Developing a method of low cycle fatigue life prediction for large-sized parts based on testing equivalent samples and applying it when improving gas turbine engine critical parts

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This paper presents the methodology of LCF equivalent tests based on the application of test samples whose stress-strain state (SSS) is similar to the stress-strain state of the part in its critical zone by using the stiffness coefficient of the stress-strain state. Requirements for manufacture of test specimens have been defined. This method was successfully tested on the example of a gas turbine engine LPC disk. The method was also used to improve the design of disk / blade root connection which resulted in a 3-fold increase of low cycle fatigue life of the disk. Method of equivalent testing is intended for prediction of cyclic fatigue life of critical large-sized parts full-scale tests of which are not possible or economically feasible.

Low cycle fatigue, equivalent tests, full-scale tests, stiffness of stress-strain state, stress concentrator, coefficient of similarity.

1. Introduction

Currently, one of the basic methods used for determining low cycle fatigue life of gas-turbine engine critical parts is the method of universal slopes proposed by S S Manson [1, 2] and improved by CIAM due to introduction of elastic and plastic strain amplitudes in a loading cycle [3, 4]. However, in some cases the results of fatigue life calculations differ significantly not only from the parts’ test results but also from test results of standard specimens as they do not fully take into account the operating conditions and complexity of stress-strain state (SSS) of the parts with stress concentrators. Furthermore, different modifications of Manson equation [5, 6] appeared, the attempts were made to improve the accuracy of low cycle fatigue prediction. However, in some cases the results of fatigue life calculations differ significantly from the results obtained during low cycle fatigue tests of the parts and also of standard specimens and specimens with stress concentrators [7, 8]. Therefore, in practice, full-scale tests are used to provide more reliable data. As the full-scale tests are complex and time-consuming and it is not possible to conduct them for large-sized parts, it is considered more reasonable to carry out equivalent tests on special specimens which simulate the stress-strain state of the examined object.

2. Scope of work

Studies [9] have found that regardless of the type of stress-strain state, before failure, localization of plastic strains occurs in a small volume of the material. The value of plastic strains determines the type of fracture – brittle fracture by tear with no or little plastic strains and ductile fracture by shear in case
of significant plastic strains. This state of material is characterized by the concept of stiffness of stress-strain state introduced by Ya B Fridman [10, 11, 12]. Therefore, stiffness of the stress-strain state not only characterizes the material’s capability for plastic strain but also determines the part’s operability under static and alternate loads. It is particularly important for parts operating in low-cycle range as their operability and fatigue life depend directly on the material’s capability for plastic deformation in a local zone – zone of stress concentrations. The proposed method of LCF life prediction based on equivalent tests of specimens with stress concentrators consists in creating identical stress-strain states of specimens and the examined parts. Based on research [13, 14] conducted at PJSC ‘UEC-Saturn’, the techniques of stress-strain state analysis and LCF life prediction were developed. They use the coefficient of stress-strain state introduced by Smirnov-Alyaev as a similarity criterion of the stress-strain state of a part and equivalent specimens [15, 16, 17]

\[
K_s = \frac{3\sigma_0}{\sigma_i} = \frac{\sqrt{2}(\sigma_1 + \sigma_2 + \sigma_3)}{\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}},
\]

where \(\sigma_0 = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3}\) – hydrostatic tension; \(\sigma_i\) – stress intensity; \(\sigma_1, \sigma_2, \sigma_3\) – principle stresses.

Based on the finite element model, the stress-strain state is reviewed in the most critical zone of the part, the types of specimen and stress concentrator parameters are selected in such a way as to ensure maximum similarity of specimen/part stresses. The required number of specimens is manufactured from the part’s material, the manufacturing process and heat treatment parameters remaining the same. Based on LCF tests, approximations are made and a low cycle fatigue life diagram is created in coordinates \(\varepsilon_i – lgN\). With full similarity of the stress-strain state of a part and its specimen, the obtained fatigue diagram of specimens characterizes the fatigue life of the examined part i.e. establishes the relation between the value \(\varepsilon_i\) of the part’s strain intensity and the relative number of cycles until fracture.

In cases where the coefficient \(K_{stif}\) of a part and its equivalent specimens is equal but it is still hard to achieve full similarity of the stress-strain state with regard to principal stresses, it is necessary to establish a coefficient of stress-strain similarity \(K_c\) between the strain intensity of a part and its specimen

\[
\varepsilon_{i,\text{part}} = K_c \cdot \varepsilon_{i,\text{sp}},
\]

If full-scale test results of development parts are available, this coefficient can be determined experimentally by comparing the strain intensity \(\varepsilon_i\) of a part and a specimen, provided they have the same fatigue life. In the absence of necessary experimental data for \(K_c\) determination, it is proposed to use the expression applied in this study which takes into account the heterogeneity of principal stresses with regard to the ratio of maximum shear stresses \(\tau_{max}\) to the intensity of stresses \(\sigma_i\) in a part – marked with index «part», and in equivalent specimens – marked with index «sp»

\[
K_h = \left[\frac{\tau_{max,\text{part}}/\sigma_{i,\text{part}}}{\tau_{max,\text{sp}}/\sigma_{i,\text{sp}}}\right]^{1/2},
\]

where \(\tau_{max} = \frac{\sigma_i - \sigma_j}{2}\), \(\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}\).

If full similarity of the stress-strain state of specimens with regard to principal stresses and stiffness coefficients cannot be achieved, the following expression can be used

\[
K_h = \left[\frac{\tau_{max,\text{part}}/\sigma_{i,\text{part}}}{\tau_{max,\text{sp}}/\sigma_{i,\text{sp}}}\right]^{1/2} \cdot \sqrt{\frac{K_{stif,\text{part}}}{K_{stif,\text{sp}}}}.
\]
In order to approbate this method, LPC Disk stg.1 of the D30KP engine made from VT3-1 alloy (Figure 1) was taken as a test object. The most loaded area of the disk is the fillet of the dovetail root (Figure 2) where fatigue cracks initiate during fatigue tests. Based on stress-strain state analysis, the value of stiffness coefficient $K_{stif} = 1.509$ is determined.

Round specimens with V-shaped concentrators were used as test specimens (Figure 3). Finite element models of test specimens and disks were made in Ansys v.18 system using Solid 186 elements. By selecting the concentrator parameters, with an angle of V-shaped groove $60^\circ$ and a radius of 0.25 mm at the top, the coefficient $K_{stif} = 1.595$ was obtained. The difference in coefficient $K_{stif}$ of specimens and parts was further taken into account in calculations. The specimens were cut-up from an unserviceable disk (Figure 4).

The specimens were tested under standard conditions in an MTS 810 testing machine, to zero-to-tension triangular cycle, performing tensile with a frequency of 2 Hz. Loads were selected in such a way as to cover the required interval of fatigue life: $7000 < N < 30000$ cycles of the examined disk. Thirty-two specimens were tested in five variants of loading, with loads in the range 3.40...5.30 kN, 6 specimens per loading variant. Statistical analysis results of test specimens, with confidence level $\alpha = 0.05$ and estimated probability $P = 0.95$, are given in Table 1 and in Figure 5 as an LCF diagram $lg (N_{ave} \pm \Delta N)$ presented in Excel by splines and for $lg N_{ave}$ – by 4- degree polynome, depending on strain intensity $\varepsilon_i$. 

![Figure 1. Model of LPC Disk stg.1.](image1)

![Figure 2. Finite-element model of Disk stg.1 element.](image2)

![Figure 3. Specimens from VT3-1 alloy with a V-shaped stress concentrator for LCF tests.](image3)

![Figure 4. Cut-up scheme of LPC disk stg.1.](image4)
To improve the accuracy of the diagram by using a polynomial of a higher degree based on Grafula.exe program, a special method was developed and the following functions were obtained for the average endurance

\[
\lg N_{\text{ave}} = 403,286 - 243,5338 \cdot x + 59,25666 \cdot x^2 - 7,176949 \cdot x^3 + 0,4326449 \cdot x^4 - 0,0103917 \cdot x^5, (4)
\]

and depending on \( \varepsilon_i = F(x) \), where \( x = \lg N_{\text{ave}} \)

\[
\varepsilon_i = -3052021,13 + 4186063,4361 \cdot x - 2389809,49965 \cdot x^2 + 726885,709444 \cdot x^3 - 124231,8027815 \cdot x^4 + 11311,86588 \cdot x^5 - 428,702508 \cdot x^6. \quad (5)
\]

Table 1. LCF test statistical analysis results for specimens with a stress concentrator, at \( \alpha = 0.05; P = 0.95 \); Student’s criterion \( t_{\alpha,k} = 2.571 \).

| No. | Load \( P, N \) | Average endurance \( N_{\text{ave}}, \) cycles | Confidence interval \( N_{\text{ave}} \pm \Delta N \) cycles | Endurance range \( N_{\text{ave}} \pm \Delta N \) cycles | \( \lg (N_{\text{ave}} \pm \Delta N) \) |
|-----|----------------|---------------------------------------------|-------------------------------------------------|------------------------------------------------|-----------------------------|
| 1   | 3400           | 59168                                      | 22828                                           | 36340...81996                                  | 4,56038...4,91379           |
| 2   | 3660           | 38830                                      | 10731                                           | 28099...49561                                  | 4,44869...4,69514           |
| 3   | 4240           | 29227                                      | 7300                                            | 21927...36527                                  | 4,34098...4,56261           |
| 4   | 5000           | 15015                                      | 4967                                            | 10048...19982                                  | 4,00208...4,30064           |
| 5   | 5300           | 10213                                      | 2896                                            | 7317...13109                                   | 3,86433...4,11757           |

To improve the accuracy of the diagram by using a polynomial of a higher degree based on Grafula.exe program, a special method was developed and the following functions were obtained for the average endurance \( \lg N_{\text{ave}} \):

Depending on \( \lg N_{\text{ave}} = F(x) \), where \( x = \varepsilon_i \cdot 10^3 \)

\[
\lg N_{\text{ave}} = 403,286 - 243,5338 \cdot x + 59,25666 \cdot x^2 - 7,176949 \cdot x^3 + 0,4326449 \cdot x^4 - 0,0103917 \cdot x^5. (4)
\]

Figure 5. LCF test results for test specimens Approximation by splines in Excel diagrams: 1 \(-\lg(N_{\text{ave}}-\Delta N)\); 2 \(-\lg N_{\text{ave}}\) and 3 \(-\lg(N_{\text{ave}}+\Delta N)\).
In order to confirm the calculated value of LCF life of the examined LPC disk, the endurance tests of three LPC stg.1 disks of D30KP engine were carried out at the test bench UIR-3 of PJSC ‘UCE-Saturn’. The tests were conducted to the trapezoidal cycle with realization of time parameters close to those of an aircraft flight cycle: acceleration from \( n_{\text{min}} = 500 \text{ rpm} \) to \( n_{\text{max}} = 5000 \text{ rpm} \) within \( \Delta t = 23 \text{ s} \), holding at \( n_{\text{max}} \) within \( \Delta t = 20 \text{ s} \), slowdown of disk rotation to \( n_{\text{min}} \) within \( \Delta t = 46 \text{ s} \) and rotation duration at \( n_{\text{min}} \) until the next acceleration was equal to \( \Delta t = 5 \text{ s} \). The average value of disk fatigue life was equal to 10763 cycles, it was taken as a basis for comparison with the results of LCF life calculation. Low cycle fatigue analysis results for \( N_{\text{calc}} \) disk given in Table 2 show that the values closest to the experiment were obtained by method of equivalent tests of specimens with a stress concentrator, stiffness coefficients being taken into account.

Table 2. Calculation results of LCF life for LPC disk stg.1.

| Calculation method                                           | \( N_{\text{calc}} \) cycles | Error \( \delta, \% \) |
|-------------------------------------------------------------|-------------------------------|----------------------|
| Experimental value                                          | 10763                         | –                    |
| Calculations to Manson equation                             | 6582                          | – 38,8               |
| Calculations to diagrams for smooth standard specimens       | 45153                         | 320                  |
| Calculations by method of equivalent testing of specimen without account for and with account for difference in \( K_\varepsilon \) coefficient: \( K_\varepsilon_{\text{specimen}} = 1,595 \) \( K_\varepsilon_{\text{disk}} = 1,509 \) | 13725 11538 | 27,5 7,2 |

This method allows us to solve a direct problem: to predict the fatigue life for large-sized parts the full-scale tests of which are either impossible due to their overall dimensions or not economically feasible as such tests are labour intensive and time consuming. The second important application of the above method – improving part design at the stages of part design and development. Based on dependency (5) for the set fatigue life \( [N] \), the permissible intensity of strain \( [\varepsilon_i] = F(\ln[N]) \) is determined for the most loaded critical zone of the part. All the other changes of the part in the process of design improvement (change of geometrical parameters) must comply with the condition \( \varepsilon_{i,\text{max}} \leq [\varepsilon_i] \). If in the process of part design development there occurs a significant change of stiffness coefficient \( K_{\text{stiff}} \), then equation (4) should be used for determining the similarity coefficient \( K_{\varepsilon} \).

As an illustration of method application, the design of the disk mentioned above has been improved in order to increase the disk’s resistance to LCF. To reduce the concentration of stresses in the most loaded area, the disk was modified. The root slot geometry was optimized by introducing a balance groove located along the slot bottom (Figure 6). In order to optimize the dimensions and

![Reworked disk slot](image-url)
location of the balance groove, stress-strain analysis has been performed to consider variants with different values of groove depth and radius, symmetrical and asymmetrical groove location relative to blade slot axis. As a result, a variant with reduction of equivalent stresses $\sigma_{eqv}$ from 818 to 748 MPa and stiffness coefficient $K_{stif}$ from 1,509 to 1,460 has been defined in the most loaded and critical zone of a disk slot. Calculations of disk LCF life obtained by traditional methods have shown an increase in fatigue life from 33870 to 52956 cycles i.e. up to ~5 times. Endurance tests of the modified disk at UIR-3 test bench showed an increase in number of cycles before fatigue crack initiation from 10763 to 29485 cycles i.e. up to ~3 times in comparison to a serial disk. Calculations obtained by method of equivalent tests given in Table 3, with the account for and with no account for stiffness of the stress-strain state, confirm the method’s efficiency: errors are 3.3 % and 8.7 % respectively.

**Table 3.** Calculation results of LCF life of the modified disk by method of equivalent tests.

| Calculation variant | Intensity of disk deformation $\varepsilon_{i,\sigma}$ $10^3$ | Similarity coefficient $K_e$ | Calculated endurance $N_{calc}$, cycles | Error $\delta$, % |
|---------------------|---------------------------------|------------------|-----------------------------|-----------------|
| Without account for stiffness of the stress-strain state | 6,2288 | 0,80710 | 30456 | 3,3 |
| With account for stiffness of the stress-strain state | 0,77218 | 26931 | – 8,7 |

The obtained results show sufficiently high efficiency of equivalent test method for specimens with a stress concentrator. It is obvious that the accuracy of part LCF prediction will depend mainly on the accuracy of test specimen fatigue life determination during endurance tests. The results of specimen LCF tests given above have significant scattering and confidence intervals which seem to depend on the state of material and quality of specimen manufacture, i.e. on technological heredity. Therefore, in order to increase accuracy of fatigue life prediction for GTE parts by reduction of confidence intervals, the influence of technological heredity of material properties on LCF has been reviewed. For this purpose, the specimens were tested in their initial state as well as after undergoing heat treatment (stabilizing annealing). Heat treatment mode: argon heating to $T = 800 - 850 ^\circ C$, holding within 60 minutes with further furnace cooling to $200 \div 250 ^\circ C$ and then argon cooling (600 torr, 30 %) to $T \leq 70 ^\circ C$.

The results of standard static tensile tests before and after heat treatment showed an insignificant (within 2...3 %) decrease in basic strength characteristics - yield strength and ultimate strength. However, the plastic properties of the material after annealing - the relative elongation $\delta$, and the reduction of the area coefficient $\psi$ have decreased by 22.37 and 29.77 % respectively.

Specimens with similar parameters of stress concentration were used for LCF tests. Tests were carried out under load $P = 5200$ N at which the specimen fatigue life was close to that of a modified disk. Table 4 shows the statistical analysis results of specimen tests at confidence level $\alpha = 0.05$, estimated probability $P = 0.95$ and Student’s criterion $t_{0.05} = 2.571$. Results of specimen endurance tests (Table 4) show that with significant reduction (~ 33 times) of the confidence interval and mean square deviation, the value of average fatigue life of specimens has reduced by 6.7 times.
Therefore, when preparing specimens for equivalent tests, it is not enough to ensure that the materials and stiffness coefficient of a part and its specimen are the same. It is essential that the manufacturing method and machining method of a specimen in the area of the stress concentrator fully correspond to those of the examined part in its critical zone with a stress concentrator.

The second important conclusion that can be made is that low cycle fatigue life of a part depends more on plastic properties of a part than on its strength properties. It means that when selecting the material for critical parts exposed to low-cycle fatigue, it is important to consider primarily the stress-strain characteristics of a part and to select a heat treatment mode that would contribute to maximum increase of the material’s plastic properties, even at the expense of its strength.

3. Conclusion
The approbation of this method has confirmed its efficiency on a relatively complex part – an LPC Disk, operating under low cycle fatigue conditions. Therefore, the method of equivalent tests of specimens with a stress concentrator can replace full-scale tests of large-sized critical parts for the purpose of assessing their low-cycle fatigue life and possibly, after additional research, the high-cycle fatigue life.

To increase resistance to LCF, the described method and characteristics of stiffness of the stress-strain state can also be used for improving the part’s design at the stages of product design and development on the basis of allowable deformations i.e. condition $\varepsilon_{i,\text{max}} \leq [\varepsilon_i]$ should be met in the critical zone of the part.

The materials chosen for the parts operating in low-cycle range should have the highest plastic properties along with necessary strength properties.

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| Calculation variant | Average fatigue life $N_{\text{ave}},$ cycles | Confidence interval $\pm \Delta N$, cycles | Unbiased estimator $S,$ cycles | Variation coefficient $v$ |
|---------------------|---------------------------------------------|-------------------------------------------|---------------------------------|--------------------------|
| In initial state    | 26495                                       | 20202                                     | 19247                           | 0,726                    |
| After heat treatment| 3955                                        | 614                                       | 585                             | 0,148                    |
| Change of characteristics, times | 6,70                                      | 32,90                                     | 32,90                           | 4,91                     |
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