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To cite this article: A Ishiwatari et al 2018 J. Phys.: Conf. Ser. 1063 012044

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Verification of accuracy of yield functions of sheet steels under shear strains in uniaxial tensile tests in multiple directions

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Abstract. Sheet steels have more or less plastic anisotropy. It is therefore important to consider the anisotropy of a sheet steel in FE analysis of press forming. There are many anisotropic yield functions proposed such as Hill’48, Blart’s Yld functions and Gotoh’s orthotropic yield function. These yield functions have material parameters to express the anisotropy of a sheet steel, and the parameters need to be calibrated based on r-values and stresses in tension tests in various directions, and/or stresses and strains in bulge tests or in biaxial tension tests. The accuracy of a calibrated yield function should be confirmed by using the results of another material test which is not used in the calibration. However, some functions have many parameters and need many calibration tests. A new method to check parameters of yield functions was developed making use of shear strains in a uniaxial tension test. In this study, shear strains were measured by DIC in uniaxial tension tests in various directions. The results, the ratios of a shear strain to the tensile strain, were compared with theoretical values from the calibrated yield functions.

1. Introduction

There are some points in FE analysis of press forming in order to predict cracks, wrinkles, and springback accurately. Material modeling is one of the most important items of them. Generally, a material model for elastic-plastic deformation is composed of a hardening rule, a yield function, and a flow rule. Either an isotropic hardening rule or a kinematic hardening rule is usually used in FE analysis. An associated flow rule is well-known as a flow rule. Although various yield functions are known, an anisotropic yield function should be used in FE analysis of press forming of an anisotropic material.

Sheet steels are more or less anisotropic. It is therefore important to consider the anisotropy of a sheet steel in FE analysis of press forming. There are many anisotropic yield functions proposed such as Hill’48[1], Blart’s Yld functions [2,3,4], Gotoh’s orthotropic yield function[5] and so on[6,7]. These yield functions have material parameters to express the anisotropy of a sheet steel, and the parameters need to be calibrated based on r-values and stresses in tension tests in various directions, and/or stresses and strains in bulge tests or in biaxial tension tests. The accuracy of a calibrated yield function should be confirmed by using the results of another material test which is not used in the calibration. However, in the case of yield functions including many parameters such as Yld2000-2d[4], not only uniaxial test results but also biaxial test results need to be used for calibration, and the accuracy of the parameters should be checked by using another test such as a hole expansion test and a press forming test. But the results of FE analysis of such a test cannot reflect indeterminate factors such as lubrication. And FE analysis itself cannot express the test exactly. The check of the parameters is not trustworthy in the case of such a test. Another new method is needed to evaluate the parameters.
A new method with shear strain, which is theoretically predicted in a uniaxial tensile test in a direction other than the rolling direction, was developed.

2. Theoretical prediction method of shear strain ratio in tensile tests

If isotropic hardening is assumed, the yield function \( f \) can be described as

\[
    f = \phi - R(\varepsilon_p)
\]

where \( R \) is the flow stress which depends on the equivalent plastic strain \( \varepsilon_p \), and \( \phi \) is a yield function such as Hill’48 and Gotoh’s. The Hill’48 yield function \( \phi_{\text{Hill}} \) is given by

\[
    \phi_{\text{Hill}} = \left\{ F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\sigma_{yx}^2 + 2M\sigma_{zx}^2 + 2N\sigma_{xy}^2 \right\}^{1/2} \equiv \bar{\sigma}
\]

In equation (2), \( x-, y-, \) and \( z- \) axes correspond to the rolling, transverse, and thickness directions, respectively.

Gotoh’s orthotropic yield function \( \phi_{\text{Gotoh}} \) has the form of

\[
    \phi_{\text{Gotoh}} = \left( A_1\sigma_x^2 + A_2\sigma_y^2\sigma_x + A_3\sigma_z^2\sigma_x + A_4\sigma_x\sigma_y + A_5\sigma_y^2\sigma_x + A_6\sigma_x^2\sigma_y + A_7\sigma_x\sigma_y + A_8\sigma_y^2\right)^{1/4}
\]

Yld2000-2d yield function \( \phi_{\text{Yld}} \) is given by

\[
    \phi_{\text{Yld}} = \left( X_{xx}'^2 + 4X_{xy}'^2 \right)^{M/2} + \left( (X_{yy}')^2 + 4X_{xy}'^2 \right)^{M/2} - \left( \frac{2\alpha_1}{3} - \frac{\alpha_1}{3} - \frac{3}{3} \right) \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix} \]

\[
    \begin{pmatrix} X_{xx}' \\ X_{yy}' \\ X_{xy}' \end{pmatrix} = \begin{pmatrix} -2\alpha_3 + 2\alpha_4 + 8\alpha_5 - 2\alpha_6 \\ 4\alpha_3 - 4\alpha_4 - 4\alpha_5 + 4\alpha_6 \\ 0 \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix}
\]

If the associated flow rule and plane stress condition are assumed, strain increments are given by

\[
    (d\varepsilon_{pp}^x \quad d\varepsilon_{pp}^y \quad d\varepsilon_{xy}^p)^T = d\lambda \begin{pmatrix} \frac{\partial f}{\partial \sigma_x} \\ \frac{\partial f}{\partial \sigma_y} \\ \frac{1}{2} \frac{\partial f}{\partial \sigma_{xy}} \end{pmatrix}^T
\]

The shear strain ratio \( d\varepsilon_{xy}^p/d\varepsilon_{xx}^p \) can be calculated using these equations.

On the other hand, strain increments in 1-2-z coordinate system which is rotated by \( \theta \) [rad] around the z-axis, can be converted from those in x-y-z coordinate system by using an equation of

\[
    \begin{pmatrix} d\varepsilon_{p11}^p \\ d\varepsilon_{p12}^p \\ d\varepsilon_{p22}^p \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d\varepsilon_{xx}^p \\ d\varepsilon_{yy}^p \\ d\varepsilon_{xy}^p \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}^T
\]

An equivalent expression of strain increments in 1-2-z coordinate system is

\[
    \begin{pmatrix} d\varepsilon_{p11}^p \\ d\varepsilon_{p22}^p \\ d\varepsilon_{p12}^p \end{pmatrix} = \begin{pmatrix} \cos^2 \theta & \sin^2 \theta & 2 \cos \theta \sin \theta \\ \sin^2 \theta & \cos^2 \theta & -2 \cos \theta \sin \theta \\ -\cos \theta \sin \theta & \cos \theta \sin \theta & \cos^2 \theta - \sin^2 \theta \end{pmatrix} \begin{pmatrix} d\varepsilon_{xx}^p \\ d\varepsilon_{yy}^p \\ d\varepsilon_{xy}^p \end{pmatrix}
\]

Combining equations (5) and (7) leads to the expression of strain increments by the partial differentials of the yield function,
\[
\begin{pmatrix}
\varepsilon_{11}^P \\
\varepsilon_{22}^P \\
\varepsilon_{12}^P
\end{pmatrix} =
\begin{pmatrix}
\cos^2 \theta & \sin^2 \theta & \cos \theta \sin \theta \\
\sin^2 \theta & \cos^2 \theta & -\cos \theta \sin \theta \\
-\cos \theta \sin \theta & \cos \theta \sin \theta & \frac{1}{2}(\cos^2 \theta - \sin^2 \theta)
\end{pmatrix}
\begin{pmatrix}
\frac{\partial f}{\partial \sigma_x} \\
\frac{\partial f}{\partial \sigma_y} \\
\frac{\partial f}{\partial \sigma_{xy}}
\end{pmatrix}
\]

In the case of Hill’48, partial differentials of the yield function are given by

\[
\frac{\partial f}{\partial \sigma_x} = \frac{1}{\sigma} \left\{ (G + H) \sigma_x - H \sigma_y - G \sigma_z \right\},
\frac{\partial f}{\partial \sigma_y} = \frac{1}{\sigma} \left\{ (F + H) \sigma_y - F \sigma_x - H \sigma_z \right\},
\frac{\partial f}{\partial \sigma_{xy}} = \frac{2}{\sigma} N \sigma_{xy}
\]

When a uniaxial stress \(\sigma_{11}\) in the 1-2-z coordinate system is loaded on the specimen, the stresses observed in the x-y-z coordinate system are calculated by

\[
(\sigma_x \; \sigma_y \; \sigma_{xy})^T = \sigma_{11}(\cos^2 \theta \; \sin^2 \theta \; \cos \theta \sin \theta)^T
\]

As a result, the shear strain ratio \(d\varepsilon_{12}^P/d\varepsilon_{11}^P \equiv \varepsilon_{12}^P/\varepsilon_{11}^P\) in a uniaxial tensile test in the direction at an angle \(\theta\) to the rolling direction can be calculated theoretically from equations (7) to (10). Both each stress and partial differential of the yield function is proportional to \(\sigma_{11}\) in the x-y-z coordination system. That means the shear strain ratio does not depend on \(\sigma_{11}\) but depend on \(\theta\) and the parameters in Hill’48; F, G, H, and N.

3. Experiment for the parameters of yield functions

Tensile tests in multiple directions and a bulge test were carried out on a 590MPa grade steel sheet with 1.2mm thickness. The directions of the tensile tests were at angles of 0.0, 22.5, 45.0, 67.5, and 90.0 degree from the rolling direction. The results of the tensile tests are shown in table 1.

| Tensile angle | Yield stress [MPa] | Tensile stress [MPa] | r value | Flow stress [MPa] at 40 \(\times 10^{-3}\) [J/mm\(^3\)] plastic work |
|---------------|-------------------|---------------------|---------|---------------------|
| 0.0           | 465               | 613                 | 0.54    | 631.3               |
| 22.5          | 437               | 604                 | 0.81    | 619.5               |
| 45.0          | 442               | 590                 | 1.25    | 605.2               |
| 67.5          | 446               | 609                 | 1.06    | 623.5               |
| 90.0          | 486               | 625                 | 0.76    | 640.8               |

The dependency of r-value on the tensile test direction suggests that this steel has relatively strong anisotropy. When the plastic work reached \(40 \times 10^{-3}\) [J/mm\(^3\)] in bulge test, the equi-biaxial tensile stress (\(\sigma_b\)) was 626 [MPa].

The parameters of Hill’48, Yld2000-2d, and Gotoh’s yield functions were calibrated based on the experimental results. Two types of parameter calibration were conducted in the case of Hill’48. One is based on flow stresses at the same plastic work in three directions: at 0, 45, and 90 degree to the rolling direction. The other calculates the parameters in Hill’48 model using r-values in the same 3 directions. The optimized parameter sets are shown in table 2. Hill’48-3sigma means the calibration with the flow stresses, and Hill’48-3r means the calibration with r-values.

| Table 2. Parameters of Hill’48 |
|-------------------------------|
| F     | G     | H     | N     |
| Hill’48-3σ | 0.517 | 0.547 | 0.453 | 1.644 |
| Hill’48-3r | 0.458 | 0.650 | 0.350 | 1.936 |
The two sets of parameters for the quadruple (M=4) and sixth degree (M=6) Yld2000-2d models of equation (4) were calibrated with the flow stresses and r-values in the 5 directions as shown in table 1, and the results of the bulge tests. The calibrations were carried out by minimizing an evaluation function of

\[ O = \sum_{\theta} \left( w_s \left( \frac{\sigma^* - \sigma_{\theta}}{\sigma_{\theta}} \right)^2 + w_r \left( \frac{r^*_{\theta} - r_{\theta}}{r_{\theta}} \right)^2 \right) + w_b \left( \frac{\sigma^*_b - \sigma_b}{r_{b}} \right)^2 \]

(11)

where \( w_s, w_r, \) and \( w_b \) are weight. In calibration, the initial values of parameters of \( \alpha_1 \) to \( \alpha_8 \) are set to 1.0. The Yld2000-2d model is an isotropic function with the initial parameters of 1.0. The optimized parameters are shown in table 3.

**Table 3. Parameters of Yld2000-2d**

| M=4 \( \alpha_1 \) | \( \alpha_2 \) | \( \alpha_3 \) | \( \alpha_4 \) | \( \alpha_5 \) | \( \alpha_6 \) | \( \alpha_7 \) | \( \alpha_8 \) |
|----------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.830 | 1.002 | 0.884 | 0.987 | 1.070 | 1.027 | 1.071 | 1.021 |

| M=6 \( \alpha_1 \) | \( \alpha_2 \) | \( \alpha_3 \) | \( \alpha_4 \) | \( \alpha_5 \) | \( \alpha_6 \) | \( \alpha_7 \) | \( \alpha_8 \) |
|----------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.872 | 1.021 | 1.000 | 0.9985 | 1.036 | 0.976 | 1.059 | 1.041 |

The calibration of the parameters in Gotoh’s function was done in the same way as Yld2000-2d. The initial values of the 9 parameters, \( \alpha_1 \) to \( \alpha_9 \), were set to 1.0, -2.0, 3.0, -2.0, 1.0, 6.0, -6.0, 6.0, and 9.0, respectively. The optimized parameters are shown in table 4.

**Table 4 Parameters of Gotoh’s function**

| \( \alpha_1 \) | \( \alpha_2 \) | \( \alpha_3 \) | \( \alpha_4 \) | \( \alpha_5 \) | \( \alpha_6 \) | \( \alpha_7 \) | \( \alpha_8 \) | \( \alpha_9 \) |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1.000 | -1.393 | 2.124 | -1.640 | 0.941 | 5.960 | -5.894 | 6.253 | 11.558 |

Flow stresses obtained in the experiments and calculated with the yield functions are compared in figure 1. Hill’48-3r whose parameters were calibrated with r-values does not express experimental results well. On the other hand, Hill’48-3sigma calibrated with flow stresses, Yld2000-2d, and Gotoh’s functions are in good agreement with experimental flow stresses. Figure 2 shows r-values calculated with the yield functions compared with experimental ones. Except for the Hill’s-3σ function, the other functions have similar results to experimental r-values. In summary, Yld2000-3d type and Gotoh’s functions can express both the experimental flow stresses and r-values well, although Hill’48 type functions cannot describe both the experimental values.

Yield loci of the above mentioned yield functions are shown in figure 3. The loci of Hill’48-3r and Hill’48-3σ are compared with Gotoh’s locus in figure 3 (a). The Hill’48-3r locus is different from the Gotoh’s locus, especially in y-axis and equi-biaxial areas. The Hill’s48-3σ locus has better agreement with the Gotoh’s function, but there is a slight gap between them in the biaxial area. The loci of Yld2000-
3d are compared with the Gotoh’s locus in figure 3 (b). The Yld2000-3d loci are in good agreement with the Gotoh’s locus, in particular, in the case of M=4.

4. Theoretical shear strain ratio and measurement of shear strain by DIC in tensile tests

Theoretical shear strain ratios are shown in figure 4 calculated with the yield functions mentioned in section 3. The shear strain ratio by Hill’48-3r is far from the other functions. The Hill’48-3σ, Yld2000-2d, and Gotoh’s functions give similar shear strain ratios. There are no noticeable difference between Yld200-2d (M=4 and 6) and Gotoh’s functions.

Shear strains of a 590MPa steel sheet were measured in tensile tests in various directions by digital image correlation (DIC). The schematic cutting layout of the tensile specimens is shown in figure 5: the specimens were cut out in the directions which are from the rolling to the transverse directions with an angle interval of 11.25 degree. Figure 6 shows the distribution of the shear strain in the tensile specimen in the direction at an angle of 67.5 degree to the rolling direction under a tensile strain of 10%. A uniform distribution is observed at the center of the specimen. The open points plotted in figure 4 show the

![Figure 3. Comparison of yield loci.](image1)

![Figure 4. Theoretical and experimental shear strain ratios under tensile strain of 10%.](image2)
measured shear strain ratios at the center of the tensile specimens under the tensile strain of 10% in the various directions.

The Hill’48-3r function cannot describe the experimental shear strain ratios as shown in figure 4. The Hill’48-3sigma function which cannot express the r-values in figure 2, is consistent with the experimental shear strain ratios in figure 4. The Yld2000-2d functions (M=4 or 6) and the Gotoh’s function are in good agreement with all the experimental results as shown in figure 1, 2, and 4. The good agreement of the Yld2000-2d and Gotoh’s functions with experimental results suggests that a measured strain ratio could be used for the verification of the accuracy of a yield function calibrated using flow stresses and r-values in various directions, and that the use of the associated flow rule would be valid.

5. Conclusions

A new method for the verification of the accuracy of a yield function was investigated using shear strains in a uniaxial tensile test.

(1) Parameters in the yield functions of Hill’48, Yld2000-2d, and Gotoh were calibrated based on tensile flow stresses and r-values in tensile tests in various directions and results of a bulge test.

(2) The Yld2000-2d and Gotoh’s yield functions can predict shear strain ratios in tensile tests in various directions by using the associated flow rule.

(3) The results of this study suggest that a measured strain ratio could be used for the verification of the accuracy of a yield function, and that the use of the associated flow rule would be valid.

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