TURBULENCE IN WEAKLY IONIZED PROTOPLANETARY DISKS

M. FLOCK1,2, TH. HENNING2, and H. KLAHR2

1 CEA Irfu, SAP, Centre de Saclay, F-91191 Gif-sur-Yvette, France
2 Max Planck Institute for Astronomy, Königstuhl 17, D-69117 Heidelberg, Germany

Received 2012 August 21; accepted 2012 October 1; published 2012 November 30

ABSTRACT

We investigate the characteristic properties of self-sustained magneto-rotational instability (MRI) turbulence in low-ionized protoplanetary disks. We study the transition regime between active and dead zones, performing three-dimensional global non-ideal MHD simulations of stratified disks covering a range of magnetic Reynolds numbers between 2700 ≤ Rm ≤ 6600. We found converged and saturated MRI turbulence for Rm ≳ 5000 with a strength of αSS ≳ 0.01. Below Rm ≲ 5000, the MRI starts to decay at the midplane at first because the Elsasser number drops below 1. We find a transition regime between 3300 ≲ Rm ≲ 5000 where the MRI turbulence is still sustained but damped. At around Rm ≲ 3000, the MRI turbulence decays but could be reestablished due to the accumulation of the toroidal magnetic field or the radial transport of the magnetic field from the active region. Below Rm ≪ 3000, the MRI cannot be sustained and is decaying. Here, hydrodynamical motions, like density waves, dominate. We observe anti-cyclonic vortices in the transition between the dead zone and the active zone.

Key words: accretion, accretion disks – dynamo – magnetohydrodynamics (MHD)

1. INTRODUCTION

Magneto-rotational instability (MRI) is a candidate for driving turbulence and enabling the accretion of matter onto a central object (Balbus & Hawley 1991, 1998; Hawley & Balbus 1991). Depending on the ionization degree, the MRI-generated turbulence will be reduced to a low turbulence regime called the dead zone (Blaes & Balbus 1994; Jin 1996; Sano et al. 2000). Various studies show that a certain level of resistivity suppresses MRI activity (Fleming et al. 2000; Sano & Stone 2002a, 2002b; Fleming & Stone 2003; Inutsuka & Sano 2005; Turner et al. 2007, 2010; Turner & Sano 2008). One of the most important dimensionless numbers that characterizes the coupling between gas and magnetic fields is the magnetic Reynolds number, Rm, which relates the timescale of advection to magnetic diffusion. We consider here the ohmic diffusion term, which is most important at the midplane. Until now, there was no clear prescription for which values of Rm the MRI turbulence is sustained in disks. Fleming et al. (2000) found sustained MRI with a zero-net flux magnetic field for Rm > 104. A recent study by Simon et al. (2011) used stratified local box simulations to investigate the turbulence level for different values of the magnetic Reynolds and Prandtl numbers. They found a so-called low state, a state where turbulence is partly suppressed, but sustained due to a dynamo process. In addition, they predicted a critical Reynolds number Rm crit in the range of 3200 < Rm crit < 6000. A similar region was investigated by Oishi & Mac Low (2011) in which they found Rm crit ≃ 3000. This critical Reynolds number is important for modeling the surface density of active layers in the protoplanetary disk as was recently done by Martin et al. (2012).

In our study, we will search for the critical magnetic Reynolds number in global zero-net flux stratified MRI simulations. Here the MRI turbulence criterion, the Elsasser number ν2Ω/η, should become unity. In contrast to the magnetic Reynolds number, the Elsasser number gives one clear threshold independent of the magnetic geometry or the stratification. We will also investigate the hydrodynamical motions, which become important in the dead-zone region (Oishi & Mac Low 2009). We concentrate on the magnetic Reynolds number regime proposed by Simon et al. (2011) and Oishi & Mac Low (2011). For our simulations, we use only explicit resistivity. Oishi & Mac Low (2011) found well-ionized MRI turbulence scales independent of Pm if Rm > Rm crit. In addition, as the molecular viscosity is very small in protoplanetary disks, we expect Prandtl numbers of Pm ≪ 1 and we focus on this low Prandtl number regime.

2. SETUP

The initial conditions for density, pressure, and azimuthal velocity follow the hydrostatic equilibrium. We set the density ρ to

ρ = ρ0R−3/2 exp \left( \frac{\sin(\theta) - 1}{(H/R)^2} \right) \quad (1)

with ρ0 = 1.0, the scale height to radius H/R = 0.07, and R = r sin(θ). The pressure locally follows an isothermal equation of state: P = c_s^2 ρ with sound speed c_s = c_0/√R. The azimuthal velocity is set to

V_\phi = \sqrt{\frac{1}{r}} \left\{ 1 - \frac{2.5}{\sin(\theta)} \frac{c_\phi}{c_0} \right\}. \quad (2)

The initial velocities V_r and V_\theta are set to a white noise perturbation amplitude of V^{init}_{r,0} = 10^{-4} c_\phi. We start the simulation with a pure toroidal magnetic seed field with a constant plasma beta of β = 2 P/B^2 = 25. To obtain a range of magnetic Reynolds numbers, we keep the magnetic dissipation value constant in the disk. We use three different values of η, η_1 = 2 × 10^{-6} AU^2 yr^{-1}, η_2 = 2.6 × 10^{-6} AU^2 yr^{-1}, and η_3 = 3.2 × 10^{-6} AU^2 yr^{-1}.

Rm = \frac{c_\phi H}{\eta} = 2450 \left( \frac{H/R}{0.07} \right)^2 \left( \frac{R}{1 \text{ AU}} \right)^{0.5} \left\{ \frac{\eta}{2 × 10^{-6}} \right\}. \quad (3)

Here, the hydrodynamic viscosity is small compared to the magnetic diffusivity ν ≪ η.
In order to estimate the numerical diffusion, we run ideal MHD simulations with different resolutions as a reference. The radial domain extends from 1 to 10 AU. The $\theta$ domain covers $\pm 4.3$ disk scale heights, or $\theta = \pi/2 \pm 0.3$. For the azimuthal domain, we use $2\pi$ for the L models and $\pi/2$ for the H models. We use a uniform grid in spherical coordinates. The L models have a resolution of $N_r = 384$, $N_\theta = 192$, and $N_\phi = 768$, and the H models have a resolution of $N_r = 384$.

All models resolve the radial scale height with 9–22 grid cells for the inner and outer radii. The vertical scale height is resolved by 22 grid points. In the L models, the azimuthal scale height is resolved by nine grid cells. The H models have a higher resolution of 17 per scale height in the azimuth. They are calculated with the FARGO MHD to reduce the numerical dissipation even more (Mignone et al. 2012). The simulation models are summarized in Table 1. We note that model L$^1$ is special. Here, the numerical dissipation cannot be neglected. This model uses no FARGO, and the MRI modes are not well resolved. Thus, in addition to physical magnetic diffusion, there is a numerical diffusion, so that effectively, $R_m < 3000$. Compared with the results of the H models, the L$^1$ model shows a magnetic Reynolds number below the value used in H$^1$. This model establishes a large dead-zone region. Here, hydrodynamical motions become important. Buffer zones extend from 1 to 2 AU as well as from 9 to 10 AU. In the buffer zones, we use a linearly increasing resistivity (up to $\eta = 10^{-3}$) that reaches the boundary. This damps the magnetic field fluctuations and suppresses boundary interactions. For our analysis, we use the range between 3 and 8 AU, which is not affected by the buffer zones. Our outflow boundary condition projects the radial gradients in density, pressure, and azimuthal velocity onto the radial boundary and the vertical gradients in density and pressure at the $\theta$ boundary. For all runs, we employ the second-order scheme in the PLUTO code with the HLLD Riemann solver (Miyoshi & Kusano 2005), piece-wise linear reconstruction and second-order Runge–Kutta time integration. We treat the induction equation with the “constrained transport” (CT) method in combination with the upwind CT method described in Gardiner & Stone (2005), using explicit resistivity. A more detailed description of the physical setup can be found in Flock et al. (2011).

### 3. RESULTS

According to Equation (3), we obtain a specific magnetic Reynolds number at each radius, resulting in a specific value of turbulence. In order to compare the results at different specific radii, the timescale of radial mixing is important to consider. Due to low magnetic dissipation, the magnetic diffusion timescale is very long compared to the turbulent mixing timescale. The maximum radial mixing scale for an alpha value of 0.01 after 1000 years at the outer radius at 8 AU is around $\delta x^\text{mix} = \sqrt{2\alpha_{\text{SS}} c_s H} \sim 0.5$ AU. For our analysis, we use specific radial positions with separation distances larger than $\delta x^\text{mix}$. In Section 3.5, we will investigate how the radial transport of the magnetic field affects the local evolution. The $\alpha_{\text{SS}}$ value at a specific radial position is calculated with

$$
\alpha_{\text{SS}}(r) = \left( \frac{\int r^2 \rho (V_{\phi}^2 r^2)^2 + \int r^2 \rho (V_r^2 r^2)^2 + \int r^2 \rho (V_\theta^2 r^2)^2}{\frac{R_m}{4\pi\rho c_s^2} \int r^2 \rho dV} \right)^{1/2}.
$$

The integral is done for each radius separately; $\int r^2 dV = \int_{r_1}^{r_2} r^2 \sin \theta dr d\theta d\phi$ with $r_1$ and $r_2$ is the left and right radial cell boundary. The $\alpha_{\text{SS}}$ is mass weighted to recover the correct total value. In Section 3.1, we compare results that have the same local time period. Here, we average $\alpha_{\text{SS}}(r)$, the turbulent velocities $V_{\text{RMS}} = \sqrt{(\delta v_r)^2 + (\delta v_\theta)^2 + (\delta v_\phi)^2}$, and the dynamo $\alpha_{\phi\phi}$ between 25 and 60 local orbits. This ensures that we compare the same dynamical evolution at a specific radius and Reynolds number. The space average of $V_{\text{RMS}}$ and $\alpha_{\phi\phi}$ is done at the midplane region between 0 and 1.5 scale heights.

In Section 3.2, we concentrate on the long-term evolution for different values of $R_m$. There we include the two MRI criteria: the Elsasser number

$$
\Lambda = \frac{B^2}{\rho \eta \Omega}
$$

and the $Q$ factor

$$
Q = \frac{\lambda_c}{\Delta \phi} = 2\pi \sqrt{\frac{16 B^2}{\phi c^2} \frac{H}{R \Delta \phi}}.
$$

The $Q$ factor is the ratio of the MRI fastest growing azimuthal wavelength to the azimuthal cell size. The value should be larger than 8 (Flock et al. 2010). In Section 3.3, we concentrate on the run L$^1$ having the largest dead zone. Here, we present the results of the dominant hydrodynamical motions.

#### 3.1. Time-averaged Statistics

Due to the change of the rotation period $\Omega$ as the radius changes, the comparison of the turbulence statistics at different radii is limited to the number of rotations at the outer radius. We analyze the average turbulent properties between 25 and 60 local orbits at different radial locations corresponding to different $R_m$; see Equation (3). We note that in this period the initial net magnetic flux has already vanished and we have a zero-net flux MRI turbulence.

Figure 1 combines the results of $\alpha_{\text{SS}}$, the turbulent rms velocity $V_{\text{RMS}}$, and the dynamo $\alpha_{\phi\phi}$ value as a function of the magnetic Reynolds number. For all turbulent quantities, the results show a saturation for magnetic Reynolds numbers around 5000 and above. Below $R_m = 5000$ the MRI is damped and the turbulence decreases. We observe a slightly higher turbulence level for high magnetic Reynolds numbers compared to a fully ideal MHD run.

There is a saturation of $\alpha_{\text{SS}}$ around the 0.015 value for magnetic Reynolds numbers greater than 5000; see Figure 1, top. Below $R_m < 5000$, $\alpha_{\text{SS}}$ drops down to 0.005 at around $R_m \sim 3300$. Even though one could see a flattening around $R_m = 3300$, the values are still dominated by Maxwell stress, which indicates that the MRI is still operating. The turbulent velocity scales roughly with the square root of $V_{\text{RMS}} \sim \sqrt{\alpha_{\text{SS}}}$.

### Table 1

| Model Name | Resolution, Domain Size, FARGO-MHD, and Range of Magnetic Reynolds Number for the Models Studied |
|------------|--------------------------------------------------------------------------------------------------|
| L$^{\text{Ideal}}$ | $384 \times 192 \times 768$ | $10^{-6}$–$10^{-2} \pi$ | FARGO |
| L$^{\text{IdealFARGO}}$ | $384 \times 192 \times 768$ | $10^{-6}$–$10^{-2} \pi$ | Yes | Ideal |
| H$^{\text{IdealFARGO}}$ | $384 \times 192 \times 384$ | $10^{-6}$–$6 \pi/2$ | Yes | Ideal |
| H$^1$ | $384 \times 192 \times 384$ | $10^{-6}$–$6 \pi/2$ | Yes | 4300–6300 |
| H$^2$ | $384 \times 192 \times 384$ | $10^{-6}$–$6 \pi/2$ | Yes | 3300–4800 |
| H$^3$ | $384 \times 192 \times 384$ | $10^{-6}$–$6 \pi/2$ | Yes | 2700–4300 |
| L$^1$ | $384 \times 192 \times 768$ | $10^{-6}$–$10^{-2} \pi$ | No | $<3000$ |

---

The Astrophysical Journal, 761:95 (8pp), 2012 December 20

---

Flock, Henning, & Klahr
found in recent local box simulations of dead zones (Okuzumi & Hirose 2011). We observe a saturation around $V_{\text{RMS}} = 0.15 \, c_\ast$ (Figure 1, middle). Below $R_m < 5000$, $V_{\text{RMS}}$ drops down to 0.09 at around $R_m \sim 3000$. We note again that these values represent the turbulent motions at the midplane; in the corona, the turbulent velocity increases. The dynamo action could be expressed by showing the correlations $\tilde{E} M^2 = \alpha_{\phi\phi} B^2$ with $EM^2_\phi = v'_\phi B'_\phi - v'_\phi B'_\phi$. $\alpha_{\phi\phi}$ is normalized over the local sound speed. We follow the analysis described in Section 3.7 in Flock et al. (2012). The dynamo action in the northern hemisphere is shown in the bottom of Figure 1. $\alpha_{\phi\phi}$ saturates at around 0.004 and decreases only down to 0.002 at $R_m = 3000$. In all the runs, the sign of $\alpha_{\phi\phi}$ in the southern hemisphere is negative with a similar amplitude as the corresponding value in the northern hemisphere. The blue crosses in Figure 1 represent the radial profile of $\alpha_{\phi\phi}$, $\nu_{\text{RMS}}$ and $\alpha_{\phi\phi}$ in model $L^1$. The peculiarity of model $L^1$ is that the dead zone is stretched over the complete radial domain, with active layers in the corona (see Figure 6). Figure 1 shows that the turbulent properties are constant with radius in model $L^1$. Judging from the level of turbulence, we suggest that the numerical dissipation shifts effectively $R_m < 3000$ in $L^1$.

The results of the ideal MHD runs show convergence. Here, the FARGO MHD method plays an important role in decreasing numerical dissipation as it is presented in Mignone et al. 2012. $\alpha_{SS}$ converges around 0.01. We note again that the results presented in Figure 1 mainly reflect the turbulence level at the midplane. In the next section, we will concentrate on the long-term evolution over height for specific Reynolds numbers.

3.2. Long-term Evolution

In this section, we focus on the long-term evolution, including turbulence over height. For each model, we perform the analysis in the middle of the domain. Here, $H^1$ has $R_m = 5500$, $R_m = 4300$ for model $H^2$, and $R_m = 3300, 3000$ for model $H^3$. In model $L^1$, we expect to have a Reynolds number around $R_m \lesssim 3000$ by comparison with model $H^3$.

We plot the time evolution of $\alpha_{SS}$ in Figure 2. The analysis shows that both ideal runs as well as resistive runs down to $R_m = 4300 \, (H^1 \text{ and } H^2)$ show a steady turbulent evolution with $\alpha_{SS} \sim 0.01$. The models $H^2$ and $L^1$ with $R_m \leq 3000$ reach an oscillatory steady state only after 600 yrs. For these models, an extended simulation runtime was needed. We mark the regions with dominating Reynolds stress in thick blue and green solid lines. The configuration of the Maxwell stress $\alpha_{M} = B'_\theta B'_\phi / 4\pi \rho c^2$ over height is presented in Figure 3. After around 200 inner orbits (years), the initial net azimuthal flux is lost and the flux starts to oscillate around zero (Fromang & Nelson 2006;
Figure 3. Evolution of the Maxwell stress $\alpha_M$ over height. Color bars show $\log_{10}(\alpha_M)$. For model H1 at $R_m = 5500$ (top), H2 at $R_m = 4300$ (second from top), H3 at $R_m = 3300$ (third from top), and L1 at $R_m \lesssim 3000$ (bottom). The black solid lines show the Elsasser numbers 10, 1, and 0.1. The red solid line shows $Q = 8$. The yellow line shows the region with dominating Reynolds stress.

Beckwith et al. (2011; Flock et al. 2011). For a magnetic Reynolds number above $R_m = 5500$, the ionization is sufficient to sustain the MRI turbulence at each height. In the midplane region, we found a saturation of $\alpha_{SS} \sim 0.01$ at around $R_m \gtrsim 5000$. In model H1 (the top, panel in Figure 3) the Elsasser number is between 1 and 10 in the midplane region. If the $R_m$ decreases below $R_m \gtrapprox 5000$, the Elsasser number reaches unity at the midplane (see also Figure 4). In model H2 (the second panel in Figure 3), there is a small region at the midplane where $0.1 < \Lambda < 1$. The total $\alpha_{SS}$ is still around 0.01
(compare Figure 2) but with higher fluctuations. Locally the magnetic turbulence starts to vanish at the midplane. At around $R_m \approx 3300$ (model H$^3$) the total $\alpha_{ss}$ gets affected and decreases (compare Figure 2). In model H$^3$, the Elsasser number $\Lambda = 1$ point is around 1 scale height. At the midplane, $\Lambda$ drops below 0.1. We confirm that the $\Lambda = 0.1$ regime is equivalent to the $\lambda_{res} = H$ definition by Okuzumi & Hirose (2011; see their Figure 1). At this point, the azimuthal MRI wavelength becomes unresolved ($Q < 8$, solid red line). At the midplane, the Reynolds stress dominates over the Maxwell stress (Figure 3, solid yellow line). At $R_m \sim 3000$, the MRI gets strongly damped. For models L$^1$ and H$^3$, after around 500 years, the poloidal magnetic fields decrease until the Elsasser number drops below 0.1, and the azimuthal MRI wavelength becomes unresolved (Figure 3, solid red line). Even though we chose a constant resistivity, the turbulence in the H$^3$ and L$^1$ runs is similar to what is present in dead zones. Upper layers of the disk are usually active (Turner & Sano 2008; Dzyurkevich et al. 2010; Bai 2011). In these active layers, the turbulent mixing and active channels can reach into the dead zone. Figure 4 presents a two-dimensional $r-\theta$ slice of the radial magnetic field overplotted with the $\Lambda = 1$ line for models H$^1$ and H$^2$ after 1000 years. At around $R_m = 5000$, the dissipation starts to damp the MRI at the midplane while there is still MRI turbulence in the corona of the disk.

In summary, for magnetic Reynolds numbers above $R_m > 5000$, we observe a saturated and converged MRI turbulence in zero-net flux stratified simulations down to the midplane region. Below $R_m < 5000$, the turbulence starts to decay at the midplane. The Elsasser number drops below 1 and regions with dominating Reynolds stress appear. At around $R_m \approx 3000$, the Elsasser number drops below 0.1 at the midplane. Long-period oscillations become visible with MRI activation and decay. Below $R_m < 3000$, the MRI cannot operate. Here, Reynolds stress dominates over Maxwell stress. The results show that for magnetic Reynolds numbers down to $R_m \approx 4300$, the MRI can sustain the turbulence with $\alpha_{ss} \approx 0.01$. Models H$^3$ and L$^1$ with $R_m < 3000$ show a decrease in turbulence and eventually a dominating Reynolds stress. From this result, we conclude that the critical magnetic Reynolds number should be around $R_m^{crit} \approx 3000$. Oishi & Mac Low (2011) also found a critical magnetic Reynolds number around $R_m \sim 3000$ in local box simulations. In the next section, we concentrate on the L$^1$ runs, which present similar conditions as they are present in dead zones.

### 3.3. Dead-zone Oscillation

Before we move to the hydrodynamical motions in the dead zones, we want to investigate the long-term oscillations of the Maxwell stress that we observed in the previous section. This oscillation could be connected to some sort of dynamo process. Simon et al. (2011) presented in local box simulations a sudden transition from a “low” turbulent state to a “high” turbulent state. We observe a similar effect in models H$^3$ and L$^1$ for $R_m \sim 3000$. The results are combined in Figure 5 showing the mean toroidal magnetic field overplotted with the turbulent EMF$_d$ (times 1000), the Elsasser number, and the $Q$ value. For this analysis, we chose the southern hemisphere between 0 and 1.5 scale heights. After around 200 years, the initial mean toroidal magnetic field (black solid line) vanishes and starts to oscillate. There is a clear correlation between the turbulent electromotive force (EMF) (red dotted line) and the mean field, which indicates a working $\alpha \Omega$ dynamo. In both models, after around 500 years the MRI from azimuthal magnetic fields becomes unresolved ($Q < 8$, blue dotted line below the black dotted line). Here, the Elsasser number drops below unity, which still allows for damped MRI growth. After 800 years, the Elsasser number drops below 0.1. At this stage, there is no MRI working and we are in the dead-zone stage. Here, the oscillations of the mean field and turbulent EMF also stop. In model H$^3$, the MRI switches on again after less than 200 years (27 local orbits) with a larger oscillation period. In model L$^1$, we observe a late switch on at 1100 years for only 150 years (20 local orbits). Afterward, the Elsasser number reaches nearly $10^{-3}$ and we do not expect a relaunch again. The sudden increase of $\Lambda$ is due to a sudden increase in the poloidal magnetic field. This sudden increase could be connected with the accumulation of toroidal magnetic fields (black solid line in the bottom panel of Figure 5) in the dead zone as well as to the radial transport of the magnetic field (see Section 3.5). In Figure 6, we present an azimuthal slice of the toroidal magnetic field in the $r-\theta$ plane after 1500 years for model L$^1$. In the dead zone ($\Lambda < 0.1$), we observe locally strong magnetic fields with a $Q$ factor of eight, which corresponds to a plasma beta below 50. In general, the line of the resolved azimuthal field $Q < 8$ matches the one of decaying MRI $\Lambda < 0.1$ (compare Figure 3).
H3 - 3.8 AU - Rm = 3000

Figure 5. Time evolution of mean azimuthal field (black solid line), turbulent EMF′φ (red dotted line), Λ (blue solid line), and the Q value (blue dotted line) for model H3 at Rm = 3000 (3.8 AU; top) and for model L1 at 3.8 AU (bottom). Values are calculated in the southern hemisphere between (0 and 1.5 scale heights). The black dotted line shows the value of Q = 8. In model L1, we observe a relaunch of MRI between 1200 and 1300 years in the dead zone. For model H3, we observe only short-period switch-off between 800 and 900 years.

L1 - 3.8 AU - Rm<= 3000

Figure 6. Contour plot of azimuthal magnetic field in the r–θ plane after 1500 years overplotted with the Elsasser number Λ = 0.1 red solid line and the Q value = 8 green solid line. We observe in the dead-zone relative strong (resolved) toroidal magnetic fields.

3.4. Hydrodynamical Motions

This dead-zone region is also interesting in terms of hydrodynamical motions. A close look at the radial velocity in the r–φ midplane in Figure 7 reveals velocity amplitudes around 0.2 c_s. On the first look, these hydrodynamical waves appear to be linear waves with a single mode. But a closer look at the Fourier spectra of the radial velocity reveals a more detailed picture. We calculated the spectra in the dead-zone region between 4 and 5 AU, time averaged between 1300 and 1500 years in model L1. In Figure 8, we compare this spectra with the one obtained in a fully ionized disk from model L Ideal FARGO. In the dead zone, there is a peak at m = 6. The turbulence at lower scales (green solid line) is around one order of magnitude below the value in the fully MRI turbulent region. Beside the density waves, we also find long-lived anti-cyclonic vortices (Figure 7, between 6 and 7 AU), which create large extended spiral arms. They are produced at around 8 AU when there is a positive density slope due to the buffer zones. We calculated the relative vorticity ω = ((∇ × V)θ - (∇ × V_{Kepler})θ)/(∇ × V_{Kepler})θ between -0.8 and -0.5 in the large vortex. The growth of vortices at the border of the dead zone could be due to Rossby wave instability as proposed by Varnière & Tagger (2006) and recently investigated by Lyra & Mac Low (2012). The vortex structure is presented in Figure 9. The vertical extension is around ±2 SH, the radial extension is 2 H′ ~ 1 AU, and the azimuthal
one is around 10 $H^\phi \sim 4$ AU. The vortex is dragged by the hydrodynamical surroundings, which gives it the concave (top panel in Figure 9, dominant radial inward velocity) and convex (bottom panel in Figure 9, $V^\phi > V^\rho$) shape, respectively. The magnetic fields (Figure 9, bottom black vectors) are not present inside the vortex but are in the surrounding layers. We measure a plasma beta of $\sim 10^5$ inside the vortex compared to $\sim 100$–1000 in the surrounding region.

3.5. Radial Mixing

We already mentioned the effect of radial transport due to turbulent mixing. In this subsection, we want to focus on the importance of the radial transport of magnetic fields. We calculate the divergence of the radial Poynting flux $\int \nabla \cdot S_r dt$ integrated over time for the total $\theta$ and $\phi$ domain at 4.6 AU using 1 scale height in the radius. We integrate the divergence of the radial Poynting flux over every output ($dt = 0.1$ local orbits) and compare it with the total magnetic energy. Figure 10 shows how much magnetic energy is transported at each output (0.1 local orbits). In the fully turbulent layers, the net radial transport is around zero with fluctuations of around 5%. In the transition zone and the dead zone, the radial transport becomes more important and these fluctuations reach peaks of 10%–20%.

4. DISCUSSION

4.1. Turbulence between Active and Dead Zones

In our work, we find that the MRI turbulence saturates around $R_m = 5000$ with $\alpha \sim 0.01$. Here the Elsasser number is around $\Lambda \geq 10$. This turbulence level is sustained for Elsasser number values of $1 < \Lambda < 10$. Stratified converged ideal MHD local simulations (Davis et al. 2010) presented similar values of $\alpha_{SS}$. Still, our results indicate a higher turbulence level using small explicit resistivity. High-resolution simulations using explicit resistivity are needed to confirm the convergence of the accretion stress. In addition, there are mean-field mechanisms, which can increase the temporal value of $\alpha_{SS}$ (Flock et al. 2012).

For magnetic Reynolds numbers around the critical value of $R_m \sim 3000$, we observe long-period oscillations of the accretion stress as well as an irregular switch off and on of MRI activity. Such oscillations were also reported by Simon et al. (2011) in local box simulations. We see indications that these oscillations are correlated with dynamo activity as locally there are accumulations of strong mean toroidal fields in the dead zone. Also radial transport of the magnetic field could play a role here. In such configurations, the corona is still active while in the midplane the MRI is switched off. Here, the Reynolds stress dominates over the Maxwell stress at $\alpha < 0.1$. A similar classification of turbulent regions in protoplanetary disks was done by Okuzumi & Hirose (2011).

4.2. Numerical, Explicit, and Turbulent Dissipation

In such turbulent simulations, there are three kinds of important dissipation sources: the numerical dissipation due to the finite grid size, the explicit resistivity, and the turbulent dissipation. In model L1, numerical dissipation plays an important role. In addition, one has to note that by using a uniform grid, the numerical dissipation will be larger at the inner part than in the outer part. Here, a logarithmic increasing grid would help. Then, there is of course the explicit dissipation, which is included in the induction equation in the code. One should be sure that the explicit one is higher than the numerical one. The turbulent dissipation is proportional to the level of turbulence. Here, it is still unclear how this dissipation process works in detail (see also Figure 9 in Fromang & Papaloizou (2007)).
5. SUMMARY

We performed three-dimensional global ideal and non-ideal MHD simulations to study MRI turbulence in low-ionized protoplanetary disks using an initial toroidal magnetic field for a range of magnetic Reynolds numbers. With our global simulations, we are able to investigate the transition regime between the active and the dead zones in protoplanetary disks. We define three different disk regimes depending on the magnetic Reynolds numbers:

1. **Above a Reynolds number of $R_m \gtrsim 5000$.** In this regime, MRI turbulence saturates and is sustained. We find steady and converged $\alpha_{\rm SS}$ values around 0.01 in ideal and non-ideal simulations. The Maxwell stress dominates in the corona and the midplane. The turbulent velocities reach values around $V_{\text{RMS}} = 0.15 c_\gamma$. The $\alpha \Omega$ dynamo is operating and we find $\alpha \Omega = 0.004$ with a positive sign in the northern hemisphere. The Elsasser number $\Lambda$ stays above 1 in the midplane and corona.

2. **Reynolds number between $R_m \approx 3000$ and 5000.** The MRI starts to decay at the midplane. The Elsasser number $\Lambda$ drops below 1. Simulations with $3300 < R_m < 5000$ show a still sustained turbulence supported by the radial transport of magnetic fields. We expect a critical magnetic Reynolds number around $R_m \approx 5000$.

3. **Reynolds number around $R_m \approx 3000$ and below.** The MRI does not operate at the midplane anymore. The Elsasser number $\Lambda$ drops below 0.1. The turbulence is dominated by the Reynolds stress. We observe long-period oscillations of MRI activity and MRI decay. There is an accumulation of toroidal magnetic fields below $\beta < 50$ in the dead zone. The turbulence at the midplane is supported by active channels in the corona, which pump sound waves into the midplane region. The velocity spectra at the midplane reveal a drop of one order of magnitude at the scale of H in the dead zone compared to the fully ionized turbulent region. We observe anti-cyclonic vortices in the transition regime that create spiral arms in the dead zone. Magnetic fields are not present in the inner part of the vortex.

We thank Sebastien Fromang for helpful comments on the global models. We thank Natalia Dzyurkevich for suggestions regarding the manuscript. We also thank Neal Turner, Satoshi Okuzumi, and Heloise Meheut for their revision. We thank Andrea Mignone for supporting us with the PLUTO code. The research leading to these results received funding from the European Research Council under the European Union’s Seventh Framework Programme (FP7/2007–2013)/ERC Grant agreement No. 258729. Parallel computations have been performed on the Theo cluster of the Max-Planck Institute for Astronomy Heidelberg located at the computing center of the Max-Planck Society in Garching.

**REFERENCES**

Bai, X.-N. 2011, ApJ, 739, 50
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Balbus, S. A., & Hawley, J. F. 1998, Rev. Mod. Phys., 70, 1
Beckwith, K., Armitage, P. J., & Simon, J. B. 2011, MNRAS, 416, 361
Blaes, O. M., & Balbus, S. A. 1994, ApJ, 421, 163
Davis, S. W., Stone, J. M., & Pessah, M. E. 2010, ApJ, 713, 52
Dzyurkevich, N., Flock, M., Turner, N. J., Klahr, H., & Henning, T. 2010, A&A, 515, A70
Fleming, T., & Stone, J. M. 2003, ApJ, 585, 908
Fleming, T. P., Stone, J. M., & Hawley, J. F. 2000, ApJ, 530, 464
Flock, M., Dzyurkevich, N., Klahr, H., & Mignone, A. 2010, A&A, 516, A26
Flock, M., Dzyurkevich, N., Klahr, H., Turner, N., & Henning, T. 2012, ApJ, 744, 144
Flock, M., Dzyurkevich, N., Klahr, H., Turner, N. J., & Henning, T. 2011, ApJ, 735, 122
Fromang, S., & Nelson, R. P. 2006, A&A, 457, 343
Fromang, S., & Papaloizou, J. 2007, A&A, 476, 1113
Gardiner, T. A., & Stone, J. M. 2005, J. Comput. Phys., 205, 509
Hawley, J. F., & Balbus, S. A. 1991, ApJ, 376, 223
Inutsuka, S., & Sano, T. 2005, ApJ, 628, L153
Jin, L. 1996, ApJ, 457, 798
Lyra, W., & Mac Low, M.-M. 2012, ApJ, 756, 62
Martin, R. G., Lubow, S. H., Livio, M., & Pringle, J. E. 2012, MNRAS, 420, 3139
Mignone, A., Flock, M., Stute, M., Kolb, S. M., & Muscianisi, G. 2012, A&A, 545, A152
Miyoshi, T., & Kusano, K. 2005, J. Comput. Phys., 208, 315
Oishi, J. S., & Mac Low, M.-M. 2009, ApJ, 704, 1239
Oishi, J. S., & Mac Low, M.-M. 2011, ApJ, 740, 18
Okuzumi, S., & Hirose, S. 2011, ApJ, 742, 65
Sano, T., Miyama, S. M., Umebayashi, T., & Nakano, T. 2000, ApJ, 543, 486
Sano, T., & Stone, J. M. 2002a, ApJ, 570, 314
Sano, T., & Stone, J. M. 2002b, ApJ, 577, 534
Simon, J. B., Hawley, J. F., & Beckwith, K. 2011, ApJ, 730, 94
Turner, N. J., Carbillado, A., & Sano, T. 2010, ApJ, 708, 188
Turner, N. J., & Sano, T. 2008, ApJ, 679, L131
Turner, N. J., Sano, T., & Dzyurkevich, N. 2007, ApJ, 659, 729
Varnière, P., & Tagger, M. 2006, A&A, 446, L13