Comparison of weak signal detection performance between chaotic system and FFT

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Abstract: Chaotic system can be used to detect weak signal in strong noise background, but is the detection performance of chaotic system prior to the detection performance of FFT? The signals under the same noise power and the same signal-to-noise ratio are detected by FFT and chaotic detection system, and the detection rate is compared by introducing the concept of detection rate of chaotic system. The results of analysis show that the detection performance of FFT is only related to the signal-to-noise ratio (SNR) with the same signal length and noise power. And the signal can be detected accurately when the SNR is -40dB, but it can’t be detected when the SNR is -60dB and -80dB. The signal can be detected sometimes, sometimes can’t be detected by the chaotic system in the same condition. Therefore, the detection probability of weak signal exists when the weak signal detected by chaotic oscillator.

1. Introduction

Duffing oscillator is a typical chaotic oscillator, which is sensitive to initial value, strong anti-noise and high sensitivity. It is sensitive to the signal with small frequency difference, but immune to the noise and the signal with large frequency difference. Therefore, it is widely used in weak signal detection[1-3].

Several improved methods of chaotic system are proposed in order to improve the performance of duffing chaotic oscillator in detecting noisy signal[4-7]. The effect of noise on the detection performance of chaotic systems has also been studied[8,9]. But few literatures compare the detection performance of chaotic oscillators with the weak signal detection performance of traditional FFT spectrum detection method.

In this paper, the concept of weak signal detection rate of chaotic system is introduced to compare the detection performance of chaotic system and FFT under the same noise intensity and signal-to-noise ratio.

2. The basic principle of weak signal detection in chaotic system

A duffing chaotic detection system which can be used to detect weak signal at any frequency is introduced in this paper. And the duffing equation is:

\[ \ddot{x} = -k\omega \dot{x} + \omega^2 \left(x - x^3 + \gamma \cos(\omega t) \right) \] (1)
Where, $\omega$ and $\gamma$ are the frequency and amplitude of the signal; $k$ is the Damping Coefficient, $k = 0.5$; $(x-x^3)$ is the nonlinear term; $\dot{x}$ and $\ddot{x}$ is the first and second order differential of $x$ respectively.

The phase trajectory of the system will change as the value of $\gamma$ changes. And the system will enter the chaotic critical state when the value of $\gamma$ reached the chaotic critical threshold $\gamma_d$. A duffing equation will be get if a signal that has the same frequency as the built-in signal of a chaotic system input. The following equation is followed:

$$x = -k_0x + \omega^2 (x-x^3 + \gamma_d \cos(\omega t) + S(t) + n(t))$$

Where, $\gamma_d$ is critical threshold of chaos; $S(t)$ is test signal; $n(t)$ is noise; $a$ is amplitude of the signal to be measured, and $a < < \gamma_d$.

The system remains chaotic when pure noise is input, and the system quickly changes from chaotic state to large-scale periodic state when the input signal contains the same frequency as the built-in signal in theory. Therefore, a weak signal detection can be performed by using the phase trajectories of the measured periodic signal to change the behavior of the System dynamics from a chaotic critical state to a large scale periodic state.

### 3. Determination of critical threshold

The determination of chaos critical threshold is very important when a weak signal is detected by duffing chaotic system. The state and chaos critical value of the system can be precisely determined by using Lyapunov characteristic exponent. The basic idea is that the largest Lyapunov characteristic exponent is greater than zero and the system is in a chaotic state. When the largest Lyapunov characteristic exponent of the system changes from greater than zero to less than zero, then the system jumps from a chaotic state to a periodic state, the value of the built-in signal amplitude corresponding to the moment of sign transformation of the largest Lyapunov characteristic exponent should be the system critical threshold[10]. The specific calculation method of Lyapunov characteristic exponent and the method of calculating critical threshold of the system by using the Lyapunov characteristic exponent are given in reference [10].

If $k = 0.5$, $\omega = 1$ in Equation (1), and the initial value is $[x, \dot{x}] = [1, 1]$, 100 sampling points, which is the value of $\gamma$, are taken at equal intervals on the interval $[0, 1]$. The largest Lyapunov characteristic exponent for each sampling point is obtained, and the corresponding relation curve is shown in fig.1.

![Fig. 1 relationship curve between largest Lyapunov characteristic exponent and $\gamma$](image)

The range of the threshold can be estimate from the results of figure 1, $\gamma_d \in [0.82, 0.83]$. If the threshold precision of chaos is two decimal places, the threshold value is 0.82. A more accurate critical value can be obtained through calculation of the corresponding Lyapunov characteristic exponents of each sampling point, which obtained by bisected $[0.82, 0.83]$ with the step size of 0.001, and the results shown in Fig.1 are obtained. The chaotic critical threshold of 3 decimal places is $\gamma_d = 0.825$ according to Tab.1. To obtain a more accurate threshold, $[0.825, 0.826]$ is bisected by 0.0001, and the Lyapunov exponents of each sampling point are obtained as shown in Fig.2.

According to the Lyapunov characteristic exponent of Tab.2, the critical threshold of 4 decimal places is 0.8258. By further subdivision, a more accurate critical threshold can be obtained. If you take 8
decimal places, you get a critical threshold of $\gamma_d = 0.82582725$.

4. Comparison of weak signal detection performance between chaotic system and FFT

In Equation (1), $k=0.5$, $\omega=1$, sampling frequency is 1000Hz, the initial value is $[x, \dot{x}]=[1, 1]$, time length of sampling is 500s, critical threshold value is 0.8258. The signal of input contain a noisy, where the variance of noise is $10^{-6}$. Fig.4 and 5 show the detect results of the signal using chaotic system and FFT respectively with a signal-to-noise ratio (SNR)=-40dB.

Fig2. curve of Lyapunov characteristic exponent and $\gamma$

Fig3. curve of Lyapunov characteristic exponent and $\gamma$

Fig.4 results of FFT and chaos detection(noise power: $10^{-6}$, SNR=-40dB)

(a)Global Diagram of FFT detection results

(b) Local diagram of FFT detection result

(c) Time Domain Diagram of chaotic system

(d) Phase Diagram after system stabilization
Fig. 5 results of FFT and chaos detection (noise power: $10^{-6}$, SNR = -40dB)

Fig. 6 and 7 show the results of an experiment using chaotic oscillator and FFT respectively when SNR = -60dB, fig. 8 and 9 show the results of an experiment using chaotic oscillator and FFT respectively under SNR = -80dB. The simulation results also show that under the same experimental conditions, the results of the two detection methods are random and different.

Results of analysis from Fig. 4-Fig. 9 show that, when the signal length is 500s, the noise power is $10^{-6}$ and the SNR is -40dB, the signal can be detected accurately by FFT method, and sometimes it can be detected and sometimes it can not be detected by chaotic system. When the SNR are -60dB and -80dB, the FFT method can’t detect the signal, while the chaotic system method can sometimes. Therefore, the concept of detection probability exists when a weak signal with very low signal-to-noise ratio is detected.
by chaotic system.

Fig. 7 results of FFT and chaos detection (noise power: $10^{-6}$, SNR = -60dB)

Fig. 8 results of FFT and chaos detection (noise power: $10^{-6}$, SNR = -80dB)
In the chaotic critical state, the detection rate of the input signal is followed as:

$$\text{DR} = \frac{N_{Ly}}{N_t} \times 100\%$$

(3)

Where, DR is the detection rate; $N_{Ly}$ is the times of the largest Lyapunov characteristic exponent greater than Zero; $N_t$ is the times of simulation experiment.

Tab.1 Performance comparison of chaotic system and FFT detection methods

| $\gamma_d$ | Noise power | method       | DR     |
|-----------|-------------|--------------|--------|
|           |             |              | -20dB  | -30dB  | -40dB  | -50dB  | -60dB  | -70dB  | -80dB  |
| 0.8258    | $10^{-4}$   | chaotic oscillator | 100%  | 100%   | 99%    | 40%    | 11%    | 5%     | 4%     |
|           | FFT         | 100%         | 100%   | 90%    | 1%     | 0%     | 0%     | 0%     |
| 0.8258725 | $10^{-6}$   | chaotic oscillator | 97%   | 72%    | 28%    | 7%     | 5%     | 8%     | 8%     |
|           | FFT         | 100%         | 100%   | 90%    | 3%     | 0%     | 0%     | 0%     |
|           | $10^{-8}$   | chaotic oscillator | 0%    | 0%     | 0%     | 0%     | 0%     | 0%     | 0%     |
|           | FFT         | 100%         | 100%   | 90%    | 1%     | 0%     | 0%     | 0%     |

Tab.1 presents the results of several typical cases of noise intensity and signal-to-noise ratio detected by FFT and chaotic system respectively. The number of simulation experiments is 100. The results show that the detection performance of FFT detection method is only related to the SNR, that is, the relative value between signal power and noise power affects the FFT detection performance. The phase change of the system can be induced when the sum of the selected critical threshold and the amplitude of the signal to be measured is greater than the true critical threshold. And the low SNR signal with noise can be detected by the chaotic oscillator in these conditions. As can be seen from tab.3, in some cases, the detection performance of chaotic oscillator is better than FFT, but in some cases, the detection performance is not as good as FFT.
5. Conclusion

The detection performance of FFT method and chaotic system detection method under the same noise power and the same signal-to-noise ratio is compared by introducing the concept of chaotic system detection rate in this paper. The results show that the detection performance of FFT is only related to the SNR with the same signal length and noise power. And the signal can be detected accurately when the SNR is -40dB, but it can’t be detected when the SNR is -60dB and -80dB. Therefore, it cannot simply be said that the detection performance of a chaotic system must be better than FFT, but analysis should be carried out based on specific signals. The chaotic system detection method is better than FFT in some cases, but in some cases, the detection performance is not as good as FFT.

References:

[1] XIA Lan-lan, LI Chun-lan, WANG Cheng-bin, et al. Summary of the Research on Weak Signal Detection Based on Duffing Chaotic System [J]. Modern Computer, 2016, 08:32-34.
[2] CHEN Jun. Application research and discussing of detective systems based on chaos theory [J]. Journal of Gansu Gaoshi, 2013, 18(2):21-25.
[3] ZHANG gang, HU Tao, WANG Ying. The weak signal detection based on duffing system and melnikov function [J]. Electronic Measurement Technology, 2015, 38(1):109-112.
[4] SANG Song. Based on wavelet transform and chaos theory of weak signal detection method [D]. Master dissertation, Northeast Agricultural University, 2013: 6-7.
[5] Rui Guosheng, Liu Linfang, Zhang Song. Duffing oscillator weak signal detection method based on Signal Preprocessing [J]. Electronic Measurement Technology, 2016, 39(4):129-132.
[6] Wang Xiaodong, Yang Shaopu, Zhao Zhihong. Research of Weak Signal Detection Based on The Improved Duffing Oscillator [J]. Journal of Dynamics and Control, 2016, 14(3):283-288.
[7] WANG Huiwu, CONG Chao. A New Signal Detection and Estimation Method by Using Duffing System[J]. Acta Electronica Sigica, 2016, 44(6):1452-1457.
[8] SHI Min, GUO Yinghua, CHEN Xiaohui. Influence of Noise on Detection Performance of Chaotic System [J]Ship Electronic Engineering, 2017, 37 (3):118-121.
[9] SUN Wenjun, RUI Guosheng, ZHANG Song, et al. Research on noise immunity of weak signal chaotic detection algorithm [J]. Radio Communications Technolgy, 2012, 38(1):59-62.
[10] ZHANG Bin. Study of the Algorithms of Lyapunov characteristic exponents and its application in chaos detection for weak signals [D]. Master dissertation, Ji Lin University, 2004: 19-37.