Research on the Superposition of Harmonic Loss Considering Skin Effect

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Abstract. Power system harmonic will cause extra power loss. The higher the harmonic order, the more obvious the skin effect, which means current density becomes larger near the surface of conductor. When several harmonics with different frequency exist, whether the current density distribution of each harmonic is independent, and whether the total harmonic loss can be regarded as the sum of each harmonic loss, need further research. In this paper, based on the basic principle of electromagnetic field, the expressions of the current density distribution and power loss under multiple harmonics background are deduced, and the superposition of harmonic loss considering skin effect is also proved, which can provide theory basis of harmonic loss calculation.

1. Introduction

With the development of electric power industry, a large scale of non-linear loads which are mainly power electronics, such as rectifier, frequency control device, electric arc furnace, rolling mill, switched-mode power supply, are connected to the power grid. The threat of harmonics to the power grid is increasingly serious. Not only cause the distortion of the voltage waveform on bus bar which harms the normal operation of electrical equipment, harmonics also cause additional power loss [1,2]. Harmonics suppression can improve the quality of power supply while conserving energy in the power grid to improve the level of economic operation and energy-saving benefits. Because the harmonic loss generated by the nonlinear loads is mostly consumed in the low-voltage distribution network, only a small part is fed back into the high-voltage network. Therefore, the study of harmonic loss should be focused on the distribution network[3]. Both comprehensive and accurate evaluation of the harmonic loss in the distribution network is the beginning of harmonic suppression and energy-saving analysis. Therefore, it is necessary to further study the theoretical analysis method of harmonic loss.

At present, there are many researches on harmonic loss, and many achievements have been made in theoretical calculation and experimental researches [4-8]. In general, considering the skin effect, the harmonic loss on conductor can be calculated using the following formula

\[ P = I_1^2 R_1 + \sum_{n=2}^{\infty} I_n^2 R_n = I_1^2 R_1 \left(1 + \sum_{n=2}^{\infty} \frac{I_n^2 R_n}{I_1^2 R_1}\right) \]  \hspace{1cm} (1)

where \(R_n\)=the equivalent resistance under nth harmonic.
The calculation method above directly adds fundamental current and each harmonic current, that is, the total loss generated by non-sinusoidal current. It is easy to see that the equivalent resistance of a conductor calculated by fundamental current using (1) can be described as beneath.

\[ R = R_1 (1 + \sum_{n=2}^{\infty} \frac{I_n^2 R_n}{I_1^2 R_1}) \tag{2} \]

The correctness of this method is obvious. When ignoring the skin effect, however, when the skin effect is considered, the distribution of current density at different frequencies is different in the cross section of the conductor. Under the multi-harmonic background, whether the distribution of the current density of each frequency remains the same as they were alone and whether harmonic loss can be simply added, need further study.

Based on the general principle of electromagnetic field, this paper proved that under multiple harmonics background the distribution of current density of each frequency is consistent with its sole existence. Based on this, this paper deduces the expression of the distribution of current density and the instantaneous power loss of the conductor under multiple harmonics background, indicating that power loss related to the sum and difference of frequencies were generated between different order harmonics. This kind of power loss are all alternating, thus their average power equals to zero. Therefore, the superposition of harmonic loss considering the skin effect is still valid.

2. Calculation of Current Density Distribution under Multi-harmonic Background

Neglecting the influence of displacement current, the electromagnetic field in the conductor can be approximated as the Magnetoquasistatic field, which satisfies the following basic equations [9,10]:

\[
\begin{align*}
\nabla \times H &= \gamma E \\
\nabla \cdot B &= 0 \\
\nabla \times E &= -\frac{\partial B}{\partial t} \\
\nabla \cdot D &= 0
\end{align*}
\tag{3}
\]

It can be derived that

\[ \nabla^2 E = \mu\gamma \frac{\partial E}{\partial t} \tag{4} \]

According to the differential form of Ohm's law:

\[ J = \gamma E \tag{5} \]

Combing (4) and (5), the following equation can be obtained.

\[ \nabla^2 J = \mu\gamma \frac{\partial J}{\partial t} \tag{6} \]

Assume a one-dimensional field shown in figure 1, which is a semi-infinite surface of the conductor where \( x > 0 \). If the current flows in the direction of \( y \)-axis, (6) can be rewritten as

\[ \frac{\partial^2 J}{\partial x^2} = \mu\gamma \frac{\partial J}{\partial t} \tag{7} \]
Figure 1. A semi-infinite one-dimensional field.

Assuming that current density is only related to $x$ and time $t$, thus we can use the separation of variables

$$J(x,t) = X(x)T(t)$$  \hspace{1cm} (8)

Combining (7) and (8), it can be transformed into (9) where the left side of the equal sign is only with respect to $x$ and left side of the equal sign is only with respect to $t$.

$$\frac{1}{\mu \gamma X} \frac{d^2X}{dx^2} = \frac{1}{T} \frac{dT}{dt}$$  \hspace{1cm} (9)

Because this equation holds, so it can only equals to a constant. Assuming this constant is $C$, the following equation is obtained

$$\frac{d^2X}{dx^2} = C \mu \gamma X$$  \hspace{1cm} (10)

$$\frac{dT}{dt} = CT$$  \hspace{1cm} (11)

 Adopting the phasor representation of sine quantities, for a sinusoidal current at a certain frequency, $T(t)$ can be written as

$$T(t) = e^{j(\omega t + \phi)}$$  \hspace{1cm} (12)

Combining (11) and (12), constant $C$ can be solved.

$$C = j \omega$$  \hspace{1cm} (13)

Thus we can also solved (10), the form of solution should be

$$X(x) = Ae^{-px} + Be^{px}$$  \hspace{1cm} (14)

where $p = \sqrt{j\omega \mu \gamma} = \sqrt{\frac{\omega \mu \gamma}{2}} (1 + j)$.

Considering when $x \to \infty$, $X$ should be finite. Therefore, $B$ equals to 0. Assuming $X=J_0$ where $x=0$, (14) can be rewritten as (15)

$$J(x,t) = J_0 e^{-\alpha x} e^{j(\omega t + \phi - \alpha x)}$$  \hspace{1cm} (15)
where $\alpha = \sqrt{\frac{\omega \mu \gamma}{2}}$.

When multi-frequency current exists in conductor, according to (8) and (12), the function of the distribution of current density in the form of phasor can be described as (16).

$$J(x,t) = \sum_{i=1}^{n} e^{j(\omega t + \phi_i)} X_i(x)$$  \hspace{1cm} (16)

where $X_i(x)$=the function that describes the distribution of current density at different frequencies. Combining (7) and (17), a equation containing $X_i(x)$ is shown as below.

$$\sum_{i=1}^{n} e^{j(\omega t + \phi_i)} \frac{d^2 X_i(x)}{dx^2} = \mu \gamma \sum_{i=1}^{n} j \omega_i X_i(x) e^{j(\omega t + \phi_i)}$$  \hspace{1cm} (17)

If the equation holds, the coefficient before the phasor left to the equal sign should be the same as the right. Therefore,

$$\frac{d^2 X_i(x)}{dx^2} = j \omega_i \mu \gamma X_i(x) \hspace{1cm} (i = 1,2,\ldots,n)$$  \hspace{1cm} (18)

Comparing (10) and (18), when multiple frequency currents exist simultaneously, the solution of distribution function under each frequency is the same as when the currents of each frequency is present alone. Thus, it is proved that the distribution of current density of each frequency under the background of multiple harmonics is consistent with its sole existence.

From the above, the corresponding trigonometric form of (16) in the time-domain can be described as following

$$J(x,t) = \sum_{i=1}^{n} J_i e^{-\alpha_i x} \cos(\omega_i t + \phi_i - \alpha_i x)$$  \hspace{1cm} (19)

It needs to be pointed out that if we separate the multi-frequency distribution function into the form of the sum of the distribution function of each frequency and put them directly into (7), the equation still holds and we can draw the same conclusion. However, this method can only prove that it is a feasible solution not unique one, it cannot prove that the distribution function of each frequency coincides with its own sole existence.

3. Calculation of Harmonic Loss and Prove of the Superposition of Harmonic Loss

As shown in figure 2, the bulk density of electric power of a current carrying bar with flat surface extends toward $x$-axis infinitely with unit width and unit height.

According to the differential forms of Joule law:

$$p' = \frac{J^2}{\gamma}$$  \hspace{1cm} (20)

The instantaneous power loss of the conductor is

$$p(t) = \int_{V} J^2 \frac{1}{\gamma} dV$$  \hspace{1cm} (21)
Figure 2. A Current Carrying Bar
Extends toward x-axis Infinitely

Considering that the integral area extends toward x-axis infinitely with unit width and unit height, the instantaneous power loss of certain frequency can be calculated using the following equation:

\[
p(t) = \frac{1}{\gamma} \int_0^\infty J_0^2 e^{-2\alpha x} \cos^2(\omega t + \alpha x)dx
\]

\[
= \frac{J_0^2}{\gamma} \left[ \frac{1}{4\alpha} \sin 2(\omega t + \phi) + \cos 2(\omega t + \phi) \right]
\]

When multiple currents with different frequency exist, then the instantaneous power loss can be expressed by the following equation:

\[
p(t) = \frac{1}{\gamma} \int_0^\infty \left( \sum_{i=1}^n J_i e^{-\alpha_i x} \cos(\omega_i t + \phi - \alpha_i x) \right)^2 dx
\]

\[
= \sum_{i=1}^n \frac{J_i^2}{\gamma} \left[ \frac{1}{4\alpha_i} \sin 2(\omega_i t + \phi) + \cos 2(\omega_i t + \phi) \right] + \sum_{i=1}^n \sum_{j=1}^n \frac{J_i J_j}{\gamma} \sin[(\omega_i - \omega_j)t + \phi - \phi] \cos[(\omega_i + \omega_j)t + \phi + \phi]
\]

Comparing (22) and (23), it can be observed that when the conductor contains multiple frequencies current, the total power loss not only includes the superposition of the power loss when the currents of each frequency exist separately, but also includes the power loss related to the sum or difference of two different frequencies which is alternating and their average power loss equals to zero.

By definition, the average instantaneous power during a period is

\[
P = \frac{1}{T} \int_0^T p(t)dt
\]

Assuming the order of harmonics is integer, if the frequency of the fundamental wave is \(\omega_f\), then the frequency of \(n\)th harmonic, so that (23) can be written as

\[
p(t) = \sum_{i=1}^n \frac{J_i^2}{\gamma} \left[ \frac{1}{4\alpha_i} \sin 2(\omega_i t + \phi) + \cos 2(\omega_i t + \phi) \right] + \sum_{i=1}^n \sum_{j=1}^n \frac{J_i J_j}{\gamma} \sin[(\omega_i - \omega_j)t + \phi - \phi] \cos[(\omega_i + \omega_j)t + \phi + \phi]
\]
The frequencies of AC component in (25) are all integer multiples of the fundamental frequency, therefore the integral time $T$ in (24) should be fundamental period. If there exists subharmonics, then the integral time $T$ in (24) should be appropriate multiples of the fundamental period, which satisfies
\[ P = \frac{1}{T} \int_0^T p(t) dt = \sum_{i=1}^{n} \frac{J_i^2}{4\alpha_i \gamma} \] (26)

This DC component reflects the irreversible flow of energy, which stands for the effective part of power loss.
Assuming that the active power loss generated by nth harmonic is $P_n$, according to (23), the following equation can be obtained
\[ P = \sum_{i=1}^{n} P_i \] (27)

It is obvious from (27) that the harmonic power loss also satisfies the stackability, which means it is feasible to calculate harmonic loss through (1) and (2).

4. Conclusion
In this paper, based on the basic principle of electromagnetic field, the expressions of the current density distribution and power loss under multiple harmonics background are deduced, and the superposition of harmonic loss considering skin effect is also proved, which can provide theory basis of harmonic loss calculation and be conducive to the development of related work. A subsection some text.

Acknowledgments
Project Supported by China Electric Power Research Institute (YD83-16-038).

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