The effect of roughness and rainfall on hydrodynamic properties of overland flow
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ABSTRACT
Overland flow is the initial driver of slope surface erosion. To discover resistance characteristics of overland flow influenced by rainfall intensity and roughness, indoor simulated rainfall experiments with six kinds of roughness, five flow discharges, and five rainfall intensities were investigated. Results showed that overland flow over rough surfaces could be considered as laminar and turbulent flow when using flow Reynolds number. According to roll waves observed, flow regimes belonged to the laminar transitional zone based on the viscosity-to-depth ratio. A critical water depth formula for overland flow was re-derived, and it showed that this test water flow consisted of supercritical flow in most cases, and subcritical flow in only a few cases. The flow resistance coefficient increased with increasing roughness, whereas it decreased as rainfall intensity increased. Considering the ‘increasing resistance’ phenomenon, this study focused on frictional resistance, thickness of the viscous sublayer, pressure drag and roll waves. Finally, a formula for sheet flow resistance was proposed based on resistance segmentation and multi-element linear regression. These findings are of significance both for understanding the characteristics and development of overland flow and overland flow dynamics.

Key words | overland flow, rainfall intensity, resistance coefficient, roughness

INTRODUCTION
Overland flow, which is formed by rainfall and snowmelt, is considered as a shallow open-channel flow moving along a slope under the influence of gravity. It is the initial driver for soil erosion evolution and an important factor causing soil destruction, transport, and deposition (Zhang et al. 2017). The depth of overland flow is generally less than a few millimeters, and the flow direction is unstable as there are continuous mass and momentum sources along the flow path (Dunkerley 2010; Ali et al. 2012). Moreover, overland flow is highly sensitive to many factors, including underlying surface conditions, splashing raindrops, and slope gradient, making it more complex and worthy of further study.

Currently, research on overland flow has focused on flow patterns and resistance characteristics. In previous studies, three flow regimes have been identified: laminar, transitional, and turbulent flow (Abrahams et al. 1986; Li et al. 1996; Wong & Chen 1997). However, there are still no accurate conclusions about flow patterns. Peng et al. (2011, 2015) adopted a creative numerical method named the lattice Boltzmann method to study shallow flows. Woolhiser et al. (1970) proposed that laminar flow is the main flow pattern of overland flow by analyzing data for a small pasture watershed during several rainstorms. The results of Pan & Shangguan (2006) and Dunkerley (2010) showed that flow patterns of overland flow could be classified into laminar subcritical flow when the underlying surface was covered with vegetation. However, overland flow is known to have the capacity to transport sediment, meaning that applying laminar flow theory may be
inappropriate. Therefore, in order to distinguish overland flow from open-channel laminar flow, Horton (1945) and Selby (1993) put forward the concept of ‘mixed laminar flow’. They agreed that slope sheet flow is a mixed flow zone with laminar flow and turbulent flow. However, Emmett (1978) found that overland flow is not completely consistent with laminar flow, transitional flow, and turbulent flow in the traditional sense, defining it as ‘disturbing flow’. Wang et al. (2012) agreed that it is difficult for overland flow to maintain laminar flow, but this is due to the transition of rolling wave flow to transitional flow in advance. Wang et al. (2018) further stated that sheet flow starts with transitional flow at the surface and finishes with turbulent flow underneath.

Regarding the resistance laws of sheet flow, flow resistance is one of the most important aspects in the study of overland flow (Abrahams & Li 1998; Smith et al. 2007). It is sensitive to multiple factors including flow velocity, flow pattern, surface conditions, sediment deposition, and tillage (Parsons et al. 1994; Lawrence 1997; Lane 2010). The effect of rain intensity on resistance is particularly controversial and has been considered as a critical factor with regard to runoff and erosion (Cerda 2002; Ziadat & Taimeh 2013). Similarly, Savat (1980) found that the ratio of rainfall resistance to total resistance is as high as 1:5 when flow on a slope is gentle and laminar, which is clearly an important constituent part. Parsons et al. (1994) proposed that rainfall resistance may provide a substantial source of resistance due to low water depth and velocity near the drainage watershed. Yoon & Wenzel (1971) and Shen & Li (1973) stated that resistance of sheet flow increased with rainfall intensity when the Reynolds number (Re) < 2,000, whereas rainfall exerted little influence when Re > 2,000. Emmett (1978) argued that flow resistance would be double under rainfall conditions. However, Dunne & Dietrich (1980) found that rainfall resistance constitutes a negligible proportion of total resistance, so it can be neglected.

With respect to the derivation of resistance formula, it has been widely accepted that resistance of different kinds equals to the sum of individual components of resistance (Rauws 1988; Weltz et al. 1992; Parsons et al. 1994; Hu & Abrahams 2005). Li (2009) considered surface objects (e.g., stone or pebbles) and rainfall as the roughness elements, and applied a linear superposition approach to define composite resistance. Similarly, Abrahams & Parsons (1994) stated that resistance to overland flow could be partitioned into several categories including grain resistance, form resistance, wave resistance, and rain resistance. Hu & Abrahams (2006, 2010) proposed that resistance should be broken up into four components without simulated rainfall: surface resistance $f_s$, form resistance $f_f$, wave resistance $f_w$, and bed-mobility resistance $f_m$. In Shen & Li's (1973) study, the Darcy–Weisbach drag coefficient when Re < 900 was expressed as the sum of the drag coefficient at ‘no rainfall’ conditions ($\lambda_0$) and the drag coefficient at ‘simulated rainfall’ conditions ($\lambda_R$). However, there are no consistent explanations of an appropriate formula. Nevertheless, exploring overland flow resistance is critical for better understanding the hydrodynamic mechanisms of soil erosion and the characteristics of overland flow.

The flow hydraulics experiments reported here used six kinds of granular surfaces with varying slope gradient and simulated rainfall characteristics. The objectives of the study were: (1) to find a better way of identifying overland flow regimes by comparing different methods; (2) to partition the contributions of rainfall intensity and roughness to total resistance; and (3) to derive a formula for overland flow resistance based on resistance segmentation.

**MATERIALS AND METHODS**

**Experimental apparatus**

This study was designed on fixed beds, which not only ensured that water flow had no effect on the bed surface, but also controlled surface roughness easily. In order to make the roughness of the bed surface uniform, artificial water sand cloth and screened quartz sand were pasted onto it. This experimental set-up consisted of a test flume, water supplier, vortex stabilizer, rainfall system, and flow-measuring devices (Figure 1). The experimental flume area was 6.0 m long, 0.5 m wide, and 0.25 m deep, with plexiglass sides. The adjustable range of the water flow feeder was 0–12, 5–15, 5–20, and 10–40 L/min. A stationary, manual rainfall device (QYJY-503) was used in this study. The placement height of the rain sprinkler was 18 meters, and the diameter of raindrops could be adjusted between 0.4 and...
6 mm. At the same time, the uniformity of simulated rain was greater than 0.80, and the precision range of adjustable rain intensity was 30–300 mm/h. This experiment was operated with a SX402 digital probe tester (Chongqing Hydrological Instrument Factory, Chongqing, China) to measure the water depth in different conditions, with 0.01-mm precision.

**Experimental design**

Two series of trials on granular surfaces and smooth surfaces were conducted under artificial simulated rainfall conditions, for a total of 145 tests. The experimental flume was adjusted at 15° to reflect a common gradient in the Loess Plateau region. Six kinds of bed conditions were used including: a smooth surface (equivalent roughness ¼ 0.009 mm); a 240 mesh, 120 mesh, and 24 mesh gauze bed (particle sizes ¼ 0.061, 0.120, and 0.700 mm, respectively); and two kinds of sand-pasted surfaces (particle size ¼ 1.770 and 3.680 mm, respectively). In addition, five unit discharges (discharge per unit width) were tested (0.167, 0.333, 0.500, 0.667, and 1.000 L/(s·m)) to reflect the range of rainfall intensity associated with erosion on the Loess Plateau. Also, these designed unit discharges ensured that the water depth was tested within a wide range. Based on the natural rainfall intensities in the Loess Plateau region, rainfall was simulated as: \( R_i = 0 \text{ mm/h}, R_i = 60 \text{ mm/h}, R_i = 90 \text{ mm/h}, R_i = 120 \text{ mm/h}, \) and \( R_i = 150 \text{ mm/h}. \) Since the roughness of the slope did not function under smooth conditions, and was significantly different from natural slope conditions, the rain intensity of 150 mm/h was excluded. There were five longitudinal cross sections for observation in the flume, positioned at 1.0, 2.0, 3.0, 4.0, and 5.0 m along the slope from top to bottom. For each cross section, five measurement points were set transversely to observe the surface velocity using the high-definition photography method with KMnO4 which has a higher precision than traditional methods (Roels 1984; Gilley et al. 1990). An SX402 digital probe tester with 0.01-mm precision was used to measure water depth.

**Data measurement and analysis**

1. According to the continuity equation of flow, the average velocity of a cross section \( (u) \) can be calculated as:

\[
    u = \frac{q}{h} \tag{1}
\]

where \( u \) is the average velocity of the cross section (m/s); \( q \) is unit discharge (m²/s); and \( h \) is average water depth (m).
2. The hydraulics dual flow Reynolds number $Re_h$ can be expressed as:

$$Re_h = \frac{uR}{v}$$

(2)

where $R$ is the hydraulic radius (m); $h$ is depth; and $v$ is the viscosity coefficient of flow (cm$^2$/s).

$v$ can be calculated by the Poiseuille equation as follows:

$$v = \frac{0.01775}{(1 + 0.0337t + 0.000221t^2)}$$

(3)

where $t$ is water temperature.

3. The Froude number is often used to judge the flow pattern in open-channel flow, and can be expressed as follows:

$$Fr = \frac{u}{\sqrt{gh}}$$

(4)

where $g$ is the acceleration due to gravity.

4. Friction velocity $u^*$ is calculated as:

$$u^* = \sqrt{gRJ}$$

(5)

where $g$ is the acceleration of gravity (=9.81 m/s$^2$); $R$ is the hydraulic radius, approximated as the depth $h$; and $J$ is the hydraulic gradient approximated as $\sin \theta$ and $\theta$ was the gradient of the sink.

5. The flow Reynolds number $Re_d$ is calculated as:

$$Re_d = \frac{u^*k_s}{v}$$

(6)

where $k_s$ is the bed surface roughness (mm). For this, when the bed surface is composed of uniform sand, the average particle size is taken.

6. The thickness of the viscous sublayer $\delta$ is calculated as:

$$\delta = \frac{11.6v}{u^*}$$

(7)

7. The Darcy–Weisbach resistance factor $\lambda$ can be calculated as:

$$\lambda = \frac{8gRJ}{u^2}$$

(8)

where $h$ is water depth (m); $u$ is mean velocity (m/s); and $g$ is acceleration due to gravity (m/s$^2$).

### RESULTS AND DISCUSSION

#### Flow regime

**Flow zone**

In this study, we considered overland flow as open-channel flow, and compared a traditional flow regime discriminant method with the roll-wave discriminant method proposed by Zhang et al. (2011). First, 145 sets of test points could be separated from the lower Reynolds number (580) and the upper Reynolds number (6,500), and the relationship between Reynolds number and resistance coefficient of sheet flow was plotted in Figure 2.

As can be seen from Figure 2, most experimental data were distributed on the left side of the laminar boundary. A few sample points were located between the laminar boundary and the turbulent boundary. It could be considered that laminar flow and transitional flow all occurred. There was no turbulent flow, and with increasing rainfall intensity, these points moved towards the transition zone. Splashing raindrops could be a main cause of this phenomenon as this made the streamline unstable. Raindrops also accelerated the flow velocity of overland flow and decreased the water depth, resulting in a lower flow resistance and more turbulent water flows. However, since there was still a transitional flow zone when the rainfall intensity was 0 mm/h, it cannot be simply considered that the rainfall was the only cause of transitional flow. During this test, the water surface was destabilized and formed waves when the hydraulic parameters reached a critical value. They had steep wave fronts and gentle wave rears that move along
the slope intermittently. As they moved in the manner of a snowball, it is called ‘rolling-wave flow’ in hydraulics. Therefore, the reason why it was difficult to maintain laminar flow could be explained by the view of Wang et al. (2012), that is, due to the early transition of rolling-wave flow to transitional flow.

Zhang et al. (2011) proposed a new method for defining the flow regime of sheet flow by using a critical parameter $\delta/h$, which defines the ratio of the thickness of the viscous sublayer to the water depth. The laminar instability zone (rolling-wave flow zone) is defined as $\delta/h \leq 0.12$. Next, experimental data were analyzed from the perspective of roll waves, as shown in Figure 3.

As shown in Figure 3, all curves for various conditions fluctuated on the right side of the critical parameter $\delta/h = 0.12$, which defines the laminar instability zone (rolling-wave flow zone). There were differences in results when using different judgments to determine the flow regime of overland flow on this slope. These two methods could be physically compared as follows.
The inertial force $F$ can be expressed as $m$ multiplied by acceleration $a$:

$$F = ma = \rho V \frac{du}{dt}$$

From a dimensional standpoint:

$$[\rho] \cdot [L^3] \cdot \frac{[v]}{[T]} = [\rho] \cdot [L^2] \cdot [v]$$

where the viscous force $T$ can be defined using Newton's law of friction:

$$T = \mu \frac{du}{dy}$$

From a dimensional standpoint:

$$[\mu] \cdot [L^2] \cdot \frac{[v]}{[L]} = [\mu] \cdot [L] \cdot [v]$$
The dimension of the ratio between inertia force and viscous force was as follows:

\[
\frac{[\rho] \cdot [L^2] \cdot [v^2]}{[\rho] \cdot [L] \cdot [v]} = \frac{[v] \cdot [L]}{[v]}
\]

(13)

where \( v \) is the characteristic velocity; \( L \) is the characteristic length; and \( \nu \) is expressed as the coefficient of kinematic viscosity.

The Reynolds number was shown in Equation (2) and its dimension was shown in Equation (13). That is, the physical meaning of the Reynolds number is the ratio of inertial force to viscous force.

The ratio of viscosity to depth is calculated as:

\[
\frac{\delta}{h} = \frac{11.6v}{u''h} = 11.6v
\]

(14)

Its dimension is the reciprocal of the Reynolds number dimension, thus the physical significance of the viscosity-depth ratio can be expressed as the ratio of viscous force to inertial force. When the Reynolds number is significantly small or \( \delta/h \) is relatively large, viscous force has a larger effect whereas inertial force has a smaller effect, and viscous force has an inhibitory effect on the motion water particles in sheet flow. In contrast, when the Reynolds number is relatively large or \( \delta/h \) is significantly small, inertial force has a larger effect and viscous force has a smaller effect. Inertial force drives particles in the thin-layer flow to move downward and intermix. It can therefore be seen that the Reynolds number and viscosity-depth ratio have the same physical essence.

However, experimental points of sheet flow were all above the laminar flow line and the smooth transition line, which can be called an ‘increasing resistance’ phenomenon of overland flow. This phenomenon is not the same as that in open-channel flow which indicates that the resistance coefficient \( k \) in the laminar flow and transitional areas has nothing to do with bed roughness \( k_g \), but is only determined by the Reynolds number. It is similar to the resistance characteristics of the turbulent zone in open-channel flow, meaning that sheet flow exhibits turbulent characteristics within the laminar flow zone, as well as in the transitional flow zone. Therefore, determining the flow regime using the hydraulic method remains controversial. In this study, it is suggested that the method based on the viscosity-depth ratio is superior to the traditional Reynolds number discriminating method. The main reason for this is that the former can reflect movements of overland flow more directly, and it is considered that roll waves can easily develop in flume experiments (Yoon & Wenzel 1971; Emmett 1978), especially when flow is barely impacted by roughness elements (Rouse 1938). Rolling waves are forms of energy expenditure and add further resistance to the flow (Li 2009). Hence, it is reasonable to analyze flow patterns of sheet flow and ‘increasing resistance’ phenomenon from the prospective of roll waves. Future research should focus on roll waves, such as the critical conditions for the generation and disappearance of roll waves, wave height, wave length, and wave frequency, to improve the roll wave discriminant method and to better explain the hydrodynamic characteristics of sheet flow.

**Flow patterns**

With respect to the different flow patterns of sheet flow, previous studies have often distinguished these using \( Fr = 1 \) as a critical value. However, it is noteworthy that the method using the Froude number is only applicable to the condition of ‘micro-waves’, but in the process of this experiment, it often appeared as ‘rolling-wave flow’. Moreover, overland flow is easily affected by a series of factors, such as surface tension and velocity distribution. Therefore, judging the flow patterns of sheet flow from the perspective of a Froude number is not appropriate. In this study, critical water depth was used to identify patterns of sheet flow. Because the deduction process of the critical depth formula in open-channel flow is not suitable for slope flow, a formula related to critical depth of sheet flow was deduced as follows:

\[
E_s = h \cos \theta + \frac{av^2}{2g}
\]

(15)

where \( h = \) average water depth (m); \( \theta = \) gradient; \( a = \) correction coefficient of kinetic energy; and \( g = \) acceleration of gravity (≈9.81 m/s²). In open-channel flow, \( \cos \theta \approx 1 \) is
often used as the downslope has a shallow gradient. However, for sheet flow, \( \cos \theta \approx 1 \) cannot be used because of a steeper slope gradient.

Critical water depth \( (h_k) \) represents the water depth corresponding to the minimum specific energy of cross sections when flow charge, cross sections’ shape and size are determined. The derivation of \( h \) is as follows:

\[
\frac{dE_s}{dh} = \frac{d}{dh}(h \cos \theta + \frac{\alpha Q^2}{2gA^3}) = \cos \theta - \frac{\alpha Q^2 dA}{gA^3 dh}
\]  

(16)

where \( Q \) = flow charge; and \( A \) = the wetted cross-sectional area. Because the width of the water surface \( b = \frac{dA}{dh} \), Equation (16) can be simplified to:

\[
\frac{dE_s}{dh} = \cos \theta - \frac{\alpha Q^2}{gA^3} B = \cos \theta - \frac{\alpha v^2}{gh}
\]  

(17)

When \( \frac{dE_s}{dh} < 0 \), \( h < h_k \), \( \cos \theta - Fr^2 < 0 \), that is, \( Fr > \sqrt{\cos \theta} \) water flow can be defined as torrent flow. Conversely, when \( \frac{dE_s}{dh} > 0 \), \( h > h_k \), \( \cos \theta - Fr^2 > 0 \), that is \( Fr < \sqrt{\cos \theta} \) water flow can be defined as subcritical flow. When \( \frac{dE_s}{dh} = 0 \), \( h = h_k \), that is \( Fr = \sqrt{\cos \theta} \), water flow can be defined as critical flow.

To determine critical depth \( h_k \), it is assumed that \( \frac{dE_s}{dh} = 0 \), therefore:

\[
\cos \theta - \frac{\alpha Q^2}{gA_k} B_k = 0
\]  

(18)

Thus:

\[
\frac{\alpha Q^2}{g} = \frac{A_k^3}{B_k} \cos \theta
\]

(19)

Because the cross section of the experimental fume was rectangular, the width of the bottom \( b = B_k \), \( A_k = bh_k \), which can be obtained by combining Equations (18) and (19) as follows:

\[
h_k = \sqrt[3]{\frac{\alpha q^2}{g \cos \theta}}
\]

(20)

where \( q = \) unit charge and \( q = Q/b \). Generally, the correction coefficient for kinetic energy ranges from 1.4 to 1.6. Considering that the velocity distribution was uniform in this experiment, it can be simplified as wide-shallow flow, therefore \( \alpha \) is equal to 1.5. The results of this experiment are shown in Figure 4 according to Equation (20).

It can be seen from Figure 4 that overland flow rarely existed in the form of subcritical flow, and most experimental points were located in the torrent zone. Figure 4(a) shows that experimental points move downward to the right with increasing roughness. That is, points move towards the subcritical flow direction of \( h_k/h < 1 \), \( h_k < h \). Figure 4(b) shows that experimental points tended to move upward to the left with an increase in rainfall intensity. That is, experimental points develop towards the torrent direction of \( h_k/h > 1 \), \( h_k > h \), but this trend is not obvious in Figure 4(b), indicating

Figure 4 | The relationship between water depth and critical depth.
that roughness played a more important role in this process compared to rainfall intensity. Compared with the open-channel formula, this critical depth formula considers the effect of slope and is more rigorous than methods using Fr. Moreover, the critical depth formula can more intuitively depict hydraulic resistance laws of overland sheet flow under different conditions. Therefore, for overland flow, the critical depth method is clearly superior to the Froude number method. In this experiment, flow patterns were studied only when the gradient was 15°. However, hydraulic characteristics also vary with the slope. Therefore, it is necessary to focus on resistance laws of sheet flow under gentle slope conditions, so as to enrich and improve current understanding.

**Influence of roughness on overland flow resistance**

Figure 3 shows that when the rainfall intensity was certain, the resistance coefficient was negatively correlated with the Reynolds number, so the regression parameter b in $\lambda = a\text{Re}^b$ obtained by previous quantitative simulation is negative (Roels 1984; Abrahams & Parsons 1994; Ogunlela & Makanjuola 2000). Because these experimental data were essentially parallel to the open-channel laminar streamline, the value b approximated as –1. The resistance coefficient increased with increasing bed roughness, which shows that roughness had an ‘increasing-resistance’ effect. Without considering the flow patterns of the test water, it can be found that if rainfall intensity and flow discharge are certain, the resistance at a roughness of $k_s = 1.770$ mm. The reason for this may be that when the roughness was 3.680 mm, the average height of sand particles on the bed was greater than water depth. This would mean that the submergence ratio is less than 1, and the water flow was submerged flow. Under this condition, the shape, as well as size of sand particles had more complex effects on the flow. Some researchers classified flow resistance on rough beds into particle resistance and form resistance, and these may change in the process of increasing roughness, so that the positive correlation between roughness and resistance coefficient may not be established indefinitely (Abrahams & Parsons 1994; Hu & Abrahams 2006).

The mechanism of increasing resistance is related to frictional resistance, the thickness of the viscous sublayer, pressure drag, and roll waves. As for roll waves, the occurrence of waves leads to an increase in the resistance coefficient, which is similar to the hydraulic jump in open channel flow, as seen in Figure 5. Savat (1980) also captured rolling waves in the laboratory and found that when rolling

| Roughness (mm) | 0      | 60     | 90     | 120    | 150 |
|---------------|--------|--------|--------|--------|-----|
| 0.009         | 1.96   | 1.67   | 1.51   | 1.46   |     |
| 0.061         | 2.04   | 1.86   | 1.90   | 1.92   | 1.82|
| 0.12          | 2.23   | 2.24   | 2.14   | 2.13   | 2.10|
| 0.7           | 2.92   | 2.93   | 2.96   | 2.96   | 2.77|
| 1.77          | 3.26   | 3.17   | 3.15   | 3.11   | 3.08|
| 5.68          | 3.24   | 3.19   | 3.11   | 3.04   | 3.21|

Figure 5 | Roll waves under a rough surface.
waves occurred, the cross section of water suddenly increased and the height of rolling waves was twice the average water depth. In the process of rolling wave migration, liquid particles on the surface and at the bottom were mixed with each other, resulting in high turbulence intensity. It also causes uneven cross-section velocity distribution and large energy loss, leading to a larger coefficient. In this paper, the mechanism of increasing resistance was mainly analyzed from three aspects: (1) frictional resistance, (2) the thickness of viscous sublayer, and (3) pressure drag.

Frictional resistance

Table 1 lists the mean water depth under different roughness conditions at a given rainfall intensity to illustrate the
‘drag-increasing’ effect of frictional resistance on the side walls. It can be seen that water depth increased with increasing roughness, so that the contact area between water flow and the side walls increased. For example, the water depth $h$ increased from 1.96 mm to 3.24 mm when the roughness rose from 0.009 mm to 3.680 mm. That means that the energy consumed in the working of water flow against resistance tended to increase, which directly led to the increasing flow resistance.

**Thickness of the viscous sublayer**

Figure 6 illustrates the relationship between the resistance coefficient and the thickness of the viscous sublayer under certain rainfall intensities. The aim here was to explore the effect of the thickness of the viscous sublayer on the ‘drag-increasing’ mechanism.

As can be seen from Figure 6, when the rainfall intensity was constant, curves tended to move upwards towards the
left with increasing roughness. That is, the greater the roughness, the smaller the thickness of the viscous sublayer, leading to a greater resistance coefficient. The ‘drag-increasing’ phenomenon in this experiment may be due to changes in the thickness of the viscous sublayer; when the sublayer is thick enough to cover the roughness of the bed (making it similar to the smooth surface), roughness will have no effects on the resistance coefficient. Conversely, when it becomes too thin to cover the roughness of the bed, rough elements will emerge and extend into the mainstream, which is equivalent to the flow of water on uneven surfaces; streamlines become displaced, making the flow path increase; water flow is more chaotic causing energy loss, meaning the resistance coefficient increases.

### Pressure drag

Pressure drag is closely related to the Reynolds number. In order to study the effect of pressure drag on the mechanism of ‘drag-increasing’, the double logarithmic relationship between drag coefficient and Reynolds number was plotted for different rainfall intensities and different roughness conditions, as shown in Figure 7.

With an increase in turbulence intensity, a trailing vortex appears and the pressure on the upstream and downstream surfaces of particles are different, resulting in pressure drag, also known as form resistance. It can be seen from Figure 7 that the flow around Reynolds number increased with increasing bed roughness except for the bed condition $k_s = 3.680$ mm, which showed a decline in the viscous resistance. Therefore, the turbulence intensity increased, meaning the pressure differences between the upstream and downstream particles increase. Pressure drag (a component of total flow resistance) accordingly rises, resulting in an increase in the resistance coefficient. When $k_s = 3.680$, this study argues that the height of roughness elements was sufficiently high so as not to be fully inundated by the flow. Lawrence (1997) said that the degree to which the roughness elements are inundated plays a dominant role in determining the resistance offered to the flow by the surface. Therefore, the condition $k_s = 3.680$ had little effect on the ‘drag-increasing’ mechanism.

### Influence of rainfall intensity on overland flow resistance

#### Relationship between resistance coefficient and rainfall intensity

It can be seen from Figure 8 that the relation is similar to that of Figure 2. When the roughness is constant, the resistance coefficient is negatively correlated with the Reynolds number, and it decreases with an increasing Reynolds number. With an increase of rainfall intensity, the distance between each point and laminar flow line, as well as smooth transition line decreases, indicating that the resistance coefficient of water diminished. That is, rainfall had a ‘reducing-resistance’ effect, but compared with smooth surface conditions, this effect was not obvious on artificially roughened bed surfaces. This indicates that bed roughness had a more important role in flow resistance than rainfall intensity.

![Figure 8](http://iwaponline.com/hr/article-pdf/50/5/1324/610998/hr0501324.pdf) | Relationship between resistance coefficient and Reynolds number under different conditions of rainfall intensity (results for $k_s = 0.170, 0.700, 1.770,$ and 3.680 are not shown as the results were almost the same as for $k_s = 0.061$).
Mechanism of ‘reducing resistance’ by rainfall intensity

It can be seen from Figure 8 that the effect of roughness on the resistance coefficient was much greater than that of rainfall intensity. Therefore, the mechanism for the ‘reducing-resistance’ effect of rainfall was examined by comparing simulated rainfall tests with a smooth surface and roughened bed. Figure 9 shows the relationship between water depth and the resistance coefficient under different rainfall intensities when the bed roughness is constant.

From Figure 9, there was a considerable drop in the drag coefficient with increasing rainfall intensity when roughness is constant. However, under the condition of artificial roughness, the decline of the drag coefficient is obviously less than that of smooth surface. Under smooth conditions, water depth had a decreasing trend with increasing rainfall intensity.
intensity, that is, the curves move to the left on the graph. However, under the condition of artificial roughness, it is difficult to see that water depth decreased with increasing rainfall intensity.

For the interpretation of the relationship between rainfall intensity and water depth, this was due to the rainfall intensity which promotes and speeds up downslope water flow. According to the continuity equation, when the flow discharge is constant, there is a decline in water depth with increasing flow velocity. This speculation is consistent with the view of Pan et al. (2010), that when a slope is greater than six degrees (the slope of this test was 15 degrees), rainfall increases the surface velocity to varying degrees. This is due to the larger raindrop momentum in the downslope flow.

Figure 10 | Relationship between underlying thickness of the viscosity and the resistance coefficient under different conditions of rainfall intensity.
direction. According to the momentum theorem, when the velocity of raindrops falling on the bed surface is larger than the surface velocity, the surface velocity of water will increase and the velocity of raindrop will decrease until the speed of the two drops is consistent. The formula of resistance coefficient can be expressed by:

\[
\lambda = \frac{8gR}{u^2}
\]  

(21)

It can be seen that when \(R\) (approximately \(= h\)) decreases and \(u\) increases with rainfall intensity, there is an apparent decline in \(\lambda\). That is, rainfall can reduce the drag coefficient theoretically, which is consistent with the experimental results. The reason for a significant decline in drag coefficient on the smooth bed is that roughness has a more obvious influence on the drag coefficient than rainfall intensity. When a bed is smooth, the ‘reducing-resistance’ effect of rainfall intensity has a greater influence on the drag coefficient. When the bed is rough, the ‘increasing resistance’ effect of roughness is dominant, and rainfall intensity becomes secondary. In addition, as shown in Figure 9, under certain conditions of slope angle, flow charges, and rainfall intensity, water depth of the smooth bed surface was smaller than that of the rough bed surface. Therefore, water flow on the smooth surface was more sensitive to the ‘reducing-resistance’ effect of rainfall intensity, leading to a smaller drag coefficient than the rough bed.

In addition, analyzing the effect of rainfall intensity on the drag coefficient considering the thickness of the viscous sublayer is helpful to understand the ‘reducing-resistance’ mechanism of rainfall. Figure 10 depicts the relationship between the thickness of viscous sublayer and the resistance coefficient under different rainfall intensities and with certain roughnesses.

As can be seen from Figure 10, the thickness of the viscous sublayer increased and drag coefficient decreased with increasing rainfall intensity with smooth conditions. That is, curves move downward to the right. However, under artificially roughed conditions, this phenomenon was less obvious than for under smooth surface conditions due to the effect of roughness on the thickness of the viscous sublayer (as described above). The thickness of the viscous sublayer was mainly affected by the bed roughness, which decreased with increasing roughness. That the above-mentioned rainfall increased the thickness of the viscous sublayer corresponds to the effect of reducing the bed roughness, so that rainfall had a ‘reducing-resistance’ effect. This is consistent with Gilley & Finkner (1991), who found that the addition of rainfall may serve to reduce random roughness.

Calculating the resistance of overland flow

Composition of drag coefficient

Based on the notion of segmentation, resistance to overland flow could be partitioned into several categories of grain resistance, form resistance, wave resistance, and rain resistance. Because all points were obtained on fixed beds and there were no changes in the bed shape during the tests, wave resistance was ignored. Thus, the resistance of overland flow could be expressed as:

\[
\lambda = \lambda_g + \lambda_f - \lambda_r
\]  

(22)

Derivation of resistance formula

Grain resistance is derived from the energy loss caused by overcoming the fixed boundary. It can also be seen as the friction resistance between water flow and the fixed bed, which is mainly affected by particle size and the Reynolds number. Emmett (1978) indicated that there is a relationship between grain resistance and Reynolds number on a smooth bed with only particle resistance. In addition, the index \(b\) of laminar flow was \(-1\). Dunne & Dietrich (1980) attributed variation of parameters \(a\) and \(b\) to the surface roughness

| Parameter b under different conditions of roughness and rainfall intensity |
|---------------------------------------------------------------|
| Roughness (mm)                                      | 0  | 60 | 90 | 120 | 150 |
| Smooth surface                                      | 1.188 | 1.180 | 1.114 | 0.849 | \ |
| 0.061                                              | 1.403 | 1.340 | 1.261 | 1.136 | 1.064 |
| 0.120                                              | 1.273 | 1.236 | 1.129 | 1.121 | 0.859 |
| 0.700                                              | 1.463 | 1.222 | 1.032 | 1.028 | 0.776 |
| 1.770                                              | 1.283 | 1.191 | 1.174 | 1.156 | 1.087 |
| 5.680                                              | 1.486 | 1.391 | 1.379 | 1.189 | 1.106 |
in the power function relationship between drag coefficient and the Reynolds number. However, Abrahams et al. (1992) thought that these changes were related to the variation of inundation of rough elements. In summary, granular resistance produced by rough elements is a substantial source of total resistance. This experimental flow belongs to the laminar transition zone, and the Reh index is about $-1.19$ (Table 2), which is consistent with Abrahams et al. (1992) and Nearing et al. (1997) ($b = -1.1$). Therefore, it is considered that grain resistance $\lambda_g$ can be expressed as:

$$\lambda_g = \frac{24}{Re_h}$$

(23)

This is consistent with the resistance coefficient ($Re < 900$) of Shen & Li's (1973) study without rainfall.
Form resistance can be expressed as:

\[
\lambda_f = C_d \rho \frac{\nu^2}{2}
\]

where \( \rho \) is the density of water; \( \nu \) is the mean velocity; \( A \) is the projection area of rough elements in the \( u \) direction; and \( C_d \) is the coefficient of drag, which is related to the shape of particles and the Reynolds number \( \text{Re}_d \).

Hirsch (1996) integrated the upper formula and Darcy–Weisbach drag coefficients, and obtained the formula for calculating form resistance as follows:

\[
\lambda_f = 4C_d \sum \frac{A_i}{A_b}
\]

where \( A_i \) is the projected area of the first rough element in the direction of the current. When \( \Lambda \geq 1 \), \( A_i = D_r D_y \), when
\[ \lambda_f = \frac{2487.42}{\text{Re}^1.25 \Lambda^{1.95}} \]  

(26)

It is found that rainfall resistance is not only related to the rainfall intensity \( R_i \) as well as Reynolds number \( \text{Re}_h \), but is also affected by the bed roughness \( k_s \), that is, the greater the \( k_s \), the smaller the contribution of rainfall to the resistance coefficient. The formula for rainfall resistance through multiple regression analysis can be expressed by:

\[ \lambda_r = 0.03 k_s^{0.30} R_i^{0.95} / \text{Re}_h^{0.33} \]  

(27)

In summary, the formula for calculating the resistance coefficient of overland flow under rainfall conditions is:

\[ \lambda = \lambda_g + \lambda_f - \lambda_r = \frac{24}{\text{Re}_h} + \frac{2487.42}{\text{Re}_d^{1.25} \Lambda^{1.95}} - \frac{0.03 k_s^{0.30} R_i^{0.95}}{\text{Re}_h^{0.33}} \]  

(28)

In order to discuss the rationality of this formula, the drag coefficient is calculated by Equation (28) and verified with the measured value, as shown in Figure 13.

It can be seen from Figure 13 that the calculated and measured drag coefficients are all around the 1:1 line, \( (R^2 = 0.8244) \), but that there is some scatter around the correlation line. This is due to the interaction of different roughness elements under the same rainfall intensity, and the rolling wave phenomenon in this test is known to affect measurement accuracy to a certain extent.

**CONCLUSIONS**

Indoor artificial simulated rainfall experiments were performed with six kinds of bed roughness, five flow discharges, and five rainfall intensities. The experimental results showed the following:

1. Overland flow can be classified as laminar flow and transitional flow using a binary Reynolds number. If the method of judging viscosity–depth ratio is adopted, overland flow belongs to the rolling wave zone. Considering the gradient, a critical water depth formula for overland flow was re-derived, and it was concluded that this water flow belonged to supercritical flow in most cases, while it was subcritical flow in only a few cases.

2. Roughness has an ‘increasing resistance’ effect. Frictional resistance, the thickness of viscous sublayer, pressure drag, and roll waves all result in an increasing drag coefficient. Rainfall has a ‘reducing-resistance’ effect. Under smooth surface conditions, water depth decreased with increasing intensity. Conversely, under artificially roughened conditions, it was almost impossible to see any changes in water depth with rainfall intensity. In addition, it was also found that rainfall can increase the thickness of the viscous sublayer, which is equivalent to...
reducing the effect of bed roughness, thus causing ‘drag reduction’.

3. According to the notion of resistance segmentation, the resistance coefficient can be divided into grain resistance $\lambda_g$, form resistance $\lambda_f$, rainfall resistance $\lambda_r$, and wave resistance $\lambda_w$. Based on this, a formula for calculating the resistance coefficient of overland flow under rainfall conditions was obtained by multiple linear regression.

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REFERENCES

Abrahams, A. D. & Li, G. 1998 Effect of saltating sediment on flow resistance and bed roughness in overland flow. Earth Surface Process and Landforms 23 (10), 953–960.

Abrahams, A. D. & Parsons, A. J. 1994 Hydraulics of interrill overland flow on stone-covered desert surfaces. Catena 25 (1–2), 111–140.

Abrahams, A. D., Parsons, A. J. & Luk, S. H. 1986 Resistance to overland flow on desert hillslopes. Journal of Hydrology 88, 343–363.

Abrahams, A. D., Parsons, A. J. & Hirsch, P. J. 1992 Field and laboratory studies of resistance to interrill overland flow on semiarid hillslopes, Southern Arizona. In: Overland Flow Hydraulics and Erosion Mechanics (A. J. Parsons & A. D. Abrahams, eds). UCL Press, London, UK, pp. 1–24.

Ali, M., Sterk, G., Seeger, M. & Stroosnijder, L. 2012 Effect of flow discharge and median grain size on mean flow velocity under overland flow. Journal of Hydrology 452–453, 150–160.

Cerda, A. 2002 The effect of season and parent material on water erosion on highly eroded soils in eastern Spain. Journal of Arid Environment 52 (3), 319–337.

Dunkerley, D. 2010 Estimating the mean speed of laminar overland flow using dye injection-uncertainty on rough surfaces. Earth Surface Process and Landforms 26 (4), 363–374.

Dunne, T. & Dietrich, W. E. 1980 Experimental investigation of overland flow on tropical hillslopes. Zeitschrift Fur Geomorphologie 35, 60–80.

Emmett, W. W. 1978 Overland flow. In: Hillslope Hydrology (M. J. Kirkby, ed.). Oxford University Press, Oxford, UK, pp. 16–54.

Gilley, J. E. & Finkner, S. C. 1991 Hydraulic roughness coefficients as affected by random roughness. Transactions of the ASAE 34 (3), 897–903.

Gilley, J. E., Kottwitz, E. R. & Simanton, J. R. 1990 Hydraulic characteristics of rills. Transitions of the ASAE 33 (6), 1900–1906.

Hirsch, P. J. 1996 Hydraulic Resistance of Overland Flow on Semiarid Hillslopes: A Physical Simulation. PhD dissertation, State University of New York at Buffalo, NY, USA.

Horton, R. E. 1945 Erosion development of streams and their drainage basins; hydrophysical approach to quantitative morphology. Geological Society of America Bulletin 56 (3), 275–370.

Hu, S. & Abrahams, A. D. 2005 The effect of bed mobility on resistance to overland flow. Earth Surface Process and Landforms 30 (11), 1461–1470.

Hu, S. & Abrahams, A. D. 2006 Partitioning resistance to overland flow on rough mobile beds. Earth Surface Process and Landforms 31 (10), 1280–1291.

Hu, S. & Abrahams, A. D. 2010 The effect of bed mobility on resistance to overland flow. Earth Surface Process and Landforms 31 (11), 1461–1470.

Lane, S. N. 2010 Roughness-time for a re-evaluation. Earth Surface Process and Landforms 30 (2), 251–253.

Lawrence, D. S. L. 1997 Macroscale surface roughness and frictional resistance in overland flow. Earth Surface Process and Landforms 22 (4), 365–382.

Li, G. 2009 Preliminary study of the interference of surface objects and rainfall in overland flow resistance. Catena 78 (2), 154–158.

Li, G., Abrahams, A. D. & Atkinson, J. F. 1996 Correction factors in the determination of mean velocity of overland flow. Earth Surface Process and Landforms 21 (6), 509–515.

Nearing, M. A., Norton, L. D., Bulgakov, D. A., Larionov, G. A., West, L. T. & Dontsova, K. M. 1997 Hydraulics and erosion in eroding rills. Water Resources Research 33 (4), 865–876.

Ogunlela, A. O. & Makanguila, M. B. 2000 Hydraulic roughness of some African grasses. Journal of Agricultural Engineering Research 75 (2), 221–224.

Pan, C. & Shangguan, Z. 2006 Runoff hydraulic characteristics and sediment generation in sloped grassplots under simulated rainfall conditions. Journal of Hydrology 351 (1), 178–185.

Pan, C., Shangguan, Z. & Lei, T. 2010 Influences of grass and moss on runoff and sediment yield on sloped loess surfaces under simulated rainfall. Hydrological Processes 20 (18), 3815–3824.

Parsons, A. J., Abrahams, A. D. & John, W. 1994 On determining resistance to interrill overland flow. Water Resources Research 30 (12), 3515–3522.

Peng, Y., Zhou, J. G. & Burrows, R. 2011 Modelling the free surface flow in rectangular shallow basins by lattice Boltzmann.
method. *Journal of Hydraulic Engineering* **137** (12), 1680–1685.

Peng, Y., Zhou, J. G., Zhang, J. M. & Burrows, R. 2015 Modeling moving boundary in shallow water by LBM. *International Journal of Modern Physics C* **24** (1), 1–17.

Rauws, G. 1988 Laboratory experiments on resistance to overland flow due to composite roughness. *Journal of Hydrology* **103** (1), 37–52.

Roels, J. M. 1984 Flow resistance in concentrated overland flow on rough slope surface. *Earth Surface Process and Landforms* **9** (6), 541–551.

Rouse, H. 1958 *Fluid Mechanics for Hydraulic Engineers*. Dover, New York.

Savat, J. 1980 Resistance to flow in rough supercritical sheet flow. *Earth Surface Process and Landforms* **5** (2), 103–122.

Selby, M. J. 1993 Hillslope materials and processes. *Transitions of the Institute of British Geographers* **19** (4), 505.

Shen, H. W. & Li, R. M. 1975 Rainfall effects on sheet flow over smooth surface. *Journal of the Hydraulics Division* **99** (5), 771–792.

Smith, M. W., Cox, N. J. & Bracken, L. J. 2007 Applying flow resistance equations to overland flows. *Progress in Physical Geography* **31** (4), 363–387.

Wang, G. Y., Liu, Y. H. & Wang, X. H. 2012 Experimental investigation of hydrodynamic characteristics of overland flow with geocell. *Journal of Hydrodynamics* **24** (5), 737–743.

Wang, G. Y., Sun, G. R. & Li, J. K. 2018 The experimental study of hydrodynamic characteristics of the overland flow on a slope with three-dimensional Geomat. *Journal of Hydrodynamics* **30** (1), 153–159.

Weltz, M. A., Arslan, A. B. & Lane, L. J. 1992 Hydraulic roughness coefficient for native rangelands. *Journal of Irrigation and Drainage Engineering* **118** (5), 776–790.

Wong, T. S. & Chen, C. 1997 Time of concentration formula for sheet flow of varying flow regime. *Journal of Hydrologic Engineering* **2** (3), 136–159.

Woolhiser, D. A., Hanson, C. L. & Kuhlman, A. R. 1970 Overland flow on rangeland watersheds. *Journal of Hydrology (New Zealand)* **9** (2), 336–356.

Yoon, Y. N. & Wenzel, H. G. 1971 Mechanics of sheet flow under simulated rainfall. *Journal of Hydraulic Engineering* **97** (9), 1367–1386.

Zhang, K. D., Wang, G. Q., Lv, H. X. & Wang, Z. L. 2011 Discussion on flow pattern determination method of shallow flow on the slope surface. *Journal of Experiments in Fluid Mechanics* **25** (4), 67–73.

Zhang, K., Wang, Z., Wang, G., Sun, X. & Cui, N. 2017 Overland-flow resistance characteristics of nonsubmerged vegetation. *Journal of Irrigation and Drainage Engineering* **143** (8), 04017021.

Ziadat, F. M. & Taimeh, A. 2013 Effect of rainfall intensity, slope, land use and antecedent soil moisture on soil erosion in an arid environment. *Land Degradation & Development* **24** (6), 582–590.

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