Buckling Behaviour of Three-Dimensional Prestressed Stayed Columns

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Abstract. Slender columns exhibit instability under compression, which causes buckling. Prestressed stayed system can be used to improve the load carrying capacity of these columns. In this study, three-dimensional prestressed stayed column was investigated through the use of commercially available finite element software- ABAQUS. Although many numerical studies into the buckling behavior of stayed columns exist, these have mainly focused on two-dimensional (2D) systems. In order to fully understand the behavior of these structural systems, three-dimensional (3D) numerical studies are essential. In this work, the buckling behavior of three-dimensional prestressed stayed columns was investigated through linear perturbation method. The critical buckling loads and modes of four-branch and three-branch columns were investigated. The effect of varying the cross-arm length on the buckling load of the four-branch column was also investigated and compared with the theoretical values. It was found that the theoretical buckling loads reported in previous research with 2D systems were conservative and largely underestimated the buckling capacities of prestressed stayed columns. Furthermore, it was observed that transition point from the symmetric to antisymmetric buckling mode occurs at a higher value of cross-arm to column length ratio for the 3D prestressed stayed columns compared to the 2D model.

1. Introduction

Columns are vertical structural members loaded in compression in the axial direction and used extensively in the construction industry. They are constructed from various kinds of materials including steel, wood, stone and concrete. A major problem with steel columns, particularly slender steel columns, is that they are sensitive to buckling under compressive loading i.e. axial loading results in horizontal deflection of the member, due to instability. Leonard Euler first derived the critical buckling load of a slender column, known as the Euler formula (Case et al. 1999) as:

\[ P_E = \frac{\pi^2 E I}{L^2} \]  (1)

where \( P_E \) is the Euler buckling load, \( E \) is the Young’s modulus of the column, \( I \) is the moment of inertia of the column and \( L \) is the column length.

Equation (1) presents the Euler load formula, which is still used today to calculate the buckling load of a slender column. The buckling behavior of columns has necessitated the development of innovative systems in an attempt to improve their performance under loading.

Prestressed stayed columns are a form of slender column which benefit from high strength and low cross-sectional area. The name prestressed comes from the fact that the cable or rod stays are pretensioned, so the whole structure is prestressed.

Due to the tensioned stays, buckling is delayed and the system exhibits enhanced buckling resistance compared to its unstayed counterpart. When the column is loaded axially, the pretension counteracts the compressive force and provides lateral stability, thereby delaying the occurrence of buckling. The stays
are connected via cross-arms, which are connected directly to the column face (Fig. 1). This type of structure can come in a number of configurations, defined by the number of cross-arms.

Fig. 1 Typical stayed column with two cross-arms about the mid plane of the column and four stays

A single, two-branch cross-arm stayed column, shown in Fig. 1, is the simplest form of this type of system. In this work, configurations with three or four branches have been investigated, as well as setups with multiple cross-arms along the length of the column (Fig. 2).

(a) Four-branch cross-arm column, (b) Three-branch cross-arm column, (c) Double cross-arm column.

Fig. 2 Different configurations of prestressed stayed column
Fig. 3 Zones of buckling behaviour of prestress stayed column (Hafez et al. 1979)

The possible buckling modes of prestressed stayed column under axial compression are shown in Fig. 4. These are the symmetric, antisymmetric and interactive buckling modes. It should be noted that interactive buckling is a post-buckling effect, which occurs as a combination of the other two modes.

Fig. 4 Buckling modes of the prestressed stayed column

(a) Symmetric buckling, (b) Antisymmetric buckling, (c) Interactive buckling

The buckling behavior of prestressed stayed columns has been studied analytically since the 1960’s (Chu and Berge, 1963, Ellis 1971). Worthy of note is the work of Hafez et al. (1979), who proposed that the buckling behavior of prestressed stayed columns can be grouped into three zones depending on the level of initial prestress (Fig. 3). The findings of this work have formed the basis for many subsequent studies into the buckling behaviour of stayed columns.

Additional analytical works were carried out by Saito and Wadee (2009), Saito and Wadee (2010), Wadee et al. (2011) and Yu and Wadee (2017). As far as the authors are aware, the work of Temple (1977) was the first attempt to understand prestressed stayed column behavior using numerical simulations. Several subsequent studies investigated the behavior of two-dimensional (2D), single cross-arm stayed columns (Wong and Temple 1982, Chan et al. 2002, Saito and Wadee 2009, Saito and Wadee 2010 and Osofero et al. 2013). These studies focused on the influence of imperfections on the buckling load, the optimal prestress level and parametric studies. However, as far as the authors are aware, the findings of these studies have not been validated for three-dimensional prestressed stayed columns.

Further numerical studies by Yu and Wadee (2017) looked at a triple bay column. Li et al. (2016) also investigated the stability of a three-branch single cross-arm column through numerical modelling using the commercially available finite element software – ABAQUS. It focused solely on the three-branch column, which is covered in the current study and expanded into the four-branch configuration. Although, Pichal and Machacek (2017) utilized ANSYS Finite Element software to examine two-
dimensional (2D) and three-dimensional (3D) columns, it employed a sliding connection between the stays and the cross-arms. Comparison with previous work is somewhat impossible due to different connection type. Previous studies have adopted fixed connection between the cross-arm and stays.

A similar structural system has also been studied by Guo et al. (2017) called a pre-tensioned cable stayed buckling-restrained brace. It differs from a 3D stayed column in that it has an inner core running along the length of the column to resist the axial load. Guo et al. (2016) investigated 3D single-cross-arm PCS-BRB using ABAQUS software. It focused on the buckling resistance of these structures. The results highlight that ABAQUS software can be used to effectively study three-dimensional geometries and that these techniques should be applied to prestressed stayed column which is the focus of this study.

2. Numerical modelling

Three-dimensional numerical models were developed for two, three and four-branch columns in ABAQUS with column length, outer diameter and thickness of 2900 mm, 42.4 mm and 3 mm respectively. The cross-arm length was varied between 50 and 600 mm with an outer diameter of 27.9 mm and thickness of 3 mm while the stay area was 60 mm². The mechanical properties of the main column, cross-arms and stay are presented in Table 1.

| Column Young’s Modulus | Cross-arm Young’s Modulus | Stay Young’s Modulus | Column Yield Stress | Cross-arm Yield Stress |
|------------------------|--------------------------|---------------------|---------------------|-----------------------|
| 209,000                | 208,000                  | 100,000             | 342                 | 421                   |

3. Numerical set-up

The 3D columns and cross-arm were modelled using the 3-noded quadratic B32 beam elements while the 2-noded linear T3D2 truss elements were used for the stays. The no-compression option was used in the stay material to prevent any compressive load being carried by the stays and to effectively simulate stay slackening. Single elements were used in the mesh for the stays so that no bending of the stays was allowed, while 10 mm mesh size was adopted for the cross-arms and column. The model had between 338 and 538 elements in total depending on the cross-arm length. A concentric point load was applied to one end of the column, and pinned boundary conditions were used at either end, allowing movement along the column axis at one end.

4. Numerical Results

Using the ABAQUS perturbation analysis, the critical buckling loads of three prestressed stayed column configurations were obtained. Four-branch, three-branch and two-branch stayed columns were investigated with varying cross-arm length. Theoretical buckling loads \( N^{\text{max}} \) for the column geometries were also calculated using Equation (2). The obtained buckling load values from the three-dimensional numerical modeling were compared with the values from a corresponding two-dimensional geometry and the theoretical values. The theoretical values of the critical buckling load, \( N^{\text{max}} \), were obtained using Equations (2) – (6) from Osofero et al. (2013), and first derived by Smith et al. (1975).

\[
N^{\text{max}} = \frac{4D^2EI}{L^2} \tag{2}
\]

Where \( E \) is the Young’s modulus, \( I \) is the second moment of area, \( L \) is the column buckling length and the values of term ‘\( D \)’ for the symmetric and antisymmetric modes can be calculated using Equations (3) and (4) respectively.

\[
\frac{2K^2}{B \sin^2 \alpha} = \frac{D^3}{D - \tan D} \tag{3}
\]
\[
\frac{B}{\sin^2 \alpha} \left( \frac{\cos^2 \alpha}{3B_a} + \frac{1}{2K_s} \right) = D - \tan \frac{D}{2} - \tan \left( \frac{4}{D} \right)
\]

(4)

\(\alpha\) is the angle between the column and cross-arm. Axial stiffness for the column, cross-arm and stays can be calculated using expressions in Equation (5) if the Young’s modulus, cross sectional area and the length of the main column \((E, A, L)\), of the cross arm \((E_a, A_a, a)\) and of the stay \((E_s, A_s, L_s)\) are known.

\[K_c = \frac{EA}{L}, \quad K_s = \frac{E_s A_s}{L_s}, \quad K_a = \frac{E_a A_a}{a}\]

(5)

The bending stiffness of the column and cross-arm \(B\) and \(B_a\) can also be calculated using Equation (6).

\[B = \frac{8EI}{L^3}, \quad B_a = \frac{E_a I_a}{a^3}\]

(6)

where, \(I\) and \(I_a\) are the second moments of area of the column and cross-arm respectively.

Theoretical buckling loads for the symmetric and antisymmetric modes obtained from Equations (2) – (6) and the corresponding optimal prestressing forces for varying cross arm lengths are presented in Table 2. It can be seen that at some value of cross-arm length, the symmetric buckling load becomes greater than the antisymmetric buckling load. This is known as the transition point i.e. when the critical buckling mode transit from the symmetric to the antisymmetric mode.

**Table 2 Theoretical buckling loads and optimum prestress forces for column length of 2900 mm and varying cross-arm length**

| a (mm) | 2a/L | \(N_{c,max}^c\) | \(T_{opt}^c\) | \(N_{c,max}^s\) | \(T_{opt}^s\) |
|--------|------|----------------|-------------|----------------|-------------|
| 50     | 0.03 | 29.19         | 1.95        | 91.35          | 6.11        |
| 100    | 0.07 | 61.17         | 4.08        | 108.36         | 7.23        |
| 130    | 0.09 | 86.23         | 5.74        | 110.69         | 7.36        |
| 150    | 0.10 | 102.19        | 6.78        | 110.77         | 7.35        |
| 160    | 0.11 | 109.12        | 7.23        | 110.53         | 7.33        |
| 175    | 0.12 | 117.69        | 7.79        | 109.99         | 7.28        |
| 180    | 0.12 | 120.05        | 7.94        | 109.77         | 7.26        |
| 190    | 0.13 | 124.07        | 8.19        | 109.28         | 7.21        |
| 200    | 0.14 | 127.30        | 8.39        | 108.73         | 7.17        |
| 250    | 0.17 | 136.13        | 8.89        | 105.80         | 6.91        |
| 300    | 0.21 | 139.66        | 9.03        | 102.90         | 6.65        |
| 350    | 0.24 | 141.42        | 9.02        | 100.34         | 6.40        |
| 400    | 0.28 | 142.44        | 8.95        | 98.01          | 6.16        |
| 420    | 0.29 | 142.73        | 8.91        | 97.17          | 6.07        |
| 600    | 0.41 | 144.06        | 8.37        | 91.42          | 5.31        |

Effect of varying cross-arm length on the buckling modes of two-dimensional and three-dimensional models is highlighted in Fig. 5. Theoretical values are also shown for comparison.
It is clear that the critical buckling mode changes at a certain cross-arm length (transition point). However, the transition point is different for the two-dimensional and three-dimensional models. At low cross-arm lengths, the symmetric mode is critical, but at the transition point, the antisymmetric mode becomes the critical buckling mode. Furthermore, the 3D model gives the highest buckling load value, conveying that the theoretical and 2D numerical model may be conservative in their estimates.

As the cross-arm length varies, the buckling mode shape changes. This is due to the change in the rotational stiffness of the cross-arm i.e. at low cross-arm length (before the transition point), the rotational stiffness of the cross-arm is greater than its axial stiffness. Hence, the critical buckling mode is symmetric with the cross arm moving longitudinally (Fig. 4a). However, as the cross-arm length increases, the rotational stiffness of the cross-arm decreases. If the cross-arm length continues to increase, a point is reached, where the rotational stiffness is now lower than the axial stiffness. At this point, which is also known as the transition point, the critical mode becomes antisymmetric, i.e. no longitudinal movement at the cross arm but rotation of the cross-arm with respect to the column surface is experienced (Fig. 4b).

Fig. 5 Comparison of buckling loads from the 2D, 3D and theoretical models

Note: TP = Transition point
Fig. 6 Symmetric buckling loads for two-dimensional and three-dimensional models at varying 2a/L.

Figures 6 and 7 highlight changes in the buckling shapes due to increase in cross-arm length. The transition point is important as the buckling load values for the symmetric and antisymmetric buckling modes are almost equal at this point. This means that interactive buckling may be induced as a combination of both buckling modes can occur.

The buckling shape of the symmetric mode changes dramatically with increase in cross-arm length. This occurs due to changes in the rotational stiffness of the cross-arm. At lower cross-arm length, the symmetric buckling mode has the greatest deflection at mid-height along the column as the rotational stiffness is greater than the axial stiffness. Whereas, as the cross-arm length increases, the mid-height becomes the least deflected point, until it is not deflected at all, as the rotational stiffness is lower than the axial stiffness.

Fig. 7 Antisymmetric buckling loads for two-dimensional and three-dimensional models at varying 2a/L.
Fig. 7 highlights changes in the shape of the antisymmetric buckling mode of a three-dimensional model as the cross-arm length increases. At low cross-arm lengths, the cross-arm deflects at the stay ends. Buckling of the cross-arms is experienced at higher cross-arm length. Representative symmetric and antisymmetric buckling modes for two-dimensional and three-dimensional models are presented in Fig. 8 to highlight the difference in buckling mode shapes.

The comparison presented in Fig. 8 highlights the similarities between the buckling shapes of the 2D and 3D systems as well as the slight differences. However, the main difference between the 2D and 3D buckling shapes is the ability of the 3D column to buckle out of the cross-arms plane. Three-branch columns, with varying angle of orientation, i.e. varying $\alpha_i$, were also investigated to establish the optimal configuration (Fig. 9).

![Symmetric and Antisymmetric Buckling Modes](image)

**Fig. 8 Comparison of symmetric and antisymmetric buckling mode shapes for 2D and 3D models**

![Orientation of Three-Branch Column](image)

**Fig. 9 Orientation of three-branch column**

Fig. 10 presents the buckling load for varying angle of orientation for both symmetric and antisymmetric buckling modes.
As can be seen, the configuration with equal angle of 120° between each arm i.e. \( \alpha_1 = \alpha_2 = \alpha_3 \) has the greatest buckling load for both the symmetric and antisymmetric buckling modes.

5. Conclusion

Results from the current work shows that buckling load values of prestressed stayed columns obtained from three-dimensional numerical modelling are consistently higher compared to the values obtained from equivalent system when modelled in two dimensions. This shows that both analytical and two-dimensional modelling underestimate the buckling capacity of this structural system which can potentially lead to it being overdesigned. In addition, it further shows that the theoretical buckling values of prestressed stayed columns are somewhat conservative and procedures for obtaining these values should be updated. It is clear that, to fully understand the buckling behaviour of prestressed stayed columns and also maximize its load carrying capacity, three-dimensional numerical modelling instead of the popular two-dimensional modelling must be employed. Although the trends obtained from the present work is encouraging, further investigation into the post buckling behaviour and the sensitivity of the system to change in key parameters, such as prestress value and column imperfection is necessary.

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