Effects of time-varying $\beta$ in SNLS3 on constraining interacting dark energy models

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It has been found that, for the Supernova Legacy Survey three-year (SNLS3) data, there is strong evidence for the redshift-evolution of color-luminosity parameter $\beta$. In this paper, adopting the $w$-cold-dark-matter ($w$CDM) model and considering its interacting extensions (with three kinds of interaction between dark sectors), we explore the evolution of $\beta$ and its effects on parameter estimation. In addition to the SNLS3 data, we also take into account the Planck distance priors data of the cosmic microwave background (CMB), the galaxy clustering (GC) data extracted from SDSS DR7 and BOSS, as well as the direct measurement of Hubble constant from the Hubble Space Telescope (HST) observation. We find that, for all the interacting dark energy (IDE) models, adding a parameter of $\beta$ can reduce $\chi^2$ by $\approx 34$, indicating that $\beta_1 = 0$ is ruled out at $5.8\sigma$ confidence level (CL). Furthermore, it is found that varying $\beta$ can significantly change the fitting results of various cosmological parameters: for all the dark energy models considered in this paper, varying $\beta$ yields a larger $\Omega_{c,0}$ and a larger $w$; on the other side, varying $\beta$ yields a smaller $h$ for the $w$CDM model, but has no impact on $h$ for the three IDE models. This implies that there is a degeneracy between $h$ and $\gamma$. Our work shows that the evolution of $\beta$ is insensitive to the interaction between dark sectors, and then highlights the importance of considering $\beta$'s evolution in the cosmology fits.

I. INTRODUCTION

Cosmic acceleration is one of the biggest puzzles in modern cosmology [1–7]. There are mainly two approaches to explain this extremely counterintuitive phenomenon: dark energy (DE) [8–22] and modified gravity (MG) [23–30]. For recent reviews, see [31–40].

Cosmological observations are of essential importance to understanding cosmic acceleration, and one of the most important observations is Type Ia supernovae (SNe Ia) [41–44]. In 2010, the Supernova Legacy Survey (SNLS) group released their three years data, i.e. SNLS3 dataset [45]. Soon after, using this dataset, Conley et al. [46] and Sullivan et al. [47] presented the SN-only cosmological results and the joint cosmological constraints, respectively. Unlike other supernova (SN) group, the SNLS team treated two important quantities, stretch-luminosity parameter $\alpha$ and color-luminosity parameter $\beta$ of SNe Ia, as free model parameters.

There are many factors that can lead to systematic uncertainties of SNe Ia. One of the most important factors is the potential SN evolution, i.e. the possibility for the redshift evolution of $\alpha$ and $\beta$. The current studies show that $\alpha$ is consistent with a constant, but the hints for the evolution of $\beta$ have been found in [48–52]. For example, in [53], using a linear $\beta(z) = \beta_0 + \beta_1 z$, Mohlabeng and Ralston studied the case of Union2.1 dataset and found that $\beta$ deviates from a constant at 7$\sigma$ confidence levels (CL). In [54], Wang & Wang found that, for the SNLS3 data, $\beta$ increases significantly with $z$ at the 6$\sigma$ CL; moreover, they proved that this conclusion is insensitive to the lightcurve fitter models, or the functional form of $\beta(z)$ assumed in [54]. Therefore, the evolution of $\beta$ is a common phenomenon for various SN datasets, and should be taken into account seriously.

It is very interesting to study the effects of a time-varying $\beta$ on parameter estimation. In [54], Wang, Li & Zhang explored this issue by considering the $\Lambda$-cold-dark-matter ($\Lambda$CDM) model, the $w$CDM model, and the Chevallier-Polarski-Linder (CPL) model. Soon after, Wang, Geng, Hu & Zhang [56] studied the case of holographic dark energy (HDE) model, which is a physically plausible DE candidate based on the holographic principle. It is found that, for all these DE models, adding a parameter of $\beta$ can reduce $\chi^2_{min}$ by $\sim 36$; in addition, considering the evolution of $\beta$ is helpful in reducing the tension between SN and other cosmological observations. It should be mentioned that, in

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phenomenological models are often used in the literature. In this paper we consider the following four cases: 

respectively. Since $\Omega_{m0}(1+z)^3$, $\rho_b = \rho_{b0}(1+z)^3$, $\rho_k = \rho_{k0}(1+z)^2$, $\Omega_r = \Omega_{r0}/(1+z_{eq})$, where $\Omega_{r0} = \Omega_{r0} + \Omega_{b0}$ and $z_{eq} = 2.5 \times 10^8 \Omega_{m0} h^2 (T_{cmb}/2.7K)^{-4}$ (here we take $T_{cmb} = 2.7255K$).

Considering the interaction between dark sectors, the dynamical evolutions of CDM and DE become

\[
\dot{\rho}_c + 3H\rho_c = Q, \quad (3) \\
\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -Q, \quad (4)
\]

where the over dot denotes the derivative with respect to the cosmic time $t$, $p_{de} = w \rho_{de}$ is the pressure of DE, $w$ is the equation of state of DE, and $Q$ is the interaction term, which describes the energy transfer term between CDM and DE. Notice that $a = \frac{1}{1+z}$ and $H = \frac{\dot{a}}{a}$, we have $\frac{d}{dz} = -H(1+z)\frac{d}{dt}$. Then Eqs. (3) and (4) can be rewritten as

\[
(1+z)\frac{d\rho_c}{dz} - 3\rho_c = -Q/H, \quad (5) \\
(1+z)\frac{d\rho_{de}}{dz} - 3(1+w)\rho_{de} = Q/H. \quad (6)
\]

The solutions of these two equations depend on the specific forms of $Q$.

So far, the microscopic origin of interaction between dark sectors is still a puzzle. To study the issue of interaction, phenomenological models are often used in the literature. In this paper we consider the following four cases:

\[
Q_0 = 0, \quad (7) \\
Q_1 = 3\gamma H\rho_c, \quad (8) \\
Q_2 = 3\gamma H\rho_{de}, \quad (9) \\
Q_3 = 3\gamma H\frac{\rho_c \rho_{de}}{\rho_c + \rho_{de}}, \quad (10)
\]

where $\gamma$ is a dimensionless coupling parameter describing the strength of interaction. For simplicity, hereafter we call them $wCDM$ model, $IwCDM1$ model, $IwCDM2$ model, and $IwCDM3$ model, respectively.

For the $wCDM$ model, the reduced Hubble parameter $E(z) \equiv H(z)/H_0$ can be written as

\[
E(z) = (\Omega_{r0}(1+z)^4 + (\Omega_{c0} + \Omega_{b0})(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{de0}(1+z)^3(1+w))^{1/2}. \quad (11)
\]
For the $\Lambda$CDM1 model, Eq. (5) has a general solution
\[
\rho_c = \rho_{c0}(1 + z)^{3(1-\gamma)}.
\] (12)

Substituting Eq. (12) into Eq. (4) and using the initial condition $\rho_{de}(z = 0) = \rho_{de0}$, we get
\[
\rho_{de} = \frac{\gamma \rho_{c0}}{\gamma + w} (1 + z)^{3(1+w)} - (1 + z)^{3(1-\gamma)} + \rho_{de0}(1 + z)^{3(1+w)}.
\] (13)

Then, substituting Eqs. (12) and (13) into Eq. (2), we obtain
\[
E(z) = \left( \Omega_{r0}(1 + z)^4 + \Omega_{b0}(1 + z)^3 + \Omega_{k0}(1 + z)^2 + \Omega_{de0}(1 + z)^{3(1+w)} + \Omega_{c0} \left( \frac{\gamma}{w + \gamma} (1 + z)^{3(1+w)} + \frac{w}{w + \gamma} (1 + z)^{3(1-\gamma)} \right) \right)^{1/2}.
\] (14)

For the $\Lambda$CDM2 model, Eq. (6) has a general solution
\[
\rho_c = \rho_{c0}(1 + z)^{3(1+w+\gamma)}.
\] (15)

Substituting Eq. (15) into Eq. (4) and using the initial condition $\rho_c(z = 0) = \rho_{c0}$, we get
\[
\rho_c = \rho_{c0}(1 + z)^3 + \frac{\gamma \rho_{de0}}{w + \gamma} (1 + z)^3 - \frac{\gamma \rho_{de0}}{w + \gamma} (1 + z)^{3(1+w+\gamma)}.
\] (16)

Then, substituting Eqs. (15) and (16) into Eq. (2), we have
\[
E(z) = \left( \Omega_{r0}(1 + z)^4 + (\Omega_{c0} + \Omega_{b0})(1 + z)^3 + \Omega_{k0}(1 + z)^2 + \Omega_{de0} \left( \frac{\gamma}{w + \gamma} (1 + z)^3 + \frac{w}{w + \gamma} (1 + z)^{3(1+w+\gamma)} \right) \right)^{1/2}.
\] (17)

For the $\Lambda$CDM3 model, the energy densities of CDM and DE satisfy
\[
\rho_c = \rho_{c0}(1 + z)^3 \left( \frac{\rho_{c0}}{\rho_{c0} + \rho_{de0}} + \frac{\rho_{de0}}{\rho_{c0} + \rho_{de0}}(1 + z)^{3(\omega+\gamma)} \right) - \frac{\gamma}{w + \gamma},
\] (18)
\[
\rho_{de} = \rho_{de0}(1 + z)^{3(1+w+\gamma)} \left( \frac{\rho_{c0}}{\rho_{c0} + \rho_{de0}} + \frac{\rho_{de0}}{\rho_{c0} + \rho_{de0}}(1 + z)^{3(\omega+\gamma)} \right) - \frac{\gamma}{w + \gamma}.
\] (19)

Substituting Eqs. (18) and (19) into Eq. (2), we get
\[
E(z) = (\Omega_{c0} C(z)(1 + z)^3 + \Omega_{de0} C(z)(1 + z)^{3(1+w+\gamma)} + \Omega_{r0}(1 + z)^4 + \Omega_{b0}(1 + z)^3 + \Omega_{k0}(1 + z)^2)^{1/2},
\] (20)

where
\[
C(z) = \left( \frac{\Omega_{c0}}{\Omega_{r0} + \Omega_{b0} + \Omega_{de0}} + \frac{\Omega_{de0}}{\Omega_{r0} + \Omega_{b0} + \Omega_{de0}}(1 + z)^{3(\omega+\gamma)} \right)^{-\frac{\gamma}{w + \gamma}}.
\] (21)

Note that in Eqs. (11), (14), (17), (20), and (21), $\Omega_{de0}$ is not an independent parameter, which is given by $\Omega_{de0} = 1 - \Omega_{c0} - \Omega_{b0} - \Omega_{r0} - \Omega_{k0}$.

**B. Observational data**

In this subsection, we introduce how to include the SNLS3 data into the $\chi^2$ analysis.

For the SNLS3 sample, the observable is $m_B$, which is the rest-frame peak B-band magnitude of the SN. By considering three functional forms (linear case, quadratic case, and step function case), Wang & Wang showed that the evolutions of $\alpha$ and $\beta$ are insensitive to functional form of $\alpha$ and $\beta$ assumed. So in this paper, we just adopt a constant $\alpha$ and a linear $\beta(z) = \beta_0 + \beta_1 z$. Then, the predicted magnitude of an SN becomes
\[
m_{\text{mod}} = 5 \log_{10} D_L(z) - \alpha(s - 1) + \beta(z) C + M,
\] (22)
where \( s \) is the stretch measure of the SN light curve shape, \( C \) is the color measure for the SN, and \( M \) is a parameter representing the absolute magnitude of a fiducial SN. The luminosity distance \( D_L(z) \) is defined as

\[
D_L(z) \equiv H_0(1 + z_{\text{hel}})r(z),
\]

(23)

where \( z \) and \( z_{\text{hel}} \) are the CMB restframe and heliocentric redshifts of SN. In addition, the comoving distance \( r(z) \) is given by

\[
r(z) = H_0^{-1} |\Omega_k|^{-1/2} \sin \left( |\Omega_k|^{1/2} \Gamma(z) \right),
\]

(24)

where \( \Gamma(z) = \int_0^z \frac{dz'}{E(z')} \), and \( \sin(x) = \sin(x) \), \( x \), \( \sinh(x) \) for \( \Omega_k < 0 \), \( \Omega_k = 0 \), and \( \Omega_k > 0 \) respectively.

For a set of \( N \) SNe with correlated errors, the \( \chi^2 \) function is

\[
\chi^2_{SN} = \Delta m^T \cdot C^{-1} \cdot \Delta m,
\]

(25)

where \( \Delta m \equiv m_B - m_{\text{mod}} \) is a vector with \( N \) components, and \( C \) is the \( N \times N \) covariance matrix of the SN, given by

\[
C = D_{\text{stat}} + C_{\text{stat}} + C_{\text{sys}}.
\]

(26)

\( D_{\text{stat}} \) is the diagonal part of the statistical uncertainty, given by \[46\]

\[
D_{\text{stat},ii} = \sigma_{mB,i}^2 + \sigma_{\text{int}}^2 + \sigma_{\text{lensing}}^2 + \sigma_{\text{host correction}}^2 + \left[ \frac{5(1 + z_i)}{z_i(1 + z_i/2) \ln 10} \right]^2 \sigma_{z,i}^2 + \alpha^2 \sigma_{s,i}^2 + 2 \alpha \sigma_{mB,i} \sigma_{mC,i} - 2 \alpha \beta(z_i) \sigma_{mC,i} - 2 \alpha \beta(z_i) C_{sC,i},
\]

(27)

where \( \sigma_{mB,i}, \sigma_{mC,i}, \) and \( \sigma_{sC,i} \) are the covariances between \( m_B, s, \) and \( C \) for the \( i \)-th SN, \( \beta_i = \beta(z_i) \) are the values of \( \beta \) for the \( i \)-th SN. Notice that \( \sigma_{z,i}^2 \) includes a peculiar velocity residual of 0.0005 (i.e., 150 km/s) added in quadrature. Following the Ref. \[46\], we fix the intrinsic scatter \( \sigma_{\text{int}} \) to ensure that \( \chi^2/dof = 1 \). Varying \( \sigma_{\text{int}} \) could have a significant impact on parameter estimation, see \[60\] for details.

We define \( \mathbf{V} \equiv C_{\text{stat}} + C_{\text{sys}} \), where \( C_{\text{stat}} \) and \( C_{\text{sys}} \) are the statistical and systematic covariance matrices, respectively. After treating \( \beta \) as a function of \( z \), \( \mathbf{V} \) is given in the form,

\[
V_{ij} = V_{0,ij} + \alpha^2 V_{a,ij} + \beta_i \beta_j V_{b,ij} + \alpha V_{0a,ij} + \alpha V_{0b,ij} - \beta_j V_{0b,ij} - \beta_i V_{0b,ji} - \alpha \beta_j V_{ab,ij} - \alpha \beta_i V_{ab,ji}.
\]

(28)

It must be stressed that, while \( V_0, V_a, V_b, \) and \( V_{0a} \) are the same as the “normal” covariance matrices given by the SNLS data archive, \( V_{0b} \), and \( V_{ab} \) are not the same as the ones given there. This is because the original matrices of SNLS3 are produced by assuming \( \beta \) is constant. We have used the \( V_{0b} \) and \( V_{ab} \) matrices for the “Combined” set that are applicable when varying \( \beta(z) \) (A. Conley, private communication, 2013).

To improve the cosmological constraints, we also use some other cosmological observations, including the Planck distance prior data \[57\], the galaxy clustering (GC) data extracted from SDSS DR7 \[58\] and BOSS \[59\], as well as the direct measurement of Hubble constant \( H_0 = 73.8 \pm 2.4 \text{km/s/Mpc} \) from the HST observations \[7\]. For the details of including Planck and GC data into the \( \chi^2 \) analysis, see Ref. \[55\]. Now the total \( \chi^2 \) function is

\[
\chi^2 = \chi^2_{\text{SN}} + \chi^2_{\text{CM}} + \chi^2_{\text{GC}} + \chi^2_{H0}.
\]

(29)

It should be mentioned that, in this paper we just use the purely geometric measurements, and do not consider the cosmological perturbations in the DE models. As analyzed in detail in Ref. \[61\], adopting a new framework for calculating the perturbations, the cosmological perturbations will always be stable in all IDE models (For a related discussion concerning the stability, see Ref. \[62\]). Therefore, the use of the Planck distance prior is sufficient for our purpose.

Finally, we perform an MCMC likelihood analysis \[63\] to obtain \( O(10^6) \) samples for each model considered in this paper.

III. RESULTS

A. Evolution of \( \beta \)

In this subsection, we explore the evolution of \( \beta \) in the frame of IDE.
TABLE I: Fitting results for various constant β and linear β(z) cases, where the SNe+CMB+GC+H₀ data are used.

| Parameters | Const β | Linear β(z) | Const β | Linear β(z) | Const β | Linear β(z) | Const β | Linear β(z) |
|------------|---------|-------------|---------|-------------|---------|-------------|---------|-------------|
| α         | 1.444 ± 0.070 | 1.423 ± 0.087 | 1.424 ± 0.094 | 1.398 ± 0.110 | 1.427 ± 0.096 | 1.421 ± 0.084 | 1.445 ± 0.082 | 1.393 ± 0.121 |
| β₀        | 3.251 ± 0.115 | 5.181 ± 0.378 | 3.272 ± 0.136 | 1.438 ± 0.372 | 3.275 ± 0.112 | 1.474 ± 0.369 | 3.248 ± 0.110 | 1.505 ± 0.362 |
| β₁        | -0.098 | 4.926 ± 0.869 | -0.098 | 5.162 ± 0.924 | -0.098 | 5.070 ± 0.819 | -0.084 | 4.886 ± 0.747 |
| ν₁         | 0.224 ± 0.010 | 0.231 ± 0.009 | 0.232 ± 0.015 | 0.244 ± 0.013 | 0.226 ± 0.012 | 0.238 ± 0.012 | 0.225 ± 0.011 | 0.244 ± 0.016 |
| ν₂         | 0.042 ± 0.002 | 0.044 ± 0.002 | 0.041 ± 0.003 | 0.040 ± 0.003 | 0.041 ± 0.002 | 0.041 ± 0.003 | 0.042 ± 0.002 | 0.042 ± 0.003 |
| β₂         | 0.00046 ± 0.0004 | 0.0032 ± 0.004 | 0.0039 ± 0.006 | 0.0057 ± 0.005 | 0.0061 ± 0.012 | 0.0192 ± 0.016 | 0.0046 ± 0.013 | 0.0194 ± 0.015 |
| γ         | -0.0028 ± 0.0043 | -0.0053 ± 0.0026 | -0.0105 ± 0.0090 | -0.0322 ± 0.0396 | -0.0195 ± 0.0752 | -0.0732 ± 0.0823 | -0.0192 ± 0.0613 | -0.0684 ± 0.0712 |
| w         | -1.111 ± 0.065 | -1.042 ± 0.068 | -1.105 ± 0.070 | -1.018 ± 0.063 | -1.124 ± 0.070 | -1.052 ± 0.068 | -1.114 ± 0.072 | -1.038 ± 0.080 |
| h         | 0.725 ± 0.014 | 0.716 ± 0.015 | 0.739 ± 0.023 | 0.743 ± 0.024 | 0.734 ± 0.025 | 0.735 ± 0.027 | 0.732 ± 0.022 | 0.729 ± 0.027 |

χ²/ν = 422.696 / 388.508 = 422.376 / 387.128 = 422.674 / 387.814 = 422.642 / 387.716

FIG. 1: 1σ confidence constraints for the evolution of β(z), given by the SNe+CMB+GC+H₀ data, for the wCDM model, the IwCDM1 model, the IwCDM2 model, and the IwCDM3 model. To make a comparison, for the wCDM model, the best-fit result of constant β case is also plotted.

In Table I we list the fitting results for various constant β and linear β(z) cases, where the SNe+CMB+GC+H₀ data are used. An obvious feature of this table is that varying β can significantly improve the fitting results: for all the models, adding a parameter of β can reduce the best-fit values of χ² by ~ 34. Based on the Wilk’s theorem, 34 units of χ² is equivalent to a Gaussian fluctuation of 5.8σ. This means that the result of β₁ = 0 is ruled out at 5.8σ confidence level (CL). As shown in Refs. [55] and [50], for the cases of various DE models (such as ΛCDM, wCDM, CPL, and HDE model) without interaction, β deviates from a constant at 6σ CL. Therefore, we further confirm the redshift-evolution of β for the SNLS3 data.

In Fig. 1 using the SNe+CMB+GC+H₀ data, we plot the 1σ confidence constraints of β(z), for the wCDM model, the IwCDM1 model, the IwCDM2 model, and the IwCDM3 model. For comparison, we also plot the best-fit result of constant β case for the wCDM model. From this figure one can see that, the 1σ regions of β(z) of all these models are almost overlapping. This shows that the evolution of β is independent of the interacting dark energy models. In addition, for all the models, β(z) rapidly increases with z. This result is consistent with the results of Refs. [57] and [50], showing that the evolution of β is insensitive to dark energy models including those with interaction between dark sectors.

It should be pointed out that the evolutionary behaviors of β(z) depends on the SN samples used. In [52], Mohlabeng and Ralston found that, for the Union2.1 SN data, β(z) decreases with z. This is similar to the case of Pan-STARRS1 SN data [52]. It is of great interest to study why different SN data give different evolutionary behaviors of β(z), and some numerical simulation studies may be required to solve this problem. We will study this issue in future works.

B. Effects of time-varying β

In this subsection, we discuss the effects of varying β on parameter estimation of IDE models.
In Fig. 2 using SNe+CMB+GC+H_0 data, we plot the 1D marginalized probability distributions of Ω_{c0}, for all the models considered in this paper. We find that varying β yields a larger Ω_{c0}: for the constant β case, the best-fit results are Ω_{c0} = 0.224, 0.232, 0.226, and 0.225, for the wCDM, the IwCDM1, the IwCDM2, and the IwCDM3 model, respectively; while for the linear β(z) case, the best-fit results are Ω_{c0} = 0.231, 0.244, 0.238, and 0.244, for the wCDM, the IwCDM1, the IwCDM2, and the IwCDM3 model, respectively. In addition, as shown in Refs. 52 and 50, for various DE models without interaction term, a time-varying β also yields a larger fractional matter density Ω_{m0} = Ω_{c0} + Ω_{h0}. Therefore, we can conclude that the effects of varying β on the present fractional matter density are insensitive to the interaction between dark sectors.

For all the models considered in this paper, the 1D marginalized probability distributions of w are plotted in Fig. 3. It is found that varying β yields a larger w: for the constant β case, w = -1.118^{+0.065}_{-0.071}, -1.105^{+0.075}_{-0.069}, -1.124^{+0.070}_{-0.062}, and -1.116^{+0.059}_{-0.072}, for the wCDM model, the IwCDM1 model, the IwCDM2 model, and the IwCDM3 model, respectively; while for the linear β(z) case, w = -1.042^{+0.068}_{-0.072}, -1.016^{+0.075}_{-0.063}, -1.052^{+0.070}_{-0.068}, and -1.038^{+0.068}_{-0.080}, for the wCDM model, the IwCDM1 model, the IwCDM2 model, and the IwCDM3 model, respectively. In other words, w < -1 is preferred at more than 1σ CL for the constant β case, while w is consistent with -1 at 1σ CL for the linear β(z) case. This means that, compared to the constant β case, the results from varying β case are in better agreement with a cosmological constant. This conclusion is consistent with the noninteracting cases 54 50, showing that the effects of varying β on w are insensitive to the interaction between dark sectors.

In Fig. 4 we plot the 1D marginalized probability distributions of h, for all the models considered in this paper. It can be seen that, for the wCDM model, varying β yields a smaller h; this result is consistent with the noninteracting cases 55 50. However, for all the IDE models, the 1D distribution results of h of the linear β case are almost same with those of the constant β case. In other words, once considering the interaction between dark sectors, varying β will not change the fitting results of h. This result is quite different from the results of Fig. 2 and Fig. 3, showing that there is a degeneracy between h and γ.

Next, we turn to the constraints on interaction parameter γ. In Fig. 5 we plot 1σ and 2σ confidence contours for {γ, h}, for all the IDE models. Again, one can see that varying β has no impact on h. In contrary, varying β yields a smaller γ: for the constant β case, the best-fit results are γ = -0.0028, -0.0105, and -0.0198, for the IwCDM1 model, the IwCDM2 model, and the IwCDM3 model, respectively; while for the linear β(z) case, the best-fit results are γ = -0.0053, -0.0322, and -0.0732, for the IwCDM1 model, the IwCDM2 model, and the IwCDM3 model, respectively. In other words, γ < 0 is slightly more favored in the linear β(z) case. This means that energy will transfer from dark matter to dark energy. In addition, we find that γ and h are anti-correlated, showing that there
is a degeneracy between \( h \) and \( \gamma \).

In Fig. 3 to make a visual comparison among three interaction forms, we plot the 2σ confidence contours for \( \{\Omega_{c0}, \gamma\} \), based on the linear \( \beta(z) \) case, for all the IDE models. From this figure one can see that, \( \gamma \) is tightly constrained in the \( \text{IwCDM}1 \) model; in contrary, \( \gamma \) cannot be well constrained in the \( \text{IwCDM}2 \) and \( \text{IwCDM}3 \) models. This result is consistent with the result of [64], in which only the constant \( \beta \) case was considered.

**IV. DISCUSSION AND SUMMARY**

In recent years, more and more SNe Ia have been discovered, and the systematic errors of SNe Ia have drawn more and more attentions. One of the most important systematic uncertainties for SNe Ia is the potential SN evolution. The hints for the evolution of \( \beta \) have been found [48–52]. For examples, Mohlabeng and Ralston [53] studied the case of Union2.1 and found that \( \beta \) deviates from a constant at 7σ CL. In [54], Wang & Wang found that, for the SNLS3 data, \( \beta \) increases significantly with \( z \) at the 6σ CL; moreover, they proved that this conclusion is insensitive to the lightcurve fitter models, or the functional form of \( \beta(z) \) assumed [54].

It is clear that a time-varying \( \beta \) will have significant impact on parameter estimation. Adopting a constant \( \alpha \) and a linear \( \beta(z) = \beta_0 + \beta_1 z \), Wang, Li & Zhang [55] explored this issue by considering the \( \Lambda \)CDM model, the \( w \)CDM model, and the CPL model. Soon after, Wang, Geng, Hu & Zhang [56] studied this issue in the frame of HDE model, which is a physically plausible DE candidate based on the holographic principle. It is found that, for all these DE models, \( \beta \) deviates from a constant at 6σ CL; in addition, considering the evolution of \( \beta \) is helpful in reducing the tension between SN and other cosmological observations. It should be pointed out that, in principle there is always an important possibility that DE directly interacts with CDM. This factor was not considered in Refs. [55] and [56].

In this paper, we extend the corresponding discussions to the case of IDE model. To perform the cosmology-fits, the \( w \)CDM model is adopted. Moreover, three kinds of interaction forms are considered: 

\[
Q_1 = 3\gamma H \rho_c, \quad Q_2 = 3\gamma H \rho_{de}, \quad Q_3 = 3\gamma H \frac{\rho_c \rho_{de}}{\rho_c + \rho_{de}}.
\]

In addition to the SNLS3 SN data, we also use the Planck distance priors data, the GC data extracted from SDSS DR7 and BOSS, as well as the direct measurement of Hubble constant from the HST observation.

We further confirm the redshift-evolution of \( \beta \) for the SNLS3 data: for all the IDE models, adding a parameter of \( \beta \) can reduce \( \chi^2 \) by \( \sim 34 \), indicating that \( \beta_1 = 0 \) is ruled out at 5.8σ CL. In addition, we find that the 1σ regions of \( \beta(z) \) of all these models are almost overlapping, showing that the evolution of \( \beta \) is insensitive to the interaction between
dark sectors. These results further verify the importance of considering the evolution of $\beta$ in the cosmology-fits.

Furthermore, we find that a time-varying $\beta$ has significant effects on the results of parameter estimation: for all the models considered in this paper, varying $\beta$ yields a larger $\Omega_c$ and a larger $w$; on the other side, varying $\beta$ yields a smaller $h$ for the $w$CDM model, while varying $\beta$ has no influence on $h$ for the three IDE models. Moreover, we find that $\gamma$ and $h$ are anti-correlated, showing that there is a degeneracy between $h$ and $\gamma$. In addition, we find that $\gamma$ is tightly constrained in the $I_wCDM1$ model, but cannot be well constrained in the $I_wCDM2$ and $I_wCDM3$ models.

In all, these results show that the evolution of $\beta$ is insensitive to the interaction between dark sectors, and highlight the importance of considering $\beta$'s evolution in the cosmology fits.

So far, only the effects of varying $\beta$ on DE models are considered. It is of great interest to study the effects of varying $\beta$ on parameter estimation in MG models. In addition, some other factors, such as the evolution of $\sigma_{int}$ [60], may also cause the systematic uncertainties of SNe Ia. These issues will be studied in future works.

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FIG. 5: The 1σ and 2σ confidence contours for \( \{ \gamma, h \} \), for the three IDE models. Both the results of constant \( \beta \) and linear \( \beta(z) \) cases are presented.

FIG. 6: The 2σ confidence contours for \( \{ \Omega_{c0}, \gamma \} \), based on the linear \( \beta(z) \) case, for all the IDE models.

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