Predictive Model for Radiatively Induced Neutrino Masses and Mixings with Dark Matter

Michael Gustafsson, 1 Jose M. No, 2 and Maximiliano A. Rivera 3

1 Service de Physique Théorique, Université Libre de Bruxelles, B-1050 Bruxelles, Belgium
2 Department of Physics and Astronomy, University of Sussex, BN1 9QH Brighton, United Kingdom and
3 Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

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A minimal extension of the standard model to naturally generate small neutrino masses and provide a dark matter candidate is proposed. The dark matter particle is part of a new scalar doublet field that plays a crucial role in radiatively generating neutrino masses. The symmetry that stabilizes the dark matter also suppresses neutrino masses to appear first at three-loop level. Without the need of right-handed neutrinos or other very heavy new fields, this offers an attractive explanation of the hierarchy between the electroweak and neutrino mass scales. The model has distinct verifiable predictions for the neutrino masses, flavor mixing angles, colliders and dark matter signals.

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The existence of a large amount of nonbaryonic dark matter in the Universe and the observation of nonzero neutrino masses may be regarded as the most direct and compelling evidence of particle physics beyond the standard model (SM). However, both the origin of neutrino masses and the nature of dark matter are still unknown. A scenario of being able to incorporate both phenomena in a unified framework would then be very attractive.

One of the best motivated dark matter scenarios is that of stable weakly interacting massive particles (WIMPs), produced as a thermal relic from the early Universe. Among the simplest realizations of this WIMP scenario is the inert doublet model [1, 2]. The SM is extended by a scalar doublet $\Phi_2$, and the dark matter scalar is made stable due to an exact $Z_2$ symmetry under which the new field has odd parity. The inert doublet model is currently constrained by results from dark matter searches as well as by particle collider data, but still a large region of its parameter space is allowed.

From the perspective of neutrino physics, currently the most popular way to generate small neutrino masses is the see-saw mechanism (see [3] for a review). In its simplest variant, it postulates the existence of very massive $SU(2)_L \times U(1)_Y$ singlet right-handed neutrinos. Other realizations involve the existence of very heavy scalar or fermionic triplets. Although elegant, this mechanism is difficult (if not impossible) to test, as the masses of the new states are typically much larger than can be experimentally probed.

Small neutrino masses can also be generated via radiative corrections. This has been explored by explicit lepton number violation in extensions of the scalar sector of the SM. As opposed to the see-saw mechanism, this approach generates small neutrino masses without relying on new particles at a very high energy scale. One simple realization of this idea is the Zee model [4], in which the SM field content is enlarged by a second scalar doublet $\Phi_2$ and a charged scalar singlet $S^\pm$. Another simple scenario is the Zee-Babu model [5], which replaces the scalar doublet $\Phi_2$ in the Zee model by a doubly charged singlet scalar field $\rho^{++}$.

However, these scenarios of radiative neutrino mass generation in [1] and [2] (together with many others, such as [6, 7] for interesting scenarios including right-handed neutrinos and dark matter particle candidates). This is for example the case in the Zee model if an odd $Z_2$ parity is assigned to $\Phi_2$. Finding a unified scenario for radiative neutrino mass generation and a dark matter particle candidate is then a nontrivial task.

In this Letter we construct a minimal model, that generates neutrino masses radiatively and provides a stable dark matter candidate, via an extended scalar sector with an exact $Z_2$ symmetry. We do this by adding to the SM two scalar singlets, $\rho^{++}$ and $S^\pm$, and a scalar doublet $\Phi_2$ with masses around the electroweak (EW) scale. The fields $S^\pm$ and $\Phi_2$ have odd $Z_2$ parity (while all other fields do not transform under this symmetry), and therefore a variation of the mentioned inert doublet model of dark matter is automatically embedded into the scenario. Due to the $Z_2$ symmetry and the field content of the model, Majorana neutrino masses are first generated at the three-loop level, naturally explaining the large hierarchy $m_\nu/v \sim 10^{-13}$ as due to the loop suppression $(g^2/16\pi^2)^3 \sim 10^{-13}$ ($g$ being an EW-sized coupling and with all masses at the EW scale $v$). This scenario then provides an intrinsic and interesting link between the stability of the dark matter candidate and the smallness of the neutrino mass scale.
I. A MODEL FOR NEUTRINO MASSES.

In addition to the SM fields, the model includes two $SU(2)_L$ singlet scalars (singly and doubly charged) $S^+$ and $ρ^{++}$, and a scalar doublet $Φ_2$. We introduce a $Z_2$ symmetry under which the $Φ_2$ and $S^+$ fields are odd, whereas $ρ^{++}$ and the SM fields are even. The $Z_2$ symmetry should be unbroken after EW symmetry breaking, so that the lightest $Z_2$-odd state remains stable and can provide a dark matter particle candidate. Given the symmetry and particle content of the model, the Lagrangian will include the following relevant terms leading to lepton number violation:

$$- ∆L = \frac{λ_5}{2} \left( Φ_1^† Φ_2 \right)^2 + κ_1 Φ_2^† iσ_2 Φ_1 S^- + κ_2 ρ^{++} S^- S^- + ξ Φ_2^† iσ_2 Φ_1 S^+ ρ^{--} + C_{ab} ξ_{IR} b_R ρ^{++} + H.c. \quad (1)$$

Here, $a, b$ denote family indices of the right-handed charged leptons $ℓ_R$, and the Yukawa couplings $C_{ab}$ form a symmetric and complex matrix, allowing for charge-parity (CP) violation in the leptonic sector.

The SM scalar doublet $Φ_1$ and the inert scalar doublet $Φ_2$ can in the unitary gauge be written as

$$Φ_1 = \frac{1}{√2} \begin{pmatrix} h \\ 0 \end{pmatrix}, \quad Φ_2 = \frac{1}{√2} \begin{pmatrix} Λ^+ \\ H_0 + i A_0 \end{pmatrix}, \quad (2)$$

where $v ≃ 174$ GeV is the vacuum expectation value of $Φ_1$. After EW symmetry breaking, and for $κ_1 ≠ 0$, the charged states $Λ^+$ and $S^+$ will mix (the mixing angle being $β$), giving rise to two charged mass eigenstates

$$H_1^+ = s_β S^+ + c_β Λ^+, \quad H_2^+ = c_β S^+ - s_β Λ^+, \quad (3)$$

with $s_β, c_β = \sin β, \cos β$, respectively. A convenient set of independent variables may be given by the five new scalar masses $m_ρ, H^0, A^0, H^+_1, H^-_1$, the mixing angle $β$ and the couplings $ξ$ and $κ_2$. All coefficients in the scalar potential should be chosen within their perturbative regime and to make the potential preserve vacuum stability [10] and fulfill dark matter constraints.

The Lagrangian in Eq. (1) breaks lepton number explicitly by two units $^1$, which generates a Majorana mass for the left-handed neutrinos. With the viable matter content neutrino masses can never be generated at one-loop order and the $Z_2$ symmetry precisely forbids all terms that would have generated neutrino masses at two-loop order. Therefore the leading contributions to neutrino masses appear first at three loops – through the “cocktail diagram” shown in Figure 1.

In the basis where charged current interactions are flavor diagonal and the charged leptons $e, μ, τ$ are mass eigenstates, the summed contributions of the six different finite three-loop diagrams shown in Figure 1 (coming from $H^+_1, A_0$ and $H_0$ running in the loop) give the Majorana neutrino mass matrix:

$$m_{ν^μ} = C_{ab} x_a x_b \frac{s_{2β}}{16π^2} (A_1 I_1 + A_2 I_2), \quad (4)$$

where $s_{2β} = \sin(2β), x_a = m_a / v$ for $a = e, μ, τ$, and

$$A_1 ≃ \frac{5M_4^4}{v^6} (∆m_2^0)^2 (∆m_3^0)^2 × ξ \frac{m_ρ}{m_0} \frac{m_0}{m_{H_0}^{1/2}} \frac{m_{H_0}}{m_0} \frac{m_{H_0}}{m_0} \frac{m_{H_0}}{m_0}, \quad (5)$$

$$A_2 ≃ -\frac{29M_4^4}{v^6} (∆m_2^0)^2 (∆m_3^0)^2 × ξ \frac{m_ρ}{m_0} \frac{m_0}{m_{H_0}^{1/2}} \frac{m_{H_0}}{m_0} \frac{m_{H_0}}{m_0} \frac{m_{H_0}}{m_0}. \quad (6)$$

The notations introduced are that $∆m_0^2 = m_0^2 - m_{A_0}^2 - m_{H_0}^2$ and $∆m_2^0 = m_2^0 - m_2^{1/2} - m_2^{1/2},$ as well as that $m_0^2 = m_0^2 + m_{H_0}^2$ and $m_2^0 = m_2^{1/2} + m_2^{1/2}$. The factors $I_{1,2}$ are dimensionless $O(1)$ numbers emerging from three-loop integrals after generic factors have been factored out. Their exact values depend on the specific mass spectrum, and we have made the full 3-loop calculation [11] with the help of SecDec [12] for numerical integrations. The normalization and the specific mass scaling with $m_0, m_0$ and $m_+$ in the above equations are found empirically; determined by fits to scans over model parameters around our benchmark point (presented below). This neutrino mass formula is typically accurate to within 30% when input parameters are allowed to vary up to at least a factor of a few from the benchmark point.

The proportionality of $m_{ν^μ}$ to the mass differences $Δm_2^0$ and $Δm_3^0$ signals a Glashow-Iliopoulos-Maiani-like (GIM-like) mechanism [13] at play in Eq. (1), which can be easily understood noticing that $Δm_2^0 ∝ λ_5$ and $Δm_3^0 ∝ κ_1$. In the limit $λ_5 → 0$ the Lagrangian in Eq. (1) conserves the lepton number and no Majorana neutrino mass can be generated, while in the limit $κ_1 → 0$, the

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1 The lepton number is conserved if either $C_{ab}$ or $λ_5$ vanish. If $κ_1 = 0$, the lepton number is still violated for nonvanishing $κ_2$ and $λ_α$, but the leading contribution to neutrino masses appears at the five-loop level.
leading contribution to $m_{\nu_{ab}}$ will appear at a higher loop order.

We now analyze the ability of the model to reproduce the observed pattern of neutrino masses and mixings. The standard parametrization for the neutrino mass matrix in terms of three masses $m_{1,2,3}$, three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$, and three phases $\alpha_1, \alpha_2, \delta$ reads

$$m' = U^T m_D U \quad \text{with} \quad m_D = \text{Diag}(m_1, m_2, m_3),$$

$$U = \text{Diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1) \times \left(\begin{array}{ccc}
c_{13}c_{12} & -c_{13}s_{12} & s_{13}s_{12} - c_{13}c_{12}s_{13}e^{i\delta} \\
c_{13}s_{12} & c_{13}c_{12} & s_{13}s_{12} - s_{13}c_{12}s_{13}e^{i\delta} \\
s_{13}e^{-i\delta} & c_{13} & c_{23}c_{13}
\end{array}\right)$$

where $s_{ij} \equiv \sin(\theta_{ij})$ and $c_{ij} \equiv \cos(\theta_{ij})$. A global fit to neutrino oscillation data after the recent measurement of $\theta_{13}$ (see for example [14]) gives $\Delta m^2_{31} \equiv m_3^2 - m_1^2 = 7.62^{+0.19}_{-0.18} \times 10^{-5} \text{eV}^2$, $\Delta m^2_{32} \equiv m_3^2 - m_2^2 = 2.55^{+0.06}_{-0.05} \times 10^{-3} \text{eV}^2$, $s_{13}^2 = 0.20 \pm 0.016$, $s_{12}^2 = 0.97$, and $s_{23}^2 = 0.46 \pm 0.03 (0.61 - 0.04)$ if the first (second) octant for $\theta_{23}$. Neutrino oscillations are not sensitive to the Majorana phases $\alpha_1$ and $\alpha_2$ nor to the absolute neutrino mass scale, while the value of the $CP$ phase $\delta$ is beyond current experimental sensitivity. In the inverted hierarchy scenario ($\Delta m^2_{31} < 0$) these experimental data lead to $m_{\nu_{e1}}^{\nu} \gtrsim 10^{-2} \text{eV}$, which cannot be accommodated by Eq. (refMajoranaMatrix) due to the $x_{e2} \sim 10^{-9}$ suppression of an already-three-loop-suppressed EW-sized mass scale $A$. Thus, a normal hierarchy pattern for neutrino mass matrices is predicted.

The fact that the entries $m_{\nu_{ee, e\mu}}^{\nu}$ in Eq. (4) are parametrically much smaller than the rest (being proportional to $x_{e2}$ and $x_{e\mu}$) results in an approximate neutrino mass texture of the form $m_{\nu_{ee}} = 0$, $m_{\nu_{e\mu}} = 0$. For given values of $\Delta m^2_{21}, \Delta m^2_{31}, s_{12}^2$ and $s_{23}^2$, the four constraints $\text{Re}[m_{\nu_{ee}^{\nu}}], \text{Im}[m_{\nu_{ee}^{\nu}}], \text{Im}[m_{\nu_{e\mu}^{\nu}}], \text{Im}[m_{\nu_{e\mu}^{\nu}}]$ can in fact only be satisfied over a certain range of $s_{13}^2 [7]$. If $\Delta m^2_{21}, \Delta m^2_{31}, s_{12}^2$ and $s_{23}^2$ are taken at their central values of the global fit in [14], then the model predicts $0.009 \lesssim s_{13}^2 \lesssim 0.015$ ($s_{13}^2 \gtrsim 0.017$) for $\theta_{23} \gtrsim \pi/4$. It thus gives a correlated prediction for $\theta_{13}$ and the deviation of $\theta_{23}$ from $\pi/4$ (maximal mixing) towards lower or larger values. Moreover, for fixed values of $\Delta m^2_{21}, \Delta m^2_{31}, s_{12}^2, s_{23}^2$ and $s_{13}^2$ (and when allowed by the mass texture) the above constraints lead to a specific prediction for $m_{\nu_{e1}}^{\nu}, m_{\nu_{e\mu}}^{\nu}$ and $m_{\nu_{e\tau}}^{\nu}$ entries of the right size (bounds on $C_{\nu_{e1}}^{\nu}$, and $C_{\nu_{e\tau}}^{\nu}$ are weaker) we need model parameters such that

$$C_{ee} s_{23} (A_1 + A_2) \simeq 1.3 \text{ TeV},$$

$$C_{\mu\mu} s_{23} (A_1 + A_2) \simeq 0.3 \text{ TeV}.$$

II. DARK MATTER.

When the lightest $Z_2$-odd states is electrically neutral the model has a WIMP dark matter candidate. For the remaining of the paper this particle will be assumed to be $H_0$ (taking $A_0$ would be equivalent). This WIMP scenario resembles the inert doublet model [2] and should share much of its phenomenology (see e.g. [15] and references therein).

The relic abundance of $H_0$ is determined by its annihilation rate at freeze-out. In the mass range $m_{H_0} = 50 - 75 \text{ GeV}$ [16] or above $520 \text{ GeV}$ [17], the correct dark matter abundance can be achieved while being compatible with existing bounds from the Large Electron-Positron collider (LEP), electroweak precision tests (EWPTs), and direct and indirect dark matter searches. The lower WIMP mass range allows us to simultaneously generate neutrino masses of the right size in our model. The correct dark matter abundance can be reached in the following situations:

(a) Annihilations into fermions via resonant SM scalars when $m_{H_0} \sim m_{h_2}/2$.

(b) Coannihilation with either $A_0$ or $H^+_1$, if the mass splitting to $H_0$ is lesser than some GeV. For fairly large mass splittings $\Delta m^2_H$ and $\Delta m^2_{H_0}$, coannihilations $H_0 - A_0$ are strongly suppressed, while coannihilations $H_0 - H^+_1$ may still be possible.

(c) The WIMP mass approaching $m_{\nu}$, where the closeness to the $WW$ threshold regulates the annihilation rate at freeze-out.

Apart from a potential signal in direct dark-matter search experiments, the model could produce a striking monochromatic gamma-ray line $\gamma$ detectable by the Fermi Large Area Telescope.

III. EXPERIMENTAL CONSTRAINTS.

Direct searches at LEP for doubly charged scalars $\rho^{++}$ decaying into same-sign dileptons set a lower bound $m_{\rho} \gtrsim 160 \text{ GeV}$ [19] (which however depends on the value of $C_{ee}$). Bounds from virtual $\rho^{++}$ exchange in Bhabha scattering lead to $C_{ee} \lesssim 9.7 \times 10^{-6} \text{ GeV}^{-2} m_{\rho}^{-2}$ [14] [20]. More stringent limits from direct searches at the Tevatron and the Large Hadron Collider (LHC) are more subtle to derive (as opposed to doubly charged scalars $\Delta^+ \tau$ from inside $SU(2)_L$ triplets, $\rho^{++}$ does not couple to W bosons). The ATLAS collaboration at the LHC searched for pair produced $\rho^{++}$ decaying into leptons and set a limit $m_{\rho} \gtrsim 400 \text{ GeV}$ [21]. For the charged states $H^+_1$, the LEP data from chargino searches can be translated into an approximate bound $m_{\mu^+} \gtrsim 70 - 90 \text{ GeV}$ (depending on $m_{H_0}$) [22]. Moreover, LEP excludes models with $m_{A_0} \lesssim 100 \text{ GeV}$ if $m_{H_0} \lesssim 80 \text{ GeV}$ and $m_{A_0} - m_{h_2} \gtrsim 10 \text{ GeV}$ [23].

The new inert fields also contribute to EWPT observables, such as the oblique parameters $S, T$ and $U$ [24].
For $m_h \simeq 126$ GeV, the most important constraint is given by $\Delta T \in [-0.04, 0.12]$ at 95% C.L. [23] (contributions to $S$ and $U$ are found to be negligible). The one-loop contribution to $T$ from the new fields is calculated to be

$$
\Delta T = \frac{1}{16\pi^2m_W^2s_W^2} \left[ c_\beta^2 \left( F_{H^+_1,H_0} + F_{H^+_2,A_0} \right) + s_\beta^2 \left( F_{H^+_2,H_0} + F_{H^+_1,A_0} \right) - 2c_\beta^2s_\beta^2 F_{H^+_1,H^+_2} - F_{H_0,A_0} \right] 
$$

with $F_{i,j} = \frac{m_i^2 + m_j^2}{2} - \frac{m_i m_j}{m_i - m_j} \ln \frac{m_i}{m_j}$, and $\theta_W$ being the Weinberg angle. EWPT constraints can be satisfied for a wide range of masses and mixing angles $\beta$ and $\delta$, as opposed to the inert doublet model [2], the present scenario allows for large mass splittings (see Figure 2). In addition, the doubly charged scalar $\rho^{++}$ mediates lepton-flavor violation (LFV) at tree level in processes such as $\mu \to 3e$ and $\tau \to 3e, 3\mu$, and at one-loop in processes like $\mu \to e\gamma$ and $\tau \to e\gamma, \mu\gamma$. This constrains the allowed values of $C_{ab}$ as a function of $m_\rho^2$, with the most stringent bounds being [20] [28]

$$
\begin{align*}
\mu^- \to 3e : & \quad |C_{e\mu} C_{e\varepsilon}| < 2.3 \times 10^{-5} (m_\rho/\text{TeV})^2 \\
\tau^- \to 3e : & \quad |C_{\mu\tau} C_{e\varepsilon}| < 9.0 \times 10^{-5} (m_\rho/\text{TeV})^2 \\
\tau^- \to 3\mu : & \quad |C_{\mu\tau} C_{\mu\nu}| < 8.1 \times 10^{-3} (m_\rho/\text{TeV})^2 \\
\tau^- \to e^+ e^- : & \quad |C_{\mu\tau} C_{e\tau}| < 6.8 \times 10^{-3} (m_\rho/\text{TeV})^2 \\
\tau^- \to e^+ e^- \mu^- : & \quad |C_{\mu\tau} C_{\mu\tau}| < 6.5 \times 10^{-3} (m_\rho/\text{TeV})^2 \\
\tau^- \to e^+ e^- \mu^- : & \quad |C_{e\tau} C_{e\tau}| < 5.2 \times 10^{-3} (m_\rho/\text{TeV})^2 \\
\tau^- \to e^+ \mu^- \mu^- : & \quad |C_{e\tau} C_{\mu\mu}| < 7.1 \times 10^{-3} (m_\rho/\text{TeV})^2 \\
\mu^+ \to e^+ \gamma : & \quad \sum_i C_{i\mu} C_{i\mu} < 3.2 \times 10^{-4} (m_\rho/\text{TeV})^2.
\end{align*}
$$

LFV constraints favor $m_\rho \geq 1$ TeV, which combined with Eq. (6) leads to large values of $\kappa_2 \gtrsim 1$ TeV and/or $\xi$, close-to-maximal mixing $\beta \sim \pi/4$ and large mass splittings $\Delta m^2, \Delta m^2_0 \sim v^2$. For such large mass splittings, satisfying the EWPT constraints requires a mass spectrum $m_{H^+_2} \gtrsim m_{A_0} \gtrsim m_{H^+_1}$, resulting in a partial cancellation of the $H^+_1$ and $H^+_2$ contributions in Eq. (8).

As a prototypical benchmark model that satisfies EWPTs (see Figure 2) and collider constraints, we take $m_{H_0} = 70$ GeV, $m_{A_0} = 475$ GeV, $m_{H^+_1} = 90$ GeV, $m_{H^+_2} = 850$ GeV and $m_\rho = 2$ TeV, with $\kappa_2 = 3$ TeV, $\xi = -2.5$ and $\beta = \pi/4$. Neutrino masses and mixings of the right size are then obtained for Yukawa couplings with absolute values: $C_{e\tau} \sim 0.32$, $C_{\mu\tau} \sim 0.066$, $C_{e\mu} \sim 3.6 \times 10^{-3}$ and $C_{\tau\mu} \sim 2.4 \times 10^{-4}$. Together with, e.g., $C_{ee} \lesssim 10^{-2}$ and $C_{He} \lesssim 10^{-3}$ these couplings fulfill at the same time all the LFV bounds. However, branching ratios for several LFV processes (like $\tau^- \to e^+ \mu^- \mu^-$ and $\mu \to e\gamma$) are predicted close to the current experimental bounds, and may be probed in the near future.

In this model, the short-distance contribution to neutrinoless double beta $0\nu\beta\beta$ decays dominates over the one coming from light-neutrino exchange (since this one is proportional to $m_{ee}$ and thus suppressed by $x_e^2 \sim 10^{-9}$). If the value of $C_{ee}$ is not too small, this could open up the possibility to test this scenario at future $0\nu\beta\beta$ decay experiments.

To conclude, we have put forward a minimal extension of the SM to include neutrino mass generation and dark matter in a unified framework, without introducing right-handed neutrinos. While giving an elegant explanation of the hierarchy $m_\nu/e$, the model predicts a small and nonzero value of $\theta_{13}$, together with a nontrivial relation between $\theta_{13}$ and the octant of $\theta_{12}$, to be tested by future neutrino experiments. It also predicts LFV, WIMP dark matter with a mass of $\sim 50-75$ GeV and new scalar states to be searched for.

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