Latent fluctuation periods and long-term forecasting of the level of Markakol lake

A S Madibekov¹, A V Babkin², A Musakulkyzy¹, A V Cherednichenko³

¹Institute of Geography, Pushkin str., 99, Almaty, 050000, Kazakhstan
²Department of Atmospheric Dynamics and Space Geosciences, Russian State Hydrometeorological University, Maloohtynskay str., 97, Saint - Petersburg, 195196, Russia
³Zhasyl Damu JSC, Seifullin str., 597, Almaty, 050022, Kazakhstan
E-mail: madibekov@mail.ru, abav@mail.ru, mus_ain@mail.ru, geliograf@mail.ru

Abstract. The analysis of time series of the level of Markakol Lake by the method of “Periodicities” reveals in its variations the harmonics with the periods of 12 and 14 years, respectively. The verification forecasts of the lake level by the trend tendency and by its combination with these sinusoids were computed with the lead time of 5 and 10 years. The estimation of the forecast results by the new independent data permitted to conclude that forecasts by the combination of the sinusoids and trend tendency are better than by the trend tendency only. They are no worse than the mean value prediction.

1. Introduction
The lake levels are very visible indicators of climate change, water balance and human influence [1-3]. The issues of studying long-term forecasting of water levels in lakes have been given a lot of scientific attention in works [4-6]. In this study, using the example of a climatically conditioned lake, a valid method is proposed that is applicable for all lakes based on the detection of periods and provides the possibility of predicting the water level with a high degree of reliability.

The water level of Markakol lake experiences significant fluctuations over time. Its rises and falls impact on various sectors of modern production associated with the utilization of its water resources. Long-term forecasting of the lake level is necessary for the socioeconomic development of its basin according to the objectives of the State Water Resources Management Program of Kazakhstan (Decree of the President of the Republic of Kazakhstan dated 04.04.2014 №786) and "Water Security of the Republic of Kazakhstan" National program, under which "Water Resources of Kazakhstan and their Utilization" Geospatial Information System is being developed and the lakes in the Republic are being studied. Forecasting of the lake level would be expedient taking into account the latent times in its vibrations.

This study presents an analysis of fluctuations in the annual values of the lake level and, in view of the identified periods, makes its verification forecasts for the short term. The time series was analyzed from 1942 - the beginning of regular observations, till 2003. Over the next 10 years its values have been used to estimate the forecasts’ results based on a new independent material. Evaluation of forecasting results has shown that for the combination of the identified sinusoids and trend tendencies, the forecasts turned out to be better than for the trend tendencies and no worse than for the mean value.
Markakol Lake is located in Kazakhstan in the eponymous bolson edged with the mountain ridges of Kurchum and Azutau. In the north-east it adjoins the Bobrovskaya depression, and in the south-west along the Kalzhyr river it has access to the Zaisan valley (figure 1). The lake is tectonic. It is the second largest lake in the basin of the Ertis river. The lake is located at an altitude of 1419.3 m above sea level and has a prolate oval shape. It extends from the northeast to the southwest. The length of the lake is 38 km, and its width is 19.0 km, its catchment area is 1180 km², its water surface is 455 km², legislation of its shoreline is 106 km, its mean depth is 14.3 m, the maximum depth is 24 m. The lake basin contains 6.5 km³ of water. By the nature of water exchange, Markakol Lake is a drainage one. There is a large number of rivers and currents of water running into the lake, including the Topolevka, Mate Bai, Urunhayka, and other rivers, and the Kalzhyr River taking off from the lake [7].

Observations of the water level and temperature near the lake shore were started in 1937, and meteorological observations at the Markakol-Zapovednik station - in 1982. The climate in the lake basin is sharply continental: the average yearly temperature is -2.4 °C, the average temperature in January, which is the coldest month, is -21.9 °C, and in July, being the warmest month, it is 14.4 °C.

Figure 1. Aerial view of Markakol Lake (Source - Google Earth).

A steady 0 °C crossing of air temperature in the spring usually occurs in the middle of April, and in the second decade of October in the fall.

2. The problem statement or Material and methods
The time series of the lake level is analyzed using "Periodicities" method [8, 9]. This method is based on the approximation of the time series of hydrometeorological characteristics by sinusoidal functions in sequence with a step-by-step change in the period. The amplitude and phase of the best approximating sinusoid, as well as the additional addend around which it oscillates, and the sum of its quadratic differences with the values in the series are calculated for each period using the least squares adjustment [10].

In the analysis of the relation of the sum of quadratic differences of the approximating sinusoid to approximation period, the local minima of this sum should be observed for individual periods. Such a minimum of the sum of the quadratic differences of approximating sinusoid and the values in the series may be a sign of the presence of periodicity here.

When adding or combining the identified sinusoids, the correlation of their sum with the time series should consistently increase, reflecting peculiarities of its oscillations. The combination of sinusoids, which can also take into account the linear trend of the series, essentially represents its pattern, which can be used as a forecast expression for calculating future values of the lake level.

The results of the long-term forecast of the lake level can be estimated based on two different criteria. The forecast of the lake level should be considered verified if the difference between its predicted and actual values is less than its permissible error Δ, equal to 0.674 of mean-square deviation of its time series σ [11]. The predicted results for a multi-year interval are estimated by the
number of verified annual forecasts and by the sum of squares of their errors $S_f$. Its actual error $\delta r$, which can be compared with the time series values of the lake level and its mean square deviation is calculated taking into account the sum of squares of the forecast errors,

$$\delta r = 0.674 \sqrt{\frac{S_f}{l}}$$

where $l$ is the verification forecast interval, 5 and 10 years, respectively. If it is required to compare the results of forecasting of different lakes levels, it is necessary to compare relative errors $\frac{\delta r}{\Delta}$, rather than their actual errors. The relation of the actual forecast error to its permissible error has no dimensionality and is calculated in fractions of 1.

The long-term forecast of the lake level on the specified interval is the better, the larger is the number of verified forecasts and the smaller is its relative error. A successful long-term lake level forecast should be no worse than a prediction based on the mean value of its time series.

3. Results and discussion

The results of approximation of the time series of the lake level are presented in table 1. The first column of this table indicates approximation periods $T$, the second column indicates the constant value around which the best approximating sinusoid of this $H_0$ period oscillates; the third and fourth columns show its amplitude $\frac{aH}{2}$ and phase $\varphi_{Ht}$, respectively. The fifth column calculates its sum of quadratic differences with $S_t$ time series of the lake level.

For certain periods, there are minima of the sums of quadratic differences of approximating sinusoids and the series values observed. Characteristics of sinusoids with the minima of sums of quadratic differences with series values are in italics. The minima of these sums are registered for the periods of 3, 6, 8, 12, 14 and 25 years respectively.

In the bottom rows of table 1, the identified periods in lake level fluctuations are presented in the decreasing order of $\eta$ correlation of their sinusoids with a time series. The highest correlation with a number of levels is observed in periods of 14 and 12 years. It was 0.390 and 0.365, respectively. We are going to use these two sinusoids with the greatest correlation with a time series exceeding 0.35 to calculate the verification forecast.

Figure 2 shows fluctuations in the Markakol Lake level [12], the identified sinusoids with the periods of 12 and 14 years, the trend tendency and a combination of these sinusoids and trend tendencies. Sequences of increased lake level values with time series maxima in 1946, 1958 and 1969 and its decreased values with the minima in 1951 and 1963, the sinusoidal extremums with the periods of 12 and 14 years are closely spaced. The sinusoidal extremum with a period of 14 years is somewhat ahead of the corresponding extremum of the 12-year-old sinusoid at the maximum lake level in 1946 and its minimum in 1951; at the maximum lake level in 1958 these extremums occur almost simultaneously.

In the set of decreased values with a minimum lake level in 1963 and increased values with a maximum in 1969, the corresponding sinusoidal extremums with a period of 14 years lag behind the extremums of harmonics with a 12-year period. The time difference of sinusoidal extremums with the periods of 14 and 12 years is becoming greater in the region of decreased level values with the minimum values in 1974 and 1982 and in the future, these sinusoids virtually oscillate in their opposite phases.

In the first half of the analyzed interval, at which the corresponding sinusoidal extremums with the periods of 12 and 14 years are closely spaced, their combination with the trend tendency well describes the groups of years of increased lake level values with the maxima in 1946, 1958 and 1969, and its decreased values with the lows in 1951 and 1963. The decreased level values time interval with the minimum values in 1974 and 1982 is described by a smaller-amplitude basin in this combination, since the time of sinusoidal extremums with the periods of 12 and 14 years significantly diverges there.
Relatively small amplitude fluctuations in the lake level after 1989 are reflected by a smooth line formed by a combination of sinusoids with the periods of 12 and 14 years, oscillating in different phases, and a trend tendency. The derived forecast equation fails to describe the peak lake level in 1988. Its correlation with the time series equals 0.516.

Table 1. Sinusoidal Approximation of the time series of water level in Markakol Lake (1942 – 2003).

| $T$, years | $H_0$, m | $\frac{\delta H}{2}$, m | $\phi_0$, radian | $S_{H^2}$, m$^2$ |
|------------|-----------|-----------------|----------------|----------------|
| 3.0        | 153.2775  | 2.2854          | 4.1756         | 4890.386       |
| 4.0        | 153.3246  | 1.1432          | -0.1532        | 5010.476       |
| 5.0        | 153.3127  | 0.1485          | 1.1332         | 5050.287       |
| 6.0        | 153.3028  | 3.1911          | 4.6408         | 4740.345       |
| 7.0        | 153.2792  | 2.0819          | 2.4486         | 4917.954       |
| 8.0        | 153.2682  | 3.3387          | 0.0314         | 4700.315       |
| 9.0        | 153.3240  | 1.4267          | 2.7949         | 4987.716       |
| 10.0       | 153.3714  | 2.4240          | 2.5629         | 4867.161       |
| 11.0       | 153.1765  | 4.4659          | 1.7497         | 4428.943       |
| 12.0       | 153.3080  | 4.7237          | 0.7937         | 4378.584       |
| 13.0       | 153.1685  | 4.5847          | 4.0762         | 4399.329       |
| 14.0       | 152.9807  | 4.9656          | 2.6031         | 4284.018       |
| 15.0       | 153.1674  | 4.5188          | -1.4637        | 4400.974       |
| 16.0       | 153.4240  | 3.6297          | -0.1930        | 4655.557       |
| 17.0       | 153.4840  | 2.2330          | 1.4720         | 4903.723       |
| 18.0       | 153.3913  | 1.1720          | -1.3958        | 5008.690       |
| 19.0       | 153.3115  | 1.4674          | 4.2768         | 4987.528       |
| 20.0       | 153.2946  | 2.1111          | -0.5672        | 4916.667       |
| 21.0       | 153.3206  | 2.5358          | 3.8733         | 4848.937       |
| 22.0       | 153.3576  | 2.7782          | -0.7113        | 4801.404       |
| 23.0       | 153.3822  | 2.9314          | -1.3789        | 4771.890       |
| 24.0       | 153.3883  | 3.0499          | 2.1825         | 4754.858       |
| 25.0       | 153.3830  | 3.1417          | 3.9539         | 4747.649       |
| 26.0       | 153.3749  | 3.1844          | 4.1530         | 4750.168       |
| 27.0       | 153.3676  | 3.1554          | 2.9593         | 4762.074       |
| 28.0       | 153.3605  | 3.0549          | 0.5248         | 4780.852       |
| 29.0       | 153.3508  | 2.9093          | 3.2639         | 4802.134       |
| 30.0       | 153.3349  | 2.7582          | -1.2763        | 4821.370       |
| 31.0       | 153.3092  | 2.6382          | -0.4325        | 4835.208       |
| 32.0       | 153.2713  | 2.5742          | -0.4075        | 4841.865       |

| $T$, years | $\eta$   |
|------------|----------|
| 14.0       | 0.3897   |
| 12.0       | 0.3649   |
| 8.0        | 0.2635   |
| 6.0        | 0.2480   |
| 25.0       | 0.2451   |
| 3.0        | 0.1783   |
Figure 2. Fluctuations in the water level of Markokol Lake – Urnai vil.: 1 - observation data (dotted line is the verification forecast interval for 2004-2013), 2 - approximating sinusoid with a period of 12 years, 3 - approximating sinusoid with a period of 14 years, 4 - linear trend tendency (its equation is presented in the lower left corner (b)), 5 - a combination of sinusoids with the periods of 12 and 14 years and trend tendency, η - its correlation with the time series.

4. Verification Forecasts of the Level of Markakol Lake

Based on the forecast equation, combinations of sinusoids with the periods of 12 and 14 years and a trend tendency - calculations were made for the lake level verification forecasts with a lead time of 5 and 10 years respectively. The forecasting results and their evaluation are presented in table 2.

The first column of this table indicates years when verification forecasts had been calculated, the second column shows the actual values of the lake level and the third column shows the difference of the mean and the actual lake level values for the analyzed period (1942-2003). The fourth and the sixth columns show lake level computation results based on the trend tendency and its combination with the sinusoids with the periods of 12 and 14 years, the fifth and the seventh columns indicate their differences with the actual level value.

The average value of the lake level time series for 1942 – 2003 was 153.31 m, with mean-square deviation of 9.03 m. The admissible forecast error was 6.08 m. The forecast for the year is thus verified if the difference in the columns 3, 5 and 7, respectively, is less than this value.
The mean value forecast however was not verified for 2008-2009 and for 2011-2013 but proved to be right for all other years. Therefore, the mean value forecast was verified 4 times for the first five years and 5 times for the entire verification period. The relative mean value forecast error calculated factored in the differences presented in column 3, was 0.703 for the first five years, and 1.206 for the entire ten-year verification period.

Table 2. Results of Verification Forecasts of the level of Markakol Lake for 2004 – 2013

| $t$, years | $H_{\text{actual}}$, m | $H_{\text{mean}} - H_{\text{actual}}$, m | $H_{\text{trend}}$, M | trend $- H_{\text{actual}}$, m | $H_{\text{combs}}$, m | $H_{\text{combs}} - H_{\text{actual}}$, m |
|------------|-----------------|----------------------------|-----------------|-----------------|-----------------|-----------------|
| 1          | 151.9           | 1.41                       | 150.92          | -0.98           | 152.46          | 0.56            |
| 2          | 158.4           | -5.09                      | 150.84          | -7.56           | 151.74          | -6.66           |
| 3          | 152.8           | 0.51                       | 150.77          | -2.03           | 150.64          | -2.16           |
| 4          | 149.2           | 4.11                       | 150.69          | 1.49            | 149.36          | 0.16            |
| 5          | 140.8           | 12.51                      | 150.62          | 9.82            | 148.22          | 7.42            |
| 6          | 146.0           | 7.31                       | 150.54          | 4.54            | 147.55          | 1.55            |
| 7          | 152.0           | 1.31                       | 150.46          | -1.54           | 147.63          | -4.37           |
| 8          | 140.0           | 13.31                      | 150.39          | 10.39           | 148.54          | 8.54            |
| 9          | 129.0           | 24.31                      | 150.31          | 21.31           | 150.17          | 21.17           |
| 10         | 166.0           | -12.69                     | 150.24          | -15.76          | 152.19          | -13.81          |

$N_5$ & 4 & 3 & 3 & 3  \\
$\frac{\bar{\delta}r}{\bar{\sigma}_5}$ & 0.703 & 0.628 & 0.506 &  \\

$N_{10}$ & 5 & 5 & 5 & 5  \\
$\frac{\Delta_{\delta r}}{\delta_{10}}$ & 1.206 & 1.105 & 1.014 &

Forecasts by trend tendency and its combination with sinusoids with the periods of 12 and 14 years did not prove to be true in 2005, 2008, 2011, 2012 and 2013. According to these schemes, 3 forecasts were verified for the period of 2004 - 2008, and 5 forecasts had proven to be true for the entire verification forecast interval of 2004-2013. The relative error of the forecast by trend tendency and its combination with the sinusoids with periods of 12 and 14 years with a lead time of five years was 0.628 and 0.506, respectively, and with a lead time of 10 years – 1.105 and 1.014.

5. Conclusions

Analysis of the Markakol Lake level time series for 1942 – 2003 using "Periodicity" method indicates the presence of harmonics with the periods of 12 and 14 years in its fluctuations. The combination of the trend tendency and these sinusoids describes well the fluctuations in the values of this series. The trend tendency and its combination with these sinusoids were used to compute verification forecasts of the lake level with a lead time of 5 and 10 years. The forecasting results were evaluated based on a new independent material and compared with the forecasts for the mean value of the time series.

Since by one of the results assessment criterion forecast was shown to be better by mean value while by another criterion it was better by trend tendency and its combination with the identified sinusoids, we can conclude that forecasting results based on mean value, trend tendency and its combination with sinusoids are of approximately the same quality. Nevertheless, the forecasts by a combination of trend tendency and sinusoids has proven to be better than those by trend tendency: with the same number of correct forecasts, the relative error is smaller in forecasts based on combination of sinusoids and the trend tendency.
When forecasting for the entire verification period of 2004 – 2013, the same number of forecasts was verified in all three schemes. Therefore, the forecast, which has a smaller relative error is a better one. Thus, the best forecast was obtained by the combination of sinusoids and trend tendency, and then by trend tendency, and the worst forecast was the one based on the mean value of the time series.

To make long-term lake level forecasts, it is necessary to analyze the time series of the levels of all lakes in the Republic of Kazakhstan, where there are ongoing continuous series. Reliable harmonics, which can be used in forecasting, should be identified for multiple, rather than one series. At the same time, the fluctuations phase for the same period should change gradually depending on the geographical location of the lakes.

For a more reliable forecasting of lake levels for the next 5-10 years, it is necessary to analyze their series for shorter time intervals, for example, for seasonal or monthly intervals.

References
[1] Van der Kamp G, Keir D and Evans M S 2008 Canadian Water Resources Journal, 33 (1) 23-38
[2] Zoljoodi M and Didevarasl A 2014 Atmospheric and Climate Sciences 4 358-368
[3] Demargne J, Wu L, Seo D J and Schaaie J 2007 Proceeding of the Symposium HS2004 at IUGG2007, Perugia, July 2007. IAHS 313
[4] Trolle D, Hamilton D P, Pilditch C A, Duggan I C and Jeppesen E 2011 Environmental Modelling & Software 26 (4) 354–370
[5] García Molinos J, Viana M, Brennan M and Donohue I 2015 J. PLoS ONE 10 (3) e0119253 doi:10.1371/journal.pone.0119253
[6] Liu L, Xu Z X, Reynard N S, Hu C W and Jones R G 2013 J. of Flood Risk Management 6 (1) 14–22
[7] Filonets P P and Omarov T R 1974 Lakes of the Northern, Western and Eastern Kazakhstan (L.: Gidrometeoizdat) 138
[8] Babkin A V 2005 Meteorology and Hydrology 11 63-73
[9] Babkin A V and Semeikin N I 2014 News of the Russian Geographical Society 1 43-48
[10] Linnik Yu V 1962 Least Square Method (Moscow: Nauka) 350
[11] Apollov B A, Kalinin G P and Komarov V D 1974 Hydrological Forecasting Course (L.: Gidrometeoizdat) 419
[12] Musakulkyzy A and Madibekov A S 2016 International Research and Practice Conference on "Water resources of Central Asia and their Utilization", on progress review for the UN "Water for Life" decade, Almaty, 360-365