Mechanisms for High-frequency QPOs in Neutron Star and Black Hole Binaries

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Accepted, Astrophys. J.

Received ___________; accepted ______________

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ABSTRACT

We explain the millisecond variability detected by Rossi X-ray Timing Explorer (RXTE) in the X-ray emission from a number of low mass X-ray binary systems (Sco X-1, 4U1728-34, 4U1608-522, 4U1636-536, 4U0614+091, 4U1735-44, 4U1820-30, GX5-1 and etc) in terms of dynamics of the centrifugal barrier, a hot boundary region surrounding a neutron star (NS). We demonstrate that this region may experience the relaxation oscillations, and that the displacements of a gas element both in radial and vertical directions occur at the same main frequency, of order of the local Keplerian frequency. We show the importance of the effect of a splitting of the main frequency produced by the Coriolis force in a rotating disk for the interpretation of a spacing between the QPO peaks. We estimate a magnitude of the splitting effect and present a simple formula for the whole spectrum of the split frequencies. It is interesting that the first three lowest-order overtones (corresponding to the azimuthal numbers \( m = 0, \ -1, \) and \(-2\)) fall in the range of 200-1200 Hz and match the kHz-QPO frequencies observed by RXTE. Similar phenomena should also occur in Black Hole (BH) systems, but, since the QPO frequency is inversely proportional to the mass of a compact object, the frequency of the centrifugal-barrier oscillations in the BH systems should be a factor of 5-10 lower than that for the NS systems. The X-ray spectrum formed in this region is a result of upscattering of a soft radiation (from a disk and a NS surface) off relatively hot electrons in the boundary layer. The typical size of the emission region should be 1-3 km, which is consistent with the time-lag measurements. We also briefly discuss some alternative QPO models, including a possibility of acoustic oscillations in the boundary layer, the proper stellar rotation, and \(g\)-mode disk oscillations.
Subject headings: accretion, accretion disks — black hole physics — radiation mechanisms: thermal — stars: neutron — X-rays: general
1. Introduction

The process of accretion onto a compact object in the neutron star (NS) and black hole (BH) binaries has many remarkable similarities. Some part of accreting matter with large angular momentum forms a disk, while another part (participating in a sub-Keplerian rotation) undergoes practically a free fall accretion until the centrifugal barrier (CB) becomes sufficient to halt the flow (see e.g. Chakrabarti & Titarchuk 1995; hereafter CT95). Thus, one may expect that two distinct zones, a disk and a barrier, can be formed in the vicinity of a compact object which are likely to be responsible for the generation of a resulting spectrum. In this paper we consider a possibility of the dynamical adjustment of a Keplerian disk to the surface of a NS.

The disk structure begins deflecting from a Keplerian one at a certain point to adjust itself to the boundary conditions at a NS surface (or at the last stable orbit, at $R_0 = 3R_s$, in the case of a BH). The transition from a Keplerian to a sub-Keplerian flow may proceed smoothly, though it is very likely that a perfect adjustment never occurs. In general, the transition should take place through the setting up of the CB (where a centrifugal force slightly exceeds the gravitational force) within the adjustment radius. Here (see §2.1 for details) we discuss the formation of kinks and shocks in the supersonic regime of accretion flow as a possible physical reason for a super-Keplerian rotation. We suggest that in a region with a super-Keplerian rotation a matter may experience the relaxation oscillations in the vertical and radial directions. These oscillations are expected to be in a resonance with the local angular velocity in a disk, and the variation in an emitting area caused by the oscillations around a transition point produces the QPOs in the X-ray flux.

The principal observational consequence of our scenario is a correlation between the X-ray flux and QPO frequency. The relatively soft disk photons in the CB region are scattered off the hot electrons thus forming the Comptonized X-ray spectrum (Sunyaev
& Titarchuk 1980; hereafter ST80). The electron temperature is regulated by the supply of soft photons from a disk, which depends on the ratio of the energy release (accretion rate) in the disk and the energy release in the CB region (see Eq. [24] for more details). For example, the electron temperature is higher for lower accretion rates (see e.g. CT95, Figures 3, 4), while for a high accretion rate (of order of the Eddington one) the CB region cools down very efficiently due to Comptonization.

In this paper we discuss the CB model as a possible mechanism for the $\sim 1$ kHz QPOs discovered by Rossi X-ray Timing Explorer (RXTE) in a number of LMXBs (see Strohmayer et al. 1996, and Van der Klis et al. 1996, hereafter S96, and VK96, respectively; Zhang et al. 1996). These observations reveal a wealth of previously unknown high-frequency X-ray variabilities which are believed to be due to the processes occurring on, or in the very vicinity of an accreting NS.

In §2 we schematically describe our CB model. We demonstrate that the main model parameter $\gamma$ (which is proportional to a ratio of accretion rate to viscosity and is in fact Reynolds number for the accretion flow) determines the QPO frequency. As the Reynolds number ($\gamma$-parameter) increases, the CB moves toward a NS surface (or toward the inner edge of an accretion disk), and the X-ray flux at higher energies declines because of the cooling of the emission region (CB). In addition, as the Reynolds number ($\gamma$-parameter) increases, the QPO frequency reaches its limiting value (§2.1). We estimate the characteristic frequency of the oscillations of the CB region in §2.2. In §2.3, we show that due to rotation of a disk the effect of the Coriolis force should result in the splitting of the main QPO frequency, and in §2.4, we present our estimate of the corresponding oscillation amplitude. We apply our CB model to the interpretation of RXTE observations of high-frequency QPOs in §3. In §4, we discuss the alternative possibilities including resonant acoustic waves in the hot boundary layer around the NS, stellar rotation,
disk nonradial oscillations for the QPOs with a frequency of \(\sim 300\) Hz and g-mode disk oscillations. Finally, we summarize our conclusions in §5.

2. Centrifugal barrier oscillations

2.1. The formulation of the problem

Let us consider a Keplerian accretion disk around a BH or a weakly magnetized NS. It is generally believed that there is a transition layer in the vicinity of a compact object where an accreting matter adjusts itself either to the surface of a rotating NS, or to the innermost boundary of an accretion disk. In the transition layer the motion is not Keplerian, and is governed by the mechanism of the angular momentum loss by an accreting matter. Thus, we may define the transition layer as a region confined between the surface of a NS (either the last stable orbit or the corotating magnetosphere) and the first Keplerian orbit. In a Keplerian disk a matter is circularly orbiting with the angular velocity

\[ \omega_K = \left( \frac{GM}{R^3} \right)^{1/2}, \]

where \(G\) is the gravitational constant, \(M\) is the mass of a compact object, and \(R\) is the radius of an orbit. The radial motion in a disk is provided by the friction and angular momentum exchange between the adjacent layers that result in the loss of initial angular momentum by an accreting matter. The radial transport of the angular momentum in a disk can be written therefore in terms of the torque of viscous forces between the adjacent layers (see e.g. Shakura & Sunyaev 1973)

\[ \dot{M} \frac{d}{dR}(\omega R^2) = 2\pi \frac{d}{dR}(W_{r\phi} R^2). \]

Here \(\dot{M}\) is the accretion rate, and \(W_{r\phi}\) is the component of a viscous stress tensor that can be expressed as

\[ W_{r\phi} = -2\eta H dR \frac{d\omega}{dR}, \]
where $H_d$ is a half-thickness of a disk, and $\eta$ is the turbulent viscosity. The only parameter entering this equation is the ratio

$$\gamma = \frac{\dot{M}}{4\pi \eta H_d}$$

which is nothing else but Reynolds number for the accretion flow. The viscosity determines the redistribution of the momentum in the flow, with both the particles and photons (in the radiation dominated region) participate in momentum transport between the shearing layers. Narayan (1992) has argued that the momentum transport (viscous diffusion) by the particles in the flow depends on the velocity of the flow if the sources supplying particles are “frozen” into the flow. Specifically, Narayan (1992) has demonstrated that if the particle sources comove with the flow at the bulk velocity $V$, and the particles diffuse with the velocity $c_s$, then the effective diffusion (viscosity) coefficient scales as $(1 - V^2/c_s^2)$. Thus, the viscosity of the supersonic flow gets effectively suppressed provided that the collective plasma effects (Tsytovich 1977) do not have enough time to develop, which seems to be the case in the limit of $V \approx c_s$ where the distribution of particles in the flow is essentially determined by the source (see Narayan 1992, Equation [3.5]).

The very important consequence of the Narayan’s result is that the regime of high $\gamma$ (Reynolds number) can occur even for the low accretion rates, $\dot{M}$, in the presence of high velocities of the flow. However, the radiative viscosity can be important under the radiation-pressure dominated conditions and in the presence of high velocities of the flow, which seems to be the case for the transition layer. The coefficient of radiative viscosity is given by

$$\eta = \frac{1}{3} m_p n_{ph} c l,$$

where $m_p$ is the proton mass, $n_{ph}$ is the photon number density, $c$ is the speed of light, and $l$ is the photon mean-free path. The photon number density is proportional to the accretion rate, $\dot{M}$. Thus, it is important that if the KHz QPO phenomenon is mostly determined by the ratio of accretion rate to the radiative viscosity of the flow, $\dot{M}/\eta$, then we would expect
it to occur in the sources with both high and low accretion rates.

We can calculate the distribution of angular velocity of matter in the transition layer by solving equation (2) with the appropriate boundary conditions. We adopt that at the inner boundary the angular velocity of matter either matches the rotation velocity of a NS (or velocity of the corotating magnetosphere at the magnetopause),

$$\omega = \omega_0 \quad \text{at} \quad R = R_0, \quad (6a)$$

or vanishes (e.g. in the case of a non-rotating BH),

$$\omega = 0 \quad \text{at} \quad R = R_0. \quad (6b)$$

The outer boundary condition for the layer must require that the orbital motion of matter matches smoothly a Keplerian motion at some radius $R_{\text{out}}$. The latter means that the angular velocity of matter and its radial derivative should be equal to those for a Keplerian disk at the same radius, i.e.

$$\omega = \omega_K \quad \text{at} \quad R = R_{\text{out}}, \quad (7)$$

and

$$\frac{d\omega}{dr} = \frac{d\omega_K}{dr} \quad \text{at} \quad R = R_{\text{out}}. \quad (8)$$

The boundary conditions (6)-(8) allow us to solve equation (2). Thus, we can unambiguously calculate a radial profile of the angular velocity in a transition layer $\omega(R)$, and derive the value of the outer radius, $R_{\text{out}}$. It is important, that the only parameter that controls the adjustment of the angular velocity in a transition layer to the Keplerian angular velocity in a disk is $\gamma-$parameter.

Let us focus on how to get a formal solution that would describe a smooth transition of the Keplerian flow into the sub-Keplerian rotation. The corresponding problem for equation (2) should be formulated as the boundary problem for the second-order differential equation
with the three boundary conditions: one at the inner boundary, and two at the adjustment point, where the flow begins to deviate from the Keplerian motion in order to adjust itself to the sub-Keplerian rotation. Thus, we have three boundary conditions for the second order differential equation which implies that the position of the adjustment point can be determined uniquely (see e.g. Korn & Korn 1961, Ch. 9.3). Such a simple formulation is quite instructive: it guarantees the uniqueness of the solution and automatically fixes the position of the outer boundary. This is important because the position of the outer boundary of the transition layer is a priori not known. Due to this very general mathematical reasoning the specific radial dependence of the coefficients in the differential equation (2) is not of principal importance.

To illustrate how one can fix the position of an adjustment radius, we shall present the solution of equation (2) for the case where the $\gamma$-parameter is a constant. Let us now introduce the dimensionless variables: angular velocity $\theta = \omega/\omega_0$, radius $r = R/R_0$ ($R_0 = 3R_s$, $R_s = 2GM/c^2$ is the Schwarzschild radius), and mass $m = M/M_\odot$. We normalize the angular frequency by the value $\omega_0 = 2\pi \times 363$ rad $\cdot$ s$^{-1}$. The main reason for this is the remarkable 363-Hz QPO discovered during the type-I X-ray bursts from 4U1728-34 (VK96), and which is believed to be associated with the rotation frequency of a NS.

In terms of these variables the Keplerian angular velocity reads

$$\theta_K = \frac{6}{mr^{3/2}}.$$  \hspace{1cm} (9)

The solution of equation (2) satisfying the boundary conditions (5)-(7) is

$$\theta(r) = D_1 r^{-\gamma} + (C_{NS} - D_1) r^{-2},$$  \hspace{1cm} (10)

where

$$D_1 = \frac{\theta_{out} - C_{NS} r_{out}^{-2}}{r_{out}^{-\gamma} - r_{out}^{-2}} \quad \text{and} \quad \theta_{out} = \theta_K(r_{out}).$$
The value of the dimensionless outer radius, \( r_{\text{out}} \), should be determined from equation (7), which can be now rewritten as

\[
\frac{3}{2} r_{\text{out}}^2 = D_1 \gamma r_{\text{out}}^{-\gamma} - 2(C_{\text{NS}} - D_1)r_{\text{out}}^{-2}.
\]  

(11)

Thus, equations (10) and (11) represent the solution comprising both a BH (\( C_{\text{NS}} = 0 \)) and a NS (\( C_{\text{NS}} = 1 \)) cases.

Editor, Please put Fig.1 here

In Figure 1 we plot a family of solutions of equation (2) for a NS case. The solid line shows a Keplerian solution described by formula (9), the dashed line shows the solution with a sub-Keplerian rotation in a transition region, and the dash-dotted line shows the solution with a super-Keplerian rotation in the upper half of a transition region. The formal explanation for these types of solutions is the fact that the occurrence of kinks in an accretion flow enhances (with respect to the Keplerian value) the absolute value of a radial derivative of the angular velocity at the outer boundary. The perfect adjustment of the Keplerian rotation to the sub-Keplerian inner boundary condition (see equation [10]) is possible only in a rather unrealistic case where at the adjustment point the angular velocity and its radial derivatives coincide with the corresponding Keplerian values. In this case one can uniquely determine the position of an adjustment point. However, if this continuous outer boundary condition (see [6] and [7]) is not fulfilled, then the solution of equation (2) subject to the inner sub-Keplerian boundary condition should necessarily have a regime corresponding to the super-Keplerian rotation.

It is important, that the kinks generally imply the presence of the discontinuities in a radial derivative of an angular velocity of the flow. Thus, when this derivative exceeds the corresponding Keplerian value, the adjustment of a flow to the inner boundary condition most likely occurs through the formation of a region with a locally super-Keplerian rotation. The physical reason for a super-Keplerian rotation in a transition region is a partial outward
transport of the angular momentum (e.g. facilitated by the formation of kinks in a flow) due to viscous stresses. As a result of this redistribution of the angular momentum, the matter, before entering a regime of an essentially sub-Keplerian rotation (in the innermost part of a transition region), gets locally involved in a super-Keplerian rotation.

Let us discuss the reasoning for the occurrence of a super-Keplerian rotation in more detail. It is well-known from the standard accretion theory (see e.g. Shakura & Sunyaev 1973, Lipunov 1992, Popham & Narayan 1992) that, depending on the value of the viscosity (see Popham & Narayan 1992 for the detailed discussion of the viscosity effects), the flow may be either subsonic or supersonic. When the flow reaches a supersonic velocity the density of matter decreases (see Landau & Lifshitz 1988, equation [83,5]), and a cooling regime changes from the free-free cooling onto the Compton cooling (see §2.3, and also Zel’dovich & Shakura 1969). As a result, the temperature of matter increases, the effects of radiative conduction overtake the effects of hydrodynamic viscosity, and a strong isothermal shock develops (Landau & Lifshitz 1988, §95). Note also, that the change in the curvature of an accretion disk caused by an asymmetric illumination of a disk by a NS (Maloney, Begelman & Pringle 1996) favors the formation of kinks. The kinks are characterized by the discontinuities in the coordinate derivatives of the velocity components. In their discussion of kinks in the hydrodynamic flows Landau & Lifshitz (1988, §96) emphasize a difference between the formation of shocks and kinks: the shocks are necessarily formed due to the specific continuous boundary conditions, while the kinks are always a result of singularities in the boundary or initial conditions. Thus, in the presence of kinks the adjustment of a Keplerian flow to the inner boundary condition can occur through a super-Keplerian motion.

The Keplerian rotation can be adjusted to the condition at the inner boundary (see Figure 1) only if the absolute value of a radial derivative of angular velocity at the radius of
kink formation exceeds the corresponding Keplerian value. When a super-Keplerian motion occurs a matter piles up in the vertical direction thus disturbing the hydrostatic equilibrium. The restoring force due to the vertical component of the gravitational force prevents matter from further accumulation in a vertical direction and supports relaxation oscillations. The radiation drag force, which is proportional to the vertical velocity component, determines the characteristic decay time of the vertical oscillations.

In our calculations illustrated in Figure 1 the dimensionless thickness of a region with a super-Keplerian rotation is about 0.2 (= 2.4 km, for a canonical NS of 1.4 $M_\odot$ mass and 10 km radius). The interpretation of the $\sim 27$ $\mu$s time lags between the softer and harder components of a spectrum observed in kHz QPO from 4U 1636-53 (Vaughan et al. 1997) in terms of the ST80 Comptonization model gives a similar estimate for a size of the hard emission region. The development of a zone with a super-Keplerian rotation may significantly affect the dynamics of a transition region: the CB may be set up on the way of a matter accreting onto a compact object. Another effect we shall address in this paper is a possibility of global oscillations of a ring-like configuration (see Figure 2 for an artistic concept of our model) formed by a matter accumulated near the CB. We will discuss these oscillations in Section 2.2 in a very schematic and semi-quantitative way. The detailed modeling of the transition region and analysis of its oscillation properties in the presence of the CB will be presented in a separate publication. We will also consider the effects of rotation on the oscillations in the case of an axisymmetric equilibrium disk structure.

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2.2. QPO frequency of the centrifugal barrier region

Figure 1 shows that in the transition region there is a layer where the centrifugal force exceeds the gravitational force. This means that in this layer matter tends to pile up before it adjusts itself to a new hydrostatically equilibrium structure in the vertical direction. It is very likely that the relaxation to a new equilibrium structure will be accompanied by the nonradial oscillations of a layer. The detailed spectrum of oscillation modes depends on the physical parameters of the unperturbed configuration and is a subject of a special analysis (see e.g. Appendix B for a general introduction). In this paper, we would like to present an order-of-magnitude estimate based on a rather simplified model. Let us consider the small displacement, \( h(R, t) \), of the gas element in a vertical direction at the radius \( R \). We assume that in hydrostatic equilibrium, the vertical gradient of the pressure \( P = P_g + P_{rad} \), where \( P_g = 2\rho kT/m_p \) is the gas pressure, and \( P_{rad} = \sigma_T \rho H_d L/m_p \) is the radiation pressure) is balanced by the vertical component of the gravity force \( F_g \approx \rho \frac{GM}{R^2} H_d (H_d/R) \).

Thus, the equilibrium condition for a small vertical displacement reads (neglecting general-relativistic corrections)

\[
\rho \frac{GMH_d^2}{R^3} = \sigma_T \rho H_d L/m_p + 2\rho kT/m_p, \tag{12}
\]

where \( L(r) \) is the local radiation flux per cm\(^2\). Note that in this equation we have neglected the vertical component of a centrifugal force which is very small compared to the vertical component of a gravitational force.

A small vertical displacement of a gas layer from the equilibrium will result in the restoring force \( f_{gr} \approx -m_p GMH/R^3 \) and the radiation drag force \(|f_r| \lesssim (4/3)\ell(\text{GMH}_d)/R^3(\dot{h}/c)\). Here \( \ell = L/L_{Ed} \) is the luminosity in Eddington units, and the inequality holds for a nonblackbody radiation field (the equality holds for the blackbody radiation field only). Thus, neglecting the density and pressure perturbations, we can write the equation describing the vertical oscillations of a layer (the equation of
motion)

\[ m_p \ddot{h} = f_{gr} - |f_r|, \quad (13) \]

where dot means a time derivative. Now, by making use of equations (12) and (13), we can rewrite the oscillation equation in the form

\[ \ddot{h} + \ell [GMH_d/cR^3] \dot{h} + [GM/R^3]h = 0. \quad (15) \]

For a harmonic perturbing force \( \propto e^{i\Omega t} \) the power spectrum of the oscillations reads

\[ < P(\Omega) > \propto \frac{\gamma_d}{(\Omega^2 - \Omega_0^2)^2 + \gamma_d^2 \Omega^2}, \quad (16) \]

where \( \Omega_0^2 = GM/R^3 \) is the eigen-frequency of the oscillator, and \( \gamma_d = \ell \Omega_0^2 H_d/c \) is the damping rate of oscillations. We can now estimate the oscillator’s Q-value:

\[ Q = \Omega_0/\gamma_d = c/(H_d \ell \Omega_0) \gtrsim 300. \quad (17) \]

The emission temperature is determined by the accretion rate (see §2.3), and therefore the Q-value which scales as

\[ Q = c/(H_d \ell \Omega_0) = \left( \frac{m_p c^2}{kT} \right)^{1/2} \frac{1}{\ell}, \quad (18) \]

should also depend (through the temperature) on the accretion rate. Namely, the Q-value should increase with increasing a centroid frequency of the QPO, since the temperature of an oscillating layer decreases as the accretion rate increases.

Editor, Please put Fig.3 here

In Figure 3 we present the QPO frequency as a function of the \( \gamma \)-parameter. This Figure implies that the QPO frequency should scale with the \( \gamma \)-parameter approximately as \( \gamma^{-0.4} \). Perhaps, this dependence manifests itself in the recent observational data on 4U 0614+091 (Ford et al. 1997), indicating that the main QPO frequency depends on the blackbody flux.
2.3. Temperature of a CB emission region

The total count rate from the source increases with an increase in an accretion rate. This is accompanied by an increase in the observed QPO frequency (if the viscosity is not affected by the radiation, see §2.1). At the same time, the X-ray spectrum becomes softer, because of an increase in a supply of soft photons from the disk illuminating the emission region of the CB.

The CB region can be treated as a potential wall at which the accreting matter releases its gravitational energy. This occurs in an optically thin region where the column density is of order of a few grams, or where the Thomson optical thickness $\tau_0 \sim$ a few. The amount of energy released per second is a fraction of the Eddington luminosity since the CB region is located in the very vicinity of a central object ($\sim 3$-6 Rs). The heating of a gas due to the gravitational energy release should be balanced by the photon emission. For the high gas temperatures, Comptonization is the main cooling channel, and the heating of electrons is due to their Coulomb collisions with protons. Under such physical conditions the energy balance can be written as (see e.g. Zel’dovich & Shakura 1969, hereafter ZS69, equation [1.3])

$$\frac{F}{\tau_0} \sim C_0 \cdot \varphi(\alpha) \varepsilon(\tau) T_e / f(T_e).$$ (19)

Here $\tau$ is the current Thomson optical depth in the emission region (e.g. in a slab), $\alpha$ is the energy spectral index for a power-law component of the Comptonization spectrum, $\varepsilon(\tau)$ is a distribution function for the radiative energy density, $f(T_e) = 1 + 2.5(kT_e/m_e c^2)$, $T_e$ is a plasma temperature in K, $C_0 = 20.2$ cm s$^{-1}$ K$^{-1}$ is a dimensional constant, and, finally, $\varphi(\alpha) = 0.75\alpha(1 + \alpha/3)$ if $\varphi(\alpha) \leq 1$, otherwise $\varphi(\alpha) = 1$. The latter formula is obtained by using the relationship between the zero- and first-order moments (with respect to energy) of the Comptonized radiation field (Sunyaev & Titarchuk 1985, §7.3, equation [30]). The distribution of the radiative energy density in the emission region $\varepsilon(\tau)$ can be obtained from
the solution of the diffusion equation (cf. ZS69, equation [1.4]),
\begin{equation}
\frac{1}{3} \frac{d^2 \varepsilon}{d \tau^2} = -\frac{(F/c)}{\tau_0},
\end{equation}
subject to the two appropriate boundary conditions.

The first boundary condition must imply that there is no scattered radiation from the outer side of the emission region, i.e.
\begin{equation}
\frac{d \varepsilon}{d \tau} - \frac{3}{2} \varepsilon = 0 \quad \text{for } \tau = 0.
\end{equation}

In our case this condition holds at the inner surface of a slab facing a central object. We note that, in reality, one can expect some additional soft flux from a NS resulting in an additional illumination of the inner surface of a slab. In the following analysis we shall neglect this effect.

The second boundary condition requires that at the outer surface of a slab the incoming flux should be equal to the external flux $H$ (not to be confused with unrelated quantity $H_d$ which is a half-thickness of the disk), i.e.
\begin{equation}
\frac{1}{3} \frac{d \varepsilon}{d \tau} = \frac{H}{c} \quad \text{at } \tau = \tau_0.
\end{equation}

The solution of equations (20)-(22) provides us with the distribution function for the energy density
\begin{equation}
\varepsilon(\tau) = \frac{F + H}{c} \{2 + 3\tau_0[\tau/\tau_0 - 0.5(\tau/\tau_0)^2F/(F + H)]\}.
\end{equation}

Thus, from equations (19) and (23) we get
\begin{equation}
\varphi(\alpha)T_e\tau_0/f(T_e) \lesssim 0.75 \cdot 10^9 \frac{F}{F + H} \text{ K}.
\end{equation}

When $H < F$ the spectral index $\alpha$ varies very little since $\alpha$ is a function of $T_e\tau_0/f(T_e)$ (see e.g. Titarchuk & Lyubarskij 1995 for the case $\tau_0 \lesssim 2$). Thus, as long as the external flux (due to the photons from the disk) is much smaller than the internal energy release (per
cm$^2$ per second) in the CB region, the spectral index is insensitive to the accretion rate in
the disk. The values of parameters consistent with equation (24) are: $\tau_0 \lesssim 5$, $T_e \lesssim 2 \times 10^8$
K, and $\alpha \lesssim 1$, which are characteristic of a hard state for the galactic BH and NS systems.

When the external flux $H$ becomes comparable to the internal energy release, $F$, the
cooling becomes more efficient not only due to Comptonization, but also due to the free-free
cooling, and therefore the electron temperature unavoidably decreases, $T_e \ll 10^9 \cdot F/(F+H)$
K (see CT95 for the numerical calculations of spectral indices and temperature).

**Editor, Please put Fig.4 here**

In Figure 4 we present the plot of a temperature in the CB emission region as a
function of the $H/F$ ratio. The latter is proportional to the mass accretion rate in a disk.
We can calculate the temperature using expression (24), then we get the following formula
for the spectral index (Sunyaev & Titarchuk 1980)

$$\alpha = (2.25 + \gamma_c)^{1/2} - 1.5,$$

(25)

where $\gamma_c = \pi^2 m_e c^2 / [3kT_e(\tau_0 + 2/3)^2 f(T_e)]$ (not to be confused with unrelated quantity $\gamma$
which is a Reynolds number of the accreting flow in the disk).

In the end of this paragraph we would like to present a rather elegant and instructive
justification for the existence of a so-called “hot” solution for the case of a large optical
depth ($\tau_0 \gg 1$) of the energy release region. We must recall that a “hot” solution was
discussed by Turolla et al. (1994) and Zane, Turolla & Treves (1997, hereafter ZTT97) in
the context of a formation and structure of a static atmosphere around a non-magnetized
steadily and spherically accreting NS. The statement regarding the existence of a “hot”
solution (Turrola et al. 1994) was quite surprising, and this solution has been even
interpreted as an artifact resulted from a perfect-reflection inner boundary condition
adopted by the authors. Here we shall demonstrate that this solution follows directly from
our equations (19), (23), and (25) and assumption that $H = 0$ at $\tau = \tau_0 \gg 1$ (the absence of an incoming flux from a boundary).

After substitution of (23) into equation (19) and in the limit of $\tau_0 \gg 1$ we get

$$f^2(T_e) \approx 2.5.$$  \hspace{1cm} (26)

This means that $kT_e = 120$ keV which perfectly matches the temperatures presented in ZTT97 (see Figure 2, there). In our derivation of expression (26) we have used the relation $\alpha = \gamma_c/3$ (see equation [25] for $\alpha \ll 1$), and we have calculated the energy density, $\varepsilon(\tau)$ (see equation [23]), for $\tau/\tau_0 \geq 1/2$. It is very interesting that in the limit of $\tau_0 \gg 1$ the derivation of equation (33) is practically independent of an optical depth of the energy release region $\tau_0$. Thus, we come up with exactly the same conclusion as in ZTT97: for a large optical depth of the energy release region $\tau_0$ the radiation and plasma are almost in an equilibrium, so that the plasma remains very hot ("hot" solution) because of the inefficient Compton cooling.

### 2.4. Effect of rotational splitting

In this Section we would like to draw a reader’s attention to the effect of rotational splitting of oscillation frequencies (Titarchuk & Muslimov 1997) that was ignored in all previous theoretical studies of QPOs.

It is well-known (see e.g. Unno et al. 1979 and references therein) that in a rotating star the nonradial oscillations are split: $\sigma_{klm} = \sigma_{kl} + m\Omega C_{kl}$, where $\sigma_{kl}$ is the oscillation frequency in the non-rotating star, $\sigma_{klm}$ is the frequency in the rotating star (in the co-rotating frame), $m$ is the azimuthal number, $\Omega$ is the stellar rotation frequency, and $C_{kl}$ is an integral that depends on the stellar structure and on the eigenfunction. Thus, in a rotating star, a nonradial oscillation drifts at a rate $m\Omega C_{kl}$ relative to a fixed longitude of
a star. The effect of rotational splitting is due to the Coriolis force and is similar to the Zeeman effect in a magnetic field.

The oscillations of a rotating disk must split as well, and the rotational splitting of the disk oscillations is particularly important for those modes whose frequencies are comparable to the frequency of a disk rotation relative to a distant observer. Our estimate (warranted when the relative correction to the eigenfrequency is small, $\sigma^{(1)}/\sigma_0 \ll 1$) of the rotational splitting for a disk (see Appendix A, equation [A11]) results in the following formula (see also Titarchuk & Muslimov 1997)

$$\Omega_{k,m} = \Omega_0 + m \left[ 1 - \frac{2}{1 + s\pi^2 k^2 + m^2} \right] \Omega, \quad (27)$$

where $\Omega_{k,m}$ is the oscillation frequency measured by a distant observer, $\Omega_0$ is the eigenfrequency, $\Omega$ is the local angular frequency of rotation of the oscillating region of a disk (as measured by a distant observer), $s \lesssim 1$ is some function depending on a disk vertical structure (see Appendix A), and $m$ and $k$ are the azimuthal and vertical mode numbers, respectively. For the lowest-order modes with $m = 0, -1, -2$, the oscillation frequencies are

$$\Omega_{k,0} = \Omega_0, \quad (28)$$

$$\Omega_{1,-1} = -(\Omega - \Omega_0) + \frac{2}{1 + s\pi^2 + 1} \Omega, \quad (29)$$

and

$$|\Omega_{1,-2}| = \Omega_0 + 2(\Omega - \Omega_0) - \frac{4}{1 + s\pi^2 + 4} \Omega, \quad (30)$$

respectively.

2.5. Oscillation amplitudes

The observed amplitudes of the flux oscillations can be explained in terms of a variable area of the surface of a region emitting the hard radiation. The latter is produced by
the upscattering of soft radiation from a disk. The amount of soft radiation from a disk intercepted by the emission region, $L_s$, is proportional to its surface area, $2\pi H_d R_{cb}$. The luminosity of hard radiation, $L_h = A(\tau_0, T_e)L_s \propto A(\tau_0, T_e)2\pi H_d R_{cb}$, where $A(\tau_0, T_e)$, the enhancement factor due to Comptonization, is determined by the optical depth, $\tau_0$, of the CB-emission region and by the electron temperature, $T_e$ (see e.g. Titarchuk 1994). Note that the CB oscillations do not affect the enhancement factor, since both the optical depth (or column density) and the temperature remain constant during the oscillations, so that their product $\tau_0 T_e \approx \text{const}$ (see equation [24]). We also point out that the radial and vertical displacements of a ring-like CB configuration should occur at the same frequency. Assuming that the volume of the CB configuration, $V_{cb} = 2\pi H_d R \Delta R$, does not change during the oscillations, it is straightforward to obtain the following relation between the vertical and radial displacements, $\delta H_d$ and $\delta (\Delta R)$, respectively,

$$\delta (R) = \delta (\Delta R) \approx -\frac{\delta H_d}{H_d} \Delta R.$$  
(31)

To obtain an order-of-magnitude estimate of a relative amplitude of the luminosity variation due to the oscillations at the main frequency $\Omega_0$ we can use the formula for a differential of the area of an emitting surface and equation (31), $S_{CB} = 2\pi RH_d$,

$$\delta S_{CB} = 2\pi (R - \Delta R) \delta H_d \approx 2\pi R \delta H_d,$$  
(32)

Then, for a relative change in the luminosity, which is proportional to a change in the area of an emitting surface, we get the following estimate

$$\delta L/L \approx \delta H_d/H_d.$$  
(33)

Thus, even small variations of a height of a cylindrical CB area $\delta H_d \sim 0.1H_d$ can produce oscillations in the hard X-ray flux at the level of $\delta L \sim 0.1L$, which is of order of the observed ones.
We must note that an exact relation between the components of a displacement vector (see Appendix A for the notations) $\xi_r, \xi_\phi$, and $\xi_z$ depends also on the azimuthal and vertical mode numbers. For example, consider an individual oscillation mode with displacement $\xi^{(0)}$ in an incompressible fluid. Then $\nabla \cdot \xi^{(0)} = 0$, and we get a linear partial differential equation that determines a relation between $\hat{\xi}_r, \hat{\xi}_\phi,$ and $\hat{\xi}_z$. Thus, in general, this relation is very complicated and should depend on the azimuthal and vertical mode numbers, $m$ and $k$, respectively.

The mechanism for the QPO emission we discuss in this paper implies that the QPO should have larger amplitudes in the hard tail of the spectrum that is produced by Comptonization in the CB region. This is in good agreement with the observations of the kHz QPO from 4U1636-536 (Zhang et al. 1996, fig. 2) for which the rms amplitude increases almost monotonically with the energy up to at least 20 keV.

3. Theory and observations of kHz QPOs

3.1. Main observational results

In this Section we shall summarize the main observational results on the kHz QPOs that need to be understood.

The launch of RXTE opened up a new era in the study of QPOs. Recently, the kHz QPOs have been discovered in the persistent fluxes of 8 LMXBs: Sco X-1 (van der Klis et al. 1996a), 4U1728-34 (Strohmayer et al. 1996), 4U 1608-52 (van Paradijs et al. 1996, Berger et al. 1996), 4U 1636-53 (Zhang et al. 1996), 4U 0614+091 (Ford et al. 1996), 4U 1735-44 (Wijnands et al. 1996), 4U 1820-30 (Smale et al. 1996), and GX5-1 (van der Klis et al. 1996b). In addition, episodic and nearly coherent oscillations have been discovered during several type-I X-ray bursts from KS 1731-260 with a frequency of 363 Hz (Strohmayer et
al. 1996), during one type-I burst from KS 1731-260 with a frequency of 524 Hz (Morgan & Smith 1996), during 3 bursts from the vicinity of GRO J1744-28 with a frequency of 589 Hz (Strohmayer et al. 1996), and during 4 type-I bursts from 4U 1636-53 with a frequency of 581 Hz (Zhang et al. 1997a). The presence of a pair of the kHz QPOs is characteristic of almost all these observations. The centroid frequency of these QPOs ranges from 400 (4U 0614+091, Ford et al. 1996) to 1171 Hz (4U 1636-53, van der Klis et al. 1996c). According to van der Klis et al. (1997) the observations of Sco X-1 also show the presence of additional two peaks at about 40 and 90 Hz simultaneously with the kHz QPO.

For Sco X-1 (van der Klis et al. 1996c), the difference in the centroid frequencies of two QPO peaks changes with time, whereas for 4U 1728-34 and 4U 0614+091 this difference is constant with time and does not depend on the count rate (Strohmayer et al. 1996 and Ford et al. 1996). In particular, for 4U 1728-34 this difference is always 363 Hz, the same as the frequency of nearly coherent oscillations observed during several bursts. For 4U 0614+091, the difference in centroid frequencies for the two QPO peaks coincides with the centroid frequency of the third QPO peak which was observed during a 1/2-hour period. In general, the centroid frequencies of such QPOs range from 400 (4U 0614+091, Ford et al. 1996) to 1171 Hz (4U 1636-53, van der Klis et al. 1996c).

There are many similarities between the QPOs observed in different sources, and the most important common features are the following:

1. For some sources (Ford et al. 1997) there is a correlation between the QPO centroid frequency and the count rate. However, Zhang et al. 1998 found that the frequency-count rate correlation is more complex in the case of Aql X-1 source. This correlation persists over a short time scale, from minutes to hours, and it apparently breaks dawn on longer time scale in this source (which has also been observed in other sources, e.g. in 4U 1608-52 [Berger et al. 1996]). However, in 4U 0614+091 (Ford et
(al. 1997) the correlation is rather good over a long time scale.

2. Large Q-values, up to $\sim 10^2$. [There is also an indication that the higher frequency QPOs become more coherent as the frequency and the total count rate increase (van der Klis et al. 1997).]

3. The $rms$ amplitudes of QPOs range from the low values (at the threshold of detectability) to a maximum of 12 % in the RXTE/PCA band (2-60 keV). [For every source, where the data are available, the $rms$ amplitudes show strong dependence on energy. For example, for 4U 1636-53 the $rms$ amplitude at 3 keV is only 4%, while at 20 keV it is as high as 16% (Zhang et al. 1996). Furthermore, the variable hard X-ray flux seems to anticorrelate with the soft X-ray count rate and with a value of the highest QPO frequency (Ford et al. 1996).]

4. In Sco X-1, there is a good correlation between the frequencies of the 6 Hz, 40 Hz, 80 Hz and the kHz QPO. Separation of two peaks anticorrelates with the kHz frequency and accretion rate. (see van der Klis et al. 1997).

5. The highest observed QPO frequencies fall in a remarkably narrow range from 1066 to 1171 Hz (Zhang et al. 1997b).

3.2. Interpretation of observational results

We suggest that a set of peaks seen in the kHz QPOs is essentially due to the effect of rotational splitting of the main oscillation frequency (see Section 2.4). This effect should be very important for the accretion disk since the characteristic frequency of the gravitational oscillations of the CB region is of order of the Keplerian frequency.

To estimate the split frequencies, we adopt that $\Omega_0 \approx \Omega$, which is a good
approximation for a nearly Keplerian accretion disk. Then, for the lowest-order modes with \( m = 0, -1 \) and \( k = 1, 2, 3, 4 \) and 5, the oscillation frequencies are:

\[
\Omega_{k,0} \approx \Omega_0, \quad \Omega_{1,-1} \approx 2\Omega_0/(\pi^2s + 2), \quad \Omega_{2,-1} \approx 2\Omega_0/(4\pi^2s + 2), \quad \Omega_{3,-1} \approx 2\Omega_0/(9\pi^2s + 2),
\]

\[
\Omega_{4,-1} \approx 2\Omega_0/(16\pi^2s + 2), \quad \text{and} \quad \Omega_{5,-1} \approx 2\Omega_0/(25\pi^2s + 2),
\]

respectively. For \( m = -2 \) and \( k = 1 \) we have \( |\Omega_{1,-2}| \approx \Omega_0[1 - 4/(\pi^2s + 5)] \).

If we take \( \nu_0 \equiv \Omega_0/2\pi = 1200 \) Hz and assume that \( s \approx 0.7 \), we get the following frequencies (arranged in ascending order): \( \nu_{5,-1} \approx 14, \nu_{4,-1} \approx 20, \nu_{3,-1} \approx 40, \nu_{2,-1} \approx 80, \nu_{1,-1} \approx 270, \) and \( \nu_{1,-2} \approx 800 \) Hz. These frequencies match the observed QPO frequencies in LMXBs. It is important that for a given mode with \( k \geq 2 \) the ratio between frequencies is practically independent of the function \( s \) and is solely determined by the quantum numbers.

As has been pointed out by Hugh Van Horn (1997) the asymmetry between modes with \( m < 0 \) (prograde modes) and modes with \( m > 0 \) (retrograde modes) in our interpretation of QPOs in terms of the rotational splitting may be akin to the result obtained by Carrol & Hansen (1982) that for the nonradial stellar oscillations the slow rotation enhances the stability of retrograde modes and makes prograde modes less stable. Note also, that the contribution of a \((k,m)\)–component to the power spectrum is determined by a smoothness of a function over the corresponding coordinates. It is well-known from the Fourier analysis that the spectral power of the \( l \)-component scales as \( l^{-n} \), where \( n \) is an order of the highest existing derivative of a perturbation with respect to the corresponding coordinate (here \( l = m \) for azimuthal components, and \( l = k \) for \( z \)-components). Thus, it is natural to expect the presence of only lowest-order azimuthal components (\( |m| = 0, 1, 2 \)) in the power spectrum, since it is very likely that the perturbations along the \( \varphi \)–coordinate are rather smooth. On the contrary, the vertical perturbations should be essentially discontinuous, and their Fourier spectra should contain a large number of components with \( k > 1 \) whose contribution to the power spectrum \( \sim k^{-2} \). It is also worth noting that the spectral power of a \((k,m)\)–component is related with the spectral power of a disk at the frequency
corresponding to this particular component. It is demonstrated (Lyubarskij 1996, Kazanas, Hua & Titarchuk 1997) that the density of the power spectrum for a Shakura-Sunyaev disk behaves as $\omega^{-1}$ at the low frequencies. Thus, because of this dependence, we can expect more power in the QPOs at lower frequencies.

Therefore, we suggest that the rotational splitting of the main oscillation frequency of the CB region in the accretion disk may be responsible for the kHz QPOs in LMXBs. The observed three frequencies, ranging from 300 to 1170 Hz, can be naturally interpreted in the following way: the lowest frequency of 200-300 Hz corresponds to a mode with $m = -1$ and $k = 1$; the higher frequency of 800 Hz corresponds to a mode with $m = -2$ and $k = 1$, and the highest frequency is the main frequency of the oscillations.

The main observational consequences of a CB model are:

1. Correlation between the total count rate and the QPO frequency (see Figure 3). The latter increases with an increase in the accretion rate or, equivalently, with an increase in the total count rate (cf. Section 3.1, item 1) provided that the viscosity is determined by the turbulent motions. In the regime where the radiative viscosity dominates the frequency-count rate dependence may be more complex, and the kHz QPOs may even disappear. Zhang et al. (1998) show that in the source Aql X-1 the total X-ray flux (the total accretion rate) does not seem to be the only factor that controls the frequency of the kHz QPOs. We argue, that this frequency-count rate dependence is rather determined by Reynolds number ($\gamma$-parameter). In other words, it is the ratio of the accretion rate to the viscosity that determines the frequency-count rate behaviour for the kHz QPOs.

2. Correlation between the Q-value and the total X-ray flux. When the illumination of the CB region (the accretion rate) increases, the temperature of the CB region should
decrease (see Figure 4 and equations [18], [24]).

3. Observed spectrum is a result of Comptonization of soft radiation from a disk in the CB region (ST80). The kHz oscillations is a manifestation of the mechanical oscillations of the hot CB region in the radial and vertical directions.

4. Partial oscillation amplitudes should be larger at higher energies [exactly this correlation has been observed by Zhang et al. 1996 (see Section 3.1 for details)]. The QPO frequency should also correlate with the total count rate and anticorrelate with the hardness of a spectrum (see Fig. 4, and also Ford et al. 1996).

5. The CB effect should be more pronounced for the higher Reynolds numbers (γ−parameters) for which the transition layer is closer to a NS surface, and for which the angular velocity in a disk Ω and therefore the characteristic frequency of gravitational oscillations of the CB region Ω₀/2π are larger.

6. Spacing between the QPO frequencies and the main oscillation frequency (the highest QPO frequency) would decrease with increasing in the accretion rate (or increase of the main oscillation frequency). In fact the parameter s (see Eq. A8) monotonically increases from 0 to 1 when the CB radius (or the oscillation) increases. The rotational splitting modes with the frequencies, 6 Hz, 40 Hz, 80 Hz, have to correlate with the main frequency 1200 Hz (see Eq. A9).

7. The QPO oscillations are likely to become highly incoherent when the oscillating region plunges into the magnetopause, where the plasma turbulence enhances the viscosity and tends to suppress the oscillations. In this case the QPO frequency should be limited by the Keplerian frequency corresponding to the Alfven radius, \( R_A \approx 1.4 \times 10^6(B/4 \cdot 10^8 \text{ G})^{-0.8}\dot{M}_{18}^{2/5}m^{-1/5} \text{ cm} \), where \( B \) is the magnetic field strength at the NS surface, \( \dot{M}_{18} = \dot{M}/(10^{18} \text{ g s}^{-1}) \) and \( m = M/M_\odot \). Note that the Alfven
radius and hence the highest frequency only slightly depend on the accretion rate. If the masses and values of the magnetic field strength for NSs in bursters are within a very narrow range, then one can expect that the highest frequencies for all these sources will also fall into a rather narrow range.

8. Morgan, Remillard & Greiner (1997) reported the detection of 67 Hz QPO from GRS 1915+105 by RXTE. They found the energy dependence of the 67 Hz QPO in their May 5 1996 observation, where the rms amplitude increased with the photon energy from a level of 1.5% below 5 keV to 6% above 13 keV. This correlation indicates that the 67 Hz QPO is associated with the most energetic spectral components visible in the PCA (Proportional Counter Array on board RXTE). In terms of our model it can be interpreted as the oscillation of the inner edge of the accretion disk (CB region), or as the $g$—mode disk oscillation in the region of radius of $4r_s$ (see for details Appendix B). Such a correlation should be more pronounced in the model accounting of the flux variation in the converging-inflow produced hard tail (Titarchuk, Mastichiadis & Kylafis 1997, Titarchuk & Zannias, 1998), because of substantial variation in the exposure of the converging inflow region (the characteristic radius of which is $2r_s$) to the soft seed photons from the disk.

The similar energy dependence is observed for 300 Hz QPO detected by RXTE (Remillard et al. 1996) from another superluminal source, GROJ1655-40. And thus we can argue that the QPOs observed from BH system detected only in the high energy bands reveal the scale of the high energy emission region and hence the nature of the high energy formation in the soft states of BH systems in terms of the converging inflow Comptonization.

4. Alternative models
4.1. Acoustic waves

The acoustic waves standing in the hot area surrounding a NS is an alternative possibility for the interpretation of the kHz QPOs. The standing shocks above a NS surface can serve as boundaries for such waves. The existing controversy about the stability of such shocks will be probably resolved by 2-D simulations (Chen et al. 1997) indicating that after an initial transition phase of rapid shock movements a system relaxes to a stable configuration comprising the quasi-stable, hot torus around a NS. These simulations show that after passing a shock, the accreting gas becomes involved in the complex, predominantly tangential vortex motions with a very small radial Mach number in the postshock region. The resulting picture is quite distinct from a simple, quasi-virial spherical accretion: it allows for the existence of a hot coronal postshock region around a NS perfectly suitable for a sound wave propagation. The acoustic oscillations of such a region can manifest themselves as the kHz QPOs. Note that the NS systems seem to be more favorable, since the standing acoustic waves may oscillate between two surfaces, the stellar surface and the outer boundary. Moreover, these surfaces can reflect, absorb and even emit sound waves.

The effects of an interaction of the shock surfaces with the sound waves are extensively discussed by Landau & Lifshitz (1989). In particular, the strong shocks are subject to corrugation instability when a shock surface becomes rippled. In the range of angles between the wave vector and a normal to the shock surface the reflection coefficient for an incident sound wave may exceed unity or even diverge. This means that the sound waves can be not only reflected but can also be spontaneously emitted by a shock (into the postshock region), so that a sound wave can receive energy from a shock. The damping of sound waves by a shock can also occur under certain circumstances. For example, for a sound wave falling normally from a postshock side onto a flat shock surface, the reflection coefficient is, in
general, < 1. Note that almost 100% reflection takes place when a gas is isothermal near the strong shock, and this may well be relevant to the physical situation under consideration. Due to Comptonization of the ionized gas the outer layers of a postshock region adjoining the shock are always isothermal. It is well-known that a similar situation occurs in the outer atmospheric layers of X-ray bursters, and in many other situations where the electrons in the outer, optically thin layers of gas clouds are heated by Comptonization and kept at a constant temperature equal to the spectral temperature of radiation emerging from a cloud. The ratio of specific heats is equal to unity and the reflection coefficient for a sound wave is almost equal to unity as well.

However, it is unlikely that all sound waves to be perfectly reflected by a boundary of a hot region, regardless of its physical nature. The accretion disk, for example, may contribute to the wave damping, while the asymmetry may, as is mentioned above, result in a spontaneous emission and, therefore, in an enhancement of a sound wave. It is important that the standing acoustic waves in a postshock region is essentially a resonance response of a dynamical system (the lobe) to the background noise, which is always present in a postshock region and contains all possible frequencies. The balance between a continuous excitation of resonant modes and dissipation of sound waves determines their finite amplitudes.

The waves under discussion are usual sound waves propagating in a fully ionized gas. The protons and electrons are coupled together by strong electrostatic forces, and the force driving the oscillations is a gradient of thermal pressure in an electron-proton mixture. In the situation characteristic of the inner boundary of an accretion disk the proton temperature, $T_p \sim \text{few keV}$ is of order of the electron temperature, $T_e$. When entering a postshock region the protons instantly get much higher temperature, of order of the virial one, $T_v \sim \text{few MeV}$. However, because of frequent collisions with much cooler electrons the protons
lose their energy very fast, over a timescale $t_{\text{p,cool}} \approx 5.8 T_e^{3/2} (M_p/m_e)^{1/2} (n_e \ln \Lambda) \text{s}$ (Spitzer 1956). Given $T_e \sim 10 \text{ keV}$, $n_e \sim 10^{17} \text{ cm}^{-3}$, and $\ln \Lambda \approx 13$, we get $t_{\text{p,cool}} \approx 2.5 \times 10^{-4} \text{s}$. The sound speed is therefore determined mostly by protons $c_s = (\partial p/\partial \rho)^{1/2} \approx 0.98 \times 10^8 T_5^{1/2} \text{ cm s}^{-1}$. Since protons and electrons are coupled by electrostatic forces, the frequent Coulomb collisions of protons determine the damping of sound oscillations, and we can use the proton mean-free path, $l_p$, in the standard expression for the wave damping in a fluid. The decay coefficient is $\gamma_d = |\dot{E}_m|/E$, where $\dot{E}_m$ is an average rate of acoustic energy dissipation, and $E$ is a total acoustic wave energy. Following Landau & Lifshitz (1989), we can write for the decay coefficient $\gamma_d \approx (2\pi \nu)^2 (l_p/c_s)$, where $\nu$ is a sound frequency.

Since the damping of sound oscillations in a hot plasma is mostly due to Coulomb collisions, the value $l_p$ can be estimated as $l_p = (H/\tau_0)(\sigma_T/\sigma_C)$, where $H$ and $\tau_0$ are the characteristic length-scale and the Thomson optical thickness of the plasma, respectively. The ratio of Coulomb to Thomson cross-sections is $\sigma_C/\sigma_T \approx 2.35 \cdot 10^5 T_5^{-2}$, where $T_5 = T/5 \text{ keV}$. The characteristic length-scale of an atmosphere, $H$, can be expressed in terms of the observed sound wave frequency as

$$H = (\pi/2)(c_s/2\pi \nu) = 2.4 \cdot T_5^{1/2} \nu_{\text{kHz}}^{-1} \cdot 10^4 \text{ cm}, \quad (34)$$

where $\nu_{\text{kHz}} = (\nu/1 \text{ kHz})$. Assuming that $H=\text{const}$, we get $\nu \propto T^{1/2}$. The proton mean free path can be estimated (by using the above relations between $l_p$ and $H$) as $l_p = 0.1 \cdot T_5^{2.5} \nu_{\text{kHz}}^{-1} \tau_0^{-1} \text{ cm}$. Thus, one can find that $\gamma_d \sim 4 \cdot 10^{-2} \nu_{\text{kHz}} T_5^{2} \tau_0^{-1} \text{ s}^{-1}$, and the characteristic decay time $t_\ast = 1/\gamma_d = 25 \tau_0 \nu_{\text{kHz}}^{-1} T_5^{-2} \text{ s}$. This means that at high temperatures the oscillations should decay very fast. Note also that the oscillation frequency may vary as a result of a changing of the boundary condition at the shock. For example, in the case of a free boundary the oscillation frequency is a factor of 2 higher than that for a fixed boundary (the estimated size of a shock is a factor of 2 larger than the characteristic length scale given by [34]). The radiation pressure and/or viscosity can also play an important role.
in damping of sound waves, despite the fact that the radiation pressure is small compared to a gas pressure at luminosities well below the Eddington limit (see §2.2).

In the low (hard) state of a NS system the temperature of a postshock region is higher, ∼ few \times 10 \text{ keV}, and the acoustic waves die off rapidly. They can, therefore, exist only in the high (soft) state where T_e ∼ few keV, and the damping is small.

The plasma oscillations usually considered in the laboratory plasmas do not propagate under the conditions we discuss here for a very simple reason. The effects of collisionless plasma like plasma waves, Landau damping of ion-acoustic waves, etc. do not apply to our situation since the plasma we are dealing with is essentially collisional. Given a short proton mean-free path, ∼ 0.1 cm, and high velocity, v_p ∼ 7 \times 10^7, \text{ cm s}^{-1}, the proton collision frequency is \nu_{\text{col}} ∼ 7 \times 10^8 \text{ Hz} that exceeds a frequency of a kHz wave.

As far as the observed amplitude of flux oscillations is concerned, it can be interpreted similarly to a CB model (Section 2), in terms of a variable area of an emitting surface.

4.2. Rotation of a NS and oscillations of a disk

Let us briefly address the issue of whether a NS itself may be a source of e.g. 363 Hz nearly coherent oscillations (S96). In general, in an oscillating system, the amplitudes of any two modes A_1e^{i\omega_1t} and A_2e^{i\omega_2t}, may be either added to or multiplied by each other. When the excitation mechanism for the modes is the same, the addition rule applies, and Fourier spectral analysis will reveal two independent peaks in the power density spectrum (PDS), at \omega_1 and \omega_2. This is the case, for instance, for a set of overtones of the same physical origin differing from each other by the mode quantum numbers only. When two oscillatory motions are of different origin, then we should multiply their amplitudes. The “beating” motion belongs to this category, including the “beat-frequency” model for the
low-frequency QPOs in LMXBs (see Lewin, Van Paradijs, & Taam 1993 for a review). In this case both \((\omega_1 + \omega_2)\) and \(|\omega_1 - \omega_2|\) peaks should be present in the PDS of a system unless \(\omega_1 = \omega_2\).

In LMXBs a NS is thought to be weakly magnetized and the lack of pulsations even in quite bright LMXBs apparently supports this idea. It is quite reasonable to assume that the nearly coherent \(\sim 363\) Hz oscillations are associated with the stellar rotation: a relatively high temporal stability of a frequency and its independence of the spectrum and flux are the main arguments in favor of such a possibility. The 363 Hz oscillations occurred only during type-I X-ray bursts from 4U1728-34 (other 3 LMXBs did not show any type-I bursts during the observations). The explanation this QPO in terms of a NS rotation encounters a difficulty that the emitting region should be seen only during a very short time (during bursts), which is, in principle, possible e.g. in a model of inhomogeneous combustion on a NS surface (see Bildsten 1995). The accretion disk trapped \(g\)-modes is another possible mechanism that may be responsible for the stable \(\omega_3\)-oscillations (see Appendix B). Similar to a NS rotation, these oscillations, when superimposed on the intrinsic oscillations of a hot region around a NS may produce multiple QPO peaks. For the type-I bursts with a relatively short recurrence time (high values of \(\dot{M}\)) the surface layers of a NS, remain convectively stable, and a combustion front propagates over the stellar surface very slowly, at \(v_{\text{comb}} \sim 300\) cm s\(^{-1}\) (Bildsten 1995). This means that a combustion inhomogeneity (e.g. associated with a more active burning in one zone where it started first) can persist for a long time, at least \(\sim 10^4\) s, provided that there is enough fuel. Thus, on a timescale of \(\sim 15\) s (typical duration of a type-I burst) a combustion inhomogeneity can manifest itself as a rather stable pattern of energy release on the stellar surface, which seems to agree with the 4U1728-34 data. The narrow PDS peak implies a high Q-value, again in agreement with the observations.
The period evolution of nearly coherent 363 Hz oscillations in 4U1728-34 during a burst can be understood in terms of the hydrodynamics of a "hot spot" produced by an accreted gas on the stellar surface (in a boundary layer). For example, as it has been suggested by Fujimoto (1988), the spreading of the accreted matter over the surface of a weakly magnetized NS accreting from a disk should be accompanied by a differential rotation in the very surface layers. This (radial) differential rotation gives rise to the hydrodynamic instabilities which will redistribute (through the development of a turbulence) the angular momentum in the surface layers of a NS. For the physical conditions characteristic of the surface layers of an accreting rapidly rotating NS the baroclinic instability (see Pedlosky 1979, Knobloch & Spruit 1982, Zahn 1983, and Fujimoto 1987 for review) is perhaps the most important one. This instability is related to the fact that in a differentially rotating star in hydrostatic equilibrium the surfaces of constant density in general do not coincide with the surfaces of constant pressure. The growth rate for this instability can be estimated as (see e.g. Fujimoto 1988) $\tau_{bc}^{-1} \lesssim 0.6\Omega_0 \cos \theta_0$, where $\Omega_0$ is the local angular velocity at the point with a colatitude $\theta_0 (< \pi/2)$. This estimate shows that the instability may develop as shortly as during 1-10 stellar revolutions. The development of the hydrodynamic instabilities means the occurrence of the largescale plasma flows and mixing of a matter in the surface layers of a NS that may eventually destroy a coherency of e.g. 363 Hz oscillations.

Two other mechanisms have been proposed in the literature to explain the kHz QPOs. Recently, Klein et al. (1996) have proposed that these QPOs result from a turbulence occurring in the settling mounds of the neutron star polar caps. Their numerical simulations produce QPOs in the kHz range with fractional $rms$ amplitudes of order of 1 %. Several QPO frequencies appear to be present in their simulations, with the highest ones are above 2 kHz. Miller, Lamb and Psaltis (1997) have proposed a model in which the QPO with higher frequency is the Keplerian frequency at the sonic point beyond which the radial
inflow velocity becomes supersonic. The position of the sonic point is determined by the radiation forces which remove the angular momentum from the accreting matter. In this model the accretion flow is also modulated by the radiation flux from a NS with its spin frequency. This accretion-flow modulation causes the Keplerian frequency to beat with a NS spin frequency to produce QPO with a lower frequency. In this model, there is an upper limit for the QPO frequencies, either the Keplerian frequency at the stellar surface, or the frequency of the marginally stable orbit.

We must admit that the variety of observational phenomena, such as e.g., the twin QPO peaks with the constant and (in some cases) variable difference between them, as well as appearance of a separate peak at the frequency corresponding to this difference need to be understood. We believe, the models of CB oscillations we attempted to address in this paper, may provide a reasonable and consistent interpretation for the most experimental facts related to the phenomena of kHz QPOs, both for the NS and BH binary systems.

It is quite possible that $\sim 200 - 400$ Hz PDS peaks manifest either the proper spin frequency of a NS, or $g$-mode oscillations in an accretion disk. The $g$-modes can be observed at frequencies of $\sim 200 - 500$ and $\sim (60 - 300) \times (5/m)$ Hz for the NS and BH systems, respectively. The disadvantage of $g$-modes from the observational point of view is that they should be localized in a rather small region of a disk with $\Delta R \sim GM/c^2$.

The crucial test of potential observability of $g$-modes in a disk around a relativistic compact object may be provided by the millisecond timing of BH transients in the high (soft) state where the luminosity is dominated by the thermal radiation from the accretion disk. Such $g$-modes should have frequencies $\sim 200 \cdot (5/m)$ Hz. The QPOs with the frequencies $\sim 67$ and $\sim 300$ Hz were recently detected by RXTE from GRS 1915+105 (Morgan et al. 1997) and from GROJ1655-40 (Remillard et al. 1996), respectively, during the very high states of these sources. These observations may suggest the occurrence
of $g$-mode disk oscillations, at least in these systems (see, also discussion in §3.2). The $g$-modes can be observationally distinguished from the CB oscillations due to the fact that the $g$-mode eigenfunctions are symmetric with respect to some radius in the disk (Nowak & Wagoner 1992), and thus the effect of rotational splitting will be smeared out for these modes (see Eq. A1), which is certainly not the case for the CB oscillations.

5. Conclusion

We propose a model that can self-consistently explain most of the observational facts on kHz QPOs. The main physical ingredients of our model are:

1. The possibility of a super-Keplerian rotation (centrifugal barrier, CB) in a boundary region between a Keplerian accretion disk and a compact object. The formation of kinks, weak discontinuities in a supersonic accretion flow, makes an adjustment of the disk rotation to the innermost boundary (NS either, magnetosphere surface or the last stable orbit around a BH) to occur through the dynamical episodes of a locally super-Keplerian rotation.

2. This dynamical adjustment is dictated by the turbulence of flow, Reynolds number, i.e. the ratio of the mass accretion and viscosity dictates the regime of the QPO behaviour.

3. The excitation of relaxation nonradial oscillations in the CB region.

4. The effect of rotational splitting of the main oscillation frequency (of order of the local Keplerian frequency) of the CB region.

We have demonstrated that the CB effect can explain various correlations and anticorrelations found in the observations of kHz QPOs (see §3.2 for details). Also, we
have shown that the splitting of the main kHz frequency due to a disk rotation produces a discrete spectrum of frequencies that perfectly match the observed QPO frequencies (see §§2.4 and 3.2 for details). In our model the QPO frequencies are inversely proportional to the mass of a compact object. We have therefore suggested that a similar phenomenon should be observed in the BH systems too, with the QPO frequencies should be a factor of 5-10 lower than in the case of the NS systems. In addition, in our model a typical size of the emission region is estimated to be $\sim 1 - 3$ km, in a good agreement with the time-lag measurements. We have briefly discussed some alternative models that may be relevant to the physics of QPOs including the standing acoustic waves in the postshock region around a NS, the effect of frequency modulation due to the proper rotation of a NS, and $g$-mode intrinsic oscillations of an accretion disk.

The authors thank NASA for support under grants NAS-5-32484 and RXTE Guest Observing Program. I.L. thanks the Isaac Newton Trust (Cambridge) and PPARC for support. The authors acknowledge discussions with Hugh Van Horn, Martin Rees, Guy Miller, Tod Strohmayer, Jean Swank, Nick White, Will Zhang. Particularly, we are grateful the anonymous referee whose constructive comments and criticism significantly improved this paper.

**APPENDIX A. ROTATIONAL SPLITTING EFFECT**

In this Section we estimate the effect of a disk rotation on the QPO frequency. The detailed theory for a slowly rotating star is given by Ledoux & Walraven (1958) and Unno et al. (1979). Here we calculate the splitting of the eigenfrequency due to Coriolis force in the case of a disk geometry.

It must be pointed out that in our analysis the correction to the main frequency
The main oscillation frequency \( \sigma_0 \) is comparable to the rotation frequency. Thus, to derive this correction, we are still allowed to exploit the linear approximation and use the unperturbed values for the displacement vectors. [The fundamental difficulty in the problem of nonradial oscillations in the presence of rotation is that the hydrodynamic equations are not t-invariant, as the Coriolis term is replaced by the corresponding term with the minus sign by time-inversion. This means that the eigenvalue problem differs from its complex conjugate, or, that the displacements \( \xi \) and \( \xi^* \) are not eigenvectors of the same eigenvalue. That is why the effect of Coriolis force on the nonradial oscillations can be taken into account only for the case of a slow rotation for which \( \xi \) and \( \xi^* \) can be approximated by the corresponding vectors determined from the eigenvalue problem without rotation, \( \xi^{(0)} \) and \( \xi^{(0)*} \), respectively.]

For the axisymmetric case (cf. Unno et al. 1979, equation [18.27]) the correction to the eigenfrequency reads

\[
\sigma^{(1)} = -i \int_{D_{cb}} [\Omega \times \xi_{k,m}^{(0)}] \xi_{k,m}^{(0)*} dD_{cb},
\]

where \( \xi_{k,m}^{(0)}(t, r, \varphi, z) \) is the (k, m)-displacement component. For a disk- (ring-) like geometry the displacement component should be calculated by using the (k, m)-harmonics of a complete set of eigenfunctions for a disk- (ring-) like configuration, \( \{u_{k,m}\} \). In equation (A1) the integration has to be taken over the disk area \( D_{cb} \) of radius \( R \), width \( \Delta R \), and half-thickness \( H_d \). The appropriate displacement component \( \xi_{k,m}^{(0)} \) can be written in cylindrical coordinates as (see e.g. Unno et al. 1979, equation [18.28])

\[
\xi_{k,m}^{(0)} = \left[ \xi_r, \xi_\varphi \frac{\partial}{\partial \varphi}, \xi_z \frac{\partial}{\partial z} \right] u_{km},
\]

where the functions \( u_{k,m} \) satisfy the free-boundary conditions at \( z = H_d \) and \( z = -H_d \) (see e.g. Morse & Feshbach 1953), and the symmetry (with respect to the disk plane) condition.
These conditions are
\[ \frac{\partial u_{k,m}(t, \varphi, H_d)}{\partial z} = \frac{\partial u_{k,m}(t, \varphi, -H_d)}{\partial z} = 0, \]  \hspace{1cm} (A3) 
and
\[ u_{k,m}(t, \varphi, z) = u_{k,m}(t, \varphi, -z), \]  \hspace{1cm} (A4) 
respectively, where \( 0 \leq z \leq H_d \). We must note that the symmetry condition is not warranted in the case of a disk tilt above the plane \( z = 0 \) on one side and below the plane on the other (Van Horn 1997).

In the case of uniform rotation, the temporal and azimuthal dependences of eigenfunctions \( u_{k,m} \) are taken as \( e^{i(m\varphi - \Omega t)} \) (Unno et al. 1979). Thus, the eigenfunction component \( u_{k,m} \) can be written as
\[ u_{k,m} = e^{im\varphi} \cos(\pi k z / H_d) e^{-i\Omega t}, \]  \hspace{1cm} (A5) 
where \( m = \pm 1, \pm 2, \ldots, k = 1, 2, \ldots \) 

The integral \( I_1 \) in the numerator of equation (A1) is given by
\[ I_1 = -2\pi H_d R \Delta R [2\Omega m \hat{\xi}_r \hat{\xi}_\varphi], \]  \hspace{1cm} (A6) 
where \( \hat{\xi}_r \) and \( \hat{\xi}_\varphi \) are the average displacements over \( r \)- and \( \varphi \)-coordinates, respectively, for a disk-(ring-) like configuration. The integral \( I_2 \) in the dominator of equation (A1) is
\[ I_2 = 2\pi H_d R \Delta R \left[ \hat{\xi}_r^2 + (\hat{\xi}_z R / H_d)^2 \pi^2 k^2 + m^2 \hat{\xi}_\varphi^2 \right], \]  \hspace{1cm} (A7) 
where \( \hat{\xi}_z \) is the average displacement over \( z \)-coordinate. Here \( \hat{\xi}_r, \hat{\xi}_\varphi, \) and \( \hat{\xi}_z \) are the average displacements for a disk configuration in radial, azimuthal, and vertical directions, respectively. To illustrate the effect of rotational splitting we may justifiably assume that \( \hat{\xi}_r \sim \hat{\xi}_\varphi \), then we arrive at
\[ \sigma^{(1)} = -\frac{2m\Omega}{1 + s\pi^2 k^2 + m^2}, \]  \hspace{1cm} (A8)
where \( s(R/H_d) = (\hat{\xi}_z R/\hat{\xi}_r H_d)^2 \) is a function determined by the vertical structure of a disk. For more or less realistic structure of a disk \( s \gtrsim 1 \) and depends on the ratio \( R/H_d \), which is either \( \gtrsim 1 \) or \( \gg 1 \).

Thus, the frequency of the oscillations as seen by a distant observer is given by (cf. Unno et al. 1979, equation [18.33])

\[
\Omega_{k,m} = \Omega_0 + m \left[ 1 - \frac{2}{1 + s\pi^2k^2 + m^2} \right] \Omega. \tag{A9}
\]

In conclusion, we must note that this formula can also be obtained as follows. If the relative correction to the main frequency produced by the Coriolis term is small, then in the equation of motion for the perturbation we can use the displacement vector corresponding to the eigenvalue problem without rotation \( \xi^{(0)} \). For a given oscillation mode the right-hand side of equation of motion (containing the gradient of a perturbed gravitational potential and the gradient of a perturbed gas pressure divided by density) can be replaced by the \(-\sigma_0^2 \xi^{(0)}\). Then, by multiplying both sides of the equation of motion by \( \xi^{(0)*} \) and integrating over the volume, we get the following quadratic equation

\[ \sigma'^2 - 2\sigma'\sigma^{(1)} - \sigma_0^2 = 0, \]

where \( \sigma' \) is the oscillation frequency in the corotating frame, and \( \sigma^{(1)} \) is the correction to the main frequency due to the Coriolis term. The solution of this equation is \( \sigma' = \sigma^{(1)} + \sigma_0[1 + (\sigma^{(1)}/\sigma_0)^2]^{1/2} \approx \sigma_0 + \sigma^{(1)} \) (if \( \sigma^{(1)}/\sigma_0 \ll 1 \)), which leads to the formula identical to (A9). Here we have ignored a solution with the minus sign, because it gives very high frequencies.

**APPENDIX B. g-MODE DISK OSCILLATIONS**

Here we calculate the characteristic oscillation frequencies for the trapped g-modes in an accretion disk exploiting the results obtained by Nowak & Wagoner 1991, 1992,
and 1993 (hereafter NW1, NW2, and NW3, respectively). The two types of oscillations that are intrinsic to the accretion disks around the compact relativistic objects and are of potential interest for our present study are \( p \)- and \( g \)-modes. The \( p \)-modes are longitudinal oscillations propagating along the disk and having displacements in the radial direction (in a disk plane). It is very unlikely that these oscillations can be observable. The \( g \)-modes are mostly transverse oscillations, and the gravity is the main restoring force for them. Their wave vector is always in the plane of a disk, but the displacements are mostly in the perpendicular, \( z \)-direction.

The essential feature of both \( p \)- and \( g \)-modes is that they are trapped in a rather narrow range of radii. The possibility of the very existence of wave motions is essentially due to the characteristic feature of the Keplerian law, the stability of orbits. In the Newtonian gravity the orbits are stable at any orbital distance. The static general-relativistic effects e.g. in the case of Schwarzschild metrics result in that the stable orbits can exist only at \( R \geq 6GM/c^2 \). For the modified Newtonian potential,

\[
\Phi = \frac{GM}{R} \left[ 1 - \frac{3GM}{Rc^2} + 12 \left( \frac{GM}{Rc^2} \right)^2 \right],
\]

the so-called epicyclic frequency (see NW1) \( \omega_e = (c^3/GM)(r_m^{-3} - 36r_m^{-5})^{1/2} \) vanishes at \( r_m = 6 \), thus reflecting the lack of stable orbits at \( r_m < 6 \). Here \( r_m \) is the radial cylindrical coordinate in units of \( GM/c^2 \), and \( M \) is the mass of a central object.

The epicyclic frequency \( \omega_e \) has a maximum at \( r_{\text{max}} = \sqrt{60} \). The \( g \)-modes are trapped near this maximum vanishing near \( r_m \simeq 6 \) and at \( r_m > r_{\text{max}} \). The characteristic width of a trapping region is therefore \( \Delta r_m \sim 1 \), and the \( g \)-modes die off exponentially outside this region over characteristic length scale \( \sim \) a disk thickness, \( h \). The trapping at large radii is due to the pressure acting against the gravitational restoring force. The elasticity coefficient in the restoring force is \( \sim 1/R^3 \), while the pressure is a much weaker function of \( R \). As a result, the effective restoring force vanishes at \( r_m \simeq r_{\text{max}} + 1 \simeq 9 \).
The equation governing the radial displacement $\delta u$ as a function of $r$ (NW2, equation [2.10]),

$$\omega^2 c_s^2 (\delta u)_{rr} = -(\omega^2 - \gamma Y \Omega^2)(\omega^2 - \omega_{epic}^2)\delta u,$$

(B2)
coupled to the corresponding equation for the oscillations in z-direction (through a slowly varying with $r$ function $Y[r]$), fully describes the main propagation properties of waves, including a characteristic radial extent of the trapping region, and effective damping of waves out of this region.

The dispersion relation for the waves describing both $p$- and $g$-modes is

$$\omega^2 c_s^2 k_r^2 = (\omega^2 - \gamma Y \Omega^2)(\omega^2 - \omega_e^2),$$

(B3)
where $\Omega$ is a Keplerian frequency, and $c_s$ is the sound speed. The values $k_r$, $Y$ and $\Omega$ in the dispersion relation are the values for the corresponding functions at $r_m \simeq r_{\text{max}} \simeq 8$ (NW2); $\omega_e(r_{\text{max}}) \simeq 0.029c^2/(GM)$, and $Y$ practically does not depend on $r$ and varies from $\sim 1$ to $\simeq 12$ for the first five eigenmodes, depending on a mode number.

The $p$-modes correspond to $\omega^2 > \gamma Y \Omega^2$, $\omega^2 > \omega_e^2$. As it has been pointed out in NW3, the $p$-modes are trapped in a very narrow region, $\Delta r_m \sim 0.1GM/c^2$, and, since they have displacements in the disk plane, these modes are unlikely to be observed. On the contrary, the $g$-modes have frequencies below $\omega_e$ and satisfy the conditions $\omega^2 < \omega_e^2$ and $\omega^2 < \gamma Y \Omega^2$. Although the eigenvalue $\omega$ can be obtained from the exact solution of equation (B2) supplemented with a corresponding equation for the z-component alone, we may reliably estimate it from the dispersion relation (B3). For this purpose, let us rewrite (B3) in the form

$$\omega^2 = \omega_e^2 + c_s^2 k_r^2 \cdot \frac{1}{1 - \gamma Y \Omega^2/\omega^2}.$$ 

(B4)
The frequencies $\omega^2 < \omega_e^2$ are allowed, and we can look for the solution of (B4) in the form $\omega = \varepsilon \cdot \omega_e$, where $\varepsilon \lesssim 1$. The hydrostatic equilibrium in the vertical direction implies that the
sound speed, $c_s^2 = \gamma P/\varrho = \gamma h^2 \Omega^2$ (SS73), satisfies the relation $c_s^2 k^2 = \gamma h^2 \Omega^2 (2\pi/\lambda)^2$, where $\lambda$ is the radial wavelength approximately equal to the size ($\simeq GM/c^2$) of a region with a mode trapping. Note that $\gamma \Upsilon [\Omega(r_{\text{max}})/\omega_e(r_{\text{max}})]^2/\varepsilon^2 \simeq (5/3)(1/12) \cdot 2.25 \cdot \varepsilon^{-2} \gg 1$, and equation (B4) yields

$$\varepsilon^2 \simeq \left[1 + \Upsilon^{-1}(2\pi h/\lambda)^2\right]^{-1}. \quad (B5)$$

For a reasonable assumption that $h \sim \lambda(<1)$, and given a typical value $\Upsilon \simeq 6$, we obtain $\varepsilon \simeq 0.4$. Thus, the value $\simeq \omega_e/2$ is a typical value for a circular frequency for one of the lowest-order $g$-modes. Given $f_e = (\omega_e/2\pi)(1 - 2/r_m)^{1/2} = 795/\mathrm{m Hz}$ (here $m = M/M_\odot$), these $g$-modes should have frequencies $f = \omega/2\pi \simeq 200 - 500$ Hz. These values perfectly fall in the range of frequencies between $\sim 250$ and $\sim 360$ Hz, observed from Sco X-1 and 4U1728-34, respectively.

**Stability and damping of $g$-mode disk oscillations**

The Q-value for the $g$-mode oscillations is $Q = f/\Delta f$, where $\Delta f$ is determined by damping. The damping of $g$-modes is due to turbulent motions. The upward energy flux along the z-axis in $g$-modes due to turbulent motions can be estimated as (Stein 1967) $F^z_{\text{grav}} \approx (20 \div 200)(l_t/H_d)^5 \varrho u_0^3/l_t$. A numerical factor in this formula depends on the assumed spectrum of turbulence. The energy density in a Keplerian disk is $E = \varrho v_\varphi^2/2 = (\varrho v_s^2/2)(R/H_d)^2$, and the damping constant can be estimated as

$$\gamma_d = \frac{F^z_{\text{grav}}}{E} = (20 \div 200) \left(\frac{l_t}{H_d}\right)^5 \frac{2\alpha^3 v_s}{l_t} \left(\frac{H_d}{R}\right)^2. \quad (B6)$$

In derivation of this formula we have used the same relation between the characteristic turbulent velocity, $u_0$, and the sound speed, $v_s$, as for the $\alpha$-disks (SS73): $u_0 = \alpha v_s$.

Also, we assumed that the length scale of turbulent eddies $l_t < H_d$. Taking into account that $v_s/H_d = v_\varphi/R = \Omega$ and $\Omega(r_m) = 9 \cdot 10^3 \cdot \mathrm{m s}^{-1}$, we get $\gamma_d = 4.3 \cdot 10^{-3}\psi \mathrm{ s}^{-1}$,
where \( \psi = m^{-1}(\alpha_2)^3[(k/H_d)_{1/3}]^4(T_1)^{1/2}(H_d/R)^2 \), and parameters \( \alpha_2 \), \( k/H_d \), and \( T_1 \) are normalized by \( 10^{-2} \), \( 1/3 \), and \( 1 \) keV, respectively. The resulting estimate is \( Q \approx 500 \text{ Hz}/\gamma_d \approx 10^5 \cdot \psi^{-1} \). The “experimental” Q-value for 4U1728-34 can be estimated from the fact that the oscillations persist for a whole duration of the burst, 15 s, which gives \( Q_{\text{exp}} \geq 360 \text{ Hz}/(1/15) = 5.4 \cdot 10^3 \).

**Luminosity variation produced by a disk \( g \)-mode oscillations**

To estimate the \( g \)-mode oscillation amplitude we can use the temperature distribution in a disk calculated by SS73. The relative contribution to the total luminosity, \( L_{\text{tot}} \), from the annulus in a disk confined by the radii from \( r_m = 7.5 \) to \( 8.5 \), where the \( g \)-modes oscillations are trapped, can be estimated as \( \Delta L/L_{\text{tot}} \approx 0.04 \). For a disk isentropic in \( z \)-direction, as discussed in NW1-3, the formula \( PV^\gamma = \text{const} \) \( (\gamma = 5/3) \) gives a relation between the temperature and thickness of a disk: \( T h^{\gamma-1} = \text{const} \). Thus, \( |\delta T/T| = (2/3)\delta h/h \). The flux from the radii between 7.5 and 8.5 will oscillate with the amplitude \( \delta(\Delta L)/\Delta L = 4\delta T/T = (8/3)\delta H_d/H_d \), and we can write

\[
\frac{\delta(\Delta L)}{L_{\text{tot}}} \approx 0.04 \frac{8}{3} \frac{\delta H_d}{H_d} \approx 0.05 \left( \frac{\delta H_d/H_d}{1/2} \right),
\]

which means that \( \sim \text{few \%} \) is an upper limit for the flux oscillations due to \( g \)-modes in a disk.
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Fig. 1.— Dimensionless angular velocity, $\theta$, versus a dimensionless radius $r$ in the disk. A family of solutions given by expression (10) (see §2.1) is presented for the case of a NS. The solid line shows the Keplerian solution described by formula (9). The dashed and dash-dotted lines correspond to the adjustment radius $r=1.3$ and 1.39, respectively. In our calculation the $\gamma$-parameter (see equation [8]) is equal to 15.

Fig. 2.— Schematic diagram illustrating the centrifugal barrier (CB) model. The CB oscillates in vertical, radial and azimuthal directions around the vicinity of the adjustment radius (see the text for details). Some fraction of the soft photons from a disk are radiated in the observer’s direction, while most of the photons from a disk illuminate the CB region and get upscattered there to higher energies.

Fig. 3.— Plot of $f \times m$ (where $f$ is the QPO frequency in Hz, and $m = M/M_\odot$ is the dimensionless mass of a compact object) versus $\gamma$-parameter ($\propto \dot{M}$).

Fig. 4.— Temperature of the CB region as a function of ratio $H/F$ (where $H$ is the external flux of soft photons from a disk that illuminate the CB region, and $F$ is the intrinsic energy flux from the CB region. We performed our calculations for the Thomson optical depth of the CB region $\tau_0 = 5$. 
Neutron Star

Sub-Keplerian disk

Keplerian disk

QPO

centrifugal barrier region

Comptonized radiation
