The Value of Summary Statistics for Anomaly Detection in Temporally-Evolving Networks: A Performance Evaluation Study

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Abstract

Network data has emerged as an active research area in statistics. Much of the focus of ongoing research has been on static networks that represent a single snapshot or aggregated historical data unchanging over time. However, most networks result from temporally-evolving systems that exhibit intrinsic dynamic behavior. Monitoring such temporally-varying networks to detect anomalous changes has applications in both social and physical sciences. In this work, we perform an evaluation study of summary statistics for anomaly detection in temporally-evolving networks by incorporating principles from statistical process monitoring. In contrast to previous studies, we deliberately incorporate temporal auto-correlation in our study. Other considerations in our comprehensive assessment include types and duration of anomaly, model type, and sparsity in temporally-evolving networks. We conclude that summary statistics can be valuable tools for network monitoring and often perform better than more complicated statistics.

1 Introduction

Recent decades have witnessed an explosion of data in the form of networks, representing important systems in various fields, e.g., physical infrastructure [Huberman and Adamic, 1999; Pagani and Aiello, 2013], social interaction [Milgram, 1967; Adamic and Glance, 2005], and biological systems [Bassett and Bullmore, 2006; Lynall et al., 2010]. Consequently, statistical modeling and analysis have become fundamental tools for studying physical and virtual networked systems, and are poised to become even more critical in the near future.

Traditionally, most research has focused on static modeling of networks, in which either a single snapshot or aggregated historical data of a system are available. However, usually networks result from time-evolving systems that exhibit intrinsic dynamic behavior. For example, often the relationships between members of a social network evolve over time due to finding new friends, collaborating with new colleagues, moving to another department, etc. Recent studies have focused on analyzing dynamic networks where a network is represented by a statistical/probabilistic model that is adaptively updated over time. However, many of these models have been developed on the premise that a system either is stationary or has smooth dynamics, and it does not experience abrupt changes.

In applications, the occurrence of sudden large changes and shocks in time-varying networks is very common. For example, resignation of a key employee or occurrence of a conflict in an organization may cause a significant change in the professional network of the employees. As another example, the occurrence of a change in the communications network of a terrorist group may indicate a high possibility of a terrorist attack that could be prevented if the change is detected quickly. Similarly, significant changes in the brain connectome network [Xia et al., 2013] of an
individual can indicate the onset of a neurological disorder like Alzheimer’s disease or epilepsy. Monitoring, change detection, and accurate estimation of the change time are crucial for effective decision-making and for taking necessary actions in a timely manner. Moreover, abrupt changes often affect a network locally. That is, only a subset of nodes and their corresponding links are altered by an event. Consequently, diagnosis, defined as identifying affected sub-networks, plays an important role in root-cause determination and action planning. For example, it is crucial to determine the group of people who might be involved in a terrorist plot, or the parts of the brain involved in a particular disease, by identifying the set of nodes that caused the change in the corresponding network.

A recent review paper by Woodall et al. [2017] provides an assessment of monitoring methods that may detect anomalies in time-evolving networks. Specifically, they reviewed statistical process monitoring methods for social, dynamic networks which fall into five broad categories: hypothesis testing (signals based on likelihood ratio tests), Bayesian methods (control limits are calculated using a Bayesian predictive distribution), scan methods (signals from a moving window based monitoring statistic), time series models (signals from large residuals), and changes in community structures/membership. While summarizing the categories, Woodall et al. [2017] highlight differences among available methods for specifying tuning parameters, such as specifying the size of moving average window, defining control limits based on ill-defined “Phase I data”, and approaches for removing seasonal effects in data. A notable point made by Woodall et al. [2017], which was re-iterated in Sengupta and Woodall [2018], is that even with variation in parameter settings comparable relevant work in the literature is not available for detecting abrupt changes in a stream of time-varying networks. Similarly, several papers have studied network monitoring under specific parametric modeling frameworks [Wilson et al., 2016; Yu et al., 2018; Zhao et al., 2018b,a]. However, such monitoring methods work under the assumption of a specific network model, and cannot be extended to the general task of network monitoring without model assumptions.

In this paper, we assess the performance of model-free summary statistics in network monitoring, such as network density, maximum degree, and their linear combinations, in anomaly detection. Such summary statistics are simple to calculate and often used in practice, but little is understood about how these summary statistics behave under varying network conditions and over time. In turn, the utility of common network summaries, such as density and maximum degree, for anomaly detection is also currently unclear. For our work, we conduct a comprehensive simulation study to assess both the successes and failures of four monitoring statistics that are functions of network density and/or maximum degree in comparison to a well-studied scan-based moving window approach [Priebe et al., 2005]. We measure success and failure for the methods based on false alarm rates, anomaly detection rates, and Area Under Curve (AUC) calculations from receiver operating characteristic (ROC) curves.

An important aspect of our work is in applying monitoring methods on temporally dependent network data to evaluate method performance in realistic scenarios. That data are simulated from well-studied network models so that we may introduce various kinds of anomalies in a controlled manner to facilitate a principled comparative evaluation of network monitoring methods. As pointed out by several authors [Woodall et al., 2017; Savage et al., 2014; Azarnoush et al., 2016], compared to real-world case studies, such controlled scenarios from synthetic networks provide a more principled testbed for performance assessment.

The remainder of the paper is as follows. The anomaly detection methods utilized, mathematical formulas, and considerations for such methods utilized are given in Section 2. In order to
better understand what types of anomalies can be detected, the network monitoring methods were applied to data generated from two popular latent variable models: the dynamic latent space model (DLSM) \cite{SewellChen2015} and the dynamic degree-corrected stochastic block model (DD-CSBM) \cite{Wilsonetal2016, MatiasMiele2017}. Brief overviews of these models are given in Section \ref{sec:models}. We reiterate that no model fitting occurred and models were only used to generate data of temporally-evolving networks. A performance evaluation of network monitoring methods on summary statistics is accomplished using a comprehensive simulation study. Settings for the simulation study as well as planted anomalies is discussed in in Section \ref{sec:simu}. Lastly, we summarize our findings and discuss future work in Section \ref{sec:concl}.

2 Summary Statistics and Network Monitoring Methods

For anomaly detection, we discuss which statistics are monitored and methods by which to monitor these select statistics. The summary statistics calculated from network data and interpretations of such quantities in a network are described in Section \ref{sec:stats}. How common techniques used in statistical process monitoring is applied to network data is described in Section \ref{sec:process}. Threshold decisions for the selected monitoring approaches are further discussed in Evaluation of monitoring approaches is discussed in Section \ref{sec:eval}

2.1 Summary Statistics

We first define our mathematical notation. Let \( n \) represent the number of nodes in a network at time \( t \), where \( t \) evolves discretely until time \( T \). Let \( Y_t \) represent an adjacency matrix at time \( t \in \{1, 2, \ldots, T\} \) and \( y_{ijt} \) represent an edge weight at time \( t \) between nodes \( i \) and \( j \), for \( i, j \in \{1, 2, \ldots, n\} \).

**Density** is defined as the sum of edges in a network divided by its total number of possible edges at time \( t \). For \( n \) nodes, the total number of possible edges is \( \left(\frac{n}{2}\right) = \frac{n(n-1)}{2} \), and for \( Y_t \), the sum of all edges at time \( t \) is \( \sum_{i=1}^{n} \sum_{j=1:j\neq i}^{n} y_{ijt} \). If we let \( W_t \) represent density at time \( t \), then

\[
W_t = \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1:j\neq i}^{n} y_{ijt}.
\]

Density is considered to be a global measure of a network; a measure that describes the entire network.

A local measure of a network might summarize individual nodes, such as degree. Let \( D_{it} \) represent the degree of node \( i \) within a network at time \( t \). In a directed network, \( D_{it} = \sum_{j=1}^{n} (y_{ijt} + y_{jit}) \); \( D_{it} \) is the sum of in- and out-degrees within directed networks. For undirected networks, degree divides \( D_{it} \) in half. To form a global measure from the local node degrees, **maximum degree** is often reported. Let \( D_t \) represent the maximum degree of a network at time \( t \);

\[
D_t = \max_i \{D_{it}\}.
\]

For this paper, we monitor network density, maximum degree, and **two linear combinations**
of density and maximum degree, denoted $M^-_t$ and $M^+_t$. We define $M^-_t$ and $M^+_t$ as follows:

$$M^-_t = \frac{1}{n} D_t - W_t$$

$$M^+_t = \frac{1}{n} D_t + W_t.$$  \hspace{1cm} (1)

We compare the effectiveness of monitoring $W_t$, $D_t$, $M^-$, and $M^+$ to detect anomalies relative to each other and a scan statistic, $S^*_t$, that was proposed by Priebe et al. [2005]. Calculations for $S^*_t$ result over moving windows of size $m$ (e.g., $m=20$) over time $t$ ($t \in \{2m + 1, ..., T\}$) and are based on the size of local neighborhoods of each node $i$. Neighborhoods within a network at time $t$ are determined from a pre-specified order $k$, e.g., $k = \{0, 1, 2\}$ in that each neighborhood $i$ ($i \in \{1, 2, ..., n\}$) is the set of all nodes (and edges between them) within $k$ edges of node $i$. Thus, the size of an order $k$ neighborhood is the number of edges contained in that neighborhood. Let $O^*(i,t)$ denote the size of an order $k$ neighborhood of $i$ and time $t$. Note, when $k = 0$, the size of an order 0 neighborhood is equivalent to degree. The calculated scan statistic for order $k$ with moving window $m$ involves a 2-step process on the sizes of order $k = \{0, 1, 2\}$ neighborhoods Zhao et al. [2018a]. We represent scan statistics based on order $k$ as $S^*_t$, for $k = \{0, 1, 2\}$ and define $S^*_t = \max\{S^{(0)}_t, S^{(1)}_t, S^{(2)}_t\}$ when reporting results.

We now overview this 2-step process as is explained in Zhao et al. [2018a] summarizing the work of Priebe et al. [2005]. First step is to standardize $O^*(i,t)$ using a previous window of size $m$. That is, the first standardization of $O^*(i,t)$ with $t > m$ is calculated by

$$O^{*(k)}_{i,t} = \frac{O^*_i(k) - \text{mean}(O^*_i(k))}{\text{max}(\text{sd}(O^*_i(k)), 1)},$$

with

$$\text{mean}(O^*_i(k)) = \frac{1}{m} \sum_{j=1}^{m} O_{i,t-j}^{(k)} \hspace{1cm} \text{and} \hspace{1cm} \text{sd}(O^*_i(k)) = \sqrt{\frac{1}{m-1} \sum_{j=1}^{m} [O_{i,t-j}^{(k)} - \text{mean}(O^*_i(k))]^2}$$

for $k = \{0, 1, 2\}$ and $i \in \{1, 2, ..., n\}$. The lower bound of 1 in the denominator of $O^*_i(k)$ prevents detection of relatively small (perhaps too small) changes in a network. This first standardization of $O^{(0)}_{i,t}$, $O^{(1)}_{i,t}$, and $O^{(2)}_{i,t}$ are done for all times $t > m$. The second step of the process involves standardizing the maxima of $O^*_i(k)$ across nodes $i$. Let $S^*_t(k) = \max_i\{O^*_1(k), O^*_2(k), ..., O^*_n(k)\}$ and calculate across nodes $i$ and time $t > 2m$, $S^{(0)}_t$, $S^{(1)}_t$, and $S^{(2)}_t$. Then, the second standardization of $S^*_t(k)$ with $t > 2m$ is calculated by

$$S^{*(k)}_t = \frac{S^*_t(k) - \text{mean}(S^*_t(k))}{\text{max}(\text{sd}(S^*_t(k)), 1)},$$

with

$$\text{mean}(S^*_t(k)) = \frac{1}{m} \sum_{j=1}^{m} S^{(k)}_{t-j} \hspace{1cm} \text{and} \hspace{1cm} \text{sd}(S^*_t(k)) = \sqrt{\frac{1}{m-1} \sum_{j=1}^{m} [S^*_t(k) - \text{mean}(S^*_t(k))]^2}.$$
for \( k = \{0, 1, 2\} \).

Finally, monitoring scan statistics, \( S^*_t \), begins at time \( t = 2m + 1 \) since the first \( 2m \) windows are needed to start the procedure. For more details on \( S^*_t \), see both Priebe et al. [2005] and Zhao et al. [2018a].

Table 1 provides a list of statistics that we study in this paper for network monitoring. We chose these statistics for their simplicity, popularity, flexibility, and relative meaning in dynamic networks. For example, network densities over time are easy to calculate and have the potential to reveal both global or local changes in networks. Globally, several nodes may increase or decrease communication (even slightly) over set time(s) in a network, and locally, a few nodes may significantly increase or decrease communication. Both such global and local changes could be reflected in changes of network densities. Note, “communication”, refers to the number or weight of edges in nodes. Similarly, maximum degree may capture global and local changes based on all or a set node behaviors. By combining measures of degree and density, e.g., with “Difference” \( (M^-_t) \) and “Sum” \( (M^+_t) \) statistics, there is additional opportunity to capture global and/or local changes in networks. For example, with binary networks where \( y_{ijt} \in \{0, 1\} \), we know \( D_t \leq n \) and \( D_t/n \leq 1 \). Thus, \( M^-_t = (\frac{1}{n}D_t - W_t) \leq 1 \), because \( 0 \leq W_t \leq 1 \). When \( M^-_t \) values are high (close to 1) there is large discrepancy between network density and the average node degree. This would suggest that individual nodes may be highly connected with other nodes (local behavior), while the overall communication of other nodes is scarce (global behavior). Finally, the scan statistic \( S^*_t \) is popular and well-cited, thus it makes sense to ground our analyses of four summary statistics by the scan statistic, \( S^*_t \). However, the scan statistic is harder to calculate than the other statistics that we study. Furthermore, for large-scale networks (e.g., social media) and real-time monitoring, computing the scan statistic can be computationally expensive and/or infeasible. Thus, in addition to comparing the effectiveness of five common network summaries, we may form opinions on the trade-offs between complex and simple monitoring approaches.

| Name (at time \( t \)) | Notation |
|-----------------------|----------|
| Density               | \( W_t \) |
| Max Degree            | \( D_t \) |
| Difference            | \( M^-_t \) |
| Sum                   | \( M^+_t \) |
| Scan Statistic        | \( S^*_t \) |

Table 1: Statistics Monitored in Dynamic Networks.

2.2 Network Monitoring Methods

Statistical process monitoring is a well-studied area, but applying these techniques to network data is not straightforward; e.g., how to set monitoring limits is unclear for network data. The general network monitoring approaches are discussed in Section 2.2 while considerations for control limits are further discussed in Sections 2.2.1 and 2.2.2.

The summary statistics calculated are monitored in two ways. One way utilizes a control chart described in Algorithm 1 and the other utilizes a moving window approach over-viewed in Algorithm 2. In the first approach, there are three main steps. Step one, establish \( T_1 \), the number of time-points required to determine baseline behavior of a network. We refer to data from \( t = 1 \)
through \( t = T_1 \) as Phase I data, where network snapshots are assumed to have no anomalies. Next, after establishing Phase I data, appropriate control limits, i.e., thresholds for acceptable non-anomalous behavior, must be determined. Third, established control limits from Phase I data are applied to remaining time-points, called Phase II data, in order to detect any anomalies or atypical behavior.

Similar to the first approach, the second approach has three main steps. First, a moving window approach requires a window length, \( m \), to be set, e.g. 20 time-points. Specifically for the scan statistic, in addition to the first set of \( m \) time-points, yet another set of \( m \) time-points is needed to start the method. Second, a threshold must be determined for acceptable behavior. Third, the remaining moving windows of length \( m \) are observed using that threshold. In both approaches, whether or not an anomaly occurred is recorded using \( A = \{ A_1, A_2, \ldots, A_{T - T_1} \} \) such that \( A_t = 1 \) when an anomaly is detected and 0 otherwise. We exemplify general monitoring approaches on \( M_t^- \) and \( S_t^* \) in Figure 1. From Figure 1(a), we observe control limits, \( CL = \hat{\mu} \pm q\hat{\sigma} \), are calculated using Phase I data, and likewise from Figure 1(b), we see the monitoring process begin at time 41 when using window size \( m = 20 \) with an acceptable threshold for the scan statistic. In this illustration, both methods signal around time-point 60.

**Algorithm 1** General Monitoring Approach for \( W_t, D_t, M_t^- \), and \( M_t^+ \).

**Input:** Temporally-evolving network data.

**Output:** \( A = \{ A_{T_1+1}, A_{T_1+2}, \ldots, A_T \} \) where \( A_t = 1 \) if anomaly is detected at time \( t \) and 0 otherwise.

1: Set number of time-points for Phase I data, \( T_1 \). Data from Phase I is assumed to be non-anomalous.
2: Use Phase I data to determine control limits for a Shewhart individuals control chart. Control limits (CL) are of the form: \( CL = \hat{\mu} \pm q\hat{\sigma} \).
3: For \( t \) in \( (T_1 + 1) : T \) {

For a summary statistic at time \( t \), denoted \( x_t \), record if an anomaly occurred or not using:

\[
A_t = \begin{cases} 
1, & x_t < \hat{\mu} - q\hat{\sigma} \\
1, & x_t > \hat{\mu} + q\hat{\sigma} \\
0, & \text{otherwise}
\end{cases}
\]

} END
Algorithm 2 General Monitoring Approach for $S_t^*$.

**Input:** Temporally-evolving network data.

**Output:** $A = \{A_{T_1+1}, A_{T_1+2}, \ldots, A_T\}$ where $A_t = 1$ if anomaly is detected at time $t$ and 0 otherwise.

1: Determine the length of moving window, i.e. set number of time-points for a window, $m$.
2: Use $2m$ windows for starting the scan method. First set of $m$ windows is for maximizing across nodes. Second set of $m$ windows is for maximizing across time.
3: Set threshold, $q$, to determine if an anomaly occurred or not, where $q \in \{1, 2, 3, \ldots\}$.
4: **For** $t$ in $(T_1 + 1) : T$ **{**

   For scan statistic at time $t$, denoted $S_t^*$, record if an anomaly occurred or not using:

   $$A_t = \begin{cases} 
   1, & S_t^* > q \\
   0, & \text{otherwise}.
   \end{cases}$$

**} END

Figure 1: Illustration of General Monitoring Approaches for (a) $M_t^-$ and (b) $S_t^*$. Plot (a) shows a Shewhart individuals control chart for $M_t^-$ where control limits are calculated using Phase I data, and plot (b) shows a moving window approach on $S_t^*$ using a pre-determined threshold. In this illustration, both methods signal around time-point 60.

In the first monitoring approach, Shewhart individual control charts are utilized for a majority of our summary statistics. In any control chart, a statistic is observed over time with control limits indicating acceptable behavior of that statistic. Since the network data are correlated snapshots over time, control limits should be adjusted for such correlation. Control limits of a Shewhart individuals control chart are $\bar{x} \pm (3MR)/d_2$ where $d_2$ is an anti-biasing constant and set to 1.13; and $MR$ is a moving range, for a process $\{x_1, x_2, \ldots, x_n\}$ so that $MR = |x_i - x_{i-1}| \ (i > 1)$ [Montgomery, 2007]. However, in the context of network data, these control limits may not directly apply. For example, using $3\sigma$ limits (3 standard deviations above or below the mean) is common practice
when the quantity monitored follows the normal distribution. Alas, the statistics we monitor here for networks may not be governed by normal distributions. In Section 2.2.1, we assess whether the standard deviation of our summary statistics are appropriately measured using $\overline{MR}/d^2$, and in Section 2.2.2, we evaluate the utility of applying 3 times the standard deviation as control limits.

2.2.1 Measuring Standard Deviation of Summary Statistics

To determine how to appropriately measure the standard deviation of our summary statistics, we compare the standard $\overline{MR}/d^2$, i.e., moving range average (AMR), with two other alternatives. First alternative is rather than the average of the moving ranges is to use the median of the moving ranges, moving range median (MMR). Second alternative is using a correlated data calculation for standard deviation (SD), $s = \sqrt{\gamma_1}$ such that $\gamma_1 = 1 - \frac{2}{(n - 1)} \sum_{\kappa=1}^{n-1} \left(1 - \frac{\kappa}{n}\right) \rho_\kappa$ and $\rho_\kappa$ is autocorrelation at lag $\kappa$ [Box et al., 2008]. These three measures of standard deviation are compared by the number of false alarms above the upper control limit, $\hat{\mu} + q \cdot \hat{\sigma}$, where $\hat{\mu} = \overline{x}$, $q \in [2, 4]$, and $\hat{\sigma}$ is $s$ for SD, $\overline{MR}/d^2$ for AMR, and median of $MR$ values scaled by $d^2$ for MMR.

Results of 200 simulations using Phase I data with 100 nodes, $T_1 = 50$ time-periods for comparing metrics, and data generated from two common network models: the Dynamic Latent Space Model (DSLAM) and the Dynamic Degree-Corrected Stochastic Block Model (DDCSBM). Details on models and model settings are described later in Section 3. Figure 2 shows four examples of false alarm rates with Figures 2(a) and (b) from a DSLM count setting using $E[W_t]$ at 3% and at 18% respectively and with Figures 2(c) and (d) from a DDCSBM count setting using $\phi$ at 0.10 and at 0.95. As seen in the figures below, false alarm rates from control limits using SD and AMR perform similarly (solid and dashed lines in Figure 2), but MMR has a larger number of false alarms when looking between 2 to 4 standard deviations above the mean (dotted lines in Figure 2). Thus, to account for the correlation in monitoring dynamic network statistics and to attain the smallest false alarm rates, the SD calculation is used as $\hat{\sigma}$ in control limits.
2.2.2 Determining Appropriate Thresholds

We ascertain if $3\sigma$ limits are appropriate in addition to suitable thresholds for monitoring the scan statistic, $S^*_t$. When monitoring the averages of batches via a Shewhart $\bar{X}$ chart and conditioning on Phase I data, there are several approaches to determine appropriate control limits [Jardim et al., 2019]. Similar principles from the $\bar{X}$ chart can be used in our context for summary statistics. In general, we aim to fix the conditional false alarm probability for a statistic $x_t$ such that $Pr(x_t > \mu + q\sigma) = p$. When monitoring the scan statistic, the method signals when the scan statistic is above a threshold [Priebe et al., 2005]. The threshold recommended by [Priebe et al., 2005] is 5, while in [Zhao et al., 2018a] the authors showed via simulation study that perhaps lowering such threshold with a tolerable amount of false alarms can improve the performance of the scan statistic. For the scan statistic, we have the conditional false alarm probability as $Pr(x_t > q) = p$. Ultimately, $p$ is determined by the practitioner, but in this work, we settle on $p = 0.03$. After calculating $\tau$, using the correlated standard deviation $s$, and setting $p$, we must calibrate an appropriate $q$.

There are several ways to calibrate an appropriate $q$ in thresholds of network monitoring. One option is if $x_t$ reasonably follows a normal distribution (or some other well-known distribution),
then \( q \) can be set using one-tail probabilities of the normal distribution, \( p \). However, this method can lead to errors when the normality (or standard distribution) assumption fails to hold. Another option is empirically obtaining false alarm rates, the number of signals for data with no anomalies. False alarm rates can be calculated from Phase I data or generated Phase II data with no anomalies. In practice, one does not know whether there is any anomaly in Phase II, and therefore only Phase I data should be used for calibrating \( q \). However, there can be a difference between nominal false alarm rates (calibrated using Phase I data) and actual false alarm rates (resulting from Phase II data) in this approach. This happens due to sampling variation as the distribution of Phase II data, even when non-anomalous, is unlikely to be an exact replicate of Phase I data. In our case, we used synthetic network data for performance assessment. Therefore, established control limits from Phase I data can be applied in Phase II data. Since data is generated, the conditional false alarm probability can be controlled to be exactly the same in Phase II data. This allows an exact calibration of false alarm rates in Phase II, which therefore allows us to accurately compare the anomaly detection performance of the various summary statistics and the scan statistic of \( \text{[Priebe et al., 2005]} \).

To determine if summary statistics relatively follow a normal distribution, empirical false alarm rates were compared to that of an upper tail of a standard normal distribution, \( Pr(Z > q) \). Here, false alarm rates are calculated using Phase II data with the same conditional false alarm probability as Phase I. Specifically in a Shewhart individuals control chart, the number of times \( x_t \) exceeds \( \bar{x} + q \sigma \) is recorded, and for the scan statistic, the number of times \( x_t \) exceeds \( q \). Calibrating \( q \) is determined empirically using false alarm rates across varying correlation and sparsity. Correlation is denoted using \( \phi \) and sparsity is average density denoted using \( E[W_t] \). Further details on models, model settings, and correlation and sparsity are discussed in Sections 2 and 4. An example in binary DLSM and DDCSBM settings is shown for monitoring the scan statistic in Figure 3. As can be seen in Figure 3, false alarm rates are generally double or triple that of the upper tail of a standard normal distribution for the scan statistic when varying the threshold of \( q \) between [2, 4]. False alarm rates tended to increase as correlation increases in the binary DLSM setting.
Figure 3: Plot of False Alarm Rates Monitoring $S^*_t$ Across Varying Correlation [(a) and (c)] and Sparsity [(b) and (d)] in Binary DLSM and DDCSBM Settings.

Examples of either binary or count networks are shown in Figures 10-13 in Appendix A.1 for $W_t$, $D_t$, $M_{-t}$, and $M_{+t}$. In general, there is less of a pattern between correlation and false alarm rates as well as sparsity and false alarm rates in the DDCSBM setting. In the DLSM setting, higher correlation tends to increase false alarm rates when monitoring these methods, while varying correlation and sparsity has little effect in the DDCSBM setting. Hence, $p = 0.03$ is used as our false alarm threshold to accommodate those statistics which do not follow a normal distribution and have higher false alarm rates than an upper one-sided tail of a standard normal distribution. Then, $q$ is calibrated from empirical false alarm rates as close as possible to $p = 0.03$.

3 Statistical Models for Temporally Evolving Networks

Our methods for anomaly detection do not rely on a parametric model, and thus, we neither have to estimate parameters of such a model nor determine if parameter estimates had changed significantly. We highlight this advantage of model-free to parametric based methods by generating dynamic network data from two popular latent variable models. Kim et al. [2018] reviewed several latent variable models including latent space models and stochastic block models in both static
and dynamic versions. Specifically, we focus on particular formulations of a dynamic latent space model (DLSM) and dynamic degree-corrected stochastic block models (DDCSBM).

3.1 Dynamic Latent Space Model

Use of latent space models relies on defining a space of latent positions using either pairwise distances or projections [Kim et al., 2018; Hoff et al., 2002] introduced both a distance based and projection based latent space model using Bayesian inference for parameter estimation via MCMC. The idea here is that the probability of forming an edge between nodes $i$ and $j$ depends on the latent positions $z_i$ and $z_j$. For example, $z_i$ and $z_j$ being relatively close together (via a distance metric) in the latent space suggests a higher probability of an edge between nodes $i$ and $j$ in the network. Hence, edges are conditionally independent given the latent positions. We focus on the formulation of Sewell and Chen [2015] who extended ideas of Hoff et al. [2002] to a dynamic version of a latent space model.

We now provide an overview of the DLSM in Sewell and Chen [2015] and more details can be found in their paper. Let $Y_t$ be an adjacency matrix at time $t \in \{1, 2, \ldots, T\}$ such that $y_{ijt} = 1$ when there is an edge between nodes $i$ and $j$ and $0$ otherwise. Let $X_t$ be a matrix of latent positions $(X_{it} = (x_{i1t}, x_{i2t})$ for node $i \in \{1, 2, \ldots, n\})$ at time $t$. For ease in plotting and visualizations, two-dimensional coordinates are used. The probability of an edge between nodes $i$ and $j$ is given by:

$$y_{ijt} \mid \Psi = \{X_{it}, X_{jt}, \beta_{IN}, \beta_{OUT}, r, \sigma^2\} \sim \text{Bern} \left( p_{ijt} = \frac{\exp(\eta_{ijt})}{1 + \exp(\eta_{ijt})} \right) \text{ s.t.}$$

$$\eta_{ijt} = \log \left( \frac{Pr(y_{ijt} = 1 \mid \Psi)}{Pr(y_{ijt} = 0 \mid \Psi)} \right) = \beta_{IN} \left( 1 - \frac{d_{ijt}}{r_j} \right) + \beta_{OUT} \left( 1 - \frac{d_{ijt}}{r_i} \right).$$

The parameters $\beta_{IN}$ and $\beta_{OUT}$ are global (system-scale) network characteristics describing popularity and social activity respectively. If $\beta_{IN} > \beta_{OUT}$ in a directed network, then this suggests the receiver is more important while the opposite scenario suggests the sender is more important. The radii in $r = (r_1, r_2, \ldots, r_n)$, are node specific characteristic describing the radius of communication in the latent space such that the radii sum to 1, i.e., $\sum_{i=1}^{n} r_i = 1$. One can imagine these radii are like a scaled degree of a node. The pairwise distance between latent positions of nodes $i$ and $j$ is denoted as $d_{ijt} = \text{dist}(X_{it}, X_{jt})$, and the spread of the latent positions are controlled by $\sigma^2$.

In order to generate data from this model, we use the prior distribution on latent positions to get $X_{1:T}$ and subsequently $d_{ijt}$ and set values for the remaining parameters, $\beta_{IN}$, $\beta_{OUT}$, $r$, and $\sigma^2$. The original prior in Sewell and Chen [2015] is $N(0^T, \tau^2 I_2)$ for the initial network and $N(X_{i(t-1)}, \sigma^2 I_2)$ for all subsequent networks. Hence, a priori, latent positions may move around the space according to some random jump (dictated by $\tau^2$ or $\sigma^2$). For meaningful networks, the variances of the latent positions must be quite small in order to achieve reasonable edge probabilities. We demonstrate how latent positions move a priori in Figure 4 using $T = 100$, $\sigma^2 = 0.0003$ and 5 clusters (defined by both color and shape). Latent positions are plotted at times $t = 2, 36, 58$, and 100. As time increases, we observe a spread of latent positions from the initial tight clusters. Perhaps intermixing of clusters is desired a priori, but the spread of latent positions has an effect on edge probabilities, $p_{ijt}$. By increasing pairwise distances of the latent positions, $d_{ijt}$, this decreases $p_{ijt}$ values over time, and in turn, decreases realizations of $p_{ijt}$ values via network density. An interpretation of Figure 4 is that people communicate less and less as they get older, which may not make practical
sense. Thus, generating latent positions from this distribution is a concern due to the spread in latent positions.  

\[
\begin{array}{c}
\begin{array}{c}
\text{Time 2} \\
\text{Time 58} \\
\text{Time 100}
\end{array}
\end{array}
\]

Figure 4: Latent Positions at Times 2, 26, 58, and 100. For \(\sigma^2 = 0.0003\) and \(T = 100\), the spread of the latent positions grow over time when starting in tight pre-defined clusters.

Our solution is to scale the spread of the latent positions, \(\sigma^2\), using a time series model, in particular, a vector auto-regression of order 1 (VAR(1)) model. Consider the following VAR(1) model:

\[
X_{i(t+1)} = \phi \cdot X_{it} + \epsilon_{t+1} \Rightarrow \begin{bmatrix} x_{i1(t+1)} \\ x_{i2(t+1)} \end{bmatrix} = \phi \ast \begin{bmatrix} x_{i1t} \\ x_{i2t} \end{bmatrix} + \begin{bmatrix} \epsilon_{1(t+1)} \\ \epsilon_{2(t+1)} \end{bmatrix},
\]

such that \(|\phi| < 1\) and \(\epsilon_{t+1} \sim N((0,0)^T, \sigma^2 I_2)\). Thus, a VAR(1) prior on latent positions has the following form:

\[
\begin{align*}
X_1 | \phi, \sigma^2 & \sim \prod_{i=1}^n N\left(0, \tau^2 = \left(\frac{\sigma^2}{1 - \phi^2}\right) I_2\right) \\
X_t | X_{t-1}, \phi, \sigma^2 & \sim \prod_{i=1}^n N\left(X_{i(t-1)}, \tau^2 = \frac{\sigma^2}{(1 - \phi^2)} I_2\right), \text{ for } t \geq 2.
\end{align*}
\]

We compare the network densities from the original prior to a VAR(1) prior when \(n = 100\), \(T = 100\), \(\beta_{IN} = \beta_{OUT} = 1\), \(r = \{1/n\}\), \(\phi = 0.3\), and \(\sigma^2 = 0.0003\) in Figure 5. Network density is plotted over time, and we observe a decay in density using the original prior whereas density using
a VAR(1) prior is within a reasonable range around an average density of 11%. Thus, using a VAR(1) prior appears able to control the spread of the latent positions a priori over time.

![Network Density using Original Prior and VAR(1) Prior on Latent Positions](image)

Figure 5: Network Density using Original Prior and VAR(1) Prior on Latent Positions with $n = 100, T = 100, \beta_{IN} = \beta_{OUT} = 1, r = \{1/n\}, \phi = 0.3$, and $\sigma^2 = 0.0003$.

By default, the model is setup as a binary network, but we can make the following modification for a count network: $y_{ijt} \mid \Psi \sim \text{Poisson}(\pi_{ijt} = \exp(\eta_{ijt}))$ s.t. $\eta_{ijt} = \log(E[y_{ijt} \mid \Psi])$ [Sewell and Chen, 2016]. Default settings of parameters in the simulation study include $\sigma^2 = (1 - \phi^2), \beta_{IN} = 1, \beta_{OUT} = 2$, and $r = \{\frac{1}{n}\}_{i=1}^n$. An outline of the data generation from a DLSM is provided in Algorithm 3 below.

### 3.2 Dynamic degree-corrected stochastic block model

Stochastic block models (SBM), on the other hand, utilize a latent community (block) assignment to assign edge probabilities [Snijders and Nowicki, 1997]. That is, edge probabilities within a community should differ from edge probabilities between communities. Matias and Miele [2017] extended an SBM model to a dynamic version which highlights evolving community memberships over time as well as any desired probability density measure on edge probabilities. A general issue with SBMs is when degree is heterogeneous among nodes [Karrer and Newman, 2011]. A tendency to group high degree nodes together and low degree nodes together forms, which may not be appropriate community assignments. Karrer and Newman [2011] developed a degree-corrected version of an SBM (DCSBM). Wilson et al. [2016] discussed a dynamic version of a DCSBM utilizing parameters to describe a propensity for a node to communicate. The DDCSBM model we use to generate data from adapts the dynamic SBM of Matias and Miele [2017] to include degree heterogeneity and the dynamic DCSBM of Wilson et al. [2016] to include correlation over time in the propensities. Ultimately, we use a model that allows for movement within community assignment, degree heterogeneity, and correlation among propensities to communicate over time.

We now provide an overview of the adapted DDCSBM used for data generation. Let $Y_t$ represent an adjacency matrix at time $t$ for $t \in \{1, 2, \ldots, T\}$. Let $K$ represent the number of communities and the probability of an edge between nodes $i$ and $j$ is defined by:

$$y_{ijt} \sim \text{Poisson}(p_{ijt} = \theta_i \theta_j \omega \omega_{ijt} Z_i Z_j).$$
Parameter \( Z_{it} \) is a latent community assignment for node \( i \) at time \( t \). The initial assignment is found via \( Z_{i0} \sim \text{Multinomial}(\alpha = \{1/K\}^K_{k=1}) \), and for all subsequent networks, community assignments can transition with probability \( \pi_{K \times K} \), where

\[
\pi_{K \times K} = \begin{pmatrix}
\pi_1 \\
\pi_2 \\
\vdots \\
\pi_K
\end{pmatrix} = \begin{pmatrix}
\pi_{11} & \pi_{1K} & \cdots & \pi_{1n} \\
\pi_{21} & \pi_{22} & \cdots & \pi_{2K} \\
\vdots & \vdots & \ddots & \vdots \\
\pi_{K1} & \pi_{K2} & \cdots & \pi_{KK}
\end{pmatrix}
\]

such that if \( Z_{it} = k, Z_{i(t+1)} \sim \text{Multinomial}(\pi_k) \) for \( k \in \{1,2,\ldots,K\} \). Next, we introduce propensity to communicate parameters, \( \Theta = \{\theta_1, \theta_2, \ldots, \theta_T\} \) such that \( \theta_i = \{\theta_{i1}, \theta_{i2}, \ldots, \theta_{iT}\} \) and \( \theta_{it} \in [1 - \delta, 1 + \delta] \) for some \( \delta \in (0,1) \) [Wilson et al., 2016]. A low propensity to communicate suggests the degree of that node will be relatively low. Parameter \( \omega_{K \times K} \) is community structure matrix such that \( \omega_{Z_{it}Z_{jt}} \in (0,1) \) and \( \text{diag}(\omega) \) are all distinct values. As noted in Matias and Miele [2017], if any intra-community communication values are the same, then distinguishing those communities remains unidentifiable. Community assignments affect propensities in the following way. Propensities of communication for a given community must be rescaled by the average propensity to preserve community structure. That is, if nodes transition between communities, then corresponding \( \theta_{it} \) will be rescaled as will the average propensity for those communities at time \( t \).

We specify the following “white noise” process to incorporate correlation. First, \( \theta_{i0}^* \sim U(-1,1) \) for all nodes \( i \). Next,

\[
\theta_{it}^* = \phi \cdot \theta_{i0}^* + (1 - \phi) \cdot \epsilon_{it} \text{ such that } \epsilon_{it} \sim U(-1,1) \text{ and } |\phi| < 1.
\] (3)
Then, \( \theta_{it} = \delta \cdot \theta_{it}^* + 1 \) so that \( \theta_{it} \in [1 - \delta, 1 + \delta] \) for some \( \delta \in (0, 1) \). In Equation 3, \( \epsilon_t \) is scaled in order to achieve \( \theta_{it} \in [1 - \delta, 1 + \delta] \). The reason for using a constant \( \phi \cdot \theta_{i0}^* \) in Equation 3 rather than a more traditional AR(1), i.e., \( \phi \cdot \theta_{i(t-1)}^* \), is that we want propensities to stay within a reasonable range of their starting value as \( |\phi| \to 1 \). After transformation when using an AR(1) model, we have the mean of the series as 1 with variance of \( \frac{(1 - \phi)^2\sigma^2}{(1 - \phi^2)} \). An issue is regardless of the starting value of a propensity, the AR(1) process tends toward the mean of 1 (after transformation). This implies, if \( \theta_{i0}^* = -0.75 \) and \( \delta = 0.98 \) so that \( \theta_{i0} = \delta \cdot \theta_{i0}^* + 1 = 0.265 \), then an AR(1) process with \( \phi = 0.90 \) for \( \theta_i \) settles around 1 rather than around \( \theta_{i0} = 0.265 \). We illustrate this tendency toward the mean of the process in Figure 6 using \( \theta_{i0} = 0.265 \) and \( \delta = 0.98 \) at a relatively low and high correlation, \( \phi = 0.30, 0.90 \). In Figure 6, we contrast an AR(1) model with our “white noise” model given in 3 using the same \( \epsilon_t \) values for both processes. Thus, low or high propensity to communicate nodes would be washed out if using a traditional AR(1) model. Therefore, scaling a “white noise” type process introduces a different kind of correlation based on the initial starting value of a propensity as can be seen in Figure 6.

The default edge type is counts for the DDCSBM, but for any threshold, \( b \in \mathbb{N} \), we can turn edge counts into a binary network. In this work, we use \( b = 1 \) to obtain a binary network such that \( y_{ijt}^* = 1 \) if \( y_{ijt} \geq m \). Settings of parameters in the simulation study include \( K = 3, \delta = 0.98, \pi = \begin{pmatrix} 0.96 & 0.02 & 0.02 \\ 0.02 & 0.96 & 0.02 \\ 0.02 & 0.02 & 0.96 \end{pmatrix} \), and \( \omega = \begin{pmatrix} 0.7 & 0.2 & 0.25 \\ 0.2 & 0.6 & 0.3 \\ 0.25 & 0.3 & 0.5 \end{pmatrix} \).
Three communities are chosen rather than two for some additional variation, $\theta_{it} \in [0.02, 1.98]$ are re-scaled within each community by the average propensity of each community respectively, relatively low transition rates to other communities are selected for $\pi$, and lastly, intra-communication was chosen to be much higher than inter-communications to help identify communities [Matias and Miele, 2017]. An outline of the data generation from a DDCSBM is provided in Algorithm 4 below. Lastly, notation used to generate data from dynamic networks is summarized in Table 2 below.

Algorithm 4 Data Generation from a DDCSBM.
1: Set $n$, $T$, $K$, $\phi$, $\delta$, $\omega$, and $\pi$, and choose a binary or count network.
2: Generate latent community assignments using $K$ and $\pi$.
   - Sample initial latent community assignments, $Z_{i0} \sim \text{Multinomial}(\alpha = \{1/K\}_{k=1}^K)$.
   - Generate $Z_{it} \sim \text{Multinomial}(\pi_k)$ for $Z_{i(t-1)} = k$.
3: Generate propensities to communicate, $\Theta$, using $\phi$ and $\delta$.
   - Generate initial values $\{\theta_{01}^*, \theta_{02}^*, \ldots, \theta_{0n}^*\} \sim U(-1, 1)$.
   - Generate $\theta_{i}^*$ using $\theta_{i}^* = \phi \cdot \theta_{0i}^* + (1 - \phi) \cdot U(-1, 1)$.
   - Recalculate $\theta_i = \delta \cdot \theta_i^* + 1$ for all nodes $i$.
4: For $t$ in $1 : T$ {
   For $i$ in $1 : n$ {
      - Scale $\theta_{it}$ by the mean of propensities in community $k$: $\theta_{it}^* = \theta_{it} / \sum_{j} \theta_{jt}$.
      - Draw $y_{ijt} \sim \text{Poisson}(p_{ijt} = \theta_{it} \theta_{jt} \omega Z_{it} Z_{jt})$ (count) or $y_{ijt}^* = \begin{cases} 0 & y_{ijt} = 0 \\ 1 & y_{ijt} \geq 1 \end{cases}$ (binary).
   } END
} END

| Symbol | Meaning |
|--------|---------|
| $K$    | Number of communities (blocks) |
| $\beta_{IN}$ | Global DLSM parameter for popularity |
| $\beta_{OUT}$ | Global DLSM parameter for social activity |
| $r$    | DLSM parameter for radius of communication of all nodes |
| $r_i$  | Radius of communication of node $i$ |
| $\mathcal{X}_t$ | Latent positions at time $t$ in a DLSM |
| $\sigma^2$ | DLSM parameter controlling spread of latent positions, $\mathcal{X}_{1:T}$ |
| $\phi$ | correlation via a time series |
| $\pi$  | DDCSBM parameter for community transition rate |
| $\Theta$ | DDCSBM parameter for propensity of communication for all nodes |
| $\theta_{it}$ | Propensity of communication for node $i$ at time $t$ |
| $\omega$ | Community structure matrix in a DDCSBM |

Table 2: List of Notation Related to Data Generated from Dynamic Models.
4 Performance Evaluation of Monitoring Methods

Based on data generated from models explained in Section 3, we conduct a comprehensive simulation study to evaluate the performances of monitoring network density, maximum degree, linear combinations of maximum degree and network density, as well as the scan statistic. A simulation study may allow for better understanding of evaluation of network monitoring approaches and types of changes detectable in dynamic network data. Performance evaluation is accomplished as follows. First, temporally-evolving network data is generated according to simulation study settings discussed in Section 4.1. Second, general network monitoring approaches are applied to such network data as was discussed in Section 2.2. Last, metrics for evaluating network monitoring output is elaborated on in Section 4.2. To help facilitate understanding of performance evaluation output, effects of varying correlation and sparsity is briefly in Section 4.3.

In our simulation study, we considered two types of anomalies: anomalies in edge probabilities and expected degree. All Monte Carlo simulations were performed using 100 nodes (n = 100) and 110 time-points (T = 110). For each type of anomaly, simulations were performed 200 times. Out of the 110 time-points, we set T1 = 50, the first 50 time-points, as Phase I data where no anomalies are embedded. The latter 60 time-points (t > 51) are used as Phase II data with embedded anomalies in edge probabilities or expected degree for some duration. Performance evaluation from each type of anomaly is discussed respectively in Sections 4.4 and 4.5.

4.1 Simulation Study Settings

To evaluate performances of summary statistics, temporally-evolving network data is simulated. Such simulated data takes into account various amounts of correlation, sparsity, duration of anomaly, model, model parameters, and network types. Correlation is observed from varying values of the VAR or white noise coefficient, $\phi$, and sparsity is observed from varying values of average density, $E[W_t]$. In particular, the values considered are

$$\phi \in \{0.1, 0.3, 0.5, 0.75, 0.9, 0.95, 0.99\} \text{ and } E[W_t] \in \{0.03, 0.06, 0.09, 0.12, 0.15, 0.18, 0.21\}.$$  

Average density is fixed at 11%, $E[W_t] = 0.11$, when varying $\phi$ in order to mimic realistic network densities of application data. Likewise, $\phi$ is fixed at 0.5 when varying $E[W_t]$ so that correlation across time is not too high nor too low. Duration of anomaly or change point length (CPL) represents consecutive time periods in which some kind of anomaly occurs, typically ranging from 5 time-points to 25 time-points throughout the study. Both binary and count network types are compared in DLSM and DDCSBM settings.

In all our scenarios, negative correlation, e.g., $\phi = -0.5$, is not included. When exploring negative correlation in the context of dynamic networks, we found similar results with summary statistics on data generated using negative correlation. While performance is similar, the interpretation of such network differs. In general, rather than nodes moving closer together or increasing the chance of a connection (form an edge), negative correlation suggests nodes may move farther apart or experience a decrease in the chance of a connection. For many brain studies and resulting dynamic networks applied on brain activity, correlation plays a big role in relating a behavior and the type of connections formed in the network [Hidalgo et al., 2009; Zabelina and Andrews-Hanna, 2016]. In both application data examples of [Hidalgo et al., 2009] and [Zabelina and Andrews-Hanna, 2016] on brain activity, negatively correlated activities in the brain tend to decrease the prevalence of a behavior or task.
For all simulations, we use the model settings described at the end of Sections 3.1 and 3.2. However, in order to achieve desired average densities, we must scale parameter(s) in network models using a scalar, $a_\ell$. A subscript $\ell$ is used to distinguish between a binary ($\ell = 1$) and count ($\ell = 2$) network. We chose to scale parameters $\sigma^2$ in a DLSM and $\omega$ in a DDCSBM. Specific settings for $a_\ell \sigma^2$ in a DLSM and $a_\ell \omega$ in DDCSBM are provided for ease of replicating results in our simulation study. When varying $\phi$ and controlling average network density at around 11%, in a DLSM, $a_1 = 0.00014$ and $a_2 = 0.00042$ for $a_\ell \sigma^2$, and in a DDCSBM, $a_1 = 0.16$ and $a_2 = 0.17$ for $a_\ell \cdot \omega$. When varying $E[W_t]$ and fixing $\phi = 0.5$, specific settings of $a_\ell$ to appropriately scale $\sigma^2$ and $\omega$ are given in Table 3.

| Model   | $a_\ell$ | Average Network Density $E[W_t]$ |
|---------|----------|----------------------------------|
| DLSM    | $a_1$    | 0.0002, 0.0002387, 0.000292, 0.000373, 0.000493, 0.000747, 0.00153 |
|         | $a_2$    | 3.5 $a_1$, 3.4 $a_1$, 3.3 $a_1$, 3.3 $a_1$, 3.3 $a_1$, 3.3 $a_1$ |
| DDCSBM  | $a_1$    | 0.35, 0.29, 0.24, 0.24, 0.18, 0.14, 0.09, 0.045 |
|         | $a_2$    | 0.32, 0.265, 0.22, 0.17, 0.13, 0.085, 0.045 |

Table 3: Scalar Settings of $a_\ell \sigma^2$ and $a_\ell \omega$ for Binary ($\ell = 1$) and Count ($\ell = 2$) Networks when Varying $E[W_t]$.

### 4.2 Performance Evaluation Metrics

Using the above mentioned settings, network data is generated and network monitoring approaches are applied as discussed in Section 2.2. For each Monte Carlo simulation, output of network monitoring approaches is a binary stream output, $A$. Performance evaluation of output from network monitoring approaches is accomplished using two measures. One such measure is detection rate (DR), which is a binary measure of whether or not an anomaly was detected at all. If an anomaly is detected that outcome is assigned a 1 and 0 otherwise. DR provides a sense of the ability of a network monitoring approach to find anomalies. To further quantify this ability, a second measure utilized is AUC calculations from ROCs. In this context, we take advantage of confusion matrices. True labels of a confusion matrix are the time periods ($t$) within the duration of an anomaly, and predicted labels are alarms found from signals of network monitoring approaches. In a resulting ROC curve, both the true positive rate (TPR) and false positive rate (FPR) must range from [0,1]. Thus, we vary $q$ from -6 to 6, i.e., $q \in [-6,6]$, in control limits $CL = \mu + q\sigma$ and threshold $q$ for the scan statistic to achieve desired FPR and TPR in [0,1]. In essence, AUC measures the number of times an anomaly is detectable. For both DR and AUC, these are calculated in each Monte Carlo simulation and later averaged over all Monte Carlo simulations.

### 4.3 Effects of Correlation and Sparsity on Summary Statistics

We explore the effect of correlation ($\phi$) and sparsity ($E[W_t]$) on means and standard deviations of summary statistics in Phase I data. Recall when monitoring summary statistics, control limits of a Shewhart individuals control chart are $\tau \pm qs$. Choosing an appropriate $q$ is discussed in Section 2.2.2 but learning effects of correlation and sparsity on $\tau$ and $s$ can aid understanding of evaluation of network monitoring. Phase I data are time-points less than 50, $t \leq 50$. Figures 14,21
are provided in Appendix A.2 displaying means and standard deviations of $W_t$, $D_t$, $M_t^-$, and $M_t^+$ across varying amounts of correlation and sparsity. Examples across varying correlation, i.e., $\phi$, are shown for a binary DLSM and count DDCSBM settings and across varying sparsity, i.e., $E[W_t]$, are shown for count DLSM and binary DDCSBM. In DLSM settings, as correlation increases, range of the means widen. In DDCSBM settings, this behavior mainly effects means of maximum degree and subsequently the sum and difference statistics. Standard deviation is less affected by correlation overall. In terms of varying sparsity, actual values of the means vary while both range and values of the standard deviation change in DLSM and DDCSBM settings. In summary, the means more so than standard deviations of $W_t$, $D_t$, $M_t^-$, and $M_t^+$ are affected by correlation and sparsity.

4.4 Performance Evaluation with Anomalies in Edge Probabilities

We design a set of simulations to ultimately affect network density by manipulating $p_{ijt}$ (edge probabilities) in simulated network data. Such manipulation is accomplished via an odds ratio, denoted $OR$. For a DLSM, $y_{ijt} \sim Bern(p_{ijt})$. Hence, for a given $OR$, we compare $p_0 = p_{ijt}$ and $p_1 = C_{ijt} \cdot p_{ijt}$ such that $OR = \left(\frac{1 - p_0}{p_0}\right) / \left(\frac{1 - p_1}{p_1}\right)$ and $C_{ijt} = \frac{OR}{(1 - p_0 + OR \cdot p_0)}$. Thus, as $OR$ increases, so does $C_{ijt}$. For a DDCSBM, $y_{ijt} \sim Pois(p_{ijt})$. Since $p_{ijt}$ is a rate, we directly compare $p_{ijt}$ and $OR \cdot p_{ijt}$. Such anomaly suggests a group of nodes all of a sudden increase (or decrease) communication. Specific scenarios are described in Table 4. Shift size refers to a relatively medium or large change in resulting density.

| Shift Size | Network Type | Number of Anomalous Nodes (N) | OR Change from 1 to |
|------------|--------------|-------------------------------|---------------------|
| Medium     | DLSM         | 33                            | 4                   |
| Medium     | DDCSBM       | 33                            | 2.5                 |
| Large      | DLSM         | 79                            | 2.5                 |
| Large      | DDCSBM       | 72                            | 1.5                 |

Table 4: Scenarios for Simulation Study Targeting Network Density via Odds Ratio.

An example using $N = 33$ nodes and OR change from 1 to 4 is shown in a binary DLSM and DDCSBM in Figure 7 with $\phi = 0.5$, $E[W_t] = 0.11$, and settings mentioned in 4. Shewhart control charts for this example are plotted for $M_t^-$ and $M_t^+$ with Phase I data as $t \leq 50$ in Figure 7. The solid red line indicates the mean from Phase I, the dashed red lines indicate $q$ standard deviations above or below the mean, the gray line separates Phase I and Phase II, and the blue dashed lines indicate the beginning and ending of the CPL. The number of standard deviations above the mean was determined by controlling the false alarm rates in the non-anomalous empirical data for $p = 0.03$ and $\phi = 0.5$. 

20
Figure 7: Shewhart Individuals Control Charts of $M^{-t}$ and $M^{+t}$ for an Anomaly in Edge Probabilities in Binary Settings with $\phi = 0.5$ and $E[W_t] = 0.11$. Plots (a) and (c) shows $M^{-t}$ and $M^{+t}$ in a DLSM setting, and plots (b) and (d) shows $M^{-t}$ and $M^{+t}$ in a DDCSBM setting. Limits $q$ are determined using setting empirical conditional false alarm probabilities at 0.03.

It is difficult to find meaningful changes to lower the odds ratio in DLSM setting since manipulating $p_{ijt}$ (edge probabilities) closer to 0 appears to be affected by the randomness of latent positions. That is, the closer two latent positions are, the higher the edge probability becomes. It could be the case that two latent positions are so close together, that even scaling such edge probability yields a $p_{ijt} > 0.2$. Thus, only scenarios increasing the odds ratio are considered.

Results of 200 simulations with $n = 100$, $T = 110$, and $CPL = 5, 10$, or 15 are summarized below in Tables 5-8 using DR and in Appendix A.3 Tables 18-21 using AUC. The method which detects the planted anomaly best are in bold. Since anomalies planted affect edge probability (and ultimately density), it would be natural for $W_t$ to detect this change the best. Results are explained first by model and network type settings and then across trends in CPL, correlation, and sparsity. In binary DLSM settings, the best performer in detection rates in the medium shift size vary between $D_t$, $M^{-t}$, and $M^{+t}$, while $W_t$ performs best in the large shift size. In nearly all count DLSM settings, $W_t$, $D_t$, $M^{-t}$, and $M^{+t}$ all detect perfectly with a DR of 1. The worst performer...
tends to be \( S^*_t \) in DLSM settings, but DR of monitoring the scan statistic does well in count DLSM settings with a large shift size. In DDCSBM settings, the best performer is almost always \( W_t \), which we would expect since the anomaly affected edge probabilities. In count DDCSBM settings, \( M^+_t \) does best, and in the large shift size, \( S^*_t \) also has a perfect detection rate. The shorter the duration, higher the correlation, and higher in sparsity, the worse most methods do in detecting an anomaly. These trends are observed more so in binary DLSM settings. From count to binary DDCSBM settings, recall, count data is transformed into binary using \( y_{ijt} = 1 \) if \( y_{ijt} \geq 1 \). In some cases, performances are better in count settings, but not in others. There is some loss of information that is either easier to capture or more difficult to capture [Zhao et al., 2018b].

We make note the design of \( S^* \) is to detect if a change occurred rather than the number of times said change occurred [Priebe et al., 2005; Zhao et al., 2018a]. That is, when monitoring the scan statistic, the method is expected to signal at least once if any anomaly occurred. In AUC results reported in Tables 18-21 in Appendix A.3, the detection amount is mainly best in \( M^-_t \) and \( M^+_t \) in DLSM settings while \( W_t \) does detect best in DDCSBM settings. The worst performer, as might be expected, is \( S^*_t \) across all settings with a decrease in AUC of about 10% to 40% compared to other summary statistics. Across duration of anomaly (CPL) and correlation (\( \phi \)), there is little difference in AUC values. However, sparsity (\( E[W_t] \)) has some effect since the sparser the network, the more difficult it could be to detect an anomaly.

Table 5: DR for DLSM with 33 Anomalous Nodes and \( OR \) from 1 to 4.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL      | \( \phi \) | \( E[W_t] \) | \( W_t \) | \( D_t \) | \( M^-_t \) | \( M^+_t \) | \( S^*_t \) | \( W_t \) | \( D_t \) | \( M^-_t \) | \( M^+_t \) | \( S^*_t \) |
| 5        | 0.5    | 0.11  | 0.320 | 0.365 | **0.400** | 0.365 | 0.210 | 0.980 | 1     | 1     | 1     | 0.545 |
| 10       | 0.5    | 0.11  | 0.560 | **0.605** | 0.595 | **0.605** | 0.405 | 0.995 | 1     | 1     | 1     | 0.560 |
| 15       | 0.5    | 0.11  | 0.700 | **0.775** | 0.730 | 0.745 | 0.460 | 1     | 1     | 1     | 1     | 0.545 |
| 10       | 0.1    | 0.11  | 0.540 | **0.650** | 0.635 | 0.590 | 0.365 | 1     | 1     | 1     | 1     | 0.630 |
| 10       | 0.9    | 0.11  | 0.340 | 0.480 | **0.515** | 0.450 | 0.275 | 0.920 | 1     | 1     | 1     | 1     | 0.290 |
| 10       | 0.5    | 0.03  | **0.530** | 0.470 | 0.430 | 0.520 | 0.350 | 0.900 | **0.970** | **0.970** | **0.970** | 0.630 |
| 10       | 0.5    | 0.21  | 0.530 | 0.725 | **0.780** | 0.680 | 0.340 | 0.990 | 1     | 1     | 1     | 1     | 0.850 |
The above scenarios concentrated on sustained anomalies in network density, and we compare such an anomaly with one that gradually increases network density over the duration of the anomaly or CPL. In the next set of scenarios, the odds ratio is gradually increased from 1 to 12 in a DLSM.
network and from 1 to 3.5 in a DDCSBM network for a sub-network of 39 nodes. CPL varies from 15, 20, to 25 with 20 as a standard duration of anomaly.

DR results are reported in Tables 9 and 10 below with AUC results in Tables 22 and 23 in Appendix A.3. Best performers are in bold, and in cases with a gradual change in the odds ratio, we see $W_t$, $D_t$, $M^-_t$, and $M^+_t$ all nearly detecting this embedded anomaly well. The scan statistic detects less so in binary cases and rather well in count settings. AUC Results from Tables 22 and 23 in Appendix A.3 are fairly similar from the sustained versions above. Therefore, between sustained and gradual changes, monitoring $M^+_t$ does fairly best in most DLSM settings while monitoring $W_t$ still detects best in DDCSBM settings in detecting (DR) and detecting well (AUC).

**Table 9: DR for DLSM with 39 Anomalous Nodes with $OR \in [1,12]$.**

| Settings | Binary | Count |
|----------|--------|-------|
| CPL  | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ |
| 15     | 0.5    | 0.11  | 0.920 | 0.990 | 0.985 | 0.990 | 0.550 | 1 | 1 | 1 | 1 | 0.975 |
| 20     | 0.5    | 0.11  | 0.965 | 0.985 | 0.985 | 0.985 | 0.645 | 1 | 1 | 1 | 1 | 0.975 |
| 25     | 0.5    | 0.11  | 0.965 | 1 | 1 | 1 | 1 | 0.665 | 1 | 1 | 1 | 1 | 0.985 |
| 20     | 0.1    | 0.11  | 0.980 | 0.995 | 0.990 | 0.995 | 0.675 | 1 | 1 | 1 | 1 | 0.965 |
| 20     | 0.9    | 0.11  | 0.735 | 0.955 | 0.955 | 0.910 | 0.520 | 1 | 1 | 1 | 1 | 0.735 |
| 20     | 0.5    | 0.03  | 0.945 | 0.920 | 0.880 | 0.950 | 0.510 | 1 | 1 | 1 | 1 | 0.955 |
| 20     | 0.5    | 0.21  | 0.980 | 1 | 1 | 1 | 1 | 0.655 | 1 | 1 | 1 | 1 | 0.980 |

**Table 10: DR for DDCSBM with 39 Anomalous Nodes with $OR \in [1,3.5]$.**

| Settings | Binary | Count |
|----------|--------|-------|
| CPL  | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ |
| 15     | 0.5    | 0.11  | 1 | 1 | 1 | 1 | 0.830 | 1 | 1 | 1 | 1 | 0.920 |
| 20     | 0.5    | 0.11  | 1 | 1 | 0.995 | 1 | 0.880 | 1 | 1 | 1 | 1 | 0.940 |
| 25     | 0.5    | 0.11  | 1 | 1 | 1 | 1 | 0.880 | 1 | 1 | 1 | 1 | 0.865 |
| 20     | 0.1    | 0.11  | 1 | 1 | 1 | 1 | 0.910 | 1 | 1 | 1 | 1 | 0.915 |
| 20     | 0.9    | 0.11  | 1 | 1 | 0.995 | 1 | 0.710 | 1 | 1 | 1 | 1 | 0.815 |
| 20     | 0.5    | 0.03  | 1 | 0.985 | 0.970 | 1 | 0.775 | 1 | 0.985 | 0.96 | 1 | 0.815 |
| 20     | 0.5    | 0.21  | 1 | 1 | 0.995 | 1 | 0.965 | 1 | 1 | 1 | 1 | 0.990 |

4.5 Performance Evaluation with Anomalies in Expected Degree

In attempts to manipulate maximum degree, we target effects of parameters in a DLSM and DDCSBM related to degree(s) of a node(s), which mainly affects expected degree. Such parameters are $r_i$ in a DLSM, which represents a radius of communication in the latent space for node $i$ that indirectly affects degree, and $\theta_{it}$ in a DDCSBM, which represents a propensity for node $i$ to communicate at time $t$ that directly affects degree. Recall, propensities of communication are
a degree-correction for the stochastic block model. First note is having radii in a DLSM time dependent neither drastically changes nor improves model fitting as discussed by Sewell and Chen [2015]. Second note is that correlation via time in a DLSM is incorporated via latent positions and not $r_i$, whereas, correlation via time for a DDCSBM is carried by degree parameters, $\Theta$. By manipulating degree parameters directly in a DDCSBM model, we cannot simply change the value of $\theta_i$ at anomalous time points. Rather, $\theta_i t$ is multiplied by some constant, $C$, throughout the anomalous time period. Table 11 summarizes considered scenarios affecting a certain number of nodes and the settings for model parameters. For a DLSM, $r_i$ are directly manipulated for a binary (B) and count (C) network, while in a DDCSBM, $\theta_i t$ are multiplied by some constant $C$. Shift size indicates a relatively medium or large change in expected degrees.

| Shift Size | Number of Nodes | $r_i = 0.1$ to $C \cdot \Theta$ |
|------------|----------------|---------------------------------|
| Medium     | 15             | 0.020(B); 0.04(C) 2.25           |
| Large      | 35             | 0.015(B); 0.0225(C) 1.75          |

Table 11: Scenarios for Simulation Study Targeting Max Degree via Node Parameters.

Let $N$ denotes number of nodes affected by a given change. An example using $N = 15$ with $r_i = 0.04(C)$ is shown in a count DLSM in Figure 8 with $\phi = 0.5$, $E[W_t] = 0.11$, and settings mentioned in 4. In Figure 8 the $N = 15$ affected nodes are enlarged and colored red with only the associated edges to and from those nodes displayed in red as well. A slight increase in communication among the $N = 15$ nodes can be viewed during an anomalous time within the change point interval. Note that not each time in the change point interval will have those nodes affected with high degree. Shewhart control charts for this example are plotted for $W_t$ and $D_t$ with Phase I data as $t \leq 50$ in Figure 9. In this example, the network density signals some with a decrease in network density in the DLSM setting, Figure 9(a). Figures 9 (c) and (d) show $D_t$ signaling during the change point interval along with some false alarms.

Figure 8: Edges of 15 Nodes in Count DLSM Networks (a) Before and (b) During an Anomaly Targeting Max Degree. A subset of 15 nodes (large red vertices) have $r_i = 0.01$ increased to $r_i = 0.04$ for the anomaly. Only edges from those 15 nodes are displayed, and an increase in communication of those 15 nodes is observed.
Figure 9: Shewhart Individuals Control Charts of $W_t$ and $D_t$ for an Anomaly Targeting Expected Degree in Count Settings with $\phi = 0.5$ and $E[W_t] = 0.11$. Plots (a) and (c) shows $W_t$ and $D_t$ in a DLSM setting, and plots (b) and (d) shows $W_t$ and $D_t$ in a DDCSBM setting. Limits $q$ are determined using setting empirical conditional false alarm probabilities at 0.03.

Results of 200 simulations with $n = 100$, $T = 110$, and $CPL = 5, 10, \text{ or } 15$ are summarized below in Tables 12 - 15 using DR and in Appendix A.4 Tables 18 - 21 using AUC. The best performing statistic is in bold. Since anomalies planted affect expected degree (and desirably maximum degree), a natural best performing statistic would be $D_t$. As with anomalies in edge probabilities, results are explained first by model and network type settings and then across trends in CPL, correlation, and sparsity. In nearly all of DLSM and DDCSBM settings, $M_t^-$ has the highest detection rates. In the medium shift size count DLSM settings, $D_t$ and $M_t^+$ detect just as well. In medium shift size DDCSBM settings, $D_t$, $M_t^-$, and $M_t^+$ all detect perfectly with a DR of 1, detection rates across the board in the large shift size tend to be lower than the medium shift size. The worst performer in all scenarios is $W_t$, with the highest DR of 35%. The second worst performer is $S_t^*$, but it performs especially well in a count DLSM with a longer CPL and count DDCSBM with a medium shift size. There is noticeably less effect from duration, correlation, and sparsity for most statistics. However, the scan statistic appears to be detect better with a longer CPL in DLSM settings. In general, most
methods outside of $W_t$ have high detection rates when planting an anomaly in expected degree. Unlike in the case where anomalies are embedded in edge probabilities, DR in DDCSBM count settings is better. This suggests the loss of information is slightly more difficult to capture in binary DDCSBM settings.

In AUC results reported in Tables 24-27 in Appendix A.4 the detection amount is mainly best in $M_t^-$ and $D_t$ in both DLSM and DDCSBM settings. The worst performer is $W_t$ with second worst being $S_t^*$ across all settings. In general $W_t$ and $S_t^*$ suffer about a 30% to 60% loss in AUC compared to other summary statistics. Across duration of anomaly (CPL) and correlation ($\phi$), there is little difference in AUC values. It appears to be difficult to detect an anomaly when there is less communication (greater sparsity) in the network. For AUC values, DLSM count settings tend to have lower AUC than in DLSM binary settings. This suggests that a change in radius of communication is not as easy to detect in count settings.

Table 12: DR for DLSM from $r_i = 0.1$ to $r_i = 0.020$ (B); 0.04 (C) for $N = 15$.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M_t^-$ | $M_t^+$ | $S_t^*$ | $W_t$ | $D_t$ | $M_t^-$ | $M_t^+$ | $S_t^*$ |
| 5 | 0.5 | 0.11 | 0.045 | 1 | 1 | 0.995 | 0.375 | 0.00 | 0.990 | 1 | 0.985 | 0.380 |
| 10 | 0.5 | 0.11 | 0.130 | 1 | 1 | 0.995 | 0.460 | 0.01 | 1 | 1 | 1 | 0.690 |
| 15 | 0.5 | 0.11 | 0.120 | 1 | 1 | 1 | 0.630 | 0.02 | 1 | 1 | 1 | 0.845 |
| 10 | 0.1 | 0.11 | 0.115 | 1 | 1 | 1 | 0.530 | 0.01 | 1 | 1 | 1 | 0.610 |
| 10 | 0.9 | 0.11 | 0.055 | 0.990 | 1 | 0.930 | 0.500 | 0.01 | 0.995 | 0.995 | 0.960 | 0.685 |
| 10 | 0.5 | 0.03 | 0.175 | 0.995 | 1 | 0.970 | 0.440 | 0.07 | 0.905 | 0.940 | 0.845 | 0.780 |
| 10 | 0.5 | 0.21 | 0.065 | 1 | 1 | 1 | 0.425 | 0.00 | 1 | 1 | 1 | 0.630 |

Table 13: DR for DLSM from $r_i = 0.1$ to $r_i = 0.015$ (B); 0.0225 (C) for $N = 35$.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M_t^-$ | $M_t^+$ | $S_t^*$ | $W_t$ | $D_t$ | $M_t^-$ | $M_t^+$ | $S_t^*$ |
| 5 | 0.5 | 0.11 | 0.095 | 0.970 | 1 | 0.845 | 0.345 | 0.065 | 0.940 | 0.995 | 0.860 | 0.465 |
| 10 | 0.5 | 0.11 | 0.130 | 0.980 | 1 | 0.920 | 0.530 | 0.075 | 0.995 | 1 | 0.960 | 0.655 |
| 15 | 0.5 | 0.11 | 0.220 | 0.995 | 1 | 0.965 | 0.620 | 0.120 | 1 | 1 | 0.980 | 0.945 |
| 10 | 0.1 | 0.11 | 0.085 | 1 | 1 | 0.955 | 0.465 | 0.095 | 0.975 | 0.990 | 0.945 | 0.715 |
| 10 | 0.9 | 0.11 | 0.055 | 0.940 | 1 | 0.805 | 0.430 | 0.050 | 0.950 | 0.990 | 0.890 | 0.680 |
| 10 | 0.5 | 0.03 | 0.205 | 0.930 | 0.950 | 0.865 | 0.495 | 0.165 | 0.745 | 0.770 | 0.705 | 0.680 |
| 10 | 0.5 | 0.21 | 0.060 | 0.995 | 1 | 0.865 | 0.375 | 0.030 | 1 | 1 | 0.985 | 0.600 |
Table 14: DR for DDCSBM from $C = 1$ to $C = 2.25$ in $C \cdot \Theta$ for $N = 15$.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M_t^-$ | $M_t^+$ | $S_t^*$ |
| 5 | 0.5 | 0.11 | 0.085 | 1 | 1 | 1 | 0.770 |
| 10 | 0.5 | 0.11 | 0.155 | 1 | 1 | 1 | 0.780 |
| 15 | 0.1 | 0.11 | 0.130 | 1 | 1 | 1 | 0.810 |
| 10 | 0.9 | 0.11 | 0.140 | 0.990 | 0.995 | 0.990 | 0.735 |
| 10 | 0.5 | 0.03 | 0.330 | 0.990 | 0.990 | 0.985 | 0.580 |
| 10 | 0.5 | 0.21 | 0.090 | 1 | 1 | 1 | 0.850 |
| 10 | 0.5 | 0.21 | 0.090 | 1 | 1 | 1 | 0.850 |

Table 15: DR for DDCSBM from $C = 1$ to $C = 1.75$ in $C \cdot \Theta$ for $N = 35$.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M_t^-$ | $M_t^+$ | $S_t^*$ |
| 5 | 0.5 | 0.11 | 0.145 | 0.950 | 0.965 | 0.935 | 0.585 |
| 10 | 0.5 | 0.11 | 0.215 | 0.975 | 0.990 | 0.965 | 0.625 |
| 15 | 0.5 | 0.11 | 0.245 | 1 | 1 | 0.985 | 0.650 |
| 10 | 0.1 | 0.11 | 0.185 | 0.970 | 0.970 | 0.955 | 0.650 |
| 10 | 0.9 | 0.11 | 0.165 | 0.985 | 0.990 | 0.975 | 0.560 |
| 10 | 0.5 | 0.03 | 0.330 | 0.845 | 0.855 | 0.815 | 0.530 |
| 10 | 0.5 | 0.21 | 0.075 | 0.980 | 0.990 | 0.965 | 0.630 |

As before in anomalies within edge probabilities, we compare sustained changes in expected degree to a gradual increase of expected degree over the duration of the anomaly (CPL). In the last set of scenarios, $r_i$ is increased from $r_i = 1/n$ to $4/n$ in DLSM setting and $C$ is increased from $C = 1$ to $5$ in a DDCSBM setting. CPL varies from 15, 20, to 25 with 20 as a standard duration of anomaly.

DR results are reported in Tables 16 and 17 below with AUC results in Tables 28 and 29 in Appendix A.4. Similar to the sustained change case, $M_t^-$ and $D_t$ detect this change the best in DLSM settings. However, $D_t$, $M_t^-$, and $M_t^+$ all detect perfectly with a DR of 1 in DDCSBM settings. The worst performance is in $W_t$, and scan statistics detect poorly in DLSM settings than in DDCSBM settings. One reason $S_t^*$ might be performing poorly is because of window contamination as discussed in Zhao et al. [2018b]. Since the same subset of nodes are increasing slowly in expected degree, this effect is already captured in a given window and subsequent moving windows. Thus, the standardization process will have already included the anomaly, which makes detecting a gradual change much more difficult. In DDCSBM settings, the same possible window contamination does not affect detection rates as in DLSM settings. AUC results are in Tables 28 and 29 in Appendix A.4. The best performer in AUC is $M_t^-$ for targeting anomalies that are aimed to affect maximum degree. As seen in the sustained case in DLSM settings, changes in radii are
not as sensitive in count networks. In some instances, AUC increases, but in others, it decreases. For DDCSBM settings, count networks tend to have higher AUC than in binary networks, which suggests the loss of information in binary networks makes the change slightly harder to detect.

Table 16: DR for DLSM from $r_i \in [1/n, 4/n]$ for $N = 20$.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL      | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ |
| 15       | 0.5    | 0.11   | 0.065 | 1     | 1     | 0.890 | 0.250 | 0.110 | 0.955 | 0.985 | 0.910 | 0.345 |
| 20       | 0.5    | 0.11   | 0.095 | 1     | 1     | 0.910 | 0.305 | 0.170 | 0.975 | 0.995 | 0.950 | 0.410 |
| 25       | 0.5    | 0.11   | 0.135 | 1     | 1     | 0.970 | 0.300 | 0.175 | 0.995 | 0.995 | 0.975 | 0.465 |
| 20       | 0.1    | 0.11   | 0.090 | 1     | 1     | 0.945 | 0.345 | 0.185 | 0.995 | 1     | 0.975 | 0.400 |
| 20       | 0.9    | 0.11   | 0.060 | 0.98  | 1     | 0.730 | 0.320 | 0.100 | 0.990 | 0.995 | 0.935 | 0.485 |
| 20       | 0.5    | 0.03   | 0.150 | 1     | 1     | 0.985 | 0.440 | 0.245 | 0.790 | 0.795 | 0.720 | 0.490 |
| 20       | 0.5    | 0.21   | 0.050 | 1     | 1     | 0.735 | 0.165 | 0.180 | 0.995 | 0.995 | 0.985 | 0.365 |

Table 17: DR for DDCSBM from $C \in [1, 5]$ in $C \cdot \Theta$ for $N = 20$.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL      | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ |
| 15       | 0.5    | 0.11   | 0.065 | 1     | 1     | 1     | 0.825 | 0.180 | 1     | 1     | 1     | 0.865 |
| 20       | 0.5    | 0.11   | 0.135 | 1     | 1     | 1     | 0.825 | 0.170 | 1     | 1     | 1     | 0.865 |
| 25       | 0.5    | 0.11   | 0.110 | 1     | 1     | 1     | 0.770 | 0.195 | 1     | 1     | 1     | 0.825 |
| 20       | 0.1    | 0.11   | 0.105 | 1     | 1     | 1     | 0.775 | 0.155 | 1     | 1     | 1     | 0.855 |
| 20       | 0.9    | 0.11   | 0.120 | 1     | 1     | 1     | 0.685 | 0.205 | 1     | 1     | 1     | 0.815 |
| 20       | 0.5    | 0.03   | 0.265 | 1     | 1     | 1     | 0.710 | 0.310 | 1     | 1     | 1     | 0.760 |
| 20       | 0.5    | 0.21   | 0.080 | 1     | 1     | 1     | 0.700 | 0.110 | 1     | 1     | 1     | 0.830 |

4.6 Overall summary of results

In the light of the results from our performance evaluation study, we make the following general observations.

1. Summary statistics like network density, maximum degree, and their linear combinations can be valuable and effective monitoring tools for detecting anomalous changes in time-evolving networks. In particular, such summary statistics can be much more powerful and accurate than more complicated and computationally expensive monitoring techniques like the scan statistic of [Priebe et al., 2005]. This remarkable fact is demonstrated throughout our study and establishes the value of summary statistics in network monitoring.

2. Network density ($W_t$) is effective in detecting changes in the odds ratio (Tables 5–10), but ineffective in detecting changes in individual node behavior (Tables 12–17). This is consistent
with what one would expect, as changes in individual node behavior do not significantly affect
the overall network density.

3. Maximum degree \((D_t)\) is effective in detecting changes in individual node behavior (Tables 12–17), which is expected. In addition, maximum degree is also effective in detecting changes in odds ratio (Tables 5–10), often performing close to or better than network density. This makes maximum degree a versatile summary statistic for network monitoring.

4. The linear combinations \((M_t^-\text{ and } M_t^+)\) can combine the strengths of network density and maximum degree. For example, when the anomaly reflects an increase in the odds ratio, there is an increase in both network density and maximum degree. In such cases, the detection rates of \(M_t^+\) are often higher than maximum degree (Tables 5–10). Furthermore, when the anomaly consists of change in individual node behavior, the detection rates of \(M_t^-\) and \(M_t^+\) are much higher than network density (Tables 12–17).

5. Another way to combine the strengths of network density and maximum degree would be to consider \((W_t, D_t)\) as a bivariate summary statistic and employ bivariate process monitoring methods. To accomplish this, we need to consider the covariance between \(W_t\) and \(D_t\) and update the calibration of \(\bar{x}, s,\) and \(q\) accordingly to construct bivariate control limits. This seems to be a very promising approach that we plan to explore in future work.

5 Conclusion

Anomaly detection in temporally-evolving networks is an active area of research, but often times, subject to a specific network model. In this work, we explore network monitoring approaches on calculated summary statistics using a comprehensive simulation study. Performance evaluations of summary statistics, density, maximum degree, and linear combinations of density and maximum degree, are compared to that of the scan statistic. To introduce interesting complexities, common temporally-evolving network models, DLSMs and DDCSBMs, were tweaked to incorporate correlation over time. This correlation better models phenomena in time-varying networks as opposed to independent snapshots over time.

In evaluating performance, metrics such as detection rates and area under a receiver-operating curve suggest that simple, relatively easy-to-compute summary statistics outperform the more sophisticated, difficult-to-implement method of the scan statistic. Albeit, the measures of success analyzed may not be best suited for documenting the advantages of a scan statistic, the scan statistic is, by construct, vulnerable to missing gradual changes in networks, i.e., window contamination. Also, the types of planted anomalies in our simulations resulted from intentional changes in edge probabilities and expected degree. Specifically, adjustments were made to the odds ratios of edge probabilities and model parameters governing expected degree. While \(W_t\) performs better to detect anomalies resulting from changes in edge probabilities, \(W_t\) performed the worst with anomalies concerning expected degree. Maximum degree \((D_t)\), however, does fairly well in both scenarios, yet linear combinations of \(W_t\) and \(D_t\), \(M_t^-\) and \(M_t^+\), perform the best in both scenarios.

The summarize, this paper demonstrates that monitoring summary statistics has clear advantages. They are simple to calculate, easy to interpret, and able to catch several types of anomalies. That is, based on results from a detailed simulation study, summary statistics showed effective in detecting anomalies under varying conditions pertaining to the following: anomaly duration,
correlation, sparsity, network types, and network models. Admittingly, summary statistics will not catch some anomalies that do not impact the statistics directly; e.g., extreme-node-switching, where two nodes that have, say, the maximum degree and minimum degrees in a network at time \( t \), swap at time \( t + 1 \). To catch such an anomaly would require detailed modeling efforts, whereas the model-free approach presented here with summary statistics saves time and fosters consistency across efforts in detecting anomalies. However, we might improve the effectiveness of summary statistic by considering multivariate, rather than univariate analytic approaches; i.e., at time \( t \), assessing density \( (W_t) \), maximum degree \( (D_t) \), difference \( (M_t^-) \), and/or sum \( (M_t^+) \) jointly. In future work, advancing univariate methods to multivariate methods should be considered.
A Simulations with No Anomaly Additional Figures

A.1 Supplemental Figures for Section 2.2.2

Figure 10: Plot of False Alarm Rates Monitoring \( W_t \) Across Varying Correlation [(a) and (c)] and Sparsity [(b) and (d)] in Count DLSM and DDCSBM Settings.
Figure 11: Plot of False Alarm Rates Monitoring $D_t$ Across Varying Correlation [(a) and (c)] and Sparsity [(b) and (d)] in Count DLSM and DDCSBM Settings.

Figure 12: Plot of False Alarm Rates Monitoring $M_t^-$ Across Varying Correlation [(a) and (c)] and Sparsity [(b) and (d)] in Count DLSM and DDCSBM Settings.
Figure 13: Plot of False Alarm Rates Monitoring $M_t^+$ Across Varying Correlation [(a) and (c)] and Sparsity [(b) and (d)] in Count DLSM and DDCSBM Settings.
A.2 Supplemental Figures for Section 4.3

Figure 14: Plot of Means in a DLSM Binary (B) Setting Across Varying Correlation.
Figure 15: Plot of Means in a DDCSBM Count (C) Setting Across Varying Correlation.

Figure 16: Plot of Means in a DLSM Count (C) Setting Across Varying Sparsity.
Figure 17: Plot of Means in a DDCSBM Binary (B) Setting Across Varying Correlation.

Figure 18: Plot of SD in a DLSM Binary (B) Setting Across Varying Correlation.
Figure 19: Plot of SD in a DDCSBM Count (C) Setting Across Varying Correlation.

Figure 20: Plot of SD in a DLSM Count (C) Setting Across Varying Sparsity.
Figure 21: Plot of SD in a DDCSBM Binary (B) Setting Across Varying Correlation.
### A.3 AUC Results for Section 4.4

Results of 200 simulations with $n = 100$, $T = 110$, and $CPL = 5, 10, \text{ or } 15$ are summarized in Tables 18-21 using AUC. The method which detects the embedded anomaly best are in bold.

**Table 18: AUC for DLSM with 33 Anomalous Nodes and OR from 1 to 4.**

| Settings | Binary | Count |
|----------|--------|-------|
| CPL | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ |
| 5 | 0.5 | 0.11 | 0.649 | 0.667 | 0.648 | **0.670** | 0.598 | 0.937 | 0.992 | **0.994** | 0.990 | 0.877 |
| 10 | 0.5 | 0.11 | 0.641 | 0.664 | 0.652 | **0.665** | 0.584 | 0.933 | 0.994 | **0.995** | 0.991 | 0.768 |
| 15 | 0.5 | 0.11 | 0.658 | 0.665 | 0.645 | **0.671** | 0.547 | 0.935 | 0.993 | **0.994** | 0.991 | 0.681 |
| 10 | 0.1 | 0.11 | 0.652 | 0.667 | 0.648 | **0.672** | 0.576 | 0.933 | 0.993 | **0.995** | 0.990 | 0.759 |
| 10 | 0.9 | 0.11 | 0.695 | 0.697 | 0.668 | **0.705** | 0.546 | 0.936 | 0.993 | **0.994** | 0.991 | 0.667 |
| 10 | 0.5 | 0.03 | 0.645 | 0.615 | 0.595 | 0.629 | 0.554 | 0.727 | 0.751 | 0.745 | **0.755** | 0.618 |
| 10 | 0.5 | 0.21 | 0.655 | 0.720 | **0.723** | 0.707 | 0.570 | 0.871 | 0.937 | 0.932 | **0.938** | 0.706 |

**Table 19: AUC for DLSM with 79 Anomalous Nodes and OR from 1 to 2.5.**

| Settings | Binary | Count |
|----------|--------|-------|
| CPL | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ | $W_t$ | $D_t$ | $M^-_t$ | $M^+_t$ | $S^*_t$ |
| 5 | 0.5 | 0.11 | **0.935** | 0.883 | 0.798 | 0.909 | 0.792 | 1 | 1 | 0.997 | 1 | 0.932 |
| 10 | 0.5 | 0.11 | **0.929** | 0.870 | 0.781 | 0.899 | 0.704 | 1 | 1 | 0.998 | 1 | 0.790 |
| 15 | 0.5 | 0.11 | **0.928** | 0.868 | 0.773 | 0.898 | 0.614 | 1 | 1 | 0.997 | 1 | 0.649 |
| 10 | 0.1 | 0.11 | **0.925** | 0.864 | 0.773 | 0.894 | 0.702 | 1 | 1 | 0.997 | 1 | 0.787 |
| 10 | 0.9 | 0.11 | **0.942** | 0.882 | 0.780 | 0.914 | 0.601 | 1 | 1 | 0.997 | 1 | 0.719 |
| 10 | 0.5 | 0.03 | **0.900** | 0.779 | 0.698 | 0.826 | 0.653 | **0.952** | 0.926 | 0.914 | 0.934 | 0.706 |
| 10 | 0.5 | 0.21 | **0.944** | 0.929 | 0.840 | 0.941 | 0.730 | **0.999** | 0.992 | 0.978 | 0.996 | 0.771 |
Table 20: AUC for DDCSBM with 33 Anomalous Nodes and OR from 1 to 2.5.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL  | \(\phi\) | \(E[W_t]\) | \(W_t\) | \(D_t\) | \(M_t^-\) | \(M_t^+\) | \(S_t^*\) | \(W_t\) | \(D_t\) | \(M_t^-\) | \(M_t^+\) | \(S_t^*\) |
| 5    | 0.5    | 0.11  | 0.982 | 0.861 | 0.772 | 0.915 | 0.822 | 0.987 | 0.898 | 0.831 | 0.936 | 0.820 |
| 10   | 0.5    | 0.11  | 0.980 | 0.853 | 0.763 | 0.909 | 0.702 | 0.988 | 0.899 | 0.833 | 0.937 | 0.705 |
| 15   | 0.5    | 0.11  | 0.977 | 0.867 | 0.781 | 0.916 | 0.588 | 0.986 | 0.899 | 0.835 | 0.936 | 0.608 |
| 10   | 0.1    | 0.11  | 0.973 | 0.868 | 0.789 | 0.916 | 0.716 | 0.982 | 0.903 | 0.847 | 0.937 | 0.704 |
| 10   | 0.9    | 0.11  | 0.975 | 0.880 | 0.804 | 0.925 | 0.664 | 0.983 | 0.911 | 0.855 | 0.943 | 0.664 |
| 10   | 0.5    | 0.03  | 0.884 | 0.752 | 0.691 | 0.805 | 0.642 | 0.976 | 0.911 | 0.855 | 0.943 | 0.664 |
| 10   | 0.5    | 0.21  | 0.992 | 0.887 | 0.777 | 0.943 | 0.717 | 0.998 | 0.952 | 0.901 | 0.976 | 0.716 |

Table 21: AUC for DDCSBM with 72 Anomalous Nodes and OR from 1 to 1.5.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL  | \(\phi\) | \(E[W_t]\) | \(W_t\) | \(D_t\) | \(M_t^-\) | \(M_t^+\) | \(S_t^*\) | \(W_t\) | \(D_t\) | \(M_t^-\) | \(M_t^+\) | \(S_t^*\) |
| 5    | 0.5    | 0.11  | 0.982 | 0.861 | 0.772 | 0.915 | 0.822 | 0.987 | 0.898 | 0.831 | 0.936 | 0.820 |
| 10   | 0.5    | 0.11  | 0.980 | 0.853 | 0.763 | 0.909 | 0.702 | 0.988 | 0.899 | 0.833 | 0.937 | 0.705 |
| 15   | 0.5    | 0.11  | 0.977 | 0.867 | 0.781 | 0.916 | 0.588 | 0.986 | 0.899 | 0.835 | 0.936 | 0.608 |
| 10   | 0.1    | 0.11  | 0.973 | 0.868 | 0.789 | 0.916 | 0.716 | 0.982 | 0.903 | 0.847 | 0.937 | 0.704 |
| 10   | 0.9    | 0.11  | 0.975 | 0.880 | 0.804 | 0.925 | 0.664 | 0.983 | 0.911 | 0.855 | 0.943 | 0.664 |
| 10   | 0.5    | 0.03  | 0.884 | 0.752 | 0.691 | 0.805 | 0.642 | 0.976 | 0.911 | 0.855 | 0.943 | 0.664 |
| 10   | 0.5    | 0.21  | 0.992 | 0.887 | 0.777 | 0.943 | 0.717 | 0.998 | 0.952 | 0.901 | 0.976 | 0.716 |

AUC results are reported in Tables 22 and 23 below for the gradual change in odds ratio.

Table 22: AUC for DLSM with 39 Anomalous Nodes with OR \(\in [1,12]\).

| Settings | Binary | Count |
|----------|--------|-------|
| CPL  | \(\phi\) | \(E[W_t]\) | \(W_t\) | \(D_t\) | \(M_t^-\) | \(M_t^+\) | \(S_t^*\) | \(W_t\) | \(D_t\) | \(M_t^-\) | \(M_t^+\) | \(S_t^*\) |
| 15   | 0.5    | 0.11  | 0.786 | 0.814 | 0.794 | 0.817 | 0.617 | 0.941 | 0.947 | 0.942 | 0.949 | 0.793 |
| 20   | 0.5    | 0.11  | 0.782 | 0.809 | 0.790 | 0.812 | 0.600 | 0.937 | 0.947 | 0.943 | 0.949 | 0.758 |
| 25   | 0.5    | 0.11  | 0.779 | 0.805 | 0.786 | 0.808 | 0.560 | 0.936 | 0.942 | 0.938 | 0.945 | 0.722 |
| 20   | 0.1    | 0.11  | 0.784 | 0.812 | 0.793 | 0.815 | 0.604 | 0.935 | 0.943 | 0.939 | 0.945 | 0.754 |
| 20   | 0.9    | 0.11  | 0.782 | 0.810 | 0.792 | 0.812 | 0.560 | 0.940 | 0.948 | 0.943 | 0.950 | 0.715 |
| 20   | 0.5    | 0.03  | 0.760 | 0.719 | 0.684 | 0.741 | 0.576 | 0.871 | 0.882 | 0.878 | 0.886 | 0.715 |
| 20   | 0.5    | 0.21  | 0.791 | 0.862 | 0.861 | 0.848 | 0.605 | 0.952 | 0.963 | 0.959 | 0.964 | 0.767 |
Table 23: AUC for DDCSBM with 39 Anomalous Nodes with $OR \in [1, 3.5]$.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL | $\phi$ | $E[W_t]$ | $W_t$ | $D_t$ | $M_t^-$ | $M_t^+$ | $S_t^*$ | $W_t$ | $D_t$ | $M_t^-$ | $M_t^+$ | $S_t^*$ |
| 15 | 0.5 | 0.11 | 0.949 | 0.869 | 0.803 | 0.902 | 0.763 | 0.952 | 0.879 | 0.829 | 0.905 | 0.771 |
| 20 | 0.5 | 0.11 | 0.947 | 0.865 | 0.795 | 0.900 | 0.715 | 0.949 | 0.872 | 0.821 | 0.899 | 0.725 |
| 25 | 0.5 | 0.11 | 0.944 | 0.857 | 0.786 | 0.893 | 0.681 | 0.944 | 0.873 | 0.823 | 0.900 | 0.692 |
| 20 | 0.1 | 0.11 | 0.938 | 0.860 | 0.799 | 0.892 | 0.717 | 0.945 | 0.878 | 0.836 | 0.901 | 0.730 |
| 20 | 0.9 | 0.11 | 0.940 | 0.868 | 0.809 | 0.897 | 0.677 | 0.946 | 0.887 | 0.847 | 0.909 | 0.697 |
| 20 | 0.5 | 0.03 | 0.888 | 0.787 | 0.733 | 0.828 | 0.670 | 0.885 | 0.793 | 0.743 | 0.830 | 0.677 |
| 20 | 0.5 | 0.21 | 0.959 | 0.885 | 0.803 | 0.920 | 0.741 | 0.962 | 0.902 | 0.858 | 0.925 | 0.759 |
### A.4 AUC Results for Section 4.5

Results of 200 simulations with \( n = 100, T = 110, \) and \( CPL = 5, 10, \) or 15 are summarized in Tables 24–27 using AUC.

#### Table 24: AUC for DLSM from \( r_i = 0.1 \) to \( r_i = 0.020 \) (B); 0.04 (C) for \( N = 15 \).

| Settings | Binary | Count |
|----------|--------|-------|
| CPL  | \( \phi \) | \( E[W_t] \) | \( W_t \) | \( D_t \) | \( M_t^- \) | \( M_t^+ \) | \( S_t^* \) | \( W_t \) | \( D_t \) | \( M_t^- \) | \( M_t^+ \) | \( S_t^* \) |
| 5 | 0.5 | 0.1 | 0.324 | 0.931 | **0.982** | 0.853 | 0.508 | 0.069 | 0.874 | **0.931** | 0.801 | 0.439 |
| 10 | 0.5 | 0.1 | 0.337 | 0.931 | **0.980** | 0.859 | 0.466 | 0.068 | 0.877 | **0.934** | 0.805 | 0.480 |
| 15 | 0.5 | 0.1 | 0.323 | 0.929 | **0.981** | 0.853 | 0.494 | 0.069 | 0.875 | **0.933** | 0.805 | 0.524 |
| 10 | 0.1 | 0.1 | 0.327 | 0.930 | **0.981** | 0.856 | 0.505 | 0.068 | 0.882 | **0.939** | 0.811 | 0.473 |
| 10 | 0.9 | 0.1 | 0.322 | 0.926 | **0.980** | 0.850 | 0.507 | 0.063 | 0.879 | **0.937** | 0.806 | 0.501 |
| 10 | 0.5 | 0.03 | 0.395 | 0.805 | **0.855** | 0.760 | 0.510 | 0.195 | 0.663 | **0.705** | 0.625 | 0.563 |
| 10 | 0.5 | 0.21 | 0.277 | 0.953 | **0.997** | 0.849 | 0.494 | 0.027 | 0.942 | **0.983** | 0.864 | 0.447 |

#### Table 25: AUC for DLSM from \( r_i = 0.1 \) to \( r_i = 0.015 \) (B); 0.0225 (C) for \( N = 35 \).

| Settings | Binary | Count |
|----------|--------|-------|
| CPL  | \( \phi \) | \( E[W_t] \) | \( W_t \) | \( D_t \) | \( M_t^- \) | \( M_t^+ \) | \( S_t^* \) | \( W_t \) | \( D_t \) | \( M_t^- \) | \( M_t^+ \) | \( S_t^* \) |
| 5 | 0.5 | 0.1 | 0.348 | 0.827 | **0.924** | 0.733 | 0.485 | 0.194 | 0.795 | **0.860** | 0.729 | 0.474 |
| 10 | 0.5 | 0.1 | 0.364 | 0.839 | **0.929** | 0.749 | 0.501 | 0.209 | 0.812 | **0.871** | 0.747 | 0.517 |
| 15 | 0.5 | 0.1 | 0.369 | 0.835 | **0.927** | 0.747 | 0.491 | 0.209 | 0.805 | **0.867** | 0.740 | 0.574 |
| 10 | 0.1 | 0.1 | 0.362 | 0.835 | **0.928** | 0.745 | 0.499 | 0.209 | 0.805 | **0.866** | 0.741 | 0.511 |
| 10 | 0.9 | 0.1 | 0.362 | 0.845 | **0.934** | 0.756 | 0.478 | 0.193 | 0.803 | **0.866** | 0.737 | 0.521 |
| 10 | 0.5 | 0.03 | 0.413 | 0.718 | **0.771** | 0.674 | 0.502 | 0.307 | 0.623 | **0.653** | 0.595 | 0.530 |
| 10 | 0.5 | 0.21 | 0.311 | 0.869 | **0.980** | 0.734 | 0.502 | 0.134 | 0.862 | **0.933** | 0.774 | 0.484 |
Table 26: AUC for DDCSBM from $C = 1$ to $C = 2.25$ in $C \cdot \Theta$ for $N = 15$.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL  | $\phi$ | $E[\text{W}_t]$ | $W_t$ | $D_t$ | $M^-$ | $M^+$ | $S^*_t$ | $W_t$ | $D_t$ | $M^-$ | $M^+$ | $S^*_t$ |
| 5     | 0.5    | 0.11  | 0.349 | 0.920 | 0.931 | 0.907 | 0.512 | 0.438 | 0.932 | 0.938 | 0.925 | 0.525 |
| 10    | 0.5    | 0.11  | 0.351 | 0.929 | 0.939 | 0.914 | 0.483 | 0.422 | 0.938 | 0.945 | 0.929 | 0.496 |
| 15    | 0.1    | 0.11  | 0.343 | 0.932 | 0.940 | 0.919 | 0.502 | 0.434 | 0.944 | 0.951 | 0.936 | 0.482 |
| 10    | 0.9    | 0.11  | 0.330 | 0.930 | 0.941 | 0.915 | 0.476 | 0.434 | 0.943 | 0.949 | 0.938 | 0.503 |
| 10    | 0.5    | 0.03  | 0.473 | 0.779 | 0.798 | 0.761 | 0.508 | 0.481 | 0.780 | 0.798 | 0.762 | 0.517 |
| 10    | 0.5    | 0.21  | 0.199 | 0.952 | 0.960 | 0.938 | 0.468 | 0.381 | 0.963 | 0.966 | 0.960 | 0.483 |

Table 27: AUC for DDCSBM from $C = 1$ to $C = 1.75$ in $C \cdot \Theta$ for $N = 35$.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL  | $\phi$ | $E[\text{W}_t]$ | $W_t$ | $D_t$ | $M^-$ | $M^+$ | $S^*_t$ | $W_t$ | $D_t$ | $M^-$ | $M^+$ | $S^*_t$ |
| 15     | 0.5    | 0.11  | 0.416 | 0.799 | 0.818 | 0.777 | 0.537 | 0.465 | 0.814 | 0.828 | 0.798 | 0.548 |
| 10     | 0.5    | 0.11  | 0.403 | 0.802 | 0.823 | 0.777 | 0.506 | 0.453 | 0.817 | 0.832 | 0.800 | 0.506 |
| 10     | 0.1    | 0.11  | 0.406 | 0.811 | 0.831 | 0.788 | 0.518 | 0.452 | 0.833 | 0.847 | 0.817 | 0.524 |
| 10     | 0.9    | 0.11  | 0.398 | 0.827 | 0.849 | 0.802 | 0.493 | 0.464 | 0.843 | 0.857 | 0.826 | 0.507 |
| 10     | 0.5    | 0.03  | 0.474 | 0.662 | 0.679 | 0.646 | 0.514 | 0.484 | 0.666 | 0.681 | 0.651 | 0.507 |
| 10     | 0.5    | 0.21  | 0.292 | 0.845 | 0.871 | 0.809 | 0.490 | 0.416 | 0.886 | 0.898 | 0.870 | 0.522 |

Results using AUC are shown in Tables 28 and 29 below for a gradual change in expected degree.

Table 28: AUC for DLSM from $r_i \in [1/n, 4/n]$ for $N = 20$.

| Settings | Binary | Count |
|----------|--------|-------|
| CPL  | $\phi$ | $E[\text{W}_t]$ | $W_t$ | $D_t$ | $M^-$ | $M^+$ | $S^*_t$ | $W_t$ | $D_t$ | $M^-$ | $M^+$ | $S^*_t$ |
| 15     | 0.5    | 0.11  | 0.156 | 0.835 | 0.951 | 0.636 | 0.394 | 0.260 | 0.776 | 0.820 | 0.726 | 0.473 |
| 20     | 0.5    | 0.11  | 0.155 | 0.838 | 0.950 | 0.633 | 0.356 | 0.256 | 0.770 | 0.814 | 0.719 | 0.447 |
| 20     | 0.9    | 0.11  | 0.163 | 0.836 | 0.945 | 0.640 | 0.318 | 0.260 | 0.772 | 0.816 | 0.724 | 0.430 |
| 20     | 0.5    | 0.03  | 0.160 | 0.839 | 0.949 | 0.638 | 0.351 | 0.263 | 0.772 | 0.816 | 0.724 | 0.448 |
| 20     | 0.5    | 0.21  | 0.153 | 0.835 | 0.946 | 0.634 | 0.359 | 0.254 | 0.770 | 0.813 | 0.720 | 0.462 |
| 20     | 0.5    | 0.03  | 0.217 | 0.840 | 0.887 | 0.775 | 0.455 | 0.350 | 0.611 | 0.635 | 0.590 | 0.475 |
| 20     | 0.5    | 0.21  | 0.131 | 0.704 | 0.965 | 0.467 | 0.293 | 0.213 | 0.827 | 0.873 | 0.765 | 0.411 |

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Table 29: AUC for DDCSBM from $C \in [1, 5]$ in $C \cdot \Theta$ for $N = 35$.

| CPL | $\phi$ | $E[W_t]$ | Binary | | Count |
|-----|-------|----------|--------|---|--------|
|     |       |          | $W_t$  | $D_t$ | $M_t^-$ | $M_t^+$ | $S_t^*$ | $W_t$ | $D_t$ | $M_t^-$ | $M_t^+$ | $S_t^*$ |
| 15  | 0.5   | 0.11     | 0.219  | 0.900 | **0.906** | 0.891 | 0.593 | 0.354 | 0.902 | **0.905** | 0.898 | 0.653 |
| 20  | 0.5   | 0.11     | 0.228  | 0.904 | **0.910** | 0.897 | 0.521 | 0.347 | 0.910 | **0.913** | 0.906 | 0.580 |
| 25  | 0.5   | 0.11     | 0.225  | 0.908 | **0.913** | 0.900 | 0.464 | 0.348 | 0.912 | **0.916** | 0.907 | 0.524 |
| 20  | 0.1   | 0.11     | 0.216  | 0.902 | **0.907** | 0.893 | 0.508 | 0.336 | 0.913 | **0.917** | 0.909 | 0.583 |
| 20  | 0.9   | 0.11     | 0.219  | 0.906 | **0.912** | 0.897 | 0.531 | 0.336 | 0.908 | **0.911** | 0.904 | 0.579 |
| 20  | 0.5   | 0.03     | 0.420  | 0.832 | **0.846** | 0.818 | 0.569 | 0.446 | 0.838 | **0.849** | 0.826 | 0.594 |
| 20  | 0.5   | 0.21     | 0.132  | 0.922 | **0.928** | 0.911 | 0.441 | 0.249 | 0.925 | **0.928** | 0.921 | 0.548 |
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