AlphaEvolve: A Learning Framework to Discover Novel Alphas in Quantitative Investment

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ABSTRACT
Alphas are stock prediction models capturing trading signals in a stock market. A set of effective alphas can generate weakly correlated high returns to diversify the risk. Existing alphas can be categorized into two classes: Formulaic alphas are simple algebraic expressions of scalar features, and thus can generalize well and be mined into a weakly correlated set. Machine learning alphas are data-driven models over vector and matrix features. They are more predictive than formulaic alphas, but are too complex to mine into a weakly correlated set. In this paper, we introduce a new class of alphas to model scalar, vector, and matrix features which possess the strengths of these two existing classes. The new alphas predict returns with high accuracy and can be mined into a weakly correlated set. In addition, we propose a novel alpha mining framework based on AutoML, called AlphaEvolve, to generate the new alphas. To this end, we first propose operators for generating the new alphas and selectively injecting relational domain knowledge to model the relations between stocks. We then accelerate the alpha mining by proposing a pruning technique for redundant alphas. Experiments show that AlphaEvolve can evolve initial alphas into the new alphas with high returns and weak correlations.

CCS CONCEPTS
• Applied computing → Computers in other domains; • Information systems → Data mining; • Computing methodologies → Symbolic and algebraic manipulation.

KEYWORDS
stock prediction; search algorithm

1 INTRODUCTION
Alphas are stock prediction models generating trading signals (i.e., triggers to buy or sell stocks). Mining alphas with high returns has been an active research topic in the field of data mining [6, 11, 24, 25]. However, high returns typically come with high risks. To buffer risk, experts from hedge funds aim at identifying a set of weakly correlated formulaic alphas with high returns. The identifying process is illustrated in Figure 1. Alphas are first designed based on experts' different perspectives and then backtested on the markets to ensure weakly correlated returns. Such alphas, as algebraic expressions, have fine properties of simplicity and good generalizability [5].

Existing AI approaches surpass the experts by designing complex machine learning alphas [3, 8, 10] or using the genetic algorithm to automatically mine a set of formulaic alphas [14, 15]. In the first approach, machine learning alphas are machine learning models that generate trading signals. They can model high-dimensional features and learn from training data to boost generalization performance. However, they are complex in their structures, and thus difficult to mine into a set of alphas with weakly correlated returns and to maintain this set [17]. Further, machine learning alphas designed with domain knowledge are based on strong structural assumptions. To be specific, the injection of relational domain knowledge assumes the returns of the stocks from the same sector change similarly [10]. However, this assumption is less likely to hold for a volatile stock market. In the second approach, the genetic algorithm has two limitations: First, the search space of the algorithm is small because only arithmetic operations are considered. In such a small search space, the algorithm can hardly evolve the initial

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1 In this paper, the return is defined as Today’s Stock Return = (Today’s Stock Price - Yesterday’s Stock Price) / (Yesterday’s Stock Price)

2 Hedge funds are institutional investors and among the most critical players in the stock markets [2].

3 In this paper, we adopt the standard for weak correlation in hedge funds: the sample Pearson correlation of 15% between portfolio returns of different alphas [13].
alphas to improve the performance. Second, the algorithm searches formulaic alphas, which only utilize short-term features (e.g., the moving average of last 30 days’ close prices).

A framework called AutoML-Zero [21] was recently proposed to discover a neural network from scratch and can thus be considered for alpha mining. Such consideration is advantageous over the genetic algorithm: AutoML-Zero expands the search space by allowing more operation types, i.e., vector or matrix operations, for feature capturing. Consequently, a domain-expert-designed alpha can be further improved. However, merely applying AutoML-zero to alpha mining suffers from the drawbacks of machine learning alphas, i.e., complex alpha structures and ineffective use of relational domain knowledge, leading to ineffective and inefficient alpha mining. Specifically, machine learning alphas typically contain complex components that are difficult to discover from scratch, e.g., graph neural network [10], attention mechanism and LSTM neural networks [3]. Even discovering a two-layer neural network requires intensive computing resources and long searching time (i.e., 10 to 1000 processes over a week) to evaluate $10^{12}$ candidate alphas [21].

To address these issues, we first introduce a new class of alphas, which can be effectively mined into a weakly correlated set of alphas. Next, we propose a novel alpha mining framework, AlphaEvolve, for generating the new alphas. To the best of our knowledge, we are the first to solve the stock prediction problem based on AutoML and the first to tackle the problem of mining weakly correlated alphas.

In this paper, we make the following contributions:

- We first introduce a new class of alphas with intriguing strengths: like formulaic alphas, these alphas can model scalar features and thus are simple to mine into a weakly correlated set, but, like machine learning alphas, they are high-dimensional data-driven models utilizing long-term features. We then propose a novel alpha mining framework, AlphaEvolve, to generate the new alphas. To the best of our knowledge, we are the first to solve the stock prediction problem based on AutoML and the first to tackle the problem of mining weakly correlated alphas.
- We enable AlphaEvolve to selectively inject relational domain knowledge without any strong structural assumption in an alpha.
- We propose an optimization technique to accelerate alpha mining by pruning redundant alphas.
- We conduct extensive experimental study on AlphaEvolve using the stock price data of NASDAQ. The results show that AlphaEvolve generates alphas with weakly correlated high returns.

The remainder of the paper is structured as follows. Section 2 defines the problem formulation. Section 3 introduces the evolutionary algorithm AlphaEvolve is based on. Section 4 elaborates on the optimization of AlphaEvolve. Section 5 analyses the experiment results. Section 6 reviews the background of the alpha mining problem. Section 7 concludes this paper.

2 PROBLEM FORMULATION

We adopt the alpha definition used by traders in hedge funds [13]. An alpha is a combination of mathematical expressions, computer source code, and configuration parameters that can be used, in
We aim to search for the best alpha from all possible alphas constrained by the maximum allowable number of operations for each component, the allowable OPs for each component, and the maximum allowable number of operands for scalars, vectors, and matrices.

An alpha is evaluated over a set of tasks \( \mathcal{F}_K \), where \( K \) is the number of tasks. Each task is a regression task for a stock, mapping an input feature matrix \( X \in \mathbb{R}^{f \times w} \) to a scalar label of return \( y \), where \( f \) is the number of feature types and \( w \) is the input time window in days. The pair of \( X \) and \( y \) defines a sample. All samples \( S \) are split into a training set \( S_{tr} \), a validation set \( S_v \), and a test set \( S_{te} \).

### 3 EVOLUTIONARY ALGORITHM FOR ALPHA MINING

AlphaEvolve is based on the evolutionary algorithm, which is an iterative selection process for the best alpha under a time budget. The process is illustrated in Figure 3:

(1) In the first iteration, AlphaEvolve is initialized by a starting parent alpha, e.g., \( A_0 \) in Figure 3. A population \( P_0 \) is generated by mutating the parent alpha. Two types of mutations are performed on the parent alpha to generate a child alpha: (1) randomizing operands or OP(s) in all operations; (2) inserting a random operation or removing an operation at a random location of the alpha.

(2) Each alpha of \( P_0 \) is evaluated on the tasks \( \mathcal{F}_K \). The evaluation process outputs a fitness score as shown on the right side of Figure 3. We use the Information Coefficient (IC) as the fitness score for alpha \( i \) (Eq. 1), where \( \hat{y}_t^{(i)} = (\hat{y}_t^{(i)}, \ldots, \hat{y}_K^{(i)}) \) is the vector of predictions at date \( t \), \( y_t \) is the vector of corresponding labels, \( \text{corr} \) is the sample Pearson correlation, \( N \) is the number of samples in \( S_v \).

\[
IC_i = \frac{1}{N} \sum_{t=1}^{N} \text{corr} \left( \hat{y}_t^{(i)}, y_t \right)
\]  

**Figure 3: The evolution and evaluation processes in alpha mining.**
(3) The alpha with the highest fitness score in a randomly selected set of fixed size, called the tournament, is selected as the new parent alpha. For example, the alpha with the highest fitness score in the tournament of \( P_0 \), \( A_3 \), is selected as the parent alpha.

(4) In the subsequent iterations, a new population is generated by adding the mutated parent alpha into the previous population and eliminating the oldest alpha. For example, \( P_i \) is generated by adding \( A_5 \) mutated from \( A_3 \) and eliminating \( A_1 \) from \( P_3 \) in Figure 3.

(5) If the training budget is exhausted, the alpha with the highest fitness score in the population is selected as the evolved alpha.

![Figure 4: An example of the execution of the RelationOp.](image)

4 OPTIMIZATION

This section introduces the proposed OPs and the pruning technique for redundant alphas to optimize the alpha search.

4.1 RelationOp and ExtractionOp

Unlike conventional AutoML frameworks where tasks are mutually independent [18, 21], our tasks \( F_K \) are related because the stocks predicted in \( F_K \) are classified into sectors and industries in a stock market. Modeling such relations in a stock market effectively improves the prediction accuracy [10]. We design a set of OPs, called RelationOps, to model such relations: they calculate an output operand based on input scalar operands calculated in the current task and other related tasks in the same sector (industry).

For the execution of an operation with the RelationOp on a sample \( s^{(a)} \in S \) where \( a \in F_K \), the input operands are the output operands calculated on the samples \( s^{(b)} \subseteq S \) where \( F \) is a set of related tasks. For the example shown in Figure 4, the execution of \( s_2 = \text{rank}(s_3) \) (in yellow) has its inputs calculated as \( s_3 = \text{norm}(m_0) \) (in orange) on the same time step from all related tasks, where \( \text{norm} \) calculates the Frobenius norm of a matrix. The output operand and \( F \) are determined by the types of RelationOps: (1) RankOp outputs the ranking of the input operand calculated on \( s^{(a)} \) among those calculated on \( s^{(b)} \); (2) Relation-RankOp outputs the ranking of the input operand calculated on \( s^{(a)} \) among those calculated on \( s^{(b)} \) where \( F_T \subset F_K \) are the tasks in the same sector (industry); (3) RelationDemeanOp calculates the difference between the input operand calculated on \( s^{(a)} \) and the mean of those calculated on \( s^{(b)} \).

We define OPs extracting a scalar feature from \( X \) and OPs extracting a vector feature from \( X \) as GetScalarOps and GetVectorOps respectively, or called ExtractionOps in general. Once an ExtractionOp is selected in a mutation step, \( X \) serves as a pool for selecting a scalar or a vector, and thus the actual input of an alpha can be a scalar or just a scalar, a column, or a row of \( X \).

![Figure 5: Examples of the redundancy pruning process.](image)

4.2 Pruning Technique

Searching for the best alpha in a large space efficiently is challenging. Earlier work [21] increases efficiency by avoiding repeated evaluations of an alpha. It fingerprints an alpha by its predictions on a small set of samples and uses the fingerprint to terminate any repeated alphas with the same predictions on the set of samples in subsequent searches. However, this method is inefficient for two reasons. First, this method evaluates redundant operations and alphas. Second, the cost of testing on the set of samples is higher in our problem because the number of tasks \( K \) is larger than the number of the original problem. Further, we cannot approximate all the tasks with a small subset because the stocks predicted in the tasks vary greatly given the noisy nature of the stock price data.

We thus propose an optimization technique by pruning redundant operations and alphas as well as fingerprinting without evaluation. Specifically, the fingerprint of an alpha is built by pruning redundant operations and alphas before evaluation, and transforming the strings of the alpha’s remaining operations into numbers. If the fingerprint is matched in the cache, the fitness score stored in the cache is reused. Otherwise, the alpha is evaluated to get its fitness score and then hashed into the cache.
The redundancy pruning process prunes the operations that do not contribute to the calculation between the input feature matrix $m_0$ and the prediction $s_1$. The process works as follows. First, we represent an alpha as a graph with operators as edges and operands as nodes. The prediction $s_1$ is the root node. Next, starting from the root node, we iteratively check a node’s redundancy by finding the operation where the node is the output operand and checking if the input operands of this operation are redundant nodes. The check returns true if a leaf is $m_0$. Finally, we prune the operation with a redundant output operand. This process is illustrated in Figure 5, where the red, blue, and green nodes are the redundant operands, the necessary operands, and $m_0$ respectively, and the dashed edge is the operator with the input operand calculated at the last time step. In Figure 5a, part of the alpha are redundant operations: The fourth operation of the Predict() component with output $s_1$, denoted by $s_1(4)$ in the graph, is redundant since it is overwritten by $s_1(8)$, which is used as the prediction; The operation with output $s_8$ is redundant since $s_8$ does not contribute to the calculation of $s_1(8)$. In Figure 5b, the alpha is redundant since $m_0$ is not used to calculate the prediction.

We discuss two typical scenarios in the evolutionary process on how the pruning technique increases search efficiency. In the early stage of the evolutionary process, an alpha usually has more redundant operations than useful ones. These redundant operations can be pruned by the pruning technique. In the later stage, an alpha with no redundancy tends to be vulnerable to random mutations, e.g., deleting a random operation would invalidate the prediction. Consequently, this alpha would possibly become a redundant alpha after random mutations and would be pruned.

5 EXPERIMENTAL STUDY

In this section, we compare the performance of AlphaEvolve among different initializations, and with baselines including the genetic algorithm and complex machine learning alphas. Then we study the effectiveness of the parameter-updating function, the selective injection of relational domain knowledge, and the pruning technique.

5.1 Dataset

We use the 5-year (2013-2017) stock price data from a major stock market NASDAQ. The 5-year data consists of 1220 days in total and is split into sets of 988, 116, and 116 days for training, validation, and test, respectively. Two types of stocks are filtered out in the preprocessing stage: (1) the stocks without sufficient samples and (2) the stocks reaching too low prices during the selected period. The first is because they are less traded and thus only bring the noise to the model, while the second is because they are too risky for investors. After filtering, there are 1026 stocks left. Each type of the features is normalized by its maximum value across all time steps for each stock.

5.2 Baselines and Settings

We use the following baselines for comparison:

1. \textit{alpha}_G is the searched alpha by the genetic algorithm, which is a popular alpha mining approach discussed in Section 1.
2. \textit{Rank}_LSTM is a variant of the LSTM model with its output mapped to a fully connected layer.
3. RSR is a variant of \textit{Rank}_LSTM by adding a graph component, which is designed with the injection of relational domain knowledge by connecting stocks in the same sector (industry). The RSR model is reported with the best performance in the dataset [10].
4. alpha\_AE\_D is the evolved alpha by AlphaEvolve, initialized with a domain-expert-designed alpha. See details of the domain-expert-designed alpha in the alpha before evolving in Figure 2.
5. alpha\_AE\_NOOP is the evolved alpha by AlphaEvolve with no initialization.
6. alpha\_AE\_R is the evolved alpha by AlphaEvolve, initialized with an alpha designed randomly.
7. alpha\_AE\_NN is the evolved alpha by AlphaEvolve, initialized with a two-layer neural network alpha.

We shall now describe the setting for each method. For AlphaEvolve, we use the population size of 100 and the tournament size of 10. The mutation probability of each operation is set to 0.9. The dimensions $f$ and $w$ for the input feature matrix $X$ are 13. The first four features are the moving averages of the close prices over 5, 10, 20, and 30 days, respectively. The next four are the close prices’ volatilities over 5, 10, 20, and 30 days, respectively. The last five are the open price, the high price, the low price, the close price, and the volume, respectively. The minimum number of the operations in each function is set to 1 and the maximum number to 21, 21, and 45, respectively. We choose the size of the maximum allowed scalar, vector, and matrix operands to be 10, 16, and 4, respectively. During the evolutionary process, we train our alpha by one epoch for fast evaluation.

For the genetic algorithm, the input and the output are the same as those of AlphaEvolve. The sizes of the generation and the tournament are the same as AlphaEvolve. Apart from that, we follow the implementation details in [15]: the probability of crossover, subtree mutation, hoist mutation, point mutation, and point replace are set to 0.4, 0.01, 0, 0.01, and 0.4, respectively.

For \textit{Rank}_LSTM and RSR, each model’s input is a vector of the close prices’ moving averages over 5, 10, 20, and 30 days for each of the input stocks, while the output is the predicted return. Following the experiment settings of [10], we fine-tune the hyper-parameters for \textit{Rank}_LSTM. To be specific, the grid for the sequence length, the number of units, and the hyperparameter balancing the loss terms are [4, 8, 16, 32], [32, 64, 128, 256], and [0.01, 0.1, 1, 10], respectively. The learning rate is set to 0.001. The best set of the hyperparameters is selected based on the performance on $S_0$. This set is used for reporting the performance of \textit{Rank}_LSTM and then getting the pre-trained embeddings for RSR following the original implementation. The average of the testing results is reported by performing 5 runs with different random seeds.

5.3 Evaluation Metrics

Apart from the IC, we use the Sharpe ratio to measure risk-adjusted returns of a portfolio built based on an alpha. We first introduce how this portfolio is built. After that, we define the portfolio return and the Sharpe ratio.
Table 1: Mining weakly correlated alpha with an existing domain-expert-designed alpha.

| Alpha        | Sharpe ratio | IC  | Correlation with the existing alpha |
|--------------|--------------|-----|-------------------------------------|
| alpha_D_0    | 1232797      | 0.067358 | NA                                  |
| alpha_AE_D_0 | 10.734581    | 0.028437 | -0.202224                           |
| alpha_G_0    | 11.858020    | 0.015012 | 0.007269                            |
| alpha_AE_D_1 | 13.580572    | 0.056703 | -0.503845                           |
| alpha_AE_D_2 | 13.935702    | 0.050688 | 0.008778                            |
| alpha_AE_D_3 | 14.175385    | 0.056209 | -0.240786                           |
| alpha_AE_D_4 | 15.067808    | 0.052364 | -0.202224                           |
| alpha_AE_D_5 | 16.209571    | 0.054962 | -0.17144                            |

5.4 Performance Evaluation

5.4.1 Alpha Mining Performance Comparisons. We run AlphaE-volve with each of the initializations and the genetic algorithm for five rounds. For AlphaE-volve, the best alpha with the highest Sharpe ratio among all initializations is selected into a set \( \mathcal{A} \) after each round. To achieve low correlation, we discard alphas correlated with any alpha in \( \mathcal{A} \) above a cutoff during the evolutionary process, where the correlation is calculated using portfolio returns on \( S_p \) and the cutoff is set at 15% by the standard in hedge funds [13]. Note that as the number of rounds and the size of \( \mathcal{A} \) increase, the difficulty of generating an alpha with low correlation increases. The same process is applied to the genetic algorithm. The time budget is set to 60 hours in each round. Notation-wise, we use the last digit number in the alpha name to represent the round number (starting from 0). In the last round, the alphas in \( \mathcal{A} \) are used as initializations, each denoted with \( B \) followed by a number referring to the round generating the alpha. Note that we do not set a cutoff with the domain-expert-designed alpha \( \text{alpha}_D \_0 \) because its Sharpe ratio and IC are too low compared to the evolved alphas and its correlations with the evolved alphas are weak. These results are observed in Table 1, in which \( \text{alpha}_A E \_D \_0 \) and \( \text{alpha}_G \_0 \) with the cutoffs set for the correlations with \( \text{alpha}_D \_0 \) are compared with \( \text{alpha}_D \_0 \).

The comparison results with the genetic algorithm are shown in Table 2, where the best alpha in a round is marked in bold. We observe that the Sharpe ratio and the IC of the genetic algorithm deteriorate with more cutoffs, showing that it does not do well in mining weakly correlated alphas. This result is due to the smaller search space of the genetic algorithm. Due to the consecutive low performances of \( \text{alpha}_G \_2 \) and \( \text{alpha}_G \_3 \), we stop the search for \( \text{alpha}_G \_4 \).

The comparison results among different initializations are shown in Table 3. \( \text{alpha}_A E \_D \_x \) shows the best results for \( x = 0 \), as well as \( \text{alpha}_A E \_B0 \_4 \) in the last round. These show that AlphaE-volve can generate good alphas by leveraging a well-designed alpha. The decrease in performance for \( \text{alpha}_A E \_D \_1 \) and \( \text{alpha}_A E \_D \_2 \) is because the cutoff is set for the correlations with \( \text{alpha}_A E \_D \_0 \), which has the same initialized alpha. For the same reason, after the cutoff is set for the correlations with \( \text{alpha}_A E \_N N \_0 \), \( \text{alpha}_A E \_N N \_x \) has dropped in performance significantly for \( x = 2,3 \). A similar drop in performance is also observed for \( \text{alpha}_A E \_R \_3 \). Such drops are observed for most initializations, but \( \text{alpha}_A E \_D \_x \) is nonetheless the second-best for \( x = 1,2 \). \( \text{alpha}_A E \_N O O P \_x \) shows the worst performances by not being the best alpha in any round due to no initialization.

The first four rounds show a decreasing trend in both the Sharpe ratio and the IC because the accumulative cutoffs increase the search difficulty. This increasing difficulty is also shown in the evolutionary trajectories of the best alphas from all rounds in \( \mathcal{A} \).

Table 2: Performance of weakly correlated alpha mining.

| Alpha        | Sharpe ratio | IC  | Correlation with the best alphas |
|--------------|--------------|-----|-------------------------------------|
| alpha_AE_D_0 | 1232797      | 0.067358 | NA                                  |
| alpha_AE_D_1 | 13.580572    | 0.056703 | -0.503845                           |
| alpha_AE_D_2 | 13.935702    | 0.050688 | 0.008778                            |
| alpha_AE_D_3 | 14.175385    | 0.056209 | -0.240786                           |
| alpha_AE_D_4 | 15.067808    | 0.052364 | -0.202224                           |
| alpha_AE_D_5 | 16.209571    | 0.054962 | -0.17144                            |

Table 3: Performance of weakly correlated alpha mining for different initializations.

| Alpha        | Sharpe ratio | IC  | Correlation with the best alphas |
|--------------|--------------|-----|-------------------------------------|
| alpha_AE_D_0 | 1232797      | 0.067358 | NA                                  |
| alpha_AE_NOOP_0 | 12.128316    | 0.064382 | NA                                  |
| alpha_AE_R_0   | 10.734581    | 0.028437 | -0.202224                           |
| alpha_AE_NN_0  | 13.580572    | 0.056703 | -0.503845                           |
| alpha_AE_D_1   | 13.580572    | 0.056703 | -0.503845                           |
| alpha_AE_NOOP_1 | 11.858020    | 0.044230 | -0.393484                           |
| alpha_AE_R_1   | 12.128316    | 0.064382 | NA                                  |
| alpha_AE_NN_1  | 14.175385    | 0.056209 | -0.240786                           |
| alpha_AE_D_2   | 15.067808    | 0.052364 | -0.202224                           |
| alpha_AE_NOOP_2 | 12.128316    | 0.064382 | NA                                  |
| alpha_AE_R_2   | 13.580572    | 0.056703 | -0.503845                           |
| alpha_AE_NN_2  | 16.209571    | 0.054962 | -0.17144                            |
| alpha_AE_D_3   | 14.901069    | 0.038437 | -0.202224                           |
| alpha_AE_NOOP_3 | 3.873076     | 0.015012 | 0.007269                            |
| alpha_AE_R_3   | 3.660071     | 0.028437 | -0.00207                            |
| alpha_AE_NN_3  | 3.070874     | 0.031879 | -0.03980                            |
| alpha_AE_B0_4  | 9.502871     | 0.032155 | 0.137851                            |
| alpha_AE_B1_4  | 3.649546     | 0.014045 | -0.003357                           |
| alpha_AE_B2_4  | 12.75912     | 0.059586 | 0.217383                            |
| alpha_AE_B3_4  | 3.803120     | 0.021081 | -0.064233                           |

\[ \text{Following [13], we set } \bar{R}_p \text{ equal to 0 for simplicity.} \]
While in the inference stage, the prediction function consists of \( S \), historically updated bound decisions based on the trends of high prices, constrained by the denominator in Eq. 2. Therefore, this alpha makes trading alpha is a special case of the new alpha with no parameters. The new alpha becomes a formula without parameters. Note that we choose \( \text{alpha}_\text{AE}_\text{B}_0 \text{.4} \) as the best alpha instead of \( \text{alpha}_\text{AE}_\text{B}_2 \text{.4} \) since the latter correlates with the previous alphas above 15%.

5.4.2 Study of The Evolved Alphas. We first study the best alphas from all rounds in \( \mathcal{A} \) and then perform the ablation study on the parameter-updating functions. To ease readability, we first change the raw output of AlphaEvolve, e.g., the form of the evolved alpha in Figure 2, into a compact set of equations. Then we divide the equations into three parts: \( M \), \( P \), and \( U \). In the training stage, the predict function is \( M \) and the parameter-updating function is \( U \), while in the inference stage, the prediction function consists of \( M \) and \( P \). \( M \) is used in both stages to pass parameters between the stages.

For \( \text{alpha}_\text{AE}_\text{D}_0 \) (Eq. 2 to Eq. 9), \( S_{t-2} \) and \( S_{t-2} \) are updated by Eq. 6 and Eq. 7 respectively in the training stage, and then passed to \( M \) as the parameters at the beginning of the inference stage. These parameters affect output operands \( S_1 \) in Eq. 2 and \( S_{t-1} \) in Eq. 3 by initializing the input operands. Then the parameter \( S_{t-2} \) is overwritten in Eq. 4. The remaining parameter \( S_{t-2} \) is used in an upper bound \( \arcsin(S_{t-2}) \) for an expression of the temporal difference (i.e., trend) of the high prices in Eq. 3. This bound will be overwritten once it is less than the trend, upon which the model becomes a formulaic alpha. This is because a formulaic alpha is a special case of the new alpha with no parameters. The prediction is the fraction with an expression of bounded trend feature on high prices as the numerator and another trend feature as the denominator in Eq. 2. Therefore, this alpha makes trading decisions based on the trends of high prices, constrained by the historically updated bound \( \arcsin(S_{t-2}) \).

\[
M: \quad S_1 = \tan(S_{t-1})/\cos(S_{t-2} - \arcsin(high\_price_{t-1})) \tag{2}
\]
\[
S_{t-1} = \min(S_{t-2} - \arcsin(high\_price_{t-1}), \arcsin(S_{t-2})) \tag{3}
\]
\[
P: \quad S_{t-2} = \arctan(\arcsin(high\_price_{t-1})) \tag{4}
\]
\[
S_{t-2} = \arctan(S_{t-2}) \tag{5}
\]
\[
U: \quad S_{t-2} = \tan(\text{heaviside}(S_{t-2})) \tag{6}
\]
\[
S_{t-2} = \arccos(\text{norm}(\text{norm}(M_{t-4}, \text{axis} = 0))) \tag{7}
\]
\[
M_{t-4} = \min(\text{abs}(\text{abs}(M_{t-4})), \text{broadcast}(\text{broadcast}(S_{t-4}, \text{axis} = 1))) \tag{8}
\]
\[
M_{t-4} = \text{matmul}(M_{t-5}, M_{t-5}) \tag{9}
\]

\[M_1 = \log(\cos(\arcsin(\text{abs}(\text{relation\_rank}(\arctan(\sin(\exp(\text{high\_price}_{t-1})))))\text{rank})), \log(\sin(\arctan(\sin(\exp(\text{high\_price}_{t-1}))))) \text{rank})) \tag{10}\]

For \( \text{alpha}_\text{AE}_\text{R}_2 \) (Eq. 11 to Eq. 16), the parameter \( M_{t-2} \) is updated recursively with an expression of the input feature matrix \( M_{t-2} \) (Eq. 14 and Eq. 15). \( S_{t-2} \) is a trend feature based on the comparison between \( \text{high\_price}_{t-2} \), and a recursively compared feature of \( \text{high\_price}_{t-2} \) (Eq. 12 and Eq. 13). Thus in the inference stage, we can observe from Eq. 11 that the alpha makes trading decision based on the volatility of the historically updated features \( M_{t-2} \), the trend feature based on high prices \( S_{t-2} \) and the recent return \( S_{t-3} \).

\[
M: \quad S_1 = \log(\cos(\arcsin(\text{abs}(\text{relation\_rank}(\arctan(\sin(\exp(\text{high\_price}_{t-1})))))\text{rank})), \log(\sin(\arctan(\sin(\exp(\text{high\_price}_{t-1}))))) \text{rank})) \tag{11}
\]
\[
P: \quad S_{t-2} = \max(\sin(S_{t-3}), \text{high\_price}_{t-4}) \tag{12}
\]
\[
S_{t-3} = \max(S_{t-4}, \max(\sin(S_{t-4}), \text{high\_price}_{t-5})) \tag{13}
\]
\[
U: \quad M_{t-2} = \min(S_{t-2} + \text{heaviside}(S_{t-2})), 1) + M_{t-2} \tag{14}
\]
\[
S_{t-2} = \text{low\_price}_{t-10} \tag{16}
\]

For \( \text{alpha}_\text{AE}_\text{D}_3 \) (Eq. 17 to Eq. 19), a lower bound of the transpose of the input feature matrix \( M_{t-2} \) is set for an expression of \( M_{t-2} \) to recursively update the parameter \( M_{t-2} \) (Eq. 19). At the beginning of the inference stage, \( M_{t-2} \) is passed to \( M \) and \( P \) as initial matrices \( M_{t-2} \) and \( M_{t-2} \) respectively (Eq. 17 and Eq. 18). Then \( M_{t-3} \) recursively compares with an expression of \( M_{t-2} \) (Eq. 18). Finally, the prediction is the standard deviation of another comparison result between \( M_{t-2} \) and an expression of another input feature matrix \( M_{t-2} \) (Eq. 17), showing that this alpha trades based on the volatility of an expression of the previous day’s features \( M_{t-1} \) bounded by the historically updated features \( M_{t-2} \). Note that once \( M_{t-3} \) is larger than \( \text{heaviside}(M_{t-2}, 1) \) (Eq. 18), this alpha becomes a formula without parameters.

\[
M: \quad S_1 = \log(\cos(\arcsin(\text{abs}(\text{relation\_rank}(\arctan(\sin(\exp(\text{high\_price}_{t-1})))))\text{rank})), \log(\sin(\arctan(\sin(\exp(\text{high\_price}_{t-1}))))) \text{rank})) \tag{17}
\]
\[
P: \quad M_{t-2} = \max(M_{t-2}, \min(\text{heaviside}(M_{t-2}, 1), M_{t-3})) \tag{18}
\]
\[
U: \quad M_{t-2} = \max(\text{transpose}(M_{t-2}), \max(M_{t-2}, \min(\text{heaviside}(M_{t-2}, 1), M_{t-3}))) \tag{19}
\]

For \( \text{alpha}_\text{AE}_\text{B}_0 \) (Eq. 20 to Eq. 22), the parameter \( M_{t-2} \) is updated recursively with an expression of \( M_{t-2} \) and an expression of \( M_{t-4} \) (Eq. 21 and Eq. 22). The prediction is based on the comparison between the inverse of \( \text{close\_price}_{t-2} \) and the expression of \( \text{M}^{30}_{t-4} \) (i.e., the moving average of the close prices over the last 30 days calculated at \( t-4 \)), and the standard deviation of \( M_{t-2} \) (Eq. 20). Thus, this alpha makes trading decisions based on the recent

![Figure 6: Evolutionary trajectories for the best alphas in all rounds.](image)
Table 4: Ablation study of the parameter-updating function.

| Alpha            | Sharpe ratio | IC  | Correlation with the best alphas |
|------------------|--------------|-----|---------------------------------|
| alpha_AE_D_0     | 21.323797    | 0.067358 | NA                              |
| alpha_AE_D_0_P   | 21.516790    | 0.057707 | NA                              |
| alpha_AE_R_2     | 18.629571    | 0.066962 | -0.177144                      |
| alpha_AE_R_2_P   | -0.344734    | 0.003149 | -0.994286                      |
| alpha_AE_D_3     | 4.910169     | 0.028437 | -0.202224                      |
| alpha_AE_D_3_P   | 5.697408     | 0.032647 | -0.241651                      |
| alpha_AE_B0_4    | 9.502871     | 0.032155 | 0.137851                       |
| alpha_AE_B0_4_P  | -0.004294    | -0.001908 | -0.097541                      |

Table 5: Performance comparisons with the complex machine learning alphas.

| Alpha            | Sharpe ratio | IC  |
|------------------|--------------|-----|
| alpha_AE_D_0     | 21.323797    | 0.067358 |
| alpha_AE_D_0_P   | 21.516790    | 0.057707 |
| Rank_LSTM        | 14.175835    | 0.065209 |
| RSR              | 5.647131     | -0.522782 |

Table 6: Efficiency of the pruning technique.

| Alpha            | Sharpe ratio | IC  | Correlation with the best alphas |
|------------------|--------------|-----|---------------------------------|
| alpha_AE_D_0     | 21.323797    | 0.067358 | NA                              |
| alpha_AE_D_0_N   | 8.898782     | 0.057817 | NA                              |
| alpha_AE_D_1     | 15.580572    | 0.045703 | -0.001845                      |
| alpha_AE_D_1_N   | 5.418189     | 0.025506 | -0.052246                      |
| alpha_AE_R_2     | 18.629571    | 0.066962 | -0.177144                      |
| alpha_AE_R_2_N   | 4.759565     | 0.032158 | 0.141613                       |
| alpha_AE_D_3     | 4.910169     | 0.028437 | -0.202224                      |
| alpha_AE_D_3_N   | 6.643422     | 0.028495 | -0.051935                      |
| alpha_AE_B0_4    | 9.502871     | 0.032155 | 0.137851                       |
| alpha_AE_B0_4_N  | 2.773826     | 0.027594 | -0.137823                      |

5.4.3 Comparisons With The Complex Machine Learning Alphas.

In Table 5, we compare the complex machine learning alphas with the generated alphas by AlphaEvolve. We observe that both the complex models fail to compete against alpha_AE_D_0. The poor performance of RSR is due to the imposition of the relational domain knowledge to the data. The NASDAQ dataset cannot be best explained by the relational domain knowledge because the NASDAQ is a more noisy stock market compared to other stock markets (e.g., the NYSE)\(^1\); it incorporates many volatile stocks with less capitalization. Therefore, the noisy stock market affected by rapid-changing information cannot be modeled with the static relational knowledge. Besides, Rank_LSTM and RSR are unstable with high standard deviations because their predictions are influenced by random effects (i.e., random seeds). In contrast, the flexibility of the domain knowledge injection by AlphaEvolve leads to the best performance of alpha_AE_D_0 without the domain knowledge, while alpha_AE_NN_1 generated with the relational domain knowledge is weakly correlated with alpha_AE_D_0 but has a relatively lower Sharpe ratio and IC.

5.4.4 Efficiency of The Pruning Technique. In the ablation study of the pruning technique, we remove this technique from AlphaEvolve but use the prediction of an alpha as the fingerprint. In Table 6, we denote each of the alphas without the pruning technique as alpha_AE_R_2 with an additional letter N. The number of searched alphas is the sum of pruned alphas and evaluated alphas. We observe that, for the baseline methods, the numbers of searched alphas are significantly less than those of AlphaEvolve, leading to ineffective and inefficient alpha mining, which proves the effectiveness of our proposed technique.

6 RELATED WORK

Previous works focus on two classes of alphas: (1) machine learning alphas with vector and matrix operations; (2) formulaic alphas with scalar operations. For the first class, various models are proposed given more efficient deep learning systems to capture high-dimensional features \([25]\). \([25]\) proposes a novel TTTV architecture to model the time-invariant/variant features for the stock index prediction. \([20]\) proposes a two-level attention mechanism to assign importance weights on time steps and stocks. \([24]\) puts up a novel State Frequency Memory (SFM) that decomposes the hidden states of memory cells into multiple frequency components to model different trading activities. \([7]\) uses a neural tensor network to extract event information from news data. \([4]\) encodes time-series images as candlestick (Box and Whisker) charts and models the image feature. \([19]\) creates a simple language of Japanese candlesticks using OHLC data. Domain knowledge injection has proven useful in designing alphas \([1, 12]\); \([10]\) injects relational knowledge through a novel graph network. In general, this class of alphas is designed by data scientists and typically too complex to mine into a weakly correlated set.

The second class of alphas is widely used in hedge funds. This class of alphas is designed by financial domain experts or mined by the genetic algorithm. \([13]\) studies alphas invested in a hedge fund and their correlation standard. \([15]\) reports alphas mined by the genetic algorithm and their performance. \([14]\) further improves the genetic algorithm by using mutual information as the fitness score to mine nonlinear formulaic alphas. \([9]\) proposes an indexing method to locate similar stock movement patterns. \([16]\) proposes N-dimensional inter-transaction rules to predict stock movements. To our best knowledge, our work is the first to mine alphas based on AutoML and first to generate the new class of alphas combining the strengths of the existing classes.

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\(^1\)https://finance.yahoo.com/news/nasdaq-vs-nyse-key-differences-200641822.html
In this paper, we introduce a new class of alphas and then propose a novel alpha mining framework AlphaEvolve based on AutoML. AlphaEvolve generates the new class of alphas which are different from previous classes of formulas and machine learning models. This class has the advantages of simplicity and generalization ability similar to formulaic alphas, and the ability to be trained by data similar to machine learning alphas. These advantages result in better performances in generating weakly correlated high returns. Consequently, AlphaEvolve provides investors with an automatic solution for low-risk investments with high returns.

7 CONCLUSIONS

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