ROTATION IN A GENERALISED DIRAC’S UNIVERSE

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Abstract

The extension of Dirac’s LNH to cover cosmological and fine-structure time-varying "constants", and the rotation of the Universe, is here analysed, including a "derivation" of the angular speed of the present Universe, and of the inflationary phase. Criticizable points on the present calculation, are clarified.

Keywords: Dirac; LNH; fine-structure; time-varying "constants"; rotation; inflation.

Resumen:

Extendiendo la Hipótesis de los Grandes Números de Dirac, para cubrir las "constantes" cosmológica y de estructura fina, y la rotación del Universo, las estudiamos, incluyendo una ”demonstración”, del cálculo de la velocidad angular del Universo actual, y del inflacionario. Puntos criticables de éste artículo, son clarificados en seguida.
1. Introduction

We shall study a generalisation of Dirac’s LNH Universe, with the introduction of time-varying speed of light, which causes a time-varying fine-structure ”constant”, and a possible rotation of the Universe, either for the present time, or for inflationary periods.

The rotation of the Universe (de Sabbata and Sivaram, 1994; de Sabbata and Gasperini, 1979) may have been detected experimentally by NASA scientists who tracked the Pioneer probes, finding an anomalous deceleration that affected the spaceships during the thirty years that they took to leave the Solar system. This acceleration can be explained through the rotation of the Machian Universe (Berman, 2007b). Berman (2008) showed that Robertson-Walker’s metric includes a rotational component, plus expansion. Berman (2008d) gave a follow up to the subject obtaining the same results. A universal spin has also been considered by Berman (2008b; 2008c).

A time-varying gravitational constant, as well as others, were conceived by P.A.M. Dirac (1938; 1974), Eddington (1933; 1935; 1939), Barrow (1990) through his Large Number Hypothesis. Later, Berman supplied the GLNH – Generalised Large Number Hypothesis (Berman, 1992; 1992a; 1994). This hypothesis arose from the fact that certain relationships among physical quantities, revealed extraordinary large numbers of the order $10^{40}$. Such numbers, instead of being coincidental and far from usual values, were attributed to time-varying quantities, related to the growing number of nucleons in the Universe. In fact, such number $N$, for the present Universe, is estimated as $(10^{40})^2$. The number is ”large” because the Universe is ”old”. At least, this was and still is the best explanation at our disposal.

The four relations below, represent respectively, the ratios among the scalar length of the causally related Universe, and the Classical electronic radius; the ratio between the electrostatic and gravitational forces between a proton and an electron; the mass of the Universe
divided by the mass of a proton or a nucleon; and a relation involving the cosmological constant and the masses of neutron and electron.

If we call Hubble’s constant $H$; electron’s charge and mass $e$, $m_e$; proton’s mass $m_p$, cosmological constant $\Lambda$, speed of light $c$, and Planck’s constant $h$, we have:

\[
\frac{cH^{-1}}{\sqrt{\frac{e^2}{m_em_e}}} \simeq \sqrt{N} \quad (1)
\]

\[
\frac{e^2}{Gm_pm_e} \simeq \sqrt{N} \quad (2)
\]

\[
\frac{\rho (cH^{-1})^3}{m_p} \simeq N \quad (3)
\]

\[
ch(m_pm_e/\Lambda)^{1/2} \simeq \sqrt{N} \quad (4)
\]

We may in general have time-varying speed of light $c = c(t)$; of $\Lambda = \Lambda(t)$; of $G = G(t)$; etc. We define the fine structure "constant" as,

\[
\alpha \equiv \frac{e^2}{\hbar c(t)} ,
\]

and consider $\alpha = \alpha(t)$, because of the time-varying speed of light.

2. Power-law variations

It is well-known that experimental data should be, by necessity, interpreted against the background theoretical framework that is satisfiable from the point of view of the majority of researchers in the field. We shall choose Robertson-Walker’s metric as such background, and power-law scale-factors.

One can ask whether the previous Section’s constant-variations could be caused by a time-varying speed of light: $c = c(t)$. We refer to Berman (2007) for information on the experimental time variability of $\alpha$. Gomide (1976) has studied $c(t)$ and $\alpha$ in such a case, which was later revived by Barrow (1998; 1998a; 1997); Barrow and Magueijo (1999); Albrecht and Magueijo (1998); Bekenstein (1992). This could explain also Supernovae observations. We refer to their papers for further information. Our framework now will be an estimate made through Berman’s GLNH. We write again the main formulae for constant
deceleration parameters, as derived originally in the literature by Berman (1983) and Berman and Gomide (1988):

\[
H = \frac{\ddot{R}}{R} = \frac{1}{m!} = \frac{1}{1+q} t^{-1}. \tag{6}
\]

\[
q = -\frac{R\dot{R}}{R^2} = m - 1. \tag{7}
\]

\[
R = (mDt)^{1/m}. \quad (m \neq 0), \tag{8}
\]

where \( R \) is the scale-factor in Robertson-Walker’s metric:

\[
ds^2 = dt^2 - \frac{R^2(t)}{(1+kr^2)^2} (dx^2 + dy^2 + dz^2). \tag{9}
\]

We express now Webb et al’s (1999; 2001) experimental result as:

\[
(\dot{\alpha}/\alpha)_{\text{exp}} \simeq -1.1 \times 10^{-5} H (1 + q). \tag{10}
\]

From (5) we find:

\[
\dot{\alpha}/\alpha = -\frac{\ddot{c}}{c}. \tag{11}
\]

Again, we suppose that the speed of light varies with a power law of time:

\[
c = At^n \quad (A = \text{constant}). \tag{12}
\]

From the above experimental value we find:

\[
n \approx 10^{-5}. \tag{13}
\]

From (12) and (13) taken care of (11), we find:

\[
\frac{\dot{\alpha}}{\alpha} = -\frac{\ddot{c}}{c} = nt^{-1}. \tag{14}
\]

From relations (1), (2), (3) and (4) we find:

\[
N \propto t^{2+6n},
\]

\[
G \propto t^{-1-3n},
\]

\[
\Lambda \propto t^{-2-4n}.
\]
\[ \rho \propto t^{1+3n} \]

We see that the speed of light varies slowly with the age of the Universe. For the numerical value (13), we would obtain:

\[ N \propto t^{2.0001} \quad (15) \]

and then:

\[ G \propto t^{-1.00005} \quad (16) \]

\[ \Lambda \propto t^{-2.0001} \quad (17) \]

\[ \rho \propto t^{-0.99995} \quad (18) \]

This is our solution, based on Berman’s GLNH, itself based on Dirac’s work (Dirac, 1938; 1974). A pre-print with a preliminary but incomplete solution was already prepared by Berman and Trevisan (2001; 2001a; 2001b).

A close analysis shows how the deceleration parameter range is situated. Both from Lunar laser ranging and Viking radar measurements by Williams et al (1976) and Reasenberg (1983) we find:

\[ \frac{\dot{G}}{G} = \sigma H \quad \text{with} \quad |\sigma| < 0.6 \]

Will (1987; 1995) and Dickey (1994) comment that these two kinds of measurements give the best limit on |\sigma|. Because in our model we have:

\[ \frac{\dot{G}}{G} \approx 1.0 \, t^{-1} \approx 1.0 \, H \, (1 + q) \quad (19) \]

we find:

\[ -0.4 > q > -1.6 \quad (20) \]

The above result is satisfactory, for a negative ”q” agrees with Supernovae observations.

We have thus shown that \[ \frac{\Delta \alpha}{\alpha} \] should really be negative, for a positive value, could mean a positive deceleration parameter. As a bonus we found possible laws of variation for \( N, G, \rho, \)
and Λ. The Λ–term time variation is also very close and even, practically indistinguishable, from the law of variation Λ ∝ t⁻².

It is clear that in this Section’s model, the electric permittivity of the vacuum, along with its magnetic permeability, and also Planck’s constant are really constant here. We point out again, that in the long run, it will be only when a Superunification theory becomes available, that our different models could be discarded, (hopefully) but one.

3. Exponential Inflation

On remembering that relations (1) and (3) carry the scale-factor of the causally related Universe, \( cH^{-1} \), we substitute it by the exponential radius,

\[
R = R_0 e^{Ht}.
\]

With the same arguments above, but, substituting, (12) by the following one,

\[
c = c_0 e^{\gamma t}, \quad (c_0, \gamma = \text{constants})
\]

we would find:

\[
N \propto e^{[H+2\gamma] t},
\]

\[
G \propto e^{-[\frac{4}{3}+\gamma] t},
\]

\[
\rho \propto e^{-2[H-\gamma] t},
\]

and,

\[
\Lambda \propto e^{-H t}.
\]

It seems reasonable that inflation decreases the energy density, and the cosmological term while \( N \) grows exponentially; of course, we take \( H > \gamma \).

4. Rotation of the Universe

A closely related issue is the possibility of a Universal spin. Consider the Newtonian definition of angular momentum \( L \),
\[ L = RMv \] \hspace{1cm} (23)

where, \( R \) and \( M \) stand for the scale-factor and mass of the Universe.

For Planck’s Universe, the obvious dimensional combination of the constants \( \bar{h}, c \), and \( G \) is,

\[ L_{Pl} = \bar{h} \] \hspace{1cm} (24)

From (23) and (24), we see that Planck’s Universe spin takes a speed \( v = c \). For any other time, we take, then, the spin of the Universe as given by

\[ L = RMc \] \hspace{1cm} (25)

In the first place, we take the known values of the present Universe:

\[ R \approx 10^{28} \text{cm} \] \hspace{1cm},

and,

\[ M \approx 10^{55} \text{grams} \] \hspace{1cm},

so that,

\[ L = 10^{93} \text{cm.cm.gram.cm/s} = 10^{120} \bar{h} \] \hspace{1cm} (26)

We have thus, another large number,

\[ \frac{L}{\bar{h}} \propto N^{3/2} \] \hspace{1cm} (27)

For instance, for the power law, as in standard cosmology, we would have,

\[ L \propto t^{3+9n} = t^{3(1+3n)} \] \hspace{1cm} (28)

For exponential inflation,

\[ L \propto e^{3[H+2\gamma]} t \] \hspace{1cm} (29)
We now may guess a possible angular speed of the Universe, on the basis of Dirac’s LNH. For Planck’s Universe, the obvious angular speed would be:

\[ \omega_{Pl} = \frac{c}{R_{Pl}} \approx 2 \times 10^{43} \text{s}^{-1}, \] (30)

because Planck’s Universe is composed of dimensional combinations of the fundamental constants.

In order to get a time-varying function for the angular speed, we recall Newtonian angular momentum formula,

\[ L = R^2 M \omega. \] (31)

In the case of power-law \( c \) – variation, we have found, from relation (27), that \( L \propto N^{3/2} \), but we also saw from (31) that \( L \propto \rho R^5 \omega \), because \( R = cH^{-1} \propto \sqrt{N} \) and \( M \propto \rho R^3 \propto N \).

Then, we find that,

\[ \omega = \omega_0 t^{1+6n} = A R^{-(1-6n)} \quad (\omega_0, A = \text{constants}) \] . \hspace{1cm} (31a)

We are led to admit the following relation:

\[ \omega \lesssim \frac{c}{R} \] . \hspace{1cm} (32)

For the present Universe, we shall find,

\[ \omega \lesssim 3 \times 10^{-18} \text{s}^{-1} \] . \hspace{1cm} (33)

It can be seen that present angular speed is too small to be detected by present technology.

For the inflationary model, we carry a similar procedure:

\[ \omega \propto \frac{N^2}{R^3 \rho^2} = e^{[\frac{-\gamma}{2}H+\gamma]} t \] . \hspace{1cm} (34)

The condition for a decreasing angular speed in the inflationary period, is, then,

\[ \gamma < \frac{\gamma}{2}H \] . \hspace{1cm} (35)
5. Pros and Cons of the present calculations

Critical appraisals of the above calculations, center on the three following arguments:

I - do the time variations, \( G(t) \), \( \rho(t) \), and \( A(t) \) proposed above, violate Einstein’s field equations?

II - if Einstein’s theory does not apply, which one does? And then, do the new equations reduce to Einstein’s in a proper limit?

III - the identification of \( R \) with \( cH^{-1} \) is a non-sense, for the former has no causal meaning while the latter is the radius of the casually connected portion of space associated with Hubble’s horizon. Each one has different time variations.

We now reply:

1st) Dirac never proposed LNH as part of GRT (General Relativity Theory), neither do I.

2nd) Robertson-Walker’s metric has been employed in most (perhaps, all) geometrical theories of Cosmology, which do reduce (i.e., are alternative), or not, to GRT in some limit. Dirac’s LNH is a foil for testing hypotheses, like the theoretical frameworks of scalar-tensor cosmologies with lambda (see for instance Berman, 2007a). In such theories, our present results may be included (Berman, 2007a).

3rd) The Robertson-Walker’s scale-factor \( R(t) \), is defined as an adimensional temporally increasing function, but nothing changes if we calibrate \( R(t) \) in such a way that it be proportional to \( cH^{-1} \). See for instance, the derivation of Robertson-Walker’s metric given in the books by Berman (2007; 2007a). Berman (1997) may have been the first one to explicit seminally such use. Anyhow, in the power-laws, the case \( m \approx 1 \), generates the presently known accelerating Universes, for it points to a possible slightly negative deceleration parameter \( q \), because in the above models, we have from (7),

\[ m = q + 1 \]

Then, from relation (8), we have,
\[ R = (mDt)^{1/m} \approx cH^{-1} = ct \quad (m \approx 1). \]

We hope to have clarified the former cons, with the latter pros.

6. Conclusions

Paraphrasing Dicke (1964; 1964a), it has been shown the many faces of Dirac’s LNH, as many as there are about Mach’s Principle. In face of modern Cosmology, the naif theory of Dirac is a foil for theoretical discussion on the foundations of this branch of Physical theory. The angular speed found by us, matches results by Gödel (see Adler et al., 1975), Sabbata and Gasperini (1979), and Berman (2007b, 2008a,b,c).

There is a no-rotation condition, for \( n = \frac{1}{6} \), in the power-law solution; likewise, with \( \gamma = \frac{3}{2}H \), this is the no-rotation condition of the inflationary angular speed formula. However, these cases are foreign to the idea of a weak time-varying formula for the fine-structure "constant".

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