Calculating scheme of ground freezing depth on the basis of data on seasonal snowfall deposition, snow cover accumulation and temperature variation

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Abstract. Soil thermal regime remains today an insufficiently studied field of environmental computing science. For the determination of the influence of air-temperature and snow cover thickness absolute values and dynamics on the ground freezing depth, and the development of root systems of trees and plants, a calculating scheme on the basis of data on seasonal snowfall deposition, snow cover accumulation, and temperature variation is considered. The calculating scheme is based on a three-layer media heat conduction problem (snow cover, frozen and thawed ground) with phase transition on the boundary between frozen and unfrozen grounds. The heat balance equation includes phase transition energy, inflow of heat from the unfrozen ground and outflow to the frozen ground, snow cover and the atmosphere. The heat flux is calculated on the basis of the Fourier law as the product of heat conductivity and temperature gradient. It is supposed that the temperature changes in each of the media linearly. An assumption is also made that snow cover consists of different layers deposited by different snowfalls and having different structure, density, and heat conductivity depending on its density. The density and heat conductivity of each layer and the whole thickness of snow cover are determined, a regional stratigraphic column for snow cover is compiled, and the ground freezing intensity and freezing depth are calculated. A comparison of the calculated and observed values of ground freezing depth for Narayan-Mar is performed, and a correlation of 0.76-0.77 is obtained.

1. Introduction
Earth’s climatic system can be described by a nonlinear dynamic system of differential equations of variables varying in time. Newton’s laws of mechanics are used for the composition of this system of differential equations on the basis of a description of the interaction in the five main components of the Earth: atmosphere, ocean, land, cryosphere, and biosphere. This system is not conservative; there is dissipation of energy. Forcing (the energy income) of the system is the income of solar radiation on the Earth’s surface.

On the basis of all these mentioned objectives for forecasts of the weather and climate, different climatic models are launched (for example, one national SL-AV model (which is semi-Lagrangian, based on Absolute Vortices, authored by M.A. Tolstykh from the Marchuk Institute of Numerical Mathematics RAS) and another one from the Main Geophysical Observatory in Saint-Petersburg). The models are launched with perturbed initial conditions, and an ensemble forecast is obtained.
This procedure is made in a similar way as for other natural systems (simplified physical models constructed from systems of differential equations). These systems of differential equations often do not have explicit solutions but are solved numerically. The solutions of these systems may present stable trajectories, between which switching at some values of variables and parameters may take place. In the vicinity of the switching (tipping) points (bifurcation) the moment of switching is determined by “noise”. In the solution behavior there may also appear “chaos”. J.M.T. Thompson and J. Sieber [1] proposed considering the tipping points of the Earth climatic system as stable points, and trajectories and switching between them similarly to the theory of stable state and forced oscillation in a nonlinear dynamic system of dumped oscillator developed by J.M.T. Thompson and H.B. Stewart [2].

A description of the tipping elements of the Earth’s climate system is taken from the paper of T.M. Lenton et al. [3] (see Figure 1).

T.M. Lenton described the properties of possible sources of the Earth’s climate system tipping points: their character and mechanism of origin and the influence on the Earth’s climate alteration. T.M. Lenton has also started to describe the early prediction of the occurrence of Earth’s climate tipping points in his work with Valeria Livina [4].

Under the main Earth’s climate tipping point sources, T.M. Lenton considered the Arctic sea-ice, the Greenland ice sheet, the west Antarctic ice sheet, the Atlantic thermohaline circulation, the El Nin˜o–southern oscillation, the Indian summer monsoon, the Sahara/Sahel and west African monsoon, the Amazon rainforest, and the boreal forest (see Figure 1).

For example, as the Arctic sea ice melts, it exposes a much darker ocean surface, which absorbs more radiation. This amplifies the warming. This area covered by ice decreases and less energy from insolation is reflected, resulting in the increasing temperature and, thus, it decreases the ice coverage. This positive feedback (it is assumed that this ice-albedo decrease is a positive feedback) may lead to the destruction of the energy balance and to instability or another stable state.

Such instability is not expected for the Southern Ocean sea ice, because the Antarctic continent covers that entire region. Some researchers think that a summer Arctic ice-loss point, if not already
passed, may be very close and a transition could occur within this century. However, Lindsay & Zhang in [5] are not so confident about that, and Eisenman & Wettlaufer in [6] argue that there probably no bifurcation for the loss of seasonal (summer) sea-ice cover; but there may be one for the year-round loss of ice cover.

The Atlantic thermohaline circulation is a conveyor, where cold and salty (more dense) waters sink to the bottom on the north of the Atlantic and fresher and warmer waters come in its place. The increased income of these fresh (and/or warm) waters on the north of the Atlantic (for example, from the Greenland glacier melt) may shut off this convection. According to the results of numerous models, this shutoff is reversible but some hysteresis is necessary. In some models this sea water convection is similar to the convection in the atmosphere, where in 1963 Lorenz observed some chaotic attractor (see [7]).

The boreal system is a complex interaction between the tree physiology, permafrost, and fire. Under climate alteration there is increase in the water stress, peak summer heat stress and increase in the mortality, vulnerability to diseases, and subsequent fire, as well as decrease in the reproduction rates, which could lead to large-scale dieback of the boreal forests [8,9], with transitions to open woodlands or grasslands. Newly unfrozen soils that drain well regionally and reduction in the amount of snow also support the drying, cause more fire and, hence, less biomass. In contrast, increased thaw depth and increased water-use efficiency under elevated CO$_2$ will tend to increase the available soil moisture, decreasing the frequency of fires and increasing the woody biomass.

So are the examples of tipping points of Earth’s climate system and the objectives of their origin. The soil thermal regime of boreal forests, despite the simplicity of measuring the soil temperature, remains today an insufficiently studied field. To determine the influence of the air-temperature and snow cover thickness absolute values and the dynamics on the ground freezing and thawing depth, develop microbiota and root systems of trees and plants, a number of models and calculation schemes have been developed. For example, V.A. Kudryavtsev [10] characterized the warming and cooling action of the snow cover on the ground depending on the snow accumulation regime and its duration, and proposed an equation for the estimation of the ground freezing depth including snow cover thickness, its thermal properties, and the amplitude of yearly air temperature oscillations. In our case the calculation scheme for ground freezing is constructed on the basis of three layer media heat conductivity problem (snow cover, frozen and thawed ground) with a phase transition on the boundary of frozen and unfrozen ground (see Figure 2).

![Figure 2. Temperature distribution in the media consisting of snow thickness (1), layer of frozen (2) and thawed (3) soil.](image)
The heat balance equation includes phase transition energy, the inflow of heat from unfrozen ground, and the outflow to frozen ground, snow cover, and atmosphere. The heat flux is calculated on basis of the Fourier law as a product of the heat conductivity and temperature gradient. It is assumed that temperature changes in each medium linearly. An assumption is also made that the snow cover consists of different layers deposited by different snowfalls and having different structure and density and heat conductivity depending on its density. Consideration of meteorological data of the air temperature, precipitation and snow thickness and snowfall intensity at the nearest meteorological station makes it possible to define the density and heat conductivity of each layer of snow cover, compile a regional stratigraphic column for the snow cover, and make more precise calculation of the ground freezing intensity and freezing depth.

2. Materials and methods
On the basis of meteorological data on the air temperature, precipitation, and snow cover thickness, we extracted data on the deposition and intensity of snowfalls at the nearest meteorological station Narayan-Mar [11] (see the distribution in Figure 3, a), and generalized stratigraphy columns were compiled for this region for the winter seasons of 1990/91-2015/16 (as in [12, 13]) (see Figure 3, b).

![Figure 3. a – average number of snowfalls of particular intensity according data of meteorological station Narayan-Mar for 1988-2008, b - generalized stratigraphic column of snow cover for meteorological station Narayan-Mar for 1988-2008.](image)

On the basis of the relation of the snow heat conductivity on density according to A.V. Pavlov’s [14] formula, estimation of the heat conductivity of separate snow layers is performed. According to the formula of heat conductivity of multilayer media, as in equation (1),

$$\frac{\lambda}{\Delta x} = \frac{1}{\frac{\Delta x_1}{\lambda_1} + \cdots + \frac{\Delta x_n}{\lambda_n}},$$

(1)

and on basis of information on the snow layers, the ground freezing depth is calculated.

The calculations of freezing of the snow-covered ground in the winter period on the basis of daily data on the air temperature, snow thickness, and heat conductivity of the snow cover allows estimating
the rate of movement of the ground freezing interface during this winter period. The rate of movement of the ground freezing interface can be expressed by the formulas:

The equation of heat balance can be written as in equation (2):

\[ F_j = cL\Delta T + F_2. \]  

(2)

\( F_j \) is the heat outflow through the snow cover and frozen ground from the ground freezing interface (W/m²);

\( cL\Delta T \) is the heat value for the phase transition in the ground, \( c \) is the ground moisture content (1-4 kg/cm³*m³), (the last value corresponds to full filling of porous medium by water for light clay with a density of 2000 kg/m³ and a porosity coefficient of 0.617 [15]);

\( L \) is the energy of H₂O phase transition (335 kJ/kg), and \( V \) is the rate of movement of the ground freezing interface (cm/s);

\( F_2 \) is the heat inflow from cooling of thawed ground in front of the ground freezing interface (W/m²).

The heat flux through the snow cover and frozen ground from the ground freezing interface is expressed according to the Fourier law by means of the temperature gradient and heat conductivity as \( F = \lambda \ (\text{grad} \ T) \). The heat conductivity and heat flux through a combination of two media (snow and frozen ground) can be expressed as in equation (3):

\[ F_j = \frac{\lambda \Delta T}{\Delta x} = \frac{\Delta T}{\left(\frac{\Delta x}{\lambda_s} + \frac{\Delta x}{\lambda_{fg}}\right)} = \frac{T_{air}}{\left(\frac{h_s}{\lambda_s} + \frac{l_{fg}}{\lambda_{fg}}\right)}. \]  

(3)

Here \( T_{air} \) is the air temperature, \( h_s \) and \( l_{fg} \) are the snow cover thickness and ground freezing depth, and \( \lambda_s \) and \( \lambda_{fg} \) are the heat conductivity of snow and frozen ground.

It is assumed that at a depth of 10 m in the ground there is a point of zero annual temperature oscillation with a temperature \( T_0 \) of about 3°C. That is why the heat inflow can be expressed as in equation (4):

\[ F_2 = \lambda_{thg} \frac{\Delta T}{\Delta x} = \frac{T_0}{10 - l_{fg}}. \]  

(4)

To validate the three-layers-calculating-scheme, an experiment of one-direction freezing of a snow-covered sand sample is performed in a refrigerated chamber under the action of negative temperatures (as in [16]). The intensity of freezing and the rate of movement of the phase transition front are determined and compared with scheme values obtained by calculation. For this, a dry sand sample with a mass of 5.2 kg, a density of 1.35 g/cm³, and less than a millimeter of grain size is placed in a plastic volume of 14*14*30 cm. One liter of distillated water is also added to the sand for its moisture content to become maximal and equal to 20%. The volume with wet sand is placed into a heat isolating cover for one direction freezing, and thermal probes at levels of -20, -10, and 0 cm from the upper sand surface level are also placed. The installation is placed in the refrigerated chamber with a negative temperature of -5°C. As the upper sand surface is cooled down, the coarse grained snow with an initial density of 0.3 g/cm³ is placed on the top of the sand, and a thermal probe at a level of +5 cm from the upper sand surface is placed close to the upper outer snow surface.

While cooling of the installation, a phase transition on the lower snow - upper ground surface takes place. Further, at the assumed heat losses of the sample at one direction freezing (1 J/s) the calculated time of freezing of the whole ground sample is 90 hours. In reality the phase transition in the ground sample finished in 84 hours. This could be explained by the heat losses through the sidewalls of the installation that were not accounted for. Therefore, the three-layers-calculating-scheme estimations are validated with experimental observations.

3. Results

The calculations of ground freezing depth were made according to the above-constructed calculating scheme on the basis of a heat conduction Stephan problem with multilayer heat conductivity. For this
reason, on the basis of knowledge about winter snowfall frequency and intensity, generalized snow stratigraphy columns for each winter season were compiled, and the heat conductivity of the multilayer system was determined.

The calculations were made with a step-size of one day. For initial conditions, it was supposed that the frozen ground thickness $l_{fg}$ was equal to 0.5 cm. For each time step (for each day) the rate of movement of the freezing interface $V$ and the frozen ground thickness $l_{fg}$ for the next day (time-step) were calculated.

According to [7], the averaged heat conductivity of thawed and frozen ground was assumed to be equal to 1.5 and 1.8 W/m °C, respectively. The results of the calculations by the estimation scheme have shown general consistency of the calculated ground freezing depth values with the observed ones. The correlation coefficient is equal to 0.76-0.77, which is quite good for such a simple calculation scheme (see Figure 4).

![Figure 4. Correlation of observed and estimated ground freezing depth in Narayan-Mar in 1988-2008.](image)

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