Odd-frequency pairing and Josephson effect at superconducting interfaces

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We demonstrate that, quite generally, the spin-singlet even-parity (spin-triplet odd-parity) pair potential in a superconductor induces the odd-frequency pairing component with spin-singlet odd-parity (spin-triplet even-parity) near interfaces. The magnitude of the induced odd-frequency component is enhanced in the presence of the midgap Andreev resonant state due to the sign change of the anisotropic pair potential at the interface. The Josephson effect should therefore occur between odd- and even-frequency superconductors, contrary to the standard wisdom. A method to probe the odd-frequency superconductors is proposed.

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It is well known that Josephson coupling occurs between superconductors with the same order parameter symmetry. Generally, symmetries with respect to momentum, spin and time are considered. On the basis of symmetry with respect to time, two classes of superconductors are introduced, referred to as odd-frequency and even-frequency superconductors. In accordance with the Pauli principle, the even-frequency state is characterized by spin-singlet even-parity or spin-triplet odd-parity order parameter, while odd-frequency superconductors belong to spin-singlet odd-parity or spin-triplet even-parity pairing state. It was suggested in Ref. \textsuperscript{2} from basic symmetry arguments that the first order Josephson coupling should be absent between odd- and even-frequency superconductors. However, as shown in the present paper, an odd-frequency component of the pair potential is quite generally induced at interfaces in superconductors with even-frequency bulk pair potential, while an even-frequency component is induced at interfaces in odd-frequency superconductors. Therefore Josephson coupling between even- and odd-frequency superconductors should be possible.

Up to now, almost all of known superconductors belong to the symmetry class of the even-frequency pairing. The possibility of the odd-frequency pairing state in a uniform system was proposed for \textsuperscript{3}He in\textsuperscript{3} and for a superconducting state involving strong correlations\textsuperscript{3,4,5,6}. Recently, the realization of the odd-frequency pairing state in inhomogeneous systems was proposed by Ref. \textsuperscript{7} in ferromagnet/superconductor heterostructures. This issue was further addressed in several related studies\textsuperscript{8,9}. Recent experiments provided evidence for such anomalous pairing states\textsuperscript{10}. Furthermore, two of the present authors predicted that the odd-frequency pairing state can be induced in a diffusive normal metal attached to a spin-triplet superconductor\textsuperscript{11}. However, in these examples the odd-frequency pairing is realized due to spin-triplet correlations. A question naturally arises whether the spin-triplet ordering is a necessary condition for the existence of the odd-frequency pairing state at superconducting interfaces. In order to address this issue, one should consider self-consistently the pair potential near the interface.

The classic example of an inhomogeneous superconducting system is a ballistic normal metal/superconductor (N/S) junction with an even-frequency superconductor. Particularly interesting are the cases when $S$ is a spin-singlet $\sigma$-wave or a spin-triplet $\varphi$-wave superconductor since the sign change of the pair potential probed by quasiparticles injected and reflected by the N/S interface can occur in such junctions. As a result, the pair potential is suppressed near the interface\textsuperscript{12} and the midgap Andreev resonant states (MARS) are formed\textsuperscript{13}. The appearance of unusual charge transport in the presence of MARS\textsuperscript{13} suggests the presence of underlying anomalous pairing states.

In this Letter we address the issue of odd-frequency pairing in the generic case of a N/S interface, where superconductor $S$ has an even-frequency pairing state in the bulk. We will use the quasiclassical Green’s function formalism where the spatial dependence of the pair potential is determined self-consistently. We will show that, quite generally, the odd-frequency component is induced near superconducting interfaces due to the spatial variation of the pair potential\textsuperscript{14}. If a superconductor has an ESE or ETO pairing state in the bulk, the order parameter at the interface has respectively an odd-frequency spin-singlet odd-parity (OSO) or an odd-frequency spin-triplet even-parity (OTE). In the absence of MARS (e.g. ESE superconductor with $s$-wave or $d_{xy} - y$-wave symmetry of the order parameter), the magnitude of the odd-frequency component of the pair amplitude decreases with a decrease of the transmission coefficient $T_m$ across the interface. On the other hand, in the presence of MARS (e.g. ETO superconduc-
tor with $p_x$-wave or ESE one with $\delta_{xy}$-wave symmetry of the order parameter), the magnitude of the odd-frequency component is enhanced with a decrease of $T_m$. Similarly, in the case of a bulk odd-frequency superconductor, an even-frequency component of the order parameter is induced at the interface. An important application of the above results is the existence of Josephson coupling between bulk odd- and even-frequency superconductors to the first order in $T_m$.

In the following, we consider an N/S junction as the simplest example of a non-uniform superconducting system without impurity scattering. Both the ESE and ETO states are considered in the superconductor. As regards the spin-triplet superconductor, we choose $S_z = 0$ for the simplicity. We assume a thin insulating barrier located at the N/S interface ($x = 0$) with $N(x < 0)$ and $S(x > 0)$ modeled by a delta function $\delta(x)$, where $\delta$ is the magnitude of the strength of the delta function potential. The reflection coefficient of the junction for the quasiparticle for the injection angle $\theta$ is given by $R = Z^2 = (Z^2 + 4 \cos^2 \theta) = 1$ in $T_m$ with $Z = 2 \sqrt{\nu_{tr}}$, where $-2 < \theta < 2$ is measured from the normal to the interface and $\nu_{tr}$ is the Fermi velocity. The quasiclassical Green’s function in a superconductor is parameterized in terms of 

$$ g = f_1 \gamma_1 + f_2 \gamma_2 + g \gamma_3 \gamma_5 g^2 = \hat{1} $$

with Pauli matrices $\gamma_1 (i = 1, 3)$ and unit matrix $\hat{1}$. Here, the index $(\cdot)$ denotes the left (right) going quasiparticles. It is possible to express the above Green’s function as $f_1 = \hat{1} g \hat{1}$ or $D = (1 D F) = (1) D F), f_2 = (F D) = (1) D F),$ and $g = (1 D D F) = (1 D D F)$, where $D$ and $F$ satisfy the Eilenberger equations in the Riccati parameterization. It is remarkable that functions $f_1 (i \gamma_1 ; i \gamma_2$ and $f_2 (i \gamma_1 ; i \gamma_2$ for any $x$. It is remarkable that functions $f_1 (i \gamma_1 ; i \gamma_2$ and $f_2 (i \gamma_1 ; i \gamma_2$ correspond to an odd-frequency and an even-frequency one of the pair amplitude, respectively.

Next, we shall discuss the parity of these pair amplitudes. The even-parity (odd-parity) pair amplitude should satisfy the relation $f_1 (i \gamma_1 ; i \gamma_2$ and $f_2 (i \gamma_1 ; i \gamma_2$ for even-parity case and $f_1 (i \gamma_1 ; i \gamma_2$ and $f_2 (i \gamma_1 ; i \gamma_2$ for odd-parity case, respectively. The resulting $f_1 (i \gamma_1 ; i \gamma_2$ and $f_2 (i \gamma_1 ; i \gamma_2$ satisfy $f_1 (i \gamma_1 ; i \gamma_2$ and $f_2 (i \gamma_1 ; i \gamma_2$ for an even-parity superconductor and $f_1 (i \gamma_1 ; i \gamma_2$ and $f_2 (i \gamma_1 ; i \gamma_2$ for an odd-parity superconductor. Note that the parity of the odd-frequency component $f_1 (i \gamma_1 ; i \gamma_2$ is different from that of the bulk superconductor for all cases.

Let us now focus on the values of the pair amplitudes at the interface $x = 0$. We concentrate on two extreme cases with $(1)$ $(1)$ and $(2)$ $(1)$. In the first case, MARS is absent since there is no sign change of the pair potential felt by the quasiparticle at the interface. Then $D = 0$ is satisfied. On the other hand, in the second case, MARS is generated near the interface due to the sign change of the pair potential. Then $D = 0$ is satisfied. At the interface, it is easy to show that $f_1 = i \gamma_1 (1 + R) D = (1 + R) D$ and $f_2 = (1 + R) D_+ = (1 + R) D_+ = (1 + R) D_+$ for Case (1) and $f_1 = i \gamma_1 (1 + R) D_+ = (1 + R) D_+$ and $f_2 = (1 + R) D_+ = (1 + R) D_+$ for Case (2), respectively, where the real number $D_+$ satisfies $D_+ < 1$ for $n \neq 0$. For Case (1), the magnitude of $f_1$ is always smaller than that of $f_2$. For Case (2), the situation is reversed. In the low transparent limit with $R = 0$, only the $f_1$ is nonzero. Namely, only the even-frequency (odd-frequency) pair amplitude exists at the interface without (with) sign change of the pair potential.
\( \tilde{f}_1, \tilde{f}_2 \) into various angular momentum component as follows,

\[
\tilde{f}_1, \tilde{f}_2 = X \sum_{m} S^{(\ell_1, 1)}_m \sin \theta \plus \sum_{m} C^{(\ell_2, 1)}_m \cos \theta
\]  

with \( m = 2l+1 \) for odd-parity case and \( m = 2l \) for even parity case with integer \( l \), where \( l \) is the quantum number of the angular momentum. It is straightforward to show that the only nonzero components are \( \ell_1 C^{(\ell_2, 1)}_1 \) and \( \ell_2 C^{(\ell_1, 1)}_{2l+1} \) for \( \ell \) or \( \ell_1 e^{i \theta} \). The absence in the Table I below, there are eight distinct cases which correspond to different combinations of the bulk pairing symmetry and the behavior of the orbital part of the bulk pair potential with respect to reflection from the interface.

Below we illustrate the above results by numerical calculations. As typical examples, we choose \( s \)-wave and \( p_x \)-wave pair potentials. Although both \( \tilde{f}_1 \) and \( \tilde{f}_2 \) have many components with different angular momenta, we focus on the lowest values of \( l \). We denote \( E_{s+} (\ell_1, n, j; x) = C^{(\ell_2, 1)}_0 \), \( E_{s+} (\ell_1, n, j; x) = C^{(\ell_1, 1)}_0 \), \( O_{s+} (\ell_1, j; x) = C^{(\ell_1, 1)}_0 \), and \( O_{s+} (\ell_1, j; x) = C^{(\ell_1, 1)}_0 \) and choose \( \ell_1 = 1 \) with temperature \( T = 0.05 T_c \). For the \( s \)-wave case, the pair potential is suppressed only for high transparent junctions (see Fig.1a). The odd-frequency component \( O_{s+} (\ell_1, j; x) \) is enhanced near the interface, it is smaller than the even-frequency one \( E_{s+} (\ell_1, n, j; x) \) (see Fig.1c). For the low transparent junction, the magnitude of \( O_{s+} (\ell_1, j; x) \) is negligible (see Fig.1b).

Let us discuss the Josephson coupling at the interface between even-frequency and odd-frequency superconductors to the first order in the interface transparency coefficient \( T_n \), assuming that spin-flip scattering at the interface is absent. According to the Table I, these are 16 possible combinations of pairing symmetries in two superconductors. Due to the difference of the spin structure of Cooper pairs, the Josephson coupling is absent for the following combinations: \( (1)-(7), (1)-(8), (2)-(7), (2)-(8), (3)-(5), (3)-(6), (4)-(5), \) and \( (4)-(6) \). The Josephson coupling is also absent for the combinations \( (1)-(5), (2)-(6), (3)-(7) \) and \( (4)-(8) \), since the odd- and even-frequency pairing states are realized on both sides of the interface. This result is consistent with the previous prediction. The remaining four combinations \( (1)-(6), (2)-(5), (3)-(8), \)
and (4)-(7) are worthy of remark. As seen from the above Table, the pairing symmetries on both sides of the interface are the same, ESE, OSO, ETO and OTE, respectively. As a result, the Josephson current can flow across the interface in these cases.

These results can be applied to actual materials. Recently, Fuseya et al. [21] predicted that the OSO state could be realized in CeCu$_2$Si$_2$ and CeRhIn$_5$ [6]. It is consistent with some experiments [22]. Here, we propose a robust check of pairing symmetry using the Josephson effect between ESE (conventional low $T_c$) and OSO (e.g. CeCoIn$_5$) superconductors. If two ESE superconductors are attached to opposite (parallel) sides of an OSO sample, the ESE order parameters induced at the two interfaces in OSO will have opposite signs. Then the structure will behave as a $\pi$-junction. Detection of a $\pi$-shift would thus be an unambiguous signature of OSO pairing symmetry in high $T_c$ cuprates [23]. As regards the OTE state, the promising system is a diffusive ferromagnet/spin-singlet $s$-wave (DF/S) hybrid structure, where OTE state is induced in DF. Recent calculation of the Josephson effect in spin-triplet $p$-wave / DF/S junctions [20] is consistent with the present prediction. From this point of view, it is of interest to study junctions between Sr$_2$RuO$_4$ [24] and DF/S hybrids.

In summary, using the quasiclassical Green’s function formalism, we have shown that the odd-frequency pairing state is generated near normal metal / even-frequency superconductor (N/S) interfaces in the absence of spin flip scattering. When the pair potential in the bulk has an even-frequency symmetry (spin-singlet even-parity ESE or spin-triplet odd-parity ETO state), the resulting order parameter at the interface has an odd-frequency symmetry (spin-singlet odd-parity OSO or spin-triplet even-parity OTE state), in agreement with the Pauli principle. On the other hand, if a superconductor has an odd-frequency (OSO or OTE) order parameter in the bulk, then, respectively, ESE or ETO pairing state should be induced near the interface. It follows from the above results that the Josephson coupling may occur between odd- and even-frequency superconductors and phase-sensitive tests can be performed to search for an odd-frequency superconducting state. Though we explicitly studied the N/S junctions only, the odd-frequency pairing state is also expected near impurities and within Abrikosov vortex cores in even-frequency superconductors. This implies that the odd-frequency pairing is not at all a rare situation as was previously considered but should be a key concept for understanding the physics of non-uniform superconducting systems.

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### Table I: The relation between the symmetry of the bulk superconductor and that of the pair amplitude at the interface in the low transparent limit. The allowed symmetry of the Cooper pair in accordance with Pauli’s rule are even-frequency spin-singlet even-parity (ESE), even-frequency spin-triplet odd-parity (ETO), odd-frequency spin-singlet odd-parity (OSO), and odd-frequency spin-triplet even-parity (OTE).

| bulk state | sign change | interface state |
|------------|-------------|----------------|
| (1) ESE (s or $d_{xz}/d_{yz}$-$s$-wave) | No | ESE |
| (2) ESE ($d_{xy}$-wave) | Yes | OSO |
| (3) ETO ($p_x$-wave) | No | ETO |
| (4) ETO ($p_y$-wave) | Yes | OTE |
| (5) OSO ($p_y$-wave) | No | OSO |
| (6) OSO ($p_x$-wave) | Yes | ESE |
| (7) OTE (s or $d_{xz}/d_{yz}$-$p_z$-wave) | No | OTE |
| (8) OTE ($d_{xy}$-wave) | Yes | ETO |

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