Influence of mutual position of a noncircular tunnel and a dynamic impact source on stress distribution in lining

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Abstract. Based on the analytical solution of the corresponding dynamic plane elasticity problem, general regularities of the formation of maximum stresses in lining of a noncircular cross-section tunnel are found and studied at different locations of a dynamic impact source. The paper shows that the closely spaced position of the source has a significant effect on the stress state of the underground structure, which in turn depends on the frequencies of the waves radiated by this source. It is revealed that when the dynamic impact source is removed deep in rock mass for a distance of the order of seven average radii of the tunnel cross section, the estimated stresses in the tunnel lining coincide with the stresses found from the solution of the problem on the wave propagation in the corresponding direction from infinity.

Application of blasting in mining and in construction is, as rule, connected with the additional safety measures to protect nearby structures since, for example, mine support (lining) is exposed in this case to static loading and essential dynamic effect. Currently, the common approach to predicting stress state in underground structures under dynamic impact is based on the analysis of rock mass and support interaction. Rock mass is simulated by a linearly deformable isotropic medium, and dynamic effects are considered as diffraction of stationary and nonstationary P- or S-waves at unsupported holes.

Figure 1. Analytical scheme.
In the framework of this approach, the original method of calculation of underground structures (mine support and lining of tunnels and buried pipelines) of arbitrary cross-section under dynamic loading has been developed at the Tula State University. The method is based on the mathematical modeling of the stress state formation in arbitrary cross-section lining and enclosing rock mass as a uniform deformable system under harmonic compression–tension or shear waves. To this effect, the solution of a dynamic plane elasticity problem of diffraction of harmonic waves propagating (including nearby the wave source) in a linearly deformable medium \( S_0 \), modeling rock mass, on a ring \( S_1 \) of the other material supporting a hole of arbitrary shape (with a single symmetry axis), modeling the lining, is used. The analytical model is shown in Figure 1.

Here, the ring \( S_1 \), modeling lining, has an arch thickness \( \Delta \) and is made of material possessing a specific weight \( \gamma \) and deformation characteristics represented by the deformation modulus \( E_1 \) and Poisson’s ratio \( \nu_1 \). The infinite uniform medium \( S_0 \), modeling rock mass, has different \( \rho_0, E_0, \nu_1 \). The ring and the medium deform jointly, i.e. the vectors of stresses and displacements at their interface \( L_0 \) are continuous. The inward interface boundary of the ring, \( L_1 \), is free from external forces. The source at the point \( 0' \) at the distance \( \bar{R}_0 \) from the origin of the coordinate system \( x0y \) emits harmonic compression wave with an angular frequency \( \bar{\omega} \) in the direction of the axis \( 0x' \) making an arbitrary angle \( \beta \) with the vertical axis \( 0x \) (the angle is counted anticlockwise).

The formulated problem is solved given the stresses and potentials are independent of the angle and the shear stresses are absent, in the descending wave, in the polar coordinate system, using the theory of functions of a complex variable, apparatus of conformal mapping of special cylindrical (Bessel) functions and complex series, the boundary disturbance method modified for the iterative process of finding complex potentials governing stresses and displacements at the points of the medium and the ring based on recurrent relations, which allows arbitrary number of approximations and enables higher accuracy of the calculation.

The described solution is implemented as a software system for efficient calculation of maximum dynamic stresses in lining under wave propagation (i.e. construction of envelopes of stress epures).

First, the obtained solution and its computer implementation were tested by means of comparing the calculated data and the available results reached by the other researchers in solving conformable dynamic problems, which can be presented as special cases in the framework of the developed method. To begin with, the results obtained by Savin [1] when determining dimensionless (related to the basic stress state in descending wave) maximum normal tangential stresses within the whole tame of the wave travel at the boundaries of circular, elliptical and square support-free holes in a infinite medium during propagation of harmonic compression wave were reproduced. Then, the results of Fotieva and Garaychuk [2] in the determination of stress at the boundary of an arbitrary cross-section tunnel in rock mass under propagation of P- and S-harmonic waves, as well as the calculated data on circular tunnel lining obtained by Erzhanov et al [3] were simulated. The actual and calculated stresses exhibited total coincidence in all occasions, unexceptionally.

Below in this paper, the calculations of tunnel lining (see the geometry in Figure 2) subjected to the dynamic impact of P-waves emitted by the source arranged in the horizontal straight matched with the axis \( 0y \) at relative distances \( \bar{r}_0 = \frac{\bar{R}_0}{R_0} = 2, 7 \) (here, \( R_0 \approx 4 \) m—average radius of the tunnel).

The calculation input data are: rock mass is composed of siltstone with \( \rho_0 = 18 \) kN/m\(^3\), deformation modulus \( E_0 = 12000 \) MPa and Poisson’s ratio \( \nu_0 = 0.3 \); lining is made of concrete V20 with \( \gamma = 24 \) kN/m\(^3\), \( E_i = 27000 \) MPa, \( \nu_i = 0.2 \). The angular rate \( \bar{\omega} = 1265 \) s\(^{-1} \) \( \approx 200 \) Hz. The calculations involve 4 approximation, the series of the zero iteration has \( N = 8 \) members.

The calculated stresses were the maximum stresses at the inward and outward boundaries of the lining for the whole time of the wave travel.
Figure 2. Geometry of the cross-section of the tunnel lining.

Figure 3. Calculated epures of the dynamic stresses $\sigma_{\text{in max}}^{(in)}$, $\sigma_{\text{in max}}^{(ex)}$ at the inward and outward boundaries of the lining of the tunnel at (a) $\tilde{r}_0 = 2$ and (b) $\tilde{r}_0 = 7$.

Figures 3a and 3b show the calculated epures of the normal tangential stresses $\sigma_{\text{in max}}^{(in)}$, $\sigma_{\text{in max}}^{(ex)}$ at the inward and outward boundaries of the lining, respectively. For the comparison, for the case when the source is at the distance $\tilde{r}_0 = 7$ (Figure 3b), the dashed lines (stress values are in brackets) show the data calculated for the compression wave propagation at infinity.
The epures of stresses shown by the solid and dashed lines in Figure 3b are almost identical. This allows drawing a conclusion that in this example with the source at the distance $r_0 \geq 7$, the dynamic effect of the emitted waves is sufficiently accurately determined as the effect of plane harmonic waves propagating at infinity.

At the second stage of the research, the multi-variant calculations revealed basic regular patterns in the formation of stress state in underground structures of noncircular cross-section under propagation of harmonic P-waves emitted by the closely located source. Special attention was given to the practically essential issue connected with the estimation of effect of the source location on the maximum stresses in the underground structure in different ground conditions.

In the second-stage calculations, the data of the support were the same as in the above example, the other data were varied: $\omega_0 = 0, 1; 2; 5; \lambda^2 = \frac{\mu_1}{\gamma_1} = 4; 50$. The source location was set by the polar coordinates $\tilde{r}_0, \beta$ in the system with the origin at the center of the cross-section of the lining. All in all, 5 alternative locations of the wave source were analyzed: in the vertical axis under the tunnel ($\beta = 0$); above the tunnel ($\beta = 180^\circ$), on the left in the horizontal axis ($\beta = 90^\circ$), and in the intermediate directions characterized by the angles $\beta = 45, 135^\circ$.

The scope of the analysis embraced the dependence of the normal tangential stresses (stress raisors) $\sigma_\theta$ at the inward boundary of the lining at the points of the arch and top of the side walls (curves 1) and in the cradle portion, including the point of the wall and cradle juncture (curves 2) on the relative source distance $\tilde{r}_0$.

The influence of the harmonic wave source location on the stress state in the tunnel in hard rock mass ($\lambda^2 = 4$) is illustrated by the curves in Figure 4 plotted by the processing data of multi-variant calculations.

![Figure 4](image_url)

**Figure 4.** Dependence of the maximum tangential stresses $\sigma_\theta$ in the lining arch and at the top of the side walls of the tunnel (curves 1) and in the cradle of the lining (curve 1) in hard rock mass on the relative distance $\tilde{r}_0$ to the spource at $\beta = 45^\circ$ (solid curves) and $135^\circ$ (dashed curves).

**Conclusion**

The implemented analysis has shown that close location of a wave source has an essential influence on stress state of an underground structure, conditioned also by the emitted wave frequency. For example,
at a low frequency $\omega_0 = 0.1$, the dependence of the maximum stresses on the dimensionless distance $\tilde{r}_0$ is maximum. With an increase in the frequency $\omega_0$, the curves become flatter and when $\omega_0 > 5$ the change in the source location from $\tilde{r}_0 = 2$ to infinity results in the change in the stress state no more than by 10%.

On the whole, the research findings imply that that increase in the distance between the source and the tunnel not always results in the decrease in the maximum stresses in the lining. At the same time, when $\tilde{r}_0 > 6$, the analyzed curves flatten out, as a rule, and the calculated stresses of the lining coincide with the stress values obtained in solving analogous problems on propagation of waves at infinity. The sources of equal capacity, placed in the symmetry axis $0\alpha$ above and below the tunnel generate the same maximum stresses in the structure. With the increase in the frequency $\omega_0$, the influence of the source distance $\tilde{r}_0$ on the maximum stresses decreases, while the effect of the wave direction (angle $\beta$) increases. Furthermore, the influence of the distance to the source is weaker in soft rocks as compared to hard rock mass.

References
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