The Pierre Auger Exotic Events and Axion Quark Nuggets

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Abstract. The Pierre Auger Observatory have reported observation of several exotic cosmic ray-like events which apparently related to thunderstorms. These events are much larger in size than conventional cosmic ray events, and they have very distinct timing features. A possible nature of the observed phenomenon is still a matter of active research and debates as many unusual features of these exotic events are hard to explain. In particular, the frequency of appearance of these exotic events is very low (less than 2 events/year), in huge contrast with a typical rate of a conventional lightning strikes in the area. We propose that the observed exotic events can be explained within the so-called axion quark nugget (AQN) dark matter model. The idea is that the AQNs may trigger and initiate a special and unique class of lightning strikes during a thunderstorm as a result of ionization of the atmospheric molecules along its path. The corresponding AQN-induced lightning flashes may show some specific features not shared by typical and much more frequent conventional flashes. We support this proposal by demonstrating that the observations, including the frequency of appearance and time duration are consistent with observations. We also comment on possible relation of AUGER exotic events with the Telescope Array bursts and the terrestrial gamma ray flashes. We list a number of features of the AQN-induced exotic events (such as specific radio pulses synchronized with these events) which can be directly tested by future experiments. We also suggest to use distributed acoustic sensing instruments to detect the acoustic pulses which must be synchronized with AUGER exotic events.

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1. Introduction

The AUGER collaboration have reported [1,3] observation of several anomalous events that cannot be explained as conventional cosmic rays (CR) events. We overview the corresponding unusual features below in details. A proposed answer [1] on the nature of these exotic events is based on the observed correlation with the lightning events. Therefore the exotic events (EE), according to [1] must be related to the lightning flashes, similar to the terrestrial gamma ray flashes (TGF) observed from satellites above thunderstorms. Still, a complete picture and explanation of these EE remains obscure as many questions and puzzles yet to be answered. In particular, one of the most pressing question is as follows. Why these exotic events are so rare such that only less than 2 events/year had been recorded while the number of the lightning flashes are much more frequent by a large factor (at least by three orders of magnitude)?

In this work we shift the question from the lightning strike per se to the question on a possible physical source which could ignite and trigger a specific class of lightning flashes, which consequently may generate the EEs. To be more precise, we advocate an idea that the observed by AUGER the rare exotic events could be result of a special class of lighting events which are initiated by the AQN dark matter particles when they randomly hit the area of the AUGER detector during the thunderstorms. We will argue in this work that our proposal is consistent with the observations [1–3], including the event rate and the time duration of EE.

One should mention here that the AQN model was invented long ago without any relation to the AUGER observations. Rather, it was invented to explain the observed similarity between the dark matter (DM) and the visible densities in the Universe, i.e. \( \Omega_{\text{DM}} \sim \Omega_{\text{visible}} \) without any fitting parameters, see recent brief review article on the AQN model [4]. The important feature of our proposal is that the AQN model involves no fine-tuning of parameters because this model is not devised to match the observation of EEs, but rather its properties are well studied and constrained by numerous unrelated phenomena in the previous studies for completely different purposes in a very different context in dramatically different systems, see overview of this model in next Sec. 2.

We now list the basic features of the EE observed by AUGER:

1. These events characterized by long lasting signals, tens \( \mu s \), and event footprints (which is defined as the total area covered by the particles emitted within this short time scale measured in tens \( \mu s \)) much larger, up to 200 km\(^2\), than those produced by the highest energy CR;
2. In the complete data sample (recorded since 2004) 23 events with the same characteristics have been identified as exotic events [1];
3. The “large” events are characterized by the involvement of a large number of stations (the number of triggered stations can reach 80 and more). The radius of large events spans from 4 to 8 km while the “small” events are characterized by radius around 2-3 km. [2];
4. Large events, coined as surface detector (SD) rings in [1] are often accompanied by
“smaller” events within 1 ms, see Fig 3 in [1]. These accompanying “small” events are likely to be generated by a common source but with a long time evolution (not compatible with the speed of light propagation);
5. There are different types of events, the so-called SD disks which also exhibit long signals (similar to SD rings), however they are characterized by smaller amplitudes, but could be large in size, see Fig 4 in [1];
6. About 70% of the small sample of “good events” consisting of 10 events are correlated in time within 1 ms with lightning strikes [2]. The correlation remains strong (∼41%) even when all EE are included, in which case the time difference between EE and strikes spans from 10 μs to 100 ms [3].
7. Some of the recorded events appear to form the clusters entering the same zone of the array within few ms, see Fig 4 in [3].

In the present work we advocate a proposal that the AQNs are the key ingredients in generation of these exotic events observed by AUGER. To be more specific, the AQNs in our proposal play the dual role. First, the AQNs may initiate and trigger a special class of lightning strikes which are much stronger and faster than a typical lightning strike. These rare AQN-induced lightning strikes generate the “large” events (as defined in item 3 above), according to the proposal. It explains a strong correlation of the EEs with the thunderstorms. Secondly, the very same AQNs in the background of electric field under thunderclouds will emit a cluster of particles. These “direct” emissions from AQNs represent the “small” events (as defined in item 3 above) which very often accompany the large events. The small events will also inevitably occur under the thunderclouds even when a lightning strike is not ignited, and consequently, a large event cannot be generated.

One should comment here that similar unusual events have been recorded previously by Telescope Array (TA) Collaboration and coined as the TA bursts [5, 6]. The “mysterious bursts” were defined as the CR-like events when at least three air showers were recorded within 1 ms. It has been observed that the TA bursts are correlated with the lightning strikes. Both these features are also present in EEs observed by AUGER, see items 6 and 4 correspondingly. We argued in [7] that these “mysterious bursts” could be a consequence of the same AQN annihilation events under the thunderstorm. It has been also proposed in [8–10] that similar unusual events can be identified with Downward TGFs. We comment on relation of the results [5–10] devoted to the TA bursts and TGFs with the AUGER exotic events in terms of the AQN proposal in Sect. 5.

Our presentation is organized as follows. In subsection 2.1 we overview the basics of the AQN model, while in subsection 2.2 we overview the features of the AQN model which will be relevant for the present work. In Sec. 3 we estimate the corresponding event rate. In Sec. 4 we describe the outcome of the AQN annihilation events and argue that the AQN may play the role of trigger which initiates and ignites very unusual (and rare) powerful lightning strike. We formulate our proposal in Sect. 5 where we identify these rare unusual lightning flashes with AUGER exotic events. In particular,
we argue that all the unusual properties listed above can be interpreted in terms of the AQN annihilation events propagating under the thunderstorm. We also make few comments on relation between TA bursts and TGFs with the AUGER exotic events within our AQN proposal. We conclude with Sec. 6 where we suggest possible tests which may support or refute our proposal.

2. The AQN DM model

We overview the basics ideas of the AQN model in subsection 2.1 while in subsection 2.2 we list some specific features of the AQNs traversing the atmosphere (such as internal temperature, level of ionization, etc). These characteristics will be important for the present study interpreting the AUGER EEs as a special class of lightnings strikes initiated by the AQNs when they enter the thundercloud regions characterized by a strong electric field.

2.1. The basics

The AQN DM model was suggested in [11] with a single goal to explain the observed similarity between the DM and the visible densities in the Universe, i.e. $\Omega_{\text{DM}} \sim \Omega_{\text{visible}}$. This feature represents a generic property of the construction [11] as both component, the visible, and the dark are proportional to one and the same fundamental dimensional constant of the theory, the $\Lambda_{\text{QCD}}$. The AQN construction in many respects is similar to the Witten’s quark nuggets, see [12–14], and review [15]. This type of DM is “cosmologically dark” as a result of smallness of the parameter relevant for cosmology, which is the cross-section-to-mass ratio of the DM particles. This numerically small ratio scales down many observable consequences of an otherwise strongly-interacting DM candidate in form of the AQN nuggets.

The original motivation for the AQN model can be explained as follows. It is commonly assumed that the Universe began in a symmetric state with zero global baryonic charge and later (through some baryon-number-violating process, non-equilibrium dynamics, and $CP$-violation effects, realizing the three famous Sakharov criteria) evolved into a state with a net positive baryon number.

As an alternative to this scenario, we advocate a model in which “baryogenesis” is actually a charge-separation (rather than charge-generation) process in which the global baryon number of the universe remains zero at all times. This represents the key element of the AQN construction.

In this model, the unobserved antibaryons comprise dark matter being in the form of dense nuggets of antiquarks and gluons in the color superconducting (CS) phase. The result of this “charge-separation process” are two populations of AQN carrying positive and negative baryon number. In other words, the AQN may be formed of either matter or antimatter. However, due to the global $CP$ violating processes associated with the so-called initial misalignment angle $\theta_0$ which was present during the early formation
stage, the number of nuggets and antinuggets will be different. This difference is always an order-of-one effect irrespective of the parameters of the theory, the axion mass $m_a$ or the initial misalignment angle $\theta_0$.

The presence of the antimatter nuggets in the AQN framework is an inevitable and the direct consequence of the $CP$ violating axion field which is present in the system during the QCD time. As a result of this feature the DM density, $\Omega_{DM}$, and the visible density, $\Omega_{\text{visible}}$, will automatically assume the same order of magnitude densities $\Omega_{DM} \sim \Omega_{\text{visible}}$ as mentioned above.

We refer to the original papers [16–19] devoted to the specific questions related to the nugget’s formation, generation of the baryon asymmetry, and survival pattern of the nuggets during the evolution in early Universe with its unfriendly environment. We also refer to a recent brief review article [4] which explains a number of subtle points on the formation mechanism, see also independent analysis [20] supporting the basic elements on the formation and survival pattern of the AQNs during the early stages of the evolution, including the Cosmic Microwave Background (CMB) and Big Bang Nucleosynthesis (BBN) epochs.

For the present studies, however, we take the agnostic viewpoint, and assume that such nuggets made of antimatter are present in our Universe today irrespective to their formation mechanism. This assumption is consistent with all presently available cosmological, astrophysical and terrestrial constraints as long as the average baryon charge of the nuggets is sufficiently large as we review below.

The strongest direct detection limit is set by the IceCube Observatory’s, see Appendix A in [22]:

$$\langle B \rangle > 3 \cdot 10^{24} \quad [\text{direct (non)detection constraint}].$$

The basic idea of the constraint is that IceCube with its surface area $\sim \text{km}^2$ has not detected any events during its 10 years of observations. In the estimate it was assumed that the efficiency of detection of a macroscopically large nugget is 100% which excludes AQNs with small baryon charges $\langle B \rangle < 3 \cdot 10^{24}$ with $\sim 3.5\sigma$ confidence level.

Similar limits are also obtainable from the ANITA and from geothermal constraints which are also consistent with as estimated in [23]. It has been also argued in [24] that that AQNs producing a significant neutrino flux in the 20-50 MeV range cannot account for more than 20% of the DM density. However, the estimates were based on assumption that the neutrino spectrum is similar to the one which is observed in conventional baryon-antibaryon annihilation events, which is not the case for the AQN model when the ground state of the quark matter is in the colour superconducting (CS) phase, which leads to the dramatically different spectral features. The resulting flux computed in [25] is perfectly consistent with observational constraints.

The authors of Ref. [26] considered a generic constraint for the nuggets made of antimatter.
antimatter (ignoring all essential specifics of the AQN model such as quark matter CS phase of the nugget’s core). Our constraints (1) are consistent with their findings including the CMB and BBN, and others, except the constraints derived from the so-called “Human Detectors”. As explained in [27] the corresponding estimates of Ref. [26] are oversimplified and do not have the same status as those derived from CMB or BBN constraints.

While ground based direct searches offer the most unambiguous channel for the detection of the conventional DM candidates such as Weakly Interacting Massive Particles (WIMP), the flux of AQNs is inversely proportional to the nugget’s mass and consequently even the largest available conventional DM detectors are incapable to exclude (or even constrain) the potential mass range of the nuggets. Instead, the large area detectors which are normally designed for analyzing the high energy cosmic rays are much better suited for our studies of the AQNs as we discuss in next section 2.2.

![AQN-structure](image)

**Figure 1.** AQN-structure (not in scale). There are several parametrically distinct scales of the problem: $R \sim 10^{-5}$ cm represents the size of the nugget filled by quark matter with $B \sim 10^{25}$ in CS phase. Much larger scale $R_{DW} \sim m_a^{-1}$ describes the axion DW surrounding the quark matter. The axion DW has the QCD substructure which has typical scale of order $R_{QCD} \sim \Lambda_{QCD}$. Finally, there is always electro-sphere which represents a very generic feature of quark nuggets, including the Witten’s original construction. In case of antimatter-nuggets the electro-sphere comprises the positrons. Its typical size strongly depends on the environment and internal temperature $T$ of the quark matter as discussed in the main text.

The absolute stability of the AQNs in vacuum is a result of the construction when the energy per baryon charge in the quark-matter nuggets is smaller than in the baryons
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| Property                              | Typical value or feature |
|---------------------------------------|--------------------------|
| AQN’s mass $[M_N]$                    | $M_N \approx 16 g (B/10^{25})$ [4] |
| baryon charge constraints $[B]$       | $B \gtrsim 3 \cdot 10^{24}$ [1] |
| annihilation cross section $[\sigma]$ | $\sigma \approx \kappa \pi R^2 \approx 1.5 \cdot 10^{-9} \text{cm}^2 \cdot \kappa (R/2.2 \cdot 10^{-5} \text{cm})^2$ |
| density of AQNs $[n_{AQN}]$          | $n_{AQN} \sim 0.3 \cdot 10^{-25} \text{cm}^{-3} (10^{25}/B)$ [4] |
| survival pattern during BBN           | $\Delta B/B \ll 1$ [28,35] |
| survival pattern during CMB           | $\Delta B/B \ll 1$ [29,35] |
| survival pattern during post-recombination | $\Delta B/B \ll 1$ [19] |

Table 1. Basic properties of the AQNs adopted from [34].

(from hadronic phase) making up the visible portion of the Universe. The same feature also holds for the original theoretical construction [12–15]. However, the difference is that, in the original model [12–15], the quark nuggets are assumed to be absolutely stable at zero pressure, while in the AQN model this stability is achieved by the additional axion domain-wall pressure, see Fig. 1 with an explanation of the AQN construction and also a brief review [4] for the details.

This difference has dramatic observational consequence- the Witten’s nugget will turn a neutron star (NS) into the quark star if it hits the NS. In contrast, a matter type AQN will not turn an entire star into a new quark phase because the quark matter in the AQNs is supported by external axion domain wall pressure, and therefore, can be extended only to relatively small distance $\sim m_a^{-1}$, which is much shorter than the NS size. However, the matter type AQNs can be accumulated in the cores of stars/planets during their long life times. This subject is of interest by itself, but it is not the topic of the present work devoted to antimatter AQNs capable to generate the AUGER exotic events. The antimatter AQNs will be completely annihilated when they hit the stars.

We conclude this brief review subsection with Table1 which summarizes the basic features of the AQNs. The parameter $\kappa$ in Table1 is introduced to account for the fact that not all matter striking the nugget will annihilate and not all of the energy released by annihilation will be thermalized in the nuggets. The ratio $\Delta B/B \ll 1$ in the Table implies that only a small portion of the (anti)baryon charge hidden in form of the AQNs get annihilated during big-bang nucleosynthesis (BBN), Cosmic Microwave Background (CMB), or post-recombination epochs (including the galaxy and star formation), while the dominant portion of the baryon charge survives until the present time. Independent analysis [20] and [26] also support our original claims as cited in the Table1 that the anti-quark nuggets survive the BBN and CMB epochs. The large mass of the nuggets along with their small sizes also implies that the direct head on AQN-AQN collisions are extremely rare events and do not modify our estimates for $\Delta B/B \ll 1$.

Finally, one should mention here that the AQN model with the same set of parameters may explain a number of other puzzling observations in dramatically different environments (Early Universe [28,29], galactic [30], Solar corona [31,32], Earth [33,34]) as highlighted in concluding section 6.
2.2. When the AQNs hitting the Earth’s atmosphere

For our present work, however, the most relevant studies are related to the effects which may occur when the AQNs made of antimatter hit the atmosphere, the annihilation processes start and a large amount of energy will be injected to surrounding material, which may be manifested in many different ways\footnote{The AQNs made of matter will not experience any annihilation processes. Therefore, their internal temperature always remains small. As a result, they do not inject much energy into the space. Therefore, the matter AQNs do not generate any strong observable effects, in contrast with antimatter AQNs, and will be ignored in the present studies.}. For example, sufficiently large (and vary rare) AQNs with $B \gtrsim 10^{27}$ entering the Earth’s atmosphere could produce infrasound and seismic acoustic waves as discussed in \cite{34,36} when the infrasound and seismic acoustic waves indeed have been recorded by dedicated instruments\footnote{A single observed event properly recorded by the Elginfield Infrasound Array (ELFO) which was accompanied by correlated seismic waves is dramatically different from conventional meteor-like events. In particular, while the event was very intense it has not been detected by a synchronized all-sky camera network (visible frequency bands) which ruled out a meteor source. At the same time this event is consistent with interpretation of the AQN-induced event because the visible frequency bands must be strongly suppressed when AQN propagates in atmosphere \cite{34}.}.

When the same AQNs enter the region under the thunderstorm the manifestation could be much more profound as even typical (and much more frequent) AQNs with $B \sim 10^{25}$ can produce very strong observable effects. It was precisely the goal of the recent work \cite{7} where it was argued that the AQN entering the thunderclouds my explain the recently observed puzzling CR like events such as mysterious bursts observed by Telescope Array \cite{5,6}. The main focus of \cite{7} was the “direct” emission by the AQNs in form of the highly energetic positrons accelerated by a strong electric field, which is known to be present during the thunderstorms.

In contrast, the focus of the present work is the studies of possible “indirect” impact of the same AQNs when the positrons and electrons (from ionized atmospheric molecules) could serve as the triggers by igniting and initiating a large and strong lightning strike, which consequently may generate the EEs observed by AUGER.

The goal of this subsection is to explain the basic features of the AQNs when they enter the dense regions of the atmosphere and the annihilation processes start. The related computations originally have been carried out in \cite{37} in application to the galactic environment with a typical density of surrounding visible baryons of order $n_{\text{galaxy}} \sim 300 \text{ cm}^{-3}$ in the galactic center, in dramatic contrast with dense region in the Earth’s atmosphere when $n_{\text{air}} \sim 10^{21} \text{ cm}^{-3}$. We review these computations with few additional elements which must be implemented in case of propagation in the Earth’s atmosphere when the density of the environment is much greater than in the galactic environment.

The total surface emissivity due to the bremsstrahlung radiation from electrosphere at temperature $T$ has been computed in \cite{37} and it is given by

$$F_{\text{tot}} \approx \frac{16}{3} \frac{T^4 \alpha^{5/2}}{\pi} \sqrt{\frac{T}{m}}, \quad (2)$$

where $\alpha$ is the fine-structure constant and $m$ is the mass of the electron.
where $\alpha \approx 1/137$ is the fine structure constant, $m = 511$ keV is the mass of electron, and $T$ is the internal temperature of the AQN. One should emphasize that the emission from the electrosphere is not thermal, and the spectrum is dramatically different from blackbody radiation. A typical internal temperature of the AQNs for very dilute galactic environment can be estimated from the condition that the radiative output of Eq. (2) must balance the flux of energy onto the nugget

$$F_{\text{tot}}(4\pi R^2) \approx \kappa \cdot (\pi R^2) \cdot (2 \text{ GeV}) \cdot n \cdot v_{\text{AQN}},$$

where $n$ represents the baryon number density of the surrounding material. The left hand side accounts for the total energy radiation from the AQN’s surface per unit time as given by (2) while the right hand side accounts for the rate of annihilation events when each successful annihilation event of a single baryon charge produces $\sim 2m_p c^2 \approx 2$ GeV energy. In Eq. (3) we assume that the nugget is characterized by the geometrical cross section $\pi R^2$ when it propagates in environment with local baryon density $n$ with velocity $v_{\text{AQN}} \sim 10^{-3} c$. The factor $\kappa$ accounts for large theoretical uncertainties related to the annihilation processes of the (antimatter) AQN colliding with atmospheric molecules.

From (3) one can estimate a typical internal nugget’s temperature in the Earth atmosphere as follows:

$$T \sim 20 \text{ keV} \cdot \left( \frac{n_{\text{air}}}{10^{21} \text{ cm}^{-3}} \right)^{\frac{1}{4}} \left( \frac{\kappa}{0.1} \right)^{\frac{1}{4}},$$

where typical density of surrounding baryons is $n_{\text{air}} \simeq 30 \cdot N_m \simeq 10^{21} \text{ cm}^{-3}$, where $N_m \simeq 2.7 \cdot 10^{19} \text{ cm}^{-3}$ is the molecular density in atmosphere when each molecule contains approximately 30 baryons. Thus, in the atmosphere the internal nugget’s temperature $T \simeq 20$ KeV.

It strongly depends on unknown parameter $\kappa$. In this work for the order of magnitude estimates we adopt $\kappa \simeq 0.1$, similar to our previous analysis performed in [34, 37]. It also depends on the altitude $z$ as the density $n_{\text{air}}$ scales with altitude as $n_{\text{air}} \propto \exp(-z/h)$, where $h \simeq 8$ km. In case if environment contains high density of ions the parameter $\kappa$ effectively increases as the AQN attracts more positively charged ions from surrounding material, which consequently may effectively increase the cross section and the rate of annihilation (resulting in larger value of $T$). All these effects are very complicated at large $T$ and the corresponding discussions are well outside the scope of the present study.

Important characteristic of the AQN propagating in the air is the number of direct collisions of the atmospheric molecules with AQN per unit time:

$$\frac{dN_{\text{collisions}}}{dt} \simeq (\pi R^2) \cdot N_m \cdot v_{\text{AQN}} \simeq 10^{18} \left( \frac{N_m}{2.7 \cdot 10^{19} \text{ cm}^{-3}} \right) s^{-1},$$

where we use the same parameters we previously used in our estimate (4) for the internal temperature of the nuggets. The dominant portion of these collisions are the elastic scattering processes rather than successful annihilation events suppressed by parameters $\kappa$ as discussed in the text after (4).
Another feature we want to mention which is relevant for our present studies is the ionization properties of the AQN. Ionization, as usual, occurs in a system as a result of the high internal temperature $T$ as mentioned above. What happens with AQN when the internal temperature $T$ becomes sufficiently high is that a large number of positrons $\sim Q$ from the electrosphere of the anti-matter AQN get excited but remain bound to the system. These positrons are very weakly bound particles which can easily leave the system as a result of elastic collisions with surrounding molecules. The corresponding parameter $Q$ can be estimated as follows:

$$Q \approx 4\pi R^2 \int_0^\infty n(z, T)dz \sim \frac{4\pi R^2}{\sqrt{2\pi\alpha}} (mT) \left(\frac{T}{m}\right)^{\frac{1}{4}},$$

where $n(z, T)$ is the local density of positrons at distance $z$ from the nugget’s surface, which has been computed in the mean field approximation in \[37\] and has the following form

$$n(z, T) = \frac{T}{2\pi\alpha (z + \bar{z})^2}, \quad \bar{z}^{-1} \approx \sqrt{\frac{2\pi\alpha}{m}} \cdot \left(\frac{T}{m}\right)^{\frac{1}{4}},$$

where $\bar{z}$ is the integration constant is chosen to match the Boltzmann regime at sufficiently large $z \gg \bar{z}$. Numerical studies \[38\] support the approximate analytical expression \[7\].

Numerically, the number of weakly bound positrons can be estimated from \[6\] as follows:

$$Q \approx 1.5 \cdot 10^{11} \left(\frac{T}{10 \text{ keV}}\right)^{\frac{3}{4}} \left(\frac{R}{2.25 \cdot 10^{-5} \text{ cm}}\right)^2.$$

These positrons from electrosphere being in the equilibrium (when the AQN experiences a relatively small annihilation rate) will normally occupy very thin layer around the AQN’s quark core as computed in \[37, 38\]. However, in our case when the AQN enters the Earth’s atmosphere a large number of non-equilibrium processes (such as generation of the shock wave resulting from large Mach number) are expected to occur. Furthermore, the positron’s cloud is expected to expand well beyond the thin layer around the core’s nugget as a result of high temperature $T$. A typical capture radius $R_{\text{cap}}(T) \gg R$ when the positrons remain to be bound to the AQN can be estimated as

$$R_{\text{cap}}(T) \simeq \frac{\alpha Q(T)}{m} \sim 2 \text{ cm} \left(\frac{T}{10 \text{ keV}}\right)^{1/4},$$

where $Q$ is estimated by \[6, 8\]. The idea of this estimate is that the typical positron’s excitation energy $\sim T$ must be equilibrated by the positron’s binding energy $\sim \alpha Q/r$ for the particles to remain loosely bound to the AQN, which represents our estimate \[9\].

Precisely some of these weakly bound positrons may get accelerated to the MeV energies in the background of strong electric field and mimic the CR events (termed as the mysterious bursts) as suggested in \[7\] and overviewed in Sect. \[4\]. These events will be coined as the “direct” emissions because these positrons can be directly detected by a surface detector.
The same positrons from electrosphere can also serve as the “seed” particles which are always required to initiate a lightning flash. These non-relativistic positrons along with the electrons (which appear as a result of ionization of the atmospheric molecules) are coined as “triggers” in this work, because they are capable to trigger and ignite a special class of the unique and powerful lightning strikes to be discussed in details in Sect. 4 and which are accompanied by the EEs observed by AUGER.

In what follows we assume that, to first order, that the finite portion of positrons $\sim Q$ leave the system as a result of the complicated processes mentioned above, in which case the AQN as a system acquires a negative electric charge $\sim -|e|Q$ and get partially ionized as a macroscopically large object of mass $M \approx m_p B$. The ratio $eQ/M \sim 10^{-14} e/m_p$ characterizing this object is very tiny such that nuggets themselves do not change momentum nor trajectory under the influence of the electric field during the thunderstorm, and will continue to propagate with conventional for DM particles velocity $v_{AQN} \sim 10^{-3} c$.

3. Frequency of appearance

Here we want to estimate the total number of events which AUGER can record within the AQN proposal during 13 years of observations (since January 2004 until 15 May 2017). One should emphasize from the very beginning that our estimates which follow are the order of magnitude estimates as there are many unknowns as we discuss in course of the text. Furthermore, the observed 23 exotic events during 13 years (less than 2 events/year) implies that statistical fluctuations could be essential. The order of magnitude estimate presented below is consistent with the observed rate. This should be considered as a highly nontrivial self-consistency check of our proposal.

Indeed, a crucial question which needs to be answered was formulated in the first paragraph of the Introduction which addresses the puzzling rareness of the exotic events. In particular, an explanation of the EEs (in terms of the correlation with the lighting flashes) is obviously not a satisfactory nor sufficient one as the dominant portion ($\sim 99.9\%$) of the lighting events is not accompanied by the AUGER exotic events.

The estimate presented below explicitly shows that the key missing factor in the event rate puzzle might be related to the rareness of the AQN dark matter events. Precisely these events when the AQNs hit the AUGER detector area under the thunderclouds ignite the special class of the extremely rare lightning strikes which generate EEs.

One should emphasize from the start that all relevant parameters such as the nugget’s size distribution or DM density $\rho_{DM} \approx 0.3 \text{ GeV cm}^{-3}$ have been used in all our

4 The authors do not provide an exact number of lighting flashes being recorded by AUGER during 13 years of the observations. In our estimate we use similar mysterious events recorded by TA collaboration when 10 bursts have been recorded during 5 years of the observations on the area where number of lighting strikes was recorded on the level $\sim 10^4$ which represent 0.1% event ratio as quoted above.
previous studies not related to the AUGER exotic events and we are not attempting to modify any parameters of the AQN model to better fit the observed rate.

The starting point is the AQN flux which eventually determines the total number of exotic events within AQN proposal is as follows:

$$\frac{d\Phi}{dA d\Omega} = \Phi = 4 \cdot 10^{-2} \left( \frac{10^{25}}{\langle B \rangle} \right) \text{ events yr}^{-1} \text{ km}^{-2},$$

(10)

where $R_\oplus = 6371 \text{ km}$ is the radius of the Earth and $\Phi$ is the total hit rate of AQNs on Earth [22]:

$$\Phi \approx 2 \cdot 10^7 \left( \frac{\rho_{\text{DM}}}{0.3 \text{ GeV cm}^{-3}} \right) \left( \frac{v_{\text{AQN}}}{220 \text{ km s}^{-1}} \right) \left( \frac{10^{25}}{\langle B \rangle} \right),$$

(11)

where $\rho_{\text{DM}}$ is the local density of DM. The expected number $N_{\text{EE}}$ of the Exotic Events within the AQN framework can be estimated as follows:

$$N_{\text{EE}} \approx A \cdot T \cdot F \frac{d\Phi}{dA d\Omega},$$

(12)

where $A \approx 3000 \text{ km}^2$ is the effective area of AUGER array, $T \approx 13$ years is total time of observations, $F$ describes the fraction of time when the effective area $A$ has been under thunderclouds, $\Delta\Omega \approx 2\pi$ for isotropic flux of AQNs. Factor $F \approx 0.25 \cdot 10^{-2}$ has been estimated for the TA mysterious bursts [7], and we keep the same numerical value for the fraction $F$ for the order of magnitude estimates in present work as well.

Collecting all these numerical factors together we arrive to the following order of magnitude estimate:

$$N_{\text{EE}} \sim 25 \left( \frac{\rho_{\text{DM}}}{0.3 \text{ GeV cm}^{-3}} \right) \left( \frac{v_{\text{AQN}}}{220 \text{ km s}^{-1}} \right) \left( \frac{10^{25}}{\langle B \rangle} \right),$$

(13)

which is amazingly close to the number of exotic events $N_{\text{obs}} = 23$ recorded by AUGER. In spite of the fact that numerically Eq. (13) is indeed very close to the observed value, one should emphasize that this estimate should be treated as the order of magnitude estimate, at the very best due to many uncertainties which enter this estimate.

First, the parameters in Eq. (13) are in fact not precisely known. Essential parameters such as $\rho_{\text{DM}}$, $\langle v_{\text{AQN}} \rangle$, and $\langle B \rangle$ only have accuracy up to order one as the local flux distribution of DM and size distribution of AQN remain unknown to date. Similarly, the fraction $F$ may also deviate by factor of order one as the thunderstorm activity in these two locations could be different by factor of 2 or so. In addition, the total number of observed EEs could be larger by a considerable factor than recorded by AUGER because the selection criteria being used in [1–3] are very conservative, and may miss some small events which we call “direct” emissions and which are not accompanied by a lightning strike as we discuss in next section 4.

Furthermore, the statistical significance of 23 events is obviously small, and there are many uncertainties in selecting procedure and measurements which may lead to misidentification of the events. The uncertainties in particular may include such problems as the lack of the signal at the center of the footprint which may or may not be physical [1–3]. In addition, some “small” events do not pass all the quality cuts
requested for a reliable reconstruction of the events because many long -lastings signals do not have the peak in the data acquisition system (DAQ), to name just a few.

In spite of all these numerous uncertainties, we consider the order of magnitude estimate \((13)\) as a highly nontrivial consistency check for our proposal as the basic numerical factors entering \((13)\) had been fixed from dramatically different physics (including solar corona heating puzzle) and can easily deviate by large factor.

Precisely this estimate \((13)\) answers (within the AQN framework) the question formulated in the first paragraph of the Introduction: why the AUGER Exotic Events are so rare? The proposed answer is that the AQNs act as the triggers to initiate and ignite a special class of strong lighting strikes which generate exotic events observed by AUGER. The rareness of these exotic events is a consequence of the rareness of the AQN events with very tiny DM flux \((10)\).

4. AQNs as the triggers of the lightning flashes

In this section we formulate the basic idea of the proposal. We shall argue that the AQNs propagating under thunderstorm can initiate and ignite the special class of lighting strikes which consequently generate EEs recorded by \([1–3]\). Our main focus in this work is the very initial moment of the lightning events. As we review below all lightning strikes require some sort of “seed” particles which initiate the strike. It is normally assumed that these “seed” particles are capable to generate a very large electron density well above the critical value \(\simeq 3 \cdot 10^8 \text{cm}^{-3}\) to initiate the lightning \([39]\), see also review papers \([40–42]\) devoted to detail study of the lightning strikes. It is often assumed that conventional CR can serve as a source of required “seed” particles which initiate the lightning events. Our goal here is to argue that the AQNs are capable to provide much stronger injection of particles to ignite and initiate a special class of the lightning events which generate EEs. We show in subsection 4.2 that the AQN will ionize the surrounding region along its path such that the local electron density may exceed the critical value which is required to ignite the lighting strike. We also argue that the number of particles which can be emitted by AQNs and which can initiate the AQN-induced lightning events is much greater in comparison with the number of “seed” particles which could provide a conventional and frequent CR. But first, in next subsection 4.1 we overview the basic requirements for the lightning process to start.

4.1. Requirements for the lightning strikes: electric field and seed particles

In what follows we overview the basic requirements for the lighting strikes to occur. It represents a short detour from our main topic. However the corresponding parameters play a key role in our arguments in following subsection 4.2 devoted to study of the AQNs under the thunderstorms. We refer to review papers \([40–42]\) devoted to detail study of the lightning strikes. Here we list the basic conditions which must be satisfied for the lightning strikes to occur \([40][42]\):
a) A sufficiently strong electric field $E \gtrsim E_c$ must exist in thunderstorms for occurrence of runaway breakdown (RB in terminology [40]) or relativistic runaway electron avalanche (RREA in terminology [41]). Numerical value for $E_c$ is given below, see (16).

The basic idea here of these processes is as follows. When the charged particles move in the background of strong electric field the rate the particles gain the energy from electric field exceeds the rate that these particles lose energy through the interaction with the air. These so-called runaway particles propagate through the air produce secondary particles by hard elastic scattering with the atomic electrons, resulting in an avalanche of runaway electrons that grows exponentially with both time and distance. The initial particles which may generate such exponential growth called seed particles, which could be electrons or positrons.

b) The spatial scale $L_E$ of a electric field $E$ in thunderstorm must substantially exceed the scale $l_a$ for the exponential growth of the avalanche, i.e $L_E \gg l_a$ as argued in [40,41]. Numerical value for $l_a$ is given below, see (17);

c) The presence of fast seed particles with energies exceeding the critical runaway energy $E > E_c$ is absolutely necessary for initiation of the RB process. The numerical value for $E_c$ is estimated in terms of the electric field $E \gtrsim E_c$ as follows:

$$E_c \approx (mc^2) \cdot \left( \frac{E_c}{2E} \right).$$

(14)

In other words, the conditions a) and b) are insufficient for RB process. It is assumed [40] that the high energetic CR particles with energy $E \gtrsim (10^{15} - 10^{16})$ eV normally generating an extensive air shower (EAS) with $(10^6 - 10^7)$ secondary electrons and positrons are capable to serve as the seed particles. These seed particles from CR will initiate and ignite the lighting strike. It was precisely the reason to coin this complicated process as the RB-EAS discharge mechanism.

The basic idea is that this process may generate the local electron density above the critical value [39]:

$$n_{\text{critical}} \simeq 3 \cdot 10^8 \text{cm}^{-3},$$

(15)

which generates a highly conductive region and initiates the lightning flash in the presence of a strong electric field.

We start by quoting the so-called critical electric field $E_c$ which must exist in thunderstorms for occurrence of RB [40] or RREA [41]:

$$E_c = (2.16 - 2.84) \frac{\text{kV}}{\text{cm}} \exp \left( -\frac{z}{h} \right), \quad h \approx 8 \text{ km}.$$  

(16)

Such strong (and even stronger) fields are routinely observed in atmosphere using e.g. balloon measurements. Another important characteristic is the avalanche scale $l_a$ and
the corresponding time scale $\tau_a$ for the exponential growth, which are numerically estimated as $[40,41]$: 

$$l_a \simeq 10^2 \text{ m}, \quad \tau_a \simeq \frac{l_a}{c} \sim \text{(fraction of) \ mu s}.$$  \hspace{1cm} (17)

The characteristic scale $l_a$ represents the minimum length scale when the exponential growth of runaway avalanche occurs. The scale $L_E \gg l_a$ essentially determines the allowed scale of the inhomogeneity and non-uniformity of a fluctuating electric field for the exponential growth to hold for sufficiently long time.

The significance of the scale (17) is related to the fact that precisely this scale determines the efficiency of the RB process as the intensity of the strike is directly related to the exponential growth of the distribution function $f$ in time $[40,42]$: 

$$f \propto \exp \left( \frac{t}{\tau_a} \right), \quad N_{\text{e-fold}} \equiv \left( \frac{t}{\tau_a} \right), \quad t \gg \tau_a$$  \hspace{1cm} (18)

when number of e-foldings $N_{\text{e-fold}}$ could assume very large values as parameter $\tau_a$ is measured in fraction of a microsecond.

While there is a consensus on typical parameters of the electric field $E_c$ and avalanche scales $l_a, \tau_a$ which are important characteristics for the lightning dynamics (items “a” and “b” above), the physics of the initial moment of lightning and the source of the required seed particles (item “c” above) remains a matter of debate, and refs $[40,41]$ represent different views on this matter, see also relatively recent articles $[44-47]$ where some specific elements of existing disagreement have been explicitly formulated and debated.

We do not wish to be involved in these ongoing debates as our main goal is not related to study of the source, dynamics and mechanisms related to the conventional and very frequent lightning events. We have nothing new to say on this matter. Rather the main goal of this work is to argue that the number of particles which can be generated by very rare AQNs traversing the thunderclouds is many orders of magnitude higher than conventional CR with energy $E \gtrsim (10^{15}-10^{16})\text{eV}$ can provide.

Therefore, for our specific purposes the disputable elements do not play any role in our studies as we simply compare the basic characteristics (such as number of injected particles, injected energy, typical time scales, etc) between AQN-generated and CR-generated “seed particles”. This comparison unambiguously shows that AQN can serve as a trigger of a very special, unique and powerful (but very rare) event as number of initially injected particles is many orders of magnitude greater than analogous characteristics which conventional CR with energy $E \gtrsim (10^{15}-10^{16})\text{eV}$ can provide.

Therefore, it is naturally to expect that such AQN-induced lightning strikes should have some very distinct features which are dramatically different from conventional and very frequent CR-induced lightning strikes. We conjecture that the AUGER exotic events $[1,3]$ precisely represent these distinct features. The first argument that this idea is at least a self-consistent proposal was presented in previous section $[3]$ where it was argued that the computed event rate agrees well with the observed rate. Below we list the basic characteristics of a conventional CR with energy $E \gtrsim (10^{15}-10^{16})\text{eV}$
which according to [40] can initiate a lighting strike. It represents the benchmark in our discussions which follow. In next subsection 4.2 we present some estimates suggesting that the dynamical characteristics of the AQNs which can serve as a trigger of a special class of the lighting events are much more powerful in comparison with these benchmark parameters listed below.

In what follows we take for simplicity a CR with energy of a primary particle $E \simeq 10^{15}$eV. This CR event is approximately characterized by the following benchmark parameters relevant for this work:

$$E \simeq 10^{15} \text{eV}, \quad N_{e^+e^-} \sim 10^6, \quad \langle E_{e^+e^-} \rangle \sim 30 \text{ MeV}. \quad (19)$$

Important point here is that the EAS could traverse the region with large electric field (16), (17) characterized by distance $L_e \gg l_a$ on a µs scale which is a typical required time duration for the RB-EAS processes to start. This process may ignite and initiate a typical lighting strike.

In next subsection 4.2 we compare parameters (19) with analogous values describing the AQN propagating under the thunderstorm. We shall see that the AQN generates much more particles in comparison with a typical CR benchmark parameters (19). Therefore, the AQN-induced lighting strikes could be dramatically different (though much more rare) events in comparison with typical CR-induced lighting flashes. We emphasize once again: the present subsection represents a short detour from our main topic and the parameters (19) will be used exclusively as a benchmark for illustrative purposes only with main intension of comparison them with similar parameters resulted from the AQN propagating under the thunderclouds.

4.2. The AQNs under a thunderstorm

In this subsection we study the dynamics of the AQN under influence of the pre-existing electric field characterized by parameters (16), (17) as reviewed in previous subsection. There is a number of phenomena which dramatically affect the initial moments of the lighting strikes as a result of sudden injection of a large number of particles (electrons and positrons) when the AQN enters the region of a strong electric field. First of all, some positrons from electrosphere will leave the system as a result of this electric field. This effect will be discussed in subsection 4.2.1 where we estimate the number of positrons being liberated from AQNs, their mean-free path, and the corresponding energy spectrum. Another profound AQN-induced effect is related to the ionization when the electrons are kicked out from the atmospheric molecules and released to the surrounding air. We estimate the rate of the ionization, the spectral features and the fate of these short lived electrons in subsection 4.2.2.

4.2.1. Fate of the AQN-induced liberated positrons. We start with the discussions of the positron’s fate because the effect of the positron’s liberation from electrosphere has been previously discussed in great details in relation to the so-called mysterious bursts, the CR-like events recorded by TA collaboration [5,6]. To be more specific, it has been
argued in [7] that the positrons will be liberated and accelerated to ∼10 MeV energy by the electric field \( E \sim \text{kV/cm} \) when the AQN propagates under the thunderstorm. These positrons can mimic the CR events and can be interpreted as the TA mysterious bursts. This interpretation is consistent with the observed intensity, timing and the frequency of appearance. In the present work we want to argue that the same AQN-induced positrons may also play another role as a trigger which initiates and ignites much larger in scale a lighting strike.

We begin our discussions with the following question: what happens to the weakly coupled positrons \([8]\) localized at distance \( R_{\text{cap}}(T) \gg R \) when the AQN enters the region of a strong electric field characterized by parameters \([16], [17]\)? The answer suggested in [7] is as follows: a sudden appearance of strong external electric field \( E \sim \text{kV/cm} \) along the AQN’s path will inject an additional energy \( \Delta E \) to the positrons estimated as

\[
\Delta E \simeq [eE \cdot R_{\text{cap}}] \sim 2 \text{ keV} \gtrsim E_{\text{bound}}. \tag{20}
\]

This energy injection of order of several keV could liberate the weakly bound positrons from the electrosphere. At this moment a finite portion of order \( Q \) of the weakly bound positrons will be liberated to atmosphere with typical kinetic energy of order \( T \sim 10 \text{ keV} \). These released positrons find themselves in the background of strong electric field \( E \) characterized by typical length scale \( l_a \simeq 10^2 \text{ m} \) according to \([17]\). This pre-existing electric field will accelerate the positrons to \( \sim 10 \text{ MeV} \) energies and even higher. Indeed,

\[
E_{\text{exit}} \simeq [eE \cdot l_a] \sim 10 \text{ MeV}. \tag{21}
\]

We expect that a finite portion of the released positrons \( (r \sim 15\%) \) is likely to be accelerated to ultra-relativistic energies \([21]\), see Appendix A.2 for the details. This portion of the positrons will move in the same direction which is entirely determined by the direction of the instantaneous electric field \( \vec{E} \) at the moment of exit. We termed in the Introduction these positrons as the “direct” ones because these energetic positrons have large mean free path and can easily reach a surface detector, producing a direct signal which can be recorded.

Precisely these positrons have been identified in [7] as a possible source of the mysterious bursts studied by the TA collaboration [5, 6]. The arguments supporting this identification are based on estimation of the total number of the released positrons.

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6 It is interesting to note that according to [41] the positrons play a key role in development of the avalanche in RREA framework due to much longer mean free path in comparison with electrons. The source of the positrons in our framework and in [41] is completely different.

7 The acceleration process can be roughly divided to two different regimes: the acceleration due to the shock waves when the positrons are accelerated from energies \( \sim 10 \text{ keV} \) to approximately relativistic velocities \( \sim 1 \text{ MeV} \), and relativistic regime when the positrons assume their high energy values \( \sim 10 \text{ MeV} \), see Appendix A.2 for the details. One should comment here that the formation of the shock waves due to the moving AQN is very generic feature of the framework because the Mach number \( M \equiv v_{\text{AQN}}/c_s \gg 1 \), where \( c_s \) is speed of sound in atmosphere. Therefore, a conventional Fermi’s mechanism of a charged particle acceleration due to the shock wave directly applies here.
\( \sim Q \) (which unambiguously predicts the intensity of the TA signal), the timing, the event rate and other characteristics which match the observations quite well. These “direct” positrons could be also the source of the small events as recorded by AUGER which are characterized by relatively small size 2-3 km on the surface similar to TA mysterious bursts, see item 3 in Introduction.

The new element here is that the very same positrons from electrosphere can also serve as the seed particles, which is absolutely required element “c” listed at the beginning of Sect 4.1. These initially \( \sim 10 \text{ keV} \) energy positrons released to atmosphere are capable to get accelerated to relativistic energies \( (21) \). As such these positrons are coined as the “triggers” below, because they are capable to trigger and ignite a special class of very powerful lightning strikes. Indeed, it has been known that the positrons can play an important role in development of the avalanche in RREA framework due to much longer mean free path (in comparison with electrons), see review \( [41] \). Our comment here is that the almost instantaneous injection of the large number of positrons may dramatically modify the dynamics of the system during the initial moments of the lighting strike. In particular, it may lead to the EEs recorded by AUGER \([1-3]\).

To put this claim in a more quantitative way one should compare the conventional characteristics of the seed particles due to the CR as given by \( (19) \) and the AQN-induced positrons. The typical number of AQN-induced particles could be as large as \( Q \sim 10^{11} \) according to \( (8) \) to be compared with CR-induced \( N_{e^+e^-} \sim 10^6 \) according to \( (19) \). While the AQN- induced mechanism suggests that initial energy of positrons will be of order \( \sim 10 \text{ keV} \), the finite portion of them could get accelerated to \( E_{\text{exit}} \sim 10 \text{ MeV} \) on a time scale of \( \mu \text{s} \), see Appendix \( \text{Appendix A} \) for the details. This should be compared with CR-induced energies \( \langle E_{e^+e^-} \rangle \sim 30 \text{ MeV} \) according to \( (19) \). Therefore, the AQN-induced positrons may dramatically modify the initial moments of the lightning strikes because the number of injected particles could be 5 orders of magnitude larger in comparison with conventional CR-induced parameters \( (19) \) with similar average energy of the particles.

One can interpret this huge enhancement factor in terms of the number \( N_{e^{-}\text{-fold}} \) of e-foldings defined by \( (18) \). The claim is that the traversing AQN can almost instantaneously inject into the system the large number of particles which is numerically equivalent to the developed RB-EAS mechanism with

\[
N_{e^{-}\text{-fold}}(e^+) \approx \ln Q \approx 25. \tag{22}
\]

A finite portion \( r \sim 0.15 \) of these positrons may even serve as seed particles as argued in Appendix \( \text{Appendix A.2} \). It obviously dramatically changes the initial moments of evolution of the lightning strike because the injection of these positrons occur almost instantaneously, in contrast with conventional relatively slow development of the exponential growth which would require relatively long time determined by \( N_{e^{-}\text{-fold}}(e^+) \).

We identify these AQN induced events with rare Exotic Events recorded by AUGER \([1-3]\) as argued in next Sect. 5.
4.2.2. Fate of the AQN-induced electrons. Now we turn to the analysis of the fate of the electrons which appear as a result of collisions of the AQN with atmospheric molecules when it propagates under the thunderstorm.

These collisions may ionize the atmospheric molecules because the AQN is a macroscopically large object with charge $Q$ estimated by $\langle 8 \rangle$. In this case the direct collisions of the AQN with atmospheric molecules can liberate the electrons such that molecules become positively charged ions. Indeed, the AQN’s negative charge $\langle 8 \rangle$ implies the presence of the internal electric field $E_{\text{AQN}}(r) \sim eQ/r^2$ surrounding the nuggets. Precisely this internal electric field due to the negatively charged quark’s core may ionize the atmospheric molecules at the moment of collision.

Indeed, when the corresponding electric potential $U_{\text{AQN}}(r)$ outside the nugget’s core becomes sufficiently strong (order of several electron-volts) the electrons from neutral molecules could be liberated and become free electrons. The corresponding $U_{\text{AQN}}(r)$ can be estimated as follows:

$$ U_{\text{AQN}}(r) \sim \frac{\alpha Q \exp \left( -\frac{r}{\lambda_D} \right)}{r} \sim T \frac{R_{\text{cap}} \exp \left( -\frac{r}{\lambda_D} \right)}{r}. \quad (23) $$

In equation (23) the internal temperature $T$ is estimated by $\langle 4 \rangle$, the capture radius $R_{\text{cap}}(T)$ is given by $\langle 9 \rangle$, while the Debye screening length $\lambda_D$ is determined, as usual, by expression

$$ \lambda_D \equiv \sqrt{\frac{T_{\text{atm}}}{4\pi\alpha N_{\text{ion}}}} \approx 0.4 \text{ cm} \sqrt{\left( \frac{T_{\text{atm}}}{300 \text{ K}} \right) \left( \frac{10^5 \text{cm}^{-3}}{N_{\text{ion}}} \right)}, \quad (24) $$

where in our numerical estimates we assume a typical (for RB atmospheric conditions under the thunderclouds) density of ions $N_{\text{ion}} \sim 10^5 \text{cm}^{-3}$, see e.g. $\langle 40 \rangle$. From this expression is quite obvious that the strong electric potential on the level of eV scale extends to relatively large distance $r \sim$ few cm where atmospheric molecules can get ionized as a result of a strong AQN-induced electric potential $\langle 23 \rangle$. Indeed, one can estimate that

$$ U_{\text{AQN}}(r) \simeq 10 \text{ eV at } \bar{r} \sim 3 \text{ cm}. \quad (25) $$

If ionization occurs closer to the nugget, the potential $U_{\text{AQN}}(r)$ becomes much stronger and liberated electrons could become much more energetic.

One should mention that the dominant portion of these liberated free electrons by the mechanism described above will obviously have typical energies in the eV range which represents a typical binding energy in atoms and molecules. These free electrons (thermal electrons in terminology $\langle 40 \rangle$) are characterized by very short life time of order $10^2 \text{ns}$ and disappear rather rapidly as a result of interaction with air molecules, see e.g. $\langle 40 \rangle$. The dominant portion of the liberated AQN-induced electrons cannot serve as seed particles, in contrast with the AQN-induced positrons discussed above because of very low initial energy of these electrons, see Appendix $\langle \text{Appendix A.3} \rangle$ with detail explanation. However, they could play a crucial role in initiating and triggering a unique and powerful lighting event (even though they cannot serve as the seed particles). This
is because the very large number of electrons being injected into the system almost instantaneously as estimated below already exceeds the critical density \( \rho_c \) without necessity for the avalanche exponential growth to develop. The intensity of this jolt and the electron density it generates can be estimated as follows.

We assume that every molecule hitting the nugget looses at least one electron by the mechanism described above. We consider it is very conservative assumption as the presence of a strong electric potential \( E \) is very generic and model independent feature of the AQN framework being a direct consequence of the internal temperature \( T \) of the nuggets. With this assumption we arrive to the following estimate for the number of electrons suddenly (on \( \mu s \) time scale) injected into the system:

\[
N(e^-) \sim \frac{dN_{\text{collisions}}}{dt} [0.1\mu s] \sim 10^{11} \left( \frac{N_m}{2.7 \cdot 10^{19} \text{ cm}^{-3}} \right),
\]

where we used formula (5) for the rate of the AQN collisions with atmospheric molecules, and \([0.1\mu s]\) as a life time (attachment time) of a free thermal electron which rapidly disappears as a result of interaction with air molecules, see [40]. The same time interval \([0.1\mu s]\) is also a typical time scale of the RB process as given by (17).

The large number of electrons \((26)\) instantaneously injected into the system can be interpreted in terms of the \( N_{e^{-}\text{-fold}} \) similar to our interpretation for the positrons from previous subsection:

\[
N_{e^{-}\text{-fold}}(e^-) \approx \ln N(e^-) \approx 25.
\]

One should emphasize that these electrons are low energy electrons and cannot serve as the seed particles as already mentioned. The formula (27) is given here exclusively for the illustrative purposes to interpret the large number suddenly injected electrons in terms of the e-folding which is conventional way to describe efficiency of the avalanche exponential growth. Sudden injection of large number of electrons obviously dramatically changes the initial moments of evolution of the lightning strike because the injection of these electrons occur almost instantaneously (on time scale of order 0.1\( \mu s \) according to (26), in contrast with conventional relatively slow development of the exponential growth which would require relatively long time determined by \( N_{e^{-}\text{-fold}}(e^+) \).

One can represent this enormously powerful jolt of low energy electrons in terms of the electron number density \( n_e(r) \) surrounding a slowly moving nugget. For an order of magnitude estimate one can use a simple picture when a moving nugget during time \( t \) is filled by electrons along its path in volume of the cylinder \((\pi \bar{r}^2) \cdot (v_{AQN}t)\). One can show that the released electrons mostly stay in the same location where they had been liberated as the diffusion coefficient is small, see Appendix Appendix A.4 with details. Therefore, we arrive to the following estimate for the average density of the electrons surrounding the nugget:

\[
\langle n_e(r) \rangle \sim \frac{[dN_{\text{collisions}}/dt] \cdot t}{(\pi \bar{r}^2) \cdot (v_{AQN}t)} \sim 10^9 \text{ cm}^{-3},
\]

where \( \bar{r} \sim 3 \text{ cm} \) as estimated in (25). One should emphasize that this is very conservative estimate and the local electron density could be much greater than (28) as already
mentioned in deriving formula (26).

It is instructive to compare the density (28) with studies [44] where it has been shown that the upper limit on low energy electron density resulting from runaway electron avalanches produced by a $10^{17}$eV CR shower lies in the window $n_e \in (10^4 - 10^7) \text{cm}^{-3}$, see Fig 9 in [44]. It is many orders of magnitude smaller than the local instantaneous density (28) produced by AQN which may ignite and initiate a very powerful (but very rare) lightning event.

Significance of the relation (28) is that the electron density surrounding the nugget exceeds the critical value (15). Such high electron density dramatically modifies the polarization processes and changes the distribution of the thunderstorm electric field. As a result, it generates a highly conductive region and initiates the lightning flash [39,44]. The RB and RREA processes of the exponential growth of the particles are not even required in this case as (28) is already sufficient to start the lightning. This jolt of electrons (26) producing enormous electron density (28) obviously must dramatically change the initial moments of evolution of the lightning strike. We identify the unusual and powerful AQN-induced lighting strikes with exotic events being recorded by AUGER.

5. AUGER Exotic Events as the AQN-induced phenomena

Our main goal here is to argue that the very unusual features of the Exotic Events as listed by items 1-7 in the Introduction could be explained within the AQN framework when these events are induced by the AQN propagating under thunderclouds. As explained in previous section we identify the Exotic Events with rare events of the AQNs propagating under the thunderclouds, i.e.

\begin{equation}
\text{AUGER Exotic Events} \equiv \text{AQN} - \text{induced events.}
\end{equation}

The corresponding annihilation events due to interaction of the anti-matter nuggets with atmospheric molecules generate enormous injection of the positrons and electrons into the surrounding region. The corresponding effects can be expressed in terms of the e-fold numbers $N_{e\text{-fold}}$ given by (22) and (27) correspondingly. Furthermore, the same annihilation events generate enormous electron density along the AQN path as given by (28). We coin these injected particles as the triggers because they can trigger, ignite and initiate a very powerful (and very distinct) lighting strike if other conditions, such as presence of a strong electric field along the AQN path, are met. If the AQN-induced lighting strike indeed does occur, it must be very distinct from conventional and typical flashes as the initial moments of the AQN-induced strike are dramatically different from typical and much more frequent lightning strikes induced by any conventional possible sources such as cosmic rays.
5.1. AQN proposal (29) confronts the observations

The first argument supporting our identification (29) is the estimation of the frequency of appearance of these events (item 2) which is consistent with our estimate presented in Sect. 3. One should emphasize that this is a very nontrivial consistency check (or “mysterious coincidence” for sceptics) as the event rate $N_{EE}$ as given by (13) is expressed in terms of apparently unrelated parameters such as the DM density. However, there are much more arguments which seemingly support this unorthodox proposal.

From our description of the physics related to the AQN propagating under the thunderclouds it is quite obvious that some events may ignite the lighting strikes, and some cannot, depending on other features of the thunderstorm at the moment the AQN hitting the area. It explains the item 6 from the list that not all the EE are 100% correlated with the lighting strikes.

The emergence of the cluster-like events representing the item 7 is also a very generic feature of the identification (29). Indeed, the AQN traversing the thunderclouds propagates just $v_{AQN} \cdot \text{ms} \sim 3 \cdot 10^2 \text{m}$ which is much shorter than a typical scale of a thunderstorm cloud system. This implies that the AQN remains in the same area where a lightning event is generated. Therefore, it is not a surprise that the cluster-like events separated by a $[\text{ms}]$ time interval are localized in the same zone of the array.

The “large” events described in item 3 are identified with events when the AQN successfully initiates the lighting strike which must be dramatically different from typical and much more frequent lightning strikes. Indeed, in this case few first lighting “steps” are initiated by enormous initial injection of the particles characterized by (22), (27) which are equivalent to well-developed phase of a conventional typical strike. The enormous electron density exceeding the critical value (28) may also play a role of igniting the rare and very unique strikes. The EE could be a consequence of these initial very powerful “steps”. The enormous scale of the “large” events is determined by the energy of the lighting itself, not by the annihilation energy released by the AQNs.

The “small” events also mentioned in item 3, on the other hand, could be related to the energetic positrons directly emitted by AQNs, similar to our interpretation [7] of the TA bursts, see next subsection 5.2 with more comments on this possible connection.

To avoid confusion with the interpretation one should emphasize that the electric field in a thunderstorm area is a strongly (temporal and spatial) fluctuating field. Therefore, the direction of the current (or the leader) of a large developed lightning event in general does not coincide with direction of the instant fluctuating electric field, which determines the direction of the energetic positrons emitted by AQNs [7]. Therefore, the “large” and “small” events are not necessary recorded at the same instant by the surface detector as the electric field generating the “small” events is likely to point in a different direction at the moment of emission. To rephrase it, an AQN plays the dual role when it propagates in the thunderclouds: it emits the very energetic positrons (“small” events), and it also triggers the lightning (“large” events). The direction of current (and hot conducting leader) in large event and direction of the bunch of relativistic particles in
small event are orientated randomly as they are induced by two different electric fields in two different regions of the thundercloud at slightly different instances separated by \( \sim ms \) time interval.

Nevertheless, one should expect some sort of correlation between “large” and “small” events. Such correlation indeed has been recorded as described in item 4. This correlation within [ms] time interval finds its natural explanation in this proposal because both phenomena are essentially originated from the same AQN within the same region of the thunderstorm activity. It could be enormous variation in scales for “large” events in intensity and in size of the footprints as these are related to the few initial “steps” of the lighting flashes. It explains the item 5. It should be contrasted with “small” events which are characterized by the internal physics of the AQNs and should not demonstrate enormous variations in shapes or scales of the footprints.

In our proposal (29) the time duration of a “large” signal is determined by the time scale of the few initial lighting “steps” which are triggered and initiated by enormous injection of the particles characterized by (22), (27) accompanied by enormous electron density along the AQN path, exceeding the critical value (28). As the inter-step intervals last some tens of \( \mu s \), the same time scale should characterize the EE which is consistent with observations listed in the item 1.

Relatively long time duration of a “small” signal on the level \((4−7)\mu s\) is determined by features of the bunch of emitted positrons in the background of the electric field, similar to our discussions in [7] in application to the mysterious bursts recorded by TA. Typical time scale for “small” events should be similar in scale (but somewhat smaller) than a time variation \(\tau_E\) of the of the electric field \(E\) in thunderclouds. This is because the acceleration of the positrons in this framework is due to this instantaneous field \(E\). The electric field \(E\) flips its direction on the time scale \(\tau_E \sim 20\mu s\) [41], which is consistent with observed time scale \((4−7)\mu s\) of “small” events.

5.2. Relation to TA bursts and downward TGF

In this subsection we want to make few remarks on possible relation between the AQN-induced events which is the topic of the present work and “mysterious bursts” [5,6] and TGFs [8,10] as observed by Telescope Array Collaboration. We want to argue that these unique, very rare, and unusual events represent different manifestations of the same phenomena when the AQNs enters the thunderstorm clouds and initiate very special, fast, powerful and unique lighting events.

The TA “mysterious bursts” are defined as the events when at least three air shower triggers were recorded within 1 ms, which would be a highly unlikely occurrence for three consecutive conventional CR hits in the same area within a radius of approximately several kilometres. Therefore, this clustering feature of the “mysterious bursts” is hard to interpret in terms of the conventional CR events. This feature in all respects is very similar to item 7 as discussed in previous subsection in connection with Exotic Events recorded by AUGER. Another distinct feature of the TA “mysterious bursts” is
the correlation with the lighting flashes, which is also very similar to item 6 from the previous subsection devoted to EE.

One should also note that the “small” events recorded by AUGER with typical size of the footprints $\sim (2 - 3)$ km (as listed in item 3) are very similar to the TA “mysterious bursts” with the same size scales of the footprints as mentioned in the previous subsection [5.1]. According to our interpretation these signals in both cases is a result of the same direct emission of the positrons by AQNs traversing the thundercloud region. Therefore, they must be characterized by the same features. In particular, these events should have well-localized footprints, which should not vary much from event to event, in contrast with “large” events as discussed in the previous subsection [5.1]. It is hard to say why TA collaboration have not recorded the signals similar to the “large” energetic events similar to EEs. It could be that TA collaboration has much lower photon detection efficiency as suggested in [2].

Further to the point on similarity between the two phenomena. The event rate of the TA “mysterious bursts” as estimated in [7] is consistent with frequency of appearance for the EE discussed in Sect. 3. Both estimates are based on one and the same formula [12] with identical hit rate determined by the DM flux $\Phi$. The consistency between both event rates is very encouraging sign that both phenomena are indeed related to the same physics of the DM nuggets.

This extremely low event rate is a very puzzling feature in both cases if one regards these events as a mere consequence of the lighting strikes because such a simplified view does not address the crucial element of the puzzling feature which is their extreme rareness in comparison with much more frequent conventional lighting strikes [8].

We now turn to the possible relation between TGFs [8–10] as observed by TA Collaboration and EE as recorded by AUGER, which is the topic of the present work. TGFs are bursts of gamma-rays initiated in the Earths atmosphere, first reported in 1994 by a satellite. Since then, a number of observations have shown that satellite-detected TGFs are produced by upward intra-cloud flashes. Similar TGFs have been also observed by TA collaboration [8–10] which are identified as the downward breakdown that occurs at the beginning of cloud to ground flashes.

The relation between TA “mysterious bursts” and TGFs has been already stated in [8–10]. Therefore, our identification of the TA bursts with the AQN-induced events automatically implies (within the same framework) that the TGFs recorded by TA must be also related to the same unique and very rare lightning strikes triggered by the AQNs traversing the thundercloud region.

The only additional comment which requires a clarification here is that the TGFs are originated from special type of flashes which are very rare and very unique, as

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8 The exact number of lightning strikes has not been recorded by AUGER. However, in case of TA mysterious events this ratio is known: during 5 years of observations the TA Collaboration recorded just 10 “mysterious events” to be compared with $\sim 10^5$ conventional lighting strikes during the same period of time recorded on the same area. This implies that 99.9% of conventional lightning strikes do not generate EE observed by AUGER.
emphasized above. The corresponding features are not shared by conventional and much more frequent lighting strikes. What makes these rare lightning strikes to become so special? Our proposal is that an instantaneous injection of large number of positrons and electrons (at the initial stage of the strikes) as represented in terms of the e-foldings (22) and (27) being accompanied by enormous electron density exceeding the critical value (28) may trigger the lighting strikes with such unusual features. Corresponding supporting arguments (suggesting that the AQN-induced lightning strikes must be dramatically different from conventional and much more frequent flashes) were presented in previous subsection 5.1.

We conclude this section with the following remark. As we already mentioned the main focus in this work is the very initial moment of the lightning events. It is not the goal of the present work to discuss the evolution (and complicated physics) during the later stages of the strikes triggered and initiated by enormous jolt of the AQN-induced particles. Such studies are obviously well beyond the scope of the present work. Our goal here is fundamentally different: we want to point out that the evolution of the lightning events (including RR [40] or RREA [41,42] processes) could be dramatically modified as a result of an initial injection of large number of positrons and electrons as given by (22) and (27) when AQNs traversing the area under thunderclouds. The generation of an enormous electron density exceeding the critical value (28) along the AQN path may also drastically alter the conventional dynamics of the lightning events. We propose that this injection of particles ignites and initiates a special and unique class of the lightning events which are identified with the AUGER Exotic Events, according to (29).

6. Conclusion and future development

Our conclusion is divided into three different subsections: first of all, in Sect. 6.1 we list the basic claims of this work, in Sect. 6.2 we describe several tests to support or refute the proposal (29). Finally, in Sect. 6.3 we list a number of other instruments which may detect the excess of emission due to the AQN annihilation events in dramatically different environments (Early Universe, galactic scale, solar corona, Earth’s atmosphere).

6.1. Basic Claims

Our basic results can be summarized as follows. We proposed that the initial injection of large number of positrons and electrons as estimated by (22) and (27) may trigger and dramatically modify the lightning flashes when AQNs traversing the area under thunderclouds. The electron density exceeding the critical value (28) along the AQN path may also ignite and initiate a unique and special class of the lightning flashes. According to our proposal this unique class of the lightning events recorded by AUGER are identified with the AQN induced events (29). The event rate as estimated by
is consistent with the number of exotic events $N_{\text{obs}} = 23$ recorded by AUGER. Therefore, the very puzzling feature of extreme rareness of the EE in comparison with much more frequent conventional lighting strikes finds its natural resolution within the AQN framework as it is determined by very tiny DM flux $\langle 10 \rangle$, see also footnote 8.

6.2. Possible future tests

In this subsection, we want to discuss possible tests which can support or refute our proposal on identification AUGER Exotic Events with the AQN-induced events (29). Some of the ideas have been already mentioned in our paper \cite{7} where we argued that the TA “mysterious bursts” is one of the manifestations of the AQN-induced events. As these bursts are identified with “small” AUGER Exotic Events, the same tests also apply here. We shall not repeat these suggestions in the present work by referring to the original paper \cite{7}.

We think that the most unambiguous test which can discriminate our proposal (29) from any other suggestions is the study of the radio pulses (as suggested in [48]) which always accompany the AQNs traversing under the thunderclouds. The corresponding radio pulses are dramatically different from conventional radio signals during thunderstorms and from CR events.

Indeed, it has been known for many years that the lightning flashes are always accompanied by the radio emission. This is a very generic feature of the the lightning discharges and it is well documented, see e.g. \cite{49} with large number of references on the original results. We want to mention some of these well known features in the text below to compare with the similar properties of the radio pulses accompanying the AQN-induced events as computed in [48].

It is known that the lightning discharges are characterized by a very large number of radio pulses which normally last in total for about 1 second. Each pulse is characterized by full width $(0.2 - 0.3)\mu s$. The electric field strength of these pulses could be as large as $|E| \sim 10^3 \text{ mV/m}$ at distance about 10 km. Most of the pulses, though, show the strength of the electric field in the $|E| \sim (100 - 200) \text{ mV/m}$ range. Another important feature of the radio emission the lightning discharges is as follows. The gaps between pulses are in the range $(10 - 10^2)\mu s$. Therefore, total number of pulses could be very large $\gtrsim 10^3$ during a single lightning event. Finally, the key element of the the radio emission during the lightning discharges is its typical frequency band. The radiation is strongly peaked in few MHz bands, while it completely diminishes for $\nu \gtrsim 10 \text{ MHz}$.

These features must be contrasted with the AQN-induced radio pulses studied in [48]. Indeed, the number of clustered radio pulses associated with the AUGER Exotic Events (and also with TA mysterious bursts) must be very few to correspond to several events which often accompany the “large” signal, see item 4 from the list. It should be contrasted with $\sim 10^3$ in case of the conventional lightning-induced radio pulses.

The most important distinct feature which discriminates the different sources of the emission is that the frequency bands of the radiation are dramatically different for
EE in comparison with conventional lightning flashes, which are strongly peaked in few MHz bands, as mentioned above. The AQN-induced radio pulse is characterized by the flat spectrum with $\nu \lesssim 200$ MHz according to [48]. The basic reason for this dramatic difference is that the electric current responsible for a typical lightning flash is dominated by the particles with $\gamma \sim 1$ while for the AQN-induced case the injection and acceleration of the large number of positrons is characterized by $\gamma \sim 20$, where $\gamma = E/m$ is conventional relativistic factor. This difference is translated into dramatic modification of the frequency bands according to [48].

Therefore, one should not expect any difficulties to discriminate the AQN-induced radio pulse (which always accompanies the EE) from a conventional radio emission during the thunderstorm. The corresponding studies of the radio pulses characterized by the flat spectrum with $\nu \lesssim 200$ MHz (and which must be synchronized with every event from the cluster of the AUGER Exotic Events) could support, substantiate or refute this proposal.

6.3. Possible future tests with other instruments

There are several other unusual and mysterious observations of the CR-like events which might be related to the AQN propagating in atmosphere. It includes, along with previously mentioned TA bursts, the ANITA observation [50, 51] of two anomalous events with noninverted polarity which can be explained within AQN framework [52]. It also includes the Multi-Modal Clustering Events observed by HORIZON 10T [53, 54] which impossible to understand in terms of the CR events, but which could be interpreted in terms of the AQN annihilation events in atmosphere as argued in [33]. Similar mysterious CR-like events can also manifest themselves in form of the acoustic and seismic signals, and could be in principle recorded if dedicated instruments are present in the same area where CR detectors are located. In this case the synchronization between different types of instruments could play a vital role in the discovery of the DM. In fact, in [34] it has been suggested to use distributed acoustic sensing (DAS) to search for a signal generated by an AQN propagating in the Earth’s atmosphere. It is interesting to note that a mysterious seismic event indeed has been recorded in infrasound frequency band by Elginfield Infrasound Array (ELFO). It has been interpreted in [34] as the AQN-induced phenomenon, see footnote 3 with details.

Our original comment here is that the acoustic signal generated by an AQN propagating in the Earth’s atmosphere under the thunderstorm must be very different from conventional acoustic waves which normally accompany the lightning strikes. In the AQN proposal the corresponding acoustic pulse must be synchronized with very initial moment of the lightning events which are triggered by enormous injection of the particles characterized by (22), (27) and (28).

The presence of the antimatter nuggets in the system implies that there will be annihilation events leading to large number of observable effects on different scales: from Early Universe to the galactic scales to the terrestrial rare events. In fact, there are many
hints suggesting that such annihilation events may indeed took place in early Universe as well as they are happening now in present epoch. In particular, the AQNs might be responsible for a resolution of the “Primordial Lithium Puzzle” [28] during BBN epoch. The AQNs may also alleviate the tension between standard model cosmology and the recent EDGES observation of a stronger than anticipated 21 cm absorption feature as argued in [29]. The AQNs might be also responsible for famous long standing problem of the “Solar Corona Mystery” [31, 32] when the so-called “nanoflares” conjectured by Parker long ago [55] are identified with the annihilation events in the AQN framework.

Another very promising alternative path to search for the AQN annihilation events is to study the excess of the radiation in the central regions of the galaxy where DM and visible matter densities are relatively high. In particular, the well-established excess of the diffuse UV emission which cannot be explained by conventional astrophysical sources has indeed been recorded in recent studies [56, 57]. As argued in [30] this puzzling diffuse UV emission can be naturally understood within the AQN framework. One should emphasize that the corresponding estimates in dramatically different environment were based on the same basic parameters of the AQN model, being used in the present work. Therefore, the corresponding estimates demonstrate at least the self-consistency of the entire framework.

If our interpretation of the Exotic Events recorded by AUGER advocated in the present work is confirmed by future studies (e.g. by analyzing the synchronized radio pulses or acoustic signals using DAS) it would represent a strong argument suggesting that the resolution of two long standing puzzles in cosmology – the nature of the DM and the matter-antimatter asymmetry of our Universe – are intimately linked. The corresponding deep connection is automatically implemented in the AQN framework by its construction.

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Appendix A. Shock Acceleration and Diffusion: application to the AQN propagating in atmosphere.

In this Appendix we overview the well known Fermi’s mechanism of a particle acceleration by a shock wave as well as diffusion features of the emitted particles.

We apply the generic properties of Fermi’s mechanism to the specific conditions which are realized for the AQN traversing in atmosphere under thunderclouds. While the mechanism is very generic in principle and widely used in astrophysics, e.g. in explanation of the cosmic rays acceleration by the shock in supernova remnants, the details are very different with dramatically different consequences. We mostly follow the textbook [58] in our formal presentation in subsection Appendix A.1 while in
subsections Appendix A.2 and Appendix A.3 we give our numerical estimates for the cases of the AQN-induced positrons and the AQN-induced electrons correspondingly. Finally, in subsection Appendix A.4 we make simple estimations for the diffusion coefficient to justify our estimate (28) given in the main body of the text for the electron’s density surrounding the AQN traversing in atmosphere.

Appendix A.1. Shock Acceleration. The basics

As we mentioned in the main text the formation of the shock waves due to the moving AQN is very generic feature of the system because the Mach number is very large, $M \equiv \frac{v_{\text{AQN}}}{c_s} \gg 1$. Therefore, a conventional Fermi’s mechanism of a charged particle acceleration due to the shock as reviewed below directly applies here. In what follows we identify the velocity of the shock front with the AQN velocity $v_{\text{AQN}} \sim 10^{-3}c$.

We start by introducing parameter $\tau$ which describes the mean time between the two consecutive occurrences of the process (crossing the shock) such that $\tau^{-1}$ describes the rate of occurrences. We also introduce factor $\beta > 1$ which describes the relative energy gain for each successful occurrence. We also need to introduce the time scale $T_{\text{out}}$ characterizing the escape process such that $P(t) = \exp\left(-t/T_{\text{out}}\right)$ describes a probability for a particle to remain in the accelerating region for time $t$. Then $P_0 = \exp\left(-\tau/T_{\text{out}}\right)$ is the probability that the particle remains within the accelerating region after a single occurrence. The number of particles $n(t)$ that remain in the accelerating region is given by

$$n(t) \approx n_0 \exp\left(-\frac{t}{T_{\text{out}}}\right), \quad \frac{dn}{dt} \approx -n \left(\frac{1}{T_{\text{out}}}\right).$$ (A.1)

The mean number of occurrences in time $t$ is $t/\tau$. Therefore, the typical energy of these particles in time $t$ is $E(t)$ is given by

$$E(t) \approx E_0\beta^t = E_0 \exp\left(\frac{t \cdot \ln \beta}{\tau}\right)$$ (A.2)

which can be inverted to express $t(E)$ as follows

$$t(E) = \tau \left[ \frac{\ln(E/E_0)}{\ln \beta} \right], \quad \frac{dt}{dE} = \frac{\tau}{E \cdot \ln \beta}.$$ (A.3)

These formulae allow us to change variables: instead of $t$ entering (A.1) one can use energy $E$ using Eq. (A.3) relating these two variables. Therefore, the number of particles $n(E)$ as a function of energy $E$ can be expressed as follows:

$$dn(E) \propto E^{-p}dE; \quad p = 1 + \frac{\tau}{T_{\text{out}} \cdot \ln \beta} = 1 - \frac{\ln P_0}{\ln \beta},$$ (A.4)

where we expressed $\tau/T_{\text{out}} \equiv -\ln P_0$ as defined above. The result (A.4) implies a power-law spectrum with a specific index $(-p)$.

Computation of the parameters $P_0$ and $\beta$ is a hard dynamical problem which requires a detail studies of the Boltzmann equations. In a simplified setting one can argue that $\ln \beta \approx -\ln P_0$ such that specific index $p \approx 2$, and $dn(E) \propto E^{-2}dE$, which
is indeed close to the observed average value \( p \approx 2.7 \) in very extended range of CR energies.

In the next subsections Appendix A.2 and Appendix A.3 we apply the Fermi’s mechanism overviewed above to the AQN traversing the atmosphere. We also give an order of magnitude estimates for the relevant parameters.

**Appendix A.2. Shock Acceleration: application to the positrons**

As we discussed in subsection 4.2.1 the typical energy of the AQN-induced positrons is order of \( T \sim 10 \text{ keV} \), which implies that their typical velocity \( v_{e^+} \sim \sqrt{2T/m} \sim 0.2c \).

The mean-free path \( \lambda_{e^+} \) and typical time scale \( \Delta t_{e^+} \) between the consecutive elastic collisions with atmospheric molecules for such positrons can be estimated as follows. The Coulomb cross section is

\[
\sigma = \frac{1}{4} \left( \frac{r_e}{v^2 \gamma} \right)^2 \frac{1}{(\sin \frac{\theta}{2})^4}, \quad r_e \equiv \frac{\alpha m}{\epsilon} \approx 2.8 \cdot 10^{-13} \text{cm}
\]  

(A.5)

Using our parameter for \( n_{\text{air}} \sim 10^{21} \text{cm}^{-3} \) as in the main body of the text in (4) we arrive to the following typical parameters for mean-free path \( \lambda_{e^+} \) and time scale \( \Delta t_{e^+} \):

\[
\lambda_{e^+} \approx (\sigma n_{\text{air}})^{-1} \sim 20 \text{ cm}, \quad \Delta t_{e^+} \approx \frac{\lambda_{e^+}}{v_{e^+}} \sim 3 \text{ ns},
\]  

(A.6)

where for the numerical estimates we use \( (\sin^2 \frac{\theta}{2}) \sim 1/2 \). Now we can estimate the mean time \( \tau_{e^+} \) between the two consecutive occurrences of the process as follows

\[
\tau_{e^+} \sim \frac{\lambda_{e^+}}{v_{\text{AQN}}} \sim 0.7 \mu s,
\]  

(A.7)

where we assume that the shock is moving with velocity \( v_{\text{AQN}} \sim 10^{-3}c \), while a particle is localized at distance \( \sim \lambda_{e^+} \) from the front after the previous occurrence. Comparing the time scales (A.6) and (A.7) one can infer that positrons elastically scatter \( \sim 10^2 \) times before the next occurrence (crossing the shock) takes place.

Our next comment is related to time scale \( T_{\text{out}} \) characterizing the escape process as expressed by (A.1). The key comment here is that the positrons obviously leave the accelerating region with typical time scale \( T_{\text{out}} \). However, in contrast with the application to the CR, the AQN produces new positrons as it propagates in the external electric field \( \mathcal{E} \). Therefore, the basic equation (A.1) is dramatically modified.

The point is that the positrons which leave the system will be replaced by new positrons from the electrosphere as the annihilation processes continue with the rate (5) such that the temperature remains the same as given by (4). The equilibration implies that number of weakly bound positrons (8) also remains the same. This important additional ingredient in the system implies that the equation (A.1) is modified as follows:

\[
\frac{dn}{dt} \approx -n \left( \frac{1}{T_{\text{out}}} - \frac{1}{T_{\text{in}}} \right),
\]  

(A.8)

where the time scales \( T_{\text{out}} \) characterizing the escape process and \( T_{\text{in}} \) characterizing the rate of newly produced positrons are approximately the same as a result of equilibration,
i.e. $T_{\text{out}} \approx T_{\text{in}}$. This modification also affects the spectral properties of the accelerated positrons, which assumes the form:

$$dn(E) \propto E^{-1}dE,$$

where we neglected the second term for specific index $p$ in eq. (A.4). We ignored this terms for our order of magnitude estimate because it is proportional to $T_{\text{out}}^{-1}$ term which we expect to be cancelled with good accuracy by $T_{\text{in}}^{-1}$ as argued above.

This dramatic simplification allows us to estimate the ratio $r$ of the positrons which can get accelerated to sufficiently high energy $E \in (0.5 - 1)$ MeV such that these positrons may become the seed particles according to criteria (14). With these assumptions the ratio $r$ is estimated as follows:

$$r \approx \frac{\int_{0.5\text{MeV}}^{1\text{MeV}} E^{-1}dE}{\int_{10\text{keV}}^{\text{MeV}} E^{-1}dE} \sim \frac{\ln 2}{\ln 10^2} \sim 0.15,$$

where with logarithmic accuracy we integrated only over energies $E \in (0.5 - 1)$ MeV because the relativistic positrons with energy $E \gtrsim$ MeV do not suffer much from re-scattering losses as the cross section $\sigma$ becomes sufficiently small and the mean free path for energetic positrons (A.6) exceeds the scale $l_a$. Therefore, the dominant portion of the positrons with MeV energy will further get accelerated to 10 MeV very quickly in fraction of $\mu$s according to (21).

To conclude: the positrons which are initially injected into the system with typical energy $\sim 10$ keV will get accelerated to $\sim 10$ MeV energy and could play a role of seed particles triggering a very powerful lightning strike as argued in Sect. 4.2.1.

Appendix A.3. Shock Acceleration: application to the electrons

The acceleration of the AQN-induced electrons by the same mechanism has dramatically different outcome in comparison with the case of positrons considered above. The crucial difference is related to the fact that the electrons are produced with typical energy $\sim \text{eV}$ as explained in Sect. 4.2.2 in contrast with typical energy of the AQN-induced positrons which is order of $\sim 10$ keV. These free thermal electrons lose its energy within a very short period of time of $\sim 10^{-8}$s and disappear rather rapidly on the time scale $\Delta t_e \sim 10^{-7}$s (attachment time) as a result of interaction with air molecules, see [40]. Therefore, it is highly unlikely that the AQN-induced eV energy electrons could accelerate to the relativistic energies by the Fermi mechanism, in contrast to positrons considered above. Therefore, the AQN-induced electrons cannot serve as seed particles, in contrast with positrons. Nevertheless, these AQN-induced electrons could play a

9 In our arguments we assume that the efficiency of acceleration is sufficiently high (the so-called first-order Fermi acceleration) such that $\ln \beta \sim v_{\text{AQN}}/c$. In this case the energy $E(t)$ can indeed reach the MeV range on time scale of few ms according to (A.2) as $\tau_{e^\pm}$ is measured in $\mu$s, see (A.7). The ms time scale for the acceleration is consistent with this proposal as the AQN propagates only few hundred meters during this ms-interval and remains deep inside the region $\sim L_\varepsilon$ where electric field $\varepsilon$ should be large according to basic requirement “b” as listed at the very beginning of section 4.1.
key dynamical role in the lightning strike as a very large number of electrons almost instantaneously get injected into the system, as equations (26) and (27) suggest.

Appendix A.4. Diffusion of injected electrons

Another way to represent the powerful jolt reflected by (26) is to estimate the electron number density \( n_e(r) \) in close vicinity of the nugget which is given by (28). However, this estimate assumes that the diffusion of the emitted electrons is negligible. In this Appendix we justify this assumption by estimating diffusion features of the emitted electrons. For the estimate we use conventional expression for the Green’s function \( G(t, r) \) which assumes the standard form:

\[
G(t, r) = \frac{1}{(4\pi Dt)^{3/2}} \exp \left( -\frac{r^2}{4Dt} \right) \rightarrow \langle r^2 \rangle \approx 6Dt
\]  

(A.11)

where \( D \) is the diffusion coefficient which has dimensionality (cm\(^2\)/s) and will be numerically estimated below. In estimate (A.11) it is assumed that the diffusion is spherically symmetric process, which is justified if typical velocities of the emitted electrons \( v_e \) are much greater than the velocity of the nugget, i.e. \( c \gg v_e \gg v_{AQN} \).

For the numerical estimation of the diffuse coefficient \( D \equiv v_e^2/(2\nu) \) we use \( \nu \sim 10^{11}\text{s}^{-1} \) where \( \nu \) is the collision frequency, see e.g. [40]. The velocity \( v_e \) of low energy electrons can be estimated as \( v_e/c \approx \sqrt{2E_e/m_0} \sim 0.6 \cdot 10^{-2} \) for \( E_e \sim 10\text{ eV} \). Collecting all these factors together we arrive to the following numerical estimates for the diffuse coefficient \( D \) and the maximal distance from the point of emission \( \langle r^2 \rangle_{\text{max}} \):

\[
D \sim 3.6 \cdot 10^5 \frac{\text{cm}^2}{\text{s}}, \quad \langle r^2 \rangle_{\text{max}} \sim 6D(\Delta t_e) \sim 0.04 \text{ cm}^2.
\]  

(A.12)

This estimate unambiguously shows that the diffusion is relatively minor effect as \( \langle r^2 \rangle_{\text{max}} \ll \bar{r}^2 \) where \( \bar{r} \sim 3\text{ cm} \) enters the estimate (28).

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