Modeling Lyman-α Forest Cross-Correlations with LyMAS

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ABSTRACT
We use the Ly-α Mass Association Scheme (LyMAS; Peirani et al. 2014) to predict cross-correlations at z = 2.5 between dark matter halos and transmitted flux in the Ly-α forest, and compare to cross-correlations measured for quasars and damped Ly-α systems (DLAs) from the Baryon Oscillation Spectroscopic Survey (BOSS) by Font-Ribera et al. (2012, 2013). We calibrate LyMAS using Horizon-AGN hydrodynamical cosmological simulations of a (100 h⁻¹ Mpc)³ comoving volume. We apply this calibration to a (1 h⁻¹ Gpc)³ simulation realized with 2048³ dark matter particles. In the 100 h⁻¹ Mpc box, LyMAS reproduces the halo-flux correlations computed from the full hydrodynamic gas distribution very well. In the 1 h⁻¹ Gpc box, the amplitude of the large scale cross-correlation tracks the halo bias b_h as expected. We provide empirical fitting functions that describe our numerical results. In the transverse separation bins used for the BOSS analyses, LyMAS cross-correlation predictions follow linear theory accurately down to small scales. Fitting the BOSS measurements requires inclusion of random velocity errors; we find best-fit RMS velocity errors of 399 km s⁻¹ and 252 km s⁻¹ for quasars and DLAs, respectively. We infer bias-weighted mean halo masses of $M_h/10^{12} h^{-1} M_\odot = 2.19_{-0.15}^{+0.16}$ and $0.69_{-0.14}^{+0.16}$ for the host halos of quasars and DLAs, with ~ 0.2 dex systematic uncertainty associated with redshift evolution, IGM parameters, and selection of data fitting range.

Key words: large-scale structure of Universe

1 INTRODUCTION

The enormous high-redshift quasar sample observed by the Baryon Oscillation Spectroscopic Survey (BOSS; Dawson et al. 2013) of SDSS-III (Eisenstein et al. 2011) SDSS = Sloan Digital Sky Survey has enabled the first empirical studies of the large scale 3-dimensional auto-correlations of transmitted flux in the Ly-α forest (e.g., Slosar et al. 2011) and of the cross-correlations between the Ly-α forest and other tracers of structure such as damped Ly-α systems (DLAs; see Font-Ribera et al. 2012 hereafter FR12) and quasars (Font-Ribera et al. 2013 hereafter FR13). Auto-correlation analyses allow high-precision measurements of the angular diameter distance and Hubble parameter via baryon acoustic oscillations (BAO; see Busca et al. 2013 Slosar et al. 2013 Delubac et al. 2015). Cross-correlation analyses provide a complementary route to BAO measurements (Font-Ribera et al. 2014), and they provide novel constraints on the properties of dark matter halos that host DLAs and quasars. On large scales, these correlations can be described by linear theory with an effective bias factor b_F and redshift-space distortion parameter $\beta_F$ for the forest that depend on the underlying cosmology and the physical parameters of the intergalactic medium (IGM; see, e.g., McDonald 2003 Seljak 2012 Arinyo-i-Prats et al. 2015). Fully exploiting these measurements, however, requires a theoretical description of Ly-α forest clustering that extends to non-linear scales. In this paper we use the Ly-α Mass Association Scheme (LyMAS; Peirani et al. 2014 hereafter P14) to predict the cross-correlation between dark matter halos and the Ly-α forest, and we use these predictions to model the measurements of DLA- and quasar-forest cross-correlations by FR12 and FR13.

Ideally one would model Ly-α forest correlations using
hydrodynamic simulations that resolve the $\sim 100$ kpc Jeans scale of the low density IGM (see [Čen et al. 1994] Zhang et al. [1995] Hernquist et al. [1996] Miralda-Escudé et al. [1996] for pioneering studies and [Lidz et al. 2010] Peeples et al. 2010] Borde et al. 2014] Arinyo-i-Prats et al. 2015] for some recent examples). However, $\sim$Gpc$^3$ simulation volumes are needed to avoid artificial box size effects and exploit the statistical precision achievable with BOSS, and calculations with the required combination of volume and resolution are not currently feasible. One approach to this challenge (e.g., Slosar et al. 2009] uses variants of the Fluctuating Gunn-Peterson Approximation (FGPA; Gunn & Peterson [1965] Croft et al. [1998] Weinberg, Katz, & Hernquist 1998] a deterministic mapping between dark matter density and Ly- $\alpha$ flux. LyMAS instead uses high-resolution hydrodynamic simulations to calibrate the conditional probability distribution $P(F_s|\delta_s)$ for the transmitted flux smoothed (in one dimension) on the scale of BOSS spectral resolution given the redshift-space dark matter density contrast $\delta_s$ smoothed (in three dimensions) on a similar scale. This conditional distribution is applied to the smoothed dark matter density field of a larger volume, lower resolution $N$-body simulation to determine the Ly- $\alpha$ forest flux along skewers through the matter distribution. Additional steps are used to impose pixel-to-pixel coherence of spectra along a given line of sight and to ensure that the 1-d power spectrum and unconditional probability distribution function (PDF) of flux matches that of the calibrating hydrodynamic simulations (see P14).

P14 calibrated $P(F_s|\delta_s)$ using the Horizon-MareNostrum simulation, a hydrodynamic simulation of a (50 h$^{-1}$ Mpc)$^3$ comoving volume with WMAP1 cosmological parameters [Spergel et al. 2003] $h \equiv H_0/100$ km s$^{-1}$ Mpc$^{-1}$). They applied this calibration to a 1024$^3$ simulation of a (300 h$^{-1}$ Mpc)$^3$ cube with a Gaussian dark matter (DM) smoothing scale $r_s = 0.3$ h$^{-1}$ Mpc and to a 1024$^3$ simulation of a (1 h$^{-1}$ Gpc)$^3$ cube with $r_s = 1.0$ h$^{-1}$ Mpc. In this work we improve the calibration using Horizon-AGN and Horizon-noAGN, two simulations of a (100 h$^{-1}$ Mpc)$^3$ volume adopting WMAP7 parameters [Komatsu et al. 2011], one with and one without feedback from active galactic nuclei (AGN), as described by [Dubois et al. 2014]. For large volumes we use one 2048$^3$ simulation of a (1 h$^{-1}$ Gpc)$^3$ cube, with dark matter smoothing $r_s = 0.5$ h$^{-1}$ Mpc, and several 1024$^3$ simulations of the same volume with $r_s = 1.0$ h$^{-1}$ Mpc. These “upgrades” improve the accuracy and precision of our predictions and give us a better handle on statistical uncertainties and the potential impact of AGN feedback.

FR12 measured the cross-correlation $\xi(\pi, \sigma)$ between DLAs and the Ly- $\alpha$ forest, as a function of line-of-sight separation $\pi$ in bins of transverse separation $\sigma$, using $\sim 60,000$ quasar spectra from the Data Release 9 (DR9; Alam et al. 2012] BOSS quasar sample [Ross et al. 2012] Paris et al. 2012]. They modeled these measurements using linear theory over transverse scales $\sigma = 1 - 60$ h$^{-1}$ Mpc (comoving), excluding scales $r = (\sigma^2 + \pi^2)^{1/2} < 0.5$ h$^{-1}$ Mpc in all bins. Adopting Ly- $\alpha$ forest linear bias parameters $b_F = -0.168$, $\beta_F = 1.0$ at effective redshift $z = 2.3$, they inferred a linear bias for DLAs of $b_{DLA} = 2.17 \pm 0.2$ at $z = 2.3$, corresponding to a characteristic halo mass $M_h = 4.5 \times 10^{12}$ h$^{-1}$ $M_\odot$ (which yields the same bias factor at this redshift). FR13 applied similar techniques to the cross-correlation of DR9 quasars with the Ly- $\alpha$ forest, extending measurements to $\sigma = 80$ h$^{-1}$ Mpc and excluding scales $r < 15$ h$^{-1}$ Mpc from their linear fits in all bins. They inferred $b_0 = 3.64^{+0.15}_{-0.15}$ for the average bias factor of DR9 quasars, with a corresponding halo mass $M_h = 3.26 \times 10^{12}$ h$^{-1}$ $M_\odot$. Similar analyses using the BOSS DR12 [Alam et al. 2015] Ly- $\alpha$ forest, DLA, and quasar samples are underway.

In §2 we give an explanation of how LyMAS is calibrated on a small-volume hydrodynamic simulation and then applied to large-scale dark matter simulations, and we briefly examine the influence of AGN feedback on the Ly- $\alpha$ forest. In §3 we present our results for the dependence of the cross-correlation on various properties and provide comparisons of our calculations to linear theory predictions for the correlation. In §4 we analyze the accuracy of LyMAS for predicting cross-correlations by examining how different smoothing lengths, box sizes, and particle numbers affect the correlation. We fit a simple, three-parameter model to the cross-correlation in §5 to determine the dependence of the strength, width, and offset of the correlation on halo mass. We compare our results to linear theory predictions for the Ly- $\alpha$ forest auto-correlation and halo-Ly- $\alpha$ cross-correlation in §6. We compare our calculations for halo-forest cross-correlations to measured cross-correlations of DLAs and quasars with the Ly- $\alpha$ forest in §7. We present our conclusions in §8.

2 SIMULATIONS

A detailed account of how LyMAS operates is given by P14, but we summarize the main points and present some changes since the original paper here. LyMAS uses PDFs for the Ly- $\alpha$ forest flux that are conditioned on the underlying dark matter density to predict the forest flux in a large-volume DM simulation. These conditional PDFs are determined from a full hydrodynamics and $N$-body simulation using the RAMSES code [Teyssier 2002] in a periodic box with side length of 100 h$^{-1}$ Mpc (50 h$^{-1}$ Mpc in P14), adopting WMAP1 cosmological parameters (WMAP1 in P14): $\Omega_m = 0.272$, $\Omega_\Lambda = 0.7284$, $\Omega_b = 0.045$, $h = 0.704$, $\sigma_8 = 0.81$, $n_s = 0.967$. The conditional PDFs are then applied to large-scale $N$-body simulations created using Gadget2 [Springel 2005], from which pseudo-spectra are extracted.

The smaller hydrodynamics simulation includes physical processes such as metal-dependent cooling, photoionization and heating from a UV background, supernova feedback, and metal enrichment. To calibrate the conditional PDFs, about one million lines of sight are extracted from this simulation, and the optical depth of Ly- $\alpha$ absorption is calculated based on the neutral hydrogen density along the line of sight. The spectra are smoothed with a 1-d Gaussian of dispersion 0.696 h$^{-1}$ Mpc, equivalent to BOSS spectral resolution at $z \approx 2.5$. DM skewers that correspond to the spectra are also extracted from this simulation, and the DM is smoothed three-dimensionally with several smoothing scales $r_s$. The optical depth along each spectrum is converted to Ly- $\alpha$ forest flux by $F = e^{-\tau}$, and PDFs for the values of the flux conditional on DM overdensity can thus be determined.

LyMAS operates on the assumption that the flux in
one pixel is not correlated with the flux in a separate pixel except through correlations in the underlying dark matter. The flux in pixel 1 is drawn from the conditional distribution \( P(F_1|\delta_1) \) without reference to the value \( F_2 \) drawn from \( P(F_2|\delta_2) \) in pixel 2, but \( F_1 \) and \( F_2 \) are correlated because of the correlation between \( \delta_1 \) and \( \delta_2 \). This ansatz, that the relation between flux and matter density is “locally stochastic,” can be expressed by the equation

\[
P(F_1, F_2|\delta_1, \delta_2) = P(F_1|\delta_1)P(F_2|\delta_2) .
\]

Along each individual line of sight, coherence among neighboring pixels is imposed by generating a “percentile field” that governs draws from \( P(F_i|\delta_i) \), rather than drawing each pixel value independently. The correct 1-d flux power spectrum is imposed, on average, by multiplying line-of-sight Fourier modes by numerically calibrated correction factors. Except for the change in the calibrating simulations themselves, our procedures here are the same as those presented by P14.

We apply LyMAS to DM simulations with side lengths 300 \( h^{-1} \) Mpc and 1 \( h^{-1} \) Gpc, the same sizes studied in P14, but with cosmological parameters matched to those of the hydro simulations. Our primary simulations have 1024\(^3\) particles for 300 \( h^{-1} \) Mpc and 2048\(^3\) particles for 1 \( h^{-1} \) Gpc, and we analyze them with DM smoothing lengths of 0.3 \( h^{-1} \) Mpc and 0.5 \( h^{-1} \) Mpc, respectively, equal to the initial mean interparticle separation. We also carried out a 1024\(^3\) simulation of the 1 \( h^{-1} \) Gpc box with the same initial conditions and two 1024\(^3\) simulations of 1 \( h^{-1} \) Gpc boxes with independent initial fluctuations, all analyzed with a 1.0 \( h^{-1} \) Mpc DM smoothing length. We apply all three smoothing lengths, \( r_s = 0.3, 0.5, 1.0 h^{-1} \) Mpc, to the 100 \( h^{-1} \) Mpc hydro simulation to calibrate conditional PDFs and to test the accuracy of LyMAS.

The hydrodynamical cosmological simulation Horizon-AGN is described in detail by Dubois et al. (2014). The initial grid for the gas distribution is 1024\(^3\), with a comoving grid cell of 0.25 \( h^{-1} \) Mpc, and adaptive mesh refinement (AMR) is used up to a maximum resolution (for hydrodynamics and gravity) of 1 proper kpc. The dark matter distribution is represented by 1024\(^3\) particles, with effective gravitational softening set by the cloud-in-cell interpolation of the particle mass distribution onto the adaptive grid. Fig. 1 shows a slice through a simulation that includes AGN feedback (Horizon-AGN, left) and the corresponding slice through a simulation without AGN feedback (Horizon-AGN, right). While the gas density field (green) is similar in both simulations, regions of high temperature (red) and high metallicity (blue) are more extended in the AGN feedback run. AGN feedback has a considerable impact on the mass function and morphologies of the simulated galaxy population (Dubois et al., 2013), but the impact on conditional flux PDFs is small, and (as we show below) the impact on halo-flux correlations is negligible. When extracting Ly-\(\alpha\) forest spectra from the simulated gas distribution, the UV background intensity is chosen to give a mean transmitted Ly-\(\alpha\) forest flux \( \bar{F} = \langle e^{-\tau} \rangle = 0.795 \), matching the metal-corrected \( z = 2.5 \) value measured from high-resolution spectra by Faucher-Giguère et al. (2008).

We identify DM halos using a friends-of-friends algorithm (Davis et al., 1985) with a linking length of 0.15. Then we compute the center of mass of each object. Once we have pseudo-spectra determined by LyMAS and the positions and masses of DM halos in the \( N \)-body simulations, we calculate the cross-correlation by randomly selecting a large number of pseudo-spectra and DM halos from each volume. We bin the DM halos by mass, and for each halo we bin the pseudo-spectra around it in transverse and line-of-sight separations \( \sigma \) and \( \pi \). We calculate the cross-correlation \( \xi \) as

\[
\xi(\pi, \sigma) = \langle F(\pi, \sigma) \rangle - 1 ,
\]

where \( F(\pi, \sigma) \) is the transmitted flux fraction (\( \pi = 1 \) for no absorption by the Ly-\(\alpha\) forest, \( \pi = 0 \) for complete Ly-\(\alpha\) absorption) at line-of-sight separation \( \sigma \) and transverse separation \( \pi \) from the center of mass of the halo, where \( \bar{F} \) is the mean Ly-\(\alpha\) forest transmitted flux in the entire box. The average of the flux (indicated by \( \langle \rangle \) ) is performed over all pixels that fall into the halo mass, \( \pi \), and \( \sigma \) bin. Equation 2 is equivalent to the cross-correlation between the halo number density field and the flux fluctuation field

\[
\delta_F = \frac{F - \bar{F}}{\bar{F}} .
\]

We also calculate the auto-correlation as

\[
\xi(\pi, \mu) = \langle \delta_F(\pi)\delta_F(\mu) \rangle ,
\]

where \( \delta_F = (F - \bar{F})/\bar{F} \), \( r \) denotes the comoving separation between the pixels where \( F_1 \) and \( F_2 \) are measured, and \( \mu = \pi/r \) denotes the cosine of the angle from the line of sight between the two pixels. All calculations are done in redshift space, incorporating the simulated peculiar velocities of the gas, DM particles, and halos.

3.1 Accuracy of LyMAS

Fig. 2 compares the halo-forest cross-correlation \( \xi(\pi, \sigma) \) computed from the gas distribution of the noAGN hydrodynamic simulation (see 2) to the cross-correlations predicted by applying LyMAS to the smoothed DM density fields of the same simulation with \( r_s = 0.3, 0.5, \) or 1.0 \( h^{-1} \) Mpc. Each panel shows the correlation as a function of line-of-sight separation \( \pi \) between the center of mass of each halo and pixels in the spectra. The three rows show three different halo-pixel transverse separation bins, \( \sigma = 1 - 4 \) \( h^{-1} \) Mpc, and the two columns are divided into mass bins for the halos, \( M_{12} \equiv M_h/10^{12} h^{-1} M_{\odot} \equiv 1.68 - 3.35 \) and 3.35 - 6.70. As expected, the strength of the cross-correlation decreases with increasing transverse or line-of-sight separation (\( \sigma \) and \( \pi \), respectively) and increases with increasing halo mass \( M_{12} \).
The four curves are nearly indistinguishable in every panel, and we find similar results for other transverse separation and halo mass bins. We conclude that halo-forest cross-correlations predicted by LyMAS at BOSS spectral resolution should be virtually identical to those that would be predicted by a full high-resolution hydrodynamic simulation of the same volume, for DM smoothing lengths up to $r_s = 1.0 \, h^{-1} \text{Mpc}$.

For representative statistical error bars, we divide the $(100 \, h^{-1} \text{Mpc})^3$ box into 16 subvolumes, with transverse size $25 \, h^{-1} \text{Mpc} \times 25 \, h^{-1} \text{Mpc}$ and line-of-sight size 100 $h^{-1} \text{Mpc}$. We calculate the cross-correlation, $\xi(\pi, \sigma)$, in each of the 16 subvolumes and compute the error on the mean by first finding the mean values in bins of $\pi$ (using the same bins as for $\sigma$), then computing the error in each $(\pi, \sigma)$ bin as the standard deviation of the mean among the 16 subvolumes in the bin. We include halo-pixel pairs that cross the subvolume divisions and assign these pairs to the subvolume that contains the center of mass of the DM halo. We always use the global mean flux $F$ when calculating $\xi(\pi, \sigma)$, instead of the mean flux within each subvolume. This procedure for calculating errors may underestimate the statistical error we would find from the dispersion among multiple independent simulations, or from multiple $(100 \, h^{-1} \text{Mpc})^3$ volumes in a larger simulation, because it underestimates the impact of coherent structure induced by large-scale Fourier modes. The multiple curves in Fig. 2 agree to much better than our estimated statistical errors in any case because they are all derived from the same underlying full-cube matter distribution.

To further analyze how well the LyMAS predictions of the halo-flux cross-correlation agree with those from the hydrodynamic simulation, Fig. 3 shows the negative of the monopole (left panel) and the quadrupole (right panel) of the cross-correlation in these simulations as functions of 3-d separation $r$ for two halo mass bins (see [3,3] for a description of how the monopole and quadrupole are calculated). Small scale deviations between LyMAS and full hydro are more evident in this representation than in the $\xi(\pi, \sigma)$ representation, which averages over a range of scales. LyMAS weakly over-predicts the strength of the monopole at $r \leq 3 \, h^{-1} \text{Mpc}$ and under-predicts the strength of the quadrupole at $r \leq 6 \, h^{-1} \text{Mpc}$, with a larger difference in the higher halo mass bin. On larger scales, the monopole and quadrupole of the LyMAS cross-correlation match the hydro correlation to well within the error bars.

As AGN feedback plays an important role in the temperature structure in high density regions (see Fig. 1), we examine its impact on the halo-forest cross-correlation. Fig. 3 compares the correlation computed from the true gas spectra in the two hydro simulations, with and without AGN feedback (as described in [2], both in a $(100 \, h^{-1} \text{Mpc})^3$ volume. Only the $\sigma = 1 - 4 \, h^{-1} \text{Mpc}$ transverse separation bin and the $M_{12} = 1.68 - 3.35$ and $3.35 - 6.70$ mass bins are shown, but results are similar in other $\sigma$ and $M_{12}$ bins. There is virtually no difference in the cross-correlation when AGN feedback is included as compared to when it is not. Previous studies have shown that AGN feedback can have a small (few percent) impact on the unconditional PDF and 1-d power spectrum of Ly-$\alpha$ transmitted flux [Viel, Schaye, & Booth 2013], and we find similar effects in our simulations. However, while AGN feedback in these simulations has a substantial effect on the properties of massive galaxies [Dubois et al. 2013], the impact on halo-forest cross-correlations is negligible at the level of the statistical errors in our simulation predictions and the observational data. We find a similarly negligible effect if we use the conditional PDFs from either the AGN or noAGN hydro simulation as our basis for applying LyMAS to larger volume N-body simulations. In the remainder of this paper, we use the noAGN simulation.

Figure 1. The same slice through a hydrodynamics simulation that includes AGN feedback (left) and one that does not (right). Blue colours trace regions of high gas metallicity, red colours trace regions of high gas temperature, and green colours trace regions of high gas density.
as the source of these conditional PDFs, but none of our conclusions would change if we used the AGN simulation instead.

We next examine the effect on the cross-correlation of using larger volume simulations. The left column of Fig. 3 shows the correlation predicted by applying LyMAS to simulation volumes of (100 $h^{-1}$ Mpc)$^3$, (300 $h^{-1}$ Mpc)$^3$, and (1 $h^{-1}$ Gpc)$^3$ with 3-d dark matter smoothing lengths of $r_s = 0.3$, 0.3, and 0.5 $h^{-1}$ Mpc, respectively. Only the $M_{12} = 1.68 - 3.35$ halo mass bin is shown. Based on Fig. 3, we are confident that differences in the calculated correlation are differences in the simulation volumes rather than smoothing scale effects. In each of the three transverse separation bins shown in the figure ($\sigma = 1 - 4, 4 - 7, 7 - 10 h^{-1}$ Mpc), there are significant differences in the strength of the correlation, most easily seen at $\xi(\pi = 0, \sigma)$. This difference is quite large in the larger transverse separation bins, where the separation is a significant fraction of the smaller simulation volumes. While these differences could be partly due to random statistical fluctuations between the boxes, the weaker correlation at large $\sigma$ in the smaller boxes is likely a systematic effect of the absence of Fourier modes larger than the box size.

The right column of Fig. 3 compares the correlation calculated by applying LyMAS to two different (300 $h^{-1}$ Mpc)$^3$ dark matter distributions (solid and dashed) that differ only in the statistical fluctuations of the initial conditions of the dark matter density. There is a significant difference in the strength of the cross-correlation between the two distributions, which means that (300 $h^{-1}$ Mpc)$^3$ is not a large enough volume to overcome substantial statistical fluctuations in the density distribution. For this reason, we use a (1 $h^{-1}$ Gpc)$^3$ box for all following analysis of the cross-correlation. Fig. 4 also makes the cautionary point that differences between simulation volumes can sometimes be much larger than that estimated by our subvolume method, though we show below that the subvolume error estimate appears reasonable for the (1 $h^{-1}$ Gpc)$^3$ box at $\sigma > 7 h^{-1}$ Mpc.
As a further examination of DM smoothing length and resolution effects, Fig. 3 shows the correlations in the (100 $h^{-1}$ Mpc)$^3$ dark matter simulation with $r_s = 0.3$ $h^{-1}$ Mpc smoothing (dashed, horizontally offset for clarity). Halo mass bins $M_{12} = 0.84 - 1.68$ (filled circles) and 3.35 – 6.70 (open circles) are shown. Error bars are computed from the standard deviation of the mean among 16 subvolumes.

Figure 4. The cross-correlation between dark matter halos and Ly-$\alpha$ forest flux calculated in the (100 $h^{-1}$ Mpc)$^3$ hydrodynamic simulations using the true gas spectra either including (blue dashed) or not including (black solid) AGN feedback. Only the $\sigma = 1 - 4$ $h^{-1}$ Mpc separation bin and $M_{12} = 1.68 - 3.35$ and 3.35 – 6.70 mass bins are shown, but the near-perfect agreement holds in other mass and separation bins. Error bars are computed from the standard deviation of the mean among 16 subvolumes.

Figure 3. The monopole (left) and quadrupole (right) of the halo-flux cross-correlation in the hydro box (solid) and for the LyMAS predictions applied to the (100 $h^{-1}$ Mpc)$^3$ dark matter simulation with $r_s = 0.3$ $h^{-1}$ Mpc smoothing (dashed, horizontally offset for clarity). Halo mass bins $M_{12} = 0.84 - 1.68$ (filled circles) and 3.35 – 6.70 (open circles) are shown. Error bars are computed from the standard deviation of the mean among 16 subvolumes.
After experimenting with different functional forms, we have found that we can achieve a visually good fit to the predicted cross-correlations in each \( \sigma \) bin for all of the mass ranges we have examined using a 3-parameter Lorentzian function

\[
\xi(\pi, \sigma) = -\alpha \frac{\gamma^2}{\pi^2 + \gamma^2} + \Delta \tag{5}
\]

where \( \alpha \) is the correlation strength at \( \pi = 0 \) and \( \gamma \) describes the width of the Lorentzian in each \( \sigma \) bin. The cross-correlation can be non-zero out to large separations, and fits are significantly more accurate if we include a large scale vertical offset \( \Delta \) as an additional parameter. The purpose of these fits is partly to allow us to report our full set of numerical results in a compact form and partly to allow us to fit observational data with a continuous model rather than interpolate across the discrete mass bins used in our computation.

We fit the Lorentzian model of Eq. (5) to the cross-correlation predicted by LyMAS in each of the transverse separation \( \sigma \) and halo mass \( M_{12} \) bins. Thick solid lines and points with error bars in Fig. 5 show the peak correlation strength, \( |\xi(\pi = 0, \sigma)| \), as a function of the average halo mass within each bin, as determined by the best-fit Lorentz profile. Errors on these points are the dispersion in the best-fit Lorentz profile among the same 16 subvolumes described in [5.1] found simply by fitting the same form to \( \xi(\pi, \sigma) \) in each subvolume. (For each of these 16 fits, the errors on values of \( \xi(\pi, \sigma) \) were the error on the mean of all pixels in the \((\pi, \sigma)\) bin.) As expected, the correlation strength increases with decreasing \( \sigma \) and with increasing halo mass. We fit a power-law of the form

\[
\xi(\pi = 0, \sigma) = A \left( \frac{M_h}{4 \times 10^{12} \, h^{-1} M_\odot} \right)^m \tag{6}
\]

in each \( \sigma \) bin; these fits are shown as the thin solid lines, with best-fit parameters and errors shown in Table A1. The powers \( m \) of halo mass are nearly identical for all transverse

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**Figure 5.** Left column shows the halo-forest cross-correlation calculated by applying LyMAS to (100 \( h^{-1} \) Mpc\(^3\)) (black dashed), (300 \( h^{-1} \) Mpc\(^3\)) (blue solid), and (1 \( h^{-1} \) Gpc\(^3\)) (green dot-dashed) dark matter volumes, in transverse separation bins \( \sigma = 1 - 4, 7 - 10, \) and 15 - 20 \( h^{-1} \) Mpc for halos with \( M_{12} = 1.68 - 3.35 \). Right column shows the cross-correlation from applying LyMAS to two different dark matter density distributions in (300 \( h^{-1} \) Mpc\(^3\)) volumes, from simulations with the same cosmological parameters but different initial conditions, in the same halo mass bin and transverse separation bins as the left column.
separation bins except the smallest, meaning that the mass
dependence of the correlation strength is nearly the same at
all transverse separations. The best-fit scaling factor
generally decreases with increasing transverse separation. Errors
on the parameters of the fit are the dispersion of the best-fit
parameter in the 16 subvolumes.

Dashed lines in Fig. 7 show the Tinker et al. (2010)
(hereafter T10) halo bias formula as a function of halo mass
at $z = 2.5$, scaled vertically to match the point at $\sim 4 \times 10^{12}$
h$^{-1} M_\odot$ for each transverse separation bin. The predicted
slope of the halo bias with mass matches the measured slope of
$\xi(\pi = 0, \sigma)$ with mass very well, except in the smallest $\sigma$
bin, where our measured slope is lower than the halo bias
prediction. The T10 bias as a function of halo mass is not
a perfect power law over the mass range plotted here, and
our highest mass points tend to fall below this relation. This
disagreement could be a consequence of our finite box size,
as even a $(1 \ h^{-1} \ Gpc)^3$ box contains relatively few of these
extreme halos at this redshift (300 halos in the largest mass
bin, compared to $\sim 1.58$ million in the smallest). As a rep-
resentation of our results, therefore, one can either take the
power-law fits reported in Table A1 or the T10 halo bias
mass dependence normalized to the reported fit amplitude
at $M_h = 4 \times 10^{12} \ h^{-1} M_\odot$ (corresponding to $\nu = 2.75$ in the
T10 formula for our adopted cosmology and $z = 2.5$). Lin-
ear theory predicts that $\xi(\pi = 0, \sigma)$ is nearly proportional to
$b_h$, but not exactly so because of the reduced redshift-space
distortion for higher halo bias (see equation (11) below).

Fig. 8 plots the dependence of the best-fit width of the
Lorentz profiles on transverse separation and halo mass. The
full width at half-minimum, $2\gamma$, of the Lorentz profile has
virtually no dependence on halo mass, so we fit a single linear
function to all bins simultaneously around a pivot point at
$\sigma = 17.5$, obtaining

$$\text{FWHM}(\sigma) = 2\gamma = (1.22\pm 0.02)(\sigma - 17.5) + (23.0\pm 0.6) \ . \ (7)$$
The relatively small errors on the slope and intercept show this fitting function works quite well for the relative width of the correlation in each transverse separation bin. Again, errors are the error on the mean of the best parameters fit to correlations in each of the 16 subvolumes.

We have so far considered correlations in narrow bins of halo mass, but quasars and DLAs likely occupy a wide range of host halos. We now examine the effect of a distribution of halo masses on the predicted correlation. We choose halos within each of the lowest four halo mass bins, $M_{12} = 0.42 - 0.84$, $0.84 - 1.68$, $1.68 - 3.35$, and $3.35 - 6.70$. We use three different mass distributions: one in which the number of halos in each bin reflects the mass distribution of all halos in the volume ("all halos", the numbers are 158,068, 61,548, 21,150, and 6,363, respectively), one in which the number of halos in each bin is the same ("flat", the number is 6,000), and one in which there are the same number of halos in the first and last bins and no halos in the middle two bins ("bimodal", the numbers are 6,000, 0, 0, and 6,000). Fig. 9 shows the halo-flux correlation for each of these distributions. Again, errors are from the subvolume method, as described in [3.1].

Also plotted in each panel of Fig. 9 is a Lorentz profile prediction for the correlation, with amplitude based on the best-fit power law in Fig. 7 (dashed lines). Within any $\sigma$ bin, the cross-correlation amplitude is approximately $M_{\text{eff}}^{0.272}$. Therefore we expect, at least for linear bias, that the clustering of the population will equal that of halos with the effective mass

$$M_{\text{h,eff}} = \left[ \frac{1}{N_{\text{halos}}} \sum_{i=1}^{N_{\text{halos}}} M_{i,272}^{0.272} \right]^{1/0.272},$$  

where $N_{\text{halos}}$ is the total number of halos used in the correlation, $M_{i,272}$ is the mass of each halo, and 0.272 is the weighted average of the best-fit powers in the power-law dependence of the correlation on halo mass for all but the smallest transverse separation bin, as given in Table 4. Equation (8) is equivalent to the condition

$$b(M_{\text{h,eff}}) = \frac{1}{N_{\text{halos}}} \sum_{i=1}^{N_{\text{halos}}} b(M_{i})$$

for a halo-bias relation $b \propto M_{\text{halo}}^{0.272}$. In Fig. 9, the amplitude of $\xi(\sigma = 0, \sigma)$ is computed from this value of $M_{\text{h,eff}}$ and the power-law fits in Table 4 and the width of the Lorentzian is determined from the linear fit of equation (7). The prediction using our best-fit parameters accurately matches the actual halo-flux correlation calculated using LyMAS in all cases. In linear theory, the cross-correlation strength should depend only on the number-weighted mean bias factor of halos. This expectation holds for the LyMAS predictions even down to small scales, indicating that bias-weighted cross-correlations can robustly constrain the mean halo mass but cannot discriminate among distributions with the same mean mass.

### 3.3 Comparison to Linear Theory

At sufficiently large scales, we expect the Ly-\(\alpha\) flux auto-correlation and the halo-flux cross-correlation to be adequately described by linear perturbation theory. In redshift-space the linear theory 3-d flux power spectrum is

$$P^F(k, \mu) = b_F^2 (1 + \beta_F \mu^2)^2 P_n(k),$$  

where $P_n(k)$ is the linear theory matter power spectrum, $\mu$ is the cosine of the angle between the wavevector $k$ and the line of sight, and $b_F$ and $\beta_F$ are the bias factor and redshift-space distortion parameter of the flux (Kaiser [1987], McDonald [2003]). For halos or galaxies one expects $\beta = f(z)/\bar{b} \approx [\Omega_m(z) h^2]^{0.55}/b$, where $f(z)$ is the fluctuation growth rate. However, for flux this relation does not hold because of the non-linear relation between flux and optical depth, and $\beta$ must be predicted separately (for more discussion see Seljak [2012], Arinyo-i-Prats et al. [2013]). For the halo-flux cross power spectrum the linear theory prediction is

$$P^{Fh}(k, \mu) = b_F b_h (1 + \beta_F \mu^2) (1 + \beta_h \mu^2) P_n(k),$$  

where the halo bias $b_h$ can be calibrated from numerical simulations (T10) and $\beta_h = f(z)/b_h$. Correlation functions can be computed from power spectra using the appropriate Fourier transform relations (e.g., equations 4.5 - 4.12 of Slosar et al. [2011]). The matter power spectrum used for our linear theory calculations has the same WMAP7 parameters adopted for the hydro and $N$-body simulations (see [2]). We measure $\xi(r, \mu)$ in bins of $\Delta \mu = 0.05$ and compute the monopole and quadrupole $\xi_0(r)$ and $\xi_2(r)$ from the sum

$$\xi_l(r) = (2l + 1) \sum_{\mu=-l}^{\mu=l} \xi_l(r, \mu) P_l(\mu) \Delta \mu,$$  

where $P_0(\mu) = 1$, $P_2(\mu) = \frac{3}{2} \mu^2 - 1$ are the Legendre polynomials of degree $l$.

Fig. 10 shows the monopole and quadrupole of the flux auto-correlation in the (1 h$^{-1}$ Gpc)$^3$ LyMAS simulation as a function of separation. Errors are the error in the mean among 16 subvolumes as described in [3.3]. The top row shows the correlation in linear space, and the bottom row is the same correlation in log-log space. The dashed and dotted lines in each panel show linear theory predictions.
with $b_F$ and $\beta_F$ determined by fitting the full range of data or only $r > 15$ $h^{-1}$ Mpc, respectively.

The $\chi^2$ for the large separations is 43.23 for 34 d.o.f. but the fit to the full data range is extremely poor, with $\chi^2 = 4593$ for 48 d.o.f. Visually one can see that the fit to $r > 15$ $h^{-1}$ Mpc gives a good description of the LyMAS monopole and quadrupole at these scales, but the LyMAS quadrupole rises above this fit at smaller $r$. At intermediate $r$ ($\approx 10$ to $50$ $h^{-1}$ Mpc), the linear theory model fit to the full range underpredicts $\xi_0(r)$ and overpredicts $|\xi_2(r)|$. For $r > 15$ $h^{-1}$ Mpc our best-fit parameters are $\beta = 0.970 \pm 0.016$, $b = -0.178$. There is significant degeneracy between these parameters, and Ly-$\alpha$ forest analyses frequently quote the combination $b(1 + \beta)$, which is better determined than $b$ or $\beta$ alone. Our $r > 15$ $h^{-1}$ Mpc fit yields $b(1 + \beta) = -0.3525 \pm 0.0011$.

Analyses of the BOSS Ly-$\alpha$ forest have reported a variety of values for $\beta$ and $b(1 + \beta)$ (e.g., Slosar et al. 2011; Busca et al. 2013; Delubac et al. 2015), with significant dependence on details of the data analysis. The most precise, and likely most robust, determination is from Blomqvist et al. (2015), who implement a new method for removing biases imprinted by the continuum determination procedure. They find $\beta = 1.39^{+0.11}_{-0.10}$ and $b(1 + \beta) = -0.374 \pm 0.007$ at $z = 2.3$. They do not fit for redshift dependence, so here we scale the value $b(1 + \beta)$ by $[(1 + 2.5)/(1 + 2.3)]^{2.9} = 1.19$ using the redshift evolution reported by Slosar et al. (2011). We have also multiplied the Blomqvist et al. (2015) predictions by a factor 0.79/0.81 to account for the slightly lower $\sigma_8$ value that they assumed. With these parameters the Blomqvist et al. (2015) fit lies above the LyMAS prediction, by factors of approximately 1.25 and 1.6 for the monopole and quadrupole, respectively. If we did not apply the redshift scaling to $b(1 + \beta)$ these factors would be approximately 0.95 and 1.25.

We leave investigation of this discrepancy in the auto-correlation function to future work. There are a number of possible contributions, and it is not clear whether one dom-
The mismatch in Fig. 10 is cause for some caution about the mass or bias values that we infer from fitting BOSS cross-correlation measurements in [15]. However, we also note that different methods for inferring $\beta$ and $b$ from BOSS have yielded a wide range of values because of the challenges of correcting for continuum determination biases, metal-line contamination, and DLA absorption, and the Blomqvist et al. (2015) values may not be the last word. There is also uncertainty in scaling from $z = 2.3$ to our simulation redshift of $z = 2.5$.

Recently, Arinyo-i-Prats et al. (2015), extending the work of McDonald (2003), have attempted to compute $b$ and $\beta$ theoretically using 3-d flux power spectra measured.

Although we find excellent agreement between hydro and LyMAS calculations of the halo-flux cross-correlation in the (100 $h^{-1}$ Mpc)$^3$ simulations (Fig. 3), the auto-correlation monopole and quadrupole differ at the $10 - 20\%$ level at $r \sim 10 h^{-1}$ Mpc, while differences at larger scales are harder to assess because the correlation function is artificially suppressed by the finite box size (W. Zhu, private communication). The mismatch in Fig. 10 is cause for some caution about the mass or bias values that we infer from fitting BOSS cross-correlation measurements in [15]. However, we also note that different methods for inferring $\beta$ and $b$ from BOSS have yielded a wide range of values because of the challenges of correcting for continuum determination biases, metal-line contamination, and DLA absorption, and the Blomqvist et al. (2015) values may not be the last word. There is also uncertainty in scaling from $z = 2.3$ to our simulation redshift of $z = 2.5$.

Recently, Arinyo-i-Prats et al. (2015), extending the work of McDonald (2003), have attempted to compute $b$ and $\beta$ theoretically using 3-d flux power spectra measured.
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from a number of hydro simulations with comoving box sizes ranging from $40 - 120 \ h^{-1}\ Mpc$. Because of the limited box size, these authors fit non-linear models to the numerical $P_F(k, \mu)$, and the extrapolated large scale values of $b$ and $\beta$ depend on the assumed non-linear models as well as on simulation and cosmological parameters. Their estimates of $\beta$ at $z \approx 2.5$ are typically $1.2 - 1.4$, clearly higher than the value we find here from LyMAS. The values of $b$ are harder to compare because they are more sensitive to cosmological parameters and the adopted form of the non-linear model. LyMAS and the model-based extrapolation method of Arinyo-i-Prats et al. [2015] represent two different strategies for deriving large scale Ly-$\alpha$ forest clustering from limited volume hydro simulations, and over time we hope that they will converge to yield consistent predictions given the same IGM and cosmological parameters.

Returning to the cross-correlation that is the focus of this paper, Fig. [11] compares the halo-flux $\xi(\pi, \sigma)$ from LyMAS to the linear theory model for two different halo mass ranges. In this case, the linear theory is that for the cross-correlation between Ly-$\alpha$ forest and halos, with redshift distortion parameter $\beta_F = 0.97$ and Ly-$\alpha$ bias $b_F = -0.178$, equivalent to our best-fit parameters to the $r > 15 \ h^{-1}\ Mpc$ fit to the LyMAS auto-correlation. For the $M_{12} = 0.42 - 0.84$ halo mass range, the T10 bias is $b_h = 2.56$, and the T10 halo bias for the larger $M_{12} = 1.68 - 3.35$ bin is $b_h = 3.71$. With these bias values the LyMAS $\xi(\pi, \sigma)$ visually matches linear theory closely even down to $\sigma = 1 - 4 \ h^{-1}\ Mpc$ and $\pi \approx 0$. However, there are statistically significant residuals for small $\pi$ at $\sigma = 1 - 4 \ h^{-1}\ Mpc$ and $\pi = 0$. However, the quadrupole rises above linear theory at $r = 10 - 20 \ h^{-1}\ Mpc$, then flattens below it at $r < 5 \ h^{-1}\ Mpc$. This is consistent with the general behavior seen for the cross-correlation at larger values of $\pi$ and $\sigma$.

Fig. [12] examines the comparison between LyMAS and linear theory in terms of the monopole and quadrupole of the halo-flux cross-correlation. We use the same $\beta_F$ and $b_F$ values as in Fig. [11] and the halo bias factors from the T10 formula evaluated at the logarithmic center of each mass bin. The monopole is well described by linear theory at $r > 3 \ h^{-1}\ Mpc$, with some deviation for the highest mass bin $M_{12} = 26.8 - 53.6$. However, the quadrupole rises above linear theory at $r = 10 - 20 \ h^{-1}\ Mpc$, then flattens below it at $r < 5 \ h^{-1}\ Mpc$. This is consistent with the general behavior seen for the cross-correlation at larger values of $\pi$ and $\sigma$.

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$h^{-1}$ Mpc. Thus, the level of agreement between LyMAS and linear theory in Fig. 11 is to some degree a consequence of the particular quantity plotted. Fig. 13 plots $\xi(\mu)$ for comoving separation $r = 5 \, h^{-1}$ Mpc. The two lowest mass bins show reasonably good accord with linear theory at all $\mu$, with LyMAS rising slightly above for separations perpendicular to the line of sight ($\mu \approx 0$) and falling slightly below for separations along the line of sight ($\mu \approx 1$). However, for the two highest mass bins LyMAS yields substantially weaker correlations at $\mu > 0.5$ ($M_{12} = 6.7 - 13.4$) or $\mu > 0.3$ ($M_{12} = 26.8 - 53.6$). The overall normalization of the linear theory curves, and to a lesser degree the $\mu$ dependence, depends on the $b_F$ and $\beta_F$ values inferred from the LyMAS flux auto-correlation, and the volume of the ($100 \, h^{-1}$ Mpc)$^3$ hydro simulation limits our ability to test the accuracy of LyMAS in predicting these quantities.

In summary, for halo masses in the range $M_{12} \approx 0.5 - 3$ found by FR12 and FR13, LyMAS predictions for $\xi(\pi, \sigma)$ in bins of $\sigma$ follow linear theory even down to small scales. However, other representations show significant deviations between LyMAS and linear theory, especially at higher halo masses and high values of $\mu$.

4 COMPARISON TO OBSERVED CROSS-CORRELATIONS

Now that we have examined how the cross-correlation depends on dark matter halo mass and separation from the halo (in both line-of-sight and transverse directions), we can compare the halo-forest correlation to other cross-correlations with the forest, such as quasars and DLAs. This will allow us to determine a characteristic mass for the host halos of these objects.

The measured cross-correlations of the Ly-α forest with DLAs and with quasars (FR12, FR13) are plotted in Fig. 14 as the red circles and green triangles, respectively. These cor-
relations were measured using ~ 60,000 quasar spectra in DR9 of BOSS; measurements from the final, DR12 BOSS sample are in progress. A surprising feature of the data points is that the DLA and quasar cross-correlations are similar in amplitude, even though FR12 and FR13 derive quite different bias factors \( \langle b_{\text{DLA}} \rangle = 2.17 \pm 0.2, b_0 = 3.64 \pm 0.13 \) for these two populations. However, the FR13 \( b_0 \) fit is driven by the large \( r = \sqrt{\sigma^2 + \sigma_0^2} \) bins, and the \( r < 15 h^{-1} \) Mpc bins are discarded, while smaller scale measurements carry much of the weight in the FR12 \( b_{\text{DLA}} \) fits. We have calculated the correlation function in LyMAS in the same transverse separation bins as the measured correlation to obtain the closest comparison. Lorentz profiles with amplitude, width, and offset predicted by our LyMAS calculation for halos of mass \( M_{12} = 0.5 \) (black solid), 2.0 (blue dashed), and 8.0 (cyan dot-dashed), are determined by the equations \( b_0 \) and \( a_0 \) and Tables A1 and A2.

Because BOSS Ly-\( \alpha \) forest spectra are calibrated to reproduce the mean Ly-\( \alpha \) absorption as a function of redshift, there is a slight bias in cross-correlation measurements, since the positive correlation over the full Ly-\( \alpha \) forest is calibrated out. FR12 and FR13 apply a Mean Transmission Correction (MTC) that removes this bias. The derivation of the MTC is laid out in Appendix A of FR12. We apply this correction to our predicted cross-correlation whenever we compare to the measured correlation of quasars or DLAs. Generally, the correction shifts correlations upward so that the cross-correlation for halos of the same mass in the Ly-\( \alpha \) forest from LyMAS at a separation of \( r = 5 h^{-1} \) Mpc is comparable to the RMS redshift error but becomes negligible at large \( |\pi| \) becomes positive instead of near zero. It is a small effect, but important for fitting the non-zero values of the measured cross-correlation.

In the \( \sigma \approx 10 - 40 h^{-1} \) Mpc transverse separation bins, it appears as though the cross-correlation for halos of average mass \( M_{12} = 0.5 \) or 2.0 fits the measurement of the cross-correlation for DLAs or quasars fairly well. However, the correlation for halos of the same mass in the \( \sigma = 1 - 4 h^{-1} \) Mpc separation bin does not fit the data. We expect that, if there is a characteristic mass distribution for halos that host DLAs or quasars, then the correlation predicted for that mass distribution should match the observations for all separation bins. The drastic overestimation of the strength of the correlation in the smaller transverse separation bins indicates a missing component in our model of the halo-flux cross-correlation.

Fig. 13 summarizes this data-model comparison by fitting our Lorentz profile form to the DLA and quasar data in each \( \sigma \) bin. We show the measured amplitude as a function of \( \sigma \) by circles and triangles for DLAs and quasars, respectively, while solid curves show the amplitude of Lorentz profiles fit to the simulation cross-correlations for three different halo mass ranges. As in Fig. 13 it is evident that the DLAs and quasars exhibit similar overall clustering strength but that any halo mass that matches the observed clustering strength at \( \sigma = 20 - 50 h^{-1} \) Mpc overpredicts it at \( \sigma = 1 - 10 h^{-1} \) Mpc. If our predictions were based on linear theory one might conclude that this trend reflects the breakdown of linear theory predictions, but LyMAS is a fully non-linear code and we have shown (Fig. 2) that it reproduces hydrodynamic simulation results essentially perfectly even at \( \sigma = 1 - 4 h^{-1} \) Mpc. The discrepancy could reflect some form of feedback or IGM physics not incorporated into our hydro simulations, but the complete insensitivity of the halo-flux predictions to the presence or absence of AGN feedback (Fig. 5) makes such an explanation unlikely.

One key difference between the simulations and the observations is that we know the redshift-space position of simulated halos exactly while the redshifts of quasars and DLAs must be estimated observationally from emission and absorption lines. Internal motions and radiative transfer effects lead to differences of many hundreds of km s\(^{-1}\) among quasar redshifts estimated from different lines (Espey et al. 1989; Richards et al. 2002; Hewett & Wild 2010). From the asymmetry of \( \xi(\pi, \sigma) \), FR13 concluded that the quasar redshifts reported by the BOSS pipeline had a systematic offset of \(-157 \) km s\(^{-1}\), which is small but not completely negligible compared to the width of \( \xi(\pi, \sigma) \) in bins of small \( \sigma \). More importantly, random redshift errors with a velocity distribution \( f(\pi) \) will convolve the predicted \( \xi(\pi, \sigma) \) in the \( \pi \) direction, increasing the width but decreasing the depth of \( \xi(\pi, \sigma) \). This convolution has a significant impact when \( \sigma \) is comparable to the RMS redshift error but becomes negligible at large \( \sigma \), so it has the correct qualitative form to reconcile our predictions with the BOSS measurements.

Fig. 13 presents a fit to the FR13 data in which we add their inferred mean \( \sigma \) offset of 157 km s\(^{-1}\) to the model predictions and add the RMS velocity error as a free parameter along with halo mass. We use the full covariance matrix that FR13 estimated analytically by treating the forest as a Gaussian random field. There are significant off-diagonal correlations of errors at different \( \pi \) in a given \( \sigma \) bin, but correlations across \( \sigma \) bins are generally weak. In our fit we adopt the Lorentz function form of equation \( 5 \) convolved with a Gaussian distribution of redshift errors, using equation \( 1 \) This statement holds true even though our \( \sigma \) bins have a finite width, a point we have tested explicitly by adding random offsets to halo positions and remeasuring \( \xi(\pi, \sigma) \). Note that an offset of \( \pi \) comoving \( h^{-1} \) Mpc corresponds to a velocity offset \( v = \pi \times (1 + z)^{-1} \times (100 \text{ km s}^{-1}) \times H(z)/H_0 \) at redshift \( z \). For our cosmology at \( z = 2.5, v = 101 \pi \text{ km s}^{-1} \).
Cross-Correlations with LyMAS

Figure 14. The cross-correlation as predicted by LyMAS (described by Lorentz profile fits, and adding the mean transmission correction) for halo masses of $M_{12} = 0.5$ (black solid), $M_{12} = 2.0$ (blue dashed) and $M_{12} = 8.0$ (cyan dot-dashed). Red circles and green triangles are the cross-correlation measured by FR12 and FR13 for DLAs and quasars, respectively. Panels show different transverse separation bins as indicated, increasing top to bottom, left to right.

It is evident from Fig. 14 that including the RMS redshift error as a parameter dramatically improves the fit to the FR13 measurements, eliminating the systematic mismatch between large and small $\sigma$ bins seen in Figs. 14 and 15. The best-fit RMS error is $399 \pm 21$ km s$^{-1}$, corresponding to $\pi = 3.8 \, h^{-1}$ Mpc for our WMAP7 cosmological parameters. This RMS error appears plausible relative to comparisons of different redshift estimators as described in the BOSS DR9 quasar catalog paper of Paris et al. (2012). The global best-fit halo mass is $M_{12} = 1.51^{+0.12}_{-0.10}$. As discussed in §2, this should be interpreted as a bias-weighted mean host halo mass of quasars, i.e., it is the mass $\bar{M}$ for which

$$b(\bar{M}) = \frac{\int_{0}^{\infty} dM b(M)p(M)dn/dM}{\int_{0}^{\infty} dM \, p(M)dn/dM}$$

(13)

where $p(M)$ is the probability that a halo of mass $M$ hosts a BOSS quasar at any given time and $dn/dM$ is the halo mass function. With the T10 halo bias relation at $z = 2.5$, our halo mass range corresponds to $b(\bar{M}) = 3.18 \pm 0.06$. Evaluated at the central redshift of the observational measurement, $z = 2.3$, the same halo mass corresponds to $b(\bar{M}) = 2.92 \pm 0.06$.

One can see from Fig. 16 that the LyMAS predictions including RMS velocity errors are a good but not perfect fit to the FR13 measurements, given the extremely small statistical errors of the latter. The overall $\chi^2$ is 290 for 162 data points, with two fitting parameters. Fig. A2 also shows residuals between the LyMAS numerical results and our Lorentzian fits to those results, but those residuals are small compared to the differences between the model predictions and the data. We have checked that adding these residu-
als (scaled in amplitude by the halo bias) to our convolved Lorenzian model — in effect, using the direct simulation results in place of the Lorentzian description of them — does not noticeably change the \( \chi^2 \) of the fit or the best-fit values of the model parameters.

Fig. 15 examines the \( \chi^2 \) contributions in greater detail, with each panel showing the \( \chi^2 \) from data points in a single \( \sigma \) bin as a function of \( M_{12} \), for our best-fit RMS velocity error of 399 km s\(^{-1}\) and for velocity errors of 100, 200, or 600 km s\(^{-1}\). With a small velocity error, the \( \sigma = 1 - 4 \) h\(^{-1}\) Mpc bin has a high \( \chi^2 \) for any \( M_{12} \), and, as seen previously in Fig. 14 the value of \( M_{12} \) that would match in this bin is strongly discordant with the values that fit larger \( \sigma \) bins. Conversely, a large velocity error makes \( \xi(\pi, \sigma) \) too broad in the innermost bins, worsening \( \chi^2 \). Even with the best-fit RMS error, the innermost bin favors a lower \( M_{12} \) than other bins. Apart from this bin, the \( \sigma \) bins with the largest \( \chi^2 \) contributions are 15 – 20 h\(^{-1}\) Mpc, 20 – 30 h\(^{-1}\) Mpc, and 40 – 60 h\(^{-1}\) Mpc, where one can see visually in Fig. 14 that our best-fit model underpredicts the measured correlation. In other bins the global best-fit \( M_{12} = 1.51 \) yields close to the minimum \( \chi^2 \) contribution, and the \( \chi^2 \) is not very far from the value 18 ± 4.2 expected for 18 data points.

The slight mismatch between our simulation output at \( z = 2.5 \) and the central redshift \( z = 2.3 \) of the FR13 observational analysis introduces some ambiguity in comparing our inferred halo bias and halo mass to the results of FR13’s linear theory fit. (Ongoing work with the much larger BOSS DR12 sample will allow tailored comparisons in fairly narrow redshift bins.) Our value of \( b(M) = 3.18 ± 0.06 \) is formally inconsistent with the \( b_0 = 3.64_{-0.15}^{+0.13} \) found by FR13, and our inferred halo mass is correspondingly lower. Using the T10 halo formula at \( z = 2.3 \), the FR13 best-fit bias corresponds to \( M_{12} = 3.26 \). Green curves in Fig. 16 show LyMAS predictions for this halo mass at \( z = 2.5 \), with our best-fit RMS velocity error. While this comparison is inexact be-cause of the redshift mismatch, it appears likely that most of the difference between our inferred bias and the somewhat higher value of FR13 arises because FR13 fit to points with \( r > 15 \) h\(^{-1}\) Mpc, thus matching the \( \sigma = 15 – 60 \) h\(^{-1}\) Mpc bins well and not penalizing the overly strong correlations predicted at small \( \sigma \) in the smaller \( \sigma \) bins. The poor global \( \chi^2 \) of our fit indicates that one should be cautious about our formal error bar. Furthermore, it is clear from Fig. 17 that higher \( M_{12} \) is disfavored mainly by the innermost \( \sigma \) bin and is therefore sensitive to our assumption of Gaussian velocity errors. If we keep the best-fit velocity error but drop the innermost bin from the mass determination we get \( M_{12} = 2.19_{-0.15}^{+0.16} \), with corresponding \( b(M) = 3.63_{-0.07}^{+0.08} \) at \( z = 2.5 \).

The mild tension between the halo mass and bias factor inferred here and in FR13 is an indication of the current level of systematic uncertainty associated with model fitting and data selection. The FR13 value has the virtue of using a model that has an acceptable \( \chi^2 \) over the range fitted (\( r > 15 \) h\(^{-1}\) Mpc) and matching the central redshift of the data. It relies on the value of \( b_0(1 + \beta_r) \) inferred from fitting the observed flux auto-correlation, and this fitting has statistical and systematic uncertainties of its own. LyMAS directly predicts the halo-flux cross-correlation and thus circumvents this step, but the mismatch between the LyMAS flux auto-correlation and the [Blomqvist et al. (2015)] linear theory parameters (Fig. 11) could indicate that the IGM structure in the hydrodynamic simulations on which LyMAS is calibrated are incorrect. The excellent match between LyMAS and the full hydrodynamic simulation predictions of the cross-correlation indicates this is not an issue with LyMAS itself, but it could be a consequence of the physics and parameters assumed in the hydrodynamic simulations. The principal strength of the LyMAS mass determination is its use of a fully non-linear model that describes \( \xi(\pi, \sigma) \) down to small scales. A hydrodynamic simulation on its own cannot make this prediction accurately because of limited volume, but the combination of hydrodynamics and LyMAS can.

Fig. 18 presents our fit to the FR12 DLA measurements including velocity errors, in the same format as Fig. 13. Here we assume no mean velocity shift, as there is nothing expected to produce a systematic offset in DLA redshifts. Our global best fit yields an RMS velocity error of 252±63 km s\(^{-1}\) and an effective host halo mass \( M_{12} = 0.69_{-0.14}^{+0.16} \) with corresponding \( b(M) = 2.66 ± 0.14 \) at \( z = 2.5 \). The inclusion of velocity errors makes an important difference in the \( \sigma = 1 - 4 \) h\(^{-1}\) Mpc bin and little difference in other bins. Our model of Gaussian velocity errors with a single RMS for DLAs is questionable, as there is likely a strong dependence of velocity errors. If we keep the best-fit velocity error but drop the innermost bin from the mass determination we get \( M_{12} = 0.45 \) and our best-fit velocity error, and one can see that the differences from our
best-fit model prediction are small compared to the observational error bars.

Analysis of the full BOSS DR12 data set will allow more detailed and more precise measurements for the cross-correlations of both quasars and DLAs, allowing narrower redshift bins for evolution measurements, dividing samples into subsets of luminosity, equivalent width, or metal-line strength, and achieving good statistical constraints at large separations. Modeling these measurements with both linear theory and non-linear predictions from LyMAS should yield numerous insights into the quasar and DLA populations, and comparisons of these model fits will either remove or sharpen the mild tensions found here for the quasar cross-correlations. With the DR12 data, it may be possible to infer the RMS velocity of DLAs within their host halos by matching the small-scale correlation for the subset of DLAs that have metal-line absorption and thus small redshift measurement errors. This measurement would be an interesting new constraint on the physics of DLA absorption.

5 CONCLUSIONS

Cross-correlation with the densely mapped Ly-α forest opens a new window on the properties of quasars, DLAs, and potentially other populations of high-redshift objects. At large scales, one expects these correlations to be described by linear perturbation theory and linear bias, and precise measurements of quasar and DLA bias can be translated (for a specified cosmological model) to precise constraints on the masses of their host dark matter halos (FR12, FR13). Interpreting small scale cross-correlations requires a fully non-linear hydrodynamic model of the Ly-α forest. However, cosmological hydrodynamic simulations with the required resolution cannot model large enough volumes to provide accurate predictions of 3-d cross-correlations.

LyMAS provides a way to bootstrap hydrodynamic simulation results onto large volume N-body simulations and thus predict halo-flux cross-correlations over the full range of observable scales. When applied to the dark matter distri-
for any of the FR12/FR13 \( \sigma \) parameters in Tables A1 and A2 to reproduce our results which can be used along with equations (6) and (7) and the by the empirically chosen Lorentzian form of equation (5), \( z \) at assumptions even at scales \( \sigma \) measure of cosmological structure is insensitive to feedback implemented in Horizon-AGN (Fig. 4), implying that this \( \xi \) range investigated here (Fig. 6). Furthermore, the predicted \( \xi \) distribution of the Horizon-AGN or Horizon-noAGN simulations, \( \xi \) reproduces the cross-correlations \( \xi(\pi, \sigma) \) predicted using the full hydrodynamic Ly-\( \alpha \) forest spectra almost exactly (Fig. 2). The LyMAS predictions are insensitive to \( \alpha \) using the full hydrodynamic Ly-\( \alpha \) forest determined by fitting the LyMAS flux auto-correlation measurement. Black curves adopt our global best-fit value of 399 km s\(^{-1}\) for the RMS velocity error, while dashed, dotted, and dot-dashed curves impose RMS velocity errors of 100, 200, and 600 km s\(^{-1}\), respectively. Filled circles mark the minimum of each curve, i.e. the halo mass preferred by the data in this \( \sigma \) bin alone for the adopted velocity error. Open black circles show the global best-fit \( M_{12} = 1.51 \), and the contribution to \( \chi^2 \) from each \( \sigma \) bin for this best-fit model is listed. There are 18 \( \xi(\pi, \sigma) \) data points in each \( \sigma \) bin.

The difference between the Lorentzian fit and the numerical results is significant at the level of our simulation’s tiny statistical errors, but it is small compared to current observational errors. Except for the \( \sigma = 1 - 4 \) h\(^{-1}\) Mpc bin, the trend of cross-correlation amplitude with halo mass follows the standard large scale \( b(M_h) \) relation from T10 (Fig. 7). Furthermore, even on small scales, \( \xi(\pi, \sigma) \) for a halo population with a range of masses depends only on the number-weighted halo bias (eq. 9). Modeling quasar or DLA observations down to these scales therefore improves constraints on the characteristic halo mass but does not provide information about the width or shape of the halo mass distribution.

![](https://example.com/figure.png)

Figure 17. \( \chi^2 \) as a function of halo mass in each \( \sigma \) bin of the FR13 quasar-flux cross-correlation measurement. Black curves adopt our global best-fit value of 399 km s\(^{-1}\) for the RMS velocity error, while dashed, dotted, and dot-dashed curves impose RMS velocity errors of 100, 200, and 600 km s\(^{-1}\), respectively. Filled circles mark the minimum of each curve, i.e. the halo mass preferred by the data in this \( \sigma \) bin alone for the adopted velocity error. Open black circles show the global best-fit \( M_{12} = 1.51 \), and the contribution to \( \chi^2 \) from each \( \sigma \) bin for this best-fit model is listed. There are 18 \( \xi(\pi, \sigma) \) data points in each \( \sigma \) bin.

| Log Halo Mass \( [M_\odot h^{-1}] \) | \( \chi^2 \) | Best-fit \( z_{\text{err}} = 399 \) km s\(^{-1}\) |
|-----------------|-----------------|-----------------|
| 11              | 20              | \( \chi_{1.51}^2 = 50.86 \) |
| 11.5            | 20              | \( \chi_{1.51}^2 = 40.40 \) |
| 12              | 20              | \( \chi_{1.51}^2 = 37.26 \) |
| 12.5            | 20              | \( \chi_{1.51}^2 = 37.63 \) |

We have compared the LyMAS results to linear theory predictions using bias parameters \( b_F \) and \( \beta_F \) for the Ly-\( \alpha \) forest determined by fitting the LyMAS flux auto-correlation, and T10 values for the halo bias. We find surprisingly good agreement in \( \xi(\pi, \sigma) \) even at \( \sigma = 1 - 4 \) h\(^{-1}\) Mpc, with maximum deviations of \( \sim 10\% \) near \( \pi = 0 \) h\(^{-1}\) Mpc.
However, in a monopole-quadrupole representation, linear theory and LyMAS differ significantly in the quadrupole at \( r < 15 \; h^{-1} \) Mpc, while agreement for the monopole is good down to \( r = 3 \; h^{-1} \) Mpc (Fig. 12). In a \( \xi(r, \mu) \) representation (Fig. 13), deviations are \( \mu \)-dependent, and they are larger for higher halo masses.

When comparing to the BOSS measurements of FR12 and FR13, we find that any choice of halo mass that fits \( \xi(\pi, \sigma) \) at \( \sigma > 7 \; h^{-1} \) Mpc overpredicts the amplitude of the cross-correlation at smaller scales (Fig. 14). The most likely explanation for this discrepancy is scatter between halo velocities and measured redshifts in BOSS, which can arise from a combination of redshift measurement errors and random peculiar velocities of quasars or DLAs within their host halos; the velocities of the halos themselves are already accounted for in the LyMAS predictions. We fit the quasar (FR13) and DLA (FR12) measurements with a model that convolves the LyMAS predictions with a Gaussian distribution of redshift offsets of RMS amplitude \( \sigma_{\text{err}} \), which removes the tension between large-scale and small-scale predictions.

For quasars, we also incorporate the \(-157 \; km \; s^{-1}\) mean redshift offset inferred by FR13 from the asymmetry of \( \xi(\pi, \sigma) \).

For quasars (Fig. 16) we find \( \sigma_{\text{err}} = 399 \pm 21 \; km \; s^{-1}\), a magnitude that is compatible with other estimates of quasar redshift errors from BOSS spectra [Paris et al. 2012]. Our best-fit halo mass is \( M_{12} = 1.51^{+0.11}_{-0.10} \), which corresponds to a halo bias \( b(M) = 3.18 \pm 0.06 \) at \( z = 2.5 \) for our WMAP7 cosmology (using the T10 halo bias relation). For DLAs (Fig. 18) we find \( \sigma_{\text{err}} = 252^{+63}_{-53} \; km \; s^{-1}\), a value that appears high for pure measurement errors but may reflect a quadrature sum of measurement errors and halo internal velocities. Our best-fit halo mass is \( M_{12} = 0.69^{+0.16}_{-0.14} \), with corresponding \( b(M) = 2.66 \pm 0.14 \) at \( z = 2.5 \).

Relative to linear theory modeling, the LyMAS results have the advantage of fitting a fully non-linear model to the full range of measurements, and (as a necessary consequence) fitting for redshift scatter as well as effective halo mass. However, there are several caveats to our numbers.
First, the slight mismatch between our simulation’s $z = 2.5$ redshift output and the $z = 2.3$ central redshift of the BOSS measurements introduces ambiguity; our best-fit halo masses correspond to bias factors that are about 10% lower at $z = 2.3$ than at $z = 2.5$ (see Fig. 10), and it is not clear which is the more appropriate characterization of our inferred halo bias. Second, the flux auto-correlation from the $1 \ h^{-1}$ Gpc LyMAS box is significantly below that from the linear theory fit of Blomqvist et al. (2015) to the BOSS auto-correlation measurements at $z = 2.3$ (Fig. 10), and our best-fit values of $\beta$ and $b_p(1 + \beta)$ do not agree with the values extrapolated from smaller volume hydro simulations by Arinyo-i-Prats et al. (2015). This difference could reflect a combination of incorrect thermal structure of the diffuse ISM in our calibrated hydro simulations (e.g., if the real universe has a different history of helium reionization), inaccurate prediction of the large scale flux auto-correlation by LyMAS, uncertainties on scaling results between $z = 2.3$ and $z = 2.5$, and errors in the Blomqvist et al. (2015) measurements, which themselves require substantial corrections for the impact of continuum determination in the observed spectra.

Finally, for quasars, our fit has a statistically unacceptable $\chi^2$/d.o.f. = 290/160. The largest contribution to $\chi^2$ comes from our innermost $\sigma$-bin, where predictions are most sensitive to our assumption of Gaussian velocity errors. If we eliminate this bin from our mass fit (but retain the best-fit $z_{\mathrm{err}}$) we find $M_{12} = 2.19^{+0.10}_{-0.16}$, corresponding $b(M) = 3.63^{+0.08}_{-0.06}$ at $z = 2.5$. We regard this as our most reliable estimate of halo mass and halo bias for BOSS quasars because of its lower sensitivity to our $z_{\mathrm{err}}$ model. The $\chi^2$/d.o.f. improves to 197/142, but it is still high, reflecting a tension between lower masses favored by the $\sigma = 4 - 15 \ h^{-1}$ Mpc data and the higher masses favored by $\sigma = 15 - 60 \ h^{-1}$ Mpc data (Fig. 17). If instead we eliminate all data points with $r < 15 \ h^{-1}$ Mpc but retain our best-fit $z_{\mathrm{err}} = 399 \ \kms$ at $z = 2.5$ the $\chi^2$/d.o.f. for this fit remains high at 221/132. For DLAs, the measurement errors are larger, and our fit (with $M_{12} = 0.69^{+0.16}_{-0.14}$ and $b(M) = 2.39^{+0.13}_{-0.11}$) has an acceptable $\chi^2$/d.o.f. = 123/126.

Our DLA halo bias is consistent with that inferred by FR12 using linear theory modeling at separations $r > 5 \ h^{-1}$ Mpc. Our quasar halo bias is lower than that inferred by FR13; an exact comparison is complicated by the slight difference in model redshift, but the best-fit FR13 parameters correspond to a predicted cross-correlation that is stronger by about $24\%$ (see Fig. 16). The main source of difference appears to be the selection of fitting range, as FR13’s elimination of $r < 15 \ h^{-1}$ Mpc separations removes the points that are favoring lower bias in our fit. The surprisingly good agreement between LyMAS and linear theory predictions of $\xi(\pi, \sigma)$ suggests that smaller separations can be retained even in a linear theory fit, provided one accounts for quasar redshift errors.

Analyses of the final BOSS data set will allow cross-correlation measurements in bins of redshift and tracer properties (quasar luminosity, DLA metallicity) while retaining useful statistical precision on large scales. Comparison of observations, linear theory fits, and LyMAS modeling across these bins will allow tests for internal consistency (e.g., expected redshift scaling) and investigations of the relation between tracer properties and halo mass. The importance of $z_{\mathrm{err}}$ in our fits highlights a new opportunity to constrain the velocity distribution of quasars or DLAs within their host halos. For quasars, one can imagine measuring the mean offset and dispersion of redshifts inferred from different emission lines as a diagnostic of gas motions and ionization states in the immediate surroundings of the central black hole. Ionizing radiation from quasars should also affect cross-correlations on small scales (the “transverse proximity effect”), and it may be possible to separate this impact from redshift dispersion effects via the expected dependence on quasar luminosity. For DLAs the comparison of RMS velocities to halo circular velocities may be a useful diagnostic for the kinematics of the metal-line absorption, perhaps distinguishing rotating disks from more chaotic gas distributions. The first studies of Ly-$\alpha$ forest cross-correlations have already yielded the most precise measurements of quasar and DLA clustering strength, and this tool holds considerable promise as a probe of the physics of high-redshift systems.

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APPENDIX A: CROSS-CORRELATION FITTING FUNCTION

Fig. A1 shows the LyMAS predictions for the cross-correlation of the $M_{12} = 0.84 - 1.68$ halos from the 2048$^3$ simulation of the $(1 \ h^{-1} \ Gpc)^3$ volume with $r_s = 0.5 \ h^{-1} \ Mpc$, for transverse separation bins that range from $\sigma = 1 - 4 \ h^{-1} \ Mpc$ up to $\sigma = 80 - 80 \ h^{-1} \ Mpc$. The correlation at $\pi = 0$ weakens steadily with increasing $\sigma$ as expected, from $\xi(\pi = 0, \sigma) \approx 0.3$ in the $\sigma = 1 - 4 \ h^{-1} \ Mpc$ bin to 0.04 at $\sigma = 10 - 15 \ h^{-1} \ Mpc$ to 0.002 at $\sigma = 60 - 80 \ h^{-1} \ Mpc$. Results for other halo mass bins are qualitatively similar, with the expected trend of stronger correlations for higher mass halos. Error bars on these points are estimated by the subvolume method. Curves in Fig. A1 show the least-squares fit of equation 5 in each $\sigma$ bin, where we have used the subvolume error bars (treated as diagonal) to fit $\alpha$, $\gamma$, and $\Delta$. Formally the fits are not often acceptable, but this may be a consequence of ignoring error covariances and/or underestimating errors from our subvolume method.

To test if the subvolume method is an appropriate measure of cosmic variance on the scale of the full volume, we compare three simulations with different initial conditions for the dark matter density field, each $(1 \ h^{-1} \ Gpc)^3$ in size, with $1024^3$ particles and a smoothing scale of $r_s = 1.0 \ h^{-1} \ Mpc$. Fig. A2 shows the residual of the measured cross-correlations $(\xi_{\text{meas}} - \xi_{\text{fit}})$ in each simulation from a Lorentz profile fit to the simulation labeled “IC 3,” which has the same dark matter density field initial conditions as our standard 2048$^3$ simulation. We plot error bars on each simulation curve computed by applying the subvolume method to that simulation. In the smallest transverse separation bins, the difference among the three curves is significantly larger than the subvolume error bars, but for $\sigma = 7 - 10 \ h^{-1} \ Mpc$ and larger the run-to-run variations are comparable in size to the error bars. This means the subvolume method underestimates the errors expected from cosmic variance in the smallest separation bins, but for larger separations the subvolume errors are reasonable. Because we do not have many $1 \ h^{-1} \ Gpc$ simulations available, we use the subvolume errors for our analysis.

Tables A1 and A2 can be used together with equations 5, 6, and 7 to accurately reproduce the $z = 2.5$ LyMAS predictions of $\xi(\pi, \sigma)$ for any value of $M_h$, with typical residuals comparable to those shown in Fig. A1. They thus provide a compact quantitative summary of our numerical results. Table A2 gives the mean value of $\Delta$ in each halo mass bin, averaged over the transverse separation bins. There does not appear to be any significant trend in $\Delta$ with transverse separation, and only a weak trend with halo mass. Nevertheless, we include it in our model, using the average values recorded in the table.
Table A1. Best-fit parameters for fits to the trend in halo-flux cross-correlation strength with halo mass $M_h$ and transverse separation $\sigma$ bin, $|\xi(\pi, \sigma)| = A(M_h/4 \times 10^{12} h^{-1} M_\odot)^m$. Errors on parameters are from the subvolume method.

| $\sigma$ bin ($h^{-1}$ Mpc) | $\log A$ | $m$ |
|-----------------------------|---------|-----|
| 1 – 4                       | $-0.45 \pm 0.003$ | $0.215 \pm 0.005$ |
| 4 – 7                       | $-0.75 \pm 0.003$ | $0.289 \pm 0.007$ |
| 7 – 10                      | $-1.00 \pm 0.004$ | $0.255 \pm 0.013$ |
| 10 – 15                     | $-1.25 \pm 0.006$ | $0.270 \pm 0.012$ |
| 15 – 20                     | $-1.50 \pm 0.008$ | $0.269 \pm 0.015$ |
| 20 – 30                     | $-1.80 \pm 0.011$ | $0.275 \pm 0.025$ |
| 30 – 40                     | $-2.10 \pm 0.020$ | $0.280 \pm 0.025$ |
| 40 – 60                     | $-2.45 \pm 0.026$ | $0.300 \pm 0.038$ |
| 60 – 80                     | $-2.80 \pm 0.036$ | $0.240 \pm 0.035$ |

Figure A1. The cross-correlation between dark matter halos and the Ly-$\alpha$ forest flux from applying LyMAS to the 2048$^3$ ($1 h^{-1}$ Gpc)$^3$ dark matter simulation with $r_s = 0.5 h^{-1}$ Mpc for halos in the range $M_{12} = 0.84 - 1.68$ (blue points, with error bars estimated from 16 subvolumes). Red curves show the best-fit Lorentzian fitting function $\xi(\pi) = -\alpha \gamma^2 / (\pi^2 + \gamma^2) + \Delta$. Panels show different transverse separation bins, increasing left to right and top to bottom, and residuals are plotted beneath each panel.

Table A2. The $\Delta$ value reported here is the mean across $\sigma$ bins in the corresponding halo mass bin, since we do not detect any significant trend in $\Delta$ with $\sigma$ bin. Errors on parameters are from the subvolume method.

| $M_{12}$ bin | $\Delta$ |
|--------------|--------|
| 0.42 – 0.84  | $0.0012 \pm 0.0003$ |
| 0.84 – 1.68  | $0.0011 \pm 0.0003$ |
| 1.68 – 3.35  | $0.0014 \pm 0.0003$ |
| 3.35 – 6.70  | $0.0014 \pm 0.0003$ |
| 6.70 – 13.40 | $0.0015 \pm 0.0003$ |
| 13.40 – 26.80| $0.0019 \pm 0.0004$ |
| 26.80 – 53.60| $0.0019 \pm 0.0004$ |
Figure A2. The residuals of the halo-flux cross-correlation from fitting a Lorentz profile to the simulation marked “IC 3.” The three curves show residuals from three 1024^3-particle (1 h^{-1} Gpc)^3 dark matter simulations with $r_s = 1.0$ h^{-1} Mpc and different sets of initial conditions for the dark matter density field (blue solid, cyan dashed, and magenta dot-dashed lines). Rows show different transverse separation bins $1 < \sigma < 4$ h^{-1} Mpc, $7 < \sigma < 10$ h^{-1} Mpc, and $15 < \sigma < 20$ h^{-1} Mpc, and columns show different halo mass bins $M_{12} = 1.68 - 3.35$ and $M_{12} = 3.35 - 6.70$. Error bars are computed by applying the subvolume method to each simulation individually.