High Temperature $\frac{\pi}{2}$—SQUID

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Abstract — A new DC-SQUID is proposed that exploits the properties of the grain boundary junctions in high $T_c$ superconductors. The orientations of the grain boundaries are chosen in such a way to establish a $\pi/2$ (rather than 0 or $\pi$) phase difference between the equilibrium phases of the two Josephson junctions in the SQUID loop. This property is exploited to increase the sensitivity and direction dependence of the SQUID for measuring magnetic flux without additional flux generating coils.

Keywords — SQUID, High $T_c$ superconductivity, Quasiclassical theory.

I. INTRODUCTION

In a usual Josephson tunnel junction, the current $I$ passing through the junction is related to the phase difference $\phi$ between the two superconductors by

$$I = I_c \sin \phi. \quad (1)$$

The equilibrium phase difference is zero in the absence of current (current is also zero when $\phi = \pi$, but this corresponds to the maximum of the Josephson energy rather than a minimum). The simple relation (1) should be replaced with a more complicated one [2], [3] in the case of constriction junctions, but the equilibrium phase difference will still be at $\phi = 0$.

A conventional DC-SQUID consists of a superconducting ring with two ordinary Josephson junctions on its opposite arms. The total current passing through the SQUID is the sum of the currents crossing each of the Josephson junctions. As a result of interference between the two currents, the total current-phase relation depends on the flux $\Phi$ threading the ring. In particular, the critical current (the maximum current allowed before going to non-stationary state with finite voltage drop across the junction) is given by

$$I_c = I_{c0} \left| \cos \left( \frac{\phi_c}{2} \right) \right|, \quad (2)$$

where $\phi_c$ is related to the external flux $\Phi$ threading the loop through $\phi_c \equiv 2\pi \Phi/\Phi_0$, with $\Phi_0 = h/2e$ being the flux quantum, $h$ the Planck constant, and $e$ the charge of electron. Here we neglect the self-inductance of the loop. Notice that $\phi_c$ in Eq. (2) has period $2\pi$ which corresponds to one flux quantum. At small $\phi_c$, the relation (2) is almost flat ($I_c \propto \phi_c^2$); consequently, the sensitivity of the measurement of small fluxes is low. Moreover, the critical current does not depend on the direction of the magnetic field. To avoid this problem, one can apply a biasing flux to move the working point of the SQUID away from the flat region. However, this biasing flux increases the noise as well as the complexity of the device and sometimes can have unwanted influence on the measured system.

Therefore, it is desirable to shift the working point of the SQUID away from the flat region without applying an external magnetic field. This is possible by replacing one of the Josephson junctions with a junction with non-zero equilibrium phase difference. A DC-SQUID with a $\pi$-phase difference between the equilibrium phases of the two Josephson junctions has already been studied [4]. Here we propose another type of SQUID (we call it a $\pi/2$-SQUID) with the equilibrium phase difference of $\pi/2$ between the junctions. The frustration caused by the two junctions produces the desired shift of the working point. In the next section we introduce such a device using high $T_c$ superconducting grain boundary junctions. In section III we study the $\pi/2$-SQUID using a simple model. Section IV is devoted to the quasiclassical calculation of the current in the $d$-wave structure and demonstrating agreement with the simple model of section II. Finally, we summarize our results in section V.

II. HIGH $T_c$ $\pi/2$—SQUID

In high $T_c$ superconductors, because of the $d$-wave symmetry of the order parameter, the current-phase relations are nontrivial and generally depend on the orientations of the order parameters on both sides of the junction [3], [4]. If we keep only the first two harmonics, we can write the current-phase relation as

$$I = I_1 \sin \phi - I_2 \sin 2\phi. \quad (3)$$
When $I_2 > I_1/2$ the equilibrium phase will occur at $\varphi = \varphi_0$, where $\varphi_0$ is neither 0 nor $\pi$. In particular, when the first harmonic vanishes, the equilibrium phase difference happens at $\varphi_0 = \pi/2$. These types of junctions, known as $\pi/2$-junctions, can be realized at the grain boundary junction between two $d$-wave superconductors with crystalline orientations of 0° and 45° with respect to the grain boundary. At such a junction (known as an asymmetric junction), there is an exact cancellation of the first harmonic and the second harmonic dominates the current \( [7] \). A symmetric junction can also be made with 45° misorientation angle by choosing -22.5° and 22.5° crystalline orientations at the two sides. This junction is very different from the asymmetric one: specifically, the exact cancellation of the first harmonic does not occur in this case \( [8] \).

In a realistic junction with non-ideal transparency and roughness at the boundary, the second harmonic gets suppressed by the boundary imperfections more strongly than the first harmonic and, as a result, the latter dominates the current. This leads to a conventional junction (0-junction) with the usual current-phase relation of Eq. (1). These junctions have also been studied experimentally \( [9] \).

By combining symmetric and asymmetric junctions, we can have a SQUID with $\pi/2$ equilibrium phase difference between the junctions. Fig. 1 illustrates such an structure. First we study this system using a simple model. The more realistic numerical simulation of the system using a quasiclassical model will come after that.

### III. Simple Model

Consider a DC-SQUID, consisting of a conventional and a $\pi/2$ Josephson junction, with current-phase relations

\[
 I_1 = I_{c1} \sin \varphi_1, \quad I_2 = -I_{c2} \sin 2\varphi_2, \quad (4)
\]

respectively. To simplify the calculations, we will assume $I_{c1} = 2I_{c2} = 2I_c$. The more general case of arbitrary $I_{c1}/I_{c2}$ can also be treated as the solutions are similar. We also neglect the self inductance of the ring in our calculations. When there is a flux $\Phi$ threading the SQUID ring, the phases at two junctions are related by $\varphi_1 - \varphi_2 = \phi_e$. The total current passing through the SQUID is then given by

\[
 I = I_{c0}[2 \sin(\varphi_2 + \phi_e) - \sin 2\varphi_2]. \quad (5)
\]

The critical current is the maximum current in this current-phase relation. To find the critical current, one has to find the solutions to the equation $dI/d\varphi_2 = 0$. These solutions are $\phi_e + 2\pi n$ and $(-\phi_e + 2\pi n)/3$, with $n$ being an integer number. Substituting into Eq. (3) we find the maximum current to be

\[
 I_c = \frac{3}{2} I_{c0} \text{Max} \left\{ \sin \frac{2}{3}(\phi_e + n\pi) \right\}. \quad (6)
\]

Here “Max” means the maximum of the three cases with $n = 0, \pm 1$.

Fig. 2 shows the flux dependence of the critical current using Eq. (3). Notice that the periodicity of the function is $\pi$ (half a flux quantum) although the period of the argument of the sine-functions in Eq. (6) is actually $3\pi$. The $\pi/\text{periodicity}$ of Eq. (6) is a result of the symmetry $I(\phi_e + \pi, \varphi_2) = I(\phi_e, \varphi_2 + \pi)$ in Eq. (3). In other words, changing the applied flux by half a flux quantum shifts the current-phase relation by $\pi$ and therefore does not change the critical current. This is evident from the ordinary SQUID for which the period is $2\pi$ (one flux quantum). The important feature here is that the $\phi_e = 0$ does not coincide with the maximum of the function as it does in usual the DC-SQUIDs. Therefore, an increase in sensitivity for measuring small fluxes is gained. Moreover, the direction of the flux is also detectable by measuring the critical current.

### IV. Quasiclassical Calculations

As was mentioned, the current-phase relations in $d$-wave grain boundary junctions does not usually have the simple forms of Eq. (3). One has to find the realistic current-phase relation using a microscopic theory, taking into account the imperfections of the boundary. Here we use a quasiclassical model to study this system. The equations we solve are the Eilenberger equations \( [10] \)

\[
 v_F \cdot \frac{\partial}{\partial \mathbf{r}} \hat{G}_\omega(\mathbf{v}_F, \mathbf{r}) + [\omega \tau_3 + \hat{\Delta}(\mathbf{v}_F, \mathbf{r}), \hat{G}_\omega(\mathbf{v}_F, \mathbf{r})] = 0, \quad (7)
\]

where

\[
 \hat{\Delta} = \begin{pmatrix} 0 & \Delta \tau_3 \\ \Delta \tau_3 & 0 \end{pmatrix}, \quad \hat{G}_\omega(\mathbf{v}_F, \mathbf{r}) = \begin{pmatrix} g_\omega & f_\omega \\ f_\omega & -g_\omega \end{pmatrix}. \quad (8)
\]

$\Delta$ is the superconducting order parameter and $\hat{G}_\omega(\mathbf{v}_F, \mathbf{r})$ is the matrix Green’s function, which depends on the electron velocity on the Fermi surface $v_F$, the coordinate $\mathbf{r}$, and the Matsubara frequency $\omega = (2n + 1)\pi T$, with $n$ being an integer number and $T$ the temperature. We also need to satisfy the normalization condition

\[
 g_\omega = \sqrt{1 - f_\omega f_\omega^*}. \quad (9)
\]
In general, $\Delta$ depends on the direction of $\mathbf{v}_F$ and is determined by the self-consistency equation

$$\Delta(\mathbf{v}_F, r) = 2\pi N(0)T\sum_{\omega>0} \langle V(\mathbf{v}_F, \mathbf{v}'_F) f_\omega(\mathbf{v}'_F, r) \rangle \mathbf{v}'_F \tag{10}$$

where $V(\mathbf{v}_F, \mathbf{v}'_F)$ is the interaction potential. Solution of matrix equation (7) together with (10) determines the current density $\mathbf{j}(r)$ in the system

$$\mathbf{j}(r) = -4\pi i eN(0)T\sum_{\omega>0} \langle \mathbf{v}_F g_\omega(\mathbf{v}_F, r) \rangle \mathbf{v}_F \tag{11}$$

In two dimensions, $N(0) = m/2\pi$ is the 2D density of states and $\langle ... \rangle = \int (d\theta/2\pi) ...$ is the averaging over directions of the 2D vector $\mathbf{v}_F$.

The transparency of the boundary is described by the parameter $D_0$. To incorporate the roughness, we assume a narrow scattering region of width $d$ between the two superconductors. The roughness is therefore parameterized by $\rho = d/l$, where $l$ is the mean free path of the quasiparticles in this scattering region. In our calculations, we use $D_0 = \rho = 0.5$ which are reasonable values for a realistic system. Different widths, $W_1$ and $W_2$, are chosen for the two arms and the ratio of the widths is denoted by $r = W_2/W_1$. The details of our numerical method for solving these equations is given in [1].

Fig. 3 displays the current-phase relation at different values of the external flux $\phi_e$ for a SQUID structure with $r = 1.4$. At $\phi_e = 0$, the graph has two local maxima, which is the signature of having a mixture of first and second harmonics. As $\phi_e$ is increased, the two maxima become closer and at some value of $\phi_e$, they will be equal. At this point, the global maximum jumps from one local maximum to another. This actually explains the sharp kinks at the minimum positions in Fig. 2 (and also in Fig. 4).

Fig. 4 shows the critical current as a function of $\phi_e$ for different values of $r$. One can immediately see the similarities between these graphs and the one obtained from our simple model in Fig. 2. Specifically, the maxima again occur with period of $\pi$, although the $\pi$-periodicity is not exact here (the real period is $2\pi$). The maximum of the critical current is shifted from $\phi_e = 0$, as was desired. One can also see from Fig. 4 that the maximum range of variation of $I_c$ is obtained at $r = 1.4$. This is actually the optimal ratio of the widths for SQUID design.

V. Conclusions

We proposed and studied a DC-SQUID which has $\pi/2$ difference between the equilibrium phases at its two junctions. The $\pi/2$ phase difference is easily obtained using the properties of high $T_c$ grain boundary symmetric and asymmetric junctions. The frustration caused by this phase difference moves the working point of the SQUID to a point at which the maximum critical current is obtained at a nonzero external flux. This will increase the sensitivity of the SQUID at small magnetic field, with a dependence on the direction of the flux, without any external biasing coil.

We introduced a simple model and compared its results with the ones obtained from a microscopic quasiclassical model. The agreement between the two models is acceptable. We found that the $\pi/2$-SQUID exhibits other new features, different from a conventional SQUID or a $\pi$-SQUID, among those is the $\Phi_0/2$ (instead of $\Phi_0$) periodicity of the critical current. We find the optimal ratio between the widths of the two arms to be $r = 1.4$.

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