Master Equation for Light emitted by correlated Atoms

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Abstract

A master equation for light generated by atoms, which states are prepared by a pumping mechanism that produces atomic correlations, is derived in the Fokker - Planck approximation. It has been found that two-particle correlations only play the role under this approach. Then the equation is applied for describing micromaser operations and light noise is discussed. We consider the correlated atomic states prepared by teleportation protocols.

1 Introduction

When atoms emit light statistics of radiation depends on the atomic noises, in particularly, a reason for which is correlation of atoms. To generate light atomic ensemble can be prepared by the various pumping mechanisms. As well known, in the case of a regular pumping atoms may generate light, which statistics of photons is sub Poisson as for the laser \[1\], \[2\] and the micromaser \[3\].

Next, combining UQCM and teleportation, it has been proposed telecloning \[10\], that allows a producing of spatially separated copies from originals by the teleportation protocol. Let \[M = 1\] then for a pure state of a qubit with the help of UQCM or teleportation one can obtain the \(N\) - particle entangled state of the GHZ (Greenberger - Horne - Zrlinger) class

\[
\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle^\otimes N + \beta|1\rangle^\otimes N
\]

where \(|\alpha|^2 + |\beta|^2 = 1\), \(|b|^\otimes N\) is the tensor product \(|b\rangle \otimes \ldots \otimes |b\rangle\), \(b = 0, 1\). In the same time copies of some quantum states can be created by teleportation without UQCM, when a multipaticle quantum channel is permitted \[11\]. By the multiparticle channel, also named the mulituser channel, an initial state is distributed with \(N\) parties, each of which has an optimal copy of the original in his hands. It is kind of telecloning. If the original is a mixed state, such type telecloning allows preparing the \(N\) - particle mixture

\[
\lambda_0|0\rangle|0\rangle + \lambda_1|1\rangle|1\rangle \rightarrow \lambda_0(|0\rangle|0\rangle)^\otimes N + \lambda_1(|1\rangle|1\rangle)^\otimes N
\]

where \(\lambda_0 + \lambda_1 = 1\). Note that in contrast to \[1\], the obtained \(N\)-particle state \(2\) is separable.

According to the no-cloning theorem \[13\] all copies are not independent, except the case \(\alpha = 0\) or \(\lambda_0 = 0\), when the state of \(N\) independent particles of the form \(|1\rangle^\otimes N\) is obtained. Let \(|0\rangle\), and \(|1\rangle\) are the lower and
the upper atomic level, then by telecloning one obtains the correlated states of $N$ atoms. The collection of these atoms can generate light, if they have inversion of the population of the working levels, when $|\beta|^2 > |\alpha|^2$, $\lambda_1 > \lambda_0$. Both states, given by (1) and (3), are quite different, but, with respect to any set of the $K < N$ particles their features are similar, if $N > 2$. The $K$-particle density matrix has the form $f(12\ldots K) = r(\langle 0|\langle 0|\otimes K + (1-r)(\langle 1|\langle 1|\otimes K$, where $r = \lambda_0/|\alpha|^2$. It means, that any atom has no coherence on the working transition, its state is mixed, and atomic correlations are purely classical. Next we also consider the state $|Z\rangle = \alpha|bb\rangle + \beta|\Psi^+\rangle$ (3) where $|\alpha|^2 + |\beta|^2 = 1$, $\Psi^+ = (|01\rangle + |10\rangle)/\sqrt{2}$ is one of the Bell states, $b = 0, 1$. This is entangled state, for which the one-particle density matrix reads $f(1) = |\alpha|^2|b\rangle\langle b| + |\beta|^2/2 + (\alpha^*\beta\langle 1|\delta_{0\beta} + |b\rangle\langle 0|\delta_{1\beta} + h.c.)/\sqrt{2}$. Indeed, there is the atomic coherence, or polarization, that results in some new features as, for example, the laser and micromaser operation without inversion of population on the working levels.

What is a difference between the presented correlated states, prepared by telecloning, from the states, produced by any “usual” pump? As for the simplest model of a pump, one can consider a collection of $N$ independent atoms, say in the state $f(1\ldots N) = f(1)^{\otimes N}$, $f(1) = \lambda_0|0\rangle\langle 0| + \lambda_1|1\rangle\langle 1|$. Let atoms are illuminated by a strong coherent field. This interaction can be described by a unitary transformation, that results in the inversion of population of the working level. But one finds, that any correlation between atoms cannot be produced. Then atoms become active, but independent, its entropy has the form $S = NH_1$, where $H_1 = -\lambda_0 \log \lambda_0 - \lambda_1 \log \lambda_1$. The entropy of the pure states (1) and (3) is equal to zero, the entropy of the mixed state (2) is $S = H_1$. Because under an unitary transformation entropy does not change, the correlated states, we consider, can't be prepared by a simple way, when independent atoms interact with a classical field.

To derive the master equation, we use formalism presented in [13] for considering the interaction of $N$ correlated atoms with a quantum electro-magnetic field. In this approach we use the standard procedure of illuminating of atomic variables assumed to be fast. It results in the Fokker - Planck equation (FPE) as example for the Glauber - Sudarshan quasiprobability widely used for the quantum optics problems. In spite of the states (1), (2) and (3), we discuss here, describe entangled and correlated atoms, in fact it is not a real interaction between atoms and they are treated as a non interacting set of particles. So that the correlation gives no additional difficulties in deriving the FPE. One of the main result followed from the used approximation is that all diffusion coefficients of the obtained FPE involve the two-atom correlations only. Then for our purpose it needs consider not less then 2 atoms, interacted with the cavity modes. In the same time we assume the number of atoms is limited, to consider the scheme close to micromaser. It is in accordance with the standard approximation of small fluctuation of field that we also use. The reason is that, the diffusion coefficients of the FPE have the contribution proportional to $N(N - 1)$, due from the initial multiatom problem, we consider here. Under the small fluctuation approximation the diffusion coefficients must be small, that is the restriction on the number of atoms.

Using the found FPE, for the case of initial state of atoms (3) we calculate the Mandel parameter of the light and its spectrum of noise for the micromaser operation. We find here that atomic correlations lead to increasing of the the light noise, but in particular case the noise can be reduced up to zero, if all atoms are prepared in the upper states $|1\rangle^{\otimes N}$. Note that the perfect reduction of the micromaser noise can be achieved also by the regular pumping as it has been demonstrated in ref. [3], when the pump results in the Fock state of the light. However this generated state has no well defined phase because of its diffusion. In contrast, if atoms are prepared in the entangled state, given by (3), we found a regime in which the light is generated without inversion of population of atomic levels and in particularly, the generated mode has steady-state phase and noise suppressed up to zero.

2 Initial equations

Let consider a collection of $N$ similar two level atoms interacting with one cavity mode having a frequency equaled to an atomic transition frequency. This system is described by the density matrix $F$ that obeys the following equation

$$\frac{\partial}{\partial t} F = [\vartheta, F]$$

$$\vartheta = -i\hbar^{-1}V$$

$$V = -i\hbar g(S_{10a} - S_{01a})$$

2
where $g = d/\sqrt{\hbar c \alpha \epsilon_0}$ is the coupling constant, $d$ is a dipole moment of a working transition, $a^\dagger$, $a$ are photon operators, $[a; a^\dagger] = 1$; atomic operators $S_{xy}$ are defined as

$$S_{xy} = \sum_{k=1}^{N} s_{xy}(k)$$

where $s_{xy}(k) = |x\rangle_k \langle y|$ 

$x, y = 0, 1$

where $|0\rangle_k$ is a lower and $|1\rangle_k$ is an upper state of an atom $k$. The relaxation processes can be taken into account by adding appropriate terms in (4).

To derive a master equation for field based on the formalism presented in [15], let introduce the representation of the density matrix $F$ over the coherent states of field

$$F = \int d^2 \alpha \Phi(\alpha)|\alpha\rangle\langle\alpha|$$

where $a|\alpha\rangle = \alpha|\alpha\rangle$. The matrix $\Phi$ is the operator with respect to the atomic variables but a c-function with respect to field. When $\Phi$ is averaged over the atomic variables, one finds the well-known Glauber-Sudarshan quasiprobability $P(\alpha) = S_{PA} \Phi$. An equation for $\Phi$ can be obtained from (4) taking into account the correspondence $[\vartheta, F] = [\vartheta_0, \Phi] + \partial_\alpha (D\Phi)$ where field operators $a$ and $a^\dagger$ are presented by complex numbers $\alpha, \alpha^*$ associated with the field amplitudes, and by the derivatives over these amplitudes as $\partial_\alpha, \partial_{\alpha^*}$. The Hamiltonian $\vartheta_0$ represented the interaction of the atoms with field. When $\vartheta$ is averaged over the atomic operators over the density matrix $f$ as $\langle S_{xy} \rangle = Sp(S_{xy} f)$ $x, y = 0, 1$. It is worth paying attention that for $f$ we get here the closed problem in which the interaction of atoms with field is described by the Hamiltonian $\vartheta_0$, where the field is given by its complex amplitude only. It looks as the problem of evolution of the atomic ensemble in a classical field

$$\frac{\partial}{\partial t} f = [\vartheta_0, f]$$

Using (4) and taking into account (8) and (11), one finds the precise equation for the correlation matrix $\Pi$. However we wish the FPE for the field quasiprobability $P$. Therefore it needs a solution for $\Pi$ in a perturbation theory over derivations to $\alpha, \alpha^*$ up to the first derivatives only, because it gives a term to the equation for $P$ (8) resulting we will have derivatives of an order not higher then two. It results in the following equation for $\Pi$

$$\frac{\partial}{\partial t} \Pi = [\vartheta_0, \Pi] - g \left[ \frac{\partial}{\partial \alpha} (S_{01} - \langle S_{01}\rangle) f + h.c \right] P$$

To derive a closed equation equation for $P$ usually it is assumed that an atomic system evolves more faster then field and introduced a rough time scale. Let $T$ is a typical time of changing of the field and consider the rough derivative over $T$ as $(P(T + t) - P(t))/T = \partial P/\partial t$. Then integrating equation (8) for $P$ over time from $t$ till $T + t$ and taking into account the fact, that the field is slowly changed in the scale time of $T$, we find the desired equation for $P$

$$\frac{\partial}{\partial t} P(t) = -\frac{g}{T} \int_{t}^{T+t} dt' \frac{\partial}{\partial \alpha} (S_{01}(t')) P(t') + S_{PA}(S_{01}(t'))$$

This is the master equation for field. Its coefficients can be found from the the problem for $f$ and $\Pi$ where the field is presented by its quasiprobability $P$ and assumed to be constant in time. Also it needs to denote the initial matrices $f(0)$ and $\Pi(0)$ are taken at an initial moment of time, say $t = 0$ and it is used as a way to introduce the pumping mechanism. As for example, consider atoms placed into cavity, which Q-quality is modulated periodically, and atoms interact with the high Q cavity mode during time $T$. Then we can describe a pumping as initial state of atoms, prepared at time $t$ as $f(t)$, and for an appropriate choosing of the time intervals we set $\Pi(t) = 0$. Also our scheme can be considered as micromaser, when a beam of atoms, that states are given by pumping, is injected into cavity, in
which $N$ atoms interacts with one mode during time $T$ and after leave the cavity. As result, the master equation describes the field behavior, arisen from summing over the all changing of the field in time $T$. Note, all contributions are independent.

3 The atomic averages

According to (11), evolution of the electromagnetic field depends on the atomic observables of two types. They are based on the atomic matrix $f$ and the correlation matrix $\Pi$, like $\langle S_p(t) \rangle = S_p f(t)$ and $S_p(\langle S_p(\Pi(t))\rangle$, where $p = 0, 1, 2, 3$ or $p = 00, 01, 10, 11$, if the binary notation of $p$ is used. Because atoms are assumed to be identical, one finds, that $\langle S_p(t) \rangle = N\langle s_p(t; 1) \rangle$, and the one-particle operator $f(t; 1) = S_{p_2...N} f(t; 1...N)$ is needed. The matrix $f$ can be obtained by solving (11), that looks as a problem of atoms in the field of the given complex amplitude $\alpha$, in other hand it can be treated as a task of interaction of atoms with a ”classical” field. The matrix $f$, as well as $\Pi$, cannot be factorized, because of the pumping, we consider, prepares the correlated atoms, in contrast the case of independent particles, discussed in Ref [13], when both $f$ and $\Pi$ are product of the one-particle operators. It is important, that there is no any interaction between atoms, in spite of the atomic correlations are created by the pumping. For the problem (11) it results in operator evolution has the factorized form and one finds

$$ f(t; 1...N) = U^{\otimes N} f(t; 0...N) U^{(U^\dagger)^{\otimes N}} $$

where $\mu = \cos(|\alpha| t)$, $\nu = -(|\alpha|/|\alpha|) \sin(|\alpha| t)$.

There is another situation for calculating the observables associated with the correlation matrix $\Pi$. Integrating (11), one finds that all values of the type $S_p(\langle S_p(\Pi(t))\rangle$ depend from the atomic variances involving pair of the atomic operators

$$ S_{p}(\langle S_p(\Pi(t))\rangle = -g \int_0^t dt' \left[ \frac{\partial}{\partial \alpha} D_{p0}(t, t') + \frac{\partial}{\partial \alpha^*} D_{10p}(t', t) \right] P(t) $$

(13)

where variances $D_{pq}(t, t')$, $p, q = 0, 1, 2, 3$ have the form

$$ D_{pq}(t, t') = \langle S_p(t) S_q(t') \rangle - \langle S_p(t) \rangle \langle S_q(t') \rangle $$

where the Heisenberg operators are introduced as $S_p(t) = (U^\dagger)^{\otimes N} S_p U^{\otimes N}$ and averaging is taken over the initial state of atoms $f(t = 0; 1...N)$. Note, that in (13) we suggest, that the atomic operators commute with all derivatives over $\alpha$. That is a typical approximation for the FPE [12]. The variances, given by (14), consist of two parts, where first is the one-particle contribution proportional to $N$, and second is the two-particle contribution proportional to $N(N - 1)$

$$ D_{pq}(t, t') = N\langle \langle s_p(t; 1) s_q(t'; 1) \rangle - \langle s_p(t; 1) \rangle \langle s_q(t'; 1) \rangle \rangle $$

(15)

and

$$ + N(N - 1) \langle s_p(t; 1) s_q(t; 2) \rangle - \langle s_p(t; 1) \rangle \langle s_q(t; 1) \rangle \rangle $$

(16)

Note, in the classical Lamb-Scully approach one term from (15) $N\langle s_p(t; 1) s_q(t'; 1) \rangle$ takes place. For calculating the atomic variances, the Heisenberg picture is suitable, it results in evolution of the one-atom operator in the form

$$ s_p(t; 1) = \sum_q R_{pq}(t) s_q(1) $$

(18)

where $p, q = 0, 1, 2, 3$, and the unitary matrix $R_{pq}$ reads

$$ R_{pq} = \begin{pmatrix} \mu^2 & -\mu \nu & -\mu \nu^* & |\nu|^2 \\ \mu \nu & \mu^2 & -\nu^2 & -\mu \nu \\ \mu \nu^* & -\nu^2 & \mu^2 & -\mu \nu^* \\ |\nu|^2 & \mu \nu & \mu \nu^* & \mu^2 \end{pmatrix} $$

(19)

Then all variances take the form

$$ D_{pq}(t, t') = \sum_{p'q'} R_{pq}(t) R_{q'p'}(t') D_{p'q'} $$

(20)

where $p, q, P, Q = 0, 1, 2, 3$ and $D_{pq} = D_{pq}(0, 0)$ is an initial correlation function, given by (14), in which $t = t' = 0$. It can be seen, that the two-particle matrix $f(t = 0; 12)$ is needed to find these variances. It means, that the two-atomic correlation only is required to specify the field behavior under the Fokker - Planck approximation.

4 The Fokker-Planck equation and noise

After introducing the polar coordinates $I = |\alpha|^2$, $\varphi = \arg \alpha$ the master equation for field (11) takes the FPE
form
\[ \frac{\partial}{\partial t} P = \left[ \sum_{u,v} \frac{\partial}{\partial u} A_u + \frac{1}{2} \frac{\partial^2}{\partial u \partial v} Q_{uv} \right] P \] (21)

All coefficients \( A_u \) at the first derivatives are denoted by the one-particle atomic polarization \( \langle s_{01}(t;1) \rangle \)
\[
A_I = -N \frac{g}{2\sqrt{TT}} \int_0^T dt \exp(-i\varphi) \langle s_{01}(t;1) \rangle + h.c. \quad (22)
\]
\[
A_\varphi = N \frac{g}{2\sqrt{TT}} \int_0^T dt i \exp(-i\varphi) \langle s_{01}(t;1) \rangle + h.c.
\]

At this step the field relaxation has to be taken into account, because the light must leave the cavity. For the model of the Lindblad type, it results in \( A_I \to A_I + CI \), where \( C \ll 1/T \) is a rate at which photons come out of the cavity. \( C \) depends on the output mirror transmitting. The diffusion coefficients are based on the atomic variances and involve the two-particle correlations
\[
Q_{uv} = \theta_{uv} \frac{g^2}{T} \int_0^T dt \int_0^t dt' q_{uv}(t,t') \] (23)

where \( \theta_{uv} = \theta_{vu}, \theta_{I\varphi} = 1, \theta_{\varphi\varphi} = 1/(4I), \theta_{II} = I, \)
\[
q_{\varphi I} = q_{I\varphi} = -i \exp(-2i\varphi) D_{11}(t,t') + c.c.
\]
\[
q_{\varphi\varphi} = D_{21}(t,t') - \exp(-2i\varphi) D_{11}(t,t') + c.c.
\]
\[
q_{II} = D_{21}(t,t') + \exp(-2i\varphi) D_{11}(t,t') + c.c.
\]

To describe noise of the light we use the Mandel parameter \( \xi \), that indicates how statistics of photons deviates from the Poisson statistics: \( \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle(1 + \xi) \), where \( n = a^\dagger a \) is the photon number operator. In the presented representation \( \xi = \langle e^2 \rangle/\langle I \rangle \), where \( e = I - \langle I \rangle \) is a fluctuation, that is the difference between "intensity" of the light \( I \) and its average value \( \langle I \rangle = \int d^2P(\alpha)|\alpha|^2 \). Assuming, that fluctuation of intensity is small and does not depend on the phase fluctuation, then it follows from (21) immediately, that the Mandel parameter is proportional to the diffusion coefficient \( Q_{II} \) as \( \xi = Q_{II}/(I\Gamma)^{-1} \), where \( \Gamma = C + \langle \partial A_I/\partial I \rangle_{I=\langle I \rangle} \) is the decay rate of the intensity fluctuations. The considered approximation of small fluctuation is the standard one for analysis the FPE and it needs \( \xi \ll \langle I \rangle \).

5 Noise of micromaser operation

To apply the obtained FPE consider the micromaser operation, when the atomic relaxation can be neglected. It is true, if the decay rates of atomic levels \( \gamma \) such that \( 1/T \gg \gamma \gg C \). Assume fluctuations of intensity are small in a sense, that the deviation \( \varepsilon = I - \langle I \rangle \ll \langle I \rangle \) slightly change near a steady state solution \( \langle I \rangle \).

First consider the pump, produced classically correlated atoms in the state, given by (1), if \( N > 2 \) and (2). Then from (21) it follows, that \( \langle I \rangle \) is the steady state of the semiclassical micromaser equation of the form
\[
\frac{\partial}{\partial t} \langle I \rangle = -C \langle I \rangle + \frac{N}{T}(\lambda_1 - \lambda_0) \sin^2 B \] (25)

where \( B = g\sqrt{\langle I \rangle}/T \). For steady state it needs an inversion of population \( \lambda_1 > \lambda_0 \) and the condition \( (1 - B/T \tan B) > 0 \) due from the requirement of the stable equilibrium.

Then using (24) under small fluctuations of intensity, one finds the Mandel parameter
\[
\xi = \frac{T}{N} \frac{Q_{II}}{(2\lambda_1 - 1) \sin^2 B(1 - B/T \tan B)} \] (26)

where the diffusion coefficient \( Q_{II} \) has the form
\[
Q_{II} = \frac{N}{T} \left(-\frac{1}{2} \sin^4 B + (2\lambda_1 - 1) B \sin B \cos B \right. + (1 - \lambda_1) \sin^2 B + 2(\lambda_1 - 1) \lambda_1 \sin^4 B
\]
\[
+ \left. 2(N - 1) \lambda_1 (1 - \lambda_1) \sin^2 B \right)
\]

In (27) the atomic correlations of both state (1) and (2) presented by the same term \( N(2N - 1)\lambda_1 (1 - \lambda_1) \sin^2 B/T \) are non negative. It means increasing of the light noise due from the initial atomic correlations. This term is equal to zero, if \( N = 1 \), or, if the pump creates independent atoms. Indeed, the phase behavior is insensitive to these atomic correlations. In accordance with the small fluctuation approximation it needs \( Q_{II} \ll \Gamma \langle I \rangle^2 \), that is the reason of restriction the number of atom \( N \).

Consider the noise of light, denoted as the spectrum of photocurrent \( i^{(2)}(\omega) \). It can be measured by a simple heterodyne scheme included a detector and spectrum analyzer. At frequencies \( \omega \approx 0 \) the spectrum of the light noise takes the form
\[
i^{(2)}(0) = 1 + 2\xi \frac{1}{1 - B/T \tan B} \] (28)
where unit indicates the shot noise level or the standard quantum limit, which is noise of the coherent field. The main features of the light statistics can be found from equation (29), (28). If all atoms are prepared independently in the state $|1\rangle^\otimes N$, then $\xi \geq -1$. The negative Mandel parameter means, that in the cavity the photon statistics is sub Poisson, and the noise of light can be suppressed below the standard quantum limit. It $\xi = -1$, the photon number variance is equal to zero, and one finds the Fock state of the light into cavity $|n\rangle$, where $n = \langle I \rangle$. However the noise reduction is small. Noise can be suppressed up to zero, when $\xi = -1/2$. These results are in agreement with results for a regular pumping, proposed in Ref. [3]. For the case of correlated atoms, we discuss here, the Mandel parameter becomes positive and the light noise increases.

In contrast, when a pump prepares atoms in the state given by (3), the atoms have the initial coherence $\langle s_{01}(0)\rangle = (\alpha^* \delta_{01} + \alpha \delta_{00})/\sqrt{2} = |\alpha\beta| \exp[i\varphi_0]$, where $\varphi_0 = (-1)^{1-b} \arg(\alpha\beta^*)$ and inversion $\langle s_{11}(0)\rangle - \langle s_{00}(0)\rangle = (-1)^{1-b}|\alpha|^2$. Here $b = 0, 1$, that indicates two version of the state (3). If $b = 0$ the inversion of population is negative, in contrast the case when $b = 1$. Then together with the small fluctuation of the intensity, assume the phase fluctuation is small in a sense, that $\mu = \varphi - \psi_0 \ll \psi_0$. Then semiclassical equations for the phase $\varphi_0$ and the intensity $\langle I \rangle$ take the form

\[
\frac{\partial}{\partial t} \psi_0 = -\sqrt{2} \frac{g}{\Gamma} |\alpha\beta| \sin(\psi_0 - \varphi_0) \tag{29}
\]

\[
\frac{\partial}{\partial t} \langle I \rangle = -C\langle I \rangle + \frac{2}{T} (-1)^{1-b}|\alpha|^2 \sin^2 B + \frac{2\sqrt{2}}{T} |\alpha\beta| \cos(\psi_0 - \varphi_0) \sin B \cos B \tag{30}
\]

In the steady state $\psi_0 = \varphi_0$ and this solution is stable. The initial atomic coherence is presented by the last term in the right hand side of (31). It result in the steady state solution for intensity, even the initial inversion of population, produced by the pump, is negative, when $b = 0$.

Noise of the light or spectrum of the photocurrent at the frequencies near zero can be written as

\[
i^2(0) = 1 + 2\xi \frac{C}{\Gamma} \tag{31}
\]

where $A = (-1)^{(1-b)}2|\alpha|^2$, $\xi = Q_{II}/(\langle I \rangle \Gamma)$. The expression for the diffusion coefficients reads, if $b = 1$

\[
Q_{II} = \frac{1}{T} [\frac{1}{2} A (\sin^2 B + 2B \sin B \cos B) + \sqrt{2} |\alpha\beta| \cos 2B (B - \sin B \cos B) - Q] \tag{32}
\]

if $b = 0$

\[
Q_{II} = \frac{1}{T} [\frac{1}{2} A \sin^2 B (\sin^2 B + 2) - 2B \cos B + \sqrt{2} |\alpha\beta| \cos 2B (B + \sin B \cos B) - 2 \sin B \cos B - Q] \tag{33}
\]

and

\[
Q = (2(1 + 2|\alpha|^2)|\beta|^2 \sin^2 B \cos^2 B + \sqrt{2} |\alpha\beta| A \sin B \cos B (1 - \cos 2B) - \frac{1}{2} A^2 \sin^4 B \tag{34}
\]

There are two types of steady states of the micromaser operation, that can be pointed. For the first of them $b = 0$, it means, that atoms are prepared initially without inversion because $\langle s_{11}(0)\rangle - \langle s_{00}(0)\rangle = |\alpha|^2$. If $|\alpha\beta| \approx 0.49$, then one finds light, which phase is well defined and statistics of photons is sub Poisson. Indeed, the light noise can be suppressed up to $i^2(0) \approx 0.27$, that is limit here. The second type of operation is obtained, when $b = 1$. In this case the pump creates inversion of population of the atomic levels and coherence. Atoms generate light, which level of noise is various. Particularly, one finds the perfect noise reduction, when $i^2(0) \approx 0$, if $|\alpha|^2 \gg |\beta|^2$. Physically, the last feature is clear, because the initial state of atoms is close to a state, of two independent atoms in the upper level. Indeed, in the micromaser operation the ideal sub Poisson light can be achieved using a regular pumping of atoms (3). However note that to do it, atoms have to be pumped by the light, which statistics of photons is already sub Poisson (6).

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