Method to improve the accuracy of the geometrical parameters measurement in stereoscopic AOTF-based spectral imagers

A V Gorevoy¹, A S Machikhin²
1Bauman Moscow State Technical University, 5/1, 2nd Baumanskaya street, Moscow, Russia 105005
2Scientific and Technological Center of Unique Instrumentation of Russian Academy of Sciences, 15, Butlerova street, Moscow, Russia 117342

Email: ¹strannik-algor@mail.ru, ²aalexanderr@mail.ru

Abstract. The problem of non-contact geometrical measurements by the stereoscopic spectral AOTF-based imagers is considered. Major factors contributing to the measurement error are analyzed for the first time. Expressions are derived for area measurement inaccuracy on arbitrary shape surfaces, caused by measurement errors of individual points’ 3D coordinates with and without a priori information about surface shape. Verification of the obtained expressions with the real experimental data showed that area measurement error for a complex figure, given by the set of points, is caused mainly by ignoring the fact that these points belong to the surface. It is shown that the use of a priori information about investigated surface shape, which is often available from the design documentation, in many cases would radically improve the accuracy of surface defects area measurement. The presented results can be effectively used for the development of new 3D spectral imagers and modernization of existing ones.

1. Introduction
Currently, one of the major problems in the analysis of materials is the automatic detection, segmentation and classification of the elements with certain physical, chemical, biological or other properties, i.e. to construct a "map of the object’s properties". The most of approaches are based either on the image processing using a priori information about the required elements or on the selection of objects with respect to certain physical properties during a particular impact on them. The first approach is applicable only when the properties of the objects are known or already obtained by another instrument. The second approach is ineffective when it is necessary to identify and classify a large number of elements with different properties. That is why the most promising solution to the problem is spectral imaging [[1]]. Allowing the simultaneous registration of the spatial and spectral information, these devices combine the advantages of both approaches. Therefore, spectral imagers are widely used in remote sensing, biomedicine, machine vision and other applications [[2]], [[3]].

It is often necessary (e.g., in a study of moving objects) and sufficient (e.g., if a priori information about the spectrum of the inspected object is available) to register spectral images not in the entire tuning range but only at the particular wavelengths. Such arbitrary spectral tuning significantly reduces the time of registration and data processing. Among the tunable optical spectral filters the most promising are acousto-optic (AOTFs). They provide unique collection of features such as
programmability, image transmission with high image quality - insignificant (less than 1%) distortion in the visible spectral range [11], [5]. Rather high spectral (up to 0,1 nm) and spatial (up to 1000×1000 dots) resolution, fast spectral tuning (~10 μs), compactness and absence of any moving elements make them a precise and ergonomic analytical tool capable of working even in out-of-lab environment [6]-[8]. Therefore AOTF-based spectral imagers are widely used in various fields of science and technology, in particular, to determine the three-dimensional (3D) structure of objects [9], [10], [11]. In this case, the use of spectral contrast allows getting more information about the position, shape and size of the volume elements of the object.

One of the methods to restore the spectral 3D structure of objects is stereoscopy [12]. Stereoscopic imaging at arbitrary wavelengths may be implemented by the optical scheme containing one AOTF in each channel (see Figure 1a) [9]. This device forms two light beams carrying the image of the object at different angles. Then stereoscopic images are registered by digital cameras. There are also other schemes of 3D spectrometers, using only one specialized AO cell (see Figure 1b) [13].

![Figure 1.](image)

**Figure 1.** Spectral stereoscopic imagers based on two (a) AO cells and one (b) AO cell
1 – inspected object, 2, 7 – diaphragms; 3, 8 – lenses, 4, 6 - polarizers, 5 – AO cells.
Arrows between optical components show the light polarization direction.

These devices are very effective, for instance, for contrast visualization of the 3D distribution of foreign inclusions or defects in non-destructive testing of various objects (see Figure 2). Usually it is not enough to find and visualize the element, but usually it is also necessary to measure its geometric parameters: length, depth, perimeter, square, etc. Measurement accuracy is highly important as it defines the quality of the whole 3D spectral imaging system.

![Figure 2.](image)

**Figure 2.** Wide-band image (a) of the pipe and narrow-band spectral stereoscopic images (b,c) in the range λ = 500÷503 nm.

The basic problem of such measurements is that in narrow spectral bands only the boundaries of the detected elements are displayed vividly and not their content (see Figure 2). In this case only 3D coordinates of points on this boundary can be determined accurately. Inside the homogeneous object correlation algorithms for determination of conjugate points lead to large errors. Even when the defect is superficial, i.e. follows the a priori known shape of the surface, the problem has no unique solution due to uncertainty of the imager’s orientation relative to the object. Therefore, in such cases, as a rule,
the shape of the object is interpolated by the set of planar triangles with vertices belonging to the boundary [[14]]. If the object has a complicated 3D shape, the error of its area measurement in this case can easily reach 30-50%.

As in the traditional stereoscopic systems, owing to many uncertainties, reliable methods to assess the accuracy of the measurement have not yet been developed. Since the 3D spectral imagers are not only observant but also the measuring devices, the estimation of the measurement error is the key issue in determining the possibility of their application. Therefore, there is a need to develop methods to estimate the accuracy of such stereoscopic measurements.

In this paper, we demonstrate an approach to solving this problem based on the model of the classical two-channel stereoscopic system (see figure 3 and figure 4) and modified algorithms for determining the three-dimensional coordinates of the object points.

2. Camera model and 3D reconstruction algorithm
The registration channels of the stereoscopic system may be represented by the simplified geometric model of the image formation $\mathbf{p}_i = P_i \circ E_i(\mathbf{x}_g)$, where $\mathbf{x}_g$ and $\mathbf{p}_i$ are 3D coordinates of object points measured in the global coordinate system (GCS) and 2D coordinates of corresponding image points on the image plane of $i$-th camera; $i = 1..N$, $N$ – number of cameras; $E_i$ is the Euclidean mapping from the GCS to the coordinate system (CS) of $i$-th camera; $P_i$ is the transformation translating points from the target camera object space onto the image plane (from $(x_i, y_i, z_i)$ to $(u_i, v_i)$ in the figure 4). We use "○" notation here to define a composition of transformations, i.e. $P_i \circ E_i(\mathbf{x}_g) \equiv P_i(E_i(\mathbf{x}_g))$. 

Figure 3. 3D spectral imaging system based on two AOTF-based imagers.
Figure 4. Simplified camera model and two-view geometry.

The mapping $P_i$ determines the unique correspondence $P : L(O, A) \rightarrow a(u, v)$ of rays $L(O, A)$ in the object space and points on the image plane. In order to allow 3D reconstruction by means of triangulation, camera model should be invertible and should provide the back-projection transformation $P^{-1} : a(u, v) \rightarrow L(O, A)$. The pinhole camera model, the pinhole model, followed by distortion, equidistant and generic polynomial camera model are commonly used [[15]]-[[17]].

The camera placement in 3D space is described by the set of transformations from one of camera-centered CS to each other. Let one of these CS be the GCS and define the set $(R_i, t_i)$ converting point coordinates $x_i = (x_i, y_i, z_i)^T$ in $i$-th CS to the global coordinates $x_g$ via following relation: $x_i = E_i(x_g) = R_i x_g + t_i$. Any transformation from $i$-th to $j$-th CS may be written as $R_{i \rightarrow j} = R_j R_i^{-1}$, $t_{i \rightarrow j} = t_j - R_j R_i^{-1} t_i$. (1)

The set of transformations $P_i$ and $E_i$ is parametrized by the vector $v = (v_1^T, \ldots, v_N^T)^T$ of intrinsic and extrinsic camera parameters, which components are to be determined during system calibration [[15]-[20]].

The reconstruction of 3D coordinates $x_g$ from $N$ projections $p_i, i = 1, N$ may be considered as the ray intersection problem $L_i = E_i^{-1} \circ P_i^{-1}(p_i)$. Because of the uncertainties in corresponding points coordinates $p_i$, these rays are skew, and the triangulation algorithm $T$ is used for estimation of $\hat{x}_g$ by minimizing a functional $C$, i.e.

$$\hat{x}_g = T(p, v) = \arg\min_{x_g} \left(C(\hat{x}_g, p, v)\right),$$

(2)

where $p = \left(p_1^T, p_2^T, \ldots, p_N^T\right)^T$.

The selection of the appropriate cost functions is not trivial in the general case, because it largely depends on a priori data about the position of target point $x_g$, its projections $p_i$, transformations $P_i$ and the statistics of coordinate measurement errors. If we assume that the deviation of the measured image coordinates $p_i$ from their true values $\hat{p}_i$ follows the Gaussian distribution $N(0, \Sigma_{p_i})$, then we
can use the Maximum Likelihood Estimator (MLE), which minimizes the Mahalanobis distance in the image plane [21]

\[ C = \sum_{i=1}^{N} \| p_i - P_i \circ E_i (\hat{x}_g) \|^2 = \sum_{i=1}^{N} (p_i - \hat{p}_i)^\top \Sigma^{-1}_{p_i} (p_i - \hat{p}_i), \]  

(3)

where \( \hat{p}_i = P_i \circ E_i (\hat{x}_g) \) is the estimated position of the 3D point \( \hat{x}_g \) projection on the image plane of \( i \)-th camera, and \( \Sigma^{-1}_{p_i} \) is the inverse (rank-constrained generalized inverse) covariation matrix of coordinate measurement error for \( p_i \), \( \Sigma_p = \text{diag}(\Sigma_{p_1}, \Sigma_{p_2}, \ldots, \Sigma_{p_N}) \). Given by nonlinear minimization of the cost function (3), the estimate \( \hat{x}_g \) is asymptotically unbiased and effective in the limit of small noise. Furthermore, this algorithm allows to achieve the theoretical bound for minimal variance (the Cramer-Rao lower bound) [18], [21].

We can use the generic mathematical model described above to evaluate 3D coordinates uncertainty of individual points \( \hat{x} \) for different types of multi-view (or equivalent to multi-view) systems, including the system presented in the Figure 3. Error evaluation of complex characteristics measurement (distances, areas or volumes) for foreign inclusions or defects may be performed similarly based on the estimated uncertainties for individual points of the detected defect contour. It is important that the individual point errors are uncorrelated in this case. However, when measuring distance and area on the surface of known shape, it becomes possible to take shape information into consideration on the triangulation stage and significantly improve accuracy. This approach requires reconsidering the above mathematical model.

3. 3D point reconstruction on the surface of known shape

Consider the case of a parametric surface in 3D space: point \( x \) belongs to the surface if its coordinates are defined by a parametric equation \( x = S(a) \) of position parameters \( a = (a, b)^\top \). Note that point coordinates \( x \) are given in the surface CS, global coordinates may be calculated as \( x_g = E_s(x) = R_s x + t_s \), camera-centered coordinates – using relation (1). Next, combine the set of parameters for \( R_s \) and \( t_s \) together with additional parameters included in function \( S(a) \) (such as radius of a cylindrical or spherical surface) and denote it as \( v_s \). Then we represent the triangulation algorithm in the following form

\[ \hat{a} = T(p, v, v_s) = \text{argmin} (C(\hat{a}, p, v, v_s)). \]  

(4)

To calculate the 3D coordinates \( x \) from \( N \) registered images, as before, the best solution is to use an algorithm that minimizes the Mahalanobis distance (3) in the image planes, i.e.

\[ C = \sum_{i=1}^{N} \| p_i - P_i \circ E_i \circ E_s (\hat{a}) \|^2 = \sum_{i=1}^{N} (p_i - \hat{p}_i)^\top \Sigma^{-1}_{p_i} (p_i - \hat{p}_i). \]  

(5)

In contrast to equations (2,3), the result of the minimization procedure according to the equations (4,5) is the estimate of parameters \( \hat{a} \), which defines the point position on the surface. Hence, we should use the known transformation \( \hat{x}_g = E_s (S(\hat{a})) \) to convert the estimated position parameters to the global coordinates.
4. Uncertainty evaluation method

The development of the uncertainty evaluation technique requires the analysis of the factors affecting the accuracy of 3D coordinates measurements for individual points. Main factors are:
- method error, caused by limitations of the considered geometric model of the image registration, simplifications and 3D reconstruction algorithm;
- random error of corresponding point positions on the image planes, which depends on the chosen correspondence technique (manual or automatic) and an image quality;
- random or systematic error of the parameters for the used mathematical model (calibration errors).

In the case of the improved triangulation algorithm (4), we should add the surface orientation and the parameters error to the factors described above. Since the analysis of the impact of the methodological error cannot be considered within the developed mathematical model, we further restrict our investigations to the second and third factors.

The estimation error of 3D coordinates $\hat{x}$ calculated by the conventional triangulation algorithm (2) and the uncertainties of image point positions $p_i$ are related via the backward propagation theorem \cite{18}, \cite{21}, \cite{22}. One can differentiate the cost function with respect to $p$ and perform some manipulations to derive the expression for the partial derivatives of the bias and the error covariance matrix for the 3D coordinates. A similar approach can be applied to evaluate the error caused by deviations of calibration parameters $v$ \cite{23}, \cite{24}.

If we consider the dependence of the estimation error of $\hat{x}$ calculated by the triangulation algorithm (4) from deviations of $p_i$, we should first solve this problem for the estimation error of position parameters $\hat{a}$. Use the same theorem as before, and, after substitution the expression for the cost function (5) and algebraic transformations, we obtain the following expressions

\[
\frac{\partial \hat{a}}{\partial p} = \left( \frac{\partial p}{\partial a} \Sigma_{p}^{-1} \frac{\partial p}{\partial a} \right)^{-1} \frac{\partial p}{\partial a} \Sigma_{p}^{-1},
\]

\[
\Sigma_{a} = \frac{\partial \hat{a}}{\partial a} \Sigma_{a} \frac{\partial \hat{a}}{\partial a}^T = \left( \frac{\partial p}{\partial a} \Sigma_{p}^{-1} \frac{\partial p}{\partial a} \right)^{-1},
\]

where

\[
\frac{\partial p}{\partial a} = \left( \frac{\partial p_1}{\partial a}^T, \frac{\partial p_2}{\partial a}^T, \ldots, \frac{\partial p_N}{\partial a}^T \right)^T,
\]

\[
\frac{\partial p_i}{\partial a} = \frac{\partial P_i \circ E_i \circ S}{\partial \hat{a}}, \quad \frac{\partial E_i \circ S}{\partial \hat{a}} = \frac{\partial P_i \circ E_i \circ S}{\partial \hat{a}}.
\]

The posterior covariance matrix $\Sigma_{a}$ for the estimate of position parameters $\hat{a}$ in the equation (7) is given for the case where system calibration performed without errors and transformations $P_i$, $E_i$ determined exactly. Furthermore, transformations $E_i$ and $S$ are assumed to be known without uncertainties too.

In order to derive the equation for the cost function (5) considering calibration errors and uncertainties of surface orientation, let the mapping $(P_i \circ E_i)$ be differentiable with respect to $v$ in a neighborhood of the true value $\bar{v}$ and let the mapping $(E_i \circ S)$ be differentiable with respect to $v_s$ in the neighborhood of $\bar{v}_s$. Then one can use the first-order approximation to write

\[
C = \sum_{i=m}^N \left| P_i - P_i \circ E_i \left( E_i \circ S (\hat{a}) \right) - \frac{\partial (E_i \circ S)}{\partial \hat{a}} (v_s - \bar{v}_s) \right| \left( \frac{\partial (P_i \circ E_i)}{\partial v} (v - \bar{v}) \right). \tag{8}
\]

After substitution and some manipulations we obtain the expression for the partial derivatives of the bias and the covariance matrix.
\[
\frac{\partial \hat{\mathbf{a}}}{\partial \mathbf{v}} = -N^{-1} \hat{\mathbf{p}}^T \Sigma_p \frac{\partial \mathbf{p}}{\partial \mathbf{v}}, \quad \frac{\partial \hat{\mathbf{a}}}{\partial \mathbf{v}_s} = -N^{-1} \hat{\mathbf{p}}^T \Sigma_p \frac{\partial \mathbf{p}}{\partial \mathbf{v}_s},
\]

\[
\Sigma_a = \frac{\partial \hat{\mathbf{a}}}{\partial \mathbf{v}} \Sigma_p \frac{\partial \mathbf{a}}{\partial \mathbf{v}} + \frac{\partial \hat{\mathbf{a}}}{\partial \mathbf{v}_s} \Sigma_v \frac{\partial \mathbf{a}}{\partial \mathbf{v}_s} + \frac{\partial \hat{\mathbf{a}}}{\partial \mathbf{v}_s} \Sigma_v \frac{\partial \mathbf{a}}{\partial \mathbf{v}_s},
\]

where \( \frac{\partial \mathbf{p}}{\partial \mathbf{v}} = \frac{\partial \mathbf{p}}{\partial \mathbf{x}_g} \frac{\partial \mathbf{x}_g}{\partial \mathbf{v}} + \frac{\partial \mathbf{p}}{\partial \mathbf{x}_s} \frac{\partial \mathbf{x}_s}{\partial \mathbf{v}} \), \( \frac{\partial \mathbf{p}}{\partial \mathbf{v}_s} = \frac{\partial \mathbf{p}}{\partial \mathbf{x}_g} \frac{\partial \mathbf{x}_g}{\partial \mathbf{v}_s} + \frac{\partial \mathbf{p}}{\partial \mathbf{x}_s} \frac{\partial \mathbf{x}_s}{\partial \mathbf{v}_s} \), are combinations of \( \frac{\partial \mathbf{p}}{\partial \mathbf{v}} \) and \( \frac{\partial \mathbf{p}}{\partial \mathbf{v}_s} \) similarly to \( \frac{\partial \mathbf{p}}{\partial \mathbf{a}} \) above.

Thus, the combination of the expression (10) and partial derivatives (6, 9) produces the final formula for the posterior covariation matrix \( \Sigma_a \) based on the main factors affecting measurement accuracy. To get a similar expression for the posterior covariation matrix \( \Sigma_{x_s} \) for the coordinates in the GCS, we differentiate \( \dot{x}_g = E(S(\hat{a})) \)

\[
\frac{\partial \dot{x}_g}{\partial \mathbf{p}} = \frac{\partial \dot{x}_g}{\partial \mathbf{a}} \frac{\partial \mathbf{a}}{\partial \mathbf{p}}, \quad \frac{\partial \dot{x}_g}{\partial \mathbf{v}} = \frac{\partial \dot{x}_g}{\partial \mathbf{a}} \frac{\partial \mathbf{a}}{\partial \mathbf{v}}, \quad \frac{\partial \dot{x}_g}{\partial \mathbf{v}_s} = \frac{\partial \dot{x}_g}{\partial \mathbf{a}} \frac{\partial \mathbf{a}}{\partial \mathbf{v}_s} + \frac{\partial \dot{x}_g}{\partial \mathbf{v}_s},
\]

\[
\Sigma_{x_s} = \frac{\partial \dot{x}_g}{\partial \mathbf{p}} \Sigma_p \frac{\partial \dot{x}_g}{\partial \mathbf{p}} + \frac{\partial \dot{x}_g}{\partial \mathbf{v}} \Sigma_v \frac{\partial \dot{x}_g}{\partial \mathbf{v}} + \frac{\partial \dot{x}_g}{\partial \mathbf{v}_s} \Sigma_v \frac{\partial \dot{x}_g}{\partial \mathbf{v}_s},
\]

where \( \frac{\partial \dot{x}_g}{\partial \mathbf{a}} = \frac{\partial E}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{a}} \), \( \frac{\partial \dot{x}_g}{\partial \mathbf{v}} = \frac{\partial E}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{v}} + \frac{\partial E}{\partial \mathbf{v}} \frac{\partial \mathbf{v}}{\partial \mathbf{v}} \).

Moreover, a number of additional factors also reduces the measurement accuracy for distances (the error of calibration parameters leads to the correlation of errors of the individual points 3D coordinates which must be taken into account when calculating the distance error), arc lengths (the sum-of-segments approximation) and areas (the sum-of-triangles approximation). As shown in [[14]], the area measurement error caused by the sum-of-triangles approximation on cylindrical and spherical surfaces is less than 1%, and is not the main contributor.

Thus, the error evaluation of complex characteristics measurement for foreign inclusions or defects requires calculation of joint partial derivatives and the joint covariance matrix for all points of the investigated object contour according to equations (6, 9–12). As an example, consider the solution of this problem for the area of triangle defined by three points \( x^1_g, x^2_g \) and \( x^3_g \) on the surface of interest.

The area of this triangle can be calculated as

\[
S = \| (x^2_g - x^1_g) \times (x^3_g - x^1_g) \| / 2.
\]

We can also obtain partial derivatives \( \frac{\partial S}{\partial x^j_g}, j = 1, 2, 3 \) from formula (13). In order to evaluate the joint covariance matrix \( \Sigma_{x_{i,j}} \) for \( x^1_g, x^2_g \) and \( x^3_g \), some modifications of the equation (12) should be done. In particular, replace \( \frac{\partial \dot{x}_g}{\partial \mathbf{p}} \) with \( \text{diag} \left( \frac{\partial \dot{x}_g^1}{\partial \mathbf{p}^1}, \frac{\partial \dot{x}_g^2}{\partial \mathbf{p}^2}, \frac{\partial \dot{x}_g^3}{\partial \mathbf{p}^3} \right) \), \( \Sigma_p \) with \( \text{diag} \left( \Sigma_{p^1}, \Sigma_{p^2}, \Sigma_{p^3} \right) \), \( \frac{\partial \dot{x}_g}{\partial \mathbf{v}} \)
with \( \mathbf{\partial X}_r \) and \( \mathbf{\partial X}_s \). Next, write the expression for the posterior variance \( \sigma_s^2 \) of the area measurement error

\[
\sigma_s^2 = \left( \frac{\partial S}{\partial x^1_g}, \frac{\partial S}{\partial x^2_g}, \frac{\partial S}{\partial x^3_g} \right) \Sigma_{g}^{x=1:3} \left( \frac{\partial S}{\partial x^1_g}, \frac{\partial S}{\partial x^2_g}, \frac{\partial S}{\partial x^3_g} \right)^T.
\] (14)

One can derive the equation for the measurement error of the area of a complex figure, which is calculated as the sum of the areas of individual triangles, similar to (14), but this is more complicated because of the presence of joint vertices of the neighboring triangles.

5. Experiments and numerical modelling

To demonstrate the proposed method we apply it for the accuracy evaluation of the point positions and the area measurements using the single-imager stereoscopic endoscope with the following parameters: CCD sensor 1/10” (752×576 elements), field of view 60° of each channel, base distance 3 mm. The probe axis coincides with the axis of the test object presented by the graph paper tube with a radius of 20 mm (see Figure 5). Analyzed points are evenly spaced on the surface of the cylinder with the step 5 mm. Thus, the area of each triangle (half of the square shown in Figure 5) is equal to 12.5 mm\(^2\).

![Figure 5](image-url)

Figure 5. Stereoscopic images of the test object. Marked squares.

We calculate the error variance of the triangles areas caused by the random uncorrelated error of the corresponding point positions with the variance \( \sigma_p^2 = 0.5 \) for both coordinates \( u, v \) and both images. The results are presented for the conventional triangulation algorithm (2) (in the Figure 6) and for the surface-adopted triangulation algorithm (4) (in the Figure 7).
Figure 6. The standard deviation of the area measurement for the conventional triangulation algorithm. The true value of the each triangle area is equal to 12.5 mm².

Figure 7. The standard deviation of the area measurement for the surface-adopted triangulation algorithm. The true value of the each triangle area is equal to 12.5 mm².

The color of each triangle represents the standard deviation $\sigma_s$ of the area measurement according to the color bar on the right side of every figure. The triangles shown in gray color are out of view for at least one of the channels. The origin of the CS in the figures is the center of probe located between two camera centers. Note that taking shape information into consideration on the triangulation stage radically improves measurement accuracy (from $\sigma_s = 8.3$ mm² down to $\sigma_s = 0.5$ mm² for the triangle placed at a distance of 50 mm from the origin).

Next, we use the developed mathematical models to examine the influence of errors of the object surface orientation relative to the probe. Below are the results of the error estimation for the 3D coordinates of the individual points and the areas of triangles caused by the random rotation with a variance $\sigma_a^2$ for both angles $\alpha_x$ and $\alpha_y$ (see Figure 8) and the random decentering with a variance $\sigma_{\Delta x}^2$ for both coordinates $x, y$ (see Figure 9).
Figure 8. The square root of the largest eigenvalues $\sigma_r$ of the covariation matrices $\Sigma_{x_t}$ for the point positions (a) and the standard deviation $\sigma_s$ of triangles’ area (b) calculated for the different values of the standard deviation $\sigma_\alpha$ of the rotation angles $\alpha_x$ and $\alpha_y$. Solid lines correspond to the surface-adopted triangulation algorithm, dotted lines correspond to the conventional one.

Figure 9. The square root of the largest eigenvalues $\sigma_r$ of the covariation matrices $\Sigma_{x_t}$ for the point positions (a) and the standard deviation $\sigma_s$ of triangles’ area (b) calculated for the different values of the standard deviation $\sigma_\Delta c$ of decentering for both coordinates $x, y$. Solid lines correspond to the surface-adopted triangulation algorithm, dotted lines correspond to the conventional one.

Figures 8a and 9a show the square root of the largest eigenvalues $\sigma_r$ of the covariation matrices $\Sigma_{x_t}$ for the points at different distances from the origin (for example, $z = 55$ mm) and the RMS value (denoted as ‘RMS’ in the figure legend) of $\sigma_r$ for all visible points on the cylindrical surfaces with $z$-coordinate in the range 10-80 mm. Hence, the values of $\sigma_r$ correspond to the major axis of the error ellipsoid, i.e. the direction along which the 3D point position has the maximum variance [25]. One can see, that in Upper Saddle River the uncertainty of the surface orientation leads to the linear growth of the errors of 3D point positions reconstructed using the
surface-adapted triangulation algorithm. Thus, it makes possible to find the largest tolerable deviation of the surface, whereby this algorithm still outperforms the conventional one (for example, find the value of \( \sigma_a \) in which a solid line crosses a dotted line in the Figure 8a).

Figures 8b and 9b show the standard deviation \( \sigma_s \) of the triangles’ area at the same distances. Note that the errors of the area measurement increase much slower than in the previous case due to the strong correlation of the individual point position errors. Consequently, the surface-adapted triangulation algorithm is preferable to the conventional one, even with the large uncertainties of surface orientation relative to the probe. This is especially evident for decentering of the imager (see Figure 9b).

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