Probing the limits of quantum theory with quantum information at subnuclear scales

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Abstract

Modern quantum engineering techniques enabled successful foundational tests of quantum mechanics. Yet, the universal validity of quantum postulates is an open question. Here we propose a new theoretical framework of Q-data tests, which recognises the established validity of quantum theory, but allows for more general — ‘post-quantum’ — scenarios in certain physical regimes. It can accommodate both models with modified wave dynamics and correlations beyond entanglement. We discuss its experimental implementation suited to probe the nature of strong nuclear interactions. In contrast to the present accelerator experiments, it shifts the focus from high-luminosity beam physics to individual particle coherent control. A successful implementation of the proposed scheme would not only provide insight into the fundamental physics but also establish new devices for quantum information processing operating at unprecedented scales.
1 Introduction

Quantum mechanics is one of the most successful scientific theories of 20th century, faithfully modelling phenomena in the micro-world. The manifestation of some of its most distinctive features — entanglement [1] and wave-particle duality [2] — require precise preparation of the system’s state and detection of individual particles. Suitable devices for quantum engineering, based on electromagentic interactions, became available only recently. On the theoretical side, the possibility of acute control of quantum states gave birth to the theory of quantum information [3]. The recognition of entanglement and coherence as resources [4] leads to tantalising technological perspectives, including quantum computation [5], quantum cryptography [6] and quantum sensing [7]. In parallel, quantum field theory emerged from the unification of quantum mechanics with the special theory of relativity [8]. It is at the core of the Standard Model of particle physics and provides an extremely accurate framework for the study of high-energy phenomena.

The tremendous success of quantum theory motivates a question about its universality and limits of validity. Could there be a ‘post-quantum’ theory violating some of the basic quantum principles? If so, in which physical regime would it become manifest? These questions have been approached from a number of different standpoints. One of them, outlined already in the 1960 by Louis de Broglie [9], assumes a nonlinear modification of the Schrödinger equation [10, 11], possibly along with a revision of the Born rule [12, 13]. A related class of theories seeks a mechanism behind the collapse of the quantum wave function [14]. Yet a different strategy, developed more recently, is based on the possibility of nonlocal correlations stronger than those predicted by quantum mechanics [15, 16, 17, 18, 19].

It is commonly expected that if there are any deviations from the standard quantum theory, then they might be related to the nature of the gravitational field. This assumption points to two physical regimes of interest. The first one is determined by extremely short distances of the order of Planck length $1.6 \times 10^{-35}$ m or exceedingly large energies around the Planck energy $1.2 \times 10^{19}$ GeV [20, 21, 22]. The second regime involves quantum superposition of macroscopic objects of size $\gtrsim 10^{-6}$ m and mass $\gtrsim 10^6$ GeV/$c^2$ [14, 23]. No experiment probing either of these domains has so far hinted at any new physics beyond the standard quantum theory [24, 25, 26].

Yet there exists another physical regime, which may hide surprises – the interior of a nucleon. It is governed by the strong nuclear force, characterised by length scales of order of $10^{-15}$ m and energies in the GeV range. Its key properties are well understood within the
modern paradigm of quantum field theory. However, all of our empirical knowledge about the nucleon’s interior is based on high-energy scattering experiments typically involving $\sim 10^{11}$ particles. In stark contrast, experiments probing the quantum foundations require precise measurements of individual particles (see [27] and references therein). Our central hypothesis assumes that new unforeseen effects might be concealed by abundant pair creation processes dominating in high energy collisions. We claim that new insights into the nature of the strong interaction could be gained via precise quantum engineering experiments at the few-particle level.

From the viewpoint of information processing a nucleon is a very intriguing system, unlike the ones encountered in atomic physics. The well-established theory of quantum chromodynamics draws a complex picture of the nucleon consisting of three valence quarks immersed in a sea of gluons and virtual quark–antiquark pairs [28]. In particular, gluons’ total angular momentum constitute up to 50% of nucleon’s spin [29, 30, 31, 32]. This provokes the question: How is the coherent quantum information distributed and processed within nucleons? Given the significant role played by the gluons, one should expect a high accumulation of degrees of freedom in a tiny volume. Our proposal aims at testing whether the nature of these degrees of freedom is quantum.

This issue cannot, however, be addressed in the standard prepare-and-measure paradigm at the level of individual particles. This is because quarks and gluons involved in the strong nuclear interactions do not exist as free particles, but are always confined within hadrons.

Nevertheless, we show here that questions about information processing at subnuclear scales can be rigorously formulated and answered in future precision experiments. To this end, we establish a new ‘black box’ formalism with quantum inputs and outputs. It is model-independent and can accommodate a large class of different post-quantum scenarios involving both modified wave dynamics and super-quantum correlations. We demonstrate on concrete examples how the protocols can serve to detect phenomena, which go beyond the standard quantum mechanical description. Our approach is thus radically different from the existing ones [33, 34, 35, 36] designed to witness the genuinely quantum feature of entanglement in high-energy phenomena.

In the second part of the article, we focus on the context of strong nuclear interactions and discuss some key elements needed for the implementation of our theoretical scheme in future experiments. The basic idea involves an orchestrated scattering of ‘quantum-programmed’ projectiles from a target followed by projective measurements of a chosen observable on individual outgoing particles. In this way one can effectively perform a quantum tomography [37, 38] of an unknown process. More advanced experimental schemes aimed at probing the strength of
correlations involve multiple, subsequent or simultaneous, elastic scatterings from the same nucleon. These would require acute trapping of individual nucleons and a precise coherent control of single projectiles with energies in the GeV range, which are beyond the state-of-art technology. We argue, however, that some insight could be gained from scattering experiments with polarized beams, routinely done in the accelerators, and meticulous quantum state reconstruction from the acquired data.

2 Theory-independent tests of post-quantumness

An elemental physical experiment results in a set of conditional probabilities \( \{ P(a_j \mid x_i) \}_{i,j} \), calculated via the standard frequency method, for given input data \( \{ x_i \}_i \) and registered outcomes \( \{ a_j \}_j \). This fact is a starting point for the “theory-independent” paradigm \([18]\), which focuses exclusively on the effective outcome probabilities, while ignoring the description of physical details of the studied systems. In such an approach the physical system under investigation is seen as “black box”, which can be probed with programmable input and controllable output systems. The latter carry the encoded information \( (x_i \text{ at the input and } a_j \text{ at the output}) \), while the box acts as an information processing device. Different theoretical schemes impose different constraints on the admissible information processing protocols executed by the box, hence lead to different predictions about some of the conditional probabilities \( P(a_j \mid x_i) \), which can be directly registered in a suitable experimental scenario.

The archetypical example of such an experiment is the Bell test, in which two space-like separated parties — Alice and Bob — perform a measurement on a particle from an entangled pair, following a choice of the measurement setting. Hence, in a Bell test the measurable probabilities take the form \( P(a, b \mid x, y) \), with \( a, b \in \{-1, +1\} \) and \( x, y \in \{0, 1\} \) denoting the outputs and inputs, respectively \( ((a, x) \text{ for Alice and } (b, y) \text{ for Bob}) \). It is well-known \([39]\) that a wide class of “local hidden variables” theories implies a bound on the quantity \( S = C(0, 0) + C(0, 1) + C(1, 0) - C(1, 1) \leq 2 \), where \( C \) is the correlation function defined as \( C(x, y) = \sum_{a,b} ab P(a, b \mid x, y) \). Multiple experiments have shown (see \([27]\) and references therein) that this inequality is violated in Nature and pointed to the validity of the quantum mechanical description, which predicts \([40]\) the bound \( S \leq 2\sqrt{2} \).

Here we put forward a conceptually new theory-independent framework founded on quantum information. We regard a physical system under investigation as a quantum-information processing device, which could be probed with controllable quantum systems. The dynamics of quantum information effectuated by such a ‘quantum-data black box’ \( (Q\text{-data box}, \text{ for short}) \)
can be modelled according to different theoretical schemes. We set only two general constraints on the admissible theoretical description of a Q-data box:

(LO) **Locality**: A Q-data box is physically bounded in space.

(NS) **No-signalling**: A Q-data box cannot facilitate superluminal communication of any information.

The demand of locality is a basic methodological assumption, which guarantees that the Q-data box is an operational notion. The boundedness in space means that any Q-data box can, in principle, be probed in isolation (i.e. “shielded”) from the rest of the Universe. Let us emphasise that the locality of a Q-data box constrains only the admissible interaction with it and not the effectuated dynamics of quantum information (see Subsections 2.1 and 2.2).

The (NS) condition is a standard one [17, 18, 19] and assures basic compatibility with relativity.

In order to unveil the properties of a given Q-data box one performs a **Q-data test**, which consists in probing a Q-data box with systems carrying the encoded quantum information (see Fig. 1). The relevant input and output data are now quantum states $\psi_{\text{in}}$, $\rho_{\text{out}}$ defined, respectively, over finite-dimensional Hilbert spaces $\mathcal{H}_{\text{in}}$ and $\mathcal{H}_{\text{out}}$. This constitutes a valid experiment when combined with an orchestrated initial quantum state preparation $P : x \rightarrow \psi_{\text{in}}$ and a final projective measurement of a chosen observable $M : \rho_{\text{out}} \rightarrow a$. Furthermore, the test can involve some classical parameters $p \in \Omega$, e.g. the energy of the ingoing system, the scattering angle or the direction of a tunable global magnetic field.

![Figure 1: The basic scheme of a Q-data test. The green rectangle illustrates a Q-data box, black straight lines depict the classical information and the blue curved ones the quantum information. The vertical arrow over the top signifies that $p$ are classical parameters of the test.](image)

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1Here we adopt a broad definition of “no-signalling”, which encompasses scenarios possibly involving non-local dynamics of correlations – see [19].
The operationality of a Q-data test hinges upon three assumptions:

(Prep) Preparation: The input state $\psi_{\text{in}}$ is pure and can be prepared arbitrarily precisely.

(Tomo) Tomography: The output state $\rho_{\text{out}}$ can be reconstructed to an arbitrary precision.

(Free) Freedom of choice: The initial state $\psi_{\text{in}}$ and the parameters $p$ can be chosen freely, i.e. independently of the state of the Q-data box.

The first two conditions are guaranteed if we recognise the validity of quantum mechanics outside of the Q-data box, whereas the “freedom of choice” is a standard assumption adopted in the black-box approach [41].

It is vital to stress the operational difference between the input and output quantum data. Whereas assumptions (Prep) and (Free) guarantee that the input state $\psi_{\text{in}}$ is fully under control, the output $\rho_{\text{out}}$ is only an effective quantum state reconstructed from the measurement outcomes $\{a_j\}_j$ via the quantum state tomography [42]. The latter is an algorithmic method for estimating an unknown quantum state from multiple repetitive measurements of observables $M_i$ from a tomographically complete set $\{M_i\}_{i=1}^{n^2}$ on $H_{\text{out}}$, where $n = \dim H_{\text{out}}$. Consequently, $\rho_{\text{out}}$ is in general expected to be mixed. Its impurity can be a combined effect of the experimental tomography imperfections and an objective indeterminacy caused, e.g. by the entanglement of the output system with the Q-data box. If $N$ denotes the number of measurements of each observable $M_i$, then assumption (Tomo) guarantees that $\rho_{\text{out}}^{[N]}$ converges in the limit $N \to \infty$ to a unique mixed state over the Hilbert space $H_{\text{out}}$.

In summary, a Q-data test yields a dataset of the form $\{\psi_{\text{in}}^{(k)}; p^{(\ell)}; \rho_{\text{out}}^{(k,\ell)}\}_{k,\ell}$ with the indices $k$, $\ell$ ranging over the input states from $H_{\text{in}}$ and parameters from the set $\Omega$, respectively. For every fixed input state $\psi_{\text{in}}^{(k)}$ and parameters’ values $p^{(\ell)}$ one needs to complete the quantum state reconstruction, which yields an effective state $\rho_{\text{out}}^{(k,\ell)}$. The more tomographic measurements $N$ are performed for each input data $\psi_{\text{in}}^{(k)}; p^{(\ell)}$, the more credible the output quantum data is.

An implementation of a Q-data test may involve a single system, prepared in an initial state $\psi_{\text{in}}$, which interacts with a given Q-data box, e.g. via scattering, and is subject to the final tomographic measurement. In different scenarios, e.g. involving absorption and subsequent emission, the outgoing system will not be the same as the ingoing one. In either case, one should keep in mind that the outgoing system will typically be correlated with the probed Q-data box. In consequence, whereas the concatenation of Q-data boxes can be probed within a single Q-data test, the latter will not, in general, be equivalent to a concatenation of Q-data tests probing individual Q-data boxes (see Fig. 2).
Figure 2: The scheme depicted in figure a) constitutes a valid Q-data test of a concatenation of two Q-boxes, but it is, in general, not equivalent to the concatenation of Q-data tests depicted in figure b). This happens because the system exiting the first Q-data box will typically be correlated with the first Q-data box. In scenario a) these correlations may affect the behaviour of the second Q-data box. In contrast, in scenario b) the second Q-data test can only depend upon the effective output state $\rho_{\text{out}}$ of the first test, which does not keep track of the correlations of the outgoing system with the first box.

Whereas the Q-data boxes are — by assumption (LO) — local, the Q-data tests need not be so. The non-locality of quantum data allows one to probe multiple Q-data boxes ‘in parallel’ using engineered input states on a product Hilbert space $\mathcal{H}_{\text{in}} = \bigotimes_n \mathcal{H}_{\text{in}}^n$ and a joined tomography over $\mathcal{H}_{\text{out}} = \bigotimes_n \mathcal{H}_{\text{out}}^R$ (see Figure 3). The involved Q-boxes can be correlated, e.g. as a result of a preceding interaction, and a non-local Q-data test can detect these correlations (see Subsection 2.3). Non-local Q-data tests can also be performed with entangled input states (see Subsection 2.4).

A particular example of a non-local Q-data test arises when a given Q-data box is probed locally by a quantum system entangled with a reference system (see Fig. 4). In such a case, the total initial state has the form $\psi_{\text{in}} = \sum_i c_i \chi_i^B \otimes \eta_i^R$, where $B$ denotes the system input into the box, $R$ is the one kept outside and $c_k$ are complex numbers determining the correlations between $B$ and $R$. The final state will has an analogous form: $\rho_{\text{out}} = \sum_j d_j \sigma_j^B \otimes \xi_j^R$. In such an extended scenario, the full structure of $\rho_{\text{out}}$, including the mutual phases between local states, is reconstructed via quantum tomography with the help of projective measurements from the tomographically complete basis on $\mathcal{H}_{\text{out}} = \mathcal{H}_{\text{out}}^B \otimes \mathcal{H}_{\text{out}}^R$.

Finally, let us explain why in a Q-data test one is allowed to use only pure states at the input. The reason is essentially the same as in the context of a standard experiment involving
Let us now turn to concrete examples of Q-data tests designed to assess the validity of different post-quantum scenarios.
2.1 Quantum process tomography

From the perspective of Q-data tests, a Q-data box effectuates some quantum-information processing protocol described, for any fixed values of external parameters’ $p \in \Omega$, by a map

$$\mathcal{E} : \mathcal{H}_{in} \rightarrow S(\mathcal{H}_{out}),$$

where $S(\mathcal{H}_{out})$ denotes the space of density operators on $\mathcal{H}_{out}$. Given a dataset collected during a Q-data test $\{\psi_{in}^{(k)}, p; \rho_{out}^{(k)}\}_k$ one can attempt to reconstruct this map.

Quantum mechanics imposes [3] rather tight constraints on the admissible form of $\mathcal{E}$. Concretely, if the Q-data box operates according to the quantum principles, then $\mathcal{E}$ must be a linear completely positive trace preserving (CPTP) map. Such a map is completely determined by $m^2(n^2 - 1)$ real parameters with $m = \dim \mathcal{H}_{in}$, $n = \dim \mathcal{H}_{out}$ and extends uniquely to a map $\tilde{\mathcal{E}} : S(\mathcal{H}_{in}) \rightarrow S(\mathcal{H}_{out})$. These parameters can be directly measured in the quantum process tomography scheme [41]. The latter consists in probing the box with $m^2$ different pure input quantum states $\psi_{in}^{(k)} \in \mathcal{H}_{in}$, which form a basis of the space $S(\mathcal{H}_{in})$, and reconstructing the corresponding output states $\rho_{out}^{(k)}$.

Let us fix such a basis $\{\psi_{in}^{(k)}\}_{k=1}^{n^2}$ and consider its unitary rotation $\{U_\delta(\psi_{in}^{(k)})\}_{k=1}^{n^2}$ with some tunable parameter $\delta$. Let $\mathcal{E}_\delta$ denote the corresponding quantum channel, which is reconstructed from the gathered data $\{U_\delta(\psi_{in}^{(k)}), p; \rho_{out}^{(k)}\}_k$. If the probed Q-data box abides by the laws of quantum mechanics then the outcome of the process tomography does not depend on the choice of the basis for input states, that is $\mathcal{E}_\delta = \mathcal{E}_{\delta'}$ for all $\delta, \delta'$.

A dependence of the reconstructed map $\mathcal{E}_\delta$ on the parameter $\delta$ would provide evidence for the post-quantum nature of information processing within the probed Q-data box. Such a deviation could be quantified with the help of any standard distinguishability measure [44], e.g. the quantum fidelity between the Choi–Jamiołkowski matrices of $\mathcal{E}_\delta$ and $\mathcal{E}_{\delta'}$.

A different scenario of a Q-data test exploits an alternative, ancilla-assisted, quantum process tomography scheme [45]. In the latter, the Q-data box is probed with a quantum system entangled with a reference system, as in Fig. 4. Now there is a single fixed input state $\psi_{in}$ and the map $\mathcal{E}$ is reconstructed from joint measurements on the system exiting the Q-data box and the reference system. If $\mathcal{E}$ is a CPTP map the two schemes of quantum process tomography are equivalent. Consequently, one can probe a Q-data box by experimentally checking this equivalence.
2.2 The Helstrom discrimination test

Consider an Alice producing one of the two quantum states \( \psi_1, \psi_2 \) with the corresponding probabilities \( p_1, p_2 \). Bob’s task is to discriminate between the two inputs in an optimal way. The probability of success is defined as

\[
P_{\text{succ}}(p_1, \psi_1, p_2, \psi_2) := \sum_{i=1}^{2} p_i P(a = i|\psi_i),
\]

where the result “\( a = 1 \)” (“\( a = 2 \)” ) corresponds to the a posteriori conclusion drawn by Bob “Alice prepared the system in a state \( \psi_1 (\psi_2) \)”.

Suppose that Bob is capable of executing any unitary quantum dynamics on the given state and performing any projective measurement on the final state. His probability of success in discriminating between the two quantum states is limited by the Helstrom bound [46]:

\[
P_{\text{succ}}(p_1, \psi_1, p_2, \psi_2) \leq \frac{1}{2} \left( 1 + \text{Tr} |p_1 \psi_1 - p_2 \psi_2| \right).
\]

The latter is strictly smaller than 1, unless the two states \( \psi_1, \psi_2 \) are orthogonal.

Suppose now that Bob probes a Q-data box with the obtained state \( \psi_i \) and registers a state \( \rho'_i \) at the output. If, using \( \rho'_i \), he can exceed the Helstrom bound (2), then the Q-data box must have effectuated some post-quantum process. Observe that even if the outgoing states \( \rho'_i \) have lower purity than the ingoing \( \psi_i \), they might still provide better distinguishability.

The violation of the Helstrom bound (2) is a generic feature of nonlinear modifications of the Schrödinger equation [47]. However, it is well known [48] that nonlinear quantum dynamics leads to superluminal signalling and hence violates one of our basic assumptions (NS) about admissible models of Q-data boxes. This happens when Alice sends to Bob a quantum particle, which is maximally entangled with another one kept at her local laboratory, say \( \Psi_{AB} = \frac{1}{\sqrt{2}} \left( |0\rangle_1 |1\rangle_0 - |1\rangle_1 |0\rangle_0 \right) \). By effectuating a projective measurement in one of the bases \( \{0, 1\} \) or \( \{+, -\} \) on her particle Alice can prepare one the two statistical ensembles \( \{\frac{1}{2} |0\rangle_0; \frac{1}{2} |1\rangle_1\} \) or \( \{\frac{1}{2} |+\rangle_1; \frac{1}{2} |-\rangle_0\} \) entering Bob’s laboratory. These ensembles yield the same quantum density matrix, but nonlinear quantum dynamics generically maps them into another pair of ensembles, which result in two different density matrices [48, 13]. Consequently, Bob could distinguish these two situations hence immediately learning Alice’s decision at a distance.

One way to save the model’s consistency is to modify the static structure of quantum mechanics [13]. In our approach this is not allowed, because we assume the validity of quantum mechanics outside of the Q-data box. Therefore, one would have to assume that the Q-data box...
introduces some stochastic element [49], for instance by effectuating a von Neumann collapse, which destroys the entanglement without modifying Alice’s local density matrix.

An experimental violation of the Helstrom bound with correctly prepared pure input states $\psi_1, \psi_2$ would provide strong evidence for the breakdown of quantum theory in the probed Q-data box. In such a case, further non-local Q-data tests involving a controlled reference system might help unveiling the correct post-quantum model.

### 2.3 Quantum random access code boxes

Let us now present an example of a Q-data test designed to probe the strength of correlations. It exploits a quantum random access code box involving both classical and quantum data [50]. The relevant Q-data box has three quantum inputs ($\Psi_0$ and $\Psi_1$ for Alice and $\omega$ for Bob) and two quantum outputs ($\sigma$ for Alice and $\rho$ for Bob), along with a two-bit external parameter $b$ on Bob’s side (see Fig. 5). The final measurement on Alice’s side yields four classical outcomes enumerated by a pair of bits $\{a_1, a_2\}$. For sake of concreteness, let us assume that the involved quantum states are qubits.

![Figure 5: An example of a quantum random access code box involving two correlated Q-data boxes with post-selection of events $a = b$ (see text for the description).](image)

In the task of quantum random access code Bob wants to learn one of the Alice’s qubits with his choice being parametrised by a random bit $x$. Alice sends to Bob her 2-bit classical output $a$. Using these as his input $b$, Bob steers his initial state $\omega_x$ into $\rho_{x,b}$. The task is successful if Bob’s quantum output $\rho_{x,b=a}$ is equal to $\Psi_x$ for both $x = 0$ and $x = 1$. In an equivalent protocol depicted in Figure 5 one generates 4 random bits $a, b$ and post-selects for events with $a = b$. 

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In [50] it was shown, that such a task can never be achieved if the Q-data box works according to the rules of quantum mechanics. On the other hand, if it involves some post-quantum correlations — the Popescu–Rohlich no-signaling boxes [15] — Bob can always recover perfectly the qubit of his choice.

The success rate of this quantum random access code can be quantified by the transmission fidelity (see [51] and references therein)

\[
f = \frac{1}{2} \sum_{x=0,1} \int d\Psi_x \langle \Psi_x | \rho_{x,b=0} | \Psi_x \rangle,
\]

where the integral is taken over a Haar measure.

Quantum mechanics imposes [52] an explicit bound \( f \leq 5/6 \). If in an experiment we can achieve a transmission fidelity \( f > 5/6 \), then we know that the Q-data box must involve some post-quantum element.

Let us note that the presented scheme for testing the strength of correlations is conceptually different than the classical Bell–CHSH test [53, 39], which also admits post-quantum scenarios [15, 17, 18]. Our test assumes the validity of quantum mechanics at the input and at the output, but admits post-quantum imprints on the input-output correlations between \( \psi_{\text{in}} \) and \( \rho_{\text{out}} \). Similar tests could involve, for instance, post-quantum steering scenarios, in which one or more parties collaborate to generate a desired quantum state at receiver’s output [54, 55].

Typically, in experiments designed to test the strength of correlations, such as the Bell test, one assumes that the two parties, Alice and Bob, are spacelike separated [18, 53]. Indeed, if Alice and Bob are close enough to interact and exchange information during the experiment, then any correlations between them can be simulated within the conventional classical + quantum paradigm. In the context of physics at subnuclear scales the spacelike separation of the inputs is not achievable. Nevertheless, any deviation from quantum bounds on correlations, such as the one presented above, would signify a highly peculiar dynamics of information, not realised naturally in any known composite quantum system.

### 2.4 Quantum no-signalling boxes

In the previous examples the input quantum states were uncorrelated. One can also conceive more general scenarios, in which both the input and output are entangled states. In this context one can ask what are the characteristic features of the dynamics of correlations imposed by the quantum theory. One specific example, the so-called no-signalling quantum boxes (NSQ), has been analysed in [56]. In this scenario one considers a special class of quantum
maps on bipartite quantum inputs. A map $\Lambda_{AB} : \rho_{AB} \mapsto \sigma_{AB}$ satisfies the condition of no-communication from Alice to Bob if for a given state $\rho_{AB}$ it maps the family of the inputs $[\Gamma_A \otimes \text{id}_B](\rho_{AB})$, with any local map $\Gamma_A$, into outputs $\sigma_{AB}(\Gamma_A, \rho_{AB})$ having the same Bob’s reduced state $\sigma_B = \text{Tr}_A\rho_{AB}$. Analogous condition of no-communication from Bob to Alice is assumed.

In [56] it was proven that the set of such NSQ maps is of volume zero in the set of all bipartite dynamics. Since the trivial, i.e. non-interacting, dynamics is also of volume zero, a randomly chosen interacting quantum dynamics violates the no-communication condition. The relevant experiment would thus consist of the quantum process tomography with different randomly chosen initial data $\rho_{AB}$ and $\Gamma_A$. In the context of subnuclear physics one expects non-trivial interactions, hence if the experiment reveals a significant percent of outcomes compatible with the no-communication condition, then it should be considered as an indication of a deviation from the standard quantum dynamics. More generally, the typicality of communication in quantum dynamics can be quantitatively calculated by means of uniform measures on CPTP maps [57].

Figure 6: A Q-data test incarnating a quantum no-communication scenario (see text for the description). The dashed line signifies that Alice’s and Bob’s Q-data boxes are correlated. The dotted curved lines point to the fact that both states $\rho_{AB}$ and $\sigma_{AB}$ can be entangled.

\[\text{Actually, if one would be able to scan over all bipartite inputs } \rho_{AB}, \text{ then } \Gamma_A \text{ would be redundant.}\]
3 Towards an experiment

The theoretical scheme of Q-data boxes presented above does not depend on the physical context. It can equally well serve to test post-quantum scenarios involving, e.g. the gravitational field \([58, 59]\).

We now sketch a general experimental setup for the implementation of a Q-data test in the context of strong nuclear interactions. To this end we regard free nucleons (or, more generally, hadrons) and nuclei as Q-data boxes, which can be probed with quantum information. The phenomenon of confinement naturally guarantees the assumption of locality.

We note that several successful implementations of quantum information processing protocols were carried out using collective quantum degrees of freedom of nuclear ensembles \([60, 61]\). These include i.a. a recent quantum interface between a single electron and a nuclear ensemble \([62]\). However, to probe the nature of strong nuclear interactions through Q-data tests one needs to address the internal quantum degrees of freedom of nuclei. A successful implementation of a Q-data test involving the internal structure of nucleons would provide a direct test of the (post-)quantum nature of the strong nuclear interaction mediated by gluons. One can also envisage an analogous test for probing the residual strong interaction binding nucleons within a nucleus.

We propose that such implementations could be achieved through scattering of quantum-programmed projectiles on nucleonic or nuclear targets. Such a scenario guarantees the existence of natural classical parameters related to the kinematics of the collision.

In the first stage the state \(\psi_{in}\) is prepared and imprinted on a projectile using standard quantum engineering techniques based on electromagnetic interactions \([63, 64, 65, 66]\). The carrier of the initial quantum information should preferably be a free electron or a photon, because of their stability, though one could in principle employ any fundamental particle, e.g. a positron or a muon. One can also conceive the preparation of the quantum state of an electron antineutrino through an orchestrated \(\beta\)-decay – see Fig. 7. A natural choice for the quantum degree of freedom encoding a qubit state \(\psi_{in}\) is the electron’s spin or photon’s polarization. Alternative procedures might include the programming of particle’s momentum, angular momentum or position quantum variables (wave packet shaping) \([67, 68]\).

During the second stage the prepared quantum state \(\psi_{in}\) is input into the Q-data box via a precise scattering process. The energy of the projectile should be large enough (\(\gtrsim 1 \text{ GeV}\)), so that it probes the internal structure of the target and not some collective degree of freedom.

The quest for the final stage of the experiment is to perform a projective measurement of a
chosen quantum degree of freedom of the scattering products. Suitable experimental schemes for projective measurements of the polarization states of individual γ-photons have recently been developed [69, 70]. Furthermore, the measurement of spins of massive projectiles could be based on the quantum Stern–Gerlach scheme [71, 72, 73, 74].

![Diagram](image)

**Figure 7:** The conceptual scheme of preparation of a single neutrino quantum state. A free neutron \( n \) prepared in a quantum state \( \Psi_n \) decays into a proton \( p \), an electron \( e \) and electron antineutrino \( \bar{\nu}_e \). The quantum information encoded in \( \Psi_n \) is transferred into an entangled state \( \Psi_{p,e,\nu} \) of the decay products. Through projective measurements on the proton and the electron one can steer the final neutrino state \( \psi_\nu \), which depends on measurement settings \( M_1, M_2 \) and outcomes \( a_1, a_2 \).

The type of the outgoing particle, together with the kinematic characteristics of the collision, and possibly other global features related e.g. to the polarization of the target, constitute the set \( \Omega \) of classical parameters of the Q-data test. For every fixed value of these parameters, \( p \in \Omega \), one needs to carry out a complete quantum tomography of the output state \( \rho_{out}^p \) on the outgoing projectile. If one expects some of the projectiles to be resulting from the same interaction vertex, then it is desirable to perform the quantum tomography of the joint state of these projectiles.

An idealistic Q-data test, akin to Bell-type experiments, should be performed with individual particles. This is, however, a formidable task.

The first stumbling block is the preparation of a target consisting of a single nucleon or a single nucleus. Recently, a single proton has been isolated in a Penning trap for sake of measuring its magnetic moment [75]. But a trap suitable for an implementation of a Q-data box should prepare a single nucleon in a sharp static position, to maximise the cross-section for the desired scattering process. This condition favours optical tweezers based on laser pulses [76], which, however, have not so far been engineered to trap single nucleons or nuclei.
The second major difficulty for such an idealistic Q-data test stems from the fact that a typical high energy collision results in an abundance of the outgoing projectiles. Hence, one should expect that the sought for quantum information will be concealed in the entire collection of the decay products. In the Bell-test language this could be seen as a “detection loophole” (aka “fair-sampling loophole”) [77, 78, 79]. In order to fathom out the input-output correlations probed in the Q-data test one should aim at clean experiments involving a manageable number of outgoing projectiles. Furthermore, the latter should be measured exclusively – i.e. the detection should involve individual projective measurements on all projectiles.

Furthermore, an implementation of a Q-data test aimed at probing the strength of correlations (see Sections 2.3 and 2.4) would require a coordinated double scattering from the same nucleon/nucleus – see Fig. 8. In the same vein, one can envisage experiments probing the nature of time-like correlations, i.e. the dynamics of quantum information at subnuclear scales. Such a scenario requires at least two subsequent scatterings from the same nucleon/nucleus (see Fig. 9), in contrast to the single-scattering scheme, which probes instantaneous interactions. An alternative scheme based on absorption and subsequent emission might be useful to probe (post-)quantum dynamical effects, the characteristic scales of which are very short – see Fig. 10.

![Figure 8: The conceptual scheme for an implementation of a Q-data test aimed at probing the strength of correlations – cf. Figs. 5 and 6](image-url)
Figure 9: The conceptual scheme for an implementation of a Q-data test probing the dynamics of quantum information. At an initial time $t_1$ the input state $\psi_{in}$ is prepared and imprinted into the Q-data box. The state $\rho_1$ encoded in the scattered projectile is subject to a projective measurement shortly afterwards. At a later time $t_2$ the state $\rho_{out}$ is read out from the Q-data box via a second scattering of the projectile carrying an ‘empty sheet’ state $\psi'_0$. Finally, the quantum state $\rho_{out}$ is measured. Note that the first scattering process must be elastic, so that the same Q-data box is probed at the second time-moment.

Figure 10: An alternative scheme for probing the dynamics of quantum information. The projectile carrying the prepared state $\psi_{in}$ is absorbed into the Q-data box. Then, the box de-excites and decays into a few products. The final state $\rho_{out}$ is reconstructed from projective measurements on the outgoing projectiles.

Nevertheless, some basic Q-data tests discussed in Sections 2.1 and 2.2 could be carried out with the state-of-art accelerator technology. Whereas quantum state engineering of individual high-energy particles poses a serious challenge, the polarized beams of electrons are routinely employed in accelerator experiments [80, 81]. Some advances towards a generation of polarized positron beams was also made [82]. In parallel, a novel technique to generate highly-polarized multi-GeV photon beams has recently been proposed [83, 84]. Such a highly energetic polarized beam should then be scattered against a nucleonic or nuclear target. We note that protonic targets can also be polarized. The direction of the target’s polarization can serve as an additional classical parameter of the test.
The key challenge for such a beam-based Q-data test, as contrasted with typical accelerator experiments, is the accomplishment of precise quantum state tomography on the outgoing projectiles. This requires multiple experiments, with the same polarization of the ingoing beam, but different projective measurements of the spin/polarization (or other quantum degree of freedom) of the decay products. As explained earlier, one should post-select the gathered data to identify the scattering processes with different values of classical parameters, \( p \in \Omega \), (particle species, scattering angle, energy etc.) and perform the reconstruction of \( \rho^p_{\text{out}} \) for every value of \( p \) – see Fig. 11.

![Figure 11: The scheme for an implementation of a Q-data test with polarized beams scattered on a nucleonic/nuclear target. The polarization of the ingoing beam fixes the initial quantum state \( \psi_{\text{in}} \). At the output one performs projective measurements of a chosen degree of freedom, e.g. spin/polarization, of all of the outgoing projectiles. The states \( \rho^p_{\text{out}} \) are reconstructed from the output data after post-selection with respect to the registered classical parameters pertaining to the kinematics of the collision and the species of the outgoing particles.](image)

Within the adopted model-independent framework there is no hint about which processes could actually exhibit some post-quantum behaviour. We only postulate that the ingoing projectiles should have energy in the GeV range in order to address the internal degrees of freedom of the target. Also it is clear that the fewer types of the outgoing projectiles, the better the chance to observe non-trivial quantum information processing at subnuclear scales. Otherwise, one should scrupulously explore the entire available space of classical parameters \( p \in \Omega \). This blind search could be enhanced by the recently developed machine learning techniques for quantum experiments [85, 86].
4 Discussion

We have put forward a research programme designed to explore the information-theoretic properties of subnuclear phenomena. The adopted theoretical scheme allows one to probe the limits of quantum mechanics from an outside — “post-quantum” — perspective. Its implementation requires a new type of experiments, which combine the energy scales available in accelerator experiments with the precise quantum engineering of single particles.

Any evidence of a deviation from quantum-mechanical behaviour would have a profound impact on our understanding of fundamental physics, calling for a revision of the basic principles underlying the Standard Model of particles. Also, it is known that certain post-quantum theories can have dramatic consequences for information processing, e.g. yielding solutions to NP-complete problems in polynomial time [87] or trivialising communication complexity [88]. Consequently, the envisaged experimental programme provides an unprecedented opportunity for direct empirical tests of various physical principles [88, 89, 90, 91], postulated for sake of taming the undesired information-processing properties of Nature.

Exploration of quantum information protocols at subnuclear scales is also of significant interest from the conventional quantum perspective. Firstly, it provides an unparalleled test of the adequacy of the quantum-field-theoretic description of fundamental interactions. Secondly, it would offer new possibilities for implementation of quantum information processing protocols at unprecedented scales and establish a solid ground for the quantum simulation [92] of nucleon’s interior on quantum computers [93, 94].

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