Quantum Cosmological Approach to 2d Dilaton Gravity†

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Abstract

We study the canonical quantization of the induced 2d-gravity and the pure gravity CGHS-model on a closed spatial section. The Wheeler-DeWitt equations are solved in (spatially homogeneous) choices of the internal time variable and the space of solutions is properly truncated to provide the physical Hilbert space. We establish the quantum equivalence of both models and relate the results with the covariant phase-space quantization. We also discuss the relation between the quantum wavefunctions and the classical space-time solutions and propose the wave function representing the ground state.

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1 Introduction

One of the most recent research directions in quantum gravity is the study of two dimensional dilaton models. Lower-dimensional gravity provides useful toy models for understanding the quantum mechanics of generally covariant theories. In two space-time dimensions, the Hilbert-Einstein action is trivial but the constant curvature condition can serve as the natural analogue of the vacuum Einstein equations [1, 2]. However, this equation cannot be derived from a local generally covariant action unless a scalar field $\Phi$ is introduced in the theory. The easiest way to obtain this equation from a local action is when the scalar field $\Phi$ plays the role of a lagrangian multiplier [1, 2].

The constant curvature equation can also be obtained from the induced 2d-gravity action [3]

$$S = \frac{c}{96\pi} \int d^2 x \sqrt{-g} \left( R \Box^{-1} R + 4\lambda^2 \right).$$  \hspace{1cm} (1.1)

This non-local action (induced by massless matter fields [3]) can be converted into a local one by introducing a dilaton scalar field $\Phi$ (we omit the coupling constant)

$$S = \frac{1}{2\pi} \int d^2 x \sqrt{-g} \left[ (\nabla \Phi)^2 + 2R\Phi + 4\lambda^2 \right].$$  \hspace{1cm} (1.2)

Recently, another manageable and intriguing model of two-dimensional gravity has been proposed by Callan, Gidding, Harvey and Strominger [4] with the crucial property of containing black holes. This model provides an excellent scenario to study black hole evaporation and back-reaction in a full quantum gravity setting (for a review see [5]). The classical action of the CGHS model (gravity coupled to a dilaton field $\Phi$ and conformal matter fields $f_i$, $i = 1, \ldots, N$) is given by

$$S = \frac{1}{2\pi} \int d^2 x \sqrt{-g} \left[ e^{-2\Phi} (R + 4(\nabla \Phi)^2 + 4\lambda^2) - \sum_{i=1}^{N} \frac{1}{2} (\nabla f_i)^2 \right].$$  \hspace{1cm} (1.3)

In a canonical framework the two-dimensional space-time manifold is usually required to have the topology of $\Sigma \times \mathbb{R}$, where the spatial section $\Sigma$ can be either $\mathbb{R}$ or $S^1$. For the non-compact case the classical solutions of (1.3) admit the standard black hole interpretation. When the spatial section is compact it is not clear whether the black hole interpretation can be maintained [6]. However, in the latter case, and when no matter fields are present, the model –and the induced 2d-gravity (1.2) as well– has the remarkable property of having all the classical solutions spatially homogeneous. This is just the main ingredient of the minisuperspace approach in quantum cosmology [7, 8] (see also the review [9]). Nevertheless, we should stress that, in contrast to 3+1-dimensional gravity, where the minisuperspace models are approximations to the full theory, in the above
models the condition of having spatially homogeneous solutions comes from the field equations themselves. Therefore, the minisuperspace approach can be an exact way to quantize the pure gravity 2d-dilaton models \[11\].

In this paper we shall study, in a parallel way, the induced 2d-gravity \((2.2)\) and the pure gravity CGHS-model for a closed spatial section in an exact canonical setting. We shall consider the ADM formulation of the 2d-dilaton models in a natural choice of the time slicing. The gauge-fixing will be spatially homogeneous and together with the diffeomorphism constraint reduces the theories to a finite number of degrees of freedom. This strategy allows us to go fairly far in solving the models. Other approaches to the quantization of 2d-gravity models (mainly using Dirac operator methods and BRST techniques) can be seen in \[11\]. In these works the supermomentum constraint is imposed at the quantum level. The possible equivalence with our results is an interesting and non-trivial point, but it is out of the scope of the present paper.

In section 2 we present the classical solutions of the induced 2d-gravity and the CGHS-model focusing on their covariant phase-space structure. We shall show in this way that both models have the same reduced phase space. In section 3 we develop the ADM formulation of the models in a generic spatially homogeneous gauge. This enables us to reduce the theory to a finite number of degrees of freedom and solve the (reduced) Wheeler-DeWitt equation exactly. The analysis of the quantum solutions of the induced 2d-gravity suggests a natural choice of the time slicing as well as a canonical transformation of the “minisuperspace” variables of the CGHS-model. The problem of the Hilbert space is considered in section 4. The space of solutions to the Wheeler-DeWitt equation is truncated to provide the proper Hilbert space. We shall also establish a quantum equivalence between both models in agreement with the equivalence predicted by the covariant phase-space quantization. In section 5, and based on the particular choice of time introduced previously, we discuss the classical behaviour of the quantum wave functions and also propose the wave function representing the ground-state.

\section{Classical solutions and covariant phase space}

\subsection{CGHS-model}

In the pure gravity CGHS-model the classical solutions for the metric and the dilaton field are given, in the conformal gauge, by

\begin{align}
 e^{-2\Phi} &= -\lambda^2 pm + \frac{M}{\lambda}, \\
 ds^2 &= \frac{\partial_+ p \partial_- m}{-\lambda^2 pm + \frac{M}{\lambda}} (-dt^2 + dx^2),
\end{align}

(2.4)

(2.5)
where \( p(m) \) is a function of \( x^+ (x^-) (x^\pm = t \pm x) \), \( \lambda = +\sqrt{\lambda^2} \) and \( M \) is a constant parameter playing the role of the black hole mass when \( \Sigma = \mathbb{R} \). When the theory is defined on a closed spatial section, \( \Sigma = S^1 \), the requirement of periodicity of the metric and the dilaton implies the following monodromy transformations of the functions \( p \) and \( m \):

\[
\begin{align*}
p(x^+ + 2\pi) &= e^r p(x^+) , \\
m(x^- - 2\pi) &= e^{-r} m(x^-) ,
\end{align*}
\]

(2.6) (2.7)

where \( r \) is an arbitrary parameter. Despite of the appearance of arbitrary functions in the general solutions, the space of non-equivalent solutions—under space-time diffeomorphisms—is finite-dimensional. This fact can be accomplished by evaluating the symplectic structure of the model

\[
\omega = \int_0^{2\pi} dx (-\delta j^0) ,
\]

(2.8)

where \( \delta \) stands for the exterior derivative on the space of classical solutions (i.e., the so-called covariant phase space \([12]\)), and \( j^\mu \) is the symplectic current potential defined, in general, as \([13]\)

\[
\delta = \partial _\mu j^\mu + \frac{\delta S}{\delta \varphi} \delta \varphi ,
\]

(2.9)

where \( S = S(\varphi) \) is the action functional.

In the present case the two-form \( \omega^0 = -\delta j^0 \) turns out to be a total derivative reflecting the absence of local degrees of freedom. More precisely, \( \omega^0 = \partial_x W \), where

\[
W = \delta \left( \frac{M}{\lambda} \right) \left( \delta \ln \partial_\pm m + \delta(-\lambda^2 pm) \delta \ln(p \partial_\pm m) \right) .
\]

(2.10)

The symplectic form (2.8) then becomes

\[
\omega = W(x + 2\pi) - W(x) ,
\]

(2.11)

and, using (2.6) and (2.7), one find that any dependence with respect to the \( p \) and \( m \) functions drops out and we are left with (see ref. \([14]\))

\[
\omega = -\delta \frac{M}{\lambda} \delta r .
\]

(2.12)

Therefore, the simple expression (2.12) captures, in an appealing way, the canonical structure of the model. We can construct explicitly the set of non-equivalent solutions by choosing appropriately the functions \( p \) and \( m \) verifying the monodromy condition (2.6) and (2.7). It is not difficult to arrive at the following
expressions (see also [13])

\[ ds^2 = \frac{1}{|\lambda|^2} \left( \frac{r}{2\pi} \right)^2 \frac{e^{\frac{r}{t}}}{M - s e^{\frac{r}{t}}} (-dt^2 + dx^2) , \]  
\[ e^{-2\Phi} = \frac{M}{\lambda} - s e^{\frac{r}{t}} , \]  

where \( s \) refers to the sign of the cosmological constant \( \lambda^2 \). In this way one immediately finds that the covariant phase space (i.e. the reduced phase space) corresponds to one single degree of freedom.

When \( M = 0 \) and the cosmological constant is negative the solution becomes the standard “linear dilaton” vacuum solution:

\[ ds^2 = -dt^2 + dx^2 , \]  
\[ \Phi = -\lambda t . \]  

### 2.2 Induced 2d-gravity

Here we present the non-equivalent classical solutions of the induced 2d-gravity (1.2) on the cylinder (for a detailed discussion see reference [13]):

\[ ds^2 = \frac{4}{|\lambda|^2} \left( \frac{r}{2\pi} \right)^2 \frac{e^{\frac{r}{t}}}{(1 - s e^{\frac{r}{t}})^2} (-dt^2 + dx^2) , \]  
\[ \Phi = \ln \alpha \left( \frac{1 - s e^{\frac{r}{t}}}{4 \sinh \frac{r}{4\pi}} \right)^2 , \]  

where \( \alpha \) is an arbitrary positive constant. The solutions (2.17) of the Liouville equation involve the hyperbolic and parabolic monodromy matrices only. The elliptic monodromy matrices are forbidden by the additional equations of motion. When the cosmological constant is negative only the hyperbolic monodromies (\( e^r 0 \\
0 e^{-r} \)) are allowed and, for \( \lambda^2 > 0 \), we also have the parabolic solution (\( 1 b \\
0 1 \)) (independent of the ‘b’ parameter) obtained as the limit \( r \to 0 \) of (2.17), (2.18),

\[ ds^2 = \frac{1}{2|\lambda|^2} \frac{1}{t^2} (-dt^2 + dx^2) , \]  
\[ \Phi = \log \alpha \frac{t^2}{\pi^2} . \]  

The symplectic structure of the model reads as [13]

\[ \omega = 4\delta \ln \alpha \delta r . \]  

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As a result of the above we can conclude that both models have the same covariant phase space. It is the cotangent bundle of two disconnected $\mathbb{R}^+(\mathbb{R})$ spaces for the induced 2d-gravity when $\lambda^2 < 0$ ($\lambda^2 > 0$) and the pure gravity CGHS-model if $\lambda^2 > 0$ ($\lambda^2 < 0$). The two sectors correspond to whether the dilaton field is expanding or contracting. In quantizing these symplectic manifolds the quantum states are represented by square integrable functions depending on the “configuration” constant of motion. We shall see in the next section how this prediction for the Hilbert space is consistent with the quantization coming from the Wheeler-DeWitt equation.

3 The Wheeler-DeWitt equation

3.1 Induced 2d-gravity

To study the 2d-dilaton gravity from a canonical point of view we first present the ADM formulation [16]. We parametrize the two-dimensional metric as

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N_1 N_1 & N_1 \\ N_1 & a^2 \end{pmatrix},$$

where $N$ and $N^1$ are the lapse and shift functions and $a^2$ plays the role of the “spatial metric”. Using the two-dimensional identity $\sqrt{-g}R = -2\partial_t(aK) + 2(a(KN^1 - a^{-2}N''))'$, where $K$ is the extrinsic curvature scalar ($K = \frac{1}{a^2N}(N_{1|1} - a\dot{a})$), it is straightforward to rewrite (1.2) in the hamiltonian form (see [17] for an earlier study):

$$S = \int d^2x (\pi_a \dot{a} + \pi_\Phi \dot{\Phi} - NC - N^1C_1).$$

The canonical momenta are

$$\pi_a = \frac{4}{N} (\Phi' N^1 - \dot{\Phi}),$$

$$\pi_\Phi = \frac{2a}{N} (\Phi' N^1 - \dot{\Phi}) + \frac{4}{N} ((aN^1)' - \dot{a}),$$

and the functions

$$C_1 = \Phi' \pi_\Phi - \pi_a a,$$

$$\mathcal{C} = \frac{1}{16} a \pi_a^2 - \frac{1}{4} \pi_a \pi_\Phi - 4a\lambda^2 - \frac{1}{a} \Phi'^2 + 4(a\Phi)'$$

are the supermomentum and hamiltonian constraints respectively.

To solve the theory we shall first reduce it removing the non-dynamical variables by gauge fixing. The canonical form of the model (3.23)-(3.27) suggests to choose the “internal” time variable as (for a review on the problem of time see [18])

$$T(\Phi, a) = \chi(t),$$

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where $\chi$ is an arbitrary function. Furthermore, we can also exploit the $\Sigma = S^1$-diffeomorphism invariance of the theory to fix the space coordinate in such a way that

$$a = a(t). \quad (3.29)$$

The above gauge conditions together with the constraint (3.26) imply that

$$\Phi = \Phi(t), \quad (3.30)$$

$$\pi_a = \pi_a(t), \quad (3.31)$$

and then the hamiltonian constraint reduces to (see also [10])

$$C = \frac{1}{16} a\pi_a^2 - \frac{1}{4} \pi_a \pi_\Phi - 4a\lambda^2, \quad (3.32)$$

(now $\pi_\Phi$ stands for $\pi_\Phi(t) \equiv \int dx \pi_\Phi(t, x)$). If we choose particular functions $\chi$ and $T$ in (3.28) it is still possible to further reduce the theory. However we prefer to maintain the function $\chi$ undetermined and go to the quantum theory by using the reduced hamiltonian constraint (3.32).

The quantum mechanics of the model is governed by the operator version of the classical constraint $C = 0$, i.e., the Wheeler-DeWitt equation. Our task is now to propose a Wheeler-DeWitt equation incorporating the usual factor ordering ambiguities and the restriction $a(t) > 0$ of the scale variable. The latter point can be incorporated in a natural way by choosing the affine variables $a$, $p_a \equiv a\pi_a$ as the basic ones for the quantization [19]. The reason is that the operator $\hat{\pi}_a = -i\hbar \frac{\partial}{\partial a}$ fails to be self-adjoint in $L^2(\mathbb{R}^+, da)$, but the operators $\hat{a} = a$, $\hat{p}_a = -i\hbar a \frac{\partial}{\partial a}$ are self-adjoint in the space $L^2(\mathbb{R}^+, \frac{da}{a})$. In terms of the classical affine variables, the constraint $C = 0$ reads

$$C = \frac{1}{16a} p_a^2 - \frac{1}{4a} p_a \pi_\Phi - 4a\lambda^2, \quad (3.33)$$

and the quantum constraint $\hat{C}$ can be written as (self-adjoint with respect to the measure $\frac{da}{a} d\Phi$)

$$\hat{C} = \left[ -\frac{1}{16} \hat{a}^{\alpha+i\beta} \hat{p}_a \hat{a}^{-1-2\alpha} \hat{p}_a \hat{a}^{\alpha-i\beta} + \frac{1}{8} (\hat{a}^{\gamma+i\sigma} \hat{p}_a \hat{a}^{-\gamma-i\sigma-1} + \hat{a}^{\gamma+i\sigma-1} \hat{p}_a \hat{a}^{-\gamma-i\sigma}) \hat{\pi}_\Phi + 4\lambda^2 \hat{a} \right], \quad (3.34)$$

where $\alpha$, $\beta$, $\gamma$ and $\sigma$ are arbitrary factor-ordering parameters.

To solve the Wheeler-DeWitt equation, $\hat{C}\Psi = 0$, we shall make use of the classical constant of motion $\pi_\Phi$. It is interesting to identify $\pi_\Phi$ on the covariant phase space. Inserting the classical solutions (2.13) and (2.14) into (3.27) we can identify the constant of motion on the covariant phase space as

$$\pi_\Phi = -\frac{r}{\pi}. \quad (3.35)$$
Expanding the wave function $\Psi$ in $\hat{\pi}_\Phi$ eigenstates

$$\Psi = \int dqe^{iqa} \Psi_q(a),$$

(3.36)

the Wheeler-DeWitt equation separates, and the functions $\Psi_q$ satisfy

$$\left(\frac{d^2}{dz^2} + \frac{1}{z}(1 - 2\zeta) \frac{d}{dz} + \frac{1}{z^2}(\zeta^2 - \nu^2) + \frac{16\lambda^2}{\hbar^2}\right)\Psi_q(z) = 0,$$

(3.37)

where $z \equiv a$ and

$$\zeta = \frac{1}{2} \left(1 + 4\frac{iq}{\hbar} + 2i\gamma\right),$$

(3.38)

$$\nu^2 = \frac{1}{4} \left(1 - 16\frac{q^2}{\hbar^2} - 8\gamma^2 + 4\alpha(\alpha + 1) + 16\frac{q}{\hbar}(\sigma - \gamma)\right).$$

(3.39)

The solutions of the above equation are

$$\Psi_q(z) = z^\zeta Z_\nu(kz),$$

(3.40)

where

$$k = 4\frac{\lambda}{\hbar},$$

(3.41)

and $Z_\nu(kz)$ are ordinary (modified) Bessel functions for $\lambda^2 > 0$ ($\lambda^2 < 0$) with order $\nu$.

We should remark now that, irrespective of normal ordering prescription of (3.34), the orders of the Bessel functions entering in the general solutions of the Wheeler-DeWitt equation are always purely real or imaginary.

To finish our analysis of the Wheeler-DeWitt equation we shall relate it with the so-called reduced phase space approach. This will be of interest by itself but in our case we shall exploit it later to propose the wave function of the “ground state”. The main point is to reduce the theory to a genuine canonical form before going to the quantum theory. This requires choosing a particular function $T$ in the time-fixing condition $T(\Phi, a) = t$ (we have set $\chi = t$ in (3.28)). The momentum conjugate to $T$ expressed in terms of $T$ and the independent canonical variables – satisfying the constraint $C = 0$ – plays the role of the hamiltonian ($H_{\text{red}}$) for the reduced phase space.

There are various useful definitions of internal time in the literature (see the review [18]). In the light of expression (3.30) a choice of time which naturally arises in the induced 2d-gravity is given in terms of the dilaton. Other (spatially homogeneous) choices of time in 2d-dilaton gravity have been considered in [20]. The choice $\Phi = T$ yields to the following reduced hamiltonian ($H_{\text{red}} = -\pi_\Phi$)

$$H_{\text{red}} = -\frac{1}{4}p_a - 4\lambda^2\frac{a^2}{p_a},$$

(3.42)
which has the property of being time-independent. The reduced degrees of freedom could then be quantized in the conventional way through the Schrödinger equation. Ignoring operator ordering ambiguities, the Schrödinger equation can be converted, in our case, into the Wheeler-DeWitt equation.

3.2 CGHS-model

Using the “homogeneous” gauge (3.28), (3.29), and arguing as in the induced 2d-gravity case, we can reduce the pure gravity CGHS-model to the mechanical action

\[ \int dt \left[ \pi_{\Phi} \dot{\Phi} + \pi_a \dot{a} - NC \right], \]  

for which the Hamiltonian constraint is given by

\[ C = \frac{1}{4} a\pi_a^2 e^{2\Phi} + \frac{1}{4} \pi_{\Phi} \pi_a e^{2\Phi} - 4a\lambda^2 e^{-2\Phi}. \]  

(3.44)

For this model the analogue of the linear constant of motion (3.35) is now

\[ \frac{1}{2} \pi_{\Phi} + a\pi_a. \]  

(3.45)

On the covariant phase space we can identify (3.45) as

\[ \frac{M}{\pi \lambda^r}. \]  

(3.46)

The constant of motion (3.43) suggests to introduce a new set of “minisuperspace” variables

\[ T = 2\Phi + \ln a, \]  

(3.47)

\[ A = ae^{-2\Phi}. \]  

(3.48)

These variables seem to be the natural ones to study the model since lead, for the time-choice \( T = T \), to a time-independent reduced Hamiltonian (\( H_{\text{red}} = -\pi_T = -\frac{1}{2}(a\pi_a + \frac{1}{2} \pi_{\Phi}) \))

\[ H_{\text{red}} = \frac{1}{3} \left[ A\pi_A \pm 2A\sqrt{\pi_A^2 + 12\lambda^2} \right]. \]  

(3.49)

In terms of the new variables the function \( C \) is given by \((p_A \equiv A\pi_A)\)

\[ C = -\frac{1}{4A}p_A^2 + \frac{1}{2A}p_A\pi_T + \frac{3}{4A}\pi_T^2 - 4A\lambda^2, \]  

(3.50)

and the quantum counterpart of the classical constraint can be written as

\[ \left[ \frac{1}{4} \hat{A}^{\alpha+i\beta} \hat{p}_A \hat{A}^{\gamma-i\sigma-1} - \hat{A}^{\alpha-i\beta} \hat{p}_A \hat{A}^{\gamma+i\sigma} + \frac{1}{4} \left( \hat{A}^{\gamma+i\sigma} \hat{p}_A \hat{A}^{\gamma-i\sigma-1} + \hat{A}^{\gamma-i\sigma-1} \hat{p}_A \hat{A}^{\gamma+i\sigma} \right) \hat{\pi}_T + \frac{3}{4} \hat{A}^{-1} \hat{\pi}_T^2 - 4\hat{A}\lambda^2 \right] \Psi = (3.51) \]
where $\alpha, \beta, \gamma$ and $\sigma$ are arbitrary factor-ordering parameters. Expanding the wave functions in $\hat{\pi}_T$ eigenstates

$$\Psi = \int dq e^{i\hat{\pi}_T q} \Psi_q(A),$$

and plugging (3.52) into (3.51) we also find the differential equation (3.37) for the function $\Psi_q(z \equiv A)$, where the parameters $\zeta$ and $\nu^2$ are now given by

$$\zeta = \frac{1}{2} \left[ 1 + 2i \frac{q}{\hbar} + 2i\beta \right]$$

$$\nu^2 = \frac{1}{4} \left[ 1 - 16 \frac{q^2}{\hbar^2} + 4\alpha(\alpha + 1) + \frac{8}{\hbar^2}(\sigma - \beta) \right].$$

As for the induced 2d-gravity, the general solution of the above equation is given in terms of different kind of Bessel functions: $\Psi_q(z \equiv A)$, where $k$ is also given by (3.41) and $z \equiv A$ but now $Z_\nu(kz)$ is a modified (ordinary) Bessel function for $\lambda^2 > 0$ ($\lambda^2 < 0$).

4 Hilbert space

To construct a suitable Wheeler-DeWitt equation we have required hermiticity of the Wheeler-DeWitt operator $\hat{\mathcal{C}}$ with respect to the standard inner product

$$<\Psi_1|\Psi_2> = \int \frac{da}{a} d\Phi \Psi_1^* \Psi_2,$$

where $\frac{da}{a} d\Phi$ is the volume element of minisuperspace. In canonical quantum gravity the use of (4.55) as the physical scalar product for the solutions of the Wheeler-DeWitt equation is rather problematic [10]. In (4.55) we are indeed integrating over the reduced minisuperspace configuration variable and some sort of “time” variable as well. From the functional integral point of view of quantum gravity [8], the expression (4.55) would have a proper meaning in terms of the standard sum over all histories with the appropriate boundary conditions. Both schemes lead to expect (4.55) to be potentially divergent and, therefore, it is natural to define the proper inner product as the regularized version of (4.55). In the following we shall determine the Hilbert space of our pure gravity 2d-dilaton models (1.2) and (1.3) in terms of normalizable solutions of the Wheeler-DeWitt equation.

4.1 Induced 2d-gravity

According to section 3, the general solution to the Wheeler-DeWitt equation, for $\lambda^2 > 0$, can be expanded as $(Re(\nu) \geq 0, Im(\nu) \geq 0)$

$$\Psi = \int dq a^{\zeta} e^{i\frac{q}{\hbar} k} (A(q)J_\nu(kz) + B(q)N_\nu(kz)), $$

(4.56)
where \( \nu \) is given by (3.39) and \( A(q), B(q) \) are arbitrary complex functions. Now we want to evaluate the norm of the wave function (4.56) with respect to (4.55). It is not difficult to arrive at the following expression (\( x = k z \))

\[
< \Psi | \Psi > = \frac{2 \pi \hbar}{k} \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dx \left( |A(q)|^2 |J_\nu(x)|^2 + |B(q)|^2 |N_\nu(x)|^2 + A^*(q)B(q)J^*_\nu(x)N_\nu(x) + A(q)B^*(q)J_\nu(x)N^*_\nu(x) \right)
\]

(4.57)

Now the difficulty is the integration over \( x \). Due to the asymptotic behaviour of the Bessel functions for large \( x \) the integrals of (4.57) are divergent. We can regularize the formal scalar product (4.57) by substituting the integration measure \( dx \) in (4.57) by \( \frac{dx}{x^\epsilon} \) (\( \epsilon > 0 \)). The resulting expression can be easily evaluated and we obtain (\( \Theta \) is the step function)

\[
< \Psi | \Psi > \sim \frac{\hbar \Gamma(\epsilon)}{k^2} \int_{-\infty}^{+\infty} dq \left\{ [\cos(\pi \nu)(|A|^2 + |B|^2) + \sin(\pi \nu)(B^*A - A^*B)]\Theta(-\nu^2) + [|A|^2 + |B|^2]\Theta(\nu^2) \right\}.
\]

(4.58)

The divergent term \( \frac{\Gamma(\epsilon)}{k^2} \) is an overall factor and can be eliminated in the definition of the physical scalar product. The Hilbert space will be made out of normalizable wave functions respect to the regularized scalar product. We shall return to this point later.

Let us analyze now the case of negative cosmological constant. We can write the general solution to the Wheeler-DeWitt equation as in (4.56) but then the ordinary Bessel functions should be replaced by the modified ones \( I_\nu \). The asymptotic behaviour of the combination \( I_{-\nu} - I_\nu \) is different from the corresponding one of \( I_\nu \). In fact the functions \( K_\nu = \frac{\pi}{2 \sin(\nu \pi)} (I_{-\nu} - I_\nu) \) decay exponentially for large \( x \) but the functions \( I_\nu \), instead, grow exponentially. This means that, even if we regularize the formal scalar product as before, the solutions \( I_\nu + I_{-\nu} \) are not normalizable. Therefore the physical wave functions should be of the form

\[
\Psi = \int dq e^{\frac{i k q}{\lambda}} a^{\frac{1}{2} + 2i \frac{\nu}{h} C(q) K_\nu(4 \frac{\lambda a}{\hbar})},
\]

(4.59)

where \( K_\nu \) are the modified Hankel functions. The point now is to determine the range of variation of the \( \nu \) parameter. If the wave functions are required to be normalizable in the regularized scalar product we find, for \( \lambda^2 < 0 \), that

\[
\nu^2 < \frac{1}{4}.
\]

(4.60)

In the light of expression (3.39) the natural choice of factor ordering, leading to the values (4.60) as \( q \) varies over the real line, is given by

\[
\alpha = \beta = \gamma = \sigma = 0.
\]

(4.61)
Therefore the relation between “$q$” and the order $\nu^2$ is

$$\nu^2 = \frac{1}{4}(1 - 16q^2)\hbar^2.$$  

(4.62)

Note that the solution $q = 0$ (i.e. $|\nu| = \frac{1}{2}$) is not normalizable. This definite choice of factor ordering will allow to establish the equivalence between the Hilbert space of normalizable solutions to the Wheeler-DeWitt equation with the Hilbert space predicted by the covariant phase space quantization (see section (4.3)).

The consistence between both approaches will also emerge for $\lambda^2 > 0$ thus supporting the above choice of factor ordering. The elementary (normalized) solutions to the Wheeler-DeWitt equation are

$$\pi \frac{\nu}{2} J_\nu(x), \quad \nu \in [0, \frac{1}{2}],$$  

(4.63)

$$\pi \frac{\nu}{2} N_\nu(x), \quad \nu \in [0, \frac{1}{2}],$$  

(4.64)

$$\left(\frac{\pi}{\cosh(\pi \text{Im}(\nu))}\right)^{\frac{1}{2}} J_\nu, \quad \text{Re}(\nu) = 0,$$  

(4.65)

$$\left(\frac{\pi}{\cosh(\pi \text{Im}(\nu))}\right)^{\frac{1}{2}} N_\nu, \quad \text{Re}(\nu) = 0.$$  

(4.66)

4.2 CGHS-model

To provide the physical scalar product and the Hilbert space for the pure gravity CGHS-model we can repeat the same arguments as for the induced 2d-gravity. The volume element in the minisuperspace is now $dA dT$. For $\lambda^2 < 0$ we find that the Hilbert space is spanned by solutions of the form

$$\Psi(T, A) = \int dq C(q)e^{\frac{iqT}{\hbar} A^\frac{1}{2} + \frac{q^2}{2\lambda} K_\nu(4\frac{\lambda}{\hbar} A)},$$  

(4.67)

where $C(q)$ is a square integrable function vanishing at $q = 0$ and the relation between $q$ and $\nu$ is still given by (4.62). For $\lambda^2 > 0$ the Hilbert space can also be described by (4.63-4.66). The corresponding wave functions are the same as for the induced 2d-gravity with the replacement $\Phi \to T$, $a \to A$.

It is clear from the above discussion that a kind of quantum equivalence between the induced 2d-gravity and the pure gravity CGHS-model appears in the homogeneous gauge and the choices of time $T = \Phi$, $2\Phi + \ln a$, respectively. We shall see in section (4.3) that this equivalence can also be understood in terms of the covariant phase-space approach.

4.3 Relation with the covariant phase space

Now we want to discuss the relation between the quantization carried out in this paper and the covariant phase space quantization \cite{23, 24}. We know (section 4)
that the reduced phase spaces of the induced 2d-gravity and the CGHS-model are the same: $T^*\mathbb{R}^+ \cup T^*\mathbb{R}^+$ or $T^*\mathbb{R} \cup T^*\mathbb{R}$ depending on the sign of the cosmological constant. In general, the quantization of the reduced phase space (symplectic manifold) requires finding a complete commuting subset of phase space variables. Due to the cotangent nature of the above phase spaces the Hilbert space is given by the space of square integrable functions on the configuration space. Therefore, the Hilbert spaces is either $L^2(\mathbb{R}^+) \oplus L^2(\mathbb{R}^+)$ or $L^2(\mathbb{R}) \oplus L^2(\mathbb{R})$.

The point now is to see how the above Hilbert spaces are related with the ones we have obtained in solving the Wheeler-DeWitt equation. Let us first consider the solutions of the form (induced 2d-gravity, $\lambda^2 < 0$) or (CGHS-model, $\lambda^2 > 0$) where $H_\nu(x)$ functions are conveniently redefined as $(\frac{2k}{\hbar}(\frac{1}{2} + \nu)\Gamma(\frac{1}{2} - \nu))^\frac{1}{2} K_\nu(x)$, $(Re(\nu) < \frac{1}{2})$. The two sectors of the Hilbert space can be defined by the natural splitting $\Psi = \Psi^+ + \Psi^-$ given by

$$\Psi^+ = \left(\frac{2k}{\hbar}\Gamma(\frac{1}{2} + \nu)\Gamma(\frac{1}{2} - \nu)\right)^\frac{1}{2} \int_{0}^{\infty} dq a^\frac{1}{2} e^{i\nu T} C^+(q) K_\nu(4\frac{\lambda}{\hbar}a)$$

$$\Psi^- = \left(\frac{2k}{\hbar}\Gamma(\frac{1}{2} + \nu)\Gamma(\frac{1}{2} - \nu)\right)^\frac{1}{2} \int_{-\infty}^{0} dq a^\frac{1}{2} e^{i\nu T} C^-(q) K_\nu(4\frac{\lambda}{\hbar}a)$$

The scalar product turns out to be

$$<\Psi|\Psi > = \int_{0}^{\infty} dq |C^+(q)|^2 + \int_{-\infty}^{0} dq |C^-(q)|^2$$

(4.70)

showing that the Hilbert space obtained from the Wheeler-DeWitt equation coincides with the one derived from the covariant phase space: $\mathcal{H} = L^2(\mathbb{R}^+, dq) \oplus L^2(\mathbb{R}^+, dq)$.

We shall extend our discussion to the case of positive cosmological constant for the induced 2d-gravity (or, equivalently, for the CGHS-model when $\lambda^2 < 0$). The Hilbert space $\mathcal{H}$ can also be split into two orthogonal subspaces

$$\mathcal{H} = \mathcal{H}^+ \oplus \mathcal{H}^-$$

(4.71)

where $\mathcal{H}^+$, $\mathcal{H}^-$ are given by solutions of the form $(Re(\nu) \geq 0$, $Im(\nu) \geq 0$)

$$\Psi^+ = \left(\frac{k\pi}{\hbar}\right)^\frac{1}{2} \int_{-\infty}^{+\infty} dq a^\frac{1}{2} e^{i\nu T} A^+(q) \left[\Theta(q) J_\nu(x) + \Theta(-q) ((\cos(\pi Im(\nu)))^\frac{1}{2} N_\nu(x) + \sin(\pi Im(\nu)) (\cos(\pi Im(\nu))^{-\frac{1}{2}} J_\nu(x))\right]$$

(4.72)

$$\Psi^- = \left(\frac{k\pi}{\hbar}\right)^\frac{1}{2} \int_{-\infty}^{+\infty} dq a^\frac{1}{2} e^{i\nu T} A^-(q) \left[\Theta(-q) J_\nu(x) + \Theta(q) ((\cos(\pi Im(\nu)))^\frac{1}{2} N_\nu(x) + \sin(\pi Im(\nu)) (\cos(\pi Im(\nu))^{-\frac{1}{2}} J_\nu(x))\right]$$

(4.73)
Using standard integrals of Bessel functions it is not difficult to check that the (regularized) scalar product is then of the form ($\Psi = \Psi^+ + \Psi^-$)

$$<\Psi|\Psi> = \int_{-\infty}^{+\infty} dq \left( |A^+(q)|^2 + |A^-(q)|^2 \right). \quad (4.74)$$

We observe again that the Hilbert space (4.71-4.74) coincides with the one predicted by the covariant phase-space quantization:

$$\mathcal{H}^+ \approx \mathcal{H}^- \approx L^2(\mathbb{R}, dq). \quad (4.75)$$

5 Wave functions and classical space-time

In this section we would like to explore the physical meaning of the quantum wave functions emphasizing their relation with the classical solutions. First, we have to point out that the parameter $k = \frac{4\lambda}{\hbar}$ in the argument of the Bessel functions plays the role of the inverse of the Planck length for these models. Therefore, for large $x$ ($x \equiv k z \gg 1$) we could expect the quantum and classical solutions should be comparable. At the Planck scale ($x \equiv k z \sim 1$) the wave functions could predict an intrinsically non-classical behaviour of space-time.

5.1 Induced 2d-gravity

On general grounds we can observe that the different behaviour of the wave functions for large $x \equiv k z$ reflects appropriately the classical solutions (2.17), (2.18). If $\lambda^2 < 0$ the universe starts with zero size ($t = -\infty$), expands to a maximum radius $a_{\text{max}} = \frac{1}{2\pi \lambda}$, and then contracts to zero size ($t = +\infty$) (the proper time between $t = -\infty$ and $t = +\infty$ is finite). The wave functions (4.59) reflects this fact through the exponential decay of the modified Hankel functions $K_\nu$ for large $x$. We can also understand the absence of the classical parabolic solution $r = 0$ (i.e. $\pi \phi = 0$, see (3.33)) due to absence of normalizable solutions for $q = 0$ (i.e. $\nu = \pm \frac{1}{2}$). For $\lambda^2 > 0$, the classical parabolic solution does exist and their quantum counterpart corresponds to the unique normalizable solution with $q = 0$, i.e. $<J_\frac{1}{2}>$.

Now we wish to understand some quantitative aspects of the quantum behaviour, as the $x^{-\frac{1}{2}}$ decay of the wave functions for large $x$ and positive cosmological constant. To this end we shall compare the quantum probability distribution defined by the wave functions ($x^{-\frac{1}{2}}|\Psi|^2$) with the “classical” one $\mathcal{P}$ defined by random sampling in the time domain

$$\mathcal{P} \Delta x \approx \Delta T. \quad (5.76)$$

To properly establish such a comparison we have to make a choice of time to define $\mathcal{P}$ in (5.76). As we have already discussed, the natural time variable for the induced 2d-gravity is the dilaton $\Phi$. 

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For $\lambda^2 > 0$ the classical solutions can be rewritten as

$$a(\Phi) = \frac{|r|}{\lambda \pi} e^{-\Phi_0} \left(1 - e^{-\frac{2(\Phi - \Phi_0)}{2}}\right)^{\frac{1}{2}},$$

(5.77)

where

$$e^{\Phi_0} = \frac{1}{4} \frac{\alpha}{(\sinh \frac{4\pi}{4})^2}.$$  

(5.78)

The classical probability ($P \approx |\frac{d\Phi}{da}|$) is given by ($x_0 \neq 0$)

$$P_{\lambda^2 > 0}^\pm(x) = \frac{1}{x_0} \left| \frac{x}{x_0} \pm \left(1 + \left(\frac{x}{x_0}\right)^2\right)^{\frac{1}{2}} \right|,$$

(5.79)

where $x_0 = \frac{2|r|}{\pi}$ and the signs $\pm$ correspond to the double possibility of having an expanding or contracting dilaton field. We have normalized (5.79) (with the positive sign) using the same regularization procedure we have already introduced in section 4.

The asymptotic behaviour of $P_{\lambda^2 > 0}^\pm \left( \sim x^{-1} \right)$ fairly reproduce the decay of the probability distribution $x^{-1}|\Psi|^2$ for large $x$. For real order the wave functions $<J_\nu>$ and $<N_\nu>$ are oscillating for large $x$ and the local average of the asymptotic form of the (normalized) probability distribution is just $x^{-1}$. In Fig. I we have plotted the functions $P_{\lambda^2 > 0}^\pm(x)$ and the probability distribution of the elementary (delta function normalized) wave functions $<J_\nu>$ and $<N_\nu>$ for $\nu = \frac{1}{2}$ ($x_0 = \sqrt{3}$). For purely imaginary order $\nu = iy$, the probability distributions corresponding to states in $H^{(+)}$ and $H^{(-)}$ oscillate between $P^-$ and $P^+$.

For $x_0 \to 0$ both $P_{\lambda^2 > 0}^+$ and $P_{\lambda^2 > 0}^-$ go to the (non normalizable) classical distribution $P_{\lambda^2 > 0} \propto \frac{1}{x}$ but at the quantum level only the solution $<J_{\frac{1}{2}}>$ is normalizable.

### 5.2 CGHS-model

The comparison between the quantum and classical solutions for the CGHS-model can also be traced along the lines of the induced gravity model. For the CGHS-model the modified Bessel functions describe the quantum states for $\lambda^2 > 0$ instead of $\lambda^2 < 0$, as for the induced gravity. However the argument of the $K_\nu(x)$ functions is now $x \equiv kA = kae^{-2\Phi}$. If we consider the classical probability distribution $P_{\lambda^2 < 0}(x)$ defined in terms of the internal time variable $T = 2\Phi + \ln a$ we shall see that the quantum solutions also reflect appropriately their classical behaviour.
Figure I: Induced 2d-gravity. Probability distribution of the elementary (normalized) wave functions $\langle J_\nu(x) \rangle$ and $\langle N_\nu(x) \rangle$ for $\nu = \frac{1}{4}$. The dotted lines are the functions $P_{\lambda^2 > 0}(x)$.

For $\lambda^2 > 0$ the classical solutions can be rewritten as

$$Ae^{Te^2} - \left( \frac{r}{2\pi} \right)^2 \frac{M}{|\lambda|^3 \sqrt{A}} e^{Te^2} + \left( \frac{r}{2\pi} \right)^2 \frac{1}{\lambda^2} = 0,$$

and the classical probability is

$$P_{\lambda^2 > 0}(x) = \frac{2}{x_{\max}} \left| \frac{1}{x} \left( \sqrt{1 - \left( \frac{x}{x_{\max}} \right)^2} - 1 \right) \right| + \frac{2}{3x_{\max}} \left| \frac{1}{x} \left( \sqrt{1 - \left( \frac{x}{x_{\max}} \right)^2} + 1 \right) \right|,$$

where $x_{\max} = kA_{\max} = k\frac{M_{\gamma\lambda^2}}{4\pi} = 2\frac{\pi T}{\hbar}$.

The expression (5.81) has been normalized with the regularization procedure of section 4. In Fig. II we have plotted $P_{\lambda^2 > 0}(x)$ and the quantum distribution $|K_\nu(x)|^2$ for $x_{\max} = 16$ and $\nu = \frac{1}{2} \sqrt{1024 - 1_i}$. In the region $x \in [0, x_{\max}]$ the wave function is essentially oscillatory and it thus predicts classical behaviour. In the classically forbidden region $x > x_{\max}$ the wave function decays exponentially. However, for $\left( \frac{x_{\max}}{2} \right)^2 < 16$, the order $\nu$ is real and the probability distribution is peaked around the origin $x = 0$ without any oscillatory (classical) region. So that, the initial point $\nu = 0$ separates two different phases of the quantum solutions. We must also note that the absence of normalizable states for $q = 0$ (i.e. $\nu = \pm \frac{1}{2}$) reflects the absence of classical
solutions with vanishing $\pi_T(=\frac{Mr}{2\pi \lambda})$ (i.e. solutions with $M = 0$). A similar discussion can be given for the induced gravity (if $\lambda^2 < 0$) and yield to the same kind of results.

We can also consider the case of negative cosmological constant. The discussion also parallels the corresponding one of the induced gravity. We only want to mention that the quantum state $\langle J_{\frac{1}{2}} \rangle$ ($\nu = \frac{1}{2}$ and therefore $q = 0$) find its classical counterpart in the solution for which $\pi_T = \frac{Mr}{2\pi \lambda}$ vanishes. This singles out the solution $M = 0$, i.e. the linear vacuum dilaton (the analogue of the classical parabolic solution of the induced gravity).

5.3 Ground state

There are several proposals in the literature [8, 21] to single out the wave function representing the ground state of the gravitational field. Next we want to put forward an acceptable proposal for the ground state of the models considered in this paper. In studying the hamiltonian form of both the induced gravity and the CGHS-model we were led to a natural choice of the internal time variable $\mathcal{T}$ ($\mathcal{T} = \Phi$ for the induced gravity and $\mathcal{T} = 2\Phi + \ln a$ for the CGHS-model). These choices yield to time-independent effective hamiltonians $(-\pi_\Phi$ (3.42) and $-\pi_T$ (3.49), respectively) and also allow an acceptable physical interpretation of the quantum wave functions. Therefore, it could also be quite natural to define the grounds states as the state of minimum “internal” energy (i.e. the state of minimum $\hat{\pi}_T$-eigenvalue in our cases). Observe that the quantum probability
distributions are not modified by the change \( q \to -q \). In fact, this transformation is equivalent to the change \( t \to -t \) (which transforms an expanding dilaton solution into a contracting one) and, therefore, we should require the ground state be defined as the state of minimum \( |\hat{\pi}_T| \). In consequence the ground state of both models \( (\lambda^2 > 0 \text{ for the induced gravity and } \lambda^2 < 0 \text{ for the CGHS-model}) \) is represented by \( < J^\pm > \). If \( \lambda^2 > 0 \) (\( \lambda^2 < 0 \)) there is no ground state for the induced gravity (CGHS-model).

It is interesting to observe that this wave function vanishes at \( x = 0 \) and it is the one having the maximum rate at which the probability density tends to zero. This is reasonably in agreement with the “no-boundary” proposal [8].

As we have already mentioned the classical counterpart of the quantum solution \( \nu = \frac{1}{2} \) is represented by the parabolic solution and the linear vacuum dilaton. We would like to note that the quantum equivalence of the induced gravity and the CGHS-model through the transformation \( a \to A, \Phi \to T \) also applies to the classical counterpart of the ground state. The parabolic solution can be converted into the linear dilaton solutions and both solutions take the form \( a \propto e^{-\frac{1}{2} \Phi} \) (\( A \propto e^{-T} \)).

6 Final comments

In this paper we have carried out a novel canonical analysis of the induced 2d-gravity and the CGHS-model on a spatially closed universe. In the canonical quantization of generally covariant theories (and gauge theories in general) one can either first quantize and then impose the constraints or first solve the constraints and then quantize. In the present work we have followed a mixed procedure. First, we have classically solved the supermomentum constraint and fixed the spatial coordinate as well. The time coordinate has been partially fixed by introducing a generic (spatially homogeneous) gauge. This means a partial identification of time before quantization. The residual reparametrization of the time coordinate is incorporated at the quantum level by the (reduced) Wheeler-DeWitt equation. We have completed the identification of time after quantization, and it has played a major role in the space-time interpretation of the quantum wave functions.

A non-trivial test of the consistence of the present approach is the full agreement we have found between the quantization obtained via the Wheeler-DeWitt equation and the one predicted by the covariant phase-space. The Hilbert spaces determined by the geometric quantization of the symplectic manifolds of the corresponding spaces of non-equivalent classical solutions are equivalent to those spanned by the (normalizable) solutions of the Wheeler-DeWitt equation. Furthermore, the quantum equivalence between the induced 2d-gravity and the CGHS-model finds its classical counterpart also in their covariant phase-space, and suggests a gauge-theoretical formulation of the induced 2d-gravity mimicking
the Poincarè extended formulation of the CGHS-model [22].

Finally we want to point out that, when the cosmological constant vanishes, the wave functions coincide exactly with the small $x$ behaviour of the wave functions in the general case. Therefore, the wave functions are not normalizable and the approach breaks down. The role of the cosmological term is thus to modify the large $x$ behaviour of the wave functions leading then to normalizable states.

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