Landmark Map: An Extension of the Self-Organizing Map for a User-Intended Nonlinear Projection

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Abstract

The self-organizing map (SOM) is an unsupervised artificial neural network that is widely used in, e.g., data mining and visualization. Supervised and semi-supervised learning methods have been proposed for the SOM. However, their teacher labels do not describe the relationship between the data and the location of nodes. This study proposes a landmark map (LAMA), which is an extension of the SOM that utilizes several landmarks, e.g., pairs of nodes and data points. LAMA is designed to obtain a user-intended nonlinear projection to achieve, e.g., the landmark-oriented data visualization. To reveal the learning properties of LAMA, the Zoo dataset from the UCI Machine Learning Repository and an artificial formant dataset were analyzed. The analysis results of the Zoo dataset indicated that LAMA could provide a new data view such as the landmark-centered data visualization. Furthermore, the artificial formant data analysis revealed that LAMA successfully provided the intended nonlinear projection associating articular movement with vertical and horizontal movement of a computer cursor. Potential applications of LAMA include data mining, recommendation systems, and human-computer interaction.

Keywords: Landmark map, Self-organizing map, Semi-supervised learning,

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1. Introduction

The self-organizing map (SOM), proposed by Teuvo Kohonen, is a type of artificial neural network that provides a nonlinear projection from a high-dimensional space to a low-dimensional discrete space [1, 2]. The SOM is trained by unique unsupervised learning using its own architecture of nodes, which is also referred to as the topology-preserving mapping algorithm [3, 4]. It is often used to visualize relationships of data via a two-dimensional contour plot called the unified distance matrix (U-matrix) [5, 6]. Owing to its convenience of visualization, the SOM has been used for data mining, especially for clustering [7]. For example, it has been used for the clustering of gait kinematics [8], farm profitability [9], and groundwater quality [10]. In addition, SOMs have been applied to classification [11]. For instance, they have been used to classify electroencephalography (EEG) signals in applications of human-computer interaction (HCI) such as a brain-computer interface (BCI) [12]. Furthermore, SOMs have also been utilized for facial recognition [13], hand and body tracking [14], gesture recognition [15], fault detection [16], mental task classification of EEG for BCI [17], EEG-based emotion recognition [18], emotion recognition using geometric facial features [19], and hazard detection of motorcycles [20]. Although SOM is useful for mining unlabeled data because of its unsupervised learning, they do not take advantage of labeled or partly labeled data in supervised or semi-supervised learning.

Some supervised and semi-supervised learning methods have been proposed for SOMs. In early studies, the class label was concatenated to the input data [21, 22, 23]. Kohonen et al. adopted learning vector quantization (LVQ), which updates only the winner node, and then fine-tuned the SOM [24]. Hagenbuchner et al. proposed a supervised SOM that assigns labels to neurons and then rejects the projected data if it has a different label [25]. Shen et al. proposed a three-
layer semi-supervised learning method for SOMs [26]. The method first trains SOM on both labeled and unlabeled data. Next, it labels nodes and trains a classifier using labeled data. Herrmann et al. introduced Zhu’s label propagation method for the SOM [27]. Thus, regardless of the relationships between the nodes and the data, many supervised and semi-supervised learning algorithms using class labels have been proposed for the SOM. However, these methods do not focus on clarifying the relationship between the original data and the location of projected nodes. If we have a clear hypothesis or objective for the projection, the data and nodes of the SOM can be associated as teachers during learning.

To obtain a user-intended nonlinear projection, this study proposes a landmark map (LAMA), which is an extension of the SOM learned by landmarks, i.e., several pairs of data and associated nodes. Such landmarks are implemented as landmark nodes having landmark data. By an alternating update method using the given data and landmark data, the LAMA is trained so that the data projected onto the landmark nodes approach the landmark data, while the properties of the SOM are retained as far as possible. To reveal the difference between the learning properties of the LAMA and the SOM, they are evaluated using two datasets, namely the Zoo dataset (UCI Machine Learning Repository, University of California, Irvine, CA, US) and an artificial formant dataset.

The LAMA differs from previously proposed supervised and semi-supervised SOM methods in that it uses several landmarks instead of teacher labels. This implies that the LAMA directly combines a small number of data and nodes as teachers without using traditional class labels. In addition, the LAMA is designed to obtain a user-intended nonlinear projection by setting several landmark nodes. On the other hand, the previously proposed supervised and semi-supervised SOM aims for classification, clustering, and dimensionality reduction. The LAMA is not designed to address those traditional supervised or unsupervised learning problems but addresses the newly defined problem. Due to the differences in the objectives and problem settings, a comparison between LAMA
and the proposed supervised or semi-supervised SOM is not straightforward. In this study, LAMA was compared with the unsupervised online SOM in terms of learning properties seen mainly in the input space.

2. Landmark map

2.1. Artificial neural network architecture of LAMA

LAMA is a type of artificial neural network that projects input data from a continuous input space to a discrete output space. LAMA learns its projection preserving arrangement of nodes in the output space (topology) and retains relationships between the landmark data in the input space and the landmark nodes in the output space. In other words, LAMA is an extension of SOMs with several landmarks that indicate connections between the landmark data and the landmark nodes. LAMA can be applied to obtain a nonlinear projection from higher-dimensional space to a lower-dimensional output space. In addition, it can provide a modified nonlinear projection between the same-dimensional input and output space.

LAMA consists of numerous standard nodes and several landmark nodes in an output layer in addition to an input node in an input layer (see Fig. 1(1)). The total number of nodes is K. The input node is connected to all the nodes in the output layer. Both standard and landmark nodes contain a codebook vector \( w_k \in \mathbb{R}^D \) and a location vector \( v_k \in \mathbb{R}^{D'} \), \( k \in \{0, 1, ..., K-1\} \) (see the size of the variables in Fig. 2). Given N data points \( x_n \in \{0, 1, ..., N-1\} \), M landmark nodes that is associated with the landmark data \( x'_m \in \{0, 1, ..., M-1\} \), and M landmark labels \( l_m \in \{0, 1, ..., K-1\} \), all the codebook vectors \( w_k, \forall k \) are trained. The combined data, codebook vectors, location vectors, landmark data, and landmark labels are denoted by \( X = (x_0, ..., x_{N-1}) \), \( W = (w_0, ..., w_{K-1}) \), \( V = (v_0, ..., v_{K-1}) \), \( X' = (x'_0, ..., x'_{M-1}) \), and \( l = (l_0, ..., l_{M-1})^T \), respectively, where \( .^T \) indicates the transpose.

The locations of the standard and landmark nodes \( V \) are structurally arranged in a discrete \( D' \)-dimensional output space (see Fig. 1(1)). For ex-
Figure 1: Overview of the landmark map (LAMA). (1) Artificial neural network architecture of the LAMA. This consists of standard nodes and landmark nodes in the output layer in addition to an input node in the input layer. The nodes in the output layer are regularly located on a grid of the output space. They have location vectors that indicate the positions of the nodes in the output space, and codebook vectors to be trained. The codebook vector indicates a location in the input space. Only the landmark nodes are associated with landmark data, which is used for semi-supervised learning. The codebook vectors are trained on data in the data-driven learning phase or landmark data in the landmark-driven phase. (2) Relationship of data, landmark data, and codebook vectors in input space. After completing the training of the LAMA, the codebook vectors cover the given data, retaining the relationships between the neighboring nodes in the output space. Note that the codebook vector of the landmark node (node 10) is the closest to the landmark data in the input space. A data point in the input space is projected onto a node in the output space, which has the closest codebook vector to the data in the input space. Thus, the landmark nodes and the landmark data teach the LAMA the user-intended nonlinear projection.

For example, 16 nodes are arranged in a $4 \times 4$ square architecture ($K = 16, k \in \{0, 1, ..., K - 1\}, (K_x \times K_y) = (4 \times 4)$) such that the distance from the neighboring nodes is 1 (no unit). In this example, node 15 has a location vector $v_{15} = (3, 3)^T$. Typically, the output space is two-dimensional, and the nodes have a square or hexagonal architecture, which enables us to easily visualize the U-matrix, which is a two-dimensional color contour plot [6]. During learning the codebook vectors, the architecture is used, for example, to find neighboring
nodes and to determine the update rate of each node.

The codebook vectors $W$ can be represented in the input space together with data $X$ and landmark data $X'$ (see Fig. 1(2)). After appropriate learning, the codebook vectors are fitted to a set of data (sometimes called manifold fitting), retaining the architecture of the nodes in the output space. For instance, the codebook vector of node 0 is located next to that of node 1. As shown in Fig. 1(2), the red lines in the input space connect the codebook vectors of the nearest neighbor nodes in the output space. Thus, the codebook vectors are learned, retaining the relationships of the neighbor nodes in the output space.

An input data point is projected onto a node in the output space by finding the node that has the smallest Euclidean norm between the codebook vector and the input data in the input space. To realize the user-intended nonlinear projection, $W$ is trained so that the codebook vector of the $m$-th landmark node $w_{lm}$ becomes the closest codebook vector to a corresponding landmark data point $x'_m$ in the input space. In addition, the LAMA is updated by $X$ so
that $W$ is fitted to the set of data retaining the architecture of $V$. Such learning is realized by an alternating update method using data and landmark data.

2.2. Alternating update method

Figure 3 shows the flowchart of training the LAMA. First, all the variables and parameters are initialized. In this study, $W$ is initialized by a uniformly distributed random number ranging from 0 to 1 in order to observe the robustness of the initial values. Further, $W$ is iteratively updated in each discrete learning step $t \in \{0, 1, ..., t_{\text{max}} - 1\}$. In each $t$, data-driven or landmark-driven phases are alternately selected in the ratio $(1 - p_{\text{th}}) : p_{\text{th}}$, where $0 \leq p_{\text{th}} < 1$. However, a user-intended projection designated by the landmark nodes cannot be obtained by simply repeating the above-mentioned procedure, due to the complexity of the parameters. Different learning functions of the two phases seem to be useful for obtaining the projection.

2.2.1. Data-driven phase

The data-driven phase updates the codebook vectors such that they fit the entire dataset (see Fig. 4(1)). In the data-driven phase, a data point $n$ is randomly selected as the input of the LAMA. Then, the node that has the closest codebook vector to the input data is selected as the winner node. Thus, the winner node $k_d$ is found such that it minimizes the Euclidean norm between $x_n$ and $w_k$:

$$k_d(n) = \arg \min_k \|x_n - w_k\|^2,$$

where $\|\cdot\|$ represents the Euclidean norm. For simplicity, the node $k_d(n)$ is also denoted as $k_d$. Next, $W$ is updated by Hebbian learning considering the distance from the winner node to a focused node $k$ in the output space in addition to the learning step. Such learning is implemented by a learning rate function called the neighborhood function, denoted by $\alpha_k(t)$:

$$\alpha_k(t) = a(t) \cdot h_a(k_d, k, \sigma(t)), \forall k.$$
Figure 3: Flowchart of learning phases. The LAMA is trained by the data-driven phase or the landmark-driven phase alternately. One of the phases is selected in each learning step ($t$).

If a randomly generated $p$ is greater than $p_{th}$, the LAMA is trained by the data-driven phase (see Fig. 4(1)). If $p$ is less than $p_{th}$, the LAMA is updated by the landmark-driven phase (see Fig. 4(2)). This process is repeated $t_{max}$ times.

Here, $a(t)$ is an exponentially descending function of $t$ that determines the maximum learning rate:

$$a(t) = a_{min} + (a_{max} - a_{min}) \exp\left(-\frac{t}{\tau_a}\right),$$

(3)

where $a_{max}$, $a_{min}$, and $\tau_a$ denote the maximum, minimum, and the time decay of the learning rate, respectively; and $h_{a}(k_d,k,\sigma(t))$ determines the spread of the learning rate by the Gaussian distribution, considering the distance between
node $k$ and winner node $k_d$:

$$h_{\alpha}(k_d, k, \sigma(t)) = \exp \left( -\frac{\|v_{k_d} - v_k\|^2}{2\sigma^2(t)} \right).$$

(4)

Here, $\sigma(t)$ is the exponentially descending function of $t$ that determines the spread of the above-mentioned Gaussian distribution:

$$\sigma(t) = \sigma_{\text{min}} + (\sigma_{\text{max}} - \sigma_{\text{min}}) \exp \left( -\frac{t}{\tau_G} \right),$$

(5)

where $\sigma_{\text{max}}$, $\sigma_{\text{min}}$, and $\tau_G$ denote the maximum, minimum, and time decay of the extent, respectively. Therefore, $\alpha_k(t)$ decays as the distance from the winner node $k_d$ to node $k$ ($\|v_{k_d} - v_k\|^2$) increases, and as $t$ increases. Finally, $W$ is updated as follows:

$$w_k \leftarrow w_k + \alpha_k(t) \cdot (x_n - w_k), \forall k.$$

(6)

During the data-driven phase, the landmark node behaves in the same way as the standard node. This implies that the landmark node can be a winner node and its codebook vector is updated in this phase without using the landmark data or landmark labels.

2.2.2. Landmark-driven phase

On the other hand, the landmark-driven phase makes the codebook vector of the landmark node close to its landmark data (see Fig. 4(2)). In the landmark-driven phase, a landmark data point $m$ is randomly selected as the input of the LAMA. Next, the corresponding landmark node is assigned to a winner node ($k_l(m) = l_m$) regardless of the distance from the landmark data point. As with the data-driven phase, the neighborhood function $\beta_k(t)$ is used, which is independent of $\alpha_k(t)$:

$$\beta_k(t) = b(t) \cdot h_{\beta}(k_l, k, \rho(t)), \forall k.$$

(7)

Here, $b(t)$ is a Gaussian function of $t$ that determines the maximum learning rate:

$$b(t) = b_{\text{min}} + (b_{\text{max}} - b_{\text{min}}) \exp \left( -\frac{\|t - t_{\text{center}}\|^2}{2\rho_b^2} \right),$$

(8)
Figure 4: Alternating update method. (1) Data-driven phase. When the data-driven phase is activated, one data point from the given data is selected randomly. Next, the winner node, i.e., the node having the codebook vector that is the nearest to the data point, is chosen. Finally, all the codebook vectors are updated using the neighborhood function. (2) Landmark-driven phase. When the landmark-driven phase is performed, a landmark data point is randomly selected. Next, the node having the landmark data point is assigned as the winner node. Finally, all the codebook vectors are updated by the neighborhood function.

where \( b_{\text{max}} \) and \( b_{\text{min}} \) denote the maximum and minimum of the learning rate, respectively; \( t_{\text{center}} \) is the step that has the maximum learning rate; and \( \tau_b \) indicates the distribution of the above-mentioned Gaussian function over the steps. Further, \( h_b(k_l, k, \rho(t)) \) assigns the spread of the learning rate by the Gaussian distribution:

\[
h_b(k_l, k, \rho(t)) = \exp \left( -\frac{\|v_{k_l} - v_k\|^2}{2\rho^2(t)} \right),
\]

(9)

Here, \( \rho(t) \) is the exponentially descending function of \( t \) that determines the extent of the above-mentioned Gaussian distribution:

\[
\rho(t) = \rho_{\text{min}} + (\rho_{\text{max}} - \rho_{\text{min}}) \exp \left( -\frac{t}{\tau_{\rho}} \right),
\]

(10)

where \( \rho_{\text{max}} \), \( \rho_{\text{min}} \), and \( \tau_{\rho} \) denote the maximum, minimum, and time decay of the extent, respectively. Therefore, \( \beta_k(t) \) decays as the distance from the winner
node $k_i$ to node $k$ increases, and as $t$ increases. Finally, $W$ is updated as follows:

$$w_k \leftarrow w_k + \beta_k(t) \cdot (x'_m - w_k), \forall k.$$  

(11)

The landmark-driven and data-driven phases have different neighborhood functions that cope with the two types of learning. An example of their parameters is shown in Fig. 5 and Fig. 6. When $t = 0$, $a(t)$ is stronger than $b(t)$ (see the example in Fig. 5). When $t = t_{\text{center}} = 1500$, $b(t)$ is larger than $a(t)$. Then, they decay monotonically. In the end ($t = t_{\text{max}} - 1 = 59999$), $a(t)$ is stronger than $b(t)$ again. Further, $\sigma(t)$ and $\rho(t)$ decay monotonically with different parameters (see the example in Fig. 6). The function $\sigma(t)$ is stronger than $\rho(t)$ when $t = 0$; however, $\sigma(t)$ decays faster.

2.3. Visualization method

The learning properties of SOM and LAMA were compared by visualizing their codebook vectors and data in the input space. As the codebook vectors and data are typically in high-dimensional input space, the dimension was reduced to two or three for the visualization. Principal component analysis (PCA) was adopted, and the first three principal components were used for the visualization. The neighboring codebook vectors were connected by lines to clarify the relationship between the neighbors.

In addition, the data labels were projected onto a two-dimensional output space and then visualized by the U-matrix. This study used a simplified U-matrix defined as follows. First, a $K_x \times K_y$ matrix $U \in \mathbb{R}^{K_x \times K_y}$ was initialized. Each component of $U$ was associated with the nodes in the output space. Focusing on a node $k$ located at $(k_x, k_y)$ in the output space, four neighbors of the node, that is, nodes that have the minimum distance from the node $k$, were selected. Then, the summation of the Euclidean distance of the codebook vectors between node $k$ and the neighboring nodes was calculated. Assuming that the number of neighboring nodes is $N_n$ and the neighbor nodes are indexed
Figure 5: Example of $a(t)$ and $b(t)$. Maximum learning rate of the neighborhood function is defined by $a(t)$ and $b(t)$ in the data-driven and landmark-driven phases, respectively, where $a(t)$ is a monotonically descending exponential function and $b(t)$ is a Gaussian function having a peak in the middle of the learning steps.

as $k_n, n \in \{1, 2, ..., N_n\}$, the component of the U-matrix at $(k_x, k_y)$ is

$$u_{k_x, k_y} = \sum_{k_n=1}^{N_n} \|w_k - w_{k_n}\|^2.$$  

(12)

This simplified U-matrix can be visualized as a contour plot. The distance between the neighbors is visualized in color.

2.4. Numerical evaluation of error indices

To evaluate the goodness of the codebook vectors of the SOM numerically, quantization and topographic errors (TE) have been applied [28]. However, they
Figure 6: Example of $\sigma(t)$ and $\rho(t)$. The stretch of the learning rate in the neighborhood function is represented by $\sigma(t)$ and $\rho(t)$ in the data-driven and landmark-driven phases, respectively. They are monotonically descending exponential functions with different slopes.

do not always explain the error of LAMA because they do not take landmarks into consideration. This study employed extended error indices to clarify the learning properties of LAMA and SOM.

2.4.1. Quantization Error of Data

The quantization error of data (QED) represents the mean distance between each data point and the corresponding winner node in input space. The QED can be calculated as follows:

$$E_{QED} = \frac{1}{N} \sum_{n=1}^{N} \left\| x_n - w_{k_d(n)} \right\|,$$

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where the winner node of the \( n \)-th data point is denoted by \( k_d(n) \) (see Eq. 1). If the QED is small, the codebook vectors are successfully distributed over the data. The QED should be increased if part of the data is far from any codebook vectors.

2.4.2. Quantization Error of Landmark

The QED represents errors between data and codebook vectors. On the other hand, the quantization error of landmark (QEL) indicates errors between landmark data and corresponding codebook vectors. It can be estimated as follows:

\[
E_{QEL} = \frac{1}{M} \sum_{m=1}^{M} \left\| x_m - w_{k_l(m)} \right\|,
\]

where the winner node of the \( m \)-th landmark data point is denoted by \( k_l(m) \). The QEL should be small if landmark data and corresponding landmark nodes are closely located in the input space.

2.4.3. Square Topographic Error

The TE verifies how well the topology is preserved. In other words, the first-place winner node of the \( n \)-th data point \( k_d(n) \), and the second-place winner node of the \( n \)-th data point \( k'_d(n) \) should be adjacent to each other. It can be computed by a function that judges whether the nodes of \( k_d \) and \( k'_d \) are larger than the minimum distance of nodes:

\[
s_{TE}(n) = \begin{cases} 
1 : & \text{if } \left\| v_{k'_d(n)} - v_{k_d(n)} \right\| > D_{TE} + \epsilon, \\
0 : & \text{otherwise,}
\end{cases}
\]

where the minimum distance to the closest node in the output space is denoted by \( D_{TE} \); and \( \epsilon = 0.01 \) indicates a margin. In this study, \( D_{TE} = 1 \). Using that function, the TE can be estimated as follows:

\[
E_{TE} = \frac{1}{N} \sum_{n=1}^{N} s_{TE}(n).
\]

The lower the TE becomes, the more the topology is preserved.
In this study, the TE is extended to consider eight neighbors of each node using square topology. The extended error index is named the square topographic error (STE). Since the maximum distance of eight neighbors is $D_{STE} = \sqrt{2} \times D_{TE}$, $s_{TE}(n)$ was modified to consider $D_{STE}$ as follows:

$$s_{STE}(n) = \begin{cases} 1 : & \|v_{k'(n)} - v_{k(n)}\| > D_{STE} + \epsilon, \\ 0 : & \text{otherwise}. \end{cases}$$  \hspace{1cm} (17)$$

Instead of TE, the following STE was employed:

$$E_{STE} = \frac{1}{N} \sum_{n=1}^{N} s_{STE}(n).$$  \hspace{1cm} (18)$$

It becomes large if the second-place winner node of a focused node is selected out of eight neighbors.

3. Zoo dataset analysis

3.1. Dataset and parameters

To reveal the differences between the learning properties of LAMA and SOM, they were applied to the Zoo dataset of the UCI Machine Learning Repository (https://archive.ics.uci.edu/ml/datasets/zoo). The dataset contains 101 animal names with 17 attributes, such as whether they have hair, feathers, and lay eggs. Note that it contains two instances of “frog” and one instance of “girl.” The attributes are described using Boolean (0 or 1) or numeric (e.g., integer) form. In this study, 16 attributions, with the exception of the type, were used. This dataset was selected because a similar animal dataset was used for the benchmark of the SOM [1]. In addition, it may explains how to represent features of, for example, food, clothes, and other products in the market.

The learning properties of LAMA and SOM were assessed by visualizing the structure of the codebook vectors in the input space. Furthermore, the location of the data and the landmark data were displayed in a U-matrix. The detailed experimental parameters used for learning are presented in Table [1]. To demonstrate the learning properties of LAMA, four different examples of
landmarks were applied. In LAMA1, a sea lion \((n = 75)\) was associated with the center node of the output space \((k = 312)\). In LAMA2, a duck \((n = 21)\) and a penguin \((n = 58)\) were assigned to a middle left node \((k = 303)\) and a middle right node \((k = 321)\), respectively. Similarly, LAMA3 contained three landmark nodes that showed the relationships between a mink \((n = 48)\) and a top center node \((k = 37)\), between a seal \((n = 74)\) and a bottom left node \((k = 552)\), and between a slowworm \((n = 80)\) and a bottom right node \((k = 572)\). Finally, LAMA4 was trained with landmark nodes located at four edges \((k = 0, 24, 600, 624)\), which were paired with the mink \((n = 48)\), the toad \((n = 89)\), the seal \((n = 74)\), and the slowworm \((n = 80)\), respectively.

In addition, numerical learning properties of SOM and LAMA were assessed using QED, QEL, and STE. SOM, LAMA1, LAMA2, LAMA3, and LAMA4 were trained 100 times each. Next, the mean values of QED, QEL, and STE for each algorithm were visualized in seven learning iterations \((t = 0, 9999, 19999, 29999, 39999, 49999, 59999)\).

3.2. Visualization of learning properties

For comparison with LAMA, SOM was trained on the Zoo dataset, and the codebook vectors and the U-matrix were then visualized. Fig. 7(1) shows the codebook vectors and the data points. The red mesh grid composed of lines between the neighboring codebook vectors cover the data smoothly, retaining the architecture of the nodes in the output space. The density of the codebook vectors seems to be similar in every part of the data. Fig. 7(2) represents the U-matrix of the SOM. The color of the U-matrix represents the summation of the distance in the input space between four immediate neighbor nodes, where the neighbor nodes were determined by the distance in the output space. The blue part of the U-matrix indicates that the neighboring codebook vectors were close, while the yellow part indicates that the neighboring codebook vectors were far. Each animal data point was projected onto the U-matrix (Fig. 7(1)). Animals with similar features, such as the platypus and the tortoise, were closely located in the U-matrix. On the other hand, animals with different features, such as the
Table 1: Parameters of SOM and LAMA used for the Zoo dataset analysis.

| Algorithm | SOM  | LAMA1 | LAMA2 | LAMA3 | LAMA4 |
|-----------|------|-------|-------|-------|-------|
| $K_x$     | 25   | 25    | 25    | 25    | 25    |
| $K_y$     | 25   | 25    | 25    | 25    | 25    |
| $t_{\text{max}}$ | 60000 | 60000 | 60000 | 60000 | 60000 |
| $a_{\text{max}}$ | 0.5  | 0.5   | 0.5   | 0.5   | 0.5   |
| $a_{\text{min}}$ | 0.15 | 0.15  | 0.15  | 0.15  | 0.15  |
| $\tau_a$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ |
| $\sigma_{\text{max}}$ | 19   | 19    | 19    | 19    | 19    |
| $\sigma_{\text{min}}$ | 0.1  | 0.1   | 0.01  | 0.01  | 0.01  |
| $\tau_\sigma$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ |
| $b_{\text{max}}$ | –    | 0.4   | 0.4   | 0.4   | 0.4   |
| $b_{\text{min}}$ | –    | 0.01  | 0.075 | 0.075 | 0.1   |
| $\tau_b$ | –    | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ |
| $t_{\text{center}}$ | –    | 15000 | 15000 | 15000 | 15000 |
| $\rho_b$ | –    | 20000 | 25000 | 25000 | 25000 |
| $\rho_{\text{max}}$ | –    | 13    | 13    | 13    | 13    |
| $\rho_{\text{min}}$ | –    | 3     | 0.7   | 1     | 1.5   |
| $\tau_\rho$ | –    | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ |
| $\rho_{\text{th}}$ | –    | 0.01  | 0.05  | 0.07  | 0.09  |
| Landmark 1 ($n : k$) | –    | 75:312 | 21:303 | 48:37 | 48:0  |
| Landmark 2 ($n : k$) | –    | –     | 58:321 | 74:552 | 89:24 |
| Landmark 3 ($n : k$) | –    | –     | –     | 80:572 | 74:600 |
| Landmark 4 ($n : k$) | –    | –     | –     | –     | 80:624 |

gorilla and the crow, were far in the U-matrix, separated by a yellow boundary.

LAMA1 provided a different view of the data. The codebook vectors of LAMA1 are shown in Fig. 8(1). The red mesh grid covered the data with its center close to the landmark data shown in yellow label. Fig. 8(2) shows the
Figure 7: Codebook vectors and U-matrix of the SOM trained from the Zoo dataset. (1) 
*Codebook vectors and data in the output space*. Nodes 0 and 24 are displayed by red labels to illustrate the rotation of the codebook vectors. (2) *Data labels in the U-matrix*. The white labels indicate the data labels projected onto the location of the U-matrix. The dot in the label implies that multiple labels are projected onto the same node. The color of the U-matrix represents the distance from the neighboring nodes. As the color turns yellow, the distance increases.

U-matrix of LAMA1. The landmark data shown in yellow were approximately centered. In the U-matrix of the SOM (Fig. 7(2)), the label was located near an edge. In addition, the data around the landmark data (sea lion) in the U-matrix of LAMA1 were widely distributed, while those of the SOM were not. Thus, LAMA1 in this example provided landmark-centered visualization.

LAMA2 shows the relationship of the data around two designated landmarks. The codebook vectors of LAMA2 are shown in Fig. 9(1). The mesh grid was shifted so that the two landmark data points were close to the center line of the grid. The density of the codebook vectors did not appear uniform, which was different from the mesh grid of the SOM (see Fig. 7(1)). Fig 9(2) shows the U-matrix of LAMA2. The two landmark data points were projected onto the middle left and the middle right, respectively. The data points between the
Figure 8: Codebook vectors and U-matrix of LAMA1 trained using the Zoo dataset. (1) Codebook vectors and data in the output space. Nodes 0 and 24 are shown by red labels to illustrate the rotation of the codebook vectors. The yellow label indicates the location of the landmark data. (2) Data labels on the U-matrix. The white labels indicate data projected onto the location of the U-matrix. The yellow label indicates the landmark data. The dot in the label implies that multiple labels are projected onto the same node. The color of the U-matrix represents the distance from the neighboring nodes. As the color turns yellow, the distance increases.

two landmark data points were clearly seen as if they were zoomed in, compared with the results of SOM and LAMA1. As seen above, LAMA2 provided enlarged data visualization focusing on two landmark data points.

Similar to LAMA2, LAMA3 showed the relationship of the data surrounded by three landmark data points. Fig 10(1) represents the codebook vectors of LAMA3. Three different landmark data points, namely, the mink, the seal, and the slowworm, were located close to the top center, bottom left, and bottom right of the mesh grid, respectively. Fig 10(2) shows the U-matrix of LAMA3. The data points encircled by these landmarks were widely displayed on the U-matrix. As the three landmark data points had different features, they were divided by the yellow boundary on the U-matrix.
Figure 9: Codebook vectors and the U-matrix of LAMA2 trained using the Zoo dataset. (1) **Codebook vectors and data in the output space.** Nodes 0 and 24 are displayed by red labels to illustrate the rotation of the codebook vectors. The yellow labels indicate the location of the landmark data. (2) **Data labels on the U-matrix.** The white labels indicate data projected onto the location of the U-matrix. The yellow labels indicate the landmark data. The dot in the label implies that multiple labels are projected onto the same node. The color of the U-matrix represents the distance from the neighboring nodes. As the color turns yellow, the distance increases.

LAMA4 was an extension of LAMA3. Fig 11(1) shows the codebook vectors of LAMA4. As all the landmark nodes were located on the edge of a square in the output space, the mesh grid were spread over the data with its edges close to the landmark nodes. Fig 11(2) shows the U-matrix of LAMA4. The landmark data points were projected close to the edges of the U-matrix. The figure showed the similarities of the data, focusing on four different landmark data points.

3.3. Evaluation of numerical error indices

Figure 12 shows the QED when analyzing the Zoo dataset. The QED of all algorithms showed similar descending curves. The lowest QED was achieved by SOM, followed by LAMA2, LAMA1, LAMA3, and LAMA4. These results
Figure 10: Codebook vectors and the U-matrix of LAMA3 trained using the Zoo dataset.

(1) Codebook vectors and data in the output space. Nodes 0 and 24 are shown by red labels to illustrate the rotation of the codebook vectors. The yellow labels indicate the location of landmark data. (2) Data labels on the U-matrix. The white labels indicate data projected onto the location of the U-matrix. The yellow labels indicate the landmark data. The dot in the label implies that multiple labels are projected onto the same node. The color of the U-matrix represents the distance from the neighboring nodes. As the color turns yellow, the distance increases.

imply that the codebook vectors of LAMA expand over the data in a similar way as SOM.

Figure 13 shows the QEL when analyzing the Zoo dataset. The QEL of the SOM was not analyzed because SOM did not have landmark nodes. Large variability can be seen between algorithms. The QEL of LAMA1, LAMA3, and LAMA4 remained below 2.6 at the end of the learning. The QEL of LAMA2 increased toward the final iteration. The increase of the QEL implies that the codebook vectors of landmark nodes are moving far from the corresponding landmark data. The differences of QEL in the Zoo dataset analysis may be due to the allocation of landmarks.

STEs are shown in Fig. 14. All algorithms show a similar tendency in terms
Figure 11: Codebook vectors and the U-matrix of LAMA4 trained on the Zoo dataset. (1) Codebook vectors and data in the output space. Nodes 0 and 24 are shown by red labels to illustrate the rotation of the codebook vectors. The yellow labels indicate the location of the landmark data. (2) Data labels on the U-matrix. The white labels indicate the data projected onto the location of the U-matrix. The yellow labels indicate the landmark data. The dot in the label implies that multiple labels are projected onto the same node. The color of the U-matrix represents the distance from the neighboring nodes. As the color turns yellow, the distance increases.

of STE. They all show sudden decreases when the number of iterations is less than 10000. All algorithms maintained a low STE at the end of the learning iteration.
Figure 12: Mean quantization error of data learned from the Zoo dataset.
Figure 13: Mean quantization error of landmark learned from the Zoo dataset.
Figure 14: Mean square topographic error of data learned from the Zoo dataset.
4. Formant dataset analysis

4.1. Dataset and parameters

This experiment provides an example of applying LAMA to an artificially generated formant dataset. Formants are important components that characterize vowels in human speech. They can be regarded as the peaks of the spectrogram of vowel sounds (see Fig. 15). The lowest peak in frequency is the first formant (F1) and the next one is the second formant (F2). In practice, the peaks are extracted from the envelope of the spectrogram by linear predictive coding. For example, the mean first and second formants (F1, F2) of the five Japanese vowels (/a/, /i/, /u/, /e/, and /o/) are (850, 1610), (240, 2400), (300, 1390), (390, 2300), and (360, 640), respectively [29]. Here, /·/ indicates phonetic transcription (broad transcription) of the speech sounds. As the changes in F1 and F2 are related to the movement of the tongue and the jaw, respectively, F1 and F2 can be used for pointing a cursor on a computer. A pointing device using formants has been proposed [30]. Such a device can be extended by a projection between the formants and the cursor movement using LAMA. This demonstration aims to explain how to apply the LAMA to HCI.

To reveal the difference between the learning properties of LAMA and SOM, they were trained on an artificially generated formant dataset, which is provided in Supplementary Material 1. The parameters, including the landmarks, are presented in Table 2. The mean F1 and F2 for each Japanese vowel were designated as the landmark data points. The landmark nodes were arranged in the output space such that the horizontal cursor movement was related to the tongue movement (/e/,/o/) while the vertical cursor movement was associated with the jaw movement (/u/,/a/). Note that the landmark data was not selected from the given dataset.

Moreover, QED, QEL, and STE were calculated for the artificial formant dataset as well as the Zoo dataset analysis.
Figure 15: Example of using LAMA for human-computer interaction. This system converts vowel sounds into vertical or horizontal movements considering articulator movement. First, the sound of the Japanese vowel /e/ is recorded via a microphone. Then, the sound is analyzed by the spectrogram. From the graph, the first and second peaks in frequency are extracted, which are called the first and the second formants (F1, F2). After correcting F1 and F2 of the Japanese vowel sounds, they are displayed in the F1-F2 plot, which indicates that the jaw position is related to F2, while the tongue position is related to F1. To exploit this relationship for cursor movement, landmarks are installed such that the tongue movement (/e/ to /o/) is associated with the horizontal cursor movement, while the jaw movement (/u/ to /a/) is related to the vertical cursor movement. If the learning is successful, the sound data /e/ is projected onto node 10, and the cursor movement in a computer is controlled as (the velocity of x, the velocity of y) = (-2, 0). Thus, the LAMA works as an interface between formants and cursor movement.

4.2. Visualization of learning properties

LAMA provided an intuitive relationship between the articulator movement and the computer cursor movement. Fig. 16(1) shows the codebook vectors of the SOM. The red mesh grid composed of codebook vectors was smoothly spread over the given data points. The horizontal movement of the tongue (movement between /e/ and /o/) was not related to horizontal movement in the output space. In addition, jaw movement (/u/, /a/) was not fit to vertical movement
Table 2: Parameters of SOM and LAMA used for the artificial formant dataset analysis.

| Algorithm | SOM | LAMA |
|-----------|-----|------|
| $K_x$     | 10  | 10   |
| $K_y$     | 10  | 10   |
| $t_{\text{max}}$ | 60000 | 60000 |
| $a_{\text{max}}$ | 0.3  | 0.3  |
| $a_{\text{min}}$ | 0.1  | 0.05 |
| $\tau_a$  | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ |
| $\sigma_{\text{max}}$ | 4    | 4    |
| $\sigma_{\text{min}}$ | 0.3  | 0.4  |
| $\tau_{\sigma}$ | $t_{\text{max}}/3 - 1$ | $t_{\text{max}}/3 - 1$ |
| $b_{\text{max}}$ | –    | 0.3  |
| $b_{\text{min}}$ | –    | 0.08 |
| $\tau_b$  | –   | $t_{\text{max}}/3 - 1$ |
| $t_{\text{center}}$ | –    | 30000 |
| $\rho_b$  | –   | 15000 |
| $\rho_{\text{max}}$ | –    | 2    |
| $\rho_{\text{min}}$ | –    | 0.8  |
| $\tau_{\rho}$ | –    | $t_{\text{max}}/3 - 1$ |
| $p_{\text{th}}$ | –    | 0.1  |

Landmark (vowel : $k$) – a:94, i:0, u:4, e:41, o:49

in the output space. The mesh grid might be rotated because the codebook vectors were randomly initialized and no constraints were given for learning. Fig. [16(2)] shows the codebook vectors of LAMA. The red mesh grid composed of codebook vectors was spread over the given data points. The mesh grid had a line between /e/ and /o/, which corresponded to the horizontal movement of the tongue. This means that the horizontal movement of the tongue is directly reflected by the horizontal movement in the output space. Furthermore, the
mesh grid contained a line between /u/ and /a/, which was related to the vertical movement of the jaw. This implies that the vertical movement of the jaw is retained in the output space. Thus, LAMA in this example provides a projection that maintains the articulator movement in the output space.

Figure 16: Codebook vectors of SOM and LAMA trained on the artificial formant dataset.
(1) Codebook vectors of the SOM and data in the input space. The blue dots represent the data points. The crossing points of the red lines indicate the codebook vectors of the SOM.
(2) Codebook vectors of LAMA and data in the input space. The yellow labels indicate the landmark data.

4.3. Evaluation of numerical error indices

Figure 17 shows the QED when analyzing the artificial formant dataset. The QEDs of both SOM and LAMA decrease monotonically. This implies that LAMA5 preserves the learning properties of SOM. Figure 18 represents the QEL of LAMA5. It increased toward the end of learning. This result implies that landmark nodes were gradually far from the landmark data during learning. Figure 19 shows the STE. Both SOM and LAMA5 show an increase of the STE. The STE of SOM maintained a low level (below 0.05) compared to that of LAMA5. This result implies that topological errors occasionally occur due to the settings of landmarks or parameters.
Figure 17: Mean quantization error of data learned from the artificial formant dataset.
Figure 18: Mean quantization error of landmark learned from the artificial formant dataset.
Figure 19: Mean square topographic error of data learned from the artificial formant dataset.
5. Discussion

Herein, LAMA was proposed, which provides the user-intended nonlinear projection by designating several landmark nodes. The experiment demonstrated that LAMA had its own learning properties compared with the SOM. In the analysis of the Zoo dataset, LAMA demonstrated that it provided varieties of landmark-oriented visualization. The QED of the SOM and LAMA showed similar tendencies and became small in the final learning condition, implying that the codebook vectors of LAMA spread over data in the same way as a SOM, while imposing constraints of some nodes and data. This study also demonstrated that the movement of the articulator was successfully maintained in the output space using LAMA. These results suggest that LAMA can be used for designing new HCI devices, in addition to data mining.

LAMA can describe the relationship between the input and output spaces by designating several landmark nodes. Thus, it is feasible to apply LAMA to HCI by converting biosignals into computer commands. For example, limited finger movements of people who have paralysis can be converted into, for example, cursor movements on a computer. Other biosignals, such as EEGs, electromyograms, and electrooculograms, can be translated into two-dimensional output space by applying a relationship between the input and output spaces.

In addition to exploiting biosignals, LAMA can be used to summarize or explore data. For example, LAMA can contribute to the design of a new recommendation system of products using popular products as landmarks. Such a system can help users search for products by some designated popular products. Once the product data are represented by features, as in the case of the Zoo dataset, LAMA can show the relationship between the given data with landmark nodes and the other data. However, the current U-matrix may not be suitable for use in a recommendation system because some data names are projected into the same node. Moreover, each item in the U-matrix is too small to visualize the image of products. A new visualization method is required for recommendation system to be more practical.
LAMA learns codebook vectors, attempting to make some codebook vectors fit the assigned landmark data and retaining the learning properties of the SOM. However, these two objectives cannot always be completely achieved simultaneously. If the landmark nodes learn stronger parameters in the landmark-driven phase, the codebook vectors of the landmark nodes may fit the landmark data with the mesh grid distorted. Such distortion will be large as the designation of the landmark nodes is different from that of the codebook vectors learned by the SOM. On the other hand, if stronger parameters are used in the data-driven phase, the learning properties are close to those of the SOM and the landmark nodes do not receive much consideration. To achieve both objectives, the parameters should be adjusted by confirming the visualization of the codebook vectors.

The learning parameters of LAMA are not determined automatically, but are manually adjusted by estimation from the visualization of the codebook vectors. Missetting of the parameters may cause the mesh grid of codebook vectors to be twisted or wrinkled. However, there is no strict rule for setting the parameters. In this study, parameters shared with the SOM were adjusted first, then all parameters were modified. By designing proper learning rates in the data-driven phase, the codebook vectors were spread over the given data. Second, the landmark nodes were arranged, and the learning rates of the landmark-driven phase and the update rate of the alternating update method were then manipulated such that the codebook vectors fit the corresponding landmark data points with the smooth mesh grid.

Setting up of landmark nodes should be based on the assumption or objective for a projection. For example, the landmark nodes for the F1-F2 projection in this study were prepared such that the articular movement is compatible with the movement in the output space. It would be preferable to assign landmark data to the projected node of the SOM because it does not require LAMA for difficult fitting. As the user-intended projection of LAMA is far from the projection of the SOM, LAMA needs to modify the mesh grid considerably, which causes wrinkles on the grid. Thus, it would be preferable to install the
landmark nodes by visualizing the data and codebook vectors of the SOM.

The arrangement of codebook vectors in the input space may rotate, depending on the way the landmarks are set. If no landmark is installed (SOM), its codebook vectors can be rotated, depending on the initial values. If a landmark is set at the center of the nodes (LAMA1), the codebook vectors are allowed to be rotated, centering the landmark. When two landmarks are designated (LAMA2), the codebook vectors can rotate, centering the axis between two landmarks. In the same way, the degree of freedom for LAMA depends on the input data and the number and location of landmarks installed.

This study employed QED, QEL, and STE to evaluate the learning properties of SOM and LAMA numerically. The optimization problem of LAMA can be explained in that it minimizes the QED and QEL simultaneously. It successfully reduced the QED while keeping the QEL low in this study. Toward better evaluation of LAMA, more comprehensive error indices, such as combined indices of QED, QEL, and STE, would be helpful. The STE was successfully reduced in the Zoo dataset analysis. However, the STE increased when LAMA5 was applied to the artificial formant analysis. This implies that the learning problem is difficult due to landmark settings, and requires several learning trials to obtain the user-intended nonlinear projection. Additional learning phases, such as a smoothing phase, may contribute to further reduction of the STE.

In future studies, LAMA can be improved by applying advanced algorithms proposed for the SOM or the generative topographic mapping (GTM). For example, the online learning method can be extended to the batch learning method [1]. Rauber et al. proposed a growing hierarchical SOM that has a dynamic architecture for improving its representation capacity [31]. Such a growing hierarchical model for LAMA may reduce difficulties in setting the parameters. In addition, there remains scope for the development of tensor analysis, such as that used in the tensor SOM and the tensor GTM [32]. Furthermore, LAMA can be cascaded similarly to the SOM of SOMs [33]. LAMA may be modeled by a continuous model such as the GTM [34]. The SOM and GTM have been evaluated on the presence of missing data [35]. The learning properties of LAMA
against such data should be clarified in future studies.

6. Conclusion

In this paper, LAMA was introduced, which provides the user-intended non-linear projection from input data in an input space to a discrete output space by setting several landmark nodes. This study demonstrated that LAMA could provide unique data visualization, for example, landmark-oriented visualization. In addition, an example was presented to demonstrate projection using an artificial formant dataset, which enable us to design an HCI application. LAMA can be applied to develop new HCI devices or recommendation systems.

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