Is there an information-loss problem for black holes?

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Abstract. Black holes emit thermal radiation (Hawking effect). If after black-hole evaporation nothing else were left, an arbitrary initial state would evolve into a thermal state ('information-loss problem'). Here it is argued that the whole evolution is unitary and that the thermal nature of Hawking radiation emerges solely through decoherence—the irreversible interaction with further degrees of freedom. For this purpose a detailed comparison with an analogous case in cosmology (entropy of primordial fluctuations) is presented. Some remarks on the possible origin of black-hole entropy due to interaction with other degrees of freedom are added. This might concern the interaction with quasi-normal modes or with background fields in string theory.

1 The information-loss problem

Black holes are amazing objects. According to general relativity, stationary black holes are fully characterised by just three numbers: Mass, angular momentum, and electric charge. This “no-hair theorem” holds within the Einstein-Maxwell theory in four spacetime dimensions. It reminds one at the properties of a macroscopic gas which can be described by only few variables such as energy, entropy, and pressure. In fact, there exist laws of black-hole mechanics which are analogous to the laws of thermodynamics (see e.g. [1] for a detailed review). The temperature is proportional to the surface gravity, \( \kappa \), of the black hole, and the entropy is proportional to its area, \( A \). That this correspondence is not only a formal one, but possesses physical significance, was shown by Hawking in his seminal paper [2]. Considering quantum field theory on the background of a collapsing star (see Fig. 1), it is found that the black hole radiates with a temperature proportional to \( \hbar \),

\[
T_{\text{BH}} = \frac{\hbar \kappa}{2\pi k_B}.
\]

(1)

The origin of this temperature is the presence of a horizon. Due to the high gravitational redshift in its vicinity (symbolised in Fig. 1 by the dashed line \( \gamma \) near the horizon \( \gamma_H \)), the vacuum modes are excited for a very long time, until the black hole has evaporated. The entropy connected with this temperature is given by the ‘Bekenstein-Hawking’ formula,

\[
S_{\text{BH}} = \frac{k_B A}{4G\hbar}.
\]

(2)
In the case of a spherically-symmetric ('Schwarzschild') black hole, one has \( \kappa = (4GM)^{-1} \), and the Hawking temperature is given by
\[
T_{\text{BH}} = \frac{\hbar}{8\pi G k_B M} \approx 6.2 \times 10^{-8} \frac{M_\odot}{M} \text{ K}.
\]
(3)

If the quantum field on the black-hole background is a massless scalar field, the expectation value of the particle number for a mode with wave number \( k \) is given by
\[
\langle n_k \rangle = \frac{1}{e^{8\pi \omega GM} - 1},
\]
(4)

with \( \omega = |k| = k \). This is a Planck distribution with temperature (3). The usual interpretation is 'particle creation': a mode with pure positive frequency will evolve (along the dashed line in Fig. 1) into a superposition of positive and negative frequencies. Therefore, an initial vacuum will evolve into a superposition of excited states ('particles'). In the Heisenberg picture, this is described by the occurrence of a non-vanishing Bogolubov coefficient \( \beta \).

An alternative point of view arises through the use of the Schrödinger picture. Taking the scalar field in Fourier space, \( \phi(k) \), the initial vacuum state can be expressed as a Gaussian wave functional. One assumes for simplicity that the full wave functional can be written as a product over independent modes, \( \Psi = \prod_k \psi_k \) (one can imagine putting the whole system into a box). Then,
\[
\psi_k \propto \exp \left[ -k|\phi(k)|^2 \right],
\]
(5)

see e.g. [3] for details about the functional Schrödinger picture. With this initial state, the functional Schrödinger equation can be solved exactly [4]. Due to the

![Fig. 1. Penrose diagram of a star to form a black hole](image-url)
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In the dynamical gravitational background, the various $\psi_k$, albeit always of Gaussian form, develop a complex function in the exponent [4,5],

$$\psi_k \propto \exp \left[ -k \coth(2\pi kGM + ikt)|\phi(k)|^2 \right].$$

This is still a pure state, but the expectation value of the particle number operator with respect to this state is of the same Planckian form as (4),

$$\langle \psi_k | n_k | \psi_k \rangle = \frac{1}{\exp[8\pi \omega GM] - 1}.$$

As a side remark I note that such a result can also be obtained from the wave functional solving the Wheeler-DeWitt equation in the WKB approximation [6].

A state such as (6) is well known from quantum optics and called a two-mode squeezed state. That Hawking radiation can be described in this terminology was first recognised – using the Heisenberg picture – in [7].

For spatial hypersurfaces that enter the horizon one must trace out the degrees of freedom which reside inside the horizon. This results in a thermal density matrix in the outside region [8], with the temperature being equal to the Hawking temperature [9].

The black-hole entropy (2) is much bigger than the entropy of a collapsing star. The entropy of the Sun, for example, is $S_\odot \approx 10^{57}$, but the entropy of a solar-mass black hole is $S_{BH} \approx 10^{77}$, i.e. twenty orders of magnitudes larger (all entropies are measured in units of $k_B$). If all matter in the observable Universe were in a single gigantic black hole, its entropy would be $S_{BH} \approx 10^{123}$. Black holes thus seem to be the most efficient objects for swallowing information.

Due to Hawking radiation, black holes have a finite lifetime. It is given by

$$\tau_{BH} \approx \left( \frac{M_0}{m_p} \right)^3 t_p \approx 10^{65} \left( \frac{M_0}{M_\odot} \right)^3 \text{ years},$$

where $m_p$ and $t_p$ denote Planck mass and Planck time, respectively. The question now arises what happens at the end of black-hole evaporation. If only thermal radiation were left behind, an arbitrary initial state (for the star collapsing to form a black hole) would evolve into a mixed state. This process does not happen in the standard quantum theory for closed systems. There, the entropy

$$S = -k_B \text{Tr}(\rho \ln \rho)$$

is conserved for the full system. Since a thermal state contains least information, one would be faced with the information-loss problem. This is, in fact, what
Hawking speculated to happen. The calculations in \[2\] are, however, restricted to the semiclassical approximation (gravity classical and matter quantum), which breaks down when the black hole approaches the Planck mass. The final answer can only be obtained within quantum gravity. The options are \[11\]

- Information is indeed lost during the evaporation, i.e. the evolution is non-unitary,
  \[\rho \rightarrow S\rho S^\dagger \neq \rho\]
- The full evolution is unitary, but this cannot be seen in the semiclassical approximation.
- The black hole leaves a remnant carrying all the information.

The state \[6\] also holds on a spatial surface that in Fig. 1 would start at the intersection of the collapsing star with the horizon and extend to spatial infinity \(i^0\), i.e. a surface that does not enter the horizon. In fact, such surfaces seem quite natural, since they correspond to constant Schwarzschild time far away from the black hole. The question then arises where the thermal nature of Hawking radiation comes from; although \[7\] is Planckian, the state \[6\] is pure and the differences to a thermal distribution can be recognised in higher-order correlation functions.

I shall argue in Sect. 3 that the thermal appearance of Hawking radiation can be understood, even for the pure state \[6\], through decoherence – the irreversible and unavoidable interaction with the environment \[10\]. For this purpose it will be appropriate to rewrite \[6\] in a form where the two-mode squeezed nature becomes explicit. In fact, one can rewrite \(\psi_k\) in the form

\[\psi_k \propto \exp \left[ -k \frac{1 + e^{2i\varphi_k} \tanh r_k}{1 - e^{2i\varphi_k} \tanh r_k} |\phi(k)|^2 \right] \equiv \exp \left[ -(\Omega_R + i\Omega_I) |\phi(k)|^2 \right], \tag{10}\]

where the squeezing parameter \(r_k\) is given by

\[\tanh r_k = \exp(-4\pi\omega GM), \tag{11}\]

and the squeezing angle \(\varphi_k\) reads

\[\varphi_k = -kt. \tag{12}\]

Thus, \(r_k \rightarrow 0\) for \(k \rightarrow \infty\) and \(r_k \rightarrow \infty\) for \(k \rightarrow 0\): modes with bigger wavelength are more squeezed than modes with smaller wavelength. At the maximum of the Planck spectrum one has \(r \approx 0.25\). This corresponds to \(\langle n_k \rangle = \sinh^2 r_k \approx 0.06\) for the expectation value of the particle number.

At \(kt = 0\) the squeezing is in \(\phi\), whereas at \(kt = \pi/2\) the squeezing is in \(p_\phi\), the momentum conjugate to \(\phi\). The ratio of the corresponding widths is \(\tanh^2 (2\pi k GM) \approx 0.37\) at the maximum of the Planck spectrum. Before I apply this to the study of decoherence, it is appropriate to review the analogous situation in inflationary cosmology. This will serve to understand the similarities to and the differences from the black-hole case.
2 Entropy of cosmological fluctuations

One of the most important advantages of an inflationary scenario for the early Universe is the possibility to obtain a dynamical explanation of structure formation (see e.g. [12] for a review). Quantum vacuum fluctuations are amplified by inflation, leading to a squeezed state (corresponding to particle creation in the Heisenberg picture). The scalar field(s) and the scalar part of the metric describe primordial density fluctuations, while the tensor part of the metric describes gravitons. The generation of gravitons is, in fact, an effect of linear quantum gravity. The primordial fluctuations can at a later stage serve as seeds for structure (galaxies and clusters of galaxies). They exhibit themselves in the CMB temperature anisotropy spectrum. The gravitons may lead to a stochastic gravitational-wave background that might in principle be observable with space-based experiments.

An important issue in the theoretical understanding of the above process is the exact way in which these quantum fluctuations become classical stochastic variables, see e.g. [13] for discussion and references. Two ingredients are responsible for the emergence of their classical behaviour. Firstly, inflation leads to a huge squeezing of the quantum state for the fluctuations and, therefore, to a huge particle creation (the number of generated particles for a mode with wave number \( k \) is \( N_k = \sinh^2 r_k \), where \( r_k \) is the squeezing parameter.) But in the limit of large \( r_k \) the quantum state becomes an approximate WKB state, corresponding in the Heisenberg picture to neglecting the part of the solution which goes as \( e^{-r_k} \) and which is called the “decaying mode”. Secondly, interactions with other, “environmental”, degrees of freedom lead to the field-amplitude basis (here called \( y_k \)) as the classical basis (“pointer basis”) with respect to which interferences become unobservable. This process of decoherence transforms the \( y_k \) into effective classical stochastic quantities. On the largest cosmological scales one finds for the squeezing parameter \( r_k \approx 100 \), far beyond any values which can be attained in the laboratory.

Whereas the quantum theory therefore does not lead to deviations from the usual predictions (based on a phenomenological classical stochastic theory) of the inflationary scenario, the entropy of the fluctuations depends on their quantum nature (on the presence of the decaying mode, albeit small), see [14]. The entropy of the squeezed quantum state is of course zero, because it is a pure state. Due to the interaction with other degrees of freedom, however, the fluctuations have to be described by a density matrix \( \rho \). The relevant quantity is then the von Neumann entropy \( \mathcal{S} \).

In the context of primordial fluctuations, different mechanisms of coarse-graining have been investigated in order to calculate the local entropy. It was found that the maximal value for the entropy is \( \mathcal{S}_{\text{max}} = 2r_k \), resulting from a coarse-graining with respect to the particle-number basis, i.e. non-diagonal elements of the reduced density matrix \( \rho \) are neglected in this basis. Consideration

\(^1\) In the following, “fluctuations” refers to primordial density fluctuations as well as gravitons.
of the corresponding contour of the Wigner ellipse in phase space shows that this would smear out the thin elongated ellipse of the squeezed state (corresponding to the high values of $\kappa$) into a big circle. Does such a coarse-graining reflect the actual process happening during inflation? Since the pointer basis in the quantum-to-classical transition is the field-amplitude basis and not the particle-number basis (which mixes the field variable with its canonical momentum), one would expect that coarse-graining should be done with respect to $y(k)$. This would then lead to $S = S_{\text{max}}/2$, which is noticeably different from maximal entropy $S_{\text{max}}$.

A crucial observation for the calculation of (9) is to note that the wavelength of the amplified fluctuations during inflation is bigger than the horizon scale, i.e. bigger than $H_{\text{I}}^{-1}$, where $H_{\text{I}}$ is the Hubble parameter of inflation (here taken to be approximately constant for simplicity). This prevents a direct causal interaction with the other, environmental, fields. However, nonlocal quantum correlations can still develop due to interaction terms in the total Hamiltonian. Since, as remarked above, the interaction is local in $y(k)$ (as opposed to its momentum), the density matrix will be of the form (suppressing $k$ for simplicity)

$$\rho_{\xi}(y, y') = \rho_0(y, y') \exp\left(-\frac{\xi}{2}(y - y')^2\right),$$  \hfill (13)

where $\rho_0(y, y')$ denotes the density matrix referring to the squeezed state, which is a Gaussian state, see e.g. [10] for a discussion. Eq. (13) guarantees that interferences in the $y$-basis become small, while the probabilities (diagonal elements) remain unchanged. The details of the interaction are encoded in the phenomenological parameter $\xi$ and are not needed for our discussion. Decoherence is efficient if the Gaussian in (13) dominates over the Gaussian from the squeezed state described by $\rho_0$. This leads to the condition

$$\frac{\xi e^{2r}}{k} \gg 1,$$  \hfill (14)

which is called the decoherence condition.

The entropy of the fluctuations is now calculated by setting $\rho = \rho_{\xi}$ in (9). For an arbitrary Gaussian density matrix, the result has been obtained in [15], see also Appendix A2.3 in [10]. With the abbreviation

$$\chi = \frac{\xi}{k}(1 + 4 \sinh^2 r),$$  \hfill (15)

the result is in our case given by the expression

$$S = -\ln 2 + \frac{1}{2} \ln \chi - \frac{\sqrt{1 + \chi}}{2} \ln \frac{\sqrt{1 + \chi} - 1}{\sqrt{1 + \chi} + 1}. \hfill (16)$$

The decoherence condition means $\chi \gg 1$, which leads for arbitrary $r$ to

$$S \approx 1 + \frac{1}{2} \ln \frac{\xi(1 + 4N)}{4k}. \hfill (17)$$
In the high-squeezing limit, $e^r \to \infty$, this yields
\[ S \approx 1 - \ln 2 + \frac{1}{2} \ln \frac{e^{2r} \xi}{k} = 1 + \frac{1}{2} \ln \frac{N \xi}{k}. \] (18)

In phase space this corresponds to $S \approx \ln A$, where $A$ is the area of the Wigner ellipse. Application of the decoherence condition (14) leads to
\[ S \ll 1 - \ln 2 \approx 0.31, \] (19)

where $\gg$ holds here in a logarithmic sense (it directly holds for the number of states $e^S$). Therefore, decoherence already occurs after few bits of information are lost. This is much less than the maximal entropy, which is obtained if the ellipse is smeared out to a big circle, corresponding to the choice $\xi/k = e^{2r}$ in (18), and leading to $S_{\text{max}} = 2r$, as has been remarked above. That only few bits of information loss can be sufficient for decoherence is well known from quantum optics [10]. The correlation between $y$ and its canonical momentum is only preserved if $S < S_{\text{max}} = 2r$ because otherwise the squeezed Wigner ellipse is no longer recognisable.

One would expect that the maximal possible entropy due to quantum entanglement alone (i.e. without dynamical back reaction) is obtained if the coarse-graining is performed exactly with respect to the field-amplitude basis $y$. As remarked above, this would lead to $S = r$. Inspecting (18), this would correspond to the choice $\xi = k$. Therefore, as long as the modes are outside the horizon, one would expect to have $\xi < k$ which has a very intuitive interpretation: the coherence length $\xi^{-1/2}$ is larger than the width of the ground state ($r = 0$), so that the environment does not spoil the property of the quantum state being squeezed in some direction compared to the ground state. It can be shown that the correlation between $y$ and the conjugate momentum remains for a sufficiently long time after the second horizon crossing (in the postinflationary phase), so that it really leads to the observed acoustic peaks (B-polarisation for gravitational waves) in the CMB. [16]. The information contained in these peaks can be interpreted as a measure for the deviation of the entropy from the maximal entropy. Therefore, coarse-grainings that lead to maximal entropy would prevent the occurrence of such peaks and would thus be in conflict with observation.

This analysis has also borne out an interesting analogy of the primordial fluctuations with a chaotic system: the Hubble parameter corresponds to a Lyapunov exponent, although our system is not chaotic, but only classically unstable [14]. In the next section the comparison of the cosmological case with the black-hole case will be made.

### 3 Hawking radiation from decoherence

As in the cosmological case, the quantum state corresponding to Hawking radiation is a two-mode squeezed state. There are, however, pronounced differences
in the black-hole case. As has been remarked at the end of Sect. 1, the squeezing parameter for the maximum of the Planck distribution is only $r_k \approx 0.25$, which is far below the values attained from inflation. High squeezing values are only obtained for very big wavelengths.

The quantum state can again be represented by the contour of the Winger ellipse in phase space. In the cosmological case the rotation of this ellipse is very slow, the corresponding time being about the age of the Universe \([10]\). This reflects the fact that it is not allowed to coarse-grain this ellipse into a big circle (Sect. 2). What about the situation for black holes? Again, the Wigner ellipse rotates around the origin, and the typical timescale is given by

$$t_k = \frac{\pi}{2k} . \tag{20}$$

This corresponds to the exchange of squeezing between $\phi$ and its conjugate momentum $p_{\phi}$, cf. the end of Sect. 1. Evaluated at the maximum of the Planck spectrum, one has

$$t_k(\text{max}) \approx 14GM \approx 7 \times 10^{-5} \frac{M}{M_\odot} \text{ s} , \tag{21}$$

which is much smaller than typical observation times. It is for this reason that a coarse-graining with respect to the squeezing angle can be performed. Squeezed states are extremely sensitive to interactions with environmental degrees of freedom \([10]\). In the present case of a quickly rotating squeezing angle this interaction leads to a diagonalisation of the reduced density matrix with respect to the particle-number basis \([17]\), not the field-amplitude basis. Thereby the local entropy is maximised, corresponding to the coarse-graining of the Wigner ellipse into a circle. The value of this entropy can be calculated along the lines of Sect. 2. In contrast to the cosmological case one finds the standard expression for a thermal ensemble,

$$S_k = \ln(1 + n_k) - n_k \ln n_k \frac{r_{k \gg 1}}{2} \ln 2 = \frac{1}{2}$$

The integration over all modes gives $S = (2\pi^2/45)T^3_{\text{BH}} V$, which is just the entropy of the Hawking radiation with temperature $T_{\text{BH}} = (8\pi GM)^{-1}$. In this way, the pure squeezed state becomes indistinguishable from a canonical ensemble with temperature $T_{\text{BH}} \equiv T$.

Independent of this practical indistinguishability from a thermal ensemble, the state remains a pure state. In fact, for timescales smaller than $t_k$ the above coarse-graining is not allowed and the difference to a thermal state could be seen in principle. For the case of a primordial black hole with mass $M \approx 5 \times 10^{14}$ g one has at the maximum of the Planck spectrum $t_k(\text{max}) \approx 1.7 \times 10^{-23}$ s. The observation of Hawking radiation at smaller times could then reveal the difference between the (true) pure state and a thermal state. It is also clear from \([20]\) that large wavelengths (much larger than the wavelength corresponding to the maximum of the Planck spectrum) have a much longer rotation time for the Wigner ellipse. This would also offer, in principle, the possibility to
distinguish observationally between pure and mixed states – provided, of course, that primordial black holes exist and can be observed.

To summarise, no mixed state for the total system has appeared at any stage of this discussion. This indicates that the full quantum evolution of collapsing star plus scalar field evolves unitarily. The thermal nature of Hawking radiation thus only emerges through coarse-graining. For hypersurfaces entering the horizon, this is achieved by tracing out degrees of freedom referring to the interior [8]. As has been emphasised here, however, a similar result holds for hypersurfaces that stay outside the horizon. Such hypersurfaces are a natural choice for asymptotic observers. The mixed appearance of the pure state for the quantum field is due to its squeezed nature. Squeezed states are very sensitive to interactions with other fields, even for very weak coupling. Such interactions lead to decoherence for the Hawking radiation. The thermal nature of the reduced density matrix is a consequence of the presence of the horizon as encoded in the particular squeezed state [6], cf. [4].

Similar conclusions hold for hypersurfaces entering the horizon [5]. A general problem is, however, that the evolution along different foliations is in general not unitarily equivalent [18]. Observations at spatial infinity cannot thus be correctly recovered from an arbitrary foliation. The precise relationship between observation and choice of hypersurfaces is not yet properly understood and deserves investigation.

Since no information loss occurs in the first place, there does not seem to be any case for an information-loss problem. The above line of argument holds, of course, only within the semiclassical regime, neglecting quantum effects of the gravitational field itself. Only a full quantum theory of gravity can give an exact description of black-hole evaporation. One would, however, not expect that a unitary evolution during the semiclassical phase would be followed by a sudden information loss at the final stage.

4 Bekenstein-Hawking entropy through decoherence?

The above discussion was concerned with Hawking radiation. I have argued that its mixed appearance is due to the decohering influence of other fields. The entropy of the Hawking radiation, Eq. (22), is the result of a coarse-graining and application of von Neumann’s formula (9). But what about the entropy of the black hole itself? In other words, can the Bekenstein-Hawking formula (2) be recovered, within the semiclassical approximation, along the same lines?

For an answer, one should be able to specify microscopic degrees of freedom of the gravitational field itself. This has been achieved, in certain situations, within tentative approaches to quantum gravity: loop quantum gravity (see e.g. [19]) and string theory (see e.g. [20]). In string theory, the derivation of (2) is achieved in an indirect way: using duality and the properties of ‘BPS states’, a black hole in the limit of strong coupling ($g \gg 1$) can be related to a bound collection of ‘D-branes’ in flat space in the limit of weak coupling ($g \ll 1$). Both configurations should have the same number of quantum states – this is the
property of BPS states. For the D-branes, standard formulas of string theory give a definite answer which coincides with the expression (2) for the duality-related black hole.

It was argued in [21] that black holes are inherently associated with mixed states, and that pure D-brane states rapidly develop entanglement with other degrees of freedom, leading to decoherence. Only the decohered D-branes should be associated with black holes. In fact, the derivation of (2) employs, at least implicitly, the use of decohered D-branes.

A related situation was investigated in [22]. There, the decay of a massive string state (i.e. a string state with high excitation number \( n \)) and small coupling was addressed. The mass is given by

\[
M^2 = \frac{n}{l_s^2}, \quad n \gg 1,
\]

where \( l_s \) is the string length. The degeneracy \( N_n \) of a level \( n \) is given by

\[
N_n \sim e^{a\sqrt{n}} = e^{aMl_s}, \quad a = 2\pi \sqrt{\frac{D-2}{6}},
\]

where \( D \) is the number of spacetime dimensions in which the string moves. The decay spectrum of a single excited state does not exhibit any thermal properties. However, if one averages over all the degenerate states with the same mass \( M \), the decay spectrum is of Planckian form, with the temperature given by the Hagedorn temperature \( T_H = (al_s)^{-1} \) [22]. It is speculated that the reason is decoherence generated by the entanglement with quantum background fields being present in the string spectrum.

It might well be that (2) can generally be justified in an analogous manner, either in loop quantum gravity or string theory. Quantitative calculations in this direction have still to be performed. They should in particular reveal the universal nature of the Bekenstein-Hawking entropy. One might expect that therein the quasi-normal modes of a black hole could play a crucial role in serving as an environment. For the Schwarzschild black hole the maximal value for the real part of the mode frequency (energy) is [23]

\[
\hbar \omega = \frac{\hbar \ln 3}{8\pi GM} = (\ln 3)kBT_{BH},
\]

This frequency seems to play a crucial role in the calculation of (2) from loop quantum gravity [24]. In quantum cosmology, the global structure of spacetime assumes classical properties through interaction with higher modes [25,10]. In a similar way one could envisage the quasi-normal modes to produce a classical behaviour for black holes – the entropy (2) would then result as the entanglement entropy of the correlated state between black hole and quasi-normal modes. Since in the corresponding calculation a sum over all modes has to be performed, one might expect the maximal frequency (25) to play a crucial role. The details, however, are far from being explored. To understand black-hole entropy as an entanglement entropy has been tried before, see e.g. [26] and the references.
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therein. A special example is induced gravity where \(2\) was recovered in the presence of non-minimally coupled fields \(27\). (This result might be of relevance to string theory.) Here it is suggested that the role of the environment is played by the quasi-normal modes, which are (for large mode number) characteristic of the black hole itself and therefore should be able to yield a universal result.

It is known that spacetime as such is a classical concept, arising from decoherence in quantum gravity (see Sect. 5 in \[10\]). The event horizon of a black hole is a spacetime concept and should therefore have no fundamental meaning in quantum gravity. It should arise from decoherence in the semiclassical limit, together with \(2\).

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