Reinterpreting the Pioneer anomaly and its annual residual

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Abstract

In addition to its long-term constancy, the Pioneer (spacecraft) anomaly appears to only exist for bodies whose mass is less than that of: planets, moons, comets, and heavy asteroids of known mass. Assuming the observational evidence is reliable and not the result of an unknown systematic effect, a violation of the Weak Principle of Equivalence is implied. This constraint is the most confronting for any prospective new physics. Any new physical mechanism that proposes an additional gravitational force, i.e. an additional spacetime curvature, is rendered unreasonable because all masses should be equally affected. This paper examines an approach that is based upon the existence of a sum of additional field energies. A finite number of tiny wavelike undulations, upon the existing gravitational field, are hypothesized. The sources of these non-Einsteinian gravitational waves are the spin-orbit coupled moons of the solar system. An excess energy arising from: a lunar orbital motion, quantum mechanical geometric phase and spacetime curvature generates these new gravitational field waves. General Relativity’s Lorentz invariance demands that these “acceleration-waves” have constant amplitude. The dissipation of these spherical waves, as they expand out from suitable moons, is seen to exist as a volumetric based reduction in the inertial mass that can sympathetically oscillate with these waves. Therefore, masses above a given wave’s cut-off mass remain completely unaffected. The full substantiation and quantification of these undulations upon the gravitational field is deferred to another paper. This article simply seeks to show that a superposition of these “acceleration-waves” results in an oscillatory or non-steady expression of spacecraft kinetic energy. The superposition of these additional oscillatory components of longitudinal motion leads to a shortfall of actual steady translational motion. With wave energy proportional to lunar mass, and some moons inactive for geometric reasons, the four Galilean moons of Jupiter and Saturn's Titan dominate this new effect. The orbital resonances of Jupiter’s Galilean moons, markedly attenuates the variation in magnitude of the Fourier-like superposition of these waves. The small variance observed for the Pioneer (acceleration) anomaly around its long-term average has created the misleading impression of a constant additional inward acceleration with associated observational noise. Additionally, Jupiter’s least orbitally resonant moon Callisto, and Saturn’s Titan, cause the Pioneer spacecraft to exhibit a (synodic) 356 day resonance, that has been misinterpreted as an Earth based ‘annual’ residual. The amplitudes of the cyclic diurnal and annual Earth-to-spacecraft motion ‘offsets’ are examined in some detail. The Earth based annual residual is found to be actually very small and incapable of figuring in the Doppler tracking observations. In order to clearly establish and scrutinize the physical validity of the hypothesis being presented, aspects of the author’s broader model are incorporated into the discussion only where necessary. Direction cosine corrections, for wavefront direction relative to spacecraft trajectory, are quantitatively neglected in order to primarily scrutinize the physical viability of the hypothesis. Finally, the hypothesis sheds promising light upon other concerns regarding the modelling of gravitation influencing ‘light’ bodies in our solar system. These include: the “Earth Flyby Anomaly”, an apparent absence of small comets, an apparent paucity of smaller bodies in the Main Belt of asteroids, and residual doubts concerning the “Migrating Planets” hypothesis - that addresses the too rapid formation of the ice giants Uranus and Neptune.

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1 Introduction

The Pioneer anomaly has failed to be explained by any systematic effects and has two awkward observational characteristics. The first is its (long term) constancy and the second is that it implies an apparent violation of the principle of equivalence — in that only spacecraft and not: planets, moons, comets or large asteroids of known mass, seem to be affected [1, p.3]. By way of these two awkward observational constraints the anomaly has resisted full explication. The latter constraint and observational evidence appear to preclude any standard force based hypothesis, such as a modification of Einstein’s General Relativity. General Relativity, as a theory of gravitation, holds impressively in the solar system for electromagnetic wave propagation and the motions of ‘heavy’ bodies.

A hypothesis concerning the re-expression of a tiny proportion of total spacecraft kinetic energy into a number of (longitudinal) oscillatory components of kinetic energy is proposed. These coexist with and in addition to the dominant steady (non-oscillatory) kinetic energy. These additional ‘spectral’ components, together, then cause an ongoing shortfall in actual speed relative to predicted spacecraft translational speed. This motion is a response to tiny harmonic\(^1\) undulations of the gravitational field.

1.1 Outlining a new approach

This write up primarily presents the ideas and mathematics that illustrate how the anomalous Pioneer ‘acceleration’ can alternatively be seen as potentially a Fourier summation of first order constant amplitude (acceleration or gravitational\(^2\)) fluctuations upon the pre-existing gravitational field. To support this it is argued that, unlike the Earth based diurnal residual, the ‘annual’ residual is spacecraft based and hence real (and not quite of 365.25 days duration either). This residual is modelled to be the only obvious resonance of a number of sinusoidal undulations on (and of) the gravitational field. The amplitudes of these sinusoidal undulations are very small compared to the strength of the gravitational field. Only in very weak gravitational fields is their presence significant.

The source of these waves shall not be fully elucidated\(^3\), other than to say, that observational evidence and the author’s model dictate that they originate from regular moons of the solar system held in spin-orbit coupling around their respective host planets (which in turn orbit the Sun)\(^4\).

This new mechanism is dominated by the bigger moons of the solar system. Of the big seven moons — Earth’s moon (Moon, or unofficially Luna) and Neptune’s Triton do not ‘generate’ acceleration waves upon the pre-existing gravitational field. Thus, Jupiter’s four Galilean moons and Saturn’s Titan are seen to dominate the mechanism’s effect in our solar system\(^5\).

2 Expanding the hypothesis

In this section the basic preliminary material required to support both: a new model, and the mathematics that follows, is presented.

2.1 A Fourier wave summation

The constancy of the Pioneer anomaly means that any Fourier-like summation of first order fluctuations of acceleration upon the gravitational field would need to be special in that: they are of constant amplitude, and together they display only a hint of abnormally large variance around a constant mean [2, Fig. 14, p.24].

\(^3\)The physical establishment of the mechanism is vital, but this involves a long write up that is primarily quantum mechanical in nature. The reader is asked to ‘bear with the hypothesis’.

\(^4\)A fully detailed model produces amplitudes of the waves, and the distribution of the cut-off mass with respect to distance from the (finite number of) sources.

\(^5\)The collision based origin of the Earth’s moon, with its large angular momentum, and the retrograde motion of Neptune’s Triton makes their relationship between third-body orbital kinematics, spacetime curvature, quantum mechanical indeterminacy, and geometric phase advance incapable of generating any atomic virtual ‘excess’ energy. Since all atoms of some moons collectively share the same virtual excess spin angular momentum based energy these virtual energies ‘sum’ to become a singular real excess energy that is expressed upon the gravitational field as a new kind of wave.

\(^1\)In order to avoid confusion with wave resonances, the expression ‘harmonic’ is not used to describe sinusoidal and co-sinusoidal functions. The term ‘sinusoidal’ is preferred.

\(^2\)To avoid confusion with Einstein’s gravitational waves, the expression ‘acceleration waves’ is preferred. These acceleration waves, like gravitation, produce spacetime curvature.

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With three of Jupiter’s four Galilean moons in orbital resonance, and Ganymede and Callisto in a subtle 7 to 3 resonance; only Titan is a free ‘agent’. These orbital resonances act to smooth out the variations in the Fourier summation of these acceleration (or gravitational field) waves. Thus a superposition of waves approaches a constant amplitude but this is never attained in the short term.

2.2 (Acceleration) Wave aspects

This section is well short of fully comprehensive. An asymmetrical interaction of: gravitational curvature of space, the quantum mechanical geometric phase of atoms, and quantum indeterminacy of energy (per orbit) in the third body of a three body celestial system leads to these ‘acceleration’ waves upon (and of) the gravitational field. Indeed they violate the usual scope of energy conservation, although, it should be noted that conservation laws rely upon symmetry.

... where there is a symmetry there is a conservation law, and with certain reservations the converse is also true [3, p.159].

Geometric phase advance in prograde celestial three body motion, involving only positive mass, is an inherently asymmetrical situation.

General Relativity’s Lorentz invariance demands constant amplitudes. The dissipation of the waves is enacted by a wave volume based reduction in the mass that these waves can influence to sympathetically ‘oscillate’ in response to them. The waves have: an overall spherical shape, a period matching the heliocentric period of orbiting moons, the propagation (of constant wave phase) is at the speed of light, and the wave’s particles are necessarily spin 0 or spin zero. The particles associated with these waves impart no momentum to the Pioneer spacecraft (or any other body.) Thus, it is only the undulations upon the gravitational field that are capable of physically influencing ('lighter') bodies in the solar system.

2.3 Regarding the equivalence principle

Regarding the equivalence principle Thibault Damour offers the following advice.

The Equivalence Principle (EP) is a heuristic hypothesis which was introduced by Einstein in 1907, and used by him to construct his theory of General Relativity. [The] EP is not one of the basic principles of Nature (like, say, the Action Principle, or the correlated Principle of Conservation of Energy). It is a “regional” principle, which restricts the description of one particular interaction [mediated by a massless spin-2 field]. An experimental “violation” of the EP would not at all shake the foundations of physics (nor would it mean that Einstein’s theory is basically “wrong”). Such a violation might simply mean that the gravitational interaction is more complex than previously assumed, and contains, in addition to the basic Einsteinian spin-2 interaction, the effect of another long-range field. (From this point of view, Einstein’s theory would simply appear as being incomplete.)

Einstein’s Principle of Equivalence is (non-locally) restricted to uniform fields, and thus it does not conflict with undulations placed upon a (predominantly static) gravitational field that are not solely of gravitational origin. Indeed, a uniform gravitational field may be seen as an oscillatory field whose frequency goes to zero, and whose period goes to infinity.

The new model being proposed, in line with the observational evidence, violates the weak principle of equivalence in that the existence or non-existence

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6 A force based model of the union of quantum mechanics and general relativity always conserves energy, a priori. By way of an overlooked energy (imbalance and subsequent) transfer, e.g. from the indeterminate micro (sub-'quantum) world to the macro (spacetime curvature) world, this force based approach could be found wanting.

7 Thus, the waves carry no angular momentum. The waves only 'carry' energy and yet their origin removes the excess virtual angular momentum per orbit shared by numerous atoms of certain moons. When such waves are generated, the momentum of a moon, as a bulk object, remains unchanged.

8 In the broadest sense, it is an interaction between general relativity and quantum mechanical (energy) indeterminacy (over a 'cycle' time) that is involved in the establishment of the undulations upon the gravitational field. An alternative name for these waves is “gravito-quantum radiation.”
of a body’s oscillatory acceleration response to the field waves is (inertial) mass dependent\(^9\).

### 2.4 Gravitation & potential energy

In General Relativity, potential energy (P.E.) as a sum of particle energies is not well defined, because potential energy lies in the geometry of spacetime itself. Additionally, we may say gravitational energy cannot be localized. The following quote from Michael Mensky reinforces this point of view.

The question of conserving energy-momentum in General Relativity (GR) always attracted much attention. One of the reasons is that covariant description of energy-momentum seems to be incompatible with the integral conservation law. Particularly, it is generally believed that no integral conservation law follows from the covariant differential conservation law for the energy-momentum tensor (EMT) of matter (i.e., from its covariant divergence being zero)\(^5\) p.261].

Today we may still think in terms of P.E. but attempting to quantitatively relate this to GR appears to be very awkward, if not ill-conceived. On the contrary, relating P.E. to classical-like waves or ‘ripples’ of acceleration, i.e. spacetime curvature undulations, is conceptually simple. The idea that a wave contains energy is familiar; thus an acceleration wave can be seen to possess some sort of potential energy — since it is physically a distortion of spacetime (in the manner of ripples on a pond).

The existence of these field undulations makes use of the curved spacetime conceptualization of GR, but at non-special-relativistic speeds.

From the above, it appears that the understanding of the links between GR and energy (and gravitation) may be accepted as incomplete.

### 2.5 Gravitational field oscillations and general relativity

Sections \(\text{2.3} \) and \(\text{2.4} \) sought to show that there appears to be nothing in the foundations of general relativity, itself, that distinctly forbids a new (gravito-quantum) mechanism from generating oscillatory fluctuations upon a pre-existing gravitational field\(^10\). When a moon rotates around a planet a similar oscillatory field effect, although non-constant with radial distance, is experienced by a point mass — in the not too near vicinity.

The generation of the waves that affect Pioneer spacecraft (S/C) are quite distinct from three body effects and general relativity’s gravitational waves. They may be thought of as type-2 gravitational waves if one prefers. Indeed, the existence of type-1 gravitational waves indirectly supports the existence of another kind of gravitational wave\(^11\).

### 2.6 Cut-off mass, convolution, and wave energy dispersion

A sharp (all or nothing) cut-off\(^12\) exists for the effect of an individual acceleration wave upon masses in motion.

Inertia involves a body’s ability to resist a change in motion. With the hypothesis being presented, it is also necessary to see inertia as associated with a wave field’s ability to ‘instill’ an oscillatory variation in the motion of a body.

This situation may be aligned to the mathematical technique of convolution, which determines a system’s output [oscillatory behaviour: yes or no] given an input signal [the acceleration wave] and the system impulse response [some function of the inertial mass of a body]\(^13\).

Although (acceleration) wave amplitudes remain constant, the (inertial) mass associated with this new kind of wave reduces in proportion to the volume the dispersing wave encompasses\(^14\). Thereby, once a type-2 gravitational wave is generated (or established), conservation of energy is obeyed\(^15\).

\(^{10}\)Certainly, the principle of special relativity is in need of some attention, and the spirit of GR seems to be under threat, but the provisional acceptance of these type-2 gravitational waves is not unreasonable.

\(^{11}\)A further source of non-uniformity for a gravitational field is tidal effects. Such effects ensure lunar spin-orbit coupling around the moon’s host planet.

\(^{12}\)Much in the manner of the Photoelectric Effect.

\(^{13}\)Paraphrasing: http://www.see.ed.ac.uk/~mjj/dspDemos/EE4/tutConv.html (square brackets content excluded).

\(^{14}\)Once again deference to a fuller elucidation of the model is necessary.

\(^{15}\)It is only the generation of the acceleration-waves that appears to disobey (expected) conservation of energy.
3 Amplitudes associated with acceleration waves on the gravitational field

This section begins the process of quantitative support for the hypothesis. Two results are established. Results pertaining to the relationships between the amplitudes of sinusoidal: acceleration, speed and range variations of a body are outlined. Since the wave amplitudes (via Lorentz invariance) are necessarily constant, the mathematics that follows is greatly simplified. Note that the solar system barycenter acts as the (quasi-)'global' (i.e. solar system) inertial frame’s reference point.

3.1 Amplitudes of the cyclic variations

The integral of a constant acceleration (of unit amplitude) over a time $\frac{t}{2}$ is simply a velocity of magnitude $\frac{a}{4}$. Now $\int_0^\frac{t}{2} \sin \theta d\theta = 1$ is a velocity amplitude related to a unit amplitude sinusoidal acceleration acting over $\frac{t}{2}$ (i.e. a quarter of a wavelength). The period of the wavelength may be either $2\pi$ or $\Delta t$. Closely related to the maximum sinusoidal acceleration (wave) amplitude $\Delta a$ and a time of $\Delta t/4$ is the maximum amplitude of a similarly sinusoidal velocity so that:

$$\Delta v = \Delta a \cdot \frac{\Delta t}{4} \cdot \frac{2}{\pi} = \Delta a \cdot \frac{\Delta t}{2\pi} = \frac{\Delta a}{\omega}$$

Notice that we let:

$$|\Delta \vec{v}| = \Delta a, \quad |\Delta \vec{v}| = \Delta v$$

In short, there is a direction and time independent relationship between the magnitudes of velocity and acceleration wave amplitudes$^{16}$. This appears in equation 50 of Anderson et al. [2, p.37] with $A_0$ replacing $\Delta a$.

Similarly, we also have for sinusoidal speed variation and associated ($\frac{1}{4}$wavelength) sinusoidal range change, the following relationship of amplitudes:

$$\Delta x = \Delta v \cdot \frac{\Delta t}{4} \cdot \frac{2}{\pi} = \Delta v \cdot \frac{\Delta t}{2\pi} = \frac{\Delta v}{\omega}$$

Note that: $\omega_{\text{diurnal}} \approx 7.3 \times 10^{-5}$ rad/s and $\omega_{\text{annual}} \approx 2.0 \times 10^{-7}$ rad/s. Additionally, by way of Ref. [2, pp.8, 15, 37], the (return trip) Doppler frequency shift ($\Delta \nu$) is determined via:

$$\frac{\Delta \nu}{\nu} = \frac{2 d\ell}{c \Delta t}$$

with the S-band downlink frequency ($\nu$) being $\sim 2.29$ GHz. Finally, for S-band Doppler: 1 Hz corresponds to 65 mm/s [2, p.18], or more pragmatically, 10 mHz corresponds to 0.65 mm/s.

3.2 Further comments

Observe that if undulations in the gravitational field are the cause of fluctuations in spacecraft (S/C) speed then (assuming pure radial motion for the S/C and a position well beyond Saturn): $\Delta \vec{v} = -\omega \Delta \vec{v}$ and $\Delta \vec{v} = \omega \Delta \vec{p}$.

Note that direction cosines are rampant in this new approach involving spherical waves emanating from a moon. Small corrections in the form of direction cosines, for the trajectories of bodies nearer the center of the solar system, are (quantitatively) neglected in this write up.

The overall write up is both quantitatively idealized and theoretically lacking full substantiation, in order to comprehensively establish the physical validity of the acceleration-wave hypothesis.

3.3 A set of annual reference values

The results of Section 3.1 applied to an annual residual allow a set of reference values to be established. For $\Delta v = 0.2$ mm/s at $\Delta \nu \approx 3.1$ mHz (a Pioneer S/C S-band Doppler frequency change), the approximate values of range and acceleration amplitude are: $\Delta x = 1$ km and $\Delta a = 0.4 \times 10^{-8}$ cm/s$^2$ respectively. These values are all linearly scalable for different magnitudes of the four physical ‘quantities’ involved ($\Delta x, \Delta v, \Delta \nu,$ and $\Delta a$).

4 Interpreting the Pioneer 10 diurnal residual

This section shall begin to examine the claim of Anderson et. al. that: “[the] annual and diurnal terms are very likely different manifestations of the same modelling problem [2, p.38].”
4.1 Introductory remarks

There are three causes of residuals arising from a method of least squares analysis. These are: observational error, approximation of parameters, and model or theory inadequacy [6, Ch.8]. Doppler data cannot clearly discern whether an oscillatory residual is Earth or spacecraft based. Only the relative acceleration, motion, or (line segment) distance between the two bodies is determined by the measurements. Assuming the observations are reliable, either an oscillatory sinusoidal residual is a result of Earth based parameter error(s), or failing that, it is an unlikely real spacecraft motion and hence beyond current gravitational theorization.

4.2 The magnitude of the diurnal residual

The diurnal residual’s interpretation is crucial to understanding the annual residual. The noise of the diurnal residual is greatest around solar conjunction, but at solar opposition, near a minimum in the solar cycle\(^{17}\), exceptionally good data is available: see [2, Fig.18, p.38].

Markwardt [7] gives a figure of 10mHz (i.e. 0.65mm/s) for the amplitude (on average) of the diurnal residuals. Ref. [2, Fig.18, p.38] implies a Nov/Dec 1996 solar opposition amplitude of 0.1376 mm/s that may be obtained from:

\[
\Delta a = \omega \Delta v
\]

and the values of \(\omega_{d.t.} = 7.2722 \times 10^{-5}\) rad/s and \(a_{d.t.} = (100.1 \pm 7.9) \times 10^{-10}\) m/s given by Ref. [2, p.38]. This gives, via \(\Delta v = \omega \Delta x\), a cyclic diurnal position offset of \(\sim \pm 1.9\) metres (i.e. \(\Delta x \approx 1.9\) m). As an angular offset at the Earth’s surface this equates to:

\[
\theta = \tan^{-1} \frac{1.89}{6378 \times 10^3} \approx 1.70 \times 10^{-5}\ deg. \approx 60\ mas
\]

where ‘mas’ is milliarcseconds. This being a small and non-problematic distance or angle, that is less than 0.1% of maximum DE 405 ephemeris error in the Earth’s orbit of 2 kilometres [8], and about 16 times 1997 Earth orientation polar motion (root-mean-square) calibration accuracy of \(\sim 12\ cm\) [2].

4.3 Diurnal residual parameter groups

Three main groups of factors or parameters affect the diurnal residual. For a diurnal effect only short term parameters and location aspects are included in the first two groups.

- Earth orientation parameters (EOP) errors. This concerns celestial polar motion offsets, and variability of the earth’s rate of spin. Let us also include here: antenna location errors for NASA’s Deep Space Network (DSN).
- Planetary ephemeris errors. These concern the location of the earth in its orbit (relative to the solar system barycenter).
- Error via miscellaneous effects. Including: ocean tides, weather, and variable atmosphere effects; troposphere and ionosphere effects (causing spectral broadening of the carrier wave frequency); and interplanetary scintillation (i.e. plasma-based fluctuations).

All of this information, and much more, goes into either JPL’s Orbital Determination Program (ODP) or the Aerospace Corporation’s Compact High Accuracy Satellite Motion Program (CHASMP). The presence of errors or inaccuracies results in a residual, or what may be termed a “spacecraft motion offset”. Anderson et. al. have found that individually the first two sources of error cannot be (solely) responsible for the residual [2, p.36].

The diurnal residual of [2, Fig.18] may be said to contain two aspects. Firstly, a stochastic (or random) aspect and secondly, a cyclic aspect. In the author’s opinion, the miscellaneous effects (discussed above), and Earth spin rate variability, predominantly produce either: very small effects or random shifts in the residual, and hence they may be neglected from an account of the cyclic residual indicated in Fig.18. It is the cyclic diurnal signature that shall now concern us, as it most closely relates to the annual residual’s interpretation.

Note that the period of the diurnal cycle is not a concern. A diurnal residual should exist in the Doppler data. The Earth is a “wobbly platform”.

\(^{17}\)There was a broad minimum in the solar cycle around May 1996.
4.4 An interpretation of the cyclic diurnal residual’s amplitude

It appears that only a combination of EOP and Ephemeris error \textit{together}, in the ODP or CHASMP, can (primarily) produce the cyclic diurnal amplitude error observed at opposition\textsuperscript{18}.

This interpretation is analogous to how celestial pole offsets arise. Celestial pole offsets are required because the model (of the Earth’s orientation) that \textit{combines} precession and nutation relies on fixed parameters for the Earth’s shape (geodesy) and internal structure, but since these are not fixed the offsets necessarily arise. Similarly, combining EOPs and the ephemeris in an orbital determination program is seen to produce a (pure) cyclic diurnal residual. This being at 0.1\% of the level of the maximum error in the DE405 ephemeris (i.e. 2 km).

This seems to indirectly agree with errors in EOPs changing the value of $a_p$ only in the 4th digit \cite{2,p.36}. The diurnal residuals indicate EOPs changing the S/C position location (only) in the 4th digit of the (Earth position) ephemeris error.

4.5 In summary

It appears that a combination of many parameters produces the diurnal residual, but a combination of two parameter ‘groups’ (EOP and ephemeris errors) dominates the production of the cyclic aspect of the diurnal residual — about a mean value.

5 Examining the Pioneer 10 annual residual (1987-1998)

An understanding of the diurnal residual allows us to now closely examine the cyclic ‘annual’ residual. The amplitude of the diurnal cyclic residual was shown to be primarily a combination of errors in two parameter groups. Anderson et. al. claim the annual residual is probably due to a similar combination of parameter errors. These being: Earth based location and (long-term) polar orientation errors, and errors in the navigation program’s ‘determination’ of spacecraft (S/C) orbital inclination to the reference frame being employed.

Note that when averages over a full day or a number of days are used, the diurnal (and short-term) parameter errors, disappear from the data. Thus, the annual residual’s amplitude is independent of the amplitude of the cyclic diurnal residual.

5.1 Orbital inclination errors and the annual residual

Jet Propulsion Laboratories (JPL), The Aerospace Corporation, and Craig Markwardt all used DE405 in their ‘best’ analyses. Interestingly, beginning with DE400 (development ephemeris 400), both the Earth orientation parameters and the Earth’s ephemeris are aligned to the ICRF (International Celestial Reference Frame). Error in the orbital inclination of a spacecraft, via an orbital determination program, is thus more closely related to EOP error than previously.

A combined outer solar system spacecraft inclination angle error with Earth polar orientation angle error, is seen to interact with Earth location (ephemeris) error to produce the annual residual. This account of the cyclic annual residual, like the cyclic diurnal residual, is based on two primary parameter groups. This account although not specifically stated, is alluded to in Ref.\cite[p.23, pp.36-38]{2}. It appears a reasonable, although sketchy, explanation. This scenario is now further investigated.

5.2 Quantifying the error associated with orbital inclination

Standish \cite[p.1166]{8} by way of the recent (2003) ephemeris DE 409 has been able to quantify errors in earlier ephemerides. For all the outer planets (short-term) inclination errors for both geocentric right ascension and declination are 0".1 (arc-seconds) for DE 200, and 0".05 for DE 405.

Since the Orbital Determination Programs (ODPs) ‘produce’ the inclination error, it is best, in this case, to use the error from DE200 which covered 1979 to 1997. In the case of a predominantly \textit{radial} motion based variation in the Doppler, the effects of S/C inclination error may

\textsuperscript{18}This conclusion is based upon email correspondence with E. Myles Standish of JPL regarding the diurnal residual. Note that Myles, whilst having no objection to this interpretation of the cyclic residual, emphasized the need to not overlook the other effects mentioned. Also see footnote 125 of Ref.\cite[p.49]{2}.
be determined\(^{19}\). Assuming radial motion for the distant Pioneer S/C, a (plus or minus) 0.1 = inclination error for a spacecraft at 55 AU (Astronomical Unit) implies an uncertainty in position, orthogonal to the spacecraft’s trajectory, of:
\[
z = (55)(150 \times 10^6) \tan(0.1) \approx 4,000 \text{ km}
\]
noting that \(\tan(0.1) \approx 4.85 \times 10^{-7}\). This orthogonal uncertainty may then be related to a line of sight uncertainty by a similar triangle, such that:
\[
\Delta x = 4 \times 10^6 \tan(0.1) \approx 2 \text{ m}
\]
where \(\Delta x\) is the magnitude of the range variation. Thus, for 0.1: \(z \approx 4,000 \text{ km}\) and \(\Delta x \approx 2 \text{ m}\); and for 1\(^{\circ}\): \(z \approx 40,000 \text{ km}\) and \(\Delta x \approx 0.2 \text{ km}\) or 200 m, whereas for 5\(^{\circ}\): \(z \approx 200,000 \text{ km}\) and \(\Delta x \approx 5 \text{ km}\).

Thus, on their own the S/C inclination errors arising from DE 200 fail to account for the ‘annual’ residual, by a long way. At its minimum the ‘annual’ range variation amplitude (\(\Delta x\)) of Pioneer 10 is approximately 500 meters via \(\Delta v \approx 0.1 \text{ mm/s}\) (see Section 5\(^{33}\) and Ref. 2 p.38)). Total orientation errors in excess of 1.5 arc-second appear to be required, i.e. above 15 times the reported orientation error of DE 200, and 30 times greater than the DE405 orientation errors of the outer planets.

5.3 Remarks on combining orbital orientation and ephemeris errors

It should be noted that errors in the Earth’s orbital position (including heliocentric radius error) will have a minimal cyclic impact upon the errors discussed in Section 5.2. Since the Earth and Pioneer 10 both lie very near the plane of the ecliptic, only errors in the Earth’s orbital position far away from Sun-Earth-Pioneer 10 conjunction or opposition will yield a (line-of-sight) Doppler range residual. This residual would show narrow peaked maximum or minimum amplitudes, rather than the smooth ‘sinusoidal’ wave observed [13, Fig. 1B].

Finally, with the earlier DE 200, the ephemerides were oriented onto their own inherent Earth’s mean equator and dynamical equinox. Thus, the additional orientation error related to the Earth’s annual orbital motion may slightly increase the error,

\(^{19}\)This, and what follows, is related to the discussion on p.74 of an article by W. G. Melbourne [10] on “space navigation”, where range error is related to an orthogonal distance (and hence angular) error.

but this increase would need to be larger than the error in the outer planets and this is unrealistic.

Subsequently, evidence to the contrary of the Ref. 2 stance on the annual residual is worth considering (this is pursued in Section 6). Additionally, let us note that Markwardt [7, p.11] believed: “...the source [of the annual residual] was ultimately undetermined.”

5.4 A stance on the ‘annual’ cyclic residual

The annual spacecraft motion offset, and hence the position offset, obtained from Doppler tracking observations — appears to be neither: a purely orbital inclination effect, nor is it feasibly a combination of this with either: Earth orbit orientation error, or Earth position error.

The reason for the ‘annual’ residual appears to be restricted to a choice between: a mixture of Earth position (ephemeris), Earth orientation, and spacecraft inclination errors; or alternatively, an inadequate model of the spacecraft’s motion. Concerns regarding the validity of an account based on parameter errors were raised in Sections 5\(^{2}\) and 5\(^{3}\). Indeed, the “orbital determination programs” need to be of a high quality (in the first place) to unambiguously obtain the Pioneer anomaly.

6 Reinterpreting the Pioneer spacecraft ‘annual’ residuals

Section 5 found that spacecraft orbital inclination error appears insufficient on its own, or together with Earth ephemeris and orientation errors, to account for the ‘annual’ residual’s amplitude measured by Doppler tracking observations. Subsequently, the residual appears to be due to either: an unrealized ephemeris error of the Earth’s position in its orbit, or the spacecraft has an unmodelled (i.e. real) annual longitudinal oscillatory motion.

Further concerns may be raised regarding firstly, the cyclic amplitude of an Earth based explanation of the ‘annual’ residual of the anomalous Pioneer ‘acceleration’; and secondly, a new concern regarding the period of this residual is raised. Beginning with the latter, these concerns are now addressed, and linkages to the acceleration-wave hypothesis are drawn upon and incorporated in the discussion.
6.1 An alternative approach to the Pioneer ‘annual’ residuals

On page 38 of the extraordinarily comprehensive Physical Review D paper [2] discussing the Pioneer anomaly, the angular velocity is given (in interval III) as $\omega_{a.t.} = (0.0177 \pm 0.0001) \text{ rad/day}$ which equates to $355 \pm 2$ days per $(2\pi)$ cycle. Fig 2. of Scherer et. al. [11] shows the real part of the autocorrelation function of the later Pioneer 10 data (1987-1995). By averaging, from the graph the clear maximum and minimum range of values, at the half and full year, a period of $\sim 355$ days (and not $\sim 365$ days) is confirmed. The shape of the real part of the autocorrelation function indicates a solitary sinusoidal-like oscillation dominates the spectral aspect of a time series representing the (long-term) Pioneer Doppler data. Markwardt [7, p.11] refers to the “‘annual’ residuals, also identifying a mean period that is far enough away from 365.25 days (i.e. 3%) to be worthy of signification.

The (heliocentric-based) orbital periods of Jupiter’s moon Callisto and Saturn’s Titan are respectively: $16.689018$ and $15.945421$ days. Remarkably, their periods will resonate every 357.9 days ($m = n + 1$ where $n \approx 21.445$)\(^{20}\). By making a synodic correction for the location of the Pioneer 10 spacecraft with respect to the motions of the host planets of these moons (1992.5 to 1998.5), the period goes to approximately 356.1 days\(^{21}\). This cycle has a significant resonance amplitude, and is the only one freely visible, involving the (proposed) lunar generated acceleration-waves. Remember, if output from only five moons dominates the Pioneer anomaly, then only the Callisto-Titan resonance is expected to be significantly under-affected by the orbital resonances of Jupiter’s Galilean moons. These resonances act to minimize the (statistical) variance of $a_p$ observations through time\(^{22}\).

6.2 Pioneer data and the ephemeris

Any inconsistency that exists between the Pioneer anomaly’s annual residual amplitude and DE405 ephemeris error in the earth’s orbital location of $\pm 1-2$ km [5, p.1171] is removed if the ‘annual’ residual is deemed real. Markwardt [7] observes a 10mHz annual amplitude\(^{23}\) which implies a 3.25 km range error (see Section 5.3 for reference values). Turyshev et. al. [13] quote an amplitude of $1.6 \times 10^{-8} \text{cm/s}^2$ implying a range error of 4 km\(^{24}\). Only later in Interval III can the annual residual’s magnitude be said to be ‘within expectations’.

Similarly, the month of data given by Anderson et. al. for the diurnal residual [2, Fig.18, p.38] covers about $30^\circ$ of the annual anomaly’s cyclic period. An amplitude increase of about 0.1 mm/s (to a maximum) over the 30 days is evident. If this implies $[\text{via } (1 - \cos 30^\circ)^{-1} = (0.134)^{-1} \approx 7.5]$ a speed sinusoid amplitude of $\sim 0.75 \text{ mm/s}$ then this roughly agrees with Markwardt’s value of 10mHz or 0.65mm/s — for the amplitude of this prospective two-wave resonance\(^{25}\).

6.3 Pioneer data and pulsar timing experiments

Chandler [14, p.108] states that: the annual signature of the Earth’s motion dominates the variations of (long term) pulse arrival times. Additionally, pulsar timing measurements are accurate to about $3 \times 10^{-8}$ seconds or 90 meters, and there has been no indication from pulsar timing experiments of any overly large error in the Earth’s ephemeris (i.e. orbital location over time) e.g. [14, pp.718-19].

The non-problematic account of a slight change in a pulsar’s location by changing from DE200 to DE405 [14], where only a 0.2 mas mismatch is ap-long term observational data of the anomalous acceleration $a_p(t)$. This is perhaps partially evident in [2, Fig.14, p.24].

\(^{20}\)For planetary ring systems at least: “Resonances are strongest when $m = n + 1$ (for example 2:1 or 43:42) and weaken rapidly as $m$ and $n$ differ more and more” [12, p.70].”

\(^{21}\)In the 6 years of interval III (1992.5 to 1998.5) Jupiter tracks, with respect to Pioneer 10’s location and trajectory, approx. +11° prograde, whereas Saturn’s position remains essentially unchanged. Callisto’s prograde progression is thus $\sim 1.8^\circ$ (relative to Titan) per 357.9 day resonant cycle. This yields a shortening of the $(360^\circ)$ resonance cycle of $\sim 1.8$ days. (See Section 5.3 for the data source used to establish these angles.)

\(^{22}\)A small amplitude, directional cosine based, 11.86 year Jupiter cycle should be present, and possibly evident, in the

\(^{23}\)Markwardt finds the rms residuals of all the ‘non-extreme’ Doppler data to be of order 8mHz — see his Table II. The annual residuals are thus of the same order of magnitude as the noise, although the noise amplitude, depending upon space and atmospheric Doppler transmission conditions, is quite variable.

\(^{24}\)The first oscillation in 1987 has an amplitude of $2.5 \times 10^{-8} \text{cm/s}^2$ indicative of a $\pm 6.25$ km range variation.

\(^{25}\)This is awkward because it is inconsistent with Ref. [4]’s stated amplitude (0.1055 mm/s) for the interval III annual sinusoid, or 1.5mHz implying a $\pm 0.5$ km range sinusoid. See Section 5.3 for further discussion.
parent, further indicates an absence of ephemeris accuracy concerns. A similar comment is made by Anderson et. al. regarding planetary (and spacecraft) ephemeris error [2, p.36].

Note that pulsar timing data lack the accuracy of Doppler diurnal residual amplitude measurements, that are approximately 2 meters (or ∼ 0.14 mm/s) around the 1996 Pioneer-Earth-Sun (solar) opposition. Also note that Doppler data precision reduces as the period of oscillations becomes longer (given a fixed oscillation amplitude of frequency variation $\Delta \nu$).

### 6.4 The Pioneer 10 and 11 ‘annual’ residual amplitudes

A combination of primarily spacecraft inclination, and also Earth orientation and ephemeris errors has been proposed as a reason for Pioneer 11’s greater ‘annual’ residual amplitude c.f. Pioneer 10 [2, p.37]. Alternatively, this feature may be related to the inclination of the spacecraft trajectories relative to the equatorial planes of Jupiter and Saturn. The planet’s equatorial plane is essentially the plane within which Jupiter’s four Galilean moons and Saturn’s Titan orbit. Jupiter’s and Saturn’s equatorial planes are tilted at 3.1 and 26.7 degrees respectively, relative to their orbital planes, with their orbital planes inclined at 1.3 and 2.5 degrees respectively, relative to the plane of the ecliptic.

Disregarding the inclinations of the planets’ orbits, the maximum possible Callisto-Titan resonance amplitude, from lunar generated acceleration waves, will thus be approximately at 15 degrees. The Pioneer 10 and 11 spacecraft trajectories are inclined at approximately 3 and 16 degrees respectively, to the plane of the ecliptic. Thus, Pioneer 11 would be expected to have a greater ‘annual’ cyclic residual amplitude, and probably a slightly different period to that measured by Pioneer 10 — if the hypothesis being outlined in this write up is viable.

If the lunar based acceleration-wave hypothesis is viable then the other (primary) orbital resonances of Jupiter’s moons will probably play some role in varying the amplitude of this (Callisto-Titan) resonance over time. This arises because the other orbital resonances ‘complicate’ the simple decomposition of this two-wave resonance out of the overall superposition effect of all the acceleration waves.

### 6.5 The fine details of the ‘reliable’ Pioneer observations

A number of details regarding the Pioneer spacecraft observations appear to be illuminated by the ‘acceleration-wave hypothesis’. All that is required is to assume the time-averaged observations are reliable and slightly more accurate than generally considered. Wave motion to spacecraft motion direction cosines are implied in all of the following fine detail aspects.

1. Why the overall values of Pioneer 10 and 11 differ slightly\(^{26}\).

2. Why the ‘annual’ residual’s amplitude is greater for Pioneer 11 c.f. Pioneer 10.

3. Why the maximum anomalous acceleration for Pioneer 10 is in early 1998.

4. Why the Pioneer 11 anomaly increases rapidly post Saturn encounter [2, Fig.7, p.18].

5. Why the magnitude of the Pioneer 10 anomaly is slightly greater in the later data, as compared to earlier data. (See Section 7.12).

6. (Possibly) Why Pioneer 10 spin rate decreases whereas Pioneer 11’s increases between manoeuvres\(^ {27}\).

Point 4 is also influenced by wave direction. At any point within Saturn’s orbit, waves may arrive from Jupiter’s Galilean moons and Saturn’s Titan at obtuse angles to each other. Waves from opposing directions will act to cancel each other out somewhat. (Section 7 shall clarify this assertion.)

Regarding point 3: from the Pioneer 10 spacecraft’s perspective, in early 1998 Jupiter and Saturn are closer in the sky to its ‘(reverse) trajectory line’ than at any other time. They lie near the spacecraft’s reverse (or negative) trajectory line\(^ {28}\).

\(^{26}\)Noting that Pioneer 10 has over 11 years of high quality data, whereas Pioneer 11’s data, with only 3\(^ {\frac{2}{3}}\) years, has had insufficient time to establish a representative ‘longer-term’ average.

\(^{27}\)Naturally gas leaks are a likely cause but “for the Pioneers there were anomalous spin-rate changes that could be correlated with changes of the exact values of the short term $a_p$. The correlations between the spin-rate changes and $a_p$ were good to $0.2 \times 10^{-8} \text{cm/s}^2$ and better [17, p.4019].”

\(^{28}\)Essentially this is the view (for a forward looking ‘passenger’) in an imaginary rear view mirror on the spacecraft.
and approach a conjunction. Hence direction cosine adjustments are minimized. Saturn actually crosses the extended (negative) trajectory vector at, or very near to, the early 1998 maximum. See [2, p.24, Fig.14] as well as planetary and spacecraft positions given by way of the “National Space Science Data Center” website — to obtain the solar ecliptic reference frame coordinates through time.\(^{29}\)

### 6.6 Damping of the annual residual

Fig. 1B in Ref. [13] shows the annual residual, as does Fig. 13 in Ref. [2, p.23]. The latter figure is divided into intervals: I, II, and III. Thus, it is reasonable to also break Fig. 1B into three intervals. The annual anomaly is most impressive in intervals I and II (Jan 1987 to July 1992), with a single sinusoidal line able to smoothly connect essentially every point (not quite the line shown). Such is not the case with Interval III (July 1992 to July 1998) which also contains amplitudes that are decidedly smaller than in intervals I and II. Thus, a fairly clear distinction exists either side of July 1992. The interval III data is also decidedly less sinusoidal than interval I and II data. Unfortunately, the qualitative detail of Pioneer 11’s annual anomaly is not discussed in Ref. [2].

A pure two-wave resonance based sinusoidal variation should not exhibit monotonic damping in its \(a_p(t)\) time series, but there are a number of mitigating circumstances. Firstly, the orbital resonances of Callisto with Jupiter’s other 3 Galilean moons, which are not rigidly fixed resonances,\(^{30}\) will be influential upon the Callisto-Titan resonance. Particularly prominent in the damping could be the method of data smoothing employed. There may also be another reason, unknown to the author, for this apparent discontinuity in data ‘quality’ between intervals II and III. Curiously, the data in interval III is actually preferred to I, II and Pioneer 11 data by Anderson et. al. [2, p.26] because the match between JPL’s ODP/\(\Sigma\) and The Aerospace Corporation’s CHASMP data is so good. Finally, it is worth mentioning that it is hard to understand how an annual residual based upon orbital inclination (angle) error could strongly diminish with increasing S/C distance from the Earth — even taking into account a reduction in Earth and spacecraft orientation errors over time.

Nevertheless, even with this rationalization the damping is a concern for the hypothesis being presented in this write up. Particularly in interval III, and to a much lesser degree in intervals I and II. This situation could be clarified by a re-processing of earlier pre-1987 data.\(^{18, 19}\)

### 6.7 ‘Annual’ residual – final remarks

Bearing in mind Section 6.6 there is still an assortment of evidence that the ‘annual’ residual cannot be a result of parameter error(s), as the diurnal residual surely is. Subsequently, the ∼ annual residual appears indicative of a modelling oversight. This model inadequacy suits the new hypothesis presented, with a Callisto-Titan acceleration wave ‘resonance’ causing a real oscillatory motion of the S/C relative to an ‘accurately’ positioned Earth — itself being relative to the solar system barycenter. The single sinusoid-like variation in the autocorrelation data of \(a_p\),\(^{11}\) implies that only an (∼ 355 ± 2) day residual is measured (in Interval III spanning 1992.5 to 1998.5), implying the (365.24 day) ‘true’ annual residual is too small to be detected. The small changes between DE200 and DE405 for both: the Pioneer anomaly, and pulsar timing experiments, add support to this assertion.

It is unlikely a spectral analysis of a very long \(a_p\) time-series would show any sign of the lesser ‘true’ cyclic annual spectral component quantified for inclination error in Section 5.2 at approximately 2 meters. Following the diurnal case (Section 4.2), let us estimate total annual error at < 40 meters. This signature absence is due to Doppler data precision diminishing as the period of oscillations becomes longer (given a fixed oscillation amplitude of frequency variation \(\Delta \nu\)). For the yearly oscillations, a 0.01 mm/s amplitude, which is around the Doppler tracking’s best level of accuracy, represents a 50 meter range amplitude (recalling Section 3.2).

The greater precision of a 21st century
sion, specifically designed to test the Pioneer anomaly \[15\], would be expected to fully clarify this issue. The inclusion of range measurements would provide a cross-reference for closely examining, and unambiguously explaining, this solitary \( \sim \) annual (S/C based), or ‘true’ annual (Earth based) cyclic residual present in the Pioneer 10 and 11 data.

7 Gravitational field undulations and energy transfer

In light of sufficient concerns regarding the current account of the annual residual of the Pioneer anomaly, we return to pursuing the (acceleration-wave) hypothesis sketched earlier. This section seeks to show that if the \((355 \pm 2)\) day cycle is real and spacecraft based (at \( \sim 356 \) days) — the origin and explanation of the Pioneer anomaly is open to a promising and progressive alternative interpretation.

7.1 Introduction

It has been mentioned by Anderson et. al. \([2, p.39]\) that a sinusoidal speed variation cannot contribute to the Pioneer spacecraft anomaly. This is true for an Earth based and/or spacecraft inclination based residual, e.g. by way of ephemeris errors and Earth orientation parameters (EOPs) errors in the orbital determination program (ODP); but this stance may be challenged if the oscillatory motion is real — and applied to ‘light’ bodies with non-zero mass\(^{31}\) e.g. the Pioneer spacecraft. We entertain and examine this possibility by way of the interaction between spacecraft ‘geodesic’ motion, and the kinematic and potential energies involved. Recall, a geodesic is the closest thing there is to a straight line in curved spacetime. When (type-2) undulations exist on the gravitational field, circumstances are different from those of a ‘static’ gravitational field, especially as regards geodesic motion.

With no local restoring ‘force’ to oppose the effects of a cyclic gravitational undulation, light bodies in celestial geodesic motion ‘go with’ the (curved spacetime based) cyclic undulations; whereas the greater inertia of ‘heavy’ bodies means they fail to respond. For light bodies, spacecraft speed and distance will oscillate around equilibrium values. A standard barycenter systemic\(^{32}\) inertial frame remains valid. With the passing ‘phase’ of an acceleration wave at the spacecraft, the local gravitational acceleration field strength varies cyclically, relative to the system’s barycenter inertial frame (and associated ‘inertial time’).

Even in the absence of a central gravitational field an oscillatory gravitational field will ‘disengage’ a spacecraft’s constant speed inertial frame, and its geodesic trajectory will acquire an additional longitudinal (speed) fluctuation — relative to a some systemic inertial reference frame.

7.2 Agenda

The aim of Section 4 is to show that the (short term\(^{33}\)) through-time Pioneer anomaly \([a_p(t)]\) can alternatively be seen as resulting from a Fourier-like summation and superposition of ‘sinusoidal’ waves\(^{34}\), i.e. gravitational field undulations (of amplitude \(\Delta a\)) upon the gravitational field. The remainder of Section 4 seeks to show, by way of total energy concerns, that the average Pioneer acceleration \((a_P = a_p(\bar{t}))\) (over a long time) is described by:

\[
a^2_p = \Sigma(\Delta a)^2 \quad \text{or} \quad a_P = \sqrt{\Sigma(\Delta a)^2}
\]

whereas the through-time the instantaneous Pioneer acceleration \([a_p(t)]\) may be expressed by way of:

\[
[a_p(t)]^2 = \Sigma[2\Delta a^2 \cos(\omega t + \phi)] \quad \text{or more formally:}
\]

\[
a_p(t) = \left\{ \sum_{i=1}^{n} [2\Delta a_i^2 \cos(\omega_i t + \phi_i)] \right\}^{\frac{1}{2}}
\]

where \(\phi_i\) is the initial (or epoch) phase of a particular moon’s wave, with orbital angular velocity \(\omega_i\) and wave amplitude \(\Delta a_i\) also moon specific.

The relationship between the energy of a spherical acceleration wave upon the gravitational field affecting a ‘point’ mass through time, and the oscillatory response of the moving mass to the wave shall be our prime concern. Recall that (systemically) \(\Delta \vec{a}\) and \(\Delta \vec{a}\) are \(180^\circ\) or \(\pi\) out of phase.

\(^{31}\)Note that the overall propagation speed of electromagnetic radiation is necessarily not retarded. It ensues that without inertial mass (by definition) the kinetic energy of E/M radiation particles cannot be incrementally ‘eroded’.

\(^{32}\)The term “systemic” is preferred to “systematic”.

\(^{33}\)Especially 1 day, 5 day, or 10 day averages.

\(^{34}\)Recall that ‘sinusoidal’ means ‘harmonic’ i.e. some function built up from sinusoidal and cosinusoidal waveforms.
7.3 Rayleigh’s energy theorem or Parseval’s theorem

Rayleigh’s energy theorem, sometimes also known as Parseval’s theorem, is an energy-conservation theorem. It says that a signal contains the same amount of energy regardless of whether that energy is computed in the space/time domain or in the Fourier/frequency domain. For a continuous function it is:

\[ \int_{-\infty}^{\infty} f^2(t) \, dt = \int_{-\infty}^{\infty} \left| g(f) \right|^2 \, df \quad (1) \]

Thus, the sum (or integral) of the square of a function is equal to the sum (or integral) of the square of its Fourier transform. Bracewell notes that: “... the theorem is true if [only] one of the integrals exists.”

Rayleigh’s theorem, by way of “the power theorem”, is also applicable to the rate of energy transfer, or power, of a ‘signal’. A power based application follows in Section 7.4. The relationship between frequency line width and Fourier transform amplitude in the energy Rayleigh identity, gives way to a clear distinction between frequency and energy in the power form of the identity that follows.

By way of the Rayleigh identity, the signal coexists in two domains: a time domain, and a frequency domain. In the application to follow, the signal shall also have two physical ‘faces’: a field face, and a moving (light or low mass) body face.

7.4 Gravitational acceleration waves and spacecraft kinetic energy (single cycle, singular wave case)

We make use of the Fourier wave based Rayleigh ‘power’ theorem identity (equation 1 of Section 7.3), but apply its frequency and particle aspect, to the new circumstance of spacecraft (S/C) geodesic motion relative to a ‘systemic’ (i.e. whole solar system) inertial frame centered at the solar system barycenter. The S/C’s geodesic motion undergoes a pure sinusoidal (or cosinusoidal) change in (gravitational) acceleration and this coexists with both: a sinusoidal change in speed (\(\Delta v\)) relative to an equilibrium mean value and, as we shall soon see, a change (per wave cycle) in barycentric reference frame speed (\(\delta v\)).

Since Einstein’s General Relativity did not succeed in making acceleration relative, this ‘real’ systemic acceleration is logically capable of representation in a (solar-)systemic inertial reference frame.

For a single cycle of a singular (or isolated) physical wave upon the gravitational field of: finite extent, fixed frequency (\(F\)), and period (\(\Delta t\)), it follows from Rayleigh’s theorem that:

\[ \int_0^{\Delta t} f^2(t) \, dt = \int_0^{F} \left| g(f) \right|^2 \, df \]

if \(f(t)\) is made an even (cosine) function. Note that \(F = (\Delta t)^{-1}\). Regarding the right hand side of the equation, the single cycle’s energy and power are easily expressed in MKS units. Thus, \(\left| g(f) \right|^2\) is an energy with units of [Joules per cycle] and \(F\) has units of [cycles per second]. Noting \(\left| g(f) \right|^2\) is fixed (per full cycle), and letting \(f(t) = \Delta a \cos(\omega t)\), we have:

\[ \int_0^{\Delta t} [\Delta a^2 \cos^2(\omega t)] \, dt = \left| g(F) \right|^2 \int_0^{F} \, df \]

Bracewell reassures us that Fourier transforms of physically real, yet finite sinusoidal waves, do mathematically exist. With the tiny physical undulations of the gravitational field (\(\Delta a\)) physically affecting a moving mass, we tentatively let the (Fourier transform) amplitude-squared term equal a specific kinetic energy so that: \(\left| g(F) \right|^2 = \frac{1}{2} (\delta v_F)^2\) (a physical specific kinetic energy). Now, since the integral of a squared sine or cosine function over one period, is half the amplitude squared multiplied by the period; upon integration and at a given frequency, we obtain:

\[ \frac{1}{2} \Delta a^2 \Delta t = \frac{1}{2} (\delta v_F)^2 F \]

For convenience, the \(F\) subscript on \(\delta v\) is dropped, and we let \(F \rightarrow f\), with \(f\) now signifying a given

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35 The terminology describing Fourier transforms is biased towards electrical engineering interests.
36 Based upon: http://research.opt.indiana.edu/Library/FourierBook/ch03.html, other internet sources, and Ref. [20, pp.119-122].
fixed wave frequency. Noting that \( \frac{1}{2}(\delta v)^2 \) is written as \( \frac{1}{2}\delta v^2 \), the identity becomes:

\[
\frac{1}{2} \delta v^2 \Delta t = \frac{1}{2} \delta v^2 f
\]  

(2)

### 7.5 Discussing a single (or unit) cycle of a singular (isolated) wave

Recalling the dimensions of the identity are \([L^2/T^3]\) which indicates a rate of specific energy transfer. The term \( \frac{1}{2} \delta v^2 \) is thus indicative of a specific kinetic energy transfer associated with the sinusoidal oscillation for a single cycle.

This specific, or mass independent, energy transfer rate is also being expressed as an unorthodox integral of squared wave acceleration (i.e. squared field undulation) over time affecting a body. Note that, at the moment, we are not directly dealing with an equality involving wave amplitudes of two physical quantities (\( \Delta a \) and \( \Delta v \)), but an equality of specific energy transfer rates at, and of, a body. Note that from the spacecraft’s local reference frame it would be the barycenter that oscillated relative to the spacecraft. Additionally, note that special relativistic effects are negligible in the solar system, while general relativistic effects are already incorporated in the comprehensive broader analysis that establishes the Pioneer anomaly.

The two domains involved for this application of Rayleigh’s (Power) theorem apply to: firstly (in the time domain), wave energy transfer rate (for one cycle) affecting a light body in the field; and secondly (in the frequency domain), the rate of oscillatory kinetic energy expressed (in a cycle) by the body. The former is an effective (specific) energy transfer rate, and the latter a (specific) energy ‘expression’ rate. Both are ‘power quantities’ determined relative to a standard systemic inertial frame (centered) at the solar system barycenter.

Just what is being physically represented, i.e. a field energy transfer rate being equal to a kinetic energy transfer rate is yet to be fully clarified.

### 7.6 Investigating potential energy

With the constant phase of the undulations upon the gravitational field propagating at the speed of light, the distance covered by the acceleration wave’s (constant) phase in a single period of Jupiter’s moon Io (for example) with an orbital period of \( \sim 1.77 \) days, is about 300 AU. With the Pioneer spacecraft travelling at about 2.5 AU per year, spacecraft are effectively stationary as far as the wave’s velocity is concerned. Thus, the variation in gravitational field strength may be idealized to simply involve the temporal cyclical variation in gravitational field strength, i.e. we assume changes in S/C distance from the barycenter are negligible. We further idealize the situation by assuming constant spacecraft mass. A varying (specific) potential energy is then simply determined by way of an oscillating gravitational field strength at the S/C or light body. This is a non-central force (or non-Schwarzschild geometry) based potential energy.

Unlike a central field where (specific) potential energy is location dependent and proportional to gravitational field strength, the specific energy of an acceleration wave is dominantly period dependent (i.e. time dependent) and proportional to the square of its amplitude. Thus, for the wave’s energy over a single oscillation, from equation (2) it follows that:

\[
e_{\text{wave}} = \frac{1}{2} \delta v^2 \Delta t^2
\]

This may be thought of as the specific undulation energy per cycle of the (‘surface’ undulatory) gravitational field influencing a moving body.

At an essentially fixed barycenter distance, in addition to the location based (primarily) static (or non-oscillatory) potential energy of a body, we now also recognize a time based oscillatory specific potential energy associated with a wave field undulation — upon the Sun dominated gravitational field of the solar system.

### 7.7 Relating wave potential energy to kinetic energy (single wave, singular cycle case)

We now conceptually examine how this additional energy carried by acceleration waves upon the gravitational field alters the expression for kinetic energy of a body in motion. We seek to confirm the specific kinetic energy associated with a single oscillation is \( \frac{1}{2} \delta v^2 \).

Recalling \( \Delta a = \omega \Delta v \) we observe how the relative oscillatory motion (\( \Delta v \), which is also some function of \( \delta v \), see Section 7.6), and potential energy fluctuations (some function of \( \Delta a \)) are \( 180^\circ \) or
π radians out of phase. Just how the expression for specific K.E. per cycle $\frac{1}{2}(\delta v)^2$, which is a response to specific P.E. fluctuations upon the field, is to be understood is now addressed.

Total energy considerations are different with type-2 undulations upon the gravitational field. When a sinusoidal acceleration wave causes a moving mass to display a sinusoidal (output) motion, it is necessary that the inertial mass ‘carrying capacity’ associated with the wave’s specific potential energy, is just sufficient to ‘excite’ the moving object, of given mass, into its sympathetic oscillatory motion response. Thus for the spacecraft:

$$(P.E.\text{-w}ave)_{s/c} = \frac{1}{2}m_{s/c}\Delta a^2\Delta t^2$$

If this is the situation, the two (inertial) masses are seen as common and equal for the wave’s effect upon the moving mass. Hence, we may discuss either: specific energy equivalence (as in equation 4) or simply energy equivalence; with the two modes of equality not being different when resonance is occurring below the cut-off mass ($m_c$) — of a particular wave at a particular location. Note that below the cut-off mass ($m_c$), the effect is independent of mass. The cut-off mass itself displays a different type of mass dependence, so that this $m_c$ is a function of the wave’s origin and evolution in space 39.

Relative to the inertial barycentric reference frame, ‘light’ bodies (e.g. spacecraft) are in both: translational motion and a cyclic (line-of-sight) motion. Since the local frames are not mechanically-forced, unlike the simple harmonic motion of a spring, there is a specific force (i.e. an acceleration) restoring back to, and driving away from, the equilibrium position. A body’s kinetic energy (K.E.) ‘reservoir’ must be drawn upon to appease the field’s oscillatory demands. It is hypothesized that: the K.E. given to the Pioneer spacecraft, post-planetary encounter, is being partially directed by P.E. oscillations (each and every cycle) into a ‘non-productive’ oscillatory response motion around an equilibrium position. In terms of translational kinetic energy, this oscillatory expression of K.E. is lost at the rate of $\frac{1}{2}m_{s/c}(\delta v)^2$ per cycle.

7.8 Relating motion shortfall to the amplitude of sinusoidal speed

Returning to the equation (2) equality at the end of Section 7.4, and substituting now for: $\Delta a = \omega \Delta v$ (from Section 3.1), $\Delta t = 1/f$ and $f = \omega/2\pi$ we obtain:

$$\frac{1}{2}(\omega \Delta v)^2(\frac{2\pi}{\omega}) = \frac{1}{2}(\delta v)^2(\frac{\omega}{2\pi})$$

which simplifies to give $\Delta v = \frac{\delta v}{2\pi}$ or:

$$\delta v = 2\pi \Delta v$$

Thus, the total loss of (solar-)‘systemic’ steady speed in a single cycle ($\delta v$), relative to predicted steady speed, equals $2\pi$ times the sinusoidal amplitude of speed variation ($\Delta v$). Note that this sinusoidal amplitude in speed is about a non-constant mean value because kinetic energy redistribution acts to slow the spacecraft’s steady-translational speed; so that at the beginning and end of the cycle the two speeds, relative to the (essentially fixed) barycenter, are different ($v_{\text{final}} - v_{\text{initial}} = -\delta v$ where $\delta v > 0$). Finally, note that $|\delta \vec{v}| = \delta v$, and that we treat $\delta \vec{v}$ predominantly as simply a scalar loss of speed.

7.9 Steady and unsteady energy of motion (singular wave case)

The total kinetic energy, possessed by the Pioneer spacecraft, can be seen as equal to a steady kinetic energy component plus an unsteady kinetic energy component. Note that for multiple waves from multiple moons, of distinct period and amplitude, there will be a number of coexisting and superpositioned unsteady component terms.

Importantly, one needs to appreciate that in the absence of any sinusoidal speed variations all the asymmetrical quantum indeterminacy collectively shared by each and every atom in a moon, expressed in the guise of a field undulation, has eventuated in this further (or secondary) new phenomenon.

39The model employs $E_{\text{wave}} = \frac{1}{2}m_c\Delta a^2\Delta t^2$ at an initial reference radius, to represent the wave’s total energy. $E_{\text{wave}}$ equals the ‘gravito-quantum’ excess energy $(E_{\text{excess}}$ ‘released’ to the gravitational field for (some) third bodies in systemic three-body motion — under the influence of weak gravitational fields.

40The violation of conservation of energy, by way of an

41‘Steady’ here means non-oscillatory as compared to oscillatory (i.e. unsteady). Naturally, the steady motion may be: linear (radial), rotational (i.e. circular or elliptical), hyperbolic or parabolic.
Consequently, the identity between $\Delta a$ and $\delta v$ (equation 2) derived from Parseval’s theorem appears to have a justifiable physical application, i.e. conceivably explaining the Pioneer anomaly. New physics, beyond the notion of the problematic extra force hypothesis\textsuperscript{43}, is arising out of a contemporary interpretation of established mathematics and physics. It is the ‘mechanism’ behind the establishment, and the associated quantification of the wave energies and $m_c$ magnitudes, that remains in need of rigorous scientifically explanation.

In this write up the waves’ existence is simply being hypothesized and then scrutinized. Partial aspects of a more detailed theoretical model have been included only to facilitate this process.

### 7.11 Multiple acceleration waves upon the gravitational field

We are now in a position to examine the effect of the superposition of acceleration waves, emanating from a number of moons, upon a spacecraft. Let us continue to idealize the situation by letting the S/C be in pure radial motion at a position far outside the solar system, and by letting the total observation time (i.e. the total data interval) be very long.

Note that the physical mechanism hypothesized, over say 100 years, has effectively fixed values of: $\Delta a, \Delta v, \delta v$ and $\Delta t = 1/f$ for any given moon.

From equation (3) (i.e. $\delta v = 2\pi \Delta v$) and $\omega = 2\pi f$ it follows that for a singular wave:

$$(\Delta v)\omega = (\delta v)f$$

where $(\delta v)f$, or if you prefer $(\delta v \cdot f)$, is the rate of speed shortfall (in MKS units) for a singular wave, over a single wavelength. Now since $\Delta a = \omega \Delta v$:

$$\Sigma(\Delta a)^2 = \Sigma(\Delta v \cdot \omega)^2 = \Sigma(\delta v \cdot f)^2 = \Sigma(\delta v / \Delta t)^2$$

Since we are dealing with energy transfer, we must continue to work with squared quantities, in this

\textsuperscript{43}An additional force, i.e. additional spacetime curvature, cannot explain the Pioneer anomaly’s apparent violation of the principle of equivalence, since the orbits of planets over a considerable period of time would indicate the presence of such a ‘generally applied’ force (i.e. the force influences all masses). A similar situation applies to the orbits of long-period (heavy-body) comets\textsuperscript{21}. Additionally, large outer solar system asteroids (of diameter $\sim 3$ km) will not exhibit the Pioneer anomaly\textsuperscript{22}. See Section 8.2.
case $\Delta a$. Thus, $\sqrt{\sum (\Delta a)^2}$ or $[\sum(\Delta a)^2]^{\frac{1}{2}}$ is used to represent the overall sum of all acceleration amplitudes. This sum equals long-term average (additional) acceleration $a_P$ or $a_P(t)$. Thus, in the idealized circumstances discussed, it is feasible to write:

$$a_P = a_P(t) = \sqrt{\sum (\Delta a)^2} = [\sum(\Delta a)^2]^{\frac{1}{2}}$$

Now since Doppler tracking measurements, and a raft of supporting science, give a value of anomalous speed shortfall ($\Delta V_P$), that over a medium-long period of time ($T_L$) indicates the action of an apparently constant anomalous acceleration, we may establish that:

$$\lim_{t \to \text{large}} [\sum (\Delta a)^2]^{\frac{1}{2}} = \lim_{t \to \text{large}} [\sum (\Delta \dot{v}/\Delta t)^2]^{\frac{1}{2}}$$

$$= \frac{\Delta V_P}{T_L}$$

$$= a_P$$

$$= \text{constant}$$

Over extended periods of time (i.e. 11.5 years) we have (on average) a ‘headline’ constant result so that: $a_P^2 = \sum(\Delta a)^2 = \text{constant}$. Whereas in the short term the $a_P(t)$ measured has a stochastic-like variation\textsuperscript{44} about the long term fixed mean value ($a_P$). Thus, upon (hypothetically) removing all signal noise, it is proposed that:

$$[a_P(t)]^2 = \sum[2\Delta a^2 \cos^2(\omega t + \phi)]$$

best matches the observational data to the hypothesized physical situation\textsuperscript{45}. Note that the positional variation in the additional (Pioneer) acceleration $[a_P(x, t)]$ has not been included, as yet, in this idealized model: where $x$ is distance from the barycenter (i.e. from the systemic inertial frame’s ‘origin’).

For a radially (outward) directed spacecraft, at a given position and a given time, we may schematically express the total (inward) acceleration acting on a ‘light’ body as:

$$a_{\text{total}}(x, t) = a_{\text{GR}}(x, t) + a_P(x, t) - a_{\text{rad}}(x, t) + \ldots$$

where $a_{\text{rad}}$ is (specific) outward time dependent solar radiation force. Note that both the: total instantaneous acceleration and this additional (or Pioneer anomalous) acceleration are now functions of both location and time. Additionally, in a three (or more) body system $a_{\text{GR}}$, or standard gravitational theorization, is also time dependent. The non-determinism of these circumstances produces a need for ephemerides.

From the above it is also clear that the additional (or Pioneer anomalous) acceleration ($a_P$) is independent of overall spacecraft speed.

### 7.12 Discussion, further remarks, and relaxing the idealization

This section has sought to illustrate that the Pioneer anomalous acceleration may (alternatively) be hypothesized as resulting from the effect of a superposition of ‘sinusoidal’ wave undulations upon a pre-existing gravitational field. Observations and the author’s wider model imply the undulations are lunar based, with only some (5 out of the 7 major) moons\textsuperscript{46} contributing for kinematical and geometric reasons. Geometry is involved by way of both spacetime curvature and quantum mechanical geometric phase.

The long term mean amplitude of all the waves together is fixed (or constant) but over shorter time periods there is necessarily variation around the long-term mean acceleration. The statistical variance of observations of $a_P(x, t)$ has been greatly moderated (i.e reduced) by the orbital resonances of the Galilean moons of Jupiter.

When spacecraft are in the ‘mid’ (10 AU) to outer solar system the spherical wavefronts are not approximately orthogonal to the direction of S/C propagation, and directional cosines play a significant role\textsuperscript{47}. The anomaly will vary with the pos-

\textsuperscript{44} Actually, the real lunar-based variation is deterministic if the wave amplitudes and cut-off masses are well described, and the planetary positions are accurately known. Even though the acceleration waves are effectively fixed in amplitude and frequency (over long-terms), the motion of their host planets and the motion of a spacecraft (or ‘light’ body), will alter the effect at the spacecraft through time. Thus, the direction-based variations in amplitude produce a deterministic time series that coexists with the effects of stochastic Doppler signal noise.

\textsuperscript{45} The factor of 2 is necessary, (tentatively) in order to make the through-time changes of the sum of the solitary waves’ individual contributions of $\delta v(t)$ — equal to $[a_P(t)]$.

\textsuperscript{46} For a variety of reasons the other prograde spin-orbit coupled moons of the solar system may be neglected. The main reason is that: $\Delta a$ is proportional to total lunar mass.

\textsuperscript{47} Within the orbit of Saturn things are different again,
tions of Jupiter and Saturn relative to the spacecraft. The effect goes to a slight asymptotic maximum as a spacecraft leaves the solar system and its heliocentric radius gets very large so that direction cosines between the direction of the moving wavefronts and a body’s trajectory approach one, i.e. unity. Thus, where \( x \) is distance from the barycenter:

\[
a_P = a_P(x, t) \approx \text{constant} \quad \text{if} \quad x \to \text{large}
\]

Large \( x \) is > 20 AU. Surprisingly, due to direction cosines and the possibility of obtuse angles inwards of Saturn, we have in general (especially for \( x < 10 \) AU):

\[
a_P \neq \text{constant} \quad \text{for all} \quad x
\]

which is supported by early Pioneer 11 observations [2, Fig.7, p.18].

A second look at the anomaly in the Galileo spacecraft’s cross solar system journey out to Jupiter (Dec 1992-July 1995) might clarify this issue. Note that a trajectory or path-based hypothesis, as compared to a Sun-based effect, involves distinctly different angles, and hence different anomalous acceleration magnitudes, at these low AU values — of 1 to 5 astronomical units.

The author’s fuller model indicates that at all times the spacecraft’s mass is many orders of magnitude below the ‘cut-off’ mass at which point a body no longer resonates with at least one of the undulations upon the gravitational field. This is touched upon in Section 8.2.

8 Further applications of the hypothesis

The hypothesized existence of: lunar generated, constant amplitude, first order fluctuations upon the gravitational field also potentially affects, in a beneficial way, other anomalous circumstances in our solar system.

8.1 The Earth flyby anomaly

Assuming the reality of this anomaly, the preceding hypothesis has a promising, easy and direct qualitative application to the anomalous increase in the velocity of spacecraft associated with some, but not all, Earth flybys [23, 24].

The kinetic energy of the inbound spacecraft will have tiny additional (longitudinal) oscillatory components of kinetic energy that are overlooked. At Earth gravity assist encounter, in the planet’s reference frame, total S/C energy remains constant (i.e. is conserved) [23] p.449]. Then, post encounter either: the change in trajectory orientation relative to Jupiter and Saturn reduces the total of the oscillatory components, or alternatively (and less likely), any longitudinal oscillatory component is yet to fully reestablish itself. Subsequently, an excess (steady) kinetic energy over predictions will be evident on occasions, and the effect will vary depending upon the pre- and post-encounter trajectories relative to the positions of (the moons hosted by) Jupiter and Saturn.

It is very difficult for any model to explain an anomalous increase in kinetic energy. Other than to cite observational errors, the hypothesis presented herein is conceivably the only reasonable non-systematic physical explanation possible.

8.2 Early solar system history

There are at least two problems concerning the outer solar system’s very early history for which a new slowing or ‘braking’ mechanism, in regions of very weak gravitational fields, may prove beneficial.

[51] After lunar formation and spin-orbit coupling had been attained, around their respective ‘host’ planets.

[52] Note that this excludes planetary ring systems.
cial. Like the (predominantly) radial Pioneer case, orbital motions (in the outer solar system) are subject to a predominantly radially directed oscillation, that in this case is now orthogonal to the trajectory of their motion. As for the Pioneer anomaly, this represents an unsteady expression of kinetic energy. The inward acceleration of the Pioneer ‘radial’ motion, for an orbiting body becomes a deceleration of magnitude $8.74 \times 10^{-9}$ cm/s$^2$ effect. Over a quarter of a million years this would produce an approximately 7 km/s slowdown of orbiting ‘light’ bodies$^{53}$, while leaving ‘heavy’ bodies (and electromagnetic radiation$^{54}$) totally unaffected. Note that Neptune’s average orbital speed is 5.43 km/s.

For ‘big rocks’ (below the cut-off mass transition), this new physical ‘method’ of braking light bodies and hence altering their motion, is much ‘quicker’ than any conceivable braking by existing conventional drag effects.

The migrating planets hypothesis that is invoked to explain the too rapid formation of the ice giants (Uranus and Neptune), in their present locations, may also possibly be overcome. This hypothesis is not without its concerns$^{55}$ and criticisms$^{26,27}$. If all small primordial bodies, in very weak gravitational fields, underwent ‘wave-braking’ by the aforementioned mechanism then they would have spiralled into outer planets or the inner solar system. Compared to other ‘braking’ mechanisms this happens very quickly and very early in the solar system’s history. Naturally, this mechanism continues to exert its influence today, with collisions of bodies generating smaller bodies ‘continuously’.

There is also an apparent total lack of ‘small’ comets observed in the solar system less than about 1km in diameter$^{28}$ Fig.2), or $\sim 5.2 \times 10^{12}$ kg (assuming a comet density of $1 \times 10^{3}$ kg/m$^3$). This implies that primordial small comets were not flung out, from the inner solar system, to the Oort cloud long-ago for some reason. The author’s fuller model produces different cut-off masses ($m_c$) for each of the five moons previously mentioned — that are considered to dominate the Pioneer anomaly. At 2.5 AU from the lunar sources, these $m_c$ values range between $3.4 \times 10^{12}$kg (Titan) to $4.5 \times 10^{13}$kg (Callisto). Whereas at 20 AU from the lunar sources, these values go from $6.6 \times 10^{9}$kg (Titan) to $8.9 \times 10^{10}$kg (Callisto). Observational evidence$^{28}$ Figs.1 and 2] implies a depletion in comets, as compared to expectations, beginning at about 5 km diameter, or a mass of $\sim 6.5 \times 10^{13}$kg. Bearing in mind that primordial comets (orbiting in the plane of the ecliptic), exist over a range of heliocentric distances with somewhat random orbital motions; the cut-off masses derived appear to qualitatively ‘match’ this situation in a very promising way$^{56}$.

In a similar vein, Kuznitcheva and Ivanov$^{28}$ conclude: “The lack of small craters on [the asteroid] Eros is the first observational evidence of a possible paucity of smaller bodies in the Main Belt$^{29}$. Additionally, there apparently exists an ‘abrupt’ edge to the present (and presumably the long-past) solar system by way of an abrupt outer edge to the Kuiper belt. This was the exact opposite of what planetary scientists had envisaged.$^{30}$

Finally, the evidence for this ongoing braking effect on ‘light’ bodies (of non-zero mass) appears to be supported by the contemporary representation of a kink in the linear relationship between the cumulative number of Earth crossing asteroids vs. the diameter of these bodies (see the log-log diagram: Fig.3 of Ref.$^{28}$). There is one straight line for big primordial bodies, and the other for ‘fall-out’ from collisions of these bigger bodies. The kink joining the two possibly illustrates both: the size level at which bigger primordial bodies start to become sparse, and (at the other end of the kink) the upper size limit of impermanent collision debris.

9 Concluding discussion

Observational evidence of the ‘through time’ behaviour of the Pioneer anomaly $a_p(t)$ is open to a new interpretation by way of hypothesizing a number of tiny (sinusoidal) undulations ($\Delta a$) on the...
gravitational field; each with its own fixed amplitude and frequency. Ignoring direction cosines:

$$a_p(t) = \sqrt{\sum_{i=1}^{n}[2\triangle a_i^2 \cos^2(\omega_i t + \phi_i)]}$$

and, on average over a long period of time, e.g. 30 years (roughly the orbital period of Saturn):

$$a_P = \bar{a_p}(t) = \sqrt{\sum(\triangle a)^2}$$

The acceleration-wave hypothesis is supported, in this write up, by partial aspects of a new model that seeks to explain the observational evidence implying at least three anomalous behaviours in our solar system. The hypothesis, at this stage, is not easy to accept, because the new physical model discussed has not been fully established. This needs to be presented to support and compliment the predominantly general presentation of a prospective model given in this article. The scope of the hypothesis is unlike other ‘singular solutions’ offered to explain a ‘constant’ Pioneer anomaly. Importantly, in this new approach the apparent violation of the Equivalence Principle is accepted and incorporated at a fundamental level in the model.

The Pioneer (acceleration) anomaly essentially becomes an oversight in predicted spacecraft motion, by way of omitting the effect of certain wave-like undulations in the gravitational field’s strength. A shortfall in motion results which is effectively a new non-conservative (specific force) ‘deceleration-wave-drag’ effect. The most impressive evidence for this effect, that is restricted to bodies whose mass is non-zero and below a certain cut-off mass zone\(^{57}\), is the Pioneer anomaly.

This article has sought to show how an ongoing braking effect, applicable only to light bodies has sculptured the contents of our solar system from its very earliest stages. Somewhat ironically, it may be conjectured that: were it not for the acceleration-wave-braking mechanism proposed herein, the Pioneer (and other) spacecraft may not even have passed safely through the asteroid belt on their journeys to Jupiter and beyond, many years ago, because it would remain strewn with collision fragments of all sizes.\(^{57}\)

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