Efficient parallel analysis of transient plane wave scattering from nonplanar doubly periodic structures

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Abstract. This paper presents a fast scheme for solving a time-domain electric field integral equation pertinent to the analysis of transient plane wave scattering from nonplanar doubly periodic structures. The proposed scheme efficiently evaluates scattered fields from periodic structures by decomposing the sources that produce them into present and past components. The fields radiated by present sources are evaluated classically. In contrast, fields generated by past sources are expanded into time domain Floquet waves with modal amplitudes that are evolved in both space and time using a spectral time-stepping scheme complemented by Huygens’ principle based boundary conditions. The kernel equations and efficient parallel evaluation associated with the scattered fields generated by past sources are validated and effectiveness of the resulting algorithm are demonstrated by a number of examples.

1. Introduction
In the past, transient plane wave scattering from doubly periodic structures has been analyzed predominately using finite difference methods [1-3]. Classically formulated finite difference schemes however cannot be applied directly to the analysis of scattering from obliquely illuminated periodic structures as they require future field values to update present ones. While several avenues for resolving this problem have been proposed [4-6], most are hard to implement and/or somewhat limited in scope. The integral equation based solvers for analyzing transient plane wave scattering form doubly periodic perfectly electrically conducting (PEC) free-standing [7] or substrate imprinted PEC elements [8] have been proposed in the past. The solvers [7][8] evaluate so-called scattered fields produced by transient periodic current constellations by using free-space Green functions, and the evaluation of the fields on/in the scatterer has to be carried out for each time step in the analysis. Hence, the solvers are rendered computationally expensive and inapplicable to the analysis of real-world structures. Indeed, it is shown that their computational complexity scales as $O(N_s^2N_t^3)$ where $N_s$ denotes the number of spatial unknowns associated with currents in one of the structure’s cells – also termed the reference or mother-cell – and $N_t$ is the number of time steps in the analysis.

To allievate this computational burden, the periodic structure solver [9] was presented and the burden was alleviated by expanding scattered fields into time domain Floquet waves (TDFWs) [10] but is limited to discretely planar structures, viz. structures comprising of a finite number of planar screens. This solver exploits the fact that fields produced by periodic and transient but quiescent current constellations can be expanded in terms of a finite number of propagating TDFWs in order to rapidly evaluate scattered fields by splitting them into two components. First, there are the fields radiated by
the sources in the mothercell and its immediate vicinity; these sources are termed present, and their fields are evaluated classically. Second, there are the fields generated by sources that are spatially removed from the mothercell; these sources are termed past, and their fields are expanded in TDFWs. Unfortunately, when sources and observers reside on different planes, the TDFW propagator no longer is time shift invariant; this fact in itself renders this technology inapplicable to the analysis of nonplanar structures. Here, to render TDFW-based TDIE solvers applicable to the analysis of transient plane wave scattering from nonplanar periodic structures, a new scheme for tracking TDFW amplitudes is proposed. The scheme evolves TDFW modal amplitudes in (one-dimensional) space and time using a spectral time-stepping scheme supplemented with Huygens’ principle based boundary conditions. Because the scheme in no way requires the TDFW propagator to be time shift invariant, it applies without difficulty to the analysis of nonplanar doubly periodic structures consisting of PEC elements and sculptured dielectric volumes, including that of PEC elements imprinted on nonplanar substrates. Furthermore, the TDFW modal amplitudes can be parallelly evaluated.

This paper is organized as follows. In Section 2, the development of the proposed Floquet-wave-based solver for doubly periodic structures described is described. Numerical results that demonstrate the capabilities and accuracy of the proposed method are presented in Section 3. Finally, Section 4 draws the conclusions.

2. Problem statement and formulation

Consider a doubly periodic structure consisting of PEC surfaces $S$ and penetrable volumes $V$. These volumes comprise isotropic though potentially inhomogeneous dielectrics with frequency independent permittivity $\varepsilon(r)$ (figure 1). The periodic arrangement $S \cup V$ resides in free-space with permittivity $\varepsilon_0$. Materials occupying $V$ are assumed nonmagnetic, i.e., of free space permeability $\mu_0$. As illustrated in figure 1, the surfaces $S$ and penetrable volumes $V$ are decomposed as $S = \bigcup_{m,n=-\infty}^{+\infty} S_{mn}$ and $V = \bigcup_{m,n=-\infty}^{+\infty} V_{mn}$, with $S_{mn}$ and $V_{mn}$, $m, n = -\infty, +\infty$, termed sub-surfaces and –volumes residing in cell $(m, n)$ anchored to vector $r_{mn} = mD_x \hat{x} + nD_y \hat{y}$, where $D_x$ and $D_y$ denote unit cell dimensions along $x$ and $y$ directions, respectively; $S_{00}$ and $V_{00}$ are said to reside in the mother cell. The structure is illuminated by a bandlimited plane wave pulse propagating along direction $\hat{k}^{inc} = -\sin \theta^{inc} \cos \phi^{inc} \hat{x} - \sin \phi^{inc} \hat{y} - \cos \theta^{inc} \hat{z}$ with electric field $E^{inc}(r, t, \hat{p}^{inc}) = \hat{p}^{inc} f(t - \hat{k}^{inc} \cdot r / c)$ where $c$ is the speed of light in free-space and $\hat{p}^{inc}$ denotes the incident field’s polarization. It is assumed that $E^{inc}(r, t, \hat{p}^{inc})$ is bandlimited to $\omega = \omega_{max}$ and vanishingly small on $S_{00}$ and $V_{00}$ for $t < 0$.

In response to the above excitation, surface currents $J^s_{mn}(r, t) = J^s_{00}(r - r_{mn}^{inc} - \hat{k}^{inc} \cdot r_{mn}^{inc} / c)$ flow on $S_{mn}$ and volumetric polarization currents $J^v_{mn}(r, t) = J^v_{00}(r - r_{mn}^{inc} - \hat{k}^{inc} \cdot r_{mn}^{inc} / c)$ arise in $V_{mn}$. Note that $J^0_{00}(r, t) = \kappa(r) \frac{\partial}{\partial t} D^{\varepsilon}_{00}(r, t)$, where $\kappa(r) = (\varepsilon(r) - \varepsilon_0) / \varepsilon(r)$ denotes the contrast ratio.

Figure 1. Illustration of the doubly periodic structure.
As detailed in [10], enforcing the (temporal derivative of the) total electric field tangential to $S_{00}$ to vanish and imposing a consistency condition on the total electric field within $V_{00}$ yields the coupled electric field integral equations for $J_{00}(r,t)$ and $D_{00}(r,t)$. The scattered fields from $J_{00}(r,t)$ and $D_{00}(r,t)$ are expressed as

$$\frac{\partial}{\partial t} E^{inc}(r,t) = \frac{-\mu_0}{4\pi c^2} \int_{S_{00}} ds' J_{00}(r',t) \star G(r,r',t) + \frac{1}{4\pi e_0} \int_{V_{00}} dv' \left[ \nabla' \cdot J_{00}(r',t) \right] \star G(r,r',t),$$

and

$$\frac{\partial}{\partial t} E^{inc}(r,t) = -\frac{\mu_0}{4\pi c^2} \int_{V_{00}} dv' \kappa(r') \frac{\partial D_{00}(r',t)}{\partial t} \star G(r,r',t) + \frac{1}{4\pi e_0} \int_{V_{00}} dv' \left[ \nabla' \cdot \kappa(r') \frac{\partial D_{00}(r',t)}{\partial t} \right] \star G(r,r',t),$$

respectively. That is, the evaluations of the scattered field associated with $J_{00}(r,t)$ and $D_{00}(r,t)$ rely on the following Green’s function for the doubly periodic structure,

$$G(r,r',t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(t - \frac{v_{inc} c}{c} \frac{r_{mn}}{c} - |r' + r_{mn} - r|/c) \frac{1}{|r' + r_{mn} - r|}.$$  

The coupled electric field equations are solved with a marching-on-in-time procedure via approximating $S_{00}$ and $V_{00}$ with planar triangular facets and tetrahedral volumes, respectively. Also, Rao-Wilton-Glisson functions [11] and their volumetric extensions proposed by Schaubert et al. [12] are chosen to represent the spatial variations of $J_{00}(r,t)$ and $D_{00}(r,t)$. And, the approximate prolate spheroidal wave functions [13] are adopted to represent the temporal variations of $J_{00}(r,t)$ and $D_{00}(r,t)$. Unfortunately, the double summation in (3) leads to a computational burden. However, this burden can be drastically relieved through the introduction of the concept of TDFWs and the use of a spectral stepping scheme complemented by Huygen’s principle based boundary conditions. In essence, the Green’s function can be expanded in terms of TDFWs [10] as

$$G(r,r',t) = \sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} \beta_{pq} \mathcal{R}e\left[ A_{pq}^{FW}(r,r',t) \right]$$

where $\beta_{pq} = 1$ for $p = 0$ and $\beta_{pq} = 2$ for $p \neq 0$, $\mathcal{R}e[.]$ selects the real part,

$$A_{pq}^{FW}(r,r',t) = \frac{e^{-j\lambda_{pq}^2(p - p')}}{2D_x D_y \sqrt{1 - \eta^2}} e^{j\eta_0^2} J_0[\eta_0 \sqrt{r^2 - r_0^2}] U(t - t_0),$$

$$r = \rho + z \hat{z}, \quad \hat{\alpha}_{pq} = \frac{\eta c}{1 - \eta^2} \hat{u}_1 \cdot \alpha_{pq}$$

$$r' = \rho' + z' \hat{z}, \quad \hat{\alpha}^2_{pq} = \frac{\alpha^2_{pq} + \alpha_{pq}^2 c^2 / (1 - \eta^2)}{c}$$

$$\eta = \sin \theta_{inc}, \quad r_0 = \sqrt{1 - \eta^2 (z - z')} / c$$

$$\alpha_{pq} = 2\pi p / D_x \hat{x} + 2\pi q / D_y \hat{y}, \quad \tau = t - t_0$$

$$\alpha_{pq}^2 = (2\pi p / D_x)^2 + (2\pi q / D_y)^2$$

$$\hat{u}_1 = \cos \phi_{inc} \hat{x} + \sin \phi_{inc} \hat{y}$$
\[ j = \sqrt{-1}, \quad J_0(\cdot) \text{ denotes the zeroth order Bessel function of the first kind and } U(\cdot) \text{ is the Heaviside step function.} \]

Although the summation in (4) comprises an infinite number of terms, upon convolution of the Green function with a bandlimited and essentially time limited signal, only finite number of so-called propagating modes should be retained provided that the field is observed no earlier than \(-(r - r') \cdot \hat{k}_0^m / c \) seconds after the signal (essentially) vanishes in the temporal dimension. Hence, the fields are evaluated via a space-time convolution of the Green function and the current sources on/in the mothercell. More specific, the fields at a given time can be expressed in terms of instantaneous and delayed field, which are respectively attributed to the present and past currents on the mothercell. The evaluation of the instantaneous fields is carried out classically with (1), while the delayed fields are computed by expanding them into a finite number of TDFWs. In essence, the fast scheme bears significant similarity to the plane wave time domain algorithm [14]. It appears that fields generated by quiescent and bandlimited periodic currents can be represented in terms of a few propagating TDFWs. A four-stage scheme is developed to evolve the amplitude of each propagating TDFW. The TDFWs are propagated like plane waves in the plane of the periodicity and are easy to manipulate. However in the direction perpendicular to the periodicity, the TDFWs are not time-shift invariant anymore. A spectral time-stepping algorithm supplemented with Huygens’ principle based boundary conditions is designed to efficiently track the TDFW modal amplitudes in the vertical direction. On the other hand, the computation of the delayed fields can be further accelerated with parallel computing owing to the TDFW modal nature.

3. Numerical results
The proposed scheme is exploited to analyze transient scattering from the following doubly periodic structures. The results obtained were compared against data from a periodic frequency domain method of moments code (P-MOM) following Fourier transformation of the time-domain currents. All the examples presented below are illuminated by a modulated Gaussian pulse parameterized by

\[ f(t) = \cos \left[ 2\pi f_c (t - t_p) \right] \exp \left[ -\left( t - t_p \right)^2 / (2\sigma^2) \right] \]

where \( f_c \) is center frequency of the incident wave, \( \sigma = 3 / (2\pi f_{bw}) \) and \( t_p = 6\sigma \) with \( f_{bw} \) termed the “bandwidth” of the signal.

![Figure 2. Power transmission coefficient plot of the doubly periodic dipole structure.](image)

The first example analyzed is the periodically imprinted dipole elements shown in the inset of figure 2. The dielectric slab has a relative permittivity \( \varepsilon_r = 3.0 \) and thickness 1 mm. The side length of the square mother cell is 1.78 cm. The PEC dipole is of dimension 1.025 cm by 0.91 mm. The structure
is discretized in terms of 2656 spatial unknowns. The incident field has $\hat{p}^{inc} = \hat{y}$, $f_i = 6$ GHz, and $f_{bw} = 6$ GHz. The time step is $\Delta t = 4.167$ ps and 512 time steps are used in the analysis. The number of Floquet modes used for field expansion is 81. The power transmission coefficient of the structure obtained via the Floquet-wave-based scheme and P-MOM scheme are compared in figure 2. The good agreement between the two data sets is observed.

The second example analyzed is the periodically imprinted dual-loop elements. The dielectric slab has a relative permittivity $\varepsilon_r = 3.0$ and thickness 0.6 cm. The side length of the mother cell is 6 cm and the dimension of the dual-loop is shown in the inset of figure 3. The structure is discretized in terms of 2398 spatial unknowns. The incident field has $\hat{p}^{inc} = \hat{x}$, $f_i = 1.7$ GHz and $f_{bw} = 1.7$ GHz. The time step is $\Delta t = 14.706$ ps and 1024 time steps are used. The number of Floquet modes used for field expansion is 73. The power transmission coefficient of the structure obtained using the proposed scheme and P-MOM scheme are compared in figure 3. The results are in good agreement.

![Figure 3](image1.png)

**Figure 3.** Power transmission coefficient plot of the doubly periodic dual-loop structure.

![Figure 4](image2.png)

**Figure 4.** Power transmission coefficient plot of the grooved doubly periodic square-loop structure.

The next example analyzed is the periodically arranged square-loop elements imprinted on a grooved dielectric slab. The grooved dielectric slab has a relative permittivity $\varepsilon_r = 3.0$. The thicknesses are 0.7 cm and 0.35 cm for the un-grooved and grooved sections, respectively. The side length of the square mother cell is 7 cm. The dimension of the square-loop is shown in the inset of figure 4. The geometry
is discretized in terms of 1908 spatial unknowns. The incident field has $\hat{\mathbf{p}}^{\text{inc}} = \hat{x}$, $f_c = 1.2$ GHz and $f_{bw} = 1.2$ GHz. The time step is $\Delta t = 20.83$ ps and 1024 time steps are used. The number of Floquet modes used for field expansion is 49. The power transmission coefficient of the structure obtained via the proposed scheme and P-MOM scheme are compared in figure 4. Again, the good agreement between the two data sets is observed.

4. Conclusions
An improved Floquet-wave-based scheme for analyzing transient scattering from nonplanar doubly periodic structures is described. It relies on evolving individual TDFWs using a spectral time-stepping scheme supplemented with Huygens’ principle based boundary conditions. Also, the modal field evaluations are accelerated by parallel computing. The scheme applies without difficulty to nonplanar doubly periodic structures comprising PEC and sculptured dielectric substrates. The efficiency and accuracy of the proposed scheme have been demonstrated via comparison against frequency domain results. Detailed formulations together with the implementations on the acceleration will be presented at the conference.

5. References
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