The fundamental understanding of Hong-Ou-Mandel effects

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Abstract

Nonlocal quantum correlation has been the main subject of quantum mechanics over the last century. Over the last few decades, various quantum technologies have been developed and applied for quantum technologies. The Hong-Ou-Mandel (HOM) effect relates to the two-photon intensity correlation on a beam splitter, demonstrating the nonclassical photon bunching phenomenon. For quantum technologies, the HOM effect has been applied to quantum interface via Bell state measurements to test nonclassicality. Here, the fundamental understanding of HOM effect is conducted using a pure classical physics of a Mach-Zehnder interferometer (MZI) via frequency manipulations of a pair of coherent photons. In this approach, a definite phase relation between the paired photons is derived, in which the photon pairs satisfy the essential requirement of indistinguishability in quantum mechanics. The analytical solution of the MZI-based HOM model is derived and proved by numerical calculations. This paper intrigues a fundamental question on quantumness of the HOM effect.

1. INTRODUCTION

Nonclassical two-photon intensity correlation is the quintessence of quantum information science [1,2]. Since 1987 [3], the Hong-Ou-Mandel (HOM) effect has been intensively studied for the anticorrelation of photon bunching [3-12]. The nonclassical feature of photon bunching of the HOM effect occurs when two indistinguishable input photons are met on a beam splitter (BS). Based on the particle nature of quantum mechanics, the HOM effect has been explained by the BS-based destructive interference between probability amplitudes of two-photon interactions without any photon-phase information [13]. Based on this interpretation, independent light sources have also been used for the evidence of the HOM effect [5,9]. Obviously, this particle nature-based approach assumes the same phase between the paired photons without any reason [13]. In that sense, it is also obvious that the same phase between two independently paired photons in ref. [9] can also be satisfied in the cavity optics. Although the quantum operator-based HOM analysis in conventional quantum mechanics results in destructive interference between two basis products, such an understanding should involve coherence features of each photon with a specific relative phase between the paired photons due to the rule of thumb in quantum mechanics that a photon never interferes with others [14]. To understand the HOM effect equivalently, it is fair that the BS system should be analyzed coherently with phase information. This is the starting point of the wave nature-based approach for the HOM effect [15].

Recently, wave nature-based interpretations have been performed to understand the fundamental physics of the HOM effect in an interferometric system [15,16], and established a complete understanding on such quantum feature of photon bunching in the HOM effect [17]. Moreover, nonlocal quantum correlation between space-like separated paired photons [18,19] has also been explained with the same wave nature of photons [20]. Based on these new interpretations of quantum features, observations of the HOM effect using different colored lights [9] or independent emitters [5] can also be understood coherently without violating quantum mechanics [15-17]. For the coherence interpretation of the HOM effect, an essential condition of the relative phase between the paired photons has been driven [15,17]. Here, a coherent model of the HOM effect is proposed and analyzed for the fundamental physics of photon bunching using coherent photon pairs that randomly generated from an attenuated laser. To implement this idea, a Mach-Zehnder interferometer (MZI) is used whose two-mode input photon pair is manipulated via a pair of synchronized acousto-optic modulators (AOMs). Eventually, indistinguishability between individual photons in each output port is established, resulting in measurement randomness. This measurement randomness is, however, has nothing to do with definiteness of relative phase between paired photons. In other words, complementarity theory in quantum mechanics does not apply for a two-photon system, as discussed in EPR paradox [21-23].
2. THEORY

Figures 1 shows schematic of the proposed coherence model of the HOM effect using a commercial laser via AOMs and linear optics. Unlike conventional HOM schemes [1,3,4,6-8] using entangled photon pairs generated from the spontaneous parametric down conversion (SPDC) processes [24,25], Fig. 1 uses the wave nature of a photon as differently demonstrated in refs 5 and 9. In SPDC-entangled photon pairs, the signal and idler photons are symmetrically frequency-detuned across the line center according to phase matching conditions of the second-order (χ(2)) nonlinear optics (see also the Inset of Fig. 1) [24-26]. The stability of center frequency (f₀) of the SPDC entangled photon pairs heavily depends on the frequency stability of a pump laser, which is essential for the HOM effect [1]. Each spatially separated input path in the typical HOM scheme include both signal and idler photons with the same probability amplitudes to satisfy the definition of an entangled state, |Ψ⟩ = (|s⟩ₐ|id⟩ₖ + |id⟩ₐ|s⟩ₖ)/√2, where |s⟩ (|id⟩) corresponds to |Eₐ⟩ (|Eₖ⟩) in Fig. 1 [13]. On the contrary of entangled photon pairs from SPDC, coherently prepared photons from an attenuated laser (L) in Fig. 1 has no such frequency detuning relation of ±δfⱼ. Instead, the spectral distribution of L can be ultranarrow far less than MHz. For the individual output measurements in both detectors (D1 and D2) in Fig. 1, a pair of delay (τ) lines (DLs) is added, where the coherently manipulated photons’ bandwidth Δ is ~million times narrower compared with that of SPDC, but much wider the bandwidth of L. Here, the coherence manipulation of Δ by AOMs is the heart of the present proposal for basis randomness. Unlike ref. 15, the τ-dependent coincidence measurements of the paired output photons in Fig. 1 result in a broad HOM dip without wavelength-dependent optical fringes (discussed in Fig. 2). Regarding the HOM effect, the proposed wave nature-based interpretation has already shown the inherent phase shift of π/2 between entangled photons [17]. This inherent π/2 phase shift has also been observed in an entangled ion pair [27]. The specific phase relation between paired entangled photons has never been discussed or driven in the particle nature-based approach for the HOM effect [1,3-13].

Fig. 1. Schematic of a coherence version of the HOM effect. L: commercial laser, OA: optical attenuator, BS: nonpolarizing 50/50 beam splitter, PBS: polarizing beam splitter, AOM: acousto-optic modulator, Q: quarter-wave plate, M: mirror, DL: delay line, D: single photon detector. Inset: AOM generating spectral bandwidth. The AOMs are synchronized.
bandwidth $\Delta$ in opposite directions across the center frequency $f_0$ of the generated bandwidth (see the Inset). Thus, the coherently modified AOM output photons exactly correspond to the signal and idler photon relation, i.e., $\pm \delta f_j$ in the SPDC [21,22]. The AOM-generated output photons are to be coincidently measured for two-photon intensity correlation of the HOM effect. Here, only coincidently generated bunched photons generated by Poisson statistics should contribute to the HOM effect, where this pair must satisfy the opposite frequency detuning relation $(\pm \delta f_j)$ by AOM manipulations as shown in the Inset of Fig. 1. Like the SPDC-generated paired photons, the $\pm \delta f_j$ relation in the Inset of Fig. 1 is random but opposite in frequency detuning. According to Poisson statistics, the generation rate of this photon pairs is ~1% at the level of mean photon number $\langle n \rangle \sim 0.04$ [28].

In Fig. 1, the coherently paired output photons are analyzed for the HOM effect with the full bandwidth of $2\Delta$, where $2\Delta$ is due to the double-pass AOMs. Using the BS matrix representation [29], the following relation is straightforwardly obtained in Fig. 1:

$$
\begin{bmatrix}
E_c \\
E_d
\end{bmatrix}_j = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & i \\
i & 1
\end{bmatrix} \begin{bmatrix}
E_a \\
E_b
\end{bmatrix}_j = \frac{E_0}{\sqrt{2}} \begin{bmatrix}
e^{-i\eta e^{-i\Delta_j}} + ie^{i\Delta_j} \\
e^{i\eta e^{i\Delta_j}} + e^{-i\Delta_j}
\end{bmatrix},
$$

(1)

where $E_a = E_0 e^{i\eta e^{-i\Delta_j}}$, $E_b = E_0 e^{i\Delta_j}$, $\Delta_j = \delta f_j \tau$, and $\eta$ is the inherent phase shift assigned to $E_a$ with respect to $E_b$. The sign of this additional phase $\eta$ must be dependent on the sign of the detuning $\Delta_j$. Thus, the following relations are obtained:

$$
I_c(\tau) = I_0(1 - \sin(\eta + 2\Delta_j)).
$$

(2)

$$
I_d(\tau) = I_0(1 + \sin(\eta + 2\Delta_j)).
$$

(3)

From the fast sweeping mode of synchronized AOMs, the detuning $\Delta_j$ for each measured pair is random. Due to the opposite and synchronized sweeping modes between $E_a$ and $E_b$, the spectral positions of $E_a$ and $E_b$ are symmetrically definite, as shown in the Inset of Fig. 1(a). Equations (2) and (3) are for the first half (e.g., positive $\delta f_j$) of the spectral distribution in the Inset.

The second half of the spectral distribution corresponds to swapping of input photons due to the sign change in $\Delta_j$, i.e., $E_a \leftrightarrow E_b$:

$$
\begin{bmatrix}
E'_c \\
E'_d
\end{bmatrix}_j = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & i \\
i & 1
\end{bmatrix} \begin{bmatrix}
E'_a \\
E'_b
\end{bmatrix}_j = \frac{E_0}{\sqrt{2}} \begin{bmatrix}
e^{i\eta e^{i\Delta_j}} + ie^{-i\Delta_j} \\
e^{-i\eta e^{i\Delta_j}} + e^{i\Delta_j}
\end{bmatrix},
$$

(4)

where $E'_a = E_0 e^{i\eta e^{i\Delta_j}}$ and $E'_b = E_0 e^{-i\Delta_j}$. Here, the sign of the inherent phase shift $\eta$ is also changed for $E'_b$ due to the sign change in the frequency detuning. Thus, the following relations are obtained:

$$
I'_c(\tau) = I_0(1 + \sin(\eta + 2\Delta_j)).
$$

(5)

$$
I'_d(\tau) = I_0(1 - \sin(\eta + 2\Delta_j)).
$$

(6)

As a result, the mean values measured in both detectors for full spectral distribution of the photon pairs are as follows:

$$
\langle I_c(\tau) \rangle = \langle I_c(\tau) + I'_c(\tau) \rangle = I_0,
$$

(7)

$$
\langle I_d(\tau) \rangle = \langle I_d(\tau) + I'_d(\tau) \rangle = I_0.
$$

(8)

The most important conclusion in Eqs. (7) and (8) is the detuning-independent uniform local measurements, which is originated in the basis-randomness of the paired photons, as in entangled photon pairs [3,23]. This means that the input photon pair of $E_a$ and $E_b$ must have the same feature of basis randomness as the entangled pairs. The quintessence of this basis randomness is in the basis-product superposition, resulting in $|\Psi\rangle_{ul} = (|E_a\rangle_u|E_b\rangle_l + |E_b\rangle_u|E_a\rangle_l)/\sqrt{2}$. Here, the subscript $u$ ($l$) represents the upper (lower) path of the MZI in Fig. 1. In other words, such entanglement relation can be coherently prepared, as differently demonstrated [5,9].

The mean two-photon intensity correlation $R_{cd}(\tau)$ between two output photons in Fig. 1, however, shows a different result from the locally measured intensities in Eqs. (7) and (8):
\[ \langle R_{cd}(\tau) \rangle = \langle l_c(\tau)l_d(\tau) + l'_c(\tau)l'_d(\tau) \rangle = \langle l'_d \rangle \langle \cos^2(\eta + 2\Delta j) \rangle. \]  

Here, the basis randomness in each detector has nothing to do with the definite relative-phase relation between individually chosen (measured) paired photons via coincidence detection, i.e., \( |E_a\rangle_u |E_b\rangle_l \) or \( |E_b\rangle_u |E_a\rangle_l \) in \( |\Psi\rangle_{ul} \). To satisfy the photon bunching phenomenon of the HOM effect, Eq. (9) must be zero at \( \tau = 0 \). Due to \( \Delta_j(\tau = 0) = 0 \) for all \( j \)th photon pairs at coincidence detection, the inherent phase shift between the paired photons must be \( \eta = \pm \pi/2 \). This is the direct result of the wave nature-based HOM interpretation that cannot be driven by the particle nature of quantum mechanics \([13]\), where the \( \pi/2 \) phase difference between paired photons is an essential condition for the HOM effect \([15-17]\). So far, this inherent phase relation between paired photons has never been discussed, even though it has been observed \([27]\). For \( \Delta_j(\tau \neq 0) \gg \pi \) or \( |\tau| \gg 0 \), the cosine term in Eq. (9) becomes saturated into the classical lower bound of \( g^{(2)}(\tau) = 0.5 \) \([15]\). Thus, the coherence model of the HOM effect is successfully demonstrated for Fig. 1 using coherently manipulated synchronized AOMs, resulting in indistinguishable photon characteristics in each pair.

3. NUMERICAL CALCULATIONS

Figure 2 shows the numerical calculations for Eqs. (7)-(9). The AOM-induced spectral bandwidth is set to \( \Delta = 10^8 \) Hz, and the corresponding decoherence time is \( t_s = 10^{-8} \) s. Equations (1) and (4) are numerically calculated for both first-order and second-order intensity correlations with the spectral range of \(-2\Delta \leq \delta f_j \leq 2\Delta\). As shown in Fig. 2, the mean intensities of the individual output photons are shown to be uniform as expected, regardless of \( \tau \) and \( \eta \) (see the blue lines). For \( \eta = \pi/2 \), the mean coincidence detection \( \langle R_{cd} \rangle \) satisfies the HOM dip with \( \tau \)-dependent decay in both wings of the HOM dip, as shown in the red curve in Fig. 2(a). Unlike ref. 15, the disappearance of \( \lambda \)-dependent fringes is due to the basis randomness caused by AOM manipulations. This side-band feature of Fig. 2(a) resembles the typical HOM dip \([1,3]\). The condition of \( \eta = 0 \), however, shows perfect coherence optics of Poisson statistics (see the red curve in Fig. 2(b)). The condition of \( \eta = \pi/4 \) shows a uniform intensity of perfect individuality in classical physics (see the red line in Fig. 2(c)). Thus, it is clear that the second-order intensity correlation of the HOM effect critically depends on \( \eta \). This fact has never been
discussed or observed in conventional HOM effects [1,3-12]. In both sides of the HOM dip in Fig. 2(a), coherence wiggles are also demonstrated, as observed with SPDC photon pairs [6,10-12]. Thus, the quantum validity of the HOM effect in the coherence model of Fig. 1 is successfully demonstrated with numerical calculations for the analytical solutions.

Figure 2(d) is for individual intensity products of Fig. 2(a). The red-circle curve indicates a particularly detuned pair, whose detuning is indicated by arrows in Fig. 2(a). As shown, all intensity products are a square of cosine function, representing the origin of the HOM effect based on the coherence feature of individual photon pairs. In an ensemble average, the two-photon intensity correlation is gradually decreased even though individual correlations do not. Regardless of detuning $\delta f_j$, the two-photon intensity product is always zero for all pairs at coincidence, $\tau = 0$. Thus, the ensemble effect of independently measured intensity products has also been successfully analyzed for Fig. 2(a) with analytical solutions of Eqs. (2) and (3).

4. Conclusion

A coherence model of HOM effect was proposed, analyzed, and numerically demonstrated for the same quantum features measured in conventional HOM effects using entangled photon pairs. In this coherence HOM model, the basis randomness-induced uniform intensity was satisfied in each output port, whereas the photon bunching of phenomenon of HOM dip was satisfied in the second-order intensity correlation via a coincidence detection. For the coherently excited random bases, a synchronized and opposite frequency sweeping technique was applied for the double-pass AOM scheme. The solution of resultant HOM dip, i.e., photon bunching, was analytically driven, and a specific phase relation between the paired photons was found. Regardless of the definite phase relationship between individually paired photons, the basis randomness-caused indistinguishability in measurements was satisfied in each output port. The essential requirement of the inherent $\pi/2$ phase difference between the randomly paired photons resulted in the HOM dip via destructive quantum interference. Unlike the particle nature-based interpretation [13], this destructive interference was from the wave nature-based quantum mechanics. The analytical solution was also numerically demonstrated for both basis randomness in the first-order and the HOM dip in the second-order of intensity correlation. The novelty of the present paper is the first complete model of the coherent photon-based HOM effect, where the measurement result cannot be distinguished from that of entangled photon-based one. As a result, the HOM effect itself cannot be either a definite test tool of entanglement or a nonclassical feature as already experimentally demonstrated via coherence manipulation of independent light sources [5,9].

Methods

For the symmetric $\pm \delta f_j$ in the Inset of Fig. 1, a pair of synchronized AOMs or electro-optic modulators can be used for fast frequency sweeping (scanning) in opposite directions. To satisfy the randomness between paired photons, the AOM frequency sweeping speed must be much faster than the acquisition rate of each detector. Due to the benefit of a narrow linewidth ($\sim$MHz) of a laser at $f_0$ in frequency, even a commercial bandwidth $\Delta$ ($\sim$100 MHz) of the AOMs does satisfy the HOM condition of symmetric detuning, where the corresponding path-length difference is $\sim$6 m. To fix the walk-off of diffracted lights from the sweeping AOMs, a double-pass AOM scheme is applied. The double-pass AOM scheme gives an additional benefit of bandwidth doubling to $2\Delta$.

For Fig. 2, Eqs. (2) and (3) are used for $-10^8 \leq \delta f \leq 10^8$ (Hz) and $-3\times10^{-8} \leq \tau \leq 3\times10^{-8}$ (s). The increment of $\delta f(\tau)$ is $2 \times 10^6$ ($2 \times 10^{-10}$). For Figs. 2(a)-(c), each individual intensity pair is calculated independently. For simplicity, a Gaussian distribution is not applied to the photon bandwidth. As shown in Fig. 2(d), individual intensity products show a square of cosine function, whose center value at $\tau = 0$ depends on $\eta$. Time-dependent decoherence is not applied for the product calculations due to small range of $\tau$ compared with coherence time.

Data availability

The data presented in the figures of this Article are available from the corresponding author upon reasonable request.
Code availability
All custom code used to support claims and analysis presented in this Article is available from the corresponding author upon reasonable request.

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Competing interests
The authors declare no competing interests.

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