Implementation of Dryden Continuous Turbulence Model into Simulink for LSA-02 Flight Test Simulation

Teuku Mohd Ichwanul Hakim¹, Ony Arifianto²

Abstract. Turbulence is a movement of air on small scale in the atmosphere that caused by instabilities of pressure and temperature distribution. Turbulence model is integrated into flight mechanical model as an atmospheric disturbance. Common turbulence model used in flight mechanical model are Dryden and Von Karman model. In this minor research, only Dryden continuous turbulence model were made.

Dryden continuous turbulence model has been implemented, it refers to the military specification MIL-HDBK-1797. The model was implemented into Matlab Simulink. The model will be integrated with flight mechanical model to observe response of the aircraft when it is flight through turbulence field. The turbulence model is characterized by multiplying the filter which are generated from power spectral density with band-limited Gaussian white noise input.

In order to ensure that the model provide a good result, model verification has been done by comparing the implemented model with the similar model that is provided in aerospace blockset. The result shows that there are some difference for 2 linear velocities (v_g and w_g), and 3 angular rate (p_g, q_g and r_g). The difference is instantly caused by different determination of turbulence scale length which is used in aerospace blockset. With the adjustment of turbulence length in the implemented model, both model result the similar output.

Introduction

Along with national needs in the area of airborne remote sensing and surveillance [1], Aeronautics Technology Center of LAPAN in cooperation with TU Berlin, Germany, develop a project to investigate the implementation of an Electronic Flight Control System (EFCS) into a light utility aircraft. The project itself started from 2014 and planned to be finish in 2019 [1]. Goal of the EFCS development in the future is to provide the control system that can help and even replace the pilot in missions that are extremely difficult, long or dangerous. In order to fulfill that goal, the EFCS have to have full authority and highly reliable. Furthermore, the EFCS shall be able to fly the aircraft to follow predefined trajectories with high precision, to stabilize the aircraft and measurement system during flight in calm and turbulence atmosphere. In the project a demonstrator aircraft which called as an LSA-02 is developed in order to investigate the flight control function and flight control laws [2]. The LSA-02 will

¹ LAPAN, Bandung Institute of Technology, TU Berlin
² Bandung Institute of Technology
use to perform research in the area of UAV technology, especially for new flight control functions. During the testing, safety pilot will be on board for safety reason and conduct the experiment.

In the aircraft, the EFCS is installed to supplement the basic mechanical flight control system. When engaged, the EFCS uses the mechanical linkage of the mechanical FCS to command control surfaces via electro-mechanical actuators. The commands are computed by Flight Control Laws (FCL). Basic FCL are implemented in the flight control computer (FCC) of the Basic Electronic Flight Control System (BEFCS). The BEFCS provides all standard autopilot modes [3].

Flight control laws (FCL) testing is an important part in the development of Electronic Flight Control System (EFCS) of an LSA-02. The testing is used to verify and validate the developed FCL against the requirements in order to ensure the functionality and safety. Verification and validation of the FCL can be performed through several type of test e.g. software in the loop simulation (SILS), hardware in the loop simulation (HILS) and flight testing (FT).

On ground test facility such as SILS and HILS, the dynamic behavior of the aircraft is simulated in flight mechanical model. The environmental disturbances e.g. wind, gust and turbulence are also simulated to provide real condition that may be faced by the aircraft during flight testing. It is not sufficient to simulate and to evaluate the aircraft only in calm condition.

Objective of the research is to provide atmospheric turbulence model in the form of Simulink model which has been verified to support the simulation. Later, the model will be integrated to the flight mechanical model to observe response of the aircraft when it is flight through turbulence field and how the FCL response to stabilize the aircraft. Figure 1 shows environment for FCL test, where the atmospheric disturbance integrated to flight mechanical model block.

![Figure 1 Environment for FCL test](image)

By definition, turbulence is a movement of air on small scale in the atmosphere that caused by instabilities of pressure and temperature distribution in clouds, near ground and in the jet-stream region. Based on [5] and [6], turbulence is defined as a stochastic process which are determined by velocity spectra. The turbulence field is assumed to be visualized as frozen in time and space.

There are 2 common continuous turbulence models that are usually used in flight mechanical model, those are:

- Dryden Continuous Turbulence Model; and
- Von Karman Continuous Turbulence Model
In practical, Dryden turbulence model is used more frequently, it is caused by the Dryden model has simpler form and the ease to produce [7]. The Dryden wind turbulence model, or it is also called as the Dryden gusts, is a mathematical model of continuous gusts which is accepted by the United States Department of Defense to be used in aircraft design and simulation applications.

**Atmospheric Turbulence Model Theory**

Dryden and Von Karman model are the common turbulence model that are used in flight mechanics. In this case, Dryden model will be used because it is easier to implement in simulation even though both of them tends to similar result. According to [8] there are four assumptions are taken in turbulence modeling for statistical properties of the turbulence

- stationary (independent of time)
- homogeneous (independent of location in space)
- isotropic (independent of direction)
- larger than the airplane (all parts of the airplane are equally affected)

In Dryden turbulence model, the linear and angular velocity components of continuous gusts treated as spatially varying stochastic processes and specifies each component’s power spectral density. It is applying shaped noise with known spectral properties as velocity and angle rate perturbations to the body axes of the vehicle, the effect of turbulence is captured during discrete time simulations [9]. The noise spectrum for each of the perturbations is described by a turbulence scale length (L), the airspeed (V), and the turbulence intensity (σ). The turbulence scale length L is the length of turbulence field which expressed in longitudinal, lateral and vertical axis (L_u, L_v and L_w). Turbulence intensity σ is the magnitude of the turbulence which expressed in longitudinal, lateral and vertical (σ_u, σ_v and σ_w). It has dimension ft/s or m/s which is root-mean-square of its magnitude.

Figure 3 shows the aircraft with velocity of V fly into a turbulence field with sine-type vertical velocity distribution with the length of L in longitudinal axis (L_u). To traverse the turbulence, the aircraft takes time

\[
T = \frac{L}{V}
\]  

(1)

The aircraft experiences the turbulence wave with the temporal frequency of

\[
\omega = \frac{2\pi}{T} = \frac{2\pi}{V/L}
\]  

(2)

The turbulence wave not just consist of one frequency, whole spectrum are exist from low to high frequency. Turbulence spectral density that are modeled in Dryden function expressed in non-dimensional frequency form as:

\[
\omega T = \omega \frac{L}{V}
\]  

(3)
Figure 2 Axes definition of turbulence linear velocity and angular rate follows body axis of the aircraft

Figure 3 Illustration of the aircraft flies into sine-type turbulence field

### Power Spectral Densities

Dryden turbulence model is characterized by following power spectral densities ($\Phi$) as the function of temporal frequency. Power spectral densities for three linear velocity component ($u_g$, $v_g$, $w_g$) are:

\[
\Phi_u(\omega) = \frac{2\sigma^2_{uL_u}}{nV} \cdot \frac{1}{1 + \left(\frac{L_u\omega}{V}\right)^2} \tag{4}
\]

\[
\Phi_v(\omega) = \frac{2\sigma^2_{vL_u}}{nV} \cdot \frac{1 + 12\left(\frac{L_v\omega}{V}\right)^2}{\left[1 + 4\left(\frac{L_v\omega}{V}\right)^2\right]^2} \tag{5}
\]

\[
\Phi_w(\omega) = \frac{2\sigma^2_{wL_u}}{nV} \cdot \frac{1 + 12\left(\frac{L_w\omega}{V}\right)^2}{\left[1 + 4\left(\frac{L_w\omega}{V}\right)^2\right]^2} \tag{6}
\]

Power spectral densities for three angular rate components ($p_g$, $q_g$, $r_g$) are:
\[ \Phi_p(\omega) = \frac{\sigma^2_w}{V_L} \cdot \frac{0.8(\frac{2\pi L_w}{4b})^{1/3}}{1 + (\frac{4b}{\pi V})^2} \]  

\[ \Phi_q(\omega) = \frac{\pm(\frac{\omega}{V})^2}{1 + (\frac{4b}{\pi V})^2} \cdot \Phi_w(\omega) \]  

\[ \Phi_q(\omega) = \frac{\pm(\frac{\omega}{V})^2}{1 + (\frac{4b}{\pi V})^2} \cdot \Phi_p(\omega) \]

where:
- \( \sigma_u, \sigma_v, \sigma_w \) indicate turbulence intensities
- \( L_u, L_v, L_w \) indicate scale length
- \( b \) indicates wing span

### Continuous Dryden Filter

Continuous Dryden filters are derived from the square roots of the Dryden power spectrum equations as it is expressed in equation 4 - 9. In order to generate turbulence signal that has similar frequency spectrum as Dryden power spectral density in the simulation, the continuous filter is used. To excite waves with full spectrum, white noise (band-limited Gaussian white noise) signal is used. It is passed through the appropriate filters. White noise is a random signal that having equal intensity at different frequencies, giving it a constant power spectral density.

Based on [10] Dryden continuous filter for velocity spectra are expressed as following transfer function:

\[ H_u(s) = \sigma_u \sqrt{\frac{2L_u}{\pi V}} \cdot \frac{1}{1 + \frac{2L_u}{V} s} \]  

\[ H_v(s) = \sigma_v \sqrt{\frac{2L_v}{\pi V}} \cdot \frac{1 + \frac{2\sqrt{3}L_v}{V} s}{(1 + \frac{2L_v}{V} s)^2} \]  

\[ H_w(s) = \sigma_w \sqrt{\frac{2L_w}{\pi V}} \cdot \frac{1 + \frac{2\sqrt{3}L_w}{V} s}{(1 + \frac{2L_w}{V} s)^2} \]

Dryden continuous filter [10] for angular rate spectra expressed as following transfer function:

\[ H_p(s) = \sigma_w \sqrt{\frac{0.8}{V (2L_w)^{1/3} (1 + \frac{4b}{\pi V})^2}} \]  

\[ H_r(s) = \frac{\mp (\frac{\pi}{V})^{1/6}}{(1 + \frac{2b}{\pi V})^{1/3}} \cdot H_p(s) \]
\[ H_q(s) = \left( \frac{\frac{1}{V}}{\frac{1}{V} + \frac{4b}{\pi V^2}} \right) \cdot H_w(s) \] 

Continuoulsy Dryden filter is a kind of low-pass filter, where the frequency that higher than certain cut-off frequency is eliminated. Cut-off frequency of the filter is determined by the ratio of turbulence scale length to the airspeed.

Application of turbulence model is clearly defined in [5] [6]. In the references there are distinction of the altitude, which are low-altitude and medium/high altitude.

Low-altitude region is defined as the altitude equal to and less than 1000 feet. In this region, following values are defined:

- Turbulence scale length is the function of altitude. According to [6], turbulence scale length for low altitude is defined as follows:
  \[ 2L_w = h \]  
  \[ L_u = 2L_v = \frac{h}{(0.177 + 0.000823h)^{1.2}} \]  

- Turbulence intensities
  \[ \sigma_w = 0.1W_{20} \]  
  \[ \sigma_u = \sigma_v = \sigma_w \frac{1}{(0.177 + 0.000823h)^{0.4}} \]  

\( W_{20} \) is typical values for wind speed at height of 20 feet, it is defined in [5] and [6]. Typical values for \( W_{20} \) defined in the Table 1.

| Turbulence Level | \( W_{20} \) |
|------------------|----------------|
| Light            | 15 knots       |
| Moderate         | 30 knots       |
| Severe           | 45 knots       |

Medium/high altitude is defined for altitude higher than or equal to 2000 feet. In this region turbulence is assumed isotropic, therefore following values are defined:

- Turbulence scale length in medium/high altitude is assumed to be constant. Relation for turbulence scale length in each axes is defined as follow:
  \[ L_u = 2L_v = 2L_w = 1750 \text{ ft} \]  

- Turbulence intensities in [5] and [6] are defined in a diagram that contains root-mean-square (RMS) turbulence intensities as the function of altitude and probability of exceedance as it is shown in Figure 4. Relation for turbulence intensities for each axes is defined as follow:
\[ \sigma_u = \sigma_v = \sigma_w \] (21)

Figure 4 Medium/high altitude turbulence intensities plot compared with [10]

Output of the filter are 3 linear velocity component \((u_g, v_g, w_g)\) and 3 angular rate component \((p_g, q_g, r_g)\). Those output will goes to the flight mechanical model as a disturbance signal.

Model Development

Model for Dryden continuous turbulence was made in Simulink. It consist of input subsystem, scale length subsystem, turbulence intensity subsystem, velocity filter subsystem and angular rate subsystem. Input values and plotting commands are defined in Matlab script. High level structure of the developed turbulence model is shown in Figure 5. Detail information of the developed model can be found in [4].

Figure 5 High level structure of developed turbulence model
In order to ensure that the Simulink models provide a good results, it is need to verify the model with other models that has been proven. Therefore the developed model is compared with Dryden continuous turbulence model which are available in Matlab Aerospace Blockset. Block structure comparison between developed models with Dryden model can be seen in Figure 6. Both model use the same input in order to provide comparable output. Those model are calculated using [6].

![Figure 6 High level structure of comparison model](image)

**Result and Discussion**

Turbulence model is simulated with the time of 100 seconds. Result of the simulation is described as follow.

Output from both model stored in the workspace and then compared in the same plot to show how close the developed model to the reference. Figure 7 shows the comparison of linear velocity ($u_g$, $v_g$ and $w_g$) between developed model and aerospace blockset model. The result shows that output of linear velocity $u_g$ is fit to the output that results from aerospace blockset. But it is not the same for output of $v_g$ and $w_g$, the magnitude are did not fit to the output of the reference. The same condition also occur for angular rate $p_g$, $q_g$ and $r_g$ as it is shown in Figure 8. Magnitude of three angular rate of developed model are different from magnitude of reference.

With the same input values given, both model should result the same output. From further analysis finally it is found that the difference between the models are the value of turbulence scale length ($L_v$ and $L_w$) that are used in calculation. Even though both models are use the same standard [6], in fact [10] use turbulence scale length as it is defined in [5]. Therefore only $u_g$ that fit together, because [5] and [6] has the same definition for $L_v$.

Reference [10] state that the use of different $L_v$ as it is defined in standard [5] and [6] is acceptable, despite it would result the difference turbulence characteristic. Turbulence length scale has direct impact to the power spectral density and magnitude of the turbulence.
Furthermore, to ensure that the developed model result the same output as aerospace blockset, then the turbulence scale length in developed model were adjusted. After adjustment in turbulence scale length, the result are shown in Figure 9 and Figure 10. In those figure all output are fit together. As well as for the power spectral density (PSD) for linear velocity and angular rate are fit together. Figure 11 shows the sample for single sided PSD with FFT for $u_g$ and $q_g$.

![Figure 7](image7.png)  
**Figure 7** Comparison results for longitudinal linear velocity $u_g$, $v_g$ and $w_g$

![Figure 8](image8.png)  
**Figure 8** Comparison results for longitudinal angular rate $p_g$, $q_g$ and $r_g
Figure 9 Comparison of linear velocity for adjusted $L_v$ and $L_w$

Figure 10 Comparison of angular rate for adjusted $L_v$ and $L_w$
Conclusion

Dryden continuous turbulence model based on [6] has been successfully developed for low altitude and medium/high altitude.

Linear interpolation has been introduced to calculate turbulence scale length and turbulence intensities for altitude between 1,000 ft and 2,000 ft to provide value of L and \( \sigma \) for turbulence calculation in those altitude range.

Verification of the Simulink model result the same output in \( u_g \) as the model from aerospace blockset. Other 2 linear velocity and 3 angular rate results in different magnitude. The differences caused by different turbulence scale length definition for \( L_v \) and \( L_w \) as it is determined in [6]. The [10] choose \( L_v \) and \( L_w \) as it is determined in [5].

The developed turbulence model is quite reasonable to be integrated to the flight mechanical model for FCL testing, because it has been result a good output compared to the reference.

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