Penrose Limit of $AdS_4 \times N^{0,1,0}$ and $\mathcal{N} = 3$ Gauge Theory

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abstract

We consider M-theory on $AdS_4 \times N^{0,1,0}$ where $N^{0,1,0} = (SU(3) \times SU(2))/ (SU(2) \times U(1))$. We review a Penrose limit of $AdS_4 \times N^{0,1,0}$ that provides the pp-wave geometry of $AdS_4 \times S^7$. There exists a subsector of three dimensional $\mathcal{N} = 3$ dual gauge theory, by taking both the conformal dimension and $R$-charge large with the finiteness of their difference, which has enhanced $\mathcal{N} = 8$ maximal supersymmetry. We identify operators in the $\mathcal{N} = 3$ gauge theory with supergravity KK excitations in the pp-wave geometry and describe how the $\mathcal{N} = 2$ gauge theory operators originating from both $\mathcal{N} = 3$ short vector multiplet and $\mathcal{N} = 3$ long gravitino multiplet fall into $\mathcal{N} = 8$ supermultiplets.
1 Introduction

The large $N$ limit of a subsector of $d = 4, \mathcal{N} = 4 \ SU(N)$ supersymmetric gauge theory is dual to type IIB string theory in the pp-wave background. This subspace of the gauge theory is described by string theory in the pp-wave background. By taking a scale limit of the geometry near a null geodesic in $AdS_5 \times S^5$, it gives rise to the appropriate subspace of the gauge theory. The operators with large $R$-charge in the subsector of $\mathcal{N} = 4 \ SU(N)$ gauge theory were identified with the stringy states in the pp-wave background. It was found that the Penrose limit of $AdS_5 \times T^{1,1}$ provides pp-wave geometry of $AdS_5 \times S^5$. Using AdS/CFT correspondence, one can identify gauge theory operators with large $R$-charge with the stringy excitations in the pp-wave geometry. Moreover, the maximal $\mathcal{N} = 4$ multiplet structure hidden in the $\mathcal{N} = 1$ gauge theory can be predicted from both a chiral operator and semi-conserved operator with large $R$-charge.

It is natural to think about the subsector of $\mathcal{N} = 2$ gauge theory in $d = 3$ in the context of $AdS_4 \times X^7$ where $X^7$ is an Einstein seven manifold. Recently the operators with large $R$-charge in the boundary field theory were obtained from the complete spectrums of 11-dimensional KK compactifications on $AdS_4 \times Q^{1,1,1}$, $AdS_4 \times M^{1,1,1}$ and $AdS_4 \times V_{5,2}$ in pp-wave limit. In old days, all the supersymmetric $\mathcal{N} = 2$ homogeneous manifolds were classified in [11]. There exist only three $\mathcal{N} = 2$ theories and they are $Q^{1,1,1}, M^{1,1,1}$ and $V_{5,2}$. The isometry of these manifolds corresponds to the global symmetry of the dual SCFT including $U(1)_R$ symmetry of $\mathcal{N} = 2$ supersymmetry.

In this paper, we consider a similar duality that is present between a certain three dimensional $\mathcal{N} = 3$ gauge theory and 11-dimensional supergravity theory in a pp-wave background with the same spirit as in [7, 8, 9, 10]. This is a continuation of previous considerations [7, 8, 9]. We describe this duality by taking a scaling limit of the duality between 11-dimensional supergravity on $AdS_4 \times N^{0,1,0}$ where $N^{0,1,0}$ was found in [11] and three dimensional $\mathcal{N} = 3$ superconformal field theory. The boundary theory in terms of $\mathcal{N} = 2$ superfields is a gauge theory with gauge group $SU(N) \times SU(N)$ with chiral fields $U_i$ transforming in the $(N, \overline{N})$ color representation and $3$ under the flavor group $SU(3)$ and $V_i$ transforming in the $(\overline{N}, N)$ color representation and $\overline{3}$ under the flavor group. The complete analysis on the spectrum of $AdS_4 \times N^{0,1,0}$ was found in [12, 13] (See also [14]). This gives the theory that lives on $N$ M2-branes at the conical singularity of a Calabi-Yau four-fold. The scaling limit is obtained by considering the geometry near a null geodesic carrying large angular momentum in the $SO(3)_R = SU(2)_R$ isometry of the $N^{0,1,0}$ space which is dual to the $SU(2)_R$ $R$-symmetry in the $\mathcal{N} = 3$ superconformal field theory.

In section 2, we review the scaling limit around a null geodesic in $AdS_4 \times N^{0,1,0}$ from the explicit metric of $N^{0,1,0}$ and obtain a pp-wave background [13]. In section 3, we identify
supergravity excitations in the Penrose limit with gauge theory operators. What we observed is the presence of a semi-conserved field in the $\mathcal{N} = 3$ long gravitino multiplet propagating in the $AdS_4$ bulk. In section 4, we summarize our results.

2 Penrose Limit of $AdS_4 \times N^{0,1,0}$

Let us start with the supergravity solution dual to the $\mathcal{N} = 3$ superconformal field theory \cite{16}. By putting a large number of $N$ coincident M2-branes at the conifold singularity and taking the near horizon limit, the metric becomes that \cite{17} of $AdS_4 \times N^{0,1,0}$

$$ds^2_{N^{0,1,0}} = ds^2_{AdS_4} + ds^2_{N^{0,1,0}},$$

where

$$ds^2_{AdS_4} = L^2 \left( - \cosh^2 \rho \; dt^2 + d\rho^2 + \sinh^2 \rho \; d\Omega_2^2 \right),$$

$$ds^2_{N^{0,1,0}} = \frac{L^2}{2} \left[ \left( \Sigma_1 - \cos \zeta \sigma_1 \right)^2 + \left( \Sigma_2 - \cos \zeta \sigma_2 \right)^2 + \left( \Sigma_3 - \frac{1}{2} \left( 1 + \cos^2 \zeta \right) \sigma_3 \right)^2 \right]$$

$$+ L^2 \left[ d\zeta^2 + \frac{1}{4} \sin^2 \zeta \left( \sigma_1^2 + \sigma_2^2 + \cos^2 \zeta \sigma_3^2 \right) \right]$$

where $d\Omega_2$ is the volume form of a unit $S^2$ and the curvature radius $L$ of $AdS_4$ is given by $(2L)^6 = 32\pi^2 \ell_p^6 N$. Topologically $N^{0,1,0}$ is a nontrivial $SO(3)$ bundle over $\mathbb{C}P^2$. The coordinate $\zeta$ and $SU(2)$ one-forms $\sigma_i$ are the same as in the $\mathbb{C}P^2$ metric and the one-forms $\Sigma_i$ are left-invariant forms on the manifold $SO(3)$. The $SU(3) \times SU(2)$ isometry group of $N^{0,1,0}$ consists of $SU(3)$ global symmetry and $SO(3)_R$ symmetry of the dual superconformal field theory of \cite{16, 17}.

Let us make a scaling limit around a null geodesic in $AdS_4 \times N^{0,1,0}$. Let us introduce coordinates which label the geodesic

$$x^+ = \frac{1}{2} \left[ t + \frac{1}{\sqrt{2}} \left( \gamma + \beta - \frac{1}{2} \psi - \frac{1}{2} \phi \right) \right], \quad x^- = \frac{L^2}{2} \left[ t - \frac{1}{\sqrt{2}} \left( \gamma + \beta - \frac{1}{2} \psi - \frac{1}{2} \phi \right) \right],$$

1The three real one-forms $\sigma_i$ are given by with the ranges, $0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi, 0 \leq \psi \leq 4\pi$

$$\sigma_1 = \cos \psi d\theta + \sin \psi \sin \theta d\phi, \quad \sigma_2 = -\sin \psi d\theta + \cos \psi \sin \theta d\phi, \quad \sigma_3 = d\psi + \cos \theta d\phi.$$ These one-forms are left-invariant under the $SU(2)$ group and satisfy $SU(2)$ algebra. Similarly the explicit form of left-invariant forms on the $SO(3)$ group manifold in terms of angular variables are

$$\Sigma_1 = \cos \gamma d\alpha + \sin \gamma \sin \alpha d\beta, \quad \Sigma_2 = -\sin \gamma d\alpha + \cos \gamma \sin \alpha d\beta, \quad \Sigma_3 = d\gamma + \cos \alpha d\beta.$$ They also satisfy similar $SO(3)$ algebra. Sometimes one uses the notation of $N(k,l)$ instead of $N^{pqr}$ due to the redundancy of last integer $r$. The parameters are related by $k/l = (3p + q)/(3p - q)$. Strictly speaking, the space $N^{0,1,0}$ we are dealing with in this paper is $N^{0,1,0}_I$ which has $\mathcal{N} = (3,0)$ supersymmetry. There exists other type of the $N^{0,1,0}$ found by Page and Pope \cite{17} and denoted by $N^{0,1,0}_I$ that has $\mathcal{N} = (1,0)$ supersymmetry. In this case, the metric is also different from \cite{2}. The squashing parameter in this case is $1/10$ not $1/2$ like as \cite{2}.
and make a scaling limit around $\rho = 0 = \theta = \alpha$ and $\zeta = \pi/2$ in the above geometry (1). By taking the limit $L \to \infty$ while rescaling the coordinates $\rho = r/L, \zeta = \pi/2 + \zeta_1/L, \theta = \zeta_2/L, \alpha = \zeta_3/\sqrt{2}L$, the Penrose limit of the $AdS_4 \times N^{0,1,0}$ becomes [13]

$$ds^2_{11} = -4dx^+dx^- + \frac{3}{4} \sum_{i=1}^3 (dr^i dr^i - r^i r^i dx^+ dx^-) + \frac{1}{4} \sum_{i=1}^3 \left( d\zeta_i^2 + \zeta_i^2 d\phi_i^2 - \sqrt{2} \zeta_i^2 d\phi_idx^+ \right)$$

where we define $\phi_1 = \frac{1}{2} (\psi + \phi), \phi_2 = -\phi, \phi_3 = \beta$ and in the last line we introduce the complex coordinates $z_i = \zeta_i e^{i\phi_i}$. Since the metric has a covariantly constant null Killing vector $\partial/\partial x^-$, it is also pp-wave metric. Note that the pp-wave geometry (3) in the scaling limit reduces to the maximally supersymmetric pp-wave solution of $AdS_4 \times S^7$ [18, 13]

$$ds^2_{11} = -4dx^+dx^- + \sum_{i=1}^3 \left( dr^i dr^i - r^i r^i dx^+ dx^- \right) + \frac{1}{4} \left( dz_i d\bar{z}_i + i\sqrt{2} (\bar{z}_i dz_i - z_i d\bar{z}_i) dx^+ \right).$$

The supersymmetry enhancement in the Penrose limit implies that a hidden $\mathcal{N} = 8$ supersymmetry is present in the corresponding subsector of the dual $\mathcal{N} = 3$ superconformal field theory. In the next section, we provide precise description of how to understand the excited states in the supergravity theory that corresponds to operators in the dual superconformal field theory.

### 3 Gauge Theory Spectrum

The 11-dimensional supergravity theory in $AdS_4 \times N^{0,1,0}$ is dual to the $\mathcal{N} = 3$ gauge theory with gauge group $SU(N) \times SU(N)$ coupled to a suitable set of hypermultiplets with Chern Simons interaction. In $\mathcal{N} = 2$ language, the field contents are given by two kinds of chiral fields $U^i, i = 1, 2, 3$ transforming in the $(\mathbf{N}, \mathbf{N})$ color representation and $V_j, j = 1, 2, 3$ transforming in the $(\mathbf{\overline{N}}, \mathbf{\overline{N}})$ color representation [16] and they transform as $\mathbf{3}$ and $\mathbf{\overline{3}}$ under the $SU(3)$ global symmetry, respectively. We identify states in the supergravity containing both short and long multiplets with operators in the gauge theory. In each multiplet, we specify a $SU(3)$ representation $\mathbf{1}$, conformal weight and $SU(2)$ isospin in $\mathcal{N} = 3$ superspace.

- $\mathcal{N} = 3$ Massless(or ultrashort) multiplets [12, 13, 16]

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\[2\] A representation of $SU(3)$ can be identified by a Young diagram and when we denote the Dynkin label $(M_1, M_2)$ so that totally we have $M_1 + 2M_2$ boxes, the dimensionality of an irreducible representation is $N(M_1, M_2) = (1 + M_1)(1 + M_2)(2 + M_1 + M_2)$. Also an irreducible representation of $SU(2)$ can be described by a Young diagram with $2J$ boxes. Its dimensionality is $2J + 1$. 

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1) One $\mathcal{N} = 3$ massless graviton multiplet $\Theta^\alpha(x, \theta^\pm, \theta^0)$: $1$, $\Delta = 3/2$, $J = 0$

This massless graviton multiplet coresponds to the lowest ($J = 0$) isospin $\mathcal{N} = 3$ superfield. It is convenient to describe it in terms of $\mathcal{N} = 2$ superfield because the structure of $OSp(2|4)$ multiplets and its description as constrained superfield are known. An $\mathcal{N} = 2$ stress-energy tensor superfield $T_{(\alpha\beta)}(x, \theta^\pm)$ corresponding to $\mathcal{N} = 2$ massless graviton multiplet satisfies the equation $D_\alpha^\pm T^{(\alpha\beta)}(x, \theta^\pm) = 0$. This $T_{(\alpha\beta)}(x, \theta^\pm)$ is a singlet with respect to the flavor group $SU(3)$ and its conformal dimension is 2. Moreover one has one $\mathcal{N} = 2$ massless gravitino multiplet characterized by its conformal dimension 3/2 and a singlet with respect to the flavor group. This corresponds to the $\mathcal{N} = 2$ conserved current $G^\alpha(x, \theta^\pm)$ relative to the third supersymmetry charge. It satisfies $D_\alpha^\pm G^\alpha(x, \theta^\pm) = 0$. The $\theta^0$ independent term of $\mathcal{N} = 3$ isospin superfield $\Theta^\alpha(x, \theta^\pm, \theta^0)$ contains $G^\alpha(x, \theta^\pm)$ and linear term in $\theta^0$ has $T_{(\alpha\beta)}(x, \theta^\pm)$. Therefore one can see the conformal dimension of $\theta^0$ is $-1/2$.

2) One $\mathcal{N} = 3$ massless vector multiplet $\Theta^\alpha_j(x, \theta^\pm, \theta^0)$: $8$, $\Delta = 1$, $J = 1$

One can construct $SO(3)_R$ triplet by taking tensor product of the isospin doublet in the fundamental representation of the flavor group $SU(3)$, $\Theta^\alpha_j(x, \theta^\pm, \theta^0)$ times its conjugate doublet $\Theta^\alpha_j(x, \theta^\pm, \theta^0)$. There exists a conserved vector current, an $\mathcal{N} = 2$ massless vector $\Sigma_j^\alpha(x, \theta^\pm)$, to the generator of the flavor symmetry group $SU(3)$ through Noether theorem satisfying the conservation equations $D_\alpha^\pm D^\pm_\alpha \Sigma_j^\alpha(x, \theta^\pm) = 0$. This $\Sigma_j^\alpha(x, \theta^\pm)$ transforms in the adjoint representation $8$ of $SU(3)$ flavor group and its conformal dimension is 1. The remaining two $\theta^0$ independent components whose third components of $J$ (or $R$-charge) are 1 and $-1$ are $\mathcal{N} = 2$ chiral superfields.

3) One $\mathcal{N} = 3$ massless vector multiplet $\Theta(x, \theta^\pm, \theta^0)$: $1$, $\Delta = 1$, $J = 1$

The $\theta^0$ independent terms in this isospin superfield have two chiral $\mathcal{N} = 2$ superfields and a linear superfield $\Sigma_{\text{betti}}^\alpha(x, \theta^\pm)$ satisfying $D_\alpha^\pm D^\pm_\alpha \Sigma_{\text{betti}}^\alpha(x, \theta^\pm) = 0$. It is known that the Betti current $\Sigma_{\text{betti}}^\alpha(x, \theta^\pm)$ of $N^{0,1,0}$ is obtained and this corresponds to additional $\mathcal{N} = 2$ massless vector multiplet related to nonperturbative baryon states carrying Betti charge.

- $\mathcal{N} = 3$ Short multiplets $^{[12, 13, 16]}$

It is known that the dimension of the scalar operator in terms of energy labels, in the dual SCFT corresponding $AdS_4 \times N^{0,1,0}$ is

$$\Delta = \frac{3}{2} + \frac{1}{2} \sqrt{1 + \frac{m^2}{4}} = \frac{3}{2} + \frac{1}{2} \sqrt{\frac{45 + \frac{E}{4} - 6\sqrt{36 + E}}{4}.}$$  \hspace{1cm} (4)$$

The energy spectrum on $N^{0,1,0}$ exhibits an interesting feature which is relevant to superconfor-

\footnote{An $\mathcal{N} = 3$ superfield $\Theta(x, \theta^\pm, \theta^0)$ is a function of the bosonic coordinate $x^\mu$ and fermionic coordinates, $\theta^\pm$ and $\theta^0$. In the expansion of $\theta^0$, the component fields are $\mathcal{N} = 2$ superfields $\Phi(x, \theta^\pm)$ where we emphasize the $\mathcal{N} = 2$ superfield by putting the dependence of $\theta^\pm$ in their arguments. Similarly for $\mathcal{N} = 3$ superfield, we insert $\theta^0$ dependence explicitly as well as $\theta^\pm$ in order not to confuse $\mathcal{N} = 2$ one.}
mal algebra and it is given by
\[ E = \frac{16}{3} \left[ 2 \left( M_1^2 + M_2^2 + M_1 M_2 + 3M_1 + 3M_2 \right) - 3J(J+1) \right] \]  

(5)

where the eigenvalue \( E \) is classified by \( SU(3) \) quantum numbers \((M_1, M_2)\) and \( SU(2) \) isospin \( J \): \( M_1 = 0, 1, 2, \cdots, M_2 = M_1 + 3j, J = j, j+1, \cdots \) and \( j \geq 0 \). The corresponding eigenmodes occur in \((M_1, M_1 + 3j)\) \( SU(3) \) representation and the angular momentum \( J SU(2) \) representation.

According to [12, 13, 16], the information on the Laplacian eigenvalues allows us to get the spectrum of \( N = 2 \) hypermultiplets of the theory corresponding to the chiral operators of the SCFT. This part of spectrum was given in [12, 13, 16] and the form of operators is
\[ \text{Tr}\Phi_c \equiv \text{Tr}(UV)^R, \quad (1 + R)^3, \quad \Delta = R, \quad J = R \geq 2 \]  

(6)

where the flavor \( SU(3) \) indices are totally symmetrized and the chiral superfield \( \Phi_c(x, \theta^\pm) \) satisfies \( D^+ \Phi_c(x, \theta^\pm) = 0 \). There exists also other type of chiral superfield \( \text{Tr}\Psi_c^I \equiv \text{Tr}(\overline{UV})^R \) with lowest value of third component of \( J \). In the \( N = 2 \) theory, it has \( R \)-charge \( -R \). The hypermultiplet spectrum in the KK harmonic expansions on \( N^{0,1,0} \) agrees with the chiral superfield predicted by the conformal gauge theory. From this, the dimension of \( UV \) should be 1 to match the spectrum. In fact, the conformal weight of a product of chiral fields equals the sum of the weights of the single components. This is due to the the relation of \( \Delta = R \) satisfied by chiral superfields and to the additivity of the \( R \)-charge.

1) One \( N = 3 \) short graviton multiplet \( \Theta^\alpha(x, \theta^\pm, \theta^0) \):
\[ (1 + R)^3, \quad \Delta = R + 3/2, \quad J = R \geq 1 \]  

(7)

Short gravitons of higher isospin can be obtained by multiplying the \( J = 0 \) massless graviton \( \Theta^\alpha(x, \theta^\pm, \theta^0) \) with chiral superfields \( \Theta(x, \theta^\pm, \theta^0) \) of any \( J \). The gauge theory interpretation of this multiplet is obtained by adding a dimension 2 singlet operator with respect to flavor group into the above chiral superfield \( \Phi_c(x, \theta^\pm) \) in the \( N = 2 \) language. We consider \( \text{Tr}\Phi_{(\alpha\beta)} \equiv \text{Tr} \left( T_{(\alpha\beta)} \Phi_c \right) \), where \( T_{(\alpha\beta)}(x, \theta^\pm) \) is a stress energy tensor and \( \Phi_c(x, \theta^\pm) \) is a chiral superfield \( \Phi_c \). In \( N = 3 \) language, this composite operator is the coefficient function appearing in the linear \( \theta^0 \).

Note that the conformal dimension of this composite operator is given by \( \Delta = (R + 3/2) + 1/2 \) which consists of the conformal dimension of \( \Theta^\alpha(x, \theta^\pm, \theta^0) \) plus 1/2. \(^4\) As before we have

\[ SD (2, \Delta, J) \longrightarrow \bigoplus_{y=J}^J SD (2, \Delta + 1/2, y) \oplus \bigoplus_{y=-J}^J SD (3/2, \Delta, y) \]

where the first element in the left hand side gives us the maximal spin. So they are 2, 3/2, 1 for graviton, gravitino and vector, respectively. In the right hand side, \( y \) is a \( U(1)_R \) charge. Therefore, \( N = 3 \) short graviton multiplet consists of two short gravitons, \( (2R - 1) \) long gravitons, two short gravitinos and \( (2R - 1) \) long gravitinos in \( N = 2 \) language.
Moreover we have other type of gauge theory object, \( \text{Tr} \Psi_R \equiv \text{Tr} (T_{(a \beta)} \Psi_c^\dagger) \) where \( \Psi_c^\dagger (x, \theta^\pm) \) is a chiral superfield and whose \( R \)-charge in the \( \mathcal{N} = 2 \) theory is \( -R \). Other component of gauge theory operator is obtained by adding a dimension 3/2 singlet operator with respect to flavor group into the above chiral superfield \( \Phi_c (x, \theta^\pm) \). That is, \( \text{Tr} \Phi_\alpha \equiv \text{Tr}(G_\alpha \Phi_c) \), corresponding to \( \mathcal{N} = 2 \) short gravitino multiplet where \( G_\alpha (x, \theta^\pm) \) satisfying \( D_\alpha^\dagger G_\alpha (x, \theta^\pm) = 0 \) is a conserved current we have seen before. There exists an operator \( \text{Tr} \Psi_\alpha^\dagger \equiv \text{Tr}(G_\alpha \Psi_c^\dagger) \) corresponding to other \( \mathcal{N} = 2 \) short gravitino multiplet. Moreover, according to the decomposition of \( OSp(3|4) \) unitary irreducible representation into \( OSp(2|4) \) one \( [13] \), there exist also long gravitons and gravitinos that have rational conformal dimensions.

2) One \( \mathcal{N} = 3 \) short vector multiplet \( \Theta (x, \theta^\pm, \theta^0) \):

\[
(1 + R)^3, \quad \Delta = R, \quad J = R \geq 2
\]  

There exist two chiral \( \mathcal{N} = 2 \) multiplets \( \text{Tr} \Phi_C \) and \( \text{Tr} \Psi_C^\dagger \) we have discussed before in this \( \mathcal{N} = 3 \) short vector multiplet. In addition to them, one can construct the following gauge theory object, based on the decomposition rule of \( [13] \), \( \text{Tr} \Phi \equiv \text{Tr} \left( \Sigma_j^i (UV)^{R-1} \right) \), where \( \Sigma_j^i (x, \theta^\pm) \) is a conserved vector current with a singlet under the flavor group. The \( R \)-charge of \( \Phi (x, \theta^\pm) \) is \( R - 1 \). The Dynkin label of \( \Sigma_j^i (x, \theta^\pm) \) is \( (1, 1) \) while those of \( (UV)^{R-1} \) is \( (R - 1, R - 1) \). Moreover we have other type of gauge theory object, \( \text{Tr} \Psi \equiv \text{Tr} \left( \Sigma_j^i (UU)^{R-1} \right) \) whose \( R \)-charge is \( -(R - 1) \). These composite operators correspond to \( \mathcal{N} = 2 \) short vector multiplets. There exist also \( \mathcal{N} = 2 \) long vector multiplets whose conformal dimensions are rational.\(^5\)

3) One \( \mathcal{N} = 3 \) short gravitino multiplet \( \Theta^\alpha (x, \theta^\pm, \theta^0) \):

\[
\frac{1}{2} (1 + R) (4 + R) (5 + 2R), \quad \Delta = R + 2, \quad J = R + 1, \quad R \geq 0
\]  

Short gravitinos of higher isospin can be obtained by multiplying the \( J = 1 \) short gravitino \( \Theta^\alpha (x, \theta^\pm, \theta^0) \) with chiral superfields \( \Theta (x, \theta^\pm, \theta^0) \) of any \( J \). From the above \( SU(3) \) representation of this multiplet, one can consider \( \text{Tr} \Phi_{a, \alpha}^{+(ijkl)} \equiv \text{Tr} \left( G_{a, \alpha}^{+(ijkl)} \Phi_C \right) \), where \( G_{a, \alpha}^{+(ijkl)} (x, \theta^\pm) \) is a conserved vector current with conformal dimension 5/2 transforming in the \( \Box \Box \Box \Box \) representation of \( SU(3) \) flavor group.\(^6\) Notice that the conformal dimension of this composite operator is made

\(^5\)The \( \mathcal{N} = 3 \) short vector multiplet by \( [8] \) decomposes as follows \( [13] \):

\[
S D \left( 1, \Delta, J \right) \rightarrow \bigoplus_{y = -J + 1}^{J-1} S D \left( 1, \Delta, y \right) \oplus S D \left( 1/2, \Delta, J \right) \oplus S D \left( 1/2, \Delta, -J \right).
\]

So it consists of two chiral multiplets, two short vector multiplets and \( (2R - 1) \) long vectors.

\(^6\)In terms of \( \mathcal{N} = 2 \) hypermultiplets, we have \( G_{a, \alpha}^{+(ijkl)} = f^{lmn}(ijkl) (U_l D^u_m U^r_j D^u_i U^r_l) \) \( [10] \).
of the one of $\Theta^\alpha (x, \theta^\pm, \theta^0)$ plus $1/2$. This is one of the two $\mathcal{N} = 2$ short gravitino multiplets. The other is the following gauge theory object $\text{Tr} \Psi^{- (ijk), \dagger}_\alpha \equiv \text{Tr} \left( G^{- (ijk)}_\alpha \Psi^\dagger_c \right)$ where $R$-charge is $-(R + 1)$. There exist two $\mathcal{N} = 2$ short vector multiplets: one is realized by $\text{Tr} \Phi^{+ (ijk)} \equiv \text{Tr} \left( \Sigma^{+ (ijk)} \Phi_c \right)$, where $\Sigma^{+ (ijk)} (x, \theta^\pm)$ is a conserved vector current of dimension 2 transforming as the $\Box \Box \Box^*$ representation of flavor group and the other by $\text{Tr} \Psi^{-(ijk), \dagger} \equiv \text{Tr} \left( \Sigma^{-(ijk)} \Psi^\dagger_c \right)$. Rational long gravitino and vector multiplets are present in this case.

$\bullet \ \mathcal{N} = 3$ Long multiplets [12, 13, 16]

Although the dimensions of nonchiral operators are in general irrational, there exist special integer values of $j$ such that for $M_1 = R, M_2 = R + 3j$ and $J = R + j$, one can see the condition, $j(j-1) = 0$ make $\sqrt{36 + E}$ be equal to $2(2R + 4j + 3)$ (See also [20, 7, 8, 9]). We consider here maximal $J = R + j$ because from the energy eigenvalues $E$ this will give us the lowest conformal dimension. Also one can generalize this analysis for nonmaximal case, $J = R + j - k$ where $k$ is some positive integers. Furthermore in order to make the dimension be rational (their conformal dimensions are protected), $45 + E/4 - 6\sqrt{36 + E}$ in [4] should be square of something. Therefore we have $\Delta = R + 2j$. This is true if we are describing states with finite $\Delta$ and $R$. Since we are studying the scaling limit $\Delta, R \to \infty$, we have to modify the above analysis. The energy eigenvalue of the Laplacian on $N^{0,1,0}$ for the supergravity mode [5] takes the form

$$E = 4 \left( R^2 + 4jR + 5j^2 + 3R + 5j \right). \quad (10)$$

One can show that the conformal weight of the $\mathcal{N} = 3$ long multiplets below becomes rational if $j = 0$ or $j = 1$.

1) One $\mathcal{N} = 3$ long gravitino multiplet:

$$\frac{(1 + R)(1 + R + 3j)(2 + 2R + 3j)}{2}, \quad \Delta = \pm \frac{3}{2} \sqrt{E + 36}, \quad j \leq J \leq j + R, \ R \geq 0, \ j \geq 2 \quad (11)$$

For finite $R$ with rational dimension, after inserting the $E$ into the above, we will arrive at the relation with same constraint:

$$\Delta - R = 2j + O \left( \frac{1}{R} \right), \quad \Delta - R = 3 + 2j + O \left( \frac{1}{R} \right). \quad (12)$$

$^7$ $\mathcal{N} = 3$ short gravitino multiplet with [4] breaks into the following scheme [3]

$$SD (3/2, \Delta, J) \longrightarrow \bigoplus_{y = -j}^j SD (3/2, \Delta + 1/2, y) \oplus \bigoplus_{y = -j}^j SD (1, \Delta, y).$$

Therefore, it consists of two short gravitinos, two short vectors, $(2R + 1)$ long gravitinos and $(2R + 1)$ long vectors in terms of $\mathcal{N} = 2$ superfields.

$^8$ In terms of two chiral fields we have explicit forms: $G^{(ijk)}_\alpha = -f^{lm(i} V^j V^{k)} (V_l D_\alpha V_m - V_m D_\alpha V_l)$ and $\Sigma^{-(ijk)} = -f^{lm(i} \nabla^j \nabla^{k)} (V_l \nabla_m - \nabla_m V_l)$. For $\Sigma^{+(ijk)}$, see the equation [14].
So the constraint $j = 0$ or $j = 1$ is not relevant in the subsector of the Hilbert space we are interested in. Candidates for such states in the gauge theory side are given in terms of semi-conserved superfields $[\mathcal{I}, \mathcal{I}, \mathcal{I}]$. According to the observation $[\mathcal{I}]$ of the decomposition of $\mathcal{N} = 3$ into $\mathcal{N} = 2$ of the multiplets, the $\mathcal{N} = 3$ long gravitino decomposes into various $\mathcal{N} = 2$ long gravitino multiplets whose conformal dimensions are greater than the above $[\mathcal{I}]$($\Delta - R = 2j + 1/2$ or $2j + 7/2$) and $\mathcal{N} = 2$ long vector multiplets whose conformal dimensions are $\Delta - R = 2j, 2j + 1, 2j + 3$ or $2j + 4$. Although they are not chiral primaries, their conformal dimensions are protected. The ones we are interested in take the following form,

$$\text{Tr}\Phi_{\text{s.c.}} \equiv \text{Tr} \left[ (\Sigma^{+(klm)})^j (UV)^R \right], \quad \Delta - R = 2j, \quad J = j + R$$

(13)

where the scalar superfields $\Sigma^{+(klm)}(x, \theta^\pm)$ transform in the $[\mathbb{I}, \mathbb{I}, \mathbb{I}]^*$ representation of flavor group $SU(3)$ and satisfy $D^\pm_a D_\mp^a \Sigma^{+(klm)}(x, \theta^\pm) = 0$ with conformal dimension 2. In this case, also we have $D^+a D^-a \Phi_{\text{s.c.}}(x, \theta^\pm) = 0$. Since the singleton superfields $U^a_b$ carry index $a$ in the $\mathbb{N}$ of $SU(N)_1$ and index $b$ in the $\overline{\mathbb{N}}$ of the $SU(N)_2$, the fields $V^b_j$ carry index $a$ in the $\overline{\mathbb{N}}$ of $SU(N)_1$ and index $b$ in the $\mathbb{N}$ of the $SU(N)_2$, one can construct the following conserved flavor current

$$\Sigma^{+(klm)} = f^{ij(k} U^l U^m) \left( V^b_j U^c_i - V^b_j U^c_i \right)$$

(14)

where the color indices are contracted in the right hand side. Note that the conformal dimension of these currents is not the one of naive sum of $U$ and $\overline{U}$ and $V$ and $\overline{V}$. As we discussed in the last section, supergravity theory in $AdS_4 \times N^{0,1,0}$ acquires an enhanced $\mathcal{N} = 8$ superconformal symmetry in the Penrose limit. This implies that the spectrum of the gauge theory operators in this subsector should fall into $\mathcal{N} = 8$ multiplets. We expect that both the chiral primary fields of the form $\text{Tr}(UV)^R$ $[\mathcal{I}]$ and the semi-conserved multiplets of the form $[\mathcal{I}]$ combine into $\mathcal{N} = 8$ multiplets in the limit. Note that for finite $R$, the semi-conserved multiplets should obey the condition $j = 0$ or $j = 1$ in order for them to possess rational conformal weights.

- $\mathcal{N} = 2$ long vectors in $\mathcal{N} = 3$ long gravitino: $\Delta - R = 2j, 2j + 1, 2j + 3, 2j + 4; J = j + R$

According to the above analysis, the semi-conserved superfield $[\mathcal{I}]$ in the gauge theory side gives rise to $\mathcal{N} = 2$ long vector multiplet whose conformal dimension $\Delta - R = 2j$. To produce a higher dimension operator with same $SU(3)$ representation and $SU(2)$ isospin representation,

$\mathcal{N} = 3$ long gravitino multiplet characterized by $[\mathcal{I}, \mathcal{I}, \mathcal{I}]$ decomposes into its $\mathcal{N} = 2$ components fields $[\mathcal{I}]$

$$SD(3/2, \Delta, J) \rightarrow \bigoplus_{y = -J}^J SD(3/2, \Delta + 1/2, y) \oplus \bigoplus_{y = -J}^J SD(1, \Delta + 1, y) \oplus \bigoplus_{y = -J}^J SD(1, E_0, y), \quad \Delta > J + 1.$$}

So it consists of $(2R + 1)$ long gravitinos and $2(2R + 1)$ long vectors.
one has to multiply a dimension 1 operator under a singlet with respect to flavor group with a semi-conserved superfield. The candidate for this is a Betti current whose third component of $J$ is zero and the dimension is 1. So we obtain the following gauge theory object $Tr\left(\Sigma_{\text{betti}}\Phi_{\text{s.c.}}\right)$ corresponding to $\mathcal{N} = 2$ long vector multiplets whose the conformal dimension is $\Delta - R = 2j + 1$.

In order to construct a gauge theory operator corresponding to an $\mathcal{N} = 2$ long vector multiplet whose conformal dimension $\Delta - R = 2j + 3$, we make $Tr\left(T_{(\alpha\beta)}^0\Phi_{\text{s.c.}}\right)$ where dimension 3 operator $T_{(\alpha\beta)}^0(x, \theta^\pm)$ with $R$-charge 0 appears in the linear $\theta^0$ term of $\Theta^\alpha(x, \theta^\pm, \theta^0)$ with $J = 1$.

Finally for last gauge theory operator in this series, by taking the quadratic expression of dimension 2 stress-energy tensor with a semi-conserved field one can make the following combination $Tr\left(T_{(\alpha\beta)}^2\Phi_{\text{s.c.}}\right)$ corresponding to $\mathcal{N} = 2$ long vector multiplet whose conformal dimension is $\Delta - R = 2j + 4$.

- $\mathcal{N} = 2$ long gravitinos in $\mathcal{N} = 3$ long gravitino: $\Delta - R = 2j + 1/2, 2j + 7/2; J = j + R$.

Since we do not have any dimension 1/2 operator whose $R$-charge is zero and a singlet under a flavor group, we have to replace a single $\Sigma^{+ (klm)}$ among the product of $(\Sigma^{+ (klm)})^j$ with dimension 3 operator $G_{\alpha}^{+ (npq)}$ by noting that the conformal dimension can be decomposed as $\Delta - R = 2(j+1) + 5/2$. Therefore one can construct the following object which satisfies the right representations: $Tr\left((\Sigma^{+ (klm)})^{j-1}G_{\alpha}^{+ (npq)}(UV)^R\right)$. Next, for the second long gravitino, one can multiply dimension 5/2 operator which is neutral under the isospin to dimension $(R + 2j + 1/2)$ operator: $Tr\left((\Sigma^{+ (klm)})^{j-1}G_{\alpha}^{+ (npq)}(UV)^RT_{(\gamma \gamma)}^0\right)$.

2) One $\mathcal{N} = 3$ long graviton multiplet:

$$\frac{(1 + R)(1 + R + 3j)(2 + 2R + 3j)}{2}, \Delta = \frac{1}{4}\sqrt{E + 36}, j \leq J \leq j + R, R \geq 0, j \geq 2 \quad (15)$$

The combination of $\Delta - R$ is given by

$$\Delta - R = \frac{3}{2} + 2j + O\left(\frac{1}{R}\right)$$

where the right hand side is definitely rational and they are integers. According to the observation \[13\] of the decomposition of $\mathcal{N} = 3$ into $\mathcal{N} = 2$ of the multiplets, the $\mathcal{N} = 3$ long graviton multiplet decomposes into various $\mathcal{N} = 2$ long graviton multiplets whose conformal dimension is $\Delta - R = 2j + 2$, long gravitino multiplets where the conformal dimensions are $\Delta - R = 2j + 3/2$ or $2j + 5/2$ and long vector multiplets whose conformal dimension is $\Delta - R = 2j + 2$. \[10\]

\[10\] $\mathcal{N} = 3$ long graviton multiplet with the condition (15) breaks into \[13\]

$$SD(2, \Delta, J) \longrightarrow \bigoplus_{y = -J}^J SD(2, \Delta + 1/2, y) \bigoplus \bigoplus_{y = -J}^J SD(3/2, \Delta, y | 2) \bigoplus \bigoplus_{y = -J}^J SD(3/2, \Delta + 1, y)$$

$$\bigoplus \bigoplus_{y = -J}^J SD(1, \Delta + 1/2, y), \quad \Delta > J + 3/2.$$ 

It consists of $(2R + 1)$ long gravitons, $2(2R + 1)$ long gravitinos, and $(2R + 1)$ long vectors.
\[ \mathcal{N} = 2 \) long gravitons in \( \mathcal{N} = 3 \) long graviton: \( \Delta - R = 2j + 2; J = j + R \)

By adding conformal dimension 2 singlet operator of neutral under the isospin to a semi-conserved current, we can construct a gauge theory operator corresponding to \( \mathcal{N} = 2 \) long graviton multiplet, \( \text{Tr} \left( T_{(\alpha \beta)} \Phi_{\text{s.c.}} \right) \).

\[ \mathcal{N} = 2 \) long gravitons in \( \mathcal{N} = 3 \) long graviton: \( \Delta - R = 2j + 3/2, 2j + 5/2; J = j + R \)

The gauge theory interpretation of this multiplet is quite simple. If we take a semi-conserved current \( \Phi_{\text{s.c.}}(x, \theta^\pm) \) defined in (13) and multiply it by a conserved current \( G_\alpha(x, \theta^\pm) \) that is a singlet with respect to the flavor group with conformal dimension 3/2 and is neutral under the isospin, namely \( \text{Tr}(G_\alpha \Phi_{\text{s.c.}}) \), we reproduce the right \( OSp(2|4) \times SU(3) \) representations of the \( \mathcal{N} = 2 \) long gravitino multiplet: \( \Delta - R = 2j + 3/2 \). Similarly, one can construct the following gauge theory operator \( \text{Tr} \left( \Sigma_{\text{betti}} G_\alpha \Phi_{\text{s.c.}} \right) \) whose conformal dimension is \( \Delta - R = 2j + 5/2 \).

\[ \mathcal{N} = 2 \) long vectors in \( \mathcal{N} = 3 \) long graviton: \( \Delta - R = 2j + 2; J = j + R \)

One can think of the following combination for this multiplet \( \text{Tr} \left( \Sigma_{\text{betti}} \Phi_{\text{s.c.}} \right) \).

3) One \( \mathcal{N} = 3 \) long graviton multiplet:

\[
\frac{(1 + R)(4 + R)(5 + 2R)}{2}, \quad \Delta = \frac{1}{4} \sqrt{E + 36}, \quad 1 \leq J \leq 1 + R, \quad R \geq 0
\]

The combination of \( \Delta - R \) with Penrose limit \( R \to \infty \) in the gauge theory side becomes \( \Delta - R = 7/2 + O \left( \frac{1}{R} \right) \). By recognizing that this is a particular case of \( j = 1 \) in previous one, we can construct corresponding gauge theory operator \( \text{Tr} \left( G_\alpha \Sigma^{+(klm)}(UV)^R \right) \) corresponding to \( \mathcal{N} = 2 \) long gravitino multiplet. For \( \mathcal{N} = 2 \) long gravitino multiplet, we have the following gauge theory operator \( \text{Tr} \left( \Sigma_{\text{betti}} \Sigma^{+(klm)}(UV)^R \right) \) or \( \text{Tr} \left( T_{(\alpha \beta)} \Sigma^{+(klm)}(UV)^R \right) \) and for long gravitino of dimension 9/2, there exists an operator \( \text{Tr} \left( \Sigma_{\text{betti}} G_\alpha \Sigma^{+(klm)}(UV)^R \right) \).

4) One \( \mathcal{N} = 3 \) long gravitino multiplet:

\[
\frac{(1 + R)(4 + R)(5 + 2R)}{2}, \quad \Delta = \frac{3}{2} + \frac{1}{4} \sqrt{E + 36}, \quad 1 \leq J \leq 1 + R[R], \quad R \geq 0
\]

where it is understood that the maximum value of \( J \) of gravitino with larger conformal dimension is \( 1 + R \) while the one with smaller conformal dimension is \( R \). In this case, for smaller conformal dimension we have to take \( J = R + j - 1 \) not \( R + j \) with \( j = 1 \). With the same value of \( M_1 \) and \( M_2 \), we get the energy eigenvalue of the Laplacian on \( N^{0,1,0} \) and arrive at one higher conformal dimension. However for larger conformal dimension, we get \( \Delta - R \) from the one in (12) by setting \( j = 1 \). Combining these we get \( \Delta - R = 3 + O \left( \frac{1}{R} \right) \) and \( \Delta - R = 5 + O \left( \frac{1}{R} \right) \).

One can describe a gauge theory operator by taking a single conserved current \( \Sigma^{+(klm)}(x, \theta^\pm) \) from a semi-conserved current \( \Phi_{\text{s.c.}}(x, \theta) \) in order to match with the conformal dimension. That is, one obtains the following new gauge theory operators related to \( \mathcal{N} = 2 \) long vectors with \( \Delta - R = 3, 4 \) are given by \( \text{Tr} \left( \Sigma_{\text{betti}} \Sigma^{0(klm)}(UV)^R \right) \) and \( \text{Tr} \left( \Sigma_{\text{betti}}^2 \Sigma^{0(klm)}(UV)^R \right) \).
repectively by imposing the condition that the highest $J$ value is $J = R$ into the above operators. When $\Delta - R = 5, 6$ one can read off corresponding gauge theory objects from the expressions in $\mathcal{N} = 2$ long vectors with general $j$ by putting $j = 1$ we have discussed before. Similarly gauge theory operator related to $\mathcal{N} = 2$ long gravitinos with $\Delta - R = 7/2$ is given by $\text{Tr} \left( \sum_{\text{betti}} G^{(mpn)}_{\alpha} (UV)^R \right)$ where we also put the right isospin charge count $J = R$.

5) One $\mathcal{N} = 3$ long graviton multiplet:

$$(1 + R)^3, \quad \Delta = \frac{1}{4} \sqrt{E + 36}, \quad 0 \leq J \leq R - 1, \quad R \geq 0$$

The combination of $\Delta - R$ with Penrose limit $R \to \infty$ in the gauge theory side becomes $\Delta - R = 5/2 + O \left( \frac{1}{R} \right)$ by taking same $M_1$ and $M_2$ values with $J = R - 1$. The gauge theory operators related to $\mathcal{N} = 2$ long gravitinos with $\Delta - R = 5/2, 3, 7/2$ are given by $\text{Tr} \left( \sum_{\text{betti}} \Sigma^j G_\alpha (UV)^{R-1} \right)$, $\text{Tr} \left( \sum_{\text{betti}} \Sigma^j T_{(\alpha \beta)} (UV)^{R-1} \right)$ and $\text{Tr} \left( \sum_{\text{betti}} \Sigma^j G_\alpha (UV)^{R-1} \right)$.

6) One $\mathcal{N} = 3$ long gravitino multiplet:

$$(1 + R)^3, \quad \Delta = \pm \frac{3}{2} + \frac{1}{4} \sqrt{E + 36}, \quad 0 \leq J \leq R[R - 1], \quad R \geq 0$$

In this case, the maximum value of $J$ is $R$ for larger conformal dimension and the one for smaller conformal dimension is $R - 1$. The combination of $\Delta - R$ with Penrose limit $R \to \infty$ in the gauge theory side becomes $\Delta - R = 1 + O \left( \frac{1}{R} \right)$ and $\Delta - R = 3 + O \left( \frac{1}{R} \right)$. Note that from the isospin representation of $\Delta - R = 1$ in the $\mathcal{N} = 3$ language, the highest $J$ value is $J = R - 1$. Therefore we have to take into account this fact. This is the only difference between general $j$ case and $j = 0$ case here. So we imposed this condition for first two cases below. The new gauge theory operators related to $\mathcal{N} = 2$ long vectors with $\Delta - R = 1, 2$ with right isospin counting are $\text{Tr} \left( \sum_{\text{betti}} \Sigma^j (UV)^{R-1} \right)$ and $\text{Tr} \left( \sum_{\text{betti}} \Sigma^j (UV)^{R-1} \right)$. For $\Delta - R = 3, 4$ we get similar expressions for previous case with $j = 0$. Finally, the gauge theory operator related to $\mathcal{N} = 2$ long gravitino with $\Delta - R = 3/2$ is $\text{Tr} \left( \Sigma^j G_\alpha (UV)^{R-1} \right)$.

4 Conclusion

We described an explicit example of an $\mathcal{N} = 3$ superconformal field theory that has a subsector of the Hilbert space with enhanced $\mathcal{N} = 8$ superconformal symmetry, in the large $N$ limit from the study of $AdS_4 \times N^{0,1,0}$. The pp-wave geometry in the scaling limit produced to the maximally $\mathcal{N} = 8$ supersymmetric pp-wave solution of $AdS_4 \times S^7$. The result of this paper shares common characteristic feature of previous cases of $AdS_4 \times Q^{1,1,1}$, $AdS_4 \times M^{1,1,1}$ and $AdS_4 \times V_{5,2}$. This subsector of gauge theory is achieved by Penrose limit which constrains strictly the states of the gauge theory to those whose conformal dimension and $R$ charge diverge

\[11\] The neutral $\Sigma^{0(ijk)}$ is given in terms of chiral fields: $\Sigma^{0(ijk)} = \sqrt{2} f^{lm}(UV)^k \left( V_l U_m - V_m U_l \right)$. 

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in the large $N$ limit but possesses finite value $\Delta - R$. We predicted for the spectrum of $\Delta - R$ of the $\mathcal{N} = 3$ superconformal field theory and proposed how the excited states in the supergravity correspond to gauge theory operators. In particular, both the chiral multiplets (8) and semi-conserved multiplets (13) should combine into $\mathcal{N} = 8$ chiral multiplets. It would be interesting to find out a Penrose limit of other types [21] of M-theory compactification, along the line of [22]. These examples have different supersymmetries and the structures of four-form field strengths are more complicated than what we have discussed.

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