Radiation content of Conformally flat initial data

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We study the radiation of energy and linear momentum emitted to infinity by the headon collision of binary black holes, starting from rest at a finite initial separation, in the extreme mass ratio limit. For these configurations we identify the radiation produced by the initially conformally flat choice of the three geometry. This identification suggests that the radiated energy and momentum of headon collisions will not be dominated by the details of the initial data for evolution of holes from initial proper separations \( L_0 \geq 7M \). For non-headon orbits, where the amount of radiation is orders of magnitude larger, the conformally flat initial data may provide a relative even better approximation.

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I. INTRODUCTION

As we get closer to the first direct detection of gravitational waves, there is a renewed interest in the construction of astrophysically relevant initial data for binary black holes, a priori thought to be the most violent source of gravitational radiation in the Universe. The traditional choice for initial data had been the classical work of Bowen and York [1]. More recently, attention has been directed toward “thin-sandwich” [2, 3] and Kerr-Schild ansätze [4, 5] initial data, and comparisons with the Bowen-York choice have been carried out on the initial slices [6, 7, 8]. Post-Newtonian limit inspired data are also beginning to appear in the literature [3, 11].

The most important question about the choice of initial data is the effect of that choice on the gravitational waveform at infinity. Answering that question is very difficult if the binary black holes have comparable mass [11, 12, 13, 14, 15], but is relatively straightforward in the particle limit. For this reason, we have previously studied the headon collision of binary black holes in the extreme mass ratio regime [16, 17, 18]. This allowed us to compare ‘on slice’ initial data and truly evolved data. In [15, 18] we focused particular attention on the importance of the choice of the extrinsic curvature for the initial data, and we considered only the usual choice for the initial three metric: conformal flatness. In this report we turn to the question omitted from [17, 18], and we look briefly at the importance of conformal flatness.

We consider time symmetric initial data so that the natural choice for the extrinsic curvature is for it to vanish. We use the Brill-Lindquist type of conformally flat initial data. (We have shown in Ref. [17] that the total radiated energy is not very sensitive to this choice.) We evolve the time symmetric initial data, compute the radiated waveforms, and try to identify what part of the radiation can be ascribed to the conformally flat choice of the initial three metric. We do this differently for large and for small initial separation of the holes. In the case of large initial separation, we can identify an early feature of the waveform that is clearly produced by the initial data. For the case of small initial separation, we introduce a more speculative measure of the radiation ascribed to the conformally flat initial data; we consider it to be the excess radiation above the minimum of a one-parameter family of initial data choices.

II. RESULTS

We consider a particle of mass \( m_0 \), to be a first-order perturbation on the background of a Schwarzschild hole of mass \( M \). The particle is initially at rest at a proper distance \( L_0 \) from the horizon of the Schwarzschild hole. For the radial infall of the particle, the odd parity perturbations of the spacetime vanish. For the even parity perturbations we use the formulae and numerical techniques described in [17] to integrate the Moncrief-Zerilli equation

\[
\left[ \partial_r^2 - \partial_t^2 - V(r) \right] \psi_\ell = \mathcal{E}_\ell(r_\nu(t), r),
\]

for the even parity wave function \( \psi_\ell(t, r) \) for each \( \ell \)-pole mode of the field.

A. radiated energies

The radiated even-parity energy at infinity is computed from

\[
\frac{dE}{dt} = \lim_{r \to \infty} \frac{1}{64\pi} \sum_{\ell=2}^{\infty} \frac{(\ell + 2)}{(\ell - 2)!} (\partial_t \psi_\ell(t, r))^2.
\]

The computed energy \( E_\ell \) for \( \ell = 2 \ldots 5 \), reported in Table I and Fig. I show the expected exponential decrease of energy with increasing \( \ell \) [16, 19]. The plot also displays an unexpected feature: For small separations there is a rise in the radiated energy. The relative importance of this feature grows with increasing \( \ell \), and for \( \ell \geq 3 \) the feature is so strong that radiation from small \( L_0/M \) is greater than that for infall from infinity.
This increase in $E_t$ for small $L_0/M$ is certainly related to the choice of initial data. As a first step in understanding this relationship, in Fig. 2 we display $\partial \psi_2/\partial t$, the quantity that dominates the energy in Table 1 and Fig. 1. The waveform of $\partial \psi_2/\partial t$, for an observer at $r^*/2M = 100$, shows the characteristic excitation and ringdown of quasinormal oscillations. But at early times, $t/2M \sim 100$, a small feature appears representing the evolution of the initial disturbance of the Schwarzschild background around the particle position. We have isolated this small feature and have computed its contribution $\delta E_2$ to the quadrupole radiated energy $E_2$. The values of $\delta E_2$ are listed in Table 1 and the circles in Fig. 3 show the fractional energy $\delta E_2/E_2$ as a function of the initial separation. The circles in Fig. 3 show a marked increase in the fractional energy as $L_0/M$ decreases, but this method of ascribing energy to the initial data can only be extended down to $L_0/M \sim 10$. For $L_0/M < 10$ the small early feature in the waveform cannot be cleanly distinguished from the initial excitation of ringing. (The error in $\delta E_2$ reaches several percent for the values tabulated in Table 1.)

One can make a speculative estimate of the small-$L_0$ initial-value energy by extrapolation. A best fit of the form $\delta E_2/E_2 = a(L_0/M)^b$ requires $a = 7.26$ and $b = -2.135$. This fitted curve reaches $\delta E_2/E_2 \approx 0.5$ for $L_0/M \approx 3.5$.

There is a completely different way to estimate the energy associated with the conformally flat initial data. To do this, we follow the approach of Martel and Poisson [20]. In the Regge-Wheeler [21] gauge for the multipole perturbations of Schwarzschild spacetime, the only nonvanishing even parity perturbations on the $t = $ constant initial hypersurface are $H_2^{l=2}$ and $K^{l=2}$. The initial value equations contain a free functional degree of freedom in $H_2^{l=2}$ and $K^{l=2}$. The usual choice, $H_2^{l=2} = K^{l=2}$, corresponds to a conformally flat perturbed three metric. Here, instead, we consider the one parameter family

$$H_2' = \alpha K^l. \quad (2.3)$$

In Ref. [20], the energy radiated has been studied as a function of $\alpha$, and the value of $\alpha$ has been found for which the radiated energy is minimum. This turns out not to be the conformally flat choice $\alpha = 1$. (See Fig. 12 in Ref. [20].) For any value of $L_0/M$, we take the $\alpha$-model of Eq. (2.3) with the minimum radiated energy as

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig1.png}
\caption{The contribution to the radiated energy at infinity of the first four multipoles, as a function of the initial location of the particle falling from rest.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig2.png}
\caption{Waveforms of $\partial \psi_2/\partial t$ at $r^*/2M = 100$. Note the small feature located around $t/2M = 100$, associated with the initial data radiation content.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig3.png}
\caption{The estimated fraction of radiated $l = 2$ energy that is due to the choice of conformally flat initial data. The circles, for $L_0/M > 9.56$ represent fractional energy in the early feature in the waveform. The squares, for $L_0/M \leq 8$, are based on the energy in excess of a baseline model of initial data that is not conformally flat.}
\end{figure}
the “baseline” model. We subtract the radiated energy for this baseline model from the radiated energy for a conformally flat model and consider the excess energy to be an artifact of the conformally flat initial data, radiation energy that is, in a sense, contained within the initial data. This excess energy, for \( \ell = 2 \) and \( L_0/M \leq 8 \), is included in Fig. 3. It may be interesting that the maximum of this excess energy is located near a proper separation \( L_0/M \approx 4.5 \), in rough agreement with the previous extrapolation.

## III. DISCUSSION

To assess the importance of the choice of the initial three geometry on the computation of radiation from black hole mergers, we have used the very simplest model: the extreme mass limit, a nonrotating hole, and radial

### TABLE I: Energy radiated in units of \( m_0^2/2M \)

| \( L_0/M \) | \( E_0/2 \) | \( E_3/2 \) | \( E_4/2 \) | \( E_5/2 \) | \( E_{\text{Total}}/2 \) | \( \delta E_2/2 \) |
|---|---|---|---|---|---|---|
| 104.3 | 0.0179 | 0.00214 | 0.000318 | 9.69E-05 | 0.0205 | 0.4922e-4 |
| 63.8 | 0.0174 | 0.00207 | 0.000298 | 5.61E-05 | 0.0198 | 0.3695e-3 |
| 33.0 | 0.0164 | 0.00194 | 0.000262 | 4.20E-05 | 0.0187 | 0.3062e-2 |
| 22.6 | 0.0157 | 0.00185 | 0.000257 | 5.31E-05 | 0.0179 | 0.7707e-2 |
| 17.3 | 0.0151 | 0.00176 | 0.000245 | 5.30E-05 | 0.0172 | 0.1536e-1 |
| 14.0 | 0.0147 | 0.00169 | 0.000234 | 4.92E-05 | 0.0167 | 0.2557e-1 |
| 12.9 | 0.0145 | 0.00166 | 0.000237 | 5.28E-05 | 0.0165 | 0.3344e-1 |
| 11.8 | 0.0143 | 0.00165 | 0.000234 | 5.28E-05 | 0.0162 | 0.3811e-1 |
| 10.7 | 0.0142 | 0.00162 | 0.000232 | 5.39E-05 | 0.0161 | 0.4549e-1 |
| 9.56 | 0.0141 | 0.00160 | 0.000227 | 5.99E-05 | 0.0160 | 0.5730e-1 |
| 7.19 | 0.0140 | 0.00170 | 0.000227 | 8.11E-05 | 0.0159 | 0.140 |
| 5.94 | 0.0154 | 0.00164 | 0.000336 | 0.000105 | 0.0174 | 0.362 |
| 5.28 | 0.0157 | 0.00181 | 0.000350 | 0.000135 | 0.0180 | 0.465 |
| 4.59 | 0.0150 | 0.00223 | 0.000375 | 0.000145 | 0.0177 | 0.509 |
| 4.23 | 0.0140 | 0.00244 | 0.000429 | 0.000144 | 0.0170 | 0.494 |
| 3.86 | 0.0125 | 0.00256 | 0.000505 | 0.000150 | 0.0157 | 0.433 |
| 3.05 | 0.00824 | 0.00219 | 0.000592 | 0.000189 | 0.0112 | 0.349 |
| 2.30 | 0.00424 | 0.00122 | 0.000383 | 0.000140 | 0.00595 | 0.354 |
| 1.29 | 0.00095 | 0.00020 | 5.24E-05 | 1.64E-05 | 0.00121 | —— |

### FIG. 4: Recoil velocity of the system for different initial proper separations of the holes. The dashed line indicates an estimate of recoil velocity for a baseline model of initial data that is not conformally flat.

\[
\frac{dP_z}{dt} = \lim_{r \to \infty} \frac{1}{32\pi} \sum_{\ell=2}^{\infty} \frac{(\ell + 3)!}{(\ell - 2)!} \frac{\partial \psi_{\ell} \partial \psi_{\ell+1}}{\sqrt{(2\ell + 1)(2\ell + 3)}}.
\]
TABLE II: Linear momentum radiated

| $r_0/M$ | $L_0/M$ | $(M/m_0)^2 c \text{ km/s}$ |
|--------|---------|-------------------------|
| 100    | 104.3   | 249                     |
| 60     | 63.8    | 248                     |
| 30     | 33.0    | 237                     |
| 20     | 22.6    | 230                     |
| 15     | 17.3    | 222                     |
| 12     | 14.0    | 218                     |
| 11     | 12.9    | 217                     |
| 10     | 11.8    | 216                     |
| 9      | 10.7    | 219                     |
| 8      | 9.56    | 221                     |
| 6      | 7.19    | 257                     |
| 5      | 5.94    | 320                     |
| 4.5    | 5.28    | 352                     |
| 4      | 4.59    | 372                     |
| 3.75   | 4.23    | 371                     |
| 3.5    | 3.86    | 358                     |
| 3      | 3.05    | 271                     |
| 2.6    | 2.30    | 144                     |
| 2.2    | 1.29    | 24                      |

...infall from rest. Our results suggest that for proper distance $L_0$ larger than around $7M$ the energy and momentum radiated are probably not significantly contaminated by the initial choice. In the astrophysically more interesting case of inspiralling binary black holes in quasicircular orbits, we expect the radiation will be affected much more by the choice of the extrinsic curvature, than the choice of the three metric. In that case, the conformally flat choice of the three-geometry seems adequate for most of the applications. A study of the initial data choices for quasicircular orbits in the small mass ratio limit is currently underway by the authors.

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[1] J. Bowen and J. W. York, Phys. Rev. D 21, 2047 (1980).
[2] E. Gourgoulhon, P. Grandclement, and S. Bonazzola, Phys. Rev. D 65, 044020 (2002), gr-qc/0106015.
[3] M. D. Hannam, C. R. Evans, G. B. Cook, and T. W. Baumgarte (2003), gr-qc/0306028.
[4] P. Marronetti and R. A. Matzner, Phys. Rev. Lett. 85, 5500 (2000), gr-qc/0009044.
[5] N. T. Bishop, R. Isaacson, M. Maharaj, and J. Winicour, Phys. Rev. D 57, 6113 (1998), gr-qc/9711076.
[6] H. P. Pfeiffer, G. B. Cook, and S. A. Teukolsky, Phys. Rev. D 66, 024047 (2002), gr-qc/0203085.
[7] W. Tichy, B. Brügmann, and P. Laguna, Phys. Rev. D 68, 064008 (2003), gr-qc/0306020.
[8] P. Laguna (2003), gr-qc/0310073.
[9] W. Tichy, B. Brügmann, M. Campanelli, and P. Diener, Phys. Rev. D 67, 064008 (2003), gr-qc/0207011.
[10] L. Blanchet (2003), gr-qc/0304080.
[11] J. Baker, B. Brügmann, M. Campanelli, and C. O. Lousto, Class. Quantum Grav. 17, L149 (2000).
[12] J. Baker, M. Campanelli, and C. O. Lousto, Phys. Rev. D 65, 044001 (2002), gr-qc/0104063.
[13] J. Baker, B. Brügmann, M. Campanelli, C. O. Lousto, and R. Takahashi, Phys. Rev. Lett. 87, 121103 (2001), gr-qc/0102037.
[14] J. Baker, M. Campanelli, C. O. Lousto, and R. Takahashi, Phys. Rev. D 65, 124012 (2002), astro-ph/0202469.
[15] J. Baker, M. Campanelli, C. O. Lousto, and R. Takahashi (2003), astro-ph/0305287.
[16] C. O. Lousto and R. H. Price, Phys. Rev. D 55, 2124 (1997), gr-qc/9609012.
[17] C. O. Lousto and R. H. Price, Phys. Rev. D 56, 6439 (1997), gr-qc/9705071.
[18] C. O. Lousto and R. H. Price, Phys. Rev. D 57, 1073 (1998), gr-qc/9708022.
[19] M. Davis, R. Ruffini, H. Press, and R. H. Price, Phys. Rev. Lett. 27, 1466 (1971).
[20] K. Martel and E. Poisson, Phys. Rev. D 66, 084001 (2002), gr-qc/0107104.
[21] T. Regge and J. Wheeler, Phys. Rev. 108, 1063 (1957).
[22] T. Nakamura and M. P. Haugan, Astrophys. J. 269, 292 (1983).
[23] Y. Kojima and T. Nakamura, Prog. Theor. Phys. 72, 494 (1984).
[24] Z. Andrade and R. H. Price, Phys. Rev. D 56, 6336 (1997), gr-qc/9611022.
[25] P. Anninos and S. Brandt, Phys. Rev. Lett. 81, 508 (1998), gr-qc/9806031.
[26] V. Moncrief, Astrophys. J. 238, 333 (1980).