Electrodynamics of Massless Charges with Application to Pulsars

Andrei Gruzinov
New York University Abu Dhabi, near Sheikh Khalifa Mosque, Abu Dhabi, United Arab Emirates

ABSTRACT

Electromagnetic field together with zero-mass charges moving in this field form a well-behaved semi-dissipative dynamical system – Electrodynamics of Massless Charges (EMC). We give equations of EMC, argue that EMC is an adequate theory for calculating pulsar magnetospheres, give an illustrative numerical calculation (showing that bolometric luminosity of an aligned rotator is approximately equal to half the spin-down power). EMC looks like a portion of the full pulsar theory that will resolve the already calculated bolometric luminosity into light curves and spectra.

1. Electrodynamics of Massless Charges (EMC)

EMC postulates that positive and negative charges move with unit, meaning speed of light, velocities:

\[ \mathbf{v}_{\pm} = \frac{\mathbf{E} \times \mathbf{B} \pm (B_0 \mathbf{B} + E_0 \mathbf{E})}{B_0^2 + E_0^2}. \] (1)

Here \( E_0 \) and \( B_0 \) are the proper electric field scalar and the proper magnetic field pseudo-scalar, defined by

\[ B_0^2 - E_0^2 = B^2 - E^2, \quad B_0 E_0 = \mathbf{B} \cdot \mathbf{E}, \quad E_0 \geq 0. \] (2)

Equations (2) mean that \( E_0 \) and \( |B_0| \) are the field magnitudes in the frame where the fields are parallel, with the sign of \( B_0 \) negative when the fields are antiparallel.

The "Aristotelean" dynamics [1], with the particle velocity rather than acceleration given by the force, results from an infinite radiation damping of massless charges. Equation [1] means that in the frame where the fields are parallel, charges move along the common direction of the fields at the speed of light. Moving along the fields nullifies the leading-order radiation damping term. And, because of curvature radiation, the parallel dynamics is also instantaneous, fixing the velocities at [1].

The full EMC system of equations is, in the 3+1 split,

\[ \dot{\mathbf{B}} = -\nabla \times \mathbf{E}, \] (3)

\[ \dot{\mathbf{E}} = \nabla \times \mathbf{B} - \rho_+ \mathbf{v}_+ + \rho_- \mathbf{v}_-, \] (4)

\[ \rho_+ + \nabla \cdot (\rho_\pm \mathbf{v}_\pm) = Q. \] (5)

Here \( \rho_\pm \) are the particle densities (multiplied by the absolute value of the charge), and the source \( Q \) represents pair creation/annihilation. The source \( Q \) can be chosen at will. This is because our massless particles do not carry energy and momentum (again because of the strong radiation damping), and therefore pairs can be created and annihilated arbitrarily. The only requirement is that \( Q \geq 0 \) if one of the densities \( \rho_\pm \) vanishes – there can be no negative particle densities.

Somewhat surprisingly, even in this incomplete form, EMC seems to be useful, because for a broad range of assumed sources \( Q \), the EMC predictions remain virtually unchanged, and one can indeed choose \( Q \) as he wishes. In pulsar applications, an
arbitrarily chosen $Q$ allows to calculate only the bolometric luminosity, while for the spectra and light curves one must calculate the source $Q$ from the actual charge-photon kinetics. This means that the photon distribution function needs to be added to the EMC system. The important point, however, is that even in this full pulsar theory, the field and charge dynamics is still given by the EMC system (3-5).

Applicability of EMC to pulsars requires instantaneous parallel dynamics at light cylinder, which gives

$$L \gg L_e \left( \frac{R}{r_e} \right)^{2/3},$$

where $L$ is the spin-down luminosity, $R$ is the light cylinder radius, $r_e = \frac{e^2}{mc^2}$ is the classical electron radius, $L_e = \frac{mc^3}{r_e} = 8.7 \times 10^{16}$ erg/s is the “classical electron luminosity”. Equation (6), for a 30ms pulsar, gives $L \gg 6 \times 10^{30}$ erg/s, and is easily satisfied.

2. Aligned rotator in EMC

To illustrate how EMC works in pulsar applications (the only application the author is aware of), we simply assume that $Q$, whatever it is, creates a prefixed quantity of high-multiplicity plasma, meaning

$$\rho_+ + \rho_- = |\rho| + f \rho_0,$$

where $\rho = \rho_+ - \rho_-$ is the charge (Goldreich-Julian) density, $f$ is the multiplicity and $\rho_0 \gtrsim |\rho|$ is a fiducial charge density greater than the absolute value of the actual charge density $|\rho|$.

In the figure we show the results for $f = 10$ and $\rho_0 = \sqrt{B^2 + E^2}/R$, where $R$ is the spherical radius. This $\rho_0$ simply represents the charge density which is capable of changing the fields by an order unity factor. The simulation technique is similar to what we have used when simulating pulsars by Strong-Field Electrodynamics (SFE) (Gruzinov 2011a). The light cylinder is at twice the stellar radius.

We get an absolute replica of the pulsar magnetosphere calculated in the high-conductivity limit of SFE. This calls for an explanation.
of §2 gives the same magnetosphere as the high-conductivity SFE.

The universality of the high-conductivity SFE magnetosphere follows from the expression for the electric current in EMC:

$$j = \frac{\rho E \times B + (\rho_+ + \rho_-)(B_0B + E_0E)}{B^2 + E_0^2}.$$  \hfill (11)

Now comparing the EMC current (11) to the SFE current (8), we see that the equilibrium state of the high-multiplicity plasma of §2 is identical to the SFE equilibrium with a certain (weird) space-dependent conductivity – but as long as the conductivity is everywhere high, its actual value is irrelevant.

The following seems to be a fair analogy. The temperature jump at the ideal gas shock can be calculated in dissipative gas dynamics with arbitrary but small viscosity. Similarly, the amount of damping of an ideal pulsar magnetosphere can be calculated in EMC with arbitrary but high multiplicity, or in SFE with arbitrary but high conductivity.

EMC and SFE give identical results for the bolometric luminosity. But only EMC seems to be a portion of the full pulsar theory, because EMC gives a faithful description of the particle density (except for creation and annihilation if any).

4. Summary

EMC – Electrodynamics of Massless Charges – is a well-behaved dynamical system. The charges carry zero energy-momentum and obey Aristotelian dynamics. The system is semi-dissipative. The electromagnetic energy can decrease, but for certain field configurations it is precisely conserved.

For pulsars, in the high-multiplicity limit, EMC gives an undamped Poynting flux everywhere except at the equatorial current layer. As the charges carry no energy, the equatorial Poynting flux damping must be interpreted as bolometric luminosity. For an aligned rotator, one gets the bolometric luminosity \(\approx 50\%\) of the spin-down power (Gruzinov 2011b). For a generic inclination angle, the bolometric luminosity has not yet been calculated. Presumably the bolometric luminosity at generic inclination is smaller than 50\% of the spin-down power, because the singular current layer is weaker in the inclined case (Spitkovsky 2006).

I thank Matt Kleban for useful discussions.

REFERENCES

Gruzinov, A., 2011a, Pulsar Magnetosphere, arXiv:1101.3100
Gruzinov, A., 2011b, Ohmic Power of Ideal Pulsars, arXiv:1101.5844
Spitkovsky, A., 2006, Time-dependent Force-free Pulsar Magnetospheres: Axisymmetric and Oblique Rotators, ApJ, 648, L51