Research of an electromagnetic vibrator with a non-linear power supply

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Abstract. Electromagnetic vibration exciters (EMW) are widely used in various sectors of the national economy (chemical, mining, engineering, oil and gas) for transportation and simultaneous processing of bulk cargo, mixing and batching, grinding, separation, etc. Recently, more and more attention has been paid to the problems of their automation. This is caused by the fact that controlled vibration machines can significantly increase and stabilize at a given level of technological processes. Automation of EMVV can be solved only by developing automatic stabilization of performance with closed feedback. Rational design of this kind of sustainability needs theoretical justification. This article discusses the analysis of self-oscillations in electromagnetic exciters with a nonlinear power source used in vibration technology. To analyze the equations, the averaging method was used. Using this method allows obtaining periodic solutions for current and mechanical vibrations. The results of the analysis can be used in the development of self-oscillating electromagnetic exciters.

In instrumentation, electroacoustic, and vibration tests of materials, parts, and structures, self-oscillating systems that generate mechanical vibrations are used [1-4]. Rational design of such devices needs a theoretical justification.

In this work, we analyze an electromagnetic vibration exciter with a nonlinear power source, covered by speed feedback. The circuit diagram is presented in Figure 1. Here is shown a linear mechanical oscillatory circuit containing a concentrated mass (electromagnet armature), a spring with a stiffness coefficient, and a damper with a coefficient of friction. It also shows the electrical part of the system, consisting of an electromagnet winding with inductance and resistance. Between the mechanical and electrical circuits is an electrodynamic connection [5-7].

The amplifier is a nonlinear element; it controls the flow of energy into the oscillatory system. Feedback on the speed of oscillations is carried out through a sensor with a sensitivity coefficient. As independent coordinates describing the behavior of the system, the displacement of mass \( m \) from its equilibrium position and current to the coil were selected \( i_L \). The equations of oscillation of the system are expressed as follows:

\[
\begin{align*}
L \frac{di}{dt} + i_L \frac{dL}{dt} + R_L i_L &= R_i \\
\frac{d^2x}{dt^2} + \alpha \frac{dx}{dt} + cx &= \frac{i_L^2}{2} \cdot \frac{dL}{dx}
\end{align*}
\]  

(1)
The inductance of an electromagnet is a function of displacement \( x \) and for the case of shockless oscillations it can be approximated by a linear function of the form:

\[
L = L_0 + \rho x
\]  

(2)

where \( L_0 \) is the inductance of the de-energized winding; 
\( \rho \) - modulation coefficient of inductance.

Given that:

\[
i_R = i_N - i_L
\]

(3)

\[
i_N = i_0 + S_1 U_y - S_3 U_y^3
\]

(4)

\[
U_y = K \dot{x}
\]

(5)

where \( S_1 \) - the steepness of the characteristics of the current source at the operating point, \( S_3 = \frac{S_1}{3U_0^2} \); 
\( U_0 \) - saturation voltage;  
\( k \) - coefficient of sensitivity of the speed sensor, we get:

\[
\frac{i_0}{R} \frac{dl_i}{dt} + bi_L = i_0 + KS_1 \dot{x} - K^3 S_3 x^3 + \frac{\rho}{R} \left( x \frac{di_L}{dt} + \dot{x} i_L \right)
\]

(6)

moreover,

\[
b = 1 + \frac{R_1}{R}; \quad U_1 = \frac{bi_0}{R}\]

Denote by:

\[
Y = i_L - \frac{i_0}{b}; \quad K_1 = KS_1 - U_1; \quad K_3 = K^3 S_3; \quad U_x = \frac{i_0 \rho}{b}
\]

we get finally:

\[
\begin{aligned}
\frac{l_0}{R} \dot{y} + by &= K_1 \ddot{x} - K_3 x^3 - \frac{\rho}{R} (x \dot{y} + \dot{x} y) \\
m \dddot{x} + \alpha \ddot{x} + cx &= U_x y + \frac{\rho}{2} \left( y^2 + \frac{\dot{y}^2}{b^2} \right)
\end{aligned}
\]

(7)

Of the family of solutions to system (7), we are only interested in periodic ones. And the task will be reduced to determining the conditions of existence and stability of periodic solutions.
To solve the system of nonlinear equations (7), we use the approximate method [8-10]. For this purpose, we assume that, to a first approximation, the indicated system admits a self-oscillating solution close to harmonic

\[
x = A \cos \psi, \dot{x} = -A \omega_0 \sin \psi, \psi = \omega t + \varphi
\]  

where is the “instantaneous” frequency value.

We substitute the solution (8) into the first equation (7) and determine the law of change of the variable (point) corresponding to the oscillation (8) We obtain the equation:

\[
\frac{L_0}{R} \ddot{y} + \frac{b}{y} = -K_1 A \omega_0 \sin \psi + K_3 A^3 \omega_0^3 \sin^3 \psi - \frac{\rho}{R} (A' \cos \psi - A \omega_0 \omega_0 \sin \psi)
\]  

Limited to the solution \(y\) for the fundamental harmonic, i.e.

\[
y = -B \sin(\psi - \delta)
\]  

we get

\[
B = \frac{A \omega_0}{b} \left( K_1 - \frac{3}{4} K_3 \omega_0 \omega_2 \right) \cos \delta
\]

\[
\tan \delta = \frac{I_0 \omega_0^2}{RB}
\]

From the expression (10) it follows that with harmonic oscillations in the mechanical subsystem, the current in the excitation winding of the electromagnet also changes harmoniously. However, current oscillations occur with respect to mechanical oscillations with an angle delay [11-14].

Taking into account expression (10), the second equation of system (7) can be transformed to the form:

\[
\dot{x} + \omega_0^2 x = -\frac{\varepsilon}{m} = -[\alpha \dot{x} + U_Z B \sin(\psi - \delta) - \beta]
\]  

where is \(\omega_0^2 = \frac{c}{m}\), \(\varepsilon\) the small parameter, \(\beta = \frac{\rho}{2} (\frac{B^2}{2} + \frac{i^2}{b^2})\)

Using solution (8), we reduce (13) to the standard form [8]

\[
\begin{cases}
\frac{dA}{dt} = -\frac{\varepsilon}{m \omega_0} [\alpha A \omega_0 \sin \psi - U_Z B \sin(\psi - \delta) - \beta] \sin \psi \\
\frac{d\xi}{dt} = \omega_0 - \omega - \frac{\varepsilon}{m \omega_0} [\alpha \omega_0 \sin \psi - U_B \sin(\psi - \delta) - \beta] \cos \psi
\end{cases}
\]  

The equations for establishing the amplitude \(A\) and phase \(\xi\) of self-oscillations are determined by the equations of the first approximation:

\[
\begin{cases}
\frac{dA}{dt} = F(A) = -\frac{\varepsilon}{2m} \left[ \alpha A - \frac{U_z}{b} A (k_1 - \frac{3}{4} k_3 \omega_0^3 A^2 \cos^2 \delta) \right] \\
\frac{d\xi}{dt} = f(A) = \omega_0 - \omega - \frac{\varepsilon U_z}{4mb} \left( k_1 - \frac{3}{4} k_3 \omega_0^3 A^2 \right) \sin 2 \delta
\end{cases}
\]  

The last equations are obtained by averaging the right-hand sides of (14) over \(\psi\). The amplitude and frequency of stationary self-oscillations can be determined from equations (15), if we put in the latter:

\[
\frac{dA}{dt} = \frac{d\xi}{dt} = 0
\]

where from

\[
A_0 = \sqrt{\frac{K_1 U_Z \omega_0^2 \cos^2 \delta - ab}{\frac{2K_3 U_Z \omega_0^5 \cos^2 \delta}}}
\]
\[ \omega^2 = \omega_0^2 + \varepsilon \frac{\omega_0^2 \alpha}{m} \tan \delta = \omega_0^2 (1 + \varepsilon \frac{\alpha L_0}{m R R_B}) \tag{18} \]

When deriving the last relation, it was taken into account that \( 2\omega_0 \equiv \omega + \omega_0 \), a \( A = A_0 \). While Formulas (17) and (18) allow us to determine the amplitude and frequency of stationary self-oscillations at fixed values of the parameters. As can be seen from formula (18), the self-oscillation frequency is \( \omega_0 \), frequency of the mechanical subsystem, since the correction for the frequency is proportional to the magnitude of the order \( \varepsilon \).

The condition for self-excitation of vibrations is equivalent to the requirement of materiality of the radical expression in equation (17), i.e.

\[ K_1 U_2 \cos^2 \delta > \alpha b. \]

If we substitute the initial notation, then the condition of self-excitation is written in the form:

\[ i_0 \rho \left( K S_1 - \frac{\rho i_0}{R + R_1} \right) \cos^2 \delta > \alpha (1 + \frac{R_1}{R})^2 \tag{19} \]

From the obtained inequality it is clear that self-oscillations are possible, first of all, provided:

\[ K S_1 > \frac{\rho i_0}{R + R_1} \tag{20} \]

Expression (20) shows that self-excitation is the easier, the more powerful the feedback sensor, the greater the steepness of the nonlinear current source and the greater the resistance shunting the working winding. At the same time, it follows from inequality (19) that self-excitation improves with a decrease in the ohmic resistance of the working winding. Further, from inequality (19) it follows that self-oscillations are impossible for \( \delta \) values equal to \( \frac{\pi}{2}, \frac{3\pi}{2}, ..., \).

The latter case is excluded by the statement of the problem:

\[ R \neq 0. \]

These findings are reflected in figure 2, which shows the self-excitation regions and the change in their amplitude and frequency, depending on the parameter \( \delta \).

![Figure 2. Frequency response auto-oscillation from angle \( \delta \).](image)

The question of the stability of oscillations reduces to the fulfillment of the following feature [2,3,11].

\[ \left[ \frac{\partial F(A)}{\partial A} \right]_{A=A_0} < 0 \tag{21} \]
We calculate the derivative $\frac{dF}{dA}$, we obtain

$$-ab - U_2 \cos^2 \delta \left( k_1 - \frac{9}{4} k_3 A^2 \omega_0^2 \right) < 0 \quad (22)$$

If we substitute the stationary amplitude (17) in condition (22), then it turns out that it is satisfied for parameter values satisfying the stationarity conditions (16). In fact, this substitution leads to a trivial inequality

$$1 < 3 \quad (23)$$

So, the stability condition for stationary amplitude is always fulfilled.

Thus, the following conclusions can be drawn:

- In real schemes, the maximum amplitude of mechanical vibrations is most easily achieved at values of $\delta$. Such values of $\delta$ are conveniently maintained, for example, by a corresponding change in shunt resistance $R$.
- The effectiveness of increasing the amplitude of self-oscillations in the considered circuit is mainly determined by the quality of manufacturing feedback elements, the sensitivity of the speed sensor, and also the available power of the control system.

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