Properties of semi-convection and convective overshooting for massive stars

C. Y. Ding$^{1,2,3}$ and Y. Li$^{1,2}$

$^1$Yunnan observatories, Chinese Academy of Sciences, Kunming 650011, China
$^2$Key Laboratory for the Structure and Evolution of Celestial Objects, Chinese Academy of Sciences, Kunming 650011, China
$^3$Graduate University of Chinese Academy of Sciences, Beijing 100049, China

Accepted 2013 November 20. Received 2013 November 6; in original form 2013 July 04

ABSTRACT

The properties of semi-convection and core convective overshooting of stars with masses of 15 and 30 M$_\odot$ are calculated in the present article. New methods are used to deal with semi-convection. Different entropy gradients are used when adopting the Schwarzschild and Ledoux methods, which are used to confine the convective boundary and calculate the turbulent quantities: \( \partial \Sigma / \partial r = -(c_s/H_P)(\nabla - \nabla_{ad}) \) when the Schwarzschild method is adopted and \( \partial \Sigma / \partial r = -(c_s/H_P)(\nabla - \nabla_{ad} - \nabla_\mu) \) when the Ledoux method is adopted. Core convective overshooting and semi-convection are treated as a whole and their development is found to present almost opposing tendencies: more intensive core convective overshooting leads to weaker semi-convection. The influence of different parameters and convection processing methods on the turbulent quantities is analysed in this article. Increasing the mixing-length parameter \( \alpha \) leads to more turbulent dynamic energy in the convective core and prolongs the overshooting distance but depresses the development of semi-convection. Adoption of the Ledoux method leads to overshooting extending further and semi-convection development being suppressed.

Key words: convection – diffusion – turbulence – stars: abundances – stars: evolution – stars: massive.

1 INTRODUCTION

Convection is commonly present in stars. The Schwarzschild criterion is valid in chemically homogeneous regions. When a chemical gradient is present, convection can only develop if the Ledoux criterion is violated. Convective overshooting and semi-convection (Taylor 1969) significantly affect the structure and evolution of massive stars. Overshooting beyond the convective core extends the mixing range of chemical elements, which supplies more nuclear fuel to central hydrogen burning. On the other hand, semi-convection affects the efficiency of chemical mixing beyond the convective core, resulting in important effects on future shell hydrogen burning.

According to the Schwarzschild criterion, the boundary of the convective core is fixed at the location where the radiative temperature gradient is equal to the adiabatic temperature gradient. However, a convective cell may have a velocity not equal to zero when it arrives at the edge of the convective core and will continue further into the stably stratified region. Such a phenomenon is referred to as ‘convective overshooting’. Outside the convective core, the turbulent velocity decelerates quickly to zero. This region is named the overshooting region. Overshooting beyond the convective core results in significant effects on the main-sequence evolution of massive stars (Chiosi & Maeder 1986; Caloi & Mazzitelli 1990; Chiosi, Bertellis & Bressan 1992; Chiosi 1998, 1999, 2007, 2009; Noels et al. 2010). Moreover, overshooting can also happen outside convective shells, but its effect has not been studied systematically. Convective overshooting should apparently be treated by non-local convection theories (Xiong 1985; Canuto 1992; Grossman et al. 1993; Canuto 2000). However, different non-local convection models usually result in different overshooting lengths. In many applications, overshooting beyond the convective core is implemented in a parametrized way: a fixed overshooting length is used and its value adjusted in order for the resulting stellar models to be in agreement with observations.

In massive stars, the stratification outside the convective core is sometimes unstable according to the Schwarzschild criterion but stable according to the Ledoux criterion. Such a region is the so-called semi-convection zone. Kato (1966) pointed out that the semi-convection zone is pulsationally unstable and suggested that the motion carries no heat but may result in incomplete mixing of the chemical elements on a thermal time-scale. Theoretical models (Spruit 1992) and two-dimensional (2D) numerical simulations (Merryfield 1995) result in quite different results for the mixing efficiency. The most popular method to treat such mixing in the...
semi-convection zone is the diffusion model (Langer, EL. Eid & Fricke 1985), which tends to recover the neutrality condition in the semi-convection zone.

Convective mixing (Stothers & CHin 2000), both in overshooting regions and in semi-convection zones, can be approximated by a diffusion process with appropriate estimation of the diffusion coefficient (Deng, Bressan & Chiosi 1996; Ventura et al. 1998). Due to the large scales of the motions and low viscosities of the stellar matter, convective motions in stars most probably run into turbulence. As a result, turbulent diffusivity dominates the convective mixing process. It is therefore desirable to investigate the turbulent properties outside the convective cores of massive stars. For example, Lai & Li (2011) recently studied the properties of turbulence below the bases of the convective envelopes for red giant branch (RGB)/asymptotic giant branch (AGB) stars by use of a turbulent convection model (TCM) proposed by Li & Yang (2007).

In this article, we apply a modified version of Li & Yang’s TCM to the convective cores of massive stars, in order to investigate the turbulent properties of the overshooting regions and semi-convection zones. In Section 2, we introduce the basic equations of the TCM used. Model parameters and the input physics of our stellar models are given in Section 3. Then we discuss the results obtained for a 15-M⊙ model in Section 4 and for a 30-M⊙ model in Section 5. We summarize our main conclusions in Section 6.

2 TURBULENCE MODELS FOR CONVECTIVE OVERSHOOTING AND SEMI-CONVERSION

The equations of the TCM we have solved are as follows:

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{k}{\varepsilon} \frac{u_i}{u_j} \frac{\partial k}{\partial r} \right) = \varepsilon + \frac{\beta g_r}{T} u_i u_j, \]

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{k}{\varepsilon} \frac{u_i}{u_j} \frac{\partial u_i}{\partial r} \right) = \frac{2}{3} \varepsilon + \frac{2 \beta g_r}{T} u_i u_j + C_k \left( \frac{\varepsilon}{k} - \frac{2}{3} \right), \]

\[ \frac{2}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{k}{\varepsilon} \frac{u_i}{u_j} \frac{\partial u_i T}{\partial r} \right) = \frac{T}{c_p} \frac{\partial}{\partial r} \left( \frac{\varepsilon}{k} \right) + C_T \left( \frac{\varepsilon}{k} + \frac{\lambda}{\rho c_p} \frac{\varepsilon^2}{k^3} \right) \frac{u_i T}{T^2} + \frac{\beta g_r}{T}, \]

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{k}{\varepsilon} \frac{u_i}{u_j} \frac{\partial T^2}{\partial r} \right) = \frac{2 T}{c_p} \frac{\partial}{\partial r} \left( \frac{\varepsilon}{k} \right) \frac{u_i T}{T^2} \]

\[ + 2 C_e \left( \frac{\varepsilon}{k} + \frac{\lambda}{\rho c_p} \frac{\varepsilon^2}{k^3} \right) \frac{T^2}{T^2}, \]

where \( \alpha \) is the so-called mixing-length parameter and \( H_p \) the local pressure scaleheight. Furthermore, the total heat flux \( F \) is composed of the radiative flux \( F_R \) and the convective flux \( F_C \):

\[ F = F_R + F_C = \frac{T \lambda}{H_p} \nabla + \rho c_p u_i u_j = \frac{T \lambda}{H_p} \nabla_{rad}, \]

where \( \nabla \) is the actual temperature gradient and \( \nabla_{rad} \) is the radiative temperature gradient. For more details please refer to Li & Yang (2007).

According to the MLT, the convection in the interior of massive stars can simply be handled as follows. First, the convective boundaries are found by applying either the Schwarzschild or the Ledoux criterion. Secondly, the temperature gradient is made equal to the adiabatic one; thirdly, the chemical elements are mixed homogeneously in the convection zones. Results are therefore different, due to the different criteria of convection to be used. According to the Schwarzschild criterion, the condition

\[ \nabla_{rad} > \nabla_{ad} \]

is used to determine the convection zones, where \( \nabla_{ad} \) is the adiabatic temperature gradient. According to the Ledoux criterion, however, the condition

\[ \nabla_{rad} > \nabla_{ad} + \nabla_{\mu} \]

is used to determine the convection zones, where

\[ \nabla_{\mu} = - \left( \frac{\partial \ln \rho}{\partial \ln T} \right)_{\mu, p} \left( \frac{\partial \ln \rho}{\partial \ln T} \right)_{\mu, p} \frac{\partial \ln \mu}{\partial \ln T}, \]

Moreover, convective overshooting and semi-convection are usually treated as two independent processes. In most massive star models, overshooting beyond the convective core is often adopted together with the Schwarzschild criterion, while semi-convection is commonly used when the Ledoux criterion is applied. However, neither the problem of convective overshooting nor that of semi-convection has been solved in basic fashion for massive star evolution (Langer 2012).

In contrast to the MLT, the TCM equations are based on the full equations of fluid hydrodynamics and have to be applied to the whole stellar interior. As a result, there is no need to determine the boundaries of the convective core and semi-convection zones. Instead, the averaged entropy gradient has to be used not only in the convective core but also in the overshooting regions and semi-convection zones. Specifically, in order to apply the TCM equations in the overshooting regions and semi-convection zones, we consider two different methods to treat the averaged entropy gradient. The first method involves using

\[ \frac{\partial T}{\partial r} = - \frac{c_p}{H_p} (\nabla - \nabla_{ad} - \nabla_{\mu}), \]

which will be referred to as the Ledoux method. It can be noticed that the chemical gradient \( \nabla_{\mu} \) is included in equation (11), in accordance with the Ledoux criterion. Inside the convective core, the element abundances are homogeneous and the chemical gradient \( \nabla_{\mu} \) is zero. Outside the convective core, however, the mean molecular weight decreases outward and the chemical gradient contributes to increasing the entropy gradient.

It can be seen, by letting the left-hand side of equation (3) be zero and inputing the resulting \( \vec{u}_i \vec{T} \) into equation (1), that the averaged entropy gradient contributes to either generation or dissipation of kinetic energy of turbulence, respectively, depending on whether its value is negative or positive, while the autocorrelation for the temperature fluctuation always contributes to the generation
of turbulent kinetic energy. In the overshooting region just outside the convective core, the radiative temperature gradient is less than the adiabatic temperature gradient, which results in the averaged entropy increasing outward. Therefore, the chemical gradient accelerates the increase of the averaged entropy gradient further. On the other hand, the radiative temperature gradient is larger than the adiabatic one in the semi-convection zone even further outside the convective boundary, but the chemical gradient can still keep the averaged entropy gradient positive. If the chemical gradient is no longer greater than the difference of radiative and adiabatic temperature gradients, the averaged entropy gradient will be negative and a full convection shell will develop. It should be noticed that the averaged entropy gradient not only determines the boundaries of the convective core and convective shells but also contributes to determining the properties of turbulence in the overshooting regions and semi-convection zones.

The second method is to use

$$\frac{\partial \pi}{\partial r} = - \frac{c_p}{H}(\nabla - \nabla_{ad}) \quad (12)$$

and it is referred to hereafter as the Schwarzschild method. This method is similar to those commonly used approaches that adopt the Schwarzschild criterion to determine the boundary of the convective core and supplement with convective overshooting beyond the convective core. The Schwarzschild method is used in the $\mu$-gradient zone, with the aim of comparison with the Ledoux method. Comparing results from these two methods, we can show more clearly the effects of the chemical gradient on the properties of the overshooting beyond the convective core and the properties of the semi-convection zones.

3 MODEL PARAMETERS AND INPUT PHYSICS

All of our stellar models were computed by a stellar evolution code h04.f, which was originally developed by Paczynski & Kozlowski and updated by Sienkiewicz (2004). Nuclear reaction rates are adopted from Bancel, Pinsonneault & Wasserburg (1995) and Harris et al. (1983). The OPAL equation of state (Rogers, Swensson & Iglesias 1996) is used. When log $T > 3.95$ (where $T$ is the temperature), the OPAL opacity tables (Rogers & Iglesias 1995; Iglesias & Rogers 1996) are used; otherwise the opacity tables from Alexander & Ferguson (1994) are used.

As the temperature gradients in massive stars are essentially independent of the kind of convection theory adopted, we apply the Schwarzschild criterion to determine the boundaries of the convection zones in either the stellar core or the stellar envelope and adopt the standard MLT to obtain the temperature gradient. The mixing-length parameter $\alpha$ is chosen to be 1.0 and 0.7, respectively.

When we have obtained the stellar structure model based on the above assumptions, we solve the TCM equations to obtain the turbulent kinetic energy in the stellar interior. In the present article, we choose small values of about $10^{-6}$ for various turbulent correlations to be the boundary conditions. There are eight parameters in the TCM model we have used: $C_s$ is the diffusion parameter for the turbulent kinetic energy; $C_{t1}$ is the diffusion parameter for the turbulent heat flux; $C_{\ell 1}$ is the diffusion parameter of the turbulent temperature fluctuation; $C_{\ell 1}$ controls the dissipation rate of the turbulent heat flux; $C_s$ controls the dissipation rate of the turbulent temperature fluctuation; $C_\ell$ measures the anisotropic degree of turbulence; $C_s$ is the diffusion parameter of turbulent mixing; $\alpha$ measures the typical length of turbulence. Their suggested values are discussed in detail by Zhang & Li (2012). In order to see the effects of model parameters on the turbulent properties, we set the values of the turbulent diffusion parameters $C_s$, $C_{t1}$ and $C_{\ell 1}$ to be equal and use a grid of values of 0.03, 0.05 and 0.07, respectively. Other parameters of the TCM are chosen as $C_T = 2.0, 3.0$ and $4.0$; $C_c = 1.00, 1.25$ and $1.50$; $C_\ell = 2.0, 2.5$ and $3.0$, respectively.

Convective mixing is assumed to be homogeneous in the convective core, as usual. However, in the overshooting region and semi-convection zones we approximate the convective mixing as a diffusion process, which is described by

$$\frac{\partial X_i}{\partial t} = \frac{\partial}{\partial m} \left[ (4\pi \rho r^2)^{-1} D_i \frac{\partial X_i}{\partial m} \right], \quad (13)$$

where $X_i$ is the abundance of species $i$ and $m$ is the mass interior to the radius $r$. In equation (13), the turbulent diffusivity is defined as

$$D_i = C_s \frac{\mu_i \mu_r}{\sqrt{k}} l, \quad (14)$$

where $C_s$ is a new model parameter and $l$ is the mixing length defined in equation (6). According to Zhang & Li (2012), we choose parameter $C_s$ to be $10^{-9}$, unless otherwise specified.

In our stellar models, the metal abundance $Z$ is fixed to be 0.02 and the initial hydrogen content $X$ is chosen to be 0.7. A stellar model is obtained as follows. The stellar structure model is computed in the usual way, using the MLT in the convective regions. The TCM equations are then solved with a given set of parameters to obtain the turbulent diffusivity. A diffusion equation is then solved to mix materials in the convective core and overshooting region, as well as in the possible semi-convection zones. We do not iterate this process, to be fully consistent with the stellar structure equations. Instead, we choose small time steps in the evolution computations to restrict the errors thus introduced. The stellar models are evolved from the zero-age main sequence until hydrogen is exhausted at the stellar centre.

4 RESULTS OF 15-M\(_\odot\) MODELS

We have calculated a series of stellar models of mass 15 M\(_\odot\) adopting the Schwarzschild method with mixing-length parameter $\alpha = 1.0$. As shown in Fig. 1, the radiative temperature gradient is always less than the adiabatic one outside the convective core, which means
that semi-convection does not develop in these stars (Mowlavi & Forestini 1994). Consequently, overshooting will be the only significant phenomenon of convection outside the central convective cores in these models.

4.1 Overshooting from the convective core

Profiles of the turbulent correlations computed using the Schwarzschild method are shown in Fig. 2 for four stellar models having a central hydrogen abundance that decreases gradually with time. It can be seen that, along with the coverage of the convective core successively shrinking back, the maximum of the turbulent velocity ($\sqrt{k}$) increases correspondingly near the stellar centre. On the other hand, the turbulent velocity does not drop to zero at the surface of the convective core but instead decays continuously in the overshooting region. This is the direct result of the turbulent diffusion described by the terms on the left-hand side of equations (1)–(4). Furthermore, it can be found with careful inspection that the decreasing speed of the turbulent velocity becomes smaller and smaller when going further and further into the overshooting region. In contrast, however, the radial kinetic energy of turbulence ($u_r u_r$) decays almost linearly in the overshooting region, which has been used in the diffusion coefficient of the convective mixing there.

It should be noted in Fig. 2 that the maximum of the autocorrelation for the temperature fluctuation ($\langle T' T' \rangle$) is not inside the convective core but located in the overshooting region just outside the convective boundary. This can be easily understood by inspecting equation (4). In the convective core, the temperature gradient is almost equal to the adiabatic one, which results in extremely small turbulent temperature fluctuations. Beyond the convective core, however, the temperature gradient is considerably smaller than the adiabatic one, which according to equation (4) contributes significantly to the generation of the turbulent temperature fluctuation. From equations (1) and (3), such a temperature fluctuation is always a production factor of turbulent kinetic energy. As a result, the decay of the turbulent velocity becomes slower in the overshooting region.

4.2 Comparisons between Schwarzschild and Ledoux methods

In Fig. 3, turbulent velocities and autocorrelations of the temperature fluctuations resulting from the Schwarzschild and Ledoux methods are compared. It can be seen that the two methods lead to exactly the same result in the convective core. Outside the surface of the convective core, however, the Ledoux method results in a decay of the turbulent velocity that is steeper at first then slower later, compared with the result of the Schwarzschild method. On the other hand, it is striking to notice that the autocorrelation of the temperature fluctuation resulting from the Ledoux method is about two orders of magnitude larger than that resulting from the Schwarzschild method.

These results can be understood by taking the effect of the chemical gradient into account in the overshooting region. The chemical gradient is shown in Fig. 4 for the above stellar model, as well as the difference in temperature gradients between the radiative and adiabatic cases. It can be seen that the chemical gradient is much larger than the difference of the radiative and adiabatic temperature gradients. According to equation (3), this fact will result directly in the depression of the turbulent velocity just beyond the surface of the convective core. On the other hand, the chemical gradient contributes dominantly, according to equation (4), to the generation

![Figure 2. Turbulent properties of 15-M⊙ models at different evolutionary stages are shown. Parameter values are the same as in Fig. 1. The Schwarzschild method is used to define the convective region and to calculate the turbulent quantities. Values on the horizontal axis are the mass fractions inside the considered points and those on the vertical axis are (from top to bottom) the turbulent dynamic energy ($u_ru'_r$) ($u_1/10^3$), the square root of the turbulent dynamic energy ($\sqrt{k}$) ($u_2/10^4$), the turbulent heat flux ($u_rT'$) ($u_3/10^5$) and the square temperature fluctuation ($T'T'$) ($u_4/10^5$). The four curves represent four different evolved phases: three of them are the same as in Fig. 1 and the last one shows the phase when the central hydrogen content is 0.02.)
Figure 3. Turbulent properties of 15-M⊙ models with different convection processing methods are shown. The evolution phase is when the core hydrogen abundance is 0.2. Parameter values are the same as in Fig. 1. The Schwarzschild and Ledoux methods are used to define the convective region and calculate the turbulent quantities, respectively, and are indicated by labels ‘sch’ and ‘led’. Values on the horizontal and vertical axes are the same as in Fig. 2, except that the units are different: $u_4/10^4$–’sch’, $u_6/10^6$–’led’.

of the temperature fluctuation in the overshooting region. As a result, the decay of the turbulent velocity slows down further in the overshooting region, where the temperature fluctuation reaches its maximum.

The turbulent velocities resulting from the Schwarzschild (upper image) and Ledoux (lower image) methods are shown in Fig. 5 for

Figure 4. Differences in temperature gradients between radiative ($\nabla_r$) and adiabatic ($\nabla_a$) cases and the chemical gradient $\nabla_{\mu}$ of 15-M⊙ models are shown in the upper panel, while the hydrogen abundance profile is shown in the lower panel. Parameter values are the same as in Fig. 1. Values on the horizontal axis are the mass fractions inside the considered points. Line of which label is ‘0’–the dotted line is a line on which the vertical ordinate values are 0.

Figure 5. The square root of the turbulent dynamic energy ($\sqrt{\kappa}$ ($u_2/10^5$)) of 15-M⊙ models at different evolution stages is shown. The Schwarzschild and Ledoux methods are used to define the convective region and calculate the turbulent quantities separately (upper: Schwarzschild, lower: Ledoux). Other parameter values are the same as in Fig. 1. Values on the horizontal axis indicate the natural logarithmic pressure. The four curves represent four different evolved phases, as explained in Fig. 2.
stellar models with decreasing central hydrogen content. It can be seen that under the effect of the chemical gradient discussed above, the e-folding distance resulting from the Ledoux method is about $0.5H_P$, similar to that obtained by the Schwarzschild method. This indicates that the effect of buoyancy in preventing motion is largely compensated for by the effect of heating from the convective heat flux against the temperature gradient in the overshooting region. As convective mixing in the overshooting region cannot be a fully mixing process, the effective mixing distance will be significantly shorter than the e-folding distance of the turbulent velocity.

4.3 Comparisons between results with different mixing-length parameters

It is well known that for massive stars the value of the mixing-length parameter $\alpha$ has little effect on the stellar structure and evolution. However, it may have a considerable effect on the turbulence properties in the stellar convective core, as shown in Fig. 6.

It can be seen that increasing the mixing-length parameter $\alpha$ from 0.7 to 1 leads to a considerable increase in turbulent velocity and radial kinetic energy. From the point of view of fluid dynamics, a larger value of $\alpha$ means that buoyancy can do work on the convective cells for a longer distance, so that the resulting kinetic energy will be larger. However, the turbulent heat flux $(\langle u'\theta' \rangle)$ remains almost the same in the stellar interior, due chiefly to the fact that the temperature gradient is basically adiabatic in the convective core. On the other hand, it is important to notice that there is a significant increment in the autocorrelation of the temperature fluctuation in the overshooting region. As a large temperature fluctuation can increase the generation of turbulent kinetic energy in the overshooting region, a larger mixing-length parameter $\alpha$ can therefore slightly increase the overshooting distance beyond the convective core.

5 RESULTS OF 30-M\(_{\odot}\) MODELS

5.1 Development of semi-convection and overshooting

By use of the Ledoux method, we have computed a series of stellar evolution models of 30 M\(_{\odot}\), with the parameter $C_x = 5 \times 10^{-9}$. The difference between radiative and adiabatic temperature gradients as well as the chemical gradient is shown in Fig. 7 for a stellar model in the late main-sequence phase. It can be seen that the radiative temperature gradient can be greater than the adiabatic one outside the convective core. This is due chiefly to the fact that the opacity, which comes mainly from free–free absorption and is proportional to the hydrogen abundance, increases outward in the chemical gradient region. As a result, semi-convection occurs in some part of the chemical gradient region. Overshooting from the convective core is always present. Consequently, these stellar models are a good sample with which to study the interaction between semi-convection and overshooting.

Profiles of turbulent correlations are shown in Fig. 8 for some stellar models with nearly equally spaced central hydrogen abundance. It can be seen that the overall properties of convection are similar to the case of stellar models with $15M_{\odot}$. The convective core shrinks continuously, along with successively increasing turbulent kinetic energy near the stellar centre. The overshooting from the convective core is significant, its extension being effectively influenced by the chemical gradient. In particular, semi-convection indeed develops near the surface of the chemical gradient region for the model with the lowest central hydrogen content, but it actually results in almost no effect on the turbulent velocity for this stellar model.

In order to see how the efficiency due to partial mixing of convection can play a significant role in the chemical gradient region, we have computed one more series of stellar evolution models with $C_x = 10^{-9}$. It can be seen in Fig. 9 that decreasing $C_x$ can significantly magnify the semi-convection, sometimes even driving it into full convection in the uppermost part of the chemical gradient region during the late stage of the main sequence. According to equation (14), the turbulent diffusivity will be smaller if $C_x$ decreases, leading to weaker mixing outside the convective core.
Semi-convection and convective overshooting

Figure 7. Differences in temperature gradient between radiative ($\nabla_r$) and adiabatic ($\nabla_a$) cases and the chemical gradient $\nabla \mu$ of 30-$M_\odot$ models are shown in the upper panel and the hydrogen abundance profile is shown in the lower panel. The turbulent diffusion coefficient $C_x$ is $5 \times 10^{-9}$. The mixing-length parameter $\alpha$ is 1.0. Values on the horizontal and vertical axes are the same as in Fig. 4.

As a result, the chemical gradients will be larger for these stellar models, as shown in Fig. 10, because it is mainly determined by the overshooting beyond the convective core in the preceding stage of main-sequence evolution. Higher hydrogen abundance in the outer part of the chemical gradient region results therefore in larger opacity and larger radiative temperature gradient, which is responsible for driving stronger semi-convection as well as the full convection observed in the late stage of the main sequence.

In contrast, however, full convection always develops in the chemical gradient regions for stellar models adopting the Schwarzschild method in the late main-sequence stage, which results in an intermediate convection zone as seen in Fig. 11 (see also Mowlavi & Forestini 1994). Due to exclusion of the chemical gradient term, semi-convection is not considered in the Schwarzschild method. It can therefore be noticed in Fig. 11 that the upward overshooting from the convective core penetrates extensively and finally meets the downward overshooting from the intermediate convection zone, resulting in much stronger mixing in almost all of the chemical gradient region.

5.2 Comparisons between Schwarzschild and Ledoux methods

The turbulent properties are compared for stellar models based on Ledoux and Schwarzschild methods. It can be seen in Fig. 12 that the Ledoux method gives a slower decay of turbulent velocity just above the boundary of the convective core than the Schwarzschild method, for a similar reason to that for the 15-$M_\odot$ model. It can be noticed that the autocorrelation of the temperature fluctuation according to the Ledoux method is again much larger than that according to the Schwarzschild method. When going into the semi-convection region, however, the turbulent velocity continues to decay according to the Ledoux method but with an even slower speed, due to the radiative temperature gradient now being larger than the adiabatic one. On the other hand, the turbulent velocity instead shows a second maximum in the same region according to the Schwarzschild method, because the stratification is now unstable against convection if the chemical gradient is omitted.
Figure 9. The square root of the turbulent dynamic energy (√k) (μ2/10^5) of 30-M☉ models at different evolution stages is shown. The Ledoux method is used. The element diffusion coefficient C_e is 10^-9. The mixing-length parameter α is 1.0. Values on the horizontal axis are the mass fraction inside the considered points and the four curves represent four different evolved phases, as explained in Fig. 8.

Figure 10. Chemical gradients V_x of 30-M☉ models with different turbulent diffusion coefficient ('1': C_t = 1 x 10^-9; '5': C_t = 5 x 10^-9) are shown in the upper panel and the hydrogen abundance profile is shown in the lower panel. The mixing-length parameter α is 1.0. The evolution phase is when the core hydrogen abundance is 0.2. Values on the x-axis are the mass fraction inside the considered points.

5.3 Comparisons between results with different mixing-length parameters

Profiles of different turbulent correlations are shown in Fig. 13 for stellar models with different mixing-length parameter α. It can be seen that increasing α from 0.7 to 1.0 results in similar increments of turbulent kinetic energy in the convective core to those for the 15-M☉ model. However, the turbulent heat flux can hardly be influenced by different choices of α, due to the actual temperature gradient being almost equal to the adiabatic one. Just beyond the convective core, the temperature fluctuation is considerably enhanced in the overshooting region by a larger value of α, which prolongs the overshooting distance of the turbulent velocity noticeably.

When going further into the semi-convection region, a larger value of α effectively suppresses convective motions there, resulting in almost no intermediate convection zone appearing. This result can be understood by considering the following two aspects. On the one hand, increasing α will lead, according to equations (5) and (6), to a smaller dissipation rate of turbulent kinetic energy. This effect slows down the decaying rate of turbulent velocity into the overshooting region and helps the convective motions to penetrate all the way into the semi-convection region. On the other hand, increasing α also produces more efficient mixing, according to equation (14), in the chemical gradient region, which will result in a smaller chemical gradient and effectively suppress the transition of the semi-convection into full convection in the chemical gradient region.

5.4 Turbulent properties among models with different turbulent parameters

Turbulent properties in the chemical gradient region are sensitively determined by the parameters in the turbulence model we have adopted. From equations (1)–(4), we note that there are six model parameters: C_t, C_1, and C_2 are diffusion parameters, C_1 and C_e are dissipation parameters and C_e is the coefficient for turbulence anisotropy.

It can be seen in Fig. 14 that on increasing the turbulent diffusion parameters C_t, C_1 and C_2, the curves of turbulent velocity become flatter and flatter, the peak values becoming smaller while the bottom values become larger. This property indicates that larger turbulent diffusion parameters lead to more significant overshooting of turbulent kinetic energy.

It can be seen in Fig. 15 that increasing the turbulent dissipation coefficient C_e results in little change for the turbulent velocity (√k) in the convective core. In the chemical gradient region, however, the turbulent velocity becomes smaller in the overshooting region just beyond the boundary of the convective core and then larger in the semi-convection region as well as the fully convective intermediate zone. From equation (4), a larger value of C_e is equivalent to a larger dissipation rate ε, which will lead to weaker overshooting in the turbulent velocity. As a result, previous mixing in the chemical gradient region is weaker, leaving a larger chemical gradient to
Semi-convection and convective overshooting

Figure 12. Turbulent properties of 30-M\(\odot\) models with different convection processing methods are shown. The evolution phase is when the core hydrogen abundance is 0.2. Parameter values are the same as in Fig. 7. The Schwarzschild and Ledoux methods are used to define the convective region and calculate the turbulent quantities separately; we use labels ‘sch’ and ‘led’. Values on the horizontal and vertical axes are the same as in Fig. 3, except that the units are different: \((u_1/10^{10}, u_2/10^5, u_3/10^6, u_4/10^6)\) – ‘sch’, \((u_4/10^6)\) – ‘led’.

In Fig. 16, it can be seen that increasing the turbulent dissipation coefficient \(C_t\) results in a similar effect to increasing \(C_c\). Overshooting beyond the convective core is suppressed, while semi-convection in the outer part of the chemical gradient region is enhanced.

It can be seen in Fig. 17 that, in the convective core, increasing the redistribution coefficient of turbulence \(C_k\) reduces the radial kinetic energy \((\overline{u_r}\overline{u_r})\), while it does not affect the turbulent velocity. This is because \(C_k\) determines how the kinetic energy is distributed among different directions of motion. In the chemical gradient
Figure 14. Turbulent properties of 30-$M_{\odot}$ models with different turbulent diffusion parameters $C_s$, $C_t1$, $C_e1$ (‘3’ is 0.03, ‘5’ 0.05 and ‘7’ 0.07) are shown. The evolution phase is when the core hydrogen abundance is 0.2. The Ledoux method is used. The mixing-length parameter $\alpha$ is 0.7, while the turbulent diffusion coefficient $C_x$ is $10^{-9}$. Values on the horizontal and vertical axes are the same as in Fig. 13, except that the vertical ordinate of the bottom diagram is the fraction of convection-transported energy to total energy ($F_c/F$).

Figure 15. Turbulent properties of 30-$M_{\odot}$ models with different turbulent dissipation coefficient $C_e$ (‘1.00’, ‘1.25’ and ‘1.50’) are shown. Other values are the same as those in Fig. 14.

region, however, increasing $C_i$ leads to a considerable increment of the turbulent velocity. This effect can be understood by considering equation (3). In the chemical gradient region, the buoyancy prevents convective motions and results in a smaller kinetic energy in the radial direction than in the other directions. A larger value of $C_i$ tends to make the turbulent motions more isotropic, so that the radial kinetic energy will be accordingly larger. From equation (3), a larger $(u'_r u'_r)$ implies a larger $\varepsilon$, leading to a smaller turbulent velocity in the overshooting region. As a result, mixing is weaker in the overshooting region, to give a larger chemical gradient in the previous evolution stage, which will be responsible for stronger semi-convection in the late stage of the main sequence.

In Figs 14–17, we can see that the temperature–velocity correlation of turbulence $(u'_r T')$ is barely influenced by different choices of
the parameters of our turbulence model and the resulting convective heat flux is essentially unaffected.

6 SUMMARY AND DISCUSSION

With reference to the Schwarzschild and Ledoux criteria, we focus on Schwarzschild and Ledoux methods, which we use to confine the convective boundary and calculate the turbulent properties. Models in this article are different from those of Deng et al. (1996) and El Eid, The & Meyer (2009) in that the turbulent velocity we used to calculate the turbulent diffusion is from the TCM and their values come from phenomenological models. Also, the entropy gradient

$$\frac{\partial T}{\partial r} = -\frac{c_p}{H_T}(\nabla - \nabla_{ad} - \nabla_{\mu}),$$

which takes the chemical gradient into account, is different from the values of Lai & Li (2011) and Zhang & Li (2012).

In the present article, turbulent quantities for several series of stellar models with masses 30 and 15 M⊙ are calculated. From our
calculations, semi-convection appears in models for stars of 30$M_\odot$ but does not appear in models for stars of 15$M_\odot$. With the central hydrogen abundance decreasing, coverage of the convective core successively shrinks and the maximum of turbulent velocity increases near the stellar centre, while turbulent velocity decays continuously in the overshooting region. Owing to the production effect of the turbulent temperature fluctuation, the decay of turbulent velocity becomes slower when going further and further into the overshooting region. For stellar models of 30$M_\odot$, semi-convection occurs in some part of the chemical gradient region due to the opacity, which comes mainly from free–free absorption increasing outward in the chemical gradient region.

We analysed the influence of different parameters on the turbulent quantities. In general, we can summarize as follows.

1. In this article, the semi-convection occurring in the late stage of the main sequence is closely related to overshooting beyond the convective core in the early stage of the main sequence. Stronger overshooting in the previous evolution results in stronger mixing in the chemical gradient region and a smaller chemical gradient, leaving a lower hydrogen abundance and a smaller radiative temperature gradient in the outer part of the chemical gradient region, which finally leads to weaker semi-convection in the late stage of the main sequence.

2. The Schwarzschild method adopted to define the convective region and to calculate the turbulent quantities is more conducive to the occurrence of semi-convection than the Ledoux method adopted. The overshooting distance is greater and the autocorrelation of the temperature fluctuation is depressed when adopting the Ledoux method. From stellar models with 15$M_\odot$, the effective mixing distance in the overshooting region is significantly shorter than 0.5$H_p$ when adopting either method.

3. Increasing the mixing-length parameter $\alpha$ leads to increasing turbulent dynamic energy in the convective core and prolongs the overshooting distance but decreases the turbulent dynamic energy in the semi-convective zone. This result can be understood by considering two aspects: on one hand, increasing $\alpha$ leads to a smaller dissipation rate of turbulent dynamic energy, which slows down the decay rate of turbulent velocity in the overshooting region; on the other hand, increasing $\alpha$ produces more efficient mixing in the chemical gradient region, which results in a smaller chemical gradient and suppresses the transition of semi-convection into full convection.

4. Increasing the turbulent diffusion parameters $(C_s, C_{1t}, C_{2t})$ flattens the curves of turbulent dynamic energy and leads to more significant overshooting of turbulent dynamic energy. When increasing the turbulent dissipation coefficient $C_t$ or $C_{1t}$, the turbulent dynamic energy is smaller in the overshooting region but larger in the semi-convective region, as well as in the intermediate convective zones. When increasing the turbulent redistribution coefficient $C_{2t}$, the radial turbulent dynamic energy is smaller but the turbulent dynamic energy does not change in the convective core, while in the chemical gradient region the turbulent dynamic energy is smaller in the overshooting region and larger in the semi-convective region as well as in the intermediate convective zones. The temperature–velocity correlation of turbulence ($w'_rT$) is scarcely influenced by different choices of the parameters of our turbulence model and the resulting convective heat flux is essentially unaffected.

ACKNOWLEDGEMENTS

We greatly appreciate our colleagues (Zhang Qian Sheng, Lai Xiang Jun, Su Jie and others) for sharing our experiences and helping in work and in life. This work is co-sponsored by the NSFC of China (Grant Nos. 11333006 and 10973035) and by the Chinese Academy of Sciences (Grant No. KJCX2-YW-T24).

REFERENCES

Alexander D. R., Ferguson J. W., 1994, ApJ, 437, 879
Bancall J. N., Pinsonneault M. H., Wasserburg G. J., 1995, Rev. Mod. Phys., 67, 781
Caloi V., Mazzitelli I., 1990, A&A, 240, 305
Canuto V. M., 1992, ApJ, 392, 218
Canuto V. M., 2000, ApJ, 534, L113
Chiosi C., 1998, in Aparicio A., Herrero A., Sánchez F., eds, Stellar Astrophysics for the Local Group: VIII Canary Islands Winter School of Astrophysics. Cambridge Univ. Press, Cambridge, p. 1
Chiosi C., 1999, in Gimenez A., Guinan E. F., Montesinos B., eds, ASP Conf. Ser. Vol. 173, Stellar structure: Theory and Test of Convection. Astron. Soc. Pac., San Francisco, p. 9
Chiosi C., 2007, in Kupka F., Roxburgh I. W., Chan K. L., eds, Proc. IAU Symp. 239, Astrophysics: Convection. Astron. Soc. Pac., San Francisco, p. 235
Chiosi C., 2009, Commun. Asteroseismol., 158, 79
Chiosi C., Maeder A., 1986, ARA&A, 24, 329
Chiosi C., Bertellis G., Bressan A., 1992, in de Jager C., Nieuwenhuijzen H., eds, Instabilities in Evolved Super and Hypergiants. North Holland, Amsterdam, p. 145
Deng L., Bressan A., Chiosi C., 1996, A&A, 313, 145
El Eid M. F., The L.-S., Meyer B. S., 2009, Space Sci. Rev., 147, 1
Grossman S. A., Narayan R., 1993, ApJS, 89, 361
Harris G. L. H., Hesser J. E., Massey P., Peterson C. J., Yamanaka J. M., 1983, PASP, 95, 607
Iglesias C. A., Rogers F. J., 1996, ApJ, 464, 943
Kato S., 1966, PASJ, 18, 374
Langer N., 2012, ARA&A, 50, 107
Langer N., EL Eid M. F., Fricke K. J., 1985, A&A, 145, 179
Lai X. J., Li Y., 2011, Res. Astron. Astrophys., 11, 1351
Li Y., Yang J.-Y., 2007, MNRAS, 375, 388
Merryfield W. J., 1995, ApJ, 444, 318
Mowlavi N., Forestini M., 1994, A&A, 282, 843
Noels A., Montalban J., Miglio A., Goadart M., Ventura P., 2010, Ap&SS, 328, 227
Rogers F. J., Iglesias C. A., 1995, Highlights Astron., 10, 573
Rogers F. J., Swenson F. J., Iglesias C. A., 1996, ApJ, 456, 902
Sienkiewicz R., 2004, Stellar Evolution Package, Version 40, available at: ftp.camk.edu.pl/camk/rv04/readme.04
Spruit S. C., 1992, A&A, 253, 131
Stothers R. B., Chin C. W., 2000, ApJ, 540, 1041
Tayler R. F., 1969, MNRAS, 144, 231
Ventura P., Zeppieri A., Mazzitelli I., D’Antona F., 1998, A&A, 334, 953
Xiong D. R., 1985, A&A, 150, 133
Zhang Q. S., Li Y., 2012, ApJ, 746, 50

This paper has been typeset from a T\LaTeX\ file prepared by the author.