Stabilizing Nonlinear Adaptive Control of Spacecraft before and after Capture

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In future space infrastructure, missions for refueling and capturing inoperative spacecraft using an orbital servicing vehicle or a space robot have been considered. To realize these missions, six-degrees-of-freedom tracking control of a chaser spacecraft is required to approach the target spacecraft. Moreover, the stability of the system connecting the chaser and target must be ensured. It is also important to suppress position and attitude errors due to disturbances. In this study, our aim is to derive an adaptive proportional-integral-derivative (PID) controller that ensures the stability of a spacecraft and target must be ensured. It is also important to suppress position and attitude errors due to disturbances. In this study, the chaser spacecraft is required to approach the target spacecraft. Moreover, the stability of the system connecting the chaser vehicle or a space robot have been considered. These missions can be considered as an extension of rendezvous docking technology, which will become a key technology for future space development. To realize the missions, a chaser spacecraft must fly around a target spacecraft to track its docking port, the position and attitude of which generally change with respect to an inertial frame according to the target’s nutation or tumbling motions. When the relative errors in the position and attitude become sufficiently small, the chaser can safely capture the target. This is the first operation (Phase 1). Next, the connected spacecraft system of target and chaser must be stabilized by energy damping and carried to some destination (e.g., the International Space Station (ISS)) by tracking another given trajectory, if necessary. After this second operation (Phase 2), the mission is completed. These control actions must be performed using only the chaser’s controller, because that of the target is most likely inoperative. In order to achieve a successful mission, the following two control issues must be considered. The first issue is the six-degrees-of-freedom (d.o.f.) nonlinear tracking control of spacecraft under external disturbances such as the aerodrag force and...

Key Words: Spacecraft, Tracking Control, Adaptive PID State Feedback Control

Nomenclature

- \{o\}: inertial frame
- \{t\}, \{c\}: target and chaser body fixed frame
- \{x\}, \{R\}: connected system and reference trajectory
- \mathbb{R}^n: linear space of real vectors of dimension \(n\)
- \mathbb{R}^{n\times m}: ring of matrices with \(n\) rows and \(m\) columns and elements in \(\mathbb{R}\)
- \mathbb{S}^q: hypersphere of dimension \(q\)
- \(t\) \(\in\) \(\mathbb{R}\): time
- \(m_c, m_t \in \mathbb{R}\): mass of chaser and target
- \(J_c, J_t \in \mathbb{R}^{3\times3}\): inertia matrix of chaser and target
- \(f_c, \tau_c \in \mathbb{R}^3\): control force and torque
- \(w_f, w_t \in \mathbb{R}^3\): disturbance force and torque
- \(r_c, r_t, r_R \in \mathbb{R}^3\): position vector of each frame
- \([e_k^T \eta_k]^T \in \mathbb{S}^3\): attitude vector (quaternion) of frame \(\{k\}\) \((k = c, t, s, R)\)
- \(v_c, v_t, v_R \in \mathbb{R}^3\): linear velocity vector of each frame
- \(\omega_c, \omega_t, \omega_s, \omega_R \in \mathbb{R}^3\): angular velocity vector of each frame
- \(p_c, p_R \in \mathbb{R}^3\): constant vector fixed frame \(\{t\}\) and \(\{R\}\)
- \(\rho_{ct}, \rho_{ct}, \rho_{sc}, \rho_{sc} \in \mathbb{R}^3\): position vector from a nominal fixed point to force input point
- \(l \in \mathbb{R}^3\): distance of center of mass between target and chaser
- \(l_{ct}, l_{ct}, l_{sc} \in \mathbb{R}^3\): position vector from a nominal fixed point to center of mass of target and chaser
- \(x^x \in \mathbb{R}^{3\times3}\): skew symmetric matrix

\[
x^x = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}
\]

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gravity-gradient torque in low-earth orbits. The second is the controller, which must simultaneously stabilize the chaser spacecraft before docking and the connected spacecraft after capture.

For nonlinear tracking control in the presence of external disturbances, most researchers have concentrated on the nonlinear $H_{\infty}$ controller, which makes the $L_2$ gain of a closed-loop system from the disturbance to controlled output less than or equal to some specified positive value. However, although control methods generally require higher feedback gains to achieve higher disturbance attenuation capability, these methods are not realizable because the maximum level of control input is constrained. Because of this problem, it is considered that $H_{\infty}$ control is not necessarily the only approach to nonlinear tracking control. On the other hand, because the time-varying rate of a disturbance in the space environment is usually very small, the disturbance can be treated as constant. To eliminate the steady-state error caused by a constant disturbance, it is well known that an integral action must be added to the controller, as with the proportional-integral-derivative (PID) controller.11)

Regarding control methods that ensure stability before and after spacecraft connection, although literature12) has proposed the Lyapunov-based tracking control law and a parameter estimation law, this study only considers angular velocity tracking and results in switching the controller before and after capture. On the other hand, we have presented a passivity-based control method without changing the controller and derived conditions for closed-loop asymptotic stability using linear matrix inequalities (LMIs) whose feedback gains are the LMI variables. However, in this control method, the influence of external disturbances is not considered. In addition, it is necessary to estimate the upper bound of the target angular velocity and the maximal eigenvalue of the inertial matrix beforehand since LMI conditions include these.

Considering the aforementioned background, this paper proposes a PID state feedback control method that guarantees asymptotic stability and tracking capability before and after capture without changing the controller. It can also effectively attenuate the constant disturbance by utilizing the integrator backstepping approach. In addition, because it is difficult to know all physical parameters precisely, the PID controller includes an adaptive scheme that can estimate physical parameters. Thus, the proposed method has the advantage of ensuring the stability of the system before and after capture with one controller. Furthermore, it does not need to estimate the upper bound of the target’s angular velocity and the maximal eigenvalue of the inertial matrix beforehand.

This paper is organized as follows. First, we describe the spacecraft model and control problem. Next, we derive an adaptive PID state feedback controller based on the backstepping approach. Then, the effectiveness of the control method is verified by numerical simulations. Finally, a conclusion is presented.

2. Modeling and Problem Description

This section describes the equations of motion and control problem both before capture and after spacecraft connection. Here, the following is assumed.

Assumption 1

The chaser and target are rigid and are connected rigidly by the docking port. Therefore, the spacecraft system is rigid before capture and after the connection.

Assumption 2

No control input is applied to the target.

In addition, this paper excludes the influence of orbital motion since the motion of spacecraft is faster than that of orbital motion. The inertial frame $\{o\}$ is therefore regarded as a reference frame in the inertial space.

2.1. Before capture

For the control problem before capture, we consider a tracking control problem in which a nominal point, $A$, fixed at $\{c\}$ tracks a target point, $B$, fixed at $\{t\}$, as shown in Fig. 1 (Phase 1). Then, the translation and rotation dynamics equation of the chaser around $A$, which is not necessarily the center of mass, in $\{c\}$, is

$$M_c \dddot{\rho}_{c} + C_c \dot{\rho}_{c} = u_c + w,$$

where

$$M_c = \begin{bmatrix} m_c & -m_c l_c^T & 0 \\ -m_c l_c^T & M_c & -J_c \\ 0 & -J_c & I \end{bmatrix},$$

$$C_c = \begin{bmatrix} m_c \omega_c^T & -m_c \omega_c^T \omega_c^T \\ m_c \omega_c^T \omega_c^T & -J_c - m_c l_c^T \omega_c^T \omega_c^T \end{bmatrix},$$

$$q_c = [r_c^T \dot{r}_c^T]^T, \quad p_c = [v_c^T \omega_c \dot{\omega}_c]^T, \quad w = [w_c^T \dot{w}_c]^T,$$

$$u_c = U_c \begin{bmatrix} f_c \\ \tau_c \end{bmatrix} = \begin{bmatrix} u_{fc} \\ u_{tc} \end{bmatrix}, \quad U_c = \begin{bmatrix} I_3 & 0 \\ 0 & \rho_{cc}^2 I_3 \end{bmatrix}.$$  

The inputs $f_c$ and $\tau_c$ can be uniquely determined after $u_{fc}$ and $u_{tc}$ are derived, because the matrix, $U_c$, satisfies det $U_c \neq 0$. The position of $A$ and the attitude of $\{c\}$ with respect to $\{o\}$ are given by the following kinematics equation if the quaternion is used for attitude parameterization in order to avoid singularities.

Fig. 1. Definition of target and chaser fixed frame and the position vector (before capturing: Phase 1).
\[
\dot{r}_s = v_s - \omega_c^x r_c,
\]
\[
\dot{\omega}_c = \frac{1}{2} \left[ \eta_c I_3 + \varepsilon_c^T \right] \omega_c = E(\theta_c) \omega_c,
\]
where \( \theta_c = [\varepsilon_c^T \eta_c]^T \in \mathbb{S}^3 \) satisfies the constraint \( ||\theta_c|| = 1 \).

On the other hand, the target dynamics and kinematics can be described as follows.

\[
M_p r + C_p \dot{r} = 0,
\]
\[
\dot{r} = v_t - \omega_t^x r_t,
\]
where

\[
M_t = \text{diag}(m_t, J_t), \quad C_t = \text{diag}(m_t \omega_t^x, \omega_t^x J_t),
\]
\[
q_t = [r_t^T \omega_t^T]^T, \quad p_t = [v_t^T \omega_t^T]^T,
\]
and \( \dot{\theta}_t = [\varepsilon_t^T \eta_t]^T \in \mathbb{S}^3 \) satisfies the constraint \( ||\theta_t|| = 1 \).

Our tracking control problem in Phase 1 is to find a controller such that

\[
\begin{align*}
\dot{r}_c &= r_t + \Delta r_c, \\
\dot{x}_c &= x_t + \Delta x_c,
\end{align*}
\]
where

\[
\Delta r_c = m_t^N [C(\omega_t^x p_t + \omega_t^x v_t) - \omega_t^x (C\omega_t - \omega_t^x C\omega_t)],
\]
\[
\Delta x_c = m_t^N [C(\omega_t^x p_t + \omega_t^x v_t) - \omega_t^x (C\omega_t - \omega_t^x C\omega_t)]
\]
\[
- \omega_t^x (C\omega_t - \omega_t^x C\omega_t) + J_s (\omega_t - \omega_t^x C\omega_t) + \omega_t^x J_s C\omega_t + (C\omega_t)^x J_s (\omega_t - \omega_t^x C\omega_t).
\]

Using the transform, the tracking control problem in Phase 1 is reduced to a regulation problem for designing the control input \( u_f \) and \( u_t \), such that

\[
\begin{align*}
r_c &= 0, \quad \varepsilon_c = 0, \quad \eta_c = 1, \quad v_c = 0, \quad \omega_c = 0
\end{align*}
\]
when \( t \to \infty \) according to Eqs. (9) and (10).

2.2. After connection

For the control problem after connection, we consider the tracking control problem in which a nominal point, \( A \), fixed at \( s \) tracks a target point, \( B \), fixed at \( \{R\} \), as shown in Fig. 2 (Phase 2). Then, the translation and rotation dynamics equation of the connected system around \( A \), which is not necessarily the center of mass, in \( s \), becomes

\[
M_p r + C_p \dot{r} = u_s + w,
\]
where

\[
\begin{align*}
M_s &= \sum_{i=1}^3 \begin{bmatrix} m_i & -m_i \omega_i^x \\ m_i \omega_i & \omega_i \omega_i^x \beta_i^x \end{bmatrix}, \\
C_s &= \sum_{i=1}^3 \begin{bmatrix} m_i \omega_i^x & -m_i \omega_i^x \omega_i \omega_i \\ \omega_i \omega_i^x \beta_i & \omega_i \omega_i \omega_i \omega_i \end{bmatrix}, \\
q_i &= [r_a^T \omega_a^T]^T, \quad p_i = [v_a^T \omega_a^T]^T, \quad w = [w_1^T w_2]^T, \\
u_s &= U_s \begin{bmatrix} f_c \\ \tau_c \end{bmatrix} = \begin{bmatrix} u_f \\ u_t \end{bmatrix}, \quad U_s = \begin{bmatrix} I_3 & 0 \\ 0 & \beta_s \end{bmatrix}.
\end{align*}
\]

As in the previous subsection, the inputs \( f_c \) and \( \tau_c \) can be uniquely determined after \( u_f \) and \( u_t \) are derived, because the matrix \( U_s \) satisfies \( \det U_s \neq 0 \). The position of \( A \) and the attitude of \( s \) with respect to \( |o| \) are given by the following kinematics equations if the quaternion is used for attitude parameterization in order to avoid singularities.

\[
\dot{r}_s = v_s - \omega_s^x r_s, \quad \dot{\theta}_s = E(\theta_s) \omega_s,
\]
where \( \theta_s = [\varepsilon_s^T \eta_s]^T \in \mathbb{S}^3 \) satisfies the constraint \( ||\theta_s|| = 1 \).

Our tracking control problem in Phase 2 is to find a con-
controller such that
\[ r_i = r_{Bi}, \quad \epsilon = \epsilon_i, \quad \eta_i = \eta, \quad v_i = v_{Bi}, \quad \omega_i = \omega_R \]
when \( t \to \infty \). The position and velocity of point \( B \) fixed at \( \{R\} \) are given by
\[ r_{Bi} = r_R + p_R, \quad v_{Bi} = v_R + \omega_R^x p_R. \quad (13) \]
In the same manner as the previous subsection, the relative equations of motion in \( \{s\} \) are described as follows.
\[ M_{se} \dot{p}_{se} + C_{se} p_{se} + \Delta_{se} = u_s + w, \quad (14) \]
\[ \dot{r}_{se} = v_{se} - (\alpha_{se} + \omega_R^x p_{se}), \quad \dot{\theta}_{se} = E(\theta_{se}) \omega_{se}, \quad (15) \]
where
\[ q_{se} = [r_{se}^T \ \dot{r}_{se}^T]^T, \quad p_{se} = [v_{se}^T \ \dot{v}_{se}^T]^T, \]
\[ \theta_{se} = [\theta_{se}^T \ \eta_{se}^T]^T, \quad M_{se} = M_s, \]
\[ C_{se} = \sum_{i=se} c_{s1i,2} \begin{bmatrix} c_{s1i,1} & c_{s1i,2} \end{bmatrix}, \quad \Delta_{se} = \sum_{i=se} \begin{bmatrix} \Delta_{s1i,1} \\ \Delta_{s1i,2} \end{bmatrix}, \]
\[ \epsilon_{se} = \eta_{se} \epsilon_i + \eta \epsilon_s + \dot{\epsilon} \epsilon_{se}, \quad \eta_{se} = \eta_i \eta_s + \epsilon \dot{\epsilon} \epsilon_{se}, \]
\[ \ddot{C} = (\eta_{Rse} - \epsilon \dot{\epsilon} \epsilon_{se}) \ddot{J}_s + 2 \dot{E}_{se} \epsilon_{se} - 2 \dot{c}_{se} \epsilon_{se}, \]
\[ r_{se} = r_i - \ddot{C}_R_s, \quad v_{se} = v_i - \dot{C}_R_s, \quad \omega_{se} = \omega_s - \ddot{C}_R_s, \]
\[ r_{ce} = r_i - \ddot{C}_R_s, \quad v_{se} = v_i - \dot{C}_R_s, \quad \omega_{se} = \omega_s - \ddot{C}_R_s, \]
\[ c_{s1i,1} = m_i (\omega_c + \ddot{C}_R_s)^x - \Delta \omega_s (\omega_s + \omega_R^x p_{se}), \quad c_{s1i,2} = -m_i (\omega_c + \ddot{C}_R_s)^x p_{se}, \]
\[ \Delta_{s1i,1} = m_i \left[ \ddot{C} (\omega_R^x p_{se} + \omega_s^x p_{se}) - I_{s1} (\ddot{C} \omega_R^x - \omega_s^x \ddot{C}_R_s) - (\omega_c + \ddot{C}_R_s)^x \right], \]
\[ \Delta_{s1i,2} = m_i (\ddot{C} (\omega_R^x p_{se} + \omega_s^x p_{se}) - I_{s1} (\ddot{C} \omega_R^x - \omega_s^x \ddot{C}_R_s) - (\omega_c + \ddot{C}_R_s)^x \right], \]
and \( \Delta_{se} \) is the quaternion of relative attitude, \( r_{se}, v_{se}, \omega_{se} \) are the relative position, linear velocity, and angular velocity, respectively, and \( \ddot{C} \) is the direction cosine matrix from \( \{R\} \) to \( \{s\} \). Using this transform, the tracking control problem in Phase 2 is reduced to a regulation problem for designing the control input \( u_{f_s} \) and \( u_{se} \) such that
\[ r_{se} = 0, \quad \epsilon_{se} = 0, \quad \eta_{se} = 1, \quad v_{se} = 0, \quad \omega_{se} = 0 \]
when \( t \to \infty \) according to Eqs. (14) and (15).

**Remark 1**
By setting the parameters with respect to the target \( m_i, l_{i}, l_{se} \) to zero and the chaser \( l_{se}, p_{se} \) to \( l_{i}, p_{i} \), and replacing the frames \( \{s\} \) and \( \{R\} \) in the relative equation of motion of the connected system Eqs. (14) and (15), as \( \{s\} \to \{c\} \), and \( \{R\} \to \{t\} \), respectively, this equation becomes the relative equation of motion of the chaser with respect to the target before capture Eqs. (9) and (10).

**3. Adaptive Controller Design**

In this section, we derive an adaptive PID controller that makes the relative error asymptotically stable under unknown physical parameters.

**Phase 1:** \( r_{se} = 0, \quad \epsilon_{se} = 0, \quad \eta_{se} = 1, \quad v_{se} = 0, \quad \omega_{se} = 0 \)

**Phase 2:** \( r_{se} = 0, \quad \epsilon_{se} = 0, \quad \eta_{se} = 1, \quad v_{se} = 0, \quad \omega_{se} = 0 \)

These conditions are obtained by applying the backstepping approach when \( w = 0 \). Regarding the target states and reference signals, the following is assumed.

**Assumption 3**
The target states \( r_i, \epsilon_i, \eta_i, v_i, \omega_i, \) and \( \omega_i \) are directly measurable, and are uniformly continuous, bounded, and known for all \( t \in [0, \infty) \).

**Assumption 4**
The reference signals \( r_s, \epsilon_s, \eta_s, v_s, \omega_s, \) and \( \omega_s \) are uniformly continuous, bounded, and known for all \( t \in [0, \infty) \).

First, the adaptive PID controller for Phase 1 is derived. Then, the following theorem can be obtained.

**Theorem 1**
Consider the following adaptive PID controller for Phase 1,
\[ u_{f_s} = -(I_3 + K_{D1} \dot{K}_{p1}) r_{se} - K_{D1} J_{i1} \dot{c}_c - K_{D1} v_{se} \]
\[ + \hat{m}_i \dot{\phi}_1 \]
\[ u_{se} = -(I_3 + K_{D2} K_{p2}) r_{se} - K_{D2} J_{i2} \dot{c}_c - K_{D2} \omega_{se} \]
\[ + \hat{m}_i \dot{\psi}_2 + \dot{\psi}_2 \dot{c}_c, \]
\[ \dot{\theta}_c = -G \bar{\psi}_1^T \dot{c}_c, \quad \dot{\beta}_c = -G \bar{\psi}_1 \dot{c}_c, \]
\[ \xi_c = \int_0^t r_{se} dt, \quad \zeta_c = \int_0^t \epsilon_{se} dt, \]
\[ \alpha_c = -K_{P1} r_{se} - K_{I1} \epsilon_{se}, \quad \alpha_c = -K_{P2} r_{se} - K_{I2} \epsilon_{se}, \]
\[ \phi_c = \dot{\alpha}_c - I_{c1} \ddot{c}_c \dot{\alpha}_c + (\omega_c + \omega_R^x c) \alpha_c \]
\[ - (\omega_c + \omega_R^x c)^T \dot{c}_c \ddot{c}_c + \dot{c}_c \dot{c}_c \alpha_c \]
\[ - l_{c1} (\omega_c + \omega_R^x c) \dot{c}_c - (\omega_c + \omega_R^x c) \dot{c}_c \alpha_c \]
\[ + \dot{c}_c \dot{c}_c \alpha_c + \dot{c}_c \dot{c}_c \alpha_c \]
\[ \dot{\psi}_c = \psi_1 \dot{\alpha}_c + \dot{\alpha}_c \psi_1 \]
\[ + \psi_2 \dot{c}_c \alpha_c + \dot{c}_c \psi_2 \]
\[ \psi_1(\alpha) \psi_1(\beta) = I_3, \quad \psi_2(\alpha) \beta = \alpha^T J b, \quad \forall a, b \in \mathbb{R}^3 \]
\[ \psi_1(\alpha) = \begin{bmatrix} a_1 & a_2 & a_3 & 0 & 0 & 0 \end{bmatrix}, \]
\[ \psi_2(\alpha) = \begin{bmatrix} a_3 b_1 & a_3 b_2 & a_3 b_3 & -a_1 b_1 & -a_2 b_2 & -a_2 b_3 \end{bmatrix}, \]
\[ \dot{a}_1 b_2 - a_2 b_3 \]
\[ a_1 b_2 0 \]
where $\hat{m}_c \in \mathbb{R}$ and $\hat{\beta}_c \in \mathbb{R}^6$ are the estimate variables, $\xi_{ce} \in \mathbb{R}^3$ ($n = 1, 2$) is the integral variable, $K_{p1}$, $K_{Du}$, $K_{ln} > 0 \in \mathbb{R}^{6 \times 3}$ are the feedback gain matrices, $J_{c1} \in \mathbb{R}$ is the positive scalar adaptive gain, $J_2 > 0 \in \mathbb{R}^{6 \times 6}$ is the adaptive gain matrix, and $J_{jk}$ ($j, k = 1, 2, 3, j \leq k$) is the $(j, k)$ element of $J_c$. Then, the state variable of the closed-loop system of Eqs. (9) and (10) with Eqs. (16)–(18) becomes

\[
(r_{ce}, \, e_{ce}, \, \eta_{ce}, \, v_{ce}, \, \omega_{ce}, \, \xi_{c1}, \, \xi_{c2}, \, \hat{m}_c, \, \hat{\beta}_c) \rightarrow (0, \, 0, \, 1, \, 0, \, 0, \, 0, \, c_1, \, c_2)
\]
as $t \to \infty$ for an arbitrary initial state when $w = 0$ ($c_1 \in \mathbb{R}$ and $c_2 \in \mathbb{R}^6$ are some constant values), where $\hat{m}_c = m_c - \bar{m}_c$ and $\hat{\beta}_c = \beta_c - \bar{\beta}_c$ are the estimate errors.

**Proof.** The proof consists of two steps as follows:

**Step 1:** Assume that $v_{ce}$ and $\omega_{ce}$ are the virtual inputs to subsystems, Eq. (10), and define the stabilizing functions such that

\[
\alpha_{c1} = -K_{p1}r_{ce} - K_{H1}\xi_{c1}, \quad \alpha_{c2} = -K_{p2}e_{ce} - K_{H2}\xi_{c2}.
\]

Now, the error variables between the state $(v_{ce}, \omega_{ce})$ and the desired control $(\alpha_{c1}, \alpha_{c2})$ are defined as

\[
\xi_{c1} = v_{ce} - \alpha_{c1}, \quad \xi_{c2} = \omega_{ce} - \alpha_{c2}.
\]

From Eq. (20), subsystems, Eq. (10), become

\[
\dot{r}_{ce} = (\xi_{c1} + \alpha_{c1}) - (\omega_{ce} + C_{a1})^T r_{ce}, \quad \dot{e}_{ce} = \theta_{ce} = (\xi_{c2} + \alpha_{c2}) + C_{a2}.
\]

The candidate Lyapunov function is defined as

\[
V_{c1} = \frac{1}{2}\|r_{ce}\|^2 + \frac{1}{2}\xi_{c1}^TK_{H1}\xi_{c1} + \|e_{ce}\|^2 + (\eta_{ce} - 1)^2
\]

\[
+ \frac{1}{2}\xi_{c2}^TK_{p2}\xi_{c2}.
\]

Using the following skew-symmetric matrix properties

\[
a^T a = 0, \quad a^T b^+ a = 0, \quad \forall a, b \in \mathbb{R}^3,
\]

the time derivative of Eq. (23) along the trajectories of the closed-loop system becomes

\[
\dot{V}_{c1} = -r_{ce}^TK_{p1}r_{ce} - e_{ce}^TK_{p2}e_{ce} + r_{ce}^T e_{c1} + e_{ce}^T e_{c2}.
\]

From Eq. (25), obviously, $r_{ce} \to 0, \, e_{ce} \to 0$ and $\xi_{ce} \to 1$ as $t \to \infty$ when $\xi_{c1} = \xi_{c2} = 0$.

**Step 2:** The adaptive controller that makes the state $z_c = [\xi_{c1}, \xi_{c2}, \eta_{ce}, \omega_{ce}]$ asymptotically stable (i.e., $z_c = 0$) when $w = 0$, is derived. From Eqs. (9), (19), and (20), the dynamics with respect to $z_c$ are

\[
M_c \ddot{z}_c + C_c z_c + \Delta_c = u_c,
\]

where

\[
M_c = M_{ce}, \quad C_c = C_{ce} + C_{cz}, \quad C_{cz} = \begin{bmatrix}
C_{c11} & C_{c12} \\
C_{c21} & 0
\end{bmatrix}, \quad C_{cz} = \begin{bmatrix}
m \phi_1 \eta_{ce} \\
m \phi_2 + \psi \beta_c
\end{bmatrix}.
\]

In addition, $\dot{\alpha}_{c1}$ and $\dot{\alpha}_{c2}$ can be easily calculated from Eqs. (18), (21), and (22). The candidate Lyapunov function is defined as

\[
V_{c2} = V_{c1} + \frac{1}{2}r_{ce}^TM_{ce}z_c + \frac{1}{2}\xi_{c1}^T \hat{m}_c^2 + \frac{1}{2}r_{ce}^TG_{2}^{-1}r_{ce}.
\]

Applying the fact that $C_{c1}$ is a skew-symmetric matrix, the time derivative of Eq. (27) along the trajectories of the closed-loop system becomes

\[
\dot{V}_{c2} = -r_{ce}^TK_{p1}r_{ce} - e_{ce}^TK_{p2}e_{ce} - \xi_{c1}^TD_{11} \hat{m}_c - \xi_{c2}^TD_{22} \hat{\beta}_c.
\]

Obviously, $\dot{V}_{c2} \leq 0$ holds. Therefore, $x_c$ is bounded, since

\[
V_{c2}(x_c(t)) \leq V_{c2}(x_c(0)), \quad \forall t \\
\dot{x}_c = [\xi_{c1}^T, \xi_{c2}^T, r_{ce}^T, e_{ce}^T, \eta_{ce}^T, v_{ce}^T, \omega_{ce}^T, m \phi_1^T, \psi \beta_c]^T,
\]

and $V_{c2}$ is radially unbounded in state space $\mathbb{R}^{22} \times \mathbb{S}^3$. Then, $\dot{x}_c$ is also bounded because the control inputs Eq. (16), and the derivative of the estimate variable are bounded by Assumption 3. It follows that $\dot{V}_{c2}$ is bounded. Therefore, it is shown that

\[
\dot{V}_{c2} \to 0 \iff r_{ce} \to 0, \quad e_{ce} \to 0, \quad \xi_{c1} \to 0, \quad \xi_{c2} \to 0
\]
as $t \to \infty$ from the Lyapunov-like lemma, and then

\[
\xi_{c1} \to 0, \quad \xi_{c2} \to 0, \quad \eta_{ce} \to 1, \quad v_{ce} \to 0, \quad \omega_{ce} \to 0
\]
as $t \to \infty$ from Eqs. (19) and (20), and $V_{c2} \to 0$. Furthermore, the closed-loop system becomes

\[
\dot{m}_c \phi_1 = 0, \quad \dot{m}_c \phi_2 + \psi \beta_c = 0
\]

when $t \to \infty$. This implies that the estimate variables converge to some constant values (i.e., $m_c \to c_1, \beta_c \to c_2$) as $t \to \infty$. This completes the proof.

**Proposition 1**

If the operator, $\phi_1$, satisfies $\|\phi_1\| \neq 0$, $\forall t \geq 0$, then $\hat{m}_c \to m_c$ as $t \to \infty$. Moreover, if $\eta_{0}$ and operator $\psi_1$ satisfy the proposition of literature (see Appendix), then $\hat{\beta}_c \to \beta_c$ as $t \to \infty$.

**Proof.** From the first formula of Eq. (30), $m \phi_1 \phi_1 = 0$, if $\|\phi_1\| \neq 0$ for all $t \geq 0$, then $\hat{m}_c$ obviously becomes $\hat{m}_c \to 0$ (i.e., $\hat{m}_c \to m_c$) as $t \to \infty$. Moreover, the second formula of Eq. (30) when $\hat{m}_c \to 0$ becomes $\hat{\beta}_c \to 0$, and the operator $\psi_1$ becomes

\[
\psi_1 = \psi_1(\phi_0) + \psi_2(\omega), \quad \phi_0 = \psi_1(\phi_0) + \omega^* \psi_1(\phi_0)
\]

from $(\xi_{ce}, \omega_{ce}, \xi_{ce}, \omega_{ce}) \to (0, 1, 0, 0)$ and $C \to I$ as $t \to \infty$. Here, $\psi_1$ is equivalent to $W(t)$ of Proposition 3 shown in Appendix. Although operator $\psi_1$ in this paper is different from operator $L$ of Appendix, operator $\psi_1$ is equal to the operator $L$ by conforming to the order of elements of $\beta_c$ to $\beta$. Therefore, if $\eta_{0}$ is periodic and $\psi_1$ satisfies the following condition

\[
\text{rank} \begin{bmatrix}
\psi_1(t_1) & \cdots & \psi_1(t_n)
\end{bmatrix}^T = 6,
\]

then $\hat{\beta}_c \to \beta_c$ as $t \to \infty$. This completes the proof.
Next, the adaptive PID controller for Phase 2 is derived. Then, the following theorem can be obtained.

**Theorem 2**

Consider the following adaptive PID controller for Phase 2,

\[
\begin{align*}
    u_t &= -(I_1 + K_{D1} K_{P1}) r_{se} - K_{D1} K_{I1} \zeta_{t1} - K_{D1} v_{se} \\
    u_c &= -(I_1 + K_{D2} K_{P2}) \xi_{se} - K_{D2} K_{I2} \zeta_{t2} - K_{D2} \omega_{se} \\
    \dot{m}_t &= -\Gamma_{c1} [\phi_{t1}^T \bar{\phi}_{t1}] z_t, \quad \dot{m}_c = -\Gamma_{c1} [\phi_{c1}^T \bar{\phi}_{c1}] z_t, \\
    \dot{\beta}_t &= -\Gamma_{p2}^T z_{t2},
\end{align*}
\]

(33)

\[
\zeta_{t1} = \int_0^t r_{se} \, dt, \quad \zeta_{t2} = \int_0^t \xi_{se} \, dt,
\]

(35)

\[
\alpha_{t1} = -K_{P1} \xi_{se} - K_{I1} \zeta_{t1}, \quad \alpha_{t2} = -K_{P2} \xi_{se} - K_{I2} \zeta_{t2},
\]

(18). Therefore, from the above-mentioned facts and Remark 2, the adaptive controller for Phase 2, Eqs. (33)–(35), is equivalent to that of Phase 1, Eqs. (16)–(18) with Eqs. (33)–(35), the adaptive controller for Phase 2, Eqs. (33)–(35), is equivalent to that of Phase 1, Eqs. (16)–(18). Therefore, the comparison of Eqs. (16)–(18) with Eqs. (33)–(35) shows that only the state variables used for feedback are different; however, the structure of the controller is the same. Furthermore, by setting the parameters and the operators with respect to the target \( m_t, J_t, I_t, \phi_1, \phi_2 \) as zero and the parameter with respect to the chaser \( l_c \), \( l_c \), and replacing the relative errors \( r_{se}, \theta_{se}, v_{se}, \omega_{se} \) and reference signals by the relative errors \( r_{se}, \theta_{se}, v_{se}, \omega_{se} \) and the target states in Eqs. (33)–(35), the adaptive controller for Phase 2, Eqs. (33)–(35), is equivalent to that of Phase 1, Eqs. (16)–(18). Therefore, from the above-mentioned facts and Remark 1, both Phases 1 and 2 can be stabilized using the same controller (i.e., without changing the controller at the docking instant). In addition, activation or scheduling of feedback gains is not necessary for stabilization.

**Remark 2**

In literature, a nonlinear tracking adaptive controller was also proposed. The estimate law in the literature was derived based on the results of the literature. However, the number of estimated parameters is more than that in the proposed method because the literature also estimates the mass and inertia of the target in addition to those of the chaser. Moreover, the control law is a PD controller and the convergence of the estimate variables were not discussed in the literature.}

4. Numerical Simulation

4.1. Mission setup

A numerical simulation is performed for a situation where the chaser tracks the docking port fixed on the target, which is in a tumbling state. After capture, in order to address the problem of transportation to the space station, the connected system tracks the reference trajectory.
4.2. Capture method of target spacecraft

To avoid collision and excessive docking impact, the following scenario is considered in Phase 1, as shown in Fig. 3. First, the chaser approaches the target from a certain distance $p_t$ (Fig. 3(a)). Then, during tracking control, the distance to the target is gradually decreased by $\delta p_t$ (Fig. 3(b) and (c)). When $p_t = 1.5\text{ m}$, the target is assumed to be captured by the chaser (Fig. 3(d)).

4.3. Physical model

The physical parameters used in the simulation are as follows.

\[
m_t = 300 \text{ kg}, \quad m_c = 200 \text{ kg},
\]

\[
J_t = \text{diag}[50, 275, 275] \text{ kgm}^2,
\]

\[
J_c = \begin{bmatrix}
75.00 & -28.13 & -28.13 \\
-28.13 & 75.00 & -28.13 \\
-28.13 & -28.13 & 75.00
\end{bmatrix} \text{ kgm}^2,
\]

\[
l_d = l_{cc} = l_a = [0 \ 0 \ 0]^T \text{ m}, \quad l_{cc} = [l \ 0 \ 0]^T \text{ m},
\]

\[
\rho_t = \rho_{cc} = \rho_a = [0 \ 0 \ 0]^T \text{ m}, \quad \rho_{cc} = [l \ 0 \ 0]^T \text{ m},
\]

where $l = 1.5\text{ m}$. The nominal point, $A$, is the chaser’s center of mass before capture; it is set as the center of mass of the target after capture. In the latter case, this means that $[t]$ and $[s]$ are the same frame.

4.4. Simulation results

The results of the numerical simulation are shown in Figs. 4–9, where the initial values, the target constant vector fixed at $[t]$, and the controller gains are set to $r_t(0) = [3 \ 3 \ 3]^T \text{ m}$, $\varepsilon_t(0) = [0 \ 0 \ 0]^T$, $\eta_t(0) = 1$, $v_t(0) = [0 \ 0 \ 0]^T \text{ m/s}$, $\omega_t(0) = [0.02 \ 0.04] - 0.04]^T \text{ rad/s}$, $r_c(0) = [10 \ 0 \ 10]^T \text{ m}$, $\varepsilon_c(0) = [0.31 \ 0.18 \ 0.31]^T$, $\eta_c(0) = 0.88$, $v_c(0) = [0 \ 0 \ 0]^T \text{ m/s}$, $\omega_c(0) = [0 \ 0 \ 0]^T \text{ rad/s}$, $\dot{\eta}_t(0)$, $\dot{\eta}_c(0) = 180 \text{ kg}$, $\dot{\eta}_e(0) = 120 \text{ kg}$, $\dot{\beta}_e(0) = [45.0 \ -16.9 \ -16.9 \ 45.0 \ -16.9 \ 45.0]^T \text{ kgm}^2$, $p_t = [3.5 \ 0 \ 0]^T \text{ m}$, $\delta p_t = 0.05 \text{ m}$, $K_{p1} = 0.2I_3$, $K_{d1} = 70I_3$, $K_{I1} = 0.008I_3$, $\Gamma_{g1} = \Gamma_{g2} = 100$, $K_{p2} = 0.2I_3$, $K_{d2} = 250I_3$, $K_{I2} = 0.008I_3$, $\Gamma_2 = 1000I_0$.

The reference trajectory that the connected system tracks is assumed to be

\[
r_e(t) = R(t)[\cos vt \ \sin vt \ 0]^T \text{ m}, \quad v = \frac{2\pi}{900} \text{ rad/s},
\]

\[
p_R = [0 \ 0 \ 0]^T \text{ m}, \quad \omega_R(t) = [0 \ 0 \ v]^T \text{ rad/s},
\]

\[
R(t) = 15[1 - \exp(-(0.01t + 0.3))] \text{ m}.
\]

This reference trajectory finally becomes a circular orbit with a radius of $15 \text{ m}$ in the $x$–$y$ plane in $[t]$. Furthermore, the following constant external disturbance is added for all $t$. 

\[
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\]

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Fig. 7. Graphical representation of the results of numerical simulation in Phase 1.

Fig. 8. Time histories of the estimated parameters $\hat{m}_c$, $\hat{m}_t$ (solid) and real parameters $m_c$, $m_t$ (dashed-dotted).

Fig. 9. Time histories of the estimated parameters $\hat{\beta}$ (solid) and real parameters $\beta$ (dashed-dotted).

Fig. 10. Time histories of operators $\phi_1$ (left) and $\phi_1$ (right).

The chaser arrives at target vector $p_t$ and gradually approaches the target by changing the reference signal under a constant external disturbance. After capturing the target at 306.0 s, the connected system tracks the reference trajectory under the constant external disturbance. From Figs. 8–10, it is found that, although all of the estimated parameters do not converge to real parameters because operator $\phi_1$ does not satisfy the conditions of Proposition 1, the stability of the systems is guaranteed under uncertain physical parameters.

5. Conclusions

This paper proposed the synthesis of a controller that ensures stability before and after the capture of an inoperative spacecraft in orbit under a constant external disturbance. For successful capture, we considered a PID state feedback controller based on integrator backstepping for six-degrees-of-freedom nonlinear spacecraft motion with parameter
adaptive schemes. The proposed controller has the advantage of ensuring the stability of the system before and after capture without changing the controller and without switching or scheduling feedback gains. The controller’s effectiveness was verified by numerical simulations. Avoiding the collision of the chaser and target before capture, extension to a case considering the orbital motion, the jump phenomenon of the states of the closed-loop system at the time of docking instant, and the application of the results of this study to a case considering the dynamics of the docking port and the estimation of target motion will be a subject of future work.

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Appendix

Proposition of literature

The adaptive attitude tracking controller listed in literature is given by

\[
\begin{align*}
\dot{\theta} &= - (K_1 \theta + I_3) e_{ce} - K_2 \omega_{ce} - (F + G) \hat{\beta} \\
\dot{\beta} &= Q^{-1} (F + G)^T (K_1 \theta + \omega_{ce}),
\end{align*}
\]

where $\hat{\beta} \in \mathbb{R}^6$ is the estimated variable, $K_1$ and $K_2 \in \mathbb{R}^{3 \times 3}$ are the positive definite and symmetric feedback gain matrices, respectively, and $Q \in \mathbb{R}^{6 \times 6}$ is the positive definite and symmetric adaptive gain matrix,

\[
F = - (\omega_{ce} + \dot{\omega}_t) \mathbf{L}(\omega_{ce} + \dot{\omega}_t) + \mathbf{L}(\omega_{ce}^* \dot{\omega}_t - \dot{\omega}_t),
\]

\[
G = - \frac{1}{2} \mathbf{L} [K_1 (\dot{e}_{ce} \omega_{ce} + \eta_{ce} \omega_{ce})],
\]

\[
\beta = [J_{ce} J \omega_t J_{s,22} J \omega_t J \omega_t J \omega_t J \omega_t]^T,
\]

\[
L(a)\beta = J \omega_t, \quad L(a) = \begin{bmatrix} a_1 & 0 & 0 & a_2 & a_3 \\ a_2 & 0 & a_3 & 0 & a_1 \\ 0 & a_3 & a_2 & a_1 & 0 \end{bmatrix}.
\]

Here, suppose that $\omega_t$ and $\dot{\omega}_t$ are bound. Then, the condition under which $\hat{\beta}$ converges to real value $\beta$ is given as follows.

**Proposition 3**

Let $\omega_t$ be periodic. Furthermore, let $0 \leq t_1 \leq t_2 \leq \cdots \leq t_n$ and assume that \[\text{rank} \begin{bmatrix} W(t_1) & \cdots & W(t_n) \end{bmatrix}^T = 6,\] (A.2) where \[W(t) = L(\omega_t(t)) + \dot{\omega}_t(t)^* L(\omega_t(t)).\] Then, under the control law given by Eq. (A.1), $\hat{\beta} \rightarrow \beta$. 

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Associate Editor