Nucleon Spin Structure. Sum Rules.

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1 The sum rules for $\Gamma_{p,n}$. Theoretical Status.

I start with the consideration of the sum rules for the first moments of the spin structure functions $g_{1p,n}(x, Q^2)$

$$\Gamma_{p,n}(Q^2) = \frac{1}{x} \int_0^1 dx g_{1p,n}(x, Q^2)$$

(1)

I will discuss the uncertainties in the theoretical predictions for $\Gamma_{p,n}$ and compare the theoretical expectations with experimental data. The aim of this consideration is to obtain from the experiment the restrictions on the uncertainties in the theoretical description of the problem.

Consider first the Bjorken sum rule \cite{1}, which now is a corner stone not only of the problem of nucleon spin structure functions, but of the whole theory of deep inelastic lepton-hadron scattering in QCD. (It is interesting to note that in his original paper \cite{1} in 1966 Bjorken had classified this sum rule as a “worthless equation”).

The Bjorken sum rule reads:

$$\Gamma_p(Q^2) - \Gamma_n(Q^2) = \frac{1}{6} g_A [1 - \frac{\alpha_s(Q^2)}{\pi}] - 3.6 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 + \frac{b_{p-n} Q^2}{Q^2},$$

(2)

where $g_A$ is the axial $\beta$-decay coupling constant and the last term in (2) represents the twist-4 correction. The perturbation QCD corrections are known up to the third order \cite{2,3} (there is also an estimate of the fourth order term \cite{4}, which is not included in (2), since the uncertainties in the included terms are larger than its contribution). The coefficients in (2) correspond to the number of flavours $N_f = 3$. (In the domain of existing experiments only three flavours of quarks are effective). In what follows I will so often transfer the data to the standard reference point $Q^2_0 = 10.5 GeV^2$ – the mean value of $Q^2$, at which EMC and SMC experiments were done.

Let us discuss the perturbative corrections in (2). Today there is a serious discrepancy in the values of $\alpha_s$ found from different experiments. The average value of $\alpha_s$, obtained at LEP is \cite{5}

$$\alpha_s(m^2_Z) = 0.124 \pm 0.007$$

(3)

In two-loop approximation this value corresponds to the QCD parameter $\Lambda_3$ for three flavours

$$\Lambda_3 = 430 \pm 100 MeV$$

(4)

On the other hand, the data on the $\Upsilon \rightarrow$ hadrons decay give \cite{5} (the first $\alpha_s$ correction \cite{6} is accounted)

$$\alpha_s(m^2_b) = 0.178 \pm 0.010$$

(5)

from which it follows

$$\Lambda_3 = 170 \pm 30 MeV$$

(6)

The small error in (6) is caused by the fact that the partial width $\Gamma(\Upsilon \rightarrow 3g)$, from which (5) was determined, is proportional to $\alpha_s^3$ and the $\alpha_s$ correction to it is small. A strong contradiction of (4) and (6) is evident.

The overall fit \cite{7} of the data of deep inelastic lepton-nucleon scattering gives in the NLO approximation $\Lambda_3 = 250 MeV$ (the error is not given). New data in the domain $m^2_Z$ indicate lower $\alpha_s(m^2_Z)$ (SLD \cite{8}: $0.118 \pm 0.013, 0.112 \pm 0.004$; OPAL \cite{9}: $0.113 \pm 0.012$), but the data of AMY \cite{10} on $e^+ e^-$ annihilation at $\sqrt{s} = 57.3 GeV$ results in $\alpha_s(m^2_Z) = 0.120 \pm 0.005$. Finally, from $\tau$- decay it was obtained \cite{11}:

$$\alpha_s(m^2_\tau) = 0.33 \pm 0.03$$

(7)
corresponding to

$\Lambda_3 = 380 \pm 60 MeV$

But the determination of $\alpha_s$ from $\tau$-decay can be criticized \cite{12} on the grounds that at such a low scale exponential terms in $q^2$ may persist in the domain of positive $q^2 = m_{\tau}^2$ besides the standard power-like terms in $q^2$ accounted in the calculation.

In such a confusing situation I will consider two options – of small and large perturbative corrections. In the first case I will take $\Lambda_3 = 200 MeV$. Then

$$\alpha_s(Q_0^2) = 0.180 \pm 0.010 \quad \Gamma_p(Q_0^2) - \Gamma_n(Q_0^2) = 0.194$$

In the second one $\Lambda_3 = 400 MeV$ and

$$\alpha_s(Q_0^2) = 0.242 \pm 0.025 \quad \Gamma_p(Q_0^2) - \Gamma_n(Q_0^2) = 0.187$$

The twist-4 contribution was disregarded in (8) and (9). There is also a discrepancy in its value. The value of $b_{p-n}$ was determined in the QCD sum rule approach by Balitsky, Braun and Koleshichenko (BBK) \cite{13}:

$$b_{p-n} = -0.015 GeV^2$$

On the other hand, the model \cite{14-16} based on connection \cite{17} of the Bjorken sum rule at large $Q^2$ with the Gerasimov, Drell, Hearn sum rule at $Q^2 = 0$ \cite{18} gives

$$b_{p-n} = -0.15 GeV^2$$

In what follows I will consider (10) and (11) as two options which correspond to small (S) and large (L) twist-4 corrections. I will discuss both approaches of determination of higher twist corrections in more details below. From my point of view, no one of these approaches is completely reliable and I will use them in comparison with experiments only as reference points.

I turn now to the sum rules for $\Gamma_p$ and $\Gamma_n$:

$$\Gamma_{p,n}(Q^2) = \frac{1}{12} \left\{ \left[ 1 - \frac{\alpha_s}{\pi} - 3.6(\frac{\alpha_s}{\pi})^2 - 20(\frac{\alpha_s}{\pi})^3 \right] \left( \pm g_a + \frac{1}{3} a_s \right) + \frac{4}{3} \left[ 1 - \frac{\alpha_s}{3\pi} - 0.55(\frac{\alpha_s}{\pi})^2 \right] \Sigma \right\} - \frac{N_f}{18\pi} \alpha_s(Q^2) \Delta g(Q^2) + \frac{b_{p-n}}{Q^2}$$

(The $\alpha_s^2$ correction to the singlet part was calculated in \cite{19}).

According to the current algebra $g_A, a_s$ and $\Sigma$ are determined by the proton (or neutron) matrix elements of flavour octet and singlet axial currents

$$-2ms_{\mu}a_s = <p, s | j_5^{(8)}\mu | p, s > \quad -2ms_{\mu} \Sigma = <p, s | j_5^{(0)}\mu | p, s >,$$

$$-2ms_{\mu}g_A = <p, s | j_5^{(3)}\mu | p, s >$$

where

$$j_5^{(8)} = \bar{u}\gamma_\mu\gamma_5u + \bar{d}\gamma_\mu\gamma_5d - 2\bar{s}\gamma_\mu\gamma_5s,$$

$$j_5^{(3)} = \bar{u}\gamma_\mu\gamma_5u - \bar{d}\gamma_\mu\gamma_5d$$

$$j_5^{(0)} = \bar{u}\gamma_\mu\gamma_5u + \bar{d}\gamma_\mu\gamma_5d + \bar{s}\gamma_\mu\gamma_5s$$

In the parton model $g_A, a_s$ and $\Sigma$ are equal to

$$g_A = \Delta u - \Delta d \quad a_s = \Delta u + \Delta d - 2\Delta S \quad \Sigma = \Delta u + \Delta d + \Delta s,$$
where

$$\Delta q = \int_0^1 [q_+(x) - q_-(x)] \quad q = u, d, s$$  \hspace{1cm} (15)$$

and $q_\pm$ are the quark distributions with the spin parallel (antiparallel) to the proton spin, which is supposed to be longitudinal (along the beam). $\Delta g$ in (12) has the similar meaning, but for gluons, as $\Delta q$ for quarks

$$\Delta g(Q^2) = \int_0^1 dx \left[ g_+(x, Q^2) - g_-(x, Q^2) \right]$$  \hspace{1cm} (16)$$

Unlike $g_A, a_8$ and $\Sigma$ which in the approximation used above are $Q^2$ independent and have zero anomalous dimensions, $\Delta g$ anomalous dimension is equal to $-1$. This means that

$$\Delta g(Q^2)_{Q^2 \to \infty} \simeq c \ln Q^2$$  \hspace{1cm} (17)$$

The conservation of the projection of the angular momentum can be written as

$$\frac{1}{2} \Sigma + \Delta g(Q^2) + L_z(Q^2) = \frac{1}{2}$$  \hspace{1cm} (18)$$

where $L_z$ has the meaning of the orbital momentum of quarks and gluons. As follows from (17),(18) at high $Q^2$ $L_z(Q^2)$ must compensate $\Delta g(Q^2)$, $L_z(Q^2) \approx -c \ln Q^2$. This means that the quark model, where all quarks are in $S$-state, failed with $Q^2$ increasing.

The gluonic contribution to $\Gamma_{p,n}$ term, proportional to $\Delta g$ in (12), was calculated in $[2,20-23]$. There was a wide discussion in the past years if gluonic contribution $\Delta g\Gamma_{p,n}$ to $\Gamma_{p,n}$ is uniquely defined theoretically or is not $[23-36]$. The problem is that gluonic contribution to the structure functions, described by imaginary part of the forward $\gamma_{\text{virt}}$-gluon scattering amplitude (Fig.1) is infrared dependent. Since in the infrared domain the gluonic and sea quark distributions

![Fig.1](image.png)

are mixed and their separation depends on the infrared regularization scheme, a suspicion arises that $\Delta g\Gamma_{p,n}$ can have any value. This suspicion is supported by the fact that in the lowest order in $\alpha_s$, the terms proportional to $\ln Q^2$ are absent in $\Delta g\Gamma_{p,n}$ and this contribution looks like next to leading terms in nonpolarized structure functions where such an uncertainty is well known.

In the framework of the operator product expansion (OPE) there is only one operator – the singlet axial current $j^{(0)}_{\mu 5}$ – corresponding to the first moment of $g_{1p} + g_{1n}$, i.e., $\Gamma_{p+n}$. This fact
can be also used as an argument in the favour that (in the $\alpha_s$ and higher orders) the separation of the terms proportional to $\Sigma$ and $\Delta g$ in (12) is arbitrary.

In order to discuss the problem consider the gluonic contribution $g_{1p}(x,Q^2)_{gl}$ to the proton structure function $g_{1p}(x,Q^2)$, described by the evolution equation

$$g_{1p}(x,Q^2)_{gl} = N_f \frac{e^2}{2} \int \frac{dy}{x} A \left( \frac{x}{y} \right) [g_+(y,Q^2) - g_-(y,Q^2)],$$

where the asymmetry $A(x_1)$ is determined by the diagrams of Fig.1. The calculation of the asymmetry $A(x_1), x_1 = -q^2/2pq$ results in appearance of integrals

$$\int \frac{d^2k_\perp}{(k_\perp^2 - x_1(1-x_1)p^2 + m_q^2)^n}, \quad n = 1, 2,$$

which are infrared dependent. To overcome this problem it is necessary to introduce the infrared cut-off (or infrared regularization), to separate the domain of large $k_\perp^2$, where perturbative QCD is reliable, from the domain of small $k_\perp^2$. The contribution of the latter must be addressed to noncalculable in perturbative QCD parton distribution. Such a procedure is legitimate because of the factorization theorem which states that the virtual photoabsorption cross section on the hadronic target $h, \sigma_h^\gamma(x,Q^2)$ can be written down in the convolution form

$$\sigma_h^\gamma(x,Q^2) = \sum_i \sigma_i^\gamma(x,Q^2,M^2) \otimes f_{i/h}(x,M^2),$$

where $\sigma_i^\gamma$ is the photoproduction cross section on the $i^{th}$ parton ($i = q, \bar{q}, g$), $f_{i/h}$ are the parton distributions in a hadron $h$, $\otimes$ stands for convolution. Both $\sigma_i^\gamma$ and $f_{i/h}$ depend on the infrared cut-off $M^2$, but the physical cross section $\sigma_h^\gamma h$ is cut-off independent. The variation of $M^2$ corresponds to redistribution among partons: the trade of gluon for sea quarks.

As follows from (19)

$$\Delta_g \Gamma_p = N_f \frac{e^2}{2} \Delta_g A(M^2),$$

where

$$A(M^2) = \int_0^1 dx_1 A(x_1,M^2)$$

The convenient way is to introduce cut off in $k_\perp^2$

$$k_\perp^2 > M^2(x_1,p^2)$$

Generally, $M^2$ may depend on $x_1$ and $p^2$. For example, the cut-off in quark virtuality in Fig.1 $-k^2 > M_0^2$ corresponds to the form (23) with $M^2 = (1 - x_1)(M_0^2 + p^2x_1)$ if $x_1 < -M_0^2/p^2$.

In the calculation of the diagrams of Fig.1 it is reasonable to neglect the light quark masses in comparison with gluon virtuality $p^2$, since we expect that $|p^2|$ is of order of characteristic hadronic masses, $|p^2| \sim 1 GeV^2$ [24]. Then, introducing the infrared cut-off (23) we have

$$A(M^2) = -\frac{\alpha_s}{2\pi} \left\{ 1 - \int_0^{x_1} dx_1(1 - 2x_1)[lnr - r] \right\}$$

where

$$r = \frac{x_1(1 - x_1)p^2}{x_1(1 - x_1)p^2 - M^2(x_1,p^2)}.$$
If \( M^2 = \text{Const} \) – a rectangular cut-off in \( k^2 \), the integral in (24) vanishes (the integrand is antisymmetric under substitution \( x_1 \rightarrow 1 - x_1 \)). Then \( \bar{A} = -\alpha_s/2\pi \) and we obtain the gluonic contribution to \( \Gamma_{p,n} \) (12). However, other forms of \( M^2(x_1, p^2) \) result in different values of \( \bar{A}(M^2) \), what supports the claim \([25]\) that \( \bar{A}(M^2) \) is cut-off dependent. Even more, if we put \( p^2 = 0, m_q^2 \neq 0 \) – the standard regularization scheme in the calculation of nonpolarized deep inelastic scattering – we will find \( \bar{A} = 0 \)\([33]\). This result is, however, nonphysical because the compensation of \(-\alpha_s/2\pi\) term in \( \bar{A} \) arises from soft non-perturbative domain of \( k^2 \sim m_q^2 \), which must be attributed to sea quark distribution.

Although generally \( \bar{A} \), as well as \( \Delta g \) and \( \Delta g \Gamma \), are infrared cut-off dependent, a special class of preferable cut-off’s can be chosen. In OPE the mean asymmetry \( \bar{A} \) is proportional to the one-gluon matrix element of axial current

\[
\Gamma_{\mu\lambda\sigma}(0, p, p) = \langle g, \epsilon_\lambda | j_{\mu5}(0) | g, \epsilon_\sigma \rangle \tag{26}
\]

at zero momentum transfer. It can be shown \([35]\) that this quantity is proportional to the divergence \( l_\mu \Gamma_{\mu\lambda\sigma}(l, p_1, p_2) \), \( l = p_1 - p_2 \) in the limit \( l^2 \rightarrow 0 \), i.e. to the one gluon matrix element of axial anomaly.

The proof is the following. The general expression of the matrix element of axial current over gluonic states with nonequal momenta has the form \([35]\)

\[
\Gamma_{\mu\lambda\sigma}(l, p_1, p_2) = F_1(l^2, p_2^2) l_\mu \epsilon_{\lambda\rho\tau\sigma} p_{1\rho} p_{2\tau} + \frac{1}{2} F_2(l^2, p_2^2) \left[ \epsilon_{\mu\lambda\rho\sigma} (p_1 + p_2)_{\rho} + \frac{p_1 \lambda}{p^2} \epsilon_{\mu\rho\sigma\tau} p_{1\rho} p_{2\tau} - \frac{p_2 \sigma}{p^2} \epsilon_{\mu\lambda\rho\sigma} p_{1\rho} p_{2\tau} \right] \tag{27}
\]

where it was assumed that \( p_1^2 = p_2^2 = p^2 < 0 \). When deriving (27) we used only Lorenz invariance, Bose symmetry of gluons and the conservation of vector currents at gluonic vertices. The formfactors \( F_{1,2}(l^2, p^2) \) have no kinematical singularities if \( p^2 \neq 0 \) and, particularly, \( F_1(l^2, p^2) \) has no pole at \( l^2 = 0 \). From (27) we have:

\[
l_\mu \Gamma_{\mu\lambda\sigma} = \left[ F_2(l^2, p^2) + l^2 F_1(l^2, p^2) \right] \epsilon_{\lambda\rho\tau\sigma} p_{1\rho} p_{2\tau} \Gamma_{\mu\lambda\sigma}(0, p, p) = F_2(0, p^2) \epsilon_{\mu\lambda\rho\sigma} p_{\rho} \tag{28}
\]

and in the limit \( l^2 \rightarrow 0 \) the mentioned above statement follows.

Let us recall the derivation of the anomaly in perturbation theory \([37]\). The anomaly corresponds to the diagrams of Fig.2.

**Fig.2**
The diagrams for the gluonic matrix element of axial current.

These diagrams have a superficial linear divergences. Cancellation of the divergences in the diagrams Fig’s 2a and 2b requires shifting of integration variable in 2b:

\[
k \rightarrow k - p_1 - p_2
\]
This shift corresponds to substitution $x_1 \rightarrow 1 - x_1$ in the integrated over $x_1$ diagrams of Fig.1, i.e. in eq.(24). We came to the conclusion that in order to retain connection of $\Gamma_{\mu\lambda\nu}(0, p, p)$ with the anomaly which follows from (28) and to preserve the standard form of the anomaly, the infrared cut-off $M^2(x_1, p^2)$ must be symmetric under $x_1 \rightarrow 1 - x_1$. In this case $\bar{A} = -\alpha_s/2\pi$ and we come to the gluonic contribution to $\Gamma_{p,n}$ given by eq.12. With such a cut-off the anomaly comes entirely from the hard momentum region and can be considered as a local probe of gluon helicity. (It must be mentioned, however, that any other forms of infrared cut-off are also possible, but they are less attractive). Some care, however, is necessary, when the calculations with this cut-off are compared with HO calculation in nonpolarized scattering, where as a rule, another regularization procedure $-p^2 = 0, m_q^2 \neq 0$ is used.

The gluonic contribution to the spin dependent proton structure function $g_{1p}(x, Q^2)_{gl}$ can be found experimentally by measuring inclusive two jets production in polarized DIS [24]. However, only large $-k_T^2$ component of Fig.1 diagrams can be determined in this way, the small $-k_T^2$ component is nonmeasurable, since in this case it is impossible to separate one jet from two jets events [35]. This fact is in complete accord with the formulated above statement about the arbitrariness of infrared cut-off.

Now about the numerical values of the constants $g_A, a_8$ and $\Sigma$ entering eq.(12). $g_A$ is known with a very good accuracy, $g_A = 1.257 \pm 0.003$ [5]. It must be mentioned, however, that eq.(13') follows from the assumption of exact isospin symmetry. In fact, one may expect its violation of the order of 1%. Therefore, the uncertainty in this number is not given by experimental error, but may be $\sim 1%$. Under assumption of SU(3) flavour symmetry in baryon decays $a_8$ is equal to

$$a_8 = 3F - D = 0.59 \pm 0.02$$

(29)

where $F$ and $D$ are axial coupling constants of baryon $\beta$-decays in SU(3) symmetry and the numerical value in the r.h.s. of (19) follows from the best fit to the data [38]. The combination $3F - D$ can be found also from any pair of baryon $\beta$-decays. The comparison of the values of $3F - D$, obtained in this way shows, that the spread is rather narrow [39], $|\delta a_8| \leq 0.05$ and at least $a_8 > 0.50$. This may be an argument in the favour that SU(3) violation is not large here. (For the recent suggestion of such violation see, however, [40]). It should be mentioned that at fixed $\Gamma_{p,n}$ the uncertainty in $a_8$ only slightly influences the most interesting quantities $\Sigma$ and $\Delta s$. As follows from (12) and (14)

$$\delta \Sigma = -\delta(\Delta s) = \frac{1}{4}\delta a_8.$$  

(30)

$a_8$ was also determined by the QCD sum rule method [41]. In this approach no SU(3) flavour symmetry was assumed and the result

$$a_8 = 0.5 \pm 0.2$$

(31)

is in agreement with (29), although the error is large.

What can be said theoretically about $\Sigma$? In their famous paper [42] Ellis and Jaffe assumed that the strange sea in the nucleon is nonpolarized, $\Delta s = 0$. Then

$$\Sigma \approx a_8 \approx 0.60$$

(32)

(The sum rule (12) in the framework of this assumption is called Ellis-Jaffe sum rule).This number is in a contradiction with the experimental data pointing to smaller values of $\Sigma$. On the other hand, Brodsky, Ellis and Karliner [43] had demonstrated that in the Skyrme model at large number of colours $N_c$, $\Sigma \sim 1/N_c$ and is small. From my point of view this argument is not very convincing: the Skyrme model may be a good model for description of nucleon periphery, but not for the internal part of the nucleon determining the value of $\Sigma$ (see also [44]).
2 Calculations of the Matrix Elements over the Polarized Nucleon by QCD Sum Rule Approach

The QCD sum rule method was used to calculate the nucleon coupling constants $g_A^{[45]}$, $g_8^{[41]}$, and the coefficients $b_{p,n}$ which determine the twist-4 contributions $^{[13]}$. The basic features of the approach are the same as originally suggested for calculation of nucleon magnetic moments $^{[46,47]}$.

We wish to find the diagonal matrix element

$$< p, s | j(0) | p, s >$$

(33)

To this end add to the QCD Lagrangian the term

$$\Delta L = j(x) S,$$

(34)

where $S$ is a constant external source. In the case of $g_A$ determination for $j$ and $S$ we substitute $j_{\mu 5}^{(3)}$ and $A_\mu^{(3)}$ - the constant external axial field etc. Consider the polarization operator of the currents $\eta$ with the nucleon quantum numbers

$$\Pi(p) = i \int d^4 x e^{ipx} < 0 | T \{ \eta(x), \bar{\eta}(0) \} | 0 >$$

(35)

$$\eta_p = \left( u^a(x) C \gamma_\mu u^b(x) \right) \gamma_\mu \gamma_5 d^c(x) e^{abc}$$

(36)

$$\eta_n = \eta_p (u \leftrightarrow d)$$

where $a, b, c = 1, 2, 3$ are colour indeces. Separate in $\Pi(p)$ the term proportional to external source $S$:

$$\Gamma(p, p, 0) S = i^2 \int d^4 x e^{ipx} S < 0 | T \{ \eta(x), j(z), \bar{\eta}(0) \} | 0 >$$

(37)

Suppose that $p^2 < 0, | p^2 | \gg R_c^{-2}$, where $R_c$ is the confinement radius, and perform OPE in $1/p^2$. Unlike the standard OPE for polarization operator, the presence of external source results in appearance of a new type of vacuum expectation values (v.e.v.), induced by external source. If the source $S$ is an axial field $A_\mu$, then, e.g.

$$< 0 | \bar{u} \gamma_\mu \gamma_5 u | 0 > |_A = c A_\mu$$

(38)

It can be easily shown that for massless quarks $m_q = 0$ ($q = u, d, s$), $c = f_\pi^2$ in the case of isospin vector $A^3_\mu$ or octet $A^8_\mu$ axial field. In the first case we can write

$$< 0 | \bar{u} \gamma_\mu \gamma_5 u | 0 > |_A = \lim_{q \rightarrow 0} \frac{1}{2} \int d^4 x e^{iqx} \times$$

$$\times < 0 | T \left\{ j_{\mu 5}^{(3)}(x), j_{\mu 5}^{(3)}(0) \right\} | 0 > |_A = \lim_{q \rightarrow 0} \Pi_{\nu \mu}^{(3)}(q) A_{\nu}^{(3)}$$

(39)

Since the axial current $j_{\mu 5}^{(3)}$ conserves

$$\Pi_{\nu \mu}^{(3)}(q) = - (\delta_{\mu \nu} q^2 - q_\mu q_\nu) \Pi(q^2)$$

(40)
The non-zero result for $\Pi_{\mu\nu}(q)$ at $q \to 0$ comes from the pole $\sim 1/q^2$ in $\Pi(q^2)$ which corresponds to the massless pion intermediate state

$$\Pi^{(3)}_{\mu\nu}(q) = \left( -\frac{q\mu q\nu}{q^2 - \mu^2} + \delta_{\mu\nu} \right) f^2_\pi$$

In (41) the pion mass in the propagator was accounted as the most important effect of nonvanishing quark masses at small $q^2$. Substituting (41) into (39) and going to the limit $q \to 0$ we obtain the desired result.

The same proof may be repeated for $A^{(8)}_\mu$ in the limit of $SU(3)$ flavour symmetry. The vacuum expectation values which enter the OPE of the r.h.s. of (36) can be classified according to their dimensions and the number of loops (the dimension of $A_\mu = A^{(3)}_\mu, A^{(8)}$ is 1, the chirality conserving structure is considered).

| Dimension | V.e.v. | Number of loops |
|-----------|--------|-----------------|
| 1         | 1.$A_\mu$ | 2               |
| 3         | $< 0 \mid \bar{q}\gamma_\mu\gamma_5 q \mid 0 >_{\pi} = f^2_\pi A_\mu$ | 1               |
| 5         | $< 0 \mid \alpha_s G^2_{\mu\nu} \mid 0 >_{\pi}$ | 2               |
| 5         | $< 0 \mid \bar{q}\gamma_\nu \tilde{G}^m_{\nu\mu} \frac{1}{2}\lambda^m q \mid 0 >_{\pi} = f^2_\pi m^2_1 A_\mu$ | 1               |
| 7         | $< 0 \mid \bar{q} q \mid 0 >_{\pi}^2 A_\mu$ | 0               |

Here $\tilde{G}_{\mu\nu} = (1/2) \epsilon_{\mu\nu\lambda\sigma} G_{\lambda\sigma}$. $m^2_1 \approx 0.2$ GeV$^2$ $^{[48]}$. The important role of induced by external field v.e.v. is evident – besides the unit operator they correspond to lowest dimensions and to minimal number of loops. The OPE for the r.h.s. of (37) can be constructed exploiting the v.e.v. given in (42).

In order to find the matrix element (33), represent the l.h.s. of (37) in terms of contributions of physical states using dispersion relations. Generally, when the momentum of external field is non-zero, $\Gamma$ is a function of three variables, $\Gamma(p_1^2, p_2^2, q^2)$, and may be represented by the double dispersion relation:

$$\Gamma(p_1^2, p_2^2, q^2) = \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho(s_1, s_2; q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} + P(p_1^2) f(p_2^2, q^2) + P(p_2^2) f(p_1^2, q^2)$$

where $P(p^2)$ is a polynomial and $f(p^2, q^2)$ is given by the ordinary dispersion relation

$$f(p^2, q^2) = \int_0^\infty \varphi(s, q^2) \frac{s}{s - p^2} + \text{subtr. terms}$$

We are interested in the limit $q \to 0, p_1^2 = p_2^2 = p^2$ and at the first sight it seems that one variable dispersion relation for $\Gamma(p^2, p^2, 0)$ can be written in this case. Indeed,

$$\int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho(s_1, s_2; 0)}{(s_1 - p^2)(s_2 - p^2)} = \int \frac{\rho(s_1, s_2; 0)}{s_1 - s_2} ds_1 ds_2 \left( \frac{1}{s_2 - p^2} - \frac{1}{s_1 - p^2} \right)$$
In the first (second) term in the r.h.s. of (45) integration over $s_1(s_2)$ can be performed and the result has the form of one-variable dispersion relation. Such transformation is, however, misleading, because, in general, the integrals

$$\int ds_1 \frac{\rho(s_1, s_2, 0)}{s_1 - s_2} = -\int ds_2 \frac{\rho(s_1, s_2, 0)}{s_1 - s_2}$$

are ultraviolet divergent. This ultraviolet divergence cannot be cured by subtractions in one-variable dispersion relation: only subtractions in the double dispersion representation (43) can be used. It is evident that the procedures which kill the subtraction terms and lead to fast convergence of dispersion integrals in standard one-variable dispersion representations like Borel transformation in $p^2$ do not help here.

Let us discuss the determination of the coupling constant $a_8$ in more details. Consider in (37) the coefficient function at the structure $(A^{(8)}_{\mu} p_{\mu}) \bar{\eta} \gamma_5$. The advantage of this structure is that it has the largest power of momenta in the numerator. Therefore, the integrals for the coefficient function converge better and the contribution of excited states in the dispersion relation (43), which is the background in our calculation, is smaller. It can be easily seen that the function $\rho$ in the r.h.s. of (43) which correspond to the bare loop diagram Fig.3 has in this case the form

$$\rho(s_1, s_2) = a s_1 s_2 \delta(s_1 - s_2), \quad (46)$$

where $a$ is a calculable constant.

Fig.3. The bare loop diagram, corresponding to determination of the coupling constants $g_A$ or $a_8$. The solid lines correspond to quark propagators, crosses mean the action of currents $\eta, \bar{\eta}$, the bubble corresponds to quark interaction with external field.

The substitution of (46) into (43) gives for the first term in the OPE in the r.h.s. of (37) at $p_1^2 = p_2^2 \equiv p^2$

$$\Gamma(p^2) = a \int_0^\infty \frac{s_1^2 ds_1}{(s_1 - p^2)^2} \quad (47)$$

In this simple example the dispersion representation is reduced to one-variable dispersion relation, but with the square of $(s_1 - p^2)$ in the denominator. Of course, by integrating by parts (47) may be transformed to the standard dispersion representation. However, the boundary term arising at such transformation must be accounted; it does not vanish even after application of the Borel transformation. This means that even in this simplest case the representation (43) is not equivalent to one-variable dispersion relation.
Let us represent $\Gamma(p^2, p^2; 0)$ in terms of contributions of hadronic states using (43) and separating the contribution of the lowest hadronic state in the channels with momentum $p^{[40]}$. As is seen from Fig.4, it is convenient to divide the whole integration region in $s_1, s_2$ into three domains: I) $0 < s_1 < W^2, \ 0 < s_2 < W^2$; II) $0 < s_1 < W^2, \ W^2 < s_2 < \infty$; $W^2 < s_1 < \infty, \ 0 < s_2 < W^2$; III) $W^2 < s_1 < \infty, \ W^2 < s_2 < \infty$. Adopt the standard in QCD sum rule model of hadronic spectrum: the lowest hadronic state nucleon plus continuum, starting from some threshold $W^2$. Then in the domain I only the lowest hadronic state $N$ contributes and

$$\rho(s_1, s_2) = a_8 \lambda^2 \delta(s_1 - m^2)\delta(s_2 - m^2)$$

where $m$ is the nucleon mass $\lambda$ is the transition constant of the nucleon in the current $\eta$: $< N \mid \bar{\eta} \mid 0 > = \lambda \bar{\nu}$ where $\nu$ is the nucleon spinor. In the domain III the higher order terms in OPE may be neglected and the contribution of hadronic states is with a good accuracy equal to the contribution of the bare quark loop (like Fig.3) with perturbative corrections. The further application of the Borel transformation in $p^2$ essentially suppresses this contribution.

The consideration of the domain II contribution is the most troublesome and requires an additional hypothesis. Assume, using the duality arguments, that in this domain also, the contribution of hadronic states is approximately equal to the contribution of the bare quark loop. The accuracy of this approximation may be improved by subtraction from each strip of the domain II of the lowest hadronic state contributions proportional to $\delta(s_1 - m^2)$ or $\delta(s_2 - m^2)$. The terms of the latter type also persist in the functions $f(p_1^2), f(p_2^2)$ in (43). They correspond to the process when the current $\bar{\eta}$ produces the nucleon $N$ from the vacuum and under the action of the external current $j$ the transition to excited state $N \rightarrow N^*$ occurs or vice versa (Fig.5).

At $p_1^2 = p_2^2 = p^2$ these contributions have the form

$$\int_{W^2}^{\infty} \frac{b(s)ds}{p^2 - m^2(s - p^2)}$$

Fig.4. The integration domains the in $s_1, s_2$ plane

Fig.5. The schematical representation of $N \rightarrow N^*$ ($N^* \rightarrow N$) transitions in the external field.
with some unknown function \( b(s) \). The term (49) will be accounted separately in the l.h.s. of (43). I stress that the term (49) must be added to the l.h.s. of (43) independently of the form of the bare loop contribution \( \rho(s_1, s_2) \). Even if \( \rho(s_1, s_2) = 0 \), when the OPE for the vertex function \( \Gamma(p^2, p^2, 0) \) with zero momentum transfer starts from condensate terms - the term (43) may persist. (49) may be written as

\[
\int_{W^2}^{\infty} ds b(s) \left( \frac{1}{p^2 - m^2} + \frac{1}{s - p^2} \right) \frac{1}{s - m^2} \tag{50}
\]

The functions \( f(p^2) \) in (43) can be represented by dispersion relation as

\[
f(p^2) = \int_0^{\infty} d(s) \frac{1}{s - p^2} ds \tag{51}
\]

The integration domain (51) may be also divided into parts \( 0 < s < W^2 \) and \( W^2 < s < \infty \). According to our model the contribution of the first part is approximated by \( N \)-state contribution, the second one by continuum. These two parts look like the contributions of the first and the second terms in the bracket in (50).

Now we can formulate the recipe how the sum rule can be written. At the phenomenological side – the l.h.s. of the sum rule – there is a contribution of the lowest hadronic state \( N \) and the unknown term (50), corresponding to nondiagonal transition \( N \to N^* \) in the presence of external field;

\[
\frac{\lambda^2 a_8}{(p^2 - m^2)^2} + \int_{W^2}^{\infty} ds b(s) \frac{1}{s - m^2} \left( \frac{1}{p^2 - m^2} + \frac{\alpha(s)}{s - p^2} \right) \tag{52}
\]

The contribution of continuum corresponding to the bare loop (or also to the higher order terms in OPE, if their discontinuity does not vanish at \( s \to \infty \)) is transferred to the r.h.s. of the sum rule. Here it is cancelled by the bare loop contribution from the same domain of integration. As a result, in the double dispersion representation of the bare loop the domain of integration over \( s_1, s_2 \) is restricted to \( 0 < s_1, s_2 < W^2 \). Finally, apply the Borel transformation in \( p^2 \) to both sides of the sum rule. In the r.h.s. – QCD side – the contribution of the bare loop has the form

\[
\int_0^{W^2} ds_1 \int_0^{W^2} ds_2 \rho(s_1, s_2) \frac{1}{s_1 - s_2} \left[ e^{-s_2/M^2} - e^{-s_1/M^2} \right]
\]

\[
= 2P \int_0^{W^2} ds_2 \int_0^{W^2} ds_1 \frac{\rho(s_1, s_2)}{s_1 - s_2} e^{-s_2/M^2} \tag{53}
\]

where \( P \) means the principal value and the symmetry of \( \rho(s_1, s_2) \) was used. The l.h.s. of the sum rule is equal to

\[
a_8 \frac{\lambda^2}{M^2} e^{-m^2/M^2} - Ae^{-m^2/M^2} + e^{-m^2/M^2} \int_{W^2}^{\infty} ds b(s) \frac{\alpha(s)}{s - m^2} e^{\exp[-(s - m^2)/M^2]} \tag{54}
\]

where

\[
A = \int_{W^2}^{\infty} ds b(s) \frac{1}{s - m^2} \tag{55}
\]

In (54) \( A \) is an unknown constant which can be determined from the same sum rule exploiting the fact that the \( M^2 \) dependence of the second term in (54) differs from the first. Of course, the calculation is reliable only if the contribution of the second term in (54) is smaller, say, less than 30% comparing with the first. The last term in (54) is of the same origin as the second
one—it comes from inelastic transitions $N \to N^*$ (Fig.5). but it is exponentially suppressed in
comparing with the second and as a rule may be neglected.

Omitting the details of the calculation, I present the result—the sum rule for determination
of the coupling constant $a_8$\,[41]

\[
\tilde{\lambda}_N^2 \left( \frac{a_8}{M^2} + A \right) e^{-m^2/M^2} = -M^4 E_2 \left( \frac{W^2}{M^2} \right) L^{-4/9} - \frac{1}{4} b E_0 \left( \frac{W^2}{M^2} \right) L^{-4/9} \\
+ \frac{a^2}{M^2} \left( -\frac{4}{3} + \frac{8}{9} \right) L^{4/9} + \frac{16}{3} \pi^2 f^2 \cdot M^4 E_1 \left( \frac{W^2}{M^2} \right) L^{-4/9} + \frac{28}{9} (2\pi)^2 f^2 m^2 E_0 \left( \frac{W^2}{M^2} \right) L^{-8/9}
\]

(56)

Here

\[
a = -(2\pi)^2 < 0 \| \bar{q}q \| 0 >= 0.55 GEV^3, \ b = (2\pi)^2 < 0 \| \frac{\alpha_s}{\pi} G^2 \| 0 >= 0.5 GEV^4
\]

\[
E_0(x) = 1 - e^{-x}, \ E_1(x) = 1 - (1 + x)e^{-x}, \ E_2(x) = 1 - \left( 1 + x + \frac{x^2}{2} \right) e^{-x}
\]

(57)

\[
L = \ln \left( \frac{M}{\Lambda} \right) / \ln \left( \frac{M}{\Lambda} \right)
\]

(58)

For the nucleon coupling constant $\tilde{\lambda}_N^2 = 32\pi^4 \lambda^2$ and continuum threshold we took the values
found in the mass sum rules\,[50,46], $\tilde{\lambda}_N^2 = 2.1 GEV^6, W^2 = 2.3 GEV^2$. The normalization point $\mu$
was chosen as $\mu = 0.5 GEV$. The sum rule (56) may be compared with the sum rule determining
the nucleon mass\,[50,46]:

\[
\tilde{\lambda}_N^2 \frac{1}{M^2} e^{-m^2/M^2} = M^4 E_2 \left( \frac{W^2}{M^2} \right) L^{-4/9} + \frac{1}{4} < 0 \| \frac{\alpha_s}{\pi} G^2 \| 0 > E_0 \left( \frac{W^2}{M^2} \right) L^{-4/9} + \frac{4}{3} a^2 M^4 L^{4/9}
\]

(59)

Adding (56), (59) and applying to the sum the differential operator

\[
(1 - M^2 \frac{\partial}{\partial M^2}) M^2 e^{-m^2/M^2}
\]

which kills the constant $A$, we get the sum rule for $a_8$:

\[
a_8 = -1 + \frac{8}{9} \frac{1}{\lambda_N^2} \left( 1 - M^2 \frac{\partial}{\partial M^2} \right) e^{m^2/M^2} \left\{ 6\pi^2 f^2 M^4 E_1 \left( \frac{W^2}{M^2} \right) L^{-4/9} + \\
+ 14\pi^2 f^2 m^2 M^2 E_0 \left( \frac{W^2}{M^2} \right) L^{-8/9} + a^2 L^{4/9} \right\}
\]

(60)

The $M^2$ dependence of the r.h.s. of (60) is plotted in Fig.6.
It is seen that the $M^2$ dependence in the interval $0.8 < M^2 < 1.3\text{GeV}^2$, where the nucleon mass sum rule holds, is remarkable. Also the constant $A$ found from (56) is not small enough, $A \simeq -0.3$. For these reasons the error in the determination of $a_s$ is large and the final result is

$$a_s = 0.5^{+0.25}_{-0.15}$$

(61)

I would like to mention that the uncertainty in the value of $a_s$ arising from proportional to $A$ term in the l.h.s. of (56) manifests itself as well in the lattice calculations of these or similar quantities. In the lattice calculations the polarization operator (35) is measured at large euclidean times $t$ and the result is proportionl to

$$\tilde{\lambda}_n^2 e^{-mt} [a_s(1 + mt) + 2Am^2]$$

(62)

(see the second reference in [46] where a similar formula was obtained in the case of nucleon magnetic moments). The last term is usually neglected in lattice calculations. As is seen from (62) and from the values of $a_s$ and $A$ presented above, in order to find $a_s$ with 10% accuracy it is necessary to go to $t > 10m^{-1} \approx 2\text{fm}$, where the whole effect is very small.

I dwell now on the attempt to determine the proton singlet axial coupling constant $a_0 = \Sigma$ by QCD sum rules [51]. The difference in comparison with $g_A$ and $a_s$ determination comes from nonconservation of singlet axial current caused by the anomaly

$$\partial_\mu j^{(0)}_{\mu 5} = \frac{3\alpha_s}{4\pi} G_{\mu\nu}^n \tilde{G}_{\mu\nu}^n + 2im_s \bar{s}\gamma_5 s$$

(63)

In the r.h.s. the strange quark mass term is accounted. whose contribution will be essential. The flavour singlet pseudoscalar meson ($\eta'$) is not a Goldstone and the used above method (eqs.(38)-(41)) of determination of induced by the axial field v.e.v.’s fails. So, a special investigation is needed in order to find the value of quark condensate induced by axial field. Like (39) we can put

$$<0 | \bar{u} \gamma_\mu \gamma_5 u | 0 >_{A= lim_{q^2 \to 0}} \Pi^{(0)}_{\nu\mu}(q) A^{(0)}_{\nu\mu}$$

(64)

where $\Pi^{(0)}_{\nu\mu}$ is defined by eq.39 with substituting $j^{(3)}_{\mu 5} \to (2/3)j^{(0)}_{\mu 5}$. But now $\Pi^{(0)}_{\nu\mu}$ contains both – transverse and longitudinal parts

$$\Pi^{(0)}_{\nu\mu}(q) = -\Pi_L(q^2)\delta_{\nu\mu} - \Pi_t(q^2)(\delta_{\nu\mu}q^2 - q_\nu q_\mu)$$

(65)

$\Pi_L(q^2)$ and $\Pi_t(q^2)$ have no poles at $q^2 = 0$ and the interesting for us quantity is

$$f_0^2 = -\tilde{\Pi}_L(0) = [\Pi_L(0) - \Pi_{L,\text{pert}}(0)]$$

(66)

The perturbative part was subtracted in (66) since it will be accounted in explicit way by perturbative calculation (practically, it is small). The constant $f_0^2$ plays here the same role as $f_\pi^2$ in (38).

In order to separate $\Pi_L(q^2)$ multiply $\Pi^{(0)}_{\nu\mu}$ by $q_\mu q_\nu$. We have

$$q_\mu q_\nu \Pi^{(0)}_{\nu\mu}(q) = -q^2 \Pi_L(q^2) = \frac{\alpha_s}{4\pi} \int d^4x \ e^{iqx} <0 | T \left\{ G_{\mu\nu}^n(x)\tilde{G}_{\mu\nu}^n(x), \right.$$ \begin{align*} &\left. \frac{3\alpha_s}{4\pi} G_{\alpha\beta}^n(0)\tilde{G}_{\alpha\beta}^n(0) + 2im_s \bar{s}\gamma_5 s(x) \right\} | 0 > \end{align*}

(67)
Represent $\Pi^{(0)}_{\mu\nu}(q)$ in terms of contributions of physical states using the dispersion relations. Then it can be easily seen that $\Pi_L(q^2)$ is contributed by pseudoscalars, the axial mesons contribute to $\Pi_t(q^2)$. Using the notations
\[
\langle 0 | \bar{q}\gamma_\nu\gamma_5 q | \eta' \rangle = ig_{\eta'}^\mu q_\nu \quad \langle 0 | j^{(0)}_{\mu5} | \eta' \rangle = if_{\eta'} q_\nu
\]
$q = u, d$ and representing $\Pi_L(q^2)$ in terms of $\eta'$ contribution and continuum, we write
\[
\Pi_L(q^2) = \frac{g_{\eta'}^u f_{\eta'} m_{\eta'}^2}{m_{\eta'}^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\beta(s')}{s' - q^2} ds'
\]
where $s_0$ is the continuum threshold. In (69) $\beta(s)$ is determined by perturbative calculation of the bare loop (Fig.7) corresponding to the r.h.s of (67).

Therefore, $\tilde{\Pi}_L(q^2)$ is given by
\[
\tilde{\Pi}_L(q^2) = \Pi_L(q^2) - \Pi_{L\text{ pert}}(q^2) = \frac{g_{\eta'}^u f_{\eta'} m_{\eta'}^2}{m_{\eta'}^2 - q^2} - \int_{0}^{s_0} \frac{\beta(s')}{s' - q^2} ds'
\]

To find the quantity $\tilde{\Pi}_L(0)$ consider the integral over the contour $C$ in Fig.8 in the complex plane $q^2$:
\[
\frac{1}{2\pi i} \int_{C} \tilde{\Pi}_L(q^2) \frac{dq^2}{q^2} = \tilde{\Pi}_L(0) = -g_{\eta'}^u f_{\eta'} + \frac{1}{\pi} \int_{0}^{s_0} \frac{\beta(s)}{s} ds
\]

The calculation of the diagram Fig.7 gives
\[
\beta(s) = \frac{3}{8\pi^3} \alpha s
\]
To find \( g_{\eta'f}\eta' \) let us write OPE for the r.h.s. of eq.(67) and use the QCD sum rule method. In OPE take into account, besides the bare loop, the contribution of the gluon condensate and of the quark-gluon condensate (Fig.9)

\[
- g < 0 \left| \bar{s}s_\alpha \gamma^\mu G^{\mu}_\alpha s \right| 0 > = m_0^2 < 0 | \bar{s}s | 0 >, 
\]

(73)

\[ m_0^2 = 0.8 GeV^2, \quad [50] \]

stemming from the term, proportional to \( m_s \) in (67).

Fig.9. Gluon (a) and quark-gluon (b) condensate contributions to the r.h.s. of eq.(67).

Let us also account the condensate \( \sim < 0 | g^3 G^3 | 0 > \), taking its estimate from Ref.52.

The sum rule for \( g_{\eta'f}\eta' \) is given by:

\[
g_{\eta'f\eta'} = \frac{1}{m_{\eta'}^2} \left[ e^{m_{\eta'}^2/M^2} \frac{3\alpha_s(M^2)}{16\pi^3} \frac{2\alpha_s(M^2)}{\pi} M^4 E_1 \left( \frac{s_0}{M^2} \right) + \right.
\]

\[
+ \left( 1 + \frac{\epsilon}{M^2} \right) + \frac{4}{3} m_0^2 m_s a_s \frac{1}{M^2} \]

(74)

Here \( \epsilon \approx 0.2 GeV^2 \) \([52]\)

\[
a_s = -(2\pi)^2 < 0 | \bar{s}s | 0 > \approx 0.8a \approx 0.44 GeV^3
\]

(75)

From (74) it numerically follows

\[
g_{\eta'f}\eta' = (4 \pm 1) \cdot 10^{-3} GeV^2
\]

(76)

and

\[
f_0^2 = -\tilde{\Pi}_L(0) = g_{\eta'f}^{u}\eta' - \frac{3\alpha_s}{8\pi^3} s_0 \approx 3.5 \cdot 10^{-3} GeV^2
\]

(77)

in comparison with the similar constant for the octet current \( f_\pi^2 = (0.133 GeV)^2 = 18.10^{-3} GeV^2 \).

Nevertheless that all looks okay in this calculation, the result is wrong. In order to demonstrate this, determine the \( s\)-quark coupling constant \( g_{\eta'f}\eta' \). The only difference with the previous calculation will be that in the r.h.s. of eq.(67) the equal-time commutator, proportional to \( m_s \bar{s}s \) will appear and, as a consequence, owing to the \( s\)-quark condensate an additional term appears in the r.h.s. of the sum rule (74)

\[
- \frac{1}{m_{\eta'}^2} e^{m_{\eta'}^2/M^2} \frac{4}{3} m_s < 0 | \bar{s}s | 0 > \approx 6.10^{-3} GeV^2
\]

(78)

Thus, our calculation results in a wrong ratio of \( \eta' \) interaction constants with \( u \) (or \( d \)) and \( s \)-quarks

\[
\frac{g_{\eta'}^{s}}{g_{\eta'}^{u}} \approx 2.5
\]

(79)
whereas it is known that $\eta'$ is a flavour singlet and this ratio must be close to 1.

We arrive at the conclusion that the sum rule for the longitudinal polarization operator with the singlet axial current does not work at the values of the Borel parameter $M^2$ of order of the $\eta'$ mass square. The only way to avoid this discrepancy is to assume that in the OPE in this case there are some important higher order terms. This conclusion is not surprising: it has been known long ago\cite{53,54} that with the standard OPE it is impossible to describe the Okubo-Zweig-fizuka rule violation in the pseudoscalar and longitudinal axial channels. It is necessary to take into account higher order terms and instantons in the direct channel were proposed as possible candidates for such terms. Now this idea is strongly supported by the calculations in the instanton liquid model\cite{55}, as well as by the lattice calculations\cite{56}.

Since the attempt to calculate the induced by singlet axial vacuum condensate fails, it is impossible to find by QCD sum rules the coupling constant $a_0 = \Sigma$ – the part of the proton spin projection carried by quarks. In fact, the situation is even worse. It can be shown\cite{51} that if the value of $\Sigma$ is taken from experiment, then the sum rule or its determination (like (60) with the induced by the field v.e.v. considered as free parameters) and the sum rule for transverse singlet axial current polarization operator $\Pi_t(q^2)$ are in contradiction. This indicates the OPE breaking down at virtualities $\sim 1 GeV^2$ in the vertex function for proton interaction with singlet axial current or and in the transverse singlet axial current polarization operator. Thus, the situation in the singlet axial channel resembles the one in the pseudoscalar channel and one may expect a noticeable violation of the OZI rule here too. A similar trouble, perhaps, faces attempts to determine $\Sigma$ using the so called "$U(1)$ Goldberger-Treiman relation" (for a review see\cite{57}). So, at this stage, the only way to find $\Sigma$ is from experiment, exploiting eq.12.

3 Twist-4 Corrections to $\Gamma_{p,n}$ from QCD Sum Rules

The general theory of twist 4 corrections to deep inelastic scattering on polarized nucleons has been developed by Shuryak and Vainstein\cite{58}. The have found (see Ref.13,Errata for corrections of errors)

$$\begin{align*}
(\Gamma_p \pm \Gamma_n)_{\text{twist}4} &= -\frac{8}{9} \frac{C^{S,NS}}{Q^2} \left[ \langle\langle U^{S,NS} \rangle\rangle - \frac{m^2}{4} \langle\langle V^{S,NS} \rangle\rangle \right] \\
&\quad + \frac{2}{9} \frac{m^2}{Q^2} \int_0^1 dx \ x^2 g_{1,p \pm n}(x)
\end{align*}$$

(80)

Here indices $S, NS$ correspond to + and - signs in the l.h.s. of (80), $C^S = 5/18, C^{NS} = 1/6$. The reduced matrix elements $\langle\langle U \rangle\rangle, \langle\langle V \rangle\rangle$ are related to the matrix elements of the operators

$$U_\mu = \bar{u} g \tilde{G}_{\mu\nu} \gamma_\nu \frac{1}{2} \lambda^\alpha u$$

(81)

$$V_{\mu\nu,\sigma} = \frac{1}{2} \bar{u} g \tilde{G}^m_{\mu\nu} \gamma_\sigma \frac{1}{2} \lambda^\alpha u + (\nu \rightarrow \sigma)$$

(82)

in the following way

$$\langle N \mid U_\mu \mid N \rangle = S_\mu \langle\langle U \rangle\rangle$$

(83)

$$\langle N \mid V_{\mu\nu,\sigma} \mid N \rangle = S_{\nu,\sigma} A_{\mu,\nu} \ s_\mu \bar{p}_{\nu} \bar{p}_\sigma \langle\langle V \rangle\rangle$$

(84)
where $s_\mu$ is the unit nucleon spin vector, $A_{\mu,\nu}$ and $S_{\nu,\sigma}$ stand for (anti)symmetrization over the given subscripts. The indeces $S, NS$ mean

$$S \rightarrow \bar{u}u + \bar{d}d + \frac{18}{5}\bar{s}s, \quad NS \rightarrow \bar{u}u - \bar{d}d$$

(85)

$\langle\langle U\rangle\rangle, \langle\langle V\rangle\rangle$ were calculated by Balitsky, Braun and Kolesnichenko (BBK) \cite{13} using the QCD sum rule approach. The result for $b_{p-n}$ was given in (10). For $b_{p+n}$ it was obtained

$$b_{p+n} = -0.022 \text{ GeV}^2$$

(86)

This result, however, cannot be considered as reliable for the following reasons:

1. BBK use the same hypothesis as Ellis and Jaffe did, i.e., assume that $s$-quarks do not contribute to the spin structure functions and instead of singlet (in flavour) operator consider the octet one.

2. When determining the induced by external field vacuum condensates, which are very important in such calculations they saturate the corresponding sum rule by $\eta$ meson contribution, what is wrong. (Even the saturation by $\eta'$-meson would not be correct since $\eta'$ is not a Goldstone).

3. One may expect that in the same way as in the calculation of $\Sigma$ by the QCD sum rule, in this problem the OPE series diverge at the scale $\sim 1 \text{ GeV}$ where the BBK calculation proceeds.

Even in the case of the Bjorken sum rule, where the mentioned above problems are absent, the value $b_{p-n}$ obtained by BBK is questionable. Let us consider this calculation in more details. The bare loop diagram for this case is shown in Fig.10.

Fig.10. The bare loop diagram for twist 4 correction to the Bjorken sum rule for deep inelastic electron-nucleon scattering. The dashed line corresponds to discontinuity over $p_1^2$ at $p_1^2 \neq p_2^2$.

In \cite{13} this diagram was calculated by introducing an ultraviolet cut off $\mu^2$. It was found that for the chosen Lorentz structure the singular in $p^2$ term is proportional to $p^4 \ln^2(\mu^2 - p^2)$. Such cut off dependence reflects the fact that the spectral function $\rho(s_1, s_2)$ in eq.(43) is not proportional to $\delta$-function. The logarithm square dependence of $\Gamma(p^2)$ on the cut-off cannot be removed by Borel transformation. For these reasons, in order to obtain physical results the authors of ref. \cite{13} considered various values of $\mu^2$ in the interval $0.1 < \mu^2 < 1 \text{ GeV}^2$ and included uncertainties arising from this procedure into the error. From the presented above point of view such an approach is not legitimate. In this case $\rho(s_1, s_2)$ is proportional to $s_1, s_2$:

$$\rho(s_1, s_2) = bs_1s_2$$

where $b$ is a constant. In the model of hadronic spectrum accepted in Sec.2, we have after the Borel transformation and using eq.(53)
\[
\Gamma(M^2) = 2P \int_0^{W^2} ds_2 \int_0^W ds_1 \frac{d(s_1, s_2)}{s_1 - s_2} e^{s_2/M^2} \\
= 2b \int_0^{W^2} sds e^{-s/M^2} \left[ W^2 + \ln \frac{W^2}{s} \right]
\]  
(87)

Eq.(27) essentially differs from the corresponding expression for the bare loop contribution in ref.\[13\]: e.g., the integrand in (87) is positive, while in \[13\] it is negative in the main region of integration. Of course, the QCD sum rule calculation in this case has a serious drawback: the continuum threshold \(W^2\) dependence of the result is not in the form of a small correction of the type \(\exp(-W^2/M^2) \ll 1\) at \(W^2 \gg M^2\), but much more strong. This is a direct consequence of high (equal to 5) dimension of the operators \(U_\mu, V_{\mu\nu}\). It is clear that the higher is the dimension of the considered operator, the stronger will be dependence on the continuum threshold and less certain the results of the QCD sum rules calculations. It must be emphasized that for operators of high dimensions the loop diagrams are in principle nonrenormalizable, the role of excited states in the physical spectrum increases and the determination of the lowest state contribution becomes impossible.

Eq.(87) must be taken instead of the contribution of the bare loop diagram, used in \[13\]. A similar procedure must be also applied in the case of other terms in the sum rules \[13\] containing an ultraviolet cut-off.

BBK accounted for operators up to dimension 8. With the corrections described above, the sum rules found by BBK have the form

\[
\langle\langle U^{NS}\rangle\rangle + A_U^{NS} M^2 = -\frac{1}{2\lambda^2_N} e^{m^2/M^2} \left\{ \frac{8}{9} M^2 \frac{\alpha_s}{\pi} \int_0^{W^2} sds e^{-s/M^2} (W^2 + s\ln\frac{W^2}{s}) \right\} \\
-\frac{1}{9} b M^4 E_1 \left( \frac{W^2}{M^2} \right) + \frac{32}{27} \frac{\alpha_s}{\pi} M^2 a^2 \ln \frac{W^2}{M^2} + \frac{8}{9} \pi^2 \Pi M^2 - \frac{2}{3} m_0^2 a^2 \right\}
\]
\[
\langle\langle V^{NS}\rangle\rangle + A_V^{NS} M^2 = -\frac{1}{2\lambda^2_N} e^{m^2/M^2} \left\{ -\frac{52}{135} M^2 \frac{\alpha_s}{\pi} \int_0^{W^2} sds e^{-s/M^2} (W^2 + s\ln\frac{W^2}{s}) \right\} \\
-\frac{1}{9} b M^4 E_1 \left( \frac{W^2}{M^2} \right) - \frac{80}{27} \frac{\alpha_s}{\pi} M^2 a^2 \left( \ln \frac{W^2}{M^2} + 0.9 \right) + \frac{8}{27} \pi^2 R M^2 - \frac{4}{9} m_0^2 a^2 \right\}
\]  
(88)
(89)

\(\langle\langle U^{NS}\rangle\rangle \approx 0.14 \text{ GeV}^2\)
\(\langle\langle V^{NS}\rangle\rangle \approx 0.254 \text{ GeV}^2\)

The substitution of (90), (91) into (80) gives the value of twist 4 correction to the Bjorken sum rule presented above in (10). The examination of the sum rules (88), (89) shows, however, that the final result (90) comes almost entirely from the contribution of the last term in OPE in (88) – the operator of dimension 8. Recently, A.Oganesian \[59\] had calculated the contribution of factorizable v.e.v.’s of the operators of dimension 10, \(<0 | G^2_{\mu\nu} | 0 > <\bar{q}q | 0 >^2\), \(g^2 <0 | \bar{q} \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a q | 0 >^2\) to the sum rule (88) and had found that by absolute value it is equal to (90) but has opposite sign. Of course, we cannot believe in this statement either: it means only that the results of the calculations are unstable and the value (10) characterizes the answer by the order of magnitude only.
Gerasimov, Drell-Hearn (GDH) Sum Rules. The Estimate of Higher Twist Corrections to $\Gamma_{p,n}$ Using Interpolation between $\Gamma_{p,n}$ and GDH Sum Rules.

The real photon-nucleon forward scattering amplitude with nucleon spin flip is expressed through one structure function. In lab. system we can write

$$e_k^{(2)}(T_{ik})_{\text{spin flip}} e_k^{(1)} = i \frac{\nu}{m^2} \epsilon_{ikl} e_k^{(2)} \epsilon_l^{(1)} s_1 S_1(\nu, 0)$$

where $e^{(1)}$ and $e^{(2)}$ are polarizations of initial and final photons. At high energies, $\nu \to \infty$ according to Regge theory the behaviour of $S_1(\nu, 0)$ is determined by the exchange of the $a_1$-Regge pole: $S_1(\nu, 0) \sim \nu^{-\alpha_{a_1}(0)-1}$

Since $\alpha_{a_1}(0) \approx -0.3 - 0.0$ the unsubtracted dispersion relation can be written for $S_1(\nu, 0)$

$$S_1(\nu) = 4 \int_0^\infty d\nu' \frac{G_1(\nu', 0)}{\nu'^2 - \nu^2}$$

where $G_1(\nu, Q^2)$ is the spin-dependent structure function. Consider the limit $\nu \to 0$ in (94). According to the F.Low theorem the terms proportional to $\nu^0$ and $\nu^1$ in the expansion in powers of $\nu$ of the photon-nucleon scattering amplitude at small $\nu$ are expressed via static characteristics of nucleon, its charge and anomalous magnetic moment.

The calculation gives

$$S_1(\nu)_{\nu \to 0} = -\kappa^2,$$

where $\kappa$ is the nucleon anomalous magnetic moment: $\kappa_p = 1.79, \kappa_n = -1.91$.

From (94), (95) the GDH sum rule follows:

$$\int_0^\infty \frac{d\nu}{\nu} G_1(\nu, 0) = -\frac{1}{4}\kappa^2$$

Till now no direct check of GDH was done. Only indirect check of (96) was performed, where in the l.h.s of (96) the parameters of resonances, obtained from the $\pi N$ scattering phase analysis, where substituted. In this way with resonances up to 1.8 GeV it was obtained $^{[15,60]}$ (cf.also $^{[61]}$)

| l.h.s of (96) | r.h.s. of (96) |
|--------------|----------------|
| proton       | -1.03          |
| neutron      | -0.83          |

| p        | 1.03 | 0.803 |
| n        | -0.83 | -0.913 |

The l.h.s. and the r.h.s. of (96) are not in a good agreement – a nonresonant contribution is needed. The direct check of the GDH sum rule would be very desirable!

An important remark: the forward spin dependent photon–nucleon scattering amplitude has no nucleon pole. This means that there is no nucleon contribution in the l.h.s. of GDH sum rule – all contributions come from excited states: the GDH sum rule is very nontrivial.

In order to connect the GDH sum rule with $\Gamma_{p,n}(Q^2)$ consider the integrals $^{[17]}$

$$I_{p,n}(Q^2) = \int_{Q^2/2}^{\infty} \frac{d\nu}{\nu} G_{1,p,n}(\nu, Q^2)$$

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It is easy to see that at large $Q^2$

$$I_{p,n}(Q^2) = \frac{2m^2}{Q^2} \Gamma_{p,n}(Q^2)$$  \hspace{1cm} (99)

and at $Q^2 = 0$ (98) reduces to the GDH sum rule. The $Q^2$ dependence of $I_{p,n}(Q^2)$ is plotted in Fig.11.

![Fig.11. The connection of the GDH sum rules with the sum rules at high $Q^2$ - qualitative $Q^2$ dependence of $I_{p,n}$ and $I_p - I_n$.](image)

The case of $I_p$ is especially interesting: $I_p$ is negative at $Q^2 = 0$ and positive at large $Q^2$, what indicates to large nonperturbative corrections. In [14] the VDM based interpolation model was suggested, describing $I_{p,n}(Q^2)$ in the whole domain of $Q^2$. The model was improved in [15], where the contributions of baryonic resonances up to $W = 1.8\text{GeV}$, taken from experiment, where accounted. The model has no free parameters, besides the vector meson mass, for which the value $\mu_V^2 = 0.6\text{GeV}^2$ was chosen. Using this model it is possible to calculate the higher twist contributions in (2),(12). The results are presented in Table 1, as the ratio of asymptotic $\Gamma^{as}$ with power corrections excluded to the experimentally measurable $\Gamma$ at given $Q^2$ [39].

**Table 1.**

Higher twist corrections in GDH sum rule + VDM inspired model.

| $Q^2(\text{GeV}^2)$ | 2     | 3     | 5     | 10    |
|---------------------|-------|-------|-------|-------|
| $\Gamma_{p}^{as}/\Gamma_p$ | 1.44  | 1.29  | 1.18  | 1.08  |
| $\Gamma_{n}^{as}/\Gamma_n$  | 1.30  | 1.20  | 1.13  | 1.06  |
| $\Gamma_{p-n}^{as}/\Gamma_{p-n}$ | 1.45  | 1.29  | 1.18  | 1.08  |
| $\Gamma_{p+n}^{as}/\Gamma_{p+n}$ | 1.47  | 1.31  | 1.19  | 1.08  |

The power corrections, given in Table 1 are essentially larger (except for the case of neutron), than the values (10),(86) found in [13]. It must be mentioned that the accuracy of the model in the domain of intermediate $Q^2$, where it is exploited, is not completely certain. So, I will consider in what follows the values of the power corrections presented in Table I as a limiting case of large higher twist corrections.
5 Comparison with Experiment.

When comparing the sum rules (2),(12) with experiment I consider two limiting variants of perturbative corrections: small with $\Lambda_3 = 200\text{MeV}$ and large with $\Lambda_3 = 400\text{MeV}$. ($\alpha_s$ is computed in 2-loop approximation, it is assumed that the number of flavours $N_f = 3$). For higher twist correction I also consider two limiting options: small (S), given by (10),(86) and large (L), determined by the data of Table 1. The contribution of gluons $\Delta g(Q^2)$ in (12) will be found in the following way. Let us assume, that at 1 GeV the quark model is valid and $L_\perp(1\text{GeV}^2) = 0$ in eq. (18). Taking $\Sigma = 0.3$, what is a reasonable average of the data, we have from (18)

$$\Delta g(1\text{GeV}^2) = 0.35$$

(100) 

The $Q^2$ dependence of $\Delta g$ can be found from the evolution equation [62]

$$\Delta g(Q^2) = \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \left\{ 1 + \frac{2N_f}{b\pi} \left[ \alpha_s(Q^2) - \alpha_s(\mu^2) \right] \right\} \Delta g(\mu^2) +$$

$$+ \frac{4}{b} \left[ \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} - 1 \right] \Sigma(\mu^2),$$

(101)

where $b = 11 - (2/3)N_f = 9$, $\mu^2 = 1\text{GeV}^2$ and $\Sigma(1\text{GeV}^2) \approx 0.3$. As the calculation shows, the change of scale at which the quark model is assumed to work (say $0.5\text{GeV}^2$ instead of $1\text{GeV}^2$) or the of use slightly different $\Sigma(\mu^2)$ in (101) only weakly influence the results for $\Sigma$ and $\Delta s$, obtained from experimental data.

I consider the following experimental data (Table 2).

Table 2.
The experimental data on $\Gamma_{p,n}$.

| Experimental group | The target | $\Gamma(p, n)$ | Mean $Q^2$(GeV$^2$) |
|-------------------|------------|----------------|---------------------|
| EMC [63]          | $p$        | $\Gamma_p = 0.126 \pm 0.010 \pm 0.015$ | 10.7 |
| SMC [64]          | $p$        | $\Gamma_p = 0.136 \pm 0.011 \pm 0.011$ | 10.5 |
| E143 [65]         | $p$        | $\Gamma_p = 0.127 \pm 0.004 \pm 0.010$ | 3 |
| E142 [66]         | $H e^3$    | $\Gamma_n = -0.022 \pm 0.011$ | 2 |
| SMC [68]          | $d$        | $\Gamma_d = 0.034 \pm 0.009 \pm 0.06$ | 10 |
|                   |            | $\Gamma_p + \Gamma_n = 0.073 \pm 0.022$ | 10 |
| E143 [67]         | $d$        | $\Gamma_d = 0.042 \pm 0.003 \pm 0.004$ | 3 |
|                   |            | $\Gamma_p + \Gamma_n = 0.0908 \pm 0.006 \pm 0.008$ | 3 |

The last column of Table 2 gives the average $Q^2$ in each experiment. In comparison with experiment the perturbative and higher twist corrections, as well as $\Delta g(Q^2)$ contributions are calculated for these $Q^2$. Experimentally, $Q^2$ are different in different $x$-bins (higher $Q^2$ at larger $x$). This effect is not accounted in the calculation. The ratio of $\alpha_s^3$ term to $\alpha_s^2$ term in perturbative corrections is of order of 1 at $\Lambda_3 = 400\text{MeV}$ and $Q^2 \approx 2 - 3\text{GeV}^2$ (as well as $\alpha_s^4/\alpha_s^3$ estimate). For this reason we introduce in these cases an additional error equal to the $\alpha_s^3$. The values of $\Sigma$ and $\Delta s$ calculated from comparison of experimental data with eq.12 are shown in Table 3.(The errors are summed in quadrature).
Table 3.
Determination of $\Sigma$ and $\Delta s$ from experimental data

| Experiment | $\Lambda_3$ MeV | High twist | $\Sigma$   | $\Delta s$   |
|------------|-----------------|------------|------------|--------------|
| EMC        | 200             | S          | 0.21 ± 0.17| −0.13 ± 0.06|
|            | 400             | S          | 0.29 ± 0.17| −0.10 ± 0.06|
|            | 200             | L          | 0.285 ± 0.17| −0.10 ± 0.06|
|            | 400             | L          | 0.37 ± 0.17| −0.07 ± 0.06|
| SMC        | 200             | S          | 0.30 ± 0.14| −0.10 ± 0.05|
| p          | 400             | S          | 0.39 ± 0.14| −0.07 ± 0.05|
|            | 200             | L          | 0.39 ± 0.14| −0.07 ± 0.05|
|            | 400             | L          | 0.47 ± 0.14| −0.04 ± 0.05|
| E143       | 200             | S          | 0.28 ± 0.10| −0.10 ± 0.03|
| p          | 400             | S          | 0.42 ± 0.10| −0.06 ± 0.03|
|            | 200             | L          | 0.57 ± 0.10| −0.006 ± 0.03|
|            | 400             | L          | 0.71 ± 0.10| 0.04 ± 0.03|
| E142       | 200             | S          | 0.60 ± 0.12| 0.003 ± 0.04|
| n          | 400             | S          | 0.57 ± 0.12| −0.005 ± 0.04|
|            | 200             | L          | 0.64 ± 0.12| 0.016 ± 0.04|
|            | 400             | L          | 0.61 ± 0.12| 0.008 ± 0.04|
| SMC        | 200             | S          | 0.27 ± 0.10| −0.11 ± 0.03|
| d          | 400             | S          | 0.33 ± 0.10| −0.09 ± 0.03|
|            | 200             | L          | 0.29 ± 0.10| −0.10 ± 0.03|
|            | 400             | L          | 0.34 ± 0.10| −0.08 ± 0.03|
| E143       | 200             | S          | 0.37 ± 0.06| −0.07 ± 0.02|
| d          | 400             | S          | 0.44 ± 0.06| −0.05 ± 0.02|
|            | 200             | L          | 0.48 ± 0.06| −0.04 ± 0.02|
|            | 400             | L          | 0.54 ± 0.06| −0.015 ± 0.02|

Remark: the contribution to $\Sigma$ of the term proportional to $\Delta g$ is approximately equal to 0.06 in the case of $\Lambda_3 = 200\,MeV$ and 0.11 in the case of $\Lambda_3 = 400\,MeV$.

If we assume that all the analysed above experiments are correct in the limits of their quoted errors (or, may be, 1.5 st.deviations), then requiring for the results for $\Sigma$ and $\Delta s$ from various experiments to be consistent, we may reject some theoretical possibilities. A look at the Table 3 shows that the variant $\Lambda_3 = 400\,MeV, L$ (a contradiction of E143, p and SMC, d results for $\Sigma$) and, less certain, the variant $\Lambda_3 = 200\,MeV, S$ (a contradiction of E142, n and SMC, d) may be rejected.

Consider now the Bjorken sum rule. For comparison with theory I choose combinations of the SMC data - on proton and deuteron, the E143 data - on proton and deuteron and the E143 data on proton and the E142 on neutron ($^3He$). The results of the comparison of the experimental data with the theory are given in Table 4.
Table 4.
Comparison of the experimental data with the Bjorken sum rule

| Combination of experiments | $(\Gamma_p - \Gamma_n)_{exper.}$ | $\Lambda_3 (MeV)$ | High twist | $(\Gamma_p - \Gamma_n)_{th}$ |
|---------------------------|-------------------------------|------------------|------------|----------------------------|
| SMC, p                    | 0.199 ± 0.038                 | 200 S            | 0.193      |
| SMC, d                    |                               | 400 S            | 0.186      |
|                           |                               | 200 L            | 0.180      |
|                           |                               | 400 L            | 0.173      |
| E143,p                    | 0.163 ± 0.010 ± 0.016         | 200 S            | 0.182      |
| E143,d                    |                               | 400 S            | 0.168      |
|                           |                               | 200 L            | 0.145      |
|                           |                               | 400 L            | 0.134      |
| E143,p recalculated to $\bar{Q}^2 = 3GeV^2$ | 0.147 ± 0.015 | 200 S | 0.182 |
|                           |                               | 400 S            | 0.168      |
|                           |                               | 200 L            | 0.145      |
|                           |                               | 400 L            | 0.134      |

From Table 4 we see again some indications for rejection of variants $\Lambda_3 = 400 MeV$, L and, less certain, $\Lambda_3 = 200 MeV$, S. (In the first case there is a contradiction of the theory with the E143,p and d data, in the second - with the E143,p, E142, n data).

At existing experimental accuracy it is impossible to choose from the data the true values of $\Lambda_3$ and twist-4 correction. My personal preference is to the variant $\Lambda_3 = 200 MeV$ and to the value of twist-4 corrections 3 times smaller than given by the GDH sum rule + VDM inspired model and, correspondingly, $b_p = 0.04$ in (12), i.e., 2.2 times larger than the BBK result. The argument in the favour of such choice is that at larger $\Lambda_3$ there will arise many contradictions with the description of hadronic properties in the framework of the QCD sum rules. The recent SLAC data [69] on $g_1(x, Q^2)$ $Q^2$-dependence indicate that $\Gamma_p^s/\Gamma_p = 1 + c_p/Q^2, c_p = 0.25 \pm 0.15$ what is compatible with the estimate above. In this case all experimental data except for E142,n, are in a good agreement with one another and the values of $\Sigma$ and $\Delta s$ averaged over all experiments, except for E142,n are

$$\Sigma = 0.35 \pm 0.05 \quad \Delta s = -0.08 \pm 0.02$$ (102)

(see also [70] where the values close to (102) were obtained).

The values of $\Sigma$ and $\Delta s$ obtained from the E142,n experiment at such a choice of $\Lambda_3$ and twist-4 corrections, are different:

$$\Sigma = 0.61 \pm 0.12 \quad \Delta s = 0.01 \pm 0.04$$ (103)

It is impossible to compete (102),(103) by any choice of $\Lambda_3$ and higher twist correction. Perhaps, this difference is caused by inaccounted systematic errors in the E142 experiment.

A remarkable feature of the result (102) (as well as of the data in Table 3) is the large value of $|\Delta s|$ - the part of the proton spin projection carried by strange quarks. This value may be compared with the part of the proton momentum carried by strange quarks

$$V_2^s = \int dx \ x [s_+(x) + s_-(x)] = 0.026 \pm 0.006 [71], \quad 0.040 \pm 0.005 [72]$$ (104)
The much larger value of $|\Delta s|$ in comparison with $V_s^2$ contradicts the standard parametrization

$$s_+(x) + s_-(x) = A x^{-\alpha} (1 - x)^\beta$$
$$s_+(x) - s_-(x) = B x^{-\gamma} (1 - x)^\beta$$

(105)

and requirement of positiveness of $s_+$ and $s_-$, if $\alpha \approx 1$ (pomeron intercept) and $\gamma \leq 0$ ($a_1$ intercept). Large $|\Delta s|$ and small $V_s^2$ means that the transitions $\bar{s}s \rightarrow \bar{u}u + \bar{d}d$ are allowed in the case of the operator $j_{\mu 5}$ and are suppressed in the case of the quark energy-momentum tensor operator $\Theta_{\mu\nu}$, corresponding to the matrix element $V_2$. Such situation can be due to nonperturbative effects and the instanton mechanism for its explanation was suggested [73]. It must be mentioned that in the suggested by Brodsky [74] more refined parametrization, which takes into account the fact that at $x \rightarrow 1$ $q_-(x)/q_+(x) \sim (1 - x)^2$, the contradiction weakens. Improvement of experimental accuracy is necessary in order to be sure that the inequality $|\Delta s| \gg V_s^2$ indeed takes place.

6 The calculations of the polarized structure functions by QCD sum rule method

I recall the basing points of the calculation of nucleon structure functions in the QCD sum rule approach (for details see [75]). Consider four-point correlator

$$T_{\mu\nu}^\pm(p,q) = -i \int d^4x d^4y d^4z \ e^{iqx} e^{ip(y-z)} \times$$

$$\times < 0 | T \left\{ \eta(y), j_\mu^+(x), j_\nu^+(0), \bar{\eta}(z) \right\} | 0 >$$

(106)

where

$$j_\mu^+ = \bar{u} \gamma_\mu(1 + \gamma_5)d, \quad j_\mu^- = \bar{d} \gamma_\mu(1 + \gamma_5)u$$

and $\eta$ is given by (36). (In order to separate $u$- and $d$ quark distributions it is convenient to consider $W^\pm$ proton scattering). Suppose that $q^2 < 0, p^2 < 0, q^2 = -Q^2$. $Q^2$ is large enough, $Q^2 \sim 10 \text{GeV}^2, Q^2 >> |p^2|$ and retain only the leading terms in the expansion over $p^2/Q^2$. (This corresponds to account of only twist 2 contributions). Assume also that $|p^2| >> R_c^{-2}$, where $R_c$ is the confinement radius and perform OPE in $1/p^2$ for the discontinuity of $T_{\mu\nu}$ in the $s$-channel

$$\text{Im} T_{\mu\nu}^\pm = \frac{1}{2i} \left[ T_{\mu\nu}^\pm(p^2, q^2, s + i\epsilon) - T_{\mu\nu}^\pm(p^2, q^2, s - i\epsilon) \right]$$

(107)

At the first sight it seems, that OPE is not legitimate here because $T_{\mu\nu}$ is the forward scattering amplitude in which large distances in the $t$-channel are of importance. This, indeed, would be the case if we would consider the exclusive object $\text{Re} T_{\mu\nu}$. But for $\text{Im} T_{\mu\nu}$ which in fact is an inclusive observable, the situation is completely different. Consider first the bare loop diagram for $\text{Im} T_{\mu\nu}$ (the contribution of dimension 0 operator in OPE) – Fig.12.
The direct calculation of the diagram gives that the virtuality of the active quark on which the scattering proceeds is equal to \( x = Q^2 / 2\nu, \nu = p q \) 

\[
k^2 = p^2 x - \frac{k_\perp^2}{1 - x}
\]  

(108) 

and \( k_\perp^2 \) is of order \( k_\perp^2 \sim |p^2| x(1 - x) \). Therefore, \( k^2 \sim p^2 x \). (Strictly speaking, this statement refers only to terms in the amplitude, singular in \( p^2 \), but just these terms are of interest for us since the Borel transformation in \( p^2 \) killing the regular in \( p^2 \) terms will be performed later). Since it was assumed that \( |p^2| >> R^{-2} \) we come to a conclusion that at not small \( x \), \( |k^2| \) and \( k_\perp^2 \) are large and OPE is legitimate. 

A general proof of the above mentioned statement follows from the fact that at large \( |p^2|, |q^2| \) the nearest to zero singularity in \( t \) of the function \( \text{Im} \ T(p^2, q^2, s, t) \) is determined by the boundary of the Mandelstam spectral function which is found to be 

\[
t = 4 \frac{p^2 q^2}{s} = -4 \frac{x}{1 - x} p^2
\]  

(109) 

So, at \( t = 0 \), large \( |p^2| \) and not small \( x \), we are far from the boundary of the spectral function in \( t \)-channel and OPE is valid.

The statement that the method does not work at small \( x \) is evident beforehand: this is the Regge domain, where OPE cannot work.

The estimate of active quark virtualities (108) have much more general meaning, beyond QCD sum rule approach. In lepton-hadron scattering \( |p^2| \) is of order of QCD scale, \( |p^2| \sim 1 \text{ GeV}^2 \). Then at small \( x \) \( |k^2| \) are small and we are in the nonperturbative domain of QCD. This means that the interaction of quarks with nonperturbative vacuum fields, especially gluonic fields, are important. The direct calculation in QCD sum rule approach supports this expectation.

The method in view is also invalid at \( x \) close to 1, \( 1 - x << 1 \). This is evident, because it is a resonance region. Finally, we restrict ourselves to intermediate \( Q^2 \sim 5 - 10 \text{ GeV}^2 \), since the evolution of the structure functions will not be accounted.

The calculation proceeds in the standard way of the QCD sum rule method. In the QCD side of the sum rule \( \text{Im} \ T_{\mu\nu}^\pm \) is calculated by OPE with the account of v.e.v.of various operators. For the nonpolarized case the gluonic condensate and the term \( \sim \alpha_s < 0 | \bar{q} q | 0 \sim 2 \) were accounted \([75]\). For polarized structure functions \( g_1 \) and \( g_2 \), besides the bare loop, only the contribution of the term \( \sim \alpha_s < 0 | \bar{q} q | 0 >^2 \) was calculated \([76]\), since it is expected that at intermediate \( x \), where the results are correct, the contribution of gluonic condensate is small. The v.e.v.< \( 0 | \bar{q} q | 0 >^2 \) do not contribute to the sum rule, because it is concentrated at \( x = 1 \) (proportional to \( \delta(1 - x) \) in twist 2 terms) – outside of the applicability domain of the method. (Only chirality conserving structure were considered in \([75,76]\)). The hadronic side of the sum rule is represented schematically by Fig.13.

Fig.12 The bare loop diagram for \( \text{Im} \ T_{\mu\nu} \). The crosses mean on mass shell propagators.
The Borel transformation is applied to both sides of the sum rule. The continuum contribution is suppressed by the Borel transformation, it is approximated by the bare loop (Fig.12) and transferred to the QCD side. The background term – Fig.13b – in its dependence on the Borel parameter $M^2$ differs from the nucleon term by an additional factor $M^2$. This circumstance permits one to kill this term in the same way, as it was killed the constant $A$ in (56).

In the case of polarized structure functions $g_1$ and $g_2$ it was shown [76] that for the bare quark loop the Bjorken and Burkhardt-Cottingham [77] sum rules are fulfilled. It was found that for the function $g_1(x)$ the results are reliable in a rather narrow domain of intermediate $x$: $0.5 \leq x \leq 0.7$. In this domain the contribution of $u$-quarks $g_1^u$ is much larger than $d$-quarks, $g_1^u \gg g_1^d$. Therefore, $g_1 \approx (4/9) g_1^u$. $g_1^u$ was calculated and for the mean value of $g_1$ in this interval it was obtained (at $Q^2 \sim 5 - 10 GeV^2$):

$$\bar{g}_1(0.5 < x < 0.7) = 0.05 \pm 50\%$$ (110)

The large uncertainty in (110) results from the large contribution of nonleading term in OPE and from large background at the phenomenological side of the QCD sum rule. The E143 proton [65] and deuteron [68] data (the latter under assumption that $g_1^u$ is small in this interval of $x$) give roughly the same values:

$$\bar{g}_1(0.5 \leq x \leq 0.7) = 0.08 \pm 0.02$$ (111)

This value is in a good agreement with the SMC result [64]

$$\bar{g}_1(0.4 < x < 0.7) = 0.08 \pm 0.02 \pm 0.01$$ (112)

and with theoretical expectation (110).

For the case of the structure function $g_2$ only $g_2^u$ can be calculated at $0.5 < x < 0.8$, the calculation of $g_2^d$ fails because of large contribution of nonleading terms in OPE. If we assume that like in the case of $g_1$, $|g_2^d| \ll |g_2^u|$, then [76]

$$g_2(0.5 < x < 0.8) = -0.05 \pm 50\%$$ (113)

The E143 data [78] in this interval of $x$ are:

$$g_2(0.5 < x < 0.8) = -0.037 \pm 0.020 \pm 0.003$$ (114)

in a good agreement with (113).

The serious disadvantages of the QCD sum rules calculations of the structure functions are:

1) large contribution of nonleading terms in OPE;
2) large background terms from inelastic transitions $N \to N^*$ (Fig.13b) at the physical side of the sum rules. In order to kill these terms we are forced to differentiate the sum rules over the Borel parameter $M^2$. This operation increases the role of nonleading terms in OPE and continuum and deteriorates the accuracy of the sum rule. A possible way to overcome this drawback of the method is to start from nonforward scattering amplitudes $q_1 \neq q_2, p_1^2 \neq p_2^2$ and to use the double Borel transformation in $p_1^2$ and $p_2^2$. The calculations in such approach will be much more complicated, but, may be the game is worth of candles.

7 Calculation of Chirality Violating Structure Function $h_1(x)$ by QCD Sum Rules

As is well known, all structure functions of the twist two - $F_1(x), F_2(x), g_1(x)$, which are measured in the deep-inelastic lepton-nucleon scattering, conserve chirality. Ralston and Soper 79 first demonstrated that besides these structure functions, there exists the twist-two chirality violating nucleon structure function $h_1(x)$. This structure function does not manifest itself in the deep inelastic lepton-hadron scattering, but can be measured in the Drell-Yan process with both beam and target transversally polarized. The reason of this circumstance is the following. The cross section of the deep inelastic electron(muon)-hadron scattering is proportional to the imaginary part of the forward virtual photon-hadron scattering amplitude.

At high photon virtuality the quark Compton amplitude dominates, where the photon is absorbed and emitted by the same quark (Fig.14a) and the conservation of chirality is evident. The cross section of the Drell-Yan process can be represented as an imaginary part of the diagram, Fig.14b. Here virtual photons interact with different quarks and it is possible, as is shown in Fig.14b, that chirality violating amplitude in Drell-Yan processes is not suppressed at high $Q^2$ in comparison with chirality conserving ones, and consequently, corresponds to twist two. This amplitude, corresponding to target spin flip, has no parton interpretation in terms of quark distributions in the helicity basis and, as was shown by Jaffe and Ji [80] can be only represented as an element of the quark-quark density matrix in this basis.

However, $h_1(x)$ can be interpreted 80 as a difference of quark densities with the eigenvalues $+1/2$ and $-1/2$ of the transverse Pauli-Lubanski spin operator $\hat{s}_\perp \gamma_5$ in the transversely polarized proton. It this basis $h_1(x)$ can be described in terms of standard parton language.

The proton structure function $h_1(x)$ can be defined in the light cone formalism as follows 80

$$i \int \frac{d^4x}{(2\pi)^4} e^{ixs} \left< p, s \left| \bar{\psi}(0) \sigma_{\mu\nu} \gamma_5 \psi(\lambda n) \right| p, s > = 2[h_1(x, Q^2)(s_{\perp \mu} p_\nu - s_{\perp \nu} p_\mu) + h_L(x, Q^2) m^2 (p_\mu n_\nu - p_\nu n_\mu) (sn) + h_3(x, Q^2) m^2 (s_{\perp \mu} n_\nu - s_{\perp \nu} n_\mu)] \right>.$$  \hspace{1cm} (115)

Here $n$ is a light cone vector of dimension (mass)$^{-1}$, $n^2 = 0, n^+ = 0, p n = 1, p$ and $s$ are the proton momentum and spin vectors, $p^2 = m^2, s^2 = -1, p s = 0$ and $s = (sn)p + (sp)n + s_\perp$, $h_L(x, Q^2)$ and
$h_3(x, Q^2)$ are twist-3 and 4 structure functions. For comparison in the same light cone notation the standard structure function $F_1(x, Q^2)$ is given by

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} < p, s | \bar{\psi}(0)\gamma_\mu\psi(\lambda n) | p, s > = 4[F_1(x, Q^2)p_\mu + M^2 f_4(x, Q^2)n_\mu]$$ \hspace{1cm} (116)

(Eqs. (115), (116) are written for one flavour).

Basing on the definitions (115),(116) an inequality was proved in \cite{80}

$$q(x) \geq h_q^1(x) ,$$ \hspace{1cm} (117)

which holds for each flavour $q = u, d, s$. (Here $h_q^1(x)$ is the flavour $q$ contribution to $h_1(x).$)

Recently, Soffer \cite{81} (see also \cite{82}) derived an inequality

$$| h_u^1 | < [u(x) + g_u^1(x)]/2$$ \hspace{1cm} (118)

$h_1(x)$ can be also represented through a $T$-product of currents \cite{83}

$$T_\mu(p, q, s) = i \int d^4xe^{iqx} < p, s | (1/2)T\{j_\mu\bar{\delta}(x), j(0) + j(x), j_\mu\bar{\delta}(0)\} | p, s > ,$$ \hspace{1cm} (119)

where $j_\mu\bar{\delta}(x)$ and $j(x)$ are axial and scalar currents.

The general form of $T_\mu(p, q, s)$ is

$$T_\mu(p, q, s) = \left( s_\mu - \frac{qs}{q^2}\bar{q}_\mu \right) \tilde{h}_1(x, Q^2) + \left( p_\mu - \frac{\nu q_\mu}{q^2} \right) (qs)l_1(x, Q^2) +$$

$$+ \varepsilon_{\mu\nu\lambda\sigma} p_\nu q_\lambda s_\sigma (qs)l_2(x, q^2)$$ \hspace{1cm} (120)

(only spin–dependent terms are retained). It can be proved \cite{83}, that

$$h_1(x, Q^2) = -\frac{1}{\pi} Im\tilde{h}_1(x, Q^2) .$$ \hspace{1cm} (121)

As is clear from (115) or (119) $h_1(x)$ indeed violates chirality.

Since $h_1(x)$ violates chirality one may expect that in QCD it can be expressed in terms of chirality violating fundamental parameters of the theory, the simplest of which (of the lowest dimension) is the quark condensate. Basing on this idea $h_1(x)$ calculation was performed \cite{83}. The main difference in comparison with the structure functions considered in Sec.6 is that in the case of $h_1(x)$ determination the chirality violating structure is studied. As a result, $h_1(x)$ was found to be proportional to the quark condensate $< 0 | \bar{u}u | 0 >$ with the correction term in OPE proportional to the mixed quark-gluon condensate $g < 0 | \bar{u}\sigma_{\mu\nu} G_{\mu\nu}^{\ast} \lambda^a u | 0 >$. It was obtained that for proton $h_{1}^u(x) \gg h_{1}^d(x)$ and as a consequence $h_1(x) \approx (4/9)h_{1}^u(x)$. The calculations in \cite{83} are valid at $0.3 < x < 0.6$. The extrapolation in the region of small $x$ can be performed using the Regge behaviour $h_1(x) \sim x^{-\alpha_{1}}$, the extrapolation in the region of large $x$, using the inequalities (117),(118). The final result of the calculation of $h_{1}^u(x)$ with the extrapolation is shown in Fig.15.
Fig.15 The $u$-quark contribution to the proton structure function $h_1(x)$ based on the QCD sum rule calculation \cite{83} at intermediate $x$, $0.3 < x < 0.6$. At $x < 0.3$ an extrapolation according to Regge behaviour was performed. The dashed line represents the Soffer inequality (118).

It is expected that the accuracy of $h_1^u(x)$ determination is about 30% at $x = 0.4$ and about 50% at $x = 0.6$. The inequality $h_1^u(x) > g_1^u(x)$ suggested in \cite{80} was confirmed. Numerically, $h_1(x)$ is rather large, that gives a good chance for its experimental study.

8 Conclusions

The experimental study of the nucleon spin structure, where very impressive results were obtained, triggered many theoretical investigations. As result, we understand now much more about the internal content of nucleon and, even generally, about the structure of QCD. We know, that Ellis-Jaffe sum rule is not the last word in the problem of the nucleon spin content: gluons and strange quarks are of importance in this problem. The connection of gluon contribution to the nucleon spin to the anomaly is clarified. More clear becomes the role of nonperturbative phenomena in QCD. In this aspect there is a very important indication that the part of the proton spin projection carried by strange quarks $|\Delta s|$ is much larger than the part of the proton momentum $V_2^s$ carried by strange quarks, $|\Delta s| \gg V_2^s$. (A confirmation of this with a better accuracy is necessary!). If confirmed, this statement indicates a nontrivial dynamics of QCD vacuum and its explanation is a challenge for a theory. It would be very desirable if experimentally it would be possible in the near future:

1. To increase the accuracy by 2-3 times.
2. To study the $Q^2$-dependence of $g_1(x, Q^2)$ (in separate bins in $x$).
3. To go to higher $Q^2$: probably this can be done at HERA. (The experiment at higher $Q^2$ will be informative if only its accuracy will not be worse than the existing ones).
4. To have better data in the domain of small $x$ – probably, this also can be done at HERA.
5. To perform measurements of two-jets events in polarized deep-inelastic scattering, which will give information about $\Delta g$.
6. To have more data on $s$-quark distribution in nonpolarized nucleon, especially at $x < 0.1$.
7. To perform new experiments on elastic $\nu p$-scattering from which one can find the combination $\Delta u - \Delta d - \Delta s$, as well as elastic $ep$ scattering with separating the $Z$-exchange term.
8. To have better data on $g_2$.
9. To measure $h_1(x)$.
10. To perform a direct check of the GDH sum rule.

The problem confronting the theory, which, I believe, could be solved in the near future, are:
1. The lattice calculation determination of the induced by external field $v.e.v$'s, especially those which cannot be calculated by the QCD sum rule approach.
2. The study of $\bar{ss}$ and $\bar{uu} + \bar{dd}$ mixing in the nucleon (on lattice and in the instanton liquid model), especially for the case of the axial and energy-momentum tensor operators.
3. Improvement of the calculation of the structure functions by the QCD sum rule method.
4. To achieve a better understanding of nonperturbative effects in the structure functions at small (but not very small) $x \sim 10^{-2}$.

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