Real decoupling ghosts quantization of CGHS model for two-dimensional black holes.*

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Abstract

A complete RST quantization of a CGHS model plus Strominger term is carried out. In so doing a conformal invariant theory with $\kappa = N/12$ is found, that is, without ghosts contribution. The physical consequences of the model are analysed and positive definite Hawking radiation is found.

1 Introduction

In the past two years a great advance in the studies of two-dimensional models of black hole has taken place. One of the most interesting approaches to this problem is certainly the model proposed by Callan, Giddings, Harvey, Strominger \footnote{Published Phys.Rev.D 51 n 4; 1995} in 1992. In that work the authors have introduced a
two-dimensional action involving gravity coupled to a dilaton field and $N$ conformal matter fields:

$$S_{cl} = \frac{1}{2\pi} \int d^2x \sqrt{-g}e^{-2\phi}(R + 4(\nabla \phi)^2 + 4\lambda^2) - \frac{1}{2} \sum_{i=1}^{N}(\nabla f_i)^2$$ (1)

In the conformal gauge $g_{++} = g_{--} = 0, g_{+-} = -\frac{1}{2}e^{2\rho}$ this becomes:

$$S_{cl} = \frac{1}{2\pi} \int d^2x [e^{-2\phi}(2\partial_+\partial_-\rho - 4\partial_+\partial_-\phi + \lambda^2 e^{2\rho}) + \frac{1}{2} \sum_{i=1}^{N}\partial_+ f_i\partial_- f_i]$$ (2)

Those authors have shown that the model admits solutions describing the formation of an evaporating black hole and that it is possible to give a semi-classical treatment of Hawking radiation. There was a great interest around this model, especially about how to quantize it to arrive to a fully quantum treatment of Hawking radiation (albeit in two dimensions). There are two kinds of approach to the quantization problem, the first of which was proposed by Bilal, Callan and de Alvis [2, 3]. They quantized the classical action (2) by generalising the form of the potential to obtain a conformal field theory, and so one loop finite. One of the problems of this approach is the lack of the linear dilaton vacuum (LDV) as a solution (the corresponding solution they find is only an asymptotic one). They also found an eternal radiating black hole and consequently the appearance of arbitrary large negative energies.

The other approach was proposed by Russo, Susskind and Thorlacius [4]. They modified the classical action (2) to obtain a theory with the linear dilaton vacuum as a solution. In order to do this, they added at one loop:

$$S_{RST} = -\frac{\kappa}{\pi} \int d^2x \phi \partial_+ \partial_- \rho$$ (3)

This is a local covariant term that allows the preservation, also at one loop, of the classically conserved current:

$$J_\pm = \partial_\pm (\rho - \phi) \quad \text{with} \quad \partial_+ \partial_- (\rho - \phi) = 0$$ (4)

Its existence allows in their model the persistence at one loop level of the LDV solution and the linking of the evaporating black hole solution to a ”shifted” LDV, terminating, in a physical reasonable way, the flux of Hawking radiation.
in finite time. The price of this is a violation of the "cosmic censorship" hypothesis, or at least the radical redefinition of it [7].

Apart from the difficulties explained above, the two models share the same problem: to obtain a conformal field theory they have to fix the value of the coefficient of the matter anomaly term $\kappa$ to be:

$$\kappa = \frac{N - 24}{12} \quad (5)$$

The shift in $\kappa$ as compared to the semiclassical value $\kappa = \frac{N}{12}$ found by CGHS, is a consequence of the missing term in these theories which would be responsible for decoupling the contribution of the reparametrization ghosts. The shift in the value of $\kappa$ induces spurious mode in Hawking radiation (a negative unphysical contribution) and causes a critical dependence on the number of matter fields ($N > 24$). Strominger [5] has proposed a method to obtain a decoupling of ghost fields by adding a term to the classical action. He proposed to improve the action with:

$$S_{\text{Ghost}} = \frac{1}{\pi} \int d^2x \left[ 2\partial_+(\rho - \phi)\partial_- (\rho - \phi) \right] \quad (6)$$

His basic point was the observation that there is no fundamental reason for choosing the same metric $g_{ij}$ to define the measure of the path integral of the graviton-dilaton-ghost as that used for the matter fields, since the difference amounts to the addition of a local function of Weyl factor. So he used a rescaled metric $\hat{g}_{ij} = g_{ij}e^{-2\phi}$ to define the measure of the graviton-dilaton-ghost system (the Faddev-Popov ghost determinant). This metric appears to be flat for all classical solutions of the original CGHS theory. The consequence is that black holes do not radiate ghosts to leading order in this theory. In conformal gauge this idea is implemented by building the graviton-dilaton-ghost Polyakov term out of $\rho - \phi$ (the action (6)) while building the matter anomaly term out of $\rho$ (the usual matter generated Polyakov term yet proposed by CGHS):

$$S_{\text{anom}} = -\frac{\kappa}{\pi} \int d^2x \partial_+ \rho \partial_- \rho \quad (7)$$

Unfortunately it appears that in quantizing this theory "à la ABC" (Bilal, Callan, de Alvis [2, 3]) the value of $\kappa$ has to be fixed, to obtain a conformal
invariant field theory, again as $\kappa = \frac{N-24}{12}$ showing that the decoupling of the ghost is not complete \[2\] .

Bilal \[6\] has explained this defect of Strominger model by the fact that the equations of motion derived from $S_{cl} + S_{anom} + S_{Ghost}$ differ, for $N = 0$, from those derived from $S_{cl}$ only by terms $\approx \partial_+\partial_-(\rho - \phi)$ that vanish by the presence of the conserved current (4). This is not true for $N \neq 0$ (in ABC quantization). So only in the absence of the matter fields $S_{Ghost}$ does not contribute to the solution of those equations.

Here I propose a RST quantization of a generalized Strominger model; in so doing the $\partial_+\partial_-(\rho - \phi) = 0$ condition is automatically satisfied also at one loop level and result is a really ghost decoupled conformal field theory. In the next section the new model will be described in detail. In sections III and IV its behaviour will be studied and in section V predictions about Hawking radiation will be made.

## 2 The Model

The starting point is the usual CGHS action (2). At one loop level this becomes in the new model:

$$S = S_{cl} + S_{anom} + S_{RST} + S_{Strom}$$

where:

$$S_{cl} = \frac{1}{2\pi} \int d^2x \left[ e^{-2\phi}(2\partial_+\partial_-(\rho - \phi) - 4\partial_+\phi\partial_-(\rho - \phi) + \lambda^2 e^{2\rho}) + \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_-(f_i) \right]$$

$$S_{anom} = -\frac{\kappa}{\pi} \int d^2x \partial_+ \rho \partial_-(\rho - \phi)$$

$$S_{RST} = -\frac{\kappa}{\pi} \int d^2x \phi \partial_+ \rho \partial_-(\rho - \phi)$$

$$S_{Strom} = \frac{1}{\pi} \int d^2x \left[ 2\partial_+ (\rho - \phi) \partial_-(\rho - \phi) \right]$$

It is easy to recognise in $S_{anom}$ the usual anomalous Polyakov term of the matter action, in $S_{RST}$ the covariant term modifying the kinetic part of the classical action ”à la RST” \[4\] (thanks to this term $\partial_+\partial_-(\rho - \phi) = 0$ also at one loop level), and in $S_{Strom}$ the above mentioned Strominger term. The
constraint equations of the conformal gauge give:

\[ T_{\pm\pm} = [e^{-2\phi} + \frac{\kappa}{4}](4\partial_{\pm}\rho \partial_{\pm}\phi - 2\partial_{\pm}^2\phi) + \frac{1}{2} \sum_{i=1}^{N} \partial_{\pm} f_i \partial_{\pm} f_i + 
\]

\[ - \kappa(\partial_{\pm}\rho \partial_{\pm}\rho - \partial_{\pm}^2\rho) + 2[\partial_{\pm}(\rho - \phi)\partial_{\pm}(\rho - \phi) - \partial_{\pm}^2(\rho - \phi)] + t_{\pm} = 0 \]

that is:

\[ T_{\pm\pm} = [e^{-2\phi} + \frac{\kappa}{4} - 1](4\partial_{\pm}\rho \partial_{\pm}\phi - 2\partial_{\pm}^2\phi) + \frac{1}{2} \sum_{i=1}^{N} \partial_{\pm} f_i \partial_{\pm} f_i + 
\]

\[ - (\kappa - 2)(\partial_{\pm}\rho \partial_{\pm}\rho - \partial_{\pm}^2\rho) + 2\partial_{\pm}\phi \partial_{\pm}\phi + t_{\pm} = 0 \]  

(9)

It is important to underline that the form of the stress energy tensor given in (9) can be determined only from the covariant expression of the action by derivations as to the components of the metric.

The function \( t_{\pm} \) appearing in (9), arises by the non locality of the Polyakov term \( \approx \int d^2x \ R(\partial^\mu \partial_\mu)^{-1} R \) that generates the anomalous part; \( t_{\pm} \) has to be determined by imposing boundary conditions.

Following Bilal and Callan’s method \[3\] we perform a redefinition of the fields to obtain a Liouville-like theory. This brings to a new relation between physical fields \( \rho, \phi \) and ”Liouville” ones \( \Omega, \chi \):

\[
\begin{align*}
\chi &= \rho \sqrt{\kappa - 2} + \frac{e^{-2\phi}}{\sqrt{\kappa - 2}} - \frac{\kappa - 4}{2\sqrt{\kappa - 2}} \phi \\
\Omega &= \frac{e^{-2\phi}}{\sqrt{\kappa - 2}} + \frac{\kappa}{2\sqrt{\kappa - 2}} 
\end{align*}
\]

(10)

With this redefinition the action is transformed into the Liouville-like form:

\[ S = \frac{1}{\pi} \int d^2x \left[ -\partial_+ \chi \partial_- \chi + \partial_+ \Omega \partial_- \Omega + \lambda^2 e^{\frac{2}{\sqrt{\kappa - 2}}(\chi - \Omega)} + \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i \right] 
\]

(11)

and the stress energy tensor (9) becomes:

\[ T_{\pm} = -\partial_+ \chi \partial_- \chi + \partial_+ \Omega \partial_- \Omega + \sqrt{\kappa - 2} \partial_+^2 \chi + \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i 
\]

(12)

From the expression of stress energy tensor (12) one can easily determine the value of \( \kappa \) required to obtain a conformal field theory. We have:
\[ c = c_\chi + c_\Omega + c_M + c_{\text{ghost}} = \left[1 - 12(\kappa - 2)\right] + 1 + N - 26 = 0 \]

for \( \kappa = \frac{N}{12} \) \hspace{1cm} (13)

This shows that the model is a conformal invariant field theory with \( \kappa \) equal to the semiclassical value found by CGHS without ghosts shift. One can immediately see that also in this case the field redefinition (10) is not "one to one" and in fact \( \Omega \) is bounded from below for \(-\infty < \phi < \infty \). Thus, in the present model too, there is a minimum for \( \Omega' = 0 \) which corresponds to

\[ \phi = -\frac{1}{2} \ln \left(\frac{\kappa}{4}\right) = -\frac{1}{2} \ln \left(\frac{N}{48}\right) \equiv \phi_c \] \hspace{1cm} (14)

It is obvious that something singular happens for this value of the field \( \phi \). It is also important to note that this behaviour of \( \Omega \) field, is common to all models cited above (ABC, RST). According to RST approach, this will bring us to fix a boundary condition that restricts the range of \( \Omega \) from plus infinity to the critical value. Of course, as pointed out by Hawking [9], in so doing we don’t have a real Liouville theory but rather a Liouville-like one, because the RST condition brings to an effectively non linear theory. The solutions of Liouville theory will only be a first approximation of the real ones.

Let us now study the physical behaviour of the model to test if it presents, as it is expected, results independent from the ghost contribution.

### 3 Solutions

The equations of motion of model (8) are:

\[
\begin{align*}
\chi : & \quad -2 \partial_+ \partial_- \chi = \frac{2 \lambda^2 e^{\frac{\kappa}{2}}(\chi - \Omega)}{\sqrt{\kappa - 2}} \\
\Omega : & \quad +2 \partial_+ \partial_- \Omega = -\frac{2 \lambda^2 e^{\frac{\kappa}{2}}(\chi - \Omega)}{\sqrt{\kappa - 2}}
\end{align*}
\] \hspace{1cm} (15)
These correspond to:

\[
\begin{align*}
\partial_+ \partial_- (\chi - \Omega) &= 0 \\
\partial_+ \partial_- (\chi + \Omega) &= -\frac{2\lambda^2 e^{\frac{x_+}{\sqrt{\kappa - 2}}} (\chi - \Omega)}{\sqrt{\kappa - 2}} \\
\end{align*}
\]

(16)

The first equation of (16) is what one expects to find thanks to the RST term, in fact now \( \partial_+ \partial_- (\rho - \phi) = 0 \) implies \( \partial_+ \partial_- (\chi - \Omega) = 0 \) as it is easy to verify using fields redefinition (10). As a result of this a gauge can be chosen in which \( \chi = \Omega, \rho = \phi \), the second equation thus reduces to:

\[
\partial_+ \partial_\Omega = -\frac{\lambda^2}{\sqrt{\kappa - 2}}
\]

(17)

Following the preceding works cited above we look for solution of (17) corresponding to flat static geometry. These are of the form:

\[
\Omega = \chi = -\frac{\lambda^2 x^+ x^-}{\sqrt{\kappa - 2}} + \frac{P\kappa}{\sqrt{\kappa - 2}} \ln(-\lambda^2 x^+ x^-) + \frac{M}{\lambda \sqrt{\kappa - 2}}
\]

(18)

Different values of P and M correspond to different solutions of the model:

1. For \( P = -\frac{1}{4}, M = 0 \) one finds the usual LDV solution:

\[
\Omega = \chi = -\frac{\lambda^2 x^+ x^-}{\sqrt{\kappa - 2}} - \frac{\kappa}{4\sqrt{\kappa - 2}} \ln(-\lambda^2 x^+ x^-)
\]

(19)

that is:

\[
e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^-
\]

(20)

2. For \( P = 0, M \neq 0 \) one finds the usual quantum black hole in thermal equilibrium with its environments:

\[
\Omega = \chi = -\frac{\lambda^2 x^+ x^-}{\sqrt{\kappa - 2}} + \frac{M}{\lambda \sqrt{\kappa - 2}}
\]

(21)

The behaviour of solutions (21) is the same as that shown in the preceding works [2, 4, 6].
4 Evaporating black holes

Following the work of RST [4], let us consider a dynamical situation where an incoming shock wave carries energy into a linear dilaton vacuum along an infalling line \( x^+ = x_0^+ \). Putting in the ++ constraints:

\[
\frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i = \frac{m}{\lambda x_0^+} \delta(x^+ - x_0^+) \tag{22}
\]

we obtain:

\[
\Omega = \chi = -\frac{\lambda^2 x^+ x^-}{\sqrt{\kappa} - 2} - \frac{\kappa}{4 \sqrt{\kappa} - 2} \ln(-\lambda^2 x^+ x^-) - \frac{m(x^+ - x_0^+)}{\lambda x_0^+ \sqrt{\kappa} - 2} \theta(x^+ - x_0^+) \tag{23}
\]

The solution (23) describes an evaporating black hole. In \( \Omega \) and \( \chi \) the singularity is not evident, but if we work in original fields \( \phi \) and \( \rho \) it is easy to see it. If we consider the curvature scalar \( R = 8e^{-2\rho} \partial_+ \partial_- \rho \) we find that it is singular for \( \Omega' = 0 \) that is \( \phi = -\frac{1}{2} \ln(\frac{\kappa}{4}) \equiv \phi_c \) mentioned before (14). So the singularity lies on the curve \( \phi = \phi_c \) where \( \Omega \) takes the value:

\[
\Omega = \Omega_c = \frac{\kappa}{4 \sqrt{\kappa} - 2} \left[ 1 - \ln \left( \frac{\kappa}{4} \right) \right] \tag{24}
\]

Introducing (24) in the dynamical solution (23) we find the same curve of the singularity \((\bar{x}^+, \bar{x}^-)\) found by RST [4]:

\[
1 - \ln \left( \frac{\kappa}{4} \right) = -\frac{4\lambda^2}{\kappa} \bar{x}^+ \bar{x}^- - \ln(-\lambda^2 \bar{x}^+ \bar{x}^-) - \frac{4m}{\lambda x_0^+ \kappa} (\bar{x}^+ - x_0^+) \theta(\bar{x}^+ - x_0^+) \tag{25}
\]

This curve asymptotically approaches the line:

\[
x^- = -\frac{m}{\lambda^3 x_0^+} \tag{26}
\]

Timelike observers with \( x^- > -\frac{m}{\lambda^3 x_0^+} \) could not escape from the singularity, so line (26) is an equivalent of the classical global event horizon for them. The classical horizon is also a curve where the dilaton field’s gradient passed from spacelike to timelike. Our region where \( \nabla \phi \) is timelike corresponds to a trapped region in higher dimensional theory. So in two dimensions we define an apparent horizon as the line where \( \nabla \phi = 0 \). So doing the apparent horizon
coincides with the event horizon of a static solution. Considering the curve $(\hat{x}^+, \hat{x}^-)$ where $\nabla \phi = 0$ we find, as RST:

$$\partial_+ \phi = 0 \implies \partial_+ \Omega = 0$$

for

$$0 = -\frac{\lambda^2 x^-}{\sqrt{\kappa - 2}} - \frac{\kappa}{4\sqrt{\kappa - 2}} \frac{-\lambda^2 x^-}{-\lambda^2 x^+ x^-} +$$

$$-\frac{m(x^- - x_0^+)}{\lambda x_0^+ \sqrt{\kappa - 2}} \delta(x^+ - x_0^+) - \frac{m\theta(x^+ - x_0^+)}{\lambda x_0^+ \sqrt{\kappa - 2}}$$

so

$$-\frac{\kappa}{4x^+ \sqrt{\kappa - 2}} = \frac{\lambda^2 x^-}{\sqrt{\kappa - 2}} + \frac{m\theta(x^+ - x_0^+)}{\lambda x_0^+ \sqrt{\kappa - 2}}$$

that is:

$$\hat{x}^+ = -\frac{\kappa}{4\lambda^2 x^-} \frac{1}{\frac{m}{\lambda^3 x_0^+}} \quad \text{for} \quad x^+ > x_0^+ \quad (27)$$

Let us now study the behaviour of the horizon. From (27) we find:

$$\hat{x}^- = -\frac{\kappa}{4\lambda^2 x^+} - \frac{m}{\lambda^3 x_0^+} \quad (28)$$

so

$$\frac{d\hat{x}^-}{dx^+} = \frac{\kappa}{4\lambda^2 x^+^2} = \frac{N}{48\lambda^2 x^+^2} > 0 \quad (29)$$

This is an important result because it tells us that the apparent horizon recedes also in this model at a rate proportional to $\kappa$ alone. What is new now is that $\kappa$ is not affected by the ghost’s contribution ($\kappa = \frac{N-24}{12}$ in ABC and RST model). The rate of contraction of the horizon is directly related to Hawking radiation so we expect to find a ghost independent energy flux as we shall prove in section V.

As shown by RST [4], the two curves (25)(27) determined above meet for a finite value of $x^+$ and from this point on evaporating black hole solution can be matched with a linear dilaton one shifted by a little amount of negative
energy. This is equivalent, in the -- constraints, to a little shock wave of matter with negative energy propagating out: the RST "Thunderpop". A brief analysis of our model shows that it is possible to reproduce identical results.

Let us now study the main problem we should like to solve in this work, that is the radiation flux of the black hole and its relations with ghosts.

5 Hawking Radiation

In evaluating the Hawking flux we shall follow the analysis of Bilal and Callan [2]. We are interested in the value of the – component of the stress energy tensor in an asymptotically minkowskian system of coordinates. We know that under a conformal trasformation the stress energy tensor trasforms as:

$$T_{\pm\pm}(w^\pm) = \left( \frac{\partial w^\pm}{\partial x^\pm} \right)^{-2} \left( T_{\pm\pm}(x^\pm) + \frac{c_{cl}}{24} D^S_{x^\pm}(w^\pm) \right)$$ (30)

where $D^S_{x^\pm}(w^\pm)$ = Schwarzian derivative = \(\frac{w^{\prime\prime\prime}}{w^\prime} - \frac{3}{2} \left( \frac{w''}{w^\prime} \right)^2\) and $c_{cl}$ = classical central charge of the system.

We have from the costraints: $T_{\pm\pm} \equiv T'_{\pm\pm} + t_\pm = T_{\pm\pm}^{\rho\phi} + T_{\pm\pm}^M + t_\pm = 0$. At the classical level only $T_{\pm\pm}^{\rho\phi}$ trasform anomalously with $c_{cl} = -12\kappa$ so we have to fix $t_\pm$ in order to cancel this anomaly and obtain a total $T_{\pm\pm}$ trasforming as a tensor and not as projective connections. This is fundamental for the condition $T_{\pm\pm} = 0$ to be a coordinate invariant statement. Thus we have:

$$t_\pm(w^\pm) = \left( \frac{\partial w^\pm}{\partial x^\pm} \right)^{-2} \left( t_\pm(x^\pm) + \frac{\kappa}{2} D^S_{\pm}(w^\pm) \right)$$ (31)

In $T_{\pm\pm}^{\rho\phi}$ only the matter generated anomalous part $T_{\pm\pm}^{\rho\phi,I} = \kappa[(\partial_\pm \rho)^2 - \partial_\pm^2 \rho]$ contributes to schwarzian anomaly and Strominger term $T_{\pm\pm}^{\rho\phi,II} = 2[\partial_\pm(\rho - \phi)\partial_\pm(\rho - \phi) - \partial_\pm^2(\rho - \phi)]$ does not influence it. The problem is that we use different metrics to define the measures of Polyakov term and the Strominger one. In the first case we adopted a Weyl metric $g_{++} = g_{--} = 0$, $g_{+-} = -\frac{1}{2}e^{2\rho}\delta_{+-}$, in the second a rescaled one $g_{++} = g_{--} = 0$, $g_{+-} = -\frac{1}{2}e^{2\rho}e^{-2\phi}\delta_{+-}$. In our RST frame of quantization we can always (even at one loop) fix the gauge $\rho = \phi$ so for every solution the Strominger term has a flat metric. In
calculating Hawking radiation we have to evaluate the stress energy tensor in an asymptotically minkowskian system of coordinates. But Schwarz term vanishes for linear transformations between minkowskian systems. This shows that Strominger term does not contribute.

With \( t_\pm \) fixed as above we have: \( T'_{\pm\pm} + t_\pm = 0 \) We are interested in the total value of the outgoing flux of energy on \( I^+_R \) but it is now evident that this will be equal to \( t_- \) in asymptotically minkowskian coordinates.

Let us now consider the dynamical solution:

\[
\Omega = \chi = -\frac{\lambda^2 x^+ x^-}{\sqrt{\kappa - 2}} - \frac{\kappa}{4\sqrt{\kappa - 2}} \ln(-\lambda^2 x^+ x^-) - \frac{m(x^+ - x_0^+)}{\lambda x_0^+ \sqrt{\kappa - 2}} \theta(x^+ - x_0^+) \tag{32}
\]

From the fields transformations and the condition \( \rho = \phi \) we have for \( x^+ > x_0^+ \):

\[
\frac{e^{-2\rho}}{\sqrt{\kappa - 2}} + \frac{\kappa}{2\sqrt{\kappa - 2}} \rho = -\frac{\lambda^2 x^+ x^-}{\sqrt{\kappa - 2}} - \frac{\kappa}{4\sqrt{\kappa - 2}} \ln(-\lambda^2 x^+ x^-) - \frac{m(x^+ - x_0^+)}{\lambda x_0^+ \sqrt{\kappa - 2}} \theta(x^+ - x_0^+) \tag{33}
\]

First we pass to an asymptotically minkowskian coordinate system:

\[
\begin{align*}
& e^{\lambda w^+} = \lambda x^+ \\
& e^{-\lambda w^-} = -\lambda x^-
\end{align*}
\tag{34}
\]

we find from (33)(for \( x^+ > x_0^+ \)):

\[
e^{-2\rho} + \frac{\kappa}{2} \rho = e^{\lambda(w^+ - w^-)} - \frac{\kappa \lambda}{4} (w^+ - w^-) - \frac{m}{\lambda^2 x_0^+} (e^{\lambda w^+} - e^{w_0^+}) \tag{35}
\]

Unlike the CGHS treatment, in the present case it is clearly not possible to find a coordinate transformation that renders the \( \rho, \phi \) fields static (i.e. only dependent on \( x^+ - x^- \)). What we can do is to find a coordinate system in which \( \rho, \phi \) are quasi-static in the \( I^+_R \) region. From the preceding equation (35) we suggest:

\[
\begin{align*}
& \tilde{w}^+ = w^+ \\
& \tilde{w}^- = -\frac{1}{\lambda} \ln\left(e^{-\lambda w^-} + \frac{m}{\lambda^2 x_0^+}\right)
\end{align*}
\tag{36}
\]
With this coordinate transformation the leading term for \( \tilde{w}^+ = w^+ \to \infty \) is \( e^{\lambda (\tilde{w}^+-\tilde{w}^-)} \) so \( \rho, \phi \) are quasi static. Then we have:

\[
D^S_{w^-}[\tilde{w}^-] = -\frac{\lambda^2}{2} \left[ 1 - \left( 1 + \frac{m}{\lambda^2 x_0^2} e^{\lambda \tilde{w}^-} \right)^2 \right] \quad (37)
\]

In the minkowskian system of coordinates we do not want to have any outcoming stress energy besides the one determined by \( T_-^M \) hence \( t_-(w^-) = 0 \). Finally we find:

\[
t_-(\tilde{w}^-) = \frac{1}{4} \lambda^2 \kappa \left[ 1 - \frac{1}{(1 + \frac{m}{\lambda^2 x_0} e^{\lambda \tilde{w}^-})^2} \right] \quad (38)
\]

This is the same result as found by CGHS [1]. It is also equal to the flux found by Bilal and Callan [2] with the difference that now one has \( \kappa = \frac{N}{12} \) and so no ghost unphysical contribution. For \( \tilde{w}^- \to +\infty \) Hawking radiation is emitted at a constant rate \( \frac{1}{4} \lambda^2 \kappa = \frac{\lambda^2 N}{48} \). For \( \tilde{w}^- \to -\infty \) it tends to zero.

Note that the flux is positive for every value of \( N \).

### 6 Conclusions

This work shows that it is possible to find for the GCHS model a frame of quantization in which one obtains a decoupling of the ghosts contribution and a conformal invariant field theory. It is found that this model predicts substantially the same results as the preceding works with the difference that now the \( \kappa \) value is not shifted relatively to its semiclassical value. It is also to be noted that the solutions and the passages used are still dependent on a shifted value \((\kappa - 2 = \frac{N-24}{12})\) but the results of physical interest are not.

Bilal and Callan concluding their work [2], underlined what they have seen as the real problems of their model. The first one was the contribution of the reparametrization ghosts to the Hawking radiation. The second, and surely more important, was the permanently continuing outgoing flux and thus ending up in negative mass solutions. The aim of this work is to show that the first problem is only a matter of opportune choice of field redefinition and,

\[\footnote{Note that this statement is unambiguous only in an asymptotic minkowskian system because transformations between this kind of system have vanishing Schwarzian derivative.}\]
in some sense, it can be considered as a technical detail. It is also evident that
the resolution of this point does not contribute to the solution of the second
one. This model presents, as RST and ABC do, the same asymptotically
constant flux independent of Bondi mass. To solve this puzzle, common to all
dilaton gravity black hole models, the author is at present only aware of two
approaches. The first is the RST one cited above: the solution is to choose
an opportune boundary condition for evaporating solutions as to match it
to a LDV [4]. The problem is that this brings to a kind of violation of the
cosmic censorship hypothesis and so compels us to a radical redefinition of
this assumption [7]. The second, more radical approach is that of Belgiorno,
Cattaneo, Fucito, Martellini [8] in which the unbounded energy problem is
solved by going beyond the usual dilaton model. Their idea is that energy
unphysical result is common to all dilaton models because it is due to a basic
”bug” in the Liouville model underlying every one of these. Those authors
propose a conformal affine Toda model. So doing they obtain a new theory
that shows a realistic behaviour of physical quantities such as energy and
temperature. These results encourage the hope that 2D quantum gravity
contains a certain amount of physical information built in and that some
unphysical behaviours of actual models can be remedied.

Note

After this article was submitted for publication, the author became aware
of a recent work by Strominger and Thorlacius [10], in which a similar con-
clusion is reached about the action here used (with the Strominger term). In
their generalized model, still certain amount of freedom is left by conformal
invariance, which is then fixed by the need of the coexistence of LDV and N
(not N − 24) dependent Hawking radiation.

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