FASHION, NOVELTY AND OPTIMALITY: 
An application from Physics

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Abstract

We apply a physical based model to describe the clothes fashion 
market. Every time a new outlet appears on the market, it can invade 
the market under certain specific conditions. Hence, the “old” outlet 
can be completely dominated and disappears. Each creator competes 
for a finite population of agents. Fashion phenomena are shown to 
result from a collective phenomenon produced by local individual im-
itation effects. We assume that, in each step of the imitation process, 
agents only interact with a subset rather than with the whole set of 
agents. People are actually more likely to influence (and be influenced 
by) their close “neighbours”. Accordingly we discuss which strategy 
is best fitted for new producers when people are either simply organ-
ised into anonymous reference groups or when they are organised in 
social groups hierarchically ordered. While counterfeits are shown to 
reinforce the first strategy, creating social leaders can permit to avoid 
them.

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1 Setting the limits

"Fashions have changed", "in fashion", "old-fashioned" are phrases frequently used in the street, on TV or in newspapers. But what fashion are we talking about? The fashion of ideas, artistic fashion or, more prosaically, dress fashion? When we talk about fashion, do we consider it to be the result of a creative process (either intellectual or industrial), or a method of communicating a certain "way of life" which would correspond in economics to the level of information? In this paper, fashion should be understood as the way a creative process entails new behaviour in a set of consumers, independently of the information structure. We apply the model to the clothes fashion market. Every time a new outlet appears on the market, it can, under certain specific conditions, invade the market. Hence, the "old" outlet can be completely dominated and disappears. Each creator competes for a finite population of agents.

As is the case for the movie industry or the industry of industrial design, fashion activity depends on the level of creativity of the designers. It also depends on the level of public recognition of this creativity. Nike certainly shares this assumption, paying Michael Jordan (a famous basket ball player in the U.S.) 20 million dollars in 1992, for endorsing Nike running shoes.

Fashion does not improve either efficiency or the marginal utility of the consumption. Clothes belong to a class of goods whose functional properties are fundamental in their consumption. It is easy to understand why I need to buy a dress, or a pair of jeans. It is harder to explain why I choose Adidas shoes, or Levi’s jeans. Moreover, it seems very difficult for an economist to explain why women ask for short skirts one year, long skirts the year after, and then short skirts again the year after that. Some theorists may talk of erratic preferences, but the desire to be in fashion could justify Janssen and Jager’s argument [1] that "...in satisfying their need for identity, people may change their behaviour without changing their preferences".

In markets in general and the fashion market in particular, people are interacting simultaneously, and constantly modifying their decisions. It is very reminiscent of the way atoms interact in inert matter. Galam and Moscovici [2] built a model of group decision making to describe the dynamics of competing effect in updating individual opinions. De Oliveira et al [3] give some interesting applications of physic tools to study economic and social problems has become more numerous. Social interactions certainly play a major role in the understanding of fashion diffusion process. By social
interactions, "we refer to the idea that the utility or payoff an individual
receives from a given action depends directly on the choices of others in that
individual’s reference group, as opposed to the sort of dependence which
occurs through the intermediation of markets" (Brock and Durlauf 1995).
Hence, we postulate that a consumer maximises his utility when he is dressed
like everybody else in his peer group.

In this paper, we consider that fashion results from the desire both to
conform and to differentiate oneself from others. We also assume that prod-
ucts are disembodied, thus any agent can switch from one brand to another
without cost. Under these assumptions, we attempt to explain how, after a
certain number of steps, a common unique behaviour can emerge from a fi-
nite population with heterogeneous behaviour. We believe that the structure
and organisation of the market are strong factors in the diffusion of fashion.

The fashion sector has changed since the 1950s. A lot of studies show that
this sector was organised into a very simple system until the 1970s (creation,
production, distribution). Since the beginning of the 1980s, the fashion sector
has developed into a quite complex and destructured system, with the emer-
gence of different levels of quality ("Fashion off the peg" and "Luxury off the
peg"). The fashion industry now follows a rationale of "filière", which im-
plies a multiplication of brands (with a tough policy of brand protection) and
sales levels. This tough policy of brand protection actually coexists with a
high level of market piracy in the design-based industry. For a long time the
"legal market" fought hard against this illegal market. It seems that things
have recently changed: An explanation could be in terms of social welfare.
Some recent analyses show how if the counterfeit industry were to disappear
it would leave the place open to criminal organization, which would clearly
be dangerous for the economy and for society. According to us, allowing
counterfeits can have positive externalities for the main brands. A high level
of piracy can be interpreted as a signal that the product is "fashionable".
In this sense, the illegal market becomes an advertising tool. The more a
brand is counterfeited, the more this brand will be seen by a wide public.
Multiplying the levels of quality can also be interpreted as an advertising
tool. The consumer side can be seen as a lattice, in which people have strong
cliquishness with their peers, in hierarchic social groups. Hence, the fashion
market seems to be extremely hierarchical both on the demand and on the
offer side.

Proving the existence of a dominant strategy for the creators could help to
explain why some fashion goods emerge, where others fail. In what follows,
we assume that the aim of a new creator is to invade the fashion market. In other terms, his product must become ”the fashion”. In this paper, we compare the different strategies a brand can use to insure the widest diffusion of its product with the minimum level of investment. We wish to explain why the fashion ”filière” is currently so diversified and hierarchically organised, why counterfeits can have positive externalities and why, therefore, creators must influence social interactions by appropriate strategies and facilitate the diffusion of their creation.

**In part 2**, we focus on a market within which people are anonymous, with homogeneous characteristics. We apply the concept and techniques of real space renormalisation group from Physics, in the line of Ma [4] to study the fashion diffusion process within hierarchical structures. In particular, we focus on the necessary conditions for a given brand to be sure to dress the majority of people in a society.

In previous papers, Galam found that majority rule voting produces a critical threshold to total power [5, 7]. The value of the critical threshold to power is a function of the voting structure, namely the size of voting groups and the number of hierarchical voting levels. We find here that when producers consider consumers as simply being organised into ”reference groups”¹ the presence of counterfeits can be helpful in invading the market.

This result is demonstrated in **part 3** where we show that in the case where consumers are anonymous, allowing counterfeits can be an efficient tool to diminish the level of investment. **In part 4**, people are no longer anonymous. They are organised into small groups, which can be interpreted as classified social groups with heterogeneous characteristics. In this situation, people can recognise each other. We postulate that they are *a priori* leaders (they ”make” the fashion) or followers. Every time a follower imitates a leader, he becomes a leader himself. We show that invasive creators can avoid counterfeits and minimise their sunk costs if they can identify organised, hierarchic social groups within the society.

**Part 5** deals with setting some quantitative conditions to invade a given market with an already existing set of references groups and leaders. **Last part** concludes.

¹The ”reference group of an individual possibly includes all the agents whom ”social psychologists” call significant others” [8]. They can be friends, relatives, neighbours, partners in business and so on.
2 From physics to fashion: The model

When sufficient agents in a consumer’s social network switch from one product to another, the preferences of this consumer are assumed to follow. But little is said about how this socialisation appears, although some authors as McCauley, Rozin and Schwartz [9] speak about cultural transmission. In what follows, we show how creators can generate social effects by making useful investments, then use these effects to invest less.

At the beginning of the game, all the consumers wear a specific brand, let us say A. In the next period, a new creator B emerges and decides to invade the market. To this end, he makes classical investments such as advertising campaigns, promotions, etc. Now, people can choose between two brands (A and B), representing two tendencies in the fashion. We assume that each individual wants to be in fashion. Reference groups can be seen as cells that will produce common social behaviour. The investment made by the new creator will be efficient if it is sufficient to allow a wide diffusion process.

At some stage, we consider all investments from both brands are stopped. There, we denote by $p_0$ the overall proportion of people wearing the B brand and $1-p_0$ the corresponding proportion wearing the A brand. From these initial conditions we study the internal dynamics of spreading or disappearance of the new brand B within the population. The underlying assumption is that each individual wants to be in fashion. We believe that it is not because they have erratic preferences that people will switch from one fashion good to another, but rather because the maximisation of their utility depends on their level of belongingness to a reference group. Consuming the same product as their peers or friends increases this feeling of belongingness.

To achieve this individual goal, we assume that each person goes through a hierarchical imitation process. Each step of this process is related to a reference social group that produces common social behaviour. Within this framework, every reference group consists of the aggregation of other smaller reference groups. Moreover, it is only when a local fashion is established at one reference group level that the higher reference group level becomes activated, as seen in Figure (1). Each increase in the reference group size is referenced with a one time unit incrementation.
2.1 The anonymous in-fashion process

We start from an anonymous situation in which the in-fashion model is driven solely by a simple individual motivation to be like the majority of the people around. There exists no leader, no intrinsic advantage to one brand over the other, only the majority effect counts. We assume the population dynamics of fashion awareness to be go within a successive set of reference groups level extending to some factor $r$ where $r$ is an integer which counts how many persons are within the first level of reference groups. Then to simplify the equations we postulate the rescaling of the reference group extension goes also by a factor $r$. It means people go in fashion first according to a local majority among $r$ persons. Once this step is completed, the in-fashion process goes up to include $r$ groups of $r$ persons each, i.e., by majority among $r^2$ persons. And so on up to include the whole population. For instance for $r = 100$ we have 100 persons by first reference group level and then it jumps to includes $r^2 = 10000$ and reach $r^3 = 1000000$, one million already after 3 levels.

To formalize above scheme we calculate the probability $p_1$ to have one initial reference group of $r$ persons to have a majority of people wearing the B brand starting from a whole larger population with a $p_0$ proportion of B wearing. All configurations of $r$ persons having from $r$ persons wearing the B brand down to m ones where $m = \frac{r+1}{2}$ for odd $r$ and $m = \frac{r}{2}$ for even $r$. 
add to yield a B majority. The case of equality between the numbers of A and B wearing persons is attributed to the B new brand as a tip to novelty. Accordingly having \( p_0 \) at \( t = 0 \) leads to \( p_1 \) at \( t = 1 \) with,

\[
p_1 = P_r(p_0)
\]

where the function \( P_r \) determines the renormalized proportion of B wearing persons. More generally, starting from one reference level \( n \) with a B proportion \( p_n \) leads to the new proportion \( p_{n+1} \) at level \( n + 1 \) with,

\[
p_{n+1} \equiv P_r(p_n) = \sum_{l=r}^{l=n} \frac{r!}{l!(r-l)!} p_n^l (1 - p_n)^{r-l}.
\]

Simultaneously the A proportion varies according to \( 1 - p_{n+1} \).

2.2 The optimal investment threshold: the necessary condition.

For a creator who wishes to invade the market, an investment threshold exists, below which investing is useless, no matter what the amount of investment. Investing more is also superfluous. The optimal strategy consists in setting the right level of investment just above the threshold to enable the full market invasion driven by the in fashion process.

To follow the dynamics of change in the density of B-dressed people we need to study the fashion function \( P_r(p_n) \) defined above in Eq.(2). One shape is shown in Figure (2) for the case \( r = 101 \). From the Figure and Equation (2) it is found that the renormalized in-fashion process produces a monotonic flow towards either one of the two stable fixed points \( p_D = 0 \) and \( p_I = 1 \). The first one corresponds to the total disappearance of the B brand with A preserving its initial monopoly. At the other extreme \( p_I \) represents a total B invasion with the A brand totally evicted.

In between these two points there exists another one \( p_c \) which is unstable since it produces the monotonic flow towards either one of the two stable fixed points. It defines the critical density below which the repeated extension in the in-fashion process leads inexorably to the total disappearance of the B brand. For any odd reference group size it is located at given by \( p_c = \frac{1}{2} \) which
Figure 2: The fashion function $P_r(p_n)$ for $r = 101$ as function of $p$. The separator is located at 50% with the two attractors at 0 and 1.

gives the threshold to B invasion at exactly 50%. Starting from $p_0 < 50\%$ leads towards 0 while the flow leads to 1 for $p_0 > 50\%$. For instance in the case of $r = 101$ we get the following series starting from $p_0 = 0.40$, $p_1 = 0.02$ and $p_2 \simeq 0$. In the case of $r = 51$ we get the series is $p_0 = 0.40$, $p_1 = 0.07$ and $p_2 \simeq 0$.

Therefore the repeated in-fashion process produces the self-elimination of any proportion of an initial B brand as long as $p_0 < 50\%$. To be completed total disappearance only two reference levels are required. Getting closer to the unstable fixed point increases slightly the number of required reference levels. With $p_0 = 0.49$ the series are $p_1 = 0.42$, $p_2 = 0.5$, $p_3 \simeq 0$ and $p_1 = 0.44$, $p_2 = 0.21$, $p_3 \simeq 0$ and for respectively $r = 101$ and $r = 51$.

At this stage, the key issue to ensure full monopoly for the initial newcomer brand is a huge investment to guarantee a starting in fashion process with more than fifty percent of the population wearing its B brand. Any value less does make the all investment pure waste with the total disappearance of the brand. Such a condition put the level of success at an almost impossible task.
3 Counterfeits as a strategic tool

When individuals on the consumer side are anonymous and simply organised into undifferentiated large size reference groups, producers cannot recognise them. In this situation, allowing counterfeits is a dominant strategy, which will implicitly share the minimum level of investment needed to invade the whole market.

However, reaching a level below the threshold is just a waste of money. To ignore the existence of a threshold level can yield to a quite expensive strategy of penetration. On the other hand to pass the threshold is also a waste of money. And moreover in the real world the exact value of the threshold is unknown. This quite difficult situation in assessing the right level of investment leads to view allowing counterfeits as a strategic attitude.

It can indeed help in pushing the initial penetration above the critical threshold, for no extra cost. When individuals on the consumer side are anonymous and simply organised into undifferentiated reference groups, producers cannot recognise them. In this situation, the best strategy is to allow counterfeits, which will implicitly share the minimum level of investment needed to invade the whole market. Therefore, allowing counterfeits can be a useful strategy when producers are facing an anonymous market, to avoid a total waste of the initial investment.

For instance, considering an investment yielding some initial $p_0 < p_c$ leads as seen before to zero. At constrast allowing counterfeits to yield some $q_0$ of persons wearing the counterfeits produces an effective proportion of B wearing $p_0 + q_0$ such that now $p_0 + q_0 > p_c$. In that case the B will invade the market thanks to the counterfeits. Although it does hold the one hundred percent of the full market it does have make a profitable investment.

In this sense, counterfeits can be considered as a diffusion vector and then, be a useful tool for lowering the threshold.

But this tool can also have dangerous effect. For example, the counterfeit producer can invade the market, creators never know exactly until where the counterfeits will invest, the presence of a high number of counterfeits can make the brand less attractive for a certain part of the population. We will now explore whether creators can use other tools to decrease the level of initial investment.
4 Imitating your neighbours: smaller size groups

We have already shown that invading a market can have a high cost, or/and be an inefficient strategy when the threshold is not reached. In this case, creators may be tempted to allow counterfeits in order to decrease their level of investment. However such a strategy does have also a high cost. On this basis we suggest another strategic tool consisting in acting on the in-fashion frame itself.

The idea is to modify the value of the unstable threshold which is instrumental in determining the all or nothing investment to invade or disappear from a market. To shift the threshold from the 50% value towards a lower value for the invader means to increase it to the in-market brand. Splitting the unstable threshold value into two different values implies to introduce some asymmetry to favor the new coming brand. This can be done naturally since we already have introduced a bias in favor of the new brand. Indeed we assume that given a reference group, in case of a tie, equal A and B wearing, the in fashion process leads to choose the new brand as the tip for novelty.

However for large reference group sizes, the occurrence of a tie is very rare and does not have much effect on the overall dynamics. At constrast smaller reference groups will exhibit more often tie situations. Therefore to implement our new strategy requires to decrease the reference size groups. Such a change can be obtained by producing clear social signs to make people able to recognize each other. We are suggesting taking out from anonymity the imitating processes to turn it personalization. In others words make people choose whom they wish to resemble. To make our task easier, we talk about "neighbours".

4.1 From anonymity to neighbourhood

For a creator who wishes to invade the market but avoid counterfeits, an optimal strategy consists in breaking down the size of reference groups by a personalization of the in fashion process. The smaller the reference groups, the lower the investment level.

Having find out where to act to produce a disymmetry in favor of the incoming new brand, we now analyse our model in the case of maximum tie effect which occurs at the smaller social group exhibiting a tie, a group of 4 persons. If at time $t = 0$, 3 or 4 people in the group are $A$-dressed, then all
the 4 people end up \textit{A-dressed} at time $t = 0 + 1 = 1$. However if only 0 or 1 people are \textit{A-dressed}, then all 4 people end up \textit{B-dressed}. In case of a tie with 2 \textit{A-dressed} and 2 \textit{B-dressed}, a bias in favour of novelty results in the new brand B being adopted by the whole group. Accordingly putting $r = 4$ in Eq. (2) gives for the first reference group level in fashion result,

$$ p_1 = P_4(p_0) = p_0^4 + 4p_0^3(1-p_0) + 6p_0^2(1-p_0)^2 ,$$

at time $t = 1$, where as before $p_0$ is the initial proportion of B brand-wearing people at $t = 0$. The following three configurations \{(2 A, 2 B), (4 A, 3 B), (0 A, 4 B)\} leads to (0 A, 4 B). While the in fashion function $P_4(p_0)$ still have the two stable fixed points zero and one, the unstable fixed point is now located at

$$ p_c = \frac{5 - \sqrt{13}}{6} \simeq 0.23 ,$$

which put the threshold to B invasion at about 23%, a much lower value than 50% as shown in Figure (3).

At the same time to keep on the whole market the A brand must always share more than 77% of the market. Starting from $p_0 < 23\%$ leads towards
0 while the flow leads to 1 for $p_0 > 23\%$.

The above people, all locally (groups of four) wearing the same brand, constitute the first level of the hierarchy reference groups denoted level 1. Then the same in-fashion process is repeated, but now within reference groups of 16 people each. Again, each whole reference group of 16 people adopts the same brand, either A or B, according to a local majority rule. A tie situation still yields a B brand choice. In terms of density the same equation holds, with

$$p_2 = p_1^4 + 4p_1^3(1 - p_1) + 6p_1^2(1 - p_1)^2,$$

(5)

since with the three types of configurations (8 A, 8 B), (4 A, 12 B), (0 A, 16 B) which can be noted as follows: 4\{(2 A, 2 B), (1 A, 3 B), (0 A, 4 B}\}

Each additional increment of time increases the size of the reference group with a new in-fashion process leading to an homogenisation on brand A or B with the density at level $n+1$ given by

$$p_{n+1} \equiv P_4(p_n) = p_n^4 + 4p_n^3(1 - p_n) + 6p_n^2(1 - p_n)^2,$$

(6)

where $p_n$ is the proportion of B brand-wearing people at level $n$ with the following three configurations $4^n\{(2 A, 2 B), (1 A, 3 B), (0 A, 4 B)\}$.

To illustrate the quantitative changes produced by the smaller size effect, we first come back to the case of an initial $p_0 = 0.40$ which goes down to zero within two reference group levels for $r = 100$ and $r = 51$. Now in the case $r = 4$ not only the series does not go to zero but instead increases toward one, i.e., the full market invasion with $p_1 = 0.53, p_2 = 0.72, p_3 = 0.93$ and $p_4 = 1$. Only 4 reference groups levels are enough to total invasion. At the fourth level only 96 people are involved instead of the one million after 3 levels in the case $r = 100$.

But now, as the threshold is at 23% we can follow what happens with an initial investment accounting for instance to a B wearing proportion $p_0 = 0.25$. We have the series $p_1 = 0.26, p_2 = 0.28, p_3 = 0.32, p_4 = 0.38, p_5 = 0.48, p_6 = 0.66, p_7 = 0.89$, and $p_8 = 1$. Eight levels of reference groups are required for a total invasion of the market. An initial value $p_0 = 0.30$ yields $p_1 = 0.35, p_2 = 0.43, p_3 = 0.58, p_4 = 0.80, p_5 = 0.97$ and $p_6 = 1$ reducing the number of reference groups required to six.

One illustration of the different outcome resulting from breaking down the neighborhood is shown in Figures (4, 5, 6, 7) for a total population of 64 persons with 21 dressing an X item and 43 a O item, X being the new
one. Figure (4) shows the in-fashion process at work for one reference group embodying at once the whole population of the 64 persons. The result is the winning of the item O initially chosen by the majority.

Figures (5, 6, 7) shows the same initial population with now the existence of 3 discrete reference groups with respectively 4, 16 and 64 people. Figure (5) shows the introduction of the first level of neighboring reference group with four persons each. X and O wearing are present on the left side. Common behavior is completed on the right side with now 24 X and 60 O.

In Figure (6) new larger reference groups of 16 people each are activated as shown on the left side. On the right side common behavior has been completed with an equality between X and O, each item being chosen by 32 persons. Figure (7) exhibits the last reference group which embodies the whole population as shown on the left side. On the right side everyone is wearing an X item. The opposite outcome of Figure (5 showing how the creation of intermediate reference group levels has been able to reverse the outcome of the in-fashion process.

We can now see why it is important for a brand to multiply quality levels and forms of marketing in its "filière". This strategy could be seen as a classical process of differentiation. The wider the scale of products, the larger the set of potential consumers. But each level can be considered as a specific means of being seen by one or more members of a reference group.

4.2 Organizing the "filière"

For a creator who wishes to invade the market, an investment threshold exists, below which investing is useless, no matter what is the amount of investment. Investing more is also superfluous. The optimal strategy consists in hierarchically organizing the "filière".

At this stage, the key issue is to determine the number of reference levels needed to ensure full monopoly for the initial newcomer brand in the case of small group sizes. For large size, the threshold is at around 50% and the level number is very small but with huge number of people involved. It is also worth to notice that increasing the size to a larger group increases the threshold value toward that value of 50%. For instance we have $p_c = 0.35, 0.40, 0.42, 0.44, 0.45, 0.46, 0.46, 0.47$ for respectively $r = 6, 8, 10, 12, 14, 16, 18, 20$.  

13
We can now calculate analytically the critical number of reference group levels $n_c$ at which $p_{n_c} = \epsilon$ with $\epsilon$ being a very small number. This determines the level of confidence of the prediction of getting the whole market. One way to evaluate $n_c$ is to expand the in-fashion function $p_n = P_r(p_{n-1})$ around the unstable fixed point $p_c$,

$$p_n \approx p_c + (p_{n-1} - p_c)\lambda_r ,$$

(7)

where $\lambda_r \equiv \frac{dP_r(p_n)}{dp_n}|_{p_c}$ with $P_r(p_c) = p_c$. Rewriting the last equation as

$$p_n - p_c \approx (p_{n-1} - p_c)\lambda_r ,$$

(8)

we can then repeat the process to get

$$p_n - p_c \approx (p_0 - p_c)\lambda_r^n ,$$

(9)

from which we get the critical number of levels $n_c$ at which $p_n = \epsilon$. Taking
Figure 5: Left: First level of reference group with four persons each. X and O wearing are present. Right: Common behavior has been completed at first level reference group with 24 X and 40 O.

the logarithm on both sides of Eq. (8) gives

\[ n_c \approx -\ln(p_c - p_0) \ln \lambda_r + n_0 , \]  

(10)

where \( n_0 \equiv \frac{\ln(p_c - \epsilon)}{\ln \lambda_r} \). As we are dealing with integers, we have to take the integer part of Eq. (10). Putting \( n_0 = 1 \) yields pretty accurate results.

5 The effective optimal threshold

As it may turn rather costly and complicated to modify both the size reference groups and the associated number of levels we now evaluate the following strategic issue. Given \( n \) reference group levels of group size \( r \), what is the initial overall proportion of B wearing required to get everybody dressed in B?

It is worth keeping in mind the fact that the dynamics for A and B are not symmetric. Here, we focus on the in-fashion dynamics from the B point
Figure 6: Left: Second level reference group is activated to include groups of 16 persons each. Common behavior has not yet been completed.
Right: Common behavior has been completed with now 32 X and 32 O.

of view. To proceed, we invert Eq. (9) to obtain

\[
p_0 = p_c + (p_n - p_c)\lambda_r^{-n} .
\]

(11)

This yields two new operative critical thresholds. The first gives the value of initial market penetration below which B is sure to disappear from the market. It is obtained from Eq. (10) by inserting \(p_n = 0\) and it yields

\[
P_{D,r}^n = p_c(1 - \lambda_r^{-n}) .
\]

(12)

In parallel, setting \(p_n = 1\) gives the second threshold \(P_{I,r}^n\), above which the B brand invades the whole market. Combining Eqs. (11) and (12) gives

\[
P_{I,r}^n = P_{D,r}^n + \lambda_r^{-n} .
\]

(13)

which shows the appearance of a strategic domain for \(P_{D,r}^n < p_0 < P_{I,r}^n\). In this region B neither disappears totally nor takes over the whole market. There is a coexistence with \(p_n\) being at equilibrium and neither 0 nor 1. This
is therefore a coexistence region in which both brands are present. No one is sure of winning. However, as seen from Eq. (13), this coexistence region shrinks as a power law $\lambda r^{-n}$ of the number $n$ of hierarchical reference group levels. Having a small number of these levels raises the threshold for total invasion but simultaneously lowers the threshold for disappearance.

The above formulas are approximated, since we have neglected corrections in the vicinity of the stable fixed points. However they give the right quantitative behavior. To get a practical feeling of what Eqs. (12) and (13) mean in terms of numbers, let us illustrate them for the case $r = 4$ where we have $\lambda = 1.64$ and $p_{c, A} = p_c = \frac{5 - \sqrt{13}}{6}$. Considering 3, 4, 5, 6 and 7 level reference systems, $P_{D,r}^n$ is equals to respectively 0.41, 0.34, 0.30, 0.27 and 0.26. In parallel $P_{I,r}^n$ equals 0.18, 0.20, 0.21, 0.22 and 0.22. These series dramatically emphasize the massive effect of the in-fashion process.
6 Conclusion

In this paper, we have discussed the different investment tools a creator can use to invade a market, in other words to become the fashion. We use the concept and techniques of real space normalisation group to study the fashion diffusion process within hierarchical structures. We show that when potential consumers are organised into large, anonymous groups producers cannot recognize them. In this case, the level of investment making it possible to invade the market is certainly high, and it may be optimal to allow counterfeits. But allowing counterfeits can also be a dangerous tool. We then show that if creators could organize people into distinctive reference levels, this would diminish the threshold and make it possible to avoid counterfeits. Simmel [10] noted that the fashion market actually seems to be the joint result of an imitation process and a search for differentiation. We show that in creating different social leaders, producers enable consumers to differentiate themselves from each other by imitating their closest social leaders. To simplify our task, we have focused on the clothes fashion market. With slight modifications, our results could be extended to other kinds of fashion such as the fashion in research ideas, for example.

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