On The Gauge-Fixed BRST Cohomology

Marc Henneaux

Physics Department, Université Libre de Bruxelles,
Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium
and
Centro de Estudios Científicos de Santiago,
Casilla 16443, Santiago 9, Chile

August 22, 2021

Abstract

A crucial property of the standard antifield-BRST cohomology at non negative ghost number is that any cohomological class is completely determined by its antifield independent part. In particular, a BRST cocycle that vanishes when the antifields are set equal to zero is necessarily exact. This property, which follows from the standard theorems of homological perturbation theory, holds not only in the algebra of local functions, but also in the space of local functionals. The present paper stresses how important it is that the antifields in question be the usual antifields associated with the gauge invariant description. By means of explicit counterexamples drawn from the free Maxwell-Klein-Gordon system, we show that the property does not hold, in the case of local functionals, if one replaces the antifields of the gauge invariant description by new antifields adapted to the gauge fixation. In terms of these new antifields, it is not true that a local functional weakly annihilated by the gauge-fixed BRST generator determines a BRST cocycle; nor that a BRST cocycle which vanishes when the antifields are set equal to zero is necessarily exact.
1 Introduction

Recently, the gauge-fixed BRST cohomology introduced and studied in [1, 2, 3] has attracted a considerable amount of interest in the context of the Pauli-Villars regularization of the antifield formalism [4, 5, 6, 7, 8]. This cohomology arises after one has made the redefinition of the antifields appropriate to the gauge-fixing procedure.

In analogy with properties that are known to hold in the standard gauge-invariant formulation of the BRST symmetry, one might conjecture that any local functional $A$ annihilated by the gauge-fixed BRST generator determines uniquely a BRST cocycle, which reduces to $A$ when one sets the antifields associated with the gauge-fixed description equal to zero. If true, this conjecture would be quite useful in many quantum calculations, which are effectively performed after the gauge is fixed.

The purpose of this paper is to clarify the structure of the gauge-fixed BRST cohomology in the space of local functionals. We show by means of explicit counterexamples that the above conjecture unfortunately does not hold. We also explain why this is not so. The difficulties arise because the differential that occurs in the gauge-fixed perturbation expansion (Koszul differential associated with the gauge-fixed equations of motion) is not acyclic in the space of local functionals, even at positive antifield number and positive ghost number. This is in sharp contrast with what happens in the usual gauge-invariant formulation, and results from the fact that the ghosts are subject to non trivial equations of motion once the gauge is fixed. Consequently, one can and does meet obstructions in trying to reconstruct the BRST-invariant extension of a given cocycle of the gauge-fixed cohomology. And furthermore, there exists non trivial cocycles that vanish when one sets the redefined antifields equal to zero. This implies that a BRST cohomological class in the space of local functionals is not uniquely determined by its antifield independent part in the gauge-fixed expansion. From the mathematical point of view, the gauge fixed perturbation expansion in the redefined antifields provides an example of homological perturbation theory with a non acyclic differential [9].
2 Standard BRST Cohomology

We first briefly recall some of the salient points of the antifield formalism necessary for the subsequent discussion.

The antifield formulation of the BRST theory \[10, 11\] appears to be one of the most powerful tools for quantizing systems with a gauge freedom \[12, 13, 14, 15, 16\]. In that approach, one replaces the original gauge invariant action \(S_0[\Phi^i]\) depending on the classical fields \(\Phi^i\) by the so-called minimal solution of the master equation,

\[ S = S_0 + \int dx \int dy \Phi^*_i(x) R^i_\alpha(x, y) C^\alpha(y) + \text{“more”} \quad (1) \]

where:

(i) \(\Phi^*_i\) are the antifields associated with the fields \(\Phi^i\);
(ii) \(C^\alpha\) are the ghosts;
(iii) \(\delta_\varepsilon \Phi^i(x) = \int dy R^i_\alpha(x, y) \varepsilon^\alpha(y)\) are the gauge symmetries, under which the action is invariant, \(\delta_\varepsilon S_0 = 0\); the bilocal objet \(R^i_\alpha(x, y)\) is a finite sum of terms, each of which involves \(\delta(x, y)\) or one of its derivatives, \(R^i_\alpha(x, y) = \sum B_{(m)} \partial^{(m)} \delta(x, y)\); the coefficients \(B_{(m)}\) depend on the fields at \(x\), say, and their derivatives up to a finite order;
(iv) “more” in \((1)\) depends on the fields \(\Phi^A \equiv (\Phi^i, C^\alpha)\) and their conjugate antifields \(\Phi^*_A \equiv (\Phi^*_i, C^*_\alpha)\), contains terms proportional to \((\Phi^*_i)^p (\Phi^*_\alpha)^q\) with \(p + 2q > 1\), and is adjusted so that \(S\) solves the classical master equation,

\[ (S, S) = 0. \quad (2) \]

We assume for simplicity that the gauge symmetry is irreducible but our consideration apply equally well to reducible gauge symmetries. We also assume that the reader is familiar with the basic principles of the antifield formalism. Besides the original work quoted above, the reader may consult the references \[3, 9, 17\] for further information. We shall follow in particular the approach of \[3, 9\], based itself on \[1, \! \! 18, \! \! 19\], where the rationale of the antifield formalism is explained from the cohomological point of view. This approach turns out to be crucial for understanding the difficulties associated with the gauge-fixed cohomology.

Because \(S\) solves the classical master equation, the left derivation defined by the equation

\[ sA = (S, A) \quad (3) \]
is a differential,
\[ s^2 = 0, \]  
the so-called BRST differential. One can thus define the BRST cohomological groups \( H(s) \) in the standard manner, as the quotient spaces \( \text{Ker}(s)/\text{Im}(s) \) of the BRST cocycles modulo the BRST coboundaries. Explicitly, the cocycle condition reads
\[ sA = 0 \text{ (s-cocycle condition)}, \]  
while the coboundary condition is
\[ A = sB \text{ (s-coboundary condition)}. \]  

One can actually consider various BRST cohomologies, depending on which functional space \( A \) and \( B \) are required to belong to. The following choices are met in practice:
(i) formal BRST cohomology \( H^\text{formal}(s) \), in which \( A \) and \( B \) are allowed to be arbitrary functionals;
(ii) BRST cohomology \( H^\text{loc.funct.}(s) \) in the space of local functionals, in which \( A \) and \( B \) are required to be local functionals, i.e., integrals of local functions (a local function is a function of the fields \( \Phi^A \), the antifields \( \Phi^*_A \) and a finite number of their derivatives);
(iii) BRST cohomology \( H^\text{local}(s) \) in the space of local functions, in which \( A \) and \( B \) are required to be local functions.

To analyse the BRST cohomology, it is convenient to introduce the antighost number, the pure ghost number and the (total) ghost number as follows,
\[
\begin{align*}
\text{antigh}(\Phi^i) &= 0, \text{ pure gh}(\Phi^i) = 0, \text{ gh}(\Phi^i) = 0, \\
\text{antigh}(C^\alpha) &= 0, \text{ pure gh}(C^\alpha) = 1, \text{ gh}(C^\alpha) = 1, \\
\text{antigh}(\Phi^*_i) &= 1, \text{ pure gh}(\Phi^*_i) = 0, \text{ gh}(\Phi^*_i) = -1, \\
\text{antigh}(C^*_\alpha) &= 2, \text{ pure gh}(C^*_\alpha) = 0, \text{ gh}(C^*_\alpha) = -2.
\end{align*}
\]

The ghost number is the difference between the pure ghost number and the antighost number.

Given \( S \), one can expand the differential \( s \) according to the antighost number. One gets
\[ s = \delta + \gamma + \sum_{k \geq 1} s_k, \text{ antigh}(\delta) = -1, \text{ antigh}(\gamma) = 0, \text{ antigh}(s_k) = k. \]
The nilpotency of $s$ implies the following relations,

$$\delta^2 = 0, \quad \delta \gamma + \gamma \delta = 0, \quad \gamma^2 + (\delta s_1 + s_1 \delta) = 0, \text{ etc.} \quad (12)$$

The first term in the expansion of $s$ is the Koszul-Tate differential associated with the gauge-invariant equations of motion. It plays a central role in BRST theory. The second term is the longitudinal exterior derivative along the gauge orbits and is related to the gauge symmetry [1, 3, 9].

It is clear that if $A$ is a BRST cocycle and has non negative antighost number, then its component $A_0$ independent of the antifields fulfills the condition

$$sA = 0 \Rightarrow \gamma A_0 + \delta A_1 = 0, \quad (13)$$

where

$$A = A_0 + \sum_{k \geq 1} A_k, \text{ antigh}(A_k) = k. \quad (14)$$

Conversely one has

**Theorem 1 :** Any solution $A_0$ of (13) determines a unique BRST cohomological class.

**Theorem 2 :** Any BRST cocycle $A$ with non negative ghost number that vanishes when the antifields are set equal to zero ($A_0 = 0$) is BRST-exact, $A = sB$.

These theorems hold equally well for $H_{\text{formal}}^\gamma(s)$, $H_{\text{loc.funct.}}^\gamma(s)$ and $H_{\text{local}}^\gamma(s)$ and are direct consequences of the perturbation expansion (11) and of the acyclic properties of the Koszul-Tate differential. They are proved in [8, 13, 20] in the Hamiltonian framework and in [1, 3] in the antifield case (see also [5], chapter 8, section 8.4 - in particular page 181 - for a general perspective independent of the precise context).

Since $\delta A_1$ in (13) is proportional to the equations of motion, the antifield independent component $A_0$ of $A$ is a $\gamma$-cocycle on-shell, i.e., is in the weak cohomology of the longitudinal derivative $\gamma$. Thus the theorems state in fact that any cohomological class of the weak longitudinal cohomology $H_{\text{weak}}^\gamma(s)$ determines uniquely a BRST cohomological class. Any representative of this BRST cohomological class is called a “BRST-invariant extension” of $A_0$ [8, 13, 1, 3, 5].
We shall not repeat the proof here, but shall only sketch the central idea by recalling how one reconstructs $A$ from $A_0$. Since $H^\text{loc.funct.}(s)$ is the trickier case, we shall from now on restrict the analysis to local functionals, $A = \int a$, where $a$ is a local $n$-form. In terms of the integrands, the $s$-cocycle and $s$-coboundary conditions (5) and (6) become respectively

\begin{align*}
sa &= dm \text{ (}s\text{-cocycle condition)}, \\
a &= sb + dn \text{ (}s\text{-coboundary condition)},
\end{align*}

for some $m$ and $n$ since the integral of a $d$-exact $n$-form is a surface term and vanishes (we assume appropriate boundary conditions). For this reason, one can identify the cohomological group $H^\text{loc.funct.}(s)$ with the cohomological group $H(s|d)$ of $s$ modulo $d$ in the space of local $n$-forms.

In the same way, the equation (13) on $A_0$ becomes

$$\gamma a_0 + \delta a_1 = dm_0.$$  

To reconstruct $a$ from a solution $a_0$ of (17) (for some $a_1$ and $m_0$), one proceeds as follows: first, one notes that (12) and (17) imply that $b_1 \equiv \gamma a_1 + s_1 a_0$ is a $\delta$-cycle modulo $d$, $\delta b_1 = dp$ for $p = \gamma m_0$. Furthermore, $b_1$ has antighost number 1. In order to be able to complete $a$ into an $s$-cocycle modulo $d$, it is necessary that $b_1$ be a $\delta$-boundary modulo $d$, $b_1 + \delta a_2 = dm_1$ for some $a_2$ and $m_1$. This ensures that $a_0 + a_1 + a_2$ fulfills the condition $sa = 0$ (modulo $d$) not just at antighost number zero - as implied by (17) - but also at antighost number one. One can then adjust successively $a_3, a_4$ etc in such a way that $a = \sum_k a_k$ is an $s$-cocycle modulo $d$ to all orders. One says that the construction of $a$ is not obstructed.

The question boils down therefore to the question of whether the cohomological groups $H_k(\delta|d)$ in which the potential obstructions could lie vanish for $k > 0$, in the space of local $n$-forms: is any $\delta$-cycle modulo $d$ with strictly positive antighost number automatically $\delta$-exact modulo $d$? As shown in [21] by means of an explicit counterexample, the answer to this question is negative. There exist obstructions, which have been related in [22] to the characteristic cohomology [23] (conserved currents, conserved $p$-forms) of the gauge-invariant field equations. However, these obstructions are not met in the above reconstruction process because $b_1$ (and the successive $b_k$’s) has strictly positive pure ghost number. As proved in [21], the cohomological
groups $H_k(\delta|d)$ vanish for $k > 0$ and strictly positive pure ghost number. The reason for this is that the ghosts are subject to no equations of motion in the gauge invariant formulation of the theory. More precisely, they are free in the homology of $\delta:$ there is no non-trivial relation among the ghosts that can be written as a $\delta$-boundary. This property is crucial and enables one to avoid the obstructions that one could otherwise meet in the reconstruction of $a$ from $a_0.$

As we shall now discuss, there is no such equivalent property in the gauge fixed formulation of the theory. Accordingly, there is no analog of Theorems 1 and 2 above for the gauge-fixed cohomology in the space of local functionals. One may (and does) meet obstructions because the ghosts are subject to equations of motion. The differential arising in the perturbative reconstruction of the BRST cocycles in the gauge-fixed description is not acyclic, even when restricted to the degrees that occur in the expansion.

## 3 Gauge-Fixed BRST Cohomology

In order to quantize the theory, it is necessary in practice to fix the gauge. To that end, one introduces non-minimal variables. The most common choice is to add antighosts $\overline{C}_\alpha$ and Nakanishi-Lautrup auxiliary fields $b_\alpha,$ with BRST transformation laws

$$s \overline{C}_\alpha = b_\alpha, \quad sb_\alpha = 0.$$  \hspace{1cm} (18)

The corresponding antifields are denoted by $\overline{C}^{*\alpha}$ and $b^{*\alpha}.$ The transformation (18) is obtained by adding the term $-\int dx \, b_\alpha \overline{C}^{*\alpha}$ to $S,$

$$S \to S - \int dx \, b_\alpha \overline{C}^{*\alpha},$$  \hspace{1cm} (19)

which preserves the master equation. Unless otherwise specified, the fields $\Phi^A$ will now collectively refer to the original classical fields, the ghosts, the antighosts and the auxiliary $b$-fields, while the antifields $\Phi^*_A$ will stand for all the conjugate antifields.

One then eliminates the antifields in a two-step procedure:

(i) First, one makes the canonical transformation

$$\Phi'^A = \Phi^A, \quad \Phi'^*_A = \Phi^*_A - \frac{\delta \Psi}{\delta \Phi^A}.$$  \hspace{1cm} (20)

7
where the odd functional $\Psi$ is called the gauge-fixing fermion; (ii) Second, one sets the new antifields $\Phi_A^*$ equal to zero. If $\Psi$ is well chosen, the resulting gauge-fixed action $S_\Psi$,  
\[ S_\Psi[\Phi^A] = S[\Phi^A, \Phi_A^* = \frac{\delta \Psi}{\delta \Phi^A}], \]  
leads to non-degenerate equations of motion, i.e., has no residual gauge invariance.

In discussing the gauge-fixed formulation of the theory, it is convenient to introduce a new degree, called the antifield number $r$, which puts all the antifields on an equal footing [2], 
\[ r(\Phi^A) = 0, \quad r(\Phi_A^*) = 1. \]  
(22)

One can expand the BRST differential and the BRST cocycles according to this new degree,
\[ s = \delta \Psi + \gamma \Psi + \sum_{k \geq 1} s'_k, \quad r(\delta \Psi) = -1, \quad r(\gamma \Psi) = 0, \quad r(s'_k) = k, \]  
\[ A = \sum_k A'_k, \quad r(A'_k) = k. \]  
(23) (24)

The differential $\delta \Psi$ is the Koszul differential associated with the gauge-fixed equations of motion [2]. It is acyclic in positive antifield degree in the space of formal functionals as well as in the algebra of local functions. However, it is not acyclic in the space of local functionals, the obstructions being related to the global symmetries (conserved currents) of the gauge-fixed action. Furthermore, one cannot use the pure ghost number as an auxiliary tool for controlling the cohomology, because the ghosts are now subject to non-trivial equations of motion. For this reason, Theorems 1 and 2 of the previous section apply to the expansion according to the antifield number (23) and (24) if one deals with the formal BRST cohomology and the local BRST cohomology, but not in the case of the BRST cohomology in the space of local functionals.

We shall exhibit explicit counterexamples in the next section. Before doing this, we note that the action of $\gamma \Psi$ on the fields can be written as 
\[ \gamma \Psi \Phi^A = (S, \Phi^A)|_{\Phi_A^* = \frac{\delta \Psi}{\delta \Phi^A}}. \]  
(25)
One easily verifies that \((\gamma \psi)^2\) is weakly zero on the fields,
\[
(\gamma \psi)^2 \approx' 0
\]  
(26)
where \(\approx'\) means “equal modulo the equations of motion \(\delta S_\psi / \delta \Phi^A = 0\) following from the gauge-fixed action”, and that the gauge-fixed action \((21)\) is invariant under the gauge-fixed BRST symmetry \(\gamma \psi\). One can then define the weak BRST cohomology \(H^{\text{weak}}(\gamma \psi)\) in the space of function(al)s of the fields by the conditions,
\[
\gamma \psi A[\Phi^A] \approx' 0 \quad (\gamma \psi\text{-cocycle condition}) \\
A[\Phi^A] \approx' \gamma \psi B[\Phi^A] \quad (\gamma \psi\text{-coboundary condition}).
\]  
(27) \hspace{1cm} (28)
Again, one may consider various cases, depending on the functional spaces to which \(A\) and \(B\) belong. Since \(sA = 0\) implies \(\gamma \psi A'_0 \approx' 0\) for the component of \(r\)-degree zero of \(A\), one sometimes refers to Theorems 1 and 2 in the context of the gauge-fixed expansion as the reconstruction theorems for the gauge-fixed cohomology \([8]\). Our main result is thus that these reconstruction theorems do not hold for local functionals.

4 The Counterexamples

4.1 The Model

The counterexamples discussed here arise in the case of the combined free Maxwell-Klein-Gordon system, with a massless scalar field. The gauge-invariant action is quadratic and equal to the sum of the free Maxwell action and the free KG action for a neutral (real) scalar field,
\[
S_0[A, \phi] = \int dx \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right]
\]  
(29)
In the Lorentz gauge enforced through the standard Gaussian average choice of gauge-fixing fermion, the gauge-fixed action is, with appropriate sign and factor conventions,
\[
S = \int dx \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \partial_\mu \overline{\psi} \partial^\mu C + b (\partial_\mu A^\mu + \frac{1}{2} b) - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \right]
\]  
(30)
The BRST symmetry reads $s = \delta_\Psi + \gamma_\Psi$, with
\[\gamma_\Psi A_\mu = \partial_\mu C, \quad \gamma_\Psi \phi = 0, \quad \gamma_\Psi C = 0, \quad \gamma_\Psi b = 0, \quad \gamma_\Psi \overline{C} = b\] (31)
and
\[\gamma_\Psi A^{*\mu} = 0, \quad \gamma_\Psi \phi^{*} = 0, \quad \gamma_\Psi C^{*} = -\partial_\mu A^{*\mu}, \quad \gamma_\Psi b^{*} = -\overline{C}^{*}, \quad \gamma_\Psi \overline{C}^{*} = 0.\] (32)
The Koszul differential $\delta_\Psi$ associated with the gauge-fixed stationary surface is given by
\[\delta_\Psi \Phi A = 0, \quad \delta_\Psi A^{*\mu} = \partial_\nu F^{\nu\mu} - \partial^\mu b, \quad \delta_\Psi \phi^{*} = \partial_\mu \phi\] (33)
and
\[\delta_\Psi C^{*} = -\partial_\mu \partial^\mu C, \quad \delta_\Psi b^{*} = \partial_\mu A^{\mu} + b, \quad \delta_\Psi \overline{C}^{*} = \partial_\mu \partial^\mu C.\] (34)

4.2 Counterexample To Reconstruction Theorem

The first counterexample is a counterexample to the reconstruction theorem. Consider the ghost number zero local function
\[-\frac{1}{2} A^{\mu} A_\mu + \overline{C} C\] (35)
It is a $\gamma_\Psi$-cocycle modulo $d$ on the surface of the gauge-fixed equations of motion since
\[\gamma_\Psi (-\frac{1}{2} A^{\mu} A_\mu + \overline{C} C) + \delta_\Psi (-b^{*} C) = \partial_\mu (-C A^{\mu}).\] (36)
Furthermore, it is non-trivial, i.e. it is not weakly $\gamma_\Psi$-exact modulo $d$. If the reconstruction theorem were correct, one could construct an $s$-cocycle modulo $d$ that starts like $a = a_0 + a_1 + a_2 + \ldots$, with $a_0 = -\frac{1}{2} A^{\mu} A_\mu + \overline{C} C$ and $a_1 = -b^{*} C$.

However, it is easy to see that there is no $a_2$ such that $sa = 0$ (modulo $d$) up to terms of antifield degree two. Indeed, one has
\[\gamma_\Psi a_1 = \gamma_\Psi (-b^{*} C) = \overline{C}^{*} C.\] (37)
Although $\overline{C}^{*} C$ is $\gamma_\Psi$-closed modulo $d$, it is not $\delta_\Psi$-exact modulo $d$, because any $\delta_\Psi$-boundary modulo $d$ necessarily vanishes when one sets all the derivatives of the fields and the $b$-field equal to zero. Thus, the construction of $a_2$
is explicitly obstructed, in this case by the generator of the global symmetry $\mathcal{C} \rightarrow \mathcal{C} + \eta C$ ($\eta$ anticommuting constant). There is no $\alpha$ which is BRST-closed modulo $d$ and which starts like $\alpha_0$, even though $\alpha_0$ is a cocycle of the gauge-fixed BRST cohomology. [One may easily check that the ambiguities in $\alpha_0$ and $\alpha_1$ cannot be used to remove the obstruction]. Note that $\mathcal{C}^* C$ has antifield number and ghost number both equal to one.

4.3 Counterexample To Theorem 2
The second example is a counterexample to Theorem 2. Let $\alpha$ be given by

$$\alpha = \mathcal{C}^* \phi - \phi^* C$$ (38)

This object is a BRST cocycle modulo $d$, has ghost number zero and is linear, homogeneous in the redefined antifields associated with the gauge-fixed description. Thus, it vanishes if one sets the redefined antifields $\mathcal{C}^*$ and $\phi^*$ equal to zero. Yet, $\alpha$ is not a BRST coboundary. The easiest way to see this is to rewrite $\alpha$ in terms of the original antifields. One has $\mathcal{C}^* = \mathcal{C}^* + \partial_\mu A^\mu + (1/2)b, \phi^* = \phi^*$ and thus $\alpha = A_\mu j^\mu - \phi^* C + s(-b^* \phi + (1/2)\mathcal{C} \phi + \partial_\mu (\phi^* \phi)), where$ $j^\mu$ is the non trivial conserved current $-\partial^\mu \phi$. Modulo an exact term, the cocycle $\alpha$ is a non trivial antifield-dependent cocycle of the form investigated in [24, 25]. This shows that there is non trivial BRST cohomology in the space of local functionals at non vanishing antifield number.

Another counterexample to Theorem 2, this time at ghost number one, can be constructed along the same lines by taking as gauge-fixing fermion $\Psi = \mathcal{C}(\partial_\mu A^\mu + (1/2)b + \phi)$, which is permissible. The $s$-cocycle modulo $d$ given by $\mathcal{C}^* C$ reads, in terms of the original antifields, $\mathcal{C}^* C = \phi C + \text{trivial terms}$, since $\mathcal{C}^*$ is now given by $\mathcal{C}^* = \mathcal{C}^* + \partial_\mu A^\mu + (1/2)b + \phi$. It is clear that $\phi C$ is a non-trivial $s$-cocycle modulo $d$, even though $\mathcal{C}^* C$ is linear, homogeneous in the redefined antifields and of ghost number one. Note that this counterexample does not assume the existence of conserved currents.

5 Conclusions
We have proved in this letter by means of explicitit counterexamples that important properties of the BRST construction that hold in the standard
gauge invariant description no longer hold when one reexpresses the theory in terms of the redefined antifields associated with the gauge fixation. This is because the redefined Koszul differential is no longer acyclic in the space of local functionals - even though it remains acyclic, of course, in the formal space of all functionals and in the algebra of local functions. The gauge-fixed description provides an example of homological perturbation theory with a non acyclic differential, in which one can - and does - meet obstructions. It follows in particular from our analysis that one cannot, in general, determine a BRST cohomological class (e.g. anomalies at ghost number one) by computing only its antifield independent part in terms of the redefined antifields although this can be done in terms of the standard antifields.

6 Acknowledgements

The author is grateful to Glenn Barnich and Joaquim Gomis for useful conversations. This work has been supported in part by research contracts with the F.N.R.S. and with the Commission of the European Community.

References

[1] J. M. L. Fisch and M. Henneaux, Commun. Math. Phys. 128 (1990) 627.

[2] M. Henneaux, in Physics, Geometry and Topology, Proceedings of the Banff Summer School in Theoretical Physics held August 14-25, 1989, H.C. Lee ed., Plenum Press (New York : 1990)

[3] M. Henneaux, Nucl. Phys. B (Proc. Suppl.) 18A (1990) 47.

[4] W. Troost, P. van Nieuwenhuizen and A. Van Proeyen, Nucl. Phys. B333 (1990) 727.

[5] A. Diaz, W. Troost, P. van Nieuwenhuizen and A. Van Proeyen, Int. J. Mod. Phys. A4 (1989) 3959.

[6] M. Hatsuda, W. Troost, P. van Nieuwenhuizen and A. Van Proeyen, Nucl. Phys. B335 (1990) 166.
[7] J. Gomis and J. París, *Nucl. Phys.* **B431** (1994) 378.

[8] J. Gomis, K. Kamimura, J. M. Pons and F. Zamora, *One Loop Anomalies and Wess-Zumino Terms for General Gauge Theories*, preprint RIMS-1020, TOHO-FP-9552, UB-ECM-PF 95/17.

[9] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems*, Princeton University Press (Princeton : 1992).

[10] C. Becchi, A. Rouet, R. Stora, *Phys. Lett.* **52B** (1974) 344; *Commun. Math. Phys.* **42** (1975) 127.

[11] I. V. Tyutin, Lebedev Preprint # 39 (1975), unpublished.

[12] J. Zinn-Justin, in *Trends in Elementary Particle Theory*, H. Rollnik and K. Dietz eds, Lecture Notes in Physics, Vol. 37, Springer-Verlag (Berlin: 1975).

[13] R. E. Kallosh, *Nucl. Phys.* **B141** (1978) 141.

[14] B. de Wit and J. W. van Holten, *Phys. Lett.* **79B** (1979) 389.

[15] I. A. Batalin and G. A. Vilkovisky, *Phys. Lett.* **B102** (1981) 27; *Phys. Rev.* **D30** (1983) 2567.

[16] B. L. Voronov and I. V. Tyutin, *Theor. Math. Phys.* **50** (1982) 218.

[17] J. Gomis, J. París and S. Samuel, *Phys. Rep.* **259** (1995) 1.

[18] M. Henneaux and C. Teitelboim, *Commun. Math. Phys.* **115** (1988) 213.

[19] J. Fisch, M. Henneaux, J. Stasheff and C. Teitelboim, *Commun. Math. Phys.* **120** (1989) 379.

[20] M. Dubois-Violette, *Ann. Inst. Fourier* **37** (1987) 45.

[21] M. Henneaux, *Commun. Math. Phys.* **140** (1991) 1.

[22] G. Barnich, F. Brandt and M. Henneaux, [hep-th/9405109](http://arxiv.org/abs/hep-th/9405109), to appear in *Commun. Math. Phys.*
[23] R.L. Bryant and P.A. Griffiths, *Characteristic Cohomology of Differential Systems (I): General Theory*, Duke University Mathematics Preprint Series, volume 1993 n^01 (January 1993).

[24] G. Barnich and M. Henneaux, *Phys. Rev. Lett.* **72** (1994) 1588.

[25] G. Barnich, F. Brandt and M. Henneaux, [hep-th/9405194](http://arxiv.org/abs/hep-th/9405194), to appear in *Commun. Math. Phys.*