Intense radiation from a relativistic electron rotating about a dielectric ball

M.L. Grigoryan*
Institute of Applied Problems in Physics
25 Hr. Nersessian Str., 375014 Yerevan, Armenia

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Abstract

The radiation from a relativistic electron uniformly rotating along an orbit in the equatorial plane of a dielectric ball was calculated taking into account the dielectric losses of energy and dispersion of electromagnetic oscillations inside the substance of ball. It was shown that due to the presence of ball the radiation from the particle at some harmonics may be several dozens of times more intense than that from the particle rotating in an infinite homogeneous (and transparent) dielectric. The generation of such a high power radiation is possible only at some particular values of the ratio of ball radius to that of electron orbit and when the Cherenkov condition for the ball material and the velocity of particle ”image” on the ball surface is met.

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1 Introduction

Due to such unique properties as the high intensity, high degree of collimation, and wide spectral range (see [1]-[6] and references therein) Synchrotron Radiation (SR) serves as an extraordinary research tool for advanced studies in both the fundamental and applied sciences. These applications motivate the importance of analyzing various controlling mechanisms of SR parameters. From this point of view it is of interest to study the influence of medium on SR.

The characteristics of high-energy electromagnetic processes in the presence of material are essentially changed by giving rise to new phenomena such as Cherenkov Radiation (CR) [7]-[9], transition radiation [10, 11], parametric X-ray radiation, channeling radiation etc. The operation of a number of devices intended for production of electromagnetic radiation is based on interactions of relativistic electrons with matter (see, e.g., [12]).

In [13] (see also [11, 9]) it was shown based on the consideration of SR from a charged particle circulating in a homogeneous medium, that the interference between SR and CR

*E-mail address: levonshg@web.am
had interesting consequences. New interesting phenomena occur in case of inhomogeneous media. A well-known example here is the transition radiation. In particular, the interfaces between media can be used for monitoring the flows of radiation emitted in various systems. In a series of papers initiated in [15]-[17] the cases of simplest geometry boundaries, namely, the boundaries with spherical [15, 16, 18, 19] and cylindrical [17] symmetries, have been investigated. As a result, nontrivial peculiarities of CR induced at such boundaries were revealed. E.g., in [18, 19] the spectral distribution of radiation intensity from a relativistic particle uniformly rotating about (or inside) a dielectric ball, in its equatorial plane was investigated. It was shown that under certain conditions the presence of ball leads to an interesting effect - at a definite frequency (harmonic) the rotating particle radiates energy greater by few orders of magnitude than that in a continuous and unbounded dielectric. However, in [18, 19] the phenomena of absorption and dispersion of electromagnetic waves were not taken into account. The present paper is meant to fill that gap.

It is organized as follows: in Section 2 the description of problem is given, in Section 3 the method of solution is described and the final analytical expression for intensity of radiation from a relativistic particle rotating about a ball is given. Numerical results are presented in Section 4 and the main results are summarized in the last section of paper.

2 The description of the problem

Now consider a relativistic particle (electron), which uniformly rotates in a magnetic field around a dielectric ball, in its equatorial plane. We shall restrict ourselves to the simplest case, when the space outside of the ball is empty. In the case under consideration the permittivity

\[
\varepsilon (r) = \varepsilon_b + (1 - \varepsilon_b) \Theta (r - r_b),
\]

where \(r_b\) is the ball radius, \(\varepsilon_b = \varepsilon_b' + i \varepsilon_b''\) is the permittivity of ball material (in general, it is a complex-valued quantity), and \(\Theta\) is a unit step function (the origin of a spherical coordinates system is located at the center of ball). The density of electrical current

\[
j(r, t) = \frac{ev}{r_e^2} e_r \delta(r - r_e) \delta(\theta - \frac{\pi}{2}) \delta(\varphi - \omega_0 t),
\]

where \(r_e\) is the electron orbit radius \((r_e > r_b)\), and \(v = r_e \omega_0\) is the linear velocity of electron, \(\theta = \pi/2\) corresponds to the equatorial plane of ball.

The uniform rotation of electron is accompanied by radiation at discrete frequencies (harmonics)

\[
\omega_k = k \omega_0,
\]

where \(k = 1; 2; 3; \ldots\). We assume that the electron braking due to the radiation is compensated by an external (e.g., electrical) force driving the particle to move uniformly around a circle. The intensity \(I_k\) of radiation averaged over a gyration period is determined by the expression

\[
I_k = \frac{c}{2\pi r \to \infty} \int \left| \nabla \times A(r, \omega_k) \right|^2 d\Omega
\]
The Fourier transform
\[ A(\mathbf{r}, \omega_k) = \frac{1}{T} \int_0^T A[\mathbf{r}, t] \exp(ik_0t) dt \] (5)
of the vector potential is determined by equation
\[ (\Delta + \frac{\omega_k^2}{c^2}) A = -\frac{1}{\varepsilon}(\nabla \varepsilon) \nabla \cdot A = -\frac{4\pi}{c} \mathbf{j} \] (6)
(see, e.g., [10], the permeability of substance being assumed to be 1) under Lorenz gauge condition:
\[ \nabla \cdot A - i\frac{\omega_k}{c} \psi = 0, \] (7)
where \( \psi(\mathbf{r}, \omega_k) \) is the time component of electromagnetic field 4-potential.

It is convenient to introduce a dimensionless quantity
\[ TI_k/\hbar \omega_k \equiv n_k, \] (8)
where \( TI_k \) is the energy radiated at frequency \( \omega_k \) during one period of electron rotation, and \( \hbar \omega_k \) is the energy of corresponding quantum of electromagnetic wave. As a result, the total energy \( W_T \) radiated in time \( T \) is determined by the expression
\[ W_T = \sum_{k=1}^{\infty} n_k \hbar \omega_k. \] (9)

If all space is filled with transparent material with real and constant \( \varepsilon \), then [9]
\[ n_k (\infty; v, \varepsilon) = \frac{n_0}{\beta \sqrt{\varepsilon}} \left[ 2\beta^2 J_2 (2k\beta) + (\beta^2 - 1) \int_0^{2k\beta} J_2 (x) dx \right], \] (10)
where
\[ n_0 = 2\pi e^2 / \hbar c \cong 0.0459, \quad \beta = v \sqrt{\varepsilon} / c, \] (11)
and \( J_k (x) \) is the Bessel function of integer order. In this formula the case of \( \varepsilon = 1 \) corresponds to synchrotron radiation in vacuum (see, e.g., [2, 25]).

We aim at the solution of equation ([9]) subject to condition that \( \varepsilon(r) \) is determined by equation ([11]). Then, using ([4]) and ([9]), we shall try to find the values of \( r_b \) and \( \varepsilon_b \) (parameters of the ball), at which the number \( n_k \) (ball; \( v, r_b/r_e, \varepsilon_b \)) of quanta generated by the rotating particle is maximum.

### 3 The method of calculations and the final formula

An arbitrary vector field \( \mathbf{S} \) can be expanded in terms of spherical vectors:
\[ \mathbf{S}(\mathbf{r}) = \sum_{\mu=1}^{3} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} S_{\mu}^{lm} (r) \mathbf{X}^{(\mu)}_{lm} (\Omega), \] (12)
where
\[ S_{\mu}^{lm} (r) = \int \mathbf{X}^{(\mu)\ast}_{lm} \cdot \mathbf{S}(\mathbf{r}) d\Omega, \] (13)
and $X_{lm}^{(\mu)}$ are spherical vectors of longitudinal ($\mu = 1$), electric ($\mu = 2$) and magnetic ($\mu = 3$) types [26]:

$$
X_{lm}^{(1)} = nY_{lm}(\Omega), \quad -l \leq m \leq l, \quad l = 0, 1, 2..., \quad n = r/r
$$

$$
X_{lm}^{(2)} = \frac{\nabla n Y_{lm}}{\sqrt{l(l+1)}}, \quad X_{lm}^{(3)} = \frac{n \times \nabla n Y_{lm}}{\sqrt{l(l+1)}}, \quad X_{00}^{(2)} = X_{00}^{(3)} = 0. \quad (14)
$$

The operator

$$
\nabla_n = r(\nabla - n\partial/r) = e_\theta \frac{\partial}{\partial \theta} + e_\varphi \frac{\partial}{\sin \theta \partial \varphi} \quad (15)
$$

acts on the $n$-dependent functions, and $Y_{lm}(\Omega)$ are spherical harmonics satisfying the equation

$$
\Delta_n Y_{lm} = -l(l+1)Y_{lm}, \quad (16)
$$

where $\Delta_n = \nabla_n \cdot \nabla_n$ is the angular part of Laplacian.

According to (12), the determination of $S(r) = A(r)$ reduces to finding the quantity $A_{\mu}^{lm}(r)$ that is independent of $\Omega = (\theta, \varphi)$ (for simplicity here and in what follows $\omega_k$ is omitted in $S(r, \omega_k)$ and analogous quantities). The calculation of $A_{\mu}^{lm}(r)$ is simplified due to the spherical symmetry of ball. Really,

$$
A_{\mu}^{lm}(r) = \sum_{\nu=1}^{3} \sum_{l_1=0}^{\infty} \sum_{m_1=-l_1}^{l_1} \int_0^{\infty} G_{\mu\nu}(r, r'; l, l_1 m_1) j_{\nu}^{l_1 m_1}(r') dr'. \quad (17)
$$

Such a writing of the solution of equation (6) is convenient because it permits to find a radiation field for an arbitrary current $j$, provided that the Green Function (GF) $G_{\mu\nu}$ of electromagnetic field is known. By direct substitution of (17), (12) and (1) into (6) one can make sure that for spherically symmetric medium

$$
G_{\mu\nu}(r, r'; l, l_1 m_1) = \delta_{ll_1} \delta_{mm_1} G_{\mu\nu}(r, r'; l) \quad (18)
$$

(GF is "dioganal" in $l$, $m$ and does not depend on $m$). Thus, in our case $G_{\mu\nu}$ is a 3x3 matrix depending only on $r$, $r'$ and $l$. This fact considerably simplifies the calculations. The problem is aggravated by the fact that in (6) the delta function

$$
\frac{\partial \varepsilon}{\partial r} = (1 - \varepsilon_b) \delta(1 - r_b) \quad (19)
$$

is entered. A rather simple and visual method of GF evaluation in similar situations may be obtained in [27] (see also [28]). The efficiency of method is due to the transition from the differential equation for GF to a relevant Lipmann-Schwinger type integral equation. Owing to the presence of delta function in (19), the Lipmann-Schwinger type integral equation is transformed into an easily solved algebraic equation, and the problem of evaluation is reduced to the solution of an ”auxiliary” differential equation without the delta function. In [15], GF of an electromagnetic field in the medium consisting of an arbitrary number of spherically symmetric layers with common center and various permittivities was determined by means of this same method. Also in [15] a corresponding formula for intensity of radiation from a charged particle arbitrarily moving in a layered spherically symmetric medium was derived. The developed method was verified for known particular
cases of the motion of charged particle (i) in a homogeneous medium and (ii) through a flat boundary between two homogeneous media (transition radiation). In [16, 18] the problem on radiation from a particle uniformly rotating about a dielectric ball in its equatorial plane is solved. In particular cases the derived analytical expressions coincide with the results known earlier [9, 25]. The radiation from an electron uniformly rotating along an orbit in the equatorial plane inside dielectric ball was studied in [19].

In the absence of absorption and ionization losses, the work of external force resisting the braking of particle motion should be equal to the radiated energy. The radiated energy was calculated also by means of this method [18, 19]. The coincidence of obtained results served as an indirect confirmation of the correctness of calculations.

Below we give only the final formula for the number of quanta generated from a relativistic particle during one revolution about a dielectric ball:

\[
    n_k \text{(ball; } v, x, \varepsilon_b) = \frac{2n_0}{k} \sum_{s=0}^{\infty} \left( |a_{kE}(s)|^2 + |a_{kH}(s)|^2 \right)
\]  

(20)

Here \(x \equiv r_b/r_e\), and

\[
    a_{kE}(s) = k b_l(E) P_l^k(0) \sqrt{(l - k)!/(l + 1)(2l + 1)(l + k)!}, \quad l = k + 2s
\]

\[
    a_{kH}(s) = b_l(H) \sqrt{(2l + 1)(l + 1)(l + k)!} dP_l^k(y) \left| \frac{dy}{y=0} \right., \quad l = k + 2s + 1
\]  

(21)

are dimensionless amplitudes describing the contributions of multipoles of electric \((E)\) and magnetic \((H)\) types respectively. In \(P_l^k(y)\) are the associated Legendre polynomials, and \(b_l(E), b_l(H)\) are the following factors depending on \(k, v, x\) and \(\varepsilon_b\):

\[
    b_l(H) = i u \left[ j_l(u) - h_l(u) \right] \frac{j_l(xu_b), j_l(xu)}{j_l(xu_b), h_l(xu)}_x, \quad u = kv/c, \quad u_b = kv\sqrt{\varepsilon_b}/c
\]

\[
    b_l(E) = (l + 1) b_{l-1}(H) - lb_{l+1}(H) + x^2 (1 - \varepsilon_b) \left[ j_{l-1}(xu_b) + j_{l+1}(xu_b) \right] \times
\]

\[
    \frac{[h_{l-1}(u) + h_{l+1}(u)]}{l(l+1)u_b j_l(xu_b)} \frac{l(l+1)u_b j_l(xu_b)}{l^2_{l-1} + (l+1) z_{l+1}^l},
\]  

(22)

where \(h_l(y) = j_l(y) + i n_l(y)\), and \(j_l(y), n_l(y)\) are spherical Bessel and Neumann functions respectively. In \(22\) we use the following notations

\[
    \{a(x\alpha), b(x\beta)\}_x \equiv a \frac{\partial b}{\partial x} - b \frac{\partial a}{\partial x}
\]

\[
    f_l^l(x) \equiv f_l(x) / \{j_l(xu_b), h_l(xu)\}_x
\]

\[
    z_{\nu}^l \equiv \frac{u_j\nu(xu_b) h_l(xu) \varepsilon_b - u_b j_l(xu_b) h_{\nu}(xu)}{u_j\nu(xu_b) h_l(xu) - u_b j_l(xu_b) h_{\nu}(xu)}
\]  

(23)

Remember that the space outside the ball is empty and \(x < 1\). The derivation of (20) is given in [16].

In case of \(x = 0\) and/or \(\varepsilon_b = 1\)

\[
    b_l(H) = i u j_l(u), \quad b_l(E) = i[(2l + 1)[u j_{l+1}(u) + j_l(u)],
\]  

(24)

and therefore, naturally, \(n_k\) does not depend on \(x\). Numerical calculations by means of formulas (20) and (10) give the same result in case of \(\varepsilon_b = \varepsilon = 1:\)

\[
    n_k \text{(ball; } \varepsilon_b = 1) = n_k \text{(vac; } \varepsilon = 1) = n_k \text{(vac)}.
\]  

(25)
4 Results of numerical calculations

Let us consider the radiation generated by electron at some harmonic \( \omega_k = k\omega_0 \), e.g., at \( k = 8 \).

Figure 1: The number \( n_k(x) \) of electromagnetic field quanta generated per one revolution of electron about a dielectric ball versus the ratio \( x = r_b/r_e \) of the ball radius to that of electron orbit. \( n_k(x) \) is calculated by formula (20). The electron energy \( E_e = 2MeV \), the radius of orbit \( r_e = 3.69cm \), the harmonic number \( k = 8 \). The radiation was emitted respectively at the wavelength of 3cm (in vacuum). The dielectric losses of energy inside the ball material (melted quartz) were taken into account. For explanations see the text.

Plotted in Fig. 1 along the axis of ordinates is the number \( n_8 \) of emitted quanta, and along the abscissa - the radius of ball, or to be more precise, - the ratio \( x = r_b/r_e \) of ball-to-electron orbit radii. The particle energy and radius of particle orbit are fixed at values \( E_e = 2MeV \) and \( r_e = 3.69cm \) respectively. For these values of \( E_e \) and \( r_e \), corresponding to the 8-th harmonic is the radiation at frequency \( \omega_8/2\pi = 10^{10}Hz \) and wavelength \( \lambda_8 = 3cm \) (in vacuum). The calculations were carried out by using formula (20) under the assumption that the ball is made of melted quartz taking into account the dielectric losses of energy and dispersion of electromagnetic waves inside the ball material:

\[
\varepsilon_b = \varepsilon_b' (\omega) + i\varepsilon_b'' (\omega) \tag{26}
\]

(for melted quartz \( \varepsilon_b' (\omega_8) = 3.78 \), and the dielectric loss angle tangent \( \varepsilon_b'' (\omega_8)/\varepsilon_b' (\omega_8) = 0.0001 \) [29, 30]).

According to (10)), the number of quanta emitted at the 8-th harmonic by electron with the same values of \( E_e \) and \( r_e \) at the rotation in a continuous, infinite and transparent medium with \( \varepsilon = 3.78 \) would have been \( n_8 (\infty) \approx 0.0274 \sim n_0 \) (see the horizontal dashed line in Fig. 1). It is easy to see [7, 9] that if the electron did not rotate, but moved rectilinearly with energy \( E_e = 2MeV \) in the same infinite medium, then during the period of time \( T = 2\pi/\omega_0 \) it would emit

\[
n_{\Delta \omega} (\infty) = (v/c - c/v\varepsilon) n_0 \approx 0.0318 \tag{27}
\]
quanta in narrow frequency band $\Delta \omega = \omega_0$. It is evident that $n_{\Delta \omega}(\infty) \sim n_8(\infty)$. In the absence of ball one has to substitute $x = 0$ (and/or $\varepsilon_b = 1$) in (20). However, in this case owing to (25) it is simpler to use formula (10) with $\varepsilon = 1$. As a result $n_8(\text{vac}) \cong 0.00475 \approx n_0/2k^{2/3}$, where $k = 8$.

According to data given in Fig. 1 $n_8(x) \sim n_8(\text{vac})$ practically for all $x$ except for $0.8 < x < 0.85$ and $0.95 < x < 1$. In each of these ranges there are peaks. In the second range the heights of peaks are greater than $n_8(\text{vac})$ and $n_8(\infty)$ by many times. For the highest peak

$$x^* = 0.9815, \quad n_8(\text{ball}; x^*) \cong 0.951, \quad n_8(\text{ball}; x^*)/n_8(\infty) \cong 35. \quad (28)$$

The corresponding value of ball radius $r_b = 3.62cm$ and, consequently, the distance between the rotating electron and the ball surface should be $r_e - r_b = 0.7mm$. Apparently,

$$n_8(\text{vac}) \ll n_{\Delta \omega}(\infty) \sim n_8(\infty) \ll n_8(\text{ball}; x^*). \quad (29)$$

Similar results are obtained for a series of other values of $k >> 1$, as well as for electrons with energies $1 \leq E_e \leq 5MeV$ and balls with $1 \leq \varepsilon'_b \leq 5$ and $\varepsilon''_b/\varepsilon'_b << 1$. In all these cases the following condition

$$v_s \sqrt{\varepsilon'_b/c} > 1 \quad (30)$$

is satisfied. Here $v_s \equiv x^*v = r_b\omega_0$. Hence, a high power radiation with $n_8(x^*) >> n_8(\infty)$ is possible only for particular values of $x = r_b/r_e$ and when the Cherenkov condition for velocity of particle ”image” on the ball surface and the real part of the permittivity of ball material is satisfied.

5 Conclusion

In the present paper the radiation from a relativistic electron at uniform rotation about a dielectric ball has been studied with due regard for dispersion and dielectric losses of energy inside the ball material. Here, in addition to the synchrotron radiation the electron may also generate Cherenkov radiation. Its appearance is attributed to the fact that the electromagnetic field coupled with electron partially penetrates the ball and rotates with the particle. In case of small distance between the relativistic particle and the ball surface, $r_e \approx r_b$, this field can propagate with a speed larger than the phase speed of light inside the ball material, and then Cherenkov radiation is to be generated.

The peculiarities in total radiation at different harmonics due to the influence of matter and radius of ball have been investigated theoretically. It was shown that in case of weak absorption ($\varepsilon''_b << \varepsilon'_b$) at some harmonics with $k >> 1$, the electron may generate $n_k \approx 1$ (see (28)) quanta of electromagnetic field during one rotation period. This value is more than 30-fold greater than the similar value of $n_k$ for an electron rotating in a continuous, infinite and transparent medium having the same permittivity as a real part of that for the ball material.

The emission of such a high power radiation is possible only when the ratio of ball radius to that of an electron orbit takes on a series of fixed values and the Cherenkov condition (30) for the speed of particle ”image” on the ball surface and the ball material is satisfied. New characteristic features and visual explanation of the phenomenon will be given in forthcoming papers.
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