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A nucleon–nucleus optical model for $A \leq 13$ nuclei at 65–75 MeV projectile energy

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Abstract

Small nuclei ($A < 16$) test the limits of optical potential theory, most recently there has been difficulty matching the helium isotopes’ analyzing powers at 71 MeV per nucleon. We provide a phenomenological optical potential that has been fit to a variety of experimental proton–nucleus and neutron–nucleus scattering data between $A = 4$ and $A = 13$ with a projectile energy between 65 and 75 MeV; the motivation being the helium isotopes. The resulting potential is dependent only on projectile isospin, neutron, and proton numbers. It fits the analyzing power of $^6$He satisfactorily only by including a non-traditional negative surface depth and a spin–orbit force which is significant at a distance larger than traditional models. We offer physical interpretations of our non-traditional results, suggest directions for microscopic calculations, and proffer calculations for future experiments.

Keywords: optical, nuclear, reaction, elastic, light, polarization

(Some figures may appear in colour only in the online journal)

1. Introduction

The first intermediate energy inverse kinematics nucleon–nucleus reaction off a polarized target was reported by Hatano et al [1–3], $^6$He$(p, p')$ at 71 MeV. The polarization has thwarted attempts to be fit by theory. This $A_p$ polarization variable dips below zero thus not mimicking its helium and lithium counter-parts. This is disappointing for it is an open question as whether elastic scattering alone can constrain structure calculations [4], the failure...
of matching this important observable challenges theorists to re-examine the limits of conventional optical model theories at low atomic number.

We first purview important theoretical investigations of this reaction. The group that produced these experimental results performed their own optical model analysis in [5, 6] using phenomenological and microscopic potentials which described some but not all of the data. The present author has also published a prediction using a microscopic full-folding optical potential in [7] which failed to describe the polarization, later we published a modified analysis [8, 9] that had improvements but still showed the inadequacies of the microscopic optical potential to reproduce \( A_y \) for the \( ^6\text{He}(p, p) \) reaction at 71 MeV. A \( g \)-matrix microscopic model was attempted by another group in [10] with similar poor results while trying to match the last data point, a question of the validity of that data point, with such high error bars, was raised. A simple microscopic model which had adjustable parameters tuned to a normalization of the volume integrals was used in [11, 12] to fit to the light nuclei (helium, lithium, and beryllium) in a systematic way. The results were respectable for the differential cross-sections but struggled to fit the helium and lithium results at higher energies including the \( ^6\text{He}(p, p) \) reaction at 71 MeV. In [13, 14] a Glauber approximation was used to develop a series of target wave functions using a variety of interactions, the fit of the polarization experiment was still poor, similarly with a coupled channel calculation done in 2011 [15]. A systematic Bruckner–Hartree–Fock study of the light nuclei was also performed and found to have large sensitivity to the NN potential and the \( A_y \) observables in [16]. The Eikonal approximation was used on the helium isotopes in [17] while a relativistic impulse approximation study was undertaken in [18] with again limited success.

Some researchers have had moderate success by starting with a traditional microscopic \( g \)-matrix model but then broadening the target density [19]. Their results are among the best fits to the elastic scattering data. Another exemplary calculation comes from [20] which uses phenomenological and semi-microscopic potentials using a variety of density and interaction models. Overall however the \( A_y \) problem still exists for the proton- \( ^6\text{He} \) reaction at 71 MeV, none of the calculations describe the complete five point \( A_y \) observable. In this work we thoroughly search the space for the best phenomenological potentials to fit the elastic scattering observables measured at 71 MeV. We develop our own local and global model which have been informed by the results of previous work.

2. Theory

Our complex phenomenological optical model potential contains the traditional volume \( (V) \), surface \( (D) \), and spin–orbit \( (SO) \) nuclear terms which are delineated using the standard Woods–Saxon form factors

\[
 f_{WS}(r, R_i, A_i) = (1 + \exp((r - R_i A_i^{1/3})/A_i))^{-1},
\]

where \( R_i \) is the radius parameter and \( A_i \) is the diffuse parameter. The \( i \) is a placeholder index for the \( V \) (volume), \( D \) (surface), and \( SO \) (spin–orbit) designations. The phenomenological optical model potential takes the standard form:
$U(r, A, N, Z, P)$
\[= -V_r(A, N, Z, P)f_{WS}(r, R_v, A_v)\]
\[-iW_r(A, N, Z, P)f_{WS}(r, R_v, A_v)\]
\[+ i4A_DW_D(A, N, Z, P)\frac{d}{dr}f_{WS}(r, R_S, A_S, P)\]
\[+ \frac{2}{r}V_{SO}(A, N, Z, P)\frac{d}{dr}f_{WS}(r, R_{SO}, A_{SO})(1 \cdot \sigma)\]
\[+ \frac{i2}{r}W_{SO}(A, N, Z)\frac{d}{dr}f_{WS}(r, R_{SO}, A_{SO})(1 \cdot \sigma)\]
\[+ f_{coul}(r, R_c, A, N, Z, P),\]

where the $V_r$ and $W_r$ are the real and imaginary potential amplitudes, respectively, and $f_{coul}$ is the coulomb term which has the following traditional format:

$$f_{coul}(r, R_c, A, N, Z, P) = \frac{1 + P \frac{Ze^2}{2}}{r}, \quad r \geq R_c,$$

(2)

$$f_{coul}(r, R_c, A, N, Z, P) = \frac{1 + P \frac{Ze^2}{2}}{R_c \left( 3 - \frac{r^2}{R_c^2} \right)}, \quad r \leq R_c.$$  

(3)

Thus for a neutron projectile this coulomb term is set to zero. The amplitudes, radii, and diffusive parameters have the following dependent variables:

- $A$—atomic number of the target nucleus,
- $N$—number of neutrons in the target nucleus,
- $Z$—number of protons in the target nucleus,
- $P$—+1 if nucleon projectile is a proton, −1 if a neutron.

This potential was put into a standard optical potential calculator which solves the Schrödinger equation for spin $\frac{1}{2}$-spin 0 scattering using a coulomb wave-function basis. A Numerov routine from [21] was used to solve the non-relativistic position space Schrödinger equation. The routine which produced the coulomb wave functions was found in [22]. A Powell routine, adopted from Numerical Recipes [23], was used to minimize a weighted reduced $\chi^2$. The approach was similar to that used in [24].

### 3. Previous work and preliminary results

We first examine notable previous research efforts. Figure 1 details past theoretical calculation by the present researchers and others. The oldest calculation is our microscopic calculation, from [7] (solid black line), published before the experiment was carried out. It manages to miss all the experimental points for the polarization variable. This author and collaborators have attempted two more microscopic calculations; first with a cluster approach (treating the target as a tightly bound alpha core and two loosely bound valence neutrons—two separate optical potential calculations), [25] (red dashed line), and later, in [8] (not shown), we included all nucleon–nucleon scattering terms needed when the target nucleus has an open $p$ shell. Our cluster calculation did manage to agree with two of the experimental points but still missed the forward angles by quite a large margin. It also did a poor job describing the differential cross-section at larger angles.
Another microscopic research effort from others which had modest success is that of Toyokawa et al [19] using a modified Melbourne g-matrix depicted as the short-dashed blue line in figure 1. They modified the strengths of the amplitudes and found their best fit by suppressing all their amplitudes except the imaginary spin–orbit which they strengthened four-fold. Their best fit was adequate for three out of the five points but the shape of the curve showed promise.

The best fits, unsurprisingly, came from phenomenological Woods–Saxon based potentials. The ansatz is unabashedly to be fit to the experimental data, we provide two previous examples from other research groups. The dotted purple line in figure 1 is from the experimental group that took this measurement [5]. Their fit has the correct shape and misses only the last point. They surmise that this calculation works well because of its exceeding distant ($r = 2.34 \text{ fm}$) but shallow spin–orbit potential [5, 6]. Although their potential was a good fit they were slightly bettered by Behairy et al [20] (the dotted–dashed green line). Again, a phenomenological fit with a good shape and a slightly better fit, especially at the last point, albeit still missing it slightly. What makes this calculation intriguing is that they loosen their constraints on the typical Woods–Saxon parameter ranges. Their imaginary central potential minimum is extremely deep (81 MeV) and falls off within the alpha core (the Woods–Saxon $R_0 = 0.52 \text{ fm}$!). They realize that the helium six nucleus may need some modifications to the standard Woods–Saxon parameter set, however the justification for this deep and short-ranged imaginary term is lacking. This is where the present research began. How much does one have to expand the traditional parameter set to get a high quality fit, and is there a rationale for these alterations?

Figure 1. The previous theoretical misses of the Rutherford reduced elastic differential cross-section and the elastic polarization by our research group and others are depicted. The solid black line and red dot–dot–dash line are microscopic calculations from our previous work ([7, 25], respectively). The dashed blue line is a microscopic calculation from others [19]. The dotted purple line and the dotted–dashed green line are examples of good phenomenological fits from others and can be found in [5, 20], respectively. All of these theoretical calculations are discussed further in the text. The experimental data depicted as circles are taken from [3], other experimental data, the squares, are from [26, 27].
4. Results

For this research we attempted to fit all five of the polarization data points of the exotic helium isotope, $^6\text{He}$. We also found that we had to search outside normal parameter limits to be successful. Our calculations are shown in figure 2 and our parameters used are in table 1. The non-traditional cases of this work include a negative surface term located between the core and valence nucleons and a long range spin–orbit term, as suggested by [5]. Note that previous good fits to the data, as presented in table 1 are missing a surface term which plays a pivotal role in this work. We also decided to keep the same radius and diffusive parameters for the real and imaginary volume elements of the optical potential. This was done in deference to dispersion relation constraints which link the two terms structurally [28–30].

Figure 2 shows the validity of the attempt, the fits for both the reduced differential cross-section and polarization are very near a reduced $\chi^2 < 1$ for both fits.

![Figure 2. The present calculations to the Rutherford reduced differential cross-section and the polarization observable. The dotted–dashed red line is a phenomenological calculation from the present work and has a negative imaginary surface depth. The dashed black line is also a phenomenological calculation from the present work and has an unusually large spin–orbit radius. The parameters for both calculations are listed in table 1. Both of these calculations are discussed further in the text. The experimental data depicted as circles are taken from [3], the squares are from [26, 27]. The reduced $\chi^2 < 1$ for both fits.](image-url)
Figure 3. Negative imaginary surface—the dotted–dashed red line is the same theoretical calculation as presented in red in figure 2, a phenomenological potential with a non-traditional negative imaginary surface term (parameters listed in table 1). The dotted blue calculation is the same potential except the negative imaginary surface term has been set to zero to elicit the sensitivity of that term. The inset depicts the central imaginary optical potential with and without the surface term. The effect of the negative surface term is to reduce the strength of this potential in the 2.0–2.5 fm range. Both of these theoretical calculations are discussed further in the text. The experimental data depicted as circles are taken from [3], the squares are from [26, 27].

Table 1. Model parameters for four local optical potentials, the first two from this research, the last two from earlier work. The coulomb radius, $R_C$ in fm, has been multiplied by $\frac{1}{3}$. The last line includes the calculation’s prediction of the reaction cross-section. The first two calculations are presented in figures 2–4. The earlier Behairy [20] and Sakaguchi [5] fits are the best fits in figure 1 and are depicted by the dotted–dashed green line and the dotted purple line. The italicized parameters are those which fall outside traditional values.

| Term  | Units    | Neg. Surf. | Ext. SO | Behairy | Sakaguchi |
|-------|----------|------------|---------|---------|-----------|
| $V$   | MeV      | 36.828     | 43.524  | 19.588  | 27.86     |
| $W$   | MeV      | 35.410     | 10.611  | 81.055  | 16.58     |
| $W_D$ | MeV      | -5.574     | 6.734   | —       | —         |
| $V_{SO}$ | MeV fm$^2$ | 2.326     | 1.719   | 2.911   | 2.020     |
| $R_V$ | fm       | 0.887      | 0.749   | 1.198   | 1.074     |
| $A_V$ | fm       | 0.696      | 0.514   | 0.583   | 0.681     |
| $R_W$ | fm       | $R_V$      | $R_V$   | 0.286   | 0.860     |
| $A_W$ | fm       | $A_V$      | $A_V$   | 0.773   | 0.753     |
| $R_D$ | fm       | 1.135      | 1.298   | —       | —         |
| $A_D$ | fm       | 0.210      | 0.620   | —       | —         |
| $R_{SO}$ | fm     | 1.134      | 1.600   | 1.018   | 1.290     |
| $A_{SO}$ | fm     | 0.881      | 0.362   | 1.063   | 0.760     |
| $R_C$ | fm       | 2.417      | 2.296   | 2.362   | 2.362     |
| $\sigma_R$ | mb  | 265        | 299     | 251     | 179       |
affects them at angles greater than 40°, thus the tail is unaffected. Its removal changes dramatically the polarization observable above 40°. In essence the negative surface lowers the polarization enough to fit the last point, it does this by increasing the elastic cross-section between the angles of 40° and 80°. At the same time it decreases the reaction cross-section by 20 mb (from 285 mb without the surface to 265 mb with the negative surface term). A plausible physical explanation for the validity of this negative term is that there is a discernible region in all Borromean halo nuclei between the core and nuclei. In this relative weakening of matter their would be a diminishing of inelastic effects that would occur, thus creating an increase in the elastic channels via the long range nuclear and coulomb forces. For helium 6 this effect is significant. In this interpretation the anomalous polarization of 6He is caused by the region between the halo and the core. This hypothesis is substantiated by examining the relative improvement for the microscopic potential of our earlier work [25] which used a cluster approach to separate the core and the halo. This result is depicted in figure 1 as the dashed red line which is one of only a few of the past microscopic optical potentials by others which managed to fit the last two data points of the polarization.

In figure 4 we examine having a distant spin–orbit potential (our spin–orbit peaks at 2.91 fm!) as first suggested by [5, 6]. Our attempt with this anomaly is shown first in figure 2 and duplicated in figure 4 as the dashed black line. The dotted brown line is where we move it into the far end of a traditional parameter value (the spin–orbit now peaks at a more modest 2.36 fm). The inset shows how the real spin–orbit potential was affected by this radius change. This modification has similar effects to the surface modification above. What we see is that the extended spin–orbit significantly affects at angles greater than 40°, thus the tail is again relatively unaffected. Its reduction to a traditional value changes the polarization observable above 50°. In essence the extended spin–orbit raises the polarization enough to fit
the third and fourth points, it does this by increasing the elastic cross-section between the angles of 45° and 85°. At the same time it does not change the reaction cross-section which stays at 299 mb. A plausible physical explanation for this spin–orbit potential at the range of ≈3 fm is that the valence neutrons are mostly responsible for most spin–orbit reactions in the ⁶He nucleus. Like a comet going around a sun, some projectiles have a close approach with the valence neutrons and are pushed away by the strong relative angular momentum between the valence neutron and the projectile. In this instance the anomalous polarization of ⁶He is caused by the halo itself. It is important to emphasize that our fits are better than [5, 6] because our spin–orbit has gone even further out (and is less diffuse) than their choice.

The question is still open whether the modifications are physically valid. In both the negative surface term and the extended spin–orbit it is important to remember that these modifications, although away from tradition, still maintain most of the usual characteristics of a nuclear reaction and the physical interpretations for the anomalies, while tentative, are also plausible. The realization that two structurally different optical potentials provided excellent fits to these experiments observables hints that elastic scattering data alone is not enough to determine structure, especially in the light exotic nuclei, as was also communicated in [4, 31, 32]. To systematically study these anomalies we will now expand our study to the nuclei near ⁶He.

5. To a global optical potential

To examine the validity of these two non-traditional potentials we thought it best to give them a stronger footing by examining the potentials of near neighbors (4 ≤ A ≤ 13) in a systematic way by developing a multi-target phenomenological potential with atomic, proton, and neutron number dependence which would be tested in a small intermediate window of 65–75 MeV nucleon projectile energies. We wanted it to be isospin dependent (to handle both proton and neutron observables), in many ways similar to our previous work of [24] but testing the lightest of nuclei. By widening our scope we can now make a stronger statistical analysis on the validity of the two non-traditional parameter choices which will now contain hundreds of degrees of freedom.

All known nucleon–nucleus reactions in the range of 4 ≤ A ≤ 13 and 65 MeV ≤ E_{Lab} ≤ 75 MeV were summed to produce one global χ². There were also a few reaction cross-sections which did not exist but were estimated by interpolation [33]. The variances in the parameters were minimized to this weighted χ² which slightly favored low atomic number (A = 4) over high atomic number (A = 13), forward angles over backward angles, and total cross-sections over reaction cross-sections. Polarizations was slightly favored over differential cross-sections when their were no error bars. Choosing the correct weightings was found to be an art form where a balance tenuously existed in which every reaction was regarded but certain significant reactions were strengthened so that the weighted χ² parameter space contained large relative minimums which the search functions could find easily.

To find these minimums a Monte Carlo preliminary gross search was done using a Sobol number generator. Since the parameter space was often over twenty-five dimensions this might include up to 1 × 10⁶ vectors in which weighted χ² were first calculated by solving the Schrödinger equation for the given potential. The lowest thirty vectors were analyzed in more fine detail by seeking the local minimums in their vicinity within the χ² parameter space. The χ² minimization program sought the steepest derivative in the multi-dimensional parameter space [23] by a Powell routine. Although the overall quality of the fits were examined
occasionally, the entire process was close to automatic. The fitting part which had to be done manually was to pick the best functional form of the atomic and excess dependence. This was done through brute trial and error until their was a clear candidate which gave good results. The functional form chosen is unorthodox but is a function of our results: that the $^6$He nucleus and to a lesser extent the $^8$He nucleus are anomalies even among the small isotopes and thus the Woods–Saxon parameter space is irregular.

Explicitly the twelve Woods–Saxon terms of this optical model are given using thirty distinct parameters where one Woods–Saxon potential term may have up to four of these parameters. The systematic polynomial formats of these terms are described below. The global optical potential is as follows, first the central ($V$ for volume, $D$ for surface, $C$ for coulomb):

\[
\begin{align*}
V_V &= 48.644 - 6.429(A - 4)f_A^4 - 3.240f_f f_A^{8/3}, \\
W_V &= 11.341 - 2.284(A - 4)f_A^4 + 9.413f_f f_A^{8/3}, \\
R_V &= 0.903 2 + 0.096 15(A - 4)f_A^4, \\
A_V &= 0.648 5, \\
W_D &= 11.272 - 1.354(A - 4)f_A^4 - 8.161f_f f_A^{8/3}, \\
R_D &= 0.540 7 + 0.328 0(A - 4)f_A^4, \\
A_D &= 0.178 3, \\
R_C &= 3.719 4 - 0.201 8(Z - 2).
\end{align*}
\] (4)

Here are the spin–orbit parameters:

\[
\begin{align*}
V_{SO} &= 3.238 - 0.250 1(A - 4)f_A^{1/3}, \\
W_{SO} &= 0.000 - 0.270 7(A - 4)f_A^{1/3} + 0.133 7f_f f_A^{1/3}, \\
R_{SO} &= 1.201 - 0.082 17(A - 4)f_A^{1/3} + 0.236 0f_f f_A^{1/4}, \\
A_{SO} &= 0.781 5,
\end{align*}
\] (5)

where $A$ is the atomic number, $P$ is twice the isospin component of the nucleon ($P = 1$ for proton, $P = -1$ for neutron), $N$ the number of neutrons, and $Z$ the number of protons in the target. The functions are: $f_A = \frac{A}{A-1}$, $f_f = |N - Z|$, and $f_f = P (N - Z)$. Note that the non-traditional $f_A$ function peaks at low $A$ and diminishes as $A$ gets larger.

Many functional forms were attempted, the best result is given. The irregularity of $^6$He and $^8$He can already be spotted by the high exponent needed on some of the $f_A$ terms which is a function that is suppressed for $A > 8$. This calculation includes both a negative surface term and a unusually long spin–orbit term in tandem which significantly exists for only the exotic helium isotopes ($^6$He and $^8$He) because of this large exponent on some of the $(N - Z) f_A$ product terms. In every trial both of these non-traditional features on these helium isotopes lead to the best fitted results. Likewise the large exponents on the $(N - Z) f_A$ product terms in the volume depths suppress the real part and build up the imaginary part for the exotic helium isotopes.

In figures 5 and 6 we display some of the calculations of our global optical potentials using a green solid line. These fits are not of the same quality as the local fits to only the $^6$He nucleus displayed in figures 2–4 since the ratio of the degrees of freedom to the dimensionality of the parameter space has increased significantly for this multi-nuclei potential (30 parameters, 715 points). Likewise the optical model is known to have better success as the atomic number increases. However the general quality is still good, especially the spin
Figure 5. Differential cross-section—the reduced Rutherford elastic differential cross-section for nine different targets, experimental measurements along with the fit as produced by the global optical potential presented here. The helium isotope data is from [3, 6, 26, 27, 35, 36]. The lithium isotope data is from [37, 38], the beryllium data is from [39], the boron data is from [38]. The neutron scattering data from carbon 12 comes from [40–42]. The proton scattering data for the carbon isotopes comes from [43–46]. The calculations and data have been adjusted to fit on one figure. The middle calculation and data on all three panes is divided by 10. The bottom calculation and data is divided by 100. For the carbon nuclei we have included calculations done previously by the author [24] using a dashed blue line.

Figure 6. Polarization—the elastic polarization for eight different targets (same as figure 5), experimental measurements along with the fit as produced by the global optical potential presented here. The data is from the same references as stated in figure 5. For the carbon nuclei we have included calculations done previously by the author [24] using a dashed blue line.
observables for $^6,^8\text{He}$ which were favored in the fitting routine. For the purposes of this research the addition of $A > 8$ nuclei to the fitting apparatus was made to put tighter constraints on the $A \leq 8$ parameters. Specifically we wanted the flexibility to extend our parameter space ranges but not uncontrollably so it was disconnected from the link to the tradition nuclei with traditional values. Thus our fits to carbon nuclei are presentable yet not stellar. We are using these nuclei as our high mass asymptotes to anchor our calculation, as we are using $^4\text{He}$ on the low mass end to do the same. The reaction and total cross-sections of figure 7 can be examined in the same context. Our calculations stay close to the experimental results thus adding validity to our choice of parameters and their functional forms It also shows that the ratio of the real interaction to the imaginary interaction are constrained realistically. The reduced $\chi^2 = 80 \pm 10$ (fitting 715 experimental points). If the points beyond 100° center of mass angle are removed in the analysis for the reduced differential cross-section and polarization data the reduced $\chi^2 = 50 \pm 10$ (fitting 645 points) thus exposing a weakness to describe the backward angles (a feature of all optical potentials). The $\chi^2$ is given as a range because in a selection of the experimental data there is unclear error. The larger reduced $\chi^2$ illustrates the difficult demands of fitting optical potentials to $^4\text{He}$ and $^{12}\text{C}$ at the same time. Obviously the best fit is a local one which fits only one nuclei as depicted in figure 2 which has a reduced $\chi^2 \approx 1$ for the $^4\text{He}$ nucleus. However the global approach is more informed and adds validity to the parameters in a way that the local fit cannot.

As a further comparison we added the results to figures 5–7 of our earlier global optical potential [24] which described $12 \leq A \leq 60$ and $30 \text{ MeV} \leq E_{\text{proj}} \leq 160 \text{ MeV}$. Overall these calculations, depicted by a blue dashed line, are better than the present result. The most significant weakness of the present model is that it overcompensates for the diffraction minimum of the differential cross-sections that the older potential describes well. The potential of [24] had three times as many parameters for a fixed energy and carbon 12 was fit under the influence of heavier stable nuclei ($^{14}\text{N},^{16}\text{O},^{19}\text{F}$, etc) which it has more in common with than the recalcitrant observables of the lithium and helium isotopes.
Our goals have been achieved: to have a working global optical model which can explain the polarization anomalies of the helium isotopes and fit within the constraints of the traditional nuclei around them including the reaction and total cross-sections for both proton and neutron scattering. Recently Farag et al [12] has achieved similar success in a global potential using a microscopic folding structure which has been renormalized to the volume integrals. Their work is energy dependent but does not cover neutron scattering. They have better success fitting the differential cross-sections while our success is better achieved with the polarization observables including the $^6$He$(p, p)$ reaction at 71 MeV which they have trouble describing.

In table 2 we explicitly display the global optical potential to three significant figures for nine different isotopes in the range $4 \leq A \leq 13$. An overview of these results show that their are many more similarities than differences. The parameters with the most variance include the imaginary amplitudes: central and surface term. The exotic isotopes show a reduced surface term (sometimes negative) and an increased imaginary central amplitude. Additionally the radius of the spin–orbit function has a large gradient which also peaks at $^4$He. The exotic nuclei which show the most anomalies are clearly the low $Z$ helium isotopes. We also calculated the volume per nucleon for the total central nuclear potential (real and imaginary) and the root-mean-squared (rms) radius for the same nuclear potential. These macroscopic sums show the exception that is $^4$He, a large imaginary volume term and rms radius. In contrast it also depicts the doubly magic $^4$He admirably with its large real volume, tight rms radii, and diminished imaginary character. The values for the volume integrals are similar to those found in [11, 12, 52]. For comparison we also show the potential terms of [24] for carbon 12 with a projectile energy of 70 MeV. Note that these parameters of our previous global optical potential are not systematically congruent with the present results. Carbon 12 was the lightest nuclei in [24] while here it is one of the heaviest. This detail epitomizes a weakness of phenomenological potentials, that even in the systematic approach of a global potential there is not a unique potential which will satisfactorily describe the observables. The disparity of these helium isotopes is further shown in a graph of a sample of the Woods–Saxon parameters (figure 8). All the isotopes except helium show a near linear relationship with each other. The helium isotopes however each have a distinct character and this is manifest in the Woods–Saxon parameters which describe them.

In our last figure, figure 9, we include a few predictions for the neutron rich $^{11}$Li, $^{12}$Be, and proton rich $^8$B nuclei using our global optical potential. The proton rich $^8$B reduced Rutherford differential cross-section calculation shows a significant strength at backward angles because of the extended coulomb potential. The polarization observables hold no surprises, they resemble nuclei with similar atomic numbers. The $^8$B calculation resembles an accentuated $^3$He polarization data set while the $^{11}$Li and $^{12}$Be resemble the carbon nuclei. The only nucleus in this global optical potential which has non-traditional values (a very negative imaginary surface term, an extended spin–orbit radius radius, and a large imaginary central depth) is $^6$He. We look forward to future experimental tests with polarized beams to test the limits of this potential and the sensitivities of elastic scattering [4].

6. Conclusion

We investigate the power of the phenomenological optical potential in position space to gain insight into the polarization observables for low $A$ nuclei being struck by a nucleon at about 70 MeV. We find that the $^4$He polarization observables can only be described with optical potential terms outside the normal constraints, namely a negative surface term and an
Table 2. Calculated values for nine possible nuclei concentrating on the helium, lithium, and carbon isotopes. The top and bottom third are Woods-Saxon parameters from equations (4) and (5), rounded to 3 significant figures, the middle third are calculated volume per nucleon and rms radius terms for the central nuclear potential. All terms in this chart assume proton scattering from the target. For reference we also include the carbon 12 potential from [24] in the 2nd column from the right. Not listed for this potential is an additional term: the real surface depth of −0.134 MeV.

| Terms | Units | He4 | He6 | He8 | Li6 | Li7 | Li9 | Li11 | C12 | C12 [24] | C13 |
|-------|-------|-----|-----|-----|-----|-----|-----|------|-----|----------|-----|
| $V_V$ | MeV   | 48.6| 29.3| 29.9| 35.8| 32.7| 31.0| 30.6 | 31.5| 28.57     | 31.2|
| $W_V$ | MeV   | 11.3| 25.6| 15.5| 6.77 | 10.6| 10.1| 8.79 | 5.25 | 7.02      | 5.55|
| $R_V$ | fm    | 0.903| 1.10| 1.13| 1.10| 1.12| 1.14| 1.16 | 1.16| 1.22     | 1.16|
| $A_V$ | fm    | 0.649| 0.649| 0.649| 0.649| 0.649| 0.649| 0.649| 0.649| 0.662    | 0.649|
| $R_C$ | fm    | 3.72 | 3.72| 3.72| 3.52| 3.52| 3.52| 3.52 | 3.52| 2.91     | 2.91|
| $W_D$ | MeV   | 11.3| −7.76| −0.338| 8.56| 4.43| 4.03| 4.73 | 7.66| 5.88     | 7.29|
| $R_D$ | fm    | 0.541| 1.20| 1.33| 1.20| 1.28| 1.36| 1.40 | 1.42| 0.916    | 1.43|
| $A_D$ | fm    | 0.178| 0.178| 0.178| 0.178| 0.178| 0.178| 0.178| 0.178| 0.512    | 0.178|
| Vol-real | MeV fm$^3$ | 471 | 340 | 332 | 416 | 373 | 338 | 328 | 328 | 337      | 319|
| Real rms rad. | fm | 2.64 | 2.85 | 2.97 | 2.85 | 2.91 | 3.02 | 3.12 | 3.17 | 3.27     | 3.20|
| Volimag | MeV fm$^3$ | 128 | 236 | 168 | 139 | 154 | 141 | 131 | 116 | 149      | 114|
| Imag. rms rad. | fm | 2.46 | 2.98 | 2.98 | 2.63 | 2.85 | 3.01 | 3.15 | 3.26 | 3.13     | 3.32|
| $V_{SO}$ | MeV fm$^2$ | 3.24 | 2.74 | 2.36 | 2.74 | 2.54 | 2.19 | 1.87 | 1.72 | 5.88     | 1.57|
| $W_{SO}$ | MeV fm$^2$ | 0.00 | −0.274 | −0.502 | −0.541 | −0.634 | −0.820 | −1.00 | −1.65 | −0.447   | −1.71|
| $R_{SO}$ | fm    | 1.20 | 1.51 | 1.30 | 1.04 | 1.11 | 1.07 | 0.963| 0.702| 0.915    | 0.682|
| $A_{SO}$ | fm    | 0.782| 0.782| 0.782| 0.782| 0.782| 0.782| 0.782| 0.782| 0.512    | 0.782|
extended spin–orbit. These outcomes implicate that a cluster model may be the best microscopic method for future success. To constrain our results we develop a global potential which covers a variety of small $A$ nuclei ($4 \leq A \leq 13$). This potential can be used as a benchmark for future exotic nucleus–nucleon polarized experiments as well as a comparable signpost for microscopic potentials.

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Figure 9. Example theoretical calculations for nuclei without experimental tests. All calculations have been drawn from our global potential using equations (4) and (5). The calculations have been adjusted to fit on one figure. The reduced Rutherford $^{11}$Li differential cross-section calculation has been divided by 10. The $^{12}$Be differential cross-section calculation has been divided by 100.

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