The dynamical condition of the vibration machine: nodes of oscillations, flexural centers, connectivity parameters

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Abstract. Methodological foundations of structural mathematical modeling are being developed in the application to the problems of forming dynamic states of the working members of technological vibration machines. The purpose of the study is to develop a method of constructing mathematical models to assess the dynamic states of the working members of vibration machines under the conditions of changing the location of application of disturbing influences. Structural mathematical models are used in the form of structural diagrams of the dynamically equivalent automatic control systems. The article shows the possibilities of estimating and forming dynamic states or the distribution of oscillation amplitudes of the working members based on the use of the transfer functions of the system, in particular, the transfer functions of interpartial constraints. A number of concepts have been introduced that reflect the peculiarities of possible relations between the movement parameters of the points of the working member and the parameters of the ratio of exciting force factors. It is shown that the position of oscillation nodes and other characteristic features of dynamic states can be corrected and formed by changing the values of the coefficients of connectivity of movements between the coordinates of points, as well as between the parameters of jointly acting vibration excitations. Analytical relations have been obtained that determine the conditions for the emergence and implementation of various dynamic modes associated with the consideration of vibration nodes and flexural centers.

1. Introduction

In recent years, much attention has been paid to the development of vibration hardening technologies, which imposes special requirements on technological machines, in which the problem of assessing, controlling and managing the dynamic state of working members becomes important. Essentially, the creation of effective vibration technologies requires the development of methods and means of forming the distribution of the amplitudes of oscillations of the points of the working members, or controlling the structure and parameters of the vibration fields of vibrating tables [1-3]. This, on the one hand, initiates the search and development of efficient design and technical solutions and, on the other hand, predetermines the elaboration of ideas about the use of new approaches based on innovative scientific concepts [4, 5]. The theoretical basis for assessing and forecasting the development of technologies for managing dynamic states are, as a rule, the methods of modern theoretical and applied mechanics, which in many cases makes it possible to evaluate with sufficient accuracy the possibilities and prospects of the developed constructive and technical proposals [6, 7].
One of the promising ways of forming dynamic states of the working members of vibrating tables is the use of additional constraints, including on the basis of specially introduced mechanisms. Among them, most attractive are solutions using specific dynamic modes characteristic of mechanical oscillatory systems. That kind of systems are traditionally used in solving the problems of the dynamics of technological vibration machines. When implementing analytical approaches, mechanical oscillatory systems with two (less often, three) degrees of freedom are usually considered under the assumption that the initial system has linear properties. In special cases, more complex systems can be used [8, 9]. The dynamic state of the working members of technological machines is created by vibration exciters of various nature, which, as a rule, is the application of several concentrated forces at some points of the working member and provides the necessary dynamic state or vibrational field. This predetermines the corresponding form of distribution of vibration amplitudes of the working member points along its length. The conditions for the excitation of vibrations of the working member are quite diverse and depend on many factors, which initiates the search and development of new methods and means of assessing, controlling and managing the dynamic states of technological vibration machines. Among the rational methods and means of exciting vibrational fields, methods for correcting dynamic states are widespread options associated with installing one vibration exciter. Approaches aimed at introducing additional constraints have gained some distinction, which is described in detail in the scientific literature [7, 10]. At the same time, positive facts in a number of areas of search for efficient solutions have not yet been detailed enough. Non-traditional methods of moving a source of vibrations, for example, a single vibration exciter along a working member, are of particular interest. The proposed article discusses the possibility of constructing a mathematical model and the corresponding technology of forming distributions of the amplitudes of oscillations of the working members of vibration machines based on the methods of structural mathematical modeling.

2. General provisions. Statement of the research task

In the tasks of evaluation, control and management of the formation of dynamic states of technological vibration machines, the definition of dynamic responses in the joints of system elements and strength calculations are focused on the possibility of representing technical objects in the form of mechanical oscillatory systems with several degrees of freedom (most often, not more than three). It is assumed that technical objects execute small oscillations and have linear properties. Taking into account that kind of approaches, the computational scheme of the technological vibration machine, shown in figure 1, can be represented as a mechanical oscillatory system of a beam type with two degrees of freedom. In this case, the working member of the machine is considered as an extended solid body (mass \( M \) and moment of inertia \( J \)) located on the supporting surface on elastic linear springs with stiffnesses \( k_1 \) and \( k_2 \). The object vibrations are caused by the external harmonic force \( Q_0 \) (Figure 1), which is applied at point \( E \), separated from the center of masses (point \( O \)) of a solid body at distance \( l_0 \). The center of masses, in turn, is located at distances \( l_1 \) and \( l_2 \) from points \( A \) and \( B \) of the working member. The system motion can be described in the systems of coordinates \( y_1, y_2 \) or \( y_0, \varphi \); between which there are relationships

\[
y_0 = ay_1 + by_2, \varphi = c(y_2 - y_1), y_1 = y_0 - l_1\varphi, y_2 = y_0 + l_2\varphi, y_{E1} = y_0 - l_0\varphi. \quad (1)
\]

where

\[
a = \frac{l_2}{l_1 + l_2}, b = \frac{l_1}{l_1 + l_2}, c = \frac{1}{l_1 + l_2}, d = \frac{l_0}{l_1 + l_2}.
\]

It is assumed that the system executes small oscillations relative to the position of static equilibrium in the practical absence of forces of resistance to movement.
In accordance with [11], the force $Q_0$ applied at p. E can be transformed into a system of two forces applied at pp. A and B:

$$Q_1 = Q_0(a + d),$$  \hspace{1cm} (2)

$$Q_2 = Q_0(b - d).$$ \hspace{1cm} (3)

The task of the research is to develop a methodological framework in the technology for assessing the dynamic states of the working member, taking into account the possibilities of varying the system parameters using the ideas about the transfer functions of the system.

3. Features of the construction of a mathematical model

The system of differential equations of motion according to Figure 1 can be represented in an operator form based on the use of known approaches [6, 7]:

$$y_1(Ma^2 + Jc^2)p^2 + y_1k_1 - y_2(Jc^2 - Mab)p^2 = Q_0(a + d),$$  \hspace{1cm} (4)

$$y_2(Mb^2 + Jc^2)p^2 + y_2k_2 - y_2(Jc^2 - Mab)p^2 = Q_2(b - d),$$ \hspace{1cm} (5)

where $p = j\omega (j = \sqrt{-1})$ is a complex variable, the $\leftrightarrow$ symbol above the variable means its Laplace transform with zero initial conditions [2, 5].

Using (4), (5), it is possible to introduce into consideration a structural mathematical model of the original system (Figure 1) in the form of a structural diagram of the dynamically equivalent automatic control system [6], which is shown in Figure 2.

4. Peculiarities of connectivity of force factors

The transfer functions of the system with the simultaneous action of two force factors $Q_1$ and $Q_2$ are determined in the coordinates $y_1$ and $y_2$ by expressions

$$W_1(p) = \frac{y_1}{Q_0} = \frac{(a + d)[(Mb^2 + Jc^2)p^2 + k_2] + (b - d)(Jc^2 - Mab)p^2}{A(p)},$$ \hspace{1cm} (6)

$$W_2(p) = \frac{y_2}{Q_0} = \frac{(b - d)[(Ma^2 + Jc^2)p^2 + k_1] + (a + d)(Jc^2 - Mab)p^2}{A(p)},$$ \hspace{1cm} (7)

where $A(p) = [(Ma^2 + Jc^2)p^2 + k_1][(Mb^2 + Jc^2)p^2 + k_2] - [(Jc^2 - Mab)p^2]^2 = 0$ – is the characteristic frequency equation of the system.
From expressions (6), (7) one can find the corresponding frequency of dynamic absorbing of oscillations:

in the coordinate \( \bar{y}_1 \) – 
\[ \omega_{\text{dyn}}^2 = \frac{k_2}{Mb(b - \alpha a) + Jc^2(1 + \alpha)} \], (9)

in the coordinate \( \bar{y}_2 \) –
\[ \omega_{\text{dyn}}^2 = \frac{k_1}{Ma(\alpha a - b) + Jc^2(1 + \alpha)} \]. (10)

In expressions (10), (11) it is assumed that
\[ \alpha = \frac{b - d}{a + d}. \] (11)

In this case, \( \alpha \) is, in essence, the coefficient of connectivity between external influences \( Q_1 \) and \( Q_2 \) when the following condition is fulfilled:
\[ \bar{Q}_2 = \alpha \bar{Q}_1. \] (12)

Work [7] considers the features of dynamic states of various technical objects within a scientific hypothesis about the possibility of practical implementation of the coefficient of connectivity of external forces in the form of \( \alpha \). In this case, the connectivity of external influences in the constructive and technical form is implemented due to the possible movement of the point \( E \) of the working member. In accordance with (12), the connectivity factor \( \alpha \) can take positive and negative values, as well as take on extreme values when \( l_0 \) changes.

5. Assessment of dynamic properties. Features of interaction of elements

From (9), (10) it follows that the frequencies of the dynamic absorbing of oscillations depend on the above-introduced coefficient \( \alpha \). In conventional approaches, when the external influence is single and is applied in one of the coordinates, that is, either at \( p. A \), or at \( p. B \), the dynamic absorbing frequency is determined by the so-called partial frequencies
\[ n_1^2 = \frac{k_1}{Ma^2 + Jc^2}, \] (13)
\[ n_2^2 = \frac{k_2}{Mb^2 + Jc^2}. \] (14)

In normal situations, when the system is acted upon by one force applied in the coordinate \( \bar{y}_1 \) or \( \bar{y}_2 \), the frequencies of the dynamic oscillation absorbing coincide with the partial frequencies defined by (13), (14).

Introduce into consideration the transfer function of interpartial relations and the coefficient of connectivity of external forces \( \alpha \):
\[ W_{12}(p) = \frac{\bar{y}_2}{\bar{y}_1} = \frac{[M(a^2 + Jc^2)p^2 + k_1] + \alpha(Jc^2 - Mb)p^2}{\alpha[Mb^2 + Jc^2)p^2 + k_2] + (Jc^2 - Mb)p^2}. \] (15)

The physical meaning of the transfer function (15) is that it gives an idea of the regularities of the distribution of the amplitudes of oscillations of the working member along its length.

Let us transform (15) as
\[ W_{12}(p) = \frac{\bar{y}_2}{\bar{y}_1} = \frac{[Ma(a - b\alpha) + Jc^2(1 + \alpha)p^2 + k_1]}{[Mb(a - \alpha a) + Jc^2(1 + \alpha)p^2 + k_2]}. \] (16)

Expression (16) can be rewritten as
\[ W_{12}(p) = \frac{\bar{y}_2}{\bar{y}_1} = \frac{A p^2 + k_1}{B p^2 + k_2}, \] (17)

where \( A = [Ma(a - b\alpha) + Jc^2(1 + \alpha)]p^2; \) (17') \( B = [Mb(a - \alpha a) + Jc^2(1 + \alpha)]p^2. \) (17'')

Using expressions (15), (16), one can elaborate the ideas about the coefficient of connectivity of oscillation amplitudes in coordinates \( \bar{y}_1 \) and \( \bar{y}_2 \), recognizing that
\[ \frac{\bar{y}_2}{\bar{y}_1} = 1, \] (18)
then from (16), (17) we obtain the expression for $\alpha$

$$
\alpha = \frac{p^2 \left[ Ma(a + ib) + Jc^2(1-i) \right] + k_i}{p^2 \left[ Mb(b + ia) + Jc^2(1-i) \right] + k_2 i}.
$$

(19)

Knowing the parameters of the system $M, Jc^2, a, b, i$ and having been given the frequency $\omega^2$, you can find the corresponding value $\alpha$.

In turn, from (19) you can also find the frequency $\omega_0^2$, on which for given $M, Jc^2, a, b$ and $i$ expression will be as follows:

$$
\omega_0^2 = \frac{k_2 i \alpha - k_i}{Mbi(b - ia)(b + ia) + Jc^2(1-i)(1+i)}.
$$

(20)

From (19) it should be noted that, at the given $\alpha$, $i, M, Jc^2, a, b, k_1$ and $k_2$ can find the frequency at which the required distribution of amplitudes of oscillations of the solid body (or working member) points.

The parameter $\alpha$, that is, the coefficient of connectivity of external influences is determined through the parameters $a, b, \alpha$, which allows us to obtain

$$
d = \frac{b - \alpha a}{1 + \alpha}.
$$

(21)

Since $d = l_0(l_1 + l_2)$, then with known $l_1$ and $l_2$, the location of the force $Q_0$ application point is also determined. When setting the parameters of the vibration field in such a way that, with the known parameters of the system, an iterative process of obtaining the necessary adjustment data can be implemented. Typically, such data is determined in the development process.

If the value of $i$ is known, that, by using similarity conditions, it is possible to find laws governing amplitude distribution of the oscillations of the points of the working member along his length.

If $\alpha$ is known, with the known $i, a, b, d$ it is possible to find the position of vibration node. Let us assume that amplitude distribution of oscillations $i = \frac{\nu_2}{\nu_1}$ is determined by a diagram in Figure 3.

**Figure 3.** The diagram of the arrangement of vibration node with $\frac{\nu_2}{\nu_1} > 1$

From similarity of triangles we will find

$$
i = \frac{\nu_2}{\nu_1} = \frac{l_1 + l_2 + l_y}{l_y},
$$

(22)

then $l_y = \frac{l_1 + l_2}{1 - i}$.

(23)

Thus, the proposed approach makes it possible to create conditions for determining the necessary data about the structure of vibration field or amplitude distribution of oscillations of the points of the working member.

6. Features of the distribution of coordinates along the length of the working member of the technological machine

The expression (17) can be converted to

$$
\omega_0^2 = \frac{k_1 - k_2 i}{A - Bi}.
$$

(24)

From (19) with the parameters of the system defined by (17), (17”), it is possible to find the excitation frequency of the working member, providing the necessary value of $i$. Consider the options...
for the formation of the distribution of the amplitudes of the points of the working member, assuming that the length $AB$ (Figure 1) is determined by the relation

$$AB = l_1 + l_2. \quad (25)$$

The general diagram of the mutual ratio of the amplitudes of oscillations of points of the solid body can be represented by the diagrams shown in Figure 4, a – h.

**Figure 4.** Variants of the relative position of the displacements in the coordinates $y_1$ and $y_2$ in stationary mode with $Q_o \neq 0$: a) the case $y_1 = 0, y_2 \neq 0, i \to \infty$; b) the case $y_1 \neq 0, y_2 = 0, i = 0$; c) the case $y_1 < 0, y_2 > 0, i < -1$, the oscillation node is p. $D, y_2 / y_1 < 0$; d) the case $y_1 > 0, y_2 < 0, -1 < i < 0$, the oscillation node is p. $D, y_2 / y_1 < 0$; e) the case $y_2 = y_1$ – the oscillation node is infinitely remote, $y_2 / y_1 = 1$; f) the case $y_2 / y_1 = -1$ oscillation node is p. $D$, the oscillation node and the p. $D$ match; g) the case of the removal of the oscillation node of the p. $A$ beyond the limits of the solid body; h) the case of removal of the node oscillations beyond the p. $B$ of the solid body.

Considering the diagrams in Figure 5 gives an idea of the regularities of the distribution of the oscillation amplitudes of the working member points. So, for the cases shown in Figure 5 in the coordinate $y_1$, the mode of dynamic oscillation damping is implemented; in Figure 5, b the oscillation mode in the coordinate $y_2$. Figure 5, c and d shows the modes of oscillations when the oscillation node (p. $D$) is located between pp. $A$ and $B$, which suggests the possibility of angular oscillations together with translational movements. If the condition $y_2 / y_1 = -1$ is fulfilled, the oscillation node (p. $D$) is in the middle of the solid body. If the oscillation node and the center of mass coincide, then the solid body loses the possibility to move translationally and executes only angular oscillations. With $y_2 / y_1 = 1$ a solid body can execute only vertical straight-line oscillations in the absence of angular ones. From (17) it follows that the coefficient of connectivity of oscillation amplitudes $i$ is implemented at a certain frequency. Using (23), it is possible to estimate the possible types of amplitude distribution at each of the external disturbance frequencies, taking into account the parameters of the systems. The connectivity coefficient $\alpha$ is taken into account in the structure of expression (9) through the coefficients $A$ and $B$ (expressions (17'), (17'')). If we accept that

$$i = \frac{\left[Ma^2 + Je^2\right]p^2 + k_1 + \alpha(Jc^2 - Mab)p^2}{\alpha(Mb^2 + Je^2)p^2 + k_2 + (Je^2 - Mab)^2}, \quad (26)$$

then $i$ will depend on the frequency of external influence; for a given $i$ and system parameters, the frequency is determined by the expression (24).
The dynamic state of the vibrating working member can be controlled by installing only one vibration exciter, which generates a concentrated force. Theoretical studies show that the concentrated force applied at some point of the working member (in this case, so-called \( E \)) can be replaced by the dynamically equivalent system of two forces that will be spaced apart in the coordinates \( y_1 \) (\( A_1 \)) and \( y_2 \) (\( B_1 \)). In the considered case of equivalent transformations, a system of two coupled forces is obtained. The coefficient of connectivity of these forces is determined by the formula \( \alpha = \frac{b - d}{a + d} \). Using the values of \( \alpha \), as well as the values of \( i \) as the coefficient of interpartial connectivity of coordinates, it is possible to determine the position of characteristic points and vibration frequencies, which determine the features of the dynamic states of the vibratory member shown in Figure 5, and also the mode of absence of angular oscillations:

\[
\alpha_{\text{numl}}^2 = \frac{k_1 - \alpha k_2}{\alpha (Mb^2 + Jc^2) + (1 - \alpha)(Je^2 - Mab) - (Ma^2 + Je^2)}.
\] (27)

7. Conclusions

The proposed method of controlling the dynamic state of a vibration technological machine is easy to implement, since the necessary distribution of the amplitudes of oscillations of the working member points is determined by the parameters of the structural and technical design of the machine that are available for evaluation.

The coefficients of the connectivity of motion coordinates \( \bar{y}_1 \) and \( \bar{y}_2 \) are quite accessible by visual access or can be determined using ordinary vibration sensors with subsequent processing of information in the control unit. A technology for constructing mathematical models based on methods of structural mathematical modeling is proposed. The utilized structural mathematical models provide the necessary transfer functions and functions of interpartial connections. All the necessary data can be obtained on the basis of analytical calculations with known parameters of the technological system. The proposed methodological basis makes it possible not only to obtain data on the choice of the location of the vibration exciter, but also to assess the scope for changing the dynamic state of the working member, including through the frequency characteristic equation, and also to determine the location of the oscillation nodes of the working members, highlighting the system stability.

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