One-step preparation of cluster states in quantum dot molecules

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Cluster states, a special type of highly entangled states, are a universal resource for measurement-based quantum computation. Here, we propose an efficient one-step generation scheme for cluster states in semiconductor quantum dot molecules, where qubits are encoded on singlet and triplet state of two coupled quantum dots. By applying a collective electrical field or simultaneously adjusting interdot bias voltages of all double-dot molecule, we get a switchable Ising-like interaction between any two adjacent quantum molecule qubits. The initialization, the single qubit measurement, and the experimental parameters are discussed, which shows the large cluster state preparation and one-way quantum computation implementable in semiconductor quantum dots with the present techniques.

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Quantum entanglement plays a crucial role in quantum information processing[1], but it is still greatly challenging to create multi-party entanglement experimentally. In 2001, Briegel and Raussendorf introduced a special kind of entangled state, the so-called cluster states[2], which are highly entangled states and can be used to realize universal measurement-based quantum computation[3]. The cluster states thereby serve as a universal resource for any quantum computation. Up to now, many physical systems such as photons, cavity quantum electrodynamics, and superconducting quantum circuits have been shown suitable for preparation of cluster states[4, 5, 6, 7]. Schemes of generating cluster states in solid system have been proposed[8, 9]. Compared with the Ising interaction based one-step cluster state construction, their Heisenberg interaction based construction needs several steps. The exact number of steps depends on the dimension of the lattice with cubic symmetry[8].

Recently, J. M. Taylor et al. proposed a fault-tolerant architecture for quantum computation, where qubits are encoded on singlet and triplet states of double coupled quantum dots, an artificial hydrogen molecule[10]. This encoding protects qubits from low-frequency noise, and suppresses the dominant source of decoherence, the effect of hyperfine interactions[11, 12, 13, 14, 15]. For this two-electron spin qubit, it is also argued that all operations for preparing, protecting, and measuring entangled electron spins can be implemented by local electrostatic gate control[13]. There has been impressive advance in the study of two-electron spin qubit[12, 13, 14, 15, 16, 17].

Following the idea of encoding qubit in two-electron state, here we propose an efficient scheme to prepare cluster states of semiconductor quantum dot molecules. As qubits are encoded on singlet and triplet state of two coupled quantum dots, the Coulomb interaction between quantum dots can be efficiently rewritten as an Ising interaction of nearest-neighbor qubits. By applying a collective electrical field or simultaneously adjusting interdot bias voltages of all double-dot molecules, we can switch on and off this interaction and generate a large cluster state of quantum molecules in just one step. The initialization, the single qubit measurement and the related experimental details for its implementation are also discussed.

Consider a quantum molecule of two coupled quantum dots as Fig. 1. The coupling between the two dots of one molecule is $T_C$. There are three kinds of charge state of this two-electron system: $(2, 0)$, $(1, 1)$, and $(0, 2)$, where

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FIG. 1: Schematic diagram of double-dot spin qubit chain. All quantum dots have same size, and the radius is $r$. The distance between two quantum dots of each quantum molecule is $a$, and the distance between two nearest-neighbor quantum molecules is $b$. A magnetic field $B$ is produced by the left solenoid. The electric field $E$ is applied across the quantum molecule to adjust energy detuning $\varepsilon$ and switch on and off effective interaction. Inset(a) Micrograph of a double-dot sample, consisting of electrostatic gates on the surface of a two-dimensional electron gas (reproduced from Fig.1 of reference[13]). Inset(b) The low energy states in the parameter ranges of interest. See the text for details.
we use notation \((n_l, n_r)\) to indicate \(n_l\) electrons on the “left” dot and \(n_r\) electrons on the “right” dot (as Fig. 1 inset(b)). In our situation, the charge state is transformed between \((0, 2)\) and \((1, 1)\), which is separated in the terms of energy scale by the charge energy \(E_c\), as shown in Fig. 1 (inset(b)). Define a relative bias parameter \(\varepsilon\) to represent the relative energy difference between the \((0, 2)\) and \((1, 1)\) charge states, which can be controlled by applying an electrical field across the two dots or by adjusting voltages on gate \(L\) and \(R\). The charge state \((1, 1)\) includes four spin states: \(|S\rangle = |↑↑\rangle - |↓↑\rangle, |T_0\rangle = |↑↑\rangle + |↑\rangle, |T_+\rangle = |↑↑\rangle, |T_-\rangle = |↓↓\rangle\). With a static magnetic field \(B\), \(T_+\) split from \(T_0\) and \(|S\rangle\) with Zeeman energy and are not considered in the following\([10]\). We encode the qubit on energy degenerate states \(|S\rangle\) and \(|T_0\rangle\). For the charge state \((0, 2)\), we can see below that only the state \(|(0, 2)S\rangle\) is involved within the case of Pauli blockade.

Due to the tunneling between the two dots, the charge states \((0, 2)\) and \((1, 1)\) hybridize. According to the reference\([17]\), we define two superposition states 
\[
|\bar{S}\rangle = \cos \theta |S\rangle + \sin \theta |(0, 2)S\rangle, |\bar{G}\rangle = -\sin \theta |S\rangle + \cos \theta |(0, 2)S\rangle.
\]

The quantum molecule, initially in the state \(|S\rangle\), will be in the adiabatic states \(|\bar{S}\rangle\), when \(\varepsilon\) is swept in a rapid adiabatic passage. To avoid state overlap, this change passage is fast with respect to spin dephasing mechanisms and nuclear-spin induced Larmor precession but slow with respect to the tunnel coupling \(T_c\). The state \(|\bar{G}\rangle\) has higher energy, and the adiabatic angle is
\[
\theta = \arctan \left( \frac{2T_c}{(\varepsilon - \sqrt{4|T_c|^2 + \varepsilon^2}} \right).
\]

In unbiased regime of \(\varepsilon = -E_c/2 \ll -|T_c|\) (\(E_c \approx 5\) meV, \(T_c \approx 0.01\) meV), the adiabatic angle \(\theta \to 0\), and the eigenstate \(|\bar{S}\rangle\to |S\rangle, |\bar{G}\rangle\to |(0, 2)S\rangle\). We introduce parameter \(J\) to indicate the energy difference between state \(|\bar{S}\rangle\) and \(|T_0\rangle\). Then in the case of \(\varepsilon = -E_c/2\), intrinsic exchange term \(J\) between \(|S\rangle\) and \(|T_0\rangle\) is about 4.8 meV. Because \(J\) is very small, it can be neglected when compared to other energy scales such as the nuclear-spin induced Larmor precession of electron spins, \(\Omega \approx 80\) neV\([17]\). At this moment, the charge state is \((1, 1)\), the energy of \((0, 2)\) \(S\) or \((0, 2)\) \(T_0\) much higher than that of \((1, 1)\), and the qubit state is mixed state of \(|S\rangle\) and \(|T_0\rangle\). In the following text, we will demonstrate how to initialize our system in this regime. When \(\varepsilon = E_c/2\), we get \(\theta \to \pi/2\), and the eigenstates \(|\bar{S}\rangle\to |(0, 2)S\rangle\), and \(|\bar{G}\rangle\to |S\rangle\). In the following, we use \(|S\rangle\) instead of \(|(0, 2)S\rangle\) and \(|T_0\rangle\) is rewritten as \(|T\rangle\). As shown in Fig. 1 (inset(b)), the molecule initially in state \(|S\rangle\) can be adiabatically evolved into the state \(|S'\rangle\) when \(\varepsilon\) is swept from \(-E_c/2\) to \(E_c/2\) in rapid adiabatic passage. Because of Pauli blockade, the molecule initially in the state \(|T_0\rangle\) cannot evolve into \(|T\rangle\)\([13, 16, 17]\).

In order to prepare cluster states of quantum molecule, all qubits are needed to be initialized in the state \((|S\rangle + |T_0\rangle)/2 = |\downarrow\rangle\) . Firstly, we set \(\varepsilon = -E_c/2\) so that the charge state of each qubit is \((1, 1)\). With static magnetic field and in the sufficient low temperature, all molecule in the charge state \((1, 1)\) can be initialized to the state \(|\uparrow\rangle\)\([20]\). When the magnetic field \(B\) is inhomogeneous, the left and right quantum dots have different Zeeman splitting. Thus we can choose to flip the electron spin in the right dot with a radio-frequency microwave field resonant with its Zeeman splitting, and change the molecule from the state \(|\uparrow\rangle\) to \(|\downarrow\rangle\).

Adiabatically sweeping \(\varepsilon\) from \(-E_c/2\) to \(E_c/2\), the molecule shifts to charge state \((0, 2)\) if the qubit state is \(|S\rangle\), but remains in charge state \((1, 1)\) if the initial state is \(|T_0\rangle\). This process can also be realized by adiabatically applying an electric field, as this electric field can produce an energy difference \(a\times E\) between the electrons in the left and right dot and change \(\varepsilon\). Here parameter \(a\) represents the distance between the two dots of one molecule. Compared to adjusting bias voltage \(\varepsilon\) by \(L\) and \(R\) gates of each qubit, this electric field can be applied collectively to all qubits.

As shown in Fig. 1 we assume there is only Coulomb interaction between nearest-neighbor quantum molecule. The interaction of non-nearest-neighbor quantum molecule can be neglected as discussed below. Initially the bias voltage \(\varepsilon = -E_c/2\), and all qubits are in the state \(|\uparrow\rangle\). In this case, each molecule is in the charge state \((1, 1)\). The Coulomb interaction between two nearest-neighbor qubits (including four electrons), for example molecule \(i\) and \(i+1\), can be directly described by the Hamiltonian (the Coulomb interaction between two electrons inside each qubit is not included\([18, 19]\)):

\[
H_{int} = \text{diag} \{H_{int_0}, H_{int_0}, H_{int_0}, H_{int_0}\}
\]

in the basis \(|TT\rangle, |TS\rangle, |ST\rangle\) and \(|SS\rangle\), where

\[
H_{int_0} = \frac{1}{4\pi\varepsilon} \left( \frac{2e^2}{b} + \frac{2e^2}{\sqrt{a^2 + b^2}} \right).
\]

where \(\varepsilon\) is dielectric constant of GaAs. When \(\varepsilon\) is adiabatically swept to \(E_c/2\), each qubit will be changed from \((|S\rangle + |T\rangle)/2\) to \((|S\rangle + |T\rangle)/2\). Then there are also four kinds of charge distribution for two nearest-neighbor molecules \(T_i T_{i+1}, S_i T_{i+1}, T_i S_{i+1}, S_i S_{i+1}\), where \(i = 1, 2, 3...\). The corresponding Coulomb interaction \(H_{int}\) can be respectively written as:

\[
H_{TT} = H_{TS'} = H_{S'T} = \frac{1}{4\pi\varepsilon} \left( \frac{2e^2}{b} + \frac{2e^2}{\sqrt{a^2 + b^2}} \right),
\]

\[
H_{S'S'} = \frac{1}{4\pi\varepsilon} \frac{4e^2}{b}.
\]

It is easy to find that \(H_{int_0} = H_{TT} = H_{TS'} = H_{S'T}\). In this case, the interaction between nearest-neighbor qubits can be described by the Hamiltonian

\[
H_{int} = \text{diag} \{H_{int_0}, H_{int_0}, H_{int_0}, H_{S'S'}\}
\]
in the basis $|TT\rangle, |TS\rangle, |S'T\rangle, |S'S\rangle$.

By sweeping $\varepsilon$, the state $|S\rangle$ population adiabatically evolves to $|S'\rangle$, and the interaction between two nearest-neighbor qubits $i$ and $i+1$ changes from $H_0$ to $H_{int} = H_0 + \Delta H_{int}$, where

$$\Delta H_{int} = E_{cc} \frac{1 - \sigma_i^z 1 - \sigma_{i+1}^z}{2}. \tag{6}$$

Here $E_{cc} = H_{S'S'} - H_{int_0}$ is the differential cross-capacitance energy between the two double-dot systems and $\sigma_i^z$ is Pauli matrix on qubit $i$. Obviously, the Hamiltonian $H_0$ can be regarded as a constant background, which contributes to a globe phase and has no effect to the required state preparation. As discussed below, the interactions between non-nearest-neighbor quantum molecules can be also taken into $H_0$ and then be neglected safely.

Then we can get an effective interaction Hamiltonian $\Delta H_{int}$ for nearest-neighbor quantum molecules, which can be switched on and off by adjusting $\varepsilon$ with a collective electrical field. This $\Delta H_{int}$ is an Ising-model Hamiltonian that can be used to create cluster states in one step. The time evolution operator for the qubit chain is then given by

$$U(t) = \exp\left(\frac{i\Delta H_{int}t}{\hbar}\right). \tag{7}$$

Choosing a proper $t_0$, we can have $E_{cc}t_0/\hbar = \pi, 3\pi, 5\pi, \ldots$. The chain of quantum molecules is prepared into cluster state

$$|\Phi_N\rangle = \frac{1}{2^{N/2}} \bigotimes_{i=1}^{N} \left(|0\rangle_i \sigma_i^{(t+1)} + |1\rangle_i \right), i = 1, 2, 3, \ldots \tag{8}$$

where we define $|0\rangle = |S\rangle$ and $|1\rangle = |T\rangle$.

Generally, the sweeping time of $\varepsilon$ should be much smaller than the time $t_0$ to neglect the state evolutions during the $\varepsilon$ shifting process. But it is noted that the effective interaction Hamiltonian and then the differential cross-capacitance energy $E_{cc}$ can be written as a function of $\theta$ as Fig.2(a)

$$E_{cc} = \frac{\sin^2 \theta}{4\pi \varepsilon} \left(\frac{2e^2}{b} - \frac{2e^2}{\sqrt{a^2 + b^2}}\right), \tag{9}$$

where $\theta$ is related with $\varepsilon$ as Eq.[11]

In most of the present experiments, the radius $r$ of the quantum dots is about several hundred nanometers ($r$ is set as 100 nm here) and the interdot distance of double-dot molecule $a$ equals to $2r$. As in Fig.[10] we assume the inter-molecule distance $b$ equals to $10a = 20r$. The distance between the next-nearest-neighbor quantum molecules is thus about $20a$. Sweeping $\varepsilon$ causes the interaction between next-nearest-neighbor quantum molecules changes by about $E_{cc}/10$. Thus the interactions between non-nearest-neighbor qubits can be approximately taken as constant background as $H_0$.[12]. When $\varepsilon$ is adiabatically swept from $-E_c/2$ to $E_c/2$ within a time $\tau_1$ of about 1 ns, the $\theta$ is changed from 0 to $\pi/2$.[13]. Before sweeping it back to $-E_c/2$, $\varepsilon$ is kept on $E_c/2$ for a time $\tau_2$. To generate cluster state, we need $\phi = \int_0^{\tau_1} + \tau_2 E_{cc}(t) dt = \pi \hbar$. As shown in Fig.2(c) $\tau_2$ is about 2 ns. We can further shorten the preparation time $\tau$ if the distance $b$ is smaller.

After the preparation, we should adiabatically sweep $\varepsilon$ back to $-E_c/2$, to return the molecules back to charge state $(1,1)$. Then the effective Hamiltonian $\Delta H_{int}$ is switched off and the cluster state is preserved except for a globe phase evolution governed by $H_0$. To implement universal quantum computation on cluster state, single qubit measurement is needed. As shown in Fig.4(b) the single-qubit measurement of the present molecule can be realized by quantum point contact (QPC) charge measurement. By adjusting individual molecule $\varepsilon$ of the measured molecule from $-E_c/2$ to $E_c/2$ by voltages on gates L and R, the current through QPC reveals the two electrons distribution and then the state of the molecule: the state $|T\rangle$ remains in charge state $(1,1)$ due to Pauli blockade; the state $|S\rangle$ evolves into state $(0,2)S'$.[13]. According to Ref.[18], arbitrary single-qubit rotations can be implemented by adjust the gate voltage of each molecule, we can make any direction measurement by a combination of a single qubit rotation and a $z$ axis measurement. As simultaneously adjusting $\varepsilon$ on nearest-neighbor quantum molecules will switch on the effective interaction between them, we should not simultaneously perform measurement on nearest-neighbor quantum qubits to avoid the backaction of the measurement.

In real system, there are unavoidable background charge noise, nuclear-spin related noise and control elec-
tunnel coupling between double dots and the electrical noise including $1/f$-type noise, which will affect the tunnel coupling between double dots and the effective interaction strength between different molecules. This kind of inhomogeneous interactions throughout the physical lattice will result in imperfect operations. Then an unwanted phase $\delta \phi$ will be added to the desired value $\pi$. The imprecise control of the preparation time $\tau$ will also induce an unwanted phase. Without losing generality, we assume the fluctuation of the phase $\delta \phi$ of each molecule has a Gaussian distribution $G(0, \sigma)$ with average value of zero and variance of $\sigma$.

According to Ref. [23], we can calculate the fidelity of N-qubit cluster states as shown in Fig. 3. Even the noise caused $\delta \phi$ has a variance of $0.03^\pi$, the fidelity of a 20-qubit cluster state can be as high as 95.7%. In addition, the preparation can be completed in a time of about 1 ns, the effect of nuclear spins can be taken as a static background [17]. Actually, this kind of non-uniformity parameter variance between different molecules (including the effect of nuclear spin effect) can be also well treated by adjusting the gate voltage of each molecule, as shown in Ref. [24].

By encoding in singlet and triplet states, qubits are protected from low-frequency noise and the effect of homogeneous hyperfine interactions for double dots. Generally, the coherence time of the singlet and triplet states can be about 10 ns, which is about two times longer than the present cluster state generation time $\tau$. In addition, recent experiments have shown that the lower bound on $T_2$ of double-dot spin qubits could be increased to about 1 $\mu$s with spin-echo techniques [13]. In the Ref. [22], there is detailed analysis for the charge noise induced decoherence for single qubit encoded in double-dot two-spin states. The generalization of the kinds of decoherence analysis to the present molecule arrays in cluster state is still under processing. However, due to the high persistency of cluster state entanglement, the decoherence of large cluster state may not be as serious as we imaged [2, 23].

In conclusion, we have proposed an efficient scheme for generating cluster states of quantum double-dot molecules in one step. By encoding in the singlet and triplet states, the Coulomb interactions are translated into an effective Ising Hamiltonian, which can be switched on and off by a collective electrical field. The discussion of the physical parameters shows that the present scheme is most within the reach of the current experimental techniques.

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FIG. 3: (a) Fidelity against the number of cluster state qubits for variance $\sigma = 0.03^\pi$. (b) Fidelity against the variance of $\delta \theta$ for 20-qubit cluster state.
state of each qubit from (1,1) to (0,2) or (2,0). Guo-Ping Guo, Xiao-Jie Hao, Tao Tu, Zhi-Cheng Zhu and Guang-Can Guo, quant-ph/0703239.

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