Rescattering effects in hadron-nucleus and heavy-ion collisions

Jianwei Qiu

Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, U.S.A.

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Abstract. We review the extension of the factorization formalism of perturbative QCD to coherent soft rescattering associated with hard scattering in high energy nuclear collisions. We emphasize the ability to quantify high order corrections and the predictive power of factorization approach in terms of universal nonperturbative matrix elements. Although coherent rescattering effects are power suppressed by hard scales of the scattering, they are enhanced by the nuclear size and could play an important role in understanding the novel nuclear dependence observed in high energy nuclear collisions.

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1 Introduction

Rescattering in hadron-nucleus and heavy-ion collisions provides an excellent tool to diagnose properties of nuclear medium and could play an important role in understanding novel nuclear dependence recently observed at relativistic heavy ion collider (RHIC) and in planning future experiments at the Large Hadron Collider (LHC). Many approaches in studying the rescattering effects have been proposed and used for calculating nuclear dependence in high energy nuclear collisions [1,2,3,4,5,6,7].

In this talk, we focus on a treatment of coherent soft rescattering associated with hard probes [2,3]. Our work is based on perturbative QCD (pQCD) factorization approach, which is different from the works of Baier et al. (BDMPS) [4] and Zakharov [5], and the reaction operator approach of Gyulassy et al. [6]. The BDMPS analysis does not require the presence of a hard scattering, but describes the coherent results of many soft scatterings. Its primary subject is induced energy loss. Our analysis requires a hard scale, and begins with the pQCD treatment of hard-scattering with emphases on momentum transfer, caused by coherent initial- and final-state soft scatterings [2,3]. Our work attempts to stay as close as possible to the pQCD factorization formalism, in which we may readily quantify high order corrections in powers of strong coupling constant $\alpha_s$, as well as corrections that decrease with extra powers of momentum transfer [8,9,10,11].

In the following we consider only initial- and final-state interaction that gives leading power in medium length ($A^{1/3}$) and in $\alpha_s$ at each scattering. We first identify the coherence length in nuclear collisions and the source of the leading medium size enhancement. We then apply pQCD factorization approach to calculate the leading nuclear dependence in several physical observables. We show that if we neglect soft rescattering off quark fields [12,13], the leading medium effects induced by multiple soft rescattering depend on only one well-defined nonperturbative matrix element, $\langle F^{++} F_{++}^{--} \rangle$, defined below. We extract its value from different physical measurements and discuss its universality. A brief summary is given at the end.

2 Coherent multiple scattering and leading nuclear $A$-dependence

A hard probe corresponds to a scattering process with a large momentum exchange $q^\mu$ whose invariant mass $Q = \sqrt{|q^2|} \gg A_{QCD}$, as sketched in Fig. 1. It can probe a space-time dimension much smaller than a nucleon at rest, $1/Q \ll 2R \sim$ fm, with nucleon radius $R$. But, the same probe might cover a whole Lorentz contracted large nucleus, if $1/Q > 2R(m/p)$ with averaged nucleon momentum $p$ and mass $m$, or equivalently, $x < x_c \equiv 1/2mR \sim 0.1$ with $x$ being an active parton momentum fraction in the scattering, $xp \sim Q$. The critical value $x_c$ corresponds to the nucleon size. If the active $x$ is much smaller than $x_c$, a hard probe could cover several nucleons in a Lorentz contracted large nucleus and interact with partons from different nucleons coherently.

Inclusive deeply inelastic scattering (DIS) on a nucleus offers an ideal example of coherent multiple scattering.
and power corrections [11]. The strength of the scattering is defined by the virtual photon momentum, \( q^\mu \). Let \( q^\mu = -x_B p^\mu + Q^2/(2x_B p \cdot n) n^\mu \) with \( n^\mu \) defined along a lightcone direction opposite to that of \( p^\mu \). While the \( Q^2 = -q^2 \) sets up the hard collision scale, the scattered quark probes the nuclear matter via multiple soft final-state interactions. When Bjorken \( x_B \equiv Q^2/2p \cdot q \ll x_c \), the multiple scatterings at a given impact parameter are coherent over entire size of the Lorentz contracted nucleus. This is best seen in the Breit frame, as shown in Fig. 2(a), where the incoming quark reverses its direction after interacting with the virtual photon and collides with the “remnants” of the nucleus at the same impact parameter. The same coherent multiple scatterings can take place in hadron-nucleus collisions along the direction of momentum exchange of the scattering [11], as shown in Fig. 2(b).

To identify the leading medium length enhanced nuclear effects, let’s consider multiple scattering contribution to inclusive DIS off a large nucleus, as sketched in Fig. 2(a). For a spin-averaged inclusive DIS cross section, there is only one large momentum scale, \( Q \), “invariant mass” of the exchanged virtual photon, and the factorization is expected to be valid for all power corrections as a consequence of operator product expansion (OPE) [11]:

\[
\begin{align*}
\frac{d\sigma^{\text{DIS}}}{dx} & = \int d^2q \frac{d\tilde{\sigma}_2}{q^2} \left[ 1 + c^{(1.2)}(1,2) \alpha_s + c^{(2.2)}(2,2) \alpha_s^2 + \ldots \right] \times \tilde{T}_{2}^{(1)} \\
& + \int \frac{d\tilde{\sigma}_3}{q^2} \left[ 1 + c^{(1.4)}(1,4) \alpha_s + c^{(2.4)}(2,4) \alpha_s^2 + \ldots \right] \times \tilde{T}_{3}^{(1)} \\
& + \int \frac{d\tilde{\sigma}_4}{q^2} \left[ 1 + c^{(1.6)}(1,6) \alpha_s + c^{(2.6)}(2,6) \alpha_s^2 + \ldots \right] \times \tilde{T}_{4}^{(1)} \\
& + \ldots
\end{align*}
\]

where \( \otimes \) represents convolutions in fractional momenta carried by partons and \( T_a \) represents a parton correlation function or a matrix element of a twist \( n \) operator. In Eq. (1), the \( \tilde{\sigma}_2 \) and \( \tilde{\sigma}_3 \) are perturbatively calculable short-distance partonic cross sections or coefficient functions, which are independent of the target size. Therefore, we need to find the nuclear size \( A^{1/3} \)-type enhancement induced by multiple rescattering from the matrix elements, if there is any.

The leading twist parton distributions, \( T_2 \)'s, represent probability densities to find a single parton in a target and can have some nuclear dependence via the input distributions to their DGLAP evolution equations. The \( A^{1/3} \)-type target-size enhancement to DIS cross section can only appear in the terms beyond the first row in Eq. (1). For definiteness, let’s consider a leading power suppressed contribution to DIS cross section, as shown in Fig. 3(a), which can be factorized into the form according to Eq. (1) [2]

\[
d\sigma^{(4)}_A = \sum_{i'j'} \int dx_1 dx_2 dx_3 T_{(i'j')}^{(4)}(x_1, x_2, x_3) \\
\times d\delta^{(4)}(x_1, x_2, x_3). \quad (2)
\]

The matrix element \( T_{(i'j')}^{(4)} \), as sketched in Fig. 3(b), is typically of the form [8],

\[
T_{(i'j')}^{(4)}(x_1, x_2, x_3) \propto \int \frac{dy_1 dy_2 dy_3}{(2\pi)^3} e^{ip^\tau (x_1y_1^- + x_2y_2^- + x_3y_3^-)} \\
\times \langle p_A|B_i(0)B_{i'}(y_3)B_{j'}(y_2)B_j(y_1)|p_A \rangle, \quad (3)
\]

where \( p = p_A/A \) and \( B_i \) is the field corresponding to a parton of type \( i = q, \bar{q}, G \). The structure of the target is manifest only in the matrix element \( T \) in Eq. (2). Each pair of fields in the matrix element Eq. (3) represents a parton that participates in the hard scattering. The \( y_i^- \) integrals cover the distance between the positions of these particles along the path of the outgoing scattered quark. In Eq. (3), integrals over the distances \( y_i^- \) generally cannot grow with the size of the target because of oscillations from the exponential factors \( e^{ip^\tau x_i y_i^-} \).

Since the kinematics of a single-scale hard collision is only sensitive to the total momentum from the target, two of the three momentum fractions: \( x_1, x_2 \), and \( x_3 \) cannot be fixed by the hard collisions. Therefore, there is always a subset of Feynman diagrams, like one in Fig. 3(a) with poles labeled by the crosses “\( x \)”, whose contribution to the partonic parts, \( \tilde{\sigma}^{(4)}_{(i'j')} \) in Eq. (2) is dominated by regions where two of the three momentum fractions vanish. The convolution over \( dx_1 dx_2 dx_3 \) in Eq. (2) is simplified to an integration over only one momentum fraction,

\[
\int \prod_{i=1}^3 dx_i \frac{p}{p_A} T_{(ij')}^{(4)}(\{x_i\}) \Rightarrow \int dx T_q(x, A) \hat{\sigma}^{(D)}(xp), \quad (4)
\]

where the partonic part \( \hat{\sigma}^{(D)} \) is finite and perturbative, with the superscript \( (D) \) indicating the contribution from
double scattering. The above matrix element, \( T_q(x, A) \), as illustrated in Fig. 3(b), has the form

\[
T_q(x, A) = \int \frac{dy^-}{2\pi} e^{ip^+ y^-} \int \frac{dy^+ dy^-}{2\pi} \theta(y^2 - y_1^2) \theta(y_2^2)
\times \frac{1}{2} \langle p_A | \bar{q}(0) \gamma^+ F^{a+}(y_2^-) F^{\alpha} q(y_1) | p_A \rangle \quad (5)
\]

where \( |p_A\rangle \) is the relevant nuclear state. The variable \( x \) here is the fractional momentum associated with the hard parton from the target that initiates the process. Similar gluon-gluon correlation function \( T_g(x, A) \) is also important \[10\]. In this type of the twist-4 parton-parton correlation functions, two integrals over the \( y^- \) and \( y_2^- \) can grow with the nuclear radius. However, if we require local color confinement, the difference between the light-cone coordinates of the two field strengths should be limited to the nuclear size and only one of the two \( y^- \) integrals can be extended to the size of nuclear target. The twist-4 parton-parton correlation functions are then proportional to the size of the target, that is, enhanced by \( A^{1/3} \).

It is important to emphasize that using a pole in the complex \( x_1 \) (longitudinal momentum) space to do the integral does not assume on-shell propagation for the scattered quark. Indeed, the \( x_1 \) integrals are not pinched between coalescing singularities at such points, and the same results could be derived by performing the \( x_1 \) integrals without going through the \( x_1 = 0 \) points \[2\].

3 Coherent power corrections in DIS

The inclusive DIS cross section on a nucleus provides an unique probe for effects of coherent multiple scatterings by varying the value of Bjorken \( x_B \). Under the approximation of one-photon exchange, the unpolarized inclusive DIS cross section probes two structure functions: \( F_T(x_B, Q^2) \) and \( F_L(x_B, Q^2) \), corresponding to the transverse and longitudinal polarization states of the virtual photon, respectively \[17\].

The structure functions at the lowest order in \( \alpha_s \) are given by \[17\]:

\[
F_T^{LT}(x_B, Q^2) = \frac{1}{2} \sum_q e_q^2 q(x_B, Q^2) + \mathcal{O}(\alpha_s) \quad (6)
\]

\[
F_L^{LT}(x_B, Q^2) = \mathcal{O}(\alpha_s) \quad (7)
\]

where (LT) indicates the leading twist contribution – the first row in Eq. (1), \( \sum_q \) runs over the (anti)quark flavors, \( e_q \) is their fractional charge, and \( q(x, Q^2) \) is the leading twist quark distribution:

\[
q(x, Q^2) = \int \frac{dy^-}{2\pi} e^{i x p^+ y^-} \langle p\bar{q}(0) \gamma^+ \frac{Q}{2} q(y^-) | p \rangle \quad (8)
\]

in the lightcone \( A^+ = n^+ A_\mu = 0 \) gauge for hadron momentum \( p^\mu = p^+ \tilde{n}^\mu \) with \( \tilde{n}^\mu = \{1, 0, 0, 1\} \) and \( n^\mu = \{0, 1, 0, 1\} \).

The Feynman diagrams in Fig. 4 give the leading tree level contributions to the lepton-nucleus DIS cross section. The cut-line represents the final state \[3\]. For transversely polarized photons Fig. 4(a) gives the leading twist partonic contribution,

\[
d\sigma^{(0)} = \frac{1}{2} e_q^2 \delta(x - x_B), \quad (9)
\]

from which \( F_T^{LT} \) in Eq. (6) is derived after convoluting with the leading twist quark distribution in Eq. (8). Diagrams with two gluons in Fig. 4(b) gives the first power correction to transverse partonic cross section \[3\] and

\[
d\theta^{(1)} = \frac{1}{2} e_q^2 \left[ \frac{1}{2N_c} \right] \frac{Q^2}{2\pi^2} \frac{d}{dx} \left[ \hat{F}^2(0) \right] x_B \quad \overset{d\sigma^{(1)}}{=} \quad (10)
\]

with the two-gluon field operator

\[
\hat{F}^2(0) = \int \frac{dy^- dy_1^-}{(2\pi)^2} \left[ F^a_{\alpha}(y_2^-) F_{\alpha}^{a+}(y_1^-) \right] \theta(y_2^-) \quad (11)
\]

and the first power correction to transverse structure function \[3\]:

\[
F_T^{(1)}(x_B, Q^2) = \left[ \frac{4\pi^2\alpha_s}{Q^2} \left( \frac{1}{2N_c} \right) \right] x_B \quad \overset{d\sigma^{(1)}}{=} \quad (12)
\]

with correlation function, \( T_q(x_B, A) \), given in Eq. (6).

The four-parton correlation functions \( T_q \)'s are nonperturbative and must be taken from experiments. To estimate the magnitude of \( T_q \)'s, we could choose a simple ansatz \[19\]:

\[
T_i(x, A) = \lambda^2 A^{1/3} \phi_i(p,q)(x, A) \quad (13)
\]

for \( i = q, g \) in terms of the corresponding twist-2 effective nuclear parton distribution \( \phi_i(p_A) \). We choose this form because we expect the \( x \)-dependence of the probability to detect the hard parton to be essentially unaffected by the presence or absence of an additional soft scattering. In Eq. (13) \( \lambda \) is assumed to be a constant with dimensions of mass. This ansatz facilitates the comparison to data \[2\].

If we further assume that a nucleus is made of a group of loosely bound color singlet nucleons, packed in a hard sphere of radius \( RA^{1/3} \), we can approximate the matrix element of nuclear state in Eq. (6) into a product of matrix elements of nucleon states \[3\]:

\[
\langle p_A | \bar{q}(0) \gamma^+ \frac{Q}{2} q(y^-) | p_A \rangle \quad \overset{d\sigma^{(1)}}{=} \quad (14)
\]

\[
\approx A \left[ \frac{4RA^{1/3}}{1 + A^{1/3}} \right] \left( F^a_{\alpha}(p) | \bar{q}(0) \gamma^+ \frac{Q}{2} q(y^-) | p \right)
\]
derive the leading $N$th coherent power corrections to the ent power correction distributions, as sketched in Fig. 5(a), the leading coherent power correction 

\[
(F^{+\alpha}F_\alpha^+) \equiv \frac{1}{p^2} \int \frac{dy^-}{2\pi p} (p|F^{+\alpha}(0)F_\alpha^+(y^-)|p) \theta(y^-).
\]

Substituting Eq. (16) into the definition of quark-gluon correlation function, \(T_q(x, A)\) in Eq. (15), we drive

\[
\chi^2 \approx \left(\frac{3}{4} R\right) \left(\frac{1}{2\pi R^3}\right) (F^{+\alpha}F_\alpha^+), \tag{16}
\]

and

\[
F_T^{(1)}(x_B, Q^2) \approx \frac{3\pi\alpha_s}{8Q^2R^2} (F^{+\alpha}F_\alpha^+) \left( A^{1/3} - 1 \right) \times \frac{1}{2} \sum_q e_q^2 x_B \frac{d}{dx_B} q_A(x_B, Q^2). \tag{17}
\]

where \(q_A\) defined in Eq. (8) with \(p = p_A/A\). In deriving Eq. (17), we used \(A^{1/3} - 1\) instead of \(A^{1/3}\) for the line integral between the two pairs of field operators in Eq. (15) so that the nuclear effect vanishes for \(A = 1\).

From Eq. (16) and known generic \(x\)-dependence of parton distributions, as sketched in Fig. 5(a), the leading coherent power correction suppresses the DIS cross section or structure functions at small \(x_B\).

If \(x_B\) is small enough, the virtual photon could interact coherently with all nucleons inside a large nucleus. From the Feynman diagrams in Fig. 5(c) with \(2N\) gluons, we derive the leading \(N\)th coherent power corrections to the transverse structure function \(3\).

\[
F_T^{(N)}(x_B, Q^2) \approx \left[ \frac{3\pi\alpha_s}{8Q^2R^2} (F^{+\alpha}F_\alpha^+) \left( A^{1/3} - 1 \right) \times \frac{1}{2} \sum_q e_q^2 x_B \frac{d}{dx_B} q_A(x_B, Q^2) \right]^N \tag{18}
\]

Summing over all leading \(A^{1/3}\)-enhanced power corrections, the first column of the right-hand-side of Eq. (18), we obtain \(3\)

\[
F_T^A(x_B, Q^2) \approx \sum_{n=0}^{N} \frac{1}{n!} \left[ \frac{e^2}{Q^2} \left( A^{1/3} - 1 \right) \right]^n x_B^n \frac{d^n}{dx_B^n} F_T^{A(LT)}(x_B, Q^2). \tag{19}
\]

with a characteristic scale of the power corrections

\[
\xi^2 \approx \frac{3\pi\alpha_s}{8R^2} (F^{+\alpha}F_\alpha^+). \tag{20}
\]

If we approximate \(N \sim \infty\) in Eq. (19), we obtain,

\[
F_T^A(x_B, Q^2) \approx F_T^{A(LT)}(x_B(1 + \Delta), Q^2) \tag{21}
\]

with \(\Delta \equiv (A^{1/3} - 1) \xi^2/Q^2\), a shift in \(x_B\). Similar expression was derived for the longitudinal structure function \(F_L^A(x_B, Q^2)\).

With only one unknown matrix element, \(\langle F^{+\alpha}F_\alpha^+ \rangle\), our calculated results can be easily tested and challenged. From Eq. (21), the nuclear dependence in structure functions should come from two distinctive sources: universal \(A\)-dependence in leading twist parton distributions and process sensitive \(A\)-dependence from power corrections.

By comparing our numerical results, evaluated with CTEQ6 PDFs, with the data, we extract the maximum size of power corrections. For \(\xi^2 = 0.09 - 0.12\) GeV\(^2\), our calculated reduction in nuclear structure functions is consistent with the \(x_B\), \(Q^2\), and \(A\)-dependence of the data as demonstrated Fig. 5.

The predictive power of factorization approach resides in the universality of unknown matrix elements. Without
based on a linear extrapolation of the $\phi$ It was shown by the NA38 and NA50 Collaborations that $4$ Power correction to Drell-Yan cross section of parton saturation from the perturbative side and for tions – coherent parton recombinations to parton evolution equation taking into account the parton recombination, like those in Fig. 7(b) and (c). A modified not remove collinear divergences involving multiple parton Eq. (1). However, DGLAP evolved parton distributions do infra-safety of all coefficient functions in the first row of Eq. [1]. However, DGLAP evolved parton distributions do not remove collinear divergences involving multiple parton recombination, like those in Fig. 7(b) and (c). A modified evolution equation taking into account the parton recombination slows down the fast growth of parton density at small $x$ [20,20], and keeps a positive density of small-$x$ gluons at low $Q^2$ [27]. All order resummed power corrections – coherent parton recombinations to parton evolution should be vary valuable for approaching the region of parton saturation from the perturbative side and for getting a reliable picture of small-$x$ partons [28,20].

4 Power correction to Drell-Yan cross section

It was shown by the NA38 and NA50 Collaborations that muon pair production for dimuon invariant mass between $\phi$ and $J/\psi$, known as the intermediate mass region (IMR), in heavy nucleus-nucleus collisions exceeds the expectation based on a linear extrapolation of the $p-A$ sources with the product of the mass numbers of the projectile and target nuclei [30]. The excess increases with the number of participant nucleons, and the ratio between the observed dimuon yield and the expected sources reaches a factor of 2 for central Pb-Pb collisions. There have been a lot of effort to attribute such an excess to the enhancement of open charm production [31], thermal dimuons production [32], and secondary meson-meson scattering in nuclear medium [33]. As shown in Ref. [30], the Drell-Yan continuum is the dominant source of the dilepton production in this mass range. An enhancement in the Drell-Yan continuum is much more effective than all other sources for interpreting the observed excess.

In hadron-nucleus and nucleus-nucleus collisions, more partons are available at a given impact parameter. Before the hard collision of producing the lepton-pair, partons from different nucleons can either interact between themselves, as sketched in Fig. 8(a) which leads to the universal nuclear dependence in parton distributions, or interact with the incoming parton, as sketched in Fig. 8(b) which gives the medium-size enhanced power corrections [34]. For the kinematics of IMR Drell-Yan process, parton distributions have a week nuclear dependence and produce a small reduction to the cross section. On the other hand, the medium size enhanced power corrections are process dependent, and actually increase the production rate and become more important when $Q^2$ decreases [34]. Let inclusive Drell-Yan cross section in nuclear collisions be approximated as

$$\frac{d\sigma_{AB}}{dQ^2} \approx AB \frac{d\sigma_{NN}^{(S)}}{dQ^2} + \frac{d\sigma_{AB}^{(D)}}{dQ^2} + \ldots$$

$$\equiv AB \frac{d\sigma_{NN}^{(S)}}{dQ^2} [1 + R_{AB}(Q)] ,$$  (22)

where superscripts $(S)$ and $(D)$ represent the single and double scattering, respectively. The $R_{AB}(Q)$ defines a nuclear modification factor of Drell-Yan continuum. Fig. 8(c) shows the calculated $R_{AB}(Q)$ from medium-size enhanced double scattering [34]. In evaluating Fig. 8(c), we used the same quark-gluon correlation function, $T_q(x, A)$, and neglected the $A$-dependence in parton distributions. Therefore, we expect that the true enhancement to the Drell-Yan continuum might be slightly smaller than $R_{AB}(Q)$ in Fig. 8(c), which is a very significant effect.

It is perhaps surprising that the medium-size enhanced power correction to DIS and Drell-Yan cross sections carries a different sign. The sign difference is a consequence of the kinematic nature of the calculated leading power
5 Transverse momentum $Q_T$ broadening

Coherent multiple rescatterings not only modify the production rate of inclusive cross sections, but also affect the momentum distribution of produced particles. Although the amount of broadening due to each soft rescattering is too weak a scale to warrant a reliable calculation, an averaged broadening in a hard collision could be a physical quantity calculable in pQCD $^2$.

In Ref. $^{35}$, Drell-Yan transverse momentum broadening was calculated in pQCD by evaluating the lowest order soft rescattering diagram in Fig. 11(a). Although the direct $Q_T$ modification from soft rescattering to $d\sigma/dQ^2dQ^2_T$ is not perturbatively calculable because of the size of small $Q_T$ kick, the $Q_T$ broadening,

$$\langle Q^2_T \rangle \equiv \int dQ^2_T Q^2_T \frac{d\sigma}{dQ^2dQ^2_T} = \int dQ^2_T \frac{d\sigma}{dQ^2dQ^2_T}, \quad (23)$$

is calculable $^2$$^8$. It was found that the Drell-Yan transverse momentum broadening in hadron-nucleus collisions can be expressed in terms of the same quark-gluon correlation function $T_q(x,A)$ (not its derivative) $^{35}$,

$$\langle Q^2_T \rangle \approx \left( \frac{4\pi^2\alpha_s}{3} \right) \sum_q e_q^2 \int dx' f_q(x') T_q(\tau/x',A)/x' \sum_q e_q^2 \int dx' f_{\bar{q}}(x') T_{\bar{q}}(\tau/x',A)/x', \quad (24)$$

where $\sum_q$ runs over all quark and antiquark flavors, $e_q$ is the quark fractional charge, and $\tau = Q^2/s$, in terms of the lepton-pair invariant mass $Q$ and hadron-hadron center of mass energy, $\sqrt{s}$.

Adopting the model in Eq. (13), the lowest order Drell-Yan transverse momentum broadening in Eq. (24) can be simplified as

$$\langle Q^2_T \rangle_{4/3} = \left( \frac{4\pi^2\alpha_s}{3} \right) \lambda^2 A^{1/3}. \quad (25)$$

By comparing Eq. (24) to data from Fermilab E772 and CERN NA10 experiments $^{36}$, $^{37}$, it was found $^{36}$ that $\lambda^2_{DY} \sim 0.01$ GeV$^2$, which corresponds to $\xi^2_{DY} \sim 0.04$ GeV$^2$ that is about a factor of 2 smaller from what was extracted from inclusive DIS data, where the leading twist nuclear dependence was not included. The difference might be caused by the difference between initial-state and final-state rescattering effects $^2$.

Momentum imbalance between two final-state jets in photoproduction off a nuclear target was calculated by evaluating the diagrams like one in Fig. 11(b), and was compared with Fermilab E683 data $^{19}$, with an assumption that the momentum imbalance between two jets is approximately the same as the momentum imbalance between two final-state partons. Again, the calculated momentum imbalance was expressed in terms of the same quark-gluon and gluon-gluon correlation function $^{19}$. By comparing with the momentum imbalance data for jet transverse momentum $p_T > 4$ GeV $^{38}$, it was found $^{19}$ that $\lambda^2_{dijet} = 0.05 - 0.1$ GeV$^2$, which corresponds to $\xi^2 \sim 0.2$ GeV$^2$ that is about a factor 2 bigger than what was found in DIS. As pointed out in Ref. $^2$, high order corrections to initial-state and final-state soft rescattering could be very different and significantly reduce the difference of the nonperturbative parameter.

6 Nuclear suppression in hadron-nucleus collisions

Dynamical nuclear effects induced by soft rescattering can be studied through the ratio of particle production rates in hadron-nucleus and hadron-hadron collisions. However, there is no observed variable, like $x_H$ in inclusive DIS, that can directly measure the coherence length of the hard scattering. All incoming parton momentum fractions are convoluted over, and strength of the collision is measured by Lorentz invariants, like Mandelstam variables, $\hat{s}$,
of the Mandelstam variable ($\hat{s}$, $\hat{t}$, and $\hat{u}$). Unlike in DIS and in Drell-Yan, there are both initial-state and final-state soft rescattering in hadronic collisions, which could in principle lead to medium size enhanced power corrections. Since the hard scales, $\hat{s}$, $\hat{t}$, and $\hat{u}$, in hadronic collisions are often much larger than a couple of GeV$^2$, effect of the medium size enhanced power corrections to hard probes in hardonic collisions is in general less significant. However, in the most forward (backward) region, the invariant $\hat{t}$ ($\hat{u}$) could be much smaller than the other invariants, so that the power corrections in $1/\hat{t}$ ($1/\hat{u}$) could become very important.\[14\]

Consider, for example, the single hadron inclusive production in hadron-nucleus collision as shown in Fig. 2(b). Once we fix the momentum fractions $x_a$ and $z_1$, the effective interaction region is determined by the momentum exchange $q^\mu = (x_a P_a - P_c/z_1)^\mu$. In the head-on frame of $q - P_b$, the scattered parton of momentum $\ell$ interacts coherently with partons from different nucleons at the same impact parameter, just like that in DIS. Interactions that take place between the partons from the nucleus and the incoming parton of momentum $x_a P_a$ and/or the outgoing parton of momentum $P_c/z_1$ at a different impact parameter are much less coherent and actually dominated by the independent elastic scattering [39]. Similar to the DIS case [11], we find [14] that resumming the coherent scattering of GeV and $\hat{t}$ in the right hand side of Fig. 13, the apparent width of data comes from the steepening of the Mandelstam variable $(-\hat{t})$. At high $p_T$, the attenuation is found to disappear in accord with the QCD factorization theorems [10]. The bottom panels of Fig. 12 show the growth of the nuclear attenuation theorem with centrality.

Dihadron correlations $C_2(\Delta \phi) = 1/N_{\text{coll}} \sum d\sigma_{\text{dijet}}/d\Delta \phi$ associated with $2 \to 2$ partonic hard scattering processes, after subtracting the bulk many-body collision background, can be approximated by near-side and away-side Gaussians. The acoplanarity, $\Delta \phi \neq \pi$, arises from high order QCD corrections and in the presence of nuclear matter - transverse momentum diffusion [39]. If the strength of the away-side correlation function in elementary N+N collisions is normalized to unity, dynamical quark and gluon shadowing in $p + A$ reactions will be manifest in the attenuation of the area $A_{\text{F率}} = R^{pA}_{h_1h_2}(b)$ [14].

The left panels of Fig. 13 show that for $p_T = 4$ GeV, $p_{T2} = 2$ GeV the dominant effect in $C_2(\Delta \phi)$ is a small increase of the broadening with centrality, compatible with the PHENIX [12] and STAR [13] measurements. Even at forward rapidity, such as $y_1 = 2$, the effect of power corrections in this transverse momentum range is not very significant. At small $p_{T1} = 1.5$ GeV, $p_{T2} = 1$ GeV, shown in the right hand side of Fig. 13, the apparent width of the away-side $C_2(\Delta \phi)$ is larger. In going from midrapidity, $y_1 = y_2 = 0$, to forward rapidity, $y_1 = 4$, $y_2 = 0$, we find a significant reduction by a factor of 3 - 4 in the strength of
7 Summary and outlook

We have briefly reviewed a pQCD factorization approach to coherent multiple scattering and argued that the $A^{1/3}$-type medium size enhanced power corrections caused by multiple soft rescattering can be consistently calculated in pQCD. We presented calculations of rescattering effects in inclusive cross sections as well as transverse momentum broadening (or the moment of particle transverse momentum distributions). By studying the coherent multiple scattering, we can probe new sets of fundamental and universal multiparton correlation functions in nuclear medium. These new functions provide new insights into the nonperturbative regime of QCD.

Initial pQCD calculations of multiple rescattering effects is very successful in understanding nuclear dependence of inclusive cross sections. For a wide range of observables, the estimate for the only unknown parameter is reasonably consistent. With a single unknown parameter, the calculations describe many observables and their nuclear dependences fairly well. The factorization approach, with its intuitive and transparent results, can be easily applied to study the nuclear modification of other physical observables in $p + A$ reactions. The systematic incorporation of coherent power corrections provides a novel tool to address the most interesting transition region between "hard" and "soft" physics in hadron-nucleus collisions.

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