Calculation Method Analysis for Structure Strengthening with External Reinforcement

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Abstract. This article describes the main provisions of the calculating method for the gain of reinforced concrete structures by the system of external reinforcement. As an example, the calculation of monolithic reinforced concrete slab with carbon fiber material using the SCAD Office software package was considered and analyzed, which most accurately reflects the geometry and physico-mechanical parameters of the structure. The calculation of reinforced concrete structures includes engineering survey to determine the existing physic-mechanical characteristics of materials, perform calibration calculations and stress calculations in reinforced concrete structures of buildings and structures, taking into account the existing stiffness characteristics, designing reinforcement engineering solutions, the calculation of the forces in the structures and verification of the structures bearing capacity after the reinforcement is completed. During the survey of reinforced concrete structures, it is determined existing loads, deformations, deflections, physico-mechanical characteristics for the subsequent selection of the composite material cross section and taking them into account after the concrete structure’s reinforcement. The external reinforcement systems is used for reinforcing concrete structures and rely on tensile stress perception, taking into account the joint deformations of the composite material used for reinforcement and the concrete structure. The calculation of reinforced concrete structures carried out on the first and second limit state. If the load after reinforcement with a composite material does not increase compared with the loads at the stage of the reinforced concrete structure operation, the calculation for the second group of limit states is not performed. The reinforced concrete structures calculation using an external reinforcement system made of composite materials is carried out from the joint operation conditions of the installed external reinforcement and the concrete base before the limit state onset. The calculation is performed in two stages: at the first stage, the reinforced concrete structure is calculated without reinforcement taking into account the existing loads and deformations, at the second stage, after the work on reinforcement, the design is calculated taking into account the efforts and deformations in the first stage.

1. Introduction
Buildings elements calculation, including monolithic reinforced concrete slabs, represents survey work in order to establish the actual physicomechanical materials characteristics, calculating stresses in the structures, taking into account existing stiffness characteristics, bearing capacity calculation, design engineering solutions for reinforcement structures, the bearing capacity verification after strengthening [1, 2, 3].
The external reinforcement system is calculated on the stretching perception, taking into account the deformations compatibility of composite material used for reinforced concrete structure strengthening. Calculation is carried out on the first and second limit state. The calculation for the second group of limit states is not performed if the load after strengthening does not increase as compared with the loads in the operation stage. The actual loads and deformations are determined to select the cross section of the composite material and take these loads and deformations into account after installing the external reinforcement system [4-8].

The calculation is performed on the deformation model in two stages. At the first stage, the reinforced concrete structure is calculated without reinforcement, taking into account the existing loads and deformations. At the second stage, after strengthening, the design is calculated taking into account the efforts and deformations at the first stage.

The work purpose considered in this article includes the SCAD Office software using for calculating monolithic reinforced concrete structures with external reinforcement with carbon fiber.

2. Task
The task is to perform an analysis of the strengthening reinforced concrete structures (slabs) calculated method to obtain a refined reinforcement characteristic.

3. Modeling
The calculation of reinforced concrete structures by external reinforcement with composite materials is performed from the conditions for the joint operation of external reinforcement and the concrete base before the limit state onset [9]. The ultimate tensile deformation $\varepsilon_f$ is determined on the basis of stresses and deformations to failure linear dependence by the formula

$$\varepsilon_f = \frac{R_f}{E_f},$$

where $R_f$ - the standard tensile resistance of the composite material, Pa;
$E_f$ - elasticity modulus of the composite material, Pa.

The design and calculation characteristics of the composite material are determined on the basis of the regulatory characteristics, taking into account the safety coefficient $\gamma_f$ and the operating conditions coefficient $C_E$, taking into account the environment influence.

The calculated tensile strength of the composite material value, is determined by the formula taking into account the working condition reliability coefficient $C_E$:

$$R_{ft} = \frac{C_E}{\gamma_f} \cdot R_f,$$

$\gamma_f$ - the reliability coefficient;
$C_E$ - the working condition reliability coefficient.

The calculated resistance of the composite first layer:

$$R_{ft} = 3,788 \cdot 10^8 \text{ Pa}$$

The elasticity modulus calculated value in tension is taken to be equal to the standard. Estimated deformation strain is determined by the formula:

$$\varepsilon_{ft} = \frac{R_{ft}}{E_f},$$

$$\varepsilon_{f,ult} = \frac{3,788 \cdot 10^8}{65 \cdot 10^3} \approx 5,83 \cdot 10^3$$
The working condition reliability coefficient $C_E$ for carbon fabrics are taken depending on the environmental conditions: 0.9 - for indoor use; 0.8 - for structures in the open air and in aggressive environments.

The safety coefficient $\gamma_f$ values for the limit states of the first group are assumed to be equal: for unidirectional carbon tapes - 1.2; for bidirectional carbon fabrics - 1.8 [7-8].

The safety coefficient $\gamma_f$ values increase by 15% when installing a composite material with fabric treatment with a binder and gluing on the adhesive layer (“wet” method).

The reliability coefficient is assumed to be 1 for calculating on the second limit state.

Joint work between external reinforcement and concrete base provides strong materials adhesion, due to the limitation of their design tensile deformations.

Limit calculated deformation of the composite material is determined by the formula:

$$\epsilon_{fu} = K_m \cdot \epsilon_{ft}$$

where $K_m$ – the working condition coefficient, depends on the reinforcement element stiffness; $\epsilon_{ft}$ - the calculated tensile strain.

The calculated tensile strength is determined by the formula:

$$R_{fu} = E_f \cdot \epsilon_{fu}$$

where $\epsilon_{fu}$ - the limiting calculated deformations of the composite material.

The canvases installation step is 1.0 m. The width of the composite is 300 mm, the thickness is 1 mm. The area of the composite $A_f$ per 1 m$^2$ is equal to 300 mm.

Plates belong to bending elements, and the calculation of structures working on bending has a number of features. The limiting stresses in a cross section perpendicular to the longitudinal axis of the reinforced concrete element are determined from the following points: the distribution of relative deformations over the height of the element cross section is linear, the calculated relative deformations reach their limit values during exfoliation $\epsilon_{fit}$ [9,11], resistance concrete tensile strength is equal to zero; the concrete resistance to compression in the ultimate state is represented by stresses equal to $R_b$ [12-16] and uniformly distributed over the height of the compressed zone, tensile and compressive stresses in steel reinforcement are equal to the calculated tensile resistances $R_s$ and compression $R_{sc}$, respectively, tensile stresses in the external reinforcement are equal the calculated tensile strength $R_{fu}$, shear strain in the adhesive layer is not taken into account [4-6].

The strength calculation of normal sections should be carried out with a limited height of the compressed zone, and the ultimate state of the structure occurs when a voltage in the composite material is equal to the calculated tensile strength $R_{fu}$ [17-20]. In this case, the compressed zone height is determined from the flat cross-section hypothesis. The relative deformation of the outermost fiber in concrete compressed is 0.0035.

The strength calculation of normal sections in bending reinforced concrete elements strengthened with composite material is made from the condition:

$$M \leq M_{ult}$$

where $M_{ult}$ – the ultimate bending moment that can be perceived by the cross section of the reinforced element kN \cdot m.

The forces in the compressed zone are perceived by the concrete and compressed rod reinforcement, and in the stretched one - by the reinforcement bar and composite material in the bending element limiting state [13].

For the bending concrete elements with rectangular cross-section, reinforced with composite material, $M_{ult}$ is determined by the formula:
Mult = Nb \cdot zb, \quad (7)

where \( Nb \) – the resultant of normal efforts in the compressed zone of concrete (ultimate force of compressed zone), kN;
\( zb \) – the distance between the ultimate force of the compressed zone and the resultant force in the tensioned reinforcement and the external reinforcement, m.

The ultimate force of the compressed zone \( Nb \) is calculated based on the equilibrium condition of the internal forces and external forces using the formula:

\[ Nb - R_s \cdot A_s - R_{fu} \cdot A_t = 0, \quad (8) \]

\( R_s \) - the design resistance of steel reinforcement to tension, Pa;
\( A_s \) - sectional area of steel reinforcement, m²;
\( R_{fu} \) - the calculated tensile strength of the composite material, Pa;
\( A_t \) - the cross-sectional area of the longitudinal reinforcement made of composite material, m².

The distance \( zb \) is determined depending on the concrete compressed zone height.

For the construction under consideration, \( Mult = 83.241 \text{ kN} \cdot \text{m} \).

Figure 1 shows the stress diagram in cross section, reinforced with carbon fiber material.
Figure 1. The stress diagram in cross section, reinforced with carbon fiber material.

The compressed zone height of concrete reinforced element is determined from the condition:

\[ X = \frac{N_b}{R_b b} \tag{9} \]

where \( N_b \) – the resultant of normal efforts in the concrete compressed zone (ultimate force of the concrete compressed zone), kN;

\( R_b \) - the concrete design resistance to axial compression, Pa.

The parameters of the slab cross section: \( b = 1000 \) mm; \( h = 300 \) mm.

4. Experiment

Force: effective moment 97.1 kN-m, which exceeds \( M_{ult} = 83.241 \) kN-m. Concrete: B25. Load action: continuous. Humidity: above 75%. Compressed reinforcement: A400 Ø10 mm; A400 Ø6 mm; \( a' = 50 \) mm. Stretched reinforcement: A500 4 Ø16 mm; B500 Ø5 mm; \( a = 50 \) mm.

Reinforcement is done with carbon ribbons. Composite parameters: FibArmTape 530 on the FibArmResin 530+ binder. The composite layer thickness is 1.0 mm. Regulatory strength (\( R_{f,n} \)) is 1100 MPa. Regulatory modulus of elasticity (\( E_{f,n} \)) is 69 GPa. Composite width is 300 mm. The layers number is 1. Strengthening is performed on the slab upper surface [14].

Table 1. The slab loads.

| No. | Element                     | Density, \( \text{r/m}^3 \) | Layer thickness, s, m | Regulatory load, \( t/m^2 \) | Reliability coefficient \( \gamma_f \) | Design load, \( t/m^2 \) |
|-----|-----------------------------|-----------------------------|-----------------------|-----------------------------|---------------------------------|--------------------------|
|     |                             |                             |                       |                             |                                 |                          |
| 1   | Reinforced concrete slab    | 2.5                         | 0.3                   | 0.75                        | 1.1                             | 0.825                    |
| 2   | Leveling screed             | 1.8                         | 0.11                  | 0.198                       | 1.3                             | 0.257                    |
| 3   | Monolithic concrete footing | 1.8                         | 0.07                  | 0.126                       | 1.3                             | 0.164                    |
| 4   | Partitions                  | -                           | -                     | 0.05                        | 1.3                             | 0.065                    |
5. Results analysis

*Definition of \( M_{x,ult} \)*

The calculation results of the stress-strain state are shown in Figure 2. The concrete compressed zone relative height is calculated by the formula:

\[
\xi = \frac{x}{h_0} = 0,151 < \xi_{RF} = \frac{x_{RF}}{h} = 0,290, \tag{10}
\]

where \( x \) - the concrete compressed zone height;
\( h_0 \) - the working section height;
\( \varepsilon_{b,2} \) – the ultimate deformation of compressed concrete. \( \varepsilon_{b,2} = 0,0035 \);
\( \varepsilon_{b,0} \) - limiting relative deformations of the composite material.

![Figure 2. Calculation results \( M_{x,ult} \).](image_url)

The concrete compressed zone height is \( x = 4.52 \) cm. Table 2 presents the calculation results of stresses in the cross section.

**Table 2.** Stresses values, MPa.

| Parameter   | Value [MPa] | Utilization factor |
|-------------|-------------|--------------------|
| Stress, \( \sigma_{b,max} \) | -0,0125 | 0,78 |
| Stress, \( \sigma_{s1} \) | 0,355 | 0,9999 |
| Stress, \( \sigma_f \) | 0,39 | 0,9999 |

There are relative deformations in Table 3.

**Table 3.** Relative deformations.

| Parameter | Value [MPa] | Utilization factor |
|-----------|-------------|--------------------|
| \( \varepsilon_{b,max} \) | -0,00101 | 0,31 |
| \( \varepsilon_{s1} \) | 0,00432 | 2,35 |
| \( \varepsilon_f \) | 0,00583 | 0,9999 |
**Definition** $M_{y, ult}$

The results of the calculation of the stress-strain state are shown in Figure 3. The concrete compressed zone relative height is calculated by the formula:

$$
\xi = \frac{x}{h_0} = 0.184 < \xi_{RF} = \frac{x_{RF}}{h} = 0.265, \quad (11)
$$

![Figure 3](image)

**Figure 3.** Calculation results $M_{y, ult}$.

The concrete compressed zone height is $x = 5.51$ cm. Table 4 presents the calculation results of stresses in the cross section.

| Parameter       | Value [MPa] | Utilization factor |
|-----------------|-------------|--------------------|
| Stress, $\sigma_{b, max}$ | -0.0095     | 0.83               |
| Stress, $\sigma_{s1}$  | 0.355       | 0.9999             |
| Stress, $\sigma_f$   | 0.32        | 0.9999             |

There are relative deformations in Table 5.

| Parameter       | Value [MPa] | Utilization factor |
|-----------------|-------------|--------------------|
| $\varepsilon_{b, max}$ | -0.00124   | 0.35               |
| $\varepsilon_{s1}$  | 0.00446     | 2.51               |
| $\varepsilon_f$   | 0.00583     | 0.9999             |

Limit values of internal forces and bearing capacity are shown in Table 6.

| No. | Value type       | $M_{\alpha}$, kN/m | $M_{\beta}$, kN/m |
|-----|------------------|--------------------|-------------------|
| 1   | Current value    | 93.4               | 88.5              |
| 2   | Limiting value  | 102.5              | 108.8             |
|     | Reserve [%]      | 97.4               | 229.0             |

**6. Calculating reinforced slab by service limit state**

Calculation of the crack opening width with the selected strengthening is performed. The calculated value of resistance of the first composite layer is: $R_f = 366.2$ MPa.

The calculation is made from the condition that $M > M_{crc}$. The calculated resistance of the first composite layer:

$$
R_f = 3.788 \cdot 10^8 \text{ Pa}
$$
The limit values of internal forces reserve for crack resistance are shown in Table 7.

Table 7. Reserves for crack resistance.

| No. | Type of value   | Crack width from $M_x$ [mm] | Crack width from $M_y$ [mm] |
|-----|-----------------|----------------------------|-----------------------------|
| 1   | Current value   | 0.201                      | 0.24                        |
| 2   | Limiting value  | 0.3                        | 0.3                         |
|     | Reserve [%]     | 33                         | 20                          |

7. Conclusions
The article reviewed and analyzed the main provisions of the method for calculating the structures strengthening, namely reinforced concrete monolithic slabs, with composite materials based on a finite element model in the SCAD Office. The considered design method of reinforced structures (floor slabs) allows to obtain a more accurate picture of the stress-strain state before reinforcement and after reinforcement. According to the calculation results, it is possible to select a more adequate amplification circuit due to a change in the geometry or stiffness characteristics of the composite material.

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