Steady State Unbalance Response Analysis of Internal Combustion Engine Crankshaft Based on Torsional Vibration

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Abstract. Crankshaft is one of the main moving parts in internal combustion engine. The instantaneous gas pressure and effective crank radius of each cylinder of the engine and the ignition interval angle of each cylinder make the torque of each crank change constantly, which causes the torsional vibration of the crankshaft. Based on the lumped parameter model, the transfer matrix method is used to analyze the steady-state unbalance response of internal combustion engine crankshaft, which has a certain reference value for mastering the influence of torsional vibration on the working state and strength safety of crankshaft.

1. Induction
Crankshaft is one of the main moving parts in internal combustion engine, its strength and reliability determine the reliability of internal combustion engine to a great extent. As the instantaneous gas pressure and effective crank radius of each cylinder of the engine change constantly, the product of the instantaneous torque gas pressure and effective crank radius also changes in each cycle. However, the firing interval angle of each cylinder makes the torque of each crank relatively change, which causes the relative motion of the crankshaft masses. This phenomenon is called "torsional vibration". Torsional vibration will make the working state of internal combustion engine unstable, which also has a negative impact on the strength and safety of crankshaft. Therefore, it is particularly important to analyze the unbalanced response of Crankshaft Based on torsional vibration, master the characteristics and laws of torsional vibration, further reduce the impact on the working stability of internal combustion engine, and eliminate the potential safety hazard of design strength.

2. Establishment of analytical model
Because the structure and stress of crankshaft are complex, when calculating the vibration characteristics of crankshaft shafting, it is generally necessary to simplify the shafting into a relatively simple mechanical model for easy solution. This paper is a lumped parameter model, in which the mass of shafting is discretized and concentrated to many concentrated points. In this model, the crankshaft system is discretized into a disk with concentrated moment of inertia and a massless elastic shaft. The moment of inertia of each disc includes the moment of inertia of crankshaft, the equivalent moment of inertia of piston and connecting rod, the moment of inertia of transmission system, shock absorber and flywheel.
The torsional vibration of crankshaft system is actually composed of crankshaft, connecting rod, pulley, timing gear, rotary inertia of each cylinder, flywheel, clutch and other parts. When the actual crankshaft system is simplified into equivalent system according to the principle of dynamic equivalence, the components related to the torsional vibration system are included in the flywheel and the parts fixed on the flywheel. Therefore, the actual crankshaft system can be simplified into an equivalent system composed of an inertia plate with only inertia but no elasticity and a shaft section with only flexibility but no inertia, as shown in Fig. 2.

3. Analysis of exciting torque of shafting
The excitation torque is the power source of torsional vibration. The excitation torque of internal combustion engine mainly comes from: the excitation torque generated by the change of gas pressure in the engine cylinder; the excitation torque generated by the gravity and inertia force of the engine crank and connecting rod mechanism; the excitation torque generated by the torque absorbed by the components receiving power is not a fixed value.

The interference torque produced by the gas pressure in the cylinder of piston engine is the main component of torsion produced by the vibration system. It varies with different models and different operating conditions. Therefore, the analysis of it should be the main purpose.

3.1. Torsional excitation moment produced by gas pressure
When the internal combustion engine works, the combustible mixture burns in the cylinder. Due to the rapid propagation rate of flame during combustion and the shock and superposition of pressure waves in the cylinder, high explosion pressure (the maximum explosion pressure of compression ignition engine is about 90-160 atmospheres; the maximum explosion force of spark ignition engine is relatively small) and pressure rise rate (the pressure rise rate of compression ignition engine is about 4-10 atmospheres) are generated near the top dead center Atmospheric pressure / crankshaft angle). These large amplitude oscillating pressure waves act on the top surface of piston and cylinder, resulting in torsional vibration of shafting and vibration of whole vehicle.

Because the combustion process is periodic, the pressure curve of each cycle can be composed by a series of sine waves with different amplitudes and phases.

\[
P_g = C_0 + \sum C_n \sin(\omega_n + \phi_n)
\]  

(1)
In the above formula, \( C_0 \) is the average indicated pressure in a cycle and \( C_n \) is \( n \) harmonic cylinder pressure component.

The exciting torque produced by the change of gas pressure in the cylinder of diesel engine is expressed by the tangential force acting on the crank pin. According to the dynamics of internal combustion engine, the tangential force is expressed as follows:

\[
P_T = \frac{\sin(\alpha + \beta)}{\cos \beta} P_g
\]

Expression (2) without \( \beta \) can be approximated as

\[
P_T \approx \left( \sin \alpha + \frac{R}{2L} \sin 2\alpha \right) P_g
\]

In the above formula, \( P_g \) is the gas pressure acting on the piston; \( \alpha \) is the crank angle; \( \beta \) is the connecting rod swing angle; \( R \) is the crank radius; \( L \) is the connecting rod length.

The gas pressure \( P_g \) varies periodically with the angle \( \alpha \). For a four stroke diesel engine, its cycle is two revolutions of the internal combustion engine. The value of gas pressure can be obtained by indicator diagram. Obviously, the tangential force is also a complex periodic function varying with the crank angle. The torque produced by tangential force on crankshaft, that is, the exciting torque of torsional vibration, is as follows:

\[
T_g = \frac{\pi}{4} D^2 R P_T
\]

The tangential force \( P_T \) is expressed in the form of Fourier series:

\[
P_T(t) = a_0 + \sum_{b=1}^{\infty} \left( a_n \cosh n\omega t + b_n \sinh n\omega t \right) = a_0 + \sum_{b=1}^{\infty} C_n \sin \left( h \omega t + \varphi_n \right)
\]

### 3.2. Phase analysis of torsional excitation torque

If the crankshaft rotates at speed \( \omega \), the firing interval angle between the kth cylinder and the first cylinder is \( \xi_{1,k} \), then the angle between the \( \gamma \) times exciting torque of the first cylinder and the top dead center of the cylinder is \( \left( \gamma \omega t + \varphi \right) \), and the angle between the \( \gamma \) times exciting torque and the top dead center of the kth cylinder is \( \left[ \gamma (\omega t + \xi_{1,k}) + \varphi \right] \). As long as the firing interval angle \( \xi_{1,k} \) of each cylinder relative to the first cylinder is known, the relative position between the \( \gamma \) times exciting torque of each cylinder and the \( \gamma \) times exciting torque of the first cylinder can be determined. For the ignition sequence of 1-5-3-6-2-4 of 6-cylinder in-line four stroke diesel engine, \( \xi_{1,5} = 120^\circ \), \( \xi_{1,3} = 240^\circ \), \( \xi_{1,6} = 360^\circ \), \( \xi_{1,2} = 480^\circ \), \( \xi_{1,4} = 600^\circ \).

### 4. Analysis results

#### 4.1. Time domain analysis
The curves in Fig. 3 show the displacement of each node changing with time. This paper assumes that the crankshaft rotates at a constant speed of 2300 r/min, and the displacement of each node is recorded once for every 15° rotation angle. For each node, 49 times and 48 time periods (i.e. the crankshaft rotates for 2 cycles) are selected to draw the curve in Figure 3.

4.2. Frequency domain analysis

The graphs in Figure 4 show the displacement response of each node at different speeds. A...J represent different speeds, the speed of 2300 r/min is taken as the initial speed, and 250 rad/min is taken as the change step. Ten different speeds are selected to measure the displacement of each node, and the curves in Figure 4 are drawn.
(c) Variation curve of node 3 displacement with rotational speed

(d) Variation curve of node 4 displacement with rotational speed

(e) Variation curve of node 5 displacement with rotational speed

(f) Variation curve of node 6 displacement with rotational speed
It can be seen that the vibration displacement is quite different at different speeds through the analysis of torsional vibration response of crankshaft of internal combustion engine. The vibration curves of different nodes are similar as well as different. The vibration intensity is different in different speed range. To a certain extent, the discrete model can meet the needs of engineering practice, but the working environment is not comprehensive enough. In the selection of external excitation torque, only the torsion torque caused by gas pressure and gravity is considered. It is necessary to consider the influence of inertia moment on crankshaft torsional vibration characteristics in the following research, and then consider the torsional vibration characteristics of the system when some cylinders "stop" running, so as to be closer to the real working condition.

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