On the Parity-Flavor-Breaking Phase in QCD With Two Flavors of Wilson Fermions

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We present further data in search of a parity-flavor-breaking phase in simulations of dynamical QCD with two flavors of light Wilson fermions in the strong coupling region. This is done on lattice sizes of \(4^4\) an up to \(10^4\) for a variety of values of \(\beta\) and \(\kappa\) as well as the coefficient, \(h\), of an explicit breaking term included in the action. We confirm the existence of a region in the \(\beta - \kappa\) plane where such a phase exists at \(\beta = 6/g^2\) smaller than 5.0 and above \(\kappa_c\).

1. Introduction

We extend our search for a parity-flavor-breaking phase in simulations of dynamical QCD with two flavors of light Wilson fermions to the strong coupling region of \(\beta\) less than 5.0. The signature used in this search is the same used in our previous work \[1\]. We introduce an explicit breaking term into the action with a small coefficient and then study the finite size behavior of its expectation value as this coefficient tends to zero.

2. Numerical Simulations

We report here on simulations done with two flavors of Wilson fermions at \(\beta = 3.0, 3.5, 3.7\) and 4.0 on volumes of \(4^4\), and up to \(10^4\) for a variety of values of \(\kappa\) ranging from less than the appropriate \(\kappa_c\) to values greater than \(\kappa_c\). Some of these simulations (denoted by the letter \(C\) in the tables) are continuing and simulations at \(\beta = 4.5\) are in progress.

We introduce into the QCD action a term of the form \(ih\bar{\psi}\gamma_5\tau_3\psi\) where \(\tau_3\) is a \(2 \times 2\) matrix representing the third element of the generators of flavor SU(2) algebra. Upon integrating the fermionic variables this is reflected in the simulation by the product of two determinants: \(\text{Det}M(h) \ast \text{Det}M(-h)\) where \(M(h)\) is given by a simple modification of the Wilson Matrix \(M_w\) as:

\[
M(h) = M_w + ih\gamma_5
\]

As pointed out by Aoki \[2\], we also have here

\[
\gamma_5 M(-h)\gamma_5 = M^\dagger
\]

and:

\[
\text{Det}M(-h) = \text{Det}M^\dagger(h).
\]

Simulations were done for the parameter \(h\) taking values ranging from 0.005 to 0.1. For the volume dependence we concentrate on the smaller values of \(h\) and in particular \(h = 0.005\) for all three volumes considered and mostly for values of \(\kappa\) greater than \(\kappa_c\).

The order parameter we compute is the expectation value of the operator \(i\bar{\psi}\gamma_5\tau_3\psi\). With our notation this is given as

\[
\text{PF} = -\text{ImTr}(\gamma_5 M^{-1}(h))
\]

3. Results

For the values of \(\beta\) considered, simulations were performed, as mentioned above, at various values of \(\kappa\) both below and above \(\kappa_c\). We shall present the data and results for each value of \(\beta\) considered separately. The results are shown in Tables 1–4.

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It is clear that if one concentrates on the behavior with volume of the simulations at \( h = 0.005 \), one may quickly note that a signature for the parity-flavor-breaking phase is indicated for the following values of \( \beta \) and \( \kappa \): (3.0, 0.25), (3.5, 0.25), (3.7, 0.25) and (4.0, 0.24). See Fig. 1 for the latter two points.

This is also confirmed by following the behavior of the order parameter as a function of \( \kappa \) at constant volume and constant \( \beta \). It is clear that all values above the order parameter goes through a relative maximum.

The behavior of the order parameter at all other values of \( \beta \) and \( \kappa \), computed in Tables 1–4, are consistent with an absence for a signature of a parity-flavor-breaking phase. This behavior is in fact very similar to the behavior for values of \( \beta \geq 5.0 \) which were discussed in ref. [1].

It is important to note here that the phase is not indicated for some points at the same \( \beta \) but larger values of \( \kappa \). In particular the phase disappears at (3.5, 0.25), at (3.7, 0.25).

It is clear from the discussion above that QCD with two flavors of Wilson fermions does exhibit a parity-flavor-breaking phase at values of \( \beta \) up to 4.0, as postulated by Aoki and collaborators. It has also been demonstrated in [1] that this phase does not seem to extend beyond \( \beta = 5.0 \).

This information is exhibited in Fig. 2. It is then possible that this phase pinches out, in a manner similar to that in the NJL model, at \( \beta < 5.0 \).

In this case, this phase would not be relevant for
the discussion of the approach to the chiral limit in QCD and the ensuing Goldstone nature of the pions for $\beta > 5.0$. In fact, all indications are such that, as shown formally sometime ago, this is simply related to the approach to zero lattice spacing and infinite volume.

Figure 1. Histogram signature of a parity-flavor-breaking phase for $\beta = 3.7$ and 4.0; $\kappa = 0.24$.

Figure 2. Parity-flavor phase diagram. Continuous line represents approximate positions for chiral and finite temperature transitions at $N_t = 4$ for comparison.

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REFERENCES

1. K. M. Bitar, Phys. Rev D 56, 2736 (1997).
2. See [reference] and references therein.