Shock waves in a rotating non-Maxwellian viscous dusty plasma

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Abstract

A theoretical model is presented to study characteristics of dust acoustic shock in a viscous, magnetized and rotating dusty plasma at both fast and slow time scales. By employing reductive perturbation technique the nonlinear Zakharov–Kuznetsov (ZK) equation has been derived for both cases when dust is inactive and dynamic (fast and slow time scales). Both electrons and ions are considered to follow kappa/Cairns distribution. It is observed that the viscosity in both cases when dust is in background and active plays as a key role in dissipation for the propagation of acoustic shock. Magnetic field and rotation are responsible for the dispersive term. Superthermality has been found to affect significantly on the formation of shock wave along with viscous nature of plasma.

The present investigation may be beneficial to understanding the rotating plasma in particular experiments being carried out.

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I. INTRODUCTION

Despite a history spanning nearly a century, research into complex (dusty) plasmas (consists of nanometers to hundreds of micrometers sized solid particles in a conventional two component plasma) has progressed significantly in last two decades mainly after the marvellous observation of dusty plasma crystals by Thomas et al. in 1994 \cite{1}. Also from more than ten years dusty plasmas under minute gravity conditions have been studied on board the International Space Station (ISS) under the joint Russian/German venture of Plasma Kristall (PK), along with PKE-Nefedov, PK-3 Plus and PK-4 2014 onwards\cite{2}.

Other than novel experimental discoveries of dusty plasma crystals, dust Mach cones \cite{3}, dust acoustic waves, dust voids \cite{4}, etc., notion of possible existence of ‘dust atoms and molecules’ was also put forward by Tsintsadze, Murtaza and Ehsan \cite{5}. Authors later reported crystallization of dust atoms in the localized region of the electromagnetic wave\cite{6}.

Importance of dusty plasma physics has been manifold, these are omnipresent in astrophysical environment like comets, interplanetary of interstellar clouds, the rings of the Giant planets like Saturn etc., whereas discharges for thin film deposition or etching, dust in tokamak are few other noteworthy technological applications \cite{4,8}. These are the reasons, a rich literature exists on the investigation of linear and nonlinear structures in a dusty plasma for instance propagation of dust-ion acoustic waves \cite{9}, dust-acoustic (DA) waves \cite{10}, dust lattice \cite{11} waves, dust Coulomb waves \cite{12}, dust ion-acoustic shock waves \cite{13} and dust-acoustic shock waves \cite{14,15}, cusp solitons \cite{16} etc. Tsytyovich, and Angelis have also contributed significantly in developing kinetic theory of dusty plasmas \cite{17,19}.

Since the time scales associated to the big sized dust particles (large mass to charge ratio) are much longer therefore these plasmas can be tracked on the individual particle level with the naked eye and so provide an excellent tool for understanding underlying physics of phase transitions and collective excitations when in solid liquid or gaseous states. It is the large mass to charge ratio of dust particles that these plasmas considered bridging key issues from several fields like warm dense matter, low-temperature physics, surface and solid-state physics etc.

Conveniently and conventionally in the past modelling of plasma systems was carried out for static frame of reference whereas actual modelling of large number of problems in astrophysics (for instance rotating magnetic stars, pulsar/Kerr black-hole magnetospheres)
and in lab (such as tokamak) physics required to be done in non inertial (in particular rotating) frames. Chandrasekhar was the first to incorporate non-intertial frames later Lehnert and Hide also contributed to it. In these pathbreaking studies it was reported that the tiny force resulted from rotation (via Coriolis force) has an effective role to play in the plasma astrophysics and in other cosmic phenomena.

Observations show that the rotating flows of magnetized plasmas are not uncommon in solar physics, it is for the reason, linear wave propagation has been studied to show the interaction of the Coriolis force in an ideal lower ionosphere. Also to understand sunspot development, the star cycle and the structure of rotating stars magnetospheres etc. non-inertial frames are inevitable. Coriolis force can also create effective magnetic-field when the ionized fluid rotates, it for the reason that features of the propagation of nonlinear acoustic waves propagating in rotating dusty plasma will be modified. Understanding magnetized dusty plasma has been a necessity and a challenge for both theoreticians and experimentalists as for the former all charge-dependent forces and fluctuating nature of dust charge can potentially be modified and latter for the complexities involved. In this regard many attempts have been made, for instance Kählert et al. (2012) proposed frictional coupling between a dusty plasma and the neutral gas to mimic the dust magnetization in a complex plasma. In this approach properties of the light species electrons and ions were not affected; however, angular momentum from a rotating gas column was transferred to a well-controlled rotation of the dust cloud. In this way the induced Coriolis force $2m(\vec{v} \times \vec{\Omega})$ acting on objects moving with velocity ($\vec{v}$) when viewed in a rotating reference frame (with frequency $\Omega$) acts in a similar manner as the Lorentz force in a magnetic field $Q(\vec{v} \times \vec{B})$ does. Needless to mention, the approach used by Kählert et al. is limited to study some particular phenomena and processes like dust charging, formation of wakefield, Coulomb shielding, modulation and filamentation, etc. cannot be studied with this. In an other attempt to study longitudinal spectrum of collective excitations for different rotation rates with a high value of magnetic induction ($\sim 3200$ T), authors spotted the onset of the magnetoplasmon-like mode in a 2D single-layer dusty plasma. Both the studies in Ref. [26 & 27] reported lower Coulomb coupling in the rotating system leading to a liquid state compared to the nonrotating case. Magnetorotational instability (MRI) also known as Velikhov-Chandrasekhar instability which is a feature of purely rotating fluids was investigated in differentially rotating dusty plasma where dust particles were considered
fixed. Conclusively magnetized and rotating dusty plasmas are crucial. It is worth mentioning to mimic laboratory and astrophysical settings group at Maryland is establishing experiments for the high-velocity rotating dusty plasmas, in particular to understand velocity limits and stability of dusty plasmas, and their relation to high-temperature magnetized plasmas.

In the past, many times basic nonlinear properties of dust acoustic waves propagating in the Maxwellian dusty plasma were studied. However data from satellite observations predict the presence of super energetic long tails (or shoulders at low energy) of nonthermal plasma particles, that is why for the more accurate study of different stable/unstable collective modes, nonthermal/non-Maxwellian distribution functions like Kappa, Cairns and generalized Lorentzian \((r, q)\) like distributions have been used. Where features of nonthermality are contained in the parameters (given as spectral indices) like \(\kappa, \alpha, r\) and \(q\) for the Kappa, Cairns and \(r, q\) distributions, respectively. or suprathermal tail.

In the present investigation, analytical model for obliquely propagating nonlinear dust acoustic wave in a rotating magnetized dusty plasma will be developed where electrons and ions will be treated as Kappa and Cairns distributed For the plasma particles in thermal equilibrium, Kappa distribution function is given as:

\[
f_s^\kappa(v_s) = \frac{n_{so}}{(2\pi)^{3/2}(\kappa - 3/2)v_{ts}^3} \frac{\Gamma(1 + \kappa)}{\Gamma(\kappa - 1/2)} \left(1 + \frac{v_s^2}{2(\kappa - 3/2)v_{ts}^2}\right)^{-(1+\kappa)}
\]

where \(n_{so}\) represents the number density of the plasma species \(s(=e, i, d\) for electron, ions and dust, respectively). The \(v_{ts} = (T_s/m_s)^{1/2}\) is the thermal speed, in which \(T_s\) and \(m_s\) denote the temperature in energy unit and mass, respectively. Where \(\kappa\) measures the superthermality and \(\Gamma\) is the well-known gamma function. Condition of \(\kappa > 3/2\) must hold in order to have the realistic thermal speeds of plasma particles. Note that for the larger values of kappa, Maxwellian distribution is retrieved. The Cairns distribution function which was originally introduced after in situ observations of Viking spacecraft and Freja satellite missions study ion-sound cavitons like structures is given as:

\[
f_s^C(v_s) = \frac{n_{so}}{(2\pi)^{3/2}(1 + \alpha)v_{ts}^3} \left(1 + \alpha \frac{v_s^4}{v_{ts}^4}\right) \exp \left(-\frac{v_s^2}{2v_{ts}^2}\right)
\]

where \(\alpha\) determines the population of nonthermal plasma particles i.e., for \(\alpha \to 0\), one can achieve Maxwellian distribution. of nonthermal energetic particles.
In this article, we will address dust acoustic shock wave which are useful for the dusty plasma experiments under microgravity research and also important from perspectives for future studies. We employ the well known reductive perturbation technique to derive the nonlinear Zakharov–Kuznetsov (ZK) equation for these waves at both fast and slow time scales. Both electrons and ions are considered to follow kappa/Cairns distribution. The solitary wave solution of this equation is obtained in Sec. IV. In Sec. V, the results are presented and discussed, and finally, the conclusion is presented in Sec. VI.

The paper is organized in the following manner: In Sec. II, the physical assumption and description of the problem to be addressed is given. Sections III and IV deal with the study of dust modified acoustic shock waves at fast time scale and derivation of the Zakharov-Kuzenkov equation for the dust acoustic shock wave at slow time, respectively. Quantitative analysis is provided in Section V, finally, Sec. VI describes the conclusions.

II. PHYSICAL ASSUMPTIONS AND DESCRIPTION OF THE MODEL

We will make the following assumptions to formulate the physical problem:

1. The plasma under consideration is magnetized, homogeneous and unbounded. The plasma constituents are electrons \( n_e \), ions \( n_i \), and negatively charged dust grains \( n_d \), and no collisions have been taken into account between the particles. Charge on dust grains is negative. The quasineutrality condition is given by

\[
n_i = n_e + Z_d n_d
\]

here \( Z_d \) is the charge of dust grain, and \( n_s = n_{s0} + \delta n_s + \delta n_s^L \), while \( n_{s0} \) represents the equilibrium density. Superscript ‘\( L \)’ refers to the ultra-low frequency for the dust acoustic wave (DAW) in comparison with the higher frequency dust modified acoustic wave (DMAW), and \( \delta n_s \) gives the density perturbation on the DAW time scale. The parameter \( s \) denotes the species dust, ions or electrons.

2. While treating this problem, we shall first consider the regime where dust is in background and it is also the fast time process which will be followed by slow time (DAW) dynamics. In the former regime, the dust mass is ignored while in the latter, the dust species is activated.
3. The magnetic field is taken along z-axis $B = B_0\hat{z}$. When dealing the slow time process, the wavelength $\lambda = 2\pi/k$ is assumed to be much smaller than the gyroradius of plasma particle which allows us to take dust as magnetized and but electrons and ions are then treated un-magnetized.

4. Also we know when $B$ is weak, the electron gyroradius is much smaller than the size of grains and therefore variations in the dust charge is too small. In this case, electrons will approach dust grain surface quite rapidly along the direction of $B$ and therefore fast electrons responsible for charging the grains may be treated as Boltzmannian. Thus for low frequency dynamics in a magnetized plasma when $\omega/k \ll v_{ts}$ the lighter species can obey Boltzmann distribution.

5. As stated in the introduction, we aim to adopt kappa and Cairns distribution for the lighter species. The normalized number density for kappa and Cairns distributed particles respectively is given by

\begin{equation}
    n_{e(i)} = \left[1 \mp \left(\kappa - \frac{3}{2}\right)^{-1} \frac{\phi}{\sigma}\right]^{-\kappa + 1/2}
\end{equation}

and

\begin{equation}
    n_{e(i)} = \left[1 \mp \frac{\beta}{\sigma} \phi \pm \frac{\beta}{\sigma^2} \phi^2\right] e^{\pm \phi}
\end{equation}

where $\kappa$ is the spectral index measuring the deviation from Maxwellian distribution, $\sigma = T_i/T_e$ and $\beta = 4\alpha/(1 + 3\alpha)$. For the electrons in (5), $\sigma = 1$. For $\kappa \to \infty(\alpha = 0)$ Maxwellian distribution is achieved. It is to be noted that (4) and (5) will be used only when dust is active in slow time scale however the first case when dust in in background (fast time phenomena) only ions dynamics play the role only Eq. (4) will be sued.

6. The time for the excitation of dust acoustic shock wave which is nonlinear processes is much smaller than required for further substantial variations in dust charge, and so dust charge can be taken constant.

7. As described in the introduction, we are considering rotating plasmas, it is worth mentioning here that we will only (in case of slow time scale) consider dust particles to be rotating whereas rotation of other constituents electrons and ions is not important for us. Also it is considered that both rotational and magnetic axis are misaligned.
III. FAST TIME SCALE PHENOMENON

Here we consider excitation of dust modified shock wave a fast time process, since, dust particles are extremely massive compared to the other constituents so they stay in the background (steady and immovable), however, their existence can be viewed via quasineutrality condition only. In the fluid for ions which are dynamic here, we will incorporate ion bulk viscosity which can only be ignored for the incompressible fluids. However, for the acoustic shock waves, the plasma compressibility is essential and therefore fluid equations are written as

\[ \frac{\partial n_i}{\partial t} + \nabla \cdot (n_i v_i) = 0 \]  \hspace{1cm} (6)

\[ \left( \frac{\partial}{\partial t} + v_i \cdot \nabla \right) v_i = -\frac{1}{Z_d \delta^{-1}} \nabla \phi + \omega_{ci} (v_i \times \hat{z}) + -\frac{\sigma}{Z_d \delta^{-1}} n_i \nabla n_i + \eta_i \nabla^2 v_i + \eta_i + \mu_i) \nabla (\nabla \cdot v_i) \]  \hspace{1cm} (7)

\[ \nabla^2 \phi = \delta^{-1} (\mu n_{e1} - \delta n_{i1}) \]  \hspace{1cm} (8)

were in Eq. (7), \( \eta_i \) and \( \mu_i \) represent the kinematic and bulk viscosity for ions (also called the second coefficient viscosity), respectively. Whereas \( \mu = n_{eo}/Z_d n_{do} = m_d/m_i \), \( \sigma = T_i/T_e \) and \( \delta = n_{io}/Z_d n_{do} \). According to the convenience of the problem being addressed, for the normalization we use dust parameters. \( \omega_{ci} = eB_0/m_i c \) has been normalized by the \( \omega_{pd} = (4\pi n_{d0} e^2 Z_d^2/m_d)^{1/2} \). The space and time variables have been normalized by the \( \lambda_d = (T_e \delta^{-1}/4\pi n_{d0} e^2 Z_d)^{1/2} \) and \( \omega_{pd}^{-1} \), respectively. Moreover, \( \eta_i \) and \( \mu_i \) are normalized by \( \omega_{pd} \lambda_d^2 \). Equations (6)-(8) in the Cartesian components for 3D nonlinear DMSW can be written as

\[ \frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_{ix})}{\partial x} + \frac{\partial (n_i v_{iy})}{\partial y} + \frac{\partial (n_i v_{iz})}{\partial z} = 0 \]  \hspace{1cm} (9)

\[ \frac{\partial v_{ix}}{\partial t} + v_{ix} \frac{\partial v_{ix}}{\partial x} + v_{iy} \frac{\partial v_{ix}}{\partial y} + v_{iz} \frac{\partial v_{ix}}{\partial z} = -\frac{1}{Z_d \delta^{-1}} \frac{\partial \phi}{\partial x} + \omega_{ci} v_{iy} - \frac{\sigma}{Z_d \delta^{-1}} n_i \frac{\partial n_i}{\partial x} + \eta_i \left( \frac{\partial^2 v_{ix}}{\partial x^2} + \frac{\partial^2 v_{ix}}{\partial y^2} + \frac{\partial^2 v_{ix}}{\partial z^2} \right) \]  \hspace{1cm} (10)

\[ \frac{\partial v_{iy}}{\partial t} + v_{ix} \frac{\partial v_{iy}}{\partial x} + v_{iy} \frac{\partial v_{iy}}{\partial y} + v_{iz} \frac{\partial v_{iy}}{\partial z} = -\frac{1}{Z_d \delta^{-1}} \frac{\partial \phi}{\partial y} - \omega_{ci} v_{iz} - \frac{\sigma}{Z_d \delta^{-1}} n_i \frac{\partial n_i}{\partial y} + \eta_i \left( \frac{\partial^2 v_{iy}}{\partial x^2} + \frac{\partial^2 v_{iy}}{\partial y^2} + \frac{\partial^2 v_{iy}}{\partial z^2} \right) \]  \hspace{1cm} (11)
\[
\frac{\partial (v_{iz})}{\partial t} + v_{iz} \frac{\partial (v_{iz})}{\partial x} + v_{iy} \frac{\partial (v_{iz})}{\partial y} + v_{iz} \frac{\partial (v_{iz})}{\partial z} = -\frac{1}{Z_d\delta^{-1}} \frac{\partial \phi}{\partial z} - \frac{\sigma}{Z_d\delta^{-1} n_i} \frac{\partial n_i}{\partial z} + \eta_i \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) v_{iz} + (\eta_i + \mu_i) \left( \frac{\partial^2 v_{ix}}{\partial z \partial x} + \frac{\partial^2 v_{iy}}{\partial z \partial y} + \frac{\partial^2 v_{iz}}{\partial z^2} \right)
\]

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \delta^{-1} \left( \mu + c_1 \phi + c_2 \phi^2 - \delta n_i \right)
\]

To study small and finite amplitude DMSW we will use reductive perturbation method [15] and introduce stretched coordinates given as:

\[
\xi = \epsilon^{1/2} x, \quad \eta = \epsilon^{1/2} y, \\
\zeta = \epsilon^{1/2} (z - \lambda_0 t) \quad \text{and} \quad \tau = \epsilon^{3/2} t
\]

where \(\lambda_0\) is the speed with which shock wave propagates and a dimensionless parameter, \(\epsilon\) \((0 < \epsilon \ll 1)\) measures the strength of nonlinearity. Further, the other variables like density, velocity, potential are expressed as:

\[
n_i = 1 + \epsilon n_1 + \epsilon^2 n_2 + \ldots, \\
v_{ix} = \epsilon^{3/2} u_1 + \epsilon^2 u_2 + \ldots, \\
v_{iy} = \epsilon^{3/2} v_1 + \epsilon^2 v_2 + \ldots, \\
v_{iz} = v_0 + \epsilon w_1 + \epsilon^2 w_2 + \ldots, \\
\phi = \epsilon \phi_1 + \epsilon^2 \phi_2 + \ldots
\]

The ion kinematic viscosity, for weakly damped systems, is assumed to be small, and can be expressed as

\[
\eta_i = \epsilon^{1/2} \eta_0, \quad \mu_i = \epsilon^{1/2} \mu_0.
\]

where \(\eta_0\) and \(\mu_0\) have dimensions of a unit. Substitution of equations (15)-(17) into (9)-(13) and collection of lowest order terms such as \(\epsilon^1\) and \(\epsilon^{3/2}\) result in

\[
w_1 = n_1 (\lambda_0 - v_o)
\]

\[
n_1 = \frac{c_i}{\delta} \phi_1
\]

\[
\frac{1}{Z_d\delta^{-1}} \frac{\partial \phi_1}{\partial \xi} + \frac{\sigma}{Z_d\delta^{-1}} \frac{\partial n_1}{\partial \xi} - \omega_{ci} v_i = 0
\]

\[
\frac{1}{Z_d\delta^{-1}} \frac{\partial \phi_1}{\partial \eta} + \frac{\sigma}{Z_d\delta^{-1}} \frac{\partial n_1}{\partial \eta} + \omega_{ci} u_i = 0
\]
From above equations, we obtain the ZKB equation describing the dust modified ion acoustic wave

$$\lambda_0 = v_0 \pm \sqrt{\frac{1}{Z_d \delta^{-1} c_1} \left( \sigma c_1 + \delta \right)} = v_0 \pm \left[ \frac{n_{i0}}{Z_d n_{i0}} \frac{T_i}{T_e} \left( 1 + \frac{T_e}{c_1 Z_d T_i} \right) \right]^{1/2}$$ (23)

above equation describes the phase speed of the waves, where \(+(-)\) sign refers to the fast (slow) modes. It can be observed from (23) that linear phase velocity of the DMSW is not affected by the magnetic field and viscosity; however, presence of the factor \(n_{i0}/Z_d n_{i0}\) shows that when dust is present in the background the phase velocity of the fast ion-acoustic mode increases whereas it is reduced for the slow mode.

Collection of higher order terms such as \(\epsilon^2\) and \(\epsilon^{5/2}\) return us

$$\left( \lambda_0 - v_0 \right) \frac{\partial u_1}{\partial \zeta} + \omega_{ci} v_2 = 0$$ (24)

$$\left( \lambda_0 - v_0 \right) \frac{\partial v_1}{\partial \zeta} = \omega_{ci} u_2$$ (25)

$$\frac{\partial^2 \phi_1}{\partial \zeta^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} = \delta^{-1} \left( c_1 \phi_2 + c_2 \phi_1^2 - \delta n_2 \right)$$ (26)

$$- \left( \lambda_0 - v_0 \right) \frac{\partial n_2}{\partial \zeta} + \frac{\partial u_2}{\partial \zeta} + \frac{\partial v_2}{\partial \eta} + \frac{\partial w_2}{\partial \zeta} = - \frac{\partial n_1}{\partial \tau} - \frac{\partial}{\partial \zeta} \left( n_1 w_1 \right)$$ (27)

$$- \left( \lambda_0 - v_0 \right) \frac{\partial w_2}{\partial \zeta} + \frac{\partial w_1}{\partial \tau} + w_1 \frac{\partial w_1}{\partial \zeta} + \frac{1}{Z_d \delta^{-1} c_1} \frac{\partial \phi_2}{\partial \zeta} + \frac{\sigma}{Z_d \delta^{-1} c_1} \frac{\partial n_2}{\partial \zeta} = \frac{\sigma}{Z_d \delta^{-1} c_1} \frac{\partial n_1}{\partial \zeta}$$

$$+ \eta_0 \left( \frac{\partial^2}{\partial \zeta^2} + \frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \zeta^2} \right) w_1 + \left( \eta_0 + \mu_0 \right) \frac{\partial^2 w_1}{\partial \zeta^2}$$ (28)

From Eqs. (24-28), we obtain the ZKB equation describing the dust modified ion acoustic shock wave

$$\frac{\partial \phi_1}{\partial \tau} + A \frac{\partial \phi_1}{\partial \zeta} + B \frac{\partial^2 \phi_1}{\partial \zeta^2} + C \frac{\partial}{\partial \zeta} \left( \frac{\partial^2 \phi_1}{\partial \zeta^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} \right) - D \left( \frac{\partial^2 \phi_1}{\partial \zeta^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} \right) - E \frac{\partial^2 \phi_1}{\partial \zeta^2} = 0$$ (29)

where

$$A = \frac{c_1}{\delta} \left[ \frac{1}{2} + \left( \lambda_0 - v_0 \right) + \frac{c_2 \delta^3}{Z_d c_1^3 \left( \lambda_0 - v_0 \right)} - \frac{T_i}{2Z_d T_e \left( \lambda_0 - v_0 \right)} \right]$$ (30)

$$B = \frac{\delta^3}{2Z_d \left( \lambda_0 - v_0 \right)}$$ (31)

$$C = \frac{\delta T_i \left( \lambda_0 - v_0 \right)}{2\omega_{ci}^2 Z_d T_e} + \frac{\delta^2}{2c_1 \omega_{ci}^2 Z_d} + \frac{\delta^3}{2c_1^2 \left( \lambda_0 - v_0 \right) Z_d}$$ (32)
\[ D = \frac{\eta_0}{2}, \quad E = \frac{\eta_0 + \mu_0}{2} \]  (33)

where \( A \) is the nonlinear coefficient, \( B \) and \( C \) are dispersive whereas \( D \) and \( E \) represent dissipation coefficients.

Presence of the factor \( n_{i0}/Z_d n_{d0} \) in (30) shows that nonlinear coefficient significantly affected by the dust in background. The reducing 4th term is small as \( \delta \) is in the denominator and the factor \( T_i/T_e \) is also less than unity in usual astrophysical and lab environments whereas it can only be larger than unity in tokamaks. It is obvious from (33) that dissipation coefficients are not affected by the presence of dust in the background.

To examine the shock like solution of ZKB equation (29), we introduce the parameter \( \chi \) as

\[ \chi = l_x \xi + l_y \eta + l_z \zeta - U_0 \tau \]  (34)

here \( l_{\alpha=x,y,z} \) are the direction cosines and \( U_0 \) is wave speed for the nonlinear propagation. Using Eq. (34) into (29) yields the following ordinary differential equation (ODE) as

\[ -U_0 \frac{d\phi_1}{d\chi} + A l_z \phi_1 \frac{d\phi_1}{d\chi} + H l_z \frac{d^3\phi_1}{d\chi^3} - G \frac{d^2\phi_1}{d\chi^2} = 0, \]  (35)

where \( H = l_z^2 B + (l_x^2 + l_y^2) C \) and \( G = E l_z^2 + D \). The shock like solution of Eq. (35) can be found using hyperbolic tangent method \[34\]. Thus, employing the condition that \( \phi_1 \) is bounded at \( \chi = \pm \infty \), we obtain shock wave solution

\[ \phi_1(\chi) = \frac{3}{25} \frac{G^2}{HA l_z^2} \left[ 2 - 2 \tanh \left( \frac{G}{10H l_z} \chi \right) + \text{sech}^2 \left( \frac{G}{10H l_z} \chi \right) \right] \]  (36)

As is evident from the above equation shock is formed due to the ion kinematic viscosity term. Here, \( 10H l_z/G \) and \( (9/25)(G^2/HA l_z^2) \) represent the width and amplitude (depends upon \( A \) the nonlinear coefficient) of the shock structure, respectively.

IV. DUST ACOUSTIC (DA) WAVE AT SLOW TIME SCALE

In this section we derive dispersion (linear and nonlinear) of the dust acoustic waves and take into account the quasi-neutrality condition \( \delta n_i \sim \delta n_e + Z_d \delta n_d \), since the time with which velocity and density of lighter species vary is much shorter than that of heavier dust i.e.,

\[ t_i \left( \sim \frac{1}{\omega_{pi}} \right) \sim v_e \left( \frac{\partial v_e}{\partial t} \right)^{-1}, \quad n_e \left( \frac{\partial n_e}{\partial t} \right)^{-1} << t_d \left( \sim \frac{1}{\omega_{pd}} \right) \]  (37)
In this case, the dynamic effects of the dust grains are included, because we are interested in the dust's time and space scales, so the fluid equations for the dust are

\[
\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{v}_d) = 0
\]  

(38)

\[
\left( \frac{\partial}{\partial t} + \mathbf{v}_d \cdot \nabla \right) \mathbf{v}_d = \delta \nabla \phi - \omega_{cd} (\mathbf{v}_d \times \hat{z}) - \frac{\sigma_d}{\delta - 1} \nabla n_d + 2\Omega_o (\mathbf{v}_d \times \hat{z}) + n_d \nabla^2 \mathbf{v}_d + (\eta_d + \mu_d) \nabla (\nabla \cdot \mathbf{v}_d)
\]  

(39)

\[
\nabla^2 \phi = \delta^{-1} (\mu n_e - \delta n_i + n_d)
\]  

(40)

where \(\sigma_d = T_d/T_e Z_d\), \(\Omega_o\) is the rotational frequency of the dust. Dust fluid velocity \(\mathbf{v}_d\) is normalized by dust acoustic speed \(c_s = (Z_d T_e \delta^{-1}/m_d)^{1/2}\). the electrostatic potential \(\phi\) is normalized by \(T_e/e\). Also, the dust kinematic viscosity \(\eta_d\) and the second coefficient of viscosity \(\mu_d\) are normalized by \(\omega_{pd}\lambda_d^2\). The ions and electrons are considered to obey Kappa and Cairns distributions, their number densities given by Eqs. (4) and (5). Above equations can be expressed in the Cartesian components form as follows

\[
\frac{\partial n_d}{\partial t} + \frac{\partial (n_d v_{dx})}{\partial x} + \frac{\partial (n_d v_{dy})}{\partial y} + \frac{\partial (n_d v_{dz})}{\partial z} = 0
\]  

(41)

\[
\frac{\partial v_{dx}}{\partial t} + v_{dx} \frac{\partial v_{dx}}{\partial x} + v_{dy} \frac{\partial v_{dx}}{\partial y} + v_{dz} \frac{\partial v_{dx}}{\partial z} = \delta \frac{\partial \phi}{\partial x} - \Omega_c v_{dy} - \frac{\sigma_d}{n_d} \frac{\partial n_d}{\partial x} + \eta_d \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) v_{dx}
\]  

+ \left( \eta_d + \mu_d \right) \left( \frac{\partial^2 v_{dx}}{\partial x^2} + \frac{\partial^2 v_{dy}}{\partial x \partial y} + \frac{\partial^2 v_{dz}}{\partial x \partial z} \right)

(42)

\[
\frac{\partial v_{dy}}{\partial t} + v_{dx} \frac{\partial v_{dy}}{\partial x} + v_{dy} \frac{\partial v_{dy}}{\partial y} + v_{dz} \frac{\partial v_{dy}}{\partial z} = \delta \frac{\partial \phi}{\partial y} + \Omega_c v_{dx} - \frac{\sigma_d}{n_d} \frac{\partial n_d}{\partial y} + \eta_d \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) v_{dy}
\]  

+ \left( \eta_d + \mu_d \right) \left( \frac{\partial^2 v_{dx}}{\partial y \partial x} + \frac{\partial^2 v_{dy}}{\partial y^2} + \frac{\partial^2 v_{dz}}{\partial y \partial z} \right)

(43)

\[
\frac{\partial (v_{dz})}{\partial t} + v_{dx} \frac{\partial (v_{dz})}{\partial x} + v_{dy} \frac{\partial (v_{dz})}{\partial y} + v_{dz} \frac{\partial (v_{dz})}{\partial z} = \delta \frac{\partial \phi}{\partial z} - \frac{\sigma_d}{n_d} \frac{\partial n_d}{\partial z} + \eta_d \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) v_{dz}
\]  

+ \left( \eta_d + \mu_d \right) \left( \frac{\partial^2 v_{dx}}{\partial z \partial x} + \frac{\partial^2 v_{dy}}{\partial z \partial y} + \frac{\partial^2 v_{dz}}{\partial z^2} \right)

(44)

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \delta^{-1} \left( (+c_{d1} \phi + c_{d2} \phi^2) - 1 + n_d \right)
\]  

(45)
where we define $\Omega_c = \omega_{cd} - 2\Omega_o$ and $\mu - \delta = 1$, comes from the charge neutrality condition and

$$c_{d1} = \begin{cases} 
(\mu - \frac{\delta}{\sigma}) (1 + \beta) \ (\text{Cairns}), \\
\frac{\mu(\kappa-1/2)}{\kappa-3/2} + \frac{\delta(\kappa-1/2)}{(\kappa-3/2)^2} \ (\text{kappa})
\end{cases}$$

$$c_{d2} = \begin{cases} 
\frac{\mu}{2} - \frac{\delta(1+4\beta)}{2\sigma^2} \ (\text{Cairns}), \\
\frac{(\mu - \frac{\delta}{\sigma^2}) \frac{\mu(\kappa-1/2)(\kappa+1/2)}{2(\kappa-3/2)^2}}{\kappa} \ (\text{kappa})
\end{cases}$$ (46)

Like in previous section, we opt standard reductive perturbation method and introduce stretched coordinates to obtain the ZK-Burgers equation

$$\xi = \epsilon^{1/2} x, \quad \eta = \epsilon^{1/2} y,$$

$$\zeta = \epsilon^{1/2} (z - \lambda_{od} t) \quad \text{and} \quad \tau = \epsilon^{3/2} t$$ (47)

where $\lambda_{od}$ is the propagation speed of the dust acoustic wave to be determined later. Furthermore, the dependent variables $n_d$, $v_d$, and $\phi$ are expanded in power series of $\epsilon$ as

$$n_d = 1 + \epsilon n_{d1} + \epsilon^2 n_{d2} + ..., $$

$$v_{dx} = \epsilon^{3/2} u_{d1} + \epsilon^2 u_{d2} + ..., $$

$$v_{dy} = \epsilon^{3/2} v_{d1} + \epsilon^2 v_{d2} + ..., $$

$$v_{dz} = v_{d0} + \epsilon w_{d1} + \epsilon^2 w_{d2} + ..., $$

$$\phi = \epsilon \phi_1 + \epsilon^2 \phi_2 + ..., $$ (48)

Similarly we express

$$\eta_d = \epsilon^{1/2} \eta_{d0}$$

$$\mu_d = \epsilon^{1/2} \mu_{d0}$$ (49)

Collection of lowest order terms return us the following equations:

$$w_{d1} = n_{d1} \ (\lambda_{do} - v_{do})$$ (50)

$$n_{d1} = -c_{d1} \phi_1$$ (51)

$$\delta \frac{\partial \phi_1}{\partial \xi} - \sigma_d \delta \frac{\partial n_{d1}}{\partial \xi} - \Omega_c v_{d1} = 0$$ (52)

$$\delta \frac{\partial \phi_1}{\partial \eta} - \sigma_d \delta \frac{\partial n_{d1}}{\partial \eta} + \Omega_c u_{d1} = 0$$ (53)
\[-(\lambda_{\text{d}_0} - v_{\text{d}_0}) \frac{\partial w_{\text{d}_1}}{\partial \zeta} - \sigma_d \delta \frac{\partial n_{\text{d}_1}}{\partial \zeta} + \delta \frac{\partial \phi_1}{\partial \zeta} = 0\]  

(54)

From equations (51-54), we obtain

\[\lambda_{\text{d}_0} = v_{\text{d}_0} \pm \sqrt{\frac{\delta(1 + \sigma_d c_{\text{d}_1})}{c_{\text{d}_1}}}\]  

(55)

Equation (55) represents the phase velocity for dust acoustic waves and ± sign for the fast and slow dust acoustic speeds. Higher order term in \(\epsilon\) gives us

\[\lambda_{\text{d}_0} - v_{\text{d}_0} \frac{\partial u_{\text{d}_1}}{\partial \zeta} - \Omega_c v_{\text{d}_2} = 0\]  

(56)

\[\lambda_{\text{d}_0} - v_{\text{d}_0} \frac{\partial v_{\text{d}_1}}{\partial \zeta} + \Omega_c u_{\text{d}_2} = 0\]  

(57)

\[\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} = \delta^{-1} (c_{\text{d}_1} \phi_2 + c_{\text{d}_2} \phi_1^2 + n_{\text{d}_2})\]  

(58)

\[-(\lambda_{\text{d}_0} - v_{\text{d}_0}) \frac{\partial n_{\text{d}_2}}{\partial \zeta} + \frac{\partial u_{\text{d}_2}}{\partial \eta} + \frac{\partial v_{\text{d}_2}}{\partial \xi} + \frac{\partial w_{\text{d}_2}}{\partial \zeta} = - \frac{\partial n_{\text{d}_1}}{\partial \tau} - \frac{\partial}{\partial \zeta} (n_{\text{d}_1} w_{\text{d}_1})\]  

(59)

Finally trivial algebra steps return us ZK-Burgers equation in the form

\[\frac{\partial \phi_1}{\partial \tau} + A' \phi_1 \frac{\partial \phi_1}{\partial \zeta} + B' \frac{\partial^3 \phi_1}{\partial \zeta^3} + C' \frac{\partial}{\partial \zeta} \left( \frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} \right) - D' \left( \frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} \right) - E' \frac{\partial^2 \phi_1}{\partial \zeta^2} = 0\]  

(61)

where

\[A' = \frac{\sigma_d c_{\text{d}_1} - 2 c_{\text{d}_2} \delta^2}{2 \delta^{-1} (\lambda_{\text{d}_0} - v_{\text{d}_0})^2}\]  

(62)

\[B' = \frac{\delta^2}{c_{\text{d}_1} (\lambda_{\text{d}_0} - v_{\text{d}_0})}\]  

(63)

\[C' = \frac{1 + \Omega_c^{-2} \delta^{-1} c_{\text{d}_1} (c_{\text{d}_1} \sigma_d + 1) (\lambda_{\text{d}_0} - v_{\text{d}_0})^2}{2 \delta^{-2} c_{\text{d}_1}^2 (\lambda_{\text{d}_0} - v_{\text{d}_0})}\]  

(64)

\[D' = \frac{\eta_0}{2}, \quad E' = \frac{\eta_0 + \mu_0}{2}\]  

(65)

Hence the Burger terms having coefficients (\(D'\) and \(E'\)) which are responsible for the generation of shock wave, originates due to the viscosity term. Also note that since dissipative terms are proportional to \(\eta_0\) and \(\mu_0\), it mean in the absence of this the dissipative terms would vanish and ZK-Burger equation will be reduced to usual ZK equation admitting nonlinear soliton solutions only.
V. STATIONARY SOLUTION AND QUANTITATIVE ANALYSIS

In this section, we numerically solve Eq. (61) to examine dust acoustic shock waves for kappa and Cairns distributed ions and electrons.

Shock like structures can be studied if the coefficients of the Burger term arising from viscous nature of plasma are positive, i.e. \( (D', E' > 0) \). To obtain the solution of ZKB equation (61), we first transform (61) into another form using \( \chi = \gamma_x \xi + \gamma_y \eta + \gamma_z \zeta - U_d \tau \).

Where \( \gamma_{s=x,y,z} \) are the direction cosines and \( U_d \) is now normalized to \( C_{sd} \). This yields:

\[
-U_d \frac{d\phi_1}{d\chi} + A' \gamma_z \phi_1 \frac{d\phi_1}{d\chi} + H' \gamma_z \frac{d^2\phi_1}{d\chi^3} - G' \frac{d^2\phi_1}{d\chi^2} = 0, \tag{67}
\]

where \( H' = \gamma_z^2 B' + (\gamma_x^2 + \gamma_y^2) C' \) and \( G' = E' \gamma_z^2 + D' \). Again employing the hyperbolic tangent (tanh) method along with the boundary conditions we found the shock wave solutions

\[
\phi_1(\chi) = \frac{3}{25} \frac{G'^2}{H' A \gamma_z^2} \left[ 2 - 2 \tanh \left( \frac{G'}{10H' \gamma_z} \chi \right) + \text{sech}^2 \left( \frac{G'}{10H' \gamma_z} \chi \right) \right] \tag{68}
\]

with, \( 10H' \gamma_z G'^{-1} \) providing the width and \( \frac{9}{25} G^2 H^{-1} A \gamma_z^{-2} \) gives the amplitude of shock waves moving with speed \( U_d \). The shock width and amplitude are dependent on the dispersive coefficients \( B' \) and \( C' \), dissipative coefficients \( D' \) and \( E' \), and direction cosines \( \gamma_x, \gamma_y, \gamma_z \). The amplitude is also dependent on the nonlinear coefficient \( A' \). The dependence of these coefficients on various plasma parameter determines the shape of the shock profile.

**Linear dispersion relation**

Now we numerically solve the Eq. (61). For illustration we have chosen some typical parameters of the dusty plasmas

\( Z_d = 50, \ n_{do} = 1cm^{-3}, \ n_{io} = 6 \times 10^3 cm^{-3}, \ l_z = 0.9, \ T_i = 2eV, \) and \( T_e = 8eV \). First of all we study how the phase velocity \( (\lambda_{do}) \) of the dust acoustic wave will be modified for the nonthermal distribution parameters \( \kappa \) and \( \alpha \) (kappa/Cairns) distributions.

It can be seen from Fig. (1) that phase velocity increases in case of fast mode upon increment in the value of \( \kappa \) and vice versa for the slow mode.

In Figs. (2) and (3) we observe that for the fixed flow speed \( v_0 = 2.3 \times 10^2 cm s^{-1} \), increasing the value of \( \alpha \) from \( 0.1 - 1.7 \) leads to enhancement in the phase velocity \( (\lambda_{do}) \) for the fast as well as slow modes of dust acoustic wave at slow time scale. Whereas in another graph [Fig. (4)], we vary the flow speed for the fast mode only such as \( v_0 = 2 \times 10^2 cm s^{-1}, \)

\( v_0 = 2.3 \times 10^2 cm s^{-1}, \) and note how this affects the phase velocity of fast mode whereas \( \alpha \) has been chosen between 0 and 0.5.

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Effect on Coefficients

It is shown in Fig. (5) that on increasing $\alpha$ between $1 - 5$, the nonlinear coefficient $A'$ increases. In another figure (6), we show how dispersive coefficient $B'$ changes with the variations in the parameter $\sigma_d = T_d/T_e Z_d$, we note that increasing this factor $\sigma_d$ causes reduction in the strength of the $B'$.

In Fig. (7), we observe how the nonlinear coefficient ($A'$) changes while changing the value of charge number that is $Z_d$. It is depicted from the figure that increasing the $Z_d$ from 20 to 72, the nonlinear coefficient decreases whereas after 72 it rises abruptly. Similarly for the values below 20, $A'$ increases.

Effect of viscosity

In Fig. (8), we examine the behavior of shock structure for different values of ion kinematic viscosity coefficient and observe that for fixed value of $\kappa$ ($= 3$), upon increasing the values of viscosity $\eta_o (= 0.10, 0.12, 0.14)$, strength of the potential is enhanced significantly. In this case the shock wave formed is of compressive in nature. However we also observe in Fig. (9) that enhancing the percentage of the suprathermal electrons i.e. $\kappa$ ($= 2, 3, 4$) for this fixed viscosity ($\eta_o = .09$), the shock amplitude increases when there is a decrease in the the kappa. This is interesting and show supprer thermalility affects shocks.

Main conclusion from here is both enhancing superthermality and kinematic viscosity both affect significantly shock waves.

In case of Cairns distribution Fig. (10) $\eta_o (= 0.06, 0.08, 0.1)$ for fixed $\alpha = 0.15$, $\Omega_c = 0.3$, we again observe a significant change in the amplitude of the shock wave however in this case shock wave is of rarefaction nature.

Effect of rotation

Analysis of the coefficients of the ZK-Burger equation shows that rotation contributes only in the dispersive coefficient. Therefore rotational frequency and the external magnetic field do not directly affect the amplitude of the shock wave; however, they do on the width of the shock waves. It can be seen in Fig. (11), as we increase both the rotation ($\Omega_c = 0.1, 0.2, 0.4, 0.6$) and keep viscosity and kappa fixed as $\eta_o = 0.10$, $\kappa = 3$, a rarefaction shock structure will be formed.

We also note in Fig. (12) that in case of Cairns distribution for $\eta_o = 0.1$ for $\alpha = 0.45$, and $\Omega_c = 0.1, 0.2, 0.6$, we observe that initially the perturbed potential $\phi_1$ increases when values of $\Omega_c$ are increased; however, later no major change has been observed.
VI. CONCLUSIONS

In this paper to study dust acoustic shock waves, we have Zakharov–Kuznetsov (ZK) equations by employing reductive perturbation technique for both cases when dust is inactive and dynamic (fast and slow time scales). When dust is active both electrons and ions are considered to follow kappa/Cairns distribution. Main conclusion is that the superthermality and viscosity in both cases when dust is in inactive and active, plays as a key role in dissipation for the propagation of acoustic shock waves. Also charge number ($Z_d$) affects the nonlinear coefficient ($A'$) such as increasing the $Z_d$ from 20 to 72, the nonlinear coefficient decreases whereas after 72 it rises abruptly. We would like to add that charge fluctuations also plays a significant role in the formation of shocks in dusty plasmas as has been reported earlier [15], in future we plan to incorporate dust variable charge into this model to understand if charge fluctuations play a dominant role, the viscosity or superthermality; however, due to complexity of the problem this is proposed for the future sequel of this paper. Magnetic field and rotation are responsible for the dispersiveness of the shock waves. To the best of the authors’ knowledge, this stationary shock solution has not been studied for the non-Maxwellian rotating viscous dusty plasma system and we believe that present findings will be useful for the experiments being established to study rotating dusty plasmas as well as for the PK-4.

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**Figure Captions**

Fig. (1): Phase velocity versus kappa ($\kappa$).

Fig. (2): Phase velocity versus $\alpha$ for the fixed flow speed for fast mode.

Fig. (3): Phase velocity versus $\alpha$ for the fixed flow speed for slow mode.

Fig. (4): Phase velocity versus flow speed.

Fig. (5): Nonlinear coefficient $A'$ versus $\alpha$.

Fig. (6): $B'$ versus parameter $\sigma_d = T_d/T_e Z_d$.

Fig. (7): Nonlinear coefficient $A'$ versus $Z_d$.

Fig. (8): Potential $\phi_1$ versus kinematic viscosity coefficient for the fixed value of $\kappa (= 3)$.

Fig. (9): For the fixed value of kinematic viscosity, $\phi_1$ versus different value of $\kappa$.

Fig. (10): Potential $\phi_1$ versus kinematic viscosity for the fixed value of $\alpha = 0.15$.

Fig. (11): Potential $\phi_1$ versus rotation ($\Omega_c$) for the fixed value of $\kappa (= 3)$ and $\eta_o = 0.1$.

Fig. (12): Potential $\phi_1$ versus rotation ($\Omega_c$) for the fixed value of $\eta_o = 0.1$ and $\alpha (= 0.45)$.