Finsler space-time in light of Segal’s principle

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ISIM(2) symmetry group of Cohen and Glashow’s very special relativity is unstable with respect to small deformations of its underlying algebraic structure and according to Segal’s principle cannot be a true symmetry of nature. However, like special relativity, which is a very good description of nature thanks to the smallness of the cosmological constant, which characterizes the deformation of the Poincaré group, the very special relativity can also be a very good approximation thanks to the smallness of the dimensionless parameter characterizing the deformation of ISIM(2).

Keywords: Very special relativity; Finsler geometry; Segal’s principle.

I. INTRODUCTION

Einstein’s two famous postulates imply the special theory of relativity and Minkowski metric of space-time only when it is assumed that space is isotropic. If we abandon this assumption, then a Finsler generalization of the special theory of relativity with the corresponding Finsler metric naturally arises, as will be shown below.

For historical reasons, this Finsler metric will be called the Lalan-Alway-Bogoslovsky metric. It is characterized by one extra dimensionless parameter $b$. When $b$ tends to zero, in general, we do not obtain the special theory of relativity as a limiting case, since a preferred light-like direction $n^\mu$ can survive this limit and the resulting space-time symmetry group will be not the Poincaré group but its subgroup ISIM(2). Cohen and Glashow suggested [1] that it is possible that just this particular ISIM(2), and not the Poincaré group, is a true space-time symmetry group. The corresponding theory is called “very special relativity” (in the bibliography we provide an incomplete list of modern references on the subject [2]).

Drawing an analogy with the cosmological constant, it can be argued that in reality $b$ is not zero, but very small. In this case, most likely, it will be impossible to detect the Finslerian nature of space-time in laboratory experiments. Nevertheless, the question of the true value of the parameter $b$ is as fundamental as the question of why the cosmological constant $\Lambda$ is so small. Perhaps both questions are just different parts of the same puzzle.

II. ANISOTROPIC SPECIAL RELATIVITY

Both Einstein [3] and Poincaré [4, 5] obtained $\lambda$-Lorentz transformations

$$x' = \lambda(V)\gamma(x - Vt), \quad y' = \lambda(V)y, \quad z' = \lambda(V)z, \quad t' = \lambda(V)\gamma\left(t - \frac{V}{c^2}x\right),$$

(1)

and then both argue that $\lambda(V) = 1$. The arguments of Einstein were physical, while Poincaré’s reasoning was more formal and was based on the analysis of the full Lorentz’s group including not only boosts, but also spatial rotations. In particular, Poincaré notes that the group property of transformations (1) (which is equivalent to the relativity principle) implies the following multiplication law (which first appeared in Poincaré’s May 1905 letter to Lorentz [6]):

$$\lambda(V_1 \oplus V_2) = \lambda(V_1)\lambda(V_2),$$

(2)

Note that (1) and (2) determine the most general form of the boost along the $x$-direction, compatible with Einstein’s two postulates. The solution of the functional equation (2) is hindered by the complexity of the relativistic velocity

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addition law. However the natural parameter for special Lorentz transformations is not the velocity $V$, but the rapidity $\psi$, defined by

$$\tanh \psi = \beta = \frac{V}{c}. \tag{3}$$

For special Lorentz transformations, rapidities are additive, and the functional equation \ref{eq:lambda_functional} takes the form of the Cauchy exponential functional equation

$$\lambda(\psi_1 + \psi_2) = \lambda(\psi_1) \lambda(\psi_2). \tag{4}$$

It is a well-known fact \cite{3} that all continuous solutions of \ref{eq:lambda_equation} have the form

$$\lambda(\psi) = e^{-b\psi} = \left(\frac{1 - \beta}{1 + \beta}\right)^{b/2}, \tag{5}$$

where $b$ is some real constant. As a result, $\lambda$-Lorentz transformations \ref{eq:lambda_transforms} take the form

\begin{align*}
x' &= \left(\frac{1 - \beta}{1 + \beta}\right)^{b/2} \gamma (x - V t), \quad y' = \left(\frac{1 - \beta}{1 + \beta}\right)^{b/2} y, \\
z' &= \left(\frac{1 - \beta}{1 + \beta}\right)^{b/2} z, \quad t' = \left(\frac{1 - \beta}{1 + \beta}\right)^{b/2} \gamma \left(t - \frac{V}{c^2} x\right). \tag{6}
\end{align*}

The special isotropy requires $b = 0$ and this is the situation corresponding to the usual special relativity. To find a generalization of the relativistic interval, which remains invariant under transformations \ref{eq:lambda_transforms}, it is convenient to introduce light-cone coordinates \ref{eq:light-cone_coords}

$$u = ct + x, \quad v = ct - x, \tag{7}$$

which transform as follows

$$u' = e^{-(1+b)\psi} u, \quad v' = e^{(1-b)\psi} v, \quad y' = e^{-b\psi} y, \quad z' = e^{-b\psi} z. \tag{8}$$

Using \ref{eq:lambda_transforms}, we can easily find that

\begin{align*}
\left(\frac{u'}{u}\right)^b u'u' &= \left(\frac{v}{u}\right)^b uv, \\
\left(\frac{y'}{y}\right)^b y'y' &= \left(\frac{v}{y}\right)^b y^2, \\
\left(\frac{z'}{z}\right)^b z'^2 &= \left(\frac{v}{z}\right)^b z^2.
\end{align*} \tag{9}

Therefore the following quantity is invariant under the $\lambda$-Lorentz transformations \ref{eq:lambda_transforms}:

$$s^2 = \left(\frac{v}{u}\right)^b (uv - y^2 - z^2) \left(\frac{u}{uv - y^2 - z^2}\right)^b = v^{2b}(uv - y^2 - z^2)^{1-b} = (ct - x)^{2b} \left(c^2 t^2 - x^2 - y^2 - z^2\right)^{1-b}, \tag{10}$$

and can be considered as a generalization of the relativistic interval. Accordingly, the space-time metric, invariant under the $\lambda$-Lorentz transformations \ref{eq:lambda_transforms}, has the form

$$ds^2 = (c^2 dt - dx)^{2b} (c^2 dt^2 - dx^2 - dy^2 - dz^2)^{1-b} = (n_\nu dx^\nu)^{2b} (dx_\mu dx^{\mu})^{1-b}, \tag{11}$$

where $n^\mu = (1, 1, 0, 0) = (1, \vec{n})$, $\vec{n}^2 = 1$, is the fixed null-vector that defines a preferred light-like direction in space-time.

As we see, in general, Einstein’s two postulates do not lead to Lorentz transformations and the Minkowski metric. If the spatial isotropy is not assumed, they lead to more general $\lambda$-Lorentz transformations and Finslerian metric \ref{eq:metric} (applications of Finsler geometry in physics is considered, for example, in \ref{10}–\ref{12}).

The obtained $\lambda$-Lorentz transformations \ref{eq:lambda_transforms} correspond to the inertial reference frame $S'$ which moves along the preferred direction $\vec{n}$. If the inertial reference frame $S'$ moves with the velocity $\vec{V} = c\vec{\beta}$ in an arbitrary direction, then the generalized Lorentz transformations that leave the Finsler metric \ref{eq:metric} invariant have the form \ref{eq:generalized_transformations}

$$x'^\mu = D(\lambda) R^\mu_\nu (\vec{n}; \alpha) L^\nu_\sigma (\vec{V}) x^\sigma, \tag{12}$$
where $L'_\sigma(\vec{V})$ represents the usual Lorentz transformations, $R^\mu_\nu(\vec{m}; \alpha)$ is a rotation about the spatial direction

$$\vec{m} = \frac{\vec{n} \times \vec{\beta}}{|\vec{n} \times \vec{\beta}|}$$

by the angle $\alpha$ such that

$$\cos \alpha = 1 - \frac{\gamma}{\gamma + 1} \frac{|\vec{n} \times \vec{\beta}|^2}{1 - \vec{n} \cdot \vec{\beta}}.$$  

(14)

For the radius vector $\vec{r}$, the result of this additional rotation is given by the Euler-Rodrigues formula [15]

$$\vec{r}' = \vec{r} + (\vec{m} \times \vec{r}) \sin \alpha + \left[ \vec{m} \times (\vec{m} \times \vec{r}) \right] (1 - \cos \alpha).$$

(15)

Finally, $D(\lambda)$ represents the dilatation

$$D(\lambda)x^\mu = \lambda x^\mu,$$

with the scale-factor

$$\lambda = \left[ \gamma(1 - \vec{\beta} \cdot \vec{n}) \right]^b.$$  

(17)

The explicit form of these generalized Lorentz transformations was obtained in [13, 14]. They have the following form

$$x'_0 = \left[ \gamma(1 - \vec{\beta} \cdot \vec{n}) \right]^b \gamma (x_0 - \vec{\beta} \cdot \vec{r}),$$

$$\vec{r}' = \left[ \gamma(1 - \vec{\beta} \cdot \vec{n}) \right]^b \left\{ \vec{r} - \frac{\vec{\beta} (x_0 - \vec{n} \cdot \vec{r})}{1 - \vec{\beta} \cdot \vec{n}} \right\},$$

$$\vec{n} \left[ \gamma \beta \cdot \vec{r} + \gamma - 1 \frac{\vec{n} \cdot \vec{r}}{1 - \vec{n} \cdot \vec{r}} \right] + \left[ \gamma - 1 \right] \vec{\beta} \cdot \vec{n} - \gamma \beta^2 x_0 \right\}.$$  

(18)

If $\vec{\beta}$ and $\vec{n}$ are parallel, and the $x$-axis is along the velocity $\vec{\beta}$, transformations (18) are reduced to the $\lambda$-Lorentz transformations (6).

The $\lambda$-Lorentz transformations [11] were first derived by Lalan [16–18]. He also recognized that the metric, invariant with respect to the $\lambda$-Lorentz transformations, was of pseudo-Finslerian type. Generalized Lorentz transformations [18] were first discovered by Alway [19]. In his article, he cites Pars [20] and mentions that the isotropic behavior of the clock, which was one of the Pars’s assumptions, is generally not guaranteed. Alway’s transformations and the corresponding Finslerian space-time metric did not attract much attention, since it was immediately recognized [21] that experimental observations severely constrain spatial anisotropy making the parameter $b$ practically zero. Some æther-drift experiments imply $|b| < 10^{-10}$, while the Hughes-Drever type limits on the anisotropy of inertia can potentially lower the limit on the Finslerian parameter $b$ up to $|b| < 10^{-26}$, albeit in the model-dependent way [22].

Generalized Lorentz transformations [18] were soon rediscovered by Bogoslovsky [13, 23] who thoroughly investigated their physical consequences [24, 25]. A particular case of the $\lambda$-Lorentz transformations [11], corresponding to $b = 1/2$, was independently rediscovered by Brown [27] and generalized to any value of $b$ by Budden [28]. Later they cite Bogoslovsky in [29, 30] and [31], but neither Lalan, nor Alway. Such studies have largely remained outside the mainstream, but recently they have gained more chances due to the advent of very special relativity.

III. VERY SPECIAL RELATIVITY AND FINSLER GEOMETRY

The Lorentz group does not have a 5-parameter subgroup. It has only one, up to isomorphism, 4-parameter subgroup, called SIM(2) (similitude group of the plane consisting of dilations, translations, and rotations of the plane) [32, 33]. If we add space-time translations to SIM(2), we get an 8-parameter subgroup of the Poincaré group called ISIM(2). It is a semi-direct product of SIM(2) with the group of space-time translations. Cohen and Glashow proposed [1] that the exact symmetry group of nature may not be the Poincaré group, but its subgroup ISIM(2). The corresponding theory, very special relativity, breaks Lorentz symmetry in a very mild and minimal way. For amplitudes with appropriate analyticity properties, SIM(2) implies $CPT$ discrete symmetry, but it violates $P$ and
$T$ discrete symmetries \[1\]. However, in any theories of particle physics in which $P$, $T$ or $CP$ is conserved, Lorentz-violating effects of very special relativity will be absent, as the inclusion of any of these discrete symmetries extends the SIM(2) subgroup to the full Lorentz group \[1\]. Since $CP$ violation is known to be small, Lorentz-violating effects in very special relativity are expected to be also small \[1\].

Generalized pure Lorentz boosts \[13\], supplemented by rotations about preferred spatial direction $\vec{n}$ and by space-time translations, form a 8-parameter group of isometries of the Finsler space-time with metric (11) called DISIM(2) in \[22\]. The very special relativity symmetry group ISIM(2) is obtained from DISIM(2) by In"{o}n"{u} and Wigner contraction \[34, 35\] in the $b \to 0$ limit. This limit is rather subtle. Indeed, although in the limit $b \to 0$ the Finsler metric (11) is reduced to the Minkowski metric, the generalized Lorentz transformations \[13\] are not reduced to the ordinary Lorentz transformations in this limit, because the preferred direction $\vec{n}$ can remain even in this limit. In this case, the resulting symmetry group will not be the Lorentz group, but its 4-parameter subgroup SIM(2), consisting of transformations \[13\], with $b = 0$, along with rotations about the preferred direction $\vec{n}$.

However, when $b = 0$, that is when the space is isotropic, we have no reason to introduce the preferred light-like direction $n^\mu$ when deriving the Lorentz transformations in the manner described above. Of course, we can calibrate the orientations of the spatial axes of inertial reference frames so that if in one of these reference frames the light beam has the direction $\vec{n}$, it will have the same direction in all inertial reference frames. In this case we will again arrive at the generalized Lorentz transformations \[13\] (with $b = 0$) instead of usual Lorentz transformations. However, the resulting theory will still be the ordinary special relativity, but with the indicated special agreement on the orientations of the spatial axes \[13\]. In this case we can choose light-like vector $n^\mu$ arbitrarily and all such choices will be physically equivalent. In fact, for \[13\] with $b = 0$ to represent Lorentz symmetry, rather than its SIM(2) subgroup, it is enough to require that the choices $\vec{n}$ and $-\vec{n}$ are equivalent. Indeed, $\vec{n} \to -\vec{n}$ corresponds to $\vec{m} \to -\vec{m}$ in \[13\], and if we require the symmetry under rotations about $-\vec{m}$, the generalized Lorentz transformations \[13\] with $b = 0$ can be easily transformed into the ordinary Lorentz transformations by an additional rotation $x'^\nu = R^\nu_\lambda(\vec{m}, \alpha) x^\lambda$, since $R^\nu_\lambda(-\vec{m}, \alpha) R^\lambda_\nu(\vec{m}, \alpha) = \delta^\nu_\mu$.

In the light of the foregoing, we come to the conclusion that there is a natural possibility that fully respects the relativity principle, about how a very special relativity can arise, instead of the special relativity, in the description of reality. Namely, space-time can be Finslerian with the Lalan-Alway-Bogoslovsky metric (11), but the parameter $b$ of this metric can be very small. The following analogy with the cosmological constant, described below, shows that this is indeed the most natural way of introducing a preferred light-like direction into space-time theory.

**IV. VERY SPECIAL RELATIVITY IN LIGHT OF SEGAL’S PRINCIPLE**

Although Minkowski, in his famous lecture “Raum und Zeit”, never mentions Klein’s Erlangen program of defining a geometry as theory of invariants of some group of transformations, a link between Minkowski’s presentation of special relativity and Erlangen program was immediately recognized by Felix Klein himself \[37\]. In 1954 Fantappiè rediscovered the connection and put forward a program which he himself called “an Erlangen program for physics”: a classification of possible physical theories through their group of symmetries \[38, 39\].

In particular, Fantappiè discovered that the group of “final relativity” is not the Poincaré group, but the De Sitter group. The Poincaré group is just the limit of the De Sitter group when the radius of curvature of the De Sitter space-time turns to infinity, much like the Galilei group, which is the limit of the Poincaré group, when the speed of light goes to infinity. In fact, these two examples are only part of a wider picture of possible kinematical groups and their interconnections within various limits, given later by H. Bacry and J. Lévy-Leblond \[40\]. This picture can be considered as a natural implementation of Fantappiè’s “Erlangen program for physics” and is based on the ideas of group contractions and deformations proposed by In"{o}n"{u} and Wigner \[34\] and by Irving Segal \[41\].

Segal’s principle \[41, 43\] states that a true physical theory must be stable against small deformations of its underlying algebraic structure. For example, the Lie algebra of the inhomogeneous Galilei group is unstable in the sense of Segal, and its deformation leads to the Lie algebra of the Poincaré group. As a result, the theory of relativity based on the Poincaré group has a larger scope of validity than the theory of relativity based on the Galilei group. However, the Poincaré Lie algebra is also unstable, and its small deformations lead to either de Sitter or anti-de Sitter Lie algebras \[40\]. In light of this, it is not surprising that it turned out that the asymptotic vacuum space-time is not Minkowski, but de Sitter space-time with non-zero cosmological constant. A really amazing question is why the cosmological constant is so small. This is a profound question of modern physics, and we still do not have a satisfactory answer to it.

As we have seen above, the very special relativity symmetry group ISIM(2) is not stable against small deformations of its structure, and a physically relevant deformation, DISIM(2), exists which leads to a Finslerian space-time. This was shown more formally in \[22\]. In light of the Segal principle, we expect that, accordingly, the very special relativity cannot be a true symmetry of nature and should be replaced by DISIM(2). Then, based on the analogy
with the cosmological constant, it can be argued that if there really is a preferred light-like direction in nature, then the Finslerian parameter $b$ will not be zero, but very small, so small that the corresponding Finslerian nature of space-time is unlikely to be detected in laboratory experiments.

V. CONCLUSIONS

In this note, we argued that if the very special relativity, and not the usual special relativity, is really implemented in nature, then most likely, space-time in the absence of gravity will have not Minkowski geometry, but Finsler geometry of the Lalan-Alway-Bogoslovsky type with the metric (11). However, in this case the anisotropy parameter $b$ is expected to be very small, like the cosmological constant $\Lambda$. So small that it will be impossible to detect the effects of Finslerian nature of space-time in laboratory experiments. Can then we repeat after Francesco Sizzi, the Florentine astronomer who opposed the discovery by Galileo of the moons of Jupiter, that such anisotropy parameter “can have no influence on the Earth, and therefore would be useless, and therefore do not exist”? Probably not, because the question of the true value of the anisotropy parameter $b$, like the cosmological constant problem, is of fundamental importance.

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[1] A. G. Cohen and S. L. Glashow, Very special relativity, Phys. Rev. Lett. 97 (2006), 021601.
[2] J. Alfaro and V. O. Rivelles, Very Special Relativity and Lorentz Violating Theories, Phys. Lett. B 734 (2014), 239-244 [arXiv:1306.1941 [hep-th]].
S. Upadhyay, M. B. Shah and P. A. Ganai, Lorentz-violating gaugeon formalism for rank-2 tensor theory, [arXiv:1906.03188 [hep-th]].
J. Alfaro, A $Sim(2)$ invariant dimensional regularization, Phys. Lett. B 772 (2017), 100-104 [arXiv:1704.02299 [hep-th]].
S. Upadhyay, M. B. Shah and P. A. Ganai, Lorentz Violating p-form Gauge Theories in Superspace, Eur. Phys. J. C 77 (2017), 157 [arXiv:1702.05735 [hep-th]].
J. Alfaro, Feynman Rules, Ward Identities and Loop Corrections in Very Special Relativity Standard Model, Universe 5 (2019), 16
S. Upadhyay, Reducible gauge theories in very special relativity, Eur. Phys. J. C 75 (2015), 593 [arXiv:1511.01063 [hep-th]].
A. Fuster, C. Pabst and C. Pfeifer, Berwald spacetimes and very special relativity, Phys. Rev. D 98 (2018), 084062 [arXiv:1804.09727 [gr-qc]].
M. B. Shah and P. A. Ganai, A Study of Gaugeon Formalism for QED in Lorentz Violating Background, Commun. Theor. Phys. 69 (2018), 166
S. Upadhyay and P. K. Panigrahi, Quantum Gauge Freedom in Very Special Relativity, Nucl. Phys. B 915 (2017), 168-183 [arXiv:1608.03947 [hep-th]].
A. Ilderton, Very Special Relativity as a background field theory, Phys. Rev. D 94 (2016), 045019 [arXiv:1605.04967 [hep-th]].
D. V. Ahluwalia and S. P. Horvath, Very special relativity as relativity of dark matter: The Elko connection, JHEP 1011 (2010), 078 [arXiv:1008.0349 [hep-ph]].
R. Bufalo and S. Upadhyay, Axion Mass Bound in Very Special Relativity, Phys. Lett. B 772 (2017), 420-425 [arXiv:1707.01345 [hep-th]].
A. C. Nayak and P. Jain, Phenomenological Implications of Very Special Relativity, Phys. Rev. D 96 (2017), 075020 [arXiv:1610.01826 [hep-ph]].
C. Y. Lee, Quantum field theory with a preferred direction: The very special relativity framework, Phys. Rev. D 93 (2016), 045011 [arXiv:1512.09175 [hep-th]].

[3] A. Einstein, Zur Elektrodynamik bewegter Körper, Annalen Phys. 17 (1905), 891-921, [Annalen Phys. 14 (2005), 194-224].
[4] H. Poincaré, Sur la dynamique de l’électron, Comptes rendus des séances de l’Académie des sciences 140 (1905), 1504-1508.
[5] H. Poincaré, Sur la dynamique de l’électron, Rendiconti del Circolo Matematico di Palermo 21 (1906), 129-175. An English translation (by G. Pontecorvo) with modern notations can be found in A. A. Logunov, On the articles by Henri Poincaré: On the dynamics of the electron (JINR, Dubna, 2001).
[6] G. Weinstein, Poincaré’s Dynamics of the Electron - A Theory of Relativity? [arXiv:1204.6576 [physics.hist-ph]].
[7] J. M. Lévy-Leblond and J. P. Provost, Additivity, Rapidity, Relativity, Am. J. Phys. 47 (1979), 1045-1049.
[8] E. Hewitt and K. Stromberg, Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable (Springer-Verlag, New York, 1975).
[9] Y. S. Kim and M. E. Noz, Dirac’s light-cone coordinate system, Am. J. Phys. 50 (1982), 721-724.
[10] R. S. Ingarden, On physical applications of Finsler geometry, Contemp. Math. 196 (1996), 213-223.
[11] P. L. Antonelli, R. S. Ingarden and M. Matsumoto, The Theory of Sprays and Finsler Spaces with Applications in Physics and Biology (Kluwer Academic Publishers, Dordrecht, 1993).
[12] G. S. Asanov, Finsler Geometry, Relativity and Gauge Theories (D. Reidel Publishing Company, Dordrecht, 1985).
[13] G. Yu. Bogoslovsky, A special-relativistic theory of the locally anisotropic space-time. I: The metric and group of motions of the anisotropic space of events, Nuovo Cim. B 40 (1977), 99-115.
[14] O. Chashchina, N. Dudisheva and Z. Silagadze, Voigt transformations in retrospect: missed opportunities?, arXiv:1609.08647 [physics.hist-ph].
[15] H. Cheng and K. C. Gupta, An Historical Note on Finite Rotations, J. Appl. Mech. 56 (1989), 139-145.
[16] V. Lalan, Sur les postulats qui sont à la base des cinématiques, Bull. Soc. Math. Fr. 65 (1937), 83-99.
[17] S. Sonego and M. Pin, Foundations of anisotropic relativistic mechanics, J. Math. Phys. 50 (2009), 042902.
[18] M. Pin, Real and conventional anisotropy, generalized Lorentz transformations and physical effects, Ph.D. Thesis, university of Udine, 2005.
[19] G. Alway, Generalization of the Lorentz Transformation, Nature 224 (1969), 155-156.
[20] L. A. Pars, The lorentz transformation, Phil. Mag. 42 (1921), 249-258.
[21] J. Strnad, Generalization of the Lorentz Transformation, Nature 226 (1970), 137-138.
[22] G. W. Gibbons, J. Gomis and C. N. Pope, General very special relativity is Finsler geometry, Phys. Rev. D 76 (2007), 081701.
[23] G. Yu. Bogoslovsky, On a special relativistic theory of anisotropic space-time, Dokl. Akad. Nauk SSSR Ser. Fiz. 213 (1973), 1055-1058 (in Russian).
[24] G. Yu. Bogoslovsky, A viable model of locally anisotropic space-time and the Finslerian generalization of the relativity theory, Fortsch. Phys. 42 (1994), 143-193.
[25] G. Yu. Bogoslovsky, Theory of Locally Anisotropic Space-Time (Moscow University Press, Moscow, 1992) (in Russian).
[26] H. R. Brown, Does the principle of relativity imply Winnie’s (1970) equal passage times principle? Phil. Sci. 57 (1990), 313-324.
[27] T. Budden, The relativity principle and the isotropy of boosts, PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association 1 (1992), 528-541.
[28] T. Budden, Galileo’s Ship and Spacetime Symmetry, Brit. J. Phil. Sci. 48 (1997), 483-516.
[29] T. Budden, A star in the Minkowskian sky: Anisotropic special relativity, Stud. Hist. Phil. Sci. B 28 (1997), 325-361.
[30] H. R. Brown, Physical relativity : space-time structure from a dynamical perspective (Oxford University Press, Oxford, 2005).
[31] P. Winternitz and I. Friš, Invariant expansions of relativistic amplitudes and subgroups of the proper Lorentz group, Sov. J. Nucl. Phys. 5 (1965), 636-643.
[32] R. Shaw, The subgroup structure of the homogeneous Lorentz group, Quart. J. Math. 21 (1970), 101-124.
[33] E. Inönü and E. P. Wigner, On the Contraction of groups and their represenations, Proc. Nat. Acad. Sci. 39 (1953), 510-524.
[34] H. J. Saletan, Contraction of Lie Groups, J. Math. Phys. 2 (2004), 1-21.
[35] D. R. Finkelstein, General quantization, Int. J. Theor. Phys. 45 (2006), 1397-1427.
[36] F. Slocum, Galileo Galilei, a pioneer in scientific research, Publ. Astron. Soc. Pac. 45 (1933), 103-126.