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Influence of foundation mass and surface roughness on dynamic response of beam on dynamic foundation subjected to the moving load

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Abstract. In this paper, Improved Moving Element Method (IMEM) is used to analyze the dynamic response of Euler-Bernoulli beam structures on the dynamic foundation model subjected to the moving load. The effects of characteristic foundation model parameters such as Winkler stiffness, shear layer based on the Pasternak model, viscoelastic dashpot and characteristic parameter of mass on foundation. Beams are modeled by moving elements while the load is fixed. Based on the principle of the publicly virtual balancing and the theory of moving element method, the motion differential equation of the system is established and solved by means of the numerical integration based on the Newmark algorithm. The influence of mass on foundation and the roughness of the beam surface on the dynamic response of beam are examined in details.

Keywords: Improved Moving Element Method, dynamic foundation model, mass on foundation, roughness amplitude, roughness wavelength, moving load.

1. Introduction
The beam type structures on foundation is one of the most widely used in many fields of engineering like civil, industry, traffic infrastructures,… Especially, the structural elements which support the moving load of transportation means such as the foundation under moving load of vehicles of transport, railway…etc. Most published research papers, as consideration of analytical solutions for beam resting on foundation, the researchers have mainly employed the Winkler-Type Foundation [1]. This is the most classic model known as the one-parameter model assuming the soil below foundation is replaced by non-mass springs, linear elastic springs and springs are considered independent of each other. One of the short comings of the Winkler hypothesis is that assumes the foundation to closely spaced independent linear springs which are not affected beyond loaded region, thus the deformation of foundation is just limited load without taking into account for the affect of adjacent regions.
Consequently, it creates the interruption between loading and unloading foundation, but in reality the surface of soil foundation is not indicated any interruptions. From that point, these foundations have yet reflected the real characteristic response of soil foundation subjected to moving load ‘Figure 1’.

![Figure 1](image1.png)

(1) Winkler foundation  (b) Practical soil foundation

**Figure 1.** Displacement of elastic foundation under uniform pressure.

One of the ways to overcome the drawbacks in the Winkler model is to find a way describing the continuous interaction among the springs by adding to the above surface of non-mass spring layer such as bending beams, tensile stresses, shear layer or in consideration of the possible sliding of soil foundation, these layers have unchanged parameters and the characteristics of the continuous interaction of the springs in the Winkler model, the parameter is so called the second parameter. Igniting this idea proposed by Filonenko-Borodich [2] described the continuous interaction of springs by introducing a thin elastic tensile membrane under a constant tensile force T. Following this proposal, Pasternak [3] hypothesized that the top surface of the springs are fully connected to a beam whose beam is only subjected to sloping deformation with shear modulus G. The common characteristics of those foundation models which are regardless to the affects of soil foundation mass rest on response of the upper structure. In recent years, Q Do Kien and T Khong Trong [4] used the experimental results show that the foundation mass involved in oscillation has a significant effect on the dynamic behavior of the plate. Then T Pham Dinh et al. [5] also employed the experimental results to determine the effect parameters of the foundation parameters of foundation mass Mr. Hoang Phuong Hoa (2016)[5] has also employed the experimental results to define the affect mass on the kinetic behavior of a free-step system. The results also show that the influence of the characteristic parameters on the effect of the foundation mass αF on oscillation has a significant effect.

P Nguyen Trong et al. [6] systematized the foundation models and proposed a new foundation model used in the behavioral analysis of interaction structure on foundation. The author proposed a new foundation model incorporating the complete parameters such as elastic stiffness and stiffness of shear layer based on the Pasternak-Type Foundation, the coefficient of viscoelastic foundation and in particular, considering the effect of foundation mass on the behavior of the beam structure called a “dynamic foundation model”. Following up the afore-mentioned studies, P Nguyen Trong et al. [7] analyzed the affect of foundation mass parameter in the dynamic foundation model placed on the beam’s separate oscillation. The results show that the foundation mass parameters have a significant effect on the dynamic response of the beam, which increases the overall vibration mass of the beams, thereby reducing the oscillation frequency of the system.

Recently, many models of structures resting on viscoelastic and Pasternak foundation have been developed. H Luong Van et al. [8] and P Phung Van et al. [9] analyzed dynamics response of composite plates resting on viscoelastic foundation. P Phung Van et al. [10] analyzed dynamics response of Mindlin plates on viscoelastic foundation subjected to a moving sprung vehicle. T Nguyen Thoi et al. [11] also analyzed the dynamics response of composite plates on the Pasternak foundation subjected to moving mass. P Lou and F T K Au [12] have studied the response of Euler-Bernoulli beam under moving mass vehicles by employing an Finite Element Method (FEM). FEM has been used widely to solve many complicated problems, but encountered issues when the mass moves to the margin of the elements and also from one element to another while vector of moving mass must be updated at every time step. So as to make good those shortcomings.

C G Koh et al. [13] has proposed to put a moving coordinate to solve the proposed a moving mass of railway track. This method is called Moving Element Method (MEM). In this method, the railway is considered as an infinite Euler-Bernoulli resting on beam on Winkler foundation and the train is simplified by a “mass-spring-dashpot” system. T Tran Minh et al. [14] has employed MEM to study
the dynamic response of express railway under inconstant speed of moving mass. K K Ang et al. [15] has studied a calculation to employ MEM to examine the dynamic response of the rail on viscoelastic foundation with moving mass. K K Ang and J Dai [16] analyzed the reaction of the high-speed railway on foundation which has inconstant stiffness, the author employed the Moving Element Method to have analytical solutions for the response of the train. K K Ang et al. [17] has used MEM to research the dynamic response of the railway system. The railway model is as a mass spring system which includes train body, cross section and wheels. Recently, T Tran Minh et al. [18] also utilized the Moving Element Method to analyze the dynamics of the express railway. In which the railway track is modelled based on Euler-Bernoulli beam on the elastic two-parameter, the impacts of reducing velocity process and the roughness levels of railway track are also investigated. MEM has a lot of advantages such as the load would never approach the margin because the limited elements system always moves, and the moving load would not have to move from this element to another, so it avoids updating the mass vector. This methods enable the limited elements with different lengths and each interaction distance can be divided more effective. However, the weak point of MEM is that must be updated the the stiffness matrix and dashpot matrix at every time step. It resulted in increasing the volume of calculation, prolonging the time of analysis and wasting the resources.

T Nguyen Van [19] has recently analyzed the dynamics of structural beam on Pasternak foundation utilized the Improved Moving Element Method (IMEM). The author represented a new method based on the MEM, an aim to provide the method of solving the main differential equations in a faster manner and saving resources.

Within the context of this paper, the author will use the IMEM has been proposed to investigate the two-parameter viscoelasticity under moving load in consideration of the simultaneous effect of characteristic parameter for mass foundation and the influence of the roughness of beam surface. The mass matrices, stiffness matrices and damping matrices for moving elements are illustrated in details afterward. The obtained results are helpful documents for studying and designing the structural beam under practical moving load.

2. Theoretical basis
The two parameter viscoelastic foundation includes the effect of mass foundation referred to “the dynamics foundation”. The dynamics foundation model has investigated the complete mentioned-above parameters such as elastic stiffness $\bar{K}_w$, cross section, dashpots $\bar{c}$, foundation mass $m$.

The dynamic response of a finite length Euler-Bernoulli beam with Young’s elastic modulus $E$, moment of inertia $I$ and mass per unit length of the rail beam $m$ as shown in ‘Figure 2’.

The continuity in the dynamics foundation model is specified by the parameter of shear layer $k_s$ based on the cut layer of Pasternak foundation model. According to P Nguyen Trong et al. [6], The equation of motion expressing the relation between force and displacement at each position foundation under the effect of the load $q(x,t)$ which can be expressed as follows:

$$q(x,t) = \bar{k}_w w(x,t) - k_s \frac{d^2 w(x,t)}{dx^2} + \bar{c} \frac{dw(x,t)}{dt} + (\bar{m} + m) \frac{d^2 w(x,t)}{dt^2}$$  

(1)

![Figure 2. Beam model on dynamic foundation.](image-url)
The beam model on dynamic response under moving load pursuant to P. Nguyen Trong et al. [6] combined with the overhanging mass system proposed by C G Koh et al. [13] paper model is shown in the following:

**Figure 3.** The Beam resting on the dynamic foundation model subjected to a moving load.

The concentrated mass $m$, dynamic response of mass on oscillating foundation model can be written as follows:

$$m = \alpha_F \rho_F H_F$$

(2)

In which

- $\alpha_F$ experimental parameters characterize the effect of the mass foundation parameter;
- $\rho_F$ density mass foundation;
- $H_F$ depth of foundation;
- $\beta_F = \alpha_F H_F$ characteristic parameter affected by depth of foundation $H_F$ and experimental parameters $\alpha_F$ describe the continuous interaction of elastic springs Winkler model. The Eq. (2) can be re-written as follows:

$$m = \beta_F H_F$$

(3)

When applied to a system of force $F$ such that the displacement of a section $\Delta l = 1$ unit, can be expressed as follow:

$$F = \Delta l k = k$$

(4)

The moving vehicles have three mass: $m 1 \ m 2 \ m 3$ and denote one direction so it has three numeral degrees of freedom.
Figure 4. The diagrams define the mass matrices, stiffness, dashpot of vehicle motion.

Table 1. Determining the stiffness and dashpot of vehicle.

| Diagram 1 | Diagram 2 | Diagram 3 |
|-----------|-----------|-----------|
| Stiffness | Dashpot   | Stiffness | Dashpot   | Stiffness | Dashpot   |
| $k_{11}=k_1$ | $c_{11}=c_1$ | $k_{12}=-k_1$ | $c_{12}=-c_1$ | $k_{13}=0$ | $c_{13}=0$ |
| $k_{21}=-k_1$ | $c_{21}=-c_1$ | $k_{22}=k_1+k_2$ | $c_{22}=c_1+c_2$ | $k_{23}=-k_2$ | $c_{23}=-c_2$ |
| $k_{31}=0$ | $c_{31}=0$ | $k_{32}=-k_2$ | $c_{32}=-c_2$ | $k_{33}=k_2+k_3$ | $c_{33}=c_2+c_3$ |
| $k_{d1}=0$ | $c_{d1}=0$ | $k_{d2}=0$ | $c_{d2}=0$ | $k_{d3}=-k_3$ | $c_{d3}=-c_3$ |

Mass matrices can be expressed in the following:

$$M_{\text{vehicle}} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$ \hspace{1cm} (5)

$$C_{\text{vehicle}} = \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1+c_2 & -c_2 \\ 0 & -c_2 & c_2+c_3 \end{bmatrix}$$ \hspace{1cm} (6)

$$K_{\text{vehicle}} = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_3 \end{bmatrix}$$ \hspace{1cm} (7)

According to the coordinates in the Figure 3, the general equation of the car model Q Do Kien and H Luong Van [21] can be expressed mathematically as follows

$$m_1 \ddot{u}_1 + c_1 (\dot{u}_1 - \dot{u}_2) + k_1 (u_1 - u_2) = -m_1g$$ \hspace{1cm} (8)

$$m_2 \ddot{u}_2 + c_2 (\dot{u}_2 - \dot{u}_3) + c_1 (\dot{u}_1 - \dot{u}_2) + k_2 (u_2 - u_3) - k_1 (u_1 - u_2) = -m_2g$$ \hspace{1cm} (9)

$$m_3 \ddot{u}_3 - k_2 (u_2 - u_3) - c_2 (\dot{u}_2 - \dot{u}_3) = -m_3g + F_c$$ \hspace{1cm} (10)
in which 

$m_1, m_2, m_3; c_1, c_2, c_3; k_1, k_2, k_3$

in turn are mass, dashpots of the vehicle, vertical springs and wheels;

$u_1, u_2, u_3, \ddot{u}_2, \ddot{u}_3; u_5, u_6, u_7$

in turn are vertical displacements, velocity, car body acceleration, and wheel-axle;

$g$

the gravitational acceleration;

$F_c$

the contact force between wheels and beam, caused by the non-flat of the beam or the roughness of the beam.

The contact force $F_c$ (with the roughness at the contact point between the moving load and the beam) is defined according to C G Koh et al. [13] as follows:

$$F_c = c_3 (\ddot{u}_d - \ddot{u}_3) + k_3 (u_d - u_3) + F_i$$

(11)

where: $F_i = c_3 \ddot{y}_d + k_3 y_i$ the track force, produced by the roughness of the beam;

$u_d$ denotes the vertical displacement at the contact point of the beam;

$u_3$ denotes the vertical displacement of the wheel-axle;

$y_i$ denotes the magnitude of the track irregularity at the contact point, according to C G Koh et al. [13], the track irregularity profile can be written in terms of a sinusoidal function as follows:

$$y_i = a_i \sin \frac{2\pi S}{\lambda_i}$$

(12)

where $a_i, \lambda_i$ denotes the amplitude and the roughness wavelength on beam, respectively;

$S$ denotes the road section that the object is moving.

In the moving element method, C G Koh et al. [13] which uses x-y coordinates. Where x axis is the beam course. The moving r-y coordinates whose origin is attached to the contact force as in Figure 5. Therefore, this coordinates moves along with the velocity $V$ as a moving load.

$$x = r + s$$

$$y = y$$

(13)

where: $x$ = fixed axis; $r$ = movable axis; $s$ = displacement ; $V(a,t)$ = velocity function; $t$ = moving time; $a$ = acceleration.

The connection between the derivative operators of the coordinates when the load moves with various velocities as follows:

$$\frac{\partial^4 w(x,t)}{\partial x^4} = \frac{\partial^4 w'(r,t)}{\partial r^4}$$

(14)

$$\frac{\partial^2 w(x,t)}{\partial x^2} = \frac{\partial^2 w'(r,t)}{\partial r^2}$$

(15)
\[
\frac{\partial^2 w(x,t)}{\partial t^2} = \frac{\partial^2 w(r,t)}{\partial t^2} - a \frac{\partial w(r,t)}{\partial r} \frac{\partial}{\partial t} + V \frac{\partial^2 w(r,t)}{\partial r^2} - 2V \frac{\partial^2 w(r,t)}{\partial r \partial t} \tag{16}
\]

\[
\frac{\partial w(x,t)}{\partial t} = \frac{\partial w(r,t)}{\partial t} + \frac{\partial w(r,t)}{\partial r} \frac{\partial}{\partial t} - V \frac{\partial^2 w(r,t)}{\partial r^2} \tag{17}
\]

where \( w(x,t) \) is the transverse deflection of the beam in the \( x-y \) axial coordinates; \( w(r,t) \) is the deflection of the beam in \( r-y \) coordinates.

By applying the principle of virtual work and using displacement functions \( N \), we can write \( M^e, C^e, K^e \) as the generalized mass, dashpots, and stiffness matrices of the beam as follows:

\[
M^e = \left( \bar{m} + m \right) \int_0^r N^T \mathbf{N} \, dr \tag{18}
\]

\[
C^e = \bar{c} \int_0^r N^T \mathbf{N} \, dr \tag{19}
\]

\[
K^e = EI \int_0^r \left( N_{.,rr} \right)^T N_{.,r} \, dr + \bar{k} \int_0^r N^T \mathbf{N} \, dr - k_i \int_0^r N^T N_{.,r} \, dr \tag{20}
\]

\[
F_1' = 2 \left( \bar{m} + m \right) V \int_0^r N^T \mathbf{N} \, dr \tag{21}
\]

\[
F_2' = \left[ \left( \bar{m} + m \right) a + \bar{c} V \right] \int_0^r N^T \mathbf{N} \, dr - \left( \bar{m} + m \right) V^2 \int_0^r N^T N_{.,r} \, dr \tag{22}
\]

\[
P = \int_0^r F_i' \mathbf{N}^T \, dr \tag{23}
\]

with \( .,r \) and \( .,rr \) in turn are the first derivative and second derivative of \( r \).

To elements of the beam, the Hermition interpolation \( N \) is written as follows:

\[
N_1^e = \frac{1}{(l^e)^3} \left[ 2r^3 - 3r^2 l^e + (l^e)^3 \right] \tag{24}
\]

\[
N_2^e = \frac{1}{(l^e)^3} \left[ r^3 l^e - 2r^2 (l^e)^2 + r(l^e)^3 \right] \tag{25}
\]

\[
N_3^e = \frac{1}{(l^e)^3} \left[ -2r^3 + 3r^2 l^e \right] \tag{26}
\]

\[
N_4^e = \frac{1}{(l^e)^3} \left[ r^3 l^e - r^2 (l^e)^2 \right] \tag{27}
\]

Based on finite element method and using the numerical degree of freedom technique respectively to matrices of the general coordinates elements, the moving equation of the whole beam model on the foundation is written as follows:

\[
M \ddot{\mathbf{z}} + (C - F_1) \dot{\mathbf{z}} + (K - F_2) \mathbf{z} = \mathbf{P} \tag{28}
\]
where: $M$, $C$, $K$, $P$ respectively are global mass, damping and stiffness matrices and the global load vector; $F_1$, $F_2$ denote those elements which depends on time, $F_1$ and $F_2$ are not forces but have the force unit so they can be considered to be Pseudo-force.

The Eq. (28) is the main differential equation of the traditional MEM, in the Eq. (28), we can see the left side is comprised of elements which change over time, those elements are the Pseudo-force $F_1$ and $F_2$ matrices. Therefore, when solving the problem we need to update the global mass, damping and stiffness matrices and this prolongs the processing time.

To fix this limitation of the traditional MEM, we like to move the Pseudo-forces from the left side of the Eq. (28) to the right side. This idea is called Improved Moving Element Method. After the moving, the Eq. (28) is written as follows:

$$M\ddot{z} + C\dot{z} + Kz = P + F_1\dot{z} + F_2\ddot{z}$$

(29)

Solving the differential motion Eq. (29) is put to act upon the help of computer which is based on Newmark algorithm. This algorithm is a calculation program written by Matlab language and the reliability as well as the calculation method of the program are compared to the results of other authors which are available in the reference.

3. Survey sample

3.1. Verifying the calculation program

In this part, the article examines some numerical examples to verify the correctness and the reliability of the Matlab program. The results are compared to those of other authors. Here is the verification of the high speed train moving on beam with hanging mass which is used by C G Koh et al. [13] ‘Figure 3’. The parameters of the train, the beam and the foundation are demonstrated in Table 2 and Table 3.

| Table 2. Vehicle parameters. |
|-----------------------------|
| Car Body |
| Bogie |
| Wheel-axle |
| $m_1$ | 3500 kg |
| $m_2$ | 250 kg |
| $m_3$ | 350 kg |
| $k_1$ | $1.41\times10^5$ N/m |
| $k_2$ | $1.26\times10^6$ N/m |
| $k_3$ | $8\times10^9$ N/m |
| $c_1$ | $8.87\times10^3$ Ns/m |
| $c_2$ | $7.1\times10^3$ Ns/m |
| $c_3$ | $6.7\times10^5$ Ns/m |

| Table 3. Beam and Foundation parameters. |
|-----------------------------|
| Beam |
| Foundation |
| $\bar{m}$ | 60 kg/m |
| $\bar{k}_w$ | $1\times10^7$ N/m |
| $E$ | $2\times10^{11}$ N/m$^2$ |
| $l$ | $3.06\times10^{-5}$ m$^4$ |
| $L$ | 50 m |
| $c$ | 4900 Ns/m |

In the first example, the beam is displaced while the train is moving on the beam with constant velocity, without consideration of the second foundation parameter affection. (velocity $V=20$m/s, roughness amplitude margin $a=0.5$mm and roughness wavelength $\lambda=0.5$m).
In the next verification, the beam is displaced when the train moves on the beam with changeable velocity, without consideration of the second foundation parameter affection (first velocity $V_0=0$m/s, then moving with constant acceleration $a_{max}=10$m/s$^2$, after 2 seconds it reaches the velocity $V_{max}=20$m/s, then it moves with constant deceleration $a_{min}=-10$m/s$^2$, and it stops after 2 seconds. The total analyzing time is $t=6$s, without consideration of the beam roughness).

From these surveyed results, compared to those of other authors and the results of the paper indicates that is well-matched with others as stated in the references. It proves the calculation program is reliability. Thence, we have the groundwork to continue to analyze the affection of foundation parameters, mass model, the roughness of the beam surface meeting the dynamic response of beam.

3.2. Numerical survey result
In the paper, the overhanging mass parameters and beam parameters are shown in Table 2 and Table 3. They will be used to investigate the paper’s problems.

**Case 1:** Survey influence of mass foundation on dynamic response of beam when the vehicle is moving with constant velocity, regardless to the affect of the beam roughness.
In the first survey, the paper researches the moving vehicles with velocity $V=20$m/s. The characteristic parameters of dynamics model proposed by T Pham Dinh et al. [22] written as: $k_v=1\times10^5$N/m, $k_s=66.7\times10^5$N, $c=1.5\times10^4$Ns/m and $\rho_f=1800$kg/m$^3$. The characteristic parameters $\beta_F$ show the simultaneous influence of vertical depth foundation $H_F$ and experimental parameters $\alpha_F$; they are in turn to investigate such as $\beta_F=0; \beta_F=0.5; \beta_F=1; \beta_F=1.5; \beta_F=2; \beta_F=2.5$ and $\beta_F=3$. 
Figure 8. The displacement of beam at interaction point.

From the results as shown Figure 8 which displays the characteristic parameters for mass foundation $\beta_F$ has significant influence on oscillation of beam, with the increase in $\beta_F$ mass which will increase in mass foundation $m$ participating the oscillation that is equal to the jump of participating the overall oscillation of beam, which will weaken the beam or in other way, it will soften the beam. Thus the displacement of the beam will also jump up and reach out the absolute value.

Case 2: Conducting survey of simultaneous effect of mass foundation and load velocity of dynamic behaviour of beam.

In the second survey, the paper researches the moving vehicles on beam with constant velocity, in turn: $V=10\text{m/s}$; $20\text{m/s}$; $30\text{m/s}$; $40\text{m/s}$; $50\text{m/s}$; $60\text{m/s}$; $70\text{m/s}$; $80\text{m/s}$; $90\text{m/s}$; $100\text{m/s}$; $110\text{m/s}$; $120\text{m/s}$; $130\text{m/s}$; $140\text{m/s}$; $150\text{m/s}$; $160\text{m/s}$; $170\text{m/s}$; $180\text{m/s}$; $190\text{m/s}$; $200\text{m/s}$. All characteristic parameters of dynamic foundation model is derived the same as Case 1, the characteristic parameters alone $\beta_F$ for the affect of mass foundation will take turn to be conducted survey as $\beta_F=0$; $\beta_F =0.5$; $\beta_F =1$; $\beta_F =1.5$ and $\beta_F =2$.

Figure 9. The max displacement of beam at interaction point.

In this problem, the effect of characteristic parameters of foundation model and dynamic response of beam subjected to moving load as given the different values of velocity parameters to be
considered. The results of analysis as shown ‘Figure 9’. The moving velocity is recognized $V=10\text{m/s}$ up to $V=70\text{m/s}$. The larger mass on oscillating foundation is, The higher displacement of beam is, on the contrary, when velocity above $V=80\text{m/s}$ up, if mass on oscillating foundation on beam system is high then the displacement of beam is reduced and proximity to a certain value.

The results as shown ‘Figure 9’ indicate the mass parameters on foundation has considerable influence on the dynamic characteristics of the system and from the point, it increases dynamic response of beam which is equivalent to the increase in characteristics parameters affected by mass on foundation. Simultaneously, the results obtained also to indicate that the characteristic parameters of moving vehicle as the velocity of movement which has a significant effect on the dynamic response of the beam structure.

**Case 3:** Conducting survey of simultaneous effect of the mass on foundation and the roughness amplitude on beam of dynamic behaviour of beam.

In the following survey, the paper surveys the moving vehicle on beam with the velocity $V=60\text{m/s}$, the parameters on dynamic foundation model is derived from the Case 1, the specific parameters $\beta_F$ offer the effect of mass on foundation which will take turn to be surveyed as $\beta_F =0; \beta_F =0.5; \beta_F =1; \beta_F =1.5$ and $\beta_F =2$. In addition, the roughness on the beam is represented as a function of time regulation on the surface of beams $y_t=at\sin(2\pi S/\lambda t)$ with $\lambda t$ in terms of amplitude and wavelength of the roughness of the beam surface. In this problem keeps the whole roughness wavelength on beam $\lambda t=0.5\text{m}$, the change in the roughness amplitude in turn $at=0.5\text{mm}; 1.5\text{mm}; 2.0\text{mm}; 2.5\text{mm}; 3.0\text{mm}; 3.5\text{mm}$ and $4.0\text{mm}$.

![Figure 10](image)

**Figure 10.** Maximum displacement of beam keeps the whole roughness wavelength on beam $\lambda_t=0.5\text{m}$ changing the specific parameter on foundation mass $\beta_F$ and the roughness amplitude.

**The effect of the roughness amplitude on beam:** Based on the results obtained in ‘Figure 10’ when keeping the whole roughness wavelength on beam $\lambda_t=0.5\text{m}$ and the characteristic parameters of the foundation mass $\beta_F$ only changes in the amplitude of the roughness. The results show that as increasing the value of roughness amplitude on beam the value of displacement of beam increase. The bigger value $a_t$ is, the bigger value of beam displacement is. This proves that the displacement of beam depends very large on the amplitude of roughness on beam, when the roughness amplitude inches up, then the value of displacement on beam also inches up at the almost linear rate.

**The effect of the mass characteristic parameter of foundation $\beta_F$ considering the roughness on the surface of beam:** The analyzed results as shown ‘Figure 10’ indicates that if considering the effect of roughness amplitude on beam then dynamic response of beam is specified as follows: the roughness amplitude within the approximately from $a_t=0.5\div1.5\text{mm}$ when increasing the value of characteristic
parameters $\beta_F$ of foundation mass, the displacement value also ascents. On the contrary, when the roughness amplitude from 1.5mm up above. As the increase in value of characteristic parameters $\beta_F$ of mass foundation, then the displacement value also decreases, the higher value of $\beta_F$ is, the lower value of displacement of beam is. But when the mass has reached the certain value then the displacement increases insignificantly. Thus, the mass parameters of foundation is fairly important, it will increase the dynamic response of system as the beam surface is flat or the roughness of amplitude is small. On the contrary, decreasing the dynamic response of system when the beam surface has greater roughness of amplitude.

**Case 4:** Conducting survey of simultaneous effect of the mass on foundation and the roughness amplitude on beam of dynamic behaviour of beam.

In the survey, the parameters of the problem is derived from the Case 3, the roughness alone of the problem still keeps the whole roughness amplitude on beam at $a_t=0.5\text{mm}$, the alternation of the roughness wavelength $\lambda_t=0.5\text{m}; 1.0\text{m}; 1.5\text{m}; 2.0\text{m}; 2.5\text{m}; 3.0\text{m}; 3.5\text{m}; 4.0\text{m}$ and $4.5\text{m}$.

![Figure 11. The maximum displacement of beam as the roughness amplitude on beam is kept as a whole by $a_t=0.5\text{mm}$, the change in the characteristic parameters of mass on foundation $\beta_F$ and the roughness wavelength.](image)

Based on the results of Figure 11 with the value of roughness wavelength on beam is big then the displacement of beam is reducing, on the other hand, when the wavelength reaches a certain value, the displacement diagram tends to move sideways to a certain value, specially as $\beta_F=0.5$ then the displacement value will be $ud=-1.5140\text{mm}；-1.4394\text{mm}；-1.4207\text{mm}；-1.4116\text{mm}；-1.4053\text{mm}；-1.4004\text{mm}；-1.3965\text{mm}$ and $-1.3932\text{mm}$ corresponding to the wavelength value which in turn $\lambda_t=0.5\text{m}; 1.0\text{m}; 1.5\text{m}; 2.0\text{m}; 2.5\text{m}; 3.0\text{m}; 3.5\text{m}$ and $4.0\text{m}$.

However, the value of roughness wavelength on beam reaches at a certain fixed value, if the value of characteristic parameters of mass on foundation then the displacement value of beam will increase, the bigger value $\beta_F$ is, the increasing of displacement value of beam is, it is specified as the roughness of wavelength $\lambda_t=1.5\text{m}$, the value of displacement: $ud=-1.4767\text{mm}；-1.4207\text{mm}；-1.4637\text{mm}；-1.5446\text{mm}$ and $-1.6273\text{mm}$ respectively to the characteristic parameters of mass on foundation in turn as written $\beta_F =0; \beta_F =0.5; \beta_F =1.0; \beta_F =1.5$ and $\beta_F =2.0$. 

12
4. Conclusions

The paper has presented the generalized results of dynamic beam motion analysis on moving load considering the concurrent influence of the characteristic parameters for the effect of foundation mass and the roughness of beam surfaces as well as the velocity of the moving load. The motion equation of the structural system is used based on Improved Moving Element Method. The results are obtained to display that the mass on oscillating foundation has the influence on the oscillation of beam. With the increase in the characteristic parameters of foundation mass $\beta F$ will increase the mass on oscillation foundation $m$, it is equivalent to the increase in the general mass on oscillation foundation of system, from that point, the system will be weakened, consequently the displacement amplitude of beam will be also raised up.

On the other hand, if considering the effect of the roughness amplitude on beam then the dynamic response of beam will be greatly effected. Specially the roughness amplitude increases from the zero (flat) to $at=1.5\text{mm}$. The higher parameter value $\beta F$ of the mass on foundation is, the bigger displacement value of beam is. In reverse, when the roughness amplitude is $1.5\text{mm}$ bigger as increasing the characteristic parameter value $\beta F$ of the mass on foundation the displacement value of beam is decreased but when the mass reaches to a certain value, the displacement is increased insignificantly. Therefore, the parameter mass on foundation is extremely important, it creates the increase in dynamic response of system when the surface of beam is almost smooth (flat) or the roughness amplitude is small. In reserve, dynamic response of system is decreased when the beam surface has a bigger amplitude of roughness.

In addition, the velocity of moving load also has influence considerately on dynamic behaviour of beam, the study results obtained to show that the higher velocity of motion is, the lower displacement of beam is.

The value of roughness wavelength on beam is rising then the displacement of beam is going down and when the wavelength reaches up to a certain value, the displacement diagram tends to move horizontally asymptotically to a certain value.

The analytical solution results of the paper presents the study of determining accurately the parameter values of mass on oscillating foundation which is meaningful and proper with the practical application, particularly its application in analytical problem meeting the requirements on the dynamic response of the structural system interacting with the foundation models under moving loads. Besides, the unevenness of the structure surface and the element motion of velocity of load which is absolutely necessary to be considered and correct to its best feature of the structural beam.

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