

**Y(4320) and Y(4390) as the candidate for \(\psi(3^3D_1)\) charmonium**

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**I. INTRODUCTION**

By analyzing the precise cross sections for \(e^+e^- \rightarrow \omega \chi_{c0}[1]\), \(e^+e^- \rightarrow \pi^+\pi^- J/\psi [2]\), \(e^+e^- \rightarrow \pi^+\pi^- h_c [3]\) and \(e^+e^- \rightarrow \pi^+D^0D^{*-}[4]\), the BESIII collaboration reported a series of vector charmonium-like states, which are \(Y(4220), Y(4320)\) and \(Y(4390)\). The charmonium-like state \(Y(4220)\) have been reported in \(\chi_{c0}(2430), \pi^+D^0D^{*-}\), \(\pi^+\pi^- h_c\) and \(\pi^+\pi^- J/\psi\) channels at present. Its width were reported to be around 40 MeV by analyzing the cross sections for \(e^+e^- \rightarrow \chi_{c0}\) and \(e^+e^- \rightarrow \pi^+\pi^- J/\psi\), while it were measured to be about 70 MeV in the cross sections for both \(e^+e^- \rightarrow \pi^+\pi^- h_c\) and \(e^+e^- \rightarrow \pi^+D^0D^{*-}\) processes. As for \(Y(4320)\), it was a broad charmonium-like state and only reported in \(\pi^+\pi^- J/\psi\) process. The charmonium-like state \(Y(4390)\) is also a broad state and was observed in both \(\pi^+\pi^- h_c\) and \(\pi^+D^0D^{*-}\) channels. The resonances parameters of \(Y(4220), Y(4320)\) and \(Y(4390)\) are presented in Fig. 1.

These newly observed charmonium-like states make resonances with \(J^{PC}=1^{--}\) between 4.0 – 4.5 GeV overcrowed and the nature of the these charmonium-like states becomes a intriguing questions. As for \(Y(4220)\), it has been observed in various channels. In the \(\pi^+\pi^- J/\psi\) channel, a structure, \(Y(4260)\) was firstly reported by BaBar Collaboration [5] and then confirmed by Belle Collaboration [6]. More precise analysis from BESIII Collaboration indicates the structure \(Y(4260)\) should contain two charmonium-like state, \(Y(4220)\) and \(Y(4320)\) [2], the former one is consistent with the one observed in the channels of \(\chi_{c0}\), \(\pi^+\pi^- h_c\), and \(\pi^+D^0D^{*-}\). Since \(Y(4260)/Y(4220)\) is close to the threshold of \(D_1(2420)\), thus it could be considered as a molecular state composed of \(D_1(2420)\) \(D_1(2420)\) [7–12]. While, the QCD sum rule estimations indicate that \(Y(4260)\) could be a mixed charmonium-tetraquark state [13, 14].

Before the observations of \(Y(4220)\), we predicted a narrow \(\psi(4S)\) around 4.2 GeV in Ref. [15], while \(\psi(4415)\) was considered as \(\phi(5S)\). After the observation of \(Y(4220)\) in the \(\chi_{c0}\) channel, we investigated \(Y(4220) \rightarrow \chi_{c0} J/\psi\) with \(Y(4220)\) assigned as \(\psi(4S)\), we find the charmed meson loops contributions are essential in understanding the hidden charm decays of \(\psi(4S)\) [16]. With the same mechanism, the branching ratio of \(\psi(4S) \rightarrow \eta J/\psi\) was predicted to be less than \(1.9 \times 10^{-3}\) [16]. This prediction is consistent with the combined fit of the \(\psi(4S) \rightarrow \eta J/\psi\) in \(B^+ \rightarrow \eta J/\psi K\) and \(e^+e^- \rightarrow \eta J/\psi\) processes [17].

With the precise measurements of Belle Collaboration [18], we find the evidence of the \(\psi(4S)\) in the cross sections for \(e^+e^- \rightarrow \pi^+\pi^- \phi(2S)\) [19] and the fit results are in line with the newly measurements from BES III Collaboration [20, 21].

As for \(Y(4320)\), it is also observed in the \(\pi^+\pi^- J/\psi\) channel. Actually, in the \(\pi^+\pi^- \phi(2S)\) channel, there exist a charmonium-like state \(Y(4360)\) near the newly observed \(Y(4320)\) [22, 23]. The mass of \(Y(4360)\) was fitted to be

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\(^1\) The charge conjugate states are implied throughout this work
4324 ± 24 MeV by BaBar Collaboration [22], which is consistent with the one of Y(4320). In addition, with recent precise data, the analysis in Ref. [20] also indicates that the charmonium-like states Y(4360) in the π⁺π⁻ψ(2S) and Y(4320) in the π⁺π⁻J/ψ should be the same state. In both π⁺π⁻h₁, and π⁺D⁺D⁻ channel, the charmonium-like state Y(4390) were observed, however, this state is absent from the π⁺π⁻J/ψ channel. In Ref. [24, 25], the possibility of Y(4390) as a D⁺D⁻(2420) molecular state were investigated. While in Ref. [26], the lineshapes of the cross sections for e⁺e⁻ → π⁺π⁻J/ψ, π⁺π⁻h₁, π⁺D⁺D⁻ could be well reproduced by the interferences of the well established charmonia ψ(4160) and ψ(4415) as well as Y(4220).

To date, the intrinsic structures of the charmonium-like states, Y(4220), Y(4320) and Y(4390) are still under debate. In our previous work [15, 16, 19], we categorized Y(4220) as ψ(4S) and ψ(4415) as ψ(5S), thus, there are no room for Y(4320) and Y(4390) in the S wave vector charmonium. However, in the D wave charmonium sector, the ψ(3770) and ψ(4160) are ψ(3D₁) and ψ(2D₁), respectively. The higher D wave charmonium have not been established. The mass of ψ(3D₁) was predicted to be 4519 MeV by the relativistic quark model [27]. However, for the higher charmonia, the coupled channel effects will shift their mass to the open charm threshold [28], thus the predicted mass of ψ(3D₁) in Ref. [27] should be too large since the coupled channel effects are not included. In Ref. [29], a screened potential model were employed to depict the coupled channel effect in the charmonium and the mass of ψ(3D₁) was predicted to be 4317 MeV. Considering the uncertainty of the quark model, both Y(4320) and Y(4390) could be the candidate for ψ(3D₁) state. To further test this possibility, we estimate the open charm decays of Y(4320) and Y(4390) with the assignment of ψ(3D₁) state, which is the main task of the present work.

This paper is organized as follow. After introduction, the formula of open charm decays of ψ(3D₁) states are presented in Section II. Our numerical results are given in Section III. Section IV is devoted to summary.

II. QUARK PAIR CREATION MODEL AND OPEN CHARM DECAYS OF ψ(3D₁) CHARMONIUM

A. review of quark pair creation model

Here, we adopt the quark pair creation (QPC) model (also named 3P₀ model since the JPC quantum numbers of the quark pair created from the vacuum are 0⁺⁺) to estimate the open charm decays of the vector D wave charmonia. The QPC model was first proposed by Micu [30–33] and then widely used to estimate the OZI allowed strong decay processes [30–39]. In the QPC model, the related S – matrix of A → BC process reads,

$$\langle BC | S | A \rangle = I - i2\pi \delta(E_J - E_i)(BC | T | A),$$  \hspace{1cm} (1)

where the transition operator T is,

$$T = -3\gamma \sum_m \langle 1m | 1 - m | 00 \rangle \int d{k}_1 d{k}_2 d{k}_3 d{k}_4 \delta^3(k_3 + k_4) \times \mathcal{Y}_{lm} \left(\begin{array}{c} k_1 - k_2 \\ k_3 - k_4 \\ 2 \end{array}\right) \chi_{1,m}^{34}(x, y, z, \alpha, \beta, \gamma, \delta) \Pi_{1,2}^{34}(x, y, z, \alpha, \beta, \gamma, \delta).$$  \hspace{1cm} (2)

where \(\mathcal{Y}_{lm}(k) = |k|Y_{lm}(0, \phi), \chi_{1,m}^{34} = (u \bar{d} + d \bar{u} + s \bar{s})/\sqrt{3}\) and \(\omega_{0}^{(3)} = \delta_{\alpha\gamma(\beta\delta)}\) are the space, spin, flavor and color parts of the wave functions, respectively. \(\alpha, \beta, \gamma, \delta\) are the color index of the created quark pair. In the QPC model, the parameter \(\gamma\) is introduced to represent the strength of the quark-antiquark pair creation from the vacuum and it could be fixed by fitting the decay data. In the present work, we take \(\gamma = 6.3\) for the up/down quark pair and \(\gamma_s = \gamma/\sqrt{3}\) for strange quark pair creation [36, 37].

In the initial rest frame, the matrix element of the transition operator is

$$\langle BC | T | A \rangle = \sqrt{8E_A E_B E_C} \gamma \sum_{M_{L_A} M_{S_A} M_{L_B} M_{S_B} M_{L_C} M_{S_C}} \langle 1m, 1 - m | 00 \rangle \times \langle L_A, M_{L_A}, S_A M_{S_A} | J_A, M_A \rangle \langle J_B, M_{L_B}, S_B M_{S_B} | J_B, M_B \rangle \times \langle L_C, M_{L_C}, S_C M_{S_C} | J_C, M_C \rangle \langle \phi_R \phi_C \phi_A \phi_B \rangle \times \langle \Lambda_{S_B M_{S_B} S_{L_B}}^{12} \Pi_{S_B M_{S_B} L_{L_B}}^{34} | 1_{S_A M_{S_A} L_{L_A}}^{12} \Pi_{S_A M_{S_A} L_{L_A}}^{34} \delta^{3}(x, y, z) \rangle.$$  \hspace{1cm} (3)

where \(\langle \Lambda_{S_B M_{S_B} S_{L_B}}^{12} \Pi_{S_B M_{S_B} L_{L_B}}^{34} | 1_{S_A M_{S_A} L_{L_A}}^{12} \Pi_{S_A M_{S_A} L_{L_A}}^{34} \delta^{3}(x, y, z) \rangle\) are the flavor matrix element and spin matrix element, respectively. While the color matrix element (\(\omega_0^{(3)}\)) cancels out the factor 3 in the transition operator defined in Eq. (2). The matrix element of the spatial part reads

$$\langle \Lambda_{S_B M_{S_B} S_{L_B}}^{12} \Pi_{S_B M_{S_B} L_{L_B}}^{34} | 1_{S_A M_{S_A} L_{L_A}}^{12} \Pi_{S_A M_{S_A} L_{L_A}}^{34} \delta^{3}(x, y, z) \rangle = \frac{1}{2} \sum_{M_{L_A} M_{S_A} M_{L_B} M_{S_B} M_{L_C} M_{S_C}} \times \delta^{3}(k_B - k_1 - k_3) \delta^{3}(k_C - k_2 - k_4) \times \Psi_{n_{L_C} M_{S_C}}^{*} (k_2, k_4) \Psi_{n_{L_B} M_{S_B}} (k_1, k_3) \times \mathcal{Y}_{1m} \left(\begin{array}{c} k_1 - k_2 \\ k_3 - k_4 \\ 2 \end{array}\right).$$  \hspace{1cm} (4)

which reflect the overlap of the spatial wave functions of the initial state and final states. The amplitude of the decay process is

$$\langle BC | T | A \rangle = \delta^{3}(k_B + k_C - k_1 - k_3) M_{M_{A} M_{B} M_{C}}.$$  \hspace{1cm} (5)

By the Jacobi-Wick rotation, the amplitude can be transformed into partial wave amplitude, which is,

$$M^{UL}(A \rightarrow BC) = \sqrt{2L + 1} \sum_{M_{A} M_{B} M_{C}} \langle L_{O} J_{A} M_{A} | J_{B} M_{B} \rangle \times \langle J_{B} M_{B} J_{C} M_{C} | J_{A} M_{A} \rangle M_{M_{A} M_{B} M_{C}}.$$  \hspace{1cm} (6)

In terms of the partial wave amplitude, the partial width is

$$\Gamma = \pi^2 |K| \sum_{L} |M^{UL}|^2,$$  \hspace{1cm} (7)

where \(|K| = \lambda^{(3)}(M_{A}^2, M_{B}^2, M_{C}^2)\) with the Källen function \(\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz\).
B. open charm decays of $\psi(3^3D_1)$ charmonium

| Channel | $\psi(3770)$ | $\psi(4160)$ | $Y(4320)$ | $Y(4390)$ |
|---------|-------------|-------------|------------|------------|
| \(D\bar{D}\) | ✓ | ✓ | ✓ | ✓ |
| \(D_s^+D_s^-\) | ✓ | ✓ | ✓ | ✓ |
| \(D_s^0\bar{D}_s^0\) | ✓ | ✓ | ✓ | ✓ |
| \(D_s^+\bar{D}_s^-\) | ✓ | ✓ | ✓ | ✓ |
| \(D_s^{*+}\bar{D}_s^{*-}\) | ✓ | ✓ | ✓ | ✓ |
| \(D_s^{*0}\bar{D}_s^{*0}\) | ✓ | ✓ | ✓ | ✓ |

To test the reliability of the QPC model, we first investigate the open charm decays of the ground and first radial excited \(3^3D_1\) charmonia, which are \(\psi(3770)\) and \(\psi(4190)\). These two \(D\) wave vector charmonia have been well established and we can compared the QPC estimations with the corresponding experimental data. And then apply the same model to study the open charm decays of \(\psi(3^3D_1)\) state. The decay modes of \(\psi(3770)\), \(\psi(4160)\) and \(\psi(3^3D_1)\) are listed in Table I.

For \(Y(4320)\) and \(Y(4390)\), their mass are above the threshold of \(D_1(2430)\bar{D}\) and \(D_1(2420)\bar{D}\). The charmed meson \(D_1(2430)\) and \(D_1(2420)\) are the mixture of the \(1^3P_1\) and \(1^1P_1\) states and the mixing scheme is,

\[
\begin{pmatrix}
[D_1(2430)] \\
[D_1(2420)]
\end{pmatrix}
= \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
[|1^3P_1\rangle] \\
[|1^1P_1\rangle]
\end{pmatrix},
\]

where the mixing angle \(\theta = -54.7^\circ\), which is determined by the heavy quark limit [40–42]. The partial wave amplitude of the involved open charm decays are given in Table II.

III. NUMERICAL RESULTS AND DISCUSSIONS

With above preparations, we could investigate the open charmed decays of the \(D\) wave vector charmonia. In Eq. (4), the spatial wave functions of the mesons are involved. In principle, these mesons wave functions could be estimated by the constituent quark model. However, as we discussed in the first section of the present work, there exist some uncertainties in the quark models. Thus, in the present work, we employ the simple harmonic oscillator wave function to simulate the spatial distribution of the quark-antiquark in meson. The detailed form of the spatial wave function is

\[
\Psi_{nf_{m_f}}(R, k) = \frac{(-)^n(-)^lR^{3/2}}{\sqrt{\pi}} \sqrt{\frac{2^{\ell+n+2}}{n!(\ell+1)!}} (kR)^l
\times F\left(-n, \ell + \frac{3}{2}, \frac{R^2k^2}{2}\right) e^{-R^2k^2/2} Y_{\ell m_f}(\hat{k})
\]

where \(n, \ell\) and \(m_f\) are the radial, angular momentum and magnetic quantum numbers, respectively. \(F(-n, \nu, x)\) and \(Y_{\ell m_f}(\hat{k})\) indicate the hypergeometric function and spherical harmonic function, respectively.

In the spatial wave function, a parameter \(R\) is introduced. As for the lowest charmed mesons, the predictions of the relativistic quark model are well consistent with the experimental measurements. Thus, in the present work, the values of parameter \(R\) for the charmed and charmed-strange mesons are fixed such that it reproduces the root mean square radius estimated by the relativistic quark model [40]. In Ref. [35–39], the simple harmonic oscillator wave function with a parameter \(R\) has been used to investigate the decay behavior of mesons and the estimated results could well reproduce the corresponding experimental data, which proves such an approach is reliable to investigate the strong decays of the hadrons. As for the charmonia, the \(R\) values are quite different in different quark model, such as in the relativistic quark model, the \(R\) values for \(\psi(3770), \psi(4160)\) and \(\psi(3^3D_1)\) are estimated to be 1.84, 2.09, and 2.24 GeV\(^{-1}\), respectively, while in the screened potential model, these \(R\) values are 2.59, 3.12 and 3.59, respectively, thus in the present work, we vary the \(R\) values of the charmonia to check the \(R\) dependence of the decay widths. The masses and \(R\) values of the involved mesons are presented in Table III. In addition, the constituent quark masses for the charm, up/down, strange quarks are adopted to be 1.60, 0.22 and 0.419 GeV, respectively [34, 35].

![FIG. 2: (Color online). The partial width of \(\psi(3770) \to DD\). The cyan band indicates the PDG average of the corresponding partial width.](image)
TABLE II: The partial wave amplitude of the open charm decays. Here $\alpha = 1/ \sqrt{3}$ is the factor resulted from the flavor matrix element.

| Decay Channel | Amplitude |
|---------------|-----------|
| $^3D_1 \rightarrow S_0^1 S_0$ | $M^{01} = \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( \sqrt{l_{00}^{11} - l_{00}^{00}} \right)$ |
| $^3D_1 \rightarrow S_1^1 S_0$ | $M^{11} = \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( \sqrt{l_{00}^{11} - 3l_{00}^{01}} \right)$ |
| $^3D_1 \rightarrow S_1^3 S_1$ | $M^{01} = \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( l_{00}^{11} - \sqrt{l_{00}^{11} - l_{00}^{01}} \right)$ |
| $^3D_1 \rightarrow S_0^1 S_1^1$ | $M^{11} = \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( l_{00}^{11} - \sqrt{l_{00}^{11} - l_{00}^{01}} \right)$ |
| $^3D_1 \rightarrow S_0^1 P_1$ | $M^{00} = \cos \theta \left[ \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( -\sqrt{l_{00}^{00} + \sqrt{l_{00}^{00} + l_{00}^{01} + l_{00}^{11} + 3l_{01}^{01} + 3\sqrt{l_{01}^{01}}} \right) \right]$ |
| $^3D_1 \rightarrow S_0^1 P_1$ | $+ \sin \theta \left[ \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( \sqrt{l_{00}^{00} - \sqrt{l_{00}^{00} + l_{00}^{01} + l_{00}^{11} + 3l_{01}^{01} + 3\sqrt{l_{01}^{01}}} \right) \right]$ |
| $^3D_1 \rightarrow S_0^1 P_1$ | $M^{10} = \cos \theta \left[ \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( -\sqrt{l_{00}^{00} + \sqrt{l_{00}^{00} + l_{00}^{01} + l_{00}^{11} + 3l_{01}^{01} + 3\sqrt{l_{01}^{01}}} \right) \right]$ |
| $^3D_1 \rightarrow S_0^1 P_1$ | $+ \sin \theta \left[ \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( \sqrt{l_{00}^{00} - \sqrt{l_{00}^{00} + l_{00}^{01} + l_{00}^{11} + 3l_{01}^{01} + 3\sqrt{l_{01}^{01}}} \right) \right]$ |
| $^3D_1 \rightarrow S_0^1 P_1$ | $M^{00} = \cos \theta \left[ \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( -\sqrt{l_{00}^{00} + \sqrt{l_{00}^{00} + l_{00}^{01} + l_{00}^{11} + 3l_{01}^{01} + 3\sqrt{l_{01}^{01}}} \right) \right]$ |
| $^3D_1 \rightarrow S_0^1 P_1$ | $+ \sin \theta \left[ \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( \sqrt{l_{00}^{00} - \sqrt{l_{00}^{00} + l_{00}^{01} + l_{00}^{11} + 3l_{01}^{01} + 3\sqrt{l_{01}^{01}}} \right) \right]$ |
| $^3D_1 \rightarrow S_0^1 P_1$ | $M^{10} = \cos \theta \left[ \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( -\sqrt{l_{00}^{00} + \sqrt{l_{00}^{00} + l_{00}^{01} + l_{00}^{11} + 3l_{01}^{01} + 3\sqrt{l_{01}^{01}}} \right) \right]$ |
| $^3D_1 \rightarrow S_0^1 P_1$ | $+ \sin \theta \left[ \frac{2\alpha}{\sqrt{3}} \sqrt{E_A E_B E_C} \left( \sqrt{l_{00}^{00} - \sqrt{l_{00}^{00} + l_{00}^{01} + l_{00}^{11} + 3l_{01}^{01} + 3\sqrt{l_{01}^{01}}} \right) \right]$ |

TABLE III: The masses and $R$ values of the involved mesons. Here (±) and (0) indicate the charge of the mesons.

| Meson | Mass (MeV) [2, 3, 43] | $R$ (GeV$^{-1}$) [40] |
|-------|----------------------|-----------------------|
| $D^-$ | 1864.83(0), 1869.58(±) | 1.52 |
| $D^*$ | 2006.85(0), 2100.26(±) | 1.85 |
| $D_0$(2400) | 2318(0)2351(±) | 1.85 |
| $D_1$(2420) | 2420.8(0), 2423.2(±) | 2.00 |
| $D_1$(2430) | 2427(0), 2427(±) | 2.00 |
| $D_2$(2460) | 2460.7(0), 2465.4(±) | 2.00 |
| $D_s$ | 1968.28(±) | 1.41 |
| $D_s^*$ | 2112.1(±) | 1.69 |
| $\psi(3770)$ | 3773.13 | — |
| $\psi(4160)$ | 4191 | — |
| $Y(4320)$ | 4320.0 | — |
| $Y(4390)$ | 4391.6 | — |

value of $\psi(4160)$ is a bit larger than the one of $\psi(3770)$, which is consistent with the expectation. In this $R$ range, our results indicate the $\psi(4160)$ dominantly decays into $D\bar{D}$, $D^*\bar{D}$ and $D^{(*)}\bar{D}^{(*)}$, while the partial widths of $D_s^-D_s^-$ and $D_s^+D_s^-$ are less than 1 MeV. In this $R$ range, the ratios of the partial widths of open charmed processes are estimated to be

$$\frac{\Gamma(\psi(4160) \rightarrow D\bar{D})}{\Gamma(\psi(4160) \rightarrow D^*\bar{D}^*)} = 0.71 \sim 1.72$$

These ratios are evaluated to be 0.46/0.01 and 0.2/0.05 by the QPC model with relativistic quark model and linear potential model, respectively[44, 45]. In Ref. [46], by using the Connell coupled-channel mode, the ratios are determined to be 0.08 and 0.16. On the experimental side, the BaBar collaboration performed a measurement of the exclusive production of $D\bar{D}$, $D^*\bar{D}$ and $D^{(*)}\bar{D}^{(*)}$, the ratios were measured to be $0.02 \pm 0.03 \pm 0.02$ and $0.34 \pm 0.14 \pm 0.05$ [47], respectively, which is different from the QPC model estimations in the present work. It should be noticed that in Ref. [47], the data are fitted with three charmonia with fixed mass and width, which are $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$. From the current situation, there should exist more vector states in this energy range and thus the fitted results will be changed if more states are included. Moreover, in the analysis, the mass and width of $\psi(4160)$ are fixed to be 4153 MeV and 103 MeV, respectively [48]. The values of the resonance parameters used in

FIG. 3: (Color online). The open charm decay widths of $\psi(4160)$. The cyan band indicates the PDG average of the total width of $\psi(4160)$. 


Ref. [47] are much different from latest PDG average, which are 4191 MeV and 70 MeV, respectively [43]. We expect the new precise measurement and analysis of the open charm decays of $\psi(4160)$ at BESIII and BelleII could determine these ratios and test the results in the present work.

Our estimations of $\psi(3770)$ and $\psi(4160)$ decays indicate that the QPC model is reliable to study the open charm decays of the charmonia. Especially, the total width of these two established $D$ wave charmonia could be well reproduced in a proper $R$ range, which are $R = 1.56 \sim 1.76$ GeV$^{-1}$ and $R = 1.82 \sim 1.97$ GeV$^{-1}$ for $\psi(3770)$ and $\psi(4160)$, respectively. Thus, we could apply this model to study the decays of $\psi(3^3D_1)$ state to evaluate the possibility of $Y(4320)$ and $Y(4390)$ as $\psi(3^3D_1)$ state. Considering the $R$ values of $\psi(3770)$ and $\psi(4160)$ determined by the total widths, we take $R = 2.0 \sim 3.0$ GeV$^{-1}$ for $\psi(3^3D_1)$ to study its open charm decays.

The $R$ dependent total and partial widths of the open charm decays of $Y(4320)$ are presented in Fig. 4, where $Y(4320)$ is assigned as $\psi(3^3D_1)$ charmonium. Our estimations indicate that when $R = 2.49 \sim 2.92$ GeV$^{-1}$ the evaluated total width are consistent with the measured one from the BESIII Collaboration [2]. This $R$ value for $\psi(3^3D_1)$ is larger than the one of $\psi(4160)$ and $\psi(3770)$, which is consistent with our expectation. In this $R$ range, $\psi(3^3D_1)$ dominantly decays into $D\bar{D}$, $D^*\bar{D}$ and $D^*\bar{D}^*$. The partial width of $\psi(3^3D_1) \rightarrow D\bar{D}$ weakly depend on the parameter $R_A$, and in the range $R_A = 2.49 \sim 2.92$ GeV$^{-1}$, we find $\Gamma(\psi(3^3D_1) \rightarrow D\bar{D}) = 45.2 \sim 48.0$ MeV. And in this $R_A$ range, $\Gamma(\psi(3^3D_1) \rightarrow D^*\bar{D}) = 24.3 \sim 46.0$ MeV and $\Gamma(\psi(3^3D_1) \rightarrow D^*\bar{D}^*) = 4.9 \sim 11.9$ MeV. The ratios of the partial widths of these dominant decay channels are predicted to be

$$\frac{\Gamma(Y(4320) \rightarrow D\bar{D})}{\Gamma(Y(4320) \rightarrow D^*\bar{D})} = 4.0 \sim 9.2$$

$$\frac{\Gamma(Y(4320) \rightarrow D^*\bar{D})}{\Gamma(Y(4320) \rightarrow D^*\bar{D}^*)} = 3.8 \sim 4.5,$$

which could be tested by the precise measurement at BES III and Belle II.

We present the $R$ dependence of the total and partial widths of the open charm decay of $Y(4390)$ in Fig. 5, where $Y(4390)$ is assigned as $\psi(3^3D_1)$ charmonium. The width of $Y(4390)$ reported in $e^+e^- \rightarrow \pi^+\pi^- h_c$ and $e^+e^- \rightarrow \pi^+D^0D^-$ are $R = 2.58 \sim 2.78$ GeV$^{-1}$ and $R = 2.83 \sim 3.00$ GeV$^{-1}$, respectively. In the range $R = 2.58 \sim 3.00$ GeV$^{-1}$, $Y(4390)$ also dominantly decays into $D\bar{D}$, $D^*\bar{D}$, $D^*\bar{D}^*$ and $D_1(2430)\bar{D}$. The ratios of these four channels are predicted to be

$$\frac{\Gamma(Y(4390) \rightarrow D\bar{D})}{\Gamma(Y(4390) \rightarrow D^*\bar{D})} = 3.1 \sim 4.1$$

$$\frac{\Gamma(Y(4390) \rightarrow D^*\bar{D})}{\Gamma(Y(4390) \rightarrow D^*\bar{D}^*)} = 2.5 \sim 3.5.$$ 

$$\frac{\Gamma(Y(4390) \rightarrow D_1(2430)\bar{D})}{\Gamma(Y(4390) \rightarrow D^*\bar{D}^*)} = 1.0 \sim 3.7. \quad (10)$$

In this $R$ range, our estimations indicate the partial widths for $D^*_1\bar{D}^*_1$, $D^*_3\bar{D}^*_3$, $D^*_3\bar{D}_0$, and $D^*_3\bar{D}(2420)$ could be several MeV, while the $D^*_1\bar{D}^*_1$ and $D^*_2\bar{D}^*_2$ decay modes could be ignored.

From the results in the present work, one can find that $D_1(2430)\bar{D}$ is one of dominant decay channels of $Y(4390)$ and the width of $D_1(2420)\bar{D}$ is also about several MeV. Both $D_1(2420)$ and $D_1(2420)$ primarily decay into $D^*\pi$, which indicates the $\pi^+D^0D^-$ may be dominated from the cascade decay $Y(4390) \rightarrow D^0\bar{D}_1(2430), D^0\bar{D}_3(2420) \rightarrow D^0(\pi^+D^-)$, which could be test by BES III and Belle II.

FIG. 4: (Color online). The open charm decay widths of $Y(4320)$, where $Y(4320)$ is assigned as $\psi(3^3D_1)$ state. The cyan band is the total width of $Y(4320)$ reported by BESIII Collaboration [2].

FIG. 5: (Color online). The same as Fig. 4 but for $Y(4390)$. The cyan bands indicate the width of $Y(4390)$ reported in the $e^+e^- \rightarrow \pi^+\pi^- h_c$ and $e^+e^- \rightarrow \pi^+D^0D^-$ by the BES III Collaboration [3, 4].
IV. SUMMARY

The observations of the vector charmonium-like states in the $e^+e^-$ annihilation processes make the states between 4.0 and 4.5 GeV overcrowded. Besides the higher excited state of $J/ψ$, these charmonium-like states could also be higher $ψ(3D_1)$ states. In the present work, we evaluate the possibility of $Y(4320)$ and $Y(4390)$ as $ψ(3D_1)$ charmonium by investigate the open charm decays of $ψ(3D_1)$ in a QPC model.

Before we apply the QPC model to investigate the decays of $ψ(3D_1)$ charmonium, we employ this model to calculate the open charm decays of $ψ(3770)$ and $ψ(4160)$, which are well established $ψ(1^3D_1)$ and $ψ(2^3D_1)$ charmonia. Our estimations indicate that the partial width of $ψ(3770) → DD$ and total width of $ψ(4160)$ could be well reproduced in a reasonable $R$ range, which are $R = 1.56 \sim 1.76$ GeV$^{-1}$ for $ψ(3770)$ and $R = 1.82 \sim 1.97$ GeV$^{-1}$ for $ψ(4160)$.

The success in the study of open charm decays of the low-lying $D$ wave vector charmonia indicate the reliable of the QPC model and encourage us to apply the same model to calculate the open charm decays of $ψ(3D_1)$ state. The mass of $Y(4320)$ is well consistent with the one of $ψ(3D_1)$ charmonium predicted by the screened potential model and the total width estimated in the present work could overlap with the experimental measurement when $R = 2.49 \sim 2.92$ GeV$^{-1}$. In the range $R = 2.49 \sim 2.92$ GeV$^{-1}$, $Y(4320)$ dominantly decays into $DD, D^*D$ and $D^*D^*$. As for $Y(4390)$, the width reported from $e^+e^- → \pi^+D^0D^-$ is larger than the one reported from $e^+e^- → \pi^+\pi^-h_c$. The total width estimated in the present work could overlap with the those from $e^+e^- → \pi^+\pi^-h_c$ and $e^+e^- → \pi^+D^0D^-$ in the range $R = 2.58 \sim 2.78$ GeV$^{-1}$ and $R = 2.83 \sim 3.0$ GeV$^{-1}$. In the range $R = 2.58 \sim 3.00$ GeV$^{-1}$, the dominant decay modes of $Y(4390)$ are also $DD, D^*D, D^*D^*$ and $D(2430)D$. The observation of $Y(4390)$ in the $π^+D^0D^-$ could be well interpreted by the large decay width of $D(2430)D$ and $D^*_1(2420)D$ modes.

To summarize, in present work, we evaluate the possibility of $Y(4320)$ and $Y(4390)$ as $ψ(3D_1)$ state by study the open charm decays in a QPC model. Our results indicate that the total width of $ψ(3D_1)$ could be consistent with the one of $Y(4320)$ and moreover, the mass of $ψ(3D_1)$ predicted by the screened potential model is also in line with the one of $Y(4320)$, thus $Y(4320)$ could be a good candidate of $ψ(3D_1)$. As for $Y(4390)$, the estimated width of $ψ(3D_1)$ could overlap with the reported width from $e^+e^- → \pi^+\pi^-h_c$ and $e^+e^- → \pi^+D^0D^-$ in different $R$ range. Our estimations also find that $π^+D^0D^-$ may result from the large widths of $D(2430)D$ and $D^*_1(2420)D$. Thus, the possibility of $Y(4390)$ as $ψ(3D_1)$ could not be rule out. To test the estimations in the present work, we expect the further precise searches for $Y(4320)$ and $Y(4390)$ in more channels, especially in the open charm channels, which could be performed by BES III and Belle II.

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