Tilting the Brane, or Some Cosmological Consequences of the Brane Universe

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Abstract

We discuss theories in which the standard-model particles are localized on a brane embedded in space-time with large compact extra dimensions, whereas gravity propagates in the bulk. In addition to the ground state corresponding to a straight infinite brane, such theories admit a (one parameter) family of stable configurations corresponding to branes wrapping with certain periodicity around the extra dimension(s) when one moves along a noncompact coordinate (tilted walls). In the effective four-dimensional field-theory picture, such walls are interpreted as one of the (stable) solutions with the constant gradient energy, discussed earlier [1, 2]. In the cosmological context their energy “redshifts” by the Hubble expansion and dissipates slower than the one in matter or radiation. The tilted wall eventually starts to dominate the Universe. The upper bound on the energy density coincides with the present critical energy density. Thus, this mechanism can become significant any time in the future. The solutions we discuss are characterized by a tiny spontaneous breaking of both the Lorentz and rotational invariances. Small calculable Lorentz noninvariant terms in the standard model Lagrangian are induced. Thus, the tilted walls provide a framework for the spontaneous breaking of the Lorentz invariance.

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1 Introduction

Nonobservation of low-energy supersymmetry may be due to the fact that we live on a non-BPS topological defect, or brane, embedded in higher-dimensional space-time \([1]\). While the external space may or may not be supersymmetric, the effective low-energy theory on the non-BPS brane is not supersymmetric.

The idea of the brane Universe is especially motivated by the solution of the hierarchy problem through lowering of the fundamental scale of quantum gravity down to TeV \([3, 4]\) (see also \([5, 6]\)). In the model suggested in Ref. \([3]\) the standard-model particles are localized on a topological defect, or 3-brane, embedded in space with large extra compact dimensions of size \(R\), in which gravity can propagate freely. In other words, the original \((4+N)\)-dimensional space-time is assumed to be split into \(\mathcal{M}_4 \times \mathcal{M}'_N\), where \(\mathcal{M}_4\) is the four-dimensional Minkowskian space while \(\mathcal{M}'_N\) is a compact manifold. We will refer to it as to the external space. Within the scenario \([3]\) one can lower the fundamental scale of gravity \(M_{\text{Pl}}\) down to TeV, or so. The observed weakness of gravity at large distances is then due to a large volume of the extra space \(R \gg M_{\text{Pl}}^{-1}\). The relation between the fundamental \((M_{\text{Pl}})\) and four-dimensional \((M_{\text{P}})\) Planck scales can be derived \([3]\) by virtue of the Gauss law,

\[
M_{\text{P}}^2 = M_{\text{Pl}}^{N+2} V_N, \tag{1}
\]

where \(V_N\) is the volume of the external space with radius \(R\). Although this relation is valid for any values of \(M_{\text{Pl}}\) and \(R\), the case most interesting phenomenologically is \(M_{\text{Pl}} \sim \text{TeV}\). (For \(R \sim 1\) mm, this implies that the extra space is two-dimensional.)

The most important “technical” question to be addressed is dynamical localization of the standard-model particles on the brane. In the field-theoretic context the fermions can be localized due to an index theorem, as was suggested in \([7]\), whereas the localization of the gauge fields requires the outside medium to be confining \([8]\). In particular, this implies that free charges cannot exist in the bulk.

On the other hand, in the string-theoretic context, the most natural framework for the brane-world picture is through the \(D\)-brane construction (for a review, see e.g. \([9]\)). In this context the standard-model particles can be identified with the open string modes stuck on the brane, whereas gravity comes from the closed string sector propagating in the bulk \([4, 10, 11]\). For the purposes of the present paper the precise nature of localization will be unimportant, since we will exploit the low-energy effective field-theory approach for which the high-energy nature of the brane is beyond “resolution”.

In this paper we will present new theoretical observations regarding the branes on the manifolds \(\mathcal{M}_4 \times \mathcal{M}'_N\) where \(\mathcal{M}'_N\) is compact. Then we discuss some possible cosmological consequences of the brane Universe with the low-scale quantum gravity. We point out that these theories admit stable solutions which could manifest themselves through a tiny spontaneous breaking of the Lorentz and rotational invariances.
In four-dimensional field theory the solutions above can be interpreted as a recently discovered new class of stable vacuum configurations in supersymmetric theories, with the constant gradient energy, which may or may not break supersymmetry \[ \text{[1, 2]} \]. In the latter case these solutions generalize the notion of the BPS saturation to infinite values \[ \text{[1, 2]} \] of the central charge \[ \text{[1, 2]} \]. Below we will show how these solutions can naturally emerge in the brane Universe picture.

Before proceeding, let us note that other possible cosmological implications of branes can be due to “brane inflation” driven by a displaced set of branes \[ \text{[12]} \] or due to nonconservation of global quantum numbers in the brane Universe \[ \text{[13]} \]. In a very different context a class of time-dependent cosmological solutions was discussed \[ \text{[14]} \] within the Ho\v{r}ava-Witten approach \[ \text{[15]} \]. Some aspects of thermal cosmology were considered in \[ \text{[16]} \]. These issues are not directly related, however, to the present work.

For definiteness assume \((4 + N)\)-dimensional space-time to be \( \mathcal{M}_4 \times \mathcal{T}_N \), where \( \mathcal{M}_4 \) is four-dimensional Minkowskian space while \( \mathcal{T}_N \) is an \( N \)-torus with the radius \( R \). The standard-model particles are the modes living on a 3-brane embedded in this space-time. Let the brane tension (the energy per unit three-volume) be \( T \). Throughout the paper we will assume that \( T \approx M^4_{\text{Pl}} \) (the only fundamental mass scale in the theory). Obviously, \( T > 0 \) if the brane is to be regarded as a dynamical object (the wall), since otherwise it will be unstable. Then, in the absence of matter and radiation, the effective four-dimensional energy density \( \rho_{\text{eff}} \) has two contributions: a positive one from the brane tension, and a (must-be) negative one from the bulk

\[
\rho_{\text{eff}} = T + \Lambda_{\text{bulk}} V_N , \tag{2}
\]
as seen in experiments at distances \( \gg R \). Here \( \Lambda_{\text{bulk}} \) is the negative bulk cosmological constant. In the lowest-energy state these two must conspire to cancel each other. This is a usual fine-tuning of the cosmological constant, about which we have nothing new to say. The lowest-energy state is achieved in the above picture when the brane is straight, its “surface” area is minimal per unit three-volume of the space (Fig. 1).

However, since the brane is a dynamical object it can bend or curve in the external space and, in general, it will do so. It can be tilted with respect to \( \mathcal{M}_4 \times \mathcal{T}_N \). Note that this curvature \( a \text{ priori} \) has nothing to do with the gravitational curvature of the whole (brane + bulk) effective four-dimensional space in the Friedmann cosmological equations emerging at distances \( \gg R \). The latter will be assumed to vanish. Bending the brane in the external space, or tilting, it will produce an energy excess, or an effective energy density of the Universe, which can be estimated as

\[
\rho_{\text{brane}} \approx M_{\text{Pl}}^4 \left( \frac{R}{r} \right)^2 , \quad r \approx R , \tag{3}
\]

where \( r \) is a typical curvature radius of the brane, \( not \) to be confused with the Friedmannian curvature radius, which we take infinite. (In the case of the tilted

\[ \text{[1]} \]

By the infinite central charges we do not mean trivial infinities associated with the area of the wall or the length of the string.
brane $r$ is its longitudinal dimension, $r \sim L$. In this case Eq. (3) can be obtained as follows:

$$\rho_{\text{brane}} \sim M_4^4 \alpha^2,$$

(4)

provided that the tilt angle $\alpha \sim R/L \ll 1$, see below.) This energy provides an effective force resisting to bending; the force tends to straighten out the brane. What would be the cosmological significance of this excess energy?

To estimate its impact we have to know $r$. It is natural to assume that in the scale smaller than the Hubble size the brane is straightened out (no excessive crumpling). Whatever mechanism solves the horizon and isotropy problems, it would also help to this straightening. So, it seems reasonable to assume that

$$r^2 \gtrsim H^{-2} \gtrsim \rho_c^{-1} M_P^2,$$

(5)

where $\rho_c$ is the critical density of the Universe today. This estimate obviously refers also to the longitudinal dimension of the tilted brane, $L \gtrsim H^{-1}$. Now, substituting this in Eq. (3) and using Eq. (1) with $N \geq 2$ we get

$$\rho_{\text{brane}} \lesssim \rho_c.$$  

(6)

The upper bound which can only, but not necessarily, appear for $N = 2$ is intriguing since it suggests that the brane can serve as some sort of dark matter in Universe. We will study the nature of its energy below.

The potential importance of the domain walls in the cosmological considerations was recognized long ago [17]. In Ref. [17] it was shown that were the domain walls within our universe, serious (potentially terminal) cosmological problems might arise in the theory. Situation dramatically changes if our Universe itself is a wall.
2 What is the tilted wall?

To illustrate the idea we will consider, for simplicity, the four-dimensional space $\mathcal{M}_4$ compactified in $\mathcal{M}_3 \times \mathcal{T}$. The situation is general and does not depend on particular details.

Let us start from $\mathcal{M}_4$ and assume that the dynamical theory under consideration has multiple discrete degenerate vacua. The simplest example is the theory of the real scalar field with the potential of the double-well type. The simplest supersymmetric example is the minimal Wess-Zumino model with the cubic superpotential. The field configuration depending only on one coordinate (call it $z$) that interpolates between one vacuum at $z \to -\infty$ and another at $z \to \infty$ is the domain wall. The width of the wall $\delta$ in the $z$ direction is of order $\delta \sim M^{-1}$ where $M$ is the mass scale of the field(s) of which the wall is made. It is assumed that $\delta$ is much smaller than any other relevant scale of dimension of length.

On $\mathcal{M}_4$ it is meaningless to speak of the tilted wall – the direction $z$ can be chosen arbitrarily. Now, we compactify $z$ and consider the theory on the cylinder $\mathcal{M}_3 \times \mathcal{T}$. The underlying dynamical theory must be modified accordingly, in order to allow for the existence of the walls. In the case at hand it is sufficient to assume that the wall-forming field $\Phi$ lives on a circle, i.e. one can consider the model of the sine-Gordon type or its supergeneralizations. Both, the superpotential and the Kähler potential must be periodic in $\Phi$, with commensurate periods. For simplicity we assume these periods to be $2\pi$. Following an old tradition, we rename the compact coordinate, $z \to X$. The non-compact coordinates (including time) will be denoted by $x_\mu$. We look for the topologically nontrivial solutions of the soliton type on the cylinder, depending on one coordinate only. The solution of the type $\Phi_0(X)$, which is independent of $x_\mu$, is the straight wall, see $A$ in Fig. 1. It satisfies the condition $\Phi_0(X + 2\pi R) = \Phi_0(X) + 2\pi$. This wall is aligned “parallel” to the cylinder. The tilted wall ($B$ in Fig. 1) is a solution of the type $\Phi_\alpha(X \cos \alpha + x \sin \alpha)$ where $\alpha$ is the tilt angle. Note that the function $\Phi_\alpha$ does not coincide with $\Phi_0$, generally speaking. In the limit of small $\alpha$ the difference between $\Phi_0$ and $\Phi_\alpha$ is $O(\alpha^2)$. The condition $\Phi_\alpha((X + 2\pi R) \cos \alpha + x \sin \alpha) = \Phi_\alpha(X \cos \alpha + x \sin \alpha) + 2\pi$ must be satisfied. It is not difficult to see that the tilted wall solution exists, if so does the straight wall solution. The tilted wall is stable provided the solution is “nailed” at the points 1, 2 at the boundaries (see Fig. 1). Alternatively, one can glue the boundaries of the cylinder converting it into a two-torus. The tilted wall solution is automatically stable then if the wall winds around the two-torus.

It is not difficult to prove that the domain walls on the cylinder cannot be BPS-saturated, strictly speaking. However, if $\delta/R \ll 1$ the straight walls may be very close to the BPS saturation, achieving the BPS saturation in the limit $\delta/R \to 0$, when the wall becomes the “genuine brane”. The tilted walls are never BPS-saturated, their tension being larger than that of the straight wall. This effect – the increase of the internal tension $T$ of the tilted wall compared to that of the straight wall – is proportional both to $\alpha$ and to $(\delta/R)^2$. It can be made arbitrarily
small in the limit $\delta/R \to 0$. We will neglect it in what follows, using one and the same tension $T$ for the tilted and straight walls. The tilting does produce an impact on $\rho$ since the area of the tilted wall per unit length of the cylinder in the $x$ direction is larger. This effect is most conveniently described by the effective low-energy theory of the zero modes on the wall. The corresponding discussion is presented in the next section.

3 Calculating $\rho_{\text{brane}}$ for non-vanishing tilt angles

To deal with the long wave-length ($\lambda \gg \delta$) deformations of the straight wall we will use an effective four-dimensional field theory emerging for the zero modes. The wall solution spontaneously break the translational invariance in one direction. Correspondingly, in the simplest case there arises one zero mode which is the Goldstone boson of the spontaneously broken symmetry. In more complicated models (see below) there may arise several zero modes $\phi_A$. In the absence of gravity the effective Lagrangian is

$$\mathcal{L} = M^2_{Pl} \int d^4x \frac{\partial \phi_A}{\partial x^\mu} \frac{\partial \phi_B}{\partial x^\nu} \eta^{AB}(\phi),$$

(7)

where $\eta^{AB}$ is the external metrics depending on the structure of the manifold on which the fields $\phi$ live. For instance, in the case of the domain wall in Minkowski space $\eta^{AB} = \delta^{AB}$. All dimensional constants in the low-energy theory of the zero modes are related to the order parameter, which is the brane tension $T \sim M_{Pl}^4$. The dynamics of the Goldstone bosons on the brane is described by (3 + 1)-dimensional field theory which may or may not be supersymmetric (in the latter case the brane must be BPS saturated). If we deal with the supersymmetric theory $\eta^{AB}$ is obtained from the Kähler potential.

Let us assume for the time being that we have only one Goldstone boson and $\eta^{AB} = 1$. Then, the solution $\phi = \alpha x$ goes through the equations of motion of the theory (7). This is the constant energy density vacuum, discussed in Refs. [1, 2]. To make contact with the discussion above, we note that the vacuum $\phi = \alpha x$ represents the tilted wall described by the solution $\Phi_\alpha$ in the full theory. The additional contribution to $\rho$ compared to the straight brane is obviously

$$\Delta \rho_{\text{brane}} = \frac{T \alpha^2}{2}.$$

(8)

This result has a very transparent interpretation in the full theory. It exactly reproduces the increase of the brane surface per unit length of the cylinder for a non-vanishing tilt angle $\alpha$, see Fig. 2. Given this interpretation, one might ask why one needs to consider the effective low-energy theory at all. The point is that at the next step we want to switch on gravity in the bulk. Having the low-energy theory of the zero modes, describing matter on the brane, helps analyze the impact of gravity.
In the language of the effective field theory (7), the bending of the brane is equivalent to $\phi^A$ acquiring some $x_\mu$ dependence in the vacuum. The generic $\phi^A(x)$ configuration is unstable and will decay producing $\phi$ waves (the sound waves on the brane). This is the mechanism of eliminating foldings on the brane. Since these waves travel with the speed of light, the brane will iron itself out in the horizon scale. Eventually it will evolve to a tilted brane.

In the state with an arbitrarily bent brane we will distinguish two components. One can be viewed as a collection of all possible Goldstone waves traveling with the speed of light, which “redshift” away like ordinary matter. Another component is the vacuum solution (more precisely, a family of solutions) that would be stable if it were not for the expansion of the Universe. This configuration “redshifts” away slower than matter and can be called the tilted brane configuration (or wrapped, if there are several windings on the length of the cylinder $L$). It can only “redshift” through the “stretching” triggered by the Universe expansion.

In the case of the generic massless fields $\phi_A$ with the flat $\eta^{AB}$, the solution $\phi = \alpha x$ is stable under any localized deformations. If $\phi_A$’s are the Goldstone bosons arising due to the spontaneous breaking of some compact symmetry, $\phi_A$’s are periodic (in fact, they are phases defined modulo shifts). This is exactly what happens in our case since the extra dimensions are assumed to be compact. Then the solution $\phi = \alpha x$ must be modified appropriately, see Sec. 2. Here we will add a comment regarding the issue of stability.

To explain the point we will use a simple example of the Goldstone field pro-

\[\text{Figure 2: The map of the cylinder of Fig. 1. The ratio of the surfaces of the branes B to A per unit length in the x direction is } 1 + \frac{\alpha^2}{2}.\]
duced as a result of breaking of some global $U(1)$ symmetry (a similar model was treated in Sec. 6 of [1]). Start from the Lagrangian

$$\mathcal{L} = |\partial_\mu \Phi|^2 - \frac{\lambda^2}{2} \left(|\Phi|^2 - v^2\right)^2.$$  

(9)

The equations of motion have a solution

$$\Phi = c e^{i\mu x}, \quad c = \sqrt{v^2 - \frac{\mu^2}{\lambda^2}},$$  

(10)

which corresponds to winding of the phase with the period $2\pi/\mu$ as one moves along $x$. This solution has both the gradient and potential energies (cf. the solutions discussed in [2]). The potential term scales as $\sim \mu^4/\lambda^2$. It is seen that at $\lambda \to \infty$ the solution (10) becomes pure gradient energy ($v$ must scale accordingly, of course, i.e. $v \sim \lambda^{-1}$). This is because in this limit, the $x$-dependence can not affect the order parameter and, thus, the potential energy. In the case of the tilted brane this would mean that the brane with a constant tilt carries purely gradient energy in the limit when bending can not affect its tension. This is true for any brane in the limit when one can ignore its thickness. The corresponding configuration can not decay into the $\phi$ waves. Its energy is reduced only through the Friedmann expansion of the Universe and, thus, scales as $\sim a^{-2}$, the scale factor in the Friedmann Universe. This means, in turn, that the state at hand will sooner or later dominate over both, the radiation and the matter densities. We will discuss the observational implications of this fact in Section 5. Before, however, let us discuss the effect of higher dimensional bulk gravity.

4 Switching on gravity in the bulk

The effective Lagrangian for the zero modes localized on the brane becomes

$$\mathcal{L}_{\text{brane}} = M_P^2 \ g^{\mu\nu} \left( \partial_\mu \phi_A \partial_\nu \phi_B \right) \eta^{AB} + \text{fermions},$$  

(11)

where $\eta_{AB}$ and $g$ are the external and induced metrics, respectively. The fermion terms are relevant in the supersymmetric case.

So far, we have discussed the brane dynamics on the cylinder neglecting gravity. Now we want to take it into account. In the scenario under consideration gravity is not confined to the wall, but, rather, propagates in the bulk. The components of the graviton belonging to $\mathcal{M}_4$ and $\mathcal{T}^N$ require separate treatment. The effect which is most important for us is due to the higher-dimensional components of the graviton (the so-called graviphotons). For simplicity we will consider only a single extra dimension compactified on a circle parametrized by the coordinate $X$. The zero mode component of $g_{\mu 5}$ is the graviphoton $A_\mu(x)$. Viewed as a four-dimensional neglect the coupling to gravity for the time being, due to the large sizes of extra dimension(s).
gauge field, $A_\mu(x)$ gauges the translation in $X$, i.e. $X \to X + f(x_\mu)$, which, from the four-dimensional standpoint, is an internal U(1) gauge symmetry. Since the brane spontaneously breaks the translational invariance, the corresponding gauge symmetry is realized nonlinearly. The Goldstone mode $\phi$ is eaten up by the gauge field $A_\mu$ (which gets a mass of order $(1 \text{ mm})^{-1}$). As a result, the tilted wall solution $\phi = \alpha x$ we have considered previously is pure gauge. It can be compensated by the gauge field $A_x = \alpha$ and presents no physically observable effect. Thus, if we have only one zero mode on the wall, the tilted wall is indistinguishable from the straight one in the presence of gravity. To make the idea work we must have two or more zero boson modes on the wall. Then the tilted wall (the constant gradient energy solution) will lead to a physically observable excess in $\rho$.

Let us explain this in more detail. Our solution can be made physical by “projecting out” the graviphoton, provided the compact manifold breaks translational invariance in the extra space. The simplest dynamical realization is as follows. Consider topologically stable winding configurations [1]. (Such configurations may anyway be needed for supersymmetry breaking [1] or for stabilizing radii at large distances [19]). Consider a five-dimensional scalar field $\xi(x_\mu, X)$ transforming under an internal U(1) symmetry as $\xi \to e^{i\alpha}\xi$. Assume that a potential forces the condensate $\langle \xi \rangle \neq 0$ to develop. The simplest choice can be $V = (|\xi|^2 - v)^2/m$, or any other similar function. Then the vacuum manifold is a circle, and there are topologically stable winding configurations

$$\xi = w e^{inX/R}, \quad w = \sqrt{v - n^2m/R^2}, \quad (12)$$

with integer $n$. They correspond to giving a vacuum expectation value to different Kaluza-Klein modes and, therefore, are topologically stable due to the mapping of the vacuum circle on the external compact space. Thus we are free to choose any of these states as the ground state. Configurations with nonzero $n$ break spontaneously the $X$ translations and, thus, give mass to the graviphoton $A_\mu$. As a result, the graviphoton field can be integrated out in the low-energy effective theory. Then the brane Goldstone $\phi$ remains a physical field.

More precisely, the picture is as follows. The theory at hand has two U(1) symmetries from the very beginning: the internal U(1)$_I$ and “external” U(1)$_E$ gauge symmetry under translations in the extra dimension. However, from the point of view of a four-dimensional observer living on the brane, both of them are internal symmetries, one global and another gauged by $A_\mu$. The condensate $\langle \xi \rangle \neq 0$, with $n \neq 0$, breaks both U(1)’s down to a global U(1) describing the change of a relative position of the brane in the extra space. The brane breaks the latter down to nothing, but since there is no gauge degree of freedom left, the corresponding Goldstone is physical.

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3Strictly speaking, we have to restrict ourselves to $n^2 < vR^2/m$ since larger winding numbers force the field to vanish, and the solution will “unwind”. For our purposes, however, it is sufficient to consider small enough $n$. 

8
A related mechanism of “projecting out” the graviphoton while still keeping at least one Goldstone mode may be provided by multicomponent walls suggested in Ref. [20]. It was noted that in supersymmetric theories with multiple degenerate vacua, under certain conditions there exists a variety of the BPS-saturated walls. Some of them may coexist together; then, the tension of such “multilayer” configuration is exactly equal to the sum of the tensions of the individual walls, independently of the distance between the layers (individual walls). A similar effect takes place for the \( D \)-branes which are the BPS-states in the limit of unbroken supersymmetry, and the net force between two straight parallel branes vanishes (see [9] for a review). This means that, apart from the zero mode corresponding to the overall translation, there are extra zero modes corresponding to shifting the layers with respect to each other, without changing the position of the center of mass. As was mentioned, on the cylinder the exact BPS saturation can be achieved only in the limit \( \delta/R \to 0 \). Hence, only the first zero mode – the one related to the overall translations – is exactly zero, others become quasi-zero. The zero mode is eaten up by the graviphoton, eliminating it from the game. The quasi-zero modes remain physical, their dynamics is described by a low-energy Lagrangian. The supersymmetry breaking in general will give mass to this pseudo-Goldstones since the net force between branes (or walls) is non-zero.

If there are no other massless states in the bulk, at large distances the only force between the branes is due to gravity which gives a small \( r^{-N} \)-suppressed mass to the Goldstones (\( r \) is the interbrane separation). This mass can be made arbitrarily small if the separation is large and, practically, it can be neglected. All dynamical consequences are the same as discussed in Sec. 3.

### 5 Implications for cosmology

Now let us discuss cosmology in more detail. We will treat the problem from the standpoint of the effective four-dimensional field theory at scales \( \gg \gg R \). As was said above, in this picture the fluctuations of the brane in the external space \( \phi_A(x_\mu) \) can be described by the Goldstone modes which, at energies \( \ll T \) (almost flat brane), behave as free particles. Canonically normalized fields are \( \chi_A \sim (T)^{1/4} \phi_A \).

For simplicity, consider a fluctuation in one transverse direction only. Then, very roughly, the issue is reduced to the behavior of a free field \( \chi \) in an expanding Universe. The question is what are the initial conditions for such a field? The general solution of its equation of motion (the comoving coordinates are assumed)

\[
\partial^2 \chi = 0
\]

has the form

\[
\chi = \sqrt{T} cx + \chi_\mu e^{ipx}
\]

(modulo the Lorentz transformations), where \( c \) is a constant and the second term is some collection of massless plane waves. The energy stored in the second term will
just “redshift” away, like massless matter. However, the first term produces a rather
different contribution. In the present context this term describes the tilted brane
with the tilt angle given by $c \sim R/r$. In other words, when we move along $x$, the
brane is wrapping around the extra dimension with the period $\sim r$. In the absence
of gravity, in the limit of an infinitely thin brane, this is a stable configuration for
any $c$. In the presence of gravity, however, its energy “redshifts” away as $\sim a^{-2}$, due
to the Friedmann expansion. Therefore, eventually this energy will dominate over
matter. When this happens actually depends on the initial condition for $r(c)$ and on
the subsequent evolution of the scale factor. Assume that initially $c \sim R H_{in}$, where
$H_{in}$ is the Hubble parameter at that time. In other words, we assume that, when
the brane was “formed”, it wrapped around the extra dimension once (on average)
per the causally connected region $\sim H_{in}^{-1}$. Now, at present time, this region must
have had evolved into a volume comparable to the present Hubble size (or larger).
This is required by whatever mechanism solving the horizon problem. This means
that

$$r_{\text{today}} \sim \frac{a_{\text{today}}}{H_{\text{in}} a_{\text{in}}} \gtrsim H_{\text{today}}^{-1}.$$  \hspace{1cm} (15)

Thus, the energy density of a tilted brane would be $\ll \rho_c$. In particular, it is sufficient
to have a period of inflation with the number of $e$-foldings

$$N \simeq \ln \frac{H_{\text{in}} M_P}{T_{\text{today}} T_{R}}.$$  \hspace{1cm} (16)

and the subsequent reheating temperature $T_R$, where $T_{\text{today}} \sim 3 K^o$. For instance, a
brief period of the “brane inflation” \footnote{The wrapped brane can provide an additional (time-dependent) force stabilizing $R$. This may have implications for the early cosmology \cite{21}.} can do the job

In reality, however, we expect inflation to have more $e$-foldings and, thus, $\rho_{\text{brane}} \ll \rho_c$ is rather natural.

\section{The Lorentz Symmetry Breaking}

The important fact is that the solution at hand spontaneously breaks the Lorentz and
rotational invariances in four dimensions. This would result in a global anisotropy
in the expansion if $\rho_{\text{brane}}$ were to dominate the Universe. Thus, we must require
$\rho_{\text{brane}}$ to be subdominant today, but it can become dominant at any time in the
future. In this respect any observational evidence of a global anisotropy would be
extremely important for constraining $\rho_{\text{brane}}$.

The tilted brane would induce rotational (or Lorentz) noninvariant terms in
the effective four-dimensional standard-model Lagrangian. The important fact is
that the brane Goldstones (or pseudo-Goldstones, which are similar in this respect)
necessarily couple to all particles living on the brane through an induced metric on
the brane. Thus, the operators of the form
\[
\frac{\partial \phi}{\partial x_\mu} \frac{\partial \phi}{\partial x_\nu} \psi \gamma_\mu \partial x^\nu \psi + \frac{\partial \phi}{\partial x_\mu} \frac{\partial \phi}{\partial x_\nu} F_{\mu \alpha} F^{\nu \alpha} + ...
\]
will appear (accompanied by the appropriate powers of \( T \)). Here, \( \psi \) are the matter fermions and \( F_{\mu \alpha} \) stands for the matter gauge fields. In the background of a titled brane \( \phi = \alpha x \), these terms will induce effective Lorentz-violating interactions among the standard-model fields.

Can one have tilted branes and still avoid the anisotropy and violations of the rotational invariance? In principle, the answer is positive. To this end one must deal with more complicated manifolds, such that the topology of the manifold \( M'_N \) is the same as that of our 3-dimensional space \( M_3 \) (of course, in any case, locally \( M_3 \) must be Minkowskian). In other words, \( M'_N \) must include unshrinkable surfaces \( C_3 \) that are isomorphic to \( M_3 \). In this case the brane that corresponds to a point on \( C_3 \) can move around when one moves along the brane in \( M_3 \). The topologically nontrivial configuration of interest emerges when this motion maps \( C_3 \) onto \( M_3 \). For such a configuration, one can find a preferred reference frame for which rotations on \( M_3 \) will not be broken, but the Lorentz invariance will be broken in the arbitrary reference frame. To be more specific, let us give an example. Imagine that we have an external space, with three extra dimensions, which forms a manifold \( K \) of radius \( R \) (say, one can think of \( K \) as of a three-sphere). Assume that “our” three-dimensional space is also a very large manifold \( K \), its radius is much larger than \( H^{-1} \). Let us call these two manifolds “small” and “big”, respectively. Clearly, at human scales the “large” manifold (plus time) is identical to \( M_4 \), but not globally. Now, our brane corresponds to a point on the “small” \( K \). Imagine a configuration such that when one moves in “large” \( K \), the point in “small” \( K \) also moves in the same way. Thus, we get a mapping \( K \to K \), which is certainly topologically stable. Roughly speaking, the brane wraps isotropically in all directions. Such a wrapping is isotropic and stable.

What is the low-energy picture corresponding to this construction? Since \( K \) has three dimensions, the brane breaks three translational invariances. Thus, there are three massless Goldstone modes in the effective low-energy theory, \( \Phi^A, \ A = 1, 2, 3 \). Let \( x^A \) be three coordinates on our large \( S_3 \) which locally look as the Cartesian coordinates in our Minkowski space. Then, the solution is \( \Phi^A = \alpha x^A \). This solution is isotropic because of the spherical symmetry of the problem. Its energy density will still scale \( \propto 1/a^2 \) because this is essentially the same gradient energy solution we have discussed previously.

### 7 Conclusions

The idea of confining our Universe to a wall which ensures an appropriate supersymmetry breaking seems to be promising. At the very least, it deserves further
investigation. Being combined with the idea of compactification of the extra dimensions and allowing gravity to propagate in the bulk it leads to potentially realistic and reach phenomenology. In this paper we have shown that the walls on $\mathcal{M}_4 \times \mathcal{M}'_N$ where $\mathcal{M}'_N$ is compact generate peculiar theoretical effects due to tilting. The situation becomes especially interesting when gravity is switched on. The appropriate theoretical framework for its analysis is provided by the effective low-energy theories of the Goldstone modes on the brane. After gravity is switched on one of these modes is eaten up by the graviphoton (making it massive and eliminating it from the massless particle spectrum). We presented models where there are residual physical Goldstone (or pseudo-Goldstone) modes.

These solutions produce a framework for the spontaneous breaking of the Lorentz and rotational invariance and may have observable consequences.

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