Accelerometer-based estimation and modal velocity feedback vibration control of a stress-ribbon bridge with pneumatic muscles

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Abstract. Lightweight footbridges are very elegant but also prone to vibration. By employing active vibration control, smart footbridges could accomplish not only the architectural concept but also the required serviceability and comfort. Inertial sensors such as accelerometers allow the estimation of nodal velocities and displacements. A Kalman filter together with a band-limited multiple Fourier linear combiner (BMFLC) is applied to enable a drift-free estimation of these signals for the quasi-periodic motion under pedestrian excitation without extra information from other kinds of auxiliary sensors. The modal velocities of the structure are determined by using a second Kalman filter with the known applied actuator forces as inputs and the estimated nodal displacement and velocities as measurements. The obtained multi-modal velocities are then used for feedback control. An ultra-lightweight stress-ribbon footbridge built in the Peter-Behrens-Halle at the Technische Universität Berlin served as the research object. Using two inertial sensors in optimal points we can estimate the dominant modal characteristics of this bridge. Real-time implementation and evaluation results of the proposed estimator will be presented in comparison to signals derived from classical displacement encoders. The real-time estimated modal velocities were applied in a multi-modal velocity feedback vibration control scheme with lightweight pneumatic muscle actuators. Experimental results demonstrate the feasibility of using inertial sensors for active vibration control of lightweight footbridges.

1. Introduction
In the last decades, lightweight and elegant structures such as stress-ribbon footbridges, which are prone to vibrations, are designed. To show the potential of high-strength Carbon Fiber Reinforced Plastics (CFRP), a 13 m span stress ribbon bridge with CFRP ribbons (see Figure 1) was built in the laboratory of the Department of Civil and Structural Engineering at the Technische Universität Berlin by Schlaich and Bleicher [1]. The combination of low extensional stiffness using CFRP for the ribbons and a lightweight bridge deck leads to considerable dynamic responses caused by pedestrian loads.

In comparison to traditional passive dampers like Tuned Mass Dampers (TMDs), which increase the mass of the bridge and should be tuned exactly to the critical natural frequency and kinetic equivalent mass of the bridge, Active Vibration Control (AVC) could overcome...
the shortage of passive methods through the use of actuators and real-time control systems. The use of pneumatic muscle actuators (PMAs) for active vibration control for the stress-ribbon footbridge in Berlin was investigated and proposed in [2, 3, 4, 5]. Displacement encoders between the bridge and the ground were applied in the laboratory set-up for the estimation of the modal states of the stress-ribbon bridge.

In this work, inertial sensors (accelerometers) will be used instead of displacement sensors. This requires a modification of the modal velocity observer while the remaining control system, originally described in [5], remains identically. Inertial sensors are more suitable for real world applications than displacement encoders due to their increased flexibility in installation on structures and due to their lower cost when using MEMS technology.

The estimation of velocities or displacements by means of inertial sensors is usually through the numerical integration of measurement data, however this step inherently causes errors that grow with time, commonly known as integration drift. To solve this well known problem in velocity/displacement estimation based on inertial sensors, the use of additional information from other auxiliary sensors [6] or the use of prior knowledge on the expected motion [7] are suggested. In [7], a drift-free position estimation approach was proposed for periodic or quasi-periodic motion using inertial sensors. The method uses a linear filtering stage in form of a Kalman filter coupled with an adaptive filtering stage to remove drift and signal attenuation. It is assumed that the motion is limited in approximated frequency bands. One possible adaptive filtering stage is the band-limited multiple Fourier linear combiner (BMFLC).

In this paper, the estimated nodal velocities and displacements are used to determine the multi-modal velocities which are applied in active vibration control of the lightweight footbridge. Section 2 will introduce the nodal and modal state-space models of the stress-ribbon bridge as well as the modal velocity observer based in form of a Kalman filter. Estimates of nodal velocities and displacements are required in this Kalman filter as measurement information. Section 3 will outline how drift-free estimates of these signals can be obtained by means of inertial sensors based on another Kalman filter combined with a BMFLC (BMFLC-KF). In the fourth section,

Figure 1. The stress-ribbon bridge with active vibration control system in TU Berlin [5].
multi-mode modal velocity feedback control will be concisely explained and the estimated modal velocities will be shown in comparison to the ones derived from classical displacement encoders. Finally, the proposed estimation method will be verified by real-time experiments in active vibration control of the footbridge.

2. State space model and modal velocity estimation of the stress-ribbon bridge

2.1. State space model of the bridge

In order to design the active vibration control system, a state space model is used, which was developed from a 8-plates rigid body model of the stress-ribbon bridge in [2]. The bridge model shown in Figure 2 includes sensors (two displacement encoders Y01, Y02 and two inertial sensors S01, S02) and three pneumatic muscle actuators (PMAs).

![Figure 2. Simplified 8-plates model of the stress ribbon bridge.](image)

For the state vector $X$, seven generalized coordinates $q_i, i = 1, 2, \ldots, 7$ and their first derivatives are selected:

$$X = [x_1 \ x_2 \ \cdots \ x_{14}]^T = [q_1 \ \dot{q}_1 \ q_2 \ \dot{q}_2 \ \cdots \ q_7 \ \dot{q}_7]^T$$

(1)

The nonlinear state space model is derived by using the Euler-Lagrange formalism. By linearization at the equilibrium point (bridge without additional load and non-active control inputs, states are zero), the linear state space description

$$\dot{X} = AX + BU, \quad y = CX = [q_1 \ \dot{q}_1 \ q_4 \ \dot{q}_4]^T$$

(2)

in nodal form is obtained, where $A$ is the system matrix, and $B$ is the input matrix. The output vector $y$ consists of the displacements $q_1$ and $q_4$ as well as the corresponding velocities $\dot{q}_1$ and $\dot{q}_4$. The input vector $U = [u_1 \ u_2 \ u_3]^T$ contains the forces introduced by the PMAs.

The displacement sensors directly measure the displacements $q_1$ and $q_4$ while the corresponding velocities $\dot{q}_1$ and $\dot{q}_4$ are obtained via numerical differentiations. The measurements with the two inertial sensors (accelerometers) are $y_{S01} = \ddot{q}_1 + \gamma_1$ and $y_{S02} = \ddot{q}_4 + \gamma_2$, where $\gamma_1$ and $\gamma_2$ are unknown slowly time-varying sensor biases. And the output vector $[\hat{q}_1 \ \dot{\hat{q}}_1 \ \hat{q}_4 \ \dot{\hat{q}}_4]^T$ is estimated with the method as described in Section 3.

2.2. Modal state space model

To design a model based controller for specific modes it is more convenient to use the modal state space representation [8]. The modal state space model can be written as

$$\dot{X}_m = A_m X_m + B_m U, \quad y = C_m X_m, \quad X_m = T^{-1} X$$

(3)
\[ A_m = T^{-1} A T = \text{diag}(A_{m1}, \ldots, A_{m7}), \quad B_m = T^{-1} B, \quad C_m = C T \]  
\[ A_{mi} = \begin{bmatrix} 0 & 1 \\ -\omega_i & -2\zeta_i \omega_i \end{bmatrix} \quad i = 1, 2, \ldots, 7 \]  

where \( \omega_i \) and \( \zeta_i \) are the natural frequency and damping ratio of the \( i \)th mode, respectively. This representation is obtained from the already derived nodal state space representation by transformation using the modal matrix \( T \). Experiments of free vibrations without control have been conducted to verify the dynamic behavior of the model. The natural frequencies of the first three modes of the 8-plate model agree very well with the measured frequencies [2].

Observability Gramians (see e.g. [8]) have been calculated in [5] to show that the first 7 modes of the bridge can be observed by either the pair of displacement sensors \((Y01,Y02)\) or the pair of accelerometers \((S01,S02)\) at the sensor locations displayed in Figure 2.

2.3. Modal state estimation based on a Kalman filter

To estimate the modal states of the bridge a Kalman filter model can be employed. By the measurements from two accelerometers or two displacement encoders (see Figure 2) the vertical symmetric and asymmetric modes can be observed. The dynamics of the estimator is given by

\[ \dot{\hat{X}}_m = A_m \hat{X}_m + B_m U + L(y - C_m \hat{X}_m) \]  

where \( \hat{X}_m \) is the estimate modal state vector. The output \( y \) is obtained via the two displacement encoders [2], or is estimated from the two accelerometers with the adaptive estimation method described in Section 3. The observer gain \( L \) is determined by solving the matrix Riccati equation taking measurement and system noise into account.

3. Real-time estimation of velocity/displacement with an accelerometer based on BMFLC-KF

Instead of a simple numerical integration of high-pass filtered acceleration measurements, a real-time estimation method based on a BMFLC-KF is proposed to estimate the drift-free velocity/displacements from the measured accelerometer data.

3.1. Band-limited multiple Fourier linear combiner

A Fourier linear combiner (FLC) was proposed in [14], which estimates adaptively the Fourier coefficients of a known base frequency and its harmonics based on the least mean squares (LMS) algorithm. For the estimation of an unknown signal within a limited band, in [15] a pre-defined frequency band \([\omega_1 - \omega_n]\) is considered and divided into \( n \) finite divisions. Then \( n \) FLC are combined to form the BMFLC to estimate band-limited signals. The frequency resolution \( \Delta f = \Delta \omega / 2\pi \) should be chosen to achieve a tradeoff between computational cost and performance requirement.

Considering that the measurement with an accelerometer consists of two parts, quasi-periodic motion and slowly time-varying “static” bias, we introduce the following BMFLC-based signal model:

\[ r_k = \gamma_k + \sum_{r=1}^{n} a_{rk} \sin(\omega_r k T) + b_{rk} \cos(\omega_r k T) \]  

where \( r_k \) denotes the estimated signal at the sampling instant \( k \), \( \gamma_k \) is the time-varying bias of an inertial sensor at the sampling instant \( k \) and the constant \( T \) is the time interval between

1 The modal damping ratios are experimentally identified as outlined in [2] and introduced in the modal model after transformation from nodal form.
adjacent samples. $a_{rk}$ and $b_{rk}$ represent the adaptive weights corresponding to the frequencies within the given band $[\omega_1 - \omega_n]$.

The reference input $X_k$ and the corresponding adaptive weight $W_k$ can be written as vectors:

$$X_k = \begin{bmatrix} \sin(\omega_1 kT) & \sin(\omega_2 kT) & \ldots & \sin(\omega_n kT) \\ \cos(\omega_1 kT) & \cos(\omega_2 kT) & \ldots & \cos(\omega_n kT) \\ 1 \end{bmatrix}^T$$  

$$W_k = \begin{bmatrix} a_{1k} & a_{2k} & \ldots & a_{nk} \\ b_{1k} & b_{2k} & \ldots & b_{nk} \\ \gamma_k \end{bmatrix}^T$$  

Hence, the signal model can be written as

$$r_k = W_k^T X_k.$$  

### 3.2. BMFLC with Kalman filter

The Kalman filter [16] is an important tool to estimate the states in a dynamic system in a recursive fashion. The formulation of the Kalman filter is generally described in the state-space form. To estimate the weights of the BMFLC, the adaptive weight vector $W_k$ is considered to be a state vector. The state transition can be modeled as a random walk model:

$$W_{k+1} = W_k + \eta_k$$  

where $\eta_k$ is a Gaussian white noise signal with zero mean. Using Equations (8-10) and introducing the measurement noise $v_k$, the output equation of the state-space model becomes

$$y_S = r_k + v_k = X_k^T W_k + v_k.$$  

Figure 3. Structure of the accelerometer-based estimation model.
The state-space model describes the accelerometer sensor output $y_S$. It is assumed that the measurement noise $v_k$ and the state noise $\eta_k$ are uncorrelated, zero mean, Gaussian white noise processes with covariances $R$ and $Q$.

A Kalman filter can be applied to estimate the state of the dynamical system at any time instant $k$ with the measurement sequence $y_S$ of an accelerometer. The following steps need to be performed recursively:

(i) Compute the Kalman filter gain $K_k$

$$K_k = P_k X_k (X_k^T P_k X_k + R)^{-1}$$

(ii) Update the estimated BMFLC weight vector

$$\hat{W}_{k+1} = \hat{W}_k + K_k (y_S - X_k^T \hat{W}_k)$$

(iii) Update covariance matrix

$$P_{k+1} = [I - K_k X_k^T] P_k + Q.$$ 

The initial conditions $\hat{W}_0$ and $P_0$ must be chosen. In Eq. (14), $\hat{y}_S = X_k^T \hat{W}_k$ represents the predicted output of the accelerometer by the Kalman filter.

### 3.3. Calculation of displacements and velocities

To obtain drift-free velocity/displacement information based on accelerometer sensing, analytical integration of the signal model (7) with the KF estimated parameters is performed, after removing the offset term:

$$y_{dis} = - \sum_{r=1}^{n} [a_{rk} \sin(\omega_r kT) + b_{rk} \cos(\omega_r kT)] / (\omega_r)^2$$

$$y_{vel} = - \sum_{r=1}^{n} [a_{rk} \cos(\omega_r kT) - b_{rk} \sin(\omega_r kT)] / \omega_r.$$ 

### 3.4. Time-Frequency characteristics of the vibrating bridge

As for multiple dominant frequencies of the stress-ribbon bridge, the BMFLC-based algorithm should perform better than other existing methods such as FLC. As described in Section 3.1, the rough time-frequency characteristics of the bridge should be known in order to define the frequency band and choose an appropriate frequency gap in the BMFLC algorithm. The frequency characteristics can be shown with the single sided amplitude spectrum. However, the time-frequency characteristics, which shows the existence of multiple dominant frequencies and their dynamic changes in the time domain, cannot be analyzed with the single sided amplitude spectrum. For further analysis of the time-frequency characteristics, the Short-Time Fourier Transform (STFT) is used to obtain the time-frequency map. By construction, STFT divides the time domain signal into individual time-frequency components by providing a high-resolution time-frequency map of the signal.

Bridge vibration under random pedestrian walking as investigated. The results are shown in Figure 4. The analysis was performed offline based on the measured acceleration in the middle span of the bridge during 30 seconds. The excited frequencies are all in a low frequency range below 12 Hz, which can be clearly recognized in the single sided amplitude spectrum. Consistently with [5], the first seven natural frequencies of the stress-ribbon bridge are also located within this frequency range. In order to get a better view of the amplitude changes, a zoomed frequency range of 0–8 Hz was chosen for the time-frequency map. It can be seen that under pedestrian loads the bridge vibrates in multiple dominant frequencies, whose amplitudes change over time. In this condition, the BMFLC-based algorithm should be applied to estimate the quasi-periodic motion of the stress-ribbon bridge.
4. Experimental verification

For experiments, the observer and control structures were implemented on a PC running the Ubuntu operating system. The real-time code was generated with Simulink Embedded Coder using the Linux ERT target\(^2\). Two inertial measurement units (Xsens MTi, Xsens Technologies B.V., The Netherlands) have been used to measure the accelerations. The remaining setup and the applied control design concept remained identical to the ones described in [5].

4.1. Modal velocity feedback control of the bridge with pneumatic muscle actuator

The pulling-only pneumatic muscle actuator consists of a flexible tube inflated with compressed air. It works by expanding in radial direction and contracting in longitudinal direction. Detailed information about modeling of pneumatic actuators including mass flow, pressure dynamics and force characteristics are given in Hildebrandt et al. [10].

Delayed modal velocity feedback control of the stress-ribbon bridge was described in [3, 5] to control the first three mode vibrations of the bridge. The closed-loop transfer functions for each controlled mode \(i\) can be written as

\[
1 + G_A(s) \cdot G_{SISO,mi}(s) \cdot \tilde{K}_{mi}(s) = 0, i = 1, 2, 3 \tag{18}
\]

where \(G_A(s)\) is the dynamics of the force-controlled pneumatic muscle actuator, and \(G_{SISO,mi}(s)\) is the dynamics of the \(i^{th}\) mode of the stress-ribbon bridge and \(\tilde{K}_{mi}(s)\) is the \(i^{th}\) controller dynamics.

The dynamics of the force-controlled pneumatic muscle actuator \(G_A(s)\) can be approximately described by a first order delay transfer function

\[
G_A(s) = \frac{1}{1 + T_T s} = \frac{a_0}{s + \frac{a_0}{T_T}} \tag{19}
\]

where \(T_T\) and \(a_0\) are constant values [5].

The transfer functions of first three modes of the stress-ribbon bridge are derived with Laplace transform of the modal state-space model for each mode respectively

\[
G_{SISO,mi}(s) = \frac{s}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}, i = 1, 2, 3. \tag{20}
\]

\(^2\) http://lintarget.sourceforge.net
The controller dynamics of the $i^{th}$ mode can be expressed as

$$\tilde{K}_{mi}(s) = k_{mi,v} \cdot e^{-sT_{di,v}}, i = 1, 2, 3$$

(21)

where the control parameters $k_{mi,v}, i = 1, 2, 3,$ and $T_{di,v}, i = 1, 2, 3,$ are chosen for each mode from a numerically calculated root locus plot of the corresponding mode. The time delay $T_{di,v}$ of the $i^{th}$ mode is chosen in such a way, that the modal control signal and the modal velocity get in phase. And the control gain $k_{mi}$ is assumed for positive feedback of the observed modal velocity of the $i^{th}$ mode. The optimal control parameters can be found by maximizing the damping of the controlled system.

As described in detail in [3, 5], the controller outputs will be transformed into reference forces for the three force-controlled PMAs.
The estimated modal velocities of the bridge with AVC were compared with those without AVC. The vibration response at the resonance mode remained low amplitudes during AVC. No spill-over effect was observed in the experiments.

4.3. Active vibration control of the stress-ribbon bridge with accelerometer-based estimation

Furthermore, the proposed estimation method was verified in active vibration control experiments with the stress-ribbon bridge. In the condition of resonance vibrations, the real-time estimated modal velocities of the bridge with AVC are compared with that of the bridge without AVC. In order to excite the resonance vibrations of the bridge, the pneumatic muscle actuators worked firstly as exciter, that means the bridge was excited by periodic forces generated from the PMAs. For the first resonance mode, the excitation signals for all PMAs were a sinusoidal force signal with a frequency of 1.3 Hz and an amplitude of 1000N. For the second resonance mode, the PMA at one fourth span of the bridge was given a sinusoidal force reference signal with a frequency of 2.5 Hz and an amplitude of 600N, while the PMA at another fourth span was applied with an inverted signal with the same frequency and amplitude. Afterwards the induced vibrations were actively damped by the pneumatic muscle actuator in AVC mode. For the purpose of comparisons, the experiments without AVC after excitations with PMAs were also conducted and the modal velocities were estimated.

In Figure 7, the estimated modal velocity with AVC (in blue) and that without AVC (in red) are plotted together for resonance vibrations at the first mode (left) and the second mode (right). It was observed in both resonance vibration experiments that the control strategy efficiently dampened the vibration response at the resonance mode. All modal velocities \( \dot{\tilde{x}}_{m_i}, i = 1, 2, 3 \) at the first 3 modes remained with low amplitudes during AVC. No spill-over effect was observed in the experiments.
5. Conclusions
In this work, accelerometer-based real-time estimation of modal state for active vibration control of a stress-ribbon bridge was investigated. The adaptive algorithm in form of a BMFLC-KF together with an analytical integration approach was applied to obtain drift-free estimates of nodal displacements/velocities from accelerometer sensing. The real-time estimated displacements/velocities were then used in modal velocity estimation by means of a Kalman Filter for the purpose of active vibration control. To confirm the feasibility of the proposed accelerometer-based estimation in a real-time environment, full-scale experiments were conducted on the bridge. The results from experiment show that accelerometer-based estimation can be applied in real-time to estimate the modal state of lightweight bridge structures without time delay.

In future, the accelerometer-based estimation could be applied for real-time estimation of the entire modal state of the stress ribbon bridge including torsional and horizontal modes.

6. Acknowledgments
The first author would like to thank the program TU Reisebeihilfen zu Tagungen und Konferenzen of Technische Universität Berlin for providing the travel grants.

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