Determination of CP violation parameter using neutrino pair beam

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ABSTRACT

Neutrino oscillation experiments under neutrino pair beam from circulating excited heavy ions are studied. It is found that detection of double weak events has a good sensitivity to measure CP violating parameter and distinguish mass hierarchy patterns in short baseline experiments in which the earth-induced matter effect is minimized.

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1 Introduction

As is pointed out in our previous paper [2], when excited ions with a high coherence are circulated, neutrino pair emission rates become large with neutrino energies extending to the GeV region. Produced neutrino beam is a coherent mixture of all pairs of neutrinos, $\nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$. This gives a CP-even neutrino beam, providing an ideal setting to test fundamental symmetries of particle physics [3], in particular, to measure the CP violating (CPV) phase in the neutrino sector [4].

In the present work sequel to the previous one, we investigate observable quantities at detection sites away from heavy ion synchrotron, including the location of the facility.

Our main physics objectives are

1. CPV $\delta$ phase measurement (excluding the ones intrinsic to the Majorana neutrino),
2. NH vs IH distinction.

We shall demonstrate that double neutrino detection is necessary to achieve these objectives. Furthermore, in order to avoid possible contamination of earth-induced effects that mimic CPV parameter dependence, it is wise to conduct oscillation experiments at a short baseline. Our results show that a location within $\sim 50 \text{ km}$ away from the synchrotron can do an excellent job.

The rest of this paper is organized as follows. In the second section we explain special features of neutrino oscillation experiments under CP-even neutrino pair beam after a brief summary of neutrino pair emission at synchrotron. The conclusion on experimental means is that one should measure double weak events at detector for CPV parameter determination. The neutrino pair beam is found insensitive to CPV phases intrinsic to the Majorana neutrino. In Section 3 we discuss short baseline experiments in which the earth-induced matter effect is neglected. We demonstrate that both CP-even and CP-odd quantities can provide a sensitive measurement of CPV parameter with high precision. Distinction of normal and inverted hierarchical mass patterns is shown to be possible in short baseline experiments of the pair beam. In Section 4 the earth matter effect is discussed and shown to give large influence on determination of CPV parameter.

Throughout this work we use the natural unit of $\hbar = c = 1$.

2 Neutrino experiments under coherent pair beam

We consider measurements of neutrinos at a distance $L$ under the coherent neutrino pair beam of all mixtures of $\nu_{a}\bar{\nu}_{a}, a = e, \mu, \tau$ produced at a heavy ion synchrotron. It is important to calculate detection rates by treating the whole event quantum mechanically, since produced neutrino pairs are not detected at the synchrotron site.

We shall first recapitulate main features of the coherent neutrino pair beam proposed in our previous paper [2]. The neutrino pairs are produced from excited heavy ions of a boost factor $\gamma$ circulating in a ring of radius $\rho$. Its production rates are enormous, given by

$$\Gamma_{2\nu} \sim 3.1 \times 10^{21}\text{Hz} \frac{N|\rho_{eg}|^2}{10^8} \frac{(\rho/4\text{km})^{1/2}}{\gamma} \left(\frac{\epsilon_{eg}}{50\text{keV}}\right)^{11/2},$$

where $\rho_{eg}$ is the ionic coherence between the excited and the ground levels of spacing $\epsilon_{eg}$ required to be substantial, and we assumed in this estimate a number $10^8$ when it is multiplied by the total available ion number $N$. Relation between the pair production amplitude $P_{bb}(1,2)$ of a neutrino pair $\nu_{a}\bar{\nu}_{b}, b = e, \mu, \tau$ with kinematical variables collectively denoted by 1,2 (angles measured from the ion tangential direction), and its rate $R_{bb}(1,2)$ is [2]

$$R_{bb}(1,2) = \frac{2|P_{bb}(1,2)|^2}{T}, \quad T = \frac{\sqrt{\pi}}{2^{1/4}}\sqrt{pF^{-1/4}},$$

$$F = (E_1 + E_2)\left(\frac{\epsilon_{eg}}{\gamma} - \frac{E_1 + E_2}{2\gamma^2}\right) - \frac{1}{2}(E_1^2 \psi_1^2 + E_2^2 \psi_2^2) - \frac{E_1 E_2}{2}(\theta_1 - \theta_2)^2 - \frac{\epsilon_{eg}}{2\gamma}(E_1 \theta_1^2 + E_2 \theta_2^2).$$
$T/2$ typically of order 10 ps is the effective time of neutrino pair emission, which is more precisely a function of neutrino energies and their emission angles. Vertical emission angles $\psi_i, i = 1, 2$ and the opening angle of a pair, $\sin^{-1}(\cos \psi_1 \cos \psi_2 \cos(\theta_1 - \theta_2))$, are both limited by the boost factor $1/\gamma$. These angles are of order 100$\mu$radian $10^4/\gamma$.

The probability amplitude of the entire process consists of three parts: the production, the propagation, and the detection due to charged current (CC) interaction, each to be multiplied at the amplitude level. Thus, one may write the amplitude for double neutrino quasi-elastic scattering (with $J$ the nucleon weak current) as

$$\sum_b \frac{G_F}{\sqrt{2}}^2 \bar{\nu}_a \gamma_\alpha (1 - \gamma_5)\nu_e J^{\alpha} \bar{c} \gamma_\beta (1 - \gamma_5)\nu_e (J^{\beta})^\dagger \langle \bar{a} | e^{-iHL} | b \rangle \langle c | e^{-iHL} | b \rangle P_{bb}(1, 2),$$

(4)

where $H$ is the hamiltonian for propagation including earth-induced matter effect [5], [6], [7], which is in the flavor basis

$$H = U \left( \begin{array}{ccc} \frac{m_e^2}{2E} & 0 & 0 \\ 0 & \frac{m_\mu^2}{2E} & 0 \\ 0 & 0 & \frac{m_\tau^2}{2E} \end{array} \right) U^\dagger \mp \sqrt{2}G_F n_e \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right),$$

(5)

with $U = (U_{ai})$, $a = e, \mu, \tau, i = 1, 2, 3$ the neutrino mixing matrix. The sign $\mp$ refers to neutrino (-) and anti-neutrino (+).

Let $V$ and $\bar{V}$ are unitary $3 \times 3$ matrices that diagonalize the hamiltonian $H$ for neutrino and $\bar{H}$ for anti-neutrino, including the earth matter effect. We shall denote three eigenvalues by $\lambda_i$ for neutrinos, and $\bar{\lambda}_i$ for anti-neutrinos. The propagation amplitude is then

$$\langle c | e^{-iHL} | b \rangle = \sum_i V^*_{ci} \bar{V}_{bi} e^{-i\lambda_i L}, \quad \langle \bar{a} | e^{-iHL} | \bar{b} \rangle = \sum_i \bar{V}^*_{ai} \bar{V}_{bi} e^{-i\bar{\lambda}_i L},$$

(6)

$$\sum_b \langle \bar{a} | e^{-iHL} | \bar{b} \rangle \langle c | e^{-iHL} | b \rangle c_b = \frac{1}{2} \sum_{ij} V^*_{ci} \bar{V}_{cj} \xi_{ij} e(\bar{\lambda}_j, \lambda_i), \quad (c_b) = \frac{1}{2} (1, -1, -1),$$

(7)

$$\xi_{ij} = \bar{V}_{ej} V_{ei} - \bar{V}_{\mu j} V_{\mu i} - \bar{V}_{\tau j} V_{\tau i}, \quad e(\bar{\lambda}_j, \lambda_i) = \exp[-iL(\lambda_i + \bar{\lambda}_j)].$$

(8)

The factor $c_b$ arises from the production amplitude $P_{bb}(1, 2)$. The precise relation between neutrino and anti-neutrino eigenvalue problem is given by

$$\bar{\lambda}(G_F) = \lambda(-G_F), \quad \bar{V}^*_a(G_F) = V_{ai}(-G_F).$$

(9)

An important question of the Majorana CPV phase (MP) dependence of the neutrino propagation amplitude $\langle a | e^{-iHL} | b \rangle$ and its anti-neutrino counterpart is worked out as follows, using the parametrization [8]. First, the eigenvalue equation $\det(\lambda - H) = 0$, when explicitly written out, indicates that $\lambda_i, \bar{\lambda}_i$ are independent of MP, $\alpha, \beta$. Define MP-independent mixing matrix by $U = U P^\dagger$, $P = (1, e^{i\alpha}, e^{i\beta})$. The hamiltonian in the mass eigen-state basis $U^\dagger H U$ has a simple MP phase dependence $P^\dagger \bar{H} P$, $\bar{H}$ being MP-independent. Diagonalization of $\bar{H}$ can be done, $\bar{H} = \bar{V}^\dagger \bar{H} P \bar{V}$ by MP-independent matrix $\bar{V}$. The unitary matrix $V$ for $H$ diagonalization is then MP-independent, since $V = \bar{V} P U^\dagger = \bar{V} P P^\dagger U^\dagger = \bar{V} U^\dagger$. This proves that $\langle a | e^{-iHL} | b \rangle$ is MP-independent.

More general formulas relating these to re-phasing invariant quantities are given in [7].

We now discuss prospects of single neutrino events in which one of pair neutrinos go undetected. The rate of neutrino $\nu_e$ undetected (and $\bar{\nu}_e$ detected) contains the squared propagation factor,

$$\sum_c | \sum_{ij} V^*_{ci} \bar{V}_{pj} \xi_{ij} e(\bar{\lambda}_j, \lambda_i)|^2 = \sum_{ijkl} V^*_{ci} V_{ck} \bar{V}_{pj} \bar{V}_{pl} \xi_{ij} \xi_{kl} e(\bar{\lambda}_j, \lambda_i) e^*(\bar{\lambda}_k, \lambda_k)$$

$$= \sum_{jkl} \bar{V}_{pj} \bar{V}_{pl} e(\bar{\lambda}_j, \lambda_i) e^*(\bar{\lambda}_k, \lambda_k) \sum_i \xi_{ij} \xi_{kl} = \sum_{jkl} \bar{V}_{pj} \bar{V}_{pl} e(\bar{\lambda}_j, \lambda_i) e^*(\bar{\lambda}_k, \lambda_i) \delta_{jkl} = 1,$$

(10)
since $e(\bar{\lambda}_j, \lambda_i)e^*(\bar{\lambda}_i, \lambda_j)$ is $i-$independent. This result relies on the unitarity of mixing matrices alone.

A conclusion drawn from this is that when only one of neutrinos in the pair is detected and its partners are undetected, oscillation patterns disappear, hence there is no way to measure CPV parameter.

We thus consider double events in what follows. The single event may be used to monitor the coherence $\rho_{eg}$ which might be otherwise not easy to measure.

3 Neutrino interaction away from synchrotron: short baseline experiments

An important question that arises in the coherent neutrino pair beam is whether the coherence of two neutrino of the pair present at the production site is maintained or not. The degree of phase de-coherence increases with travel distance of the neutrino pair. The most important de-coherence interaction is forward scattering caused by atomic electrons [5] when the neutrino passes through the earth. This introduces a phase of order, $$\sqrt{2G_FN_eL} \sim 1 \times \frac{n_e}{1.4 \times 6 \times 10^{23}\text{cm}^{-3}} \frac{L}{1860\text{km}},$$ (11)

(for the simple earth-model made of pure SiO$_2$ of mass density 2.8 g cm$^{-3}$) and its fluctuating component destroys the phase coherence necessary to apply the idea of the coherent pair beam. Thus, we may divide the nature of the pair beam into coherent and incoherent regimes, its boundary being roughly estimated at of order 2000 km.

Double event detection probability of quasi-elastic scattering (QES) producing two charged leptons $\mu^+$ and $c(= e, \mu)$ is determined from the product of three factors,

$$|\sum_{ij}V_{\bar{c}i}V_{\bar{\mu}j}^*\xi_{ij}e^{(i\bar{\lambda}_j, \lambda_i)}|^2 \frac{d^4\Gamma}{dE_1dE_2d\Omega_1d\Omega_2} \frac{d^2\sigma}{dE_{\mu}d\sin\psi_\mu} \frac{d^2\sigma}{dE_\mu d\sin\psi_\mu}. \tag{12}$$

$\mp$ corresponds to neutrino ($\nu_c$) and anti-neutrino ($\bar{\nu}_\mu$) events. One can assume for parent neutrino pairs that $i = 1$ for $\bar{\nu}_a, a = \mu$ and $i = 2$ for $\nu_c$.

The probability of double detection is roughly estimated as follows. First, the detection probability of a single neutrino event is estimated by the factor $\sigma n_Nl$ where $n_N$ is the nucleon number density and $l$ is the detector’s size along the neutrino beam. The cross section is of order $10^{-39} \sim 10^{-38}$ cm$^2$ for a 1 GeV neutrino, which gives $\sigma n_Nl \sim 10^{-11} \sim 10^{-10}$ for a single detection of weak process with $\sim 100$ m detector size. The double detection probability is then, with a perfect acceptance, $10^{-22} \sim 10^{-21}$ for a 100 kt class of detectors, which gives the double rate of order 10 $\sim 100$ mHz, using the formula (11). This rate is not too small. Actual experimental design, which we do not discuss in this paper, must take into account detailed rate calculation including detector geometry, location etc. as well as possible backgrounds.

In this section we shall consider short baseline oscillation experiments ignoring earth-induced matter effects, hence the propagation factors of eq. (12) becomes

$$P_{ac} = |\sum_{ij}U_{\bar{c}i}U_{\bar{\mu}j}^*\xi_{ij}e^{-i\bar{L}(m_1^2/E_1+m_2^2/E_2)}|^2, \quad \xi_{ij} = U_{\tau i}U_{\tau j}^* - U_{\mu i}U_{\mu j}^* - U_{\tau i}U_{\tau j}^* \tag{13}.$$
A typical CP-odd quantity, CPV asymmetry for the rate difference of $\bar{\nu}_a\nu_c$ and $\bar{\nu}_c\nu_a$ events, is defined by the ratio of rate difference to the rate sum:

$$A(\delta) = \frac{d\Gamma(\delta : G_F) - d\Gamma(-\delta : -G_F)}{d\Gamma(\delta : G_F) + d\Gamma(-\delta : -G_F)}.$$  \hspace{1cm} (14)

The change $G_F \rightarrow -G_F$ is necessary when the earth matter effect is included in the next section.

Computed oscillation patterns given by $P_{\mu^+e^-}$ and asymmetries calculated from this quantity are illustrated in Fig(1) $\sim$ Fig(4). For these figures we assume the normal hierarchical (NH) mass pattern of the vanishing smallest neutrino mass, using oscillation parameters as given in [8]. In these computations reaction cross sections are not multiplied, and we did cut off the lowest neutrino energies at much larger than $m_e$ and $m_\mu$ to ignore threshold effects of charged current (CC) interactions.

These results indicate expected behaviors of oscillation patterns and CPV asymmetry in short baseline experiments limited to distances shorter than $\sim 100 \text{ km}$:

1. CPV asymmetry is large and of order unity near the synchrotron site, while CP-even rates of $\bar{\nu}_\mu\nu_e + \bar{\nu}_e\nu_\mu$ becomes larger further away from the site.
2. $\bar{\nu}_\mu\nu_e$ double events, in particular their asymmetric events of $E_\mu \gg E_e$, have larger CPV asymmetry than $\bar{\nu}_\mu\nu_\mu$ events.

![Figure 1: Oscillation pattern given by $P_{\mu e}$ of eq.(13) (in solid black) and asymmetry (in dashed red) at various distances for $\bar{\nu}_\mu\nu_e$ CC double events. $\delta = \pi/4$, $E_{\bar{\nu}_\mu} = 500\text{MeV}$, $E_{\nu_e} = 5\text{MeV}$.](image)

Finally, we show differences of normal hierarchical mass pattern (NH) and inverted hierarchical pattern (IH) in Fig(5). There is no problem of distinction between these two cases in short baseline experiments of neutrino pair beam.

Requirement for an effective detector to measure these quantities is a good separation of $\mu^\pm$ charges, and a good position detection of $e^\pm$ showers.

### 4 Comparison with long baseline experiments

It is well known that the earth matter effect fakes CPV measurement, and we shall examine this issue in experiments under the coherent neutrino pair beam. In our analysis we use correction to mass eigenvalues.
Figure 2: Asymmetry at various distances for $\bar{\nu}_e \nu_e$ CC double events. $\delta = \pi/4$, $E_{\bar{\nu}_\mu} = 500\text{MeV}$ and $E_{\nu_e} = 5\text{MeV}$ in solid black, 50 MeV in dashed red, and 500 MeV in dash-dotted blue (much smaller than the other two cases). NH of smallest mass zero is assumed.

Figure 3: Asymmetry vs electron neutrino energy for $\bar{\nu}_\mu \nu_e$ CC double events. $E_{\bar{\nu}_\mu} = 500\text{MeV}$ and $\delta = \pi/6$ in solid black, $\pi/4$ in dashed red, and $\pi/2$ in dash-dotted blue. NH of smallest mass zero is assumed.
Figure 4: Asymmetry vs CPV $\delta$ for $\bar{\nu}_\mu \nu_e$ CC double events. $E_{\bar{\nu}_\mu} = 500$ MeV, $E_{\nu_e} = 5$ MeV at 10 km away in solid black, 50 km in dashed red, and 100 km in dash-dotted blue. NH of smallest mass zero is assumed.

Figure 5: NH vs IH distinction at 10 km away from the synchrotron, given by asymmetric energy combinations: $P_{\mu e}$ is plotted for $E_{\bar{\nu}_\mu} = 500, 200$ MeV, fixed and variable $E_{\nu_e}$. NH in blacks, 500 MeV in solid and 200 MeV in dotted lines, and IH in colored, 500 MeV, in dashed red and 200 MeV in dash-dotted blue. $\delta = 0$. 
and eigen-vectors due to earth matter effect:
\[
\lambda_1 \sim \frac{m_1^2}{2E} - \sqrt{2} G_F n_e |U_{e1}|^2, \quad \lambda_2 \sim \frac{m_2^2}{2E} - \sqrt{2} G_F n_e |U_{e2}|^2, \quad \lambda_3 \sim \frac{m_3^2}{2E} - \sqrt{2} G_F n_e |U_{e3}|^2,
\]
(15)
\[
\langle \lambda_1 | a \rangle \sim \left( U_{a1} + 2\sqrt{2} G_F n_e E U_{e1} \frac{U_{a2} U_{e2}^*}{\delta m_{21}^2} + \frac{U_{a3} U_{e3}^*}{\delta m_{31}^2} \right),
\]
(16)
\[
\langle \lambda_2 | a \rangle \sim \left( U_{a2} + 2\sqrt{2} G_F n_e E U_{e2} \frac{U_{a1} U_{e1}^*}{\delta m_{12}^2} + \frac{U_{a3} U_{e3}^*}{\delta m_{32}^2} \right),
\]
(17)
\[
\langle \lambda_3 | a \rangle \sim \left( U_{a3} + 2\sqrt{2} G_F n_e E U_{e3} \frac{U_{a1} U_{e1}^*}{\delta m_{13}^2} + \frac{U_{a2} U_{e2}^*}{\delta m_{23}^2} \right).
\]
(18)

Note the trivial relation \( \delta m_{ij}^2 = -\delta m_{ji}^2 \).

Result of numerical computations is illustrated in Fig(6), which indicates a great sensitivity of coherent pair beam experiments to the earth matter effect. One might say that the matter effect contaminates CPV effects, and it would be wise to conduct oscillation experiments in detectors placed on earth. In order to avoid the flux reduction caused by emission angles away from the beam tangential, a useful site distance is limited to order 50 km.

The great sensitivity to the earth matter density of oscillation patterns is an obstacle against a clean measurement of CPV parameter, but it might open a possibility of devising a method of earth tomography by means of the coherent neutrino pair beam, which we hope to discuss in future.

Figure 6: Effects of earth matter on oscillation patterns given by \( P_{\bar{\mu} e} \). Oscillation without matter effect in solid black, with matter effect of earth-model made of pure SiO\(_2\) in dashed red, and its electron number density 20% made larger in dash-dotted blue. \( \delta = \pi/4 \) and energy combination \((E_{\bar{\nu}_\mu}, E_{\nu_e}) = (500 \text{ MeV}, 50 \text{MeV})\) for \( \bar{\mu} e \) events.

In summary, we showed that coherent neutrino pair beam can provide an excellent chance of measuring CPV parameter and distinction of the mass hierarchical patterns if double weak events are detected.

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We use the parametrization as given by

$$ (U_{ai}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P, $$

$$ P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}, \quad a = e, \mu, \tau, i = 1, 2, 3, $$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. 

