The Swampland Spectrum Conjecture in Inflation

ROLF SCHIMMRIGK

Dept. of Physics
Indiana University at South Bend
Mishawaka Ave., South Bend, IN 46634

Abstract

The quantum gravity conjectures that aim to separate the landscape from the swampland among the low energy theories were originally formulated in the context of scalar field spaces spanned by moduli. Because these conjectures have implications for cosmology they have recently been considered in a more general context for scalar field theories with potentials, in particular inflation. From an effective field theory perspective the presence of a potential induces a natural metric that changes the distance measure along scalar field trajectories. This suggests a modified formulation of those conjectures that involve trajectory distances.
1 Introduction

Recent discussions in the literature have attempted to address the possible implications of the conjectures that aim to separate low energy theories that admit UV completions from those that do not. The former are said to belong to the quantum gravity landscape, the latter to the swampland [1, 2, 3, 4, 5]. Two of these conjectures are concerned with the distance that scalar fields traverse in some field space $X$. The first of these is the infinite diameter conjecture [1, 6], which posits that the target space should admit trajectories of infinite length. Building on this is a second conjecture that has been the focus of much attention and is in the earlier literature referred to as the swampland conjecture, and more recently, after the advent of [4] as the swampland distance conjecture. This conjecture is concerned with the mass spectrum of the theory as the fields evolve and will here be referred to simply as the spectrum conjecture to distinguish it from the infinite diameter conjecture on which it builds.

The formulation of the spectrum conjecture has not stabilized yet, but very roughly it states that for a given low-energy field theory the Lagrangian remains valid only if the evolution traverses a distance smaller than the Planck scale. The picture assumed here is that the mass spectrum changes as the scalar field evolves from a point $p_0$ in the field space $X$ to a point $p$ at distance $\mathcal{D}(p_0, p)$ from $p_0$ because light particles appear whose masses scale with the distance. In the original asymptotic formulation for a single scalar field with a flat target space the distance dependence is taken to be

$$m(p) \cong M_{Pl} e^{-\alpha \mathcal{D}(p_0, p)/M_{Pl}},$$  

(1)

where $\alpha$ is an undetermined positive parameter that at present has to be estimated in a model dependent way and $M_{Pl}$ is the reduced Planck scale. The main issue that is unresolved in this formulation is when the exponential behavior should be expected to set it. In the simplest case the spectrum conjecture is formulated as an asymptotic statement for a single scalar field.
φ, i.e. the functional form of the masses is assumed to be valid in the limit in which the flat target space distance $D = \Delta \phi$ diverges, and in this limit an infinite tower of light particles is envisioned to appear \cite{2}. An attempt to encode the local structure of the moduli space has been made in \cite{7}, where a further unknown function is introduced in addition to the exponential factor in order to parametrize the current ignorance of the local effects.

Originally the formulation of the landscape vs. swampland discussion was framed in the context of string theory, where the main focus is on scalar fields $\phi^I$ that are moduli (see e.g. \cite{7, 8, 9, 10, 11, 12, 13, 14, 15, 16} for recent work and additional references). In this context the distance $D(p_0, p)$ is measured with the metric on the moduli space, given by the metric $G_{IJ}$ of the kinetic term

$$L_{\text{kin}} = -\frac{1}{2} G_{IJ}(\phi^K) g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J.$$  \hspace{1cm} (2)

Recently however both the infinite diameter conjecture and the associated spectrum conjecture have been considered in the context of more general field theories, in particular in the framework of inflation, where one of the issues that has been discussed extensively is concerned with large field inflation. The spectrum conjecture has been addressed both in the singlefield framework \cite{17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27} and in the multifield generalization \cite{28, 29, 30, 31}. In this more general case the existence of a potential changes the nature of the trajectory because the inflaton rolls down the potential surface, which in an $n$-component scalar field theory is a hypersurface embedded in an $(n+1)$-dimensional space. In this framework the potential induces a natural metric on the potential surface that leads to a modified distance formula, involving both the field space metric $G_{IJ}$ as well as a term that is induced by the gradients of the potential $V(\phi^I)$. The purpose of this note is to formulate the resulting modified distance $D_V$ and to discuss its implications in the context of the quantum gravity vs. swampland conjectures.

\section{Field space trajectories}

In curved multifield theories the target space is a configuration space $X$ that is equipped with a Riemannian metric $G_{IJ}$ and the fields are constrained by a potential $V(\phi^I)$, where the number of fields $I, J = 1, ..., n$ is arbitrary. The Klein-Gordon evolution of the background field $\phi^I(t)$

$$D_t \phi^I + 3H \phi^I + G^{IJ} V_J = 0$$  \hspace{1cm} (3)

then leads to trajectories in the field space $X$. Here $D_t$ is the covariant derivative, defined on a vector field $W^I$ as $D_t W^I = \partial_t W^I + \Gamma^I_{JK} \phi^J W^K$, where the connection is assumed to be of Levi-Civita type, and the connection coefficients $\Gamma^I_{JK}$ are the associated Christoffel symbols.
The distances in the target space $X$ are given by the standard length formula

$$D = \int_{t_0}^{t_\epsilon} dt \sqrt{G_{I,J}(\phi^K)\dot{\phi}^I \dot{\phi}^J}. \quad (4)$$

A concrete illustration of this picture can for example be given within the class of automorphic inflation [32, 33], in which case the target space is obtained via group quotients that endow $X$ with a discrete symmetry that is constrained by the fact that it contains the shift symmetry. In the special case of modular inflation the field space is the complex upper halfplane $\mathcal{H}$, spanned by an inflaton doublet $(\phi^1, \phi^2)$, on which the geometry is determined by the Poincaré metric

$$G_{I,J} = \frac{1}{(\text{Im } \tau)^2} \delta_{I,J} \quad (5)$$

where the dimensionless variables $\tau' = \dot{\phi}/\mu$ introduce a mass scale $\mu$ that is constrained by CMB data. Potentials in this framework are given by a modular invariant function, for example the $j$-inflation potential $V = \Lambda^4 |j|^2$, where $j(\phi')$ is the absolute modular invariant [32, 34, 35]. Modular invariance suggests to combine the inflaton multiplet into the dimensionless complex variable $\tau$.

Inflationary trajectories in the modular inflation target space $X = \mathcal{H}$ are shown in Fig. 1 for the potential of $h_2$-inflation. Their length is obtained by computing

$$D = \mu \int dt \frac{1}{\text{Im } \tau} \sqrt{\left(\frac{d\tau^1}{dt}\right)^2 + \left(\frac{d\tau^2}{dt}\right)^2} \quad (6)$$

along the inflaton path. These paths start in a region of the inflaton space where the slow-roll parameters are small and they continue until the parameter $\epsilon$ approaches unity.

**Fig. 1.** Phenomenologically consistent inflaton trajectories $(\tau^1, \tau^2)$ on the target space $X = \mathcal{H}$ of the upper halfplane for $j$-inflation.
3 Trajectory distances with potentials

In the case of scalar field theories with a potential $V(\phi')$ the trajectories in the target space $X$ are not the trajectories that encode the physics of the theory because they represent a lower-dimensional projection of the path traversed by the inflaton on the potential surface associated to $V$. Thinking of $V$ as a function that is defined by the field target space $X$ and takes values in the set of real numbers $\mathbb{R}$ shows that the presence of the potential adds another dimension, leading to the space $X \times \mathbb{R}$ as the relevant configuration space. The potential surface is to be viewed as a hypersurface embedded in this space. For the case of flat target spaces considered in many papers on multifield inflation with $n$ fields this product space simply corresponds to the $n$-dimensional euclidean space $\mathbb{R}^{n+1}$. In multifield theories with curved targets $X$ the embedding space usually has a different topology, depending on the symmetries of the models.

The presence of a potential suggests that the physical length of the inflaton paths is naturally measured on the surface to which the trajectories are constrained. Depending on the precise structure of the potential the projection of the hypersurface trajectories onto the target space trajectories can provide a good approximation of the actual paths on the potential surface when the gradients of the potential are small over the whole time interval. If on the other hand the variation of the potential normal to the target space is significant then the hypersurface trajectory lengths can be quite different from the distances traversed by the scalar field multiplet in the target space $X$. Current CMB observations [36, 37] are consistent with slow-roll inflation, in which case at the beginning of inflation the slow-roll parameters

$$\epsilon_I = \frac{V_{,I}}{V}$$

are small, hence the gradients of the potential are small. If however inflation ends when the slow-roll parameter $\epsilon$ approaches unity, as it does in many classes of theories, the parameters are no longer small.

An example of a potential hypersurface is shown in Fig. 2 for a model in the class of modular inflation theories considered in [32, 34, 35]. The target space is again the complex upper halfplane $X = \mathcal{H}$ with the above projected 2D trajectories in Fig. 1, but now the inflaton components are shown as they evolve on the potential hypersurface. Fig. 2 illustrates that the length of the inflaton trajectory can be quite different from the length of its projection in the target space, depending on the structure of the potential. It exemplifies that the gradients $V_{,I}$ of the potential can vary considerably along the inflaton trajectory in the space $X \times \mathbb{R}$. It is therefore natural to take the effect of the potential into account and consider the distances on the potential hypersurface as the fundamental quantities. The geometry of such hypersurfaces...
is classical and traces in the simplest case of surfaces in flat three-dimensional space back to the beginnings of differential geometry with Gauß.

![Fig. 2. Trajectory with N = 60 e-folds and Planck compatible observables on the potential surface of j-inflation with V = Λ^4|j|^2.](image)

Given a multifield inflationary model with a target space metric $G_{IJ}$ and a defining potential $V(\phi^I)$ as a function on the target space, a choice has to be made about how precisely to parametrize the resulting ambient space. A useful way to think about potentials in arbitrary multifield scalar theories is by separating an overall scale $\Lambda$ and to write the potential as a product of $\Lambda^4$ and a dimensionless function. The target space $X$ is parametrized by the inflaton multiplet $\phi^I$ and the trajectory surface $S^V$ associated to the potential $V$ in the space $X \times \mathbb{R}$ is defined by the vector

$$f^V(\phi^I) = (\phi^I, V(\phi^I)/\Lambda^3).$$

This surface leads to a metric $G_{ij}^{\text{em}}$ that is induced by the metric of the embedding space $X \times \mathbb{R}$ as

$$G_{i,j}^{\text{em}} = \langle f^V_i, f^V_j \rangle,$$

where the derivatives are with respect to the inflaton components $\phi^I$. The inner product depends on the structure of initial field theory. If the metric $G_{IJ}$ is flat then this product is just the euclidean product, while in the curved case it is no longer flat but induces the embedding space metric

$$G_{ij}^{\text{em}} = G_{ij} + \frac{1}{\Lambda^6} V_{,i} V_{,j}.$$
This leads to a modified distance formula that is induced by both the target space metric and
the potential as
\[ D_V(t_i, t_e) = \int_{t_i}^{t_e} dt \sqrt{G_{IJ} \dot{\phi}^I \dot{\phi}^J + \frac{1}{\Lambda^6} (\dot{\phi}^I V_I)^2}. \]  
(11)

This formula has an intuitively natural structure in that the larger the gradient is of the
potential the more the target space distance \( D \) will be different from \( D_V \). This is precisely as
expected from our conceptual discussion above.

In the context of multifield slow-roll inflation the distance \( D_V \) can be expressed purely as a
function of the potential and hence is amenable to an analytic treatment. It is convenient to
write the slow-roll approximation \( D^e_V \) of \( D_V \) in terms of the slow-roll parameters \( \epsilon_I \) introduced
above, which leads to
\[ D^e_V = \int_{t_i}^{t_e} dt \sqrt{V/3 G^{IJ} \epsilon_I \epsilon_J \left( 1 + \frac{1}{\Lambda^6} \frac{V^2}{M^4_{Pl}} (G^{IJ} \epsilon_I \epsilon_J) \right)}. \]  
(12)

In the above discussion the simplest possible choice was adopted for the embedding surface in
the sense that the metric on \( X \times \mathbb{R} \) was taken to be the metric on \( X \) and the flat metric on \( \mathbb{R} \).
In principle it is of course possible to consider a warped metric on the reals and furthermore to
introduce off-diagonal components of the metric, should the physical situation warrant this.

### 4 The modified spectrum conjecture of quantum gravity

The discussion of the previous section suggests to consider the distance \( D_V \) of eq. (11) in the
context of those conjectures that are affected by the distance measurements, most immediately
the infinite diameter conjecture [1, 6, 10, 11, 38] and the spectrum conjecture [2, 11, 12, 13]. A
theory that is in compliance with the infinite diameter conjecture relative to the target space
metric automatically satisfies the conjecture relative to \( D_V \) because the additional term in \( D_V \)
is positive. For the spectrum conjecture on the other hand the additional term matters because of its more discriminating quantitative nature. For the asymptotic formulation of the statement
the modification of the distance measure leads to a mass spectrum that introduces new light
states according to the relation
\[ m \simeq M_{Pl} \exp \left( -\frac{\alpha}{M_{Pl}} D_V(\phi_i, \phi_e) \right). \]  
(13)

In the slow-roll approximation this factors for a small potential gradient term as
\[ e^{-\alpha D_V/M_{Pl}} \simeq C_V e^{-\alpha D/M_{Pl}}, \]  
(14)
where $C_V$ encodes the potential gradient term. The numerical factor $\alpha$ is not known, but has been conjectured to be of order one, thereby making the assumption that no factors like $4\pi^2$ will appear [7]. While the mass scale considered in the spectrum conjecture is canonically chosen to be the Planck mass $M_{Pl}$, the natural scale in quantum gravity, one might expect that there is a local mass scale $m(\phi_0)$ that enters, as considered in [7], where the local structure of the moduli space is furthermore parametrized in the context of a single scalar field by an additional factor $\Gamma(\phi_0, \Delta \phi)$. In the present context this local factor takes the form $\Gamma(\phi_0, D_V(\phi^I))$ with $D_V(\phi^I)$ as in eq. (11).

Current CMB observations [36, 37] are consistent with slow-roll inflation, for which the relative gradients $\epsilon_I = M_{Pl}V^I/V$ are small. Furthermore, these parameters should vary slowly so as to achieve the standard range of e-folds, usually required to be bounded by $N_e \in [50, 70]$. In many models inflation ends eventually and the end is reached as the slow-roll parameter $\epsilon$ approaches unity. This means that the slow-roll approximation will fail toward the end of the trajectory and for those regions the potential term in $D_V$ will be significant.

The spectrum conjecture can be related to the weak gravity conjecture [39] under the assumption that the gauge coupling varies with the scalar field evolution [7]. This relation builds on the general expectation that in a quantum gravity context continuous coupling parameters are determined by vevs of scalar fields, an assumption that has been encoded as conjecture zero in the swampland literature [2]. In its simplest and most robust form the WGC states that in any consistent theory of quantum gravity with a U(1) gauge group there exists a particle whose mass is bounded by

$$m \leq qgM_{Pl}, \quad (15)$$

where $g$ is a dimensionless measure of the U(1) coupling parameter and $q$ is the relative charge. If the coupling parameter is a function of some scalar fields $\phi^I$ then the evolution of these fields of course changes the mass of at least this particular particle. While in our universe this relation holds with a very wide margin, indicating that a further ingredient of the gravity story is missing, this conjecture has been explored extensively for its possible implications for large field inflation. In a general formulation of the weak gravity conjecture the distance enters and therefore the consideration of $D_V$ also has implications for the conceptual development of the weak gravity conjecture. Finally, the spectrum conjecture has very recently also been related to the de Sitter gradient conjecture in ref. [5].

Acknowledgement.

It is a pleasure to thank Monika Lynker for discussions. This work was supported in part by a Faculty Research Grant at Indiana University South Bend.
References

[1] C. Vafa, *The string landscape and the swampland*, arXiv: hep-th/0509212

[2] H. Ooguri and C. Vafa, *On the geometry of the string landscape and the swampland*, Nucl. Phys. B766 (2007) 21 – 33, arXiv: hep-th/0605264

[3] T.D. Brennan, F. Carta and C. Vafa, *The string landscape, the swampland, and the missing corner*, arXiv: 1711.00864 [hep-th]

[4] G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, *de Sitter and the swampland*, arXiv: 1806.08362

[5] H. Ooguri, E. Palti, G. Shiu and C. Vafa, *Distance and de Sitter conjectures on the swampland*, arXiv: 1810.05506 [hep-th]

[6] M.R. Douglas and Z. Lu, *Finiteness of volume of moduli spaces*, arXiv: hep-th/0509224 [hep-th]

[7] D. Klaewer and E. Palti, *Super-Planckian spatial field variations and quantum gravity*, JHEP 01 (2016) 088, arXiv: 1610.00010 [hep-th]

[8] R. Blumenhagen, I. Valenzuela and F. Wolf, *The swampland conjecture and F-term axion monodromy inflaton*, JHEP 07 (2017) 145, arXiv: 1703.05776 [hep-th]

[9] E. Palti, *The weak gravity conjecture and scalar fields*, JHEP 08 (2017) 034, arXiv: 1705.04328 [hep-th]

[10] A. Hebecker, P. Henkenjohann and L.T. Witkowski, *Flat monodromies and a moduli space size conjecture*, JHEP 12 (2017) 033, arXiv: 1708.06761 [hep-th]

[11] T. Grimm, E. Palti and I. Valenzuela, *Infinite distances in field space and massless towers of states*, JHEP 08 (2018) 143, arXiv: 1802.08264

[12] R. Blumenhagen, D.Klaewer, L. Schlechter and F. Wolf, *The refined swampland distance conjecture in CY moduli spaces*, JHEP 06 (2018) 052, arXiv: 1803.04989 [hep-th]

[13] R. Blumenhagen, *Large field inflation/quintessence and the refined swampland distance conjecture*, PoS Corfu2017 (2018) 175, arXiv: 1804.10504 [hep-th]

[14] A. Landete and G. Shiu, *Mass hierarchies and dynamical field range*, Phys. Rev. D98 (2018) 066012, arXiv: 1806.01874 [hep-th]
[15] S.-J. Lee, W. Lerche and T. Weigand, *Tensionless strings and the weak gravity conjecture*, arXiv: 1808.05958 [hep-th]

[16] S.-J. Lee, W. Lerche and T. Weigand, *A stringy test of the scalar weak gravity conjecture*, arXiv: 1810.05169 [hep-th]

[17] Q.-G. Huang, *Constraints on the spectral index for the inflation models in the string landscape*, Phys. Rev. **D76** (2007) 061303(R)

[18] P. Agrawal, G. Obied, P.J. Steinhardt and C. Vafa, *On the cosmological implications of the string swampland*, Phys. Lett. **B784** (2018) 271, arXiv: 1806.09718 [hep-th]

[19] A. Kehagias and A. Riotto, *A note on inflation and the swampland*, arXiv: 1807.05445 [hep-th]

[20] M. Dias, J. Frazer, A. Retolaza and A. Westphal, *Primordial gravitational waves and the swampland*, Fortschr. Phys. (2018) 180063, arXiv: 1807.06579 [hep-th]

[21] H. Matsui and F. Takahashi, *Eternal inflation and swampland conjectures*, arXiv: 1807.11938 [hep-th]

[22] W.H. Kinney, S. Vagnozzi and L. Visinelli, *The zoo plot meets the swampland: mutual (in)consistency of single-field inflation, string conjectures, and cosmological data*, arXiv: 1808.06421 [astro-ph.CO]

[23] S. Brahma and M.W. Hossain, *Avoiding the string swampland in single-field inflation: excited initial conditions*, arXiv: 1809.01277 [hep-th]

[24] S. Das, *A note on single-field inflation and the swampland criteria*, arXiv: 1809.03962 [hep-th]

[25] M. Kawasaki and V. Takhistov, *Primordial black holes and the string swampland*, arXiv: 1810.02547 [hep-th]

[26] M. Motaharfar, V. Kamali and R.O. Ramos, *Warm way out of the swampland*, arXiv: 1810.02816 [astro-ph.CO]

[27] A. Ashoorioon, *Rescuing single field inflation from the swampland*, arXiv: 1810.04001 [hep-th]

[28] A. Nicolis, *On super-Planckian fields at sub-Planckian energies*, JHEP **07** (2008) 023, arXiv: 0802.3923 [hep-th]
[29] T.C. Bachlechner, C. Long and L. McAllister, *Planckian axions and the weak gravity conjecture*, JHEP 01 (2016) 091, arXiv: 1503.07853

[30] A. Achucarro and G.A. Palma, *The string swampland constraints require multi-field inflation*, arXiv: 1807.04390 [hep-th]

[31] I. Ben-Dayan, *Draining the swampland*, arXiv: 1808.01615 [hep-th]

[32] R. Schimmrigk, *Automorphic inflation*, Phys. Lett. B748 (2015) 376, arXiv: 1412.8537 [hep-th]

[33] R. Schimmrigk, *A general framework for automorphic inflation*, JHEP 05 (2016) 140, arXiv: 1512.09082 [hep-th]

[34] R. Schimmrigk, *Modular inflation observables and phenomenology of $j$-inflation*, JHEP 09 (2017) 043, arXiv: 1612.09559 [hep-th]

[35] R. Schimmrigk, *Multifield reheating after modular $j$-inflation*, Phys. Lett. B782 (2018) 193, arXiv: 1712.09669 [hep-ph]

[36] N. Aghanim et al. (PLANCK Collab.), *Planck 2018 results. VI. Cosmological parameters*, arXiv: 1807.06209 [astro-ph.CO]

[37] Y. Akrami et al. (PLANCK Collab.), *Planck 2018 results. X. Constraints on inflation*, arXiv: 1807.06211 [astro-ph.CO]

[38] B. Heidenreich, M. Reece and T. Rudelius, *Emergence and the swampland conjectures*, Phys. Rev. Lett. 121 (2018) 051601, arXiv: 1802.08698 [hep-th]

[39] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, *The string landscape, black holes and gravity as the weakest force*, JHEP 06 (2007) 060, arXiv: hep-th/0601001