Modelling of heat and mass transfer processes of capillary-porous bodies

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Abstract. Rational nature management, recognized throughout the world as the dominant trend in the development of the economy, involves the most complete use of plant raw materials consumed by humans. For example, in the nutritional structure of people in most countries, plant proteins predominate over animal ones. In Japan, this ratio is 78.3 and 21.7; in Ukraine - 72.3 and 27.7; in the USA - 65.3 and 32.9; in Great Britain – 61 and 32.4; in Germany - 65.3 and 34.7; in France - 60 and 40; in Canada - 68.4 and 31.6; in China –87.3 and 12.7; in Italy - 74.6 and 25.4. A significant contribution to solving this problem is made by the correct storage of the collected agricultural raw materials at the stage of preparing them for processing. Given the significant amount of root crops, grains and seeds in edible plant raw materials, the interest of food producers to the conditions and modes of their storage becomes clear. Modern storages used for these purposes are equipped with various means to provide the necessary heat and humidity conditions; they even can regulate the composition of the environment in the premises of direct placement of stored mounds of root crops, grain and seeds. The paper considers the possibilities of the analytical assessment of the conditions for such storage of these mounds that would ensure their maximum preservation.

1. Introduction
Storage temperature has a significant effect on weight loss and rot loss. At high temperatures, the intensity of respiration and evaporation of water increases, microorganisms develop intensely. However, the temperature can be lowered to certain limits. The lower boundary is limited to temperatures causing functional disturbances or freezing. The freezing temperature of most root crops, grains and seeds ranges from minus 4 to minus 1°C. Storage at this temperature is rarely used, since low temperatures can cause irreversible changes [1-2].

The general requirement for an optimal storage temperature is also the absence of sudden changes in temperature and relative humidity, since even with a slight increase in temperature, condensation can form on the walls, ceiling of the store and on the products. It was established that the temperature fluctuation in the storage at 1°C causes a change in relative humidity of 5-6%. With increasing temperature, the relative humidity decreases, resulting in increased product mass loss [3-4]. Humidity
is an equally important parameter that must be controlled during storage, especially for root crops, grains and seeds, since they have quite significant shrinkage. The optimum air humidity for different types of raw materials varies. Exceeding it can lead to the intensive development of damage. With low oxygen content or an increased concentration of CO₂ in conditions of high humidity, the predisposition to physiological disorders increases. Variations in the relative humidity of the air can be associated with a violation of the regulation of refrigeration units, frequent openings of chamber doors and repeated unloading of products, insufficient vapour barrier, and prolonged operation of refrigeration equipment and fans. This enhances the intensity of respiration; and it is the reason for the increase in moisture loss of products and severe icing of evaporators.

To reduce weight loss due to insufficient moisture, it is recommended to moisten the container, carefully insulate the walls and floor of the storehouse, cover the stacks with products and moisten the air when laying it in storage [5-6]. An important factor affecting the effective storage of fruits and vegetables is the movement of air (air exchange) in the storage rooms. It is necessary for heat generated during respiration by plant objects, the uniform distribution of the incoming cooled air into the chambers, to prevent a significant temperature difference in the production and to remove ethylene, which stimulates tissue maturation and aging. It is necessary to properly regulate the air movement, since at high intensity the drying of the raw material increases, and at low, the occurrence of foci of damage in the bulk due to significant temperature fluctuations is possible [7].

2. Problem statement
For a two-layer mound of food products such as tubers of root crops, grain or seeds, modelled by a spherical capillary-porous “shell-core” medium, the system of differential equations of joint heat and mass transfer is applicable [8]:

\[
\frac{\partial [r_k(r,\tau)]}{\partial \tau} = a_{qk} \frac{\partial^2 t_k(r,\tau)}{\partial r^2} + \varepsilon_k \rho_k \frac{\partial [ru_k(r,\tau)]}{\partial \tau},
\]

\[
\frac{\partial [ru_k(r,\tau)]}{\partial \tau} = a_{mk} \frac{\partial^2 [ru_k(r,\tau)]}{\partial r^2} + a_{mk} \cdot \delta_k \frac{\partial^2 [r_k(r,\tau)]}{\partial r^2}
\]

The following notation is accepted here:

- \(k\) – layer number, \(k=1\) (core); 0<\(r<R_1\); \(k=2\) (shell); \(R_1<r<R_2\); \(t_k(r,\tau)\) – temperature of the \(k\)-th mound layer; \(r\) – current coordinate; \(R_k\) – ball radius; \(\tau\) – time, \(\tau>0\); \(u_k(r,\tau)\) – moisture content; \(a_{qk}\) – thermal diffusivity coefficient; \(a_{mk}\) – coefficient of potential conductivity (moisture conductivity); \(c_{qk}\) – specific heat; \(\varepsilon_k\) – phase transformation criterion; \(\rho_k\) – specific heat of phase transformation; \(\delta_k\) – thermogradient coefficient.

For the zonal calculation system, the thermophysical characteristics of the material are constant, different for different layers.

The temperature and moisture content of the environment during the conditioning of the mound vary slightly. In addition, in the process under consideration, the thermal transfer of a substance is not significant, i.e. \(\delta_k = 0\). By the time conditioning of the embankment of the aforementioned products begins, a uniform distribution of temperature and moisture is established in them.

All the noted features of drying and wetting the product mound lead to the solution of a system of differential equations

\[
\frac{\partial t_k}{\partial \tau} = a_{qk} \frac{\partial}{\partial r} \left( r^2 \frac{\partial t_k}{\partial r} \right) + \varepsilon_k \rho_k \frac{\partial u_k}{\partial \tau},
\]

\[
\frac{\partial u_k}{\partial \tau} = a_{mk} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u_k}{\partial r} \right)
\]
under the following boundary conditions:

\[
t_c(r, 0) = t_0 = \text{const}; \\
u_c(r, 0) = u_0 = \text{const}; \\
t_1(R, \tau) = t_2(R, \tau); \\
u_1(R, \tau) = u_2(R, \tau); \\
\lambda_{q1} \frac{\partial t_1(R, \tau)}{\partial r} = \lambda_{q2} \frac{\partial t_2(R, \tau)}{\partial r}; \\
\lambda_{m1} \frac{\partial u_1(R, \tau)}{\partial r} = \lambda_{m2} \frac{\partial u_2(R, \tau)}{\partial r}; \\
\frac{\partial t_1(0, \tau)}{\partial r} = \frac{\partial u_1(0, \tau)}{\partial r}; \\
t_1(0, \tau) = t_2(0, \tau) = \text{const}; \\
u_1(0, \tau) = u_2(0, \tau) = \text{const},
\]

where \( t_0 \) – seed initial temperature; \( t_c \) – ambient temperature; \( u_0 \) – initial moisture content; \( u_c \) – equilibrium moisture content; \( R \) – ball radius; \( \lambda_{qk} \) – coefficient of thermal conductivity; \( \lambda_{mk} \) – mass conductivity coefficient.

The equalities written above represent the initial conditions and boundary conditions of the fourth kind, which consist in the equality of temperatures, moisture contents, and heat and moisture fluxes at the interface between the core of an individual element of the mound and its shell.

The last three relations determine the conditions of symmetry and physical limitation of temperatures and moisture content, as well as boundary conditions of the 1st kind, which specify the temperature \( t_c \) and moisture content \( u_c \) on the surface of the mound element.

3. Results and discussion

The problem is solved by the method of Laplace integral transforms. The distribution of the mass and heat transfer potential fields is obtained as follows:

\[
\Theta_{k1} = \frac{u_1(r, \tau) - u_c}{u_0 - u_c} = \frac{2R}{X} \sum_{n=1}^{\infty} c_n \zeta_k \exp\left(-\mu_n^2 F_0\right); \\
\Theta_{k2} = \frac{t_1(r, \tau) - t_c}{t_0 - t_c} = \frac{2\varepsilon \kappa_2 \zeta_k \zeta_2 R}{X(I - \mu \mu_2)} \times \left[ \sum_{i=1}^{\infty} c_i \zeta_i \exp\left(-\mu_i^2 F_0\right) - \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} c_{ij} \zeta_{jm} \exp\left(-\mu_n^2 F_0\right) \right]
\]

where \( \mu_n \) – successive positive roots of the characteristic equation

\[
\sqrt{a_m ucrg\left(a_m(R-1)\mu\right)} + 1 + \lambda_{m}(\mu ctg\mu - 1) = 0; \\
c_n = \frac{1}{\varphi_1} \sin \mu_n;
\]
\[ z_1 = \sin \left( \sqrt{a_m(R-1)\mu_n} \right) \sin \left( \mu_n X \right) ; \]

\[ z_2 = \sin \mu_n \sin \left( \sqrt{a_m(R-X)\mu_n} \right) ; \]

\[ \varphi_{1n} = k_m \mu_n \sin^2 \left( \sqrt{a_m(R-1)\mu_n} \right) + \frac{a_m(R-1)\mu_n}{a_m\mu_n} - \frac{1 - \sqrt{a_m k_m}}{\sqrt{a_m \mu_n}} \sin^2 \mu_n \sin^2 \left( \sqrt{a_m(R-1)\mu_n} \right) \]

\( \mu_j \) - successive positive roots of the characteristic equation

\[ \sqrt{a_q Lu_1 \mu \operatorname{ctg} \left( \sqrt{a_q Lu_1 (R-1)\mu} \right) + 1 + \lambda_q \times \left( \sqrt{Lu_1 \mu \operatorname{ctg} \left( \sqrt{Lu_1 \mu} \right) - 1} \right)} = 0 ; \]

\[ c_j = \frac{1}{\varphi_1 \varphi_2} \sin \left( \frac{1 - Lu_2 (1 - e_2 K_0) \varphi_{2j}}{e_2 K_0 Lu_2} \varphi_{2j} + \frac{\lambda_q (b - \lambda) \varphi_{3j}}{\varphi_{2j}} \sin \left( \sqrt{a_q Lu_1 (R-1)\mu_j} \right) \sin \left( \sqrt{a_m (R-1)\mu_j} \right) \right) ; \]

\[ z_{11} = \sin \left( \sqrt{a_q Lu_1 (R-1)\mu} \right) \sin \left( \sqrt{Lu_1 \mu X} \right) ; \]

\[ z_{21} = \sin \left( \sqrt{Lu_1 \mu} \right) \sin \left( \sqrt{a_q Lu_1 (R-X)\mu} \right) ; \]

\[ z_{11} = \sin \left( \sqrt{a_q Lu_1 (R-1)\mu} \right) \sin \left( \sqrt{Lu_1 \mu X} \right) ; \]

\[ z_{21} = \sin \left( \sqrt{Lu_1 \mu} \right) \sin \left( \sqrt{a_q Lu_1 (R-X)\mu} \right) ; \]

\[ c_1 = \frac{\sin \mu_n}{\varphi_{2n}} ; \quad c_2 = \frac{-(b - \lambda)}{\varphi_{1n} \varphi_{2n}} \frac{\varphi_{3n} \sin^2 \mu_n}{\varphi_{2n} \sin \left( \sqrt{Lu_1 \mu} \right) \sin \left( \sqrt{a_q Lu_1 (R-1)\mu} \right) \sin \left( \sqrt{a_m (R-1)\mu} \right) \sin \left( \sqrt{Lu_1 \mu X} \right) ;} \]

\[ c_1 = \frac{\sin \mu_n}{\varphi_{2n}} ; \quad c_2 = \frac{-\lambda_q (b - \lambda) \varphi_{3n} \sin^2 \mu_n}{\varphi_{1n} \varphi_{2n} \sin \left( \sqrt{a_q Lu_1 (R-1)\mu} \right) \sin \left( \sqrt{a_m (R-1)\mu} \right) \sin \left( \sqrt{Lu_1 \mu X} \right) ;} \]

\[ z_{11a} = \sin \left( \sqrt{a_m (R-1)\mu} \right) \sin \left( \mu_n X \right) ; \]

\[ z_{12a} = \sin \left( \sqrt{a_m (R-1)\mu} \right) \sin \left( \sqrt{Lu_1 \mu X} \right) ; \]

\[ z_{21a} = \sin \left( \mu_n \sin \left( \sqrt{a_m (R-X)\mu} \right) \right) ; \]

\[ z_{21a} = \sin \left( \sqrt{a_m (R-1)\mu} \right) \sin \left( \sqrt{a_q Lu_1 (R-X)\mu} \right) \sin \left( \sqrt{a_m (R-1)\mu} \right) \sin \left( \sqrt{Lu_1 \mu X} \right) ; \]

\[ \varphi_{1j} = \sqrt{a_m \mu \operatorname{ctg} \left( \sqrt{a_m (R-1)\mu} \right) + 1 + \lambda_m \mu \operatorname{ctg} \mu_j - 1} ; \]

\[ \varphi_{2j} = k_q \sqrt{Lu_1 \mu} \sin^2 \left( \sqrt{a_q Lu_1 (R-1)\mu} \right) + \sqrt{a_q Lu_1 (R-1)\mu} \sin^2 \left( \sqrt{Lu_1 \mu} \right) + \frac{1 - \sqrt{a_q k_q}}{\sqrt{a_q Lu_1 \mu}} \sin^2 \left( \sqrt{a_q Lu_1 \mu} \right) \sin^2 \left( \sqrt{a_q Lu_1 (R-1)\mu} \right) ; \]

\[ \varphi_{3j} = \mu \operatorname{ctg} \mu_j - 1 - d \left( \sqrt{Lu_1 \mu \operatorname{ctg} \left( \sqrt{Lu_1 \mu} \right) - 1} ; \right) ; \]
\[ \varphi_{2n} = \frac{a_q}{a_q} \mu_n \cotg \left( \frac{a_q \mu_n (R-1) \mu_n}{1 + \lambda_q \left( \sqrt{a_q \mu_n (R-1) \mu_n} - 1 \right) \mu_n} \right) \]
\[ \varphi_{3n} = d \left[ \frac{a_q}{a_q} \mu_n \cotg \left( \frac{a_q \mu_n (R-1) \mu_n}{1 + \lambda_q \left( \mu_n \cotg \mu_n - 1 \right) \mu_n} \right) \right] \]
\[ \varphi_{4n} = \mu_n \cotg \mu_n - 1 - d \left( \sqrt{\mu_n \cotg \mu_n} - 1 \right) \mu_n \]

Where: \( \Theta_{ki} \) – dimensionless moisture content; \( \Theta_{k2} \) – dimensionless temperature; \( K_{O_2} = \frac{\rho \varphi \Delta t}{c_q \Delta t} \) – Kossovich heat transfer number; \( Lu_k = \frac{a_{mk}}{a_{qk}} \) – Lykov number; \( F_0 = \frac{\alpha_{m1} \tau}{R_i} \) – Fourier number;

\[ a_q = \frac{a_q}{a_q}; a_m = \frac{a_m}{a_m}; \lambda_q = \frac{\lambda_q}{\lambda_q}; \lambda_m = \frac{\lambda_m}{\lambda_m}; R = R \frac{R}{R}; b_i = \left( 1 - \frac{1}{Lu_i} \right) \frac{c_i}{\varepsilon \rho_i} \right) \cdot b \right)_{01} ; i = 1,2 ; b = b \frac{b}{b} \]

If \( \sqrt{a_m (R-1)} = \frac{\beta}{\alpha} \cdot \sqrt{a_q (R-1)} = \delta \), where \( \alpha, \beta, \gamma, \delta \) – integers, then in addition to solution (5), the expression
\[ \frac{2R}{X \pi (\alpha + \beta k_m)} \sum_{\omega=1}^{\infty} \frac{1}{2k_m} \sum_{i=1}^{\infty} Y \exp \left( - \alpha^2 \pi^2 \omega^2 F_0 \right) \]
has to be added, where

\[ Y_i = \left( -1 \right)^{\alpha + \beta \omega} \cdot \sin \left( \alpha \pi \omega X \right) ; \]
\[ Y_2 = k \cdot \cos \left( \beta \pi \omega (X - R) \right) \] and to the decision (6) – expression
\[ \frac{2 \varepsilon \rho K_{O_2} Lu \cdot R}{X (1 - Lu)} \left( \frac{1}{Lu} \sum_{\omega=1}^{\infty} \sum_{i=1}^{\infty} \left( -1 \right)^{\omega} C \exp \left( - \mu^2 F_0 \right) \right) \]
\[ + \frac{1}{\alpha + \beta k_m} \sum_{\omega=1}^{\infty} \sum_{i=1}^{\infty} \left( -1 \right)^{\omega} Y \exp \left( - \mu^2 F_0 \right) \]

where \( c_{11} = \left[ \left( -1 \right)^{\alpha \omega} \frac{1 - Lu \left( 1 - \varepsilon \rho K_{O_2} \right)}{v \varepsilon \rho K_{O_2} Lu \cdot \mu} + \frac{1}{\mu} \sin \left( a_m \sqrt{R \left( R-1 \right) \mu} \right) \right] \]
\[ c_{12} = k \left[ \left( -1 \right)^{\alpha \omega} \frac{1 - Lu \left( 1 - \varepsilon \rho K_{O_2} \right)}{v \varepsilon \rho K_{O_2} Lu \cdot \mu} - \frac{1}{\mu} \sin \left( a_m \sqrt{R \left( R-1 \right) \mu} \right) \right] \]
\[ Y_1 = \sin \left( \sqrt{Lu \cdot \mu} \right) ; Y_2 = \sin \left( a_q \sqrt{Lu \cdot \mu} \right) ; \]
\[ C_{110} = \frac{b}{\pi \omega} ; Y_{110} = \sin \left( \mu (X - 1) \right) ; \]
\[ C_{210} = - \frac{\alpha \lambda_q (b - \lambda)}{\phi_2 \sin \left( \sqrt{Lu \cdot \mu} \right) \sin \left( \sqrt{Lu \cdot \mu} \right)} ; Y_{210} = \sin \left( \sqrt{Lu \cdot \mu} \right) ; \]
\[ C_{12\omega} = \frac{k_m}{\pi \omega} ; \quad Y_{12\omega} = \sin\left(\sqrt{a_m \mu_{\omega}} (X - 1)\right) ; \]
\[ C_{22\omega} = -\frac{\alpha \lambda_b (b - \lambda)}{\varphi_{2\omega}} \sin\left(\sqrt{a_q L u_1 \mu_{\omega}} (R - 1)\right) ; \]
\[ Y_{22\omega} = \sin\left(\sqrt{a_q L u_1 \mu_{\omega}} (R - X)\right) ; \]
\[ \mu_r = \pi / \nu ; \quad \mu = \alpha \pi \omega . \]

Under mild drying conditions of the mound \((t \leq 50 ^\circ C)\), moisture transfer inside it occurs only in the form of a liquid \((\epsilon = 0)\). In this case, the original system of equations (1) - (2) turns into an unconnected system of heat and mass transfer equations, the solution of which together with the previous boundary conditions leads to known solutions [8, 9].

4. Conclusion
Thus, the boundary-value problem of moisture and heat transfer, set in relation to the conditioning processes of the mound of root crops, grain or seeds in terms of humidity and temperature in the first (at a constant speed) period, has been solved.

The obtained analytical solution makes it possible to continue research (using the appropriate software packages for computers), allowing to establish the duration of the periods of conditioning processes that occur at a constant speed, to develop measures to intensify the conditioning processes of the mound of root crops, grain or seeds by humidity and temperature.

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