Intensity and polarization of radiation reflected from accretion disc

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Abstract We consider the reflection of non-polarized radiation from the point-like sources above the accretion discs both the optically thick and optically thin. We investigate the dependence of the polarization of reflected radiation on the aperture angle of incident radiation. The aperture angle is determined by the radius of accretion disc and the height of the source above the disc. For optically thick accretion discs we show that, if the aperture angle is smaller 70 grad, then the wave electric field oscillations of reflected radiation are parallel to the accretion disc plane. For aperture angle greater than 70 grad the electric field oscillations are parallel to the plane "normal to accretion disc—the line of sight". The latter also holds for reflection from the optically thin accretion disc independent of the aperture angle value.

Keywords Radiative transfer · Scattering · Polarization · Accretion discs

1 Introduction

The black holes (BH) demonstrate the presence of accretion discs both optically thick and optically thin. About 300 supermassive black holes (SMBH) exist in active galactic nuclei (AGN). There are ~20 BH in X-ray stellar binary systems (see Cherepashchuk 2006). The structure of accretion discs is considered both without the relativistic corrections and with them (see Shakura and Sunyaev 1973; Novikov and Thorne 1973; Riffert and Harold 1995; Hubeny and Hubeny 1997 etc.). The standard nonrelativistic model (Shakura and Sunyaev 1973; Urpin 1983; Pariev and Colgate 2007) is most popular. It should be noted that the models of accretion discs demonstrate the large difference in temperature and density as the function of the distance r from the center of a disc.

The observed linear polarization from (AGN) in Seyfert galaxies frequently demonstrate two types of polarization. In the first type the wave electric field E-oscillations are parallel to the plane of accretion disc. In the second type the E-oscillations are parallel to the plane (nN), where N is the normal to the disc plane and n is the line of sight. It appears the same effect is in X-ray binary systems (see, for example, Fabrika 2004 and references therein). The most often observed polarization is parallel to the disc plane. In Seyfert-1 AGNs the polarization frequently corresponds to oscillations in the plane (nN), but there are polarizations with intermediate values of position angles. Note that the measurement of linear polarization from AGN is one of very effective technique to obtain the information about the structure of AGN. The polarimetric results are given in many papers (see, for example, Impey et al. 1991; Goodrich and Miller 1994; Smith et al. 2002, 2004; Marin 2014).

The presence of different types of jets is widely adopted in literature (see, for example, Blandford and Konigl 1979; the review of Blandford 1990, 2003; Hawley et al. 2007; Krivoshieyev et al. 2009). The various models of jets give rise to the spectrum of radiation—from X-ray up to optical region. These radiations escape from different heights in the jet. The mechanisms of jets can be different. The main problem of construction of jets models is how jet arises in the center of accretion disc. Usually one takes into account the influence of magnetic field and the velocity of accretion discs rotation. Besides, it is necessary to use the observed radiation from the jet (if they are observed).
The goal of our paper is investigation of the radiation reflection from accretion disc both optically thick and optically thin. Note that the height $H$ of considered radiation source depends on wavelength. We demonstrate that the intensity and polarization of reflected radiation depend on aperture angle $\Theta$ of the incident radiation. We also present the emerging radiation going from optically thick layers of the accretion disc (the Milne problem).

2 Statement of problem

Chandrasekhar (1960) derived the radiative transfer equation for the intensity $I(\tau, \mu)$ and the Stokes parameter $Q(\tau, \mu)$ for an atmosphere consisting of free electrons. This equation, which we use in our calculations, has the matrix phase function $\hat{P}(\mu, \mu')$. Here $\mu = nN = \cos \theta$, $\mu' = n'N = \cos \theta'$, where the angles $\theta$ and $\theta'$ characterize the line of sight $n$ and the direction of incident radiation $n'$, respectively (see Fig. 1). The factorization of the matrix phase function $\hat{P}(\mu, \mu')$, i.e. the presentation of this matrix as a product of two matrices $\hat{P}(\mu, \mu') = \hat{A}(\mu)\hat{A}^T(\mu')$, plays a very important role in radiative transfer theory. The subscript $T$ stands for the matrix transpose. We use the factorization presented in many papers (see, for example, Ivanov 1995; Silant’ev et al. 2017). Note that we restrict ourselves by conservative axially symmetric atmosphere.

The radiative transfer equation for the vector (column) with the components $(I, Q)$ can be written in the form:

$$(n\nabla)I(\mu, \mu) = -\alpha I(\mu, \mu) + \alpha \hat{A}(\mu)K(\mu),$$

where $\alpha$ is the extinction factor. We use the cylindrical frame of reference $r = (z, \rho, \varphi)$. The source term describes the single scattered incident radiation from the point-like source located above the accretion disc. Recall, that we consider single or more scattered light, i.e. the diffused radiation (see Chandrasekhar 1960).

For point-like source of non-polarized light with the aperture of incident photon $\theta'$ ($\mu' = \cos \theta' = (z + H)/\sqrt{(z + H)^2 + \rho^2}$) we have:

$$\hat{s}(\mu) = L_0(\mu') \frac{\exp(-\tau/\mu')}{\rho^2 + (z + H)^2} \hat{A}^T(\mu').$$

$L_0(\mu')$ is the luminosity of the source. In general case the luminosity can be anisotropic. The value $H$ is the height of the source above the surface of accretion disc. The axis $Z$ is directed inside the accretion disc. The value $z = 0$ corresponds to the surface of accretion disc.

In our case the source has the spot-like form. Far from this “spot” the intensity $I$ and parameter $Q$ tend to zero. A telescope is located far from the accretion disc and observes the integral radiation.

The accretion disc, as a whole, looks like point source of radiation. The integration over all surface $(2\pi \rho d\rho)$ gives rise to usual radiative transfer equation for these integrated values (we use the same notions for them).

Introducing the optical depth $d\tau = adz$, we obtain the equation:

$$\mu dI(\tau, \mu)/d\tau = -I(\tau, \mu) + \hat{A}(\mu)K(\tau) + \hat{A}(\mu)\hat{s}(\tau)\begin{pmatrix}1 \\ 0 \end{pmatrix}.$$  (3)

Equation (3) is appears as the usual radiative transfer equation in plane parallel atmosphere (Sobolev 1963).

The vector $K(\tau)$ has the form:

$$K(\tau) = \frac{1}{2} \int_{-1}^{1} d\mu \hat{A}^T(\mu)I(\tau, \mu).$$  (4)

The factorization matrix $\hat{A}(\mu)$ (see Ivanov 1995; Silant’ev et al. 2017) is equal to:

$$\hat{A}(\mu) = \begin{pmatrix}1, \sqrt{C}(1 - 3\mu^2) \\ 0, 3\sqrt{C}(1 - \mu^2) \end{pmatrix}.$$  (5)

Here $C = 1/8 = 0.125$.

Using the relation $\rho d\rho/[(\rho^2 + (z + H)^2)] = -d\mu'/\mu'$, we obtain the following formula for $\hat{s}(\tau)$:

$$\hat{s}(\tau) = \int_{\cos \Theta}^{1} \frac{d\mu'}{\mu'} L_0(\mu')\hat{A}^T(\mu') \exp(-\tau/\mu').$$  (6)

The value $\Theta$ is the total aperture of incident radiation, depending on the radius $\rho_0$ of accretion disc and the height $H$ of the point-like source: $\cos \Theta = H/\sqrt{H^2 + \rho_0^2}$. If the radius of the accretion disc tends to infinity, then $\Theta \to 90^\circ$. This case is considered in the papers Grinin and Domke (1971) and Silant’ev and Gnedin (2008). In this case the source $\hat{s}(\tau)$ is independent of $H$. For accretion discs in...
X-ray stellar binary systems the radius $r_0$ is the distance between the components. Note that reverberation technique, used by Fabian et al. (2009a, 2009b); Zoghbi and Fabian (2011), gave the estimation of the height $H \approx 10r_g$ for the sources of X-ray radiation. Here $r_g$ is the gravitational radius of black hole. Such small $H$ gives the $\Theta \approx 90^\circ$. Recall, that in this case the reflected radiation is independent of the height $H$ and the polarization corresponds to $E$-oscillations in the plane ($n N$).

In principle, the total aperture can be determined as the angle inside of which all the radiation falls on the disc.

3 The solution by resolvent technique

The radiative transfer equation (3) can be written in more compact form:

$$\frac{dI(\tau, \mu)}{d\tau} = -I(\tau, \mu) + \hat{A}(\mu)S(\tau), \quad (7)$$

$$S(\tau) = K(\tau) + \hat{s}(\tau) \left[ \begin{array}{c} 1 \\ 0 \end{array} \right]. \quad (8)$$

Using the formal solution of Eq. (7), we can derive the integral equation for $S(\tau)$:

$$S(\tau) = \hat{s}(\tau) \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] + \int_{0}^{\infty} d\tau' \hat{L}(|\tau - \tau'|)S(\tau'), (9)$$

where the matrix kernel $\hat{L}(|\tau - \tau'|)$ has the form:

$$\hat{L}(|\tau - \tau'|) = \int_{0}^{1} \frac{d\mu}{\mu} \hat{\Psi}(\mu) \exp(-|\tau - \tau'|/\mu), \quad (10)$$

$$\hat{\Psi}(\mu) = \frac{1}{2} \hat{A}^T(\mu) \hat{A}(\mu) \equiv \hat{\Psi}^T(\mu). \quad (11)$$

The general theory for calculation of the vector $S(\tau)$ is given in Silant'ev et al. (2015). According to this theory, Eq. (9) has solution through the resolvent matrix $\hat{R}(\tau, \tau')$ (see Smirnov 1964; Sobolev 1969):

$$S(\tau) = \hat{s}(\tau) \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] + \int_{0}^{\infty} d\tau' \hat{R}(\tau, \tau') \hat{s}(\tau') \left[ \begin{array}{c} 1 \\ 0 \end{array} \right], \quad (12)$$

where the resolvent matrix obeys the integral equation:

$$\hat{R}(\tau, \tau') = \hat{L}(|\tau - \tau'|) + \int_{0}^{\infty} d\tau'' \hat{L}(|\tau - \tau''|) \hat{R}(\tau'', \tau'). \quad (13)$$

$\hat{R}(\tau, \tau')$ can be calculated if we know the matrices $\hat{R}(\tau, 0)$ and $\hat{R}(0, \tau)$. The kernel $\hat{L}(|\tau - \tau'|)$ of equation for $\hat{R}(\tau, \tau')$ is symmetric: $\hat{L} = \hat{L}^T$. This gives rise to the relation $\hat{R}(\tau, \tau') = \hat{R}(\tau', \tau)$. Note (see Silant'ev et al. 2015), that the double Laplace transform of $\hat{R}(\tau, \tau')$ has the form:

$$\hat{R}(1, 1) = \frac{1}{\mu + \mu'} \left[ \hat{R}(1, 0) + \hat{R}(0, 1) \right] + \frac{1}{\mu} \hat{R}(0, 0) \hat{R}(1, 1/\mu + 1/\mu'). \quad (14)$$

The Laplace transform of $R(\tau, 0)$ with the parameter $1/\mu$ plus the unit matrix $\hat{E}$ is known as $\hat{H}(\mu)$—matrix:

$$\hat{H}(\mu) = \hat{E} + \hat{R}(1, 0). \quad (15)$$

From Eqs. (14) and (15) we obtain the following nonlinear equation:

$$\hat{H}(\mu) = \hat{E} + \mu \int_{0}^{1} d\mu' \hat{H}(\mu') \hat{H}^T(\mu') \hat{\Psi}(\mu'), \quad (16)$$

4 The emerging radiation from optically thick accretion discs

According to Eq. (7) the emerging radiation $I(0, \mu)$ has the form:

$$I(0, \mu) = \hat{A}(\mu) \int_{0}^{\infty} \frac{d\tau}{\mu} S(\tau) \exp(-\tau/\mu) = \hat{A}(\mu) \int_{0}^{1} d\mu' L(0, \mu') \hat{H}(\mu) \hat{H}^T(\mu') \hat{\Psi}(\mu') \mu + \mu'/ \mu \mu', \quad (17)$$

Recall, that $\mu = \cos \theta$, where $\theta$ is the angle between the line of sight $n$ and the normal $N$ to the accretion disc, i.e. this is the inclination angle. We emphasize that $I(0, \mu)$ depends on the aperture $\Theta$ of incident radiation. For numerical calculations it is better introduce the new matrix $\hat{D}(\mu) = \hat{A}(\mu) \hat{H}(\mu)$, which obeys the equation:

$$\hat{D}(\mu) = \hat{D}(\mu) \hat{A}(\mu), \quad (18)$$

The kernel of this equation does not depend on $\mu^4$, i.e. Equation (18) is simpler than Eq. (16). The technique of numerical calculation of $\hat{D}(\mu)$ is given in Silant'ev et al. (2017). Expression (17) in new matrix takes the form:

$$I(0, \mu) = \int_{\cos \theta}^{1} d\mu' L(0, \mu') \hat{D}(\mu) \hat{D}^T(\mu') \left[ \begin{array}{c} 1 \\ 0 \end{array} \right]. \quad (19)$$
Table 1 Functions $a(\mu)$, $b(\mu)$, $c(\mu)$, $d(\mu)$

| $\mu$ | $a(\mu)$ | $b(\mu)$ | $c(\mu)$ | $d(\mu)$ |
|------|----------|----------|----------|----------|
| 0.01 | 1.0370   | 0.3536   | 0.0075   | 1.0607   |
| 0.02 | 1.0670   | 0.3823   | 0.0126   | 1.1280   |
| 0.03 | 1.0946   | 0.3919   | 0.0170   | 1.1310   |
| 0.04 | 1.1208   | 0.4001   | 1.0208   | 1.1470   |
| 0.05 | 1.1460   | 0.4071   | 0.0244   | 1.1616   |
| 0.06 | 1.1704   | 0.4133   | 0.0276   | 1.1746   |
| 0.07 | 1.1941   | 0.4186   | 0.0306   | 1.1864   |
| 0.08 | 1.2174   | 0.4231   | 0.0334   | 1.1971   |
| 0.09 | 1.2402   | 0.4270   | 0.0361   | 1.2070   |
| 0.10 | 1.2626   | 0.4303   | 0.0386   | 1.2159   |
| 0.15 | 1.3703   | 0.4378   | 0.0492   | 1.2493   |
| 0.20 | 1.4728   | 0.4325   | 0.0574   | 1.2667   |
| 0.25 | 1.5716   | 0.4155   | 0.0637   | 1.2702   |
| 0.30 | 1.6675   | 0.3874   | 0.0683   | 1.2611   |
| 0.35 | 1.7611   | 0.3486   | 0.0714   | 1.2400   |
| 0.40 | 1.8526   | 0.2995   | 0.0731   | 1.2075   |
| 0.45 | 1.9422   | 0.2401   | 0.0735   | 1.1639   |
| 0.50 | 2.0302   | 0.1705   | 0.0726   | 1.1096   |
| 0.55 | 2.1166   | 0.0909   | 0.0705   | 1.0446   |
| 0.60 | 2.2015   | 0.0014   | 0.0672   | 0.9692   |
| 0.65 | 2.2851   | −0.0981  | 0.0627   | 0.8834   |
| 0.70 | 2.3673   | −0.2074  | 0.0571   | 0.7874   |
| 0.75 | 2.4482   | −0.3266  | 0.0503   | 0.6812   |
| 0.80 | 2.5279   | −0.4555  | 0.0425   | 0.5650   |
| 0.85 | 2.6063   | −0.5943  | 0.0335   | 0.4387   |
| 0.90 | 2.6835   | −0.7428  | 0.0234   | 0.3024   |
| 0.95 | 2.7698   | −0.7737  | 0.0213   | 0.2739   |
| 0.99 | 2.8434   | −1.0347  | 0.0025   | 0.0320   |
| 1.00 | 2.8344   | −1.0691  | 0.0000   | 0.0000   |

It is of interest, that directly from Eq. (18) one can obtain that zero’s moments $a_0 = 2$ and $b_0 = 0$ (see Silant’ev et al. 2017). The numerical calculation confirms this. The functions $a(\mu)$, $b(\mu)$, $c(\mu)$ and $d(\mu)$ are given in Table 1.

A telescope observes the direct radiation flux from the source $F_{\text{direct}} = L_0(\mu)/R^2$, where $R$ is the distance to the telescope. The flux of scattered radiation is equal to $F_{\text{diff}}(\mu) = \mu I(0, \mu)/R^2$. Note that there exists the relation:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \hat{A}(\mu) \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (20)$$

Substitution relation (20) in Eq. (19) gives rise to the formula for $F_{\text{diff}}(\mu)$:

$$F_{\text{diff}}(\mu) = \frac{\mu}{R^2} \int_{0}^{\pi} d\cos\theta \frac{L_0(\mu') \hat{D}(\mu') \hat{D}^*(\mu') \hat{A}(\mu')}{\mu + \mu'} \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (21)$$

According to Eq. (18) for isotropic source ($L_0(\mu) = L_0$) and the aperture $\Theta = 90^\circ$ formula (21) takes the simple form:

$$F_{\text{diff}}(\mu) = \frac{2L_0}{R^2} \left( \hat{D}(\mu) - \hat{A}(\mu) \right) \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (22)$$

Below we restrict ourselves by the isotropic source and different values of aperture $\Theta$. We present the total fluxes $F_I(\mu) = F_{I(\text{diff})}(\mu) + F_{I(\text{direct})}$ and $F_Q(\mu) = F_{Q(\text{diff})}(\mu)$ ($F_Q(0) = 0$) without the common factor $L_0/R^2$. For this reason $F_I(0) = 1$ and $F_Q(0) = 0$, because at $\mu = 0$ the flux $F$ consists of the non-polarized direct radiation from the source. According to Eq. (22), the total $F_I(\mu)$ and $F_Q(\mu)$ can be given in the form:

$$F_I(\mu) = \frac{L_0}{R^2} (2a(\mu) - 1), \quad F_Q(\mu) = \frac{2L_0}{R^2} c(\mu), \quad F_I(0) = \frac{L_0}{R^2}. \quad (23)$$

In Table 2 we give the angular dependence $J(\mu) = F_I(\mu)$ and the polarization degree $p(\mu) = F_Q(\mu)/F_I(\mu)$ in % for aperture $\Theta = 90^\circ$. Note that $p(\mu)$ for $\Theta = 90^\circ$ is positive. This corresponds to the wave electric field oscillations parallel to the plane (nN) (see Table 2). For comparison we give the angular dependence $J_M(\mu)$ and polarization degree $p_M(\mu)$ for the Milne problem:

$$J_M(\mu) = \frac{a(\mu) + b(\mu)}{a(0) + b(0)} s, \quad p_M(\mu) = \frac{c(\mu) + d(\mu)}{a(\mu) + b(\mu)} s \quad (24)$$

where $s = -0.10628$ is the solution of homogeneous equation for $K(0)$ (see Silant’ev et al. 2017).

Recall, that the Milne problem describes the diffusion of thermal radiation from the sources located far below the surface of the optically thick accretion disc. In the Milne problem the polarization $p(\mu)$ is negative. This corresponds to the wave electric field oscillations parallel to the accretion disc plane. Note that the reflected radiation has the maxi-
on the relative values of intensities $F_Q(\mu)$ and the Milne problem intensity $F_M(\mu)$. Tables 1 and 2 can be used for estimations of observed intensity and polarization for different relative values of $F_Q(\mu)$ and $F_M(\mu)$. So, these Tables allow us to estimate the intensities and polarization in different models.

Figure 2 presents the angular distribution of the emerging radiation $J(\mu)$ and the degree of polarization $p(\mu)$ in % for a number of values of aperture $\theta$. We see that for $\theta < 70^\circ$ the $E$—oscillations are parallel to the accretion disc plane. The values $\theta > 70^\circ$ correspond to oscillations parallel to the plane $(nN)$, which frequently occur in Seyfert-1 AGNs. Further we will explain this behavior.

## 5 The emerging radiation from optically thin accretion disc

The optically thin accretion disc is characterized by small optical depth $\alpha z = \tau \ll 1$. In this situation we can neglect by multiple scattering of light. Instead of Eq. (17) for reflected radiation wave the expression:

$$I(0, \mu) = \hat{A}(\mu) \int_0^\tau \frac{d\tau}{\mu} s(\tau) \exp(-\tau/\mu)$$

$$= \hat{A}(\mu) \int_0^\tau \frac{d\tau}{\mu} \exp(-\tau/\mu) \int_{\cos \theta}^1 \frac{d\mu'}{\mu'} L_0(\mu')$$

$$\times \hat{A}^T(\mu') \exp(-\tau/\mu') \left( \begin{array}{c} 1 \\ 0 \end{array} \right).$$

Here we used the expression (6) for $s(\tau)$. After integration over $d\tau$ we obtain the relation:

$$I(0, \mu) = \hat{A}(\mu) \int_{\cos \theta}^1 d\mu' L_0(\mu') \hat{A}^T(\mu')$$

$$\times \frac{1 - \exp(-\tau(1/\mu' + 1/\mu))}{\mu + \mu'} \left( \begin{array}{c} 1 \\ 0 \end{array} \right).$$

The expression (26) takes into account the single scattered radiation. The flux of radiation in a telescope is equal to:

$$F(0, \mu) = \frac{1}{R^2} \left[ \mu I(0, \mu) + L_0(\mu) \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \right],$$

where $R$ is a distance to telescope.

In Figs. 3 and 4 we give the angular distribution $J(\mu)$ and polarization degree $p(\mu)$ for accretion optical depths $\tau = 0.1$ and 0.3. Recall, that the scattered radiation intensity is proportional to optical depth $\tau$ and the value of aperture $\theta$. In this situation the intensity of direct radiation from the source is greater than the intensity of reflected radiation. The degree of polarization $p(\mu)$ in % is more distinct. Note

| $\mu$ | $J_M(\mu)$ | $-p_M(\mu)$% | $J(\mu)$ | $p(\mu)$% |
|-------|------------|--------------|----------|------------|
| 0.01  | 1.037      | 1.074        | 1.388    | 0          |
| 0.02  | 1.066      | 1.134        | 2.223    | 0          |
| 0.03  | 1.094      | 1.189        | 2.854    | 0          |
| 0.04  | 1.120      | 1.242        | 3.358    | 0          |
| 0.05  | 1.146      | 1.292        | 3.772    | 0          |
| 0.06  | 1.170      | 1.341        | 4.118    | 0          |
| 0.07  | 1.194      | 1.388        | 4.411    | 0          |
| 0.08  | 1.218      | 1.435        | 4.661    | 0          |
| 0.09  | 1.241      | 1.480        | 4.876    | 0          |
| 0.10  | 1.264      | 1.525        | 5.060    | 0          |
| 0.15  | 1.375      | 1.741        | 5.656    | 0          |
| 0.20  | 1.482      | 1.946        | 5.904    | 0          |
| 0.25  | 1.587      | 2.143        | 5.945    | 0          |
| 0.30  | 1.690      | 2.335        | 5.851    | 0          |
| 0.35  | 1.791      | 2.522        | 5.663    | 0          |
| 0.40  | 1.892      | 2.705        | 5.406    | 0          |
| 0.45  | 1.991      | 2.884        | 5.096    | 0          |
| 0.50  | 2.091      | 3.060        | 4.745    | 0          |
| 0.55  | 2.189      | 3.233        | 4.361    | 0          |
| 0.60  | 2.287      | 3.403        | 3.949    | 0          |
| 0.65  | 2.385      | 3.570        | 3.513    | 0          |
| 0.70  | 2.483      | 3.735        | 3.058    | 0          |
| 0.75  | 2.580      | 3.896        | 2.584    | 0          |
| 0.80  | 2.677      | 4.056        | 2.094    | 0          |
| 0.85  | 2.774      | 4.213        | 1.590    | 0          |
| 0.90  | 2.870      | 4.367        | 1.072    | 0          |
| 0.91  | 2.890      | 4.398        | 0.967    | 0          |
| 0.92  | 2.909      | 4.428        | 0.862    | 0          |
| 0.93  | 2.928      | 4.458        | 0.756    | 0          |
| 0.94  | 2.947      | 4.489        | 0.649    | 0          |
| 0.95  | 2.967      | 4.519        | 0.542    | 0          |
| 0.96  | 2.986      | 4.549        | 0.435    | 0          |
| 0.97  | 3.005      | 4.579        | 0.327    | 0          |
| 0.98  | 3.024      | 4.609        | 0.218    | 0          |
| 0.99  | 3.044      | 4.639        | 0.109    | 0          |
| 1     | 3.063      | 0            | 4.669    | 0          |
that all the forms of all Figures are similar. The difference exists with the scales. It appears this is the consequence that they describe the single scattering in accretion discs.

6 Discussion of the calculation results

We investigated the reflection of non-polarized radiation from point-like sources above the reflecting accretion disc. The optically thick and optically thin accretion discs were considered. We found two types of polarization in reflected radiation. The first type corresponds to the wave electric field E-oscillations parallel to the plane of accretion disc. The second type corresponds to E-oscillations parallel to the plane (nN), which is perpendicular to the accretion disc plane. Recall, that n is the line of sight direction, and the unit vector N is perpendicular to the accretion disc.

Why arise these types of polarization? Note that the first type corresponds to Chandrasekhar’s solution of the Milne problem. This type is named as the usual (positive) polarization. This polarization holds in many cases. The second type of polarization occurs rarely and was named as “negative” polarization. Of course, there are intermediate directions of the wave electric field oscillations between the positive and negative polarizations. They are characterized by concrete positional angles.

Recall, that E-oscillations of single scattered non-polarized radiation are perpendicular to the plane of scattering (nn_0), where n_0 is the direction of incident radiation. If n ⊥ n_0, then the scattered radiation is 100% polarized. The degree of polarization \( p(\mu_0) \sim \sin^2 \mu_0 \), where \( \mu_0 \) is the cosine of the angle between vectors n and n_0.

The case of optically thin accretion disc is most simple. Let us consider two characteristic regions of scattering. In the first region the plane of scattering coincides with the plane (nN). Here the scattered radiation has polarization \( \sim \sin \theta \) and the E-oscillations are parallel to the plane of accretion disc. In the second region the plane of scattering is perpendicular to plane (nN). Here the angle of scatter-
Fig. 3 The angular distribution $J(\mu)$ and polarization degree $p(\mu)$ in % of observed radiation flux from optically thick accretion disc, consisting of free electrons. The numbers denote the apertures of point-like sources.

Note that the total radiation with the wavelength $\lambda$ can also have other sources, not only from a jet (see, for example, Krivosheyev et al. 2009).

7 Conclusion

In this paper we study the reflection of non-polarized radiation from the point-like sources located above the reflecting accretion disc. The optically thick and optically thin accretion discs are considered. We also take into account the intensity of radiation going to a telescope directly from a source. For apertures $\Theta < 70^\circ$ the polarization of total radiation, reflected from optically thick disc, corresponds to wave electric field oscillations parallel to the plane of accretion disc. For apertures $\Theta > 70^\circ$ the polarization corresponds to oscillations parallel to the plane ($nN$), where $N$ is the normal to the disc and $n$ is the line of sight. Such polarization is frequently observed in active galactic nuclei of
Fig. 4 The angular distribution $J(\mu)$ and polarization degree $p(\mu)$ in % of observed radiation flux from optically thick accretion disc, consisting of free electrons. The numbers denote the apertures of point-like sources.

Seyfert-1 galaxies. For optically thin accretion discs the reflected radiation has the polarization corresponding to the $E$-oscillations parallel to the plane $(\mathbf{nN})$ independently of the aperture. We considered the accretion discs consisting of free electrons.

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