PHOTON-INDUCED ENTANGLEMENT OF DISTANT MESOSCOPIC SQUID RINGS

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Abstract  
An experiment that involves two distant mesoscopic SQUID rings is studied. The superconducting rings are irradiated with correlated photons, which are produced by a single microwave source. Classically correlated (separable) and quantum mechanically correlated (entangled) microwaves are considered, and their effect on the Josephson currents is quantified. It is shown that the currents tunnelling through the Josephson junctions in the distant rings, are correlated.

Keywords: Entanglement, nonclassical microwaves, mesoscopic SQUID, Josephson devices.

1. Introduction

A fundamental property of superconducting quantum interference devices (SQUIDs) is that they exhibit quantum coherence at the macroscopic level [1]. This property may be used for the purposes of quantum information processing [2, 3].

A lot of research on superconducting devices investigates their interaction with classical microwaves. On the other hand the use of nonclassical microwaves makes the system fully quantum mechanical and interesting quantum phenomena arise. For example, in this paper we show that entangled two-mode microwaves produce correlated currents in distant SQUID rings.

Nonclassical electromagnetic fields at low temperatures \((k_B T \ll \hbar \omega)\) have been studied for more than twenty years both theoretically and experimentally [4]. The interaction of SQUID rings with nonclassical microwaves has been studied in the literature [5, 6].

In previous publications [7] we have studied the effects of entangled electromagnetic fields on distant electron interference experiments. In this paper we review and extend further this work in the context of SQUID rings. We
consider two mesoscopic SQUID rings, which are far from each other and are irradiated with entangled microwaves, produced by a single source (Fig. 1). It is shown that the Josephson currents in the distant SQUID rings are correlated. The photon-induced correlations between the currents are quantified. It is shown that the current correlations depend on whether the photons are classically correlated (separable) or quantum mechanically correlated (entangled). The difference between separable and entangled microwave density matrices [8] is in the nondiagonal elements; and the effect of these nondiagonal elements on the Josephson currents is explicitly calculated.

2. Interaction of a single SQUID ring with nonclassical microwaves

In this section we consider a single SQUID ring and study its interaction with both classical and nonclassical microwaves.

For irradiation with classical microwaves, the Josephson current is
\[ I_A = I_1 \sin \theta_A \], where \( \theta_A = 2e\Phi_A \) is the phase difference across the junction due to the total flux \( \Phi_A \) through the ring. We assume the external field approximation, where the back reaction (the additional flux induced by the SQUID ring current) is neglected; i.e., the flux \( L I_A \), where \( L \) is the self-inductance of the ring, is negligible in comparison to \( \Phi_A \). The magnetic flux has a linear and a sinusoidal component:
\[ \Phi_A = V_A t + \phi_A-m(\omega_1 t) \].

Consequently the observed current is
\[ I_A = I_1 \sin(\omega_A t + q \exp(i\omega t)) \]; \( \omega_A = 2eV_A \).

We now consider the interaction of a SQUID ring with nonclassical microwaves, that are carefully prepared in a particular quantum state and are described by a density matrix \( \rho \). The dual quantum variables of the nonclassical field are the vector potential \( A_i \) and the electric field \( E_i \). Integrating these over the SQUID ring we obtain the magnetic flux and the electromotive force operators
\[ \hat{\phi} = \oint_C A_i dx_i, \hat{V}_{EMF} = \oint_C E_i dx_i \].

In the external field approximation the flux operator evolves as
\[ \hat{\phi}(t) = \xi^{-1/2}[\hat{a} \exp(i\omega t) + \hat{a}^\dagger \exp(-i\omega t)], \]
where \( \xi \) is a parameter proportional to the area of the SQUID ring and the \( \hat{a}^\dagger, \hat{a} \) are the photon creation and annihilation operators. Consequently the phase difference \( \theta_A \) is the operator
\[ \hat{\theta_A} = \omega_A t + q[\hat{a}^\dagger \exp(i\omega t) + \hat{a} \exp(-i\omega t)], \quad q = \sqrt{2e}\xi; \]
and the current also becomes an operator,
\[ \hat{I}_A \] = \( I_1 \sin \hat{\theta}_A \). Expectation values of the current are calculated by taking its trace with respect to the density matrix \( \rho \), which describes the nonclassical electromagnetic fields,
\[ \langle \hat{I}_A \rangle = \text{Tr}(\rho \hat{I}_A) = I_1 \text{Im}[\exp(i\omega_A t)\hat{W}(\lambda_A)], \]
\[ \lambda_A = iq \exp(i\omega_A t). \]
$W(x)$ is the Weyl function [9] given by

$$W(x) = \text{Tr}[\rho D(x)]; \quad D(x) = \exp(x\hat{a}^\dagger - x^*\hat{a})$$

(7)

where $D(x)$ is the displacement operator. Higher moments of the Josephson current quantify the quantum statistics of the electron pairs tunnelling through the junction.

Figure 1. Two distant mesoscopic SQUID rings A and B are irradiated with nonclassical microwaves of frequencies $\omega_1$ and $\omega_2$, correspondingly. The microwaves are produced by the source $S_{BM}$ and are correlated. Classical magnetic fluxes $V_A t$ and $V_B t$ are also threading the two rings A and B, correspondingly.

3. Interaction of two distant SQUID rings with entangled microwaves

In this section we consider two mesoscopic SQUID rings far apart from each other, which we refer to as A and B (Fig. 1). They are irradiated with correlated microwaves. Let $\rho$ be the density matrix of the microwaves, and $\rho_A = \text{Tr}_B\rho$, $\rho_B = \text{Tr}_A\rho$, the density matrices of the microwaves interacting with the two SQUID rings A, B, correspondingly. When the density matrix $\rho$ is factorizable as $\rho = \rho_A \otimes \rho_B$, the two modes are not correlated. If it can be written as $\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$, where $p_i$ are probabilities, it is called separable and the two modes are classically correlated. Density matrices which cannot be written in one of these two forms are entangled (quantum mechanically correlated) [8].

The currents in the two SQUIDs are

$$\langle \hat{I}_A \rangle = I_1 \text{Tr}(\rho_A \sin \theta_A), \quad \langle \hat{I}_B \rangle = I_2 \text{Tr}(\rho_B \sin \theta_B).$$

(8)

The expectation value of the product of the two current operators is given by:

$$\langle \hat{I}_A \hat{I}_B \rangle = I_1 I_2 \text{Tr}(\rho \sin \theta_A \sin \theta_B).$$

(9)

In general $\langle \hat{I}_A \hat{I}_B \rangle$ is different from $\langle \hat{I}_A \rangle \langle \hat{I}_B \rangle$ and we calculate the ratio

$$R = \frac{\langle \hat{I}_A \hat{I}_B \rangle}{\langle \hat{I}_A \rangle \langle \hat{I}_B \rangle}.$$ 

(10)
For factorizable density matrices \( \rho_{\text{fact}} = \rho_A \otimes \rho_B \) we easily see that \( R_{\text{fact}} = 1 \).

For separable density matrices \( \rho_{\text{sep}} \) the ratio \( R_{\text{sep}} \) is not necessarily equal to one and numerical results for various examples are shown below.

We also calculate the second moments

\[
\langle \hat{I}_A^2 \rangle = I_A^2 \text{Tr}[\rho_A \sin^2 \theta_A], \quad \langle \hat{I}_B^2 \rangle = I_B^2 \text{Tr}[\rho_B \sin^2 \theta_B].
\] (11)

The statistics of the photons affects the statistics of the tunnelling electron pairs, which is quantified with the \( \langle \hat{I}_A \hat{I}_B \rangle, \langle \hat{I}_A^2 \rangle, \langle \hat{I}_B^2 \rangle \) (and also with the higher moments).

### 3.1 Microwaves in number states

We consider microwaves in the separable (mixed) state

\[
\rho_{\text{sep}} = \frac{1}{2} (|N_1 N_2\rangle \langle N_1 N_2| + |N_2 N_1\rangle \langle N_2 N_1|),
\] (12)

where \( N_1 \neq N_2 \). We also consider microwaves in the entangled state \( |s\rangle = 2^{-1/2}(|N_1 N_2\rangle + |N_2 N_1\rangle) \), which is a pure state. The density matrix of \( |s\rangle \) is

\[
\rho_{\text{ent}} = \rho_{\text{sep}} + \frac{1}{2} (|N_1 N_2\rangle \langle N_2 N_1| + |N_2 N_1\rangle \langle N_1 N_2|),
\] (13)

where the \( \rho_{\text{sep}} \) is given by Eq. (12). It is seen that the \( \rho_{\text{ent}} \) and the \( \rho_{\text{sep}} \) differ only by the above nondiagonal elements.

In this example, the reduced density matrices are the same for both the separable and entangled states:

\[
\rho_{\text{sep},A} = \rho_{\text{ent},A} = \rho_{\text{sep},B} = \rho_{\text{ent},B} = \frac{1}{2} (|N_1\rangle \langle N_1| + |N_2\rangle \langle N_2|).
\] (14)

Consequently in this example \( \langle \hat{I}_A \rangle_{\text{sep}} = \langle \hat{I}_A \rangle_{\text{ent}}, \) and also \( \langle \hat{I}_B \rangle_{\text{sep}} = \langle \hat{I}_B \rangle_{\text{ent}}. \)

For the density matrix \( \rho_{\text{sep}} \) of Eq. (12) we find

\[
\langle \hat{I}_A \rangle = \frac{I_A}{2} \exp \left( \frac{-q^2}{2} \right) [L_{N_1}(q^2) + L_{N_2}(q^2)] \sin(\omega_A t),
\] (15)

\[
\langle \hat{I}_B \rangle = \frac{I_B}{2} \exp \left( \frac{-q^2}{2} \right) [L_{N_1}(q^2) + L_{N_2}(q^2)] \sin(\omega_B t),
\] (16)

where the \( L_n(x) \) are Laguerre polynomials. The currents \( \langle \hat{I}_A \rangle, \langle \hat{I}_B \rangle \) are in this example independent of the microwave frequencies \( \omega_1, \omega_2. \)

The expectation value of the product of the two currents [Eq. (9)] is

\[
\langle \hat{I}_A \hat{I}_B \rangle_{\text{sep}} = I_1 I_2 \exp(-q^2) L_{N_1}(q^2) L_{N_2}(q^2) \sin(\omega_A t) \sin(\omega_B t).
\] (17)

Consequently the ratio \( R \) of Eq. (10) is

\[
R_{\text{sep}} = \frac{4L_{N_1}(q^2) L_{N_2}(q^2)}{(L_{N_1}(q^2) + L_{N_2}(q^2))^2}.
\] (18)

In this example the \( R_{\text{sep}} \) is time-independent.
Entanglement of distant SQUID rings

The moments of the currents, defined by Eq. (11), are also calculated:

\[
\langle \hat{I}_A^2 \rangle = \frac{I_A^2}{2} \left\{ 1 - \frac{1}{2} \exp(-2q^2) [L_{N_1}(4q^2) + L_{N_2}(4q^2)] \cos(2\omega_A t) \right\},
\]

\[
\langle \hat{I}_B^2 \rangle = \frac{I_B^2}{2} \left\{ 1 - \frac{1}{2} \exp(-2q^2) [L_{N_1}(4q^2) + L_{N_2}(4q^2)] \cos(2\omega_B t) \right\}.
\]

For the case of \( \rho_{\text{ent}} \) the \( \langle \hat{I}_A \rangle, \langle \hat{I}_B \rangle \) are the same as in Eqs. (15), (16); and the \( \langle \hat{I}_A^2 \rangle, \langle \hat{I}_B^2 \rangle \) are the same as in Eqs. (19), (20). However the \( \langle \hat{I}_A \hat{I}_B \rangle \) is

\[
\langle \hat{I}_A \hat{I}_B \rangle_{\text{ent}} = \langle \hat{I}_A \hat{I}_B \rangle_{\text{sep}} + I_{\text{cross}},
\]

where

\[
I_{\text{cross}} = -\frac{I_A I_B}{2} \exp(-q^2) L_{N_1}^{N_2-N_1}(q^2) L_{N_2}^{N_1-N_2}(q^2) \cos(\omega_A t + \omega_B t)
\]

\[-(-1)^{N_1-N_2} \cos(\omega_A t - \omega_B t) \cos(\Omega t),
\]

\[
\Omega = (N_1 - N_2)(\omega_1 - \omega_2).
\]

It is seen that the effect of entangled microwaves on Josephson currents is different from the effect of separable microwaves. In this case the ratio \( R \) of Eq. (10) is

\[
R_{\text{ent}} = R_{\text{sep}} + \frac{I_{\text{cross}}(t)}{\langle \hat{I}_A \rangle \langle \hat{I}_B \rangle},
\]

which is a time-dependent quantity oscillating around the \( R_{\text{sep}} \).

3.2 Microwaves in coherent states

We consider microwaves in the classically correlated state

\[
\rho_{\text{sep}} = \frac{1}{2}(|A_1 A_2 \rangle \langle A_1 A_2 | + |A_2 A_1 \rangle \langle A_2 A_1 |),
\]

where \( |A_1 \rangle \), \( |A_2 \rangle \) are microwave coherent states. We also consider the entangled state \( |u \rangle = |N \rangle (|A_1 A_2 \rangle + |A_2 A_1 \rangle) \), with density matrix

\[
\rho_{\text{ent}} = 2\mathcal{N}^2 \rho_{\text{sep}} + \mathcal{N}^2 (|A_1 A_2 \rangle \langle A_2 A_1 | + |A_2 A_1 \rangle \langle A_1 A_2 |),
\]

where the normalization constant is given by

\[
\mathcal{N} = \left[ 2 + 2 \exp \left( -|A_1 - A_2 |^2 \right) \right]^{-1/2}.
\]

For microwaves in the separable state of Eq. (25) the reduced density matrices are

\[
\rho_{\text{sep},A} = \rho_{\text{sep},B} = \frac{1}{2}(|A_1 \rangle \langle A_1 | + |A_2 \rangle \langle A_2 |),
\]

and hence the current in A is

\[
\langle \hat{I}_A \rangle_{\text{sep}} = \frac{I_A}{2} \exp(-q^2) \{ \sin[\omega_A t + 2q|A_1 | \cos(\omega_1 t - \theta_1)]
\]+ \sin[\omega_A t + 2q|A_2 | \cos(\omega_1 t - \theta_2)] \}.
\]
where \( \theta_1 = \arg(A_1) \), and \( \theta_2 = \arg(A_2) \). A similar expression yields the current in B. We have also calculated numerically the ratio \( R_{\text{sep}} \).

For microwaves in the entangled state of Eq. (26) the reduced density matrices are

\[
\rho_{\text{ent},A} = \rho_{\text{ent},B} = \mathcal{N}^2(A_1|A_1| + |A_2\rangle\langle A_2| + \tau|A_1\rangle\langle A_2| + \tau^*|A_2\rangle\langle A_1|),
\]

where \( \tau = \langle A_1|A_2\rangle = \exp(-|A_1|^2/2 - |A_2|^2/2 + A_1^*A_2) \).

The current in A is

\[
\langle I_A \rangle_{\text{ent}} = 2A^2(\langle I_A \rangle_{\text{sep}} + \mathcal{N}^2EF_1 \exp \left( -\frac{q^2}{2} \right) I_1),
\]

where \( E = \exp[-|A_1|^2 - |A_2|^2 + 2|A_1A_2|\cos(\theta_1 - \theta_2)] \), and

\[
F_1 = \left[ \exp(q|A_1|S_{A,1} - q|A_2|S_{A,2}) + \exp(-q|A_1|S_{A,1} + q|A_2|S_{A,2}) \right] \sin(\omega_At + q|A_1|C_{A,1} + q|A_2|C_{A,2}),
\]

with \( S_{A,1} = \sin(\omega_1t - \theta_1) \), \( S_{A,2} = \sin(\omega_2t - \theta_2) \), \( C_{A,1} = \cos(\omega_1t - \theta_1) \), \( C_{A,2} = \cos(\omega_2t - \theta_2) \). A similar expression yields the current in B, and we have also calculated numerically the ratio \( R_{\text{ent}} \).

### 3.3 Numerical results

In the numerical results of Figs. 2 and 3 the microwave frequencies are \( \omega_1 = 1.2 \times 10^{-4} \), \( \omega_2 = 10^{-4} \), in units where \( k_B = \hbar = c = 1 \). The critical currents are \( I_1 = I_2 = 1 \). The other parameters are \( \xi = 1 \), \( \omega_A = \omega_1 \), \( \omega_B = \omega_2 \), \( N_1 = 1 \), \( N_2 = 4 \), and \( \theta_1 = \theta_2 = 0 \). For a meaningful comparison between microwaves
Entanglement of distant SQUID rings

Figure 3. (a) $\langle \hat{I}_A \rangle_{\text{sep}} - \langle \hat{I}_A \rangle_{\text{ent}}$, and (b) $\langle \hat{I}_2^2 \rangle_{\text{sep}} - \langle \hat{I}_2^2 \rangle_{\text{ent}}$ against $(\omega_1 - \omega_2)t$ for the coherent state $\rho_{\text{sep},A}$ of Eq. (28) and $\rho_{\text{ent},A}$ of Eq. (30) with $A_1 = 1, A_2 = 2$. The photon frequencies are $\omega_1 = 1.2 \times 10^{-4}$ and $\omega_2 = 10^{-4}$, in units where $k_B = \hbar = c = 1$.

4. Discussion

We have considered the interaction of two distant SQUID rings A and B with two-mode nonclassical microwaves, which are produced by the same source. The flux, the phase difference and the Josephson currents are operators and their expectation values with the density matrix of the nonclassical microwaves give the physically observed quantities. We have assumed the external field approximation, where the electromagnetic field created by the Josephson currents (back reaction) is neglected and we have calculated various quantities.

It has been shown that the Josephson currents in the two rings are correlated in the sense that $\langle I_A I_B \rangle$ is different from $\langle I_A \rangle \langle I_B \rangle$ (for non-factorizable
We have considered examples where the photons are classically correlated and quantum mechanically correlated; and we have shown that the non-diagonal terms in the latter case affect the Josephson currents. Further work in this direction could be the formulation of Bell-type inequalities for the Josephson currents, which are obeyed in the case of separable microwaves and violated in the case of entangled microwaves.
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