HIGH DIMENSIONAL STOCHASTIC LINEAR CONTEXTUAL BANDIT WITH MISSING COVARIATES

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ABSTRACT
Recent works in bandit problems adopted lasso convergence theory in the sequential decision-making setting. Even with fully observed contexts, there are technical challenges that hinder the application of existing lasso convergence theory: 1) proving the restricted eigenvalue condition under conditionally sub-Gaussian noise and 2) accounting for the dependence between the context variables and the chosen actions. This paper studies the effect of missing covariates on regret for stochastic linear bandit algorithms. Our work provides a high-probability upper bound on the regret incurred by the proposed algorithm in terms of covariate sampling probabilities, showing that the regret degrades due to missingness by at most $\zeta_{\min}$, where $\zeta_{\min}$ is the minimum probability of observing covariates in the context vector. We illustrate our algorithm for the practical application of experimental design for collecting gene expression data by a sequential selection of class discriminating DNA probes.

Index Terms— Contextual bandit, missing values, regret analysis

1. INTRODUCTION
High-dimensional linear stochastic bandits have become of increasing interest in recommendation systems, healthcare, and experimental design. Particular attention has been on linear bandits where only a small subset of the covariates is correlated with rewards. In a linear bandit, the learner observes a context variable $X_t \in \mathbb{R}^{K \times d}$ at round $t$, where each arm $i$ is associated with a given feature vector $X_{t,i} \in \mathbb{R}^d$, the $i$-th row of $X_t$. Then, based on the chosen arm $a_t$ at time $t$, the learner observes a noisy reward $\hat{r}_{t,i} = X_{t,a_t}^\top \beta^* + \epsilon_t$ for a fixed and unknown parameter vector $\beta^* \in \mathbb{R}^d$. The objective is to determine a policy for choosing $a_t$ to maximize the expected reward based on past observations $\{X_{t,a_t}, \hat{r}_{t,i}\}_{i=1}^t$.

Frequently, an additional sparsity assumption is imposed on $\beta^*$. Sparsity is an effective constraint when the feature space is high-dimensional, but only a subset of features are correlated with the expected reward. In this case, it is natural to adopt the lasso regression framework where the reward parameter $\beta^*$ satisfies $s_0 = \|\beta^*\|_0 \ll d$. This results in variants on the lasso bandit including the sparse agnostic (SA) bandit [1], which is then extended to the sparse agnostic with missingness (SAM) bandit in this paper.

Missing values are common in contextual bandits [2], motivating this work from a practical standpoint. For example, in a clinical application of warfarin dosing with patient data [3], missing values for certain genotypes are prevalent in the context vectors. These can be imputed based on demographic or other genotype information when available.

This work adopts the regression with missing data framework [4] to unbias the covariance matrix when there are missing values. Specifically, we adopt the missing completely at random (MCAR) model. In this model, variables in the context vector $X_{t,i} \in \mathbb{R}^d$ of the $i$-th arm at time $t$ are observed with probabilities $\zeta = [\zeta_1, \ldots, \zeta_d] \in [0, 1]^d$. Mathematically, the observed context with missing entries can be written as $Z = X \odot U$, where $\odot$ is a Hadamard product and $U_{t,i} \in \{0, 1\}^d$ is a Ber($\zeta_i$) independent of both $X_{t,i}$ and $\epsilon_t$.

The difficulty in establishing convergence of lasso bandits has been highlighted in [1, 5, 6]. This is due to the fact that the observation noise $\epsilon_t$ associated with successive pulls of the chosen arms is not i.i.d. As first addressed in [5], the sequence $\epsilon_t X_t$ can be shown to be a Martingale difference sequence under mild conditions. This enables the application of tail inequalities to the lasso estimator. The authors of [6] used the Martingale approach to perform regret analysis of the Doubly-Robust (DR) Lasso bandit, and [1] applied them to the sparse agnostic (SA) Lasso bandit.

The convergence results [1] relied on technical conditions, namely the compatibility condition and the restricted eigenvalue condition. In order to establish these conditions in bandit problems, the oracle lasso convergence theory in [7] was modified using a Martingale concentration inequality. This requires positive-definiteness of the context covariance matrix, a condition that is often violated when there are missing values in the observed contexts. This paper relaxes this requirement, allowing us to prove convergence when there are missing values in the context variables in stochastic linear bandit problems.
To the best of our knowledge, this is the first paper to propose a lasso bandit with missing covariates in addition to providing theoretical guarantees. Our work establishes restricted lower- and upper-restricted eigenvalue (RE) conditions on the adjusted sample covariance matrix when the observation noises $\varepsilon_t$ are dependent on the past observations. We show that missingness in covariates inflates the regret bounds by a factor inversely proportional to the squared minimum sampling probability $\xi_{min}$, which is inversely proportional to missingness.

1.1. Related Work

Sparse Linear Contextual Bandit: Interest in sparse linear bandits in high-dimension began with [8], [9], and continued with [10] and [5]. Recently, [6] applied sparse linear structure to the stochastic linear bandit problem and incorporated a doubly-robust approach to prove convergence. The regret analysis in [6] mainly adopts the procedure in [5] that extended the standard lasso convergence results to online regression with non-i.i.d. samples. The authors of [1] introduced sparse agnostic Lasso bandit under missing data and motivate the proposed missingness (SAM) bandit.

Missing data in regression and covariance estimation: Traditional methods introduced in [11] worked with the EM algorithm to perform statistical inference for missing data. However, even in the batch setting where they do converge, EM algorithms often converge slowly. M-estimators developed in [4] were designed in mind to cope with missing and corrupted data by making a simple adjustment to the sample covariance matrix. The adjusted sample covariance estimates, however, are not necessarily positive semi-definite, which makes the resulting likelihood non-convex. In [4], it was proved that the local and global optima of the non-convex lasso problem have comparable mean squared error. Thus, a simple projected gradient algorithm for the non-convex lasso objective under missing data is sufficient to guarantee the convergence of these estimators. We apply the lasso framework of [4] to obtain an integrated and principled solution to the sparse agnostic with missingness (SAM) bandit.

2. PROBLEM SETUP

We introduce our formulation of the linear contextual bandit problem under missing data and motivate the proposed estimator.

Missing covariates: In a sparse stochastic linear bandit, the reward for pulling the $i$-th arm at time $t$ is of the form $r_{t,i} = X_{t,i}^\top \beta^* + \varepsilon_t$, for $i \in [K]$, given the covariates $X_{t,i} \in \mathbb{R}^d$, called context variables. Over the time steps $[T]$, the learner estimates the unknown regression parameter $\beta^* \in \mathbb{R}^d$, which is assumed to be sparse. In a typical stochastic linear bandit problem, after pulling arm $a_t$, we observe a reward $\hat{r}_t$, linked with context vector $X_{t,a_t} \in \mathbb{R}^d$, via the noisy linear model

$$\hat{r}_t = X_{t,a_t}^\top \beta^* + \varepsilon_t, \quad t \in [T], \ a_t \in [K],$$  \hspace{1cm} (1)

where $\varepsilon_t \in \mathbb{R}$ is the observation noise independent of $X_{t,i}$. Instead of directly observing $X_{t,i}$, we observe $Z_{t,i} = [Z_{t,i1}, \ldots, Z_{t,id}]$ with missing entries defined as follows

$$Z_{t,i} = \begin{cases} X_{t,i} & \text{if the entry is not missing} \\ 0 & \text{if the entry is missing} \end{cases}.$$  \hspace{1cm} (2)

Equivalently, the learner observes the context for the $i$-th arm at time $t$ as $Z_{t,i} = X_{t,i} \odot U_{t,i}$, where $U_{t,i} \in \{0, 1\}^d$ and $\odot$ is the Hadamard product. Each entry $U_{t,i}$ is an independent Bernoulli random variable with sampling probability parameter $\zeta_j$ for the $j$-th covariate of the context vector $X_{t,i}$. The estimation goal is to recover $\beta^*$ as we sequentially receive the context vectors $Z_{t,i}$ with missing entries.

Sparse Agnostic with Missingness bandit: The learner must decide which of $K$ available arms to pull based on the observed contexts $Z_{t,i} = X_{t,i} \odot U_{t,i} \in \mathbb{R}^d$, $i \in [K]$. That is, at time $t$, we observe $Z_t \in \mathbb{R}^{K \times d}$ matrix-variate data and the missingness pattern $U_t \in \mathbb{R}^{K \times d}$, where the $i$-th row of these variables corresponds to the $i$-th arm. Based on the context variables, the learner pulls an arm and incurs a reward. Note that when $\zeta_j = 1$ for all covariates $j \in [d]$, our problem setting is the same as the fully observed case of [6] and [1].

The policy we adopt in this paper is to pull the arm maximizing the estimated reward

$$a_t = \arg \max_{i \in [K]} (Z_{t,i} \odot \hat{\zeta}) \hat{\beta}_{t-1}, \quad t \in [T],$$  \hspace{1cm} (3)

where $\odot$ is element-wise division, $\hat{\zeta} \in \mathbb{R}^d$ is an empirical estimate of the sampling probabilities based on the observed $U_t$'s, and $\hat{\beta}_t$ is an estimate of $\beta^*$. The policy defined by (3) is called the plug-in-estimation policy [4].

We define the optimal arm at time $t$ as

$$a^*_t = \arg \max_{1 \leq i \leq K} X_{t,i}^\top \beta^*, \quad t \in [T],$$  \hspace{1cm} (4)

and the regret($t$) as the difference between the expected reward of the optimal arm and the expected reward of the chosen arm at time $t$ based on (3).

$$\text{regret}(t) = \mathbb{E}[r_{t,a^*_t} - r_{t,a_t} \mid \{X_{t,i}\}_{i=1}^K, a_t, U_t] = X_{t,a_t}^\top \beta^* - X_{t,a_t}^\top \beta, \quad t \in [T],$$

where $r_{t,a^*_t}$ and $r_{t,a_t}$ are the maxima achieved in (3) and (4). The learner aims to minimize the cumulative regret over $T$ steps.

For the rest of the paper, we define the filtration $\mathcal{F}_{t-1}$ as the union of all observations up to time $t-1$ including rewards, missingness patterns, and contexts:

$$\mathcal{F}_{t-1} = \{(Z_{t}, \hat{r}_{t,a_t}, U_t)\}_{t=1}^{t-1}.$$

Given $\mathcal{F}_{t-1}$, the learner selects the arm $a_t$ according to the current estimate $\hat{\beta}_t$. 


3. SA LASSO BANDIT WITH MISSING COVARIATES

We introduce covariate missingness into the SA Lasso bandit [1] when the contexts are corrupted with missing values. We analyze the regret when the plug-in-estimation policy (3) is used in SA with missingness (SAM) bandit algorithm. The same plug-in adjustment procedure and regret analysis can be applied to the lasso bandit and to the DR bandit with missingness (DRM) but we omit the details here.

3.1. Bandit with missing covariates

Compared to [6], where the uneven sampling of the covariates is addressed by taking the average of the contexts at each round and calculating the corresponding pseudo-rewards, [1] introduced the SA Lasso bandit by making an additional assumption on the distribution of the contexts such that the covariance matrix \( \Sigma_t = \mathbb{E}[X_t^T X_t | F_{t-1}] \) behaves sufficiently well to converge to the marginal context covariance matrix \( \Sigma = \mathbb{E}[X_T^T X_T] \). More specifically, [1] showed that

\[
\sum_{i=1}^k \mathbb{E}[X_i | X_t = 1] (X_{t,i} = \arg \max_{X \in X_t} \langle \beta, X \rangle) \approx (2\nu C_{X})^{-1} \Sigma,
\]

under the balanced covariance assumption (Assumption 5) stated later in this paper, where \( X_t = [X_{t,1}, \ldots, X_{t,K}]^T \) (See Lemma 3 and Lemma 10 of [1] for more details). Here we will show that under the balanced covariance assumption, we can achieve a similar result to [1] even in the presence of missing covariates.

For the SA Lasso bandit with missing covariates algorithm, we directly use the observed rewards \( \tau_t \) and the incompletely observed contexts \( Z_{t,a} = X_{t,a} \odot U_{t,a} \). Thus, we define \( Z_t = [Z_{t,a_1}, \ldots, Z_{t,a_K}] \in \mathbb{R}^{K \times d} \) and \( \tau_t = [\tau_1, \ldots, \tau_t] \in \mathbb{R}^t \), where \( \tau_{r,a} = X_{r,a} \odot U_{r,a} \) and \( \tau_r = X_{r,a} \beta + \epsilon_r \).

3.2. Lasso estimation with adjusted covariance matrix

Based on the observed \( Z_t \), we optimize

\[
\hat{\beta}_t \in \arg \min_{\|\beta\|_1 < R} \left\{ \frac{1}{2} \beta^T \hat{\Gamma}_{\text{miss},t} \beta - \langle \hat{\gamma}_{\text{miss},t}, \beta \rangle + \eta_t \|\beta\|_1 \right\},
\]

where \( \|\cdot\|_1 \) is an \( \ell_1 \) norm and

\[
\hat{\Gamma}_{\text{miss},t} = \left( \frac{1}{t} \sum_{i=1}^t Z_{i,t}^T Z_{i,t} \right) \odot \hat{M}
\]

\[
\hat{\gamma}_{\text{miss},t} = \left( \frac{1}{t} \sum_{i=1}^t \beta^T \tau_{i,t} \right) \odot \hat{\xi},
\]

\[
\hat{M}_{ij} = \begin{cases} 
\hat{\xi} & \text{if } i = j \\
\hat{\xi}_i \hat{\xi}_j & \text{if } i \neq j
\end{cases}
\]

\[
\hat{\xi} = \mathbb{E}[X|X_t = 1] \text{ is the sampling probability of each of the covariates.}
\]

Due to the missing value, \( \hat{\Gamma}_{\text{miss},t} \) contains negative eigenvalues that make (5) non-convex and unbounded from below. In order to circumvent this problem, [4] introduced the additional \( \ell_1 \) constraint on \( \beta \) for noisy and missing data. Our analysis adopts this estimator for the high-dimensional stochastic linear bandit problem. [12] further extended this problem for non-convex regularization in the batch setting.

While applying the approach of [4] to the bandit problem seems simple enough, several challenges arise due to the sequential-decision making setting of the linear bandit. As the noise \( \epsilon_r \) is not i.i.d., we cannot directly apply the convergence results from [4] to the lasso bandit with covariate missingness. Nonetheless, in Theorem 5.1 below, we can establish convergence.

4. ALGORITHM

Algorithm 1 solves the lasso bandit problem under the covariate missingness. The key differences compared to the fully-observed counterpart are 1) the use of adjusted plug-in estimators \( \hat{\Gamma}_{\text{miss},t} \) and \( \hat{\gamma}_{\text{miss},t} \) and 2) the theoretically justified regularization parameter \( \eta_t \). An additional tuning parameter \( R \) is introduced due to the non-convexity of the problem (5) in order to constrain \( \beta \) to be in an \( \ell_1 \) ball. This is motivated by a similar condition proposed in [4] and [13].

**Algorithm 1: SA with missingness (SAM) bandit**

**Input:** \( \eta_t, R \)

Initialize \( \beta_0 = 0, \hat{\xi}_0 = 1 \)

for \( t = 1, \ldots, T \)

Observe contexts \( Z_t \sim \mathcal{P}_{K \times d} \) and the missing pattern \( U_t \)

Update \( \hat{\xi}_t = \hat{\xi}_{t-1} + \frac{1}{t} \left( \frac{\hat{\Gamma}}{R} \sum_{i=1}^K U_{t,i} - \hat{\xi}_{t-1} \right) \)

Pull arm \( a_t = \arg \max_{i \in [K]} \langle Z_{i,t}, \hat{\xi}_t \rangle \hat{\beta}_t \)

Observe \( \tau_{t,a} \) for the arm \( a_t \)

Update \( \eta_t = \eta_t \sqrt{\frac{4 \log(4 t \|\hat{\gamma}_{\text{miss},t}\|_2^2 + \log d)}{2 \|\hat{\xi}\|_2}} \times \tau_{\text{min}} \)

Updated \( \hat{\Gamma}_{\text{miss},t} \) and \( \hat{\gamma}_{\text{miss},t} \) based on (6)

Update \( \hat{\beta}_t \) based on (5)

end
algorithm for (7) converges within the statistical tolerance of the global optimum even when \( \hat{\Gamma}_{\text{miss},t} \neq 0 \).

5. REGRET ANALYSIS UNDER MISSING DATA

Here we give an upper bound on the regret of the proposed SAM bandit defined by Algorithm 1. This analysis equally applies to the DRM bandit. The distributional assumptions on the covariates and the reward noise include the bounded feature set and reward function parameter (A1), i.i.d. on the context variables \( X_i \in \mathbb{R}^{K \times d} \) (A2), and the sub-gaussianity of the error \( \varepsilon_t = f_{t,i} - X_{t,i} \beta^* \) (A3). A1-3 are standard assumptions in the stochastic linear bandit literature [5, 6, 1] to derive the upper bound for the cumulative regret.

We additionally involve the compatibility condition on the true Gram matrix \( \Sigma := \frac{1}{n} \mathbb{E}[X^TX] \). This is a standard assumption in high-dimensional regression literature [7] and stochastic linear bandit problems [5, 6, 1, 14]. Before we define the compatibility condition, we first define the active set \( S_0 = \{ j : \beta_j^* \neq 0 \} \) as the set of indices that correspond to non-zero values of \( \beta_j^* \). Thus, the true \( \beta^* \) can be divided into \( \beta_{j,S_0} = \beta_j 1_{j \in S_0} \) and \( \beta_{j,S_0^c} = \beta_j 1_{j \notin S_0} \).

Let \( C(S_0) \) be the set of vectors \( \beta \in \mathbb{R}^d \) defined as

\[
C(S_0) = \{ \beta \in \mathbb{R}^d | \| \beta_{S_0^c} \|_1 < 3 \| \beta_{S_0} \|_1 \}. \tag{8}
\]

Then, we can define the compatibility condition.

A3 (Compatibility Condition): For an active set \( S_0 \), there exists a compatibility constant \( \phi^2 > 0 \) such that \( \phi_0^2 \| \beta_{S_0} \|^2 \leq s_0 \beta^T \Sigma \beta \) \( \forall \beta \in C(S_0) \).

A4 (Relaxed symmetry): For a joint distribution \( P_X \), there exists \( \nu \) such that \( \frac{P_X(-x)}{P_X(x)} \leq \nu \) for all \( x \in \mathbb{R}^d \). The relaxed symmetry assumption is satisfied by a wide range of distributions including the Gaussian distribution and the uniform distribution. Note that for symmetric distributions, A4 is satisfied with \( \nu = 1 \).

A5 (Balanced covariance): Consider a permutation \( (i_1, \ldots, i_K) \) of \( (1, \ldots, K) \). For any integer \( k \in \{2, \ldots, K-1\} \) and fixed vector \( \beta \), there exists \( C_X \) such that

\[
E \left[ X^T_{i_k}X_{i_{k+1}}1_{(X_{i_k} \beta^* < \cdots < X_{i_k} \beta^*)} \right]
\leq C_X E \left[ (X^T_{i_k}X_{i_k} + X^T_{i_{k+1}}X_{i_k})1_{(X_{i_k} \beta^* < \cdots < X_{i_k} \beta^*)} \right].
\]

In bandit problems, the sample covariance governing the observation at time \( t \) is \( E[X^TX|F_{t-1}] \). As we are selecting arms \( a_t = \arg \max_{i \in [K]} (Z_{t,i} \otimes \hat{\zeta}_t) \beta_t \) based on the current estimate \( \hat{\beta}_t \), the SA bandit algorithm may undersample part of the distribution. As introduced in [1], the balanced covariance assumption implies that we can control the covariance matrix based on the extreme selections of the arms.

For example, if the arms are completely correlated, \( C_X \) is constant independent of dimensions. In a more general setting, [1] proved that the balanced covariance condition is satisfied with \( C_X = \binom{K-1}{K-2} \) with \( K_0 = \lceil \frac{K-1}{2} \rceil \) when the arms are independent and identically distributed from Gaussian distribution. The balanced covariance condition was first introduced in [1], and the value of \( C_X \) gives insight into the behavior of the population covariance matrix.

Theorem 5.1 (SA Lasso bandit with missing values) Suppose Assumption 1, 2, 3, 4 and 5 hold. Then, for some constant \( c_0, c_1 > 0 \), the cumulative regret of the SA Lasso bandit with missing values is \( O \left( \frac{1}{c_{\min}} \sqrt{s_0 T \log(dT)} \right) \) with probability at least \( 1 - c_0 \exp(-c_1 \log d) \).

As pointed out in [1], the learner does not have to go through the exploration phase to achieve this convergent regret. This is due to the balanced covariance assumption (A5). Note that, compared to the fully observed setting, the regret is increased by \( \frac{1}{c_{\min}} \) due to the missing covariates. This matches our intuition that extra arm pulls are required to accurately estimate the off-diagonal entries of \( \hat{\Gamma}_{\text{miss},t} \) when the covariates are missing.

6. SIMULATION STUDY

Simulation: We conduct simulations to evaluate the improvement in cumulative regret based on our modifications to the lasso bandit, the DR lasso bandit, and the SA lasso bandit (Figure 1). We set \( K = 20, d = 200 \) and the missing probability \( 1 - \zeta_j \in \{0.65, 0.9\} \) for all \( j \in [K] \). The sparsity was set to \( s_0 = cv^2 \), where \( c \) is constant. At each round \( t \), we generate the true contexts \( X_t \in \mathbb{R}^{K \times d} \), where \( X_{t,i} \sim N(0, \Sigma) \) and \( \Sigma \) is a Toeplitz matrix. The learners observe \( Z_t = X_t \otimes U_t \), where all entries \( U_{t,i,j} \sim \text{Ber} (\zeta_j) \). We could have also varied the sampling probability \( \zeta_j \) for each covariate, but Theorem 5.1 shows that only the minimum sampling probability plays a role in the convergence rate. In this setting, the Gaussian model for \( X_t \) satisfies the symmetry condition (A4). Lastly, we generate \( \varepsilon_t \) from the normal distribution, and the reward is observed based on the approximation \( Z_t \otimes \hat{\zeta}_t \) of the context \( X_t \). Figure 2 shows the regret performance of our algorithm over varying missing probability \( 1 - \zeta_j \in [0.65, 0.9] \).

Case Study: Experimental Design: Understanding the interactions among microbiomes in microbial communities is important to applications in health, ecology, and antibiotic design. Recently, [15] introduced a model microbiome community for the rhizosphere, called THOR or BFK, that combines three microbial species Bacillus cereus (B), Flavobacterium johnsoniae (F), and Pseudomonas koreensis (P) to study the complex interactions between them under different conditions (classes). We use data from this model to illustrate the application of the proposed contextual bandit with missingness to the sequential design of experiments for discovering the gene probes that best discriminate between two experimental conditions: a BFK community having a wildtype strain of K (class 1) vs a community having a mutant strain of K (class 2).

Gene probe Selection Problem: Often only a few key genes among the thousands of genes play important roles in
We aim to establish proof of concept that such discriminative regret after normalization according to proposed modifications of the lasso bandit, the DR Lasso and processed in the lab of one of the co-authors. We per-

final data as log-transformation to these expression data and denote the gene expression datasets for species $B$, $F$, $K$ and selects probes that do not necessarily yield high rewards. In the early phase, the learner explores the covariate space and selects probes that do not necessarily yield high rewards. After a short period of time, the set of probes selected by the learner quickly narrows down to the subset of statistically significant probes. Figure 3 shows the pull success rate for $F$ selected probes for $K$ of selected probes for $B$, $F$, $K$ data, and the bandit quickly discriminates this in its early set of replicates (samples) for conditions (classes) 1 and 2, respectively. There are 6179 gene probes for Bacillus, 5198 genes in Flavobacterium, and 5864 genes in Pseudomonas with $m_1 = 38$ and $m_2 = 34$.

**Bandit Formulation:** We formulate the sequential gene selection problem with the contextual bandit having the following components

- **Arms:** The arms correspond to the genomes of $B$, $F$, $K$ for the three species, denoted $X \in \mathbb{R}^{k \times (m_1 + m_2)}$ where $k$ represents the number of selectable DNA probes, which depends on $B$, $F$, or $K$.
- **Covariates:** The covariates of the $i$-th arm at time $t$ $X_{t,i} \in \mathbb{R}^{m_1 + m_2}$ are the gene expressions of the $i$-th probe for the set of samples for both experimental conditions.
- **Reward:** The reward is the observed discrimination provided by the selected gene probe (arm). We define the reward as the log of the $p$-value of Welch’s $t$-statistic for two-sample $t$-test to measure discrimination. The reward is defined as

$$\hat{r}_{t} = \log \left( \frac{1 - p_{a_t}}{p_{a_t}} \right) + \epsilon_t = X_{a_t}\beta^* + \epsilon_t$$  \hspace{1cm} (9)

where $p_{a_t}$ denotes the $p$-value of the Welch’s test of the null hypothesis that the arm $a_t$ is a non-discriminative gene whose means are identical in each class.

**Evaluation and Results:** Based on the reward function (9), we apply our contextual bandit and treat the zero expression as missing entries. As the goal of the bandit problem is to select the most discriminating DNA probes at each time point, we evaluate the fraction of the probe selections that correctly lead to a statistically significant ($\alpha = 0.05$). To simulate noisy rewards in terms of $p$-value, each probe is sampled with replacement at each time $t$.

In the early phase, the learner explores the covariate space and selects probes that do not necessarily yield high rewards. After a short period of time, the set of probes selected by the learner quickly narrows down to the subset of statistically significant probes. Figure 3 shows the pull success rate for our proposed bandit. For each species, the success rate rapidly reaches around 0.95 for SAM. The difference between the class distributions is the largest for Flavobacterium in the overall data, and the bandit quickly discriminates this in its early set of selected probes for $F$ (shown in orange in Figure 3).

**7. CONCLUSION**

We introduced a solution to the missing value problem for a sparse linear contextual bandit. Missing covariates often result from the high cost of collecting context vectors or probe sensor failures. A modification of the SA lasso bandit was...
presented when the context covariates may be incompletely observed. We modeled the problem as missing completely at random but with possibly different covariate missingness probabilities. Even in this simple setting, the missingness plug-in lasso estimator results in a non-convex objective function, and we have adopted and extended the regression solution proposed in [4] to bandit problems in this paper.

A natural extensions of our work include 1) the missing-not-at-random framework and 2) the best-k identification problem, which allows multiple arms to be pulled at once. A missingness extension to this problem is still an open problem.

8. REFERENCES

[1] Min-hwan Oh, Garud Iyengar, and Assaf Zeevi, “Sparsity-agnostic lasso bandit,” in International Conference on Machine Learning. PMLR, 2021, pp. 8271–8280.

[2] Ambuj Tewari and Susan A Murphy, “From ads to interventions: Contextual bandits in mobile health,” in Mobile Health, pp. 495–517. Springer, 2017.

[3] International Warfarin Pharmacogenetics Consortium, “Estimation of the warfarin dose with clinical and pharmacogenetic data,” New England Journal of Medicine, vol. 360, no. 8, pp. 753–764, 2009.

[4] Po-Ling Loh and Martin J Wainwright, “High-dimensional regression with noisy and missing data: Provable guarantees with nonconvexity,” The Annals of Statistics, vol. 40, no. 3, pp. 1637–1664, 2012.

[5] Hamsa Bastani and Mohsen Bayati, “Online decision making with high-dimensional covariates,” Operations Research, vol. 68, no. 1, pp. 276–294, 2020.

[6] Gi-Soo Kim and Myunghee Cho Paik, “Doubly-robust lasso bandit,” in NeurIPS 2019 : Thirty-third Conference on Neural Information Processing Systems, 2019, pp. 5877–5887.

[7] Sara A Van De Geer, Peter Bühlmann, et al., “On the conditions used to prove oracle results for the lasso,” Electronic Journal of Statistics, vol. 3, pp. 1360–1392, 2009.

[8] Yasin Abbasi-Yadkori, David Pal, and Csaba Szepesvari, “Online-to-confidence-set conversions and application to sparse stochastic bandits,” in Artificial Intelligence and Statistics, 2012, pp. 1–9.

[9] Alexandra Carpentier and Rémi Munos, “Bandit theory meets compressed sensing for high dimensional stochastic linear bandit,” in Artificial Intelligence and Statistics. PMLR, 2012, pp. 190–198.

[10] Davis Gilton and Rebecca Willett, “Sparse linear contextual bandits via relevance vector machines,” in 2017 International Conference on Sampling Theory and Applications (SampTA). IEEE, 2017, pp. 518–522.

[11] Nicolas Stüdler and Peter Bühlmann, “Missing values: sparse inverse covariance estimation and an extension to sparse regression,” Statistics and Computing, vol. 22, no. 1, pp. 219–235, 2012.

[12] Po-Ling Loh and Martin J Wainwright, “Regularized m-estimators with nonconvexity: Statistical and algorithmic theory for local optima,” The Journal of Machine Learning Research, vol. 16, no. 1, pp. 559–616, 2015.

[13] Mark Rudelson, Shuheng Zhou, et al., “Errors-in-variables models with dependent measurements,” Electronic Journal of Statistics, vol. 11, no. 1, pp. 1699–1797, 2017.

[14] Xue Wang, Mingcheng Wei, and Tao Yao, “Minimax concave penalized multi-armed bandit model with high-dimensional covariates,” in International Conference on Machine Learning, 2018, pp. 5200–5208.

[15] Gabriel L Lozano, Juan I Bravo, Manuel F Garavito-Diago, Hyun Bong Park, Amanda Hurley, S Brook Peterson, Eric V Stabb, Jason M Crawford, Nichole A Broderick, and Jo Handelsman, “Introducing thor, a model microbiome for genetic dissection of community behavior,” MBio, vol. 10, no. 2, pp. e02846–18, 2019.