A Recursive Least Squares Method with Double-Parameter for Online Estimation of Electric Meter Errors

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Abstract

In view of the existing verification methods of electric meters, there are problems such as high maintenance cost, poor accuracy, and difficulty in full coverage, etc. Starting from the perspective of analyzing the large-scale measured data collected by user-side electric meters, an online estimation method for the operating error of electric meters was proposed, which uses the recursive least squares (RLS) and introduces a double-parameter method with dynamic forgetting factors $\lambda_a$ and $\lambda_b$ to track the meter parameters changes in real time. Firstly, the obtained measured data are preprocessed, and the abnormal data such as null data and light load data are eliminated by an appropriate clustering method, so as to screen out the measured data of the similar operational states of each user; Then equations relating the head electric meter in the substation and each users’ electric meter and line loss based on the law of conservation of electric energy are established. Afterwards, the recursive least squares
algorithm with double-parameter is used to estimate the parameters of line loss and the electric meter error. Finally, the effects of double dynamic forgetting factors, double constant forgetting factors and single forgetting factor on the accuracy of estimated error of electric meter are discussed. Through the program-controlled load simulation system, the proposed method is verified with higher accuracy and practicality.

**Keywords**

Electric Meter; Error Estimation; Line Loss; RLS; Double Forgetting Factors

**Abbreviations**

RLS- Recursive Least Squares; AMI- Advanced Metering Infrastructure; CT- Current Transformer; PT- Phase voltage Transformers; MAPE- Mean Absolute Percent Error; RMSE- Root Mean Square Error; PLC- Power Line Carrier

**Introduction**

With the construction and development of smart grids, the power industry has entered an era of big data. Electric meter is an important part of acquiring big data which have received wide attention. According to reports [1], before and after the World Metrology Day on May 20, 2018, the State Grid Corporation of China has installed more than 457 million electric meters, covering the 99.57% of the user service area. Facing the huge amounts of electric meters with complex application sites, how to improve electric meters’ self-diagnosis and verification, and improve their status evaluation and analysis capabilities, has become the focus of power grid companies. At present, the main way for power companies to verify the accuracy of electric meters is to use professionals that regularly carry instruments and equipment to the site for periodic sampling inspection [2,3]. The existing calibration mode has some disadvantages such as high working intensity and long calibration cycle and low managed efficiency, so it is difficult to meet the requirements of the maintenance and replacement of
electric meter status. The measured results are directly related to grid security and whether the trade settlement between the two parties is fair and reasonable. However, the global power industry has not found yet a practical theory and technology that can accurately measure and monitor the operating errors of electric meters in real time. Therefore, in order to realize the change of the electric meter from the periodic replacement to the state replacement and to judge the operational error’s status of electric meter, it is imperative to find an efficient and accurate method for estimating online the operational errors of electric meters.

With the development of smart grids and especially the popularization of Advanced Metering Infrastructure (AMI), power companies have acquired large-scale measured data. In recent years, some achievements have been made in the application of large-scale data measured from electric meters. In the research on anti-theft methods, the measured data of electric meters are used in gray correlation analysis [4,5], support vector machine and local anomaly factor algorithm [6] and estimated of loss in distribution line [7]. Through these methods, the identification of the substation and phase sequence, as well as the detection of the power stealing of the user substation, can effectively realize the location of abnormal power users. In addition, researchers have used measured data of electric meters to build a power consumption prediction model based on artificial neural networks, aiming at achieving accurate demand forecasting in the smart grid system as well as acquiring power consumption profiles for demand response purposes [8]. With the use of smart meters in smart cities, a data-driven probabilistic peak demand estimation framework using fine-grained smart meter data and sociodemographic data of the consumers was proposed in [9], which drives fundamental electricity consumption across different categories.

In the field of analysis of electric meter operational errors, there are relatively few studies in academic and industrial fields. In [10], Korhonen used automatic meter reading data to deduce a calculation method for the operating errors of electric meters. The applicability of the method depends on the user's power-
consumption level, the number of user's meters and other factors and the method will not be able to accurately estimate the errors of an electric meter if the loss between the head meter and the users’ meters is large. Guo [11] proposed a method to solve the errors of electric meter by the generalized law of energy conservation, but the method of decomposing the reading matrix into the upper and lower triangular matrix is verbose and matrix is prone to failure. Due to factors such as data size and quality, data in some unit measurement periods can’t satisfy the requirements of independence and orthogonality, and the method lacks real-time performance. In [12] researchers described a method of online smart meter error calibration using meter reading data. The approach of data analysis using sum meter reading and branch meter reading in a tree topology grid was studied. An algorithm based on the combination of K-means clustering and regularization theory is proposed to evaluate smart meter errors precisely. The proposed method has a better solution, but there may still be small solution errors caused by random factors such as the inaccurate estimation of energy losses. A mathematical model of On-Off-Key (OOK) dynamic load current was built in [13], a mathematical model of dynamic load energy sequences is proposed and three dynamic load power modes are defined: transient, short-term and long-term, based on which, an algorithm for measuring the dynamic errors of electric meter was proposed. Results indicate that the dynamic errors of electric meters are closely related to the dynamic load power mode of driving and the characteristics of dynamic errors are quite different among different kinds of electric meter. In [14], an error verification device for harmonic electric meters was proposed, that can output different amplitudes, different phases, different frequencies of the voltage and current through a power source, then transform the data detected by the measured harmonic meters and from a reference standard error calculator, it can verify the metering errors of harmonic meters quickly. In [15], an estimation method for electric meter errors based on a parameter degradation model was proposed. The comprehensive influence of various error factors such as temperature, humidity and load under actual working conditions are considered, but the error parameters of electric meter can only be estimated in a short time and real-time
estimated tracking can’t be achieved. In [16], the adaptive variable weight method is introduced in the fuzzy analytic hierarchy process and the state evaluation model of the meter is established to solve the problem that the weight of the index remains unchanged, which ensures the influence of each index on the measurement error of the electric meter is dynamically reflected. Aiming at solving the electric meter failure prediction problem and based on historical failure data of electric meters in some regions, a smart meter fault identification model is proposed based on a C5.0 algorithm in [17] which shows the accuracy of the failure prediction model for smart meters is higher, achieving good prediction effects. Reference [18] adopts a stratified sampling method to sample the electric meters in typical areas with high humidity and heat and severe cold that have been running for one year and a sampling life tolerance model is established. Considering the effect of temperature and humidity in errors, based on the bathtub curve, a Weibull lifetime distribution model is established. In addition, some related electric power workers have analyzed the factors affecting the accuracy of electric meter measurement from the perspectives such as communication transmission harmonic characteristics, ambient temperature, operating load size, and data sampling method [19–22].

In summary, although some research results have been obtained in the application of electric meter measured data and online estimation of operational errors, there are still some shortcomings in estimating accuracy and real-time performance. Based on the existing research, an online estimation method for solving the operational error of electric meters by using the recursive least squares algorithm with double-parameter is proposed. First, the measured data in similar operational states of each power user is selected. Then the recursive least squares algorithm with double-parameter is applied to calculate the operational error parameter of the electric meter and the line loss parameter of the low-voltage substation simultaneously. As explained throughout this paper, the key contributions of this paper can be summarized as follows:
(1) Compared with the existing methods, the proposed method realizes online estimation of line loss parameters and electric meter error parameters simultaneously by using the double-parameter recursive least squares algorithm with double dynamic forgetting factors.

(2) In addition, due to the introduction of double dynamic forgetting factors in the recursive least-squares algorithm, the flexible correction ability of the new data to the double-parameter estimation is ensured. The forgetting factors are adjusted according to the frequency of measured data collection system, which enhances the real-time performance and the parameter changes can be better tracked, so a large number of electric meters can be checked online.

The remainder of this paper is organized as follows: Section 2 introduces the overall estimation framework of the solution of the operational error and the method of processing the data that will be useful for obtaining the measured data to satisfy the requirements of the estimation model. Section 3 constructs the theoretical model for estimating the operational error of electric meters, and the error parameter is calculated based on the double-parameter recursive least squares algorithm. Section 4 presents the evaluation metrics and the methods to be compared in the case studies. Finally, the conclusions are presented in Section 5.

**Acquisition and Processing of Information of Electric Meters**

**Implementing Scheme for Online Estimation of Electric Meter Error**

The online estimated method of electric meter operational error mainly includes four steps: firstly, information, which includes profile information from power marketing information systems, (this system is composed of people, computers and computer programs, for power market decision makers to collect, select, analyze, evaluate and distribute the marketing information timely and accurately) and measured data from electric meters (user-side and substation) should be acquired. Then the acquired
data is preprocessed by clustering to get rid of the abnormal measured data such as null data and light load data. Next, an online estimation model for the operational errors of electric meters is established and the electric meter errors based on the proposed double-parameter RLS algorithm are calculated. Finally, we simulate real data of electric meters by using a program-controlled load simulation system (the whole simulation system can simulate the power consumption of real substations, and support the adjustment of different power consumption conditions, different line losses and different error conditions) and using Mean Absolute Percent Error (MAPE) and Root Mean Square Error (RMSE) as the evaluation metrics for checking the accuracy of the estimated error result of the electric meter. The whole process is shown in Figure 1.

![Diagram](image-url)

**Figure 1:** Online estimation process of electric meter errors.

**Required Electric Meter Information**

The information needed for online estimation of electric meter errors is as follows:

(1) The profile information contains five types:
   - User’s information such as user number, name, address, category of power consumption, etc.
   - Electric meter’s information such as electric meter’s number, name, type, address, current transformer (CT)/phase voltage transformer (PT) ratio, etc.
   - Metering point record information of the user information collection system: metering point identification, number, name, address, classification, nature, main purpose type, metering point side, voltage level, etc.
   - Electrical parameters of the head meter in the low-voltage substation, such as voltage, average power factor, active energies, reactive energies, etc.
   - Main electrical characteristic parameters related to the load, such as load rate, operating load type, quality and
proportion of the power consumption, terminal voltage of users.

(2) Measured data of electric meters which is collected by electric meters data acquisition system based on AMI.

Though system schemes may be slightly different for different regions, a typical scheme is shown in Figure 2. The distribution transformer station as a unit forms the acquisition system, and the concentrator is installed under the common distribution transformer, which realizes the power consumption information collection of all users under the substation with a communication system, such as power line carrier (PLC), RS485 or micro-power wireless. At the same time, it gathers calibrated meter data of the distribution transformer to realize the collection of the distribution transformer. According to the overall design scheme of the state grid corporation in China, the users’ collection structure of electricity information is a small-scale centralized collection, with a unified upload to the master station [23].

![Electric meter data acquisition physical architecture based on AMI.](image)

**Figure 2:** Electric meter data acquisition physical architecture based on AMI.

**Preprocessing Data Measured by Electric Meter**

The operational error of the electric meter generally refers to the relative error between the measured value and the actual value
collected during the operation, which can be deduced by the following formula [24]:

$$\zeta = \frac{z_{\text{real}} - z}{z}$$

(1)

where $\zeta$ is the measurement error of the electric meter; $z_{\text{real}}$ is the actual electric energy value in the unit measured period, which can be understood as the actual electricity consumption (true value) through the electric meter; $z$ is the increment (view value) of the electric meter reading during the unit measured period.

In order to meet the requirements of the error estimation model established and method proposed, which will be elaborated in Section 3, it is important to ensure the operational error parameters of the electric meter and line loss all be close to a certain value during the unit measurement period.

However, in reality, the difference of the load level in the user terminal will change the current on the distribution line, the voltage at the delivery end of the substation side, and the voltage on the user side of the power supply, which causes the line loss rate of the distribution area not being constant and changing as the users’ load levels fluctuate.

In addition, according to the working principle of the electric meter, on the one hand, the current and voltage fluctuation of line load will affect current and voltage of the sampling circuit of electric meters and the operation of the computing chip, such as the value of the load current and voltage cause the sampling circuit power consumption and heating changes and the harmonics of the load current and voltage affect the frequency characteristics of the sampling circuit [25]. These factors discussed above will affect the measured value of the electric meter. From this we can conclude that the errors generated by the electric meter in the work are not constant values, and the operational error of the electric meter becomes larger as the power factor decreases and increases as the relative value of the voltage and current amplitude changes. On the other hand, it is
called light load when the operational load current is below 5%~10% of the rated current. The light load affects the creep performance of the electric meter and the working state of the current transformer, so that the accuracy of the electric meter is greatly affected and the measured data error is higher and it is necessary to remove the data measured under light load conditions.

Therefore, in order to reduce the influence of the fluctuation of the load rate on the measurement error of the electric meter and line loss rate, it is necessary to ensure that the meter error and the line loss rate in the unit measured period have a certain value. It is also indispensable to cluster the collected electric meter measured data for the purpose of obtaining the measured value of the electric meter in a similar operating state.

Thence, preprocessing the measured data collected by the electric meter combined with the improved fuzzy clustering algorithm [26] in the process of online estimation of operational error, in which the abnormal measured data such as null data and light load data are rejected. The clustering process for selecting the measured data in the similar operational states of each power user is shown in Figure 3.

We sort the preprocessed data in the measured chronological order to form a head meter and every user meters’ matrix of the measured data, which are used as input variable for recursive least squares with double-parameter method.
Start

Read meters’ measured data: 
\[ z_1(t), z_2(t), \ldots, z_i(t), \ldots, z_m(t), y(t) \]

Determine the number of clusters based on the hill climbing method

Calculate the initial clustering centers \( V_0 \), weighed index \( w \) and membership degree matrix \( U_0 \)

Update the clustering centers \( V \) and membership degree matrix \( U \)

Calculate \( J_w(U, V) \)

\[ \Delta J_w(U, V) \leq \varepsilon \]

Final cluster center and membership matrix

End of clustering

Figure 3: Flow chart of our improved fuzzy clustering algorithm clustering for measured data.

The Solution Method of Electric Meter Operational Error

Establishing an Estimated Model of Operational Error

The typical power distribution topology is shown in Figure 4. A head electric meter is installed under a concentrator which is connected to a common distribution transformer. In the situation shown in the figure, the number of user meters is \( m \). We consider the fact that the head electric meter under each distribution transformer has been calibrated, so the head electric meter is assumed standard. In addition, there is line loss in the
line topology of the power distribution area. Based on the law of conservation of energy, the reading of the head electric meter in the low-voltage substation is equal to the sum of the true values of the electric meters of each user plus any line loss of the power distribution area during the unit measured period (line loss refers to the loss of electric energy in the form of thermal energy in the process of transmission, substation, distribution and marketing from power plant to power user, generally referred to as active loss. The term $w_{\text{loss}}$ mentioned in (2) refers to the entire process of power loss from the main transformer to the user electric meter). For any unit measured period, the electric meters reading in the station have the following relationship:

$$y_0(t) = \sum_{i=1}^{m} y_i(t) + w_{\text{loss}}(t),$$

(2)

where $y_0(t)$ is the power supplied by the head standard meter during the $t^{th}$ measured period; $w_{\text{loss}}(t)$ is power loss between the head meter and each users’ meter during the $t^{th}$ measured period; $m$ is total number of users’ electric meters in the substation; $y_i(t)$ is the real power consumption of the $i^{th}$ user during the $t^{th}$ measured period, according to Equation (1), using $y_i(t)$ instead of $z_{\text{real}}(t)$, the relationship between actual electricity consumption (true value) through the electric meters and the increment (view value) of the electric meter reading during the $t^{th}$ measured period as follows:

$$y_i(t) = z_i(t)(1 + \zeta_i(t)),$$

(3)

where, $z_i(t)$ is power consumption of the $i^{th}$ user measured by electric meter during the $t^{th}$ measured period; $\zeta_i(t)$ is measurement error of the $i^{th}$ electric meter.

The specific calculation process of $w_{\text{loss}}(t)$ is as follows [27]:

$$w_{\text{loss}}(t) = \frac{E(t) \cdot \Delta U(t)}{100} \times K_p(t),$$

(4)
where $E$ (MW·h) is the active power of the head electric meter in the low-voltage substation; $\Delta U$ is the loss rate of voltage in low voltage lines (the voltage loss is the difference between the amplitude of the voltage at the beginning and the ending, and its value is approximately equal to the vertical component of the voltage drop when the voltage phase difference between the two ends is very small); $K_p$ is the ratio of percentage of power loss to percentage of voltage loss which is related to the load power factor and the selected phase angle difference $\gamma$ between current and voltage of the head electric meter.

$\Delta U$ can be defined as follows:

$$\Delta U(t) = \frac{U_1(t) - U_2(t)}{U_1(t)} \times 100\%,$$

where $U_1$ (kV) is the outlet voltage of the distribution side, usually taking the average value of three-phase electricity; $U_2$ (kV) is the lowest point voltage on the user-side, if the low voltage is a single phase load, several low voltage averages must be measured.

$K_p$ can be written as follows:

$$K_p(t) = \frac{1 + (tg\gamma(t))^2}{1 + (\frac{X}{R}(t))tg\gamma(t)},$$

where $X/R$ is the ratio of wire reactance to resistance; $\gamma$ is the phase angle difference between current and voltage of the head electric meter, which is the power factor angle and $tg\gamma(t)$ can be written as follows:

$$tg\gamma(t) = \frac{Q(t)}{E(t)},$$

where, $Q$ (Mvar·h) is reactive power of the head electric meter in the substation.
Figure 4: Topology of typical power distribution area.

Parameter Estimation of Meter Error and Line Loss

According to Equations (2), (4) and (6), using $\varphi(t)$ and $\hat{\theta}_b(t)$ replace $\frac{E(t)\Delta U(t)}{100}$ and $K_p(t)$ respectively, we obtain the recursive least squares with double-parameter equation as follows:

$$y_0(t) = Z^T(t)\hat{\Theta}_a(t) + \varphi(t)\hat{\theta}_b(t),$$

where, $Z^T(t) = [z_1(t), z_2(t), \ldots, z_m(t)]$ is the matrix of measured data of each user meter during the $i^{th}$ measured period; $\hat{\Theta}_a(t) = [\hat{a}_1(t), \hat{a}_2(t), \ldots, \hat{a}_m(t)]^T$ is the matrix of error parameter to be estimated in each user’s meter, and define the electric meter operational error $\hat{\zeta}_i(t)$ as follows:

$$\hat{\zeta}_i(t) = \hat{a}_i(t) - 1,$$

Through direct estimation of unknown parameters $\hat{\Theta}_a(t) = [\hat{a}_1(t), \hat{a}_2(t), \ldots, \hat{a}_m(t)]^T$ and $\hat{\theta}_b(t)$ by recursive least squares with double-parameter algorithm. The electric meter operational error $\hat{\zeta}_i(t)$ and the ratio of wire reactance to resistance $X/R$ can be indirectly obtained.
A Recursive Least Square Scheme with Double-Parameter

When researching the measurement error of the electric meter and the loss of electric energy of the distribution line in the low-voltage distribution area, two limitations in the RLS algorithm based on a single forgetting factor are noticed [10]:

1. The electric meter error parameter changes with the line loss parameter at the same rate.
2. In the formulation of the loss-function defined in the RLS algorithm of a single forgetting factor and the resulting recursive scheme in the subsequent formula, the error due to all parameters is classified as a single scalar term.

Therefore, the algorithm has no way to realize whether the error is caused by one or more parameters. As a result, if there is drift in one of the parameters, then all parameters that cause the estimated overshoot or undershoot will be corrected in the same order. If the drift continues for a while, it may cause the overall performance of the estimate to deteriorate and may even cause the so-called estimate to be tightened or amplified, so the goal is to conceptually ‘separate’ the errors caused by each parameter and then apply a suitable forgetting factor for each parameter.

Therefore, an estimated method is proposed which is based on a double forgetting factor. The recursive least squares with double parameter which can not only use the real-time measured information to modify the estimated result repeatedly but also can adapt to the situation where the different parameters change speed in the multi-parameter estimated is different. The method can simultaneously estimate the operational error of the Electric meter and line loss. The electric meter operational error estimated model established above shows that there are two unknown parameters \( \hat{\Theta}_a(t) \) and \( \hat{\Theta}_b(t) \) need to be estimated, so two forgetting factors \( \lambda_a \) and \( \lambda_b \) are introduced. And the residual cost function defining this estimated model is as follows:
With this definition for the residual cost function, the first term on the right side of equation (10) only represents the error of the step \( t \) due to first parameter estimate, \( \hat{\Theta}_a(t) \) and the second term corresponds to the second parameter estimate, \( \hat{\Theta}_b(t) \). Now, each of these errors can be discounted by an exclusive forgetting factor. Notice that \( \Theta_a(j) \) and \( \theta_b(j) \) are unknown, and we will later replace them with their estimates \( \hat{\Theta}_a(t) \) and \( \hat{\Theta}_b(t) \). The swapping between the estimated and the actual parameters allows us to formulate the proposed modification to the classical Least Squares (LS) method with forgetting factors.

Here, \( \lambda_a \) and \( \lambda_b \) are forgetting factors for the first and second parameters, respectively. Incorporating multiple forgetting factors provides more degrees of freedom for tuning the estimator and, as a result, parameters with different rates of variation could possibly be tracked more accurately. The optimal estimates are those that minimize the loss function and are obtained as follows [28]:

\[
\frac{\partial J}{\partial \Theta_a(t)} = 0 \Rightarrow \sum_{j=1}^{J} \lambda_a^{-j} \left( y_o(j) - Z^T(j) \hat{\Theta}_a(t) - \varphi(j) \theta_a(j) \right) = 0
\]

(11)

Rearranging Equation (11), \( \hat{\Theta}_a(t) \) is found to be:

\[
\hat{\Theta}_a(t) = \left( \sum_{j=1}^{J} \lambda_a^{-j} Z^T(j) Z(j) \right)^{-1} \left( \sum_{j=1}^{J} \lambda_a^{-j} \left( y_o(j) - \varphi(j) \theta_a(j) \right) \right).
\]

(12)

Similarly, \( \hat{\Theta}_b(t) \) will be:

\[
\hat{\Theta}_b(t) = \left( \sum_{j=1}^{J} \lambda_a^{-j} \left( \varphi(j) \right)^2 \right)^{-1} \left( \sum_{j=1}^{J} \lambda_a^{-j} \left( y_o(j) - Z^T(j) \Theta_a(j) \right) \right).
\]

(13)
For real time estimated, a recursive form is required. Using the analogy that is available between Equations (12), (13) and the classical form (8), the recursive form can be readily deduced:

\[
\hat{\Theta}_a(t) = \hat{\Theta}_a(t-1) + K_a(t)\left(y_a(t) - Z^\top(t)\hat{\Theta}_a(t-1) - \varphi(t)\hat{\theta}_b(t)\right),
\]

where:

\[
K_a(t) = P_a(t-1)\varphi(t)\left(L_a + \varphi^\top(t)P_b(t-1)\varphi(t)\right)^{-1},
\]

\[
P_a(t) = (1 - K_a(t)\varphi(t))P_a(t-1)\frac{1}{\lambda_a},
\]

and similarly:

\[
\hat{\theta}_b(t) = \hat{\theta}_b(t-1) + K_b(t)\left(y_b(t) - Z^\top(t)\Theta_a(t) - \varphi(t)\hat{\theta}_b(t-1)\right),
\]

where:

\[
K_b(t) = P_b(t-1)\varphi(t)\left(L_b + \varphi^\top(t)P_b(t-1)\varphi(t)\right)^{-1},
\]

\[
P_b(t) = (1 - K_b(t)\varphi(t))P_b(t-1)\frac{1}{\lambda_b}.
\]

In the two aforementioned equations, \(\Theta_a(j)\) and \(\theta_b(j)\) are unknown, so we replace them with their estimates, \(\hat{\Theta}_a(t)\) and \(\hat{\theta}_b(t)\), as is customary in similar situations, such as the ‘separation principle’ in optimal control. The substitution is also justified when the actual and the estimated values are very close to each other or within the algorithm region of convergence. A convergence proof, or conditions for convergence of the algorithm under this assumption, remains open for future research. Upon substitution for \(\Theta_a(j)\), \(\theta_b(j)\) and rearranging equations (14) and (17), we obtain:

\[
\hat{\Theta}_a(t) + K_a(t)\varphi(t)\hat{\theta}_b(t) = \hat{\Theta}_a(t-1) + K_a(t)\left(y_a(t) - Z^\top(t)\hat{\Theta}_a(t-1)\right),
\]

\[
K_a(t)Z^\top(t)\hat{\Theta}_a(t) + \hat{\theta}_b(t) = \hat{\theta}_b(t-1) + K_b(t)\left(y_b(t) - \varphi(t)\hat{\theta}_b(t-1)\right),
\]

for which the solution is:
\[
\begin{bmatrix}
\hat{\Theta}_s(t) \\
\hat{\Phi}_s(t)
\end{bmatrix} = 
\begin{bmatrix}
1 & K_s(t) \\
K_s(t)Z^T(t) & 1
\end{bmatrix}
\begin{bmatrix}
\hat{\Theta}_s(t-1) + K_s(t)(y_s(t) - Z^T(t)\hat{\Theta}_s(t-1)) \\
\hat{\Phi}_s(t-1) + K_s(t)(y_s(t) - \phi(t)\hat{\Phi}_s(t-1))
\end{bmatrix},
\tag{22}
\]

Using the fact that \( P_a(t) \) and \( P_b(t) \) are always positive, it can be proved that the determinant of the matrix is always non-zero and therefore the inverse always exists. With some more mathematical manipulations, Equation (22) can be written as follows:

\[
\hat{\Theta}(t) = \hat{\Theta}(t-1) + K(t)\left(y_s(t) - \Phi(t)\hat{\Theta}(t-1)\right),
\tag{23}
\]

where, \( K(t) \) is defined as follows:

\[
K(t) = \frac{1}{1 + \frac{P_a(t-1)(Z^T(t))}{\lambda_a} + \frac{P_b(t-1)(\phi(t))}{\lambda_b}} \begin{bmatrix}
P_a(t-1)Z^T(t) \\
P_b(t-1)\phi(t)
\end{bmatrix},
\tag{24}
\]

where, \( \lambda_a \) and \( \lambda_b \) are the forgetting factors corresponding to the two parameters \( \xi(t) \) and \( X/R \) to be estimated, respectively, and the value range is \([0, 1]\). Considering that \( \xi(t) \) is a fast variable, in order to ensure that the estimated result of \( \xi(t) \) has better tracking performance, it should happen that \( \lambda_a > \lambda_b \).

Real-Time Adjustment of Forgetting Factors

The abovementioned parameter estimation method uses the constant forgetting factor, which can only be used in slow time-varying systems. However, in the actual problem of electric meter error estimation, the dynamic characteristics of the system do not always change according to the same law, and sometimes change rapidly. Sometimes the change is very slow, and sometimes there is a mutation. If a small forgetting factor is selected according to the fast change of the parameter, the information obtained from the data is less when the parameter changes slowly which will cause the parameter estimation error to increase exponentially, which is very sensitive to interference. If we choose a large forgetting factor based on the slow change
of parameters, it can memorize data that is far away and it will be insensitive to sudden changes in the system parameters. Therefore, if a constant forgetting factor is chosen, satisfactory results cannot be obtained.

According to the characteristics of the estimation parameters required, appropriate automatic adjustment methods are selected for the forgetting factors $\lambda_a$ and $\lambda_b$, respectively. The specific algorithm is as follows [29,30]:

Forgetting factor $\hat{\lambda}_a$ corresponding to the electric meter operational error estimated parameter:

$$\hat{\lambda}_a(t) = 1 - \left(1 - \frac{Z(t)P_a(t-1)Z^T(t)}{1 + Z(t)P_a(t-1)Z^T(t)}\right) \frac{e^2(t)}{R},$$

(25)

where, $0 < \hat{\lambda}_a(t) < 1$, $R \in (0,1)$ is the observed noise variance, in this paper, its value is 0.5, $e^2(t)$ is the variance of the estimated value.

Forgetting factor $\hat{\lambda}_b$ corresponding to the line loss estimated parameter:

$$\hat{\lambda}_b = R\left[1 + (\hat{\theta}_a(t) - \hat{\theta}_a(t-1))\right]^{-1},$$

(26)

**Case Analysis**

**Accuracy of Error Estimated under Different $\lambda$**

In order to verify the validity of the proposed method and the necessity of adopting the dynamic double forgetting factors method, we select a typical electricity low-voltage substation of urban residential users as an analysis case. The research area contains a head electric meter and three hundred user-side electric meters. The actual measured electric meter data from January 2016 to December 2018 in one substation are analyzed. The collection frequency of electricity consumption of ordinary residential users is 24 h. We eliminate abnormal data such as light load data and null data by appropriate clustering methods
and obtaining the measured data preprocessed as the analysis samples. The operational error recursive estimated curves of some electric meters solved by the proposed method are shown in Figure 5. The y-axis represents the error rate of the electric meter (error rate is the percentage form of \( \hat{\xi}_i(t) \), which represents the extent that the measured value of the electric meter deviates from the true value of the electric meter; when the error rate exceeds 2%, the electric meter is defined as an error-over meter), x-axis represents the frequency of data acquisition, the output frequency of the error analysis result is as the same as the frequency of the measured data acquisition, which is generated once a day.

\[ \text{error rate of No.33} \]
\[ \text{error rate of No.74} \]
\[ \text{error rate of No.100} \]
\[ \text{error rate of No.180} \]

**Figure 5:** The operational error recursive estimation curves. (a) represents the error rate recursive estimation curve of No.33 electric meter. (b) represents the error rate recursive estimation curve of No.74 electric meter. (c) represents the error rate recursive estimation curve of No.100 electric meter. (d) represents the error rate recursive estimation curve of No.180 electric meter.

It can be seen from Figure 5 that the estimated consequence of the double dynamic forgetting factors recursive least squares algorithm is shown by the blue line which is closest to the real error of the electric meter, and the convergence speed of the algorithm is faster than the other two algorithms; The estimated consequence of the double constant forgetting factors algorithm is shown by the red line. The estimated accuracy is not as high
as the double dynamic forgetting factors algorithm. Because different estimated parameters change at different rates, the constant forgetting factors can’t be made according to the change rule of the parameters and the adjustment also causes the convergence speed of the algorithm to be slower. The estimated effect of the single forgetting factor algorithm is shown by the black line. As can be seen from the figure, the calculation accuracy of the single forgetting factor is the lowest because the line loss rate is regarded as a certain value so that the total power consumption of each user’s electric meter in the unit measured period generates a large error, thereby causing a large interference to the estimated error of the user electric meter and resulting in greatly reduced accuracy of the estimated operational error of the electric meter. It also leads to recursive estimated curves are unstable with the circumstances of large noise.

Therefore, the forgetting factor should be automatically adjusted as the dynamic characteristics of the estimated parameters change. When the system parameter changes are abrupt, the small forgetting factor should be automatically selected to improve the sensitivity of the identification. When the system parameters change slowly, a large forgetting factor should be automatically selected in order to improve the recognition accuracy based on the memory length.

**Distribution of Meter Error Rate under Different λ.**

In order to verify the effectiveness of the proposed method and analyze the influence of forgetting factors in different situations on the estimated value of the meter errors we use the typical urban residential station mentioned in Section 4.1 as the research object. The measurement data collected by 300 electric meters in this area are used as the input variable of the algorithm, and the simulation results are shown in the following figure.

Figure 6 shows the online estimated results of the operational errors of all the user electric meters under a station. From Figure 6, we can conclude that the estimated results based on the double dynamic forgetting factors recursive least squares
algorithm have the highest accuracy, the double forgetting factors is the second, and the single forgetting factor is the worst. Therefore, the double dynamic forgetting factor are introduced by simultaneously estimating the operating error parameters of the electric meter and line loss parameters for ideal real-time online estimation of the operational error of large-scale user electric meters under a power substation can be obtained.

![Graphs showing operational error rates under different λ values](image)

**Figure 6:** Result of operational error of 300 meters under different states of λ.

**Estimation of Line Loss Rate**

There are many factors affecting grid power loss. For distribution grids below 380/220 V, the users’ power load has a significant impact on the line loss rate of the power distribution area. The user electrical load is the most important power consumption factor of the power supply system. Therefore, the power load determines the losses of the power supply system. The numerical distribution of the load and the spatial distribution of each load point affect the loss of the multi-branch
The load curve reflects the variation of the load within a certain time interval. It is inevitable to increase the equipment capacity to meet the safe and stable power supply if the electric load curve fluctuates greatly, which resulting in an increase in line loss.

For three types of electricity users in industrial, commercial, and urban residents, choosing their typical distribution area as the analysis object to calculate the network loss rate respectively, and still selecting the measured data of electric meter from January 2016 to December 2018 as data source.

Figure 7 shows the estimated results of the line loss rate of different type power distribution stations. The black curve represents the type of the substation occupied by industrial users. Since industrial users are in high-load power state all the year round and the load curve fluctuates greatly, the result that line loss rate of this type distribution area is the largest, which is between 3.3% and 3.8%; The blue curve represents the type of commercial-based area, the electrical load characteristics are similar to industrial stations, but the overall electricity consumption is lower than that of the industry, which is between 2.4% and 3.1%; The red curve represents the type of the power distribution stations are main occupied by ordinary residents. When compared with industrial users and commercial users, we could get the result that the residential electricity load is the lowest, but resulting in a large fluctuation in the line loss rate of the distribution area due to the influence such as season, holidays and other factors have created the large fluctuations in the power load. The line loss rate ranging from 1.5% to 2.6%.

Figure 7: The estimated line loss rate results.
Error Detection Rate Analysis

In order to further analyze the accuracy of the error rate of electric meters estimated by the proposed method, the actual operation state of the typical station user can be fully simulated in the laboratory environment through the programmable control load simulation system of the substation. The entire simulation system can simulate the power of the real substation and support the adjustment such as different power usage conditions, different line losses, and different electric meter error conditions. Therefore, the power consumption and line loss status of four types which include typical industrial users, commercial users, urban residents and rural residents are simulated respectively by using this simulation system and the real error of the electric meter are obtained.

In practice, the power consumption levels of different industries are quite different. The power consumption of industrial areas is the highest, followed by commercial areas. Compared with rural areas, in developed cities, residents consume more electricity. Therefore, the simulation system is used to simulate the power consumption of four type users: industrial, commercial, rural residents and urban residents. In the simulation process, it is guaranteed that all factors except the power consumption of different types of users are the same, such as the frequency of data collection and the number of users. In the simulation, set the data collect frequency to 1 hour, and the number of user’s electric meters is $p$.

Then MAPE and RMSE are used as the index for evaluation. In the online estimation of electric meter error, the smaller MAPE and RMSE values indicate the higher accuracy of the estimated error parameters. MAPE and RMSE can be expressed as:

$$MAPE = \frac{1}{p} \sum_{i=1}^{p} \left| \frac{\hat{\xi}_i - \xi_i}{\xi_i} \right| \times 100\% ,$$

$$RMSE = \sqrt{\frac{1}{p} \sum_{i=1}^{p} (\hat{\xi}_i - \xi_i)^2} ,$$

(27)  

(28)
where, \( p \) is the total number of users in the simulation; \( \hat{\zeta}_i \) and \( \zeta_i \) are the estimated value and actual value of the measurement error of the \( i \)th electric meter respectively.

As shown in Figure 8, under the situation that multiple simulation experiments are performed for each type of user, the missed detection rate and over-detection rate of the error estimation method proposed in this paper are analyzed. Through statistical calculation and analysis, the missed detection rate and over-detection rate of industrial users and commercial users are lower than for ordinary residents and the overall rate of missed detection is lower than the over-detection rate. The detection rate is below 1%. It can be inferred that the proposed method is applicable to the real-time online estimation of the operational errors of electric meters with high precision, and can also realize the estimation of the operational error for large-scale electric meters.

![Figure 8: The false detection rate of electric meter errors.](image-url)
Conclusions

In order to study how to estimate online the errors of electric meters, this paper proposes a double-parameter recursive least squares estimation method and a double-varying forgetting factor strategy that is in line with the development trend of AMI. The case analysis results show that the estimated performance of the double-varying forgetting factor is better than other estimation methods, such as the double-constant forgetting factor and single constant forgetting factor, and its false detection rate is below 2%. Moreover, the proposed method can simultaneously estimate the parameters of the electric meter error and line loss, and improve the accuracy of the online estimation of electric meter errors. In addition, the estimation method proposed is based on the elimination of abnormal data such as light load data and null data, although how to reduce the above discussed effects in the process of the data processing and algorithm solving needs further study.

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