A Picture of $D$-branes at Strong Coupling II. Spinning Partons

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ABSTRACT

We study the Born-Infeld $D$-brane action in the limit $g_s \to \infty$. The resulting actions is presented in an arbitrary background and shown to describe a foliation of the world-volume by strings. Using a recently developed “degenerate” supergravity the parton picture is shown to be applicable also to supersymmetric $D$-branes.
1 Introduction

In a recent paper a picture of $D$-branes at strong coupling was presented \cite{1}. There it was shown that $D$-branes can be viewed as bound states of string-partons in a certain limit. More precisely, the limit when the string coupling $g_s$ goes to infinity with $\alpha'$ kept fixed corresponds to the limit when the $Dp$-brane tension $T_p$ goes to zero. In this limit the Born-Infeld action for the $Dp$-brane is replaced by

$$S = \frac{1}{4} \int d^{p+1}\xi V^i W^j (\gamma_{ij} + F_{ij}), \quad (1.1)$$

where $\gamma_{ij} + F_{ij}$ is the induced metric and world-volume field strength plus antisymmetric tensor, and $V^i$ and $W^i$ are world-volume vector densities replacing the auxiliary metric. The solutions to the field equations that follow from (1.1) can be shown to describe strings in a particular gauge where two of the world-volume coordinates act as string coordinates and the rest just become labels. It is this “foliation” we have in mind when we say that $D$-branes at strong coupling are described by string-partons.

Now, in \cite{1} only the bosonic part of the $D$-branes was considered. In general one would like to carry out a similar analysis for the full space-time supersymmetric theory. This would presumably lead to super-strings as partons. Equivalently these could be viewed as spinning (world-sheet supersymmetric) strings. Note that although there is no reason to expect that the supersymmetric $D$-branes have an equivalent spinning version this is nevertheless true for the string-partons. It will therefore serve as a consistency check of our picture if we can find a “spinning” version of (1.1). We use quotation marks since we are not talking about a standard $(p+1)$-dimensional supergravity version, but rather a “degenerate” type of supergravity\footnote{Degenerate in that the moving frames are non-invertible.} which leads to the partons being spinning strings. The problem very much resembles that encountered in finding the spinning version of the tensionless (fundamental) string \cite{3}. In a certain gauge, the tensionless string may be
viewed as a collection of massless particles moving subject to a constraint. World-sheet supersymmetrization is effected by coupling the string fields to a “degenerate” 2D-supergravity and leads to the spinning tensionless string being described by a collection of spinning particles [3]. This procedure is best described in a superspace formulation, and in this letter we present a similar approach to the $D$-brane limit [1.1].

The plan of the paper is as follows: Section 2 contains our main results, including the action for a spinning version of $D$-branes at strong coupling. In Section 3 we recapitulate the derivation of the $T_p \to 0$ limit of [1] and extend it to include a nontrivial background. In Section 4 we give a short presentation of the new supergravity and apply it to construct the spinning version of the $D$-branes. Section 5 contains our conclusions.

## 2 Results

Some time ago one of the authors showed how to write a first order version of the Born-Infeld action for a bosonic $p$-brane,

$$S^2_{BI}(T_p) = T_p \int d^{p+1} \xi \sqrt{-\det(\gamma_{ij} + F_{ij})},$$  \hspace{1cm} (2.2)

where $\gamma_{ij} \equiv \partial_i X^\mu \partial_j X^\nu G_{\mu \nu}(X)$ is the metric on the world-sheet induced from a background metric $G_{\mu \nu}$ and $F_{ij}$ is a world-volume field strength, which in this article we will take to be

$$F_{ij} \equiv \partial_i A_j + B_{ij}.$$

(2.3)

Here $B_{ij} \equiv \partial_i X^\mu \partial_j X^\nu B_{\mu \nu}$ is the pull-back of the background Kalb-Ramond field. The first order action is [1]:

$$S^1_{BI}(T) = \frac{1}{2} T \int d^{p+1} \xi \sqrt{-s} \left( s^{ij} (\gamma_{ij} + F_{ij}) - (p-1) \right),$$  \hspace{1cm} (2.4)
where \( s^{ij} \) is a general second rank world-volume tensor, (no symmetry assumed)\(^4\). We take (2.4) to represent the D brane action, thus disregarding the overall dilaton factor \( e^{-\phi} \) in the Lagrangian, since it will play no role for our considerations. If one wishes, it is readily reinserted in all our formulae.

In [1] (2.4) was derived from (2.2) via a Hamiltonian formulation. This process also allowed the limit \( T \to 0 \) to be taken and thus the formulation (1.1) was found. An equivalent formulation of the tensionless D-brane action was also given:

\[
S_{BI}^1(0) = \frac{1}{4} \int d^{p+1}\xi (\eta^{AB} e^i_A e^j_B - \varepsilon^{AB} e^i_A e^j_B) (\gamma_{ij} + F_{ij}),
\]

where \( A, B = 0, 1, e_A \equiv e^i_A \partial_i \) are “degenerate”, (for \( p > 1 \)), zweibeins corresponding to a 2D Lorentzian “tangent space”. In that tangent space the Minkowski metric is \( \eta^{AB} \) and \( \varepsilon^{AB} \) is the epsilon symbol. It is this form that will serve as our starting point for supersymmetrization. The form of the actions (2.5) and (1.1) clearly show the nature of the strong coupling limit of D-branes that we are considering: In this limit the D-brane dynamics is governed by actions that involve a “degenerate” metric of rank 2, \( \propto e^i_A e^j_B \eta^{AB} \).

Supersymmetrization of (2.4) entails promoting the target space coordinates \( X^\mu(\xi) \) to superfields \( X^\mu(\xi, \theta) \) and coupling them to a supergravity. The supergravity we use is a recently developed “degenerate” supergravity [8] whose basic superfield-densities are \( E_M^A \) with \( M \in \{+, -, i\}, i = 0, \ldots, p \) and \( A \in \{+, -, \#, =\} \). The bosonic densities in (2.5) are given by

\[
e_-^i \equiv |E_-^i|
\]

where \( | \) denotes “the \( \theta \)-independent part of”. The spinning version of (2.5) reads

\[
S_{BI} = -\frac{1}{4} \int d^{p+1}\xi d^2\theta \nabla_+ X^\mu \nabla_- X^\nu \mathcal{E}_{\mu\nu}(X),
\]

\(^4\)This form of the action has recently been used to discuss the geometry of D-branes [9], to investigate the rigid symmetries of D-branes [10] and as a starting point for a Hamiltonian discussion of D-strings [11].
where $\nabla_\pm \equiv E_\pm^M \partial_M + \omega_\pm M$, with $M$ the 2D Lorentz generator, and $E_{\mu\nu} \equiv G_{\mu\nu} + B_{\mu\nu}$. Below we shall once again restrict to a flat background. Note that there is no $A_i$ field in (2.7). In fact, a feature of the supergravity constraints is that the $A_i$ field equations that follow from (2.5) are automatically satisfied. The action (2.7) is thus the supersymmetization of (2.5) with the $A_i$ field integrated out. We relegate the details of this model to Section 4 below and end this section by presenting the component version of (2.7):

$$S_{BI} = \frac{1}{4} \int d^{p+1}\xi \{ \partial_+ X \cdot \partial_- X + 2\chi_+ \partial_+ \chi_- \partial_- X \cdot \Psi_- + 2\chi_- \partial_+ \partial_- X \cdot \Psi_- + i\Psi_+ \cdot \partial_+ \Psi_- - i\Psi_- \cdot \partial_+ \Psi_+ - 2(\Psi_+ \cdot \Psi_-) (\chi_+ \chi_-^*) \}$$

$$= \frac{1}{4} \int d^{p+1}\xi \left\{ \eta^{AB} e_A^i e_B^j \partial_i X \cdot \partial_j X + 2\chi_A \gamma^i \gamma^j \chi_B \cdot \partial_i X \right\} + \bar{\Psi} \cdot \gamma^i \partial_\alpha \Psi - \frac{1}{2} \left( \bar{\Psi} \cdot \Psi \right) \left( \chi_A \gamma^B \chi_B \right)$$

$$+ \mathcal{F}_{\alpha\beta} \cdot \mathcal{F}^{\alpha\beta} - \varepsilon^{AB} A_i \partial_j (e_A^i e_B^j) \} . \quad (2.8)$$

Here $\chi$ is the supergravity spinor (density) field, $\Psi$ and $\mathcal{F}$ are the spinor and auxiliary partners of $X$ and \cdot denotes target space contraction with the flat metric. Note that we have reintroduced $A_i$ as a Lagrange multiplier to be able to work with unconstrained $e_A^i$'s. To facilitate comparison to the spinning string we have also included the covariant version, introducing the 2D $\gamma$-matrices and denoting 2D spinor indices by $\alpha, \beta, \ldots$ The only field that is not a density is $X$. For the special case of $p = 1$ we do indeed recover the spinning string, albeit with redefined fields to take care of their density characters.

3 Derivation of the bosonic model

In this section we briefly recapitulate the derivation of (2.4) from (2.2) for arbitrary $p$, extending it to include a non-trivial background. We follow the

5Here $\mathcal{F}_{\alpha\beta} = \varepsilon_{\alpha\beta} \mathcal{F}$.

6 In [1] the derivation is given for $G_{\mu\nu} = \eta_{\mu\nu}$ and $B_{\mu\nu} = 0$. 

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procedure originally presented in [9], and described in this context, e.g., in [10], i.e., we derive the momenta, the constraints and then the Hamiltonian. Integrating out the momenta from the phase space Lagrangian we then obtain a configuration space action with the Lagrange multipliers for the constraints among the variables. We finally identify those multipliers with geometric objects on the world volume.

The generalized momenta that follow from (2.2) are (dropping the $p$ index on $T$)

$$\Pi_\mu = \frac{1}{2} T \sqrt{-\det(\sigma_{kl})} \partial_\mu X^\sigma \left[ \sigma^{(i0)} G_{\sigma\mu} + \sigma^{[i0]} B_{\sigma\mu} \right]$$

$$P^a = \frac{1}{2} T \sqrt{-\det(\sigma_{kl})} \sigma^{[a0]}$$

$$P^0 = 0,$$  \hspace{1cm} (3.9)

where $\sigma_{ij} \equiv \gamma_{ij} + F_{ij}$ and $\sigma^{ij}$ is its inverse. The primary constraints are

$$\tilde{\Pi}_\mu \partial_a X^\mu + P^b F_{ab} = 0,$$

$$P^0 = 0,$$

$$\tilde{\Pi}_\mu G^{\mu\nu} \tilde{\Pi}_\nu + P^a \gamma_{ab} P^b + T^2 \det(\sigma_{ab}) = 0,$$  \hspace{1cm} (3.10)

where $a, b, \ldots = 1, \ldots, p$ are transversal indices and $\tilde{\Pi}_\mu \equiv \Pi_\mu + P^a B_{a\mu}$. With the substitution $\Pi \to \tilde{\Pi}$ and the full background $F_{ij}$ from (2.3), the constraints (3.10) are formally identical to those in a trivial background. The corresponding Hamiltonian is

$$\mathcal{H} = P^i \partial_i A_0 + \chi P^0 + \rho^a \left( \tilde{\Pi}_\mu \partial_a X^\mu + P^b F_{ab} \right) + \lambda \left( \tilde{\Pi}_\mu G^{\mu\nu} \tilde{\Pi}_\nu + P^a \gamma_{ab} P^b + T^2 \det(\sigma_{ab}) \right),$$  \hspace{1cm} (3.11)

where $\chi, \rho^a$ and $\lambda$ are Lagrange multipliers for the constraints. Including a secondary ”Gauss’ law” type constraint, the final Hamiltonian is just the

\footnote{The Hamiltonian analysis of [1] was used in covariant quantization of D-branes [11] and the phase space Lagrangian was used in studying solitonic solutions of branes within branes [12].}
sum of the constraints, in agreement with the diffeomorphism invariance of the original Lagrangian, and the phase space action may be written

\[ S_{PS} = \int d^{p+1}\xi \left[ \tilde{\Pi}_\mu \partial_0 X^\mu + P^a F_{0a} - \chi P^0 
- \lambda \left( \tilde{\Pi}_\mu G^{\mu\nu} \tilde{\Pi}_\nu + P^a \gamma_{ab} P^b + T^2 \text{det}(\sigma_{ab}) \right) 
- \rho^a \left( \tilde{\Pi}_\mu \partial_a X^\mu + P^b F_{ba} \right) \right]. \]  

(3.12)

This is identical to the \( B_{\mu\nu} = 0 \) phase space action presented in [1], (with \( \Pi \rightarrow \tilde{\Pi} \)). The analysis proceeds as in that paper from here on: integrating out \( \tilde{\Pi} \) and \( P \), linearizing the resulting action and finally identifying the Lagrange multipliers with \( s_{ij} \) we obtain \( \text{(2.4)} \).

The derivation of the \( T \rightarrow 0 \) limit also follows directly as in [1]: the result is \( \text{(1.1)} \) or, equivalently, \( \text{(2.3)} \) (now including the full background).

We would also like to recapitulate how the string-parton picture arises. To contribute something new, we will again assume a non-trivial background. The equations of motion that follow from variation of \( \text{(1.3)} \) with respect to \( W^i, V^i, A_i \) and \( X^\mu \) are

\[
\begin{align*}
V^i (\gamma_{ik} + F_{ik}) &= 0, \\
(\gamma_{ik} + F_{ik}) W^k &= 0, \\
\partial_i \left( V^{[i} W^{k]} \right) &= 0, \\
d_t \left[ (V^{[i} W^{k]} G_{\mu\nu} + V^{[i} W^{k]} B_{\mu\nu}) \partial_k X^\nu \right] \\
&- V^{i} W^{j} \partial_i X^\rho \partial_j X^\nu (G_{\rho\nu} + B_{\rho\nu}) \gamma_{\mu} &= 0, \\
\end{align*}
\]

(3.13)

The first two equations of motion can be reduced to

\[
\begin{align*}
V^i V^k \gamma_{ik} &= 0, \\
W^i W^k \gamma_{ik} &= 0, \\
\end{align*}
\]

(3.14)

which say that \( V \) and \( W \) are null-like vector fields in the induced metric. The third equation can be written as \( [V, W]^i = (\partial \cdot W) V^i - (\partial \cdot V) W^i \), which, after

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8This change of coordinates in phase space has a trivial Jacobian.
choosing the gauge $\partial \cdot V = \partial \cdot W = 0$ says that $V$ and $W$ commute and define good coordinates. The coordinates thus defined coordinatize two dimensional submanifolds of the world volume. They will be the world sheets of the constituent strings. On each world-sheet we have the differential operators

\[
\partial_+ = V^i \partial_i, \\
\partial_- = W^i \partial_i. 
\]  

(3.15)

Using these and the gauge choice we see that the equations of motion reduce to

\[
\gamma_{++} = 0, \\
\gamma_{--} = 0, \\
\partial_+ \partial_= X^\mu + \Gamma^\mu_{\nu\rho} \partial_+ X^\rho \partial_= X^\nu = 0, 
\]  

(3.16)

where $\Gamma^\mu_{\nu\rho}$ is the torsionful connection expressed in terms of $G_{\mu\nu}$ and $B_{\mu\nu}$.

The equations (3.16) are exactly the conformal gauge equations of motion for a string in a non-trivial background. The additional coordinate dependence of $X^\mu$ now becomes a label distinguishing different string world-sheets.

For the special case when $V^i$ and $W^i$ are parallel, an analogous analysis shows that the world volume splits into a collection of massless particles.

4 Derivation of the supersymmetric model

In this Section we briefly present the “degenerate” $(p + 1)$-dimensional supergravity. The structure is reminiscent of, but still distinctly different from the usual 2D superspace supergravity, as described in, e.g., [13], [14]. Details can be found in [8]. We also discuss the supersymmetrisation of (2.5) leading to (2.7) in some detail.

The defining relations for our supergravity are

\[
\nabla_{\pm} = E^+_{\pm} \partial_+ + E^-_{\pm} \partial_- + E^i_{\pm} \partial_i + \omega_{\pm} M, 
\]  

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\[ \nabla_+ = e^+_i \partial_i + \chi_+^+ \partial_+ + \chi_+^- \partial_- + \omega_+ M, \tag{4.17} \]

along with the constraints
\[
\{ \nabla_+, \nabla_- \} + \Gamma_+(\nabla_-) = RM, \\
\nabla_+^2 + \Gamma_+ \nabla_+ = \pm i \nabla_+. \tag{4.18} \]

Here the additional “connections” are given by
\[
\Gamma_\pm \equiv \partial_i E^i_\pm + \partial_+ E^+_\pm + \partial_- E^-_\pm \pm \frac{1}{2} \omega_\pm \equiv 1 \cdot \nabla_\pm. \tag{4.19} \]

All fields are superfields and depend on the superspace coordinates \(\xi^i, \theta^+, \theta^-\). The partial derivatives are defined with respect to those coordinates. A novel feature is that the \(\theta\)’s transform as (weight \(\frac{1}{4}\)) densities under \(\xi\) diffeomorphisms. Diffeomorphisms, \((\sigma^i)\), supersymmetry, \((\epsilon^\pm)\) and Lorentz, \((\Lambda)\), transformations are coded into the superfield \(K\) defined by
\[
K \equiv \sigma^i \partial_i + \epsilon^+ \partial_+ + \epsilon^- \partial_- + \Lambda M, \tag{4.20} \]

and the transformations of the derivatives in (4.17) are given by
\[
\delta \nabla_\pm = [\nabla_\pm, K] - \frac{1}{2} (1 \cdot \tilde{K}) \nabla_\pm, \\
\delta \nabla_\pm = [\nabla_\pm, K] - (1 \cdot \tilde{K}) \nabla_\pm, \tag{4.21} \]

where
\[
1 \cdot \tilde{K} \equiv \partial_i \sigma^i - \partial_+ \epsilon^+ - \partial_- \epsilon^- \tag{4.22} \]

These are the appropriate transformations for densities of weights \(\frac{1}{4}\) and \(\frac{1}{2}\) respectively. From these relations the transformations of the components may be derived. To display the physical content of the theory it is convenient to work in a Wess-Zumino (WZ) gauge which we define as follows:
\[
\nabla_\pm | = \partial_\pm, \\
[\nabla_\pm, \nabla_\mp] | + \Gamma_\pm \nabla_\mp | = 0. \tag{4.23} \]

From the constraints and the Bianchi identities it follows that only certain component fields are independent. We define components by projection
and use the same notation for the supergravity superfields and their lowest components:

\[
\begin{align*}
e_±^i & \equiv e_±^i|, \quad \chi_±^± & \equiv \chi_±^±|, \quad \omega_± & \equiv \omega_±|,
\end{align*}
\]

\(R \equiv R|, \quad \rho_± \equiv \nabla_± R|.
\)

(4.24)

We shall also need the components of a scalar superfield \(X\) (in WZ-gauge),

\[
X^\mu \equiv X^\mu|, \quad \Psi_±^\mu \equiv \partial_± X^\mu|, \quad \mathcal{F}_±^\mu \equiv \partial_± \partial_- X^\mu|,
\]

(4.25)

where \(\mu\) is a target space index. Note that the density character of \(\theta\) leads to \(\Psi\) and \(\mathcal{F}\) being densities.

Under \((p+1)\)-dimensional diffeomorphisms the components transform as specified by their density weights, and under Lorentz transformations according to their Lorentz charge. The local supersymmetry transformations of the supergravity fields are

\[
\begin{align*}
\delta e_±^i &= \mp i \epsilon_± \chi_±^± e_±^i \pm i \epsilon_± (2 \chi_±^± e_±^i - \chi_±^± e_±^i) \\
\delta \chi_±^± &= \partial_± \epsilon_± - \epsilon_± \left(\frac{1}{2} \partial_± e_±^i \pm i \chi_±^± \chi_±^± \pm \omega_±\right) + \epsilon_± \left(2 i \chi_±^± \chi_±^± \mp \frac{1}{2} \chi_±^± \chi_±^± \pm \frac{1}{2} R\right) \\
\delta \chi_±^∓ &= \partial_± \epsilon_± \pm \frac{3}{2} i \epsilon_± \chi_±^± \chi_±^± - \epsilon_± \left(\frac{1}{2} \partial_± e_±^i \mp \frac{3}{2} \chi_±^± \chi_±^∓\right) \\
\delta \omega_± &= -\epsilon_± (\pm i \chi_±^± \omega_± \pm \chi_±^± R) \\
&\quad \pm \epsilon_± \left(2 i \chi_±^± \omega_± \mp i \chi_±^± \omega_± \mp i \rho_±\right), \\
\delta R &= -\epsilon_± \rho_+ - \epsilon_- \rho_-.
\end{align*}
\]

(4.26)

where \(\epsilon\) is the lowest component of the corresponding superfield. The matter field transformations are

\[
\begin{align*}
\delta X^\mu &= - \epsilon_± \Psi_±^\mu - \epsilon_- \Psi_-^\mu, \\
\delta \Psi_±^\mu &= \mp i \epsilon_± \partial_± X^\mu \pm \epsilon_± \mathcal{F}_±^\mu \mp \frac{1}{2} \left(\epsilon_± \chi_±^± \pm \epsilon_± \chi_±^∓\right) \Psi_±^\mu \\
&\quad \mp i \epsilon_± \chi_±^± \Psi_±^\mu,
\end{align*}
\]
\[ \delta \mathcal{F}^\mu = \ i e^- \partial_+ \Psi_\mu^+ + i e^+ \partial_+ \Psi_\mu^- - \lambda^+ \Psi_\mu^+ - \lambda^- \Psi_\mu^-, \]  
\[ (4.27) \]

where \( \lambda^\pm \equiv \partial_+ \partial_- \epsilon^\pm \).

As mentioned in Section 2, one of the consequences of the Bianchi identities in conjunction with the constraints (4.18) is that

\[ \partial_i \left( e^i_+ e^j_+ \right) = 0, \]  
\[ (4.28) \]
i.e., the \( A_i \) field equations of (2.5). At the superspace level, the locally supersymmetric \( \sigma \)-model (2.7) that we consider is therefore a generalization of (2.5) with the \( A_i \) field integrated out. To have unconstrained fields in our action we then reintroduce the \( A_i \) field as a Lagrange multiplier in the component action (2.8). The relation (4.28) is preserved by supersymmetry transformations and (2.8) is thus locally supersymmetric if we take \( A_i \) to transform as a singlet.

Just like for the purely bosonic case, we may use the gauge \( \partial_i e^i_+ = 0 \) to choose special coordinates. In other words, in this gauge the 2D Lorentzian “tangent space” may be identified with a subspace of the \( (p+1) \)-dimensional tangent space to the bosonic coordinate space and there are coordinates \( \tilde{\xi}^i \in \{ \tilde{\xi}^a, \tilde{\xi}^+, \tilde{\xi}^- \} \) with \( a = 1, \ldots, p-1 \), such that \( e^i_\pm = \partial \xi^i / \partial \tilde{\xi}^\pm \). As is customary we also gauge away \( \chi_A \) using local supersymmetry and the typical 2D-invariance of the action under

\[ \delta \chi_A = \gamma_A \kappa, \]  
\[ (4.29) \]

where \( \kappa(\xi) \) is the transformation parameter. In this gauge, the field equations that follow from (2.7) may be written

\[ \gamma_{\pm \pm} = \mp i \Psi_{\pm} \cdot \partial_{\pm} \Psi_{\pm}, \]  
\[ \Psi_{\pm} \cdot \partial_{\pm} X = 0, \]  
\[ \partial_+ \partial_- X^\mu = 0, \]  
\[ i \partial_+ \Psi_\mu^+ = 0, \]  
\[ (4.30) \]
where we only display the $\pm$ components of the $\delta e_A^i$ equations. The equations (4.30) are precisely those of a spinning string in conformal gauge. Again we find a parton picture with the additional coordinates labeling a foliation of the world volume.

5 Discussion

We have extended the bosonic results of [1] both to include non-trivial backgrounds for the bosonic theory and to allow for spinning string partons. The latter result is necessary if one is to believe in this limit of $D$-branes which are after all (space-time) supersymmetric. Ideally, we should have started from a space-time supersymmetric $D$-brane action, derived the strong coupling limit and shown that this could be viewed as being built from superstrings. Since space-time and world-sheet supersymmetric strings are different formulations of the same theory, we choose instead to construct a world-volume supersymmetrization of our bosonic result. We view the emerging picture of spinning string partons as strong evidence that the strong coupling limit of $D$-branes can be viewed as a model with strings as partons.

In this context it is gratifying to note that it has been shown by Hull [15] that the only solutions to supergravity at strong coupling are strings and particles.

As mentioned in Section 3, our discussion allows for (spinning) particles as partons too. This corresponds to the case when $e^i_+\pm e^i_-$ are proportional to each other.

In [1] it was shown that for the special case of $p = 1$ the action (1.1), (or (2.3)), unifies tensile and tensionless fundamental strings. The string tension $T$ is an integration constant for the $A_i$ equation. This generalizes immediately to the spinning model presented here. For $p = 1$ (2.8) leads to

\footnote{A discussion of $D$-branes as being built from particles was recently given in [16].}

\footnote{Similar results were previously discussed in [17].}
the tensile or tensionless fundamental spinning string depending on the value
of the same integration constant.

We end by some speculative comments on the applicability of our results. The
strong coupling limit of Type IIA strings is described by $11D$ super-
gravity (the low energy limit of $M$-theory). In our strong coupling limit we
see nothing that indicates an extra dimension appearing. In fact, since we
only include the common (bosonic) sector in the background, we cannot even
discriminate between type IIA and Type IIB, say. We hope to be able to
discuss the full background later using the Hamiltonian description of super
$D$-branes in ref. [18]. In any case, for the common sector discussed here the
background seems to enter in a trivial way in the limit $g_s \to \infty$ for a single
$D$-brane. To see the appearence of an extra dimension, one would like to
be able to take the limit in the $D$-brane action and its background simul-
taneously. Apart from this problem, one might perhaps also question the
description in terms of a standard action in the strong coupling limit. Since
higher loop quantum effects become more and more important, perhaps we
need to be able to treat infinite genus Riemann surfaces for the open strings
describing the $D$-brane fluctuations. This might require a novel description
that includes new effects at such strong coupling.

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