Probing Time-Dependent Fundamental Constants with Nucleosynthesis in Population III Stars

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Abstract: Variations of fundamental physical constants have been sought for many years using various astronomical objects because their discovery can be key to developing beyond-standard physics. In particular, nuclear reaction rates are sensitive to fundamental constants, so nucleosynthetic processes can be used as a probe. We calculate the evolution and nucleosynthesis of massive Population III stars with the time-dependent nucleon–nucleon interaction $\delta_{NN}$, which may have left traces in elemental abundances in extremely metal-poor stars. The results are compared with the abundances in the most iron-poor star that has ever been discovered, namely, SMSS J031300.36-670839.3. It is found that calcium production in Population III stars is very sensitive to variations of the triple-$\alpha$ reaction rate and hence $\delta_{NN}$. We conclude that variations of the nucleon–nucleon interaction are constrained as $-0.002 < \delta_{NN} < 0.002$ at the redshift $z \sim 20$, assuming that calcium in SMSS J031300.36-670839.3 originates from hydrogen burning in a massive Population III star.

Keywords: Population III star; supernova; metal-poor star; nucleosynthesis; triple-$\alpha$ reaction

1. Introduction

Since the large number hypothesis was proposed by Dirac [1], a lot of laboratory experiments and astronomical observations have been performed to detect variations of fundamental constants [2–4]. The discovery of such variations, which are predicted by some beyond-standard theories, would be a breakthrough for new physics. For example, in the superstring theory, coupling constants are written as vacuum expectation values of the dilaton field, so they are regarded as dynamical variables [5].

One of possible tools that can be used to probe the variations of fundamental constants is stellar evolution and nucleosynthesis (e.g., [6–9]). In particular, stars formed from primordial gas, which are called Population III stars, reflect physics in the early Universe at the redshift of $z \sim 20$. It has been reported that the evolution of Population III stars is sensitive to the variation of the nucleon–nucleon (N–N) interaction through the effects on the triple-$\alpha$ reaction [7]. Ref. [7] constrained the variation of the N–N interaction from the composition of a CO core, showing that the production of carbon and/or oxygen becomes extreme with a different N–N interaction.

Although survivors of Population III stars have not been discovered, nucleosynthesis in first stars may have left traces in elemental abundances in an extremely metal-poor star, which may be formed from the gas polluted by ejecta from a Population III supernova.
metal-poor star called SMSS J031300.36-670839.3 (hereafter SMSS J0313-6708) was discovered [10]. Iron lines have not been detected in the spectra of this star, and the upper limit on its metallicity is 

\[ [\text{Fe/H}] < -7.00 \ (1\sigma) \] [11]. This star is the most iron-poor star that has ever been discovered so far, so it is the most promising candidate for a second-generation star.

The origin of SMSS J0313-6708 is, however, still under discussion. Some argue that the star is a survivor of Population III stars that were polluted by the interstellar medium [12]. Others argue that the star is a second-generation star, although the nature of the preceding Population III star is uncertain. It has been reported that a jet-induced explosion of a massive Population III star can reproduce the elemental abundances of SMSS J0313-6708 [13], while a spherical explosion can explain the abundances, too [10,14,15].

One of the differences between a jet-induced and a spherical explosion is the origin of calcium in their ejecta, which is detected in SMSS J0313-6708. In the jet-induced explosion, the observed calcium is produced in hydrostatic and explosive oxygen burning and incomplete silicon burning [13]. However, calcium is also produced by the breakout of the hot-CNO cycle during hydrogen burning because of the high temperature that is characteristic of Population III stars [10,14,15]. The calcium production in hydrogen burning is highly sensitive to temperature, which depends on the triple-\(\alpha\) reaction rate and hence on the N–N interaction. In this study, we assume that the observed calcium mostly originated from hydrogen burning in a massive Population III star.

The aim of this work is to calculate nucleosynthesis in Population III stars and compare it with the chemical composition of the iron-free star to constrain time-dependent fundamental constants in the early Universe. Although the evolution of Population III stars with variations of fundamental constants was followed in a previous work [7], detailed nucleosynthesis has not been calculated.

This paper is organized as follows. Section 2 describes the triple-\(\alpha\) reaction rate with the time-dependent N–N interaction and the stellar models adopted in this study. Section 3 describes the results of the calculations. Section 4 is devoted to the discussion and summary.

2. Methods

2.1. Triple-\(\alpha\) Reaction

The triple-\(\alpha\) reaction is responsible for the carbon production in stars. Carbon is believed to be an essential element for the existence of intellectual observers, so Hoyle predicted an excited state of \(^{12}\text{C}\) with an anthropic argument [16], which was confirmed by experiments at 7.7 MeV [17,18]. The triple-\(\alpha\) reaction rate is sensitive to the resonance energy of the Hoyle state.

The effects of the time-dependent N–N interaction on the triple-\(\alpha\) reaction rate are described in Ref. [7]. They solved the Schrödinger equation with the potential

\[
V(r_{ij}) = V_C(r_{ij}) + (1 + \delta_{NN})V_N(r_{ij}),
\]

where \(r_{ij}\) is the relative distance, and \(V_C\) and \(V_N\) are the potentials that originate from the Coulomb interaction and the nuclear interaction, respectively, using a cluster approximation [19]. Here, \(\delta_{NN}\) represents modifications to the potential. The N–N interaction is described by the Minnesota force [20]. Ref. [7] reports that the solution of the Schrödinger equation shows that the resonance energies can be approximated as

\[
E_R(^{8}\text{Be}) \approx E_{R0}(^{8}\text{Be}) - 12.208\delta_{NN} \ [\text{MeV}], \ E_R(^{12}\text{C}) \approx E_{R0}(^{12}\text{C}) - 20.412\delta_{NN} \ [\text{MeV}],
\]

where \(E_R(^{8}\text{Be})\) is the energy of the ground state of \(^{8}\text{Be}\) with respect to the \(\alpha + \alpha\) threshold, and \(E_R(^{12}\text{C})\) is the energy of the Hoyle state with respect to the \(\alpha + ^{8}\text{Be}\) threshold. The subscripts “0” show the values with \(\delta_{NN} = 0\).
The triple-α reaction rate in low temperatures is calculated as [21,22]

\[ N_A^2 \langle \sigma v \rangle = 3N_A \left( \frac{8\pi \hbar}{2\pi^2} \right) \left( \frac{\mu_{aa}}{2\pi k_B T} \right)^\frac{3}{2} \int_0^\infty \frac{\sigma_{aa}(E)}{\Gamma_a(E)} e^{-\frac{E}{k_B T}} N_A \langle \sigma v \rangle_\alpha^8 Be dE, \tag{3} \]

where \( \mu_{aa} \) is the reduced mass of two \( \alpha \) particles, \( E \) is the energy whose origin is at the \( \alpha + \alpha \) threshold and \( \Gamma_a(E) \) is the width of the \( ^8Be \) decay. The cross-section of the \( \alpha + \alpha \) elastic scattering is given by the Breit–Wigner formula [23],

\[ \sigma_{aa}(E) = \pi \lambda(E)^2 \left( \frac{2\Gamma_a(E)^2}{(E - E_{R}(^8Be))^2 + \frac{\Gamma_a(E)^2}{4}} \right), \tag{4} \]

where \( \lambda \) is the de Brügge wavelength.

The rate \( \langle \sigma v \rangle_\alpha^8Be \) is written as

\[ \langle \sigma v \rangle_\alpha^8Be = N_A \left( \frac{8\pi}{2\pi^2} \right) \left( \frac{\mu_{a8Be}}{2\pi k_B T} \right)^\frac{3}{2} \int_0^\infty \sigma_{a8Be}(E', E)e^{-\frac{E'}{k_B T}} dE', \tag{5} \]

where \( \mu_{a8Be} \) is the reduced mass of \( \alpha + ^8Be \) particles, and \( E' \) is the energy whose origin is at the \( \alpha + ^8Be \) threshold. The cross-section is given by

\[ \sigma_{a8Be}(E', E) = \pi \lambda(E')^2 \frac{\Gamma_a(E')\Gamma_E(E' + E)}{(E' + E - E_R(^{12}C) - E_R(^8Be))^2 + \frac{\Gamma_a(E')^2}{4}}, \tag{6} \]

where \( \Gamma_\gamma(E' + E) \) and \( \Gamma_a(E') \) are the partial widths of the \( \alpha \)- and \( \gamma \)-channels, respectively. \( \Gamma = \Gamma_\gamma + \Gamma_a \) is the total width.

The particle widths depend on the energy as

\[ \Gamma_a(E) = \Gamma_a(E_R) \frac{P_0(E)}{P_0(E_R)}, \tag{7} \]

where \( P_0(E) \) is the Coulomb penetration factor for the angular momentum \( l = 0 \). The \( \gamma \)-decay width \( \Gamma_\gamma(E) \) for the E2 transition is proportional to \( E^5 \).

Table 1 shows the resonance parameters we adopt, which are compiled in recent literature [24,25]. We perform the integration in Equation (3) numerically to calculate the triple-\( \alpha \) reaction rate. Figure 1 shows the reaction rates with various \( \delta_{NN} \). It is seen that the reaction rate increases as a function of \( \delta_{NN} \) because \( E_R \) decreases with larger \( \delta_{NN} \).

![Figure 1](image_url)  
**Figure 1.** The triple-\( \alpha \) reaction rates as a function of the temperature with different \( \delta_{NN} \). They are normalized by the standard reaction rate with \( \delta_{NN} = 0 \).
Table 1. Resonance parameters adapted in this study [24,25].

| Nucleus | \(J^\pi\) | Resonance Energy | \(\alpha\)-width | \(\gamma\)-width |
|---------|-----------|-----------------|-----------------|-----------------|
| \(^{8}\text{Be}\) | \(0^+\) | \(E_{\text{R}}(^{8}\text{Be}) = 91.84\) keV | 5.57 eV | |
| \(^{12}\text{C}\) | \(0^+\) | \(E_{\text{R}}(^{12}\text{C}) = 287.47\) keV | 9.3 eV | 3.81 meV |

2.2. Stellar Models

The stellar models adopted in this study are described in detail in Ref. [14]. We summarize the setup of the models in this section.

We use a one-dimensional stellar evolution code, Modules for Experiments in Stellar Astrophysics (MESA; [26–30]) version 10398. The stellar masses are 40, 80 and \(120\,M_\odot\), and the initial composition is from a recent Big Bang nucleosynthesis (BBN) calculation [31]. Mass loss is ignored because of the absence of atomic lines of heavy elements and dusts that drive mass loss. The evolution is followed until the end of central helium burning. The nuclear reaction network includes 306 isotopes up to gallium. The parameter for the mixing length theory is set to \(\alpha_{\text{MLT}} = 1.68\) [32], and the overshooting parameter is \(f_{\text{ov}} = 0.004\).

3. Results

3.1. Evolution and Nucleosynthesis of Population III Stars with \(\delta_{\text{NN}}\)

Massive Population III stars with \(> 20\,M_\odot\) are supported by the CNO cycle during the main sequence because the CNO cycle is more sensitive to temperature than the pp-chain [33–37]. CNO catalysts, which are needed to ignite the CNO cycle, are not contained in the initial composition, but they are produced by the triple-\(\alpha\) reaction on-site. The central temperature reaches \(\sim 10^8\) K to produce \(^{12}\text{C}\) before the CNO cycle starts.

Figure 2 is the Hertzsprung–Russell diagram, which follows evolution until the end of central helium burning. The solid lines show the fiducial models with \(\delta_{\text{NN}} = 0\), and the other lines show the models with variations of \(\delta_{\text{NN}}\). It is seen that the initial contraction stops earlier in models with larger \(\delta_{\text{NN}}\) because larger triple-\(\alpha\) reaction rates create \(^{12}\text{C}\), which is necessary to start the CNO cycle with lower temperatures. The changes in temperature with different \(\delta_{\text{NN}}\) can also be seen in Figure 3 (left panel), which shows the central temperature in the \(40\,M_\odot\) models during hydrogen burning as a function of the central proton fraction. The black line shows the fiducial result with \(\delta_{\text{NN}} = 0\), and the others show the results with different \(\delta_{\text{NN}}\) in increments of 0.001. The central temperature becomes smaller by \(\sim 0.025\) dex when \(\delta_{\text{NN}}\) becomes smaller by \(\sim 0.001\).

The high temperature \(\sim 10^8\) K achieved during hydrogen burning causes the breakout of the hot-CNO cycle. The nucleosynthetic flow proceeds along proton-rich paths and produces \(^{40}\text{Ca}\). As studied in detail in Ref. [14], calcium production is sensitive to the temperature. Figure 3 (right panel) shows the \(^{40}\text{Ca}\) abundances during hydrogen burning in the \(40\,M_\odot\) models. The black line shows the fiducial result with \(\delta_{\text{NN}} = 0\), where the \(^{40}\text{Ca}\) mass fraction reaches \(X(^{40}\text{Ca}) \sim 3 \times 10^{-10}\). It is seen that the calcium abundance is highly sensitive to the temperature. Especially in the case of \(\delta_{\text{NN}} = 0.004\), \(X(^{40}\text{Ca})\) is \(\sim 8\) orders smaller than that in the case of \(\delta_{\text{NN}} = 0\).

The variations in \(\delta_{\text{NN}}\) affect the composition of the CO core, too. The C/O ratio in the core is dependent on the rates of the triple-\(\alpha\) reaction and \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\). Figure 4 shows the mass fractions of carbon and oxygen in the CO core as a function of \(\delta_{\text{NN}}\). The solid lines show the carbon abundances, and the broken lines show the oxygen abundances. The carbon abundances increase as a function of \(\delta_{\text{NN}}\), while the oxygen abundance decreases when \(\delta_{\text{NN}} > 0\). This is because the triple-\(\alpha\) reaction becomes faster with larger \(\delta_{\text{NN}}\), and the \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reaction becomes relatively minor. It is noted that \(X(\text{C}) + X(\text{O}) < 1\) when \(\delta_{\text{NN}} < 0\). In these cases, \(^{20}\text{Ne}\) and \(^{24}\text{Mg}\) are produced in a significant amount through \(^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}\) and \(^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}\). These reaction rates are enhanced because of the
higher temperatures achieved by the lower triple-α reaction rates. This result is qualitatively consistent with previous studies [7,38,39].

**Figure 2.** The Hertzsprung–Russell diagram. The solid lines show the fiducial models with \( \delta_{NN} = 0 \). The broken lines adopt \( \delta_{NN} = -0.002 \) (red), \( \delta_{NN} = 0.002 \) (purple) and \( \delta_{NN} = 0.004 \) (green).

**Figure 3.** Evolution of the central temperature (upper) and the \( ^{40} \text{Ca} \) mass fraction (lower) during hydrogen burning as a function of the \( ^{1} \text{H} \) mass fraction with different \( \delta_{NN} \) in increments of 0.001.
Mass Fraction
\( \delta_{\text{NN}} \)
C(40M\(_{\odot}\))
C(80M\(_{\odot}\))
C(120M\(_{\odot}\))
O(40M\(_{\odot}\))
O(80M\(_{\odot}\))
O(120M\(_{\odot}\))
Figure 4. The mass fractions of carbon and oxygen in the CO core as a function of \( \delta_{\text{NN}} \). The solid lines show the carbon abundances, and the broken lines show the oxygen abundances.

3.2. Compositions of the Ejecta and Chemical Abundances in the Most Iron-Poor Star

Nucleosynthesis in Population III stars may have left traces in chemical abundances in extremely metal-poor stars. In this section, we compare the calcium production in our models and the observed abundance in SMSS J0313-6708, which is the most metal-poor star ever discovered [10,11].

Massive Population III stars with \( \gtrsim 20M_{\odot} \) are believed to form a black hole as a remnant after a supernova explosion. Because of the strong gravitation, a large amount of mass will fall back on the black hole, although its dynamics is still uncertain. In order to calculate the chemical composition of the ejecta into the interstellar medium, we adopt a parameter called a mass cut \( M_{\text{cut}} \). The mass of an element in the ejecta is written as

\[
M_{\text{element}} = \int_{M_{\text{cut}}}^{M} X(M_r) dM_r,
\]

where \( M \) is the stellar mass, \( M_r \) is the mass coordinate and \( X(M_r) \) is the mass fraction of the element at \( M_r \).

The integral in Equation (8) is evaluated to acquire the elemental abundances. Figure 5 shows \([\text{Ca}/C]\) in the ejecta as a function of \( M_{\text{cut}} \). The gray band is the observed value \([\text{Ca}/C] = -4.39\) in SMSS J0313-6708 [11]. The black lines show the fiducial models with \( \delta_{\text{NN}} = 0 \). It is seen that \([\text{Ca}/C]\)’s in \( M_{\text{cut}} > M_{\text{CO}} \) are larger than those in \( M_{\text{cut}} < M_{\text{CO}} \), where the CO core masses are \( M_{\text{CO}} \sim 12M_{\odot}, 31M_{\odot}, \) and \( 52M_{\odot} \) for the 40\(M_{\odot}, 80M_{\odot}, \) and \( 120M_{\odot} \) models, respectively. This reflects the fact that carbon is much richer in the CO core than in the H/He envelope. As reported in Ref. [14], in the fiducial models, \([\text{Ca}/C]\) is larger than \( -4.39 \) when \( M_{\text{cut}} > M_{\text{CO}} \) and smaller than \( -4.39 \) when \( M_{\text{cut}} < M_{\text{CO}} \). This implies that the mass cut should be in the H/He envelope to avoid overproducing carbon, and mixing during the supernova explosion should lower \([\text{Ca}/C]\) to the observed value.

Following this argument, we can see that the models with \( \delta_{\text{NN}} = -0.002 \) and 0.004 cannot reproduce the observation. In the case of \( \delta_{\text{NN}} = -0.002 \), which is shown in the blue lines in Figure 5, \([\text{Ca}/C]\) is higher than the observation regardless of the stellar mass and the mass cut. This overproduction of calcium originates from the lower rates of the triple-\(\alpha\) reaction, which lead to the higher temperature that enhances calcium production. On the other hand, in the case of \( \delta_{\text{NN}} = 0.004 \), which is shown in the orange lines, calcium is always underproduced. Therefore, it is concluded that the change in the N–N interaction is constrained to

\[
-0.002 < \delta_{\text{NN}} < 0.004.
\]
Figure 5. The [Ca/C] ratio in the ejecta for 40\(M_\odot\), 80\(M_\odot\) and 120\(M_\odot\) models as a function of the mass cut. The gray band is the observed abundance in SMSS J0313-6708.

The calcium production in our models is summarized in Figure 6. The upper branch is [Ca/C] with the mass cut in the envelope, and the lower branch is [Ca/C] with the mass cut in the CO core. Because of mixing during a supernova explosion, [Ca/C] in the ejecta should be between these branches. The red band shows the observed value of [Ca/C]. One can see that the models with \(\delta_{\text{NN}} = -0.002\) and 0.004 are inconsistent with the observation. The values of [Ca/C] with \(M_{\text{cut}} < M_{\text{CO}}\) do not decrease monotonously as a function of \(\delta_{\text{NN}}\), though the calcium production during hydrogen burning decreases. This is because neutron-rich isotopes of calcium are significantly produced during helium burning through neutron capture reactions.
The upper branch shows the results with the mass cut set in the envelope, and the lower branch shows the results with the mass cut set in the CO core. The red band is the observed abundance in SMSS J0313-6708.

Ejecta from the supernova explosion is diluted by the hydrogen-rich interstellar medium before forming next-generation stars, so [Ca/H] in the envelope of the Population III star models should be smaller than [Ca/H] observed in metal-poor stars. Figure 7 shows [Ca/H] in the models as a function of \( \delta_{\text{NN}} \). The green horizontal line shows the observed value. It is seen that [Ca/H] decreases as a function of \( \delta_{\text{NN}} \), and the theoretical values are larger than the observed values only when \( \delta_{\text{NN}} < 0.02 \). This implies that the Ca abundance in SMSS J0313-6708 cannot be explained if \( \delta_{\text{NN}} > 0.02 \). Combining this result and Equation (9), we get

\[
-0.002 < \delta_{\text{NN}} < 0.002
\]  

as a constraint on \( \delta_{\text{NN}} \) in the early Universe, assuming that calcium in SMSS J0313-6708 originates from the breakout of the hot-CNO cycle in massive Population III stars.

4. Discussion and Conclusions

In this study, we find that calcium production during hydrogen burning in massive Population III stars is highly dependent on variations of the N–N interaction. In order for the calcium production in our models to be consistent with the chemical abundance in the most metal-poor star SMSS J0313-6708,
the variations of the N–N interaction are constrained to $-0.002 < \delta_{NN} < 0.002$, although the origin of calcium in the star is still an open question.

We can constrain the time-dependence of fundamental physical constants from the constraint on $\delta_{NN}$. Ref. [7] solved the Schrödinger equation associated with the potential of Equation (1) to relate $\delta_{NN}$ to the binding energy of deuteron $B_D$. Their result was $\Delta B_D/B_D = 5.716 \delta_{NN}$. On the other hand, Ref. [40] calculated the current quark mass dependence of $B_D$ from the Dyson–Schwinger equation with the AV18 [41] and UIX [42] interactions. They showed that $\Delta B_D/B_D = -1.39 \Delta m_q/m_q$, where $m_q$ is the quark mass. Using these relations, we get the constraint

$$ \frac{\Delta m_q}{m_q} < 8 \times 10^{-3}. \tag{11} $$

The quark mass can be constrained from BBN, too [43–46]. Ref. [44] concluded that $-5 \times 10^{-3} < \Delta m_q/m_q < 7 \times 10^{-3}$ by comparing between BBN models and observations of primordial abundances. Equation (11) gives a weaker but independent constraint on $\Delta m_q$.

The constraint shown in Equation (9) can also be used to constrain variations of the fine structure constant $\alpha_{EM}$. Ref. [47] deduces a relation $\Delta B_D/B_D = 18 R \Delta \alpha_{EM}/\alpha_{EM}$, where $R$ is a parameter that depends on grand unification models. Adopting $R = 36$, which is the simplest model predicts, we get the constraint

$$ \frac{\Delta \alpha_{EM}}{\alpha_{EM}} < 2 \times 10^{-5}. \tag{12} $$

Recent observations of zinc and chromium quasar absorption lines with the Keck and Very Large Telescopes constrain $\Delta \alpha_{EM}$ as $\Delta \alpha_{EM}/\alpha_{EM} = (0.4 \pm 1.4_{\text{stat}} \pm 0.4_{\text{sys}}) \times 10^{-6}$ [48]. The redshifts of the quasars are $z = 1.0–2.4$, so the constraints from Population III stars at $z \sim 20$ and quasar absorption lines are complementary to each other, although the constraint shown in Equation (12) is $\sim 5$ times weaker.

In this study, we only take into account the triple-$\alpha$ reaction, but variations of the N–N interaction may affect $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ and $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$, which determine the C/O ratio in the core. Because it is a non-resonant reaction, the $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$ reaction is not expected to be affected by variations of the N–N interaction. The $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction may be more sensitive because of the subthreshold states of $^{16}\text{O}$. It is expected that smaller $\delta_{NN}$ leads to larger $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rates and hence a larger [Ca/C] ratio in the CO core [49]. This implies that the constraint on $\delta_{NN}$ becomes tighter if the effect of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction is taken into account.

Stellar models have uncertain parameters. In the case of non-rotating models, the largest uncertainties originate from mass loss and convective parameters, but the mass loss is negligible in Population III stars. The convective parameters change the core mass and the distribution of elements, but the elemental abundances during hydrogen burning would be changed only by factors. Variations of the N–N interaction discussed in this work change the abundances by many orders of magnitude, so our conclusion (Equation (10)) would not be affected qualitatively. The effects of rotation and the magnetic field are not clear, so further investigations are desirable.

In order to calculate the triple-$\alpha$ reaction rates, we adopt the Minnesota force [20], which is a phenomenological two-body potential. However, the real nuclear potential includes other components (e.g., the tensor force and the $L \cdot S$ force). More realistic potentials may affect the results, so it is desirable for the reaction rates to be calculated using modern theories, including the chiral effective field theory [50]. This is beyond the scope of this study.

To date, SMSS J0313-6708 is the only known star with $[\text{Fe/H}] < -7.0$, but next-generation telescopes (e.g., Thirty Meter Telescope [51]) are planned and expected to discover more samples of such mega metal-poor stars. Chemical abundances of the stars will enable us to perform statistical studies, which will unveil the origin of mega metal-poor stars and possibilities of beyond-standard physics in the early Universe.
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Abbreviations
The following abbreviations are used in this manuscript:

MESA Modules for Experiments in Stellar Astrophysics
N–N nucleon–nucleon
BBN Big Bang nucleosynthesis

References
1. Dirac, P.A.M. The Cosmological Constants. Nature 1937, 139, 323, doi:10.1038/139323a0.
2. Martins, C.J.A.P. The status of varying constants: A review of the physics, searches and implications. Rep. Prog. Phys. 2017, 80, 126902, doi:10.1088/1361-6633/aa860e.
3. Uzan, J.P. Varying Constants, Gravitation and Cosmology. Living Rev. Relativ. 2011, 14, 2, doi:10.12942/lrr-2011-2.
4. Chiba, T. The Constancy of the Constants of Nature: Updates. Prog. Theor. Phys. 2011, 126, 993–1019, doi:10.1143/PTP.126.993.
5. Damour, T.; Polyakov, A.M. The string dilation and a least coupling principle. Nucl. Phys. B 1994, 423, 532–558, doi:10.1016/0550-3213(94)90143-0.
6. Huang, L.; Adams, F.C.; Grohs, E. Sensitivity of carbon and oxygen yields to the triple-alpha resonance in massive stars. Astropart. Phys. 2019, 105, 13–24, doi:10.1016/j.astropartphys.2018.09.005.
7. Ekström, S.; Coc, A.; Descouvemont, P.; Meynet, G.; Olive, K.A.; Uzan, J.P.; Vangioni, E. Effects of the variation of fundamental constants on Population III stellar evolution. Astron. Astrophys. 2010, 514, A62, doi:10.1051/0004-6361/200913684.
8. degl’Innocenti, S.; Fiorentini, G.; Raffelt, G.G.; Ricci, B.; Weiss, A. Time-variation of Newton’s constant and the age of globular clusters. Astron. Astrophys. 1996, 312, 345–352.
9. Demarque, P.; Krauss, L.M.; Guenther, D.B.; Nydam, D. The Sun as a Probe of Varying G. Astrophys. J. 1994, 437, 870, doi:10.1086/175048.
10. Keller, S.C.; Bessell, M.S.; Frebel, A.; Casey, A.R.; Asplund, M.; Jacobson, H.R.; Lind, K.; Norris, J.E.; Yong, D.; Heger, A.; et al. A single low-energy, iron-poor supernova as the source of metals in the star SMSS J031300.36-670839.3. Nature 2014, 506, 463–466, doi:10.1038/nature12990.
11. Nordlander, T.; Amarsi, A.M.; Lind, K.; Asplund, M.; Barklem, P.S.; Casey, A.R.; Collet, R.; Leenaarts, J. 3D NLTE analysis of the most iron-deficient star, SMSS0313-6708. Astron. Astrophys. 2017, 597, A6, doi:10.1051/0004-6361/201629202.
12. Komiya, Y.; Suda, T.; Fujimoto, M.Y. The Most Iron-deficient Stars as the Polluted Population III Stars. Astrophys. J. 2015, 808, L47, doi:10.1088/2041-8205/808/2/L47.
13. Ishigaki, M.N.; Tominaga, N.; Kobayashi, C.; Nomoto, K. Faint Population III Supernovae as the Origin of the Most Iron-poor Stars. Astrophys. J. 2014, 792, L32, doi:10.1088/2041-8205/792/2/L32.
14. Mori, K.; Nomoto, K.; Ishigaki, M.N. Calcium Production in Population III Stars and Constraints on the Origin of the Most Metal-poor Stars. 2019, in preparation.
15. Bessell, M.S.; Collet, R.; Keller, S.C.; Frebel, A.; Heger, A.; Casey, A.R.; Masseron, T.; Asplund, M.; Jacobson, H.R.; Lind, K.; et al. Nucleosynthesis in a Primordial Supernova: Carbon and Oxygen Abundances in SMSS J031300.36-670839.3. Astrophys. J. 2015, 806, L16, doi:10.1088/2041-8205/806/1/L16.
16. Hoyle, F. On Nuclear Reactions Occurring in Very Hot STARS.I. the Synthesis of Elements from Carbon to Nickel. Astrophys. J. Suppl. Ser. 1954, 1, 121, doi:10.1086/190005.
17. Cook, C.W.; Fowler, W.A.; Lauritsen, C.C.; Lauritsen, T. B12, C12, and the Red Giants. Phys. Rev. 1957, 107, 508–515, doi:10.1103/PhysRev.107.508.
18. Dunbar, D.N.F.; Pixley, R.E.; Wenzel, W.A.; Whaling, W. The 7.68-Mev State in C\textsuperscript{12}. Phys. Rev. 1953, 92, 649–650, doi:10.1103/PhysRev.92.649.

19. Korennoy, S.; Descouvemont, P. A microscopic three-cluster model in the hyperspherical formalism. Nucl. Phys. A 2004, 740, 249–267, doi:10.1016/j.nuclphysa.2004.05.013.

20. Thompson, D.; Lemere, M.; Tang, Y. Systematic investigation of scattering problems with the resonating-group method. Nucl. Phys. A 1977, 286, 53–66, doi:10.1016/0375-9474(77)90007-0.

21. Nomoto, K.; Thielemann, F.K.; Miyaji, S. The triple alpha reaction at low temperatures in accreting white dwarfs and neutron stars. Astron. Astrophys. 1985, 149, 239–245.

22. Angulo, C.; Arnould, M.; Rayet, M.; Descouvemont, P.; Baye, D.; Leclercq-Willain, C.; Coc, A.; Barhoumi, S.; Aguer, P.; Rolfs, C.; et al. A compilation of charged-particle induced thermonuclear reaction rates. Nucl. Phys. A 1999, 656, 3–183, doi:10.1016/S0375-9474(99)00030-5.

23. Iliadis, C. Nuclear Physics of Stars; John Wiley & Sons: Hoboken, NJ, USA, 2007; doi:10.1002/9783527692668.

24. Kelley, J.; Purcell, J.; Sheu, C. Energy levels of light nuclei A = 12. Nucl. Phys. A 2017, 968, 71–253, doi:10.1016/j.nuclphysa.2017.07.015.

25. Tilley, D.; Kelley, J.; Godwin, J.; Millener, D.; Purcell, J.; Sheu, C.; Weller, H. Energy levels of light nuclei A = 8, 9, 10. Nucl. Phys. A 2004, 745, 155–362, doi:10.1016/j.nuclphysa.2004.09.059.

26. Paxton, B.; Smolec, R.; Schwab, J.; Gautschy, A.; Bildsten, L.; Cantiello, M.; Dotter, A.; Farmer, R.; Goldberg, J.A.; Jermn, A.S.; et al. Modules for Experiments in Stellar Astrophysics (MESA): Pulsating Variable Stars, Rotation, Convective Boundaries, and Energy Conservation. Astrophys. J. Suppl. Ser. 2019, 243, 10, doi:10.3847/1538-4365/ab2241.

27. Paxton, B.; Schwab, J.; Bauer, E.B.; Bildsten, L.; Blinnnikov, S.; Duffell, P.; Farmer, R.; Goldberg, J.A.; Marchant, P.; Sorokina, E.; et al. Modules for Experiments in Stellar Astrophysics (MESA): Convective Boundaries, Element Diffusion, and Massive Star Explosions. Astrophys. J. Suppl. Ser. 2018, 234, 34, doi:10.3847/1538-4365/aaa5a8.

28. Paxton, B.; Marchant, P.; Schwab, J.; Bauer, E.B.; Bildsten, L.; Cantiello, M.; Dessart, L.; Farmer, R.; Hu, H.; Langer, N.; et al. Modules for Experiments in Stellar Astrophysics (MESA): Binaries, Pulsations, and Explosions. Astrophys. J. Suppl. Ser. 2015, 220, 15, doi:10.1088/0067-0049/220/1/15.

29. Paxton, B.; Cantiello, M.; Arras, P.; Bildsten, L.; Brown, E.F.; Dotter, A.; Mankovich, C.; Montgomery, M.H.; Stello, D.; Timmes, F.X.; et al. Modules for Experiments in Stellar Astrophysics (MESA): Planets, Oscillations, Rotation, and Massive Stars. Astrophys. J. Suppl. Ser. 2013, 208, 4, doi:10.1088/0067-0049/208/1/4.

30. Paxton, B.; Bildsten, L.; Dotter, A.; Herwig, F.; Lesaffre, P.; Timmes, F. Modules for Experiments in Stellar Astrophysics (MESA). Astrophys. J. Suppl. Ser. 2011, 192, 3, doi:10.1088/0067-0049/192/1/3.

31. Pitrou, C.; Coc, A.; Uzan, J.P.; Vangioni, E. Precision big bang nucleosynthesis with improved Helium-4 predictions. Phys. Rep. 2018, 754, 1–66, doi:10.1016/j.physrep.2018.04.005.

32. Girardi, L.; Bressan, A.; Bertelli, G.; Chiosi, C. Evolutionary tracks and isochrones for low- and intermediate-mass stars: From 0.15 to 7 M\textsubscript{sun} and from Z = 0.0004 to 0.03. Astron. Astrophys. Suppl. Ser. 2000, 141, 371–383, doi:10.1051/0004-6361:2000126.

33. Heger, A.; Woosley, S.E. Nucleosynthesis and Evolution of Massive Metal-free Stars. Astrophys. J. 2010, 724, 341–373, doi:10.1088/0004-637X/724/1/341.

34. Ohkubo, T.; Nomoto, K.; Umeda, H.; Yoshida, N.; Tsuruta, S. Evolution of Very Massive Population III Stars with Mass Accretion from Pre-main Sequence to Collapse. Astrophys. J. 2009, 706, 1184–1193, doi:10.1088/0004-637X/706/2/1184.

35. Heger, A.; Woosley, S.E. The Nucleosynthetic Signature of Population III. Astrophys. J. 2002, 567, 532–543, doi:10.1086/338487.

36. Marigo, P.; Girardi, L.; Chiosi, C.; Wood, P.R. Zero-metallicity stars. I. Evolution at constant mass. Astron. Astrophys. 2001, 371, 152–173, doi:10.1051/0004-6361:20010309.

37. Cassisi, S.; Castellani, V. An Evolutionary Scenario for Primeval Stellar Populations. Astrophys. J. Suppl. Ser. 1993, 88, 509, doi:10.1086/191831.

38. Tur, C.; Heger, A.; Austin, S.M. On the Sensitivity of Massive Star Nucleosynthesis and Evolution to Solar Abundances and to Uncertainties in Helium-Burning Reaction Rates. Astrophys. J. 2007, 671, 821–827, doi:10.1086/523095.

39. Schlattl, H.; Heger, A.; Oberhummer, H.; Rauscher, T.; Csótó, A. Sensitivity of the C and O production on the 3\alpha rate. Astrophys. Space Sci. 2004, 291, 27–56, doi:10.1023/B:ASTR.0000029953.05806.47.
40. Flambaum, V.V.; Wiringa, R.B. Dependence of nuclear binding on hadronic mass variation. *Phys. Rev. C* **2007**, *76*, 054002, doi:10.1103/PhysRevC.76.054002.

41. Wiringa, R.B.; Stoks, V.G.J.; Schiavilla, R. Accurate nucleon-nucleon potential with charge-independence breaking. *Phys. Rev. C* **1995**, *51*, 38–51, doi:10.1103/PhysRevC.51.38.

42. Pudliner, B.S.; Pandharipande, V.R.; Carlson, J.; Wiringa, R.B. Quantum Monte Carlo Calculations of $A \leq 6$ Nuclei. *Phys. Rev. Lett.* **1995**, *74*, 4396–4399, doi:10.1103/PhysRevLett.74.4396.

43. Mori, K.; Kusakabe, M. Roles of $^7\text{Be}(n, p)^7\text{Li}$ resonances in big bang nucleosynthesis with time-dependent quark mass and a possible Li reduction. *Phys. Rev. D* **2019**, *99*, 083013, doi:10.1103/PhysRevD.99.083013.

44. Cheoun, M.K.; Kajino, T.; Kusakabe, M.; Mathews, G.J. Time-dependent quark masses and big bang nucleosynthesis revisited. *Phys. Rev. D* **2011**, *84*, 043001, doi:10.1103/PhysRevD.84.043001.

45. Bedaque, P.F.; Luu, T.; Platter, L. Quark mass variation constraints from Big Bang nucleosynthesis. *Phys. Rev. C* **2011**, *83*, 045803, doi:10.1103/PhysRevC.83.045803.

46. Berengut, J.; Flambaum, V.; Dmitriev, V. Effect of quark mass variation on big bang nucleosynthesis. *Phys. Lett. B* **2010**, *683*, 114–118, doi:10.1016/j.physletb.2009.12.002.

47. Coc, A.; Nunes, N.J.; Olive, K.A.; Uzan, J.P.; Vangioni, E. Coupled variations of fundamental couplings and primordial nucleosynthesis. *Phys. Rev. D* **2007**, *76*, 023511, doi:10.1103/PhysRevD.76.023511.

48. Murphy, M.T.; Malec, A.L.; Prochaska, J.X. Precise limits on cosmological variability of the fine-structure constant with zinc and chromium quasar absorption lines. *Mon. Not. R. Astron. Soc.* **2016**, *461*, 2461–2479, doi:10.1093/mnras/stw1482.

49. Oberhummer, H.; Csótó, A.; Schlattl, H. Stellar Production Rates of Carbon and Its Abundance in the Universe. *Science* **2000**, *289*, 88–90, doi:10.1126/science.289.5476.88.

50. Machleidt, R.; Entem, D. Chiral effective field theory and nuclear forces. *Phys. Rep.* **2011**, *503*, 1–75, doi:10.1016/j.physrep.2011.02.001.

51. Skidmore, W.; TMT International Science Development Teams & TMT Science Advisory Committee. Thirty Meter Telescope Detailed Science Case: 2015. *Res. Astron. Astrophys.* **2015**, *15*, 1945, doi:10.1088/1674-4527/15/12/001.

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