Controllable non-reciprocal transmission of single photon in Möbius structure

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We propose a controllable non-reciprocal transmission model. The model consists of a Möbius ring, which is connected with two one-dimensional semi-infinite chains, and with a two-level atom located inside one of the cavities of the Möbius ring. We use the method of Green function to study the transmittance of a single photon through the model. The results show that the non-reciprocal transmission can be achieved in this model and the two-level atom can behave as a quantum switch for the non-reciprocal transport of the single photon. This controllable non-reciprocal transmission model may inspire new quantum non-reciprocal devices.

I. INTRODUCTION

According to Maxwell’s electromagnetic equations, in most conditions, the propagation of electromagnetic wave is reciprocal, that is, the transmission of light from A to B is the same as that from B to A [1]. However, because reciprocity greatly limits our processing of optical signals, we need to find a way to break this symmetry. With the development of physics, it has been recognized that quantum physics can provide a fertile ground where non-reciprocity concepts can be fruitfully exploited [2] and metamaterials with negative refraction can be produced [3]. Optical non-reciprocity refers to the phenomenon that the propagation of light from one direction is different from the propagation from the opposite direction. It has a great important role in protecting sensitive optical components [4], constructing quantum networks, and quantum signal processing [5]. Therefore, it is necessary to explore the optical devices which may achieve non-reciprocity [6–10]. Traditional non-reciprocal devices mainly include Faraday isolators which are based on the magneto-optical effect [11] and some micro-resonators which utilize the nonlinearity of materials [12]. However, the non-reciprocal devices which are based on the magneto-optical effect usually have large size because of the property of the materials. Moreover, non-linear devices may be still reciprocal for some weak signals [13]. Therefore, people have been exploring alternative strategies to realize non-reciprocal transmission, for example, the use of atomic thermal motion [14] and optical power coupling [15], etc. Here, we hope to find an effective, convenient and safe device with non-reciprocity.

In recent years, there has been a surge of interest in the theoretical and experimental study of the topological properties of quantum system [16–18], topologically-nontrivial structures has gradually attracted our attention. Furthermore, the molecule-based devices have motivated many investigations because of their application prospect [19–22]. Because of the unusual properties and potential applications [22], Möbius structure has recently been under great focus both experimentally and theoretically, for example, the realization of negative refraction by using Möbius structure [24] and observing various topological effects in Möbius structure [25]. Molecules with different structures can exhibit various novel physical properties. Previous work have found that the transmission through Möbius ring exhibits obvious differences from that of ordinary ring—the transmission through the Möbius ring is suppressed in one direction [25]. As Möbius structure has shown its development and application potential, this inspires us to use Möbius structure to realize non-reciprocity.

In this paper, we propose a controllable non-reciprocal transmission model with single photon incidence. The model consists of a Möbius ring connecting two semi-infinite chains. Both Möbius ring and semi-infinite chains are composed of a series of cavities. And a two-level atom is embeded in one of the cavities of the Möbius ring. This atom behaves as a quantum switch that can control the non-reciprocal transport of the single photon. Our research may inspire new quantum non-reciprocal devices.

The paper is organized as follows. In Sec. II, we introduce our model firstly, and then we consider two cases. One of the cases corresponds to that a two-level atom is added into one of the cavities of the Möbius ring. And the other one case corresponds to that there is no atom in the model. Then we calculate the transmittance of a single photon through the system by using Green function. In Sec. III we show and analyze the results of our calculation. Finally, we discuss the prospect and conclusions in Sec. IV.

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II. MODEL

In this paper, we consider a Möbius ring coupled with two semi-infinite chains, and a two-level atom is embedded in one of the cavities of the Möbius ring as shown in Fig. 1. The photon can transport through the chains and the tunnel between the chains and the Möbius ring. Let $a_j^\dagger (a_j)$ and $b_j^\dagger (b_j)$ be the creation(annihilation) operator of the $j$th cavity of the upper and lower ring of Möbius ring, respectively.

The Hamiltonian of the Möbius ring reads

\[
H_M = \sum_{j=0}^{N-1} \left[ \varepsilon (a_j^\dagger a_j + b_j^\dagger b_j) - V a_j^\dagger b_j - \xi (a_j a_{j+1} + b_j b_{j+1}) \right] + \text{h.c.},
\]

(1)

where $V$ represents the coupling strength between $a_j$ and $b_j$ sites, $\xi$ is the coupling strength between two adjacent cavities, and $\varepsilon$ represents the frequency of the cavity in the Möbius ring. We assume that there are $2N$ cavities located at the upper and lower ring of Möbius ring. And the Möbius ring has boundary condition that $a_N = b_0$, $b_N = a_0$, which is different from ordinary rings. This shows that the Möbius ring does not obey the periodic boundary condition. By using the local unitary transformation $c_R = \frac{1}{\sqrt{2}} (e^{-i\phi / 2} a_j - e^{i\phi / 2} b_j)$ and $c_L = \frac{1}{\sqrt{2}} (a_j + b_j)$, the Hamiltonian becomes

\[
H_M = \sum_{j=0}^{N-1} \left[ \varepsilon (c_R^\dagger c_R + c_L^\dagger c_L) + e^{i\phi / 2} c_R^\dagger c_L - c_L^\dagger c_R - \xi (c_R^\dagger c_{R+1} + c_L^\dagger c_{L+1}) \right] + \text{h.c.},
\]

(2)

where $\phi = \frac{2\pi}{M} - \frac{\pi}{2}$ and $\phi_j = j \frac{2\pi}{M}$. The operator $c_R^\dagger (c_L^\dagger)$ satisfies the periodic boundary condition. Then we use Fourier transformation $C_k = \frac{1}{\sqrt{N}} \sum_j e^{i\eta j} c_j^\dagger$ to diagonalize the Hamiltonian and transform the Hamiltonian from real space to momentum space. By Fourier transformation, the Hamiltonian becomes

\[
H_M = \sum_k (E_{k\uparrow} C_k^\dagger C_k + E_{k\downarrow} C_k^\dagger C_k),
\]

(3)

where $E_{k\uparrow} = \varepsilon + V - 2 \xi \cos(k - \frac{\pi}{M})$, $E_{k\downarrow} = \varepsilon - V - 2 \xi \cos(k)$, $k = j\delta$. In our calculation, we set $\varepsilon = 0$. According to the above calculation, we can draw the energy band diagram of the Möbius ring, as shown in Fig. 2. From Fig. 2 we can see that the lower energy band $E_{k\downarrow}$ is axially symmetric with respect to $k = 0$. However, compared with the lower band, the symmetry axis of the upper band $E_{k\uparrow}$ is shifted.

The Hamiltonian of the semi-infinite chains is

\[
H_C = \omega \sum_{l=-\infty}^{l=+\infty} c_l^\dagger c_l + \sum_{l=1}^{\infty} c_l^\dagger c_{l+1} - \zeta [ \sum_{l=-\infty}^{l=+\infty} c_l^\dagger c_{l+1} + \sum_{l=1}^{\infty} c_l^\dagger c_{l+1}],
\]

(4)

where $\omega$ describes the frequency of the cavity in the chain, $\zeta$ is the coupling strength between two adjacent cavities. By using Fourier transformation $c_k = \frac{1}{\sqrt{M}} \sum_j e^{ikj} c_j$, the Hamiltonian can be rewritten as

\[
H_C = \sum_k [\omega - 2 \xi \cos(k)] c_k^\dagger c_k,
\]

(5)

where $M$ is the total number of $l$.

The interaction Hamiltonian of the two semi-infinite chains and the Möbius ring is

\[
H_{MC} = \kappa (c_{-1} a_{L'} + c_{l} a_{R'} + \text{h.c.}),
\]

(6)
where $\kappa$ represents the coupling strength between the two semi-infinite chains and the Möbius ring, the subscript $L'$ ($R'$) represents the connection cavity between the left (right) chain and the Möbius ring. In the same way, by Fourier transformation we can also transform this Hamiltonian into the momentum space.

When a two-level atom is added into the nth cavity of the Möbius ring, the total Hamiltonian of the system can be expressed as

$$H_S = H_M + H_C + H_{MC} + H_A + H_{AM},$$  \hspace{1cm} (7)\]

where $H_A$ is the Hamiltonian of the atom and $H_{AM}$ is the interaction Hamiltonian of the atom and the Möbius ring. Here

$$H_A = \Omega_A a_d^\dagger a_n,  \hspace{1cm} (8)$$

$$H_{AM} = \gamma (a_d^\dagger a_n + a_n^\dagger a_d),  \hspace{1cm} (9)$$

with $a_n = \frac{1}{\sqrt{2}}(e^{\frac{i\phi}{\sqrt{\pi}}} C_{k\uparrow} + c_j)$, where $\Omega_A$ describes the atom's frequency, and $\gamma$ represents the coupling strength between the atom and the Möbius ring and the subscript $n$ represents that the atom is added into the nth cavity. After Fourier transformation we can obtain that

$$H_{AM} = \gamma \sqrt{2} \sum_k [\frac{\sin(\xi_k)}{\xi_k}] c_{k\uparrow}^\dagger C_{k\downarrow} + C_{k\downarrow}^\dagger c_{k\uparrow}] + d_d^\dagger \frac{\sin(\xi_k)}{\xi_k} [\frac{\sin(\xi_k)}{\xi_k}] C_{k\downarrow} + C_{k\downarrow}].$$  \hspace{1cm} (10)$$

We use the method of Green function to analyze the transmission of the single photon through the system\cite{31}. The Green Function is

$$G = \frac{1}{(E + i0^+)(I - H_a - \Sigma_{L,R})},$$  \hspace{1cm} (11)$$

where $E$, $H$, $\Sigma_{L,R}$ correspond to the photon’s incident frequency, the Hamiltonian of the Möbius ring and the self-energies, respectively. By Fourier transformation, the self-energy in the momentum space can be expressed as

$$\Sigma_{L(R)} = -i\kappa e^{ik'} \sum_k \frac{1}{2N} \sin(e^{i\phi} \frac{\pi}{2N}) (C_{k\uparrow} + C_{k\downarrow}) ,$$  \hspace{1cm} (12)$$

Then the level broadenings can be expressed as

$$\Gamma_1 = i [\Sigma - \Sigma']_L,  \hspace{1cm} (13)$$

$$\Gamma_2 = i [\Sigma - \Sigma']_R.  \hspace{1cm} (14)$$

The transmission coefficient can be obtained by\cite{31}

$$T = \text{Tr}[G_1 G_2 G_1^\dagger].$$  \hspace{1cm} (15)$$

III. RESULTS

We first consider the case that the photon is incident from the upper band of the Möbius ring and the system without atoms. According to Eq. (15), we plot the photon’s transmittance $T$ with respect to the momentum $k$, as shown in Fig. 3. Fig. 3 (a1), (a2), (a3) describe the change of transmittance $T$ with $k$ when $N$ is taken as 3, 5, 7, respectively. In these figures, the connection cavity between the left (right) chain and the Möbius ring is $a_{L'} = a_0$ ($a_{R'} = a_{N-1}$). Fig. 3 (b1), (b2) correspond to $N = 4$ and $N = 6$, respectively. In these two figures, the connection cavity between the right chain and the Möbius ring is $a_{R'} = a_{N-1}$. The red solid line and blue dashed line represent two opposite photon incident directions ($k > 0$ or $k < 0$). From Fig. 3(a) we can see that when $N$ is an odd number (3, 5, 7) and the photon is incident from the upper band that the non-reciprocity occurs. This is because the upper band of the Möbius ring is asymmetric with respect to $k = 0$. As shown in Fig. 2, it is also shows that when $N$ is an even number, there is non-reciprocity at $k = \frac{\pi}{2N}$ and with the increase of $N$, the effect of non-reciprocity decreases. This is because when $N$ decreases, the shift of the symmetry axis of the upper energy band $(E_{k\uparrow} = \varepsilon + V - 2\xi \cos(k - \frac{\pi}{2N}))$ will increase, and the asymmetry of Möbius ring will become more obvious.

Moreover, from Fig. 3(b1), (b2) we can find that when $N$ is an even number and the connection cavity between the right chain and the Möbius ring is $a_{R'} = a_{N-1}$, no matter what the value of $k$ is, there is no non-reciprocity. The insets of Fig. 3 (b1), (b2) reduce the range of $T$ values. From the insets, we can see that the difference between the two transmission curves can be seen only when $T$ is reduced to about the order of $10^{-10}$, and there is still no non-reciprocity at the point $k = \frac{j\pi}{2}$ with $j = 0, 1, ..., N - 1$. Based on Fig. 3 (b2), we change the connection cavity between the right chain and the Möbius ring to $a_{R'} = a_{N-1}$, and the curve of $T$ versus $k$ is shown in Fig. 3 (b3). We can see that there is obvious non-reciprocity in this case. We can explain this phenomenon from the structure of the model. When $N$ is an even number and the connection cavity between the left (right) chain and the Möbius ring is $a_{L'} = a_0$ ($a_{R'} = a_{N-1}$), the structure of the model is symmetric, so there is no clear non-reciprocal behavior in this case, cf. Fig. 3 (b1), (b2). When we change the connection cavity to $a_{R'} = a_{N-1}$, it is equivalent to breaking the structural symmetry of the model, so there is non-reciprocity, cf. Fig. 3 (b3).

Then we consider the case that the photon is incident from the lower band of the Möbius ring and the system without adding atoms. Still keep the connection cavity of the left and right chains and the Möbius ring as $a_{L'} = a_0$, $a_{R'} = a_{N-1}$. The calculation results are shown in Fig 4. We can find that when photon is incident from the lower band, no matter weather $N$ is an odd or even number, there is no non-reciprocity under different $k$ values. This is mainly because the lower band of the
where they represent two opposite photon incident directions. \(a_4\) and \(k > 0\) and the blue dashed line represents the case of \(N = 4\), respectively. And in these two cases, the connection cavity between the left chain and the M"obius ring is \(a_{L'} = a_0\), and that between the right chain and the M"obius ring is \(a_{R'} = a_{N-1}\). (b1), (b2) correspond to \(N = 6\) and the connection cavity between the right chain and the M"obius ring change to \(a_{R'} = a_{N-1}\). The red solid line represents the case of \(k > 0\) and the blue dashed line represents the case of \(k < 0\), where they represent two opposite photon incident directions. 

M"obius ring is symmetric with respect to \(k = 0\) as shown in Fig. 2.

Then, we add a two-level atom into one of the cavities of the M"obius ring. We plot the transmittance \(T\) of single photon with respect to \(\Delta\), where \(\Delta\) represents the detuning between the frequency of incident photon and the frequency of the atom. In Fig. 5, we consider the case that the photon is incident from the upper band of the M"obius ring and \(N\) is taken as an odd number. For different \(N\), we take the momentum of the incident photon as \(k = \pm \frac{\pi}{N(N-1)}\). The positive and negative signs represent the different incident directions of the photon. Fig. 5(a) corresponds to the situation that the atom is added into the connection cavity between the right chain and the M"obius ring, that is, \(a_n = a_{R'} = a_{N-1}\). Fig. 5(b) corresponds to the case that the connection cavity between the right chain and the M"obius ring unchanged, which is \(a_{R'} = a_{N-1}\), but the atom is added into the cavity \(a_n = a_{1}\). Fig. 5(c) corresponds to that the cavity of the atom is added is unchanged, which is \(a_{R'} = a_{N-1}\), but the connection cavity between the right chain and the M"obius ring change to \(a_{R'} = a_{N+1}\) from Fig. 5(a) we can see that for different \(N\), except for \(\Delta = 0\), there is non-reciprocity in other positions, and the smaller \(N\) is, the stronger the non-reciprocity is. The red line in the figure corresponds to the cases that
We think that the reason for this phenomenon may be non-reciprocity occurs, as shown in the inset of Fig. 6. When the position of the atom is changed from one of the cavities of the Möbius ring to the right chain, the system will also affect the non-reciprocity of the system. Moreover, different positions of the atom will bring different results. In addition, the atom is changed from \( a_n = a_{N/2} \) to \( a_n = a_1 \), we can see that for \( N = 7, 9, 13 \), the transmission corresponding to \( k = \frac{(N-1)\pi}{N} \) is no longer zero under different \( \Delta \) values, and the non-reciprocity also appears at the point of \( \Delta = 0 \). Comparing Fig. 6(a) with Fig. 6(b), we can see that changing the position of the atom will affect the non-reciprocity of the system. When the position of the atom is changed from \( a_n = a_{N/2} \) to \( a_n = a_1 \), we can see that for \( N = 7, 9, 13 \), the transmission corresponding to \( k = \frac{(N-1)\pi}{N} \) is no longer zero under different \( \Delta \) values, and the non-reciprocity also appears at the point of \( \Delta = 0 \). Similarly, comparing Fig. 6(a) with Fig. 6(c), we can see that changing the connection cavity between the right chain and the Möbius ring will also affect the non-reciprocity of the system. Fig. 6(c) also shows that for \( N = 3, 7 \), the transmission corresponding to \( k = \frac{(N-1)\pi}{N} \) is no longer zero under different \( \Delta \) values, and the non-reciprocity also appears at the point of \( \Delta = 0 \). According to Fig. 6 we can conclude that adding a two-level atom into one of the cavities of the Möbius ring will affect the non-reciprocity of the system. Therefore, the atom can still be used as a quantum switch to control the non-reciprocity of the system.
IV. CONCLUSION

In this paper, we explore the method of realizing non-reciprocity in the system based on Möbius structure. We first analyze the transmission of a single photon through the system without atoms. We find that when the photon is incident from the upper band of the Möbius ring and the number of cavities in the Möbius ring is odd, the non-reciprocal transmission can be realized, but not when the number of cavities is even. We also find that with the increase of \( N \), the effect of non-reciprocity decreases, cf. Fig. 4(a). This is because when \( N \) decreases, the shift of the symmetry axis of the upper energy band \( E_{k'} \) will increase, and the asymmetry of the Möbius ring will become more obvious. Then, we consider the case that the number of cavities in the Möbius ring is even. By calculation, we find that when \( N \) is an even number and the connection cavity between the right chain and the Möbius ring is changed to \( a_{k'} \neq a_{k} \), the non-reciprocity occurs, cf. Fig. 4(b3). The reason for this phenomenon can be explained from the structure of the model, that is, the change of the connection cavity between the right chain and the Möbius ring destroys the structural symmetry of the model. However, because the lower band of the Möbius ring is symmetric with respect to the momentum \( k = 0 \), whether \( N \) is an even or odd number, the non-reciprocal transmission can not be found when the photon is incident from the lower band, cf. Fig. 4(b2). We then consider the case that a two-level atom is added into one of the cavities of the Möbius ring. By analyzing the photon’s transmittance \( T \) with respect to detuning between the frequency of the incident photon and the frequency of the atom, we find that adding a two-level atom into one of the cavities of the Möbius ring will affect the non-reciprocity of the system, and different adding positions of the atom will bring different results. Therefore, the atom can act as a quantum switch to control the non-reciprocity of the system. We also show that when \( N \) is an even number and the symmetry of the model structure is kept, i.e. \( a_{k'} = a_{k} \), adding a two-level atom into one of the cavities of Möbius ring \( (a_{k} \neq a_{k'}) \) leads to the non-reciprocity of the system. Therefore, we can conclude that no matter whether \( N \) is an odd or even number, the atom can be used as a quantum switch to control the non-reciprocity of the system. This controllable non-reciprocal transmission model may inspire new quantum non-reciprocal devices.

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