Research on subsystem division scheme of overlapping decentralized control strategy

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ABSTRACT

The overlapping decentralized control strategy is a solution to the decentralized control of large-scale systems based on the inclusion principle. Its design process can be summarized as three stages of expansion-decoupling-contraction. In this paper, the subsystem division scheme of the overlapping decentralized control strategy is studied for building structure systems under earthquake. Using the energy-to-peak algorithm, we mainly analyse the influences of the location of the overlapping layer, the number of overlapping layers and the number of subsystems on the control effect of the structural system, and compare them with the traditional energy-to-peak centralized control method. The numerical results indicate that the dynamic response of the structure can be effectively suppressed by raising the location of the overlapping layer, increasing the number of overlapping layers, and increasing the number of subsystems. Compared with the centralized control strategy, the control effect is equivalent, or even better, verifying the feasibility of this research scheme.

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1. Introduction

With the improvement of living standards, people’s demand for living environment and building quality is becoming increasingly high. Under the premise of structural safety and stability, modern buildings with novel shape, larger scale and more functions can better meet the needs of people, thus, ensuring that complex structures can live comfortably and safely under disturbance is a major concern. Structures tend to have a large dynamic response under excitation, especially seismic excitation. Therefore, the vibration response of the building needs to be controlled within a reasonable range to ensure the safety and comfort of the building. Substantial work has been done in the field of vibration control for structures at present (Doroudi & Hosseini Lavassani, 2021; Drgoňa et al., 2017; Javanmardi et al., 2019; Saaed et al., 2013; Spencer & Nagarajaiah, 2003). For active control strategy, centralized control is the most commonly used method. However, as the research progresses, one gradually find that there exists some shortcomings in the centralized control scheme of ‘decentralized acquisition and centralized processing’, such as the large number of sensors, long information transmission lines, and the lag of signal feedback. In addition, once individual sensors fail, it can easily lead to the failure of the entire control system (Bakule et al., 2016; Kang et al., 2021; Li & Adeli, 2018). Therefore, the decentralized control strategy, which can effectively solve the problems of high system dimension, feedback lag, and poor system stability faced by traditional centralized control, has received widespread attention.

The meaning of decentralized control is to decompose the large-scale system into several subsystems in a certain way, the controller of each subsystem only implements control according to local information, and the information exchange between adjacent subsystems depends on the different decomposition ways, to finally realize the control of the whole structure. Rubió-Massegú et al. (2020) designed an inter-story fluid viscous damper system. The controller of this system was obtained by the decentralized static feedback H-infinity control method, which reached the level of optimal active feedback control. Warsewa et al. (2019); Warsewa et al. (2020a, 2020) applied a decentralized distributed observer to the monitoring and control of large-scale adaptive structures, introduced a decentralized state estimation control method for adaptive high-rise structures and a decentralized linear quadratic Gaussian control method for adaptive structures, and analysed the robustness of the decentralized LQG controller. Wagner et al. (2020) forced the nonlinear model to the dynamics of a local linear target system by proposing a decentralized control scheme.
for structures that can handle nonlinearities through tension-only elements in the control strategy. Amini et al. (2017) obtained an appropriate alternative to the centralized control strategy by combining an embedded Luenberger observer with a substructure derived from the decentralized control strategy. Hu et al. (2018) proposed an output feedback control law only requiring the local sensor information, which is designed based on the decentralized simple adaptive control method. Palacios-Quiñonero et al. (2018) presented an effective strategy to produce high-performance decentralized controllers with partial local-state information for vibration control of large structures. With an application of the long short-term memory neural network, Li et al. (2022) designed a long short-term memory intelligent decentralized control method for seismic response control of high-rise buildings.

Currently, decentralized control strategies are mainly divided into three types, namely, fully decentralized control, partially decentralized control, and hierarchically decentralized control (Lynch & Law, 2002). Palacios-Quiñonero et al. (2011) designed overlapping and multi-overlapping decentralized controllers based on the inclusion principle describing the whole process of expansion, decoupling, and contraction of large systems. Numerical simulation of a simplified model of a five-story building under seismic excitation using overlapping and multi-overlapping decentralized LQR control methods, has achieved better control results. Xu et al. (2022) proposed an overlapping decentralized guaranteed cost hybrid control strategy for adjacent structures with uncertain parameters, by combining the overlapping decentralized control strategy with the guaranteed cost control algorithm. Bakule et al. (2015) assessed the performance of the overlapping decentralized control design by analysing the natural frequency and time responses of both pre-earthquake and post-earthquake high-fidelity benchmark models for a given benchmark evaluation criteria. Kang et al. (2022) obtained an effective control by passive fault-tolerant overlapping decentralized control method based on the H-infinity control algorithm.

The overlapping decentralized control method based on the inclusion principle can effectively decouple the interconnected subsystems in a large-scale system and realize the decentralized control of the large-scale system. Since the extended decoupling and coordinated contraction of the system are performed under the conditions related to the inclusion principle, the overlapping controller can ensure that the performance of the original system remains constant throughout the design process. Although current literatures (Bakule et al., 2015; Kang et al., 2022; Palacios-Quiñonero et al., 2011; Xu et al., 2022) consider a variety of algorithms to achieve overlapping decentralized control of building structures, the effect of subsystem division is not explored in depth. Therefore, based on reference (Palacios-Quiñonero et al., 2011), this paper discusses the subsystem division scheme for the structural model of a five-story building using the overlapping decentralized energy-to-peak algorithm. The main contributions of this paper are summarized as follows:

1. Decentralized control strategy is a trend to solve the control problems of large systems. The optimization of subsystem division scheme helps to improve the control performance of the system and provides reference for engineering applications.
2. The location of the overlapping layer, the number of overlapping layers and the number of subsystems all have an impact on the control effect of the system. Optimizing these factors can improve the control efficiency and achieve better control results.
3. The energy-peak algorithm is used to design the subsystem controller so that the computational efficiency and control performance of the control system can be effectively guaranteed.
4. The LIM method is used to solve the energy-peak index, which reduces the complexity of the calculation.

The main content of the paper is organized as follows. In section 2, the design idea and computational procedure of the overlapping decentralized state feedback energy-to-peak control algorithm are introduced. In section 3, a five-story building model under seismic excitation is used as an example to analyse the effects of the selected overlapping layer location, the selected number of overlapping layers and the selected number of subsystems on the structural dynamic response, and the control effectiveness of each scheme is evaluated. In section 4, some conclusions and future research directions are briefly discussed.

## 2. Description of overlapping decentralized control method

### 2.1. Description of the building system

The building motion under seismic excitation can be described by

\[ M \ddot{q}(t) + C \dot{q}(t) + Kq(t) = B_\omega \omega(t) + D_\omega \dot{\omega}(t), \]

where \( M, C, K \) are the matrix of mass, stiffness, and damping, respectively; \( q(t), \dot{q}(t), \ddot{q}(t) \) represent the vector of displacement, velocity, and acceleration for each story,
respectively, $B_u$ is the vector of control location, $u(t)$ indicates the control force matrix, $D_s$ is the matrix of external disturbance input, $\omega(t)$ denotes the vector of external excitation.

The matrices of mass, stiffness, and damping have the following forms:

$$M = \text{diag}(m_1, m_2, \ldots, m_n),$$

$$K = \begin{bmatrix}
k_1 + k_2 & -k_2 & 0 & \cdots \\
k_2 & k_2 + k_3 & -k_3 & \cdots \\
0 & -k_3 & k_3 + k_4 & \cdots \\
\vdots & \vdots & \vdots & \ddots \\
0 & 0 & 0 & \cdots \\
0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \ddots \\
-k_{n-1} & k_{n-1} + k_n & -k_n & 0
\end{bmatrix},$$

$$C = \begin{bmatrix}
c_1 + c_2 & -c_2 & 0 & \cdots \\
c_2 & c_2 + c_3 & -c_3 & \cdots \\
0 & -c_3 & c_3 + c_4 & \cdots \\
\vdots & \vdots & \vdots & \ddots \\
0 & 0 & 0 & \cdots \\
0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \ddots \\
-c_{n-1} & c_{n-1} + c_n & -c_n & 0
\end{bmatrix},$$

using the inter-story actuation scheme, the form of $B_u$ is

$$B_u = \begin{cases}
(B_u)_{ij} = -1 & (i = 1, 2, \ldots, n) \\
(B_u)_{ij} = 1 & (i = 1, 2, \ldots, n-1) \\
(B_u)_{ij} = 0 & \text{else}
\end{cases}$$

$$D_s = -M[1]_{n \times 1},$$

where $A_1, B_1, E_1$ represent the matrix of state, control, and disturbance input, respectively, the specific forms are

$$A_1 = \begin{bmatrix} \mathbf{0}_{(n \times n)} & I_n \\ -M^{-1}K & -M^{-1}C \end{bmatrix},$$

$$B_1 = \begin{bmatrix} \mathbf{0}_{(n \times 1)} \\ M^{-1}B_u \end{bmatrix},$$

$$E_1 = \begin{bmatrix} \mathbf{0}_{(1 \times n)} \\ -[1]_{(1 \times 1)} \end{bmatrix}.$$
where $\star$ represents the transpose of the element in the symmetric position, $X > 0$, $Y = GX$, $\eta > 0$. If an optimal $\hat{y}$ and the corresponding matrices $X_s$, $\hat{X}_s$ can be obtained at the same time, then the control feedback matrix can be computed as
\[
\hat{G}_s = \hat{Y}_s \hat{X}_s^{-1},
\]
(15)
mimely, the optimal $\gamma_G$ can be obtained by
\[
\gamma_G^{*} = \hat{y}^{1/2}(t).
\]
(16)

### 2.3. Overlapping decentralized state feedback energy-to-peak control algorithm

Based on the inclusion principle (Palacios-Quiñonero et al., 2011), for a pair of linear system models
\[
\begin{align*}
S : \dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t),
\end{align*}
\]
(17)
\[
\begin{align*}
\tilde{S} : \dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t) \\
\tilde{y}(t) &= \tilde{C}x(t),
\end{align*}
\]
(18)
where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C_y \in \mathbb{R}^{n \times n}$, $\tilde{A} \in \mathbb{R}^{\tilde{n} \times \tilde{n}}$, $\tilde{B} \in \mathbb{R}^{\tilde{n} \times m}$, $\tilde{C}_y \in \mathbb{R}^{\tilde{n} \times \tilde{n}}$.

For the linear system $S$, if there is a set of expansion matrices $V, R, T$ and the corresponding contraction matrices $U, Q, H$, then the expanded system $\tilde{S}$ can be written as.
\[
\tilde{A} = VAV + MA_{\tilde{B}} = VBQ + N_{\tilde{B}}, \tilde{C}_y = TC_yU + L_C,
\]
(19)
where $M_{\tilde{B}}, N_{\tilde{B}}, L_C$ are the corresponding complementary matrices (Palacios-Quiñonero et al., 2011).

The state, input, and output vector of the expanded system $\tilde{S}$ can be expressed as.
\[
\begin{align*}
\tilde{x}^T &= (x_I^T, x_2^T, \ldots, x_{\tilde{n}}^T) \\
\tilde{u}^T &= (u_1^T, u_2^T, \ldots, u_{\tilde{n}}^T) \\
\tilde{y}^T &= (y_1^T, y_2^T, \ldots, y_{\tilde{n}}^T).
\end{align*}
\]
(20)
The entire system $S$ is extended using an overlapping decentralized strategy, which yields $L$ overlapping subsystems
\[
\begin{align*}
\tilde{S}_{D_i} : \dot{\tilde{x}}(t) &= \tilde{A}_i \tilde{x}(t) + \tilde{B}_i \tilde{u}(t) \\
\tilde{y}(t) &= \tilde{C}_y(t),
\end{align*}
\]
(21)
where $\tilde{A}_i, \tilde{B}_i$, and $\tilde{C}_y$ are the known constant matrices describing the subsystems. Also, we can define a decoupled extension system
\[
\begin{align*}
\tilde{S}_D : \dot{\tilde{x}}(t) &= \tilde{A}_D \tilde{x}(t) + \tilde{B}_D \tilde{u}(t) \\
\tilde{y}(t) &= \tilde{C}_y(t),
\end{align*}
\]
(22)
with $\tilde{A}_D = \text{diag}(\tilde{A}_{11}, \tilde{A}_{22}, \ldots, \tilde{A}_{LL}), \tilde{B}_D = \text{diag}(\tilde{B}_{11}, \tilde{B}_{22}, \ldots, \tilde{B}_{LL}), (\tilde{C}_y)_{D} = \text{diag}(\tilde{C}_{y_{11}}, \tilde{C}_{y_{22}}, \ldots, \tilde{C}_{y_{LL}})$.

For obtaining the controller of the whole system, it is necessary to calculate the controllers $\tilde{G}_D$ of each subsystem by using the overlapping decentralized energy-to-peak algorithm, and write them in the form of a diagonal matrix, so the extended system control feedback matrix is
\[
\tilde{G}_D = \text{diag}(\tilde{G}_{D_1}, \tilde{G}_{D_2}, \ldots, \tilde{G}_{D_L}),
\]
(23)
Finally, based on the contraction principle, the control feedback matrix of the original system is
\[
G_0 = Q \tilde{G}_D V.
\]
(24)

### 3. Numerical simulations and analysis

The five-story building with $m_i = 2.156 \times 10^5 \text{kg}$, $k_i = 1.5 \times 10^8 \text{N/m}$ in reference (Palacios-Quiñonero et al., 2011) is taken as an example. The damping matrix is determined by the Rayleigh damping and the damping ratio is chosen to be 0.05. The actuation scheme of structure adapts the inter-story actuation scheme. An evanescent wave El Centro and a far-field ship wave Northridge are selected, the peaks are adjusted to $3.0 \text{m/s}^2$, the duration is 30 s, and the sampling step is 0.02 s.

To deeply study the influence of the subsystem division on the effectiveness of overlapping decentralized control strategy, three situations are analysed, i.e. different locations of overlapping layer, different numbers of overlapping layers and different quantities of subsystems, then compared with the energy-to-peak centralized control strategy.

Case 1: Centralized control

The centralized control strategy is to design its active controller for the whole structure, as shown in Figure 1. To compute the full-state feedback energy-to-peak controller, the corresponding matrices under the El Centro wave are
\[
C_z = \begin{bmatrix}
I_0 & 0_{5 \times 10} \\
0_{10 \times 5} & I_5
\end{bmatrix},
\]
\[
D_z = \begin{bmatrix}
0_{10 \times 5} \\
I_5
\end{bmatrix} \times 10^{-8.2}.
\]

**Figure 1.** Centralized control.
The corresponding controlled output matrices under the Northridge wave is chosen as

\[
C_2 = \begin{bmatrix} I_{10} \\ D_2 \end{bmatrix} = \begin{bmatrix} 0_{10 	imes 5} \\ I_5 \end{bmatrix} \times 10^{-8}.
\]

The LMI optimization problem in Equation (14) is solved to obtain the centralized state feedback controller, which is substituted into the state-space model to acquire the seismic response of the structure.

### 3.1. Overlapping layer with different locations

The structure is extended and decoupled into two substructures using the overlapping decentralized control strategy. The overlapping layer between subsystems is a single story, and when considering the different locations of the selected overlapping layer, the overlapping scheme is divided into three kinds, as shown in Figure 2.

**Case 2:** \(S_D^{(1)} = \{1, 2\}, S_D^{(2)} = \{2, 3, 4, 5\}\). The overlapping layer is the second story.

**Case 3:** \(S_D^{(1)} = \{1, 2, 3\}, S_D^{(2)} = \{3, 4, 5\}\). The overlapping layer is the third story.

**Case 4:** \(S_D^{(1)} = \{1, 2, 3, 4\}, S_D^{(2)} = \{4, 5\}\). The overlapping layer is the fourth story.

For the subsystems obtained after the expanded decoupling, the corresponding matrices of the three schemes are as follows:

**Case 2:**

\[
C_{z1} = \begin{bmatrix} I_4 \\ 0_{4 \times 2} \end{bmatrix}, D_{z1} = \begin{bmatrix} 0_{4 \times 2} \\ I_2 \end{bmatrix} \times 10^{-8.5};
\]

\[
C_{z2} = \begin{bmatrix} I_8 \\ 0_{8 \times 4} \end{bmatrix}, D_{z2} = \begin{bmatrix} 0_{8 \times 4} \\ I_4 \end{bmatrix} \times 10^{-8}.
\]

**Case 3:**

\[
C_{z1} = \begin{bmatrix} I_6 \\ 0_{6 \times 3} \end{bmatrix}, D_{z1} = \begin{bmatrix} 0_{6 \times 3} \\ I_3 \end{bmatrix} \times 10^{-8.5};
\]

\[
C_{z2} = \begin{bmatrix} I_6 \\ 0_{6 \times 3} \end{bmatrix}, D_{z2} = \begin{bmatrix} 0_{6 \times 3} \\ I_3 \end{bmatrix} \times 10^{-8}.
\]

**Case 4:**

\[
C_{z1} = \begin{bmatrix} I_8 \\ 0_{8 \times 4} \end{bmatrix}, D_{z1} = \begin{bmatrix} 0_{8 \times 4} \\ I_4 \end{bmatrix} \times 10^{-8.5};
\]

\[
C_{z2} = \begin{bmatrix} I_4 \\ 0_{4 \times 2} \end{bmatrix}, D_{z2} = \begin{bmatrix} 0_{4 \times 2} \\ I_2 \end{bmatrix} \times 10^{-8}.
\]

According to Section 2.3, the overlapping controllers of the three schemes are solved, and the seismic response of each scheme is generated by substituting them into the state-space model.

Figures 3–5 present the dynamic response and maximum control force of the controlled system for the building structure under various schemes, being compared with the centralized control.

Observed from Figures 3 and 4 that cases 1, 2, 3, and 4 have obvious suppression effects on the dynamic response of the structure. The inter-story displacement control rates of the structure under El Centro wave are 84%, 85%, 86%, and 89%, respectively, and the acceleration control rates are 71%, 69%, 76%, and 81%, respectively. Under Northridge wave, the inter-story displacement control rates are 55%, 71%, 75% and 81%.

**Figure 2.** Decentralized control with different locations of overlapping layer.

**Figure 3.** Maximum inter-story displacement for different locations of overlapping layer.
respectively, and the acceleration control rates are 62%, 67%, 76% and 82%, respectively. It is evident that the overlapping decentralized control strategy can achieve a good control effect; At the same time, as the location of the overlapping layer in the structure rises, the control effect is getting better and better. Among them, the control effects of cases 3 and 4 are better than the centralized control strategy.

3.2. Overlapping layers with different quantities

When the overlapping decentralized control strategy is used to decouple the structure extension into two substructures, and the overlapping layers between the sub-systems are multiple stories, the overlapping scheme is divided into three kinds, as shown in Figure 6.

Case 5: $\tilde{S}_D^{(1)} = \{1, 2, 3\}$, $\tilde{S}_D^{(2)} = \{2, 3, 4, 5\}$. The second and third stories are selected as overlapping layer.
Case 6: $\tilde{S}_D^{(1)} = \{1, 2, 3, 4\}$, $\tilde{S}_D^{(2)} = \{3, 4, 5\}$. The third and fourth stories are selected as overlapping layer.
Case 7: $\tilde{S}_D^{(1)} = \{1, 2, 3, 4\}, \tilde{S}_D^{(2)} = \{2, 3, 4, 5\}$. The second, third and fourth stories are selected as overlapping layer.

For the subsystems gained after extended decoupling, the corresponding matrices of the three schemes are

$$
\begin{align*}
\text{Case 5:} & & C_z1 = \begin{bmatrix} I_6 \\ 0_{3 \times 6} \end{bmatrix}, D_z1 = \begin{bmatrix} 0_{6 \times 3} \\ I_5 \end{bmatrix} \times 10^{-8.5}; \\
\text{Case 6:} & & C_z1 = \begin{bmatrix} I_8 \\ 0_{4 \times 8} \end{bmatrix}, D_z1 = \begin{bmatrix} 0_{8 \times 4} \\ I_4 \end{bmatrix} \times 10^{-8}; \\
\end{align*}
$$

In the following Section 2.3, the overlapping controllers of the three schemes are calculated, and the seismic response of each scheme is obtained by substituting them into the state-space model.

The dynamic response and maximum control force of the controlled system of the building under each scheme are given in Figures 7–9 and compared with the centralized control.

Figures 7 and 8 reveal that the inter-story displacement control rates under the El Centro wave are 86%,
89% and 88%, respectively, and the acceleration control rates are 74%, 80% and 78%, respectively. The inter-story displacement control rates of the structure under the Northridge wave are 74%, 80%, and 78%, respectively, and the acceleration control rates are 74%, 81%, and 78%, respectively. It can be seen that cases 5, 6, and 7 have obvious effects on suppressing the dynamic response of the structure, and they are superior to the centralized control strategy. Meanwhile, it is discovered that when the starting location of overlapping layers is the same, the control effect becomes better and better with the increasing number of overlapping layers.

### 3.3. Subsystems with different numbers

The overlapping decentralized control strategy is adopted to decouple the structural expansion into more than two subsystems. When the overlapping layer between subsystems is a single layer, there are four overlapping schemes, as described in Figure 10.

Case 8: $\tilde{S}_D^{(1)} = \{1, 2\}, \tilde{S}_D^{(2)} = \{2, 3, 4\}, \tilde{S}_D^{(3)} = \{4, 5\}$. The overlapping layers are the second and third stories, respectively.

Case 9: $\tilde{S}_D^{(1)} = \{1, 2, 3\}, \tilde{S}_D^{(2)} = \{3, 4\}, \tilde{S}_D^{(3)} = \{4, 5\}$. The overlapping layers are the third and fourth stories, respectively.

Case 10: $\tilde{S}_D^{(1)} = \{1, 2, 3\}, \tilde{S}_D^{(2)} = \{2, 3\}, \tilde{S}_D^{(3)} = \{3, 4\}, \tilde{S}_D^{(4)} = \{4, 5\}$. The overlapping layers are the second, third and fourth stories, respectively.

For the subsystems gained after the expanded decoupling, the corresponding matrices of the four schemes are

Case 8:

$C_2 = 3 \times [l_4 \ 0_{2 \times 4}], D_{z1} = \begin{bmatrix} 0_{4 \times 2} & I_2 \end{bmatrix} \times 10^{-8.5};$

$C_2 = 3 \times [l_4 \ 0_{2 \times 4}], D_{z2} = \begin{bmatrix} 0_{4 \times 2} & I_2 \end{bmatrix} \times 10^{-8};$

$C_2 = 3 \times [l_6 \ 0_{3 \times 6}], D_{z3} = \begin{bmatrix} 0_{6 \times 3} & I_3 \end{bmatrix} \times 10^{-8}.$

Case 9:

$C_2 = 3 \times [l_4 \ 0_{2 \times 4}], D_{z1} = \begin{bmatrix} 0_{4 \times 2} & I_2 \end{bmatrix} \times 10^{-8.5};$

$C_2 = 3 \times [l_6 \ 0_{3 \times 6}], D_{z2} = \begin{bmatrix} 0_{6 \times 3} & I_3 \end{bmatrix} \times 10^{-8};$

$C_2 = 3 \times [l_4 \ 0_{2 \times 4}], D_{z3} = \begin{bmatrix} 0_{4 \times 2} & I_2 \end{bmatrix} \times 10^{-8}.$

Case 10:

$C_2 = 3 \times [l_6 \ 0_{3 \times 6}], D_{z1} = \begin{bmatrix} 0_{6 \times 3} & I_3 \end{bmatrix} \times 10^{-8.5};$

$C_2 = 3 \times [l_4 \ 0_{2 \times 4}], D_{z2} = \begin{bmatrix} 0_{4 \times 2} & I_2 \end{bmatrix} \times 10^{-8};$

$C_2 = 3 \times [l_4 \ 0_{2 \times 4}], D_{z3} = \begin{bmatrix} 0_{4 \times 2} & I_2 \end{bmatrix} \times 10^{-8}.$

Figure 9. Maximum control force for different numbers of overlapping layer.

Figure 10. Decentralized control with different subsystem numbers.
Case 11:

\[
C_1 = 3 \times \begin{bmatrix} I_4 \\ 0_{2 \times 4} \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0_{4 \times 2} \\ l_2 \end{bmatrix} \times 10^{-8.5};
\]

\[
C_2 = 3 \times \begin{bmatrix} I_4 \\ 0_{2 \times 4} \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0_{4 \times 2} \\ l_2 \end{bmatrix} \times 10^{-8};
\]

\[
C_3 = 3 \times \begin{bmatrix} I_4 \\ 0_{2 \times 4} \end{bmatrix}, \quad D_3 = \begin{bmatrix} 0_{4 \times 2} \\ l_2 \end{bmatrix} \times 10^{-8};
\]

\[
C_4 = 3 \times \begin{bmatrix} I_4 \\ 0_{2 \times 4} \end{bmatrix}, \quad D_4 = \begin{bmatrix} 0_{4 \times 2} \\ l_2 \end{bmatrix} \times 10^{-8}.
\]

By solving the overlapping controllers of the four schemes according to Section 2.3, the seismic response of each scheme can be obtained by substituting them into the state-space model.

The dynamic response and maximum control force of these cases are displayed in Figures 11–13 and compared with the centralized control.

As shown in Figures 11 and 12, the inter-story displacement control rates of the structure under the El Centro wave are 90%, 90%, 96%, 90%, respectively, and acceleration control rates are 84%, 84%, 90%, and 84%, respectively. Under the Northridge wave, the inter-story displacement control rates are 93%, 93%, 96% and 94% respectively, and the acceleration control rates are 85%, 85%, 91% and 86% respectively. Clearly seen that cases 8, 9, 10 and 11 are all effective in suppressing the dynamic response of the structure and better than the centralized control strategy. Meanwhile, it is found that the increase...
in the number of subsystems is beneficial to the control effect.

### 3.4. Comprehensive analysis

In order to compare the control effects of the overlapping decentralized strategies of different schemes proposed in this paper more intuitively, the following evaluation indices $J_1$, $J_2$ and $J_3$ of the structural response are provided (Ohtori et al., 2004). Among them, $J_1$, $J_2$ are the norm indices of inter-story displacement and absolute acceleration, respectively, and $J_3$ is the peak output of the actuator. The expressions are

\[
J_1 = \max \{ (||d(t)||)/||d_{ui}(t)||) \},
\]

\[
J_2 = \max \{ (||\ddot{x}_{ai}(t)||)/||\ddot{x}_{ui}(t)||) \},
\]

\[
J_3 = \max \left\{ \left( \max_{i,j} |F_{ij}(t)| \right) / W \right\},
\]

where $d_i$ and $\ddot{x}_{ai}$ are the inter-story displacement and absolute acceleration of the $i$th story of the structure, respectively; $d_{ui}$ and $\ddot{x}_{ui}$ are the inter-story displacement and absolute acceleration of the $i$th story of uncontrolled structure, respectively; $F_{ij}$ and $W$ are the control force of the actuator on $i$th story and the total weight of the structure, respectively, and $|| \cdot ||$ is the corresponding 2-norm.

The results of the evaluation indices for each case are given in Table 1, and Figure 14 depicts the histogram of evaluation indices. From the evaluation indices, it is evident that all the schemes proposed in this paper can suppress the dynamic response of the structure well.

The evaluation index under the El Centro wave is chosen to compare the evaluation index of each scheme with

| Evaluation indices | Seismic input | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 | Case 6 | Case 7 | Case 8 | Case 9 | Case 10 | Case 11 |
|--------------------|--------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|---------|
| $J_1$              | El Centro    | 0.16   | 0.15   | 0.13   | 0.10   | 0.14   | 0.11   | 0.12   | 0.11   | 0.03   | 0.11    |         |
|                    | Northridge   | 0.47   | 0.30   | 0.25   | 0.19   | 0.26   | 0.20   | 0.22   | 0.07   | 0.08   | 0.05    | 0.07    |
| $J_2$              | El Centro    | 0.30   | 0.37   | 0.29   | 0.22   | 0.31   | 0.24   | 0.26   | 0.19   | 0.19   | 0.12    | 0.18    |
|                    | Northridge   | 0.37   | 0.37   | 0.28   | 0.21   | 0.30   | 0.22   | 0.24   | 0.17   | 0.17   | 0.11    | 0.16    |
| $J_3$              | El Centro    | 0.30   | 0.27   | 0.29   | 0.30   | 0.29   | 0.30   | 0.29   | 0.29   | 0.29   | 0.29    | 0.29    |
|                    | Northridge   | 0.30   | 0.26   | 0.27   | 0.26   | 0.27   | 0.26   | 0.26   | 0.29   | 0.29   | 0.30    | 0.29    |
the centralized control strategy (case 1), if the ratio is less than 1, it means that the control effect is superior to the centralized control. The evaluation indices $J_1$ and $J_2$ represent the overall displacement and acceleration control of the structure under seismic excitation, while $J_3$ represents the maximum peak of the control force. The comparison graphs of the evaluation indices $J_1$, $J_2$, and $J_3$ indicate that the control effects of cases 4, 6, and 10 are relatively better, and the control effect of case 10 is optimal.

Comparing the cases with overlapping decentralized energy-to-peak control strategy, conclusions can be made: with the location rising of the overlapping layer, which means the number of stories in the lower subsystem increases and the number of stories in the upper subsystem decreases, the control effect of the structure is better; when the initial location of overlapping layers is the same, with the increase in the number of the overlapping layer, the information obtained by the subsystem is more comprehensive, and the structure can obtain better control effect. Meanwhile, the control effect of the building can be improved significantly by increasing the number of subsystems.

4. Conclusion

Aiming at the changes of overlapping layers and subsystems in the overlapping decentralized energy-to-peak control strategy, this paper studies the control effects of various schemes of overlapping decentralized energy-to-peak control strategy, and draws the following conclusions:

1. The design method of the overlapping decentralized energy-to-peak control strategy is feasible in the vibration control system of the building structure, and various schemes have achieved good control effects, partially overlapping decentralized control strategy have better control effect than the centralized control strategy.

2. The best way to obtain the optimal control effect for structural vibration control is increasing the number of stories in the lowest subsystem while using the overlapping decentralized control energy-to-peak strategy. When the starting locations of the overlapping layers are the same, the structure can obtain a superior control effect by increasing the number of overlapping layers or overlapping subsystems. At the same time, the research demonstrates that different external excitation will have a certain influence on the control effect of the controlled system.

3. The overlapping decentralized control strategy provides a novel perspective to solve the vibration control problem of structures. The location and number of overlapping layers and the number of subsystems should be considered comprehensively in the controller design to obtain a superior control effect. Future research directions include studying the effect of external excitation on subsystem partitioning schemes, such as stochastic seismic excitation or wind excitation.

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