Asymptotic Quasinormal Frequencies of Brane-Localized Black Hole

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Abstract

The asymptotic quasinormal frequencies of the brane-localized $(4 + n)$-dimensional black hole are computed. Since the induced metric on the brane is not an exact vacuum solution of the Einstein equation defined on the brane, the real parts of the quasinormal frequencies $\omega$ do not approach to the well-known value $T_H \ln 3$ but approach to $T_H \ln k_n$, where $k_n$ is a number dependent on the extra dimensions. For the scalar field perturbation $\text{Re}(\omega/T_H) = \ln 3$ is reproduced when $n = 0$. For $n \neq 0$, however, $\text{Re}(\omega/T_H)$ is smaller than $\ln 3$. It is shown also that when $n > 4$, $\text{Im}(\omega/T_H)$ vanishes in the scalar field perturbation. For the gravitational perturbation it is shown that $\text{Re}(\omega/T_H) = \ln 3$ is reproduced when $n = 0$ and $n = 4$. For different $n$, however, $\text{Re}(\omega/T_H)$ is smaller than $\ln 3$. When $n = \infty$, for example, $\text{Re}(\omega/T_H)$ approaches to $\ln(1 + 2 \cos \sqrt{5}\pi) \approx 0.906$. Unlike the scalar field perturbation $\text{Im}(\omega/T_H)$ does not vanish regardless of the number of extra dimensions.

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The stability problem of the black holes, when perturbed by the external fields, is a long-standing issue in the context of the general relativity [1–4]. It is well-known that all perturbations are radiated away, which is characterized by the quasinormal modes [5]. These quasinormal modes are defined as solutions of the perturbation wave equation, belonging to complex-characteristic frequencies and satisfying the boundary conditions for the purely outgoing waves at infinity and purely ingoing waves at the horizon, \textit{i.e.}

\[ \Psi \sim e^{i\omega z} \quad \text{as} \quad z \rightarrow -\infty \]
\[ \Psi \sim e^{-i\omega z} \quad \text{as} \quad z \rightarrow \infty \]

where \( z \) is an appropriate “tortoise” coordinate and the time dependence of the fields is taken as \( e^{i\omega t} \).

In our notation the quasinormal frequencies \( \omega \) should satisfy \( \text{Im}(\omega) \geq 0 \). As a consequence, the quasinormal modes diverge exponentially at both boundaries. This makes it extremely difficult to determine the quasinormal frequencies numerically. This is a main reason why only very few frequencies with moderate imaginary parts were known [6].

About two decades ago Leaver [7] found a possibility to compute the quasinormal frequencies without having to deal with the corresponding quasinormal modes numerically. Instead of the solutions he used the recursion relation which provides an infinite continued fraction. However, this method has a technically problem of convergence when \( \text{Im}(\omega) >> \text{Re}(\omega) \). This defect of the continued fraction method was mostly removed by Nollert [8] by computing the “remaining” infinite continued fraction.

For the case of the 4d Schwarzschild black hole Nollert showed using his improved numerical method that the asymptotic quasinormal frequencies for the scalar field and gravitational perturbations become

\[ \omega = \frac{\bar{n} + 1/2}{2} i + 0.0874247 \quad (\bar{n} = 0, 1, 2, \cdots) \]  

with, for simplicity, assuming \( r_h = 1 \) where \( r_H \) is an horizon radius. This was confirmed by Andersson [9] by the phase integral method, which is an improved WKB-type technique.
After few years Hod [10] claimed surprisingly that the numerical number in Eq.(2) is identified as
\[ 0.0874247 \rightarrow \frac{\ln 3}{4\pi} = T_H \ln 3 \] (3)
where \( T_H \) is an Hawking temperature. This identification and the Bohr’s correspondence principle naturally imply that the minimal quantum area is \( 4 \ln 3 \), one of the values \( 4 \ln k \) suggested in Ref. [11]. This is intriguing from the loop quantum gravity [12] point of view because it suggests that the gauge group should be \( SO(3) \) rather \( SU(2) \) [13]. Subsequently, the identification (3) was analytically shown in Ref. [14,15]. Especially in Ref. [15] the authors transformed the boundary condition of the quasinormal modes at the horizon into the monodromy in the complex plane of the radial coordinate. We will use this method to compute the asymptotic quasinormal frequencies of the brane-localized \((4+n)\)-dimensional Schwarzschild black hole. For the quasinormal modes of the other asymptotically flat\(^1\) black holes see Ref. [18–21] and references therein.

Recently, much attention is paid to the higher-dimensional black holes. Besides its own theoretical interest the main motivation of it seems to be the emergence of the \( TeV \) scale gravity arising in the brane-world scenarios [22–25], which opens the possibility to make a tiny black holes factory in the future high-energy colliders such as LHC [26–30]. In this reason the absorption and emission problems of the higher-dimensional black holes were extensively explored recently [31–34].

The lower quasinormal frequencies for the brane-localized 5-dimensional rotating black holes were recently computed numerically [35]. In this letter we would like to go further for the study of the brane-world black holes by examining the asymptotic quasinormal frequencies of the brane-localized Schwarzschild black holes. Since the metric of the brane-localized Schwarzschild black hole is induced by the higher-dimensional bulk metric, it is not a vacuum solution of the Einstein equation defined on the brane. We will show that this fact

\(^1\)See also Ref. [16,17] for asymptotically non-flat black holes
makes the real part of the asymptotic quasinormal frequencies not to be the well-known value $T_H \ln 3$ but to be $T_H \ln k_n$, where $T_H$ is an Hawking temperature of the higher-dimensional black hole and $k_n$ is a number dependent on the extra dimensions.

We start with the $(4+n)$-dimensional Schwarzschild black hole whose metric is given by [36,37]

$$ds_B^2 = -h(r)dt^2 + h^{-1}(r)dr^2 + r^2d\Omega_{n+2}^2$$

(4)

where

$$h(r) = 1 - \left(\frac{r_H}{r}\right)^{n+1}$$

(5)

and $r_H$ is an horizon radius. In Eq. (4) the subscript “B” stands for “bulk”. For the consideration of the brane-localized black hole we should consider the 4d-induced metric of $ds_B^2$, which is

$$ds_4^2 = -h(r)dt^2 + h^{-1}(r)dr^2 + r^2(\theta^2 + \sin^2\theta d\phi^2)$$

(6)

Now, it is easy to show that the equation $\Box \Phi = 0$, which governs the scalar field perturbation leads to the following radial equation:

$$h(r) \frac{d}{dr} \left[ r^2 h(r) \frac{dR}{dr} \right] + \left[ \omega^2 - h(r) \frac{\ell(\ell + 1)}{r^2} \right] R = 0$$

(7)

When deriving Eq. (7), we used the factorization condition $\Phi = e^{i\omega t} R(r) Y_{\ell,m}(\theta, \phi)$. Defining $R = \Psi / r$, one can show that Eq. (7) reduces to the following Schrödinger-like equation

$$\left( h(r) \frac{d}{dr} \right)^2 \Psi + \left[ \omega^2 - V_S(r) \right] \Psi = 0$$

(8)

where $V_S(r)$ is an effective potential of the scalar field perturbation, which is given by

$$V_S(r) = h(r) \left( \frac{\ell(\ell + 1)}{r^2} + \frac{1}{r} \frac{dh}{dr} \right)$$

(9)

Now, we would like to derive the wave equation which governs the gravitational perturbation. In order to derive it we should change the metric itself, say $ds_4^2 \rightarrow \delta s_4^2 = ds_4^2 + \delta s^2$ where
\[ \delta s^2 = [H_0(r)dt d\phi + H_1(r)dr d\phi] e^{i\omega t} \sin \theta \frac{dP_t}{d\theta} (\cos \theta) \]  

(10)

with assuming \( H_0, H_1 << 1 \) for the linearization\(^2\). Then, it is straightforward to derive the Ricci tensor \( \tilde{R}_{\mu\nu} \) and curvature scalar \( \tilde{R} \).

It is worthwhile noting that since the metric \( ds^2_4 \) in Eq.(6) is an induced one from the higher-dimensional bulk metric, it is not an exact vacuum solution of 4\( d \) Einstein equation. Thus, it may satisfy the non-vacuum Einstein equation \( \mathcal{E}_{\mu\nu} = T_{\mu\nu} \) where \( \mathcal{E}_{\mu\nu} = \tilde{R}_{\mu\nu} - g_{\mu\nu}\tilde{R}/2 \) and \( T_{\mu\nu} \) is an energy-momentum tensor. Since the perturbation in general changes both the geometry of the spacetime and the energy-momentum tensor, adding \( \delta s^2 \) to \( ds^2_4 \) should makes the Einstein equation as \( \mathcal{E}_{\mu\nu} + \delta \mathcal{E}_{\mu\nu} = T_{\mu\nu} + \delta T_{\mu\nu} \) where \( \delta \mathcal{E}_{\mu\nu} \) and \( \delta T_{\mu\nu} \) are order of \( H_0 \) or \( H_1 \). Although we can not compute \( \delta T_{\mu\nu} \) by conventional method because we do not know the matter source nature, we can compute it as following. Firstly, we note that the nonzero components of \( \delta T_{\mu\nu} \) are only \( \delta T_{t\phi}, \delta T_{r\phi} \) and \( \delta T_{\theta\phi} \). This can be easily conjectured by computing the Einstein tensor. One constraint is a covariant conservation of the energy-momentum tensor \( (T^{\mu\nu} + \delta T^{\mu\nu})_{\mu\nu} = 0 \). When \( \nu = t, r \) and \( \theta \), this is automatically satisfied. Thus, this conservation law generates one constraint. Second constraint comes from the fact that the effective potential which will be derived later should coincide with the well-known Regge-Wheeler potential when \( n = 0 \) limit. The final constraint comes from the fact that the \((t, \phi)\) component of the Einstein equation should be derived from \((r, \phi)\) and \((\theta, \phi)\) components. These constraints uniquely determine \( \delta T_{\mu\nu} \) and the final expressions of the Einstein equation are

\[
\begin{align*}
-\frac{1}{4} h(r) \frac{d^2 H_0}{dr^2} + \left[ \frac{\ell(\ell + 1) - 2}{4r^2} + \frac{1}{2r^2} h(r) + \frac{1}{2r} \frac{dh}{dr} + \frac{1}{4} \frac{d^2 h}{dr^2} \right] H_0 \\
+ \frac{i\omega}{4} h(r) \left( \frac{dH_1}{dr} + \frac{2}{r} H_1 \right) = -\frac{1}{4i\omega} h^2(r) \left( \frac{2}{r^2} \frac{dh}{dr} + \frac{4}{r} \frac{d^2 h}{dr^2} + \frac{d^3 h}{dr^3} \right) H_1 \\
-\frac{i\omega}{4} h^{-1}(r) \left( \frac{dH_0}{dr} - \frac{2}{r} H_0 \right) + \left[ \frac{\ell(\ell + 1) - 2}{4r^2} - \frac{\omega^2}{4} h^{-1}(r) + \frac{1}{2r} \frac{dh}{dr} + \frac{1}{4} \frac{d^2 h}{dr^2} \right] H_1 = 0
\end{align*}
\]

(11)

\(^2\)The metric change in Eq.(10) corresponds to the odd-parity gravitational perturbation. For detail see Ref. [4]
\[ i\omega h^{-1}(r)H_0 - h(r) \frac{dH_1}{dr} - \frac{dh}{dr} H_1 = 0. \]

Eliminating \( H_0 \) in Eq.(11) appropriately, one can derive a second-order differential equation for solely \( H_1 \) in the form

\[
\frac{d^2 H_1}{dr^2} + \left( 3h^{-1}(r) \frac{dh}{dr} - \frac{2}{r} \right) \frac{dH_1}{dr} + \left[ \left( \omega^2 + \left( \frac{dh}{dr} \right)^2 \right) h^{-2}(r) - \left( \frac{\ell(\ell + 1) - 2}{r^2} + \frac{4dh}{r dr} \right) h^{-1}(r) \right] H_1 = 0. \tag{12}
\]

Defining \( H_1 \equiv r h^{-1}(r)\Psi(r) \), we can derive the following Schrödinger-like equation:

\[
\left( h(r) \frac{d}{dr} \right)^2 \Psi + \left[ \omega^2 - V_G(r) \right] \Psi = 0 \tag{13}
\]

where \( V_G(r) \) is an effective potential of the gravitational perturbation, which is given by

\[
V_G(r) = h(r) \left( \frac{\ell(\ell + 1)}{r^2} + \frac{1 dh}{r dr} - \frac{2}{r^2} (1 - h(r)) + \frac{d^2 h}{dr^2} \right). \tag{14}
\]

Using \( dh/dr = (n + 1)(1 - h(r))/r \), one can express the effective potentials \( V_S(r) \) and \( V_G(r) \) as following

\[
V_{\text{eff}}(r) = h(r) \left[ \frac{\ell(\ell + 1)}{r^2} + \frac{\sigma_n}{r^2} (1 - h(r)) \right] \tag{15}
\]

where

\[
\sigma_n = \begin{cases} 
  n + 1 & \text{for scalar field perturbation} \\
  -(n + 1)^2 - 2 & \text{for gravitational perturbation}
\end{cases} \tag{16}
\]

Thus we can treat the scalar and gravitational perturbation in an unified way.

Now, we define a tortoise coordinate \( z \) as

\[
\frac{d}{dz} \equiv h(r) \frac{d}{dr} = \left[ 1 - \left( \frac{r_H}{r} \right)^{n+1} \right] \frac{d}{dr}. \tag{17}
\]

Integrating Eq.(17), we can make an explicit expression of \( z \) in the form

\[
z = r + \frac{r_H}{n + 1} \sum_{j=0}^{n} e^{i2\pi j/(n+1)} \ln \left[ 1 - \frac{r}{r_H} e^{-i2\pi j/(n+1)} \right]. \tag{18}
\]
When deriving Eq. (18), we fixed the integration constant by imposing that $r = 0$ corresponds to $z = 0$, which is crucial [15] for the calculation of the asymptotic quasinormal frequencies. From this tortoise coordinate and the boundary condition (1) it is straightforward to compute the monodromy around the singularity $r = 1$

$$\mathcal{M}(1) = e^{\omega/T_H}$$

where $T_H$ is an Hawking temperature of the $(4 + n)$-dimensional black hole, i.e. $T_H = (n + 1)/4\pi r_H$. Following Ref. [15] we will compute another expression of the monodromy by following a contour given in Fig. 3 of Ref. [15] and using an analytic continuation. Equating these two expressions enables us to calculate the asymptotic quasinormal frequencies. For the following calculation it is important to note that near the naked singularity $r \sim 0$ the tortoise coordinate $z$ is proportional to $r^{n+2}$ as

$$z \sim -\frac{r_H}{n+2} \left(\frac{r}{r_H}\right)^{n+2}.$$

Adopting a method used in Ref. [15], we can now compute the asymptotic quasinormal frequencies. The explicit expression of the effective potential given in Eq. (15) is

$$V_{\text{eff}}(r) = \left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right] \left[\frac{\ell(\ell + 1)}{r^2} + \frac{\sigma_n}{r^2} \left(\frac{r_H}{r}\right)^{n+1}\right].$$

Near the naked singularity $r = 0$, therefore, the effective potential can be written by its dominant term

$$V_{\text{eff}}(r) \sim -\frac{\sigma_n r_H^{2n+2}}{r^{2n+4}} = \frac{\sigma_n/(n + 2)^2}{z^2}.$$  

Thus, the wave equation

$$\frac{d^2\Psi}{dz^2} + \left(\omega^2 - V_{\text{eff}}(r)\right)\Psi = 0$$

simply provides a solution

$$\Psi_{0,A} \sim \sqrt{2\pi \omega z} [A_+ J_\nu(\omega z) + A_- J_{-\nu}(\omega z)]$$

in this regime where
\[
\nu = \sqrt{\frac{1}{4} - \frac{\sigma_n}{(n + 2)^2}}.
\] (25)

Taking a \(\omega z \to \infty\) limit in \(\Psi_{0,A}\), we can obtain one asymptotic solution
\[
\Psi_{\infty,A} \sim \left[ A_+ e^{i\pi(1+2\nu)/4} + A_- e^{i\pi(1-2\nu)/4} \right] e^{-i\omega z}
\] (26)

with a constraint
\[
A_+ e^{-i\pi(1+2\nu)/4} + A_- e^{-i\pi(1-2\nu)/4} = 0.
\] (27)

To follow the contour given in Ref. [15] we should turn to an angle \(3\pi\) around \(z = 0\) in \(\Psi_{0,A}\). Then we can derive another solution in the \(r \sim 0\) regime
\[
\Psi_{0,B} \sim \sqrt{2\pi \omega z} e^{i3\pi} \left[ A_+ J_\nu(\omega z e^{i3\pi}) + A_- J_{-\nu}(\omega z e^{i3\pi}) \right].
\] (28)

Since \(J_\nu(z) = z^\nu \varphi(z)\) where \(\varphi(z)\) is an even analytic function in the entire region of the complex plane, it is easy to show that \(\Psi_{0,B}\) reduces to
\[
\Psi_{0,B} \sim e^{i3\pi/2} \sqrt{2\pi \omega z} \left[ A_+ e^{i3\pi \nu} J_\nu(\omega z) + A_- e^{-i3\pi \nu} J_{-\nu}(\omega z) \right].
\] (29)

Taking \(\omega z \to -\infty\) limit in \(\Psi_{0,B}\), we can obtain another asymptotic solution
\[
\Psi_{\infty,B} \sim \left[ A_+ e^{i5\pi(1+2\nu)/4} + A_- e^{i5\pi(1-2\nu)/4} \right] e^{-i\omega z} + \left[ A_+ e^{i7\pi(1+2\nu)/4} + A_- e^{i7\pi(1-2\nu)/4} \right] e^{i\omega z}.
\] (30)

Now, we can follow the contour to return to the initial point through the asymptotic region. Since the second term in Eq.(30) is exponential small in this region, the monodromy can be written as
\[
\mathcal{M}(1) = \frac{A_+ e^{i5\pi(1+2\nu)/4} + A_- e^{i5\pi(1-2\nu)/4}}{A_+ e^{i\pi(1+2\nu)/4} + A_- e^{i\pi(1-2\nu)/4}}.
\] (31)

With an aid of the constraint (27) this simply reduces to
\[
\mathcal{M}(1) = -(1 + 2 \cos 2\pi \nu).
\] (32)

Thus equating Eq.(19) with Eq.(32) gives
\[ e^{\omega/T_H} = -(1 + 2 \cos 2\pi \nu). \]  

(33)

Now, we turn to the scalar field perturbation. In this case \( \nu \) becomes

\[ \nu_S = \sqrt{\frac{1}{4} - \frac{n + 1}{(n + 2)^2}}. \]  

(34)

Note that \( \nu_S = 2/3 \) when \( n = 4 \), which makes the rhs of Eq.(33) to be zero. When \( n < 4 \), the asymptotic quasinormal frequencies become

\[ \frac{\omega_S}{T_H} = 2\pi i \left( \bar{n} + \frac{1}{2} \right) + \ln(1 + 2 \cos 2\pi \nu) \quad (\bar{n} = 0, 1, 2, \cdots). \]  

(35)

Of course, \( \nu_S = 0 \) when there is no extra dimension, which gives a well-known result \( Re(\omega_S/T_H) = \ln 3 \). When extra dimensions exist, the real parts of the asymptotic quasinormal frequencies are summarized in Table I^3.

| \( n \) | \( Re(\omega_S/T_H) \) |
|---|---|
| 0 | \( \ln 3 = 1.09861 \) |
| 1 | \( \ln 2 = 0.69315 \) |
| 2 | 0 |
| 3 | \( \ln((3 - \sqrt{5})/2) = -0.96242 \) |
| 4 | \( -\infty \) |

Table I

It is interesting to note that \( Re(\omega/T_H) \) vanishes when there are two extra dimensions. When \( n > 4 \), the rhs of Eq.(33) becomes positive. Thus the asymptotic quasinormal frequencies become

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^3In Ref. [35] it is shown that when \( n = 1 \), the real part of the lowest quasinormal frequency is 0.273 which is much larger than the asymptotic value \( T_H \ln 2 = 0.055 \)
\[ \frac{\omega_S}{T_H} = \ln(-1 - 2 \cos 2\pi \nu_S) \] (36)

In this case, therefore, we get \( \text{Im}(\omega_S/T_H) = 0 \). If there are infinite extra dimensions, \( \nu_S \) becomes 1/2, which makes \( \text{Re}(\omega_S/T_H) = 0 \).

In the gravitational perturbation \( \nu \) becomes

\[ \nu_G = \sqrt{\frac{1}{4} + \frac{(n + 1)^2 + 2}{(n + 2)^2}}. \] (37)

It is interesting to note that \( \nu_G = 1 \) when \( n = 0 \) and \( n = 4 \). Thus in addition to the case of no extra dimension we get a well-known result \( \text{Re}(\omega_G/T_H) = \ln 3 \) when there are four extra dimensions. Unlike the case of the scalar field perturbation, however, the rhs of Eq.(33) is always negative regardless of \( n \). Thus, the asymptotic quasinormal frequencies are always given by

\[ \frac{\omega_G}{T_H} = 2\pi i \left( \bar{n} + \frac{1}{2} \right) + \ln(1 + 2 \cos 2\pi \nu_G) \quad (\bar{n} = 0, 1, 2, \cdots). \] (38)

The real part of \( \omega_G/T_H \) is plotted in Fig. 1.
FIG. 1. The $n$-dependence of $Re(\omega_G/T_H)$. It is interesting to note that the well-known result $Re(\omega_G/T_H) = \ln 3$ is reproduced when $n = 0$ and $n = 4$. For other value of $n$ $Re(\omega_G/T_H)$ is shown to be smaller than $\ln 3$.

Fig. 1 shows that $Re(\omega_G/T_H)$ becomes the well-known result $\ln 3$ when $n = 0$ and $n = 4$ as mentioned before. When $n$ is other number\textsuperscript{4}, $Re(\omega_G/T_H)$ becomes smaller than $\ln 3$. In particular, if there are infinite extra dimensions, $\nu_G$ becomes $\sqrt{5}/2$, which gives $Re(\omega_G/T_H) = \ln(1 + 2 \cos \sqrt{5} \pi) \approx 0.906$.

In this letter we have computed the asymptotic quasinormal frequencies of the brane-localized $(4 + n)$-dimensional Schwarzschild black hole. For the case of the scalar field perturbation the well-known result $Re(\omega_S/T_H) = \ln 3$ is reproduced when there is no extra dimension. When $0 < n < 4$, $Re(\omega_S/T_H)$ is shown to be smaller than $\ln 3$ When $4 < n$, $Im(\omega_S/T_H)$ becomes zero. In the case of the gravitational perturbation, however,

\textsuperscript{4}When $n = 1$, Ref. [35] shows that the real part of the lowest quasinormal frequency is 0.805 which is much larger than the asymptotic value 0.086.
\( Im(\omega_G/T_H) \) does not vanish regardless of the number of the extra dimensions. In particular, the well-known result \( Re(\omega_G/T_H) = \ln 3 \) is obtained when \( n = 0 \) and \( n = 4 \).

It is of interest to extend our computation to the brane-localized \((4 + n)\)-dimensional Reissner-Nordström and Kerr black holes. It is known that the asymptotic quasinormal frequencies of the 4\(d\) non-extremal Reissner-Nordström black hole have real part \( T_H \ln 2 \) unlike the Schwarzschild black hole case [15]. It is of interest to examine how the existence of the extra dimensions changes the quasinormal frequencies. In the case of the Kerr black hole the superradiance effect seems to play an important role [19]. Since it is proven generally that the superradiance modes exist in the brane-localized and bulk rotating black holes [38,39], it seems to be interesting to examine how these modes affect the asymptotic quasinormal spectrum. It is also interesting to confirm our result by adopting an appropriate numerical technique.

The most interesting issue at least for me is to explore how the \( n \)-dependent quasinormal modes affect the Hawking radiation of the black holes. Similar issue was studied long ago by York [40]. We would like to study this issue in the future.

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