Models of Light Singlet Fermion and Neutrino Phenomenology

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Abstract

We suggest that a singlet fermion $S$ exists beyond the standard see-saw structure. It mixes with light neutrinos via interactions with the right-handed neutrino components, so that $\nu_e \to S$ conversion solves the solar neutrino problem. Supersymmetry endowed with R-symmetry is shown to give a natural framework for existence, mass scale and mixing ($\sin^2 2\theta_{es} \sim (0.1 - 1.5) \cdot 10^{-2}$) of such a fermion. Models with an approximate horizontal symmetry are constructed, which embed the fermion $S$ and explain simultaneously solar, atmospheric, hot dark matter problems as well as may predict the oscillation $\bar{\nu}_\mu \to \bar{\nu}_e$ in the region of sensitivity of KARMEN and LSND experiments.
1 Introduction

The solar neutrino problem [1], the deficit of muon neutrinos in atmospheric neutrino flux [2], the large scale structure of the Universe [3] and possible candidate events in a search for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations [4] (see however [5]) give indications on non-zero neutrino masses and lepton mixing. Simultaneous explanation of all (or some) of these problems may call for the existence of more than three light neutrinos which mix among themselves [6]. Strong bounds on the number of neutrino species both from the invisible $Z^0$–width and from primordial nucleosynthesis (NS) [7] require the additional neutrino to be sterile (singlets of $SU(2) \times U(1)$). The right-handed (RH) components of known neutrinos are natural candidates for such sterile states. However, in such a case one has to depart from the conventional see-saw mechanism which implies large masses to the RH components.

A number of schemes with light sterile neutrinos has been suggested [8]–[12]. Most of them are based on radiative mechanism of mass generation or on some hybrid schemes which include both the elements of the see-saw and radiative mechanisms. In these schemes sterile neutrino is considered on the same footing as the usual neutrinos. The lepton number is broken typically at the electroweak scale.

We will consider another possibility. We suggest that usual see-saw mechanism works with all three right-handed neutrinos having large Majorana masses: $M \sim 10^{10} – 10^{12}$ GeV. At the same time the theory contains an additional singlet fermion $S$ which has its origin beyond the standard lepton structure. The singlet $S$ is very light and mixes with neutrinos.

Supersymmetry (SUSY) can provide a natural justification for the existence of $S$. Many extensions of the standard model contain singlet scalar fields: singlet majoron [13], invisible axion [14], or scalars for spontaneous generation of the $\mu$–term [15], etc.. The supersymmetric partners of such scalars could be identified with $S$. Moreover, SUSY can play a crucial role in the determination of mass scales in the singlet sector.

In this paper we consider possible origin of light fermion $S$, its mass and mixing with light neutrinos. The models with $S$ are constructed so that they can simultaneously explain the above mentioned neutrino anomalies.
2 Light singlet fermion and the solar neutrino problem

Primordial nucleosynthesis (as well as the data from SN87A) gives strong bound on the oscillation of active neutrinos into sterile neutrino \([16]\). This practically excludes \(\nu_\mu \rightarrow S\) oscillations as a solution of the atmospheric neutrino problem. The singlet fermion with mass in the eV–range could be considered as a candidate for hot dark matter (HDM) \([6]\). However, if its density satisfies the NS bound on the number of additional neutrino species: \(\delta N_\nu < 0.1\), it can not reproduce the optimal parameters \([3]\) for the large scale structure formation in the Universe: \(m_s \sim (2 - 5)\) eV and \(\Omega_s \simeq 0.2\), where \(\Omega_s\) is the energy density of \(S\) in the Universe in the unit of the critical density. Therefore it may happen that the only place where singlet fermion plays a role is the solar neutrino problem.

Let us find the region of parameters \(\Delta m^2\) and \(\sin^2 2\theta_{es}\), where \(\theta_{es}\) is the mixing angle of \(\nu_e\) with \(S\) for which the resonance conversion \(\nu_e \rightarrow S\) inside the Sun can explain the existing data. It is instructive to compare the sterile (\(\nu_e \rightarrow S\)) and the active neutrino (\(\nu_e \rightarrow \nu_f\)) cases. The \(\nu_e \rightarrow S\) solution of the solar neutrino problem differs from the \(\nu_e \rightarrow \nu_\mu (\nu_\tau)\) solution in two ways.

1. The effective density, \(\rho_s\), for \(\nu_e \rightarrow S\) conversion is smaller than that, \(\rho_f\), for \(\nu_e \rightarrow \nu_\mu\) conversion:
   \[
   \frac{\rho_s}{\rho_f} = \frac{Y_e - \frac{1}{2} Y_n}{Y_e}.
   \]
   Here \(Y_e\) and \(Y_n\) are the number densities of the electron and the neutron per nucleon, respectively. In the center of the Sun one gets \(\rho_s/\rho_f \simeq 0.76\) \([17]\). The central density \(\rho_c\) determines position of the adiabatic edge of the suppression pit: \((E/\Delta m^2)_a \propto 1/\rho_c\). Consequently, in the \(\nu_e \rightarrow S\) case the adiabatic edge is shifted to larger \(E/\Delta m^2\) in comparison with the flavour case: \((E/\Delta m^2)_s = (E/\Delta m^2)_f \cdot \rho_f/\rho_s\). The position of the nonadiabatic edge depends on \(\dot{\rho}/\rho\) and the difference between the flavour and sterile cases is practically negligible.

In the region of small mixing solutions \([4]\) the allowed values of \(\Delta m^2\) are determined essentially by \((E/\Delta m^2)_a\) and by Gallium experiment data \([18]\). Therefore the shift of the adiabatic edge for \(\nu_e \rightarrow S\) to larger \(E/\Delta m^2\) results in corresponding shift of \(\Delta m^2\) to smaller values:
   \[
   \Delta m^2|_s \simeq \frac{\rho_c}{\rho_f} \Delta m^2|_f \sim 0.76 \Delta m^2|_f .
   \]

2. The fermion \(S\) has no weak interactions, and therefore \(S\) flux from \(\nu_e \rightarrow S\) conversion does not contribute to the Kamiokande signal (\(\nu e \rightarrow \nu e\) scattering) in contrast with flavour

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\(^1\)Large mixing domain is excluded by primordial nucleosynthesis data
case, where $\nu_\mu$ interacts via neutral currents. This influences the allowed region of mixing angles. Indeed, for unfixed original Boron neutrino flux (which has the largest, $\sim 50\%$, theoretical uncertainties) the bound on $\sin^2 2\theta_{es}$ is determined by the “double ratio” \cite{18}:

$$R_{H/K} \equiv \frac{R_{Ar}}{R_{\nu_e}},$$

where $R_{Ar} \equiv Q^{obs}_{Ar}/Q^{SSM}_{Ar}$ and $R_{\nu_e} \equiv \Phi^{obs}_{B}/\Phi^{SSM}_{B}$ are the suppressions of signals in Cl–Ar and Kamiokande experiments, respectively. Here $Q^{SSM}_{Ar}$, $\Phi^{SSM}_{B}$ are the predictions in the reference model (e.g. \cite{19}) and $Q^{obs}_{Ar}$, $\Phi^{obs}_{B}$ are the observable signals. Due to the $\nu_\mu$ ($\nu_\tau$)–effect, $R^s_\nu$ in the sterile case is smaller than $R^f_\nu$ in the flavour case, and since $R_{Ar}$ is the same in both cases one gets $R^s_{H/K} > R^f_{H/K}$. With diminishing of $\theta_{es}$, the suppression of $\Phi_B$ due to conversion weakens and the effect of $\nu_\mu$ ($\nu_\tau$) decreases, therefore $R^s_{H/K}$ approaches $R^f_{H/K}$. As a consequence, the lower bound on $\theta_{es}$ coincides practically with that for flavour conversion: $\sin^2 2\theta_{es} \gtrsim (0.8 - 1.0) \cdot 10^{-3}$ \cite{18}. On the contrary, with increase of $\theta_{es}$ the suppression of $\Phi_B$ due to the conversion becomes stronger, so that $R_{Ar} \to 0$ and $R^s_{\nu_e} \to 0$; at the same time due to the neutral current effect of $\nu_\mu$ ($\nu_\tau$), $R^f_{\nu_e} \to 0.16$. Therefore $R^f_{H/K} \to 0$, whereas $R^s_{H/K}$ does not change strongly. We have found $R^s_{H/K} \simeq 0.77, 0.74, 0.72, 0.69$ for $\sin^2 2\theta_{es} \simeq 2 \cdot 10^{-3}, 5 \cdot 10^{-3}, 10^{-2}, 2 \cdot 10^{-2}$, respectively. The experimental value of the double ratio is $R_{H/K} = 0.67 \pm 0.11$. However for large $\sin^2 2\theta_{es}$ the original flux of Boron neutrinos should be large (to compensate for strong suppression effect). If we restrict $\Phi_B \leq 1.5 \Phi^{SSM}_{B}$, then the bound on the mixing angle becomes: $\sin^2 2\theta_{es} \lesssim 1.5 \cdot 10^{-2}$. This also satisfies the NS bound \cite{16}.

Resonance conversion implies that $m_S > m_{\nu_e}$ and if there is no fine-tunning of masses, $m_S \simeq \sqrt{\Delta m^2}$. Thus using (1) and known results for flavour conversion as well as bounds on $\sin^2 2\theta_{es}$ discussed above we get the following range of the parameters:

$$m_S \simeq (2 - 3) \cdot 10^{-3} \text{eV} \qquad \sin \theta_{es} \simeq \tan \theta_{es} \simeq (2 - 6) \cdot 10^{-2}. \quad (2)$$

3 Mass and mixing of singlet fermion via right-handed neutrino

Let us consider the following Lagrangian,

$$\mathcal{L} = \frac{m_e}{\langle H_2 \rangle} L_e \nu^c_e H_2 + \frac{M_e}{2} \nu^c_e \nu^c_e + m_{es} \nu^c_e S,$$ \quad (3)

where $L_e$ is the lepton doublet, $H_2$ is the Higgs doublet and $\nu^c_e$ is the right-handed neutrino component. We suggest that there is no direct coupling of $S$ with $L_e$ due to a certain symmetry,
and the mass term $SS$ is absent or negligibly small. The Dirac mass $m_e$ and the mixing mass $m_{es}$ are much smaller than the Majorana mass $M_e$: $m_e, m_{es} \ll M_e$. The Lagrangian (3) leads to the mass matrix in the basis $(S, \nu_e, \nu^c_e)$:

$$
\mathcal{M} = \begin{pmatrix}
0 & 0 & m_{es} \\
0 & 0 & m_e \\
m_{es} & m_e & M_e
\end{pmatrix} .
$$

(4)

The diagonalization of (4) is straightforward: one combination of the $\nu_e$ and $S$,

$$
\nu_0 = \cos \theta_{es} \nu_e + \sin \theta_{es} S ,
$$

is massless, and the orthogonal combination,

$$
\nu_1 = \cos \theta_{es} S - \sin \theta_{es} \nu_e ,
$$

acquires a mass via the see-saw mechanism:

$$
m_1 \simeq -\frac{m_e^2 + m_{es}^2}{M_e} .
$$

(5)

The mass of the heavy neutrino is $\simeq M_e$. The $\nu_e-S$ mixing angle is determined by

$$
\tan \theta_{es} = \frac{m_e}{m_{es}} ,
$$

(6)

and correspondingly $\sin^2 2\theta_{es} = 4[m_{es} m_e / m_{es}^2 + m_e^2]$. Taking for $m_e$ the typical Dirac mass of the first generation: $m_e \sim (1 - 5) \text{MeV}$, and suggesting that $\nu_e \rightarrow S$ conversion explains the solar neutrino problem with $m_1 = m_S$ as in (2), we find

$$
m_{es} = \frac{m_e}{\tan \theta_{es}} \simeq (0.02 - 0.3) \text{GeV} .
$$

(7)

According to (3) the RH mass scale is

$$
M_e \simeq \frac{m_{es}^2}{m_1} = \frac{m_e^2}{m_1 \tan^2 \theta_{es}} \simeq (10^8 - 3 \cdot 10^{10}) \text{GeV} .
$$

(8)

Consider now the models which lead to the Lagrangian (3) with parameters (3) and (8).

The simplest possibility is to use the $U(1)$ symmetry of lepton number and to generate the masses in (3) by VEV $\langle \sigma \rangle$ of the scalar singlet, $\sigma$. Prescription of the lepton charges $(1, -1, -3, 2)$ for $(\nu_e, \nu^c_e, S, \sigma)$ admits the following interactions in the singlet sector:

$$
\mathcal{L} = h \nu^c_e \nu_e \sigma + h^l \nu^c_e S \frac{\sigma^2}{M_{Pl}} + h^u SS \frac{\sigma^3}{M_{Pl}^2} ,
$$

(9)
where \( M_{Pl} \) is the Planck mass. The Lagrangian (9) reproduces the mass terms of (3) with \( M_e = h\langle \sigma \rangle \) and \( m_{es} \simeq h'\langle \sigma \rangle^2/M_{Pl} \). The desired values of \( M_e \) and \( m_{es} \) (3), (8) can be achieved with e.g., \( \langle \sigma \rangle \simeq 10^{10} \) GeV, \( h \simeq 1 \) and \( h' \simeq 10^{-2} \). The last term in (9) generates the Majorana mass of \( S \), \( m_{SS} = h''\langle \sigma \rangle^3/M_{Pl} \), so that all the neutrinos are massive. For \( h'' < \sim 10^{-4} \) one gets \( m_1 \simeq m_{es}^2/M_e \) as before, whereas the smallest mass is \( \simeq m_{es}^2/m_{SS} < m_1 \).

Let us consider the possible role of supersymmetry in the appearance of the singlet fermion and in the determination of its properties. In principle, \( S \) can be a superpartner of the goldstone boson which appears as a result of spontaneous violation of a certain global symmetry like lepton number or Peccei-Quinn symmetry. In this connection, let us consider a SUSY model with spontaneous violation of lepton number. The superpotential of the singlet majoron model is

\[
W = \frac{m_e}{\langle H_2 \rangle} L_e \nu^c e H_2 + f \nu^c \nu^c \sigma - \lambda (\sigma \sigma' - M^2) y ,
\]

where lepton numbers of the superfields \((L_e, \nu^c e, \sigma, \sigma', H_2, y)\) are \((1, -1, 2, -2, 0, 0)\). Lepton number is spontaneously broken by non-zero VEV’s of \( \sigma \) and \( \sigma' \). As the result, the majoron and its fermionic partner, the majorino, are massless in the supersymmetric limit.

The identification of the majorino with \( S \) requires, however, the following complication of the model.

(1) Supersymmetry breaking results in appearance of non-zero VEV of \( y \) which generates the mass of the majorino \( S \): \( m_{SS} = \lambda \langle y \rangle \). The soft-breaking terms \( \lambda (A_y \sigma \sigma' - B_y M^2) y + \text{h.c.} \), where \( A_y, B_y \simeq \mathcal{O}(m^3/2) \) are soft-breaking parameters give

\[
\langle y \rangle \simeq \frac{1}{2\lambda} (A_y - B_y) ,
\]

and consequently too big value of \( m_{SS} \simeq (A_y - B_y)/2 \sim \mathcal{O}(m^3/2) \), whereas \( A_y - B_y \lesssim 10^{-3} \) eV is needed. One can get \( A_y - B_y = 0 \) at tree level in no-scale supergravity or in the case of the non-minimal kinetic term discussed below. However, non-zero value of \( A_y - B_y \) will be generated due to renormalization group evolution of soft-terms. In order to suppress the mass below the solar neutrino mass scale (3), tuning of parameters is needed: \( f \lesssim 10^{-5} \), if all three generation of leptons are taken into account.

(2) Mixing of neutrinos with the majorino implies violation of R-parity. In (10) the mixing can be induced by the second term if sneutrino \( \tilde{\nu}^c \) gets non-zero VEV \( \langle \tilde{\nu}^c \rangle \). The latter requires the introduction of terms like \( \nu^c F(X_i) + W'(X_i) \), where \( F(X_i) \) and \( W'(X_i) \) are the functions of new superfields \( X_i \). They should be arranged in such a way that in the global SUSY limit
F does not get a VEV: \( \langle F(X_i) \rangle = 0 \), and after lepton number breaking linear term \( \sim M^2 \nu^c_e \) appears in the superpotential. Then the corresponding soft-term will give the mixing mass. One can find that the additional sector requires at least several new fields with non-zero lepton numbers which leads to further complication. R-parity violation is a general feature of models in which \( S \) is identified with fermionic superpartner of scalars acquiring non-zero VEV as in models for majoron, axion and \( \mu \)–term.

The above problems can be avoided in models with R-parity conservation. In this case, the lightest supersymmetric particle can be served as cold dark matter of the Universe. To preserve R-parity one should place the singlet \( S \) in the superfield with zero VEV. Consider the superpotential:

\[
W = \frac{m_e}{\langle H_2 \rangle} L_e \nu^c_e H_2 + f \nu^c_e \nu^c_e \sigma + f' \nu^c_e S y - \frac{\lambda}{2}(\sigma^2 - M^2)y.
\]

(12)

Its structure is determined by the R–symmetry under which the fields carry the R–charges:

\[ (1, 1, -1, 2, 0, 0) \] for \( (L_e, \nu^c_e, S, y, \sigma, H_2) \).

Note that the R–symmetry forbids the bare mass terms \( SS \) as well as the coupling \( SS\sigma \). Since lepton symmetry is explicitly broken no majoron appears. In the global SUSY limit, \( \sigma \) gets non-zero VEV \( \langle \sigma \rangle \sim M \sim 10^{11} \) GeV which generates the Majorana mass of \( \nu^c_e \): \( M_e = f \langle \sigma \rangle \).

SUSY breaking induces the following soft-breaking terms in the scalar potential:

\[
V_{\text{soft}} = \left\{ A_L \frac{m_e}{\langle H_2 \rangle} L_e \nu^c_e H_2 + f A_y \nu^c_e \nu^c_e \sigma + f' A_S \nu^c_e S y - \frac{\lambda}{2}(A_y \sigma^2 - B_y M^2)y + \text{h.c.} \right\} + \sum_i m^2_i |z_i|^2,
\]

(13)

where \( z_i \) denotes the fields appearing in the superpotential (12) and \( A_L, \) etc., are the soft-breaking parameters. Minimization of the potential shows the following:

1. The fields \( L_e, \nu^c_e, S \) do not develop VEV and therefore R-parity is unbroken.
2. The field \( y \) acquires non-zero VEV due to the soft-breaking terms as in (11). Consequently, the mixing mass for \( S \) and \( \nu^c_e \) appears:

\[
m_{es} = \frac{f'}{2\lambda}(A_y - B_y)
\]

(14)

Since \( m_{es} >> m_1 \), no strong tuning of \( A_y - B_y \) is needed as in the previous case (11). At \( A_y - B_y \sim O(m_3/2) \), the desired value of \( m_{es} \) (11) can be obtained by choosing \( f'/\lambda \sim 10^{-3} - 10^{-2} \). However, more elegant possibility is that \( A_y = B_y \) at the Planck scale but a
non-zero value for $A_y - B_y$ is generated due to renormalization group evolution through the differences in interactions of $\sigma$ and $y$. In this case one expects

$$m_{es} \sim \frac{\bar{\lambda}^2}{16\pi^2} m_{3/2},$$

(15)

where $\bar{\lambda}$ represents a combination of the constants $\lambda, f$ and $f'$. As a consequence, the value $m_{es} \sim 0.1$ GeV does not require smallness of $\bar{\lambda}$ or $f'$.

The equality $A_y = B_y$ at the Planck scale can be achieved by the introduction of non-minimal kinetic term with mixings between the observable and hidden sectors. Let us introduce the following Kähler potential:

$$K = \mathcal{C} \overline{\mathcal{C}} + \mathcal{C} \mathcal{C} (a \frac{Z}{M_{Pl}} + \overline{\sigma} \frac{Z}{M_{Pl}}) + Z \overline{Z},$$

(16)

where $C$ and $Z$ represent an observable and hidden sector field, respectively. Then usual assumption that the observable sector has no direct coupling to the hidden sector in superpotential, $W = W(C) + W(Z)$, leads to the universal soft-terms:

$$V_{soft} \sim m_{3/2} W(C) + \text{h.c.},$$

(17)

provided $\overline{a} = \langle W(Z) \rangle / \langle M_{Pl} \partial W / \partial Z + W(Z) Z / M_{Pl} \rangle$. Note also that the field $C$ does not acquire a soft-breaking mass. This mechanism can be generalized to arbitrary number of observable sector fields. For our purpose $C \equiv \sigma, y$, i.e., we couple $\sigma$ and $y$ to the hidden sector field $Z$ with the above-mentioned choice for $a$.

Note that $\sigma$ field plays two-fold role in the model: it gives Majorana mass of $\nu^c$ and it also generates mixing of $\nu^c$ with $S$ by inducing a VEV for $y$ after the SUSY breakdown. Moreover, $\sigma$ can be used to generate the $\mu$–term via the non-renormalizable interaction:

$$\frac{\sigma^2}{M_{Pl}} H_1 H_2.$$

(18)

The $\mu$–term can also be generated through the renormalizable interaction: $yH_1 H_2$ in the case of $\langle y \rangle \simeq \mathcal{O}(m_{3/2})$.

It is easy to incorporate the spontaneous violation of lepton number or/and Peccei-Quinn symmetry into the model. As in (14) one should introduce the superfield $\sigma'$ with lepton number $-2$ and zero R-charge and replace the $\sigma^2$ term of (12) by $\sigma \sigma'$. In this way the $\mu$–term (18) can be naturally related to the solution of the strong-CP problem via Peccei-Quinn mechanism [21], and the majoron will coincide with the invisible axion [22].
4 Models with light singlet fermion

Two other neutrinos, $\nu_\mu$ and $\nu_\tau$, can be included in the scheme by adding to (3) analogous terms with $L_\alpha$ and $\nu_\alpha^c$ ($\alpha = \mu, \tau$). Then as in (3), mixing of these neutrinos with singlet $S$ is determined by $\tan \theta_{\alpha s} = m_{\alpha s}/m_\alpha$, where $m_{\alpha s}$ and $m_\alpha$ are the corresponding mixing and Dirac masses.

Primordial nucleosynthesis gives strong bounds on the angles $\theta_{\alpha s}$ and/or on masses of light neutrino components: $\sim m_{\alpha s}^2/M_\alpha$. Suppose that $S$ is family blind, and its couplings with all neutrinos are universal: $m_{es} \simeq m_{\mu s} \simeq m_{\tau s} \simeq (0.02 - 0.3)$ GeV. Note that this mass scale (motivated by solar neutrinos) is of the order of Dirac masses in the second generation. Then with $m_{\mu s} \sim m_\mu \sim 0.3$ GeV, one gets $\tan \theta_{\mu s} \simeq 1$, and if $m_2 \simeq m_{\mu s}^2/M_\mu \simeq 0.1$ eV, the oscillation $\nu_\mu \rightarrow S$ could explain the deficit of atmospheric neutrinos. However, this possibility is strongly disfavoured by NS data. For $m_\mu \simeq 1$ GeV one has $\sin^2 2\theta_{\mu s} \simeq (0.2 - 4) \cdot 10^{-2}$, and the NS bound [16] is satisfied if $\Delta m^2 \simeq (10^{-4} - 10^{-3})$ eV$^2$, or $m_2 \simeq 3 \cdot 10^{-2}$ eV. For the third generation ($m_\tau \sim 100$ GeV), analogous figures are: $\sin^2 2\theta_{\tau s} \simeq (0.2 - 5) \cdot 10^{-6}$ and $m_3 < 3$ eV. Therefore, the cosmologically interesting masses of $\nu_\tau$ are admitted. Note that the bound on $m_2$ form NS and values of $m_1$ and $m_3$ desired by solar and HDM problems can be reproduces by moderate mass hierarchy of the RH neutrinos: $M_\alpha \simeq 10^{10} - 10^{12}$ GeV.

To have simultaneously neutrinos as HDM and the solution of the atmospheric neutrino problem via $\nu_\mu \rightarrow \nu_\tau$ oscillations one needs $m_2 \simeq m_3 \simeq 2$ eV. In this case ($\Delta m^2 \simeq 4$ eV$^2$) the NS bound: $\sin^2 2\theta_{\mu s} \simeq 10^{-6}$ implies $m_{\mu s}/m_\mu < 5 \cdot 10^{-4}$ or $m_{\mu s} \simeq 0.5$ MeV at $m_\mu \sim 1$ GeV, i.e. the coupling of $S$ with $\nu_e$ should dominate: $m_{es} >> m_{\mu s}$.

Both the dominance of $S-\nu_e^c$ coupling and the near degeneracy of neutrinos corresponding to the second and the third generations ($m_2 \simeq m_3$) can arise as consequences of some family (horizontal) symmetry.

Let us consider $U(1)^h$–symmetry with charge prescription $(0,1,-1)$ for the first, the second and third generations of leptons, respectively. Each generation includes the left-handed doublet $L_\alpha$ and the right-handed $\nu_\alpha R, e_\alpha R$. Higgs doublets as well as new particles $S, \sigma, \sigma', y$ have zero charges. In the limit of exact symmetry, the Higgs doublet and the singlet fermion $S$ can couple only with the electron neutrino, reproducing the matrix (3). The couplings for the second and third generations allowed by $U(1)^h$:

$$W = \frac{m_\mu}{\langle H_2 \rangle} L_\mu \nu_\mu^c H_2 + \frac{m_\tau}{\langle H_2 \rangle} L_\tau \nu_\tau^c H_2 + \frac{M}{\langle \sigma \rangle} \nu_\mu^c \nu_\tau^c \sigma$$

(19)
lead to a mass matrix in \((\nu_\mu, \nu_\tau, \nu_\mu^c, \nu_\tau^c)\) basis

\[
\begin{pmatrix}
0 & 0 & m_\mu & 0 \\
0 & 0 & 0 & m_\tau \\
m_\mu & 0 & 0 & M_{\mu\tau} \\
0 & m_\tau & M_{\mu\tau} & 0
\end{pmatrix}.
\tag{20}
\]

The mass matrix of charged leptons is diagonal. The diagonalization of (20) results in ZKM-type (Zeldovich-Mahmoud-Konopinsky) light neutrino formed by \(\simeq \nu_\mu\) and \(\simeq \nu_\tau\) components with mass

\[
m_2 = -m_3 = \frac{m_\mu m_\tau}{M_{\mu\tau}}.
\tag{21}
\]

For \(m_\mu \sim 1\) GeV, \(m_\tau \sim 100\) GeV and \(M_{\mu\tau} \sim 3 \cdot 10^{16}\) GeV one gets \(m_2 \simeq 3\) eV which is required for the HDM components. In the limit of exact horizontal symmetry \(\nu_e - S\) and \(\nu_\mu - \nu_\tau\) form two unmixed blocks and in particular, \(m_{\mu s} = m_{\tau s} = 0\).

Family symmetry can be conserved at high scale but can be explicitly broken by interactions with Higgs doublets. Such breaking could be induced spontaneously also by introducing new Higgs doublets with non-zero \(U(1)^h\) charges (±1 or 2) or by non-renormalizable interactions of the type: \(L_\epsilon \nu_\epsilon^c H_2 \sigma_\mu/M\), where \(\sigma_\mu\) has the charge +1 and acquire the VEV at large scale, \(\langle \sigma_\mu \rangle \sim 10^{-4}M\).

Violation of \(U(1)^h\) leads to mass splitting in \(\nu_\mu - \nu_\tau\) system as well as to mixing between \(\nu_e - S\) and \(\nu_\mu - \nu_\tau\) blocks. Consider the phenomenological consequences of introducing \(U(1)^h\) violation separately in different sectors of the model.

1. The non-diagonal Dirac mass terms \(m_{\mu\tau}\nu_\mu \nu_\tau^c + m_{\tau\mu}\nu_\tau \nu_\mu^c\) result in mass-squared difference

\[
\Delta m_{23}^2 \simeq 4 \frac{m_{\mu\tau} m_2}{m_\mu}.
\tag{22}
\]

For the atmospheric neutrinos one needs \(\Delta m_{23}^2 \simeq 10^{-2}\) eV\(^2\), then for \(m_2 \sim 2\) eV and \(m_\mu \simeq 1\) GeV, it follows from (22) that \(m_{\tau\mu}\) should be very small: \(\simeq (0.5 - 1)\) MeV. Mixing of \(\nu_\mu\) and \(\nu_\tau\) is practically maximal.

2. The introduction of a diagonal element in the Majorana sector; e.g., \(M_\tau \nu_\tau^c \nu_\tau^c\), gives

\[
\Delta m_{23}^2 \simeq 2 \frac{m_\mu}{m_\tau} \left( \frac{M_\tau}{M_{\mu\tau}} \right) m_2^2,
\tag{23}
\]

and to have \(\Delta m_{23}^2 \simeq 10^{-2}\) eV\(^2\) with \(m_\mu/m_\tau \sim 2 \cdot 10^{-2}\), one needs \(M_\tau/M_{\mu\tau} \sim 0.1\).
(3) To get $\nu_e-\nu_\mu$ mixing one can introduce the Dirac mass terms $m_{e\mu}\nu_e\nu_\mu^c + m_{\mu\mu}\nu_\mu\nu_e^c + \text{h.c.}$. Present sensitivity region of KARMEN and LSND: $\sin^2 2\theta_{e\mu} \sim (3 - 5) \times 10^{-3}$ corresponds to $m_{e\mu}/m_\mu \simeq 3 \times 10^{-2}$, and consequently to $m_{e\mu} \simeq 30$ MeV. In this case $\nu_\mu-S$ mixing will also be generated with $\tan \theta_{\mu s} \sim (m_{es}m_{\mu e})/(m_\mu m_\tau) \sim 3 \times 10^{-5}$ which is far below the NS bound.

(4) Violation of $U(1)^h$-symmetry implies in general a non-diagonal mass matrix for the charged leptons. In this case the lepton mixing matrix is the product, $V = V_\nu \cdot V_l^\dagger$, where $V_\nu$ and $V_l$ diagonalize the mass matrices of neutrinos and charged leptons, respectively. Let us suppose for simplicity that the effects of $U(1)^h$ violation come from $V_l$ only ($V_\nu$ has two-block structure as before), and moreover $V_l$ mixes essentially the first and the second generation with the angle $\theta_l$. Then the oscillations $\nu_e \leftrightarrow \nu_\mu$ are expected with $\Delta m^2 \simeq m_2^2$ and the depth $\sin^2 2\theta_{e\mu} \simeq \sin^2 2\theta_{es} \cdot \sin^2 \theta_l$.

For $\theta_{es}$ and $\theta_l$ fixed by solar neutrino data and the LSND/KARMEN sensitivity, one finds $\sin^2 2\theta_{e\mu} \simeq (1 - 5) \times 10^{-6}$ which can satisfy the NS bound. This model realizes the scenario described in [3].

The $\nu_\mu-\nu_\tau$ mass splitting can be generated without explicit $U(1)^h$ violation. The modified $U(1)^h$ charge prescription in the model [12] with $\sigma': (-1,2,0,-2,2,1,0)$ for $(\nu_e^c, \nu_\mu^c, \nu_\tau^c, \sigma, \sigma', S, y)$ allows for the superpotential,

$$W = \frac{m_e}{\langle H_2 \rangle} \nu_e^c \nu_e^c H_2 + \frac{m_\mu}{\langle H_2 \rangle} \nu_\mu^c \nu_\mu^c H_2 + \frac{m_\tau}{\langle H_2 \rangle} \nu_\tau^c \nu_\tau^c H_2$$

$$+ \frac{M_e}{2\langle \sigma' \rangle} \nu_e^c \nu_e^c \sigma' + \frac{M_\mu}{\langle \sigma \rangle} \nu_\mu^c \nu_\mu^c \sigma + \frac{M_\tau}{2\langle y \rangle} \nu_\tau^c \nu_\tau^c + \frac{m_{es}}{\langle y \rangle} \nu_e^c Sy. \quad (24)$$

For $\nu_e-S$ it reproduces the matrix (4), whereas for $\nu_\mu-\nu_\tau$ system one gets the matrix (20) with non-zero $M_\tau \nu_\tau^c \nu_\tau^c$ term, thus generating mass splitting (23). However, the blocks $\nu_e-S$ and $\nu_\mu-\nu_\tau$ remain decoupled, and thus no observable effect in KARMEN/LSND is expected.

5 Conclusion

We suggest that a light singlet fermion $S$ whose existence is hinted by some neutrino observations may have its origin beyond neutrino physics. Such a fermion can however be incorporated
into the standard see-saw picture, where interactions of \( S \) with the heavy right-handed neutrinos can generate its mixing with the light neutrinos. Such a mixing allows an understanding of the lightness of \( S \) without ad hoc introduction of very light scale. The mixing mass parameter \( m_{es} \simeq (0.02 - 0.3) \text{ GeV} \) leads to the mass of the singlet and its mixing with electron neutrino in the region \( m_1 \simeq (2 - 3) \cdot 10^{-3} \text{ eV} \) and \( \sin^2 2\theta_{es} \simeq (1 - 15) \cdot 10^{-3} \), where the \( \nu_e \rightarrow S \) resonance conversion gives a good fit of all solar neutrino data.

Supersymmetry can provide a framework within which the existence and the desired properties of such a light fermion follow naturally. There is a number of models with singlet scalars which acquire VEV and are introduced to break symmetries such as lepton number and Peccei-Quinn symmetry, or to generate \( \mu \)-term, etc.. However, identifying \( S \) with the fermionic superpartner of such scalars implies violation of R-parity, and further complication of model. We have considered a specific example with \( S \) identified as the majorino. It may be possible to suppress the mass of \( S \) generated after SUSY breakdown by introducing non-minimal Kähler potentials.

The conservation of R-parity requires for the fermion \( S \) to be a component of singlet superfield which has no VEV. This allows to construct simple model (12) in which the properties (mass and mixing) of \( S \) follow from the conservation of R-symmetry. The singlet field is mixed with RH neutrinos by the interaction with the field \( y \) which can acquire VEV radiatively after soft SUSY breaking. The model can naturally incorporate the spontaneous violation of Peccei-Quinn symmetry or/and lepton number. The fields involved can spontaneously generate the \( \mu \)-term.

Approximate horizontal (family) \( U(1)^h \) symmetry as in (19) provides simultaneous explanations for the predominant coupling of \( S \) to the first generation (thus satisfying the NS bound) and for the pseudo-Dirac structure of \( \nu_\mu - \nu_\tau \) needed in solving the atmospheric neutrino and hot dark matter problem. Breaking of \( U(1)^h \) can be arranged in such a way that the parameters of \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) oscillations are in the region of sensitivity of LSND and KARMEN experiments.

Future solar neutrino experiments will allow to prove or reject the hypothesis of the \( \nu_e \rightarrow S \) conversion in the Sun [23] and thus to test the models elaborated in this paper.

*Note added:* When our work was practically accomplished we encountered the paper [24] discussing non-supersymmetric model based on discrete symmetry in which sterile neutrino
mixes with usual light neutrinos via RH components. Our results have been reported at XXX Rencontres de Moriond, March 11-18 (1995), Les-Arcs Savoie, France (to be published).

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