Resummation of target mass corrections in two-photon processes: twist-two sector.

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Abstract

We develop a formalism for the resummation of target mass corrections in off-forward two-photon amplitudes given by a chronological product of electromagnetic currents, arising in e.g. deeply virtual Compton scattering. The method is based on a relation of composite operators with a definite twist to harmonic tensors, which form an irreducible representation of the Lorentz group. We give an application of the framework for the matrix elements of twist-two operators.

Keywords: two-photon processes, target mass corrections, harmonic tensors, double distributions, skewed parton distributions

PACS numbers: 11.10.Hi, 12.38.Bx, 13.60.Fz
1 Hadron mass corrections.

The leading twist approximation to hard processes in QCD is affected by a number of multiplicative and additive corrections. To the first class one obviously attributes radiative corrections in the strong coupling, while the latter encodes higher twist contributions, which provide power suppressed corrections. In the case when a given large scale $Q$, that controls the factorization of a reaction, becomes rather low, one has to take care of power suppressed effects since they can significantly modify the scaling behaviour and the magnitude of the leading twist prediction for the corresponding cross section. Power corrections can be divided into two classes according to their origin: dynamical and kinematical ones. The first one comprises of multiparton correlations inside hadrons [1] and gives rise to new non-perturbative functions. The second one arises from a separation of composite operators into parts that have definite transformation properties w.r.t. the Lorentz group and thus possess a well defined geometrical twist, i.e. dimension minus spin. This decomposition provides the so-called Wandzura-Wilczek type contributions [2] arising from components having definite symmetry properties as well as target mass corrections [3] stemming from the subtraction of trace terms in the afore mentioned operators.

In the present paper we develop a formalism for the resummation of the target mass corrections for off-forward two-photon processes whose amplitudes are given by a chronological product of electromagnetic currents. This method is indispensable for a study of hadron mass effects in the cross section of e.g. deeply virtual Compton scattering [4, 5, 6], — the processes sensitive to the generalized parton distributions (GPDs). The present day facilities can offer the $Q^2$ of the order of a few (2-4) GeV$^2$ (for the reactions in question) and one has to account for power suppressed contributions to the twist-two and -three observables arising from the ratio of the nucleon mass to the hard momentum transfer. Unfortunately, the formalism for resummation of target mass corrections invented by Georgi and Politzer [3] for the deep inelastic scattering (see also [7] for a first discussion of the topic), and accepted in all consequent generalizations and applications to forward electroweak scattering amplitudes, is not directly applicable to off-forward processes in the context of GPDs. The reason for that is the appearance of towers of new Lorentz structures in the matrix elements of local operators, arisen due to the non-zero $t$-channel momentum transfer, which cannot be handled in the fashion proposed in [3]. Note that a discussion of mass effects in exclusive processes with a much simpler kinematics can be found in Ref. [8].

In the present context, instead of dealing with single variable functions, e.g. GPDs, we use spectral functions, the so-called double distributions (DDs) [4, 6], which depend on two variables. This allows to resum the mass corrections in a straightforward manner and to derive all-order results in a compact form. After this is accomplished one can in principle use an inverse integral representation to express DDs by means of GPDs, however, this transformation requires the
knowledge of the support of GPDs including an unphysical region. Therefore, this transformation can be hardly used for a successful phenomenological application.

The consequent presentation runs as follows. In the next section we give a short introduction to a procedure of the trace subtraction by means of harmonic polynomials. In Section 3 we resum the target mass effects resulting from the trace subtraction in matrix elements of twist-two operators for zero and spin-\(\frac{1}{2}\) targets. Then it is shown that the expansion of these results in powers of \((M^2/q^2)^j\) can be cast in a conventional form of GPDs and be used for numerical predictions of target mass effects in physical cross sections. Finally, we conclude.

2 Harmonic polynomials and twist decomposition.

The goal of our study is an evaluation of the off-forward Compton amplitude keeping target mass effects stemming from the twist-two contributions in the light-cone expansion of the off-forward matrix element of the chronological product \(\tilde{T}_{\mu\nu}(\frac{x}{2}, -\frac{x}{2}) = \langle P_2 | T \{ j_{\mu}(\frac{x}{2}) j_{\nu}(-\frac{x}{2}) \} | P_1 \rangle\) of electromagnetic currents \(j_{\mu} = \bar{\psi} \gamma_{\mu} \psi\). Since these are entirely kinematical effects, we consider dynamical higher twist effects given by multiparticle operators as independent and, therefore, we safely neglect them. Note, however, that the trace subtraction in twist-three contributions in the Wandzura-Wilczek approximation will generate kinematical mass effects accompanied by leading twist operators. We will comment on this issue in the concluding section. In the approximation, we are considering, the Fourier transformed two-photon amplitude reads

\[
T_{\mu\nu} = \int d^4x \, e^{ixq} \tilde{T}_{\mu\nu}(\frac{x}{2}, -\frac{x}{2}) = \frac{1}{\pi^2} \int d^4x \, \frac{x_{\sigma}}{[-x^2 + i0]^2} \times \langle P_2 | \cos(x \cdot q) S_{\mu\nu,\rho\sigma} V_{\rho}(\frac{x}{2}, -\frac{x}{2}) + \sin(x \cdot q) \epsilon_{\mu\nu\rho\sigma} A_{\rho}(\frac{x}{2}, -\frac{x}{2}) | P_1 \rangle,
\]

where \(S_{\mu\nu,\rho\sigma} = g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\rho\sigma}\) and for the totally antisymmetric tensor we use the normalization \(\epsilon^{0123} = 1\). The vector \(q\) is a semi-sum of incoming \(q_{1\nu}\) and outgoing \(q_{2\mu}\) photon momenta, \(q = \frac{1}{2}(q_1 + q_2)\). The non-local operators in Eq. (1) are \(V_{\rho}(x, -x) = \bar{\psi}(-x)[-x, x] \Gamma_\rho \psi(x)\) with a Dirac matrix \(\Gamma_\rho\) which equals \(\gamma_\rho\) or \(\gamma_\rho\gamma_5\) for vector (V) or axial-vector (A) sectors, respectively. The path ordered exponential \([-x, x]\) will be dropped everywhere in the following formulae.

We are aiming now in a decomposition of \(O_\rho\) into traceless operators \(R^2_\rho\) and \(R^3_\rho\) with definite geometrical twist-two and -three, respectively:

\[
O_\rho(x, -x) = R^2_\rho(x, -x) + R^3_\rho(x, -x) + R^r_\rho(x, -x).
\]

To this end, we expand first the non-local operator \(O_\rho(x, -x)\) in a Taylor series

\[
O_\rho(x, -x) = \sum_{j=0}^{\infty} \frac{(-i)^j}{j!} x_{\mu_1} \cdots x_{\mu_j} O_{\rho, \mu_1 \cdots \mu_j}, \quad \text{with} \quad O_{\rho, \mu_1 \cdots \mu_j} = \bar{\psi} \Gamma_\rho i \slashed{D}_{\mu_1} \cdots i \slashed{D}_{\mu_j} \psi.
\]
where \( \overrightarrow{\mathcal{D}}_\mu = \overrightarrow{\mathcal{D}}_\mu - \overrightarrow{\mathcal{D}}_\mu \) with the covariant derivative \( \mathcal{D}_\mu = \partial_\mu - ig B_\mu \), and then we perform the twist decomposition for the local operators \( \mathcal{O}_{\rho;j} = x_{\mu_1} \ldots x_{\mu_j} \mathcal{O}_{\rho;\mu_1 \ldots \mu_j} \). An effective way of trace subtraction is to solve the sufficient condition \( \partial^2 H^3 \left( x^2, \partial^2 \right) T_j(x) = 0 \) [9, 10, 11], where \( T_j \) results from the contraction of a rank-\( j \) tensor with the product of four-vectors \( x_{\mu_1} \ldots x_{\mu_j} \), and \( H^3 \) is a subtraction operator. The solution is given by the so-called harmonic polynomials [4]

\[
H^3 \left( x^2, \partial^2 \right) = \sum_{k=0}^{[j/2]} \frac{\Gamma(j-k+1)}{k! \Gamma(j+1)} \left( -\frac{x^2}{4} \right)^k \left( \partial^2 \right)^k.
\] (4)

For operators having a free vector index one has an extra condition so that in total the tracelessness implies \( \partial^2 \mathcal{R}^i_{\rho;j} = 0 \) and \( \partial_\mu \mathcal{R}^i_{\rho;j} = 0 \), for \( i = 2, 3 \), where the twist-two operators are completely symmetric while the twist-three operators are antisymmetric in a pair of their Lorentz indices. Employing the equation of motion for the massless Dirac field, one can show that \( \mathcal{R}^r \) is proportional to total derivative, \( \nabla \), squared, apart from multiparticle operators, \( \mathcal{R}^r_\rho \propto -\nabla^2 \mathcal{R}^2_\rho + \) multiparticle operators. When it is sandwiched between hadronic states, it generates contributions proportional either to the \( t \)-channel momentum transfer squared or given by multi-parton correlations functions. Both of them will be neglected in the following. Thus, our considerations will be limited by the condition \( M^2 \gg \Delta^2 \), with \( M^2 = P_1^2 = P_2^2 \) and \( \Delta^2 \), being the hadron mass and \( t \)-channel momentum transfer, respectively.

A straightforward algebra leads to the desired decomposition in operators with definite geometrical twist:

\[
\mathcal{O}_{\rho;j} = \mathcal{R}^2_{\rho;j} + \frac{2j}{j+1} \mathcal{R}^3_{\rho;j} + \mathcal{R}^r_{\rho;j},
\] (5)

where the traceless twist-two and -three operators read (cf. [11])

\[
\mathcal{R}^2_{\rho;j} = \frac{1}{j+1} \partial_\mu H^{j+1} \left( x^2, \partial^2 \right) x_\sigma \mathcal{O}_{\sigma;j}, \quad j \geq 0,
\] (6)

\[
\mathcal{R}^3_{\rho;j} = \frac{1}{2j} \left[ g_{\rho\sigma} x \cdot \partial - x_\sigma \partial_\rho \right] \left[ g_{\sigma\tau} - \frac{1}{j+1} x_\sigma \partial_\tau \right] H^j \left( x^2, \partial^2 \right) \mathcal{O}_{\tau;j}, \quad j \geq 1,
\] (7)

and the remainder is

\[
\mathcal{R}^r_{\rho;j} = \frac{1}{j+1} \left\{ \partial_\rho \left[ 1 - H^{j+1} \left( x^2, \partial^2 \right) \right] x_\sigma + \left[ g_{\rho\sigma} x \cdot \partial - x_\sigma \partial_\rho \right] \left[ 1 - H^j \left( x^2, \partial^2 \right) \right] \right\} \mathcal{O}_{\sigma;j}.
\]

It is obvious that the twist-two operators [8] are traceless. Using \( \partial^2 H^j \left( x^2, \partial^2 \right) \mathcal{O}_{\mu;j} = 0 \) and Euler theorem \( x \cdot \partial \partial_\mu H^j \left( x^2, \partial^2 \right) \mathcal{O}_{\mu;j} = (j-1) \partial_\mu H^j \left( x^2, \partial^2 \right) \mathcal{O}_{\mu;j} \), it is an easy task to check that the operators [7] are traceless too.
3 Resummation within DD formalism.

In this section we will give a general framework for the resummation of kinematical mass corrections. For simplicity we consider here only the twist-two sector. The major steps consist of: a parametrization of the off-forward matrix elements of symmetric local operators by means of moments of DDs, the trace subtraction with the harmonic projectors $H_j$, then a Fourier transformation to the momentum space and, finally, a resummation of infinite series.

In order to demonstrate the main features of our formalism, let us consider first matrix elements of twist-two operators sandwiched between states of spinless target $\langle P_2|R_2^{\rho,j}|P_1 \rangle$. Here only the matrix elements of the parity even sector are relevant and read

$$
\langle P_2|\mathcal{O}^{\text{sym}}_{\rho;\mu_1...\mu_j}|P_1 \rangle = P_1 \rho\mu_1...\mu_j B_{j+1,j+1} + \cdots + \Delta \rho \Delta \mu_1...\Delta \mu_j B_{j+1,0} + \cdots .
$$

They are build from the two vectors $P = P_1 + P_2$ and $\Delta = P_2 - P_1$, \{\ldots\} denotes symmetrization of corresponding indices and the ellipsis stands for possible terms containing the metric tensor. Since we act on this expression with $H^j$ we can neglect the latter. After projection with harmonic polynomials via Eq. (6) one gets

$$
\langle P_2|\mathcal{R}^{\rho,j}_{\mu_1...\mu_j}|P_1 \rangle = \frac{1}{j+1} \partial_\rho H^{j+1} \left( x^2, \partial^2 \right) \sum_{k=0}^{j+1} (x \cdot P)^{j+1-k} (x \cdot \Delta)^k B_{j+1,j+1-k} 
$$

with the operation $\mathbf{S}$ standing for symmetrization and trace subtraction. The first line gives a compact expression for subtracted operator which is extremely convenient for consequent considerations and is the basis of our formalism.

On the other hand we can deduce the twist-two operator from Eq. (8) by a contraction of all Lorentz indices with a light-like vector $n$ so that the trace terms vanish identically. This tells us that the coefficients $B_{j+1,j+1-k}$ in front of the Lorentz tensors are related to the reduced matrix elements of light-cone operators and can be represented in terms of moments of conventional leading twist GPDs or DDs via

$$
B_{j+1,j+1-k} = \frac{1}{k!} \frac{\partial^k}{\partial \eta^k} \left|_{\eta=0} \right. \int_{-1}^{1} dx x^{j-1} B(x, \eta) = \binom{j}{k} \int_{\Omega} dy dz y^{j-k} z^k f(y, z)
$$

where $0 \leq k \leq j$, $1 \leq j$ and the integration domain is $\Omega = \{-1 \leq y \leq 1, -1 + |y| \leq z \leq 1 - |y|\}$. Consequently, the parametrization of the matrix element for the non-local symmetric operator

\footnote{This is not the definition given in Refs. [4, 6] which suffered from inconsistencies due to omitted Lorentz structures as noted in [12]. Our solution of this problem differs also from [12] where a two-component parametrization has been suggested.}
and the light-ray operators coincide up to terms proportional to $x_\rho$ or $x^2$:

$$\langle P_2 | V^{\text{sym}}_{\rho}(x,-x) | P_1 \rangle = \int_{\Omega} dy \, dz \, f(y,z) \mathcal{P}_\rho \, e^{-ix \cdot \mathcal{P}} + \ldots,$$

where we introduced a new shorthand notation $\mathcal{P}_\mu \equiv y P_\mu + z \Delta_\mu$.

The resummation of matrix elements of traceless local operators can be performed in the formalism of DD $f(y,z)$. By means of Eq. (11) it is easy to see that the finite sum w.r.t. $k$ in Eq. (4) provides

$$\langle P_2 | V R^2_{\rho\mu j} | P_1 \rangle = \int_{\Omega} dy \, dz \, f(y,z) \frac{1}{j+1} \partial_\rho H^{j+1}(x^2, \partial^2) (x \cdot \mathcal{P})^{j+1}.$$  

The evaluation of projection of harmonic polynomials results into conventional Chebyshev polynomials related to the irreducible representation of the orthogonal group $SO(4)$

$$H^j(x^2, \partial^2)(x \cdot \mathcal{P})^j = \left(\frac{x^2 \mathcal{P}^2}{4}\right)^{j/2} U_j \left(\frac{x \cdot \mathcal{P}}{\sqrt{x^2 \mathcal{P}^2}}\right).$$

This result when plugged into Eq. (12) gives our final expression for the twist-two traceless local operators. We will use it after we perform the Fourier transformation in Eq. (1). We accomplish it making use of the following general formula

$$\int d^4 x e^{ix \cdot q} \frac{x_{\mu_1} \cdots x_{\mu_j}}{[-x^2]^k} = (-i)^{j+1} 2^{4-2k+j} \pi^2 \frac{\Gamma(2-k+j)}{\Gamma(k)} \frac{q_{\mu_1} \cdots q_{\mu_j}}{[-q^2]^{2-k+j}},$$

where on the r.h.s. we dropped all terms involving the metric tensor $g_{\mu_1 \mu_2}$, since they vanish when contracted with a traceless tensor. Thus, we get

$$\int d^4 x e^{ix \cdot q} \frac{x_{\sigma} \, x_{\mu_1} \cdots x_{\mu_j}}{[-x^2]^2} \frac{2}{q^2} \langle P_2 | V R^2_{\rho\mu_1 \cdots \mu_j} | P_1 \rangle$$

$$= i^{j+2} \pi^2 \Gamma(j+1) \frac{1}{q^2} \Pi_{\sigma \mu_1} \frac{q_{\mu_2}}{q^2} \cdots \frac{q_{\mu_j}}{q^2} \langle P_2 | V R^2_{\rho \mu_1 \cdots \mu_j} | P_1 \rangle,$$

with projector

$$\Pi_{\mu \nu} \equiv g_{\mu \nu} - 2 \frac{q_\mu q_\nu}{q^2}, \quad \Pi_{\mu \rho} \Pi_{\rho \nu} = g_{\mu \nu}.$$

Then the Fourier transformation of the twist-two contribution to the Compton amplitude gives

$$\int d^4 x e^{ix \cdot q} \frac{x_{\sigma} \, x_{\mu_1} \cdots x_{\mu_j}}{[-x^2]^2} \langle P_2 | V R^2_{\rho}(x, \frac{x}{2}) | P_1 \rangle$$

$$= 2\pi^2 \left\{ \frac{d_\sigma}{q^2} \sum_{j=0}^{\infty} \langle P_2 | V R^2_{\rho j} | P_1 \rangle - \frac{1}{2} \Pi_{\sigma \tau} \frac{\partial}{\partial q_\tau} \sum_{j=1}^{\infty} \frac{1}{j} \langle P_2 | V R^2_{\rho j} | P_1 \rangle \right\}.$$  

Here we used a new convention

$$\tilde{R}^2_{\rho j} = \frac{q_{\mu_1}}{q^2} \cdots \frac{q_{\mu_j}}{q^2} R^2_{\rho \mu_1 \cdots \mu_j}.$$
and exploited an identity

\[
\frac{\partial}{\partial q^2} = q^2 \Pi_{\sigma \tau} \frac{\partial}{\partial q_\tau}.
\]

To evaluate the sums in (17) we substitute (13) in (12) and replace \(x_\mu\) by \(q_\mu/q^2\). This gives the local traceless matrix elements \(\langle P_2|V^2_R_{p;\ j}|P_1\rangle\) in terms of Chebyshev polynomials. The summation in Eq. (17) can be now done making use of the generating function for the Chebyshev polynomials

To get a required \(j\)-dependent coefficient in the series we do integration(s) of both sides with appropriate weight. The sums required in the calculation of (17) read

\[
\sum_{j=0}^{\infty} \frac{a^{j+1}}{(j+1)} U_{j+1}(b) = \int_0^a \frac{da'}{a'} (G_U(a', b) - 1),
\]

\[
\sum_{j=1}^{\infty} \frac{a^{j+1}}{j(j+1)} U_{j+1}(b) = \int_0^a \int_0^{a'} \int_0^{a''} \frac{da''}{(a'')^2} (G_U(a'', b) - 1 - 2a''b).
\]

Here \(a = \frac{1}{2} \sqrt{P^2/q^2}\) and \(b = q \cdot \mathcal{P}/\sqrt{q^2 P^2}\). This immediately leads to

\[
\sum_{j=0}^{\infty} \langle P_2|V^2_R_{p;\ j}|P_1\rangle = q^2 \Pi_{\rho \tau} \frac{\partial}{\partial q_\tau} \int_\Omega dy \, dz \, f(y, z)
\]

\[
\times \left\{ \frac{1 - \sqrt{1 + \mathcal{M}^2}}{2 \sqrt{1 + \mathcal{M}^2}} \ln \left( 1 + \frac{1 - \sqrt{1 + \mathcal{M}^2}}{2 \Xi} \right) - \frac{1 + \sqrt{1 + \mathcal{M}^2}}{2 \sqrt{1 + \mathcal{M}^2}} \ln \left( 1 + \frac{1 + \sqrt{1 + \mathcal{M}^2}}{2 \Xi} \right) \right\},
\]

where we introduced the conventions

\[
\Xi \equiv - \frac{q^2}{q \cdot \mathcal{P}}, \quad \mathcal{M}^2 \equiv - \frac{q^2 P^2}{(q \cdot \mathcal{P})^2}.
\]

Inserting our findings (20) and (21) into Eq. (17) the next step is a mere differentiation w.r.t. \(q_\mu\), which is done by the formula

\[
q^2 \Pi_{\mu \nu} \frac{\partial}{\partial q_\nu} \tau (\mathcal{M}^2, \Xi) = \left\{ 2 \mathcal{M}^2 (q_\mu + \Xi \mathcal{P}_\mu) \frac{\partial}{\partial \mathcal{M}^2} - \mathcal{P}_\mu \frac{\partial}{\partial \Xi} \right\} \tau (\mathcal{M}^2, \Xi)
\]

for a test function \(\tau\). Finally, we contract the resulting equation with the tensor \(S_{\mu \nu ; \rho \sigma}\) which leads to the hadronic tensor

\[
T^2_{\mu \nu} = -\frac{1}{q^2} \int_\Omega dy \, dz \, f(y, z) \left\{ (q^2 g_{\mu \nu} - q_\mu q_\nu) \mathcal{F}_1 + \left( \mathcal{P}_\mu + \frac{q_\mu}{\Xi} \right) \left( \mathcal{P}_\nu + \frac{q_\nu}{\Xi} \right) \mathcal{F}_2 \right\},
\]
with mass-dependent coefficient functions

\[
F_1 = \frac{4\Xi - \mathcal{M}^2 (1 - \Xi)}{\Xi [4\Xi (1 + \Xi) - \mathcal{M}^2]} + \frac{\mathcal{M}^2 (2\Xi - \mathcal{M}^2)}{4\Xi (1 + \mathcal{M}^2)^{3/2}} \ln \left( \frac{1 - \sqrt{1 + \mathcal{M}^2 + 2\Xi}}{1 + \sqrt{1 + \mathcal{M}^2 + 2\Xi}} \right) + (\Xi \to -\Xi),
\]

\[
F_2 = \frac{\Xi [4\Xi - \mathcal{M}^2 (1 - \Xi)]}{(1 + \mathcal{M}^2) [4\Xi (1 + \Xi) - \mathcal{M}^2]} + \frac{3\Xi \mathcal{M}^2 (2\Xi - \mathcal{M}^2)}{4(1 + \mathcal{M}^2)^{5/2}} \ln \left( \frac{1 - \sqrt{1 + \mathcal{M}^2 + 2\Xi}}{1 + \sqrt{1 + \mathcal{M}^2 + 2\Xi}} \right) + (\Xi \to -\Xi).
\]

In order to get the right \(i0\)-prescription, so as to pick up a correct sheet of the Riemann surface for the logarithm, we notice that in \(13\) we have to restore the suppressed Feynman prescription as follows \(q^2 \to q^2 + i0\).

Let us comment on our result. As we observe, the leading order massless (generalized) Callan–Gross-type relation (see \(13\)) \(F_2 = \Xi^2 F_1\) is violated by target mass corrections. We checked that our result coincides in the forward limit with the well-known ones \(3\). Obviously, we have \(q_\mu T^2_{\mu\nu} = 0\) and therefore current conservation is fulfilled in the forward case. Note that this is only the case in the sum of the direct and the crossed amplitudes. However, the gauge invariance in the off-forward kinematics is violated, i.e. \(q_\mu T^2_{\mu\nu} \neq 0\) and, as it was previously studied in the case of massless amplitudes, it is restored once higher twist corrections are accounted for \(14, 15, 16, 17\).

This must also persist to the case of traceless operators which are not projected on the light cone.

Now let us generalize the formalism to a spin-\(\frac{1}{2}\) target. The matrix elements \(\langle P_2 | \mathcal{R}^2_{\rho\mu} | P_1 \rangle\) are build now from the vectors \(P, \Delta\) and a set of (independent) Dirac bilinears, characterizing the spin content of the target. For our purposes it is convenient to use the following structures

\[
(h_{\mu}, \tilde{h}_{\mu}) = \bar{U}(P_2) \gamma_\mu (1, \gamma_5) U(P_1), \quad (b, \tilde{b}) = \bar{U}(P_2) (1, \gamma_5) U(P_1),
\]

(26)

for (vector, axial) sectors, respectively. Thus, we parametrize the matrix element of e.g. vector operator according to

\[
\langle P_2 | \mathcal{O}^{\text{sym}}_{\rho\mu_1 \ldots \mu_j} | P_1 \rangle = h_{(\rho} P_{\mu_1} \cdots P_{\mu_j)} A_{j+1,j+1} + \cdots + h_{(\rho} \Delta_{\mu_1} \cdots \Delta_{\mu_j)} A_{j+1,1}
\]

(27)

\[
+ \frac{b}{2M} \left\{ P_{(\rho} P_{\mu_1} \cdots P_{\mu_j)} B_{j+1,j+1} + \cdots + \Delta_{(\rho} \Delta_{\mu_1} \cdots \Delta_{\mu_j)} B_{j+1,0} \right\} + \ldots,
\]

where again terms proportional to the metric tensor are not needed. The non-local twist-two vector operator reads

\[
\langle P_2 | \mathcal{R}^\rho_{\rho}(x, -x) | P_1 \rangle = \int d^2 y d^2 z \left\{ f_A(y, z) h \cdot \partial^2 + f_B(y, z) \frac{b}{2M} \mathcal{P} \cdot \partial^2 \right\} \times \partial_\rho \sum_{j=1}^\infty \frac{(-i)^j}{j!(j+1)^2} \left( \frac{x^2 \mathcal{P}^2}{4} \right)^{(j+1)/2} \mathcal{U}_{j+1} \left( \frac{x \cdot \mathcal{P}}{\sqrt{x^2 \mathcal{P}^2}} \right).
\]

(28)

Here we have generated the factor of \(x\) in \(h \cdot x\) by a differentiation w.r.t. \(\mathcal{P}\). The same we also did with the \(b\) form factor for the purpose of a uniform representation, as well as to have a cross check on
our previous calculations. Obviously, this step is not required at all and we can readily borrow the results already found for a (pseudo) scalar target. The same equation holds for axial operator with a trivial dressing of symbols with tildes.

The reduced matrix elements $B_{j+1,j+1-k}$ are represented in terms of GPDs or DDs as in Eq. (11), — replace $f(y, z)$ by $f_B(y, z)$. For the reduced matrix elements $A_{j+1,j+1-k}$ we write

$$A_{j,k} = \frac{1}{k!} \left[ \frac{\partial^k}{\partial \eta^k} \right]_{\eta=0} \int_{-1}^{1} dx \, x^{j-k} A(x, \eta) = \left( \begin{array}{c} j-1 \\ k \end{array} \right) \int_{\Omega} dy \, dz \, y^{j-k} z^k f_A(y, z),$$

where $j \geq 1$ and the index $k$ now varies in the intervals $0 \leq k \leq j-1$. There is an immediate consequence of the fact that $j$-th moment of the function $A(x, \eta)$ is a polynomial of order $\eta^{j-1}$ only. Namely, translating it to the conventional Ji’s distributions, $H = A + B$, $E = -B$, we immediately find that

$$\frac{\partial}{\partial \eta^2} \int_{-1}^{1} dx x^{j-1} \{H(x, \eta) + E(x, \eta)\} = 0.$$ 

As a particular example we recall that for the $j = 2$ moment, which corresponds to Ji’s sum rule, the $\eta$-dependence drops off as it was noted before [4]. For the axial channel the GPDs $\tilde{A}$ and $\tilde{B}$ are identical to the conventional $\tilde{H}$ and $\tilde{E}$ and this implies that the $j$-th moment of $\tilde{H}$ ($\tilde{E}$) has the expansion to order $j - 1$ ($j$) in $\eta$.

The Fourier transform and the resummation are done in the same way as before and result in

$$T_{\mu \nu}^2 = -\frac{1}{q^2} \int_{\Omega} dy \, dz \left\{ \left( \tilde{f}_A(y, z) \tilde{h} \cdot \partial^P + \tilde{f}_B(y, z) \frac{\tilde{b}}{2M} \, \tilde{P} \cdot \partial^P \right) i\epsilon_{\mu \nu \rho \sigma} q_\rho \, \partial^P \right\} G_1$$

\[+ \left( f_A(y, z) h \cdot \partial^P + f_B(y, z) \frac{b}{2M} \, P \cdot \partial^P \right) \times \left( q^2 g_{\mu \nu} - q_\mu q_\nu \right) F_1 + \left( P_\mu + \frac{q_\mu}{\Xi} \right) \left( P_\nu + \frac{q_\nu}{\Xi} \right) F_2, \]

with massive coefficient functions

\[F_1 = \frac{2\Xi}{4 \Xi} \left( 1 + 2M^2 \right) - \frac{M^4 L_-}{4 \Xi} - \frac{L_+}{2 \left( 1 + M^2 \right)} - \frac{M^2 L}{2 \left( 1 + M^2 \right)^{3/2}} + (\Xi \rightarrow -\Xi), \]

\[F_2 = \frac{\Xi^2 \left( 1 + 4M^2 \right)}{4 \left( 1 + M^2 \right)^{5/2}} - \frac{\Xi^2 \left( 1 - 2M^2 \right) L_-}{4 \left( 1 + M^2 \right)^2} - \frac{3\Xi^2 M^2 L}{2 \left( 1 + M^2 \right)^{5/2}} + (\Xi \rightarrow -\Xi), \]

\[G_1 = \frac{\Xi L_-}{2 \left( 1 + M^2 \right)^{1/2}} - \frac{\Xi L_+}{2 \left( 1 + M^2 \right)} - \frac{\Xi M^2 L}{2 \left( 1 + M^2 \right)^{3/2}} - (\Xi \rightarrow -\Xi), \]

where

$$L_\pm \equiv \ln \left( \frac{1 - \sqrt{1 + M^2}}{2 \Xi} \right) \pm \ln \left( \frac{1 + \sqrt{1 + M^2}}{2 \Xi} \right),$$

$$L \equiv \text{Li}_2 \left( -\frac{1 - \sqrt{1 + M^2}}{2 \Xi} \right) - \text{Li}_2 \left( -\frac{1 + \sqrt{1 + M^2}}{2 \Xi} \right).$$
Here \( \text{Li}_2 \) is the Euler dilogarithm \( \text{Li}_2(x) = -\int_0^1 \frac{du}{y} \ln(1-y) \).

As we mentioned before, the resummed mass corrections can not be expressed in terms of GPDs (at least we did not succeed in doing this). However, every term in the mass expansion, \( M^2/q^2 \), can be converted into the conventional GPD representation. Let us demonstrate it for the expanded Compton form factor \( F_1 \) for a (pseudo) scalar target given in Eq. (25). To the first non-trivial order, i.e. to \( \mathcal{O}(M^4/q^4) \) accuracy, the latter reads in DD form

\[
F_1 = \int dy \, dz \, f(y, z) \left\{ \left( C_1^{(0)}(\xi - i 0) + C_1^{(0)}(-\xi - i 0) \right) \right.
- \left. \frac{M^2}{q^2} y^2 \xi^2 \left( C_1^{(1)}(\xi - i 0) + C_1^{(1)}(-\xi - i 0) \right) \right\},
\]

where in our approximation, \( M^2 \gg \Delta^2 \), we set \( \mathcal{M}^2 = -4 \frac{M^2}{q^2} y^2 \xi^2 \) and introduced the coefficient functions \( C_1^{(0)}(\xi^{-1}) = -(1+\xi)^{-1} \) and \( C_1^{(1)}(\xi^{-1}) = \frac{1}{(1+\xi)^2} + 2 \ln \left( \frac{\xi}{1+\xi} \right) \). To cast the DD into the usual GPD representations, which are related by the equation \( x \int_\Omega dy \, dz \, f(y, z) \delta(x-y-\eta z) \equiv B(x, \eta) \), we need the following general result

\[
\int dy \, dz \left\{ \frac{z^n}{y^n} \right\} \mathcal{F}(\xi^{-1}) f(y, z) = \int dx \mathcal{F}(\xi) \int dx' V_1^{(n)}(x, x') \left\{ \Pi_{k=0}^{n-1} \left( \frac{\partial}{\partial \eta} \right)^k \right\} B(x', \eta),
\]

where we used a shorthand notation for the differential operator \( \hat{d}(x, \eta) = x \frac{\partial}{\partial x} - \eta \frac{\partial}{\partial \eta} \) and the kernels

\[
V_1^{(n)}(x, x') = \frac{(x'-x)^{n-1}}{(n-1)!} \Theta_{11}^0(x, x-x'), \quad \text{with} \quad \Theta_{11}^0(x, y) = \frac{\theta(x) - \theta(y)}{x - y}.
\]

Substitution of these expressions back into Eq. (32) results into

\[
F_1 = \int_{-1}^1 dx \int_{-1}^1 dx' \left\{ \left( C_1^{(0)}(\xi - i 0) + C_1^{(0)}(-\xi - i 0) \right) \delta(x - x') \right.
- \left. \frac{M^2}{q^2} \left( C_1^{(1)}(\xi - i 0) + C_1^{(1)}(-\xi - i 0) \right) V_1^{(2)}(x, x') \left( \hat{d}(x', \eta) - 1 \right) \hat{d}(x', \eta) \right\} B(x', \eta),
\]

which has now the desired form. Completely analogous manipulations produce the GPD representation for all other form factors.

4 Conclusions.

In the present note, we have presented a formalism for the resummation of target mass corrections stemming from the twist-two approximation to the off-forward Compton scattering amplitude. The machinery is based on the group-theoretical content of the procedure of trace subtraction.
from local operators with definite symmetry property – the operators with a well defined twist are given by a harmonic projection of the latter. This allows to resum target mass corrections to all orders in a compact form in terms of DDs. However, once the amplitudes are expanded in $M^2/q^2$ series, they can be cast in the form involving GPDs.

An important feature of the result is that the mass corrections enter with additional powers of the scaling variable $\xi$. Thus, we expect that they are suppressed in the small $\xi (x_{Bj})$ region. Since there still might be a numerical enhancement due to coefficient functions, a numerical model dependent study of the target mass effects to the deeply virtual Compton scattering process for the kinematics of HERMES and Jefferson Lab experiments is necessary.

In the twist-two approximation the current conservation is violated and must be restored by taking into account contributions with higher geometrical twists. To make the former manifest, one has to decompose those higher twist operators in a linear independent set by means of the QCD equation of motion \[13, 16, 17\]. This procedure gives us a part which is expressed in terms of total derivatives acting on operators with a lower twist. Of course, the latter when combined together with the leading prediction must render a manifestly gauge invariant result. However, the trace subtraction and the application of the QCD equation of motion do not commute and one can easily fail to reproduce an amplitude which respects current conservation. This problem goes beyond the scope of the present study and requires further investigations.

We would like to thank A.V. Radyushkin for a clarifying discussion of Ref. \[17\].

References

[1] H.D. Politzer, Nucl. Phys. B 172 (1980) 349.
[2] S. Wandzura, F. Wilczek, Phys. Lett. B 72 (1977) 195.
[3] H. Georgi, H.D. Politzer, Phys. Rev. D 14 (1976) 1829.
[4] D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, J. Hořejší, Fortsch. Phys. 42 (1994) 101.
[5] X. Ji, J. Phys. G 24 (1998) 1181.
[6] A.V. Radyushkin, Phys. Rev. D 56 (1997) 5524.
[7] O. Nachtmann, Nucl. Phys. B 63 (1973) 237.
[8] P. Ball, V.M. Braun, Nucl. Phys. B 543 (1999) 201; P. Ball, JHEP 9901 (1999) 010.
[9] N.J. Vilenkin, *Special functions and the theory of group representations*, Translation of Mathematical Monographs, Vol. 22, American Mathematical Society, (Providence, Rhode Island, 1968).

[10] I.I. Balitsky, V.M. Braun, Nucl. Phys. B 311 (1989) 541.

[11] B. Geyer, M. Lazar, D. Robaschik, Nucl. Phys. B 559 (1999) 339.

[12] M.V. Polyakov, C. Weiss, Phys. Rev. D 60 (1999) 114017.

[13] A.V. Belitsky, D. Müller, A. Kirchner, A. Schäfer, *Twist-three analysis of photon electroproduction off pion*, hep-ph/0011314.

[14] I.V. Anikin, B. Pire, O.V. Teryaev, Phys. Rev. D 62 (2000) 071501.

[15] A.V. Belitsky, D. Müller, Nucl. Phys. B 589 (2000) 611.

[16] N.A. Kivel, M.V. Polyakov, A. Schäfer, O.V. Teryaev, Phys. Lett. B 497 (2001) 73.

[17] A.V. Radyushkin, C. Weiss, Phys. Lett. B 493 (2000) 332; *DVCS amplitude at tree level: Transversality, twist-3, and factorization*, hep-ph/0010296.