Magnetotransport Study in the Layer Imbalanced
\( \nu = 1 \) Bilayer Quantum Hall State.

Y Ogasawara\(^1\), A Fukuda\(^2,3\), K Iwata\(^1\), T Sekikawa\(^1\), T Arai\(^3\), Y Hirayama\(^4\), Z F Ezawa\(^4\), and A Sawada\(^3\)

\(^1\)Graduate School of Science, Department of Physics, Kyoto University, Kyoto 606-8502, Japan
\(^2\)Department of Physics, Hyogo College of Medicine, Nishinomiya 663-8501, Japan
\(^3\)Research Center for Low Temperature and Materials Sciences, Kyoto University, Kyoto 606-8502, Japan
\(^4\)Graduate School of Science, Department of Physics, Tohoku University, Sendai 980-8578, Japan

E-mail: yo-ogasawara@scphys.kyoto-u.ac.jp

Abstract. We carried out magnetotransport measurements of the charge imbalanced bilayer quantum Hall state (QHS) at the total Landau level filling factor \( \nu = 1 \) under the titled magnetic fields. Changing the total electron densities \( n_T \) and the layer imbalance parameter \( \sigma = (n_f - n_b)/(n_f + n_b) \), where \( n_f(n_b) \) indicates the density of the front (back) layer, we investigated magnetoresistance near the phase transition point between the charge transferable bilayer QHS and the 1/3+2/3 compound QHS at large in-plane magnetic fields \( B_\parallel \). We measured the thermal activation energy \( \Delta \) for various \( n_T \) and \( \theta \), and constructed detailed phase diagram in the \( n_T-\theta \) plane at \( \sigma = 0.33 \), including commensurate (C)-like, incommensurate (IC)-like and the compound state. Results indicate that each transition between the compound QHS and the C-like or IC-like phases reflects the pseudospin symmetries.

1. Introduction
The high mobility two-dimensional electron gases show extremely characteristic magnetotransport features under the strong magnetic fields and at low temperature. It is known as the quantum Hall (QH) effect, and recently bilayer QH states (QHSs) become one of the center of researcher’s attention. This system is abundant in novel quantum transport phenomena[1]. We can describe the layer degree of freedom as pseudospin. The tunneling energy between the two layers works as the pseudo-Zeeman term. The bilayer QHS at a total Landau filling factor \( \nu = 1 \) is the most ideal system to investigate the pseudospin ferromagnetism[1]. Because of existence of the tunneling of electrons between the two layers, the energy level of the system splits into the symmetric and asymmetric state and the tunneling energy gap works as the pseudo-Zeeman term, which stabilizes the bilayer QHS. We call this QHS as the charge transferable bilayer QHS (CTQHS). Requiring finite energy gap to excite the ground states, the QHS is an incompressive state. When total electron density \( n_T \) increases, the normalized layer separate length \( d/l_B \), where \( l_B = \sqrt{\hbar/eB} \) is a magnetic length and \( d \) is a bare layer separation, becomes large and effective interlayer Coulomb interaction decreases. As a result, the incompressive bilayer QHS ”melts” into the compressive layer independent state. The incompressive QHS is broken into the pseudospin wave (PSW) state and no more QHSs can survive [2,3]. The incompressive-
compressive quantum transition is very important to understand the bilayer QHSs. In addition, the layer charge imbalance parameter $\sigma = (n_f - n_b)/(n_f + n_b)$, where $n_f(n_b)$ is the density of the front (back) layer, can be introduced as the vertical component of pseudospin. When the layer imbalance parameter $\sigma$ increases, the incompressible QHS becomes stable [4,5]. At $\sigma = 1/3$, the bilayer system constructs the $1/3+2/3$ compound QHS (CPQHS), where each layer independently behaves like a monolayer $1/3$ and $2/3$ fractional QHS. Thus, a transition occurs from the CTQHS to the CPQHS around $\sigma = 1/3$. However, details of the transition between the CTQHS and the CPQHS are not clear yet.

When an in-plane magnetic field $B_{\|}$ is introduced to the bilayer $\nu = 1$ QHS, $B_{\|}$ affects the Aharonov-Bohm phase of the system. The in-plane field causes the commensurate-incommensurate (C-IC) phase transition in the charge balanced bilayer $\nu = 1$ QHS. The C-IC phase transition is first experimentally measured by the observation of an activation energy $\Delta$[6]. When $B_{\|}$ is small, the ground states is a commensurate (C) phase where pseudospin rotates proportional to the position. On the other hands, when $B_{\|}$ is quit large enough, an incommensurate (IC) phase appears, where all pseudospins point to a certain direction. Thus, pseudospin structure of these systems is dramatically different between C and IC phase. Around the C-IC phase transition points, existence of pseudospin domain wall i.e. pseudospin soliton has pointed out [7] and observations of pseudospin soliton such as an anisotropic magnetoresistance $R_{xx}$ peak are reported [8].

A lot of works of the bilayer QHSs have been done in the past, but there exist few measurements that report the influence of $B_{\|}$ in the layer imbalanced bilayer $\nu = 1$ QHSs. This work is devoted to reveal the effects of $B_{\|}$ to the layer-imbalanced bilayer $\nu = 1$ QHS, and especially focused on incompressive-compressive quantum phase transition in the layer imbalanced bilayer $\nu = 1$ QHS. We constructed the phase diagram in the $n_T$-$B_{\|}$ plane at $\sigma = 0.33$, and found effects of $B_{\|}$ to the transitions between the CTQHS and the CPQHS at $\sigma = 1/3$. We also discuss the behaviors of $\Delta$ around the phase transition points.

2. Experiments

We carried out the magnetotransport measurements under high magnetic fields in a dilution refrigerator with minimum temperature of 40 mK. Magnetoresistance $R_{xx}$ and Hall resistance $R_{xy}$ were measured using standard low-frequency lock-in techniques with the source-drain current $I_{SD} = 10$ nA and frequency $f = 17.7$ Hz. We used the sample of GaAs/Al$_{0.33}$Ga$_{0.67}$As double quantum wells with 20 nm well width separated by a 3 nm thick barrier. The tunneling energy of this sample estimated as $\Delta_{SAS} = 11$ K. We used the modulation-doped sample and low temperature mobility is $\mu = 1.5 \times 10^6$ cm$^2$/Vs at $n_T = 1.0 \times 10^{11}$ cm$^{-2}$. The front and back layer charge density can be controlled independently by applying the front and back Shottkey gate voltage, and thus we can set up $n_T$ and $\sigma$ freely. We employ the sample rotation machinery using stepping motor, by which we can adjust a tilting angle $\theta$. An in-plane magnetic field is introduced by tilting the sample in the magnetic field with an angle $\theta = \arctan(B_{\|}/B_{\perp})$. We derived the activation energy $\Delta$ from the slope of the longitudinal resistance with respect to the temperature $T$: $R_{xx} = R_{0}\exp(-\Delta/T)$, where $R_0$ is the magnetoresistance at $T = \infty$.

3. Results and Discussions

We measured the magnetoresistance $R_{xx}$ for various $n_T$ and $\sigma$ at $\nu = 1$ (Fig. 1). Figure 1(a) shows $R_{xx}$ as functions of $n_T$ and $\sigma$ at fixed $\theta = 36^\circ$. At $\sigma = 0$, when $n_T$ is small, $R_{xx}$ vanishes because this region is the CTQHS. As $n_T$ increases, $R_{xx}$ rapidly increases at $n_T(\sigma = 0) \geq 2.1 \times 10^{11}$ cm$^{-2}$ and the CTQHS is collapsed, where two layers are independent because the intralayer interaction dominates the interlayer one. Therefore, the abrupt change of $R_{xx}$ follows incompressive-compressive phase transition [2]. Theoretically, the CTQHS is stabilized by the layer imbalance [3]. In Fig. 1(a), the CTQHS becomes more stable monotonically as $\sigma$ increases at $\sigma > 0.33$, more stable monotonically as $\sigma$ increases at $\sigma > 0.33$, more stable monotonically as $\sigma$ increases at $\sigma > 0.33$, more stable monotonically as $\sigma$ increases at $\sigma > 0.33$, more stable monotonically as $\sigma$ increases at $\sigma > 0.33$, more stable monotonically as $\sigma$ increases at $\sigma > 0.33$. 
For $\theta = 36^\circ$, we plotted larger than those at $\theta = 50^\circ$. Solid line is on $\sigma = 0.33$. Temperature is 150mK. (c) $R_{xx}$ at $\sigma = 0.33$ in the $\nu = 1$ QHS as a function of $n_T$ at $\theta = 36^\circ$ and $\theta = 50^\circ$. Temperature is 125mK.

Figure 2. (a) Activation energy gap $\Delta$ at $\nu = 1$ and $\sigma = 0.33$ as a function of $n_T$ at $\theta = 36^\circ$ and $\theta = 50^\circ$. (b) Activation energy $\Delta$ in the $\nu = 1$ QHS as a function of $\theta$ at $\sigma = 0.33$. (c) Phase diagram of the bilayer $\nu = 1$ QHS in the $n_T$-$B_\parallel$ plane at $\sigma = 0.33$, including the commensurate (C)-like, incommensurate (IC)-like and the compound phase.

supporting the theory. However, at small $\sigma$, behaviors of the $R_{xx}$ are quit different. As $\sigma$ increases from the balanced points, the CTQHS becomes unstable at once, but becomes stable again. Particularly at $\sigma = 0.33$, $R_{xx}$ is very small. In the high $n_T$ region at $\sigma = 0.33$, two layers become independent, and the CPQHS ($1/3+2/3$) is formed. At low $n_T$ region, on the contrary, the system is the CTQHS. In Fig. 1(a), the phase transition point may have a minimum of these gaps and maximum of $R_{xx}$. In Fig. 1(b) at larger tilting angle $\theta = 50^\circ$, the $R_{xx}$ becomes larger than those at $\theta = 36^\circ$ near the transition point between the CTQHS and the CPQHS. In Fig. 1(c), we plotted $R_{xx}$ as a function of $n_T$ for $\theta = 36^\circ$ and $\theta = 50^\circ$. The $R_{xx}$ peak appears at $\theta = 50^\circ$ much larger than one at $\theta = 36^\circ$. We also carried out the same kind of experiments for various $\theta$, which indicate that the large $R_{xx}$ at the transition point is strongly related to effects of the in-plane magnetic field $B_\parallel$.

To investigate the effects of $B_\parallel$ quantitatively, we measured an activation energy $\Delta$ at $\sigma = 0.33$ for $\theta = 36^\circ$ and $\theta = 50^\circ$ as a function of $n_T$ (Fig. 2(a)). First, we discuss the data at $\theta = 36^\circ$. When $n_T$ is small, $\Delta$ slowly decreases as $n_T$ increases and the CTQHS is predominant. At large $n_T$, the system corresponds to the CPQHS, $\Delta$ increases gradually as $n_T$ increases because monolayer $\nu = 1/3$ and $\nu = 2/3$ QH state is stabilized by increasing $n_T$ and Zeeman energy, respectively. $\Delta$ takes a minimum at the phase transition point (indicated by the arrow a in Fig. 2(a)). $\Delta$ changes sharply and $\partial \Delta / \partial n_T$ seems to be discontinuous at the phase transition point. When $\theta$ increases, $n_T$ that gives the phase transition point shifts to lower side of $n_T$ and
the $\Delta$ becomes smaller. Next, we argue about the data at $\theta = 50^\circ$. The system transits from the CTQHS to the CPQHS at $n_T^b = 1.6 \times 10^{11}$ cm$^{-2}$ (indicated by the arrow $b$ in Fig. 2(a)) as $n_T$ increases. At $n_T^b$, however, the minimum of $\Delta$ is broad and the slope of $\Delta$ changes gradually. To investigate the difference, we focus on the other characteristic point at $n_T^c = 1.0 \times 10^{11}$ cm$^{-2}$ where the slope of $\Delta$ changes dramatically (indicated by the arrow $c$ in Fig. 2(a)). To clear up the origin of the change, we measured $\Delta$ for various $\theta$ at $\sigma = 0.33$ as a function of $\theta$. In Fig. 2(b), as $\theta$ is increased, $\Delta$ steeply decreases at small $\theta$ and stays almost constant at large $\theta$ even at a finite $\sigma$, which is similar to the C-IC phase transition in the balanced configuration ($\sigma = 0$). Therefore we call these two states as C-like phase and IC-like phase, respectively, even in the layer imbalanced QHS.

By collecting these phase transition points, we determine the phase boundary between the C-like and IC-like phase. In Fig. 2(b), $\theta$ that gives the transition point between the C-like and the IC-like phase at fixed $n_T = 1.0 \times 10^{11}$ cm$^{-2}$ is $50^\circ$. Therefore the point indicated by the arrow $c$ in Fig. 2(a) is identified as the transition point between the C-like and IC-like phases. We illustrate the phase boundaries among the C-like, IC-like phase and CPQHS (1/3+2/3) in the $n_T-\theta$ plane at $\sigma = 0.33$ in Fig. 2(c). From Fig. 2(c), we can see that the system transit from the C-like phase to the CPQHS at $\theta = 36^\circ$, whereas transit from the IC-like phase to the CPQHS at $\theta = 50^\circ$, as $n_T$ increases.

Finally, we discuss the contribution of spin and pseudospin degree of freedom to $\Delta$. Both of the layers in the CPQHSs are in spin-polarized states because of strong magnetic field, the spin structure may not change at the transition point between the CTQHS and the CPQHS. However the pseudospin structures may be different between the C-like and IC-like phases. Thus, changes of the pseudospin structure may affect the behaviors of $\Delta$ around the transition points.

In conclusion, we carried out the magnetotransport measurements in the charge imbalanced bilayer QHS and constructed the phase diagram in $n_T-\theta$ plane at $\sigma = 0.33$ including the compound, C-like and IC-like QHS. We found that behaviors of $R_{xx}$ and $\Delta$ at the phase transition points between the C-like phase and the compound QHS slightly different from ones between the IC-like phase and the compound QHS. This suggests the different continuity of the pseudospin structure around the phase transition point between the CTQHS and the CPQHS (1/3+2/3).

Acknowledgements
We are grateful to T. Saku for growing the heterostructures. This research was supported in part by Grants-in-Aid for the Scientific Research (Nos. 14010839, 18740181, 18340088, 18043012 and 20654028).

References
[1] Z. F. Ezawa, Quantum Hall Effects, Field Theoretical Approach and Related Topics. World Scientific, Singapore, 2000.
[2] J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett., 69:3804, 1992.
[3] D. Levesque, J. J. Weis, and A. H. MacDonald, Phys. Rev. B, 30:1056, 1984.
[4] Y. N. Jogalekar and A. H. MacDonald, Phys. Rev. B, 64:155315, 2001.
[5] Jinwu Ye, Phys. Rev. Lett., 97:236803, 2006.
[6] S. Q. Murphy, J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett., 72:728, 1994.
[7] N. Read, Phys. Rev. B, 52:1926, 1995.
[8] A. Fukuda, D. Terasawa, M. Morino, K. Iwata, S. Kozumi, N. Kumada, Y. Hirayama, Z. F. Ezawa, and A. Sawada, Phys. Rev. Lett., 100:016801, 2008.
[9] A. Fukuda, M. Morino, K. Iwata, D. Terasawa, S. Kozumi, N. Kumada, Y. Hirayama, Z.F. Ezawa, A. Sawada, Physica E, 40:1255, 2008.