Stochastic stability assessment of a semi-free piston engine generator concept

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Abstract. Small engines, as power generators with low-noise and vibration characteristics, are needed in two niche application areas: as electric vehicle range extenders and as domestic micro Combined Heat and Power systems. A recent semi-free piston design known as the AMOCATIC generator fully meets this requirement. The engine potentially allows for high energy conversion efficiencies at resonance derived from having a mass and spring assembly. As with free-piston engines in general, stability and control of piston motion has been cited as the prime challenge limiting the technology’s widespread application. Using physical principles, we derive in this paper two important results: an energy balance criterion and a related general stability criterion for a semi-free piston engine. Control is achieved by systematically designing a Proportional Integral (PI) controller using a control-oriented engine model for which a specific stability condition is stated. All results are presented in closed form throughout the paper. Simulation results under stochastic pressure conditions show that the proposed energy balance, stability criterion, and PI controller, operate as predicted to yield stable engine operation at fixed compression ratio.

1. Introduction

The CO₂ reduction targets required of motor vehicles can be reached in only two ways: by switching entirely away from fossil fuels to carbon-neutral energy in all-electric vehicles, or by continuously improving hybrid vehicle technologies [1]. Both options pose economic and technical challenges. The most pressing need for all-electric vehicles is for range improvement through increased battery capacity and charge-rate, and a reduction in battery weight and cost. The challenge for hybrid vehicles on the other hand lies with increased internal combustion engine efficiency alongside an even greater reduction in cost, weight, and complexity [2].

Free piston engines as range extenders offer a promising option that is currently being examined by many automotive companies and research institutions [3, 4, 5]. Characterized by the absence of a crank mechanism, the piston ‘freely’ moves unrestricted. An integrated generator converts the piston motion into electrical power. These devices offer several potential advantages over their conventional crankshaft counterparts: mechanical simplicity and compact design, owing to an integrated generator; reduced friction losses, and therefore improved efficiency, owing to fewer moving parts; reduced in-cylinder heat transfer loss as a result of a faster expansion stroke allowed by the absence of a crank mechanism [6]; and variable compression ratio allowing for optimized loading. The literature is abundant with several design variations [7] but common to most, is a free piston with a rebound mechanism in form of a gas chamber, in place of a connecting rod and crankshaft.
Recently, an innovative dual-mode *semi-free* piston range extender design combining both kinetic energy recovery and storage as well as electric power generation has been proposed [8]. Because its rebound device is a resilient spring, as opposed to the typical gas chamber, the piston is semi-free. This spring-mass system potentially allows for operation at resonance thereby yielding the highest possible energy conversion efficiencies together with its deliberate mechanical design for low noise and vibration. Despite the vast design architectures, free piston engines are yet to receive wide application. This has largely been attributed to the difficulty in controlling and stabilizing the unrestricted piston motion – the same property that has made these engines very attractive. Linear controllers e.g. the PI controller have been shown to be effective [9, 10, 11, 12]. However, the results are not in closed form, and reported experimental success seems to require a great deal of calibration. Some interesting and more analytical nonlinear control strategies such as in [13, 14] still don’t go quite far enough to establish strict stability margins. Stability analysis has very recently been attempted in [15, 16] using solution approximation methods but unfortunately the engine dynamics proved too nonlinear to warrant definite stability ranges. In this paper we address stability of a semi-free piston engine and test our results under stochastic conditions. We derive and formally state a fundamental energy balance criterion followed by a general stability criterion for the device. These results hold for the relationship between in-cylinder pressure, generator output, and piston speed at constant amplitude piston oscillations. For practical realization, we systematically design a PI controller using a control-oriented engine model and state the associated stability condition in terms of controller parameters. We further show that either top-dead-centre or bottom-dead-centre needs to be regulated – not both, as with similar gas chamber designs. This makes the control problem less complicated and therefore easier to implement in practice. Simulation results under stochastic combustion pressure conditions show the efficacy of the proposed controller and verify the proposed energy balance and stability criteria.

### 2. Fundamentals

#### 2.1. Mode of Operation

An idealized schematic of a two-stroke semi-free piston engine generator is shown in Fig. 1. A piston-translator assembly is attached to a linear spring; the spring drives the piston through a cylinder of centre $x = 0$ from a nominal bottom dead centre position $x_{BDC}$ to a nominal top dead centre position $x_{TDC}$. Combustion occurs in the clearance volume $V_c$, driving the piston back to $x_{BDC}$. Piston kinetic energy is converted into electric energy by a linear generator whose ‘rotor’ is integrated in the translator.

![Figure 1](image-url). Ideal design layout of a semi-free piston engine generator concept. Left of centre $x = 0$ is positive whereas right to the centre is negative.
We base the engine operation on the ideal Otto or Spark Ignition cycle (Fig. 2) whose annotations are referenced throughout the paper. Note that the volumes $V_1 = V_2$ and $V_0 = V_3$ will generally vary from cycle to cycle.

![Figure 2. Ideal Otto Cycle PV diagram.](image)

2.2. Mathematical Model

Consider the engine system in Fig. 1. A piston-translator assembly of mass $m$ with piston head cross-sectional area $A$, attached to a spring of stiffness $k$, undergoes a displacement $x$. It is common to model the generator force as proportional to piston speed i.e. with constant parameter $b$ such that as a result of in-cylinder pressure $P$, the governing system equation (derived from Newton’s second law) is:

$$m\ddot{x} + bx + kx = -AP$$

The in-cylinder pressure $P$ at instantaneous cylinder volume $V$ is modelled as an isentropic process with compression and expansion constants $K_{com} = PV_0^{\gamma_{com}}$ and $K_{exp} = PV_0^{\gamma_{exp}}$ respectively, where $\gamma_{com}$ and $\gamma_{exp}$ are the corresponding ratios of gas specific heat capacities. As such we have the pressure as:

$$P = \begin{cases} 
K_{com}V^{-\gamma_{com}}, & \dot{x} \leq 0 \text{ (compression)} \\
K_{exp}V^{-\gamma_{exp}}, & \dot{x} > 0 \text{ (expansion)} 
\end{cases}$$

Referring to Fig. 2, the constants $K_{com}$ and $K_{exp}$ for a given cycle are given by:

$$K_{com} = P_0^{\gamma_{com}}$$
$$K_{exp} = P_2^{\gamma_{exp}} = (P_1 + \Delta P)V_2^{\gamma_{exp}} = (P_1 + \frac{\kappa}{V_1}m_{\text{fuel}})V_2^{\gamma_{exp}}$$

where $m_{\text{fuel}}$ is an input fuel mass for a given cycle, and $\kappa$ is a constant relating $m_{\text{fuel}}$ to a combustion pressure rise $\Delta P$. Finally, the instantaneous cylinder volume is given by:
\[ V = V_c + A(\bar{x}_{TDC} - x) \]  

(4) 

3. Energy Balance and Stability

Sustained engine operation with constant amplitude oscillations or steady compression ratio can be achieved if the engine maintains a closed thermodynamic cycle as the one shown in Fig. 2. For such a cycle where the system returns to its original thermodynamic state, a general energy balance directly derived from the first law of thermodynamics applies [17]:

\[ \Delta E = E_{out} - E_{in} = 0 \]  

(5) 

Equation (5) means that there is no change in the system energy over the cycle. In other words, all energy added to the system at the start of a cycle must match all the energy converted by the system at the end of the cycle. It turns out this idea can be formally stated in terms of work done by a so-called generalized damping force in the engine system equation.

Assuming the input \( m_{fuel} \) is constant or feedback dependent in the form \( m_{fuel}(x, \dot{x}) \) for all cycles, system (1) is autonomous, and can generally be represented as:

\[ m\ddot{x} + b\dot{x} + kx = -AP(x, \dot{x}) \]  

(6) 

which can be recast as:

\[ m\ddot{x} + F_d + kx = 0 \]  

(7) 

where a generalized damping force \( F_d \) has been realized as:

\[ F_d = \left( A \frac{P(x, \dot{x})}{\dot{x}} + b \right) \dot{x} \]  

(8) 

As there is no net change in system energy over a closed cycle according to (5), the work done by this force over closed cycle is zero i.e.:

\[ W_{F_d | \text{cycle}} = \int_{\text{cycle}} F_d \cdot dx = \int_{\text{cycle}} F_d \cdot \frac{dx}{dt} dt = \int_{\text{cycle}} F_d \cdot \dot{x} dt = 0 \]  

(9) 

Using equations (9) and (8), we are now in position to state an energy balance criterion for engine operation in closed thermodynamic cycles i.e. constant amplitude oscillations or fixed compression ratio.

**Energy Balance Criterion:** For constant amplitude oscillations of period \( T \), the relationship between in-cylinder pressure \( P(x, \dot{x}) \), the generator parameter \( b \), and piston speed \( \dot{x} \), is given by:

\[ \int_{T} (AP(x, \dot{x}) + b\dot{x})\dot{x} dt = 0 \]  

(10) 

Validity of this criterion is verified by simulation in Fig. 7. Although not quite extensively studied, the idea of an energy balance has been recently mentioned in hydraulic free piston engine analysis as
critical for stability [16, 12, 15]. In this work we relate the energy balance concept to closed thermodynamic cycle operation, deriving a criterion that states the fundamental relationship between in-cylinder pressure, piston speed and generator output.

It is possible to formulate a general stability criterion for the engine based on the energy balance criterion (10). But first, we ensure during operation, the following necessary design requirements:

(i) Combustion occurs at every end-of-compression position \( x_{TDC} \). This is especially feasible for a computer-controlled spark ignited engine.

(ii) A combustion event always causes a non-zero piston displacement. This is possible if the loading is maintained below a maximum threshold.

(iii) Scavenging occurs for every end-of-expansion position \( x_{BDC} \). This can be achieved if the exhaust port is in the piston head and not into the cylinder wall, allowing for flexible exhaust gas venting at all \( x_{BDC} \) positions.

The design requirements (i)-(iii) are all practically feasible and effectively imply no-stall engine operation. Therefore, engine instability may only be manifested by way of boundless growth of piston oscillations. By defining constant amplitude oscillations as a requirement for stable engine operation, we can formulate a general stability criterion from the energy balance (10) as follows:

**General Stability Criterion:** Assuming the design requirements (i)-(iii) above are in place, the engine system (1) is stable by way of constant amplitude oscillations of period \( T \) if, as \( t \to \infty \),

\[
\int_{\tau}^{\infty} \left( AP(x, \dot{x}) + b \dot{x} \right) \dot{x} dt \to 0
\]

where \( \tau \) is a possibly varying period from cycle-to-cycle in the transient phase before steady state at which \( \tau = T \).

Criterion (11) means that when not zero, the work done by the generalized damping force must tend to zero with progressive oscillations in order to achieve constant-amplitude oscillations. This tendency to zero, or for practical purposes, nearness to zero, is verified with simulation in Fig. 7. It is of course a general criterion – while true, it is quite abstract. A more specific stability condition of system (1) in terms of system and controller parameters will be derived and stated in the following section.

4. Control System Design
We now embark on a systematic approach to develop a linear controller for stable engine operation. The complexity of (1) is prohibitive for controller design. We instead devise an alternative descriptive control-oriented model suitable for controller development. An accompanying stability condition is stated. Verification simulation results follow, showing good control performance.

4.1. Control Objective
Free piston engines can operate at variable compression ratios depending on the piston end-of-compression position \( x_{TDC} \) and end-of-expansion position \( x_{BDC} \) from cycle-to-cycle. The maximum compression ratio is realized when \( x_{TDC} = \overline{x}_{TDC} \) and \( x_{BDC} = \overline{x}_{BDC} \). In this section we show that the main control objective is to regulate either \( x_{TDC} \) or \( x_{BDC} \) (but not both as with configurations involving a gas chamber [18, 6, 19]).
The dynamic compression ratio is given by: 
\[
\frac{V_i}{V_f}
\]
(Fig. 2). In general, it is desirable to operate at a high fixed compression ratio for better thermal efficiency [20, 21] and regular power generation. The maximum permissible compression ratio \( r_c \) occurs when \( V_i = V_f \) such that:
\[
\frac{r_c}{V_f} = \frac{V_i}{V_f}
\]  
(12)

At this compression ratio, \( x_{TDC} = \overline{x}_{TDC} \) and \( x_{BDC} = \overline{x}_{BDC} \). If we set \( \overline{x}_{TDC} \), we can use equation (4) to find the corresponding \( \overline{x}_{BDC} \) at this compression ratio as:
\[
\overline{x}_{BDC} = \frac{V}{A} (1 - r_c) + \overline{x}_{TDC}
\]  
(13)

The following energy balance governs the compression phase:
\[
P_{E_{0,spring}} = 0 W_{1,\text{gas}} + P_{E_{1,\text{spring}}} + 0 E_{1,\text{loss}}
\]  
(14)

where \( P_{E_{0,spring}} \) and \( P_{E_{1,\text{spring}}} \) are the potential energy of the spring before and after compression respectively (subscripts 0 and 1 are adopted in reference to Fig. 2); \( W_{1,\text{gas}} \) is the work done to compress the in-cylinder gas from 0 to 1 (Fig. 2), and \( 0 E_{1,\text{loss}} \) represents all energy losses including generator power output and frictional loss from 0 to 1 (Fig. 2). Explicitly writing down the energy terms in equation (14) yields:
\[
\frac{1}{2} k \overline{x}_{BDC}^2 = \frac{P V_i - P_0 V_f}{(1 - \gamma_{\text{com}})} + \frac{1}{2} k \overline{x}_{TDC}^2 + 0 E_{1,\text{loss}}
\]  
(15)

Using equations (15), (14), and (2), we find the spring stiffness required to send the piston from \( \overline{x}_{BDC} \) to \( \overline{x}_{TDC} \) as:
\[
k = \frac{2 K_{\text{com}} V_c^{1 - \gamma_{\text{com}}} (1 - r_c^{1 - \gamma_{\text{com}}}) + 2 (\gamma_{\text{com}} - 1) 0 E_{1,\text{loss}}}{(\gamma_{\text{com}} - 1) \left[ \frac{V}{A} (1 - r_c) + \overline{x}_{TDC} \right]^2 - \overline{x}_{TDC}^2}
\]  
(16)

It therefore follows that regulation of \( x_{TDC} \) at \( \overline{x}_{TDC} \) implies regulation of \( x_{BDC} \) at \( \overline{x}_{BDC} \) and vice versa. This is possible owing to a fixed restoring force parameter \( k \) of the spring that requires no control, unlike a gas chamber where the gas pressure must be actively regulated [18, 6, 19]. The loss term \( 0 E_{1,\text{loss}} \) in equation (10) is probably impossible to find analytically owing to the nonlinear dynamics of system (1). Nonetheless, a numerical simulation can yield a good estimate of this term.

It is now apparent that the main control objective is to regulate only one dead centre variable at a position corresponding to a desired compression ratio. To satisfy this objective we require a relationship or model, (e.g. a transfer function \( G \)) between the input (\( m_{\text{fuel}} \)) and the output (\( x_{TDC} \) or \( x_{BDC} \)), as illustrated in Fig. 3.
4.2. Control-Oriented Model Structure

For illustration purposes, we choose to regulate $x_{TDC}$ at $\bar{x}_{TDC}$. A new $x_{TDC}$ position can be thought of as being dependent on current and previous $x_{TDC}$ positions, as well as current and previous inputs. We define the input and output variables as:

$$u = m_{fuel}$$
$$y = x_{TDC}$$

Assuming linearity, the relationship between $y$ and $u$ on a cycle-by-cycle discrete time scale is expressed as a difference equation with constant coefficients:

$$y(k) + a_1 y(k-1) + a_2 y(k-2) + ... a_n y(k-n) = b_1 u(k-1) + ... + b_n u(k-n) + \epsilon(k)$$

where $k = 1, 2, ...$ is a cycle counting index (not to be confused with spring constant). The coefficients $a_1, ... , a_n, b_1, ... , b_n$ are estimated using measured input/output data of the true process by well-developed system identification [22]. The term $\epsilon(k)$ is used to account for the fitting error of model (18) to measured data. For simplicity, we assume a first order relationship, where the coefficients are estimated with sufficient accuracy so that $\epsilon(k)$ is effectively negligible i.e.:

$$y(k) + a_1 y(k-1) = b_1 u(k-1)$$

Indeed, it turns out this first order relationship suffices for control of system (1) as simulation results show in the following section. Being a discrete-time system, define the delay operator $z^{-1}$ as:

$$y(k-n) = y(k)z^{-n}$$

which we apply to equation (15) to realize the transfer function $G(z^{-1})$ as:

$$\frac{y(k)}{u(k)} = G(z^{-1}) = \frac{b_1 z^{-1}}{1 + a_1 z^{-1}}$$

Figure 3. Relationship between input $m_{fuel}$ and output $x_{T/BDC}$ abstracted by model $G$. 
4.3. Controller Development

We now proceed to develop a PI controller for the engine system (1) using control-oriented model transfer function (21).

\[ r = x_{TDC} \]
\[ C(z^{-1}) \]
\[ u = m_{fuel} \]
\[ G(z^{-1}) \]
\[ y = x_{TDC} \]

**Figure 4.** Schematic of a closed-loop engine control system with negative feedback.

The generic negative feedback control loop (Fig. 4) with controller transfer function \( C(z^{-1}) \), plant transfer function \( G(z^{-1}) \), reference \( r = x_{TDC} \), and output \( y = x_{TDC} \) has the transfer function:

\[
\frac{y(k)}{r(k)} = \frac{C(z^{-1})G(z^{-1})}{1 + C(z^{-1})G(z^{-1})} \tag{22}
\]

By defining the error: \( e(k) = r(k) - y(k) \), a PI controller with positive gain parameters \( k_p, k_i \) has the control law:

\[
u(k) = k_p e(k) + k_i I(k) \tag{23}\]

where \( I(k) = e(k) + I(k-1) \), whose transfer function is realized as:

\[
\frac{u(k)}{e(k)} = C(z^{-1}) = \frac{k_p + (k_i - k_p)z^{-1}}{1 - z^{-1}} \tag{24}\]

The closed-loop transfer function is achieved by putting transfer functions (24) and (21) into (22) to give:

\[
\frac{y(k)}{r(k)} = \frac{k_p b_1 z^{-1} + (b_1 k_i - k_p b_1)z^{-2}}{1 + (k_p b_1 + a_1 - 1)z^{-1} + (b_1 k_i - b_k k_p - a_1)z^{-2}} \tag{25}\]

Transfer function (25) has two poles from its characteristic equation:

\[
\alpha z^2 + \beta z + \gamma = 0 \tag{26}\]

where \( \alpha = 1, \beta = (k_p b_1 + a_1 - 1), \) and \( \gamma = (b_1 k_i - b_k k_p - a_1) \). It is well-known that for stability of discrete-time systems, both poles must lie within the unit circle in the complex \( z \) plane [23]. Hence we can now state the pertinent stability condition of the PI controlled system as follows:
**Stability Condition:** The engine system (1) is stable under PI control if the pole locations $z$ satisfy the following condition:

$$z = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha \gamma}}{2\alpha} < 1$$

(27)

Thus, the PI controller gain parameters must be selected to satisfy condition (27) in order to achieve stable engine operation. The controller, in its standard form of (23), may be modified so as to reduce the response overshoot. Overshoot of the piston beyond a desired dead centre position can be destructive to the engine. A possible modification is to add a static component $m_{\text{fuel}} \geq 0$ as follows:

$$u(k) = m_{\text{fuel}} + k_p e(k) + k_i I(k)$$

(28)

The newly added static component is a base amount of fuel estimated to be as close as possible to the theoretical quantity needed for sustained piston oscillations under a given loading.

5. **Simulation Test of Stability Under Stochastic Conditions**

Effectiveness of the proposed PI controller under stochastic in-cylinder pressure conditions is verified by numerical simulation. The noise is modelled as an additive Gaussian white noise process with a signal-to-noise ratio of 30dB. Figure 5 shows a noisy pressure trace for the first two engine cycles when it follows an ideal Otto cycle. Table 1 details the simulation parameters for the engine system (1). The nominal bottom dead centre $\bar{x}_{BDC}$ was computed with (13). Assuming negligible frictional effects, spring stiffness $k$ in (16) was computed exactly by setting generator parameter $b$ to zero (i.e. the generator switched off) during compression to yield $E_{i, \text{loss}} = 0$. At simulation start, the piston had an initial offset of 3% above $\bar{x}_{BDC}$. The relevant thermodynamic constants were extracted from standard internal combustion engine reference texts such as [21, 20].

**Table 1.** Engine simulation parameters.

| Parameter                  | Symbol | Units | Quantity |
|----------------------------|--------|-------|----------|
| compression ratio          | $r_c$  | -     | 12.0     |
| Piston-translator mass     | $m$    | kg    | 6.5      |
| piston head area           | $A$    | $m^2$ | 0.005    |
| clearance volume           | $V_c$  | $m^3$ | 0.0001   |
| nominal top dead centre    | $\bar{x}_{TDC}$ | $m$ | 0.10     |
| generator parameter        | $b$    | $kg \cdot s^{-1}$ | 174.0 |

The control-oriented model parameters $q_i, \dot{b}_i$ were estimated with event-based sampled input/output data collected during a system identification experiment of (1). The method of least square estimation [22] yielded the parameter values: $a_i = 0.6995, b_i = 0.5346$. Parameters $k_p = 1, k_i = 0.3$ in controller (28) were selected to ensure the control system remains stable according to stability condition (27). The quantity $m_{\text{fuel}}$ was estimated as 0.0558 kg.
Figure 5. Noisy pressure trace of the first two engine cycles. The sharp pressure rise is the combustion event whereas the sharp pressure drop is scavenging. The lowest pressure point is the standard atmospheric pressure, as the engine is naturally aspirated.

Figure 6. Top dead centre error goes to near-zero values below 1%. Hence engine operation is stable amidst the noise levels in the system. The values were selected to meet stability condition (27).

As expected, the top-dead-centre error in Fig. 6 descends to near zero and stays there when the parameters $k_p, k_i$ are selected according to stability condition (27). The error remains below 1% amidst the noise levels in the system, showing stable engine operation. Fig. 7 shows two plots: cycle-by-cycle work done by generalized damping force $F_d$, and cycle-by-cycle work done by the generator load force. Indeed, the cycle-by-cycle work done by $F_d$ goes to zero and stays there during stable engine operation as expected from criterions (10) and (11). The work done by the generator load force is positive, representing useful electric energy conversion by the engine. Finally, stable operation means a steady
compression ratio. As expected, the plot in Fig. 8 shows the engine dynamic compression ratio remaining very close to the designed value of 12.0.

![Figure 7](image1.png)

**Figure 7.** Work done by the generalized damping force on a cycle by cycle basis goes to zero with progressing oscillations thereby verifying criteria (10) and (11). Work done by the generator load force stays positive, representing useful electric energy conversion.

![Figure 8](image2.png)

**Figure 8.** Engine compression ratio remains near the designed value of 12.0 during stable operation.

6. **Conclusions**

Stability of a semi-free piston engine generator has been studied and tested under stochastic in-cylinder pressure conditions. A novelty of this work stems from proposing two important criteria relating in-cylinder pressure, generator output, and piston speed using the idea of work done by a defined generalized damping force. Also, a rigorous PI controller analysis has been undertaken and an accompanying useful stability condition given in explicit form. All proposed results have been successfully verified by numerical simulation at significant noise levels.
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