Stress loading and losses of power in the pin-roller gearing with clearances

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Abstract. In planetary cycloid gears the motion is transmitted simultaneously to many pin-rollers and teeth of cycloid disk. The use of schemes with many parallel contacts requires measures to ensure the assembly of the gears due to the introduction of malleable elements or guaranteed gaps in the contacts into the design. In the paper the analytical decision of the problem of the gap impact in gearing of planetary cycloid gear to the load distribution on the teeth of the cycloid disk, the maximum forces acting on them and friction losses in the contact of the teeth of the cycloid disk and pin-rollers is reviewed. The calculated dependences for the coefficients of the load increase due to the gaps and the effect of the gaps in the gearing on the losses on the sliding friction are obtained. The results of calculation of the load increase coefficient in gearing and the clearance influence coefficient on friction losses in gearing are presented. This decision allows to take into account the possible increase in the load of the contacts and the change of the coefficient of loss in gearing at the design stage.

1. Introduction

Planetary cycloid gears have recently been increasingly used due to their inherent positive properties. The study of gear is given much attention. One of the first is the work [1], in which the geometry and statics of the planetary and cycloid gears are considered. It identifies the forces in the gearing and the exit mechanism in the precise manufacture of the gear. In [2], also for precise manufacturing, the method of geometric calculation was considered and 3D modelling of the cycloid gear was performed. In [3], the stress state of cycloid discs was considered using numerical and experimental methods for the most critical case in which the entire load is transferred by one pair of pin-roller and tooth of the cycloid disc. The influence of design parameters on forces and contact stresses are presented in [4] of S.K. Malhotra and M.A. Parameshvaran. Another method of force analysis of a cycloid reducer is presented by L. Lixing and others in [5].

Clearances in gearing and W mechanism of planetary cycloid gears reduce the contact zone and change the nature of the distribution of forces on the contact pairs. This, in turn, affects the forces transmitted from
the clearance and pin-rollers on the satellite support. In [6], a method for calculating the deformations of the teeth and clearances between the teeth of the cycloidal disk and the rollers is presented. The effect of profile correction on deformations and clearances, as well as on the number of teeth that simultaneously transfer the load, was considered. In [7], a kinematic analysis of a new design of a single-stage cycloid speed reducer was carried out using Autodesk Inventor and Solidworks software. J. G. Blanche and D. C. H. Yang investigated the effect of machining tolerances on the clearance and torque ripple [8]. M. Blagoevich presented a method for calculating the deformations of the teeth of a cycloid disk and clearances arising between the cycloid disk and the gear ring [9]. It is worth noting [10, 11] devoted to the study of contact forces and stresses in cycloid gears.

Studies have established that the forces and stresses in the gear are influenced by various factors, and, above all, the manufacturing accuracy and the mechanism load itself. Due to the constructive and geometrical nonlinearity of the planetary cycloid gear, they are sensitive to structural changes in the contact zones with increasing load. This determines the nature of the load distribution over the contact pairs and the stresses arising in the details. These phenomena are not studied enough. The aim of the work is to analyse the influence of the clearance on the contact zones, forces and friction losses in the gearing.

2. Analytical decision

Clearances affect the complex of interrelated quantities: backlash and kinematic error of rotation, forces acting in gearing and total forces transmitted to the cycloid disc support, losses in the gearing, torsional stiffness of the gear. In general, clearances are determined by a combination of a large number of factors and must be determined on the basis of the calculation of dimensional chains. Components of chains can be scalar quantities; vector values or clearances when the parts occupy any relative position within the clearance, or are completely selected in a certain direction, as a rule, in the direction of the applied force.

The noted uncertainty in the definition of clearances is not analysed in this work. As a rule, the clearances provided for assembly are much larger than those that occur during manufacture. Therefore, in the first approximation, the case of identical initial clearances in \( \Delta \) gearing is considered further.

In a precisely manufactured planetary cycloid gear, all the pin-rollers (rollers) are in simultaneous contact with the teeth of the cycloid disc. Due to the clearances without load, the movement must be transmitted by one pair of pin-roller-tooth of the cycloid disc. However, under load, other contact pairs come into operation, a contact zone is formed and the load is transferred by all contact pairs of this zone. The size of the zone depends on many factors, and first of all, on the initial clearance in the gearing, the stiffness of the contact pairs and the transmitted load.

Figure 1 shows the relative location of the gear links. When determining the clearances, deformations and forces in the contact, it is usually assumed [1] that when a wheel is stationary, the cycloid disc rotates through \( \beta \) angle under the action of the moment applied to it. In this case, in the \( i \)-th contact pair, a convergence along the normal to the surfaces of the contact occurs and \( \delta_i = \beta l_i - \Delta \) deformation appears in some pairs. \( l_i \) value that enters this expression is the distance from the centre of the \( \Omega_i \) cycloid disc to the normal to the contact surfaces, which goes from the centre of the pin-roller to \( P_z \) pole of the link.
It can be defined by the following formula:

\[ l_i = r_{w1} \sin \theta_i = r_{w1} \sin \left( \tau_i + \arctan \left( \frac{\sin \tau_i}{1 - \cos \tau_i} \right) \right), \]

where \( r_{w1} \) is the radius of the initial circumference of the cycloid disc in the gearing with the pinwheel, \( r_{w1} = z_2 a_w \), \( z_i \) is the number of cycloid disc teeth; \( a_w \) is gear centre distance; \( \tau \) is the angle of the pin-roller on the pinwheel, which is counted counter clockwise from the pin-roller located on the vertical axis of symmetry; \( \lambda \) is coefficient of shortening of epicycloids.

Normal forces in contact pairs at \( \delta_i > 0 \)

\[ F_i = c_{iz} \delta_i, \]

where \( c_{iz} \) is stiffness coefficient of contact pair. Available data allow us to consider the stiffness of all contact pairs equal.

Torque on the cycloid disc, created by the forces acting on the teeth of the cycloid disc

\[ T_c = \sum F_i l_i. \]  \( \quad (1) \)

In gearing without clearance [1]

\[ F_i = F_{max} \frac{l_i}{r_{w1}} = F_{max} \sin \theta_i. \]  \( \quad (2) \)

For it

\[ T_c = F_{max} r_{w1} \sum \sin^2 \theta_i, \quad i = 0, 1, ..., z_2/2. \]  \( \quad (3) \)
In gearing with clearance

\[ F_{ig} = F_{\text{max}} g \frac{\beta r_{w1} \sin \theta - \Delta}{\beta r_{w1} - \Delta}. \]  

(4)

We will obtain from (5) and (2)

\[ T_s = F_{\text{max}} g \frac{\sum (\beta r_{w1}^2 \sin^2 \theta - \Delta r_{w1} \sin \theta)}{\beta r_{w1} - \Delta}. \]

(5)

Summing in formula (5) is carried out for the contact zone:

\[ \theta = \arcsin \frac{\Delta}{\beta r_{w1}} < \theta < \pi - \arcsin \frac{\Delta}{\beta r_{w1}} = \theta_2. \]

We shall introduce the coefficient of increase of the maximum load in the gearing

\[ k_z = \frac{F_{\text{max}} g}{F_{\text{max}}}. \]  

(6)

where \( F_{\text{max}} g \), \( F_{\text{max}} \) are maximum forces acting during the same torque on the cycloid disc in the gearing with clearance and without clearance. From (3) and (5) we shall have:

\[ k_z = \frac{(\beta r_{w1} - \Delta) \sum \sin^2 \theta}{\sum (\beta r_{w1} \sin^2 \theta - \Delta \sin \theta)}. \]  

(7)

The sums in the numerator and denominator (7) are replaced by integrals. After the transformations we get:

\[ k_z = \frac{\pi \left(1 - \frac{\Delta}{\beta r_{w1}}\right)}{\pi - 2 \arcsin \frac{\Delta}{\beta r_{w1}} - 2 \Delta \left[1 - \left(\frac{\Delta}{\beta r_{w1}}\right)^{2n/2}\right]}. \]

Sliding speeds in contact pairs are proportional to the distance of \( K \) contact point from \( P_z \) gearing pole. The closer the contact pair to the pole, the slower the slip speed in it. Losses in the gearing consist of sliding friction losses between the teeth of the cycloid disc and pin-rollers and rolling friction between the teeth and the pin-rollers over each other. Sliding friction power in \( i \)-th contact pair:

\[ P_{k_z} = \bar{f} \omega_{\text{rel}} F_i K P_z, \]  

(8)

where \( \bar{f} \) is reduced friction coefficient, \( \omega_{\text{rel}} \) – the angular speed of the relative rotation of the cycloid disc and pinwheel; \( F_i \) – normal force in the contact; \( KP_z \) – the distance from the contact point of the cycloid disc tooth and \( K \) pin-roller to \( P_z \) gearing pole.

For the gearing without clearance taking into consideration (2)
\[ P_{wi} = f_0 \omega_{rot} F_{max} r_2 \left( \sin \tau - \frac{r_t}{r_2} \frac{\sin \tau_i}{\sqrt{1 - 2\lambda \cos \tau_i + \lambda^2}} \right). \]

Summing over all contacts and replacing sums with integrals, we obtain for the entire contact zone

\[ P_{wi} = f_0 \omega_{rot} F_{max} r_2 \left( \int_1^\pi \left( 1 - \frac{r_t}{r_2} \right) \right). \]

In the gearing with clearance taking into account formulas (4), (6), (8)

\[ P_{wi} = k_f f_0 \omega_{rot} F_{max} r_2 \frac{\sin \tau - \frac{r_t}{r_2} \sin \tau_i}{\sqrt{1 - 2\lambda \cos \tau_i + \lambda^2}} \frac{- \Delta}{\beta r_{w1}} \sqrt{1 - 2\lambda \cos \tau_i + \lambda^2} + \frac{\Delta}{\beta r_{w1}} \frac{r_t}{r_2}. \]

For the whole contact zone \((\tau_1, \tau_2)\), if \(\cos \tau_{1,2} = \lambda \left( \frac{\Delta}{\beta r_{w1}} \right)^2 + \left( 1 + \frac{\Delta}{\beta r_{w1}} \right)^2 - 1 \)

\[ P_{wi} = A k_f f_0 \omega_{rot} F_{max} r_2 z_3 \left( 2 \pi \left( 1 - \frac{\Delta}{\beta r_{w1}} \right) \right), \]

where

\[ A = (\cos \tau_1 - \cos \tau_2) - \frac{r_t}{r_2} \sqrt{1 - 2\lambda \cos \tau_2 + \lambda^2} - \sqrt{1 - 2\lambda \cos \tau_1 + \lambda^2} - \frac{4\Delta}{\beta r_{w1}} \left( \cos (\tau_1/2) - \cos (\tau_2/2) + \frac{(1 - \lambda)^2}{9\lambda} \ln \left| \frac{\tau_2}{\tau_1} \right| - \ln \left| \frac{\tau_2}{\tau_1} \right| \right) + (\tau_2 - \tau_1) \frac{\Delta}{\beta r_{w1}} \frac{r_t}{r_2}. \]

The value \(k_p = \frac{P_{wi}}{P_k}\) will be called clearances influence coefficient to the power of the sliding friction in the gearing.

3. Decision interpretation

Figure 2 presents the results of the calculation of the coefficients \(k_z\) and \(k_p\) for a number of values \(\Delta\) and \(\beta\). As it can be seen from the figure, an increase in \(\Delta\) initial clearance leads to \(k_z\) increase, and, therefore, to the maximum force in the gearing. It also follows from figure 2 that \(k_z\) increases when \(\beta\) decreases. This is explained by the fact that this also reduces the contact area, and, consequently, increases the maximum force in the gearing with the same torque. This can happen together with high stiffness of contact pairs.
An increase in initial clearance results in $k_p$ decrease and a loss in sliding friction, despite an increase in the maximum force in the gearing. This can be explained by the fact that teeth come out of the contact, removed from the gearing pole, where there are large sliding speeds. With an increase in $\beta$ angle, and, consequently, of the torque, $k_p$ coefficient increases nonlinearly, which is explained by the entry into operation of contact pairs remote from the pole, where the slip velocity is higher.

![Figure 2](image)

**Figure 2.** Diagrams of the ratio of the increase in the maximum force $k_z$ (a) and the slip power loss coefficient $k_p$ (b) in gearing with clearance from the initial clearance $\Delta$ and the angle of cycloid disc rotation $\beta$.

4. Conclusions
The results of the analysis show that the maximum force in the planetary cycloid gearing increases with an increase in the clearance, and the loss for sliding friction decreases. With increasing load due to the constructive and geometric nonlinearity of the gearing, $k_p$ coefficient increases nonlinearly. This decision allows taking into account the possible changes in stress loading and friction losses in the gearing depending on the expected clearance at the design stage.

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