Hard exclusive production of a vector meson

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The processes of a light neutral vector meson, $V = \rho^0, \omega, \phi$, electroproduction and a heavy quarkonium, $V = J/\Psi, \Upsilon$, photoproduction are studied in the framework of QCD factorization. We derive a complete set of hard-scattering amplitudes which describe these processes at next-to-leading order (NLO).

1. INTRODUCTION

The strong interest in the processes of elastic neutral vector meson electroproduction on a nucleon,

\[ \gamma^*(q) N(p) \rightarrow V(q') N(p') , \]

where $V = \rho^0, \omega, \phi$, or heavy quarkonium $V = J/\Psi, \Upsilon$, is related with the possibility to constrain the gluon density in a nucleon\cite{112}.

The large negative virtuality of the photon, $q^2 = -Q^2$, or (in a case of quarkonium photoproduction) the heavy quark mass, $m$, provides a hard scale for the process which justifies the application of QCD factorization methods that allow to separate the contributions to the amplitude coming from different scales. The amplitude of light vector meson electroproduction is given by a convolution of the nonperturbative meson distribution amplitude (DA) and the generalized parton densities (GPDs) with the perturbatively calculable hard-scattering amplitudes\cite{8}.

The direct application of factorization theorem\cite{3} to the production of a heavy meson is restricted to the region of very large virtualities, $Q^2 \gg m^2$, where the mass of the heavy quark may be completely neglected. In contrast, in photoproduction or electroproduction at moderate virtualities the heavy quark mass provides a hard scale and the nonrelativistic nature of heavy meson is important. In this case, according to nonrelativistic QCD (NRQCD) which provides a systematic nonrelativistic expansion, a factorization formalism must be constructed in terms of matrix elements of NRQCD operators. They are characterized by their different scaling behavior with respect to $v$, the typical velocity of the heavy quark. In the leading approximation only the matrix element $\langle O_1 \rangle_V$ contributes, which describes in NRQCD the leptonic meson decay rate\cite{4}

\[ \Gamma[V \rightarrow l^+l^-] = \frac{2e^2 \pi \alpha^2}{3} \frac{\langle O_1 \rangle_V}{m^2} \left( 1 - \frac{8\alpha_S}{3\pi} \right)^2. \]

Here $\alpha$ is the fine-structure constant and $m$ and $e$ are the pole mass and the electric charge of the heavy quark ($e_c = 2/3$, $e_b = -1/3$) and $\alpha_S$ is the strong coupling constant.

In this contribution we present our recent results for the hard scattering amplitudes at next-to-leading order both for heavy quarkonium photoproduction\cite{5} and for light vector meson electroproduction\cite{6} processes. The account of the radiative correction to the hard scattering amplitudes allows to reduce the theoretical uncertainty of the factorization approach, the dependence on the factorization, $\mu_F$, and renormalization, $\mu_R$, scales, which is especially important at high energies, since in this case (i.e. in the small $x$ region) the dependence of the parton distributions on the factorization scale is very strong.

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2. QUARKONIUM PRODUCTION

In the leading order of the relativistic expansion the meson mass can be taken as twice the heavy quark pole mass, \((q')^2 = M^2\) and \(M = 2m\). The photon polarization is described by the vector \(e_\gamma\), \((e_\gamma, q) = 0\). The invariant c.m. energy is \(s_{\gamma p} = (q + p)^2 = W^2\). We define

\[
\Delta = p' - p, \quad P = \frac{p + p'}{2}, \quad t = \Delta^2, \\
(q - \Delta)^2 = (q')^2 = M^2, \quad \zeta = \frac{M^2}{W^2}, \quad (3)
\]

At \(|t| \ll M^2\) the factorization formula reads

\[
\mathcal{M} = \frac{4\pi \sqrt{4\pi \alpha}}{N_c \xi} \left( \frac{(O_1)/V}{m^3} \right)^{1/2} \int_{-1}^{1} dx \times \\
\left[ T_g(x, \xi) F^g(x, \xi, t) + T_q(x, \xi) F^{g,S}(x, \xi, t) \right], \\
F^{q,S}(x, \xi, t) = \sum_{q=u,d,s} F^q(x, \xi, t). \quad (4)
\]

as the sum of the gluon, \(F^g(x, \xi, t)\), and the quark, \(F^q(x, \xi, t)\), GPDs contributions. GPDs are defined as a functions which parametrized the matrix elements of the renormalized light-cone quark and gluon operators. The polarization vector of quarkonium is \(e_V\), variable \(\xi = \zeta/(2 - \zeta)\) parametrizes the non vanishing longitudinal momentum transfer in the process.

The hard scattering amplitude \(T_g(x, \xi)\) (or \(T_q(x, \xi)\)) represents essentially the on-shell parton amplitude for the scattering of a pair of gluons (quarks) which are collinear to the proton momentum and have the fractions \((x + \xi)/(1 + \xi)\) and \((x - \xi)/(1 + \xi)\). Calculated in the dimensional regularization method these one-loop amplitudes contain poles, the infrared collinear and the ultraviolet singularities. The full renormalization procedure includes mass counterterm diagrams, the renormalization of the heavy quark field and the renormalization of the strong coupling. The factorization of collinear singularities is achieved by the replacement of the bare GPDs by the renormalized ones. This procedure leads to the finite results for \(T_g(x, \xi)\) and \(T_q(x, \xi)\) at NLO

\[
T_g(x, \xi) = \frac{\alpha_S^2(\mu_R)C_F}{2\pi} f_q \left( \frac{x - \xi + i\varepsilon}{2\xi} \right), \quad (5)
\]

\[
T_q(x, \xi) = \frac{\xi}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \times \left[ \alpha_S(\mu_R) + \frac{\alpha_S^2(\mu_R)}{4\pi} f_g \left( \frac{x - \xi + i\varepsilon}{2\xi} \right) \right], \quad (6)
\]

here \(C_F = 4/3\). The functions \(f_q\) and \(f_g\) (see [5]) contain terms \(\sim \ln(m^2/\mu^2_F)\).

The dependence of NLO hard scattering amplitudes on \(\mu_F\) compensates partially the effect of the evolution of GPDs with factorization scale (the dependence of \(T_g\), \(T_q\) and GPDs on \(\mu_F\) in [6] is not shown for shortness). That leads to the substantial reduction of the scale ambiguity of the theoretical predictions in NLO in comparison with leading order (LO), see Fig. 1

![Figure 1](image_url)

Figure 1. The cross section of the \(\Upsilon\) photoproduction; the predictions at LO (upper figure) and NLO (lower figure) for the scales \(\mu_F = \mu_R = 1.3, 7\) GeV. The data are from ZEUS [7] and H1 [8]. For the \(t\)– dependence we assumed exponential with the slope parameter \(b = 4.4\) GeV\(^{-2}\).
3. LIGHT MESON PRODUCTION

We refer the reader to [6,9] for the complete set of analytical results and for some numerical predictions for the light vector meson electroproduction at NLO. The new features in comparison to quarkonium production are the contribution of the quark GPD at LO and the appearance of a light meson DA in the factorization formulae (instead of NRQCD matrix element).

Similarly to the quarkonium production case we observe that in HERA kinematic range the NLO corrections are large and mostly of the opposite signs than the corresponding Born terms, consequently the final values of the amplitude are the result of a strong cancelation between the LO and NLO parts. Leading contribution to the NLO correction comes from the integration region $\xi \ll |x| \ll 1$, simplifying the gluon hard-scattering amplitude in this limit we obtain the estimate

$$M_{\gamma^* L N \to V L N} \approx$$

$$\frac{-2i \pi^2 4\pi \alpha \alpha_S f_V Q \sqrt{4\pi \alpha S N c}}{N_c Q \xi} \int_0^1 dz \frac{\phi_V(z)}{z(1-z)} F^g(\xi, \xi, t)$$

$$+ \frac{\alpha_S N_c}{\pi} \ln \left( \frac{Q^2 (1-z)}{\mu_F^2} \right) \int_\xi^1 dx \frac{F^g(x, \xi, t)}{x},$$

here $f_V$ is a meson coupling constant known from $V \to e^+e^- \to e^+e^-$ decay, $N_c = 3$, factor $Q_V$ depends on the meson flavor content [9], $\xi = x_B^H/(2-x_B^H)$.

Given the behavior of the gluon GPD at small $x$, $F^g(x, \xi, t) \sim const$ unless one chooses the value of the factorization scale sufficiently lower than the kinematic scale. For the asymptotic form of meson DA, $\phi_V^{as}(z) = 6z(1-z)$, the last term in [9] changes the sign at $\mu_F = \frac{Q}{\xi}$, for the DA with a more broad shape this happens at even lower values of $\mu_F$.

Our leading twist results, see Fig. 2, obtained with NLO hard-scattering amplitudes and NLO GPDs [10] (which were adjusted to describe HERA deeply virtual Compton scattering data) are in qualitative agreement with the measured at HERA $\rho$ meson electroproduction cross section.

![Figure 2. $\sigma(Q^2, W = 95\text{GeV})$ as a function of $Q^2$ for two NLO GPDs: MRST2001 (M) and CTEQ6M (C). The factorization scale $\mu_F = Q$. For the solid lines $\mu_R = \mu_F$, for the dashed lines $\mu_R = Q/\sqrt{\xi}$. The data points are taken from [11].](image)

Without account of NLO terms the predictions would be substantially above the data.

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