On the entropy of matrix black holes

Miao Li and Emil Martinec
Enrico Fermi Institute and Department of Physics, University of Chicago, 5640 S Ellis Avenue, Chicago, IL 60637, USA

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Abstract. We compute the entropy of 5D black holes carrying up to three charges using matrix theory.

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1. Introduction

Matrix theory [1] appears to provide a nonperturbative definition of quantum gravity. As such, it should resolve the conceptual issues surrounding the quantum mechanics of black holes [2]. Four-dimensional (4D) and five-dimensional (5D) black holes with four and three charges \( Q_i \), respectively, appear to be ideal testbeds in this regard. Near extremality and in the weak coupling limit \( g_s Q_i \ll 1 \), it has been shown that configurations of \( D \)-branes can account for the Bekenstein–Hawking entropy [3], Hawking temperature [4, 5], emission and absorption cross sections [6, 7], and grey-body factors [8]. The 5D case in particular was examined from the point of view of matrix theory in [9, 10]. In [9], the authors analysed the behaviour of probes in the background geometry, and gathered evidence for an appealing scenario for black-hole dynamics in matrix theory; closely related ideas were presented in [10], where contact was made with the weak-coupling \( D \)-brane calculations mentioned above. In this note, we present an alternative derivation of the Bekenstein–Hawking entropy of these 5D black holes, using only a plausible assumption about the light degrees of freedom of \( 5+1 \) super Yang–Mills on a 5-torus, and the special kinematics laid out in [9]. Borrowing an analysis of this situation from Maldacena [11] (which arises in the context of a \( D \)-brane calculation), the entropy follows immediately. We argue that the leading term in the entropy is protected by nonrenormalization theorems.

2. The classical solution

The 5D black hole with three charges can be realized as the dimensional reduction of a six-dimensional (6D) black string, where one of the charges is the momentum travelling down the string (cf [5] and references therein). This black string with travelling wave can be realized in \( M \)-theory as a collection of intersecting 2-branes and 5-branes, with gravitational waves (which we shall call 0-branes) bound to the one-dimensional (1D) intersection [12]. In an obvious notation, the configuration is

\[
\begin{bmatrix}
\cdot & 6 & 7 & 8 & 9 & 11 \\
\cdot & \cdot & \cdot & \cdot & \cdot & p_{11} \\
5 & \cdot & \cdot & \cdot & 11
\end{bmatrix}
\]
The general nonextremal metric is [5,13,14]

\[
ds^2 = T^{1/3} F^{2/3} \left( T^{-1} F^{-1} \left( -K^{-1} h \, dr^2 + K \, dx_{11}^2 \right) \right.
\]
\[+ T^{-1} \, dx_1^2 + F^{-1} (dx_2^2 + \ldots + dx_5^2) h^{-1} \, dr^2 + r^2 \, d\Omega_5^2 \biggr] ,
\]
where \( dx_{11} = dx_{11} + (K' - 1) \, dt \), and \( r^2 = x_1^2 + \ldots + x_5^2 \). The various functions entering (2.2) are

\[
K = 1 + \frac{Q_0}{r^2}, \quad Q_0 = r_0^2 \sinh^2 \alpha
\]
\[
K' = 1 - \frac{Q_0}{r^2} K^{-1}, \quad Q_0' = r_0^2 \sinh \alpha \cosh \alpha
\]
\[
T = 1 + \frac{Q_2}{r^2}, \quad Q_2 = r_0^2 \sinh^2 \sigma
\]
\[
F = 1 + \frac{Q_5}{r^2}, \quad Q_5 = r_0^2 \sinh^2 \gamma
\]
\[
h = 1 - \frac{r_0^2}{r^2}.
\]

The extremal limit is \( r_0 \to 0 \), \( \alpha, \gamma, \sigma \to \infty \), with the charges held fixed. Dimensional reduction gives the 5D Einstein metric

\[
\begin{align*}
\lambda^2 (r) h(r) \, dr^2 + \lambda^{-1} (r) \left[ h^{-1} (r) \, dr^2 + r^2 \, d\Omega_5^2 \right],
\end{align*}
\]

where

\[
\lambda (r) = (TFK)^{-1/3} = \frac{r^2}{(r^2 + Q_0^2)(r^2 + Q_2^2)(r^2 + Q_5^2)}^{1/3}.
\]

Metrics such as (2.2) were given a brane interpretation in [15], which proposed an identification of the various charges with numbers of branes and antibranes (translated here into M-theory equivalents)

\[
N_0 = \frac{VR_5 R^2}{4l_p^2} r_0^2 e^{2\alpha}, \quad N_0' = \frac{VR_5 R^2}{4l_p^2} r_0^2 e^{-2\alpha}
\]
\[
N_2 = \frac{V}{4l_p^6} r_0^2 e^{2\sigma}, \quad N_2' = \frac{V}{4l_p^6} r_0^2 e^{-2\sigma}
\]
\[
N_5 = \frac{R_5}{4l_p^3} r_0^2 e^{2\gamma}, \quad N_5' = \frac{R_5}{4l_p^3} r_0^2 e^{-2\gamma}.
\]

Here \( V \) is the volume of the 4-torus spanning \( x_6, \ldots, x_9 \); \( R_5 \) is the radius of the \( x_5 \) circle; and \( R \) is the radius of the \( x_{11} \) circle. A useful relation is

\[
r_0^2 = 4R^2 \left( \frac{N_2 N_2 N_5 N_5}{N_0 N_0'} \right)^{1/2},
\]

allowing one to relate the different brane numbers in an interesting way:

\[
\left( \frac{N_0}{R} \right) \left( \frac{N_0'}{R} \right) = \left( \frac{N_2 R R}{l_p^2} \right) \left( \frac{N_2 R R}{l_p^2} \right) = \left( \frac{N_5 R V}{l_p^5} \right) \left( \frac{N_5 R V}{l_p^5} \right).
\]

The first of these quantities has an obvious interpretation—it is the invariant mass (in Planck units) of the gas of massless gravitons on the intersection string. Since the other two are \( U \)-dual to a gas of massless particles, we will call them the ‘invariant masses’ of the 2-branes and 5-branes. Equation (2.8) then states that there is an equipartition of ‘invariant masses’
or component energies of the black hole. In [9, 10], it was argued that the proper way to embed these solutions in matrix theory is to take the light-front directions \( x^\pm = t \pm x_1 \), and boost it to the infinite momentum frame (IMF). In this process, \( N \equiv N_0 \to \infty \) and \( N_\bar{0} \to 0 \).

3. Matrix realization and state counting

Matrix theory compactified on a transverse \( T^5 \) is thought to be equivalent to \( 5 + 1 \) super Yang–Mills theory on the dual torus \( \tilde{T}^5 \) [1, 16]. In this construction, a 5-brane wrapped on the longitudinal direction is an instanton; a longitudinal membrane is represented by momentum flux \( T_0 \); and \( N \) units of 0-brane charge are realized using \( U(N) \) as the Yang–Mills gauge group. For configuration (2.1), we take the instanton flux along \( 6789 \), and the momentum flux in the 5 direction. The instanton in the \( 5 + 1 \) gauge theory is a solitonic string, whose collective coordinates consist of four bosons and four fermions describing the transverse oscillations. The instanton charge on a torus can split into \( N \) pieces, hence the effective string tension is decreased by a factor of \( N \). For sufficiently large values of the charges, these ‘instanton strings’ dominate the entropy [11]. Because there are both winding and antiwinding strings (see figure 1(a)), representing 5-branes and (anti-)5-branes, it is possible for an instanton–anti-instanton pair to annihilate. This takes place locally by a joining–splitting interaction, figure 1(b). The winding energy is temporarily converted into kinetic energy; further joinings/splittings will transfer some of this energy back into winding. The equilibrium configuration will have a string gas with a rough equipartition between kinetic and potential energy.

The statistically preferred set of configurations (figure 1(c)) has a single long string carrying essentially all the energy—the instanton strings are in the Hagedorn phase (note that the formulation is microcanonical, so this is well defined). Thus, to compute the entropy, we need to determine the number of states available to a single long string with \( c_{\text{eff}} = 6 \), winding charge along the 5 direction \( (N_5 - N_\bar{5}) \), and momentum charge along the 5 direction \( (N_2 - N_\bar{2}) \).

The ADM energies carried by the various charges are

\[
\begin{align*}
    I_p E_0 &= \frac{V R R_5}{4 l_p^8} r_0^2 e^{2\alpha} = \left( \frac{N_0 N_\bar{0} l_p^2}{R^2} \right)^{1/2} e^{2\alpha} \\
    I_p E_2 &= \frac{V R R_5}{4 l_p^8} r_0^2 e^{2\alpha} = \left( \frac{N_0 N_\bar{0} l_p^2}{R^2} \right)^{1/2} e^{2\alpha} \\
    I_p E_5 &= \frac{V R R_5}{4 l_p^8} r_0^2 e^{2\gamma} = \left( \frac{N_0 N_{\bar{5}} l_p^2}{R^2} \right)^{1/2} e^{2\gamma},
\end{align*}
\]

(3.1)

with corresponding antibrane expressions obtained by flipping the sign in the exponentials. The total ADM energy is thus

\[
    I_p E_{\text{ADM}} = (N_0 + N_\bar{0}) \frac{I_5}{R} + (N_2 + N_\bar{2}) \frac{R R_5}{I_p^2} + (N_5 + N_\bar{5}) \frac{R V}{l_p^5}.
\]

(3.2)

† It seems that the counting of string states is a common thread in entropy computations; see [17] for an intriguing discussion and further references.
Figure 1. Maldacena’s picture of the gas of ‘instanton strings’. Through repeated joining/splitting interactions, the energy is collected into the entropically preferred state—one large string.

The energy of the system not carried by 0-branes is available to the string, since the IMF energy equals

$$E_{LC} = p_+ = E_{ADM} - \frac{N}{R}.$$  (3.3)

Note also that this energy is the Hamiltonian of the 5 + 1 gauge theory. The energy available to oscillators of the instanton string is reduced by the constraint that the black hole carries net 2-brane and 5-brane charge, which are carried on the string as momentum $l_p \mathcal{P} = (N_2 + \bar{N}_2)RR_5/l_p^2$ and winding $l_p \mathcal{W} = (N_5 - \bar{N}_5)RV/l_p^2$. We will defer until the end of this section all questions about string interactions, as well as the range of parameter space over which our discussion is valid. Meanwhile, we will treat the instanton string as noninteracting. Then the left and right excitation numbers are

$$n_{L,R} = \alpha'_{\text{eff}} [E_{LC}^2 - (\mathcal{P} \pm \mathcal{W})^2]$$

$$= \frac{\alpha'_{\text{eff}}}{l_p^2} \left[ \frac{VRR_5}{4l_p^2 r_0^2} \right]^2 \left[ (\text{ch}2\sigma + \text{ch}2\gamma)^2 - (\text{sh}2\sigma \pm \text{sh}2\gamma)^2 \right]$$

$$= \frac{\alpha'_{\text{eff}}}{l_p^2} \left[ \frac{VRR_5}{4l_p^2 r_0^2} \right]^2 4\text{ch}^2(\sigma \mp \gamma).$$  (3.4)

The entropy is now evaluated as

$$S_{\text{BH}} = 2\pi \left[ \sqrt{\frac{1}{2} c_{\text{eff}}n_L} + \sqrt{\frac{1}{2} c_{\text{eff}}n_R} \right]$$

$$= 2\pi \left[ \frac{\alpha'_{\text{eff}} VRS_5 R^2}{T_p l_p^2} \right]^{1/2} (\sqrt{N_2} + \sqrt{\bar{N}_2})(\sqrt{N_5} + \sqrt{\bar{N}_5}).$$  (3.5)

One must have $\alpha'_{\text{eff}} = N l_p^9 / VRS R^2$ to match the entropy. Naturally, the energy per unit length of an instanton string in 5 + 1 gauge theory is

$$T_\text{eff} = \frac{4\pi^2}{g_{\text{YM}}^2 N} = \frac{VRS R^2}{2\pi N l_p^9}$$  (3.6)

(the $1/N$ arises from the charge fractionalization mentioned above). Then with the standard relation $T_\text{eff} = (2\pi \alpha'_{\text{eff}})^{-1}$, the Hagedorn gas of instanton strings precisely accounts for the Bekenstein–Hawking entropy. The combination $g_{\text{YM}}^2 N$ appearing in (3.6) suggests that conventional large-$N$ techniques might be useful for the study of the instanton string gas. It is important to note that the factors in the entropy cannot be ascribed to particular
branes/antibranes; everything gets mixed up in the ‘plasma’ of light excitations, as we see from figure 1. Another important feature is that the tension (3.6) is finite in the limit \(N, R \to \infty, N/R^2\) fixed that characterizes the large \(N\) limit with fixed longitudinal momentum density and fixed entropy per unit length.

It would be interesting to calculate the temperature \(\beta_H^{-1}\) of Hawking radiation from the present viewpoint; one has

\[
\beta_H = \frac{\pi}{a} \sqrt{N(\sqrt{N_2} + \sqrt{N_2})(\sqrt{N_3} + \sqrt{N_3})}
\]

(3.7)

\[
l_p a = \frac{2}{l_p} \left( \frac{R S}{l_p} \right)^{1/2} = \frac{2}{l_p} \left( \frac{R V}{l_p} \right)^{1/2} \left( \frac{N_3 N_3}{N_3} \right)^{1/2}.
\]

Note that the ‘invariant mass’ \(2a = T_H S_{BH}\) would appear to be the difference between the energy and the free energy. One can compare this temperature with that of the 1D gas of excitations on the string:

\[
\beta_{1d} = \frac{\pi}{2a} \left[ \frac{\text{ch} \gamma \text{ch} \sigma}{\text{ch} 2\gamma + \text{ch} 2\sigma} \right].
\]

(3.8)

Using the relation \(\beta_H = 2\pi (\alpha'_\text{eff})^{1/2} \text{ch} \gamma \text{ch} \sigma\), which may be deduced from (3.7), one finds

\[
\beta_H = \beta_{1d} [4(\alpha'_\text{eff})^{1/2} E_{\text{LIC}}].
\]

(3.9)

This reduces [11] to the relation \(\beta_H = \beta_{\text{Hagedorn}}\) in the near-extremal limit.

Another property of the black hole that we can motivate from the instanton string picture is the relation (2.8) between the invariant masses of 2-branes and 5-branes. If in equation (3.4) we introduce adjustable parameters for the amount of momentum and winding, \(P = a \sinh 2\sigma, W = b \sinh 2\gamma, E_{\text{LIC}} = (a \cosh 2\sigma + b \cosh 2\gamma)\), then one has

\[
E_{L,R} = [E_{\text{LIC}}^2 - (P \pm W)^2] = (a^2 + b^2 + 2ab \cosh[2(\sigma \mp \gamma)]).
\]

(3.10)

The entropy (3.5) turns out to depend only on the ratio \(t = a/b\); a variation of this parameter gives

\[
\delta S = \pi \left( \frac{1}{\sqrt{N_L}} + \frac{1}{\sqrt{N_R}} \right) N N_2 N_3 e^{-2\sigma - 2\gamma} (1 - 1/t^2) \delta t,
\]

(3.11)

which determines the stationary point \(t = 1\). This is just the balancing formula

\[
R S l_p^{-2} \sqrt{N_2 N_2} = R V l_p^{-5} \sqrt{N_3 N_3}.
\]

(3.12)

However, there is a puzzle: the second derivative \(S''(t = 1) > 0\), suggesting the system is unstable. We are confused about the interpretation of this fact.

Let us pause now to compare the derivation of \(S_{\text{BH}}\) given here with that in [10]. In [10], the \(x_5\) circle is shrunk to a sub-Planckian size in order to recover a matrix description of a regime where one can make a comparison with string perturbation theory. The 1+1D field theory on the dual circle gives a matrix description of IIA strings bound to NS5-branes. The infrared description of this field theory is an \(N = (4, 4)\) supersymmetric sigma model on a target space

\[
S_{\text{BH}}^{N_0 N_4 T^4},
\]

(3.13)

and the entropy is effectively the density of states of this sigma model at level \(N_2\), \(S_{\text{BH}} = \sqrt{N_0 N_4} (\sqrt{N_2} + \sqrt{N_2})\). The interactions of the matrix strings are marginal operators in this effective field theory, but this will not affect the central charge and hence the leading term in the entropy. One is performing a rather similar calculation to the one above which resulted in equation (3.5). However, the 1+1 field theory in [10] is restricted to a kind
of static gauge description of the strings, where the spatial direction along the string is identified with $x_5$. Our approach is somewhat more covariant in this respect. This is necessary, since in the Hagedorn regime describing ‘fat’ black holes (those where all the brane numbers (2.6) are macroscopic), the string wanders ergodically, with many overhangs so that the static gauge breaks down. From (2.8) one can see that if one takes $R_5$ small and $V$ large, then one can have large numbers of (anti-)2-branes at minimal cost; however (anti-)5-branes are expensive, therefore they will annihilate as far as possible. In other words, we have recovered the picture of [10], where there are only the instanton strings winding in one direction (the large dual circle to $x_5$ with radius $\sim R_5^{-1}$), with a gas of momentum modes travelling in both directions. A second possibility (realizing part of the $U$-duality of the entropy) involves taking the vanishing size of the transverse $T^4$ of volume $V$ on which the 5-branes are compactified, while making $R_5$ large. Then the situation is reversed: the (anti-)5-branes have low cost, while the (anti-)2-branes annihilate as far as possible. We imagine that other limits might be possible by exploiting more of the duality group, cf [18–21].

The central charge and hence the leading term in the entropy is extremely robust. We are dealing with the lightest excitations of a non-BPS state in a maximally supersymmetric gauge theory. These light excitations are described by an action with $N = (4, 4)$ two-dimensional (2D) supersymmetry, and so the entropy is protected by nonrenormalization theorems for the central charge of this theory (in fact the central charge is quantized for this much supersymmetry). The effective string tension, which governs the relation between spacetime and worldsheet energies, is determined by the $F^2$ term in the gauge theory and is also not renormalized. This feature opens up exciting new possibilities for solving problems such as the cosmological constant. The vacuum we live in is a nonsupersymmetric state in a supersymmetric theory; if the effective matrix dynamics is governed by a supersymmetric theory, the cosmological constant could be protected even though in most other respects there is no supersymmetry.

One possible cause for concern in the analysis leading to (3.5) is the effect of string interactions, which we have been ignoring. However, their chief influence is to change the connectivity of the Hagedorn string locally, without altering its position (i.e. the string occupies the same 1D locus immediately before and after interaction). Thus, if anything, the string merely explores its phase space more rapidly and efficiently; the phase-space distribution and hence the entropy should not be significantly altered. Indeed, in the calculation of [10] it was proposed that the main effect was simply to shift the target space background (3.13) by resolving its orbifold singularities, leaving the entropy unchanged.

Until now, we have not discussed the range of validity of our calculation, for instance how far from extremality it can be trusted. One limitation might come from the UV behaviour of the $5 + 1$ Yang–Mills theory, which is not well understood (for recent discussions in the context of matrix theory, see [18–21]). In the present situation, although the energy density is large, this is achieved by macroscopically populating the longest wavelength modes of the system (rather than putting the system at finite temperature, for instance, whereby high momentum modes would be excited). Therefore we need not worry about short-distance effects. One needs to work in a regime where there are enough charges $N_2, N_\bar{2}, N_5, N_\bar{5}$ so that one is in the asymptotic region of the level density of the instanton string. From (2.8), one sees that $g_{YM}^2 N$ becomes large unless either $R_5$ or $V$ is small, corresponding to the perturbative regime accessible to D-brane technology. Thus, the difficulty with ‘fat’ black holes is not that the Yang–Mills theory is dominated by ultraviolet behaviour, but rather that it becomes strongly coupled in a different way: $g_{YM} \to 0, N \to \infty$, but $g_{YM}^2 N \to \infty$. In this limit, the tension of the instanton string goes to zero.
The remarkable fact about the entropy formula (3.5) is that it seems to be universally valid, suggesting that the density of states is always controlled by a string theory. At high density, any piece of instanton string finds itself close to many other pieces. Now recall that the instanton string is the matrix theory representation of the wrapped 5-brane; wrapped 5-branes approaching one another generate ‘tensionless’ strings [22, 23] (see [24] for the large $N$ limit). Could it be that ‘tensionless’ here refers merely to the effective tension of a string in its Hagedorn phase? In any event, we would like to interpret the black-hole entropy (3.5) as informing us that there is a string theory controlling the properties of large $N$ $5+1$ super Yang–Mills theory in the regime appropriate to large black holes, and that (3.5) is its nonperturbative density of states. A rigorous demonstration of this fact would show us how string theory accommodates large black holes as quantum states. Remarkably, questions about the nature of these objects have been transformed in the infinite momentum frame into issues of the renormalization group in field theory.

4. Discussion

The entropy counting confirms the picture of [9], wherein the black hole was viewed as a plasma of excitations in the super Yang–Mills theory. The special property of the 5D black holes considered here is that one can identify the light excitations of the $5+1$ super Yang–Mills plasma with the collective modes of a string. It should be possible to count the entropies of other M-theory black holes (for a survey, see [13]), provided we can identify the light excitations of the associated Yang–Mills theory in states carrying the relevant charges. In order to embed such black holes in matrix theory, one wants one of the charges carried by the hole to be momentum, so that this can be identified with 0-brane charge.

This was seen to be possible for the 4D black hole with three charges in [9]; black holes in $6 \leq d \leq 9$ can be realized as boosted 2-branes [13], and hence can also be realized in matrix theory. Thus, black holes in six dimensions and above might be understood in a similar fashion, by identifying the relevant soft modes of the Yang–Mills plasma. The difference with the 4D and 5D cases is that, in $6 \leq d \leq 9$, the entropy vanishes as one approaches extremality, as a power of $r_0$. From formulae analogous to (2.7) and (2.8), the entropy will have to have a power of antibrane charge in it. Looking at the ADM mass [13]

$$E_{\text{ADM}} \propto r_0^{D-3} \left[ \frac{2}{D-3} + \cosh 2\sigma + \cosh 2\alpha \right],$$

it would appear that there is a (fraction of a) constituent for which an appropriate charge cannot be turned on, or perhaps the constituents have a nontrivial interaction energy. It would, of course, be interesting to understand the specific form of the soft modes of the plasma that gives the required scaling properties of entropy as a function of mass.

The 4D case is a collection of intersecting 5-branes carrying momentum on their intersection strings. In the $6+1$ Yang–Mills theory on the dual torus, the instantons describing the longitudinal 5-branes are membrane-like objects extended over the orthogonal dual coordinates. It would be interesting to see if one can extract the entropy from the dynamics of a single space-filling (Hagedorn-like) membrane carrying the wrapping charges along three mutually orthogonal planes. Hanany and Klebanov [26] (see also [27]) suggested that the properties of intersecting 5-branes are related to a noncritical string living in the three-dimensional (3D) intersection. Here one of these three dimensions is $x_{11}$, so in the IMF one is looking for a string inside a 2-brane. Perhaps this is the effective object which controls the density of states. Note also that it is not properly understood how to compactify
$M$-theory in this situation \cite{18}. Turning this around, we might use the properties of black holes to gain a window into the relevant high-dimensional field theories.

It is important that the excitations of the system are of long wavelength, and have an energy scale which is suppressed by $1/N$ \cite{25, 18, 28–30}. This property is responsible for recovering perturbative string theory and its exponential density of states in the large $N$ limit, when a circle is shrunk to sub-Planckian size. It is the collective motions of the various objects (instantons, torons, etc) that dominates the entropy of black holes. In order to account for the entropy of fat black holes, the UV (high-frequency) properties of the system should never become important, or else the system could dissolve into the dynamics of field theory. It would be a disaster if that did happen, as one would not be able to account for the exponential density of states of fat black holes.

One can now see the outline of a calculation of scattering from a large black hole, once we understand enough about the $5+1$ supersymmetric field theory. The matrices describing the black-hole/probe system are initially on the Coulomb branch, having the approximate structure

$$
\begin{bmatrix}
\text{BH} & 0 \\
0 & \text{probe}.
\end{bmatrix}
$$

(4.1)

The energy of the probe is in the kinetic energy of the Higgs field, the lower right block of the matrix. As the clump of matter comes in, the difference in the scalar vev’s goes to zero; the energy is absorbed by the instanton string gas, being deposited in gauge field modes over the whole matrix. In the tails of the probability distribution, it may occur that a fluctuation has the off-diagonal fields almost vanishing in some row(s) and column(s), while the diagonal entries of the scalars have some kinetic energy; a block of the matrix can then re-emerge onto the Coulomb branch—in other words, a Hawking particle escapes.

There may be some useful analogies to exploit between the present kinematical set-up (black string stretched along the longitudinal $x^+-x^-$ plane) and the parton picture of QCD. We have seen that the black-hole entropy is carried by the ‘instanton string’ representing the effects of 2-brane and 5-brane constituents. These are like the valence quarks that carry the quantum numbers of the system (they carry the entropy and all gauge charges, including the angular momentum \cite{11}). The 0-branes play the role of ‘wee partons’ \cite{1}. The Hawking radiation spectrum consists of such wee partons, since it is dominated by the emission of massless particles carrying small longitudinal momentum. Indeed, calculations of the emission of charged scalars \cite{31, 8} (here charge is momentum along $x_{11}$) confirm that the radiation is peaked near zero longitudinal momentum. Thus, the Hawking radiation spectrum can be regarded as a prediction of the distribution of wee partons in the black hole, as a function of longitudinal momentum fraction. It would be very interesting to verify this distribution function by a calculation in the present context.

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