Forgetting in order to Remember Better

Hang Yu*, Ziyi Liu*, and Jiansheng Wu†
Shenzhen Institute for Quantum Science and Engineering and Department of Physics,
Southern University of Science and Technology, Shenzhen 518055, P.R. China
*These authors contributed equally to this work, and
† Corresponding author. E-mail: wujs@sustc.edu.cn

In human memory, forgetting occurs rapidly after the remembering and the rate of forgetting slowed down as time went on. This is so-called the Ebbinghaus forgetting curve. There are many explanations of how this curve occur based on the properties of the brains. In this article, we use a simple mathematical model to explain the mechanism of forgetting based on rearrangement inequality and get a general formalism for short-term and long-term memory and use it to fit the Ebbinghaus forgetting curve. We also find out that forgetting is not a flaw, instead it is help to improve the efficiency of remembering when human confront different situations by reducing the interference of information and reducing the number of retrievals. Furthermore, we find that the interference of information limits the capacity of human memory, which is the "magic number seven".

PACS numbers:

In 1885, Herman Ebbinghaus experimentally investigated the properties of human memory quantitatively, and he found that forgetting occur most rapidly after the remembering and the rate of forgetting slowed down as time went. He plotted the retention of nonsense symbols in his memory as a function of time, and this is so-called the Ebbinghaus forgetting curve[1]. This is the first experiment to investigate the human memory quantitatively. The forgetting curve can be roughly fitted by an exponential function or a power law function quantitatively. On the other hands, modern psychological and neural science show that there are four mechanisms of forgetting: storage failure (the lost of memory mark in neural system), motivated forgetting (forgetting due to emotional reasons, for example, traumatic experiences), interference (failure to recall one information due to the exist of similar information)[2, 3], retrieval failure (inability to locate a specific memory although it is known to exist). Then an interesting question arise that the exponential decay of human memory are caused by which of the four possible mechanism, and can we get the forgetting curve from these mechanism. The purpose of this article is to answer this question.

One simple answer is that the exponential decay is due to the storage failure. For example, if the memorized information of a certain situation (We call anything remembered and to be remembered “situation”, which may include any kinds of events and objects) $M_{n+1}$ at $n + 1$ is proportional to the previous one $M_n$, i.e. $M_{n+1} = sM_n$. And the ratio $s$ is less than one due to the lost of memory mark in neural system. We can get $M_{n+1} = s^nM_1 = M_1 \exp[-n \ln(1/s)]$. This answer means that forgetting is a flaw of neural system. If there are no other advantage of forgetting, such a flaw shouldn’t exist after hundreds of thousand years of evolution of human being. In this article we view the forgetting from another angle: What is the advantage of forgetting?

Memory is the foundation of human thinking and reasoning. People remember situations because remembering will increase the efficiency if one has meet these situations before then one knows how to response them in proper ways. In order to recall corresponding information when confront different situations in human lives in a timely and efficient manner, we hope to memorize as much information as possible. But there are two constraints on the memory. One is the limitation of cognitive resource, that is, human memory has finite capacity. The other is that efficiency of retrieval is low and the interference will occur if too many informations are stored. So one best way is to forgetting in order to improve the efficiency of retrieval and lower the interference effect. What is the best way to forget as time going?

Models. The basic advantage of memory for human being is that if one remembers certain situation, one know how to response it in a quick manner, otherwise one has to spend more time and effort. So memory help to improve efficiency. But the situation becomes more complicated if one has many informations and one’s memory capacity is limited. The problem of memory can be modeled as followings: How should one use one’s finite memory capacity to remember different situations one confronts to increase the efficiency to deal with those situations. Here we assume there are $N$ possible situations and the $i$-th situation appear with probability $P_i$ and we have already arrange to probability in descending order ($P_i \geq P_{i+1}$). If one meets a situation, one firstly searches one’s memory according the descending order of the possible situations. In this way, one can minimize the expectation of search time. The time of try-out is,

$$E_{T.O.} = \sum_i iP_i \leq \sum_i iP_{\sigma_i}$$

where $P_{\sigma_i}$ is a rearrangement of sequence $P(i = 1, 2, ..., N)$. And the inequality is validated due to the rearrangement inequality.

To illustrate, first we consider a binary case with $P_1 \geq P_2$ here we have $E_{\text{min}} = P_1 + 2P_2$. Thus if the possibility of $P_1 \geq P_2$ is $p$, then the expected value of $E_{\text{min}}$ is,

$$E_{\text{min}} = p(P_1 + 2P_2) + (1 - p)(2P_1 + P_2)$$
FIG. 1: An example of how networks in memory are simplified into a
tree and a binary tree. In (a) we choose a network of words beginning
with “wol-”, in (b) we generate a tree graph that “wolf” is chosen as
root, and “wolfram” are chosen as the subject. By treating all other
situations as noise, we see a binary tree as shown in (c).

For cases involving three or more situations, the probability
is a bit complicated, but we can still extend the above to N
possible situation case,

\[ E_{\text{min}} = \sum_{\sigma} P(P_{\sigma_1} \geq P_{\sigma_2} \geq \ldots \geq P_{\sigma_N}) \cdot (P_{\sigma_1} + 2P_{\sigma_2} + \ldots + NP_{\sigma_N}) \]

where \( \sigma \) represent a cyclic of the elements 1, 2, ..., \( N \) and \( \sigma_i \)
represent the \( i \)-th elements of the cyclic.

In real daily lives of human being, we confront different situations
which may form a network instead of a list as above. But we can use a
tree to organize these different situations. As shown in Fig(1), for a network of words
beginning with “wol-”, we generate a tree graph that “wolf” is chosen as root
and others as the subjects (illustrated in Fig.(1b)). In this way, we
have a list of different situations. In this article, we mainly
consider the binary scenario as it is representative and it is
the simplest case in mental process. For example, the tree in
Fig(1b) can be further simplified as a binary tree: The “wolf”
and “wolfram” are chosen as root and subject respectively and
all other situations are treated as noise. So the binary tree is
with two branches. one branch (denoted as situation \( f \)) is our
focus to which an response can be activated in a proper way,
and the other branch (denoted as situation \( n \)) is the noise to
which no proper response can be activated.

Suppose one meet an incoming situation, and one searches
one’s memory for situation 1 (the focused) and then for situa-
tion 2 (the noise) if the probability such that \( P_f \geq P_n \). If the
situation 1 is the incoming situation \( f \), then he can response
in a proper way. But if the situation 1 is not the incoming sit-
uation \( n \), then he fails to do that. So the probability for one to
response in a proper way is,

\[ p = \int_{P_f > P_n} \rho(P_f, P_n) dP_f dP_n \]

where \( \rho(P_f, P_n) \) are the joint probability of two situations \( f \)
and \( n \).

Suppose during duration \( t \), a situation was recorded \( n_0^* \)
times totally and \( n_0 \geq 1 \) since we need to confront it at
least one time to remember it. We call \( n_0 = n_0^* - 1 \) the
memory frequency and it is one of the elementary factors
which affect the strength of memory positively. In most of
the case in daily lives, \( n_0 \) satisfies the Poisson distribution
with frequency \( \lambda \), i.e.,

\[ P(n_0|\lambda) = \frac{(\lambda t)^{n_0} e^{-\lambda t}}{\Gamma(n_0 + 1)} \]

But the frequency \( \lambda \) is unknown. Using the Bayes statistics
with a constant prior distribution \( \rho(\lambda) \), we can estimate \( \lambda \) from
\( n_0 \) by

\[ P(\lambda|n_0) = P(n_0|\lambda)\rho(\lambda)/\rho(n_0) = \frac{A \rho(n_0+1) e^{-\lambda t}}{\Gamma(n_0 + 1)} \]

Here, in the binary tree, the branch of the focus is the in-
coming situation with frequency \( \lambda \) and the branch of noise is
a noise assumed to appear at frequency \( k \), so the probability
that the situation can be recalled, is the followings,

\[ P_M(n_0, kt) = \int_0^\infty P(\lambda|n_0) d\lambda = \frac{\Gamma(n_0 + 1, kt)}{\Gamma(n_0 + 1)} \]

where \( \Gamma(n_0 + 1, kt) = \int_0^\infty \rho(n_0, t) e^{-\lambda t} d\lambda = \int_0^\infty \lambda^{n_0+1} e^{-\lambda} d\lambda \)
is the upper incomplete \( \Gamma \)-function and the \( P_M \) function is the
regularized upper incomplete \( \Gamma \)-function. This function is ac-
tually the probability over a duration time \( t \) of which a situa-
tion (which has appeared \( n_0 \) times) can be recalled. \( P_M \) is
equal to 1 at the beginning (\( t = 0 \). It work as the retention
strength of memory as a function of time, i.e., the memory
function. And it equals to 1 if there exist no noise (\( k = 0 \)) as
well. We can see here the forgetting of memory partially are
due to the interference of noise.

If an event is only encountered at the initial and doesn’t
appear again, i.e. \( n_0 = 0 \), then we have \( P_M = e^{-kt} \) which
is the exponential function usually used to fit the Ebbinghaus
forgetting curve. Here \( k \) is the frequency of the noise event
(assumed to satisfy the Poisson process as well). And we can
see the larger the frequency of the noise events, the faster the
memory strength decay. We can see from Eq.1 that the larger
the \( n_0 \), the slower the decay ratio at small \( t \) which means that
the memory strength can last for a longer time. For much
larger \( n_0 \), we can see the memory last longer which is still
around 1 when \( kt \leq n_0 \). \( P_M(k, t) \) can be expanded as,

\[ P_M(n_0, kt) = \frac{\Gamma(n_0 + 1, kt)}{\Gamma(n_0 + 1)} = e^{-kt} \sum_{s=0}^{n_0} \frac{(kt)^s}{\Gamma(s + 1)} \]

which is the cumulative distribution function for Poisson ran-
dom variables: If \( X \) is a Poisson(\( k \)) random variable with
frequency \( k \) then \( P(X < n_0 + 1) = P_M(n_0, kt) \). So we can interpret-
tate the forgetting curve in another way: If a situation appear
\( n_0 + 1 \) times and \( n_0 + 1 \) is greater than the number of time
that noise situation appear, we can remember it. Furthermore,
from the above reasoning, we can see how the memory function behavior depend on the distribution of the incoming situations. For different distribution, we can get different memory function.

If the incoming situation appear according the binomial process with total number of sampling \( N = i/\tau \) and the situations occur \( n_0 + 1 \) times with probability \( k\tau \) (\( \tau \) is the duration when a situation appear and \( k \) is the frequency of the noise situation), then the memory function should be

\[
P_M^B(n_0, k, \tau, t) = \frac{B(k\tau, n_0 + 1, N - n_0 - 1)}{B(n_0 + 1, N - n_0 - 1)},
\]

where \( B(a, b) \) is the Beta function and \( B(x, a, b) \) is the incomplete Beta function.

Some situations are special. For examples, some situations happen one or several times in a life time (for example, marriage), they appear with probability \( p \) and it doesn’t varies with time. The forgetting curve is

\[
P_M^C(p, t) = \Theta(p - k).
\]

where \( k \) is probability of such kind of situations other than the focused one and \( \Theta(x) \) is a step function s. t. \( \Theta(x) = 1 \) for \( x \geq 0 \) and \( \Theta(x) = 0 \) for other case. So the memory function is a constant. Once such situation happen, it is remembered and it is forgotten once a new noise situation of such kind happen.

It was believed that long-term memory is from the consolidation of short-term memory, so-called memory consolidation[23]. However, different from Atkinson Shiffrin memory model[11], studies of patients with perisylvian cortex damage and inferior parietal cortex damage show that these patients had deficits of short-term memory but long-term memory was preserved[12-14]. If memory consolidation of short-term memory is the only way to form long-term memory, the above situation shouldn’t exist. And the protein kinase C, zeta (PKC zeta)[19] which might be important for creating and maintaining long-term memory and not important for short-term memory suggests that long-term memory is different from short-term memory. So there are three parts of memory, including short-term memory (it is also called working memory), long-term memory not from memory consolidation and long-term memory from consolidation (memory consolidation)[13]. Here we can use \( P_M \) with small \( n_0 \) as short-term memory function and those with large \( n_0 \) as long-term memory function. At the same time, they can have different value of noise frequency \( k \). Then it is reasonable that we assume arbitrary memory function (or forgetting curve) can be decomposed as

\[
P_{LM}(k, t) = \sum_{n=1}^{\infty} C_n \frac{\Gamma(n + 1, kt)}{\Gamma(n + 1)}
\]

where \( C_n \) are constant s.t. \( \sum_{n=1}^{\infty} C_n = 1 \) which grantee the unitary at \( t = 0 \). To fit Ebbinghaus forgetting curve, we need two sets of \( n_0 \) and \( k \) values and three forgetting functions to well fit the curve shown in the next section.

For long-term memory, we take relative small constant \( k_l \) because the noise of environment is stabilized at long time scale. Based on the study of interference theory[3, 17, 18] and serial-position effect[16], the former and latter information can affect the memory of middle information negatively. We consider there is an increment of noise \( k_n \) for short-term memory and when more situations are confronted. Assuming that the noise frequency is \( k_{i0} \) for short-term memory for first incoming situation, the environmental noise for the \( i \)-th incident situation is \( ik_{i0} \).

Using the memory function with \( n_0 = 0 \) as an approximation, the expected number of situation (memory capacity) that can be remembered during \( \Delta \) and the total number of try-out are the followings respectively,

\[
E_C = \sum_{i=0}^{\infty} p_i = \sum_{i=0}^{\infty} e^{-ik_{i0}\Delta} = \frac{1}{(1 - e^{-k_{i0}\Delta})}
\]

\[
E_{T.O} = \sum_{i=0}^{\infty} p_i = \frac{1}{\sum_{i=0}^{\infty} e^{-ik_{i0}\Delta}} = \frac{e^{-k_{i0}\Delta}}{(1 - e^{-k_{i0}\Delta})} = (E_C - 1)
\]

It is interesting to notice that the try-out numbers is increasing with the memory capacity expanding. At the same time to reduce the number of try-out, the working space should be limited so that one can response to a situation in a quick and efficiency manner.

Results. The first experimental study on human memory was done in late 19th century by Ebbinghaus. It demonstrated the basic characteristics of how memories fading over time. As is proposed in the article, Ebbinghaus chose exponential model to fit his results, a power law and later logarithm law. And better fitting models are appearing since then[18]. Although the experiment used percentage of time reduction instead of percentage of retention, here we directly take the two representation of memory strength as equal. Here we use our model to fit the data of experiments by Ebbinghaus[11], Mack and Seitz[20] and Dros[8] in Fig.3. Our function is

\[
p = 0.68 \Gamma(n_s + 1, k_{s0}t) \Gamma(n_s + 1) + 0.13 \Gamma(n_s + 1, k_l t) \Gamma(n_s + 1) + 0.19 \Gamma(n_s + 1, k_l t) \Gamma(n_s + 1)
\]

Here we choose two \( n_0 \) and two \( k \) values to construct the function of forgetting curve. That is, \( n_s = 0 \) characterizes working memory, \( n_l = 13 \) characterizes the long-term memory, \( k_{s0} = 300/day \), and \( k_l = 1/day \). We can see long-term memory is characterized by larger \( n_0 \) and lower noise frequency and the short-term memory is opposite. The second term is for the memory consolidation, which means the transformation from short-term memory to long-term memory as mentioned above. It is very interesting that in this memory consolidation term, a good fitting need a low noise frequency as long-term memory and a \( n_0 \) value as short-term memory instead of a noise frequency as short-term memory and a \( n_0 \) value as long-term memory. It is counterintuitive. The intuitive thinking is
that the repeating of short-term memory is to increase the $n_0$ value. But it is not the case here. The function of the repeating of the short-term memory is to lower the noise frequency from short-term memory level to long-term memory level.

For memory capacity of short-term memory, we take $k_{s0} = 300/\text{day}$ from function $p(t)$ and the increment of $k_s$ happens during a short time scale in our model. The period of short-term memory $\Delta$ can be considered as the duration of the attention or test [21], so we take $\Delta$ from 30 seconds to 1 min. We get $E_c \in [4.3, 9.1]$. This is close to “the magic number 7” which is the average number of non-correlated items that people can remember in short-term memory [25].

In conclusion, by using a simple rearrangement inequality, we get a general formalism for both short-term and long-term memory function. How fast the forgetting occur depends on the number that a remembered situation occurs and the noise frequency of the noise situation. The Ebbinghaus’s forgetting curve can be well fitted by three memory function: short-term memory, long-term memory and memory consolidation. The memory consolidation term shows that the impact of repeating of short-term memory is to reduce the noise frequency. The forgetting is not a flaw, on the contrary, it can help to increase the efficiency of remembering and this is consistent with some recent psychological experiments [24]. It has two folds of impacts. One is that it reduce the interference of informations. The second is that it reduces the number of retrievals. It is shown in our paper that the interference of information lim-