Generalized ghost dark energy in Brans-Dicke theory

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It was argued that the vacuum energy of the Veneziano ghost field of QCD, in a time-dependent background, can be written in the general form, \( H + O(H^2) \), where \( H \) is the Hubble parameter. Based on this, a phenomenological dark energy model whose energy density is of the form \( \rho = \alpha H + \beta H^2 \) was recently proposed to explain the dark energy dominated universe. In this paper, we investigate this generalized ghost dark energy model in the setup of Brans-Dicke cosmology. We study the cosmological implications of this model. In particular, we obtain the equation of state and the deceleration parameters and a differential equation governing the evolution of this dark energy model. It is shown that the equation of state parameter of the generalized ghost dark energy can cross the phantom line \( (w_D = -1) \) in some range of the parameters spaces.

I. INTRODUCTION

Nowadays, it is a general belief that our Universe is currently experiencing a phase of accelerated expansion. Various cosmological observations confirm this acceleration. The first significant evidence was given from measurements of type Ia supernovae [SNeIa] 1, 2. These results have been confirmed repeatedly by several other observations such as measurement of the anisotropies of the cosmic microwave background (CMB), spectrum by the Wilkinson Microwave Anisotropy Probe (WMAP) 3, and by the measurement of the baryon acoustic oscillations (BAO) in the Sloan Sky Digital Survey (SDSS) luminous galaxy sample 4.

To explain such a phase of acceleration in the framework of Einstein gravity we need to assume that the universe is filled with an unknown type of energy component which is called dark energy (DE). This component of energy has a negative equation of state parameter (EoS) \( w = p/\rho < -\frac{1}{3} \) and is responsible for such an acceleration. One main task for the theoretical physicists is to identify the nature of such DE. The simplest candidate is the famous cosmological constant with \( w = -1 \), which is still one of the best, among various models, in agreement with observations. However, this candidate suffers the so called fine-tuning and the coincidence problems 5. Further observations favor alternatives whose EoS parameter change with time. Simplest example of this class is scalar fields which have time varying EoS parameters. An incomplete list of the scalar filed or time varying EoS parameter models can be found in 6–15. In addition to the DE approach, one can also explain the late time acceleration of the universe with modified gravity 16 or inhomogeneous cosmology 17. In these approaches one assumes that the underlying theory of gravity should be modified in such a way that the acceleration of the universe expansion can be derived naturally from the theory.

Since any new model of DE has many unknown features and can lead new problems in the literature, our prior choice is to handle the DE problem without introducing new degrees of freedom beyond what are already known. Recently one class of such models has been attracted a lot of attentions, the so called “ghost dark energy” (GDE). In this approach the Veneziano ghost field is responsible for the recent cosmic acceleration. The Veneziano ghost field was recently proposed to explain the U(1) problem in QCD. The U(1) problem is that the Lagrangian of QCD has, in the massless limit, a global chiral U(1) symmetry, which does not seem to be reflected in the spectrum of light pseudoscalar mesons. The ghost field has no contribution in the vacuum energy in Minkowski spacetime but in the time dependent or non-limit, a global chiral U(1) symmetry, which does not seem to be reflected in the spectrum of light pseudoscalar mesons.

For example, the following model has been proposed

\[
\rho = \alpha H + \beta H^2
\]

where \( \alpha \) and \( \beta \) are constants. This model is called GDE model. It is shown that the equation of state parameter of the generalized ghost dark energy can cross the phantom line \( (w_D = -1) \) in some range of the parameters spaces.

\[
\alpha = H + \frac{\beta}{3} H^2
\]

It was argued that the vacuum energy of the ghost field can be written as

\[
\rho = \alpha H + \beta H^2
\]

This term can also lead to a better agreement with observations 16–23.

On the other hand, in recent years, scalar tensor theories have been reconsidered extensively. The reason comes

\[
H = H + \frac{\beta}{3} H^2
\]

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from the fact that scalar fields appear in different branches of theoretical physics as a consistency condition. For example, the low energy limit of the string theory leads to introducing a scalar degree of freedom. One important example of the scalar tensor theories is the Brans-Dicke (BD) theory of gravity which was presented by Brans and Dicke in 1961 to incorporate the Mach’s principle in the Einstein’s theory of gravity \cite{28}. This theory also passed the observational tests in the solar system domain \cite{29}. Since the generalized GDE (GGDE) model have a dynamic behavior it is more reasonable to consider this model in a dynamical framework such as BD theory. It was shown that some features of original GDE in BD cosmology differ from Einstein’s gravity. For example while the original GDE is instable in all range of the parameter spaces in standard cosmology \cite{23}, it leads to a stable phase in BD theory \cite{30}. All the above reasons motivate us to investigate the GGDE model with subleading term $H^2$ in the framework of BD theory.

This paper is organized as follows. In the next section, we review the GGDE model in standard cosmology. In section III we extend the study to the framework of BD cosmology. We finish with closing remarks in section IV.

**II. GENERALIZED GHOST DARK ENERGY MODEL**

For flat FRW universe filled with GDE and pressureless dark matter, the first Friedmann equation may be written as

$$ H^2 = \frac{1}{3M_p^2}(\rho_M + \rho_D), $$

(1)

where $\rho_m$ and $\rho_D$ are, respectively, the energy densities of pressureless matter and ghost dark energy. The generalized ghost energy density is \cite{26}

$$ \rho_D = \alpha H + \beta H^2, $$

(2)

where $\alpha$ is a constant with dimension $[\text{energy}]^3$, roughly of order $\Lambda_{\text{QCD}}^3$ and $\Lambda_{\text{QCD}} \approx 100 MeV$ is QCD mass scale, and $\beta$ is another constant with dimension $[\text{energy}]^2$. We define the dimensionless density parameters as usual,

$$ \Omega_m = \frac{\rho_m}{\rho_{cr}}, \quad \Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{\alpha + \beta H}{3M_p^2 H}, $$

(3)

where the critical energy density is $\rho_{cr} = 3H^2M_p^2$. Thus, the Friedmann equation can be rewritten as $\Omega_m + \Omega_D = 1$. The conservation equations read

$$ \dot{\rho}_m + 3H \rho_m = 0, $$

(4)

$$ \dot{\rho}_D + 3H \rho_D(1 + w_D) = 0. $$

(5)

Integrating Eq. (4), we find

$$ \rho_m = \rho_{m0}(1 + z)^3, $$

(6)

where $z = 1/a - 1$ is the redshift function. Thus, Friedmann equation (1) can be written

$$ H^2 \Gamma - \alpha H = \rho_{m0}(1 + z)^3, $$

(7)

where $\Gamma = 3M_p^2 - \beta$. Solving the above equation we find

$$ H(z) = \frac{1}{2\Gamma} \left( \alpha \pm \sqrt{\alpha^2 + 4\Gamma \rho_{m0}(1 + z)^3} \right). $$

(8)

Using the fact that $\rho_{m0} = 3M_p^2H_0^2\Omega_{m0} = (\Gamma + \beta)H_0^2\Omega_{m0}$, the above equation can be further rewritten as

$$ H(z) = \frac{1}{2\Gamma} \left( \alpha \pm \sqrt{\alpha^2 + 4\Gamma(\Gamma + \beta)H_0^2\Omega_{m0}(1 + z)^3} \right). $$

(9)

Using the energy density ratio relation $u = \rho_m/\rho_D$, the Friedmann equation can be also written as

$$ 3M_p^2H^2 = (1 + u)\rho_D. $$

(10)
Taking the time derivative of relation (2) and using the Friedmann equation we find \( \dot{\rho}_D = (\alpha + 2\beta H)\dot{H} \). Also, from the Friedmann equation as well as continuity equations we have

\[
\dot{H} = -\frac{1}{2M_p^2}(1 + u + w_D)\rho_D. \tag{11}
\]

Substituting \( \dot{\rho}_D \) into Eq. (15), after some simplifications, we find the EoS parameter of the GGDE as

\[
w_D = \frac{1}{2} \left( \frac{1}{1 + \xi H} \right), \tag{12}
\]

where \( \xi = \beta/\alpha \). It is easy to see that at the early time where \( \Omega_D \ll 1 \) we have

\[
w_D = -\frac{1}{2} \left( 1 + \xi H \right), \tag{13}
\]

while at the late time where \( \Omega_D \rightarrow 1 \) the GGDE mimics a cosmological constant, namely \( w_D = -1 \). This behavior is similar to the original GDE [21]. When \( \xi = \beta = 0 \), one recovers the EoS parameter of the original GDE [21]

\[
w_D = -\frac{1}{2} \left( 1 - \Omega_D \right). \tag{14}
\]

Also, the deceleration parameter is obtained as

\[
q = -1 - \frac{\dot{H}}{H^2} = -1 + \frac{3}{2} \Omega_D (1 + u + \omega_D), \tag{15}
\]

where we have used Eq. (11) and relation \( \rho_D = 3M_p^2 H^2 \). Substituting \( w_D \) from (12), we can further simplify \( q \) as

\[
q = \frac{1}{2} - \frac{3}{2} \Omega_D \left[ 2 - \Omega_D + 2\xi H (1 - \Omega_D) \right]^{-1}. \tag{16}
\]

At the late time where the dark energy dominates (\( \Omega_D \rightarrow 1 \)) we have \( q = -1 \). Taking the time derivative of \( \Omega_D \) in Eq. (3), we find

\[
\dot{\Omega}_D = \frac{\alpha}{3M_p^2} \frac{\dot{H}}{H^2} = \frac{\Omega_D H}{1 + \xi H} (1 + q) \tag{17}
\]

Using relation \( \dot{\Omega}_D = H \frac{d\Omega_D}{d\ln a} \), we obtain

\[
\frac{d\Omega_D}{d\ln a} = \frac{\Omega_D}{1 + \xi H} (1 + q) \tag{18}
\]

\[
= \frac{3}{2} \left( \frac{\Omega_D}{2 - \Omega_D} + 2\xi H (1 - \Omega_D) \right). \tag{19}
\]

This is the equation of motion governing the evolution of GGDE. The evolution of \( H \) in Eqs. (16) and (19) is given by Eq. (19).

### III. GENERALIZED GHOST DARK ENERGY IN BD COSMOLOGY

The action of BD theory, in the canonical form, can be written [31]

\[
S = \int d^4x \sqrt{g} \left( -\frac{1}{8\omega} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + L_M \right), \tag{20}
\]

where \( R \) is the scalar curvature and \( \phi \) is the BD scalar field. Varying the above action with respect to FRW metric, \( g_{\mu\nu} \), and the BD scalar field \( \phi \), we obtain the following field equations

\[
\frac{3}{4\omega} \phi^2 \left( H^2 + \frac{k}{a^2} \right) - \frac{1}{2} \phi^2 + \frac{3}{2\omega} H \dot{\phi} \phi = \rho_m + \rho_D, \tag{21}
\]

\[
-\frac{1}{4\omega} \phi^2 \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) - \frac{1}{\omega} H \dot{\phi} \phi - \frac{1}{2\omega} \ddot{\phi} - \frac{1}{2} \left( 1 + \frac{1}{\omega} \right) \phi^2 = p_D, \tag{22}
\]

\[
\ddot{\phi} + 3H \dot{\phi} - \frac{3}{2\omega} \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) \phi = 0, \tag{23}
\]
Hereafter, we consider the flat FRW universe, thus we set $k = 0$. At this point the system of our equations is not closed and we still have a freedom to choose the scalar field. In the framework of BD cosmology the BD scalar field $\phi$ is usually assumed to has a power law relation in terms of scale factor, namely

$$\phi = \phi_0 a(t)^\varepsilon. \quad (24)$$

A case of particular interest is that when $\varepsilon$ is small whereas $\omega$ is high so that the product $\varepsilon \omega$ results of order unity [34]. This is interesting because local astronomical experiments set a very high lower bound on $\omega$ [35]: in particular, the Cassini experiment implies that $\omega > 10^4$ [29, 36]. Taking the time derivative of relation (24), we obtain

$$\frac{\dot{\phi}}{\phi} = \varepsilon \frac{\dot{a}}{a} = \varepsilon H. \quad (25)$$

Using Eqs. (24) and (25), the first Friedmann equation (21) can be rewritten ($k = 0$)

$$H^2 (1 - 2\omega \varepsilon^2 + 2\varepsilon) = \frac{4\omega}{3\phi^2} (\rho_D + \rho_m). \quad (26)$$

We introduce the fractional energy densities corresponding to each energy component as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{4\omega \rho_m}{3\phi^2 H^2}, \quad (27)$$
$$\Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{4\omega \rho_D}{3\phi^2 H^2}, \quad (28)$$

where we have defined the critical energy density as

$$\rho_{cr} = \frac{3\phi^2 H^2}{4\omega}. \quad (29)$$

The reason for this definition comes from the fact that in BD theory, the non-minimal coupling term $\phi^2 R$ replaces with the Einstein-Hilbert term $R/G$ in such a way that $G_{eff}^{-1} = 2\pi \phi^2 / \omega$, where $G_{eff}$ is the effective gravitational constant as long as the dynamical scalar field $\phi$ varies slowly.

Using generalized ghost energy density (2) we can rewrite Eq. (28) as

$$\Omega_D = \frac{4\omega}{3\phi^2 H} (\alpha + \beta H), \quad (30)$$

Using definitions (27) and (28), Eq. (26) can be expressed as

$$\Omega_D + \Omega_m = \gamma, \quad (31)$$

where we have defined

$$\gamma = 1 - \frac{2\omega}{3} \varepsilon^2 + 2\varepsilon. \quad (32)$$

Clearly for $\varepsilon = 0$ ($\omega \rightarrow \infty$) we have $\gamma = 1$. In this case the BD scalar field becomes constant and Einstein gravity is restored. Taking the time derivative of Friedmann equation (26) we find

$$2H \dot{H} \gamma = \frac{4\omega}{3\phi^2} (\dot{\rho}_D + \dot{\rho}_m) - \frac{8\omega}{3\phi^2} (\rho_D + \rho_m). \quad (33)$$

Using the continuity equations, as well as Eq. (26) we get

$$2\dot{H} \gamma = -\frac{4\omega \rho_D}{\phi^2} (1 + u + \omega_D) - \frac{8\omega}{3\phi^2} \rho_D (1 + u). \quad (34)$$

Dividing by $H^2$, we have

$$\frac{2\dot{H} \gamma}{H^2} = -3\Omega_D (1 + u + \omega_D) - 2\varepsilon \Omega_D (1 + u). \quad (35)$$
Using the fact that \((1 + w)\Omega_D = \gamma\), we obtain
\[
\frac{\dot{H}}{H} = -H \left( \varepsilon + \frac{3}{2} + \frac{3 \Omega_D \omega_D}{\gamma} \right).
\] (36)

Finally for the time derivative of ghost energy density, we obtain
\[
\dot{\rho}_D = (\alpha + 2\beta H) \dot{H} = -H^2 \left( \varepsilon + \frac{3}{2} + \frac{3 \Omega_D \omega_D}{\gamma} \right) (\alpha + 2\beta H).
\] (37)

Inserting this relation in Eq. (3), it is a matter of calculation to show that
\[
w_D = \frac{\gamma \left( \frac{2}{3} \varepsilon - 1 \right) + \frac{4\xi H \gamma}{3}}{(2\gamma - \Omega_D) + 2\xi H (\gamma - \Omega_D)}.
\] (38)

When \(\beta = 0\) one recovers
\[
w_D = \frac{\gamma}{(2\gamma - \Omega_D)} \left( \frac{2}{3} \varepsilon - 1 \right).
\] (39)

which is the EoS parameter of the original GDE in BD theory presented in [22]. On the other hand, in the absence of BD scalar field \(\varepsilon = 0\) \((\gamma = 1)\) we obtain the result of the previous section, namely
\[
w_D = \frac{-1}{(2 - \Omega_D) + 2\xi H (1 - \Omega_D)}.
\] (40)

The solar-system experiments give the lower bound for the value of \(\omega\) to be \(\omega > 40000\) [29]. However, when probing the larger scales, the limit obtained will be weaker than this result. It was shown [36] that \(\omega\) can be smaller than 40000 on the cosmological scales. Also, Wu and Chen [37] obtained the observational constraints on BD model in a flat universe with cosmological constant and cold dark matter using the latest WMAP and SDSS data. They found that within 2\(\sigma\) range, the value of \(\omega\) satisfies \(\omega < -120.0\) or \(\omega > 97.8\) [37]. They also obtained the constraint on the rate of change of \(G\) at present
\[
-1.75 \times 10^{-12} \text{yr}^{-1} < \frac{\dot{G}}{G} < 1.05 \times 10^{-12} \text{yr}^{-1},
\] (41)
at 2\(\sigma\) confidence level. As a result in our case with assumption [24] we get
\[
\frac{\dot{G}}{G} = \frac{\dot{\phi}}{\phi} = \varepsilon H < 1.05 \times 10^{-12} \text{yr}^{-1}.
\] (42)

This relation can be used to put an upper bound on \(\varepsilon\). Assuming the present value of the Hubble parameter to be \(H_0 \simeq 0.7\), we obtain
\[
\varepsilon < 0.01.
\] (43)

The GGDE model in BD framework has an interesting feature compared to the GDE model in Einstein’s gravity. It was shown that in standard cosmology based on Einstein’s theory, the EoS parameter of the noninteracting GGDE cannot cross the phantom line \(w_D = -1\) and at the late time where \(\Omega_D \to 1\) approaches \(-1\) [21]. Choosing \(\Omega_D0 = 0.72\), \(H_0 = 0.7\) for the present time and a suitable choice of \(\xi\), this inequality valid provided we take \(\varepsilon < 0.01\) in a narrow range which is consistent with observations. This indicates that one can generate a phantom-like EoS for the GGDE in the BD framework. One should note that increasing \(\xi\) can exclude crossing the phantom line which indicates a negative contribution of the subleading term \(H^2\) with respect to the leading term \(H\) in the energy density.

Since the dynamic of the universe should be discussed in term of the effective EoS parameter thus in addition to the EoS parameter of the GGDE we also study the effective EoS parameter, \(w_{\text{eff}}\), which is defined as
\[
w_{\text{eff}} = \frac{\rho_t}{\rho_t + \rho_M},
\] (44)

where \(\rho_t\) and \(P_t\) are, respectively, the total energy density and total pressure of the universe. As usual we assumed the dark matter is in the form of pressureless fluid \((P_M=0)\). Using relation [35] for the flat case one can find
\[
w_{\text{eff}} = \frac{\Omega_D}{\gamma} w_D = \frac{\Omega_D \left( \frac{2}{3} \varepsilon - 1 \right) + \frac{4\xi H \Omega_D}{3}}{(2\gamma - \Omega_D) + 2\xi H (\gamma - \Omega_D)}.
\] (45)
FIG. 1: In these figures $w_{eff}$ and $q$ for GGDE and GDE are plotted against $\Omega_D$. Solid curve corresponds to GGDE and dashed ones belong to GDE. In both of these figures we take $\omega = 10000, \varepsilon = 0.003, H_0 = 0.7, \xi = 0.5$.

It is also interesting to study the behavior of the deceleration parameter defined as

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}. \tag{46}$$

Substituting Eq. (11) in the above relation one can easily reach

$$q = \frac{1}{2} + \varepsilon + \frac{3}{2} \frac{\Omega_D w_D}{\gamma}. \tag{47}$$

Inserting Eq. (38) into (47) yields

$$q = \frac{1}{2} + \varepsilon + \frac{3}{2} \frac{\Omega_D \left(\frac{2}{3} \varepsilon - 1\right) + 4\varepsilon H \Omega_D}{\gamma (2 \gamma - \Omega_D) + 2 \varepsilon H (\gamma - \Omega_D) + \varepsilon}. \tag{48}$$

When $\beta = 0$ we obtain

$$q = \frac{1}{2} + \varepsilon + \frac{3}{2} \frac{\Omega_D \left(\frac{2}{3} \varepsilon - 1\right)}{\gamma (2 \gamma - \Omega_D)}. \tag{49}$$

In the limiting case $\varepsilon = 0 \ (\omega \to \infty)$ we have $\gamma = 1$ and hence the BD scalar field becomes trivial; as a result Eq. (48) reduces to its respective expression in flat standard cosmology obtained in the previous section

$$q = \frac{1}{2} - \frac{3}{2} \Omega_D \left[2 - \Omega_D + 2 \xi H (1 - \Omega_D)\right]^{-1}. \tag{50}$$

Let us study some special cases of interest for the deceleration parameter $q$. If we take $\Omega_{D0} = 0.72, H_0 \simeq 0.7$ and for the present time and choosing $\varepsilon = 0.002, \xi = 0.1$ and $\omega = 10000$ we obtain $q_0 = -0.35$, which is consistent with the present value of the deceleration parameter obtained in [38]. Transition from deceleration to acceleration occurs at $\Omega_D = 0.46$. It is worth noting that GGDE results in a smaller rate of acceleration in comparison with the GDE then this models leads a delay in different epoches of the cosmic evolution with respect to the original GDE (for instance the GDE with choice of a same set of parameters lead $q_0 = -0.37$). For a better insight about these two models we plotted $w_{eff}$ and $q$ for both of models in Fig1.

Finally, we obtain a differential equation governing the evolution of GGDE from early deceleration to the late time acceleration. Following the method of the previous section we find

$$\dot{\Omega}_D = \frac{\alpha \Omega_D H}{\alpha + \beta H} (q + 1) - 2 \Omega_D \varepsilon H. \tag{51}$$

$$= \Omega_D H \left(\frac{1 + q}{1 + \xi H} - 2 \varepsilon\right). \tag{52}$$

Inserting $q$ from (48) and using relation $\dot{\Omega}_D = H\Omega_D'$, we obtain

$$\dot{\Omega}_D = \Omega_D \left(\frac{1 + q}{1 + \xi H} - 2 \varepsilon\right), \tag{53}$$

where the prime denotes derivative with respect to $x = \ln a$ and $q$ is given by Eq. (48). In the limiting case $\beta = 0$ one recovers the result obtained in [22].
IV. CLOSING REMARKS

A phenomenological GGDE model whose energy density is of the form $\rho = \alpha H + \beta H^2$ was recently proposed to explain the observed acceleration of the universe expansion. This model originates from the fact that the vacuum energy of the Veneziano ghost field in QCD is of the form, $H + O(H^2)$. It was shown that the difference between the vacuum energy of quantum fields in Minkowski space and in FRW universe can play the role of observed dark energy.

In this paper we first studied the cosmological implications of the GGDE model in standard cosmology. We can classify our achievements in two categories. The first is that in the BD framework the GGDE can cross the phantom line with suitable choice of the free parameters while in the standard cosmology we found a de Sitter phase as the fate of the universe. The second result is that the subleading term $H^2$ can lead a delay in different epoches of the cosmic evolution. We also discussed this result explicitly by a numerical evaluation and showed that taking a same set of parameters result $q_0 = -0.35$ for the GGDE while for GDE gives $q_0 = -0.37$. It is easily seen from Fig.1 that GDE (dashed curve) enters the acceleration phase sooner than the GGDE (solid curve). This point was also addressed in standard cosmology \cite{26} as the negative contribution of the subleading term $H^2$ in the energy density.

Finally, we would like to mention that in the present work we only considered the mathematical presentation of the GGDE model in the framework of Brans-Dicke cosmology. Other aspects of this model will be addressed in the separate works. For example, recently, we have studied the stability of the GGDE model against perturbations in the FRW background. Based on the square sound speed analysis, due to the existence of a free parameter in this model, the GGDE model is theoretically capable to lead a dark energy dominated stable universe \cite{39}. The extension of this study to the Brans-Dicke cosmology is under investigation and will be addressed elsewhere.

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