Compilation-based Solvers for Multi-Agent Path Finding: a Survey, Discussion, and Future Opportunities

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Abstract
Multi-agent path finding (MAPF) attracts considerable attention in artificial intelligence community as well as in robotics, and other fields such as warehouse logistics. The task in the standard MAPF is to find paths through which agents can navigate from their starting positions to specified individual goal positions. The combination of two additional requirements makes the problem computationally challenging: (i) agents must not collide with each other and (ii) the paths must be optimal with respect to some objective.

Two major approaches to optimal MAPF solving include (1) dedicated search-based methods, which solve MAPF directly, and (2) compilation-based methods that reduce a MAPF instance to an instance in a different well established formalism, for which an efficient solver exists. The compilation-based MAPF solving can benefit from advancements accumulated during the development of the target solver often decades long.

We summarize and compare contemporary compilation-based solvers for MAPF using formalisms like ASP, MIP, and SAT. We show the lessons learned from past developments and current trends in the topic and discuss its wider impact.

1 Introduction
Compilation is one of the most prominent techniques used across variety of computing fields ranging from theory to practice. In the context of problem solving in artificial intelligence, compilation is represented by reduction of an input instance from its source formalism to a different usually well established formalism for which an efficient solver exists. The reduction between the formalisms is usually fast so that obtaining the instance in the target formalism and interpreting the solution back to the source formalism after using the solver consumes only small part of the total runtime while the most of time is assumed to be spent by the solver. The key idea behind using compilation in problem solving is that the solving process benefits from the advancements in the solver for the target formalism, often accumulated over decades.

The compilation-based solving approach has been applied successfully applied in solving combinatorial problems like plannig [Ghallab et al., 2004], verification [Bradley and Manna, 2007], or scheduling [Blazewicz et al., 2004] where the target formalism is often represented by Boolean satisfiability (SAT) [Barrett et al., 2009], mixed integer linear programming (MIP) [Jünger et al., 2010; Rader, 2010], answer set programming (ASP) [Lifschitz, 2019] or constraint satisfaction (CSP) [Dechter, 2003]. Significant advancements has been achieved in compilation-based approaches in specific domains, namely in multi-agent path finding (MAPF) [Silver, 2005; Ryan, 2008; Standley, 2010] that we are focusing on in this paper.

The standard variant of MAPF is the problem of finding collision-free paths for a set of agents from their starting positions to individual goal positions. Agents move in an environment which is usually modeled as an undirected graph $G=(V,E)$, where vertices represent positions and edges the possibility of moving between positions. Agents in this abstraction are discrete items, commonly denoted $A=\{a_1,a_2,\ldots,a_k\}$, $k \leq |V|$, placed in vertices of the graph, moving instantaneously between vertices provided that there is always at most one agent in a vertex and no two agents traverse an edge in opposite directions.

There are numerous applications of the MAPF problem in warehouse logistics [Li et al., 2020b], traffic optimization [Mohanty et al., 2020], multi-robot systems [Preiss et al., 2017], and computer games [Snae et al., 2012] to name few representatives, for more real-life applications see [Felner et al., 2017].

Relatively simple formulation of the MAPF problem is an important factor that made it an accessible target of various solving methods including compilation-based approaches. The simultaneous existence of diverse methods for MAPF has fostered mutual cross-fertilization and deeper understanding. Current state-of-the-art compilation-based solvers for MAPF go even beyond the standard single shot reduction-solving-

1Different movement rules exist such permitting the move into a vacant vertex only. There is also large body of works dealing with related problems like pebble motion in graphs [Kornhauser et al., 1984], token swapping and token permutations in graphs [Bonnet et al., 2018] using similar movement primitives.
interpreting loop coined for classical planning by the SAT-
Plan algorithm [Kautz and Selman, 1992] where Boolean satis-
fiability has been used as the target formalism and the SAT
solver has been treated merely as a black-box solver. In-
tensive cross-fertilization between the dedicated search-based
methods for MAPF, that solve the problem directly, and com-
piilation techniques led to numerous improvements in encod-
ings of the problem in target formalisms but also how the
solver is used, treating it less like a black-box.

The effort culminated recently in a combination of compi-
ilation and lazy conflict resolution introduced in Conflict-
based search algorithm (CBS) [Sharon et al., 2015] resulting
in approaches that construct the target encoding lazily in close
cooperation with the solver. The solver in these lazy schemes
suggests solutions for incomplete encodings of the input in-
stance that do not specify it fully. After checking the inter-
preted solution against the original specification, that is if it
is a valid MAPF solution (agents do not collide, do not jump,
do not disappear, and do not appear spontaneously, etc.), the
high-level part of the MAPF solver suggests a refinement of
the encoding and the process is repeated. This scheme
has been implemented using SAT [Surynek, 2019] and MIP
[Gange et al., 2019; Lam et al., 2019] as target formalisms.

We briefly summarize in this paper how the research in compi-
ilation for MAPF came to this point and discuss in more
details opportunities and limitations of contemporary
best SAT-based and MIP-based compilation schemes.

2 An Overview of Target Formalisms

We will first give a brief overview of popular formalisms that
are often used as the target of the compilation process.

Constraint satisfaction problem (CSP) is a tuple \((X, D, C)\)
where \(X\) is a finite set of variables, \(D\) is a finite domain of
values that can be assigned to variables, and \(C\) is a set of
constraints in the form of arbitrary relations over the vari-
able, that is a constraint is a subset of Cartesian product
\(D \times D \times ... D\) (the arity corresponds to how many variables
participate in the constraint). A solution of CSP is an assign-
ment of values from \(D\) to all variables such that all constraints
are satisfied by this assignment (that is, constraints are treated
as a conjunction).

In the MAPF for example, we can express positions of
agents \(a_t\) at time step \(t\) using a variable \(X_i^t \in V\) (in-
dicating that the domain is \(V\)) [Ryan, 2010]. Then con-
straints expressing the MAPF rules such as: \(X_i^t \neq X_i^{t+1}\)
\(\rightarrow \{X_i^t, X_i^{t+1}\} \in E\), agents use edges (do not skip); and
\(\text{Distinct}(X_i^t, X_j^t, ..., X_k^t)\), agents do not collide in vertices,
etc. In addition to MAPF rules, constraints bounding the ob-
jectives are added to obtain encoding of the bounded MAPF.

The CSP paradigm provides powerful complete search
algorithms coupled with constraint propagation and do-
main filtering techniques that prune the search space
[Jussien et al., 2000]. One of the most significant advan-
tages of CSP in contrast to other paradigms are global con-
straints, dedicated filtering algorithms often based on match-
ings for special relations like \(\text{Distinct}\) (variables should
take different values) or more general cardinality con-
straints, that enforce consistency across large set of vari-
ables [Régin, 1994]. Using global constraints it is easy to
detect that \(\text{Distinct}(X_i^t, X_j^t, X_k^t)\) cannot be satisfied when
\(D = \{v_1, v_2\}\) (there is no matching between the variables
and values of size 3) but it is hard to see it looking on the
individual inequalities \(X_i^t \neq X_j^t\) separately.

Mixed integer linear programming (MIP). A linear pro-
gram (LP) is a finite list of linear inequalities plus linear ob-
jective, the task is to minimize the objective such that the in-
equalities hold. Geometrically the inequalities define a poly-
tope and the objective function defines a gradient so the task
is to find a some boundary point of the polytope that min-
imizes the gradient. Formally, the task is to minimize \(c^\top x\)
subject to \(Px \leq b, x \geq 0\), where \(x\) is a real vector represent-
ing the decision variables, \(P\) is a matrix of real coefficients
of linear inequalities, \(b\) is another real-valued vector. An
optimal solution to linear program can be found in polyno-
tial time however in discrete decision problems like MAPF
it not much convenient to use fractional assignment of deci-
sion variables.

That is why often some or all decision variables are de-
clared to be integers making LP an integer program (IP) or
mixed integer program (MIP) respectively. Introducing in-
teger decision variables improves the expressive power for
discrete problems but on the contrary in makes the problem
NP-hard.

The important advantage of MIP is natively supported
arithmetic hence optimizing with respect to various cumula-
tive objectives used in MAPF can be modeled easily when
MIP is used as the target formalism.

Boolean satisfiability (SAT) problem consists in deciding
whether there exists truth-value assignment of variables that
satisfies a given Boolean formula. The formula is often
specified using the conjunctive normal form (CNF), it is a
conjunction of clauses where each clause is a disjunction
of literals, a literal is either a variable or a negation of a
variable. SAT is often considered to be a canonical NP-
complete problem [Cook, 1971]. A formula in CNF can be
regarded through the CSP formalism so that clauses repre-
sent individual constraints. Such view actually catalyzes mu-
tual cross fertilization between solving algorithms for SAT
and CSP. Modern SAT solvers are based on the conflict-
driven clause learning algorithm [Eén and Sörensson, 2003;
Audemard and Simon, 2018] that actually implement con-
straint propagation and back-jumping techniques known from
CSP.

The significant challenge when SAT is used as the target
formalism is bridging the original representation of the prob-
lem and the yes/no environment of SAT. In the context of
MAPF, it is difficult to handle objectives that use arithmetic.

The disadvantage in the lack of expressiveness caused by
the fact that formulae are often far from being human read-
able, is well balanced by efficiency of SAT solvers enabled
by numerous techniques such as learning, restarts, etc.

Answer set programming (ASP). The difficulty of encoding
problems in the SAT formalism at the low-level led to de-
velopment of various high-level languages that enable encoding
of problems more intuitively. One such notable higher level
approach used in MAPF is ASP [Erdem et al., 2013]. The
problem in ASP is described as a logic program which is au-
automatically translated to a Boolean formula. The formula is solved by the SAT solver and the outcome is propagated back at the logic program level where it is already in the human readable form.

We show a part of the ASP program for MAPF to illustrate how the ASP paradigm works, see [Erdem et al., 2013] for the full program. We have atoms \(path(i, t, v)\) whose interpretation is that agent \(a_i \in A\) is at vertex \(v\) at time step \(t\). These atoms are defined recursively using the rules of a logic program:

\[
path(i, 0, v) \leftarrow \text{start}(i, v) \tag{1}
\]

\[
1 \{path(i, t + 1, u), path(i, t + 1, v) : \text{edge}(u, v)\} \leftarrow path(i, t, v) \tag{2}
\]

The program says that agents’ paths start at starting positions 1 and then either traverse edge or wait in a vertex 2. Ones around the rule specify the number of atoms selected for the answer of the rule, the lower and the upper bounds are specified. Here one of the two instances of \(path\) is selected.

ASP allows to specify constraints which are the rules without the head, eliminating answers that satisfy the body. Constraints can be used to eliminate conflicts for MAPF as follows:

\[
\leftarrow path(i, t, v), path(j, t, v) \tag{3}
\]

The ASP program is solved via reduction to SAT and by using the SAT solver. The important advantage of ASP in contrast to plain SAT-based approach is that ASP provides high level language to describe problems and one does not need to construct the target Boolean formula. The formula is constructed automatically by the ASP solver from the logic program.

3 From Classical Planning to MAPF

One of the pioneering works that used compilation for problem solving is the SATPlan algorithm reducing the classical planning problem to Boolean satisfiability [Kautz and Selman, 1992]. Classical planning is the task of finding a sequence of actions that transforms a given initial state of some abstract world to a desired goal state. States are for problem solving is the SATPlan algorithm reducing the classical planning problem to Boolean satisfiability [Kautz and Selman, 1992]. Classical planning is the task of finding a sequence of actions that transforms a given initial state of some abstract world to a desired goal state. States are constructed automatically by the ASP solver from the logic program.

3.1 Time Expansion

Another important concept significantly used by SATPlan is a time expansion that enables representing states at individual time steps inside the target formalism. Decision variables representing atoms are indexed with time steps up to \(t_{max}\); the variable indexed with \(0 < t \leq t_{max}\) is set to \(\text{TRUE}\) if and only if the corresponding atom holds at time step \(t\). In addition to state variables, there are action variables for each time step \(t\) indicating whether given action takes place at \(t\). Constraints ensure that if an action variable at \(t\) is set to \(\text{TRUE}\) then precondition atoms must hold in state variables at time step \(t\) and effects must hold in state variables at time step \(t + 1\).

Consider an action that moves agent \(a\) from vertex \(v_1\) to vertex \(v_2\): \(\text{precond}(\text{move}) = \{\text{at}(a, v_1)\}, \text{effect}^-(\text{move}) = \{\text{at}(a, v_2)\}, \text{effect}^+(\text{move}) = \{\text{at}(a, v_1)\}\) for which we need to introduce following constraints \(\forall t \in \{1, 2, ..., t_{max} - 1\}:

\[
\text{move}^t \rightarrow \text{at}(a, v_1)^t \land \text{at}(a, v_2)^{t+1} \land \lnot \text{at}(a, v_1)^{t+1} \tag{4}
\]

Such encoding can be easily constructed as well as read off in the interpretation phase. The pseudo-code of SATPlan is shown as Algorithm 1.

3.2 Encodings of MAPF

The early compilation-based approaches to MAPF were rooted in the CSP formalism [Ryan, 2010]. Even early SAT-based approaches used CSP-like variables for expressing the positions of agents with domains consisting all possible vertices [Surynek, 2012]. These decision variables were represented as a vector consisting of the logarithmic number of bits (Boolean variables) with respect to the size of the domain log-space encoding [Petke, 2015]. The disadvantage of log-space encodings in the context of MAPF seems to be weak Boolean constraint propagation and the fact that reachability analysis does not prune out variables from the encoding (removing values from decision variables corresponds to forbidding certain combination of settings of bit vectors but Boolean variables representing bits remain).

This is why all following SAT-based compilations of MAPF used direct encoding where there is a single Boolean variable \(X_{a_i}^{t, v}\) for each agent \(a_i \in A\), each vertex \(v \in V\), and relevant time step \(t\). Despite these decision variables are somewhat redundant they provide a good support for constraint propagation and in reachability analysis they can be completely removed from the formula.

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| Algorithm 1: Framework of the SATPlan algorithm. |
|--------------------------------------------------|
| 1. SATPlan\((P)\)                                  |
| 2. \(t_{max} \leftarrow 1\)                      |
| 3. while \(\text{TRUE}\) do                       |
| 4. \(F \leftarrow \text{encode-SAT}(P, t_{max})\) |
| 5. \(\text{assignment} \leftarrow \text{consult-SAT-Solver}(F)\) |
| 6. if \(\text{assignment} \neq \text{UNSAT}\) then |
| 7. \(\text{plan} \leftarrow \text{interpret}(P, \text{assignment})\) |
| 8. \(\text{return plan}\)                        |
| 9. \(t_{max} \leftarrow t_{max} + 1\)            |

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4 Milestones Making Compilation Competitive

Although modern SAT solvers are powerful search tools they alone are not sufficient and the way how the MAPF instance is presented to the SAT solver is equally important. From my personal perspective there were two milestones that made compilation for MAPF a competitive alternative to search-based methods: (1) reachability analysis and (2) introducing laziness.

4.1 Reachability Analysis

The disadvantage of SAT-based compilation in planning is often the big size of the formula because variables for all ground atoms and all ground actions must be included. That is why improvement such as reachability analysis has been introduced. Reachability analysis in classical planning is connected with planning graphs [Blum and Furst, 1997]. The planning graph is a structure representing time expansion of the set of possibly valid atoms across discrete time steps. Starting with the initial state at the first time step, atoms for the next time step are generated first by considering all applicable actions in the previous time step and by adding their positive effects together with atoms from the previous time step. To mitigate the growth of the set of atoms the concept of mutual exclusion (mutex) for pairs of atoms and actions are considered. Whenever two atoms are mutex they cannot satisfy the preconditions of an action and the action is hence pruned out. Mutexes originate at pairs of conflicting actions and propagate to atoms in their effects forward in the planning graph.

Reachability analysis analogous to planning-graphs in the domain of MAPF is done via multi-valued decision diagrams (MDDs) [Andersen et al., 2007]. MDDs were surprisingly not used in compilation-based approach for the first time but they were originally applied in the Increasing-cost tree search algorithm (ICTS) [Sharon et al., 2013]. The idea of MDD is to represent all paths of the specified cost. The disadvantage of SA T-based compilation in planning is often the big size of the formula because variables for all ground actions in vertices can be ruled out by the following constraint for every \( v \in V \) and timestep \( t \):

\[
\sum_{a_i \in A | (v,t) \in MDD_i} X^t_{i,v} \leq 1 \tag{5}
\]

There are various ways how to translate the constraint using propositional clauses. One possible way is to introduce \( \neg X^t_{i,v} \lor \neg X^t_{j,v} \) for all possible pairs of \( a_i \) and \( a_j \).

Following constraints ensure that directed paths are taken in MDDs. If agent \( a_i \) appears in \( u \in V \) at time step \( t \) then it has to leave through exactly one edge connected to \( u \):

\[
X^t_{i,u} \rightarrow \bigvee_{[(u,t);(v,t+1)] \in MDD_i} E^t_{i,u,v} \tag{6}
\]

\[
\sum_{(v,t+1) | [(u,j);(v,t+1)] \in MDD_j} E^t_{i,u,v} \leq 1 \tag{7}
\]

4.2 Introducing Laziness

Conflict-based search (CBS) [Sharon et al., 2015] is currently the most popular approach for MAPF. It is due to its simple and elegant idea which enables to implement the algorithm relatively easily, its good performance, and openness to various improvements via using heuristics.

From the compilation perspective, the CBS algorithm should be understood as a lazy method that tries to solve an underspecified problem and relies on to be lucky to find a correct solution even using this incomplete specification. There is another mechanism that ensures soundness of this lazy approach, the branching scheme. If the CBS algorithm is not lucky, that is, the candidate solution is incorrect in terms of MAPF rules, then the search branches for each possible refinement of discovered MAPF rule violation and the refinement is added to the problem specification in each branch. Concretely, the MAPF rule violations are conflicts of pairs of agents such as collision of \( a_i \in A \) and \( a_j \in A \) in \( v \) at time step \( t \) and the refinements are conflict avoidance constraints for single agents in the form that \( a_i \in A \) should avoid \( v \) at time step \( t \) (for \( a_j \) analogously).

While in CBS, the branching scheme and the refinements must be explicitly implemented, in the compilation-based approach we can just enforce MAPF rule violation by adding a new constraint into the problem specification and leave branching to the solver for the target formalism. In the SAT case, conflict can be avoided by adding following disjunctive constraints representing both branches in CBS:

\[
\neg X^t_{i,v}(a_i) \lor \neg X^t_{i,v}(a_j) \tag{8}
\]
In this way, the encoding of MAPF (or any other problem) is built dynamically and eventually may end up by the complete specification of the problem as done in MDD-SAT (or in SATPlan in the context of classical planning).

This approach is commonly known as lazy encoding and is often used in the context of satisfiability modulo theories (SMT) and also in ILP and MIP. In the context of SMT, we are interested in decision procedures for some complex logic theory T, that is decomposed into the Boolean part given to the SAT solver and decision procedure for the conjunctive fragment of T, denoted \( \text{DECIDE}_T \). The SAT solver and \( \text{DECIDE}_T \) cooperate in solving logic formula in T. SAT solver chooses literals, \( \text{DECIDE}_T \) verifies and suggests refinements. In this sense, CBS is analogous to this approach where the role of \( \text{DECIDE}_T \) is represented by conflict checking procedure.

Generally, SMT provides a view that starting constraints can be different than those defining correct paths and omitting conflicts. How do such different starting constraint perform is yet to be verified.

As shown by experiments a solution or a proof of that it does not exists is often found for incomplete specification that is, well before all constraints are added to the encoding. Surprisingly intuitive explanation why this is possible comes from the geometry of linear programming. The finite set of inequalities define a polytope of feasible solutions as intersection of half-spaces, the boundary of the polytope are made by planes. Optimal feasible solution is an element of some of the planes defining the boundary but not an element of all of them (in other words some inequalities are satisfied because optimal solution is deep inside their half-space). Hence, to specify the optimal solution one does not need all the constraints. Similarly if there is no solution, the polytope is empty. Again this may happen by intersecting only some of the half-spaces.

The CBS inspired SAT-based compilation algorithm for MAPF known as SMT-CBS is shown as Algorithm 2.

### Algorithm 2: MAPF solving via SAT compilation (SMT-CBS).

```plaintext
1 Lazy-SAT-MAPF(M)
2 SoC ← lower-Bound(M)
3 conflicts ← ∅
4 while TRUE do
5   (paths, conflicts) ← Solve-Bounded(M, SoC, conflicts)
6   if paths ≠ UNSAT then
7     return paths
8   SoC ← SoC + 1
9
10 Solve-Bounded(M, SoC, conflicts)
11 while TRUE do
12   F' ← encode-SAT(M, SoC, conflicts)
13   assignment ← consult-SAT-Solver(F')
14   if assignment ≠ UNSAT then
15     paths ← interpret(M, assignment)
16     conflicts' ← validate(M, paths)
17     if conflicts' = ∅ then
18       return (paths, conflicts)
19     for each c ∈ conflicts’ do
20       F' ← F' \ eliminate-Conflict(c)
21     conflicts ← conflicts \ conflicts'
22 return (UNSAT, conflicts)
```

The program represents an incomplete encoding of the input instance as collisions between agents are not forbidden initially. Collision resolution is done lazily similarly as in the SAT-based approach. However the important difference here is that the pool of candidate paths is extended together with adding collision avoidance constraints.

Once a vertex collision in \( v \in V \) is detected in current (fractional) solution \( \{ \lambda_{i,\pi} \} \), that is the following inequality holds for \( v \):

\[
\sum_{a_i \in A} \sum_{\pi \in \Pi(a_i)} s_v^\pi \lambda_{i,\pi} > 1,
\]

where \( s_v^\pi \in \{0, 1\} \) indicates selection of vertex \( v \) by path \( \pi \) (1 means the vertex is selected), corresponding collision elimination constraint for vertex \( v \) is included into the encoding: that is, the following constraint is added:

\[
\sum_{a_i \in A} \sum_{\pi \in \Pi(a_i)} s_v^\pi \lambda_{i,\pi} \leq 1,
\]

Now cost penalties are introduced for agents using vertex \( v \) where the collision happened and the pool of paths is extended with at least one better path for each agent considering the paths’ costs and the penalties. Compared to the SAT-based approach, the extension of the set of candidate paths is more general, more fine grained, and provides room to integrate heuristics that include more promising paths first. The SAT-based compilation increases the bound of the sum-of-costs (or other objective) instead, which allows considering more paths satisfying the bound in future iterations, but such
extension adds many new paths all at once without distinguishing if some of them are more promising than others.

On the other hand, the MIP-based approach compared to SAT-based compilation requires to deal with fractional solutions, which adds non-trivial complexity to the high level solving process. As shown in the framework of the MIP-based approach, Algorithm 3, elimination of fractionality of solutions requires to implement branch-and-bound search represented in the code by a non-deterministic choice (line 18). In contrast to this, all non-polynomial time effort is left to the SAT solver in the SAT-based approach.

Experiments show that the method performs well even in its vanilla variant. This can be attributed to the strength of the target MIP solver and to the design of decision variables that are particularly suitable for it.

6 SAT-based vs. MIP-based Compilation

As the pseudo-codes of the MIP-based and SAT-based compilation schemes suggest there are different opportunities how to enhance each approach with MAPF specific heuristics and pruning techniques such as symmetry breaking [Li et al., 2020a] or mutex reasoning [Zhang et al., 2020]. In this regard, MIP-based scheme is more open for integration of domain specific improvements as more decisions are made at the high-level, namely branching strategy where heuristics can be included, paths pool refinement that could further guide the search can be modified at the high level. Altogether, the MIP solver is treated more like a white-box.

In contrast to this, SAT-based approach still leaves lot of decisions on general purpose SAT-solver into which it is difficult to include any MAPF specific heuristics without changing the implementation of the solver. On the other hand, to role of the solver is bigger as it solves without any intervention from the high-level entire NP-hard component of the problem. In the MIP-based approach, the MIP solver solves independently a linear problem which is done in polynomial time while the hard exponential-time part is solved in cooperation with the high-level.

Significant weak point in SAT-based compilation is arithmetic and consequently difficult handling of the MAPF objectives in the target formalism. Bounding the objective in the yes/no environment of SAT must be done via modeling arithmetic circuits inside the formula which has twofold difficulties. The circuit is often large in the number of variables we need to bound and Boolean constraint propagation is usually less efficient across the circuit. MIP on the other hand can express common objectives used in MAPF as its own objective in straightforward way.

7 Conclusion

We summarized and compared main ideas of recent compilation-based approaches to MAPF, the MIP-based and SAT-based solvers. We also describe and analyze important milestones advancing the field to the current state-of-the-art. Our summary highlights important common features of both approaches such as lazy conflict elimination, but also focuses on significant differences such as the need in SAT to deal with cost bounds at the level of Boolean formula or the need in MIP to eliminate fractional values of decision variables.

As it is clear from aspects we focus on, we attribute great importance to design choices of how the compilation algorithms operate (e.g. eager vs. lazy style). In our perspective, algorithmic aspects of compilation approach is has more significant impact than concrete encoding of rules MAPF in the target formalism. More important is the design of solving algorithm that uses the encoding as its building block.

7.1 Future Prospects

The concept of laziness makes compilation flexible so it is applicable in various generalized variants of MAPF such as MAPF with continuous time [Andreychuk et al., 2019] as shown for SAT [Surynek, 2020]. Adapting the MIP approach for MAPF with continuous time is likely possible too.

Another room for future improvements comes from the fact that the solver both in the SAT-based and MIP-based compilation runs uninterrupted until the solution to the encoded instance is found or it is proved that there is no solution. This could be inefficient in case of the incomplete encodings since there could be a conflict induced by partial assignment of decision variables. Checking partial assign-
ments for conflicts could be the next step in compilation-based approaches. The framework for SAT is already known as DPLL(T) [Katz et al., 2016].

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