Enhancing the Closed Loop Performance of Semi-active Suspension System with I-PD Controller

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Abstract. This paper aims to investigate for the control quarter car semi-active suspension system using PID controller. In the study, simulation and transfer function models are constructed and used for the control of relative displacement of semi-active suspension system. The conventional PID controller is designed by using Ziegler-Nichols method and it used for the control of suspension system. The closed loop performance of PID controller is improved through reducing the derivative gain value. It shows that, the peak overshoot of closed loop responses are sluggishly reduced. The modified forms PID controller such as PI-D and I-PD controllers are furthermore used for the control of suspension system. From the results, the I-PD controller deliver better closed loop performance compared with PID and PI-D controllers. The control of suspension system using I-PD controller provide superior closed loop performance by means of minimization of peak overshoot and settling time.

1. Introduction
The controllers are used in various engineering applications such as chemical, electrical, and mechanical engineering. The controller is designed and applied for the control of wide range of physical systems from electrical circuits to guided missiles in robots. The main objectives of closed loop system are to control, measure, and monitor the behaviour of a system. The output is controlled by measuring its output and compared with desired value through feedback. From the comparison, an error signal is generated and controller is reducing the error. The output of the system is to reach the desired value when error value is zero and it is the main function of a controller in closed loop system. The proportional-integral (PI) and proportional-integral-derivative (PID) controllers are commonly used in process industries and research laboratory because of their simple form, convenient design approach and robustness [1]. In more than 90% of the industries are controlled still PID controller owing to their simplicity, clear functionality, and ease of use [2]. The PID controller is designed and properly tuned for majority of industrial control loops and it is delivered sufficient closed loop performance [3]. The PID controller parameters are tuned with vast methods based on the closed loop performance. The most common and well known methods are Ziegler-Nichols and Cohen-Coon methods. The PID controller gains are obtained by matching the frequency response of the closed-loop control system [4]. In recently, the setting the PID parameters are obtained by using a simple and systematic approach and applied for the control of nonlinear uncertain systems [5]. The transfer function model of a process is essential to design a PI/ PID controller [6]. Generally, the first-order plus time delay (FOPTD) system parameters are needed for tuning PID controller parameters [7, 8].

In recently, PID controller is used for the control semi-active suspension system [9]. Generally, the vehicle suspensions are classified into passive, semi-active and an active suspension, which is categorised based on the mode of operation mode and performances [10]. Usually, the conventional control method is used for the control of passive without on-line feedback [11, 12]. On contrary, active suspension is also a control technique with online feedback action. Therefore, active suspension has been extensively studied and various approaches have been used for the control [13]. The
effective controller designing for the control of semi-active suspension received must attention. Since they can achieve desirable performance than passive suspension and consume less power than active suspensions. Semi-active control with MR dampers for vehicle suspensions have been studied in [14,15] and it have been evaluated in terms of their applicability in practice.

The PID controller is used for the control of vehicle semi-active suspension system and it has more advantages as compared with the conventional passive approach [16, 17, 18]. The control suspension system is to reduce motions of the vehicle body and road disturbances. An effective controller is used for the control semi-active suspension system which is reducing the vertical forces conveyed to the passenger, and to exploit the tyre-to-road contact for supervision and protection. The better ride quality, sophisticated braking, reduced weight transfer and handling performances are improved in an active suspension system with the use of feedback control of the actuator [19]. In this work, the I-PD controller structure is designed and applied for the control of semi-active suspension system. The proposed I-PD controller provides better closed loop performance with minimizing the peak over shoot in closed loop response. The paper is organized as follows. Section two presents the mathematical modelling of quarter car semi-active suspension system. Section three is about review of PID controller and their modified structures. Section four deals that, the designing of PID, PI-D and I-PD controllers and analyses their performances for the control of semi-active suspension system. Conclusions are drawn in Section five.

2. Mathematical Modelling of Semi-active suspension system

The mathematical model of a semi-active suspension system is developed based on the following assumptions. The suspension system is a linear system; effects due to insignificant forces are neglected and damping property. These observation and assumption are considered and a quarter car semi-active suspension system is shown in Fig. 1.

![Figure 1. Quarter car semi-active suspension model](image)

The force balance equation for the suspension of mass \((M_s)\) is given in Eq. (1),

\[
M_s \dddot{z}_s + K_s(z_s - z_v) + C_s(\dot{z}_s - \dot{z}_v) = U(t)
\]

(1)

The Eq. (2) is represented the force balance equation for mass of tyre and suspension \((M_3)\) and it is given by,
Where, \( M_S \) is mass of vehicle body
\( M_U \) is mass of the tyre and suspension
\( K_S \) is coefficient of suspension spring
\( K_t \) is coefficient of tyre material
\( C_s \) is damping coefficient of the dampers

The displacements of suspension system \( Z_S \), \( Z_U \) and \( Z_r \) are mass suspension (car body travel), mass of tire and road disturbance respectively.

The Laplace transform of the above force balance equation Eq. (1) is,
\[
M_S s^2 Z_S + K_S (Z_S - Z_U) + C_s (sZ_S - sZ_U) = U(s)
\]  
(3)

The simplified form of above equation Eq. (3) is,
\[
M_S s^2 Z_S = -K_S (Z_S - Z_U) - C_s (sZ_S - sZ_U) - U(s)
\]  
(4)
\[
M_S s^2 Z_S = -K_S (Z_S - Z_U) - C_s (sZ_S - sZ_U) - U(s)
\]  
(5)
\[
M_S s^2 Z_S = -K_S Z_S + K_S Z_U - C_s Z_S + C_s Z_U - U(s)
\]  
(6)
\[
(M_S s^2 + C_s s + K_s) Z_S = (C_s s + K_s) Z_U - U(s)
\]  
(7)
\[
(M_S s^2 + C_s s + K_s) Z_S - (C_s s + K_s) Z_U = -U(s)
\]  
(8)

Apply the Laplace transform of Eq. (2) is,
\[
M_U s^2 Z_U + K_U Z_U - C_u Z_U - C_u Z_U - K_s Z_U = U(t)
\]  
(9)

The above equation Eq. (8) is simplified and given below,
\[
M_U s^2 Z_U + K_U Z_U - K_s Z_U - C_u Z_U - C_u Z_U - K_s Z_U = U(t)
\]  
(10)
\[
-(C_u s + K_s) Z_U + (M_U s^2 Z_U + (C_u s + C_u) Z_U + (K_s + K_s) Z_U - (C_u s + K_s) Z_U = -U(t)
\]  
(11)

The input-output relation of the system is obtained by using the Eq. (8) and Eq. (11),
\[
\begin{bmatrix}
M_S s^2 + C_s s + K_s \\
-(C_s s + K_s) \\
M_U s^2 Z_U + (C_u s + C_u) s + (K_s + K_s) s
\end{bmatrix}
\begin{bmatrix}
Z_S \\
Z_U \\
U(t)
\end{bmatrix} =
\begin{bmatrix}
0 \\
C_s s + K_s \\
-1
\end{bmatrix}
\]  
(12)

The semi-active suspension system model contains two inputs and two outputs. The simplified block diagram of semi-active suspension system is shown in Fig. 2.

![Figure 2](image-url)

**Figure. 2** The simplified model of semi-active suspension model
The performance of a suspension system is related to the difference between $Z_s$ and $Z_U$. The transfer functions model of the suspension system is obtained based on the inputs $U(t)$ and $Z_r$. The transfer functions of the suspension system are given below.

The transfer function $G_U(s)$ is obtained with the road disturbance $Z_r$ is zero and it is given below.

$$ G_U(s) = \frac{Z_s - Z_U}{U(s)} \quad Z_r = 0 \quad (13) $$

The transfer function $G_U(s)$ is represented by,

$$ G_U(s) = \frac{(M_U + M_s)s^2 + C_s s + K_r}{(M_s s^2 + C_s s + K_s)(M_U s^2 Z_U + (C_s + C_i)s + (K_s + K_i)) - (C_s s + K_s)^2} \quad (14) $$

The transfer function $G_Z(s)$ is obtained with the road disturbance $U(t)$ is zero and it is given below.

$$ G_Z(s) = \frac{Z_s - Z_U}{Z_r} \quad U(s) = 0 \quad (15) $$

The transfer function $G_Z(s)$ is represented by,

$$ G_Z(s) = \frac{-M_s C_s s^3 - M_s K_s s^2}{(M_s s^2 + C_s s + K_s)(M_U s^2 Z_U + (C_s + C_i)s + (K_s + K_i)) - (C_s s + K_s)^2} \quad (16) $$

The block diagram of the semi-active suspension system with obtained transfer functions is shown in below.

![Block diagram of transfer function model of semi-active suspension system](image)

**Figure 3.** Block diagram of transfer function model of semi-active suspension system

The simulation and transfer function models are developed for the control of quarter car semi-active suspension system. The following system parameters are used for the development of a simulation and transfer function models which is given in Table 1.

| S. No. | Parameters                      | Values    |
|-------|---------------------------------|-----------|
| 1     | Mass of vehicle body (M_s)     | 540.5 kg  |
| 2     | Mass of the tyre and suspension (M_u) | 62 kg     |
| 3     | Co-efficient of suspension spring (K_s) | 13100 N/m |
| 4     | Co-efficient of tyre material (K_t) | 252000 N/m |
| 5     | Damping co-efficient of the dampers (C_s) | 400 N-s/m |
3. Review of PI/PID controller structures
The terms P, I and D are proportional, integral and derivative terms respectively. The proportional control signal is proportional to the error signal which is represented by [20],

\[ u(t) = K_p \epsilon(t) \]  

(17)

The PI and PID control signals [20] are represented by Eq. (18) and Eq. (19),

\[ u(t) = K_p \left( \epsilon(t) + \frac{1}{T_i} \int \epsilon(t) \, dt \right) \]  

(18)

\[ u(t) = K_p \left( \epsilon(t) + \frac{1}{T_i} \int \epsilon(t) \, dt + T_a \frac{d \epsilon(t)}{dt} \right) \]  

(19)

The control signal of PI-D and I-PD controllers are given in Eq. (20) and Eq. (21) respectively [21, 22].

\[ u(t) = K_p \left( \epsilon(t) + \frac{1}{T_i} \int \epsilon(t) \, dt - T_a \frac{d \epsilon(t)}{dt} \right) \]  

(20)

\[ u(t) = K_p \left( \epsilon(t) - \frac{1}{T_i} \int \epsilon(t) \, dt - T_a \frac{d \epsilon(t)}{dt} \right) \]  

(21)

The following relations are used to find the integral and derivative constants.

The integral gain,

\[ K_i = \frac{K_p}{T_i} \]

The derivative gain, \( K_d = K_i \cdot T_d \)

Where, \( K_p \) is proportional gain, \( T_i \) and \( T_d \) are integral time and derivative time respectively. The PID controller parameters are obtained by using Ziegler-Nichols closed loop method [23]. The tuning equations are given below.

\[ K_p = 0.6 \cdot K_u \]  

(22)

\[ T_i = 0.5 \cdot P_u \]  

(23)

\[ T_d = 0.125 \cdot P_u \]

Using the above equations, obtain the parameters of PID controller.

4. Results and discussion
The simulation model of a quarter car semi-active suspension is developed by using Eq. (1) and Eq. (2). These equations are reformed and it is given by,

\[ \ddot{Z}_5 = \frac{1}{M_s} \left( -K_s (Z_5 - Z_v) - C_s (\dot{Z}_5 - \dot{Z}_v) + U(t) \right) \]  

(24)

\[ \ddot{Z}_v = \frac{1}{M_s} \left( K_s (Z_5 - Z_v) + C_s (\dot{Z}_5 - \dot{Z}_v) + K_i (Z_5 - Z_v) + K_d (\dot{Z}_5 - \dot{Z}_v) - U(t) \right) \]  

(25)

The above reformed Eq. (24) and Eq. (25) are used to develop a simulation model of quarter car semi active suspension model. The values system parameters are given in Table 1 and these values used in a simulated model. Using the simulation model, the open loop response \( Z_5 - Z_v \) is obtained when
excited with unit step change and disturbance Z is 0.2. The open loop response of a quarter car semi-active suspension system is shown in Fig. 4.

![Figure 4: Open loop response of semi-active suspension system (Simulation model).](image)

From the above responses, the vehicle body is reached the steady state at 1.8 sec with peak overshoot. The magnitude of this type of input is not change with respect to time and it is considered as road surface is linear. The position and acceleration of vehicle body is reached the steady state with peak overshoot. The above figure (Fig. 5) is obtained with simulation model and the responses of model of the suspension system also obtained in this work for the determination of model confirmation. The transfer functions of a quarter car semi-active suspension system are determined by using Eq. (14) and Eq. (16). Table 1 parameters are used in the transfer functions $G_v(s)$ and $G_z(s)$ are given in Eq. (26) and Eq. (27) respectively.

$$G_v(s) = \frac{410s^2 + 2500s + 255000}{21000s^4 + 231000s^3 + 116450000s^2 + 1005000000s + 11475000000}$$  \hspace{1cm} (26)

$$G_z(s) = \frac{-8750000s^3 - 89250000s^2}{21000s^4 + 2310000s^3 + 116450000s^2 + 1005000000s^2 + 11475000000}$$  \hspace{1cm} (27)

The above transfer functions are used in the transfer function model of a suspension system and obtain the open loop response. Fig. 6 shows that the validation of both simulation model and transfer function model.
Figure 6 Open loop response of semi-active suspension system (-----Simulation model and ________Transfer function model)

From this response it is clearly identified that, the open loop responses of both simulation model (---- line) and transfer function model (_______ line) are identical. It shows that the developed models are validated and these models are used for designing the controller.

In Ziegler-Nichols closed loop method, PID controller parameters are obtained by using the closed loop system with PID controller and system model. In the closed loop system, the gain values of I and D are set to be zero and by varying the $K_P$ value gradually until the time response produce a sustain oscillation. From the oscillatory output, we can calculate the period of the oscillation (ultimate period) and the corresponding value of $K_P$ is the ultimate gain value.

In a closed loop response of semi-active suspension model is yield sustained oscillations with $K_U$ is 4167. From this oscillatory response, the ultimate period $P_U$ is calculated which is 0.83 sec. The PID controller parameters are obtained by using the ultimate period and gain values. The controller tuning parameters are given in Table 2.

| Sl. No. | Controllers | $K_P$ | $K_I$ | $K_D$ |
|--------|-------------|-------|-------|-------|
| 1      | P Controller| 2080  | -     | -     |
| 2      | PI Controller| 1890 | 4500  | -     |
| 3      | PID Controller| 2500 | 6000  | 3000  |

The closed loop response of the semi-active suspension system is obtained using the designed PID controller. The relative motion of suspension system is controlled by using negative feedback mechanism with PID controller and the closed loop response is shown in Fig. 7.
From this figure, the response of PID controller is provided the peak overshoot and the controlled variable reached the set-point about 2 sec. The performance of designed PID controller is evaluated by decreasing the derivative gain value. The amount of peak overshoot is gradually decreased with the effect of minimizing the derivative gain.

The performance of PID controller performance is evaluated further decreasing derivative gain values and the responses are shown in Fig. 8. It shows that, the peak overshoot is over decreasing and also the settling time is slowly decreased. The effect derivative gain minimization is an important for the control semi-active suspension system. In this work, the modified form of PID controller structures such as PI-D and I-PD controllers are used and control the relative motion of a vehicle suspension system. The closed loop responses of PI-D and I-PD controller are compared with convention PID controller are shown in Fig. 9.
Figure 9 Closed loop responses with PID, PI-D and I-PD controllers.

From the above Fig. 9 shows that the closed loop performances of PID, PI-D and I-PD controllers for the control of relative motion of suspension system \((Z_s - Z_u)\). The conventional PID controller is delivered closed loop responses with peak overshoot. The modified form PI-D controller also provided response with peak overshoot. The I-PD controller structure is conveyed the closed loop responses with lesser amount of peak overshoot and also settling time is also reduced. The I-PD controller is used for the control of quarter car semi-active suspension system; it delivered better closed loop performance such as response with reduced the amount of peak overshoot and settling time.

5. Conclusion
The PID, PI-D and I-PD controllers are designed for the control of quarter car semi-active suspension system. The simulation and transfer function model of semi-active suspension system are developed and the open loop responses are confirmed. The closed loop response of relative displacement is obtained with PID controller which comprises the peak-overshoot. The derivative effect is minimized through reducing the derivative gain \((K_d)\) values in a designed PID controller and the corresponding closed loop responses are obtained. From these results, the peak overshot is gradually minimized by the effect minimized derivative gain. The modified structure of PID controller such as PI-D and I-PD controllers are used for the control of relative displacement of suspension system. From the results, IPD controller provided better closed loop performance in terms peak overshoot and settling time are minimized.

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