Supersymmetric And Smooth Hybrid Inflation

In The Light Of WMAP3

Mansoor ur Rehman, ∗ V. N. Şenoğuz, † and Qaisar Shafi, ‡
Bartol Research Institute, Department of Physics and Astronomy,
University of Delaware, Newark, DE 19716, USA

In their minimal form both supersymmetric and smooth hybrid inflation yield a scalar spectral index $n_s$ close to 0.98, to be contrasted with the result $n_s = 0.951^{+0.015}_{-0.019}$ from WMAP3. To realize better agreement, following Ref. [1], we extend the parameter space of these models by employing a non-minimal Kähler potential. We also discuss non-thermal leptogenesis by inflaton decay and obtain new bounds in these models on the reheat temperature to explain the observed baryon asymmetry.

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I. INTRODUCTION

Supersymmetric (SUSY) hybrid inflation models [2, 3], through their connection to the grand unification scale, provide a compelling framework for the understanding of the early universe. SUSY hybrid inflation is defined by the superpotential

$$W = \kappa \hat{S} (\hat{\Phi} \hat{\Phi} - M^2),$$

where $\hat{S}$ is a gauge singlet and $\hat{\Phi}$, $\hat{\Phi}$ are a conjugate pair of superfields transforming as nontrivial representations of some group $G$. A simple example of the gauge group $G$ can be provided by the standard model gauge group supplemented by a gauged $U(1)_{B-L}$, which requires, from the anomaly cancellations, the presence of three right handed neutrinos. The

∗Electronic address: rehman@udel.edu
†Electronic address: nefer@udel.edu
‡Electronic address: shafi@bxclu.bartol.udel.edu

1 This superpotential was considered in the context of electroweak symmetry breaking in Ref. [5].
Kähler potential can be expanded as

\[ K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 + \kappa S^4/m_p^2 + \cdots \]  

(2)

where \( S, \Phi \) and \( \overline{\Phi} \) are the bosonic components of the superfields, and \( m_p = 2.4 \times 10^{18} \) GeV is the reduced Planck mass.

In these models, if the Kähler potential is assumed to be minimal, the scalar spectral index \( n_s \approx 0.985 \) for the dimensionless coupling \( \kappa \) in the superpotential \( \sim 10^{-2} \), and larger for other values of \( \kappa \). The running of the spectral index \( dn_s/d\ln k \) and the tensor to scalar ratio \( r \) is negligible \([6, 7, 8]\). On the other hand, for negligible \( r \) the WMAP three year central value for the spectral index is \( n_s \approx 0.95 \), and SUSY hybrid inflation with a minimal Kähler potential is disfavoured at a \( 2\sigma \) level \([9]\).

It was recently shown that the spectral index for SUSY hybrid inflation can be substantially modified in the presence of a small negative mass term in the potential. This can result from a non-minimal Kähler potential, in particular from the term proportional to the dimensionless coupling \( \kappa_S \) above \([1]\). Ref. \([1]\) presents the results for \( \kappa \) values \( \gtrsim 10^{-3} \). In this paper we will explore the possible extension of the range of \( \kappa \) to lower values depending on \( \kappa_S \). As we will see increasing the value of \( \kappa_S \) increases the range of \( \kappa \) to lower values, consistent with the measured value of the curvature perturbation amplitude \( R = 4.86 \times 10^{-5} \). This in turn extends the range of other parameters like the symmetry breaking scale \( M \), the inflaton mass \( m_{\text{inf}} \) and the reheat temperature \( T_r \).

The outline of the paper is as follows. In section II we consider SUSY hybrid inflation with a non-minimal Kähler potential. Using the standard constraints, we present our numerical results for the allowed range of \( \kappa, n_s \) and \( M \) for different values of \( \kappa_S \). In section III we consider smooth hybrid inflation, an extension of SUSY hybrid inflation which avoids potential problems associated with topological defects. We again present how the parameters change with \( \kappa_S \). In section IV we discuss non-thermal leptogenesis by inflaton decay and show that enough matter asymmetry can be generated with lower values of reheat temperature for nonzero \( \kappa_S \) in both SUSY and smooth hybrid inflation. We then conclude by reviewing our results in section V.

\[2\] Note however that it is claimed the error contours are in fact considerably larger than shown in Ref. \([9]\), with \( n_s \approx 0.985 \) only disfavoured at a \( 1\sigma \) level \([10]\).
II. SUSY HYBRID INFLATION WITH NON-MINIMAL KÄHLER POTENTIAL

Non-minimal supersymmetric hybrid inflation may be defined by the superpotential given in Eq. (1), together with a general Kähler potential

$$K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 + \kappa_S \frac{|S|^4}{4m_p^2} + \kappa_{S\phi} \frac{|S|^2 |\Phi|^2}{m_p^2} + \kappa_{S\overline{\Phi}} \frac{|S|^2 |\overline{\Phi}|^2}{m_p^2} + \kappa_{SS} \frac{|S|^6}{6m_p^4} + \cdots$$ \hspace{1cm} (3)

The SUGRA scalar potential is given by

$$V_F = e^{K/m_p^2} \left( K^{-1} D_{z_i} W D_{z_j} W^* - 3m_p^{-2} |W|^2 \right)$$ \hspace{1cm} (4)

with $z_i$ being the bosonic components of the superfields $\hat{z}_i \in \{ \hat{\phi}, \hat{S}, \cdots \}$ and where we have defined

$$D_{z_i} W \equiv \frac{\partial W}{\partial z_i} + m_p^{-2} \frac{\partial K}{\partial z_i} W$$

$$K_{ij} \equiv \frac{\partial^2 K}{\partial z_i \partial z_j}$$

and $D_{z_i} W^* = (D_{z_i} W)^*$. In the D-flat direction $|\Phi| = |\overline{\Phi}|$, and using Eqs. (1, 3) in Eq. (4), we get \[11, 12\]

$$V_F = \kappa^2 M^4 \left( 1 - \kappa_S \frac{|S|^2}{m_p^2} + \gamma_S \frac{|S|^4}{2m_p^4} + \cdots \right) + \kappa^2 |\Phi|^2 \left( 2 \left( |S|^2 - M^2 \right) + \cdots \right) + \cdots$$

where $\gamma_S = 1 - \frac{\tau_{SS}}{2} + 2\kappa_S^2 - 3\kappa_{SS}$.

In the following discussion and calculations, we will set all the couplings in the Kähler potential except $\kappa_S$ to zero. The only coupling except $\kappa_S$ which could have a significant effect is $\kappa_{SS}$, since if $\kappa_{SS}$ is large and positive the quartic term becomes negative. The potential in this case is lifted by a higher order term for large values of $\kappa$.

Assuming suitable initial conditions the fields get trapped in the inflationary valley of local minima at $|S| > S_c = M$ and $|\Phi| = |\overline{\Phi}| = 0$, where $G$ is unbroken. The potential is dominated by the constant term $V_0 = \kappa^2 M^4$. Inflation ends when the inflaton drops below its critical value $S_c = M$ and the fields roll towards the global SUSY minimum of the potential $|S| = 0$ and $|\Phi| = |\overline{\Phi}| = M$. In the inflationary trajectory the potential is

$$V_F = \kappa^2 M^4 \left( 1 - \kappa_S \frac{|S|^2}{m_p^2} + \gamma_S \frac{|S|^4}{2m_p^4} + \cdots \right).$$
Taking also into account the radiative correction and soft SUSY breaking terms, the potential is of the following form

$$V \approx V_F + \Delta V_{\text{loop}} + V_{\text{soft}}$$

$$= \kappa^2 M^4 \left( 1 - \kappa_S \left( \frac{M}{m_p} \right)^2 x^2 + \gamma_S \left( \frac{M}{m_p} \right)^2 x^4 + \frac{\kappa^2 N}{8\pi^2 F} \right) + a\kappa M^3 x + a^2 M^2 x^2$$

where

$$\Delta V_{\text{loop}} = \frac{1}{64\pi^2} \text{Str} \left[ M^4(S)(\ln \frac{M^2(S)}{Q^2} - \frac{3}{2}) \right] = \left( \frac{\kappa M}{8\pi^2} \right)^4 N F(x)$$

and

$$V_{\text{soft}} = a\kappa M^3 x + a^2 M^2 x^2$$

with

$$F(x) = \frac{1}{4} \left( (x^4 + 1) \ln \left( \frac{x^4 - 1}{x^4} \right) + 2x^2 \ln \frac{x^2 + 1}{x^2 - 1} + 2 \ln \frac{\kappa^2 M^2 x^2}{Q^2} - 3 \right)$$

and

$$a = \frac{m_{3/2}}{2} \frac{|2 - A| \cos[\arg S + \arg(2 - A)]}{z_0}.$$ 

Here $N$ is the dimensionality of the representation of the fields $\Phi$ and $\Phi$, $Q$ the renormalization scale and $x = |S|/M$. In our numerical calculations we will take $a = 1 \text{ TeV}$.

The number of e-folds after the comoving scale $l$ has crossed the horizon is given by

$$N_l = 2 \left( \frac{M}{m_p} \right)^2 \int_1^{x_l} \frac{V}{\partial_x V} \, dx$$

where $|S_l| = x_l M$ is the value of the field at the comoving scale $l$. During inflation, the comoving scale corresponding to $k_0 = 0.002 \text{ Mpc}^{-1}$ exits the horizon at approximately

$$N_0 = 53 + \frac{1}{3} \ln \left( \frac{T_r}{10^9 \text{ GeV}} \right) + \frac{2}{3} \ln \left( \frac{\sqrt{\kappa M}}{10^{15} \text{ GeV}} \right)$$

where $T_r$ is the reheating temperature, and the subscript ‘0’ indicates that the values are taken at $k_0$.

The amplitude of the curvature perturbation is given by

$$\mathcal{R} = \frac{M}{\sqrt{6\pi m_p^3}} \left( \frac{V^{3/2}}{\partial_x V} \right)_{x = x_0} = 4.86 \times 10^{-5}$$

which is the WMAP normalization at $k_0$. 

FIG. 1: The region in the $\kappa$ and $\kappa_S$ plane satisfying $R = 4.86 \times 10^{-5}$.

For small values of $\kappa$, $x_0$ becomes practically equal to 1 and the radiative term becomes negligible. The soft mass term $a^2M^2x^2$ is likewise negligible. Eq. (11) then yields

$$R = \frac{\kappa^2}{\sqrt{6\pi}} \left( \frac{M}{m_p} \right)^4 \left( \frac{1}{2\kappa\gamma_S \left( \frac{M}{m_p} \right)^5 - 2\kappa\kappa_S \left( \frac{M}{m_p} \right)^3 + \frac{a}{m_p}} \right).$$

Maximizing this expression with respect to $M$ gives us the lower bound on $\kappa$ from $R_{\text{max}} = 4.86 \times 10^{-5}$. For small values of $\kappa_S$ the quartic term is dominant over the quadratic term and these terms become equal for $\kappa_S \sim 2 \times 10^{-4}$. For greater values of $\kappa_S$ the quadratic term becomes dominant. Numerically, we obtain the lower bounds on $\kappa$ as shown in Fig. 1 with

$$\kappa \gtrsim c_1^{5/6} \left( 1 - c_1^{1/3} c_2 \right)^{5/6} \text{ for } \kappa_S < 6 \times 10^{-5},$$

$$\kappa \gtrsim \left( \frac{1}{c_2} \right)^{5/2} \left( 1 - \frac{1}{c_2^3 c_1} \right)^{5/2} \text{ for } \kappa_S > 6 \times 10^{-5},$$

$$\kappa \gtrsim 9.1 \times 10^{-6} \text{ for } \kappa_S \sim 6 \times 10^{-5},$$

where $c_1 = \frac{5b\sqrt{6\pi}R_{\text{max}}}{\left( \frac{2b}{\gamma_S} \right)^{4/5}}$, $c_2 = \frac{4\kappa_S}{5b} \left( \frac{2b}{\gamma_S} \right)^{3/5}$, and $b = \frac{a}{m_p}$.
FIG. 2: $M$ as a function of $\kappa$ for different values of $\kappa_S$ ($N = 1$).

For values of $\kappa$ and $M$ such that we can ignore both the quartic and the loop terms in the potential, $\mathcal{R}$ becomes\(^3\)

$$ \mathcal{R} \approx \frac{\kappa^2}{\sqrt{6\pi}} \left( \frac{M}{m_p} \right)^4 \left( \frac{1}{-2\kappa \kappa_S \left( \frac{M}{m_p} \right)^3 + \frac{a}{m_p}} \right), \quad (13) $$

which for $\kappa_S > 0$ gives the expression for $M$:

$$ M \approx \left( \frac{b}{2\kappa \kappa_S} \right)^{1/3} \left( 1 - \frac{\kappa^2}{b\sqrt{6\pi}\mathcal{R}} \left( \frac{b}{2\kappa \kappa_S} \right)^{4/3} \right)^{1/3} m_p. $$

Maximizing Eq. (12) with respect to $M$, we find $M = (2b/\kappa \gamma_S)^{1/5} m_p$ at the lower bound on $\kappa$. The numerical values of $M$ obtained using Eqs. (5–10) is shown in Fig. 2.

The slow-roll parameters may be defined as Eqs. (14) is shown in Fig. 2

$$ \epsilon = \frac{m_p^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = m_p^2 \left( \frac{V''}{V} \right), \quad \xi^2 = m_p^4 \left( \frac{V'V''}{V^2} \right), \quad (14) $$

where $V'$ denotes the derivative with respect to the normalized real field $\sigma \equiv \sqrt{2}Mx$.

Assuming the slow-roll approximation is valid (i.e. $\epsilon \ll 1, \eta \ll 1$), the spectral index $n_s$ and

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\(^3\) Here we again have $x_0 \approx 1$. 
FIG. 3: $n_s$ as a function of $\kappa$ for different values of $\kappa_S (N = 1)$. The red and pink bands correspond to the WMAP 1$\sigma$ and 2$\sigma$ range [9].

The running of the spectral index $dn_s/d\ln k$ are given by

$$n_s \approx 1 - 6\epsilon + 2\eta, \quad (15)$$

$$\frac{dn_s}{d\ln k} \approx 16\epsilon\eta - 24\epsilon^2 - 2\xi^2. \quad (16)$$

Using Eq. (15), we calculate $n_s$ as a function of $\kappa$ for different values of $\kappa_S$ (Fig. 3). In the $\kappa$ range where the quartic and loop terms are subdominant, from Eq. (13) $n_s$ is approximated to be

$$n_S = 1 - 2\kappa_S + 6\gamma_S \left( \frac{M}{m_p} \right)^2 x_0^2 + \left( \frac{m_p}{M} \right)^2 \left( \frac{\kappa^2 N}{8\pi^2} \right) \partial_x^2 F \bigg|_{x = x_0} \approx 1 - 2\kappa_S.$$ 

This range is represented by the horizontal sections in Fig. 3. For still smaller values of $\kappa$, the quartic term becomes important and the curves bend upward, with $n_s$ becoming greater than 1.\footnote{There is also an upper branch of solutions for $M$ and $n_s$ as functions of $\kappa$, where $n_s$ remains > 1 \cite{4}. We do not display this branch of solutions since it is disfavoured by the WMAP results.} We also plot $n_s$ versus $\log [V^{1/4}/\text{GeV}]$ and $n_s$ for different values of $N$ in Figs. 4, 5. The running of the spectral index is negligible in SUSY hybrid inflation, with $|dn_s/d\ln k| \lesssim 10^{-3}$. 
FIG. 4: $n_s$ as a function of log[$V^{1/4}$GeV] for different values of $\kappa_S$ (N = 1). The red and pink bands correspond to the WMAP 1σ and 2σ range [9].

FIG. 5: $n_s$ as a function of $\kappa$ for different values of $\mathcal{N}$ ($\kappa_S = 0.01$). The red and pink bands correspond to the WMAP 1σ and 2σ range [9].
FIG. 6: An example to show that the change in $\theta \equiv \arg S$ can be controlled by taking suitable initial conditions, $\theta = \theta_0$ and $x = 10x_0$ ($\arg(2 - A) = 0$, $\kappa = 10^{-4}$, $\kappa_S = 0$).

It should be noted that for large enough values of $\kappa_S$ or small enough values of $\kappa$, the potential develops a false minimum at $\sigma > \sigma_0$. Successful inflation then requires the $\sigma$ field to have just enough kinetic energy so that it reaches the local maximum of the potential with negligible kinetic energy. This seems rather improbable since it is realized only for a very narrow band of initial values. Furthermore, we assume that the $A$-term ($a\kappa M^3 x$) in the potential is positive. Since this depends on the value of $\theta \equiv \arg S$, it should be checked whether the change in $\theta$ is small. As displayed in Fig. 6 this is possible but again requires specific initial conditions.

5 Alternatively, if the field is trapped in the false minimum it should tunnel to a point just beyond the local maximum, but this is exceedingly improbable.

6 For large values of $\kappa$ ($\kappa \gtrsim 5 \times 10^{-4}$ for $a = 1$ TeV), the $A$-term in the potential does not play a significant role. However, for smaller values of $\kappa$, the $A$-term is important and its derivative determines $R$ together with the derivative of the radiative term. If the $A$-term is negative it should be subdominant with respect to the radiative term. Since the derivative of the radiative term has a lower bound at $x = 1$ depending on $\kappa$, this condition puts a lower bound on $\kappa \approx 3 \times 10^{-4}$ ($5 \times 10^{-4}$) for $\kappa_S = 0$ ($\kappa_S = 0.01$). There is also a different branch of solutions with higher values of $M$ where instead of the radiative term the quartic term is important. These solutions allow smaller values of $\kappa$, but with the quartic term dominating the spectral index is greater than unity.
Here some discussion of initial conditions is in order. The initial values of the fields can vary in different regions of the universe. Furthermore, the couplings in the Kähler potential are determined by the vacuum expectation values (VEVs) of moduli fields, which can also vary in different regions. The regions with VEVs such that the inflaton mass is suppressed will inflate more and become exponentially large compared to other regions. In this sense, for negative values of $\kappa_S$ so that the potential has a positive $(\text{mass})^2$ term, the smallness of $|\kappa_S|$ ($\kappa_S \ll 1$) can be regarded as a selection effect (see Ref. [13] for a discussion).\footnote{It is worth noting that inflation can be realized using only the MSSM fields, with an apparent tuning of parameters that can be similarly justified [14].}

However, for positive values of $\kappa_S$ the potential has a negative $(\text{mass})^2$ term which can lead to a local maximum. Once the inflaton field is sufficiently close to this local maximum (with negligible kinetic energy), eternal inflation is realized. It would then seem that the regions satisfying the conditions for eternal inflation would always dominate, since even if they are initially rare, their volume will increase indefinitely [15]. It is, however, also possible that there are no regions satisfying these conditions. Alternatively, eternal inflation could occur not only close to the local maximum mentioned above, but also at higher energies regardless of the value of $\kappa_S$. It then becomes notoriously difficult, if not impossible, to compute the probability distribution of observables such as $n_s$, even if the initial distribution of $\kappa_S$ is assumed to be known.\footnote{See Ref. [16] for a recent review of progress in defining probabilities in an eternally inflating spacetime, and Ref. [17] for discussion and computational examples.}

To summarize, it is not clear whether the parameter range explored in this paper is less likely to be observed compared to the minimal Kähler potential or negative $\kappa_S$ cases. Even if we only consider $\kappa_S$ small enough so that the potential remains monotonic, $n_s$ can still be significantly lower compared to the minimal Kähler case for large values of $\kappa$, with $n_s \simeq 0.95$ for $\kappa \gtrsim 0.1$ and $\kappa_S \simeq \kappa/9$.

Finally, we note that for SUSY hybrid inflation there are additional constraints if the symmetry breaking pattern produces cosmic strings [7]. For example, strings are produced when $\Phi, \bar{\Phi}$ break $U(1)_R \times U(1)_{B-L}$ to $U(1)_Y \times Z_2$ matter parity, but not when $\Phi, \bar{\Phi}$ are $SU(2)_R \times U(1)_{B-L}$ doublets. In this section we assumed that cosmic strings are not produced.
III. SMOOTH HYBRID INFLATION

A variation on SUSY hybrid inflation is obtained by imposing a $Z_2$ symmetry on the superpotential, so that only even powers of the combination $\Phi \bar{\Phi}$ are allowed [6, 18]:

$$W = S \left( -v^2 + \frac{(\Phi \bar{\Phi})^2}{M_*^2} \right),$$  \hspace{1cm} (17)

where the dimensionless parameter $\kappa$ is absorbed in $v$. The resulting scalar potential possesses two (symmetric) valleys of local minima which are suitable for inflation and along which the GUT symmetry is broken. The inclination of these valleys is already non-zero at the classical level and the end of inflation is smooth, in contrast to SUSY hybrid inflation. An important consequence is that potential problems associated with topological defects are avoided. This ‘smooth hybrid inflation’ model is similar to the ‘mutated hybrid inflation’ model considered in Ref. [19] and generalized in Ref. [20].

The common VEV at the SUSY minimum $M = |\langle \nu^c_H \rangle| = |\langle \bar{\nu}^c_H \rangle| = (v M_*)^{1/2}$. For $\sigma^2 \gg M^2$, the inflationary potential is given by

$$V \approx v^4 \left[ 1 - \frac{2}{27} \frac{M^4}{\sigma^4} + \frac{\sigma^4}{8 m_p^4} \right],$$  \hspace{1cm} (18)

where the last term arises from the SUGRA correction for a minimal Kähler potential [6]. The soft terms in this case do not have a significant effect on the inflationary dynamics. If we set $M$ equal to the SUSY GUT scale $M_{\text{GUT}} = 2 \times 10^{16}$ GeV, we get $v \approx 1.4 \times 10^{15}$ GeV and $M_* \approx 2.8 \times 10^{17}$ GeV. (Note that, if we express Eq. (17) in terms of the coupling parameter $\kappa$, this value corresponds to $\kappa \sim O(v^2/M_{\text{GUT}}^2) \sim 10^{-2}$.) The value of the field $\sigma$ is $1.1 \times 10^{17}$ GeV at the end of inflation (corresponding to $\eta = -1$) and $\sigma_0 \approx 2.4 \times 10^{17}$ GeV at $k_0$. In the absence of the SUGRA correction (which is small for $M \lesssim 10^{16}$ GeV), $\sigma_0 \propto M^{2/3} m_p^{1/3}$, $R \propto M^{10/3}/(M_*^2 m_p^{4/3})$ and the spectral index is given by [18]

$$n_s \approx 1 - \frac{5}{3 N_0} \approx 0.97.$$  \hspace{1cm} (19)

The SUGRA correction raises $n_s$ from 0.97 to above unity for $M \gtrsim 1.5 \times 10^{16}$ GeV [6].

One problem with this model is that the cutoff scale $M_*$ is close to the inflaton field value $\sigma_0$ for $M \simeq M_{\text{GUT}}$. $M_*$ becomes smaller than $\sigma_0$ for $M \lesssim 10^{16}$ GeV, for which the effective field theory is in general no longer valid. However, with a negative mass term that could result from a non-minimal Kähler potential larger values of $M_*$ are possible. Also, as in SUSY hybrid inflation, the spectral index can have lower values.
FIG. 7: $M_*$ (solid) and $\sigma_0$ (dashed) as functions of the gauge symmetry breaking scale $M$ for smooth hybrid inflation.

For a Kähler potential $K = |S|^2 + |\Phi|^2 + |\Phi|^2 + \lambda|S|^4/4M_*^2 + \ldots$, the potential is obtained as

$$V \approx v^4 \left[ 1 - \frac{2}{27} \frac{M^4}{\sigma^4} - \frac{\kappa_S}{2} \frac{\sigma^2}{m_p^2} + \frac{\gamma_S}{8} \frac{\sigma^4}{m_p^4} \right].$$ (20)

Here we have defined $\kappa_S \equiv \lambda m_p^2/M_*^2$ to express the potential in a form similar to that of the previous section. The $M_*$ and $\sigma_0$ values for different values of $\kappa_S$ is displayed in Fig. 7.

The spectral index $n_s$ for different values of $\kappa_S$ is displayed in Fig. 8. Note that for $\kappa_S = 0$, requiring $\sigma_0 < M_*$ constrains $n_s \gtrsim 0.99$. Having a non-zero $\kappa_S$ allows smaller values of $n_s$ in better agreement with the WMAP3 results. For large enough values of $\kappa_S$ or small enough values of $M$ (the dashed sections in the figure), the potential develops a false minimum at $\sigma > \sigma_0$ as in SUSY hybrid inflation. Again, even with $|\kappa_S|$ small enough so that there is no such false minimum, $n_s$ can be as low as 0.95.
FIG. 8: The spectral index $n_s$ as a function of the gauge symmetry breaking scale $M$ for smooth hybrid inflation. The dashed sections indicate that the field is initially close to a local maximum.

IV. REHEAT TEMPERATURE AND THE GRAVITINO CONSTRAINT

After the end of inflation, the fields fall toward the SUSY vacuum and perform damped oscillations about it. The VEVs of $\Phi$ and $\overline{\Phi}$, along their right handed neutrino components $\nu^c_H$, $\overline{\nu^c}_H$, break the gauge symmetry. The oscillating system, which we collectively denote as $\chi$, consists of the two complex scalar fields $(\delta \nu^c_H + \delta \overline{\nu^c}_H)/\sqrt{2}$ (where $\delta \nu^c_H$, $\delta \overline{\nu^c}_H$ are the deviations of $\nu^c_H$, $\overline{\nu^c}_H$ from $M$) and $S$, with equal mass $m_{\text{inf}}$.

We assume here that the inflaton $\chi$ decays predominantly into right handed neutrino superfields $N_i$, via the superpotential coupling $(1/m_P)\gamma_{ij}\overline{\Phi}\Phi N_iN_j$ or $\gamma_{ij}\overline{\Phi}N_iN_j$, where $i, j$ are family indices. Their subsequent out of equilibrium decay to lepton and Higgs superfields generates lepton asymmetry, which is then partially converted into the observed baryon asymmetry by sphaleron effects.$^9$

The right handed neutrinos, as shown below, can be heavy compared to the reheat temperature $T_r$. Note that unlike thermal leptogenesis, there is then no washout factor since

$^9$ Baryogenesis via leptogenesis was considered in Ref. 21. Non-thermal leptogenesis by inflaton decay was considered in Ref. 22, and for SUSY hybrid inflation in Ref. 23.
lepton number violating 2-body scatterings mediated by right handed neutrinos are out of equilibrium as long as the lightest right handed neutrino mass \( M_1 \gg T_r \) \[24\]. More precisely, the washout factor is proportional to \( e^{-z} \) where \( z = M_1/T_r \) \[25\], and can be neglected for \( z \gtrsim 10 \). Without this assumption, generating sufficient lepton asymmetry would require \( T_r \gtrsim 2 \times 10^9 \) GeV \[26\], and as discussed below this is hard to reconcile with the gravitino constraint.

GUTs typically relate the Dirac neutrino masses to that of the quarks or charged leptons. It is therefore reasonable to assume that the Dirac masses are hierarchical. The low-energy neutrino data indicates that the right handed neutrinos in this case will also be hierarchical in general. As discussed in Ref. \[27\], setting the Dirac masses strictly equal to the up-type quark masses and fitting to the neutrino oscillation parameters generally yields strongly hierarchical right handed neutrino masses \( (M_1 \ll M_2 \ll M_3) \), with \( M_1 \sim 10^6 \) GeV. The lepton asymmetry in this case is too small by several orders of magnitude. However, it is plausible that there are large radiative corrections to the first two family Dirac masses, so that \( M_1 \) remains heavy compared to \( T_r \).

A reasonable mass pattern is therefore \( M_1 < M_2 \ll M_3 \), which can result from either the dimensionless couplings \( \gamma_{ij} \) or additional symmetries. The dominant contribution to the lepton asymmetry is from the decays with \( N_3 \) in the loop, as long as the first two family right handed neutrinos are not quasi-degenerate. Under these assumptions, the lepton asymmetry is given by \[6, 12\]

\[
n_L/s \lesssim 3 \times 10^{-10} \frac{T_r}{m_{\text{inf}}} \left( \frac{M_i}{10^6 \text{ GeV}} \right) \left( \frac{m_{\nu_3}}{0.05 \text{ eV}} \right),
\]

where \( M_i \) denotes the mass of the heaviest right handed neutrino the inflaton can decay into.

From the experimental value of the baryon to photon ratio \( \eta_B \approx 6.1 \times 10^{-10} \) \[9\], the required lepton asymmetry is found to be \( n_L/s \approx 2.5 \times 10^{-10} \) \[28\]. Since \( m_{\text{inf}} > 2M_i \), Eq. \[21\] then yields

\[
T_r \gtrsim 1.6 \times 10^6 \text{ GeV} \left( \frac{0.05 \text{ eV}}{m_{\nu_3}} \right).
\]

This is a general bound valid for non-thermal leptogenesis by inflaton decay, assuming hierarchical right handed neutrinos that are heavy compared to \( T_r \).\[10\] More specific bounds can be

\[10\] Having quasi-degenerate neutrinos increases the lepton asymmetry per neutrino decay \( \epsilon \) \[29\] and thus
FIG. 9: \( M \) as a function of \( m_{\text{inf}} \) for different values of \( \kappa_S \) (\( N = 1 \)).

obtained using the inflaton decay rate \( \Gamma_\chi = (1/8\pi)(M_i^2/M^2)m_{\text{inf}}. \) The reheat temperature \( T_r \) is given by

\[
T_r = \left( \frac{45}{2\pi^2 g_*} \right)^{1/4} (\Gamma_\chi m_p)^{1/2} \approx 0.063 \left( \frac{m_p m_{\text{inf}}}{M} \right)^{1/2} M_i. \tag{23}
\]

For SUSY hybrid inflation the values of \( m_{\text{inf}} = \sqrt{2}\kappa M \) are shown in Fig. 9. Eq. (23) yields the result that \( M_i \) is about 200 (6) times heavier than \( T_r \), for \( \kappa = 10^{-5} \) (10^{-2}) with \( \kappa_S = 0 \). \( M_i/T_r \) decreases slightly for non-zero \( \kappa_S \), with \( M_i/T_r \approx 150 \) (5) for the same \( \kappa \) values and \( \kappa_S = 0.01 \). Thus, small values of \( \kappa \) are consistent with ignoring washout effects as long as the lightest right handed neutrino mass \( M_1 \) is also \( \gg T_r \).

Using the required value of \( n_L/s \) along with Eqs. (21, 23), we can express the \( T_r \) sufficient to generate the observed matter asymmetry in terms of the symmetry breaking scale \( M \) and

allows lower values of \( T_r \) corresponding to lighter right handed neutrinos. Provided that the neutrino mass splittings are comparable to their decay widths, \( \epsilon \) can be as large as 1/2. The lepton asymmetry in this case is of order \( T_r/m_{\text{inf}} \) and sufficient lepton asymmetry can be generated with \( T_r \) close to the electroweak scale.
the inflaton mass $m_{\text{inf}}$:

$$T_r \gtrsim 1.6 \times 10^7 \text{ GeV} \left(\frac{10^{16} \text{ GeV}}{M}\right)^{1/2} \left(\frac{m_{\text{inf}}}{10^{11} \text{ GeV}}\right)^{3/4} \left(\frac{0.05 \text{ eV}}{m_{\nu 3}}\right)^{1/2}. \tag{24}$$

We show the lower bound on $T_r$ calculated using this equation in Fig. 10 (taking $m_{\nu 3} = 0.05 \text{ eV}$). The limit in Eq. (22) is saturated at $\kappa \approx 3 \times 10^{-7}$, where $m_{\text{inf}} = 2M_t$. For smaller values of $\kappa$, sufficient lepton asymmetry cannot be obtained unless the asymmetry is enhanced by having quasi-degenerate neutrinos.

For smooth hybrid inflation, $m_{\text{inf}}$ is given by $2\sqrt{2}v^2/M$. The value of $m_{\text{inf}}$ is shown in Fig. 11. From Eq. (23), $M_t/T_r$ is about 10 (40) for $\kappa_S = 0$ (0.01). We show the lower bound on $T_r$ (taking $m_{\nu 3} = 0.05 \text{ eV}$) in Fig. 12.

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11 Note that the cosmological bound on the sum of the neutrino masses leads to the limit $m_{\nu 3} \lesssim 0.2 \text{ eV}$. 

FIG. 11: The inflaton mass $m_{\text{inf}}$ vs. the symmetry breaking scale $M$ for smooth hybrid inflation. Only those sections satisfying $M > \sigma_0$ and $0.9 < n_s < 1.02$ are shown.

FIG. 12: The lower bound on the reheat temperature $T_r$ vs. the symmetry breaking scale $M$ for smooth hybrid inflation. Only those sections satisfying $M > \sigma_0$ and $0.9 < n_s < 1.02$ are shown.
An important constraint on supersymmetric inflation models arises from considering the reheat temperature $T_r$ after inflation, taking into account the gravitino problem which requires that $T_r \lesssim 10^6$–$10^{10}$ GeV [31]. This constraint on $T_r$ depends on the SUSY breaking mechanism and the gravitino mass $m_{3/2}$. For gravity mediated SUSY breaking models with unstable gravitinos of mass $m_{3/2} \simeq 0.1$–$1$ TeV, $T_r \lesssim 10^6$–$10^9$ GeV [32], while $T_r \lesssim 10^{10}$ GeV for stable gravitinos [33]. In view of these bounds, smooth hybrid inflation is relatively disfavoured compared to SUSY hybrid inflation since $T_r \gtrsim 10^9$ for $M = M_{\text{GUT}}$.\footnote{A new inflation model related to smooth hybrid inflation is discussed in [34] (see also [12]), where the energy scale of inflation $v$ is lower and consequently lower reheat temperatures are allowed.}

Besides the thermal production of gravitinos which puts an upper bound on $T_r$, there are also constraints from gravitinos directly produced by inflaton decay. It was recently pointed out that these constraints can be rather severe for SUSY and smooth hybrid inflation [35], although since the gravitino production depends on the SUSY breaking sector the models are still viable. As displayed in Fig. 9 significantly lower values of $m_{\text{inf}}$ can be obtained with a non-minimal Kähler potential for SUSY hybrid inflation. This extends the allowed range of parameters where the gravitino constraint can be evaded. For smooth hybrid inflation $m_{\text{inf}}$ tends to be higher (Fig. 11).

Finally we note that our estimates for the reheat temperature and matter asymmetry may be affected due to MSSM flat directions delaying the thermalization of inflaton decay products or dominating the energy density of the Universe [36], although it has been argued that the flat directions can decay rapidly due to non-perturbative effects [37]. Also, there can be additional sources of baryon asymmetry such as ‘coherent baryogenesis’ [38].

V. CONCLUSION

We considered supersymmetric hybrid inflation and smooth hybrid inflation models using a general (non-minimal) Kähler potential. The parameter space of the models are extended compared to the minimal Kähler potential case. With a negative mass term in the potential, it is possible to obtain values of the spectral index in the central WMAP3 range. Also, sufficient matter asymmetry can be generated with lower values of the reheat temperature.

In most of the parameter range we consider, the potential develops a false minimum at large field values and successful inflation is then only possible with specific initial conditions.
However, since these initial conditions lead to eternal inflation, it is not clear whether this parameter range is less likely to be observed than the minimal Kähler potential case. Even if we only consider the range for which the potential is monotonic, it is still possible to obtain a spectral index as low as 0.95 with a negligible tensor to scalar ratio. For supersymmetric hybrid inflation this requires $\kappa \gtrsim 0.1$ while the gravitino problem favors smaller values of $\kappa$.

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