Precision Measurements of Collective Oscillations in the BEC-BCS Crossover

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We report on precision measurements of the frequency of the radial compression mode in a strongly interacting, optically trapped Fermi gas of ⁶Li atoms. Our results allow for a test of theoretical predictions for the equation of state in the BEC-BCS crossover. We confirm recent quantum Monte-Carlo results and rule out simple mean-field BCS theory. Our results show the long-sought beyond-mean-field effects in the strongly interacting BEC regime.

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Ultracold, strongly interacting Fermi gases [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] have attracted considerable attention over the past few years, serving as unique model systems to create, control, and investigate novel states of quantum matter. Experimentally, the availability of such systems has opened up exciting possibilities to study many-body quantum phenomena like molecular Bose-Einstein condensation (BEC) [3] and the crossover to study many-body quantum phenomena like molecular Bose-Einstein condensation (BEC) [3] and the crossover to molecular superfluids [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. These experiments may also lead to a better understanding of strongly interacting quantum systems in different areas of physics, ranging from high-Tc superconductors to neutron stars and the quark-gluon plasma.

A degenerate two-component Fermi gas undergoes the BEC-BCS crossover [14] when the s-wave scattering length a is varied from positive to negative values across a scattering resonance. In the crossover region, where a is comparable with or larger than the interparticle spacing, the equation of state is governed by many-body effects. Understanding the equation of state is a fundamentally important challenge and constitutes a difficult task for many-body quantum theories, even in the zero-temperature limit. Mean-field BCS theory [14] provides a reasonable interpolation between the well-understood limits. More sophisticated crossover approaches [15] yield quantitatively different results in certain regimes, none of them however providing a complete description of the problem. The most advanced theoretical results were obtained by numerical calculations based on a quantum Monte-Carlo (QMC) approach [16].

On the BEC side of the crossover, there is an interesting competition in the equation of state between the strong interactions in a Bose gas and the onset of fermionic behavior. For a strongly interacting Bose gas, one can expect quantum depletion to increase the average energy per particle. To lowest order, this beyond-mean-field effect leads to a correction to the equation of state predicted by Lee, Huang, and Yang (LHY) almost 50 years ago [17]. Beyond mean-field effects are expected to reduce the compressibility of a strongly interacting Bose gas as compared to the weakly interacting case. However, when approaching the resonance, fermionic behavior emerges and the system loses its purely bosonic character, which increases the compressibility of the strongly interacting gas. Mean-field BCS theory does not contain beyond-mean-field effects and the LHY correction is absent there. However, the QMC results predict beyond-mean-field effects to be visible on the BEC-side of the crossover [16].

In this Letter, we report on precision measurements of the radial compression mode in an optically trapped, strongly interacting Fermi gas of ⁶Li atoms. The mode serves as a sensitive probe for the compressibility and thus the equation of state of a superfluid gas in the BEC-BCS crossover [18, 19]. We reach a precision level that allows us to distinguish between the predictions resulting from mean field BCS theory and QMC calculations. Previous experiments on collective modes, performed at Duke University [8, 11] and at Innsbruck University [9], showed frequency changes in the BEC-BCS crossover in both the slow axial mode and the fast radial compression mode of a cigar-shaped sample. The accuracy, however, was insufficient for a conclusive test of the different many-body theories in the strongly interacting regime.

We prepare a strongly interacting, degenerate gas of ⁶Li atoms in the lowest two internal states as described in our previous publications [4, 8, 10]. The broad Feshbach resonance centered at a magnetic field of B = 834 G facilitates precise tuning of the scattering length a [20]. Forced evaporative cooling is performed in a 1030-nm near-infrared laser beam focussed to a waist of 54 µm at 764 G. This results in a deeply degenerate cloud of \( N = 2.0(5) \times 10^5 \) atoms. By adiabatically increasing the trap laser power after cooling, the sample is recompressed to achieve nearly harmonic confinement. In the axial direction the gas is magnetically confined in the curvature of the field used for Feshbach tuning with an axial trap frequency of \( \omega_z/2\pi = 22.4 \) Hz at 834 G. The experiments reported here are performed at two different final values of the laser power of the recompressed trap. At 135 mW (540 mW), the trap is 1.8 µK (7.3 µK) deep and the radial trap frequency is \( \omega_r \approx 2\pi \times 290 \) Hz (590 Hz). The Fermi energy of a non-interacting cloud is calcu-
4 ms after release, we determine the mean cloud radius. Fermi profiles to images of the expanding cloud taken from the trap. From fits of two-dimensional Thomas-Fermi radii after expansion. Here we obtain $\omega_c/2\pi = 1185$ Hz.

A typical set of measurements is shown in Fig. 2. A fit of a damped harmonic oscillation to such data yields the frequency $\omega_c$ and damping rate $\gamma$ of the radial compression mode.

Our experiments are performed close to the limit of an elongated harmonic trap potential with cylindrical symmetry. This elementary case is of great general relevance for many quantum gas experiments in optical and magnetic traps (see, e.g., [22]), and collective excitations are conveniently normalized to the trap frequency $\omega_r$. [18, 19]. The compression mode frequency can then be written as $\omega_c = f_c \omega_r$, where $f_c$ is a dimensionless function of the interaction parameter $1/k_Fa$ and is related to an effective polytropic index $\Gamma$ [18, 19] of the equation of state by $\omega_c^2 = 2(\Gamma + 1)\omega_r^2$.

In order to compare our experimental results with theory, we consider the quantity $f_c$, i.e. the normalized compression mode frequency of the ideal, cylindrically symmetric, elongated trap. We find, that for our experimental conditions, $f_c$ is approximated by the ratio $\omega_c/\omega_\perp$ of the measured compression mode ($\omega_c$) and mean sloshing mode ($\omega_\perp$) frequencies to better than one percent. On the desired accuracy level of 10$^{-3}$, however, two small effects have to be taken into account: the residual trap ellipticity and the anharmonicity of the radial potential in combination with the spatial extension of the trapped sample. We thus introduce two small corresponding corrections, expressing $f_c$ in the form $f_c = (1 - \kappa c^2 + ba)\omega_c/\omega_\perp$.

For the ellipticity correction $\kappa c^2$, a straightforward solution of the hydrodynamic eigenfrequency equation [21] yields $\kappa = (2 + \Gamma)/4\Gamma$, where $\Gamma$ can be approximated by $\Gamma = (\omega_c/\omega_\perp)^2/2 - 1$. For the anharmonicity correction, the parameter $\alpha = \frac{1}{2}m\omega_r^2 r_{\text{rms}}^2/U_0$ relates the potential energy associated with the root-mean-square radius $r_{\text{rms}}$ [23] of the trapped cloud to the trap depth. The coefficient $b$ results from the differential anharmonicity shifts in the compression and sloshing modes and can be calculated according to [21, 24, 25]. We obtain [21] $b = 0.167$ and 0.280 in the limits of BEC and unitarity, respectively.

Our measurements on the sloshing and compression modes are summarized in Table I, including the two small corrections. For the data in the strongly interacting...
TABLE I: Experimental data on radial collective modes in the BEC-BCS crossover. The data in the upper seven (lower eight) rows refer to the sets of measurements taken in the shallower (deeper) trap with \( U_0 = 1.8 \mu \text{K} \) and \( E_F = 500 \text{nK} \) \( (U_0 = 7.3 \mu \text{K} \) and \( E_F = 800 \text{nK} \) ). The values in parentheses indicate 1\( \sigma \) fit uncertainties of individual measurements. Note that a systematic scaling uncertainty of \(~4\%\) for \( 1/k_F a \) results from the uncertainty in the atom number \( N = 2.0(5) \times 10^6 \).

| \( B \) \( 1/k_F a \) (G) | sloshing \( \omega_{s}/2\pi \) (Hz) | \( \epsilon \) | compression \( \omega_{c}/2\pi \) (Hz) | corr. \( \kappa^2 \hbar \alpha \) \( (10^{-4}) \) |
|-----------------|-----------------|-----|-----------------|-----------------|
| 727.8 \( 2.21 \) | 292.7(5) 0.083(3) | 596.3(6) 0.007(2) | 48 20 |
| 735.1 \( 1.96 \) | 298.6(5) 0.091(3) | 602.8(8) 0.008(3) | 60 26 |
| 742.5 \( 1.75 \) | 294.5(5) 0.067(3) | 593.2(7) 0.005(2) | 33 28 |
| 749.8 \( 1.55 \) | 296.3(4) 0.073(3) | 599.0(7) 0.006(2) | 38 28 |
| 760.9 \( 1.27 \) | 296.0(4) 0.088(2) | 592.3(7) 0.009(2) | 58 24 |
| 771.9 \( 1.03 \) | 293.6(7) 0.074(5) | 586.2(8) 0.007(3) | 41 27 |
| 834.1 \( 0 \) | 287.5(7) 0.073(5) | 519.4(9) 0.014(3) | 55 94 |
| 757.2 \( 1.07 \) | 605.9(9) 0.065(3) | 1210.9(12) 0.010(2) | 32 13 |
| 768.2 \( 0.87 \) | 592.5(7) 0.069(2) | 1186.6(12) 0.012(2) | 36 16 |
| 775.6 \( 0.75 \) | 590.2(4) 0.060(1) | 1170.2(21) 0.007(4) | 28 14 |
| 782.2 \( 0.64 \) | 604.8(9) 0.061(3) | 1187.1(16) 0.006(3) | 29 16 |
| 801.3 \( 0.38 \) | 588.7(8) 0.063(2) | 1135.2(12) 0.010(2) | 33 24 |
| 812.3 \( 0.24 \) | 585.5(7) 0.058(2) | 1106.9(16) 0.014(3) | 30 33 |
| 834.1 \( 0 \) | 596.3(9) 0.070(3) | 1089.0(12) 0.010(2) | 48 40 |
| 849.1 \( -0.14 \) | 583.2(7) 0.052(2) | 1046.7(37) 0.007(2) | 29 47 |

BEC regime \((1/k_F a \gtrsim 1)\) we used the weaker trap with \( \omega_{s}/2\pi \approx 290 \text{ Hz} \) to minimize unwanted heating by inelastic collisions. Closer to resonance \((1/k_F a \lesssim 1)\) inelastic processes are strongly suppressed, but the increasing cloud size introduces larger anharmonicity shifts. Here we chose the deeper trap with \( \omega_{s}/2\pi \approx 590 \text{ Hz} \). On the BCS side of the resonance we observed increased damping as a precursor of the breakdown of hydrodynamics \([9,11]\). We thus restricted our measurements to magnetic fields below 850 G to ensure low damping rates \((\gamma/\omega_{s} < 0.01)\) and superfluid hydrodynamics.

At a given magnetic field, a set of measurements on the sloshing and compression modes typically takes a few hours. To minimize uncertainties from slow drifts and day-to-day variations we always took the sloshing mode reference measurement right before or after the compression mode data. By repeating measurements under identical settings we found a typical remaining fractional uncertainty for the normalized compression mode frequencies of \(5 \times 10^{-3}\), which is about 2-3 times larger than the fit errors of individual measurements.

In Fig. 3 we show our final results on the normalized compression mode frequency \( f_c \) versus interaction parameter \( 1/k_F a \). The experimental data include the small corrections for trap ellipticity and anharmonicity and can thus be directly compared to theory in the limit of an elongated harmonic trap with cylindrical symmetry. The open and filled circles refer to the measurements listed in Table I for \( \omega_{s}/2\pi \approx 290 \text{ Hz} \) and \( 590 \text{ Hz} \), respectively. The error bars indicate the typical scatter of the data points. The filled triangle shows a zero-temperature extrapolation of the measurements displayed in Fig. 3. The theory curves refer to mean-field BCS theory (lower curve) and QMC calculations (upper curve) and correspond to the data presented in Ref. [19]. The horizontal dashed lines indicate the values for the BEC limit \((f_c = 2)\) and the unitarity limit \((f_c = \sqrt{10}/3 = 1.826)\).

FIG. 3: Normalized compression mode frequency \( f_c \) versus interaction parameter \( 1/k_F a \). The experimental data include the small corrections for trap ellipticity and anharmonicity and can thus be directly compared to theory in the limit of an elongated harmonic trap with cylindrical symmetry. The open and filled circles refer to the measurements listed in Table I for \( \omega_{s}/2\pi \approx 290 \text{ Hz} \) and \( 590 \text{ Hz} \), respectively. The error bars indicate the typical scatter of the data points. The filled triangle shows a zero-temperature extrapolation of the measurements displayed in Fig. 3. The theory curves refer to mean-field BCS theory (lower curve) and QMC calculations (upper curve) and correspond to the data presented in Ref. [19]. The horizontal dashed lines indicate the values for the BEC limit \((f_c = 2)\) and the unitarity limit \((f_c = \sqrt{10}/3 = 1.826)\).
shifts in collective mode measurements, let us comment on the previous experiments performed in Innsbruck [9] and at Duke University [8, 11]. We have reanalyzed our old data on the radial compression mode and identified a previously undetermined ellipticity of $\epsilon \approx 0.2$ as the main problem in our data interpretation [29]. The fact that we had normalized the compression mode frequency to the vertical trap frequency ($\omega_c/\omega_y$) led to a substantial downshift in the hydrodynamic regime, but not in the collisionless regime. We furthermore believe that significant temperature shifts were present in the previous collective mode experiments. In particular for the strongly interacting BEC regime, temperature shifts in our old data on the axial mode [9] and the Duke data on the radial mode [11] provide a plausible explanation for these measurements being closer to the predictions of mean-field BCS theory than to the more advanced QMC results.

In conclusion, our work shows that collective modes allow for precision tests of many-body theories in strongly interacting quantum gases. In future experiments, the observation of collective oscillation modes will serve as a powerful tool to investigate strongly interacting superfluids in a more general context, e.g. in mixtures of fermionic quantum gases.

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