Study of the Concrete Production Process - A graph theoretic approach

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Abstract : The selection of concrete is a challenging task for civil engineers. The choice of concrete is normally arrived upon according to the correlation of the main factors and its co parameters. The civil engineers have to be rationalised in choosing the appropriate concrete for construction with the particular material for the application in order to produce low cost concrete. In this paper, graph-theoretic approach is used to find the best concrete process among the available different processes. The fishbone diagram represents the factors and its cofactors that are involved in the production process of concrete. A digraph characteristic is drawn which represents the factors like fibers, materials, method of curing and Admixture with their cofactors affecting the selection process. Identifying the interdependency of the factors and their inheritances and representing them by using numerical values with a matrix form are carried out in this work for three different processes of production of concrete. Plain Cement Concrete (PCC), Self Compacting Concrete (SCC) and High Strength Concrete (HSC) are considered in order to achieve the quality, strength and cost. This technique is illustrated with an example for decision-making problems when there are multiple alternatives to multiple interdependent factors.

Keywords: Concrete, Curing method, Graph Theory, Fish-bone, VPF

1. INTRODUCTION

Concrete is the principal ingredient for the construction of buildings, bridges, piers, abutments etc. The process of producing the concrete with desired mix to enhance the properties like workability, strength, water-cement ratio, durability and cement-binder ratio, use to admixtures and fibers etc. The concrete with desirable strength mainly depends on the suitable selection of ingredients, processing techniques and also type of fiber and admixtures used. A methodology which amalgamates concurrent design concepts, graph theory, matrix algebra, and permanent multinomial is proposed to design the mechatronic system which involves various conceptual designs and analysis to get high quality product. They also proposed a step-by-step procedure for developing a new algorithm for software coding [1]. An attempt was made to develop an integrated systems model for the structure of the composite product system in terms of its constituents and interactions, molding processes, curing kinetics, etc. using graph theory and matrix algebra. This approach will be used to correlate composite performance characteristics with the composite structural model [2]. Only a few studies [3-5] have
been reported in the system approach for different applications like automobile, tribology, failure analysis, reliability etc. Digraph models are a well-established system approach and it has been applied in the various fields of science and technology [6-10].

The root problem of the construction delay that happened in several tasks at PT Rekayasa project was analysed using fishbone diagram and the result showed that the most powerful delay is caused by human error [11]. Many public projects are extensions of previous ones, and inaccuracies in estimating project cost and construction time can result in improper sequencing of related projects or phasing within projects, thus delaying much needed improvements [12]. The relative importance of the causes of the construction project failure was investigated by three techniques like Ishikawa diagrams, Pareto diagrams and 5 – why techniques [13].

The civil engineers have to be rationalised in choosing the appropriate concrete for construction with the particular material for the application in order to produce low cost concrete. In this paper the attributes and its cofactors to achieve the mechanical properties of different types of concrete are identified based on the requirement. Using graph theory approach, the dependence and interdependence factors of the concrete are indexed.

2. FISHBONE DIAGRAM FOR MECHANICAL PROPERTIES OF CONCRETE

Figure 1 shows the fish bone diagram to represent the factors which influence the mechanical properties of concrete. The mechanical strength of fiber reinforced concrete is represented as digraph; the four factors identified are Fiber (W₁), Material (W₂), Admixture (W₃), and Method of Curing (W₄) taken into consideration. The interactions between the factors are shown as a digraph plotted as in the Figure 2. The value of the assigned factor for qualitative measure of the various parameters are listed in Table 1 and Table 2.
Figure 2. Interaction diagram for the factors

Table 1. Value of interdependence of factors ($W_{ij}$)

| S.No | Qualitative Measure of interdependency | Assigned Value |
|------|---------------------------------------|----------------|
| 1    | Very strong                           | 5              |
| 2    | Strong                                | 4              |
| 3    | Medium                                | 3              |
| 4    | Weak                                  | 2              |
| 5    | Very Weak                             | 1              |

Table 2. Value of interdependence of factors ($W_{ij}$)

| S.No | Qualitative Measure of interdependency | Assigned Value |
|------|---------------------------------------|----------------|
| 1    | Extremely low                         | 1              |
| 2    | Low                                   | 2              |
| 3    | Below average                         | 3              |
| 4    | Average                               | 4              |
| 5    | Above average                         | 5              |
| 6    | High                                  | 6              |
| 7    | Extremely High                        | 7              |

2.1. Digraph Representation for mechanical properties

2.1.1 Fiber based digraph

The Fiber factor $W_1$, has five cofactors such as Type($F_1$), Tensile Strength($F_2$), Dosage($F_3$), Aspect ratio($F_4$), Orientation($F_5$). To determine the interdependency between them, a Fiber based digraph is represented as shown in Figure 3.
2.2. Material based digraph

The factor $W_2$, Material is having four co-factors: Grain Size($M_1$), Fineness($M_2$), Specific Gravity($M_3$) and Chemical composition($M_4$) and to determine the interdependency between them, a material based digraph is represented as shown in Figure 4.

![Figure 4. Digraph for concrete materials](image)

2.3. Admixture based digraph

The admixture factor $W_5$, has three co-factors: Type($A_1$), Dosage($A_2$) and water-cement ratio ($A_3$) and to determine the interdependency between them, the admixture based digraph is represented as shown in Figure 5.

![Figure 5. Digraph for Admixtures](image)

2.4. Curing based digraph

The Curing factor $C_4$, has three co-factors: Type($C_1$), period of curing($C_2$) and Curing temperature($C_3$) and to determine the interdependency between them, the admixture based digraph is represented as shown in Figure 6.

![Figure 6. Digraph for Curing Types](image)
3. CONCRETE SELECTION

The selection of concrete is done based on the experiences of the civil engineers. Production of concrete depends on a number of parameters. The choice of concrete is normally arrived according to the correlation of the main factors and its co parameters. To illustrate clearly the three process of production of concrete like Plain Cement Concrete (PCC), Self Compacting Concrete (SCC) and High Strength Concrete (HSC) are considered in order to achieve the quality, strength and cost. The digraph of the concrete production and its correlation are shown in Figure 7. Generally, production of concrete is associated with ingredients of fine aggregate, coarse aggregate, water cement ratio, admixtures, method of curing, addition of fibres etc.

![Digraph of the concrete production](image)

**Figure 7.** Digraph of the concrete production

3.1. Plain Cement Concrete (PCC)
Plain cement concrete is the mixture of binding material (cement), fine aggregate (sand) and coarse aggregate (gravel) without reinforcing steel. Normally concrete is strong in compression and weak in tension, to overcome this steel as reinforcement can be used in concrete to take tensile load. PCC is the important component of a building which is laid on the surface to soil in order to avoid the contact of soil and water to the steel reinforcement. The proportioning of PCC may be 1:2:4 or 1:1.5:3 which is based on the requirement. The batching of the ingredients can be made by weight or volume. The mixing of concrete can be either hand or machine mixing based on the quality of the concrete. The PCC are placed in the form of layers not more than 150 mm thick, which can be compacted by hand (tamping rod) and machine (needle vibrator). Finally curing of concrete has to be done by pouring water, making a pond with water and placing wet sand bags.

3.2. Self Compacting Concrete (SCC)
The Self Compacting Concrete is a flow able concrete which does not require any compaction, it gets compacted by its own weight without any segregation of aggregates. This concrete is made to flow like a liquid by means of chemical and mineral admixtures. The consistency of SCC can be measured by flow rate rather than the slump cone test for normal concrete. Properly designed mix of SCC does not segregate; maintain durability and excellent stability characteristics. This type of concrete can be used in the place of congested reinforcement (Beam – Column Joint). Water cement ratio and cement binder ratio are the major parameters which decide the strength of the self compacting concrete when compared to traditional vibrating concrete. SCC concrete has to be placed at height taller than 5 meter without segregation of aggregate. It can also mix at site and pumped through a piping system to the place of concreting which is 200 meters away. Use of SCC reduces the overall construction cost.

3.3. High Strength Concrete (HSC)
The mix design of high strength concrete is influenced by the properties of cement, sand, gravel and water-cement ratio in order to achieve the compressive strength greater than 40 MPA. To get high compressive strength it is necessary to lower the water-cement ratio which invariably affects the workability of concrete. This type of concrete can be used to construct the high rise buildings and
bridge piers to resist more loads. The ingredients used and concept involved in increasing the compressive strength must be clearly studied in order to obtain the desired properties. Testing of materials is an integral step in the production process, since quality control studies show that slight change in the mix proportion leads to large change in the compressive strength of concrete. After completion of mix design, extra consideration for workability and other related properties has to be given.

4. MATRIX REPRESENTATION AND VPF VALUE

Each factor and its corresponding sub factors can be represented in the form of a digraph for easier visualization, but the increasing number of factors result in making the digraph representation complex and hard to understand. So, to simplify the representation, the factors and their interdependence are represented by a matrix. An $N \times N$ matrix represents $N$ factors along the main diagonal and each element $b_{ij}$ represents the dependence of factor $j$ on $i$.

$$M = \begin{pmatrix}
B_1 & B_{12} & B_{13} & B_{14} \\
B_{21} & B_2 & B_{23} & B_{24} \\
B_{31} & B_{32} & B_3 & B_{34} \\
B_{41} & B_{42} & B_{43} & B_4
\end{pmatrix}$$

The dependency factors represented in the matrix format give a good picture of the importance of different factors, but cannot help much in taking decisions. In order to simplify the matrix to take productive decisions, the permanent function representation is used. The Variable Permanent Function(VPF) is calculated similar to that of a determinant but with a positive sign.

The VPF value of the given matrix is calculated in the given manner.

$$M = \begin{pmatrix}
B_1 & B_{12} & B_{13} & B_{14} \\
B_{21} & B_2 & B_{23} & B_{24} \\
B_{31} & B_{32} & B_3 & B_{34} \\
B_{41} & B_{42} & B_{43} & B_4
\end{pmatrix}$$

$$VPF = B_1B_2B_3B_4 + B_{12}B_{23}B_4 + B_{13}B_{24}B_3 + B_{14}B_{23}B_2 + B_{21}B_3B_4 + B_{22}B_3B_4 + B_{23}B_4B_3 + B_{24}B_4B_2 + B_{31}B_2B_4 + B_{32}B_2B_3 + B_{33}B_2B_2 + B_{34}B_2B_1 + B_{41}B_2B_3 + B_{42}B_2B_2 + B_{43}B_2B_1 + B_{44}B_2B_1 + B_{12}B_{34}B_4 + B_{13}B_{24}B_3 + B_{14}B_{23}B_2 + B_{21}B_{34}B_3 + B_{22}B_{34}B_2 + B_{23}B_{34}B_1 + B_{24}B_{34}B_1 + B_{31}B_{24}B_2 + B_{32}B_{24}B_1 + B_{33}B_{24} + B_{34}B_{24} + B_{41}B_{24}B_1 + B_{42}B_{24} + B_{43}B_{24} + B_{44}B_{24}$$

5. GRAPH THEORETIC APPROACH FOR CONCRETE SELECTION

The graph theoretic approach of calculating Variable Permeability Function can be used in deciding the best concrete that can be used for construction based on the mechanical properties of concrete and their interdependence. The weights assigned for correlation of factors are decided by domain experts or by employees with long term experience.

The correlation matrices have been created for relating the subfactors of the 4 major contributors to the mechanical properties of concrete, i.e. Fibers, Admixture, Material and Method of Curing.

5.1. Weight Matrix for Mechanical Properties of Concrete

According to the fishbone diagram, the major factors contributing to the mechanical properties are Fibers($W_1$), Material($W_2$), Admixture($W_3$) and Method of Curing($W_4$). The matrix also is formed on the basis of the digraph of the factors, as shown in figure 2.
5.2. Fiber based Weight Matrix
According to the fishbone diagram, the co-factors contributing to the fiber based mechanical properties are Type($F_1$), Tensile Strength($F_2$), Dosage($F_3$), Aspect ratio($F_4$) and Orientation($F_5$). The interdependence is determined from the digraph in figure 3 and the weight matrix is formed as given below.

$$F = \begin{pmatrix}
F_1 & 5 & 4 & 5 & 4 \\
4 & F_2 & 4 & 5 & 3 \\
3 & 4 & F_3 & 4 & 0 \\
3 & 5 & 3 & F_4 & 0 \\
1 & 2 & 0 & 0 & F_5
\end{pmatrix}$$

5.3. Material based Weight Matrix
According to the fishbone diagram, the co-factors contributing to the material based mechanical properties are Grain Size($M_1$), Fineness($M_2$), Specific Gravity($M_3$) and Chemical composition($M_4$). The interdependence is determined from the digraph in figure 4 and the weight matrix is formed as given below.

$$M = \begin{pmatrix}
M_1 & 4 & 2 & 1 \\
4 & M_2 & 4 & 1 \\
0 & 4 & M_3 & 2 \\
4 & 1 & 1 & M_4
\end{pmatrix}$$

5.4. Admixture based Weight Matrix
According to the fishbone diagram, the co-factors contributing to the fiber based mechanical properties are Type($A_1$), Dosage($A_2$) and water-cement ratio($A_3$). The interdependence is determined from the digraph in figure 5 and the weight matrix is formed as given below.

$$A = \begin{pmatrix}
A_1 & 4 & 3 \\
4 & A_2 & 4 \\
4 & 3 & A_3
\end{pmatrix}$$

5.5. Curing based Weight Matrix
According to the fishbone diagram, the co-factors contributing to the fiber based mechanical properties are Type($C_1$), period of curing($C_2$) and Curing temperature($C_3$). The interdependence is determined from the digraph in figure 6 and the weight matrix is formed as given below.

$$C = \begin{pmatrix}
C_1 & 3 & 3 \\
0 & C_2 & 2 \\
2 & 2 & C_3
\end{pmatrix}$$

6. MATRIX REPRESENTATION OF THE PROCESS OF PRODUCTION

The three selections of concrete under study are plain cement concrete (TC$_1$), self compacting concrete (TC$_2$) and High strength concrete (TC$_3$). The digraph of the production process is given in Figure 7. The individual weight matrices of the three selections are as given below.
The three main steps of applying graph theory for the selection of the better type of concrete are. Permanent Function Representation of the weight matrices is the most important and decisive step in selecting the type of concrete required. The VPF of the individual matrices is calculated and concrete with highest VPF is selected. The most important step in this part is the selection of attributes that will affect the concrete properties significantly. Only these attributes and their co-factors are considered during the VPF calculation.

The most significant factors and their co-factors which affect the mechanical properties of the concrete the most are as follows:

1. Fibers - Type, Tensile Strength, Dosage, Aspect ratio, Orientation
2. Material - Grain Size, Fineness, Specific Gravity, Chemical composition
3. Admixture - Type, Dosage and water-cement ratio

| Plain Cement Concrete | Self Compacting Concrete | High Strength Concrete |
|-----------------------|--------------------------|------------------------|
| TC_{PCC} = \begin{pmatrix} TC_1 & 4 & 5 \\ 0 & TC_2 & 2 \\ 0 & 4 & TC_3 \end{pmatrix} | TC_{SCC} = \begin{pmatrix} TC_1 & 3 & 3 \\ 0 & TC_2 & 2 \\ 0 & 3 & TC_3 \end{pmatrix} | TC_{HSC} = \begin{pmatrix} TC_1 & 4 & 4 \\ 0 & TC_2 & 3 \\ 0 & 4 & TC_3 \end{pmatrix} |

The diagonal elements are determined by the selected attributes. TC_1 is calculated from the Fiber based matrix representation.

\[
TC_1 = \begin{pmatrix} 4 & 5 & 4 & 4 \\ 4 & 3 & 4 & 5 & 3 \\ 3 & 4 & 5 & 4 & 0 \\ 3 & 5 & 3 & 3 & 0 \\ 1 & 2 & 0 & 0 & 3 \end{pmatrix}
\]

VPF = 25528

\[
TC_1 = \begin{pmatrix} 2 & 5 & 4 & 5 & 4 \\ 4 & 3 & 4 & 5 & 3 \\ 3 & 4 & 5 & 4 & 0 \\ 3 & 5 & 3 & 3 & 0 \\ 1 & 2 & 0 & 0 & 3 \end{pmatrix}
\]

VPF = 22840

\[
TC_1 = \begin{pmatrix} 3 & 5 & 4 & 5 & 4 \\ 4 & 4 & 4 & 5 & 3 \\ 3 & 4 & 5 & 4 & 0 \\ 3 & 5 & 3 & 3 & 0 \\ 1 & 2 & 0 & 0 & 3 \end{pmatrix}
\]

VPF = 25147

TC_2 is determined by the second attribute i.e Material. The VPF is calculated from the material based matrix.
TC\_2 = \begin{pmatrix} 3 & 4 & 2 & 1 \\ 4 & 4 & 4 & 1 \\ 0 & 4 & 3 & 2 \\ 4 & 1 & 1 & 3 \end{pmatrix}

VPF = 1021

TC\_2 = \begin{pmatrix} 4 & 4 & 2 & 1 \\ 4 & 4 & 4 & 1 \\ 0 & 4 & 2 & 2 \\ 4 & 1 & 1 & 3 \end{pmatrix}

VPF = 898

TC\_2 = \begin{pmatrix} 4 & 4 & 2 & 1 \\ 4 & 5 & 4 & 1 \\ 0 & 4 & 4 & 2 \\ 4 & 1 & 1 & 3 \end{pmatrix}

VPF = 1352

TC\_3 is determined by the third attribute i.e Admixture. The VPF is calculated from the admixture based matrix

TC\_3 = \begin{pmatrix} 4 & 4 & 3 \\ 4 & 3 & 4 \\ 4 & 3 & 4 \end{pmatrix}

VPF = 296

TC\_3 = \begin{pmatrix} 5 & 4 & 3 \\ 4 & 5 & 4 \\ 4 & 3 & 5 \end{pmatrix}

VPF = 384

TC\_3 = \begin{pmatrix} 5 & 4 & 3 \\ 4 & 4 & 4 \\ 4 & 3 & 2 \end{pmatrix}

VPF = 280

The calculated values of TC\_1, TC\_2 and TC\_3 are substituted in TC\_pcc, TC\_scc and TC\_hsc matrices and their VPF values are calculated

\begin{align*}
TC\_pcc &= \begin{pmatrix} 25528 & 4 & 5 \\ 0 & 1021 & 2 \\ 0 & 4 & 296 \end{pmatrix} \\
VPF &= 7715174272
\end{align*}

\begin{align*}
TC\_scc &= \begin{pmatrix} 22840 & 3 & 3 \\ 0 & 898 & 2 \\ 0 & 3 & 384 \end{pmatrix} \\
VPF &= 7876099920
\end{align*}

\begin{align*}
TC\_hsc &= \begin{pmatrix} 25147 & 4 & 4 \\ 0 & 1352 & 3 \\ 0 & 4 & 280 \end{pmatrix} \\
VPF &= 9519950084
\end{align*}

On calculating the Variable Permanent Function (VPF) values for each selection of concrete, it can be observed that

- VPF (Plain Cement Concrete) = 7715174272
- VPF (Self Compacting Concrete) = 7876099920
- VPF (High Strength Concrete) = 9519950084

Thus, it is observed that High Strength Concrete is the best for usage when compared to Plain Cement Concrete and Self Compacting Concrete.

8. CONCLUSIONS

The paper introduces a graph theoretic approach to select one among three types of concrete based on their mechanical properties. The mechanical properties that are considered to affect the concrete are Fibers, Material, Admixture and Method of Curing. The co-factors for each major factor are listed in the fishbone diagram of mechanical properties of concrete. These factors are represented in digraphs. The digraphs are converted into weighted matrices with the help of the qualitative metrics. The study is conducted to find the better type of concrete among three types: Plain Cement Concrete(PCC), Self Compacting Concrete(SCC) and High Strength Concrete(HSC). The Variable Permanent Function values are found for each factor under the types of concrete. On observing the VPF values, it is concluded that High Strength Concrete has significantly higher VPF value when compared to PCC and SCC, and thus is preferred for usage.
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