A note on inflationary scenario with non-minimal coupling

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(Dated: May 2, 2014)

We consider first order perturbation theory for a non-minimally coupled inflaton field without assuming an adiabatic equation of state. In general perturbations in non-minimally coupled theory may be non-adiabatic. However under the slow-roll assumptions the perturbation theory may look like adiabatic one. We show in the frame-work of perturbation theory, that our results of spectral index and bound on no-minimal coupling parameter agree with the results obtained using the adiabatic equation of state by the earlier authors.

I. INTRODUCTION:

Inflationary paradigm has become extremely useful in solving many problems with the standard big-bang theory and very successful in predicting the fluctuations in the observed cosmic microwave background radiation. However, the nature of the inflaton potential remains to be uncertain and as a consequence there has been a variety of models through which the inflationary paradigm can be implemented. One of the important class of such models that has been extensively investigated in recent time is that of a non-minimally coupled scalar fields. In most of the model of the inflation the mass of the scalar field is considered to be around $10^{13}$ GeV and the extremely small value for the strength of the quartic self-coupling $\lambda \approx 10^{-16}$ is consistent with the results obtained by earlier workers.

As mentioned earlier, for the case of chaotic inflation in Jordan frame that the density perturbation on Jordan frame are constrained by the ratio $\frac{\Delta n}{n}$ and this may allow for the values $\xi > 1$. However no such bound was required for the chaotic inflation scenario. In this work we show that in general there exist two branches, in one branch $\dot{\varphi}$ (inflaton velocity) is negative and $\dot{\varphi}$ is positive in the other branch depending on the nature of the potential. For those classes of potential where $\varphi > 0$ one must have an upper bound on $\xi$. But the other branch may allow a large value of $\xi$ which is consistent with the already known results.

The action in Jordan frame is given by

$$ S = \int d^4 x \sqrt{-g} \left[ f(\varphi) R - \frac{1}{2} \varphi^{,\mu} \varphi^{,\mu} + V(\varphi) \right], \quad (1) $$

in case of non-minimal coupling $f(\varphi) = 1 + \xi \varphi^2$, where $\varphi$ is the scalar field, $\xi$ is a constant, $R$ is the Ricci scalar and $V(\varphi)$ is the potential. Here we have considered $M_{pl} = 1$. 

(For more details see Refs. [13, 22].)
The field equation in Jordan frame is given as
\[ f(\varphi) \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = \frac{1}{2} \left( \varphi_{,\mu} \varphi_{,\nu} - \frac{1}{2} g_{\mu\nu} \varphi_{,\alpha} \varphi_{,\alpha} \right) + \frac{1}{2} g_{\mu\nu} V(\varphi) + f(\varphi) \varphi_{,\mu} \varphi_{,\nu} - g_{\mu\nu} \Box f(\varphi) \]  
(2)

Jordan frame action can be transformed into the Einstein frame action
\[ S = \int d^4x \sqrt{-\hat{g}} \left[ \frac{1}{2} \hat{R} - \frac{1}{2} \hat{\sigma}_{\mu\nu} \hat{\sigma}^{\mu\nu} + \hat{V}(\sigma) \right], \]  
(3)

by transforming the metric and the scalar field in the following way,
\[ \hat{g}_{\mu\nu} = 2f(\varphi) g_{\mu\nu}, \quad \frac{d\sigma}{d\varphi} = \frac{1}{2f} \left( 1 + 3 \frac{f^2}{f'} \right). \]  
(4)

where \( \hat{\cdot} \) represents the Einstein frame and \( \hat{V}(\sigma) = \frac{V(\varphi)}{f^2} \).

Using the field equation in Eq. (2) can be shown to be transformed into the Einstein equations. This is true for the perturbed field equations also. In Einstein frame the equation of motion for the comoving curvature perturbation \( \hat{\mathcal{R}} = \hat{\psi} + \hat{H} \frac{d\varphi}{d\sigma} \) can be written as
\[ \hat{\mathcal{R}}'' + [\ln \hat{\gamma}]' \hat{\mathcal{R}}' + k^2 \hat{\mathcal{R}} = 0 \]  
(5)

where \( \hat{\gamma} = \frac{\partial^2 \sigma^2}{\partial \sigma^2} \), \( \hat{\psi} \) and \( \hat{H} \) are metric perturbations and Hubble parameter in Einstein frame. In Einstein frame spectral index \( \hat{n}_R \) can be expressed in terms of slow roll parameters as
\[ \hat{n}_R - 1 = -4\epsilon_{\hat{V}} - 2\delta. \]  
(6)

where \( \epsilon_{\hat{V}} = \frac{3}{2} \frac{\dot{\varphi}^2}{f^2} \) and \( \delta = 1 - \frac{\ddot{\varphi}}{f} \) are the slow roll parameters in the Einstein frame.

II. SLOW ROLL PARAMETERS IN JORDAN FRAME:

Before we calculate the equation of motion of \( R \) we define the slow-roll parameters in the Jordan frame. Equation for the scalar field \( \varphi \) in Jordan frame is given by
\[ \varphi'' + 2H \varphi' + a^2 f \varphi R + a^2 V(\varphi) = 0, \]  
(7)

where \( R = -\frac{\dot{\varphi}}{\dot{\varphi}} \left( \hat{H}^2 + \hat{H}^2 \right) \). Here \( \dot{\varphi} \) denotes the derivative with respect to conformal time \( \eta \), this is related to natural time as \( d\eta = \frac{dt}{a} \). \( \hat{H} \) is the Hubble parameter in conformal time defined as \( \hat{H} = \frac{\dot{a}}{a} \) and \( a \) is the scale factor. Friedman equations in Jordan frame gives us the following relations:
\[ \hat{H}^2 (1 + \beta) = \frac{1}{6f} \left( \frac{\dot{\varphi}^2}{2} + a^2 V(\varphi) \right), \]  
(8)
\[ \hat{H}^2 - \hat{H}' = \frac{\varphi^2}{4f} + \frac{f'}{2f} - \hat{H} \frac{f'}{f}, \]  
(9)

In the case of the standard inflation we may say that the inflaton is slowly rolling down the potential, therefore the assumptions of slow roll are
\[ \epsilon_{\hat{V}} = \frac{3}{2} \frac{\dot{\varphi}^2}{f^2} \ll 1, \quad \delta = 1 - \frac{\ddot{\varphi}}{\dot{\varphi}} \ll 1. \]  
(10)

In case of inflation driven by the scalar field minimally coupled to gravity \( -\frac{\ddot{H}}{H^2} = 1 - \frac{\dot{H}^2}{H^2} = \epsilon \), therefore \( \epsilon \ll 1 \) implies the smallness of \( \dot{H} \) compared to \( H^2 \), i.e. \( H \) remains almost constant during inflation. But for the inflationary scenario in the Jordan frame smallness of \( \epsilon \) alone does not ensure that \( H \) will remain constant during inflation. Finally using Eq. (8) we can write Eq. (9) as:
\[ \left( 1 - \frac{\dot{H}'}{H^2} \right) = \epsilon_{\hat{V}} (1 + \beta) + \frac{\alpha \beta}{2} - \beta. \]  
(11)

Here we have defined \( \beta = \frac{f}{f'} \) and \( \alpha = \frac{f''}{f'} \). At this point if we consider \( \epsilon_{\hat{V}} \ll 1, \alpha \ll 1 \) and \( \beta \ll 1 \) and neglect the second and third term on the right hand side of the above expression, still we have
\[ -\frac{\dot{H}'}{H^2} = \left( 1 - \frac{\dot{H}'}{H^2} \right) \approx \epsilon_{\hat{V}} - \beta. \]  
(12)

So, we can see that smallness of \( \beta \) and \( \alpha \) are required to drive inflation. Therefore the end of inflation is not determined by \( \epsilon_{\hat{V}} \) only. Inflation ends when \( \epsilon_{\hat{V}} - \beta \sim 1 \).

III. PERTURBED EQUATIONS:

In this work we are writing the total metric (background and perturbed) in Newtonian gauge as,
\[ g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu} = a^2 \left( (1+2\phi) \otimes \otimes \{-1 + 2\psi \} \delta_{ij} \right). \]  
(13)

It is useful to note that metric perturbations, and \( \mathcal{H} \) follow the transformations
\[ \hat{\psi} = \psi - \frac{f_1}{2f}, \quad \hat{\phi} = \phi + \frac{f_1}{2f}, \quad \hat{\mathcal{H}} = \mathcal{H} + \frac{f'}{2f}. \]  
(14)

when we go to Einstein frame from Jordan frame, where \( f_1 = \delta f = f_{,\varphi} \delta \varphi \). Perturbing the Eq. (4) we get the following equations. Time-space component of field equation is:
\[ \psi' + \mathcal{H} \dot{\phi} = \frac{\dot{\varphi}^2}{4Hf} \left( \frac{\hat{H} \delta \varphi}{\hat{H}} \right) - \mathcal{H} \frac{f_1}{2f} - \phi \frac{f'}{2f} + \frac{f'_1}{2f}. \]  
(15)

Off diagonal space-space component is,
\[ \psi - \phi = \frac{f_1}{f}. \]  
(16)

Time-time component of the field equation is,
\[ (3H^2) f_1 + 2 |\Delta \psi - 3 \mathcal{H} (\psi' + \mathcal{H} \phi) | f = \frac{1}{2} \phi' \delta \varphi' + \Delta (f_1) - 3 \mathcal{H} f_1' + 3 \psi' f' + \frac{1}{2} a^2 V_{,\varphi} \delta \varphi + 6 \mathcal{H} \phi f' - \frac{1}{2} \phi \varphi'^2. \]  
(17)
The above equations are written in Jordan frame. Using the transformations defined by equation (4) along with (14), equations (15), (16) and (17) transform into the corresponding equations in the Einstein frame considered in the literature, for example Ref. [22]. Next, we write down some useful relations which will be used frequently in later discussion. From the definition of \( f_1 \) given above in terms of \( \beta \) we can write,

\[
\frac{f_1}{f} = \beta \left( \frac{\mathcal{H} \delta \varphi}{\varphi'} \right)
\]

\[
\frac{f_1}{f} \simeq \mathcal{H} \beta \frac{\gamma}{\mathcal{H}} \left( \frac{\gamma}{\mathcal{H}} - 1 \right) \mathcal{H} \delta \varphi + \beta \left( \frac{\mathcal{H} \delta \varphi}{\varphi'} \right)',
\]

(18)

where we define \( \gamma \equiv \frac{\beta \gamma}{\mathcal{H}} \). Note that \( \gamma \) is not a slow roll parameter. Here we have dropped the terms proportional to second order in slow roll parameters \( \beta \) and \( \alpha \). Similarly \( \Theta \) can be written as

\[
\mathcal{H}^2 - \mathcal{H}' \simeq \mathcal{H} \gamma - \mathcal{H}^2 \beta.
\]

(19)

At this point let us define two variables \( Y \) and \( \mathcal{R} \) as following

\[
Y = \frac{a^2}{\mathcal{H}} \psi, \quad \mathcal{R} = \psi + \frac{\mathcal{H} \delta \varphi}{\varphi'}.
\]

(20)

It should be noted that \( \mathcal{R} \) and \( \mathcal{R} \) are invariant under the transformations [4]. Substituting the expression of \( \psi \) in (15), in linear order of \( \beta \) we can write (15) in terms of \( Y \) and \( \mathcal{R} \) as,

\[
Y' \simeq \frac{a^2 \gamma}{\mathcal{H}} \mathcal{R} - \frac{a^2 \beta}{2} \left( 2 \frac{\mathcal{H}}{a^2} Y - \frac{\mathcal{R}'}{\mathcal{H}} \right),
\]

(21)

where we have used (15) and (19).

Using the background equation of motion of \( \varphi \) we can eliminate \( V_\varphi \) from (17) and finally using (16), (18) and (19) we can write (17) as

\[
\Delta \left( \frac{\mathcal{H} Y}{a^2} \right) \simeq \gamma (1 - \beta) \mathcal{R}' - \frac{\beta \gamma}{2} \left( \gamma \mathcal{R} - \frac{\mathcal{H}}{a^2} Y' \right) + \frac{\beta}{2} (\mathcal{R}' \mathcal{R}).
\]

(22)

Using the above equations one can calculate the power spectrum in the usual way. Equations (21) and (22) are coupled equations. Decoupling these two equations would lead to the following equation for \( \mathcal{R} \):

\[
\mathcal{R}'' + \left[ \ln \left( \frac{a^2}{\mathcal{H}} \gamma (1 - \beta) \right) \right]' + \mathcal{H} \beta \mathcal{R}' + k^2 \left[ 1 + \beta - \left( \frac{\beta a^2}{2 \mathcal{H}} \right)' \frac{\mathcal{H}}{a^2 \gamma} \right] \mathcal{R} = 0.
\]

(23)

As \( \gamma \), \( k^2 \) as the square of sound speed of perturbation namely \( C_s^2 \),

\[
C_s^2 \simeq 1 + \beta - \left( \frac{\beta a^2}{2 \mathcal{H}} \right)' \frac{\mathcal{H}}{a^2 \gamma}.
\]

(24)

It is possible to write the last term in the above expression in terms of slow roll parameters and one can write the expression for \( C_s^2 \) as

\[
C_s^2 = 1 + \beta - \frac{\beta}{2 - 2\epsilon_V}.
\]

(25)

Here it should be noted that the value of \( \frac{\beta}{2 - 2\epsilon_V} \) can’t exceed \( \frac{1}{2} \) as \( \beta < \epsilon_V \) is required to be satisfied when \( \beta \) is positive (see section (VI)).

IV. POWER SPECTRUM AND SPECTRAL INDEX:

To calculate the power spectrum and spectral index, we follow the standard procedure given by Mukhanov [27]. We first substitute \( \mathcal{R} = m \nu \) into Eq. (23), where \( m \) is a function of \( \nu \) only, i.e., \( m = m(\eta) \) and \( \nu = v(\eta, k) \).

Removing the \( \nu' \) term by setting the coefficient of \( \nu' \) to be zero, which yields \( \frac{\nu'}{\nu} = -\frac{k^2}{2} \), we get the equation of \( v \)

\[
v'' + \left[ C_s^2 k^2 - \frac{A'}{2} - \frac{A^2}{4} \right] v = 0.
\]

(26)

Here \( A = \left[ \frac{a^2 \gamma (1 - \beta)}{\mathcal{H}} + \mathcal{H} \beta \right] \). Now this equation can be written as Bessel differential equation and can be solved exactly in terms of Hankel functions once we write \( A' \) and \( A^2 \) as \( \frac{1}{\nu^2} \). \( A \) and \( A' \) can be expressed in terms of slow roll parameters as

\[
A = 2aH (1 + \epsilon_V - \beta + \delta), \quad A' = 2a^2 H^2 (1 - \epsilon_V + \beta) (1 + \epsilon_V - \beta + \delta).
\]

(27)

Here we have used (12). From (12) we can also notice that in this present case we can write \( aH \simeq -\frac{1}{4\epsilon_1}. \) Using this expression we can write \( \frac{\nu'}{\nu} + \frac{\nu^2}{4} = \frac{1}{4} (\nu^2 - \frac{1}{4}) \), where \( \nu = \frac{1 + \epsilon_V - \beta + \delta}{\nu} + \frac{1}{2} \simeq \left( \frac{3}{4} + 2\epsilon_V - 2\beta + \delta \right) \). Therefore, we finally write Eq. (26) as

\[
v'' + \left[ C_s^2 k^2 - \frac{1}{\eta^2} \left( \nu^2 - \frac{1}{4} \right) \right] v = 0.
\]

(28)

Considering the fact that \( C_s \) is a constant we can express the solution of this equation in terms of the Hankel functions,

\[
v_k = (-\eta)^{1/2} \left[ A_1 H^{(1)}_\nu (-C_s k \eta) + A_2 H^{(2)}_\nu (-C_s k \eta) \right]
\]

(29)
Matching this solution of \( v \) with the free quantum field for \( \frac{C_{\phi k}}{\pi H} \gg 1 \) (short wave length limit), we choose
\[
A_1 = \frac{\sqrt{\pi}}{2} \exp \left( \frac{\pi}{2} \left( \nu + \frac{1}{2} \right) \right), \quad A_2 = 0. \tag{30}
\]
For long wavelength (\( \frac{C_{\phi k}}{\pi H} \ll 1 \)) we find that \( v_k \) behaves as
\[
v_k \propto k^{-\nu}. \tag{31}
\]
Next writing this solutions in terms of \( R \) we get the power spectrum as
\[
P_R \propto k^{3-2\nu}. \tag{32}
\]
Spectral index is defined as
\[
n_R - 1 = \frac{d \ln P_R}{d \ln k}. \tag{33}
\]
Therefore we get the expression of spectral index as
\[
n_R - 1 = 3 - 2\nu. \tag{34}
\]
Substituting the expression of \( \nu \) in the above equation we get the spectral index as
\[
n_R = 1 - 4\epsilon + 4\beta - 2\delta. \tag{35}
\]
In this expression we get an additional term \( 4\beta \) along with the standard terms we get from the minimal coupling case.

V. VARIOUS VALUES OF \( \xi \):

A. Spectral index:

In the appendix the first order perturbation theory is done and expression of spectral index is given \[35\] in Jordan frame. In this calculation we have not assumed any specific equation of state unlike the previous authors \[17, 18\]. It is shown later that expression (\( n_R = 1 - 4\epsilon + 4\beta - 2\delta \)) obtained here matches exactly with Ref. \[17, 18\] in the region \( \xi \varphi^2 \gg 1 \). WMAP 7 years data suggests that \( n_R = 0.968 \pm 0.012 \) \[28\]. In this case we let consider that \( \delta = 0 \). Therefore the expression of \( n_R \) suggests that \( \epsilon_V - \beta \sim 10^{-2} \). This condition is consistent with the condition that \( 0 < -\frac{\dot{H}}{H^2} \ll 1 \). The lower bound is necessary to have accelerated expansion. The expression of \( \beta \) can be written as:
\[
\beta = -\frac{\dot{H}}{H^2} = \frac{2\xi \varphi}{\sqrt{f}} \frac{\dot{\varphi}}{H \sqrt{f}}. \tag{36}
\]
Using the expression of \( \epsilon_V \) \[10\] and the Friedmann equation \[8\] one can write the following identity:
\[
\frac{\dot{\varphi}^2}{f H^2} \approx 4\epsilon_V. \tag{37}
\]
Therefore for any potential in general case one can have two values of \( \beta \) in the region \( \xi \varphi^2 \gg 1 \):
\[
\beta = \pm 4\sqrt{\xi} \sqrt{\epsilon_V}. \tag{38}
\]
As \( \beta \) can be written in terms of the scalar field \( \varphi \) (\( \beta = 2\sqrt{\frac{\xi}{f}} \)), which sign to be picked will be dependent entirely on the dynamics of the scalar field for any specific potential.

Using the Friedmann equation and writing the equation of motion \[17\] in cosmic time one can find the expression (2.13) in Ref. \[9\]:
\[
3H\dot{\varphi} \approx \frac{1}{1 + \xi \varphi^2 (1 + 6\epsilon)} [4\xi \varphi V - (1 + \xi \varphi^2) \varphi V] \tag{39}
\]
Therefore \( \beta \) becomes negative if
\[
4\xi \varphi V - (1 + \xi \varphi^2) V\varphi < 0, \tag{40}
\]
\( V = \frac{1}{4} \lambda \varphi^4 \) is an example of this branch. Whereas \( \beta \) becomes positive if
\[
4\xi \varphi V - (1 + \xi \varphi^2) V\varphi > 0, \tag{41}
\]
\( V = \frac{1}{4} m^2 \varphi^2 \) is an example for this branch. When condition \[10\] is satisfied one can write
\[
-\frac{\dot{H}}{H^2} = \epsilon_V + 4\sqrt{\xi} \sqrt{\epsilon_V}. \tag{42}
\]
In this case to have \(-\frac{\dot{H}}{H^2} \sim 10^{-2} \) one can choose a large value of \( \xi \), provided \( \epsilon_V \) is very small. For example, if we choose \( \xi \sim 10^4 \) then \( \epsilon_V \sim 10^{-8} \). It turns out that \( \lambda \varphi^4 \) potential in the power-law class of potentials can give us \( \epsilon_V \) that small (\( \sim 10^{-8} \)).

In case of the other branch with positive \( \beta \) one can write the expression \[12\] as
\[
-\frac{\dot{H}}{H^2} = \epsilon_V - 4\sqrt{\xi} \sqrt{\epsilon_V}. \tag{43}
\]
In this case to have \(-\frac{\dot{H}}{H^2} > 0 \) along with \( \beta \ll 1 \) we have to have an additional condition on \( \beta \): \( \beta < \epsilon_V \). This condition gives us as upper bound on the non-minimal coupling parameter \( \xi < \frac{\epsilon_V}{4\beta} \). For a typical value of \( \epsilon_V \sim 10^{-2} \) we have \( \xi < 10^{-3} \). In this branch one can not make \( \epsilon_V \) very large by choosing a smaller value \( \epsilon_V \) like the previous case, as this may make \(-\frac{\dot{H}}{H^2} < 0 \) violating the condition for accelerated expansion.

Further one may ask the question whether the formalism used here and the “gauge-invariant” formalism presented in Refs. \[10, 22\] are equivalent? In other words one can ask the question if the results obtained in this formalism is same as the result obtained by previous authors. In what follows we address this question: The expression of spectral index in those references is given by
\[
n_s = 1 - 4\epsilon - 2\delta + 2\beta - 2\gamma, \tag{44}
\]
where $\epsilon = -\frac{H}{H'}$, $\delta = \frac{\dot{\phi}}{H \dot{\phi}}$, $\gamma = \frac{\dot{\xi}}{2H\dot{\xi}}$, $\beta = \frac{\dot{\xi}}{H\dot{\xi}}$ and $E = (f + \frac{3}{2}f_\phi^2)$. Using the expression of $\dot{H}$ and $H^2$ given in [19] one can write $\epsilon = \frac{\dot{\phi}}{f H_\phi} + \frac{2f_{\phi}^2}{f H_\phi}$. The second term is a second order term and can be ignored. Next, one can write $\epsilon \simeq \epsilon_V - \beta$ where $\epsilon_V = \frac{v^2}{2f_\phi^2}$. So, $\gamma$ can be written as $n_s = 1 - 4\epsilon_V - 2\beta + 6\delta - 2\gamma$. Next using the definition of $E$ one can write

$$\gamma = \beta \frac{(1 + 3 f_{\phi f}) f}{f + \frac{3}{2}f_\phi^2}. \quad (45)$$

Therefore one can see that $\gamma$ and $\beta$ are not independent parameters. Using the definition of $f$ it can be found that in the region we are interested in $\xi \phi^2 \gg 1$, $\gamma = \beta$. Therefore we find $n_s = 1 - 4\epsilon_V - 2\beta + 4\beta$. The parameter $\beta$ is similar to the parameter $\beta$ we have used in this manuscript. Therefore we get the similar kind of bound from earlier results also.

The last exponential term is the standard term that comes in the minimally coupled theory and the first exponential term is the contribution of the non-minimal coupling. In general three cases can arise: (i) when $\beta = 0$ the last term decays and one gets $R = \text{Constant}$. This case is nothing but the minimal coupling scenario. (ii) In case of negative value of $\beta$, the first term again exponentially decays to give us a constant $R$ in the growing mode. However (iii) if the parameter $\beta$ is positive then it is required that $\beta \ll 1$ to stop the first exponential term from evolving and one gets $R = \text{Constant}$ here also. From the time-time component of perturbed field equation one gets at $k \to 0$ limit:

$$R' \simeq \beta^2 H \psi \simeq 0. \quad (49)$$

as throughout the calculation it is assumed that $\beta \ll 1$.

From the above discussion it is clear that when $\beta$ is positive then $\beta \ll 1$ is required to stop the curvature perturbation from evolving on super horizon scale. But as we have shown that $\beta \ll 1$ may not be a sufficient condition to have the spectral index in the observed range. We have already shown in section (V A) that an additional condition $\beta < \epsilon_V$ is required to satisfy in order to have $n_s$ in the observed range in the branch where $\beta$ is positive. Arbitrarily large value of $\xi$ (inconsistent with $\beta < \epsilon_V$) are not allowed in this case.

In conclusion we have done an explicit first order gauge invariant perturbation theory in the Jordan frame. Unlike the work done by previous authors where adiabatic equation of state was assumed, we have done the calculations without assuming any form of equation of state. We show that the results like spectral index and the bound on non-minimal coupling constant $\xi$ is in good agreement with the known facts. For those classes of potentials where the inflaton velocity $\dot{\phi} > 0$ we must have a bound on the non-minimal coupling constant $\xi$ so that the condition for the Hubble expansion rate $0 < -\frac{H}{H'} \ll 1$ is respected. Whereas in the other case when $\dot{\phi} < 0$ the coupling constant $\xi$ can be very large provided the slow-roll parameter $\epsilon_V$ is very small.

B. Freezing out of curvature perturbation:

Now let us consider the evolution equation of $R$ in the super horizon scale. In $k \to 0$ limit equation (23) can be written as:

$$R'' + AR' = 0. \quad (46)$$

where $A = 2aH (1 + \epsilon_V - \beta - \delta)$. The general solution of the equation (46) is:

$$R = C_1 + C_2 \int \exp \left[ - \int A d\eta \right] d\eta, \quad (47)$$

where $C_1$ and $C_2$ are constants of integration. Substituting the expression of $A$ in equation (47) one can rewrite the expression of $R$ as:

$$R = C_1 + C_2 \int \left[ \exp \left( 2 \int aH \beta d\eta \right) \times \exp \left( -2 \int aH (1 + \epsilon_V + \delta) d\eta \right) \right] d\eta. \quad (48)$$

[1] A. H. Guth, Phys. Rev. D23, 347 (1981)
[2] A. Riotto (2002), arXiv:hep-ph/0210162
[3] A. D. Linde, Lect. Notes Phys. 738, 1 (2008), arXiv:0705.0164 [hep-th]
[4] T. Futamase and K.-i. Maeda, Phys. Rev. D39, 399 (Jan 1989)
[5] E. Komatsu and T. Futamase, Phys. Rev. D59, 064029 (1999), arXiv:astro-ph/9901127 [astro-ph]
[6] N. Sakai and J. Yokoyama, Phys. Lett. B456, 113 (1999), arXiv:hep-ph/9901336 [hep-ph]
[7] V. Faraoni, Phys. Rev. D53, 6813 (1996), arXiv:astro-ph/9602111
[8] N. Makino and M. Sasaki, Prog. Theor. Phys. 86, 103 (1991)
[9] E. Komatsu and T. Futamase, Phys. Rev. D58, 023004 (Jun 1998)
[10] J. Cervantes-Cota and H. Dehnen, Phys. Rev. D 51, 395 (1995), arXiv:astro-ph/9412032 [astro-ph]
[11] G. Gupta, E. N. Saridakis, and A. A. Sen, Phys. Rev. D 79, 123013 (2009), arXiv:0905.2348 [astro-ph.CO]
[12] M. Setare and E. Saridakis, JCAP 0903, 002 (2009), arXiv:0811.4253 [hep-th]
[13] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B 659, 703 (2008), arXiv:0710.3755 [hep-th]
[14] R. Fakir and W. G. Unruh, Phys. Rev. D 41, 1783 (1990)
[15] M. Das and S. Mohanty (2011), arXiv:1111.0799 [hep-ph]
[16] R. H. Dicke, Phys. Rev. 125, 2163 (Mar 1962)
[17] J. chan Hwang, Class. Quantum Grav. 7, 1613 (1990)
[18] J. chan Hwang, Phys. Rev. D 42, 2601 (1990)
[19] J.-c. Hwang and H. Noh, Phys. Rev. D 54, 1460 (Jul 1996)
[20] J.-c. Hwang and H. Noh, Phys. Rev. Lett. 81, 5274 (Dec 1998)
[21] N. Sugiyama and T. Futamase, Phys. Rev. D 81, 023504 (Jan 2010)
[22] D. I. Kaiser (1995), arXiv:astro-ph/9507048
[23] D. I. Kaiser, Phys. Rev. D 52, 4295 (1995), arXiv:astro-ph/9408044 [astro-ph]
[24] I. A. Brown and A. Hammami (2011), arXiv:1112.0575 [gr-qc]
[25] S. Carloni, P. K. Dunsby, and C. Rubano, Phys. Rev. D 74, 123513 (2006), arXiv:gr-qc/0611113 [gr-qc]
[26] J. White, M. Minamitsuji, and M. Sasaki (2012), arXiv:1205.0656 [astro-ph.CO]
[27] V. Mukhanov, Physical Foundations of Cosmology (Cambridge University Press, 2005)
[28] E. Komatsu et al. (WMAP), Astrophys. J. Suppl. 192, 18 (2011), arXiv:1001.4538 [astro-ph.CO]