Characterizing the dynamic properties of the solar turbulence with 3-D simulations: Consequences in terms of $p$-mode excitation.

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Abstract. A 3D simulation of the upper part of the solar convective zone is used to derive constraints about the averaged and dynamic properties of solar turbulent convection. Theses constraints are then used to compute the acoustic energy supply rate $P$ injected into the solar radial oscillations according to the theoretical expression in Samadi & Goupil (2001). The result is compared with solar seismic data.

Assuming, as it is usually done, a gaussian model for the frequency component $\chi_k(\nu)$ of the model of turbulence, it is found that the computed $P(\nu)$ is underestimated compared with the solar seismic data by a factor $\sim 2.5$.

A frequency analysis of the solar simulation shows that the gaussian model indeed does not correctly model $\chi_k(\nu)$ in the frequency range where the acoustic energy injected into the solar $p$-modes is important ($\nu \simeq 2 \text{–} 4 \text{ mHz}$). One must consider an additional non-gaussian component for $\chi_k(\nu)$ to reproduce its behavior. Computed values of $P$ obtained with this non-gaussian component reproduce better the solar seismic observations. This non-gaussian component leads to a Reynolds stress contribution of the same order than the one arising from the advection of the turbulent fluctuations of entropy by the turbulent motions.

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1. Introduction

Turbulent motions in the upper convective zone of solar like stars, generate acoustic energy, which in turn is injected at the rate $P(\nu)$ into $p$-modes oscillations ($\nu$ is the frequency of a given mode). Solar-type oscillations are therefore meant as stochastically excited by turbulent convection. This type of excitation process concerns low massive stars with an outer convective zone.

Measurement of the rate of acoustic energy, $P(\nu)$, injected into solar-like oscillations is one of the goals of future space seismic missions COROT (Baglin et al 1998) and Eddington (Favata et al 2000). These seismic data will then make it possible to constrain the theory of excitation and damping of solar-type oscillations, which in turn will provide valuable information about the properties of stellar convection zones.

Theoretical formulations for $P(\nu)$ offer the advantage of testing separately several properties entering in excitation mechanism. Here we consider the formulation by Samadi & Goupil (2001, see section 2) and also Samadi (2001) for a detailed summary).

Computation of $P(\nu)$ requires an accurate knowledge of the dynamic properties of turbulence in stars. Unfortunately, the current observations of the solar granulation cannot provide a precise enough description of the turbulent spectrum properties.

In the present work we use a 3D simulation of the upper part of the solar convective zone to determine the dynamic properties of solar turbulence (see section 3) which are necessary to compute $P(\nu)$. Our calculations are then compared with the helioseismic constraints from the GOLF/SOHO instrument (see section 4).

2. A theoretical formulation for $P(\nu)$

The calculation of the rate $P(\nu)$ at which a given $p$-mode is excited results from an integration over the stellar mass ($m$) and local integrations over distance ($r$) and time ($t$) of the mode eigenfunction ($\xi$) and the correlation product of the excitation source ($< \vec{S} \vec{S} >$). This expression can be written in a schematic form as:

$$P(\nu) \propto \int dm \int dr \int dt \xi \cdot < \vec{S} \vec{S} > \cdot \vec{\xi}$$

The excitation source ($\vec{S}$) has two identified origins: the turbulent Reynolds stress and the advection of the turbulent entropy fluctuations by the turbulent motions. $< \vec{S} \vec{S} >$ is expressed in terms of the turbulent kinetic energy spectrum $E(k, \nu)$ and the spectrum of the entropy fluctuations $E_s(k, \nu)$ where $k$ is the wavenumber of a given turbulent element.

Detailed expressions for $P(\nu)$ and $< \vec{S} \vec{S} >$ are given in Samadi & Goupil (2001).

Following Stein (1967), $E(k, \nu)$ is split into a spatial component $E(k)$ and a frequency component $\chi_k(\nu)$ as

$$E(k, \nu) = E(k) \chi_k(\nu).$$

The same decomposition is assumed for $E_s(k, \nu)$.

A gaussian shape for $\chi_k(\nu)$ is usually assumed in the calculation of $P(\nu)$ (e.g. Goldreich & Keeley 1977). This is equivalent to suppose that two distant points in the turbulent medium are uncorrelated. In section 3 below, the gaussian hypothesis is compared with the properties of $\chi_k(\nu)$ inferred from a 3D simulation of the solar convective zone. Section 4 discusses consequences of either choice on the excitation rate $P(\nu)$. 
3. Constraints from a 3D simulation of the upper part of the solar convective zone.

We study a 3D simulation of the upper part of the solar convective zone obtained with the 3D numerical code developed at the Niels Bohr Institute for Astronomy, Physics and Geophysics (Copenhagen, Denmark). Physical assumptions are described in Stein & Nordlund (1998).

The simulated domain is 3.2 Mm deep and its surface is 6 x 6 Mm². The grid of mesh points is 256 x 256 x 163, the total duration 27 mn and the sampling time 30s.

The simulation data are used to determine the quantities $E(k, z)$, $E_s(k, z)$ and $\chi(k, z)$ involved in the theoretical expression for the excitation rate $P(\nu)$.

We proceed in two steps (more details will be given in Samadi et al 2002a and Samadi et al 2002b):

- We compute at each layer $z$ the 2D Fourier transform, along horizontal plans, of the velocity field $\mathbf{u}$ and the entropy $s$ and perform integrations over circles with radius $k$. This provides $\hat{u}(k, z, t)$ and $\hat{s}(k, z, t)$ where $k$ is the wavenumber along the horizontal plan. We finally time average each quantities $\hat{u}$ and $\hat{s}$ over the time series. This provides $\hat{u}(k, z)$, $\hat{s}(k, z)$ and then time averaged kinetic energy spectrum $E(k) \equiv \hat{u}^2(k, z)$ and the time averaged spectrum of the entropy fluctuations $E_s(k) \equiv \hat{s}^2(k, z)$.

- At 3 different layers of the simulated domain, we compute the 3D Fourier transform, with respect to time and along the horizontal plan, of the velocity field $u$. We next perform integrations over circles with radius $k$. This yields $\hat{u}(k, z, \nu)$ and therefore $E(k, \nu, z) \equiv \hat{u}^2(k, \nu, z)$ and - using Eq.(2) - $\chi(k, \nu, z)$.

The dependence of $\chi_k$ with frequency is plotted at the top of the superadiabatic region -where the $p$-modes excitation is the largest - (Fig. 1), 0.3 Mm deeper (Fig. 2) and 0.6 Mm deeper (Fig. 3).

We compare $\chi_k(\nu)$ obtained from the simulation analysis with three analytical functions. These are the Gaussian function (GF hereafter):

$$\chi_k(\nu) = \frac{1}{\nu_k} e^{-\left(\nu/\nu_k\right)^2},$$

the Gaussian plus an Exponential function (GEF hereafter):

$$\chi_k(\nu) = \frac{1}{2} \left( \frac{1}{\nu_k \sqrt{\pi}} e^{-\left(\nu/\nu_k\right)^2} + \frac{1}{2\nu_k} e^{-|\nu/\nu_k|} \right),$$

and the Gaussian plus a Lorentzian function (GLF hereafter):

$$\chi_k(\nu) = \frac{1}{2} \left( \frac{1}{\nu_k \sqrt{\pi}} e^{-\left(\nu/\nu_k\right)^2} + \frac{1}{\pi \nu_k} \frac{1}{1 + \left(\nu/\nu_k\right)^2} \right),$$

where $\nu_k$ is the line-width at half maximum of $\chi_k$. $\nu_k$ is related to $\tau_k$ the characteristic correlation time-scale of an eddy with wavenumber $k$ as

$$\nu_k \equiv (\pi \tau_k)^{-1}$$

and $\tau_k$ is related to the velocity $u_k$ of an eddy with wavenumber $k$ as

$$\tau_k \equiv \lambda (k u_k)^{-1},$$
Figure 1. The solid curve with dots represents $\chi_k(\nu)$ obtained from the simulation at the top of the superadiabatic region ($z = 0.04$ Mm) and for the wavenumber $k$ at which $E(k)$ is maximum. The solid curve represents the Gaussian function (GF, Eq. 3), the dashed curve the Gaussian Exponential function (GEF, Eq. 4) and the dots-dashed curve the Gaussian Lorentzian function (GLF, Eq. 3).
Figure 2. Same as Fig 1, at a deeper layer (z=0.34 Mm).
Figure 3.  Same as Fig 1, at a deeper layer (z=0.64 Mm).
The velocity $u_k$ is obtained from the kinetic energy spectrum $E(k)$ (Stein 1967) as

$$u_k^2 = \int_k^{2k} dk E(k).$$  \hspace{1cm} (8)

The parameter $\lambda$ in Eq. (7) accounts for our lack of precise knowledge of the time correlation $\tau_k$ (or $\nu_k$) in stellar conditions. We find however that in most part of the excitation region, the line width of $\chi_k$ is satisfactorily reproduced with $\lambda = 1$.

At the top of the superadiabatic region, the GF and GLF do not correctly model $\chi_k(\nu)$ whereas the GEF is in better agreement (see Fig. 1). However the discrepancies between the GF (or the GLF) and the simulation data occur mostly above the solar cut-off frequency ($\nu \sim 5.5$ mHz). Discrepancies between the GF (or the GLF) and the simulation data have then minor consequences for the $p$-modes excitation in this region.

This is not the case deeper in the simulation where the largest discrepancies between the GF and data occur in the frequency range where the larger amount of acoustic energy is injected into the $p$-modes ($\nu \sim 2 - 4$ mHz). The GEF and GLF reproduce better than the GF the $\nu$-variation of $\chi_k$ below the top of the superadiabatic region ($z < 0$ Mm, see Fig. 2 & 3).

4. Consequences in terms of $p$-mode excitation

We compute $P(\nu)$ according to Eq. (1):

- The eigenfunctions ($\xi$) and their frequencies ($\nu$) are computed with Balmforth’s (1992) non-adiabatic code for a solar 1D mixing-length model based on Gough’s (1977) non-local time-dependent formulation of convection.

- The $k$-dependency of $E(k, z)$, is modeled as following:

$$E(k) \propto (k/k_0)^{+1} \text{ for } k_0 > k > 0.17 k_0$$

$$E(k) \propto (k/k_0)^{-5/3} \text{ for } k > k_0$$  \hspace{1cm} (9)

where $k_0 = 2\pi/\beta \Lambda$, $\Lambda = \alpha H_p$ is the mixing-length, $H_p$ the pressure scale height and $\alpha$ the mixing-length parameter. The value of $\alpha$ is this of the 1D solar model for consistency. The value of $k_0$ hence of $\beta$ is obtained from the simulation. This analytical $k$-dependency of $E$ reproduces the global features of $E$ arising from the simulation. Same model is considered for $E_s(k, z)$.

- $E(k, z)$ and $E_s(k, z)$ verifies the normalization conditions:

$$\int dk E(k, z) = \frac{1}{2} < u^2 - < u >^2 > (z)$$

$$\int dk E_s(k, z) = \frac{1}{2} < s^2 - < s >^2 > (z)$$  \hspace{1cm} (10)

where $< . >$ denotes time and horizontal average. The total energy contained in $E(k, z)$ and $E_s(k, z)$ and their depth dependences are then obtained from the simulation according to Eq. (10).

- For the frequency component $\chi_k(\nu)$, we assume successively the GF, the GEF and the GLF (see section 3).
Figure 4. The curves correspond to computed $P(\nu)$ in which we assume different analytical functions for $\chi_k(\nu)$: the GF (solid curve), the GEF (dashed curve) and the GLF (dots-dashed curve). The dots represent $P(\nu)$ derived from the amplitudes and line widths of the $\ell = 0$ $p$-modes measured by GOLF/SOHO instrument and kindly provided by F. Baudin (Baudin et al 2003, see also Thiery et al 2000). No significant difference for our purpose here are observed between the GOLF/SOHO data and Libbrecht (1988) observations.
Results are presented in Fig 4. Using the GF leads to a significant over-estimation of $P(\nu)$. This is a consequence of the large discrepancy between the GF and the 3D simulation frequency dependence of $\chi_k(\nu)$. Using the GLF yields a power $P(\nu)$ which is much larger than the observations in particular at high frequency. This is because at the top of the superadiabatic region - where the excitation is the largest - the GLF over-estimates $\chi_k(\nu)$.

The best agreement between computed and observed $P(\nu)$ is found when assuming the GEF. This is a direct consequence of the rather good agreement between the GEF and the dynamic behavior of the solar turbulence inferred from the simulation (see Sect. 3).

5. Conclusions

Our investigation demonstrates the non-gaussian character of the stochastic excitation of solar $p$-modes. Indeed, the gaussian function (GF) as a model for $\chi_k(\nu)$ leads to an important over-estimation of $P(\nu)$ whereas the Gaussian Exponential Function (GEF), which decreases more slowly with $\nu$ than the GF does, results in a better agreement between $P(\nu)$ and the seismic solar observations. The maximum value of $P(\nu)$ is now reproduced without any adjustment of free parameters in contrast with previous approaches.

The non-gaussian character of stochastic excitation causes the Reynolds stress contribution to be of the same order as the contribution arising from the advection of the turbulent fluctuations of entropy by the turbulent motions (not shown here, see Samadi et al 2002b). This last result is in better agreement with recent results by Stein & Nordlund (2001) and contrasts with previous results by Samadi et al (2001) based on the gaussian assumption for $\chi_k(\nu)$ and also with the estimation carried out by Goldreich et al (1994).

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