Supersymmetric Grand Unified Theories and Global Fits to Low Energy Data

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Abstract

We present a self-consistent $\chi^2$ analysis of several supersymmetric (SUSY) grand unified theories recently discussed in the literature. We obtain global fits to low energy data, including gauge couplings, fermion masses and mixing angles, gauge boson masses and $BR(b \rightarrow s\gamma)$. One of the models studied provides an excellent fit to the low energy data with $\chi^2 \sim 1$ for 3 degrees of freedom, in a large region of the experimentally allowed SUSY parameter space. We also discuss the consequences of our work for a general MSSM analysis at the $Z$ scale.
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1 Introduction

There is no doubt that the Standard Model (SM) describes physical processes at the highest available energies with very good precision. Yet it contains some 18 independent parameters which remain completely undetermined and await derivation from a more fundamental theory. 13 of these parameters are related to the fermion mass sector which is clearly distinguished by an amazingly simple regularity in the hierarchy of masses and mixing angles of the three fermionic families.

This observed pattern may provide a clue to a more fundamental theory at some higher scale $M$ ($M \sim M_{\text{Planck}}$ or $M_{\text{string}}$ or $M_{\text{GUT}}$). Our main hypothesis is that only a small set of effective mass operators below $M$ dominates in the Yukawa matrices, subsequently leading to the fermion mass matrices at the scale of electroweak symmetry breaking. These effective operators are of utmost interest. They can lead to the reconstruction of the full effective field theory at the scale $M$ and provide the matching conditions for a fundamental string theory, believed to be the ultimate quantum theory incorporating gravity together with the gauge interactions of quarks and leptons.

SO(10) supersymmetric theories are excellent candidates for such a theory below the string scale. They maintain the successful prediction for gauge coupling unification and provide a powerful framework for predictive theories of fermion masses and mixing angles. This is because all the fermions of a single family are contained in the 16 dimensional representation of SO(10) - thus fermion mass matrices are related by symmetry. In the most predictive theories, the ratio of Higgs vevs $- \tan \beta$ is large, and the top quark is naturally heavy as found experimentally. However, as a consequence of large $\tan \beta$ there are potentially large supersymmetric weak scale threshold corrections which could play an important role in fitting the fermion masses and mixings. Thus a self-consistent analysis necessarily includes the dimensionful soft SUSY breaking parameters, in addition to the dimensionless gauge and Yukawa couplings.

2 Connecting GUT Scale Physics with Low Energy Observables

Here we present the results of such a complete top-bottom analysis. It starts at the GUT scale $M_G$, which is a free parameter itself, with unified gauge coupling $\alpha_G$, $n_\gamma$ free parameters entering the Yukawa matrices, and with five universal soft SUSY breaking parameters $\mu$, $m_0$, $M_{1/2}$, $A_0$ and $B_\mu$. In addition, we introduce $c_3$ as a one loop GUT threshold correction to $\alpha_3(M_G)$, and non-universal Higgs masses $m_{H_d}$ and $m_{H_u}$. The dimensionless (dimensionful) couplings are run down to the $Z$ scale using two (one) loop renormalization group equations (RGEs) of MSSM. We have checked at a few selected points that the corrections to our results obtained by using two loop RGEs for dimensionful couplings are, in fact, insignificant. At the $Z$ scale we match the MSSM directly to the $SU(3)_c \times U(1)_{EM}$, thus leaving out the SM as an effective theory on our way down to the experimentally measured low energy data listed in table. At the $Z$ scale we use the tree level conditions for electroweak symmetry breaking to fix $v$ and $\tan \beta$. Then, still within the MSSM, we calculate the amplitude for the process $b \to s\gamma$, one loop corrected $W$ and $Z$ masses.

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$^a$Clearly, $n_\gamma$ is model dependent.

$^b$By definition, $M_G$ is to be understood as the scale where $\alpha_1$ and $\alpha_2$ are the same, and equal to $\alpha_G$.  

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corrections to the $\rho$ parameter from new physics outside the SM, and one loop corrected $G_\mu$, where we leave out the SUSY vertex and box contributions to the $\Delta r$ parameter. When crossing the $Z$ scale, we compute the complete one loop threshold corrections to $\alpha_s$ and $\alpha_{EM}$, whereas only those one loop threshold corrections to the fermion masses and mixings enhanced by $\tan \beta$ are computed. Masses and couplings which require further running are renormalized down to their appropriate scales using three loop QCD and one loop QED RGEs. The branching ratio $\text{BR}(b \to s\gamma)$ is renormalized down to the scale $M_0$ using the leading log approximation. Note that some observables entering the $\chi^2$ function (see table 1) are known so well that we have to assign a theoretical, instead of experimental, error as their standard deviation. This is true for $M_Z$, $M_W$, $\alpha_{EM}$, $G_\mu$ and the charged lepton masses. We estimated conservatively our theoretical error to be 0.5% based on the uncertainties from higher order perturbation theory and from the performance of our numerical analysis. The error on $G_\mu$ is estimated to be within 1% due to the fact that in addition to the uncertainties mentioned above we neglect SUSY vertex and box corrections to $\Delta r$. Also note that $\epsilon_K$, the observable of $CP$ violation, has been replaced by a less precisely known hadronic matrix element $\hat{B}_K$. Thus our theoretical value of $\hat{B}_K$ is defined as that value needed to agree with $\epsilon_K$ for a set of fermion masses and mixing angles derived from the GUT scale. Finally note, that the light quark masses are replaced by their ratios (at the scale 1GeV), and $m_c(M_c)$ by the difference $M_b - M_c$, since the latter quantities are known to better accuracy. $Q^{-2} = (m_d^2 - m_u^2)/m_s^2$ is the Kaplan-Manohar-Leutwyler ellipse parameter.

In addition, the $\chi^2$ function is increased significantly by a special penalty whenever a sparticle mass goes below its current experimental limit.

### 3 Model 4 of ADHRS

We have analyzed several models of fermion masses. First, we have studied the best working model of ADHRS\textsuperscript{[7]}, model 4. The model is defined by four effective operators at the GUT scale ($A$’s stand for adjoint states and $S$ for a singlet state): $O_{33} = 16_3 10 16_3$, $O_{23} = 16_2 \frac{A}{\sqrt{3}} 10 \frac{A}{\sqrt{3}} 16_3$, $O_{12} = 16_1 \left( \frac{A}{\sqrt{3}} \right)^3 10 \left( \frac{A}{\sqrt{3}} \right)^3 16_2$ and by one of six possible $O_{22}$ operators (they all give the same 0:1:3 Clebsch relation between up quarks, down quarks and charged leptons responsible for the Georgi-Jarlskog relation). Each operator enters with its own complex coefficient, and we can rotate away three independent phases. Hence in model 4 $n_{\phi} = 5$; the number of initial parameters for the $\chi^2$ analysis is 15 and we are left with 5 degrees of freedom. Fig. 2 shows the contour lines of constant $\chi^2$ in the $m_0 - M_{1/2}$ plane for different values of $\mu$. No substantial improvement of the performance of model 4 can be achieved by neglecting one out of the twenty low energy data given in table 1. On the other hand we found that a significant improvement is possible by adding one new operator, contributing to the 13 and 31 elements of the Yukawa matrices.

### 4 Model 4c

Next, we analyzed two models derived from the complete SO(10) SUSY GUTs discussed recently by Lucas and Rab\textsuperscript{[11]}. The models were constructed as simple extensions of model 4. Different label (a,b,...f) refers to the different possible 22 operators.
In the extension to a complete GUT the different 22 operators lead to inequivalent theories due to different U(1) charge assignments. When one demands “naturalness”, i.e. includes all terms in the superpotential consistent with the symmetries of the theory one finds only one new operator (O_{13}) for models 4a and 4c. Model 4b, on the other hand, has no new operator and thus is equivalent to model 4 (above). The 22 and 13 operators of model 4c are \( O_{22} = 16_2 \frac{2}{3} 10 \frac{1}{3} 16_2 \) and \( O_{13} = 16_1 \left( \frac{4}{3} \right)^3 10 \frac{4}{3} 16_3 \). With the 13 operator \( n_\mu = 7 \), which implies 3 degrees of freedom. The results of the global analysis are given in Fig. 2, and in table 2 for a selected point II marked on Fig. 2. Model 4a is defined by different 22 and 13 operators and gives the fits with the best \( \chi^2 \approx 4-6 \) in most of the SUSY parameter space. (It yields also \( \chi^2 \approx 3 \), but only for the corner in the SUSY parameter space with large \( m_0, M_{1/2} \) and \( \mu \).)

5 Discussion and Conclusions

The results of our analysis, as well as the whole project presented here, may be understood from two different perspectives.

The emphasis of this work has been put on the study of the origin of fermion mass matrices, within the context of minimal SO(10) SUSY GUTs. The best working model of ADHRS, with just four effective operators in the Yukawa sector, can be assigned a confidence level of only 3-4%, in spite of the fact that each observable is within 2\( \sigma \) of its experimental value. The addition of a 13 operator may improve the performance of the model. Substantial improvement however, is not automatic, as was mentioned in the example. 

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Note: models d, e and f have the second family 16_2 coupled directly to 10_1 and a heavy 16. If this coupling is as large as the third generation Yukawa coupling, then we would obtain excessively large flavor changing neutral current processes, such as \( \mu \rightarrow e\gamma \). Thus these models were not considered in ref. [5].
A table 2: Model 4c - Results at Point II. Initial parameters: 
$1/\alpha_G = 24.36, \alpha_G = 3.17 \times 10^{-16} \text{GeV}, \epsilon_1 = 4.89\%$, $A = 0.807, B = 5.44 \times 10^{-4}, C = 1.15 \times 10^{-4}, D = 4.94 \times 10^{-4}, \delta = 5.71, E = 1.31 \times 10^{-2}, \Phi = 1.04, \mu = 80 \text{GeV}, m_0 = 700 \text{GeV}, M_{1/2} = 240 \text{GeV}, m_{H_d}/m_0 = 1.42, m_{H_u}/m_0 = 1.24, A_0 = 458.35 \text{GeV}, B_\mu = 120.66 \text{GeV}^2$. The last column displays SUSY threshold corrections in %.

| Observable | Computed | Partial $\chi^2$ | S.t.c. |
|------------|----------|-----------------|-------|
| $M_Z$      | 91.12    | 0.02            |       |
| $M_W$      | 80.38    | <0.02           |       |
| $G_\mu$    | 1.166 \times 10^{-5} | <0.02   |       |
| $\alpha_{EM}^{-1}$ | 137.0    | <0.02           | 1.43  |
| $\alpha_s(M_Z)$ | 0.1151  | 0.34            | 12.78 |
| $\rho_{new}$ | +1.87 \times 10^{-4} | 0.09   |       |
| $M_t$      | 175.7    | <0.02           | 0.74  |
| $m_b(M_b)$ | 4.287    | 0.06            | 5.43  |
| $M_b - M_c$ | 3.440    | 0.04            | 7.56  |
| $m_s$      | 189.0    | 0.03            | 3.68  |
| $m_t/m_s$  | 0.0502   | <0.02           | 0.00  |
| $Q^{-2}$   | 0.00204  | <0.02           | 1.78  |
| $M_\tau$   | 1.776    | <0.02           | -2.08 |
| $M_\mu$    | 105.7    | <0.02           | -1.50 |
| $M_e$      | 0.5110   | <0.02           | -1.50 |
| $V_{us}$   | 0.2205   | <0.02           | 0.00  |
| $V_{cb}$   | 0.0400   | 0.07            | 1.58  |
| $V_{ub}/V_{cb}$ | 0.0772 | <0.02           | 0.00  |
| $B_K$      | 0.8140   | <0.02           | -3.18 |
| BR($b \rightarrow s\gamma$) | 2.382 \times 10^{-4} | <0.02   |       |
| TOTAL $\chi^2$ | 0.7306    |                  |       |

of model 4a. Nevertheless, we showed that model 4c provides an excellent fit to all 20 observables of table [4], with the confidence level better than 68% in a large region of the experimentally allowed SUSY parameter space. Note that the best fits extend to the region with very large masses and mixings is suppressed by large squark and slepton masses. As a result, in this region the effective number of degrees of freedom in the fermionic sector is actually larger than 3, since there are 7 parameters ($A, B, C, D, \delta, E, \Phi$) in the Yukawa matrices determining the 13 low energy masses and mixings of the fermions. This means that the Yukawa sector of model 4c does actually a much better job than appears at first glance. Whether or not this particular model is close to the path Nature has chosen remains to be seen. One important test will be via the CP violating decays of the $B$ meson. Model 4c predicts a value for $\sin 2\alpha \approx 0.95$ which is insensitive to the SUSY breaking parameter, whereas in the SM the value of $\sin 2\alpha$ is unrestricted. Another important test may come from nucleon decay rates.

When one disregards the origin of the Yukawa matrices at the GUT scale, this work can also be viewed as an MSSM global fit in the large $\tan \beta$ regime. From this perspective, a special feature of our approach is that we are getting complete $3 \times 3$ fermion and sfermion mass matrices at the SUSY breaking scale, instead of just the leading 33 elements. Viewed as MSSM global fits, our results suggest that there is no narrow, strongly preferred region in the SUSY parameter space. Hence, at present one cannot make strong conclusions (with $\tan \beta$ large) about the masses of sfermions, which leaves open many channels for Tevatron and LEPII experiments. It is also interesting to note that the best fits at a number of points give a low pseudoscalar Higgs mass and low value of $\alpha_s(M_Z)$ leaving the door open for a natural explanation of the 1-$\sigma$ increase in the partial width $Z \rightarrow b\bar{b}$, without any tuning of the initial parameters to obtain this effect. Finally, by keeping the complete $3 \times 3$ mass matrices for both fermions and sfermions one can study flavor dependent processes in a theory which fits the low energy data; for example, rare $B$ and $K$ decays, $B \rightarrow \bar{B}$ mixing, lepton flavor violating processes or even $Z \rightarrow b\bar{b}$.

At the present time, when direct evidence for physics beyond the SM evades experimental observations, global analysis serves as the best test of new physics. Moreover, our results show that minimal SUSY SO(10) models remain among the best candidates for the theory of nature beyond the Standard Model.

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