Evidence of oblique electron acoustic solitary waves triggered by magnetic reconnection in Earth’s magnetosphere

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Abstract
Motivated by the recent Magnetospheric Multiscale (MMS) observations of oblique electron acoustic waves, we addressed the generation mechanism of the observed waves by utilizing the reductive perturbation technique. A nonlinear Zakharov-Kuznetsov (ZK) equation is derived for a collisionless, magnetised plasma composed of cool inertial background electrons, cool inertial electron beam, hot inertialess suprathermal electrons; represented by a κ-distribution, and stationary ions. Moreover, the instability growth rate is derived by using the small-k perturbation expansion method. Our findings revealed that the structure of the electrostatic wave profile is significantly influenced by the external magnetic field, the unperturbed hot, cool, and electron beam densities, the obliquity angle, and the rate of superthermality. Such parameters also have an effect on the instability growth rate. This study clarifies the characteristics of the oblique electron solitary waves that may be responsible for changing the electron and ion distribution functions, which alter the magnetic reconnection process. Moreover, the increase of the growth rate with the plasma parameters could be a source of anomalous resistivity that enhances the rate of magnetic reconnection.

1. Introduction

The phenomenon of magnetic reconnection occurs when the frozen-in condition is violated, resulting in the large scale magnetic topology, and the magnetic energy is explosively transformed into kinetic energy that thermalizes the plasma charged particles to high energy. Magnetic reconnection allows plasmas that were previously restricted to regions of different orientations of magnetic fields to mix by changing the magnetic topology [1]. A magnetic reconnection setup can be represented by a separatrix, an X point, an inflow region, an outflow region, and ion and electron diffusion regions. The separatrices include a combination of plasma from both edges of the current sheet and it refers to the newly reconnected magnetic lines. Two separatrices can form an X point. The latter creates an X line during three-dimensional (3-D) magnetic reconnection. Ions and electrons are not magnetic in the region of ion and electron diffusion. Moreover, the electron (ion) diffusion region has a width equals to the electron (ion) skin depth ($d_{ej} = c/\omega_{pe,ej}$), where $c$, $\omega_{pe}$, and $\omega_{pi}$ are the speed of light, electron plasma frequency, and ion plasma frequency, respectively [2, 3].

Addressing the dynamics of magnetic reconnection is important owing to its impact on the space weather [4, 5] and fusion reactors [6]. The energetic particles resulting from the incidence of magnetic reconnection can harm spacecraft equipment and potentially put people at risk, particularly those that are in deep space, where they are not protected by Earth’s magnetic field. It’s critical to understand how these charged particles are
accelerated to such great speeds to predict the short warning time. Further, unexpected sawtooth oscillations and disturbances may result from magnetic reconnection in fusion processes.

Plasma waves are a good candidate to study the characteristics of collisionless magnetic reconnection since they can regulate the particles flux, provide information about the density and composition of the plasma, and carry messages from far plasmas like heliospheric plasma [7–9]. Several types of waves have been observed in the regions of magnetic reconnection. In 2002, a wide frequency spectrum containing electron plasma waves was reported in and near the X-line reconnection region [10]. The Geotail satellite and Cluster spacecraft have also detected additional electrostatic waves in the Earth’s magnetotail at the separatrix of the reconnection diffusion area, such as solitary waves, Langmuir modes, and electron cyclotron waves [11, 12]. Several types of plasma waves have been observed by different space missions such as Geotail and Cluster in the region of Earth’s magnetopause associated with magnetic reconnection [13]. Wei et al. [14] discussed the Cluster mission’s observation of waves in the Earth’s magnetotail. They demonstrated that the waves are in the range of whistler frequency and that the waves’ excitation may be caused by hot electron beams. In order to study the microphysics of magnetic reconnection, the Magnetospheric Multiscale Mission (MMS) is launched in 2015 [15]. The electron diffusion region at the dayside magnetopause has been successfully encountered by the mission. At the reconnection region in the dayside magnetopause, MMS observed the propagation of different plasma waves such as whistler waves which are accompanied by the electrostatic solitary modes [16, 17]. High-frequency electrostatic waves have also been recorded by MMS in the area of the dayside magnetopause’s reconnection ion diffusion region [18].

Since waves are believed to introduce anomalous resistivity; i.e. wave particle interaction, in the electron diffusion area; increasing the rate of magnetic reconnection and thermalizing charged particles, the effect of plasma waves on magnetic reconnection is of significant interest to researchers [19]. Drake et al. [20] documented that electron holes generated by the Buneman instability can cause anomalous resistivity close to the x-line, which can enhance the rate of magnetic reconnection. Although recent observations have shown the formation of electron holes near the x-line [21], there is no evidence that plasma waves have a direct impact on magnetic reconnection. Furthermore, waves have been proposed to contribute to the onset of magnetic reconnection [14], but this has not yet been experimentally confirmed. Graham et al. [22] utilized the observed parameters and the high-resolution fields obtained by MMS to measure cross-field electron diffusion, viscosity, and anomalous resistivity; i.e. wave-particle interaction, caused by lower hybrid waves in the Earth’s magnetopause. Their results showed that plasma waves have no contribution to the onset of collisionless magnetic reconnection. However, by making the density gradient steeper, it could impact the rate of magnetic reconnection. Spacecraft typically goes through reconnection regions quickly owing to the fast motion of the reconnecting field lines. The effects of waves on the reconnection process, and in particular the reconnection rate, are consequently exceedingly challenging to experimentally measure. Therefore, determining the impact of electrostatic waves on magnetic reconnection is challenging. However, we can verify more aspects of the analytical models and simulations using spacecraft data, which will reinforce our trust in their models.

Observations of space plasmas revealed the existence of a population of hot, energetic electrons with suprathermal distributions [23, 24]. For suprathermal populations, a tail was seen on the velocity distribution function, and its kinetic energy is significantly larger than the thermal energy of the background cold inertial electrons. It has been demonstrated that a family of $\kappa$ distribution functions accurately describes these suprathermal hot electrons [25]. The spectral index $\kappa$ represents the departure from a Maxwellian distribution; low values of $\kappa$ are connected to a considerable superthermal, whereas a Maxwellian distribution is regained in the limit $\kappa \rightarrow \infty$ [26–28]. Theoretical models of electrostatic acoustic waves have recently been updated to include superthermal and cold electrons [29–33].

Electrostatic solitary waves are frequently linked to magnetic reconnection at the magnetopause and magnetotail, which can cause the unstable electron distributions needed to form electrostatic solitary waves. Therefore, one of the signs of magnetic reconnection is the presence of electrostatic solitary waves. For the first time, Cattell et al. [34] reported the observation of electrostatic solitary waves in the Earth’s magnetopause. They observed that soliton waves propagated at various rates are equivalent to the fast and slow thermal speeds of electrons and ions, respectively. They argued the difference between soliton modes speeds to the mixing of magnetospheric, magnetosheath, and ionosphere particles that can exist in the magnetopause current layer. Reconnection is very asymmetric at the magnetopause because the reconnecting plasmas have different densities, temperatures, and magnetic field strengths. As a result, simulations demonstrate that the instabilities that cause electrostatic solitary waves to propagate during asymmetric reconnection can be different from those during symmetric reconnection [35]. In 2015, Graham et al. [36] showed that the observed electrostatic waves during the occurrence of magnetic reconnection in the Earth’s magnetopause are characterized by distinct speeds in coincidence with Cattell et al. [34] observations. They suggested that the Buneman instability; i.e. instability resulting from a relative drift between ions and electrons, bump–on–tail; i.e. instability resulting when an electron beam with a velocity width lower than the beam speed interacts with
Maxwellian plasma, and bistreaming instabilities; i.e. instability resulting from two populations of counterstreaming electrons, are reasonable candidate mechanisms for the propagation of electrostatic waves in regions of magnetic reconnection. Recently, the characteristics of ion acoustic electrostatic solitary waves that are observed in the Earth’s ionosphere have been studied [37]. The results showed that the profile of the solitary waves is observed to be affected by changing the plasma parameters. Moreover, the generation of electrostatic waves by counter electron beam in the Earth’s magnetosphere and the observation of solitary waves in the solar wind had been investigated [38].

Several observations documented the mixing of cold and warm plasma in the Earth’s magnetosphere. Su et al [39] reported the observation of cold plasma in the magnetopause near the region of magnetic reconnection. McFadden et al and André et al [40, 41] showed that cold plasma could affect the structure of magnetic reconnection in the Earth’s magnetopause. The Alfvén velocity and reconnection rate can both be altered by the cold ion plume’s influence on reconnection [42, 43]. A numerical simulation revealed how the mixing of hot exhaust and cold sheath electrons in the diffusion region modifies the electron distribution functions and shapes the pattern of exhaust flow [44]. Graham et al [45] investigated the mechanism behind the heating of cold ions near regions of magnetic reconnection in the Earth’s magnetopause. Their results indicated that waves like lower hybrid drift waves close to the X line heat cold ions, entraining reconnection. Using observations of electrostatic waves in the diffusion region by the Magnetospheric Multiscale mission, Ergun et al [46] employed kinetic simulation to address the generation mechanism of these waves. Their numerical results revealed that warm magnetosheath electrons should flow into the magnetosphere on a newly reconnected plasma line. The electron flow could excite a two stream instability, giving rise to electron and ion acoustic waves. A retarding electric field is anticipated to form in order to slow the flow of electrons since the magnetosheath ions move at a slower rate. The magnetosphere’s cold plasma may be accelerated into a beam by the same decelerating electric field, and this beam can then be used to generate large amplitude waves. The magnetosheath plasma temperature and density, as well as those of the cold plasma in the magnetosphere, are what determine the wave modes. MMS has also discovered kinetic Alfvén waves and large-amplitude electrostatic waves at the magnetopause, driven by the mixture of cold and warm plasma close to the electron diffusion area. ([47] and references therein). In the present work, our effort is to address the mechanism behind the observation of oblique modes in the Earth’s magnetopause near the region of magnetic reconnection [46]. At present, there is a lack of significant studies of oblique modes at Earth’s magnetopause. As a potential mechanism to produce the oblique modes, we suggested two-stream instability, which results from mixing warm plasma from the magnetosheath with cold magnetospheric plasma, as a possible mechanism to generate the oblique modes as depicted in figure 1.

We employ the reductive perturbation technique to derive the Zakharov-Kuznetsov equation (ZK) that can be used to investigate three dimensional problems of oblique plasma waves in different plasma arrangements [48–50]. The solution of ZK provides information about the impact of obliqueness, magnetic field, electron superthermality, etc., on the properties of electrostatic solitary waves. Addressing the properties of solitary pulses is a useful complement to electrostatic analyzer measurements on the Magnetospheric Multiscale Mission. This is because the electrostatic analyzer can not distinguish between photoelectrons produced by solar radiation and naturally cold electrons.

This work is arranged as follows: the physical model and the derivation of ZK have been presented in sections 2 and 3, respectively. The solution and the stability of oblique electron soliton waves have been discussed in sections 4 and 5, respectively. Section 6 clarifies the numerical analysis of the current proposed model. Finally, our findings were concluded in section 7

2. Basic equations

We investigate the generation of electrostatic pulses in a four-component collisionless, magnetized plasma of cool inertial electron beam with temperature $T_b$, cool inertial background electrons with temperature $T_e$, hot inertialess suprathermal electrons represented by a $\kappa$-distribution of temperature $T_h$ (the condition $T_h \gg T_b$ and $T_e$ is satisfied), and uniformly distributed stationary ions. The external magnetic field $B_0$ is considered to be along the z-direction, i.e., $B_0 = EB_0$. At equilibrium, the quasineutrality requirement in such a plasma system is $n_{e0} = n_{b0} + n_{c0} + n_{h0}$, where $n_{e0}$ is the equilibrium density of the sth species ($s = h, c, b,$ and $i$ for hot superthermal electrons, cold electrons, beam electrons, and ions, respectively). The following normalized equations regulate the fluid model [51]
where $u$ is the fluid electron velocity that normalized by the hot electron thermal velocity $V_{th} = (k_B T_h/m_e)^{1/2}$ and $s = c$ and $b$ for cold background electrons and cold electron beam, respectively. The electrostatic potential $\phi$ is normalized by the thermal potential $k_B T_e/e$. The electron gyro-frequency in normalized form can be written as $\Omega = \omega_c/\omega_{pe}$ with $\omega_c = eB/m_e$ and $\omega_{pe} = (n_e e^2/e_0 m_e)^{1/2}$. Also, the quantities $\theta_c = T_c/T_e$ and $\theta_b = T_b/T_e$ are the temperature ratio of cold to hot electrons and beam to hot electrons, respectively. Therefore, for the stationary ions we have $Zn_i/n_e = 1 + \rho_{b,c} + \rho_{h,c}$, where $Z$ is the atomic number and $n_i$ is the equilibrium number density of ions. The time variable $t$ is normalized by the inverse of cold electron plasma frequency $\omega_{pe}^{-1} = (n_e e^2/e_0 m_e)^{1/2}$ and space variable is normalized by the characteristic Debye length $\lambda_D = (e^2/n_e e_0 e^2)^{1/2}$, with $k_B$ is the Boltzmann constant and $e$ is the electric charge. It is critical to note that $\kappa > 3/2$ is required for a meaningful particle distribution function [32], and the superthermal index $\kappa$ lies in the range $\infty > \kappa > 3/2$ [33, 52, 53], where $\kappa \rightarrow 3/2$ is for extremely superthermal situation and $\kappa \rightarrow \infty$ is for Maxwellian case.

Ions are considered static to neutralize the background plasma. In fact, ions are massive in comparison with electrons. When both possess the same energy, the movement of the ions becomes negligible in comparison with electron movement, where the velocity is inversely proportional to the particle mass. Although considering the dynamics of ions leads to more instabilities, such as the Bunemann instability that could generate electrostatic solitary waves, the fluid model cannot figure out the transition from two-stream instability to Bunemann instability [52].

In this study, the electrostatic solitary waves depend on the characteristics of cold inertial background electrons. Thus, low frequency electrostatic solitary waves will be excited with a phase velocity larger than the cold electron thermal speed and lower than the hot electron thermal speed, indicating that Landau damping is unlikely to occur [55]. Moreover, Landau damping is reduced if $0.2 \leq n_i/(n_e + n_i) \leq 0.8$ [56–58].

Figure 1. A sketch of the mixing of warm magnetosheath electrons with cold magnetosphere electrons.
sustain the excitation of EAWs when the electron beam penetrates the plasma, we investigate the region
$0.25 \leq \rho_{el} \leq 4$ and $\rho_{el} \leq 0.01$ where the linear waves are not substantially damped \cite{55}. If $T_{th}/T_c \gg 10$ the linear waves withstand Landau damping \cite{56–58}. Therefore, we consider the region $\theta_c \ll 0.1$ where the waves are not significantly dampened.

3. Derivation of the ZK equation

To obtain the ZK equation, we utilise the reductive perturbation method and define the stretched independent variables as follows:

$$X = e^{\frac{1}{2}x}, \quad Y = e^{\frac{1}{2}y}, \quad Z = e^{\frac{1}{2}(L_z z - V_p t)}, \quad \tau = e^{\frac{1}{2}t}. \quad (2a)$$

The phase velocity of EAWs is denoted by $V_p$. The physical quantities appearing in equations (1) are expanded in a power series in $\epsilon$ about their equilibrium values as:

$$n_e = (1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \epsilon^3 n_e^{(3)} + ...),$$
$$n_b = (1 + \epsilon n_b^{(1)} + \epsilon^2 n_b^{(2)} + \epsilon^3 n_b^{(3)} + ...),$$
$$u_{cx} = (\epsilon^2 u_{cx}^{(2)} + \epsilon^3 u_{cx}^{(3)} + ...),$$
$$u_{cy} = (\epsilon^2 u_{cy}^{(2)} + \epsilon^3 u_{cy}^{(3)} + ...),$$
$$u_{cz} = (\epsilon^2 u_{cz}^{(2)} + \epsilon^3 u_{cz}^{(3)} + ...),$$
$$u_{bx} = (\epsilon^2 u_{bx}^{(2)} + \epsilon^3 u_{bx}^{(3)} + ...),$$
$$u_{by} = (\epsilon^2 u_{by}^{(2)} + \epsilon^3 u_{by}^{(3)} + ...),$$
$$u_{bz} = (\epsilon^2 u_{bz}^{(2)} + \epsilon^3 u_{bz}^{(3)} + ...),$$
$$\phi = (\epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + ...). \quad (2b)$$

Using the expansion and the stretching given by equations (2a) and (2b), respectively, in the basic equations (1), we obtain the following first order:

$$n_e^{(1)} = \frac{L_z^2}{3\theta_{el} L_z^2 - V_p^2} \phi^{(1)},$$
$$n_b^{(1)} = \frac{L_z^2}{3\theta_{el} L_z^2 - V_p^2} \phi^{(1)},$$
$$u_{cx}^{(1)} = \frac{V_p^2}{\Omega(3\theta_{el} L_z^2 - V_p^2)} \frac{\partial}{\partial Y} \phi^{(1)},$$
$$u_{cy}^{(1)} = \frac{-V_p^2}{\Omega(3\theta_{el} L_z^2 - V_p^2)} \frac{\partial}{\partial X} \phi^{(1)},$$
$$u_{cz}^{(1)} = \frac{L_z V_p}{3\theta_{el} L_z^2 - V_p^2} \phi^{(1)},$$
$$u_{bx}^{(1)} = \frac{V_p^2}{\Omega(3\theta_{el} L_z^2 - V_p^2)} \frac{\partial}{\partial z} \phi^{(1)},$$
$$u_{by}^{(1)} = \frac{-V_p^2}{\Omega(3\theta_{el} L_z^2 - V_p^2)} \frac{\partial}{\partial x} \phi^{(1)},$$
$$u_{bz}^{(1)} = \frac{L_z V_p}{3\theta_{el} L_z^2 - V_p^2} \phi^{(1)}. \quad (3)$$

After plugging equations (3) into Poisson’s equation, we get the following expression for the phase velocity relation:
dependent on many system factors, as shown by equation (4). The phase speed of the slow acoustic wave is

$$V_p = L_c \left\{ \frac{((2\kappa - 3)(1 + 3\theta_x + 3\theta_y)(1 + \rho_{he}c) + 12\rho_{he}c(\kappa - 1)(\theta_x + \theta_y))}{-12((2\kappa - 3)(1 + \rho_{he}c) + 4\rho_{he}c(\kappa - 1))} + ((2\kappa - 3)(\theta_x \rho_{he}c + \theta_y(1 + 3\theta_x(1 + \rho_{he}c))) + 12(\kappa - 1)(\theta_x \theta_y \rho_{he}c)) + (2(2\kappa - 3)(1 + \rho_{he}c) + 8(\kappa - 1)\rho_{he}c)^2}{2(2\kappa - 3)(1 + \rho_{he}c) + 8(\kappa - 1)\rho_{he}c} \right\}$$

The presence of two dispersion curves in equation (4) indicates that the plasma model under consideration may transmit cyclotron waves (as fast modes) as well as acoustic waves (as slow modes). We investigate the influence of important plasma parameters on slow modes because we are interested in analysing the characteristics of acoustic modes in the plasma system under consideration. The phase speed of the slow acoustic wave is dependent on many system factors, as shown by equation (4). Collecting of the second order equations from the next higher-order of $\epsilon$ and solving them, to get

$$u^{(2)}_{cx} = \frac{V_p^3}{\Omega^2(3\theta_x L_z^2 - V_p^2)^3} \frac{\partial^2}{\partial X^2} \phi^{(1)}$$
$$u^{(2)}_{cy} = \frac{V_p^3}{\Omega^2(3\theta_y L_z^2 - V_p^2)^3} \frac{\partial^2}{\partial Y^2} \phi^{(1)}$$
$$u^{(2)}_{bx} = \frac{V_p^3}{\Omega^2(3\theta_x L_z^2 - V_p^2)^3} \frac{\partial^2}{\partial X^2} \phi^{(1)}$$
$$u^{(2)}_{by} = \frac{V_p^3}{\Omega^2(3\theta_y L_z^2 - V_p^2)^3} \frac{\partial^2}{\partial Y^2} \phi^{(1)}$$

The next higher order of equations leads to the following second order perturbed quantities as

$$\frac{\partial}{\partial Z} Z^{(2)}_c = \frac{1}{\Omega^2(3\theta_x L_z^2 - V_p^2)^3} \left\{ 3\Omega^2 L_z^4 \left( \theta_x + V_p^2 \right) \frac{\partial}{\partial Z} \phi^{(1)} \right\}$$
$$\frac{\partial}{\partial Z} Z^{(2)}_b = \frac{1}{\Omega^2(3\theta_y L_z^2 - V_p^2)^3} \left\{ 3\Omega^2 L_z^4 \left( \theta_y + V_p^2 \right) \frac{\partial}{\partial Z} \phi^{(1)} \right\}$$

The ZK equation may be built by equating these variables in Poisson’s equation and after some algebraic adjustments, yield the following ZK equation

$$\frac{\partial \phi_1}{\partial t} + A \frac{\partial \phi_1}{\partial Z} + B \frac{\partial^2 \phi_1}{\partial Z^2} + C \left( \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} \right) \phi_1 = 0,$$ (7)

where the nonlinear coefficient (A), the dispersion coefficient (B), and (C) are given as:

$$A = \frac{(3\theta_x L_z^2 - V_p^2)^2(3\theta_y L_z^2 - V_p^2)^2}{2L_z^2 V_p(3\theta_x L_z^2 - V_p^2)^2 + \rho_{he}c(3\theta_y L_z^2 - V_p^2)^2} \times \left( 1 + \rho_{he}c + 3L_z^4 \right)$$
$$B = \frac{(3\theta_x L_z^2 - V_p^2)^2(3\theta_y L_z^2 - V_p^2)^2}{2L_z^2 V_p(3\theta_x L_z^2 - V_p^2)^2 + \rho_{he}c(3\theta_y L_z^2 - V_p^2)^2}$$

$$C = \frac{(3\theta_x L_z^2 - V_p^2)^2(3\theta_y L_z^2 - V_p^2)^2}{2L_z^2 V_p(3\theta_x L_z^2 - V_p^2)^2 + \rho_{he}c(3\theta_y L_z^2 - V_p^2)^2}$$
\[ C = \frac{(3\theta L_z^2 - V_p^2)(3\theta L_z^2 - V_p^2)^2}{2L_z^4 V_p (3\theta L_z^2 - V_p^2)^3 + \rho_{bc}(3\theta L_z^2 - V_p^2)^3} \left( 1 + \frac{V_p^2}{\Omega^2} \left( \frac{1}{(3\theta L_z^2 - V_p^2)^2 + \rho_{bc}} \right) \right) \]  

(8c)

4. Solitary wave analysis

The solitary wave solution of equation (7) results from the balance between nonlinearity and dispersive effects, we will first investigate the transformation of the independent variables by rotating the spatial axes \((X, Z)\) by an angle, \(\theta\) to study the properties of the solitary waves propagating in a direction making an angle \(\theta\) with the \(Z\)-axis, i.e., with the external magnetic field and lying in the \((X-Z)\) plane, and renaming \(Y\) and \(T\), the coordinate transformations are described as follows [59–61]:

\[
\begin{align*}
\zeta &= X \cos \theta - Z \sin \theta, \\
\xi &= X \sin \theta + Z \cos \theta, \\
\eta &= Y, \\
\tau &= T.
\end{align*}
\]

(9)

Introducing these new independent variables into ZK equation (7), we get

\[
\begin{align*}
\frac{\partial \phi^{(1)}}{\partial \tau} + R_1 \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + R_2 \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + R_3 \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + R_4 \frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} + R_5 \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \eta} + R_6 \frac{\partial^3 \phi^{(1)}}{\partial \eta^3} &= 0,
\end{align*}
\]

(10)

where the coefficients in equation (10) are given as:

\[
\begin{align*}
R_1 &= A \cos \theta, \\
R_2 &= B \cos^3 \theta + C \sin^2 \theta \cos \theta, \\
R_3 &= -A \sin \theta, \\
R_4 &= -B \sin^3 \theta - C \cos^2 \theta \sin \theta, \\
R_5 &= 2C(\sin \theta \cos^2 \theta - \frac{1}{2} \sin^3 \theta) - 3B \cos^2 \theta \sin \theta, \\
R_6 &= -2C(\sin^2 \theta \cos \theta - \frac{1}{2} \cos^3 \theta) + 3B \sin^2 \theta \cos \theta, \\
R_7 &= C \cos \theta, \\
R_8 &= -C \sin \theta.
\end{align*}
\]

(11)

The steady-state solution of the ZK equation is as follows:

\[ \phi^{(1)} = \phi_0(\rho), \]

where \(\rho = \xi - M \tau\), and \(M\) is the Mach number normalized by the ion-acoustic speed \(c_s\). So, equation (10) can be expressed as [62]

\[ -M \frac{d\phi_0}{d\rho} + R_1 \phi_0 \frac{d\phi_0}{d\rho} + R_2 \frac{d^3 \phi_0}{d\rho^3} = 0. \]

(12)

The EASWs pulse solution can be obtained by integrating and applying suitable boundary conditions as

\[ \phi_0(\rho) = \varphi_m \text{sech}^2 \left( \frac{\rho}{w} \right), \]

(13)

where \(\varphi_m\) and \(w\) are the solitary wave amplitude and width, respectively, as given by

\[ \varphi_m = 3M / R_0 \text{and} w = 2\sqrt{R_5 / M}. \]

(14)

The accompanying electric field might be deducted and have the following form:

\[ E_0(\rho) = -\nabla \phi_0(\rho) = \frac{2\varphi_m}{w} \text{sech}^2 \left( \frac{\rho}{w} \right) \tan \left( \frac{\rho}{w} \right). \]

(15)

It is clear from equations (11) and (14) that both the amplitude and the width of the solitary wave are dependent on the cold, and beam electrons densities and temperatures. Also, the superthermality and obliqueness have considerable effects. The wave amplitude is an essential element to describe the energy of the wave according to the following equation [63, 64]:

\[ E_n = \int_{-\infty}^{\infty} \frac{\phi_0^2(\rho)}{\chi^2} d\rho, \]

(16)

\[ E_n = \frac{4w\varphi_m^2}{3\chi^2}. \]

It displays the amount to which the confined charged particles obtain energy from the wave.
5. Instability analysis

We use the small-k expansion perturbation approach to investigate the stability of this solution. We assume the following expression for the perturbed electric potential:

\[ \phi^{(1)} = \phi_0(\rho) + \Phi(\rho, \zeta, \eta, \tau), \]

where the inclined plane spreading long-wavelength-wave takes the symbol \( \Phi \) that can be represented as

\[ \Phi(\rho, \zeta, \eta, \tau) = \psi(\rho) \exp[i(k_\perp l_\perp + k_\parallel l_\parallel + k_\perp l_\perp) - \gamma \tau], \]

in which \( l_\perp^2 + l_\parallel^2 + l_\perp^2 = 1 \), \( \psi(\rho) \), and \( \gamma \) can be enhanced by applying small values of \( k \) to the form

\[ \gamma = k_{\parallel} + k_{\perp}^2 + \ldots \]

The linearized ZK equation can be obtained by substituting equations (17) into (10) to obtain

\[ \frac{\partial \Phi}{\partial \tau} - M \frac{\partial \Phi}{\partial \rho} + R_0 \frac{\partial \Phi}{\partial \rho} + R_2 \frac{\partial^3 \Phi}{\partial \rho^3} + R_3 \frac{\partial \Phi}{\partial \rho} + R_4 \frac{\partial^3 \Phi}{\partial \rho^3} + R_5 \frac{\partial^3 \Phi}{\partial \rho^3 \partial \zeta^3} + R_6 \frac{\partial^3 \Phi}{\partial \rho^3 \partial \eta^3} + R_7 \frac{\partial^3 \Phi}{\partial \rho^3 \partial \zeta^3 \partial \eta^3} = 0. \]

Substituting equations (18) and (19) into (20) and equating the same-power coefficients of \( k \), resulting in

\[ (-M + R_0) \psi_0 + R_2 \frac{d^2 \psi_0}{d \rho^2} = C', \]

\( C' \) denotes the integration constant. The homogeneous part of equation (21) has two independent linear solutions which can be expressed as [61]:

\[ f = \frac{d \psi_0}{d \rho}, \]

\[ g = f \int_{\rho}^{\rho_0} \frac{d \rho}{f^2}. \]

As a result, the generic solution might take the following form

\[ \psi_0 = C_1 f + C_2 g - C f \int_{\rho}^{\rho_0} \frac{d \rho}{S_2} + C g \int_{\rho}^{\rho_0} \frac{d \rho}{S_2}, \]

where \( C_1 \) and \( C_2 \) are the constants of the integration. Finally, the zeroth-order solution may be reduced to the following one

\[ \psi_0 = C_1 f. \]

The first and second-order equations may be found from equations (18)–(20) and the dispersion relation can be represented as follows

\[ \gamma_1 = \Delta - M l_\perp + \sqrt{\Delta^2 - \Gamma}, \]

where

\[ \Delta = \frac{2}{3} (\mu_1 \phi_m - 2 \mu_2 / w^2), \]

\[ \Gamma = \frac{16}{45} (\mu_1^2 \phi_m^2 - 3 \mu_1 \mu_2 \phi_m / w^2 - 3 \mu_2^2 / W^4 + 12 \mu_3 \mu_3 / w^4), \]

\[ \mu_1 = (R_l l_\perp + R_\parallel l_\parallel), \mu_2 = (3 R_l l_\perp + R_\parallel l_\parallel), \]

and \( \mu_3 = (3 R_l l_\perp^2 + 2 R_\parallel l_\parallel l_\perp + R_\parallel l_\parallel^2). \)

As a result of equation (25), we can see that instability arises when the condition \( \Gamma - \Delta^2 > 0 \) is met. The instability growth rate, \( Gr \), which is given by

\[ Gr = \sqrt{\Gamma - \Delta^2}. \]

The impact of the plasma parameters on the growth rate will be discussed in the next section.

6. Numerical analysis and discussion

In this section, we employ the plasma parameters observed by the MMS mission in the Earth’s magnetopause, to address the generation mechanism and the instability of oblique electron plasma waves close to the magnetic reconnection domain [46, 66]. Using the reductive perturbation technique, we deduced the dispersion relation then the nonlinear analysis yields the 3 + 1 dimension ZK equation in a four-component collisionless,
magnetised plasma composed of cold inertial background electrons, cool inertial electron beam, hot inertialess suprathermal electrons, and evenly distributed stationary ions. The solution to this developed equation yields a solitary wave, then we obtained its associated electric field and energy. The small-\( k \) expansion approach was used to investigate the multidimensional instability of the earlier ZK equation \[66, 67\].

The results of this investigation may be summarized as follows: The phase velocity, \( V_p \), of the considered plasma model indicates the presence of two modes. These ensure access to acoustic waves and cyclotron waves to propagate in slow and fast modes, respectively. This phase velocity is affected by the ratio of hot to cool electron number density, \( \rho_h, c \), the ratio of beam to cool electron number density, \( \rho_b, c \), the ratio of cool to hot electron temperature, \( \theta_c \), the ratio of beam to hot electron temperature, \( \theta_b \), the obliquity angle, \( \theta \), and the superthermal parameter, \( \kappa \). Figure 2(a) shows the variation of phase velocity with the obliquity angle at different values of superthermality. It is clear that the phase velocity of EASWs decreases as the obliquity angle is increased. However, increasing the value of superthermality leads to an increase in the phase velocity, following the same behaviour as with the variation of the obliquity angle. The effect of the beam to hot electron temperature ratio is dominant at its small values while its increase leads to the propagation of oblique ESWs with distinguishable velocities at certain values of the cold electron temperature ratio as depicted in figure 2(b). This means that at a certain value of the cold electron beam temperature, the instability grows, giving rise to the ability to move the energy from the electron beam to the propagated electrostatic waves. We can observe from figure 2(c) that the phase velocity has a slight decrease with the increase of the beam to cold electron number density ratio. Investigating this result at different values of the hot to cold electron number density ratio showed a significant reduction in the wave phase velocity. This means that different instabilities are responsible for the generation of oblique ESWs. Since ions are considered to be stationary to neutralize the background plasma, the two-stream instability will be excited owing to the mixing of warm magnetosheath electrons with cold magnetosphere electrons. This instability gives rise to the generation of linear electrostatic modes that are growing to produce electrostatic solitary waves. It is convenient to note that the Bunemann instability could be exist if the dynamics of mobile ions are taken into account \[54\]. Therefore, at very small values of the hot to cold electron number density, the two stream instability is a reasonable source of the electrostatic waves. While increasing the value of hot to cold electron number density with respect to the beam to electron number density ratio causes the bump-on-tail instability to be dominant. Therefore, the oblique ESWs could propagate with different velocities near the regions of magnetic reconnection depending on the type of instability in accordance with results by Graham et al \[68\].

**Figure 2.** The variation of the phase velocity \( V_p \), represented by equation (4) (a) versus \( \theta \) for different values of \( \kappa \) at \( \rho_b, c = 1.5, \rho_h, c = 0.003, \theta_h = 0.25 \), (b) against \( \theta_b \) for different values of \( \theta_c \) at \( \rho_h, c = 1.5, \rho_b, c = 0.003, \kappa = 3 \) with \( \theta = 5.0 \), (c) against \( \rho_b, c \) for different values of \( \rho_h, c \) at \( \kappa = 3.0, \theta = 5.0, \theta_b = 0.03 \) with \( \theta_c = 0.15 \).
The nonlinear term ($A$) controls the steepening and polarity of the generated waves. Figure 3 depicts the dependency of the nonlinear coefficient, ($A$), on the previously indicated parameters. It is shown that ($A$) can be either positive or negative depending on the plasma parameters. The positive ($A$) is related with compressive EASWs, whereas the negative ($A$) is associated with rarefactive EASWs. We discovered that increasing both the obliquity angle and beam to cold electron number density lead to a decrease in the coefficient ($A$) at different values of superthermality and hot to cold number density ratio according to figures 3(a) and (c), respectively. As illustrated in figure 3(b), for high levels of beam to hot electron temperature ratio, the coefficient ($A$) becomes negative, and its absolute value grows as the cold electron temperature increases.

The effects of $\kappa$, $\theta_c$, and $\rho_{he}$ on the properties of longitudinal dispersion coefficient ($B$) during the variation with $\theta_c$, $\theta_h$, and $\rho_{he}$ are manifested in figures 4(a), (b), and (c), respectively. We noticed that the dispersion coefficient ($B$) decreases as the obliquity angle increases, while it increases as ($\theta_h$ and $\rho_{he}$) increase, then it becomes constant. Figures 5 and 6 present the variation of solitary wave amplitude ($\phi_m$) and width ($w$) with the prior factors, respectively. Such electrostatic waves develop when the nonlinear and dispersion coefficients are balanced. Since the amplitude depends on the nonlinear term, ($A$), the amplitude can be positive or negative depending on the values of the system parameters. Figure 5(a) shows that the amplitude decreases with increasing the obliquity angle for different values of the superthermality parameter. For smaller values of the cool to hot electron temperature ratio and the hot to cool electron number density, the amplitude becomes smaller as presented by figures 5(b) and (c). On the other hand, the pulse width increases with increasing $\theta_c$ and $\rho_{he}$ for different values of $\kappa$ and $\theta_c$ as presented by figures 6(a) and (b). Moreover, the pulse width ($w$) is suppressed by increasing $\rho_{he}$ for different values of $\rho_{he}$ as shown in figure 6(c).

The results obtained from figures 5 and 6 are confirmed by the solitary wave profiles and the associated electric field as shown in figures 7 and 8, respectively. It is clear from figures 7(a), (b), and (c) that both the wave amplitude and width increase with increasing $\kappa$, while increasing $\theta_h$ and $\rho_{he}$ leads to decrease the pulse amplitude. Further, we can observe from figure 7(d) that increasing the electron gyro-frequency leads to an increase in the pulse width without a significant change in the pulse amplitude. The associated electric field presented by figure 8 had the same behavior. Figure 9 shows the variation of wave energy, which is mostly determined by the wave amplitude with the same parameters impact ($\phi_m$). It is clear from figure 9(a) that the energy increases or decreases with increasing $\kappa$ or $\theta_c$ as presented by figures 9(a) and (b), respectively. While increasing $\rho_{he}$ leads to increase the soliton energy as shown in figure 9(c). As a result, the energy incorporates the impacts of the preceding parameters on $\phi_m$. 

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![Figure 3](image-url)
Figure 4. The variation of the dispersive term $B$, represented by equation (8b) (a) against $\theta$ for different values of $\kappa$ at $\rho_{hc} = 1.5, \rho_{bc} = 0.003, \theta_i = 0.05$ with $\theta_t = 0.05$, (b) against $\theta_t$ for different values of $\theta_i$ at $\rho_{hc} = 1.5, \rho_{bc} = 0.003, \kappa = 3$ with $\theta = 5.0$, (c) against $\rho_{bc}$ for different values of $\rho_{hc}$ at $\kappa = 3.0, \theta = 5.0, \theta_i = 0.03$ with $\theta_t = 0.08$.

Figure 5. The variation of the EASW’s amplitude $\phi_m$, represented by equation (8b) (a) against $\theta$ for different values of $\kappa$ at $\rho_{hc} = 2.5, \rho_{bc} = 0.004, \theta_i = 0.04$ with $\theta_t = 0.15$, (b) against $\theta_t$ for different values of $\theta_i$ at $\rho_{hc} = 1.5, \rho_{bc} = 0.006, \kappa = 3$ with $\theta = 5.0$, (c) against $\rho_{bc}$ for different values of $\rho_{hc}$ at $\kappa = 3.0, \theta = 5.0, \theta_i = 0.01$ with $\theta_t = 0.14$. 

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The positive solitary waves and related electric field are shown in figures 10 and 11, respectively. As can be seen from figure 10(a), the pulse amplitude decreases little as $\kappa$ increased, whereas increased $\theta_c$ results in a significant drop in the pulse profile, as can be seen in figure 10(b). However, increasing $\rho_{hc}$ leads to increase the amplitude of the soliatry waves according to figure 10(c), as long as increasing the electron gyro-frequency had no impact on the pulse energy as depicted in figure 10(d). Figure 11: similar to how the pulse amplitude behaved, the associated electric field for the positive solitary waves did as well.

Figure 6. The variation of the EASWs width $w$, represented by equation (8b) (a) against $\theta$ for different values of $\kappa$ at $\rho_{hc} = 2.5$, $\rho_{hc} = 0.004, \theta_b = 0.15$, (b) against $\theta_b$ for different values of $\theta_c$ at $\rho_{hc} = 1.5$, $\rho_{hc} = 0.006, \kappa = 3$ with $\theta = 5.0$, (c) against $\rho_{hc}$ for different values of $\rho_{hc}$ at $\kappa = 3.0, \theta = 5.0, \theta_c = 0.03$ with $\theta_b = 0.08$.

Figure 7. The evolution of $\phi_0$ of EASWs that represented by equation (13) with $\rho$ at $\theta = 5$ for different values of (a) $\kappa$ with $\rho_{hc} = 2.5, \Omega = 0.5, \rho_{hc} = 0.004, \theta_b = 0.15, \theta_c = 0.04$, (b) $\theta_c$ with $\Omega = 0.5, \rho_{hc} = 1.5, \rho_{hc} = 0.006, \theta_b = 0.15$, and $\kappa = 3.0$, (c) $\rho_{hc}$ with $\kappa = 3.0, \Omega = 0.5, \rho_{hc} = 0.004, \theta_b = 0.14, \theta_c = 0.01$, (d) $\Omega$ with $\rho_{hc} = 2.5, \rho_{hc} = 0.004, \theta_b = 0.15, \theta_c = 0.04, \kappa = 3.0$. The positive solitary waves and related electric field are shown in figures 10 and 11, respectively. As can be seen from figure 10(a), the pulse amplitude decreases little as $\kappa$ increased, whereas increased $\theta_c$ results in a significant drop in the pulse profile, as can be seen in figure 10(b). However, increasing $\rho_{hc}$ leads to increase the amplitude of the soliatry waves according to figure 10(c), as long as increasing the electron gyro-frequency had no impact on the pulse energy as depicted in figure 10(d). Figure 11: similar to how the pulse amplitude behaved, the associated electric field for the positive solitary waves did as well.
The variation of the instability growth rate (Gr) against $\theta$, $\theta_c$, and $\rho_{b,c}$ is illustrated in figure 12. It is obvious from figure 12(a) that (Gr) decreases as $\theta$ increases, and it reaches zero at a certain value of $\theta$. This critical value depends on the superthermal parameter. Figure 12(b) shows that the reduction of (Gr) becomes sharp after $\theta_c$ attains a certain value. This critical value of $\theta_c$ depends on $\theta_b$. Moreover, figure 12(c) demonstrates that the growth rate (Gr) increases with increasing $\rho_{b,c}$ at a certain value of $\rho_{b,c}$. However, increasing the value of $\rho_{b,c}$ leads
to reduce the growth rate. Therefore, regions with excess cold electrons could be a source of anomalous resistivity that could affect the activation of magnetic reconnection ([69–72] and references therein).

7. Conclusions

In this work, we employed the multifluid model to address the characteristics of oblique electron acoustic solitary waves observed by the MMS mission in Earth’s magnetopause near the regions of magnetic reconnection. Our efforts are also to investigate the connection between electrostatic waves and the rate of
magnetic reconnection. We derived the ZK equation to explain nonlinear small-amplitude electron-acoustic waves in a collisionless, magnetized plasma with four components: cold inertial background electrons; a cool inertial electron beam; hot inertialess suprathermal electrons; and evenly distributed stationary ions. The solution of the ZK equation has been used to investigate the properties of EASWs. Our results may be summed up as follows:

- This study provides the possibility of the generation of both positive and negative oblique electron acoustic waves in the Earth’s magnetopause.

- We observed that the plasma parameters such as the obliquity angle, the ratio of beam to hot electron temperature, the ratio of cool to hot electron temperature, the ratio of beam to cold electron number density, and the ratio of hot to cold electron number density had a significant effect on the structure of the soliton profile.

- We discovered that the oblique soliton waves could propagate with different velocities close to regions of magnetic reconnection. The two-stream instability, which is related to the properties of the electron beam velocity, and the bump-on-tail instability, which is related to the density of the background plasma, are responsible for the difference in the phase velocity of the oblique ESWs.

- The influence of plasma parameters on the instability growth rate has been explored. It has been demonstrated that increasing the ratio of cold to hot electron density and obliqueness can slow the rate of instability progression. While increasing the ratio of beam to cold electron number density leads to an increase in the instability development rate.

- Regions with excess cold electrons can be a source of anomalous resistivity, which might have an impact on the rate of magnetic reconnection.

Finally, the two fluid model has a number of drawbacks, including the following: (i) the microphysics such as trapping of cold and warm particles in a potential well cannot be addressed, (ii) although the two stream instability is basically a linear dispersion relation, the growing of linear modes may growing until they reach a nonlinear modes like solitary waves which is challenging to describe in purely analytic techniques, and (iii) the growth rate of linear perturbations is insufficient to describe the complex structure related to magnetic reconnection. Therefore, we stress that the comprehensive quantitative dependence on fluid technique that we
have processed probably won’t be precise, but that the results could be subjectively correct because the fundamental model from which we start has certain limitations for dynamic concerns. Accordingly, our findings should only be considered a starting point, and more research is necessary. Moreover, The multi-fluid model will be utilized in the future to address the possible polarized modes that are ignored in the kinetic theory based on the observations from the literature because it is hard to clearly identify all the polarized modes from the full kinetic model. The role of electrostatic solitary waves in generating whistler waves in regions of magnetic reconnection and the microphysics of the connection between electrostatic solitary waves and the activation of magnetic reconnection will also be addressed in the future.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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