An Autonomous Distributed Admission Control Scheme for IEEE 802.11 DCF

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ABSTRACT

Admission control as a mechanism for providing QoS requires an accurate description of the requested flow as well as already admitted flows. Since 802.11 WLAN capacity is shared between flows belonging to all stations, admission control requires knowledge of all flows in the WLAN. Further, estimation of the load-dependent WLAN capacity through analytical model requires inputs about channel data rate, payload size and the number of stations. These factors combined point to a centralized admission control whereas for 802.11 DCF it is ideally performed in a distributed manner. The use of measurements from the channel avoids explicit inputs about the state of the channel described above. BUFFET, a model based measurement-assisted distributed admission control scheme for DCF proposed in this paper relies on measurements to derive model inputs and predict WLAN saturation, thereby maintaining average delay within acceptable limits. Being measurement based, it adapts to a combination of data rates and payload sizes, making it completely autonomous and distributed. Performance analysis using OPNET simulations suggests that BUFFET is able to ensure average delay under 7ms at a near-optimal throughput.

Categories and Subject Descriptors
C.2.1 [Computer-Communication Networks]: Network Architecture and Design

General Terms  
Performance

Keywords  
Admission Control, Measurements, Wireless LANs, Analytical Models, Simulations

1. INTRODUCTION

With the widespread use of WLANs based on IEEE 802.11 distributed coordination function (DCF), efforts are on to improve the Quality of Service (QoS) offered by WLANs. The most important component of QoS is the delay experienced by packets. While real-time flows have strict requirements on delays, all applications remain sensitive to high and variable delays.

The proposed QoS-oriented 802.11e standard provides prioritized access through the Enhanced Distributed Channel Access (EDCA), but 802.11e devices are not widely available. Moreover, the QoS provision of 802.11e EDCA depends on appropriate configuration of the tunable parameters and admission control, otherwise its performance degrades to that of DCF. On the other hand, it has been argued [13] that DCF is capable of providing acceptable delays as long as the load on WLAN is maintained within the capacity of the WLAN.

Provision of QoS in communication systems necessarily involves maintaining the load within the finite capacity of the system. This task is performed by the call admission control (CAC) mechanism based on a selected criterion. Admission control can be performed on a per-flow or per-host basis, either before admitting an entity, or, in some cases, even after admission, if it is clear that the desired (or guaranteed) QoS requirements can not be met.

Various models proposed for DCF and EDCA help predict the achievable throughput and delay [4, 7]. However, the application of these models for admission control requires an exact description of the traffic parameters such as packet arrival rate, average packet size, as well as WLAN parameters. Our previous experience [10] suggests that describing the packet stream at the link layer is difficult due to the diverse application characteristics as well as control overheads of the intermediate layers.

In addition, since the WLAN capacity is shared between all stations, the capacity computation requires the statistics of all flows in the WLAN that in turn lends itself to a centralized mode of admission control.

In order to preserve the advantages of the distributed operation of DCF, our endeavor is to design an autonomous,
distributed admission control that requires minimal inputs
and is able to deduce current state of the WLAN from rel-
relevant measurements. The use of measurements from the
channel or the WLAN interface will serve two purposes:

- help the station estimate the aggregate statistics for its
  admitted flows that are difficult to characterize.
- help the station deduce traffic statistics for other sta-
tions’ flows without using any message passing.

In this paper, we present one such distributed admission
control scheme named BUFFET. In the next section we
present a summary of related work that motivates the need
for current work. Section 3 presents the admission control
problem and the analytical framework for the solution. Sec-
tion 4 describes the algorithm in detail as well as the two
competing approaches we use for performance comparison.
Performance analysis of BUFFET and other approaches is
presented in Section 5. We conclude with a discussion of
performance results and future work in Section 6.

2. RELATED WORK
Bianchi and Tinnirello [5] use the collision probability $p$
derived from the measured transmission probability to esti-
mate the number $n$ of competing stations. Pong and Moors
[11] propose a call admission control for EDCA based on
the saturation model in [4]. Per-flow measurement of the
collision probability $p$ is used to estimate the throughput
achievable by the flow. A limitation of saturation model
based CAC is that the model exaggerates the effect of con-
tention, especially at higher $n$.

The centralized CAC for EDCA proposed by Kong et al. [9]
uses the measured channel utilization to estimate the achiev-
able bandwidth (fair share of the throughput) for the re-
quested flow based on a non-saturation model. The CAC
for EDCA proposed by Bai et al. [2] attempts to keep the
queue utilization ($\rho$) below a threshold. $\rho$ is computed using
regression analysis and an analytical model from the mea-
sured value of $\tau$ (the transmission probability by a station
in a slot) and load specification.

An important shortcoming of the CAC mechanisms listed
above is that they require exact specification of packet ar-
ival rates (except saturation model based CACs) and pay-
load size for all flows. It implies a centralized CAC mech-
anism that uses and stores this information for admission
decision.

It is possible that a flow obtains more than its fair share
of bandwidth (WLAN throughput/$n$) without violating the
QoS of other flows as long as the WLAN is not saturated.
A uniform throughput sharing assumption [9, 11] results in
rejecting such flows, even if there is spare capacity.

Channel utilization (fraction of channel time used by trans-
missions) threshold based CAC has been explored in [3, 6,
8, 12]. The CAC proposed by Chou et al. [6] maintains the
allocated airtime below a threshold, but the airtime com-
putation excludes the overheads of contention mechanism.
Admission control for DCF proposed in [12] combines chan-
nel utilization (including the requested flow) threshold based
CAC for real-time traffic and rate control for best-effort traf-
fic. The CAC scheme for EDCA in [8] uses the measured
utilization to decide on accepting a new flow or stopping
low-priority flows. The work in [3] evaluates two threshold-
based schemes for infrastructure WLANs, based on channel
utilization and access point queue size respectively.

Performance of threshold based CAC schemes is contingent
on the selection of the correct threshold especially in realistic
WLAN scenarios because the optimum value of the thresh-
hold depends on payload, channel data rate, and number of
stations.

3. ADMISSION CONTROL IN 802.11 WLANS

The link capacity of a 802.11 WLAN varies depending on
traffic parameters such as the number of stations, packet
size, and channel data rate [13]. The admission control for
WLANs is further complicated by the requirement of pre-
dicting the capacity or the delay at the 802.11 MAC. The
admission control objective in this paper is to keep the av-
average delay for all flows within acceptable limits. Thus the
admission control provides statistical QoS guarantees.

3.1 Requirements and desired properties of
distributed CAC

We start with the following design objectives for the dis-
tributed CAC mechanism:

- The algorithm is to run at every wireless station without
  requiring any centralized control and coordination.
- No knowledge of global parameters would be assumed;
  measurements are made locally at the WLAN interface.
- The measurements or the algorithm will not necessitate
  any change to the 802.11 protocol.

The following are the desired properties of a measurement-
based admission control algorithm:

- The algorithm should be responsive to changing load
  and number of stations.
- It should adapt to varying data rates selected by sta-
tions based on channel quality.
- It should not depend on accurate statistical character-
istics of all flows.
- It should be scalable with respect to the number of flows
  as well as stations.

3.2 Using analytical model of 802.11 MAC to
predict saturation

The delay experienced by a packet is the queuing delay at
WLAN interface plus the time to transmit the packet (in-
cluding contention and collisions, if any). This queue can be
modeled as an $M/G/1$ queue assuming Poisson arrival pro-
cess. The service rate of the queue is however dependent on
the arrival rate $\lambda$. As illustrated by the delay vs. load curve
in Fig. 1, the delay is close to nominal packet transmission
time at moderate loads whereas it increases by an order of
magnitude after the WLAN saturates (e.g., at 29% load in
Thus prevention of WLAN saturation has the desired effect of maintaining average delay within acceptable limits.

\[ \text{Fig. [1]} \]

Avoiding saturation requires predicting it in real time given the state of current load, requested load, and WLAN parameters. For this purpose, we use a Discrete Time Markov Chain (DTMC) based analytical model we have developed. It is an extended version of the model in [4] and is applicable to saturated as well as non-saturated WLANs. We add a state called idle to the single-station DTMC in [4]. A station, after completing a post-backoff (mandatory backoff after a successful transmission), examines the state of the interface queue. We define a probability \( \gamma \) as the probability of queue being empty with which the station enters idle state after post-backoff. The estimated \( \gamma \) after accounting for an incoming flow can be used as an indicator of saturation (as illustrated in Fig. [1]). \( \gamma \) can be obtained from the model as a function \( \Psi(\lambda, n, T_s) \) (equation (6)) of three variables \( \lambda \) (packet arrival rate), \( n \) and \( T_s \) (duration of successful transmission). The details of the DTMC and computation of \( \gamma \) are presented in the Appendix. Based on this argument, the CAC algorithm follows.

\section{Model Based Distributed Admission Control (BUFFET)}

In a distributed CAC scheme, a station may not be aware of the flows belonging to other stations, neither will it know the instantaneous data rates adopted by individual stations. However, the station is capable of listening to all transmissions and derive inference about the current load on the WLAN as well as WLAN parameters. The load/population dependent inputs to the model are generated by combining the measurements with the flow specifications provided by the application for the requested flow. The expected value of \( \gamma \) computed using the model is used to decide on accepting/rejecting the flow. The point of departure for BUFFET as compared to other techniques is that no external inputs other than the flow specification for the requested flow are required.

\subsection{WLAN interface measurements}

We follow the convention of denoting the measurement samples by \( \hat{\text{-}} \) (e.g., \( \hat{T} \)) and running averages by \( \bar{\text{-}} \) (e.g., \( \bar{T} \)).

\subsection{Frame transmission rate \( R_{tx} \)}

As the station has no means of measuring the packet arrival rate at other stations, we derive the aggregate packet arrival rate to the WLAN, \( \lambda_{MAC} \) from the measured rate of packet transmissions (successful and collisions) \( \bar{R}_{tx} \).

\subsection{Average transmission slot duration \( \bar{T}_{tx} \)}

The throughput of a non-saturated WLAN is greatly influenced by the average duration of a frame transmission which in turn depends on the average frame size for all frames (including higher layer control frames) and PHY data rates used by the transmitting station.

A radio interface is capable of measuring the average duration \( \bar{T}_{tx} \) of transmission. This single measurement abstracts out the effect of the two important variable parameters mentioned above and it suffices because the model requires just the duration of successful and collided transmissions (\( T_s \) and \( T_c \) respectively).

\subsection{The number of stations with active flows, \( n \)}

The number of active stations (\( n \)) is determined from the number of unique transmitters on the channel.

The measurement samples are updated every \( T_{update} \). In order to reduce the effect of short-term dynamics of traffic and channel conditions, we maintain their exponential weighted average with smoothing parameter \( \alpha \).

\[ \bar{T}_{tx} = \alpha \bar{T}_{tx} + (1 - \alpha) \bar{T}_{tx} \]

\[ \bar{R}_{tx} = \alpha \bar{R}_{tx} + (1 - \alpha) \bar{R}_{tx} \]

Assuming that the new flow is from an inactive station, \( n \leftarrow n + 1 \)

\subsection{Input flow specification}

The description of the traffic offered at the link-layer (referred to as FlowSpec) by a new flow will be provided by specifying the following parameters:

- packet arrival rate \( \lambda_{flow} \)
- average payload size in bits, \( PAYLOAD_{flow} \)

It should be noted that BUFFET makes use of the declared FlowSpec only while admitting that particular flow. For the previously admitted flows, the aggregate flow statistics are derived from channel measurements and thus inaccuracy as well as change in FlowSpec will be automatically adjusted before admitting the next flow.

\subsection{Deriving model inputs from measurements and FlowSpec}

For a moderately loaded WLAN in a steady state, all arrived packets at the interface queues are successfully transmitted on the channel. However, as we are considering random packet arrival processes, momentary queue buildup can happen when collisions occur. Therefore we approximate packet arrival rate to the WLAN to be:

\[ \lambda_{MAC} = R_{succ} + R_{coll} = \bar{R}_{tx} \]
All admitted flows are able to obtain their required throughput that may be different from their fair share as long as the WLAN throughput is less than the capacity. As an approximation, the model assumes a uniform $\lambda$ at every station. This approximation does not affect the accuracy of the results as we are not using a station’s fair share of throughput for admission decision. Thus, accounting for the new flow being admitted, $\lambda$ per station is then averaged as

$$\lambda_{new} = \frac{\lambda_{MAC}}{n} + \frac{\lambda_{flow}}{n} \quad (1)$$

For a non-saturated WLAN, we ignore the effect of collision on the measured frame duration. We factor the payload and data rate for the new flow by calculating $T_d^{flow}$, $T_s$ and $T_c$ as follows:

$$T_d^{flow} = DIFS + PHY_{HDR}$$
$$+ (MAC_{HDR} + PAYLOAD_{flow})/R$$
$$+ SIFS + PHY_{HDR} + ACK$$

$$T_s = \frac{\lambda_{MAC} T_{tx}}{n} + \frac{\lambda_{flow} T_d^{flow}}{n} \quad (2)$$

$$T_c = T_s - (PHY_{HDR} + ACK + SIFS)$$

The $PHY_{HDR}$ and $ACK$ in the above equations are expressed as their respective durations and $R$ is the PHY data rate used by the station.

4.4 Admission Criterion

As described earlier, a non-zero value of $\gamma$ indicates that the WLAN is not saturated. We use $\gamma_{new}$ predicted by the model as an indicator of saturation:

$$\gamma_{new} = \Psi(\lambda_{new}, n, T_s) \quad (3)$$

We admit a new flow only if the predicted value of $\gamma_{new}$ is non-zero. The BUFFET algorithm is illustrated in Fig. 2.

**Figure 2: Admission control flowchart**

4.5 Description of other CAC mechanisms for performance comparison

We compare BUFFET with centralized CAC mechanisms belonging to two other categories, namely saturation model based CACs and threshold based CACs. Although they are originally proposed for EDCA, we adapt them to DCF by considering only one access category as described next.

4.5.1 Call Admission Control based on saturation model (TPUTSAT)

According to the CAC mechanism proposed by Pong and Moors [11] based on Bianchi’s saturation model [4] each station computes the probability $\tau$ of a transmission in a slot from the measured probability $p$ of collision as

$$\tau = \frac{2(1 - 2p)}{(1 - 2p)(W + 1) + pW(1 - (2p)^m)}$$

from which $P_{tr}$, $P_s$ and $T_{slot}$ are obtained. A flow is admitted if the achievable throughput by a station is sufficient to meet the throughput demand:

$$S_{flow} = \frac{(1 - \tau)^{(n-1)} PAYLOAD_{flow}}{T_{slot}}$$

Admit if $S_{flow} >= Requested$ throughput

The packet arrival rate $\lambda$ does not need to be supplied for computation of $S_{flow}$ as it is the throughput at saturation.

4.5.2 Threshold based admission control (AIRTIME)

An airtime allocation and admission control is proposed in [6]. Without consideration for parameterized QoS, the airtime required per second by a flow $f$ from station $i$ is

$$r_{i,f} = \frac{s_{i,f}}{R_i}$$

where $s_{i,f}$ is throughput requirement of flow $f$ and $R_i$ is the PHY data rate used by station $i$. Assuming the knowledge about all admitted flows, a new flow $q$ from station $p$ is admitted if:

$$r_{p,q} + \sum_i \sum_j r_{i,j} \leq EA$$

where $EA$ is the effective airtime ratio or airtime threshold that excludes the control overhead of the resource allocation mechanism.

5. PERFORMANCE ANALYSIS

We analyze the performance of BUFFET, TPUTSAT and AIRTIME through simulations using the OPNET 11.5 modeler [1] according to the parameters given in Table 1. In each scenario, a new station requests a flow every 10 seconds. All the flows have fixed payload size and Poisson packet arrivals unless mentioned otherwise. We compare the number of admitted flows (throughput) and average delay after the time when either admission control kicks in or saturation sets in.

The delay vs. simulation time curves in Fig. 4 with and without CAC (BUFFET) illustrate the working of the CAC mechanism. At 170 seconds, BUFFET determines that the requested flow would cause saturation and hence starts rejecting flows. Accepting flows beyond this point causes the delay to rise sharply.
BUFFET is able to maintain the average delay under 7ms for all scenarios. More importantly, this consistent delay performance is achieved at a throughput close to the optimum. For example, in scenario-1, BUFFET admits 27 flows; AIRTIME with a threshold of 0.09 admits 30 flows but at the cost of WLAN saturation. This aspect is pictorially depicted by Fig. 4 for scenario-3 which shows that BUFFET achieves high utilization at low delays, managing a good balance between delay and utilization.

Table 2 suggests that the delay and throughput for AIRTIME is a function of the airtime threshold. The optimum threshold itself is variable across scenarios due to the effect of payload size and channel data rate on resource allocation overheads. Therefore, setting a correct threshold is essential for good performance of AIRTIME.

On the other hand, both BUFFET and TPUTSAT avoid saturation and provide low delays without depending on a threshold. TPUTSAT being based on a saturation model provides marginally lower delays but conservatively admits fewer number of flows. This effect is more pronounced for higher $n$ when saturation models tend to overestimate the effect of collision and contention. For instance, for a WLAN size of 60 stations (scenario-5) TPUTSAT admits 40% fewer flows than BUFFET.

For CBR flows (Table 1), BUFFET conservatively admits fewer flows than TPUTSAT owing to the assumption of Poisson packet arrival. The loss of throughput is however marginal. Lower measured probability of collision due to regular packet arrivals helps TPUTSAT admit more flows.

Table 3: BUFFET with for non-uniform flows

| Flow type-1 | Flow type-2 |
|-------------|-------------|
| Flow Rate | Payload (B) | Flow Rate | Payload (B) | Flow Rate | Payload (B) | Flow Rate | Payload (B) | Flow Rate | Payload (B) | Flow Rate | Payload (B) | Flow Rate | Payload (B) | Flow Rate | Payload (B) | Flow Rate | Payload (B) |
| 11 | 500 | 2 | 100 | 30 | 4.83 | 11 | 500 | 100 | 100 | 29 | 7.03 |
| 2 | 500 | 33 | 11 | 1500 | 172 | 30 | 6.76 | 11 | 100 | 32 | 11 | 1500 | 172 | 28 |
| 11 | 500 | 100 | 11 | 100 | 32 | 29 | 4.06 |

BUFFET is therefore ideal for realistic WLAN deployments with diverse applications and channel conditions, providing a fully distributed, zero-configuration autonomous setup.

### 6. CONCLUSION

In this work, we propose an autonomous distributed admission control named BUFFET for 802.11 DCF that is based on an analytical model. In order to keep the average delay within acceptable limits, BUFFET admits a flow only if it does not lead to WLAN saturation, an indicator of which is a parameter $\gamma$ predicted by the model. BUFFET is able to derive all inputs to the model from the measurements (frame transmission rate, average transmission duration and number of stations) and requested FlowSpec.

Performance analysis through OPNET simulations suggests that BUFFET is able to provide consistent sub-7ms delay while achieving near-optimal throughput. We also compare the performance of BUFFET with two other admission control schemes, one based on saturation throughput (TPUTSAT) and the other based on airtime threshold (AIRTIME). TPUTSAT is found to be too conservative in admitting load sizes as well as data rates through $T_{tx}$ measurements.
flows, especially for higher number of stations. Configuration of correct threshold (which itself is widely variable based on load and data rate) is essential for correct operation of AIRTIME.

The fully distributed nature of BUFFET, wherein it is able to deduce information about already admitted flows, coupled with its ability to work correctly for a combination of diverse data rates and payload sizes makes it ideal for zero-configuration self-regulating distributed WLAN setup.

We are currently implementing BUFFET for Atheros chipset based 802.11g WLAN cards on GNU/Linux systems. Applying the algorithm to 802.11e EDCA by extending the model and using similar measurements per access category is another future direction we are pursuing.

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The state of the DTMC is defined by the tuple \( \{i, k\} \) as defined in [4] or idle. Similar to [4], we assume a constant and independent conditional collision probability \( p \) and ignore the relevant retry limits (ShortRetryLimit or LongRetryLimit). In addition, we assume that packet arrival to the MAC are Poisson with rate \( \lambda \). We define \( P_{aa}, P_{ac}, P_{at} \) and \( P_{a} \) as the probabilities of a packet arrival during successful transmission slot, collision slot, idle slot and a generic slot respectively.

\[
\begin{align*}
P_{as} &= 1 - e^{-\lambda T_s} \\
P_{ac} &= 1 - e^{-\lambda T_c} \\
P_{at} &= 1 - e^{-\lambda a} \\
P_{a} &= P_{tr}P_{aa} + P_{tr}(1 - P_{a})P_{ac} + (1 - P_{tr})P_{at}
\end{align*}
\]

\footnote{We follow the same terminology as that used in [4].}

Accordingly, there are four possibilities after completion of post-backoff:

- Station already has packets in its queue; it transmits back-to-back packets after post-backoff without going in idle state.
- A packet arrived in idle state, and during a silent slot on the channel is transmitted in the next slot.
- A packet arrived in idle state and during a slot corresponding to a transmission, before the transmission could be sensed (initial CCA Time). The station is required to backoff in this case, similar to post-backoff.
- A packet arrived in idle state and during a slot corresponding to a (successful or collided) transmission but after the transmission has been sensed. The station is allowed to transmit after the end of the ongoing transmission and the subsequent DIFS silence.

The state of the DTMC is defined by the tuple \( \{s(t), b(t)\} \) as defined in [4] or idle. Similar to [4], we assume a constant and independent conditional collision probability \( p \) and ignore the relevant retry limits (ShortRetryLimit or LongRetryLimit). In addition, we assume that packet arrival to the MAC are Poisson with rate \( \lambda \). We define \( P_{as}, P_{ac}, P_{at} \) and \( P_{a} \) as the probabilities of a packet arrival during successful transmission slot, collision slot, idle slot and a generic slot respectively.

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P_{at} &= 1 - e^{-\lambda a} \\
P_{a} &= P_{tr}P_{aa} + P_{tr}(1 - P_{a})P_{ac} + (1 - P_{tr})P_{at}
\end{align*}
\]

\footnote{We follow the same terminology as that used in [4].}

We model the behavior of a single station using a Discrete Time Markov Chain (DTMC). The model is based on the model proposed by Bianchi [4] for saturated WLANs using a DTMC. To account for non-saturated conditions we add a state called idle to this DTMC (Fig. 5). The state after completing a packet transmission performs a mandatory backoff with random backoff counter picked from \( (0, CW_{MIN} - 1) \) \( \left[ \text{Post-backoff} \right] \). If the MAC transmission queue is empty after post-backoff, it goes in idle state. It remains in this state till the end of the slot corresponding to the first packet arrival. We define the probability \( \gamma \) as the probability that the station queue is non-empty after the post-backoff. At saturation, this probability becomes zero.

The single-step transition probabilities are:

\[
\begin{align*}
P_{\{i, k \mid i, k + 1\}} &= 1, \quad k \in (0, W_i - 2), \quad i \in (1, m) \\
P_{\{0, k \mid i, 0\}} &= \frac{(1 - p)}{W - 1}, \quad k \in (1, W - 1), \quad i \in (0, m) \\
P_{\{i, k \mid i - 1, 0\}} &= \frac{p}{W_i}, \quad k \in (0, W_i - 1), \quad i \in (1, m) \\
P_{\{m, k \mid m, 0\}} &= \frac{p}{W_m}, \quad k \in (0, W_m - 1) \\
P_{\text{idle} \mid \text{idle}} &= 1 - P_a \\
P_{\{0, k \mid \text{idle}\}} &= P_{tr}(1 - e^{-\lambda \text{CCA}}) \quad k \in (1, W - 1) \\
P_{\{0, 0 \mid \text{idle}\}} &= P_{tr}P_a(e^{-\lambda \text{CCA}} - e^{-\lambda T_s}) \\
&+ P_{tr}(1 - P_a)(e^{-\lambda \text{CCA}} - e^{-\lambda T_c}) \\
&+ (1 - P_{tr})P_{at} + P_{tr}(1 - e^{-\lambda \text{CCA}}) \quad \frac{1}{W} \\
P_{\text{idle} \mid 0, 1} &= \gamma \\
P_{\{0, 0 \mid 0, 1\}} &= 1 - \gamma \\
P_{\{0, k \mid 0, k + 1\}} &= 1, \quad k \in (1, W - 1)
\end{align*}
\]

This is an aperiodic, irreducible Markov chain for which steady state probabilities are known to exist. Applying the normalization condition

\[
1 = \sum_{i=0}^{m} \sum_{k=0}^{W_i} b_{i,k} + b_{idle}
\]

and solving the chain, the probabilities \( \tau, P_{tr}, P_a \) and throughput \( S \) are obtained similar to [4].

**A.1 Calculation of MAC service time** \( (D_{MAC}) \)

The MAC service time is the duration from the time a packet becomes head of the queue to the time it is acknowledged. The number of transmissions before the packet is successfully transmitted follows a modified geometric distribution
with parameter $p$. The delays and probabilities for the four possibilities after successful transmission mentioned above are calculated as follows:

$$D_{b2b} = T_s + \frac{T_c p}{1-p} + BO_{slots},$$

$$P_{b2b} = 1 - \gamma$$

$$D_{nob2b\_idle} = D_{b2b} - \frac{W T_{slot}}{2}$$

$$P_{nob2b\_idle} = \frac{\gamma(1 - P_s) P_a}{P_a}$$

$$D_{nob2b\_tx} = D_{b2b} - \frac{W T_{slot}}{2} + \left[ P_s T_s + (1 - P_s) T_c \right]$$

$$P_{nob2b\_tx} = \gamma \left[ P_{tr} P_s (e^{-\lambda CCA} - e^{-\lambda T_s}) \right] + \gamma \left[ P_{tr} (1 - P_s) (e^{-\lambda CCA} - e^{-\lambda T_c}) \right]$$

$$D_{nob2b\_cca} = D_{b2b} + P_s T_s + (1 - P_s) T_c$$

$$P_{nob2b\_cca} = \gamma P_{tr} \frac{(1 - e^{-\lambda CCA})}{P_a}$$

where $BO_{slots} = \frac{(1-p) W T_{slot}}{2} \left[ \frac{2(1-2p)^m}{1-2p} + \frac{[(2p)^m(2-p)(1-p)]}{(1-p)^2} \right]$ and $T_{slot}$ is the average duration of a logical slot. The average MAC service time, $D_{MAC}$, is the conditional average of the above.

**A.2 Calculation of $\gamma$**

Treating the WLAN interface as an $M/G/1$ queue, from the definition of queue utilization ($\rho$) and $\gamma$, we obtain

$$\rho = \min(1, \frac{\lambda}{\mu}) = \min(1, \lambda D_{MAC})$$

$$\gamma = P\{MAC \text{ queue is empty} \} = 1 - \rho$$

(6)

The values of $\gamma$, $\tau$ and $D_{MAC}$ are obtained numerically through successive iterations. For convenience, we express $\gamma$ as a function of three load-dependent variables:

$$\gamma = \Psi(\lambda, n, T_s)$$

(7)