SEDs of Flared Dust Disks

Radiation Transfer Model Versus 2-Layer Model

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Abstract
A radiation transfer model for the top-layer of a flared dust disk around a low mass star is presented. The disk structure, temperature distribution and spectral energy distribution are compared with the results of the analytical two-layer model of Chiang & Goldreich. We find that both, a proper iterative calculation of the main parameters and proper radiation transfer calculations, are important to not overestimate the disk structure and its emission especially at long wavelengths.

Introduction
Many T Tauri stars are observed with a flat spectral energy distribution (SED) in the near infrared (e.g. Beckwith et al. 1990). This is in contradiction with the standard disk which is usually modeled as a flat blackbody. The flat spectrum in the near infrared hints to an additional hot dust component in the circumstellar region of young stars. Several models have been proposed, e.g. a dusty halo (e.g. Miroshnichenko et al. 1999) or a dust shell beyond the outer edge of the disk (e.g. Gregorio-Hetem & Hetem 2002). Both models can fit observed SEDs. However, it is not clear how such a configuration of the circumstellar dust should form.

We are dealing here with a different location of the hot dust component, namely the surface of the circumstellar disk. For the surface to become much hotter than the interior, it must be inclined with respect to the infalling stellar radiation, i.e. the disk must be flared. Such a flaring can happen naturally when the disk is in hydrostatic equilibrium in z-direction with a temperature distribution that falls off more slowly than $r^{-1}$ (Kenyon & Hartmann 1987).
Chiang & Goldreich (1997, hereafter CG) developed an analytical two-layer model. Their flared dust disk contains an optically thick isothermal mid-layer and a superheated surface layer. CG provide simple analytical expressions to calculate the SEDs which are indeed flat in the near infrared, and their model is used by many people to fit the SEDs of young stellar objects.

Our intention was to study the reliability of these analytical two-layer models. We developed a radiation transfer code for the upper disk part to calculate the parameters that determine the disk structure and temperature distribution self-consistently, and we compare our results with the CG model. Note that we compare our results to the original analytical CG model. A comparison between an improved two-layer model with a radiation transfer model has recently been done by Dullemond & Natta (2003).

1. Description of the dust disk model

Our disk consists of an optically thick ($\tau_{\text{mid}}^V \gg 1$) mid-layer which is isothermal in $z$-direction. The mid-layer is sandwiched by two toplayers which are marginally optically thick at visual wavelengths but still optically thin at IR wavelengths, i.e. $\tau_{\text{top}}^V \geq 1$ and $\tau_{\text{IR}}^\text{top} \leq 1$. Consequently, the infalling stellar light is completely absorbed within the top-layer. The dust particles re-radiate the energy at IR wavelengths, so half the redistributed energy leaves the disk into space, and the other half penetrates the mid-layer and heats it. As a third distinct region we define the disk photosphere which encloses the uppermost part of the top-layer and whose location is defined by the parameter $h$ (see Sect. 1.2).

Further, we make the following assumptions: the disk is in hydrostatic equilibrium in $z$-direction, gas and dust are well mixed throughout the disk, and in the mid-layer gas (g) and dust (d) are in thermal equilibrium at the same temperature $T_g = T_d = T_{\text{mid}}$. The emission and absorption of the gas component of the disk is ignored.

We restrict the discussion to passive disks, which means that the only heating source is the star which illuminates the disk surface. In addition, our (gas) disk extends down to the stellar surface with no inner hole. The existence of such a hole would lead to a puffed-up inner rim and self-shadowing effects of the disk (see Dullemond et al. 2001 and Dullemond 2002).

1.1 Stellar illumination of the disk surface

For a razor-thin passive disk around a star with effective temperature $T_\ast$ and radius $R_\ast$, the monochromatic flux entering the disk perpendicular
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Figure 1 Sketch of a flared disk. The flaring term of the grazing angle is indicated. The inner radius of the dust disk is \( r_0 \).

to the surface at distance \( r \) from the star is

\[
\mathcal{f}_\nu^\perp = B_\nu(T_\star) \left[ \arcsin \frac{R_\star}{r} - \frac{R_\star}{r} \sqrt{1 - \left( \frac{R_\star}{r} \right)^2} \right]
\]

where the star is assumed to emit a Planck spectrum. This flux can be parametrized in terms of the solid angle \( \Omega \) under which the star is seen from a disk surface element at distance \( r \) and the so-called grazing angle \( \alpha_{\text{gr}} \), i.e. the mean angle of incidence of the stellar flux

\[
\mathcal{f}_\nu^\perp = \alpha_{\text{gr}} \Omega B_\nu(T_\star)
\]

The grazing angle of a razor-thin disk is therefore

\[
\alpha_{\text{gr}}^{\text{razor}} = \frac{1}{\Omega} \left[ \arcsin \left( \frac{R_\star}{r} \right) - \frac{R_\star}{r} \sqrt{1 - \left( \frac{R_\star}{r} \right)^2} \right]
\]

which for large distances reduces to the handy formula used by CG

\[
\alpha_{\text{gr}}^{\text{razor}} \gg \frac{R_\star}{3\pi} \quad r \quad \frac{4}{\pi} \frac{R_\star}{3\pi} \quad r \simeq 0.4 \frac{R_\star}{r}
\]

This relation also holds for a wedge-shaped disk as long as its opening angle is small.

In a flared disk the surface is curved and the grazing angle increases with distance from the star (see the the solid lines in Fig.1). Therefore, the flared disk intercepts more stellar light and is heated more efficiently. The grazing angle of the flared disk is by the amount \( \alpha_{\text{gr}}^{\text{flare}} \) greater than for a flat disk (Fig. 1), and the total grazing angle becomes simply \( \alpha_{\text{gr}} = \alpha_{\text{gr}}^{\text{razor}} + \alpha_{\text{gr}}^{\text{flare}} \). This flaring term of the grazing angle, \( \alpha_{\text{gr}}^{\text{flare}} \), can be computed from

\[
\alpha_{\text{gr}}^{\text{flare}} = \arctan \left( \frac{dh}{dr} \right) - \arctan \left( \frac{h}{r} \right) \simeq r \frac{d}{dr} \left( \frac{h}{r} \right)
\]

With this new grazing angle, which is a function of the location \( h \) of the photosphere, we can calculate the flux penetrating the flared disk in \( z \)-direction.

\[ h \geq r \]
1.2 The location of the disk photosphere

The photosphere encloses the uppermost part of the top-layer. Its onset is described by the parameter \( h \), which is defined as the height \( z \) above the mid-plane where the visual optical depth along the direction of the infalling stellar light, i.e. along the grazing angle, equals 1. This leads to a vertical optical depth of

\[
\tau^\perp_V = \sin \alpha_{\text{gr}} .
\]  

(6)

Since the disk is assumed to be in hydrostatic equilibrium, \( \tau^\perp_V \) can be calculated

\[
\tau^\perp_V = \kappa_V \rho_0(r) \int_{h}^{\infty} e^{-\frac{\tau^\perp_V}{2H}} \, dz
\]  

(7)

where \( \rho_0 \) is the density in the mid-plane and \( H \) is the scale height of the disk given by

\[
H = \sqrt{\frac{kT_\odot}{GM_* \mu}} r^{3/2}
\]  

(8)

Here, \( M_* \) and \( \mu \) are the stellar mass and the mean molecular weight.

From equaling Eqs. (6) and (7) \( h \) can be determined, but it is a function of temperature and grazing angle.

1.3 The temperature of the mid-layer

The downwards and upwards directed fluxes, \( F^\downarrow \) and \( F^\uparrow \), are in equilibrium throughout the passive disk. We can calculate these two fluxes explicitly at the boundary between the isothermal mid-layer and the top-layer. We assume that half the incident stellar flux is re-radiated into space and the other half penetrates the mid-layer, leading to

\[
F^\downarrow = \frac{1}{2} f^\perp = \frac{1}{2} \int \alpha_{\text{st}} \Omega B_\nu(T_\star) \, d\nu
\]  

(9)

The flux leaving the mid-layer in upward direction is given by

\[
F^\uparrow = 2\pi \int \int B_\nu(T_{\text{mid}}) \left( 1 - e^{-\tau_\nu/\mu} \right) \mu \, d\mu \, d\nu
\]  

(10)

where \( T_{\text{mid}} \) is the isothermal temperature of the mid-layer, and we set \( \mu = \cos \theta \) with \( \theta \) as the angle measured from the \( z \)-axis. The visual optical depth of the mid-layer in vertical direction can be written in the form

\[
\tau^\text{mid}_V(r) = \tau^\text{mid}_V(r_0) \left( \frac{r}{r_0} \right)^{-s}
\]  

(11)
with the visual optical depth at the inner edge, \( \tau_{\text{mid}}^{\text{V}}(r_0) \), and the exponent \( s \) as free parameters. The temperature of the mid-layer, \( T_{\text{mid}} \), follows from equaling Eqs. (10) and (9). This temperature is, however, a function of the grazing angle.

We have now three important parameters, \( \alpha_{\text{gr}}, h, \) and \( T_{\text{mid}} \), but none of them can be computed independently, instead they must be calculated iteratively.

1.4 Radiation transfer within the top-layer

We have to specify the visual optical depth in the top-layer in \( z \)-direction, \( \tau_{\text{V}}^{\text{top}} \). It should be high enough so that first, the infalling stellar radiation is completely absorbed and second, the dust grains at the bottom of the top-layer reach a temperature close to that of the mid-layer in order to guarantee a smooth transition. On the other hand, \( \tau_{\text{V}}^{\text{top}} \) must be small enough to allow the dust emission, which occurs at infrared wavelengths, to escape the top-layer.

The radiation transfer equation of a plane-parallel slab,

\[
I_{\nu}(\mu, \tau_{\nu}) = I_0 \frac{\tau_{\nu}}{\mu} - \int S_{\nu}(t)e^{-\frac{(t-\tau_{\nu})}{\mu}} \frac{dt}{\mu}
\]  

with the source function \( S_{\nu} \), the incident intensity \( I_0 \) and \( \mu = \cos \theta \) is split into up-streams, \( I_{\nu}^{+} \), that penetrate the top-layer from the mid-layer, and down-streams, \( I_{\nu}^{-} \), that cross the top-layer starting from the surface. The incident intensity is either the stellar radiation for downwards directed streams (but only for angles \( \theta \) under which the star can be seen, else it is zero), or the emission of the mid-layer, \( B_{\nu}(T_{\text{mid}})(1 - e^{-\tau_{\text{V}}^{\text{mid}}/\mu}) \), for the upwards directed streams.

With the help of the Feautrier parameters

\[
u_{\nu} = \frac{1}{2} (I_{\nu}^{+} + I_{\nu}^{-}) \quad \text{and} \quad u_{\nu} = \frac{1}{2} (I_{\nu}^{+} - I_{\nu}^{-})
\]  

the mean intensity \( J_{\nu} \) becomes

\[
J_{\nu} = J_{\nu}(\tau_{\nu}) = \int u_{\nu}(\mu, \tau_{\nu}) \, d\mu
\]  

which is needed to calculate the source function and the emission of the grains. The source function itself determines the up- and down-streams of the intensity; the radiation field calculation must therefore be iterated.

Finally, the temperature of the grains at each location in the top-layer follows from balancing their absorption from the surrounding radiation field, \( J_{\nu} \), and their emission at their equilibrium temperature, \( T_{\text{d}} \).
2. Results

Since we want to compare our results with those of CG we took the same parameters as they did: for the star, \( T_* = 4000 \text{ K}, L_* = 1.44 L_\odot, \) \( R_* = 2.5 R_\odot, M_* = 0.5 M_\odot; \) for the disk, \( r_{\text{in}} \simeq 0.07 \text{ AU}, r_{\text{out}} = 270 \text{ AU}, \) \( \tau_{\text{V mid}}(r) \simeq 4 \cdot 10^5 r_{\text{AU}}^{1.5}; \) for the dust grains, spheres of radius \( a = 0.1 \mu\text{m} \) with mass density \( \rho_d = 2 \text{ g cm}^{-3} \) and absorption efficiency \( Q = 1 \) for \( \lambda \leq 2\pi a \) and \( Q \simeq 2\pi a/\lambda \) for \( \lambda \geq 2\pi a. \) The latter implies an absorption coefficient \( \kappa_V \simeq 4 \times 10^4 \text{ cm}^2 \text{ per gram of dust}. \) Scattering is absent and we use \( \tau_{\text{V top}} = 3. \) The disk is seen face-on.

We first compare our height \( h \) of the photosphere (Fig. 2) with the CG results. The ratio \( h/r \) which determines \( \alpha_{\text{gr}} \) can be written in terms of the scale height \( H \)

\[
\frac{h}{r} = \frac{h}{H} \frac{H}{r} \quad (15)
\]

According to CG the ratio \( h/H \) drops from 5 at 3 AU to 4 at 100 AU but they use \( h/H = 4 \) throughout their calculations which leads to an extremely thick flared disk with \( h(r = 270 \text{ AU}) = 270 \text{ AU}. \) Our calculations show, however, that \( h/H \) drops from about 6 at \( r_0 \) to 2.8 at the outer edge, and our disk stays much flatter at large distances. In
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Figure 4. Temperature distribution within the top-layer at \( r_o \) (top) and at \( r = 250 \) AU (bottom). The star is the CG value, square and triangle are our and the CG mid-layer value.

Figure 5. Comparison between CG and our results for the top-layer (top), mid-layer (middle) and total SED (bottom).

a recent work Chiang et al. (2001) corrected their old results for the disk thickness and their corrected photosphere almost agrees with our results.

The temperature in the mid-layer, \( T_{\text{mid}} \) (Fig. 3), is not so different in the two models. Our mid-layer is slightly hotter especially in the inner parts (\( r \leq 20 \) AU) but comparable with CG for larger distances. Our temperature calculations in the top-layer result in a temperature gradient between the very hot surface and the bottom of the top-layer. In Fig. 4 we compare our temperature distribution with the CG surface temperature as well as with our and the CG mid-layer temperatures at two different distances. We cannot reproduce the very hot surface temperature of CG, but on the bottom of the top-layer (at \( \tau_{V}^{\text{top}} = 3 \)) the temperature approaches everywhere the value of the mid-layer (see bottom panel of Fig. 3).

The differences in the disk and temperature structure between the CG model and our more detailed radiation transfer calculations lead of course also to differences in the SED (Fig. 5). The emission from the CG
surface layer which is much hotter than our top-layer is shifted to shorter wavelengths. The emission of the CG mid-layer comes from a narrower wavelength region than in our model and is too low. The total SED of CG therefore deviates from our results especially in the long wavelength region ($\lambda \geq 30 \mu m$).

3. Conclusions
A radiation transfer model for the upper disk part of a flared disk is presented and the results are compared with the analytical two-layer model of CG. We show that the major parameters that determine the structure of the disk, $\alpha_{gr}$, $h$, $T_{mid}$, must be calculated iteratively. In addition we perform detailed radiation transfer calculations in the top-layer to find the temperature structure in the upper disk part. At the bottom of the top-layer the temperature agrees very good with the values of the mid-layer. The very hot surface temperature found by CG could not be reproduced, and our total SED deviates from the one of CG especially in the long wavelength regime. Although the two-layer model gives handy formulae to calculate the SED of a flared dust disk, the results should only be used as a ‘first guess’ for the structure and temperature distribution of the disk and radiation transfer calculations should be performed for a better characterization of the circumstellar dust disk.

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