Optimization of Two-Dimensional Finite Element on Primary Bone Type-7 Fracture Model

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Abstract. Optimization works is an important method for Finite Element (FE) analysis to get better accuracy in simulation study before proceed for further analysis. In this present work, convergence study of local crack tip meshing involving radius of first row element, \(a/n\) (DELR) and number of crack tip element (NTHET) is employed on single edge crack in homogenous properties of human cortical bone. Mode I and Mode II Type -7 penetration were determined by using FE analysis and compared with the experimental results. Based on the results, a good agreement is found between numerical and experimental results.

1. Introduction
A Finite Element (FE) analysis requires a proper modelling in terms of specimen geometry, material properties and meshing strategies. The concept of Linear Elastic Fracture Mechanics (LEFM) concerns small scale yielding stresses at the local crack tip which is characterized by single parameter called SIFs. This parameter is important to determine the material strength of defect structure in terms of engineering application of fracture mechanics problems. The refinement meshing involving Stress Intensity Factor (SIF) has been done by Miranda et al., using difference crack geometries and mesh refinements to quantify the meshing performance particularly in very small elements around crack tip [1]. In his simulation strategies, the excessive refinement meshing may degrade the accuracy in crack problems. In another attempt, Gope et al., [2] used FE numerical method to compute SIFs of collinear cracks and compared with photo elastic experimental with good agreement between numerical and experimental. Gope et al., employed mesh refinement studied to compute stress field in Mode I loading condition. On the other hand, the evaluation of SIFs by meshing strategies has been done by Guinea et al., [3] involving variable of element sizes, shapes and mesh arrangement to compute SIFs. The results obtained by Guinea et al, show KI is a good agreement with energy estimations.

Conversely, for cortical bone refinement meshing, the LEFM theory is used which approached the fiber matrix composite materials which actual mechanical components in engineering field [4]. To ensure the model used is true, the refinement meshing need to be done either local or global meshing element with applied of boundary conditions. Commonly the fracture analysis has been solved using shear/ stress analysis without microcracks, and conversely with disturbance of stress/ shear using the edge dislocation solution as Green’s function theorem. Furthermore, Gauss-Chebyshev method has been used for numerical simulations of singular integral equations to compute the SIFs [2-3]. On the
other hand previous studied performed fracture analysis of two dimension cortical bone using Franc 2D with triangular elements meshing with eight nodes in meshing structures [7]. To present the singular strain at the crack tip, a modified isoparametric elements is introduced as suggested by Barsoum [8]. This paper analyses the influence of radius of first row element (DELR) and number of elements around crack tip (NTHET) on the numerical values of KI obtained by J-integral method, using the cortical bone two-dimension structure of FE analysis. The precision results obtained by normalization technique between FE analysis and empirical formulations which show the closest to unity is the most accurate stress field estimation.

2. Bone Modelling Meshing (Type-7)

The cortical bone is modelled in two-dimension schematic model of 1 mm$^2$ (height x width) using FE software ANSYS with thickness was kept 1 mm. The cortical bone Type- 7 crack penetration model was studied in plane strain, isoparametric with quadrilateral elements (PLANE 83). The validation of meshing is been normalized with the analytical equations from Brown and Srawley, Gross and Brown and Tada.

The details of cortical bone dimension and mechanical properties for Type- 7 crack penetration are shown in Table 1. The most standard often used for mechanical testing in cortical bone is a tensile test and the bending test. The tensile test can be in a beam shape either in bar or rod with constant shape and area and can be in spherical, square and rectangular. Here, the 2D FE model is developed as a rectangular shape with height and width at 1 mm$^2$.

| Table 1. Mechanical properties and dimension used in convergence analysis in CINT method (Dimension: W x H =1 mm x 1 mm). |
|---------------------------------------------------------------|
| **Type-7** ($a = 0.125$mm) | **Mechanical properties** |
| Primary bone | Homogenous | Anterior | Posterior | Medial | Lateral |
| Crack direction | Transverse | 13.2 | 9.92 | 14.67 | 11.18 |
| | Longitudinal | 23.15 | 19.29 | 21.13 | 15.14 |

The refinement meshing was concentrated at local concentration key point using J-integral method to calculate SIFs. The mesh density is modified using number of elements of first row as, $n = 8, 10, 12, 16$ and $20$ and keeping the radius of first row element as (DELR), $a/n$ where $a$ is the crack length at initial crack, $a = 0.125$mm. FE analysis undergoes the meshing strategies until converge to closest unity (least error). Figure 1 show the optimization Type- 7 crack penetration in this study.
3. Results and Discussion

The results tabulated in Table 2 shows that there are small differences between CINT method less than 0.2 % for \( a/8 < a/n < a/20 \). In this research, the CINT algorithm is mainly preferred to use in order to compute SIFs and SERR fracture parameter due to smallest error when taking the average for these three empirical formulations in different DELR (\( a/n \)) mm. In reference to Equation 1, the error analysis is based on the work of [9] where the percentage error is expressed as:

\[
\text{Mode I % error of } K_{\text{ITP}}(A,P,M,L): = \frac{K_{\text{ITP}}(A,P,M,L) - K_{\text{ref}}(BS,GB,T)}{K_{\text{ref}}(BS,GB,T)} \times 100\% \tag{1}
\]

\[
\text{Mode I % error of } K_{\text{ILP}}(A,P,M,L): = \frac{K_{\text{ILP}}(A,P,M,L) - K_{\text{ref}}(BS,GB,T)}{K_{\text{ref}}(BS,GB,T)} \times 100\% \tag{2}
\]

Based on standard SIFs formulation [10] stress field \( \sigma_{ij} \) yielding near the crack tip [6-8] can be expressed as simple relationship between factor of SIFs, \( (K_{1j},K_{1j}) \) and geometrical shape correction factor \( (Y_{j},Y_{1j}) \) for transverse and longitudinal primary cortical bone, respectively.

\[
K_{\text{ITP}} = Y_{\text{ITP}}(A,P,M,L)\sigma_{ij} \sqrt{\pi a(a/W)} \tag{3}
\]

\[
K_{\text{ILP}} = Y_{\text{ILP}}(A,P,M,L)\sigma_{ij} \sqrt{\pi a(a/W)} \tag{4}
\]

Table 2 Comparisons SIFs of J-integral between Brown and Srawley, Gross and Brown and Tada. Tabulated in Table 3 shows the normalization between \( K_{1j} \) FE analysis and empirical formulations for DELR and Table 4 the normalization of NTHET. The details results are shown in Figure 2-6.
Table 2. Normalized in DELR between three empirical formula.

| DELR (mm) = a/n | a/8   | a/10  | a/12  | a/16  | a/20  |
|-----------------|-------|-------|-------|-------|-------|
| K\textsubscript{o}/BS | 0.999722 | 1.000287 | 1.000287 | 1.000261 | 1.000278 |
| K\textsubscript{o}/GB | 0.983737 | 1.000026 | 1.000026 | 1.000000 | 0.999991 |
| K\textsubscript{o}/T | 1.000017 | 1.000026 | 1.000026 | 1.000000 | 0.999991 |

Table 3. Normalized in NTHET between three empirical formula.

| NTHET | 0    | 8    | 12   | 16   | 20   |
|-------|------|------|------|------|------|
| K\textsubscript{i}/BS | 1.048 | 1.048 | 1.048 | 1.049 | 1.049 |
| K\textsubscript{i}/GB | 1.032 | 1.032 | 1.032 | 1.032 | 1.032 |
| K\textsubscript{i}/T | 1.043 | 1.043 | 1.044 | 1.044 | 1.044 |

Based on J-integral method, theoretically, K\textsubscript{i} is computed from the relation given for single edge crack. Based on Figure 2, the radius of first row element (DELR) show that the optimization at n = 10, where n is number of wedges. Thus, the DELR for a/10 is chosen and keep constant by varying number of wedges, n = 8, 10, 12, 16 and 20 (NTHET).

![Radius of first row element, DELR (a/n) mm](image)

Figure 2. Effect SIFs of first crack tip radius of first row element (DELR).

The results obtain from Figure 3 are the combination SIFs for K\textsubscript{i} normalization between K\textsubscript{i} theories K\textsubscript{i}/K\textsubscript{o} = F\textsubscript{i}(F\textsubscript{E}) in FE analysis and theoretical Brown and Srawley F\textsubscript{i}(F\textsubscript{E}/BS\textsubscript{T}), Gross and Brown F\textsubscript{i}(F\textsubscript{E}/GB\textsubscript{T}) and F\textsubscript{i}(F\textsubscript{E}/T\textsubscript{100}) against number of crack tip (a/n).
Whereas, in specific decimal points configuration as shown in Figure 4-6, show the number of elements around the crack tip NTHET as 16 ($n = 16$), yields least error which closest to unity amongst the other mesh configuration. Thus, afterwards analysis, will be conduct using number of first row element, DELR ($a/n$) mm = $a/10$ mm and number of elements, NTHET at $n = 16$.

**Figure 3.** Effect SIFs of number of crack tip element for three empirical formulations.

**Figure 4.** Effect SIFs of number of crack tip element for Brown and Srawley.
Figure 5. Effect SIFs of number of crack tip element for Gross and Brown.

Figure 6. Effect SIFs of number of crack tip element for Tada.

4. Conclusion

Overall, this study presents the formulation in singularity meshing is set to be $\alpha/n = 0.0125$ mm for DELR $a/n = 10$ and keep constant for NTHET with various number of elements around the crack tip, $n = 8, 10, 12, 16$ and 20 which yield as 16 ($n = 16$) due to least error which closest to unity in the entire normalised Mode I. Those formulations are continued for details and comprehensive FE simulation for primary and secondary Type-7 cracking direction of cortical bone.
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