A Proposed Alternative Low Energy Quantum Field Theory of Gravity Based on a Bose-Einstein Condensate Effect

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Abstract

An alternative quantum field theory for gravity is proposed for low energies based on an attractive effect between contaminants in a Bose-Einstein Condensate rather than on particle exchange. In the “contaminant in condensate effect,” contaminants cause a potential in an otherwise uniform condensate, forcing the condensate between two contaminants to a higher energy state. The energy of the system decreases as the contaminants come closer together, causing an attractive force between contaminants. It is proposed that mass-energy may have a similar effect on Einstein’s space-time field, and gravity is quantized by the same method by which the contaminant in condensate effect is quantized. The resulting theory is finite and, if a physical condensate is assumed to underly the system, predictive. However, the proposed theory has several flaws at high energies and is thus limited to low energies. Falsifiable predictions are given for the case that the Higgs condensate is assumed to be the condensate underlying gravity.
1 Proposed Theory on a Scalar Field

Within Bose-Einstein Condensates [1], the authors predict in a separate paper [2] the existence of a new effect which causes an attractive force between two contaminants, the “contaminant in condensate” (CIC) effect. It is proposed that contaminants act as a potential within the condensate. This causes the condensate in between two contaminants to jump to a higher energy state than if no contaminants existed. By assuming that the condensate behaves as a massive scalar field governed by:

\[
\frac{1}{c^2} \frac{\partial^2 \varphi(t, x)}{\partial t^2} - \frac{\partial^2 \varphi(t, x)}{\partial x^2} + \frac{m^2 c^2}{\hbar^2} \varphi(t, x) = 0
\]  

(1)

with induced standing waves between contaminants governing the condensate superstate given by:

\[
\varphi_n^{(\pm)}(t, x) = \sqrt{\frac{c}{a \omega_n}} e^{\pm i \omega_n t} \sin k_n x
\]  

(2)

\[
\omega_n = \sqrt{\frac{m^2 c^4}{\hbar^2} + c^2 k_n^2}
\]  

(3)

\[
k_n = \frac{\pi n}{a}, n = 1, 2, ..., \]

(4)

the expectation value of energy associated with the superstate over all energy levels is determined to be:

\[
E(a) \approx - \frac{mc^2}{4} - \frac{\pi \hbar c}{24a} + \frac{\hbar c}{23 \pi a} \mu^2 \ln \mu
\]  

(5)

with \(\mu \equiv mca/\hbar\) in the case \(\mu \ll 1\). Note for a massless field this becomes:

\[
E(a) = - \frac{\pi \hbar c}{24a}
\]  

(6)

These results were derived by the same method by which the Casimir effect is derived [3].
However, in a physical Bose-Einstein condensate, energy levels are so low that, as argued in [2], induced superstates are likely always in their lowest energy state available. In order to make a more accurate model of the force associated with the CIC effect, one must therefore find the energy associated with the creation of a superstate and its change to a different size. To simplify both the scattering calculations and the creation of an S-matrix to describe the CIC effect, the approach taken to determining the energy of an induced superstate is to associate a scalar particle propagator with the condensate superstate:

\[
\frac{1}{k^2 - m^2 + i\varepsilon}. \tag{7}
\]

Again, as all energy states are not integrated over as in the Casimir effect, it is safe to manipulate one state at a time in calculations.

The superstate “propagates” in the space of distances rather than physical space however. That is, a superstate is said to “propagate” from one distance to another, as describing a condensate superstate by a single point would incompletely describe its position. Taking into account this philosophical point, the standard machinery of QFT is used. A force between two particles is then produced by the creation of a superstate and its movement. For a massless field, this results in a force:

\[
F = -\frac{1}{4\pi a^2} \tag{8}
\]

as usual [4] [5].

Note two particles can occupy the same point in distance-space, preventing superstate interaction, and the superstate does not need to interact with any particles to create a force. As this occurs in the CIC effect, it thus fulfills two physical requirements that are required of a QFT of the CIC effect.

Also it should be noted that with no interactions among particles allowed, the Feynman rules for our theory are trivial.
A finite quantum field theory has thus been defined, essentially by fiat. All interactions which could cause a divergence have been eliminated as only creation, propagation, and annihilation are allowed. The model can further be extended to a spin-2, massless tensor field to find a finite, though not predictive, quantum theory of gravity, as will be demonstrated below.

In condensates, though, only energy outside of the condensate will serve as a potential. Thus if a physical condensate is used as the source of gravity in our quantum field theory, higher order self-interaction terms can be ignored as unphysical. A predictive theory of gravity can thus be created. (Note that if the Higgs condensate is assumed to underly gravity as will be the case in section (3), this model would explain why the Higgs condensate has no apparent “weight” and its energy density is not observed, a problem noted in [6].)

In order to preserve relativity, all particles interacting through a condensate must be separated by either a time-like distance or light-like distance. Also, the energy for the creation of a superstate of many particles comes from each individual particle, lowering the temperature of the system as a whole.

The resulting theory starts with the graviton propagator in the harmonic gauge as usual [4], but redefines it in distance space so it can apply to a particle superstate rather than a particle. This gives:

\[
D_{\mu\nu,\lambda\sigma}(k) = \frac{1}{2} \frac{\eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\sigma}}{k^2 + i\varepsilon} \tag{9}
\]

where \(\eta\) is the Minkowski metric and \(g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\) where \(h_{\mu\nu}\) are deviations from the Minkowski metric. This couples to the stress-energy tensor \(T^{\mu\nu}\) defined according to the variation of the matter action \(S_M\) by:

\[
T^{\mu\nu}(x) = -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}(x)} \tag{10}
\]

and gives the scattering amplitude:
\[ GT_{(1)}^{\mu\nu}D_{\mu\nu,\lambda\sigma}(k)T_{(2)}^{\lambda\sigma} = \frac{G}{2k^2}(2T_{(1)}^{\mu\nu}T_{(2)\mu\nu} - T_{(1)}T_{(2)}). \]  

(11)

Between non-relativistic matter, this becomes \( \frac{G}{2k^2}T_{(1)}^{00}T_{(2)}^{00} \). As usual, the Fourier transform gives the interaction potential:

\[ G \int \int d^3x d^3x' T^{(1)00}(x)T^{(2)00}(x') \int d^3k e^{i \mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')} \frac{1}{k^2} \]

(12)

where a change is made from the graviton-exchange model and one integrates over distances rather than positions. This reduces to the Newtonian potential \( \frac{GM_1 M_2}{r} \).

This is an identical result to a graviton-exchange model [4], but interactions which cause divergences are not predicted.

It should be noted that the above model bears resemblance to Sakharov’s “Induced Gravity” model [7], as both speculate gravity to arise from underlying quantum fluctuations rather than as a fundamental force. However, the mechanism by which this is thought to occur is different in the model above. Vacuum energy is not presumed as a basis for gravity. Rather, superstates of a physical condensate mediate the gravitational interaction.

### 3 Physical Predictions

In order for a theory of gravity to exist which works by the massless mechanism in section (2), there must exist a field associated with a spin-2, massless, tensor particle that energy forms a potential in (perhaps associated with the Einstein’s space time field). A condensate of these particles would then be sufficient to cause a gravity-like interaction. The theory thus requires and therefore predicts the existence of some form of massless condensate in which energy forms a potential.

The theory would also predict that there is no self-interaction correction to
the strength of the gravitational interaction. However, this prediction would only occur at energies currently not testable.

Alternatively, a condensate associated with gravity consisting of massive particles, such as the Higgs condensate, would produce testable effects. Note that there is nothing preventing a condensate of massive particles from producing an effect such as that described above which could be associated with gravity. This is because the information about a superstate would still travel at the speed of light. There are two chief effects predicted in this case, which are described below.

It can be heuristically argued that the CIC effect in a condensate composed of a massive field should behave no differently than in a condensate composed of a massless field. This is because of the argument that information about the condensate is massless, and thus the CIC effect would still behave as though it were occurring in a massless medium. The results below disregard this argument and strictly follow the mathematics of the proposal.

### 3.1 A Universal Repulsive Force and its role in Early Universe Cosmology

As described in section (1), the energy caused by two contaminants in a massive, scalar condensate is equal to, if we sum over all excitation modes of a potential supersate:

\[
E(a) \approx -\frac{mc^2}{4} - \frac{\pi \hbar c}{24a} + \frac{\hbar c}{23\pi a} \mu^2 \ln \mu. \tag{13}
\]

This results in a force including arbitrary constants \(b_n\):

\[
F(a) \approx -\frac{b_a}{a^2} - b_1 \ln b_2 a - b_3. \tag{14}
\]

In the early universe, uniform extreme high energy conditions could potentially cause induced superstates to obtain higher energies. It is thus proposed
that the repulsive component of equation (14) may play a role in inflationary cosmology. If the Higgs condensate induces a gravity-like effect in the CIC mechanism, then the following inflationary potential is predicted to have occurred after symmetry breaking:

\[ V(\phi) = b_1 \phi \ln(b_2 \phi) + b_3. \]  

(15)

This results in “slow roll” parameters [8]:

\[ \epsilon = \frac{m_{pl}^2}{16\pi} \cdot \frac{V'(\phi)}{V(\phi)} = \frac{m_{pl}^2}{16\pi} \cdot \frac{b_1 \ln(b_2 \phi) + b_1}{\phi \cdot b_1 \ln(b_2 \phi) + b_3} \]  

(16)

\[ \eta = \frac{m_{pl}^2}{8\pi} \cdot \frac{V''(\phi)}{V(\phi)} = \frac{m_{pl}^2}{8\pi} \cdot \frac{b_1}{\phi^2 \cdot b_1 \ln(b_2 \phi) + b_3} \]  

(17)

and a predicted number of e-foldings [9]:

\[ N = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{V(\phi)}{V'(\phi)} d\phi = \int_{\phi_i}^{\phi_f} \frac{\phi \cdot b_1 \ln(b_2 \phi) + b_3}{b_1 \ln(b_2 \phi) + b_1} d\phi. \]  

(18)

### 3.2 Universal Repulsive Force in the present day?

As there is no reason for this potential to disappear after the inflationary period (when the potential energy of the field predominates over its kinetic energy) is over, there should thus be a universal repulsive force between particles of order \(O(\ln a)\) presently. However, if the assumption that a condensate will always be in its lowest energy state available is used, which is perhaps more accurate, then this effect is not predicted. In fact, the energy of massive condensate superstate creation and movement is predicted by the path integral method to be \(-\frac{1}{4\pi a} e^{-ma}\) which is clearly a physically untenable potential.

It is still proposed however, that this potential force, with an appropriately small coupling constant, could be a physical justification for the apparent cosmological constant [10]. However, for the proposed repulsive force to be
produced by this mechanism, the perhaps unphysical assumption that a permeating condensate exists in arbitrarily high energy states must be made. Also, unfortunately, we have found no satisfactory method to incorporate this effect in Einstein’s equation as of the time that this is being written. This is a major flaw in the theory of a massive condensate inducing gravity and it may well be that there is no method of successfully incorporating it into Einstein’s equation. It is presently the subject of ongoing work. It does, though, seem promising that some form of potentially testable prediction can be made for certain variations of a gravity as CIC effect theory.

4 Conclusion

We have attempted to show in as brief and straightforward a manner possible that if there exists a field (such a spin-2 tensor field or the Higgs field) associated with a particle that forms a condensate which permeates space and in which mass-energy forms a potential, a finite, predictive quantum field theory of gravity can be developed by assuming gravity to be a CIC effect in the condensate. A CIC effect in the Higgs condensate could produce a finite, predictive quantum field theory of gravity with falsifiable predictions, primarily a universal repulsive force of order $O(\ln a)$. However, there are serious problems with incorporating the results of the predictions with Einstein’s equations.

Unfortunately, there are inherent problems with the CIC approach approach at high energies, which is why it is only proposed as a low energy theory. First, since predicted effects at very high, early universe energies cannot be incorporated into Einstein’s equations at this time, it does not seem to be reducible to Einstein’s theory. Second, it is not background independent.
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