Simulation of Contact Characteristics for Expandable Mechanism of Shelter Based On Multi-Grid Method

Linlin Li*, Wenli Jin, Baoyun Qi
The 28th Research Institute of China Electronics Technology Group Corporation, Nanjing 210007, China

*Corresponding author e-mail: lilinlin0962@163.com

Abstract. With the increasing demands of the diversified expandable shelter and its reliability of structure in modern society, the requirements of the transmission mechanism that affects the deploying and withdrawing of shelter are also getting higher. At present, there are few studies on the mechanical characteristics of its contacts. In this paper, the contact model of the expandable mechanism of shelter under the elastohydrodynamic lubrication (EHL) condition is established based on the multigrid method. The algorithm is implemented on the platform of MATLAB, and effects of the load, velocity and lubricant viscosity on the contact characteristics of the expandable mechanism are analysed. The results show that the performances can be improved by reducing the external load and increasing properly the velocity and lubricant viscosity. The research in this paper can also provide theoretical reference for design of the expandable mechanism of shelter.

1. Introduction
The expandable shelter is widely used in the large mobile command post, display vehicles and other places requiring the large working space. It can provide the loose and comfortable working environment for equipment and operators, because the space after deployment can reach 1 or nearly 2 times of that of the ordinary shelter. Usually, the maximum weight of integrated shelter can reach more than 10 ton, and the transmission mechanism will directly affect mobility, reliability and safety of shelter [1]. Thus, it is necessary to study deeply the contact characteristics of the expandable mechanism of shelter.

There have been many studies on the structural design and application forms of the expandable shelter in recent years. Tang and Zhu [2] proposed a combined expandable shelter, focusing on the performance of key structures and rapid deployment. Chang et al. [3] proposed a hydraulic lifting scheme for the expandable shelter, which realized the expansion of shelter along the height and width. Tai [4] carried out the finite element analysis of the expandable shelter from the perspective of structure statics.

From the above, there is a lack of research on the contact characteristics of the transmission system of the expandable shelter. Therefore, based on the multi-grid algorithm and MATLAB as the programming platform, the motion characteristics of the expandable mechanism in the EHL condition are studied in this paper, and influences of the actual working conditions on the contact characteristics of the expandable mechanism are analyzed.
2. Mathematical model

2.1. Transmission principle
The expandable shelter often uses a motor-driven gear-rack transmission mechanism to achieve the deploying and withdrawing of the shelter, as shown in Figure 1. The motor transmits power to the gear shaft through the reducer, and then drives the side shelter to move along the rack. It can be seen that the gear-rack transmission is an important part of the expandable mechanism of shelter, and its contact characteristics will directly affect the transmission efficiency and stability of system. In fact, a layer of film will be formed during movement of the gear and rack, that is, the gear and rack work under the condition of the EHL. Therefore, based on the theory of EHL, this paper analyses influences of the actual working conditions on the kinematic characteristics of the expandable mechanism.

2.2. EHL model for gear-rack transmission mechanism
For the isothermal EHL problem, it is usually assumed that the lubricants are the Newtonian fluid, and the Reynolds equation can be expressed as

\[
\frac{\partial}{\partial x} \left( \frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) = u \frac{\partial (\rho h)}{\partial x} + \frac{\partial (\rho h)}{\partial t}
\]  

(1)

Where \( p \) is the film pressure, \( h \) is the film thickness, \( \rho \) and \( \eta \) denote the density and viscosity of lubricants, \( u \) denotes the entrainment velocity of the gear and rack.

In equation (1), there is a strong coupling relationship between the film pressure and the film thickness, density, and viscosity, and the solution is complicated. In order to decrease the number of variables and reduce the difficulty of solving, equation (1) needs to be dimensionless. Introduce the following dimensionless parameters:

\[
X = \frac{x}{b}, H = \frac{hR_1}{b^2}, \bar{p} = \frac{p}{\rho_0}, \bar{\eta} = \frac{\eta}{\eta_0}, P = \frac{p}{p_h}, T = \frac{ut}{b}, U = \frac{\eta_0\mu}{E'R_x}, W = \frac{w}{E'R_x l}, G = \alpha E'
\]  

(2)

Here, \( H, P, U, \bar{p}, \bar{\eta} \) and \( W \) denote the dimensionless film thickness, pressure, velocity, density, viscosity and load, \( \rho_0 \) and \( \eta_0 \) denote the ambient density and viscosity, \( b, R_1, p_h, E' \), and \( l \) denote the contact half-width, the effective radius of the gear and rack, the maximum Hertz pressure, the effective Young’s modulus of materials and the effective contact length. Based on line contact Hertz theory, \( b, p_h, E' \) and \( R_1 \) can be written as

\[
b = \sqrt{\frac{8\mu R_1}{\pi l E'}}, \quad p_h = \frac{2w}{\pi bl}
\]  

(3)

\[
R_x = R_1, \quad \frac{2}{E'} = \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2}
\]  

(4)

Here, \( R_1 \) is the radius of curvature of the gear, \( E_1, E_2 \) and \( \mu_1, \mu_2 \) are the Young’s modulus and Poisson’s ratio of materials of the gear and rack. Therefore, the dimensionless Reynolds equation can be written as
\[
\frac{\partial}{\partial X} \left( \frac{b^3 p_h}{12 \eta_0 uR_x^2} \frac{\bar{p}H^3}{\bar{\eta} \phi} \frac{\partial P}{\partial X} \right) = \frac{\partial (\bar{p}H)}{\partial X} + \frac{\partial (\bar{\eta}H)}{\partial T} \tag{5}
\]

The dimensionless boundary conditions for equation (5) can be written as

\[
P(X_i, T) = P(X_o, T) = 0 \tag{6}
\]

\[
P(X, T) \geq 0 \tag{7}
\]

Where \(X_i\) and \(X_o\) are the dimensionless inlet and outlet location of film. The dimensionless film thickness in equation (5) can be written as

\[
H(X, T) = H_0 + \frac{X^2}{2} - \frac{1}{\pi} \int_{X_i}^{X_o} P(X', T) \ln |X - X'| dX' \tag{8}
\]

Where, \(H_0\) is the dimensionless central film thickness? Under the EHL condition, the density and viscosity is a function of pressure [5-6], and the dimensionless forms can be written as

\[
\bar{\rho} = 1 + \frac{0.6 \times 10^{-9} p_h P}{1 + 1.7 \times 10^{-9} p_h P} \tag{9}
\]

\[
\bar{\eta} = \exp \left( 9.67 + \ln \eta_0 \right) \left[ \left( 1 + 5.1 \times 10^{-9} p_h P \right)^2 - 1 \right] \tag{10}
\]

\[
z = \alpha f \left[ 5.1 \times 10^{-9} \left( 9.67 + \ln \eta_0 \right) \right]
\]

When the gear and rack are subjected to the load \(w\), the dimensionless load balance equation can be expressed as

\[
\int P(X, T) dX = 2/\pi \tag{11}
\]

### 3. Numerical method

Considering that Reynolds equation is a complex set of non-linear equations, it is difficult to obtain analytical solutions. In this paper, equations (1) to (11) are discretized by the finite difference method, and the Reynolds equation is solved iteratively by the multi-grid method [7]. In order to solve the film pressure and elastic deformation, a five-layer grid is used to calculate the film pressure and thickness on the top grid. A multi-grid framework is constructed based on \(W\)-cycle. At each time step, the convergence errors of the film pressure and film thickness on the highest grid can be expressed as

\[
\sum_i |P_i - \bar{P}_i| / \sum_i P_i < 10^{-4} \tag{12}
\]

\[
\left| \sum_i P_i - \pi/2 \right| < 10^{-4} \tag{13}
\]
Where, $P_i$ and $P_e$ are the starting and ending film pressure in each $W$-cycle.

4. Results and discussion
Taking the gear-rack contact pair of the expandable shelter as an example, effects of the working conditions on its motion characteristics are analyzed. The parameters of the gear and rack are as follows: $E_1 = E_2 = 206\text{GPa}$, $\mu_1 = \mu_2 = 0.3$, $\alpha = 2.19 \times 10^{-8} \text{m}^2/\text{N}$, $\eta_0 = 0.08\text{Pa} \cdot \text{s}$, the gear modulus $m_z = 3\text{mm}$, the number of teeth $z = 21$.

In order to verify the reliability of the mathematical model and algorithm, the dimensionless minimum film thickness calculated by the present method is compared with that calculated by Hamrock-Jacobson’s minimum film thickness formula [8] in the Figure 2. It can be seen that the dimensionless minimum film thickness increases with the increase of dimensionless velocity, and the numerical results in this paper are in good agreement with those calculated by Hamrock-Jacobson’s minimum film thickness formula, which verifies the reliability of the model and algorithm in this paper.

Figure 3 shows the dimensionless film pressure and film thickness distribution of the gear-rack contact pair under the different load. It can be seen that as the external load of the gear-rack increases, the secondary pressure peak and necking position move toward the film outlet direction, and the film thickness decreases continuously. Therefore, the larger load of the transmission mechanism is, the worse the lubrication characteristics, and it will eventually affect the stability of the gear transmission.

Figure 4 shows the variations of dimensionless minimum film thickness when the velocity of the gear is 0.01m/s, 0.03 m/s, 0.07 m/s, 0.12 m/s, 0.2 m/s and 0.3 m/s, respectively. As can be seen from Figure 4, the minimum film thickness increases significantly with the increase of velocity. As result, the smaller the velocity, the more unfavorable to the formation of film in the gear-rack contact area. Where possible, increasing properly the velocity of the gear can reduce the wear and increase the transmission efficiency.

Figure 5 shows the variations of the dimensionless minimum film thickness when the ambient viscosity of lubricants used in gear and rack is 0.0272Pas, 0.059Pas, 0.08Pas and 0.13Pas, respectively. It can be seen from Figure 5 that the film thickness increases significantly with the increase of viscosity. Therefore, the high viscosity lubricant is beneficial to the film forming of the contact area between the gear and rack, which can reduce the friction between the two. While increasing the motion stability of the gear pair, it will also improve significantly the life of the mechanisms. Of course, the high viscosity of lubricants will increase the cost, and the suitable grade of lubricants can be selected according to the actual situation.

![Figure 1. Gear-rack transmission.](image-url)
Figure 2. Comparison of present results with classical formula.

Figure 3. Effects of loads on film pressure and film thickness.

Figure 4. Variation of film thickness with velocity

Figure 5. Variation of film thickness with viscosity.

5. Conclusion
Based on the multi-grid method, the contact characteristics of the expandable mechanism of shelter under the EHL condition are studied. The effects of the load, velocity and lubricant viscosity on the pressure and film thickness are analyzed. Through the above analysis, the following conclusions can be drawn:
(1) Under certain conditions, the lubrication performance of gear transmission can be improved by reducing the load and increasing the deploying and withdrawing velocity of the shelter.

(2) Increasing the lubricant viscosity can increase the stability and life of the gear transmission. At the same time, the cost will increase accordingly, but the lubricants can be chosen according to the actual situation.

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