Frozen up Dilaton and the GUT/Planck Mass Ratio

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By treating modulus and phase on equal footing, as prescribed by Dirac, local scale invariance can consistently accompany any Brans-Dicke ω-theory. We show that in the presence of a soft scale symmetry breaking term, the classical solution, if it exists, cannot be anything else but general relativistic. The dilaton modulus gets frozen up by the Weyl-Proca vector field, thereby constituting a gravitational quasi-Higgs mechanism. Assigning all grand unified scalars as dilatons, they enjoy Weyl universality, and upon symmetry breaking, the Planck (mass)\(^2\) becomes the sum of all their individual (VEV)\(^2\)s. The emerging GUT/Planck (mass)\(^2\) ratio is thus \(\sim \omega g_{\text{GUT}}^2/4\pi\).

Critical local Weyl invariance

The Brans-Dicke theory \([1]\) is described by the action

\[
I_{BD} = -\int d^4x \sqrt{-g} \left( \phi^2 R + 4\omega g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} \right). \tag{1}
\]

The theory, characterized by a dimensionless parameter \(\omega\), the coefficient of the dilaton kinetic term, is invariant under the combined global scaling transformation

\[
g_{\mu\nu}(x) \to e^{-2\chi(x)} g_{\mu\nu}(x), \quad \phi(x) \to e^{\chi(x)} \phi(x). \tag{2}
\]

Consistent with the latter global symmetry is the quartic scalar potential term \(V(\phi) = \lambda \phi^4\). As is well known, it is only the critical case \(\omega = -\frac{1}{2}\) which further enjoys the full local scale symmetry

\[
g_{\mu\nu}(x) \to e^{-2\chi(x)} g_{\mu\nu}(x), \quad \phi(x) \to e^{\chi(x)} \phi(x). \tag{3}
\]

In the ‘unitary’ gauge, often called Einstein gauge, defined by fixing \(\phi(x) = v\), for some arbitrary constant \(v\), the theory resembles general relativity characterized by an arbitrary Planck mass \(M^2_\text{Pl} = 16\pi v^2\), and furthermore accompanied by a matching cosmological constant \(\lambda_E = \frac{1}{2} \lambda v^2\). This by itself, however, does not make Einstein theory of gravity a gauge-fixed version of the critical Brans-Dicke theory.

Non-critical local Weyl invariance

Recalling the profound success of the standard electroweak theory, a local scale symmetry is most welcome and currently quite popular \([2,3]\). The relative minus sign between the gravitational and the kinetic scalar terms, a characteristic feature of the critical Brans-Dicke theory, appears to be problematic on ghost related grounds. However, as we were guided by Dirac \([4]\), local scale invariance can be extended to accompany any Brans-Dicke ω-theory, including in particular the \(\omega > 0\) (no ghost) branch. The Dirac prescription reads

\[
I_D = -\int d^4x \sqrt{-g} \left( \phi^2 R^\star + 4\omega g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} \right), \tag{4}
\]

instructing us to replace the various tensors involved by their (starred) co-tensor substitutes. The procedure requires the presence of the Weyl vector field \(\kappa_{\mu}\), subject to the familiar transformation law

\[
\kappa_{\mu}(x) \to \kappa_{\mu}(x) - \chi(x)_{,\mu}. \tag{5}
\]

An optional universal coupling constant has been momentarily absorbed within \(\kappa_{\mu}\) redefinition (to be justified later on universality grounds).

The Ricci scalar replacement \(R^\star\) takes the explicit form

\[
R^\star = R - 6g^{\mu\nu} \kappa_{\mu\nu} + 6g^{\mu\nu} \phi_{,\mu} \phi_{,\nu}. \tag{6}
\]

Under scale transformations it behaves as a co-scaler of power -2, that is \(R^\star \to e^{2\chi(x)} R^\star\). While the dilaton field \(\phi\) is by construction a co-scaler of power -1, its covariant derivative \(\phi_{,\mu}\) is not a co-vector at all. It is only the co-covariant Weyl derivative \([5]\)

\[
\phi_{\mu\nu} = \phi_{,\mu} + \kappa_{\mu} \phi, \tag{7}
\]

which constitutes a co-vector of power -1, thereby making the kinetic term replacement \(g^{\mu\nu} \phi_{,\mu} \phi_{,\nu}\) a legitimate co-scaler of power -4. The Weyl co-covariant derivative conceptually differs from the Stueckelberg \([6]\) covariant derivative \(\phi_{\mu\nu} + m_{\mu\nu}\), but is in full analogy with the Maxwell covariant derivative \(\phi_{\mu\nu} + ieA_{\mu} \phi_{,\nu}\) of an electrically charged (and hence necessarily complex) scalar field. The imaginary electromagnetic coupling constant \(ie\) has been forcefully traded for a real (currently absorbed as noted earlier) coupling constant.

A mandatory ingredient is a kinetic term for the Weyl vector field. Truly, it is not directly required on plain local scale symmetry grounds, but in its absence \(\kappa_{\mu}\) would have stayed non-dynamical in nature. The transformation law eq.\((5)\) dictates the exact Maxwell structure, with the corresponding anti-symmetric differential 2-form given by

\[
X_{\mu\nu} = \kappa_{\mu\nu} - \kappa_{\nu\mu}. \tag{8}
\]

Altogether, up to the total derivative \(6(\phi^2 \kappa^{\mu})_{,\mu}\), and a full re-arrangement of the various terms floating around, the non-critical (arbitrary \(\omega\)) local Weyl invariant theory can be described in a somewhat more familiar language by the action

\[
I = -\int d^4x \sqrt{-g} \left[ \phi^2 (R - 6g^{\mu\nu} \kappa_{\mu\nu}) + 4\omega g^{\mu\nu} D_\mu \phi D_\nu \phi + \lambda \phi^4 + \frac{4}{3} g^{\mu\nu} g^{\lambda\sigma} X_{\mu\lambda} X_{\nu\sigma} \right], \tag{9}
\]
where we have used the shorthand notation
\[ s = \frac{3 + 2\omega}{2\omega} \neq 1. \] (10)

The latter action eq.(9) looks deceptively conventional, so a word of caution is necessary. Note that
\[ D_\mu \phi = \phi_{,\mu} + sk_{\mu} \phi \] (11)
is in fact a fake co-variant derivative, and should not be confused with the genuine co-variant derivative eq.(7).

In the Einstein gauge \( \phi(x) = v \) (with \( v \) still being an arbitrary constant at this stage), the theory resembles a particular Einstein-Proca [3] theory accompanied as before by a cosmological constant \( \Lambda_E = \frac{1}{2} \lambda v^2 \) (note that the corresponding Proca/Planck mass ratio is \( v \)-independent). However, this by itself does not make Einstein-Proca theory of gravity a gauge-fixed version of the non-critical Bran-Dicke theory. A gravitational Higgs-like mechanism capable of singling out the ‘unitary’ Einstein gauge on physical (local scale) symmetry breaking grounds is in order. In this paper, however, only a pseudo-Higgs mechanism is offered.

**A fake scale symmetry?**

The notion of a fake symmetry has been coined by Jackiw and Pi [3] to address a situation where the conserved Noether/Weyl current vanishes identically. Their assertion was that in certain cases the corresponding Weyl symmetry does not actually have any dynamical role. The critical Brans-Dicke theory, as well as some of its currently proposed derivative models [4], fall into such a category. Following the Jackiw-Pi analysis, we now calculate the Noether/Weyl current stemming from the action eq.(9). The result is non-trivial, owing to the presence of \( \kappa_{\mu} \), thereby implying in our case a genuine local Weyl symmetry.

First, without using the Euler-Lagrange equations, we perform the variation with respect to the combined symmetry transformations eqs.[3],[8], and find
\[ \delta \mathcal{L} = \mathcal{L}_{;\mu} \sqrt{-g} + L^\mu = -6\phi^2 g^{\mu\nu} \chi_{;\nu} . \] (12)
Utilizing a previous Jackiw-Pi calculation, the Weyl vector field \( \kappa_{\mu} \) and its antisymmetric derivative \( X_{\mu\nu} \) simply do not enter at this stage. Next, however, we do invoke the equations of motion, and following the Noether procedure, conventionally use them to eliminate \( \frac{\partial \mathcal{L}}{\partial \phi}, \frac{\partial \mathcal{L}}{\partial \kappa_{\mu}} \), and \( \frac{\partial \mathcal{L}}{\partial q_{\mu\nu}} \) from the variation. Doing so, we arrive at an alternate divergence formula for \( \delta \mathcal{L} \) which can be written in the form
\[ \delta \mathcal{L} = (L^\mu + J^\mu)_{;\mu} \sqrt{-g} . \] (13)
Equating eqs.(12,13), the Weyl/Noether conservation law makes its appearance
\[ J^\mu_{;\mu} = 0 . \] (14)
But this time, contrary to the fake symmetry case, the classically conserved symmetry current \( J^\mu \) does not vanish. To be specific, it is explicitly given by
\[ J^\mu = 8\omega \phi^2 k^\mu \chi + s X^{\mu\nu} \chi_{;\nu} , \] (15)
emphasizing the role played by the Weyl gauge field. Associated with the Lagrangian eq.(9) is thus a genuine local scale symmetry.

**Frozen up dilaton**

A spontaneously scale symmetry breaking mechanism in four (generically in more than two) dimensions is still at large. The emergence of the Planck mass scale within the framework of a theory which does not tolerate the introduction of any dimensional parameter at the level of thebare Lagrangian is quite challenging [10]. For a recent attempt, based on a Coleman-Weinberg like mechanism in a framework similar to ours, see Ref.(11). With this in mind, we leave the kinetic part of the Lagrangian absolutely intact, and thus fully scale symmetric, and supplement the potential part by a soft scale symmetry breaking piece. By ‘soft’ we mean

- Terms whose coefficients have a positive power of mass,
- Terms whose transformation law do not involve derivatives of the gauge function \( \chi(x) \).

In particular, while a scalar field mass term is welcome, a vector field mass term will not do. Following ’tHooft [12], adding a non-conformal part such as a scalar field mass term does not have any effect on the dangerously divergent term in the effective action. The more so in the context of this paper, adding a scalar field mass term should be regarded merely a technical tool primarily designed to single out the Einstein gauge. We will show that the corresponding classical solution, if it exists, cannot be anything else but general relativistic.

We thus return to the action eq.(19), and would like to trade the strictly quartic potential \( \lambda \phi^4 \) for a more general potential of the type \( V(\phi) = \lambda \phi^4 + p \phi^2 + q \). On pedagogical grounds, however, to appreciate the fact that our results are generic, and are not that sensitive to the exact structure of the potential, we keep momentarily working with a general \( V(\phi) \). Associated with the non-critical Weyl invariant Lagrangian contaminated by a general scalar potential \( V(\phi) \) are the following field equations, corresponding to variations with respect to \( \phi, \kappa_{\mu}, g_{\mu\nu} \), re-
spectively:
\[ 4\omega g^{\mu
u} \partial_\mu \phi \partial_\nu \phi = \frac{1}{2} V'(\phi) + \phi R + 4\omega s g^{\mu\nu}(\kappa_\mu \kappa_\nu - \kappa_{\mu\nu}) , \quad (16a) \]

\[ X_{\mu\nu} = 4s \phi (\phi^2 + 2\kappa_\mu \phi^2) , \quad (16b) \]

\[ \phi^2 \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = -\phi^2 \gamma + 6g_\mu g^{\beta\gamma} \phi^2 + \frac{1}{2} g_\mu V(\phi) - \gamma_{\mu\nu} + \frac{3}{2} g_\mu g^{\alpha\beta} \gamma_{\alpha\beta} - \frac{1}{2} X_{\alpha\beta} X_{\nu\lambda} + \frac{1}{8} g_{\mu\nu} X^\alpha_{\beta\gamma} X_{\alpha\beta} , \quad (16c) \]

where we have used the notation
\[ \gamma_{\mu\nu} = -6s\phi^2 \kappa_\mu \kappa_\nu + 4\omega D_\mu \phi D_\nu \phi . \quad (17) \]

We can now trace eq. (16a) and subsequently substitute the Ricci scalar \( R \) into eq. (16a). Re-organizing the various terms, we arrive at a generalized Klein-Gordon equation for \( \phi^2 \), namely
\[ g^{\mu\nu} \left( \phi^2 \gamma + 2\kappa_\mu \phi^2 \right) = \frac{\partial W_{\text{eff}}(\phi^2)}{\partial \phi^2} . \quad (18) \]

The effective potential \( W_{\text{eff}}(\phi^2) \) which governs the \( \phi^2 \)-evolution, defined by means of
\[ \frac{\partial W_{\text{eff}}(\phi^2)}{\partial \phi^2} = \frac{1}{3 + 2\omega} \left( \frac{1}{2} \phi V'(\phi) - 2V(\phi) \right) , \quad (19) \]
is known to play a central role [13] in scalar-tensor theories. One may verify that, owing to its conformal nature, the quartic term \( \lambda \phi^4 \) in \( V(\phi) \) does not contribute to \( W_{\text{eff}}(\phi^2) \).

By no coincidence, the same current which sources the kinetic part of the Lagrangian, the \( \kappa_\mu \) field equation does not directly 'know' about \( V(\phi) \). In particular, its associated conserved current remains independent of \( V(\phi) \). Hence, the \( \kappa_\mu \) equation still captures the full local scale invariance of the kinetic part of the Lagrangian. (ii) Treating eq. (20) as a differential equation for \( V(\phi) \) results in the conformal \( V(\phi) = \lambda \phi^4 \). But once a different potential enters the game, affecting only the \( \phi, g_{\mu\nu} \)-equations of motion, the local scale symmetry identity turns an algebraic constraint which is solely respected by the Einstein gauge.

Clearly, the local structure of \( V(\phi) \) is irrelevant at this level. It is only a global feature, namely the discrete spectrum of the \( \frac{\partial W_{\text{eff}}(\phi^2)}{\partial \phi^2} \) roots, which actually matters. Insisting on soft scale symmetry breaking, we restrict ourselves to the class of bi-quadratic polynomials. It is practical to parametrize the potential as follows
\[ V(\phi) = \lambda \phi^4 + (2\Lambda - \nu^2)(2\phi^2 - \nu^2) . \quad (25) \]
Such a potential has the further advantage that its effective potential companion (up to a non-physical additive constant)
\[ W_{\text{eff}}(\phi^2) = \frac{\lambda_1^2 - 2\Lambda}{3 + 2\omega} (\phi^2 - v^2)^2, \]
admits a single extremum (as a function of \( \phi^2 \)). Note that the positivity of \( W \) is correlated with the negativity of the \( \phi^2 \) mass term added. While classically, as explained, we cannot tell a minimum from a maximum, quantum mechanical stability would require \( \Lambda < \frac{1}{2} \lambda v^2 \) for a ghost free (positive) \( \omega \). This includes the special \( \Lambda = 0 \) case.

**CP-violating Weyl-Maxwell mixing**

By construction, the local scale invariant non-critical Brans-Dicke theory can easily accommodate a complex dilaton field. The action eq. (3) is simply traded for
\[ \mathcal{I} = -\int d^4x \sqrt{-g} \left[ \phi^\dagger \phi \left( R - 8\mu g^{\mu\nu} \kappa_{\nu} \right) + 14 \omega g^{\mu\nu} (D_\mu \phi)^2 + \lambda (\phi^\dagger \phi)^2 + \frac{2}{3} g^{\mu\nu} g^{\lambda\sigma} X_{\mu\lambda} X_{\nu\sigma} \right], \]
leaving the door open for the incorporation of Abelian and non-Abelian gauge fields. Using the notation
\[ \phi(x) = \rho(x) e^{i\theta(x)}, \]
with eq. (11) becoming now
\[ D_\mu \phi = e^{i\theta} (\rho_{\mu} + s \rho \kappa_{\mu} + i \rho \theta_{\mu}). \]
In turn, the action eq. (27) takes now the exact form of eq. (9), with \( \rho \) replacing \( \phi \) of course, to which the generalized kinetic term \( 4 \omega \rho^2 g^{\mu\nu} \theta_{\mu} \theta_{\nu} \sqrt{-g} \) is added. Note that the realization of the Einstein gauge \( \rho(x) = v \) is achieved without restricting \( \theta(x) \) whatsoever, leaving the latter to play the role of a free massless scalar field in the Einstein frame. Still, using the notion of a Goldstone boson, while quite tempting, is unjustified here since no global symmetry has actually been spontaneously violated.

The subsequent incorporation of a \( U(1) \) gauge interaction is achieved naturally and flawlessly. However, from an obvious reason (soon to be clarified), it should be emphasized from the outset that it cannot be electromagnetism we are talking about. Associated with a \( U(1) \)-charged dilaton is now the unified co-derivative \( \phi_{\ast \mu} = \phi_{\mu} + \kappa_{\mu} \phi + i e A_\mu \phi. \) By the same token, the modified fake co-derivative derivative \( A_{\mu\nu} \) differs from the covariant derivative \( A_{\mu\nu} \) only by a symmetric term, namely
\[ A_{\mu\nu} = A_{\mu\nu} + \kappa_{\mu} A_{\nu} + \kappa_{\nu} A_{\mu} - g_{\mu\nu} \kappa^2 A_\lambda, \]
so that also \( F_{\mu\nu} \equiv A_{\mu\nu} - A_{\nu\mu} \) stays power zero.

The Maxwell kinetic term, on the other hand, can be accompanied by a novel mixed kinetic term
\[ \frac{1}{4} g^{\mu\nu} g^{\lambda\sigma} X_{\mu\lambda} X_{\nu\sigma} + \frac{1}{4} g^{\mu\nu} g^{\lambda\sigma} F_{\mu\lambda} F_{\nu\sigma} + \frac{\xi}{2} g^{\mu\nu} g^{\lambda\sigma} F_{\mu\lambda} X_{\nu\sigma}, \]
parametrized by a dimensionless coefficient \( \xi \), which cannot be ruled out solely on local symmetry grounds. On group theoretical grounds, a non-Abelian analogue simply cannot exist. Note that a \( U(1) \otimes U(1) \) kinetic mixing has already been studied by Holdom [17] as a mechanism for shifting electromagnetic charges by a calculable amount.

Soft scale symmetry breaking, the advocated mechanism for singling out general relativity at the classical level, is governed now by the \( U(1) \)-invariant scalar potential
\[ V(\phi) = \lambda (\phi^\dagger \phi)^2 + (2\Lambda - \lambda v^2)(2\phi^\dagger \phi - v^2). \]
A closer inspection reveals that once the dilaton modulus \( \rho(x) \) gets frozen up and the Goldstone boson \( \theta(x) \) eaten up, one encounters a diagonal (mass)² matrix for the two vector fields involved, namely
\[ m_\rho^2 = 2(3 + 2\omega) v^2, \quad m_A^2 = 4 e v^2. \]
Notably, unlike the conventional Higgs mechanism, the physical spectrum does not contain a massive free scalar particle, to be regarded a fingerprint of the pseudo-Higgs mechanism. The non-diagonal kinetic mixing eq. (11) gives rise to the Holdom effect. The equations of motion involve two conserved currents
\[ X_{\mu\nu} + \xi F_{\mu\nu} = 8\omega s \left[ \frac{1}{2} (\phi^\dagger \phi)_{;\mu} + \kappa_{\mu} \phi^\dagger \phi \right], \]
\[ F_{\mu\nu} + \xi X_{\mu\nu} = 4 e g^{\mu\nu} \left[ -i \phi^\dagger \nabla_{\mu} \phi + 2 e A_\mu \phi^\dagger \phi \right]. \]
In particular, a residual non-vanishing \( U(1) \) source current, proportional to \( (1 - \xi^2)^{-1} \xi e v^2 \kappa_{\mu} \), survives the \( e \to 0 \) limit. Moreover, contrary to the Holdom mixing, the Weyl-Maxwell mixing is in fact CP-violating. This comes about when noticing [13], as indicated by the structure of the conserved currents, that under a CP transformation
\[ A_\mu \rightarrow -A_\mu, \quad \kappa_\mu \rightarrow \kappa_\mu. \]

The masses \( m_{\rho, \lambda} \) are proportional to a common VEV, the one which sets the Planck scale \( M_{Pl}^2 = 16\pi v^2 \) in the present theory. This is the reason why \( A_\mu \) cannot represent here electromagnetism. From the same reason, the standard model Higgs doublet cannot serve as a dilaton.
(like in the Higgs inflation scenario \[19\]). It is more likely that a grand unified theory (GUT) is involved. In which case, \( m^2_{\text{GUT}} \) and \( M_{\text{Pl}}^2 \) share a common origin, and hence acquire the one and the same mass scale. Up to some potentially large group theoretical factor \( \sim 10^{2-3} \), associated with the dimensions and multiplicity of the scalar field representations involved, the typical (mass)\(^2\) ratio should be

\[
\frac{m^2_{\text{GUT}}}{M_{\text{Pl}}^2} \propto \frac{g^2_{\text{GUT}}}{4\pi}
\]  

(38)

where \( g_{\text{GUT}} \) stands for the coupling constant of the grand unifying group.

**Weyl universality and the Planck scale**

A grand unified theory generically introduces a variety of scalar fields. And once local scale symmetry joins the game, the question is whether such a unified theory can tolerate the coexistence of several kinds of dilatons \( \phi_i \), differing from each other not only by their grand unified representation \( r_i \) but also by their scaling powers. The answer of course is negative, and the reason is quite obvious. We may have several scalars at our disposal, but just one underlying metric to govern the dynamics of the spacetime they live in. To be specific, owing to the identical structure of their kinetic terms (exhibiting a single \( g^{\mu\nu} \)), all minimally coupled scalar fields must constitute co-scalars of order \(-1\). By a similar token, all massless fermions involved constitute co-spinors of order \(-3/2\), transforming according to \( \psi \rightarrow e^{\frac{2i}{3} \omega} \). Their kinetic terms

\[
\int d^4x \sqrt{-g} \bar{\psi} \gamma^\mu (x) \left[ \partial_\mu + \Gamma_\mu (x) \right] \psi ,
\]

(39)

with \( \Gamma_\mu (x) \) denoting the Levi-Civita spin connection, have the further advantage that they are automatically conformally invariant. In turn, unlike the universal minimal coupling of the scalar fields, *fermions simply do not couple to the Weyl gauge field \( \kappa_\mu \).*

We now attempt to go one step further and suggest a variant grand unified theory where all scalar fields are in fact dilatons. Following the Dirac prescription, their individual local scale symmetric contributions to the kinetic term in the Lagrangian sum up into

\[
- \int d^4x \sqrt{-g} \left[ R^i + \sum_i \phi_i^\dagger \phi_i + 4g^{\mu\nu} \sum_i \omega_i \phi_i^\dagger \phi_i \phi_i^{*\mu} \phi_i^{*\nu} \right] ,
\]

(40)

where \( \phi_i^{*\mu} = (\nabla_\mu + i \kappa_\mu + i g T^k_i A_\mu^k) \phi_i \). Note that local scale symmetry can tolerate \( \omega_i \neq \omega_j \) for \( i \neq j \). Such an arbitrariness in the Weyl sector reminds us, in a remote way, of a similar arbitrariness which characterizes the Yukawa sector. While the latter formula is just a straightforward generalization of eq. (4), its overall message is pleasing and is by no means conventional: The Planck (mass)\(^2\) which governs the general relativistic coupling of matter to geometry is nothing but the sum over all individual (VEV)\(^2\)'s which have been invoked to give mass to the variety of particles in the first place. To be more specific,

\[
M_{\text{Pl}}^2 = 16\pi \sum_i v_i^4
\]

(41)

This formula is clearly in accord with the GUT/Planck mass ratio eq. (38) and by being sensitive to the underlying group theoretical structure (expressed via the sum), can hopefully be used to tell one such grand unified theory from the other. A particular \( SO(10) \) grand unified model incorporating the latter idea is currently in the make.

**Epilogue**

Local scale symmetry and its celebrated relative local *phase* symmetry should be treated on equal theoretical footing. As far as the Brans-Dicke theory is concerned, the fact that local scale symmetry is not restricted any more to the critical case \( \omega = -\frac{3}{2} \), but can virtually accompany any \( \omega \) variant, opens the door for revisiting a full range of imaginative theoretical ideas. A leading such idea views general relativity as a spontaneously generated theory of gravity. Its spontaneous scale symmetry breakdown mechanism is however still unknown, at least at the practical level. In this respect, the soft (explicit, but without affecting the divergent term in the effective action) scale symmetry breaking hereby invoked, while being just a poor man’s alternative, it is nonetheless sophisticated enough to provide us with a novel gravitational quasi-Higgs mechanism where the dilaton modulus gets frozen up (thereby defining the Newton constant) by the Weyl-Proca vector field. The latter acquires the Planck mass scale and becomes the physical fingerprint of the emergent general relativity.

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