Universality of the Collins-Soper-Sterman
nonperturbative function
in vector boson production

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Abstract

We revise the $b_\ast$ model for the Collins-Soper-Sterman resummed form factor to improve description of the leading-power contribution at nearly nonperturbative impact parameters. This revision leads to excellent agreement of the transverse momentum resummation with the data in a global analysis of Drell-Yan lepton pair and $Z$ boson production. The nonperturbative contributions are found to follow universal quasi-linear dependence on the logarithm of the heavy boson invariant mass, which closely agrees with an estimate from the infrared renormalon analysis.

Transverse momentum distributions of heavy Drell-Yan lepton pairs, $W$, or $Z$ bosons produced in hadron-hadron collisions present an interesting example of factorization for multi-scale observables. If the transverse momentum $q_T$ of the electroweak boson is much smaller than its invariant mass $Q$, $d\sigma/dq_T$ at an n-th order of perturbation theory includes large contributions of the type $\alpha_s^m\ln^m(q_T^2/Q^2)/q_T^2$ ($m = 0, 1 \ldots 2n - 1$), which must be summed through all
orders of $\alpha_s$ to reliably predict the cross section [1]. The feasibility of all-order resummation is proved by a factorization theorem, first formulated for $e^+e^-$ hadroproduction [2,3], stated by Collins, Soper, and Sterman (CSS) for the Drell-Yan process [4], and recently proved by detailed investigation of gauge transformations of $k_T$-dependent parton densities [5,6].

The heavy bosons acquire non-zero $q_T$ mostly by recoiling against QCD radiation. The CSS formalism accounts for both the short- and long-wavelength QCD radiation by means of a Fourier-Bessel transform of a resummed form factor $\tilde{W}(b)$ introduced in impact parameter ($b$) space. The perturbative contribution, characterized by $b \lesssim 0.5$ GeV$^{-1}$, dominates in $W$ and $Z$ boson production at all values of $q_T$. The nonperturbative contribution from $b \gtrsim 0.5$ GeV$^{-1}$ is not negligible at $q_T < 20$ GeV in the precision measurements of the $W$ boson mass $M_W$ at the Tevatron and LHC [7]. The model for the nonperturbative recoil is the major source of theoretical uncertainty in the extraction of $M_W$ from the experimental data. This uncertainty must be reduced in order to measure $M_W$ with accuracy of about 30 MeV in the Tevatron Run-2 and 15 MeV at the LHC. The nonperturbative model presented below approaches the level of accuracy desired in these measurements.

The nonperturbative component [described by the function $\mathcal{F}_{NP}(b, Q)$ given in Eq. (4)] can be constrained in a few experiments by exploiting process-independence, or universality, of $\mathcal{F}_{NP}(b, Q)$, just as the universal $k_T$-integrated parton densities are constrained with the help of inclusive scattering data. The universality of $\mathcal{F}_{NP}(b, Q)$ in unpolarized Drell-Yan-like processes and semi-inclusive deep-inelastic scattering (SIDIS) follows from the CSS factorization theorem [5]. In the study presented here, we carefully investigate agreement of the universality assumption with the data in a global analysis of fixed-target
Drell-Yan pair and Tevatron $Z$ boson production. We revise the nonperturba-
tive model used in the previous studies [8,9] and improve agreement with the
data without introducing additional free parameters. Renormalization-group
invariance requires $F_{NP}(b, Q)$ to depend linearly on $\ln Q$ [3,4]. With our lat-
est revisions put in place, the global $q_T$ fit clearly prefers a simple function
$F_{NP}(b, Q)$ with universal $\ln Q$ dependence. The new $F_{NP}(b, Q)$ has reduced
dependence on the collision energy $\sqrt{S}$ comparatively to the earlier fits. The
slope of the $\ln Q$ dependence found in the new fit agrees numerically with its
estimate made with methods of infrared renormalon analysis [10,11].

The function $F_{NP}(b, Q)$ primarily parametrizes the “power-suppressed” terms,
i.e., terms proportional to positive powers of $b$. When assessed in a fit, $F_{NP}(b, Q)$
also contains admixture of the leading-power terms (logarithmic in $b$ terms),
which were not properly included in the approximate leading-power function
$\tilde{W}_{LP}(b)$ [cf. Eq. (4)]. In contrast, estimates of $F_{NP}(b, Q)$ made in the infrared
renormalon analysis explicitly remove all leading-power contributions from
$F_{NP}(b, Q)$ [11]. While the recent studies [9,10,11,12,13] point to an approx-
imately Gaussian form of $F_{NP}(b, Q)$ [$F_{NP}(b, Q) \propto b^2$], they disagree on the
magnitude of $F_{NP}(b, Q)$ and its $Q$ dependence. The source of these differences
can be traced to the varying assumptions about the form of the leading-power
function $\tilde{W}_{LP}(b)$ at $b < 2$ GeV$^{-1}$, which is correlated in the fit with $F_{NP}(b, Q)$.
The exact behavior of $\tilde{W}(b)$ at $b > 2$ GeV$^{-1}$ is of reduced importance, as
$\tilde{W}(b)$ is strongly suppressed at such $b$. The new improvements described here
(excellent agreement of $F_{NP}(b, Q)$ with the data and renormalon analysis)
result from modifications in the model for $\tilde{W}_{LP}(b)$ at $b < 2$ GeV$^{-1}$. The im-
provements are preserved under variations of the large-$b$ form of $\tilde{W}_{LP}(b)$ in a
significant range of the model parameters.
Our paper follows the notations in Ref. [9]. The form factor $\tilde{W}(b)$ factorizes at all $b$ as [2,3,4]

$$\tilde{W}(b) = \sum_{j=q,\bar{q}} \frac{\sigma_j^{(0)}}{S} e^{-S(b,Q)} \mathcal{P}_j(x_1, b) \mathcal{P}_j(x_2, b),$$

(1)

where $\sigma_j^{(0)}/S$ is a constant prefactor [4], and $x_{1,2} \equiv e^{\pm y} Q/\sqrt{S}$ are the Born-level momentum fractions, with $y$ being the rapidity of the vector boson. The $b$-dependent parton densities $\mathcal{P}_j(x, b)$ and Sudakov function

$$S(b, Q) \equiv \int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ A(\alpha_s(\bar{\mu})) \ln \left( \frac{Q^2}{\mu^2} \right) + B(\alpha_s(\bar{\mu})) \right]$$

(2)

are universal in Drell-Yan-like processes and SIDIS [5]. When the momentum scales $Q$ and $b_0/b$ (where $b_0 \equiv 2e^{-\gamma_E} \approx 1.123$ is a dimensionless constant) are much larger than 1 GeV, $\tilde{W}(b)$ reduces to its perturbative part $\tilde{W}_{\text{pert}}(b)$, i.e., its leading-power (logarithmic in $b$) part evaluated at a finite order of $\alpha_s$:

$$\tilde{W}(b) \bigg|_{Q, b_0/b \gg 1 \text{GeV}} \approx \tilde{W}_{\text{pert}}(b) \equiv \sum_{j=q,\bar{q}} \frac{\sigma_j^{(0)}}{S} e^{-S_F(b,Q)} \times \left[ \mathcal{C} \otimes f \right]_j (x_1, b; \mu_F) \left[ \mathcal{C} \otimes f \right]_j (x_2, b; \mu_F).$$

(3)

Here $S_F(b, Q)$ and $[\mathcal{C} \otimes f]_j (x, b; \mu_F) \equiv \sum_a \int_x^1 d\xi/\xi \mathcal{C}_{ja}(x/\xi, \mu_F) f_a(\xi, \mu_F)$ are the finite-order approximations to the leading-power parts of $S(b, Q)$ and $\mathcal{P}_j(x, b)$. $f_a(x, \mu_F)$ is the $k_T$-integrated parton density, computed in our study by using the CTEQ6M parameterization [14]. $C_{ja}(x, \mu_F)$ is the Wilson coefficient function. We compute $S_F(b, Q)$ up to $O(\alpha_s^2)$ and $C_{ja}$ up to $O(\alpha_s)$.

In $Z$ boson production, the maximum of $b\tilde{W}(b)$ is located at $b \approx 0.25$ GeV$^{-1}$, and $\tilde{W}_{\text{pert}}(b)$ dominates the Fourier-Bessel integral. In the examined low-$Q$ region, the maximum of $b\tilde{W}(b)$ is located at $b \approx 1$ GeV$^{-1}$, where higher-order
corrections in powers of $\alpha_s$ and $b$ must be considered. We reorganize Eq. (1) to separate the leading-power (LP) term $\tilde{W}_{LP}(b)$, given by the model-dependent continuation of $\tilde{W}_{pert}(b)$ to $b \gtrsim 1 \text{ GeV}^{-1}$, and the nonperturbative exponent $e^{-\mathcal{F}_{NP}(b,Q)}$, which absorbs the power-suppressed terms:

$$\tilde{W}(b) = \tilde{W}_{LP}(b) e^{-\mathcal{F}_{NP}(b,Q)}.$$  

(4)

At $b \to 0$, the perturbative approximation for $\tilde{W}(b)$ is restored: $\tilde{W}_{LP} \to \tilde{W}_{pert}$, $\mathcal{F}_{NP} \to 0$. The power-suppressed contributions are proportional to even powers of $b$ [10]. Detailed expressions for some power-suppressed terms are given in Ref. [11]. At impact parameters of order $1 \text{ GeV}^{-1}$, we keep only the first power-suppressed contribution proportional to $b^2$:

$$\mathcal{F}_{NP} \approx b^2 (a_1 + a_2 \ln(Q/Q_0) + a_3 \phi(x_1) + a_3 \phi(x_2)) + \ldots,$$  

(5)

where $a_1$, $a_2$, and $a_3$ are coefficients of magnitude less than $1 \text{ GeV}^2$, and $\phi(x)$ is a dimensionless function. The terms $a_2 \ln(Q/Q_0)$ and $a_3 \phi(x_j)$ arise from $S(b,Q)$ and $\ln[P_j(x_j,b)]$ in $\ln[\tilde{W}(b)]$, respectively. We neglect the flavor dependence of $\phi(x)$ in the analyzed region dominated by scattering of light $u$ and $d$ quarks. $\mathcal{F}_{NP}$ is consequently a universal function within this region. The dependence of $\mathcal{F}_{NP}$ on $\ln Q$ follows from renormalization-group invariance of the soft-gluon radiation [3]. The coefficient $a_2$ of the $\ln Q$ term has been related to the vacuum average of the Wilson loop operator and estimated within lattice QCD as $0.19^{+0.12}_{-0.09} \text{ GeV}^2$ [11].

The preferred $\mathcal{F}_{NP}$ is correlated in the fit with the assumed large-$b$ behavior of $\tilde{W}_{LP}$. We examine this correlation in a modified version of the $b_*$ model [3,4]. The shape of $\tilde{W}_{LP}$ is varied in the $b_*$ model by adjusting a single parameter
Continuity of \( \tilde{W} \) and its derivatives, needed for the numerical stability of the Fourier transform, is always preserved. We set \( \tilde{W}_{LP}(b) \equiv \tilde{W}_{\text{pert}}(b_*) \), with \( b_*(b, b_{\text{max}}) \equiv b(1 + b^2/b_{\text{max}}^2)^{-1/2} \). \( \tilde{W}_{LP}(b) \) reduces to \( \tilde{W}_{\text{pert}}(b) \) as \( b \to 0 \) and asymptotically approaches \( \tilde{W}_{\text{pert}}(b_{\text{max}}) \) as \( b \to \infty \). The \( b_* \) model with a relatively low \( b_{\text{max}} = 0.5 \text{ GeV}^{-1} \) was a choice of the previous \( q_T \) fits \([8,9]\). However, it is natural to consider \( b_{\text{max}} \) above \( 1 \text{ GeV}^{-1} \) in order to avoid \textit{ad hoc} modifications of \( \tilde{W}_{\text{pert}}(b) \) in the region where perturbation theory is still applicable. To implement \( \tilde{W}_{\text{pert}}(b_*) \) for \( b_{\text{max}} > 1 \text{ GeV}^{-1} \), we must choose the factorization scale \( \mu_F \) such that it stays, at any \( b \) and \( b_{\text{max}} \), above the initial scale \( Q_{\text{ini}} = 1.3 \text{ GeV} \) of the DGLAP evolution for the CTEQ6 PDF’s \( f_a(x, \mu_F) \).

We keep the usual choice \( \mu_F = C_3/b_*(b, b_{\text{max}}) \), where \( C_3 \sim b_0 \), for \( b_{\text{max}} \leq b_0/Q_{\text{ini}} \approx 0.86 \text{ GeV}^{-1} \). Such choice is not acceptable at \( b_{\text{max}} > b_0/Q_{\text{ini}} \), as it would allow \( \mu_F < Q_{\text{ini}} \). Instead, we choose \( \mu_F = C_3/b_*(b, b_0/Q_{\text{ini}}) \) for \( b_{\text{max}} > b_0/Q_{\text{ini}} \), \textit{i.e.}, we substitute \( b_0/Q_{\text{ini}} \) for \( b_{\text{max}} \) in \( \mu_F \) to satisfy \( \mu_F \geq Q_{\text{ini}} \) at any \( b \). Aside from \( f_a(x, \mu_F) \), all terms in \( \tilde{W}_{\text{pert}}(b) \) are known, at least formally, as explicit functions of \( \alpha_s(1/b) \) at all \( b < 1/\Lambda_{QCD} \). We show in Ref. \([15]\) that this prescription preserves correct resummation of the large logarithms and is numerically stable up to \( b_{\text{max}} \sim 3 \text{ GeV}^{-1} \).

We perform a series of fits for several choices of \( b_{\text{max}} \) by closely following the previous global \( q_T \) analysis \([9]\). We consider a total of 98 data points from production of Drell-Yan pairs in E288, E605, and R209 fixed-target experiments, as well as from observation of \( Z \) bosons with \( q_T < 10 \text{ GeV} \) by CDF and DØ detectors in the Tevatron Run-1. See Ref. \([9]\) for the experimental references. Overall normalizations for the experimental cross sections are varied as free parameters. Our best-fit normalizations agree with the published values within the systematical errors provided by the experiments, with the
Fig. 1. The best-fit values of $a(Q)$ obtained in independent scans of $\chi^2$ for the contributing experiments. The vertical error bars correspond to the increase of $\chi^2$ by unity above its minimum in each $Q$ bin. The slope of the line is equal to the central-value prediction from the renormalon analysis [11].

exception of the CDF Run-1 normalization (rescaled down by 7%).

To test the universality of $F_{NP}$, we individually examine each bin of $Q$. We choose $F_{NP} = a(Q)b^2$ and independently fit it to each of the 5 experimental data sets to determine the most plausible normalization in each experiment. We then set the normalizations equal to their best-fit values and examine $\chi^2$ at each $Q$ as a function of $a(Q)$. For $b_{max} = 1 - 2 \text{ GeV}^{-1}$, the best-fit values of $a(Q)$ follow a nearly linear dependence on $\ln Q$ [cf. Fig. 1]. The slope $a_2 \equiv da(Q)/d(\ln Q)$ is close to the renormalon analysis expectation of $0.19 \text{ GeV}^2$ [11]. The agreement with the universal linear $\ln Q$ dependence worsens if $b_{max}$ is chosen outside the region $1-2 \text{ GeV}^{-1}$. Since the best-fit $a(Q)$ are found independently in each $Q$ bin, we conclude that the data support the universality of $F_{NP}$, when $b_{max}$ lies in the range $1 - 2 \text{ GeV}^{-1}$. In another test, we find that each experimental data set individually prefers a nearly quadratic dependence on $b$, $F_{NP} = a(Q)b^{2-\beta}$, with $|\beta| < 0.5$ in all five experiments.
Fig. 2. The best-fit $\chi^2$ and coefficients $a_1$, $a_2$, and $a_3$ in $\mathcal{F}_{NP}(b, Q)$ for different values of $b_{max}$, $C_3 = b_0$ (stars) and $C_3 = 2b_0$ (squares). The size of the symbols approximately corresponds to $1\sigma$ errors for the shown parameters.

To further explore the issue, we simultaneously fit our model to all the data. We parametrize $a(Q)$ as $a(Q) \equiv a_1 + a_2 \ln \left[ Q/(3.2 \text{ GeV}) \right] + a_3 \ln \left[ 100x_1x_2 \right]$. This parametrization coincides with the BLNY form [9], if the parameters are renamed as $\{g_1, g_2, g_1g_3\}$ (BLNY) $\rightarrow \{a_1, a_2, a_3\}$ (here). It agrees with the generic form of $\mathcal{F}_{NP}(b, Q)$ in Eq. (5), if one identifies $\phi(x) = \ln(x/0.1)$. We carry out two sequences of fits for $C_3 = b_0$ and $C_3 = 2b_0$ to investigate the stability of our prescription for $\mu_F$ and sensitivity to $\mathcal{O}(\alpha^2_s)$ corrections. The dependence on $C_3$ is relatively uniform across the whole range of $b_{max}$, indicating that our choice of $\mu_F$ for $b_{max} > b_0/Q_{ini}$ is numerically stable.

Fig. 2 shows the dependence of the best-fit $\chi^2$, $a_1$, $a_2$, and $a_3$ on $b_{max}$. As $b_{max}$ is increased above 0.5 GeV$^{-1}$ assumed in the BLNY study, $\chi^2$ rapidly decreases, becomes relatively flat at $b_{max} = 1 - 2$ GeV$^{-1}$, and grows again at $b_{max} > 2$ GeV$^{-1}$. The global minimum of $\chi^2 = 125(111)$ is reached at $b_{max} \approx 1.5$ GeV$^{-1}$, where all data sets are described equally well, without major tensions among the five experiments. The improvement in $\chi^2$ mainly
ensues from better agreement with the low-$Q$ experiments (E288, E605, and R209), while the quality of all fits to the $Z$ data is about the same. This is illustrated by Fig. 3, which shows the differences between the measured and theoretical cross sections, divided by the experimental errors $\delta_{\text{exp}}$, as well as the values of $\chi^2$ in each experiment, in two representative fits for $b_{\text{max}} = 0.5 \text{ GeV}^{-1}$, $C_3 = b_0$ (squares) and $b_{\text{max}} = 1.5 \text{ GeV}^{-1}$, $C_3 = 2b_0$ (triangles).

The data are arranged in bins of $Q$ (shown by gray background stripes) and $q_T$, with both variables increasing from left to right. For $b_{\text{max}} = 1.5 \text{ GeV}^{-1}$, the (Data – Theory) differences are reduced on average in the entire low-$Q$ sample, resulting in lower $\chi^2$ in three low-$Q$ experiments. Two outlier points in the E605 sample (the first point in the second $Q$ bin and fifth point in the fifth $Q$ bin) disagree with the other E288 and E605 data in the same $Q$ and $x$ region and contribute $15–25$ units to $\chi^2$ at any $b_{\text{max}}$. If the two outliers were removed, one would find $\chi^2/\text{d.o.f.} \approx 1$ at the global minimum.

The magnitudes of $a_1$, $a_2$, and $a_3$ are reduced when $b_{\text{max}}$ increases from 0.5 to 1.5 GeV$^{-1}$. In the whole range $1 \leq b_{\text{max}} \leq 2 \text{ GeV}^{-1}$, $a_2$ agrees with the
Fig. 4. The best-fit form factors $b \tilde{W}(b)$ in (a) Tevatron Run-2 $Z$ boson production; (b) E605 experiment. In the E605 case, $b \tilde{W}(b)$ are divided by the best-fit normalizations $N_{fit}$ for the E605 data, and the form factor in the Qiu-Zhang parametrization [12] for $b_{QZ}^{max} = 0.3$ GeV$^{-1}$ is also shown.

renormalon analysis estimate. The coefficient $a_3$, which parametrizes deviations from the linear $\ln Q$ dependence, is considerably smaller ($< 0.05$) than both $a_1$ and $a_2$ ($\sim 0.2$). A reasonable quality of the fit is retained if $a_3$ is set to zero by hand: $\chi^2$ increases by $\approx 5$ in such a fit above its minimum in the fit with a free $a_3$. In contrast, $\chi^2$ increases by $> 200$ units if $a_3 = g_1 g_3$ is set to zero at $b_{max} = 0.5$ GeV$^{-1}$, as it was noticed in the BLNY study.

The preference for the values of $b_{max}$ between 1 and 2 GeV$^{-1}$ indicates, first, that the data do favor the extension of the $b$ range where $\tilde{W}_{LP}(b)$ is approximated by the exact $\tilde{W}_{pert}(b)$. In $Z$ boson production, this region extends up to $3 - 4$ GeV$^{-1}$ as a consequence of the strong suppression of the large-$b$ tail by the Sudakov exponent. The fit to the $Z$ data is actually independent of $b_{max}$ within the experimental uncertainties for $b_{max} > 1$ GeV$^{-1}$. The best-fit form factors $b \tilde{W}(b)$ for $b_{max} = 0.5$ and 1.5 GeV$^{-1}$ in $Z$ boson production are shown in Fig. 4(a).
In the low-$Q$ Drell-Yan process, continuation of $b\tilde{W}_{\text{pert}}(b)$ far beyond $b \approx 1$ GeV$^{-1}$ raises objections, since $b\tilde{W}_{\text{pert}}(b)$ has a maximum and is unstable with respect to higher-order corrections at $b \approx 1.2 - 1.5$ GeV$^{-1}$. The dubious large contributions to $\tilde{W}_{\text{pert}}(b)$ in this $b$ region would deteriorate the quality of the fit. The $b_*$ prescription with $b_{\text{max}} < 2$ GeV$^{-1}$ reduces the impact of the dubious terms on $\tilde{W}(b)$: for $b_{\text{max}}$ small enough, the maximum of $\tilde{W}_{\text{pert}}(b_*)$ is only reached at $b \gg 1.2$ GeV$^{-1}$, where it is suppressed by $e^{-F_{NP}(b,Q)}$. The best-fit form factors for the E605 kinematics, divided by the best-fit normalizations of the E605 data $N_{\text{fit}}$, are shown in Fig. 4(b).

If a very large $b_{\text{max}}$ comparable to $1/\Lambda_{QCD}$ is taken, $\tilde{W}_{\text{LP}}(b)$ essentially coincides with $\tilde{W}_{\text{pert}}(b)$, extrapolated to large $b$ by using the known, although not always reliable, dependence of $\tilde{W}_{\text{pert}}(b)$ on $\ln b$. Similar, but not identical, extrapolations of $\tilde{W}_{\text{pert}}(b)$ to large $b$ are realized in the models [12,13], which describe the $Z$ data well, in accord with our own findings. In $Z$ boson production, our best-fit $a(M_Z) = 0.85 \pm 0.10$ GeV$^2$ agrees with 0.8 GeV$^2$ found in the extrapolation-based models, and it is about a third of 2.7 GeV$^2$ predicted by the BLNY parametrization. Our results support the conjecture in [12] that $a_3$ is small if the exact form of $\tilde{W}_{\text{pert}}(b)$ is maximally preserved. To describe the low-$Q$ data, the model [12] allowed a large discontinuity in the first derivative of $\tilde{W}(b)$ at $b$ equal to the separation parameter $b^{Z}_{\text{max}} = 0.3 - 0.5$ GeV$^{-1}$, where switching from the exact $\tilde{W}_{\text{pert}}(b)$ to its extrapolated form occurs [cf. Fig. 4(b)]. In the revised $b_*$ model, such discontinuity does not happen, and $\tilde{W}_{\text{LP}}(b)$ is closer to the exact $\tilde{W}_{\text{pert}}(b)$ in a wider $b$ range at low $Q$ than in the model [12]. The two models differ substantially at $b \approx 1$ GeV$^{-1}$, as seen in Fig. 4(b).
To summarize, the extrapolation of $\tilde{W}_{\text{pert}}(b)$ to $b > 1.5$ GeV$^{-1}$ is disfavored by the low-$Q$ data sets, if a purely Gaussian form of $\mathcal{F}_{NP}$ is assumed. The Gaussian approximation is adequate, on the other hand, in the $b_*$ model with $b_{\text{max}}$ in the range $1 - 2$ GeV$^{-1}$. Here variations in $b_{\text{max}}$ are compensated well by adjustments in $a_1, a_2,$ and $a_3$, and the full form factor $b\tilde{W}(b)$ stays approximately independent of $b_{\text{max}}$. The best-fit parameters in $\mathcal{F}_{NP}$ are quoted for $b_{\text{max}} = 1.5$ GeV$^{-1}$ as \{\begin{align*}
 & a_1, a_2, a_3 \} = \{0.201 \pm 0.011, 0.184 \pm 0.018, -0.026 \pm 0.007\} \text{ GeV}^2 \\
 & \text{for } C_3 = b_0, \quad \{0.247 \pm 0.016, 0.158 \pm 0.023, -0.049 \pm 0.012\} \text{ GeV}^2 \text{ for } C_3 = 2b_0. 
\end{align*}\}
In Ref. [15], the experimental errors are propagated into various theory predictions with the help of the Lagrange multiplier and Hessian matrix methods, discussed, e.g., in Ref. [14]. We find that the global fit places stricter constraints on $\mathcal{F}_{NP}$ at $Q = M_Z$ than the Tevatron Run-1 $Z$ data alone. Theoretical uncertainties from a variety of sources are harder to quantify, and they may be substantial in the low-$Q$ Drell-Yan process. In particular, $\chi^2$ for the low-$Q$ data improves by 14 units when the scale parameter $C_3$ in $\mu_F$ is increased from $b_0$ to $2b_0$, reducing the size of the finite-order $\tilde{W}_{\text{pert}}(b)$ at low $Q$. The best-fit normalizations $N_{fit}$ also vary with $C_3$. The dependence of the quality of the fit on the arbitrary factorization scale $\mu_F$ indicates importance of $\mathcal{O}(\alpha_s^2)$ corrections at low $Q$, but does not substantially increase uncertainties at the electroweak scale. Indeed, the $\mathcal{O}(\alpha_s^2)$ corrections and scale dependence are smaller in $W$ and $Z$ production. In addition, the term $a_2 \ln Q$, which arises from the soft factor $S(b, Q)$ and dominates $\mathcal{F}_{NP}$ at $Q = M_Z$, shows little variation with $C_3$ [cf. Fig. 2c]. Consequently, the revised $b_*$ model with $b_{\text{max}} \approx 1.5$ GeV$^{-1}$ reinforces our confidence in transverse momentum resummation at electroweak scales by exposing the soft-gluon origin and universality of the dominant nonperturbative terms at collider energies.
We thank C.-P. Yuan for his crucial contribution to the setup of the fitting program, and T. Londergan, A. Szczepaniak, S. Vigdor, and CTEQ members for the helpful discussions. This work was supported by the NSF grants PHY-0100348 and PHY-0457219, and DOE grant W-31-109-ENG-38.

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