DETECTING PLANETS AROUND COMPACT BINARIES WITH GRAVITATIONAL WAVE DETECTORS IN SPACE

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ABSTRACT

I propose a method to detect planets around compact binaries that are strong sources of gravitational radiation. This approach is to measure gravitational wave phase modulations induced by the planets, and its prospect is studied with a Fisher matrix analysis. I find that, using LISA, planets can be searched for around \( \sim 3000 \) Galactic double white dwarfs with detection limit \( \sim 4 \, M_\odot \) (\( \sim 2 \times 10^{30} \) g; the Jupiter mass). With its follow-on missions, planets with mass \( \gtrsim 1 \, M_\odot \) might be detected around double neutron stars even at cosmological distances \( z \sim 1 \). In this manner, gravitational wave observation has the potential to make interesting contributions to extrasolar planetary science.

Subject headings: binaries: close — gravitational waves — planetary systems

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1. INTRODUCTION

Since gravitational wave (GW) detectors have omnidirectional sensitivity, many sources can be simultaneously observed without adjusting detectors for individual ones. While this might appear advantageous for astrophysical studies, it also has downsides. Depending on the number of GW sources, overlaps of signals in data streams of detectors become a significant problem, especially in the low-frequency regime probed by space GW detectors. For example, LISA (Bender et al. 1998) will detect \( \sim 3000 \) Galactic double white dwarf binaries above \( \sim 3 \) mHz (see, e.g., Nelemans 2006; Ruiter et al. 2007). Without removing the foreground GWs made by these numerous binaries, it might be difficult to observe weak interesting signals, such as extreme-mass-ratio-inspiral (EMRI) events. In this respect, extensive efforts are being paid to numerically demonstrate how well both strong and weak signals can be analyzed, using mock LISA data (Arnaud et al. 2007).

In this Letter, I study a method to search for planets (more generally substellar companions) orbiting ultracompact binaries. The proposed approach is to observe binaries’ wobble motions caused by the planets and imprinted as GW phase modulations from the binaries. This approach is close to the eclipse timing method (see, e.g., Deeg et al. 2008) to detect planets around binaries, and the underlying technique is similar to the planet search around pulsars with radio telescopes (Wolszczan & Frail 1992; see also Dhurandhar & Vecchio 2001). As the expected modulations due to planets are small, the ongoing numerical efforts for LISA have direct relevance to the prospects of the detection method proposed in this Letter.

Here I briefly discuss the significance of this method on extrasolar planetary science. In the last 15 years, its rapid progress has largely been led by theoretically unanticipated discoveries, such as those of the hot Jupiters (Mayor & Queloz 1995) or the pulsar planets (Wolszczan & Frail 1992). However, at present, observational studies for circumbinary planets are in a very preliminary stage (Udry et al. 2002; Metterspaugh et al. 2007; Deeg et al. 2008). In addition, the impacts of stellar evolution processes, including giant star phases or supernova explosions, are still highly uncertain (see, e.g., Villaver & Livio 2007; Silvotti et al. 2007 for recent studies). Since ultracompact binaries such as double white dwarfs are end products of stellar evolution, the proposed method to search for planets around them would provide us with important clues to these unclear issues. While the probability of finding a planet around a compact binary is uncertain, the large numbers of available binaries (e.g., with LISA) are advantageous for various statistical analyses, such as estimating the mass distribution of planets by separating the information on orbital inclination \( i \).

2. PHASE MODULATION BY A PLANET

To begin with, I discuss GWs from a detached double white dwarf binary on a circular orbit without a planet (Cutler 1998; Takahashi & Seto 2002). I write its almost monochromatic waves around frequency \( f_{gw} \), as

\[
h_0(t) = A \cos [2\pi f_{gw} t + \varphi_0 + D_0(t)] = A \cos \varphi(t),
\]

where the term \( \sim f_{gw} \) represents the intrinsic frequency evolution with \( f_{gw} \gg f_{gw0} \) (\( f_{gw0} \): observational time \( \sim 10 \) yr). The term \( D_0(t) = 2\pi f_{gw} \Omega_0 c^2 \sin \theta \cos (\phi(t) - \phi_0) \) represents the Doppler phase modulations due to revolution of a detector around the Sun (\( \Omega_0 = 1 \) AU) with its orbital phase \( \phi(t) = 2\pi t/1 \) yr + const. The angular parameters (\( \theta, \phi_0 \)) are the direction of the binary on the sky in the ecliptic coordinate. In equation (1) I have neglected amplitude modulation by rotation of the detector. To determine the direction of the binary, this effect is less important than the Doppler modulation \( D_0(t) \) at \( f_{gw} \gg cR_0 \sim 1 \) mHz (Takahashi & Seto 2002). In relation to this, I do not explicitly deal with the orientation parameters of binaries. This is just for simplicity. These parameters determine the polarization states of the waves. The orientation-averaged amplitude of the waves is given as

\[
A = \frac{8}{\sqrt{3}} G^{5/3} M^{5/3} \pi^{1/3} f_{gw}^{-1/3} \frac{r}{c^4} \left( \frac{M}{0.45 \, M_\odot} \right)^{5/3} \left( \frac{f_{gw}}{3 \, \text{mHz}} \right)^{2/3} \left( \frac{r}{8.5 \, \text{ kpc}} \right)^{-1}
\]

with the chirp mass \( M = M_1^{5/3} M_2^{1/3} (M_1 + M_2)^{-5/3} \) (\( M_1 \) and \( M_2 \): the two masses of the binary). In this equation, I put the chirp mass at \( M = 0.45 \, M_\odot \) (Farmer & Phinney 2003) and use the distance to the Galactic center \( r = 8.5 \) kpc as the typical distance to Galactic binaries. The matched filtering technique is an ad-
vantageous method for GW observation and the signal-to-noise ratio of the binary is evaluated in the standard manner as

$$(S/N)_0 = \frac{A_{2\pi f_p}}{f_p} \cdot \left( \frac{6.6 \times 10^{-23}}{1.2 \times 10^{-20} \text{ Hz}^{-1.5}} \right) \cdot \left( \frac{T_{\text{obs}}}{10 \text{ yr}} \right)^{1/2}, \quad (3)$$

with LISA detector noise level $h_N$ that is within 15% around $1.2 \times 10^{-20} \text{ Hz}^{-1/2}$ in the frequency regime 3–10 mHz relevant for the present analysis (Bender et al. 1998). Here I assumed LISA has two independent data streams with identical noise spectra.

When the binary has a circular orbit with mass $M_p$ and orbital frequency $f_p$, the observed waveform $h_\alpha(t)$ has an additional phase shift $\Delta\phi(\alpha)$ due to the binary’s wobble induced by the planet, and I set the waveform by $h_\alpha(t) = A \cos [\phi(t) + \Delta\phi(\alpha)]$. For a planet on a circular orbit, the phase shift is given by $\Delta\phi = -\pi f_p t + \phi_0$ with the orbital phase $\phi_0$, (orbital phase constant) and the amplitude $\Psi_p = (2\pi)^{1/3} f_p^{-1/3} f_{gw}^{-1/3} M_p^{-1/3} M_\ast \sin i$. For $i$: inclination of the planet’s orbit or explicitly

$$\Psi_p = 0.054 \left( \frac{M_p \sin i}{M_\ast} \right)^{2/3} \left( \frac{M_T}{1.04 M_\odot} \right)^{-2/3} \left( \frac{f_p}{3 \text{ mHz}} \right)^{-1/3} \left( \frac{f_{gw}}{0.33 \text{ yr}^{-1}} \right)^{-2/3}. \quad (4)$$

For a system at a cosmological distance with redshift $z$, the amplitude $\Psi_p$ is given by multiplying a factor $(1 + z)$ to equation (4) with the intrinsic (not redshifted) orbital frequency $f_{gw}$ and the observed (redshifted) GW frequency $f_{gw}$. Note that the redshift $z$ can be estimated from the observed luminosity distance (Schutz 1986). As I want to know the smallest mass $M_p \sin i$ detectable with GW observation and it is easier to find a planet with a larger amplitude $\Psi_p$, I hereafter assume $\Psi_p \leq 1$. Then the modulated signal $h_\alpha(t)$ is expressed as

$$h_\alpha(t) = h_0(t) + h_\perp(t) + h_\parallel(t) + O(\Psi_p^2) \quad (5)$$

with two new components

$$h_\perp(t) = \frac{A \Psi_p \sin [\phi(t) \pm \phi_0(t)]}{2}$$

$$= \frac{A \Psi_p \sin [2\pi(f_{gw} \pm f_p)t^2 + \phi_0 \pm \phi_0]}{2}. \quad (6)$$

A simple interpretation can be made for equation (5). In addition to the original signal $h_0(t)$ given in equation (1), the motion of the planet produces two replicas $h_\perp, h_\parallel$ [smaller by a factor of $\Psi_p/2$ than $h_0(t)$] at nearby frequencies $f_{gw} \pm f_p$. Because of the coupling with the binary’s rotation, the orbital frequency $f_{gw}$ of the planet is now up-converted into a band that might be observed with GW detectors. Here it is important to note that the GW signal of each replica $h_\perp, h_\parallel$ itself is described with a nearly monochromatic waveform for a standard Galactic binary (including dependencies on angular parameters). This fact is important for data analysis, as seen later. In this Letter I only study a planet on a circular orbit, but this analysis can be straightforwardly extended for multiple planets or eccentric orbits that produce other small replicas at frequencies $f_{gw} \pm n f_p (n = 2, 3, \ldots)$, not only with $n = 1$ (Durandh & Vecchio 2001).

Based on the simple interpretation of the modulated signal $h_\alpha(t)$, I can naively define the signal-to-noise ratio for the two small replicas $h_\perp, h_\parallel$ by

$$X_p = \frac{A \Psi_p}{h_p} \left( \frac{T_{\text{obs}}}{10 \text{ yr}} \right)^{1/2} = 5.3 \left( \frac{\Psi_p}{0.054} \right) \left( \frac{S/N)_0}{138} \right). \quad (7)$$

as for the original one $h_0$ given in equation (3). If the parameters $\alpha = (A, f_p, f_{gw}, \phi_0, \theta, \phi_0)$ are well determined with the strong original one $h_0$, they can be used to estimate the three additional parameters $\alpha = (f_p, \Psi_p, \phi_0)$ for the small replicas $h_\perp, h_\parallel$. The expected observational errors for the three new parameters $\alpha_p$ are evaluated by a $3 \times 3$ Fisher matrix (see, e.g., Takahashi & Seto 2002), and I obtain the asymptotic results at $T_{\text{obs}} f_p \gg 1$ as

$$\left( \frac{\Delta \Psi_p}{\Psi_p} \right)_3 = X_p^{-1}, \quad \left( \Delta f_p \right)_3 = \frac{1}{\sqrt{3}} X_p^{-1/2} T_{\text{obs}} X_p^{-1}. \quad (8)$$

where suffix “3” represents fitting only three new parameters.

The actual observational situation is more complicated. For example, the frequency resolution is given by $\sim T_{\text{obs}}^{-1}$, and with a short observational period $T_{\text{obs}}$, there must be significant interference between the weak signals $h_\perp(t)$ and the strong one $h_0(t)$. Furthermore, the triplet $(h_0, h_\parallel, h_\perp)$ needs to be identified in the presence of thousands of other binaries. To study the interference within the triplet, I first discuss signal analysis only with a single binary-planet system and detector noises. I numerically evaluated the observational errors expected for simultaneous fitting of all nine parameters $\alpha$, and $\alpha_p$ listed above, and obtained the magnitudes of the errors $(\Delta \Psi_p, \Delta f_p)_3$ for various sets of input parameters (suffix S: simultaneous fitting). I found that for a given observational time $T_{\text{obs}} \geq 1 \text{ yr}$, these results depend strongly on the orbital frequency $f_p$, weakly on the phase $\phi_0$, and negligibly on other parameters. As shown in Figure 1, the errors $(\Delta \Psi_p, \Delta f_p)_3$ become much larger than the previous simple estimations $(\Delta \Psi_p, \Delta f_p)_3$ for frequencies

$$f_p T_{\text{obs}} \lesssim 2 \text{ or } |f_p - 1|T_{\text{obs}} \lesssim 1. \quad (9)$$

In the latter regime, two phase modulations $D_\perp(t)$ and $D_\parallel(t)$ become highly degenerated. Outside these two bands, the replicas $h_\perp, h_\parallel$ are well separated from the original one $h_0$ in the frequency space, and the simple estimation in equation (8) becomes reliable. In these preferable frequency regimes, the mass $M_p \sin i$ can be estimated within 10% error (at the same time, the naive $S/N X_p \sim 10$ for a planet with $M_p \sin i \gtrsim 5.4 M_\odot / (3 \text{ mHz})^{1/3} (f_0 / 0.33 \text{ yr}^{-1})^{1/3}$. Here I used equations (4), (7), and (8), and the following typical parameters: $r = 8.5 \text{ kpc}$, $M_\odot = M_2 = 0.52 M_\odot$, $T_{\text{obs}} = 10 \text{ yr}$, and $h_p = 1.2 \times 10^{-20} \text{ Hz}^{-1/2}$.

Now I study circumbinary planet searches among GWs from other binaries. For simplicity, I pick up a binary-planet system at $f \gtrsim 3 \text{ mHz}$ and outside the interfering frequency regimes (eq. [9]). I consider the following two-step data analysis: (1) detecting the individual signals $h_0, h_\perp, h_\parallel$, and (2) identifying a triplet combination caused by a planet. The frequency distribution of Galactic white dwarf binaries is modeled as $dN/df = 0.08(N_0/3000) (f_{gw}/3 \text{ mHz})^{-1/3} \text{ yr}^{-1}$ with the total number $N_0 \sim 3000$ at $f_{gw} \gtrsim 3 \text{ mHz}$ (see, e.g., Bender et al. 1998). A similar density is expected for Galactic AM CVn stars (Nelemans 2006). For observational period $T_{\text{obs}} \sim 10 \text{ yr}$, the oc-
Fig. 1.—Planet search sensitivity around white dwarf binaries with LISA. Estimated observational errors are presented for the orbital frequency of the planet ([Δf]_p; long-dashed curves) and for the amplitude of the GW phase modulation ([ΔΨ/Ψ]_p; solid curves) induced by the planet. These errors are normalized by their asymptotic values ([Δf]_p/W)_{pS} derived with a simple interpretation for the signal modulation (see eq. [8]). Thick curves are for integration period T_{obs} = 3 yr and thin ones for T_{obs} = 10 yr. It is difficult to find a planet with a low orbital frequency due to the poor frequency resolution. Two phase modulations and induced T_p = 10 f_p/3 H_{11002}^2 D(t)_{obs} by motions of LISA and the planet degenerate at f_p/2 H_{11506}^2. Outside these two bands the new signal by the planet can be well separated from the strong original one h_p(t), and the planet search works efficiently. These results depend very weakly or negligibly on source parameters other than Δf_p. [See the electronic edition of the Journal for a color version of this figure.]

cupation number T_{obs}^{-1} dN/df of binaries per frequency bin will be much smaller than 1. For a planet search, it is crucial to detect replicas h_{p±} whose signals are weak but individually fitted with standard Galactic binary waveforms. Identification of weak binary signals is currently one of the most important topics in LISA data analysis. While the situation is somewhat different, Crowder & Cornish (2007) demonstrated that many (but not all) binaries can be detected down to S/N ∼ 7 (corresponding to X_r/√2 ∼ 7 for each replica) even under more crowded conditions, i.e., a larger occupation number T_{obs}^{-1} dN/df (see their §§ 4.2 and 4.3). They also showed that the Fisher matrix analysis provides a reasonable prediction for parameter estimation errors. These results are very encouraging for a planet search that might conversely provide another motivation for ongoing activities for LISA data analysis.

Next I discuss an outline for identifying a triplet signal by a binary-planet system. The first task is to search for a potential pair h_p± from a list of resolved binaries, using the fact that the pair should have the same direction (and orientation) parameters with similar frequencies. The second task is to confirm the existence of another replica h_p± whose parameters can be estimated only with the h_p± pair. Considering the expected binary density dN/df, this discrimination method will work well. In this manner the triplet can be identified among other binaries with a small extension of the standard Galactic binary search. Then coherent analysis can be performed for the modulated signal h_{p±} to improve the quality of parameter estimation.

For unambiguous detections of planets, other effects that produce similar waveforms should be closely examined. From the arguments about the triplet structure, it is expected that the phase modulation D_p(t) can be easily separated from other small modulations at higher frequencies ≫ f_p that also generate small replicas but with larger frequency differences. Meanwhile, because of the geometrical nature of GW generation, an observed waveform depends on angular parameters describing configuration of a binary. For example, it is shown that, for an eccentric binary in the LISA band, an triplet waveform can be produced by the periastron advance with a frequency difference O(1 yr^{-1}) (Seto 2001; Willems et al. 2008). But the triplet structure is different from the planet case. Precession of orbital plane of a binary (by the spin-orbit coupling) can also generate a triplet waveform, and might be important for double neutron stars with BBO/DECIGO. But it has different amplitude patterns.
(or equivalently polarization states), and has a larger frequency difference.

In Figure 2 I plot the detectable planet on the semimajor axis–mass plane. Here I use the relation $a = 1(f_p/1 \text{ yr})^{-3/5} \times (M_*/1 M_\odot)^{1/3}$ AU for the orbital frequency $f_p$ and the semimajor axis $a$. The planets around $f_p = 1 \text{ yr}^{-1}$ (corresponding to $1.01 \text{ AU}$ for $M_*/1.04 M_\odot$) are excluded due to the degeneracy discussed before.

3. DISCUSSION

The follow-on missions to LISA, such as the Big Bang Observer (BBO) (Phinney et al. 2003) or the Decihertz Interferometer Gravitational Wave Observatory (DECIGO) (Seto et al. 2001; Kawamura et al. 2006) were proposed primarily to detect stochastic GW background from inflation in the band with $f_{\text{min}} \approx f \approx f_{\text{max}}$ with $f_{\text{min}} \sim 0.2 \text{ Hz}$ and $f_{\text{max}} \sim 1 \text{ Hz}$. At the lower frequency regime $f \approx f_{\text{min}}$ the foreground GWs by extra-Galactic white dwarf binaries would fundamentally limit sensitivity for GW observation (Farmer & Phinney 2003). In contrast, at $f \approx f_{\text{max}}$ a deep GW window is expected to be opened. To this end, it is crucial to resolve and remove foreground GWs generated by cosmological double neutron star binaries (NS+NSs) whose estimated merger rate is $\sim 3 \times 10^5 \text{ yr}^{-1}$. In addition to NS+NSs, there might be double black hole binaries or black hole–neutron star binaries, while their merger rates are highly uncertain. Here I provide a brief sketch for planet searches around cosmological NS+NSs with the follow-on missions. I fix the masses of NS+NSs at $M_1 = M_2 = 1.4 M_\odot$.

In the observational band $[f_{\text{min}}, f_{\text{max}}]$, a NS+NS is on its final stage before merger. The time left before the merger is $1(f_p/0.2 \text{ Hz})^{-8/3}(1 + z)^{-5/3} \text{ yr}$, which severely limits the observable orbital frequency $f_p$ of a planet. Using the restricted 1.5-order post-Newtonian waveform (Cutler & Harms 2006), I evaluated the expected observational errors in the scenario wherein all the parameters are simultaneously fitted, including two phase shifts $D_i(t)$ and $D_j(t)$. For various sets of input parameters, I examined the observational error for the amplitude $\Psi_{pl} = \Psi_{pl}(f_p = 1 \text{ Hz})$, and found that by observing at least three orbital cycles [namely $f_p \approx f_{\text{inl}} \equiv 3(f_{\text{min}}/0.2)^{8/3}(1 + z)^{-5/3} \text{ yr}^{-1}$] the relative error is given as

$$\left(\frac{\Delta \Psi_{pl}}{\Psi_{pl}}\right) \sim \frac{\Delta (M_p \sin i)}{M_p \sin i} \sim (1 + z)^{-1} \frac{2.3}{(S/N)_b} \left(\frac{M_p \sin i}{3 M_\odot}\right)^{-1} \left(\frac{f_p}{3 \text{ yr}^{-1}}\right)^{2/3} \tag{10}$$

with the signal-to-noise ratio $(S/N)_b$ for the observed NS+NS. Here I assumed a nearly flat noise spectrum (in units of $\text{Hz}^{-1/2}$) in the band $[f_{\text{min}}, f_{\text{max}}]$ (Phinney et al. 2003; Kawamura et al. 2006). For a given orbital frequency $f_p$ and signal-to-noise ratio $(S/N)_b$, the mass resolution is better than the previous results for LISA. This is because of the higher frequencies $f_p$ used in the present case. For $f_p \approx f_{\text{inl}}$ (less than three orbital cycles in the observational band), the observational error $\Delta (M_p \sin i)/(M_p \sin i)$ becomes significantly larger than equation (10).

Due to a limitation of estimated computational power available at the time of the follow-on missions $\sim 2025$, the minimal noise level of detectors required to remove NS+NSs corresponds to $(S/N)_b \sim 100$ for NS+NSs at $z = 1$ (Cutler & Harms 2006). For $z = 1$ the critical orbital frequency becomes $f_{\text{inl}} = 19 \text{ yr}^{-1}$ (semimajor axis $\sim 0.23 \text{ AU}$ for $M_*/1.4 M_\odot$), and the mass $M_p \sin i$ can be measured within $10\%$ error for a planet with $M_p \sin i \geq 1.21(f_p/19 \text{ yr}^{-1})^{2/3}(S/N)_b/(100)^{1/3} M_\odot$. The range of detectable planets is shown in Figure 2. If detected at $z = 1$, the planet is $10^4 \text{ yr}$ times as distant as those currently found in our galaxy. Note that the estimated merger rate of NS+NSs around $z \sim 1$ is $\sim 10^4 \text{ yr}^{-1}$. The bottom edge of the shaded region moves to $(0.39 \text{ AU}, 0.52 M_\odot)$ for NS+NSs at $z = 0.5$. I would like to thank an anonymous referee for helpful comments which improved the draft.

REFERENCES

Arnaud, K. A., et al. 2007, Classical Quantum Gravity, 24, 529
Bender, P. L., et al. 1998, LISA Pre-Phase A Report (Greenbelt: GSFC)
Crowder, J., & Cornish, N. J. 2007, Classical Quantum Gravity, 24, S575
Cutler, C. 1998, Phys. Rev. D, 57, 7089
Cutler, C., & Harms, J. 2006, Phys. Rev. D, 73, 042001
Deeg, H. J., et al. 2008, A&A, in press (arXiv:0801.2186)
Dhurandhar, S. V., & Vecchio, A. 2001, Phys. Rev. D, 63, 122001
Farmer, A. J., & Phinney, E. S. 2003, MNRAS, 346, 1197
Kawamura, S., et al. 2006, Classical Quantum Gravity, 23, 125
Mayor, M., & Queloz, D. 1995, Nature, 378, 355
Muterspaugh, M. W., et al. 2007, preprint (arXiv:0705.3072)
Nelemans, G. 2006, in AIP Conf. Proc. 873, Laser Interferometer Space Antenna, ed. S. M. Merkowitz & J. C. Livas (New York: AIP), 397
Phinney, E. S., et al. 2003, The Big Bang Observer, NASA Mission Concept Study
Ruiter, A. J., et al. 2007, preprint (arXiv:0705.3272)
Schutz, B. F. 1986, Nature, 323, 310
Seto, N. 2001, Phys. Rev. Lett., 87, 251101
Seto, N., Kawamura, S., & Nakamura, T. 2001, Phys. Rev. Lett., 87, 221103
Silvotti, R., et al. 2007, Nature, 449, 189
Takahashi, R., & Seto, N. 2002, ApJ, 575, 1030
Udry, S., et al. 2002, A&A, 390, 267
Villaver, E., & Livio, M. 2007, ApJ, 661, 1192
Willems, B., Vecchio, A., & Kalogera, V. 2008, Phys. Rev. Lett., 100, 041102
Wolszczan, A., & Frail, D. A. 1992, Nature, 355, 145