Research Article

Multi-Attribute Decision-Making Method Based Distance and COPRAS Method with Probabilistic Hesitant Fuzzy Environment

Haifeng Song, Zi-chun Chen

School of Sciences, Xihua University, Chengdu, Sichuan, 610039, China

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ABSTRACT

As an extension of hesitant fuzzy set, the probabilistic hesitant fuzzy set (PHFS) can more accurately express the initial decision information given by experts, thus the decision method based on PHFS is more true and reliable. In this paper, multi-attribute decision-making (MADM) method is proposed under probabilistic hesitant fuzzy environment, which is based the new distance measures of probabilistic hesitant fuzzy elements (PHFEs) and the COmplex PRoportional ASsessment (COPRAS) method. Firstly, the existing problems of some distances are analyzed and we propose some new distance measures including new Hamming distance, new Euclidean distance and new generalized distance under probabilistic hesitant fuzzy environment. Secondly, a maximizing deviation method based on the new Hamming distance measure is proposed to obtain the attribute weights in probabilistic hesitant fuzzy information. Then, the COPRAS method is extended to solve MADM problems under probabilistic hesitant fuzzy environment. Finally, compared other methods, an example is given to demonstrate the effectiveness of the proposed method.

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1. INTRODUCTION

Multi-attribute decision-making (MADM) problems are becoming more and more common in human social activities. For example, it could be applied to site selection assessment [1], hotel selection [2], passenger demand and service quality of High-speed rail [3,4], sustainable supplier selection [5], and so on. Decision makers (DMs) solve MADM problems based on their own values, preferences and knowledge system. However, when DMs evaluate the MADM problems, they probably cannot give a specific evaluation value but a vague range due to the uncertainty and hesitation of expressing information. To solve this problem, hesitant fuzzy set (HFS) proposed by Torra [6] solves this problem well, which allows experts to take several possible values as membership degree. After that, many scholars domestic and overseas have made a lot of achievements in HFS theory [7–15]. HFS allows multiple membership degrees in an element, but these membership degrees have the same importance by default. However, in most cases, due to the preference and quantity of experts, different membership degrees actually correspond to different importance. In order to solve this problem, the probabilistic hesitant fuzzy set (PHFS) [16] is generated, which adds corresponding probability information for each membership degree and thus expresses different importance. Based on the fuzzy set and its extended forms, PHFS as a practical and effective tool that can improve the rationality and credibility of the result. In recent years, scholars have done some works to extend PHFS theory [17–23]. It is worth noting that the sum of the probabilities of all membership degrees in the same probabilistic hesitant fuzzy element (PHFE) is required to be one in these studies. In fact, in many practical decision-making problems, this requirement cannot be usually met. Because of this, Zhang et al. [24] reduced the conditions for probabilistic information and perfect the definition of PHFS.

For MADM problem, alternatives are evaluated by their attributes, so attribute weights become an important part of the ranking of alternatives. There are several ways to obtain attribute weights and they can be divided into three categories [25]: objective, subjective and integrated subjective and objective methods. The objective method [26–29] determines attribute weights based on objective decision information, mainly including maximum deviation method and entropy method. Subjective method [30–35] determines the attribute weights according to the DMs’s subjective preference information, mainly including eigenvector method, weighted least squares method and card method. The synthetic subjective and objective method [36–38] determines attribute weights by combining the subjective preference information with the objective decision information provided by the DMs. In addition, there are still many new challenges in obtaining attribute weights. Up to now, little research has been focused on extracting the weight of evaluation attributes in probabilistic hesitant environment. In
In this sense, it is necessary to develop an effective method to obtain attribute weights to solve the probabilistic hesitant MADM problem.

In the decision-making process, distance is one of the basic tools to measure the difference or similarity between PHFEs. Differences between PHFEs include length, membership degree and probability. In probabilistic hesitant fuzzy environment, most existing distances have some disadvantages. For example, distance proposed by Wu and Xu [39] does not satisfy the reflexivity of the definition of distance measure; Hamming distance proposed by Zhang et al. [40] and the Euclidean distance proposed by Ding et al. [41] do not consider the difference in length and probability of PHFEs; The Euclidean distance proposed by Li et al. [21] is based on the normalized PHFEs, which is not the original PHFEs. Therefore it has a certain influence on the decision result. As a result, the research on distance measures in probabilistic hesitant fuzzy environment is not enough and needs to be improved. Therefore, it is necessary to establish effective distance to make up for the lack of distance in the literature, so as to solve the problem of probabilistic hesitant fuzzy decision problems.

The COmplex PRoportional ASsessment (COPRAS) is one of the latest MADM methods. COPRAS method is a compensation method, which is easy to calculate. Besides, it converts qualitative attributes into quantitative attributes. It can calculate the maximum and minimum index values respectively in the evaluation process, and considers the influence of maximization and minimization of attributes on the evaluation results respectively. Utilizing the COPRAS approach, many researchers have achieved some results. Manik Chandra Das et al. [42] used a comprehensive model combining fuzzy Analytic Hierarchy Process and fuzzy COPRAS to evaluate and rank the preferences of stakeholders in Seven Indian Institutes of Technology. Zheng et al. [43] established an indicator system for evaluating chronic obstructive pulmonary disease severity from systems engineering and practical clinical experience, and proposed a hesitant fuzzy linguistic COPRAS method to solve decision-making problems in a fuzzy language environment. Harish Garg and Nancy [44] proposed two algorithms based on COPRAS method and aggregation operators method to solve the decision-making problems based on possibility linguistic single-valued neutrosophic set information. Harsh S. Dhiman and Dipankar Deb [45] evaluated the sequencing of fuzzy COPRAS and fuzzy Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) and applied them to select the best sites in three hybrid wind farms. Therefore, although COPRAS method has achieved good results, there is hardly any extension for MADM under probabilistic hesitant fuzzy environment. Based on this, we propose COPRAS method in probabilistic hesitant fuzzy environment.

In this study, in order to overcome the shortcomings of distance in the literature, the focus of this research is to introduce several new distances to obtain attribute weights. Meanwhile, the COPRAS approach is extended into probabilistic hesitant fuzzy environments. The main motivations and contributions of our work are summarized as follows.

1. In order to make up for these existing distances [21,39–41] that do not take into account the differences in length, membership degrees and probability between PHFEs, this study proposes some new distances based on PHFEs.

2. In order to obtain attribute weights, the newly proposed distance is applied to the maximizing deviation method to establish a nonlinear programming model under the probabilistic hesitant fuzzy environment.

3. In order to make up for the shortcomings that the traditional COPRAS method cannot handle MADM problems with PHFE evaluation information, the COPRAS method is extended. COPRAS method is extended to the probabilistic hesitant fuzzy environment.

The structure of this paper is as follows. Section 2 reviews the basic definitions and operations of HFS and PHFS. In Section 3, the existing distance measures of PHFEs under probabilistic hesitant fuzzy environment are discussed, and new distance measures are proposed. In Section 4, a new maximizing deviation method based on the new distance measure is presented in the probabilistic hesitant fuzzy environment, and the extended COPRAS method is proposed to solve the probabilistic hesitant MADM problem. In Section 5, an example of energy selection is used to prove the effectiveness of the proposed method. Finally, this paper was summarized in Section 6.

2. PRELIMINARIES

In this section, we review some basic definitions on HFS, PHFS.

2.1. Hesitant Fuzzy Sets

Definition 1. [6] Let $\mathcal{X}$ be a finite universe of discourse, an HFS $A$ on $\mathcal{X}$ is defined as:

$$A = \{ (x, h_A(x)) \mid x \in \mathcal{X} \},$$

where $h_A(x)$ is a set of some values in $[0, 1]$, denoting the possible membership degrees of the element $x \in \mathcal{X}$ to the set $A$. Xia and Xu [46] called $h = h_A(x)$ a hesitant fuzzy element (HFE).

Definition 2. [46] Given three HFEs $h_1$, $h_2$, and a positive number $\varepsilon$, it holds that:

1. $h^\varepsilon = \cup_{\varepsilon \in h} \{ \varepsilon^+ \}$;
2. $\varepsilon h = \cup_{\varepsilon \in h} \{ 1 - (1 - \varepsilon)^\varepsilon \}$;
3. $h_1 \oplus h_2 = \cup_{\varepsilon_1 \in h_1, \varepsilon_2 \in h_2} \{ \varepsilon_1 + \varepsilon_2 - \varepsilon_2 \}$;
4. $h_1 \otimes h_2 = \cup_{\varepsilon_1 \in h_1, \varepsilon_2 \in h_2} \{ \varepsilon_1 \varepsilon_2 \}$.

Definition 3. [46] For a HFE $h$, the score function of $h$ can be defined as:

$$S(h) = \frac{1}{t_\varepsilon} \sum_{\varepsilon \in h} \varepsilon$$

where $t_\varepsilon$ is the number of all $\varepsilon$ in $h$. 
Definition 4. [47] For a HFE $h$, we define the deviation degree:

$$DF(h) = \left[ \frac{1}{h} \sum_{\xi \in h}(\zeta - S(h))^2 \right]^{\frac{1}{2}}$$

Definition 5. [47] Let $h_1$ and $h_2$ be two HFEs, $S(h_1)$ and $S(h_2)$ are the scores of $h_1$ and $h_2$, respectively, and $D(h_1)$ and $D(h_2)$ the deviation degrees of $h_1$ and $h_2$, respectively, then:

1. If $S(h_1) < S(h_2)$, then $h_1 < h_2$;
2. If $S(h_1) = S(h_2)$, then
   - If $D(h_1) = D(h_2)$, then $h_1 = h_2$;
   - If $D(h_1) > D(h_2)$, then $h_1 < h_2$;
   - If $D(h_1) < D(h_2)$, then $h_1 > h_2$.

2.2. Probabilistic Hesitant Fuzzy Sets

Definition 6. [16, 44] Suppose that nonempty set $\mathcal{X} = \{x_1, x_2, \ldots, x_n\}$ be a fixed set, then an PHFS $H$ on $\mathcal{X}$ is defined as:

$$H = \{ (x, h_x(p)) \mid x \in \mathcal{X} \},$$

where $h_x(p)$ is a set of some values in $[0, 1]$, $h_x$ denotes the possible membership degrees of the element $x \in \mathcal{X}$ to the set $H$. $p$ a set of some values in $[0, 1]$, $p$ represents a set of probabilities with respect to $h_x$.

A PHFE can be described as $h(p) = h_x(p) = \{ \zeta_i(p_x) ; \tau = 1, 2, \ldots, \tau \sum_{r=1}^{\tau} p_r \leq 1 \}$.

where $\zeta_i$ denote the possible membership degree of the PHFE $h(p)$, $p_r$ represent the probability with respect to $\zeta_i$. $\tau$ is the number of all $\zeta_i(p_x)$ in $h(p)$.

Definition 7. [24, 48] Given three PHFEs $h(p)$, $h_1(p)$ and $h_2(p)$, and a positive number $\epsilon$, then

1. $h(p)^\epsilon = \bigcup_{\zeta \in h} \{ \zeta^\epsilon_i(p_x) \}$;
2. $\epsilon h(p) = \bigcup_{\zeta \in h} \{ (1 - (1 - \zeta)^\epsilon)_{p_x} \}$;
3. $h_1(p) \oplus h_2(p) = \bigcup_{\zeta_1, \zeta_2 \in h_1, h_2} \{ \zeta_1 + \zeta_2 - \zeta_1 \zeta_2 \} (p_1 \cup p_2) / \sum_{r=1}^{\tau} (p_1 \cup p_2) \}$;
4. $h_1(p) \otimes h_2(p) = \bigcup_{\zeta_1, \zeta_2 \in h_1, h_2} \{ \zeta_1 \zeta_2 \} (p_1 \cup p_2) / \sum_{r=1}^{\tau} (p_1 \cup p_2) \}$.

Definition 8. [24] Let $h(p)$ be a PHFE, the score function of $h(p)$ can be defined as:

$$S(h(p)) = \frac{\sum_{r=1}^{\tau} \zeta_i p_r}{\sum_{r=1}^{\tau} p_r}$$

Definition 9. [24] For a PHFE $h(p)$, we define the deviation degree:

$$DF(h(p)) = \sum_{r=1}^{\tau} ((\zeta_i - S(h(p))) p_r)^2 / \sum_{r=1}^{\tau} p_r.$$
Distance is one of the basic tools to measure the difference or similarity between PHFEs. It can be widely used in MADM. The existing distance measures of PHFE are analyzed, and the new distance measures are proposed.

### 3.1. The Existing Distance Measures Under PHFEs

Based on the definition of distance measure, this section analyzes the existing distance measures of PHFEs.

**Definition 12.** [41,49] Let $H = \{ (x, h_i(p)) | x \in X \}$ be a PHFS, $h_1$, $h_2$ and $h_3$ are three PHFSs on $H$, then the distance measure between $h_1$ and $h_2$ is defined as $d(h_1, h_2)$, which satisfies the following properties:

1. $0 \leq d(h_1, h_2) \leq 1$
2. $d(h_1, h_2) = 0$ if and only if $h_1 = h_2$
3. $d(h_1, h_2) = d(h_2, h_1)$
4. $d(h_1, h) + d(h, h_2) \geq d(h_1, h_2)$

The existing distance measures of PHFEs.

Wu and Xu [39] proposed the distance between $h_1(p)$ and $h_2(p)$ as follows:

$$d_h(h_1(p), h_2(p)) = \sum_{i=1}^{m} \left| p_i \cdot \sum_{i=1}^{n} \left| \tau_i - \tau_i' \right| \right|_P$$

(1)

where $\tau_i$ is the number of the elements in $h_1(p)$, $\tau_i' (p_{ri})$ is the value in PHFE $h_1(p)$, $i = 1, 2$.

For two PHFEs $h_1(p)$ and $h_2(p)$, the distance measures of $d(h_1(p), h_2(p))$ among them should satisfy the following properties:

1. Nonnegative: $0 \leq d(h_1(p), h_2(p)) \leq 1$;
2. Reflexivity: $d(h_1(p), h_2(p)) = 0$ if and only if $h_1 = h_2$;
3. Commutativity: $d(h_1(p), h_2(p)) = d(h_2(p), h_1(p))$;
4. Triangle inequality: $d(h_1(p), h_2(p)) + d(h_2(p), h_3(p)) \geq d(h_1(p), h_3(p))$.

Unfortunately, the distance described in Definition 12 does not have this reflexivity. Let's take the following example.

**Example 1.** Let $h_1(p) = h_2(p) = \{ 0.4 \left( \frac{1}{2} \right), 0.85 \left( \frac{1}{2} \right) \}$ are two equal PHFEs. The distance between them is $d(h_1(p), h_2(p)) = 0.1$. Obviously, by Definition 12, the reflexivity doesn't work because $d(h_1(p), h_2(p)) = 0.1 \neq 0$.

Zhang et al. [40] defined the Hamming distance between $h_1(p)$ and $h_2(p)$, its specific definition is as follows:

$$d_{h'}(h_1(p), h_2(p)) = \frac{1}{i} \sum_{i=1}^{n} \left| p_i \cdot \tau_i (p_{1i}) - p_i \cdot \tau_i (p_{2i}) \right|$$

(2)

where $t_1$ and $t_2$ are the number of the elements in $h_1(p)$ and $h_2(p)$.

Ding et al. [41] defined a Euclidean distance, let $h_1(p)$ and $h_2(p)$ are two PHFEs and the Euclidean distance among them was defined as follows:

$$d_e(h_1(p), h_2(p)) = \left( \frac{1}{i} \sum_{i=1}^{n} \left| p_i \cdot x_i (p_{1i}) - p_i \cdot x_i (p_{2i}) \right|^2 \right)^{\frac{1}{2}}$$

(3)

where $t_i$ is the number of the elements in $h_i(p)$, $i = 1, 2$ and $t_i = t_i = t_i$. $x_i (p_{1i})$ and $x_i (p_{2i})$ are the $rth$ lowest values in $h_i(p)$ and $h_2(p)$.

Li et al. [21] defined a distance between $h_1(p)$ and $h_2(p)$, then the Euclidean distance among them was developed as follows:

$$d_{EU}(h_1(p), h_2(p)) = \sum_{i=1}^{n} \left( x_i (p_{1i}) - x_i (p_{2i}) \right)^2 p_i$$

(4)

where $x_i (p_{1i})$ and $x_i (p_{2i})$ are the $rth$ values in $h_i(p)$ and $h_2(p)$. $p_i$ is the probabilities with respect to $x_i (p_{1i})$, $i$ is the number of the elements in $h_i(p)$, $i = 1, 2$.

Li et al. [21] proposed the distance after normalization of PHFEs, so it is not the original PHFEs after normalization. PHFEs are different before and after normalization, so the distance will be different.

Differences among PHFEs include differences in their lengths, differences in their values, and differences in their probabilities. Therefore, in order to study the differences among PHFEs, we should consider their membership degrees, their length and probability. Otherwise, distance measures will lead to unreasonable results. However, all distance measures proposed in the literature do not take into account the effect of PHFEs’ length and the difference of probability and 1.

**Example 2.** Suppose that there are two patterns, which are represented by the PHFEs $h_1(p) = \{ 0.55(0.3), 0.85(0.2) \}$ and $h_2(p) = \{ 0.45(0.2), 0.55(0.2), 0.65(0.1), 0.75(0.1) \}$. Now I have a sample to recognize, which is represented by a PHFE $h(p) = \{ 0.50(0.3), 0.65(0.2) \}$, known by the principle of the minimum distance measure of the PHFEs,

$$d(h_0(p), h(p)) = \min \{ d(h_1(p), h(p)), d(h_2(p), h(p)) \}$$

Then we can decide that the sample $h(p)$ belongs to the pattern $h_0(p)$.

We can see that the difference of membership degrees and probability values between $h(p)$ and $h_i(p)$ and the difference of membership degrees and probability values between $h(p)$ and $h_i(p)$ are almost the same. Where the number of $h(p)$ is the same as $h_i(p)$ and the probability distribution is similar, but different from $h_i(p)$. This means that $h(p)$ reflects roughly the same hesitant as $h_i(p)$, but not the same hesitant as $h_2(p)$. Then, it's easy to understand that $h(p)$ should belong to pattern $h_0(p)$. However, by applying the above distance measures Eqs. (2) and (3), then $d_{h'}(h(p), h_1(p)) = 0.0625$, $d_{h'}(h(p), h_2(p)) = 0.055$; $d_e(h(p), h_1(p)) = 0.0785$, $d_e(h(p), h_2(p)) = 0.0588$; and $h(p)$ belongs to the pattern $h_2(p)$. This is in stark contrast to our intuition.
Therefore, we believe that it is necessary to further consider the distance measures between PHFEs. The following, we propose some new distances that can overcome the above shortcomings by considering the hesitation of each PHFE.

### 3.2. Proposed the New Distance Measures Under PHFEs

Before coming up with the new distance measures, we still have some work to do. In order to fully study the differences between PHFEs, we should consider the differences of membership degrees, probability and their lengths. Based on this, we have the following definition.

**Definition 13.** Let \( h(p) = \{ \zeta_i(p_r), r = 1, 2, \ldots, t \} \) be a PHFE, we can get:

\[
Z(h(p)) = 2 - \frac{4}{2 + \left(1 - \frac{1}{t}\right) + \left(1 - \sum_{r=1}^{t} p_r\right)}
\]

where \( Z(h(p)) \in [0, 1] \) is the overall hesitancy of the PHFE \( h(p) \). \( t(i \geq 1) \) is the length of \( h(p) \), \( p_r \in [0, 1] \) represents probability with respect to \( \zeta_r \). \( 1 - \frac{1}{i} \) reflects the hesitancy degree of \( h(p) \), the more elements in a PHFE, the greater the hesitancy. \( 1 - \sum_{r=1}^{t} p_r \) is incompleteness of the probability information of PHFE \( h(p) \). The incompleteness of the probability information can also reflect the hesitancy degree of PHFE. The higher the probability, the less the incompleteness, and the less the hesitancy of PHFE. When \( Z(h(p)) = 0 \), then we can get \( i = 1 \) and \( p = 1 \). In this case, we use the distance measures defined by Eq. (2). Now, we're not going to talk about \( Z(h(p)) = 0 \).

Based on the above definition, some new distance measures under PHFEs are proposed.

**Definition 14.** Let \( h_1(p) \) and \( h_2(p) \) are two PHFEs and the new generalized distance measure between them is defined as follows:

\[
d_{ng}(h_1(p), h_2(p)) = \left[ \frac{1}{2} \left( \frac{1}{t} \sum_{r=1}^{t} \left| p_{1r} \zeta_{1r}(p_{1r}) - p_{2r} \zeta_{2r}(p_{2r}) \right|^4 \right)^{\frac{1}{2}} + \left| Z(h_1(p)) - Z(h_2(p)) \right| \right]^{\frac{1}{2}}
\]

When \( \lambda = 2 \), the new Euclidean distance between \( h_1(p) \) and \( h_2(p) \) is defined as:

\[
d_{eu}(h_1(p), h_2(p)) = \left[ \frac{1}{2} \left( \frac{1}{t} \sum_{r=1}^{t} \left| p_{1r} \zeta_{1r}(p_{1r}) - p_{2r} \zeta_{2r}(p_{2r}) \right|^2 \right)^{\frac{1}{2}} + \left| Z(h_1(p)) - Z(h_2(p)) \right| \right]^{\frac{1}{2}}
\]

where \( Z(h_i(p)) = 2 - \frac{4}{\left(1 - \frac{1}{t}\right) + \left(1 - \sum_{r=1}^{t} p_r\right)} \), \( \lambda > 0 \), \( t_1 \) and \( t_2 \) are the length of \( h_1(p) \) and \( h_2(p) \), \( t = \max \{ t_1, t_2 \} \) \( \zeta_i(p_r) \) is the \( rth \) lowest value in \( h_i(p) \), \( p_r \) is the probability with respect to \( \zeta_r \), \( i = 1, 2 \).

**Theorem 1.** \( d_{ng}(h_1(p), h_2(p)), d_{nh}(h_1(p), h_2(p)) \) and \( d_{eu}(h_1(p), h_2(p)) \) meet Definition 12.

**Proof.** (1) Nonnegative:

According to the definition of PHFS, we can get:

\[
\zeta_i, p_r \in [0, 1], \left| p_{1r} \zeta_{1r}(p_{1r}) - p_{2r} \zeta_{2r}(p_{2r}) \right| \in [0, 1],
\]

\[
Z(h_i(p)) \in [0, 1], \ i = 1, 2.
\]

So, \( d_{ng}(h_1(p), h_2(p)) \in [0, 1] \).

(2) Sufficient necessity:

\[
d_{ng}(h_1(p), h_2(p)) = 0 \iff \left| p_{1r} \zeta_{1r}(p_{1r}) - p_{2r} \zeta_{2r}(p_{2r}) \right| = 0, \ t_1 = t_2, \text{ and } \left(1 - \sum_{r=1}^{t_1} p_{1r}\right) = \left(1 - \sum_{r=1}^{t_2} p_{2r}\right) \iff h_1(p) = h_2(p).
\]

(3) Commutativity:

by Definition 13,

\[
d_{ng}(h_1(p), h_2(p)) = \left[ \frac{1}{2} \left( \frac{1}{t} \sum_{r=1}^{t} \left| p_{1r} \zeta_{1r}(p_{1r}) - p_{2r} \zeta_{2r}(p_{2r}) \right|^4 \right)^{\frac{1}{2}} + \left| Z(h_1(p)) - Z(h_2(p)) \right| \right]^{\frac{1}{2}}
\]

\[
= \left[ \frac{1}{2} \left( \frac{1}{t} \sum_{r=1}^{t} \left| p_{2r} \zeta_{2r}(p_{2r}) - p_{1r} \zeta_{1r}(p_{1r}) \right|^4 \right)^{\frac{1}{2}} + \left| Z(h_2(p)) - Z(h_1(p)) \right| \right]^{\frac{1}{2}}
\]

\[
= d_{ng}(h_2(p), h_1(p)).
\]

(4) Triangle inequality:

\[
\left| p_{1r} \zeta_{1r}(p_{1r}) - p_{2r} \zeta_{2r}(p_{2r}) \right|^4 \leq \left| p_{1r} \zeta_{1r}(p_{1r}) \right|^4 + \left| p_{2r} \zeta_{2r}(p_{2r}) \right|^4 + \left| p_{1r} \zeta_{1r}(p_{1r}) \right|^4 - \left| p_{1r} \zeta_{1r}(p_{1r}) \right|^4 + \left| p_{2r} \zeta_{2r}(p_{2r}) \right|^4
\]

Then \( d_{ng}(h_1(p), h_2(p)) \leq d_{ng}(h_1(p), h_3(p)) + d_{ng}(h_3(p), h_2(p)) \)

Similarly, it can be proved that \( d_{nh}(h_1(p), h_2(p)) \) and \( d_{eu}(h_1(p), h_2(p)) \) also satisfy the four properties of Definition 12.
4. THE COPRAS METHOD DISTANCE-BASED UNDER PROBABILISTIC HESITANT FUZZY ENVIRONMENT

In this section, the COPRAS method is extended to solve the probabilistic hesitant MADM problems.

The MADM problems is explained as follows:

- A is the alternatives represented by \( A = \{ A_1, A_2, \ldots, A_m \} \).
- C is the attributes denoted by \( C = \{ C_1, C_2, \ldots, C_n \} \).
- \( W \) is the weights with respect to attribute expressed by \( W = \{ W_1, W_2, \ldots, W_n \} \).
- The evaluation of the alternative \( A_i \) with respect to the attribute \( C_j \) is denoted by \( h_{ij}(p) \) which is the hesitant fuzzy number \( \tau_{ij} \) as follows:

\[
\tau_{ij} = \left\{ p_{ij}^{(c)} \mid i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n; \ \tau = 1, 2, \ldots, i \right\}.
\]

4.1. The Classical COPRAS Method

The COPRAS method is used to appraise the maximizing and minimizing indexes, and considers the influence of maximization and minimization of attributes on the evaluation results respectively. It is a compensation method, attributes are independent and it converts qualitative attributes into quantitative attributes. It is important to note that the classical COPRAS method can only be adapted when the attributes information are crisp numbers. Attribute weights [\( W_1, W_2, \ldots, W_n \)] are given by the DM. The MADM steps for the classical COPRAS method are described as follows:

Step 1: Obtain the decision matrix \( X = \{ h_{ij} \}_{n \times m} \) and normalize decision matrix \( X \) into \( X^* = \{ s_{ij} \}_{n \times m} \).

where \( s_{ij} \) is the evaluation value of \( i \)th alternative in \( j \)th attribute, expressed in the form of crisp number, \( s_{ij}^* \) is the normalization of \( s_{ij} \), \( i \in M \), \( M = \{ 1, 2, \ldots, m \} \), \( j \in N \), \( N = \{ 1, 2, \ldots, n \} \).

\[
s_{ij}^* = \frac{s_{ij}}{\sum_{i=1}^{m} s_{ij}}; j = 1, 2, \ldots, n
\]

Step 2: Calculate the weighted values of \( X^* \) through the following expression:

\[
\bar{s}_{ij} = s_{ij}^* W_j, \ i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n
\]

Step 3: Compute the aggregated values of the weighted values of \( X^* \) for positive or negative type attributes by the following expression:

\[
\begin{align*}
PN_{+i} &= \sum_{j=1}^{n} \bar{s}_{ij}^*; i = 1, 2, \ldots, m \\
PN_{-i} &= \sum_{j=m+1}^{n} \bar{s}_{ij}^*; i = 1, 2, \ldots, m
\end{align*}
\]

where \( r \) reflects the number of positive type attributes and \( n - r \) indicates the number of negative type attributes, and \( P_{n;i} \) describes the maximizing or minimizing indexes of \( i \)th attribute.

Step 4: Calculate the relative priority value or significance value \( RV_i \) of each alternative by the following formula:

\[
RV_i = \frac{\min \{ PN_{+i}, \sum_{j=1}^{m} \min_{j: PN_{-i}} P_{n;j} \}}{\max \{ PN_{-i}, \sum_{j=1}^{m} \min_{j: PN_{+i}} P_{n;j} \}}
\]

It can also be written as follows:

\[
RV_i = \frac{\sum_{j=1}^{m} PN_{+j}}{\sum_{j=1}^{m} \frac{1}{PN_{-j}}}
\]

Step 5: The alternatives are ranked in descending order according to the relative importance value.

4.2. Determine the Weights of Evaluation Attributes

In this section, by the maximizing deviation method [50], we propose a new distance measure Eq(6) to obtain the evaluation attribute weights under probabilistic hesitant fuzzy environments.

Firstly, calculate the new Hamming distance between the alternative \( A_i \) and other alternatives \( A_k \) \( (k = 1, 2, \ldots, m, k \neq i) \) with respect to the attribute \( C_j \) by the following expression:

\[
D_{ij} = \sum_{k=1,k \neq i}^{m} d_{nh}(h_{ij}(p), h_{kj}(p)),
\]

\[
i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n
\]

where

\[
d_{nh}(h_{ij}(p), h_{kj}(p)) = \frac{1}{7} \left[ \sum_{\tau=1}^{4} \left| P_{ij}^{(\tau)} - P_{kj}^{(\tau)} \right| + 2 \left( \frac{1}{2+\left( 1-\frac{1}{\bar{b}_{ij}^*} \right)} + \frac{4}{\sum_{\tau=1}^{4} b_{ij}^{(\tau)} \bar{b}_{ij}^{(\tau)}} \right) - 2 \left( \frac{1}{2+\left( 1-\frac{1}{\bar{b}_{kj}^*} \right)} + \frac{4}{\sum_{\tau=1}^{4} b_{kj}^{(\tau)} \bar{b}_{kj}^{(\tau)}} \right) \right]
\]
Secondly, for attribute \( C_j \), calculate the total distance of all alternatives is calculated as follows:

\[
D_j = \sum_{i=1}^{m} D_{ij}
\]

Let

\[
D(W) = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1, k \neq i}^{m} d_{ab}(h_{ij}(p_j), h_{kj}(p_k)) W_j.
\]

where \( D(W) \) represents the weight distance function.

Because of the attribute weights are completely unknown; so, we can construct a nonliner programming model to gain the weights as follows:

\[
\begin{aligned}
&\text{max } D(W) = \sum_{j=1}^{n} \sum_{i=1}^{m} W_j D_{ij} \\
&\text{s.t. } W_j \geq 0, \sum_{j=1}^{n} W_j^2 = 1, j = 1, 2, \ldots, n.
\end{aligned}
\]

To solve the above model, we construct the Lagrange function as follows:

\[
f(W, \eta) = D(W) + \eta \left( \sum_{j=1}^{n} W_j^2 - 1 \right)
\]

where \( \eta \) is a real number, expressing the Lagrange multiplier variable. Then the partial derivatives of \( f \) are computed as:

\[
\frac{\partial f}{\partial W_j} = \sum_{i=1}^{m} \sum_{k=1, k \neq i}^{m} d_{ab}(h_{ij}(p_j), h_{kj}(p_k)) + \eta W_j = 0 \tag{8}
\]

\[
\frac{\partial f}{\partial \eta} = \frac{1}{2} \left( \sum_{j=1}^{n} W_j^2 - 1 \right) = 0 \tag{9}
\]

Next, it follows from Eq. (8) that

\[
W_j = \frac{-\sum_{i=1}^{m} \sum_{k=1, k \neq i}^{m} d_{ab}(h_{ij}(p_j), h_{kj}(p_k))}{\eta} \tag{10}
\]

Substituting Eq. (10) into Eq. (9), we have

\[
\eta = -\sqrt{\sum_{j=1}^{n} \left( \sum_{i=1}^{m} \sum_{k=1, k \neq i}^{m} d_{ab}(h_{ij}(p_j), h_{kj}(p_k)) \right)^2} \tag{11}
\]

Obviously, \( \eta < 0 \). Then, combine Eqs. (10) and (11), we can get

\[
W_j = \frac{\sum_{i=1}^{m} \sum_{k=1, k \neq i}^{m} d_{ab}(h_{ij}(p_j), h_{kj}(p_k))}{\sqrt{\sum_{i=1}^{m} \sum_{k=1, k \neq i}^{m} d_{ab}(h_{ij}(p_j), h_{kj}(p_k))^2}}
\]

Lastly, the attribute weight \( W_j \) can be rewritten as:

\[
W_j = \frac{D_j}{\sqrt{\sum_{j=1}^{n} D_j^2}}
\]

By normalizing \( W_j (j = 1, 2, \ldots, n) \), we can get:

\[
\frac{W_j}{\sum_{j=1}^{n} W_j}
\]

4.3. The Extended COPRAS Method Under Probabilistic Hesitant Fuzzy Environment

The COPRAS method has the ability to consider positive and negative attributes, and can be independently evaluated in during the evaluation process. The main advantage of this approach is that it can be used to calculate the utility value of a given alternative, thereby showing how better or worse one alternative is than other alternative. The following steps summarize the method based on the previous Hamming distance in the probabilistic hesitant fuzzy environment.

Step 1: Obtain the evaluation decision matrix \( H = \{ h_{ij}(p_j) \}_{m \times n} \) and normalize it to \( H = \{ h'_{ij}(p_j) \}_{m \times n} \).

Step 2: We go to obtain the weight of the attribute \( W_j (j = 1, 2, \ldots, n) \) by (4.2).

Step 3: Calculate the weighted evaluation value \( \tilde{h}_{ij} \) of each attribute.

\[
\tilde{h}_{ij} = \left\{ \begin{array}{l}
\tilde{c}_{ij}^{(t)} \left( \tilde{p}_{ij}^{(t)} \right), \tau = 1, 2, \ldots, l \\
\tilde{c}_{ij}^{(r)} \left( \tilde{p}_{ij}^{(r)} \right), i = 1, 2, \ldots, m; j = 1, 2, \ldots, n.
\end{array} \right.
\]

Step 4: According to Definition 11, the aggregated value of the weighted value \( \tilde{h}_{ij} \) is calculated for the positive or negative type...
attributes, as shown below:

\[ PN_{i+} = \bigoplus_{j=1}^{r} h_{ij} \]

\[ PN_{i-} = \bigoplus_{j=r+1}^{n} h_{ij} \]

\[ = \bigcup_{r=1}^{l} \left\{ 1 - \prod_{j=1}^{r} \left( 1 - \frac{S_{ij}}{P_{ij}} \right) \left( \sum_{j=1}^{r} \frac{S_{ij}}{P_{ij}} / \sum_{j=r+1}^{n} \frac{S_{ij}}{P_{ij}} \right) \right\} \]

where \( r \) reflects the number of positive type attributes and \( n - r \) indicates the number of negative type attributes, and \( PN_i = \left\{ \frac{S_{ij}}{P_{ij}} / \left( \frac{S_{ij}}{P_{ij}} \right) \right\} \) describes the maximizing or minimizing indexes of \( i \)th attribute.

**Step 5:** Calculate the score values of the aggregated values based on the negative and positive type attributes.

\[ S(PN_{i+}) = \sum_{r=1}^{l} S_{ij} / P_{ij} \]

\[ S(PN_{i-}) = \sum_{r=1}^{n} S_{ij} / P_{ij} \]

**Step 6:** Integrate the scores of the negative and positive attributes of each alternative. The relative priority value or significance value \( RV_i \) of each alternative \( A_i \) is calculated by the following formula:

\[ RV_i = S(PN_{i+}) + \frac{\min S(PN_{i-}) \sum_{i=1}^{m} S(PN_{i-})}{S(PN_{i-}) \sum_{i=1}^{m} S(PN_{i-})} \]

It can also be written as follows:

\[ RV_i = S(PN_{i+}) + \frac{\sum_{i=1}^{m} S(PN_{i-})}{S(PN_{i-}) \sum_{i=1}^{m} S(PN_{i-})} \]

**Step 7:** Rank the alternatives based on the relative priority values or significance values. The higher the relative values \( RV_i \) is, the greater the alternative is.

### 5. CASE STUDY

#### 5.1. Instance Profile

It is suppose that there are five sources of energy \( A_i, i = 1, 2, \ldots, 5 \) to choose. DM use four attributes to assess energy, including advanced technology \( C_1 \), degree of impact on the environment \( C_2 \), potential market value \( C_3 \) and energy service life \( C_4 \). Among them, \( C_1, C_3 \) and \( C_4 \) are positive attributes, \( C_2 \) is negative attribute. The attribute weights unknown. Because of the uncertainty of information and the form of PHFEs, the evaluation value of each energy under the corresponding attribute is given. The DMs give evaluation information \( H_{ij}(p_{ij}) \) in Table 1.

### 5.2. The Decision-Making Process

**Step 1:** Normalize the evaluation information \( H \) into \( \sum_{i=1}^{n} h_{ij}(p_{ij}) \) in Table 2.

**Step 2:** We go to obtain the weight of the attribute \( W_j(j = 1, 2, 3, 4) \) by (4.2), then the result of attribute weights calculation are as follows: \( W_1 = 0.2366, W_2 = 0.215, W_3 = 0.2052, W_4 = 0.3432 \).

**Step 3:** Table 3 indicates the weighted evaluation value \( h_{ij}(p_{ij}) \) of each attribute.

**Step 4:** The aggregated value of \( h_{ij}(p_{ij}) \) is obtained in Table 4.

**Step 5:** The score function values of aggregated value are shown in Table 5.

**Step 6:** The relative priority value or significance value of each alternative is calculated as follows:

\( RV_1 = 0.6150, RV_2 = 0.6460, RV_3 = 0.7666, RV_4 = 0.5742, RV_5 = 0.5306 \)

**Step 7:** Regarding the relative values: \( RV_1 > RV_2 > RV_3 > RV_4 > RV_5 \)

and alternatives are ranked as follows: \( A_1 > A_2 > A_1 > A_4 > A_5 \)

### Table 1: Evaluation information.

|   | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) |
|---|---|---|---|---|
| \( A_1 \) | 0.3(0.1), 0.8(0.8) | 0.3(0.1), 0.6(0.7) | 0.6(0.7) | 0.3(0.4), 0.4(0.3), 0.6(0.2) |
| \( A_2 \) | 0.5(0.6), 0.7(0.4) | 0.4(0.8) | 0.3(0.4), 0.5(0.4) | 0.6(0.9) |
| \( A_3 \) | 0.2(0.4), 0.7(0.4), 0.9(0.1) | 0.4(0.3), 0.6(0.4) | 0.5(0.3), 0.6(0.6) | 0.8(0.7) |
| \( A_4 \) | 0.4(0.2), 0.5(0.3), 0.6(0.4) | 0.3(0.1), 0.5(0.5) | 0.3(0.2), 0.4(0.1), 0.6(0.5) | 0.5(0.4), 0.7(0.2) |
| \( A_5 \) | 0.4(0.3), 0.6(0.1), 0.7(0.2) | 0.3(0.3), 0.4(0.2), 0.5(0.4) | 0.3(0.4), 0.5(0.3), 0.6(0.3) | 0.1(0.2), 0.5(0.7) |

### Table 2: Normalize the evaluation information.

|   | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) |
|---|---|---|---|---|
| \( A_1 \) | 0.3(0.1), 0.55(0), 0.8(0.8) | 0.3(0.1), 0.45(0), 0.6(0.7) | 0.6(0), 0.6(0), 0.6(0.7) | 0.3(0.4), 0.4(0.3), 0.6(0.2) |
| \( A_2 \) | 0.5(0.6), 0.6(0), 0.7(0.4) | 0.4(0), 0.4(0), 0.4(0.8) | 0.3(0.4), 0.4(0), 0.5(0.4) | 0.6(0), 0.6(0), 0.6(0.9) |
| \( A_3 \) | 0.2(0.4), 0.7(0.4), 0.9(0.1) | 0.4(0.3), 0.5(0), 0.6(0.4) | 0.5(0.3), 0.55(0), 0.6(0.6) | 0.8(0), 0.8(0), 0.8(0.7) |
| \( A_4 \) | 0.4(0.2), 0.5(0.3), 0.6(0.4) | 0.3(0.1), 0.4(0), 0.5(0.5) | 0.3(0.2), 0.4(0.1), 0.6(0.5) | 0.5(0.4), 0.6(0), 0.7(0.2) |
| \( A_5 \) | 0.4(0.3), 0.6(0.1), 0.7(0.2) | 0.3(0.3), 0.4(0.2), 0.5(0.4) | 0.3(0.4), 0.5(0.3), 0.6(0.3) | 0.1(0.2), 0.3(0), 0.5(0.7) |
5.3. Comparative Analysis

In order to illustrate the effectiveness of the proposed method, on the basis of the above case study, comparative analysis with other methods are carried out.

Case 1 Comparing with the classical COPRAS method.

In the classical COPRAS method, the attributes information are crisp numbers. Based on this, we convert the probabilistic hesitant fuzzy evaluation information into the crisp numbers using the score function, and then compare it with the extended COPRAS method. By (4.1), we calculate the relative significance values or priority values and the results are as follows: \( R_{V_1} = 0.1954, R_{V_2} = 0.2042, R_{V_3} = 0.2257, R_{V_4} = 0.1927, R_{V_5} = 0.1797. \)

Thus, \( A_1 > A_2 > A_3 > A_4 > A_5. \)

Case 2 The proposed method is compared with the aggregating operator-based methods.

Zhang et al. [24] improved the properties and operation rules of PHFEs and proposed some aggregation operators. Here the weighted geometric aggregation operator and weighted averaging operator are used to integrate the probabilistic hesitant fuzzy information.

The second, aggregate the evaluation values of the same alternative with probabilistic hesitant fuzzy weighted averaging operator (PHFWA), and the score values of the alternatives are as follows:

\[
S(\text{PHFWA}(A_1)) = 0.5748, S(\text{PHFWA}(A_2)) = 0.5854, S(\text{PHFWA}(A_3)) = 0.6995, S(\text{PHFWA}(A_4)) = 0.5715, S(\text{PHFWA}(A_5)) = 0.4949, \text{and finally the ranking is obtained according to the descending order of the score of the alternatives.}
\]

Thus, \( A_3 > A_2 > A_1 > A_4 > A_5. \)

Case 3 Comparing with the possibility degree formula-based method for PHFEs.

Song et al. [51] proposed a charting technique to analyze the structure of PHFEs, and then they proposed a new possibility degree formula to sort the PHFEs. This comparison method is more accurate, especially when facing different PHFEs with the same or intersecting values. At the same time, the proposed possibility degree formula can realize the optimal ranking in probabilistic hesitant fuzzy environment. Here, we use the possibility degree to sort the above case. Specific steps are as follows:

First, we can use the PHFWA operator to aggregate the evaluation values of the same alternative. The results are as follows:

\[
A_1 = [0.48(0.18), 0.52(0.09), 0.63(0.73)],
A_2 = [0.53(0.29), 0.57(0), 0.61(0.71)],
A_3 = [0.61(0.31), 0.68(0.13), 0.75(0.56)],
A_4 = [0.5(0.31), 0.54(0.14), 0.62(0.55)],
A_5 = [0.39(0.35), 0.49(0.18), 0.58(0.47)].
\]

After that, calculate the possibility degree that each alternative has priority over other alternatives through the possibility degree formula, as shown in Table 6.
In order to compare these methods more intuitively, the ranking results are obtained in Table 9.

Table 9 shows that the ranking results obtained by the classical COPRAS method and Zhang et al. [24] and Song et al. [51] are the same as those obtained by the proposed method, indicating that the proposed method is effective.

The possible reasons for using different methods to obtain the same ranking are as follows: these methods obtain the alternative ranking according to the same steps, use the same normalized method to integrate the evaluation values, and then aggregate the evaluation value to obtain the score value of each alternative, and finally rank according to the score value or possibility degree. When the method based on the aggregation operator is compared with the proposed one, we can find that the calculation based on the aggregation operator is more complex. On the other hand, compared with the proposed method, the classical COPRAS method has the same ranking result, but the proposed method can better reflect the gap between alternatives, while the calculation results of the classical method show that the alternatives are similar. Finally, compared with the proposed method, although the method based on the possibility degree has obtained the same result, the difference between these alternatives are very weak.

From the Table 9, we can also see that the distance-based method differs from the proposed method in sorting. This is because of their differences in their basic theory. The method based on distance is to compare the alternatives according to the distance from the NIS and choose the alternative farthest from the NIS as the best alternative. Moreover, the alternative is compared according to the hesitant fuzzy information, and some shortcomings, such as whether the choice of PIS and NIS is appropriate, still need to be considered. The reason for this result may be due to the choice of normalization method and distance measures. Therefore, it is normal to differ from the method mentioned.

Comparative analysis shows that the MADM method proposed in this study has the following advantages over other methods.

1. Based on the proposed new distance measures, this paper constructs a nonlinear model to obtain attribute weights, which makes the results more objective and accurate.
2. Compared with the method based on aggregation operator, the proposed method is easier to calculate, and the aggregation operator is not suitable for solving multiattribute problems with a large number of attribute indexes.
3. Compared with the method based on the classical COPRAS method and the possibility degree-based method, the proposed method can better reflect the differences between alternatives, and the calculation is objective. Moreover, it is reasonable to use the improved COPRAS method to select the optimal alternative under the probabilistic hesitant fuzzy environment.

6. CONCLUSION

PHFS is generated by the preference of DM and can reflect the importance of different membership degrees. In other words, it can reflect the characteristics of different evaluations in MADM problems. In this study, a MADM method with probabilistic hesitant
The PIS and the NIS are determined by the PHFEs. The proposed by Wang et al. [52] can be used for reference. When evaluation information is proposed. First, the definitions and operation rules of HFS and PHFS are discussed. Second, the existing distance measures of PHFEs are discussed and the new distance between PHFEs are proposed. Subsequently, combine the newly proposed distance with the maximizing deviation method to establish a nonlinear programming model to obtain attribute weights. Then, the COPRAS method is extended into probabilistic hesitant fuzzy environments. Lastly, the problem of selecting energy source facing membership degrees and probability information at the same time, the existing research usually chooses to merge the two information in a certain way and then deal with it according to the original method. Therefore, in future research, we need to find a new method to make the probability information can be processed more reasonably when establishing the measure of probabilistic hesitant fuzzy information.

## CONFLICTS OF INTEREST

The authors declare they have no conflicts of interest.

## AUTHORS’ CONTRIBUTIONS

All authors have contributed equally to the paper.

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### Table 7 | The PIS and the NIS are determined by the PHFEs.

|   | C₁ | C₂ | C₃ | C₄ |
|---|----|----|----|----|
| PIS | 0.3(0.1), 0.55(0.0), 0.8(0.8) | 0.3(0.1), 0.45(0.0), 0.6(0.7) | 0.6(0.0), 0.6(0.0), 0.6(0.0) | 0.3(0.4), 0.4(0.3), 0.6(0.2) |
| NIS | 0.2(0.4), 0.7(0.4), 0.9(0.1) | 0.3(0.1), 0.45(0.0), 0.6(0.7) | 0.6(0.0), 0.6(0.0), 0.6(0.0) | 0.3(0.4), 0.4(0.3), 0.6(0.2) |

NIS, negative ideal solutions; PHFEs, probabilistic hesitant fuzzy elements; PIS, positive ideal solutions.

### Table 8 | Distance between each alternative to the PIS and NIS.

|   | C₁ | C₂ | C₃ | C₄ |
|---|----|----|----|----|
| A₁ | 0.1667 | 0 | 0.0433 | 0 |
| A₂ | 0.21 | 0.0967 | 0 | 0.0433 |
| A₃ | 0.1667 | 0 | 0.06 | 0.09 |
| A₄ | 0.2 | 0.0933 | 0.0333 | 0.0567 |
| A₅ | 0.2167 | 0.1033 | 0.0967 | 0.12 |

NIS, negative ideal solutions; PIS, positive ideal solutions.

### Table 9 | The ranking results of the different methods.

| Method | Ranking Values | Ranking |
|---|---|---|
| The classical COPRAS method | 0.1954 0.2042 0.2257 0.1927 0.1797 | A₁ > A₂ > A₃ > A₄ > A₅ |
| Method of Zhang et al. [24] PHFWA | 0.5748 0.5854 0.6995 0.5715 0.4949 | A₁ > A₂ > A₃ > A₄ > A₅ |
| PHFWG | 0.5411 0.5706 0.6106 0.5397 0.4382 | A₁ > A₂ > A₃ > A₄ > A₅ |
| Method of Xu and Zhang [50] | 0.4185 0.4574 0.6228 0.3653 0.3484 | A₁ > A₂ > A₃ > A₄ > A₅ |
| Method of Song et al. [51] | 0.2308 0.2367 0.2646 0.1590 0.1084 | A₁ > A₂ > A₃ > A₄ > A₅ |
| Proposed method | 0.6150 0.6460 0.7666 0.5742 0.5306 | A₁ > A₂ > A₃ > A₄ > A₅ |

COPRAS, COMplex PROportional Assessment; PHFWA, probabilistic hesitant fuzzy weighted averaging operator; PHFWG, probabilistic hesitant fuzzy weighted geometric operator.
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REFERENCES

[1] Z. Turskis, E.K. Zavadskas, J. Antuchevidiene, N. Kosareva, A hybrid model based on fuzzy AHP and fuzzy WASPAS for construction site selection, Int. J. Comput. Commun. 10 (2015), 873–888.
[2] S.M. Yu, J. Wang, J.Q. Wang, L. Li, A multi-criteria decision-making model for hotel selection with linguistic distribution assessments, Appl. Soft Comput. 67 (2018), 741–755.
[3] Z.S. Chen, X.L. Liu, R.M. Rodriguez, X.J. Wang, K.S. Chin, K.L. Tsui, L. Martinez, Identifying and prioritizing factors affecting in-cabin passenger comfort on high-speed rail in China: a fuzzy-based linguistic approach, Appl. Soft Comput. 95 (2020), 106558.
[4] Z.S. Chen, X.L. Liu, K.S. Chin, W. Pedrycz, K.L. Tsui, M.J. Skibniewski, Online-review analysis based large-scale group decision-making for determining passenger demands and evaluating passenger satisfaction: case study of high-speed rail system in China, Inf. Fusion. 69 (2021), 22–39.
[5] A. Arabsheybani, M.M. Paydar, A.S. Safaei, An integrated fuzzy MOORA method and FMEA technique for sustainable supplier selection considering quantity discounts and supplier’s risk, J. Clean. Prod. 190 (2018), 577–591.
[6] V. Torra, Hesitant fuzzy sets, Int. J. Intell. Syst. 25 (2010), 529–539.
[7] Z.S. Chen, K.S. Chin, Y.L. Li, Y. Yang, Proportional hesitant fuzzy linguistic term set for multiple criteria group decision making, Inf. Sci. 357 (2016), 61–87.
[8] Z.S. Chen, Y. Yang, X.J. Wang, K.S. Chin, K.L. Tsui, Fostering linguistic decision-making under uncertainty: A proportional interval type-2 hesitant fuzzy TOPSIS approach based on Hamacher aggregation operators and andness optimization models, Inf. Sci. 500 (2019), 229–258.
[9] M. Iordache, D. Schitea, M. Deveci, Z. Akyurt, I. Iordache, An integrated ARAS and interval type-2 hesitant fuzzy sets method for underground site selection: seasonal hydrogen storage in salt caverns, J. Pet. Sci. Eng. 175 (2019), 1088–1098.
[10] D. Liang, M. Wang, Z. Xu, D. Liu, Risk appetite dual hesitant fuzzy three-way decisions with TODIM, Inf. Sci. 507 (2020), 585–605.
[11] R.M. Rodriguez, B. Bedregal, H. Bustince, Y.C. Dong, B. Farhadinia, C. Kahraman, et al., A position and perspective analysis of hesitant fuzzy sets on information fusion in decision making. Towards high quality progress, Inf. Fusion. 29 (2016), 89–97.
[12] R.M. Rodriguez, A. Labella, G.D. Tre, L. Martinez, A large scale consensus reaching process managing group hesitation, Knowl.-Based Syst. 159 (2018), 86–97.
[13] J. Tang, F. Meng, Calculating priority weights from interval-valued multiplicative hesitant fuzzy preference relations, Soft Comput. 23 (2019), 11573–11592.
[14] J. Tang, F. Meng, S. Zhang, Q. An, Group decision making with interval linguistic hesitant fuzzy preference relations, Expert Syst. Appl. 119 (2019), 231–246.
[15] C. Zhang, D. Li, J. Liang, Multi-granularity three-way decisions with adjustable hesitant fuzzy linguistic multi-granulation decision-theoretic rough sets over two universes, Inf. Sci. 507 (2020), 665–683.
[16] B. Zhu, Decision Making Methods and Applications Based on Preference Relations, Southeast University, Nanjing, China, 2014.
[17] H. Garg, G. Kaur, A robust correlation coefficient for probabilistic dual hesitant fuzzy sets and its applications, Neural Comput. Appl. 32 (2020), 8847–8866.
[18] R. Krishankumar, K.S. Ravichandran, S. Kar, P. Gupta, M.K. Mehlawat, Interval-valued probabilistic hesitant fuzzy set for multi-criteria group decision-making, Soft Comput. 23 (2019), 10853–10879.
[19] J. Li, J.Q. Wang, J.H. Hu, Multi-criteria decision-making method based on dominance degree and BWM with probabilistic hesitant fuzzy information, Int. J. Mach. Learn. Cyb. 10 (2019), 1671–1685.
[20] J. Li, J.Q. Wang, Multi-criteria decision making with probabilistic hesitant fuzzy information based on expected multiplicative consistency, Neural Comput. Appl. 31 (2019), 8897–8915.
[21] J. Li, Q. Chen, L.I. Niu, Z.X. Wang, An ORESTE approach for multi-criteria decision-making with probabilistic hesitant fuzzy information, Int. J. Mach. Learn. Cyb. 11 (2020), 1591–1609.
[22] M. Lin, Q. Zhan, Z. Xu, R. Chen, Group decision making with probabilistic hesitant multiplicative preference relations based on consistency and consensus, IEEE Access. 6 (2018), 63329–63344.
[23] B. Zhu, Z. Xu, Probability-hesitant fuzzy sets and the representation of preference relations, Technol. Econ. Dev. Econ. 24 (2018), 1029–1040.
[24] S. Zhang, Z. Xu, Y. He, Operations and integrations of probabilistic hesitant fuzzy information in decision making, Inf. Fusion. 38 (2017), 1–11.
[25] Y. Dong, Y. Liu, H. Liang, F. Chiclana, E. Herrera-Viedma, Strategic weight manipulation in multiple attribute decision making, Omega. 75 (2018), 154–164.
[26] G.W. Wei, Maximizing deviation method for multiple attribute decision making in intuitionistic fuzzy setting, Knowl.-Based Syst. 21 (2008), 833–836.
[27] J. Wang, G.W. Wei, C. Wei, J. Wu, Maximizing deviation method for multiple attribute decision making under q-rung orthopair fuzzy environment, Def. Technol. 16 (2020), 1073–1087.
[28] J. Yuan, X. Luo, Approach for multi-attribute decision making based on novel intuitionistic fuzzy entropy and evidential reasoning, Comput. Ind. Eng. 135 (2019), 643–654.
[29] F. Xiao, EFMCMD: evidential fuzzy multicriteria decision making based on belief entropy, IEEE Trans. Fuzzy Syst. 28 (2020), 1477–1491.
[30] L. Dong, D. Xiao, Y. Liang, Y. Liu, Rough set and fuzzy wavelet neural network integrated with least square weighted fusion algorithm based fault diagnosis research for power transformers, Electr. Power Syst. Res. 78 (2008), 129–136.
[31] X. Gu, Q. Zhu, Fuzzy multi-attribute decision-making method based on eigenvector of fuzzy attribute evaluation space, Decis. Support Syst. 41 (2006), 400–410.
[32] M.R. Mosavi, A. Khavari, A. Tabatabaei, M.J. Rezaei, Jamming mitigation using an improved fuzzy weighted least square method in combined GPS and GLONASS receiver, AEU-Int. J. Electron. C. 76 (2017), 107–116.
[33] A.K. Shah, D.M. Adhyaru, Parameter identification of PWARX models using fuzzy distance weighted least squares method, Appl. Soft Comput. 25 (2014), 174–183.
[34] J. Tian, D. Yu, B. Yu, S. Ma, A fuzzy TOPSIS model via chi-square test for information source selection, Knowl.-Based Syst. 37 (2013), 515–527.

[35] Y.M. Wang, Z.P. Fan, Z. Hua, A chi-square method for obtaining a priority vector from multiplicative and fuzzy preference relations, Eur. J. Oper. Res. 182 (2007), 356–366.

[36] G. Deng, Y. Jiang, J. Fu, Monotonic similarity measures between intuitionistic fuzzy sets and their relationship with entropy and inclusion measure, Inf. Sci. 316 (2015), 348–369.

[37] J. Ma, Z.P. Fan, L.H. Huang, A subjective and objective integrated approach to determine attribute weights, Eur. J. Oper. Res. 112 (1999), 397–404.

[38] X. Xu, A note on the subjective and objective integrated approach to determine attribute weights, Eur. J. Oper. Res. 156 (2004), 530–532.

[39] Z. Wu, J. Xu, A consensus model for large-scale group decision making with hesitant fuzzy information and changeable clusters, Inf. Fusion. 41 (2018), 217–231.

[40] W. Zhang, J. Du, X. Tian, Finding a promising venture capital project with TODIM under probabilistic hesitant fuzzy circumstance, Technol. Econ. Dev. Econ. 24 (2018), 2026–2044.

[41] J. Ding, Z. Xu, N. Zhao, An interactive approach to probabilistic hesitant fuzzy multi-attribute group decision making with incomplete weight information, J. Intell. Fuzzy Syst. 32 (2017), 2523–2536.

[42] M.C. Das, B. Sarkar, S. Ray, A framework to measure relative performance of Indian technical institutions using integrated fuzzy AHP and COPRAS methodology, Socio-Econ. Plan. Sci. 46 (2012), 230–241.

[43] Y. Zheng, Z. Xu, Y. He, H. Liao, Severity assessment of chronic obstructive pulmonary disease based on hesitant fuzzy linguistic COPRAS method, Appl. Soft Comput. 69 (2018), 60–71.

[44] H. Garg, Nancy, Algorithms for possibility linguistic single-valued neutrosophic decision-making based on COPRAS and aggregation operators with new information measures, Measurement. 138 (2019), 278–290.

[45] H.S. Dhiman, D. Deb, Fuzzy TOPSIS and fuzzy COPRAS based multi-criteria decision making for hybrid wind farms, Energy. 202 (2020), 117755.

[46] M. Xia, Z. Xu, Hesitant fuzzy information aggregation in decision making, Int. J. Approx. Reason. 52 (2011), 395–407.

[47] N. Chen, Z. Xu, M. Xia, The ELECTRE I multi-criteria decision making method based on hesitant fuzzy sets, Int. J. Inf. Tech. Decis. 14 (2015), 621–657.

[48] W. Wu, Y. Li, F. Jin, Z. Ni, X. Zhu, Group decision making model based on probabilistic hesitant fuzzy information aggregation operations, PR & AI. 30 (2017), 894–906.

[49] W. Takahashi, Nonlinear Functional Analysis: Fixed Point Theory and its Applications, Yokohama Publishers, Yokohama, 2000.

[50] Z. Xu, X. Zhang, Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information, Knowl.-Based Syst. 52 (2013), 53–64.

[51] C. Song, Z. Xu, H. Zhao, A novel comparison of probabilistic hesitant fuzzy elements in multi-criteria decision making, Symmetry. Basel. 10 (2018), 177.

[52] L. Wang, H. Garg, N. Li, Pythagorean fuzzy interactive Hamacher power aggregation operators for assessment of express service quality with entropy weight, Soft Comput. 25 (2021), 973–993.