Adaptive formation control for autonomous surface vessels with prescribed-time convergence

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Abstract
This article is dedicated to solving the problem of predefined-time cooperative control for autonomous surface vessels encountering model uncertainties and external perturbations. By virtue of the prescribed-time stable theory, a robust formation controller is constructed, with which the settling time of the cooperative system can be prescribed in advance. The controller is developed under the backstepping framework, where the dynamic surface control is applied to generate the real-time command. Considering the unmodeled autonomous surface vessel dynamics, the neural network-based nonlinear approximator is incorporated with minimum-learning-parameter technique. Under this scenario, the real-time control can be pursued with one parameter being estimated. Finally, comparative simulation examples are provided to exhibit the effectiveness and advantages of designed control strategies.

Keywords
Autonomous surface vessels, predefined-time cooperative control, dynamic surface control, neural network, minimum-learning-parameter

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Introduction
In the past, formation control has become a hot topic in the ocean engineering industry and been known as one of the most promising technologies. Compared with the single vehicle, formation enjoys a variety of appealing features for ocean surveillance, resource detection, and geochemical sampling, to just name a few. However, the ever-increasing complexity of marine missions and fragile onboard devices render it a tough work to devise cooperative controller. To overcome these obstacles, numerous methods have been resorted to, involving backstepping methodology,¹⁻³ neural networks (NNs) control,⁴⁻⁷ and adaptive control.⁸⁻⁹

Although there is substantial research that has been undertaken, the problem of model uncertainties is fundamentally ignored in the studies of Fu and Yu¹⁰ and Xiao et al.¹¹ Considering the imprecise model parameters, extensive uncertainties are usually resident in autonomous surface vessels’ (ASVs) model dynamics. To enhance ASV systems’ adaptation, adaptive control approaches,⁷⁻¹³ NNs,¹⁴⁻¹⁵ and observer technologies¹⁶⁻¹⁸ were widely employed. For the path-following problem of a single surface vessel, Wang et al.¹⁹ revisited the above problem via a novel adaptive-based NNs strategy, which possesses the
disadvantage of enormous weight matrix to be estimated. To obtain less computation burden, Lu et al.\(^6,7\) proposed minimum-learning-parameter (MLP)-based control strategies for multiple ASVs to reinforce the robustness despite of uncertainties. However, it still needs further investigation on how to solve the time-consuming problem of NNs for formation systems.

It is somewhat defective that only the asymptotical stability was achieved in the majority of foregoing mentioned approaches. Compared with the asymptotical controller,\(^20,21\) the unique character of finite-time control lies in the fast response.\(^22,23\) However, the positive correlation between the initial states and convergence rate renders it no more practical to some extent. This calls for the appearance of the fixed/predefined-time control.\(^{17,27–29}\) In this regard, a new-type of performance function with predefined-time convergence is introduced by Wang et al.,\(^20\) and Zhang et al.\(^31\) have investigated a fixed-time tracking control for a family of ASVs with a novel bounded function. Nevertheless, one notable caveat is that the problem of “complexity explosion” in backstepping is not considered in the study by Zhang and Yang.\(^31\) With this in mind, there still needs research on the backstepping control problem considering “complexity explosion.”

Motivated by the above observations, a predefined-time distributed cooperative controller for ASV formation is constructed in this article. Comparing to the current literatures, the main contributions of the proposed controller are summarized as follows:

(i) Compared with the existing finite-time and asymptotic control methods,\(^7,13,20\) predefined-time stability of the closed-loop signals is effectively ensured under the backstepping framework, which means that the formation achieves the asymptotic tracking with prescribed-time performance.

(ii) The unmodeled ASV dynamics are well addressed with radial basis function neural networks (RBF NNs). Simultaneously, the excessive time-consuming defect of NNs is well modified synthesizing with the MLP algorithm, by which the estimation dimensions of online parameters decrease.

(iii) In the existing adaptive controllers for formation issues,\(^4,6,23,32\) the centralized framework is deployed. Totally different from such strategy, a weighted directed graph is introduced into the controller design process to circumvent the single-point failure.

The remainder of this article is structured as follows. The mathematical model and the interact topology of ASVs formation are described in the second section. The cooperative controller with corresponding stability analysis and control performance are given in the third and fourth sections, respectively. Finally, conclusions are drawn in the last section.

Preliminaries and problem formulation
Interaction graph
Consider a group of \(N\) fully actuated ASVs in \(\mathbb{R}^2\). The directed interaction among ASVs can be given by a weighted directed graph \(G = (V, E)\), which consists of a vertex set \(V = \{1, \ldots, N\}\) and an edge set \(E \subseteq V \times V\). A direct edge \((i, j) \in E\) represents that \(i\)-th ASV can receive the information from \(j\)-th ASV. If \((i, j) \in E\), then we regard \(j\)-th ASV is a neighbor of \(i\)-th ASV. In this setting, the neighbor set of \(i\)-th ASV is defined as \(\mathcal{N}_i = \{j \in V : (i, j) \in E\}\). Besides, the weighted adjacent matrix \(A = \{a_{ij}\}\) is defined such that \(a_{ij} = 1\) for \((i, j) \in E\) and \(a_{ij} = 0\) otherwise. Particularly, there is no self-loop in graph, that is, \(a_{ii} = 0\). Accordingly, one can define the Laplacian matrix \(L = \{l_{ij}\}\) as \(l_{ij} = -a_{ij}\), \(i \neq j\) and \(l_{ii} = \sum_{k=1}^{N} a_{ik}\). A directed path from \(i\)-th ASV to \(j\)-th ASV is a sequence of edges of the form \((i_1, i_2), (i_2, i_3), \ldots\). A directed tree is a directed graph, where every node has exactly one parent except for one node, called the root, and the root has directed paths to every other node.

Lemma 1. The Laplacian matrix \(L\) is positive definite, provided that the interaction graph \(G\) has a directed spanning tree.\(^7\)

Mathematical model of the ASVs
For a group of ASVs suffering from external disturbances, the mathematical model of the \(i\)-th \((i = 1, 2, \ldots, N)\) vehicle can be expressed as

\[
\begin{align*}
\dot{\eta}_i &= R(\psi_i) v_i \\
M \dot{v}_i + C(v_i) v_i + D(v_i) v_i + g_i &= \tau_{\text{act}} + \tau_i
\end{align*}
\]  
(1)

where the \(\eta_i\) and \(v_i\) are defined under the earth-fixed frame and the body-fixed frame, respectively, and \(M\) and \(D(v_i)\) are both symmetric and positive definite. \(R(\psi_i)\) is the rotation matrix. More related parameters and variables can be referred to the nomenclature table in Appendix 1.

Function approximation based on RBF NNs
Lemma 2. For any given real continuous function \(f(\mathbf{x}) : \Omega \rightarrow \mathbb{R}\), where \(\mathbf{x} \in \mathbb{R}^n\) is the input vector with \(\Omega\) being a compact subset of \(\mathbb{R}^n\), there exists an ideal weight matrix \(W \in \mathbb{R}^p\) such that

\[
f(\mathbf{x}) = W^T H(\mathbf{x}) + e
\]  
(2)

where \(n\) is the dimension of the input vector, \(p\) is the number of neurons, \(e\) is the bounded function approximation error, and \(H(\mathbf{x}) = [H_1(\mathbf{x}), H_2(\mathbf{x}), \ldots, H_p(\mathbf{x})]^T\) is the Gaussian basis function vector which satisfies

\[
H_i(x) = \exp\left(-\frac{||x - c_i||^2}{2\sigma_i^2}\right), i = 1, 2, \ldots, p
\]  
(3)
where \( c_i \in \mathbb{R}^n \) is the center of the receptive field and \( \sigma_i \in \mathbb{R} \) is the width of the Gaussian function.

**Notations**

Throughout this article, \( (\cdot)^T \) expresses the transpose of a matrix; \( |\cdot| \) denotes either the Euclidean norm of a vector or the induced two-norm of a matrix. \( \lambda_{\max}(\cdot) \) and \( \lambda_{\min}(\cdot) \) are the maximum and minimum eigenvalues of a positive matrix, respectively. For an arbitrary vector \( \zeta = [\zeta_1, \zeta_2, \ldots, \zeta_n]^T \), \( \text{sig}(\zeta) = \text{diag}(\text{sign}(\zeta_1)|\zeta_1|, \ldots, \text{sign}(\zeta_n)|\zeta_n|)^T \), where \( \text{sign}(\cdot) \) and \( \text{diag}(\cdot) \), respectively, denote the standard sign function and the diagonal matrix.

**Control objective**

Consider a group of \( N \) fully actuated follower ASVs and a directed interaction graph \( G \). Define \( \eta_r \) as the trajectory of the virtual leader, then the control objective of this article is to design control law \( \Psi_i \) such that \( \eta_r \) will be tracked and a relative formation shape \( \Delta_i \in \mathbb{R}^3 \) could be constituted within prescribed time. Mathematically speaking, we have

\[
\lim_{t \to T_c} ||\eta_r - \eta_i|| = 0, \quad \forall i = 1, 2, \ldots, N
\]

(4)

where \( T_c \) is the user-defined settling time, \( \rho \) is a positive constant, and \( \Delta_i \) denotes the formation configuration of each follower expressed in the earth-fixed coordinate. Previous to presenting the main results of this article, some important assumptions and lemmas must be introduced as follows:

**Assumption 1.** For the reference trajectory \( \eta_r \in \mathbb{R}^3 \), it assumes that there exists a positive constant \( \rho_M \) satisfying

\[
||\eta_r|| \leq \rho_M.
\]

**Assumption 2.** The external disturbances possess an upper bound, that is, \( ||\tau_{wi}|| \leq \tau_d \) with \( \tau_d > 0 \).

**Lemma 3.** For any scalar \( 0 < b < 1 \) and \( x_i \in R (i = 1, 2, \ldots, n) \), the following inequality holds\(^{10}\)

\[
(|x_1| + \cdots + |x_n|)^b \leq |x_1|^b + \cdots + |x_n|^b
\]

(5)

**Lemma 4.** The following inequality holds true for any given vector \( p^T, q^T \in \mathbb{R}^n \)

\[
p^T q \leq \frac{1}{2} (p^T p + q^T q)
\]

(6)

**Lemma 5.** For the nonlinear system (1), if the radially unbounded Lyapunov function \( V: \mathbb{R}^n \to \mathbb{R} \) exists and satisfies\(^{27}\)

\[
\dot{V} \leq -\frac{1}{\theta \gamma (1 - p)} V^{\theta - p - 1} (p^\sigma + \gamma)^2 - p + \sigma
\]

(7)

with \( 0 < p < 1, \gamma > 1, 0 \leq \sigma < \infty, \) exponential coefficient \( 0 < \theta < 1 \) and predefined time \( T_c > 0 \), the global stability of the system can be achieved within prescribed time \( T_c \).

**Lemma 6.** The first-order filter defined in the study by Lu et al.\(^7\) is employed here to obtain the filtered virtual command, which is described as the following equation

\[
e_i \hat{\alpha}_{i1} + \bar{\alpha}_{i1} = \alpha_{i1}
\]

(8)

where \( e_i \) is a diagonal positive definite matrix and satisfies \( \lambda_{\min}(\varepsilon) < 1 \). \( \alpha_{i1} \) is the virtual control signal and filtered command \( \bar{\alpha}_{i1} \) is obtained by letting \( \alpha_{i1} \) pass through the low-pass filter. As seen from the study of Huang et al.\(^3\), the time derivative of filtered error \( z_{i2} = \bar{\alpha}_{i1} - \alpha_{i1} \) is introduced with positive scalar \( b_1 \)

\[
\dot{z}_{i2} = \hat{\alpha}_{i1} - \bar{\alpha}_{i1} = -e_i^{-1} z_{i2} + B_i(\cdot)
\]

(9)

**Controller design**

This section focuses on designing a controller for the ASV formation using the directed graph theories, NNS approximation, and backstepping design. By virtue of prescribed-time theory, the tracking missions can be completed while maintaining desired formation shape. The design process could be divided into two parts, that is, the virtual commands design and the nominal controller design, as shown in Figure 1.

**Step 1.** Previous to devising the virtual command, we define the formation error \( z_{i1} \), filtering error variable \( z_{i2} \), and tracking error \( z_{i3} \) as follows

\[
\begin{align*}
z_{i1} &= \eta_i - \Delta_i - \eta_r, \\
z_{i2} &= \bar{\alpha}_{i1} - \alpha_{i1}, \\
z_{i3} &= v_i - \bar{\alpha}_{i1}
\end{align*}
\]

(10)

where \( z_{ij} = [z_{ijx}, z_{ijy}, z_{ijz}]^T (j = 1, 2, 3) \), \( \alpha_{i1} = [\alpha_{i1x}, \alpha_{i1y}, \alpha_{i1z}]^T, \bar{\alpha}_{i1} = [\bar{\alpha}_{i1x}, \bar{\alpha}_{i1y}, \bar{\alpha}_{i1z}]^T; \alpha_{i1} \) is the virtual control law that will be designed later. In terms of equations (1) and (10), the error dynamics can be obtained as equation (12) with \( f_i \) denoting as

\[
\begin{align*}
f_i &= -C_i(w_i)v_i - D_i(v_i)w_i - g_i + M_i \hat{\alpha}_{i1} \\
\dot{z}_{i1} &= R(\psi_i)(z_{i2} + z_{i3} + \alpha_{i1}) - \dot{\eta}_r, \\
\dot{z}_{i2} &= \hat{\alpha}_{i1} - \bar{\alpha}_{i1}M_i, \\
\dot{z}_{i3} &= f_i + \tau_{wi} - \tau_i
\end{align*}
\]

(11)

Then, we define the cooperative tracking error as \( s_{i1} \), which is given by the following expression

\[
s_{i1} = \sum_{j=1}^{N} a_{ij}[(\eta_j - \Delta_j) - (\eta_j - \Delta_j)] + d_i(\eta_i - \Delta_i - \eta_r)
\]

(12)

Here, \( d_i = 0 \) holds when the \( i \)-th follower has no access to the leader vehicle and otherwise \( d_i = 1 \). We can regard
the leader vehicle as a node, with which the edges are constructed and the interaction graph \( G \) is enlarged as \( \hat{G} \). We define the Laplacian matrix \( \hat{G} \) as \( \hat{L} \). To stabilize the error \( s_{11} \) in prescribed time \( T_c \), the virtual command is constructed as

\[
\alpha_i = -R^T(\psi) \left[ \frac{2^{\theta_p-\theta_p-1}}{\theta (1-p) T_c} \sin^{2 \theta_p - 2 \theta_p + 1} (z_1) \right] \frac{1}{2} ||z_1||^{2 \theta + \gamma} + k_{11}s_{11}
\]

where \( k_{11} > 0, i = 1, 2, ..., N \). The following definitions are presented for simplicity concern

\[
\begin{align*}
z_i &= [z_{i1}, ..., z_{iN}]^T, \quad v = [v_1, ..., v_N]^T, \quad \delta = \text{diag} \{ \delta_i \}, \\
\bar{\tau} &= [\bar{\tau}_1, ..., \bar{\tau}_N]^T, \quad M = \text{diag} \{ M_i \}, \quad C = \text{diag} \{ C_i \}, \\
D &= \text{diag} \{ D_i \}, \quad s_i = [s_{i1}, ..., s_{iN}]^T \quad i = 1, 2, ..., N \quad (11)
\end{align*}
\]

Thus, based on the foregoing simplification and the interaction graph, the virtual command is rearranged in a compact form as

\[
\alpha_i = -R^T_0 \left[ \frac{2^{\theta-\theta_p-1}}{\theta (1-p) T_c} \sin^{2 \theta_p - 2 \theta_p + 1} (z_1) \right] \frac{1}{2} ||z_1||^{2 \theta + \gamma} + k_i (\bar{L} \otimes \mathbf{I}) z_1
\]

where \( \mathbf{I}_3 \) devotes the three-dimensional identity matrix. Under the virtual control signal, the error valuable \( z_1 \) could be forced to a region containing the origin within specified time. To better illustrate this fact, we choose the Lyapunov function \( V_1 \) as follows

\[
V_1 = \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_2^T z_2
\]

By the application of equations (9), (11), (12), virtual command (15), and Lemma 6, \( \dot{V}_1 \) satisfies

\[
\dot{V}_1 = z_1^T [R_0 (z_2 + z_1 + \alpha_1) - 1 \otimes \dot{\eta}_r] + z_1^T \dot{z}_2
\]

\[
\leq - \| R_0 \|^2 \left[ \frac{1}{\theta (1-p) T_c} \left( \frac{1}{2} z_1^T z_1 \right)^{\theta-\theta_p+1} \right] \left( \frac{1}{2} z_1^T z_1 + \gamma \right) + z_1^T k_1 (\bar{L} \otimes \mathbf{I}) z_1 + z_1^T[R_0(z_2 + z_3)] - z_1^T \otimes \dot{\eta}_r - z_2^T \dot{\eta}_r - z_2^T \dot{\eta}_r - z_2^T b \quad (17)
\]

Since \( \| R_0 \| = 1 \) holds. Based on Assumption 1 and Lemma 4, it can be further deduced that

\[
\dot{V}_1 \leq - \frac{1}{\theta (1-p) T_c} \left( \frac{1}{2} z_1^T z_1 \right)^{\theta-\theta_p+1} \left( \frac{1}{2} z_1^T z_1 + \gamma \right) k_1 (\bar{L} \otimes \mathbf{I}) - \left( 1 + \frac{N^2}{2} \right) \mathbf{I}_3
\]

\[
z_1 - (e^{-1} - \mathbf{I}_3) z_2 + \frac{1}{2} z_3^T z_3 + \frac{v^2}{2} + \frac{b^2}{2} \quad (18)
\]

The guidelines about the parameter turning are intuitively exhibited in equation (18), from which one can deduce that \( k_1 \) and \( \epsilon \) should satisfy \( \lambda_{\min}(k_1) \geq \frac{2+\lambda}{2\lambda_{\min}(L)} \) and
\( \lambda_{\text{min}}(\varepsilon) < 1 \). Consequently, equation (18) is further developed as equation (20) with 
\( q_0 = \frac{1}{2}(p_1^2 + b_2^2) \)
\[
\dot{V}_1 \leq -\frac{1}{\theta_\gamma(1-p)T_c} \left( \frac{1}{2} z_1^T z_1 \right) ^{\theta_p-\theta+1} \left( \frac{1}{2} z_1^T z_1 + \gamma \right) ^{2-p} \]
\[
- (e^{-1} - I_3) z_1^T z_2 + \frac{1}{2} z_1^T z_3 + \sigma_0 \tag{19}
\]

**Remark 1.** Compared with the coordinated controller proposed in the study by Lu et al., the major superiors of equation (15) can be pursued in that the prescribed-time stability can be achieved for \( z_1 \) rather than asymptotic stability, which means that formation tracking missions could be accomplished within a given settling time. Furthermore, the major drawback inherited in backstepping is eliminated by exploiting the dynamic surface control approach.

**Step 2.** In this step, the control law \( \tau_i \) of each vessel is designed. Previous to designing the control law, we select the Lyapunov function \( V_2 \) as follows
\[
V_2 = V_1 + \frac{1}{2} z_3^T M z_3 \tag{20}
\]

Considering the fact that \( \dot{M} = 0 \) and equation (11), the following expression is obtained with
\[
\dot{V}_2 = \dot{V}_1 + z_3^T M \dot{z}_3 = \dot{V}_1 + z_3^T (\tau_i + \tau_f + f) \tag{21}
\]
where \( f = [f_1^T, \ldots, f_N^T]^T = -C v - D \dot{v} - g - M \dot{\alpha}_1 \). Actually, model uncertainties are frequently encountered, which render it impossible to obtain the accurate information of \( f_i \). To this end, the unmodeled dynamics will be approached by MLP-based NNs, which can be expressed as
\[
||f_i|| = ||W_i^T H_i + e_i|| \leq ||W_i|| ||H_i|| + ||e_i|| \leq \mu_i h_i + \varepsilon_{\alpha_0} \tag{22}
\]
where \( ||e_i|| \leq \varepsilon_{\alpha_0} < \infty \) with \( \varepsilon_{\alpha_0} > 0, \mu_i = ||H_i|| \) and \( \mu_i \) is an unknown parameter. As a result, equation (21) is further calculated as the following expression
\[
\dot{V}_2 \leq \dot{V}_1 + \sum_{i=1}^N z_3^T \tau_i + \sum_{i=1}^N \mu_i h_i ||z_3|| + \sum_{i=1}^N (\tau_{di} + \varepsilon_{\alpha_0}) ||z_3|| \tag{23}
\]

For convenience, we define \( D_i = \tau_{di} + \varepsilon_{\alpha_0} \). Then, the cooperative control law for each formation member can be constructed as
\[
\tau_i = -\left[ \frac{2^{\theta-p}-1}{\theta_\gamma(1-p)T_c} \right] \frac{\sign(2^{\theta_p-2\theta+1}(z_3))}{(2)^{\theta_p-2\theta+1}(z_3)} \right] \frac{1}{(2)^{\theta_p-2\theta+1}(z_3) + \gamma} \tag{24}
\]
\[
+ k_{i2} z_3 + \frac{k_{i3} h_i \tanh(\hat{\mu}_i) z_3 \|z_3\|}{\|z_3\|} + \frac{k_{i4} \tanh(\hat{D}_i) z_3 \|z_3\|}{\|z_3\|} \tag{24}
\]

where \( \varepsilon_{\alpha_1} = \frac{b_1}{N(1+k_1 h_i \tanh(\hat{\mu}_i))} \) and \( \varepsilon_{\alpha_2} = \frac{b_2}{N(1+k_1 h_i \tanh(\hat{D}_i))} \), \( b_{in}(n = 1, 2) \) and \( k_{ij}(j = 2, 3, 4) \) are positive constants satisfying \( k_{i2} > \frac{1}{2} \) and \( \hat{\mu}_i \) and \( \hat{D}_i \) are the estimations of \( \mu_i \) and \( D_i \). Adaptive updating laws for \( \mu_i \) and \( D_i \) are designed as follows with \( \varepsilon_{\alpha_1}(n = 1, 2) \) being positive constants
\[
\dot{\hat{\mu}}_i = \frac{\varepsilon_{\alpha_1}}{k_{i1}} \cosh^2(\hat{\mu}_i) h_i ||z_3||, \quad \dot{\hat{D}}_i = \frac{\varepsilon_{\alpha_2}}{k_{i4}} \cosh^2(\hat{D}_i) ||z_3|| \tag{25}
\]

Additionally, estimation errors of \( \hat{\mu}_i \) and \( \hat{D}_i \) are defined as
\[
\hat{\mu}_i = \mu_i - k_{i3} \tanh(\hat{\mu}_i), \quad \hat{D}_i = D_i - k_{i3} \tanh(\hat{D}_i) \tag{26}
\]

**Remark 2.** Control parameters \( \varepsilon_{\alpha_1}, n = 1, 2 \) are introduced for the purpose of chattering reducing. To elaborate this point, these parameters can be set as \( \varepsilon_{\alpha_1} = 0 \). Then, equation (24) are rewritten as
\[
\tau_i = -\left[ \frac{2^{\theta-p}-1}{\theta_\gamma(1-p)T_c} \right] \frac{\sign(2^{\theta_p-2\theta+1}(z_3))}{(2)^{\theta_p-2\theta+1}(z_3) + \gamma} \tag{24}
\]
\[
+ k_{i2} z_3 + \frac{k_{i3} h_i \tanh(\hat{\mu}_i) z_3 ||z_3||}{||z_3||} + \frac{k_{i4} \tanh(\hat{D}_i) z_3 ||z_3||}{||z_3||} \tag{27}
\]

Obviously, chattering phenomenon is inevitable attributing to the discontinuous term \( \frac{q_i}{\|z_3\|} \) and it can be alleviated by increasing the parameter \( \varepsilon_{\alpha_1} \) in equation (24). However, a larger \( \varepsilon_{\alpha_1} \) will increase the tracking errors. Thus, designers must make a trade-off between the control precision and chattering problem in order to obtain satisfactory performance.

**Remark 3.** In equation (25), these adaptive updating laws are structured to provide upper bounds for \( \tau_i \). Taking \( \hat{\mu}_i \) as an example, one can find that \( \hat{\mu}_i \) will appear in \( \tau_i \) with the form of \( \frac{k_{i3} \tanh(\hat{\mu}_i) z_3}{||z_3||} \). Recalling the property of the hyperbolic tangent function, we know that \( \tanh(\hat{\mu}_i) \) is always bounded regardless of the value of \( \hat{\mu}_i \). Simultaneously, the estimation errors are defined as equation (26) to further ensure the boundedness for \( \hat{\mu}_i \) and \( \hat{D}_i \).

**Theorem 1.** For the formation system (1) under Assumptions 1 and 2, if the enlarged graph \( \mathcal{G} \) has a directed spanning tree, the control parameters satisfy the condition and the adaptive control scheme is developed as equations (24) and (25), prescribed-time stability will be achievable for the formation errors \( z_1, z_3 \) and the uniformly ultimately bounded of \( z_2, \hat{\mu}_i \) and \( \hat{D}_i \) can be ensured.

**Proof.** The illustration of the prescribed-time stability will be carried out in two steps, that is, to prove the uniformly ultimately bounded of estimation errors, and then to demonstrate the prescribed-time stability of tracking errors.
\( z_1 \) and \( z_3 \). Two auxiliary variables \( \hat{\mu}_i \) and \( \hat{D}_i \) are introduced here satisfying \( \hat{\mu}_i \geq \mu_i \), \( \hat{\mu}_i \geq \hat{\mu}_i \), and \( \hat{D}_i \geq D_i \). From the definitions of \( \hat{\mu}_i \) and \( \hat{D}_i \), it deduces that \( \hat{\mu}_i \) and \( \hat{D}_i \) always exist to satisfy \( \hat{\mu}_i \geq \mu_i \) and \( \hat{D}_i \geq D_i \). Observing estimation errors (26), one finds that inequalities \( |\hat{\mu}_i| \leq \mu_i + k_i3 \) and \( |\hat{D}_i| \leq D_i + k_i4 \) hold, which means that \( \hat{\mu}_i \) and \( \hat{D}_i \) always exist to satisfy \( \hat{\mu}_i \geq \mu_i \) and \( \hat{D}_i \geq D_i \). Consequently, there exist positive constants \( \hat{\mu}_i \) and \( \hat{D}_i \) simultaneously satisfying \( \hat{\mu}_i \geq \mu_i \), \( \hat{\mu}_i \geq \hat{\mu}_i \) and \( \hat{D}_i \geq D_i \). To show the validity of Theorem 1, the Lyapunov function \( V_3 \) is defined as

\[
V_3 = V_2 + \sum_{i=1}^{N} \left[ \frac{1}{c_{i1}} \left( \hat{\mu}_i - k_i3 \tanh(\hat{\mu}_i) \right)^2 + \frac{1}{c_{i2}} \left( \hat{D}_i - k_i4 \tanh(\hat{D}_i) \right)^2 \right]
\]

(28)

By resorting to equation (23) and the designed controller in equation (24), the derivative of equation (28) is calculated as the following expression

\[
\dot{V}_3 \leq \dot{V}_1 - \sum_{i=1}^{N} \left[ \left( \frac{2^\theta}{\theta \gamma (1 - p) T_c} \right)^{\theta - \theta_1 - 1} \right] \left( \frac{1}{2^\theta} \left\| z_{i3} \right\|^{2\theta} + \gamma \right)^{2 - p} + k_i2 \left\| z_{i3} \right\| + \left( \frac{k_i3 h_i \tanh(\hat{\mu}_i)}{\left\| z_{i3} \right\| + \varepsilon_1} \right) \left( \hat{\mu}_i - k_i3 \tanh(\hat{\mu}_i) \right) + \frac{k_i4 \tanh(\hat{D}_i)}{\left\| z_{i3} \right\| + \varepsilon_2} \left( \hat{D}_i - k_i4 \tanh(\hat{D}_i) \right)
\]

\[
\sum_{i=1}^{N} \left( - \frac{2 k_i3 c_{i1} \cosh(\hat{\mu}_i)}{c_{i1} \cosh(\hat{\mu}_i)} \right) \left( \frac{\hat{\mu}_i - k_i3 \tanh(\hat{\mu}_i)}{\left\| z_{i3} \right\|} \right) + \sum_{i=1}^{N} \left( \frac{2 k_i4}{c_{i2} \cosh(\hat{\mu}_i)} \right) \left( \frac{\hat{D}_i - k_i4 \tanh(\hat{D}_i)}{\left\| z_{i3} \right\|} \right)
\]

(29)

Combining with \( \dot{V}_1 \) in equation (19), the updating laws (25) and Lemma 3, equation (29) is rewritten as

\[
\dot{V}_3 \leq - \frac{1}{\theta \gamma (1 - p) T_c} \left( \frac{1}{2^\theta} \left\| z_{i3} \right\| \right)^{\theta - \theta_1 - 1} \left( \frac{1}{2^\theta} \left\| z_{i3} \right\| + \gamma \right)^{2 - p} - (\varepsilon - I_3) z_3^T z_2 + \frac{1}{2} z_2^T z_3 + \Delta_0 + \sum_{i=1}^{N} \mu_i h_i \left\| z_{i3} \right\| + \sum_{i=1}^{N} D_i \left\| z_{i3} \right\| - \frac{1}{\theta \gamma (1 - p) T_c} \left( \frac{1}{2^\theta} \left\| z_{i3} \right\| \right)^{\theta - \theta_1 - 1}
\]

\[
\sum_{i=1}^{N} \left( \frac{k_i3 h_i \tanh(\hat{\mu}_i)}{\left\| z_{i3} \right\| + \varepsilon_1} \right) \left( \hat{\mu}_i - k_i3 \tanh(\hat{\mu}_i) \right) + \frac{k_i4 h_i}{\left\| z_{i3} \right\| + \varepsilon_2} \left( \hat{D}_i - k_i4 \tanh(\hat{D}_i) \right)
\]

\[
\sum_{i=1}^{N} \left( \frac{k_i3 h_i \tanh(\hat{\mu}_i)}{\left\| z_{i3} \right\| + \varepsilon_1} \right) \left( \hat{\mu}_i - k_i3 \tanh(\hat{\mu}_i) \right) + \frac{k_i4 h_i}{\left\| z_{i3} \right\| + \varepsilon_2} \left( \hat{D}_i - k_i4 \tanh(\hat{D}_i) \right)
\]

\[
\sum_{i=1}^{N} \left( \frac{k_i4 h_i}{\left\| z_{i3} \right\| + \varepsilon_2} \right) \left( \hat{D}_i - k_i4 \tanh(\hat{D}_i) \right)
\]

(30)

For estimation variables \( \hat{\mu}_i \), if their initial values are set as positive constants, \( \hat{\mu}_i > 0 \) will always exist. Thus, \( \frac{k_i3 h_i \tanh(\hat{\mu}_i)}{\left\| z_{i3} \right\| + \varepsilon_1} \) can be calculated as

\[
\frac{k_i3 h_i \tanh(\hat{\mu}_i) z_{i3}}{\left\| z_{i3} \right\| + \varepsilon_1} \leq -k_i3 h_i \tanh(\hat{\mu}_i) \left\| z_{i3} \right\| + \frac{k_i3 h_i h_i}{N}
\]

(31)

Upon the employment of equation (31), \( \hat{D}_i \geq D_i \), \( \hat{D}_i \geq D_i \) and \( \hat{\mu}_i \geq \mu_i \), \( \hat{\mu}_i \geq \hat{\mu}_i \), one can deduce that

\[
\dot{V}_3 \leq - \frac{1}{\theta \gamma (1 - p) T_c} \left( \frac{1}{2^\theta} \left\| z_{i3} \right\| \right)^{\theta - \theta_1 - 1} \left( \frac{1}{2^\theta} \left\| z_{i3} \right\| + \gamma \right)^{2 - p} - (\varepsilon - I_3) z_3^T z_2 - \frac{1}{\theta \gamma (1 - p) T_c} \left( \frac{1}{2^\theta} \left\| z_{i3} \right\| \right)^{\theta - \theta_1 - 1} \left( \frac{1}{2^\theta} \left\| z_{i3} \right\| + \gamma \right)^{2 - p} - (\varepsilon - I_3) z_3^T z_2
\]

\[
\sum_{i=1}^{N} \left( \frac{k_i3 h_i \tanh(\hat{\mu}_i)}{\left\| z_{i3} \right\| + \varepsilon_1} \right) \left( \hat{\mu}_i - k_i3 \tanh(\hat{\mu}_i) \right) + \frac{k_i4 h_i}{\left\| z_{i3} \right\| + \varepsilon_2} \left( \hat{D}_i - k_i4 \tanh(\hat{D}_i) \right)
\]

\[
\sum_{i=1}^{N} \left( \frac{k_i3 h_i \tanh(\hat{\mu}_i)}{\left\| z_{i3} \right\| + \varepsilon_1} \right) \left( \hat{\mu}_i - k_i3 \tanh(\hat{\mu}_i) \right) + \frac{k_i4 h_i}{\left\| z_{i3} \right\| + \varepsilon_2} \left( \hat{D}_i - k_i4 \tanh(\hat{D}_i) \right)
\]

(32)

Since the parameters \( \gamma \) and \( \rho \) defined in Lemma 5 satisfy \( \gamma > 1 \) and \( 2 - \rho > 1 \), one can deduce that the following equation holds: \( \left( (1/2^\theta z_{i3})^{\theta} + \gamma \right)^{2 - p} > 1 \). And then, considering \( \varepsilon_{\text{min}}(e) < 1 \), equation (32) is further simplified as

\[
\dot{V}_3 \leq - \frac{1}{\theta \gamma (1 - p) T_c} \left( \frac{1}{2^\theta} \left\| z_{i3} \right\| \right)^{\theta - \theta_1 - 1} - 2(\varepsilon - I_3) \left( \frac{1}{2^\theta} \left\| z_{i3} \right\| \right)^{\theta - \theta_1 - 1}
\]

(33)
can deduce that the uniformly ultimately bounded of expression of \( p \) and \( \sim \) guarantee a better control accuracy. At the preceding section and adaptive technique are required to

Remark 4. 

prescribed-time convergence of \( J \) is confirmed, which is really helpful in illustrating the designer-appointed convergence of formation system with prescribed-time stability and robust disturbance attenuation.

\[
V_4 = \frac{1}{2} \zeta_i^T \zeta_1 + \frac{1}{2} \zeta_3^T M \zeta_3
\]  

(34)

Benefitting from equations (18), (21), and (32), the time derivative of \( V_4 \) can be readily calculated as

\[
\dot{V}_4 = \zeta_i^T \dot{\zeta}_1 + \zeta_3^T \dot{M} \zeta_3 \leq -\frac{1}{\theta \gamma (1-p) T_c} \left( \frac{1}{2} \zeta_3^T \zeta_3 \right) \theta \gamma (1-p) T_c
\]

\[
\left( \left( \frac{1}{2} \zeta_i^T \zeta_1 \right)^\theta + \gamma \right) - \frac{1}{\theta \gamma (1-p) T_c} \left( \frac{1}{2} \zeta_3^T \zeta_3 \right) \theta \gamma (1-p) T_c
\]

\[
\left( \left( \frac{1}{2} \zeta_3^T \zeta_3 \right)^\theta + \gamma \right) + \sum_{i=1}^N h_i \left( \mu_i - k_{13} \tanh (\mu_i) \right) \frac{1}{N} \left( \zeta_i^T \zeta_i \right)
\]

\[
+ \sum_{i=1}^N \left| \zeta_i \right| \left| \left( \tilde{D}_i - k_{14} \tanh (\tilde{D}_i) \right) + \frac{1}{2} \zeta_3^T \zeta_3 + \frac{\beta_0^2}{2} \right.
\]

\[
+ \sum_{i=1}^N \frac{1}{N} \left( k_{13} \zeta_i \zeta_i + k_{14} \zeta_i \zeta_i \right) \leq
\]

where \( \theta_1 = \min \left( \frac{1}{\theta \gamma (1-p) T_c}, \frac{1}{\theta \gamma (1-p) T_c}, \frac{\sqrt{c_i h_i} ||z_i||, \sqrt{\tilde{c}_i ||z_i||}} {\sqrt{c_i h_i} ||z_i||, \sqrt{\tilde{c}_i ||z_i||}} \right) \) and \( \sigma_1 = \sigma_0 + \sum_{i=1}^N \frac{1}{N} \left( k_{13} h_i, k_{14} \zeta_i \zeta_i \right) \). Owing to the fact that the inequality \( 0 < \theta_0 - \theta + 1 < 1 \) holds true for any \( 0 < \theta < 1 \) and \( 0 < \theta < 1 \), one can deduce that the uniformly ultimately bounded of \( z_1, z_2, z_3, \mu_i, \) and \( \tilde{D}_i \) can be pursued. Moreover, learning from the intact expression of \( \sigma_1 \), it is intuitive that the tracking accuracy would not deteriorate as the formation members increase.

Remark 4. Since the filtering error \( \zeta_2 \) and estimate errors \( \tilde{\mu}_i \) and \( \tilde{D}_i \) do exist as the formation maneuvers, the proper parameter selection and adaptive technique are required to guarantee a better control accuracy. At the preceding section of the proof, the boundedness of \( z_2, \mu_i, \) and \( \tilde{D}_i \) are confirmed, which is really helpful in illustrating the prescribed-time convergence of \( z_1 \) and \( z_3 \).

Then, we introduce the following Lyapunov function to validate the designer-appointed convergence of formation errors \( z_1 \) and \( z_3 \)

\[
V_4 = \frac{1}{2} \zeta_i^T \zeta_1 + \frac{1}{2} \zeta_3^T M \zeta_3
\]

(35)

where \( \sigma_3 = \sum_{i=1}^N h_i \left( \mu_i - k_{13} \tanh (\mu_i) \right) ||z_i|| + \sum_{i=1}^N \frac{1}{N} \left( k_{13} h_i, k_{14} \zeta_i \zeta_i \right) \) and \( \tilde{D}_i \) can be treated as a bounded scalar satisfying \( 0 < \sigma_3 < \infty \). By virtue of Lemma 5, it is evident that the proposed formation coordinated controller is of prescribed-time stability within given time \( T_c \). Thus, the validity of Theorem 1 is completely illustrated.

Remark 5. Upon the employment of the devised prescheduled-time control architecture, fast system response and high reliability are acquired when compared to the existing formation methods. Specifically speaking, the control architecture \( \Psi \) performs the function of characterizing the formation system with prescribed-time stability and robust disturbance attenuation.

Simulations

In this section, numerical simulation examples are presented for the purpose of illustrating the appealing performance of the devised controller. The simulations are implemented on the MATLAB R2018a platform using a fixed-step Runge–Kutta solver (100 Hz). Suppose that the ASV formation consists of three follower ASVs and a virtual leader, which is arranged as shown in Figure 2.

The simulation system is mainly composed of five portions, including (1) ASV model, (2) control module, (3) adaptive module, (4) RBF NNs, and (5) external disturbances. Under the controller expressed as equation (24), the formation system governed by equation (1) is able to keep three followers tracking a virtual leader while remaining a predefined formation shape despite the impact of the external perturbations. To achieve this control objective, control parameters are strictly selected. For fair
The parameter of the Gaussian function

Number of neurons 15 Inputs

The parameter of the low-pass filter

ε

Table 1. Control parameters.

| Parameter                  | Value                  | Parameter                  | Value                  |
|----------------------------|------------------------|----------------------------|------------------------|
| $k_1$                      | diag(7, 10, 9)         | $k_2$                      | diag(8, 9, 10)         |
| $(c_1, c_2)$                | (0.1, 0.1)             | $(b_1, b_2)$               | (1, 1)                 |
| $(k_3, k_4)$                | (20, 20)               | $(\theta, \gamma, \rho)$  | (0.5, 2, 0.5)          |
| The parameter of the low-pass filter | εi diag(0.01, 0.01, 0.01) |

| Number of neurons | 15 | Inputs | $[1.3 \ 1.3] \times [1.3 \ 1.3]$ |
|-------------------|----|--------|----------------------------------|
| The parameter of the Gaussian function | $\sigma_i = 2$ |

comparison, the adaptive cooperative formation controllers proposed in the studies of Lu et al.\textsuperscript{7} and Huang et al.\textsuperscript{34} are simulated additionally under the same condition. Parameters of the formation system are adopted as the same as in the study of Lu et al.\textsuperscript{7} that is, the mass of each ASV is 23.8 kg with its length being 1.255 m and width being 0.29 m. The external disturbances of the three dimensions are presented as

$$\tau_{wi} = \begin{bmatrix} -2\cos(0.5t)\cos(t) + 0.3\cos(0.5t)\sin(0.5t) - 3 \ N \\ 0.01\sin(0.1t)N \\ 0.6\sin(1.1t)\cos(0.3t)N \cdot m \end{bmatrix}$$

The trajectory of the leader vessel is given as $\eta_c = [0.2 + 4.5 m, 0 m, 0 rad]^T$. The configuration vectors of the formation members are defined as $\Delta_1 = [2 m, 0 m, 0 rad]^T$, $\Delta_2 = [0 m, 2 m, 0 rad]^T$, and $\Delta_3 = [0 m, -2 m, 0 rad]^T$. Initial values of each formation member's states are given as $\eta_1(0) = [-1 m, 0 m, -\frac{\pi}{6} rad]^T$, $\eta_2(0) = [-2 m, 3 m, -\frac{\pi}{3} rad]^T$, $\eta_3(0) = [-1 m, -3 m, \frac{\pi}{6} rad]^T$, $v_1(0) = [0, 0, 0]^T$, $v_2(0) = [0, 0, 0]^T$, and $v_3(0) = [0, 0, 0]^T$. In addition, initial values of adaptive updating laws are set as zero. In equation (32), one can intuitively learn how the control parameters would influence the steady-state error of the formation system and the response of each state. Therefore, by virtue of equation (32) and constant trial and error, control parameters are presented in Table 1.

### Simulation results of the prescribed-time controller

Simulation results under the proposed adaptive prescribed controller (24) are shown in Figures 3–6 as prescribed time $T_c = 10 s$. The trajectory of ASV formation in $x$–$y$ plane is depicted in Figure 3(a). It is evident that the trajectory of virtual leader is well tracked by the follower ASVs. Simultaneously, the target formation can be achieved within prescribed time and keep unaltered during the tracking mission. The time responses of formation errors are exhibited in Figures 3(b) and 4(b). From their zoomed portions, it can be concluded that the designed formation system is able to accomplish the fast-tracking maneuver within 10s despite the parameter variations and external disturbances. Figure 4(a) elaborates the detailed information of the filtering errors. Obviously, this kind of deviation does exist in the adopted filter, but remains bounded rapidly.

Control torques and estimation variables are presented in Figures 5 and 6, respectively. From these results, we have found that smooth control command will be deduced by this controller and all the estimated valuables will remain bounded. It follows from these related pictures that

![Figure 3](image-url)  
Figure 3. Formation trajectory of ASVs in $x$–$y$ plane (a) and the corresponding tracking errors $\bar{z}_{ii}$ (b) under the devised controller. ASV: autonomous surface vessel.
trajectory tracking and configuration remaining are fulfilled with satisfactory performance in prescribed time when the devised control scheme takes effect.

**Comparative simulation results**

For a better presentation of the high reliability and fast response ability of the devised controller in this article, comparative simulations are conducted in this subsection. Since the works of Lu et al.\(^7\) and Huang et al.\(^\text{34}\) are both about the cooperative formation control of multiple ASVs against the time-varying environmental disturbances, they are therefore deemed as the appropriate comparative examples. Lu et al.\(^7\) proposed a disturbance-rejection controller, while a finite-time control strategy for multiple ASVs was proposed in the study by Huang et al.\(^\text{34}\). To illustrate the robustness of the proposed control protocol, the following three types of piecewise disturbances are utilized to imitate the ocean waves and winds, which can be expressed as follows:

![Figure 4](image1.png)

**Figure 4.** Filtering error \(z_{2i}\) (a) and velocity error \(z_{3i}\) (b) under the devised controller.

![Figure 5](image2.png)

**Figure 5.** Control torque \(τ_i\) and its partial enlarged pictures under the devised controller.

![Figure 6](image3.png)

**Figure 6.** Estimation variables \(\hat{D}_i\) (a) and \(\hat{ρ}_i\) (b) under the devised controller.
Corresponding results are shown in Figure 7. To show the
superiority of the proposed control strategy, the prescribed
tial states and disturbances under Case 1, involving the same ini-
herent from previous subsection, are between 0 and 1.

As seen from Figure 9(a), the output of controller in the
channel z as displayed in Figure 7(b). Even under Case 2 and Case 3, the continuous growing disturbances
is no longer an obstacle to the formation tracking system
(Figure 8).

As seen from Figure 9(a), the output of controller in the study by Lu et al. [7] under Case 1 has changed a lot at the
Figure 7. Formation error $z_{ii}$ of the proposed controller (a) and that of Huang et al.\textsuperscript{34} (b) under Case 1.

Figure 8. Formation error $z_{ii}$ under Case 2 (a) and Case 3 (b) under the devised controller.

Figure 9. Control torque $τ_i$ of Lu et al.\textsuperscript{7} (a) and the devised strategy (b) under Case 1.
initial stage and simultaneously possessed a huge amplitude extended to $10^6$, which is deemed to be unrealistic in practical. From these aspects, one can deduce that the presented controller (29) enjoys better control performance with a smaller control torque, as shown in Figure 9(b).

**Conclusion**

In this brief, the predefined-time tracking control problem is addressed for multiple ASVs exposed to model uncertainties and external perturbations. Three unique features distinguish the devised algorithm from the existing ones are: (i) The predefined-time control protocol is developed, which augments the feasibility of the proposed strategy. (ii) The adverse impact enticed by external disturbances is tackled considering “explosion of terms.” (iii) Rapid system response is provided with MLP algorithm. Since the tracking accuracy is purely characterized by the designed parameters, future work will be dedicated toward removing this constraint in a user/designer-friendly way.

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Appendix 1

Some of the related nomenclatures are given in Table 1A.

| Number | Symbol | Description | Unit          |
|--------|--------|-------------|---------------|
| 1      | $x$    | Position in surge | m            |
| 2      | $y$    | Position in sway  | m            |
| 3      | $\psi$ | Yaw angle     | rad          |
| 4      | $u$    | Velocity in surge | m/s         |
| 5      | $v$    | Velocity in sway | m/s         |
| 6      | $r$    | Yaw velocity  | rad/s        |
| 7      | $M$    | Inertial matrix | —            |
| 8      | $C(v)$ | Centripetal and Coriolis matrix | —            |
| 9      | $D(v)$ | Hydrodynamic damping matrix | —            |
| 10     | $g$    | Unmodeled dynamics | —            |
| 11     | $\eta = [x, y, \psi]^T$ | Generalized position vector | [m, m, rad] |
| 12     | $\nu = [u, v, r]^T$ | Generalized velocity vector | [m/s, m/s, rad/s] |
| 13     | $\tau_w = [\tau_{wu}, \tau_{wv}, \tau_{wr}]^T$ | External disturbances | [N, N, N] |
| 14     | $\tau = [\tau_u, \tau_v, \tau_r]$ | Control torque | [N, N, N] |
| 15     | $W$    | Ideal weight matrix | —            |
| 16     | $H(x)$ | Gaussian basis function | —            |
| 17     | $\varepsilon$ | Approximation error | —            |
| 18     | $\eta_i$ | The desired trajectory and heading vector | [m, m, rad] |
| 19     | $\Delta_i$ | The desired formation shape | [m, m, rad] |