Nonlinear buffer layers relevant for reduced nonlinear effects in HTS microwave devices

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Abstract. Microwave devices made of a High-Temperature Superconductor (HTS) exhibit a nonlinear response as the microwave power increases. The HTS nonlinearities generate a nonlinear inductance \( L_d(i_{rf}) \) and a nonlinear resistance \( R_d(i_{rf}) \) in a device. \( L_d(i_{rf}) \) and \( R_d(i_{rf}) \) are responsible for an increase of the device loss, a small frequency dispersion as well as the generation of spurious signals like Intermodulation Distortion (IMD). Nevertheless, the HTS nonlinearities in a microwave device can be reduced using a nonlinear dielectric like a ParaElectric Material (PEM). This assumption has recently been demonstrated theoretically. In a microwave device made of a HTS and a PEM, the nonlinear contribution to the capacitance \( C_d(v_{rf}) \) from the PEM acts oppositely to the nonlinear contribution to \( L_d(i_{rf}) \). This may cancel the effect of the HTS inductive nonlinearities. The PEM also produces a nonlinear conductance \( G_d(v_{rf}) \) in a device. All these nonlinear terms contribute to the IMD output power and the nonlinear quality factor \( Q_0 \) of a resonant passive microwave device. In this paper, the dependence of the different nonlinear contributions on frequency and applied dc bias voltage \( (V_{dc}) \) is investigated. The relevance to employ PEM in order to reduce the nonlinearities in HTS microwave devices is discussed.

1. Introduction
In a microwave passive device made of a High-Temperature Superconductor (HTS) and a ParaElectric Material (PEM), two different sources of nonlinearities are active. On the one hand, the HTS produces conducting nonlinearities whereas, on the other hand, the PEM generates dielectric nonlinearities. HTS nonlinearities are characterized by a nonlinear inductance \( L_d(i_{rf}) \) and a nonlinear resistance \( R(i_{rf}) \). The nonlinear inductance \( L_d(i_{rf}) \) comes from the nonlinear variation of the pairs densities \( n_s \) as a function of the microwave current, \( i_{rf} \). For a \( d \)-wave superconductor, the pair breaking effect sets the intrinsic limit of the HTS nonlinear response [1]. For weak nonlinearities, the quadratic form given in (1) and (2) usually fits closely the experimental \( L_d(i_{rf}) \) and \( R_d(i_{rf}) \).

\[
L_d(i_{rf}) = L_{d0} + \Delta L |i_{rf}|^2 \\
R_d(i_{rf}) = R_{d0} + \Delta R |i_{rf}|^2
\]  

Extrinsic contributions from defects at moderate \( i_{rf} \) are also described by the quadratic form [2].

Besides, the dielectric constant \( (\varepsilon) \) and the loss tangent \( (\tan\delta) \) of PEM depend nonlinearly on an applied dc or rf voltage. This property is advantageous to build agile devices controlled by an external...
$dc$ bias voltage ($V_{dc}$). At microwave frequencies, $\varepsilon_r(v_{rf})$ and $\tan\delta(v_{rf})$ generate a nonlinear capacitance $C_d(v_{rf})$ and a nonlinear conductance $G_d(v_{rf})$ in a device [3]:

$$C_d(v_{rf}) = C_{d0} - \Delta C \cdot |v_{rf}|^2$$  \hspace{1cm} (3)

$$G_d(v_{rf}) = G_{d0} + \Delta G \cdot |v_{rf}|^2$$  \hspace{1cm} (4)

Extrinsic causes might also contribute to $C_d(v_{rf})$ and $G_d(v_{rf})$. At low enough $v_{rf}$ they are also modeled by the quadratic dependence. The terms defined in (1)-(4) are given per unit length.

2. Expression for the nonlinear signal

In this section the nonlinear response of a Coplanar Transmission Line (CTL) resonator is investigated. The assumption is made that the PEM is integrated as a thin layer between the substrate and the HTS (Figure 1). The PEM is supposed to be very thin to minimize the dielectric loss [4].

![Figure 1 Schematic cross-section of a CTL made of a thin PEM layer grown onto a low-loss substrate. The conductor is a HTS material grown on top of the PEM.](image)

In a microwave TL, the output signals include a contribution of each distributed element [4]. The nonlinear telegrapher’s equations written for a model CTL resonator yield an expression of the output current (Eq. 1) and power (Eq. 2) at the IMD frequencies ($\omega_{12}$) [5]:

$$I_{12} = -\frac{9}{16} \frac{Q_i}{\omega_{12} L_{12}} I_{12}^4 \left[ (\Delta R + j\omega_{12} \Delta L) + \left| Z_{12} \right| \left( \left| Z_{01} \right|^2 \Delta G + j\omega_{12} \Delta C \right) \right]$$  \hspace{1cm} (5)

$$P_{12} = \frac{1}{4} \beta^2 \left| R_{12} \right|^2 \left| I_{12} \right|^2 \left[ \left| Z_{12} \right|^2 + \left| Z_{01} \right|^2 \left| I_{12} \right|^2 \right]$$  \hspace{1cm} (6)

Where $I_i$ is the current in each tone, $Q_i$ the loaded quality factor, $\beta$ a coupling factor and $l$ is the resonator length. From the general definition of the unloaded quality factor ($Q_0$) of a resonant TL, the following analytic expression is obtained:

$$Q_0 = \frac{\omega}{2} \frac{L_{10} + C_{10}}{R_{10} + G_{10}} \left| Z_{10} \right|^2 + \frac{3}{4} \left( \Delta L + \Delta C \left| Z_{10} \right|^2 \right) I_{max}^2$$  \hspace{1cm} (7)

The expressions (5)-(7) depend on all the nonlinear contributions. In (6) the absolute value of the sum of all nonlinear terms gives the IMD signal level. In (7), the nonlinear $Q_0$ is a function of the ratio between the nonlinear reactive contributions and the nonlinear resistive ones. Experimental observations have shown that, at microwave frequencies, $\Delta R$, $\Delta L$ [6] and $\Delta G$ are positive contributions to the output signals [3]. $\Delta C$ is given a negative sign in (3) as suggested by the frequency variation according to $v_{rf}$ for a resonator integrating a PEM thin film [3]. This is the fundamental condition to observe reduced reactive nonlinearities in a HTS device. It is worth noting that the resistive
nonlinearities could not cancel as $\Delta R$ and $\Delta G$ have the same sign. $\Delta R$ and $\Delta G$ are usually small compared to $\omega \Delta L$ and $\omega \Delta C$ for very high quality HTS and PEM films.

3. Frequency and dc voltage dependence of the nonlinear parameters
In this section, the frequency and dc voltage ($V_{dc}$) dependence of the nonlinear parameters $\Delta A$ ($A = R, L, C$ or $G$) is investigated. In the case a CTL with the cross-section presented in figure 1, the thickness of the PEM is much smaller than the low-loss dielectric substrate. The dielectric losses are then considered to be low and the approximation $\omega \varepsilon_{PEM} \gg \gamma_{PEM} \tan \delta_{eff}$ is valid. The HTS conducting losses are also assumed to be low and the approximation $\omega \Delta L \gg R_{d0}$ applies. With these approximations the characteristic impedance can be written as: $Z_0 = \left(\frac{L_{d0}}{C_{d0}}\right)^{1/2}$.

The nonlinear terms $\Delta A$ of the expressions (1)-(4) are the second elements of a Taylor series. They are related to the second derivative of the parameter $A_d(\alpha_{rf})$ relative to $\alpha_{rf}$, with $\alpha = i, v$. In the following, it is assumed that the PEM is far from saturation. Close to saturation, $\varepsilon_{r_{PEM}}$ and $\tan \delta_{PEM}$ are constant with respect to the applied $\text{dc}$ or $\text{rf}$ voltage.

3.1 Dependence of $\Delta L$ and $\Delta R$ on frequency and bias voltage
$L_d$ is a function of the penetration depth ($\lambda$). It does not depend on frequency for intrinsic nonlinearities, as the intrinsic $\lambda$ is frequency independent. However, $L_d$ might have implicit frequency dependence if extrinsic sources of nonlinearities like vortex flow are active. Some extrinsic contributions to $\lambda$ might make $\Delta L$ frequency dependent. Anyway, the nonlinear HTS reactive contribution ($\omega \Delta L$) is explicitly frequency dependent. The resistance per unit length $R_d$ is geometrically related to the surface resistance ($R_s$). $R_s$ depends on the square of the frequency and on the conductivity [7]. $\Delta R$ is then frequency dependent. On another hand, $\Delta R$ and $\Delta L$ should not depend on an applied $V_{dc}$, as $L_d$ and $R_d$ are not supposed to vary according to $V_{dc}$. Nonetheless, a small variation is expected due to the frequency shift as $V_{dc}$ is applied to the PEM and because of the frequency dependence of $\Delta R$ and $\Delta L$.

3.2 Dependence of $\Delta G$ and $\Delta C$ on frequency and bias voltage
The intrinsic properties of a PEM are strongly dependent on the polarization state of the material. The capacitance $C_d$ of a CTL integrating a thin PEM layer can be written as [8]:

$$C_d = C_0 \times (q_0 + q_{sub} \varepsilon_{sub} + q_{PEM} \varepsilon_{PEM})$$

(8)

Where $\varepsilon_{sub}$ and $\varepsilon_{PEM}$ are the dielectric constants of the substrate and the PEM respectively, the $q_i$ are the filling factors for each dielectric, and $C_0$ is the air contribution to $C_d$. For a resonant microwave device, $Q_0$ includes contributions from the conductor and from the dielectric substrate [10]:

$$\frac{1}{Q_0} = (\gamma_s^* R_s + \gamma_d^* \tan \delta_{eff})$$

(9)

Comparing (9) with the linear part of $Q_0$ given in (7), a formulation for $G_d$ can be obtained:

$$G_d = \omega C_d (\gamma_s^* R_s + \gamma_d^* \tan \delta_{eff})$$

(10)

Where the $\gamma_i$ are geometrical factors, $R_s$ is the surface resistance of the HTS film and $\tan \delta_{eff}$ is the effective loss tangent of the PEM buffered substrate [9]:

$$\tan \delta_{eff} = q_{sub} \varepsilon_{sub}^* \tan \delta_{sub} + q_{PEM} \varepsilon_{PEM}^* \tan \delta_{PEM}$$

(11)
In the case of PEM single crystals, \( \varepsilon_{\text{r, PEM}} \) does not depend on frequency until \( f \sim 100 \) GHz. However, a nonlinear frequency dispersion is often observed experimentally in the case of PEM thin films [10], likely caused by strains and defects. The frequency dependence of \( \varepsilon_{\text{r, PEM}} \) is then related to the quality of the PEM thin layer and the \( \Delta C \) dependence on frequency is function of that of \( \varepsilon_{\text{r, PEM}} \). Anyway, the PEM reactive contribution to the nonlinearities \( \omega \Delta C \) is explicitly frequency dependent.

On another hand, the analytic expression giving the variation of \( \varepsilon_{\text{r, PEM}} \) as a function of an applied dc bias field shows that it can be derived infinitely [11]. \( \varepsilon_{\text{r, PEM}} \) is also strongly dependent on the applied rf voltage and \( C_d(v_{\text{rf}}) \) can be developed as a Taylor series with infinite orders, which makes \( \Delta C \) voltage dependent [12].

The dependence of \( \Delta G \) can be determined from (10). \( \Delta G \) is related to the second derivative of \( G_d(v_{\text{rf}}) \) with respect to the applied microwave voltage. In (10) the geometric factors are constant and \( R_s \) is not supposed to change according to \( V_{\text{dc}} \), although it has a quadratic frequency dependence.

\[
\frac{\partial G_d}{\partial V_{\text{rf}}} = C_d \left( \gamma R_s + \gamma_s \tan \delta V_{\text{rf}} \right) \frac{\partial \omega}{\partial V_{\text{rf}}} + \omega \left( \gamma R_s + \gamma_s \tan \delta V_{\text{rf}} \right) \frac{\partial C_s}{\partial V_{\text{rf}}} + \omega \frac{C_s}{V_{\text{rf}}} \tan \delta V_{\text{rf}} \right) \frac{\partial \delta V_{\text{rf}}}{\partial V_{\text{rf}}} (12)
\]

The first derivative of \( \Delta G \) given in (12) shows that its second derivative with respect to \( V_{\text{rf}} \) includes terms that depend on frequency and \( V_{\text{dc}} \), as long as the PEM is not saturated. We can thus assume that \( \Delta G \) will be frequency and \( V_{\text{dc}} \) dependent. This gives the possibility to change the strength of \( \Delta G \) via an applied dc bias field. \( \Delta G > 0 \) enforces the dependence of \( Q_0 \) on the microwave power, increasing simultaneously the IMD output power level. If \( \Delta G \) can be changed applying a \( V_{\text{dc}} \), the nonlinear response of the structure presented in figure 1 would be modified. In that sense, it is interesting to investigate how the \( Q_0 \) power dependence varies with \( \Delta G \). Figure 2 presents the \( Q_0 \) power dependence given in (7) in the case of a 50 Ohms CTL resonator with the cross-section of figure 1. The simulation results are plotted for several values of \( \Delta G \), versus the circulating power defined as \( P_{\text{circ}} = 1/2Z_0* I_{\text{max}}^2 \).

They show improved power handling for the smallest \( \Delta G \) values. \( I_{\text{max}} \) is the peak current at the center of the resonator. \( \Delta C \) was chosen constant in the simulation of figure 2.

![Figure 2](image_url)

Figure 2 Variation of the nonlinear \( Q_0 \) normalized to its corresponding maximal low-power value for different values of the nonlinear reactance \( \Delta G \): solid line: \( -\Delta G \), \( -10*\Delta G \), \( -\Delta G/10 \)

Besides, the frequency dependence of the different nonlinear terms makes the conditions to observe a compensation frequency dependent. This dependency makes the design of high power handling HTS devices challenging, as the PEM thin films properties are significantly modified by the presence of defects and strains.
4 Conclusions

The frequency and $V_{dc}$ dependence of the different nonlinear terms of a PEM buffer layer and an HTS thin film used to fabricate microwave passive devices were investigated. Under defined conditions the PEM nonlinear reactive contribution may cancel the reactive nonlinearities from the HTS. However, the conditions for cancellation are frequency dependent. This makes difficult the exploitation of the properties of a PEM buffer layer in order to reduce the nonlinearities of the HTS film. On another hand, the nonlinear contribution of the PEM buffer also depends on $V_{dc}$. This dependency might be interesting to diminish the contribution of the nonlinear conductance $\Delta G$ or to adjust the nonlinear capacitance $\Delta C$ to a value in compliance with the condition for cancellation. However, these dependencies make the design of a device integrating the two materials challenging. The properties of the PEM are often dependent on the sample quality. Extrinsic contributions from defects and strains, as well as the grain size in the PEM layer, modify significantly its properties and its dependence on $V_{dc}$. Preliminary characterizations of the PEM nonlinearities or highly reproducible PEM thin layers are necessary to take advantage of the properties of these materials in the design of a high power handling HTS device.

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