Optimal Power Control over Fading Cognitive Radio Channels by Exploiting Primary User CSI

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Abstract

This paper is concerned with spectrum sharing cognitive radio networks, where a secondary user (SU) or cognitive radio link communicates simultaneously over the same frequency band with an existing primary user (PU) link. It is assumed that the SU transmitter has the perfect channel state information (CSI) on the fading channels from SU transmitter to both PU and SU receivers (as usually assumed in the literature), as well as the fading channel from PU transmitter to PU receiver (a new assumption). With the additional PU CSI, we study the optimal power control for the SU over different fading states to maximize the SU ergodic capacity subject to a new proposed constraint to protect the PU transmission, which limits the maximum ergodic capacity loss of the PU resulted from the SU transmission. It is shown that the proposed SU power-control policy is superior over the conventional policy under the constraint on the maximum tolerable interference power/interference temperature at the PU receiver, in terms of the achievable ergodic capacities of both PU and SU.

Index Terms

Cognitive radio, ergodic capacity, fading channel, power control, spectrum sharing.

I. INTRODUCTION

In this paper, we are concerned with the newly emerging cognitive radio (CR) type of wireless communication networks, where the secondary users (SUs) or the so-called CRs communicate over the same frequency band that has been allocated to the existing primary users (PUs). For such scenarios, the SUs usually need to deal with a fundamental tradeoff between maximizing the secondary network throughput and minimizing the performance loss of the primary network resulted from the SU transmissions. One commonly known technique used by the SUs to protect the PUs is opportunistic spectrum access (OSA) [1], whereby the SUs decide to transmit over the channel of interest only if the PU transmissions are detected to be off. Many algorithms have been reported in the literature for detecting the PU transmission status, in general known as spectrum sensing (see, e.g., [2]-[4] and references therein). In contrast to OSA, another general operation model of CRs is known as spectrum sharing (SS) [5], where the SUs are allowed to transmit simultaneously with the PUs provided that the interferences from the SUs to PUs
will cause the resultant PU performance loss to be within an acceptable level. Under the SS paradigm, the “cognitive relay” idea has been proposed in [6], [7], where the SU transmitter is assumed to know \textit{a priori} the PU’s messages and is thus able to compensate for the interferences to the PU receiver resulted from the SU transmission by operating as an assisting relay to the PU transmission. As an alternative method for SS, the SU can protect the PU transmission by regulating the SU to PU interference power level to be below a predefined threshold, known as the \textit{interference temperature} (IT), [8], [9]. This method is perhaps more practical than the cognitive relay method since only the SU to PU channel gain knowledge is required to be known to the SU transmitter.

In this paper, we focus our study on the SS (as opposed to OSA) -based CR networks. It is known that in wireless networks, \textit{dynamic resource allocation} (DRA) whereby the transmit powers, bit-rates, bandwidths and/or antenna beams of users are dynamically allocated based upon the channel state information (CSI) is essential to the achievable network throughput. For the case of single-antenna PU and SU channels, adaptive power control for the SU is an effective means of DRA, and has been studied in, e.g., [10], [11], to maximize the SU’s transmission rate subject to the constraint on the maximum tolerable average IT level at the PU receiver. In [12], the authors proposed both optimal and suboptimal spatial adaptation schemes for the SUs equipped with multiple antennas under the given IT constraints at different PU receivers. DRA in the multiuser CR networks based on the principle of IT has also been studied in, e.g., [13], [14]. It is thus known that most existing works in the literature on DRA for the SUs are based on the IT idea. Although IT is a practical method to protect the PU transmission, its optimality on the achievable performance tradeoff for the SU has not yet been carefully addressed in the literature, to the author’s best knowledge.

In this paper, we consider a simplified fading CR network where only one pair of SU and PU is present and all the terminals involved are equipped with a single antenna. We assume that not only the CSI on the SU fading channel and that from the SU transmitter to PU receiver is known to the SU (as usually assumed in the literature), but is also the CSI on the PU fading channel (a new assumption made in this paper). In practice, CSI on the SU’s own channel can be obtained via the classical channel training and feedback methods, while CSI from the SU transmitter to PU receiver can be obtained by the SU transmitter via estimating the reversed channel from PU receiver under the assumption of channel
reciprocity. The more challenging task is perhaps on obtaining the CSI on the PU fading channel by the SU, for which more sophisticated techniques are required, e.g., via eavesdropping the feedback from the PU receiver to PU transmitter [15], or via a cooperative secondary node located in the vicinity of the PU receiver [16]. With the additional PU CSI, the SU is able to transmit with large powers when the PU fading channel is in inferior conditions such as deep fading, since under such circumstances the resultant PU performance loss is negligible, nearly independent of the exact interference level at the PU receiver. In contrast, for the case where the IT constraint is applied, the SU’s transmit power is determined by the channel gain from the SU transmitter to PU receiver, while it is independent of the PU CSI. Motivated by these observations, this paper shows that there in fact exists a better means to protect the PU transmission as well as to maximize the SU transmission rate than the conventional IT constraint or more specifically, the average interference power constraint (AIPC) over the fading states at the PU receiver. This new proposed method ensures that the maximum ergodic capacity loss of the PU resulted from the SU transmission is no greater than some prescribed threshold, thus named as primary capacity loss constraint (PCLC). Clearly, the PCLC is more directly related to the PU transmission than the AIPC. In this paper, we will formally study the optimal power-control policy for the SU under the new proposed PCLC, and show its performance gains over the AIPC in terms of improved ergodic capacities of both PU and SU.

The rest of this paper is organized as follows. Section II presents the system model. Section III presents the conventional power-control policy for the SU based on the AIPC. Section IV introduces the new PCLC, and derives the associated optimal power-control policy for the SU. Section V provides numerical examples to evaluate the achievable rates of the proposed scheme. Finally, Section VI gives the concluding remarks.

Notation: $|\cdot|$ denotes the Euclidean norm of complex number. $\mathbb{E}[\cdot]$ denotes the statistical expectation. The distribution of a circularly symmetric complex Gaussian (CSCG) random variable (RV) with mean $x$ and variance $y$ is denoted by $\mathcal{CN}(x, y)$, and $\sim$ means “distributed as”. $(\cdot)^+ = \max(0, \cdot)$. The notations $|\cdot|$, $(\cdot)^T$, and $(\cdot)^H$ denote the matrix determinant, transpose, and conjugate transpose, respectively. $I$ denotes an identity matrix. $\|\cdot\|$ denotes the Euclidean norm of complex vector.
II. SYSTEM MODEL

As shown in Fig. 1, we consider a SS-based CR network where a SU link consisting of a SU transmitter (SU-Tx) and a SU receiver (SU-Rx) transmits simultaneously over the same narrow band with a PU link consisting of a PU transmitter (PU-Tx) and a PU receiver (PU-Rx). The fading channel complex gains from SU-Tx to SU-Rx and PU-Rx are represented by $\tilde{e}$ and $\tilde{g}$, respectively, and from PU-Tx to PU-Rx and PU-Rx by $\tilde{f}$ and $\tilde{o}$, respectively. We assume a block-fading (BF) channel model and denote $i$ as the joint fading state for all the channels involved. Furthermore, we assume coherent communications for both the PU and SU links and thus only the fading channel power gains (amplitude squares) are of interest. Let $e_i$ denote the power gain of $\tilde{e}$ at fading state $i$, i.e., $e_i = |\tilde{e}(i)|^2$; similarly, $g_i$, $f_i$, and $o_i$ are defined. It is assumed that $e_i$, $g_i$, $f_i$, and $o_i$ are independent RVs each having a continuous probability density function (PDF). It is also assumed that the additive noises at both PU-Rx and SU-Rx are independent CSCG RVs each $\sim CN(0, 1)$. Since we are interested in the information-theoretic limits, it is assumed that the optimal Gaussian codebook is used by both the PU and SU.

Consider first the PU link. Since the PU may adopt an adaptive power control based on its own CSI, the PU’s transmit power at fading state $i$ is denoted by $q_i$. It is assumed that the PU’s power-control policy, $P_p(f_i)$, is a mapping from the PU channel power gain $f_i$ to $q_i$, subject to an average transmit power constraint represented by $E[q_i] \leq Q$. Examples of PU power control are the “constant-power (CP)” policy

$$q_i = Q, \ \forall i$$  \hspace{1cm} (1)

and the “water-filling (WF)” policy [17], [18]

$$q_i = \left( d_p - \frac{1}{f_i} \right)^+$$  \hspace{1cm} (2)

where $d_p$ is a constant “water-level” with which $E[q_i] = Q$. In this paper, we assume that the PU is oblivious to the existence of the SU, and any interference from SU-Tx is treated as additional Gaussian noise at PU-Rx. Thus, the ergodic capacity of the PU channel is expressed as

$$C_p = \mathbb{E} \left[ \log \left( 1 + \frac{f_i q_i}{1 + g_i p_i} \right) \right]$$  \hspace{1cm} (3)
where \( p_i \) denotes the SU’s transmit power at fading state \( i \) (to be more specified later). It is worth noting that the maximum PU ergodic capacity, denoted as \( C_p^{\text{max}} = \mathbb{E} [\log (1 + f_i q_i)] \), is achievable only if

\[
\frac{f_i q_i}{1 + g_i p_i} = f_i q_i, \quad \forall i.
\] (4)

From (4), it follows that \( g_i p_i = 0 \) if \( f_i q_i > 0 \) for any \( i \). In other words, to achieve \( C_p^{\text{max}} \) for the PU, the SU transmission must be off when the PU transmission is on, which is the same as the OSA with perfect spectrum sensing.

Next, consider the SU link. The SU is also known as CR since it is aware of the PU transmission and is able to adapt its transmit power levels at different fading states based on all the available CSI between the PU and SU to maximize the SU’s average transmit rate and yet provide a sufficient protection to the PU. In this paper, we assume that the CSI on both \( g_i \) and \( f_i \) is perfectly known at SU-Tx for each fading state \( i \). For notational convenience, we combine the Gaussian-distributed interference from PU-Tx with the additive noise at SU-Rx, and define the equivalent SU channel power gain \( h \) at fading state \( i \) as

\[
h_i : = \frac{e_i}{1 + g_i q_i},
\]
which is also assumed to be known at SU-Tx for each \( i \). Thus, the SU’s power-control policy can be expressed as \( P_s(h_i, g_i, f_i) \), subject to an average transmit power constraint, \( \mathbb{E}[p_i] \leq P \).

By assuming that SU-Rx treats the interference from PU-Tx as additional Gaussian noise, the SU’s achievable ergodic capacity is then expressed as

\[
C_s = \mathbb{E} [\log (1 + h_i p_i)].
\] (5)

Note that the maximum SU ergodic capacity, denoted by \( C_s^{\text{max}} \), is achievable when \( P_s \) maximizes \( C_s \) with no attempt to protect the PU transmission. In this case, the optimal \( P_s \) is the WF policy expressed as \( p_i = \left( d_s - \frac{1}{h_i} \right)^+ \), where \( d_s \) is the water-level with which \( \mathbb{E}[p_i] = P \).

Remark 2.1: It is important to note that in the assumed system model, we have deliberately excluded the possibility that the PU’s allocated power at fading state \( i \) is a function of the received interference power from SU-Tx, \( g_i p_i \). If not so, the SU’s power control needs to take into account of any predictable reaction of the PU upon receiving the interference from SU-Tx, e.g., the PU may change transmit power that will also result in change of the interference power level at SU-Rx. Such feedback loop over the SU’s and PU’s power adaptations will make the design of the SU transmission more involved even for the deterministic channel case. This interesting phenomenon will be studied in the future work.
III. SU Power Control Under AIPC

Existing prior work in the literature, e.g., [10], has considered the peak/average interference power constraint over fading states at PU-Rx as a practical means to protect the PU transmission. In this section, we first present the SU power-control policy to maximize the SU ergodic capacity under the constraint that the average interference power level over different fading states at PU-Rx must be regulated below some predefined threshold, thus named as average interference power constraint (AIPC). The associated problem formulation is similar to that in [10], but with an additional constraint on the SU’s own transmit power constraint, and is expressed as

\[
\text{(P1): } \begin{align*}
\text{Maximize} & \quad \mathbb{E}[\log (1 + h_i p_i)] \\
\text{Subject to} & \quad \mathbb{E}[g_i p_i] \leq \Gamma \\
& \quad \mathbb{E}[p_i] \leq P \\
& \quad p_i \geq 0, \ \forall i
\end{align*}
\]

where \(\Gamma \geq 0\) is the predefined threshold for AIPC. It is easy to verify that (P1) is a convex optimization problem, and thus by applying the standard Karush-Kuhn-Tucker (KKT) optimality conditions [20], the optimal solution of (P1), denoted as \(\{p_i^{(1)}\}\), is obtained as

\[
p_i^{(1)} = \left( \frac{1}{\nu^{(1)} g_i + \mu^{(1)}} - \frac{1}{h_i} \right)^+ 
\]

where \(\nu^{(1)}\) and \(\mu^{(1)}\) are non-negative constants.

Note that the power-control policy given in (9) is a modified version of the standard WF policy [17], [18]. Compared with the standard WF policy, (9) differs in that the water-level is no longer a constant, but is instead a function of \(g_i\). Interestingly, similar variable water-level WF power control has also been shown in [19] for multi-carrier systems in the presence of multiuser crosstalk. If \(g_i = 0\), (9) becomes the standard WF policy with a constant water-level \(1/\mu^{(1)}\), since in this case the SU transmission does not interfere with PU-Rx. On the other hand, if \(g_i \to \infty\), from (9) it follows that the water-level becomes zero and thus \(p_i^{(1)} = 0\) regardless of \(h_i\), suggesting that in this case no SU transmission is allowed since any finite SU transmit power will result in an infinite interference power at PU-Rx.

\[^1\text{Numerically, } \nu^{(1)}\text{ and } \mu^{(1)}\text{ can be obtained by, e.g., the ellipsoid method [21]. This method utilizes the sub-gradients } \Gamma - \mathbb{E}[g_i p_i[n]]\text{ and } P - \mathbb{E}[p_i[n]]\text{ to iteratively update } \nu[n+1]\text{ and } \mu[n+1]\text{ until they converge to } \nu^{(1)}\text{ and } \mu^{(1)}\text{, respectively, where } \{p_i[n]\}\text{ is obtained from (9) for some given } \nu[n]\text{ and } \mu[n]\text{ at the } n\text{th iteration.}\]
Furthermore, it is observed from (9) that the power control under AIPC does not require the PU CSI, $f_i$, which is desirable from an implementation viewpoint. However, there are also drawbacks of this power control explained as follows. Supposing that $f_iq_i = 0$, i.e., the PU transmission is off, the SU can not take this opportunity to transmit if $g_i$ happens to be sufficiently large such that (9) results in that $p_i^{(1)} = 0$. On the other hand, if $f_iq_i$ happens to be a large value, suggesting that a substantial amount of information is transmitted over the PU channel, such transmission may be corrupted by a strong interference from the SU if in (9) $g_i$ and $h_i$ result in a large interference power $g_ip_i^{(1)}$ (though it is upper-bounded by $1/n$) at PU-Rx. Clearly, the above drawbacks of the AIPC-based power control are due to the lack of joint exploitation of all the available CSI at SU-Tx, which will be overcome by the proposed power-control policy in the next section.

It is worth mentioning that although the AIPC-based power control is non-optimal, the AIPC still guarantees an upper bound on the maximum PU ergodic capacity loss, as given by the following theorem:

**Theorem 3.1:** Under the given AIPC threshold, $\Gamma$, the PU ergodic capacity loss due to the SU transmission, defined as $C_p^{\max} - C_p$, is upper-bounded by $\log(1 + \Gamma)$, regardless of $\mathcal{P}_p(f_i), \mathcal{P}_s(h_i, g_i, f_i)$, and the distributions of $f_i$, $h_i$, and $g_i$.

**Proof:** The proof is based on the following equality/inequalities:

\[
C_p \overset{(a)}{=} \mathbb{E} \left[ \log \left(1 + \frac{f_iq_i}{1 + g_ip_i}\right) \right] \\
\overset{(b)}{\geq} \mathbb{E} \left[ \log (1 + f_iq_i) \right] - \mathbb{E} \left[ \log (1 + g_ip_i) \right] \\
\overset{(c)}{\geq} \mathbb{E} \left[ \log (1 + f_iq_i) \right] - \log (1 + \mathbb{E}[g_ip_i]) \\
\overset{(d)}{\geq} C_p^{\max} - \log(1 + \Gamma)
\]

where $(a)$ is due to (3); $(b)$ is due to $g_ip_i \geq 0, \forall i$; $(c)$ is due to the concavity of the function $\log(1 + x)$ for $x \geq 0$ and Jensen’s inequality (see, e.g., [18]); and $(d)$ is due to the definition of $C_p^{\max}$ and the inequality (6).

**IV. SU Power Control Under PCLC**

In this section, we propose a new SU power-control policy by utilizing all the CSI on $h_i$, $g_i$, and $f_i$, known at SU-Tx. This new policy is based on an alternative constraint of AIPC to protect the PU
transmission, named as *primary capacity loss constraint* (PCLC). PCLC and AIPC are related to each other: From Theorem 3.1 it follows that the AIPC in (6) with a given $\Gamma$ implies that $C_p^{\text{max}} - C_p \leq \log(1 + \Gamma)$, while the PCLC directly applies the constraint $C_p^{\text{max}} - C_p \leq C_\delta$, where $C_\delta$ is a predefined value for the maximum tolerable ergodic capacity loss of the PU resulted from the SU transmission. The ergodic capacity maximization problem for the SU under the PCLC and the SU’s own transmit power constraint is expressed as

\[
(P2): \quad \text{Maximize } \{p_i\} \quad \mathbb{E} \left[ \log \left( 1 + h_i p_i \right) \right] \\
\text{Subject to } \quad C_p^{\text{max}} - C_p \leq C_\delta \tag{10}
\]

Note that (P1) and (P2) only differ in the constraints, (6) and (10). Since $C_p^{\text{max}}$ is a fixed value given $Q$, the distribution of $f_i$, and the PU power control $P_p(f_i)$, using (3) we can rewrite (10) as

\[
\mathbb{E} \left[ \log \left( 1 + \frac{f_i q_i}{1 + g_i p_i} \right) \right] \geq C_0 \tag{11}
\]

where $C_0 = C_p^{\text{max}} - C_\delta$. Unfortunately, the constraint (11) can be shown to be non-convex, rendering (P2) to be also non-convex. However, under the assumption of continuous fading channel gain distributions, it can be easily verified that the so-called “time-sharing” condition given in [23] is satisfied by (P2). Thus, we can solve (P2) by considering its Lagrange dual problem, and the resultant duality gap between the original and the dual problems is zero. Due to the lack of space, we skip here the detailed derivations and present the solution of (P2) directly as follows:

\[
p_i^{(2)} \left( p_i^{(2)} \right) = \left( \frac{1}{\lambda_i \left( p_i^{(2)} \right) \nu_i^{(2)} g_i + \mu_i^{(2)} \nu_i^{(2)} g_i + \mu_i^{(2)}} - \frac{1}{h_i} \right)^+ \tag{12}
\]

where similarly like (P1), $\nu^{(2)}$ and $\mu^{(2)}$ are nonnegative constants that can be obtained by the ellipsoid method. Compared to the AIPC-based power-control policy in (9), the new policy in (12) based on PCLC has an additional multiplication factor in front of the term $\nu_i^{(2)} g_i$, which is further expressed as

\[
\lambda_i \left( p_i^{(2)} \right) = \frac{f_i q_i}{\left( 1 + g_i p_i^{(2)} \right) \left( 1 + g_i p_i^{(2)} + f_i q_i \right)} \tag{13}
\]

\(^2\)Since most communication systems in practice employ some form of “power margin” and/or “rate margin” (see, e.g., [22]) for the receiver to deal with unexpected interferences, the PCLC is a valid assumption if the PU belongs to such systems.
Note that $\lambda_i$ is itself a (decreasing) function of the optimal solution $p_i^{(2)}$. Thus, the power-control policy \( (12) \) can be considered as a *self-biased* WF solution. From \( (12) \) and \( (13) \), we obtain

**Theorem 4.1:** The optimal solution of \( (P2) \) is

$$
p_i^{(2)} = \begin{cases} 
0 & \text{if } \frac{\lambda_i(0)\nu^{(2)}g_i + \mu^{(2)}}{\lambda_i(z) \nu^{(2)}g_i + \mu^{(2)}} - \frac{1}{h_i} \leq 0 \\
z_0 & \text{otherwise,}
\end{cases}
$$

where $z_0$ is the unique positive root of $z$ in the following equation:

$$z = \frac{1}{\lambda_i(z) \nu^{(2)}g_i + \mu^{(2)}} - \frac{1}{h_i}.
$$

An illustration of the unique positive root $z_0$ for the equation \( (15) \) is given in Fig. 2. Note that $F(z) \triangleq \frac{1}{\lambda_i(z) \nu^{(2)}g_i + \mu^{(2)}}$ is an increasing function of $z$ for $z \geq 0$, and $F(0) = \frac{1}{h_i}$, $F(\infty) = \frac{1}{\mu^{(2)}}$. As shown, $z_0$ is obtained as the intersection between a 45-degree line starting from the point \((0, \frac{1}{h_i})\) and the curve showing the values of $F(z)$. Numerically, $z_0$ can be obtained by a simple bisection search [20].

Some interesting observations are drawn on the PCLC-based power control \( (14) \) as follows:

First, from \( (12) \) and \( (13) \) it is observed that what is indeed required at SU-Tx for power control at each fading state is the received signal power at PU-Rx, $f_iq_i$, instead of the exact PU channel CSI, $f_i$. Note that $f_iq_i$ may be more easily obtainable by the SU than $f_i$ in some cases, e.g., when $f_iq_i$ is estimated and then sent back by a collaborate secondary node in the vicinity of PU-Rx.

Second, from \( (13) \) and \( (14) \) it is inferred that $p_i^{(2)} > 0$ for any $i$ if and only if $\frac{\lambda_i(0)\nu^{(2)}g_i + \mu^{(2)}}{\lambda_i(z) \nu^{(2)}g_i + \mu^{(2)}} < \frac{1}{h_i}$. For given $\nu^{(2)}$, $\mu^{(2)}$, and $h_i$, it then follows that $p_i^{(2)} > 0$ only when $g_i$ and/or $f_iq_i$ are sufficiently small. This is intuitively correct because they are indeed the cases where the SU will cause only a negligible PU capacity loss. In the extreme case of $g_i = 0$ and/or $f_iq_i = 0$, the condition for $p_i^{(2)} > 0$ becomes $\frac{1}{\mu^{(2)}} > \frac{1}{h_i}$, the same as the standard WF policy.

V. **Numerical Examples**

The achievable ergodic capacity pairs of PU and SU, denoted by $(C_p, C_s)$, over realistic fading channels are presented in this section via simulation. $\tilde{f}$, $\tilde{e}$, $\tilde{g}$, and $\tilde{o}$ are assumed to independent CSCG RVs $\sim \mathcal{C}\mathcal{N}(0, 1)$, $\mathcal{C}\mathcal{N}(0, 1)$, $\mathcal{C}\mathcal{N}(0, 0.5)$, and $\mathcal{C}\mathcal{N}(0, 0.01)$, respectively. It is also assumed that $P = Q = 10$, corresponding to an equivalent average signal-to-noise ratio (SNR) of 10 dB for both PU and SU channels (without the interference between PU and SU). The following cases of $(C_p, C_s)$ are then considered:
• “PCLC”: The SU employs the proposed power-control policy (14). $C_s$’s are obtained from (5) by substituting $p_i$’s that are solutions of (P2) with different values of $C_s$, while the corresponding $C_p$’s are obtained from (3).

• “AIPC”: The SU employs the conventional power-control policy (9). $C_s$’s are obtained from (5) by substituting $p_i$’s that are solutions of (P1) with different values of $\Gamma$, while the corresponding $C_p$’s are obtained from (3).

• “AIPC, Lower Bound”: The SU employs the AIPC-based power-control policy (9). $C_s$ is obtained same as that in the second case, while for a given value of $\Gamma$, $C_p$ is obtained as $C_p^{\text{max}} - \log(1 + \Gamma)$. Note that $\log(1 + \Gamma)$ is shown in Theorem 3.1 to be a capacity loss upper bound for the PU and, thus, $C_p$ in this case corresponds to a PU capacity lower bound.

• “MAC, Upper Bound”: An auxiliary 2-user fading Gaussian multiple-access channel (MAC) is considered here to provide the capacity upper bounds for the PU and SU. In this auxiliary MAC, all the channels are the same as those given in Fig. 1 except that PU-Rx and SU-Rx are assumed to be collocated such that the received signals from PU-Tx and SU-Tx can be jointly processed. Let $h_p(i) = [\tilde{f}(i) \tilde{o}(i)]^T$ and $h_s(i) = [\tilde{g}(i) \tilde{e}(i)]^T$. Considering the auxiliary MAC, for a given PU power-control policy, $P_p(f_i)$, it can be shown that the upper bounds on the PU and SU achievable rates belong to the following set (24)

$$\bigcup_{P_s(h_i, g_i, f_i): \mathbb{E}[p_i] \leq P} \left\{ (C_p, C_s) : C_p \leq \mathbb{E} \left[ \log(1 + q_i \|h_p(i)\|^2) \right], C_s \leq \mathbb{E} \left[ \log(1 + p_i \|h_s(i)\|^2) \right], C_p + C_s \leq \mathbb{E} \left[ \log \left| I + q_i h_p(i) h_p^H(i) + p_i h_s(i) h_s^H(i) \right| \right] \right\}$$

which can be efficiently computed by applying the methods given in [25] and the details are thus omitted here for brevity.

Fig. 3 and Fig. 4 show the achievable PU and SU ergodic capacities for the aforementioned cases when the PU power-control policy is the CP policy in (1) and the WF policy in (2), respectively. It is observed that in both figures, the capacity gains by the proposed SU power-control policy using the PCLC are fairly substantial over the conventional policy using the AIPC. For example, when the PU capacity loss resulted from the SU transmission is 5%, i.e., $C_p = 0.95 \cdot C_p^{\text{max}}$, the SU capacity gain by the proposed policy over the conventional policy is around 28% in the CP case, and 50% in the
WF case. Another interesting observation is that for the proposed SU power control, the SU capacity reaches its minimum value of zero in the CP case when the PU capacity attains its maximum value, $C_p^{\text{max}}$, while in the WF case the SU capacity has a non-zero value at $C_p^{\text{max}}$. In general, capacity gains of the proposed SU power control are more significant in the case of WF over CP PU power control. This is because the WF policy results in variable PU transmit powers based on the PU CSI and thus makes the proposed SU power control that is designed to exploit the PU CSI more beneficial.

VI. CONCLUDING REMARKS

In this paper, we studied the fundamental capacity limits for spectrum sharing based cognitive radio networks over fading channels. In contrast to the conventional power-control policy for the SU that applies the interference-power or interference-temperature constraint at the PU receiver to protect the PU transmission, this paper proposed a new policy based on the constraint that limits the maximum permissible PU ergodic capacity loss resulted from the SU transmission. This new policy is more directly related to the PU transmission than the conventional one by exploiting the PU CSI. It was verified by simulation that the proposed policy can lead to substantial capacity gains for both the PU and SU over the conventional policy.

Many extensions of this work are possible. First, this paper considers the ergodic capacity as the performance limits for both PU and SU, while similar results can be obtained for non-ergodic PU and SU channels where the outage capacity should be is a more appropriate measure. Second, results in this paper can also be extended to the cases with imperfect/quantized PU CSI. Last, the proposed scheme in this paper is applicable to the general parallel Gaussian channel with sufficiently large number of sub-channels, e.g., the broadband channel that is decomposable into a large number of parallel narrow-band channels via multi-carrier modulation and demodulation.

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Fig. 1. System model for the CR network.

Fig. 2. Illustration of the unique positive root $z_0$ for the equation (15).
Fig. 3. Achievable rates of PU and SU when the PU power-control policy is the CP policy given by $I$. 
Fig. 4. Achievable rates of PU and SU when the PU power-control policy is the WF policy given by (2).