Response prediction method for tension piles considering progressive soil deformation

LIU Nianwu1, LIANG Yaoying2,*, YU Jitao1, SHAO Peng2 and ZHANG Qianqing3

1 China Railway Eryuan Engineering Group CO. LTD., Chengdu, 610031, China.
2 Institute of Foundation and Structure Technologies, Zhejiang Sci-Tech University, Hangzhou, Zhejiang, 310018, China
3 Geotechnical and Structural Engineering Research Center, Shandong University, Jinan, Shandong, 250061, China, zjuzqq@163.com.
*Corresponding author’s e-mail: liangyaoying1994@163.com.

Abstract. This paper presents a analytical approach for the analysis of the response of a single pile and pile group subjected to tension load by the load-transfer approach. The shaft displacement of a tension pile is assumed to comprise the relative displacement of disturbed soil around a pile and the elastic vertical soil displacement. Considering the interactive effects among piles, a new load transfer function of the individual pile in a pile group is proposed. In addition, an effective calculate method is developed for analyzing the response of a single pile and a pile group subjected to tension load. Comparative analyses are then performed to demonstrate the veracity of the proposed method. The comparative analysis results show that the proposed approach is reliable for the analysis of the response of single piles as well as pile groups subjected to tension load.

1. Introduction
The capacity of an uplift pile mainly depends on skin friction between the pile and the adjacent soil and the self-weight of the pile. Several methods have studied the behavior of piles subjected to uplift load, including experimental and numerical methods (Goel and Patra, 2007; Zhang et al., 2011; Zhang et al., 2015a, 2018a). These studies significantly contributed to predicting the behavior of single piles subjected to tension load and evaluating the capacity of an uplift pile.

In this study, a new load transfer function of a single pile was proposed to simulate the relationship between shaft displacement and unit skin friction. Considering the ‘reinforcing effect’ of a pile on the soil continuum in pile groups, a new load transfer function of the individual pile in a pile group was also proposed. Furthermore, an effective iterative computer program was developed by using the load-transfer approach to analyze the response of a single pile and a pile group subjected to tension load.

2. Theory and calculation
For piles with tension load, a hyperbolic nonlinear model can be selected to estimate the relationship between unit skin friction and the relative displacement between soil and piles. (Zhang et al., 2015a, 2015b). The hyperbolic relationship is as follows:
\[
\tau_{sz} = \frac{S_{sz}}{a + bS_{sz}} \tag{1}
\]

where \(a\) and \(b\) are empirical coefficients; \(S_{sz}\) is the relative displacement between soil and piles at depth \(z\); and \(\tau_{sz}\) is the unit skin friction at depth \(z\). The value of \(a, b\) were obtained from Zhang (2015a, 2015b).

\(S_z\), the total shaft displacement at depth \(z\), is assumed to be composed of \(S_{sz}\), relative displacement, and \(\Delta S_{sz}\), the purely elastic displacement of soil far from the pile shaft, and can be computed as:

\[
S_z = S_{sz} + \Delta S_{sz} \tag{2}
\]

The purely elastic displacement of soil away from the pile, \(\Delta S_{sz}\), is only related to skin friction, and can be calculated as follows (Randolph and Wroth, 1979):

\[
\Delta S_{sz} = \frac{r_0}{G_{s}} \ln \left( \frac{r_m}{r_0} \right) \tau_{sz} = c\tau_{sz} \tag{3}
\]

where \(c\) is the reciprocal of soil stiffness around a single pile. For a single pile subjected to tension load, the values of \(c\) and \(a\) are identical.

By substituting Eq (1) and Eq (2) into Eq. (3), the value of \(\tau_{sz}\) can be written as follows:

\[
\tau_{sz} = \frac{(a + c + bS_z) - \sqrt{(a + c + bS_z)^2 - 4bcS_z}}{2bc} \tag{4}
\]

The new established load transfer function (see Eq.(4)) used in the analysis of the behaviors of a single pile subjected to tension load can also be extended to the analysis of the response of a pile group by accounting for the interaction between different piles.

Considering interactive effects in a group of \(n\) piles subjected to tension load, the elastic displacement of surrounding soil \(i\), \(\Delta W_{szij}\), induced by the skin friction of pile \(j\), \(\tau_{szj}\) (\(j = 1\) to \(n\), and \(j \neq i\)) can be written as:

\[
\Delta W_{szij} = \sum_{j=1}^{n} \sum_{j \neq i}^{n} \frac{\tau_{szj}r_0^2}{G_{s}r_{ij}} \ln \left( \frac{r_m}{r_{ij}} \right) \tag{5}
\]

In consideration of the interactive effects among piles, the elastic displacement of soil surrounding individual pile \(i\), \(\Delta S_{szi}\), can be calculated as follows:

\[
\Delta S_{szi} = \Delta S_{szi} + \Delta W_{szij} = \Delta W_{szij} - \Delta W_{szij}^{'} = \frac{\tau_{szj}r_0}{G_{s}} \ln \left( \frac{r_m}{r_0} \right) + \sum_{j=1}^{n} \frac{\tau_{szj}r_0}{G_{s}} \ln \left( \frac{r_m}{r_{ij}} \right) - \sum_{j=1}^{n} \frac{\tau_{szj}r_0^2}{G_{s}r_{ij}} \ln \left( \frac{r_m}{r_{ij}} \right) \tag{6}
\]

In order to develop a simplified and computationally efficient solution method, in a pile group subjected to tension load, assume \(\tau_{szi} = \tau_{szj} = \tau_{sz}\). The justification for such an important simplification has been proposed by Lee and Xiao (2001). The value of \(\Delta S_{szi}\) presented in Eq. (6) can be rewritten as:

\[
\Delta S_{szi} = \sum_{j=1}^{n} \frac{r_0}{G_{s}} \ln \left( \frac{r_m}{r_{ij}} \right) - \sum_{j=1}^{n} \frac{r_0^2}{G_{s}r_{ij}} \ln \left( \frac{r_m}{r_{ij}} \right) \tau_{sz} = c\tau_{sz} \tag{7}
\]

where \(c\) is the reciprocal of soil stiffness around an pile in a pile group.

Considering the self-weight of the pile, \(dP_z\), can be computed from Eq. (4):

\[
dP_z = \frac{U \left[ (a + c + bS_z) - \sqrt{(a + c + bS_z)^2 - 4bcS_z} \right] + 2bc\gamma_p A_p}{2bcP_z} \text{d}S_z \tag{8}
\]
Regarding the analysis of an individual pile \( i \) in a pile group, the value of \( c \) should be replaced by the value of \( c \) presented in Eq. (7).

Based on the proposed method, the response of a single pile or the pile in a pile group subjected to tension load can be analyzed. Herein taking a single pile subjected to tension load as an example, the algorithm for the response of a tension pile considering progressive displacement of the pile-soil system is given as follows.

1. Assume a tension pile is divided into \( n \) segments.
2. Assume a small pile end displacement, \( S_{bn} \); the pile end load, \( P_{bn} \), is hypothesized to be zero.
3. A vertical displacement, \( S_{cn} \), at the middle of pile segment \( n \) is hypothesized (for the first trial, assume \( S_{cn}=S_{bn} \)). Based on the load transfer function given in Eq. (4), the unit skin friction of pile segment \( n \), \( \tau_{sn} \), can be calculated using the hypothesized value of \( S_{cn} \).
4. The axial force increment of pile segment \( n \), \( \Delta P_{n} \), the pile head load of pile segment \( n \), \( P_{tn} \), and the average axial force of pile segment \( n \), \( \overline{P}_{n} \), can be calculated from Eq. (9). One obtains:
\[
\begin{align*}
\Delta P_{n} = & U \left[ (a+c+bS_{bn}) - \sqrt{(a+c+bS_{bn})^2 - 4bcS_{tn}} \right] + 2bc\gamma_{p}A_{p} \left( \frac{E_{p}A_{p}}{L_{n}} \right) \\
\overline{P}_{n} = & P_{bn} + \Delta P_{n} \\
\overline{P}_{t} = & P_{bn} + 0.5\Delta P_{n} 
\end{align*}
\]
where \( L_{n} \) is the length of pile segment \( n \).
5. Assuming a linear variation of load in pile segment \( n \), the elastic deformation at the midpoint of pile segment \( n \), \( S_{tn} \), and the pile head displacement of pile segment \( n \), \( S_{tn} \), can be calculated by:
\[
S_{tn} = \left( S_{bn} + S_{cn} \right) / 2
\]
6. A new formula of axial force increment of pile segment \( n \), \( \Delta P_{n}' \), can be calculated from Eqs. (10), (11), and (12). One obtains:
\[
\Delta P_{n}' = \left\{ \begin{array}{l}
U \left[ (a+c+bS_{bn}) - \sqrt{(a+c+bS_{bn})^2 - 4bcS_{tn}} \right] + 2bc\gamma_{p}A_{p} \left( \frac{E_{p}A_{p}}{L_{n}} \right) \\
2bcP_{n}
\end{array} \right\}
\]
where \( P_{n} \) is the modified average axial force of segment \( n \).
(7) Based on Eq (10) and Eq (11), another expression of axial force increment of pile segment \( n \), \( \Delta P_{n}'' \), can also be obtained.
\[
\Delta P_{n}'' = \left\{ \begin{array}{l}
U \left[ (a+c+bS_{bn}) - \sqrt{(a+c+bS_{bn})^2 - 4bcS_{tn}} \right] + 2bc\gamma_{p}A_{p} \left( \frac{E_{p}A_{p}}{L_{n}} \right) \\
2bcP_{n}
\end{array} \right\}
\]
where \( S_{cn}' \) is a modified vertical movement at the middle height of pile segment \( n \).
8. Compare the axial force increment of pile segment \( n \), \( \Delta P_{n} \), with the axial force increment of pile segment \( n \), \( \Delta P_{n}'' \).
- **①** If the value of \( \left| \Delta P_{n}' - \Delta P_{n}'' \right| / \Delta P_{n} \) is smaller than a specified tolerance, select \( \Delta P_{n}' \) as the axial force increment of segment \( n \).
- **②** If the value of \( \left| \Delta P_{n}' - \Delta P_{n}'' \right| / \Delta P_{n} \) is larger, assume \( \Delta P_{n}' = \Delta P_{n}'' \). Repeat steps 4 to 6 until the value of \( b \left| \Delta P_{n}' - \Delta P_{n}'' \right| / \Delta P_{n} \) is within the assumed tolerance.
(9) Calculate the load and displacement at the top of segment \( n \), \( P_{tn} \) and \( S_{tn} \), respectively. And the method is as follows,

\[ S_{tn} = S_{bn} + S_{cn}, \quad P_{tn} = P_{bn} + \Delta P_n \]  

(10) Following the suggestion of \( P_{tn} = P_{b(n-1)} \) and \( S_{tn} = S_{b(n-1)} \), repeat steps 4 to 9 from pile segment \( n \) to pile segment 1 until a load-settlement relationship developed at the pile head is obtained.

(11) Repeat the procedure from steps 2 to 10 with different values of \( S_{bn} \) until a series of load-displacements is obtained.

Regarding the response of the pile in a pile group subjected to tension load, the parameter \( c \) presented in Eqs. (12) and (13) is replaced by \( c' \), and the response of an individual pile can then be analysed.

3. Case study

Sowa (1970) has tested a reinforced concrete pile with soft clay. The ground water level was 1.2 m below the ground surface. The pile properties and soil properties of Case one are shown in Table 1. In this calculation, following the suggestion of Zhang et al. (2010), Poisson’s ratio of the soil was assumed to be 0.5, the value of \( \delta/\varphi \) was adopted as 0.75, and the value of \( K/K_0 \) was taken as 2. The unit skin friction of tension pile is estimated from Eq. (3), and the value of \( \lambda \) was adopted as 0.7. The calculated value for limiting unit skin friction of the soil is shown in Figure 1. Note that the single pile is divided into 30 segments of 0.4 m each, and the ultimate unit skin friction of each pile segment can be adopted as an average value of the unit skin friction.

The computed pile response for the measured single pile load-settlement curve given by Sowa (1970) and derived using the proposed method was compared with the calculated values given by Zhang et al. (2015b) and the computed results of Goel and Patra (2007), as shown in Figure 2.

Figure 2 shows that up to the pile head load of 250 kN, the load-displacement curves at the pile head derived using the proposed method are in good agreement with the calculated values suggested by Zhang et al. (2015b). The values calculated using the proposed method are slightly larger than those of Sowa (1970) and Goel and Patra (2007) at all loading levels, they are rather satisfactory considering the measured values and the computed results suggested by Goel and Patra (2007). It can also be concluded from Figure 2 that the pile displacement estimated using the proposed approach increases with an increasing value of \( R_{ef} \) under the same load. The reliability of the proposed method for analyzing the response of a single pile subjected to tension load is verified by this case study.
The present calculated values $R_{sf} = 0.85$, $R_{sf} = 0.90$, $R_{sf} = 0.95$.

Figure 2. Measured and calculated load-displacement curves of a single uplift pile

Table 1. Pile and soil properties in the case study (after Sowa (1970))

| Natural unit weight of the soils (kN/m³) | Effective unit weight of the soils (kN/m³) | Angle of shearing resistance of the soils (°) | Shear modulus of the soils (MPa) | Pile diameter (m) | Pile length (m) | Pile elastic modulus (GPa) |
|----------------------------------------|------------------------------------------|---------------------------------------------|--------------------------------|------------------|---------------|-------------------------|
| 18.4                                   | 8.4                                      | 30                                          | 2.0                           | 0.53             | 12            | 30                      |

4. Conclusions

This paper presents a simple analytical method for analyzing the response of a single pile and a pile group subjected to tension load by using the load-transfer approach. Through theoretical analysis, a highly effective iterative computer program was developed for calculating the response of a single pile and a pile group subjected to tension load. Compared with other methods, this method is more efficient and practical.

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