Spin effects in $p\bar{p}$ interaction and their possible use to polarize antiproton beams.

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Abstract

Low energy $p\bar{p}$ interaction is considered taking into account the polarization of both particles. The corresponding cross sections are calculated using the Paris nucleon-antinucleon optical potential. Then they are applied to the analysis of the polarization buildup which is due to the interaction of stored antiprotons with polarized protons of a hydrogen target. It is shown that, at realistic parameters of a storage ring and a target, the filtering mechanism provides a noticeable polarization in a time comparable with the beam lifetime.
I. INTRODUCTION

An extensive physical program with polarized antiprotons has been proposed recently by the PAX Collaboration [1]. This program has initiated a wide discussion on the possibility to use a polarized hydrogen gas target to polarize stored antiprotons (see [1, 2] and literature therein). Various modifications of the filtering method first proposed in Ref. [3] have been considered. The filtering method exploits the dependence of the scattering cross section on the orientations of the target and projectile proton (antiproton) spins. Due to this dependence beam protons with positive spin projections on the direction of the target polarization scatter out of the beam (scattering angle $\vartheta$ larger than the acceptance angle $\theta_{acc} \ll 1$) at a rate different from protons with a negative spin projection. As a result the beam becomes polarized. This method can also be used for the antiproton beam.

In Refs. [4, 5], it was shown that, for $\vartheta < \theta_{acc}$ (a proton or antiproton remains in the beam), the polarization buildup is completely due to the spin-flip transitions. The corresponding cross sections turn out to be negligibly small for both proton-proton and proton-electron scattering. For a pure electron target, filtering mechanism also does not provide a noticeable polarization, Ref. [4]. Thus, it is necessary to study in detail the filtering method for the antiproton beam using a hydrogen gas target with the proton polarization.

The method suggested in [3] has been realized in the experiment [6], where 23-MeV stored protons scattered on an internal gas target of the polarized hydrogen atoms. The measured polarization degree is in reasonable agreement with the theoretical predictions based on the known $pp$ scattering cross section, see discussion in Refs. [2, 4, 5]. Theoretical prediction for the rate of polarization buildup of the antiproton beam is essentially more complicated problem because the spin-dependent part of the $p\bar{p}$ scattering cross section is not well known both theoretically and experimentally. Therefore, it was necessary to apply various phenomenological approaches in order to explain numerous experimental data, see, e.g., Refs. [7, 8, 9, 10, 11, 13] and recent reviews [14, 15]. However, some parameters of the models determining the spin-dependent part of the cross sections are not well defined from the experimental data available [16].

In the present paper, we calculate the cross section of polarized proton and antiproton
interaction using the Paris nucleon-antinucleon optical potential $V_{N\bar{N}}$ with the parameters given in Refs. \[10, 11, 12\]. This potential has the form:

$$V_{N\bar{N}} = U_{N\bar{N}} - i W_{N\bar{N}},$$

(1)

where the real part $U_{N\bar{N}}$ is the $G$-parity transform of the well-established Paris $NN$ potential for the long- and medium-ranged distances ($r \gtrsim 1$ fm), and some phenomenological part for shorter distances. The absorptive part, $W_{N\bar{N}}$, of the optical potential takes into account the inelastic channels of $N\bar{N}$ interaction, i.e. annihilation into mesons, being important at short distances. Our knowledge of $W_{N\bar{N}}$ is more restricted than that of $U_{N\bar{N}}$.

II. CROSS SECTIONS

It is convenient to calculate the cross sections of $p\bar{p}$ scattering in the center-of-mass frame where antiproton and proton have the momenta $p$ and $-p$, respectively. We assume that $p \ll M$ (where $M$ is the proton mass) and perform calculations in the nonrelativistic approximation, so that the momentum of antiproton in the lab frame is $p_{\text{lab}} = 2p$. Let us direct the polar axis $z$ along the unit vector $\nu = p/p$. Kinetics of polarization depends on the cross section

$$\sigma = \sigma_{\text{ann}} + \sigma_{\text{ex}} + \sigma_{el};$$

(2)

where $\sigma_{\text{ann}}$ is the annihilation cross section, $\sigma_{\text{ex}}$ is the charge-exchange cross section of the process $p\bar{p} \rightarrow n\bar{n}$, and $\sigma_{el}$ is the elastic cross section of $p\bar{p}$ scattering summed up over final spin states, integrated over the azimuth angle, and over the scattering angle $\vartheta$ from the acceptance angle $\theta_{\text{acc}}$ to $\pi$. We remind that $\theta_{\text{acc}}$ is defined in the center-of-mass frame. In the lab frame the acceptance angle is $\theta_{\text{acc}}^{(l)} = \theta_{\text{acc}}/\sqrt{2}$. The cross section $\sigma$ has the form

$$\sigma = \sigma_0 + (\zeta_1 \cdot \zeta_2) \sigma_1 + (\zeta_1 \cdot \nu)(\zeta_2 \cdot \nu) (\sigma_2 - \sigma_1),$$

(3)

where $\zeta_1$ and $\zeta_2$ are the unit polarization vectors of the proton and antiproton, respectively. For example, $\sigma = \sigma_0 + \sigma_2$ for $\zeta_1 \parallel \zeta_2$ and $\zeta_1 \parallel \nu$; $\sigma = \sigma_0 + \sigma_1$ for $\zeta_1 \parallel \zeta_2$ and $\zeta_1 \perp \nu$.

We choose the quantization axes along the vector $\nu$ and express the spin wave function of the initial $p\bar{p}$ state via the spin wave function, corresponding to the total spin $S = 1$ and projection $S_z = \mu$, and the spin wave function, corresponding to the total spin $S = 0$. Then
we obtain

\[
\begin{align*}
\sigma_0 &= \frac{1}{2} \Sigma_{11} + \frac{1}{4} (\Sigma_{10} + \Sigma_{00}), \\
\sigma_1 &= \frac{1}{4} (\Sigma_{10} - \Sigma_{00}), \\
\sigma_2 &= \frac{1}{2} \Sigma_{11} - \frac{1}{4} (\Sigma_{10} + \Sigma_{00}),
\end{align*}
\]

(4)

where \( \Sigma_{1\mu} \) is the cross section calculated for the total spin \( S = 1 \) and \( J_z = \mu \), \( J_z \) is the projection of the total angular momentum, \( \Sigma_{00} \) is the cross section calculated for the total spin \( S = 0 \) (so that \( J_z = 0 \)). Note that \( \Sigma_{11} = \Sigma_{1-1} \).

Our potential is a sum of the nucleon-antinucleon optical potential \( V_{NN} \) and the Coulomb potential \( V_C(r) = -e^2/r \), where \( e \) is the proton charge. The hadronic amplitude can be represented as a sum of a pure electromagnetic amplitude and the strong amplitude. We emphasize that the latter does not coincide with the strong amplitude calculated without account for the electromagnetic interaction. The strong amplitude is not singular at small scattering angle \( \vartheta \). In the nonrelativistic limit, the triplet \( F_{C1} \) and singlet \( F_{C0} \) electromagnetic amplitudes coincide with the amplitude \( f_{C} (\vartheta) \) where

\[
\begin{align*}
f_C(\vartheta) &= \frac{\alpha}{4v p \sin^2(\vartheta/2)} \exp\{i(\alpha/v) \ln[\sin(\vartheta/2)] + 2i\chi_0\}, \\
\chi_0 &= \arg \Gamma(1 - i\frac{\alpha}{2v}),
\end{align*}
\]

(5)

where \( v = p/M \) is the nucleon (antinucleon) velocity in the center-of-mass frame, \( \alpha = e^2 \) is the fine structure constant. It was shown in Ref. [4] that, for time comparable with the beam life-time, the polarization degree of the antiproton beam, \( P_B \), is determined by the ratio \( \sigma_{1,2}/\sigma_0 \). The pure Coulomb cross section is spin independent and, therefore, contributes only to \( \sigma_0 \) that diminishes the ratio \( \sigma_{1,2}/\sigma_0 \). Therefore, we do not consider very small c.m. energies \( E < 5 \) MeV because in this region the Coulomb cross section becomes essentially larger than the nuclear cross section. Note that the interference of the Coulomb amplitude and the spin-dependent part of the strong amplitude, corresponding to elastic scattering without spin flip, may be important [17].

In our calculation, we use the standard partial-wave analysis (see, e.g., Refs. [11, 13]). For the strong elastic triplet scattering amplitude \( (p\bar{p} \rightarrow p\bar{p}) \), we have

\[
F_{1\mu}^{cl} = \frac{i\sqrt{4\pi}}{2p} \sum_{m,L,J} Y_{Lm}(\vartheta, \varphi) C^{J\mu}_{Lm,1\mu-m} R^J_{L\mu},
\]
\[ R_{L\mu}^J = \sum_{L'} (-1)^{\frac{J+J'}{2}} \sqrt{2L' + 1} C_{L'0,1\mu}^{J} \exp(i\chi_L + i\chi_{L'}) \left( \delta_{LL'} - S_{LL'}^J \right), \]

\[ \chi_L = \arg \Gamma(L + 1 - \frac{i\alpha}{2v}), \] \( \tag{6} \)

where summation over \( L, L' \) is performed under the conditions \( L, L' = J, J \pm 1 \) and \(|L-L'| = 0, 2\). The strong singlet elastic scattering amplitude is given by

\[ F_{0}^{el} = \frac{i\sqrt{4\pi}}{2p} \sum_{L} \sqrt{2L + 1} \sum_{L} Y_{L0}(\vartheta, \varphi) \exp(2i\chi_L) \left( 1 - S_L \right). \] \( \tag{7} \)

In Eqs. (6) and (7), \( S_{LL'}^J \) and \( S_L \) are the partial elastic triplet and singlet scattering amplitudes, respectively, \( Y_{Lm}(\vartheta, \varphi) \) are the spherical functions, \( C_{Lm,1\mu-\mu}^{J} \) are Clebsch-Gordan coefficients.

The charge exchange amplitude \((p\bar{p} \rightarrow n\bar{n})\) is given by

\[ F_{1\mu}^{cex} = -\frac{i\sqrt{4\pi}}{2p} \sum_{m, L, J} Y_{Lm}(\vartheta, \varphi) C_{Lm,1\mu-\mu}^{J} \tilde{R}_{L\mu}^J, \]

\[ \tilde{R}_{L\mu}^J = \sum_{L'} (-1)^{\frac{J+J'}{2}} \sqrt{2L' + 1} C_{L'0,1\mu}^{J} \exp(i\chi_{L'}) \tilde{S}_{LL'}^J, \] \( \tag{8} \)

for the triplet contribution and

\[ F_{0}^{cex} = -\frac{i\sqrt{4\pi}}{2p} \sum_{L} \sqrt{2L + 1} Y_{L0}(\vartheta, \varphi) \exp(i\chi_L) \tilde{S}_L \] \( \tag{9} \)

for the singlet one. In Eqs. (8) and (9), \( \tilde{S}_{LL'}^J \) and \( \tilde{S}_L \) are the partial charge exchange triplet and singlet scattering amplitudes, respectively.

The cross sections \( \Sigma_{1\mu} \) and \( \Sigma_{00} \) can be represented as a sum of pure Coulomb contributions, \( \Sigma_{1\mu}^C \) and \( \Sigma_{00}^C \), hadronic contributions, \( \Sigma_{1\mu}^h \) and \( \Sigma_{00}^h \), and the interference terms, \( \Sigma_{1\mu}^{int} \) and \( \Sigma_{00}^{int} \). For Coulomb contributions, we have

\[ \Sigma_{1\mu}^C = \Sigma_{00}^C = \sigma^C = \frac{\pi\alpha^2}{(vp\theta_{acc})^2}, \] \( \tag{10} \)

where a smallness of \( \theta_{acc} \) is taken into account. It follows from the optical theorem that the total hadronic cross sections are

\[ \Sigma_{1\mu}^h = \frac{2\pi}{p^2} \sum_{L, J} \sqrt{2L + 1} C_{L0,1\mu}^{J} \Re \left[ R_{L\mu}^J \right], \]

\[ \Sigma_{00}^h = \frac{2\pi}{p^2} \sum_{L} (2L + 1) \Re \left[ \exp(2i\chi_L)(1 - S_L) \right]. \] \( \tag{11} \)
The terms in the cross section corresponding to the interference of the Coulomb and the strong elastic $p\bar{p}$ amplitudes read

$$\\Sigma^{\text{int}}_{1\mu} = -\frac{2\pi\alpha}{vp^2} \log \left( \frac{2}{\theta_{\text{acc}}} \right) \sum_{L,J} \sqrt{2L+1} C_{L0,1\mu}^J$$

$$\times \left\{ \text{Im} \left[ \exp(-2i\chi_0) R_{L\mu}^J \right] + \frac{\alpha}{2v} \log \left( \frac{2}{\theta_{\text{acc}}} \right) \text{Re} \left[ \exp(-2i\chi_0) R_{L\mu}^J \right] \right\} ,$$

$$\Sigma^{\text{int}}_{00} = -\frac{2\pi\alpha}{vp^2} \log \left( \frac{2}{\theta_{\text{acc}}} \right) \sum_{L} (2L+1) \left\{ \text{Im} \left[ \exp \left( 2i(\chi_L - \chi_0) \right)(1 - S_L) \right] \right\} + \frac{\alpha}{2v} \log \left( \frac{2}{\theta_{\text{acc}}} \right) \text{Re} \left[ \exp \left( 2i(\chi_L - \chi_0) \right)(1 - S_L) \right] \right\} .$$

(12)

The hadronic contributions to the elastic cross section of $p\bar{p} \rightarrow p\bar{p}$ process have the form:

$$\Sigma^{\text{el}}_{1\mu} = \frac{\pi}{p^2} \sum_{L,J} |R_{L\mu}^J|^2 ,$$

$$\Sigma^{\text{el}}_{00} = \frac{\pi}{p^2} \sum_{L} (2L+1) |1 - S_L|^2 .$$

(13)

The cross sections of the charge exchange process $p\bar{p} \rightarrow n\bar{n}$ are:

$$\Sigma^{\text{cex}}_{1\mu} = \frac{\pi}{p^2} \sum_{L,J} |\tilde{R}_{L\mu}^J|^2 ,$$

$$\Sigma^{\text{cex}}_{00} = \frac{\pi}{p^2} \sum_{L} (2L+1) |\tilde{S}_L|^2 .$$

(14)

III. NUMERICAL RESULTS

Using the results obtained above we can discuss the kinetics of the polarization buildup. Let $P_T$ be the target polarization vector and $\zeta_T = P_T/P_T$. The arising polarization $P_B(t)$ of the antiproton beam is collinear to $\zeta_T$. The general solution of the kinetic equation describing the polarization buildup is given in [4]. As shown in this paper, under certain conditions which are usually fulfilled in storage rings, the quantity $P_B(t)$ and the number of particles in the beam $N(t)$ have the form

$$P_B(t) = \tanh \left[ \frac{t}{2} (\Omega_+^{\text{out}} - \Omega_-^{\text{out}}) \right] ,$$

$$N(t) = \frac{1}{2} N(0) \left[ \exp \left( -\Omega_-^{\text{out}} t \right) + \exp \left( -\Omega_+^{\text{out}} t \right) \right] ,$$

(15)

where

$$\Omega_{\pm}^{\text{out}} = nf \left\{ \sigma_0 \pm P_T \left[ \sigma_1 + (\zeta_T \cdot \nu)^2(\sigma_2 - \sigma_1) \right] \right\} .$$

(16)
Here $n$ is the areal density of the target, $f$ is a revolution frequency of the beam. The function $P_B(t)$ in Eq. (15) contains $\Omega^{out}_\pm$ only in the combination $\Omega^{out}_- - \Omega^{out}_+$. For $|\sigma_2| > |\sigma_1|$, this difference is maximal at $\zeta_T \parallel \nu$. For $|\sigma_2| < |\sigma_1|$, the difference is maximal at $\zeta_T \perp \nu$.

Our estimation shows that $|\Omega^{out}_- - \Omega^{out}_+| \ll (\Omega^{out}_- + \Omega^{out}_+)$, and below this relation is assumed to be fulfilled. Then the beam lifetime, $\tau_b$, due to the interaction with a target is

$$\tau_b = \frac{2}{(\Omega^{out}_- + \Omega^{out}_+)}.$$  

(17)

Note that the Figure Of Merit, $FOM(t) = P_B^2(t)N(t)$, is maximal at $t_0 = 2\tau_b$ when the number of antiprotons is $N(t_0) \approx 0.14N(0)$. For the polarization degree at $t_0$ we have

$$P_B(t_0) = \begin{cases} 
-2P_T\sigma_1/\sigma_0, & \text{if } \zeta_T \cdot \nu = 0 \\
-2P_T\sigma_2/\sigma_0, & \text{if } |\zeta_T \cdot \nu| = 1
\end{cases}$$  

(18)

The positive (negative) value of $P_B(t_0)$ means that the beam polarization is parallel (antiparallel) to $\zeta_T$.

Predictions for the total and elastic cross sections, obtained using the Paris potential with the parameters from Refs. [10, 11, 12], are in good agreement with the experimental data. However, the spin-dependent parts, $\sigma_1$ and $\sigma_2$, of the cross section are rather small in comparison with the total one. Therefore, we can not exclude that the accuracy in our predictions for $\sigma_1$ and $\sigma_2$ would be worse than that for the total cross section. The dependence of $\sigma_1$ and $\sigma_2$ on the antiproton kinetic energy in the lab frame, $T$, is shown in Fig. 1 in the interval $20 \div 100 MeV$ for a several values of the acceptance angle.

**FIG. 1:** The cross sections $\sigma_1$ (mb) and $\sigma_2$ (mb) as a function of the kinetic energy $T$ (MeV) in the lab frame. The acceptance angles in the lab frame are $\theta^{l}_{acc} = 10$ mrad (solid curve), $\theta^{l}_{acc} = 20$ mrad (dashed curve), and $\theta^{l}_{acc} = 30$ mrad (dashed-dotted curve)
The dependence of $\sigma_{1,2}$ on $\theta_{acc}$ is completely due to interference of the Coulomb and the strong elastic amplitudes. This interference was very important for describing the proton beam polarization buildup due to $pp$ scattering, significantly diminishing both spin-dependent contributions ($\sigma_{1,2}$) to the cross section, see Refs. [4, 17]. Interference turns out to be even more important in the $p\bar{p}$ case, drastically modifying $\sigma_{1,2}$ as compared with the pure strong contribution. The corresponding quantities, $\sigma_{1,2}^{int}$, are shown in Fig. 2. It is seen that $\sigma_{1}^{int}$ is rather close to $\sigma_{2}^{int}$.

![Graph showing contributions $\sigma_{1}^{int}$ and $\sigma_{2}^{int}$ to the cross sections $\sigma_{1}$ and $\sigma_{2}$, respectively, as a function of the kinetic energy $T$ (MeV) in the lab frame.](image)

FIG. 2: The contributions $\sigma_{1}^{int}$ (mb) and $\sigma_{2}^{int}$ (mb) to the cross sections $\sigma_{1}$ and $\sigma_{2}$, respectively, as a function of the kinetic energy $T$ (MeV) in the lab frame. The acceptance angles as in Fig. 1.

The dependence of $P_{B}(t_{0})$ on $T$ is shown in Fig. 3 for $P_{T} = 1$, $\zeta_{T} \cdot \nu = 0$ ($P_{\perp}$), and $|\zeta_{T} \cdot \nu| = 1$ ($P_{\parallel}$). It is seen that $P_{\perp}$ has a maximum at energies $50 \div 70$ MeV. The position of this maximum shifts to the left with the increasing acceptance angle. The maximal value of $P_{B}(t_{0})$ also increases with growing $\theta_{acc}$ though this growth becomes slower at larger $\theta_{acc}$. For $|\zeta_{T} \cdot \nu| = 1$, the corresponding beam polarization $P_{\parallel}$ becomes noticeable only for sufficiently large $T$ where $t_{0}$ would be too large. The dependence of $t_{0} = 2\tau_{b}$, Eq. (17), on $T$ is shown in Fig. 4 for $n = 10^{14}$ cm$^{-2}$, $f = 10^{6}$ sec$^{-1}$, and several values of the acceptance angle. It is seen that for these realistic values of the density $n$ and acceptance angle $\theta_{acc}^{l}$, the polarization time is rather reasonable.

In conclusion, using the Paris nucleon-antinucleon optical potential, we have calculated the spin-dependent part of the cross section of $p\bar{p}$ interaction and the corresponding degree of the beam polarization. Our results indicate that a filtering mechanism can provide a noticeable beam polarization in a reasonable time.
FIG. 3: The polarization $P_B(t_0)$ at $P_T = 1$ as a function of the kinetic energy $T$ (MeV) in the lab frame for $\zeta_T \cdot \nu = 0$ ($P_\perp$) and $|\zeta_T \cdot \nu| = 1$ ($P_\parallel$). The acceptance angles as in Fig. 1.

FIG. 4: The dependence of $t_0$ (hour) on $T$ (MeV) for $n = 10^{14}$ cm$^{-2}$, $f = 10^6$ sec$^{-1}$. The acceptance angles as in Fig. 1.

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