ASYMMETRIES OF SOLAR $p$-MODE LINE PROFILES

DOUGLAS ABRAMS AND PAWAN KUMAR

Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139

Received 1996 February 26; accepted 1996 June 12

ABSTRACT

Recent observations indicate that solar $p$-mode line profiles are not exactly Lorentzian but rather exhibit varying amounts of asymmetry about their peaks. We analyze $p$-mode line asymmetry by using both a simplified one-dimensional model and a more realistic solar model. We find that the amount of asymmetry exhibited by a given mode depends on the location of the sources exciting the mode, the mode frequency, and weakly on the mode spherical harmonic degree but not on the particular mechanism or location of the damping. We calculate the dependence of line asymmetry on source location for solar $p$-modes and provide physical explanations of our results in terms of the simplified model. A comparison of our results to the observations of line asymmetry in velocity spectra reported by Duvall et al. for modes of frequency $\sim 2.3$ mHz suggests that the sources for these modes are located more than 325 km beneath the photosphere. This source depth is greater than that found by Kumar for acoustic waves of frequency $\sim 6$ mHz. The difference may indicate that waves of different frequencies are excited at different depths in the convection zone. We find that line asymmetry causes the frequency obtained from a Lorentzian fit to a peak in the power spectrum to differ from the corresponding eigenfrequency by an amount proportional to a dimensionless asymmetry parameter and to the mode line width.

Subject headings: line: profiles — Sun: oscillations

1. INTRODUCTION

Most work in helioseismology has relied on the assumption that solar $p$-mode line profiles are Lorentzian, which is expected for the power spectrum of randomly forced, damped harmonic oscillators. However, recent results indicate that this may be only approximately true. Duvall et al. (1993) have presented observational evidence from their South Pole data that $p$-mode line shapes are not exactly Lorentzian but rather exhibit varying amounts of asymmetry about their peaks. This is a difficult observation and has not yet been reproduced by other groups. However, several authors have investigated the problem theoretically and have found that line profiles are asymmetric whenever a localized excitation source is present (Gabriel 1992, 1993, 1995; Roxbourgh & Vorontsov 1995; see also Fig. 2 of Kumar et al. 1990). Therefore we believe that the line-asymmetry observations of Duvall et al. will be validated more accurately when more precise measurements are available.

The excitation of $p$-modes is thought to occur in a thin layer near the top of the solar convection zone. All previous work on the asymmetry indicates that the degree of asymmetry depends on the depth of the excitation sources. Therefore $p$-mode line shapes may provide a means of locating the acoustic sources responsible for exciting modes of various frequencies.

In this paper, we present a detailed analysis of $p$-mode line profiles for both a simplified one-dimensional model and a realistic solar model, previously used by Kumar et al. (1994) to study peaks in the power spectrum above the acoustic cutoff frequency. For the one-dimensional model problem, we analyze the dependence of line asymmetry on source depth, mode dissipation (including line width and location of damping), mode frequency, and mode degree $\ell$ and also investigate the error introduced in $p$-mode frequency determination by Lorentzian fits to the power spectrum. Our results for the one-dimensional problem are contained in § 2. For the realistic solar model, we investigate the dependence of line asymmetry on source depth, mode frequency, and $\ell$ and also examine errors in eigenfrequency determination. These results are contained in § 3. We summarize our main conclusions in § 4.

2. MODEL PROBLEM

The following (homogeneous) one-dimensional wave equation describes adiabatic solar oscillations in the Cowling approximation (Deubner & Gough 1984):

$$\frac{d^2 \psi}{dr^2} + \left[\frac{\omega^2}{c^2} - V(r)\right] \psi = 0 , \quad (1)$$

where $\psi = \rho^{1/2} c^2 V \cdot \xi$, $\xi$ is a Fourier component of the fluid displacement, $\rho$ is the equilibrium density, and $c$ is the sound speed. The effective acoustic potential is given by

$$V(r) \approx \ell(\ell + 1) r^2 + \omega_{ac}^2/c^2 , \quad (2)$$

where $\omega_{ac}$ is the acoustic cutoff frequency. The first term in the potential determines a mode’s lower turning point, while the second term peaks near the temperature minimum and causes acoustic waves to be reflected. Solar $p$-modes can be modeled reasonably well by setting the sound speed to 1 everywhere and using the following simple form for the potential (see also Kumar & Lu 1991; Kumar et al. 1994):

$$V(r) = \begin{cases} \infty , & \text{for } r \leq 0 , \\ 0 , & \text{for } 0 < r < a , \\ a^2 (\text{const}) , & \text{for } r \geq a , \end{cases} \quad (2)$$

where $a$ is the sound-travel time from the lower to the upper turning point of a given mode and $a$ is the acoustic cutoff frequency at the temperature minimum. Waves of angular frequency less than $a$ are trapped below $a$, while waves of angular frequency greater than $a$ can propagate to infinity; thus, for frequencies less than $a$, there is a discrete spectrum of real eigenfrequencies. Neglecting a weak dependence on mode frequency, different values of $a$ correspond to modes of different degree $\ell$.  

1 Alfred P. Sloan Fellow and NSF Young Investigator.
We now add a damping term and a source term to equation (1). Although the damping processes that affect solar $p$-modes are quite complex, we assume for the present purposes that they may be modeled by a viscous damping force. Upon adding these two terms, we obtain

$$\frac{d^2}{{dr}^2} \psi + i \omega \gamma(r) \psi + \left[ \omega^2 - V(r) \right] \psi = f_o(r),$$

(3)

where $f_o(r)$ and $\gamma(r)$ are the Fourier components of the source function and of the coefficient of the damping term, respectively. As long as the region of excitation is much smaller than the wavelengths considered, we may use $f_o(r) = S_o \delta(r - r_o)$. Since the source power spectrum $S_o$ varies negligibly over a typical mode line width, it does not affect our calculations of asymmetry, and we set it to unity for all frequencies. We consider two forms for $\gamma(r)$. One case we consider is $\gamma(r) = \Gamma_o$ (independent of $r$), where $\Gamma_o$ is the (frequency dependent) line width. The other is $\gamma(r) = \Gamma_o \delta(r - r_o)$, which is a better approximation to solar $p$-mode dissipation. These two extreme cases should demonstrate whether line asymmetry has any dependence on the details of the damping process.

Equation (3) is easily solved to obtain the power spectrum seen by an observer in the photosphere, and individual peaks in spectra generated in this manner do indeed exhibit varying degrees of asymmetry (see Fig. 1). In the next two subsections, we examine the asymmetry when the source is inside or outside the well. In § 2.3, we examine the effects of asymmetry on the accuracy of determining the system’s eigenfrequencies using Lorentzian fits to the peaks.

Before proceeding, we introduce a method of quantifying line asymmetry, which will be used for the rest of this paper. We decompose the observed power spectrum in the neighborhood of a peak corresponding to a mode $\alpha$ into even and odd functions. Since the odd function is zero at the peak and again far from the peak, its magnitude has a maximum at some intermediate distance from the peak, typically less than one line width. The ratio of the maximum magnitude of the odd function to the maximum magnitude of the even function is a dimensionless measure of the asymmetry, which we denote by $\eta_p/100$. Then $\eta_p$ is the percentage line asymmetry of mode $\alpha$. The sign of $\eta_p$ is taken to be positive or negative according to whether there is more power on the high- or the low-frequency side of the peak, respectively.

2.1. Source inside the Well

When the source is inside the well and damping is uniform [i.e., $\gamma(r) = \Gamma_o$ everywhere], the solution to equation (3) is

$$\psi(r) = -\frac{\sin kr}{k \cos ka + k_1 \sin ka} e^{-k_i(r - r_o)},$$

(4)

where $k = (\omega^2 + i \omega \Gamma_o)^{1/2}$ and $k_1 = (\omega^2 - \omega - i \omega \Gamma_o)^{1/2}$. When $\Gamma_o \ll \omega$ (as is the case for solar $p$-modes) and $\omega \approx \omega_s$, an eigenfrequency, the velocity power spectrum seen by an observer at location $r$ is

$$P_o(r) \equiv c^2 |\psi_o(r)|^2 \approx \frac{\sin^2 r_o \omega}{(\Delta \omega)^2 + \Gamma_o^2/4},$$

(5)

where $\Delta \omega = \omega - \omega_s$. The line profile depends on the source’s position through the factor $\sin^2 r_o \omega$. When $\sin r_o \omega_s \approx 1$, the variation of the numerator with frequency over the mode line width is small, and the line profile is nearly Lorentzian. (In all cases, the variation over a line width of the factor outside the square brackets is negligible.) However, when $\sin r_o \omega_s \approx 0$, the peak is quite asymmetric.

![Fig. 1.—Two peaks with values of the asymmetry parameter $\eta_p$ of $-5\%$ and $-10\%$ (see § 2 for the definition of $\eta_p$). Both spectra were calculated using the one-dimensional model with $a = 3275$ s of sound-travel time and $v = 1.5$ mHz. For the peak with $\eta_p = -10\%$, the source was placed at the upper turning point; for the other, the source was placed 300 km below the upper turning point. Line profiles with the same value of $\eta_p$ look essentially identical, regardless of source location, frequency, cavity length, line width, or type of damping.](image-url)
Note that, when the source lies inside the well, the most asymmetric peaks therefore correspond to modes that are excited to small amplitudes.

For the discussion that follows, it will be useful to rewrite the power spectrum in the neighborhood of a mode frequency as

$$P_\omega(r) \approx C \sin^2 \left( \delta + r_s \Delta \omega \right) \frac{(\Delta \omega)^2 + \Gamma_0^2/4}{(\Delta \omega)^2}, \quad (6)$$

where $C$ is approximately constant, $\delta = \omega_r (r_s - r_M)$, and $r_0$ is the closest number to $r_s$ such that $\sin r_0 \omega_r = 0$. (Note that $r_0$ may be greater than $r_s$; in particular, it need not correspond to a node of the eigenfunction in question.) It is apparent from equation (6) and the discussion above that the asymmetry parameter $\eta_r$ has the same sign as $\delta$ and that the magnitude of $\eta_r$ is maximized (i.e., peaks are very asymmetric) when $\delta$ is small. When $\delta = 0$, however, the source lies exactly at a node, and it is meaningless to speak of line asymmetry since the Lorentzian peak is completely suppressed.

We have checked the dependence of line asymmetry on source location, mode frequency, line width, and $\ell$ for the sources that lie within 1000 km (100 s of sound-travel time) of the upper turning point (see Fig. 2). Moving the source deeper causes $\eta_r$ to become more positive except when the source passes through a node, in which case $\eta_r$ jumps discontinuously from a large positive to a large negative value. This follows from the dependence of $\delta$ on $r_s$; as the source is moved deeper into the well, $\delta$ decreases monotonically except when $\eta_r$ passes through zero, in which case $\delta$ jumps discontinuously from $-\pi/2$ to $\pi/2$.

The behavior of the asymmetry as a function of mode frequency depends on the source’s location: for source locations within 400 km of the upper turning point, the magnitude of $\eta_r$ is greatest for low-frequency modes while, for some deeper source locations (e.g., 800 km depth), it is greatest for high-frequency modes. (Since the asymmetry for a given mode depends on the locations of its eigenfunction’s nodes relative to the source, the modes with the most asymmetric power spectra continue to change as the source is moved deeper still.)

Increasing the line width while keeping other parameters fixed increases the magnitude of the asymmetry, since the variation of the numerator in equation (6) over the extent of the peak is effectively increased. Finally, with the source restricted to lie near the top of the cavity, the magnitude of the asymmetry increases with increasing cavity length (decreasing $\ell$-value) since the numerator in equation (6) varies more rapidly. These dependences of the asymmetry on the line width and effective cavity length are quite general and do not depend on the detailed nature of the potential.

We have also considered the case in which the damping is localized as a delta function 0–250 km below the upper turning point; we find the dependence of line shape on the nature and location of damping to be very weak. In order to make meaningful comparisons between the cases of global and local damping, we match the imaginary parts of the respective eigenfrequencies. For cavity lengths corresponding to low degrees ($\ell \approx 5$), we consider line widths of up to 15 $\mu$Hz while, for cavity lengths corresponding to higher degrees, we consider larger line widths; for $\ell \approx 350$, we consider line widths of up to 50 $\mu$Hz. In this parameter regime, we find that if $1\% \leq |\eta_r| \leq 10\%$, changing the damping location changes $\eta_r$ by less than 10% of its total value; we also find that the results are similar to those obtained by using uniform damping.\(^2\)

Thus, when the source lies just beneath the upper turning point, line shapes depend strongly on source location, mode frequency, line width, and $\ell$ but only weakly on the type and location of damping. Placing the source in the evanescent region leads to similar conclusions; this case is discussed next.

2.2. Source outside the Well

When damping is uniform throughout the well, the solution of equation (3) for the amplitude seen by an observer at location $r$, due to a point source at $r_s (r > r_s > a)$, is

$$\psi_{\omega}(r) = \left[ \frac{2k \cos ka \sin k_r (r_s - a)}{k \cos ka + k_1 \sin ka} + \frac{2k_1 \sin ka \cosh k_r (r_s - a)}{k \cos ka + k_1 \sin ka} \right] e^{-k_1 (r - a)/2k_1}, \quad (7)$$

where $k$ and $k_1$ are defined as before. However, we gain more insight in this case by considering separately the contributions to the total amplitude due to waves that travel along different paths from the source to the observer. (This is the approach taken by Duvall et al. 1993.) In particular, the total observed amplitude is given by the sum of three parts: (1) the wave that travels directly outward from the source to the observer (hereafter the direct wave), (2) the wave that first travels toward the cavity and is then reflected back toward the observer at the potential step (hereafter the reflected wave), and (3) the sum of the infinite sequence of waves that arises as a result of multiple reflections in the cavity (hereafter the cavity wave).

The total amplitude seen by an observer is thus

$$\psi_{\omega, \text{tot}}(r) = \psi_{\omega, \text{dir}}(r) + \psi_{\omega, \text{ref}}(r) + \psi_{\omega, \text{cav}}(r) \quad = -\frac{e^{-k_1 (r - r_s)}}{2k_1} + \frac{e^{-k_1 (r + r_s - 2a)}}{2k_1} \left( k_1 + ik \right)^{-1},$$

$$+ \frac{2k \cos ka \sin k_r (r_s - a) + 2ka e^{-k_1 (r + r_s - 2a)}}{2kk_1 + ik(k_1 - k^2)} \left[ 1 + e^{2\pi a \left( \frac{ik_1}{ik_1 - k} \right)} \right]^{-1}. \quad (8)$$

The first two terms are roughly constant in magnitude and phase over a typical line width (up to 50 $\mu$Hz), while the third term (the cavity wave) exhibits resonant behavior. In particular, constructive interference occurs when the waves due to multiple reflections in the cavity arrive at the observer in phase; the frequencies at which this occurs are the eigenfrequencies. Therefore, for all cases of interest, the amplitude of the cavity wave is much greater than the amplitudes of the direct and reflected waves at an eigenfrequency. The magnitude of the cavity wave in the neighborhood of a mode eigenfrequency varies symmetrically about the peak. However, interference between the cavity wave and the sum of the direct and reflected waves causes the total power seen by an observer to be asymmetric about the peak.

For small line widths (specifically, in the limit that $k$ and $k_1$ are real), the cavity wave leads the direct wave by $\pi/2$ in phase when $\omega$ is equal to an eigenfrequency; this phase
difference is due to the phase shifts experienced by a wave upon entering or leaving the well, plus the phase shift due to travel from the top of the cavity to the bottom and back with an inverting reflection at the bottom. Since the reflected wave is smaller in magnitude than the direct wave, it follows that at resonance the cavity wave always leads the sum of the direct and reflected waves in phase. Furthermore, the phase of the cavity wave is a monotonically increasing function of frequency. Thus the observed asymmetry is always negative, while the magnitude of asymmetry depends on the differences in amplitude and phase between the three interfering terms. (It suffices to evaluate these at the source's location since the three waves experience the same phase change and attenuation in traveling from the
source to the observer.) The relative amplitudes and phases of the three waves near a mode frequency are shown in Figure 3. These conclusions are unaltered by the presence of damping.

We have investigated the dependence of the asymmetry on source location, mode frequency, mode degree, and line width (see Moving the source away from the cavity Fig. 2). Moving the source away from the cavity leads to more asymmetric line profiles because the amplitude of the cavity wave (evaluated at the source's location) decreases as a result of attenuation in the evanescent zone while the amplitude of the direct wave is unaffected; the interference of the direct wave with the cavity wave therefore has a more pronounced effect. Increasing the mode frequency while keeping the line width fixed, on the other hand, leads to less asymmetric profiles because the amplitude of the cavity wave at resonance increases with increasing mode frequency compared to the sum of the direct and reflected waves. We find that increasing the cavity length (decreasing $l$) while keeping the line width fixed yields more asymmetric profiles. Finally, increasing the line width yields more asymmetric line profiles since increasing the damping decreases the amplitude of the cavity wave without significantly affecting the amplitudes of the direct or reflected waves.

As when the source is inside the well, line profiles depend very weakly on the nature and location of damping: the numbers quoted in this respect in apply here as well.

We have shown that line asymmetry, when the source is above or below the upper turning point, depends strongly on source location, line width, mode frequency, and $l$ but only weakly on the nature and location of damping. Line asymmetry also leads to errors in determining the system's eigenfrequencies by using Lorentzian fits to the peaks; this is discussed next.

2.3. Errors in Eigenfrequency Determination

Errors in eigenfrequency determination by use of Lorentzian fits to observed power spectra occur for two reasons: first, the observed peak in a given line profile may be shifted from the corresponding eigenfrequency; second, the frequency of the best fit may differ from the frequency that corresponds to the observed peak. We have tested the eigenfrequency-determination error in the model problem by fitting Lorentzian curves to profiles generated for different source locations, mode frequencies, cavity lengths, and line widths, including cases in which the source is inside or outside the well and the damping is global or local.

We find a simple relationship between percentage asymmetry and the amount $\delta \nu_s$ by which the frequency obtained from a Lorentzian fit to the power spectrum of a mode $\nu$ differs from the corresponding eigenfrequency. Expressed as a percentage of the corresponding line width ($\Gamma_s$), $\delta \nu_s$ is proportional to the percentage asymmetry, $\eta_s$:

$$\delta \nu_s = (b/100)\Gamma_s \eta_s,$$

where $b$, the constant of proportionality, depends only on the type of damping used and not on mode frequency, line width, $l$-value, or source location. For spectra generated with global damping, $b \approx 1.6$ whereas, for spectra generated with local damping, $b \approx 1.1$ (see Fig. 5 of § 3.3).

3. LINE ASYMMETRY FOR $p$-MODES OF THE SUN

We have also analyzed line asymmetry by using a solar model due to Christensen-Daalsgard (1991). Our calculations of $p$-mode power spectra include radiative damping and stochastic mode excitation due to turbulent convection but ignore dissipation caused by interaction of modes with turbulent convection. Based on the calculations described

Fig. 3.—Ratio of the amplitude of the cavity wave to the amplitude of the direct wave (this ratio varies symmetrically about its peak), phase of the cavity wave with respect to the direct wave (in radians), and ratio of the amplitude of the reflected wave to the amplitude of the direct wave. All three curves were calculated using the one-dimensional model, with the source outside the cavity, for a peak with $\eta_s = -10\%$.
in § 2, we do not expect our results to depend on the type of damping used; however, because the line widths that we calculate are smaller than those observed, we must take into account the dependence of asymmetry on line width found for the simple model of § 2 when interpreting our numerical results.

Theoretical power spectra are computed by solving for Green’s functions of the nonadiabatic oscillation equations in the Cowling approximation. The details of this calculation are described by Kumar (1994). As described there, the mean velocity power spectrum due to stochastic excitation by turbulent convection is given by

\[ \langle P_\alpha \rangle \approx \frac{\omega^2}{R_0} \int dr \int dr_0 \rho(r_0) \frac{d}{dr_0} G_\alpha(r; r_0) ^2 S_\alpha(r_0), \]  

(10)

where \( G_\alpha(r; r_0) \) is the Green’s function for a source term in the momentum equation and \( S_\alpha(r_0) \) is the source strength, which we consider to be the Reynolds stress. The outer integral is over the region in the photosphere where the optical line is formed, and the inner integral is over the source region. The \( p \)-mode power spectrum calculated using equation (10) depends on the derivative of the Green’s function because we consider only quadrupole sources of sound waves, which correspond to the derivative of the Reynolds stress; this derivative is transferred to the Green’s function after integration by parts. The power spectrum for dipole sources, obtained by using equation (10) with \( G_\alpha \) in place of \( dG_\alpha / dr \), yields different line shapes.

The source strength for quadrupole acoustic emission is given by Kumar (1994):

\[ S_\alpha(r_0) \sim \frac{H^4 v_H^3}{1 + (\tau_H \omega)^{-1}}, \]  

(11)

where \( H \), \( \tau_H \), and \( v_H \) are, respectively, the correlation (“mixing”) lengths, correlation times, and velocities of energy-bearing convective eddies at radius \( r_0 \) \( (v_H \approx H \tau_H) \). As a consequence of its strong dependence on convective velocity, the source strength is expected to be a sharply peaked function of position (see Fig. 1 of Kumar 1994). We treat the unknown source strength as a Gaussian distribution of width 50 km (this is roughly what is expected if we take the convective velocity as given by standard mixing-length theory) and vary it peak position within the top 1200 km of the convection zone. We compute the \( p \)-mode line profiles and asymmetries, which, when compared with observations, should yield the radial location of the acoustic sources. We note that the equilibrium solar model was calculated using standard mixing-length theory.

In the next two subsections, we present the results of our solar-model calculations for the dependence of line asymmetry on source location, mode frequency, and mode degree and explain our results in terms of the simplified model. In § 3.3, we briefly discuss the errors in eigenfrequency determination that are introduced in line asymmetry.

### 3.1. Solar Model Results

We have calculated solar \( p \)-mode power spectra for \( \ell \) between 5 and 500, frequencies in the range \( \sim 1-5 \) mHz, and source location in the top 1200 km of the convection zone. Peaks exhibit varying amounts of asymmetry, depending on source location, mode frequency, and mode degree (see Fig. 4). Moving the source deeper causes the asymmetry parameter \( \eta_\ell \) to become more positive unless the source passes through a node, in which case \( \eta_\ell \) jumps from a large positive to a large negative value. (There is some deviation from this pattern when the source lies in the top 30 km of the convection zone.)

When the source depth is less than 400 km, the most asymmetric peaks (with \( |\eta_\ell| \approx 10% \)) correspond to low-frequency modes, while peaks corresponding to modes above 3 mHz show very little asymmetry \((|\eta_\ell| < 4\%)\). The asymmetry for mode frequencies below 3 mHz is negative, i.e., there is more power on the lower frequency side of the peak. In contrast, for some deeper source locations (e.g., between 800 and 1000 km), the most asymmetric peaks correspond to high-frequency modes, while peaks that correspond to modes below 3 mHz show little asymmetry. In this case, the asymmetry of the most asymmetric peaks is positive. In general, which modes have the most asymmetric power spectra, and the sign of the corresponding asymmetries, depends on the location of the source and changes as the source is moved deeper still.

For any source location and mode frequency, the magnitude of the asymmetry is a weakly increasing function of \( \ell \); the parameter \( \eta_\ell \) changes by no more than a few percentage points over the range \( \ell = 5-500 \).

Duvall et al. (1993) found negative asymmetry in \( p \)-mode velocity spectra corresponding to modes of frequency \( 2.2-2.5 \) mHz and \( \ell \) in the range 157–221; from their data, we find that \( \eta_\ell = -2.5\% \) for these modes. Allowing an uncertainty in \( \eta_\ell \) of \( \pm 0.5\% \), our numerical calculations reproduce this result provided that we assume that the sources for modes of this frequency lie at a depth of 325–525 km beneath the photosphere. However, our calculated line widths are less than the observed line widths in this frequency range, which implies that, for any source location, we underestimate the magnitude of \( \eta_\ell \). Since \( |\eta_\ell| \) for these modes is a decreasing function of source depth for locations within the top few scale heights of the convection zone, this technique therefore yields a lower bound of 325 km on the source depth for these modes.

Duvall et al. found positive line asymmetry in the intensity power spectra of the same modes. This is a puzzling result. At any given location in the solar atmosphere, the dynamical oscillation equations may be solved for the pressure and density perturbations \((\delta \rho \) and \( \delta \rho \)) in terms of the radial displacement function \((\xi_r)\), radial wavenumber, and equilibrium properties of the atmosphere. In the model problem of § 2, it can be shown analytically that \( \xi_r \) is the only one of these quantities that varies appreciably over a mode line width, and numerical calculations show this to be the case for the solar model. As a result, we find that the perturbation to any thermodynamic quantity is essentially proportional to the radial displacement function. Any reversal of asymmetry must therefore be due to a subtle detail of the process by which the observed flux is modulated by pulsation.

### 3.2. Explanation of Solar-Model Results

The behavior described above is in qualitative agreement with the results obtained by using the simplified model of § 2, except for the dependence of asymmetry on source posi-
Asymmetry in the shear model (in terms of $\eta_1$) as a function of source depth (measured relative to the top of the convection zone) for modes with $\ell$ approximately equal to (a) 5, (b) 100, (c) 250, and (d) 375. Points where $\eta_1$ changes abruptly from positive to negative (with increasing depth) correspond to nodes of the respective eigenfunctions.

In the simple model problem, the asymmetry depends strongly on the cavity length (the effective mode degree) while, in the solar model, the asymmetry is a very weak function of degree. The difference is accounted for by the increase of line width with degree in the solar model, which cancels the effect on the asymmetry of decreasing the effective cavity length. (The dependence of line width on $\ell$ calculated with the solar model is similar to the observed dependence; therefore we do not expect this result to be affected significantly by the discrepancies between calculated and observed line widths described earlier.)
As mentioned previously, the derivative used in calculating power spectra from the solar model has an important
effect on the shapes of lines in the spectra (see eq. [10]). In
calculating power spectra with the simple model of § 2, no
derivative is explicitly used. However, the dependent variable
used there is \( \psi = \rho^{1/2}c^2V \cdot \xi \), which is approximately
equal to \( \rho^{1/2}c^2 \frac{d\xi}{dr} \) near the surface. As a result of the
derivative present in the definition of \( \psi \), the shapes of lines
in spectra calculated using the simple model agree qualitatively
with those in spectra calculated using the solar model. (Note that the choice of variables determines the
form of the acoustic potential. For the set of variables used
here, the effective potential for \( p \)-modes may be approximated by the square-well potential of eq. [2]; for other sets
of variables not involving derivatives—e.g., the set used by
Gabriel 1992 and Roxburgh & Vorontsov 1995—the
acoustic potential bears little resemblance to the square-well potential.)

3.3. Errors in Eigenfrequency Determination
As in the one-dimensional model problem, we find that
for a mode \( a \) the frequency shift (see expressed as a
percentage of the line width \( \Gamma \), is proportional to the per-
centage asymmetry, \( \eta_a \) (see Fig. 5). The constant of propor-
tionality \( b \), defined by equation (9), is found to be 1.5 in the
solar case. The linear relationship holds regardless of
source location, mode frequency, or mode degree, as in the
one-dimensional problem.

4. SUMMARY AND DISCUSSION
The shapes of lines in \( p \)-mode power spectra are, in
general, not symmetric about the peak. We find that the
magnitude and sense of asymmetry depend strongly on the
location of the sources and mode frequency and weakly on
mode degree (\( l \)).

We define a dimensionless measure of asymmetry for a
given mode by decomposing its power spectrum in the
neighborhood of the peak into even and odd components;
the ratio of the peak amplitudes of the two components,
expressed as a percentage, measures the magnitude of the asymmetry. We define the asymmetry to be positive (or
negative) when there is more power on the high-frequency
(or low-frequency) side of the peak.

We have calculated the line asymmetry for \( p \)-modes of a
simplified one-dimensional problem, as well as a solar
model, for various source locations. Line asymmetry is
found to have similar behavior in both cases. In particular,
we find that moving the source deeper causes the asym-
metry to become more positive unless the source passes
through a node of the mode eigenfunction, in which case the
asymmetry changes discontinuously from a large positive to
a large negative value. The magnitude of the asymmetry
increases weakly with mode degree: for fixed source loca-
tion and approximately constant mode frequency, the
absolute value of the asymmetry parameter increases by no
more than a few percent over the range \( r = 5-500 \). For the
one-dimensional problem, the asymmetry has no significant
dependence on the location or nature of damping as long as
mode line width is held fixed.

Physically, line asymmetry may be understood in terms
of a wave interference model, in which waves travel along
different paths from the source to the observer, as suggested
by Duvall et al. However, we find that the line asym-
metry does not depend significantly on phase shifts due to
nonadiabatic effects in evanescent regions, as Duvall et al.
suggested.

By comparing the asymmetries of theoretically calculated
profiles to the asymmetries of the profiles observed by
Duvall et al. (1993) for modes of frequency \( \sim 2.3 \) mHz, we
set a lower bound on the source depth, for these modes, of
325 km (measured with respect to the bottom of the

![Fig. 5. Frequency shifts (expressed as percentages of corresponding line widths) of best-fit frequencies from eigenfrequencies vs. \( \eta_a \). These points correspond to the peaks, calculated using the solar model, whose asymmetries are plotted in Fig. 4. The slope is \( \sim 1.5 \).](image-url)
photosphere). This is greater than the depth of 140 $\pm$ 60 km found by Kumar (1994) for the sources exciting acoustic waves above the acoustic cutoff frequency ($\sim$ 5.3 mHz). Based on the theory of wave generation by turbulent convection and standard mixing-length theory, we expect the emission of acoustic waves to occur deeper in the convection zone for waves of lower frequency. The difference between the present results and the results of Kumar (1994) may be a confirmation of this prediction.

We find that line asymmetry causes the frequency obtained from a Lorentzian fit to a given peak in the power spectrum to differ from the corresponding eigenfrequency by an amount proportional to the mode line width and to the asymmetry of the peak. This holds for both the square-well potential model and the more realistic solar model. As a percentage of the line width, this frequency shift for the solar model is $\sim$ 1.5 times the percent asymmetry of the peak.

Duvall et al. reported that the sense of line asymmetry is different in velocity and intensity power spectra; in particular, for modes of frequency below 3 mHz, they found negative asymmetry in velocity spectra and positive asymmetry in intensity spectra. This is a puzzling result, for which we have no explanation.

We thank Tom Duvall for sending us the data used in § 3.1 and Eliot Quataert for many useful comments and discussions. This work was supported by NASA grant NAGW-3936.

REFERENCES

Christensen-Dalsgaard, J. 1991, in Lecture Notes in Physics, 388, Challenges to Theories of the Structure of Moderate-Mass Stars, ed. D. Gough & J. Toomre (Berlin: Springer), 11
Deubner, F. L., & Gough, D. 1984, ARA&A, 22, 593
Duvall, T. L., Jr., Jefferies, S. M., Harvey, J. W., Osaki, Y., & Pomerantz, M. A. 1993, ApJ, 410, 829
Gabriel, M. 1992, A&A, 265, 771
------. 1993, A&A, 274, 935
------. 1995, A&A, 299, 245
Kumar, P. 1994, ApJ, 428, 827
Kumar, P., Duvall, T. L., Jr., Harvey, J. W., Jefferies, S. M., Pomerantz, M. A., & Thompson, M. J. 1990, in Lecture Notes in Physics, 367, Proc. Oji Int. Semin., Progress of Seismology of the Sun and Stars, ed. Y. Osaki & H. Shibahashi (Berlin: Springer), 87
Kumar, P., Fardal, M. A., Jefferies, S. M., Duvall, T. L., Jr., Harvey, J. W., & Pomerantz, M. A. 1994, ApJ, 422, L29
Kumar, P., & Lu, E. 1991, ApJ, 375, L35
Roxburgh, I. W., & Vorontsov, S. V. 1995, MNRAS, 272, 850