Quantum correlation between nodes in a network which consist of several independent sources of entanglement and in multipartite entanglement systems are important for general understanding of the nature of nonlocality, quantum information processing and communication. In previous years, demonstrations of network nonlocality for bilocal scenarios have been in the focus. Yet, it has been found that the seminal protocols do not certify entanglement between end nodes, which otherwise require all sources in the network to be of nonlocal nature. This has motivated for development of even stronger concept, called full network nonlocality. Here, we experimentally demonstrate full network nonlocal correlations in a network. Specifically, we use two pairs of polarization entangled qubits created by two separate and independent entanglement sources and a partial Bell state measurement. Our results may pave the way for the use of nonlocality test in quantum communication protocols with a full network nonlocality certification.

Quantum entanglement is one of the fundamental properties in nature, which leads to nonlocal correlations between distance objects and has no intuitive explanation in classical physics. In the wake of Bell’s theorem [1], which shows incompatibility between the local hidden variable (LHV) model and the predictions stipulated by quantum mechanics, there has been a stream of experiments falsifying LHV models by violating the Bell inequality. However, it is only recently we have witness loophole-free confirmation of Bell nonlocality [2], showing the need of further development of quantum technologies to realize quantum entangled-based applications. Entanglement across multiple nodes [3–5] promise to enable a wide range of applications like secure communication, distributed quantum computing, cryptography, quantum key distribution, random number generation and quantum sensing [6–10]. Connection of multiple entangled systems into a network structure is a backbone of future quantum internet [11] which, by using entanglement-swapping mechanism [12], can in principle enable long-distance quantum key generation for secure communication [13, 14]. To realize the potential of quantum internet, it is important to establish reliable certification methods of non-local correlations between nodes in the network. Historically, entanglement swapping protocols based on Event-ready Bell test [12] led to the first bipartite quantum network where conditional successful outcome from an intermediate node together with correlation outcome between terminal nodes is used to disprove an LHV model (see Fig. 1a). Naturally, a general quantum network consists of truly independent sources, hence it is important to investigate a different scenario based on several independent hidden variables [15]. Recent experiments have considered this scenario to extend LHV model into bilocal hidden variable model (BLHV) proposed by [16], further investigated by [17–21] and experimentally verified by
thus form a BLHV model [16]: described by two independent hidden variables \( \lambda \) two independent sources, we can form a three node network reduces to original Bell’s notion of LHV [1, 30]. Extending to \( \lambda \) that connect to node \( k \) and inputs receptively, \( \bar{\lambda} \) not certify entanglement between the end nodes of a bilocal Bell-like inequality, simple counterexample shows that it can reduce to nonlocality arising from different quantum network structures [16, 27–29] are interesting to investigate due to their importance for future communication and certification protocols. Generally, for a bilocal network composed of two sources and two communication parties, each of whom selects a private input \( x_k \) and produces an output \( a_k \), the correlations follow a network local model if they can describe by each source independently emitting a local hidden variables \( \lambda_j \) [26]:

\[
p(\bar{a}|\bar{x}) = \int d\lambda_1 \mu_1(\lambda_1) \int d\lambda_2 \mu_2(\lambda_2) 
\times p(a_1|x_1, \bar{\lambda}_1)p(a_2|x_2, \bar{\lambda}_2),
\]

where \( \bar{a} = \{a_1, a_2\} \) and \( \bar{x} = \{x_1, x_2\} \) are sets of output and inputs receptively, \( \mu_1, \mu_2 \) are probability density functions and \( \bar{\lambda}_k \) is a set of local variables associated with the source that connect to node \( k \). Independence of \( \lambda_j \) emerges naturally as a construction of independent source in a network. In its simplest form, with a single source (\( \lambda_1 = \lambda_2 \)), the model reduces to original Bell’s notion of LHV [1, 30]. Extending to two independent sources, we can form a three node network described by two independent hidden variables \( \lambda_1 \) and \( \lambda_2 \), thus form a BLHV model [16]:

\[
p(a, b, c|x, y, z) = \int d\lambda_1 d\lambda_2 \mu_1(\lambda_1) \mu_2(\lambda_2) 
\times p(a|x, \lambda_1)p(b|y, \lambda_1)p(c|z, \lambda_2).
\]

The bilocal inequalities obtained from BLHV model (2) are proof of network nonbiolocality as it rules out any bilocal model. Although it is conceptually novel way to violate Bell-like inequality, simple counterexample shows that it can not certify entanglement between the end nodes of a bilocal network [23]. In that case we need to embrace an even stronger concept in notion of network nonlocality where all sources are to be of nonlocal nature. Introduced by [26] we define full network nonlocality (FNN) if the network cannot be described by allowing at least one source in the network to be of a local nature, while the rest are characterized to be independent nonlocal (see Fig. 1c). Formalizing this concept to bilocal case scenario, the non-FNN correlations are given by

\[
p(a, b, c|x, y, z) = \int d\lambda \mu(\lambda)p(a|x, \lambda)p(b, c|y, z, \lambda).
\]

By considering swapping of the sources, it has been shown that produced conditional distributions, given by \( p(a, b, c|x, y, z) \), leads to following FNN witnesses where both need to be violated simultaneously [26]:

\[
R_{C-NS} = 2 \langle A_0B_1C_0 \rangle - 2 \langle A_0B_1C_1 \rangle + 2 \langle A_1B_0C_0 \rangle - \langle B_0 \rangle + \langle C_1 \rangle \leq 3
\]

and

\[
R_{NS-C} = 2 \langle A_0B_1C_0 \rangle - 2 \langle A_0B_1C_1 \rangle + 2 \langle A_1B_0C_0 \rangle + 2 \langle A_1B_0C_1 \rangle - \langle B_0 \rangle - \langle C_1 \rangle - \langle A_1 \rangle \leq 3,
\]

where the expectation values are computed in accordance to [17]:

\[
\langle A_2B_0C_2 \rangle = \sum_{a, c} -1^{a+c} [p(a, 0, c|x, z) + p(a, 1, c|x, z) - p(a, 2, c|x, z)]
\]

and

\[
\langle A_2B_1C_2 \rangle = \sum_{a, c} -1^{a+c} [p(a, 0, c|x, z) - p(a, 1, c|x, z)].
\]

In our experiment the two entanglement sources emit singlet state (\( \psi^- \)). Bob performs a partial BSM, with three different outcomes that are parameterized by ternary bit \( \{0, 1, 2\} \in b, \) corresponding to state \( \phi^+ \), \( \phi^- \) and the third corresponding to undistinguishing between the two remaining Bell states (\( \psi^+ \) and \( \psi^- \)). Alice and Charlie have to choose between two possible dichotomic measurements. \( \{x, z\} \in \{0, 1\} \), corresponding to observables \( A_0 = \sigma_x \), \( A_1 = \sigma_z \), \( C_0 = \frac{\sigma_x + \sigma_z}{\sqrt{2}} \) and \( C_1 = \frac{\sigma_x - \sigma_z}{\sqrt{2}} \) and are producing binary outputs \( \{a, c\} \in \{0, 1\} \). The resulting distribution leads to the violation of \( R_{C-NS} = R_{NS-C} = 5/\sqrt{2} \approx 3.5355 \).

The schematics of our setup is presented in Fig. 2. A femtosecond pulsed laser with repetition rate of 80 MHz producing ultraviolet light (390 nm) which is split and focused into two different setups acting as our two independent sources of entanglement. The first source consists of a crossed configuration of two 2 mm thick beta barium borate (BBO) nonlinear crystals producing polarization-entangled photon pairs through the process of second order degeneration.
Experimental setup for testing FNN model in a bilocal network configuration: The stations of Alice, Bob, and Charlie are highlighted in blue shading and the two entanglement sources S1 and S2, which connect them, are highlighted in orange. Both sources are pumped by ultraviolet light centered at a wavelength of 390 nm with a repetition rate of 80 MHz. Source in blue shading and the two entanglement sources S1 and S2, which connect them, are highlighted in orange. Both sources are pumped by a moving motor which receive rotation coordinates from a quantum random number generator (QRNG) connected to a variable phase shifter (see further in SM). Alice and Charlie implement their measurements using half–wave plates, polarizing beam splitters (PBS) and couplers to single photon detectors (actively quenched Si-avalanche photodiodes, DET) via single mode fibers. Bob implements his partial BSM, which has two input ports, using a single PBS and two half-wave plates. By using fiber 50:50 beam splitter (BS) to split each of the four output ports, Bob is able to do partial photon-number resolving detection and therefore gain information about the two unresolvable Bell states. The observable four-photon coincidence events, which are generated by one simultaneous click for the sources 1 and 2 respectively. It is well known that type-1 spontaneous parametric down conversion (SPDC). The second source consists of one 2 mm thick type-2 BBO crystal generating non-collinear polarization-entangled photons at 780 nm. To further insure independency between the sources we introduce decoherence by a movable glass plate in one of the pumping pulsed laser paths. The glass plate is steered by a moving motor which receive rotation coordinates from a quantum random number generator, described further in supplementary material. One photon from each pair is sent to a BSM analyzer, where they are interfered. To get a high-quality interference it is necessary for the photons from both paths to be indistinguishable in their temporal, spatial, and spectral modes. To increase spectral overlap we use 3 nm narrowband interference filters (IF). We verify indistinguishability by checking four-photon coincidence in a Hong-Ou-Mandel (HOM) experiment (see supplementary material). The measured HOM dip gave us a visibility value of $v = (89 \pm 1.3)\%$. We have measured for each individual source the visibility in diagonal polarization bases which resulted in visibility of $(99.1 \pm 0.1)\%$ and $(98.0 \pm 0.1)\%$ for the sources 1 and 2 respectively. It is well known that with linear optics one cannot distinguish between all four Bell states, leading us to distinguish only two of them, $\psi^{\pm}$. However, by using beam splitters at the outputs of BSM analyzer we can detect, with 50% probability the presence of other two Bell states, $\psi^\pm$, although without possibility to distinguish them apart [31]. The photons were detected with twelve avalanche photodiodes (APD) with an average efficiency of approximately 55-60 % and the output signals from these detectors sent to a field programmable gate array (FPGA) based coincidence counting unit. The total experiment lasted for approximately 15 hours at low pumping power to avoid creation of multiple photons pairs. The produced correlations from the experiment rendered FNN witnesses of $R_{C-NS} = 3.17 \pm 0.05$ and $R_{NS-C} = 3.17 \pm 0.05$, more than three standard deviations above FNN inequality (4) and (5) (see supplementary material for further details).

In summary, we have for the first time demonstrated an experiment to show FNN for bilocal scenario. The resulted violations of $R_{NS-C} = 3.17 > 3$ and $R_{C-NS} = 3.17 > 3$ are well above the classical bound. However, obtained values are lower than predictions $R_{NS-C} = R_{C-NS} \approx 3.5355$ [26] can be explained by an imperfect HOM-dip visibility. Similarly to any Bell-type inequalities, our experiment of FNN is subject to detection and locality loopholes. From a more practical perspective, our findings open up for
considering complex quantum networks and its connection to FNN. This involves configurations in a hybrid variant where more than two nodes are connected by nonlocal source together with one or several distance nonlocal sources. This will arguably be of great interest from the quantum information point of view as it also provides a certification method to guarantee entanglement between two distance parties just sharing a BSM device.

METHODS

Experimental details. Correlated photons pair were generated by two separated and independent SPDC sources. The first source composed of two 2 mm thick BBO nonlinear crystals in a configuration where one crystal was rotated 90 degrees relative to the other. The second source composed of two separated and independent SPDC sources. The pumping fields with wavelength of λ = 780 nm were adapted in power to generate approximately equal amount of correlated photons from each source. After spatial and temporal walkoff compensation and narrowband filtering (10nm at Alice and 3 nm at Bob’s and Charlie’s) the photons were collected by three different stations using single mode fibers coupled to avalanche photodetectors. At Alice’s station, the observable $A_1 = \sigma_x$ corresponds to half-wave plate rotated by $\theta_1^A = 22.5^\circ$ and $A_2 = \sigma_z$ corresponds to half-wave plate rotated by $\theta_1^A = 0^\circ$. At Charlie’s measuring station $C_1 = \frac{\sigma_x + \sigma_z}{\sqrt{2}}$ corresponds to half-wave plate rotated by $\theta_1^C = 11.25^\circ$ and $C_2 = \frac{\sigma_x - \sigma_z}{\sqrt{2}}$ corresponds to half-wave plate rotated by $\theta_1^C = -11.25^\circ$. Bob’s station has only one input, performing a partial BSM which compose of a polarisation beam splitter together with two half-wave plates at 22.5°. Each measuring configuration takes about 3.5 hours to perform and we use fair-sampling assumption.

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