Elastic scattering of alpha particles from $^9$Be in the framework of optical model

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Abstract. The analysis of the available data on the $\alpha + ^9$Be elastic scattering in the energy range from 18.4 to 104 MeV, including recent measurements at energies of 40 and 90 MeV is carried out. The experimental data on elastic scattering were analyzed within the framework of the optical model using Woods-Saxon potential and the double folding one.

1. Introduction

The interest in investigating the properties and dynamics of interaction between light weakly bound nuclei continues against the background of notable progress in the use of high intensity secondary beams. It has been shown that in light nuclei the nucleons tend to group into clusters, relative motion of which defines to a large extent some properties of these nuclei. The $^9$Be nucleus is a unique example of a nuclear system presenting a cluster structure while remaining a stable particle. The $^9$Be nucleus consists of two $\alpha$ particles and a neutron, each pair is unbound, but, taken together, the constituents of the nucleus form a stable system. Such a structure, which one sometimes refers to as a Borromean structure, is characteristic of many nuclei in the vicinity of the neutron drip line. It is natural to expect that the $^9$Be nucleus will exhibit some properties of so-called exotic nuclei. Recently experimental studies explicitly confirm the cluster structure of $^9$Be [1,2]. In addition to the cluster structure, in [3], it was suggested that the first excited state $1.68$ MeV ($1/2^+$) in $^9$Be has an increased radius as nuclei with neutron halo.

Several papers have reported investigations on elastic and inelastic scattering of deuterons [4-6], $^3$He [7,8] and $\alpha$ particles [9-11] on $^9$Be. For example, in the following papers, the processes of interaction of deuterons [5,6] and $\alpha$ particles [11] with $^9$Be nuclei were analyzed using distorted wave Born approximation (DWBA) and coupled-channel (CC) models with phenomenological potentials, later in the work [7,8], in the framework of the CC method with both phenomenological and folding potentials, certain excited states of $^9$Be were analyzed.

In this work, we analyze experimental data [3,10-15] on elastic scattering between $\alpha$ particles on $^9$Be nuclei that exhibit cluster properties. Optical-model analyses of the elastic-scattering...
data were carried out in conjunction with analyses of other $^9$Be($\alpha$, $\alpha$)$^9$Be differential cross-section data in the energy range 18.4-104 MeV [3, 10-15]. This paper is part of our extensive study of the exotic excited states of the $^9$Be and their cluster structure.

2. Results and discussion

Firstly, let us consider the analysis of the elastic scattering cross section. The optical potential was chosen in the usual Woods–Saxon (WS) form and WS potential consisted of the real and imaginary (with volume absorption) parts. Our total real potential for these cases consists of the nuclear ($V_{\text{nucl}}$) and the Coulomb ($V_{\text{C}}$) potentials, respectively

\[ U(r) = V_{\text{nucl}}(r) + V_{\text{C}}(r) \]  

where, the nuclear potential is assumed to have a Wood-Saxon shape:

\[ V_{\text{nucl}}(r) = V_0[1 + \exp\left(\frac{r - R_w}{a_v}\right)]^{-1} + iW[1 + \exp\left(\frac{r - R_w}{a_w}\right)]^{-1}. \]  

and Coulomb potential of a uniform charged sphere

\[ V_{\text{C}}(r) = \frac{Z_p Z_t e^2}{2R_C}(3 - \frac{r^2}{R_C^2}), \text{ for } r \leq R_C \]
\[ V_{\text{C}}(r) = \frac{Z_p Z_t e^2}{r}, \text{ for } r > R_C \]

with radius

\[ R_i = r_i\left(A_1^{\frac{1}{3}} + A_3^{\frac{1}{3}}\right), \quad i = V, W, SO, C \]

here, $V_0$ is the Woods-Saxon potential depth, R the potential radius and $a$ the diffuseness parameter which determines the sharpness of the potential surface. Larger values of $a$ giving a softer surface. $Z_p$ and $Z_t$ are the proton numbers of the projectile and target of system, respectively.

The microscopic nuclear potential that we have also used to analyze the experimental data for $\alpha$+$^9$Be system was based on the double folding (DF) model [16]. DF potential is calculated by using the nuclear matter distributions of both projectile and target nuclei together with an effective nucleon-nucleon interaction potential ($\nu_{NN}$). Thus, the DF potential is

\[ V^{DF}(R) = \int dr_1 \int dr_2 \rho_p(r_1)\rho_t(r_2)\nu_{NN}(r_{12}) \]  

$\rho_p(r_1)$ and $\rho_t(r_2)$ are the nuclear matter density distributions of both the projectile and target nuclei, respectively. Gaussian density distributions (GD) have been used for both nuclei [17] defined as:

\[ \rho(r) = \rho(0) \exp(-\beta r^2) \]  

where $\beta$ is adjusted to reproduce the experimental values for the root-mean-square radii of $^4$He=1.68 fm and $^9$Be=2.50 fm [18]. $\rho(0)$ values can be obtained from the normalization condition

\[ \int \rho(r)r^2dr = \frac{A}{4\pi} \]  

where A is the mass number.

The effective nucleon-nucleon interaction, $\nu_{NN}$, is integrated over both density distributions. Several nucleon-nucleon interaction expressions can be used for the folding model potentials. We have chosen the most common one, the M3Y (Michigan-3-Yukawa) realistic nucleon-nucleon
Table 1. Potential parameters obtained for elastic scattering of α particles from $^9$Be at concerned energies.

| E, MeV | Set | $V$, MeV | $r_V$, fm | $a_V$, fm | $W$, MeV | $r_W$, fm | $a_W$, fm | N  |
|--------|-----|----------|-----------|-----------|----------|-----------|-----------|----|
| 18.4   | WS  | 93.60    | 1.38      | 0.73      | 13.68    | 1.05      | 0.91      |
|        | DF A|          |           |           | 13.68    | 1.05      | 0.91      | 1.35 |
| 29     | WS  | 99.90    | 1.38      | 0.73      | 25.19    | 1.05      | 0.94      |
|        | DF A|          |           |           | 25.19    | 1.05      | 0.94      | 1.05 |
| 40     | WS  | 91.37    | 1.38      | 0.73      | 44.23    | 1.05      | 0.84      |
|        | DF A|          |           |           | 44.23    | 1.05      | 0.84      | 1.05 |
| 48     | WS  | 81.83    | 1.38      | 0.73      | 31.46    | 1.05      | 1.06      |
|        | DF A|          |           |           | 31.46    | 1.05      | 1.06      | 0.95 |
| 50     | WS  | 81.05    | 1.38      | 0.73      | 33.41    | 1.05      | 1.08      |
|        | DF A|          |           |           | 33.41    | 1.05      | 1.08      | 1.45 |
|        | DF B|          |           |           | 81.05    | 1.38      | 0.73      | 0.40 |
| 65     | WS  | 80.32    | 1.38      | 0.73      | 37.20    | 1.05      | 1.06      |
|        | DF A|          |           |           | 37.20    | 1.05      | 1.06      | 1.50 |
|        | DF B|          |           |           | 80.32    | 1.38      | 0.73      | 0.40 |
| 90     | WS  | 79.52    | 1.38      | 0.73      | 38.20    | 1.05      | 1.06      |
|        | DF A|          |           |           | 38.20    | 1.05      | 1.06      | 1.50 |
|        | DF B|          |           |           | 79.52    | 1.38      | 0.73      | 0.43 |
| 104    | WS  | 78.70    | 1.38      | 0.73      | 38.32    | 1.05      | 1.05      |
|        | DF A|          |           |           | 38.32    | 1.05      | 1.05      | 1.01 |
|        | DF B|          |           |           | 78.70    | 1.38      | 0.73      | 0.46 |

Interaction. The M3Y has two forms, one corresponds to M3Y-Reid [19] and another is based on the so-called M3Y-Paris interaction [20]. In the present work, we use the former form with the relevant exchange correction term due to the Pauli principle, given by

$$\nu_{NN}(r) = 7999 \exp(-4r) - 2134 \exp(-2.5r) + J_{00}(E)\delta(r) MeV, \quad (6)$$

where $J_{00}(E)$ represents the exchange term, since nucleon exchange is possible between the projectile and the target.

In this case while the real part of the optical model has been obtained by using the above-described DF model (set DF A in the table 1), we have adopted the following WS form for the imaginary potential. In other hand, to describe the experimental data at rear angles the imaginary part of the optical model has been obtained by using the DF model (set DF B in the table 1), and we have adopted the following WS form for the real potential.

Therefore, for the nucleon–nucleon-DF potential case, the nuclear potential consists of a real and an imaginary part:

$$U^{DF}(r) = N(V_{DF}(r) + iW(r)). \quad (7)$$

where $N$ is the normalization factor, which is determined by the fit of the optical model (OM) calculation to the experimental data. Calculations of differential cross-sections of elastic scattering were performed within the framework of OM by using the WS and DF potentials in FRESCO code [21].
The comparison between the experimental data and the theoretical predictions for $^9\text{Be}(\alpha, \alpha)^9\text{Be}$ at the 18.4, 29, 40, 48, 50, 65, 90 and 104 MeV energies [3, 10-15] are shown in figures 1 and 2 based on the potential parameters, which are listed in table 1. In figures 1 and 2 the abbreviation WS corresponds to the calculations of the optical model with Woods-Saxon potential. DF A corresponds to the calculations of the optical model with folding potential for real part and imaginary potential was taken from WS A and DF B corresponds to the calculations of the optical model with folding potential for imaginary part and real part potential was taken from WS A.

The global potential of Avrigeanu et.al. [22] is taken as the starting potential, but in our case we changed some parameters of potential. To reduce the discrete ambiguity in determining the optical potential (OP), in set WS the radii of the real ($r_V$) and imaginary ($r_W$) parts and diffusions of the real part ($a_V$) of the potential were fixed. Fitting the theory to the experiment were carried out with a variation of the remaining 3 parameters of OP ($V$, $W$ and $a_W$) by $\chi^2$ minimization. In the OM calculations the Coulomb radius $r_c=1.28$ fm was taken.

In the case of DF A calculations (red dashed lines), the normalization factor ($N$) was in the range 0.95 - 1.5. To obtain a fit of the calculations with the experimental data, the $N$ and the imaginary potential parameters must be adjusted. This agreement might be improved by adjusting the $N$ the imaginary potential parameters better.
Figure 2. Comparison between the experimental data and the calculated differential cross section for elastic scattering of alpha from $^9$Be at energies 50, 65, 90 and 104 MeV using Woods-Saxon (WS) and double folding (DF A and B) potentials.

A good description of the experimental data in the full angular range within the framework of OM is one of the difficult problems, especially considering the rise of the cross section at the rear angels. But we still tried to describe the data well. As mentioned above, to describe the differential cross sections at the rear angles at energies of 45, 50, and 65 MeV (figure 2), double folding calculations for the imaginary part of the potential (DF B, purple dashed lines) were performed. The optimal agreement between theory and experiment was achieved by varying the parameters of the real part and the normalization coefficient $N$ of the imaginary part of the potential, found from analysis in the framework of the OM (WS) (table 1). It should be noted that description based on the DF B is satisfactory at 45, 50 and 65 MeV energies.

3. Summary
The data on elastic scattering were analyzed using two approaches Woods-Saxon (Phenomenological) and double folding (semi-microscopic) potentials. Literature elastic scattering data together with data received by us were analyzed. The obtained potential parameters gives a good description of the angular distributions of the elastic scattering, agrees with the experimental data on the reaction cross sections. It is shown that both potentials correlate with each other and give a similar description of the experimental data. The potentials obtained will be useful in the study of differential reaction cross sections involving the studied nuclei.
We plan to analyze the experimental data on inelastic scattering of other excited states of $^9\text{Be}$ at energies 29 and 40 MeV and transfer reactions induced by the $\alpha+^9\text{Be}$ system, using the present optical potential parameters in future.

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4. References

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