Scattering of GWs on wormholes: foreshadow and afterglow/echoes from binary merges

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We study specific features of the scattering of gravitational waves on wormholes and normal matter objects. We derive and solve the GW energy transport equation and show that the scattered signal lies in the same frequency spectrum bands as the basic signal. The scattering forms specific long living tails which always accompany the basic signal and have a universal form. The scattering on normal matter objects forms tails which have always the retarding character, while wormholes lead to advanced tails as well. In addition, wormholes may produce considerably stronger effect when the total energy in tails exceeds the energy of the incident wave by the factor up to $10^3$. In both cases the retarding tails have a long living character when the mean amplitude behaves with time as $h \sim 1/\sqrt{t} + R/c$. For a single GW event the echo tails give only a tiny contribution to the mean amplitude. However such tails accumulate with events and may be observed by their contribution to the noise produced by the stochastic GW background.

I. INTRODUCTION

The gravitational-wave (GW) event GW150914 [1] has opened a new window in the Universe that gave rise to the born of the gravitational wave astronomy. Further observations [2] and new GW projects launched (e.g., see Ref. [3]) promise new and more detailed information about the structure and different processes in our Universe. In particular, several different groups have recently claimed tentative evidence for repeating echoes in the LIGO/Virgo observations of binary black hole mergers [4]-[7]. The last group [7] claims the significance to reach $4.2\sigma$ at $f \simeq 72$ Hz. We leave aside numerous possible mechanisms of origin of such echoes discussed in literature (e.g., see references in the above papers) and, in the present paper, explore properties of echoes which are generated by the scattering of the basic GW signals on relic wormholes.

The simplest natural echo events may appear simply by lensing effects. Indeed, in considering the scattering of GWs on normal astrophysical objects the most rough estimate gives for the cross-section the value of the order $\sigma_g \propto r_g^2$, where $r_g = 2GM$ is the gravitational radius of an object. This will cause the deflection angle $\theta \sim 2r_g/r$ ($r$ is the distance from the ray to the object) and the time delay for additional images of the source $\delta t/\Delta t \sim \theta^2$ ($\Delta t$ is the time passed from the emission to the detection). In general, for compact objects all such values are extremely small. A considerable effects may appear in two cases, when the distance from the ray to the object is sufficiently small, or when the mass is sufficiently big. On cosmological distances the time delay accumulates and becomes random. The accumulated random delays affect also the primary ray, which sets additional difficulties in observations of such effects. This means that in the first place the GW astronomy allows to probe only very large astrophysical objects (lensing on galaxies, clusters etc.).

Despite to the fact that the creation of relic wormholes in the very early Universe from quantum spacetime foam is a rather natural process, wormholes are still assumed to be exotic astrophysical objects. This happens, in the first place, due to the absence of a direct observational evidence for wormholes. Unlike normal objects wormholes possess the cross-section of the order $\sigma_w \propto b^2$, where $b$ is the throat radius, which can take very big values $b \gg r_g$. In addition to lensing, it is possible to observe a more detailed diffraction picture which appears due to the scattering on a single wormhole. In this case the necessary condition is to have a wormhole on the line of sight between the source of GW radiation and the observer. That seems to be an extremely rare situation. However, if we consider a distribution of relic wormholes, then the scattering of GWs on wormholes should give a noticeable contribution to the scattered signal. Indeed, in the models in which dark matter phenomenon appears due to a distribution of wormholes

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In [11], the gravitational perturbations of a Morris-Thorne wormhole are approached. Further details can be found in [18], [19] and [20].

The total scattered signal may exceed the direct GW signal on the factor up to $\tau_w \sim 10^3$, where $\tau_w$ is the mean optical depth for GWs. We recall that for the scattering of GWs on normal astrophysical objects (e.g., stars) the optical depth is negligible $\tau_s \sim \sigma_s n_s \ell \ll 1$, where $\sigma_s < r_0^2$ is the mean cross-section for a typical star, $n_s$ is the mean density of stars, and $\ell$ is the traversed path along the ray, and the Universe is completely transparent. In the presence of wormholes however the Universe may be transparent only for some particular directions/rays, while the mean value of $\tau_w$ can be estimated by the dark to luminous matter ratio (e.g., in low surface brightness galaxies the mass to luminosity ratio $M/L$ may reach $10^3$ already at the edge of the optical disk). We also point out that it is not difficult to suggest parameters of a single wormhole that produce the scattered signal exceeding or having the same order as the direct GW signal. The only problem here is that the arrival times will be considerably separated (up to millions of light years).

The estimate $\sigma_w \propto b^2$ follows from the geometrical optics and it is valid for all types of radiation scattered by wormholes (CMB, GWs, Cosmic Rays, radiation emitted by galaxies, etc.) and, therefore, results presented in the present paper may equally applied to any type of radiation. Effects of scattering of radiation (different from GWs) on wormholes mix with analogous effects produced by normal matter [10]. It is very difficult to separate them. In this sense GWs give a unique tool to probe the existence of wormholes in space, since they are weakly scattered by ordinary matter and ordinary astrophysical objects. In the present paper we demonstrate that the scattering of the basic GW signal from a binary merge forms a specific halo of secondary sources and at the Earth every GW impulse will have heavy long living tails. In the case of wormholes there are both - advanced and retarded tails, while the total energy flux emitted by the halo and the energy contained in the tails exceeds that in the basic signal. However this energy is widely distributed in time.

The two basic features of such a scattering are that the scattered signal gets into the same frequency window as the basic signal and that the mean GW amplitude $h (h \sim \sqrt{T_0}$, where $I_0 \sim \langle \hat{h}^2 \rangle = \frac{1}{T_0} \int_0^T (\hat{h}^2 + \hat{h}_0^2) dt$, $T = 2\pi/\omega$, and $\hat{h} = \partial h / \partial t$ in the tail decays with time very slowly $I_0 \sim 1/(t + R/c)$ and, therefore, $h(t) \sim 1/\sqrt{t + R/c}$. The amplitude ratio $\delta I/I^0$ has the order $\tau_0 c/dt \ll 1$, where $\tau$ is the optical depth and $dt$ is the duration of the basic signal and, therefore, the direct detection of such a tail is not possible. However, when taking into account all different binary merge events, tails accumulate which gives an additional multiplier $N \sim \nu R/c \gg 1$, where $\nu$ is the rate of events and $R$ is the mean distance to the source. Therefore, we should predict essentially enhanced level of the stochastic GW background in the given frequency band (roughly by the factor

$$
\sigma(\nu R/c) \sim \frac{\delta I/I^0}{\nu R/c} \sim \frac{T_0}{\nu R/c}
$$

as compared with the measured/predicted rate of merges, where $\rho_{DM}$ and $\rho_b$ are the dark matter and baryon densities).

II. WORMHOLES MODELIZATIONS

The problem of scattering by a single wormhole involves several difficulties. There existing models of wormholes involving exotic matter, i.e. the cross-section depends essentially on the structure of such a wormhole (matter properties, structure of the throat etc.). Some features should not depend on the exact structure of wormholes and the matter supporting the wormhole. Moreover, since GW sources (binary merges) are rather rear events, it is an extremely rare chance to observe the scattering on a single wormhole (it should have the position on the line of sight). Part of this problem is treated in [11]-[12].

It is possible to find a metric, for which the features of the wormhole do not depend on the kind of matter supporting it. These kinds of metrics do not even depend on the particular structure of the wormhole. After [13], [14], [15], [16], and, as summarized, for example, in [17], for a Schwarzschild-spherically-symmetric Morris-Thorne wormhole, the metric chosen corresponds to the line element

$$
ds^2 = -e^{2\beta(r)} dt^2 + e^{2\alpha(r)} dr^2 + r^2 (d\theta^2 + \sin \theta d\phi^2)
$$

(1)

where the shape function $b(r)$ is defined as

$$
e^{2\alpha(r)} \equiv \frac{1}{1 + \beta(r) r}.
$$

(2)

At the minimum radius $r_0$, the wormhole is defined by the function $\beta(r)$ assuming the value $\beta(r_0) \equiv r_0$ (throat measure), with the vertical asymptotics

$$
\lim_{r \to r_+} \alpha(r) = +\infty
$$

(3)

Further details can be found in [18], [19] and [20]. In [11], the gravitational perturbations of a Morris-Thorne wormhole are approached.
In [12], the Teukolski equations in the Newman-Penrose formalism are specified.

The scattering of Gaussian gravitational wave packets by a single Morris-Thorne wormhole has been studied in [21], where the transverse polarization component and the traceless polarization components have been analyzed, both for an odd potential and for an even one. As a result, only a very narrow model-dependent band peak has been demonstrated to be observable.

For a single Kerr-like wormhole, whose perturbations are ruled by a symmetric potential, the echoes of a scattered signal have been analyzed with similar results in [22]; rotation has been demonstrated to modify the emission forms. The observational differences between the scattering of gravitational radiation on a single wormhole and that of ultra-compact stars by the differences of the quasi-normal modes [23] by means of the inverse-spectrum method for the detection of gravitational waves, for which non-negligible differences arise in the non-negligible experimental errors for next generation gravitational wave detectors in the reconstruction of the quasi-normal spectrum.

For single ultra-static wormholes, non-Schwarzschild symmetric wormhole metrics have been demonstrated to reduce the necessity for the explanation of the wormhole properties by exotic matter, and the guidelines to eliminate such dependence on the specific matter content rather than on the solution to the Einstein field equations for the Schwarzschild-symmetric case have been outlined [24].

A. Modelization of the noise

It is possible to get some estimates of the measurements of the scattering of gravitational waves on wormholes-like objects, and to compare it with other gravitational-wave signals and with other wormhole signals.

A lensing for the shadow of Misner-Thorne wormholes is proposed in [25], in the case of spherical symmetry. The edge of the lensing allows one to distinguish the wormhole from other celestial bodies. The analysis of the lensing is also suited for the analysis of the parameters qualifying the solutions to the EFE’s. The strong gravitational lensing for static wormhole geometries is analyzed in [26]. In particular, the parameters available in the lensing effect provide one with the possibility to determine different wormhole geometries and different celestial-bodies geometries.

In [27], the differences in the observational evidence for a Schwarzschild wormhole, an Ellis-Bronnikov wormhole, a Schwarzschild blackhole and a blackhole are studied. The estimations of the parameters characterizing the solutions for the EFE’s of an Ellis-Bronnikov wormhole are constrained by experimental accuracy and the criteria to keep it distinguished from a Schwarzschild blackhole are determined in the gravitational strong-lensing regime after the study of the quasi-normal modes. The case of a Kerr blackhole is also discussed.

In [28], the gravitational lensing for an Ellis wormhole is calculated. The case of a collection of wormholes is also faced. In particular, the radius of the throat of the wormhole is taken as a determining factor for the determination of the number density of the collection of wormholes for identifying it form other compact objects in the geometrical-optics approximation. The observational methods of femto-lensing analysis [21–31], micro-lensing and or the astrometric-image-centroid- displacements analysis [32] are compared and found not suited for the analysis of such macroscopic celestial configurations, as they do not resolve the modulation of the light curve or the displacements in the time domain.

In [33], the geodesics and the deflection angle for a charged Lorentzian wormhole are calculated.

In [34], the observables for the strong gravitational lensing of wormholes are determined. The Ellis-Bronnikov solutions is analyzed. Different locations for the locations are proposed in the vicinity of the wormhole, which are however not compatible with Earth-based experiments and/or with satellite-based experiments.

In [35], the signal analysis is constructed to discriminate between linear stationary, non-linear stationary noise and non-stationary noise. In particular, it is possible to separate the linear part of the gravitational wave from the non-linear stationary noise and form the non-stationary contribution, the linear-stationary noise arising as a particular case. The gravitational-wave perturbation $h$ can be therefore described as

$$h(t) = \eta(t) + F(w_i(\tau < t))$$

with $\eta$ the uncorrelated and untracktable noise background, being $F$ the other contributions. It is possible to subtract the non-stationary parameterized noise contributions after solving for each frequency the spectral analysis and transforming the spectral analysis in the time domain. For the detection of gravitational-wave signals of Astrophysical origin, the non-stationarity signals can be subtracted without the modification of the evaluated parameters. For a Morris-Thorne wormhole, in [21], the ECOs are calculated by the geometrical-optic approximation and evaluated as produced only in a narrow band, for which the detection would perturb the signal by adding noise only in the asymptotical signal (outside the bandwidth evaluated).
In resonant-mass detectors (antennas), in a paramagnetic environment, the impedance of astrophysical objects which in general have a rather complex form (couples of two-dimensional surfaces that all physical fields are continuous at throats. On sky such boundaries will be seen as couples of specific/exotic boundaries at which we should specify specific boundary conditions. Boundary conditions follow simply from the fact along the minimal sections of wormhole throats. As the result we get the space manifold with a set of couples of two angles on the Sky, or any other coordinates of the flat Friedman model). This is achieved by making cuts map which is commonly used to describe points in the homogeneous and isotropic Universe (e.g., the red shift and map. In astrophysical applications however it is convenient to describe wormholes in terms of the single coordinate produced a fluctuating radiation-pressure forces scaling as \( \omega \), with \( \omega \) the resonant frequency, \( Q_\epsilon \) the electrical \( Q \)-factor of the transducer resonator, \( C \) the capacitance, \( x \) the distance between the two masses.

The limit \( Q_\epsilon \geq \omega_p/\omega_a \) can be considered. In interferometers, the perturbation on the end of the hand mirrors produces a fluctuating radiation-pressure forces scaling as \( \sqrt{N} \), while the input laser power \( P \) increases causing a relative photon-counting error decrease as \( \sqrt{N}/N \), while the radiation-pressure error increases as \( \sqrt{N} \). It is possible to minimize the total error (including the capacitor contributions) by minimizing the the total error with respect to the power \( P \) of the laser field.

\[
\beta = \frac{1}{2m\omega_a^2}\frac{CV_\epsilon^2E}{x^2},
\]

III. TOPOLOGICAL STRUCTURE OF SPACE WITH A DISTRIBUTION/GAS OF RELIC WORMHOLES

In general relativity topological structure of space is determined by onset, as additional initial conditions (see also recent discussion in Ref. [42]). The most natural initial conditions correspond to a fractal topological structure of space, i.e., to a fractal distribution of wormholes. Indeed, lattice quantum gravity models predict fractal properties of space at sub-planckian scales (e.g., see Refs. [43, 44]), while the inflationary phase in the past should enormously stretch all scales and temper such a structure as initial conditions.

We point out that the space which contains a distribution of wormholes cannot be covered by a single coordinate map. In astrophysical applications however it is convenient to describe wormholes in terms of the single coordinate map which is commonly used to describe points in the homogeneous and isotropic Universe (e.g., the red shift and two angles on the Sky, or any other coordinates of the flat Friedman model). This is achieved by making cuts along the minimal sections of wormhole throats. As the result we get the space manifold with a set of couples of boundaries at which we should specify specific boundary conditions. Boundary conditions follow simply from the fact that all physical fields are continuous at throats. On sky such boundaries will be seen as couples of specific/exotic astrophysical objects which in general have a rather complex form (couples of two-dimensional surfaces \( S_A \), where the index \( A = 1,..N \) numerates wormhole throats). Those surfaces/objects may move, rotate, possess equal masses and magnetic poles. The boundary conditions are induced by the cutting procedure. For example, if we consider a couple of such surfaces \( S_k \) which correspond to the same wormhole, then the internal region of space restricted by \( S_+ \) (or, in general, some part of it) admits a one-to-one map on a portion of the outer region for the conjugated surface \( S_- \). This defines the boundary conditions in a unique way.

In general case throat sections (their space-like part) have the form of a sphere with \( n \) handles \( S_n^2 \). As it was discussed in Ref. [42] the simplest wormhole has the section which has the form of a torus. We recall that such wormholes do not require the presence of exotic forms of matter and they do evolve. In open Friedman models their rate of evolution exactly coincides with the common cosmological expansion and they are stable. In the case of the flat space the rate of their evolution is still not investigated properly. Some encouraging results were obtained in the limit when one radius of a torus-like throat tends to infinity and the geometry acquires cylindrical configuration. In particular, in a series of papers [45, 48] static and stationary cylindrical wormhole solutions were found and it was demonstrated that asymptotically flat wormhole configurations do not require exotic matter violating the weak energy condition. We point out that if such a torus-like wormhole rotates in space, this surely should prevent it from very fast collapse. The thorough investigation of such objects represents too complex problem which still waits for its investigation.

In the present paper we, for the sake of simplicity, assume spherical throats of wormholes. Some words to approve the use of such an approximation worth adding. Indeed, spherical wormholes collapse very quickly and, therefore, they cannot be distinguished from ordinary black holes. Stable traversable spherical wormholes may exist only in the presence of exotic forms of matter and in modified theories, where the role of the exotic matter is played by...
an appropriate modification \[49, 51\]. So far, there is no any rigorous experimental evidence for the existence of exotic forms of matter, or for the presence of any modification of general relativity (we leave aside possible quantum corrections which work only in the quasi-classical region, e.g., at Planck scales). This means that the approximation of spherical throats should be very rough. Nevertheless, in considering collective effects of GW scattering on a distribution of wormholes, such an approximation works rather well. Owing to the fact that more general wormhole throats have random orientations in space and assuming that they have an isotropic distribution, the final result always contains the averaging over orientations. When we perform such an averaging, then every throat restores the spherical symmetry. The spherical wormhole, in turn, is much more simple object to work with and it admits a much more simple consideration.

In conclusion of this section we consider the exact illustrative model of the scattering on a single torus-like wormhole in the open model (see the exact description of the appropriate metric in Ref. \[12\]) and construct the map of geodesic lines at boundaries for a spherical wormhole.

A. Echoes from a single wormhole

Consider first the open Freedman model. Then the space-like part represents the standard Lobachevsky space. The simplest wormhole which connects two Lobachevsky spaces is obtained by a factorization over a discrete subgroup of the group of motions. The discrete subgroup \(G\) is determined by a couple of generators \(T_a(l_1)\) and \(T_b(l_2)\) which describe two shifts of the space in orthogonal directions (\(l_1\) and \(l_2\) denote two orthogonal geodesics) on distances \(r_1 = a\) and \(r_2 = b\). Any element \(g_{mn} \in G\) is merely \(g_{mn} = T_{a}^m T_{b}^n\) with \(m, n = 0, \pm 1, \pm 2, \ldots\). The minimal section of the throat has the form of a torus with two radii \(r_1\) and \(r_2\). As it was demonstrated in Ref. \[12\] it is possible to introduce such coordinates on the Lobachevsky space \((\chi_1, \chi_2, \chi_3)\) that the shift is merely \(g_{mn} (\chi_1, \chi_2, \chi_3) = (\chi_1 + 2\pi m, \chi_2 + 2\pi n, \chi_3)\), while perturbations of the metric \(\delta g_{\alpha\beta}(x)\) obey the “periodic” property

\[
\delta g_{\alpha\beta}(\chi_1, \chi_2, \chi_3) = \delta g_{\alpha\beta}(\chi_1 + 2\pi m, \chi_2 + 2\pi n, \chi_3).
\]

Consider now the retarded Green function \(G_F(x, t; x', t')\) for the open Friedman model which describes the propagation of a single GW impulse. The above periodicity means that in the presence of the wormhole any source in the Friedman space also obeys the periodic conditions and, therefore, in terms of the unrestricted Friedman space it induces multiple additional images

\[
\frac{1}{\sqrt{\gamma}} \delta_w(x - x') = \frac{1}{\sqrt{\gamma}} \delta_F(x - x') + \sum_{m, n \neq 0} \frac{1}{\sqrt{\gamma}} \delta_F(x - g_{mn}(x')).
\]

Then in the presence of the wormhole the true Green function has the obvious structure

\[
G_w(x, t; x', t') = G_F(x, t; x', t') + \sum_{m, n \neq 0} G_F(x, t; g_{mn}(x'), t').
\]

The same structure acquires any signal emitted by binary mergers. Here the first term corresponds to the direct GW signal, while the sum corresponds to echoes. The amplitudes of echoes depend on the position of the source (with respect to the wormhole center) and the parameters of the wormhole (radii \(a\) and \(b\)). We point out that the general structure of echoes \[3\] appears also for spherically symmetric wormholes in the asymptotically flat space, e.g., see Refs. \[21, 22\]. Therefore, it is not a specific feature of the model discussed. When we have a distribution of such wormholes the general structure remains the same, while the GW signal becomes much more complicated. Moreover, part of echoes overwrite the basic signal. In general amplitudes of echoes are extremely small and phases become random.

It was shown in Refs. \[8, 9\] that additional images of an actual source \(7\) may play the role of dark matter. In this case the distribution of DM in the Universe is determined by the distribution of wormholes, while the observed in galaxies rigid relation between visible and dark matter components acquires the most natural explanation. This means also that the multiple echoes have the same origin and the intensity of echoes relates somehow to the distribution of dark matter. This feature will be used for estimates of the intensity of the stochastic GW background.

B. Geodesic map for spherical wormholes

As it was pointed out the space with wormholes cannot be covered by a single coordinate atlas. For practical aims it is convenient to have a single atlas (e.g., a part of the Friedman space). In this case we cut the wormhole
throat by the minimal section of the throat. Then the wormhole represents a couple of surfaces \( \Sigma_{\pm} = S_{\pm} \times R \) (which are the direct product of a sphere and the time axis) whose internal regions are removed, while surfaces of the spheres are glued. From the topological standpoint this corresponds to the standard Minkowski space with boundaries. In general case for a remote observer such boundaries move in space and are glued by the Poincare motion \( x^\alpha_\pm - X^\alpha_\pm = \Lambda^\alpha_\beta (x^\beta - X^\beta) \). The gluing means that when the ray \( x(t) \) reaches one such a boundary \( \Sigma_\pm \) at some finite value \( t_0 \), i.e., \( x^\alpha_\pm(t_0) \in \Sigma_\pm \) and particular values of \( k^\alpha_\pm(t_0) \), its continuation comes from the other boundary \( \Sigma_\mp \) with new initial data \( x^\alpha_\mp(t_0) \in \Sigma_\mp \) and \( k^\alpha_\mp(t_0) \) related by the Poincare map \( x^\alpha_\pm - X^\alpha_\pm = \Lambda^\alpha_\beta (x^\beta - X^\beta) \) and \( k^\alpha_\pm = \Lambda^\alpha_\beta k^\beta_\pm \).

In general, there is some back reaction (some change in parameters \( X^\alpha_\pm, X^\beta_\pm \), and \( \Lambda^\alpha_\beta \)) as described in Ref. [9]. In the present paper we neglect the back reaction. In the reference frames in which the throat entrances are at rest the map corresponds simply to the inversion of the spheres. This can be easily seen for the simplest Ellis-Bronnikov metric (EB-wormhole) \( ds^2 = dt^2 - f(r)dr^2 \), where \( f(r) = 1 + b^2/r^2 \) and \( dt^2 = dr^2 + r^2(\sin^2 \theta d\theta^2 + d\theta^2) \) is the standard line element of the flat space. The inversion map \( r' = b^2/r \) simply interchanges the inner and outer regions of the sphere \( r = b \) which is the minimal throat section of the EB-wormhole. Close to the minimal section \( r = b \) almost any spherical metric with a smooth source can be reduced to this case.

First, for the sake of simplicity we assume that the space is flat (this is not an approximation since in the neighborhood of any point of the surfaces \( \Sigma_{\pm} \) the metric can be taken as a flat Minkowski metric, the only exclusion case is the so-called thin-shell wormholes). Second, we assume \( X^\alpha_\pm = (t, R_{\pm}) \) (the shift of time is absent), and velocities \( \frac{dt}{dR_{\pm}} = V_{\pm} < c \) (velocities of centers of spheres \( S_{\pm} \)). Then \( \Lambda^\alpha_\beta \) is a composition of a space rotation \( U^\alpha_\beta \) and Lorentz boost. Consider the incident wave \( (\omega_{in}, k_\pm) \). Let it falls on the throat \( S_- \) at the point \( x'_- = bn'_- + R_- \) (where \( bn'_- = \xi'_- = x'_- - R_- \)). Since the throat \( S_- \) moves in space with the velocity \( V_- \) the frequency and the wave number in the coordinate system in which the throat \( S_- \) is at rest are (we assume \( 1 - (V_-/c)^2 \approx 1 \))

\[
\omega'_- \approx \omega_{in} - (V_- k_\pm), \quad k'_- \approx k_\pm - \frac{\omega_{in}}{c} V_-.
\]

This wave (ray) is absorbed by the throat \( S_- \) and re-radiates from \( S_+ \) at the point \( x'_+ = bn'_+ + R_+ \) which relates to \( x'_- \) by the relation (rotation)

\[
n'_+ = U^l_m n'^m_-. \]

In the reference system in which \( S_+ \) is at rest the frequency and the wave number of the outgoing (re-radiated) wave are

\[
\omega'_+ = \omega'_-, \quad k'^l_+ = U^l_m (k'^m_+ - 2n'^m_- (n'_- k'_-)).
\]

Thus in the initial coordinate system we find the re-radiated values \( (\omega_{out}, k_{out}) \)

\[
\omega = \omega_{out} \approx \omega'_+ - (V_+ k'_+), \quad k = k_{out} \approx k'_+ - \frac{\omega'_+}{c} V_+.
\]

**IV. ENERGY TRANSPORT EQUATION**

The incoherent nature of the scattered signal shows that the exact form of the wavefront is not important for observations. It may be important only for observing the scattering on a particular single wormhole. If we a so happy to make all the necessary conditions meet, then we will need the detailed structure of the scattered signal. However the situation is such that in the nearest future we may hope to detect only collective effects of their scattering. To this end the most convenient way to use the equations for the transport of energy. Such an equation comes out from the standard kinetic equation for the number of gravitons \( N(k,r) \) in the phase space \( \Gamma = (k,r) \). Indeed, in the case when phases of GWs are random the energy of the GWs can be written as follows

\[
W = \sum_j \int W_{k,j} d^3k d^3r = \sum_j \int \hbar \omega N_{k,j} d^3k d^3r.
\]

In the above expression the index \( j \) stands for polarizations. In what follows we will omit the index \( j \). In equation \( W_k \) is the spectral energy density (energy in a unit volume of space and a unit volume of wave numbers). In the isotropic case (more generally, when the number of gravitons depends only on the frequency \( N_\omega \)) the spectral energy
is described by \( W_k d^3 k = W_k k^2 dk d\Omega = W_k k^2 \left| \frac{dk}{d\Omega} \right| d\omega d\Omega = W_\omega d\omega d\Omega \) When considering radiation it is commonly used the spectral intensity of GW radiation (Poynting vector or the energy flux)

\[
S_k = W_k \frac{d\omega}{dk} \quad I_k = |S_k|, \tag{12}
\]

where \( \frac{d\omega}{dk} = V_\omega \) is the group velocity. In general relativity \( |V_\omega| = c \), while in different modified theories its value may change. We see that intensity of waves relates to \( W_k, W_\omega \) simply as \( I_k = c W_k \).

Consider the number of gravitons/photons \( (W_k = h\omega(k, r) N_k) \)

\[
N_{k,r} = \sum_a \frac{1}{\sqrt{\gamma}} \delta \left( r - r_a(t) \right) \delta \left( k - k_a(t) \right). \tag{13}
\]

The equation for \( N_k \) may be obtained from the geometrical optics for simplest case of a set of gravitons. One may consider gravitons as massless (spin-2) particles with the momenta \( p_\mu = \hbar k_\mu \) and the dispersion relation \( k_\mu k_\nu g^{\mu\nu} = 0 \) which defines the energy \( \omega(k, r, t) \). Here \( \gamma_{ij} \) is the space metric, \( (r(t), k(t)) = \Gamma_k(t) \) corresponds to a particular isotropic geodesic line and \( \omega(k, r, t) \) obeys to

\[
g^{\alpha\beta} \frac{\partial \psi}{\partial x^\alpha} \frac{\partial \psi}{\partial x^\beta} = g^{\alpha\beta} k_\alpha k_\beta = 0. \tag{14}
\]

Then the kinetic equation has the form (e.g., see Ref. [9])

\[
\frac{1}{\sqrt{\gamma}} \frac{d\sqrt{\gamma} N_k}{dt} = \tilde{\alpha}_k + \int \Sigma (\Gamma, \Gamma_w, \Gamma', \Gamma'_w) N (\Gamma') N_w (\Gamma'_w) d\Gamma' d\Gamma'_w d\Gamma_w \tag{15}
\]

where we use the definition

\[
\frac{1}{\sqrt{\gamma}} \frac{d\sqrt{\gamma} N_k}{dt} = \frac{1}{\sqrt{\gamma}} \frac{\partial \sqrt{\gamma} N_k}{\partial t} + \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial r} \left( \frac{dr}{dt} \sqrt{\gamma} N_k \right) + \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial k} \left( \frac{dk}{dt} \sqrt{\gamma} N_k \right), \tag{16}
\]

\( \tilde{\alpha}_k \) is the emission of particles/gravitons in the unite time and unite volume, \( \Sigma \) is the scattering matrix

\[
\Sigma (\Gamma, \Gamma_w, \Gamma', \Gamma'_w) = c \sigma (\Gamma, \Gamma_w, \Gamma', \Gamma'_w). \tag{17}
\]

In the above expression \( \sigma (\Gamma, \Gamma_w, \Gamma', \Gamma'_w) \) is the cross-section of the scattering on a wormhole \( \Gamma, \Gamma_w \to \Gamma', \Gamma'_w \) (here \( \Gamma \) and \( \Gamma_w \) are parameters before the scattering (incident) and \( \Gamma' \) and \( \Gamma'_w \) are parameters after the scattering) which are determined by (9)-(10). \( N_w (\Gamma_w) \) is the number of wormholes in the configuration space \( \Gamma_w \), and \( \Gamma_w = (R_k, V_k, b, U, \ldots) \) are all the parameters of the wormholes. In what follows we will assume the case when wormhole are infinitely heavy objects (neglect the back reaction), then we get

\[
\sigma (\Gamma, \Gamma_w, \Gamma', \Gamma'_w) = \tilde{\sigma} (\Gamma, \Gamma'_w) \delta (\Gamma_w - \Gamma'_w). \tag{18}
\]

In the general case it is given by (e.g., see Ref. [1], [53])

\[
\sigma_- (\Gamma, \Gamma_w, \Gamma', \Gamma'_w) = \delta (\xi_+ - b) \delta (\Gamma' - \Gamma_+) \delta (\Gamma'_w - \Gamma_+ - \delta (\xi_- - b) \delta (\Gamma' - \Gamma) \delta (\Gamma'_w - \Gamma'_w) \tag{18}
\]

where \( \Gamma' = \Gamma_{in} = (x_{in}, k_{in}) \) and \( \Gamma = \Gamma_{out} = (x_{out}, k_{out}) = (r, k) \). The sign \( \sigma_- \) means here that GW ray falls on \( S_- \). The second term in (18) corresponds to the absorption on \( S_- \) and the first term corresponds to the re-radiating of the absorbed signal from \( S_+ \). Analogous term \( \sigma_+ \) corresponds to the wave which falls on \( S_+ \) and re-radiates from \( S_- \). In the final expression (54) it gives only the factor 2 (due to the symmetry between entrances).

The equation for the energy transport is found to be

\[
\frac{1}{\sqrt{\gamma}} \frac{d\sqrt{\gamma} W_k}{dt} - \left( \frac{1}{\omega} \frac{d\omega}{dt} \right) W_k = \alpha_k - \int \mu (\Gamma_k, \Gamma'_k) W (\Gamma'_k) d\Gamma'_k \tag{19}
\]

where

\[
\mu (\Gamma_k, \Gamma'_k) = -\int \frac{\omega}{\omega} \Sigma (\Gamma, \Gamma', \Gamma_w) N_w (\Gamma_w) d\Gamma_w. \tag{20}
\]
Σ is defined by (17), and the energy density \( W_k \) for a set of gravitons is

\[
W_k = \sum_a \frac{\hbar \omega(k_a, r_a, t)}{\sqrt{\gamma}} \delta(\mathbf{r} - \mathbf{r}_a(t)) \delta(\mathbf{k} - \mathbf{k}_a(t)).
\]

In equation (19) the term \( \alpha_k = \hbar \omega \tilde{\alpha}_k \) describes spontaneous radiation and \( \mu \) describes adsorption/re-radiation (induced radiation) of the GW radiation in a unit volume of the medium.

We point out that equation (19) works also in the case when the medium consists of normal matter objects (stars or black holes, gas, etc.). In such a case however the scattering matrix \( \Sigma \) should be determined separately and in (20) \( N_w \rightarrow N_s \). There exist however a phenomenological way to get estimates for the GW scattering on stars or black holes. It corresponds to the limit when the separation of wormhole entrances vanishes (20).

The first term in r.h.s. in (19) describes the change of the energy flux due to the non-stationarity of the background. In particular, in an expanding Universe the frequency changes according to the cosmological shift only

\[
\frac{d \nu}{dt} = -H
\]

(here the dependence on time goes through \( \lambda(t) \sim r = a(t)x \)) and the change of the volume element is

\[
\frac{dV}{dt} = \sqrt{\gamma} \frac{dV}{dt} = 3H,
\]

where \( H \) is the Hubble constant (\( \sqrt{\gamma} = a^3(t) \)). If we neglect the effects of the expansion (the red shift of the frequency), then (19) reduces merely to

\[
\frac{\partial W_k}{\partial t} + V_0 \frac{dW_k}{dt} = \alpha_k - \int \mu W_k d\Gamma_k
\]

where we used the relation \( \frac{dV}{dt} = V_g = |\frac{d\nu}{d\kappa}| \), \( \ell \) is the natural parameter along the ray (the length), and \( \frac{d\nu}{dt} = \frac{d\nu}{d\sigma} + \frac{\partial k}{\partial t} \frac{\partial k}{\partial \kappa} \).

V. SCATTERING OF GRAVITATIONAL WAVES

A. Direct signal

Let the spacetime be flat \( \omega = ck \) and let us take the source in the form

\[
\alpha_k = w_0(k) \delta(\mathbf{r} - \mathbf{r}'') \delta(t - t'').
\]

Then at the moment \( t'' \) we have radiation with the spectrum \( w_0(k) = L \delta(\omega - \omega_0(t'')) \) from the point \( x'' \). We point out that binary mergers form the source in the form

\[
\alpha_k = \frac{d w(\omega, t)}{d t} \delta(\mathbf{r} - \mathbf{r}'') \quad w(\omega, t) = L(t) \delta(\omega - \omega_0(t))
\]

where the amplitude \( L(t) = Q(t)^2 \) and \( \omega_0(t) \) correspond to the chirp signal [52]. The real signal can be simply taken as the sum of type (22) signals.

In the leading order, without taking into account the scattering on wormholes, we find the solution of (21) in the form (see Appendix VIII D)

\[
W_k^0(x, t) = \frac{w_0(\omega)}{c |\mathbf{r} - \mathbf{r}''|^2} \delta \left( t'' - t + \frac{|\mathbf{r} - \mathbf{r}'|}{c} \right) \delta(\cos \theta - \cos \theta') \delta(\phi - \phi')
\]

where \( \theta', \phi' \) relate to the direction of the velocity \( \mathbf{V}_s \). The chirp signal (23) forms the energy density as follows

\[
W^0(k, r, t) = \frac{|w(\omega, t)|_{\text{ret}}}{c |\mathbf{r} - \mathbf{r}'|^2} \delta(\cos \theta - \cos \theta') \delta(\phi - \phi')
\]

where we denote \( |f(t)|_{\text{ret}} = f \left( t - \frac{|\mathbf{r} - \mathbf{r}'|}{c} \right) \).
B. Damping and echoes: the halo of secondary sources

The additional signal comes from the additional sources (the so-called a distributed halo)

$$\delta \alpha_k = \int \frac{\omega}{\omega'} \Sigma (\Gamma, \Gamma_w', \Gamma') W(\Gamma') N_w(\Gamma'_w) d\Gamma' d\Gamma'_w d\Gamma_w. \quad (26)$$

If the distribution of wormholes in terms of the matrix $U$ is isotropic, then re-radiation will come out in an isotropic way (omnidirectional flux). Then we should follow only the frequency shift which is given by [9, 11]

$$\omega_{\text{out}} \approx \omega_{\text{in}} \left( 1 - \frac{1}{c} (\mathbf{V}_- m_{\text{in}}) - \frac{1}{c} (\mathbf{V}_+ m_{\text{out}}) \right). \quad (27)$$

We point out that $\omega_{\text{in}}$ and $\omega_{\text{out}}$ states are symmetric. Here the unit vector $m_{\text{in}} = (\mathbf{R}_- - \mathbf{x}') / |\mathbf{R}_- - \mathbf{x}'|$ and $m_{\text{out}} = (\mathbf{x} - \mathbf{R}_+) / |\mathbf{x} - \mathbf{R}_+|$ points to the observer.

Let us evaluate the term which corresponds to the absorption of radiation. We point out that the absorption term in [16] defines simply damping of the intensity and has the structure in [21]

$$\int \mu W'_n d\Gamma'_k = \mu(k) W_k \quad (28)$$

where

$$\mu(\omega) = 2c \int \sigma(b, \omega) n(r, b) db. \quad (29)$$

Above $n(r, b)$ is the number density of wormholes at the point $r$ and with the throat radius $b$, $\sigma(b, \omega)$ is the total cross-section of such wormholes (in the most general case it depends on $\omega$), and the multiplier 2 comes from taking into account absorption on both throats $S_+$ and $S_-$ (both give equal contribution due to the symmetry $+ \leftrightarrow -$). The term [23] simply defines damping along the ray $W_k \sim e^{-\tau} W_k$, where $\tau = \frac{1}{c} \int \mu d\ell$ is the optical depth. It is important that in the case of the GW propagation through the normal matter the damping is determined by the same term [23] with $\mu(\omega) = c \sigma_s n_s(r)$, where $\sigma_s$ and $n_s$ are the cross-section and the number density of respective objects (stars, black holes, gas, or any other objects).

The additional radiation capability (secondary sources) is more complex term which is

$$\delta \alpha_k = 2c \int \frac{\omega}{\omega'} \delta (\xi_+ - b) \delta (\Gamma'_w - \Gamma_{\text{out}}) W'_n(\Gamma'_w) N_w(\Gamma'_w) d\Gamma' d\Gamma_w. \quad (30)$$

Now if we use the approximation (due to $|R_- - x'| \gg \xi_+$)

$$\delta (\xi_+ - b) \approx \pi b^2 \delta^3 (\mathbf{x} - \mathbf{R}_+) \quad (31)$$

which means that in the first approximation throat is a point source at the position $\mathbf{R}_+$, and using the assumption that throat re-radiates in isotropic way (due to averaging over the rotation matrix $U$), then we find

$$\delta \alpha_w \approx 2 \int \frac{\pi b^2}{|R_+ - x'|^2} \frac{\omega_{\text{in}}}{\omega_{\text{out}}} \left[ w \left( \omega_{\text{in}}, t \right) \right]_{\text{ret}} \delta^3 (\mathbf{x} - \mathbf{R}_+) N_w(\Gamma_w) d\Gamma_w \quad (32)$$

Using different distributions for wormholes $N_w(\Gamma'_w)$ we may get different answers. The simplest distribution seem to be (recall that due to the property $\mathbf{V}_+ = U \mathbf{V}_-$ only $\mathbf{V}_+$ is independent and $V_+^2 = V_-^2$)

$$N_w(\Gamma_w) = \frac{n_w}{4\pi \Lambda^2} \delta (\Lambda - |R_+ - R_-|) \delta (b - b_0) f(\mathbf{V}_-), \quad (33)$$

where $f(\mathbf{V}_-) = \left( \frac{1}{2\pi \tau} \right)^{3/2} \exp \left( -\frac{V_-^2}{2\tau} \right)$ and $\int N_w(\Gamma_w) db d^3 R_+ d^3 R_- d^3 V_- = N_w = n_w V$ (V is the volume of space). Such a distribution corresponds to the case when wormholes are homogeneously distributed in space and all wormholes have the same throat radius $b = b_0$ and the same distance between entrances $|R_+ - R_-| = \Lambda$.

The fact that density $\alpha_k$ and, therefore, $W_k(\mathbf{x}, t)$ depend only on $\omega$ simplifies essentially the re-radiation terms. Let $\sigma_k^2 \to 0$ (i.e., $V_+^2 = V_-^2 = 0$ and throats are static), then $\omega_{\text{in}} = \omega_{\text{out}}$ and we find

$$\delta \alpha_w \approx 2 \int \frac{\pi b_0^2}{|R_+ - x'|^2} \left[ w \left( \omega, t \right) \right]_{\text{ret}} \delta^3 (\mathbf{x} - \mathbf{R}_+) \frac{n_w}{4\pi \Lambda^2} \delta (\Lambda - |R_+ - R_-|) d^3 R_+ d^3 R_- \quad (34)$$
which gives
\[
\delta \omega \sim 2 \int \frac{\pi b_0^2 [w(\omega, t)]_{ret} \cdot n_{\omega}}{|R_+ - x|^2} \frac{n_{\omega}}{4\pi L^2} \delta (A - |x - R_-|) \, d^3 R_-.
\] (35)

Then for the additional energy density we find \( n_+ = (x - R_+)/|x - R_+| \) and \( n_{\omega}' = k/k \)
\[
\delta W_\omega \sim 2 \int \frac{\pi b_0^2}{|R_+ - x'|^2} \frac{w}{c} \left( \frac{t - |R_+ - x|}{c} - \frac{|R_+ - x'|}{c} \right) \delta^2 (n_+ - n_{\omega}') \, N_{\omega} d^3 R d^3 R_-.
\] (36)

where \( N_{\omega} = \frac{\omega}{4\pi c} \delta (A - |R_+ - R_-|) \) and the retarding terms. Integrating this over the directions \( \mathbf{k}/k = n_{\omega}' \) gives the spectral energy by the relations \( \delta W_\omega \, d^3k = k^2 \delta W_\omega \, dk \, d\Omega' = \frac{\omega^2}{c} \delta W_\omega \, d\omega \, d\Omega' \) and, therefore, we define the spectral energy flux \( \delta I_\omega = \frac{\omega^2}{c^2} \int \delta W_\omega \, d\Omega' \) as
\[
\delta I_\omega \sim 2 \frac{\omega^2}{c^2} \int \frac{\pi b_0^2}{|R_+ - x'|^2} \frac{w}{c} \left( \frac{t - |R_+ - x|}{c} - \frac{|R_+ - x'|}{c} \right) N_{\omega} d^3 R d^3 R_-.
\] (37)

In agreement to \( [5] \) for a discrete set of wormholes the energy flux can be written as the sum
\[
\delta I_\omega \sim \frac{\omega^2}{c^2} \sum |R_{j+} - x'|^2 \frac{w}{c} \left( \frac{t - |R_{j+} - x|}{c} - \frac{|R_{j+} - x'|}{c} \right) + (+ \leftrightarrow -),
\] (38)

where the sum is taken over wormholes numbered by \( j \). Here all cross terms which may arise from \( [5] \) disappear due to loss of coherence. Comparing this to the basic energy flux \( [25] \) of the direct signal
\[
I_\omega(r, t) = \frac{\omega^2}{c^2} \left( \frac{w}{c} \right) \left( \frac{t - |x'|}{c} \right)
\] (39)

we see that every echo signal, i.e., every particular term in \( [35] \), has the same spectral composition and the same form. The difference appears only in the arrival times, directions to the source, and amplitudes.

C. Corrections

The above expressions assume that the space is flat and the red shift is absent. The redshift can be straightforwardly accounted for, it gives in \( [35] \) and \( [36] \) the additional multiplier \((1 + z)^{-3}\), the shift of frequencies, i.e., the replacement \( \omega \to (1 + z)\omega \) in the function \( w(\omega) \) (e.g., in the case when \( w = w_0 \delta (\omega - \omega_0) \) we should replace it with \( \frac{w_0}{(1 + z)^3} \delta (\omega - \frac{\omega_0}{1 + z}) \)) and the replacement \( t \to a(t) \int dt/a \). Moreover, they assume also that wormholes have negligible length of throats, while the substitution \( [31] \) produces an error in the time delay of the order \( \eta \sim 2b/c \). In a long throat however the actual flux (it’s amplitude) does not decrease but remains almost constant. From the other side the spherical symmetry assumes that wormhole metric remains conformally flat (for the space-like part of the metric). Therefore, formally the behavior of the energy density scattered by a single wormhole remains the same \( \delta W_{\omega,A} \sim 1/R_{A}^4 \). All what is actually changed is the retarding time. This means that the main effect is the adding an additional retarding time \( \eta_A \) to every particular wormhole. From the rigorous standpoint \( \eta_A \) can be found by considering geodesics with the metric \( [11] \) and it explicitly depends on all wormhole parameters and the positions of the source \( x' \) and the observer \( x, \), i.e., \( \eta_A = \pi(x, x', \Gamma_w) \). Phenomenologically, however, such a quantity can be added as an additional parameter to \( \Gamma_w \), while \( [35] \) transforms to
\[
\delta I_\omega \sim \frac{\omega^2}{c^2} \sum \frac{\pi b_0^2}{|R_{j+} - x'|^2} \frac{w}{c} \left( \frac{t - |R_{j+} - x|}{c} - \frac{|R_{j+} - x'|}{c} - \eta_j \right) + (+ \leftrightarrow -).
\] (40)

Every particular term in the above expression describes a particular echo signal. On practice, however, all such signals merge and form a long-living tails for the basic signal. The structure and the dependence on time of the tail we consider in the next section.
VI. TAILS: TIME STRUCTURE OF THE SCATTERED SIGNAL

Since durations of actual GW signals from binary mergers are very short, in the leading order every such a signal can be approximated by delta-like impulse. Consider the source in the form \( w(\omega, t) = w_0(\omega) \delta(t - t') \). Then the spectral energy flux \( F_\omega^0(x, t) \) in the direct signal is

\[
F_\omega^0(x, t) = \frac{\omega^2 w_0(\omega)}{c^3} R \frac{d}{dt} \left(t - t' - \frac{R}{c}\right),
\]

where \( R^2 = |x - x'|^2 \), which determines the values

\[
\Phi_\omega^0 = \int F_\omega^0 dt = \frac{\omega^2 w_0(\omega)}{c^3}, \quad \Phi_\omega = \int \Phi_\omega^0 d\omega = \frac{4\pi F}{R^2},
\]

where \( 4\pi F = 4\pi \int \frac{\omega^2}{c^3} w_0(\omega) d\omega \) has the sense of the total energy emitted by the source.

The distribution of wormholes we take in the simplest form (33) but for static wormholes, i.e., \( \sigma_V \to 0 \) and \( N_w = \frac{4\pi}{2\Lambda^2} \delta(\Lambda - |R_+ - R_-|) \delta(b - b_0) \). For simplicity we assume that the delay parameter \( \eta_k = \text{const.} \) Such a distribution corresponds to the case when all wormholes have the same throat radius \( b = b_0 \) and the same distance between entrances \( |R_+ - R_-| = \Lambda \). The more general case one obtains by averaging results with an additional distribution \( p(\Lambda, b_0) \) (which has sense of the probability density for wormholes \( \int p(\Lambda, b_0) d\Lambda db_0 = 1 \)).

Let us define the multiplier

\[
\beta_{\omega} = \frac{2\pi b_0^2 N_w}{4\pi\Lambda^2} R^2 \Phi_\omega^0.
\]

Then the additional energy flux \( \delta I_\omega = c\delta \vec{W}_\omega \) (spectral energy which falls on a unite square per unit time) can be cast to the form

\[
\delta I_\omega \simeq \beta_{\omega} \int \delta \left(\frac{t - |R_+ - x|}{c} - \frac{|R_- - x'|}{c} - \eta - t'\right) \frac{\delta(\Lambda - |R_+ - R_-|) d^3R_+ d^3R_-}{|x - R_+|^2 |R_- - x'|^2}.
\]

First, we determine the total energy in the tails.

A. Total energy in tails

The total energy in the tail which falls on a unit square is

\[
\delta \Phi_\omega = \int \delta I_\omega dt = \beta_{\omega} \int \frac{\delta (\Lambda - |R_+ - R_-|)}{|x - R_+|^2 |R_- - x'|^2} R^2 d^3R_+ d^3R_-.
\]

Now using the substitutions \( R_+ = X + R_- \) and the set of variables \( X = Xn, R_- = rI + x'' \), \( I^2 = n^2 = 1 \), we get

\[
\delta \Phi_\omega = \frac{(4\pi)^2 \Lambda^2 \beta_{\omega}}{R} C(\chi),
\]

where we have defined variables \( R = |x - x''|, \chi = \frac{1}{\Lambda}, \) and the function \( C(\chi) \) is determined by the integral

\[
C(\chi) \simeq \frac{1}{(4\pi)^2} \int \frac{d^2\Omega_n d^2\Omega_{\chi}}{|\chi n + y|^{17/2}} dy.
\]

In the above integral we rescale the integration variable \( y = \frac{r}{\chi}, \) and \( m = (x - x'')/R \). This function is determined in Appendix [VIII A] and has the form

\[
C(\chi) = \frac{C(0)}{(2\chi)} ((\chi + 1) - |\chi - 1|) = \frac{C(0)}{\chi} \begin{cases} \chi, & \text{as } \chi < 1 \\ 1, & \text{as } \chi > 1 \end{cases}.
\]
where the numerical constant is $C(0) = 2.4674$. This determines the total energy flux in the tail as
\[
\frac{\delta \Phi_w}{\delta \omega} = 4\pi C_0 \left( \Omega \right),
\]  
where $\tau = 2\pi l_0^2 n_{w} R$. We see that the ratio of amplitudes is proportional to $\tau = \tau_w = 2\pi \left\langle b^2 \right\rangle R n_w$, which has sense of the optical depth, i.e. the mean number of wormholes contained in the volume $2\pi l_0^2 R$. In the case of stars $\tau = \tau_s = \pi \left\langle r_g^2 \right\rangle R n_s$ and we get $\tau_s \ll 1$ (the multiplier $2$ appears in $\tau_w$ since every wormhole has two entrances). The mean optical depth determines damping of the basic signal \cite{28} when propagating in all possible directions and it reaches values $\tau_w \gg 1$. It is necessary to stress that the value $\tau_w \gg 1$ does not mean that the basic signal does not reach the observer, since in general case wormholes are distributed rather irregularly (or even by a fractal law).

Therefore, there are always directions in which the Universe is transparent (there are no wormholes on the line of sight). However if we accept that wormholes are responsible for dark matter phenomenon, then e.g., in Low Surface Brightness galaxies, $\tau_w \sim 10^3$, while for mean value in the Universe it has at least the order $\tau_w \sim \rho_{DM}/\rho_b$ (the ratio of dark matter to baryon densities).

**B. The dependence on time**

In the variables $R = X + R_-$ and $R_- = R$ the energy flux \cite{11} reads
\[
\delta I_w \simeq \beta \omega \int \frac{\delta \left( t - \frac{|X+R-x|}{c} - \frac{|R-x''|}{c} - \eta - t'' \right)}{|X+R-x|^2 |R-x''|^2} \delta (\Lambda - |X|) d^3 X d^3 R.
\]
Using coordinates $X = x n$, $n^2 = 1$ and integrating over $x$ we get
\[
\delta I_w (x-x'', t-t'') \simeq \Lambda^2 \beta_0 \int \frac{\delta \left( t - t'' - \frac{|Y+\Lambda n-R m|}{c} - \frac{Y_s - \eta}{c} \right)}{|Y+\Lambda n-R m|^2 Y^2} d^2 \Omega_n d^2 Y,
\]
where we use the set of variables as $R = Y + x''$, $x = x'' = R m$, and $R = |x-x'|$. This can be re-written as
\[
\delta I_w (x-x'', t-t'') \simeq c \beta_0 \chi^2 J(\xi, \chi) = \psi \frac{c}{R} \Lambda^2 \Omega_n d^3 Y, \]
where we have denoted $\xi = \frac{(t-t''-\eta)c}{R}$, $\chi = \frac{\Lambda}{R}$, and (we use the variable $y = Y/R$ in the integral)
\[
J(\xi, \chi) = \int \frac{\delta (|y-(m-\chi n)| + y - \xi)}{|y-(m-\chi n)|^2} \frac{d^3 y}{y^2} d^2 \Omega_n.
\]

The exact form of $J(\xi, \chi)$ is determined in the Appendix \textit{VIII B} and \textit{VIII C}.

First, we consider the tail for the scattering by normal matter $\chi = 0$. In the approximation $\chi < 1$ and for the region $\xi > 1 + \chi$ eq. \cite{49} gives
\[
J(\xi, \chi) \simeq J(\xi, 0) = 8\pi^2 \frac{\Omega_n}{\xi} \ln \left( \frac{\xi + 1}{\xi - 1} \right).
\]

The plot of $J(\xi, 0)$ is given on Fig. 11. In the limit $\chi \rightarrow 0$ the distance between throats vanishes and the advanced signal is merely absent (e.g., in the region $0 < \xi < 1$ we get $J(\xi, 0) = 0$). All the scattered signal comes with a retarding. The infinite value of $J$ at $\xi = 1$ does not mean that the echo is very strong, since it should be compared with the delta impulse \cite{11}. In considering the incident signal of a finite duration $\delta t$, the tail smoothes and takes a finite value. Indeed, for very small times $\xi - 1 \ll 1$ we have an approximation $\xi = 1 + \Delta \xi$ and therefore $J(\xi, 0) = 16\pi^2 \left( \ln 2 - \ln \Delta \xi + \frac{\Delta \xi}{2} \ln \frac{2\Delta \xi}{\Delta t} + ... \right)$. Integrating this over $t'' (dt'' = \Delta t d\xi$, where $\Delta t = R/c$) gives already the finite value
\[
< J(\xi = 1, 0) > = \frac{1}{\delta t} \int_0^{\delta t} J(\xi, 0) dt'' \approx 16\pi^2 (1 + \ln \frac{2\Delta t}{\delta t}).
\]
Figure 1: The plot of $J(\xi,0)$ which corresponds to the scattering on normal matter. All the tail lies in the retarded region $\xi > 1$. Tails in the case $\chi \ll 1$ have similar form but contain an advanced part.

Figure 2: The plot of $J_{\text{max}}(\chi) = J(1 + \chi, \chi)$. This function determines the maximal possible amplitude in echoes.

Qualitatively, the smoothed function $< J(\xi,0) >$ behaves as $J(\xi,\chi)$ for $\chi \ll 1$ which we describe below. The case $\chi = 0$ corresponds to the scattering on normal matter when re-radiation has an isotropic character. In this case however the optical depth $\tau_s \ll 1$ and $\delta L_\omega / L_0 \to 0$. We point out that the value $\tau_s$ has here the statistical character. In the case when there is an object, on the way of the ray, the echo signal may be considerable. This case is described by lensing or scattering on a single object.

Consider now the case of wormholes. For non-vanishing values $\chi$ the advanced signal does exist in the region $\xi > 1 - \chi$ and increases till the maximal value $J_{\text{max}}$ which is reached at the point $\xi = 1 + \chi$. This gives

$$J_{\text{max}}(\chi) = J(1 + \chi, \chi) = 16\pi^2 \left( \frac{\ln (\chi + 1)}{\chi} - \frac{\ln \chi}{\chi + 1} \right).$$

The plot of $J_{\text{max}}$ is presented on Fig. 2. From (48) we see that $J_{\text{max}}$ determines the maximal possible amplitude in the echo. Different forms of tails $J(\xi,\chi)$ for different values of $\chi$ are presented on Fig. 3. Dashed line corresponds to the case $\chi \to 0$. The advanced signal corresponds to the region $\xi < 1$ and it does exist only for values $\chi < 2$. In the region $\xi > 2$ the tail is completely retarded. In the approximation $\chi \gg 1$ we find

$$J(\xi,\chi) \simeq \frac{1}{\chi^2} J(\xi,0) = \frac{8\pi^2}{\chi^2} \frac{2}{\xi} \ln \left( \frac{\xi + \chi}{\xi - \chi} \right) \theta (\xi - \chi).$$

Here the advanced signal is also absent (since the travel time to wormhole entrances exceeds the value $\Delta t = c/R$).

As we see from Fig. 3 for sufficiently big $\xi$ the decrease of tails with time as $\sim 1/\xi \sim 1/t$ is the common feature of tails for all values of $\chi$. 
Figure 3: The form of tails $J(\xi, \chi)$ for a set of values of $\chi$. Advanced part of tails corresponds to $\xi < 1$. For values $\chi > 2$ tails possess only the retarded part.

C. Structure of general GW signals with tails

In the case of a delta-like impulse the total signal is described by
\[ I^{\text{tot}}_{\omega} = \Phi_{\omega}^0 \left[ \delta \left( t - t'' - \frac{R}{c} \right) + \frac{c \tau_w}{R} \frac{\pi}{4} J(\xi, \chi) \right], \]
where $\Phi_{\omega}^0$ is determined by (42). An arbitrary GW emission we obtain, if we replace $w_0(\omega)$ with a function $w_{0}(\omega, t'')$ and integrate the above equation over $t''$. This defines the spectral energy flux as
\[ I_{\omega}^{\text{tot}}(x, t) = \frac{\omega^2}{c^3} \frac{w_{0}(\omega, t'')}{R^2} + \int \frac{\omega^2}{c^3} \frac{w_{0}(\omega, t'')}{R^2} \frac{c \tau_w}{R} \frac{\pi}{4} J(\xi, \chi) dt''. \]

The typical duration of emission is very short, while the function $J(\xi, \chi)$ is very slow function (recall that $\xi = (t - t'' - \eta)/c$ almost does not change with a small change of $t''$, i.e., $\Delta \xi = c \Delta t''/R \ll 1$). This means that the multiplier $\frac{c \tau_w}{R} \frac{\pi}{4} J(\xi, \chi)$ can be taken at the moment of the start of emission $t'' = t_0$ and can be removed from the integration. As it was pointed out above in the case $\chi = 0$ one should replace $J$ with $< J >$ from (51). Then integrating this over frequencies $I = \int I_{\omega} d\omega$ we find
\[ I(t) = I^0(t) + \frac{\tau_w}{4\pi} J(\xi_0, \chi) \int I^0(t) dt. \] (54)

We see that tails have indeed a universal structure. For a short impulse $\int I^0 dt \sim I^0 \delta t \sim 2\pi I^0/\omega_0$, where $\delta t$ is the duration of the basic signal. The value $R/c = \Delta t$ is the propagation time, and therefore we find
\[ \frac{\delta I}{I^0} \sim \frac{\delta t \tau_w}{\Delta t 4\pi} J(\xi_0, \chi) \ll 1. \] (55)

The typical ratio $\frac{\delta I}{I^0}$ is extremely small and, therefore, the direct detection of the tail signal is hardly possible. Indeed, the energy flux relates to the amplitude $h$ as follows
\[ I = \frac{c^3}{32\pi G T} \int (h_x^2 + h_+^2) dt, \]
where $h_x$ and $h_+$ correspond to the two independent polarizations. Assuming the periodic signal $\sim e^{-i\omega_0 t}$ we find
\[ h^2 + \delta h^2 = \frac{16\pi G}{c^3 \omega_0^2} I^0(t) \left( 1 + \frac{\delta I}{I^0} \right). \]

This explicitly shows that the tail contribution is $\delta h \ll h$. This means that the echo signals reported in Refs. [4]-[7] cannot be explained as echoes from wormholes or ordinary matter objects. However if the duration of the direct
signal $h$ is very short, the tail part $\delta h^2$ decays with time very slowly $J(\zeta_0, \chi) \sim \Delta t/(\Delta t + t) \sim 1$ and the total energy in the tail, according to (47), may exceed the energy in the basic signal (i.e., $\int \delta h^2 dt \gg \int h^2 dt$). This means that the presence of such a heavy tail may be nevertheless observed by the increase of the noise level. The tails live for an extremely long period of time, at least $J(\zeta_0, \chi) \approx 1$ for $t \sim \Delta t$, and during this time all such tails from different binary merges accumulate. This roughly gives the additional multiplier $N = \nu \Delta t$, where $\nu$ is the mean rate of events and this gives already the factor $\Delta t/\Delta t \geq 1$ in (55). Therefore the total noise level can be considerable and the reported in [4]-[7] echoes may simply detect the stochastic background GW radiation.

VII. OUTLOOK

The topological bias for a distribution of wormholes can therefore be evaluated for Misner-Thorne wormhole by considering the neighborhood of each wormhole in the distribution after the modelization of the Dirac distribution for point-like sources after the characteristic sizes of the throats of each wormhole in the distribution, and by considering the related approximations for the $\beta$ function. Under this modelization, the emission of particles has to be expected within the same range of emission can be considered not much far form the range of absorption.

The shift on the frequencies is estimated not higher than $10^{-3}$, so that it can be neglected. But this estimate is very simple. First, peculiar velocities in galaxies (at edges) are around $300\text{km/s}$, which gives $v/c \sim 10^{-3}$. This estimate works both for spirals and ellipticals. Second, there are peculiar velocities of clusters (observed by kinematic Sunyaev-Zel’dovich effect) which do not exceed $1500\text{km/s}$. Rotation curves of spiral galaxies can be observed through several properties, such as Hubble type, structure, activity, and environment.

A review on the literature about galaxy rotation curves can be found in [55], while the possible observational errors are reviewed in [56]. In [57], the measurements of velocities of clusters due to kinematic Sunyaev-Zel’dovich effect are evaluated, for which the systematic errors are examined in [58]. Thus the Doppler shift expected does not exceed 0.5%, i.e. also at low frequencies [60]. The experimental verification can be modeled at different Astrophysical scales, i.e. also at satellite-distances scales [62].

In the performed analysis, throats have been assumed to participate in the peculiar motions.

The shift due to the expansion can be estimated to be very small as well. The behavior of an gravitational wave in several models of expanding universes was studied in [63], while the behavior of a gravitational wave in an expanding Universe in the presence of a point-mass was schematized in [64].

Higher-order effects arising from Special Relativity and GR [65], [66], [67], are discussed in [68] for comparison with constraints with more general metric theories of gravity, in [69], [70] for comparison with constraints on dark-matter like contributions.

The noise can be therefore estimated to appear in the same range of the original ingoing signal.

VIII. APPENDIX

A. Function $C(\chi)$

The function $C(\chi)$ is determined by

$$C (\chi) \approx \frac{1}{1\pi^2} \int \frac{d^2\Omega_s d^2\Omega_t}{|\chi n + y l - m|^2} dy,$$

where $d^2\Omega$ is the solid angle. Assuming $\chi = \frac{\lambda}{R} \ll 1$ we get $C(\chi) \approx C(0)$

$$C (0) = \frac{1}{2} \int_0^\infty \int_0^\pi \frac{d(\cos \theta)}{(y^2 - 2ycos \theta + 1)^2} dy = \frac{1}{2} \int_0^\infty \frac{1}{2y} \ln \frac{(y + 1)^2}{(y - 1)^2} dy = 2.4674.$$

The opposite case $\chi \gg 1$ reduces to the analogous expression

$$C (\chi) \approx \frac{1}{\chi} C (0).$$

(57)
In the general case we define \( \mathbf{m} - \chi \mathbf{n} = \gamma \mathbf{k} \), where \( k^2 = 1, \gamma = \sqrt{\chi^2 - 2\chi \cos \alpha + 1} \), and \( \cos \alpha = (\mathbf{m} \mathbf{n}) \), then we get

\[
C(\chi) = \frac{1}{(4\pi)^2} \int \left( \int \frac{d^2l}{|r1 - k|^2} dr \right) \frac{d^2\Omega_n}{\gamma},
\]

(59)

where \( r = \frac{r1}{\gamma} \). This transforms the integral into

\[
C(\chi) \simeq \frac{1}{(4\pi)^2} \int \frac{2\pi d(\cos \theta)}{(r^2 - 2r \cos \theta + 1)} \left( \int \frac{d^2\Omega_n}{\gamma} - C(0) \int \frac{d^2\Omega_n}{\gamma} \right)
\]

(60)

which gives

\[
C(\chi) = \frac{C(0)}{2} \int_{-1}^1 \frac{d(\cos \alpha)}{\sqrt{\chi^2 - 2\chi \cos \alpha + 1}}.
\]

(61)

Finally we find the expression

\[
C(\chi) = \frac{C(0)}{(2\chi)} ((\chi + 1) - |\chi - 1|) = \frac{C(0)}{\chi} \left\{ \begin{array}{ll}
\chi, & \text{as } \chi < 1 \\
1, & \text{as } \chi > 1.
\end{array} \right.
\]

(62)

B. Function \( J(\xi, 0) \)

In the approximation \( \chi \to 0 \) eq. (49) reduces to

\[
J(\xi, 0) = \int \frac{\delta (|y - m| + u - \xi)}{|y - m|^2} \frac{d^2y}{y^2} d^2\Omega_n.
\]

(63)

Let us use \( x = \cos \theta \), then the above integral gives

\[
J(\xi, 0) = 8\pi^2 \int_0^1 \int_0^\infty \frac{\delta (y + \sqrt{y^2 + 2yx + 1} - 1)}{dy dx} dy dx.
\]

(64)

Let us use new variable \( u = \sqrt{y^2 + 2yx + 1} > 0, dx = \frac{y}{u} du \) whose range is \( |y| < u < y + 1 \). Then we get

\[
J(\xi, 0) = 8\pi^2 \int_0^\infty \left( \int_{|y|}^{y+1} \frac{\delta (y + u - \xi)}{dy} dy \right) dy.
\]

(65)

This integral splits in two parts which should be considered separately

\[
J_1 + J_2 = \int_0^1 \left( \int_{1-y}^{1+y} \frac{\delta (y + u - \xi)}{dy} dy \right) dy + \int_0^\infty \left( \int_{y-1}^{y+1} \frac{\delta (y + u - \xi)}{dy} dy \right) dy.
\]

(66)

Consider the first part \( J_1 \). In the regions \( \xi < 1 \) and \( \xi > 3 \) the delta function has no roots and we find simply \( J_1 = 0 \). In the rest region \( 1 < \xi < 3 \) the delta function possesses roots only for \( y > \frac{1}{\xi} \) and therefore

\[
J_1 = \int_{\frac{1}{\xi} - 1}^1 \left( \int_{1-y}^{1+y} \frac{\delta (u - (\xi - y))}{dy} dy \right) dy,
\]

which defines

\[
J_1(\xi) = \int_{\frac{1}{\xi} - 1}^1 \frac{dy}{(\xi - y) y} = \left\{ \begin{array}{ll}
\frac{1}{\xi} \ln \left( \frac{\xi+1}{\xi-1} \right), & \text{as } \xi < 3 \\
0, & \text{as } \xi \notin [1, 3]
\end{array} \right.
\]

(67)

Now let us consider the second part \( J_2 \). Again in the region \( \xi < 1 \) the delta function has no roots and \( J_2 = 0 \). In the region \( 1 < \xi < 3 \) roots exist for \( y < \frac{1}{\xi} (\xi + 1) \) and we find

\[
J_2 = \int_{\frac{1}{\xi} (\xi + 1)}^{1} \frac{dy}{(\xi - y) y} = \frac{1}{\xi} \ln (\xi + 1), \text{ as, } 1 < \xi < 3.
\]

(68)
In the region \( \xi > 3 \) we find the upper and lower limits for \( y \) as \( y = \frac{1}{2} (\xi \pm 1) \) which gives

\[
J_2 = \int_{\frac{1}{2}(\xi-1)}^{\frac{1}{2}(\xi+1)} \frac{dy}{(\xi-y)y} = \frac{1}{\xi} \ln \left( \frac{(\xi+1)^2}{(\xi-1)^2} \right), \quad as, \ \xi > 3.
\] (69)

Now collecting all we find

\[
J(\xi,0) = 8\pi^2 \frac{\theta(\xi-1)}{\xi} \ln \left( \frac{\xi+1}{\xi-1} \right),
\] (70)

where \( \theta(\xi-1) \) is the step function. We also point out that \( \int J(\xi,0)d\xi = (4\pi)^2 C(0) = 8\pi^2 \times 4.9348. \)

**C. Function \( J(\xi, \chi) \)**

Consider the the function \( J(\xi, \chi) \) which is determined by the integral 49,

\[
J(\xi, \chi) = \int \frac{\delta \left( y - (m - \chi n) + y - \xi \right)}{|y - (m - \chi n)|^2} \frac{d^3y}{y^2}d^2\Omega_n.
\]

If we define new variables as \( m - \chi n = \gamma k \) with \( \gamma = \sqrt{\chi^2 - 2\chi \cos \alpha + 1}, \cos \alpha = (mn), \) and \( r = y/\gamma, \) then the integral reduces to the function \( J(\xi,0) \) as

\[
J(\xi, \chi) = \int \left( \int \frac{\delta \left( r - k + r - \frac{\gamma}{\xi} \right)}{|r - k|^2} \frac{d^3r}{\gamma^2} \right) d^2\Omega_n \gamma^2 = \frac{1}{2} \int_1^\chi J\left( \frac{\xi}{\gamma}, 0 \right) \frac{d(\cos \alpha)}{\gamma^2}. \] (71)

Now by means of introducing \( \gamma \) as a new integration variable instead of \( \cos \alpha \) we find

\[
J(\xi, \chi) = \frac{1}{2\chi} \int_{|\xi-1|}^{|\chi+1|} J\left( \frac{\xi}{\gamma}, 0 \right) \frac{d\gamma}{\gamma}. \] (72)

Substituting here (70) we get

\[
J(\xi, \chi) = \frac{8\pi^2}{\chi} \int_{\frac{\xi}{\gamma}}^{\frac{\chi+1}{\gamma}} \ln \left( \frac{1+x}{1-x} \right) \theta (1-x) dx,
\] (73)

where \( \theta (1-x) \) is the step function. This integral has different forms for different regions.

**First region** is \( \xi < |\chi-1| \). It gives \( x > 1 \) and, therefore,

\[
J(\xi, \chi) = 0, \quad as \ \xi < |\chi-1| \] (74)

**The second region** is \( |\chi-1| < \xi < \chi + 1 \). The step function defines the upper limit \( x = 1 \) and we get

\[
J(\xi, \chi) = \frac{8\pi^2}{\chi} \left( 2 \ln 2 - \ln \left( \frac{\xi^2 - (\chi - 1)^2}{\xi^2} \right) - \frac{|\chi-1|}{\xi} \ln \left( \frac{\xi + |\chi-1|}{|\xi-1|} \right) \right). \] (75)

**The last region** is \( \xi > \chi + 1 \). Then \( \theta (1-x) = 1 \) and we find

\[
J(\xi, \chi) = \frac{8\pi^2}{\chi} \left( \ln \frac{\xi^2 - (\chi + 1)^2}{\xi^2 - (\chi - 1)^2} + \frac{\xi + 1}{\xi} \ln \left( \frac{\xi + \chi + 1}{\xi - \chi - 1} \right) - \frac{|\chi-1|}{\xi} \ln \left( \frac{\xi + |\chi-1|}{|\xi-1|} \right) \right). \] (76)

Consider the case when \( \chi \ll 1 \). Then we find \( J(\xi, \chi) = 0 \) as \( \xi < 1 - \chi \). In the region \( 1 - \chi < \xi < 1 + \chi \) we find the approximate expression

\[
J(\xi, \chi) = \frac{8\pi^2}{\xi} \int_0^{\xi-1+\chi} \ln \left( \frac{2\xi - t}{t} \right) dt \approx \frac{8\pi^2}{\chi} \left( \xi - (1 - \chi) \right) \left( 1 + \ln \frac{2\xi}{\xi} \right). \] (77)

In the region \( \xi > 1 + \chi \) we get the expression as \( J(\xi, \chi) \simeq J(\xi,0) \) whose plot is presented on Fig. 1.
D. Free motion

For a point-like source $\alpha_k = w_0(\omega) \delta(x - x'') \delta(t - t'')$ and in the absence of the scattering on wormholes (i.e., in the case of free motion) eq. (19) can be solved by the characteristics method. Indeed the geodesic motion of particles is described by rays

$$ r = r_0 + V_g(t - t''), \quad k = k_0, \quad V_g = \frac{\partial \omega}{\partial k} = \frac{\omega}{k} \quad (78) $$

Then, if we take $(k_0, r_0)$ as new coordinates, the energy density becomes $W[k(k_0, r_0, t), r(k_0, r_0, t)] = W(k_0, r_0, t)$ and (19) reads

$$ \frac{dW(k_0, r_0, t)}{dt} = \alpha_k = w_0(\omega) \delta(r_0 - r'') \delta(t - t''), \quad (79) $$

which has the obvious solution in the form

$$ W(k_0, r_0, t) = w_0(\omega) \delta(r_0 - r'') \theta(t - t''), \quad (80) $$

where $\theta(t - t'')$ is the step function ($\theta(x) = 1$ as $x > 0$). Now to return to the initial coordinates $(k, r)$ we should simply replace back

$$ r_0 = r - V_g(t - t''), \quad k_0 = k, \quad (81) $$

which gives (we assume $t > t''$)

$$ W(k, r, t) = w_0(\omega) \delta(r - r'' - V_g(t - t'')). \quad (82) $$

In terms of the spherical coordinates $V_g = (c, \theta', \phi')$ and $r - r'' = (|r - r''|, \theta, \phi)$ the above expression transforms to

$$ W(k, r, t) = \frac{w_0(\omega)}{c|r - r''|^2} \delta\left(\frac{|r - r''|}{c} - t + t''\right) \delta(c \cos \theta - c \cos \theta') \delta(\phi - \phi'). \quad (83) $$

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