Computational analysis for electrical breakdown in air due to streamer discharge in rod-to-plane arrangement

Maha F Abd Alameer and Thamir H Khalaf *
Department of Physics, College of Science, University of Baghdad, Iraq.
Corresponding author e-mail: drthamirhameed@gmail.com

Abstract: In addition to the theoretical and experimental, the numerical simulation is the third way to study the electrical discharge phenomenon. In this work, a numerical simulation was executed within a rod-to-plane electrodes arrangement to track the streamer discharge between the two electrodes for different rod dimensions and air gap length distances. The simulation results show the path of the streamer channels, and their branches, from the rod to the plane. Also, the electric potential and field distributions were presented as contour plots. The plots show clearly the sites of the streamer initiation and growth and how the distributions were affected by the streamer growth between the two electrodes. The dc applied voltage was varied, and the minimum value that caused the streamer to bridge the air gap and touch the plane was indicated as the breakdown voltage. It was found that, the breakdown voltage increased with the increasing of the air gap length distance, the electric field threshold value, the rod diameter, and less of that with the increasing of the rod lengths.

Key words: Streamer discharge, Numerical Simulation, Breakdown voltage, Air gap breakdown.

1. Introduction
There is a wide attention to study the electric breakdown in air, mainly, in two directions. The first is to prevent the occurrence because the air is a good natural and low cost insulator used in transmission lines and high voltage equipments. The second is to produce plasmas can be used in different applications. In order to explain the phenomenon of low pressure, Townsend proposed the gas breakdown theory. In low pressure, Townsend discharge theory can be a good method to describe gas breakdown. But when Pd is large (near atmospheric pressure), the growth of breakdown voltage is no longer a linear relationship, and Paschen’s law cannot be used in conditions with a large Pd. So that, Meek, Leob and Reather established a theory about streamer discharge development to illustrate high pressure gas discharge, such as air breakdown at atmospheric pressure [1].
A streamer electric discharge may be formed when a sufficiently high voltage is applied to a gas filled electrodes’ gap. It is an electrical discharge phenomenon characterized by a narrow and weakly ionized non-thermal plasma channel with a self-propagating head consisting of a thin curved layer of either net positive or negative charge [2, 3]. In air at ground pressure, streamers are formed through electron avalanches in electric fields greater than about 30 kV/cm and propagate with typical speeds of 0.1–10 mm/ns [2].
The streamers can be distinguished by the starting place to positive streamer and negative streamer: when the applied voltage on the discharge gap is sufficiently high, then if electrons leave the cathode and ionize the gas molecules in the gap, the ionized electrons would accelerate by the electric field and go on to ionize other gas molecules. After the original electrons move a distance in the direction of the applied electric field, they generate new electrons with a very rapid process. The new electrons will accumulate to build a single electron avalanche. As the electron avalanche moves forward, the ionization process on its
head will be more intense. When the electron avalanche is through the whole space, the space charge density in the head part has been so great that the electric field of the tail is greatly strengthened to radiate a lot of photons around, photons cause space photoionization and the newly formed photoelectrons are attracted by the positive space charge of the head part of the main electron avalanche. The photoelectrons create a new electronic collapse in the electrical field strengthened by distortion, which is called the secondary electron avalanche. The secondary electron avalanche moves to the main electronic avalanche and the electrons of the secondary electron avalanche head part move into the positive space charge region of the main electron avalanche head part (the electrons of the main electron avalanche have gone into the anode mostly). Due to the smaller field strength here, mostly electrons form as negative ions. A large number of positive and negative charged particles constitute a plasma stream which is called a positive streamer. Positive flow starts from the anode to the cathode after the initial avalanche head arrives at the anode. When the streamer reaches the cathode, the entire gap was run through by the plasma channel with good conductivity. So the gap breakdown is completed. Described above is a situation where the electron avalanche needs to go through the entire gap to form the streamer when the voltage is low. This voltage is the breakdown voltage [1].

Streamer discharge has many industrial applications because of its ability to enhance the activity of many chemical reactions without using a heat. Examples include production of ozone, water cleaning and removal of pollutants such as SO$_2$, NO$_x$, and fly ash [4, 5, 6]. Streamers also appear in nature in the form of so-called sprites which forms high above thunderclouds at an altitude of about 80 km. They are known to be the first self-propagating stage of a lightning discharge [7].

In last decades, numerical simulation of streamer discharges has got a wide interest, due to fast numerical algorithms and more powerful computers [8]. Numerical simulation makes it possible to obtain a much deeper understanding of streamer dynamics than what can be obtained only through experiments [2]. In addition, an accurate performing of experiments requires a cost measuring equipments and particularly designed set-ups. So that, numerical simulations offer a low-cost, valuable and multilateral option increasingly being desirable in industry.

The details of microphysics knowledge for streamer channels were reached, such as models of electron energy [5], and of transport coefficients and cross-sections of the most reactions, at least the main for air and other gases. On the other hand, we barely understand most macroscopic processes in a fully developed corona or streamer tree involving hundreds or thousands of mutually interacting plasma filaments [9]. Studies for the growth of electrical discharge trees are mostly based on the dielectric breakdown model that was proposed by Niemeyer et al [10] to explain the fractal properties of some electrical discharges. In their model, a discharge tree expands in discrete time-steps by the stochastic addition of new segments with a probability that depends on the local electric field. The original dielectric breakdown model assumed that the channels in the tree are perfectly conducting, but there is strong experimental evidence that the electric potential decreases along a discharge channel. A. Luque et al [11] modeling for streamer discharges assuming the streamer channels as imperfect conductors.

In this paper, we assume the streamer channel as weakly ionized plasma channel or a conductor has high resistance. i.e. there is a voltage drop cross it. And simulate the streamer discharge to indicate the breakdown voltage cross air gaps.

2. Modeling

Many studies have proposed the streamer discharge channels as good conductors and the breakdown occurs when the streamer crosses the electrodes gap. The main parameter that affects the streamer discharge is the electrostatic external or applied field, in addition to some stochastic probabilities. The model, here, based on stochastic models, [12,13], and follows the following considerations:
1- The streamer initiates at a site in air when the local electric field value at that site $E_{thr} \geq 26$ kV/cm [14]. The local electric field can be evaluated at each streamer step using Laplace’s equation as follows:

$$\varepsilon \nabla^2 V = 0$$  \hspace{1cm} (1)

$V$ is the potential in each point between the two electrodes at the surrounding domain in Volt (V), and $\varepsilon$ is the dielectric permittivity (in air, $\varepsilon \equiv \varepsilon_0$). The value of the local electric field intensity, can be given as:

$$E = -\nabla V$$  \hspace{1cm} (2)

Or, in 2-D the electric field intensity is calculated in terms of the variation of the electrical potential as:

$$E_{loc} = -\left(\frac{\partial V}{\partial x} i + \frac{\partial V}{\partial y} j\right)$$  \hspace{1cm} (3)

2- The evolution of the streamer from step to the other step spending a stochastic time $\tau$ given as [14]:

$$\tau = -\frac{\ln(\delta)}{r(E)}$$  \hspace{1cm} (4)

Where $r$ is a field depended growth rate function and given as:

$$r(E) = A \left(\frac{E_{loc}}{V/d}\right)^n$$  \hspace{1cm} (5)

The parameter $A$ is a constant and during the simulations took the value $3.7 \times 10^5$ sec$^{-1}$ [15]. $n$ is a number that controls the variation of the growth rate with the local electric field $E_{loc}$, $V$ is the applied potential at the anode and $d$ is the gap distance between the electrodes.

3- The streamer channels were assumed as conductors have high resistance, and the voltage drops, $E_{s}$ along in magnitude of (4-5) kV/cm [9,15].

4- The main channel of the streamer will branch into only two branches at certain condition given as [17]:

$$MxQ \geq 2CrQ$$  \hspace{1cm} (6)

Where

$$MxQ = \max \int_0^\xi (\alpha - \eta)dx$$  \hspace{1cm} (7)

And the parameter $CrQ$, is defined as the natural logarithm of the charge $N_c$ at which the avalanche is able to convert itself to a streamer, and given as:

$$N_c = \exp \left[\int_0^\xi (\alpha - \eta)dx\right]$$  \hspace{1cm} (8)

Where $\alpha$ is the Townsend’s primary ionization coefficient and $\eta$ is the coefficient of attachment.

5- Branches of the streamer will be followed for only one step, because they usually will decay and only the main will bridge the gap between electrodes.

3. Implementation of the Model

3.1 Configurations

The model was implemented within Rod-to-plane electrodes arrangement, figure 1, with different dimensions of the plane and the rod, with air gaps distances between electrodes start from 1 cm to 3 cm in a 0.5 cm step increment are used for measurement of air breakdown voltages. The rod electrode is a cylinder long enough ($\ell$), ended with hemisphere tip of radius $r$. The plane electrode is a disk plane, with
diameter $D$. One electrode (rod electrode) is stressed by Positive voltage is applied and the other (plane electrode) is grounded. This electrodes arrangement is producing a non-uniform field because the surfaces of both the electrodes are not similar.

Figure 1. Rod-to-plan electrodes configuration.

3.2 Numerical Solution

As mention before, the electric field is main parameter that affects the streamer discharge behavior. To determine the distribution of the electric field within the electrodes configuration, firstly the potential distribution must be determined. In other words, Laplace’s equation must be solved. finite element method is a numerical procedure for solving this type of problems. This is because the method basically relies on solving a large set of algebraic equations and entails considerable manipulation. The fundamental concept of the finite element method is that any region is made up of elements; therefore, the general behavior of a system can be determined by considering the behavior of its component (subsystem). The finite element analysis of any problem involves basically four steps:

- Discretizing the solution region into a number of sub regions or elements.
- Deriving the governing equations for typical elements.
- Assembling of all elements in the solution region.
- Solving the system of all equations that were obtained.

3.3. Simulation Procedure

The implementation of the model within the configuration in figure 1, or in other words, the simulation of the streamer discharge phenomenon was done as in following procedure

1- Mesh generation: In 2D geometries the domains are discretized into triangular elements. When curves are included in the geometry, especially on the boundaries, there is a risk that the surrounding area will not be properly divided and the resulted solution, even if it will be acceptable, it will not be the optimal. A fix to this is to try to use mesh structures that smoothly
change in size and resolution near these boundaries, where the solution is exposed to more steep changes and certainly is an area of higher interest [14]. AUTOMESH 2D Multi-Domain V1.5 package was used to generate the suitable meshes for the cases of 1cm, 1.5cm, 2cm, 2.5cm, and 3cm air gaps for the different diameters. The meshes were designed to have high density elements around the tip of the rod and low density far away because of the expected high variation of the voltage and the electric field around this region. An example of a generated meshes for the geometry of rod-plane electrodes is shown in figure 2. The division of the domains seems quite uniform, with a small change of the distribution of the elements in the vicinity of the electrode.

Figure 2. The discretized mesh as examples for two cases.

2- Solving of Laplace’s Equation: Based on the meshes that was generated as described in (1), the simple_2d solver [17] was used to solve Laplace’s equation numerically in the two dimensions’ area of the longitudinal cross section of the electrodes arrangement. The electric potential of all nods of the mesh that belong to the dielectric (air) is calculated by solving the Laplace equation with the boundary conditions on the electrodes and the discharge pattern.

3- Tracking the first step of the streamer between the electrodes: it was done as follows,

a) Using the potential at the nodes of each element, the electric field at the center of each element was calculated as $E_{loc}$ for each element.

b) The values of $E_{loc}$ for all elements was compared with the threshered value $E_{thr}$ and the elements that have values equal or greater than $E_{thr}$ were determined.

c) The stochastic time $\tau$ was calculated for all elements that were determined in (b).

d) The site of the streamer initiation was selected at the element which have the smallest $\tau$.

e) The branching condition in equation (6) was checked to indicate if there is a branch or not.

f) The voltage drop cross the segment, from the selected element in (d) to the tip of the rode, was calculated and subtracted from the applied voltage at the rod electrode. The result voltage was given for the nodes of the selected element as the site of the streamer.

4- Tracking the other steps of the streamer between the two electrodes: that requires updating the boundary conditions by add the nodes of the selected elements to the nodes of the rod electrode and repeating the procedure from 2 to 4 until the streamer reach the plane electrode.

5- The minimum applied voltage, which caused the streamer to reach the plane, was indicated as the breakdown voltage of a specified air gap within the electrodes arrangement.

The above procedure was executed by a program written in Fortran lange.
4. Results and Discussions
The results, that were presented here, are outputs of the execution of the Fortran program. It is simulated the streamer discharge Phenomenon within air gap between rode and plane electrodes, figure 1, in different dimensions. The air is considered as dry air at atmospheric pressure and room temperature. The simulation was tracked the time evolution of streamer steps from the rod to the plane and indicated the minimum applied voltage can have caused the breakdown.

4.1. Streamer Tracking
The streamer initiation and growth was tracked within the solution region between the two electrodes. The streamer initiated at the elements that have electric field values greater than $E_{thr}$. In the rode-plane configuration, the highest values, as expected, was found around the tip of the rod. The model was implemented different rod diameters.

Figure 3 shows the streamer path between the two electrodes when the rod length of 2.0 cm and diameter of 0.1 cm, 0.8 cm, and 1.0 cm. The drop voltage was averaged as 4.5 kV/cm, electric field threshold $E_{thr}$ of 26 kV, and the air gap distance length of 1.5 cm. The plane electrode is a disk has diameter of 7.5 cm.

![Figure 3](image)

**Figure 3.** Enlargement of the streamer initiation and growth from the rod to the plane electrodes when the rode diameter of a) 0.1 cm, b) 0.8 cm, and c) 1.0 cm.

From the figure above, one can note the effect of the increasing of the rod diameter on the behavior of the streamer in path, branching and the reaching time. The reaching time, which was indicated below each subfigure, increased with the increasing of the diameter which means the streamer velocity decreases with the increasing of the diameter. That can be explained by the effect of distributions of electric potential and field within the simulation region in the section below.

4.2. Electric Potential and Field within the Simulation Region
Figures 4 and 5, show a contour plotting for the effect of the streamer growth on the distributions of the voltage and electric field magnitudes of the rod-to-plane configuration for different diameters of the rod, at minimum breakdown voltage in the air gap of 1.5 cm length distance.
The sites that have high voltages and the high electric fields between the electrodes are the weakest insulation. In this case, the weak sites that was prepared for streamer initiation and growth is identified to be the sites where the magnitude of the electric field is the highest. According to figures 4 and 5, one can observe that the streamer moves according to the high voltage/E field sites from the rod down to the plane electrode.

**Figure 4.** The potential distribution within the electrode arrangement according to the streamer growth from the rod to the plane when the rode diameter of a) 0.1 cm, b) 0.8 cm, and c) 1.0 cm.

**Figure 5.** The electric field distribution within the electrode arrangement according to the streamer growth from the rod to the plane when the rode diameter of a) 0.1 cm, b) 0.8 cm, and c) 1.0 cm.

### 4.3. Effects of the Rod Dimensions

The rod has the important effect on the streamer discharge and then the values of breakdown voltage for any gap. That is because it is the site of the applied high voltage. Therefore, the simulation was expanded to show the effect of the rod diameter and length on the breakdown voltage values for different air gap lengths and different values of the electric field threshold that found in literatures.

The same producer above was followed and the breakdown voltage was indicated as the minimum applied voltage that caused the streamer to bridge the gap and reach the plane. The results for different diameters (0.1cm, 0.4cm, and 1.0) cm were presented in table 1 below:
Table 1. The breakdown voltage for different diameters of the rode electrode.

| Gap length (cm) | Breakdown Voltage (kV) |
|-----------------|------------------------|
|                 | Rod Diameter (cm)      |
|                 | 0.1 | 0.4 | 1.0 |
|                 | Thr (kV/cm) | Thr (kV/cm) | Thr (kV/cm) |
| 21.3 | 25 | 26 | 30 | 31 | 21.3 | 25 | 26 | 30 | 31 | 21.3 | 25 | 26 | 30 | 31 |
| 1    | 8.6 | 9.8 | 10.2 | 10.8 | 11.3 | 10.1 | 11.8 | 12.3 | 14.2 | 14.7 | 14.6 | 17.1 | 17.8 | 20.6 | 21.2 |
| 1.5  | 12.0 | 13.1 | 13.8 | 15.0 | 16.1 | 12.7 | 14.9 | 15.5 | 17.8 | 18.4 | 18.5 | 21.7 | 22.5 | 26.0 | 27.0 |
| 2    | 14.9 | 16.7 | 17.4 | 18.8 | 19.6 | 15.1 | 17.7 | 18.4 | 21.3 | 22.0 | 22.6 | 26.5 | 27.6 | 31.8 | 32.9 |
| 2.5  | 18.7 | 21.1 | 22.0 | 21.9 | 24.7 | 17.2 | 20.2 | 20.8 | 24.3 | 25.1 | 27.6 | 32.4 | 33.7 | 38.8 | 40.0 |
| 3    | 21.8 | 24.5 | 25.2 | 25.1 | 28.9 | 19.0 | 22.4 | 23.2 | 26.8 | 27.7 | 30.2 | 35.4 | 36.8 | 42.5 | 43.9 |

From table 1., one can note that the breakdown voltage, in general and as expected, increased with the increasing of gap length and threshold electric field. Also, it is clear that, the breakdown voltage increased with the increasing of the diameter of the rod electrode. That can be explained as the increasing of the diameter of the rod or the diameter of the hemisphere means decreasing in the electric field in the shortest distance between the electrodes which require higher applied voltage to causes the streamer to bridge the gap and the breakdown occurrence.

Also, the length of the rod was examined for the different diameters. The results show no considerable effect on the breakdown voltage values, as presented in table 2, because the little effect on the electric field distribution between the electrodes.

Table 2. The breakdown voltage for different diameters of the rode electrode.

| Rode diameter (cm) | Breakdown Voltage |
|--------------------|-------------------|
|                    | Rod length (cm)   |
|                    | 1.0 | 1.5 | 2.0 |
| 0.1                | 17.6 | 17.7 | 17.9 |
| 0.2                | 18.2 | 18.7 | 18.8 |
| 0.4                | 18.8 | 19.0 | 19.5 |
| 1.0                | 25.8 | 26.3 | 26.4 |
| 1.2                | 27.1 | 28.1 | 28.4 |
5. Conclusions
From the simulation results, one can indicate the following conclusions

1. The physical shape of the electrodes gives a variation in the breakdown voltages.
2. The breakdown voltage of the rod-to-plane gaps with the positive dc voltage linearly increases with the gap spacing.
3. The distribution of the electric field in a rod-plane gap is strongly affected by the geometry (shape and dimensions) of the electrodes and the arrangements of the air gap.
4. The threshold electric field value $E_{thr}$ exerts a strong influence on the value of the minimum breakdown voltage.
5. The values of the breakdown voltage influenced by the dimensions of the rod especially the rod diameter.

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