Pretrial Negotiations Under Optimism

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Abstract

We develop a tractable and versatile model of pretrial negotiation in which the negotiating parties are optimistic about the judge’s decision and anticipate a possible arrival of public information about the case prior to the trial date. We derive the agreement dynamics and show that negotiations result in either immediate agreement, a weak deadline effect — settling sometime before the deadline, a strong deadline effect — settling at the deadline, or impasse, depending on the level of optimism. We show that the distribution of the settlement times has a U-shaped frequency and a convexly increasing hazard rate with a sharp increase at the deadline, replicating stylized facts about such negotiations.

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1 Introduction

Costly settlement delays and impasse are common in pretrial negotiations. While only about 5% of the cases in the United States go to court, the parties settle only after long, costly delays.\(^1\) Excessive optimism has been recognized as a major cause of delay and impasse in pretrial negotiations, especially when the optimistic parties learn about the strength of their case as they negotiate.\(^2\) In this paper, we develop a tractable model of pretrial negotiations in which the optimistic parties may receive public information about the strength of the case. We determine the dates at which a settlement is possible and obtain sharp patterns of behavior as an outcome of our model. Our analysis predicts some well-known stylized facts and also makes a few novel predictions.

We build on the canonical pretrial negotiation model. A plaintiff has filed a case against a defendant, and they negotiate over an out-of-court settlement. At each date, one party is randomly selected as a proposer; the proposer proposes a settlement amount and the other party accepts or rejects. If the parties cannot settle by a given deadline, a judge decides whether the defendant is liable. If the judge determines that the defendant is liable, then the defendant pays a fixed amount \(J\) to the plaintiff; otherwise, he does not pay anything. Delays are costly, in that each party pays a daily fee until the case is closed and pays an additional cost if they go to court. Unlike in the standard model, we assume that the plaintiff assigns a higher probability to the defendant’s being found liable in court than the defendant does; the difference between the two probabilities is the level of optimism, denoted by \(y\). In our baseline model, we further assume that as time passes, the negotiating parties may learn the strength of their respective cases by observing an arrival of new public information: a decisive piece of evidence that arrives according to a Poisson process, revealing what the judge will decide. Note that, for consistency, the probability that the evidence would reveal

\(^1\)Average settlement delay in malpractice insurance cases is reported to be 1.7 years (Watanabe (2006)). The legal cost of settlement delays in high-stake cases are sometimes on the order of tens of millions of dollars. In the well-known case of Pennzoil v. Texaco, the legal expenses were several hundreds of millions of dollars, and the case was settled for 3 billion dollars after a long litigation process (see Mnookin and Wilson (1989) and Lloyd (2005)). In commercial litigations, ongoing litigations also have indirect costs due to uncertainty, delayed decisions, missed business opportunities, and suppressed market valuation, and these costs may dwarf the legal expenses above.

\(^2\)See Neale and Bazerman (1985), Babcock et al. (1995), and Babcock and Loewenstein (1997) for the empirical evidence for excessive optimism among negotiators; see Landes (1971), Posner (1972), Farber & Katz (1979), and Shavell (1982) for classical static models in which optimism causes disagreement; See Yildiz (2004) for a dynamic model of bargaining in which learning causes delays in bargaining. See Yildiz (2011) for an extensive review of the literature on bargaining under optimism.
a verdict in favor of the plaintiff (that is, a verdict that finds the defendant liable) upon arrival is equal to the probability that the court would find in favor of the plaintiff at the court date, and hence the level of optimism about the nature of the information is also $y$. It is also worthwhile to emphasize that our baseline model assumes the American Rule: each party incurs its own costs regardless of the outcome. Accordingly, the plaintiff can withdraw her case at any time, and hence she cannot commit to continue pursuing her case once it is revealed that the court will find in favor of the defendant.

The basic dynamics and the logic of delay in our model are as follows. After the arrival of information, our model is identical to the standard bilateral bargaining model, and the parties agree immediately. The settlement amount depends on the nature of the information. If the information reveals a verdict in favor of the plaintiff, a settlement amount $S$ is determined by splitting the savings from negotiation and litigation costs according to the probability of making an offer for each party, which reflects that party’s bargaining power in the standard model. If the information reveals a verdict in favor of the defendant, then the settlement amount is zero. Since $S$ is non-negative, the players are optimistic about the settlement amount after an arrival of information in equilibrium, and so, the difference between the expected settlements after an information arrival is $yS$. Such optimism turns out to be the main force towards a delay. In equilibrium, the parties strategically settle without an information arrival at a given date if and only if the expected benefit from waiting for $yS$ through a future arrival of information is lower than the total cost of waiting. Hence, we can determine the dynamics of strategic settlement by simply analyzing the settlement amount $S$ in the standard model.

It turns out that the resulting pattern of behavior relies heavily on which party has the stronger bargaining power. When the plaintiff has the stronger bargaining power, $S$ is an increasing function of the remaining time until the court date, since the plaintiff gets some of the defendant’s cost savings in the settlement. Hence, the incentive for delay increases as negotiations get farther from the deadline. This results in a sharp prediction for the timing of the settlement. For high values of optimism $y$, the players never settle strategically. They go to court if information does not arrive, no matter how far the court date is. We call such an outcome *impasse*. For intermediate values of optimism $y$, they wait for an information arrival until the last possible day for settlement and strategically settle at the deadline. Such an agreement on the steps of the courthouse is commonly observed in real-world negotiations and is referred to as the *deadline effect* in the literature, as we discuss below. For low values of optimism, equilibrium is characterized by a weaker version of the deadline effect: the
parties wait until a fixed number of days before the deadline to settle strategically.

On the other hand, if the defendant has the stronger bargaining power, then $S$ is a decreasing function of the time remaining until the deadline. In that case, the incentive to delay decreases as we go away from the deadline. In equilibrium, the parties strategically settle either at the beginning or at the deadline, but never in between. Then, except for a settlement due to information arrival, the only possible outcomes are: immediate agreement (for low values of $y$ or long deadlines), the deadline effect (for intermediate values of optimism) and impasse (for high values of optimism).

Our model leads to sharp empirical predictions on the distribution of settlement times, which is a combination of settlements due to information arrival and strategic settlement. There are point masses at the beginning and at the deadline—due to immediate agreement and the deadline effect, respectively. In between, the overall frequency of settlements is decreasing, yielding a U-shaped pattern. This is in line with empirical regularities. The frequency of settlements decreases in the duration of negotiations (e.g. see Kessler (1996)), but a significant number of cases settle at the deadline in studies that keep track of the court date. For example, Williams (1983) reports that 70% of civil cases in Arizona were settled within 30 days of the court date and 13% were settled on the court date itself. A U-shaped distribution of settlements also arises in some bargaining models with incomplete information (Spier (1992) and Fanning (2014)). A more subtle parameter that is considered in the empirical literature is the hazard rate of settlement, which measures the frequency of settlements among cases that are ongoing. The hazard rate in our model is increasing and convex—with a point mass at each end. The empirical studies that we are aware of are mixed: Fournier and Zuehlke (1996) estimates a convexly increasing hazard rate as in here, while Kessler (1996) reports a mildly decreasing hazard rate.

Our model is highly versatile and can be adapted to various different environments. To illustrate this, we study the reform of switching from the American Rule to the English Rule. As in our baseline model, the American Rule, which is used in most of the United States, requires each party to pay its own legal costs regardless of the outcome of the trial. The English Rule, which is used in most of England and Canada, requires that the loser of the trial pay all of the legal fees incurred in relation to the case if a case reaches court. Since each rule has widespread use, the merits of imposing one over the other in different situations is widely debated among policy makers. For example, there have been trials of imposing forms of the English Rule in Florida and Alaska, and as of July 2015, there is a standing bill in the United States congress to impose the English Rule for litigation relating to computer
hardware and software patents.\(^3\)

We show that under the English rule, there is more disagreement and there are longer delays among the cases that do settle. Thus, litigation for cases raised is longer and costlier overall. The logic of our result is straightforward. In our model, switching from the American rule to the English rule is mathematically equivalent to adding to the judgment amount \(J\), the total costs \(C\) incurred— both to be paid by the loser under the English Rule. Such an increase in \(J\) unambiguously increases the incentive to delay under optimism. This is because such an increase in stakes translates directly to an increase in optimism about the future. The reform adds the costs \(C\) to the settlement \(S\), increasing the level of optimism about a future settlement to \(y(S + C)\) from \(yS\), increasing the incentive to wait. Our result is consistent with the existing results from static models, which show that the English rule causes a higher fraction of cases to go to trial (see, for example, Shavell (1982) for a static model of heterogeneous beliefs and Bebchuk (1984) for a static model of asymmetric information). Our model adds a dynamic dimension to this analysis, and shows that in addition to more trials, the English rule also causes longer delays in settlement for the cases that do not make it to trial.\(^4\)

Although empirical comparisons across countries are difficult due to endogenous differences, evidence from the Florida and Alaska experiments provides some support for our predictions. Snyder and Hughes (1990) and (1995) find that medical malpractice suits that were not dropped were more likely to go to court rather than be settled prior to court under the English Rule than under the American Rule, while Rennie (2012) finds no statistically significant difference in lawsuits pursued between district courts that use the English Rule and those that use the American Rule.

We also extend our model by allowing the rate of information arrival to vary over time. As an application, we study the impact that the timing of a period of discovery, in which information arrives at a higher rate, might have on the frequency of agreement. A forthcoming discovery period increases the incentive for the bargaining parties to wait for information, potentially causing delays in settling. Taking the case that the plaintiff has a stronger position in bargaining as an example, we illustrate why discovery periods should be scheduled early in negotiations in order to avoid additional delays in agreement and induce early settlement due to uncovered information.

Our paper is closely related to the literature on the dynamics of bargaining under op-

\(^3\)See for example, Bill H.R. 9 of the Innovation Act of 2015.

\(^4\)A commonly discussed benefit of the English rule is that it deters cases that are unlikely to succeed from being litigated in the first place (Shavell (1982)); we rule out such cases in our analysis.
timism. Yildiz (2003) introduces a dynamic model of bargaining under optimism in which the parties are optimistic about their probability of making offers in the future, which is the main source of bargaining power in those models. Yildiz (2003) shows that optimism alone cannot explain the bargaining delays: there is immediate agreement whenever the parties remain sufficiently optimistic for sufficiently long. Within the framework of Yildiz (2003), Yildiz (2004) shows that optimism causes substantial delays when the parties expect the arrival of new information in the future. They wait without settling in the hopes that they will persuade the other party as new information arrives. A persuasion motive for bargaining delays such as this is also central in our paper. Our key difference is that we study a more descriptive model of pretrial negotiations in which the optimism is about the judge's decision, rather than the probability of making an offer as an abstract measure of bargaining power.

More closely to our paper, Watanabe (2006) develops a detailed model of pretrial negotiations, in which the parties are optimistic about the judges decision. His model is more general than ours: multiple partially informative pieces of evidence arrive according to a Poisson process (while we have a single decisive piece of evidence), and the timing of the plaintiff's submission of her case and thereby the court date, is endogenous (while both are exogenously fixed in our paper). By focusing on a less general and more tractable model, we are able to analytically derive the agreement dynamics, finding simple explicit formulas for the timing of the settlements and the cutoffs on the level of optimism that determine whether there is immediate agreement, a weak deadline effect, a strong deadline effect, or an impasse. This further allows us to derive the distribution of settlement times analytically. In contrast, Watanabe (2006) mainly focuses on structurally estimating his more general model on his dataset.

Finally, Simsek and Yildiz (2014) show that optimism about the future probability of making an offer leads to a deadline effect. However, the logic of the two results are quite different. In the Simsek-Yildiz model, the impact of bargaining power is largest at the deadline because the cost of delay is highest there. Hence, parties that are optimistic about their bargaining power wait until that moment in order to realize the large gain there. In our model, what entices the players to wait is optimism about the information that may arrive within the next moment instead. They wait as long as the impact of such information, which is measured by $S$, is sufficiently large. In the case of a strong plaintiff, $S$ actually decreases as the deadline approaches, and this is what causes the deadline effect. Our players wait until $S$ becomes sufficiently small or they hit the deadline, where the cost-benefit analysis is different.
because of the different costs and information revelation mechanism in the courthouse.

In the next section we present our baseline model. We present the agreement dynamics in our baseline model in Sections 3-5. In Section 6, we extend our model by allowing the rate of information arrival to vary over time. We derive the distribution of settlement times and resulting empirical predictions of our model in Section 7. In Section 8, we study an alternative model in which the plaintiff is committed to continuing to pursue her case regardless of information arrival. We study the impact of adopting the English Rule in Section 9. We present a generalization in which signals are partially informative in Section 10. Section 11 concludes. The proofs are relegated to the appendix.

2 Model

We consider the canonical pre-trial negotiation model, but assume that the parties have optimistic views about the case and they receive public information about the outcome of the case during the negotiation. We fix a time interval \( T = [0, \bar{t}] \) for some \( \bar{t} > 0 \) and consider a plaintiff and a defendant, both risk neutral. The plaintiff has sued the defendant for a damage that she has incurred, and a judge will decide whether the defendant is liable at \( \bar{t} \). They negotiate over an out-of-court settlement in order to avoid the litigation costs that would be incurred in advent of and during court.

There are two states of the world: one in which the defendant is liable, denoted by \( L \), and one in which the defendant is not liable, denoted by \( NL \). At the beginning, the parties do not know the true state, and have differing subjective beliefs about the state: the plaintiff and the defendant assign probability \( q_P \) and \( q_D \) on state \( L \), respectively. We assume that the parties are optimistic, i.e., \( q_P > q_D \), and write

\[
y = q_P - q_D
\]

for the initial level of optimism.

As time passes, they may learn the state by observing public information. In particular, we assume that a decisive piece of evidence arrives according to a Poisson process with positive arrival rate \( \lambda > 0 \) throughout \( T \), revealing the true state. That is, if the defendant is liable, then an arrival proves his liability; similarly, if the defendant is not liable, an arrival extricates him from liability. If the parties do not settle out of court, the true state will be revealed in court. The arrival rate \( \lambda \) is assumed to be independent of the state.

We consider the following standard random-proposal bargaining model. The parties can strike a deal only on discrete dates \( t \in T^* \equiv \{ n\Delta | n\Delta < \bar{t}, n \in \mathbb{N} \} \) for some fixed positive
where $\Delta$, where $\mathbb{N}$ is the set of natural numbers. We will write $\hat{t} = \max T^*$ for the last date they can strike a deal. At each $t \in T^*$, one of the parties is randomly selected to make an offer where the plaintiff is selected with probability $\alpha \in [0,1]$ and the defendant is selected with probability $1 - \alpha$. The selected party makes a settlement offer $S_t$, which is to be transferred from the defendant to the plaintiff, and the other party accepts or rejects the offer. If the offer is accepted, then the game ends with the enforcement of the settlement. If the offer is rejected, the Plaintiff decides whether to remain in the game or drop the case, in which case there will not be any payment. If the offer is rejected and the plaintiff does not drop the case, we proceed to the the next date $t + \Delta$. At date $\tilde{t}$, there is no negotiation, and the judge orders the defendant to pay $J$ to the plaintiff if he is found liable and to pay nothing if he is found not liable, where $J > 0$ is fixed.

Both negotiation and the litigation are costly. If the parties settle at some $t < \tilde{t}$, then the plaintiff and the defendant incur costs $c_P t$ and $c_D t$, respectively, yielding the payoffs

$$u_P = S_t - c_P t \quad \text{and} \quad u_D = -S_t - c_D t$$

for the plaintiff and the defendant, respectively. If they go to court, the plaintiff and the defendant pay additional litigation costs $k_P$ and $k_D$, respectively. We will write $c \equiv c_P + c_D$ and $k \equiv k_P + k_D$ for the total costs of negotiation and litigation, respectively. Note that the payoff vector is $(u_P, u_D) = (J - c_P \tilde{t} - k_P, -J - c_D \tilde{t} - k_D)$ at state $L$ and $(u_P, u_D) = (-c_P \tilde{t} - k_P, -c_D \tilde{t} - k_D)$ at state $NL$. In this paper, we will analyze the subgame-perfect equilibrium of the complete information game in which everything described above is common knowledge.

**Remark 1** In the baseline model, we will assume that $c/\lambda \ll k$. That is, the cost $k$ of litigation is much larger than the expected cost of negotiation $c/\lambda$ until the information arrives. In particular, we will assume that

$$\frac{c}{\lambda} \leq k \frac{J + \alpha k - k_P}{J}.$$  \hspace{1cm} (1)

Furthermore, we restrict ourselves to cases in which the plaintiff can credibly threaten to go to court at the deadline. Thus, we will additionally assume that $q_P J > k_P + c_P \Delta$.

Since we have a deadline, backward induction leads to a unique subgame-perfect equilibrium—up to multiple best responses in the knife-edge cases in which the parties are indifferent between agreeing or delaying the agreement; in those cases we stipulate that the parties agree, for simplicity. The equilibrium is Markovian in that the equilibrium behavior at a
given date does not depend on the actions in the previous dates, depending only on whether information has arrived and the content of the information. Hence, we will divide histories in three groups:

(L) the true state revealed to be $L$;

(NL) the true state revealed to be $NL$;

(∅) no information has arrived.

We will write $L$, $NL$, or $∅$ as the arguments of the equilibrium actions depending on the history. We will also write $V_{t,P}$ and $V_{t,D}$ for the continuation values of plaintiff and the defendant, respectively, if they do not reach an agreement at $t$ or before, ignoring the costs incurred until $t$. Finally, we will write $P(t|t_0) = e^{-λ(t-∆-t_0)} - e^{-λ(t-t_0)}$ for the probability of arrival just before date $t ∈ T^*$ conditional on not having arrival by $t_0$.

Remark 2 In our model, the parties may observe public information regarding the outcome of the case. One can think of many such sources of uncertainty. One important piece of information is the identity of the judge, as judges often have a known bias (or judicial philosophy). Jury selection is another similar case of information arrival. Discovery is also a major source of information revelation. The parties often have private information about the outcome of the discovery, but discovery may also reveal some public information (to both parties) because one can never guess what kind of e-mails, depositions, or private notes will be uncovered throughout the duration of litigation.

Remark 3 Here the assumption that the arrival rate is independent of the state is made only for the sake of simplicity of exposition. If the arrival rate depended on the state, then the parties’ beliefs would change as they wait for information. For example, if the only possible evidence is evidence for liability and information arrives only in state $L$, then non-arrival of information is evidence of the state $NL$, and the probability that $NL$ increases for both players and the level of optimism drops with time as they wait for information. One can, of course, still use backward induction to solve for equilibrium, but the analysis would be more complicated, due to the changing of beliefs over time. For a model of optimism in which information is available only at one of the states, see Thanassoulis (2010), who analyzes a bilateral trade model with optimism about the market conditions.

Remark 4 We assume that the evidence arriving is decisive. This assumption substantially simplifies our analysis by allowing us to use the standard theory to find the solution after
information arrival, but it is not critical for our results. In Section 10 below, we generalize our results to the case that evidence is not decisive, given one possible arrival. Moreover, we show that the more informative the evidence is, the longer the delay in agreeing. When the evidence is not decisive, one could also envision a situation in which new evidence may arrive. This complicates the analysis substantially but we expect it to yield qualitatively similar results.

3 Agreement and Disagreement Regimes

In this section, we derive the subgame-perfect equilibrium of the pre-trial negotiations game and explore the dates at which the parties reach an agreement and the dates at which they must disagree.

After an arrival of information, there is no difference of opinions between the negotiating parties, and the analysis is standard: in equilibrium, there is an agreement at each date after the arrival. Using standard arguments, one can easily show that there will not be any payment if the defendant is revealed not to be liable:

\[ S_t(NL) = 0 \] (2)

for all \( t \). It is crucial for this observation that the plaintiff has the option to drop the case. In this case, she cannot commit to pursuing a costly negotiations process knowing that there will not be any payment at the end. After an arrival of information that indicates that the defendant is liable, one can also easily show that the parties settle at any given date \( t \) for a settlement amount

\[ S_t(L) = \max \{ J + \alpha (\bar{c} (\bar{t} - t) + k) - (c_P (\bar{t} - t) + k_P) , 0 \} . \] (3)

To see this, note that the plaintiff should get the present value of her disagreement payoff, which is \( J - (c_P (\bar{t} - t) + k_P) \), plus the \( \alpha \) fraction of the total cost of disagreement, which is \( c (\bar{t} - t) + k \).\(^5\) If this present value is negative, then she gets 0 because she has the option to drop the case. In the rest of the paper, we will focus on the histories without arrival.

Without an arrival, the parties may or may not settle depending on their expectations of the future. When \( V_{t,P}(\emptyset) + V_{t,D}(\emptyset) < 0 \), there is a strictly positive gain from trade: aside from sunk costs, the total payoff from a settlement is zero while the total payment from

\(^5\)This is the expected value of the settlement. If the plaintiff makes an offer at date \( t \), the settlement is \( S_{t+\Delta}(L) + c_D \Delta \), while if the defendant makes an offer, the settlement is \( S_{t+\Delta}(L) - c_P \Delta \).
delaying agreement further is \( V_{t,P}(\emptyset) + V_{t,D}(\emptyset) \). In this case, the players reach an agreement in equilibrium at \( t \), when the settlement amount is \(-V_{t,D}\) if the plaintiff makes an offer and \( V_{t,P} \) if the defendant makes an offer. On the other hand, when \( V_{t,P}(\emptyset) + V_{t,D}(\emptyset) > 0 \), there cannot be any settlement that satisfies both parties' expectations, and they disagree in equilibrium. In the knife-edge case in which \( V_{t,P}(\emptyset) + V_{t,D}(\emptyset) = 0 \), both agreement and disagreement are possible in equilibrium, and we focus on the equilibrium with agreement. (All equilibria are payoff equivalent.)

**Definition 1** We say that there is an agreement regime at \( t \) if and only if \( V_{t,P}(\emptyset) + V_{t,D}(\emptyset) \leq 0 \). We say that there is a disagreement regime at \( t \) otherwise.

Next we formally define the earliest date at which the parties reach a settlement.

**Definition 2** The earliest date \( t^* \) with an agreement regime is called the settlement date without information. The (stochastic) date of arrival of information is denoted by \( \tau_A \). The settlement date is

\[
\tau^* = \min \{ t^*, \tau_A \}.
\]

Note that there are two reasons for a settlement: arrival of information and reaching a date with an agreement regime. The parties agree in equilibrium at whichever date comes first. Here, \( t^* \) is a function of the parameters of the model. In contrast, \( \tau_A \) and \( \tau^* \) are random variables.

Whether there is an agreement regime at a given date depends on the parties’ optimism about a settlement due to a future arrival and the expected cost of waiting. We next introduce these concepts formally.

Fix an arbitrary date \( t_0 \) and suppose that \( t_1 \) is the earliest date with an agreement regime after \( t_0 \). If there is an arrival at some \( t \in (t_0, t_1] \), the parties reach an agreement. The expected value of such a settlement for the plaintiff and the defendant is \( q_P S_t(L) \) and \( q_D S_t(L) \), respectively, as the settlement value is zero when the information indicates no liability. Hence, the players have optimistic beliefs about such a settlement, and the level of optimism is \( y S_t(L) \). On the other hand, if they agree at \( t_1 \) without an arrival, there is no optimism or pessimism about such a settlement because the value \( S_{t_1}(\emptyset) \) of a settlement is known in equilibrium and the parties have identical expectations about such a contingency. Hence, the total optimism about the future settlements is

\[
Y(t_0, t_1) = \sum_{t \in T^*, t_0 < t \leq t_1} P(t|t_0) y S_t(L).
\]
This is the expected value of a settlement due to an arrival of information, multiplied by the optimism level $y$. On the other hand, the expected total cost of waiting is

$$C (t_0, t_1) = e^{-\lambda(t_1-t_0)}c(t_1-t_0) + \sum_{t \in T^*, t_0 < t} P(t|t_0)c(t-t_0).$$

(5)

Here, the first term is the cost $c(t_1-t_0)$ paid for waiting until $t_1$ to settle, multiplied by the probability that information does not arrive before $t_1$, and the second term is the expected cost of waiting if there is an arrival at a date in the interval between $t_0$ and $t_1$. Because of the constant arrival rate, the cost adds up to

$$C(t_0, t_1) = \frac{\Delta}{1 - e^{-\lambda \Delta}}c \left(1 - e^{-\lambda(t_1-t_0)}\right).$$

(6)

Here, $1 - e^{-\lambda(t_1-t_0)}$ is the conditional probability of arrival before $t_1$ at $t_0$, and $\Delta c / (1 - e^{-\lambda \Delta})$ approaches the expected cost $c/\lambda$ of waiting for arrival as $\Delta \to 0$. Hence, the total cost of waiting is approximately the expected cost of waiting for an arrival, normalized by the probability of arrival before the parties give up on waiting.

If there is no agreement regime after $t_0$, we take $t_1 = \bar{t}$. In this case, we simply add $e^{-\lambda(i-t_0)} y J$ to the optimism from future settlements to incorporate the optimism about the judge’s decision:

$$Y (t_0, \bar{t}) = e^{-\lambda(i-t_0)} y J + \sum_{t \in T^*, t_0 < t} P(t|t_0) y S_t (L).$$

(7)

We also include the litigation costs in the expected total cost:

$$C(t_0, \bar{t}) = e^{-\lambda(i-t_0)} [c(\bar{t}-t_0) + k] + \sum_{t \in T^*, t_0 < t} P(t|t_0)c(t-t_0).$$

(8)

The next result states that there is an agreement regime at a given date if and only if the total amount of optimism about future settlements does not exceed the expected total cost of waiting.

**Lemma 1** There is an agreement regime at $t_0$ if and only if

$$Y (t_0, t_1) \leq C (t_0, t_1)$$

(9)

where $t_1$ is the earliest date with an agreement regime after $t_0$ when there is such a date and it is $\bar{t}$ otherwise.

Lemma 1 provides a simple cost-benefit analysis for determining the agreement regimes. In particular, whether there is an agreement regime at a given date is independent of the
settlement amounts without an arrival. Using this fact, we can identify the dates with agreement and disagreement regimes in a straightforward fashion, without computing the equilibrium settlements.

The next lemma provides a simple cutoff $s^*$ for the settlement $S_t(L)$ in case of proven liability that determines whether there is an agreement regime in the previous date. The cutoff $s^*$ will play a central role in our analysis.

**Lemma 2**  For any $t_0 \in T^*$, there is a disagreement regime at $t_0$ whenever

$$S_{t_0+\Delta}(L) > c \frac{\Delta}{1-e^{-\lambda\Delta}} \equiv s^*.$$  \hspace{1cm} (10)

Conversely, there is an agreement regime at $t_0$ whenever $S_{t_0+\Delta}(L) \leq s^*$ and there is an agreement regime at $t_0 + \Delta$.

Lemma 2 provides a simple cutoff $s^*$ for $S_t(L)$ that determines whether there is an agreement regime at $t - \Delta$. If $S_t(L)$ is to remain above $s^*$ over an interval $[t_0,t_1]$ with an agreement regime at $t_1$, then the parties disagree at $t_0 - \Delta$, waiting for an arrival of information, in hopes that information will yield a more advantageous settlement. On the other hand, if $S_t(L) \leq s^*$, the optimism for waiting for information for one more period does not justify the cost of delaying agreement one more period to $t$. If, in addition, the parties are not so optimistic at $t$ that they would rather wait for further information, then their overall optimism at $t - \Delta$ is so low that they reach an agreement. Using Lemma 2, we will next characterize the settlement date in equilibrium.

**Remark 5** The assumption that the arrival rate $\lambda$ is constant over time is not critical to our results. In Section 6 below, we extend Lemma 2 to the case that $\lambda$ changes deterministically from date to date, reflecting that information is most likely to arrive in a period with some court activity (such as a jury selection session or a pre-trial hearing) and unlikely to arrive at other times.

## 4 Agreement Dynamics

In this section, we determine the dates with an agreement regime. The dynamics of the settlement crucially rely on which party has the higher bargaining power. When the plaintiff is stronger, under moderate optimism levels, we will show that the parties disagree until just before the deadline and reach an agreement at the steps of the courthouse. This is
emblematic of the well-known empirical phenomenon called the *deadline effect*. Under a low level of optimism, a weaker form of the deadline effect emerges: the parties wait for a date near the deadline to settle, regardless of how far the deadline is. When the parties are extremely optimistic, they will never settle and go to the court, leaving the decision to the judge. On the other hand, when the defendant is stronger, the settlement date without information also depends on the deadline. If the deadline is far away, then they reach an agreement immediately. Otherwise, depending on the level of optimism, they either wait until the deadline to settle, exhibiting the strong form of deadline effect, or go to court and let the judge make a decision.

The relative bargaining power of the parties affects the settlement dynamics through the following mechanism. As Lemma 2 shows, the dynamics of agreement and disagreement regimes are determined by the size $S_t(L)$ of settlement after an arrival of information indicating liability. Moreover, by (3), $S_t(L)$ may increase or decrease as we go away from deadline, depending on which party has more bargaining power. In particular, when the relative bargaining power of the plaintiff is higher than her share of the negotiation cost, i.e.,

$$\alpha > \frac{c_P}{c} \equiv \alpha^*, \quad (11)$$

the coefficient of $(\bar{t} - t)$ in (3) is positive, showing that $S_t(L)$ increases linearly as we go away from the deadline. In this case, even if the parties settle near the deadline, they must disagree at some dates when the deadline is far away because an agreement at $t$ implies a disagreement at $t - \Delta$. On the other hand, when the relative bargaining power of the plaintiff is below $\alpha^*$, $S_t(L)$ decreases as we go away from the deadline. In that case, when the deadline is sufficiently far away, they must reach an agreement because $S_t(L)$ becomes zero.

**Remark 6** Alternatively, we could measure the bargaining power with cost, so that the plaintiff has stronger bargaining power when the cost $c_P$ is low: $c_P < \frac{\alpha}{1 - \alpha} c_D$. When the parties make offers with equal probabilities, this reduces to $c_P < c_D$.

In the next subsections we will determine the dynamics of agreement and disagreement regimes separately for these two cases. Towards stating our results, we first determine two important cutoff values for the level of optimism. The first cutoff determines whether the parties go to the court or settle at the deadline $\hat{t} \equiv \max T^*$ (as in the static models of pretrial bargaining with optimism): there is an agreement regime at $\hat{t}$ if and only if \(^6\)

$$y \leq \frac{k + (\bar{t} - \hat{t}) c}{J} \equiv \hat{y}. \quad (13)$$

\(^6\)To see this, note that $V_{i,P}(\emptyset) = q_P J - k_P - c_P (\bar{t} - \hat{t})$ and $V_{i,D}(\emptyset) = -q_D J - k_D - c_D (\bar{t} - \hat{t})$. 

13
Note that $\hat{y}$ is approximately $k/J$ when $\Delta$ is small. The second cutoff determines whether an agreement regime at the deadline implies an agreement $\hat{t} - \Delta$: by Lemma 2, if there is an agreement $\hat{t}$, then there is an agreement regime at $\hat{t} - \Delta$ if and only if

$$y \leq y^* \equiv \frac{c}{S_\hat{t}(L)} \frac{\Delta}{1 - e^{-\lambda \Delta}}.$$ 

Here, $y^*$ is determined by setting $S_\hat{t}(L) = s^*$, where the cutoff $s^*$ is defined in Lemma 2. The cost assumption (1) ensures that $y^* \leq \hat{y}$ when $\Delta$ is sufficiently small. We will assume in the following that this is indeed the case. We will refer to optimism levels $y > \hat{y}$ as excessive optimism and the optimism levels $y \in [y^*, \hat{y}]$ as moderate optimism. (See the discussion of the continuous time limit in the next section for further remarks on these cutoffs.)

### 4.1 Agreement Dynamics with Powerful Plaintiff

We now assume that the plaintiff has high bargaining power, by assuming that $\alpha \geq \alpha^*$. In this case, regardless of how far the deadline is, the parties wait until near the deadline to settle, or they go all the way to court, as established in the following result.\(^7\)

**Proposition 1** Assume $\alpha > \alpha^*$ and $\hat{y} \geq y^*$. When $y > \hat{y}$, there is a disagreement regime at every $t \in T^*$. When $y < \hat{y}$, there is a disagreement regime at every $t < \hat{t}$ and there is an agreement regime at every $t \geq \hat{t}$ where

$$t^* = \max \{t \in T^* | S_t(L) > s^* \}.$$

In particular, when $y^* \leq y \leq \hat{y}$, as long as they do not receive an information, the parties wait until the deadline and settle there in equilibrium.

**Proof.** First consider the case that $y > \hat{y} \geq y^*$. Since $y > y^*$, it follows that $S_\hat{t}(L) > s^*$. Moreover, since $\alpha \geq \alpha^*$, $S_t(L)$ is decreasing in $t$. Hence, $S_t(L) > s^*$ for every $t \leq \hat{t}$. Therefore, by Lemma 2, there is a disagreement regime at each $t < \hat{t}$. On the other hand, since $y > \hat{y}$, there is also a disagreement regime at $\hat{t}$. Now consider the case $y \leq \hat{y}$, so that there is an agreement regime at $\hat{t}$. By inductive application of the second part of Lemma 2, there is an agreement regime at each $t \geq \hat{t}$ because $S_{t+\Delta}(L) \leq s^*$. Moreover, by the first part of Lemma 2, there is a disagreement regime at each $t < \hat{t}$ because $S_{t+\Delta}(L) > s^*$. \(\blacksquare\)

---

\(^7\)Note, also, that there is only a settlement if the plaintiff will credibly go to court in the absence of a settlement, since the plaintiff cannot commit not to drop the case. This is true when $V_{\hat{t},P}(\emptyset) \geq 0$ or $q_p \geq (k_P + c_P(\hat{t} - \hat{t}))/J$.  

\(^7\)In the proposition we use the convention that max of empty set is $-\infty$.  

14
Figure 1: Agreement dynamics under a strong plaintiff

As we have discussed before, the parties settle when there is an arrival, which eliminates any difference of opinion. For contingencies without an arrival, Proposition 1 presents an intuitive pattern of behavior, depending on the level of optimism, as summarized in Figure 1. When the parties are excessively optimistic, i.e., when \( y > \hat{y} \), the negotiations result in an impasse, and a judge determines whether or not the defendant is liable after a costly litigation process. This case is illustrated in the right panel of Figure 1. Even at \( t = \hat{t} \), \( S_t(L) > s^* \) and there is no agreement at the deadline. As we consider earlier dates, \( S_t(L) \) increases, and so we cannot have an agreement regime at any of these dates either. When the parties are moderately optimistic, i.e., when \( y^* \leq y \leq \hat{y} \), a strong form of the deadline effect is exhibited in equilibrium: the parties wait until the deadline, \( \hat{t} \), and reach an agreement at the steps of the courthouse. This case is illustrated in the middle panel of Figure 1. Since \( y < \hat{y} \), there is an agreement regime at the deadline. However, \( S_t(L) > s^* \) and so there is a disagreement regime at \( t = \hat{t} - \Delta \), and as we consider earlier dates, \( S_t(L) \) increases, so that there is no other date with an agreement regime. Finally, when the optimism level is low, i.e., when \( y < y^* \), a weak form of the deadline effect is exhibited in equilibrium: the parties wait for a date \( t^* \) near the deadline, and settle at \( t^* \). They would have agreed at any date after \( t^* \) as well. This case is illustrated in the left panel of Figure 1. Since \( y < \hat{y} \), there is again an agreement regime at the deadline. Since \( y < y^* \), \( S_t(L) \leq s^* \), and so as we consider earlier dates, there is also an agreement regime at every date until \( t^* \). Since \( S_t(L) \) is increasing as we consider earlier dates, \( S_t(L) > s^* \) for every date \( t \) before \( t^* \), and so there is no agreement regime any such date. It is crucial to observe that neither the cutoffs \( y^* \) and \( \hat{y} \) nor the length \( \hat{t} - t^* \) of the interval of agreement regimes is a function of \( \hat{t} \). No matter how far the deadline is, the parties wait for a fixed neighborhood of the deadline to reach an agreement.
4.2 Agreement Dynamics with Powerful Defendant

We now assume that the defendant has a stronger position in bargaining by assuming that
\( \alpha < \alpha^* \). We then determine whether there is an agreement or a disagreement regime at
any given \( t \). In particular, we establish that there is either an immediate agreement, or the
strong form of the deadline effect (in which the parties wait until the deadline to settle) or
an impasse.

Toward stating our result, we define the expectation of the settlement \( S_t \) conditional on
\( (t_0, t_1] \) as a function of \( t_0 \) and \( t_1 \):

\[
f(t_0, t_1) = E[S_t(L) \mid t_0 < t \leq t_1] = \sum_{t_0 + \Delta}^{t_1} \text{P}(t \mid t_0)
\]

(12)

Whether there is a agreement regime at \( t_0 \) depends on whether \( f(t_0, t_1) \) exceeds \( s^* \),
where \( t_1 \) is the first date with an agreement regime after \( t_0 \). Using this fact, the next result
establishes a sharp characterization of the dates with an agreement regime.

**Proposition 2** Assume \( \alpha < \alpha^* \) and \( \hat{y} \geq y^* \). The players either agree immediately in
equilibrium, or there is no agreement regime before the deadline. In particular, when \( y \leq \hat{y} \),
there is an agreement regime at every \( t \in \{0, \Delta, \ldots, t^{**}, \hat{t}\} \) and a disagreement regime at
every \( t \) with \( t^{**} < t < \hat{t} \) where

\[
t^{**} \equiv \max \{ t \in T^* \mid f(t, \hat{t}) \leq s^* \}.
\]

(13)

When \( y > \hat{y} \), there is there is an agreement regime at every \( t \leq t^{***} \) and a disagreement
regime at every \( t \) with \( t > t^{***} \) where

\[
t^{***} \equiv \max \{ t \in T^* \mid f(t, \hat{t}) \leq s^{**}(t) \}
\]

(14)

and

\[
s^{**}(t) = s^* - \frac{e^{-\lambda(t-\hat{t})}}{1 - e^{-\lambda(t-\hat{t})}} \left[ J - \left( c(\bar{t} - \hat{t}) + k \right) / y \right]
\]

**Proof.** Lemma 2 directly implies that in equilibrium, either the players agree immediately
or they wait for the deadline. If there is a date \( \bar{t} < \hat{t} \) with an agreement regime, then by
Lemma 2, \( S_{\bar{t}+\Delta}(L) \leq s^* \). Moreover, since \( \alpha < \alpha^* \), \( S_t(L) \) is increasing in \( t \), implying that
\( S_{\bar{t}+\Delta}(L) \leq s^* \) for each \( t \leq \bar{t} \). Since there is an agreement regime at \( \hat{t} \), together with Lemma
2, this implies that there is an agreement regime at each \( t \leq \hat{t} \). In particular, there is an
agreement regime at \( t = 0 \), and the players reach an agreement immediately in equilibrium.
To prove the second statement, assume that \( y \leq \hat{y} \), so that there is an agreement regime at \( \hat{t} \). Then, one can show that, for any \( t_0 \geq t^{**} \),

\[
Y(t_0, \hat{t}) = yf(t_0, \hat{t}) \left(1 - e^{-\lambda(\hat{t} - t_0)}\right)
\]

and

\[
C(t_0, \hat{t}) = ys^* \left(1 - e^{-\lambda(\hat{t} - t_0)}\right) + e^{-\lambda(\hat{t} - t_0)}yJ
\]

Hence, by Lemma 1, there is an agreement regime at \( t_0 \) if and only if \( f(t_0, \hat{t}) \leq s^* \). In particular, there is a disagreement regime at each \( t_0 \) with \( t^{**} < t_0 < \hat{t} \), and there is an agreement regime at \( t^{**} \). The previous paragraph also implies then that there is an agreement regime at each \( t \leq t^{**} \).

Finally, when \( y > \hat{y} \), there is a disagreement regime at \( \hat{t} \), and for each \( t_0 \geq t^{**} \),

\[
Y(t_0, \hat{t}) = yf(t_0, \hat{t}) \left(1 - e^{-\lambda(\hat{t} - t_0)}\right) + e^{-\lambda(\hat{t} - t_0)}yJ
\]

and

\[
C(t_0, \hat{t}) = ys^* \left(1 - e^{-\lambda(\hat{t} - t_0)}\right) + e^{-\lambda(\hat{t} - t_0)}[(c(\hat{t} - \hat{t}) + k)].
\]

Note that \( Y(t_0, \hat{t}) > C(t_0, \hat{t}) \) if and only if the inequality in (14) holds. Moreover, \( f(t, \hat{t}) - s^{**}(t) \) is increasing in \( t \). Thus, at any \( t > t^{***} \), we have \( f(t, \hat{t}) > s^{**}(\hat{t}) \), and there is disagreement. Similarly, there is an agreement regime at every \( t \leq t^{***} \). ■

The dynamics in Proposition 2 is as in Figure 2. Consider the case \( y^* < y < \hat{y} \) and large \( \tilde{t} \). There is an agreement regime at \( \tilde{t} \), but there are disagreement regimes before \( \tilde{t} \) because \( y > y^* \) and hence \( S_{\tilde{t}}(L) > s^* \). There is a disagreement regime so long as \( S_{\tilde{t}}(L) \) remains above \( s^* \). Nonetheless, since \( S_{\tilde{t}}(L) \) is decreasing as we go away from the deadline, \( S_{\tilde{t}}(L) \) eventually goes below \( s^* \). There may still be a disagreement regime although \( S_{\tilde{t}}(L) \) is below, as \( f(t, \tilde{t}) \) may remain above \( s^* \). As we decrease \( t \), \( f(t, \tilde{t}) \) also goes below \( s^* \) (at \( t^{**} \)), and we have an agreement regime again. When \( y > \hat{y} \), there is no agreement regime at \( \tilde{t} \), and the cutoff for \( f(t, \tilde{t}) \) is different, but the picture remains the same qualitatively. There is an agreement regime at the beginning only when \( \tilde{t} \) is large.

For the case of a powerful defendant, Proposition 2 establishes that there are only three possible patterns of behavior when there is no arrival: immediate agreement, strong deadline effect, and impasse. When \( y \leq y^* \), there is an agreement regime at each date, leading to an immediate agreement. For other values, the settlement dates depend on the deadline \( \bar{t} \). For every \( y > y^* \), there exists \( \bar{t} \leq \infty \) such that there is immediate agreement whenever \( \bar{t} \geq \bar{t} \). When \( \bar{t} < \bar{t} \), there is strong deadline effect if \( y^* < y \leq \hat{y} \), and there is impasse if \( y > \hat{y} \).
Figure 2: Agreement dynamics under a powerful defendant when the deadline is sufficiently far.

5 Agreement Dynamics in Continuous Time–A Discussion

In this section we discuss the equilibrium behavior in greater detail for the continuous time limit \( \Delta \to 0 \). We focus on the case that \( \alpha > \alpha^* \) for ease of exposition. The case that \( \alpha < \alpha^* \) follows similarly and is fully worked out in the appendix. In the continuous-time limit, the relevant values in equilibrium (namely \( \hat{y}, y^*, s^*, \) and \((\bar{t} - t^*)\)) take very simple intuitive form.

First, the cutoff \( \hat{y} \) for impasse simply becomes

\[ \hat{y} \equiv k/J, \]

i.e., the ratio of the court cost \( k \) to the judgment \( J \). As in the static models, when the optimism level exceeds this ratio, the bargaining results in an impasse, and the parties settle otherwise. The cutoff \( y^* \) for the strong and the weak forms of the deadline effect becomes

\[ y^* \equiv \frac{c}{\lambda} \cdot \frac{1}{J + \alpha k - k_P}. \]

Here, \( c/\lambda \) is the expected cost of negotiation if the parties waited for an arrival of information to settle their case, while \( J + \alpha k - k_P \) is the settlement at the deadline if information arrives at the deadline and shows that the defendant is liable. In this discussion we maintain the cost assumption in (1), which holds for example when the litigation is costlier than negotiation (i.e. \( k > c/\lambda \)) and the bargaining power \( \alpha \) of the plaintiff exceeds her share \( k_P/k \) in the litigation costs. In that case, the cutoff \( y^* \) is lower than \( \hat{y} \).
As stated in Proposition 1, in equilibrium, for the moderate values \( y \in [y^*, \hat{y}] \) of optimism, the parties exhibit a strong form of the deadline effect by waiting exactly until the deadline to settle, and for the lower values \( y < y^* \) of optimism, they exhibit a weak form of deadline effect by waiting near deadline to settle. For the extreme values \( y > \hat{y} \) of optimism, the negotiation results in impasse.

We will next illustrate the agreement dynamics and derive an explicit simple formula for the strategic settlement date \( t^* \).\(^8\) As shown in Figure 1, \( t^* \) is determined by the intersection of \( S_t(L) \) with \( s^* \), which simply becomes

\[
\begin{align*}
  s^* &\equiv \frac{c}{\lambda} \cdot \frac{1}{y},
\end{align*}
\]

the ratio of the total expected cost \( c/\lambda \) of waiting for information to the level \( y \) of optimism, in the continuous-time limit. By Lemma 2, an agreement regime at \( t \) carries over to a previous date if and only if \( S_t(L) \) is lower than this ratio. When \( y < y^* \), \( S_t(L) < s^* \) and the agreement at the deadline carries over to previous dates as shown in the figure. As we go away from deadline, we have agreement regimes until \( S_t(L) = s^* \) in which case, an agreement regime at \( t \) implies a disagreement regime at an instant before in the continuous time limit by the same lemma. As established by Proposition 1, there cannot be an agreement regime at earlier dates and the parties settle at \( t^* \) in equilibrium:

\[
S_{t^*}(L) = s^*.
\]

By substituting (3) and (15) in the above equality, we obtain

\[
\begin{align*}
  t^* &= \bar{t} - \frac{c}{\lambda y} \cdot \frac{(J + \alpha k - k_P)}{(\alpha - \alpha^*) c},
\end{align*}
\]

which can also be written as

\[
\begin{align*}
  t^* &= \bar{t} - \frac{(y^* - y)}{(\alpha - \alpha^*) c} S_{t}(L) .
\end{align*}
\]

Note that the difference between the strategic settlement date \( t^* \) and the deadline \( \bar{t} \) is independent of the deadline. No matter how far the deadline is, the parties wait for information until they reach a fixed neighborhood of the deadline and settle there regardless of the arrival of information.

How close will they come to the deadline? This depends on several factors. First, the more optimistic they are, the closer they get to the deadline: \( \bar{t} - t^* \) is proportional to

\(^8\)Recall that the realized settlement date is the minimum of \( t^* \) and the date of information arrival, which is stochastic.
\((y^* - y)/y\). As the level of optimism approaches to the cutoff, the length \(\bar{t} - t^*\) shrinks to zero, and the parties exhibit nearly strong form of deadline effect. On the other hand, for arbitrary small values of optimism, they can settle arbitrarily far away from the deadline: \(\bar{t} - t^* \to \infty\) as \(y \to 0\). In particular, they reach an immediate agreement when

\[
y < y_{\text{min}} \equiv \frac{c/\lambda}{(\alpha - \alpha^*) \bar{c}t + (J + \alpha k - k_p)}.
\]

Here, \(y_{\text{min}}\) is the smallest level of optimism under which there is delay in equilibrium. Interestingly, \(y_{\text{min}}\) is decreasing in \(\bar{t}\) and approaches zero as \(\bar{t} \to \infty\). That is, no matter how small the optimism is, there will be some amount of delay due to the weak form of deadline effect when the deadline is sufficiently far. A similar delay occurs when the expected cost \(c/\lambda\) of negotiation while waiting for information is small. The other two factors that determine the length \(\bar{t} - t^*\) of agreement regimes are \(S_{\bar{t}}(L) = J + \alpha k - k_p\) and the strength \(\alpha - \alpha^*\) of the bargaining position of the plaintiff. By (16), \(\bar{t} - t^*\) is decreasing in \(S_{\bar{t}}(L)\) and shrinks to zero as \(S_{\bar{t}}(L)\) approach the ratio \(\frac{c}{\lambda y}\) \(\equiv s^*\). Similarly, \(\bar{t} - t^*\) is decreasing in \(\alpha - \alpha^*\). In summary, the deadline effect gets stronger with the level of optimism \(y\), the bargaining power of the plaintiff \((\alpha - \alpha^*)\) and \(S_{\bar{t}}(L) = J + \alpha k - k_p\).

6 Time-Varying Arrival Rates

In our baseline model, we assume that the arrival rate of evidence is static throughout the bargaining process. In reality, the rate of evidence might vary across time. For example, the probability of evidence might be higher during periods of discovery or jury selection. In this section, we show that our framework for determining periods of agreement and disagreement can be naturally extended to cover this case. In particular, we extend our baseline model to allow the rate of arrival to be arbitrarily time varying, according to any well-behaved function \(\lambda(t)\).

To do this, take any integrable function \(\lambda(t)\) and write \(\Lambda(t_0) = 1 - e^{-\int_{t_0}^{t_0+\Delta} \lambda(t) dt}\) for the probability of an arrival in any period \(t_0\). We extend Lemma 2 as follows:

**Lemma 3** For any \(t_0 \in T^*\), there is a disagreement regime at \(t_0\) whenever

\[
S_{t_0+\Delta}(L) > \frac{c}{y} \frac{\Delta}{\Lambda(t_0)} \equiv s^*(t_0).
\]

Conversely, there is an agreement regime at \(t_0\) whenever \(S_{t_0+\Delta}(L) \leq s^*(t_0)\) and there is an agreement regime at \(t_0 + \Delta\).
Lemma 3 extends the characterization for agreement by making the cutoff for agreement time dependent. The only change is that $\Lambda(t_0)$ is now a function of time, rather than a fixed parameter, while all the other parameters such as $S_t(L)$ and $c/y$, which are independent of $\lambda(t_0)$, remain as in the static case. Note that as in the static case, the cutoff $s^*(t_0)$ becomes

$$s^*(t_0) = \frac{c}{\lambda(t_0)y}$$

in the continuous time limit. That is, the same formula applies, except that the particular value of $\lambda$ depends on the time $t_0$ in which it is being considered. Note also that the cutoff $s^*(t_0)$ is proportional to $1/\lambda(t_0)$.

![Graph showing agreement dynamics](image)

**Figure 3:** Agreement dynamics under a powerful plaintiff with a time-varying rate of arrival.

As in previous sections, one can use Lemma 3 to determine the agreement dynamics in specific situations by comparing the settlement $S_t(L)$ to $s^*(t_0)$. As an illustrative example, consider the case of a powerful plaintiff as in Section 4.1 and suppose that $y < \hat{y}$ so that there is agreement at the deadline. Suppose, further, that $\lambda(t_0)$ is some function such that $s^*(t_0)$ is given by the red curve in Figure 3. Since there is agreement at the deadline, and $s^*(\hat{t})$ is greater than $S_t(L)$, there is also agreement at the interval of periods $t_3 < t < \hat{t}$ where $t_3$ is the first period before the deadline at which $s^*(t) = S_t(L)$. By Lemma 3, there is disagreement during the periods between dates 0 and $t_1$, and between $t_2$ and $t_3$. However, our lemma is silent on the periods between dates $t_1$ and $t_2$.

To determine whether there is agreement during these dates, we would need to compare $s^*(t_0)$ against $E[S_t(L)|t_0 \leq t \leq t_3]$ where the expectation of $S_t(L)$ is calculated analogously to the static case, as given in equation (12). That is, there is an agreement at $t_0$ if $f(t_0, t_3) < \ldots$
$s^*(t_0)$ where

$$f(t_0, t_3) = E[S_t(L)|t_0 \leq t \leq t_3] = \frac{\sum_{t=t_0+\Delta}^{t_3} S_t(L)P(t|t_0)}{1 - e^{-\int_{t_0}^{t_3} \lambda(t) \, dt}},$$

and the probability $P(t|t_0) = e^{-\int_{t_0}^{t} \lambda(t) \, dt} \Lambda(t - \Delta)$ is computed using the time varying function $\lambda(t)$.

Figure 4: Plot of the arrival rate function $\lambda(t)$ given by a baseline rate $\lambda_B$ and a discovery rate $\lambda_D$, under different schedules for the discovery period $T_D$.

![Figure 4](image)

Figure 5: Agreement dynamics in an extension of the left-most panel of Figure 1, in which a discovery period $T_D$ with higher arrival rate $\lambda_D$ is scheduled as in Figure 4.

![Figure 5](image)

This might have direct implications when choosing when to schedule periods of discovery, during which $\lambda(t)$ is particularly high. As a simple case, imagine that $\lambda(t)$ can take two values—a baseline rate $\lambda_B$ and a discovery rate $\lambda_D$ where $\lambda_D \gg \lambda_B$, and that discovery takes place over some fixed interval of periods $T_D$. Our model shows that the choice of when $T_D$ will take place can influence the settlement dynamics.

For example, consider the case of a powerful plaintiff. As discussed in Section 4.1, without any discovery there will be a strategic settlement at $t^*$ and never before. If, however, there is a discovery interval prior to $t^*$, as shown in panel 1 of Figures 4 and 5, during which the
cutoff $s^*(t_0)$ is below the baseline level, then by Lemma 3, there will be no difference in the settlement dynamics: the parties will wait until $t^*$ to agree unless there is an arrival of information. However, since the rate of arrival is higher during $T_D$, the probability of an arrival of information—and therefore, of agreement due to information arriving—is higher during this period. Thus, scheduling $T_D$ to be as early as possible increases the aggregate probability of early settlement.

In contrast, if $T_D$ begins after $t^*$, as in panel 2 of Figures 4 and 5, then there will be disagreement prior to $t^*$ as before, but now there will be disagreement during $T_D$ as well. Moreover, the higher rate of arrival during discovery will entice the parties to wait for $T_D$, creating further incentive for disagreement prior to $T_D$. Depending on when $T_D$ is placed, this may postpone the first strategic settlement date to the end of the discovery period all together.

7 Distribution of Settlement Times

In this section, we explore the empirical distribution of settlement date implied by our model. Towards this goal, we analyze the distribution of the equilibrium settlement date $\tau^*$ when the level $y$ of optimism is a random variable.

We mainly consider the case when there is a strong plaintiff, that is, $\alpha > \alpha^*$. As we saw in the discussion of agreement dynamics, there can be three different trajectories of agreement in this case, depending on the optimism. There is an impasse whenever the parties are excessively optimistic ($y > \hat{y}$), there is a strong deadline effect whenever the parties are moderately optimistic ($y^* \leq y \leq \hat{y}$), and there is a weak deadline effect when optimism is low ($y < y^*$). Thus, we exhibit the likelihood of continuing negotiations across time, as a function of optimism $y$. We consider $y$ to be exogenous to the model, and for convenience of exposition, uniformly distributed on the interval $[0, 1]$. This assumption is formally stated as follows.

**Assumption 1** $\alpha > \alpha^*$; $\hat{y} \geq y^*$, and $y$ is uniformly distributed on the interval $[0, 1]$ and is stochastically independent from arrival of information.

Note that, since $y$ is a random variable, the settlement date $t^*$ without information (i.e. the first date of an agreement regime) is also a random variable, defined over $T$. Note also that we have constrained $y$ to be positive, although it can be negative in practice. We do this for convenience of exposition, and all of the following results hold with minor changes
in the more general case. We define the function,
\[ \beta(t) = \frac{c}{\lambda} \frac{(\alpha - \alpha^*) (\bar{t} - t) c + J + \alpha k - k_p}{\alpha^2 - \alpha} \]
which will give the distribution function of \( t^* \) away from boundaries, as we will see next. Using Proposition 1, we can compute the distribution of \( t^* \) from the distribution of \( y \) via equation (16) as follows.

**Lemma 4** Under Assumption 1, the cumulative distribution function of the settlement date \( t^* \) without information is given by
\[ F_{t^*}(t) = \begin{cases} 
\beta(t) & \text{if } t < \hat{t} \\
\hat{y} & \text{if } t = \hat{t}.
\end{cases} \]

Note that at \( t = \hat{t} \), \( \beta(\hat{t}) = y^* \). Hence, \( F_{t^*} \) has a discontinuity of size \( \hat{y} - y^* \) at \( \hat{t} \), yielding a point mass at \( t = \hat{t} \). This is because in the event \( y \in [y^*, \hat{y}] \), there is a strong deadline effect and parties agree at the deadline. Recall that the actual settlement date
\[ \tau^* = \min \{t^*, \tau_A\} \]
depends not only \( t^* \) but also the stochastic date \( \tau_A \) at which information arrives. The distribution of the settlement date is derived next.

**Lemma 5** Under Assumption 1, the cumulative distribution function of the settlement date \( \tau^* \) is given by
\[ F_{\tau^*}(t) = (1 - e^{-\lambda t}) + e^{-\lambda t} F_{t^*}(t). \]
For \( t < \hat{t} \), the probability density function of the settlement date \( \tau^* \) is
\[ f_{\tau^*}(t) = \lambda e^{-\lambda t} \left[ ((\alpha - \alpha^*) \beta(t)^2 - \beta(t) + 1 \right], \]
and there is a point mass of \( e^{-\lambda t} (\hat{y} - y^*) \) at \( \hat{t} \). For \( t < \hat{t} \), the hazard rate of the settlement date \( \tau^* \) is
\[ H(t) \equiv \frac{f_{\tau^*}(t)}{1 - F_{\tau^*}(t)} = \lambda \left[ \frac{(\alpha - \alpha^*) \beta(t)^2}{1 - \beta(t)} + 1 \right]. \]

The settlement date \( \tau^* \) is a combination of two variables. The first variable is the information arrival, which is assumed to have a constant hazard rate, resulting in a decreasing frequency. The second variable strategic settlement, namely \( t^* \). The strategic settlement has an increasing frequency and hazard rate, with a point mass at the deadline. In general, the frequency and the hazard rate of the actual settlement date \( \tau^* \) depends on which of the two variables dominates. It turns out that the qualitative properties of the distribution of the actual settlement \( \tau^* \) is independent of the parameters, as stated formally next.
Proposition 3 Under Assumption 1, the density function $f_{\tau^*}$ of the settlement date is strictly decreasing (up to $\hat{t}$ where there is a point mass) and the hazard rate $H(t)$ of the settlement date $\tau^*$ is strictly increasing for $t < \hat{t}$.

The general distributional properties of the settlement date is as plotted in Figure 6. The cumulative distribution function (on the right panel) is concave up to the deadline, where there is a point mass. Hence, the density function of the settlement date is decreasing up to the deadline and has a point mass at the deadline (as in the middle panel). This results in a U-shaped frequency of settlements, decreasing for the most part of the negotiation with spike at the end—in line with empirical regularities. A more subtle parameter that is considered in the empirical literature is the hazard rate $H$ of the settlement, which measures the frequency of the settlement conditional on the cases that have not settled yet. The hazard rate in our model is increasing convexly with a point mass at the end (as in the right panel). The empirical studies that we are aware of are mixed. Fournier and Zuehlke (1996) estimates that the hazard rate $H(t)$ is proportional to $t^\gamma$ where $\gamma$ is about 1.9, showing that the hazard rate is increasing and convex. On the other hand, Kessler (1996) reports a mildly decreasing hazard rate.

![Figure 6: The distribution of settlement date $\tau^*$ under a strong plaintiff.](image)

When the defendant is stronger (i.e. $\alpha < \alpha^*$), the settlement time $\tau^*$ has a degenerate distribution, allowing only immediate agreement, strong deadline effect, and impasse. This immediate corollary of Proposition 2 is formally stated next.

Proposition 4 For any $\alpha < \alpha^*$ and any distribution of $y$, the settlement time $\tau^*$ has a point mass at $t = 0$ (immediate agreement), at $t = \hat{t}$ (the deadline effect) and at $\bar{t}$ (impasse). Moreover, the hazard rate of the settlement is constant: $H(t) = \lambda$ for all $t \in (0, \hat{t})$. The probability of immediate agreement is increasing in $\hat{t}$ and decreasing in $\alpha$.

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9See Kessler (1996) and Williams (1983) for some example of empirical studies.
The qualitative properties of the settlement distribution under a powerful defendant mirror the case of a powerful plaintiff with the following exceptions. First, although the frequency of the settlement has still a U shape, there is now a point mass at the beginning. That is, a significant portion of cases settle immediately, and the portion of such cases gets larger when the deadline is pushed away (perhaps because of the backlog of cases in the court) or when the defendant gets a stronger bargaining position (e.g. if the cost $c_D$ of negotiation for the defendant decreases or the cost of plaintiff increases). Second, the hazard rate in the interior is constant, rather than convexly increasing. This is simply because any such settlement must be due to an information arrival, which is assumed to have a constant hazard rate.

If the dataset includes both cases with powerful defendants and with powerful plaintiffs, then the settlement date $\tau^*$ still has a point mass at the beginning ($t = 0$), at the deadline ($t = \hat{t}$) and at the impasse ($t = \bar{t}$). For $t \in (0, \hat{t})$, the hazard rate is convexly increasing and the frequency $f_{\tau^*}$ of the settlements is decreasing. We formally state this below.

**Proposition 5** When $\alpha$ is randomly chosen from the full support $[0, 1]$, the density function $f_{\tau^*}$ of the settlement date is strictly decreasing from $t = 0$ up to $\hat{t}$, and has point masses at $t = 0$, $t = \hat{t}$ and $t = \bar{t}$. The hazard rate $H(t)$ of the settlement date $\tau^*$ is strictly increasing for $t < \hat{t}$.

When there is a possibility of having either a powerful plaintiff or a powerful defendant, the density of settlements is a weighted combination of the settlement densities in each case. The density, and therefore the frequency, of settlement is decreasing for $t \in (0, \hat{t})$, as it is driven by the density of settlements with powerful plaintiffs with zero density among settlements with powerful defendants. At the points $t = 0$, $t = \hat{t}$ and $t = \bar{t}$, there are point masses stemming from both cases, so that the point masses generated by the settlement densities in cases of powerful defendants are amplified by those from the densities of powerful plaintiffs. The resulting average density is qualitatively identical to that of a powerful plaintiff. Thus, the prediction of a U-shaped frequency of settlements is robust to assumptions about a particular party being more powerful. Similarly, the aggregate hazard rate is convexly increasing as in the case of a powerful plaintiff, so that the prediction of a convex hazard rate is also robust to assumptions about the powerful party.

All in all, the empirical implications of our model are similar to those of bargaining with incomplete information. For example, Spier (1992) establishes a U shaped distribution as

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10 Such a point mass occurs even under a powerful plaintiff when the probability $\Pr(y < 0)$ of pessimism is positive.
in the case of powerful defendant above. The main difference is that the relative bargaining power of the parties play an important role in our model, and the qualitative implications depend on which party has a stronger bargaining position.

8 Commitment

In our main model, we have assumed that the plaintiff can drop his lawsuit at any point in the negotiations up to the court date. This assumption, which we made in the spirit of modeling reality more closely, provides an inherent asymmetry in the bargaining power of the plaintiff and the defendant. In particular, it implies that $S_t(NL) = 0$. That is, upon discovery that the defendant is not liable, neither the plaintiff nor the defendant are liable for any further costs. Here we explore the consequences of a rule by which once a plaintiff has initiated a lawsuit, he must commit to the case and cannot leave it without either a settlement or a court decision.

**Assumption 2** Once a lawsuit is initiated, it must terminate either in a settlement or in a court decision, regardless of the information that arrives in the interim.

With this alternative assumption, the balance of bargaining power between the plaintiff and the defendant does not play a role in agreement dynamics. Depending on the level of optimism, we can have either immediate agreement, a strong deadline effect, or impasse.

Under the alternative assumption, upon the arrival of decisive evidence that the defendant is not liable, the parties settle at any date $t$ for the settlement amount

$$S_t(NL) = \alpha (c(\bar{t} - t) + k) - (c_P(\bar{t} - t) + k_P).$$

Note that this is the $\alpha$ fraction of the total cost of disagreement added to the present value of the plaintiff’s disagreement payoff given that the defendant is known to be not liable. We no longer have that $S_t(NL) = 0$ because since the plaintiff cannot drop the lawsuit unilaterally, he can commit to negotiating and force both sides to pay legal fees until a settlement or the court date comes about. Thus, the cost of disagreement is binding, and any settlement must account for it. If $\alpha$ is high, this may result in a positive settlement $S_t(NL)$. Since the defendant may also force incurring the cost of further negotiation, $S_t(NL)$ may also be negative.

If decisive evidence that the defendant is liable arrives instead, then the parties settle at any date $t$ for the settlement amount

$$S_t(L) = J + \alpha (c(\bar{t} - t) + k) - (c_P(\bar{t} - t) + k_P).$$
Note that in a departure from our main model, the settlement amount here does not depend on whether or not $S_t(L)$ is positive or negative as the plaintiff no longer has the outside option of leaving negotiations with payoff 0.

Observe that the difference between the settlements in the two cases is constant:

$$S_t(L) - S_t(NL) = J.$$ 

Hence, the difference between the expectations of settlements is always $yJ$, regardless of whether it is agreed upon given an arrival of information or decided in the court. This implies that the parties wait if and only if waiting is efficient in the sense that the expected cost of waiting is lower than the expected value of realizing the extra gain of $yJ$. In the last date $\hat{t}$, there is an agreement if and only if

$$y \leq \hat{y} = \frac{k + c (\hat{t} - \hat{t})}{J} \approx \frac{k}{J}.$$

At any earlier date, whether there is an agreement is determined by whether the difference $S_t(L) - S_t(NL) = J$ is above or below the cutoff $s^*$, defined before. Note that the difference is constant $J$ here, while it was the function $S_t(L)$ of time and relative bargaining powers in the main model. The cutoff for agreement is determined by setting $J = s^*$, which yields

$$y_c^* = \frac{c}{J} \left( \Delta \right) \frac{1}{1 - e^{-\lambda \Delta}} \approx \frac{c}{\lambda} \cdot \frac{1}{J}.$$ 

This produces the following dynamics.

**Proposition 6** Under Assumption 2, when $y < y_c^*$, we have immediate agreement. When $y_c^* < y \leq \hat{y}$, we have a strong deadline effect: the parties wait until the last date $\hat{t}$ to settle. When $y > \hat{y}$, there is impasse, and the court decides the case.

Unlike the previous sections, the dynamics of agreement in Proposition 6 do not rely on $\alpha$ or the amount of time before the deadline. This suggests that the ability to costlessly drop the lawsuit in the case of evidence in favor of the defendant has a significant effect on bargaining. When the plaintiff does not have this ability, initiating a lawsuit commits the plaintiff to paying a portion of the projected costs of bargaining all the way to court regardless of the outcome. As such, changing expectations about the probability of arrival of evidence does not affect the agents’ willingness to settle before the deadline – if they are willing to agree at the start of negotiations, they do so, and if they are sufficiently optimistic so as to begin bargaining, they will never settle except possibly at the deadline.
Figure 7: The distributions of the strategic settlement date $t^*$ under commitment and no commitment when $\alpha < k_P/k$ and $\alpha > k_P/k$. When $\alpha = k_P/k$, the cutoffs for agreement with and without commitment are the same.

Recall that in the absence of commitment, $y^* = \frac{c}{\lambda(J+\alpha k - k_P)}$. Thus, when $\alpha k > k_P$, the cutoff for immediate agreement with commitment $y_C^*$ is higher than the cutoff without commitment $y^*$, so that there is immediate agreement for a larger range of levels of optimism. If on the other hand, $\alpha k \leq k_P$ so that the plaintiff expects to pay more of the aggregate court fees than he expects to obtain as a fraction of the value saved by settling outside of court, then the cutoff for agreement is lower with commitment than without and there is immediate agreement for a smaller range of levels of optimism. The distributions of the strategic settlement date $t^*$ in each of these cases are demonstrated in Figure 7. Under commitment, $t^*$ is zero for all $y \leq y_C^*$, and $\hat{t}$ for all higher values of $y$ for which $t^*$ is defined. Without commitment, $t^*$ is increasing convexly for $y \in (0, y^*)$, and is $\hat{t}$ for all higher values of $y$ for which $t^*$ is defined.

9 Policy Exercise: American vs. English Rule

A common application for the study of pretrial negotiations is its use in evaluating different payment shifting rules. In the literature two main payment systems have been studied. The first one, known as the American Rule since it is used in most of the United States, requires each party to pay its own legal costs regardless of the outcome of the trial. The second one, known as the English Rule, which is used in most of England and Canada, requires that the loser of the trial must pay all of the legal fees incurred in relation to the case if a case reaches court. In our main model we have focused on the American Rule. In this section,
we present the analysis under the English Rule, and show that there is more disagreement and longer delays under the English Rule than under the American Rule.

We establish our result in two different treatments. We consider the case that negotiation costs are tangible legal fees so that the English Rule applies to both court fees and negotiation costs. In this case, it is natural to assume that the plaintiff must commit to carry on negotiations even after learning that there is no liability since he is responsible for legal costs incurred up to that point by the defendant. We also consider the case that the negotiation costs are intangible, such as non-pecuniary costs and the negative effect of impending litigation on the company performance. In this case, the English Rule only applies to court fees so that it is more natural to assume that the parties cannot commit (as in our main model). In both cases, the settlement dynamics under the English Rule are qualitatively identical to those under the American Rule. Rather, the English Rule raises the effective judgment fee from $J$ under the American Rule to $J + \bar{c}t + k$. When there is commitment, the English Rule lowers the the cutoff for impasse, causing disagreement with even lower levels of optimism. When there is no commitment, the English Rule shifts up the settlement amount under knowledge of liability $S_t(L)$, lowering the cutoff for delays and impasse, and causing longer delay at each level of optimism. For the sake of completeness, we also present the case of tangible costs without commitment, in which case, the English rule causes even more disagreement and delay.

### 9.1 Tangible Costs with Commitment

We first consider the case that the costs $c_P$ and $c_D$ are tangible legal fees and the plaintiff must commit to a case once he starts it. In this case, the English Rule yields the payoff vector $(u_P, u_D) = (J, -J - c\bar{t} - k)$ in state $L$ and $(u_P, u_D) = (-c\bar{t} - k, 0)$ in state $NL$. The payoff difference between the two states is now $J + c\bar{t} + k$, as opposed to the payoff difference $J$ under American rule. This increased payoff difference is the only source of the difference between the two rules. Now, at the last day $\hat{t}$, the parties agree rather than going to court if the total optimism $y(J + c\bar{t} + k)$ about their prospects exceeds the cost $k + c(\hat{t} - \hat{t})$ of going to the court, i.e.,

$$y \leq \hat{y}_{EC} = \frac{k + c(\hat{t} - \hat{t})}{J + c\bar{t} + k} \approx \frac{k}{J + c\bar{t} + k}.$$  

Recall that the corresponding threshold under the American rule was $\hat{y}_C \approx k/J$ as the optimism was only about $J$. Clearly, the increased stakes under the English rule leads to an impasse on a wider range of optimism levels.
As in the analysis under the American rule with commitment, the payoff difference above is reflected in the settlement amounts under the two states:

\[ S_t^E(L) - S_t^E(NL) = J + ct + k, \]

where \( S_t^E(L) \) and \( S_t^E(NL) \) are the settlement amounts under information in favor of the plaintiff and the defendant, respectively, at time \( t \).

Under the American rule, the difference is just \( J \). That is, under the English Rule, the difference is shifted up by the amount contested at the court, and this increases the incentive for delay. Indeed, we now compare the larger difference \( S_t^E(L) - S_t^E(NL) \) to the same cutoff \( s^* \), as defined in Lemma 2, to determine whether there is an agreement before the deadline. This leads to the cutoff

\[ y_{EC}^* = \frac{c}{J + ct + k} \frac{\Delta}{1 - e^{-\lambda \Delta}} \approx \frac{c}{\lambda} \cdot \frac{1}{J + ct + k}, \]

which is the solution to \( J + ct + k = s^* \). Recall that the corresponding threshold under the American Rule was \( y_C^* \approx \frac{c}{\lambda} \). As under the American Rule with commitment, when \( y < y_{EC}^* \), there is immediate agreement. When \( y_{EC}^* < y \leq \hat{y}_{EC} \), there is a strong deadline effect and the parties wait until the last date \( \hat{t} \) to settle. When \( y > \hat{y}_{EC} \), there is impasse, and the court decides the case. Since the difference between the English rule and American rules stem from the increased stakes under the English rule, the cutoffs are all proportionally smaller than their counterparts under the American Rule:

\[ \frac{y_{EC}^*}{y_C^*} = \frac{\hat{y}_{EC}}{\hat{y}_C} = \frac{J}{J + ct + k}. \]

Figure 8 illustrates the settlement dynamics across different levels of optimism under the English and American Rules. When the level of optimism \( y \) is lower than \( y_{EC}^* \), there is immediate agreement under both systems, but for the range \( y \in (y_{EC}^*, y_C^*) \), our model predicts immediate agreement under the American Rule, but a strong deadline effect under the English Rule. Similarly, when the level of optimism is in the range \( y \in (\hat{y}_{EC}, \hat{y}_C) \), our model predicts that a settlement will be struck at the deadline under the American Rule, but that settlement negotiations will result in impasse and the case will go to court under the English Rule. Thus, under our model, we would expect more cases to go to court under the English Rule than under the American Rule, as is in line with the predictions generated by standard incomplete information models such as Bebchuck (1984). Moreover, our model also predicts that more cases will have lengthy delays of settlement under the English rule.

\footnote{One can also compute that \( S_t^E(G) = J + \alpha(c(\bar{t} - t) + k) + c_G t \) and \( S_t^E(NG) = -(1 - \alpha)(c(\bar{t} - t) + k) - c_D t \).}
than the American rule so that greater negotiation costs are incurred and litigation processes are longer and more costly over all.

Note that the sizes of the differences between $y^*_E$ and $y^*_C$ and between $\hat{y}_E$ and $\hat{y}_C$ are proportional to the costs of negotiation and going to court. To see this observe that, $y^*_C - y^*_E = \frac{ct+k}{J+c+ct+k}y^*_C$ and $\hat{y}_C - \hat{y}_E = \frac{ct+k}{J+c+ct+k}\hat{y}_C$ so that the greater the cost of disagreement $ct+k$ is relative to the judgment fee $J$, the greater the proportion of cases that we would expect to delay settlement and to go to court under the English Rule than under the American Rule. This suggests that the effect of the English Rule on settlement dynamics is particularly sensitive to the choice of court fee.

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**Figure 8:** A comparison of the settlement dynamics under the American and the English rules.

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### 9.2 Intangible Costs Without Commitment

We now consider the case of the English Rule with intangible negotiation costs. In this case, it is more natural to consider the case in which the plaintiff is not forced to commit to a lawsuit once he initiates it, as the losing party does not need to pay the other party’s costs. In this setting, the English Rule yields the payoff vector $(u_P, u_D) = (J - c_P\bar{\tau}, -J - c_D\bar{\tau} - k)$ in state $L$ and $(u_P, u_D) = (-c_P\bar{\tau} - k, -c_D\bar{\tau})$ in state $NL$. We assume that $q_P \geq \frac{K+c_P\Delta}{K+J}$ so that the plaintiff can credibly threaten to go to court at the deadline, as in our base model analysis. When it is known that the defendant is liable, the difference between the American and the English rule is simply the difference in the payment: under the English rule, the defendant pays $J+k_P$ instead of $J$. Since the settlement amount is 0 at state $NL$ under both rules, the equilibrium behavior under the English rule is the same as the behavior under the American rule with the judgment $\tilde{J} = J + k_P$, instead of $J$. In particular, the settlement
amount upon the arrival of evidence of liability is

\[ S_E^t(L) = \max\{ J + \alpha (\bar{t} - t) + k - c_P (\bar{t} - t), 0 \}, \]

where we add \( k_P \) to the first term in the definition of \( S_t(L) \). Whenever \( S_t(L) > 0 \), we have

\[ S_E^t(L) - S_t(L) = k_P. \]

That is, the English Rule merely shifts the settlement amount under the American Rule by some constant positive amount, so that the agreement dynamics in this case are qualitatively analogous to those of our main model. In particular, unlike the case of tangible costs with commitment, either a strong plaintiff or strong defendant case may arise with behavior following that described in Propositions 1 and 2.

We can also obtain the cutoffs for various patterns of equilibrium behavior by replacing \( J \) with \( J + k_P \). The threshold of the level of optimism needed for agreement at the deadline becomes \( \hat{y}_E \approx \frac{k}{J + k_P} \), as opposed to the threshold \( \hat{y} \approx \frac{k}{J} \) under the American Rule. Similarly, the threshold for the level of optimism needed for agreement at a date \( t \) before the deadline becomes \( y_E^* \approx \frac{c}{\lambda(J + \alpha k)} \), as opposed to the threshold \( y^* \approx \frac{c}{\lambda(J + \alpha k - k_P)} \) under the American Rule. As in the case of tangible costs, our model predicts more delays and more impasse under the English Rule than under the American Rule.

We illustrate the settlement dynamics for this case, when the plaintiff is strong in Figure 9. When the defendant is strong, the dynamics are analogously shifted. The settlement function under information in favor of the plaintiff with the English Rule with intangible costs, \( S_E^t(L) \), is parallel to the settlement function under plaintiff-favoring information with the American Rule, \( S_t(L) \), only shifted up by the plaintiff’s court fee \( k_P \). Thus, the earliest date of agreement under the English Rule, \( t^*_E \), is later than the earliest date of agreement under the American Rule, \( t^* \). Similarly, when the defendant is strong, the earliest date of disagreement is earlier under the English Rule than under the American Rule.

10 Partially Informative Evidence

In our baseline model, we assume that the evidence is decisive, in that it reveals the outcome of the court. In this section, we consider a more general model in which the evidence is only partially informative and may not fully reveal the outcome of the court. We generalize our baseline model by allowing the piece of evidence to be a binary signal with the following
probability table:

| evidence \ state | L   | NL   |
|------------------|-----|------|
| L                | $\pi$ | $1 - \pi$ |
| NL               | $1 - \pi$ | $\pi$ |

where $\pi \in [1/2, 1]$ is the precision of the signal. When the state is liable (namely $L$), the evidence says liable—denoted by $L$—with probability $\pi$ and not liable—denoted by $NL$—with probability $1 - \pi$; likewise it says liable with probability $1 - \pi$ and not liable with probability $\pi$ when the state is not liable. The baseline model corresponds to the extreme case, $\pi = 1$. We demonstrate that our results remain qualitatively intact when $\pi$ is sufficiently high so that the parties settle when a liable (plaintiff-favoring) signal arrives and the case is dropped when a not-liable (defendant-favoring) signal arrives. In that case, we also have an intuitive comparative static: as the precision $\pi$ increases, the settlement amount under the liable signal increases, increasing the incentive for waiting, and resulting in longer delays and more cases with impasse. In our exposition, for simplicity, we will focus on the continuous-time limit and the powerful plaintiff case with $\alpha k - k_P \geq 0$ and $\hat{y} > y^*$; the other cases are adjusted similarly.

With partially informative evidence, when the evidence suggests liability, the parties increase their assessments of the probability of liability, but they are still uncertain about the outcome, and there is still optimism about the outcome of the court, although it is reduced. In particular, after observing a signal that suggests liability, the plaintiff and the defendant update their beliefs over the likelihood of liability to

$$q_{P,L} = \frac{q_P \pi}{q_P \pi + (1 - q_P) (1 - \pi)}$$

and

$$q_{D,L} = \frac{q_D \pi}{q_D \pi + (1 - q_D) (1 - \pi)},$$
respectively. The optimism level reduces to

\[ y_L = a_L y \]

where

\[ a_L = \frac{\pi (1 - \pi)}{(1 - \pi + q_P (2 \pi - 1)) (1 - \pi + q_D (2 \pi - 1))}. \]

Observe that \( q_{PL} \) and \( q_{DL} \) are increasing in \( \pi \) (approaching 1 as \( \pi \to 1 \)), while the residual optimism level \( y_L \) is decreasing in \( \pi \) (approaching 0 as \( \pi \to 1 \)). Similarly, when they observe a signal suggesting no liability, the plaintiff and the defendant decrease their assessments of liability to

\[ q_{PLNL} = \frac{q_P (1 - \pi)}{q_P (1 - \pi) + (1 - q_P) \pi} \quad \text{and} \quad q_{DLNL} = \frac{q_D (1 - \pi)}{q_D (1 - \pi) + (1 - q_D) \pi}, \]

respectively, and the optimism level reduces to

\[ y_{NL} = a_{NL} y \]

where

\[ a_{NL} = \frac{\pi (1 - \pi)}{(\pi + q_P (1 - 2 \pi)) (\pi + q_D (1 - 2 \pi))}. \]

We focus on the case that

\[ q_{PLNL} < k_P / J. \] (19)

In that case, the plaintiff drops the case instead of going to court if the evidence says not liable, yielding the settlement amount

\[ S_t (NL) = 0 \]

as in the baseline model. This case arises when the precision \( \pi \) of the signal is high.\(^\text{12}\)

On the other hand, when the evidence says liable, the parties settle at \( \hat{t} \) if and only if

\[ y_L = a_L y \leq \hat{y}, \]

where \( \hat{y} = k / J \) is as in the baseline model. When \( \pi \) is sufficiently high, the residual optimism level \( y_L \) will be below \( \hat{y} \), and the parties will settle after the arrival of evidence of liability. We will assume throughout that \( \pi \) is sufficiently high that both (19) and (20) hold.

\(^{12}\)In particular, when

\[ \pi > \frac{q_P (1 - k_P / J)}{(1 - q_P) k_P / J + q_P (1 - k_P / J)}. \]
The settlement amount after a signal of liability at $\hat{t}$ is

$$S_t(L) = q_{P,L}J - k_P + \alpha (k - y_L J) = J(q_{P,L} - \alpha y_L) + \alpha k - k_P.$$ 

To see this, observe that the gain from settlement is $k - y_L J$, which is reduced from $k$ because of optimism about the case. The plaintiff gets his expected payoff from going to court $q_{P,L}J - k_P$, plus an $\alpha$ fraction of the gain from trade $k - y_L J$. The settlement at any earlier date $t$ is

$$S_t(L) = S_t(L) + (\alpha c - c_P)(\hat{t} - t)$$

as in the baseline model. Since we are in the powerful plaintiff case (i.e., $\alpha > c_P/c$), we have that $S_t(L) > S_t(L)$. Observe that with respect to the case of decisive evidence (with $\pi = 1$), $S_t(L)$ is lower. Indeed, as $\pi$ decreases, both the expected payment $q_{P,L}J$ and the gain from trade $k - y_L J$ decrease. This decreases $S_t(L)$ and $S_t(L)$ by the same amount for all $t$.

This shift in the settlement amount is the only change with respect to our baseline model. It does not affect our qualitative results, but it implies less delays to settlement. Formally, as in the baseline model, an agreement regime at a date $t$ carries over to the date $t - \Delta$ if and only if $S_t(L) \leq s^*$, where $s^* = \frac{c}{\lambda y}$.

Observe that although the dynamics are precisely as in the baseline case for any given $S_t(L)$, the settlement amount $S_t(L)$ itself is lower due to the imprecision of the evidence, and this results in different cutoff levels. In particular, an agreement at $\hat{t}$ carries over to $\hat{t} - \Delta$ if and only if

$$y < y^*(\pi) = \frac{c}{\lambda} \frac{1}{S_t(L)} = \frac{c}{\lambda} \frac{1}{J(q_{P,L} - \alpha y_L) + \alpha k - k_P}.$$  

This leads to the following extension of Proposition 1 to the case of partially informative evidence.

**Proposition 7** Assume $\alpha > \alpha^*$, $\hat{y} \equiv k/J > \max \{y_L(\pi), y^*(\pi)\}$, and $q_{P,NL}(\pi) < k_P/J$.

When $y > \hat{y}$, there is a disagreement regime at every $t \in T^*$. When $y < \hat{y}$, there is a disagreement regime at every $t < t^*$ and there is an agreement regime at every $t \geq t^*$ where

$$t^*(\pi) = \max \{t \in T^* | S_t(L) > s^*\}.$$ 

In particular, when $y^*(\pi) \leq y \leq \hat{y}$, as long as they do not receive any information, the parties wait until the deadline and settle there in equilibrium. Finally, $y^*(\pi)$ is decreasing and $t^*(\pi)$ is weakly increasing in $\pi$.

\[13\]To see this, assume that there is an agreement regime at $t$, so that they reach an agreement at $t$ without an information arrival. Hence, without any information arrival by $t - \Delta$, the sum of the parties’ expected payoffs from waiting until $t$ is $y \left(1 - e^{-\lambda \Delta}\right) S_t(L) - c\Delta$, as in the baseline model. The parties agree at $t - \Delta$ if and only if $y \left(1 - e^{-\lambda \Delta}\right) S_t(L) - c\Delta \leq 0$, or equivalently $S_t(L) \leq s^*$. 

36
This extends Proposition 1 verbatim to the current setup where the cutoff $y^*(\pi)$ for the strong deadline effect and the settlement date $t^*(\pi)$ depend on $\pi$, subsuming Proposition 1 as the special case of $\pi = 1$. It further establishes that as evidence becomes more informative (i.e. as $\pi$ increases), the cutoff $y^*(\pi)$ decreases, resulting in a strong deadline effect for a larger range of optimism, and $t^*(\pi)$ increases, resulting in longer delays when there is a weak deadline effect. The fact that $y^*(\pi)$ is decreasing in $\pi$ is evident from (21), which defines $y^*(\pi)$ for the continuous-time limit, where $q_{P,L}$ is increasing and $y_{L}$ is decreasing. The fact that $t^*(\pi)$ is increasing is illustrated in Figure 10: as $\pi$ increases, $S_{L}(L)$ shifts up, and the intersection $t^*(\pi)$ of $S_{L}(L)$ and $s^*$ increases. Indeed, when $y \leq y^*(\pi)$, we have

$$t^* = \hat{t} - \frac{s^* - S_{L}(L)}{\alpha c - c_{P}},$$

which depends on $\pi$ through $S_{L}(L)$: as $\pi$ increases, $S_{L}(L)$ also increases, increasing $t^*$.

## 11 Conclusion

Costly settlement delays and impasse are common in pretrial negotiations. A prominent explanation for these delays is excessive optimism about the outcome of the trial. This has been well established for the case of impasse in traditional static models of negotiation. In this paper, we develop a tractable dynamic model of negotiations in which the parties can learn about the strength of their respective cases as negotiations go on. With our model, we are able to analytically derive whether and when the parties reach an agreement, and how the dynamics of such an agreement are affected by a variety of factors, such as the balance of bargaining power between the two parties, the parties’ ability to commit, and the extent
of each party’s liability.

The underlying rationale for delay is as follows: the parties’ optimism about the trial translates into optimism about the settlement amount that would be agreed upon if information in their favor arrives. The parties wait without settling if and only if their optimism about a future settlement exceeds their costs of waiting an additional day, and they settle as soon as the cost of waiting outweighs their prospects for future gains. Therefore, the dynamics of such strategic settlements is driven by the dynamics of the settlement amount when the outcome of the trial is known. By analyzing the effect of the various factors on the settlement amount, we are able to understand the effects of different policies currently considered in public discourse, in the presence of optimism. It is well known in the literature how different factors affect $S_t(L)$. Thus, our model enables an analysis of how these factors would affect agreement dynamics as well.

For example, under the baseline model with the “American Rule”, under which the parties cannot commit, the difference in the settlement amount $S_t(L) - S_t(NL)$ increases as we go away from the deadline, when the plaintiff has more bargaining power. In this case, the incentive to wait under optimism also increases as we go away from the deadline, yielding straightforward agreement dynamics. If the level of optimism $y$ is so excessive that the parties would not agree at the deadline, then they will never agree prior to the deadline either, so that the negotiations will end in impasse. This happens when the level of optimism $y$ exceeds the cutoff $\hat{y}$ in Table 1, which equates the optimism about the trial outcome ($J\hat{y}$) with cost of going to trial ($k$). On the other hand, if the level of optimism is moderate so that $y$ is between $y^*$ and $\hat{y}$, then the parties settle at the last possible day of negotiations, but will not agree before that. This is because the level of optimism $yS_t(L)$ exceeds the effective cost of delaying negotiations an additional day $c/\lambda$. This deadline effect is a well known empirical regularity. If the level of optimism is low, so that $y < y^*$, then the parties wait until the date $t^*$, at which the effective cost of optimism is equal to the effective cost of waiting.

By contrast, if the defendant has more bargaining power, the settlement amount decreases as we go away from the deadline, decreasing the incentive for the delay. This leads to a qualitatively different set of agreement dynamics in which the parties either agree immediately, or they wait until the deadline. Likewise, if the plaintiff is able to commit to maintaining his case, then the difference in settlement amounts $S_t(L) - S_t(NL)$ is constant, irrespective of the time from the deadline and the balance of bargaining power. Thus, agreement is either immediate or at the deadline, depending on whether the level of optimism exceeds $y^*_C$, which
Table 1: Summary of Cutoffs for Optimism

| Rule                        | Cutoff for Impasse$^{15}$ | Strategic Settlement Date$^{16}$ |
|-----------------------------|---------------------------|----------------------------------|
| Weak Deadline               |                           |                                  |
| American Rule (Baseline)    | $y^* = \frac{c}{\lambda J + \alpha k - k_p}$ | $\hat{y} = \frac{K}{J}$          |
| American Rule (Commitment)  | $y^*_C = \frac{c}{\lambda J}$ | $\hat{y}_C = \frac{K}{J}$        |
| English Rule (Baseline)     | $y^*_E = \frac{c}{\lambda J + \alpha k}$ | $\hat{y}_E = \frac{K}{J + k_p}$  |
| English Rule (Commitment)   | $y^*_{EC} = \frac{c}{\lambda J + \epsilon t + k}$ | $\hat{y}_{EC} = \frac{K}{J + \epsilon t + k}$ |

equates the effective cost of delaying negotiations with optimism about the trial.

While the ability to commit does not impact whether or not there is impasse, under the American rule, it does affect the timing of an interior settlement. When $\alpha > k_p/k$, the cutoff for a weak deadline effect is smaller under the baseline model than in a model with commitment under the American rule. As we have shown, when the plaintiff’s bargaining power exceeds his share of the costs of litigation, commitment strictly reduces delay simply because the difference in the settlement amounts under the two outcomes is lower everywhere.

By the same token, we have shown how a policy change from the American rule to the English rule would affect settlement delays. Such a reform is equivalent to adding the plaintiff’s effective court cost to the judgment amount. This causes the difference in settlement amounts $S_t(L) - S_t(NL)$ to be higher, regardless of commitment, thereby increasing the incentive for both parties to delay. Consequently, such a reform would result in longer delays and higher rates of impasse. This is clearly illustrated in Table 1. The cutoff for a weak deadline is smaller under the English rule than under the American rule, regardless of whether there is commitment ($y^*_{EC} < y^*_C$) or not ($y^*_E < y^*$). Analogously, for the cutoff of impasse, we have that $\hat{y}_{EC} \leq \hat{y}_E \leq \hat{y}_C = \hat{y}$ so that it is lowest with commitment under the English rule. Along similar lines, our model is versatile enough to study the impact of various other policy changes in the bargaining environment.

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$^{14}$This is the value of optimism s.t. if $y \leq y^*$, then an agreement at $t$ implies an agreement at $t - \Delta$. Hence, there is a weak deadline whenever $y \leq y^*$, and a strong deadline whenever $y > y^*$. Note that there is no weak deadline effect under commitment.

$^{15}$This is the value of optimism s.t. if $y \leq \hat{y}$, then there is agreement at the deadline $\hat{t}$. If $y > \hat{y}$, then there is impasse.

$^{16}$This is the first date at which there is an agreement without information arrival, when there is a weak deadline effect. Note that under commitment, agreement is either immediate or at the deadline, so that $t^{past}$ is never in the interior of the bargaining period.
**Omitted Proofs**

**Proof of Lemma 1.** Consider the case that there is an agreement regime at \(t_1\) (the other case is proved similarly). The continuation value of the plaintiff is

\[
V_{t,P}(\emptyset) = \sum_{t \in T^*, t_0 < t \leq t_1} P(t|t_0)(q_PS_t(L) - c_P(t - t_0)) + e^{-\lambda(t_1 - t_0)}(S_{t_1}(\emptyset) - c_P(t_1 - t_0)).
\]

To see this note that, if there is an arrival at \(t\), the parties reach an agreement where the settlement is \(S_t(L)\) if the information indicates liability and 0 otherwise, yielding expected settlement payment of \(q_PS_t(L)\) to the plaintiff. She also pays costs depending on the settlement date. Similarly,

\[
V_{t,D}(\emptyset) = \sum_{t \in T^*, t_0 < t \leq t_1} P(t|t_0)(-q_DS_t(L) - c_D(t - t_0)) + e^{-\lambda(t_1 - t_0)}(-S_{t_1}(\emptyset) - c_D(t_1 - t_0)),
\]

where she expects to pay only \(q_DS_t(L)\) if there is an arrival at \(t\). Adding up the two equations, we obtain that \(V_{t,P}(\emptyset) + V_{t,D}(\emptyset) \leq 0\) if and only if

\[
\sum_{t \in T^*, t_0 < t \leq t_1} P(t|t_0)[(yS_t(L) - (c_P + c_D)(t - t_0))] \leq e^{-\lambda(t_1 - t_0)}(c_P + c_D)(t_1 - t_0),
\]

as in the lemma.

**Proof of Lemma 2.** This lemma follows from Lemma 3, which we prove next.

**Proof of Lemma 3.** Consider any \(t\). Without any information arrival, the continuation values of the Plaintiff and the Defendant at \(t\) are

\[
V_{t,P}(\emptyset) = p\Lambda(t)S_{t+\Delta}(L) + (1 - \Lambda(t))V_{t+\Delta,P}(\emptyset) - c_P\Delta
\]

\[
V_{t,D}(\emptyset) = -q\Lambda(t)S_{t+\Delta}(L) + (1 - \Lambda(t))V_{t+\Delta,D}(\emptyset) - c_D\Delta,
\]

respectively. To see this, observe that there will be an arrival with probability \(\Lambda(t)\) until the next period. According to the Plaintiff, with probability \(p\) the information will point to liability, leading to settlement \(S_{t+\Delta}(L)\); the settlement will be zero with the remaining probability \(1 - p\). With probability \(1 - \Lambda(t)\), there is no information arrival, and the continuation values are as at \(t + \Delta\), minus the cost of waiting. By adding up these equalities, we obtain

\[
V_{t,P}(\emptyset) + V_{t,D}(\emptyset) = y\Lambda(t)S_{t+\Delta}(L) + (1 - \Lambda(t))(V_{t+\Delta,P}(\emptyset) + V_{t+\Delta,D}(\emptyset)) - c\Delta.
\]

Observe also that the sum of continuation values is always non-negative:

\[
V_{t+\Delta,P}(\emptyset) + V_{t+\Delta,D}(\emptyset) \geq 0;
\]

40
it is positive if there is disagreement at \( t + \Delta \) and zero if there is agreement at \( t + \Delta \). Hence,

\[
V_{t,P}(\emptyset) + V_{t,D}(\emptyset) \geq y_A(t) S_{t+\Delta}(L) - c\Delta.
\]

Now, if \( S_{t+\Delta}(L) > \frac{c\Delta}{y_A(t)} \), then \( y_A(t) S_{t+\Delta}(L) - c\Delta > 0 \), showing that \( V_{t,P}(\emptyset) + V_{t,D}(\emptyset) > 0 \). There is a disagreement regime at \( t \) in that case, proving the first part. To prove the second part, observe that if there is agreement at \( t + \Delta \), then \( V_{t+\Delta,P}(\emptyset) + V_{t+\Delta,D}(\emptyset) = 0 \), yielding

\[
V_{t,P}(\emptyset) + V_{t,D}(\emptyset) = y_A(t) S_{t+\Delta}(L) - c\Delta.
\]

If in addition \( S_{t+\Delta}(L) \leq \frac{c\Delta}{y_A(t)} \), then \( y_A(t) S_{t+\Delta}(L) - c\Delta \leq 0 \), showing that there is an agreement regime at \( t \).

**Proof of Lemma 4.** Since \( y \) is uniformly distributed on \([0,1]\), we can compute the distribution of \( t^* \) from (16) as follows. By (16), for any \( t < \hat{t} \),

\[
F_{t^*}(t) = \Pr\{t^* \leq t\} = \Pr\{y \leq \frac{c/\lambda}{(t-t)(\alpha - \alpha^*)c + (J + \alpha k - kP)}\} = \beta(t),
\]

where the last equality is by substituting the values of \( A \) and \( B \) and by the fact that \( y \) is uniformly distributed. At the deadline \( \hat{t} \), by Proposition 1, there is an agreement regime if \( y \leq \hat{y} \), and there is impasse otherwise. Therefore, \( F_{t^*}(\hat{t}) = \hat{y} \).

**Proof of Lemma 5.** Since \( y \) and \( \tau_A \) are independent, by definition,

\[
F_{\tau}(t) = \Pr\{\min\{t^*, \tau_A\} \leq t\}
= \Pr\{\tau_A \leq t\} + \Pr\{t^* \leq t\} \Pr\{\tau_A > t\}
\]

yielding the expression for \( F_{\tau^*}(t) \) in the proposition. It is straightforward to obtain the expressions for the density and the hazard rate from \( F_{\tau^*}(t) \).

**Proof of Proposition 3.** Take any \( t < \hat{t} \). From Lemma 5,

\[
\frac{\partial f_{\tau^*}(t)}{\partial t} = \lambda \left(-\lambda e^{-\lambda t} \left[(\alpha - \alpha^*) \beta(t)^2 - \beta(t) + 1\right] + e^{-\lambda t} \left[2(\alpha - \alpha^*) \beta(t) \beta'(t) - \beta'(t)\right]\right).
\]

Hence \( f_{\tau^*}(t) \) is decreasing if and only if the above expression is negative:

\[
-\lambda \left[(\alpha - \alpha^*) \beta(t)^2 - \beta(t) + 1\right] + \left[2(\alpha - \alpha^*) \beta(t) \beta'(t) - \beta'(t)\right] < 0.
\]

This further simplifies to

\[
\lambda \left[2(\alpha - \alpha^*)^2 \beta(t)^3 - 2(\alpha - \alpha^*) \beta(t)^2 + \beta(t) - 1\right] < 0.
\]

This is indeed the case when \( (\alpha - \alpha^*) \) is positive and \( \beta(t) \in [0,1] \). Indeed, varying \( \beta \in [0,1] \) and \( \alpha - \alpha^* \geq 0 \), the only root of \( \lambda \left[2(\alpha - \alpha^*)^2 \beta^3 - 2(\alpha - \alpha^*) \beta^2 + \beta - 1\right] \) is given by \( \beta = 1 \) and
$\alpha - \alpha^* = 0$. Since the above expression is $-\lambda$ for $\beta = 0$, this shows that it is negative at all feasible values of $\alpha - \alpha^*$ and $\beta$.

Now, consider the hazard rate, $H(t)$. One can compute that

$$\frac{\partial H(t)}{\partial t} = \frac{\lambda^2 (\alpha - \alpha^*)^2 \beta(t)^3}{(1 - \beta(t))^2} + \frac{\lambda^2 (\alpha - \alpha^*)^2 \beta(t)^3}{(1 - \beta(t))} > 0.$$  

(Both terms are positive because $\beta > 0$ and $\alpha - \alpha^* > 0$.)

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Supplementary Appendix on Further Extensions and Discussion (Not For Publication)

B.1 Continuous Time with a Powerful Defendant

Assume $\alpha < \alpha^*$. Then, Proposition 2 establishes that there are only three possible patterns of behavior when there is no arrival: immediate agreement, strong deadline effect, and impasse. As we have discussed above, immediate agreement arises whenever $y \leq y^*$ or $\bar{t} > \tilde{t}$. Strong deadline effect is exhibited in equilibrium when $y^* < y \leq \hat{y}$ and $\bar{t} < \tilde{t}$. Finally, when $y > \hat{y}$ and $\bar{t} < \tilde{t}$, negotiation results in impasse. We will now provide explicit formula for $\tilde{t}$ and explain how it varies with key variables.

To this end, observe that, in the limit $\Delta \to 0$,

$$f(0, t) = \frac{1}{1 - e^{-\lambda t}} \int_0^t (J + \alpha k - kp - (\alpha^* - \alpha) c (t - x)) \lambda e^{-\lambda x} dx$$  \hspace{1cm} (22)

$$= \frac{c}{\lambda} \left[ (\alpha^* - \alpha) \left( 1 - \frac{\lambda \hat{t}}{1 - e^{-\lambda \hat{t}}} \right) + \frac{1}{y^*} \right].$$

By Proposition 2, when $y^* < y \leq \hat{y}$, the cutoff is determined by the equation $f(0, \tilde{t}) = s^*$. Substituting the values of $f$ and $s^*$, one can rewrite this equation as

$$\frac{\lambda \tilde{t}}{1 - e^{-\lambda \tilde{t}}} = 1 + \frac{1}{\hat{y}^*} - \frac{1}{\hat{y}} \frac{\alpha^* - \alpha}{\alpha^* - \alpha}.$$  \hspace{1cm} (23)

The left-hand side is the ratio of time to the cumulative probability of arrival when the time is normalized by taking the expected arrival date as the unit of time. It is 1 when $\tilde{t} = 0$ and is increasing in $\tilde{t}$. Hence, $\tilde{t}$ is positive and increasing with the right-hand side. In particular, $\tilde{t}$ is increasing in the level of optimism $y$. This is intuitive as the delay in our model is derived by optimism. Moreover, as $y \to y^*$, $\tilde{t}$ approaches 0. Hence, the immediate agreement result for $y \leq y^*$ extends continuously to the case of $y > y^*$. Likewise, $\tilde{t}$ is decreasing in $\alpha^* - \alpha$, and it goes to infinity as $\alpha$ approaches the cutoff $\alpha^*$. This shows that the strong deadline effect for the case of powerful plaintiff extends to the case of powerful defendant continuously under moderate optimism.

When $y > \hat{y}$, there is a disagreement regime at time 0 if and only if

$$f(0, \tilde{t}) > s^* - (J - k/y) e^{-\lambda \tilde{t}} / \left( 1 - e^{-\lambda \tilde{t}} \right) = s^* - (1 - \hat{y}/y) J e^{-\lambda \tilde{t}} / \left( 1 - e^{-\lambda \tilde{t}} \right).$$

Comparing this to (22), one can conclude that there is impasse for all values of $\tilde{t}$ whenever $(1 - \hat{y}/y) J/c \geq \alpha^* - \alpha$, i.e., whenever

$$y \geq \frac{\hat{y}}{1 - (\alpha^* - \alpha) c/J} \equiv \bar{y}.$$
When \( y \geq \bar{y}, \tilde{t} = \infty \). When \( y < \bar{y}, \tilde{t} \) is determined by the equality
\[
\frac{\lambda \tilde{t}}{1 - e^{-\lambda \tilde{t}}} = 1 + \frac{(1 - \hat{y}/y) J/c + (1/y^* - 1/y)}{(\alpha^* - \alpha) - (1 - \hat{y}/y) J/c}.
\]
Once again, \( \tilde{t} \) is continuous at \( y = \hat{y} \) and increasing in \( y \), approaching infinity as \( y \to \bar{y} \). Note also that \( \tilde{t} \) is increasing in \( \alpha \), and approaches infinity as \( \alpha \to \alpha^* \) (while \( \bar{y} \) approaches \( \hat{y} \) in the mean time).

Since \( \tilde{t} \) is increasing in \( y \), for any fixed \( \tilde{t} \), there exists a unique \( \tilde{y} \in (y^*, \bar{y}) \), such that there is immediate agreement whenever \( y < \tilde{y} \). The players wait until the deadline to settle when \( y \in (\tilde{y}, \hat{y}) \), and go all the way to the court when \( y > \max \{\hat{y}, \bar{y}\} \).

**B.2 Cost Assumptions**

In the previous sections, we have assumed that
\[
\frac{c}{\lambda} \leq k \frac{J + \alpha k - k_P}{J}.
\]
This is the case when the cost \( k \) of litigation is much larger than the expected cost \( c/\lambda \) of negotiation until the information arrives (i.e. \( c/\lambda \ll k \)) and that the plaintiff is willing to go to the court when it is known that the defendant is liable (i.e. \( J > k_P \)). This assumption allowed us to assume that \( y^* \leq \hat{y} \), so that if \( y > \hat{y} \), then there is complete impasse, and no settlement ever materializes. When there is a powerful defendant, this assumption does not affect the qualitative results. When there is a powerful plaintiff, the agreement dynamics are as follows when this assumption fails.

**Proposition 8** Assume that \( \alpha > \alpha^* \) and \( \hat{y} < y^* \). When \( y^* < y \), there is a disagreement regime at every \( t \in T^* \). When \( y < \hat{y} \), there is a disagreement regime at every \( t < t^* \) and there is an agreement regime at every \( t \in [t^*, \hat{t}] \) where, as before, \( \hat{t} = \max T^* \) and
\[
t^* = \max \{t \in T^* | S_t(L) > s^*\}.
\]

When \( y^* > y > \hat{y} \), there is an agreement regime for every \( t \in (t^*, t^{**}) \) and a disagreement regime for every \( t < t^* \) and \( t \in [t^{**}, \hat{t}] \) where
\[
t^{**} = \max \left\{ t \in T^* | f(t_0, \tilde{t}) \leq s^* + \frac{e^{-\lambda(t_0-t)}}{1 - e^{-\lambda(t_0-t)}} \left[ (c(\tilde{t} - t) + k - yJ)/y \right] \right\}.
\]

The dynamics of Proposition 8 in the case that \( y^* > y > \hat{y} \) are illustrated in Figure 11. When \( y > y^* \), it follows that \( y > \hat{y} \) and so the dynamics are just as in the first case discussed in Proposition 1 and illustrated in Figure 1: there is an impasse and the agents go to court. When \( \hat{y} > y \), it is
also true that \( y^* > y \), and so the dynamics are as in the third case demonstrated in Figure 1: there is a weak deadline effect with disagreement until date \( t^* \), and agreement thereafter at any point. That is, when the level of optimism is on an extreme end of the cutoffs for agreement during negotiations and at the deadline, the settlement behavior is unchanged by the relative sizes of the cost of litigation and the expected cost of negotiation before information arrives. However, while there is weak deadline effect in the case that \( \hat{y} > y > y^* \), the dynamics are quite different when \( c > k \frac{J + \alpha k - k P}{J} \), and \( y^* > y > \hat{y} \). There is disagreement from the start of negotiations until date \( t^* \), as in the case of the weak deadline effect in Proposition 1, and there is immediate agreement at date \( t^* \). However, while the agents are willing to agree for some time after \( t^* \), they refuse to do so after date \( t^{**} \), at which point the agents’ optimism about future settlements becomes smaller than their expected cost of waiting to settle. Following \( t^{**} \), if the agents have not yet settled, they will disagree through the deadline, and go to court.

**Figure 11:** Agreement dynamics under a powerful plaintiff when \( y^* > y > \hat{y} \).

**B.3 English Rule with Tangible Costs and No Commitment**

For the sake of completeness, we also illustrate the case of the English Rule with tangible costs and no commitment in an online appendix. As we show in the main text, allowing the plaintiff to drop the lawsuit simply gives him more bargaining power and so creates further delays in settlement. In this case, the effective judgment is \( J + k_P + c\bar{t} \) instead of \( J \). Under information indicating liability, the settlement is \( S_{ET}(L) = \max\{J + \alpha (c(\bar{t} - t) + k) + cP, 0\} \), while \( S_{ET}(NL) \) remains 0. Notice that \( S_{ET}(L) \) is also parallel to \( S_{i}(L) \) only now shifted up by \( k_P + cP\bar{t} \), the total (tangible) cost of disagreement at the deadline. Thus, the settlement dynamics in this case are analogous to those in the case of the English Rule with intangible costs and no commitment, with the earliest date of agreement \( t_E^{ET} \), even later in the case of a strong plaintiff (and the earliest date of disagreement...
even earlier in the case of a strong defendant).

Figure 12: Agreement dynamics with tangible costs and no commitment under the English Rule.