A cosmological AMR MHD module for Enzo

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Abstract. Magnetic fields play an important role in almost all astrophysical phenomena including star formation. But due to the difficulty in analytic modeling and observation, magnetic fields are still poorly studied and numerical simulation has become a major tool. We have implemented a cosmological magnetohydrodynamics package for Enzo which is an AMR hydrodynamics code designed to simulate structure formation. We use the TVD solver developed by S. Li as the base solver. In addition, we employ the constrained transport (CT) algorithm as described by D. Balsara. For interpolation magnetic fields to fine grids we used a divergence free quadratic reconstruction, also described by Balsara. We present results from several test problems including MHD caustics, MHD pancake and galaxy cluster formation with magnetic fields. We also discuss possible applications of our AMR MHD code to first star research.

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INTRODUCTION

Adaptive mesh refinement(AMR) cosmological hydrodynamics simulations play an important role in the study of structure formation of different scales from galaxy clusters to first stars in the past ten years. Its ability to achieve very high resolution in large scale simulations with relatively small computer resources has helped us to understand the first stars formed in our Universe. The possible effects of magnetic fields have been largely ignored. It is well established that magnetic fields are present on different scales, from intracluster medium to interstellar medium. The origin and evolution of these magnetic fields and their role on the structure formation are still unclear. So, it is imperative to include magnetic fields in current hydrodynamics AMR cosmology code. In this paper, we present the newly developed MHD version of the Enzo, which is wildly used in the study of first stars [1,14,15].

MHD WITH ENZO

The MHD equations in the comoving coordinates are:

\[
\frac{\partial \rho}{\partial t} + \frac{1}{a} \nabla \cdot (\rho v) = 0
\]

\[
\frac{\partial \rho v}{\partial t} + \frac{1}{a} \nabla \cdot (\rho v + \mathbf{p} - \mathbf{BB}) = -\frac{\dot{a}}{a} \rho v - \frac{1}{a} \rho \nabla \phi
\]

\[
\frac{\partial E}{\partial t} + \frac{1}{a} \nabla \cdot [\mathbf{v} (\mathbf{p}) + \mathbf{E} - \mathbf{B} (\mathbf{B} \cdot \mathbf{v})] = -\frac{\dot{a}}{a} (\rho v^2 + 3p + \frac{B^2}{2}) - \frac{\rho}{a} \nabla \cdot \nabla \phi
\]

with

\[
E = \frac{1}{2} \rho v^2 + \frac{p}{\gamma - 1} + \frac{1}{2} \mathbf{B}^2
\]

\[
\mathbf{p} = p + \frac{1}{2} \mathbf{B}^2
\]

where all variables have their usual meaning, a is the expansion parameter. To track the pressure more accurately in the supersonic region, we have also implemented the modified entropy equation given in Ryu et al. [16] and the internal energy equation given in Bryan et al. [4] in our code.

The MHD solver used for all the test problems here is a high-order Godunov-type finite-volume numerical solver developed by S.T. Li [7,8]. This solver was recently successfully used to study magnetic jet problems [9,12,13].

We used a constrained transport(CT) scheme flux CT [3] to maintain divergence-free magnetic fields. For the AMR hierarchy in Enzo, we used a modified divergence-free reconstruction scheme original proposed by Balsara [2] including second order accurate divergence-free restriction and prolongation for magnetic fields. Details of the CT and AMR of magnetic fields in the MHD Enzo can be found in Collins et al. [5].

The MHD module has been tested and shown to be compatible with other physics packages installed in Enzo, such as radiative cooling, star formation and feedback.
TESTS

MHD Caustic and Pancake

The MHD Caustic test is taken from Li et al. [10] which generalizes the HD test of [16]. The initial sinusoidal velocity field in the x-direction has the peak value $1/2\pi$, the initial density and pressure has been set to be uniform with $\rho = 1$ and $p = 10^{-10}$. Then caustics will be formed because of the compression by the velocity field. An initial uniform magnetic field of $10^{-3}$ in code units in the y direction was added to the simulation. Figure 1 compares the density and $B_y$ at $t = 3$ of unigrid and AMR runs. AMR run had 256 cells in the root grid and 2 level refinements by 2. The AMR solution is indistinguishable from a uniform grid solution with 1024 cells.

Pancake is another standard test problem of cosmological hydrodynamics simulation [15]. We have run the collapse of a one-dimensional pancake in a purely baryonic universe with $\Omega = 1$ and $h = 0.7$. Initially, at $t_i = 1$, which corresponds to $z_i = 20$ in this test, a sinusoidal velocity field with the peak value $0.65/(1 + z_i)$ in the normalized unit has been imposed in a box with the comoving size $64 h^{-1} Mpc$, so that the shock forms at $z = 1$. The initial baryonic density and pressure have been set to be uniform with $\rho = 1$ and $p = 6.2 \times 10^{-8}$ in the normalized code units. We applied initial uniform magnetic fields $B_x = 2.0 \times 10^{-5} G$, $B_x = B_z = 0$ in the simulation. We did the calculations with unigrid with 1024 cells and AMR run with 256 cells of root grid and 2 level refinements. Figure 2 shows the density and $B_y$ at $z = 0$.

Cluster Formation with Magnetic Fields

Cluster formation is one of the problems most widely studied by Enzo [11]. We have done this problem and compared with results from Enzo-ppm to test our new code in large scale structure formation. The simulation is a ΛCDM model with parameters $h = 0.7$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $\sigma_8 = 0.928$. The survey volume is $256 h^{-1} Mpc$ on a side. The simulations were computed from a 128$^3$ root grid and 2 level nested static grids in the Lagrangian region where the cluster forms which gives an effective root grid resolution of $512^3$ cells ($0.5 h^{-1}$ Mpc) and dark matter particles of mass $1.49 \times 10^5 M_\odot$. AMR is allowed only in the region where the galaxy cluster forms, with a total of 8 levels of refinement beyond the root grid, for a maximum spatial resolution of $7.8125 h^{-1} kpc$.

We first present the results of no initial magnetic fields and compare them with the results from Enzo-ppm. The cluster parameters from the MHD code is almost identical to those from Enzo-ppm. The virial radii are $2.229 Mpc$ from hydro and $2.226 Mpc$ from MHD while the virial masses are $1.265 \times 10^{15} M_\odot$ for hydro and $1.260 \times 10^{15} M_\odot$ for MHD. Figure 3 compares the projections of the baryon density and temperature.

![Figure 1](image1.png)

**FIGURE 1.** Plots of density and y component of magnetic fields of MHD caustics at t=3. The initial magnetic field is $10^{-3}$.

![Figure 2](image2.png)

**FIGURE 2.** Plots of density and y component of magnetic fields of MHD pancake. The initial magnetic field is $2 \times 10^{-5} G$.

![Figure 3](image3.png)

**FIGURE 3.** Logarithmic projected gas density(top) and logarithmic projected X-ray weighted temperature(down) at $z = 0$ of adiabatic simulations. The images cover the inner $4 h^{-1} Mpc$ of cluster centers. The left panels show results from the PPM solver and the right panels show results from the MHD solver. The units of density and temperature are $M_\odot/Mpc^3$ and Kelvin respectively.

We have performed simulation with initial magnetic fields, $B_x = B_z = 0$, $B_y = 1.0 \times 10^{-9} G$ with radiative cooling, star formation and star formation feedback. Figure 4 shows the images of baryon density, temperature, magnetic energy density and Faraday rotation measurement of the cluster center. The rotation measurement is integral along the projection direction.
Another simulation we performed is without initial magnetic field but with the Biermann battery effect. The induction equation is modified by adding an additional battery term:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \nabla \times \left( \frac{c \mathbf{v} \mathbf{p}_e}{n_e e} \right)$$

(7)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{c m_H}{e} \frac{1}{1 + \chi} \nabla \times \left( \frac{\mathbf{p}}{\rho} \right)$$

(8)

where $c$ is speed of light, $p_e$ is the pressure of electron, $n_e$ is the electron number density, $e$ is the electron charge, $m_H$ is the hydrogen mass and $\chi$ is the ionization fraction. We took $\chi = 1$ constant in space in our simulation. We performed this computation with radiative cooling. Figure 5 shows the projection of logarithmic baryon density and the magnetic energy density of the cluster center.

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