Optimal three-loop cascade PI-P-PI controller for electro-hydraulic power steering system

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Abstract. The present paper presents an optimally tuned proportional integral differential (PID) system which calculates the driving signal for the switching valves in order to achieve steering trajectory tracking. The designed cascade control algorithm consists of three PID controllers respectively for the spool valve position, for the effective flow rate and for the steering cylinder position. The stages of the cascade control algorithm synthesis are presented. The goals of control algorithm are to achieve fast transients with minimal overshoot and steady state error. Controllers’ are tuned consequently by starting from the innermost loop based on an identified single-input multiple output (SIMO) model of the steering system by optimization algorithm with a quadratic cost. After one of the controllers is successfully tuned the model of the corresponding closed loop system is approximated and employed in the optimization procedure for the next control loop from the cascade system. The developed control algorithm is programmed into a specialized vehicle microcontroller and tested on a steering system test bench in the laboratory.

1. Introduction

Most of future off-road construction and agriculture machines will be fully automatized and operating without continuous human presence. Such development of autonomous and remote-controlled vehicles is dependent on implementation of precise and reliable steering servo system. The steering system is essential for machine operation since machine functional capacity cannot be effectively employed if the machine is held stationary. Moreover, the usual operational environment for such machines is off-road so the mobility of the machine is extremely important. The considerable mass of construction machines and the harsh off-road environment require powerful steering mechanism. Therefore, the hydraulic transmission is employed which works as an amplifier of the operator command [1].

In conventional (non-electronic) steering system the force amplification is achieved with a spool-sleeve valve (figure 1). It is a rotary kind of valve which is coupled to the operator’s steering handle (the steering wheel). A small valve displacement initiated by the machine operator directs a portion of flow rate to the left or to the right chamber of the main steering cylinder. Since the cylinder piston is mechanically linked to the wheels the vehicle begins to turn in the commanded direction.

However, in order to steer the vehicle with an electronic signal a proportional spool valve, pilot operated with switching micro-valves [2] replaces the rotary spool-sleeve valve (figure 2). Switching micro-valves (also known as digital valves) are not the only option to achieve electronic steering. Its alternatives are direct operated directional spool valve or steer-by-wire strategy. The leading
manufactures of hydraulic steering systems favour the switching valves due to its mechanical simplicity and low cost. Usually four micro-valves are connected in a bridge configuration where two of the valves are normally opened and the other two are normally closed. A solenoid based electric circuit drives each micro-valve between its two states. The micro-valve bridge is connected to the left and right endpoints of a translational spool valve which direct the flow from the pump to the main cylinder. In order to drive the spool valve to the left the upper and lower micro-valves from the right side of the bridge are switched.

Various problems arise in automatic control of the discussed pilot operated steering system. First the switching of the valves is a nonlinear discrete-event process which induce pressure and flow rate oscillations through the hydraulic system with relatively high frequency (around 40÷100 Hz). The oscillations cannot be compensated with the software controller since they are inherent in the system.
architecture. The second problem is positive overlap in the spool valve which means that minimal amount of translation is required to open the valve in each direction. The valve overlap is mechanically designed to prevent spontaneous steering drifts which could compromise the safety. From control point of view the positive valve overlap looks like a dead band - a static nonlinearity in the loop. The third problem is the nonlinear relationship between flow rate and pressure variables which is caused by the varying geometrical shapes of the valve openings and by the continuity equation for throttle. Hence even if the spool valve position is precisely controlled the generated flow would not vary linearly with its translation. The fourth problem is unpredictable nature of the steering loads acting on the wheels and translated to the piston of the main cylinder. The control system must guarantee the commanded steering angle despite the external load.

The article proposes a complicated cascade control system which addresses these problems. It is composed of three linear controllers connected such that the output of one controller is a reference signal for the next (a cascade interconnection). Such configuration is advantageous when a single control action has to account for several output variables of the plant [3, 4]. The investigated system is of SIMO type (single input multiple output). The manipulated variable is the duty cycle of the pulse-width modulated signal determining the average openings of the switching valves. The first output signal is the spool valve position measured with a linear variable differential transducer (LVDT) sensor. The second output is the consumed flow rate of the system measured with a geared flow meter. The last output is the piston position of the steering cylinder. The goal of the system is to tracking fast enough a generated reference trajectory for the steering.

2. Mathematical model of the steering system

To investigate quantitatively the power steering system an accurate enough mathematical model of its dynamic response is necessary. The two main approaches to modelling of electrohydraulic steering systems are the analytic analysis and the identification from experimental data [5]. This article uses a black-box state-space discrete-time model obtained with identification procedure [6] presented in details in a previous authors’ article in press. Here only the resultant model is summarized which has the following form

\[
\begin{align*}
    x(k+1) &= Fx(k) + Gu(k) + K_v e(k) \\
    y(k) &= Cx(k) + Du(k) + e(k),
\end{align*}
\]  

where \( x(k) \) is a state vector with components \( x_1(k), x_2(k) \) and \( x_3(k) \) at a time instant \( t = kT_s \) of the system temporal evolution (\( T_s = 25ms \) is the sampling time). The state variables represent linear combinations of unknown but fixed internal system variables [7]. The \( u(k) \) is the manipulated variable which in our case is the duty cycle applied to the solenoids of the switching valve bridge. The signal \( u(k) \) is in the range \( \pm 4V \), with the positive values activate the right upper and lower valves and the negative values active the left upper and lower valves. The output signals (spool position \( y_{spool} \), flow rate \( y_{flow} \) and piston position \( y_{piston} \)) are collected into the vector \( y(k) \). The matrices \( F, G, C, D \) and \( K_v \) determine the structure and parameterization of the linear dynamic model. Their values are estimated from experimentally collected data when a random binary input signal is generated. Their values are

\[
F = \begin{pmatrix} -1.05 & 0.19 & 6.27 \\ -0.04 & 1 & 0.198 \\ -0.371 & 0.033 & 2.044 \end{pmatrix}, \quad G = 10^{-3} \begin{pmatrix} -0.37 \\ -0.034 \\ -0.23 \end{pmatrix}, \quad C = \begin{pmatrix} -31 & 0.67 & -6.58 \\ -0.584 & 0.007 & -0.25 \\ 0.059 & -2.019 & 0.125 \end{pmatrix},
\]

\[
D = 10^{-3} \begin{pmatrix} 0.117 \\ 0.014 \\ 0.0021 \end{pmatrix}, \quad K_v = \begin{pmatrix} -0.025 & -0.023 & -0.022 \\ -0.002 & 0.0066 & -0.429 \\ -0.007 & -0.01 & 0.0033 \end{pmatrix}.
\]

The signal \( e(k) \) is a vector with three components which represent the residual in the model calculated as a difference between the experimentally recorded output signal and the predicted from the model. It is described as a white noise random signal with a zero mean and covariance
Parameters of the matrices are tuned such that the residual covariance to be minimized meaning that the model is approximating the experimental data from the plant. As can be seen from the covariance matrix the model is most accurate on its third output (the piston position), followed by the flow rate and spool position. The mathematical model can be represented also as a transfer functions in frequency domain

\[ W_{\text{spool}}(z) = \frac{y_1(z)}{u(z)} = \frac{0.013z^2 + 0.01z - 0.022}{z^2 - 2z + 1.19z - 0.186}, \quad (4) \]
\[ W_{\text{flow}}(z) = \frac{y_2(z)}{u(z)} = 10^{-4} \frac{2.4z^2 + 1.7z - 4.3}{z^2 - 2z + 1.19z - 0.186}; \quad (5) \]
\[ W_{\text{piston}}(z) = \frac{y_3(z)}{u(z)} = 10^{-5} \frac{1.4z^2 + 5.1}{z^2 - 2z + 1.19z - 0.186}, \quad (6) \]

where the operator \( z = e^{j\omega} \) with \( \omega \) the frequency in rad/s and \( z^{-1} \) corresponding to an unit delay in the temporal domain. It can be noted that the denominators of all the transfer functions are the same. The difference is in the zeros and the static gain.

The step response of the model is presented in the figure 3. The response of the spool position is aperiodic since a mechanical spring counteracts the pressure generated form the valve bridge along the spool. The flow rate response is almost immediate with the spool translation but requires a little more time to settle. It is aperiodic too since the pump works with a constant pressure. And the third output is the steering piston stoke which is linearly increasing as expected. The figure contains also the upper and the lower bound for each signal calculated from the residual covariance. These bounds represent the uncertainty of the model and for the presented system they are small enough. However, this uncertainty indicates that a closed-loop control is required in order to achieve the desired reference tracking.

![Figure 3. Output signals when unit signal is applied to the input. \( y_1 \) is the proportional spool position, \( y_2 \) is the flow rate and \( y_3 \) is the steering cylinder position.](image-url)
3. Design of the control system

The presented mathematical model is used to tune the proposed control structure which is based on a three-loop cascade of PI regulator for spool position, P regulator for the flow rate and again PI regulator for the piston position. It is summarized with the diagram from figure 4. The spool controller is selected as PI since its response should be as fast as possible. The equation for the spool PI regulator in discrete-time domain

\[ u(k) = K_{sp,p} \left( r_{sp}(k) - y_{spool}(k) \right) + K_{sp,i} x_{sp,i}(k) \]

(calculates the control signal \( u(k) \) at time instant \( kT_s \) based on the deviation from the set-point \( r_{sp}(k) \) and its integrated value according to forward Euler approximation

\[ x_{sp,i}(k) = x_{sp,i}(k-1) + T_s \left( r_{sp}(k) - y_{spool}(k) \right) + w_{sp}(k), \]

where \( w_{sp}(k) = K_{w,sp}(u_{sat}(k) - u(k)) \) is the anti-windup term preventing over-integration in \( x_{sp,i}(k) \) which would delay the settling of the spool position transient to its set-point due to saturation \( u_{sat}(k) \) of the control action \( u(k) \) (i.e. wind-up effect). The anti-windup \( K_{w,sp} \) is tuned enough up to prevent the controller from hitting the lower saturation bound immediately after the upper saturation bound is reached for positive value of the set-point. The real saturation bounds of the control signal \( u(k) \) are \( \pm 4V \) but if they are actually reached the switching valve electronics would raise an exception and would stop working, therefore, the practical bounds are between \( \pm 3.5V \). The parameters of the PI regulator which have to be tuned by optimization are its proportional gain \( K_{sp,p} \) and its integral gain \( K_{sp,i} \).

![Figure 4. Three-loop cascade control system for the electrohydraulic power steering.](image)

The spool position controller would guarantee the following of the set-point signal \( r_{sp}(k) \) corresponding to setting the directional spool valve to a desired position which would cause the development of the flow rate \( y_{flow}(k) \) leading to actual translation of the steering cylinder piston. However, the mathematical relationship between the spool valve opening and flow rate is not linear due to geometrical shape of the valve. Also the design of the valve is dependent on the safety requirement for minimal spool valve translation before actual opening of the valve (positive overlap). The identified linear model of the process is focused on the transient time-constants but such nonlinearities [8] cannot be accounted for with linear transfer function. So additional flow controller is designed as a P controller

\[ r_{sp}(k) = K_{flow,p} \left( r_{flow}(k) - y_{flow}(k) \right), \]

where the parameter \( K_{flow,p} \) is optimized up to minimize the error between required flow rate \( r_{flow}(k) \) and actually measured \( y_{flow}(k) \). Such controller leads to “linearization” of the relationship between valve opening and flow which makes the tuning of the piston position regulator easier [9]. The value of the \( K_{flow,p} \) is limited from going to high because the controller also amplifies the measurement noise from the gear flowmeter device. The amplification of noise in detected in the optimization procedure by measuring the standard deviation of the control action and weighting it accordingly. The bounds for the flow rate controller \( r_{sp}(k) \) are determined from the spool valve range of motion which is \( \pm 2.5 \) mm.
The last controller from the chain is the steering piston position controller which calculated the flow rate set-point such that the desired steering angle to be achieved faster

\[ r_{low}(k) = K_{cyl,p} (r(k) - y_{piston}(k)) + K_{cyl,i} x_{cyl,i}(k), \]  

where \( r(k) \) is commanded position of the steering cylinder from the operator or remote control system and \( K_{cyl,p} \) and \( K_{cyl,i} \) are the controller parameters for tuning. Since there are limits for the flow rate set-point \( r_{low}(k) \) which are \( \pm 10 \) l/min the integral term

\[ x_{cyl,i}(k) = x_{cyl,i}(k - 1) + T_s \left( r(k) - y_{piston}(k) \right) + w_{cyl}(k) \]  

also include an anti-windup term.

### 3.1. Spool controller optimization

The main problem with the presented controller structure remains the tuning of the controller parameters which are \( \theta = (K_{sp,p}, K_{sp,i}, K_{f,low}, K_{cyl,p}, K_{cyl,i}) \). First the spool valve controller parameters \( K_{sp,p} \) and \( K_{sp,i} \) are tuned such that cost function value

\[ J_{sp}(K_{sp,p}, K_{sp,i}) = \sum_{k=0}^{100} \left( 1 - y_{spool}(k, K_{sp,p}, K_{sp,i}) \right)^2 \]  

is minimized according to nonlinear-least squares method. The minimization is an iterative process which requires multiple simulations of the model for different values of the optimized parameters \( (K_{sp,p}, K_{sp,i}) \). After each simulation the value of the cost function \( J_{sp} \) is evaluated and based on the gradient its derivative a new guess for parameters is examined which would minimize further \( J_{sp} \). This optimization procedure is presented on figure 5. The results from the optimization are shown in table 1 for initial values of the parameters \( K_{sp,p} = 0.01 \) and \( K_{sp,i} = 1.67 \).

| Iteration | Function evaluations | New cost value | Norm of step |
|-----------|----------------------|----------------|--------------|
| 0         | 3                    | 7.56323        |              |
| 1         | 6                    | 4.8853         | 1.73024      |
| 2         | 9                    | 3.74238        | 2.48575      |
| 3         | 12                   | 3.37197        | 2.27601      |
|           |                      |                |              |
| 8         | 27                   | 3.32255        | 0.00878      |
| 9         | 30                   | 3.32255        | 0.00275      |

The final values of the controller parameters are \( K_{sp,p} = 7.7464 \) and \( K_{sp,i} = 1.2736 \). These values are reached after nine iterations and 30 cost function evaluations. The cost function value is reducing from 7.56 to 3.32 or 56%. The optimization is stopped because the change of the cost function value between two consecutive iterations is below a predefine absolute tolerance. As can be seen the norm of the step is decreasing too which indicate the approaching of the function local minimum. Simulation results of the spool valve controller are presented in figure 6 for the output signal (spool valve translation) and in figure 7 for the corresponding control signal. The figures compare the performance of the initial controller parameters with the controller parameters after the optimization. The optimized controller is faster in reaching the set-point. The overshoot in both controller is comparable but its area is smaller after the optimization.
3.2. Flow rate and piston position controllers’ optimization

The second optimization procedure is executed to tune the parameter $K_{flow}$ of the flow rate controller and the parameters $K_{cyl,p}$ and $K_{cyl,i}$ of the piston position controller (figure 8). These parameters are tuned such that the following cost function

$$J_{sp}(K_{sp,p}, K_{sp,i}) = \sum_{k=0}^{100} \left( 1 - y_{spool}(k, K_{sp,p}, K_{sp,i}) \right)^2 + 0.1u(k)$$

(13)

is minimized according to nonlinear-least squares method. The results from the optimization are shown in table 2 for initial values of the parameters $K_{cyl,p} = 1$, $K_{cyl,i} = 0.5$ and $K_{flow} = 7$. The final values of the controller parameters are $K_{cyl,p} = 15.8$, $K_{cyl,i} = 0.044$ and $K_{flow} = 10.7$. These values are reached after nine iterations and 120 cost function evaluations. The cost function value is reducing from 10.0482 to 9.1321 or 9%. The optimization is stopped because the change of the cost function value between two consecutive iterations is below a predefine absolute tolerance. As can be seen the norm of the step is decreasing too which indicate the approaching of the function local minimum.

Simulation results of the spool valve controller are presented in figure 9 for the output signal and in figure 10 for the corresponding control signal. The figures compare the performance of the initial controller parameters with the controller parameters after the optimization. Both controllers have similar performance but the optimized controller offer less oscillatory control action.
Table 2. Iterations for optimal tuning of flow rate and piston position controller.

| Iteration | Function evaluations | New cost value | Norm of step |
|-----------|----------------------|----------------|--------------|
| 0         | 4                    | 10.0482        |              |
| 1         | 8                    | 10.0122        | 0.2316       |
| 2         | 12                   | 9.9996         | 0.1781       |
| 3         | 16                   | 9.4736         | 0.2515       |
| 28        | 116                  | 9.1326         | 0.1907       |
| 29        | 120                  | 9.1321         | 0.1947       |

Figure 8. Optimal tuning of flow rate and piston position controllers.

Figure 9. Piston position closed-loop step response.

Figure 10. Piston position closed-loop control signals.

4. Experimental results

4.1. Embedded code generation

The designed cascade controller is evaluated on a laboratory system for examination of electrohydraulic systems [10]. It is composed of steering cylinder with 300 mm stroke range of motion, Danfoss OSPE electrohydraulic steering unit and Danfoss PVES switching valve block. The target microcontroller is Danfoss MC012-022 suitable for vehicle automation and diagnostic applications. The microcontroller is programmed in PLUS 1 IDE with CFC and ST languages. The cascade controller is developed as a Simulink block diagram. Then Simulink PLC coder facilitates the automatic code generation of ST function block corresponding to the Simulink diagram of the control algorithm.
4.2. Recorded signals

Presented experimental results compare the performance of the initial controller (tuned manually) and the optimized controller. The generated reference signal drives the steering piston 10 cm to the left for 20 s followed by a command for 10 cm to the right for 20 s. Such signal is kind of extremal and allows transients in the closed-loop control system to be fully investigated and clearly compared. The response of the laboratory steering system is shows on figure 11. It is interesting to note that when moving in a positive direction both controllers show similar performance. But when moving in the opposite direction the optimized controller reaches the reference signal two times faster. The parameters of the optimized controller are increased with respect to the initial controller so faster response is expected. However, when moving in positive direction the laboratory system seem irresponsible to the rising of the parameter values. Figure 12 present the corresponding control actions. The amplitudes of the control signal of the optimized controller are slightly increased.

Figures 13 and 14 compares the flow rate reference tracking for the initial and for the optimized controller. Actually these figures show the performance of the internal flow rate P controller. Both controllers achieve good reference tracking [11]. The measured flow rate signal is disturbed by a sensor noise so the value of $K_{flow}$ should be limited. Figures 15 and 16 compare the spool valve position reference tracking. Both controllers achieve good tracking when the reference is below 0.6mm and present some steady error for higher levels. This effect is related to the reactivity of the passing flow

Figure 11. Experimental step response of the closed-loop system.

Figure 12. Experimental control action of the closed-loop system.

Figure 13. Flow rate – initial controller.

Figure 14. Flow rate – optimized controller.
5. Conclusion
The article presented a PI-P-PI cascade control system for electrohydraulic steering of heavy duty machines. The three controllers are cascaded such that the output of one controller is a reference signal for the next. Several output variables of the plant are utilized in calculation of the effective control action which is the duty cycle of PWM signal [12] driving a switching valve bridge. The first output signal is the spool valve position measured with a linear variable differential transducer (LVDT) sensor. The second output is the consumed flow rate of the system measured with a geared flowmeter. The last output is the piston position of the steering cylinder. After manual initial tuning the performance of the closed loop system is increased by a nonlinear least squares optimization procedure. The comparative analysis of the performance of the system tuned manually and optimized control system is done. Results shows that the parameters of the optimized controller are increased with respect to the initial controller so faster response is expected. This statement is experimentally confirmed not only by the control signal records but also by the cylinder piston tracking records when the system working with the both different controllers.

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