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On the six-vertex Model’s free energy. (English) Zbl 07606703
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Summary: In this paper, we provide new proofs of the existence and the condensation of Bethe roots for the Bethe Ansatz equation associated with the six-vertex model with periodic boundary conditions and an arbitrary density of up arrows (per line) in the regime \( \Delta < 1 \). As an application, we provide a short, fully rigorous computation of the free energy of the six-vertex model on the torus, as well as an asymptotic expansion of the six-vertex partition functions when the density of up arrows approaches 1/2. This latter result is at the base of a number of recent results, in particular the rigorous proof of continuity/discontinuity of the phase transition of the random-cluster model, the localization/delocalization behaviour of the six-vertex height function when \( a = b = 1 \) and \( c \geq 1 \), and the rotational invariance of the six-vertex model and the Fortuin-Kasteleyn percolation.

MSC:
82Bxx Equilibrium statistical mechanics
60Kxx Special processes
05Cxx Graph theory

Full Text: DOI arXiv

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