Transverse Forced Vibrations of the Plates, the Dissipative Properties of Which are Described Memory Functions

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Abstract. The fundamentals of theoretical methods for engineering calculations of bending vibrations of thin plates, the material of which has hereditary properties, are presented. The manifestation of the hereditary properties of the material of the plate under consideration at a given stress-strain state can be judged by the relaxation core of the material. The dependence of the creep and relaxation kernels on the time difference corresponds to the fact that the “memory” of the material about the force effect produced at a given moment is determined by the elapsed time interval. In particular, this means that if the force action on the elastic-viscous body is cyclic, then the deformation of the body will also be cyclic with some phase shift. A technique for solving forced vibrations of plates under the influence of harmonic loads is proposed. An exact solution to the problem of forced vibrations of a supported rectangular plate, whose dissipative properties are described by memory functions, is constructed.

Keywords: plate, forced vibrations, hereditary properties, relaxation kernel, creep kernel.

1. Introduction

As you know, in vibration calculations of elastic and viscous elastic systems, the main difficulties arise in choosing a model that would most reliably simulate the mechanism of inelastic resistance of the system at a given stress-strain state. These difficulties arise because in mechanical systems, as noted in [1,2,3], the sources of energy dissipation are extremely diverse and each type of energy loss depends on many factors. During bending vibrations of thin plates, the material of which has pronounced memory properties, energy dissipation occurs mainly due to irreversible processes in the plate material itself. These energy losses can mainly be taken into account in engineering calculations if we use the Boltzmann-Voltaire theory of heredity [4,5,6], the linear theory of elasticity [6,7,8] and the method described below.
2. Methods
2.1. Some general considerations and relationships

According to the linear theory of heredity in one-dimensional deformation of bodies, the relationship between stresses and deformations is expressed by the following equations:

\[
\sigma(t) = E \left[ \varepsilon(t) - \frac{t}{0} \int R(t-s) \varepsilon(s) ds \right]; \tag{1}
\]

\[
\varepsilon(t) = E^{-1} \left[ \sigma(t) + \frac{t}{0} \int K(t-s) \sigma(s) ds \right]; \tag{2}
\]

where \(K(t-s)\) - material creep core, \(\varepsilon(t)\) - linear deformation, \(\sigma(t)\) - voltage, \(E\) - instant modulus of elasticity, \(t\) - time counted from the start loaded, \(S\) - integration parameter, \(R(t-s)\) - core of relaxation. Dependency of kernels \(K(t-s)\) and \(R(t-s)\) from the difference corresponds to the fact that the "memory" of the material about the force effect produced at the moment \(s\), determined by elapsed time \(t-s\). In particular, this means that if the force action on an elastic-viscous body is cyclic, then the deformation of the body will also be cyclic with some phase shift [4]. With bending vibrations of the plates, this pattern will be described by more complex memory functions than with static deformation, since energy dissipation in the material is closely related to the accumulation of microdamages in the process of cyclic deformation of the material, generally speaking, in all three directions of the plate. These energy losses will be the greater, the more structural defects in the material and microdamages arising in the process of vibrations of the plate [7,8,9]. Considering the complexity of the mechanism of inelastic resistances during vibrations of mechanical systems in general, and of thin plates in particular, it seems that the most reliable and forgiven energy losses in the process of vibrations are determined by vibrograms of free vibrations of the studied plates.

Let us write equations (1), (2) in operator form similar to Hooke's law

\[
\sigma(t) = \tilde{E} \varepsilon(t); \quad \varepsilon(t) = \tilde{E}^{-1} \sigma(t) \tag{3}
\]

and apply them to study bending vibrations of thin plates in the framework of the linear Kirchhoff-Love theory [10, 11, 12], assuming the Poisson's ratio to be constant \(\nu = \text{const}\), here and everywhere in what follows, we will use linear-time operators over functions in the notation:

\[
\begin{align*}
\tilde{E} f(x,y,z) & = \tilde{E} \left[ f(x,y,z) - \int_0^1 R(t-s) f(x,y,z,s) ds \right], \\
\tilde{E}^{-1} f(x,y,z,t) & = \tilde{E}^{-1} \left[ f(x,y,z,t) + \int_0^1 K(t-s) f(x,y,z,s) ds \right], \\
\tilde{F}(x,y,z,t) & = f(x,y,z,t) \pm \int_{\mathbb{R}} f(x,y,z,s) ds.
\end{align*}
\tag{4}
\]

With bending vibrations of plates [13,14,15] in the framework of the linear theory of elasticity and heredity, the relationship between the components of deformation \(\varepsilon_{ij}, \gamma_{ij}\) deflection \(\omega(x,y,t)\) the middle surface of the plate and stress components \(\sigma_{ij}, \tau_{ij}\) based on the generalized Hooke's law and formulas (4), can be written in the form

\[
\begin{align*}
\tilde{\varepsilon}_{xx}(x,y,z,t) & = -\tilde{E}^{-1} \frac{\partial^2 w(x,y,t)}{\partial x^2} + \int_0^t K(t-s) \frac{\partial^2 w(x,y,s)}{\partial x^2} ds, \\
\tilde{\varepsilon}_{yy}(x,y,z,t) & = -\tilde{E}^{-1} \frac{\partial^2 w(x,y,t)}{\partial y^2} + \int_0^t K(t-s) \frac{\partial^2 w(x,y,s)}{\partial y^2} ds, \\
\tilde{\varepsilon}_{xy}(x,y,z,t) & = -\tilde{E} \frac{\partial^2 w(x,y,t)}{\partial x \partial y} + \int_0^t K(t-s) \frac{\partial^2 w(x,y,s)}{\partial x \partial y} ds. \tag{5}
\end{align*}
\]
here: $\varepsilon_{xy}, \varepsilon_{y}$ - components of the relative linear deformation along the coordinate axes $OX, OY$, located in the median plane of the plate, $\gamma_{xy}$ - angular deformation in planes parallel to the plane of the plate, $Z$ - distance of points of the plate from its middle surface, $h$ - plate thickness.

Normal and tangential stresses determined by formulas (6) are the main components of bending stresses in a thin plate, the material of which has hereditary memory properties. Stress components determined by formulas (7) are minor components of stresses during bending of thin plates [16,17,18].

2.2. Problem statement and solution methods

A rectangular hereditarily elastic plate supported along the edges is considered. The differential equation of bending vibrations of a homogeneous isotropic plate, taking into account the mass forces of its material, can be written in operator form as

$$\frac{\partial \ddot{Q}_x}{\partial x} + \frac{\partial \ddot{Q}_y}{\partial y} + \rho h \frac{\partial^2 w}{\partial t^2} = 0$$

(8)

where $\ddot{Q}_x$ and $\ddot{Q}_y$ - unit shear forces determined through stresses (7) by the formulas

$$\ddot{Q}_x = \int_{\frac{h}{2}}^h \ddot{r}_{xx} dz; \quad \ddot{Q}_y = \int_{\frac{h}{2}}^h \ddot{r}_{xy} dz.$$

(9)

If from formula (7) we substitute the values $\ddot{r}_{xy}, \ddot{r}_{xy}$ into expressions (9) and the result obtained is introduced into equation (8), then the latter, after simple transformations, can be represented as an integro-differential equation

$$D_0 \nabla^2 \nabla^2 w(x,y,t) - D_0 \int_{-\infty}^{t} R(t-s) D^2 \Delta^2 w(x,y,s) ds + \rho h \frac{\partial^2 w}{\partial t^2} = q_0(x,y) \sin pt.$$

(10)

where $\rho$ - unit volume density of a plate, $w$ - deflection, $E_0$ - instant modulus of elasticity, $h$ - plate thickness, $D = \frac{E_0 h^2}{12(1-v^2)}$ - instantaneous cylindrical stiffness of the plate, $\nabla^2 \nabla^2 = \frac{\partial^2}{\partial x^2} + 2 \frac{\partial^2}{\partial x \partial y} + \frac{\partial^2}{\partial y^2}$ the biharmonic Laplace operator, and the boundary conditions

$$w(x,y,t) = w(l,y,t) = w_x(0,y,t) = w_y(0,y,t) = 0,$$

$$w(x,0,t) = w(x,m,t) = w_{yy}(x,0,t) = w_{yy}(x,m,t) = 0$$

(11)

The solution to problem (10) - (11) is sought in the form

$$w(x,y,t) = \sum_{i,j=1}^{\infty} a_{ij} \xi_{ij}(x,y) \sin (pt + \phi_{ij}).$$

(12)

Here $\xi_{ij}(x,y) = \sin \frac{nx}{l} \sin \frac{ny}{m}$,

$$a_{ij} = \frac{q_{ij}}{\rho p^2 - Dn^2 \left( \frac{x^2}{l^2} \right)^2 \left( 1 - \tau_2 \right)^2 + Dn^4 \tau_2 \left( \frac{x^2}{l^2} \right)^2},$$

(13)
\[
\phi_{ij} = \arctg \frac{ar_{ij} \left( \frac{r^2_{ij}}{l^2} \right)^2}{\rho \rho^{2-\rho} \pi^2 \left( \frac{r^2_{ij}}{l^2} \right)^2 \left( 1 - r^2_{ij} \right)^2}
\]

Where \( q_{0ij} \) - expansion coefficients \( q_{0}(x, y) \) by function \( \zeta_{ij} \):

\[
q_{ij} = \frac{4}{im} \int_{0}^{l} \int_{0}^{m} q_{0}(x, y) \sin \frac{i\pi x}{l} \sin \frac{i\pi y}{m} dxdy .
\]

\[
\Gamma_{x} = \int_{0}^{\infty} R(s) \cos p sds, \quad \Gamma_{z} = \int_{0}^{\infty} R(s) \sin p sds .
\]

Solution (12) satisfies Eq. (10) and boundary conditions (11), which can be verified by substituting (12) in (10) and (11) taking into account the identity

\[
\int_{0}^{\infty} R(t - s) \sin p sds = \Gamma_{z} \sin pt - \Gamma_{z} \cos pt .
\]

It should be noted that many works have been devoted to the determination of eigenforms, in particular [19,20], therefore, in what follows, we will assume that the vibration modes of plates made of elastic-viscous materials are known.

3. Numerical results

As the relaxation kernel of a viscoelastic material, we take the three-parameter Rzhanizyn – Koltunov kernel \( R(t) = \frac{Ae^{-\beta t}}{t^{1-\alpha}} \) [6], which has a weak singularity, where \( A, \alpha, \beta \) -are the parameters of the materials [6]. Let’s take the following parameters: \( A = 0.048; \ \beta = 0.05; \ \alpha = 0.1, \ \nu = 0.35 \). The calculation results were obtained on the basis of the developed C ++ program. The change in the displacement amplitude is shown in Fig. 1. at various values of viscosity. Figure 1 shows graphs of changes in the displacement amplitude depending on the frequency of external loads at different values of the relaxation core amplitude 1) 0.00048; 2) 0.0048; 3) 0.048. It is seen that with an increase in the displacement amplitude, the corresponding amplitudes gradually decrease.

![Figure 1. Change in the amplitude of displacements depending on the frequency of external loads:](image)

Equation (10) describes the process of forced vibrations of plates in the presence of inelastic resistances in a material with hereditary properties. This equation in form is the equation of motion of a one-dimensional mechanical system, the dissipative properties of which are described by a fractional-exponential memory function [21]. It can be seen from this equation that each specific plate
has its own “own” structural damping, i.e., damping due to the physical and mechanical properties of the plate material, its geometric dimensions and specified boundary conditions. Apparently, only geometrically similar samples of plates, with the same boundary and initial conditions, will have the same vibrograms of free vibrations if the material of the samples has the same physical and mechanical properties. If there is no external load, then natural damped oscillations of the plate are considered.

4. Conclusions
If, for example, the material of the samples is absolutely elastic and the supports are ideal, then the process of free vibrations will be harmonic with a frequency, the value of which will depend on its geometric dimensions and boundary conditions. In this case, the integral term of equation (10) will be equal to zero.

The described technique for studying the dissipative properties of monolithic plates can also be applied in vibration calculations of plates of layered structures.

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