Distributed tracking control problem of Lipschitz multi-agent systems with external disturbances and input delay

Ahmadreza Jenabzadeh and Behrouz Safarinejadian
School of Electrical and Electronic Engineering, Shiraz University of Technology, Shiraz, Iran

ABSTRACT
This paper addresses the distributed tracking control problem for Lipschitz nonlinear multiagent systems (LNMASs) in the presence of external disturbances and input delay. For this end, a distributed tracking control algorithm is proposed for LNMASs that guarantee each agent can estimate and track a nonlinear target. The suggested algorithm is developed based on a future state predictor and a finite time estimator to cope with the tracking control problem for LNMASs with input delay and external disturbances. Finally, the efficiency of the proposed algorithms is illustrated by simulation results.

ARTICLE HISTORY
Received 21 February 2018
Accepted 2 June 2018

KEYWORDS
Distributed tracking control; multiagent systems; Lipschitz; delay; external disturbances

1. Introduction
In the past decade, distributed tracking control (DTC) of a group of mobile agents has attracted researchers’ attention due to its wide applications in many fields including formation control, leader-follower problem, target tracking, flocking, intelligent transportation and so on (Fang, Wei, Chen, & Xin, 2017; Hong, Yu, Yu, Wen, & Alsaedi, 2017; Miao, Wang, & Fierro, 2017; Miranbeigi, Moshiri, & Rahimi Kian, 2016; Tian, Zuo, & Wang, 2017; Wang, Gao, Alsaadi, & Hayat, 2014; Yu, Yan, & Li, 2017; Zhao & Jia, 2016). DTC of multiagent systems includes two aspects of control and estimation. The control aspect plans to design a distributed controller for each agent to track the states of the target. The estimation aspect aims to propose a distributed estimator for each agent to estimate the states of the target.

DTC problem can be investigated for multiagent systems with linear and nonlinear dynamics. Most of the existing algorithms on DTC focus on multiagent systems with linear dynamics (such as Olfati-Saber & Jalalkamali, 2012; Su, Chen, Chen, & Wang, 2016; Su, Li, & Chen, 2017), while practical multiagent systems and targets usually have nonlinear dynamics. Therefore, it is important to consider the DTC algorithms of nonlinear multiagent systems. In Yu, Dong, et al. (2017), a distributed control law has been proposed for second order nonlinear multiagent systems to cope with DTC problem. Li, Dong, and Nguang (2017) has studied the DTC problem for third order nonlinear multiagent systems and presented some algorithms for every agent to track a target with third order nonlinear dynamics. In Li (2015), some distributed controllers have been designed that ensure multiagent systems with high order dynamics can track a target under a directed graph. In Zhao, Guan, Li, Zhang, and Chen (2017), two control laws have been presented for multiagent systems with nonholonomic dynamics to guarantee collision avoidance among agents and tracking a target with unknown dynamics under fixed and switching topologies.

In the recent years, study of LNMASs has received great attention due to the fact that this kind of multiagent systems includes many practical multiagent systems. Li, Liu, Fu, and Xie (2012) presented a distributed control law for LNMAS to ensure each follower (agent) can track a leader (target) with Lipschitz nonlinear dynamics. The suggested controller has been developed for LNMAS with switching topologies (Wen, Duan, Chen, & Yu, 2014). In Wen, Yu, Xia, Yu, and Hu (2017), a DTC algorithm has been introduced for LNMAS to estimate and track the leader’s states with Lipschitz nonlinearities. It is worth noting that the DTC algorithms in Wen et al. (2017), Wen et al. (2014), and Li et al. (2012) have been designed based on the assumption that the states of the targets are available or at least one of the agents is connected to the target. This assumption does not hold in the DTC problem and the states of the target should be estimated by a distributed estimator. Therefore, the algorithms of Wen et al. (2017), Wen et al. (2014), Li et al. (2012) cannot be used for the DTC
of LNMAS. It should be noted that there are some distributed estimators which can estimate the states of the targets with nonlinear dynamics (Jenabzadeh & Safarinejadian, 2017; Jenabzadeh, Safarinejadian, & Mohammadnia, 2017; Li, Shen, Wang, & Alsaadi, 2017; Li, Shen, Wang, & Alsaadi, 2018; Shen, Wang, & Liu, 2011; Wei, Liu, Wang, & Wang, 2016). In Shen et al. (2011), Li, Shen, et al. (2017), Jenabzadeh and Safarinejadian (2017), and Jenabzadeh et al. (2017), the authors have proposed some estimators for the targets whose nonlinear part satisfy a special constraint and cannot be applied to the target with Lipschitz nonlinearities. Furthermore, Wei et al. (2016) has suggested a distributed filter based on an event-triggered mechanism to estimate the states of a class of time varying discrete-time nonlinear systems with Lipschitz-like condition. Moreover, the distributed state estimators of Li et al. (2018) like the presented filter in Wei et al. (2016) have been presented to estimate the states of a class of discrete systems and cannot be used for the targets with continuous-time dynamics. Based on these explanations, it is necessary to introduce a DTC algorithm for LNMASs in which the states are not available to any of the agents explicitly.

In reality, some inevitable destructive factors affect on system performance and may lead to instability. For instance, external disturbances are the source of poor performance and instability of multiagent systems. Moreover, time delay exists in practical multiagent systems and degrades the system performance. One type of time delay in multiagent systems is the input delay which is caused by processing and connecting time for the packets arriving at each agent. Based on these explanations, it is important to design some algorithms to decrease the effect of external disturbances and input delay on the performance of multiagent systems. To the best of the authors’ knowledge, the DTC problem in LNMASs subject to external disturbances and input delay in which the target’s states are not available is still open in the literature. This motivates us for this study.

In this paper, the DTC problem of LNMASs is solved in the presence of external disturbances and input delay. For this goal, a DTC algorithm is firstly established and analyzed for LNMASs in the absence of external disturbances and input delay. Then, the proposed algorithm is extended for LNMASs with external disturbances and input delay. The main difficulties in the design of the distributed estimator and controller in this paper are as follows: (1) The agents should track a target whose states are not available. It means that the distributed controller requires the estimated states of the target. Consequently, stability analysis of estimation errors and tracking errors should be carried out simultaneously. Since the target as well as the multiagent system have nonlinear dynamics, stability analysis is much more difficult than that of existing work in which multiagent systems have linear dynamics. (2) Due to input delay, the proposed controllers cannot track the target. It means that the controllers must be modified such that the states of agents with input delay converge to the states of the target. Since there is no controller for the DTC of LNMASs in the literature, a novel distributed controller is required for LNMASs with input delay. This controller is designed based on a future state predictor. The proposed predictor is one of the first predictors which can estimate the future states of the multiagent systems with nonlinear dynamics. (3) In this paper, the design of distributed estimator and controller are also carried out in the presence of external disturbances. Most of the existing works have solved the DTC problem in the presence of external disturbances for multiagent system with linear dynamics. Therefore, solving the DTC problem in the presence of external disturbances for LNMASs can be one of the first attempts for DTC of multiagent systems with nonlinear dynamics. The main contributions of this paper are as follows:

- Compared with the distributed control laws of Yu, Dong, et al. (2017), Li, Dong, et al. (2017), Li (2015), Zhao et al. (2017), Li et al. (2012), Wen et al. (2014) and the distributed estimator of Shen et al. (2011), Wei et al. (2016), Li, Shen, et al. (2017), Jenabzadeh and Safarinejadian (2017), Jenabzadeh et al. (2017), and Li et al. (2018) which consider one of the aspects of the DTC, both aspects of the DTC is considered simultaneously in this paper and a DTC algorithm is obtained for LNMASs.

- In the design of the distributed control law for DTC, it is assumed that the states of the targets are not available and the target is not connected to any agent in this paper, while the results in Wen et al. (2017), Wen et al. (2014), and Li et al. (2012) assumed that the states of the targets are available or at least one of the agents is connected to the target.

- A distributed estimator is proposed in this paper that can estimate the states of a general class of continuous targets with Lipschitz condition, while the estimator of Shen et al. (2011), Wei et al. (2016), Li, Shen, et al. (2017), Jenabzadeh and Safarinejadian (2017), Jenabzadeh et al. (2017), and Li et al. (2018) can estimate the states of some limited targets with some special constraints or with discrete dynamics.

- A DTC algorithm is suggested for multiagent systems with Lipschitz nonlinear dynamics that can be used for many practical multiagent systems, whereas the results of Olafati-Saber and Jalalkamali (2012), Su et al. (2016) and Su et al. (2017) are limited to linear
multiagent systems that cannot be applied to nonlinear multiagent systems.

- A DTC algorithm is obtained for LNMASs so that they can work appropriately in the presence of external disturbance and input delay.

The rest of this paper is organized as follows: the problem definition is presented in Section 2. Some DTC algorithms are designed for LNMASs in the absence and presence of input delay and external disturbances in Section 3. The simulation results are given in Section 4. Finally, Section 5 concludes the paper.

Notation: \( I_u \) represents an \( v \times v \) identity matrix. \( R^n \) denotes the \( n \) dimensional Euclidean space. \( R^{u \times p} \) represents the set of all \( u \times p \) real matrices. \( ||.|| \) denotes the Euclidean vector norm. \( \lambda_2(L) \) indicates the second smallest eigenvalue of the matrix \( L \). The matrix inequality \( v > 0 \) expresses that \( v \) is a symmetric positive definite matrix.

Graph Theory. To model a multiagent system, an undirected graph \( G = (Y, F, Q) \) is used, where \( Y = \{1, 2, \ldots, N\} \) expresses the set of agents, \( F \in Y \times Y = \{(i,j) : i,j \in Y\} \) denotes the set of links among agents and \( Q = [q_{ij}] \in R^{N \times N} \) represents the adjacency matrix. The Laplacian matrix is defined as \( L = Z - Q \) where \( Z \in R^{N\times N} \) is a diagonal matrix in which \( Z_{ii} = \sum_{j \in N_i} q_{ij}, N_i = \{j \in Y : (i,j) \in F, j \neq i\} \) expresses the neighbours set of the agent \( i \). If agents \( i \) and \( j \) are connected, the agent \( i \) is the neighbour of agent \( j \) and \( q_{ij} > 0 \). Moreover, if there exists at least one path between every two arbitrary agents, then \( G \) is called a connected graph.

2. Problem definition

Consider a LNMAS with the \( i \)-th agent dynamics given as:

\[
i_d(t) = A r_i(t) + \psi(r_i(t)) + B u_i(t - h) + D_i(t), \quad i = 1, \ldots, N \tag{1}
\]

where \( r_i(t) \in \mathbb{R}^m \) and \( u_i(t) \in \mathbb{R}^5 \) are state and input of the \( i \)-th agent, respectively. \( A \in \mathbb{R}^{m \times m} \) and \( B \in \mathbb{R}^{m \times 5} \) are constant matrices with \( (A, B) \) being controllable. \( h \) is a given positive input delay. \( D_i(t) = \begin{pmatrix} D_{i1}(t) \\ \vdots \\ D_{im}(t) \end{pmatrix} \) is an external disturbance which satisfies

\[
|D_{ix}(t)| \leq D_{ix}, \quad x = 1, \ldots, m
\]

where \( D_{i1}, \ldots, D_{im} \) are positive constants. The target dynamics is described by:

\[
s(t) = As(t) + \psi(s(t)) + Cw(t), \quad C \in \mathbb{R}^m \tag{2}
\]

where \( s(t) \in \mathbb{R}^m \) is the state of the target. The \( i \)-th agent has the following sensing model:

\[
y_i(t) = h_i(s(t)) + r_i v_i, \quad i = 1, \ldots, N
\]

where \( w \) and \( v_i \) are Gaussian white noises. We assume that there are positive constants \( \zeta_1 \) and \( \zeta_2 \) such that for \( q, p \in \mathbb{R}^m \):

\[
||\psi(q) - \psi(p)|| \leq \zeta_1 ||q - p|| \tag{3}
\]

\[
||h_i(q) - h_i(p)|| \leq \zeta_2 ||q - p||. \tag{4}
\]

The main goals of this paper are twofold:

- To obtain a DTC algorithm such that the LNMAS (1) with external disturbances \( (h = 0) \) can estimate and track the states of the target (2).
- To design a DTC algorithm such that the LNMAS (1) with input delay \( (D_i(t) = 0) \) can estimate and track the states of the target (2).

Before designing a DTC algorithm for the above two cases, it will be derived for external disturbances and input delay free case \( (D_i(t) = 0, h = 0) \). In this case, a distributed estimator and a control law are needed for every agent to design a DTC algorithm for LNMAS (1). The following distributed estimator (DE) is proposed for the LNMAS (1) to estimate the states of the target (2):

\[
\dot{\hat{s}}_i(t) = A\hat{s}_i(t) + \psi(\hat{s}_i(t)) + X_i(y_i(t) - h_i(\hat{s}_i(t)))
\]

\[
+ \gamma_i \Omega_i^{-1} \sum_{j \in N_i} q_{ij}(\hat{s}_j(t) - \hat{s}_i(t)), \quad i = 1, \ldots, N \tag{5}
\]

where \( \hat{s}_i(t) \in \mathbb{R}^m \) is the estimation of \( s(t) \), \( X_i \in \mathbb{R}^m \) is the gain matrix, \( \Omega_i > 0 \in \mathbb{R}^{m \times m} \) is the estimator matrix which is used to derive the sufficient conditions for the existence of the DE (5), and \( \gamma_i > 0 \) is the consensus gain which is used to adjust the consensus time of the states of DE (5) when the estimator stability holds. It is worth noting that the DE (5) includes three terms. The term \( A\hat{s}_i(t) + \psi(\hat{s}_i(t)) \) is used to reconstruct the dynamics of the target (2). Furthermore, the term \( (y_i(t) - h_i(\hat{s}_i(t))) \) is the part of the measurement that contains new information about the target’s states and is used to update the state estimate. Since the estimator (5) should be a distributed estimator which uses the information of the estimators of the neighbouring agents to estimate the states of the target (2), the term \( \sum_{j \in N_i} q_{ij}(\hat{s}_j(t) - \hat{s}_i(t)) \) is added to the DE (5). The DE (5) is a modified version of DE of Jenabzadeh et al. (2017). Indeed, Jenabzadeh et al. (2017) introduced a DE to estimate the states of targets with a pseudo Lipschitz condition. In this paper, DE of Jenabzadeh et al. (2017) is
extended for targets with Lipschitz condition that can be used for many practical targets.

To track the states of the target \( z(t) \), a distributed controller (DC) is suggested for LNMAS (1) as follows:

\[ u_i(t) = B^T K_i^{-1} \left( \sum_{j \in N_i} q_{ij}[r_j(t) - r_i(t)] - (r_i(t) - \hat{s}_i(t)) \right) \]

where \( K_i \geq 0 \in R^{m \times m} \).

### 3. Main results

In this section, the stability of the proposed DTC algorithm including DE (5) and DC (6) is analyzed. Then, the proposed algorithm is developed for the DTC problem in the presence of input delay and external disturbances.

**Assumption 3.1:** The graph \( G \) related to the LNMAS (1) is undirected and connected.

**Theorem 3.1:** Consider LNMAS (1) which utilizes DE (5) and DC (6). Under Assumption 1, LNMAS (1) can estimate and track the states of the target \( z(t) \) in the absence of external disturbances and input delay if the following inequalities are satisfied:

\[
\begin{align*}
AK_i + K_i A^T + \xi_1^2 l_m + K_i^2 - BB^T &< 0, \quad (7) \\
\Omega_i A + A^T \Omega_i + \xi_1^2 \Omega_i + \xi_2^2 \Omega_i X_i X_i^T \Omega_i + (2 - \rho_i) l_m + K_i^{-1} BB^T K_i^{-1} &< 0 \quad (8)
\end{align*}
\]

where \( \rho_i \) is a positive scalar which satisfies \( \rho_i \leq 2\gamma \lambda_2(L) \).

**Proof:** Let \( e_i = s(t) - \hat{s}_i(t) \) and \( \sigma_i = r_i(t) - s(t) \). Then, one gets (without noise):

\[
\begin{align*}
\dot{\sigma}_i &= A \sigma_i(t) + \psi(r_i(t)) - \psi(s(t)) \\
&+ BB^T K_i^{-1} \left( \sum_{j \in N_i} q_{ij}[r_j(t) - r_i(t)] - (r_i(t) - \hat{s}_i(t)) \right) \\
\dot{e}_i &= A \sigma_i + \psi(s(t)) - A \hat{s}_i(t) - \psi(\hat{s}_i(t)) - X_i \sigma_i(t) - h_i(\hat{s}_i(t)) - \gamma_1 \Omega_i^{-1} \sum_{j \in N_i} q_{ij}[\hat{s}_i(t) - \hat{s}_i(t)].
\end{align*}
\]

A Lyapunov function is selected as follows:

\[
V = V_o + V_e = \sum_{i=1}^{N} \sigma_i^T K_i^{-1} \sigma_i + \sum_{i=1}^{N} e_i^T \Omega_i e_i
\]

where \( \sigma = [\sigma_1^T, \ldots, \sigma_N^T]^T \) and \( e = [e_1^T, \ldots, e_N^T]^T \). One obtains the derivative of \( V \) as follows:

\[
\dot{V} = \sum_{i=1}^{N} 2\sigma_i^T K_i^{-1} \left( A \sigma_i + \psi(r_i(t)) - \psi(s(t)) \\
+ BB^T K_i^{-1} \left( \sum_{j \in N_i} q_{ij}[r_j(t) - r_i(t)] - (r_i(t) - \hat{s}_i(t)) \right) \\
+ \sum_{i=1}^{N} 2e_i^T \Omega_i \left( A \sigma_i + \psi(s(t)) - \psi(\hat{s}_i(t)) - X_i \sigma_i(t) - h_i(\hat{s}_i(t)) - \gamma_1 \Omega_i^{-1} \sum_{j \in N_i} q_{ij}[\hat{s}_i(t) - \hat{s}_i(t)] \right) \right).
\]

Utilizing inequalities

\[
\alpha^T \vartheta = |\alpha^T \vartheta| \leq ||\alpha|| |\vartheta|| \quad (10)
\]

\[
2||\alpha|| |\vartheta|| \leq \alpha^T \alpha + \vartheta^T \vartheta \quad (11)
\]

and conditions (3) and (4), one can obtain

\[
2\sigma_i^T K_i^{-1} \left( \psi(r_i(t)) - \psi(s(t)) \right) \leq 2\gamma_1 \| K_i^{-1} \sigma_i \| |\sigma_i| |
\]

\[
\leq 2\gamma_1 \left( \xi_1^2 (K_i^{-1})^2 + \rho_i \right) |\sigma_i| |
\]

\[
2e_i^T \Omega_i \left( \psi(s(t)) - \psi(\hat{s}_i(t)) \right) \leq 2\gamma_1 \| K_i^{-1} \sigma_i \| |\sigma_i| |
\]

\[
\leq 2\gamma_1 \left( \xi_1^2 \Omega_i^2 + l_m \right) |\sigma_i| |
\]

\[
-2e_i^T \Omega_i X_i \sigma_i(t) - h_i(\hat{s}_i(t)) \leq 2\gamma_2 \| X_i^T \sigma_i \| |\sigma_i| |
\]

\[
\leq 2\gamma_2 \left( \xi_2 \Omega_i \| X_i^T \sigma_i \| + l_m \right) |\sigma_i| |
\]

Substituting (12) in (9) and using \( \dot{\hat{s}}_i(t) - \hat{s}_i(t) = e_i - e_j, r_j(t) - r_i(t) = - (\sigma_i - \sigma_j) \), we have

\[
\dot{V} \leq \sum_{i=1}^{N} \sigma_i^T \left( K_i^{-1} A + A^T K_i^{-1} + \xi_1^2 (K_i^{-1})^2 + l_m \right) \sigma_i \\
- 2 \sum_{i=1}^{N} \sum_{j \in N_i} q_{ij} \sigma_i^T K_i^{-1} BB^T K_i^{-1} \sigma_i - \sigma_j \\
- \sum_{i=1}^{N} 2\sigma_i^T K_i^{-1} BB^T K_i^{-1} (r_i(t) - \hat{s}_i(t)) \\
+ \sum_{i=1}^{N} e_i^T \left( \Omega_i A + A^T \Omega_i + \xi_1^2 \Omega_i^2 + \xi_2^2 \Omega_i X_i X_i^T \Omega_i \\
+ 2l_m e_i - 2\gamma \sum_{i=1}^{N} \sum_{j \in N_i} q_{ij} e_i^T [e_i - e_j].
\]
Using \((r_i(t) - s_i(t)) = (r_i(t) - s(t) + s(t) - s_i(t)) = \varepsilon_i + \varepsilon_i,\) one has
\[
\dot{V} \leq \sum_{i=1}^{N} \sigma_i^T (K_i^{-1}A + A^T K_i^{-1} + \frac{1}{2} \varepsilon_i^2 (K_i^{-1})^2 + l_m) \sigma_i
\]
\[
-2 \sum_{i=1}^{N} \sum_{j \in N_i} q_{ij} \sigma_i^T K_i^{-1} B B^T K_i^{-1} [\varepsilon_i - \varepsilon_j]
\]
\[
-2 \sigma_i^T K_i^{-1} B B^T K_i^{-1} \varepsilon_i + \sum_{i=1}^{N} -2 \sigma_i^T K_i^{-1} B B^T K_i^{-1} \varepsilon_i
\]
\[
+ \sum_{i=1}^{N} \varepsilon_i^T (\Omega_i A + A^T \Omega_i + \frac{1}{2} \varepsilon_i^2 \Omega_i^2 + \frac{1}{2} \varepsilon_i^2 \Omega_i X_i T \Omega_i
\]
\[
+ 2l_m) \varepsilon_i - 2 \gamma_i \sum_{i=1}^{N} \sum_{j \in N_i} q_{ij} \varepsilon_i^T [\varepsilon_i - \varepsilon_j].
\]
Applying (10) and (11) for \(-2 \sigma_i^T K_i^{-1} B B^T K_i^{-1} \varepsilon_i,\) one gets
\[
\dot{V} \leq \sum_{i=1}^{N} \sigma_i^T (K_i^{-1}A + A^T K_i^{-1} + \frac{1}{2} \varepsilon_i^2 (K_i^{-1})^2 + l_m
\]
\[
-K_i^{-1} B B^T K_i^{-1}) \sigma_i - 2 \sum_{i=1}^{N} \sum_{j \in N_i} q_{ij} \sigma_i^T K_i^{-1} B B^T K_i^{-1} [\varepsilon_i - \varepsilon_j]
\]
\[
+ \sum_{i=1}^{N} \varepsilon_i^T (\Omega_i A + A^T \Omega_i + \frac{1}{2} \varepsilon_i^2 \Omega_i^2 + \frac{1}{2} \varepsilon_i^2 \Omega_i X_i T \Omega_i
\]
\[
+ 2l_m + K_i^{-1} B B^T K_i^{-1} - 2 \gamma_i \varepsilon_i]
\]
\[
-2 \sigma_i^T K_i^{-1} B B^T K_i^{-1} \varepsilon_i + \sum_{i=1}^{N} -2 \sigma_i^T K_i^{-1} B B^T K_i^{-1} \varepsilon_i
\]
\[
+ \sum_{i=1}^{N} \varepsilon_i^T (\Omega_i A + A^T \Omega_i + \frac{1}{2} \varepsilon_i^2 \Omega_i^2 + \frac{1}{2} \varepsilon_i^2 \Omega_i X_i T \Omega_i
\]
\[
+ 2l_m + K_i^{-1} B B^T K_i^{-1} - 2 \gamma_i \varepsilon_i].
\]
Using graph theory and defining \(\sigma_i' = K_i^{-1} \sigma_i,\) we have
\[
\dot{V} \leq \sum_{i=1}^{N} \sigma_i'^T (A K_i + K_i A^T + \frac{1}{2} \varepsilon_i' l_m + K_i^2
\]
\[
- (1 + 2\lambda_2 (L)) B B^T) \sigma_i'
\]
\[
+ \sum_{i=1}^{N} \varepsilon_i'^T (\Omega_i A + A^T \Omega_i + \frac{1}{2} \varepsilon_i' \Omega_i^2 + \frac{1}{2} \varepsilon_i' \Omega_i X_i T \Omega_i
\]
\[
+ 2l_m + K_i^{-1} B B^T K_i^{-1} - 2 \gamma_i \lambda_2 (L) l_m) \varepsilon_i'.
\]
By choosing \(\rho_i\) so that \(\rho_i \leq 2 \gamma_i \lambda_2 (L)\) and utilizing Assumption 1, (7) and (8), one can obtain
\[
A K_i + K_i A^T + \frac{1}{2} \varepsilon_i' l_m + K_i^2
\]
\[
- (1 + 2 \lambda_2 (L)) B B^T
\]
\[
\leq A K_i + K_i A^T + \frac{1}{2} \varepsilon_i' l_m + K_i^2 - B B^T < 0,
\]
One can conclude from (13) and (14) that \(\dot{V} < 0.\) It implies that \(\varepsilon_i\) and \(\sigma_i\) converge to zero asymptotically for \(i = 1, \ldots, N\) and LNMAS (1) with (5) and (6) can asymptotically estimate and track the target (2) in the absence of external disturbances and input delay.

3.1. Distributed tracking control problem in the presence of input delay

In the presence of input delay, DC (6) is transformed into
\[
u_i(t - h) = B^T K_i^{-1} \sum_{j \in N_i} q_{ij} [\hat{r}_j(t - h) - r_i(t - h)]
\]
\[
- (r_i(t - h) - \hat{s}_i(t - h)).
\]
LNMAS (1) with DC (15) cannot track the target (2) because of the delay \(h.\) To solve this problem, the following DC is proposed:
\[
u_{wi}(t) = B^T K_i^{-1} \sum_{j \in N_i} q_{ij} [\hat{r}_j(t) - \hat{r}_i(t)] - (\hat{r}_i(t) - \hat{s}_i(t))
\]
(16)
where \((\hat{r}_j(t), \hat{r}_i(t), \hat{s}_i(t))\) are estimations of \((r_j(t + h), r_i(t + h), \hat{s}_i(t + h)).\) When \((\hat{r}_j(t), \hat{r}_i(t), \hat{s}_i(t))\) becomes equal to \((r_j(t + h), r_i(t + h), \hat{s}_i(t + h),\) (16) with delay \(h\) has the same performance of DC (6) and DTC is achieved in the presence of input delay. DC (15) includes the states of the agents and the target. Therefore, two predictors are needed to predict the future states of the agents and the target. Since agents and target have the same dynamics, a predictor is designed to predict future states of the agents and the target. The local future states predictor (FSP) is described by:
\[
\hat{r}_i(t) = A \hat{r}_i(t) + \psi \hat{r}_i(t) + u_{wi}(t) + \phi_i(r_i(t) - \hat{r}_i(t - h),
\]
(17)
where \(\phi_i > 0.\)

**Theorem 3.2:** Consider FSP (17). The state of \(\hat{r}_i(t)\) asymptotically converges to \(r_i(t + h)\) if the following condition holds:
\[
\Sigma_i = \left( \begin{array}{cc}
\Pi_i A + A^T \Pi_i + \frac{1}{2} \Pi_i^2 + l_m + M_i & -\Pi_i \phi_i \\
-\phi_i \Pi_i & -M_i
\end{array} \right) < 0
\]
where \(\Pi_i > 0\) and \(M_i > 0.\)
Proof: Let \( \dot{e}_i(t) = r_i(t+h) - \hat{r}_i(t) \). Then, one can obtain
\[
\dot{e}_i(t) = Ae_i(t) + \psi(r_i(t+h)) - \psi(\hat{r}_i(t)) - \phi_i e_i(t-h).
\]
Choose the Lyapunov function \( V_1(e_i(t)) = V_2 + V_3 \) where
\[
V_2 = \dot{e}_i^T(t) \Pi \psi(r_i(t+h)) - \psi(\hat{r}_i(t))
\]
and
\[
V_3 = \dot{e}_i^T(t) \Pi \psi(r_i(t+h)) - \psi(\hat{r}_i(t)) + \phi_i e_i(t-h).
\]
The time derivative of \( V_1(e_i(t)) \) is as follows:
\[
\dot{V}_1 = 2\dot{e}_i^T(t) \Pi \psi(r_i(t+h)) - \psi(\hat{r}_i(t)) + \phi_i e_i(t-h)
\]
where \( \dot{V}_1 \) and \( \phi_i \) are elements of the vector \( K \).

Using (3) and (4), we have
\[
2\dot{e}_i^T(t) \Pi [\psi(r_i(t+h)) - \psi(\hat{r}_i(t))] \leq 2\zeta_1||\Pi||e_i(t)||e_i(t)||
\]
\[
\leq \dot{e}_i^T(t) \Pi \Xi e_i(t).
\]
Putting (20) into (19), one can obtain
\[
V_1(e_i(t)) \leq \dot{e}_i^T(t) \Pi [\Pi + \Pi + \xi_1^2 \Pi + \Pi + \Pi] e_i(t)
\]
\[
+ \dot{e}_i^T(t)(-\Pi \Pi) e_i(t-h)
\]
\[
+ \dot{e}_i^T(t) e_i(t-h)
\]
\[
+ \dot{e}_i^T(t)(-\Pi - \Pi) e_i(t-h) = \chi_i^T(t) \Sigma_i \chi_i(t)
\]
where \( \chi_i(t) = [\dot{e}_i^T(t), e_i(t-h)]^T \).

Remark 3.1: To obtain parameters of the DE (5), DC (6) and FSP (17) (\( K_i, \Omega_i, X_i \)
and \( \phi_i \)), inequalities (7), (8) and (18) should be solved. To this goal, the inequalities are converted to the LMI
using Schur’s Complement (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994) as follows.

\[
A K_i + K_i A^T + \xi_1^2 \Pi + \Pi - \Pi - \Pi \leq 0, \quad \Sigma_i \leq 0
\]

where \( \Pi = \Omega_i X_i \) and \( \Sigma_i \) are elements of the vector \( \Pi \psi(r_i(t+h)) - \psi(\hat{r}_i(t)) \).

Remark 3.2: The existence of controller (6) depends on the feasibility of LMI (22). Using Finsler’s Lemma (Iwasaki and Skelton (1994)), it can be concluded that there is a \( K_i > 0 \) such that LMI (22) holds if and only if there is a \( K \) such that \( (A + BK_i) \hat{K}_i + (A + BK_i)^T + L_i + \xi_1^2 \Pi \leq 0 \)

where \( \hat{K}_i = K_i / \xi_1^2 \), which is dual of the observer design problem for the Lipschitz systems in Rajamani and Cho (1998) and Rajamani (1998). It implies that LMI (22) is feasible and also there exists controller (6) if and only if \((A, B)\) is controllable. Similarly, it can be proved that LMI (23) is feasible when \( \lambda_2(L) \) is positive, i.e. the graph related to the multiagent systems (1) is connected. Furthermore, LMI (24) is feasible if the matrix \( A \) is Hermit of the Boyd et al. (1994). Based on these explanations, if the mentioned conditions hold, then the LMI (22), (23) and (24) are feasible and the parameters of DE (5) and DC (6) can be computed. Therefore, the multiagent systems (1) can estimate and track the target (2) using the DE (5) and DC (6) with the computed parameters.

3.2. Distributed tracking control problem in the presence of external disturbance

It is evident that LNMAS (1) with DC (6) do not track the target (2) in the presence of external disturbance \( D_i(t) \). To solve this problem, the following finite time estimator (FTE) is used to estimate \( D_i(t) \) for each agent in a finite time (Shiessel, Shkolnikov, & Levant, 2007):

\[
\begin{align*}
\hat{r}_1(t) &= \hat{g}_1(r_1(t)) + \hat{u}_{id1}(t) - \hat{\xi}_1 \hat{D}_{11}^{1/2} \hat{\sigma}g
\times (\hat{r}_1(t) - r_1(t)) \hat{r}_1(t) - r_1(t) \hat{D}_{11}(t)
\hat{D}_{11}(t) &= -\beta_1 \hat{D}_{11} \hat{g}(\hat{\xi}_1) \hat{D}_{11}^{1/2} \hat{\sigma}g(\hat{r}_1(t) - r_1(t))
\times (\hat{r}_1(t) - r_1(t))^{1/2}
\end{align*}
\]

where \( \hat{D}_{11}(t) \) and \( \hat{D}_{11}(t) \) are estimations of \( \hat{D}_{11}(t) \) and \( \hat{D}_{11}(t) \).
Theorem 3.3: Consider LNMAS (1) which uses the DE (5) and the following DC
\[
\ddot{u}_{id}(t) = B^T K^{-1}_{i} \left( \sum_{j \in N_i} q_j [\tilde{r}_j(t) - r_j(t)] - (r_i(t) - \dot{s}_i(t) + \tilde{D}_i(t)) \right).
\]

Under Assumption 1, the agents can estimate and track the states of the target (2) asymptotically in the presence of external disturbance \( D_i(t) \).

Proof: We should prove that DC (26) in the presence of \( D_i(t) \) has the same performance of DC (6). To this end, assume that the estimation errors of FTE (25) are as follows:
\[
e_{r1i} = \tilde{r}_{1i}(t) - r_{1i}(t),
\]
\[
e_{d1i} = \tilde{D}_{1i}(t) - D_{1i}(t),
\]
\[
e_{rim} = \tilde{r}_{im}(t) - r_{im}(t),
\]
\[
e_{dim} = \tilde{D}_{im}(t) - D_{im}(t).
\]

Then, one obtains
\[
\dot{e}_{r1i} = -\xi_1 \tilde{D}_{1i}^{1/2} \text{sig}(e_{r1i})|e_{r1i}|^{1/2} + e_{d1i}
\]
\[
\dot{e}_{d1i} = -\beta_1 \tilde{D}_{1i} \text{sig}(e_{d1i} - \dot{e}_{r1i}) + [-\tilde{D}_{1i}, \tilde{D}_{1i}]
\]
\[
\vdots
\]
\[
\dot{e}_{rim} = -\xi_{im} \tilde{D}_{im}^{1/2} \text{sig}(e_{rim})|e_{rim}|^{1/2} + e_{dim}
\]
\[
\dot{e}_{dim} = -\beta_{im} \tilde{D}_{im} \text{sig}(e_{dim} - \dot{e}_{rim}) + [-\tilde{D}_{im}, \tilde{D}_{im}].
\]

Shtessel et al. (2007) proved that there exists a finite time \( T_1 \) such that the states of the system (27) converge to zero. Thus, for \( t \geq T_1 \), one has \( \tilde{D}_i(t) \equiv D_i(t) \). It implies that the DC (26) has the same performance as DC (6) in the presence of external disturbance \( D_i(t) \). Therefore, one can conclude that LNMAS (1) with DC (26) and DE (5) in the presence of \( D_i(t) \) can estimate and track the states of the target (2) asymptotically.

Remark 3.3: In this paper, the tracking control problem is solved for LNMASs with input delay or external disturbances. The proposed DTC algorithms cannot be used when there exist input delay and external disturbances simultaneously in the dynamics of LNMASs. The reason for this is the structure of DTC algorithms in both cases. Actually, in the presence of external disturbances, controller (26) is proposed based on the finite time estimator (25) (Shtessel et al. 2007) which can estimate external disturbances of the systems in the absence of input delay. This means that the controller (26) cannot be used in the presence of input delay. On the other hand, in the presence of input delay, the control law (16) is designed based on the FSP (17). In this case, the existence of external disturbances make us not to be able to prove asymptotically converges of the FSP (17). Therefore, the condition (18) and the parameters of FSP (17) cannot be obtained. This implies that the controller (16) cannot be applied to a LNMAS with external disturbances.

4. Simulation results

In this section, the efficiency of DE (5), DCs (6), (16) and (26) are validated. For this purpose, the following LNMAS is considered:
\[
\dot{r}_i(t) = \begin{pmatrix}
-1.175 & 0.9871 \\
-8.458 & -0.8776
\end{pmatrix} r_i(t) + \begin{pmatrix}
0 \\
0.33 \sin(f_2(t))
\end{pmatrix}
\]
\[
+ \begin{pmatrix}
-0.194 \\
-19.29
\end{pmatrix} u_i(t - h) + D_i(t), \quad i = 1, \ldots, 4.
\]

The topology among agents is demonstrated in Figure 1. The target is given by
\[
\dot{s}_i(t) = \begin{pmatrix}
-1.175 & 0.9871 \\
-8.458 & -0.8776
\end{pmatrix} s_i(t) + \begin{pmatrix}
0 \\
0.33 \sin(s_2(t))
\end{pmatrix}
\]
\[
+ \begin{pmatrix}
0.1 \\
0.1
\end{pmatrix} w(t).
\]

The sensing model of the \( i \)th agent is represented by:
\[
y_i = s_{1i}(t) + s_{2i}(t) + 0.1 v_i(t),
\]
where \( w(t) \) and \( v_i(t) \) are independent white noises with variances 0.1 and 0.01. The simulation parameters are as follows:
\[
\Omega_i = \begin{pmatrix}
2.2490 & -0.2213 \\
-0.2213 & 0.5481
\end{pmatrix},
\]
\[
X_i = \begin{pmatrix}
0.1875 \\
0.4319
\end{pmatrix},
\]
\[
\Phi_i = \begin{pmatrix}
0.1435 & 0.0319 \\
0.1345 & 0.3835
\end{pmatrix}, \gamma_i = 1, h = 0.5.
\]

Figure 1. The topology of LNMAS (28).
Figure 2. The tracking error of LNMAS (28) without input delay and external disturbance.

Figure 3. The estimation errors of DE (5) of LNMAS (28) without input delay and external disturbance.

Figure 4. The tracking error of LNMAS (28) with input delay.
Figure 5. The estimation errors of DE (5) of LNMAS (28) with input delay.

Figure 6. The tracking error of LNMAS (28) with external disturbances.

Figure 7. The estimation errors of DE (5) of LNMAS (28) with external disturbances.
\[ K_t = \begin{pmatrix} 1.5306 & -1.0239 \\ -1.0239 & 13.6092 \end{pmatrix}, \quad \zeta_1 = 0.33, \]
\[ \zeta_2 = 1.7321, q_{ij} = 1, \quad \lambda_2(L) = 2, \]
\[ D(t) = \begin{pmatrix} 1.5 \sin t \\ \sin t \end{pmatrix}, \quad \bar{D}_i = \begin{pmatrix} 1.5 \\ 1 \end{pmatrix}, \quad \xi_{1i} = 1.63, \]
\[ \xi_{2i} = 4, \quad \beta_{1i} = 1.33, \quad \beta_{2i} = 4. \]

At first, using the above parameters, LNMAS (28) with DC (6) and DE (5) are simulated in the absence of external disturbance and input delay \((D(t) = 0, h = 0)\). Figures 2 and 3 show tracking and estimation errors of LNMAS (28). These results verify the desired performance of DC (6) and DE (5) where the agents of LNMAS (28) estimate and track the states of the target (29) in the absence of external disturbance and input delay. In the following, to validate DC (16) and DE (5), LNMAS (28) with DC (16) and DE (5) are simulated in the absence of external disturbance \((D(t) = 0)\). Figures 4 and 5 illustrate tracking and estimation errors of LNMAS (28). These results show that the agents of LNMAS (28) estimate and track the states of the target (29) in the presence of input delay.

Finally, LNMAS (28) with DC (26) and DE (5) are simulated in the absence of the input delay \((h = 0)\) to validate the DC (26) and DE (5). Figures 6 and 7 demonstrate the tracking and estimation errors of LNMAS (28). These figures show that the agents of LNMAS (28) are successful to estimate and track the states of the target (29) at a desired time in the presence of external disturbances.

5. Conclusions

In this paper, a distributed tracking control problem has been investigated for LNMAs with external disturbances and input delay. Some novel DTC algorithms have been suggested and proved that every agent can estimate and track a Lipschitz target. The simulations have shown the effectiveness of the presented algorithms in distributed tracking control of a LNMAS.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory* (Vol. 15). Philadelphia, Pennsylvania, USA: Siam.
Fang, H., Wei, Y., Chen, J., & Xin, B. (2017). Flocking of second-order multiagent systems with connectivity preservation based on algebraic connectivity estimation. *IEEE Transactions on Cybernetics*, 47(4), 1067–1077.
Hong, H., Yu, W., Yu, X., Wen, G., & Alsaeedi, A. (2017). Fixed-time connectivity-preserving distributed average tracking for multiagent systems. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 64(10), 1192–1196.
Iwasaki, T., & Skelton, R. E. (1994). All controllers for the general H∞ control problem: LMI existence conditions and state space formulas. *Automatica*, 30(8), 1307–1317.
Jenabzadeh, A., & Safarinejadian, B. (2017). A Lyapunov-based distributed consensus filter for a class of nonlinear stochastic systems. *Automatica*, 86, 53–62.
Jenabzadeh, A., Safarinejadian, B., & Mohammadnia, F. (2017). Distributed consensus filter for a class of continuous-time nonlinear stochastic systems in sensor networks. *Asian Journal of Control*, 19(4), 1284–1294.
Li, W. (2015). Distributed output tracking of high-order nonlinear multi-agent systems with unstable linearization. *Systems & Control Letters*, 83, 67–73.
Li, C., Dong, L., & Nguang, S. K. (2017). Cooperative control of multi-agent systems with variable number of tracking agents. *IET Control Theory & Applications*, 11(12), 1922–1927.
Li, Z., Liu, X., Fu, M., & Xie, L. (2012). Global H∞ consensus of multi-agent systems with Lipschitz nonlinear dynamics. *IET Control Theory & Applications*, 6(13), 2041–2048.
Li, Q., Shen, B., Wang, Z., & Alsaadi, F. E. (2017). A sampled-data approach to distributed H∞ resilient state estimation for a class of nonlinear time-delay systems over sensor networks. *Journal of the Franklin Institute*, 354(15), 7139–7157.
Li, Q., Shen, B., Wang, Z., & Alsaadi, F. E. (2018). An event-triggered approach to distributed H∞ state estimation for state-saturated systems with randomly occurring mixed delays. *Journal of the Franklin Institute*, 355(6), 3104–3121.
Miao, Z., Wang, Y., & Fierro, R. (2017). Cooperative circumnavigation of a moving target with multiple nonholonomic robots using backstepping design. *Systems & Control Letters*, 103, 58–65.
Miranbeigi, M., Moshiri, B., & Rahimi Kian, A. (2016). Application of distributed control on a large-scale production/distribution/inventory system. *Systems Science & Control Engineering*, 4(1), 68–77.
Olfati-Saber, R., & Jalalkamali, P. (2012). Coupled distributed estimation and control for mobile sensor networks. *IEEE Transactions on Automatic Control*, 57(10), 2609–2614.
Rajamani, R. (1998). Observers for Lipschitz nonlinear systems. *IEEE Transactions on Automatic Control*, 43(3), 397–401.
Rajamani, R., & Cho, Y. M. (1998). Existence and design of observers for nonlinear systems: Relation to distance to unobservability. *International Journal of Control*, 69(5), 717–731.
Shen, B., Wang, Z., & Liu, X. (2011). A stochastic sampled-data approach to distributed H∞ filtering in sensor networks. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 58(9), 2237–2246.
Shtessel, Y. B., Shkolnikov, I. A., & Levant, A. (2007). Smooth second-order sliding modes: Missile guidance application. *Automatica*, 43(8), 1470–1476.
Su, H., Chen, X., Chen, M. Z., & Wang, L. (2016). Distributed estimation and control for mobile sensor networks with coupling delays. *ISA Transactions*, 64, 141–150.
Su, H., Li, Z., & Chen, M. Z. (2017). Distributed estimation and control for two-target tracking mobile sensor networks. *Journal of the Franklin Institute*, 354(7), 2994–3007.
Tian, B., Zuo, Z., & Wang, H. (2017). Leader–follower fixed-time consensus of multi-agent systems with high-order integrator dynamics. *International Journal of Control*, 90(7), 1420–1427.

Wang, Q., Gao, H., Alsaadi, F., & Hayat, T. (2014). An overview of consensus problems in constrained multi-agent coordination. *Systems Science & Control Engineering: An Open Access Journal*, 2(1), 275–284.

Wei, G., Liu, S., Wang, L., & Wang, Y. (2016). Event-based distributed set-membership filtering for a class of time-varying non-linear systems over sensor networks with saturation effects. *International Journal of General Systems*, 45(2), 532–547.

Wen, G., Duan, Z., Chen, G., & Yu, W. (2014). Consensus tracking of multi-agent systems with Lipschitz-type node dynamics and switching topologies. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 61(2), 499–511.

Wen, G., Yu, W., Xia, Y., Yu, X., & Hu, J. (2017). Distributed tracking of nonlinear multiagent systems under directed switching topology: An observer-based protocol. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 47(5), 869–881.

Yu, J., Dong, X., Li, Q., Liu, F., Ren, Z., & Ni, H. (2017). Distributed adaptive practical time-varying tracking control for second-order nonlinear multi-agent system using neural networks. In *Control & Automation (ICCA), 2017 13th IEEE International Conference on* (pp. 976–981). IEEE.

Yu, M., Yan, C., & Li, C. (2017). Event-triggered tracking control for couple-group multi-agent systems. *Journal of the Franklin Institute*, 354(14), 6152–6169.

Zhao, X. W., Guan, Z. H., Li, J., Zhang, X. H., & Chen, C. Y. (2017). Flocking of multi-agent nonholonomic systems with unknown leader dynamics and relative measurements. *International Journal of Robust and Nonlinear Control*, 27(17), 3685–3702.

Zhao, L., & Jia, Y. (2016). Neural network-based adaptive consensus tracking control for multi-agent systems under actuator faults. *International Journal of Systems Science*, 47(8), 1931–1942.