Low-energy enhancement and fluctuations of γ-ray strength functions in $^{56,57}$Fe: test of the Brink-Axel hypothesis

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Abstract. Nuclear level densities and γ-ray strength functions of $^{56,57}$Fe have been extracted from proton-γ coincidences. A low-energy enhancement in the γ-ray strength functions up to a factor of 30 over common theoretical E1 models is confirmed. Angular distributions of the low-energy enhancement in $^{57}$Fe indicate its dipole nature, in agreement with findings for $^{56}$Fe. The high statistics and the excellent energy resolution of the large-volume LaBr$_3$(Ce) detectors allowed for a thorough analysis of γ strength as function of excitation energy. Taking into account the presence of strong Porter-Thomas fluctuations, there is no indication of any significant excitation-energy dependence in the γ-ray strength function, in support of the generalized Brink-Axel hypothesis.

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1. Introduction

One of the long-standing experimental and theoretical challenges within nuclear physics is the determination of the nucleus’ available quantum levels and the decay properties of these levels in the excitation-energy region between the ground state and up to the particle threshold(s). In this intermediate excitation-energy region, often called the quasicontinuum, the nuclear level density (NLD) and the average, reduced γ-decay probability, i.e. the γ-strength function (γSF), shed light on the dynamic behavior of the nucleus. Apart from providing information on basic nuclear properties, these quantities are also indispensable input for calculations of, e.g., neutron-capture cross sections. These cross sections are of great importance for applications such as the astrophysical heavy-element nucleosynthesis [1, 2] and modeling of next-generation nuclear power plants [3, 4].

Amongst a handful of experimental techniques, the Oslo method [5] has been established as one of the promising approaches to obtain experimental information on the NLD and γSF. The advantage of the Oslo method compared to other techniques is that both these quantities can be extracted from one and the same experiment, utilizing typically a charged-particle reaction to record particle-γ coincidences, in which the structural shape of the NLD and the γSF can be determined. By measuring the energy of the outgoing charged particle, the initial excitation energy of the residual nucleus is determined. The γ rays de-exciting this initial excitation energy are recorded in coincidence, thus obtaining γ spectra as function of initial excitation-energy.

In 2004, an unexpected enhancement of the γSF for low transition energies ($E_\gamma \lesssim 3$ MeV) was discovered in the iron isotopes $^{56,57}$Fe [6]. This feature was not predicted by any theoretically derived γSFs; in fact, the γSF data showed an enhancement of more than a factor of 10 compared to typical models for the E1 strength [6]. In the following years this enhancement, also called upbend, was found in many medium-mass nuclei, including $^{43–45}$Sc [7, 8], $^{60}$Ni [9], $^{73,74}$Ge [10], and Mo isotopes [11 12 13]. To date, the heaviest nuclei where the upbend has been seen are $^{138,139}$La [14] and $^{151,153}$Sm [15]. The upbend was experimentally shown to be of dipole nature in $^{56}$Fe [16]. Moreover, it has been demonstrated [17] that such a low-energy enhancement in the γSF could significantly increase radiative neutron-capture rates of relevance for the r-process – if found to be present in very neutron-rich nuclei.

In 2012, the upbend was independently confirmed in $^{95}$Mo [12] using a different technique. This triggered theoretical investigations of the origin of this phenomenon. Within the thermal-continuum quasiparticle random-phase approximation (TCQRPA), the upbend was explained as due to E1 transitions caused by thermal single-quasiparticle excitations in the continuum [18], with its strength depending on the nuclear temperature. On the other hand, shell-model calculations [19 20] show a strong increase in $B(M1)$ strength for low-energy M1 transitions. At present, $^{60}$Ni is the only case where experimental data favor a magnetic character of the upbend [9]. More experimental information is needed in order to determine whether the upbend is
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In this work, we present NLDs and $\gamma$SFs of $^{56,57}$Fe extracted from (p,$p'\gamma$) coincidences, to be compared with data from other reactions using heavier projectiles, inducing higher initial spins. We analyze systematic errors in the normalization procedure and compare our results to available data in the literature. For the first time, we present angular distributions of the upbend in $^{57}$Fe, as well as $\gamma$SFs as function of excitation energy to investigate the so-called generalized Brink-Axel hypothesis for $^{56,57}$Fe. This hypothesis has up to now only been validated for the heavy nucleus $^{238}$Np [21].

This article is organized as follows. In section 2, we give experimental details and the main steps of the Oslo-method analysis. In section 3, the NLDs and $\gamma$SFs are shown and the normalization uncertainties are discussed. Further, in section 4, angular distributions are presented for $^{57}$Fe, while section 5 deals with $\gamma$SFs as function of excitation energy and implications for the generalized Brink-Axel hypothesis. Finally, a summary and outlook are given in section 6.

2. Experimental details and data analysis

The experiments were performed at the Oslo Cyclotron Laboratory (OCL). A 16-MeV proton beam with intensity of $\approx 0.5$ nA impinged on self-supporting targets of 99.9% enriched $^{56}$Fe and 92.4% enriched $^{57}$Fe. Both targets had mass thickness of $\approx 2$ mg/cm$^2$. Accumulating times were $\approx 85$h and $\approx 92$h for $^{56,57}$Fe, respectively.

The charged ejectiles were measured with the Silicon Ring particle-detector system (SiRi) [22] and the $\gamma$ rays with the CACTUS array [23]. The SiRi system consists of eight $\Delta E - E$ telescopes. Each telescope is composed of a 130-$\mu$m thick front detector segmented into eight strips (angular resolution of $\Delta \theta \approx 2^\circ$), and a 1550-$\mu$m thick back detector. In total, SiRi has 64 individual detectors and a solid-angle coverage of $\approx 6\%$. For these experiments, SiRi was placed in forward angles with respect to the beam direction, covering $40 - 54^\circ$. From the measured energy of the ejectiles and the reaction kinematics, the excitation energy of the residual nucleus is deduced.

In this experiment, the CACTUS array contained 22 collimated 5 in. $\times$ 5 in. NaI(Tl) detectors and six collimated 3.5 in. $\times$ 8 in. LaBr$_3$(Ce) detectors from the Milan HECTOR$^+$ array [24,25]. The NaI detectors were placed on the CACTUS frame with six different angles $\theta$ with respect to the beam direction (37.4, 63.4, 79.3, 100.7, 116.6, and 142.6 degrees), while the LaBr$_3$ crystals covered four angles (63.4, 79.3, 100.7, and 116.6 degrees). The $\gamma$-energy thresholds were $\approx 400$ keV and $\approx 800$ keV for the NaI and LaBr$_3$ detectors, respectively. Particle-$\gamma$ coincidences were recorded event-by-event, with the overlap of the $\Delta E$ and $E$ detectors of SiRi as mastergate for the analog electronics. To obtain reasonable statistical error bars, i.e. $\approx 50\%$ or better on the extracted NLD and $\gamma$SF, about 40,000 coincidences are needed. In total, after background subtraction of random coincidences (about 10% of the prompt time peak), about 65 million coincidences were obtained for the NaI detectors and about 12 million
coincidences for the LaBr$_3$ detectors with the $^{56}$Fe target. Correspondingly, for $^{57}$Fe, about 15 million and 2.1 million coincidences were recorded for the NaI and LaBr$_3$ detectors, respectively. The time resolution of the SiRi-NaI detectors was 14.4(5) ns and for the SiRi-LaBr$_3$ detectors 6.3(3) ns.

In figure 1, the proton spectrum of SiRi in coincidence with $\gamma$ rays from the present experiment is compared to the $\alpha$ spectrum from the previous experiment reported in Ref. [6]. The significant improvement in energy resolution is clear; the proton spectra have a full width at half maximum (FWHM) of $\approx 90$ keV compared to the $\alpha$ spectra where FWHM $\approx 500$ keV. The main reason for this improvement is the segmentation of the $\Delta E$ detectors in SiRi compared to the old setup with a non-segmented $\Delta E$ detector. The segmentation allows for a much more precise determination of the scattering angle and thus the recoil energy. Also, using a proton beam instead of a $^3$He beam gives a smaller recoil energy to the residual nucleus. For more details, we refer to [22].

The proton-$\gamma$ coincidence matrices for the NaI and LaBr$_3$ detectors are displayed in figure 2. The superior energy resolution for the LaBr$_3$ spectra relative to the NaI ones is evident, as well as diagonals for which the excitation energy $E$ equals the $\gamma$ energy $E_\gamma$ corresponding to decay to the ground state. Other diagonals are also clearly visible, for example the direct decay to the first-excited $2^+$ state in $^{56}$Fe.

It is also very interesting to note the "triangles" in the $^{57}$Fe matrix where the $\gamma$ intensity suddenly drops, see for example at $E_\gamma \approx E \approx 8.5$ MeV in figure 2c,d. One would naively think that the $\gamma$ intensity would be significantly reduced as soon as the

![Figure 1](image-url)
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Figure 2. (Color online) $\gamma$-ray energy versus excitation energy before unfolding for (a) $^{56}$Fe, NaI detectors; (b) $^{56}$Fe, LaBr$_3$ detectors; (c) $^{57}$Fe, NaI detectors; (d) $^{57}$Fe, LaBr$_3$ detectors. Energy bins are 14 keV/channel.

neutron separation energy $S_n$ is reached; however, this is well above $S_n = 7.646$ MeV. This feature is explained by considering the average spin $\langle J \rangle$ populated at high excitation energies. From $\gamma$ transitions in coincidence with protons, we identify the decay from the $6^+$ level at $E = 3.39$ MeV in $^{56}$Fe as well as other levels with spins 2, 3, 4, 5 \cite{16}. Levels with these spins will be hindered in decaying through $s$-wave neutron emission to the $0^+$ ground state in $^{56}$Fe. This hindrance is studied in detail for $^{95}$Mo and applied in a novel technique to determine spins in $^{20}$.

In order to obtain the correct $\gamma$-energy distribution for each excitation-energy bin, the signals from the NaI and LaBr$_3$ detectors must be corrected for the detector response. We applied the unfolding technique described in \cite{27}, which is an iterative procedure using a strong smoothing of the Compton part of the spectrum. In order to construct response functions for the NaI and LaBr$_3$ detectors, we used in-beam measured transitions from $^{56}$Fe, $^{28}$Si, $^{13}$C, and $^{16}$O \cite{28}.

Moreover, we made use of a subtraction technique \cite{29} to extract the distribution of primary $\gamma$ rays, i.e. the first $\gamma$ rays emitted in the decay cascades, for each excitation-
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energy bin. This distribution contains information on the NLD and the $\gamma$SF as deduced from Fermi’s Golden Rule [30, 31]:

$$\lambda = \frac{2\pi}{\hbar} |\langle f | H' | i \rangle|^2 \rho_f,$$

where $\lambda$ is the decay rate between initial state $i$ and final state $f$, $H'$ is the transition operator and $\rho_f$ is the density of final states. Similarly, the distribution of primary $\gamma$ rays as function of $E$ depends on the level density at $E_f = E - E_\gamma$ and the $\gamma$-transmission coefficient $T$ for the $\gamma$ transition with energy $E_\gamma$. The $\gamma$-transmission coefficient is directly proportional to the $\gamma$SF. Our ansatz is [5]:

$$P(E_\gamma, E) \propto \rho(E_f) T(E_\gamma),$$

where $P(E_\gamma, E)$ is the matrix of primary $\gamma$ rays, representing relative intensities or branching ratios for a given transition energy $E_\gamma$ at a given initial excitation energy $E$.

The primary $\gamma$-ray matrices $P(E_\gamma, E)$ for $^{56,57}$Fe are shown in figure 3. They are normalized for each excitation-energy bin so that $\sum_{E_\gamma} P(E_\gamma, E) = 1$. This means that the probability for $\gamma$ decay from a given bin is 1, and that the intensity of a given $\gamma$-ray energy reflects the branching ratio for that particular transition energy.

These matrices are used as input for the extraction of the NLD and $\gamma$SF for the four data sets. The expression in equation 2 is valid for statistical decay, i.e. where the decay is independent of the formation of the compound state [34]. This is fulfilled at rather high excitation energies where the initial NLD is high, typically above $\approx 2\Delta$ where the pair-gap parameter $\Delta \approx 12A^{-1/2}$ [34]. Note that $T$ is a function only of $E_\gamma$ and not $E$ or $E_f$, in accordance with the generalized Brink-Axel hypothesis [32, 33]. This will be discussed in detail in section 5.

The functional form of the NLD and $\gamma$SF is determined through a least-$\chi^2$ fit to the $P(E_\gamma, E)$ matrices as described in [3]. The 3D landscapes as shown in figure 3 are used in the fit. The sum of all primary transitions for each $E$ bin is normalized to unity. As the $P(E_\gamma, E)$ matrices contain many more data points (“pixels”) than the free parameters (the vector elements of $\rho(E_f)$ and $T(E_\gamma)$), the solution is uniquely determined and the fit routine converges fast, typically within 10-20 iterations.

Some considerations need to be made before extracting the NLD and $\gamma$SF from the data. First, a low-energy limit for the excitation energy is applied to avoid the discrete region at low $E$, for which the condition of a compound-nucleus decay is highly questionable. Further, an upper limit $E_{\text{max}}$ must be given, which typically corresponds to $S_n$, as neutrons are not measured or discriminated in the present experimental setup. Finally, a low-energy limit on the $\gamma$ energy, $E_{\gamma,\text{low}}$, is determined to exclude eventual higher-generation transitions not properly subtracted in the primary-distribution extraction, as discussed in detail in [35]. The chosen energy limits for the extraction procedure are: $E_{\gamma,\text{low}} = 2.1$ MeV, $E_{\text{min}} = 6.6$ MeV, and $E_{\text{max}} = 11.3$ MeV for $^{56}$Fe; correspondingly, $E_{\gamma,\text{low}} = 1.4$ MeV, $E_{\text{min}} = 5.0$ MeV, and $E_{\text{max}} = 8.2$ MeV for $^{57}$Fe. The neutron separation energies $S_n$ are 11.197 MeV and 7.646 MeV for $^{56,57}$Fe, respectively. The reason why we are able to put $E_{\text{max}}$ higher than $S_n$ in the
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Figure 3. (Color online) Distribution of primary $\gamma$ rays energy versus excitation energy for (a) $^{56}$Fe, NaI detectors; (b) $^{56}$Fe, LaBr$_3$ detectors; (c) $^{57}$Fe, NaI detectors; (d) $^{57}$Fe, LaBr$_3$ detectors. Energy bins are 124 keV/channel for $^{56}$Fe and 120 keV/channel for $^{57}$Fe. Note the different energy scales for the lower and upper panels.

case of $^{57}$Fe, is that the first-excited state in $^{56}$Fe is at 847 keV, allowing in principle for $E_{\text{max}} = (7.65 + 0.85)$ MeV = 8.5 MeV as we are requiring proton-$\gamma$ coincidences. Similarly, for $^{57}$Fe, the upper limit is $\approx 100$ keV above $S_n$.

To test the quality of the fit, which is based on all primary spectra included in the extraction procedure, we take the obtained $\rho(E_f)$ and $\mathcal{T}(E_\gamma)$ functions and use them to generate primary $\gamma$ spectra to be compared with the input spectra bin by bin. This is shown in figure 4. Error bars in the primary spectra reflect statistical uncertainties, and systematic uncertainties stemming from the unfolding procedure and the extraction of the primary $\gamma$ rays [5].

As can be seen from figures 4–7, the overall agreement between the data and the calculated primary spectra is very good. It should be noted that Porter-Thomas fluctuations [36] of the decay strengths are not taken into account. These fluctuations are expected to be large when the final level density $\rho_f$ is low. This is clearly visible e.g. in the decay to the first-excited level in $^{56}$Fe, see figure 4a and the peak at $E_\gamma \approx 6.5$ MeV, where data points are several standard deviations off the calculated $\rho \times \mathcal{T}$. Here, there is only one final level and the relative decay strength is seen to fluctuate strongly
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Figure 4. (Color online) Comparison of experimental primary γ spectra for $^{56}$Fe (black points, NaI detectors) with the calculated ones (blue histogram) from the extracted $\rho$ and $T$ functions for a set of initial excitation-energy bins as indicated in the panels. Energy bins are 124 keV/channel.

Figure 5. (Color online) Same as figure 4 for $^{56}$Fe, using data from the LaBr$_3$ detectors.

for different initial excitation energies.
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3. Level density and $\gamma$ strength

3.1. Normalization

As only the functional form of the NLD and $\gamma$SF can be deduced from the primary $\gamma$ spectra, the slope and absolute normalization must be determined from auxiliary data.

Figure 6. (Color online) Same as figure 4 for $^{57}$Fe measured with NaI detectors. Energy bins are 120 keV/channel.

Figure 7. (Color online) Same as figure 4 for $^{57}$Fe using data from the LaBr$_3$ detectors.
It is shown in [5] that any solution $\tilde{\rho}_f$ and $\tilde{T}$ will give an equally good $\chi^2$ fit to the primary-$\gamma$ data through the transformations

$$\rho(E - E_{\gamma}) = \mathcal{A} \exp[\alpha(E - E_{\gamma})] \tilde{\rho}(E - E_{\gamma}),$$

(3)

$$T(E_{\gamma}) = B \exp(\alpha E_{\gamma}) \tilde{T}(E_{\gamma}),$$

(4)

where the parameters $\mathcal{A}$, $B$, are the absolute normalization of the NLD and the $\gamma$-transmission coefficient, respectively, and $\alpha$ is the common slope parameter.

For the NLD, the parameters $\mathcal{A}$ and $\alpha$ are found by fitting our data to known levels from the literature [37] at low excitation energy and to neutron-resonance spacing data from [38] at $S_n$. The discrete levels are binned with the same bin width as our experimental data. For $^{56}$Fe, there is no information from neutron-resonance experiments as $^{55}$Fe is unstable. For this case, we have estimated the NLD at $S_n$ from systematics in the following way:

(i) To estimate the lower-limit NLD, we calculate the total level density from the $s$-wave neutron resonance spacing $D_0$ for Fe isotopes where this value is available from [38] according to the expression

$$\rho(S_n) = \frac{2\sigma^2}{D_0} \left( \frac{1}{(J_t + 1) \exp[-(J_t + 1)^2/2\sigma^2] + J_t \exp[-J_t^2/2\sigma^2]} \right),$$

(5)

assuming equally many positive- and negative-parity states. Here, $J_t$ is the ground-state spin of the target nucleus in the neutron-resonance experiment and $\sigma$ is the spin cutoff parameter. We make use of the phenomenological spin cutoff parameter suggested in [40]:

$$\sigma^2(E) = 0.391 A^{0.675} (E - 0.5 P a')^{0.312}.$$  

(6)

Here, $A$ is the mass number and $P a'$ is the deuteron pairing energy as defined in [40]. This approach gives a low value for the spin cutoff parameter and thus a low limit for the level density. Further, we calculate $\rho(S_n)$ from the global systematics [40] directly. By taking the $\chi^2$ fit of the semi-experimental $\rho(S_n)$ with the values from systematics in the same fashion as done for $^{89}$Y in [39], one obtains an estimate for the $^{56}$Fe $\rho_{\text{low}}(S_n)$. All parameters are given in table A1. This normalization is referred to as norm-1 in the following.

(ii) To estimate the upper-limit NLD, we apply the same procedure as in (i) but with the spin cutoff parameter given by the rigid-body moment of inertia approach as parameterized in [41]:

$$\sigma^2(E) = 0.0146 A^{5/3} \frac{1 + \sqrt{1 + 4a(E - E_1)}}{2a}.$$  

(7)

Here, $a$ is the level-density parameter and $E_1$ is the excitation-energy backshift determined from global systematics of [41]. All parameters are given in table A2. We refer to this normalization as norm-2.

For $^{57}$Fe, we use the $D_0$ value given in [38] and estimate $\rho(S_n)$ using equation 5 again with spin cutoff parameters both from [40] and [41]. Consistent with the approach
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For $^{56}$Fe, the lower limit is obtained with the spin cutoff parameter in equation 6, and the upper limit with the one in equation 7, also including the uncertainties in $D_0$. All parameters are listed in table A1 and A2 in Appendix A.

As our data reach up to $E_{\text{max}} - E_{\gamma,\text{low}}$, we must interpolate between the estimated $\rho(S_n)$ and our upper data points. This is done using the constant-temperature formula of Ericson [42, 43]:

$$\rho_{\text{CT}}(E) = \frac{1}{T} \exp \frac{E - E_0}{T}.$$  \hspace{1cm} (8)

The applied parameters $T$ and $E_0$ are given in table A3 for the various normalization options, giving the best fit to our data in the regions $E = 8.2-9.2$ MeV and $E = 6.2-6.6$ MeV for $^{56,57}$Fe, respectively. The normalized level densities are shown in figure 8.

With the normalized NLDs at hand, and assuming equal parity [44], we normalize the $\gamma$-ray transmission coefficient $T$ to the average, total radiative width $\langle \Gamma_{\gamma,0} \rangle$ taken from [38] (see table A1) according to [44]

$$\langle \Gamma_{\gamma,0}(S_n, J_t \pm 1/2, \pi_t) \rangle = \frac{B}{4\pi \rho(S_n, J_t \pm 1/2, \pi_t)} \int_{E_{\gamma}=0}^{S_n} dE_\gamma T(E_\gamma)$$
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\begin{equation}
\times \rho(S_n - E_\gamma) \sum_{J_t = -1}^{1} g(S_n - E_\gamma, J_t \pm 1/2 + J), \tag{9}
\end{equation}

where $J_t$ and $\pi_t$ are the spin and parity of the target nucleus in the $(n, \gamma)$ reaction and $\rho(S_n - E_\gamma)$ is the experimental NLD. Note that the experimental transmission coefficient in principle includes all types of electromagnetic transitions: $T_{E1} + T_{M1} + T_{E2} + ...$; however, dipole transitions are found to be dominant for decay in the quasicontinuum (e.g., $^{16}$ [45]). The sum in equation 9 runs over all final states with spins $J_t \pm 1/2 + J$, where $J = -1, 0, 1$ from considering the spins reached after one primary dipole transition with energy $E_\gamma$ (see also equation 3.1 in [45]). Note that the factor $1/\rho(S_n, J_t \pm 1/2, \pi_t)$ equals the neutron resonance spacing $D_0$. From the normalized transmission coefficient, the γSF is determined by

\begin{equation}
f(E_\gamma) = \frac{T(E_\gamma)}{2\pi E_\gamma^2}. \tag{10}
\end{equation}

Again, $^{56}$Fe lacks neutron resonance data and we have therefore estimated $\langle \Gamma_{\gamma0} \rangle$ from a linear fit to the values of the other Fe isotopes taken from [38], see table 2.1. The normalized γSFs for the different normalization options for the level densities are shown in figure 4. The error band includes uncertainties in $D_0$, spin cutoff parameters, and $\langle \Gamma_{\gamma0} \rangle$. We see that the γSFs have a distinct U-like shape, independent on the choice of normalization. There is a characteristic increase in strength at low transition energies, which is very similar in shape and magnitude to recent predictions from large-scale shell-model calculations [20].

At the highest γ-ray energies, we observe a drop in strength, which could be due to the reaction populating spins at high excitation energies that on average are higher than the (close-to) ground-state spin(s), and/or a small overlap with the wave functions for the initial and final levels. In particular, for $^{56}$Fe, only $1^-$ and $1^+$ levels contribute to the dipole strength to the ground state. For lower transition energies, a broad range of levels is available as the final level density is much higher. One should therefore note that the upper data points ($E_\gamma > 9.5$ and 7.2 MeV for $^{56,57}$Fe, respectively) do not represent a general, averaged γSF in the quasicontinuum. The rather peculiar behavior of these data points indicate a possible (strong) dependence on the initial and final level(s), as well as significant Porter-Thomas fluctuations. This will be further investigated and discussed in section 5.

3.2. Comparison with other data

There exist data on the NLDs of $^{56,57}$Fe from previous experiments at the OCL [6], using the $^3$He-induced reactions $^{57}$Fe($^3$He,α$\gamma$)$^{56}$Fe and $^{57}$Fe($^3$He,$^3$He$'\gamma$)$^{57}$Fe. Moreover, level densities have also been inferred from particle-evaporation spectra of the reactions $^{55}$Mn(d,n)$^{56}$Fe [40], $^{59}$Co(p,α)$^{56}$Fe [47], $^{58}$Fe($^3$He,α)$^{57}$Fe [48], and $^{60}$Ni(n,α)$^{57}$Fe [49]. Reactions involving heavier projectiles and/or ejectiles, as well as detection angles in backward direction where the compound-reaction mechanism is dominant, populate...
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higher initial spins than for the $(p,p')$ reaction in forward angles. This would help nailing down eventual reaction dependencies on the final results.

Figure 10 shows the comparison of the present data and previous results on the NLDs. We find that the overall agreement is very good, although there are some differences between the data sets. For $^{56}$Fe, we see that the particle-evaporation data give a higher NLD between $E \approx 5 - 7.5$ MeV, which is interpreted as due to the higher spins reached in these experiments compared to the proton inelastic scattering. The absolute normalization of our data is rather uncertain due to the lack of neutron-resonance data as discussed before; however, there is a significant boost in the number of levels at $E \approx 6$ MeV for all data sets relative to the known, discrete levels. For $^{57}$Fe, a similar increase is taking place at $E \approx 4$ MeV. This could be caused by two factors: a quenching of pair correlations due to breaking of nucleon Cooper pairs, and sufficient energy to cross the $f_{7/2}$ shell gap with more than one particle (neutron or proton) into the $p_{3/2}, f_{5/2}, p_{1/2}$ orbitals.

We note that there is a significant deviation between the data of [48] and [49] above $S_n$ for $^{57}$Fe. It would be highly desirable to perform new experiments in this
energy region to clarify whether this is due to different spins populated, the particle transmission coefficients used in the analyses or issues with their absolute normalization to the discrete levels.

For the $\gamma$SF, there are to our knowledge no photonuclear data available for $^{56,57}$Fe. We have therefore compared our data to photoneutron ($\gamma,n$) cross sections of $^{55}$Mn and $^{59}$Co [50], and also an evaluation of the $^{56}$Fe($\gamma,n$) cross section [51]. The photoneutron cross section $\sigma_{\gamma n}$ maps out the shape of the Giant Dipole Resonance (GDR) [52], and is converted to $\gamma$ strength by the relation [53]

$$f(E_\gamma) = \frac{1}{3\pi^2\hbar^2c^2} \frac{\sigma_{\gamma n}(E_\gamma)}{E_\gamma}.$$ (11)

Moreover, data from threshold ($\gamma,n$) neutron-time-of-flight experiments on $^{57}$Fe [54, 55, 56] provide an estimate for the $E1$ and $M1$ strength function at $E_\gamma \approx 7.7$ MeV; the sum of these are also compared to our data. The result is shown in figure 11, where we show only our normalizations for norm-1 and norm-2 for clarity. In general, we observe a very good agreement with the previous $^3$He-induced data below $S_n$. We note that for $^{56}$Fe, the slope of the $\gamma$SF of [6] is somewhat steeper, leading to an overall lower strength for $E_\gamma < 4$ MeV and a higher strength above $E_\gamma \approx 7$ MeV. This is likely due to a different (steeper) NLD normalization as seen in figure 10a. For $^{57}$Fe, a significant difference is only seen for $E_\gamma$ above $\approx 6$ MeV, where the $^3$He-induced data undershoot the present results. The reason for this discrepancy is not clear; it could be related partly to different normalizations of the NLD, and/or strong transitions to the ground band for the $(p,p')$ reaction that are less pronounced for the $^3$He inelastic scattering.
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Finally, we note that our data show a natural continuation of the GDR tail towards lower γ energies. We stress again that the strength of the highest transition energies close to the neutron threshold is very likely to be strongly dependent on the initial and final state(s). Hence this is not quasicontinuum decay, but is still a real effect due to nuclear structure and spin selection rules. For another similar case, namely $^{89}$Y($p,p'$) [39], there exist also inelastic photon scattering data [57], which display the same pattern as our data at high transition energies. This suppressed strength could be relevant also for e.g. ($n,\gamma$) cross-section calculations, if the neutron-capture reaction populates the same low initial spins. Thus one would expect a suppression of primary transitions to e.g. the ground state in the neutron-capture reaction as well.

According to the principle of detailed balance [58], one expects that the photoabsorption ("upward") strength equals the decay ("downward") strength. However, as discussed by Bartholomew et al. [53], the principle of detailed balance is only strictly fulfilled within the extreme statistical model. Hence, one would assume that this is only valid at high excitation energies where the NLD is high and the wave functions are strongly mixed. Moreover, the Brink hypothesis has been used [32] for calculating radiative widths at $S_n$, again assuming that the GDR tail extrapolated from photonuclear data to low γ energies could appropriately describe the decay process. Despite rather large uncertainties, it is fair to say that the present experimental data

Figure 11. (Color online) Comparison of γSFs from different reactions for (a) $^{56}$Fe and (b) $^{57}$Fe. Photonuclear data are taken from [50], and the evaluated $^{56}$Fe($\gamma,n$) cross section from [51]. Also the $^{57}$Fe $E1+M1$ strength at $E_\gamma \approx 7.7$ MeV from resonant ($\gamma,n$) neutron time-of-flight measurements are shown [54, 55, 56]. For $^{56}$Fe, the present work provides the γSF for $2.1 \leq E_\gamma \leq 11.3$ MeV, while data from [6] cover $1.0 \leq E_\gamma \leq 10.3$ MeV. Correspondingly, for $^{57}$Fe, the present work covers the range $1.4 \leq E_\gamma \leq 8.2$ MeV, and data from [6] $1.0 \leq E_\gamma \leq 7.6$ MeV. The photonuclear data from [50] are for $E_\gamma > 10.2$ MeV.
from different reactions agree reasonably well and provide a quite consistent picture on
the general shape of the γSF.

3.3. Comparison with theory

Although there are many phenomenological models and some more microscopic
calculations available, they typically deviate considerably both in shape and magnitude.
In Fig. 12 a selection of frequently used models are compared to the data. Note that
we have used the global parameterization for the NLD models of [40] and [41] to test
their predictive power. For the NLD, none of the models reproduce the data over
the full energy range. Clearly, only the microscopic approach is able to grasp some
of the structures seen in the experimental results. Apparently, all the NLD models
overshoot the data at high excitation energy. This could have severe consequences for
for e.g. calculations of reaction cross sections.

Also for the γ SF models, the situation is rather confusing. Again, there is no model
that can capture all features for the full γ-energy range. The smooth, phenomenological
E1 models are quite appropriate close to the GDR, but are missing the upbend
at low transitions energies. Even for the GLO model, which actually overshoots our
56Fe data at \( E_\gamma \approx 4 - 7 \text{ MeV} \), underestimates the strength by a factor of \( \approx 3 \) at
\( E_\gamma = 2.1 \text{ MeV} \). This might not be a surprise, considering the possible \( M1 \) nature of
the low-energy enhancement [19, 20], or an enhancement due to strong \( E1 \) continuum
single-particle transitions [18], none of which are incorporated in the phenomenological
models. Also, the microscopic models are undershooting the low-energy data as well—the
56Fe data show a factor of \( \approx 30 \) more strength than the quasi-particle random-phase
approximation (QRPA) E1 strength [61]. However, the microscopic approaches do show
structural features rather similar to the data between \( 7 - 10 \text{ MeV} \). Clearly, the situation
is at present far from satisfactory, and more theoretical work is required to understand
in depth both NLDs and γSFs, preferably within the same theoretical framework.

4. Angular distributions, 57Fe

In [16], it was shown that the low-energy upbend in 56Fe is dominated by dipole
transitions. Here, we apply the same type of analysis for the so-far unexplored 57Fe
upbend.

We use the various angles for which the NaI detectors are placed and extract
angular distributions by sorting the data into \((E_\gamma, E)\) matrices according to \( \theta \) of the
NaI detectors relative to the beam direction. As the LaBr3 detectors were placed at
only four angles, and had a rather high \( E_\gamma \) threshold, these were not used for this
analysis. From the intensities as a function of angle, we can fit angular-distribution
functions of the form [62, 63]

\[
W(\theta) = A_0 + A_2 P_2(\cos \theta) + A_4 P_4(\cos \theta),
\]  

(12)
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Figure 12. (Color online) Comparison of data and theoretical calculations for (a) $^{56}$Fe NLD, (b) $^{57}$Fe NLD, (c) $^{56}$Fe $\gamma$SF and (d) $^{57}$Fe $\gamma$SF. Microscopic level densities within the Hartree-Fock-Bogoliubov plus combinatorial approach are from [59]. Parameters for the CT model and the back-shifted Fermi-gas (BSFG) model are from [40] and [41], respectively. For the $\gamma$SF models, GDR parameters for the standard Lorentzian (SLO) [32] and the Generalized Lorentzian (GLO) [45] are from [56]. The microscopic $\gamma$SF calculations are from [60] (taken at $T = 1.4$ MeV) and [61].

where $P_k(\cos \theta)$ is a Legendre polynomial of degree $k$.

The normalized angular-distribution coefficients are given by $a_k = Q_k \alpha_k A_k / A_0$, where $Q_k \approx 1$ is the geometrical attenuation coefficient due to the finite size of the $\gamma$ detectors, and $\alpha_k$ is the attenuation due to partial alignment of the nuclei relative to the beam direction. We estimate uncertainties in the intensities according to $\sigma_{\text{tot}} = \sqrt{\sigma_{\text{stat}}^2 + \sigma_{\text{syst}}^2}$. The statistical errors are given by $\sqrt{N}$ where $N$ is the number of counts, and the systematic errors are deduced from the relative change in $N$ for each symmetric pair of angles $(37.4^\circ, 142.6^\circ)$, $(63.4^\circ, 116.6^\circ)$, and $(79.3^\circ, 100.7^\circ)$. The statistical errors are typically $\approx 4\%$ or smaller, and the systematic errors are thus the dominant source of uncertainty. More details about the angular distributions are given in Appendix B.
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Figure 13. (Color online) Angular distributions for (a)–(c) single transitions and (d) the upbend region from primary transitions in $^{57}$Fe.

Table 1. Angular-distribution coefficients of transitions measured in the present experiment. The theoretical $a_k^{\text{max}}$ coefficients for complete alignment are taken from Ref. [63].

| $E$ (keV) | $E_\gamma$ (keV) | $I_i \rightarrow I_f$ | $XL$ | $\delta$ | $a_k^{\text{max}}$ | $a_2$ | $a_4^{\text{max}}$ | $a_4$ |
|----------|------------------|----------------------|------|------|-----------------|------|-----------------|------|
| 706      | 692              | 5/2$^-$ → 3/2$^-$   | $M1 + E2$ | $-0.465$ | $-1.068$ | $-0.80(20)$ | $0.12$   | $-0.11(11)$   |
| 1627     | 1261             | 3/2$^-$ → 3/2$^-$   | $M1 + E2$ | $-0.35$  | $-0.127$ | $0.35(5)$    | $0.00$   | $-0.05(14)$   |
| 1990     | 1283             | 9/2$^-$ → 5/2$^-$   | $E2$  | $-$     | $0.476$  | $0.28(20)$   | $-0.29$ | $-0.21(16)$   |

In figure 13 we show the angular distributions of known transitions in $^{57}$Fe, and how they compare with the theoretical $a_k^{\text{max}}$ values. All numbers are given in table 1. The comparison with the experimentally extracted $a_2$ coefficients and the theoretical maximum values for the known transitions shown in figure 13a,b, indicates an attenuation $a_k \approx 0.6 - 0.75$.

The behavior of the $E_\gamma = 1261$ keV non-stretched M1 + E2 transition is somewhat puzzling, as [65] gives a rather large mixing parameter of $-0.35$ (see figure 13c). The shape of our data indicates a stronger contribution from the non-stretched M1 part, although we do have a large uncertainty in the $a_4$ parameter. Nevertheless, assuming a

† Transitions are called stretched for a maximum change in the angular momentum of the nuclear states, and non-stretched if the change is less than the maximum allowed for the given multipolarity.
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pure $M1$ transition, one finds $a_2^{\text{max}} = 0.400$, which is close to the experimental value of 0.35(5). We note that all $a_4$ parameters are consistent with 0 within their error bars.

For the upbend, we have fitted equation 12 to the primary spectra for the range $E = 5.4 - 7.6 \text{ MeV}$ and $E_\gamma = 1.4 - 3.6 \text{ MeV}$ with $a_2$ and $a_4$ as free parameters, obtaining $a_2 = 0.11(6)$ and $a_4 = -0.06(6)$ (see figure 13d). The uncertainty in $a_4$ is very large, but its value is small, indicating that contributions from stretched $E2$ transitions are not dominant. Moreover, we have made a fit of the data to the sum of Legendre polynomials for $J_i = 3/2 - 11/2$, with a weighting coefficient for the stretched and the non-stretched part. Here, we obtain 65(12) and 35(6)% for the non-stretched and the stretched transitions, respectively. Note that possible contributions from other spins and $E2$ transitions could modify these numbers, which should only be taken as a qualitative guidance.

However, when we fit only the sum of the stretched $E2$ Legendre polynomials for $J_i = 3/2 - 11/2$, we find a significantly worse agreement, see figure 13d. This is true also for the fit including the maximum experimental attenuation of $\approx 0.60$. Fitting a sum of stretched and non-stretched dipoles and stretched $E2$ transitions yields a fit similar to that of only the attenuated $E2$s. Also a fit of only stretched dipole transitions is clearly not reproducing the data. The best fit (more than a factor of 5 better $\chi^2$) is obtained with a sum of stretched and non-stretched dipole transitions, although small $E2$ contributions e.g. from $M1 + E2$ mixing cannot be ruled out.

To study the angular-distribution coefficients for the upbend in $^{57}$Fe in more detail, we make individual fits of equation 12 to eight 300-keV wide excitation-energy cuts in the primary $\gamma$-ray matrix in the range $E = 5.4 - 7.6 \text{ MeV}$, $E_\gamma = 1.4 - 3.6 \text{ MeV}$. The resulting $a_2$ and $a_4$ coefficients are shown in figure 14. We obtain $a_2 = 0.10(2)$ and $a_4 = -0.05(2)$, in excellent agreement with the simultaneous fit to the whole region as shown in figure 13d.

The same trend was found in theoretical $\langle B(M1) \rangle$ values from shell-model calculations of $^{57}$Fe [20], where non-stretched $M1$ transitions contributed most to the low-energy enhancement. Also, stretched $M1$ transitions dominated both experimentally [16] and theoretically [20] in the case of $^{56}$Fe, bringing together a consistent picture, at least qualitatively. Hence, we conclude that the upbend structure in $^{57}$Fe is also caused by dipole transitions, but for this case the non-stretched transitions seem to dominate.

5. Generalized Brink-Axel hypothesis: γSF as function of excitation energy

As the LaBr$_3$ detectors have excellent energy resolution and efficiency for high-energy γ rays, we make use of the technique described in [11, 21, 66] to extract the γSF as function of excitation energy.

We start with the primary $\gamma$-ray matrix $P(E_\gamma, E)$ obtained in section 2. We will now make the assumption that the NLD is the one determined in section 3, but the transmission coefficient $T$ is now allowed to be dependent on both excitation energy
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Figure 14. Extracted $a_2$ and $a_4$ coefficients from independent fits of 300-keV excitation-energy cuts in the $^{57}$Fe primary matrices for the six CACTUS angles.

and γ-ray energy, $T(E_\gamma, E)$. As $\rho(E_f)$ is known, we can in principle determine $T(E_\gamma, E)$ for each excitation-energy bin just by dividing the primary γ matrix with the NLD: $T(E_\gamma, E) \sim P(E_\gamma, E)/\rho(E_f)$, using our ansatz in equation 2. Specifically, we have

$$\rho(E - E_\gamma)T(E_\gamma, E) = N(E)P(E_\gamma, E),$$

(13)

where $N(E)$ is a normalization factor in units MeV$^{-1}$, depending only on the initial excitation energy.

Now, this game can be played in two ways:

(a) We investigate $T$ as function of initial excitation energy through the relation

$$T(E_\gamma, E) = N(E) \frac{P(E_\gamma, E)}{\rho(E - E_\gamma)}.$$  (14)

We determine $N(E)$ by

$$N(E) = \frac{\int_0^E T(E_\gamma)\rho(E - E_\gamma) dE_\gamma}{\int_0^E P(E_\gamma, E) dE_\gamma}.$$  (15)

Note that $T(E_\gamma)$ is the normalized transmission coefficient from section 3. However, it will not influence the shape of the extracted $T(E_\gamma, E)$ as it acts as a constant after integrating over all $E_\gamma$. Hence, it only serves to provide an approximate absolute normalization of $T(E_\gamma, E)$.

(b) We can also find $T$ as function of final excitation energy by

$$T(E_\gamma, E_f) = N(E_\gamma + E_f) \frac{P(E_\gamma, E_f + E_\gamma)}{\rho(E_f)},$$  (16)

where we keep in mind that $E_f + E_\gamma = E$. Again, we assume that $T(E_\gamma)$ gives a good estimate of the absolute value and we can approximate the normalization for
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![Graph](image_url)

Figure 15. (Color online) Extracted $\gamma$SFs as function of initial excitation energy for $^{56}$Fe. Bins are 248 keV/channel for $E$ and 124 keV/channel for $E_\gamma$.

For a given final excitation energy $E_f$ and for a specific $E_\gamma$ fulfilling $E = E_f + E_\gamma$ by

$$N(E_\gamma + E_f) = \frac{\int_0^{E_f + E_\gamma} T(E'_\gamma) \rho(E_f) dE'_\gamma}{\int_0^{E_f + E_\gamma} P(E'_\gamma, E_f + E'_\gamma) dE'_\gamma}. \tag{17}$$

The $\gamma$SF as function of excitation energy is then easily calculated from the transmission coefficient by use of equation 10. The results are shown for $^{56,57}$Fe in figures 15 and 16 respectively.

We observe that the decay strength to the ground state increases as function of both $E$ and $E_\gamma$, which is fully consistent with the $\gamma$SF determined previously in section 3 and the expected influence of the tail from the GDR. Moreover, we find that the $\gamma$SF varies with initial excitation energy, but that the general shape is preserved: there is always an upbend at low $E_\gamma$ and a rather flat distribution of strength in the middle $E_\gamma$ region, before it again increases for high $E_\gamma$.

To investigate the fluctuations, following [66], we compare the average $\gamma$SF for all initial excitation energies with the $\gamma$SF obtained for a specific excitation-energy bin. We find that the fluctuations relative to the average $\gamma$SF can be large, more than 100% for some $\gamma$-ray energies and $E$. Also, the fluctuations are in some cases significantly larger than the error bars. Therefore, it seems that although the overall shape of the
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$\gamma$SF is indeed preserved in agreement with the generalized Brink-Axel hypothesis, the $\gamma$SF for a specific transition energy and excitation energy could have a large deviation, in particular when the excitation-energy bin is narrow and containing rather few levels.

Finally, we also investigate the $\gamma$SF for a specific final excitation energy. We have chosen the ground state in $^{56}$Fe and the ground-state band ($1/2^-$, $3/2^-$, $5/2^-$) in $^{57}$Fe. The $\gamma$SF for this $E_f$ is then compared to a typical $\gamma$SF at a high initial $E$, see figures [17 and [18]. Again, we observe that the general trend is preserved, although significant deviations are present, for example for the $^{56}$Fe strength at $E_\gamma \approx 9.7$ MeV. This is interpreted to be caused by Porter-Thomas fluctuations, which are expected to be large when the final and/or the initial NLD is low [21].

6. Summary and outlook

We have presented data on $^{56,57}$Fe from $(p,p'\gamma)$ reactions using NaI and LaBr$_3$ crystals simultaneously. We confirm the upbend in these isotopes, which represents an increase in strength of a factor $\approx 3 - 30$ relative to commonly used models for the $E1$ strength.
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Figure 17. Extracted $\gamma$SF for the $^{56}$Fe ground state as $E_f$ (black points) and for $E = 10.9$ MeV.

Figure 18. Extracted $\gamma$SF for the $^{57}$Fe ground-state band, $E_f = 0.0 - 0.25$ MeV (black points) and for $E_i = 7.3$ MeV.

at low transition energies. Moreover, external data involving heavier projectiles and/or
ejectiles are typically within the error bars. The lower NLD in $^{56}$Fe for $E \approx 5 - 7.5$ MeV compared to the external data is attributed to the lower spins reached in the $(p, p')$ reaction.

We have shown angular distributions of the upbend for $^{57}$Fe for the first time. Our results indicate a mix of stretched and non-stretched dipoles contributing to the upbend, in agreement with recent shell-model calculations. Moreover, we have investigated the excitation-energy dependence of the $\gamma$SF. The data show that the general trends are preserved in accordance with the Brink-Axel hypothesis. However, we also encounter large fluctuations, which seem to be due to strong Porter-Thomas fluctuations caused by the low level density in these light nuclei.

Currently, the CACTUS array is in the process of being replaced by OSCAR (Oslo SCintillator ARRay), for which all the NaI detectors are replaced with 3.5 in. × 8 in. LaBr$_3$(Ce) detectors, and a new frame and target chamber are being built as well. This new array will open up a wealth of new opportunities, such as discriminating against neutrons above $S_n$ to extract the $\gamma$SF at even higher energies, gating on discrete transitions to study the feeding pattern and thus spin dependencies of the NLD and $\gamma$SF, and many more. We expect to be able to study the upbend and Porter-Thomas fluctuations in much more detail with this new equipment.

Comparing our data with frequently used NLD and $\gamma$SF models clearly shows the need for better theoretical predictions. After all, here we present data on stable nuclei; the predictive power for unstable, highly exotic nuclei involved in e.g. the nucleosynthesis is definitely unsatisfactory. To improve the situation, more data on both stable and unstable nuclei are required to help testing and constraining available calculations, as well spurring new theoretical approaches and methods in nuclear physics.

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Appendix A. Data tables for normalizations

Table A1. Neutron resonance parameters $D_0$ and $\langle \Gamma_{\gamma 0} \rangle$ from [38], and spin cutoff parameters from global systematics of [40]: $A_f$ is the final nucleus following neutron capture, $J_t$ is the ground-state spin of the target nucleus, $S_n$ is the neutron-separation energy, $D_0$ is the s-wave level spacing [38], $\sigma$ is the spin-cutoff parameter from equation [8], $Pd'$ is the deuteron shift as defined in [40], and $\rho(S_n)$ is the total level density calculated from equation [5]. Finally, $\rho_{\text{syst}}$ is the total level density at $S_n$ as predicted from the global systematics of [40]. †Estimated from systematics.

| $A_f$ | $J_t$ | $S_n$ (MeV) | $D_0$ (keV) | $\sigma(S_n)$ (MeV) | $Pd'$ (MeV) | $\rho(S_n)$ ($10^3$ MeV$^{-1}$) | $\rho_{\text{syst}}(S_n)$ ($10^4$ MeV$^{-1}$) | $\langle \Gamma_{\gamma 0} \rangle$ (meV) |
|-------|-----|---------|---------|----------------|---------|------------------|------------------|------------------|
| 55Fe  | 0   | 9.298   | 20.5(14)| 3.41           | 0.463   | 1.19(9)          | 1.28             | 1600(700)         |
| 56Fe  | 3/2 | 11.197  | 3.36(124)† | 3.47           | 2.905   | 2.18(59)†        | 2.94             | 1900(600)†        |
| 57Fe  | 0   | 9.298   | 25.4(22) | 3.35           | 0.211   | 0.926(80)        | 1.14             | 920(410)          |
| 58Fe  | 1/2 | 10.044  | 7.05(70) | 3.44           | 2.874   | 1.81(18)         | 3.49             | 1850(500)         |
| 59Fe  | 0   | 6.581   | 21.6(26) | 3.30           | 0.470   | 1.06(13)         | 1.01             | 1130(110)         |
Table A2. Neutron resonance parameters $D_0$ from [38], and spin cutoff parameters from global systematics of [41]: $A_f$ is the final nucleus following neutron capture, $J_i$ is the ground-state spin of the target nucleus, $S_n$ is the neutron-separation energy, $\sigma$ is the spin-cutoff parameter from equation (7), $D_0$ is the $s$-wave level spacing [38], $\alpha$ and $E_1$ are the level density parameter and energy shift from [41], and $\rho(S_n)$ is the total level density calculated from equation (5). Finally, $\rho_s$ is the total level density at $S_n$ as predicted from the global systematics of [41]. $^\dagger$ Estimated from systematics.

| $A_f$ | $J_i$ | $S_n$ (MeV) | $D_0$ (keV) | $\sigma(S_n)$ (1/MeV) | $\alpha$ (MeV) | $E_1$ (MeV) | $\rho(S_n)$ (10$^3$ MeV$^{-1}$) | $\rho_s$ (10$^3$ MeV$^{-1}$) |
|-------|-------|-------------|-------------|---------------------|--------------|-----------|----------------|----------------|
| $^{55}$Fe | 0     | 9.298       | 20.5(14)    | 4.02                | 5.817        | -0.524    | 1.62(11)      | 2.00            |
| $^{56}$Fe | 3/2   | 11.197      | 3.30$^{+0.9}_{-0.6}$ | 4.05               | 6.196        | 0.942     | 2.87(68)      | 4.22            |
| $^{57}$Fe | 0     | 9.298       | 25.4(22)    | 3.83                | 6.581        | -0.523    | 1.20(10)      | 1.62            |
| $^{58}$Fe | 1/2   | 10.044      | 7.05(70)    | 3.93                | 6.936        | 0.942     | 2.32(23)      | 4.66            |
| $^{59}$Fe | 0     | 6.581       | 21.6(26)    | 3.70                | 7.297        | -0.424    | 1.32(16)      | 1.38            |

Table A3. Parameters for the constant-temperature interpolation for the different normalization options. Both parameters $T$ and $E_0$ are given in MeV.

| Nucleus | Lower $T$ | Middle $T$ | Upper $T$ | Lower $E_0$ | Middle $E_0$ | Upper $E_0$ | Lower $T$ | Middle $T$ | Upper $T$ | Lower $E_0$ | Middle $E_0$ | Upper $E_0$ |
|---------|-----------|-----------|-----------|-------------|--------------|-------------|-----------|-----------|-----------|-------------|--------------|-------------|
| $^{56}$Fe | 1.41      | 0.320     | 1.40      | -0.034      | 1.38         | -0.169      | 1.40      | -0.070    | 1.35      | 0.045       | 1.30         | 0.232       |
| $^{57}$Fe | 1.32      | -1.618    | 1.30      | -1.575      | 1.29         | -1.601      | 1.31      | -1.882    | 1.29      | -1.829      | 1.28         | -1.848      |

Appendix B. More details for the angular distributions

For the Legendre polynomials we have

$$P_2(\cos \theta) = \frac{1}{2} \left[ 3(\cos \theta)^2 - 1 \right],$$

$$P_4(\cos \theta) = \frac{1}{8} \left[ 35(\cos \theta)^4 - 30(\cos \theta)^2 + 3 \right].$$

In the case of a fully aligned state with respect to the beam direction ($\alpha_k = 1$), the $a_k^{\text{max}}$ coefficients are given by [63]

$$a_k^{\text{max}}(J_i LL' J_f) = \frac{B_k}{1 + \delta^2} \left[ F_k(J_f LL J_i) + 2\delta F_k(J_f LL' J_i) + \delta^2 F_k(J_f LL' L' J_i) \right].$$

Here, $J_i, J_f$ are the spins of the initial and final level, $L, L'$ are transition multipoarities, $\delta$ is the mixing ratio between the multipoarities defined according to [64]:

$$\delta = \frac{\langle J_f || E(L + 1) || J_i \rangle}{\langle J_f || M(L) || J_i \rangle}.$$  

Here, $E(L + 1)$ is the electric transition operator for multipolarity $L + 1$, and $M(L)$ is the magnetic transition operator for multipolarity $L$. Further, the $B_k, F_k$ coefficients are defined in [63], where also values for the product $B_k F_k$ are tabulated.
We investigate known transitions in $^{57}$Fe, such as the 692-keV $\gamma$ ray decaying from the level at 706 keV, where $J_i = 5/2^-$ and $J_f = 3/2^-$, and the transition is known to be of $M1 + E2$ type with a mixing ratio $\delta = -0.465$. We get

$$a_2^{\text{max}} = \frac{1}{1 + 0.465^2} \left[ B_2 F_2(3/2, 1, 1, 5/2) + 2 \cdot (-0.465) \cdot B_2 F_2(3/2, 1, 2, 5/2) + 0.465^2 B_2 F_2(3/2, 2, 2, 5/2) \right].$$

From [63] we have $B_2 F_2(3/2, 1, 1, 5/2) = -0.400$, $B_2 F_2(3/2, 1, 2, 5/2) = 1.014$, and $B_2 F_2(3/2, 2, 2, 5/2) = 0.204$, giving $a_2^{\text{max}} = -1.068$. For $a_4^{\text{max}}$, we find

$$a_4^{\text{max}} = \frac{1}{1 + 0.465^2} \left[ 0.465^2 B_4 F_4(3/2, 2, 2, 5/2) \right];$$

with $B_4 F_4(3/2, 2, 2, 5/2) = 0.653$, we get $a_4^{\text{max}} = 0.116$. Similarly, we get for an $E2$ transition with $J_i = 9/2^-, J_f = 5/2$ and no mixing ($\delta = 0$), $a_2^{\text{max}} = 0.476$ and $a_4^{\text{max}} = -0.286$. 