WMMSE RESOURCE ALLOCATION FOR FD-NOMA

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ABSTRACT

Power and channel allocation in interference-limited systems is a key enabler for beyond 5G (B5G) technologies, such as multi-carrier full duplex non-orthogonal multiple access (FD-NOMA). In FD-NOMA systems power allocation is a very computationally intense non-convex problem due to the presence of strong interference and the integrality condition on channel allocation. In this paper, we propose an iterative power allocation algorithm based on the minimization of the weighted mean square error, which converges to a feasible allocation of the original problem. Experimental results show that the proposed algorithm has by far the lowest complexity among other state-of-the-art solutions. Moreover, they assess the validity of our approach showing performance close to the theoretical optimum.

Keywords Full-duplex, NOMA, WMMSE.

1 Introduction

Full duplex (FD) and non-orthogonal multiple access (NOMA) are among the most promising technologies to be adopted for future wireless communication systems [1]. These two technologies achieve superior performance by relaxing the conventional requirement of orthogonal access to the wireless channel. NOMA and FD have been recently combined in a novel scheme, which has the advantage of achieving enhanced flexibility, user fairness, and increased spectral efficiency [2]. The price to pay is that the system performance is severely affected by the presence of large multi-user interference so that power and channel allocation [3,4] are fundamental for its successful implementation and deployment. As a matter of fact, even in a single carrier scenario, power allocation in FD-NOMA is not convex due to the mutual interference among users. Hence, its solution requires the use of advanced and complex algorithms, e.g., see [5]. In a multi-carrier scenario, the complexity increases since one has to jointly establish the allocation and the pairing order on each subcarrier. To the best of our knowledge, only a few works in the literature have addressed the problem of resource allocation for FD-NOMA systems [6,9]. All these studies consider the SISO case, i.e., when both the base station and the nodes have a single antenna. On the other hand, in the MIMO case users can be separated by a beamforming precoder, and hence NOMA can be applied to users belonging to the same beam [10,14]. Hence, the same SISO paradigm may be applied to users belonging to the same beam. The FD-NOMA solutions proposed so far envisage optimization algorithms having large to very large computational loads. The main reason for their complexity is that: a) the objective function is not convex; b) the allocation problem involves a set of mixed allocation variables: binary, the channel assignment, and continuous, the power allocation. In [7,8] the binary assignment variables have been addressed by first relaxing the integrality condition and then assigning a penalty method for non-integer variables. Using an alternative perspective, in [9] we have proposed a scheme that solves the problem in the dual Lagrangian domain, following a block coordinate descent approach.

In this work, we propose a low-complexity resource allocation algorithm for the rate maximization in multi-carrier SISO FD-NOMA systems, based on the minimization of the weighted mean square error (MSE) [15]. At the equilibrium, the weighted minimum MSE (WMMSE) solution is also a local maximizer of the sum-rate, and its iterative formulation, based on block coordinate descent, allows to address very complex continuous problems. In the case of NOMA, the WMMSE formulation is made problematic by the non-linear nature of the interference cancellation process, which requires a fixed ordering of the users and a corresponding allocation phase. One of the main contributions

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of the proposed algorithm is that it succeeds in almost completely removing the need for binary assignment variables so to focus mainly on power allocation, thus noticeably reducing the problem complexity. The relaxation of the allocation variables does not influence the optimality of the solution, so that numerical simulations show performance close to the theoretic optimum.

2 System Model

We consider an OFDMA FD-NOMA system, where single-antenna user equipments (UEs) are served by a single-antenna base station (BS). Due to hardware limitations, the FD technology is implemented only at the BS, which is able to cancel a large fraction of the self-interference that it generates. The NOMA paradigm is implemented at both the BS and the UEs, which are able to cancel a certain number of interfering signal of the same type (uplink or downlink) on each channel \( f \in \mathcal{F} \), the set of available channels, through successive interference cancellation (SIC). To correctly perform SIC, we assume that a) the signal to be cancelled is perfectly reconstructed, condition assumed for the remainder of the paper, and b) absence of detection errors, condition discussed in Section 2.1.

We denote by \( \mathcal{U} \) and \( \mathcal{D} \) the sets of the \( M = |\mathcal{U}| \) uplink and \( N = |\mathcal{D}| \) downlink users in the system, respectively. The information symbols \( s_{i,f} \) transmitted (or received) by user \( i \) on subcarrier \( f \) are modelled as zero-mean complex random variables with with unitary power, the signal is then scaled by a factor \( \sqrt{P_{i,f}} \), so that the signal power is \( P_{i,f} \), where the index \( i \) denotes the transmitter if \( i \in \mathcal{U} \) or the receiver if \( i \in \mathcal{D} \). In the FD-NOMA scenario all users can transmit on any subcarrier without any orthogonality requirements and uplink and downlink transmissions are both affected by uplink and downlink interference. This is a particularly challenging scenario since uplink users might cause serious interference to downlink users if proper countermeasures are not in place. In this setting, the signal intended for user \( i \in \mathcal{U} \cup \mathcal{D} \) can be generally expressed by

\[
y_{i,f} = h_{i,i}(f)\sqrt{P_{i,f}}s_{i,f} + \sum_{j \in (U \cup D) \setminus i} h_{j,i}(f)\sqrt{P_{j,f}}s_{j,f} + z_{i,f},
\]

where \( z_{i,f} \) is the zero-mean thermal noise at the receiver with variance \( \sigma^2 = \mathbb{E}\{|z_{i,f}|^2\} \), \( h_{i,i} \) represents the direct channel between UE and the BS and \( h_{j,i} \) describes the multi-user (MUI) due to NOMA and co-channel interference (CCI) due to FD. Accordingly, depending on the combination of uplink and downlink users there are four different cases for the interference a) if \( i \in \mathcal{D} \) and \( j \in \mathcal{U} \), \( h_{j,i}(f) \) represents the cross-channel gains between uplink and downlink users (CCI); b) if \( i \in \mathcal{U} \) and \( j \in \mathcal{D} \), \( h_{j,i}(f) \) denotes the residual gain due to non perfect self-interference cancellation at the BS (CCI); c) if \( i, j \in \mathcal{U} \), it is \( h_{j,i}(f) = h_{j,j}(f) \) (MUI) and d) if \( i, j \in \mathcal{D} \), it is \( h_{j,i}(f) = h_{i,i}(f) \) (MUI).

2.1 Interference cancellation

For practical reasons, we assume that for any subcarrier the message of at most two users can be sent in any direction and of those two users, according to the NOMA paradigm, only one is able to cancel interference coming from the same direction. For both uplink and downlink, we single out a strong and a weak user. Their definition is asymmetric: in the downlink, strong users cancel the contributions of the other users, while in the uplink the contribution of the strong user can be easily canceled by the BS so that the weak users are not interfered by it. Accordingly, we introduce the binary allocation variable \( x_{i,f} \in \mathcal{X} \), which is set to 1 if user \( i \) is the strong one on channel \( f \) and 0 otherwise. The set \( \mathcal{X} \) is defined consistently with our assumptions as \( \mathcal{X} = \{x_{i,f} \in \{0,1\} | i \in \mathcal{U} \cup \mathcal{D}, \sum_{i \in \mathcal{U}} x_{i,f} \leq 1, \sum_{i \in \mathcal{D}} x_{i,f} \leq 1 \} \). After SIC the received signal on subcarrier \( f \) for user \( i \) is

\[
y_{i,f} = h_{i,i}(f)\sqrt{P_{i,f}}s_{i,f} + \sum_{j \in \mathcal{T}^{(x)}_{i,f}} h_{j,i}(f)\sqrt{P_{j,f}}s_{j,f} + z_{i,f},
\]

where \( \mathcal{T}^{(x)}_{i,f} \) is the set of the potential users interfering with user \( i \) on subcarrier \( f \), and it is defined as

\[
\mathcal{T}^{(x)}_{i,f} = \begin{cases} 
\mathcal{U} \setminus \{i\} & i \in \mathcal{U}, x_{i,f} = 1 \\
\mathcal{U} \cup \mathcal{D} \setminus \{i, s\} & i, s \in \mathcal{U}, x_{s,f} = 1 \\
\mathcal{U} & i \in \mathcal{U}, x_{i,f} = 1, \\
\mathcal{U} \cup \mathcal{D} \setminus \{i\} & i \in \mathcal{D}, x_{i,f} = 0.
\end{cases}
\]

In practice, the number of actively interfering users on channel \( f \) is smaller than the cardinality of \( \mathcal{T}^{(x)}_{i,f} \). For example, user \( k \), which sets \( P_{k,f} = 0 \), does not interfere with \( i \) on channel \( f \) even if it belongs to \( \mathcal{T}^{(x)}_{i,f} \). The advantage of the
formulation (2) is that does not require an explicit allocation of the weak user but depends only on power allocation and as such is helpful for the derivation of the main algorithm in Section 3.1.

Let us denote by \( P_D = \{ P_{i,f}, f \in \mathcal{F}, i \in \mathcal{D} \} \) and \( P_U = \{ P_{i,f}, f \in \mathcal{F}, i \in \mathcal{U} \} \) the vectors collecting the transmit powers for all downlink and uplink users in the system and by \( P = [P_D, P_U] \), from (2), the signal to interference-plus-noise ratio (SINR) for user \( i \) on subcarrier \( f \) is

\[
\gamma_{i,f}(P,x) = \frac{|h_{i,i}(f)|^2 P_{i,f}}{\sum_{j \in X_{i,f}^i} |h_{j,i}(f)|^2 P_{j,f} + \sigma^2}.
\]

so that the rate of \( i \) on \( f \) is \( R_{i,f}(P,x) = \log (1 + \gamma_{i,f}(P,x)) \).

In the uplink, interference cancellation is straightforward: the BS receives all the data streams and, hence, can always successfully cancel the strong user. Conversely, in the downlink the data stream intended for the weak user’s receiver is canceled at the strong user’s receiver. Thus, if \( s \in \mathcal{D} \) is the strong user and \( k \in \mathcal{D} \) is the weak one on the downlink channel \( f \), the condition for perfect cancellation of \( k \) is

\[
R_{k,s,f}(P,x) > R_{k,f}(P,x)
\]

where \( R_{k,s,f}(P,x) \) is the achievable rate of user \( k \) measured at the receiver \( s \). This condition is equivalent to (7)

\[
\Gamma_{k,s}(f) = \sum_{j \in U} \Theta_{j,f}^{(k,s)} P_{j,f} + \Delta_f^{(k,s)} \leq 0,
\]

with \( \Theta_{j,f}^{(k,s)} = |h_{k,k}(f), h_{j,s}(f)|^2 - |h_{k,s}(f), h_{j,k}(f)|^2 \) and \( \Delta_f^{(k,s)} = (|h_{k,k}(f)|^2 - |h_{k,s}(f)|^2) \sigma^2 \). An interesting feature of (6) is that it depends only on the channel gains of downlink users but not on their powers.

3 Max sum-rate algorithm for FD-NOMA

To formulate the max-rate optimization problem we introduce also the binary allocation variable \( t_{i,f} \in \mathcal{T} \), which is set to 1 if user \( i \) is the weak one on channel \( f \) and 0 otherwise. Taking into account that there are at most two users per channel direction and that a user can not be at the same time weak and strong on a given subcarrier, \( \mathcal{T} \) is defined as the set \( \mathcal{T} = \{ t_{i,f} \in \{0,1\} \mid i \in \mathcal{U} \cup \mathcal{D}, t_{i,f} = 0 \text{ if } x_{i,f} = 1, \sum_{i \in \mathcal{U}} t_{i,f} \leq 1, \sum_{i \in \mathcal{D}} t_{i,f} \leq 1 \} \). Our objective is to allocate power and channels to the users to maximize the overall weighted sum-rate \( U(x,t,P) = \sum_{f \in \mathcal{F}} \sum_{i \in \mathcal{U} \cup \mathcal{D}} x_{i,f} \alpha_s R_{i,f}(P,x) + t_{i,f} \alpha_w R_{i,f}(P,x) \)

\[
\max_{P \succeq 0, x \in X, t \in \mathcal{T}} U(x,t,P)
\]

subject to

\[
\sum_{f \in \mathcal{F}} P_{i,f} \leq P_U, \quad \forall i \in \mathcal{U}\tag{7a}
\]

\[
\sum_{i \in \mathcal{D}} \sum_{f \in \mathcal{F}} P_{i,f} \leq P_D\tag{7b}
\]

\[
\Gamma_{k,s}(f) \leq 0, \quad \forall k,s \in \mathcal{D} \mid x_{s,f} = 1, t_{k,f} = 1, \quad \forall f \in \mathcal{F}.	ag{7c}
\]

The positive weights \( \alpha_s \) and \( \alpha_w \) in the objective function are employed to enforce a certain degree of fairness among strong ad weak users, constraints (7a) and (7b) are the power budget for the uplink and downlink users and constraint (7c) guarantees successful pairwise SIC at downlink strong user \( s \) when the weak user \( k \) is actually transmitting on the same subcarrier. Unfortunately, (7) is a mixed integer programming problem because of the simultaneous presence of binary and continuous variables and this, together with non-linear interference cancellation constraint, makes it extremely complex to solve. Accordingly, we choose a heuristic approach to breaking the solution of the optimization in three phases:

1. Allocate the strong users: in this phase one strong uplink and one strong downlink user is selected for each subcarrier;
2. Relax the constraint that only one weak user is allowed for subcarrier: we remove any limit to how many weak users can be allocated to one subcarrier. The actual number of weak users active on a channel is given by the number of users whose power is different from 0: channel assignment is the result of power allocation. Anyhow, each weak user still needs to satisfy (6) to be feasible.

3. Power allocation: having fixed \( x^\star \in X \) and relaxed \( t \), (7) becomes the power allocation problem

\[
\max_{P \succeq 0} \sum_{f \in F} \sum_{i \in U \cup D} x_{i,f} \alpha_i R_{i,f}(P, x) + (1 - x_{i,f}) \alpha_w R_{i,f}(P, x)
\]

subject to (7.a), (7.b), (7.c).

The only part where binary assignment is needed is in the first step of the algorithm, i.e., the strong users allocation part, which is a pure full-duplex allocation problem solvable with state-of-the-art approaches such as [16].

The fact that more than one weak user is allowed per subcarrier simplifies the formulation of problem (8) but could potentially raise many practical problems. Nevertheless, we can formulate the following theorem.

**Theorem 1.** Any local optima for (8) allows at most two NOMA users per channel in each direction.

**Proof.** See Appendix A.

Thus, the solutions of the relaxed problem (8) are compliant with the exclusive integer constraints on channel assignment.

### 3.1 A WMMSE formulation for (8)

Even after removing the dependence on the binary allocation variables \( x \) and \( t \), the power allocation (8) is not convex: in the sum-rate expression the power of a given user \( k \) appears both at the numerator of the SINR when \( k \) is the desired user and at the denominator when \( k \) is seen as interference. To address the non-convexity, we reformulate the sum-rate maximization problem in the presence of interference as weighted MSE minimization [15]. Regardless of the transmit direction, multiplying the received signal \( y_{i,f} \) by a scaling factor \( g_{i,f} \) yields the MSE

\[
e_{i,f} = \mathbb{E}_{s,z} \left\{ |g_{i,f} y_{i,f} - s_{i,f}|^2 \right\} = |1 - g_{i,f} h_{i,i}(f) \sqrt{P_{i,f}}|^2 + \sum_{j \in I_{i,f}} |g_{i,f} h_{j,i}(f)|^2 P_{j,f} + \sigma^2.
\]

By differentiating (9) and setting the derivative to zero, the value of \( g_{i,f} \) that minimizes the MSE is

\[
g_{i,f} = \frac{h_{i,i}^*(f) \sqrt{P_{i,f}}}{|h_{i,i}(f)|^2 P_{i,f} + \sum_{j \in I_{i,f}} |h_{j,i}(f)|^2 P_{j,f} + \sigma^2},
\]

and the correspondent value for the minimum MSE is

\[
e_{i,f}(P) = (1 + \gamma_{i,f}(P, x))^{-1}.
\]

The weighted minimum MSE problem is defined as

\[
\min_{P \succeq 0, \omega \succeq 0, g} \sum_{f \in F} \sum_{i \in U \cup D} \alpha_{i,f} [w_{i,f} e_{i,f}(P) - \log(w_{i,f})]
\]

subject to (7.a), (7.b), (7.c).

where \( \alpha_{i,f} = x_{i,f} \alpha_s + (1 - x_{i,f}) \alpha_w \) is either \( \alpha_s \) if user \( i \) is the strong one on channel \( f \) or \( \alpha_w \) otherwise, \( w_{i,f} \) are positive weights and \( w \) and \( g \) are the vectors collecting all values of \( w_{i,f} \) and \( g_{i,f} \), respectively. We can now formulate this theorem that allows us to solve the power allocation problem as a weighted MSE minimization.

**Theorem 2.** Solving (12) is equivalent to solving (8), i.e. (12) yields a local optimum for (8).

**Proof.** See Appendix B.
Problem (12) is still not convex, but can be solved by adopting an iterative block coordinate descent (BCD) scheme. BCD converges to a local minimum if the optimization over each block of variables is convex and differentiable [17]. Thus, we split the original problem (12) into four different sub-problems obtained by fixing any three out of the four sets \( \{P_D\}, \{P_U\}, \{w\} \) and \( \{g\} \) of optimization variables and solve (12) with respect to (w.r.t.) the remaining set of variables. By doing so, each of the sub-problems is convex and differentiable and, iterating the solution of the problems sequentially, the algorithm converges to a local optimum of (12), which, because of Theorem 2, yields a power distribution that locally maximizes (8) as well. The four sub-problems are

**Optimizing w.r.t. g.** Given the power allocations \( P, g_{i,f} \forall i \in U \cup D, f \in F \) can be evaluated as in (10).

**Optimizing w.r.t. w.** Given \( P \) and \( g \), we can compute \( e_{i,f} \) as in (9). Differentiating the objective function w.r.t. \( w \) and setting the result to zero yields

\[
w_{i,f} = e_{i,f}^{-1} \forall i \in U \cup D, \forall f \in F.
\]

Since \( 0 < e_{i,f} \leq 1 \), the positive constraints on \( w \) are always met.

**Optimizing w.r.t. P_D.** Given the values of \( g, w \) and \( P_U \), the power allocation problem for the downlink users is convex and at such can be solved in the dual domain. Neglecting irrelevant terms, the Lagrangian dual function is

\[
\min_{P \succeq 0} \sum_{f \in F, k \in D} \beta_{k,f} e_{k,f}(P) + \mu P_{k,f},
\]

where, to ease the notation, \( \beta_{k,f} = \alpha_{k,f} w_{k,f} \) and \( \mu \) is the positive Lagrangian multiplier associated to the constraint (7b). Constraint (7b) is active only for the weak downlink users, translating in an on/off condition: given channel \( f \) and the strong downlink user \( s \), a weak user \( k \) can transmit only if it is \( \Gamma_{k,s}(f) \leq 0 \). This is captured in (14) by the use of the set \( D_f = D_f^w \cup \{s \mid x_{s,f} = 1\} \), which includes the strong downlink user \( s \) and \( D_f^w = \{k \mid x_{k,f} = 0 \& \Gamma_{k,s}(f) \leq 0\} \), the set of the weak users for which (7b) is satisfied.

The optimal power for user \( i \in D_f \) is

\[
P_{i,f} = \left[ \frac{\beta_{i,f} g_{i,f} h_{i,i}(f)}{\beta_{i,f} g_{i,f} h_{i,i}(f)^2 + \sum_{l \in \mathcal{C}_{i,f}^{(x)}} \beta_{l,f} g_{l,f} h_{l,l}(f)^2 + \mu} \right]^2
\]

and it is zero for all the other users. In (15), we have used the set \( \mathcal{C}_{i,f}^{(x)} \), which represents the set of users that potentially receive interference from \( i \in U \cup D \) on subcarrier \( f \), i.e.

\[
\mathcal{C}_{i,f}^{(x)} = \begin{cases} D_f, & i \in U, \ x_{i,f} = 1, \\ U \cup D \setminus i, & i \in U, \ x_{i,f} = 0, \\ U \cup D \setminus i, & i \in D, \ x_{i,f} = 1, \\ U \cup D \setminus \{i, s\}, & i, s \in D, \ x_{s,f} = 1. 
\end{cases}
\]

**Optimizing w.r.t. P_U.** Given the values of \( g, w \) and \( P_D \), the problem evaluating the uplink power coefficients is convex. The extra NOMA condition (7c), which is active only for the uplink users, is linear in \( P_U \) and the Lagrangian dual function is

\[
\min_{P_U \succeq 0} \sum_{j \in U, f \in F} \beta_{j,f} e_{j,f} + \mu_j P_{j,f} + \sum_{k \in D_f^w} \mu_{k,f} \Theta_{j,f}^{(k,s)} P_{j,f},
\]

where \( \mu_j \) and \( \mu_{k,f} \) are positive Lagrangian multipliers associated to constraint (7a) and (7c), respectively and \( s \) is the strong downlink user on \( f \), i.e., \( s \in D \mid x_{s,f} = 1 \).

The power coefficient for user \( i \in U \) can be evaluated as

\[
P_{i,f} = \left[ \frac{w_{i,f} g_{i,f} h_{i,i}(f)}{w_{i,f}^2 g_{i,f}^2 h_{i,i}(f)^2 + \sum_{l \in \mathcal{C}_{i,f}^{(x)}} w_{l,f}^* g_{l,f}^* h_{l,l}(f)^2 + \lambda_i} \right]^2
\]

where, \( \lambda_i = \mu_i + \sum_{k \in D} \mu_{k,f} \Theta_{i,f}^{(k,s)} \), and \( \mathcal{C}_{i,f}(x) \) is given in eq. (16). The evaluation of \( \mu_i \) and \( \mu_{k,f} \) is obtained through well known ellipsoid method [19].
4 Numerical Results

We consider a single cell scenario with \( F = 6 \) subcarriers, and with the same number of uplink and downlink users \( M = N \). The number of channels and users is set to test the system in overloaded conditions [9]. The cell radius is 100 m [8]. The path loss exponent is 4, while the shadowing is log-normally distributed having standard deviation 8 dB. The SI cancellation factor at the BS is set to a constant value of 110 dB, as in [7]. The strong users needed to initialize the WMMSE algorithm are selected by maximizing the overall sum-rate neglecting the interference due to NOMA paradigm. This problem is solved running the full-duplex allocation algorithm given in [16]. The value of the weights for the strong users is set to \( \alpha_s = 1 \), while the value of \( \alpha_w \) varies depending on the simulations. For all the results, the maximum power for each uplink user is \( P_U = 14 \) dBm and \( P_D = 20 \) dBm for the BS.

We use as benchmark the theoretic optimum (REF) and the sub-optimal (SCA) approach proposed in [7]. The complexity of the various schemes, computed considering the term that dominates the number of elementary operations, is presented in Table 1. The WMMSE approach, whose load includes the allocation of strong users, is the lightest, being linear w.r.t. the total number of users.

Fig. 1 shows the performance of the proposed and benchmark algorithms in terms of the weighted sum-rate \( U \), for different numbers of users in the cell, and with \( \alpha_w = 2 \). We plot the performance of the proposed approach (WMMSE), and of the pure full-duplex approach (FD-OMA) [16], as a function of the number of iterations. We also show the performances attained at the converge by REF and SCA benchmark algorithms [7]. For WMMSE, \( U \) increases monotonically after each iteration as proven in [19]. Since after the firsts 100 iterations the increment of performances is negligible, we fix to 200 the maximum number of iterations, justifying the value of Table 1. At convergence, the results are very close to the ones obtained by the more complex FD-NOMA algorithms and notably outperform the FD-OMA scheme.

![Figure 1](image-url)

**Figure 1:** \( U \) vs number of iterations, for different number of users \( N + M \) (blue for 10, red for 30 and magenta for 50 users).

Fig. 2 refers to an instance of the simulation scenario with \( M = N = 3 \). The strong user is \( s = 3 \) and Fig. 2 plots as straight lines the rates \( R_{k,f} \) of all downlink weak users \( k \in D^f_w = \{1, 2\} \) and as dashed lines the rates \( R_{k,s,f} \) of the same users measured at the downlink strong user \( s = 3 \) for a given channel \( f \) vs the iteration number. Consistently with Theorem 1, the number of active downlink weak users on channel \( f \) converges to one, compliant with the exclusivity of resource assignment. Moreover, the dashed lines are clearly above the straight lines, thus fulfilling constraint (7. c). Results collected for any other instances show the same behaviour.
Figure 2: Rates $R_{k,f}$ (solid lines) and $R_{k,s,f}$ (dashed lines) vs number of iterations for a specific simulation instance.

Fig. 3 shows the impact of different values of $\alpha_w$ vs the number of users $N + M$ by plotting the Jain’s fairness index (solid line, left y-axis) and the spectral efficiency $R/F$ (dashed line, right y-axis). The rate considered here is not weighted, i.e., $R = \sum_{i} R_{i,f}(P, x)$, showing the effect of the increased fairness between strong and weak users in terms of the achievable throughput. As expected, a higher $\alpha_w$ leads to higher fairness and lower spectral efficiency. For $N + M > 40$, the fairness of the system does not depend on $\alpha_w$.

Figure 3: Jain’s fairness (solid lines) and spectral efficiency (dashed lines) vs $N + M$, for different values of $\alpha_w$.

5 Conclusions and Future Work

In this paper, power and channel allocation problem for multi-carrier non-orthogonal multiple access full duplex systems has been investigated. We have proposed a solution based on the minimization of the weighted mean square error, which benefits of the insights obtained by the problem decomposition. The proposed approach achieves performance close to the optimum at a fraction of the complexity. Future work will be focused on how to extend the proposed algorithm to a FD-NOMA MIMO setting, enabling the allocation of more than two active users per direction.

A Proof of Theorem 1

Theorem 1. Any local optima for (8) allows at most two NOMA users per channel in each direction.

Proof. We adopt a reductio ad absurdum argument and assume that (8) yields a solution where the rate on a given downlink channel, say $f_0$, is maximized by more than two users. The proof for the uplink direction is similar and is omitted for lack of space. By construction, for each channel there is at most one strong user, so we assume that at the equilibrium there are one strong user $s$ and two weak users $w_1$ and $w_2$. To simplify the notation, we omit the channel index and indicate with $\bar{R}_i$ and $P_i$ the rate and the power of user $i$ on channel $f_0$. We indicate with

$$\bar{\sigma}^2 = \frac{\sum_{j \in \mathcal{U}} |h_{j,i}(f_0)|^2 P_{j,f_0} + \sigma^2}{|h_{i,i}(f_0)|^2}$$
the normalized noise-plus-(uplink-)interference for user $i$ and with $\tilde{P}_0 = \tilde{P}_s + \tilde{P}_{w_1} + \tilde{P}_{w_2}$ the total power transmitted

on downlink channel $f_0$. Consistently with these definitions, it is

$$R_s = \log_2 \left(1 + \frac{\tilde{P}_s}{\sigma_s^2}\right), \quad \tilde{R}_{w_1} = \log_2 \left(1 + \frac{\tilde{P}_{w_1}}{\tilde{P}_s + \tilde{P}_{w_2} + \sigma_{w_1}^2}\right), \quad \tilde{R}_{w_2} = \log_2 \left(1 + \frac{\tilde{P}_{w_2}}{\tilde{P}_s + \tilde{P}_{w_1} + \sigma_{w_2}^2}\right).$$

To prove our theorem we make the non-restrictive assumption that $\tilde{\sigma}_{w_2}^2 > \tilde{\sigma}_{w_1}^2$, and we need to show that the solution with the three users $s, w_1, w_2$ can not be an equilibrium point because, freezing all other parameters of the system, we can find a solution that yields a higher weighted rate, by allocating only users $s$ and $w_1$. In particular, we show that it is more beneficial to allocate all the power $\tilde{P}_0 - \tilde{P}_s = \tilde{P}_{w_1} + \tilde{P}_{w_2}$ to user $w_1$, so that its rate is $\log_2 \left(1 + \frac{\tilde{P}_0 - \tilde{P}_s}{\tilde{P}_s + \tilde{\sigma}_{w_1}^2}\right)$, rather than sharing it between $w_1$ and $w_2$. Accordingly, the inequality to prove is

$$\alpha_s R_s + \alpha_w (\tilde{R}_{w_1} + \tilde{R}_{w_2}) < \alpha_s R_s + \alpha_w \log_2 \left(1 + \frac{\tilde{P}_0 - \tilde{P}_s}{\tilde{P}_s + \tilde{\sigma}_{w_1}^2}\right),$$

which, after few simplifications and exploiting the properties of the logarithm, becomes

$$\frac{\tilde{P}_0 + \tilde{\sigma}_{w_1}^2}{\tilde{P}_s + \tilde{P}_{w_2} + \tilde{\sigma}_{w_1}^2} \left(1 + \frac{\tilde{P}_{w_2}}{\tilde{P}_s + \tilde{P}_{w_1} + \tilde{\sigma}_{w_1}^2}\right) < \frac{\tilde{P}_0 + \tilde{\sigma}_{w_1}^2}{\tilde{P}_s + \tilde{\sigma}_{w_1}^2}. (19)$$

In turn, (19) is equivalent to

$$1 + \frac{\tilde{P}_{w_2}}{\tilde{P}_s + \tilde{P}_{w_1} + \tilde{\sigma}_{w_1}^2} < \frac{\tilde{P}_s + \tilde{P}_{w_2} + \tilde{\sigma}_{w_1}^2}{\tilde{P}_s + \tilde{\sigma}_{w_1}^2} = 1 + \frac{\tilde{P}_{w_2}}{\tilde{P}_s + \tilde{\sigma}_{w_1}^2},$$

which is always true because it is $\tilde{\sigma}_{w_1}^2 < \tilde{P}_{w_1} + \tilde{\sigma}_{w_1}^2$. The solution with three users on the downlink channel $f_0$ can not be an equilibrium point for (8) and the hypothesis is false.

**B Proof of Theorem 2**

**Theorem 2.** Solving (12) is equivalent to solving (8), i.e. (12) yields a local optimum for (8).

**Proof.** Let us add to the variables involved in (12) the apex $(\tau)$ to indicate that they have been computed at the $\tau$-th iteration. Hence, we indicate with $e^{(\tau)}_{i,f} = e_{i,f}(\mathbf{P}^{(\tau)}, g_{i,f}^{(\tau)})$ the MSE computed for some given values of $\mathbf{P}^{(\tau)}$ and $g_{i,f}^{(\tau)}$. Owing to the optimality of each subproblem, we have

$$e_{i,f}^{(\tau+1)} \leq e_{i,f}^{(\tau)}(\mathbf{P}^{(\tau+1)}, g_{i,f}^{(\tau)}) \leq e_{i,f}^{(\tau)}.$$ (20)

Hence, since $1 + \log(r) \leq r, \forall r \in \mathbb{R}_+$ we have

$$\alpha_i, f \left[1 + \log \left(w_{i,f}^{(\tau)} e_{i,f}^{(\tau+1)}\right)\right] \leq \alpha_i, f w_{i,f}^{(\tau)} e_{i,f}^{(\tau+1)} (21)$$

Summing (21) over $i \in U \cup D$ and $f \in \mathcal{F}$ and combining the result with (20) yields to:

$$\sum_{i,f} \alpha_i, f + \alpha_i, f \log \left(w_{i,f}^{(\tau)}\right) \leq \sum_{i,f} \alpha_i, f w_{i,f}^{(\tau)} e_{i,f}^{(\tau)} - \alpha_i, f \log \left(e_{i,f}^{(\tau+1)}\right)$$ (22)

Replacing (13) in (22), we obtain

$$\sum_{i,f} -\alpha_i, f \log \left(e_{i,f}^{(\tau)}\right) \leq \sum_{i,f} -\alpha_i, f \log \left(e_{i,f}^{(\tau+1)}\right),$$ (23)

or, equivalently, replacing (11) in (23)

$$U \left(\mathbf{P}^{(\tau)}\right) \leq U \left(\mathbf{P}^{(\tau+1)}\right).$$ (24)

In words, the weighted sum rate provably increases at each iteration. Moreover $\mathbf{P}^{(\tau)}$ is a feasible solution of problem (12), $\forall \tau > 0$, and so of problem (8). Therefore, the proposed approach is guaranteed to have a robust convergence to a local optimum solution of the original power allocation problem (8).
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