Evaluating Topological Optimized Layout of Building Structures by Using Nodal Material Density Based Bilinear Interpolation

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Abstract

This study presents a boundary representation of Eulerian-type in order to achieve conceptual structural layouts of building structures by relieving material discontinuities of structural topology optimization in comparison with classical ones. According to the partition of unity concept, a bilinear interpolation can be employed by nodal densities and shape functions with the use of discretization of four-node square elements and then material properties, i.e. here, densities in finite elements are not constant but variational values. As a result, the improvement of material continuity can be established as optimal topologies, and numerical singularity and zigzag material boundaries which may occur in classical topology optimization design are relieved. Numerical applications verify the efficiency of this method by evaluating this conceptual structural layout approach for designing building structures.

Keywords: material topology optimization; bilinear interpolation; nodal density; conceptual structural layout

1. Introduction

Recently topology optimization has become one of the most popular and interesting topics in the expanding field of structural optimal design. Numerous research papers (Yan et al., 2014; Lee et al., 2012; Leskovar and Premrov, 2012) concerning new methods in this field have appeared in journals and been presented at conferences. The possibility that the results of topology optimization results may be applied to practical building and civil engineering designs (Liang et al., 2000; Lee et al., 2010), and not only the simple conceptual stages in this study, are very suggestive to engineers and designers.

In the conventional topology optimization introduced by Bendsøe and Kikuchi (1988) the distribution and boundaries of the material are represented on fixed grids such as finite elements with the boundaries represented by a Eulerian method. This representational method is computationally more efficient than the Lagrangian method, which requires generations of moving grids to be meshed. However, one of the most severe computational difficulties in finite element based topology optimization is caused by solid ground elements connected only through a corner node. This situation results in the discontinuity of boundaries between voids (white) and solid (black) regions as shown in Fig.1. This configuration may appear with checkerboard patterns or jagged boundaries and cause artificial stress singularities, which can have an influence on structural analyses. The material discontinuity problem of the corner contacts may be overcome by (a) a more accurate finite element analysis using several simple finite elements per ground element (Rozvany and Zhou, 1991) or higher order element (Sigmund and Petersson, 1998), (b) modification of the original problem by using filtering sensitivity (Sigmund, 2001). However, computational costs may become an issue in (a), even though it improves the resolution of topology optimization. The solution presented in (b) is less than optimal because the original topology optimization problem is changed by filtering. Most of all, both (a) and (b) leave some diagonal chains in the solution.

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An alternative is to use adaptive mesh generations in order to explicitly achieve continuity in the material boundaries in the ranges of topology optimization. Maute (1995) provided the adaptive topology optimization that is typically used. It starts with conventional material based topology optimization. After several optimization steps a smooth boundary is determined. Additional topology optimization steps with a new discretization are performed within this domain and the result is improved by a shape optimization procedure which adjusts the smooth boundary. However, this approach is computationally burdensome due to its use of Lagrangian-type boundary representation.

Another alternative in order to remove the discontinuity in the material boundaries is the level set method, which has been recently introduced by Sethian and Wiegmann (2000) and Allaire et al. (2004). In the optimal solution smooth boundaries are represented by zero level sets of signed distance functions even on fixed meshes, which also maintain computational efficiency. However, the classical level set method has some limitations (Burger et al., 2004). The first drawback is that the optimal smooth shapes absolutely depend on the initial topology. If the initial topology is not appropriate, the resulting optimal shape may be not valid. The second one is related to a level set equation which couples the evolution of the level set functions to the movement of the boundaries. For numerical stability, the length of every time-step during the integration of the level set equation has to be limited so that the level sets do not move more than one grid length, following the CFL condition. Because of this rigorous restriction, the speed of convergence may be slowed substantially, although narrow band and re-initialization schemes are often used as solutions for level set equations. Another problem is related to the nature of boundary evolution. Level sets are designed to describe the propagation of interfaces by shape or local derivatives, for example applying speed functions. Therefore, it may be difficult to create new holes in material regions or to seed materials in previously empty regions.

Considering the limitations of these topology optimization methodologies, the present approach has been devised in terms of continuous material boundaries for computational efficiency. This approach arrives at the same boundary as the conventional topology optimization methods because boundary propagations use fixed grids and design parameter updates using gradient-based OC or MMA. According to the partition of unity concept (Melenk and Babuška, 1996), it is possible to generate arbitrary point densities located in an element through bilinear interpolation and then the density does not take a constant but a variational distribution. This redistribution of design parameters results in a layout that is much more detailed than using constant element densities, and it reduces numerical singularity near corner contacts. However, the material discontinuity of the optimal solutions does not disappear completely since grid density distributions based on arbitrary point densities in the elements also depend on the geometry of the grid similar to conventional element density distributions. In order to solve these problems, the 3D point density function, which results from bilinear interpolation, is directly related to the extraction of practically continuous material boundaries. A 0.5 cut-off level of iso-lines yields completely obvious boundaries between voids and solids. Note that material volumes surrounded by only 0.5 iso-lines identify approximate volume fraction constraints.

The outline of this study is as follows: The geometric Eulerian type boundary representation is described in Section 2. In Section 3, formulations of topology optimization using nodal material density based bilinear interpolation are presented including sensitivity analysis of optimization design variables. Section 4 shows a promising numerical application generating a design layout of a beam-to-column connection structure, which is the most commonly used in the field of building engineering by using the present nodal material density based topology optimization. Section 5 presents the conclusions of this study.

2. Geometric Eulerian Type Boundary Representation

2.1 Definition of Design Domain

The design domain for material distribution problems of topology optimization is divided into solid (1) and void (0) regions using the signs of Heaviside function. Eulerian-type boundary represented by the zero level sets of a signed distance function is well known as the level set method (Sethian and Wiegmann 2000; Allaire et al., 2004). In this study, the 0.5 iso-line (Lee, 2007) is utilized for smooth boundary representation. Note that the 0.5 cut-off level is appropriate since this solid region is satisfied with volume constraint. When the densities of finite elements are smaller than 0.5, the regions are considered to be void phases in the design domain. Otherwise, the regions whose densities are over 0.5 indicate solids.

2.2 Approximation of Discontinuities

First, consider the construction of an approximation that is itself discontinuous, which is often called a strong discontinuity. The mesh is completely independent of the geometry or location of the discontinuity.

Although the boundaries can be represented by any technique, for convenience let's describe the boundaries of the discontinuity by 3-dimensional continuous density function. Suppose that square finite elements with four nodes are being considered discretization. The shape function at node is denoted by \( N_i \) and the corresponding nodal density, by \( \Phi_i \). The grid point density function, \( \Phi_{\text{sub}}(x) \) is usually approximated by bilinear interpolation. The approximation will be of the following form:
where \( x \) is an arbitrary location of grid point density. It can be seen that this is an application of the partition of unity concept (Melenk and Babuška, 1996). The representation of the discontinuity by Eq. (1) enables it to be completely described by nodal data.

Fig. 2. shows the grids and a 3-dimensional grid point density function located in one finite element. It should be noted that the one finite element is detailed by a 3x3 grid, whose properties are defined by the nodal data, i.e. the grid point density data of the 3-dimensional grid point density function.

While the grid point densities are determined during the optimization processes, the boundaries by 0.5 isolines are propagated with changes to the topologies such that it is possible to create voids in solids or solids in voids.

\[
\Phi_{sub}(x) = \sum_{j=1}^{4} N_j(x) \phi_j(x) \tag{1}
\]

3. Topology Optimization Problem
3.1 Problem Statements

In this paper, a 2-dimensional linear elastostatic problem is considered for structural topology optimization. In continuous formulations of the topology optimization problem, the design is given by a continuous scalar function \( 0 \leq \Phi \leq 1 \), in the fixed design domain \( \Omega \subseteq \mathbb{R}^n \). The schematic of a topologically optimized solid structure with specified field and boundary conditions is shown in Fig. 3.

The problem statements of the structural topology optimization are specified by the objective function and constraints. According to a principle of minimum potential, the objective function is written as a minimal strain energy form as follows:

\[
f = \min \left[ -\frac{1}{2} \int_{\Omega} \delta \varepsilon^T C \varepsilon \; d\Omega \right] \tag{2}
\]
After discretization, the continuous material tensor $C$ is dependent on the density-stiffness relationship of the typical SIMP approach.

The inequality optimization constraint is $0 \leq \Phi \leq 1$. A linear elastostatic equilibrium is an equality constraint and written as

$$
\int_{\Omega_i} \delta \epsilon^{T} \sigma \, d\Omega_i = \int_{\Omega_i} \delta \epsilon^{T} b \, d\Omega_i + \int_{\Gamma_i} \delta \epsilon^{T} t \, d\Gamma_i
$$

(3)

where it is defined as the work of body forces $b$ and traction forces $t$ on the displacement field $u$.

Another equality constraint is a limit on the required amount of materials in terms of the constant volume $V_0$ of the design domain as follows:

$$
\int_{\Omega_i} d\Omega_i - V_0 = 0
$$

(4)

### 3.2 SIMP Approach

After the discretization process of the continuous design domain $\Omega_i$, the material density $\Phi$ is constantly assigned as $\Phi_i$ for each finite element $i$ in order to complete the structural analyses at the level, for example in the finite element method. The element density is defined by applying a penalty contour to the design variable field, i.e., as in the so-called "power law approach" or SIMP approach. According to this approach, material density distribution affects element stiffness. Thus, the element stiffness-density relationship may be expressed in terms related to Young’s modulus $E$ i.e. $E_i$ is assigned by the updated element density $\Phi_i$ and is defined as

$$
E_i(\Phi_i) = E_0 \left( \frac{\Phi_i}{\Phi_0} \right)^k;
$$

(5)

where $E_0$ and $\Phi_0$ are the nominal values of Young’s modulus and material density, respectively. The penalty parameter $k \geq 1$, penalizes intermediate material densities.

Fig.4. shows the penalty relationship between Young’s modulus and element density according to various penalty parameters.

For example, and without loss of generality, an isotropic material model with a plane stress (such as a wall structure) is used here, so that

$$
C_i = \frac{E_i(\Phi_i)}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{bmatrix}
$$

(6)

where $C_i$ is a material tensor of each finite element $i$ and includes the updated term of Young’s modulus $E_i$ defined by the updated element density $\Phi_i$. $\nu$ is Poisson’s ratio.

### 3.3 Discrete Formulation of Variational Sensitivity Analysis

In general, the sensitivity (Jin and Kim, 2013) of the optimization problems such as objective functions or constraints can be calculated by analytical or numerical methods. Since sensitivity errors of the numerical sensitivity method may become great, the method is often used to verify solutions. The analytical method is usually used for the sensitivity of optimization problems due to the small errors in its solutions. The analytical sensitivity method is distributed as both a discrete and variational approach. In the discrete approach, optimization problems are at first discretized and then the derivative is calculated. However, the variational approach first differentiates continuous optimization problems, and second, the derivative is discretized.

The analytical sensitivity method of the variational approach is utilized here, since the variational method is numerically more efficient than the discrete method in certain optimization problems and the large numbers of design parameters, which have been stated by Schwarz (2001). Since continuous displacement fields depend on the design variables, $s$ (for instance, material densities), the total differential form of the objective function consists of parts of an explicit partial derivative and an implicit partial derivative, and is defined by Haug et al. (1986).

$$
\nabla_s f^e = \nabla_s f + \nabla_u f^e \nabla_u u
$$

(7)

The total partial derivative is written as

$$
\nabla_s f = \frac{1}{2} \int_{\Omega_i} e^{T} \nabla_s C(\Phi) e \, d\Omega_i + \frac{1}{2} \int_{\Gamma_i} e^{T} C(\Phi) \nabla_s e \, \nabla_u u \, d\Gamma_i
$$

(8)

By using a derivative of the equilibrium that satisfies the field and boundary conditions, the term of a derivative of the continuous displacement fields $\nabla_u u$ by design variables can be written as

$$
\int_{\Omega_i} \delta \epsilon^{T} C(\Phi) L \nabla_u u \, d\Omega_i = \int_{\Omega_i} \delta \epsilon^{T} \nabla \bar{b} \, d\Omega_i + \int_{\Gamma_i} \delta \epsilon^{T} \nabla \bar{t} \, d\Gamma_i.
$$
In order to calculate the derivative of the continuous displacement fields $\nabla u$, an adjoint method is used here. It is not necessary for the adjoint method to use a great amount of computational resources to directly calculate the continuous displacement fields. A new objective function $f$ is defined as

$$f = f - \lambda \left[ \delta u^{T} \sigma \, d\Omega_{s} - \int_{\partial \Omega_{s}} \delta u^{T} \mathbf{b} \, d\Gamma_{s} - \int_{\partial \Omega_{s}} \delta u^{T} \mathbf{i} \, d\Gamma_{s} \right]$$

(10)

where the renewed objective function has an additional 0-term of the static equilibrium, which is multiplied by a Lagrangian multiplier $\lambda$. The derivative of the Lagrangian multiplier disappears because of the 0-term. Therefore the derivative of Eq. (10) is written as

$$\nabla_{u} f = \nabla_{u} f - \lambda \left[ \delta u^{T} \sigma \, d\Omega_{s} - \int_{\partial \Omega_{s}} \delta u^{T} \mathbf{b} \, d\Gamma_{s} - \int_{\partial \Omega_{s}} \delta u^{T} \mathbf{i} \, d\Gamma_{s} \right]$$

(11)

The Lagrangian multipliers take arbitrary values. A specific Lagrangian multiplier value can be determined in Eq. (11) in order to remove the derivative of the continuous displacement fields, which is numerically very expensive. Therefore, a specific equation ($a = 0$ in Eq. (14) is written to include the specific Lagrangian multiplier value. After discretization of the continuous design domain, the specific equation with a satisfactory Lagrangian multiplier is expressed as

$$\hat{u}^{T} \left[ B^{T} C(\Phi) B \right] \hat{u} \, d\Omega_{s} - \lambda \left[ \delta u^{T} \sigma \, d\Omega_{s} - \int_{\partial \Omega_{s}} \delta u^{T} \mathbf{b} \, d\Gamma_{s} - \int_{\partial \Omega_{s}} \delta u^{T} \mathbf{i} \, d\Gamma_{s} \right] = 0$$

(12)

Through Eq. (12), the required Lagrangian multiplier value is obtained and the value is substituted for the discrete form of Eq. (11). Under the assumptions that body force $\mathbf{b}$, traction force $\mathbf{t}$, differential matrix $L$ and Jacobi matrix $J$ are independent of design variables $s$, the total partial derivative of the objective function can be simply rewritten as follows:

$$\nabla_{u} f = \frac{1}{2} \hat{u}^{T} \left[ B^{T} C(\Phi) B \right] \hat{u} \, d\Omega_{s}$$

(13)

4. Numerical Applications and Discussion

4.1 Topology Optimization of Beam-To-Column Connections with Loads

This example shows how the boundary discontinuity of an optimal topology and shape can be removed in order to perform continuous material topology optimization. In this example, the design of a beam-to-column connection is considered, where each loading case $P_{1} = 1 \, kN$ and $P_{2} = 1 \, kN$ are studied. Fig.5. illustrates the layout of initial domain with $L$, $H = 2m$, and a simple support at the bottom. The material properties are given by Young's modulus $E = 1.0 \, MPa$ and Poisson's ratio $\nu = 0.3$. A plane stress state is assumed.

An objective function is a minimal strain energy or maximal stiffness, and a volume constraint of 15% of the total volume is treated as an optimization problem.
Fig. 6. Topological Optimized Material Density Distribution at Iteration 50 and Loading Case $P_1 = 1 \text{kN}$

(a) Element-based density distribution
(b) Grid-based density distribution

Fig. 7. Topological Optimized Material Density Distribution at Iteration 50 and Loading Case $P_2 = 1 \text{kN}$

(a) Element-based density distribution
(b) Grid-based density distribution

Fig. 8. Descriptions of 0.5 Iso-Line Boundary for Topological Optimized Layouts of Beam-To-Column Connections with Respect to Loading Conditions

(a) 0.5 iso line boundary at loading case of $P_1$
(b) 0.5 iso-line boundary at loading case of $P_2$
are an improvement over the classical element-based topology optimization method.

In order to extract complete continuous boundaries, the 0.5 iso-lines of the 3-dimensional grid point density function can be applied to the topology as shown in Fig.9. This 0.5 iso-line boundary in the case of both loading cases is shown in Fig.8. Here the weak vertical material areas shown in Fig.6. disappear in Fig.8. due to the use of 0.5 iso-line.

The convergence histories of the objective function and volume constraint are illustrated in Fig.10. The volume constraints of the 0.5 iso-lines are gradually converged during the iterations.

5. Conclusions
In this work, the bilinear interpolation method and a 0.5 iso-line were used in order to yield continuous material boundaries instead of the discontinuous boundaries of the classical topology optimization method. The relationship between both concepts (the bilinear interpolation method and 0.5 iso-line) was demonstrated using a 3-dimensional grid point density function, which is a variation of the distribution in the design domain.

The volumes in the material boundaries, which are constructed by 0.5 iso-lines, are satisfied with the limited volumes as the volumes are gradually constrained during the optimization iterations. This justifies the use of the 0.5 level in the grid-point density function in order to generate completely continuous optimal shapes.

From this study, the kinds of material discontinuity shown in Fig.11. were classified. Fig.11. (a), as mentioned in Fig.1. in Section 1, denotes very strong material discontinuity, for example, in the material boundaries of classical element-based topology optimization. This difficulty can be reduced by using the filtering sensitivity introduced by Sigmund and the results are shown in Fig.11. (b), i.e., strong material discontinuity. The approach used in this paper resulted in weak material discontinuity as shown in Fig.11. (c).
Using this new methodology with other engineering problems (non-linear Solid Mechanics, Fluid Mechanics, Electromagnetism, etc.) that have general material discontinuity is straightforward. They will be explored in future works.

The practical use of the present structural layout design method by using the present computational software topology optimization is adaptable to arbitrary geometrical and loading situations. In addition, there is a considerable potential for applying it to practical mega structures such as bridges and buildings for easy application of this simple and clear computational program with graphical input and output routines which could replace the traditional drawing board methods for developing building design and engineering.

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References

1) Bendsøe, M. P., Kikuchi, N. (1988) Generating optimal topologies in optimal design using a homogenization method, Computational Methods in Applied Mechanics and Engineering, 71, pp.197-224.
2) Leskovar, V. Z., Premrov, M. (2012) Design Approach for the Optimal Model of an Energy-Efficient Timber Building with Enlarged Glazing Surface on the South Facade, Journal of Asian Architecture and Building Engineering, 11(1), pp.71-78.
3) Lee, D. K., Starossek, U., Shin, S. M. (2010) Topological optimized design considering dynamic problem with non-stochastic structural uncertainty, Structural Engineering and Mechanics, 36(1), pp.79-94.
4) Xia, L. J., Wu, J. M., Jin, X. D. (2002) Study on topology optimum design of engineering structures, Journal of Ship Mechanics, 6(6), pp.107-113.
5) Rozvany, G. I. N., Zhou, M. (1991) Applications of the COC algorithm in layout optimization, (In: Eschenauer, H., Matteck, C. and Olhoff, N. (Eds.) Engineering Optimization in Design Processes, Proceeding International Conference held in Karlsruhe, Germany, Sep. 1990), pp.59-70.
6) Sigmund, O., Petersson, J. (1998) Numerical instabilities in topology optimization: A survey on procedures dealing with checkerboards, mesh-dependencies and local minima, Structural Optimization, 16, pp.68-75.
7) Sigmund, O. (2001) A 99 topology optimization code written in Matlab, Structural Multidisciplinary Optimization, 21, pp.120-127.
8) Maute, K., Ramm, E. (1995) Adaptive topology optimization, Structural Optimization, 10, pp.100-112.
9) Sethian, J. A., Wiegmann, A. (2000) Structural boundary design via level set and immersed interface methods, Journal of Computational Physics, 163, pp.489-528.
10) Allaire, G., Jouve, F., Toader, A. M. (2004) Structural Optimization using Sensitivity Analysis and A Level-set Method, Journal of Computational Physics, 194(1), pp.363-393.
11) Burger, M., Hackl, B., Ring, W. (2004) Incorporating Topology Derivative into the Level Set Methods, Journal of Computational Physics, 194, pp.344-362.
12) Melenk, J. M., Babuška, I. (1996) The partition of unity finite element method: basic theory and applications, Research Report, 96-01, Seminar für Angewandte Mathematik Eidgenössische Technische Hochschule, Switzerland.
13) Schwarz, S. (2001) Sensitivitätsanalyse und Optimierung bei Nichtlinearer Strukturverhalten, Ph.D. Thesis, University of Stuttgart, Germany.
14) Haug, E. J., Choi, K. K., Komkov, V. (1986) Design Sensitivity Analysis of Structural Systems, Academic Press, Orlando, New York.
15) Lee, D. K. (2007) Combined Topology and Shape Optimization of Structures using Nodal Density as Design Parameter, Journal of Asian Architecture and Building Engineering, 6(1), pp.151-158.
16) Yan, X., Huang, X., Zha, Y., Xie, Y. M. (2014) Concurrent topology optimization of structures and their composite microstructures, Computers and Structures, 133(3), pp.103-110.
17) Lee, D. K., Kim, J. H., Starossek, U., Shin, S. M. (2012) Evaluation of Structural Outrigger Belt Truss Layouts for Tall Buildings by using Topology Optimization, Structural Engineering and Mechanics, 43(6), pp.711-724.
18) Jin, E. M., Kim, J. T. (2013) Analysis of a Sensitivity Evaluation of the Exterior PV System for a House, Journal of Asian Architectural and Building Engineering, 12(1), pp.101-108.