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Ubiquity of Linear Resistivity at Intermediate Temperature in Bad Metals

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Bad metals display transport behavior that differs from what is commonly seen in ordinary metals. One of the most significant differences is a resistivity that is linear in temperature and rises to well above the Ioffe-Regel limit (where the mean-free path is equal to the lattice spacing). Using an exact Kubo formula, we show that a linear resistivity naturally occurs for many systems when they are in an incoherent intermediate-temperature state. First, we provide a simple analytic model to give intuition for this phenomenology. Then, we verify the analytic arguments with numerical calculations for a simplified version of the Hubbard model which is solved with dynamical mean-field theory. Similar features have also been seen in Hubbard models, where they can begin at even lower temperatures due to the formation of resilient quasiparticles.

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I. INTRODUCTION

Transport properties of strongly correlated materials, such as oxides in the families of vanadates, cobaltates or cuprates, Kondo semiconductors such as FeSi, FeSb, CeB$_6$ or SnB$_6$, and organic charge transfer salts are poorly understood, despite an overwhelming amount of experimental work which established non-Fermi-liquid behavior for these systems. One common feature of these vastly different materials is that they are formed by doping away from a Mott-Hubbard insulating state. Starting from this observation, and the ubiquity of quasi-linear non-Fermi liquid materials, we provide a simple explanation of the experimental data at moderate to high temperature.

We begin by deriving the transport coefficients using an analytic approach, in the spirit of Mahan and Sofo’s work on the best thermoelectrics, where the optimization of transport properties was calculated based on a simplified ansatz for the (vertex-corrected) transport relaxation time which then allowed one to perform the optimization. Here, we work in a similar vein, but consider the temperature dependence of the resistivity based on a general discussion of the properties of the transport relaxation time for a strongly correlated metal. By modeling this simplest form for correlated transport, the results should hold for a wide range of materials, and thereby explain the ubiquity of the linear resistivity at intermediate temperature. In the second part, we substantiate the phenomenological results by calculating the resistivity of a non-trivial model of strongly correlated electrons propagating on a d-dimensional lattice. We use the Falicov-Kimball model which, like the Hubbard or periodic Anderson model, has a gap in the excitation spectrum and, unlike these other models, admits an exact solution for the resistivity at arbitrary doping and temperature.

II. MODEL-INDEPENDENT PHENOMENOLOGY

Our starting point is the Kubo formula for the conductivity which reads,

$$\sigma_{dc}(T) = \sigma_0 \sum_\sigma \int d\omega \left( -\frac{df(\omega)}{d\omega} \right) \tau_\sigma(\omega),$$

(1)

where $\sigma_0$ is a material specific constant with units of conductivity, $-df(\omega)/d\omega$ is the derivative of the Fermi function that is sharply peaked around the chemical potential $\mu$, so that the integral is cut-off outside the Fermi window $|\omega| \geq k_B T$. The summation is over the spin states $\sigma$ and $\tau_\sigma(\omega)$ is the exact transport relaxation time which includes the velocity factors, averaged over the Fermi surface, and all the effects of vertex corrections, if present. We set $k_B = \hbar = 1$ and measure all energies with respect to $\mu$.

Since $\tau_\sigma(\omega)$ is nonnegative and vanishes for energies outside the band, it must have at least one maximum within the band. In a Fermi liquid, $\tau_\sigma(\omega)$ diverges as $T \to 0$ and $\omega \to 0$, and the resistivity, $\rho(T) = 1/\sigma_{dc}(T)$, follows a $T^2$ law at low temperature. If there is residual scattering, due to disorder for example, the divergence gets cut-off and the Fermi-liquid form no longer holds. In a pure strongly correlated metal, for temperatures above
the low-temperature coherence scale, the transport relaxation
time typically has two maxima, located in the upper
and the lower Hubbard bands, and neither the shape
nor the position of these broad maxima, relative to μ(T),
change appreciably with temperature. The transport re-
 laxation time of the Hubbard model, Falicov-Kimball
model, Anderson model, and other effective models of
strong correlations, exhibits these features. Since the
chemical potential of a strongly correlated metal is within
one of the two Hubbard bands, we calculate the resistiv-
ity focusing on τσ(ω) with just a single broad maximum
at ω0, neglecting the excitations across the gap.

The conductivity given by Eq. (1) crucially depends on
the overlap between −dψ(dω) and τσ(ω), i.e., on tem-
perature and doping. Temperature broadens the Fermi
window where the integrand is appreciable, while doping
changes the number of carriers, so that μ gets shifted
with respect to ω0. The value and the shape of τσ(ω)
around ω0 can also be doping dependent.

To estimate the resistivity we expand τσ(ω) around its
maximum at ω0,

\[ \tau_\sigma(\omega) \approx \tau_0 - \tau_1(\omega - \omega_0)^2, \]

where \( \tau_0 = \tau_\sigma(\omega_0), \) \( \tau_1 = -d^2\tau_\sigma(\omega)/2d\omega^2 \) \( \omega \to \omega_0 \), and we
use a simple model in which \( \tau_\sigma(\omega) \) is approximated by the
parabolic form in Eq. (2) for \( \Lambda_- < \omega < \Lambda_+ \) and \( \tau_\sigma(\omega) = 0 
\) otherwise; this form properly has a maximum, and shows
linear behavior as one approaches the band edges, as
expected for a three-dimensional material. The cutoffs \( \Lambda_\pm \)
are obtained by setting \( \tau_\sigma(\omega) = 0 \) in Eq. (2). This yields
\( \Lambda_\pm = \omega_0 \pm x_0 \), where \( x_0 = \omega_0/\tau_1 \) is inversely proportional
to the curvature of \( \tau_\sigma(\omega) \) at \( \omega_0 \) and \( x_0 \) has dimensions
of energy. Since the high-energy part of \( \tau_\sigma(\omega) \) does not
contribute much to the conductivity, \( x_0 = \omega_0 - \Lambda_0 \) often
defines an effective bandwidth relevant for transport of
a doped Mott insulator.

To evaluate the integral in Eq. (1), we introduce di-

dimensionless variables, \( \nu = \omega/x_0 \) and \( \tilde{T} = T/x_0 \),
and write the relaxation time as, \( \tau_\sigma(\nu)/\tau_0 = 1 - (\nu - \nu_0)^2 \)
where \( \nu_0 = \omega_0/x_0 \). Integrating by parts, and using
\( \tau_\sigma(\Lambda_-) = \tau_\sigma(\Lambda_+) = 0 \), yields

\[ \sigma_{dc}(\tilde{T}) = 2\tau_0\sigma_0 \int_{\nu_0-1}^{\nu_0+1} d\nu \frac{f(\nu)}{\nu} \frac{d\tau(\nu)}{d\nu}, \]

where \( f(\nu) = 1/[1 + \exp(\nu/\tilde{T})], \) \( d\sigma_d/d\nu = -2(\nu - \nu_0), \)
and we took the spin degeneracy into account. The inte-
grand is a regular function and the numerical evaluation
is straightforward. The renormalized resistivity, \( \rho(T)/\rho_0 \),
is shown in panel (b) of Fig. 1 for several characteristic values of \( \nu_0 \).

Panel (a) shows \( \tau_\sigma(\nu)/\nu_0 \) used for each of the resistivity curves. The data
indicate three types of behavior, depending on the rela-
tive position of \( \mu \) and \( \omega_0 \). Here \( \mu \) is fixed as a function of
\( T \), but as seen below, fixing the density instead, produces
similar results.

For \( \nu_0 \geq 1 \), when the chemical potential is close to
the band-edge, the resistivity decreases rapidly as tem-
perature increases from \( T = 0 \). At about \( T \approx \omega_0/2 \),
the resistivity drops to a minimum and then, increases
with temperature, assuming at about \( T \approx \omega_0 \) a linear
form. Such a behavior is typical of lightly doped Mott
insulators. For \( \nu_0 \leq 1 \), when the chemical potential is
just above the band edge, the low-temperature resistiv-
ity is metallic. It starts from a finite value, at \( T = 0 \),
and grows to a well pronounced maximum, which is re-
duced and shifted to lower temperature as \( \nu_0 \) is reduced.
The minimum still occurs at about \( T \approx \omega_0/2 \) and, for
\( T \geq \omega_0 \), the resistivity becomes a linear function in a
broad temperature range. Such a behavior is typical of
bad metals. For \( \nu_0 < 1 \), the chemical potential is close to
the maximum of \( \tau_\sigma(\nu) \) and \( \rho(T) \) increases parabolically

FIG. 1. (color online) Panel (a): Rescaled relaxation time
\( \tilde{\tau}_\sigma(\nu) \) plotted as a function of rescaled frequency \( \nu = \omega/x_0 \)
relative to the chemical potential, \( \mu \), which is indicated by the
vertical line at \( \nu = 0 \). (For definition of the scaling factors
see the text.) The different curves show \( \omega_\nu(\nu) \) shifted with
respect to \( \mu \) by \( \nu_0 = 0.1, 0.5, 0.75, 0.9, \) and 1.1, respectively.
Curve (a) corresponds to a dirty metal, curves (b), (c), and
(d) to a bad metal, and curve (e) to a slightly doped Mott
insulator. Panel (b): The rescaled resistivity obtained from
Eq. (3) plotted as a function of rescaled temperature \( \tilde{T} =
T/x_0 \). The different curves are obtained for \( \tilde{T}_0 \) as defined
in the panel (a).
The interaction of the conduction electrons with localized electrons is \( U \) and \( t^* \) is the hopping integral scaled so that we can properly take the \( d \to \infty \) limit. We work on both a hypercubic and Bethe lattice using units where \( t^* = 1 \). We maintain the paramagnetic constraint, \( \rho_{\sigma \bar{\sigma}} = \rho_{\bar{\sigma} \sigma} = \rho_c \), by equating the conduction and localized densities. For hole doping, we have \( \rho_c = \rho_f = 1 - \delta \leq 1 \), where \( \delta \) is the concentration of the holes in the lower Hubbard band, while for electron doping, \( \rho_c = 1 + \delta \geq 1 \), where \( \delta \) is the concentration of electrons in the upper Hubbard band.

The model is solved using DMFT in the infinite dimensional limit \( d \to \infty \), such that the self-energy \( \Sigma(\omega) \) is a functional of the local conduction electron Green’s function, \( G_{\text{loc}}(\omega) \), and the full lattice problem is equivalent to a single-site model with an electron coupled self-consistently to a time-dependent external field. Several reviews, whose notation we adopt, now exist both on DMFT generally and on the exact DMFT for the Falicov-Kimball model. We find \( \Sigma(\omega) \), \( G_{\text{loc}}(\omega) \), and the local density of conduction states \( \rho_{\text{loc}}(\omega) = -\text{Im} G_{\text{loc}}(\omega)/\pi \) numerically using methods described elsewhere.

For \( \rho_c = 1 \), \( \rho_{\text{loc}}(\omega) \) is symmetric and, for large enough \( U \), we have a Mott insulator in which a filled lower Hubbard band is separated from an empty upper Hubbard band by a band gap with the chemical potential in the middle of the gap \( (U_c = \sqrt{2} \) for the hypercubic lattice and \( U_c = 2 \) for the Bethe lattice). Away from half-filling, \( \rho_{\text{loc}}(\omega) \) is asymmetric and for electron doping, which is the case we consider, the chemical potential is in the upper Hubbard band. Its distance from the lower band edge \( \Delta_c \) is determined by charge conservation \( \delta = 2 \int d\omega f(\omega)\rho_{\text{loc}}(\omega) - 1 \).

For \( d \to \infty \), the vertex corrections to the conductivity vanish and explicit formulas can be found for the relaxation time. On the Bethe lattice, this yields:

\[
\tau_{\sigma}(\omega) = \frac{1}{3\pi^2} \text{Im}^2\left[G_{\text{loc}}(\omega)\right] \left(\frac{|G_{\text{loc}}(\omega)|^2}{|G_{\text{loc}}(\omega)|^2 - 1} - 3\right). \tag{5}
\]
while on the hypercubic lattice, we have\(^{16}\):

\[
\tau_\sigma(\omega) = \frac{1}{4\pi^2} \frac{\text{Im} \, G_{\text{loc}}(\omega)}{\text{Im} \, \Sigma(\omega)} + \frac{1}{2\pi^2} \{1 - \text{Re} [(\omega + \mu - \Sigma(\omega)) G_{\text{loc}}(\omega)]\}.
\]

For fixed \(\rho_f\), the shape of \(\tau_\sigma(\omega)\) is independent of temperature. In a Fermi liquid, where one can approximate\(^{15}\)

\[\tau_\sigma(\omega) \simeq \frac{\text{Im} \, G_{\text{loc}}(\omega)/\text{Im} \, \Sigma(\omega)}{\text{Im} \, \Sigma(\omega) \rightarrow 0} \rightarrow 0,\]

the relaxation time \(\tau_\sigma(\omega)\) diverges as \(\omega \rightarrow 0\). In the Falicov-Kimball model, however, \(\text{Im} \, \Sigma(0)\) does not vanish and \(\tau_\sigma(0)\) remains finite. For large \(U\), the width of the single-particle excitations exceeds their energy leading to overdamped excitations rather than with quasiparticles, such that the Fermi liquid description is not applicable.

The transport relaxation time of the Falicov-Kimball model due to such overdamped excitations, obtained for a fixed value of \(U\) and several values of \(\delta\), is shown in Fig. 2. The left and right panel show the results for the hypercubic and Bethe lattice, respectively. Note the similarity to the inverse quadratic approximation used in the first part. The transport relaxation time vanishes below the band edge \(\Lambda_-\) and has a peak at the energy \(\omega_0\) in the upper Hubbard band (for electron doping). As \(\delta\) increases, \(\omega_0\) and \(\Lambda_-\) decrease but the difference \(\omega_0 - \Lambda_-\) remains approximately constant. The resistivity obtained for the same set of parameters is shown in Fig. 3. The doping dependence of \(\rho(T)\) follows from the observation that \(\delta\) reduces \(\omega_0\) and that, for \(\Lambda_- < \mu < \omega_0\), the Fermi window removes the contribution of the high-energy part of \(\tau_\sigma(\omega)\). Close to half-filling (very small \(\delta\)), where \(\mu \simeq \Lambda_- \ll \omega_0\), the resistivity exhibits a low-temperature peak, then, drops to a minimum at about \(T \approx \omega_0/2\) and, eventually, becomes a linear function of \(T\), for \(T \geq \omega_0\). An increase of \(\delta\) brings \(\omega_0\) closer to \(\mu\), which reduces the resistivity maximum and brings the onset of the linear region to lower temperatures. For a sufficiently large \(\delta\), the maximum is completely suppressed and the resistivity is a monotonically increasing function of temperature. For \(\delta \approx 0.2\), we find \(\omega_0 \approx \mu\) and obtain a resistivity with a well defined \(T^2\) term at the lowest temperatures. Note, the crossover between different regimes can also be induced by pressure which modifies the hopping integrals and shifts \(\omega_0\) with respect to \(\mu\).

**IV. CONCLUSIONS**

The results obtained for the Falicov-Kimball model are in complete agreement with the phenomenological theory presented in the first part of the paper. Hence, the analytic model is verified as providing the generic behavior of a doped Mott insulator at intermediate \(T\). The central result of this paper is that the linear resistivity seen in strongly correlated materials at intermediate \(T\) is governed by the appearance of a maximum in \(\tau_\sigma(\omega)\) above the chemical potential. The slope of the linear resistivity does not vary much for a range of chemical potentials near the maximum, so the temperature dependence of \(\mu(T)\) does not change this behavior. In other correlated models like the Hubbard model, the linear resistivity will disappear when \(T\) is reduced below the renormalized Fermi-liquid scale, but it appears that the resilient quasiparticle picture\(^{25}\) allows the linear region to be brought down to even lower \(T\)'s than seen in the Falicov-Kimball model. In the very high \(T\) limit, where \(T\) is bigger than the bandwidth, general arguments\(^{21}\) show that the resistivity is linear for the Bethe lattice, but saturates at a constant for the hypercubic lattice. Those results are complementary to the general linear resistivity here, found for temperatures much less than the bandwidth.

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