Mechanical analysis of enhancement in tensile performance of thin-walled circular tubes by internal support

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Abstract
Fillers can improve the tensile performance of ductile tubes. Mechanical analyses of tensile tubes with filler have generally focused on experimental and numerical studies on concrete-filled steel tube (CFST) components, while mechanical performance of tensile tubes with flexible supporting fillers has rarely been investigated. In the current research, we have proposed a “steel tube + pre-stressed flexible internal support” structure. Meanwhile, strengthening of tensile thin-walled tubes with internal supports was studied in terms of stress and deformation. The trends of yield strength and ultimate strength of tensile tubes were determined and calculation equations of yield strength and normal ultimate strength of tensile tubes with internal support were derived. Strengthening coefficient variations as functions of radius-thickness ratios of steel tubes and elastic moduli of internal supports as well as the optimized internal support \(p\) corresponding to maximum increment of tensile performance of tubes were also determined. It was experimentally verified that the initial supporting pre-stress of internal support on steel tubes could be achieved by cold shrink fitting technique. Experimental results revealed that the developed composite structure significantly enhanced the mechanical performance, fracture toughness, and energy consumption characteristics of tensile tubes. Hence, the proposed structure was confirmed to have promising applications.

Keywords
Internal support, thin-walled circular tubes, tensile performance, mechanical analysis, fracture experiment

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Introduction
Due to their excellent mechanical performance and good workability, thin-walled tubes with internal support (e.g. concrete-filled steel tube (CFST)) have been extensively employed in the constructions of residential buildings, long span bridges, and power transmission facilities. In practical engineering, some parts of steel tube components or component sections such as lower chord of truss bridges and the tensile sides of power transmission towers under strong wind loads may be tensile. Mechanical analyses of tensile tubes with fillers have focused on tensile tests and numerical simulations of CFST components.

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International design specifications have been proposed for CFST. CFST design specifications of Japan, America, and Europe (AIJ2008; AISC2005; CEN2004) neglect concrete effect and equate tensile CFST components with tensile hollow steel tubes, leading to relatively conservative designs. However, experimental and theoretical studies have indicated that when steel tubes were under tensile stress, radial contraction was limited by core concrete, resulting in the generation of confining force at the interface of two materials and making steel tubes in the two-dimensional tensile condition, which improved CFST tensile bearing capacity by 10%. Based on GB 50936-2014 Technical Code for Concrete Filled Steel Tubular Structures, tensile strengths of CFST components are 10% higher than those of hollow steel tubes.

Experimental and theoretical investigations on tensile CFST components have been performed by researchers around the world. Han et al. and Li et al. studied CFST component stress states under tension along axial direction and found that the presence of concrete fillers improved the tensile strength of CFST components by 11% and 14%, respectively. Also, the simplified calculation equations of tensile strength were derived. Han et al. explored the time-dependent behaviors of CFST tensile members subjected to coupled long-term loading and chloride corrosion. Their findings revealed that the filling of core concrete increased ultimate tensile strength by an average of 12.3%. Chen et al. investigated the mechanical performance of CFST specimens with reinforcing bars or angles under axial central and eccentric tensile and derived calculation equations for tensile strengths of this type of CFST components. It was experimentally found that the tensile strength of CFST components was improved by 14.6% on average compared to corresponding hollow tubes. Ye et al. performed experimental studies on the mechanical behaviors of tensile concrete-filled stainless-steel tube (CFSST) components and found that the effective combination of concrete filler and stainless-steel tube could increase the tensile strength of CFSST components by 5%–10% compared to corresponding hollow stainless-steel tubes. Regarding the ultimate tensile strength of concrete-filled double skin steel tube (CFSST) members, Li et al. and Tao and Yu considered that the tensile strength was contributed from both outer and inner tubes, and the coefficient of the strength enhancement of outer tube was considered to be 1.1. Han et al. conducted axial tension tests on concrete-encased CFST (CECFST) and angle-encased CFST (AE CFST) components, analyzed the load transfer mechanisms of these components, and derived corresponding simplified calculation equations for tensile strength. Wang et al. performed comprehensive research works on the behaviors of CFRP externally-reinforced circular CFST members under combined tension and bending, and investigated CFRP enhancement effect. Zhou et al. conducted experimental studies on the confining-strengthening, confining-stiffening, and tension-stiffening effects of axially tensile circular concrete-filled steel tube (CCFST) and squared concrete-filled steel tube (SCFST) components. They concluded that the tensile strengths of CCFST and SCFST were 10.2% and 5.2% higher than corresponding hollow tubes, respectively. Xu et al. developed an analysis model for the strength and stiffness enhancement of CCFST under axial tensile loads. Their model considered the surface performance of steel tube and concrete, offered simplified design equations for strength coefficient, and revealed that tensile strength was 15.6% higher than corresponding hollow tubes.

Few studies have been reported on the tensile performance of steel tubes with flexible supporting fillers. Zhu conducted axial compression experiments and tensile numerical simulations on PU-filled steel tube (PST) components and investigated confinement effect of steel tubes on polyurethane (PU), which revealed the in-depth working mechanism of PST composite tubes under axial loads. Hang evaluated the deformation and destruction properties of PU-filled double skin steel tubes (PFST) under axial compression, carried out numerical simulation of the PFST member under axial tension using finite element software, and adopted finite element regression method to derive calculation equations for ultimate tensile bearing capacity of PFST components. Gao et al. performed tensile fracturing tests on 304 stainless steel thin-walled circular tube components under flexible supporting operating condition and obtained the change rules of ultimate tensile strength, elongation, and fracturing energy versus loading rate and internal support elastic modulus.

Research on CFST revealed that the tensile strengths of CFST members were higher than those of reference hollow steel tubes due to the “composite action” between outer steel tube and infilled concrete. They also derived corresponding calculation equations. Research on the calculation of tensile strength of steel tubes with fillers has focused on CFST field. Domestic and international CFST design standards stipulate the tensile strength of steel tubes. Some researchers have performed experimental, numerical, and theoretical studies on the tensile strengths of different CFSTs. There are few reports on the calculation equations of tensile strengths of steel tubes with flexible supporting materials. Using numerical method, the tensile properties of PST and PFST have been studied and the strengthening coefficients of ultimate tensile bearing capacity of corresponding components have been obtained using finite element regression. In Zhu and Hang, the enhancement of tensile strength of steel tubes was mainly due to the bonding effect between
steel tubes and PU filler. Most of these calculation equations are empirical equations derived from tensile experiments or numerical simulations of specific components, most of which rely on different deformation versions of enhancement factor 1.1 and superposition equations of the strength of different components, lacking deep study in theoretical level. Table A1 in the Appendix 2 summarizes the calculation equations of the tensile strengths of different steel tubes with fillers.

In the current research, stress states of steel tubes and internal supports were theoretically studied. The stress and deformation increments of thin-walled tensile tubes with internal support were coupled and the changing trajectory of yield strength and ultimate strength of tensile tubes was given. The calculation equation of yield and normal ultimate strengths of composite structures was derived, and the changing rule of strengthening coefficient with radius-thickness ratio of steel tubes and elastic modulus of internal support was summarized. The optimized internal support corresponding to maximum increment of tensile performance was obtained, and applicability of cold shrink fitting technique to realize initial supporting pre-stress of internal support on steel tubes was further evaluated and its feasibility was experimentally verified. Experiments demonstrated that the developed composite structure considerably improved the mechanical performance, fracture toughness, and energy consumption properties of tensile tubes. Therefore, the developed structure was considered to have promising applications.

**Stress analysis of tube wall**

**Basic hypotheses**

This article follows the following basic hypotheses in the derivation process:

1. The yield condition of the steel tube meets the Mises yield criterion.
2. There was no bonding between the outer surface of the internal support and the inner wall of the thin-walled tube, so the friction shear stress between the internal support and the thin-walled tube was not considered.
3. The thin-walled tube material satisfied the isotropic linear hardening theory and the single curve hypothesis.
4. During the stretching process, the thin-walled tube undergone elastoplastic deformation, while the internal support material only undergone elastic deformation.

## Mises yield criterion

When a material is under certain stress conditions, it yields and this condition is called yield condition, which is generally represented by yield function. Mises yield condition is one of the most commonly applied yield conditions and its physical significance is that when the distortion energy corresponded to the stress condition of a certain point within an object reaches a certain value $C$, the material yields at this spot. At yield state, second invariant of deviatoric stress of tensile hollow tube $J_2$ can be stated as:

$$ J_2 = \frac{1}{6} \left( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right) = C $$  \hspace{1cm} (1)

Mises yield condition can be applied to describe the yield behaviors of metallic materials with good ductility. When material constant is standardized according to uniaxial elongation experiments of thin-walled circular tubes, one would have:

$$ J_2 = \frac{1}{6} \left( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right) = \frac{\sigma_2^2}{3} $$  \hspace{1cm} (2)

In principal stress space, Mises yield surface is expressed as a cylindrical surface along the direction of hydrostatic compressive axis (as shown in Figure 1(a)). The intersecting line of this yield surface and $\sigma_1 - \sigma_2$ plane is a ellipse (as shown in Figure 1(b)), which could be expressed as:

$$ \sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 - \sigma_3^2 = 0 $$  \hspace{1cm} (3)

From Figure 1(b): When the tube wall material in the bidirectional tension state reaches the initial yield, with the increase of $\sigma_2$, $\sigma_1$ first increases and then decreases. When $\sigma_2$ takes a specific value, $\sigma_1$ can achieve a maximum value.
Stress analysis of tube wall at yield state

Under the action of axial tension, the steel tube would shrink laterally. Due to the supporting effect of the concrete, the lateral necking of the steel tube was restricted. The steel tube wall was in a three-way stress state, namely: longitudinal tension, circumferential tension, and radial compression. The Poisson effect of tensile thin-walled structures with filler (e.g. CFST) results in the generation of inward contraction, and fillers can provide thin-walled structures with different support levels. Figure 2 shows the typical stress state of tensile thin-walled structures with fillers.

Outer surface of tube wall. The outer surface of tube wall is under plane stress with zero radial stress; that is, $\sigma_r = 0$. Its axial stress is $\sigma_l$ and hoop stress $\sigma_t$ is related to compressive stress $p$ on the inner wall of thin-walled tubes.

Since walls are relatively thin, wall thickness $t$ is a small value compared to internal radius $r$ of circular tubes. By neglecting the gradient of $\sigma_t$ along the direction of radius, according to the principle of force balance (as shown in Figure 3), one would have:

$$\sigma_r = \frac{\int_0^\pi rp \sin \phi d\phi}{2t} = \frac{pr}{t}$$  \hspace{1cm} (4)

which is the same as that calculated with the Laplace equation for thin-walled tubes under pressure.

By substituting axial stress $\sigma_l$, hoop stress $\sigma_t$ and equation (4) into equation (3):

$$\sigma_l^2 - \sigma_l \frac{pr}{t} + \left(\frac{pr}{t}\right)^2 - \sigma_t^2 = 0$$  \hspace{1cm} (5)

Defining radius-thickness ratio as $\lambda = \frac{r}{t}$ and substitute it into equation (5) gives:

$$\left(\frac{\sigma_l}{\sigma_t}\right)^2 - \frac{\sigma_l}{\sigma_t} \frac{p}{\sigma_t} \lambda + \lambda^2 \left(\frac{p}{\sigma_t}\right)^2 - 1 = 0$$  \hspace{1cm} (6)

Using $\frac{p}{\sigma_t}$ and $\frac{\sigma_l}{\sigma_t}$ as independent and dependent variables, respectively, the trajectory of equation (6) would be a series ellipses with the change of $\lambda$. Ellipse trajectory in the first quadrant is shown in Figure 4 indicating that $\sigma_l$ of outer surface was first increased and then decreased with the increase of $p$. The ellipse became more and more flat with gradual increase of $\lambda$. If $\frac{p}{\sigma_t} = \frac{1}{\sqrt{\lambda}}$, the maximum value of $\frac{\sigma_l}{\sigma_t}$ is $2\frac{1}{\sqrt{\lambda}} \approx 1.155$.

Inner surface of tube wall. The inner surface of tube wall is under stress state along three directions. The three principal stresses include axial stress $\sigma_l$, hoop stress $\sigma_t$,
and radial stress \(-p\), among which hoop stress depends on radial stress. The friction shear stress \(\tau_f\) of the inner surface of tube walls is neglected.

With inner surface of tube walls at yield state:

\[
J_2^f = \frac{1}{6} \left[ (\sigma_t - \lambda p)^2 + (\sigma_t + p)^2 + (\lambda p + p)^2 \right] = \frac{\sigma_t^2}{3} \quad (7)
\]

Equation (7) can be rearranged as:

\[
\left( \frac{\sigma_t}{\sigma_s} \right)^2 + (1 - \lambda) \frac{\sigma_t}{\sigma_s} \frac{p}{\sigma_s} + (\lambda^2 + \lambda + 1) \left( \frac{p}{\sigma_s} \right)^2 - 1 = 0
\]

Using \(\frac{p}{\sigma_s}\) and \(\frac{\sigma_t}{\sigma_s}\) as independent and dependent variables, respectively, the trajectory of equation (8) is a series ellipses for different values of \(\lambda\). Figure 5 shows ellipse trajectory in the first quadrant indicating that the \(\sigma_t\) of inner surface was first increased and then decreased with the increase of \(p\). The ellipse became more and more flat with the increase of \(\lambda\). The maximum value of \(\frac{p}{\sigma_s}\) was obtained when \(\frac{\sigma_t}{\sigma_s} = \frac{\lambda - 1}{(\lambda + 1)\sqrt{3(\lambda^2 + \lambda + 1)}}\) and the maximum value of \(\frac{\sigma_t}{\sigma_s}\) increased with \(\lambda\) to the extremum of \(\frac{1}{\sqrt{2}} \approx 1.155\).

Figure 6 shows the comparison and analysis of inner and outer surfaces of tube wall. At constant value of \(\lambda\), the increment of inner surface was smaller than that of outer surface. With the increase of \(\lambda\), the yield trajectories of inner and outer surfaces gradually became closer.

**Stress analysis of tube wall at extreme state**

According to the isotropic hardening theory, \(J_2^f\) of steel tubes with filler at extreme state is equal to the corresponding \(J_2^u\) of the extension of hollow tubes.

Isotropic hardening under complex stress states is shown in Figure 7. Isotropic characteristics are maintained after material hardening and the anisotropy induced by plastic deformation is neglected. Based on isotropic hardening theory, it could be concluded that the expression of \(J_2^u\) is unchanged. According to single curve assumption, \(J_2^u\) of steel tubes with filler at extreme state is equal to the corresponding \(J_2^u\) of the extension of hollow tubes.

Based on isotropic hardening theory and single curve assumption, the corresponding \(J_2^u\) could be found when the tube broke under the condition of internal support:

\[
J_2^u = \frac{1}{6} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 \right] = \frac{\sigma_u^2}{3} \quad (9)
\]
Equation (9) could be rearranged as:

\[
\left( \frac{\sigma_1}{\sigma_u} \right)^2 - \frac{\sigma_1 \sigma_2}{\sigma_u^2} + \left( \frac{\sigma_2}{\sigma_u} \right)^2 - 1 = 0
\]  
(10)

The case of biaxial tension (first quadrant) and the independent stresses along two directions were considered. Derivation of equation (10) is taken on \( \sigma_2 \). It is found that when \( \frac{\sigma_2}{\sigma_u} = \frac{1}{\sqrt{3}} \frac{\sigma_1}{\sigma_u} \) has the maximum value of \( \left( \frac{\sigma_1}{\sigma_u} \right)_{\text{max}} = \frac{2}{3} \approx 1.155 \). In other words, under the condition of biaxial tensile state, the ultimate tensile stress can be increased by about 15.5%.

The stress state analyses of inner and outer surfaces of tube walls under extreme state are similar to corresponding analyses under yield state, which is not further elaborated.

**Deformations analysis of tube wall**

**Cross-section area of tensile hollow tube wall at elastic stage**

According to the generalized Hooke law, the strain state of thin-walled circular tubes at elastic stage under axial tensile load could be stated as:

\[
\begin{align*}
\varepsilon_t &= \frac{\sigma_t}{E} - \frac{\mu(\sigma_t + \sigma_r)}{E} \\
\varepsilon_r &= \frac{\sigma_r}{E} - \frac{\mu(\sigma_t + \sigma_r)}{E} \\
\varepsilon_p &= \frac{\sigma_p}{E} - \frac{\mu(\sigma_t + \sigma_r)}{E}
\end{align*}
\]  
(11)

When hollow tube is uniaxially stretched, axial tensile stress is \( \sigma_t \), hoop stress is \( \sigma_r = 0 \), and radial stress is \( \sigma_p = 0 \). By substituting these parameters into equation (11):

\[
\begin{align*}
\varepsilon_t &= \frac{\sigma_t}{E} \\
\varepsilon_r &= -\frac{\mu \sigma_t}{E} \\
\varepsilon_p &= -\frac{\mu \sigma_t}{E}
\end{align*}
\]  
(12)

For tensile hollow tubes, the ratio of the cross-section area of tube wall \( S_1 \) and original cross-section area of tube wall \( S_0 \), \( \frac{S_1}{S_0} \), is expressed as:

\[
\frac{S_1}{S_0} = \left( 1 - \frac{\sigma_t}{E} \right)^2
\]  
(13)

**Cross-section area of tube walls with internal supports at elastic stage**

Regarding steel tubes with internal supports, the stress states of inner and outer surfaces are different such that outer surface is under biaxial tensile stress \( (\sigma_t, \sigma_r) \), while inner surface is exposed to tension in two directions and compression in one direction \( (\sigma_t, \sigma_r, -p) \).

**Deformations of outer surface of tube walls.** Substitution of \( \sigma_t, \sigma_r = \frac{p}{r} = \lambda p \), and \( \sigma_r = 0 \) into equation (11) gives:

\[
\begin{align*}
\varepsilon_t &= \frac{1}{E} \left( \sigma_t - \mu \lambda p \right) \\
\varepsilon_r &= \frac{1}{E} \left( \lambda p - \mu \sigma_t \right) \\
\varepsilon_p &= -\frac{1}{E} \mu (\sigma_t + \lambda p)
\end{align*}
\]  
(14)

At elastic stage, the ratio of the cross-section area of tube wall with internal support \( (S_1) \) to the original cross-section area of tube wall \( (S_0) \), \( \frac{S_1}{S_0} \), is obtained as:

\[
\frac{S_1}{S_0} = (1 + \varepsilon_t)(1 + \varepsilon_r)
\]

\[
= \left( 1 + \frac{\lambda p}{E} - \frac{\mu}{E} \sigma_t \right) \left( 1 - \frac{\mu}{E} \sigma_t - \frac{\mu}{E} \lambda p \right)
\]  
(15)

According to equations (13) and (15), \( \frac{S_1}{S_0} \) is derived as:

\[
\frac{S_1}{S_0} = \frac{(1 + \frac{\lambda p}{E} - \frac{\mu}{E} \sigma_t)(1 - \frac{\mu}{E} \sigma_t - \frac{\mu}{E} \lambda p)}{(1 - \frac{\mu}{E} \sigma_t)^2}
\]  
(16)

At elastic stage, the Poisson’s ratio of steel tubes \( \mu \) is considered to 0.3. \( \frac{S_1}{S_0} \) is defined where \( C_e \) is the elastic part in longitudinal total stress. Substitution of \( C_e \) into equation (16):

\[
\frac{S_1}{S_0} = \frac{\left( 1 - \mu \frac{C_e}{E} + C_e \lambda \frac{p}{E} \right)(1 - \mu \frac{C_e}{E} - C_e \lambda \frac{p}{E})}{(1 - \mu \frac{C_e}{E})^2}
\]  
(17)

When the outer surface of tube wall reaches yield state, \( \left( \frac{S_1}{S_0} \right)_{y} \), becomes:

\[
\left( \frac{S_1}{S_0} \right)_{y} = \frac{\left( 1 - \mu \frac{C_e}{E} + C_e \lambda \frac{p}{E} \right)(1 - \mu \frac{C_e}{E} - C_e \lambda \frac{p}{E})}{(1 - \mu \frac{C_e}{E})^2}
\]  
(18)

\( C_e \) is different when different material reaches the yield state. With different \( C_e \) values, the value of outer surface \( (S_1/S_0)_{y, \text{max}} \) is summarized in Table A2 in the Appendix 2.

As can be seen in Table A2 in the Appendix 2, \( (S_1/S_0)_{y} \) is a parabola which is changed with \( \frac{C_e}{\sigma_t} \). Changing trajectories corresponding to \( C_e \) values of 0.001, 0.002, 0.005, and 0.01 are shown as solid curves in Figure 8(a) to (d), respectively. When \( \frac{C_e}{\sigma_t} \) takes \( \frac{1}{\sigma_t} \), \( (S_1/S_0)_{y} \) takes the maximum value. Even when the elastic strain reaches 1%, the relative increase in cross-sectional area is only 0.7%. Under normal circumstances, the elastic strain of steel is only on the order of...
1000th, so the relative increase in cross-sectional area of tube wall outer surfaces can be ignored.

Deformations of inner surface of tube walls. Substitution of $\sigma_i = \frac{\lambda p}{E}$ and $\sigma_r = -p$ into equation (11) gives:

$$
\begin{align*}
\varepsilon_i &= \frac{\sigma_i}{E} - \frac{\mu(\lambda p - p)}{E} \\
\varepsilon_r &= \frac{\lambda p}{E} - \frac{\mu(\sigma_i - p)}{E} \\
\varepsilon_r &= -\frac{p}{E} - \frac{\mu(\sigma_i + \lambda p)}{E}
\end{align*}
(19)
$$

At elastic stage, the ratio of cross-section area of tube wall with internal support $S_i^e$ to the original cross-section area of tube wall $S_0^e$ is derived as:

$$
\frac{S_i^e}{S_0^e} = (1 + \varepsilon_i)(1 + \varepsilon_r) = \left(1 + \frac{\lambda p}{E} - \frac{\mu\sigma_i}{E} + \frac{\mu p}{E}\right)
\left(1 - \frac{p}{E} - \frac{\mu\sigma_i}{E} - \frac{\mu\lambda p}{E}\right)
(20)
$$

Based on equations (13) and (20), $\frac{S_i^e}{S_0^e}$ is obtained as:

$$
\frac{S_i^e}{S_1^e} = \frac{1 + \frac{\lambda p}{E} - \frac{\mu\sigma_i}{E} + \frac{\mu p}{E}}{(1 - \frac{p}{E} - \frac{\mu\sigma_i}{E} - \frac{\mu\lambda p}{E})}
(1 - \frac{\mu\sigma_i}{E})^2
(21)
$$

At elastic stage, Poisson’s ratio of steel tube $\mu$ is assumed to be 0.3.

Substitution of $C_e$ into equation (21) gives:

$$
\frac{S_i^e}{S_1^e} = \frac{1 - \mu C_e + C_e(\lambda + \mu)\frac{\sigma_i}{E}}{(1 - \mu C_e)^2}
\left[1 - \mu C_e - C_e(1 + \mu)\frac{\sigma_i}{E}\right]
(1 - C_e)^2
(22)
$$

Figure 8. Increment of cross-section areas of inner and outer surfaces of tube walls with internal support at elastic stage: (a) $C_e = 0.001$, (b) $C_e = 0.002$, (c) $C_e = 0.005$, and (d) $C_e = 0.01$. 
When the inner surface of tube walls reach yield state, \( \left( \frac{S_i^e}{S_i^l} \right)_s \) becomes:

\[
\left( \frac{S_i^e}{S_i^l} \right)_s = \frac{1 - \mu C_e + C_e(\lambda + \mu)\frac{p}{\sigma_i}}{(1 - \mu C_e)^2} \left[ 1 - \mu C_e - C_e(1 + \mu\lambda)\frac{p}{\sigma_i} \right]
\]

(23)

Different materials have various \( C_e \) values at yield state. The values of inner surface \( \left( S_i^e/S_i^l \right)_{y, \text{max}} \) for different \( C_e \) values are given in Table A3 in the Appendix 2.

As given in Table A3 in the Appendix 2, \( \left( S_i^e/S_i^l \right)_s \) is a parabola which is changed with \( \frac{p}{\sigma_i} \). Changing trajectories corresponding to \( C_e \) values of 0.001, 0.002, 0.005, and 0.01 are shown as dashed curves in Figure 8(a) to (d), respectively. When \( \frac{p}{\sigma_i} \) takes \( \frac{1}{\lambda} \) and \( \lambda \) tends to infinity, \( \left( S_i^e/S_i^l \right)_s \) takes the maximum value. Even when the elastic strain reaches 1%, the relative increase in cross-sectional area is only 0.7%. Under normal circumstances, the elastic strain of steel is only on the order of 1000th, so the relative increase in cross-sectional area of tube wall inner surfaces can be ignored.

**Cross-section areas of tube walls with internal supports at strengthening stage**

When Mises material reaches strengthening stage, its strain is divided into elastic and plastic strains. For elastic strain part, Poisson’s ratio of tube \( \mu \) is considered as \( \mu = 0.3 \), still satisfying generalized Hooke law. That is to say, elastic strain part complies with the equations obtained in Section 3.2. Due to the plastic flow characteristics, for the plastic strain part, Poisson’s ratio of tube \( \mu \) is considered as \( \mu = 0.5 \).\(^{25,26}\) On the basis of volume invariant principle and homogeneous deformation theory, for plastic strain part, the ratio of cross-section area of tube wall with internal support to that of tensile hollow tube wall \( \left( \frac{S_i^e}{S_i^l} \right)_s \) at strengthening stage is derived as:

\[
\left( \frac{S_i^e}{S_i^l} \right)_s = 1
\]

(24)

When Mises material enter strengthening deformation stage, for which the stress-strain curves conform to “bilinear isotropic hardening” model (Figure 9), the following relationship is obtained for \( C_e \) and \( \varepsilon_l \):

\[
C_e = \varepsilon_l \frac{h}{E} + \frac{\sigma_u}{E} \left( 1 - \frac{h}{E} \right)
\]

(25)

\( S_i^e \) of tube wall outer surface can be related to \( p/\sigma_i \) by substituting equation (25) and \( \sigma_l = \sigma_u \) into equation (17):

\[
\begin{align*}
\left( \frac{S_i^e}{S_i^l} \right)_u &= \frac{S_i^e}{S_i^l} = \frac{C_e}{\varepsilon_l} + \frac{S_i^P}{S_i^l} + \frac{\varepsilon_l - C_e}{\varepsilon_l} \\
&= f_{\text{out}} \left( \frac{p}{\sigma_u} \right) + \frac{C_e}{\varepsilon_l} + 1*\frac{\varepsilon_l - C_e}{\varepsilon_l}
\end{align*}
\]

(26)

Based on equations (24)–(26), the ratio of the cross-section area of outer surface of tube wall to that of tensile hollow tube wall at extreme state is obtained as:

\[
\left( \frac{S_i^e}{S_i^l} \right)_u = \frac{S_i^e}{S_i^l} + \frac{C_e}{\varepsilon_l} + \frac{\varepsilon_l - C_e}{\varepsilon_l}
\]

(28)

Based on equations (24), (25) and (27), the ratio of the cross-section area of inner surface of tube wall to that of tensile hollow tube wall at extreme state is obtained as:

\[
\left( \frac{S_i^e}{S_i^l} \right)_{y, \text{max}} = \frac{S_i^e}{S_i^l} + \frac{C_e}{\varepsilon_l} + 1*\frac{\varepsilon_l - C_e}{\varepsilon_l}
\]

(29)

**Coupling of stress and deformation analyses**

As discussed in Section 3.2, the change of stress state significantly improves the carrying capacity of steel tubes.

As discussed in Section 3.3, the relative increase of cross-section area can decrease real stress. For
commonly applied steel types, elastic strain part is about millesimal order. As can be seen from Figure 8, for elastic strain parts with millesimal order, the increment of tube wall cross-section area is only 0.07%.

Since \( \frac{S_2}{S_1} = 1 \), \( \frac{S_2}{S_1} \) is decreased with the increase of \( \varepsilon_t \) in equations (28) and (29). In other words, the relative increment of tube wall cross-section area will be further decreased in extreme state. Therefore, under either yield or extreme states, the relative increment of tube wall cross-section area is very small and the corresponding decrease of real stress in tube wall can be neglected.

When coupling effect of stress and deformation analyses is considered, relative increment of cross-section area can be neglected and only the stress improvement brought by stress state change is taken into account. Based on equations (6) and (8) and Figure 6, under yield and extreme states, optimized support \( p \) and maximum stress increment of tube walls with different radius-thickness ratios are summarized in Table 1.

### Calculation equations of the strengths of steel tubes with flexible internal supports

#### Calculation equations of yield strength

With flexible supporting materials, the typical stress states of the inner surfaces of tensile steel tubes and internal supports are shown in Figure 10. The friction shear stresses of the inner surfaces of tube walls \( \tau_f \) can be neglected and tube walls are assumed to be under triaxial stress state (axial stress \( \sigma_t \), hoop stress \( \sigma_r \), radial stress \( -p \)). Because of the compressive effect of steel tubes, internal support bears confining compressive stress \( p \).

According to the generalized Hooke law,

\[
\varepsilon_t = \frac{\sigma_t}{E} - \frac{\mu(\sigma_t + \sigma_r)}{E} \quad (30)
\]

\[
\varepsilon_{t, in} = \frac{\sigma_{t, in}}{E_{in}} - \frac{\mu_{in}(\sigma_{t, in} + \sigma_{r, in})}{E_{in}} \quad (31)
\]

Since steel tubes and internal supports satisfy the force balance and compatibility of deformation, one can consider that:

\[
\varepsilon_t = \varepsilon_{t, in} \quad (32)
\]

For steel tubes,

\[
\begin{align*}
\sigma_t &= \lambda p \\
\sigma_r &= -p
\end{align*} \quad (33)
\]

and for internal support,

---

| Yield state | Extreme state |
|-------------|---------------|
| \( \lambda - 1 \) | \( \lambda + 1 \) |
| \( \lambda + 1 + \sqrt{3}(\lambda + 1 + 1) \) | \( \lambda + 1 + \sqrt{3}(\lambda + 1 + 1) \) |

| Optimized support \( p \) on inner surface | Average \( p \) on outer surface |
|---------------------------------|-------------------|
| \( \frac{S_2}{S_1} \) | \( \frac{S_2}{S_1} \) |

| Table 1: Optimized support \( p \) and maximum stress increment of tube wall at yield and extreme states. |
Taking \( m \) is substituted into equation (36) and (37) gives:

\[
\frac{\sigma_t - \mu(\sigma_t - p)}{E} = \frac{1}{(1 - \mu_{in})} \frac{-p}{E_{in}}
\]

Considering equations (33) and (34), \( p \) in equation (35) is eliminated to give:

\[
\sigma_t = \frac{\lambda \mu}{(\lambda + \mu) + \frac{E}{E_{in}}(1 - \mu_{in})} \sigma_l
\]

Define

\[
\omega_s = \frac{\lambda \mu}{(\lambda + \mu) + \frac{E}{E_{in}}(1 - \mu_{in})}
\]

Substitution of \( \sigma_r \) and \( \sigma_t \) into equation (3) gives:

\[
\sigma_l^2 - \sigma_l \sigma_t + \sigma_t^2 = \sigma_s^2
\]

Yield strength of steel tubes with fillers can be calculated by substituting equations (36) and (37) into equation (38):

\[
\sigma_l = \sqrt{\frac{1}{1 - \omega_s + \omega_s^2}}
\]

Taking \( \mu = 0.3, \mu_{in} = 0.4, \) and \( E = 210 \) GPa, Figure 11 presents the changing trajectories of strengthening coefficient \( \sqrt{1/(1 - \omega_s + \omega_s^2)} \) for different values of \( \lambda, E_{in} \). From Figure 11, it is concluded that: (1) At yield state, increase of the elastic modulus of internal support increases strengthening coefficient; (2) Under constant internal support conditions, increase of the radius-thickness ratio of steel tube increases strengthening coefficient; (3) If \( E_{in} \to \infty \) and \( \lambda \to \infty, \)

\[
\sqrt{1/(1 - \omega_s + \omega_s^2)} \leq 1.125 < 1.155; \text{therefore, theoretical maximum (1.155) cannot be reached by adjusting the elastic modulus of internal support}; \text{(4) Strengthening coefficient } \sqrt{1/(1 - \omega_s + \omega_s^2)} \text{ is increased by increasing Poisson’s ratio of internal support } \mu_{in}, \text{but it is not sensitive to the change of } \mu_{in}.
\]

**Calculation equations of normal ultimate strength**

At strengthening stage, the Poisson’s ratio of steel \( \mu \) tubes approaches 0.5.\textsuperscript{25,26} Strain is related to not only stress state, but also deformation history. Therefore, the constitutive relation of steel tubes at strengthening stage was investigated based on incremental theory.

Assume that during steel tube tensile, radius-thickness ratio \( \lambda \) remains unchanged. The steel tubes are under the stress state of biaxial stretching \( (\sigma_l, \sigma_t, \sigma_s) \) and uniaxial compression \( (\sigma_l, \sigma_r, \sigma_s) \). Since \( \sigma_r \) is much smaller than \( \sigma_l \) and \( \sigma_t, \sigma_s \) can be neglected giving rise to:

\[
\begin{align*}
\sigma_1 &= \sigma_l \\
\sigma_2 &= \sigma_t = \lambda \sigma_p \\
\sigma_3 &= \sigma_s = 0
\end{align*}
\]

The deviators of stress are:

\[
\begin{align*}
s_1 &= \frac{2}{3} \sigma_l - \frac{1}{3} \sigma_t \\
s_2 &= -\frac{1}{3} \sigma_l + \frac{2}{3} \sigma_t \\
s_3 &= -\frac{1}{3} \sigma_l - \frac{1}{3} \sigma_t
\end{align*}
\]
According to equation (41), the second invariant of deviatoric stress $J_2$ is derived as:

$$J_2 = \frac{2}{3} (\sigma_i^2 - \sigma_i \sigma_t + \sigma_t^2)$$  \hspace{1cm} (42)

By defining plastic scale factor as $dX$ and according to the rule of orthogonal flow we have: \(^{24}\)

$$dX = dX \left( \frac{\partial f}{\partial \sigma_2} \right)$$  \hspace{1cm} (43)

and from Chen, \(^{24}\)

$$\begin{align*}
\left\{ dX = \frac{1}{h} \left( \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} \right) \\
\frac{\partial f}{\partial \sigma_{ij}} = \frac{\sqrt{3}}{2} \sqrt{J_2} s_{ij}
\end{align*}$$  \hspace{1cm} (44)

Substitution of equation (44) into equation (43) gives:

$$dX = \frac{3}{4h^2 J_2} (s_1 s_2 d\sigma_t + s_2^2 d\sigma_s)$$  \hspace{1cm} (45)

and substitution of equations (40)–(42) into equation (45):

$$dX = \frac{1}{8h} (\sigma_i^2 - \sigma_i \sigma_t + \sigma_t^2)$$

$$\left\{ -(2\sigma_i^2 - 5\sigma_i \sigma_t + 2\sigma_t^2) d\sigma_t + (\sigma_i^2 - 4\sigma_i \sigma_t + 4\sigma_t^2) d\sigma_s \right\}$$  \hspace{1cm} (46)

Assuming that inner flexible supporting materials are always at elastic stage, one could conclude $h_{in} = E_{in}$.

Similar to the process of obtaining the $dX$, the plastic strain increment of flexible internal support can be derived as:

$$dX_{in} = \frac{1}{2E_{in}}$$  \hspace{1cm} (47)

Based on deformation compatibility relation between inner surfaces of tube wall and outer surface of internal support, equations (46) and (47) are combined to give:

$$dX = dX_{in}$$  \hspace{1cm} (48)

Equation (48) gives the relationship between $\sigma_t$ and $\sigma_t$ at strengthening stage. Then, combining equation (10) the relationship between $\sigma_t$ and $\sigma_t$ at strengthening stage can be obtained, further getting the expression of the strengthening coefficient of ultimate strength. However, the solving process of the above equation is very complex. Therefore, Han et al. \(^6\) developed 5000 micro strain method. A total of 5000 micro strain method considers the following three assumptions: (1) For most steel types, when longitudinal tensile strain reaches 5000 $\mu e$, steel tubes enter yield state; (2) In the following stage, tensile strain shows significant development trend while tensile load grows slowly; (3) Corresponding tensile stress when the longitudinal tensile strain of steel tubes reaches 5000 $\mu e$ is normal ultimate strength, but not the ultimate strength of elongation at break.

Based on the above three assumptions, tensile stress corresponding to the longitudinal tensile strain of the steel tube reaching 5000 $\mu e$ is defined as its normal ultimate strength.

When the longitudinal tensile strain of steel tubes reaches 5000 $\mu e$, the overall deformation of steel tubes is small. Under the condition of small deformation, calculation equation (36) can still be used as:

$$\sigma_t = \frac{\lambda \mu}{(\lambda + \mu)} + \frac{\lambda}{E_{in}} (1 - \mu_{in}) \sigma_t$$  \hspace{1cm} (49)

Define

$$\omega_{in} = \frac{\lambda \mu}{(\lambda + \mu)} + \frac{\lambda}{E_{in}} (1 - \mu_{in})$$  \hspace{1cm} (50)

Substitution of $\sigma_t$ and $\sigma_t$ into equation (10) gives:

$$\sigma_t^2 - \sigma_t \sigma_t + \sigma_t^2 = \sigma_t^2$$  \hspace{1cm} (51)

The normal ultimate strength of steel tubes with fillers can be obtained by substituting equations (49) and (50) into equation (51) as:

$$\sigma_t = \frac{1}{1 - \omega_{in} + \omega_{in} \sigma_{in}}$$  \hspace{1cm} (52)

**Discussion**

**Optimization of $p$**

Based on equation (36), the relation of internal support pressure at elastic stage ($p$) and elastic modulus of internal support ($E_{in}$) can be expressed as:

$$p = \frac{\mu \sigma_t}{(\lambda + \mu)} + \frac{\lambda}{E_{in}} (1 - \mu_{in})$$  \hspace{1cm} (53)

Steel tubes are stretched to initial yield point. Steel tubes gain higher internal support pressure $p$ at higher values of elastic modulus of internal support $E_{in}$. When $E_{in}$ approaches infinity, $p_{max}$ can be stated as:

$$p_{max} = \frac{\mu}{\lambda + \mu} \sigma_t = \frac{0.3}{\lambda + 0.3} \sigma_t$$  \hspace{1cm} (54)

where the Poisson’s ratio of steel tubes $\mu$ is assumed to be 0.3.

As can be seen from Table 1, optimized $p$ of steel tube at yield state $p_{best}$ can be obtained by:
According to generalized Hooke law,

\[ \varepsilon_{r,\text{in}} = \frac{(1 - \mu_{\text{in}}) P_{\text{best}}}{E_{\text{in}}} \]  \hspace{1cm} (58)

where \( E_{\text{best}} \) depends on radius-thickness ratio of steel tubes \( l \), Poisson’s ratio of internal support \( \mu_{\text{in}} \) and radial contraction rate \( \varepsilon_{r,\text{in}} \) of internal support at certain low temperature. The application of the above method can retrodict the optimized elastic modulus of internal support and identify the material with corresponding modulus. According to trail, the optimized elastic modulus of internal support for commonly applied tubes is in the range of hundreds to a few thousand MPa. A variety of flexible polymer fillers, including nylon, PTFE, PU, and ABS can be found in this range. Therefore, using flexible internal support with cold shrink fitting technique, prestress can be generated on steel tubes by internal support and initial optimized supporting pressure \( P_{\text{best}} \) can be obtained.

**Exploratory experiment**

Figure 12(a) shows the dimensions of the 304 stainless steel thin-walled circular tube standard components prepared according to ISO6892-1 Metallic materials-Tensile testing and Figure 12(b) shows hollow tube component. Also, the component with PTFE filler is presented in Figure 12(c). The measured parameters of

\[ p_{\text{best}} = \left[ \frac{1 - \frac{1}{2\lambda + 1} \sqrt{3(\lambda^2 + \lambda + 1)}}{2\lambda + 1} \right] + \frac{1}{2\sqrt{3\lambda}} \]  \hspace{1cm} (55)

\[ \sigma_x = \frac{1}{2\sqrt{3(1 + \frac{1}{\lambda})^2}} \sigma_x \]  \hspace{1cm} (56)

If \( \lambda = 2 \), \( f(\lambda) \approx 0 \); \( \frac{1}{2\sqrt{3(1 + \frac{1}{\lambda})^2}} \sigma_x \) and \( \frac{0.3}{\lambda + 0.3} \sigma_s \) are the increasing and decreasing functions of \( \lambda \), respectively. However, radius-thickness ratio of thin-walled circular tubes is much greater than 2. Therefore, \( f(\lambda) > 0 \) and in turn \( p_{\text{best}} < p_{\text{best}} \) are always true. At elastic stage, optimized \( p \) cannot be obtained by adjusting the elastic moduli of internal supports. Rigid internal supports outperform flexible ones in improving yield strength.

It is worth noting by the researchers that how to realize the optimized \( p \) to reach the largest enhancement of yield strength. Herein, we have proposed two solutions (micro-expansion concrete and cold shrink fitting technique) to achieve optimized pre-stress of internal supports.

Adding micro-expansion concrete into steel tubes is extensively applied in engineering.27–29 Lots of experimental studies have been conducted on the deformation ability of micro-expansion concrete and the bonding properties of steel-concrete interface.30–33 However, micro-expansion concrete needs long maintenance times. Therefore, we used cold shrink fitting technique as an example to realize pre-stressed internal support of steel tubes.

Experiments demonstrated that low-temperature cooling could lead to the contraction of polymers, including nylon, polytetrafluoroethylene (PTFE), polyurethane (PU), and acrylonitrile butadiene styrene (ABS) plastic. Then, the cooled internal support material is inserted into the steel tube. When the internal support material returns to room temperature, initial supporting prestress is produced on steel tubes by internal support.

According to generalized Hooke law,

\[ \varepsilon_{r,\text{in}} = \frac{(1 - \mu_{\text{in}}) P_{\text{best}}}{E_{\text{in}}} \]  \hspace{1cm} (57)

At yield state, the supporting pressure generated by internal supporting material on steel tube is optimized support \( P_{\text{best}} \) and the radial contraction rate of internal support is \( \varepsilon_{r,\text{in}} \):

Figure 12. (a) Size of 304 stainless steel thin-walled circular tube standard components, (b) tensile hollow tube components, and (c) tensile components with PTFE filler.
304 stainless steel thin-walled circular tubes are: original gauge length $L_0 = 130$ mm, parallel length $L_c = 150$ mm, total length of components $L_t = 270$ mm, outer diameter $D_0 = 30.05$ mm, wall thickness $t = 2.08$ mm, and elastic modulus $E = 210$ GPa. PTFE was selected as flexible internal support in the experiment and its elastic modulus was measured to be $E_{in} = 231$ MPa. Under the condition of cooling by liquid nitrogen for 3 min, the radial contraction was 0.25 mm.

Loading equipment was Shenzhen Wance 100-ton HUT106D microcomputer-controlled electrohydraulic servo universal testing machine (located at the Army Engineering University of PLA in Nanjing, China) with axial tensile speed of 15 mm/min. Under each working condition, four effective components were stretched and the macroscopic features of components after break are presented in Figure 13. Also, tensile load-displacement curve of components under different working conditions are shown in Figure 14. Specimen datas under different working conditions are compared in Table 2. As given in Table 2, the supporting effect of prestressed flexible filler retards the transverse necking of tensile circular tubes toward symmetry axis, significantly enhancing the mechanical performance, fracture toughness, and energy consumption of measured tensile components. The average increments of yield force, ultimate bearing capacity, elongation at break, and fracturing energy of steel tubes are 10.81%, 15.35%, 19.33%, and 47.38%, respectively. Great increase of mechanical performance of tensile tubes confirms the effectiveness and feasibility of the proposed cold shrink fitting technique.
Applications and prospects

Research on tubes with internal support has generally focused on the mechanical performance of tensile CFST, while the effects of flexible supporting materials on the mechanical performances of tubes have rarely been studied. In the current research, we have proposed the “steel tubes + pre-stressed flexible internal support” structure with high toughness, convenience, light weight, and low price which significantly improved the mechanical performance and fracture toughness of tensile tubes, making it very suitable for the fabrication of steel tube composite structures.

Pre-stressed flexible internal supports and thin-walled tensile tubes can form highly efficient composite structures and the elastic modulus of internal support is applicable to a wide range. Compared with bulky CFSTs, lightweight flexible internal supports have evident advantages of lower weight and higher convenience in construction. This study provides new technological path for improving the comprehensive mechanical performance of steel tube composite structures, which is of vital importance in improving structure design security.

As previously mentioned, improvement of the mechanical performance of tensile steel tubes by flexible internal supports is illustrated from theoretical and experimental points of view. However, the following issues need further attention by researchers:

1. In real scale engineering components, by finding cheap internal support materials with suitable elastic modulus, the manufacturing cost and structure weight can be further decreased to realize the structural industrialization, which is the noteworthy direction.

2. This study suggests that the prestress of flexible internal supports on steel tubes can be realized by cold shrink fitting technique. Quantitative relationship between the cold contraction and temperature of different internal supports needs to be further defined.

3. In our experiments, only one specification of 304 stainless steel tube was experimentally studied and the effects of material properties of tubes and length-to-radius ratio of components on internal supports were not taken into account. Further studies are required on components with different materials, section forms, and sizes.

4. This study has focused on the influence of internal support on the tensile mechanical performance of tubes, while the improvement of mechanical performance of tubes under compression, bending, torsion, local buckling, and more complex stress conditions needs to be further investigated.

Conclusions

In the current research, the following conclusions were drawn:

1. Strengthening of tensile tube relies on the coupling of stress and deformation. In fact, change of stress state is the main factor in the improvement of yield and ultimate tensile strengths, while the enhancement due to deformation is negligible. The theoretical yield strength of the tube with internal support is 15.5% higher than that of hollow tube.

2. The calculation equations of yield and normal ultimate strengths of tensile tubes with internal supports were theoretically deduced. The trends of strengthening coefficient as a function of $\lambda$, $E_{in}$, and $\mu_{in}$ were determined. At yield state, strengthening coefficient is proportional to $E_{in}$. By keeping internal support constant, strengthening coefficient is proportional to $\lambda$. Theoretical maximum increment can barely be achieved by changing $E_{in}$. Strengthening coefficient is also proportional to $\mu_{in}$, although the changes are negligible.

3. Yield and extreme states are distinguished and optimized internal support $p$ values corresponding to the tensile performance of tubes at yield and extreme states were deduced. This study proves the application of flexible internal supports with cold shrink fitting technique to
realize the initial supporting pre-stress of internal supports on steel tubes, and experimentally verifies the effectiveness and feasibility of this method.

(4) A “steel tubes + pre-stressed flexible internal support” structure is proposed. This structure significantly improves the mechanical performance, fracture toughness, and energy consumption properties of tensile tubes. In addition, it is easy to assemble and its weight and cost are low, which is very suitable for the fabricated steel tube composite structures with the characteristics of high toughness, convenience, light weight, and economy, making it a great candidate for many applications.

Author contributions
Conceptualization, YG and FS; methodology, YG and FS; software, QX and PF; validation, QX and PF; formal analysis, LG and LB; investigation, LG and LB; resources, XY and HZ; data curation, YG and FS; writing – original draft preparation, YG and FS; writing – review and editing, YG and FS; visualization, QX and PF; supervision, LG and LB; project administration, XY and HZ; funding acquisition, YG and FS. All authors have read and agreed to the published version of the manuscript.

Data availability
The data presented in this study are available on request from the corresponding author.

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Appendix I

Notations

| Abbreviation | Description                                      |
|--------------|--------------------------------------------------|
| CFST         | Concrete-filled steel tube                       |
| CFSSST       | Concrete-filled stainless steel tube             |
| CCFST        | Circular concrete-filled steel tube              |
| SCFST        | Square concrete-filled steel tube                |
| CFDFST       | Concrete-filled double skin steel tube           |
| CECFST       | Concrete-encased concrete-filled steel tube      |
| AECFST       | Angle-encased concrete-filled steel tube         |
| CFRP         | Carbon fiber reinforced polymer                  |
| PFST         | PU-filled steel tube                             |

| Symbol       | Description                                      |
|--------------|--------------------------------------------------|
| $E$          | Elastic modulus of steel tube (MPa)              |
| $E_{in}$     | Elastic modulus of internal support (MPa)        |
| $h$          | Plastic modulus of tube (MPa)                    |
| $h_{in}$     | Plastic modulus of internal support (MPa)        |
| $\sigma_s$   | Yield stress of tube’s steel (MPa)               |
| $\sigma_u$   | Ultimate stress of tube’s steel (MPa)            |
| $\sigma_t$   | Longitudinal tensile stress of tube (MPa)        |
| $\sigma_{t, in}$ | Longitudinal tensile stress of internal support (MPa) |
| $\sigma_{r}$ | Radial compressive stress of tube (MPa)          |
| $\sigma_{r, in}$ | Radial compressive stress of internal support (MPa) |
| $\sigma_{1/2/3}$ | First/second/third principal stress of tube (MPa) |
| $\sigma_{1/2/3}$ | First/second/third deviator stress of tube (MPa) |
| $\tau_f$     | Frictional shear stress on the inner surface of tube (MPa) |
| $p$          | Contact compressive stress (MPa)                |
| $D_0$        | Original external diameter of tube (mm)         |
| $d_0$        | Original internal diameter of tube (mm)         |
| $r$          | Original internal radius of tube (mm)           |
| $t$          | Wall thickness of tube (mm)                     |
| $S_0$        | Original cross-sectional area of tube (mm$^2$)   |
| $S_1$        | Cross-sectional area of hollow tube when stretched (mm$^2$) |
| $S_2$        | Cross-sectional area of tube with internal support when stretched (mm$^2$) |
| $\lambda$    | Radius to thickness ratio of tube (−)           |
| $\mu$        | Poisson’s ratio of tube (−)                     |
| $\mu_{in}$   | Poisson’s ratio of internal support (−)         |
| $\varepsilon_t$ | Longitudinal strain of tube (−)                |
| $\varepsilon_{t, in}$ | Longitudinal strain of internal support (−)     |
| $\varepsilon_r$ | Hoop strain of tube (−)                        |
| $\varepsilon_{r, in}$ | Hoop strain of internal support (−)             |
| $\varepsilon_r$ | Radial strain of tube (−)                       |
| $\varepsilon_{r, in}$ | Radial strain of internal support (−)           |

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Appendix 2

Table A1. Calculation equations of tensile strengths of different steel tubes with fillers.

| Criterion or Reference | Components type | Increment (%) | Research method | Calculation formula |
|------------------------|-----------------|---------------|-----------------|---------------------|
| AIJ2008¹               | CFST            | 0             | /               | N_CFST, Design      |
| AIISC2005²             | CFST            | 0             | /               | N_CFST, Design      |
| CEN2004³               | CFST            | 0             | /               | N_CFST, Design      |
| Pan and Zhong⁴         | CFST            | 10            | Experiment      | N_CFST, Design      |
| GB50936-2014⁵          | CFST            | 10            | /               | N_CFST, Design      |
| Han et al.⁶            | CFST            | 12.3          | Experiment      | N_CFST, Design      |
| Chen et al.⁷           | CFST            | 14            | Experiment      | N_CFST, Design      |
| Ye et al.¹¹            | CFST            | 5–10          | Experiment      | N_CFST, Design      |
| Tao and Yu⁴⁴           | CFDST           | /             | /               | N_CFST, Design      |
| Li et al.¹²            | CFDST           | 20.8          | Experiment      | N_CFST, Design      |
| Han et al.¹⁵           | CFDST           | /             | /               | N_CFST, Design      |
| Xu et al.¹⁶            | AECFST          | /             | /               | N_CFST, Design      |
| Wang et al.¹⁷          | CFRP + CFST     | /             | Experiment      | N_CFST, Design      |
| Zhou et al.¹⁸          | CCFST           | 10.2          | Experiment      | N_CFST, Design      |
| Zhou et al.¹⁹          | SCFST           | 5.2           | Experiment      | N_CFST, Design      |
| Xu et al.²⁰            | CCFST           | 15.6          | Experiment      | N_CFST, Design      |
| Zhu²¹                  | PFST            | 23–29⁺        | FEA             | N_CFST, Design      |
| Hang²²                 | PFDST           | 29.7–40.8⁺    | FEA             | N_CFST, Design      |

¹ PFST and PFDST consider the surface bonding force between PU and steel tubes, so the increment is relatively large.

Table A2. ($S_2/S_1$)$_{l, max}$ of tube wall outer surfaces with different Cε values.

| Cε  | Expression of ($S_2/S_1$)$_{l}$ | $\frac{p}{\sigma_t}$ | ($S_2/S_1$)$_{l, max}$ |
|-----|---------------------------------|----------------------|------------------------|
| 0.001 | $-\frac{30}{9997} \lambda^2 \left( \frac{p}{\sigma_t} \right)^2 + \frac{7}{9997} \lambda \frac{p}{\sigma_t} + 1$ | $\frac{1}{\lambda}$ | 1.0007 |
| 0.002 | $-\frac{30}{9997} \lambda^2 \left( \frac{p}{\sigma_t} \right)^2 + \frac{7}{9997} \lambda \frac{p}{\sigma_t} + 1$ | $\frac{1}{\lambda}$ | 1.0014 |
| 0.005 | $-\frac{30}{1997} \lambda^2 \left( \frac{p}{\sigma_t} \right)^2 + \frac{7}{1997} \lambda \frac{p}{\sigma_t} + 1$ | $\frac{1}{\lambda}$ | 1.0035 |
| 0.01  | $-\frac{30}{997} \lambda^2 \left( \frac{p}{\sigma_t} \right)^2 + \frac{7}{997} \lambda \frac{p}{\sigma_t} + 1$ | $\frac{1}{\lambda}$ | 1.007 |

Table A3. ($S_2/S_1$)$_{l, max}$ of tube wall inner surfaces with different Cε values.

| Cε  | Expression of ($S_2/S_1$)$_{l}$ | $\frac{p}{\sigma_t}$ | ($S_2/S_1$)$_{l, max}$ |
|-----|---------------------------------|----------------------|------------------------|
| 0.001 | $\frac{1}{997} \left( 10\lambda + 3 \right) \frac{p}{\sigma_t} + \frac{7}{997} \lambda + 1$ | $\frac{1}{\lambda}$ | $\lambda \rightarrow \infty$ | 1.0007 |
| 0.002 | $\frac{1}{4997} \left( 10\lambda + 3 \right) \frac{p}{\sigma_t} + \frac{7}{4997} \lambda + 1$ | $\frac{1}{\lambda}$ | $\lambda \rightarrow \infty$ | 1.0014 |
| 0.005 | $\frac{1}{1997} \left( 10\lambda + 3 \right) \frac{p}{\sigma_t} + \frac{7}{1997} \lambda + 1$ | $\frac{1}{\lambda}$ | $\lambda \rightarrow \infty$ | 1.0035 |
| 0.01  | $\frac{1}{997} \left( 10\lambda + 3 \right) \frac{p}{\sigma_t} + \frac{7}{997} \lambda + 1$ | $\frac{1}{\lambda}$ | $\lambda \rightarrow \infty$ | 1.007 |