Influence of gauge fluctuations on fermion pairing order parameter

Yu-Liang Liu

Center for Advanced Study, Tsinghua University, Beijing 100084, People's Republic of China

Abstract

Using a prototype model, we study the influence of gauge fluctuations on fermion pairing order parameter which has the gauge symmetry, and demonstrate that the gauge fluctuations can destroy the long range order of the fermion pairing order parameter, and make it only have short range correlation. If this parameter is a superconducting order parameter, we show that the Meissner effect of the system keeps intact, and the system is in the superconducting state even though the long range order of the superconducting order parameter is destroyed by the gauge fluctuations. Our calculations support that the pseudo-gap region of the high Tc cuprate superconductivity is a spin pseudo-gap region rather than an electron pre-paired region.
After the discovery of the high Tc cuprate superconductor [1], it has been soon reached a common consensus that the high Tc cuprate superconductors are a two-dimensional strongly correlated system, and low energy behavior is determined by their copper-oxide plane(s) [2-5]. However, so far a complete theory representing this system does not appear, and there are still a lot of controversies and unsolved questions in this field. This strongly correlated electron system has novel physical properties of normal and superconducting states. In normal state, the system shows non-Fermi liquid behavior [3,6-8], and has strong antiferromagnetic fluctuations in low doping and optimal doping regions. Moreover, in the low doping region there is a pseudo-gap region, and stripe phase fluctuations may also appear. These phenomena cannot be clearly explained by an unified theory. In superconducting state, the system has a d-wave symmetry superconducting order parameter, but it is not clear which mechanism can produce this order parameter. There is another puzzle question that single electron excitation spectrum keeps intact as the system going to the superconducting state from the pseudo-gap region [9-11]. This phenomenon seems to mean that the pseudo-gap region is an electron pre-paired region [12].

Usual perturbation methods are hard to treat strongly correlated systems, because in which there is not a controllable small parameter, and electrons have strong correlation. For enough strong electron correlation, there are not well-defined quasi-particles near the Fermi surface, and the systems show non-Fermi liquid behavior. On the other hand, these perturbation methods cannot directly describe the electron correlation. However, the electron correlation is a key parameter hidden in the strongly correlated systems. The eigenfunctional bosonization method [13] is a good candidate for treating the strongly correlated systems, because in this method there naturally appears a phase field, where its imaginary part represents the electron correlation.

In this paper, we introduce a prototype model, which represents a two-dimensional spin-1/2 fermion system with transverse gauge fields, to study the influence of the gauge fluctuations on the fermion pairing order (spin pseudo-gap) parameter, here we assume that the fermion pairing order parameter does not violate the gauge symmetry, and these transverse
gauge fields originate from some other strong fermion interactions, such as the Hubbard model with large on-site electron Coulomb interaction, or the electron single occupied constraint of the t-J model. In the absence of the gauge fields, if the fermion pairing parameter is a constant (long range order), we demonstrate that after turning on the gauge fields, the correlation function and the average value of the fermion pairing order parameter go to zero with anomalous exponential asymptotic behavior. The transverse gauge fields provide a strong phase fluctuation factor to the fermion pairing parameter by phase fields. If these fermions are electrons, we demonstrate that the Meissner effect of the system keeps intact, and the system is still in the superconducting state, even though the long range order of the superconducting order parameter is destroyed by the transverse gauge fields. This result is similar to that of a two-dimensional superconductor with long range Coulomb interaction, where the long range order of the superconducting parameter is destroyed by the long range Coulomb interaction, but the Meissner effect of the system leaves intact \[14\]. However, there is a little difference between these two cases, for the long range Coulomb interaction, the correlation function of the electron pairing order parameter shows power-law asymptotic behavior.

We consider a prototype model represented by the Hamiltonian with transverse gauge fields \(A(x)\) \((\nabla \cdot A(x) = 0)\),

\[
H = \int d^2 x \left\{ \psi_1^\dagger(x) \left[ \frac{1}{2m} (-i \nabla - gA)^2 - \mu \right] \psi_1(x) \\
+ \psi_2^\dagger(x) \left[ \frac{1}{2m} (-i \nabla + gA)^2 - \mu \right] \psi_2(x) \\
+ \Delta(x) \psi_1(x) \psi_2(x) + \Delta^*(x) \psi_2^\dagger(x) \psi_1^\dagger(x) \right\} \tag{1}
\]

where the fermion field \(\psi_1(x)\) represents the fermions with spin-up, the fermion field \(\psi_2(x)\) represents the fermions with spin-down, and \(\Delta(x) = -u <\psi_2^\dagger(x) \psi_1^\dagger(x)>\) is fermion pairing order parameter (spin pseudo-gap) induced by other interactions, where \(u\) is the interaction strength. It is noted that the fermion pairing terms do not violate the gauge symmetry of the transverse gauge fields. At \(A(x) = 0\), for simplifying our following calculations, we simply take this parameter as a real constant, \(\Delta(x)|_{A=0} = \Delta_0\). The system described by the
Hamiltonian (1) is a strongly correlated system, as $\Delta(x) = 0$ it shows the non-Fermi liquid behavior [15,16].

With the Hamiltonian (1), the action of the system can be written as a simple form,

$$S = \int dtd^2x \Psi^\dagger(x,t) \hat{M}(x,t) \Psi(x,t)$$

where $\Psi^\dagger(x,t) = (\psi^\dagger_1(x,t), \psi_2(x,t)), \hat{M} = \hat{M}_0 + \hat{\phi}(x,t)$ and

$$\hat{M}_0 = \begin{pmatrix}
i \partial_t + \mu + \frac{\nabla^2}{2m}, & \Delta_0 \\
\Delta_0, & i \partial_t - \mu - \frac{\nabla^2}{2m}
\end{pmatrix}$$

$$\hat{\phi}(x,t) = \begin{pmatrix}
-\frac{ig}{m} A(x,t) \cdot \nabla - \frac{g^2}{2m} A^2(x,t), & \Delta^*(x,t) - \Delta_0 \\
\Delta(x,t) - \Delta_0, & \frac{ig}{m} A(x,t) \cdot \nabla + \frac{g^2}{2m} A^2(x,t)
\end{pmatrix}$$

We have separated the propagator operator $\hat{M}(x,t)$ into two parts, one is a free term, and another one depends on the transverse gauge fields $A(x,t)$. In this form, we first exactly solve the eigen-equation of the operator $\hat{M}_0$, then we determine the differential equations of phase fields induced by the transverse gauge fields.

Integrating out the fermion fields $\psi_1(x,t)$ and $\psi_2(x,t)$, we obtain the effective action (omitting constant term),

$$S[A] = -i \int_0^1 d\xi \int dtd^2x tr \left( \hat{\phi}(x,t) G(x,t; x', t', [\phi]) \right) |_{x' \rightarrow t}$$

where the Green’s functional $G(x,t; x', t', [\phi])$ can be represented by eigen-functionals of the propagator operator $\hat{M}(x,t)$,

$$G(x,t; x', t', [\phi]) = \sum_i \sum_{k, \omega} \frac{1}{E_{k\omega}[\phi]} \Psi_{k\omega}^{(i)}(x,t,[\phi]) \Psi_{k\omega}^{(i)*}(x', t', [\phi])$$

The eigen-functional equation of the operator $\hat{M}(x,t)$ reads,

$$\hat{M}(x,t) \Psi_{k\omega}^{(i)}(x,t,[\phi]) = E_{k\omega}[\phi] \Psi_{k\omega}^{(i)}(x,t,[\phi])$$

where $i = 1, 2$ and the eigenvalues are $E_{k\omega}^{(1,2)}[\phi] = \omega \pm E_k + \Sigma_k^{(1,2)}[\phi]$, where $E_k = \sqrt{\epsilon^2(k) + \Delta_0^2}$, $\epsilon(k) = k^2/(2m) - \mu$, and $\Sigma_k[\phi] = \int_0^1 d\xi \int dtd^2x \Psi_{k\omega}^{(i)*}(x,t,[\phi]) \hat{\phi}(x,t) \Psi_{k\omega}^{(i)}(x,t,[\phi])$ is a smooth
function, and independent of $\omega$. Due to the non-zero fermion pairing parameter $\Delta_0$, the energy spectrum of the fermions splits into two branches, one represents unoccupied states (conducting band), and another one represents occupied states (valence band). The fermion pairs are in the top of the valence band. The eigen-functionals $\Psi^{(1,2)}_{k\omega}(x,t,[\phi])$ represent these occupied and unoccupied states, respectively, and they can be written as,

$$
\Psi^{(1)}_{k\omega}(x,t,[\phi]) = A_k \left( \frac{1}{TL^2} \right)^{1/2} \begin{pmatrix} u_k e^{Q_k(x,t)} \\ -v_k e^{\tilde{Q}_k(x,t)} \end{pmatrix} e^{ikx - i(\omega + \Sigma_k^{(1)}[\phi])t}
$$

$$
\Psi^{(2)}_{k\omega}(x,t,[\phi]) = A_k \left( \frac{1}{TL^2} \right)^{1/2} \begin{pmatrix} u_k e^{F_k(x,t)} \\ -v_k e^{\tilde{F}_k(x,t)} \end{pmatrix} e^{ikx - i(\omega + \Sigma_k^{(2)}[\phi])t}
$$

where $u_k = \sqrt{(1 + \epsilon(k)/E_k)/2}$ and $v_k = \sqrt{(1 - \epsilon(k)/E_k)/2}$ are coherence factors, $A_k$ is a normalization constant, and $T$ and $L$ are the time and space length scales of the system, respectively. At $A(x,t) = 0$, the eigen-functionals in (6) are the exact solutions of the operator $\hat{M}_0$, where $A_k = 1$. The fermion pairing parameter now can be approximately written as near the Fermi surface ($k \sim k_F$),

$$
\Delta(x,t) = \Delta_0 e^{\hat{Q}_k(x,t) + \hat{Q}^*_k(x,t)}
$$

(7)

where $\Delta_0$ is determined by the self-consistent equation $\Delta_0 = \frac{e^\pi}{2L^2} \sum_k |A_k|^2 \Delta_0 / E_k$. The phase fields $Q_k(x,t)$ and $\tilde{Q}_k(x,t)$ satisfy Eikonal-type equations [13]. If we only keep linear terms, we can obtain that,

$$
\begin{cases}
[i\partial_t + \frac{k \cdot \nabla}{m} + \frac{\nabla^2}{2m} |Q_k(x,t)| + \frac{g}{m} k \cdot A(x,t) = -\Delta_0 [\tilde{Q}_k(x,t) + \tilde{Q}^*_k(x,t)] \\
[i\partial_t - \frac{k \cdot \nabla}{m} - \frac{\nabla^2}{2m} |\tilde{Q}_k(x,t)| - \frac{g}{m} k \cdot A(x,t) = -\Delta_0 [Q_k(x,t) + Q^*_k(x,t)]
\end{cases}
$$

(8)

Under this approximation, we have the same effective action as that obtained by usual random-phase approximation (RPA) (see below). By solving these equations, we have the relations between the phase fields and the transverse gauge fields,

$$
\begin{cases}
Q^R_k(q,\Omega) = -\frac{gq^2}{m^2D_-} k \cdot A(q,\Omega) \\
\tilde{Q}^R_k(q,\Omega) = -\frac{gq^2}{m^2D_+} k \cdot A(q,\Omega)
\end{cases}
$$

$$
\begin{cases}
Q_k^I(q,\Omega) = -\frac{2g(\Omega - \frac{k \cdot q}{m})}{mD_+} k \cdot A(q,\Omega) \\
\tilde{Q}_k^I(q,\Omega) = \frac{2g(\Omega + \frac{k \cdot q}{m})}{mD_+} k \cdot A(q,\Omega)
\end{cases}
$$

(9)
where $D_{\pm} = (\Omega \pm k \cdot q/m)^2 - q^4/(2m)^2 = \Delta_0 q^2/m$, $Q_k(q, \Omega) = Q^R_k(q, \Omega) + Q^I_k(q, \Omega)$, and $\tilde{Q}_k(q, \Omega) = \tilde{Q}^R_k(q, \Omega) + \tilde{Q}^I_k(q, \Omega)$. The imaginary parts of the phase fields do not contribute to the effective action, but they represent the fermion correlation induced by the transverse gauge fields. Only the real parts of the phase fields contribute to the effective action of the system.

With equations (4) and (9), we have the effective action,

$$S[A] = \frac{1}{TL^2} \sum_{q, \Omega} \Pi_{ij}(q, \Omega) A_i(-q, -\Omega) A_j(q, \Omega)$$

(10)

where $\Pi_{ij}(q, \Omega) = (i \Gamma(\Delta_0) \Omega/q + \chi(\Delta_0) q^2)(\delta_{ij} - q_iq_j/q^2)$, and $\Gamma(\Delta_0) = \gamma(\Delta_0) + 2m \Delta_0 (\gamma + \gamma(\Delta_0))/q^2$. The parameters $\gamma(\Delta_0)$ and $\chi(\Delta_0)$ are the smooth functions of the gap $\Delta_0$. At $\Delta_0 = 0$, we have $\gamma(0) = \gamma$ and $\chi(0) = \chi$, and have the same effective action of the transverse gauge fields $A(x, t)$ as that of Refs. [6,7], where it was obtained by usual RPA method. We would like to point out that in the eigen-functional bosonization method it is naturally to introduce the phase field, which is a key parameter hidden in strongly correlated systems, because its imaginary part represents the fermion correlation. However, it is absent in previous perturbation methods. With the phase field, we can easily study the influence of fermion interactions on the low energy behavior of the system, and calculate a variety of correlation functions.

At $\Delta_0 = 0$, the system shows non-Fermi liquid behavior produced by the strong fluctuations of the transverse gauge fields [15,16]. It can be easily shown by present representation. With equation (4) and taking functional average of the transverse gauge fields $A(x, t)$, we can obtain single fermion Green’s function,

$$G_1(x - x', t - t') = i < G_{11}(x, t; x', t', [\phi]) >_A$$

$$= \frac{1}{L^2} \sum_k \theta(-\epsilon(k))e^{i\epsilon(k)(x-x')-ie(k)(t-t')}e^P^I_k(x-x', t-t')$$

(11)

where $P^I_k(x, t)$ comes from the contribution of the imaginary part of the phase field $Q_k(x, t)$, and the contribution from the real part of the phase field $Q_k(x, t)$ can be neglected. Near the Fermi surface $k \sim k_F$, we have the relation,
\[ P_{k_F}^I(0, t) \simeq -i^{1/3} a g^2 t^{1/3} \]  

where \( a = \frac{k_F}{4\pi^2 m} (\chi/\gamma)^{1/3} \int dx (1 - \cos(x)) x^{-4/3} \). This result is basic same as that of Ref. \[15,16\]. Due to the time dependence of the phase factor \( P_{k_F}(0, t) \), the single fermion Green’s function shows a singular low energy dependence which violates the quasi-particle excitation assumption of the Landau Fermi liquid. Therefore, the system shows non-Fermi liquid behavior.

We now study the influence of gauge fluctuations on the fermion pairing order parameter. From equation (7), we see that the gauge fields \( A(x, t) \) provide a phase fluctuation factor to the fermion pairing parameter \( \Delta(x, t) \) by the phase fields \( Q_k(x, t) \) and \( \tilde{Q}_k(x, t) \). Therefore, the gauge fields strongly affect the fermion pairing parameter. By taking functional average of the transverse gauge fields, we have the following relations,

\[ < \Delta(x, t) \Delta^*(x, t) >_A \simeq \Delta_0^2, \quad < \Delta(x, t) \Delta^*(x', t) >_A \simeq \Delta_0^2 e^{-b|x-x'|}, \]
\[ < \Delta(x, t) \Delta^*(x, t') >_A \sim e^{-c|x-t'|^{1/5}}, \quad < \Delta(x, t) >_A \sim e^{-d/\omega_0^{1/5}}, \]  

(13)

where \( b \sim \Delta_0^{-1/5}, c \sim i^{1/5} \frac{k_F}{4\pi^2 m} [\chi(\Delta_0)/(2m \Delta_0 (\gamma + \gamma(\Delta_0)))]^{1/5}, 2d \sim c \) is a constant, and \( \omega_0 \) is a characteristic energy scale of the transverse gauge fields, in general it is very small (\( \omega_0 \sim 1/L \)). These relations are our central results, we now explain them. The first expression means that the real parts of the phase fields \( Q_k(x, t) \) and \( \tilde{Q}_k(x, t) \) have little influence on the spin pseudo-gap, and the last three relations mean that the imaginary parts of the phase fields strongly suppress the correlation of the fermion pairing parameter \( \Delta(x, t) \), and make the long range order of the fermion pairing parameter be a short range order. Therefore, the fluctuations of the transverse gauge fields can destroy the long range order of the fermion pairing parameter, and make it only have short range correlation.

If these fermions are electrons, the pairing parameter \( \Delta(x, t) \) is a superconducting order parameter. At \( A(x, t) = 0 \), the Hamiltonian (11) represents a two-dimensional superconductor with superconducting gap \( \Delta_0 \). The transverse gauge fluctuations make the system have a short range pairing parameter \( \Delta(x, t) \), but they do not destroy the superconducting state.
of the system. This can be justified by calculating the Meissner effect of the system after turning on an external magnetic field $B(x,t) = \nabla \times A_e(x,t)$. The current functional reads,

$$J(x,t, [A]) = -\psi_1^\dagger(x,t) \frac{i\nabla}{m}\psi_1(x,t) - \psi_2^\dagger(x,t) \frac{i\nabla}{m}\psi_2(x,t)$$
$$- \frac{1}{m}[(A_e(x,t) + gA(x,t))\psi_1^\dagger(x,t)\psi_1(x,t) + (A_e(x,t) - gA(x,t))\psi_2^\dagger(x,t)\psi_2(x,t)]$$

(14)

Taking functional average of the transverse gauge fields, we have the relation,

$$J(x,t) = <J(x,t, [A])>_A$$
$$= \left( \frac{a(\Delta_0)}{m^2} - \frac{\rho(\Delta_0)}{m} \right) A_e(x,t)$$

(15)

where $\rho(\Delta_0) = -(1/L^2) \sum_k \theta(-\epsilon(k))\epsilon(k)/E_k$, and $a(\Delta_0) = (1/L^2) \sum_k \frac{k^2}{2E_k} \frac{E_k E_{k'}}{E_k + E_{k'}} |k'\to k|$. At $\Delta_0 = 0$, we have $J(x,t) = 0$. This equation shows that even though the transverse gauge fields destroy the long range superconducting order, the Meissner effect of the system keeps intact, i.e., the system still is in the superconducting state. This result is consistent with that of equation (13). The Meissner effect is mainly determined by the coherence factors $u_k$ and $v_k$ of the superconducting state, and is insensitive to the phase factor of the electron pairing order parameter. However, the fluctuations of the transverse gauge fields can destroy the long range order of the electron pairing order parameter. One must remember that these gauge fields are not electromagnetic gauge fields, they originate from some strong electron interactions, and the superconducting state of the system does not violate this gauge symmetry.

Even though all our calculations are based on the s-wave symmetry of the pairing parameter, they are also qualitatively valid for the d-wave symmetry of the pairing parameter. The above results may provide some useful informations for studying the high Tc cuprate superconductors, where their low energy physical properties are determined by the two-dimensional strongly correlated electron system in the copper-oxide plane(s). It is well-known that this strongly correlated system can be well described by the t-J model [4] or the Hubbard model with large on-site Coulomb interaction [2]. Electron single occupation constraint of the t-J model in fact represents the strong electron correlation, which can
induce the gauge interaction among the fermions and holons in the slave boson/fermion representation of the electron operator \[17,19\], or the gauge interaction of the electrons due to the Hilbert space of the system is strongly suppressed by this constraint. We now only focus on the pseudo-gap region in the low doping region. If this region is an electron pre-paired region, and the change of electron pairing order parameter from long range order to short range order is induced by the gauge fluctuations, our calculations show that under an external magnetic field, the Meissner effect of the system will be observed, and the system is in the superconducting state, which contradicts with present experimental data. In fact, the system is in a normal state, and there is not the Meissner effect in this region. However, our calculations support that the pseudo-gap region is a spin pseudo-gap region, and the fermions do not have electric charges. The transport property of the normal state is determined by the holons (carrying electric charges). Due to strong gauge fluctuations, the system still shows non-Fermi liquid behavior even though there is the spin pseudo-gap in spin excitation spectrum. This is qualitatively consistent with present experimental observations. How does the system go into the superconducting state? There may have two ways, one is usual Bose-Einstein condensation of the holons \[7,17\], however, in this case there is too high onset temperature of superfluidity. Another one is that in the low temperature limit the gauge fluctuations become enough strong to re-combine a fermion and a holon into an electron, and the fermion pairing order parameter becomes the superconducting order parameter. In this case, the superconducting order parameter has the same symmetry (d-wave symmetry) and the same modulus with the fermion pairing order parameter, which is qualitatively consistent with the angle-resolved photoemission experiments \[9,11\], where single electron excitation spectrum nearly has no change as the system going to the superconducting state from the spin pseudo-gap region.

In summary, with the prototype model, we have studied the influence of the transverse gauge fields on the fermion pairing order parameter which keeps the gauge symmetry, and demonstrated that the gauge fluctuations destroy the long range order of the fermion pairing order parameter, and make it only have the short range correlation. If this order parameter
is the superconducting order parameter, we demonstrated that the Meissner effect of the system keeps intact, and the system is in the superconducting state even though the long range order of the superconducting order parameter is destroyed by the gauge fluctuations. Therefore, our calculations support that the pseudo-gap region in the low doping region of the high Tc cuprate superconductivity is the spin pseudo-gap region rather than the electron pre-paired region, because the high Tc cuprate superconductivity is a strongly correlated electron system, where the Hilbert space of the electron states is strongly suppressed, the gauge fluctuations are existing.
REFERENCES

[1] J.G.Bednorz and K.A.Müller, Z. Phys. b64, 189(1986).

[2] P.W.Anderson, Science 235, 1196(1987).

[3] P.W.Anderson, The Theory of Superconductivity in the High-Tc Cuprates, (Princeton Univ. Press, Princeton, NJ, 1997).

[4] F.C.Zhang and T.M.Rice, Phys. Rev. B37, 3759(1988).

[5] G.Baskaran, Z.Zou and P.W.Anderson, Solid State Commun. 63, 973(1987); I.Affleck and J.B.Marston, Phys. Rev. B37, 3774(1988); G.Kotliar and J.Liu, Phys. Rev. B38, 5142(1988).

[6] G.Baskaran and P.W.Anderson, Phys. Rev. B37, 580(1988); L.Ioffe and A.Larkin, Phys. Rev. B39, 8988(1989).

[7] P.A.Lee and N.Nagaosa, Phys. Rev. B45, 966(1992).

[8] C.M.Varma et. al., Phys. Rev. Lett. 63, 1986(1989).

[9] D.G.Marshall et al., Phys. Rev. Lett. 76, 4841(1996).

[10] A.G.Loeser et al., Science 273, 3235(1996).

[11] H.Ding et al., Nature (London) 382, 51(1996).

[12] V.Emery and S.Kivelson, Nature (London) 374, 434(1995); see also, Y.Uemura et al., Phys. Rev. Lett. 66, 2665(1991); M.Randeria, N.Trivedi, A.Moreo and R.Scalettar, Phys. Rev. Lett. 69, 2001(1992).

[13] Y.L.Liu, preprints, cond-mat/0011253; cond-mat/0011254.

[14] Y.L.Liu and T.K.Ng, Phys. Rev. Lett. 83, 5539(1999); T.K.Ng, cond-mat/9706033.

[15] D.V.Khveshchenko and P.C.E.Stamp, Phys. Rev. Lett. 71, 2118(1993).
[16] H.J.Kwon, A.Houghton and J.B.Marston, Phys. Rev. B52, 8002(1995).

[17] X.G.Wen and P.A.Lee, Phys. Rev. Lett. 76, 503(1996); P.A.Lee, N.Nagaosa, T.K.Ng and X.G.Wen, Phys. Rev. B57, 6003(1998).

[18] D.H.Lee, Phys. Rev. Lett. 84, 2694(2000).

[19] L.Balents, M.P.A.Fisher and C.Nayak, Int. J. Mod. Phys. B12, 1033(1998).