Mathematical and computer modelling of the nonlinear dynamics of particle beams in cyclic accelerators

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Abstract. The article explores the methods of mathematical and computer modelling of the nonlinear dynamics of charged particle beams in cyclic accelerators in terms of the matrix formalism using both numerical and symbolic representations of the corresponding nonlinear differential equations. This paper shows how the matrix formalism approach allows the use of modern computational algorithms and artificial intelligence methods for more efficient and flexible modelling of particle beam dynamics in accelerators. Also, the article proposes a method of representing control elements using LEGO objects, which makes it possible to increase the efficiency of using control elements from a computational point of view and to facilitate the process of modelling particle dynamics.

1. Introduction
Modern computational experiments require to perform numerical modelling of the problem under investigation using various methods in order to determine the optimal solution, analyze the system from different angles, and evaluate the performance and computational complexity. This article provides a comparative analysis of existing approaches for mathematical and computer modelling of nonlinear dynamics of charged particle beams.

2. The matrix formalism instead of tensor description
In the article [1] methods of mathematical and computer modelling of nonlinear dynamics of particle beams in cyclic accelerators are considered in terms of the matrix representation of the corresponding nonlinear differential equations. The proposed approach differs from traditional representations of nonlinear equations in the form of Taylor series. We use the representation of the coefficients in the form of two-dimensional matrices. The similar approach allows us to significantly reduce the time

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spent on the dynamics of the model beam, and also use symbolical mathematics for computation of necessary two-dimensional matrices.

It should be noted that traditionally the Taylor series are used for each trajectory of the beam particles, this is what leads to computational significant time and resource (from a computational point of view) costs. In the proposed approach, a set of two-dimensional matrices can be built up to the desired nonlinearity order (instead of the tensor representation). The traditional approach is based on the numerical representation of the corresponding beam particle transformations, taking into account the configuration of the control elements (dipoles, quadrupoles, etc.), that significantly reduce in the case of using of parallel and distributed computing systems and technologies.

The use of matrix formalism (using both numerical and symbolic representation) also makes it possible to apply new effective methods of numerical modelling, in particular using the methods of artificial intelligence [2]. It should be noted that in this case, we can not only use effective computational algorithms but also use modern high-performance computing resources.

3. Using LEGO-objects for symbolic presentation of nonlinear dynamic systems

The particular attention it is necessarily paid to problems of correct and effective (from a computational point of view) use of control elements (for example, including taking into account fringe fields), which leads to the need to present control elements using LEGO-objects. It should also be noted that it is the matrix formalism that allows the calculation of such objects in the symbolic forms [3] stored in the corresponding databases. In particular, the use of the concept of LEGO-objects allows us to consider the fringe fields as additional objects and also carry out necessary optimization procedures. We also note that proposed method is in good agreement with A.Dragt's concept [4] and demonstrates the effectiveness both for problems of dynamics and optimization of control systems, as well as for estimating the influence of various effects on beam dynamics (including taking into account the spin). Using the tools of symbolic computation not only significantly increases the computational efficiency of the method, but also allows the creation of databases of "finished" transformations (Lego-objects), which greatly simplify the process of modelling particle dynamics.

It should be specially noted that if we terminate the Taylor series, we break the symplectic property, which leads to incorrect of the long-time evolution of particles (for example, in colliders). In this case, within the framework of the proposed approach for truncating the series, we should carry out the procedure of symplectification for corresponding matrices up-to necessary order of nonlinearity. The proposed approach has been tested for some number of projects, including the NICA project [5] and the JEDI project.

4. Software for modelling of beam dynamics in accelerators

Now the spectrum of software packages for the physics of beams of charged particles is extremely wide. As the most popular and effective systems, we should mention programs such as COZY Infinity [6], MAD X [7], Polymorphic Tracking Code (PTC), MARYLIE can be used.

Existing programs can be divided into several types. Some of them describe the ray tracing (OXRAY, RAYTRACE, TURTLE), determine the particle trajectories through internal and external electromagnetic fields using numerical integration. These programs are reliable and easy to use, but the results of their work in the form of ray coordinate values do not give complete information about the process in progress.

Programs of another kind are used to describe the motion of a particle beam in the phase space (TRANSPORT, COZY Infinity, MAD, MARYLIE). In these programs, elements of the transition matrices for the optical systems are calculated, that gives a more detailed representation of the investigated system.

One of the most widely used approaches for calculating the dynamics of beam physics is the differential algebra, that use algebraic relations instead the calculations of derivatives with difference schemes (MXYZTPLK, ZLIB, MARYLIE, COZY Infinity). A common disadvantage of these
solutions is that they are poorly parallelized, since the ideology of differential algebra is used in the concept of the tensor calculations. This complicates the implementation of algorithms on parallel computing architectures.

The theoretical basis for the other approaches related to modern numerical methods for integrating ODEs is the use of the Taylor models in constructing the Lie maps. Among the programs that use this ideology for calculating, it should be noted the MARYLIE, MAD, TRANSPORT. Programs that use Lie methods: COZY Infinity, BMAD, MAD, LEGO, UAL, SAD, MARYLIE, SIXTRACK.

The diverse individual formats of input and output data, the lack of a common and convenient interface, and enough narrow specialization of these programs lead to some number of problems for modern researching. This significantly reduces the computational efficiency and quality of the corresponding experiments. The paper [8] describes a universal tool for automating and improving the computational efficiency of computational experiments. As an example authors consider a method for developing a concept and prototype of a corresponding software package (figure 1) that would combine the advantages of existing (primarily non-commercial) software packages. The corresponding software is based on the unification of the format of input and output data for the corresponding programs, and visualizes information in various ways, including reference and training information for "beginners". This framework will allow researchers to simulate complex systems, identify the features of the influence of certain misalignments in the control system via the graphical interface. The results obtained within the framework of the developed structure can be used for both to the development of new both numerical and symbolical methods for solving evolutionary nonlinear equations.

Figure 1. The main window of the BOSS program: 1 – the palette of the control elements, 2 - the system modeling area, 3 - the system global parameters bar, 4 - the software package selection bar, 5 - the data visualization.

5. Conclusion
This article discusses the methods of mathematical and computer modeling of the charged particle beams dynamics. The analysis in this scientific field allowed us to identify the strengths and weaknesses of available approaches and research methods.

6. References
[1] Andrianov S., Ivanov A., Kulabukhova N., Krushinevskii E., Sboeva E., “Dynamic Equations: The Matrix Representation of Beam Dynamic Equations Instead of Tensor Description”, presented at IPAC’18, Vancouver, Canada, April - May 2018, paper THPAK134
[2] Ivanov A., Sboeva E., Krushinevskii E., Andrianov S., Kulabukhova N., “Matrix Representation of Lie Transform in TensorFlow”, Proceedings of IPAC2018, Vancouver, BC, Canada, April - May 2018, paper THPAK088
[3] Sboeva E., Krushinevskii E., Andrianov S., Ivanov A., Kulabukhova N., “Symbolic Presentation of Nonlinear Dynamic Systems in Terms of LEGO-objects”, Proceedings of IPAC2018, Vancouver, BC, Canada, April - May 2018, paper THPAK090
[4] Dragt A.J., Douglas D.R., Particle tracking using lie algebraic methods, Vol. 215, 122–127 (1984)
[5] NICA, http://nica.jinr.ru
[6] Berz M., Makino K. COSY INFINITY 9.1 Programmer’s Manual. — Michigan State University, 2011
[7] MAD X, http://mad.web.cern.ch/mad
[8] Krushinevskii E., Andrianov S., Ivanov A., Kulabukhova N., Sboeva E., “Software-computing System for Numerical Modelling of Beam Dynamics in Accelerators”, Proceedings of IPAC2018, Vancouver, BC, Canada, April - May 2018, paper THPAK087