THE DYNAMICAL GENERATION OF CURRENT SHEETS IN ASTROPHYSICAL PLASMA TURBULENCE

GREGORY G. HOWES

Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52242, USA

Received 2016 June 27; revised 2016 July 18; accepted 2016 July 25; published 2016 August 16

ABSTRACT

Turbulence profoundly affects particle transport and plasma heating in many astrophysical plasma environments, from galaxy clusters to the solar corona and solar wind to Earth’s magnetosphere. Both fluid and kinetic simulations of plasma turbulence ubiquitously generate coherent structures, in the form of current sheets, at small scales, and the locations of these current sheets appear to be associated with enhanced rates of dissipation of the turbulent energy. Therefore, illuminating the origin and nature of these current sheets is critical to identifying the dominant physical mechanisms of dissipation, a primary aim at the forefront of plasma turbulence research. Here, we present evidence from nonlinear gyrokinetic simulations that strong nonlinear interactions between counterpropagating Alfvén waves, or strong Alfvén wave collisions, are a natural mechanism for the generation of current sheets in plasma turbulence. Furthermore, we conceptually explain this current sheet development in terms of the nonlinear dynamics of Alfvén wave collisions, showing that these current sheets arise through constructive interference among the initial Alfvén waves and nonlinearly generated modes. The properties of current sheets generated by strong Alfvén wave collisions are compared to published observations of current sheets in the Earth’s magnetosheath and the solar wind, and the nature of these current sheets leads to the expectation that Landau damping of the constituent Alfvén waves plays a dominant role in the damping of turbulently generated current sheets.

Key words: plasmas – solar wind – turbulence – waves

1. INTRODUCTION

The ubiquitous presence of turbulence impacts the evolution of many space and astrophysical plasma environments, mediating the transport of energy from violent events or instabilities at large scales down to the small scales at which the energy is ultimately converted to heat of the protons, electrons, and minor ions. It is widely believed that plasma turbulence plays an important role in heating the solar corona to millions of degrees Kelvin, accelerating the solar wind to hundreds of kilometers per second, regulating star formation, transporting heat in galaxy clusters, and affecting the injection of particles and energy into the Earth’s magnetosphere. At the forefront of plasma turbulence research is the effort to identify the physical mechanisms by which the turbulent fluctuations are damped and their energy converted to plasma heat or some other energization of particles.

In contrast to the intermittent filaments of vorticity that arise in hydrodynamic turbulence (She et al. 1990), intermittent current sheets are found to develop in plasma turbulence (Matthaeus & Montgomery 1980; Meneguzzi et al. 1981). Recent work investigating the statistics of these coherent structures, self-consistently generated by the plasma turbulence, has demonstrated that the dissipation of turbulent energy is largely concentrated in these current sheets (Uritsky et al. 2010; Osman et al. 2011; Zhdankin et al. 2013). Since current sheets are associated with enhanced dissipation, illuminating their origin and nature is critical to identifying the dominant physical mechanisms of dissipation in plasma turbulence.

How current sheets develop in plasma turbulence is a longstanding question in the study of space and astrophysical plasmas (Parker 1972; Pouquet 1978; Priest 1985; van Ballegooijen 1985; Antiochos 1987; Zweibel & Li 1987; Biskamp & Welter 1989; Longcope & Strauss 1994; Cowley et al. 1997; Spangler 1999; Biskamp & Müller 2000; Merrifield et al. 2005; Greco et al. 2008). Early work focused on the study of solar coronal loops, asking if the continuous motion of line-tied footpoints in ideal MHD would lead an initially smooth magnetic field to develop a tangential discontinuity (Parker 1972), necessarily supported by a sheet of finite current, according to Maxwell’s equations. It was argued that continuous footpoint motion cannot generate a discontinuous magnetic field (van Ballegooijen 1985; Antiochos 1987; Zweibel & Li 1987), but later shown that current layers of finite but arbitrarily small thickness were realizable through continuous footpoint motion (Longcope & Strauss 1994; Cowley et al. 1997). Of course, in the more complete kinetic plasma description, current layers generally have structure at both characteristic ion and electron length scales; kinetic simulations of plasma turbulence indeed observe the development of current sheets of finite thickness (Wan et al. 2012; Karimabadi et al. 2013; TenBarge & Howes 2013).

Recent spacecraft measurements of current sheets in the near-Earth solar wind have lead to fundamental questions about their origin and their influence on plasma heating. Do the measured current sheets represent advected flux tube boundaries (Borovsky 2008, 2010), or are they generated dynamically by the turbulence itself (Boldyrev et al. 2011; Zhdankin et al. 2012)? In the past few years, vigorous activity has focused on the spatial localization of plasma heating by the dissipation of turbulence in current sheets through statistical analyses of solar wind observations (Borovsky & Denton 2011; Osman et al. 2011, 2012, 2014; Perri et al. 2012; Wang et al. 2013; Wu et al. 2013) and numerical simulations (Wan et al. 2012; Karimabadi et al. 2013; TenBarge & Howes 2013; Wu et al. 2013; Zhdankin et al. 2013).

To understand the origin of coherent structures, we must investigate how the turbulent nonlinear interactions govern their development (Howes 2015). In plasma turbulence, the Alfvén wave represents the fundamental response of the plasma
to an applied perturbation. Early research on incompressible MHD turbulence in the 1960s (Iroshnikov 1963; Kraichnan 1965) emphasized the wave-like nature of turbulent plasma motions, suggesting that nonlinear interactions between counterpropagating Alfvén waves—or Alfvén wave collisions—mediate the turbulent cascade of energy from large to small scales. The Alfvén wave remains central to modern theories of MHD turbulence that provide explanations for the anisotropic nature of the turbulent cascade (Goldreich & Sridhar 1995) and the dynamic alignment of velocity and magnetic field fluctuations (Boldyrev 2006).

In this Letter, we demonstrate that the generation of current sheets in plasma turbulence is a natural consequence of strong Alfvén wave collisions. Furthermore, we present a first-principles explanation for this current sheet development in terms of the nonlinear dynamics, showing that the current sheet can be accurately reconstructed from a linear superposition of the interacting Alfvén waves and a surprisingly small number of nonlinearly generated modes. The properties of the resulting current sheet are compared to previously published observations of current sheets in turbulence in the Earth’s magnetosheath (Retinò et al. 2007; Sundkvist et al. 2007) and the solar wind (Perri et al. 2012). Finally, we discuss implications for the damping of current sheets generated by strong Alfvén wave collisions.

2. ALFVÉN WAVE COLLISIONS

Although the incompressible MHD equations lack a number of physical effects that occur in realistic space and astrophysical plasmas, they do contain the minimal ingredients that lead to the anisotropic cascade and development of current sheets in plasma turbulence. Expressed here in the symmetric Elsässer form, these equations are

\[
\frac{\partial z^\pm}{\partial t} + \mathbf{v}_A \cdot \nabla z^\pm = -\zeta^\pm \cdot \nabla z^\pm - \nabla P / \rho_0,
\]

and \( \nabla \cdot z^\pm = 0 \). Here, \( \mathbf{v}_A = \mathbf{B}_0 / \sqrt{4\pi \rho_0} \) is the Alfvén velocity due to the equilibrium field \( \mathbf{B}_0 = B_0 \hat{\mathbf{z}} \) where \( \mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B} \). \( P \) is total pressure (thermal plus magnetic), \( \rho_0 \) is mass density, and \( z^\pm = \mathbf{u} \pm \mathbf{B} / \sqrt{4\pi \rho_0} \) are the Elsässer fields which represent waves that propagate up or down the mean magnetic field. The nonlinear term, \( \zeta^\pm \cdot \nabla z^\pm \), governs the nonlinear interactions between counterpropagating Alfvén waves, or Alfvén wave collisions. The *nonlinearity parameter*, the ratio of the nonlinear to the linear term magnitudes in Equation (1), \( \chi \equiv |\zeta^\pm \cdot \nabla z^\pm| / |\mathbf{v}_A \cdot \nabla z^\pm| \), characterizes the strength of the nonlinearity. The limit of strong incompressible MHD turbulence occurs when \( \chi \sim 1 \), in which the nonlinear energy transfer timescale is comparable to the linear wave period, a condition known as critical balance (Goldreich & Sridhar 1995).

Following significant previous studies on weak incompressible MHD turbulence (Sridhar & Goldreich 1994; Ng & Bhattacharjee 1996; Galtier et al. 2000), the nonlinear energy transfer in Alfvén wave collisions has recently been solved analytically in the weakly nonlinear limit \( \chi \ll 1 \) (Howes & Nield 2013), confirmed numerically with gyrokinetic numerical simulations (Nield et al. 2013), and verified experimentally in the laboratory (Howes et al. 2012), establishing Alfvén wave collisions as the fundamental building block of astrophysical plasma turbulence. Here, we briefly review the details of the nonlinear energy transfer in the weak turbulence limit, \( \chi \ll 1 \).

Consider the nonlinear interaction between two perpendicularly polarized, counterpropagating Alfvén waves with wavevectors \( \mathbf{k}_1^\pm = k_1^\pm \hat{x} - k_2^\pm \hat{z} \) and \( \mathbf{k}_2^\pm = k_3^\pm \hat{y} + k_4^\pm \hat{z} \), as shown in Figure 1 (red circles), where \( k_1 \) and \( k_2 \) are positive constants. The lowest-order nonlinear interaction in the asymptotic solution creates an inherently nonlinear, purely magnetic mode with wavevector \( \mathbf{k}_3^{(0)} = k_3^0 \hat{x} + k_4^0 \hat{y} \) (green triangle). This secondary mode has \( k_2 = 0 \) and frequency \( \omega = 2k_1|\mathbf{v}_A| \), but does not grow secularly in time. At the next order, the primary modes then interact with this secondary \( k_2 = 0 \) mode to transfer energy secularly to two nonlinearly generated Alfvén waves (blue squares), where \( \mathbf{k}_1^+ \) transfers energy to an Alfvén wave with \( \mathbf{k}_1^+ = 2k_1^0 \hat{x} + k_3^0 \hat{y} - k_4^0 \hat{z} \), and \( \mathbf{k}_1^- \) transfers energy to \( \mathbf{k}_1^- = k_1^0 \hat{x} + 2k_3^0 \hat{y} + k_4^0 \hat{z} \). This process is the fundamental mechanism by which turbulence transfers energy anisotropically from large to small scales (Howes & Nield 2013).

3. STRONG ALFVÉN WAVE COLLISION SIMULATIONS

Although the analytical solution for the dynamics of Alfvén wave collisions is calculated in the MHD approximation, we employ here a gyrokinetic code for two reasons: (1) to demonstrate that the physical mechanism is not altered under the weakly collisional plasma conditions relevant to many space and astrophysical plasma environments and (2) to enable the direct comparison of current sheet profiles from our simulations to spacecraft measurements in the weakly collisional solar wind. We employ the Astrophysical Gyrokinetics code *AstroGK* (Numata et al. 2010) to perform a gyrokinetic simulation of the nonlinear interaction between two counterpropagating Alfvén waves in the strongly nonlinear limit, referred to as a *strong Alfvén wave collision*.

*AstroGK* evolves the perturbed gyroaveraged distribution function \( h_s(\mathbf{x}, \mathbf{y}, z, \mathbf{v}) \) for each species \( s \), the scalar potential \( \mathbf{\varphi} \), the parallel vector potential \( A|| \), and the parallel magnetic...
field perturbation $\delta B_i$ according to the gyrokinetic equation and the gyroaveraged Maxwell’s equations (Frieman & Chen 1982; Howes et al. 2006). Velocity space coordinates are $\lambda = v_i^2 / v^2$ and $\varepsilon = v^2 / 2$. The domain is a periodic box of size $L_x \times L_y \times L_z$, elongated along the straight, uniform mean magnetic field $B_0 = B_0 \hat{z}$, where all quantities may be rescaled to any parallel direction, $z$. Collisions employ a fully conservative, linearized collision operator with energy diffusion and pitch-angle scattering (Abel et al. 2008; Barnes et al. 2009).

To set up the simulation of an Alfven wave collision, following Nielsen et al. (2013), we initialize two perpendicularly polarized, counterpropagating plane Alfven waves, $z^+ = z_+ \cos (k_i x - k_i z - \omega_0 t) \hat{y}$ and $z^- = z_- \cos (k_i x + k_i z - \omega_0 t) \hat{x}$, where $\omega_0 = k_i \nu_A$, $k_\perp = 2\pi / L_\perp$, and $k_\parallel = 2\pi / L_\parallel$. We specify a balanced collision with equal counterpropagating wave amplitudes, $z^+ = z^-$, so the nonlinearity parameter is $\chi = k_\parallel z_+ / (k_i \nu_A)$. To study the nonlinear evolution in the limit $k_\parallel \rho_i \ll 1$, we choose a perpendicular simulation domain size $L_\perp = 40\pi \rho_i$ with simulation resolution $(n_x, n_y, n_\perp, n_z, n_t) = (64, 64, 64, 32, 16, 2)$. The fully resolved perpendicular range in this dealiased pseudospectral method covers $0.05 \leq k_\parallel \rho_i \leq 1.05$. Here, the ion thermal Larmor radius is $\rho_i = \nu_i / \Omega_i$, the ion thermal velocity is $v_\parallel^2 = 2T_i / m_i$, the ion cyclotron frequency is $\Omega_i = q_i B_0 / (m_i c)$, and the temperature is given in energy units. The plasma parameters, relevant to near-Earth solar wind conditions, are $\beta_i = 1$ and $T_i / T_e = 1$.

To demonstrate that the dominant physical pathway of nonlinear energy transfer, elucidated analytically in the weak turbulence limit $\chi \ll 1$ (Howes & Nielsen 2013), persists as the turbulence reaches the strong turbulence limit $\chi \rightarrow 1$, we compare the analytical prediction for the nonlinear evolution of the parallel vector potential for mode $k_\parallel$; $A_i(k_\parallel)$, to simulation results with increasingly strong nonlinearity. In Figure 2, we plot the real (blue) and imaginary (red) parts of the complex parallel vector potential $A_i(k_\parallel)$ for Alfven wave collision simulations with (a) $\chi = 1/4$, (b) $\chi = 1/2$, and (c) $\chi = 1$. Although the analytical solution (Howes & Nielsen 2013) is strictly valid only for weak nonlinearity $\chi \ll 1$, even for the moderately strong nonlinearity of $\chi = 1/2$, the solution remains relatively accurate; at $\chi = 1$, the solution ceases to be quantitatively correct (since the primary modes lose significant energy through the vigorous nonlinear energy transfer), but the general picture of the secular transfer of energy mediated by nonlinearly generated $k_\parallel = 0$ modes to nonlinearly produced daughter Alfven waves remains qualitatively correct.

4. CURRENT SHEET FORMATION

The primary numerical result reported here is the discovery that, as the amplitudes of the colliding Alfven waves increase to the strong turbulence limit, $\chi \rightarrow 1$, the transient development of a current sheet is a natural consequence. The current sheet resulting from a nonlinear gyrokinetic simulation of a strong Alfven wave collision with $\chi = 1$ is plotted in Figure 3(a), where the normalized parallel current $j_z / j_0$ (color bar) and positive (black) and negative (white) contours of the parallel vector potential, $A_i$, are plotted across a plane perpendicular to the equilibrium magnetic field at $z = -L_\perp / 4$ and $t = 1.30 T_A$, where the Alfven wave period is $T_A = L_\parallel / V_A$ and $j_0 = n_0 q_i v_i L_\parallel / L_\perp$.

The current sheet in the upper right quadrant may be characterized by its width $w$ and thickness $\delta$ in the perpendicular plane, its length along the equilibrium magnetic field $l$, and its lifetime $\tau$. The width $w$, thickness $\delta$, and length $l$ are determined using the FWHM extent of the current density in each of these directions. The lifetime $\tau$ is estimated as the time over which its peak current density exceeds half the global maximum in the simulation domain. We find width $w \approx 2L_\perp / 3 \approx 90\rho_i$ and thickness $\delta \approx 9\rho_i$, yielding an aspect ratio of $w / \delta \approx 10$. The parallel length is $l \approx L_\parallel$, and the total lifetime from the beginning of current sheet formation to the end of its decay is $\tau \approx 3T_A / 4$.

So, what controls the self-consistently generated current sheet’s lifetime and morphology? The length $l$ along the equilibrium magnetic field and its width $w$ in the perpendicular plane are determined by the parallel and perpendicular components of the wavelengths of the original interacting Alfven waves. The lifetime of this current sheet is related to the period of the original (large-scale) Alfven waves, a time much longer than the Alfven crossing time across the thickness $\delta$ of the current sheet. The thickness $\delta$ appears to approach the smallest resolvable perpendicular scale in the simulation. However, in this particular simulation, the thickness $\delta$ also happens to be approximately the scale $k_\parallel \rho_i \sim 1$ where the linear physics becomes dispersive, in other words the thickness is coincident with the perpendicular wavelength where the non-dispersive Alfven waves convert to dispersive kinetic Alfven waves. Further exploration is required to determine whether the current sheet thickness is bounded by the simulation resolution or by the decoupling of ions from the electromagnetic fluctuations at $k_\parallel \rho_i \sim 1$.\
Note that plasma turbulence simulations ubiquitously show the development of current sheets ([Matthaeus & Montgomery 1980; Meneguzzi et al. 1981; Wan et al. 2012; Karimabadi et al. 2013; TenBarge & Howes 2013; Wu et al. 2013; Zhdankin et al. 2013]), which sometimes appear to persist for a long time relative to other turbulent fluctuations on the scale of the thickness of the current sheet. This mechanism for current sheet generation may explain why these current sheets appear to persist for a long time because their lifetime is governed by the interaction of much larger-scale Alfvén waves. The period of those large-scale Alfvén waves is much longer than the period of the turbulent fluctuations on the scale of the current sheet thickness, resulting in a persistent current sheet.

5. PHYSICAL MECHANISM OF CURRENT SHEET DEVELOPMENT

The key conceptual result presented here is an explanation for this current sheet development in terms of the nonlinear dynamics of strong Alfvén wave collisions. In the weakly nonlinear limit, \( \chi \ll 1 \), the nonlinearly generated Alfvén waves \( k_+^2 \) are the dominant recipients of the energy transferred secularly to smaller scales, where the self-consistently generated \( k_0^{(0)} \) mode, which has \( k_z = 0 \), mediates the transfer. However, as the colliding Alfvén wave amplitudes increase to the strongly nonlinear limit, \( \chi \to 1 \), the asymptotic expansion of the equations of evolution ceases to be well-ordered, so higher-order terms—terms that can safely be neglected in the weakly nonlinear limit—begin to contribute significantly. Nonetheless, the dominant pathway of the nonlinear energy transfer mediated by self-consistently generated \( k_z = 0 \) modes, found in the weakly nonlinear limit, persists as one approaches and reaches the strongly nonlinear limit, \( \chi \to 1 \), as shown in Figure 2.

This persistence of the nonlinear energy transfer mechanism is due to the fact that the phases and amplitudes of all of the nonlinearly generated modes are determined by the mathematical form of the nonlinear term in Equation (1), even in the strongly nonlinear limit. If the nonlinearly generated modes rise to sufficient amplitudes, as occurs in the strongly nonlinear limit, they may constructively interfere with the primary Alfvén waves to create a coherent structure, in this case a current sheet.

A simple analogy for the development of a coherent structure by the interference of a sum of sinusoidal modes is the square wave, given by the infinite sum of Fourier modes,

\[ f(x) = \sin x + \sin(3x)/3 + \sin(5x)/5 + \cdots. \]

Current sheet development in plasma turbulence is similar, but involves the sum of Fourier modes spanning the two-dimensional plane perpendicular to the local mean magnetic field, as depicted in Figure 1. The fluctuations arising from the turbulent cascade, although they may appear random, in fact have phase and amplitude relationships determined by the nonlinear terms in the governing nonlinear equations of evolution. It is these nonlinearly determined phase and amplitude relationships that give rise to the coherent structures that arise from what may appear to be random turbulent fluctuations. A consequence of this insight is that the current sheet development can be predicted analytically, as is demonstrated below by a direct comparison between the analytical calculation and a nonlinear numerical simulation for the moderately nonlinear case of \( \chi = 1/2 \).
development is a result of interference between just five complex Fourier modes in the perpendicular plane, and can be predicted analytically from a rigorous first-principles calculation, supports the hypothesis that interference between the primary and nonlinearly generated Fourier modes is responsible for the development of current sheets in Alfvén wave collisions.

However, the qualitative properties of the nonlinear energy transfer imply a further major simplification of the nonlinear evolution. For the particular wavevectors specified for the two primary counterpropagating Alfvén waves, \( k_z^\parallel \), the mathematical form of the nonlinear term in Equation (1) fixes the value of \( k_z \) for all nonlinearly generated modes to be constant along lines of slope 1 (thin diagonal arrows) on the \((k_x, k_y)\) plane in Figure 1. Since the energy transfer is mediated by self-consistently generated \( k_z = 0 \) modes, energy is preferentially transferred to Fourier modes falling along the \( k_z = \pm k_y \), \( k_z = 0 \), and \( k_z = -k_y \) lines (Howes & Nielson 2013), so it is just these relatively few modes that govern the current sheet development. In Figures 3(b)–(f), we plot a successively increasing number of modes just along these three diagonal lines, filtering out all other modes from the \( \chi = 1 \) simulation: (b) just the two primary Alfvén waves \((k_x/k_z, k_y/k_z) = (0, 1)\) and \((1, 0)\); (c) adding the self-consistently generated \( k_z = 0 \) mode at \((1, 1)\) for 3 total modes; (d) adding \((2, 1), (2, 2),\) and \((1, 2)\) for 6 modes; (e) adding \((3, 2), (3, 3),\) and \((2, 3)\) for 9 modes; and (f) adding \((4, 3), (4, 4),\) and \((3, 4)\) for 12 total modes. The successive addition of these modes in this figure confirms that constructive interference among these modes leads to the observed current sheet structure. By comparing the results of the full simulation in (a) to the plot with just 12 modes in (f)—only 1.3% of the 945 possible complex Fourier modes in the simulation—the structure of the current sheet from the full simulation is quite accurately reproduced, showing only minor quantitative differences.

6. COMPARISON TO SPACECRAFT OBSERVATIONS

Do the current sheets generated by strong Alfvén wave collisions have similar properties to those measured in space plasmas? To address this important question, we sample the current sheet in our simulation along the trajectory given by the black line in Figure 3(a). In Figure 5, we plot the resulting
profiles of the normalized magnetic field $\hat{B}_i = (\delta B_i / B_0)(L_i / L_0)$ in both simulation $(x, y, z)$ and minimum variance $(L, M, N)$ coordinates, normalized current $\hat{j}_i = j_i / j_0$, normalized electric field $\hat{E}_i = (\delta E_i / \delta B_0)(L_i / L_0)$, and normalized $\hat{j} \cdot \hat{E}$. These profiles compare favorably with published measurements of the magnetic field variation in minimum variance coordinates and of $\hat{j} \cdot \hat{E}$ across current sheets from turbulence in the magnetosheath (Retinò et al. 2007; Sundkvist et al. 2007) and solar wind (Perri et al. 2012). This preliminary comparison motivates future efforts to make a more detailed statistical comparison of the properties of current sheets arising in strong Alfvén wave collisions with those measured by spacecraft missions.

7. CONCLUSION

The nonlinear dynamics of strong Alfvén wave collisions provides a natural explanation for the ubiquitous development of current sheets in plasma turbulence. The discovery that current sheets arise transiently through constructive interference among the primary waves and nonlinearly generated waves provides valuable insight into the physical mechanisms by which the turbulent fluctuations are damped. Because the dominant constructively interfering modes are $k_z = +k_\parallel$ ($k_z = -k_\parallel$) Alfvén waves that propagate in the $+\hat{z}$ ($-\hat{z}$) direction and $k_\parallel = 0$ modes nonlinearly generated by the interactions between these counterpropagating waves, collisionless damping of the constituent Alfvén waves via the Landau resonance with protons and electrons is expected to play an important role in the dissipation of the current sheets, as previously suggested (TenBarge & Howes 2013). Of course, other mechanisms exist that can produce current sheets, such as particular flow and magnetic field geometries, like the Orszag-Tang vortex. A final question that remains to be answered is whether such alternative mechanisms play any role in the development of current sheets in solar wind turbulence, or are strong Alfvén wave collisions sufficient to account for all current sheets observed in the turbulent solar wind?

This work was supported by NSF grant PHY-10033446, NSF CAREER Award AGS-1054061, and NASA grant NNX10AC91G. This work used the Extreme Science and Engineering Discovery Environment (XSEDE), which is supported by National Science Foundation grant number ACI-1053575.

REFERENCES

Abel, I. G., Barnes, M., Cowley, S. C., Dorland, W., & Schekochihin, A. A. 2008, PhPl, 15, 122309
Antiochos, S. K. 1987, ApJ, 312, 886
Barnes, M., et al. 2009, PhPl, 16, 072107
Biskamp, D., & Müller, W.-C. 2000, PhPl, 7, 4889
Biskamp, D., & Welter, H. 1989, PhFB, 1, 1964
Boldyrev, S. 2006, PhRvL, 96, 115002
Boldyrev, S., Perez, J. C., Borovsky, J. E., & Podesta, J. J. 2011, ApJL, 741, L19
Borovsky, J. E. 2008, JGR, 113, A08110
Borovsky, J. E., & Denton, M. H. 2011, ApJL, 739, L61
Cowley, S. C., Longcope, D. W., & Sudan, R. N. 1997, PhR, 283, 227
Friedman, E. A., & Chen, L. 1982, PhPl, 25, 502
Galitser, S., Nazarenko, S. V., Newell, A. C., & Pouquet, A. 2000, JPlPh, 63, 447
Goldreich, P., & Sridhar, S. 1995, ApJ, 438, 763
Greco, A., Chuychai, P., Mattheeus, W. H., Servidio, S., & Dmitruk, P. 2008, GeoRL, 35, 19111
Howes, G. G. 2015, RSP, 373, 20140145
Howes, G. G., Cowley, S. C., Dorland, W., et al. 2006, ApJ, 651, 590
Howes, G. G., Drake, D. J., Nielson, K. D., et al. 2012, PhRvL, 109, 255001
Howes, G. G., & Nielson, K. D. 2013, PhPl, 20, 072302
Iroshnikov, R. S. 1963, Astron. Zh., 40, 742; English Translation: Sov. Astron., 7, 566 (1964)
Karimabadi, H., et al. 2013, PhPl, 20, 012303
Kraichnan, R. H. 1965, PhFl, 8, 1385
Longcope, D. W., & Strauss, H. R. 1994, ApJ, 437, 851
Mattheeus, W. H., & Montgomery, D. 1980, NYASA, 357, 203
Meneguzzi, M., Frisch, U., & Pouquet, A. 1981, PhRvL, 47, 1060
Merrifield, J. A., Müller, W.-C., Chapman, S. C., & Dendy, R. O. 2005, PhPl, 12, 022301
Ng, C. S., & Bhattacharjee, A. 1996, ApJ, 465, 845
Nielson, K. D., Howes, G. G., & Dorland, W. 2013, PhPl, 20, 072303
Numata, R.,Howes, G. G.,Tatsuno, T., Barnes, M., & Dorland, W. 2010, JCoPh, 229, 9347
Osman, K. T., Mattheaus, W. H., Gosling, J. T., et al. 2014, PhRvL, 112, 215002
Osman, K. T., Mattheaus, W. H., Greco, A., & Servidio, S. 2011, ApJL, 727, L11+ 
Osman, K. T., Mattheaus, W. H., Wan, M., & Rappazzo, A. F. 2012, PhRvL, 108, 261102
Parker, E. N. 1972, ApJ, 174, 499
Perri, S., Goldstein, M. L., Dorelli, J. C., & Sahraoui, F. 2012, PhRvL, 109, 191101
Pouquet, A. 1978, JFM, 88, 1
Priest, E. R. 1985, RPPh, 48, 955
Retinò, A., Sundkvist, D., Vaivads, A., et al. 2007, NatPh, 3, 236
She, Z.-S., Jackson, E., & Orszag, S. A. 1990, Nat, 344, 226
Spangler, S. R. 1999, ApJ, 522, 879
Sridhar, S., & Goldreich, P. 1994, ApJ, 432, 612
Sundkvist, D., Retinò, A., Vaivads, A., & Bale, S. D. 2007, PhRvL, 99, 025004
TenBarge, J. M., & Howes, G. G. 2013, ApJL, 771, L27
Uritsky, V. M., Pouquet, A., Rosenberg, D., Mininni, P. D., & Donovan, E. F. 2010, PhRvE, 82, 056326
van Ballegooijen, A. A. 1985, ApJ, 298, 421
Wan, M., et al. 2012, PhRvL, 109, 195001
Wang, X., Tu, C., He, J., Marsch, E., & Wang, L. 2013, ApJL, 772, L14
Wu, P., et al. 2013, ApJL, 763, L30
Zhdankin, V., Boldyrev, S., Mason, J., & Perez, J. C. 2012, PhRvL, 108, 175004
Zhdankin, V., Uzdensky, D. A., Perez, J. C., & Boldyrev, S. 2013, ApJL, 771, 124
Zweibel, E. G., & Li, H.-S. 1987, ApJ, 312, 423