Towards the global fit of the TMD gluon density in the proton from the LHC data

A.V. Lipatov\textsuperscript{1,2}, G.I. Lykasov\textsuperscript{2}, M.A. Malyshev\textsuperscript{1}

November 8, 2022

\textsuperscript{1}Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, 119991, Moscow, Russia
\textsuperscript{2}Joint Institute for Nuclear Research, 141980, Dubna, Moscow region, Russia

Abstract

We propose a new analytical expression for the Transverse Momentum Dependent (TMD, or unintegrated) gluon density in the proton. Essential phenomenological parameters are extracted from the LHC data on inclusive hadron production in \(pp\) collisions at low transverse momenta, \(p_T \sim 1 \text{ GeV}\). The latter are described in the framework of modified soft quark-gluon string model, where gluonic state and non-zero transverse momentum of partons inside the proton are taken into account. To determine the parameters important at moderate and large \(x\) we used measurements of inclusive \(b\)-jet and Higgs boson production at the LHC as well as latest HERA data on proton structure functions \(F_2^c(x, Q^2)\) and \(F_2^b(x, Q^2)\) and reduced cross sections \(\sigma_{\text{red}}^c(x, Q^2)\) and \(\sigma_{\text{red}}^b(x, Q^2)\). The Catani-Ciafaloni-Fiorani-Marchesini evolution equation is applied to extend the initial gluon distribution into the whole kinematical region. We have achieved simultaneous description of all considered processes with \(\chi^2/d.o.f. = 2.2\), thus moving forward to the global fit of TMD gluon density from collider data. The obtained TMD gluon distribution in a proton is available for public usage and implemented into the TMDLIB package and Monte-Carlo event generator PEGASUS.

Keywords: small-\(x\) physics, high-energy factorization, CCFM evolution, TMD gluon density in the proton
1 Introduction

It is known that theoretical description of any physical observables, measured in the collider experiments, is mainly based on different factorization theorems in Quantum Chromodynamics (QCD). These theorems provide the necessary framework to separate hard partonic physics, described with the perturbative QCD expansion, from soft hadronic physics, described in terms of parton density functions (PDFs). The latter contain information on the non-perturbative structure of a hadron (proton). The most popular framework is provided by the conventional (so-called collinear) QCD factorization. In this approach, gluon and quark densities depend only on the longitudinal momentum fraction \( x \) of the proton momentum carried by a parton. An appropriate QCD evolution describing the dependence of PDFs on the resolution scale \( \mu^2 \) is given by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [1]. Such formalism is usually successful for sufficiently inclusive processes, like deep-inelastic lepton-hadron scattering (DIS), if a few higher-order terms in perturbative QCD expansion are taken into account.

However, in order to describe less inclusive processes proceeding at high energies with large momentum transfer and/or containing multiple hard scales the Transverse Momentum Dependent (TMD, or unintegrated) parton densities \( f_a(x, k_T^2, \mu^2) \) with \( a = q, g \) are required (for more information see, for example, review [2] and references therein). These quantities encode additional transverse momentum and polarization degrees of freedom and satisfy the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [3] or Catani-Ciafaloni-Fiorani-Marchesini (CCFM) [4] evolution equations. In this way one can effectively resum large logarithmic terms proportional to \( \alpha_s^n \ln^s \frac{s}{\Lambda_{\text{QCD}}^2} \sim \alpha_s^n \ln^s \frac{1}{x} \) which are expected to become equally (or even more) important in comparison with conventional DGLAP contributions proportional to \( \alpha_s^n \ln^s \frac{\mu^2}{\Lambda_{\text{QCD}}^2} \). Such high-energy factorization [5], or \( k_T \)-factorization formalism was formulated and it is becoming a widely exploited tool in high energy physics. A certain advantage of this approach is that one can quite easily take into account a large piece of higher-order pQCD corrections into the calculations. Several Monte-Carlo event generators based on the \( k_T \)-factorization formalism, like as CASCADe [7], KATIE [8] and PEGASUS [9] are developed and a number of corresponding phenomenological applications is known in the literature. Thus, the \( k_T \)-factorization approach becomes an essential tool which allows one to make theoretical predictions for future experiments at modern (LHC) and future (FCC, EiC, EicC) colliders.

In this sense, a special interest is connected with the selection of the TMD parton density in the proton best suited to describe the currently available collider data and which, therefore, can be used to generate the necessary realistic predictions. However, in contrast with a great amount of our knowledge about the conventional PDFs accumulated in theoretical and experimental studies over past years, the TMD parton densities are still poorly known quantities. There are some popular approaches to evaluate the latter, for example, the Kimber-Martin-Ryskin (KMR) prescription [10,11], CCFM-based formalism [12] and Parton Branching (PB) approach [13,14]. Variety of currently available TMD sets are collected in the TMDLIB package [15], which is a C++ library providing a framework and an interface to the different parameterizations.

It is known that an important role in derivation of TMD parton densities in a proton plays the appropriate choice of the non-perturbative input \( f_a^{(0)}(x, k_T^2, \mu_0^2) \), which is used as the initial condition for subsequent QCD evolution [16,19]. In fact, its influence on the description of experimental data can be significant [20,23]. Similar to collinear PDFs, starting TMD parton distributions are usually parameterized in a rather general form (see Section 2 below) and then fitted to some experimental data. Such procedures were carried out for the CCFM [12] and PB [24] approaches with xFITTER tool [25], where
latest precision HERA measurements of the proton structure function $F_2(x, Q^2)$ were used. In contrast, in our previous studies \cite{20,23} the modified soft quark-gluon string model (QGSM) \cite{26,27} was applied to determine parameters of an analytical expression for the starting TMD gluon density in a proton, $f_g^{(0)}(x, k_T^2, \mu_0^2)$. In the modified QGSM both the longitudinal and transverse motion of quarks and gluons \cite{28,29} as well as the saturation effects at small $x$ and low scales can be taken into account. The essential phenomenological parameters were obtained from the best description of RHIC and LHC data on the inclusive spectra of hadrons produced in $pp$ and $AA$ collisions at low transverse momenta, and the CCFM evolution equation was applied to extend the proposed TMD gluon density in the whole kinematical region. It was shown that such an approach is able to describe HERA data on proton structure functions $F_2^c(x, Q^2)$, $F_2^g(x, Q^2)$ and $F_L(x, Q^2)$ and LHC data on several processes, in particular, single top production and inclusive Higgs boson production at $\sqrt{s} = 8$ and 13 TeV.

In the present paper we continue our study and recalculate $f_g^{(0)}(x, k_T^2, \mu_0^2)$ more accurately using the modified QGSM. Moreover, we determine the parameters of the initial gluon density using the LHC data on soft hadron (kaon and pion), inclusive $b$-jet and Higgs boson production in $pp$ collisions at different energies as well as latest HERA data on proton structure functions $F_2^c(x, Q^2)$ and $F_2^g(x, Q^2)$ and reduced cross sections $\sigma_{\text{red}}^c(x, Q^2)$ and $\sigma_{\text{red}}^b(x, Q^2)$. Thus, we perform a step forward to the global fit of TMD gluon density from the collider data, that significantly improves our earlier analyses \cite{20,23}.

The paper is organized as follows. In Section 2 we shortly describe our theoretical input and discuss the basic steps of calculations of soft hadron spectra in the modified QGSM. Then, from best description of the LHC data on the latter we derive an updated analytical expression for the initial TMD gluon density in a proton. In Section 3 we perform a fit of several phenomenological parameters from the LHC and HERA data and compare our results with the known ones. We give conclusions in Section 4.

2 The model

Similar to conventional PDFs, a construction of the TMD parton distributions in a proton starts from the input densities, which are further used as the initial conditions for subsequent non-collinear QCD evolution. As it was mentioned above, usually the initial TMD gluon density at some starting scale $\mu_0^2$ (which is of order of hadron scale) is taken in the rather general empirical form with factorized Gauss smearing in transverse momentum $k_T^2$ (see, for example, \cite{12}):

$$f_g^{(0)}(x, k_T^2, \mu_0^2) = a_1 x^{a_2}(1 - x)^{a_3} e^{-k_T^2/\sigma_0^2},$$  \hspace{1cm} (1)

where all the parameters have to be extracted from the experimental data. Alternatively, a more physically motivated non-factorized expression for $f_g^{(0)}(x, k_T^2, \mu_0^2)$ can be taken from the Golec-Biernat-Wüsthoff (GBW) approach \cite{30,31} based on color dipole picture for deep inelastic scattering (DIS):

$$f_g^{(0)}(x, k_T^2, \mu_0^2) = c_g R_0^2(x) k_T^2 e^{-R_0^2(x) k_T^2}, \quad R_0(x) = \frac{1}{Q_0} \left( \frac{x}{x_0} \right)^{\lambda/2},$$  \hspace{1cm} (2)

where $c_g = 3 \sigma_0/(4\pi^2\alpha_s)$, $\sigma_0 = 29.12$ mb, $\alpha_s = 0.2$, $Q_0 = 1$ GeV, $x_0 = 4.1 \cdot 10^{-5}$ and $\lambda = 0.277$. In this approach, the effect of saturation of $q\bar{q}$ dipole cross section at large distance $r$ between quark and anti-quark in the dipole or small $\mu$ is assumed. This saturation of the dipole cross-section is a direct consequence of the saturation of the
cross-section of virtual photon-proton scattering \( (\gamma^* p) \) \( [30] \). It leads to scale-independent behavior of the TMD gluon density \( f_g^{(0)}(x, k_T^2, \mu^2_0) \) at \( \mu < \mu_{\text{sat}} \), where \( \mu_{\text{sat}} \) is the saturation scale. The GBW model was successfully applied to both inclusive and diffractive \( ep \) scattering at HERA. However, it meets some difficulties in accurate description of several hard LHC processes. In our previous studies \( [20–22] \) to describe successfully LHC data on soft hadron production in \( pp \) collisions we modified the starting form of the gluon density \( [2] \) as the following:

\[
f_g^{(0)}(x, k_T^2, \mu^2_0) = c_0 c_1 (1 - x)^b \times \left[ R_0^2(x) R_0^2(x) k_T^2 + c_2 \left( R_0^2(x) k_T^2 \right)^{a/2} \right] \exp \left( -R_0(x) |k_T| - d \left[ R_0^2(x) k_T^2 \right]^{3/2} \right).
\] (3)

Then, to extend the consideration to a region of larger \( p_T \) we added to (3) some function dependent on \( k_T \) and low \( x \), which was matched with the exact solution \( [32] \) of the BFKL equation outside of the saturation region. As it was mentioned above, in this way one could achieve reasonably well description \( [23] \) of some HERA and LHC data using the proposed analytical expression (non-factorized with respect to \( x \) and \( k_T^2 \)) for the initial TMD gluon density with parameters obtained in the modified QGSM approach \( [28, 29] \).

However, in the present paper we suggest a new approach for calculation of \( p_T \) spectra of soft hadron production. Below we discuss our suggestion based on the scale independent behavior of the starting gluon density \( f_g^{(0)}(x, k_T^2, \mu^2_0) \) at \( \mu \leq \mu_0 \) and low \( x \). After that we found corresponding phenomenological parameters from the best description of soft hadron spectra measured at different LHC energies. For the reader’s convenience, below we recall shortly the basic formulas with a brief review of calculation steps.

### 2.1 Hadron spectra at low \( p_T \) in the mid-rapidity region

As is well known, the soft hadron production in \( pp \) collisions at small momentum transfer and large Feynman variables \( x_F \) can be analyzed successfully within the soft QCD models, such as quark-gluon string model (QGSM) \( [26, 27] \) or dual parton model (DPM) \( [33] \). It is based on the Regge behaviour of the cross section at large \( x_F \). In the QGSM, the interaction dynamics is based on two colorless strings formed between the quark/diquark \( (q/qq) \) and diquark/quark \( (qq/q) \) of the colliding protons\(^1\). At their breaking, the quark-antiquark and diquark-antidiquark pairs are created in the chromo-static QCD field and then they fragmentate into final hadrons \( h \). Corresponding quark and diquark distribution functions and their fragmentation functions into hadrons were calculated \( [26, 27] \). Such approach allows one to describe the experimental observables at non zero \( x_F \) and low transverse momentum \( p_T \) quite satisfactorily.

In the mid-rapidity region, according to the Abramovsky-Gribov-Kancheli cutting rules (AGK) \( [34] \), only one-pomeron Mueller-Kancheli diagrams contribute to the inclusive hadron spectrum. However, it has some difficulties in the description of inclusive hadron spectra measured in this kinematical region. In fact, the predicted hadron transverse momentum distributions fall down very fast with increase of \( p_T \) compared to the data \( [29] \). To avoid these difficulties the QGSM was modified \( [28, 29] \). So, it was suggested that there are soft gluons in a proton which split into \( q\bar{q} \) pairs and, therefore, give additional contribution to the hadron spectrum. The contribution of the one-Pomeron exchange graph between gluons in the colliding protons and contribution of one-Pomeron Mueller-Kancheli diagrams to inclusive \( p_T \) spectrum were taken into account. However,

\(^1\)These two colorless strings can be stretched between valence quarks and diquarks corresponding to the one-Pomeron exchange between colliding protons. Also many strings can be stretched between sea quarks and antiquarks in the interacting protons, which corresponds to \( n \)-Pomeron exchanges.
the application of the AGK cutting rules in the case, when soft gluons as well as quarks are included in the calculation is very questionable. In these contributions of quarks, diquarks and gluons were calculated separately, independently of each other assuming that the contribution of gluons to the spectrum vanishes at zero transverse momenta of produced hadrons. Additionally to that the splitting function of gluons into \( q\bar{q} \) pairs was calculated ignoring the dependence of gluon distribution on the transverse momentum \( k_T \). Therefore, in this paper we recalculate the inclusive \( p_T \) spectra of charge hadrons produced in \( pp \) collisions at mid-rapidity and small \( p_T \).

As mentioned above, the data on \( \gamma^*p \) cross-section show its saturation at low \( Q^2 \) and low \( x \). It leads to the saturation of the dipole cross section and scale independent behavior of the starting gluon density at low \( Q^2 \) less than the saturation transfer square \( Q^2_{\text{sat}} \). Therefore, the colliding protons at low \( Q^2 \) can be considered as two systems consisting of three valence quarks and gluon environment with the wave function \( \Psi_g \), its square is related to the starting gluon distribution as \( |\Psi_g|^2 \sim f_g^{(0)}(x, k_T^2, \mu_0^2) \). Then, the \( pp \) interaction amplitude can be presented in the simple spectator form \( F_{pp} = f_g^{(0)}(x, k_T^2, \mu_0^2) \), where \( f_g^{(0)} \) is the amplitude of interaction of two \( 3q \) systems. To calculate the inclusive spectrum \( \rho(x, p_T) \equiv E d^2\sigma/dx dp \) of hadrons \( h \) we have to calculate the sum of the quark contribution \( \rho_q \) and the gluon one \( \rho_g \), i.e.,

\[
\rho(x, p_T) = \rho_q(x, p_T) + \rho_g(x, p_T).
\]

The first term in (4) was calculated within the QGSM using only the one-Pomeron exchange or the cylinder graph because in the mid-rapidity and small \( x_T = 2p_T/\sqrt{s} \) the multi-Pomeron exchanges result in the negligibly small contributions, as it was shown in [29]. It is presented in the following form:

\[
\rho_q(s, x, p_T) = \sigma_1 \phi_q(s, x, p_T),
\]

where \( \sigma_1 \) is the cross section of the one-Pomeron exchange, see [26] and references therein:

\[
\sigma_1 = \frac{\sigma_P}{z} (1 - e^{-z}), \quad \sigma_P = 8\pi\gamma_P (s/s_0)^\Delta, \quad z = \frac{2C\gamma_P (s/s_0)^\Delta}{R^2 + \alpha'_P \ln s/s_0}.
\]

All the parameters in (6) are found from experimental data on the total and differential cross sections of elastic \( pp \) and \( p\bar{p} \) scattering at high energies: \( \gamma_P = 1.27 \text{ GeV}^{-2}, \Delta = 0.156, C = 1.8, R^2 = 4.0 \text{ GeV}^{-2}, \alpha'_P = 0.25 \text{ GeV}^{-2} \). The function \( \phi_q(s, x, p_T) \) is calculated within the QGSM and presented in Eq. (8). The second one \( \rho_g(x, p_T) \) is the convolution of the modified gluon distribution \( F_g(x, k_T) \) with the fragmentation function of gluons to hadrons \( D_g \rightarrow h \) multiplied by the integral from \( |f_g^{(0)}|^2 \) over the intrinsic phase space, which results in approximately the inelastic \( pp \) cross section \( \sigma_{\text{in}}^{pp} \), because the LHC data described in this paper exclude the elastic \( pp \) collisions. The modified gluon distribution \( F_g(x, k_T) \) as well as the modified quark and diquark distributions \( F_q(x, k_T), F_{q\bar{q}}(x, k_T) \) are calculated taking into account the energy-momentum conservation law, see Eqs. (15, 20).

\[
\rho_g(x, p_T) = F_g \otimes D_g \rightarrow h \times \sigma_{\text{in}}^{pp}.
\]

Finally the hadron spectrum at low \( x \) and low \( p_T \) can be presented as the following:

\[
\rho(x, p_T) = \sigma_1 \phi_q(s, x, p_T) + \sigma_{\text{in}} \phi_g(s, x, p_T).
\]
First and second terms in Eq. \(8\) represent the quark/diquark and gluon contributions, respectively:

\[
\phi_q(s, x, p_T) = \left\{ \Phi_q(x_+, p_T)\Phi_q(x_+, p_T) + \Phi_{qq}(x_+, p_T)\Phi_q(x_-, p_T) \right\},
\]

\[
\phi_g(s, x, p_T) = \left\{ \Phi_g(x_+, p_T) + \Phi_g(x_-, p_T) \right\},
\]

where

\[
x_\pm = \frac{1}{2} \left( \pm x + \sqrt{x^2 + 4(m_h^2 + p_T^2)/s} \right), \quad x = 2\sqrt{m_h^2 + p_T^2} \sinh y,
\]

\(m_h\) is the produced hadron mass and phenomenological parameters \(C_q\) and \(C_g\) have to be determined from the data. Keeping in mind that the proton consists of two up and one down quarks and taking into account the non-zero parton transverse momenta, the contributions of quark, diquark and gluon fragmentations within the modified QGSM approach are calculated as convolutions \(28, 29\):

\[
\Phi_q(x, p_T) = \frac{1}{x} \int_0^\infty d\xi \int_0^{2\pi} d\phi \int_0^{2\pi} \left[ \frac{2}{3} F_a(\xi, k_T^2)G_{a\rightarrow h}(z, |p_T - zk_T|) + \frac{1}{3} F_d(\xi, k_T^2)G_{d\rightarrow h}(z, |p_T - zk_T|) \right],
\]

\[
\Phi_{qq}(x, p_T) = \frac{1}{x} \int_0^\infty d\xi \int_0^{2\pi} d\phi \int_0^{2\pi} \left[ \frac{2}{3} F_{ua}(\xi, k_T^2)G_{ud\rightarrow h}(z, |p_T - zk_T|) + \frac{1}{3} F_{uu}(\xi, k_T^2)G_{uu\rightarrow h}(z, |p_T - zk_T|) \right],
\]

\[
\Phi_g(x, p_T) = \frac{1}{x} \int_0^\infty d\xi \int_0^{2\pi} d\phi \int_0^{2\pi} F_g(\xi, k_T^2)G_{g\rightarrow h}(z, |p_T - zk_T|),
\]

where \(k_T\) is the transverse momentum of quark, diquark and/or gluon, \(z = x/\xi\) and \(\phi\) is the azimuthal angle between \(p_T\) and \(k_T\). Quark, diquark and gluon fragmentation functions to hadrons (namely, to pions and kaons), \(G_{a\rightarrow h}(z, |p_T|)\) with \(a = q, qq\) or \(g\), were calculated in the QGSM at leading (LO) and next-to-leading (NLO) orders \(38\). Corresponding analytical expressions are collected in Appendix A. Functions \(F_a(x, k_T^2)\) involved in \(12\) — \(14\) are related to the TMD parton distributions in a proton taken at some scale determined by the produced hadron transverse momentum\(^2\). The functions \(F_q(x, k_T^2)\), \(F_{qq}(x, k_T^2)\) and \(F_g(x, k_T^2)\) were calculated using the energy-momentum conservation law for quark, diquark and gluon. So, for example,

\[
F_q(x, k_T^2) = \int_{x_\pm}^1 d\xi_1 d\xi_2 1/(1 - \xi_1 - \xi_2) \int d^2p_T d^2q_T \delta^{(2)}(k_T + p_T + q_T) \times
\]

\[
\times \tilde{f}_q(x, k_T^2)\tilde{f}_{qq}(\xi_1), p_T^2\tilde{f}_g(\xi_2, q_T^2),
\]

As it was mentioned above, this scale could be considered as the starting scale for subsequent QCD evolution due to small hadron transverse momentum, \(p_T \sim 1\) GeV.
where \( \tilde{f}_u(x, k_T^2) \equiv f_u(x) g_u(k_T^2) \) and \( \tilde{f}_d(x, k_T^2) \equiv f_d(x, k_T^2) / x \) are the \( k_T \)-dependent quark, diquark and gluon densities, respectively (see below). Performing integration over \( \xi_1 \) and \( p_T^2 \) in (15), one can easily obtain:

\[
F_u(x, k_T^2) = \tilde{f}_u(x) g_q(k_T^2) \int_{x^\pm}^{1-x} d\xi_2 \int_0^\infty dq_T^2 \int_0^{2\pi} d\varphi \times \nonumber
\times \tilde{f}_{ud}(1 - x - \xi_2) g_{qq}(|k_T + q_T|^2) \tilde{f}_g(\xi_2, q_T^2),
\]

(16)

\[
F_d(x, k_T^2) = \tilde{f}_d(x) g_q(k_T^2) \int_{x^\pm}^{1-x} d\xi_2 \int_0^\infty dq_T^2 \int_0^{2\pi} d\varphi \times \nonumber
\times \tilde{f}_{ud}(1 - x - \xi_2) g_{qq}(|k_T + q_T|^2) \tilde{f}_g(\xi_2, q_T^2),
\]

(17)

where \( \varphi \) is the azimuthal angle between \( k_T \) and \( q_T \). Similar to that one can derive expressions for \( F_{qq}(x, k_T^2) \):

\[
F_{ud}(x, k_T^2) = \tilde{f}_{ud}(x) g_{qq}(k_T^2) \int_{x^\pm}^{1-x} d\xi_2 \int_0^\infty dq_T^2 \int_0^{2\pi} d\varphi \times \nonumber
\times \tilde{f}_u(1 - x - \xi_2) g_{qq}(|k_T + q_T|^2) \tilde{f}_g(\xi_2, q_T^2),
\]

(18)

\[
F_{uu}(x, k_T^2) = \tilde{f}_{uu}(x) g_{qq}(k_T^2) \int_{x^\pm}^{1-x} d\xi_2 \int_0^\infty dq_T^2 \int_0^{2\pi} d\varphi \times \nonumber
\times \tilde{f}_u(1 - x - \xi_2) g_{qq}(|k_T + q_T|^2) \tilde{f}_g(\xi_2, q_T^2).
\]

(19)

For \( F_g(x, k_T^2) \) we have the following formula including the charge and isotopic invariance:

\[
F_g(x, k_T^2) = \tilde{f}_g(x, k_T^2) \int_{x^\pm}^{1-x} d\xi_2 \int_0^\infty dq_T^2 \int_0^{2\pi} d\varphi \times \nonumber
\times \left\{ \frac{2}{3} \tilde{f}_u(1 - x - \xi_2) g_q(|k_T + q_T|^2) \tilde{f}_{ud}(\xi_2) g_{qq}(q_T^2) + \right. \\
\left. + \frac{1}{3} \tilde{f}_d(1 - x - \xi_2) g_{qq}(|k_T + q_T|^2) \tilde{f}_{uu}(\xi_2) g_{qq}(q_T^2) \right\}.
\]

(20)

The distributions \( f_u, f_d, f_{uu}, f_{ud} \) and \( g_q, g_{qq} \) as functions of \( x \) and \( k_T \) respectively are presented in the Appendix A. Our choice for the TMD parton densities involved in (16) — (20) is particularly discussed in Section 2.2.

Finally, the inelastic \( pp \) cross section \( \sigma_{in} \) can be calculated as the difference between the total and elastic \( pp \) scattering cross sections: \( \sigma_{in} = \sigma_{tot} - \sigma_{el} \), where \( \sigma_{tot} \) should satisfy to Regge asymptotic, \( \sigma_{tot} \sim (s/s_0)^{\alpha_p-1} \). However, it was shown [39] [40] that \( \sigma_{tot} \) and \( \sigma_{el} \) can be parametrized in the following way:

\[
\sigma_{tot} = 21.7(s/s_0)^{0.0808} + 56.08(s/s_0)^{-0.4525} \text{ mb}, \quad (21)
\]

\[
\sigma_{el} = 11.84 - 1.617 \ln s + 0.1359 \ln^2 s \text{ mb}. \quad (22)
\]
where \( s_0 = 1 \) GeV. Therefore, we will use these expressions in the numerical calculations.

Using the expressions above, one can calculate the cross sections for soft hadron production in \( pp \) collisions. Some technical details are given in Appendix B. To perform numerical multidimensional integration, we employ a Monte-Carlo technique implemented into the VEGAS tool [41].

### 2.2 TMD parton distributions in a proton at low scale

Concerning the TMD gluon density, \( f_g(x, k_T^2) \), here we will follow our previous considerations [20–23] where the different expressions based on the GBW saturation model were tried. In fact, it was demonstrated that overall description of collider data could be significantly improved if usual GBW gluon distribution given by (2) is modified. In the present analysis we update the modification proposed earlier [20, 21] and take into account certain quark-gluon sum rule

\[
\sum_a \int_0^1 dx \int d^2 k_T x \tilde{f}_a(x, k_T^2) = 1.
\]  

(23)

where \( a = u, d, uu, ud, g \).

We will consider the data on charged hadron (pion and kaon) production at small transverse momenta \( p_T \leq 1 \) GeV taken at different energies, namely, \( \sqrt{s} = 0.9, 2.36, 7 \) and 13 TeV [42–44]. We find that in order to describe these data the most appropriate expression for the starting gluon density \( f_g(x, k_T^2, \mu_0^2) \), hereafter labeled as LLM gluon, is the following:

\[
f_g(x, k_T^2) = c_g(1 - x)^{b_g} \sum_{n=1}^{3} (c_n R_0(x)|k_T|)^n e^{-R_0(x)|k_T|},
\]  

(24)

where \( R_0(x) \) is defined in [2] and we kept \( x_0 = 4.1 \cdot 10^{-5} \) and \( \lambda = 0.22 \). Our best fit for phenomenological parameters leads to \( c_1 = 5, c_2 = 3, c_3 = 2 \) and \( Q_0 = 1.233 \) GeV. However, the measured hadron spectra [42–44] refer to relatively small \( x \) region and appear to be mostly insensitive to \( b_g \) value, which plays a role at essentially larger \( x \). So, we determined the latter from the LHC data on several hard processes, as described in Section 3. Following [45,46], we treat \( b_g \) as a running parameter at \( k_T^2 \geq Q_0^2 \):

\[
b_g = b_g(0) + \frac{4 C_A}{\beta_0} \ln \frac{\alpha_s(Q_0^2)}{\alpha_s(k_T^2)},
\]  

(25)

where \( b_g(0) = 5.854, C_A = N_c, \) and \( \beta_0 = 11 - 2/3 N_f \) is the first coefficient of the QCD \( \beta \)-function. At small \( k_T^2 < Q_0^2 \), the fixed value \( b_g = b_g(0) \) is used. A similar approach was applied in the investigation of EMC effect from the study of shadowing at low \( x \) to antishadowing at \( x \sim 0.1 - 0.2 \) [47].

The experimental data on charged hadron production involved into the fit are compared with our predictions in Fig. 1 (left panel). One can see that good agreement is achieved in a wide range of energies.

---

3The TMD gluon density used in [22,23] has a very high \( k_T^2 \) tail, that leads to sizeble value of gluon average transverse momentum. The latter, of course, should have a significant perturbative component unwanted for our purposes.
2.3 Saturation dynamics

As it is assumed in [30, 31], the effective dipole cross-section, as a function of the distance $r$ between $q$ and $\bar{q}$ is saturated at large $r$. It is presented in the following form:

$$\hat{\sigma}(x,r^2) = \sigma_0 \left\{ 1 - \exp\left( -\frac{r^2}{4R_0^2(x)} \right) \right\},$$

(26)

where $R_0$ is determined in (2). The normalization $\sigma_0$, the parameters $\lambda$ and $x_0$ were found from a fit of all inclusive DIS data [30, 31]. The relation of the TMD gluon distribution to the dipole cross section $\hat{\sigma}(x,r^2)$ was calculated [31] within the two gluon exchange approximation between $q, \bar{q}$ and the nucleon debris. It has the following form:

$$\hat{\sigma}(x,r^2) = \frac{4\pi^2}{3} \int \frac{dk_T^2}{k_T^2} \left\{ 1 - J_0(k_{TT}) \right\} \alpha_s(\mu_0^2)f_g^{(0)}(x,k_T^2,\mu_0^2),$$

(27)

where $J_0(k_{TT})$ is the cylindrical special function of order 0. Comparing (26) to (27) one can get immediately the expression (2).

Inserting our gluon distribution (LLM) at the initial $\mu_0$ presented in (24) to (27) one can get the dipole cross section at different values of $x$ as a function of $r$, which is proportional to $2/Q_s^2$, according to (26). According to Fig. 2 (left panel), the GBW dipole cross section $\hat{\sigma}(x,r^2)$ calculated using (2) to our calculation (24) is presented as a function of $r$ at different $x$. One can see that the saturation of the dipole cross section at large $r$ strongly depends on $x$ and on the TMD gluon density. The GBW gluon results in the saturation scale $r_s \sim 2/R_0$, according to (26) [31]. According to Fig. 2 the saturation scale corresponding to the GBW gluon density at $x = 4.2 \times 10^{-5}$ is $Q_s \sim 2/r_s \sim 0.8$ GeV, whereas LLM gluon results in $Q_s \sim 2/r_s \sim 0.4$ GeV at the same $x$. It means that at $Q^2 < Q_s^2$ the dipole cross section and starting gluon density do not depend on scale $Q^2$.

Let us note that the GBW and LLM gluon densities vanish at $k_T \to 0$. It is due to the neglect of the initial gluon mass $m_g$ in the gluon propagator, see (27) and [31]. With the inclusion of the gluon mass the TMD gluon distributions does not vanish at the zero gluon transverse momentum. However, $f_g^{(0)}(x,k_T^2,\mu_0^2)$ does not saturate even at $m_g = 100$ MeV, according to Fig. 2 (right panel).

2.4 CCFM evolution

Being sure that proposed TMD gluon density in a proton is able to reproduce well
the collider data in a soft kinematical region, one can consider the expression \cite{24} as the starting condition for subsequent QCD evolution.

As it was mentioned above, we will apply the CCFM equation \cite{4}. It resums both large logarithms $\alpha_s^n \ln^n 1/x$ and $\alpha_s^n \ln^n 1/(1 - x)$ and introduces angular ordering condition to treat correctly gluon coherence effects. In the limit of asymptotic high energies (i.e. small $x$), it is almost equivalent to BFKL \cite{3}, but also similar to usual DGLAP evolution \cite{1} for large $x$. Therefore, it provides a suitable tool for our purposes.

In the leading logarithmic approximation (LLA), the CCFM equation for TMD gluon density
\footnote{Hereafter, we denote the evolution variable as $\mu^2$. Another notation, namely, $\mu^2$, is also often used in the literature.} $f_g(x, k_T^2, \mu^2)$ can be written as

\begin{equation}
\begin{split}
f_g(x, k_T^2, \mu^2) &= f_g^{(0)}(x, k_T^2, \mu_0^2) \Delta_s(\mu, \mu_0) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(\mu - zq) \Delta_s(\mu, zq) \tilde{P}_{gg}(z, k_T^2, q^2) f_g \left( \frac{x}{z}, k_T^2, q^2 \right),
\end{split}
\end{equation}

where $k'_T = q(1 - z) + k_T$ and $\tilde{P}_{gg}(z, k_T^2, q^2)$ is the CCFM splitting function:

\begin{equation}
\begin{split}
\tilde{P}_{gg}(z, k_T^2, q^2) &= \tilde{\alpha}_s(q^2(1 - z)^2) \left[ \frac{1}{1 - z} + \frac{z(1 - z)}{2} \right] + \tilde{\alpha}_s(k_T^2) \left[ \frac{1}{z} - 1 + \frac{z(1 - z)}{2} \right] \Delta_{ns}(z, k_T^2, q^2).
\end{split}
\end{equation}

The Sudakov and non-Sudakov (or Regge) form factors read:

\begin{equation}
\ln \Delta_s(\mu, \mu_0) = - \int \frac{d\mu'^2}{\mu_0^2} \int_0^{1 - \mu_0/\mu' \frac{1}{\zeta}} \frac{d\zeta}{\zeta} \tilde{\alpha}_s(\mu'^2(1 - \zeta)^2) \frac{1}{1 - \zeta},
\end{equation}

\begin{equation}
\ln \Delta_{ns}(z, k_T^2, q_T^2) = -\tilde{\alpha}_s(k_T^2) \int_0^1 \frac{d\zeta'}{\zeta'} \int \frac{dq^2}{q^2} \Theta(k_T^2 - q^2) \Theta(q^2 - z'^2) q_T^2,
\end{equation}

where $\tilde{\alpha}_s = 3\alpha_s/\pi$. The first term in (28) is the initial TMD gluon density $f_g^{(0)}(x, k_T^2, \mu_0^2)$ determined at the scale $\mu_0^2$ multiplied by the Sudakov form factor, describing the contribution of non-resolvable branchings between the starting scale $\mu_0^2$ and scale $\mu^2$. In our
calculations, the expression (24) will be used as the initial gluon density. The second term represents the details of the QCD evolution expressed by the convolution of the CCFM gluon splitting function $P_{gg}(z, k_T^2, q^2)$ with the TMD gluon density and Sudakov form factor. The angular ordering condition is introduced with the theta function, so the evolution scale $\mu^2$ is coming from the maximum allowed angle for any gluon emission determined by the hard scattering subprocess: $\mu^2 = s + Q_T^2$, where $Q_T$ is the net transverse momentum entering into the hard subprocess with center-of-mass energy $s$. This choice for scale $\mu^2$ is usually considered as a built-in property of the CCFM evolution (see, for example, [12] and references therein).

The CCFM equation can be solved numerically using the UPDFEvolv routine [48], so that the TMD gluon density can be obtained in the whole kinematical range. In this way, the TMD gluon distribution is tabulated in a commonly recognized format (namely, grid of $50 \times 50 \times 50$ bins in $x$, $k_T^2$ and $\mu^2$) which is used in the TMDLib tool [15].

3 Fitting the essential parameters

There are phenomenological parameters in the initial TMD gluon density (24) which are not predicted by the theory and therefore have to be extracted from the collider data. Our fitting strategy is based on the splitting the overall procedure into two almost independent parts, where each of them is referring to the regions of low and large $x$, respectively. The low $x$ region has been already considered above, in Section 2.2. Now we turn to moderate and large $x$ and refine the behaviour of proposed gluon distribution by extracting the $c_q$ and $b_q(0)$ parameter from measured cross sections of some hard processes. We will use the CMS data on inclusive $b$-jet production in $pp$ collisions at $\sqrt{s} = 7$ TeV [49] and recent data on inclusive Higgs boson production in different decay modes taken by the ATLAS [50,51] and CMS [52] Collaborations at $\sqrt{s} = 13$ TeV. The cross sections of these processes are governed by gluon-gluon fusion subprocesses and receive an essential contribution, in particular, from appropriate $x$ region

In fact, the CMS data on $b$-jet production refer to the kinematical region defined by $18 < p_T(b) < 196$ GeV and rapidity $|y(b)| < 2.2$ [49]. The ATLAS data on inclusive Higgs production in diphoton decay mode were obtained at $p_T^\gamma/m^\gamma\gamma > 0.35$ (0.25) for the leading (subleading) decay photon, pseudorapidity $|\eta^\gamma| < 2.37$ for the both photons and the invariant mass $105 < m^\gamma\gamma < 160$ GeV [50]. The CMS data refer to a similar kinematical region: $p_T^\gamma/m^\gamma\gamma > 1/3$ (1/4) for the leading and subleading photons, $|\eta^\gamma| < 2.5$ and $100 < m^\gamma\gamma < 180$ GeV [52]. The ATLAS measurement [51] performed in the $H \rightarrow ZZ^* \rightarrow 4l$ decay mode requires at least four leptons in the event with at least one lepton having $p_T > 20$ GeV, another lepton having $p_T > 15$ GeV and remaining ones having $p_T > 10$ and 5 GeV, respectively. All leptons must have the pseudorapidity $|\eta(l)| < 2.7$, the leading pair invariant mass $m_{12}$ must be $50 < m_{12} < 106$ GeV and the subleading one should be $12 < m_{34} < 115$ GeV. Finally, the four-lepton invariant mass $m_{4l}$ must satisfy $105 < m_{4l} < 160$ GeV cut. Thus, the typical $x$ values probed in these analyses, $x \sim 2\mu/\sqrt{s}$, are varied from $x \sim 5 \cdot 10^{-3}$ to $x \sim 2 \cdot 10^{-2}$, where the scale $\mu$ of considered processes is determined, for example, by the transverse masses of produced particles. Additionally, we use the latest HERA data on the charm and beauty contributions to inclusive proton structure functions $F_2^c(x, Q^2)$, $F_2^b(x, Q^2)$ [54,56] and reduced cross sections $\sigma_{\text{red}}(x, Q^2)$ and $\sigma_{\text{red}}^b(x, Q^2)$ [57], obtained at $Q^2 > 2.5$ GeV$^2$ in a wide region of $x$. The DIS reduced cross section of heavy quark Q

\[\text{We will not consider the ATLAS data on} \quad b\text{-jet production since they refer to extremely large } b\text{-jet transverse momenta, where effects of parton showers and/or hadronization corrections play an important role.}\]
angles of anti-leptons from the two decay $Z$ decay angle with respect to the beam axis (in the four-lepton rest frame) and production on Higgs decay photon helicity angle (in the Collins-Soper frame), leading lepton pair considered several angular correlations in Higgs boson production, namely, distributions their different rapidities, Higgs boson transverse momentum and rapidity spectra. We also

Figure 3: The TMD gluon densities in a proton $f_g(x, k_T^2, \mu^2)$ calculated as a function of longitudinal momentum fraction $x$ at different values of transverse momentum $k_T^2$ and hard scale $\mu^2$. Shaded bands represent the uncertainties of $b_g(0)$ fitting procedure. Note that the gluon densities calculated at $\mu^2 = 10^4$ GeV$^2$ are multiplied by factor of 100.

can be presented as

$$\sigma^Q_{red}(x, Q^2) = F_2^Q(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L^Q(x, Q^2),$$

where $y = Q^2/(x S)$ and $F_L^Q(x, Q^2)$ is the contribution of heavy quark $Q$ to the proton longitudinal structure function $F_L(x, Q^2)$. All these observables are governed by photon-gluon fusion subprocess and therefore also very sensitive to the gluon content of a proton.

We extracted the $c_g$ and $b_g(0)$ values from best simultaneous description of several observables, in particular, distributions on leading $b$-jet transverse momenta measured at their different rapidities, Higgs boson transverse momentum and rapidity spectra. We also considered several angular correlations in Higgs boson production, namely, distributions on Higgs decay photon helicity angle (in the Collins-Soper frame), leading lepton pair decay angle with respect to the beam axis (in the four-lepton rest frame) and production angles of anti-leptons from the two decay $Z$ bosons, where these angles are defined relatively to the $Z$ direction. The fitting procedure is rather standard and straightforward. Technically, applying the {\sc updevolv} routine [48], we solved numerically the CCFM equation for a (large) number of fixed guessed $b_g(0)$ values in a wide (but still reasonable) range $3 < b_g(0) < 8$. Then, using each of the generated TMD gluon densities in the proton, we calculated the cross sections of all considered processes according to previous evaluations [58, 60]. Best simultaneous description of the experimental data for all observables above is achieved at $c_g = 0.1731$ and $b_g(0) = 5.854^{+1.920}_{-1.553}$ with $\chi^2/d.o.f. = 2.2$, see Fig. 1

Note that we took into account contributions to the Higgs production cross sections from weak boson fusion ($W^+W^- \rightarrow H$ and $ZZ \rightarrow H$), associated $HZ$ or $HW^\pm$ production and associated $t\bar{t}H$ production. These contributions are essential at high transverse momenta and have been calculated in the conventional NLO pQCD. We take them from [50, 52].
The TMD gluon densities in a proton \( f_g(x, k^2_T, \mu^2) \) calculated as a function of transverse momentum \( k^2_T \) at different values of longitudinal momentum fraction \( x \) and hard scale \( \mu^2 \). Shaded bands represent the uncertainties of \( b_g(0) \) fitting procedure. Note that the gluon densities calculated at \( \mu^2 = 10^4 \) GeV\(^2\) are multiplied by factor of 100.

The TMD gluon densities in a proton calculated with fitted value of \( b_g(0) \) and \( c_g \) are shown in Figs. 3 and 4 as functions of proton’s longitudinal momentum fraction \( x \) and gluon transverse momentum \( k^2_T \) for different values of hard scale \( \mu^2 \). The shaded bands represent the uncertainties of our fitting procedure. As one can see, these uncertainties become important at \( x \geq 10^{-1} \). For comparison, we also show the CCFM-evolved TMD gluon distributions from [12], namely, JH’2013 set 2, which is often used in the phenomenological applications. In contrast with our approach, the \( x \) dependence of JH’2013 set 2 input has a general form given by [11] with parameters derived from the high-precision HERA data on the proton structure functions \( F_2(x, Q^2) \) and \( F_2^c(x, Q^2) \) at \( x < 5 \times 10^{-3} \) and \( Q^2 > 3.5 \) GeV\(^2\). We find that both these gluon densities have a remarkably different \( x \) and \( k^2_T \) behaviour, especially in the region of small \( k^2_T \), see Fig. 4. Some phenomenological consequences of the latter we demonstrate here.

So, the experimental data involved in our fit are compared with the obtained predictions in Figs. 5 — 11. The shaded bands represent the theoretical uncertainties of our calculations. For comparison, we also used here the JH’2013 set 2 gluon distribution. One can see that our fit leads to a good agreement with the experimental data practically for all considered observables. The HERA data on structure functions \( F_2(x, Q^2) \), \( F_2^c(x, Q^2) \) and reduced cross sections \( \sigma_{\text{red}}^c(x, Q^2) \), \( \sigma_{\text{red}}^b(x, Q^2) \) are reasonably well described by both considered TMD gluons within the uncertainties. However, we find that the JH’2013 set 2 gluon density provides a bit worse description of \( b \)-jet and/or Higgs boson production at the LHC, especially at low transverse momenta (see Figs. 5 — 7). The better agreement of these data achieved with the proposed TMD gluon density is an immediate consequence of using the physically motivated expression (24) for input distribution. In fact, at low transverse momenta the relative small gluon \( k^2_T \) are probed, where the difference between

Figure 4: The TMD gluon densities in a proton \( f_g(x, k^2_T, \mu^2) \) calculated as a function of transverse momentum \( k^2_T \) at different values of longitudinal momentum fraction \( x \) and hard scale \( \mu^2 \). Shaded bands represent the uncertainties of \( b_g(0) \) fitting procedure. Note that the gluon densities calculated at \( \mu^2 = 10^4 \) GeV\(^2\) are multiplied by factor of 100.
Figure 5: Transverse momentum distributions of inclusive $b$-jets produced in $pp$ collisions at $\sqrt{s} = 7$ TeV at different rapidities calculated using the CCFM-evolved TMD gluon density [24] with fitted value $b_g(0) = 5.85$. Predictions obtained with the JH’2013 set 2 gluon are shown for comparison. Shaded bands represent the estimation of theoretical uncertainties of our calculations. Kinematical cuts are described in the text. Experimental data are from CMS [49].
Figure 6: Differential cross sections of inclusive Higgs boson production at $\sqrt{s} = 13$ TeV (in the diphoton decay mode) calculated as functions of diphoton transverse momentum $p_T$, rapidity $y$ and photon helicity angle $|\cos \theta^*|$ (in the Collins-Soper frame). Notation of all histograms is the same as in Fig. 5. Kinematical cuts are described in the text. Experimental data are from ATLAS [50] and CMS [52].
the considered gluon distributions becomes essential, as it is shown in Fig. 4. Moreover, significant overestimation of the measured $b$-jet and especially Higgs boson $p_T$-spectra at low $p_T$ obtained with JH'2013 set 2 gluon leads to a notable difference in absolute normalization of Higgs rapidity, decay photon scattering angle $\cos \theta^*$, invariant masses $m_{12}$, $m_{34}$ and other observables shown in Figs. 6 and 7. So, our calculations clearly demonstrate that experimental data for considered processes are strongly sensitive to the TMD gluon distribution in the proton and can be used to constrain the latter. Of course, future more precise measurements could be very useful and important to reduce uncertainties in determination of the phenomenological parameters from the data.

We performed here a step forward to the global analysis of collider data in the $k_T$-factorization approach. The obtained TMD gluon density is already available for the community. It is implemented into the Monte-Carlo event generator pegasus. Moreover, it is included in tmdlib package, which is used by the cascade and katie Monte-Carlo generators.

4 Conclusion

We have proposed a new analytical expression for the TMD gluon density in the proton valid in a soft kinematical region. Using the modified quark-quark gluon string model, where gluonic state and non-zero transverse momentum of partons inside the proton are taken into account, we have obtained some corresponding phenomenological parameters from the best description of LHC data on charged hadron (pion and kaon) spectra produced in $pp$ collisions at low transverse momenta $p_T \simeq 1$ GeV. We have shown that the new suggested TMD gluon distribution incorporates saturation effects for the dipole cross section at a scale lower than the prediction of GBW model. Then, treating the obtained TMD gluon distribution as the initial condition for the subsequent non-collinear QCD evolution, we have extended it to the whole kinematical region using the CCFM equation. Several parameters important at moderate and large $x$ have been fitted from the LHC data on inclusive $b$-jet and Higgs boson production as well as latest HERA data on proton structure functions $F_2^q(x,Q^2)$ and $F_2^q(x,Q^2)$ and reduced cross sections $\sigma_{\text{red}}(x,Q^2)$ and $\sigma_{\text{red}}(x,Q^2)$. Our fit leads to simultaneous description of all these processes with good $\chi^2/d.o.f. = 2.2$. The obtained TMD gluon distribution in a proton is available for public usage and implemented in the popular TMDLIB package and Monte-Carlo event generator PEGASUS.

Acknowledgements

We thank S.P. Baranov, A.A. Prokhorov, H. Jung, S. Turchikhin and S. Taheri Monfared for their important comments and remarks. Studies described in Section 2 were supported by the Russian Science Foundation under grant 22-22-00387. Updated release of the Monte-Carlo generator PEGASUS (version 1.07.01) was supported by the Russian Science Foundation, grant 22-22-00119.

Appendix A

In the conventional QGSM framework (neglecting the transverse momentum dependence) the quark and diquark distribution functions in a proton $f_a(x)$, where $a = u, d,$
Figure 7: Differential cross sections of inclusive Higgs boson production at $\sqrt{s} = 13$ TeV (in the $H \rightarrow ZZ^* \rightarrow 4l$ decay mode) calculated as functions of Higgs transverse momentum $p_T$, rapidity $y$, leading and subleading lepton pair invariant masses $m_{12}$ and $m_{34}$, leading lepton pair scattering angle $|\cos \theta^*|$ (in the Collins-Soper frame), first and second anti-lepton production angles $\cos \theta_1$ and $\cos \theta_2$. Notation of all histograms is the same as in Fig. 5. Kinematical cuts are described in the text. Experimental data are from ATLAS [51].
Figure 8: Structure functions $F_2^g(x, Q^2)$ measured at different scales calculated using the CCFM-evolved TMD gluon density [24] with fitted value $b_g(0) = 5.85$. Predictions obtained with the JH’2013 set 2 gluon are shown for comparison. Shaded bands represent the estimation of theoretical uncertainties of our calculations. Experimental data are from ZEUS and H1 [54, 56].
Figure 9: Structure functions $F_2^b(x, Q^2)$ measured at different scales calculated using the CCFM-evolved TMD gluon density [24] with fitted value $b_0(0) = 5.85$. Predictions obtained with the JH'2013 set 2 gluon are shown for comparison. Shaded bands represent the estimation of theoretical uncertainties of our calculations. Experimental data are from ZEUS and H1 [54,56].
Figure 10: Reduced cross sections $\sigma_{\text{red}}(x,Q^2)$ measured at different scales calculated using the CCFM-evolved TMD gluon density [24] with fitted value $b_g(0) = 5.85$. Predictions obtained with the JH'2013 set 2 gluon are shown for comparison. Shaded bands represent the estimation of theoretical uncertainties of our calculations. Experimental data are from ZEUS and H1 [57].
Figure 11: Structure functions $\sigma^b_{\text{reg}}(x, Q^2)$ measured at different scales calculated using the CCFM-evolved TMD gluon density \cite{24} with fitted value $b_g(0) = 5.85$. Predictions obtained with the JH'2013 set 2 gluon are shown for comparison. Shaded bands represent the estimation of theoretical uncertainties of our calculations. Experimental data are from ZEUS and H1 \cite{57}. 

Here overall normalization factors are given by

\[ C_u^p = C_{ud}^p = \frac{\Gamma(2 - 1/2 + 3/2)}{\Gamma(1 - 1/2)\Gamma(1 + 3/2)} = 1/1.1781, \]

\[ C_d^p = C_{uu}^p = \frac{\Gamma(2 - 1/2 + 5/2)}{\Gamma(1 - 1/2)\Gamma(1 + 5/2)} = 1/1.01859. \]

In the modified QGSM, where gluonic state in the proton and partonic transverse momentum are taken into account, the TMD quark and diquark densities can be written as

\[ f_a(x, k_T^2) = c_a f_a(x)g_a(k_T^2), \quad g_a(k_T^2) = \frac{B_a^2}{2\pi}e^{-B_1|k_T|}, \]

where \( B_q = B_{qq} = 5 \text{ GeV}^{-1} \). The normalization factors \( c_a \) (which are about of 1/2) can be included to restore QCD quark-gluon sum rules.

Here we list analytical expressions for quark, diquark and gluon fragmentation functions used in our calculations. These expressions were obtained in the QGSM at the LO [26,27]. So, for gluons we have

\[ G_{g \rightarrow h}(z, |p_T|) = 2G_{g \rightarrow \pi}(z)I_\pi^g(|p_T|) + 2G_{g \rightarrow K}(z)I_K^g(|p_T|), \]

where the coefficients 2 come from the following relations:

\[ G_{g \rightarrow \pi^+}(z) = G_{g \rightarrow \pi^-}(z), \quad G_{g \rightarrow K^+}(z) = G_{g \rightarrow K^-}(z). \]

The parametrizations of \( G_{g \rightarrow \pi}(z) \) and \( G_{g \rightarrow K}(z) \) are the following:

\[ G_{g \rightarrow \pi}(z) = 6.57z^{0.54}(1 - z)^{3.01}, \]

\[ G_{g \rightarrow K}(z) = 0.37z^{0.79}(1 - z)^{3.07}, \]

and functions \( I_\pi^g(|p_T|) \) and \( I_K^g(|p_T|) \) read:

\[ I_\pi^g(|p_T|) = I_K^g(|p_T|) = \frac{(B_{f_h}^g)^2}{2\pi}e^{-B_{f_h}^g|p_T|}, \]

with \( B_{f_h}^g = B_{f_n}^g = B_{f_K}^g = 4.5 \text{ GeV}^{-1} \). For quark fragmentation functions we have:

\[ G_{u \rightarrow \pi^+}(z, |p_T|) = [a_0(1 - z) + a_0(1 - z)^2]I_\pi^q(|p_T|), \]

\[ G_{d \rightarrow \pi^+}(z, |p_T|) = (1 - z)G_{u \rightarrow \pi^+}(z)I_\pi^q(|p_T|), \]

\[ G_{u \rightarrow K^+}(z, |p_T|) = a_k(1 - z)^{1/2}(1 + a_1z)I_K^q(|p_T|), \]

\[ G_{u \rightarrow K^-}(z, |p_T|) = a_k(1 - z)^{3/2}I_K^q(|p_T|), \]

\[ G_{d \rightarrow K^+}(z, |p_T|) = G_{u \rightarrow K^-}(z)I_K^q(|p_T|), \]

\[ G_{d \rightarrow K^-}(z, |p_T|) = G_{u \rightarrow K^+}(z)I_K^q(|p_T|). \]
Analytical forms for \( I^q_\pi(|p_T|) \) and \( I^g_K(|p_T|) \) are the same as for \( I^q_\pi(|p_T|) \), \( I^g_K(|p_T|) \), but the slopes are \( B^q_\pi = B^q_{f_K} = 7 \text{ GeV}^{-1} \). Other parameters are \( a_0 = 0.65, a_k = 0.075 \) and \( a_{1K} = 2 \). Finally, for diquarks one can write:

\[
G_{uu \to \pi^+}(z, |p_T|) = a_0(1 - z)^2 I^{qq}_\pi(|p_T|), \tag{A17}
\]

\[
G_{ud \to \pi^+}(z, |p_T|) = a_0(1 + (1 - z)^2)(1 - z)^2 I^{qq}_\pi(|p_T|), \tag{A18}
\]

\[
G_{uu \to K^+}(z, |p_T|) = a_k(1 - z)^{5/2}(1 + a_{2K} z) I^{qq}_K(|p_T|), \tag{A19}
\]

\[
G_{ud \to K^+}(z, |p_T|) = a_k(1 - z)^{7/2} I^{qq}_K(|p_T|), \tag{A20}
\]

\[
G_{ud \to K^+}(z, |p_T|) = \frac{a_k}{2}(1 - z)^{5/2}(1 + a_{2K} z + (1 - z)^2) I^{qq}_K(|p_T|), \tag{A21}
\]

\[
G_{ud \to K^+}(z, |p_T|) = \frac{a_k}{2}(1 - z)^{7/2}(1 + (1 - z)^2) I^{qq}_K(|p_T|), \tag{A22}
\]

where \( I^{qq}_\pi, K(|p_T|) = I^{q,q}_\pi, K(|p_T|) \) and \( a_{2K} = 5 \).

**Appendix B**

Here we present some details of our calculation of charged hadron spectra at low \( p_T \).

The functions \( \Phi_a \) involved into \([9]\) and \([10]\) can be presented in the following way:

\[
\Phi_a(x_\pm, p_T) = \frac{1}{2\pi} \int_0^1 dx_1 \int_0^{2\pi} d\varphi_1 \times \int_{x_\pm}^{1-x_1} dx_2 \int_0^{2\pi} d\varphi_2 \tilde{F}_a(x_1, x_2, k_{1T}^2, k_{2T}^2) G_{a \to h}(z, |p_T - z| k_{1T}|) \tag{B1}
\]

where \( a = q, qq \) or \( g \) and \( z = x_\pm/x_1 \). The kernels \( \tilde{F}_a(x_1, x_2, k_{1T}^2, k_{2T}^2) \) are

\[
\tilde{F}_q(x_1, x_2, k_{1T}^2, k_{2T}^2) = f_q(x_1) g_q(k_{1T}^2) f_{qq}(1 - x_1 - x_2) g_{qq}(x_1 k_{1T}^2 + x_2 k_{2T}^2), \tag{B2}
\]

\[
\tilde{F}_{qq}(x_1, x_2, k_{1T}^2, k_{2T}^2) = f_{qq}(x_1) g_{qq}(k_{1T}^2) f_q(1 - x_1 - x_2) g_q(x_1 k_{1T}^2 + x_2 k_{2T}^2), \tag{B3}
\]

\[
\tilde{F}_g(x_1, x_2, k_{1T}^2, k_{2T}^2) = f_g(x_1) k_{1T}^2 f_{qq}(1 - x_1 - x_2) g_{qq}(x_1 k_{1T}^2 + x_2 k_{2T}^2). \tag{B4}
\]

To simplify the integration in (B1) we perform a change of variables:

\[
x_1 = t_0(1 - x_\pm) + x_\pm, \quad k_{1T}^2 = (1 - t_1)/t_1, \quad \varphi_1 = 2\pi t_2,
\]

\[
x_2 = t_3(1 - x_1) = t_3(1 - x_\pm)(1 - t_0) + x_\pm, \quad k_{2T}^2 = (1 - t_4)/t_4, \quad \varphi_2 = 2\pi t_5. \tag{B5}
\]

The integration on all \( t \)-variables can be now performed in the range \((0, 1)\). The transition Jacobian reads:

\[
J(x_\pm, t_0, \ldots, t_5) = \frac{4\pi^2(1 - x_\pm)(1 - x_1(t_0))}{t_1^2 t_4^2} = \frac{4\pi^2(1 - x_\pm)^2(1 - t_0)}{t_1^2 t_4^2}. \tag{B6}
\]
References

[1] V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15, 438 (1972); L.N. Lipatov, Sov. J. Nucl. Phys. 20, 94 (1975); G. Altarelli, G. Parisi, Nucl. Phys. B 126, 298 (1977); Yu.L. Dokshitzer, Sov. Phys. JETP 46, 641 (1977).

[2] R. Angeles-Martinez, A. Bacchetta, I.I. Balitsky, D. Boer, M. Boglione, R. Boussarie, F.A. Ceccopieri, I.O. Cherednikov, P. Connor, M.G. Echevarria, G. Ferrera, J. Grados Luyando, F. Hautmann, H. Jung, T. Kasemets, K. Kutak, J.P. Lansberg, A. Lelek, G.I. Lykasov, J.D. Madrigal Martinez, P.J. Mulders, E.R. Nocera, E. Petreska, C. Pisano, R. Placakyte, V. Radescu, M. Radici, G. Schnell, I. Scimemi, A. Signori, L. Szymanowski, S. Taheri Monfared, F.F. Van der Veken, H.J. van Haevermaet, P. Van Mechelen, A.A. Vladimirov, S. Wallon, Acta Phys. Polon. B 46, 2501 (2015).

[3] E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP 44, 443 (1976); E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP 45, 199 (1977); I.I. Balitsky, L.N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978).

[4] M. Ciafaloni, Nucl. Phys. B 296, 49 (1988); S. Catani, F. Fiorani, G. Marchesini, Phys. Lett. B 234, 339 (1990); S. Catani, F. Fiorani, G. Marchesini, Nucl. Phys. B 336, 18 (1990); G. Marchesini, Nucl. Phys. B 445, 49 (1995).

[5] S. Catani, M. Ciafaloni, F. Hautmann, Nucl. Phys. B 366, 135 (1991); J.C. Collins, R.K. Ellis, Nucl. Phys. B 360, 3 (1991).

[6] L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys. Rep. 100, 1 (1983); E.M. Levin, M.G. Ryskin, Yu.M. Shabelsky, A.G. Shuvaev, Sov. J. Nucl. Phys. 53, 657 (1991).

[7] H. Jung, S.P. Baranov, A. Bermudez Martinez, L.I. Estevez Banos, F. Guzman, F. Hautmann, A. Lelek, J. Lidrych, A.V. Lipatov, M.A. Malyshev, M. Mendizabal, S. Taheri Monfared, A.M. van Kampen, Q. Wang, H. Yang, Eur. Phys. J. C 81, 425 (2021).

[8] A. van Hameren, Comput. Phys. Commun. 224, 371 (2018).

[9] A.V. Lipatov, M.A. Malyshev and S.P. Baranov, Eur. Phys. J. C 80, 330 (2020).

[10] M.A. Kimber, A.D. Martin, M.G. Ryskin, Phys. Rev. D 63, 114027 (2001); A.D. Martin, M.G. Ryskin, G. Watt, Eur. Phys. J. C 31, 73 (2003).

[11] A.D. Martin, M.G. Ryskin, G. Watt, Eur. Phys. J. C 66, 163 (2010).

[12] F. Hautmann, H. Jung, Nucl. Phys. B 883, 1 (2014).

[13] F. Hautmann, H. Jung, A. Lelek, V. Radescu, R. Zlebcik, Phys. Lett. B 772, 446 (2017).

[14] F. Hautmann, H. Jung, A. Lelek, V. Radescu, R. Zlebcik, JHEP 1801, 070 (2018).

[15] N.A. Abdulov, A. Bacchetta, S.P. Baranov, A. Bermudez Martinez, V. Bertone, C. Bissolotti, V. Candelise, L.I. Estevez Banos, M. Bury, P.L.S. Connor, L. Favart, F. Guzman, F. Hautmann, M. Hentschinski, H. Jung, L. Keersmaekers, A.V. Kotikov,
A. Kusina, K. Kutak, A. Lelek, J. Lidrych, A.V. Lipatov, G.I. Lykasov, M.A. Malyshov, M. Mendizabal, S. Prestel, S. Sadeghi Barzani, S. Sapeta, M. Schmitz, A. Signori, G. Sorrentino, S. Taheri Monfared, A. van Hameren, A.M. van Kampen, M. Vanden Bemden, A. Vladimirov, Q. Wang, H. Yang, Eur. Phys. J. C 81, 752 (2021).

[16] E. Avsar, arXiv:1108.1181 [hep-ph]; arXiv:1203.1916 [hep-ph].

[17] S.M. Aybat, T.C. Rogers, Phys. Rev. D 83, 114042 (2011).

[18] B.I. Ermlaev, M. Greco, S.I. Troyan, Eur. Phys. J. C 71, 1750 (2011); B.I. Ermlaev, M. Greco, S.I. Troyan, Eur. Phys. J. C 72, 1953 (2012).

[19] P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, JHEP 1509, 106 (2015).

[20] A.A. Grinyuk, A.V. Lipatov, G.I. Lykasov, N.P. Zотов, Phys. Rev. D 87, 074017 (2013).

[21] A.V. Lipatov, G.I. Lykasov, N.P. Zотов, Phys. Rev. D 89, 014001 (2014).

[22] A.A. Grinyuk, A.V. Lipatov, G.I. Lykasov, N.P. Zотов, Phys. Rev. D 93, 014035 (2016).

[23] N.A. Abdulov, H. Jung, A.V. Lipatov, G.I. Lykasov, M.A. Malyshev, Phys. Rev. D 98, 054010 (2018).

[24] A. Bermudez Martinez, P. Connor, H. Jung, A. Lelek, R. Zlebcik, F. Hautmann, V. Radescu, Phys. Rev. D 99, 074008 (2019).

[25] V. Bertone, M. Botje, D. Britzger, S. Camarda, A. Cooper-Sarkar, F. Giulii, A. Glazov, A. Luszczak, F. Olness, R. Placakyte, V. Radescu, W. Slominski, O. Zenaiev, arXiv:1709.01151 [hep-ph].

[26] A.B. Kaidalov, Z. Phys. C 12, 63 (1982); A.B. Kaidalov, Surveys High Energy Phys. 13, 265 (1999); A.B. Kaidalov, O.I. Piskunova, Z. Phys. C 30, 145 (1986).

[27] G.I. Lykasov, M.N. Sergienko, Z. Phys. C 52, 635 (1991); G.I. Lykasov, M.N. Sergienko, Z. Phys. C 56, 697 (1992); G.I. Lykasov, M.N. Sergienko, Z. Phys. C 70, 455 (1996).

[28] V.A. Bednyakov, G.I. Lykasov, V.V. Lyubushkin, Europhys. Lett. 92, 31001 (2010).

[29] V.A. Bednyakov, A.A. Grinyuk, G.I. Lykasov, M. Poghosyan, Int. J. Mod. Phys. A 27, 1250042 (2012).

[30] K. Golec-Biernat, M. Wüsthoff, Phys. Rev. D 59, 014017 (1998).

[31] K. Golec-Biernat, M. Wüsthoff, Phys. Rev. D 60, 114023 (1999).

[32] Yu.V. Kovchegov, Phys. Rev. D 61, 074018 (2000).

[33] A. Capella, U. Sukhatme, C.J. Tan, J. Tran Thanh Van, Phys. Rev. D 36, 109 (1987).

[34] V. Abramovsky, V.N. Gribov, O. Kancheli, Sov. J. Nucl. Phys. 18, 308 (1973).

[35] ZEUS Collaboration, Phys. Lett. B 487, 53 (2000).
[36] ZEUS Collaboration, Eur. Phys. J. C 21, 443 (2001).
[37] S.I. Sinegovsky, M.N. Sorokovikov, Eur. Phys. J. C 80, 34 (2020).
[38] J. Binnewies, B.A. Kniehl, G. Kramer, Phys. Rev. D 52, 4947 (1995).
[39] N. Cartiglia, arXiv:1305.6131 [hep ex].
[40] I.M. Dremin, Particles 2, 57 (2019).
[41] G.P. Lepage, J. Comput. Phys. 27, 192 (1978).
[42] ATLAS Collaboration, New J. Phys. 13, 053033 (2011).
[43] CMS Collaboration, Phys. Rev. Lett. 105, 022002 (2010).
[44] ATLAS Collaboration, Eur. Phys. J. C 76, 502 (2016).
[45] A.V. Kotikov, A.V. Lipatov, B.G. Shaikhatdenov, P. Zhang, JHEP 2002, 028 (2020).
[46] A.V. Kotikov, A.V. Lipatov, P. Zhang, Phys. Rev. D 104, 054042 (2021).
[47] A.V. Kotikov, B.G. Shaikhatdenov, P. Zhang, Phys. Rev. D 96, 1140022 (2017).
[48] F. Hautmann, H. Jung, S. Taheri Monfared, Eur. Phys. J. C 74, 3082 (2014).
[49] CMS Collaboration, JHEP 1204, 084 (2012).
[50] ATLAS Collaboration, Phys. Rev. D 98, 052005 (2018).
[51] ATLAS Collaboration, Eur. Phys. J. C 80, 941 (2020).
[52] CMS Collaboration, JHEP 1901, 183 (2019).
[53] ATLAS Collaboration, Eur. Phys. J. C 71, 1846 (2011).
[54] ZEUS Collaboration, JHEP 1409, 183 (2014).
[55] H1 Collaboration, Eur. Phys. J. C 71, 1769 (2011).
[56] H1 Collaboration, Eur. Phys. J. C 65, 89 (2009).
[57] ZEUS and H1 Collaborations, Eur. Phys. J. C 78, 473 (2018).
[58] H. Jung, M. Krämer, A.V. Lipatov, N.P. Zotov, JHEP 1101, 085 (2011); H. Jung, M. Krämer, A.V. Lipatov, N.P. Zotov, Phys. Rev. D 85, 034035 (2012).
[59] A.V. Lipatov, M.A. Malyshev, N.P. Zotov, Phys. Lett. B 735, 79 (2014); N.A. Abdulov, A.V. Lipatov, M.A. Malyshev, Phys. Rev. D 97, 054017 (2018).
[60] A.V. Kotikov, A.V. Lipatov, G. Parente, N.P. Zotov, Eur. Phys. J. C 26, 51 (2002).