Electric-Magnetic Duality, Matrices, & Emergent Spacetime

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Abstract

This is a rough transcript of talks given at the Workshop on Groups & Algebras in M Theory at Rutgers University, May 31–Jun 04, 2005. We review the basic motivation for a pre-geometric formulation of nonperturbative String/M theory, and for an underlying eleven-dimensional electric-magnetic duality, based on our current understanding of the String/M Duality Web. We explain the concept of an emerging spacetime geometry in the large $N$ limit of a $U(N)$ flavor matrix Lagrangian, distinguishing our proposal from generic proposals for quantum geometry, and explaining why it can incorporate curved spacetime backgrounds. We assess the significance of the extended symmetry algebra of the matrix Lagrangian, raising the question of whether our goal should be a duality covariant, or merely duality invariant, Lagrangian. We explain the conjectured isomorphism between the $O(1/N)$ corrections in any given large $N$ scaling limit of the matrix Lagrangian, and the corresponding $\alpha'$ corrections in a string effective Lagrangian describing some weak-coupling limit of the String/M Duality Web.

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1 Introduction

Understanding the symmetry principles and the fundamental degrees of freedom in terms of which nonperturbative String/M theory is formulated is a problem of outstanding importance in theoretical high energy physics. The Rutgers Mathematics workshop on Groups & Algebras in M Theory this summer devoted part of its schedule to an assessment of the significance of Lorentzian Kac-Moody algebras to recent conjectures for the symmetry algebra of String/M theory. The status, and future prospects, for developments in the representation theory of generic infinite-dimensional Lorentzian Kac-Moody algebras also received intense discussion. My talks at the workshop were devoted to an introductory survey of the String/M Duality web, followed by my own proposal for nonperturbative string/M theory [1, 8]: a fundamental theory of emergent spacetime, based on a matrix Lagrangian with U(N) flavor symmetry, but also distinguished by extended global symmetries, which has the pleasing consequence of yielding the different weak coupling limits of the String/M Duality Web in its myriad choices of multiple-scaled large N limit. We will enlarge upon this succinct, but weighty, one–line summary of our proposal in this transcript. Our emphasis will be on the worldsheet evidence for an 11D electric-magnetic duality in nonperturbative String/M theory [19, 20], and on clarifying the precise role played by extended global symmetries in our matrix theory formulation. Related technical details can be found in the references [1, 8, 38].

Perturbative string theory in flat spacetime backgrounds, including possible two-form background fields, is known to be both renormalizable, anomaly-free, and ultraviolet-finite. We use the term perturbatively renormalizable to describe the target spacetime string theory Lagrangian despite the presence of infinitely many couplings in the $\alpha'$ expansion, because only a finite number of independent parameters go into their determination, and these are all found at the lowest orders in the $\alpha'$ expansion. The existence of only a finite number of independently renormalized couplings is the defining criterion for the Wilsonian renormalizability of a quantum theory. Thus, from this perspective, the perturbative string theoretic unification of gravity and Yang-Mills gauge theories with chiral matter can be seen as providing a precise, and unique, gravitational extension of the anomaly-free and renormalizable Standard Model of Particle Physics.

The ultraviolet cutoff, $m_s = \alpha'^{-1/2}$, can therefore be taken to infinity while keeping the mathematical framework of weakly-coupled perturbative string theory reliable, down to arbitrarily short distances. There is no evidence of quantum corrections to the flat spacetime background geometry; the so-called semi-classical ground state is also the quantum ground state. Assuming that the supersymmetry breaking scale in Nature is far below the string scale, we then appear to have a perfectly good perturbative unification of quantum gravity and the supersymmetric Standard Model of astro-particle physics and cosmology. Note that all indications to date point to the probable weak-coupling unification in our 4D world [62]. In addition, strong-weak coupling dualities have, in many cases, enabled us to gain considerable insight into the strong coupling physics tied to supersymmetry breaking and other open questions relating to dynamics. So why do we need to go beyond?

The motivation for developing a pre-geometric formulation for nonperturbative string/M theory is not because of any evidence for a breakdown of continuum spacetime at short distances, not for any shortcomings of the string theoretic description, and not wholly because we need to understand
strong coupling and nonperturbative physics better. See my recent talk \cite{62} enlarging on these particular aspects of string theory. The essential reason we need a pre-geometric formulation is that perturbative string theory is inherently background dependent: it violates the essence of Einstein’s theory of relativity because it separates the description of matter and interactions, both of which are strings at weak coupling, from the description of the continuum target spacetime in which they exist. In the absence of a nonperturbative formulation in which both target spacetime geometry, and matter and interactions, emerge on equal footing, this fundamental gap in our understanding would remain even if we had a handle on strong coupling physics. This is the basic motivation for what we have called a fundamental theory of emergent spacetime geometry \cite{8}.

It is important to distinguish the concept of emergent geometry as introduced by us \cite{1, 8} from the set of ideas usually called quantum geometry. In our notion of a fundamental theory of emergent geometry based on zero-dimensional matrices, there is no underlying classical continuum manifold upon which we build nontrivial metrics out of quantum gravitational effects: loop quantum general relativity, branched polymer models, or quantum spacetime foam, all assume a continuum classical scaffolding \cite{37}. Nor is emergent geometry a latticization of a background spacetime geometry.\footnote{Lattice gauge theory does offer an important analogy in one respect. Ken Wilson’s latticization of Yang-Mills gauge theory was remarkably insightful in that it implemented lattice gauge invariance: it is a gauge invariant nonperturbative formulation, even away from the continuum limit. Our matrix Lagrangian achieves the same for string theory: the global symmetries of the spacetime Lagrangians of perturbatively renormalizable string theories that emerge in its large $N$ limits are already present in the matrix Lagrangian at finite $N$ \cite{8}. We should note some novel developments in the latticization of rigid SYM theories with 16 supercharges \cite{60}, and with fermionic variables living on both links and sites.}

We begin with a Lagrangian of zero dimensional matrices, and a symmetry algebra extending far beyond the usual $U(N)$. Spacetime itself, and the target space Lagrangians of perturbatively renormalizable and anomaly-free superstring theories, emerge from the myriad large $N$ limits of this matrix Lagrangian. Each large $N$ limit is a quantum ground state of the fundamental theory, and its global symmetry algebra originates in the symmetries of the matrix Lagrangian. From the perspective of quantum cosmology \cite{61}, the large $N$ ground states of the matrix Lagrangian are the quantum final states of the Universe, and each quantum state comes with its individual set of initial conditions. Within the framework of the Hartle-Hawking paradigm for a quantum theory of the Universe \cite{61}, we can associate each such “final state”, a spacetime string effective Lagrangian in specific target spacetime background, to the endpoint of a consistent history in our theory for the quantum mechanics of finite $N$ matrices. Following this epoch, the evolution of physics from the string scale down to TeV scale energies will be governed by conventional, tried-and-true, spacetime quantum field theoretic, renormalization group methods \cite{62}.

Any viable proposal for nonperturbative String/M theory must incorporate electric-magnetic duality symmetries \cite{13, 14, 31}. Let us begin by examining the evidence for an eleven dimensional origin for duality. This is a key assumption made in most discussions of the Duality Web of String/M theory and its global symmetry algebra, usually also assumed to be the source of the electric-magnetic dualities that can become manifest in lower spacetime dimensions. Electric-magnetic duality in $D$ spacetime dimensions interchanges an electrically charged $(d−p−4)$brane with a magnetically charged $(d−p−4)$brane, respectively, sources for a pair of dual rank $(p+2)$ and rank $(d−p−2)$ field strengths. The ten-dimensional perturbatively renormalizable and anomaly-free type II superstring
theories contain a full spectrum of such pbranes, with \( p \) in the range: \(-1 \leq p \leq 9\), covering D-instantons thru D9branes \([15]\). This raises the following puzzle. By a generalization of the Dirac quantization condition for electric and magnetic point charges in four spacetime dimensions, one would expect that the product of the quantum of charge for a Dpbrane, and that of its \( d \)-dimensional Poincare dual D\((d-p-4)\)brane, satisfies the relation:

\[
\nu_p \nu_{d-4-p} = 2\pi n, \quad n \in \mathbb{Z}
\]

(1)

The spectrum of values for \( p \) listed above does not cover all of the expected charges in ten dimensions: we are missing a D(-2)brane and a D(-3)brane, the 10d Poincare duals of the D8brane and D9brane, respectively. This is especially puzzling because, in a groundbreaking work \([15]\), Polchinski had shown that the quantum of Dpbrane charge could be computed from first principles using the worldsheet formalism of weakly coupled perturbative string theory, predicting also that the value of \( n \) in the Dirac quantization relation is unity. Thus, although we were expecting to find evidence for electric-magnetic duality in the full Dpbrane spectrum we appear, instead, to have found a clash with Poincare-Hodge duality in ten target spacetime dimensions.

There is a simple resolution to this puzzle, and it brings with it the key insight that the electric-magnetic duality underlying nonperturbative String/M theory is inherently eleven dimensional. It is remarkable that the smoking gun for this insight can be found in a weakly coupled perturbative type IIA string theory calculation using ordinary worldsheet methods. Having thereby discovered concrete evidence for the validity of this fundamental symmetry of the nonperturbative theory in a hands-on, bottom-up, approach using the perturbative formalism, we can proceed with confidence to examine its top-down consequences starting from elegant first principles. The details of the worldsheet calculation can be found in the references \([16, 17, 18, 19, 20]\); we will only outline the basic result in what follows.

Consider the anomaly-free and perturbatively renormalizable weakly coupled 10d type IIA string theory in the background with 32 D8branes. One of the two supersymmetries of the IIA string has been broken by the presence of orientifold planes at \( X^9=0 \), and \( X^9=R_9 \), and a T-duality transformation maps this background to an analogous background with 32 D9branes of the 10d type IIB string theory. The stack of coincident Dpbranes lies on a single orientifold plane, and carries worldvolume Yang-Mills gauge fields with gauge group \( SO(32) \). At finite string coupling, an eleventh target space “dimension” emerges, corresponding to the vacuum expectation value of the scalar dilaton field \([14]\). Consider the following gedanken experiment in the strongly coupled IIA string theory:\(^3\) we evaluate the pair correlation function of a pair of spatially separated Wilson loops wrapping the eleventh dimension. The limit of pointlike loops, and large spatial separation, corresponds to taking \( R_{11}\to0 \), and hence we recover a result in \( \text{ten-dimensional} \) type IIA supergravity. Our thought experiment must, therefore, have a precise analog in the factorization limit of a suitable weakly coupled perturbative type IIA string amplitude with the worldsheet topology of an annulus. The relevant computation is as follows: consider the Polyakov path integral summing over worldsurfaces with the topology of an annulus, and with Dirichlet boundary conditions on all ten embedding target space

\(^3\)This is only a gedanken experiment at the present time because the strongly coupled IIA string theory is, more precisely, M theory compactified on an \( S^1 \times S^1 / \mathbb{Z}_2 \), and we do not know how to calculate in that theory beyond its low energy 11-dimensional supergravity limit.
dimensions. We will require further that the boundaries are mapped to a pair of given pointlike loops, $C_i$, $C_f$, spatially separated by a distance $R$ in the $X^8$ direction, lying within the worldvolume of the stack of 32 coincident D8branes on an $O8$ plane. This augmented boundary value problem for embedded Riemann surfaces with the topology of an annulus is the precise supersymmetric type IIA analog of a computation carried out in 1986 by Cohen, Moore, Nelson, and Polchinski [16] for the bosonic string theory: the off-shell closed string tree propagator between pointlike loops. An analysis of the type IIA macroscopic loop amplitude appears in a paper by myself and Novak [18]. The factorization limit of this amplitude with pointlike loops yields the long-range interaction of a pair of supergravity sources, and the result was found by me to be consistent with the tension of a Dirichlet (-2)brane [19].

Notice that the Poincare dual of the D8brane in ten spacetime dimensions is a D(-2)brane. The field equations, and spacetime Lagrangian, for Roman’s massive type IIA supergravity [12, 21] do, in fact, include a scalar field strength, $F_0$, and the D(-2)brane can be identified as its source. Our worldsheet result therefore lays to rest any discrepancy in 10d electric-magnetic duality if we also accept that there are no consistent 10d superstring backgrounds with nonvanishing D9brane charge. A 9brane would be a supergravity source for an eleven-form field strength, so it is clearly not something expected in a ten-dimensional supergravity. From the perspective of the worldsheet calculation, notice that a Dirichlet boundary with 9+1 worldvolume coordinates is special: since the Dirichlet defect fills all of 10d spacetime, there is nowhere for the emanating “flux lines” to go except to probe an eleventh dimension. Thus, we must conclude that, strictly speaking, a D9brane with nonvanishing flux can only exist in the strongly coupled type IIA/M theory limit since it has an additional, eleventh embedding target space dimension. Happily, the magnetic dual of the D9brane in eleven dimensions is a D(-2)brane, so that the Dirichlet pbrane spectrum for which we have found evidence in the worldsheet formulation of the weakly coupled perturbative type II string theories, namely, $-2 \leq p \leq 9$, is consistent with the existence of an eleven dimensional electric-magnetic duality in the nonperturbative theory [19, 20]. It is remarkably fortuitous that a remnant of 11D electric-magnetic duality survives in the weakly coupled 10d limits. Both the electric D(-2)brane, and its magnetic D9brane dual, make an appearance in the standard worldsheet formalism of perturbative superstring theory.

Does 11d electric-magnetic duality imply that nonperturbative String/M theory is necessarily a theory formulated in 11 target space dimensions? Surprisingly, the answer turns out to be No, a point that can be made from a variety of different perspectives [1, 6, 38, 8]. The matrix formulation for a theory of emergent spacetime proposed by us in [8] achieves precisely this objective: as many as eleven spacetime coordinates can emerge in the large $N$ limits of what starts out as a zero-dimensional matrix Lagrangian with flavor $U(N)$ symmetry. Electric-magnetic duality can be built into this formalism by specifying the extended symmetry algebra $G_s \times G \times U(N)$, where $G$ is the electric-magnetic self-dual Yang-Mills gauge group, and $G_s$ is the global symmetry algebra of the supergravity sector with both electric and magnetic dual potentials treated on equal footing [6, 38, 8]. It is the extended symmetry algebra that will determine the precise form of both the matrix Lagrangian, as well as the particular large $N$ limits of interest [8], an observation first made by us in [1].
2 The Electric-Magnetic Dual Global Symmetry Algebra

In our earliest proposal of a matrix formulation of a theory of emergent spacetime [1], we alluded to the existence of a hidden symmetry algebra in the matrix Lagrangian that was larger than the obvious $U(N) \times G$. A particularly obvious indication of this fact was an $SL(10; \mathbb{R})$ symmetry in our matrix Lagrangian, but we suspected that $G_s$ was much larger than $SL(10; \mathbb{R})$ [1]. In particular, as the reader can guess from our discussion in the introduction, we would like $G_s$ to reflect the electric-magnetic duality of the pform gauge potentials in the supergravity sector. Of course, thus far we have only invoked the evidence from string perturbation theory for electric-magnetic duality in the Ramond-Ramond sector of the type II superstrings. Based on various disparate pieces of evidence from duality symmetries of the low energy field theory limit, should we not extend our conjecture to cover the pform potentials of the Neveu-Schwarz sector? And what is the evidence for electric-magnetic duality in the Yang-Mills gauge sector? All of these questions have bearing on the precise form of the extended symmetry algebra of the matrix Lagrangian, $G_s \times G \times U(N)$.

From the perspective of Eguchi-Kawai planar reduction [9], the $SL(D; \mathbb{R})$ invariance of the zero-dimensional matrix Lagrangian in [1] can be immediately identified as a remnant of the $D$-dimensional Lorentz invariance following the reduction of a higher dimensional field theory to a spacetime point. In [38, 8], we realized that the field theory Lagrangians from which the class of matrix models to which [1] belongs are descended via some spacetime reduction prescription must have a huge global symmetry algebra. It turns out [8] that they are characterized by a $U(N)$ flavor symmetry, a finite dimensional Yang-Mills gauge symmetry, as well as the hidden Cremmer-Julia symmetry of some higher dimensional supergravity theory [23]. The extended symmetry algebra of such matrix Lagrangians, and their derivation from a modification of the Eguchi-Kawai prescription for spacetime reduction from higher dimensional field theories, was explored in the recent paper [8]. We should emphasize that the higher dimensional field theory Lagrangian with its $U(N)$ flavor symmetry is of no intrinsic physics interest to us; it is simply a convenient starting point from which to derive the matrix Lagrangian we desire. In particular, our prescription for spacetime reduction is not reversible; the myriad continuum field theories which will emerge from the double, or multiple, scaled large $N$ limits of the zero-dimensional matrix Lagrangian are the physical theories of interest, and the unphysical $U(N)$ flavor symmetry is thereby erased.

What form does $G_s$ take for the matrix Lagrangian with sixteen supercharges considered in [1]?

The significance of electric-magnetic duality in the appearance of the global symmetry algebras belonging to the Cremmer-Julia sequence has been studied in a series of recent works [23, 22, 24, 6]; a review appears in [38]. In particular, as has been clearly elucidated by West [6], and in work with Schnakenburg [4, 5, 3], the mapping of the global symmetry algebras of each of the ten,

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4The anomaly-free ten-dimensional superstring theories are based on simply-laced gauge groups, which are electric-magnetic self-dual. But non-simply-laced groups do occur in the moduli spaces of CHL orbifolds [54, 53, 62], namely, supersymmetry preserving orbifolds of any perturbatively renormalizable and ultraviolet finite string compactification, and so $G$ should, strictly speaking, be replaced by $G \times G^*$, where $G^*$ denotes the dual magnetic gauge group [52, 53, 54]. For simply-laced groups, $G^*$ coincides with $G$.

5A different, and very beautiful, direction of research on electric-magnetic duality has explored the relationship to K theory, and to theories with generalized cohomology [26, 27]. The algebra of BPS states has also been explored in [47].
and eleven, dimensional supergravities with 32 supercharges to the single rank eleven Lorentzian Kac-Moody algebra, $\mathcal{E}_{11}$, relies upon the inclusion of both electric and magnetic potentials among the generators of the algebra. Thus, without manifest electric-magnetic duality, this remarkable algebraic unification of the theories with 32 supercharges would not hold.

$\mathcal{E}_{11}$ is, of course, the termination of the $\{\mathcal{E}_{11-n}\}$ Cremmer-Julia sequence of hidden symmetry groups characterizing the dimensional reductions of 11d supergravity to $n$ dimensions [23], with $n=0$, and the result is no longer a field theory, but a zero-dimensional matrix model. In practice, the procedure of field-theoretic dimensional reduction is ill-defined in two dimensions and below.\(^6\) We should emphasize that West’s recent identification of $\mathcal{E}_{11}$ as the symmetry algebra of theories with 32 supercharges in [6] does not rely on dimensional reduction, nor on the usual dualization of dimensionally-reduced fields followed by a mapping to a non-linear realization in order to identify the exponentiated group element as belonging to some known algebra. This traditional methodology [11, 23] would not work anyhow for reductions to below three dimensions. West’s new methodology for uncovering an $\mathcal{E}_{11}$ symmetry in theories with 32 supercharges is reviewed by us in [38]. We will apply it in the context of theories with 16 supercharges in what follows below.

Neither can the reduction of a higher dimensional field theory to a spacetime point a la Eguchi-Kawai result in a zero-dimensional matrix model preserving the symmetries of the Einstein supergravities, a distinction clarified in our recent paper [8]. Two significant changes are required in order to derive the zero-dimensional matrix Lagrangian proposed by us in [1] from a higher dimensional field theory by spacetime reduction. First, we must begin with a higher-dimensional supergravity-Yang-Mills Lagrangian with large $N$ flavor symmetry. It is essential that the gravitational zweibein and the Yang-Mills potential live in identical $N\times N$ representations of the $U(N)$, thereby ensuring that the extended symmetry algebra of the matrix Lagrangian takes the direct product form: $U(N)\times G\times G_s$. Supersymmetry commutes with this algebra, with the fermionic superpartners living in identical $U(N)$ representations. Note that this is also the global symmetry algebra of the higher dimensional field theory, except that there we can distinguish $U(N)$ as a flavor symmetry, $G$ as a Yang-Mills gauge symmetry, and $G_s$ as the global symmetry algebra of the supergravity sector. Next, we must modify the Eguchi-Kawai prescription which simply drops all space and time derivatives in the reduction of the field theory to a single point in spacetime. In our spacetime reduction prescription, we instead replace all fields by linearized Taylor expansions about the origin of the local tangent space at a single point in the spacetime manifold. This preserves, in particular, the notion of a first order partial derivative in the local tangent space, allowing us to assign $U(N)$ matrix-valued definitions to both the spacetime coordinates, and spacetime derivatives, consistent with the basic rules of differential geometry. The $N\times N$ matrix variables, $\{E^a_\mu, E^\mu_a; \phi\}$, with $\mu, a = 0, \cdots, 9$, encapsulate the basic information on the emergent spacetime geometries. In the large $N$ limit, as many as ten, or eleven, upon including the dilaton scalar mode, noncompact coordinates, and coordinate derivatives, can emerge from the diagonalized eigenvalue configurations of this basis of matrix variables. Details can be found in the references [1, 38, 8].

We emphasize that without a prescription to build an emergent coordinate derivative in the large $N$ limit, it is not possible for the full $G_s$ supergravity symmetry to emerge from a given matrix $G_s$ supergravity sector.\(^6\) For a review of the subtleties which originate in the ambiguity in dualizing a gauge field in less than three dimensions, we refer the reader to the discussions in [22, 23, 24, 38], which include citations to the original literature.
Lagrangian. Nor would we have the requisite tools to build up emergent curved spacetime geometries in the large $N$ limit. The difficulties in reconstructing the full nonlinear gravitational interaction of the Einstein action, beyond the nonrelativistic Newtonian potential, and of incorporating curved spacetime backgrounds in addition to flat spacetime, were two of the major obstacles to making progress on the M(atrix) theory conjecture [34, 35, 36].

What is the precise form of $G_s$ for the matrix Lagrangian proposed in [1, 8]? The question can be addressed by using a recent work of West and Schnakenburg [6], where the global symmetry algebra of the $N=1$ $d=10$ chiral supergravity has been analyzed. Recall that the $10d$ chiral supergravity with sixteen supercharges is, a priori, an anomalous field theory, but it can be self-consistently coupled to Yang-Mills gauge fields in order to achieve anomaly cancellation. The gauge, gravitational, and mixed, anomaly-free choices of gauge group are $E_8 \times E_8$ and $SO(32)$, and these are the $10d$ supergravity-Yang-Mills theories appearing in the low energy limits of the heterotic and type I superstring theories [31]. The matrix Lagrangian in [1] was proposed by us as a nonperturbative formulation of these perturbatively renormalizable and anomaly-free superstring theories.

Restricting to the fields in the $10d$ $N=1$ chiral supergravity sector alone, and using the notation of [6], we have the following symmetry generators:

$$K^a_b, R, R^{c_1 c_2}, R^{c_1 \cdots c_6}, R^{c_1 \cdots c_8}.$$ \hfill (2)

The supergravity sector contains a zero-form dilaton and an antisymmetric two-form potential, in addition to their $10d$ Poincare-Hodge duals: respectively, the magnetic dual eight-form, and six-form, gauge potentials. The $K^a_b$ are the generators of $GL(10, \mathbb{R})$; linear combinations of these generators can be shown to span the $10d$ Lorentz algebra [6]. The commutator algebra of the generators in Eq. (2) was analyzed in [3]. It takes the form:

$$[K^a_b, K^c_d] = \delta^c_b K^a_d - \delta^a_d K^c_b, \quad [K^a_b, P_c] = \delta^a_c P_b, \quad [K^a_b, R^{c_1 \cdots c_p}] = \delta^a_b R^{c_1 \cdots c_p} + \cdots , \hfill (3)$$

plus the simplified algebra of 0, 2, 6, and 8-form generators:

$$[R, R^{c_1 \cdots c_p}] = d_p R^{c_1 \cdots c_p}, \quad [R^{c_1 \cdots c_p}, R^{c_1 \cdots c_q}] = c_{p,q} R^{c_1 \cdots c_{p+q}} . \hfill (4)$$

The numerical values of the coefficients can be computed directly by requiring self-consistency with the Jacobi identities, and the equations of motion, as shown in [6]. A useful self-consistency check on this algebra was performed by us in [38]. We showed that the coefficients obtained in [6] agree precisely with those inferred from a chirality projection on the global symmetry algebra of the type IIB chiral $10d$ supergravity obtained in [5]. Setting to zero the extra forms removed by the chirality projection in Eqs. (1.1-1.3) of the latter reference, we found that the remnant non-vanishing structure constants take the simple form [38]:

$$d_{q+1} = -\frac{1}{4}(q - 3), \quad q = 1, 5, \quad c_{2,6} = \frac{1}{2} . \hfill (5)$$

The self-consistency of the global symmetry algebras is a low energy reflection of the well-known connection between the $10d$ type IIB, and type IIB, superstring theories, following the orientation projection to the symmetric combination of left-moving and right-moving modes on the worldsheet of the type IIB superstring [31]: the orientifold plane breaks half the supersymmetries of the type
IIB theory, giving a chiral \( N=1 \) supergravity with graviton-dilaton multiplet, and an antisymmetric two-form potential in the Ramond-Ramond sector. Up to field redefinitions, and rescalings of the couplings, this supergravity matches with that obtained in the low energy limit of the 10d heterotic superstring. It should be noted that the two-form potential now appears, instead, in the Neveu-Schwarz sector [31]. Finally, unlike the heterotic string, the type IIB string can accommodate additional Ramond sector background fields, but a precise correspondence can be found between the mass spectrum and couplings of the two superstring theories in all of the backgrounds with sixteen supercharges [19], as we will describe later in this paper.

How does the global symmetry algebra of the 10d heterotic-type I chiral supergravity, \( \mathcal{G}_{IB} \), relate to the symmetry algebra of theories with 32 supercharges? From our demonstration above, it is clear that \( \mathcal{G}_{IB} \in \mathcal{G}_{IIB} \). West has shown [6] that the global symmetry algebras of each of the ten, and eleven, dimensional supergravities with 32 supercharges can be mapped to the single rank eleven Lorentzian Kac-Moody algebra \( \mathcal{E}_{11} \). This algebra is also known as the rank 11 very-extension of the compact Lie algebra \( E_8 \), denoted as \( E_8^{++} = E_8^{(3)} \), where the superscript signifies the extension of the Dynkin diagram of \( E_8 \) by three nodes: affine, over, and very. The explicit construction, and some simpler aspects of the representation theory, of very-extended algebras can be found in [7]. If we compare the generators and commutation rules of \( \mathcal{G}_{IB} \) with the standard Chevalley basis for the very-extended algebra \( E_8^{(3)} \), written in either its IIA or IIB guises as shown in [6], we find that we are missing some of the positive root generators in either formulation. We have all of the generators, \( E_a = K_{a+1}^a, a=1, \ldots, 9 \), of \( GL(10, \mathbb{R}) \). In the IIA formulation, given in Eq. (4.4) of [6], we are missing the roots corresponding to the R-R one-form, and NS-NS twoform, namely, \( E_{10} = R_{10} \), and \( E_{11} = R_{910} \). In the IIB formulation, we are missing the roots labelled \( E_9 = R_{10}^{10} \), and \( E_{10} = R_2 \), arising, respectively, from the NS-NS two-form potential, and R-R scalar. It is clear we cannot build a full \( \mathcal{E}_{11} = E_8^{(3)} \) algebra from the restricted set of generators in \( \mathcal{G}_{IB} \).

In [3], it was pointed out that a different rank eleven very-extended algebra, namely, the very-extension of the \( D_8 \) compact subalgebra of \( E_8 \), can be spanned by the generators of \( \mathcal{G}_{IB} \). The authors of [3] identify the isomorphism \( E_a = K_{a+1}^a, a=1, \ldots, 9, E_{10} = R_{10}^{10}, \) and \( E_{11} = R_{5678910} \). This choice of simple roots can be shown to generate the very-extended algebra \( D_8^{(3)} \). It should be noted that the two-form and six-form potentials are Poincare-Hodge duals in ten dimensions, and the isomorphism identified in [3] includes both in the simple root basis for \( \mathcal{D}_{11} \). It is interesting that this differs in spirit from the isomorphisms mapping, respectively, \( \mathcal{G}_{IIA}, \mathcal{G}_{IIB}, \) or \( \mathcal{G}_{11} \), to the Chevalley basis of \( E_8^{(3)} \) [6]; neither employs a pair of electric and magnetic dual potentials within the simple root basis. Moreover, the generators of the rank eleven very-extension of \( D_8 \) appears to encapsulate the basic physical content of the supergravity sector of renormalizable superstring theories remarkably well: the graviton, the scalar dilaton and antisymmetric two-form potentials, plus their 10D Poincare-Hodge duals. We find the isomorphism of \( \mathcal{G}_{IB} \) to the rank eleven algebra \( \mathcal{D}_{11} \), discovered in section 1 of [3], to be remarkably compelling, and also aesthetically pleasing.

The 10d \( N=1 \) chiral supergravity is, of course, anomalous, and must be extended by a Yang-Mills sector with suitable nonabelian gauge group in order to achieve consistency as a quantum theory. This requires inclusion of the massive Kalb-Ramond term in the 10d effective action, at leading order in \( m_s^{-1} \) [2, 31]. Including the full slew of \( O(m_s^{-1}) \) corrections present in the target spacetime Lagrangian of the heterotic, or type I, superstring theories, where the mass scale, \( m_s = \alpha'^{-1/2} \), is the
tension of the fundamental closed string [31], implements the beautiful phenomenon of perturbative renormalizability in these theories. The precise form of the target spacetime Lagrangian of the heterotic string theory, inclusive of terms up to fourth order in the $\alpha'$ expansion, has been derived by Bergshoeff and de Roo in [2]. What is the global symmetry algebra of the leading terms in the $\alpha'$ expansion of this Lagrangian? The leading terms correspond to those in the 10d $N=1$ Einstein-Yang-Mills Lagrangian, and these can be obtained by the Noether method [39], checking invariance under local Lorentz and Yang-Mills gauge transformations, in addition to local supersymmetry transformations. The spacetime Lagrangian of the perturbative ten-dimensional $N=1$ superstring theories has been shown to have dual, two-form and six-form, formulations [2], evidence for at least a partial electric-magnetic duality in the nonperturbative superstring theory which shares the same low energy limit. In addition, the type IB–heterotic strong-weak coupling duality conjecture maps the D5branes of the type I string to the NS5branes of the heterotic string [58, 59]. Thus, string ground states with eight supercharges have given significant evidence and physical insight into the dynamics of heterotic fivebranes. Notice that the perturbative mass spectrum of the heterotic string only contains the massless scalar dilaton and antisymmetric two-tensor potential, both arising in the Neveu-Schwarz sector. From the perspective of the type IB superstring, the dilaton arises in the Neveu-Schwarz sector, while the antisymmetric two-form is in the Ramond sector.

Thus, although the perturbative string mass spectrum has only given us the electric potentials of this theory, a full electric-magnetic duality in the spectrum of supergravity potentials in the nonperturbative theory would imply the global symmetry algebra $D_{11} \times G$, where $G$ is the anomaly-free Yang-Mills group, respectively, $E_8 \times E_8$ or $SO(32)$. There is at least partial evidence that the spacetime Lagrangian of this theory admits both electric, and dual magnetic, formulations. Passing between the twoform and sixform formulations involves a simple set of field redefinitions, and relations between couplings [2, 39], and preliminary evidence has been given for a dual graviton formulation of the kinetic terms in the bosonic sector of 11d supergravity [4]. Thus, we can conjecture that the extended symmetry algebra underlying the matrix formulation for the nonperturbative type I-I'–heterotic string theories in [1, 8] takes the form $D_{11} \times G \times U(N)$. In the next section, we will see how the emergent spacetime backgrounds accessible as large $N$ limits of the matrix Lagrangian in [1, 8] gives rise to a large class of type I, type II, and heterotic string backgrounds in diverse spacetime dimensions, $d \leq 10$, all of which preserve sixteen supercharges, and also correspond to perturbatively renormalizable and ultraviolet finite superstring theories. Our discussion will give rise to a natural question one could frame much more broadly: should a nonperturbative formulation for String/M theory be electric-magnetic duality covariant, or merely duality invariant?

3 Large N Limits & the String/M Duality Web

The nature of the isomorphism pointed out by us in [1, 8] between large $N$ limits of the matrix Lagrangian, and weak coupling limits of the String/M Duality web, is as follows. The matrix Lagrangian in [1] has the global symmetries of the 10d $N = 1$ chiral supergravity-Yang-Mills theory [2, 39], which we will denote as $\mathcal{G}_s \times G$, in addition to the obvious $U(N)$ flavor symmetry of the generic $U(N)$ matrix model. As described in section 2, $\mathcal{G}_s \times G$ can plausibly be made as large as $D_{11} \times G$, depending on how much of the electric-magnetic duality can be made manifest in the
Lagrangian itself. Given the range of potentials represented in the matrix Lagrangian, the large $N$ limit allows for many inequivalent scalings, generalizing the two-parameter, $(g_s, N)$, double-scaling limit introduced for the $c = 1$ matrix models [10]. For the $U(N)$ matrix Lagrangian with sixteen supersymmetries in [1, 8], written in its “heterotic” two-form formulation, we have the following list of parameters to work with:

$$(g_s = e^\phi, \bar{E}_a^\mu, \bar{E}_a^{\mu}, \bar{A}_a, \bar{B}_{ab}, \bar{F}_{ab}, \bar{H}_{abc}, \bar{B}_{c1\cdots c_p}, \bar{H}_{c1\cdots c_{p-1}}), \ p = 6, 8 \ .$$

(6)

We have grouped together the electric potentials, namely, the scalar dilaton, graviton, antisymmetric two-form potential, and Yang-Mills vector potential, separate from the electric field strengths, thereby allowing for the possibility of both background fields, and background fluxes. Finally, there are the magnetic potentials, the dual eightform and sixform potentials, and their corresponding magnetic field strengths. Their presence in the matrix Lagrangian is only implicit: the Lagrangian admits a duality transformation on the “electric” matrix variables, so we are free to consider the dual matrix Lagrangian, with a dual set of large $N$ scalings, prior to taking the large $N$ limit. The existence of the dual six-form formulation of the continuum spacetime Lagrangian is well-known [2, 39, 13], and some preliminary evidence has also been given for a dual graviton formulation of the 11d supergravity Lagrangian [4].

The indices in the equation above run from 0 to 9. The $N\times N$ zweibein matrix variables, $(E_a^\mu, E_\mu^a)$, encapsulate the data specifying the background target spacetime geometry. As many as ten continuum spacetime coordinates, and ten continuum spacetime coordinate derivatives, can emerge in the large $N$ limit [8]. The respective scalings of the dilaton, and the zweibein, with powers of $N$ and $m_s = \alpha'^{-1/2}$, are determined by requiring the canonical overall normalization for the 10d Einstein action, as well as the relative normalization of the kinetic term for the dilaton. Recall that the “type IB” and “heterotic” guises of the 10d N=1 spacetime Lagrangian are distinguished by the scaling of the Yang-Mills kinetic term relative to the Einstein kinetic term [14, 31]; the Yang-Mills gauge fields appear in the open string sector in type IB, and $g_s = g_{open}^2$. In addition, the antisymmetric two-form potential appears in the Ramond sector of type IB, and the kinetic terms for Ramond sector fields do not contain powers of $[e^\phi]$ [14, 31]. These two differences will determine the relative scalings for $A_a, B_{ab}$ in the large $N$ limits that distinguish type IB from heterotic. Scaling as would be appropriate for the 10d heterotic Lagrangian, implies one of two anomaly-free choices for the gauge group, $SO(32)$ or $E_8 \times E_8$. For the type IB scaling, we must instead choose the gauge group $SO(32)$.

Let us move on to explaining the emergence of the toroidal backgrounds. Suppose we take the large $N$ limit of the matrix Lagrangian, scaling $(g_s, E_a^\mu, E_\mu^a)$ with the appropriate powers of $N$, $m_s$ to give the correctly normalized Einstein spacetime action in dimensions $d < 10$. For a toroidal compactification, the $(E_a^\mu, E_\mu^a)$ will decompose naturally into noncompact, and compact, coordinates and coordinate derivatives. Notice that the decomposition of the remaining matrix variables into components carrying a noncompact, or compact, index, proceeds in precise analogy with how one constructs the dimensional reductions of the fields in a continuum spacetime Lagrangian [14, 23]. In particular, the T-duality symmetries that characterize the moduli space for a given compactification become manifest: associating with each compactification its T-duality group, we can identify the isomorphism between large $N$ limits and moduli spaces. Finally, this prescription carries over naturally to the supersymmetry preserving heterotic CHL orbifolds [54, 55, 62]: the
orbifold projection on the string mass spectrum identifies a specific truncation of the moduli space of massless scalars, also isomorphic to a corresponding truncation of the finite $N$ matrix Lagrangian. To obtain the string effective Lagrangians that describe 9D CHL orbifolds of the type IIA, or type I$'$, string theories, preserving only 16 supersymmetries [55], we must incorporate the Ramond sector one-form potential, $\bar{C}_a$, in the definition of a distinct, and nontrivial, large $N$ scaling limit.

More generally, notice that the inequivalent multiple-scaled large $N$ limits of the matrix Lagrangian are distinguished by the individual scalings of any combination of the background parameters with appropriate powers of $N$, the string mass-scale, $m_s = \alpha'^{-1/2}$, and the dimensionless string coupling, $g_s = e^\phi$, while taking $N \rightarrow \infty$. One final comment: if $G$ is a non-simply-laced group, as happens at certain enhanced symmetry points in the CHL moduli spaces, we must include the duality-transformed vector potential of the magnetic dual group, $A^*$, among the magnetic potentials listed in Eq. (6).

Setting $\alpha'$ to zero in the multiple-scaled large $N$ matrix Lagrangian recovers the massless low energy field theory limit of some perturbatively renormalizable string ground state: this is by construction, since the choice of matrix Lagrangian was determined by its global symmetries. We have conjectured that, as a consequence, the off-diagonal $O(1/N)$ terms in the matrix Lagrangian for a particular choice of large $N$ scaling will be isomorphic to the $O(\alpha')$ corrections in a corresponding perturbatively renormalizable ground state of string theory [8]. To anybody familiar with calculating the string massive mode spectrum, mass-level by mass-level, this conjecture should come as no surprise: the primary fields of the worldsheet conformal field theory appear in the low energy massless field theory limit, and their descendants fill in the tower of massive string modes. Thus, the algebraic symmetry structure that determines the massless field theory limit will also be replicated at each mass level.\footnote{The symmetries of the string mass spectrum and perturbative S-matrix have been the focus of several previous works in string theory [40, 41, 60]. Related work on the high-energy symmetries of String Theory appears in [43, 44, 45, 46].}

This property is characteristic of the $O(1/N)$ corrections to the large $N$ limit of any matrix Lagrangian.

4 Conclusions

The Lagrangian in [1] was originally proposed by us as a matrix formulation for nonperturbative type I-heterotic string theory. It was argued that spacetime itself, and the target spacetime Lagrangians of perturbatively renormalizable string theories with sixteen supercharges, emerge from the large $N$ limits of this matrix Lagrangian [8]. The matrix Lagrangian in [1] shares the global symmetries of the chiral 10d N=1 supergravity-Yang-Mills theory, in addition to the obvious $U(N)$ symmetry of a generic matrix model.

The significance of the global symmetry algebra in determining what aspects of the electric-magnetic duality symmetries of string/M theory backgrounds in diverse spacetime dimensions should be made manifest in the matrix Lagrangian itself, is the general theme explored in sections 1 thru 3 of this paper. We reviewed the evidence from the worldsheet formalism of perturbative string theory for the eleven-dimensional electric-magnetic duality in the Ramond-Ramond
sector of the type II superstring theories and Dpbrane spectrum [15, 19, 20]. If both electric and magnetic dual generators are included in the global symmetry algebra on equal footing, then, as has been shown by West and collaborators [6, 3], the algebra extends to a rank eleven Lorentzian Kac-Moody algebra: $E_{11}$ for theories with 32 supercharges, and $D_{11}$ for the theories with sixteen supercharges. It should be noted, as was pointed out in my 1995 paper with Polchinski [54, 62], that electric-magnetic duality in theories with sixteen supercharges only becomes manifest in the four-dimensional moduli spaces, in the guise of S-duality [2, 13, 54]. Innovative formalisms have been developed for implementing fully geometric realizations of self-duality [24, 25], but no completely duality covariant formulation exists for any of the supergravities, or string theories, at the present time. The precise role of the underlying eleven-dimensional electric-magnetic duality as it operates within the String/M Duality web, the associated algebras, and the question of whether we need a duality covariant, or merely a duality invariant, formulation for nonperturbative string/M theory, therefore remain questions that will need further exploration.

The matrix proposal for nonperturbative String/M theory given by us in [1, 8] has been modestly successful in establishing self-consistency with the type I-heterotic-M strong-weak coupling duality conjectures, in a multitude of disconnected moduli spaces, and in diverse spacetime dimensions. The nature of the disconnectedness of string moduli spaces, and their physical interpretation, has been described in our recent work [62]. Our discussion in section 3 of the myriad inequivalent large N scaling limits of a single finite N matrix Lagrangian clarifies that the existence of isolated low energy Universes in string theory does not, in itself, imply an essential role for the Anthropic Principle [49, 50, 51] in quantum cosmology. At leading order in the $\alpha'$ expansion, the efficacy of this result is self-evident. We have conjectured further that the $1/N$ expansion of the matrix Lagrangian, for any given choice of scalings, provides the systematics of the $\alpha'$ corrections to the massless low energy field theory limit in some renormalizable string ground state [8]. A confirmation of this conjecture in the perturbatively renormalizable flat spacetime backgrounds would be a significant step forward in establishing the viability and uniqueness of our proposal for nonperturbative String/M theory.

Conventional methods for computing the perturbative superstring S-matrix, thereby inferring higher order terms in the target spacetime Lagrangian, although straightforward in principle have been cumbersome in practice [48, 31]. Thus, even in exactly solvable flat spacetime backgrounds where the worldsheet formalism is extremely well-developed, it is often difficult to write down precise expressions for terms at high order in the $\alpha'$ expansion. The self-consistency of an exactly solvable worldsheet conformal field theory description, when it exists, suffices to establish the existence of the corresponding perturbatively renormalizable, all-orders-in-$\alpha'$, spacetime Lagrangian. It should be kept in mind that, if the supersymmetry breaking scale in Nature is far below the string mass scale, the higher $O(1/m_s)$ corrections are in any case irrelevant for the purposes of low energy physics in such backgrounds [62]. But at intermediate values of the string coupling constant, or in genuinely nonperturbative backgrounds of string theory where we do not have an exactly solvable worldsheet formulation, the tantalizing possibility that we might nevertheless be able to compute the higher order corrections in $\alpha'$, thus also establishing renormalizablity, is intriguing, to say the least.

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References

[1] S. Chaudhuri, *Nonperturbative Type I-I’ String Theory*, hep-th/0201129v1.

[2] E. Bergshoeff and M. de Roo, *Duality Transformations of String Effective Actions*, Phys. Lett. B249 (1990) 27.

[3] I. Schnakenburg and P. West, *Kac-Moody Symmetries of Ten-dimensional Nonmaximal Supergravity Theories*, hep-th/0401189, Section 1.

[4] I. Schnakenburg and P. West, *Massive IIA Supergravity as a Nonlinear Realization*, hep-th/0204207.

[5] I. Schnakenburg and P. West, *Kac-Moody Symmetries of IIB Supergravity*, hep-th/0107181.

[6] P. West, *The IIA, IIB, and eleven dimensional theories and their common E_{11} origin*, hep-th/0402140.

[7] M. Gaberdiel, D. Olive, and P. West, *A class of Lorentzian Kac-Moody algebras*, Nucl.Phys. B645 (2002) 403, hep-th/0205068.

[8] S. Chaudhuri, *Spacetime Reduction of Large N Flavor Models: A Fundamental Theory of Emergent Local Geometry?*, Nucl. Phys. B719 (2005) 188; hep-th/0408057.

[9] T. Eguchi and H. Kawai, Phys. Rev. Lett. 48 (1982) 1063. G. Parisi, Phys. Lett. B112 (1982) 463. D. Gross and Y. Kitazawa, Nucl. Phys. B206 (1982) 440. G. Bhanot, U. Heller, and H. Neuberger, Phys. Lett. B113 (1982) 47.

[10] E. Brezin and V.A. Kazakov, *Exactly Solvable Field Theories of Closed Strings*, Phys. Lett. B236 (1990) 144. M.R. Douglas and S.H. Shenker, *Strings in less than one dimension*, Nucl. Phys. B335 (1990) 144. D.J. Gross and A.A. Migdal, *Nonperturbative Two-dimensional Quantum Gravity*, Phys. Rev. Lett. 64 (1990) 127. An update on the basic mathematics appears in P. Zinn-Justin and J. B. Zuber, *On Some Integrals over the Unitary Group and their large N Limit*, math-ph/0209019.

[11] E. Cremmer and B. Julia, *The SO(8) Supergravity*, Nucl. Phys. B156 (1979) 141. Phys. Lett. B80 (1978) 48.

[12] L. Romans, *Massive N=2A Supergravity in Ten Dimensions*, Phys. Lett. B169 (1986) 374.
[13] A. Sen, *Strong-weak Coupling Duality in Four Dimensional String Theory*, Intl. Jour. Mod. Phys. **A9** (1994) 3707. J. Schwarz and A. Sen, *Duality Symmetries of the 4d Heterotic Strings*, Phys. Lett. **B312** (1993) 105. C. Hull and P. Townsend, *Unity of Superstring Dualities*, Nucl. Phys. **B438** (1995) 109.

[14] E. Witten, *String Theory Dynamics in Various Dimensions*, Nucl. Phys. **B443** (1995) 84, hep-th/9505054.

[15] J. Polchinski, *Dirichlet-branes and Ramond-Ramond Charge*, Phys. Rev. Lett. **75** (1995) 4724.

[16] A. Cohen, G. Moore, P. Nelson, and J. Polchinski, Nucl. Phys. **B267** (1986) 143.

[17] S. Chaudhuri, Y. Chen, and E. Novak, Phys. Rev. **D62** (2000) 026004.

[18] S. Chaudhuri and E. Novak, *Supersymmetric Pair Correlation Function of Wilson Loops*, Phys. Rev. **D62** (2000) 046002.

[19] S. Chaudhuri, *Confinement and the Short Type I’ Flux Tube*, Nucl. Phys. **B591** (2000) 243.

[20] S. Chaudhuri, *11D Electric-Magnetic Duality and the Dbrane Spectrum*, hep-th/0409033.

[21] E. Bergshoeff, M. de Roo, M. Green, G. Papadopoulos, and P. Townsend, *Duality of type II 7branes and 8branes*, Nucl. Phys. **B470** (1996) 113. C. Hull, *Massive String Theories from M-theory and F-theory*, JHEP 9811 (1998) 027. I.V. Lavrinenko, H. Lu, C. Pope, K. Stelle, *Superdualities, Brane-Tensions, and Massive IIA/IIB Duality*, Nucl. Phys. **B555** (1999) 201.

[22] H. Lu and C. Pope, *T-Duality and U-Duality in Toroidally Compactified Strings*, Nucl. Phys. **B510** (1998) 139; hep-th/9701177.

[23] E. Cremmer, B. Julia, H. Lu, and C. Pope, *Dualization of Dualities I*, Nucl. Phys. **B523** (1998) 73, hep-th/9710119.

[24] E. Cremmer, B. Julia, H. Lu, and C. Pope, *Dualization of Dualities II: Twisted self-duality of doubled fields and superdualities*, hep-th/9806106.

[25] C. Hull, *A Geometry for Non-geometric String Backgrounds*, hep-th/0406102.

[26] G. Moore and E. Witten, *Self-Duality, Ramond-Ramond Fields, and K-Theory*, JHEP 0005 (2000) 032, hep-th/9912279. D. Diaconescu, G. Moore, and E. Witten, *E8 Gauge Theory, and a Derivation of K-Theory from M-Theory*, Adv.Theor.Math.Phys. 6 (2003) 1031.

[27] I. Kriz and H. Sati, *M Theory, Type IIA Superstrings, and Elliptic Cohomology*, hep-th/0404013; *Type IIB String Theory, S Duality, and Generalized Cohomology*, hep-th/0410293.

[28] M. Green and J. Schwarz, *Anomaly Cancellation in Superstring Theory*, Phys. Lett. **B149** (1984) 117; *Infinity Cancellations in the SO(32) Superstring Theory*, Phys. Lett. **B151** (1985) 21.
[29] D. Gross, J. Harvey, E. Martinec, and R. Rohm, *Heterotic String Theory I: The Free Heterotic String*, Nucl. Phys. **B256** (1985) 253; *Heterotic String Theory II: The Interacting Heterotic String*, Nucl. Phys. **B267** (1985) 75.

[30] J. Polchinski, *Evaluation of the One Loop String Path Integral*, Comm. Math. Phys. **104** (1986) 37.

[31] J. Polchinski, *String Theory*, in two volumes (Cambridge).

[32] S. Chaudhuri, *Ultraviolet Limit of Open String Theory*, JHEP 9908 (1999) 003. S. Chaudhuri and E. Novak, *Effective String Tension and Renormalizability: String Theory in a Noncommutative Space*, JHEP 0008 (2000) 027, hep-th/0006014.

[33] J. Polchinski and A. Strominger, *New Vacua for Type II String Theory*, Phys. Lett. **B388** (1996) 736.

[34] T. Banks, W. Fischler, S. Shenker, and L. Susskind, *M(atrix) Theory*, Phys. Rev. **D55** (1997) 5112. See the review by W. Taylor, *M(atrix) Theory: Matrix Quantum Mechanics as a Fundamental Theory*, Rev. Mod. Phys. **73** (2001) 419.

[35] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, *A Large N Reduced Model as Superstring*, Nucl. Phys. **B510** (1998) 158. S. Iso and H. Kawai, Space-Time and Matter in IIB Matrix Model - gauge symmetry and diffeomorphism, Intl. Jour. Mod. Phys. **A15** (2000) 651, hep-th/9903217.

[36] R. Helling, J. Plefka, M. Serrone, and A. Waldron, *Three Graviton Scattering in M-Theory*, Nucl. Phys. **B559** (1999) 184, hep-th/9905183. H. Nicolai, *On Hidden Symmetries in d=11 Supergravity and Beyond*, hep-th/9906106. H. Nicolai, D. Olive, *The Principal SO(2,1) Subalgebra of a Lorentzian Kac-Moody Algebra*, Lett. Math. Phys. 58 (2001) 141.

[37] A. Ashtekar and J. Lewandowski, *Background Independent Quantum Gravity: A Status Report*, Class. Quant. Grav. **21** (2004) R53, gr-qc/0404018. A. Iqbal, N. Nekrasov, A. Okounkov, and C. Vafa, *Quantum Foam and Topological Strings*, hep-th/0312022. R. Dijkgraaf, S. Gukov, A. Neitzke, and C. Vafa, *Topological M Theory as Unification of Form Theories of Gravity*, hep-th/0411073. J. Baez, *An Introduction to Spin Foam Models of Quantum Gravity and BF Theory*, Lect. Notes in Phys. 543 (2000) 25, gr-qc/9905087.

[38] S. Chaudhuri, *Hidden Symmetry Unmasked: Matrix Theory and E(11)*, electronic review article, hep-th/0404235.

[39] D. Z. Freedman and J. A. Schwarz, *Unification of Supergravity and Yang-Mills Theory*, Phys. Rev. **D15** (1977) 1007. E. Bergshoeff, M. de Roo, B. de Wit, and P. van Nieuwenhuizen, *Ten-dimensional Maxwell-Einstein Supergravity, its Currents, and the issue of its Auxiliary Fields*, Nucl. Phys. **B195** (1982) 97. A. H. Chamseddine, Phys. Rev. **D24** (1981) 3065. G. F. Chapline and N. S. Manton, Phys. Lett. **B120** (1983) 105. E. Bergshoeff and M. de Roo, Nucl. Phys. **B328** (1989) 439.
[40] T. Banks and M. Peskin, *Gauge Invariance of String Fields*, Nucl. Phys. **B264** (1986) 513. T. Banks, M. Peskin, C. Preitschof, D. Friedan, E. Martinec, *All Free String Theories are Theories of Forms*, Nucl. Phys. **B274** (1986) 71. M. Evans and B.A. Ovrut, *Spontaneously broken intermass level symmetries in string theory*, Phys. Lett. 231B(1989)80; *Deformations of conformal field theories and symmetries of the string*, Phys. Rev. D41(1990)3149.

[41] I. Fraenkel, H. Garland, and G. Zuckerman, *Semi-infinite cohomology and string theory*, Proc. Nat. Acad. Sci. 83 (1986) 8442.

[42] P. Goddard and D. Olive, *Algebras, lattices and strings*, in Vertex Operators in Mathematics and Physics, Proceedings of a Conference eds. J. Lepowsky, S. Mandelstam, I.M. Singer, Springer; Intl. Jour. Mod. Phys. A1(1986)303.

[43] D. Gross, *High Energy Symmetries of String Theories*, Phys.Rev.Lett. **60** 1229 (1988). D. Gross and V. Periwal, *String Theory Beyond the Planck Scale*, Nucl. Phys. **B303** 407 (1988).

[44] E. Witten and B. Zwiebach, *Algebraic Structures and Differential Geometry in 2D String Theory*, Nucl. Phys. **B377** (1992) 55, hep-th/9201056.

[45] R. Borcherds, *Generalized Kac-Moody Algebras*, J. Algebra 115(1988)501; *Monstrous Moonshine and Monstrous Lie Superalgebras*, Invent. Math. 109(1992)405.

[46] G. Moore, *Symmetries of the Bosonic String S-Matrix*, hep-th/9310026; *Finite in All Directions*, hep-th/9305139.

[47] J. Harvey and G. Moore, *On the algebras of BPS states*, Commun. Math. Phys. 197 (1998) 489.

[48] D. J. Gross and N. J. Sloane, Nucl. Phys. **B291** (1987) 41; N. Sakai and Y. Tanii, Nucl. Phys. **B287** (1987) 457. See the recent paper, K. Peeters, J. Plefka, and S. Stern, *Higher Derivative Gauge Field Terms in the M Theory Action*, hep-th/0507178.

[49] S. Weinberg, *Anthropic Bounds on the Cosmological Constant*, Phys. Rev. Lett. 59, 2607 (1987); *Theories of the cosmological constant*, astro-ph/9610044.

[50] R. Bousso and J. Polchinski, *Quantization of Four-form Fluxes and Dynamical Neutralization of the Cosmological Constant*, JHEP 0006 (2000) 006, hep-th/0004134.

[51] F. Denef and M. Douglas, *Distributions of Flux Vacua*, JHEP 0405 (2004) 072.

[52] P. Goddard, J. Nuits, and D. Olive, Nucl. Phys. B125 (1977) 1.

[53] S. Chaudhuri, G. Hockney, and J. Lykken, *Maximally Supersymmetric String Theories in D<10*, Phys. Rev. Lett. 75 (1995) 2264.

[54] S. Chaudhuri and J. Polchinski, *Moduli Space of CHL Strings*, Phys. Rev. **D52** (1995) 7168.

[55] S. Chaudhuri and D. Lowe, *Type IIA-Heterotic Duals with Maximal Supersymmetry*, Nucl. Phys. **B459** (1996) 113.
[56] J. Polchinski and E. Witten, *Evidence for Heterotic-Type I String Duality*, Nucl. Phys. B460 (1996) 525.

[57] P. Horava and E. Witten, *Heterotic and Type I String Dynamics from Eleven Dimensions*, Nucl. Phys. B460 (1996) 506.

[58] E. Witten, *Small Instantons in String Theory*, Nucl.Phys. B460 (1996) 541. M. Berkooz et al, *Anomalies, Dualities, and Topology of D=6 N=1 Superstring Vacua*, hep-th/9605184.

[59] E. Gimon and J. Polchinski, *Consistency Conditions on Orientifolds and DManifolds*, Phys. Rev. D54 (1996) 1667, hep-th/9601038. E. Gimon and C. Johnson, *K3 Orientifolds*, Nucl.Phys. B477 (1996) 715.

[60] D. Kaplan, *Recent Developments in Lattice Supersymmetry*, hep-lat/0309099. S. Catterall, *Lattice Formulation of N=4 Super Yang Mills Theory*, hep-lat/0503036. D. Kaplan and M. Unsal, *A Euclidean Lattice Construction of SYM with 16 Supercharges*, hep-lat/0503039.

[61] S. W. Hawking, *The Quantum State of the Universe*, Nucl. Phys. B239 (1984) 257. J. Hartle, *Scientific Knowledge from the Perspective of Quantum Cosmology*, gr-qc/9601046, *Anthropic Reasoning and Quantum Cosmology*, hep-th/0406104.

[62] S. Chaudhuri, *CHL Compactifications and Beyond*, talk given at the 3rd Simons Workshop in Mathematics & Physics, SUNY-Stonybrook, Jul 26, 2005; hep-th/0508147.