I discuss origin and possible experimental manifestations of charge-spin separation in 1D Luttinger and Luther-Emery liquids, the latter describing 1D Mott and Peierls insulators and superconductors. Emphasis is on photoemission where the spectral function generically shows two dispersing peaks associated with the collective charge and spin excitations, and on transport. I analyse the temperature dependences of the charge and spin conductivities of two organic conductors and conclude that most likely, charge-spin separation is not realized there and that they can be described as fluctuating Peierls insulators.

1 One-dimensional metals: the Luttinger liquid

One of the most exciting areas in the condensed-matter many-body problem is the search for metallic non-Fermi liquid states. This has been sparked by the unusual normal-state properties of the underdoped high-$T_c$ cuprates but definite confirmation of a non-Fermi liquid state there is still pending. This is so mainly because of theoretical difficulties to establish such a picture in two dimensions and provide a unifying framework which could tie together the multitude of puzzling experimental results. The situation is precisely opposite in one dimension (1D): here the Luttinger liquid has become a paradigm for a non-Fermi liquid metal but an unambiguous identification of a specific system as a Luttinger liquid has not been achieved yet. Unlike 2D, responsible for this is the wealth of quantitative theoretical information available to date which puts strong constraints to a Luttinger liquid interpretation of the otherwise unexplained exotic features of many experiments.

Shortly, a Luttinger liquid can be described as a metal whose elementary excitations are not fermionic quasi-particles but collective bosonic charge and spin fluctuations. This picture emerges because the coupling of quasi-particles to collective modes is extremely strong in 1D. Quantitatively, one can pin down two key factors to the breakdown of the Fermi liquid and measure them by four parameters which completely describe the low-energy physics in 1D: (i) Singular Peierls-type particle-hole fluctuations which interfere with superconducting fluctuations, and which lead to non-universal power-law scaling of many properties, measured by a coupling constant $K_p$: this parameter parametrizes all scaling laws between the various correlation functions. Generically, there is a similar coupling constant $K_\sigma$ for the spin fluctuations. However, spin-rotation invariance requires $K_\sigma = 1$ and consequently, this quantity is often omitted from discussion. (ii) Charge-spin separation,
generally measured by the velocities $v_{\rho,\sigma}$ of the collective modes. All gapless 1D quantum systems are conjectured to be Luttinger liquids, and no counterexample is known yet (in theory).

Candidates for Luttinger liquid correlations are quasi-1D conductors such as $(\text{TMTSF})_2\text{PF}_6$, i.e. Bechgaard salts, semiconductor quantum wires, and Quantum Hall edge states. Most experiments to date probed the power-law scaling, i.e. $K_{\rho}$. While generally consistent with theory, problems concerning the values of $K_{\rho}$ remain. An important problem therefore are diagnostic tests of Luttinger liquid behavior through charge-spin separation. Section 2 addresses this issue. Many experiments have been performed on materials which are 1D but cannot be classified as Luttinger liquids, and I will discuss the extent to which charge-spin separation remains visible there in Section 3. Section 4 discusses charge and spin transport in 1D organic conductors based on fluoranthene and perylene molecules, and to what extent transport can serve as a diagnostic tool for charge-spin separation.

2 Charge-spin separation in Luttinger liquids

To see the origin of charge-spin separation, consider a many-body system $H = T + V$ where $T$ is the kinetic and $V$ the potential energy. The ground state of $T$ alone is the filled Fermi sea. If $[T, V] \neq 0$, then the interaction can modify the ground state by virtual particle-hole excitations (this part of $V$ generates the nontrivial $K_{\rho} \neq 1$). If the whole $V$ or a part thereof commutes with $T$, this part cannot change the ground state which remains the Fermi sea, but it can lift degeneracies of excitations. In 3D, this effect is unimportant because the spacing of the eigenstates of $T$ scales with the inverse linear dimensions of the system while $V$ scales with inverse volume. However, both scales are equal in 1D (A more technical way to express this is to say that interactions are marginal in 1D.) and there is an important reconstruction of the spectrum of $H$ in 1D, leading to charge-spin separation. This can be seen rather easily in a toy problem where an additional particle is put into the second empty free-particle state above $E_F$ and then $V$ is turned on: there will be two excitation energies associated with the additional particle. In general, charge-spin separation is an asymptotic property of the low-energy subspace of $H$.

This toy problem suggests that single-particle properties are important tests of charge-spin separation. Indeed, the single-electron spectral function

$$\rho(q, \omega) = (-1/\pi) \text{Im} G(q + k_F, \omega + \mu)$$

do not have two peaks dispersing with the collective velocities $v_{\rho,\sigma}$. The peaks are power-law singularities governed by a single-particle exponent $\alpha = (1/4) \sum_{\nu=\rho,\sigma} (K_{\nu} + K_{\nu}^{-1} - 2)$. However, for reasons poorly understood to date, signatures of these spectral functions have not been found in photoemission studies of the Bechgaard salts.

3 1D Peierls and Mott insulators and superconductors

Photoemission experiments on the quasi-1D conductor $K_{0.3}\text{MoO}_3$ have shown two dispersing signals, which disperse with about the Fermi velocity resp. with $5v_F$, ...
Figure 1: Spectral function of the Luther-Emery model, assuming a spin gap $\Delta_\sigma$, for $q > 0$ (left). The main band then is at $\omega > 0$ and the shadow band at $\omega < 0$. The right part of the figure shows the dispersions of both signals in the main band.

suggesting collective character. Such a spectrum is incompatible both with a Fermi liquid picture and that of a fluctuating Peierls insulator (a standard model for the normal state of charge density wave [CDW] systems) but consistent with charge-spin separation ($v_\rho > v_\sigma \sim v_F$). The problem with a Luttinger liquid interpretation is that the material undergoes a CDW transition at 180 K while Luttinger liquids do not have dominant CDW correlations. As a corollary, one cannot go directly from a Luttinger liquid to a CDW phase. Coupling Luttinger liquids to phonons shows that CDW correlations can only dominate if a spin gap opens and the system is in the universality class of the Luther-Emery model with $K_\rho < 1$. Such a 1D spin-gapped Luttinger liquid is the precursor to CDW formation which would occur at lower temperature as a result of transverse coupling. In fact, inverting the Luttinger liquid conjecture, one would conclude that all 1D systems which are not paramagnetic metals must have a gap at least in one excitation channel (charge or spin) if not in both.

1D Mott insulators have a charge but no spin gap and also fall into the Luther-Emery universality class. Recently, photoemission results have been published for one such system, SrCuO$_2$, again with apparently two dispersing peaks. It is not clear, however, what spectral features of a Luttinger liquid carry over to the Luther-Emery model. Specifically: is charge-spin separation still visible there?

I have constructed the spectral function of the Luther-Emery model. There are three important differences with respect to those of a Luttinger liquid: (i) spectral weight is shifted to higher energies as a result of the opening of a charge or spin gap $\Delta_{\rho,\sigma}$; (ii) there are strong shadow bands translating the increased coherence which is at the origin of the gap opening; (iii) important changes occur in the lineshapes which I now discuss. In the main band $[\omega > (\omega)]$ for $q > (\omega)$ the number and structure of singularities depend on the relative magnitude of $v_\rho$ and $v_\sigma$ and on $K_\rho$. For a spin gap and $v_\rho > v_\sigma$ (as is the case for repulsive forward scattering and thus dominant CDW correlations), I find only one true singularity.
\[ \omega - \left( v_\rho^2 q^2 + \Delta_\rho^2 \right)^{1/2} \alpha^{-1/2} \] is defined in this way, since the notion of a \( K_\sigma \) does not make sense with a spin gap; the Luttinger liquid’s second (charge) singularity is shifted to \( \Delta_\rho + v_\rho q \) but, importantly, cut off to a finite maximum \( \sim \Delta_\rho^{(\alpha-1)/2} \), cf. Fig. 1. If \( v_\sigma > v_\rho \) (generally for attractive forward scattering i.e. a 1D superconductor), this becomes a true singularity \[ \omega - \Delta_\rho - v_\rho q \] while the other singularity persists.

In the case of a 1D Mott insulator, for the physically relevant case of repulsive forward scattering \( (v_\rho > v_\sigma) \), one finds two dispersing singularities in the main band with universal exponents \(-1/2\), Fig. 2. However, the two singularities only occur for wavenumbers larger than a critical one defined by the intersection of \( \varepsilon_\rho(q) = (\Delta_\rho^2 + v_\rho^2 q^2)^{1/2} \) and \( \varepsilon_\sigma(q) = \Delta_\rho + v_\sigma q \) (this also applies to the 1D superconductor: again, simply exchange all channel labels). A single singularity is observed for \( q \) below the intersection. Unlike the spin-gap case, the functional form of the shadow bands here is different: as in the Luttinger liquid, there is no emission at \(-\varepsilon_\sigma(q)\), and spectral weight sets in with an inverse-square-root singularity only at \(-\varepsilon_\rho(q)\). In addition, their intensity is suppressed by \( q \)-dependent weight factors. Numerical calculations are in qualitative agreement \[3\]. For \( v_\sigma > v_\rho \) as in the supersymmetric \( t-J \) model, there is only one singularity on the charge dispersion with exponent \(-1/2\), in agreement with a microscopic calculation \[4\]. Except in this latter case, charge-spin separation therefore is still visible in the spectral functions of 1D Peierls and Mott insulators. This is consistent with photoemission experiments where available \[9\] \[11\].

For \( K_{0.3}MoO_3 \) not only photoemission is described correctly by a Luther-Emery model but also much of the other experimental phenomenology available \[12\] \[14\]. This statement holds true despite the discussion by Gweon et al. \[9\]. The Luther-Emery model therefore might be a natural starting point for a description of the low-energy physics of some CDW materials such as \( K_{0.3}MoO_3 \). However, to obtain a
good description of the photoemission properties, one has to assume rather strong electron-electron interactions, and independent support for such a hypothesis will be required. Another problem is that the spin gap we would obtain from photoemission is much bigger than the one from susceptibility, and the reasons for this discrepancy are not understood. The magnitude of this (pseudo)gap, however, is also a problem for all competing attempts to describe these experiments.

4 Spin transport and charge spin separation in 1D conductors

With the ambiguities and the critical comments by others on the evidence in favor of charge-spin separation provided by photoemission, it is clearly of importance to search for alternative diagnostic tools, if possible less surface sensitive. I now take up an important suggestion by Q. Si¹⁵ that with charge-spin separation, and under conditions appropriate for the 2D high-$T_c$ superconductors, the charge and spin conductivities of an itinerant electron system would have different temperature dependences. Specifically, I ask if similar correlations can be found in 1D systems. We first look at available experiments.

Both conductivities have been measured by others¹⁶,¹⁷ in the quasi-1D CDW systems (FA)$_2$PF$_6$ and (PE)$_2$PF$_6$ (FA = fluoranthene, PE = perylene). These materials are distinguished by very narrow ESR lines. Using the Einstein relation $\sigma_{\text{spin}} = \chi D$ with $\chi$ the susceptibility and $D$ the spin diffusion constant, the spin conductivity can be derived from $D(T)$ measured by ESR. Using the meanwhile published data of Wokrina et al.¹⁷, the temperature dependences of the charge and spin conductivities, normalized to their values at 150 K (I have preferred this temperature for normalization to, say, room temperature because of the small error bars on the spin conductivity at 150 K), are obtained as given in Fig. 3. I have reported a similar analysis for (FA)$_2$PF$_6$ at a recent conference¹⁴. Clearly, within the error bars, charge and spin conductivity have the same temperature dependence.

Is this evidence against charge-spin separation? Not necessarily. The idea behind Si’s suggestion is that charge-spin separation, most fundamentally, is a property of the low-energy sector of the Hilbert space which takes up a product structure. It is then possible to have scattering processes active entirely within one of the two factor spaces. In this case, one obtains the different temperature dependences of charge and spin conductivities. However, this need not be so: the dissipation process may well couple charge and spin degrees of freedom despite charge-spin separation in the system. This possibility, of course, was known to Si and is relevant, e.g., for the process producing the subdominant temperature dependence in his gauge model discussion¹⁵. To illustrate the point, one can also look at different scattering processes for the excitations of a Luttinger liquid: (i) electron-electron scattering in systems with even commensurability ($\rho = N_{el}/N_{site} = r/s$ with $r,s$ integer, $s$ even) in general produces different temperature dependences, as in Si’s Luttinger example; (ii) with odd commensurability ($s$ odd), similar or dissimilar temperature dependences for charge and spin conductivities generally depend on details of the interactions (through the values of $K_\rho$); (iii) electron-phonon and electron-impurity scattering produce identical temperature dependences. For (FA)$_2$PF$_6$ and (PE)$_2$PF$_6$ we thus face the following alternatives: (1) no charge-spin separation;
Figure 3: Normalized charge (triangles) and spin conductivities (dots) of (perylene)$_2$PF$_6$ vs. temperature, from [17].

(2) transport dominated by a scattering process involving both charge and spin, independent of charge-spin separation; (3) both. I now show that most likely, (3) applies. This is different from Si’s situation: in his analysis, the known linear temperature dependence of the (charge) resistance of the high-$T_c$ copper oxides together with some additional information so strongly constrain their description that a powerful link between charge-spin separation and the temperature dependences of the charge and spin conductivities can be made there.

Here, the very similar temperature dependences of charge and spin conductivities strongly suggest similar $T$-dependence of both $D$ and $\chi$ for charge and spin. Else, differences in the $T$-dependences of charge and spin diffusion constants would have to be offset precisely by opposite differences in the susceptibilities which is un plausible (though impossible to exclude strictly). The spin diffusion constants provided by ESR generally have weak or no temperature dependence [16,17]. We then infer that the scattering is essentially elastic both for charges and spins, as we expect for electron-impurity scattering. Organic conductors generally being very pure, the most natural origin of the “impurities” are precursor fluctuations of the CDW phase formed above the Peierls temperature. Such fluctuations are particularly strong in 1D and have been described quantitatively by a Ginzburg-Landau theory, and their influence on the electronic properties is treated as correlated but static disorder [18]. One can then compute the charge and spin conductivities in this model using memory functions, and obtains $\sigma_{\text{charge}}(T) \propto \sigma_{\text{spin}}(T) \sim \xi(T)\langle \Psi^2 \rangle_T$ where $\xi$ is the temperature-dependent correlation length and $\langle \Psi^2 \rangle_T$ are the mean
squared fluctuations of the order parameter $\Psi$. These quantities can be calculated exactly for a Ginzburg-Landau model. For $T > T_P$, we systematically obtain a concave temperature dependence quite similar to that observed experimentally, Fig. 3. Within this model, the strict proportionality of charge and spin conductivities is due to its treatment of the CDW precursor fluctuations as correlated impurities. Moreover, its general neglect of electron-electron interactions from the outset eliminates any possibility of charge-spin separation.

One can also calculate the spectral functions of a fluctuating Peierls insulator, Fig. 4. While they have only one dispersing signal deep in the range of (un)occupied states, they are distinctly non-Fermi liquid in shape: they broaden as $(q, \omega) \to 0$ and develop strong shadow bands translating the incipient CDW order. Both for this model and for the Luttinger liquid, we are thus in a position to correlate transport and spectroscopy. Such a correlation in principle also exists in a Fermi liquid where transport is described by a Boltzmann equation and the spectral function ideally is dominated by a quasi-particle peak whose shape, in simple cases, is known.

Finally, it is clear that the model of a fluctuating Peierls insulator cannot apply to $K_{0.3}MoO_3$ (quite independent of what a measurement of spin conductivity would give): it cannot produce a conductivity which could describe the experimental variation $\sigma_{\text{charge}}(T) \sim T^{-1}$ observed for $T > T_P$. The difference between $K_{0.3}MoO_3$ on one side and (FA)$_2PF_6$ and (PE)$_2PF_6$ on the other is most likely due to the different electron-phonon coupling which could be comparable to the electronic interaction in $K_{0.3}MoO_3$ but much stronger in (FA)$_2(PF)_6$ and (PE)$_2PF_6$.

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