The Group of Parallel Transports in the Riemannian Space

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Abstract

We show that there is an infinite group of special automorphisms of the deformed group of diffeomorphisms, which describes parallel transports in Riemannian spaces of any variable curvature. Generators of translations of such group contain covariant derivatives, and structure functions - the curvature tensor.

Key words: deformed group of diffeomorphisms, parallel transports, curvature, covariant derivatives, Riemannian space

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1 Introduction

On the way of realization of the Klein’s Erlangen Program [1] for the geometrical structure
of a Riemannian space with arbitrary variable curvature in the paper [2] there is shown, that
Riemannian structure in a manifold $M$ may be naturally set by the deformed group of its
diffeomorphisms $T^g_{gH} = \text{Diff } M$. Information about geometrical structure is contained in
the multiplication law of the group $T^g_{gH}$, which sets the rule of parallel transports of vectors in the
tangent bundle $TM$ over $M$.

In this article we shall show that parallel transports of vectors in Riemannian spaces of any
variable curvature is described by an infinite deformed group $DT$ of special automorphisms of
the deformed group of diffeomorphisms $T^g_{gH}$, which sets this geometrical structure on manifold $M$. Here we construct this group and describe some of its properties. Specifically we show
that the covariant derivatives are among generators and the Riemann curvature tensor is among
structure functions of the group $DT$.

2 The Group of Deformations of the Generalized Gauge Groups

Let’s consider a local Lie group $G_M$ with parameters $\tilde{g}^\alpha$ (indices $\alpha, \beta, \gamma, \delta$) and the multiplication law $(\tilde{g} \cdot \tilde{g}')^\alpha = \tilde{\varphi}^\alpha(\tilde{g}, \tilde{g}')$, which acts (perhaps inefficiently) on a coordinate chart $U$ of the manifold $M$ with coordinates $x^\mu$ (indices $\mu, \nu, \pi, \rho, \sigma$) according to the formula $x'^\mu = \tilde{f}^\mu(x, \tilde{g})$. The local
infinite Lie group $G^g_M$ is parameterized by smooth functions $\tilde{g}^\alpha(x)$ which satisfy the condition

$$\det\{d_\nu \tilde{f}^\mu(x, \tilde{g}(x))\} \neq 0 \quad \forall x \in U, \quad (1)$$

where $d_\nu := d/dx^\nu$. The multiplication law in $G^g_M$ is determined with the help of functions
$\tilde{\varphi}^\alpha$ and $\tilde{f}^\mu$ which determine the multiplication law in the Lie group $G_M$ and its action on the
manifold $M$ by the formulae [3]:

$$(\tilde{g} \times \tilde{g}')^\alpha(x) = \tilde{\varphi}^\alpha(\tilde{g}(x), \tilde{g}'(x')), \quad (2)$$

$$x'^\mu = \tilde{f}^\mu(x, \tilde{g}(x)). \quad (3)$$

The formula (3) sets the action of $G^g_M$ on $M$.

Definition 1 The groups $G^g_M$, which are parameterized by smooth functions $\tilde{g}^\alpha(x)$ with property [1] and have multiplication law [2], [3] are called the generalized gauge groups.

Let’s pass from the group $G^g_M = \{\tilde{g}(x)\}$ to the group $G^g_{gH} = \{g(x)\}$ isomorphic to it in
accordance with the formula

$$g^\alpha(x) = H^a_{\alpha}(\tilde{g}(x)) \quad (4)$$
(Latin indices assume the same values as the corresponding Greek ones). The smooth maps $H_x : G_M \to G_M$ have the properties:

1) $H_x(0) = 0 \quad \forall x \in M$;

2) $\exists H_x^{-1}(g) : H_x^{-1}(H_x(g)) = g \quad \forall g \in G_M, \ x \in M$.

The multiplication law of the group $G^g_M$ is determined by its isomorphism to the group $G^g_M$ and the formulae (2) and (3):

$$ (g \ast g')(x) = \varphi^a(x, g(x), g'(x')) := H_x^a(\tilde{\varphi}(H_x^{-1}(g(x)), H_x^{-1}(g'(x')))), \quad (5) $$

$$ x'^{\mu} = f^{\mu}(x, g(x)) := \tilde{f}^{\mu}(x, H_x^{-1}(g(x))). \quad (6) $$

The formula (6) sets the action of $G^g_M$ on $M$.

**Definition 2** Transformations (4) between the groups $G^g_M$ and $G^H_M$ are called the deformations of generalized gauge groups and the groups $G^H_M$ are called the infinite (generalized gauge) deformed groups.

In the set $D = \{H_x\}$ of maps $H_x$ the multiplication law can be defined:

$$(H_1 \circ H_2)_x(g) := H_1x(H_2x(g)). \quad (7)$$

The set $D$ becomes a group according to this multiplication law.

**Definition 3** The maps $H_x : G_M \to G_M$ with properties 1H, 2H are called the deformation maps (functions $H_x^a(g)$ are called the deformation functions), the group $D = \{H_x\}$ of deformation maps with the multiplication law (7) - the group of deformations.

The functions $h(x)_{\alpha}^a := \partial_{\alpha}H_x^a(\tilde{g})|_{\tilde{g}=0}$, where $\partial_{\alpha} := \partial/\partial \tilde{g}^a$, are called deformation coefficients.

With the help of the coefficients of the expansion

$$ \varphi^a(x, g, g') = g^a + g'^a + \gamma(x)^{ab}g^b + \frac{1}{2} \rho(x)^{abc} g^a g^b g^c + \ldots \quad (8) $$

functions

$$ F(x)^a_{bc} := \gamma(x)^{ab} - \gamma(x)^{ac} \quad (9) $$

$$ R(x)^a_{abc} := \rho(x)^{abc} - \rho(x)^{acb} \quad (10) $$

are defined, which are their skew-symmetric parts. They are called the structure functions (versus the structure constants for ordinary Lie groups) and the curvature coefficients of the deformed group $G^H_M$ respectively.
Proposition 1  Generators commutators of the deformed group \( G^g_M \) are the linear combinations of generators with structure functions which are the coefficients \( \beta \):

\[
[X_a, X_b] = F(x)^c_{ab} X_c. \tag{11}
\]

The equation \( \text{(11)} \) generalize the Maurer-Cartan equation

\[
[q_a, q'_b] = \tilde{F}^\gamma_{a\beta} q'_\gamma \tag{12}
\]

for the infinite deformed groups \( G^g_M \), where \( \tilde{F}^\gamma_{a\beta} \) are the structure constants of the initial Lie group \( G_M \). The equation \( \text{(11)} \) is reduced to the equation \( \text{(12)} \) for the generalized gauge nondeformed group \( G^g_M \).

3  The Deformed Group of Diffeomorphisms and Geometrical Structure of Riemannian Space

Let \( G_M = T_M \), where \( T_M \) is the group of translations. In this case \( \tilde{t} \cdot \tilde{t}' = \tilde{t} + \tilde{t}' \) and \( x'^\mu = x^\mu + \tilde{t}^\mu \). The group \( T^g_M \) is parameterized by the functions \( \tilde{t}^\mu(x) \), which satisfy the condition \( \det \{ \delta^\mu_{\nu} + \partial_{\nu} \tilde{t}^\mu(x) \} \neq 0, \forall x \in M \). The multiplication law in \( T^g_M \) is

\[
(\tilde{t} \cdot \tilde{t}')^\mu(x) = \tilde{t}^\mu(x) + \tilde{t}'^\mu(x'), \tag{13}
\]

\[
x'^\mu = x^\mu + \tilde{t}^\mu(x), \tag{14}
\]

where \( \text{(14)} \) determines the action of \( T^g_M \) on \( M \). The multiplication law indicates that \( T^g_M \) is the group of diffeomorphisms \( Diff M \) in additive parametrization. The generators of the \( T^g_M \)-action \( \text{(14)} \) on \( M \) are simply derivatives \( \tilde{X}_\mu = \partial_\mu \) and this fact corresponds to the case of the flat space \( M \).

Suppose that the group \( T^g_M \) is deformed \( T^g_M \to T^{gH}_M : t^m(x) = H^m_x(\tilde{t}(x)) \). The multiplication law in \( T^{gH}_M \) is determined by the formulae:

\[
(t \circ t')^m(x) = \varphi^m(x, t(x), t'(x')) := H^m_x(H^{-1}_x(t(x)) + H^{-1}_{x'}(t'(x'))), \tag{15}
\]

\[
x'^\mu = f^\mu(x, t(x)) := x^\mu + H^{-1}_x(t(x)). \tag{16}
\]
Formula (16) sets the action of $T^g_M$ on $M$.

Let’s consider expansion

$$H^m_x(\tilde{t}) = h(x)^m_\mu(\tilde{t}^\mu + \frac{1}{2}\Gamma(x)^{\mu}_{\nu\rho}\tilde{t}^\nu\tilde{t}^\rho + \frac{1}{6}\Delta(x)^{\mu}_{\nu\rho\sigma}\tilde{t}^\nu\tilde{t}^\rho\tilde{t}^\sigma + \ldots).$$

(17)

Using of the formula (15), for coefficients of expansion (8) we can obtain

$$\gamma^m_{kn} = h^m_\mu(\Gamma^{\mu}_{kn} + h^n_\nu\partial_\nu h^m_\mu),$$

(18)

$$\rho^m_{kn} = h^m_\mu(\Delta^{\mu}_{kn} - \Gamma^s_{ns}\Gamma^{\mu}_{kl} - h^n_\nu\Gamma^{\mu}_{\kappa\lambda}h^k_{\lambda}h^l_{\mu}).$$

(19)

So formulae (9) and (10) for the structure functions and the curvature coefficients of deformed group $T^g_M$ yield

$$F^n_{\mu\nu} = -\partial_\mu h^n_\nu - \partial_\nu h^n_\mu,$$

(20)

$$R^\mu_{\lambda\kappa\nu} = \partial_\kappa\Gamma^\mu_{\lambda\nu} - \partial_\nu\Gamma^\mu_{\kappa\lambda} + \Gamma^\mu_{\kappa\sigma}\Gamma^\sigma_{\nu\lambda} - \Gamma^\mu_{\kappa\lambda}\Gamma^\sigma_{\nu\sigma}.$$  

(21)

In this formulae matrix $h^m_\mu$ and reciprocal to it matrix $h^m_\mu$ we use for changing Greek indices to Latin (and vice versa).

Formulae (20) and (21) show that groups $T^g_M$ contain the information about the geometrical structure of the space $M$ where they act. The generators $X_k = h^\mu_\nu\partial_\nu$ of the $T^g_M$ action (16) on $M$ can be treated as affine frames. Structure functions $F^n_{\mu\nu}$ differ from the anholonomity coefficients only by the factor $-1/2$.

Let us write the multiplication law of the group $T^g_M$ (15) for the infinitesimal second factor:

$$(t * \tau)^m(x) = t^m(x) + \lambda(x, t(x))^{m}_{n} \, \tau^n(x'),$$

(22)

where $\lambda(x, t)^m_n := \partial_\nu \varphi^m(x, t, t') \, |_{t'=0}$. Formula (22) gives the rule of the addition of vectors, which are set in different points $x$ and $x'$ or the rule of the parallel transport of the vector field $\tau$ from point $x'$ to point $x$:

$$\tau^n_m(x) = \lambda(x, t(x))^{m}_{n} \, \tau^n(x'),$$

(23)

or in the coordinate basis

$$\tau^\mu_{[\|} = \partial_\nu H^\mu_{\nu(\tilde{t})}\tau^{\nu}(x + \tilde{t})$$

(24)

where $\partial_\nu := \partial/\partial\tilde{t}^\nu$. This formula determines the covariant derivative

$$\nabla^\nu_\nu \tau^\mu(x) = \partial_\nu \tau^\mu(x) + \Gamma(x)^{\mu}_{\sigma\nu}\tau^\sigma(x),$$

(25)

where functions $\Gamma(x)^{\mu}_{\sigma\nu}$ setting the second order of the expansion (17) of deformation functions, play the role of coefficients of an affine connection in the coordinate basis. So, curvature coefficients (21) $R^\mu_{\lambda\kappa\nu}$ of the group $T^g_M$ coincide with the Riemann curvature tensor. The functions $\Gamma(x)^{\mu}_{\sigma\nu}$ are symmetric on the bottom indices, so torsion is equal to zero. Relationship (18) means...
that coefficients $γ^{m kn}$, which set the second order of the expansion \(\mathbf{8}\) of the multiplication law in the group $T_{gH}^M$, are coefficients of the affine connection in the affine basis $X_k$.

At the consecutive performance of the deformations $H_{2x}$ and $H_{1x}$ for the resulting deformation $H_{3x} = (H_1 \circ H_2)_x$ one can obtain:

$$h_{3\mu}^m = h_{1p}^m h_{2\mu}^p, \quad Γ_{3\muν}^m = Γ_{1p}^m h_{2μ}^p h_{2ν}^s + h_{1p}^m Γ_{2μν}^p.$$  \hspace{1cm} (26)

The last formula corresponds to the notion of deformations of connections \(\mathbf{4}\) and explains the term "deformations" in our case.

Suppose now, that generators $X_k = h_k^ν \partial_ν$ of the $T_{gH}^M$-action on $M$ \(\mathbf{16}\) are orthonormalized frames, i.e. $g(X_m, X_n) = η_{mn}$, where $η_{mn}$ - Euclidean metric, and infinitesimal parallel transports of vector fields lead only to their rotations, i.e.:

$$λ(x, t)^m_n ≈ δ_n^m + γ^{m kn} t^k ∈ SO(n).$$ \hspace{1cm} (27)

For coefficients $γ^{m kn}$ this gives

$$γ_{ksl} + γ_{lsk} = 0,$$ \hspace{1cm} (28)

(we fulfill lowering indices with the help of the metric: $γ_{inkl} := η_{mn}γ^{n kl}$). Together with formula \(\mathbf{15}\) the equation \(\mathbf{28}\) gives the condition of coordination of connection with the metric $g(\partial_ν, \partial_ρ) =: g_{νρ} = h^m_μ h^ν_ν η_{mn}$:

$$Γ_{μσ}^ρ + Γ_{νμσ} = ∂_σ g_{μν},$$ \hspace{1cm} (29)

With the condition of torsion vanishing, this yields that coefficients $Γ^ρ_{μν}$ may be written as

$$Γ^ρ_{μν} = \frac{1}{2} g^{ρσ}(∂_μ g_{νσ} + ∂_ν g_{μσ} − ∂_σ g_{μν}).$$ \hspace{1cm} (30)

So these coefficients coincide with the Christoffel symbols \{ρ^μ_ν\}.

**Proposition 2** The deformed group $T_{gH}^M$ of diffeomorphisms of coordinate chart $U ⊂ M$, which is obtained with satisfying the condition \(\mathbf{28}\) (or \(\mathbf{29}\)), acting on $U$ sets on it structure of a Riemannian space. Geometrical characteristics of the space $U$ (connection coefficients, curvature tensor etc.) are contained in the multiplication law of the group $T_{gH}^M$. Thus any Riemannian structure on $U ⊂ M$ may be set \(\mathbf{3}\).

This proposition realizes Klein’s Erlangen Program for the Riemannian space.

## 4 The Parallel Transports as Automorphisms of Deformed Groups of Diffeomorphisms

In the approach considered in previous section the vector fields in the curved Riemannian space were presented by infinitesimal parameters of the deformed group of diffeomorphisms. So, we can
consider the parallel transports of vector fields as certain automorphisms of the deformed group of diffeomorphisms $T^g_M$.

Let’s consider a transformation

$$\tau^m(x) := \tilde{H}^m_x((t*\tau)(x) - t(x)) = \tilde{H}^m_x(\varphi(x,t(x),\tau(x')) - t(x))$$

(31)

where

$$x'^\mu = f^\mu(x,t(x)) := x^\mu + H^{-1}_x(t(x)),$$

(32)

$\tilde{H}_x$ are variable deformation maps, and $H_x$ - the fixed deformation map, which was used for constructing the group $T^g_M$ and which defines geometrical characteristics of space where we intend to consider parallel transports. The inverse for (31) transformation is:

$$\tau^m(x) = \varphi^m(x,t^{-1}(x),t(\tilde{x}) + \tilde{H}^{-1}_x(\tau(\tilde{x}))),$$

(33)

where $\tilde{x}^\mu := f^\mu(x,t^{-1}(x))$.

Let’s pass from the group $T^g_M = \{\tau(x)\}$ to the group $T^g_M = \{\tau(x)\}$, isomorphic to it by the formula (31).

The multiplication law in the group $T^g_M$ is determined by its isomorphism (31) to the group $T^g_M$ and by the multiplication law (15), (16) in the group $T^g_M$. The group $T^g_M$ acts on the chart $U$ according to the formula

$$\tilde{x}^\mu = f^\mu(x,\tau(\tilde{x})) := f^\mu(x,\tau(x)) = f^\mu(\tilde{x},t(\tilde{x}) + \tilde{H}^{-1}_x(\tau(\tilde{x}))).$$

(34)

We should emphasize that the transformation of a point $x$ is determined by the value of functions $\tau(\tilde{x})$ (that parameterizes the group of parallel transports $T^g_M$) in another point $\tilde{x}$.

We shall name the group $T^g_M$ with infinitesimal parameter $\tau(x)$ an infinitesimal group $T^g_M$. For infinitesimal group $T^g_M$ from (31) and (34) follows:

$$\tau^m(x) = L(x)^m_p \lambda(x,t(x))_p^\alpha \tau^\alpha(x'),$$

(35)

$$h^\alpha_m(x) = h(x)^\alpha_k \lambda^{-1}(\tilde{x},t(\tilde{x}))^k_n L^{-1}(\tilde{x})^n_m,$$

(36)

where in this case $L(x)^m_p := \partial_p \tilde{H}^m_x(t)|_{t=0}$ and $h^\alpha_m(x) := \partial_m f^\mu(x,\tau)|_{\tau=0}$.

Transformations (35) (or (36)) form an infinite group $DT$ with parameters $g(x) = \{t(x), L(x)\}$ and multiplication law

$$(g * g')^m(x) = \varphi^m(x,t(x),t'(x')),$$

(37)

$$(g * g')^m_n(x) = L(x)^m_p \lambda(x,t(x))_p^\alpha \lambda'(x',t'(x'))^\alpha_t \lambda^{-1}(x,\varphi(x,t(x),t'(x')))^t_n,$$

(38)

where

$$x'^\mu = f^\mu(x,t(x))$$

(39)
and we consider that \( g^m(x) = t^m(x) \) and \( \partial^m(x) = L(x)_m^n \). This multiplication law shows, that group \( DT \) has the structure \( T^\otimes GL^g(n) \), where \( T^\otimes = \{ t(x), \lambda^{-1}(x, t(x)) \} \) and \( GL^g(n) = \{ 0, L(x) \} \) are its subgroups. Moreover, the group \( DT \) is the deformed generalized gauge group \( T \otimes GL(n) \) [3].

Formula (39) determines the action of the group \( DT \) on the chart \( U \subset M \), formula (35) on tangent vectors and (36) on affine frames \( X_m = h^\mu_m \partial_\mu \) over \( U \) respectively.

**Definition 4** The group \( DT = \{ (t(x), L(x)) \} \) of automorphisms (35) of the deformed group of diffeomorphisms \( T^\otimes \), with the multiplication law (37)-(39), which act on tangent vectors and affine frames over \( U \subset M \) according to formulae (35) and (36) respectively is called the **group of parallel transports** in the space \( U \).

Let’s consider structure functions \( F(x)_{ab} \) of the group of parallel transports \( DT \) on the condition if \( a = k, b = l \) (that corresponds to the translation parameters). For \( c = m \) from formula (37) we obtain

\[
F^m_{kl} = h^m_\mu (h^\nu_k \partial_\nu h^\mu_l - h^\nu_l \partial_\nu h^\mu_k)
\]

(40)

and for \( c = \frac{m}{n} \) from formula (38) -

\[
F^m_{nk} = -\gamma^m_{sn} F^s_{kl} + h^s_k \partial_\sigma \gamma^m_{ln} - h^s_l \partial_\sigma \gamma^m_{kn} + \gamma^m_{ks} \gamma^s_{ln} - \gamma^m_{ls} \gamma^s_{kn}.
\]

(41)

These equations show that the structure functions \( F^m_{kl} \) and \( F^m_{nk} \) of the group of parallel transports \( DT \) coincide with the structure functions \( R^m_{nkl} \) and curvature coefficients \( R^m_{nkl} \) of the deformed group of diffeomorphisms \( T^\otimes \), i.e. with anholonomity coefficients (with the factor \(-2\)) and the Riemann curvature tensor (written in the affine frame) respectively.

Generators \( X^\tau_a \) of the action (35) of the group \( DT \) on the tangent vectors for \( a = m \) are \((X^\tau_a)_l^k = \delta^k_l \delta^a_m \) and for \( a = m \) are

\[
(X^\tau_m)_l^k = X_m^\delta^k_l + \gamma^k_{ml}
\]

(42)

and coincide with covariant derivatives \( X_m = \nabla_m \) in the affine frame.

From the the generalized Maurer-Cartan equation (11) for the group of parallel transports \( DT \) follows the equation

\[
[\nabla_k, \nabla_l] = F^m_{kl} \nabla^m_n + R^m_{nkl},
\]

(43)

which is equivalent to the structure equations of the curved space of the torsion-free affine connection with the variable curvature \( R^m_n \) (if this connection satisfies condition (28) of the Riemannian space):

\[
d\omega^m = \omega^m \wedge \omega^m_n,
\]

(44)

\[
d\omega^m_n = \omega^k_n \wedge \omega^m_k + R^m_n,
\]

(45)

where \( \omega^m = h^m_\mu dx^\mu \), \( \omega^m_n = \gamma^m_\mu dx^\mu \) and \( R^m_n = \frac{1}{2} R^m_{n\mu\nu} dx^\mu \wedge dx^\nu \).
Formula (43) indicates that $R^{m}_{nkl} = 0$ is the necessary and sufficient condition that the set of translations $\{t(x), 1\}$ in the group $DT$ should form a subgroup. Gauge linear transformations $L(x)$ in the case of a curved Riemannian space are necessary for ensuring the group structure of the group $DT$.

Formula (36) describes motion of the mobile frame $X_{\parallel m} = h^\mu_n \partial_\mu$ at the transformations of parallel transports from the group $DT$. For infinitesimal translations (and finite linear transformation $L^m_n$) the formula (36) gives

$$X_{\parallel m} = \tilde{X}_m - \bar{t}^n \gamma^n_{\, sm} \tilde{X}_n,$$

(46)

where $\tilde{X}_m = L^{-1n}_m X_n$, $\bar{t}^n = L^s_n \, t^n$ and

$$\gamma^n_{\, sm} = L^n_l (\gamma^l_{\, rm} L^{-1r}_s L^{-1n}_m + L^{-1n}_s h^\sigma_n \partial_\sigma L^{-1m}_n).$$

(47)

Formula (46) can be used for the definition of covariant derivatives in the mobile frame terms:

$$\nabla_{X_s} X_{\parallel m} = \lim_{t^s \to 0} (X_m - X_{\parallel m})/t^s = \gamma^n_{\, sm} X_n.$$  

(48)

Let’s suppose that the group $T_{gH}^M$ is obtained with the fulfilling condition (28) (or (29)). In this case we can prove the next proposition.

**Proposition 3** Parallel transports of vector fields in curved Riemannian space are described by the group $DT$ of special automorphisms of the infinitesimal deformed group of diffeomorphisms $T_{gH}^M$. Translations generators of the group $DT$ are the covariant derivatives of vector fields, and the structure functions of the group $DT$ contain the curvature tensor.

The equations of structure of Riemannian space (44), (45) (Cartan equations) are the necessary and sufficient conditions of the group $DT$ existence.

The group $DT$, as well as the group $T_{gH}^M$, contains information about the structure of the Riemannian space on $U \subset M$. Generators and structure functions of group $DT$ contain this information. The structure of Riemannian space is set on $U$ at infinitesimal action of group $DT$ in the tangent bundle of space $U$ while for setting of the Riemannian structure on $U$ with the help of the group $T_{gH}^M$ it is necessary to consider its action on $U$, at least, up to the second order on translations $t$ inclusively.

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