The Origin of Proton Mass from $J/\Psi$ Photo-production Data

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The trace of the stress tensor characterizes the transformation of a theory under rescaling. In quantum chromodynamics (QCD), this trace contains contributions from the bare masses of the quarks and also from a purely quantum effect, called the QCD trace anomaly. It affects all masses in the theory. We present an estimation of the QCD trace anomaly from the near threshold $J/\Psi$ photo-production data of the GlueX experiment. We apply a vector meson dominance model to describe the photo-production of the $J/\Psi$ meson and a running strong coupling which includes nonperturbative effects in the low $\mu^2$ region. Despite large uncertainties, we find that the experimental data favors a small trace anomaly of $b = 0.07 \pm 0.17$. We report the resulting proton mass decompositions at $\mu^2 = 0.41 \text{GeV}^2$ and $\mu^2 = 4 \text{GeV}^2$.

I. INTRODUCTION

Most of the mass of the observable Universe is contained in its protons and neutrons, collectively called nucleons. However the origin of the nucleon mass is still largely not understood. The Higgs mechanism provides a relatively modest contribution. Nearly all of the rest arises from various interactions contained in QCD. QCD contains three ingredients. First, the kinetic energy and self-interactions of gluons are described by Yang-Mills theory [1]. Next, the matter is described by quarks [2].

With these two ingredients, the theory leads to divergent results. To remove these divergences, it must be regularized and renormalized. In Nobel Prize winning work this was done by ’t Hooft and Veltman [3] nearly 50 years ago, who found that an additional piece, known as the regulator, needs to be added to the theory in the process of removing the divergences. The final theory is defined by taking a certain limit of the regulator in which it does not vanish, but rather contributes a finite amount to various physical quantities. Its contribution to the trace of the stress tensor is called the trace anomaly. The stress tensor contains the Hamiltonian, which is the operator which measures the energy of a state and so determines the rest masses of all particles. Therefore the trace anomaly contributes to the masses of all particles in the theory.

The mass of a nucleon $M_N$ is decomposed into four terms: the quark mass contribution $M_m$, the quark energy contribution $M_q$, the gluon energy contribution $M_g$, and the trace anomaly contribution $M_a$ [4, 5]. The dependences of these terms on the QCD trace anomaly $b$, and the momentum fraction $a$ of the quarks at energy scale $\mu$ are [5],

$$M_q = \frac{3}{4} \left(a - \frac{b}{1 + \gamma_m}\right) M_N,$$
$$M_g = \frac{3}{4} (1 - a) M_N,$$
$$M_m = \frac{4 + \gamma_m}{4(1 + \gamma_m)} b M_N,$$
$$M_a = \frac{1}{4} (1 - b) M_N,$$

where $\gamma_m$ is the quark mass anomalous dimension, which can be calculated in perturbative QCD [6]. In searching a physical interpretation of the mass decomposition, a new proton mass decomposition is proposed [7], with a semiclassical picture viewing a proton as a hydrodynamical system of pressure. In the new mass decomposition, the pressure effect is considered, and there are only the internal energy terms of quarks and gluons.

The momentum fraction, $a$, is defined to be

$$a(\mu^2) = \sum_f \int_0^1 x[q_f(x, \mu^2) + \bar{q}_f(x, \mu^2)] dx$$

in terms of parton distribution functions $q_f$. Thanks to decades of worldwide efforts on both experimental and theoretical fronts, parton distribution functions have been well determined using a global analysis of deep inelastic scattering data. To complete the proton mass decomposition, only the trace anomaly $b$ remains poorly constrained by the experimental data.

Although the proton mass is mainly generated from chiral symmetry breaking, a small contribution does arise from the masses of the quarks. The quark mass contribution is determined from the scalar condensates and the quark masses themselves, which are related to the QCD trace anomaly $b$ via

$$b M_N = \langle N | m_u \bar{u} u + m_d \bar{d} d | N \rangle + \langle N | m_s \bar{s} s | N \rangle = m_l \langle N | \bar{u} u + \bar{d} d | N \rangle + m_s \langle N | \bar{s} s | N \rangle$$
$$= \Sigma_{\pi N} + \Sigma_{\rho N}.$$
The scalar condensates of the nucleon vacuum is vital in the searching for a weakly-interacting massive particle, one type of the dark matter candidates, since they are the important parameters for the calculation of the scattering between the nucleon and the dark matter particle [8][10]. The scalar nucleon matrix element of up and down quarks, \(\Sigma_{\pi N}\), is about 45 MeV as has been determined from the low energy \(\pi - N\) scattering amplitude [11][13] and with more data from the pionic-atom spectroscopy [14][15]. Less is known about the strange \(\Sigma\) term \(\Sigma_{sN}\). It is believed to be large as the mass of the strange quark is large. However, recent Lattice QCD calculations find that \(\Sigma_{sN}\) is comparable to \(\Sigma_{\pi N}\). In particular, the QCDSF Collaboration finds \(\Sigma_{sN} = 11 \pm 13\) MeV [16] and the \(\chi\)QCD Collaboration finds \(\Sigma_{sN} = 40 \pm 12\) MeV [17]. The strange \(\Sigma\) term is also suggested to be small around 16 MeV from an effective field theory [15].

In summary, the QCD trace anomaly \(b\) is an essential ingredient in the nucleon mass and in particular in the strange quark contribution. The goal of the present work is to report a new and relatively powerful experimental constraint on \(b\) and to describe how it affects our picture of the origin of the proton’s mass.

Low energy scattering between heavy quarkonium and a nucleon can be used to probe properties of the nucleon. It is not difficult to compute the scattering amplitude using the operator production expansion. In the quarkonium rest frame, if the incident nucleon energy is much smaller than the binding energy between the quark-antiquark pair inside the quarkonium, then the leading twist gluon field operator alone already provides a good approximation [18]. In the non-relativistic domain, the heavy quarkonium is mainly sensitive to the chromo-electric part of the gluon field of the nucleon. This is because the velocity of the heavy quark inside of the heavy quarkonium is small and the chromo-magnetic part is suppressed by powers of velocity. Hence the heavy quarkonium-nucleon scattering amplitude is determined by the strength of the nucleon color field and the quark terms in the energy-momentum tensor [15].

Y. Hatta, A. Rajan, and D.-L. Yang had tried to extract the trace anomaly based on a holographic QCD framework, from the \(J/\Psi\) photo-production data near threshold [19][20]. However their result shows that the trace anomaly is loosely constrained by the current data. It is worthwhile and necessary to look for other reliable and well-acknowledged theories to extract the QCD trace anomaly. In this work we will extract the QCD trace anomaly \(b\) from the scattering amplitude between charmonium and nucleons, using the theoretical framework [15][21] based on a Vector-Meson-Dominance (VMD) model.

II. FORWARD CROSS-SECTION OF \(J/\Psi\) PHOTOPRODUCTION ON PROTONS

We will perform our analysis using the differential cross-section data recently published by GlueX Collaboration at Jefferson Laboratory [22]. The cross-section data is on exclusive \(J/\Psi\) photo-production using a real bremsstrahlung photon of 10.72 GeV energy on average. At this energy, \(t_{\text{min}}\) of the exclusive reaction is -0.4361 GeV\(^2\). The differential cross-section as a function of \(-t\) is shown in Fig. 1. The data is fit with an exponential function \((d\sigma/dt = d\sigma/dt|_{t=0} \times e^{-kt})\) which describes the \(-t\)-dependence of the cross-section. \(d\sigma/dt|_{t=0}\) is found to be \(3.8 \pm 1.4\) nb/GeV\(^2\), and the exponential slope \(k\) is found to be \(-1.67 \pm 0.38\) GeV\(^{-2}\). Errors are determined by fixing \(\chi^2 - \chi_{\text{best fit}}^2 = 1\).

![FIG. 1. The differential cross section of \(J/\Psi\) photoproduction near threshold as measured by the GlueX Collaboration [22]. Only statistical uncertainties are shown.](image)

The dependence of the forward cross-section \(d\sigma/dt|_{t=0}\) on the QCD trace anomaly \(b\) is discussed in Sec. III. Comparing the measured forward cross-section to the \(b\)-dependent prediction yields a fit for \(b\).

III. VMD MODEL AND \(J/\Psi\) NEAR THRESHOLD PHOTOPRODUCTION

In the VMD model, the forward cross-section for \(J/\Psi\) photo-production on a nucleon is [21],

\[
\frac{d\sigma_{J/\Psi \rightarrow J/\Psi N}}{dt} \bigg|_{t=0} = \frac{3\Gamma(J/\Psi \rightarrow e^+ e^-)}{\alpha m_{J/\Psi}} \left( \frac{k_{J/\Psi N}}{k_{e^+ e^-}} \right)^2 \frac{d\sigma_{J/\Psi \rightarrow J/\Psi N}}{dt} \bigg|_{t=0}, \tag{4}
\]

where \(k^2_{ab} = [s - (m_a + m_b)^2][s - (m_a - m_b)^2]/4s\) denotes the squared center of mass momentum of the corresponding reaction, and \(\Gamma\) stands for the partial decay width of
the $J/\Psi$. The center of mass energy $\sqrt{s}$ is 4.58 GeV with an incident photon of 10.72 GeV. The decay width of the $J/\Psi$ to an electron-positron pair is 5.547 keV [23]. $\alpha$ is the fine structure constant.

The differential cross-section of the $J/\Psi-N$ interaction is

$$
\frac{d\sigma_{J/\Psi N \to J/\Psi N}}{dt}|_{t=0} = \frac{1}{64\pi} \frac{1}{m_{J/\Psi}^2} |F_{J/\Psi N}|^2,
$$

where $F_{J/\Psi N}$ denotes the invariant $J/\Psi-N$ scattering amplitude, and $\lambda = (p_{NP}/m_{J/\Psi})$ is the nucleon energy in the charmonium rest frame [21]. At low energies, the amplitude takes the form [18].

$$
F_{J/\Psi N} \simeq r_0^2 d_2 \frac{2 \pi^2}{27} \left( 2 M_N^2 - \sum_{i=u,d,s} m_i q_i |N_i\rangle \right)
\simeq r_0^2 d_2 \frac{2 \pi^2}{27} (2 M_N^2 - 2 b M_N^2)
\simeq r_0^2 d_2 \frac{2 \pi^2}{27} 2 M_N^2 (1 - b).
$$

The sum of the $\Sigma$ terms of the quarks is connected to the trace anomaly $b$. Note that Kharzeev uses a relativistic normalization of hadron states $\langle N|N\rangle = 2 M_N V$, where $V$ is a normalization volume [18]. In the chiral limit and low energy scattering, the mass of a hadron state comes purely from the quantum fluctuations of gluons. The “Bohr” radius $r_0$ of the charmonium in Eq. (6) is given by,

$$
r_0 = \left( \frac{4}{3\alpha_s} \right) \frac{1}{m_c}.
$$

The Wilson coefficient $d_2$ in Eq. (6) is defined as [18] [24],

$$
d_2^{(1S)} = \left( \frac{32 N_c}{\pi} \right)^2 \sqrt{\frac{\pi}{\Gamma(n+\frac{2}{3})}} \frac{\Gamma(n+\frac{2}{3})}{\Gamma(n+5)},
$$

where $N_c$ is the number of colors. The renormalization scale $\mu^2$ is taken to be the “Rydberg” energy squared $\epsilon_0^2$ of the bound state of the heavy quark-antiquark pair [18] [25], which is defined by,

$$
\epsilon_0 = \left( \frac{3\alpha_s}{4} \right)^2 m_c.
$$

**IV. STRONG COUPLING CONSTANT AND THE CHARM QUARK MASS**

To avoid the Landau pole of the running strong coupling constant at low $\mu^2$, we fix $\alpha_s$ using the analytic approach of Ref. [26], in which QCD nonperturbative effects are folded into the coupling. The resulting analytic expression for $\alpha_s$ is

$$
\alpha_s^{\text{ana}} = \frac{4\pi}{\beta_0} \left( \frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{Q^2 - \Lambda^2} \right).
$$

The term $\Lambda^2/(Q^2 - \Lambda^2)$ is a nonperturbative power law contribution, which cancels the Landau pole at $\Lambda^2$.

The QCD running coupling constant can be determined from a global analysis of the deep inelastic scattering data in a wide $Q^2$ range, such as GRV98 [27] and CT14 [28]. One finds $\Lambda^2 = 0.0643$ GeV$^2$ in Eq. (10) by matching the CT14(LO)’s $\alpha_s$ at the charm quark threshold $m_c^2 = 1.69$ GeV$^2$. Using the CT14(LO) analysis, $\alpha_s$ is 0.3719 at 1.69 GeV$^2$. The analytic coupling $\alpha_s^{\text{ana}} = 0.3612$ also matches the GRV98(LO)’s $\alpha_s = 0.3626$, at the chosen charm quark threshold of 1.96 GeV$^2$ according to GRV. Fig. 2 shows the strong running coupling constant $\alpha_s$ as a function of $\mu^2$.

The world average charm quark mass is 1.27 GeV according to the Particle Data Group [23]. The charm quark mass is 1.3 GeV and 1.4 in the CT14(LO) analysis [28] and the GRV98(LO) analysis [27] respectively. In our analysis, these values of the charm quark mass are all used as different QCD input parameters.

**V. TRACE ANOMALY AND PROTON MASS DECOMPOSITION**

The strong coupling constant $\alpha_s$ depends on the energy scale $\mu^2 = \epsilon_0^2$. At the same time, in Eq. (1), the energy scale $\epsilon_0$ also depends on the strong coupling $\alpha_s$. Therefore $\epsilon_0^2$ is fixed by a combination of Eq. (6) and the running strong coupling formula shown in Eq. (10). With different charm quark masses, the obtained energy scales $\epsilon_0^2$ and $\alpha_s$ are shown in Table I. With these parameters, we calculate the “Bohr” radius $r_0$, and finally obtain the
QCD trace anomaly $b$ applying Kharzeev’s method based on the VMD model described in Sec. [11]. The resulting values of the QCD trace anomaly $b$ are listed in Table [11]. Note that the formulas in Sec. [11] are written in natural units. The relevant conversion in the comparison of the theoretical forward cross-section with the experimental data is $1 \text{ GeV}^{-2} = 3.881 \times 10^9 \text{ nb}$.

TABLE I. Values of the charm quark mass $m_c$, “Rydberg” energy squared of heavy quarkonium $\epsilon_0^2$, the strong coupling constant $\alpha_s$, the “Bohr” radius, the QCD trace anomaly $b$, and the total momentum fraction carried by quarks. The reported uncertainty on $b$ reflects only the statistical uncertainties of the cross-section data. $a$ is calculated using Eq. (2) with dynamical parton distribution functions [29].

| $m_c$ [GeV] | $\epsilon_0^2$ [GeV$^2$] | $\alpha_s(\mu^2 = \epsilon_0^2)$ | $r_0$ [fm] | $b$ | $a$ |
|-------------|----------------|-----------------|----------|-----|-----|
| 1.27        | 0.0937         | 0.654           | 0.316    | 0.05±0.18 | 0.861 |
| 1.3         | 0.0962         | 0.651           | 0.310    | 0.00±0.19 | 0.858 |
| 1.4         | 0.105          | 0.641           | 0.293    | -0.20±0.22 | 0.846 |

Kharzeev estimated $m_N^2/\epsilon_0^2$ to be 2.1 in Ref. [25], i.e. $\epsilon_0^2 = 0.42 \text{ GeV}^2$. If the “Rydberg” energy $\epsilon_0$ of the $c\bar{c}$ pair is not fixed by Eq. (9), then it is better to calculate $\epsilon_0$ from the experimental data. Thinking about pulling apart a $c\bar{c}$ pair to generate a $D\bar{D}$ pair, a naive estimate of the “Rydberg” energy $\epsilon_0$ is $m_D + m_{\bar{D}} - m_{J/\Psi}$ [18]. Using the masses of the $D$ and $J/\Psi$ mesons, $\epsilon_0^{\text{exp}}$ is calculated to be 0.41 GeV$^2$. Around the low energy scale of 0.41 GeV$^2$, the charm quark mass is near the pole mass 1.67 GeV [23]. At these new “Rydberg” energies, the obtained $\alpha_s$, $r_0$, and $b$ are listed in Table [1]. The $\chi^2 = (d\sigma/dt)^{\text{exp}}_{t=0} - (d\sigma/dt)^{\text{VMD model}}_{t=0})^2 / \left( d\sigma/dt|_{t=0}^{\text{exp}} \right)^2$ as a function of $b$ is shown in Fig. 3. If we have forward cross-section data at several photon energies near the threshold, the combined uncertainty of the extracted trace anomaly can be reduced.

FIG. 3. $\chi^2$ as a function of the QCD trace anomaly using the $J/\Psi$ photo-production data at $E_\gamma \sim 10.72 \text{ GeV}$. In the VMD model, $\epsilon_0^2 = 0.41 \text{ GeV}^2$, $\alpha_s(\epsilon_0^2) = 0.494$, and $m_c(\epsilon_0^2) = 1.67 \text{ GeV}$ are applied as inputs. It is 0.541 at $\mu^2 = 4 \text{ GeV}^2$ [29]. In principle if the trace anomaly $b$ is scale-invariant, we can obtain the proton mass decomposition at any scale $\mu^2$. The proton mass decompositions at $\mu^2 = 0.41 \text{ GeV}^2$ and $\mu^2 = 4 \text{ GeV}^2$ are shown in Fig. 4 for illustration. In our calculations, the quark mass anomalous dimensions at 0.41 GeV$^2$ and 4 GeV$^2$ were evaluated to be -0.691 and -0.315, respectively. Note that the anomalous dimension $\gamma_m$ defined by Ji [14, 15] is $\gamma_m = \mu^2 d\ln(\mu^2 M_N^2)/d\mu^2$, which is twice of the definition in Ref. [6]. A Lattice QCD calculation gives $M_q = (0.33 \pm 0.04)M_N$, $M_g = (0.37 \pm 0.05)M_N$, $M_s = (0.23 \pm 0.01)M_N$, and $M_{\Sigma} = (0.09 \pm 0.02)M_N$ at $\mu^2 = 4 \text{ GeV}^2$ [30]. Our result on the proton mass decomposition is close to the result from the Lattice QCD simulation.

VI. DISCUSSIONS AND SUMMARY

We estimate the strange $\Sigma$ term to be around 21 MeV, with the QCD trace anomaly $b = 0.07$ extracted from the current data of near-threshold $J/\Psi$ photo-production and the $\Sigma_{N}$ term determined from experimental measurements of $\pi$-N scattering [11]. The estimated $\Sigma_{N}$ term is small based on our analysis. More precise data on near-threshold heavy quarkonium photo-production and higher order theoretical corrections are needed. Nonetheless, the current $\Sigma_{N}$ term extracted is more or less consistent with the results of Lattice QCD (11 ± 13 MeV and 40 ± 12 MeV) [16, 17] and the phenomenological result [15]. We find that the sum of the $\Sigma$ terms is $bM_N = 66 \text{ MeV}$. This lies between the Lattice QCD predictions 49 ± 25 MeV [16] and 86 ± 19 MeV [17] from two independent collaborations.

The trace anomaly is very sensitive to the parameter $r_0$, based on the VMD model adopted in this analysis. In theory, the “Bohr” radius $r_0$ depends on the two key QCD inputs – the strong coupling constant and
the charm quark mass. Therefore we should look for more experimental constraints on the “Bohr” radius \( r_0 \) of the heavy quark-antiquark pair, the strong coupling \( \alpha_s \), and the charm quark mass \( m_c \). Higher-twist calculations and the systematic uncertainty of the model should be investigated as well. So far, the uncertainty of the trace anomaly extracted in this work is quite large.

More statistics are needed on the experimental side for the trace anomaly. A VMD model is used [18]. \( \epsilon_0^2 \) is estimated to be \((m_B + m_{\bar{B}} - m_{\Upsilon(1S)})^2\). The charm quark mass is even smaller, compared with the charm quark mass \( m_c \). Hence the systematic uncertainty of the model is even smaller, compared with the charm quark mass \( m_c \). The charm quark mass is much smaller at a higher binding energy \( \epsilon \). The magnetic contribution to the \( \Upsilon(1S) \)-N interaction is even smaller as the velocity of the bottom quark inside \( \Upsilon(1S) \) is even smaller, compared with the \( J/\Psi \)-N. Hence the theoretical framework in this work is more suitable for near threshold \( \Upsilon(1S) \) photo-production.

Using the VMD model, Kharzeev also calculated the forward cross-section for \( J/\Psi \) photo-production in terms of the total cross-section for \( J/\Psi - N \) scattering [21]:

\[
\frac{d\sigma_{N\to J/\Psi N}}{dt}\bigg|_{t=0} = \frac{3\Gamma(J/\Psi \to e^+e^-)}{\alpha m_{J/\psi}} \times \frac{[s-(m_N+m_{J/\psi})^2][s-(m_N-m_{J/\psi})^2]}{16\pi(s-m_N^2)^2} \times (1+\rho^2)(\sigma_{J/\Psi N}^{tot})^2,
\]

where \( s \) is the center-of-mass energy squared of \( \gamma N \), and \( \rho \) is the ratio of the real part to the imaginary part of the amplitude \( M_{J/\psi N} \). The value of \( \rho \) is unknown, Sibirtsev et. al. suggest that it be set to zero as a first approximation [32]. In the following calculations, we assume \( \rho = 0 \).

With the exploitation of the QCD sum rules [33], the leading approximation to \( \sigma_{J/\Psi N}^{tot} \) can be evaluated using the gluon-\( J/\Psi \) cross-section [21]:

\[
\sigma_{J/\Psi N}^{0}(\lambda) = \frac{8\pi}{9} \left( \frac{32}{3} \right)^2 \frac{1}{\alpha_s m_c^2} \int_{\epsilon_0/\lambda}^1 dx \frac{(x\lambda/\epsilon_0 - 1)^{3/2} g(x, m_c^2)}{(x/\lambda)^5} \frac{1}{x},
\]

where \( \epsilon_0 = 0.642 \) GeV is the “Rydberg” energy, and \( \lambda \) is the nucleon energy in the \( J/\Psi \) rest frame (see Sec. II).

![FIG. 4. Decompositions of proton mass at \( \mu^2 = 0.41 \) GeV\(^2\) and \( \mu^2 = 4 \) GeV\(^2\), with \( b = 0.07 \).](image)

![FIG. 5. The gluon distributions from global analyses, and a simple parametrization](image)

TABLE III. Forward cross-sections for \( \Upsilon(1S) \) photo-production on protons near the threshold, with \( \epsilon_0^2 = 1.21 \) GeV\(^2\), \( \alpha_s(\epsilon_0^2) = 0.397 \) and \( m_b = 4.18 \) GeV. A VMD model is used [18]. \( \epsilon_0^2 \) is estimated to be \((m_B + m_{\bar{B}} - m_{\Upsilon(1S)})^2\).

| \( \sqrt{s_{\gamma N}} \) (GeV) | \( t_{\min} \) (GeV\(^2\)) | \( t_{\max} \) (GeV\(^2\)) | \( r_0 \) (fm) | \( d\sigma_{\Upsilon}/d(s, t = 0) \) (fb/GeV\(^2\)) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 12 GeV          | 0.96 GeV\(^2\)  | 51 GeV\(^2\)   | 0.158 fm        | 86 fb/GeV\(^2\) |
| 14 GeV          | 0.35 GeV\(^2\)  | 104 GeV\(^2\) | 0.158 fm        | 46 fb/GeV\(^2\) |
| 16 GeV          | 0.17 GeV\(^2\)  | 164 GeV\(^2\) | 0.158 fm        | 27 fb/GeV\(^2\) |
global fits \[27, 28\]. Fig. 5 shows various gluon distributions from the global analyses of experimental data and a simple parametrization of a gluon distribution. In Kharzeev’s calculation, he choose the gluon distribution to be \(2.5(1-x)^4\) \[21\]. The IMParton gluon distribution is a pure dynamical gluon distribution which lacks any “valence”-like gluon distribution as a nonperturbative input. The forward cross-sections of the exclusive \(J/\Psi\) photo-production are calculated using these gluon distributions. The results are shown in Table IV. The cross-section with Kharzeev’s gluon distribution is roughly two times smaller than the experimental value 3.8 nb/GeV\(^2\). That is also found by the GlueX Collaboration, and as a result in their publication they multiply the theoretical cross-section by a factor of 2.3 so that it agrees with the total cross-section data \[22\]. Nonetheless, the VMD model is a powerful tool for describing \(J/\Psi\) photo-production, even with the approximation that \(\rho\) is equal to zero.

| \(g(x, m_c^2)\) | \(m_c\) | \(\alpha_s(m_c^2)\) | \(\rho\) | \(d\sigma_{\gamma N}/dt(s,t=0)\) |
|----------------|--------|----------------|-------|------------------|
| IMParton16     | 1.27 GeV | 0.375          | 0     | 0.24 nb/GeV\(^2\) |
| GRV98(LO)      | 1.27 GeV | 0.375          | 0     | 3.41 nb/GeV\(^2\) |
| CT14(LO)      | 1.27 GeV | 0.375          | 0     | 13.6 nb/GeV\(^2\) |
| 2.5(1-x)^4     | 1.27 GeV | 0.375          | 0     | 2.44 nb/GeV\(^2\) |
| IMParton16     | 1.4 GeV  | 0.361          | 0     | 0.16 nb/GeV\(^2\) |
| GRV98(LO)      | 1.4 GeV  | 0.361          | 0     | 2.03 nb/GeV\(^2\) |
| CT14(LO)      | 1.4 GeV  | 0.361          | 0     | 8.11 nb/GeV\(^2\) |
| 2.5(1-x)^4     | 1.4 GeV  | 0.361          | 0     | 1.78 nb/GeV\(^2\) |

Using our calculations, the \(J/\Psi\) photo-production data near threshold also provides an opportunity to differentiate the various gluon distributions on the market. The gluon distribution by CT14(LO) is a little too large while the GRV’s gluon distribution is more or less reasonable, judged by the current data and the simplified calculation in this work. We also find that the variable \(\rho\) is non-zero if there is no “valence”-like gluon distribution in the proton \[29\].

In summary, we have extracted the QCD trace anomaly \(b\) from recently published JLab data. We have provided a new proton mass decomposition and suggested a small strange \(\Sigma_N\) term.

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