Throughput Analysis of IEEE 802.11 Multi-hop Wireless Networks with Routing Consideration: A General Framework

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Abstract

The end-to-end throughput of multi-hop communication in wireless ad hoc networks is affected by the conflict between forwarding nodes. It has been shown that sending more packets than maximum achievable end-to-end throughput not only fails to increase throughput, but also decreases throughput owing to high contention and collision. Accordingly, it is of crucial importance for a source node to know the maximum end-to-end throughput. The end-to-end throughput depends on multiple factors, such as physical layer limitations, MAC protocol properties, routing policy and nodes’ distribution. There have been many studies on analytical modeling of end-to-end throughput but none of them has taken routing policy and nodes’ distribution as well as MAC layer altogether into account. In this paper, the end-to-end throughput with perfect MAC layer is obtained based on routing policy and nodes’ distribution in one and two dimensional networks. Then, imperfections of IEEE 802.11 protocol is added to the model to obtain precise value. An exhaustive simulation is also made to validate the proposed models using NS2 simulator. Results show that if the distribution to the next hop for a particular routing policy is known, our methodology can obtain the maximum end-to-end throughput precisely.

Index Terms

Wireless multi-hop networks, End-to-end throughput, Maximum capacity, Routing policy, Analytical analysis.

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I. INTRODUCTION

Wireless ad hoc networks allow several nodes located outside of the transmission range of one another to communicate through intermediate nodes. Using a routing protocol, a source node can find a path toward the destination. Nodes selected by the routing protocol are responsible for forwarding the source’s packets until they reach the destination. In a case that source and destination are in the transmission range of each other, no routing is required and consequently, maximum end-to-end throughput is obtained by analyzing physical and Medium Access Control (MAC) layer properties.

In multi-hop wireless communications, achievable throughput is significantly lower than single-hop communication due to the inevitable transmission overlap of consecutive forwarding nodes. Additionally, sending more packets than maximum achievable throughput degrades the end-to-end throughput even further. Hence, the knowledge about the maximum achievable throughput, which can be exploited by source nodes, considerably improves multi-hop communications.

In the previous research [1]–[8], the analytical expression for maximum throughput was obtained for different scenarios. Nevertheless, all of these papers have assumed that all nodes placed equi-distance apart in a straight line. In practice, wireless nodes are distributed randomly and it is highly unlikely to end up with this assumption. Additionally, even if all nodes do follow such a restriction, the routing protocol may choose nodes in a way that the assumption fails. Thus, it would not be a precise analysis if the end-to-end throughput of multi-hop communication is obtained without the routing policy and node distribution.

In this paper, node distribution and routing policy as well as MAC layer limitations are taken into consideration to obtain the precise value of end-to-end throughput in multi-hop communication when there is a single flow in the network. The geometry of randomly deployed wireless nodes is commonly modeled by a Poisson Point Process (PPP), since it is equivalent to placing each node uniformly in an \( n \)-dimensional space. [9], [10]. Given a 1-dimensional (1-D) PPP, first, we obtain the maximum end-to-end throughput with perfect MAC and physical layer for two different routing policies including random neighbor routing and furthest neighbor routing. In perfect MAC and physical layer, it is assumed that a node has no negative effect on communications happening outside its interference range. Regardless of the existence of such physical layer and MAC protocol, this assumption avails us to capture the effect of node distribution and routing policy alone on the maximum end-to-end throughput.
Second, we analyze the effect of routing policy and node distribution on IEEE 802.11 MAC protocol and extend our analysis to capture the effects of MAC layer limitations. Although the formulas we obtained here are confirmed by simulation and are useful for theoretical analysis, they are too complicated to be used in wireless nodes. Therefore, we provide a simple way to approximate these formulas so that a source node can use them for flow and admission control.

Third, we obtain some promising approximation formulas for maximum end-to-end throughput in 2-D networks which is validated by simulations. Our methodology can be used for any routing policy and node distribution as long as the first and second moments of the distribution of distance between two consecutive nodes in a path is known. To the best of our knowledge, it is the first analytical results about maximum throughput in 2-D multi-hop wireless networks when IEEE 802.11 is used alongside furthest and random neighbor routing.

One direct application of our results resides within the domain of protocol design. For instance, protocols, such as TCP, have inherent flow and congestion control. However, their performance is not near optimum as a result of increasing window size more than optimum value [14]. Knowing the maximum achievable throughput can be directly used in protocol design. It can also be used for layer 2 protocol design. Given that the maximum throughput of multi-hop communication is less than single-hop communication, the maximum multi-hop throughput can be used to design layer 2 protocols that avoid greedily sending at the rate of single-hop throughput which contributes to congestion and consequently degrade multi-hop communication throughput.

The remainder of this paper is organized as follows. In section II, we briefly present related works in this area. Section III obtains end-to-end throughput with perfect MAC and physical layer in 1-D networks. Section IV considers the limitation of lower layers and obtains end-to-end throughput for IEEE 802.11. Section V obtains an approximate formula for end-to-end throughput which is simple enough to be used in wireless nodes. While it seems impossible to obtain an exact formula for maximum throughput in 2-D networks, some approximations are given in Section VI. A comprehensive performance evaluation is carried out in Section VII. Finally, Section VIII concludes our work.

II. RELATED WORK

In [11], Nelson and Kleinrock obtained the spatial capacity of a wireless network when using slotted Aloha Mac protocol. They assumed that intermediate nodes for routing are selected randomly among nodes located toward a destination. Although they considered both routing and
MAC layer in their analysis, slotted Aloha is not commonly used in ad hoc wireless networks. Moreover, it is not clear whether their approach can be generalized to any routing policy. In [12], Gupta and Kumar derived theoretical bounds for the capacity of wireless ad hoc networks. But, in practice, with wireless networks based on IEEE 802.11 MAC protocol, achievable end-to-end throughput is far from the derived bounds [8]. Taking into account the limitation imposed by carrier sensing, authors in [13] have given an insight over the fundamental discrepancy between single-hop and multi-hop communication. They have intuitively asserted that maximum end-to-end throughput of multi-hop communication is \( n \) times less than single-hop throughput where \( n \) is the number of consecutive nodes whose transmission conflicts with each other. Not only did their simulation reveal the incompleteness of their assertion, but also their simulations were only conducted for lattice network in which all nodes are placed equi-distance apart.

In [14], the effect of MAC layer on TCP performance was analyzed and two techniques were proposed to mitigate the performance of TCP. However, their analysis only holds when all nodes are placed equi-distance apart. In [15], [16], authors have studied the criteria based on which the second or more paths from a source to destination increase throughput. However, their analysis only considered the scenario where all nodes are placed equi-distance apart and they also assumed that the carrier sensing range is equal to transmission range which is not realistic [17]. In [18], the throughput of a new backoff scheme for 802.11, called TO-DCF, was analytically analyzed. Nevertheless, their analysis is only applicable for single-hop networks in which all nodes are close enough to communicate directly. Hence, no routing mechanism was studied.

Authors in [19] proposed a methodology to obtain maximum end-to-end throughput in a chain topology by creating contention graph corresponding to the topology. However, this method needs the location of all nodes. In [20], the deviation based method is proposed to obtain end-to-end throughput which uses the central limit theorem to model the throughput deviation. However, as it is shown in [21], the deviation based model fails to estimate the end-to-end throughput when the number of nodes in a route exceeds 4. Authors in [21] proposed a method to mitigate the deficiencies of the deviation based method which works for even long paths. However, this model requires the location of all nodes which may not be available. In [22], authors obtained the multi-hop throughput in wireless networks when slotted Aloha and incremental redundancy is used. They assumed that nodes are distributed following Poisson point process and two routing policies, namely furthest neighbor routing and nearest neighbor routing, are studied. They showed that
coding and retransmissions provide a fully decentralized MAC protocol. However, their analysis cannot be used for commonly use IEEE 802.11 MAC protocol.

Assuming that all nodes are placed equi-distance apart, the authors in [23] obtained the maximum end-to-end throughput of chain-topology. Many research [1]-[8], [24] used the same idea to obtain a slightly better formula for maximum end-to-end throughput in chain-topology. However, none of those works took node distribution and routing policy into account. Furthermore, it is not clear how one can extend the models to 2 dimensional networks. In [17], however, the author generalized the method used in [23] to any chain-topology when the location of nodes are known. By running a backward iterative process which is relatively complicated for a computationally limited wireless node, a source node can obtain the maximum end-to-end throughput. In [24], the model is extended to support heterogeneous networks in which nodes transmit different length frames with various loads when all nodes reside in carrier sense range of all other nodes. The model, however, cannot be extended to multi-hop networks.

In [25], throughput and stability results of single-hop CMSA/CA network were obtained, but it is restricted due to the assumption that nodes freeze their arrival process during a back-off period [26]. Furthermore, their analysis is only applicable for single-hop communication. In [27], a two-level modeling approach was used to model multi-hop wireless communication in which the first level is a slightly modified version of [28] and the higher level model consists of M/M/1/K queues representing each node in the network. The predication modeling framework also covers scenarios where two flows in opposite directions exist. Despite the claim that their model can handle any number of nodes, the scenarios they covered is limited to only four nodes. Additionally, routing is not considered in their study.

In [29], greedy distance maximization model, similar to our furthest neighbor routing policy, was proposed for 2-D randomly deployed sensor networks. They obtained a transformation of Gamma distribution to accurately model multi-hop distance distribution. However, their approach can only be used for furthest neighbor routing policy. In [30], the author provided an analytical expression for maximum end-to-end throughput by building a conflict graph of a path. Although maximum end-to-end throughput of a chain with randomly distributed nodes is given in this paper, the expression requires finding clique number of the conflict graph which itself is an NP-Hard problem. The author also suggested using the maximum forward degree of the conflict graph instead of clique number, but no simulation was given to confirm the validity of the model. Moreover, the hidden and exposed node problems, the difference between transmission
and interference range and, in general, MAC related problems were neglected. The conflict graph modeling was also used in [31] to provide a conceptually and computationally simpler model for unsaturated and single-hop wireless networks. Despite the accuracy of their model, it cannot be extended to multi-hop wireless networks and it cannot certainly deal with routing. As we mentioned earlier and to the best of our knowledge, no research has taken routing policy, node distribution and MAC layer limitation into account simultaneously. Moreover, no analytical results are available about maximum throughput in 2-D networks.

III. ANALYSIS OF ROUTING POLICY IN 1-D NETWORKS

In this section, average end to end throughput is obtained considering a perfect MAC layer. Note that in this paper throughput is defined as the end-to-end throughput of the entire flow which is measured by finding the receiving rate of a destination node.

**definition III.1.** Perfect MAC layer is defined as a MAC layer in which no hidden or exposed node problem occurs. Additionally, after an intermediate node sends its packet, it will wait until the packet reaches outside its interference range before sending another packet.

We will later extend our model to support imperfections in MAC layer in the next section. Let denote the throughput of single-hop connection by $C$. Now, we obtain the end-to-end throughput of multi-hop connection for two different routing policies referred to as random neighbor routing and furthest neighbor routing.

In random neighbor routing, next hop is selected randomly among the nodes in the transmission range of the current node toward a destination. Note that, the next hops are typically selected by a routing protocol but, in nodes point of view, next hops may have some characteristics which categorize that routing protocol as a random neighbor routing.

For example, in Fig. [1] node $i$ selects one of the nodes in a set $S = \{i + 1, i + 2, i + 3, i + 4\}$ with equal probability. On the other hand, in furthest neighbor routing, intermediate nodes select the furthest node in its transmission range toward destination, i.e. node $i + 4$ is selected as the next hop. In 1-dimensional networks, the end-to-end throughput for this routing mechanism can be used as an upper-bound.

Needless to say, in 2-D networks when several flows are being carried out, it is possible for longer routes avoiding the congested region to outperform shorter routes even if the furthest neighbor routing is used. Particularly, when a routing mechanism in which the routes tend to
Fig. 1. In random neighbor routing, node $i$ selects one of the nodes $i+1$, $i+2$, $i+3$ or $i+4$. In furthest neighbor routing, node $i$ selects $m$.

be the shortest one is used, most of the network traffic pass through the center of the network which makes the central portion of network highly congested [32]. That is why even using the furthest neighbor routing mechanism which may pass through the congested central part of the network may be less efficient than a long route avoiding the center of the network.

Note that current well-known routing protocols such as AODV are not necessarily a random neighbor routing. Although jittering mechanisms [33], [34], such as Uniform Jittering, may contribute to AODV’s randomness, the fact that middle nodes can forward a RREQ packet more than once if the newly received RREQ packet belongs to a shorter path, renders this protocol less random. If intermediate nodes were not allowed to send RREQ packets more than once and they were just allowed to forward the first RREQ they had received, AODV would be construed as a random neighbor routing. It is also quite obvious that AODV is not equal to the furthest neighbor routing either. As long as hop count is concerned, AODV is better than random neighbor routing but worse than furthest neighbor routing. This fact will be clearly shown in Section VII by simulation.

The rest of this section categorized into three parts. In the first part, a general approach towards the analysis of end-to-end throughput is proposed. Given the information about routing policy and node distribution, the proposed analysis can be used to obtain end-to-end throughput of any routing policy. More precisely, our analysis requires the average number of hop needed to reach at a distance $x$ away from a source to be known. Then, in the following subsections, the end-to-end throughput of random neighbor routing as well as furthest neighbor routing are obtained based on our general approach.
A. General Approach

Assume that node $i$ just forwarded its packet to the node $i + 2$, depicted in Fig. 1. Since they are located in the transmission range of each other, they cannot send their packets simultaneously. That is why the throughput of multi-hop transmission is less than single-hop transmission. Hence, we are interested in finding the average number of consecutive nodes following $i$ in the path that must forward the packet until the packet reaches a point where its transmission does not interfere with the transmission of $i$ so that it can send another packet.

For simplicity, we assume that all nodes have the same transmission range, $R_{tx}$, interference range, $R_i$, and carrier sensing range, $R_{cs}$, defined in [23]. Note that in a perfect MAC layer, hidden or exposed node problems do not exist. Hence, in this section, the value of $R_{cs}$ is not taken into consideration, i.e. $R_{cs} = R_i$. Consequently, two transmissions can be simultaneously carried out when the sender and receiver of two connections are not in the interference range of each other.

In Fig. 2, assume that the node $i$ has just forwarded its packet. Let $N(x)$ be the average number of nodes that must forward the packet to reach at a distance $x$ away from a node. In order for $i$ to forward its next packet, the previous packet must travel outside of the interference range of node $i + 1$. Otherwise, node $i + 1$ fails to receive packets of $i$. Hence, the total number of transmissions needed so as to send the next packet is $1 + N(R_i)$. Hence, the average value of maximum throughput is obtained from Eq. (1).

$$T_{max} = \frac{C}{1 + N(R_i)}$$  \hspace{1cm} (1)

where $C$ is a throughput of a single hop communication. In perfect MAC layer, $T_{max}$ is the maximum throughput which can be used in upper layer for regulating source’s sending rate. Needless to say, there would be no increase in end-to-end throughput if a source increases its sending rate. In reality, with imperfect MAC, increasing sending rate even deteriorates the end-to-end throughput due to high contention and collision. On the other side, if the source’s sending rate is less than $T_{max}$, the end-to-end throughput is equal to source’s sending rate. Therefore, the end-to-end throughput is obtained from Eq. (2) where $r$ is the source’s sending rate.

$$T(r) = \text{Min}\left\{r, \frac{C}{1 + N(R_i)}\right\}$$  \hspace{1cm} (2)
Table I

TABLE OF NOTATIONS.

| Symbol | Description |
|--------|-------------|
| $r$    | Sending rate of a source node |
| $C$    | Throughput of a single-hop communication |
| $T_{\text{max}}$ | Maximum throughput of multi-hop communication |
| $T(r)$ | Maximum throughput of multi-hop communication when source sending rate is $r$ |
| $R_{\text{tx}}$ (R) | Transmission range |
| $R_i$  | Interference range |
| $R_{cs}$ | Carrier sensing range |
| $N(x)$ | Average number of nodes to reach at a distance $x$ for general routing policy |
| $\bar{N}(x)$ | Approximation of $N(x)$ |
| $N_{1D}(x)$ | Average number of nodes in random neighbor routing to reach at $x$ in 1-D networks |
| $N_{FR}(x)$ | Average number of nodes in furthest neighbor routing to reach at $x$ in 1-D networks |
| $N_{R2D}(x)$ | Average number of nodes in random neighbor routing to reach at $x$ in 2-D networks |
| $N_{FR2D}(x)$ | Average number of nodes in furthest neighbor routing to reach at $x$ in 2-D networks |
| $\lambda$ | Node density |
| $d_i$ | Distance of node $i$ from the source |
| $P_{\text{col}}$ | Probability of collision |
| $x$ | Normalized air time [23] |
| $\gamma_n$ | Normalized lower incomplete Gamma function |
| $\psi(x)$ | Function defined by Eq. (16) for convenience |
| $C_n$ | Constant terms of $N_F(x)$ defined by Eq. (17) |

Now that we have the general formula to obtain the end-to-end throughput, we need to compute $N(x)$. This function depends only on routing mechanism and node distribution. In this section, we assume that location of nodes in a network follows a homogeneous 1-dimensional Poisson Point Process (PPP), that is, nodes are distributed randomly in a segment of a line as it is shown in Fig. 2.
B. Random Neighbor Routing

In the random neighbor routing, a next hop is selected from neighboring nodes toward the destination with equal probability. In Fig. 1, the intermediate node \( i \) selects one of the nodes in a set \( S = \{ i + 1, i + 2, i + 3, i + 4 \} \) with probability of \( \frac{1}{4} \). Here, we are interested in the distance distribution to the next hop in a random neighbor routing.

**Theorem III.2.** The distribution of distance to the next hop when using the random neighbor routing in a PPP network has a uniform distribution, \( U \sim [0, R_{tx}] \).

**Proof.** In random neighbor routing, the next hop is selected randomly from the set of all nodes in the transmission range of an intermediate node, i.e. toward the destination, with an equal probability. Given that there are \( n \) nodes in the transmission range of an intermediate node, we are interested in finding the distance distribution of each of them. It is known [35] that in order to have a homogeneous PPP, in a region of length \( R_{tx} \), location of points should be selected from a uniform random variable (RV), \( U \sim [0, R_{tx}] \). Hence, the distance distribution of all \( n \) nodes in the transmission range of an intermediate node are identically independently distributed (i.i.d.). In other words, next hop is selected randomly with an equal probability from a pool of \( n \) i.i.d uniform random variables. Therefore, regardless of the node selected, the distribution of the distance to that node is uniform RV, \( U \sim [0, R_{tx}] \).

Note that a next hop in the random neighbor routing is not the same as the next immediate node. In fact, it is well known that the distance between two adjacent nodes in PPP follows Exponential distribution [35]. However, the next hop in random neighbor routing can be any node located in the transmission range of node \( i \) towards the destination.

The theorem is also true for binomial point process (BPP) since PPP is equivalent to BPP, conditioned on the presence of \( n \) nodes in the transmission range [36]. Consequently, all the formulas obtained in this paper for uniform random policy in PPP is also true in BPP.

As it is shown in Fig. 2, in order for the node \( i \) to send its next packet, the last packet should be forwarded until it reaches a node whose interference range is smaller than its distance to the node \( i + 1 \). In other words, we are interested in finding how many uniform i.i.d random variables are required so that the sum of their values exceeds the interference range.

Given that the distance distribution to the next hop in random neighbor routing is uniform, \( N(x) \) is given by theorem III.3. Hereafter, we will write \( R \) instead of \( R_{tx} \) for brevity.
Theorem III.3. The expected number of i.i.d uniform random variables ($U \sim [0, R]$) whose sum exceeds $x$ is given by Eq. (3).

$$N_R(x) = \sum_{k=0}^{\left\lceil \frac{x}{R} \right\rceil - 1} \frac{(-1)^k}{k!} (\frac{x}{R} - k) e^{(\frac{x}{R} - k)}$$  \hspace{1cm} (3)

Proof. We are using induction to prove the theorem. For $0 < x \leq R$, the solution is given in [37] which is $e^{x/R}$. Now, assuming that the formula holds for $0 < x \leq nR$, we prove it also holds for $nR < x \leq (n+1)R$ in which $n$ is a positive integer. Let’s define $M(x)$ as follows [37]

$$M(x) \triangleq \min\left\{ n : \sum_{i=1}^{n} U_i > x \right\}$$  \hspace{1cm} (4)

and

$$N_R(x) = E[M(x)].$$  \hspace{1cm} (5)

$N_R(x)$ can be obtained by conditioning on $U_1$, as the first uniform random variable, as follows

$$N_R(x) = \int_0^R E[M(x)|U_1 = y] f_{U_1}(y) dy = \frac{1}{R} \int_0^R E[M(x)|U_1 = y] dy.$$  \hspace{1cm} (6)

The conditional expectation itself is obtained from Eq. (7).

$$E[M(x)|U_1 = y] = \begin{cases} 1 & \text{if } x < y \leq R \\ 1 + N_R(x - y) & \text{if } y < x \end{cases}$$  \hspace{1cm} (7)

Substituting Eq. (7) into Eq. (6) and letting $u = x - y$ results in Eq. (8).
By differentiating the last equation with respect to \( x \), i.e. Eq. (8), we are given Eq. (9).

\[
N'_R(x) = \frac{N_R(x) - N_R(x - R)}{R}.
\] (9)

Considering that \( nR < x \leq (n + 1)R \), it is easy to see that Eq. (3) holds in Eq. (9).

In [9], authors obtained a closed-form distribution of number of intermediate nodes required to reach a node at distance \( D \) away from the source using incomplete gamma function as Eq. (10).

\[
N_H \sim \gamma_n(D; \lambda, n\beta) - \gamma_n(D; \lambda, (n + 1)\beta)]
\] (10)

In Eq. (10), \( \beta = 1 + \bar{d}\lambda \), \( \lambda \) and \( D \) are node density and distance, respectively. \( \bar{d} \) is the average hop length under an arbitrary routing policy and \( \gamma_n(x; \lambda, n) \) is normalized lower incomplete Gamma function defined as Eq. (11).

\[
\gamma_n(x; \lambda, n) \triangleq \frac{1}{\Gamma(n)} \int_0^{\lambda x} t^{n-1}e^{-t}dt.
\] (11)

Note that this formula works for any routing policy, such as random neighbor routing and furthest neighbor routing. The only difference is the average hop length which clearly differs from one routing policy to another. Assuming an imaginary node is located at distance \( D \) away from the source, we can use their formula to obtain \( N_R(x) \) by finding the expected value of the distribution. Note that for random neighbor routing \( \bar{d} = \frac{1}{2}R \). The formula of [9], however, is not as accurate as our formula, as it is shown in Fig. 3. For the simulation, we used a simple C++ code which randomly distributed nodes in a line of length 1250 meter. Then, a source node was put at the beginning of the line. Finally, random neighbor routing policy was used to find how many nodes are required to reach certain points. We conducted 2000 simulation for each data point. Moreover, we omitted the simulations in which the network was disconnected which only happened at low densities. It is also clear that density does not have any effect on the number of nodes needed to reach a point as long as the network is connected. Note that it is only true for random neighbor routing.
C. Furthest Neighbor Routing

The number of nodes in an area of length $R$ in PPP is Poisson random variable with parameter $\lambda R$ in which $\lambda$ is node density \cite{35}. Hence, the complementary cumulative distribution of distance, conditioned on having at least one node in an area of length $R$ could be obtained from Eq. (12). This equation indicates the probability of having at least one node in an area of length $R - x$, conditioned on having at least one node in the whole $R$. From Eq. (12) we can conclude Eq. (13) and as a result Eq. (14) is derived, which is similar to the equation proposed in \cite{38} that did not have the density term, $\lambda$.

\[
P(X > x) = \frac{1 - e^{-\lambda(R-x)}}{1 - e^{-\lambda R}} \tag{12}
\]

\[
F_X(x) = 1 - P(X > x) = \frac{e^{\lambda x} - 1}{e^{\lambda R} - 1} \tag{13}
\]

\[
f_X(x) = \frac{\lambda e^{\lambda x}}{e^{\lambda R} - 1}. \tag{14}
\]

Here, to make our model mathematically tractable, we assume that distribution of distance to next node for node $i$ is independent of the previous nodes. However, for furthest random routing, this is not always true, particularly when the network has low density. If the furthest node of $i$ is node $i+1$ in a distance $d_{i+1}$, we know that there is no node in distance $d_{i+1} < x < R$. Hence, the furthest node of node $i+1$ cannot reside in this region. That is why in furthest neighbor routing the distribution of distance to the next hop of intermediate nodes is not independent. It is worth noting that, in a relatively dense network, this region becomes so small that it barely has any perceptible influence on the analysis.

**Theorem III.4.** The expected number of i.i.d random variables, with distribution function $f_X(x)$, whose sum exceeds $x$ is given by Eq. (15).

\[
N_F(x) = C_\left\lceil \frac{x}{R} \right\rceil e^{\psi(x)} + \left\lceil \frac{x}{R} \right\rceil \left(1 - e^{-\lambda R}\right) + \sum_{k=1}^{\left\lceil \frac{x}{R} \right\rceil - 1} \frac{(-1)^k \psi(x) \psi(k)(x-kR)}{k!} e^{\psi(x-Rk)} C_\left\lceil \frac{x}{R} \right\rceil - k, \tag{15}
\]

In Eq. (15), parameters $\psi(x)$ and $C_n$ are defined by Eq. (16) and (17), respectively.

\[
\psi(x) = \frac{\lambda x}{1 - e^{-\lambda R}}. \tag{16}
\]
\[ C_n \triangleq \begin{cases} e^{-\lambda R} & \text{if } n = 1 \\ e^{\psi(-R)}[\psi(R)C_1 + e^{-\lambda R} - 1] + C_1 & \text{if } n = 2 \\ e^{\psi(-R(n-1))}(e^{-\lambda R} - 1) + C_{n-1} + \\ \sum_{k=1}^{n-2} \frac{\psi(R(n-1))\psi^{k-1}(R(n-1-k))}{(-1)^k k! e^{\psi(R_k)}} \times (C_{n-k-1} - C_{n-k}) & \text{if } n > 2 \end{cases} \] (17)

**Proof.** Just like what was described in Theorem III.3, we use induction to prove the theorem. Let \( M(x) \) and \( N_F(x) \) be as Eq. (18) and Eq. (19), respectively.

\[ M(x) \triangleq \min \left\{ n : \sum_{i=1}^{n} f_i > x \right\} \] (18)

\[ N_F(x) = E[M(x)]. \] (19)

It is notable that \( N_F(x) \) can be obtained by conditioning on \( U_1 \) as it is shown in Eq. (20), while the conditional expectation itself is obtained form Eq. (21).

\[ N_F(x) = \int_0^R E[M(x)|f_1 = y]f_1(y)dy = \frac{\lambda}{e^{\lambda R} - 1} \int_0^R e^{\lambda y} E[M(x)|f_1 = y]dy \] (20)

\[ E[M(x)|f_1 = y] = \begin{cases} 1 & \text{if } x < y \leq R \\ 1 + N_F(x - y) & \text{if } y < x \end{cases} \] (21)

Letting \( u = x - y \) results in Eq. (22) while Eq. (23) could be obtained from multiplying both side by \( e^{-\lambda x} \).
\[ N_F(x) = 1 + \frac{\lambda e^{\lambda x}}{e^{\lambda R} - 1} \int_{x-R}^{x} e^{-\lambda u} N_F(u) du. \]  

(22)

\[ e^{-\lambda x} N_F(x) = e^{-\lambda x} \frac{\lambda}{e^{\lambda R} - 1} \int_{x-R}^{x} e^{-\lambda u} N_F(u) du. \]  

(23)

By differentiating Eq. (23) with respect to \( x \), we are given Eq. (24).

\[ N_F'(x) = \frac{\lambda N_F(x)}{e^{\lambda R} - 1} + \lambda N_F(x) - \frac{\lambda e^{\lambda R} N_F(x-R)}{e^{\lambda R} - 1} - \lambda. \]  

(24)

As a result of the definition of \( N(x) \), for \( 0 < x < R \), \( N(x-R) \) is equal to zero. Note that if the value of \( x \) is negative, there is no transmission needed to reach the distance \( x \) since the packet has already passed that point. Hence, we conclude Eq. (25) for \( 0 < x \leq R \). The solution of this equation is mentioned in Eq. (26). Since \( N_F(0) = 1 \), \( c_1 \) would be equal to \( e^{-\lambda R} \).

\[ N_F'(x) = \frac{\lambda N_F(x)}{e^{\lambda R} - 1} + \lambda N_F(x) - \lambda = \frac{\lambda e^{\lambda R} N_F(x)}{e^{\lambda R} - 1} - \lambda \]  

(25)

\[ N_F(x) = c_1 e^{\psi(x)} + 1 - e^{-\lambda R} \]  

(26)

Now, considering that the theorem holds for \( (n-1)R < x \leq nR \), it is straightforward, yet tedious, to see that Eq. (15) holds in Eq. (24) for \( nR < x \leq (n + 1)R \). Thus, we omit the remainder of the cumbersome calculation and substitution here. Note that differential equation constant, \( c_n \), is obtained by solving \( N_F(R(n - 1) + \epsilon) = N_F(R(n - 1)) \).

Eq. (10) can also be used to obtain average number of hops in furthest neighbor routing as it is suggested in [9]. The value of \( \bar{d} \), however, is different from the case of random neighbor routing. The authors suggested the following formula to obtain \( \bar{d} \) of furthest neighbor routing.

\[ \bar{d} = \frac{1}{\lambda} \ln \left( 1 - \frac{\lambda \bar{d}}{\lambda - \lambda \bar{d} - 1} \right). \]  

(27)

However, the implicit form of the equation which requires numerical evaluation increases the complexity of their solution even further [39]. Having compared our formula with the one proposed in [9], we have found that our approach is extremely accurate as it is shown in Fig. 4. Regardless of the node density, our approach always agrees with simulation.
Using Eq. (2) and Theorem III.4, a source node can obtain the end-to-end throughput for furthest neighbor routing. Unlike previous works on end-to-end throughput [1]–[8], [17] which assumed that nodes are placed equi-distance apart or assumed the location of nodes to be known by the source, our analysis obtains end-to-end throughput based on realistic node distribution and routing policy.

IV. MAC Layer Consideration

In this section, limitations of wireless MAC layer are taken into account to obtain a more accurate expression for maximum end-to-end throughput. It is obvious that the perfect MAC, as defined in previous section, is impossible in wireless networks. Therefore, it is crucial to consider MAC and phyiscal layers’ limitation alongside the routing and node distribution. Most previous research has focused on MAC layer limitations without considering node distribution and routing policy, which clearly shows the importance of analyzing MAC layer. Although analyzing MAC and routing layers simultaneously is difficult, no analysis is precise enough if it overlooks one of the aspects. Thus, we provide a method to constitute a comprehensive model containing both routing and MAC layer consideration in this section. In this section, the probability of collision and hidden node problem, which is non-existence in perfect MAC layer, are taken into account to obtain a more accurate end-to-end throughput.

In wireless networks, no perfect MAC layer protocol has been proposed so far. It is also unlikely to be able to devise one due to the inherent problems and difficulties of wireless media.
Hence, our analysis in the previous section can be used as a maximum upper bound throughput. If some information about the MAC layer is available, it is possible to improve our analysis. As an instance, if the probability of collision is known, denoted by $P_{col}$, the number of transmission needed to successfully transmit a packet would be $\frac{1}{1-P_{col}}$. Therefore, maximum throughput can be obtained from Eq. (28), which is the improved version of Eq. (1). It is worth noting that, in this equation, the assumption $R_{cs} > R_i$ is considered.

$$T_{max} = \frac{1 - P_{col}}{1 + N(R_{cs})}$$

Note that, in general, $R_{tx} < R_i < 2R_{tx} < R_{cs}$ is satisfied [17]. In this case, unlike perfect MAC layer, carrier sensing may block non-interfering transmission, referred to as exposed node problem [8]. Consequently, it limits the capacity further. Hence, in order to make sure that an intermediate node can send its next packet, the previous packet must be forwarded to a distance greater than $1 + N(R_{cs})$. That is why $N(R_{cs})$ is used in this formula instead of $N(R_i)$.

It is also notable that the value of $P_{col}$ is not independent of routing mechanism, nodes distribution and density. That is the main reason the other proposed analysis has always assumed a chain topology with constant distance or very particular structure. To obtain the value of $P_{col}$, both routing mechanism and MAC layer protocol are taken into account here. We will use an IEEE 802.11 MAC analysis based on [23] and two different routing mechanisms elaborated upon in the previous section to obtain end-to-end throughput.

In IEEE 802.11 MAC, nodes that are not located in the carrier sensing range of a sender, but located in the carrier sensing range of the receiver may cause a collision in that transmission [23]. This region containing these interfering nodes, which is shown in Fig. 5, is called hidden node area. Since the source is not aware of the transmission in this region, it may start a transmission. However, the receiver will fail to decode the packet due to the interference.

Since collision occurs due to nodes in hidden node area, we are interested in obtaining the number of nodes in this region. Given any arbitrary routing policy that selects next hop statistically independent from the previous nodes, we expect to see an equal number of nodes, chosen for routing, in each line segment of length $x$ on average. As it is shown in Fig. 5, the length of hidden node area is equal to the distance between node $i$ and $i + 1$, denoted by $E[d]$. On average, we expect to see only one node in a region of length $E[d]$ since $E[d]$ is the average distance between two consecutive nodes. Thus, the average number of nodes in hidden node
area is 1 regardless of the routing policy. Note that we are only interested in nodes selected for routing packets. There might be many non-contributing nodes in this area that does not have any influence on throughput.

Since this argument is not necessarily true for all statistical phenomena, we have computed number of forwarding nodes in each region of length $E[d]$. To obtain number of nodes in a region of length $E[d]$, one can simply evaluate $N(x + E[d]) - N(x)$. As it is shown in Fig. 6a, the number of nodes in hidden node area for random neighbor routing converges to 1 and there is no significant variance over different $x$. This is also true for furthest neighbor routing illustrated in Fig. 6b. Like random neighbor routing, the number of nodes in hidden node area converges to 1 regardless of the node density. Although for the higher value of density, it is closer to 1, even for lower densities the variance is negligible. Nevertheless, starting from the source node, the hidden nodes must reside between the source’s carrier sensing range ($R_{cs}$) and the next hop’s carrier sensing range. In other word, even at the worse case scenario where the next hop is very close to the source node, hidden nodes are located at distances which are far greater than 250m since $R_{cs} > 2R_{tx} > R_{tx} = 250m$. The most variance occurs for small value of $x$ in furthest neighbor routing. Note that as $x$ increases, $N(x)$ gets smoother, as it is depicted in Fig. 3 and 4. Hence, $N(x + E[d]) - N(x)$ also becomes smoother as $x$ increases. Hence, we
obtain the probability of collision based on the fact that there is always one node in hidden node area on average.

Let $j$ be the node that is located in the hidden node area and $x_i$ be the normalized airtime of node $i$ according to [23]. Assuming that all nodes transmit at the same rate, we omit the index of $x_i$ and, hereafter, we write $x$ instead. If any node that located in carrier sensing range of both $i$ and $j$ is transmitting a packet, $i$ and $j$ cannot start sending a packet and consequently they cannot collide in this case. Such intermediate nodes are called contending nodes. The number of contending nodes of the node $i$ located at distance $d_i$ from the source would be $N(d_i + R_{cs}) - N(d_i)$.

In order for a collision to happen, all contending nodes must be silent. Hence, this value should be removed from the sample space. The probability of collision for the node $i$, according to [23], is obtained from Eq. (29) in which $a$ is a fraction of time devoted to the data packet.

$$P_{col} = \frac{ax}{1 - [N(d_i + R_{cs}) - N(d_i)]x}$$

Here, the normalized airtime represented by $x$ is equal to $\frac{r_C}{C}$. Note that in this formula $P_{col}$ depends on the node’s distance from the source, $d_i$. However, we assumed that all nodes are statistically independent as a result of which their position does not have a considerable influence. Additionally, we will show in the next section that $N(d_i + R_{cs}) - N(d_i)$ can be considered to be constant since the derivative of $N$ converges to a constant value. Hence, we will compute $P_{col}$ for the first node, the $d_i$ of which is zero.

### V. APPROXIMATION OF $N(x)$

In this section, we aim to provide an approximation for $N_R(x)$ and $N_F(x)$, and use it to obtain $T(r)$. As it is shown in Fig. 3, it is easy to observe that for large enough $x$, $N(x)$ approaches a straight line. Additionally, it is also easy to see that, using root test, $N'(x)$ given by Eq. (9) and (25) is convergent. Hence, it is reasonable to find a linear approximation of $N(x)$.

**Theorem V.1.** If $Y_1, Y_2, \ldots$ are mutually independent, identically random variable with any arbitrary distribution (routing policy) that represents the distribution of the distance to the next hop, linear approximation of their corresponding $N(x)$ is as follows

$$\bar{N}(x) = \frac{x}{E[Y]} + \frac{E[Y^2]}{2E^2[Y]}$$
Proof. Using the same approach as it is used in Theorem (III.3), we get the following equation, independent of $Y$’s distribution, when $0 < x < R$,

$$
N(x) = 1 + \int_0^x f_Y(x - u)N(u)du.
$$

(31)

Using the Laplace transform and factoring $\mathcal{L}\{N(y)\}$, we get

$$
\mathcal{L}\{N\}(s) = \frac{1}{s(1 - \mathcal{L}\{f_Y\}(s))}.
$$

(32)

To solve this differential equation, we need to know the Laplace transform of $f_Y$. Assuming that all moments of $f_Y$ exist, Laplace transform can be obtained by moment generating property as follows

$$
\mathcal{L}\{f_Y\}(s) = \sum_{n=0}^{\infty} (-1)^n \frac{s^n}{n!} E[Y^n].
$$

(33)

Using this Laplace transform in Eq. (32), we are given

$$
\mathcal{L}\{N\}(s) = \frac{1}{s^2} \times \frac{1}{\sum_{n=1}^{\infty} (-1)^{n-1} \frac{s^n}{n!} E[Y^n]}. \quad (34)
$$

Now, since the summation is a power series, we can substitute the reciprocal of this power series as follows [40]

$$
\mathcal{L}\{N\}(s) = \frac{1}{s^2} \times \left( \frac{1}{E[Y]} + \frac{E[Y^2]}{2E^2[Y]} s + ... \right).
$$

(35)

Note that since we are interested in a linear approximation of $N(x)$, the Laplace form should be in a form $\mathcal{L}\{N\}(s) = as^{-2} + bs^{-1}$. As a result, we only consider the first two terms of the series, and the approximated Laplace transform is given by

$$
\mathcal{L}\{N\}(s) \approx \frac{s^{-2}}{E[Y]} + \frac{E[Y^2]}{2E^2[Y]} s^{-1}.
$$

(36)

It is easy to see that this equation is the Laplace transform of Eq. (30).

\[\square\]

Given the fact that $N'(x)$ approaches a constant value, the value of $[N(d_i + R_{cs}) - N(d_i)]$ becomes independent of $d_i$ and $P_{col}$ becomes equal for all intermediate nodes. Hence, Eq. (29) can be solved with $d_i = 0$. Now that a simple approximation of $N(x)$ is provided, maximum throughput can be obtained by Eq. (28).
A. Approximation of $N_R(x)$

For random neighbor routing, linear approximation is obtained as follows

$$\bar{N}_R(x) = \frac{2x}{R} + \frac{2}{3}.$$  \hspace{1cm} (37)

Curve fitting using least squares error method for $0 < x < 10R$ also gave almost the same value. The slope of the line obtained by least squares method was also 0.00798 for $R = 250$ which is almost the same as $N'_R(x)$. Note that x-intercept, unlike the slope of the approximation line, is independent of the $R$.

B. Approximation of $N_F(x)$

Using the same method as random neighbor routing, the slope of the approximation function of $N_F(x)$ is obtained by its expected value. The expected value of the distribution function $f_x$ in Eq. (14), denoted by $E[X_F]$, is shown in Eq. (38).

$$E[X_F] = \frac{e^{-\lambda R} + \lambda R - 1}{\lambda(1 - e^{-\lambda R})}. \hspace{1cm} (38)$$

The second moment of $X_F$ is as follows

$$E[X_F^2] = \frac{1}{e^{\lambda R} - 1} [R^2 e^{\lambda R} - \frac{2R}{\lambda} e^{\lambda R} + \frac{2 e^{\lambda R}}{\lambda^2} - \frac{2}{\lambda^2}]. \hspace{1cm} (39)$$

Using these average and second moment of $X_F$, we can use Eq. (30) as a linear approximation. Usefulness of our linear approximations will be shown in Section VII.

VI. Maximum Throughput in 2-D Networks

As it has been shown in previous sections, the exact formula to obtain $N(x)$ can get extremely complicated for some routing policies. Consequently, the approximation formulas introduced in Section V are shown to be fairly accurate. In 2-D networks, obtaining the exact value of $N(x)$ even gets more complicated, if possible at all. In fact, the probability distribution of the number of hops to reach the destination in 2-D networks is derived in [41] which not only does not have a closed-form structure, but also may have infinite unsolvable integrals. Hence, in this section, we obtain the approximate value of $N(x)$ which will be mathematically traceable.

As depicted in Fig. 7, The most problematic issue in 2-D networks is that the sum of distances from the first to the second node, i.e. $||a||$, and the second to the third node, i.e. $||b||$, is not equal
to the distance from the first node to the third one, i.e., $||c||$. In fact, the angle of progression (AoP), which is defined as an angle of a sector in which the next hop is selected, $\theta$, should be taken into account. However, this consideration makes these random variables dependent and consequently makes the problem of finding $N(x)$ in 2-D networks intractable. Since it is shown that routing protocols usually choose a straight line from a source to a destination if the network is dense enough [42], we assume that these random variables are independent. Note that if the angle of progression is relatively small, these random variables will act independently. This assumption also allows us to use Eq. (28) to obtain maximum throughput.

This section includes two parts which aim to approximate $N(x)$ for random neighbor routing, $N_{R2D}(x)$, and furthest neighbor routing, $N_{F2D}(x)$.

A. Approximation of $N_{R2D}(x)$

First, we need to find the probability density of distance to the next hop when random neighbor policy is used, like our 1-D analysis. The probability density function is denoted by $f_{R2D}$. Cumulative distribution function can be obtained by dividing the area of sector of length $x$ by the area of sector of length $R$, as follows

$$F_{R2D}(x) = \frac{\theta x^2}{\frac{\pi}{2} R^2} = \frac{x^2}{R^2}.$$  \hspace{1cm} (40)

Therefore, density function and expected value are obtained as follows,
\begin{equation}
 f_{R2D}(x) = \frac{dF(x)}{dx} = \frac{2x}{R^2}
\end{equation}

and

\begin{equation}
 E[X_{R2D}] = \frac{2}{3}R.
\end{equation}

The second moment is also easy to get which is

\begin{equation}
 E[X_{R2D}^2] = \frac{R^2}{2}.
\end{equation}

Hence, the approximate number of nodes required to reach the location at \(x, \bar{N}_{R2D}(x)\), is obtained as follows

\begin{equation}
 \bar{N}_{R2D}(x) = \frac{3x}{2} + \frac{9}{16}.
\end{equation}

Now, using Eq. (28) and (44), we can obtain the end-to-end throughput of random neighbor routing in 2-D networks. In the next section, we will show that this approximation works perfectly particularly when the angle of progression is small.

**B. Approximation of \(N_{F2D}(x)\)**

In this section, we will find the approximation for \(N_{F2D}(x)\). The probability density of the distance to the furthest node in 2-D networks obtained similar to the 1-D case and it is as follows

\begin{equation}
 f_{F2D}(x) = \frac{\lambda\theta x e^{\frac{\theta}{2}x^2}}{e^{\frac{\theta}{2}x^2} - 1}.
\end{equation}

The expected value is also derived from Eq. (45)

\begin{equation}
 E[X_{F2D}] = \frac{Re^{\frac{\theta}{2}x^2}}{e^{\frac{\theta}{2}x^2} - 1} - \int_0^R e^{\frac{\theta}{2}x^2} dx
\end{equation}

which needs the computation of the error function. Chebyshev integral inequality states that

\begin{equation}
 \int_a^b f(x)g(x)dx \geq \frac{1}{b-a} \left[ \int_a^b f(x)dx \right] \left[ \int_a^b g(x)dx \right].
\end{equation}

Using this inequality and assuming that \(f(x) = e^{\frac{\theta}{2}x^2}\) and \(g(x) = 2x\), both of which are monotonically increasing, we can obtain a bound for the second term in \(E[X_{F2D}]\) as follows
The linear function, \( g(x) \), is used here for convenience which allows us to integrate the left side of the inequality. In fact, any increasing function can be used, leading to a different inequality. Note that this bound decreases quickly to zero as density increases. Using this inequality, the approximated value of the expected value is obtained as follows:

\[
E[X_{F2D}] \approx \frac{Re^\frac{\theta \lambda R^2}{2}}{e^\frac{\theta \lambda R^2}{2} - 1} - \frac{2}{R \lambda \theta}. \tag{49}
\]

The difference between Eq. (46) and (49) is shown in Fig. 8. It is clear that for realistic node density, Eq. (49) accurately approximated the exact expected value. For the second moment of \( X_{F2D} \), we get

\[
E[X_{F2D}^2] = \frac{R^2 e^\frac{\theta \lambda R^2}{2}}{e^\frac{\theta \lambda R^2}{2} - 1} - \frac{2}{\lambda \theta} \approx R \times E[X_{F2D}]. \tag{50}
\]

Hence, Eq. (51) is a linear approximation of \( N_{F2D} \).

\[
\tilde{N}_{F2D}(x) = \frac{x}{E[X_{F2D}]} + \frac{E[X_{F2D}^2]}{2E^2[X_{F2D}]} = \frac{1}{E[X_{F2D}]}(x + \frac{R}{2}). \tag{51}
\]
VII. SIMULATIONS

To validate the analytical results obtained in previous sections we did an exhaustive network simulation using network simulator NS2 [43]. We intentionally changed AODV routing protocol to selects random or furthest neighbor based on the scenario. In our simulations, all nodes are distributed in a line of length $2000m$ randomly. For 2-D case, all nodes were randomly deployed in a region of $2000 \times 1000m^2$. The source and destination are located at two different sides of our network. IEEE 802.11 MAC without RTS/CTS mechanism is used in all wireless nodes. The distance model with the transmission range of $250m$ is chosen for message passing among nodes. Interference range and carrier sensing range are set as $450m$ and $500m$, respectively. Nodes are assumed to be stationary and there is no other source of traffic in the network. Each simulation scenario for each data point is conducted 1000 times, each with different seed value. The confidence interval of 99% is also shown in the figures. A single flow consisting of a pair of source and destination nodes located in two far apart edges of the network is simulated for 100 seconds.

The data rate of the wireless channel is chosen as $1Mbps$. Moreover, nodes are already aware of the maximum throughput of a single-hop transmission. To find the throughput of a single-hop communication, i.e. $C$, we evaluated the maximum throughput by simulating a transmission over 2 adjacent nodes. In our simulation, $C$ is $0.87Mbps$. Note that if the value of Short Inter Frame Space (SIFS), Distributed Inter Frame Space (DIFS), MAC header, Acknowledgment (ACK) length or any other protocol specific parameters are available at the transport or network layer protocols, $C$ could be easily obtained mathematically.

Using random neighbor routing, the sending rate of the source node has been increased to find the rate at which the end-to-end throughput reaches its maximum. As it is shown in Fig. 9a, our exact and approximate analysis of IEEE 802.11 yield almost the same result. Here, the graph of perfect MAC is given just to show how much the imperfection of a MAC layer can affect the end-to-end throughput. Note that the aim of our analysis is to find the maximum achievable throughput. Thus, we are not particularly concerned about the values our model gives for sending rates greater than the maximum. The reason why our model fails to predict values greater than maximum is that our model overestimates the probability of collision in this case. When a sending rate of the source node is above the maximum, the majority of collisions happen at the first nodes. Hence, packets that are forwarded successfully out of this region can successfully
reach the destination. Note that intermediate nodes receive fewer packets than the second or third nodes and as a result their forwarding rate is less than the maximum throughput. It is worth mentioning that when a source node’s sending rate is less than the maximum end-to-end throughput, almost no collision or contention will occur. As a result, as you can see in Fig. 9, the confidence interval is very close to the average value. However, as the sending rate gets closer to the maximum value, variance over end-to-end throughput increases as the probability of collision and contention increases.

Fig. 9b illustrates end-to-end throughput for node density equals to 0.04 using furthest neighbor routing. This means that each node has 10 neighboring nodes on average. As it is shown in Fig. 9b, our exact analysis predicts the maximum end-to-end throughput with high precision. Although the approximate approach slightly underestimates the maximum end-to-end throughput, the difference is negligible and it is still possible to use the approximate value in practice.

It is worth noting that in furthest neighbor routing, the number of neighbors is of paramount importance. Since the routing policy enforces a node to select the furthest neighbor, increasing the node density increases the chance of selecting a node closer to $R_{tx}$. As a result, the value of $N_F(x)$ decreases as node density increases. Hence, we expect an improvement in end-to-end throughput as node density increases. As it is shown in Fig. 10, the end-to-end throughput increases until it reaches a point in which the distance to the next hop is always close to $R_{tx}$.

In Fig. 11 end-to-end throughput of AODV is shown. As it is mentioned earlier, the throughput of AODV should be between random neighbor routing and furthest neighbor routing. The simulation whose results are shown in this figure clearly affirms our argument of Section III.
Fig. 10. Effect of node density on maximum end-to-end throughput in 1-D networks.

Fig. 11. AODV routing protocol in comparison with random and furthest neighbor routing.

Fig. 12. End-to-end throughput of random neighbor routing in 2-D networks ($\lambda=0.00015$).
Fig. 12 illustrates the end-to-end throughput of random neighbor routing in 2-D networks. Our analytical expression almost meets the simulation results when the angle of progression (AoP) is small. Note that since the route to the destination usually approaches a straight line [42], [44], it is reasonable to assume that the AoP is smaller than 60 degrees in reality. Comparing Fig. 9a and 12, we have noticed that the end-to-end throughput of random neighbor routing in 2-D networks is greater than 1-D networks. This interesting phenomenon stems from the fact that in 2-D networks random neighbor routing tends to select a more distant node than in 1-D. In 1-D networks, the number of nodes distributed in each small region at a distance of $x$ from the source node are statistically equal. On the other hand, in 2-D networks, the number of nodes located at a distance of $x$ from the source is increased as $x$ gets bigger. Note that nodes at a distance $x$ from the source are located at an arc whose length is increased as $x$ gets bigger. For this reason, the end-to-end throughput of random neighbor routing increases as the dimension of space increases. As it is shown in Fig. 12 the end-to-end throughput decreases as AoP increases. The reason is that when AoP is smaller the forwarding nodes are more likely to be in a straight line towards the destination. However, when AoP is larger, the number of nodes in the hidden node area becomes more than one. As we discussed in section VI in 2-D networks, when the next hops are not chosen in a relative straight line to the destination, the sum of distances from the first node to the second node, and the second to the third node is not equal to the distance from the first node to the third one, shown in Fig. 7. That means that nodes are generally closer together. As a result, contention is higher and the number of nodes in the hidden node area is also greater than one. That is why our model fails to predict the maximum end-to-end throughput when AoP is large.

Next, we conducted a series of simulations to validate our analysis of furthest neighbor routing, which is plotted in Fig. 13. In this figure, Analytical and Perfect MAC Analytical are obtained using Eq. (46), and Analytical (Approximation) is obtained from Eq. (49). As it is shown, unlike random neighbor routing, the angle of progression has a less dramatic effect on the maximum throughput. Moreover, as Angle of progression increases, the upper bound in Eq. (48) vanishes to zero and consequently the approximation function gets closer to the exact function.

Note that in this figure, the density is 0.0002, which means that there are about 40 nodes in the transmission range of each node. However, the number of nodes in the angle of progression is meaningfully lower. For instance, when AoP is 60 degree, i.e. $\pi/3$, there are only 6.5 nodes available to be selected in the routing process. In fact, we encountered some simulations in
which there were no path at all between the source and destination. Hence, our approximation estimated the maximum value of throughput even for reasonable node density.

To capture the effect of node density on maximum throughput, Fig. 14 is provided. As node density or AoP increases, our analytical formula approaches the simulation value. Due to the bound expressed in Eq. (48), it is clear that our approximation underestimates the maximum throughput when both density and AoP are low. In most cases, however, the difference is negligible.

VIII. CONCLUSION

This paper has presented a novel analytical approach to obtain end-to-end throughput. The routing policy along with its effect on the distance between intermediate nodes disregarded
completely in previous works. In this work, we have taken both into consideration to obtain a more precise value for maximum end-to-end throughput. We have shown that routing policy has an important role in end-to-end throughput which cannot be overlooked. It has an influence upon the number of forwarding nodes in crucial regions such as transmission range, interference range as well as carrier sensing range. Given a perfect MAC layer in which no hidden or exposed node problem occurs, the maximum end-to-end throughput has been obtained for two routing policy, including random neighbor routing and furthest neighbor routing. Then, the model has been extended to consider imperfection of IEEE 802.11 MAC layer. Due to the relative complexity of the model, the approximation method has been also presented so as to render this model practical for computationally limited wireless nodes. We have also extended our approximation to include 2-D networks. The validity of our analytical approach has been validated by simulation. We believe that the proposed model can be used in wireless nodes for admission control and flow control. Given the distribution to the next hop for a particular routing policy, our methodology also could be used to obtain the maximum end-to-end throughput.

As a future work, we will investigate how delay can be obtained in such networks. Due to the shared medium of wireless networks, when a node is transmitting, it affects the communication of neighboring nodes. Classical queue-based approaches cannot deal with this dependency. Therefore, it is more difficult to obtain the end-to-end delay analytically. It is also worth investigating how a different and more realistic channel model can be considered in obtaining end-to-end throughput. Moreover, our analysis can only predict a single flow. When there are multiple flows in the networks whose nodes are close enough to interfere, the end-to-end throughput will be considerably lower. In this scenario, further analysis is needed to find the number of hidden nodes and consider the effect of high contention.

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