Topological memory in nonlinear driven-dissipative photonic lattices

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We theoretically propose a topological memory in a photonic lattice of nonlinear lossy resonators subjected to a coherent drive, where the system remembers its topological phase. Initially, the system is topologically trivial. After the application of an additional coherent pulse, the intensity is increased, which modifies the couplings in the system and then induce a topological phase transition. However, when the effect of the pulse dies out, the system does not go back to the trivial phase. Instead, it remembers the topological phase and maintains its topology acquired during the pulse application. We further show how the pulse can be used as a switch to trigger amplification of the topological modes. Our work can be useful in triggering the different functionalities of the active topological photonic devices as well as in storing information with topological protection.

The intriguing properties of topological photonics have enabled widespread applications in modern optical devices, such as robust signal transport [1–5], optical delay line [6], quantum interface [7], quantum light source [8], robust splitters [9], topological lasers [10–14], etc. Topological photonics is also promising for optical information processing technologies. For example, valley photonic crystals are identified as an excellent candidate for robust information transfer in next-generation devices [3–5]. Similar to transferring the information, the ability to store them in memory, is an equally important task in information processing. However, optical memories along with topological protection have not been explored till now.

Nonlinearity is at the core of memory devices. Interplay between the nonlinearity and the topology has made way for many novel effects such as topological solitons [15–19], high harmonic generation [20, 21], topological phase transitions [22–26], and others [27–32] (see Ref. [33] for a comprehensive review). However, none of the previous works can show the memory feature: once the key ingredient, that induces the functionalities, is removed from the scheme, the systems can no longer continue to exhibit such effects.

In this work, we introduce for the first time a topological memory in a lattice of lossy resonators having local onsite Kerr-nonlinearity, where the system remembers its topological phase. The lossy nature of the system leads to a steady-state in the presence of a coherent drive $F$. However, due to the nonlinearity, our system subjected to a properly designed $F$ shows not only one but two steady states: low and high-intensity states. Our system is topologically trivial and after $F$ is introduced, it attains the low-intensity steady-state. The introduction of an additional coherent pulse increases the intensity of the system. At higher intensities, the nonlinear interaction modifies the couplings and the otherwise-trivial system becomes topological. However, at longer times when the effect of the pulse dies, the system does not go back to its previous trivial phase. Instead, it remembers the topological phase and maintains its topology acquired during the pulse application. As an application of the topological memory, we show a unique amplification phenomenon, where the amplification is triggered by a pulse.

We start by considering a nonlinear optical resonator subjected to a coherent drive $F$ (see Fig. 1(a)), which is represented by the following nonlinear schrödinger equation (NLSE)

$$i \frac{\partial \psi}{\partial t} = (\omega_0 - i \Gamma) \psi + |\psi|^2 \psi + F \exp(-i \omega_p t).$$

FIG. 1. (a) Schematic of a coherently driven nonlinear resonator. (b) Analytically and numerically calculated bistable curves in green and red, respectively. Parameters: $\Delta = -3$, $\Gamma = 1$.

Here $\omega_0$ is the onsite potential and $\Gamma$ is the linear decay. The next term represents the defocusing Kerr nonlinearity, where the nonlinear coefficient is set to 1. $F$ is a coherent drive having frequency $\omega_p$. The time dependence in the driving term can be removed by moving to the rotating frame by transforming $\psi \to e^{-i \omega_p t} \psi$. For the steady-state $\psi_s$, where $\frac{\partial \psi_s}{\partial t} = 0$, one can obtain

$$|F|^2 = \left[ (\Delta + |\psi_s|^2)^2 + \Gamma^2 \right] |\psi_s|^2,$$

where $\Delta = (\omega_0 - \omega_p)$. The above expression captures the nonlinear nature of the system, where the dependence of

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the output intensity $|\psi_1|^2$ on the input drive $|F|^2$ is a third-degree polynomial, suggesting under suitable conditions for each value of $|F|^2$, there exist three different values of $|\psi_1|^2$. In Fig. 1(b), the green curve shows that within the gray region for each $|F|^2$ indeed three possible $|\psi_1|^2$ exist. However, in practice, the middle branch is not stable. This can be confirmed by numerically solving Eq. (1) with the $F$ first slowly increasing and then decreasing with time [34]. The red dashed curve obtained numerically follows the analytical green curve, however, the middle branch does not appear. The system shows bistability by allowing both the low and high-intensity stable states for a fixed value of $F$ within the gray region.

An important characteristic of bistability is their ability to mimic the memory: the state of the system is not only determined by the current factors (such as $F$) but also by its previous state. For example, let us consider a system that is initially in the low-intensity state “$A$” as shown in Fig. 1(b). An additional coherent drive $\Delta F$ is added such that the system moves to a high-intensity state “$B$”. Now if $\Delta F$ is removed, the system does not return back to its original state “$A$”, instead, it chooses the high-intensity state “$C$”. While determining the final state, the system memorizes the information (high-intensity) about the intermediate state “$B$” and in the case where “$B$” is a low-intensity state, the system would return to the original state “$A$” upon removing $\Delta F$.

Next, we arrange the nonlinear resonators in a 2D lattice. In order to be close with experiments, we model the dynamics of the system using the NLSE in the continuum limit (where the space is taken as continuous). Without loss of generality, we work with the dimensionless NLSE, which is expressed as

$$i \frac{\partial \psi(x,y)}{\partial t} = \left[ -\nabla^2 + V(x,y) - i\Gamma \right] \psi(x,y) + |\psi(x,y)|^2 \psi(x,y) + F(x,y)e^{-i\omega_p t} + F_p(x,y) \exp \left[ -\frac{(t-t_0)^2}{2\tau^2} \right] e^{-i\omega_p t}. \quad (3)$$

Here $\nabla^2 \equiv (\partial^2/\partial x^2 + \partial^2/\partial y^2)$ is the transverse Laplacian operator, $V$ is the external potential profile corresponding to the resonators, $\Gamma$ is the linear decay, $F$ is a position-dependent coherent pump having frequency $\omega_p$, and $F_p$ is a coherent pulse having duration $\tau$ centered at time $t_0$. Next, we consider circular resonators having diameter $d_m$, which we call main resonators. Two main resonators separated by $L$ are coupled via an auxiliary larger resonator having diameter $d_a$, where $d_a > d_m$ (see Fig. 2(a)). The potential is taken as $V = 0$ inside and $V = V_0 > 0$ outside the resonators.

To capture the role of nonlinearity, we first consider two main resonators connected by an auxiliary resonator as shown in Fig. 2(a). The ground state wave function $\psi_0$ is mainly localized at the auxiliary resonator (see Fig. 2(b)), whereas the first ($\psi_1$) and second ($\psi_2$) excited states are localized at the main resonators (see Figs. 2(c-d)). The coupling strength $J$ between the main resonators can be estimated from the difference in the eigenvalues of the symmetric ($E_2$) and asymmetric ($E_1$) eigenstates, where $J = (E_2 - E_1)/2$. The important feature that plays a key role and signifies the nonlinear effect in this work is the ability to control the coupling between the main resonators by changing the intensity of the auxiliary resonator. This is captured by choosing an effective potential $V_{eff} = V + g|\psi_0|^2$, where $g$ corresponds to the peak value of the ground state intensity, and obtaining the coupling in the nonlinear regime $J_{NL}$ in a self-consistent way. Fig. 2(e) shows the enhancement of $J_{NL}$ compared to $J$ as a function of $g$.

Now that we have all the ingredients, we proceed to study the topological phase in a 2D square lattice formed by the above-mentioned resonators, where between any two main resonators there is an auxiliary resonator. Recalling that the intensity of the auxiliary resonators enhances the coupling between the main resonators, we choose the coherent pump profile in such a way that the inter-cells are coupled strongly similar to the 2D Su-Schrieffer–Heeger (SSH) model [35]. The spatial profile of the coherent pump is expressed as

$$F(x,y) = P_0 \sum_{X_n,Y_n} \exp \left[ -\frac{(x-X_n)^2 + (y-Y_n)^2}{2\sigma^2} \right], \quad (4)$$

where $P_0$ is the strength of the pump and $\{X_n,Y_n\}$ are the coordinates of the center of the pumped auxiliary resonators as shown in Fig. 3(a). Such pump profiles are readily achieved in practice using the spatial light modulators [36, 37]. We choose the value of $P_0$ such that the auxiliary resonators subjected to the pump are bistable [34]. To show the bistable behaviour of the whole system we solve Eq. (3) without the pulse ($F_p = 0$) and take zeros as the initial condition. While plotting the spatial profiles, we keep the background potential to distinguish the intensities between the main and auxiliary resonators.
resonators. Fig. 3(b) shows the steady-state of the system before the application of the pulse, where intensity is mainly localized at the pumped auxiliary resonators. The coupling between the resonators results in slightly non-identical bistability curves of the resonators placed at different positions [34]. Due to this, the intensity among the pumped resonators varies a little, but they remain in the low-intensity state, where \(|\psi|^2\) is negligible.

Next, we apply a gaussian-shaped coherent pulse \(F_p\) centered at the bulk as shown by the green circle in Fig. 3(b). The addition and removal of the additional pump \(\Delta F\) in Fig. 1(b) is played by the pulse \(F_p\) here. In Fig. 3(c) the steady-state of the system after the application of the pulse is shown. The system indeed remembers the high intensity created by the pulse and once the effect of the pulse dies out the system chooses to stay at the high-intensity state. Compared to the low-intensity state in Fig. 3(b), a much larger intensity outside the pumped auxiliary resonators exists, which signifies the enhancement of the coupling due to significant \(|\psi|^2\). In Fig. 3(d) the total intensity of the system, \(I(t) = \int \psi(x,y,t) \, dx \, dy\), where the integration is over the whole system, is shown as a function of time, which shows the bistable behaviour of the system. The full dynamics of the system before and after the application of the pulse are shown in movie1.

Having established the memory effect in the 2D lattice, here we show the topology associated with it. In Fig. 4(a) the eigenfrequencies of the linear system are shown, which can be found by putting the nonlinear and pumping terms to zero in Eq. (3) and diagonalizing its corresponding Hamiltonian. The lower band (shown in gray) has the main contributions from to the auxiliary resonators, whereas the upper band has the main contributions from the main resonators. For the rest of the work, we shall focus on the main resonator band, which is topologically trivial and gapless in the linear regime. To include the nonlinear effect, we study the Bogoliubov fluctuations on top of the steady-state [38]:

\[
\psi(x,y) = \psi_s(x,y) + u_n(x,y)e^{-i\omega_n t} + v^*_n(x,y)e^{i\omega_n t}. \tag{5}
\]

Here \(\psi_s\) represent the low and high-intensity steady states shown in Figs. 3(b-c), respectively. \(u_n\) and \(v_n\) represents the fluctuations having frequency \(\omega_n\). Substituting Eq. (5) into Eq. (3) and by ignoring the higher-order terms in \(u_n\) and \(v_n\), we obtain the following eigenvalue equation

\[
\begin{pmatrix}
H_0 + 2|\psi_s|^2 & \psi_s^2 \\
-\psi_s^2 & -H_0 - 2|\psi_s|^2
\end{pmatrix}
\begin{pmatrix}
u_n \\
v_n
\end{pmatrix} = \omega_n
\begin{pmatrix}
u_n \\
v_n
\end{pmatrix}, \tag{6}
\]

where \(H_0 = [-\nabla^2 + V(x,y) - i\Gamma - \omega_p]\), which is rescaled with respect to the pump frequency \(\omega_p\).

The fluctuation Hamiltonian has particle-hole symmetry, which makes the eigenfrequencies appear in pairs \((\omega_n, -\omega_n^*)\). The Hilbert space of the fluctuation Hamiltonian is double in size compared to the linear one. Consequently, for better visualization we show the eigenfrequencies near the main resonator band and for \(\Re(\omega_n) > 0\). Fig. 4(b) shows the eigenfrequencies of the fluctuations before the pulse is applied. Similar to the linear case, the band is gapless. This is understandable as for the low-intensity state, the nonlinearity is not significant enough to induce the topological transition. After the application of the pulse positioned at the bulk, the system switches to the high-intensity state. In this case, the nonlinear effect becomes significant and the coupling between the main resonators connected by a pumped auxiliary resonator increases. The system goes through a topological phase transition from trivial to second-order topological phase, where a bulk band gap opens and four topological corner modes appear. An effective tight-binding model based on the strong coupling induced by the pump can reproduce the topological corner modes [34]. The topological corner modes are protected by the bulk polarization, which corresponds to the non-zero value of the Wannier center. The Wannier center for the fluctuations lies at (0.5,0.5), which is topologically nontrivial [34].

In Fig. 4(c) the eigenfrequencies of the fluctuation after the application of the pulse are shown, where the topological corner modes are marked in red and the edge modes are shown in green. Figs. 4(d-e) show the spatial...
FIG. 4. (a-c) Real eigenfrequencies for different cases. In (a) the auxiliary resonator band is shown in gray and the main resonator band is shown in black. Red and green dots in (c) correspond to the corner and edge modes, respectively. The blue line in (a) represents $\omega_p$, with respect to which (b-c) are rescaled. (d-e) Spatial profiles of an edge and a corner mode, respectively. All the parameters are kept the same as those in Fig. 3.

FIG. 5. (a-c) Imaginary versus real eigenfrequencies for different cases. Red and green in (c) correspond to the topological corner and edge modes, respectively. Steady states from Figs. 3(b-c) are used for obtaining (b-c). (d-e) Steady states of the system with gain at the corners before and after the pulse is applied, respectively. The green circle in (d) shows the position and width of the pulse. Parameters: Peak value of the gain $G_0 = 3\Gamma$. All other parameters are kept the same as those in the Fig. 3.

profiles of one of the topological edge modes ($n = 146$) and topological corner modes ($n = 142$), respectively. In experiment, the topological modes will be hidden in the high-intensity steady state. However, they can be probed using a weak additional coherent pump followed by the frequency filtration to subtract the steady-state.

As an application, we use topological memory to control the functionality of an active topological photonic device. We introduce gain at the main four corner resonators of the 2D lattice. The gain is modeled by adding a term $+iG(x, y)\psi(x, y)$ at the right-hand side of Eq. (3), where $G(x, y)$ is composed of four Gaussians centered at the four corners having width $\sigma$ and peak value $G_0$. In this case, $H_0$ in Eq. (6) is updated to $H_0 \rightarrow H_0 + iG(x, y)$. To signify the role of nonlinearity, it is important that the gain alone can not induce lasing in the linear regime. Consequently, we obtain the imaginary versus the real eigenfrequencies for the linear system. The system stays below the lasing threshold $\text{Im} \omega_n < 0$ (see Fig. 5(a)). Next, we take the steady states corresponding to Figs. 3 (b-c) and obtain the same plot for the fluctuations before and after the pulse is applied. Similar to the linear case, the modes have $\text{Im} \omega_n < 0$ before the application of the pulse (see Fig. 5(b)). However, after the pulse is applied topological corner modes appear and due to the significant overlap with the gain, they have $\text{Im} \omega_n > 0$ indicating instability (see Fig. 5(c)). This has a significant effect on the steady-states of the system shown in Figs. 5(d-e). As predicted from the complex eigenfrequencies, the gain at the four corners does not alter the steady-state before the pulse, which is the same as the one obtained without the gain in Fig. 3(b). However, after the pulse is applied, a large intensity at the corners is observed along with the high-intensity steady state at the bulk (see movie2). By moving to the frequency domain it is possible to show that the amplified states are at a different frequency than $\omega_p$, implying frequency conversion and that they indeed have the profiles of the corner modes [34]. This amplification, which is triggered by a pulse, is different from all the previous cases. For example, in lasers if the pumping term (or any other ingredient), which induces amplification (lasing) is removed, understandably the amplification would stop. Due to the memory effect, the functionality in our scheme remains, although the pulse, which triggers the amplification disappears.

The NLSE in Eq. (3) is generally used to describe the topological physics in photonic waveguide arrays [17, 18]. The Bogoliubov fluctuations in Eq. (6) can be arranged using the parametric down-conversion [39–42]. Alter-
natively, the system of exciton-polaritons, where cavity photons exhibit Kerr-nonlinearity by coupling strongly with the quantum well excitons, is also a promising platform for realizing our scheme. Bistability is well-established for exciton-polaritons [43, 44] and under the mean-field approximation, they can be described by the NLSE (also known as the Gross-Pitaevskii equation) [45]. Bogoliubov fluctuations naturally arise in polariton system [19, 46]. By choosing the proper physical units, our present parameters can be related to the exciton-polariton lattices [34].

To conclude, we have presented a new concept, where the topological phase can be induced as a memory. In particular, we show that a nonlinear system of photonic lossy resonators goes through a topological phase transition under the application of a coherent pulse. The system continues to maintain its topology, although the effect of the pulse disappears. This scheme is independent of the dimension and similar effects can be found in 1D lattices [34]. Our scheme can be useful for storing information as well as in triggering different functionalities of active topological photonic devices and making them effectively self-sustainable.

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