Anti de Sitter Holography via Sekiguchi Decomposition

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Abstract In the present paper we start consideration of anti de Sitter holography in the general case of the \((q+1)\)-dimensional anti de Sitter bulk with boundary \(q\)-dimensional Minkowski space-time. We present the group-theoretic foundations that are necessary in our approach. Comparing what is done for \(q = 3\) the new element in the present paper is the presentation of the bulk space as the homogeneous space \(G/H = SO(q,2)/SO(q,1)\), which homogeneous space was studied by Sekiguchi.

1 Introduction

For the last fifteen years due to the remarkable proposal of [1] the AdS/CFT correspondence is a dominant subject in string theory and conformal field theory. Actually the possible relation of field theory on anti de Sitter space to conformal field theory on boundary Minkowski space-time was studied also before, cf., e.g., [2–7]. The proposal of [1] was further elaborated in [8] and [9]. After that there was an explosion of related research which continues also currently.

Let us recall that the AdS/CFT correspondence has 2 ingredients [1,8,9]:

1. the holography principle, which is very old, and means the reconstruction of some objects in the bulk (that may be classical or quantum) from some
objects on the boundary; 2. the reconstruction of quantum objects, like 2-point functions on the boundary, from appropriate actions on the bulk.

Our focus is on the first ingredient. We note that until recently the explicit presentation of the holography principle was realized in the Euclidean case, i.e., for the group $SO(q+1,1)$ relying on Wick rotations of the final results, cf., e.g. [9, 10]. Yet it is desirable to show the holography principle by direct construction in Minkowski space-time, i.e., for the conformal group $SO(q,2)$.

This was done for the case $q=3$ in detail in [11]. In the present paper we start consideration of the general case of the $(q+1)$-dimensional anti de Sitter bulk with boundary $q$-dimensional Minkowski space-time. Actually, here we only lay the group-theoretic foundations that are necessary in our approach while the actual construction is postponed to [12]. As historical remark we mention that this approach originated in the construction of the discrete series of unitary representations in [13, 14], which was then applied in [15] for the Euclidean conformal group $SO(4,1)$. A different approach was applied to the general Euclidean case $SO(N,1)$ in [10]. Also the nonrelativistic Schrödinger algebra case was considered in [16].

The new element in the present paper is the presentation of the bulk space as the homogeneous space $G/H = SO(q,2)/SO(q,1)$. For this we use the Sekiguchi decomposition [17]

$$G \cong \text{loc } \tilde{N}AH$$

where $A$ is the subgroup of dilatations, $\tilde{N}$ is isomorphic to the subgroup of translations. The above means that the subgroup $\tilde{N}AH$ is an open dense set of $G$, and thus the homogeneous space $G/H$ is locally isomorphic to bulk space $\tilde{N}A$.

## 2 Preliminaries

We need some well-known preliminaries to set up our notation and conventions. The Lie algebra $G = so(q,2)$ may be defined as the set of $(q+2) \times (q+2)$ matrices $X$ which fulfil the relation:

$$^tX\eta + \eta X = 0, \quad (1)$$

where the metric $\eta$ is given by

$$\eta = (\eta_{AB}) = \text{diag}(-1,1,\ldots,1,-1), \quad A,B = 0,1,\cdots,q+1 \quad (2)$$

Then we can choose a basis $X_{AB} = -X_{BA}$ of $G$ satisfying the commutation relations

$$[X_{AB},X_{CD}] = \eta_{AC}X_{BD} + \eta_{BD}X_{AC} - \eta_{AD}X_{BC} - \eta_{BC}X_{AD}. \quad (3)$$
We list the important subalgebras of $\mathcal{G}$:

- $\mathcal{K} = \text{so}(q) \oplus \text{so}(2)$, generators: $X_{AB} : (A, B) \in \{1, \ldots, q\}, \{0, q + 1\}$, maximal compact subalgebra;
- $\mathcal{Q}$, generators: $X_{AB} : A \in \{1, \ldots, q\}, B \in \{0, q + 1\}$, non-compact completion of $\mathcal{K}$;
- $\mathcal{A} = \text{so}(1, 1)$, generator: $D \cong X_{q,q+1}$, dilatations;
- $\mathcal{M} = \text{so}(q-1, 1)$, generators: $X_{AB} : (A, B) \in \{0, \ldots, q - 1\}$, Lorentz subalgebra;
- $\mathcal{N}$, generators: $T_\mu = X_{\mu q} + X_{\mu,q+1}$, $\mu = 0, \ldots, q-1$, translations;
- $\tilde{\mathcal{N}}$, generators: $C_\mu = X_{\mu q} - X_{\mu,q+1}$, $\mu = 0, \ldots, q-1$, special conformal transformations;
- $\mathcal{A}_0 = \text{so}(1,1) \oplus \text{so}(1,1)$, generators: $X_{0,q-1}, X_{q,q+1}$;
- $\mathcal{M}_0 = \text{so}(q-2)$, generators: $X_{AB} : (A, B) \in \{1, \ldots, q - 2\}$;
- $\mathcal{N}_0$, generators: $T_\mu, \mu = 0, \ldots, q-1$, $T'_\mu = X_{\mu 0} + X_{\mu,q-1}, \mu = 1, \ldots, q-2$, extended translations;
- $\tilde{\mathcal{N}}_0$, generators: $C'_\mu, \mu = 0, \ldots, q-1$, $C'_\mu = X_{\mu 0} - X_{\mu,q-1}, \mu = 1, \ldots, q-2$, extended special conformal transformations;
- $\mathcal{H} = \text{so}(q,1)$, generators: $X_{AB} : (A, B) \in \{0, \ldots, q\}$.

The last subalgebra is the analog of the maximal compact subalgebra $\text{so}(q+1)$ of the Euclidean conformal algebra $\text{so}(q+1,1)$ of $q$-dimensional Euclidean space. Thus, it may result from the Wick rotation of the Euclidean conformal algebra $\text{so}(q+1,1)$ to the Minkowskian conformal algebra $\text{so}(q,2)$.

Thus, we have several decompositions:

- $\mathcal{G} = \mathcal{K} \oplus \mathcal{Q}$, Cartan decomposition;
- $\mathcal{G} = \mathcal{K} \oplus \mathcal{A}_0 \oplus \mathcal{N}_0$, (also $\mathcal{N}_0 \rightarrow \tilde{\mathcal{N}}_0$), Iwasawa decomposition;
- $\mathcal{G} = \mathcal{N}_0 \oplus \mathcal{M}_0 \oplus \mathcal{A}_0 \oplus \tilde{\mathcal{N}}_0$, minimal Bruhat decomposition;
- $\mathcal{G} = \mathcal{N} \oplus \mathcal{M} \oplus \mathcal{A} \oplus \tilde{\mathcal{N}}$, maximal Bruhat decomposition;
- $\mathcal{G} = \mathcal{H} \oplus \mathcal{A} \oplus \mathcal{N}$, (also $\mathcal{N} \rightarrow \tilde{\mathcal{N}}$), Sekiguchi decomposition [17].

The subalgebra $\mathcal{P}_0 = \mathcal{M}_0 \oplus \mathcal{A}_0 \oplus \tilde{\mathcal{N}}_0$ is a minimal parabolic subalgebra of $\mathcal{G}$. The subalgebra $\mathcal{P} = \mathcal{M} \oplus \mathcal{A} \oplus \tilde{\mathcal{N}}$ is a maximal parabolic subalgebra of $\mathcal{G}$.

Finally, we introduce the corresponding Lie groups:

- $G = SO_0(0,2)$ with Lie algebra $\mathcal{G} = \text{so}(q,2)$, $H = SO(q,1)$ with Lie algebra $\mathcal{H} = \text{so}(q,1)$, $K = SO(q) \times SO(2)$ is the maximal compact subgroup of $G$, $A_0 = \exp(A_0) = SO_0(1,1) \times SO_0(1,1)$ is abelian simply connected, $\mathcal{N}_0 = \exp(\mathcal{N}_0) \cong \tilde{\mathcal{N}}_0 = \exp(\tilde{\mathcal{N}}_0)$, are abelian simply connected subgroups of $G$ preserved by the action of $A_0$. The group $M_0 \cong SO_0(q-2)$ (with Lie algebra $\mathcal{M}_0$) commutes with $A_0$. Further $A = \exp(A) = SO_0(1,1)$ is abelian simply connected, $N = \exp(N) \cong \tilde{N} = \exp(\tilde{N})$, are abelian simply connected subgroups of $G$ preserved by the action of $A$. The group $M \cong SO_0(q-1,1)$ (with Lie algebra $\mathcal{M}$) commutes with $A$.

We mention also some group decompositions:
\[ G = KA_0N_0, \text{ (also } N_0 \to \tilde{N}_0), \text{ Iwasawa decomposition; } \tag{4a} \]
\[ G \cong |_{\text{loc}} \tilde{NA}MN, \text{ maximal Bruhat decomposition; } \tag{4b} \]
\[ G \cong |_{\text{loc}} \tilde{NA}H, \text{ (also } \tilde{N} \to N), \text{ Sekiguchi decomposition } \tag{4c} \]

In (4b,c) the groups on the RHS are open dense subsets of \( G \). We should note that in [17] was studied the more general case \( SO_0(q,r+1)/SO(q,r) \).

The subgroup \( P_0 = M_0A_0N_0 \) is a \textit{minimal parabolic subgroup} of \( G \). The subgroup \( P = MAN \) is a \textit{maximal parabolic subgroup} of \( G \). Parabolic subgroups are important because the representations induced from them generate all admissible irreducible representations of semisimple groups [18, 19]. The group (algebra) \( SO_0(q,2) \ (so(q,2)) \) has one more maximal (cuspidal) parabolic subgroup (subalgebra) which we do not give here for the lack of space, cf., e.g., [20, 21] for \( q = 4 \).

### 3 Elementary representations

We use the approach of [22] which we adapt in a condensed form here. We work with so-called \textit{elementary representations} (ERs). They are induced from representations of the parabolic subgroups. Here we work with the maximal parabolic \( P = MAN \), where we use (non-unitary) finite-dimensional representations \( \lambda \) of \( M = SO(q-1,1) \) in the space \( V_{\lambda} \), (non-unitary) characters of \( A \) represented by the conformal weight \( \Delta \), and the factor \( N \) is represented trivially. For further use we give explicit parametrization of \( \lambda \):

\[ \lambda = (\lambda_1, \ldots, \lambda_{\hat{q}}), \quad \hat{q} = \left\lfloor \frac{q}{2} \right\rfloor \tag{5} \]

where \( \lfloor w \rfloor \) is the largest integer not greater than \( w \). The numbers \( \lambda_i \) are all integer or all half-integer and they fulfill the following conditions:

\[ |\lambda_1| \leq \lambda_2 \leq \cdots \leq \lambda_{q/2}, \quad \text{for } q \text{ even} \tag{6} \]
\[ 0 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_{(q-1)/2}, \quad \text{for } q \text{ odd} \]

The data \( \lambda, \Delta \) is enough to determine a weight \( \chi \in \mathcal{H}_G^* \), where \( \mathcal{H}_G \) is the Cartan subalgebra of \( G \), cf. [22]. Thus, we shall denote the ERs by \( C_{\chi} \). Sometimes we shall write: \( \chi = [\lambda, \Delta] \). The representation spaces are \( C^\infty \) functions on \( G/P \), or equivalently, on the locally isomorphic group \( N \) with appropriate asymptotic conditions (which we do not need explicitly, cf., e.g. [21, 22]). We recall that \( \tilde{N} \) is isomorphic to \( q \)-dimensional Minkowski spacetime \( \mathcal{M} \) whose elements will be denoted by \( x = (x_0, \ldots, x_{q-1}) \), while the corresponding elements of \( \tilde{N} \) will be denoted by \( n_x \). The Lorentzian inner product in \( \mathcal{M} \) is defined as usual:

\[ \langle x, x' \rangle = x_0x_0' - \cdots - x_{q-1}x_{q-1}', \tag{7} \]
and we use the notation $x^2 = \langle x, x \rangle$.

The representation action is given as follows:

\[(T^x(g)\varphi)(x) = y^{-\Delta} D^\lambda(m) \varphi(x')\]  

the various factors being defined from the local Bruhat decomposition (4b)

\[G \cong_{\text{loc}} \tilde{N} AMN : \]

\[g^{-1} \tilde{n}_x = \tilde{n}_{x'} a^{-1}_y m^{-1} n^{-1}, \]

where $y \in \mathbb{R}_+$ parametrizes the elements $a \in A$, $m \in M$, $D^\lambda(m)$ denotes the representation action of $M$ on the space $V_\lambda$, $n \in N$.

On these functions the infinitesimal action of our representations looks as follows:

\[T_{\mu} = \partial_{\mu}, \quad \partial_{\mu} \equiv \frac{\partial}{\partial x^\mu}, \quad \mu = 0, \ldots, q - 1,\]

\[D = -\sum_{\mu=0}^{q-1} x^\mu \partial_{\mu} - \Delta,\]

\[X_{a0} = x_0 \partial_a + x_a \partial_0 + s_{0a}, \quad a = 1, \ldots, q - 1,\]

\[X_{ab} = -x_a \partial_b + x_b \partial_a + s_{ab}, \quad 1 \leq a < b \leq q - 1,\]

\[C_{\mu} = -2\eta_{\mu\nu} x^\mu D + x^2 \partial_{\mu} - 2 \sum_{\nu=0}^{q-1} x^\nu s_{\mu\nu},\]

where $s_{\mu\nu}$ are the infinitesimal generators of $D^\lambda(m)$.

We recall several facts about elementary representations [15, 22]:

- The Casimir operators $C_i$ of $G$ have constant values on the ERs:

\[C_i(\{X\}) \varphi(x) = \chi_i(\lambda, \Delta) \varphi(x), \quad i = 1, \ldots, \text{rank } G = [\frac{1}{2}q] + 1, \]

where $\{X\}$ denotes symbolically the generators of the Lie algebra $G$ of $G$, the action of which is given in (10).

- On the ERs are defined the integral Knapp-Stein $G_\chi$ operators which intertwine the representation $\chi$ with the representation $\tilde{\chi} \doteq [\tilde{\lambda}, q - \Delta]$, where $\tilde{\lambda}$ is the mirror image of $\lambda$. We recall that the mirror image $\tilde{\lambda}$ is equivalent to $\lambda$ when $q$ is odd, while for $q$ even and $\lambda$ parametrized as in (5): $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_{q/2})$ we have $\tilde{\lambda} = (-\lambda_1, \lambda_2, \ldots, \lambda_{q/2})$.

- The representations $\chi$ and $\tilde{\chi}$ are called partially equivalent due to the existence of the intertwining operator $G_\chi$ between them. The representations are called equivalent if the intertwining operator $G_\chi$ is onto and invertible.

- We also recall that the Casimirs $\chi_i$ have the same values on the partially equivalent ERs:

\[\chi_i(\lambda, \Delta) = \chi_i(\tilde{\lambda}, q - \Delta)\]
In the above general definition $\varphi(x)$ are considered as elements of the finite-dimensional representation space $V^\lambda$ in which act the operators $D^\lambda(m)$. The representation space $C^\chi$ can be thought of as the space of smooth sections of the homogeneous vector bundle (called also vector $G$-bundle) with base space $G/P$ and fibre $V_\lambda$, (which is an associated bundle to the principal $P$-bundle with total space $G$). Actually, we do not need this description, but following [22] we replace the above homogeneous vector bundle with a line bundle again with base space $G/P$. The resulting functions $\hat{\varphi}$ can be thought of as smooth sections of this line bundle.

In the case when the representation $\lambda$ is of symmetric traceless tensors of rank $\ell$, i.e., $\lambda = (0, \ldots, 0, \ell)$, we can be more explicit following [15]. Namely, the functions $\hat{\varphi}$ are scalar functions over an extended space $\mathcal{M} \times \mathcal{M}_0$, where $\mathcal{M}_0$ is a cone parametrized by the variable $\zeta = (\zeta_0, \ldots, \zeta_{q-1})$ subject to the condition:

$$\zeta^2 = \langle \zeta, \zeta \rangle = \zeta_0^2 - \cdots - \zeta_{q-1}^2 = 0. \quad (13)$$

The functions on the extended space will be denoted as $\hat{\varphi}(x, \zeta)$. The internal variable $\zeta$ will carry the representation $D^\lambda$. Thus, on the functions $\hat{\varphi}$ the infinitesimal generators $s_{\mu\nu}$ from (10) are given as follows:

$$s_{0a} = \zeta_0 \frac{\partial}{\partial \zeta_a} + \zeta_a \frac{\partial}{\partial \zeta_0}, \quad s_{ab} = -\zeta_a \frac{\partial}{\partial \zeta_b} + \zeta_b \frac{\partial}{\partial \zeta_a}, \quad (14)$$

4 Bulk representations

It is well known that the group $SO(q, 2)$ is called also anti de Sitter group, as it is the group of isometry of $(q+1)$-dimensional anti de Sitter space:

$$\xi^A \xi^B \eta_{AB} = 1, \quad A, B = 0, \ldots, q + 1. \quad (15)$$

There are several ways to parametrize anti de Sitter space. For $q = 3$ in the paper [11] was utilized the same local Bruhat decomposition (4b) that we used in the previous section. In the present paper we shall use the Sekiguchi decomposition (4c), i.e., the factor-space $G/H \cong \tilde{N}A$. In fact, we use isomorphic (w.r.t. [11]) coordinates $(x, y) = (x_0, \ldots, x_{q-1}, y)$, $y \in \mathbb{R}_+$. In this setting anti de Sitter space is called bulk space, while $q$-dimensional Minkowski space-time is called boundary space, as it is identified with the bulk boundary value $y = 0$.

It is natural to discuss representations on anti de Sitter space $\tilde{N}A$ which are induced from the subgroup $H = SO(q, 1)$. Namely, we consider the representation space:

$$\hat{C}_\tau = \{ \phi \in C^\infty(\mathbb{R}^q \times \mathbb{R}_{>0}, \hat{V}_\tau) \} \quad (16)$$
where $\tau$ is an arbitrary finite-dimensional irrep of $H$, $\hat{\mathcal{V}}_\tau$ is the finite-dimensional representation space of $\tau$, with representation action:

$$(\hat{T}^\tau(g)\phi)(x,y) = \hat{D}^\tau(h)\phi(x',y')$$  \hspace{1cm} (17)$$

where the Sekiguchi decomposition is used:

$$g^{-1}\vec{n}_x a_y = \vec{n}_x' a_y' h^{-1}, \quad g \in G, \ h \in H, \ n_x, n_x' \in N, \ a_y, a_y' \in A$$  \hspace{1cm} (18)$$

and $\hat{D}^\tau(k)$ is the representation matrix of $\tau$ in $\hat{\mathcal{V}}_\tau$. For later use we give the parametrization of the relevant subgroups:

$$H = \left\{ h = \left[ \begin{array}{cc} h' & 0 \\ 0 & \pm 1 \end{array} \right] | h \in SO_0(q,2), \ h' \in SO_0(q,1), \right\} \cong SO(q,1)$$  \hspace{1cm} (19)$$

$$A = \left\{ a_y = \left[ \begin{array}{cc} \eta_q & 0 \\ 0 & \cosh(s) \sinh(s) \end{array} \right] | y = e^s, \ s \in \mathbb{R} \right\}$$  \hspace{1cm} (20)$$

$$\vec{N} = \left\{ \tilde{n}_x = \left[ \begin{array}{ccc} \eta_1 & 0 & t \\ 0 & \eta_{q-1} & s^1 \\ t & -s & 1 + \frac{x^2}{2} - \frac{t^2}{2} \end{array} \right] | (t,s) = \frac{x}{\sqrt{2}} \in \mathbb{R}^q \right\}$$  \hspace{1cm} (21)$$

The infinitesimal generators of (17) are given as follows:

$$\hat{T}_\mu = \partial_\mu, \quad \mu = 0, \ldots, q - 1$$  \hspace{1cm} (22)$$

$$\hat{D} = -\sum_{\mu=0}^{q-1} x_\mu \partial_\mu - y \partial_y,$$

$$\hat{X}_{ab} = x_a \partial_a + x_a \partial_b + \gamma_{ab}, \quad a = 1, \ldots, q - 1$$

$$\hat{X}_{ab} = -x_a \partial_b + x_b \partial_a + \gamma_{ab}, \quad 1 \leq a < b \leq q - 1$$

$$\hat{C}_\mu = -2\gamma_{\mu\nu} x_\mu D + (x^2 + y^2) \partial_\mu - 2 \sum_{\nu=0}^{q-1} x_\nu \gamma_{\mu\nu} - 2 y \Gamma_\mu,$$

where $\gamma_{\mu\nu}, \Gamma_\mu$ are infinitesimal generators of $\hat{D}^\tau(h)$, such that (due to the compatibility of $\lambda$ and $\tau$) $\gamma_{\mu\nu} = \frac{1}{4}[\Gamma_\mu, \Gamma_\nu]$, $[\gamma_{\mu\nu}, \Gamma_\rho] = \gamma_{\mu\nu} \Gamma_\rho - \gamma_{\mu\rho} \Gamma_\nu$.

Note that the realization of $so(q,2)$ on the boundary given in (10) may be obtained from (22) by replacing $y \partial_y \rightarrow \Delta$ and then taking the limit $y \rightarrow 0$.

What is important is that, unlike the ERs, the representations (17) are highly reducible. Our aim is to extract from $\hat{C}_\tau$ representations that may be equivalent to $C_\chi$, $\chi = [\lambda, \Delta]$. The first condition for this is that the $M$-representation $\lambda$ is contained in the restriction of the $H$-representation $\tau$ to $M$, i.e., $\lambda \in \tau|_M$. Another condition is that the two representations would have the same Casimir values $\lambda_i(\lambda, \Delta)$. 
This procedure is actually well understood and used in the construction of the discrete series of unitary representations, cf. [13,14], (also [11,15] for $q = 4$). The method utilizes the fact that in the bulk the Casimir operators are not fixed numerically. Thus, when a vector-field realization of the anti de Sitter algebra so($q,2$) (e.g., (22)) is substituted in the bulk Casimirs the latter turn into differential operators. In contrast, the boundary Casimir operators are fixed by the quantum numbers of the fields under consideration. Then the bulk/boundary correspondence forces eigenvalue equations involving the Casimir differential operators. Actually the 2nd order Casimir is enough for this purpose. That corresponding eigenvalue 2nd order differential equation is used to find the two-point Green function in the bulk which is then used to construct the boundary-to-bulk integral intertwining operator. This operator maps a boundary field to a bulk field. For our setting this will be given in detail in [12].

Having in mind the degeneracy of Casimir values for partially equivalent representations (e.g., (12)) we add also the appropriate asymptotic condition. Furthermore, from now on we shall suppose that $\Delta$ is real.

Thus, the representation (partially) equivalent to the ER $\chi$ is defined as:

$$\hat{C}_\chi^\tau = \{ \phi \in \hat{C}_\tau : C_i(\{\hat{X}\}) \phi(x,y) = \lambda_i(\lambda,\Delta) \phi(x,y), \quad \forall i, \quad \lambda \in \tau|_M,$$

$$\phi(x,y) \sim y^\Delta \varphi(x) \text{ for } y \to 0 \}$$

(23)

where $\hat{X}$ denotes the action (22) of $G$ on the bulk fields.

In the case of symmetric traceless tensors of rank $\ell$ for both $M$ and $H$ we can extend the functions on the bulk extended also with the cone $\Omega_0$. These extended functions will be denoted by $\phi(x,y,\zeta)$. On these functions we have the infinitesimal action given by (22) with $s_{\mu\nu}$ are given by (14), while $\Gamma_{\mu}$ are certain finite-dimensional matrices which we shall give in [12].

5 Two parametrizations of bulk space

As we mentioned in [11] we used as parametrization of the bulk space the coset $G/MN = |_{loc} \tilde{N}A$. The local coordinates of this coset come from the Bruhat decomposition:

$$g = \{g_{AB}\} = \tilde{n}_a y_m n$$

(24)

which exists for $g \in G$ forming a dense subset of $G$. The local coordinates of the above bulk are:

$$y = \frac{1}{2}(g_{qq} + g_{q,q+1} + g_{q+1,q} + g_{q+1,q+1})$$

$$x_\mu = \frac{g_{\mu,q} + g_{\nu,q+1}}{g_{qq} + g_{q,q+1} + g_{q+1,q} + g_{q+1,q+1}}, \quad \mu = 0, \ldots, q - 1$$

(25)
The parametrization used in the present paper for the bulk space is the coset $G/H = |_{\text{loc}} \tilde{N}A$. Certainly, it is isomorphic to the bulk above, however, the local coordinates are different, namely, the latter come from the Sekiguchi decomposition:

$$ g = \{g_{AB}\} = \tilde{n}_x a_y h $$

(26)

Explicitly, they are given as follows:

$$ y = \left| g_{q+1,q + 1} \right| $$

(27)

$$ x_\mu = \frac{g_{\mu,q+1}}{g_{q+1,q} + g_{q+1,q+1}}, \quad \mu = 0, \ldots, q - 1 $$

Comparing the two parametrizations (25) and (27) we see that the latter is simpler and thus easier to implement. Thus, in the follow-up paper [12] we shall use the Sekiguchi decomposition (26).

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