{k_x + k_y}{\alpha} \). As the spin flip is determined by the component of the EMF which is perpendicular to the spin polarization, therefore different direction of the spin polarization experiences different EMF: for electron spin along the direction \( \hat{z} \), it feels the total EMF \( \mathbf{h}(\mathbf{k}) \); for the one along \( \hat{n}_2 \), it feels \( h_{1(2)}(\mathbf{k}) \). For narrow QW’s, the linear term is much larger than the cubic terms and it appears only in \( h_{1}(\mathbf{k}) \). Therefore the spin along the direction \( \hat{n}_1 \) feels only the cubic terms and hence experiences a much weaker spin relaxation; whereas the spin along the other two directions feels the linear term and shows much shorter and similar SRT’s.

It is noted that the difference of the time evolutions of the spin polarizations between the helix and collinear statistics is marginal in all directions: The evolutions of spin polarization along \( \hat{n}_1 \) are almost identical while those along \( \hat{n}_2 \) show marginal differences after the initial fast spin relaxation. This is because the spin-orbit couplings along the latter two directions are much stronger than the one along \( \hat{n}_1 \). Nevertheless, it is shown that even for the very strong spin-orbit coupling strengths used in our calculation, the difference of the time evolution of the spin polarization between the helix and the collinear statistics is too marginal to affect the value of the SRT effectively. Therefore, for the spin polarized system, the statistics used in the scattering do not affect the SRT too much. We will further show this in the next subsection focusing on the spin relaxation along \( \hat{n}_1 \) as we are interested in the longest SRT along this direction.

B. Comparison of the SRT along direction (1,1,0) with helix and collinear statistics

In this subsection we compare the SRT’s along the special direction \( \hat{n}_1 \) in the helix and collinear statistics with the electron-nanomagnetic impurity and the electron-LO phonon scattering included. In Fig. 2 we plot the SRT as a function of the temperature for different impurity densities \( N_i = 0.1N_e, 0.5N_e \) and \( 1.0N_e \) with the electron density \( N_e = 4 \times 10^{11} \) cm\(^{-2} \). The solid curve: with the helix statistics; Dashed curve: with the collinear statistics.

![FIG. 1: The temporal evolutions of spin polarizations along (a): \( \hat{z} \); (b): \( \hat{n}_2 \); (c): \( \hat{n}_1 \) with the electron-LO phonon and the electron-impurity scattering included at temperature \( T = 200 \) K in the system with identical strengths of the Rashba and Dresselhaus coupling. The impurity density \( N_i \) is \( 0.5N_e \). Solid curves: in the collinear statistics; Dashed ones: in the helix statistics.](image)

![FIG. 2: Spin relaxation time vs. the temperature for different impurity densities \( N_i = 0.1N_e, 0.5N_e \) and \( 1.0N_e \) in the two statistics with the electron density \( N_e = 4 \times 10^{11} \) cm\(^{-2} \). Solid curve: with the helix statistics; Dashed curve: with the collinear statistics.](image)
It is seen from the figure that in both statistics the SRT’s along \( \hat{n}_1 \) decrease with increase of the temperature or with decrease of the impurity density. These results are consistent with our previous calculations\(^{18,19,31} \) as the inhomogeneous broadening induced by the EMF here depends cubically on the momentum \( k \). We have shown in our previous work\(^{18,31} \) that in GaAs QW’s when the cubic term of the Dresselhaus is dominant, the SRT decreases with the temperature.

From the figure one further finds that the SRT along \( \hat{n}_1 \) in the two statistics are almost the same. This is consistent with the result in the previous subsection. Thus it is suitable to study the SRT with one specific statistics, e.g., in the collinear statistics as the scattering can be treated more precisely and efficiently.

C. SRT along the direction (1,1,0) with the electron-electron scattering included in the collinear statistics

In the previous two subsections, we draw the conclusion that the SRT in the two statistics are almost the same. Although we did not include the electron-electron Coulomb scattering above as it is extremely difficult to compute the coulomb scattering quantitatively in the helix statistics numerically to meaningfully compare the SRT from the collinear statistics for the reason given in the previous section, we still believe inclusion of the Coulomb scattering would not bring a vast change to the SRT from the two statistics, especially for the polarization along \( \hat{n}_1 \) where the spin-orbit coupling is weak. Moreover, recent studies of spin kinetics have shown that the Coulomb scattering plays a crucial role in spin R/D.\(^{11,12,14,18,19} \) Therefore in this subsection we investigate the properties of the SRT along \( \hat{n}_1 \) in the collinear statistics with the Coulomb scattering explicitly included.

In Fig. 3 we plot the SRT versus the electron density with and without impurities at different temperatures \( T=120 \text{K} (\triangledown), 200 \text{K}(\bullet) \) and \( 300 \text{K}(\triangle) \). The solid curves are for case without impurity and the dashed ones are for the case with impurity \( (N_i = 0.5N_e) \). We find the impurity and the temperature dependence of the SRT with the electron-electron Coulomb scattering included is all the same with that in Sec. IIIB without the Coulomb scattering. By comparing the values at the same conditions in Fig. 2, it is easy to find that the SRT with the electron-electron scattering is larger, which is consistent with the previous results.\(^{17,18,19} \) It is also seen from the figure that the SRT decreases with the increase of the electron density and the rate of decrease is slower with increase of the temperature. It can be understood from the view of the inhomogeneous broadening.\(^{32} \) For higher electron density, electrons tend to populate to higher momentum space. Therefore the inhomogeneous broadening induced by the EMF becomes larger which results in a shorter SRT. Moreover, when the temperature is high enough, electrons prefer to distribute in high momentum space even at low electron densities. Therefore the inhomogeneous broadening becomes less sensitive to the change of the electron density. This explains why the SRT becomes insensitive to the electron density at high temperatures.

In the previous two subsections, we turn to the well width dependence of the SRT. It is noted that when the well width is changed, the relation \( \beta = \gamma \left( \frac{a}{a_0} \right)^2 \) is maintained by changing the external gate voltage. In Fig. 4 we plot the SRT versus the width of the QW for different electron densities \( N_e = 1 \times 10^{11} \) cm\(^{-2} \) (solid curves) and \( 4 \times 10^{11} \) cm\(^{-2} \) (dashed curves) at different temperature 120 K (curves with •) and 300 K (curves with ▲). The impurity density \( N_i = 0 \).
cm$^{-2}$ and $4 \times 10^{11}$ cm$^{-2}$ at $T = 120$ K and 300 K. The impurity is taken to be zero in the calculation. Contrary to the previous results in (001) QW’s with only the Dresselhaus spin-orbit coupling,\textsuperscript{18,19} here one finds that the SRT decreases with the increase of the well width. This is because in the present case with identical strengths of the Rashba and Dresselhaus spin-orbit coupling, the component of the EMF which can flip electron spin along $\hat{n}_1$-direction is only the cubic term of the Dresselhaus coupling, which is independent on the well width. Nevertheless, the linear EMF along the direction $\hat{n}_1$, electron spins prefer to line along $\hat{n}_1$. Therefore spin flipping along $\hat{n}_1$ is suppressed and thus the inhomogeneous broadening which leads to the spin relaxation along $\hat{n}_1$ is suppressed. Consequently the SRT increases with decrease of the well width.

IV. CONCLUSIONS

In conclusion, we study the spin relaxation under the identical Rashba and Dresselhaus coupling strengths in (001) GaAs QW by the kinetic spin Bloch equations with different statistics: the collinear and the helix statistics. We show that the SRT’s calculated from both the helix statistics and collinear statistics differ marginally in all directions. This is very important for the study of the spin R/D as most spin R/D calculations are performed with the collinear statistics and the numerical evaluation of the scattering within the collinear statistics is much more accurate and faster.

We show that when the cubic term of the Dresselhaus spin-orbit coupling is ignored, the SRT’s along (1,1,0) direction in both statistics are infinite. However, with the inclusion of the cubic term, the spin relaxation along this direction becomes finite, although still much longer than along the other two perpendicular directions. We study impurity density, temperature, electron density and the well width dependence of the SRT along this direction. We find the SRT increases with increase of the impurity density and decreases with increase of the temperature or the electron density. These properties are all the same with case of (001) QW with only the Dresselhaus spin-orbit coupling. However, the SRT decreases with increase of the well width, which is contrary to the previous results. All these behaviors are explained following our previous theories.

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APPENDIX A: THE KINETIC SPIN BLOCH EQUATIONS IN THE HELIX SPIN SPACE

In the helix spin space, the physics meaning of each matrix elements $\rho_{k,q',q}^h = \langle \eta' | \rho_{k} | \eta \rangle$ is not well defined as the helix spin state is a mixture of spin-up and spin-down states and is $k$ dependent [Eq. (2)]. Here the superscript $h$ denote the quantity in the helix spin space. The density matrices and the kinetic spin Bloch equations in the helix spin spaces can be transformed from that in the collinear spin space by a unitary transformation: $\rho_k^h = U_k^\dagger \rho_k U_k$, with

$$U_k = \frac{1}{\sqrt{2}} \left( \frac{1}{|\mathbf{h}(k)|} \mathbf{1} - \frac{i}{|\mathbf{h}(k)|} \right).$$

(A1)

The coherent terms are

$$\dot{\rho}_{k,+,-|\text{coh}} = -i\rho_{k,+,-|\text{coh}} + 2\text{Im}[P_{k,+}^h \rho_{k,+,-}],$$

(A2)

$$\rho_{k,+,-|\text{coh}} = -iP_{k}^h(\rho_{k,+}^h - \rho_{k,-}^h) - i\Delta_{k}^h \rho_{k,+,-},$$

(A3)

in which

$$P_{k}^h = -\sum_{q} V_{q} \{ \text{Re}[\rho_{k-q,-}^h] + i \text{Re}[h'(k,q)] \text{Im}[\rho_{k-q,+}^h] + i \text{Im}[h'(k,q)](f_{k-q,+}^h - f_{k-q,-}^h)/2 \},$$

(A4)

$$\Delta_{k}^h = 2|\mathbf{h}(k)| - \sum_{q} V_{q} \{ \text{Re}[h'(k,q)](f_{k-q,+}^h - f_{k-q,-}^h) - \text{Im}[h'(k,q)]\text{Im}[\rho_{k-q,+}^h] \}. $$

(A5)
In these equations \( h'(k, q) = \tilde{h}^*(k)\tilde{h}(k - q) / ||\tilde{h}(k)|| ||\tilde{h}(k - q)|| \). The scattering terms are

\[
\rho_{k|\text{scatt}}^h = \left\{ \pi N_i \sum_q |U_q|^2 \sum_{\xi, \xi_2} \delta(\varepsilon_{k, q, \xi} - \varepsilon_{k, \xi_2}) S_{k, k - q} T_{k - q, \xi}^h (S_{k - q, k} T_{k - q, \xi}^h - \rho_{k - q, k}^h) T_{k, \xi_2}^h \right. \\
+ \pi \sum_{q q', \lambda} |g_{qq', \lambda}|^2 \sum_{\xi, \xi_2} S_{k, k - q} T_{k - q, \xi}^h \\
\times \delta(\varepsilon_{k, q, \xi} - \varepsilon_{k, \xi_2} + \Omega_{qq', \lambda}) \left[ (N_{qq', \lambda} + 1)(1 - \rho_{k, q}^h) - N_{qq', \lambda} S_{k, q} (1 - \rho_{k, q}^h) \right] \\
+ \delta(\varepsilon_{k, q, \xi} - \varepsilon_{k, \xi_2} - \Omega_{qq', \lambda}) \left[ (N_{qq', \lambda} + 1)(1 - \rho_{k, q}^h) - N_{qq', \lambda} S_{k, q} (1 - \rho_{k, q}^h) \right] \right\} \\
+ \left\{ \ldots \right\} \quad (A6)
\]

with \( S_{k, k - q} = U_{k - q}^\dagger U_{k - q} = \frac{1}{2} [1 + h'(k, q)] + (1 - h'(k, q)) \sigma_z \) and \( T_{k, \xi}^h = U_{k, \xi}^\dagger T_{k, \xi} U_k \). In the helix spin space, \( T_{k, \xi}^h = T_{k, \eta}^h = 1/2 \) for the collinear statistics and \( T_{k, \xi}^h = T_{k, \eta}^h = \frac{1}{2} [1 + \eta \sigma_z] \) for the helix statistics.

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