Optimal radiation pattern synthesis of mutually coupled antenna array using an efficient compensation method

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Abstract
An efficient compensation method is proposed for the reduction of mutual coupling among the elements of an antenna array using the optimum current amplitude excitation weights. The performance of an antenna array degrades heavily due to mutual coupling. The mutual coupling effect can neither be eliminated nor be ignored in the context of the antenna array design. Several analytical and numerical techniques have been proposed in the past few decades for the calculation and compensation of mutual coupling effect. Here, the mutual coupling is compensated by reducing the driving impedance of each identical array element in the array by adopting grey wolf optimization and moth flame optimization algorithms, respectively. The sum of the driving impedances of each element of the antenna array is reduced efficiently by adjusting the feed current amplitudes of all the individual array elements. Compared to the existing known methods, the proposed method is less complex, cost-effective and easy to implement for commercial and industrial applications to achieve better performance.

1 | INTRODUCTION
Antenna arrays have a wide range of applications in the field of wireless communication and signal processing systems. Exhaustive research works for the synthesis of the radiation pattern in different areas of wireless communication, for example, electronic beam steering, estimation of direction of arrival, diversity technique and so on, are in progress over the last few years. The effect of mutual coupling [10–16] is an indispensable part of all the practical application areas of the antenna array. For practical design issue, the physical size reduction of the antenna array is desirable; however, it will increase the mutual coupling between the elements of the array. The performance of the antenna array degrades severely due to the mutual coupling between the elements of the array, which causes the impedance mismatch, deviation of the radiation pattern, distortion and so on. Various analytical and numerical methods have been applied to calculate and compensate for the mutual coupling effect between the two radiating elements [12–14]. A survey on the effect of mutual coupling in array antennas is reported in [10].

Ample research had been done in the past for the synthesis of the radiation pattern of different configurations of antenna arrays using optimisation techniques [3–6]. However, most of the techniques are not suitable for practical applications as they do not consider the mutual coupling effect in the radiation pattern calculation.

Here, the mutual coupling is compensated by optimising and controlling the feed current amplitudes of array elements using the grey wolf optimization (GWO) and moth flame optimization (MFO) algorithms. Here, the driving impedance of each array element is calculated using a simple set of equations using the concept of the two-port network, which can even be extended up to n-ports.

The driving impedance of each array element is referred to as the input impedance of the array in the presence of other elements or obstacles. The driving impedance of each array element is computed using the principle of the induced emf method [1]. It shows that the driving impedance consists of the current ratio, mutual impedance and self-impedance when radiating into an unbounded medium. It also indicates that changing any one of the parameters will not affect the other two parameter values even though the driving impedance value will change proportionally. The driving impedance of dipole and monopole antennas can be very accurately predicted by the induced emf method [1, 17]. Most of the earlier literature...
mainly directed to the mutual impedance for mutual coupling reduction and compensation technique. Here, the authors propose a new controlling method of the current ratio for mutual coupling compensation using the evolutionary optimization techniques called GWO [18, 19] and MFO [7–9]. To demonstrate and validate the proposed method, 10 elements and 16 elements monopole antennas are employed as a discrete dipole element (DDE) [17] for the application of 5G communication.

2 | COST FUNCTION FORMULATION

To exemplify the general use of the proposed technique, a new array factor (AF) is synthesised [17] by replacing the individual elements in an antenna array [20–22] with a pair of elements called DDEs.

The geometry of the DDE array is obtained by folding a linear array into two parallel arrays, as shown in Figure 6. The synthesised AF of DDE is calculated similar to that of [17] and is given in Equation (1), where $DF_n$ is the dipole factor of each DDE and is given in Equation (2).

$$AF = \sum_{n=1}^{N} DF_n \exp[j(n-1)(kd\cos \theta + \beta)]$$ (1)

$$DF_n = \sum_{n=1}^{N} I_n \cos(kl_n \sin \theta + \eta)$$ (2)

The intra-element spacing within a DDE is represented by $l_n$, and the constant phase shift is indicated by $\eta$; the progressive phase difference between two discrete dipoles is $\beta$, and the inter-element spacing between two consecutive DDE is represented by $d$; $\beta$ is considered here as zero and $\eta = -kl_n$ for broadside configuration; $N$ shows the total number of discrete dipoles in the array. $I_n$ is the current amplitude excitation weight for the $n$th element of the DDE; $k$ and $\theta$ represent the propagation constant and the angle of propagating electromagnetic (EM) wave, respectively. Directivity (DIR) has a significant role in the design of DDEs. It is defined as the ratio of the maximum power density of the designed antenna and the power density emitted from the anisotropic antenna. DIR is calculated similarly to that of [1]. Here, the current amplitude excitation weights and the inter-element distance are the two optimising parameters that are employed to achieve the maximally reduced side lobe levels of mutually coupled antenna array using the cost function (CF) as given in Equation (3).

$$CF = W_1 \times \frac{|AF(\theta_{ms1}, I_n) + AF(\theta_{ms2}, I_n)|}{|AF(\theta_0, I_n)|} + W_2 \times Z_{DCR}$$ (3)

where $Z_{DCR}$ is the driving impedance of DDE of monopole antenna due to the current ratio and is given by Equation (4),

$$Z_{DCR} = \left[\sum_{m=1}^{n} \frac{\sum_{n=1}^{N} I_n}{I_m} - n\right]$$ (4)

where $W_1$ and $W_2$ are the weighting factors of values 1 and 0.5, respectively. The peak value of $\theta$ is represented by $\theta_0$, whereas $\theta_{ms1}$ and $\theta_{ms2}$ represent the peak side lobe angles below and above $\theta_0$, respectively. $I_n$ represents the current amplitude excitation weight of the $n$th element. The current ratio is represented by $I_m$, where $n$ shows the number of DDE elements. The first term of Equation (3) is used to reduce the side lobes, whereas, the second term is used for the reduction of mutual coupling.

Here, two recently developed evolutionary optimisation techniques known as GWO [18, 19] and MFO [7–9] that are independently employed to determine the optimum current amplitude excitation weights and the inter-element distance between 10 and 16 elements DDE of monopole antennae. The best results achieved using these different evolutionary optimisation techniques are validated using a widely used EM field simulator known as computer simulation technology-microwave studio (CST-MS).

3 | RESULTS AND DISCUSSION

The optimal radiation pattern synthesis of symmetrical 10 and 16 elements linear antenna array (LAA) of the dipole antenna array design using the GWO and MFO algorithms have been consecutively executed 50 times with the control parameters as shown in Table 1, using MATLAB 7.5 version and Intel Core 2 processor with 3 GHz and 3 GB RAM to achieve the near-global optimal solution. After 50 time execution of the GWO and MFO algorithms, the best optimal results are reported and are compared here. The best-achieved results are also validated by using EM field simulator CST-MS and are also experimentally verified.

| Parameters | GWO | MFO |
|------------|-----|-----|
| Population size ($n$) | 100 | 100 |
| Iteration cycle | 500 | 500 |
| $b$ | – | 1 |
| $t$ | – | $[-1,1]$ |
| $r$ | – | $[-1,-2]$ |
| $a$ | 2.0 | – |
| $r_1$ | (0,1) | – |
| $r_2$ | (0,1) | – |
| $A$ | (−1,1) | – |
| $C$ | (0,2) | – |

Table 1: Control parameters of employed algorithms

Abbreviations: GWO, grey wolf optimisation; MFO, moth flame optimisation.
### 3.1 Numerical results

Table 1 shows the control parameters of the employed algorithms for the design of 10 and 16 element DDE of a monopole antenna, where the inter-element distance is kept at $\lambda/2$, and the resonance frequency and, wire radii of the monopole are 3.5 GHz and $0.001\lambda$, respectively.

The mutual coupling between the DDE elements depends on the inter-element spacing and the length of the monopole [15]. Table 2 shows the current amplitude excitation weights, maximum value of side lobe level (SLL), DIR and the execution time taken for the synthesis of DDE of 10 and 16 element monopole antenna with uniform spacing ($d = \lambda/2$) obtained by employing the GWO and MFO algorithms.

The maximum SLL, DIR, and execution time taken for the design of 10 elements DDE of monopole antennas by employing the GWO algorithm are $-26.17$ dB, 8.36 dBi and 3.54 min, respectively. The same parameters, obtained by employing the MFO algorithm, for the design of 10 elements DDE of monopole antennas are $-26.82$ dB, 8.82 dBi and 2.56 min, respectively. In case of 16 elements DDE design of monopole antennas, the values of the SLL, DIR and execution time taken by employing the GWO and MFO algorithms are $-39.46$ dB, 9.68 dBi and 4.51 min and $-40.84$ dB, 10.07 dBi and 3.59 min, respectively.

The results, as shown in Table 2, justify that the MFO-based results perform better for the design of 10 and 16 elements DDE of monopole elements.

The GWO- and MFO-based results for the design of 10 and 16 elements DDE of monopole antennas are compared with the recently published results [3–6, 26], where 10 and 16 elements LAA are realised with isotropic elements without considering the mutual coupling effect.

Hence, those results reported in [3–6, 26] are not practically suitable because, in real-world applications, the antennae are not isotropic, and the effect of mutual coupling cannot be avoided for any practical antenna array design issue as the mutual coupling is an indispensable part for all practical antenna arrays. Table 3 clearly shows that despite considering the mutual coupling effect in the 10 element and 16 element DDE formed with practical antenna-like monopole antennas, the GWO and MFO algorithms based results are far improved ones for the maximum SLL reduction as compared to the recently published results [3–6, 26]. The radiation pattern plots for the design of 10-element DDE obtained using different algorithms are shown in Figure 1.

The radiation pattern plots of Figure 1 show that GWO and MFO based results for the maximum reduction of SLL outperform the results obtained using the ant lion optimization (ALO) [26], symbiotic organisms search (SOS) [3], biogeography-based optimisation (BBO) [6], particle swarm optimisation (PSO) [5] and Taguchi optimisation methods [4].

Figure 2 plots the radiation patterns for the design of 16 elements DDE obtained by using different algorithms which confirms that the performances for maximum SLL reduction by employing the GWO and MFO algorithms are far better compared to the results obtained using the ALO [26], SOS [3], BBO [6], PSO [5] and Taguchi optimisation methods [4]. The results, as given in Table 3, and the radiation pattern plots of Figures 1 and 2 justify that both GWO and MFO algorithms perform exceptionarlly well for the design of 10 and 16 elements DDE of monopole antennas.

### Table 2 Different parameters obtained by employing GWO and MFO for the design of DDE of monopole antennas

| No. of elements | Algorithm | $I_n$ (Amplitude) | Maximum SLL (dB) | Directivity (dBi) | Execution time (min) |
|-----------------|-----------|-------------------|------------------|-----------------|---------------------|
| 10              | GWO       | 0.8739 0.7592     | $-26.17$         | 8.36            | 3.54                |
|                 |           | 0.6381 0.3956     |                  |                 |                     |
|                 | MFO       | 1.0008 0.8969     | $-26.82$         | 8.82            | 2.56                |
|                 |           | 0.6967 0.4942     |                  |                 |                     |
|                 |           | 0.2966            |                  |                 |                     |
| 16              | GWO       | 0.8879 0.8365     | $-39.46$         | 9.68            | 4.51                |
|                 |           | 0.7210 0.5904     |                  |                 |                     |
|                 | MFO       | 1.0002 0.9348     | $-40.84$         | 10.07           | 3.59                |
|                 |           | 0.8129 0.6541     |                  |                 |                     |
|                 |           | 0.4860 0.3205     |                  |                 |                     |
|                 |           | 0.1929 0.1006     |                  |                 |                     |

Abbreviations: DDE, discrete dipole element; GWO, grey wolf optimization; MFO, moth flame optimization; SLL, side lobe level.
between two employed algorithms, it is observed that MFO-based design performs better compared to the results obtained by using the GWO algorithm.

3.2 | Accuracy test of GWO and MFO

The statistical parameters of CF values obtained by employing the GWO and MFO algorithms after 50 independent runs for the synthesis of 10 and 16 elements DDE of the monopole antennas are given in Table 4. The results presented in Table 4 show that the variations of CF values for the GWO and MFO algorithm-based designs are trivial enough for both the cases of 10 elements and 16 elements DDE synthesis, which prove the robustness and stability of the proposed approach.

A two-sample t-test [24] is performed between GWO/MFO to statistically establish the difference between the results.
achieved by employing the two algorithms. The $t$-value is defined in Equation (5) and is discussed in details in [24].

$$
t = \frac{\bar{a}_2 - \bar{a}_1}{\sqrt{\frac{\sigma_1^2}{\beta+1} + \frac{\sigma_2^2}{\beta+1}}}$$  \hspace{1cm} (5)

where $\bar{a}_1$ and $\bar{a}_2$ denote the mean values of the first and second algorithm, respectively; $\sigma_1$ and $\sigma_2$ represent the standard deviation of the first and the second algorithm, respectively. $\beta$ shows the degree of freedom. The results of the $t$-test conducted between the GWO/MFO pairs are shown in Table 5.

![Diagram](image.png)

**FIGURE 2** Radiation pattern of the 16-element DDE of monopole elements. DDE, discrete dipole element

| No. of elements | Algorithms | Minimum CF | Maximum CF | Mean CF | Standard deviation |
|-----------------|------------|------------|------------|---------|-------------------|
| 10              | GWO        | 0.4043     | 0.8004     | 0.6914  | 0.1452            |
|                 | MFO        | 0.2386     | 0.6486     | 0.5186  | 0.1311            |
| 16              | GWO        | 0.4839     | 0.8484     | 0.7201  | 0.1290            |
|                 | MFO        | 0.3302     | 0.6702     | 0.5193  | 0.1168            |

Abbreviations: CF, cost function; GWO, grey wolf optimisation; MFO, moth flame optimisation.

**TABLE 4** Statistical performance of CF values achieved by employing GWO and MFO

| No. of elements | Algorithms | $t$-Test values |
|-----------------|------------|-----------------|
| 10              | GWO/MFO    | 5.3536          |
| 16              | GWO/MFO    | 7.6343          |

Abbreviations: GWO, grey wolf optimization; MFO, moth flame optimization.

The obtained $t$-value is higher than 2.15 for $\beta = 49$ which signifies the difference between the GWO/MFO optimisation techniques with a 98% confidence level. Thus, statistical results also confirm the superiority of the MFO algorithm.

Box and whisker plots [25] are also portrayed here to compare the statistical performance of the GWO and MFO algorithm based results for the 10 elements and 16 elements DDE design of monopole antennas. The GWO and MFO algorithms are employed here, which are executed 50 times to obtain the best results. All the CF values achieved in every execution are employed in box and whisker plots. Figures 3 and 4 show the box and whisker plots achieved by employing the GWO and MFO algorithms for the synthesis of 10 and 16 elements DDE of monopole antennas.

The top and bottom of the boxes represent the 75th and 25th percentile, respectively. Median is shown as a triangle. The median value of CF achieved by employing the GWO and MFO algorithms for the design of 10 elements DDE of monopole antennas are 0.6122 and 0.4298, respectively. The range of variation of CF obtained by using MFO is 0.2386–0.6486, which is better compared to the variation of CF obtained by using GWO where the range of variation is 0.4043–0.8004.
The median values of CF achieved by employing the GWO and MFO algorithms for the design of 16 elements DDE of monopole antennas are 0.6632 and 0.5123, respectively. The range of variation of CF obtained by using MFO is 0.3302–0.6702, which is better compared to the variation of CF obtained by using GWO, where the value is 0.4839–0.8484.

All the above-mentioned box and whisker plots show that the range of variation of CF is more extensive in MFO compared to the variation of CF achieved by employing GWO. Thus, the box and whisker plots depict that both the algorithms used for the design of 10 and 16 elements DDE of monopole antennas offer robust and stable results. However, the performance of MFO-based results shows marginally superior to the results achieved using GWO for achieving lower CF and its related statistical parameter values.

### 3.3 Convergence profile plots of GWO and MFO

The convergence profiles of GWO and MFO algorithm based designs for 10 and 16 elements DDE of monopole antennas are shown in Figure 5, which shows the plots of the CF values with the iteration cycle.

#### 3.4 MFO-based results validated using electromagnetic simulation and measurements

CST-MS is the widely used EM field simulation software based on the finite integration technique, which can be applied from low frequency to high frequency structure design. The
TABLE 6  Comparison of numerical and EM simulated results of 10 elements and 16 elements DDE of monopole antennas

| No. of elements | Numerical and EM simulated result | Maximum SLL (dB) | FNBW (°) | Directivity (dBi) |
|-----------------|-----------------------------------|------------------|----------|------------------|
| 10              | MATLAB                            | −26.82           | 33.16    | 8.82             |
|                 | CST-MS                            | −27.40           | 24       | 8.42             |
| 16              | MATLAB                            | −40.84           | 27       | 10.07            |
|                 | CST-MS                            | −41.40           | 19       | 9.77             |

Abbreviations: CST-MS, computer simulation technology-microwave studio; DDE, discrete dipole element; EM, electromagnetic; FNBW, first null beam width; SLL, side lobe level.

FIGURE 6  Front and back panel of substrate integrated 16 elements monopole antenna array: (a) front panel and (b) back panel

Numerical results and the accuracy tests conducted between two employed algorithms reveal that MFO based results perform better compared to the GWO algorithm based results for the design of 10 and 16 elements DDE of monopole antennas. Hence, the numerical results achieved by employing the MFO algorithm are validated by using CST-MS and are also experimentally verified by using the vector network analyser. The current amplitude excitation weights and the inter-element spacing values are the same as those of the numerical results obtained by employing the MFO algorithm for the synthesis of 10 and 16 elements DDE of monopole elements. The numerical and EM field simulation-based results are compared in Table 6 which shows the maximum SLL, first null beam width (FNBW) and the DIR values obtained by employing MATLAB simulation for 10 elements DDE of monopole antennas are −26.82 dB, 33.16° and 8.82 dBi, respectively; whereas, the results obtained using CST-MS are −27.4 dB, 24° and 8.42 dBi, respectively.

The maximum SLL, FNBW and the DIR values obtained by employing MATLAB simulation for 16 elements DDE of monopole antennas are −40.84 dB, 27° and 10.07 dBi, respectively, whereas the results obtained using CST-MS are −41.4 dB, 19° and 9.77 dBi, respectively.

The simulated results are also experimentally verified by employing a pre-fabricated antenna array [17], where the antenna elements are assembled using substrate integrated λ/4 monopoles, and single-sided copper-clad PTFE (εr = 2.4, 1.6 mm) printed circuit board as employed for EM field simulation for far-field radiation pattern synthesis of 16 elements DDE of monopole antennas. It is worth mentioning that l = 14.35λ and separation between two elements within a DDE is λ/2. Figure 6 shows the front and back panel of the fabricated antenna array of DDEs containing 16 monopole antennae.

A comparison between the simulated and the measured results for the return loss characteristics of 16 elements DDE of monopole antennae is given in Figure 7, which endorses a good agreement between the simulated and measured results.

The simulated and measured results of the radiation pattern for 16 elements DDE of monopole antennae are plotted in Figure 8. The above plots of simulated and measured results of 16 elements DDE of monopole antennae, as shown in Figure 8, ensure the validation of the proposed method. The observed difference between the simulated and measured results is due to the influence of the coaxial connectors, which are not included in the EM field simulation.

4  CONCLUSION

The method proposed here for the compensation of mutual coupling effect among the array elements using optimal current amplitude excitation weights is straightforward, efficient and accurate for the radiation pattern synthesis of antenna arrays. Two different evolutionary optimisation techniques, namely the GWO and MFO algorithms, are employed here to determine the optimal solutions for the design of 10 and 16 elements DDE of monopole antennae design considering the
mutual coupling effect. The performance and accuracy of the employed algorithms are compared to determine the best-suited algorithm for the proposed optimisation problem. Finally, the best results achieved by employing the evolutionary optimisation technique (MFO) are validated using a widely used EM field simulation software known as CST-MS. The results show an excellent covenant between the numerical analysis obtained by employing the MFO algorithm and the results obtained using an EM field simulator named CST-MS. Thus, the obtained MFO based results confirm that the proposed method performs exceptionally well for the design of DDE of monopole antennae considering the mutual coupling by employing the MFO algorithm.

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