New observables of the cosmic microwave background

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Abstract
We introduce new observables of the cosmic microwave background radiation, which can be measured through the detection of high order modes excited within an antenna feed system, coherently combined with those currently detected by space observatories. The use of such observables could potentially further constrain the validity of cosmological theories.

Keywords CMB · High order mode · Polarization · Space observatory

1 Introduction
The measurement of the temperature of the cosmic microwave background (CMB) radiation is the key for the development and verification of any cosmological theory. After its experimental discovery by Penzias and Wilson in 1965 [1], it has been, and will be, the subject of several experiments conducted from Earth and in space. A historical review of past experiments is provided in [2].

When focusing our attention to space observatories, which are especially relevant to this work, we can recall the RELIKT-1 mission [3] operating during 1983–1984, the Cosmic Background Explorer [4] (COBE, 1989 to 1993), the Wilkinson Microwave Anisotropy Probe [5] (WMAP, 2001 to 2010) and the Planck mission [6] (2009 – 2015). LiteBIRD will be launched in 2028 and will operate for three years at the Sun-Earth Lagrangian point L2 [7]. In all such experiments the temperature of the CMB is measured over an angular grid across the celestial sphere, with ever increasing level of accuracy, angular resolution and angular coverage. When grouping the many specificities of each mission (calibration methods, angular and spectral coverage etc.), into one long-term effort, it can be stated that a common principle is followed, to measure the power of the radiation observed by an antenna system,
and to relate such measurement to the structure of the temperature field in the angular surrounding of the antenna boresight. Multiple feeds and associated detection mechanisms ensure the coverage of several frequency bands as well as orthogonal polarisations, and the measurement is conducted over a grid of angular directions, allowing the reconstruction of the whole temperature field.

Complementarily to the above space observatories, it is important to mention that several experiments, from the past into the future, are based on ground measurements, either from radio telescopes or balloons based. Even though the space observatories are generally superior for what concerns impact from measurement noise, frequency coverage (and consequent ability to separate foregrounds) and removal of systematic errors, large ground telescopes (or arrays of telescopes) can provide superior angular resolution at an affordable cost. We mention here future experiments like the Simons observatory, which will include one 6 m and three 0.5 m telescopes, to cover angular scales from few degrees to arcminutes, in a broad range of frequencies from 27 to 280 GHz [8], and the CMB-S4 experiment, made of 21 telescopes of different apertures at the South Pole and in the Atacama desert in Chile, exploiting cryogenically-cooled superconducting detectors [9]. On the side of balloons based experiments, we mention the planned Large-Scale Polarization Explorer (LSPE), aimed at measuring the polarization of the CMB at large angular scales, and focused on the rotational component of CMB polarization (B-modes) [10].

The goal of the present work is to introduce additional observables, primarily for future space observatories, which can also be related to the structure of the temperature field. The approach is to detect high order modes in the feed system, which are excited, together with the dominant mode (the one used already), by the impinging radiation and that typically do not propagate beyond the feed system. Such modes carry information about the radiation, they can be extracted and coherently combined with the dominant mode to form useful observables. Such high order modes are already used in other fields, e.g., for tracking systems used to identify and track the direction of arrival of radio emitters, and the present paper shows how their use can be extended to the characterisation of the CMB.

The paper is organised as follows: In Sect. 2 the current methodology is discussed, with a focus on the main principles and ideas. The objective is to recall few key assumptions which are necessary when interpreting the measured observable, and which are also required for using the newly proposed ones. In Sect. 3 the new observables are introduced. Section 4 describes the approach for a sample antenna system, limiting the discussion to two high order modes and to a single polarisation. Section 5 shows how the important aspect of polarisation is incorporated in the new approach, from an experimental point of view. Implementation challenges due to non-ideal behaviour of the involved components are reported and briefly discussed in Sect. 6, concluding remarks and hints for future work are reported in Sect. 7.

### 2 Considerations about the current CMB observable

Space observatories characterise the CMB by measuring the power received by an antenna system over a grid of angular directions, with a time integration required to suppress the measurement noise. In order to discuss the methodology,
let us focus on one individual measurement conducted over a fixed orientation, and define a reference frame solidly with the antenna system as depicted in Fig. 1, with angular coordinates $\theta, \phi$ such that for example the zero of the polar angle $\theta$ corresponds to the antenna boresight. Similarly, the CMB temperature field (and in particular its fluctuation $\Delta T$ around a mean) can be expressed as a function of the two above angular coordinates $\theta$ and $\phi$, possibly after a coordinate transformation from a standard (e.g., cosmological) reference frame

$$T(\theta, \phi) = \overline{T} + \Delta T(\theta, \phi).$$

In the above definition $\overline{T}$ is the mean value, measured to be $2.72548 \pm 0.00057\text{K}$ [11], and $\Delta T$ is the fluctuation about the mean. We assume that the antenna feed system has two output ports, reacting to orthogonal polarisation states of the incoming radiation. When random unpolarised radiation is observed, the power is split equally across the two ports, however this is not necessarily the case and it depends upon the physical properties of the source.

The measurement of the CMB temperature is performed through the conceptual setup depicted in Fig. 1, consisting of a low noise power amplifier connected to a feed output port, followed by filtering, frequency down conversion to complex baseband and power measurement (all above duplicated to account for two orthogonal polarisations). The described setup is a simplified and idealised version of actual experiments, like the Low Frequency Instrument (LFI) on-board Planck [12], and may differ from other methods, e.g. the connection of bolometers to each feed output port for direct power measurement [13], without need of frequency down-conversion. Furthermore, use of calibration loads is required for the LFI, to take into consideration $1/f$ noise from the electronics [12]. However, provided that one is considering an idealised system, the various methods of measuring the signal power within a pre-selected bandwidth are equivalent, and the merit of the one illustrated
in Fig. 1 is to allow an extension to the coherent detection of high order modes, which is the actual subject of this paper, and is discussed in the next section.

The signal available at the power measurement point shown in Fig. 1 is made up of a contribution due to the internal noise from the feed and electronics, and the contribution due to the CMB temperature. We assume for the moment that the CMB temperature is different from zero only at a point source along the direction \( \theta, \phi \) within a small solid angle \( \delta \Omega \), within the antenna reference frame. In such scenario the received power is expressed as follows

\[
\left[ \text{received power} \right]_{\text{point source}} = g_x W k \left( \frac{1}{4\pi} T(\theta,\phi) G(\theta,\phi) \delta \Omega + T_R \right),
\]

where \( k \) is the Boltzmann constant, \( W \) is the bandwidth within which the power is measured. The black-body spectral radiance is assumed to follow the Rayleigh–Jeans approximation. The term \( G(\theta,\phi) \) represents the value of the antenna gain in the direction \( \theta, \phi \), essentially the antenna far-field radiation pattern in its own reference frame, and finally the noise temperature \( T_R \) represents the noise introduced by the lossy feed and by the full electronics chain, within the measurement bandwidth. The term \( g_x \) denotes the power gain of the electronics, from the input of the low noise amplifier down to the power measurement point.

Even though the antenna and electronics gains are real numbers in the power Eq. (2), they have to be considered, within the frame of a complex formalism used to represent narrowband signals, as the absolute modulus squared of complex numbers, with given amplitude and phase, in the case of the antenna gain dependent on both angular coordinates \( \theta, \phi \).

\[
G(\theta,\phi) = \left| \sqrt{G(\theta,\phi)} e^{i\Phi(\theta,\phi)} \right|^2,
\]

\[
g_x = \left| \sqrt{g_x e^{i\varphi_x}} \right|^2.
\]

The amplitude and phase components in Eq. (3) may be frequency dependent, however here we assume an ideally flat frequency response within the measured bandwidth (furthermore, for the phase we neglect any linear variation in frequency associated to a propagation delay within the system). The power on the left side of Eq. (2) therefore is due to a narrowband noise-like signal being received by the feed, which can be represented, after frequency down-conversion to complex baseband, as follows (time dependency is shown only below, and is omitted in the rest unless it is not obvious)

\[
[x(t)]_{\text{point source}} = v(t) \sqrt{g_x e^{i\varphi_x}} \sqrt{WkT_R} + u(t) \sqrt{g_x e^{i\varphi_x}} \sqrt{\frac{1}{4\pi} WkT(\theta,\phi) \delta \Omega} \sqrt{G(\theta,\phi)} e^{i\Phi(\theta,\phi)}.
\]

In the above equation we have denoted with \( u \) and \( v \) uncorrelated realizations of complex zero-mean, stationary stochastic processes with spectrum contained in \( W \) and with unitary power

\[
E[|u|^2] = E[|v|^2] = 1,
\]

\[
E[v^*u] = 0.
\]
with \((\cdot)^*\) denoting complex conjugation, and where \(E[\cdot]\) denotes statistical ensemble expectation. In a physical scenario the radiation is received from all directions, and therefore the total received signal \(x\) is obtained by summing the components from all directions \(\theta_i, \phi_i\)

\[
x = v \sqrt{g_x e^{i\varphi_x}} \sqrt{W_k T_R} + \sqrt{\frac{g_x W_k}{4\pi}} e^{i\varphi_x} \sum_i u_i \sqrt{G(\theta_i, \phi_i)} e^{i\phi(\theta_i, \phi_i)} \sqrt{T(\theta_i, \phi_i) \delta \Omega_i},
\]

(6)

with \(u_i\) representing the random time evolution of each component, fulfilling the same properties of \(u\) in Eq. (5). The power of the received signal \(x\) is equal to \(E[|x|^2]\), and can be computed based on Eq. (6); furthermore, if the temperature field is spatially incoherent, we can assume

\[
E[u_i^* u_j] = \delta_{ij},
\]

(7)

where \(\delta_{ij}\) is equal to one if \(i = j\), and zero otherwise. When using the assumption of spatial incoherence, we get, after few manipulations

\[
E[|x|^2] = g_x W_k T_R + \frac{g_x W_k}{4\pi} \sum_i T(\theta_i, \phi_i) G(\theta_i, \phi_i) \delta \Omega_i.
\]

(8)

When transforming the above sum into an integral and when assuming that the spatial incoherency of the CMB temperature continues to hold in such a limit, we get the following expression

\[
E[|x|^2] = g_x W_k \left( T_R + \frac{1}{4\pi} \int T(\theta, \phi) G(\theta, \phi) d\Omega \right).
\]

(9)

Before continuing, we underline that the proportionality factor \(g_x W_k\) in Eq. (9), even though important for implementation aspects, is conceptually irrelevant. We therefore assign it equal to one, assuming it has been perfectly calibrated, and redefine Eq. (9) as follows

\[
T_x = T_R + \frac{1}{4\pi} \int T(\theta, \phi) G(\theta, \phi) d\Omega.
\]

(10)

The physical quantity of interest in the above equation is of course the temperature provided by the integral component. The reason for “proving” (or rather motivating) the above well-known Eq. (10), is to emphasize the fact that it relies upon the hypothesis of spatial incoherency of the CMB radiation. The equation establishes a definite relation between a measurable quantity \((T_x)\), a quantity whose value (possibly slowly time-varying) can be accurately calibrated \((T_R)\), a function that can be measured “once for all” \((G, the antenna radiation pattern)\) and the quantity of physical interest \(T(\theta, \phi)\). Obviously, the effects of the angular resolution of the measurement, linked to the beamwidth of the antenna, and of the finite time integration have to be taken into account in the post-processing of the raw data. The second effect, which determines the sensitivity of the measurement
process, translates unchanged into the new approach and is not addressed in this paper, whereas the aspect of angular resolution is discussed in the following Sect. 3, when introducing the use of high order modes in a unified terminology.

3 Use of high order modes

Let us now refer to the Fig. 2. The setup is equivalent to the one of Fig. 1 for what concerns the power/temperature measurement, which is covered by the blocks in grey, once again duplicated to cover two orthogonal polarisations. As reported in the previous section, the power measurement could be performed after frequency down-conversion to complex baseband in the “correlations” block, or without frequency down-conversion by use of bolometers. However, an element has been added to the antenna system, to be able to couple to a high order mode within the feed, \textit{i.e.}, a mode with a spatial configuration which is different from the one of the dominant mode used for power detection. The amplitude of the high order mode typically decays along the direction of guided propagation, beyond the feed system.\(^1\) Indeed, infinite modes are excited within the feed system by the impinging radiation, and as we will see they encode information about such radiation, which can be retrieved. Even though the high order coupling element reacts to both orthogonal polarisations, in the following we analyse one polarisation only (\textit{e.g.} polarisation 1 in Fig. 2, leading to the signals \(x, y\)) and we postpone polarisation-related discussions to

\(^1\) We are not considering here multi-mode feeds like those described \textit{e.g.} in [14] and [15], in which several modes propagate and are collectively (and incoherently) detected with the main purpose of shaping the antenna power radiation pattern.
Sect. 5. Let us consider a high order mode and associated radiation pattern \( H_\mu(\theta, \phi) \) (defined within the same reference frame adopted for the “main” radiation pattern \( G \) and for the CMB temperature field) once again meant to be the absolute modulus squared value of a complex number

\[
H_\mu(\theta, \phi) = \left| \sqrt{H_\mu(\theta, \phi)} e^{i \psi_\mu(\theta, \phi)} \right|^2. \tag{11}
\]

The index \( \mu \) is equal to 1, 2, 3, ..., with \( \mu = 1 \) representing the dominant mode (associated to the radiation pattern \( G(\theta, \phi) \) defined in the previous section), and the subsequent \( \mu \) identifying high order modes.

By following the same steps applied in the previous section, the signal \( x \) observed through the main radiation pattern \( H_1 = G \) can be expressed as in Eq. (6), and its power can be measured as previously discussed. The above means that the proposed setup preserves the functionality of that shown in Fig. 1 (the presence of the coupling device implies an additional loss on the main path, which however can be made small in practical implementations). In addition to the signal \( x \) however, a signal \( y \) is also available, received through the pattern \( H_\mu \), which, following the same reasoning leading to Eq. (6), can be expressed as follows

\[
y(t) = z(t) \sqrt{g_x e^{i \varphi_x}} \sqrt{W k T_{R,y}} + \sqrt{\frac{g_y W k}{4\pi}} \sum_i u_i(t) \sqrt{H_\mu(\theta_i, \phi_i)} e^{i \psi_\mu(\theta_i, \phi_i)} \sqrt{T(\theta_i, \phi_i)} \delta \Omega, \tag{12}
\]

with the measurement bandwidth \( W \) being the same as on the main path. It is important to remark that, within the current assumption of identical polarisation for \( x \) and \( y \), each normalised time evolution \( u_i \) is the same in Eqs. (6) and (12) (of course if “\( i \)” scans through the same set of angular directions in the two equations), because the two involved electromagnetic modes differ among each other only in terms of spatial distribution of the field within the feed system. Furthermore, in the above equation the power gain of the electronics has been denoted with \( g_y \), with an associated phase \( \varphi_y \). Finally, \( z \) is the realization of a stochastic band-limited process of unitary power, uncorrelated with \( v \) and \( u_i \) (it is related to dissipative effects in the feed components and subsequent electronics associated to the detection and amplification of the high order field, collectively labelled with \( T_{R,y} \)). The quantity of interest now, of which the correlator of Fig. 2 provides an estimate, is the cross-correlation between the signals \( x \) and \( y \), equal to \( E[x^* y] \). The computation of such cross-correlation proceeds along the same lines as in the previous section for the computation of the power of \( x \). In particular, the spatial incoherency of the CMB radiation must once again be invoked to obtain, in integral form, the following expression

\[
E[x^* y] = \sqrt{g_y g_x} W k e^{i(\varphi_y - \varphi_x)} \frac{1}{4\pi} \int T(\theta, \phi) \sqrt{H_\mu(\theta, \phi) H_1(\theta, \phi)} e^{i(\psi_\mu(\theta, \phi) - \psi_1(\theta, \phi))} d\Omega. \tag{13}
\]

As done before, we ignore complications linked to the gain of the electronics for both paths of Fig. 2 and assign the complex pre-factor to one, and redefine the observable as a (complex) temperature as follows
where we have defined the synthesized radiation pattern
\[ \Pi_{\mu,1}(\theta, \phi) = \sqrt{H_{\mu}(\theta, \phi)H_1(\theta, \phi)}e^{i(\psi_\nu(\theta, \phi) - \psi_1(\theta, \phi))}, \] (15)
which is in general complex, with the exception of \( \Pi_{1,1} = H_1 = G \) which is real and positive. The above Eq. (14) is fully general and describes a set of observables including the current one represented by Eq. (10) when setting \( \mu = 1 \), i.e.
\[ T_x = T_R + T_{1,1}. \] (16)

However, Eq. (14) does not offer a hint for exploitation yet, and another step is required in order to identify a potential application. Indeed, it is opportune to expand both the temperature field and the synthesized radiation patterns in spherical harmonics
\[ T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{l,m} Y_{l,m}(\theta, \phi), \]
\[ \Pi_{\mu,1}(\theta, \phi) = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} b_{l,m}^\mu Y_{l,m}(\theta, \phi), \] (17)
with the following definition and normalisation properties
\[ Y_{l,m}(\theta, \phi) = (-1)^m (Y_{l,-m}(\theta, \phi))^* = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos \theta)e^{im\phi}, \]
\[ \int Y_{\lambda,\mu}(\theta, \phi) Y_{\lambda,\mu}^*(\theta, \phi) \, d\Omega = \delta_{\lambda,\lambda} \delta_{\mu,\mu}, \]
and where \( P_l^m(\cdot) \) is the associated Legendre polynomial of degree \( l \) and order \( m \). Concerning the synthesized radiation pattern, once this is known the coefficients of the expansion (17) can be computed according to
\[ b_{l,m}^\mu = \frac{1}{4\pi} \int \Pi_{\mu,1}(\theta, \phi) Y_{l,m}(\theta, \phi)^* \, d\Omega. \] (19)

With respect to the CMB literature, we have designated the coefficients of the temperature expansion in Eq. (17) as \( a_{l,m} \) to indicate the fact that these coefficients are defined in the antenna reference frame, and therefore they do not coincide with the (un-primed) coefficients obtained over a standard reference frame. However, it is obvious that the above representation of the temperature field is unambiguously

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2 The indexing in definitions (14) and (15) seems redundant as one could have indexed the quantities with just an integer, without the mentioning to “1”. However, in a more general approach one can correlate the mode \( \mu \) with a mode \( \nu \) which is different from the dominant one, e.g. leading to the observable \( T_{\mu,\nu} \). Additional indexing has to be added when wishing to correlate modes from orthogonal polarisations, which is avoided here to keep the exposition simple. A brief discussion about these generalisations is deferred to Sect. 5.

3 The definition of the radiation pattern expansion is not unique. The selected definition leads to the behaviour of the \( b \)-coefficients shown in Fig. 3.
related to the representation in another frame. Due to the fact that the CMB temperature is a real quantity, the following property holds

$$a_{l,m} = (-1)^m (a_{l,m}^*)^*.$$  \hspace{1cm} (20)

When using the expansions of Eq. (17) in Eq. (14), and when using the properties (18), we immediately obtain

$$T_{\mu,1} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} (a_{l,m}^*)^* b_{l,m}^{\mu,1}.$$  \hspace{1cm} (21)

The above equation illustrates how the measurement of high order modes can introduce constraints on linear combinations of the coefficients $a_{l,m}$, with the weights of such combinations being represented by coefficients of the involved synthesized radiation pattern expansion. As it will be clear later (and as it is well known for the currently exploited $T_{1,1}$) such observables will be able to explore only degrees $l$ up to a given maximum order $l_{\text{max}}$ due to the finite aperture of the observing system, introducing a filtering action on the angular spectrum of the observed temperature field. In the following we select the radiation patterns of a sample antenna system, clarifying the above aspects and also allowing to estimate the amount of signal which can be detected from each observable. Furthermore the selected radiation patterns will lead to the possibility of directly testing intrinsic features of the CMB, as we are going to show.

Fig. 3 Behaviour of the absolute value of the beam coefficients as a function of the selected synthesized radiation pattern, of the half-power beamwidth $h$ and of the angular order $l$. The figure on the bottom-right shows the coefficients properly scaled in order to collapse the curves into one for each synthesized pattern. For the selected case of left hand circular polarisation for dominant and high order mode, the phase of the complex coefficients is 0, $\pi/2$ and $\pi$ radians for $\mu = 1, 2, 3$. 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Behaviour of the absolute value of the beam coefficients as a function of the selected synthesized radiation pattern, of the half-power beamwidth $h$ and of the angular order $l$. The figure on the bottom-right shows the coefficients properly scaled in order to collapse the curves into one for each synthesized pattern. For the selected case of left hand circular polarisation for dominant and high order mode, the phase of the complex coefficients is 0, $\pi/2$ and $\pi$ radians for $\mu = 1, 2, 3$.}
\end{figure}
4 Sample system

We make reference to radiation patterns obtained by use of a corrugated circular waveguide feed of internal radius $r_1$, for circular polarised radiation, addressed in [16] and [17], in the latter reference corresponding to the “uniform phase” approximation in Eq. (6) within the paper. The reason for selecting a corrugated circular waveguide feed is simply that closed formulas are available for the radiated field. The extension to a corrugated conical horn is conceptually straightforward, and methods for computing the radiated field, requiring numerical integration, are reported in the same [17] and also in [18]. The reason for selecting corrugated structures (either waveguide or horn) is due to the possibility of obtaining pattern symmetry with respect to the azimuthal angle $\phi$. When referring to [17], the following expressions of the radiated fields are available for the balanced modes $HE_{\mu 1}, \mu = 1, 2, 3 \ldots$ for the two circular polarisations (denoted with L and R, for left and right hand circular polarisation respectively)

$$E_\phi = \pm iE_\theta \propto \frac{1 + \cos \theta}{2} \cdot \frac{J_{\mu - 1}(\beta_0 r_1 \sin \theta)}{(\beta_0 r_1 \sin \theta)^2 - \zeta_{\mu - 1}^2} e^{\pm i \mu \phi} \frac{L}{R},$$

where $\zeta_{\mu}$ is the first zero of the Bessel function of the first kind and order $\mu$, and the proportionality symbol is meant to be via a complex coefficient depending upon order and polarisation. Furthermore $\beta_0 = 2\pi / \lambda$ with $\lambda$ being the centre wavelength of the observed narrowband spectrum. The left and right hand circular polarised components are built as follows

$$E_{1 \lambda} = \frac{E_\theta \mp iE_\phi e^{\pm i \phi}}{\sqrt{2}} = \sqrt{2}E_\theta e^{\pm i \phi}.$$

Based on the above we can build the synthesized radiation patterns of Eq. (15). For the case of left hand circular polarisation for both dominant and high order modes (more general considerations to follow in Sect. 5), we get

$$\Pi_{\mu 1}(\theta, \phi) = e^{i \xi_{\mu 1}} \frac{e^{i \frac{c_i}{h^2}(1 + \cos \theta)^2}}{h^2} \frac{J_{\mu - 1}(\beta_0 r_1 \sin \theta) J_0(\beta_0 r_1 \sin \theta) e^{i(\mu - 1) \phi}}{\left[ (\beta_0 r_1 \sin \theta)^2 - \zeta_{\mu - 1}^2 \right] \left[ (\beta_0 r_1 \sin \theta)^2 - \zeta_0^2 \right]},$$

where $h \simeq 4.152/(\beta_0 r_1)$ is the half-power beamwidth of the antenna in radians, the constants $c_i$ ensure proper normalisation of each individual radiation pattern independently from the specific value of $h$, and $\xi_{\mu 1}$ is a constant phase term depending on the two involved radiation patterns. Based on the above Eq. (24), the coefficients $b$ of Eq. (19) can be computed for each considered synthesized pattern. Due do the complex exponential term in Eq. (24), the non-zero coefficients for the synthesized pattern $\Pi_{\mu 1}$ are only those with azimuthal index $m = \mu - 1$. As a consequence, the observable $T_{\mu 1}$ of Eq. (21) will only depend upon the coefficients $a_{l,\mu - 1}^\mu$, each weighted by the beam coefficient $b_{l,\mu - 1}^\mu$. 
The behaviour of the non-zero beam coefficients is shown in the following Fig. 3.

We can use Eq. (25) to get an idea of the magnitude (say \( |\delta T| \)) that can be expected from the various observables. Indeed, by assuming statistical independence of the coefficients of the CMB expansion of Eq. (17), we obtain

\[
T_{\mu,1} = \sum_{l=0}^{\infty} \left( a'_{l,\mu-1} \right)^* b_{l,\mu-1}^{\mu,1}. \tag{25}
\]

The behaviour of the non-zero beam coefficients is shown in the following Fig. 3.

We can use Eq. (25) to get an idea of the magnitude (say \( \delta T \)) that can be expected from the various observables. Indeed, by assuming statistical independence of the coefficients of the CMB expansion of Eq. (17), we obtain

\[
(\delta T_{\mu,1})^2 \triangleq E \left[ \left| T_{\mu,1} \right|^2 \right] = \sum_{l=2}^{\infty} E \left[ \left| a'_{l,\mu-1} \right|^2 \right] \left| b_{l,\mu-1}^{\mu,1} \right|^2 = \sum_{l=2}^{\infty} C_l \cdot \left| b_{l,\mu-1}^{\mu,1} \right|^2, \tag{26}
\]

where

\[
C_l = E \left[ \left| a'_{l,\mu-1} \right|^2 \right]. \tag{27}
\]

is the so called CMB power spectrum. The above Eq. (26) has a clear intuitive meaning: the various components of the power spectrum are simply weighted through the beam coefficients, as through a filtering action (see for example the related discussion in [19] related to the current observable). The behaviour of \( C_l \) as a function of the multipole moment \( l \) is omnipresent in literature, here the values from [20] have been used (converted from the value reported in the file \( D_l = l(l+1)C_l/2\pi \)). The result is shown in Table 1, for three different half-power beamwidths. As expected, the signal from the synthesized pattern \( \Pi_{1,1} \) (the current power detection method) is larger the smaller \( h \), whereas this is not the case for the other observables which, in this very coarse analysis, show a peak behaviour for \( h \) around 0.6 degrees.

The other remark is that, even though the signal from \( \Pi_{1,1} \) is dominant in all scenarios, the other two signals are of not negligible magnitude, especially the one from \( \Pi_{2,1} \); indeed, for the “best” \( h \) of around 0.6 degrees, the signals from \( \Pi_{2,1}, \Pi_{3,1} \) are a non-negligible fraction of the corresponding dominant signal.

It is also important to emphasize that the observables defined in Eq. (25) are orthogonal when assuming statistical independence of the coefficients \( a'_{l,m} \)

\[
E \left[ \left( T_{\mu,1} \right)^* T_{m,1} \right] = 0, \mu \neq m, \tag{28}
\]
which follows immediately from the fact that their linear combinations involve coefficients $a'_{l,m}$ with different azimuthal index. The above Eq. (28) therefore allows us testing a key feature of the CMB, i.e. the statistical independence of the coefficients $a'_{l,m}$ with different azimuthal index.

5 Polarisation

Mainstream investigations of the CMB concern its polarisation anisotropies (e.g. [7, 10, 21]). In the previous section we have analysed in detail the correlation between dominant and high order modes for one specific polarisation. However, as emphasized in Fig. 2, the two orthogonal polarisations can be made available for the dominant and for the selected high order mode, leading to multiple possibilities of correlation. A more generic observable than the one of Eq. (14) can be defined as follows

$$T_{\mu,\text{pol}_x,\mu,\text{pol}_y} \propto E[x^\dagger y]$$

(29)

where $\mu_x,\text{pol}_x$ are the mode order and polarisation associated to the signal $x$, and similarly for $y$. In this way several observables can be formed, spanning over the available modes and polarisations. For example the mode $T_{1L,1L}$ is equal to the previously computed $T_{1,1}$ of Eq. (16) corresponding to a simple power detection on the left hand-circular polarisation, whereas the mode $T_{1L,2R}$ is obtained by correlating the high order mode signal in right hand circular polarisation, with the fundamental mode in the orthogonal polarisation. Not all observables are independent of course, e.g. $T_{\mu L,\mu R}$ and $T_{\mu R,\mu L}$, and not all of them are expected to be mutually orthogonal, like $T_{1L,1L}$ and $T_{2L,2L}$. The observables can be put in relation with components of the polarisation tensor introduced in [22], and they will probe one azimuthal index in the respective spherical harmonics expansion

$$\text{azimuthal index}(T_{\mu,\text{pol}_x,\mu,\text{pol}_y}) = \pm(\mu_y - 1) \mp (\mu_x - 1) \frac{L}{R},$$

(30)

As an example, the mode $T_{2L,2R}$ will select the azimuthal index 2. For unpolarised radiation, and for a perfectly calibrated system, the observables of Eq. (29) will either vanish or will be equal in pairs whereas any deviation from the unpolarised scenario will bring information which can be related to the CMB polarisation tensor. As a final remark, all above observables will be expressed through angular filtering functions similar, and in some cases equal, to those displayed in Fig. 3. The exhaustive computation of all window functions is not conducted within this paper.

6 Implementation aspects

The approach presented in the previous sections is based on the ideal setup of Fig. 2, and on the ideal behaviour of the involved system components. The aim of this section is to present important challenges which should be tackled in order to implement a practical system, even though a detailed discussion aiming at solving
such challenges cannot be attempted here, as it should be the subject of a dedicated work. An incomplete list of topics includes the following: 1) efficient extraction of required high order modes; 2) excitation of unwanted modes; 3) noise performance, 4) bandwidth, 5) polarisation discrimination and 6) return loss optimisation.

Concerning the first point, the whole approach relies on the ability to detect high order modes, to be correlated with the fundamental mode in order to extract the new observables. It should be remarked that in principle one could extract a mixture of high order modes, whose correlation would anyway lead to a useful observable, consisting in a linear combination of coefficients \( a \) with distinct azimuthal numbers \( m \) in Eq. (21), not necessarily collapsing into the simplified expression of Eq. (25), related to the correlation of a pure high order mode. However, assuming that it would be desirable to extract a pure high order mode, \( e.g. \) to ensure orthogonality between observables, the challenge of selecting the desired mode has to be addressed. The detection of pure high order modes which can be used for monopulse autotracking is widely described in literature. Examples are provided in [23], related to a corrugated horn feed for ground applications operating at 8 – 8.5 GHz, where HE21 is extracted, in [24] (30–50 GHz, extraction of the mode TE21) and in [25] (94 GHz with extraction of TM01), the latter reference revealing the technological difficulty of increasing the operating frequency towards the THz range for such devices.\(^4\) The extraction of higher order modes, like HE31, is not addressed in monopulse autotracking literature, due to the fact that the correlation of such mode with the fundamental leads to information which cannot be obviously interpreted (apart from the use proposed within this paper). However it should be feasible to extract such modes with similar engineering methodologies as applied \( e.g. \) in [23], with a higher level of complexity and more stringent manufacturing tolerance, the larger the order and the operating frequency.

Even when being able to detect the desired high order mode, unwanted modes may be excited, as exemplified in [26] where impedance mismatches in the conversion from TE11, TE21 modes into HE11 and HE21 can lead to excitation of higher order modes. The generation of unwanted modes would be detrimental for the approach presented in this paper, as it would ultimately lead to “distorted” observables, and should be mitigated by proper design and manufacturing techniques.

Concerning measurement noise, the described technique relies upon coherent detection of the received signal [27], where the (complex) waveform is preserved, after frequency down-conversion to complex baseband, for further post-processing (correlation between fundamental and high order mode), as opposed to non-coherent detection where only the power of the incoming signal is detected. The implication is that state-of-the-art non-coherent detectors with ultimate noise performance like Superconducting Transition Edge Sensor (TES) bolometers (planned for LiteBIRD [28]) or emerging Microwave Kinetic Inductance Detector (MKID) technology [29], are not applicable to the presented approach. Coherent and non-coherent detection

\(^4\) The examples from [24] and [25] are not directly applicable to CMB detection, due to the poor symmetry of the resulting radiation patterns. However they exemplify the general issue of detecting and isolating high order modes at high frequencies.
might be not mutually exclusive, because non coherent detection of the fundamental signal and coherent correlation of fundamental and high order mode may co-exist on different paths, however the practical feasibility of such architecture is not verified in this account.

The achievable detection bandwidth is an important parameter for CMB detection. Ultimately the sensitivity in Kelvin is inversely proportional to the square root of the product of bandwidth and (constrained) time integration, which implies enlarging the bandwidth as much as possible. The extraction of a high order mode is the fruit of resonant behaviours within the feed system, which are only effective within a narrow bandwidth around a given centre frequency, especially if the phase relation between fundamental and high order mode has to be preserved [30]. When looking at available literature, like the quoted [23] and [25], one could estimate the available bandwidth as few percent of the centre frequency.

An important challenge is related to the ability to detect both orthogonal polarisations of the incoming radiation (as conceptually depicted in Fig. 1 and Fig. 2), with proper discrimination. The retrieval of both orthogonal polarisations is feasible for the dominant mode as well as for the higher order modes, as exemplified in [23] and [25], with cross-polar discrimination between 25 and 30 dB over the supported frequency range for the dominant mode. For the high order mode, due to the presence of a null in the boresight direction, the amount of cross-polarisation is more difficult to define. In [25] the simulations show overall good cross-polar performance. The main issue however is that the retrieval of the required orthogonal polarisations for the high order mode may require a complex network of waveguide components, with potential accommodation issues for a space born experiments, maybe less of a concern for ground based systems.

Finally we mention the difficulty of optimising the return loss, a performance affecting the overall sensitivity, for both the dominant mode and the high order mode, as exemplified in [25] where adequate performance is achieved for the dominant mode (better than 23 dB in the supported frequency range), and worse performance is obtained for the high order mode (better than 16 dB).

In summary, there are important technological challenges linked to the proposed approach, which have to be tackled in order to get measurements of adequate quality, when compared to those currently obtained. For some metrics, like sensitivity, inferior performance should be expected due to coherent detection and lower available bandwidth. However the diverse, complementary and orthogonal nature of the new observables could be a balancing factor, and the incorporation of the technique within a space experiment could be traded at system level.

7 Conclusions and future work

We have introduced new observables of the CMB: they can be formed by detecting, within the antenna system of a space observatory, high order modes of the received electromagnetic field, to be coherently combined with the main mode already used for power detection. The new observables provide distinct linear combinations of the coefficients of the expansion of the CMB temperature, of a reasonable magnitude to
be detected. It is a legitimate question to ask whether or not, and at which extent, the new observables will improve the characterisation of the CMB with respect to those currently detected. This question is not easy to answer, however two preliminary considerations could be made. On one side it is clear that the sensitivity of the newly introduced observable is expected to be inferior to the one obtained with current observables through non coherent power detection, as illustrated in Table 1 and also discussed in Sect. 6. From another perspective, the validity of Eq. (28) in Sect. 4, showing the orthogonality of two observables with different mode index, follows from widely accepted assumptions about the statistical independence of the coefficients $a_{l,m}$. Any measured and confirmed non-orthogonality, would lead to reconsider such assumptions and associated implications. From a practical point of view it would be sufficient to measure simultaneously two observables, e.g. $T_{1l,1l}$ and $T_{2l,1l}$ through narrowband channels over several orientations, and establish whether, and within which accuracy, the average of Eq. (28) is zero.

This work is only the first step of a roadmap which should include 1) a deeper analysis of the new observables, to link them to the existing theoretical framework in cosmology and to assess their potential usefulness 2) the detailed definition of the experimental setup, e.g. deepening on polarisation aspects, sizing the half-power beamwidth for best response, selecting high order modes and feed structure, etc. 3) manufacturing a test feed system to verify the practical feasibility of the approach and finally 4) flying an observatory including the measurement of new observables in addition to the currently used.

All in all, we believe that, in order to balance the efforts devoted to theoretical advances in cosmology, it is essential to enrich the experimental dataset, not only by increasing the performance of current observables, but also by introducing diverse and complementary ones.

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**Declarations**

**Conflict of interest** The authors declare that there are not conflicts of interest.

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