Optimum Location of Thermal Radiation Shield in Superconducting Rotating Machines

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Abstract. Superconducting rotating machines have rotor maintained at low temperatures, below the critical temperature of the superconductor. This establishes large temperature difference between the cold rotor and surroundings, resulting in large heat leak into rotor through conduction, convection and radiation. Minimizing this heat leak is essential to reduce the power expense of cryogenic cooling system. A radiation shield is anchored at a suitable location on torque tube to minimize the radiation heat leak into the cold rotor. This paper presents a methodology to determine the optimum location of this anchor-point of radiation shield for a given geometry, which minimizes the total heat leak into cold rotor. The location of radiation shield is found to be depending on emissivity of cold rotor.

1. Introduction
Superconducting rotating machines have a cold rotor maintained at a temperature below the critical temperature of superconductor resulting in a large temperature difference between the rotor and surroundings. This causes a heat-in-leak into the system which is removed though external cooling by evaporation of liquid cryogen or by circulation of a cold gas \cite{1, 2}. Heat-in-leak occurs essentially through, 1) Conduction from supports & electrical connections, 2) Convection though surrounding medium and 3) Radiation from enclosing surfaces. Minimizing this is essential to reduce the energy expense for refrigeration, which is reflected in power consumption of the cryo-cooler or quantity of cryogenic fluid required.

In superconducting rotating machines the cold rotor which is maintained at cryogenic temperatures is connected to the room temperature shaft through structural elements called torque tube for transmission of torque. Further, the torque tubes are subjected to large temperature difference across the ends. Design of these torque tubes is challenging because of the conflicting requirements between mechanical and thermal performances. Mechanical integrity of the torque tube calls for a larger cross sectional area and shorter length. This is in direct conflict with thermal design which calls for lower cross sectional area and longer length \cite{3}. Electrical connections like the current leads also contribute to conduction heat transferred. Conduction heat leak through torque tubes and elements like electrical connections is one of the dominant modes of heat transfer into the cold rotor. Coolant fluid exiting the rotor is occasionally used to intercept part of this heat from entering into cold space \cite{5, 6, 7}. Convection may be eliminated by evacuating the enclosure. By ensuring a vacuum below a certain limit ($<10^{-4}$ mBar) heat transfer through residual gas conduction can be neglected. Heat transfer through radiation is reduced by providing a radiation shield anchored at a suitable location on the torque tubes. In principle any number of radiation shields can be provided, however owing to other mechanical and design complexities, superconducting rotating machines have one or sometimes no...
radiation shield. Figure 1 below shows schematic arrangement of radiation shield anchored onto a torque tube.

Figure 1. Schematic of Rotor Assembly with Radiation Shield Mounted on Torque Tube.

In this paper a method is presented to determine the optimum anchor point location of radiation shield on torque tube, which minimizes the heat-in-leak into the cold rotor. It is demonstrated that for a given size of the machine and temperature requirement of cold rotor, optimum anchor point depends on the emissivity of cold rotor.

2. Heat Transfer Model

2.1. Conduction

The conduction heat transfer through torque tube is modelled as shown in figure 2. The hot end and cold end of the support structure are assumed to be maintained at temperatures $T_H$ (300K, ambient) and $T_L$ (30K, typical for a High Tc Superconducting rotating machine) respectively. For a constant cross sectional area, the hot end and cold end heat transfer rates $Q_H$ and $Q_L$ in the torque tube are given by the following expressions.

$$Q_H = \int_0^{T_H} k(T) \,dT$$

$$Q_L = \int_{T_L}^L k(T) \,dT$$

Here, $A$ is the cross sectional area of torque tube and $k(T)$ is the thermal conductivity of torque tube material which is a function of temperature. SS304 is considered as torque tube material for present analysis and its variation with thermal conductivity is taken from [4].
The rate at which heat is transferred by the radiation shield into torque tube at the anchor point can be obtained by energy balance across the torque tube as

\[ Q_{c} = Q_{h} - Q_{\text{r}} \]  

(3)

It is convenient to work with non-dimensional versions of above equations by defining the following dimensionless parameters

\[ \phi = \frac{Q}{Q_{\text{max}}}; \quad K = \frac{1}{\alpha \int \frac{T}{T_{i}} k(T) \text{d}T}; \quad \eta = \frac{x}{L} \]  

(4)

Here, \( Q_{\text{max}} \) is the conduction heat leak through torque tube without any radiation shield, which is given as

\[ Q_{\text{max}} = \frac{A}{L} \int_{T_{i}}^{T_{f}} k(T) \text{d}T \]  

(5)

and \( \alpha \) is the integrated thermal conductivity across the end temperatures of the torque tube given by

\[ \alpha = \frac{A}{L} \int_{T_{i}}^{T_{f}} k(T) \text{d}T \]  

(6)

Using equation (4) non dimensional form of equations (1), (2) and (3) can be written as

\[ \phi_{\text{c}} = \frac{1 - K}{\eta} \]  

(7)

\[ \phi_{h} = \frac{K}{1 - \eta} \]  

(8)

\[ \phi_{r} = \phi_{c} - \phi_{h} \]  

(9)

2.2. Radiation

The radiation heat transfer from enclosing surfaces to the cold space is modelled as shown in figure 3. The subscripts \( v, r \) and \( c \) refer to the vacuum vessel, radiation shield and cold rotor respectively. All the bodies are assumed to be gray and emit diffusely.

![Figure 3. Radiation Heat Transfer from Enclosures.](image)

The radiation shield is assumed to be at a constant temperature which is the temperature of torque tube at anchor point. Also the surface of the torque tube from hot end to anchor point (area \( A_{\text{rhi}} \)) is assumed to behave identical to surface of vacuum vessel for radiation calculations. Similarly surface
of torque tube from anchor point to cold space (area - $A_{TL}$) is assumed to behave identical to the surface of radiation shield. This assumption greatly simplifies the radiation heat transfer calculations and is reasonable when same material is employed for torque tube, vacuum vessel and radiation shield.

The radiation heat transfer from vacuum vessel to radiation shield ($Q_v$) and from radiation shield to cold space ($Q_c$) are given by

$$Q_v = \sigma A_{T_v} (T_{vl}^4 - T^4) \quad (10)$$

$$Q_c = \sigma A_{T_c} (T^4 - T_{vl}^4) \quad (11)$$

Here, $T_{vl}$ (taken as 300K) is temperature of vacuum vessel, $T_c$ (taken as 30K) is the temperature of cold rotor. $T$ is the temperature of radiation shield surface which is assumed to be same as the temperature of torque tube at anchor point of radiation shield. Radiation shape factors $F_{rv}$ and $F_{cr}$ are geometry dependent and under assumptions stated earlier they can be shown to be equal to

$$F_{rv} = \left[ \frac{1}{\epsilon_v} + \frac{A_{L_v}}{A_v + A_{TL}} \left( \frac{1}{\epsilon_v} - 1 \right) \right]^{-1} \quad (12)$$

$$F_{cr} = \left[ \frac{1}{\epsilon_c} + \frac{A_{L_c}}{A_c + A_{TL}} \left( \frac{1}{\epsilon_c} - 1 \right) \right]^{-1} \quad (13)$$

The emissivity $\epsilon$ is a function of material, its surface condition and its surface temperature. For present study SS304 is considered as material for vacuum vessel, radiation shield and torque tube. A linear variation of emissivity is considered for the range of temperatures studied. The variation of emissivity with temperature for SS304 is taken from [8] and a linear curve is fitted as

$$\epsilon = (7.2222 \times 10^{-4})T + 0.07439 \quad (14)$$

The heat transfer rate from radiation shield into the torque tube can be obtained by energy balance across the radiation shield, given by

$$2Q_R = Q_v - Q_c \quad (15)$$

For a given location of radiation shield $x$, the temperature of anchor point can be found iteratively by matching $Q_R$ from the equations (3) and (15). Non-dimensional values of heat transfer from equations (10), (11) and (15) can be obtained by dividing with $Q_{max}$ as:

$$\phi_v = \frac{Q_v}{Q_{max}} \quad \phi_c = \frac{Q_c}{Q_{max}} \quad (16)$$

$$2\phi_R = \phi_v - \phi_c \quad (17)$$

The combined heat-in-leak into cold rotor ($Q_{CS}$) by conduction and radiation heat transfer is given as

$$Q_{CS} = Q_L + Q_C \quad \phi_{CS} = \frac{\phi_{CS}}{\phi_{max}} \quad (18)$$
2.3. Geometry:
For a given geometry of superconducting rotor assembly, above formulation can be employed for identifying the optimum location of anchoring point on torque tube. In the present analysis, a typical high temperature superconducting rotor assembly (see figure 1) is considered with dimensions as given in Table 1 below.

Table 1. Dimensional details of torque tube and radiation shield employed for the present study

| Dimension                      | Symbol | Value (mm) |
|-------------------------------|--------|------------|
| Outer Diameter of Torque Tube  | D_{to} | 560        |
| Inner Diameter of Torque Tube  | D_{ti} | 540        |
| Length of Torque Tube          | L      | 550        |
| Diameter of Radiation Shield   | D_{r}  | 600        |
| Diameter of Vacuum Vessel      | D_{v}  | 625        |
| Diameter of Cold Rotor         | D_{c}  | 580        |
| Length of Cold Rotor           | L_{c}  | 1150       |

3. Results and Discussion

3.1. Anchor point temperature ($T_{ap}$):

![Graph showing variation of anchoring point temperature with anchoring point location for various cold rotor emissivities](image)

The variation of anchoring point temperature ($T_{ap}$) with anchoring point location ($\eta$) is shown in figure 4. As the anchor point is moved from hot end to cold end of torque tube, the temperature of...
anchor point reduces gradually, after a certain location nearer to cold end, there is a steep decrease. Without the radiation shield and under constant thermal conductivity, the above temperature distribution is simply a straight line joining (0, 300) and (1, 30). Introducing a radiation shield increases the temperature of the anchor point on torque tube as it adds heat into the torque tube ($Q_k$) at anchor point. The steep decrease in the temperature near the cold end is attributed to steep decrease in the thermal conductivity of torque tube material (SS304) below 200K and consequent decrease in the emissivity of the radiation shield ($\varepsilon_r$) with temperature.

3.2. Combined heat-in-leak into cold rotor ($Q_{cs}$)

For a given cold rotor emissivity ($\varepsilon_c$), the variation of non-dimensional combined heat-in-leak ($Q_{cs}$) due to conduction ($Q_L$) and radiation ($Q_C$) into the cold rotor with anchor point location ($\eta$) is shown in the figure 5.

![Figure 5: Variation of combined ($Q_{cs}$), conduction ($Q_L$) and radiation ($Q_C$) heat-in-leaks with Anchor Point Location.](image)

As the anchoring point is moved from hot end to cold end, the radiation heat transfer ($Q_C$) reduces sharply due to decrease in anchor point temperature ($T_{ap}$) and emissivity of radiation shield ($\varepsilon_r$), whereas the conduction heat transfer increases marginally. This results in a local minimum of the combined heat-in-leak ($\phi_{cs}$). Further, as the anchoring point is moved towards the cold end, there is a decrease in the conduction and radiation heat transfers which results in a local maximum of the $\phi_{cs}$. This decrease is due to steep decrease in emissivity of the radiation shield ($\varepsilon_r$), arising due to steep decrease in temperature of the anchor point ($T_{ap}$) as shown in figure 4. This decrease in emissivity reduces the heat transfer from radiation shield into the torque tube at anchor point ($\phi_k$), which in turn reduces the combined heat-in-leak into the cold rotor.
3.3. Optimum location of Radiation Shield

Figure 6 shows the variation of combined conduction and radiation heat-in-leak into the cold rotor ($\Phi_{CS}$) with anchoring point location ($\eta$), for various emissivity values of cold rotor ($\epsilon_c$). For high values of $\epsilon_c$ (1, 0.4) the heat-in-leak reduces monotonically with $\eta$ and location of anchoring point for minimum $\Phi_{CS}$ is at cold end, which is the optimum location. For low values of $\epsilon_c$ (0.04, 0.001) there exists a local minimum and local maximum of $\Phi_{CS}$ and the optimum anchor point location is closer to the hot end of torque tube. There is a critical value of $\epsilon_c$ below which the optimum $\eta$ is close to hot end. Above this value, the optimum location is very close to cold end.

![Figure 6: Variation of Combined Heat-in-leak ($\Phi_{CS}$) with Anchor Point Location ($\eta$) for various Cold Rotor Emissivities.](image)

4. Conclusions

A heat transfer model of the rotor assembly of high temperature superconducting motor is developed, considering the conduction heat transfer through supports and radiative heat transfer from enclosures. Optimum location of the thermal radiation shield, which minimizes the combined heat-in-leak due to conduction and radiation into the cold rotor is studied. The following observations are made from the present study:

- Optimum location of radiation shield anchoring point on torque tube of high Tc superconducting motors depends on the emissivity of the cold rotor maintained at low temperatures.
- At high emissivity values of cold rotor the optimum location of radiation shield should be very close to the cold end of the torque tube.
- At very low emissivity values of the cold rotor optimum location of radiation shield is close to the hot end of the torque tube, i.e. no radiation shield is recommended.
There exists a critical cold rotor emissivity above which the optimum location of anchor point is always at cold end. Below this critical value, optimum location suddenly jumps closer towards hot end of torque tube.

5. References
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