Vibration Mode Induced Shapiro Steps and Backaction in Josephson Junctions

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A model of a superconducting tunnel junction coupled to a mechanical oscillator is studied at zero temperature in the case of linear coupling between the oscillator and tunneling electrons. It is found that the Josephson current flowing between two superconductors is modulated by the motion of the oscillator. Coupling to harmonic oscillator produces additional Shapiro steps in $I-V$ characteristic of Josephson junction whose position is tuned by the frequency of the vibration mode. We also find a new velocity dependent term originating from the backaction of the ac Josephson current. This term is periodic in time and vanishes at zero bias voltage.

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The coupling of the charge carriers to vibrational modes and localized spins in electronic devices has been a subject of intense investigation recently. Vibrational modes and spins possess dynamic internal degrees of freedom, much unlike static impurities or defects. As a consequence, they affect the electronic dynamics in these devices. Interesting $I$-$V$ characteristics (i.e., peaks in differential tunneling conductance) in molecular electronics $[1, 2, 3, 4, 5, 6]$ may indicate strong influence from the electronic-vibrational coupling. A step structure (rather than peak structure) in the differential tunneling conductance has also been observed in the STM-based inelastic tunneling spectroscopy around a local vibrational mode on surfaces $[7]$. The vibrational effects on the conductance of molecular quantum dots were also examined $[8, 9, 10, 11, 12, 13]$. Spin detection and manipulation is crucial in spintronics and quantum information processing. The modulation of the current by precessing spins may be used for electrons in the left and right superconducting leads. The motion of the oscillator modulates the electron tunneling between the superconductors. The chemical potentials on both sides differ by the applied voltage, $\mu_L - \mu_R = eV$.

These additional peaks are the result of coupling of ac Josephson current at $\omega_J$ and oscillation at $\omega_0$. (ii) Electron tunneling through the junction leads to a novel time dependent change in oscillator energy. If no voltage bias is applied across the junction (i.e., dc Josephson effect in equilibrium), the oscillator energy is time independent. When a nonzero voltage bias applied, the oscillator will display time-dependent energy variations.

Our model is illustrated in Fig. 1. It consists of two ideal superconducting leads coupled to each other by a tunneling barrier, which contains a mechanical oscillator. The Hamiltonian

$$H = H_L + H_R + H_T. \quad (1)$$

The first two terms are, respectively, the Hamiltonians for electrons in the left and right superconducting leads of the tunnel junction,

$$H_{L(R)} = \sum_{k(p), \sigma} c_{k(p),\sigma}^\dagger c_{k(p),\sigma} \epsilon_{k(p),\sigma} + \sum_{k(p)} |\Delta_{L(R)}| c_{k(p)}^\dagger c_{-k(-p)} + H.c., \quad (2)$$

where we denote the electron creation (annihilation) operators in the left,right (L,R) leads by $c_{k(p),\sigma}^\dagger$ ($c_{k(p),\sigma}$) and $c_{\sigma}^\dagger$ ($c_\sigma$) respectively. $k$ ($p$) are momenta and $\sigma$ is the spin index, $\epsilon_{k(p),\sigma}$ and $\Delta_{L(R)}$ are, respectively, the single par-
ticle energies of conduction electrons, and the pair potential (the gap function) in the leads. Without loss of generality, we assume that the superconductors are of a spin-singlet s-wave pairing symmetry, and consider the Josephson tunneling at zero temperature. The third term in Eq. (1) depicts the tunneling between the superconductors:

\[ H_T = \sum_{k,p,\sigma} [T_{kp} c_{k\sigma}^\dagger c_{p\sigma} + \text{H.c.}] , \]

where the tunneling matrix elements \( T_{kp} \) transfer electrons through an insulating barrier. When a local vibrational mode is embedded into the tunneling barrier then, in the linear coupling regime,

\[ T_{kp} = T_{kp}^{(0)} [1 + \alpha u], \]

where \( \alpha \) describes the coupling between the tunneling electrons and vibrational mode. The quantity \( u \) is the displacement operator for the oscillator.

Henceforth, we denote by \( M \) and \( K \) the mass and spring constant of the mechanical oscillator. As the energy associated with the vibrational mode, \( \omega_m = \sqrt{K/M} \sim 10^{-1} - 10^{-6} \text{ eV} \) is much smaller than the typical electronic energy on the order of \( 1 \text{ eV} \), the mechanical oscillation is very slow as compared to the time scale of electronic processes. This allows us to apply the Born-Oppenheimer approximation to treat the electronic degrees of freedom as if the local oscillator is static at every instantaneous location. We will treat the dynamics of the mechanical oscillator including the back action from the tunneling electrons.

Whenever a voltage bias is applied across the junction, the Josephson current

\[ I_J(t) = e \int_{-\infty}^{t} d\tau [e^{i e V (t + \tau)} \langle [A(t), A(t')]_\pm \rangle - e^{-i e V (t + \tau)} \langle [A(t), A(t')]_\pm \rangle], \]

where the operator \( A(t) = \sum_{k,p,\sigma} T_{kp} c_{k\sigma}^\dagger (t) c_{p\sigma}(t) \). Here \( c_{k(p)\sigma}(t) = e^{i K_{L(R)} t} c_{k(p)\sigma} e^{-i K_{L(R)} t} \) with \( K_{L(R)} = H_{L(R)} - \mu_L(R) N_{L(R)} \) and \( N_{L(R)} = \sum_{k(p),\sigma} c_{k(p)\sigma} c_{k(p)\sigma} \). The unequal chemical potentials of the two superconductors lead to a voltage bias \( \mu_L - \mu_R = eV \). Hereafter, we set \( \hbar = 1 \). A little algebra yields:

\[ I_J(t) = J_S^{(0)}(eV)(1 + \alpha u)^2 \sin(\omega_m t) + \alpha \Gamma_S(eV)(1 + \alpha u) \frac{\partial u}{\partial t} \cos(\omega_m t). \]

Here we assume that the two superconductors are identical and set the constant phase difference between them \( \phi_0 = 0 \). The Josephson frequency is given by \( \omega_J = 2 eV \). The quantity \( J_S^{(0)} \) is the amplitude of the Josephson current in the absence of coupling to the vibrational mode (that is, \( \alpha = 0 \)), which is found to be,

\[ J_S^{(0)}(eV) = e \sum_{k,p} \frac{|\Delta|^2 T_{kp}^{(0)}|^2}{E_k E_p} \left( \frac{1}{eV + E_k + E_p} - \frac{1}{eV - E_k - E_p} \right), \]

with \( |\Delta| \) is the superconducting energy gap and \( E_{k(p)} = \sqrt{(\epsilon_{k(p)} - E_F)^2 + |\Delta|^2} \). The amplitude

\[ \Gamma_S(eV) = e \sum_{k,p} \frac{|\Delta|^2 T_{kp}^{(0)}|^2}{E_k E_p} \left[ \frac{1}{(E_k + E_p - eV)^2} - \frac{1}{(E_k + E_p + eV)^2} \right]. \]

An order of magnitude estimate gives \( \Gamma_S \omega_J/J_S^{(0)} \sim (eV/|\Delta|)^2 \ll 1 \).

From Eq. (1), we may construct the coupling modulated part of the Josephson junction energy

\[ H_J = E_J(1 + \alpha u)^2 [1 - \cos(\omega_m t)] + \frac{\Gamma_S}{2e} (1 + \alpha u) \frac{\partial u}{\partial t} \sin(\omega_m t), \]

where \( E_J = J_S^{(0)}/2e \). The derivative of \( H_J \) with respect to the phase yields the supercurrent in Eq. (2). Eq. (3) captures the Josephson backaction effect - it vividly illustrates how the electronic degrees of freedom influence the mechanical oscillator of Fig.1 coupled to them. We may obtain this result by also employing other standard techniques. For instance, integrating out the fermionic degrees of freedom, we obtain an effective action \( S_{eff} \) for \( u(\tau) \) in Matsubara time. When the standard mechanical part of the oscillator is added, the action

\[ S = \frac{1}{2} \int d\tau \int d\tau' |T_{kp}^{(0)}|^2 (1 + \alpha u(\tau))(1 + \alpha u(\tau')) K(\tau, \tau') + \frac{1}{2} \int d\tau [K u''^2 + M (\frac{du}{d\tau})^2]. \]

The non-locality in time present in the first term reflects how electronic correlations between different times influence the oscillator coordinates and lead to the Josephson backaction effect. Here, \( K(\tau, \tau') = F_L(\tau - \tau') F_R(\tau' - \tau) \exp[eV(\tau + \tau')] \), with \( F \) the Matsubara-Gorkov function. As the resulting classical action is not time translationally invariant, energy is not conserved. Within the Born-Oppenheimer approximation \( u(\tau') \approx u(\tau) \) \( \tau' - \tau \partial u/\partial \tau \), the equation of motion

\[ 0 = \frac{\delta S}{\delta u(\tau)} = M \frac{d^2 u}{d\tau^2} + Ku + \alpha |T_{kp}^{(0)}|^2 \int d\tau' K(\tau, \tau') [1 + \alpha u(\tau) + \alpha \frac{\partial u}{\partial \tau'}(\tau' - \tau)] \]

(11)
leading to
\[ M \frac{d^2 u}{dt^2} + \gamma_S(t) \frac{du}{dt} + Ku = F(t). \]  

(12)

Here, the driving force
\[ F(t) = -2\alpha E_J (1 + \alpha u) \left\{ 1 - [1 + \Gamma_S \omega_J/4eE_J] \cos(\omega_J t) \right\}, \]

(13)

with \( \Gamma_S \omega_J/4eE_J \approx (eV/|\Delta|)^2 \ll 1 \), and the time-dependent energy non-conserving
\[ \gamma_S(t) = -(\alpha^2 \Gamma_S/e) \sin(\omega_J t). \]

(14)

These relations are also very transparent within the real time approach of Eq. (9), where the total oscillator Hamiltonian \( H_{osc} = H_J + P^2/2M + Ku^2/2 \). A more detailed Keldysh contour analysis yields the same results [24]. In Eq. (13), the first, linear in \( u \) term, may, alternatively, be lumped into the spring moduli \( K \) and regarded as a Josephson stiffness- a shift of the spring constant resulting from electronic correlations. The oscillatory part of the driving force and the time dependence of the Josephson backaction generated \( \gamma_S(t) \) (Eq. (14)) lead to interesting experimental consequences which we will now elaborate on. Equations (9), (12), (13), and (14) constitute the central result of this work.

Josephson Backaction. As shown by Eq. (14) and Eq. (2), the velocity coefficient (\( \gamma_S(t) \)) originates from the coupling of the mechanical oscillator to the tunneling electrons. It is quadratically proportional to the coupling constant \( \alpha \), which is similar to the case of normal metal tunnel junctions [24]. This term has two novel features: (i) \( \gamma_S \) depends on the voltage bias. At zero voltage bias (the dc setting), \( \gamma_S \) vanishes since \( \Gamma_S \) is zero. \( \gamma_S \) is finite only when a finite voltage bias is applied across the junction (i.e., ac case). In the low voltage limit (\( eV \ll |\Delta| \)), \( \gamma_S \) is linearly proportional to the voltage bias. (ii) Once a finite voltage bias is applied, \( \gamma_S \) is also a periodic function of time with the Josephson frequency \( \omega_J \). Both of these features are absent in the normal metal tunnel junctions [24]. These properties are unique to the coupling of the mechanical oscillator to the superconductors. In the normal metal, there is no quasiparticle energy gap on the Fermi surface. Matters become far richer when the oscillator is coupled to the superconductor. On the one hand, in the superconductor, there exists an energy gap on the Fermi surface and the quasiparticles are depleted below this energy. This leads to the quenching of the single particle tunneling channel; no contribution to the dissipation of the oscillator due to normal quasiparticles is possible. On the other hand, due to macroscopic quantum coherence in the superconductor, Cooper pairs can tunnel through the barrier between the two superconductors with a probability comparable to that of single particle tunneling in a normal metal junction. When a static voltage bias is applied, the tunneling of a single Cooper pair requires an energy \( 2eV \) to overcome the potential barrier. The energy carried by Cooper pair upon tunneling can be transferred to oscillator. For zero bias voltage, the tunneling pairs do not acquire/lose energy (the effective electronic only action is time independent, no effective external sources are present, and energy is preserved at all times). In the ac setting, \( \sin \omega_J t \) is odd under time reversal and, as a consequence, the term \( \sin \omega_J t \frac{d}{dt} \) is allowed in the effective Hamiltonian Eq. (9).

Shapiro steps. To calculate the Josephson current, we need to solve Eq. (12) for the displacement field \( u(t) \). In the weak coupling limit and in view of the fact that \( \Gamma_S \) is much smaller than \( J_S^{(0)} \), the main physics can be captured by neglecting the damping terms and the \( \alpha i \Gamma_S \) and \( \alpha^2 \) terms in the driving force. In this limit,
\[ u(t) = u_0 \cos(\omega_0 t) + \frac{2\alpha E_J}{M(\omega_0^2 - \omega_J^2)} \cos(\omega_J t) - \frac{2\alpha E_J}{K}, \]

(15)

and the Josephson current
\[ I_J(t) = J_S^{(0)} \left\{ (1 - \frac{4\tilde{\alpha}^2}{K}) \sin(\omega_J t) + 2\tilde{\alpha} \cos(\omega_0 t) \sin(\omega_J t) \right\} \]

+ \frac{2\tilde{\alpha}^2 \sin^2(2\omega_J t)}{K(1 - \omega_J^2/\omega_0^2)} + \tilde{\alpha}^2 \cos^2(\omega_J t) \sin(\omega_J t), \]

(16)

where \( \tilde{\alpha} = \alpha \omega_0 \) and \( K = K \omega_0^2/E_J \). Equation (16) demonstrates clearly that the Josephson current not only oscillates with time with a frequency \( \omega_J \), but is also modulated by the vibrational mode of the mechanical oscillator with a frequency \( \omega_0 \). In Fig. we plot the absolute mag-
plitude of the Fourier transform of the Josephson current given by Eq. \(\omega_0\) for various values of the vibration mode frequency \(\omega_j\). The spectrum shows a main peak at the frequency \(\omega_j\). In addition, the coupling of the mechanical oscillator and tunneling electrons generates new side peaks at frequencies \(\omega_j \pm \omega_0, \omega_j \pm 2\omega_0\), and \(2\omega_j\). The intensity of these peaks is proportional to the coupling constant. Note that the peak at \(2\omega_j\) originates from the second term in \(u(t)\) given by Eq. \(\omega_0\), which is a direct manifestation of the feedback effect from the Josephson tunneling. Our calculation implies that a dc component arises if the voltage bias has one of the Shapiro step values \(\omega_0\) and \(2\omega_0\). When higher-order effects are taken into account, the equation-of-motion for the oscillator can only be solved numerically. The main conclusions presented here remain qualitatively unchanged.

**Conclusion.** We studied the Josephson junction coupled to a mechanical oscillator between its two superconducting leads. We found that the Josephson current flowing between two spin-singlet pairing superconductors is modulated by the motion of the oscillator. The coupling of an oscillator of eigenfrequency \(\omega_0\) to an ac junction of characteristic frequency \(\omega_j\) leads to beats. We find novel Shapiro steps induced at \(\omega_j \pm \omega_0, \omega_j \pm 2\omega_0\), and \(2\omega_j\). This differs from the case of a precessing spin \(\omega_j\), where due to the sum rule of tunneling through different spin channels, the Josephson current is not modulated if the spin-orbit coupling mechanism does not exist. The electrons tunneling through the superconducting junction lead to a novel non-energy conserving effect. If a voltage bias is applied, this time dependent effect arises from the back-action of the supercurrent on the oscillator dynamics. As far as we know, no measurements of Josephson current through a vibrational mode between two superconductors have been reported yet. Recent progress in molecular electronics \(\omega_j\) and nanomechanical resonators \(\omega_0\) holds great promise in attaching single molecules to superconducting leads, or even tune the tunnel barrier of the superconducting junctions by a mechanical cantilever. Our predictions are, potentially, within the realm of present technology. Another possible experiment concerns atomically sharp superconducting tip in low temperature STM in both the quasiparticle tunneling \(\omega_j\) and Josephson tunneling regimes \(\omega_0\) (coined “Josephson STM” or JSTM \(\omega_0\)) on conventional superconductors. It is very interesting to extend the JSTM technology by using a superconducting tip to study the Josephson current in the vicinity of a local vibrational mode on the superconducting surface, which may provide a new detection technique.

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