Assignment of single particle configurations in odd-A nuclei near A~100 with angular correlation measurements

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Abstract. The multipole mixing ratios of $\Delta I = 1$ transitions between levels in rotational bands built on single-particle states in odd neutron nuclei are dependent on the configurations of the states. In particular, the mixing ratio can be used to distinguish between several possible single-particle configurations if interpreted with the particle plus axial-rotor model (PRM). This work features the first determination of the ground-state configurations of $^{109,111}$Ru. The single-particle structures of the ground states of $^{101}$Zr and $^{103,105,107}$Mo as well as excited states in $^{103,105}$Mo are also investigated, with a new result found in $^{107}$Mo.

1. Introduction
The single particle configurations for odd-A deformed nuclei are based on either Nilsson shell model calculations or by comparing the states with the states in neighboring nuclei where shell model calculations are performed. The multipole mixing ratios of $\Delta I = 1$ transitions between levels in rotational bands built on single-particle states in odd neutron nuclei are dependent on the configurations of the states. In particular, the mixing ratio can be used to distinguish between several possible single-particle configurations if interpreted with PRM. The neutron single-particle configurations of $^{101}$Zr, $^{103,105,107}$Mo, and $^{109,111}$Ru are determined by measuring the mixing ratios of $\Delta I = 1$ transitions in their rotational bands. The mixing ratios are interpreted in terms of PRM. These nuclei are known to be highly deformed [1], with the $h_{11/2}$ neutron orbital playing a role in driving the deformation although the proton-neutron interaction may be also important. These nuclei are produced in spontaneous fission of $^{252}$Cf. In this paper, we described the technique of measuring angular correlations with large gamma detector arrays and discussed the assignment of single particle configurations. The details on angular correlation techniques and the measurements can be found in references [2,3].

The advantages of doing angular correlation experiments with large detector arrays such as Gammasphere as opposed to traditional setups with 2–4 detectors are major increase in angular resolution and detection efficiency. However, there are many difficulties for measuring angular correlations with such large detector arrays. Specifically, the number of pairs increases as the square of the number of detectors. Since the detector

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efficiency and the solid angle correction factors are different for each pair, handling a problem of this magnitude requires careful analysis techniques, and we describe these below. Following the description of our method, we show results for unattenuated angular correlations.

2. Experimental technique and analysis procedure

The data for this analysis were taken by using the Gammasphere detector array, which was located at Lawrence Berkeley National Laboratory. A $^{252}$Cf spontaneous fission source with an $\alpha$ activity of 62 $\mu$Ci was sandwiched between two iron foils. The foils were thick enough to stop the fission fragments, eliminating the need for a Doppler correction. Approximately $5.7 \times 10^{11}$ three- and higher-fold $\gamma$ coincidence events were recorded.

The coincidence intensity between a cascade of two $\gamma$ rays as a function of angle is fitted to Eq. (1).

$$W(\theta) = 1 + G_2 A_2(\delta)P_2(\cos\theta) + G_4 A_4(\delta)P_4(\cos\theta)$$  (1)

In this equation the $A_k$ s are calculable coefficients depending on the spin sequence of the levels involved and the multipolarities and mixing ratios of the transitions between them. The $G_k$ coefficients relate to attenuations of the correlations and are equal to unity unless some interaction is present during the lifetime of the intermediate state of the cascade that can cause precession of the nuclear spin. For the cascades involved in this work, the nuclear hyperfine field in iron is less than about 50 T. Unless the life-time of the intermediate state is greater than ~0.7 ns, the $G_k$s are close to unity.

The mixing ratios of $\Delta I = 1$ transitions within a rotational band are given by

$$\delta \left( \frac{E_2}{M1} \right) = \sqrt{\frac{1}{12} \frac{E_f}{E_i} \frac{Q_0}{2^{12} g_s K} \frac{\langle T_1 K0 | l | K \rangle}{\langle T_1 K0 | l | K \rangle}}$$  (2)

where $Q_i$ is the quadrupole moment; $g_s$ and $g_R$ are the intrinsic and rotational $g$ factors, respectively; and $E_i$ is the $\Delta I = 1$ transition energy in keV. For these calculations, $2g_s$ is taken to be equal to $Z/A$ and the quadrupole moment $Q_s$ is calculated from the deformation parameter, $\beta$, using the relation

$$\beta = 91.7 \frac{Q_0}{ZA^{2/3}}$$  (3)

with $\beta$ assumed to take a common value, $\beta = 0.3$. The single-particle $g_s$ factor depends on the Nilsson configuration of the particle and can be calculated by using the $M1$ matrix elements tabulated in Ref. [4] for various single-particle configurations by

$$g_K = g_l + \frac{g_s - g_l}{2K} \sum (\alpha_{lK-\frac{1}{2}}^2 + \alpha_{lK+\frac{1}{2}}^2)$$  (4)

where the $\alpha$'s are Nilsson coefficients tabulated in Ref. [4] as a function of the deformation parameter $\beta$, and $g_s$, $g_l$ are the orbital and spin $g$ factors. In these calculations, which involve neutron states, the values $g_s=0$ and $g_l^{eff} = 0.6g_l^{ref} = -2.296$ are used.

Gammasphere consists of 110 germanium detectors, corresponding to 5995 unique detector pairs. The detectors are placed at 17 different azimuthal angles and 60 various polar angles. For each azimuthal angle, there are 5–10 detectors placed symmetrically with respect to the polar angle. Due to the symmetries of Gammasphere, the angle between any two detectors will be one of only 64 possible values, and each of these 64 angle bins has many pairs of detectors. For our experiment, only 101 detectors were present, so the number of detectors in each bin is slightly less than if Gammasphere were at full capacity. The angular properties of Gammasphere and the process of determining angle bins are given in greater detail in [5]. The angle bin information for our experiment is given in Table 1. For the angular correlation, we sorted our triple coincidence data into 64 two dimensional histograms corresponding to the 64 angle bins. The histograms were produced by reading the full data set event by event. For each $\gamma - \gamma$ coincidence, the angle between the detectors which detected $\gamma_1$ and $\gamma_2$ was calculated and
the event was added to the appropriate histogram. For each bin, \( n \), the histogram was then fit to find the number of coincidences of the cascade of interest, \( N_n \). This \( N_n \) must be corrected for relative detector efficiency, the number of detectors in the particular bin, and the detector response functions, all discussed below. The value for the corrected \( N_n \) is given by

\[
N_n = \varepsilon_n(E_1, E_2) \int_0^\pi W(\theta) R_n(\theta, E_1, E_2) \sin(\theta) \, d\theta,
\]

where \( \varepsilon_n(E_1, E_2) \) is the function describing the relative efficiency of the pair and \( R_n(\theta, E_1, E_2) \) is the response function of the pair. The 64 histograms can also be made with additional gates when added selectivity is needed. Figure 3 shows the response function of each angle bin. The response functions for each pair can then be summed to find the response function of each detector. For a given detector pair, the response function describes the distribution of possible angles about the central angle of the pair as a function of energy. The response functions for each pair can then be summed to find the response function of each angle bin.

To calculate the \( \varepsilon_n(E_1, E_2) \)'s, a linear \( \gamma \) spectrum was constructed for each detector. A separate linear spectrum was constructed for the sum of all detectors. The summed spectrum was divided by the number of working detectors and then the individual detector spectra were divided by the normalized sum spectrum. The resulting spectra give the relative efficiency curve for each detector.

The number of coincidences in each angle bin is corrected for the relative efficiencies of the detector pairs. The efficiencies of the individual detectors of Gammasphere may vary greatly compared to the mean efficiency. If the detector pairs \( i,j \) belong to bin \( n \), the efficiency of the bin is given by

\[
\varepsilon_n(E_1, E_2) = \frac{1}{2} \sum_{i,j} \varepsilon_i(E_1) \varepsilon_j(E_2) + \varepsilon_i(E_2) \varepsilon_j(E_1).
\]

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For a typical angular correlation measurement, it is necessary to calculate a solid angle correction factor \( Q_k \) for each parameter \( \lambda_k \). However, for very low intensity transitions, the sensitivity of the angular correlation measurement can be improved by determining the detector response function \( R_k(0, E_1, E_2) \) for each pair of detectors. For a given detector pair, the response function describes the distribution of possible angles about the central angle of the pair as a function of energy. The response functions for each pair can then be summed to find the response function of each angle bin.

We calculated the response function using a simple Monte Carlo simulation, with the \( \gamma \)-ray transport simulated up to the first collision. This is equivalent to the traditional calculation of \( Q_k \). The mean free path, \( \lambda(E) \), of \( \gamma \)-rays.

| Bin | \( \cos(\theta) \) | Num. Pairs | Bin | \( \cos(\theta) \) | Num. Pairs | Bin | \( \cos(\theta) \) | Num. Pairs | Num. Pairs |
|-----|------------------|------------|-----|------------------|------------|-----|------------------|------------|------------|
| 1   | 0.939            | 203        | 23  | 0.309            | 56         | 45  | -0.391           | 90         |            |
| 2   | 0.934            | 58         | 24  | 0.298            | 100        | 46  | -0.471           | 52         |            |
| 3   | 0.840            | 47         | 25  | 0.282            | 101        | 47  | -0.495           | 89         |            |
| 4   | 0.823            | 45         | 26  | 0.269            | 104        | 48  | -0.500           | 56         |            |
| 5   | 0.815            | 104        | 27  | 0.175            | 100        | 49  | -0.577           | 116        |            |
| 6   | 0.809            | 56         | 28  | 0.168            | 44         | 50  | -0.580           | 101        |            |
| 7   | 0.764            | 100        | 29  | 0.139            | 23         | 51  | -0.594           | 90         |            |
| 8   | 0.762            | 45         | 30  | 0.064            | 92         | 52  | -0.613           | 104        |            |
| 9   | 0.755            | 50         | 31  | 0.057            | 104        | 53  | -0.656           | 50         |            |
| 10  | 0.745            | 30         | 32  | 0.000            | 273        | 54  | -0.745           | 50         |            |
| 11  | 0.656            | 50         | 33  | -0.057           | 104        | 55  | -0.755           | 50         |            |
| 12  | 0.613            | 104        | 34  | -0.064           | 90         | 56  | -0.762           | 45         |            |
| 13  | 0.594            | 88         | 35  | -0.139           | 23         | 57  | -0.764           | 100        |            |
| 14  | 0.580            | 100        | 36  | -0.168           | 44         | 58  | -0.809           | 56         |            |
| 15  | 0.577            | 116        | 37  | -0.175           | 101        | 59  | -0.815           | 104        |            |
| 16  | 0.500            | 56         | 38  | -0.269           | 104        | 60  | -0.823           | 44         |            |
| 17  | 0.495            | 88         | 39  | -0.282           | 101        | 61  | -0.840           | 44         |            |
| 18  | 0.471            | 52         | 40  | -0.298           | 100        | 62  | -0.934           | 58         |            |
| 19  | 0.391            | 91         | 41  | -0.309           | 56         | 63  | -0.939           | 196        |            |
| 20  | 0.357            | 58         | 42  | -0.327           | 46         | 64  | -1.000           | 46         |            |
| 21  | 0.333            | 60         | 43  | -0.333           | 60         |      |                  |            |            |
| 22  | 0.327            | 44         | 44  | -0.357           | 58         |      |                  |            |            |
was calculated using the known Gammasphere detector properties. The energy dependence of $R_n(\theta,E_1,E_2)$ is negligible, and so only $R_n(\theta)$ was calculated.

To determine the angular correlation, each of the 64 histograms was fit to find the intensity of the peak of interest, $N_n$. Our fitting method is based on the analysis of two dimensional $\gamma-\gamma$ coincidence spectra. To fit a given histogram, a window within the histogram is selected. Within this window, the positions of the peaks are defined by using projections of the histogram on the two axes. The surface is then approximated by three types of background and the sum of the two dimensional $\gamma$ peaks found in the projections. The three types of background consist of a smooth background and two series of ridges parallel to the axes corresponding to $\gamma$ lines in the $x$ and $y$ direction. The last part of the fitting procedure is to solve a well known NNLS (non-negative least square) problem [6].

Using the $N_n^{\text{Exp}}$ found from fitting the histograms, we can estimate the two parameters $A_2$ and $A_4$ using the minimization expression

$$\min \sum_n \frac{(N_n^{\text{Exp}}(\theta_n)-N_n(\theta_n))^2}{\delta_n^{\text{Exp}}+\delta_n^{\text{model}}}$$  (7)

The experimental uncertainty in the $\gamma$ peak intensity is always much greater than the model error, so $\delta_{\text{model}}$ was neglected. The 64 angle bins are further grouped into 17 larger bins, thereby increasing the statistics of each bin. Since we have to determine only three parameters, 17 data points are more than adequate.

3. Experimental results

One example of angular correlation measurements is shown in Fig. 1. In Fig 1, $^{138}\text{Ba}$ has the $2^+$ level with a very short lifetime of 0.202(8) ps which indicates that the correlation should be unattenuated. Then, we obtained the expected value of $G_2 \equiv 1$. The consistency of this result with the expected value shows the viability of our method for measuring angular correlations with Gammasphere.

The relevant partial level schemes of odd-A Mo and Ru nuclei are shown in Fig. 2. In Table 2 are shown the $A_2$ and $A_4$ coefficients and also the extracted $\delta$ values for the $\Delta I=1$ transitions within the rotational bands. To interpret the mixing ratios of the $\Delta I=1$ transitions within rotational bands, the experimental values were compared to calculations made with a particle plus axial rotor model for various single-particle states. In the fifth and sixth columns in Table 2 are shown the calculated $\delta$ values for various assumed configurations.
Orlandi et al. [7] established the 3/2[411] assignment of the ground band of $^{101}\text{Zr}$, which is also supported in this work. They found the ground band in $^{103}\text{Mo}$ to have the same configuration, which is also supported by this work. They further showed the negative parity 5/2$^+$ bandhead at 347 keV in $^{103}\text{Mo}$ to have the 5/2[532] configuration and identified this configuration as the ground-state band in $^{105}\text{Mo}$. The present results in $^{105}\text{Mo}$ support this assignment. In $^{105}\text{Mo}$, the configuration of the ground state was assigned by Urban et al. [8] to be 5/2[413] based on the measurement of $E2/M1$ branching ratios in the rotational band. Two independent values of the mixing ratio of the 152.1-keV transition between the lowest levels of this band in $^{105}\text{Mo}$, are reported in this

| Nucleus | Cascade | $A_2^{exp} , A_4^{exp}$ | $\delta$ (exp) | $\delta$ (cal) | Configuration |
|---------|---------|------------------------|---------------|---------------|--------------|
| $^{101}\text{Zr}$ | 9/2$^+\rightarrow$ 5/2$^+\rightarrow$ 3/2$^+$ | -0.12(2), -0.00(4) | -0.16(9) | -0.13 | 3/2[411] |
| | 310.0-(98.2) | | | | |
| | 7/2$^-\rightarrow$ 5/2$^+\rightarrow$ 3/2$^+$ | 0.08(2), 0.02(3) | -0.13(10) | | |
| | 222.9-(98.2) | | | | |
| | 7/2$^-\rightarrow$ 5/2$^+\rightarrow$ 3/2$^+$ | 0.09(1), 0.01(2) | -0.19(5) | -0.15 | 3/2[411] |
| | 251.4-(102.8) | | | | |
| | 11/2$^-\rightarrow$ 7/2$^-\rightarrow$ 5/2$^+$ | -0.05(1), 0.01(2) | g factor | | |
| | 144.5-251.4 | | | | |
| | 13/2$^-\rightarrow$ 9/2$^-\rightarrow$ 7/2$^+$ | -0.23(3), -0.04(5) | -0.29 to -1.94 | -0.15 | 5/2[532] |
| | 372.3-(124.9) | | | | |
| | 9/2$^-\rightarrow$ 7/2$^-\rightarrow$ 5/2$^+$ | +0.25(4), -0.07(6) | -0.49$^{+0.14}_{-0.22}$ | -2.66$^{+0.92}_{-1.40}$ | |
| | 124.9-251.4 | | | | |
| $^{105}\text{Mo}$ | 11/2$^-\rightarrow$ 7/2$^-\rightarrow$ 5/2$^+$ | -0.12(1), 0.02(2) | -0.12(3) | -0.15 | 5/2[532] |
| | 283.2-(95.3) | | | | |
| | 13/2$^-\rightarrow$ 9/2$^-\rightarrow$ 7/2$^+$ | -0.17(1), -0.01(2) | -0.25(4) | -0.17 | 5/2[532] |
| | 390.6-(138.3) | | | | |
| $^{107}\text{Mo}$ | 9/2$^-\rightarrow$ 7/2$^-\rightarrow$ 5/2$^+$ | 0.17(1), -0.00(3) | -0.44 to -1.44 | -0.20 | 5/2[402] |
| | 306.4-(152.1) | | | | |
| | 11/2$^-\rightarrow$ 7/2$^-\rightarrow$ 5/2$^+$ | -0.21(3), -0.01(4) | -0.31 to -1.76 | | |
| | 414.8-(152.1) | | | | |
| | 9/2$^-\rightarrow$ 7/2$^-\rightarrow$ 5/2$^+$ | 0.14(3), 0.06(4) | -0.18(9) | -0.16 | 7/2[523] |
| | (110.2)-348.3 | | | | |
| $^{109}\text{Ru}$ | 11/2$^+\rightarrow$ 7/2$^+\rightarrow$ 5/2$^+$ | -0.16(2), 0.01(3) | -0.25(6) | -0.26 | 5/2[402] |
| | 472.8-(185.1) | | | | |
| | 13/2$^+\rightarrow$ 9/2$^+\rightarrow$ 7/2$^+$ | -0.20(3), -0.03(4) | -0.35$^{+0.09}_{-0.12}$ | -0.24 | 5/2[402] |
| | 540.7-(222.7) | | | | |
| $^{111}\text{Ru}$ | 9/2$^-\rightarrow$ 7/2$^-\rightarrow$ 5/2$^+$ | 0.15(2), 0.03(2) | -1.29$^{+0.04}_{-0.34}$ | | |
| | 166.6-(150.2) | | | -0.48(5) | -0.21 | 5/2[402] |
| | 7/2$^-\rightarrow$ 7/2$^-\rightarrow$ 5/2$^+$ | -0.37(1), -0.00(2) | -0.32(2) | | |
| | 103.8-(150.2) | | | | | -1.73(9) |
| | 11/2$^-\rightarrow$ 7/2$^-\rightarrow$ 5/2$^+$ | -0.22(1), -0.03(4) | -1.23(16) | | |
work. They are both negative and, taking the stronger $M_1$ limit from the experimental fit ranges, give a $\delta$ around $-0.5$, showing strong preference for the $5/2[402]$ assignment over the $5/2[413]$ for which the predicted $\delta$ value is positive and close to unity. It would seem that Urban et al. [8], who obtained $\delta$, without sign, chose $\delta$ positive contrary to the present direct sign measurement. For the negative-parity band-head at 348.4 keV, the configuration was assigned $7/2[523]$ [8], which is supported by the present result on mixing in the 110.2-keV transition. In $^{109,111}$Ru, Wu et al. [9] attempted to determine the single-particle configurations of the ground-state bands based on $E2/M1$ branching ratios but were not able to distinguish between the $5/2[413]$ and $5/2[402]$ configurations. In this work angular correlation data gives clear evidence that the ground states of $^{109,111}$Ru both have a $5/2[402]$ configuration. Calculations for a $5/2[402]$ state yield $\delta = -0.26$ and $\delta = -0.21$, respectively, in excellent agreement with the measured values of $-0.25(6)$ and $-0.32(2)$, whereas calculations for the $5/2[413]$ configuration yield large positive values.

Fig. 2.

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