Vibration response of defect-ball-defect of rolling bearing with compound defects on both inner and outer races

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Abstract. Aiming at the problem that the existing compound defects model of rolling bearings under radial load is difficult to reflect the actual contact between rolling elements and defects. A new model is proposed to accurately reflect the simultaneous or sequential contact between inner and outer race defects and rolling elements. Considering the coupled excitation between shaft and bearing and pedestal, time-varying displacement excitation, and radial clearance, a four degree-of-freedom vibration model of rolling bearing with compound faults on both inner and outer races is built. The vibration equations are calculated by the method of numerical way, and the model is verified by experiment. The vibration response characteristics of the Defect-Ball-Defect model are studied, which renders a theoretical criterion for bearing fault diagnosis.

Keywords: Compound defects, Defect-Ball-Defect model, dynamic model, vibration response characteristics, fault diagnosis.

1. Introduction

As one of the core components of rotating machinery, the bearings are used widely in modern industrial machinery. In the long-term operation, the bearing will have some problems such as pitting, spalling, cracking, and accidental loss of materials in the contact area, threatening the safety and reliability of mechanical equipment operation. The analyze on the dynamic model and vibration characteristics of the defects bearing is beneficial for structural design and optimization, diagnose bearing faults, and life calculation of rolling element bearing [1].

The vibration signal is a powerful means to diagnose the defect of the bearing. Lots of the rolling ball bearing models with composite defects were constructed to predict this signal. Patel et al. [2, 3] established a 6-DOF dynamics model of the bearing system to explore the vibration response of the rolling element bearing with local and multiple defects. Gao et al. [4] investigated the influence of the angle between fault and fault, the amount of faults, and the position of fault on the dynamic response of the ball bearings. Cao et al. [5] illustrated the vibration response modeling ways of cylindrical roller bearings with local fault, multiple faults, and composite faults. Wang et al. [6] investigated the influence of compound faults on the dynamic vibration response under different speeds, loads and sizes of faults, respectively. A dynamic model with multiple-point defects of the inner race was found by Yaqub et al. [7] to simulate the effect of the spalling position distribution on the rolling ball bearing...
vibration. Zhu et al. [8] introduced the Lempel-Ziv complexity measure into the model to indicate the disorderly of vibration signals of local and compound faults. Zhang et al. [9] set up the vibration model of the rolling bearings with compound faults on the outer race and rolling element and analyzed the variation of amplitude under different speed, defective size and load.

Although many efforts have been made in the composite defects resulted in the vibration response, there is little research about the vibration mechanism of ball bearing when the inner and outer race defects contact the ball simultaneously or successively. In this article, considering the coupled excitation between the shaft and bearing and pedestal, time-varying displacement and radial clearance, a 4-DOF vibration model of the bearing is established. The vibration equations are calculated by the numerical value method, and the model is verified by the experiment on the test rig of MFS. The vibration response characteristics of the Defect-Ball-Defect model are studied, which supplied a fresh idea for exploring the vibration response mechanism of compound defects between the rolling bearing inner race and outer race.

2. Establishment of the dynamic model

2.1. Model simplification

The interaction relationship of bearings is hugely complicated, so the bearing vibration model is built according to some reasonable assumptions: (a) The contact between rolling elements and races meets Hertz's contact condition. (b) The rolling elements are distributed evenly around the shaft. (c) Ignore the influence caused by inertial force and gyro moment. (d) The speed of the inner ring rotates is the same as motor, and the outer ring is fixed on the pedestal.

Based on the above assumptions, the bearing system is further simplified as the dynamic model presented in figure 1(a). Figure 1(b) shows that the contact between the inner ring-shaft and outer ring-pedestal are simplified as spring-damping connections. The collision between the ball and raceways is equivalent to linear spring connections.

![Figure 1](image)

**Figure 1.** Simplified bearing model. (a) Simplified model of outer ring-pedestal and inner ring-shaft. (b) Simplified model of the rolling elements and races.

2.2. Time-varying displacement excitation

In the compound defects model, one part of the time-varying displacement is given rise to the variable stiffness vibration of the bearing, and the other part is caused by the defects of the bearing races. So, the total radial deformation of compound defects can be given as follows:

\[
\delta_{usi} = (X_i - X_p) \cos(\theta_{k_1}) + (Y_i - Y_p) \sin(\theta_{k_1}) - C_s - \lambda_{usi}
\]

where \(C_s\) represents the radial clearance, \(\lambda_{usi}\) refer to the additional displacement of rolling elements through inner and outer race defects, \(\theta_{k_1}\) denotes the position angle of k-th rolling element at t time, can be obtained by equation (2)

\[
\theta_{k_1} = 2\pi(k - 1)/N_s + \omega t + \theta_{k_1}^0, \quad k = 1, 2, \cdots, N_s
\]
where $\theta_0$ is the position of the first rolling element at $0$ s, $N_b$ is the number of the ball, and $w_c$ refers to the angular velocity of the cage, can be given by equation (3)

$$w_c = 0.5(1 - D_b / D_m)w_c$$

where $D_b$ is the diameter of the rolling element, $D_m$ is the pitch dimension, and $w_c$ refers to the rotation speed of the shaft.

2.3. Contact force

In accordance with Hertz's contact condition, the time-varying contact force can be calculated [10].

$$F = K_b \delta^{1.5}$$

where $\delta$ refers to the deformation amount, and $K_b$ references the equivalent contact stiffness coefficient, which can be written as following [10]:

$$K_b = \left[1 / (1 / K_i^{1.5} + 1 / K_o^{1.5})\right]^{1.5}$$

where $K_i$ and $K_o$ represents the contact stiffness between the rolling elements and inner/outer races, and see Harris et al. ’s research for calculation [10].

The contact force could be written by equation (6)

$$\begin{align*}
F_x &= K_b \sum_{i=0}^{N_b} (\eta\delta_{i,s})^{1.5} \cos \theta_{i,s} \\
F_y &= K_b \sum_{i=0}^{N_b} (\eta\delta_{i,s})^{1.5} \sin \theta_{i,s}
\end{align*}$$

where $\eta$ is used to judge whether rolling elements enter the load area.

$$\eta = \begin{cases} 
1 & \delta_{i,s} > 0 \\
0 & \delta_{i,s} \leq 0
\end{cases}$$

2.4. Dynamic equation

From the analysis in section 2.2 and 2.3, and combined with figure 1, a 4-DOF dynamic equation can be expressed as follow:

$$\begin{align*}
M_x \ddot{X}_x + C_x \dot{X}_x + K_x X_x + F_x &= W_x \\
M_y \ddot{Y}_y + C_y \dot{Y}_y + K_y Y_y + F_y &= W_y \\
M_p \ddot{X}_p + C_p \dot{X}_p + K_p X_p - F_x &= 0 \\
M_p \ddot{Y}_p + C_p \dot{Y}_p + K_p Y_p - F_y &= 0
\end{align*}$$

where $W_x$ and $W_y$ are the external loads applied by the bearing system. The vibration equations are calculated by the Rung-Kutta method. The time step is $10^{-6}$ s, the rotation speed is 1200 r/min, the radial load is 50 N, and the axial load is 0 N. The initial displacement is $X_p=Y_p=X_o=Y_o=1 \times 10^{-6}$ mm, and the initial velocity is 0.

3. Defect-Ball-Defect solution model

The compound defects model can analyze the vibration signals of bearing under normal circumstances, but it can't explain the vibration response characteristics of Defect-Ball-Defect, quantitatively. Therefore, a solution model for the Defect-Ball-Defect contact condition is present.
As shown in figure 2(a), the model builds a coordinate with the center of the outer ring as the origin, setting the initial defect position of the outer raceway as 270° direction, setting the initial defect position of the inner race freely, and setting the initial position of the 1st ball as 0°. The ball rolls over the outer ring defect when the bearing starts to run. According to $\omega_b > \omega_c$, the rolling element will contact the inner and outer race defect simultaneously or after another. This model is defined as the Defect-Ball-Defect model.

The half-angle of the balls in the defect area can be calculated by equation (9)

$$\theta_{\text{defect-ball}} = \arcsin(L_d / D_b) \quad (9)$$

The half-angle of the inner race in the spalling zone can be expressed as follow:

$$\theta_{\text{defect-i}} = \arcsin(L_i / D_i) \quad (10)$$

4. Experiment validation and model analysis

To further verify the rationality of the vibration model of rolling ball bearing built in this article. ER-16k bearing is selected as experimental bearing, a rectangular defect with a width of 0.6 mm and a depth of 0.25 mm was fabricated on the inner and outer races of the rolling bearing by Laser ablation to simulate the fault, and the basic parameters of the ER-16k bearing are displayed in Table 1. Figure 3 is the test rig for collecting acceleration signals of bearing.

![Figure 2. The solution model of Defect-Ball-Defect. (a) The position of defects and balls at 0 s. (b) Defect zone. (c) Angle is relative to the defect zone.](image)

![Figure 3. MFS test rig.](image)

The rotating frequency is 20 Hz when the speed of shaft is 1200 r/min. The theoretical fault frequencies of the outer ring is 71.44 Hz and inner ring is 108.6 Hz, respectively, and see Harris et al.’s research for calculation [10].

| Parameters                  | Value  | Parameters     | Value  |
|-----------------------------|--------|----------------|--------|
| Inner race dimension $D_i$ (mm) | 30.59  | Pitch dimension $D_m$ (mm) | 39.65  |
| Outer race dimension $D_o$ (mm)  | 46.47  | Ball dimension $D_b$ (mm)  | 7.94   |
| Poisson ratio $\mu$ (/)       | 0.3    | Number of ball $N_b$ (/)   | 9      |
Young modulus $E$ (GPa) 219  
Radial clearance $C_r$ (mm) 0.0055

The data of the simulation bearing agree with the experimental. As shown in figure 4, the simulated signals of the rolling ball bearing with the composite defects and figure 4(b) is a time-domain amplified signal from 1.2 s to 1.3 s. The three equally spaced impulses can be seen in 1.228-1.27 s during the collision between the ball and outer raceway defect. Three equally spaced impulses can be seen in 1.23-1.257 s during the interaction between the rolling element and the inner race defect. Furthermore, the intervals are approximately 0.009 s and 0.014 s for the peak of each shock wave, which is equal to $1/f_{BPFI}$ and $1/f_{BPFO}$, respectively. It can be shown from figure 4(c) that the fault frequency of the shaft, outer and inner ring is 19.84 Hz, 71.41 Hz and 108.6 Hz, respectively.

**Figure 4.** Simulated signals. (a) Acceleration waveform of time-domain. (b) Enlarged acceleration waveform of time-domain. (c) Acceleration waveform of frequency-domain.

Figure 5 shows the experimental signals of the rolling ball bearing with the compound defects. Figure 5(b) is a time-domain amplified signal. It can have three equally spaced impulses seen in 1.244-1.283 s during the collision between the ball and outer race defect. Three similarly spaced impulses can be seen in 1.253-1.28 s during the ball impacts inner race defect. What's more, the intervals are approximately 0.009 s and 0.014 s for the peak of each shock wave, which is equal to $1/f_{BPFI}$ and $1/f_{BPFO}$, respectively.
**Figure 5.** Experimental signal. (a) Acceleration waveform of time-domain. (b) Enlarged acceleration waveform of frequency-domain.

In figures 4(a) and 5(a), the obvious periodic impulses appear to the experimental and simulated signal. In figure 4(c) and Figure 5(c), $f_{BPFO}$, $f_{BPFI}$, $2 \times f_{BPFO}$, $3 \times f_{BPFO}$, $2 \times f_{BPFI}$ and $3 \times f_{BPFI}$ can be seen obviously. In figure 5(c), the fault frequencies of the outer ring and inner ring are 72.42 Hz and 109.5 Hz for the experimental signals in the frequency domain, respectively. The errors of $f_{BPFO}$ and $f_{BPFI}$ between measured and simulated results are 1.39% and 0.82%, respectively. The main reason for the relative errors is the sliding of rolling elements and the vibrations from the shaft and pedestal.

As a result, the simulated acceleration signal agrees with experimental results and confirms the accuracy and effectiveness of the proposed fault model.

5. **Vibration characteristic of Defect-Ball-Defect contact condition**

The defects width of the inner and outer races are 0.5 mm, the depth is 0.25 mm, the applied load is 50 N, and the rotating speed of the shaft is 1200 r/min. The initial position of the inner race fault is 270° direction. Figure 6(a) shows the contact condition of bearing in the defect area within 0 to 0.5 s. It can be seen the Defect-Ball-Defect contact condition occurred in the running process of the 6th ball. Figure 6(b) is an enlarged view of the contact situation of the 6th ball. It can be seen from the drawing that the time of the 6th ball to roll over the fault zone is 0.1492s-0.1517 s, and the inner race defect to roll over the fault area is 0.1499-0.1501 s. Figure 6(c) is the time-domain signal, where $t_{mi1}$, $t_{mi2}$ and $t_{mi3}$ are the impulse signals generated by the collision between the 7th ball, 6th ball and 5th ball after entering the outer race defect, respectively. $t_{ai1}$, $t_{ai2}$ and $t_{ai3}$ are impulse signals generated by the collision of the defective trailing edge of the inner race with the 8th ball, 7th ball and 6th ball, respectively.

**Figure 6.** Defect-Ball-Defect contact condition. (a) The contact condition of the balls and inner ring defect within 0.5 s. (b) Enlarged the contact state for the 6th ball. (c) An enlarged time-domain diagram in this case.

The 6th ball starts entering the outer race defect at $t_{ai1}$, reducing stress and showing a step signal in the time domain. Before the 6th ball collided with the defective trailing edge of the outer race, the inner race defect rolled over the 6th ball. $t_{i1}$ to $t_{i2}$ is the theoretical time for the inter race defect rolled over the 6th ball, but the time is $t_{ai1}$ to $t_{ai2}$ when the inner race defect rolled over the 6th ball in the simulation signal. It is 0.0002 s ahead compared with the theoretical value, caused by the deformation of the race caused by the variable stiffness vibration of the bearing in the load zone. The 6th ball collides the trailing edge of the defect of outer race at $t_{mi2}$, which produced a high-frequency impulse signal in the time domain. Its acceleration amplitude is higher than other impulse signals. The reason is that the
previous impulse signal had not wholly disappeared and the next impulse signal increased the acceleration amplitude. When time is $t_{62}$, the 6th ball leaves the defect, regarded as a pressure recovery process, and the vibration signal gradually returns to stability.

6. Conclusions
Considering the effects of the coupled excitation between the shaft and bearing and pedestal, time-varying displacement excitation and radial clearance, a 4-DOF vibration model of rolling ball bearing with compound faults on both inner and outer races are found. The built model was validated by experiment. The results show that the simulation results agree with the test results, which proves the correctness and feasibility of the model.

Based on the compound faults model, the vibration response characteristics of the races defects and rolling elements contact simultaneously or successively are obtained by the Defect-Ball-Defect solution model. The results show that the time for the inner ring to impact the rolling element is 0.0002s earlier than the theoretical calculation value in the Defect-Ball-Defect contact condition. The coupling of vibration signals can be observed clearly, which increases the amplitude of vibration acceleration and accelerates the damage of bearings.

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