Students’ misconceptions on the algebraic prerequisites concept: causative factors and alternative solutions

Dea Permata, P Wijayanti and Masriyah
Universitas Negeri Surabaya, Jl. Ketintang, Surabaya, 60231, Indonesia

Corresponding author: deapermata12006051@gmail.com

Abstract. Mastery of the concept of number operations is the main prerequisite for understanding the algebraic concepts. The misconception of integer operations is still widely found in students. This research aimed to describe students’ misconceptions on the algebraic prerequisites concept, namely the operation of integers, causative factors and alternative solutions. This research was qualitative descriptive research using diagnostic test methods and interview. Three students who did the most and varied misconceptions were chosen to be the subject of this research. The results showed the misconceptions experienced by the subject occurred in the integer addition, integer subtraction, integer division and multiplication with zero numbers. The factors that cause misconception were wrong intuition and incorrect or incomplete reasoning and alternative solutions were re-explain and cognitive conflict. For further research, other researchers can conduct research on students’ misconceptions on the algebraic prerequisites concept besides number operations.

1. Introduction
One branch of mathematics that must be mastered by students from elementary school to high school is algebra. Students who lack understanding in algebra will have difficulty understanding other mathematical material [1]–[4] and other lessons [1]–[3]. Before understanding algebra, the concept of algebraic prerequisites must be understood first. Numbers and numerical operations, ratios and proportions, the order of operations, equality, patterning, algebraic symbolism and letter usage, algebraic equations, functions, and graphing are concepts of algebraic prerequisites [5]. Mastery of the concept of number operations is the main prerequisite for understanding the algebraic concepts.

Students’ understanding of algebra is still low [1], [2], [6], [7]. This can be one of the main barriers for many students in learning mathematics. Repeated errors in solving algebraic concepts can lead to misconceptions. The misconception is a symptom of the cognitive structure that causes errors even though there are other sources that cause errors [8], [9]. The misconception is an idea or view that is wrong about a concept someone has that is different from the concept that is considered true by experts [10]–[12]. Misconception is the result of a lack of understanding or error in applying rules or mathematical generalizations [9], [13]. The misconception is an error that results from a lack of understanding of a concept that appears repeatedly and continuously [1], [10], [14]–[16]. The misconception is an idea or view that is not true about a concept that is owned by someone who is different from the concept that is considered true by experts, errors in applying rules or mathematical generalizations and errors in understanding concepts that occur repeatedly and continuously.

Misconceptions differ from errors. The error is the use of procedures or concepts that are wrong in solving mathematical problems [9], [17]. The error is the wrong answer because of inappropriate and
unsystematic planning that appears unstable in solving mathematical problems [8], [16]. Based on the description above, it can be seen that there are differences between misconceptions and errors. The error is the wrong answer because improper and unsystematic planning that appears unstable while misconception is cognitive symptoms that cause errors, which occur repeatedly and continuously.

Some examples of misconceptions in algebra experienced by students are 7th graders in Indonesia in completing \( x - 9 = 13 \) by means of \( 13 = x + 9 \) [2], 8th grades in Malang complete \( 8 + 4x = 12x \) [18] and 8th grades students in Semarang in completing \( 2x + 3y = 5xy \) [19]. These findings indicate that students have not mastered the algebraic prerequisites concept, one of them is the operation of integers. The misconception of integer operations is still widely found in students, such as misconceptions carried out by 7th grade students in Surakarta who understand \( -90 - 9 = 81 \) [20], 8th graders in Bangkalan Regency understand \(-3 + (-10) = 13, -15 + (-8) = 7, -12 - 7 = -5, -12 - 7 = 5, 10 + (-7) = 17, 8 - (-14) = -22 \) and \( 8 - (-14) = 6 \) [21]. 7th graders in Palembang misconception integer addition and subtraction operations namely \(-24 - 8 = 16, 19 + (-6) = -25 \) and \(-31 + (-8) = 39 \) [22] and student's misconception of multiplication with zero numbers, \( 9 \times 0 \times 8 \) is understood as \( 9 \times 8 \) resulting in 72 [23].

Misconceptions that occur in students are important to overcome because remembering misconceptions can hinder the acceptance of new students 'material and influence students' success in solving mathematical problems [24]–[27]. Finding misconceptions, causal factors and suitable alternative solutions can be done to overcome student misconceptions [11], [13], [14], [28]–[30]. Factors causing misconception can come from the student itself. These factors are preconception, associative thinking, incomplete or incorrect reasoning, wrong intuition, cognitive developmental stages, ability and learning interest [11]. The alternative solutions that can be done by the teacher in overcoming misconceptions were re-explain and cognitive conflict strategy [31]. Re-explain can be done again by explaining part of each concept or procedure that students have not understood, giving an example of correct solutions and giving examples or non-examples concept. Cognitive conflict strategy can encourage students to reevaluate the mistakes made because students can identify the contradictions of the mathematical principle between the original answers and the students' answers. Cognitive conflict strategies generate dissatisfaction with the concepts students have so that it helps students to change their wrong concepts into concepts based on scientific concepts [32], [33].

Table 1 below shows the possible misconceptions on the algebraic prerequisites concept that occur toward the students.

| Concept   | Possible Misconceptions that Occur                                                      |
|-----------|------------------------------------------------------------------------------------------|
| Integer   | **Addition**                                                                              |
|           | 1. Stating the addition of positive integers with negative integers is done by subtracting positive integers with the inverse of the negative integer but not understanding the sign used for the answer. |
|           | 2. Stating that the result of adding two negative numbers is positive.                    |
|           | 3. Stating that the result of adding two negative numbers is by subtracting large numbers by small numbers. |
| Integer   | **Subtraction**                                                                           |
|           | 1. Wrong in completing the subtraction followed by a negative sign.                       |
|           | 2. Solve the problem by subtracting large numbers by small numbers.                      |
| Integer   | **Division**                                                                              |
|           | 1. Ignores the negative sign when completing the division of negative integers with positive integers. |
|           | 2. Read the division sign as a subtraction when completing the division of negative integers with positive integers. |
|           | 3. Read division operations such as multiplication operations.                           |
| Multiplication | **with zero numbers**                                                                    |
|           | 1. Stating that 0 does not represent anything.                                           |
|           | 2. Read multiplication operations such as exponent operations.                           |
|           | 3. Put 0 behind the multiplied number.                                                    |
Based on the description above, it is necessary to know what the misconceptions that the students experienced on the algebraic prerequisites concept, namely the operation of integers, causative factors and alternative solutions, so that this study aimed to describe students’ misconceptions on the algebraic prerequisite concepts, namely the operation of integers, causative factors and alternative solutions.

2. Method
This research was qualitative descriptive research using diagnostic test methods and interview. This research was conducted at SMPN 1 Koba, class VIII A. This class consists of 32 students, chosen purposively from six available classes. The diagnostic test provided consists of 7 questions about the operation of integers. The subjects in this research were three students who did the most and varied misconceptions in completing the diagnostic test. Students who had misconceptions of 4, 5 and 7 questions from 7 questions to be the subject of this research. The semi-structured interview was conducted on the subject of the research to find out more about students’ misconceptions on the algebraic prerequisites concepts and causative factors.

3. Result and Discussion
   3.1. The Misconceptions made by Student 1
Figure 1 (a) showed that student 1 was wrong in solving integer addition and subtraction questions. Student 1 understands the addition and subtraction of integers by subtracting large numbers by small numbers and giving a sign for answers by looking at the sign of a small number of the question. Student 1 answered questions number 1 spontaneously by subtracts 9 with 2 produces 7 and gave a negative sign for 7 because student 1 assumed that the sign for the answer was obtained by looking at the sign of a small number of question, (-2), so the answer is -7, as well as questions number 2, 3 and 4. Student 1 answered questions number 2, 3 and 4 in the same way as number 1. In accordance with previous research that students stated the addition of positive integers with negative integers and negative integers with negative integers by subtracting large numbers with small numbers and not understanding the sign used for the answer [34], [35] as well as for the subtraction of positive integers with negative integers and negative integers with positive integers [35]–[37].

Student 1 was also wrong in solving integer division questions, can be seen in Figure 1 (b). Student 1 understands integer division by subtracting the first number by the second number. Student 1 subtracts 36 with 9 produces 27 and subtracts 27 with 3 produces 24. Agree with previous research, namely students read the sign of division as a subtraction operation [37]. Misconceptions made by student 1 in completing addition, subtraction and integer division due to student 1 experiencing wrong intuition. Wrong intuition can cause misconception because students spontaneously express their views on the question without first understanding how to solve the problem [11].

Figure 1 (c) shows that student 1 understands multiplication with zero numbers by putting point 0 then jumps 20 times so that 0 does not represent anything. Any number multiplied by 0 remains the number itself. Students thought that 0 did not represent anything [23]. The cause of this misconception was that students experienced incorrect or incomplete reasoning of the information obtained. This was because student experienced incorrect or incomplete reasoning of the information obtained so that created misinterpretation [11].

Alternative solutions that can be used to overcome misconceptions were re-explain and cognitive conflict strategy. Re-explain the concept of addition and subtraction of integers can be done using a number line or using positive beads and negative beads. Example 9 + (-2).
Re-explain the concept of integer division can be done by explaining that $36 \div 9$ is subtraction 9 to 36 repeatedly until it is not left over. Re-explain the concept of multiplication with zero numbers to students can be done by explaining that $20 \times 0 = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0$. Add 0 to itself 10 times.

Cognitive conflict strategy can be done in way (1) ask students about the addition and subtraction of integers, integer division and multiplication with zero numbers, (2) ask students to give examples and non examples, (3) linking concepts with examples given by students, (4) ask students to analyze existing concepts with examples or non-examples given by students, (5) give other examples that have not been given by students in accordance with the scientific concepts of experts, (6) ask students to analyze the examples given by students so that students will experience cognitive conflict and (7) providing instructions for ending conflict.

3.2. The Misconceptions made by Student 2
Figure 2 (a) showed that student 2 was wrong in solving the integer addition and subtraction questions. Student 2 subtracted large numbers with small numbers and signs for answers were searched by multiplying the sign of the first number by the second number. For example, question number 1, student

![Figure 1](image-url)
2 answers by subtracting 9 by 2. Then because the sign for the first number was positive and the sign for the second number is negative, positive multiplied by negative results are negative, so student 2 answers with the answer -7 and so on for questions number 2, 3 and 4. This is consistent with previous research that subtracting large numbers with small numbers and used the wrong sign for answering, namely multiplying the signs of the number in the question was a misconception in the addition of positive integers and negative integers and negative integers with negative integers [34], [35] as well as for the subtraction of positive integers with negative integers and negative integers with positive integers [35]-[37]. The factors causing misconceptions by student 2 subjects in completing addition and subtraction integers were wrong intuition. Student’s wrong intuition can cause misconceptions [11]. Student 2 answered the question number 6, (-27) ÷ 3 = 9, can be seen in Figure 2 (b). Student 2 ignored the negative sign for the answer. Agree with previous research that students ignored the negative signs when dividing negative numbers with positive numbers [37]. This was probably due to student 2 reasoning that was incorrect or incomplete from the information that she obtained from the concept of dividing negative integers with positive integers. Incorrect or incomplete reasoning for the information obtained can cause misconceptions [11].

Re-explain the concept of addition and subtraction of integers can be done using a number line or using positive beads and negative beads. Example -6 + (-3).

Because positive meets negative, the result will be negative. So the result of 9 + (-2) = -7.

Because negative meets negative, the result will be positive. So the result of -6 + (-3) = 3.

Because positive meets negative, the result will be negative. So the result of 3 − (-8) = -5.

Because negative meets positive, the result will be negative. So the result of -5 − 2 = -3.

Use the opposite of division, 3 × 9 = 27. So the result is 9.

Re-explain the concept of integer division can be done by explaining that (-27) ÷ 3 is there are -9 a three times in -27.

Cognitive conflict strategy can be done in way (1) ask students about the addition and subtraction of integers and integer division, (2) ask students to give examples and non examples, (3) linking concepts with examples given by students, (4) ask students to analyze existing concepts with examples or non-examples given by students, (5) give other examples that have not been given by students in accordance
with the scientific concepts of experts, (6) ask students to analyze the examples given by students so that students will experience cognitive conflict and (7) providing instructions for ending conflict.

3.3. The Misconceptions made by S3

Student 3 was wrong in solving integer subtraction questions, can be seen in Figure 3 (a). Student 3 understood integer subtraction by subtracting large numbers with small numbers and giving a sign for an answer by looking at the sign of a large number from the question. Student 3 spontaneously subtracts 8 with 3 produces 5 and gives a negative sign for 5 because student 3 assumed that the sign for the answer was obtained by looking at the sign of a large number of question, (-8), so the answer is -5. In accordance with previous research that students stated the subtraction of positive integers with negative integers and negative integers with positive integers by subtracting large numbers with small numbers and used the wrong sign for the answer [35]–[37]. Figure 3 (b) showed that student 3 understands integer division by subtracting the first number by the second number. This agreed with previous research, namely student read the division sign as a subtraction operation [37]. Figure 3 (c) showed that student 3 answers 20 × 0 = 20 (multiplication with zero numbers) by stating that 0 does not represent anything. Any number multiplied by 0 remains the number itself. Agree with previous research which stated that students thought that 0 did not represent anything when multiplying with zero numbers [23]. The causes of misconceptions by student 3 in completing the subtraction and division of integers and multiplications with zero numbers was wrong intuition. Wrong student intuition because spontaneously expressing his views on a concept can cause misconceptions [11].

Alternative solutions that can be used to overcome misconceptions are re-explain and cognitive conflict strategy. Re-explain the concept of addition and subtraction of integers can be done using a number line or using positive beads and negative beads. Example -5 – 2.

$$-12 \quad -11 \quad -10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1$$
Re-explain the concept of integer division can be done by explaining that \((-27) \div 3\) is there are -9 a three times in -27. Re-explain the concept of multiplication with zero numbers to students can be done by explaining that \(20 \times 0 = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0\). Add 0 to itself 10 times.

Cognitive conflict strategy can be done in way (1) ask students about the subtraction of integers, integer division and multiplication with zero numbers, (2) ask students to give examples and non examples, (3) linking concepts with examples given by students, (4) ask students to analyze existing concepts with examples or non-examples given by students, (5) give other examples that have not been given by students in accordance with the scientific concepts of experts, (6) ask students to analyze the examples given by students so that students will experience cognitive conflict and (7) providing instructions for ending conflict.

4. Conclusion
Misconceptions in the concept of addition and subtraction of integers made by students were subtracting a large number from a small number and using the wrong sign for an answer by looking at the sign of a small number of questions, multiplying the sign of first number by the second number and looking at the sign of a large number of questions. The misconceptions of the concept of integer division were to subtract the first number by the second number and ignores the negative sign when completing a negative integer division. The misconception in the concept of multiplication with zero numbers was stating that 0 does not represent anything. The causative factor of student misconception are wrong intuition and incorrect or incomplete reasoning. Alternative solutions that can be used to overcome misconceptions are re-explain and cognitive conflict.

5. Acknowledgment
The authors thank to Universitas Negeri Surabaya for supporting this research. The authors also thank to the head department to our master program, Dr. Tatag Yuli Eko Siswono, M.Pd., who teaches and motivates the researchers to learn more about research methodology on research in mathematics education. Furthermore, the authors also thank to the teachers in SMPN 1 Koba who helped in this research and the students who were willing to be research subjects.

6. References

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