SONOLUMINESCENCE AND THE HEIMLICH EFFECT

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The phenomenon of sonoluminescence (SL), originally observed some sixty years ago, has recently become the focus of renewed interest, particularly with the discovery that one can trap a single bubble and induce it to exhibit SL stably over a large number of acoustical cycles.

In a typical experimental situation, a bubble of gas (usually just air, possibly doped with a noble gas) in a liquid (usually just water) is made to expand and then to contract violently under the influence of an applied acoustic field. During this motion, the bubble emits a very sharp pulse of light, after which it expands again and oscillates about its equilibrium radius, until stability is regained. The process then reoccurs in the next cycle.

Some features to note are: (1) the pulse is extremely narrow, probably not more than 10 ps and possibly much less, whereas the acoustic frequency is on the order of 30 kHz and the relevant scale for bubble collapse is perhaps 10 ns; (2)
the photon energies are typically at least of order a few eV, and may be greater. (The water is opaque to photons with energies beyond 6 eV); (3) The intensity of SL varies considerably depending on a number of parameters (intensity of the sound field, temperature of the water, composition of the bubble, etc.) but under optimal conditions pulses with several million photons are routinely achieved \( \text{[5, 8]} \); (4) SL represents a remarkable concentration of energy: the acoustic energy per atom is typically eleven or twelve orders of magnitude less than the energies of the individual photons that are emitted.

On the theoretical side, the problem of understanding SL resolves itself into three coupled components: the dynamics of the bubble driven by the sound wave; the dynamics of the gas within the bubble; and the radiative process that produces the photons. The first and second of these \( \text{[2, 7]} \) would appear to be classical problems governed by well-known equations (which is not to say that everything has been understood), whereas the third is undoubtedly a quantum phenomenon whose origin is still very much in dispute.

In this work we shall adopt a version of the provocative suggestion \( \text{[10]} \) put forward by Schwinger: the mechanism responsible for the radiation in SL is a dynamic version of the Casimir effect. It has been known since Casimir’s original work in 1948 \( \text{[11]} \) that the zero-point energy of quantum fields can be modified by the presence of boundaries, and that these modifications generate observable effects. For example, in Casimir’s original work, the quantum fluctuations of the electromagnetic field in the presence of a pair of uncharged, parallel, perfectly conducting plates were shown to give rise to an attractive force between the plates.

Schwinger invites us to consider a generalization of the situation, in which
the boundary is that between a dielectric medium (the water) and, essentially, the vacuum (the gas inside the bubble). Here, of course, the geometry is spherical, which already makes the computation more difficult, and an additional but clearly crucial complication is that the location of the boundary depends on time. Under these circumstances, one may expect that instead of (or perhaps in addition to) the static Casimir force, one will observe the radiation of the quanta of the electromagnetic field, which will constitute the sonoluminescence pulse.

The challenge is to present a calculation of this effect that is simple enough to be tractable, and yet captures the essential physics. Schwinger struggled with this problem over the course of seven telegraphic communications \[11, 12\] to the Proceedings of the National Academy. Eberlein \[13\] has done a computation based on an analogy with the Unruh effect \[14\], in which the adiabatic approximation is used to permit quantization in a bubble of fixed radius, and then the photon emission amplitude is computed to lowest order in the velocity of the bubble. Her results illustrate an inherent problem with the dynamic Casimir effect: Casimir energies tend to be quite small, and in order to reproduce the observed pulse intensity one must invoke bubble velocities that are rather higher than seem physically reasonable, even perhaps exceeding that of light. Milton \[15\] has done a careful investigation of the static situation and has pointed out similar difficulties. Related computations, in a simplified dynamical model, have been performed by Sassaroli, Srivastava and Widom. \[16\]

In this work we consider a model that neglects the volume effect, i.e. the fact that light propagates with different velocities in the bubble and in the medium, and concentrates on the surface effect, i.e. the fact that a boundary condition must be
imposed on the photon field at the surface of the bubble. Furthermore, we adopt the attitude that, at least for the purposes of an initial investigation, it should not matter precisely which boundary condition is chosen. Thus the choice of a particular form for the term that enforces the boundary condition will be motivated more by computational convenience than by an appeal to underlying physical principles, leaving open the future possibility of finding a more realistic choice.

Thus we consider the action

$$S = -\frac{1}{4} \int d^4x (F_{\mu\nu} F^{\mu\nu} + f(x) F_{\mu\nu} \tilde{F}^{\mu\nu}).$$

Here $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$, $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$ and $f(x)$ is a dimensionless function that represents the coupling of the photon to the boundary of the bubble, located at $r = R(t)$, where $R(t)$ is an externally prescribed function. We note that $F_{\mu\nu} \tilde{F}^{\mu\nu} = 2 \partial_{\mu} (\epsilon^{\mu\nu\rho\sigma} A_{\nu} \partial_{\rho} A_{\sigma})$ so the second term of $S$ may be written $\frac{1}{2} \int d^4x \partial_{\mu} f(x) \epsilon^{\mu\nu\rho\sigma} A_{\nu} \partial_{\rho} A_{\sigma}$. If we choose $f(x) = f_0 \theta(R(t) - r)$, we shall obtain a strictly local coupling of the photon to the surface. Classically $S$ describes a system obeying the equations of motion

$$\partial^{\mu} F_{\mu\nu} + \partial^{\mu} f \tilde{F}_{\mu\nu} = 0,$$

which, for our choice of $f$, is solved by a freely propagating electromagnetic wave subject to the boundary condition

$$n^{\mu} \tilde{F}_{\mu\nu} = 0.$$
on the surface, where \( n_\mu \) is the four-dimensional normal, \( n_\mu = \frac{(\dot{R}, \hat{r})}{\sqrt{1-R^2}} \).

At the quantum level, the simplest thing to do is to treat the \( F\tilde{F} \) term in \( S \) as a perturbation, that is, to assume that \( f_0 \) is a small parameter. Whether this is physically reasonable can only be checked a posteriori, by fitting \( f_0 \) to the data and seeing if it is indeed small.

In this note, we shall follow this approach, and compute some relevant amplitudes in lowest order perturbation theory. Diagramatically, the basic vertex is

![Diagramatic representation of the basic vertex](image)

where the dot represents the action of the external source \( f(x) \). It is actually more convenient to work in momentum space, and therefore we require the Fourier transform

\[
f(p) = \frac{1}{(2\pi)^4} \int d^4x e^{ip\cdot x} f(x).
\]

When \( f(x) = f_0 \theta(R(t) - r) \), we find (assuming that \( R(t) = R(-t) \), which is true in Eberlein’s model but which does not really fit the data)

\[
f(p_\mu) = \frac{-2f_0}{(2\pi)^3 \sqrt{p}} \frac{\partial}{\partial p} \text{Im} g(p, p_0),
\]

where \( g(p, p_0) = \int_{-\infty}^{\infty} dt e^{ipR(t)} e^{ip_0 t} \) and \( p = |\vec{p}| \).
As a simple example where we shall be able to evaluate $g$ explicitly, we may consider

$$R(t) = R_0, \ |t| > T$$

$$R(t) = R_0 + v(|t| - T), \ |t| < T.$$  

This then yields

$$\text{Im}g = pv \left[ \frac{\cos(pR_0 + p_0T) - \cos(pR_0 - p_0T)}{p_0(p_0 + pv)} + \frac{2\cos(pR_0 - pvT)}{p^2v^2 - p_0^2} \right].$$

Among the quantities of physical interest that we may compute is the average number of photons that are produced due to the action of the source:

$$\langle N \rangle = \pi^3 \int d^4 P \theta(P_0) \theta(P_0^2 - P^2) \ |f(P, P_0)|^2 (P_0 - P^2)^2.$$  

The challenge is to see whether the left-hand side can be of order $10^6$, even though $f_0^2$, which appears on the right-hand side, is a small number. Unfortunately, one sees that since $f$ falls off as $p_0^2$ for large $p_0$, the right-hand side actually diverges. To ameliorate this, one should include some or all of the following effects:

(a) instead of a sharp boundary function, $\theta(R(t) - r)$ one should presumably smooth the boundary over a small distance $\Delta$. This will result in a modification of $f$:

$$f(p, p_0) \rightarrow f(p, p_0)e^{-p\Delta}$$
which will not directly solve the large $p_0$ divergence problem for $\langle N \rangle$, but will insure that the $p$ integral remains finite in this and other expressions;

(b) As Casimir pointed out in his original work \[11\] the boundary that is represented by $f(x)$ essentially disappears for high frequencies, because high energy photons do not interact with the boundary as a whole, but only, if at all, with the individual constituents. Thus one should insert a high frequency cutoff on physical grounds;

(c) The experimental data are cut off by the fact that water absorbs all photons with energies greater than about 6 eV. Thus the experimentally measured $\langle N \rangle$ is only for photons with energies less than 6 eV, and hence one should integrate only up to $P_0^2 \sim 12$ eV or so (the $P_0$ in the integral represents the energy of a pair).

Of course, one does not want the water to absorb large numbers of high energy photons: these would have observable effects that are not seen.

In addition to the total number of produced photons, it is also possible to compute other quantities of interest in the same approximation, such as the spectrum of produced particles. It seems wise, however, to concentrate first on $\langle N \rangle$, since too large an $f_0$ will vitiate the approximation. This and related computations are currently in progress. \[17\]

We turn next to the consequences of a point we remarked upon earlier: independent of whatever the radiation mechanism turns out to be, $SL$ can be viewed as the conversion of acoustic energy, which is distributed diffusely throughout the liquid, into a burst of electromagnetic energy which is concentrated spatially into a small region, perhaps a few microns, at the center of the bubble, and which invests
a typical photon with an energy of at least a few eV, eleven or twelve orders of magnitude more than the acoustic energy of an atom in the liquid. Furthermore, this process is reasonably efficient, in that the energy carried off by the photons is comparable to the energy required to combat the viscosity of the fluid. 

These features suggest that SL might, with a lot of technological development, be a candidate for a mechanism of particle acceleration. After all, the one indispensable property of an accelerator is not just the energy, but rather the ability to transfer energy that is macroscopically generated to individual microscopic particles. Whatever the mechanism, this is an ability that SL is observed to possess.

To pursue this idea within the framework of the kind of model that we have been discussing, we must add to the action the terms, familiar from QED, describing the electron and its interaction with the electromagnetic field:

\[ S_f = \int \! d^4x [\bar{\psi} (i\partial_\mu \gamma^\mu - m) \psi + eA^\mu \bar{\psi} \gamma_\mu \psi]. \]

Then, combining the QED interaction with the photon-bubble interaction, we shall have diagrams describing the acceleration of the electron. To lowest order in the electron-photon interaction, we have

\[ q \]

\[ p \rightarrow p' \]

in which the bubble creates a pair and one of the photons then is absorbed by the
electron. The probability of starting with an electron of momentum $p$ (say, at rest) and measuring an electron of momentum $p'$, together with a photon of momentum $q$, is given by

$$\left| \langle p' q | p \rangle \right|^2 = \frac{e^2 m^2 K(p, p', q)}{(p' - p)^4 4\pi q_0 E E'} \left| f(p' + q - p) \right|^2$$

where the bar indicates an average over the spin of the electron and the polarization of the photon, and $K$ is a kinematical factor,

$$K = \left( \frac{p' \cdot p}{m^2} - 1 \right) ((p \cdot q)^2 + (p' \cdot q)^2) - (q \cdot (p' - p))^2.$$  

To obtain the total amplitude for acceleration, one should then integrate this expression over $q$.

We refer to this process as the ”Heimlich effect”, because, at a rather different length scale, it produces the same result as the well-known Heimlich maneuver [18]: a bubble is squeezed, and a particle pops out.

We note that the SLC accelerator at SLAC imparts an energy of about 15 eV/micron to the electrons. Since the size of the sonoluminescent region is of order a micron, and since the energies are in the 1-10 eV range, we appear to have the potential to achieve similar results. Of course, SLAC is 2 miles long whereas so far SL has been confined to one micron-sized bubble at a time. It is premature to speculate on how difficult it might be to improve this situation.

In summary, we have presented a phenomenological model that we believe captures the essence of Schwinger’s suggestion about the mechanism behind the $SL$
radiation process. To confront this model with the data, one must achieve reliable numerical estimates first of $\langle N \rangle$ and then of other quantities such as the photon spectrum, not only for the simple $R(t)$ chosen here but for more realistic choices as well. Furthermore, one should be prepared to extend the analysis beyond lowest order perturbation theory, and to take advantage of the quadratic nature of the photon-bubble interaction in order perhaps to obtain non-perturbative results that will more stringently test the model. [17]

We have also suggested, regardless of the validity of our model, that the phenomenon of sonoluminescence may provide the tentative first step toward a new method of particle acceleration that may be of increasing relevance as the currently dominant species of accelerator faces inevitable decline and perhaps even extinction in the twenty-first century.

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