Experience of application the computer program based on a simplified mathematical model for industrial centrifugal compressors candidates

A F Rekstin¹, K V Soldatova¹ and Yu B Galerkin¹

¹R&D Laboratory “Gas dynamics of turbo machines” Peter the Great St.Petersburg Polytechnic University, Polytechnical st. 29, St.Petersburg, Russia

Email: rekstin2k7@mail.ru

Abstract. Various engineering techniques are used to calculate and design centrifugal compressors, one of which is the Universal Modelling Method developed at SPbPU. Variants’ of centrifugal compressors analysis is the first step in gas-dynamic design. Brief information on the simplified mathematical model of efficiency is presented. It is possible to compare the variants of compressors with different number of shafts, shafts RPM, the number and location of intercoolers, the type of impellers and diffusers. Examples of a variants’ calculation of a pipeline compressors, a compressor for underground gas storage and a general-purpose air compressor are presented. It is shown that single-shaft compressors for underground gas storages should be optimized by selecting the maximum possible number of stages, by selecting the ratio of loading factors and the diameters of the impellers. It is shown that the single-shaft scheme of a general-purpose air compressor is inferior in efficiency to a two-shaft scheme with larger number of intercoolers. The presented samples have demonstrated that computer program based on a simplified mathematical model for industrial centrifugal compressors provides reliable and sufficient information for different compressors variants’ analysis.

1. The object and task of the study. Basic considerations

The compressor parameters required by the design specification can be substantiated using an almost innumerable number of compressor variants. In general, the choices for the designer are:

- the number of compressor rotors,
- the number of stages on each rotor,
- the frequency of rotation of each rotor,
- number and position of intermediate coolers,
- diameters of impellers depending on impeller loading factor,
- types of impellers (2D or 3D),
- types of diffusers (vane, vaneless).

Industrial compressors are powerful machines with a long service life. Usually, they serve continuously with minimal downtime for maintenance. The cost of energy needed to power up the compressor drive is many times more than the investment and maintenance costs. Therefore, most often the goal of gas-dynamic design is to create a compressor with minimal energy consumption, i.e. with the maximum efficiency.
Computer programs of the Universal Modeling Method [1 - 6] allow to calculate the gas dynamic characteristics of any of the possible variants of a fully designed compressor. But comparing a large number of options would be too costly. For variants’ comparison an easier method is needed. The author of [7] developed the idea and proposed the simplified model. The authors of [8] modernized the model. The idea of the model in short is presented below.

Stage characteristics are functions of compressor’s flow path configuration and similarity criteria [4]:

\[
\eta^*, \pi^* = f \left( \bar{F}, \Phi, M_u, k, \text{Re}_u \right),
\]

flow rate coefficient:

\[
\Phi = \frac{\bar{m}}{\rho_0^* 0.785 D_2^2 u_z},
\]

impeller Mach number:

\[
M_u = \frac{u_2}{\sqrt{kRT_0}},
\]

impeller Reynolds number:

\[
\text{Re}_u = \frac{u_2 D_2}{\mu_0^* \rho_0^*},
\]

where \( \eta^* \) is efficiency (total pressure and temperature), \( \pi^* \) is pressure ratio (total pressure), \( \bar{F} \) is symbol of flow part configuration, \( \rho_0^* \) is gas density at an impeller inlet, \( D_2 \) is impeller diameter, \( u_2 \) is blade velocity, \( k \) is isentropic coefficient, \( R \) is gas constant, \( \mu_0^* \) is dynamic viscosity of gas at an impeller inlet.

In accordance with design procedure in [4], configuration of a stage flow part \( \bar{F} \) depends on:
- flow rate coefficient at design flow rate \( \Phi_{\text{des}} \),
- impeller loading factor at design flow rate \( \psi_{\text{f,des}} \):

\[
\psi_{\text{f,des}} = \left( \frac{C_{u2}}{u_2} \right)_{\text{des}},
\]

where \( C_{u2} \) is circumferential component of the absolute velocity at an impeller exit,
- mechanical constrain a hub ratio \( \bar{D}_h = D_h / D_2 \),
- compressibility criteria \( M_u, k \).

The parametrical formula demonstrates only specific features of the design method:

\[
\bar{F} = f \left( \Phi_{\text{des}}, \psi_{\text{f,des}}, \bar{D}_h, M_u, k \right).
\]
In [8] the system of the renewed simplified math model equations with empirical coefficients is presented:

$$
\eta_{\text{des}}^* = 1 - X_i K_{\nu} K_{\psi} K_{D_{\text{h}}} K_{M_{\text{b}}} + \Delta \eta_{\text{VD}} - \Delta \eta_{\text{IN}}.
$$

By flow rate coefficients $\phi_{\text{des}} \leq 0.085$:

$$
K_\phi = 1 + X_3 \left[X_5 \left(0.085 - \phi_{\text{des}}\right)\right]^{X_9},
$$

at $\phi_{\text{des}} > 0.085$:

$$
K_\phi = 1 + X_3 \left(\phi_{\text{des}} - 0.085\right)^{X_9} \left(1 + X_i D_{\text{h}}\right),
$$

where $X_i$ is empirical coefficient in math model.

Correction coefficient for influence of a loading factor:

$$
K_{\psi_r} = 1 + X_9 \left(\psi_{\text{tdes}} - 0.5\right)^{X_9},
$$

if $\psi_{\text{tdes}} < 0.5$, then:

$$
K_{\psi_r} = 1.
$$

Correction coefficient for influence of hub ratio:

$$
K_{D_{\text{h}}} = 1 + X_1 D_{\text{h}}^{X_9} \left(1 + \phi_{\text{des}}\right)^{X_9}.
$$

Correction coefficient for influence of Mach number

$$
K_{M_{\text{b}}} = 1 + X_4 (M_u - 0.5)^{X_9} (\phi - 0.01)^{X_9}.
$$

The loss of efficiency in stage next to inlet nozzle:

$$
\Delta \eta_{\text{IN}} = X_{17} \frac{\phi_{\text{des}}}{\psi_{\text{tdes}}},
$$

An increase in efficiency when vane diffuser is applied is calculated like this:

$$
\Delta \eta_{\text{VD}} = X_{19} \left(\psi_{\text{tdes}} - 0.5\right)^{X_9},
$$

if $\psi_{\text{tdes}} < 0.50$ then $\Delta \eta_{\text{VD}} = 0$.

The equations given in [8] show that such compressor parameters as impeller diameters, flow rate coefficient, blade velocity and impeller Mach number, efficiency of stages and compressor as a whole can be easily calculated using an iterative process.
2. PC programs for compressor variants analysis

To calculate compressor variants the design specification must include:
- mass flow rate,
- inlet total temperature,
- inlet total pressure,
- delivery total pressure,
- gas constant,
- isentropic coefficient,
- dynamic viscosity at an inlet.

The only constructive constraint that is taken into account is hub ratio. If its value is known, it is input directly by the user. Otherwise, hub ratio $h_D$ for flexible and rigid shafts can be calculated using an empirical equation [9]:

$$
\tilde{h}_D = 0.00044(i_\text{st} + 2.3)n^{0.5} \text{ for flexible shaft;}
$$

$$
\tilde{h}_D = 0.000813(i_\text{st} + 2.3)n^{0.5} \text{ for rigid shaft,}
$$

where $i_\text{st}$ is number of impellers on the rotor, $n$ is RPM.

For evaluation of a compressor housing volume, the system of empirical equations was presented in the original version of the simplified math model [7]:

$$
V_{hs} = \frac{\pi}{4} L_{hs} \tilde{D}^2_{hs} \tilde{D}^3_{2(1)},
$$

(18)

where: $L_{hs} = 1 \tilde{b}_1 + 4.5 \sum_{i} \tilde{D}_i \tilde{b}_i$ is compressor housing length, $\tilde{D}_i = \sqrt{\tilde{D}_h^2 + 1.26\Phi_{des}^{2/3}}$ is blade inlet relative diameter, $\tilde{b}_i = \frac{\tilde{D}_i^2 - \tilde{D}_h^2}{4\tilde{D}_1}$ is relative impeller blade height at inlet, $\tilde{D}_4 = 1.57$ if $\Phi_{des(1)} > 0.055$, $\tilde{D}_4 = 1.57 - 2.5(0.055 - \Phi_{des(1)})$ if $\Phi_{des(1)} \leq 0.055$, $\tilde{D}_{max} = \tilde{D}_4 + 4\tilde{b}_1$, $\tilde{D}_{2(1)}$ is the impeller diameter of the first stage.

To calculate the variants of a single-shaft compressor without intermediate cooling, it is sufficient to know the efficiency of each of the compressor stages. Its calculation is the task of the simplified mathematical model. The simplified math model is incorporated in the Universal Modeling Method PC programs:
- “Optimal design of a single-rotor no intercooler centrifugal compressor”,
- “Optimal design of a single-rotor intercooler centrifugal compressor”.

When applying the PC program “Optimal design of a single-rotor no intercooler centrifugal compressor” the designer should choose the number of rotor revolutions per minute if it is provided in the technical specifications. Otherwise the flow rate coefficient of the 1st stage must be chosen by the designer.

The program automatically calculates variants with different number of stages. Variants with different loading factors $\psi_{T,des} = 0.30\text{-}0.80$ are calculated for each number of stages. For compressor parameters in the technical specifications number of stages $i_\text{st}$ and $\psi_{T,des}$ defines $\Phi_{des}$ of all stages. The validity of the simplified math model is proven in range $\Phi_{des} = 0.015 \text{-} 0.15$. Iteration algorithm defines the flow rate coefficient of the 1st stage if RPM is specified, or vice versa. Additionally, it defines flow rate coefficients of all stages, the impeller diameters, blade velocity, impeller Mach number, stages’ efficiency, compressor efficiency and internal power. There is an option to compare variants with different diameters of impellers and different $\psi_{T,des}$. 


In the PC program “Optimal design of a single-rotor no intercooler centrifugal compressor” the position of the gas coolers should be selected. It is necessary to insert the loss of the total pressure in the gas cooler with respect to the inlet pressure, and the difference in temperature at the outlet of the gas cooler and the temperature at the inlet to the compressor. For multi-rotor compressors the designer would choose either RPM, or the flow rate coefficient of the 1st stage on each rotor. Then, each of the shafts is calculated as a single-shaft compressor.

Below we will provide additional details of the software’s working principles and analytic capabilities using samples of variant analysis.

3. Sample of variant analysis #1. Pipeline compressor 25 MW, delivery pressure 7.45 MPa, 4850 RPM

The pipeline compressors with delivery pressure 7.45 MPa and pressure ratio $\pi_{des} = 1.44$ are widely applied in Russia. Variants’ analysis is presented graphically on Figure 1.

![Graphical representation of variant analysis](image)

**Figure 1**– PC programs “Optimal design of a single-rotor no intercooler centrifugal compressor”. Pipeline compressor 25 MW delivery pressure 7.45 MPa. Variants with 1, 2, 3 stages and different loading factors

The authors design experience shows [1 - 4] that for pipeline compressors with comparatively low blade velocity (in fact, for all industrial compressors) low loading factors are preferable. Advantages are higher efficiency, better surge limit, maximum power flow rate getting closer to design flow rate. The last is important for gas turbine drives. Therefore, variants with $\psi_{des} > 0.55 – 0.57$ are not acceptable.
But one-stage variant with $\psi_T = 0.55$ has low flow rate coefficient $\Phi = 0.0174$ that leads to unacceptable efficiency 0.73 and blade velocity of 357 m/s that is too high for pipeline compressors.

Two-stage variants have parameters that are close to some successful designs of the authors [1]. At $\psi_T = 0.50$ the efficiency exceeds 0.85. In accordance with design experience, the surge limit of this variant may be $m_{\text{crit}} = (0.45 - 0.50) \bar{m}$, which is very good for a centrifugal compressor.

Three-stage variant with $\psi_T = 0.45$ promises the highest efficiency and better surge limit. The result of lesser impeller diameters of three stages leads to practically the same volume of the housing as in the two-stage variant. Big flow rate coefficients make obligatory use of 3D impellers, while for two-stage variant simpler 2D impellers are possible.

4. Sample of variant analysis #2. Pipeline compressor 25 MW, delivery pressure 9.91 MPa, 4850 RPM

Modern long-distance pipelines in Russia are operating at pressure of 9.91 MPa. Volumetric flow rate is lesser in comparison with the sample #1. The variants with one stage are absolutely impossible due to too low $\Phi$. Parameters of variants with two and three stages are presented on Figure 2.

![Figure 2](image-url)  
**Figure 2**– PC programs “Optimal design of a single-rotor no intercooler centrifugal compressor”. Pipeline compressor 25 MW delivery pressure 9.91 MPa. Variants with 2 and 3 stages and different loading factors

Two-stage variants also have non-optimal flow rate coefficients. Narrow channels of the flow part and impeller parasitic losses limit efficiency to non-acceptable level. The optimal is the three-stage variant with $\psi_T = 0.50$, that promises the highest efficiency. The authors’ design experience shows that for this variant with flow rate coefficients of stages $\Phi \leq 0.085$ the costly 3D impellers are not obligatory.

5. Sample of variant analysis #3. Compressor for underground gas storage 7.8 MW, delivery pressure 12.3 MPa, 9000 RPM

The necessary pressure ratio is $\pi = 3.0$. Variants with different number of stages and loading factors are presented on Figure 3.
Figure 3 – PC program “Optimal design of a single-rotor no intercooler centrifugal compressor”. Compressor for gas storage 7.8 MW delivery pressure 12.3. Variants with 6 - 9 stages and different loading factors

Evident disadvantage of the compressor is too low RPM of the drive. The variant with 6 stages and $\psi_{T \; des} = 0.50 - 0.55$ cannot be highly effective due to low $\Phi_{des}$ of stages. If we are using 7 stages and impellers have high loading factors, efficiency must be close to 80%, but the surge limit will be rather close to the design flow rate. Nine stages variant would be the good compromise of flow rate coefficient and loading factor. But 9 stages is hardly acceptable because of rotor dynamics.

The variant with 8 stages was additionally studied with lesser diameters and loading factor of 4 – 8 stages. The efficiency of 81% is possible if the impellers #1-4 loading factor is 0.72, and impellers #5-8 loading factor is 0.648. These values of $\psi_{T \; des}$ are high, and good characteristics would not be possible. The radical improvement is possible with a turbine drive with higher RPM.

6. Sample of variant analysis #4. Single-rotor air compressor for pneumatic station, volumetric flow rate 160 $m^3$/min, delivery pressure 9 MPa (abs), 18000 RPM

These parameters are typical to many pneumatic stations of industrial enterprises. Single-rotor solution is outdated now. In Russia since 1950s there were installed many single-shaft compressors with 6 stages and two intercoolers between three two-stage sections.

For these variants the drive is a synchronous electric motor and 1:6 gearbox.

Variants were calculated with 5% loss of inlet pressure in intercoolers. The temperature after an intercooler exceeds the ambient temperature by 15K. Figure 4 presents the menu with the compressor technical specification and data of one of the variants.
Figure 4 – PC program “Optimal design of a single-rotor intercooler centrifugal compressor”. Technical specification and information on variant’s scheme and stages

Variant with equal impeller diameters and equal $\psi_{T_{des}} = 0.65$ has isothermal efficiency of 65%. Flow rate coefficient of the 1st stage is 0.167. It is too much for highest efficiency of the stage. After calculation several other variants the best appeared to be the variant with diminishing diameters of #3, 4 and #5, 6 impellers and different loading factors $\psi_{T_{des}} = 0.65$ (impellers #1, 2), $\psi_{T_{des}} = 0.60$ (impellers #3, 4), $\psi_{T_{des}} = 0.55$ (impellers #5, 6). Its isothermal efficiency is 0.673, which is not bad for a single-shaft scheme. As loading factors are smaller than in other cases, the better surge limit could be expected. The impeller diameter of the 1st stage is 0.3 m, blade velocity is 286 m/s, which is comparatively small. It is the positive moment – the lower is the velocity the less are maintenance problems.

7. Sample of variant analysis #5. Two-rotor air compressor for pneumatic station, volumetric flow rate 160 m$^3$/min, delivery pressure 9 MPa (abs)

Modern solution for a pneumatic station is a gear compressor – the central gear and two rotors are in the same housing with four impellers at the rotors opposite ends. Each rotor can have optimal RPM, three intercoolers are easily disposed after the 1st, 2nd and 3rd stages. In comparison with the previous single-shaft compressor the number of stages is less. It leads to higher blade velocity. The impellers are 3D without shrouds. Figure 5 presents the menu of the PC program “Optimal design of an arbitrary scheme centrifugal compressor”. There is presented information on the variant with the loading factor for all stages $\psi_{T_{des}} = 0.75$.

Figure 5 – PC program “Optimal design of an arbitrary scheme centrifugal compressor”. Variant with the same loading factor for all stages $\psi_{T_{des}} = 0.75
Isothermal efficiency of this variant is 68%, but surge must be close to design flow rate due to big loading factor. After numerous variants’ comparison the chosen variant is presented in the Table 1.

### Table 1. PC program “Optimal design of an arbitrary scheme centrifugal compressor”. Data of the chosen variant of the two-rotor compressor

| Variant # | Type       | \( \Phi \) | \( M_u \) | \( \psi_T \) | \( Re_u \) | \( u_2 \) | \( D_2 \) | \( \eta_{it} \) | \( T_{in}^* \) | \( T_{out}^* \) |
|-----------|------------|------------|-----------|-------------|------------|---------|---------|------------|-------------|-------------|
| 1         | 3D+VD     | 0.1000     | 0.8796    | 0.75        | 7.13E+05   | 299.29  | 0.3416  | 0.8154     | 288.15      | 358.37      |
| 2         | 3D+VD     | 0.0576     | 0.8618    | 0.6900      | 1.240E06   | 299.29  | 0.3416  | 0.8476     | 300.15      | 364.76      |
| 3         | 3D+VD     | 0.1000     | 0.8618    | 0.6700      | 1.230E06   | 299.29  | 0.1972  | 0.8300     | 300.15      | 362.88      |
| 4         | 3D+VD     | 0.0594     | 0.8618    | 0.6500      | 2.080E06   | 299.29  | 0.1972  | 0.8541     | 300.15      | 361.01      |

Rotor # 1 \( n=16734.06 \) D\( h_b=0 \)

Rotor # 2 \( n=28990.57 \) D\( h_b=0 \)

Thermal Efficiency, \( \eta_{comp} = 0.695 \)

Power consumption, \( N=843.65 \) kW

The isothermal efficiency of 69.5% is high for the compressor with comparatively small dimensions. The loading factors are lesser in comparison with the variant above. The surge limit could be better due to it. Blade velocity of 300 m/s is not high for this kind of a compressor.

8. Conclusion

The simplified math model is the instrument for choice of the better variant among many others. The predicted efficiency does not have to be equal to the efficiency of the final project as the simplified math model does not deal with the designed flow path. It is only statistically proven. The simplified math model does not calculate volumetric and mechanical losses of real compressors. To compare variants’ efficiency with the efficiency of some real compressor would be not correct. The simplified model is adequate if it reflects influence of design parameters properly. But of course the variants’ efficiency must be as close as possible to the efficiency level of real compressors. The plant tests of pipeline compressors after the Authors design have maximum efficiency 0.85 – 0.875 depending on design constrains. The simplified model estimates the same level of efficiency for these compressors. The same is in case of underground gas storage compressors and air compressors. The Authors’ design practice shows that the simplified mathematical model provides reliable and sufficient information.

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