The orbital evolution of planets in disks

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Abstract. The orbital parameters of the observed extrasolar planets differ strongly from those of our own solar system. The differences include planets with high masses, small semi-major axis and large eccentricities. We performed numerical computations of embedded planets in disks and follow their mass growth and orbital evolution over several thousand periods.

We find that planets do migrate inwards on timescales of about $10^5$ years on nearly circular orbits, during which they may grow up to about 5 Jupiter masses. The interaction of the disk with several planets may halt the migration process and lead to a system similar to the solar planetary system.

1. The model

The evolution of protoplanets can only be studied simultaneously with that of the protostellar disk in which the planets are still embedded early in their lifetime. The disk is assumed to be flat and non-self gravitating, and is modelled by the planar two-dimensional $(r - \varphi)$ Navier-Stokes equations. The mutual gravitational interaction of the planets and the star, and the gravitational torques of the disk acting on the planets and the central star are included. The time dependent hydrodynamic equations are integrated by using a finite difference scheme (Kley 1998, 1999). The computations cover the radial range from 1.2 to 20 AU. The initial surface density distribution is axisymmetric and follows $\Sigma \propto r^{-1/2}$; to speed up the computations an annular gap is imposed (Kley 1999). The material orbits the star with Keplerian speed. The planets of one Jupiter mass are initially placed at typical distances of several AU from the star. The whole system (disk, star and planets) is then evolved for several thousand orbital periods.

2. Results of a single Planet

The presence of the planet perturbs the axisymmetric density by generating trailing spirals (Fig. 1). In the situation where the tidal torques are greater than the internal viscous torques in the disc, and the disc response becomes non linear, an annular gap, or surface density depression, will form at the radius of the planet. The radial width of the gap is determined by the equilibrium of gravitational (gap opening) torques and viscous (gap closing) torques. For a
one Jupiter mass planet and a typical disk viscosity of $\alpha \approx 10^{-3}$ the width of the gap is about two Roche-lobe sizes of the planet.

Although the density around the planet is greatly reduced in the gap region, material from upstream may enter the Roche-lobe and eventually be accreted onto the planet (Fig. 2). Typical values for the viscosity ($\alpha \approx 10^{-3}$) and mass of the disk ($M_{\text{disk}} = 0.01 M_\odot$) yield a mass accretion rate onto the planet of $4 \times 10^{-5} M_{Jup}/yr$ (Kley 1999). The rate increases with larger disk viscosity and decreases when the mass of the planet grows. For very low disk viscosity and larger planetary masses the mass accumulation eventually terminates, and the maximum mass the planet may reach is about 5 Jupiter masses ($M_{Jup}$), see also Bryden et al. (1999), and Nelson et al. (2000).

![Figure 1. Gray scale plot of the surface density after 200 orbits of the planet, which is located at $x = -5.2, y = 0.0$. The gravitational torques due to the planet acting on the disk lead to the excitation of spiral density waves.](image)
Figure 2. The flow field in the vicinity of the embedded protoplanet in the corotating frame. The blue line indicates the size of the Roche-lobe of the planet.

The material in the disk acts gravitationally on the planet and disturbs the orbit. The net torque onto the planet, obtained by summation over the different annuli, is negative which leads to an inward migration on typical timescales of $10^5 \text{yrs}$ (Nelson et al. 2000).

3. Evolution of two planets

As it is believed that more eccentric orbits are caused by the interaction of several objects, we have performed a run with two embedded Jupiter-type planets. Initially they were located at $1a_J$ and $2a_J$ in opposition to each other, i.e. separated by $\Delta \varphi = 180^\circ$. The two planets were assumed to be on circular orbits initially.
Two planets create a much more complicated pattern of wave-like disturbances in the density distribution of the disk than just one planet does. In a calculation of one planet on a fixed circular orbit, the wave pattern induced in the disk is stationary in the co-rotating frame. As seen in Fig. 3, in case of two planets (the Roche-lobes are indicated) it changes strongly with time. As both planets accrete essentially all of the mass which enters their Roche-lobes, their masses grow in time. The material located initially radially between the planets is consumed by the planets within the first few hundred orbital periods.

![Figure 3. Surface density distribution in $r - \varphi$ display at four evolutionary times. The panels are labelled consecutively by the number of orbits of the inner planet.](image_url)

Finally, for the inner planet, the only gas available is that which flows through the gap created by the outer planet. Hence the mass of the inner planet grows at a smaller rate than the outer one, see Fig. 4 (bottom). At the
same time, the gravitational interaction of the two planets, the star and the disk lead to changes in the orbital elements of the objects. The change in the semi-major axis of each planet is given in Fig. 4 (top). During the evolution, the fluctuations of the radial distances to the star strongly increase, indicating a growth in eccentricity of the planet.

Figure 4.  **Top:** Evolution of the semi-major axis of the two planets.  **Bottom:** Evolution of the masses of the two planets (1: inner planet, 2: outer planet). The inferred mass accretion rates during the long term evolution are indicated. Time is given in units of the initial period of the inner planet.
4. Conclusions

By initially considering one planet on a fixed circular orbit, it was shown that even though the planet opens up an annular gap in disk it is nevertheless able to accrete more mass from its surroundings. As more massive planets induce a wider and deeper gap the mass accumulation essentially terminates, which puts the upper limit to the mass of the planet mass at about 5 to 10 $M_{\text{Jup}}$, in good agreement with the observations.

By considering the evolution of a system consisting of two Jupiter-type planets, we show (Kley 2000) that by mutual gravitational interaction the inward motion of the inner planet may come to a halt. As the outer planet continues to move inward and approaches the inner, the gravitational interaction between the two planets increases. The resulting configuration may be unstable, and lead to systems similar to $\upsilon$ And.

If the protoplanetary nebula has dissipated before the planets come very close to each other, one is left with a system of massive planets at several AU distance, similar to our own Solar System.

References

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