Automated Assessment of the Value of Private Equity Funds on the Basis of an Optimization Software Package

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Abstract. With the development of private equity market in Russia, the creation of a private equity fund (PEF) optimizing the distribution of investment resources in it remains a relevant task. The object of the study in this paper is PEF, the subject is an automated assessment of the value of PEF on the basis of the author's economic-mathematical model (EMM). The aim of this paper is to study the possibility of using the optimization software for this assessment and its application to the real PEF portfolio of projects. The optimization-based economic-mathematical model underlying the calculations makes it possible to identify the economic potential of the fund's portfolio (in the form of its net present value), which makes it possible to optimally allocate investments in projects of the real sector of the economy, taking into account the state of the regulatory environment and the interests of investors. As methods of the study in this paper are used: economic-mathematical modeling; numerical experiment using an optimization financial-analytical package of applied programs. The results of the research can be used by fund managers to solve operational, investment and financial problems from the standpoint of increasing the value of funds and optimal investment allocation in various sectors of the economy.

1. Introduction
The development of economic relations in Russia, aimed at expanding production, creating new jobs, diversifying the economy generates the need for the creation of private equity institutions. Accumulation of funds of investors who prefer to invest in companies in the real economy, and financing of growing companies whose shares are not traded in the stock market, is possible through private equity funds [1; 2]. In our opinion, in the conditions of the country's economy oriented to the development of the real sector, in the near future such a method of direct investment as the creation of PEF will become quite popular in various regions. Companies in the real economy will have access to an additional source of financing, especially when banking products or stock market instruments are not available to them. Thus, there is a need to develop and improve tools for assessing the value of PEF, as well as to create mechanisms for the effective management of such funds, taking into account the specifics of their creation and functioning.

PEF in Russia is usually created in the organizational and legal form of a closed unit investment fund for qualified investors and an investment partnership for unqualified investors [3]. The regulator
establishes the main requirements for the subject of evaluation and the periodicity of such an assessment [4]. It is advisable to conduct an evaluation of PEF, regardless of the organizational and legal form, at least three times, using different approaches: at the stage of creation and monitoring, using a revenue approach; at the stage of closing PEF - comparative one [5-9]. The value of PEF can be determined by summing the values of all companies included in the portfolio of PEF and net present value (Figure 1).

![Figure 1. Illustration of the constituent elements of the cost of PEF.](image)

Currently, economic science offers myriad opportunities for assessing the effectiveness of investment and the formation of an optimal investment portfolio in the stock market. Despite the fact that the number of models for determining the optimal portfolio is large, the main ones are based on the paper of the American economist H. Markowitz [10], in which one of the first mathematical model of an optimal investment portfolio was proposed. Russian theories of optimizing investment portfolios in the vast majority are also based on foreign models of the formation of securities portfolios and derivative financial instruments [11-17].

The activity of PEF differs from the activity of funds of securities in the way that investments directly flow into the real sector of the economy, and not into securities. In this regard, in our view, it is necessary, in one way or another, to take into account the specifics of production and sales of real sector projects (capital productivity, depreciation of fixed assets, demand, etc.) included in the portfolio of PEF. In addition, it is obvious that a large amount of information needs to be processed due to a significant number of analyzed projects in the portfolio. Therefore, the development of automated methods for their analysis, allowing to maximize the portfolio NPV, taking into account the features of the creation and operation of PEF in Russia, are also relevant tasks.

2. Meaningful statement of the problem
Suppose that the portfolio of the fund includes projects for the production of (n) types of products (industries, areas of economic activity, etc.). These projects are characterized by different levels of their effectiveness, which, in particular, can be determined by the average profitability, the average capital productivity of fixed assets and / or other indicators. Suppose that the projects in the portfolio of PEF can be invested at different points of time. Moments of launching and termination of the projects are arbitrary and may not match with the closing time of the entire fund. It is assumed that PEF projects can be implemented for a long period (3-15 years), which is stipulated by Russian legislation. The payment to the managers of the asset management company are also taken into consideration. The fee amount is fixed in the fiduciary management agreement [18]. It is necessary to determine the optimal amount of investment in each PEF project in accordance with the criterion of
maximizing the discounted cash flow from operating and investment activities by the sum of all projects in the PEF portfolio.

3. Solution of the problem
To solve the formulated problem, a mathematical model of PEF in the form of a multiparameter linear programming problem was developed [19]. This model takes into account such features of PEF, distinguishing it from the portfolio of financial assets, such as the availability of restrictions on the level of demand for manufactured products, the ability to take into account the production features of PEF projects due to the existence of a corresponding group of required variables. In addition, the proposed model, through the complex structure of discounting factors, takes into account the effects of the duration of project operations, and also takes into account directly the interests of PEF management through an annual reward for managing its assets. The type of the proposed model and the presence of an end-user-oriented system of automated information processing in it make it possible to effectively apply this tool in real conditions for PEF, including a significant number of investment projects. In this paper we consider a particular case of the model presented in [19], when a number of risk components of PEF functioning are not taken into account. At the same time, these components, if there are appropriate methods, can be taken into account traditionally, in the discount rates of the fund projects.

To perform calculations based on the proposed model, we used an optimization software package, described in detail in [20].

This complex is a set of automated information system for input, processing and analysis of input financial and economic information, a solver and a multiparameter graph analyzer for solving one- and multi-objective linear programming problems (MLPP). The package is targeted at different users. For a specialist in mathematics, it allows to create and correct mathematical models in the form of the MLPF, as well as to control the correctness of information entry. For an economist-analyst, for a businessman - in a convenient mode to create own configuration of the business project (allocating blocks of characteristics of assets, products, external environment of the project, financial block, etc.) and enter statistical and expert input information. And for both of them - to present the results of calculations both in the form of graphs of multiparameter dependencies, and in the form of Pareto-sets in the criterial space (two or three criteria). The package provides two modes of operation - 1) model designer; 2) the automated workplace of the financial analyst. The model constructor allows to create custom models in a visual mode, forming the model matrices block-by-block. In this case, the user is able to edit vector-matrix information, having in front of him as an image of the entire matrix, and its arbitrary blocks. The package is also equipped with a debugger, which allows to detect errors or inconsistencies in the input of numerical model information. The automated workplace of the business analyst, after the introduction of numerical information from the domain, allows to get the results of calculations and perform their graphical analysis. This mode of the package requires minimal training for the user - knowledge of the meaningfulness of the numerical parameters of the model, it gives an opportunity to students or specialists with a relatively small set of knowledge in the subject area to easily master the work with the package.

When solving the tasks set in the paper, the key advantages of this complex are:
- possibility of multi-parameter analysis of the linear programming problem (LLP) in a user-oriented mode;
- a visual editor of the diagrams of LLP, which presented block-by-block, that allows to minimize the errors of information entry into the model, especially in condition of large dimensions typical for the active PEF portfolios;
- high speed of calculations (a graph containing information obtained by consecutively solving 100 LLP is constructed within no more than one second);
- construction and simultaneous parametric analysis of a large number of dependency curves from any model parameters in one and several figures;
- flexible adjustment of the type of graphs (legend, labels, values in points);
4. Approbation of the economic-mathematical model

The approbation of the proposed toolkit was carried out in relation to PEF created in the form of investment partnership managed by Patriot42 in the territory of the Kemerovo Region. The characteristics of projects with a specified minimum investment in each of them, presented in Table 1.

Table 1. Main characteristics of portfolio of projects.

| Parameter                        | Project 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|----------------------------------|-----------|---|---|---|---|---|---|
| δₖ, ROA, %                       | 16.1      | 13.8 | 6.8 | 6.8 | 5.2 | 4.5 | 3.2 |
| Moment the project starts in PEF, unit of time | 1 | 2 | 2 | 1 | 3 | 3 | 1 |
| Moment of completion of the project in PEF, unit of time | 7 | 7 | 5 | 4 | 6 | 6 | 5 |
| Cost capacity of the project production market, RUB million | 3200 | 800 | 120 | 4000 | 5000 | 1200 | 300 |
| Discount rate, %                 | 15        | 20 | 18 | 15 | 15 | 15 | 15 |
| Min vol. of investments, RUB million | 1433 | 50 | 35 | 19 | 350 | 26 | 70 |
| NPV with min investments, RUB million |            | | | | | | |

With a minimum volume of investments in the projects of PEF portfolio (RUB 1,983 million), the estimated NPV was RUB 229 million. Then the cost of PEF will be determined as the sum of investment costs and added value – RUB 2,212 million. (RUB 1,983 + 229 million).

Table 2. Results of calculation of optimal investments and portfolio NPV.

| Project | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total | Max NPV, RUB million |
|---------|---|---|---|---|---|---|---|-------|----------------------|
| Min vol. of investments δₖ, RUB million | 1434 | 50 | 3 | 19 | 350 | 26 | 7 | 1983 | 229 |
| Additional vol. of investments, RUB million | 1034 | 703 | 0 | 263 | 0 | 0 | 0 | 2000 | 871 |
| Optimal vol. of investments, RUB million | 2467 | 753 | 3 | 282 | 350 | 260 | 7 | 398 | 1100 |
Table 2 shows the results of calculations that allow to maximize the value of the fund: the distribution of the additional volume of investments; NPV when additional investments are made, as well as total NPV with minimal and additional investments.

Based on the data presented (Table 2), we can conclude that an additional optimal amount of investment in the portfolio of PEF in the amount of RUB 2 billion should be distributed between projects No. 1, No. 2 and No. 4. When implementing minimum and additional investments in the fund's portfolio, the cost of PEF will be the maximum possible amount of RUB 5,083 million (RUB 3,983 + 1,100 million). And the return on assets for 7 years of the activity of the fund will be 27.6% (RUB 1,100 million / RUB 3,983 million).

5. Conclusion
The results of modeling and numerical analysis of considered PEF allow to draw the following conclusions, which should be taken into account when creating and making investment decisions:
- to maximize the cost of PEF it is required to attract an additional investment in the amount of RUB 2 billion;
- to recommend the management company to allocate an additional amount of investment between projects No. 1, No. 2 and No. 4;
- in case of attracting additional volume of investments in the amount of RUB 2 billion the estimated value of the fund in seven years will be RUB 5,083 million (RUB 3,983 + 1,100 million).

The application of the proposed toolkit (an optimization economic-mathematical model and a package of applied programs for its automated analysis) to evaluate the cost of PEF, in our opinion, will facilitate the end user - the investment analyst, the PEF manager, etc. - to take optimal investment decisions, which will contribute to the development institutions of direct investment, increasing the volume of direct investment in the real sector of the economy.

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