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To cite this version:
F. Acerbi, Valérie V. Guillard, M. Saubanère, C. Guillaume, Nathalie Gontard. Modelling CO₂ transfer in foil ripened semi-hard Swiss-type cheese. Journal of Food Engineering, Elsevier, 2018, 222, pp.73 - 83. 10.1016/j.jfoodeng.2017.10.025. hal-01645481

HAL Id: hal-01645481
https://hal.archives-ouvertes.fr/hal-01645481
Submitted on 26 May 2020

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Accepted Manuscript

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PII: S0260-8774(17)30458-2
DOI: 10.1016/j.jfoodeng.2017.10.025
Reference: JFOE 9058

To appear in: Journal of Food Engineering

Received Date: 30 May 2017
Revised Date: 27 October 2017
Accepted Date: 28 October 2017

Please cite this article as: Acerbi, F., Guillard, V., Saubanere, M., Guillaume, C., Gontard, N., Modelling CO₂ transfer in foil ripened semi-hard Swiss-type cheese, Journal of Food Engineering (2017), doi: 10.1016/j.jfoodeng.2017.10.025.

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Modelling $\text{CO}_2$ transfer in foil ripened semi-hard Swiss-type cheese

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Abstract

Eye growth in cheese with intense CO$_2$ production during ripening mainly depends on CO$_2$ production and transfer properties. Despite CO$_2$ production and diffusion during ripening of semi-hard Swiss-type cheese eyes in such cheeses are considered as important quality parameters, the research concerning key gas production and transfer in cheese remains widely overlooked. In this study, experimentally assessed CO$_2$ production was coupled with transfer coefficients in a mathematical model in order to predict CO$_2$ gradients formed inside the cheese during ripening. The permeability coefficient of CO$_2$ through the multilayer barrier packaging which wraps the cheese during ripening was also included in the model. The presented model was validated by assessing the CO$_2$ concentration in the cheese and its partial pressure in the packaging headspace. CO$_2$ production rate was found to be the most important input parameter affecting CO$_2$ gradients formed in cheese during ripening whereas the other input parameters (solubility, diffusivity, permeability) had little effect on the total CO$_2$ gradient.

Keywords: solubility, diffusivity, permeability, CO$_2$, cheese ripening
Nomenclature

\( \alpha \)  Level of statistical significance
\( A \)  Surface (m\(^2\))
\( Bi \)  Biot number (non-dimensional)
\( br \)  Bottom rind position of the cheese
\( C \)  Concentration (kg/m\(^3\))
\( ch \)  Relative to cheese
\( D \)  Effective diffusivity coefficient of \( \text{CO}_2 \) in cheese (m\(^2\) s\(^{-1}\))
\( exp \)  Experimental values
\( HS \)  Packaging headspace
\( I \)  Relative to the interface
\( j \)  Relative to a gas specie
\( k \)  External mass transfer coefficient (m/s)
\( K \)  Degree in Kelvin, with \( K = 273.15 + \) degree in Celsius
\( KP \)  Gas permeability through the packaging film (cm\(^3\) \( \mu \)m\(^{-1}\) m\(^{-2}\) d\(^{-1}\) bar\(^{-1}\))
\( l \)  Cheese thickness (m)
\( M \)  Molar mass (kg/mol)
\( p \)  Partial pressure (Pa)
\( P_T \)  Total pressure (atm)
\( Pe \)  Gas permeability through the packaging film (mol m\(^{-1}\) s\(^{-1}\) Pa\(^{-1}\))
\( PAB \)  Propionic Acid Bacteria
\( \text{pred} \)  Predicted values
\( \text{PTATN} \)  ratio of phosphotungstic acid soluble nitrogen on total nitrogen (g/100g)
\( S \) or \( \text{SCO}_2 \)  Solubility coefficient of \( \text{CO}_2 \) in cheese (mol m\(^{-3}\) Pa\(^{-1}\)) or mmol kg\(^{-1}\) atm\(^{-1}\)) or \( \text{CO}_2 \) solubility
\( \text{SM} \)  Salt in moisture ratio (g/100g)
\( t \)  Time (s)
\( T \)  Temperature (K)
\( R \)  Universal gas constant: 8.314 (J mol\(^{-1}\) K\(^{-1}\))
\( \text{CVRMSD} \)  Coefficient of variation of the root mean squared deviation
\( V \)  Volume (m\(^3\))
\( x \)  Position in the cheese (m)
\( y \)  Experimental or predicted variable
\( \mu \)  average
\( \varphi \)  Mass flow (kg s\(^{-1}\))
\( \nu \)  Production rate (kg m\(^{-3}\) s\(^{-1}\))
\( \infty \)  Relative to the surrounding atmosphere
1. Introduction

In food engineering, mathematical modelling of physical mechanisms such as heat or mass transfer was used for many years to simulate and optimize/control operation units such as drying or salting (Bona et al., 2007; Mayor and Sereno, 2004; Payne and Morison, 1999; Santapaola et al., 2013). More recently these models were coupled to biological ones such as Mickaëlis Menten equation that formalize the aerobic respiration (Ho et al., 2011; Guillard et al., 2012), chemical reaction for oxidation (Bacigalupi et al., 2013) or predictive microbiology models in order to better represent the evolution of the packed food during storage (Chaix et al., 2015).

This modelling approach coupling equations coming from different disciplinary fields was recently applied to semi-hard Swiss-type cheese in order to model the eyes’ growth during ripening (Laridon, 2014; Laridon et al., 2016). CO$_2$-source in this type of cheese is propionic acid bacteria (PAB) which means that CO$_2$-production rate and amount in such cheese is much higher compared to that of lactic acid bacteria. The ripening of these cheeses includes a step during which cheeses are stored at warm temperature (about 1-2 weeks at 20-25°C) for stimulating the PAB fermentation. During this step, CO$_2$ is intensively produced, leading to the growth of eyes (Fröhlich-Wyder and Bachmann, 2004). Cheeses are often foil ripened and a plastic packaging is used to wrap the cheeses during the whole ripening period for avoiding excessive loss of gaseous compounds. The modelling approach proposed by Laridon (2014) therefore concomitantly took into account mass transfer of gas (CO$_2$ produced by bacteria and responsible of eyes’ growth), production of the CO$_2$ and mechanical constraint imposed to cheese paste by this production. This model was based on experimentally assessed rheological parameters (stress) and CO$_2$ production rate in simplified condition, but some input parameters such as CO$_2$ diffusivity and solubility into the cheese were estimated from the literature and assumed constant throughout the cheese paste and the ripening age. The sensitivity analysis performed by these authors revealed...
afterwards the higher influence of CO$_2$ production and diffusion parameters compared to the rheological ones in semi-hard cheese (Laridon, 2014).

Faced to the importance of CO$_2$ diffusion and CO$_2$ production rate in the ripening of semi-hard Swiss-type cheese, this paper aimed at deepening these two phenomena by coupling them in a mathematical model in order to simulate and predict evolution with time of CO$_2$ gradients in the cheese paste and in the packed cheese (including CO$_2$ permeation through the ripening foil). In this purpose, the aim of this study was to develop and experimentally validate in various conditions, mimicking the ripening conditions of Swiss-type cheese, a mathematical diffusion-reaction model to predict CO$_2$ gradients in packed cheeses during ripening.

2. Materials and methods

2.1 Cheese and packaging

Semi-hard cheese blocks of 1 kg were kindly supplied by a cheese company. The cheeses included about 42% w/w moisture and 26.5% w/w fat, with pH varying from 5.50 to 5.70 from 14 to 28 days from renneting. The cheeses used for the model validation without CO$_2$ production (same batch) did not include any intentionally added CO$_2$ producing bacteria (PAB and hetero-fermentative lactic acid bacteria or LAB) and was produced from pasteurized milk to avoid any sources of hetero-fermentative LAB or PAB. The cheeses used for the validation with CO$_2$ production included PAB (10$^6$ CFU ml$^{-1}$ cheese milk) as main source of CO$_2$ production. They were produced from a second, dedicated batch from pasteurized milk. After brining, all cheeses were packed in a multilayer barrier packaging film made of PE/PVDC/PE. Indicatively, the permeability to CO$_2$ at 19°C equalled 40000 cm$^3$ µm $^{-2}$ d$^{-1}$ bar$^{-1}$ (Acerbi et al., 2016c). The ripening schedule of all ripened cheeses was as follows: 13°C until 14 days from renneting, 19°C
for 14 days, 2–3 days at 13°C, longer storage at 6°C. The target chemical composition of cheeses used in this study is described in Table 1.

Cheese blocks were about 15 to 8 cm square in shape (see Supplementary Material A) and were sampled in their core, orthogonally to the interface exposed to headspace/surrounding atmosphere (Figure 1), at least 3 cm away from side rinds, resulting in a cylinder of 8 cm height and about 2 cm of diameter. The sampling region was then cut in thin slices of minimum 0.5 cm of thickness for assessment of chemical composition gradient or CO$_2$ gradient.

Table 1

2.2 Chemical analyses

The chemical composition (fat, moisture, pH, total nitrogen (TN) and fractions of TN, salt, organic acids) of three cheeses per production was measured, in order to verify that the cheese production was on target. Dry matter and sodium chloride were measured according to the ISO 5534 (Anonymous, 2004a) and ISO 5943 (Anonymous, 2006) standards. Total nitrogen (TN), water soluble nitrogen (SN) and phosphotungstic acid soluble nitrogen (PTA-N) were measured using Kjeldahl based methods according to ISO 8968-3 standard (Anonymous, 2004b; Bütikofer et al., 1993). The ratio of PTA-N on TN (g/100g) was considered as a good proteolytic indicator. pH and fat content were characterized according to respectively, FD V04-035 (Anonymous, 2009) and NF V04-287 (Anonymous, 2002) French standards. Organics acids were assayed using internal method based on high-performance liquid chromatography.

CO$_2$ determination in cheese was carried out with the protocol described in Acerbi et al. (2016b). The protocol included a first step where CO$_2$ was extracted from the cheese sample by immersing it in acidic solution. The extracted CO$_2$ was then scavenged by a soda solution of known
molarity, which was finally titrated with hydrochloric acid. The difference between the initial and final soda molarity related to the moles which reacted with CO₂.

2.3 Microbiology analyses

Propionic acid bacteria were enumerated with a method based on the count of the diluted colonies grown on agar plates enriched mainly with sodium lactate and yeast extract after 1 week of incubation at 30°C. The results were expressed in colony-forming unit per gram of cheese (CFU g⁻¹).

2.4 Gas chromatography for headspace analyses

The partial pressure of N₂, O₂ and CO₂ was measured (resolution: 0.005%) by injecting 10 µl of headspace of cheese packs inside a micro gas chromatography unit including a thermal conductivity detector (MicroGC 3000, SRA Instruments). Homogeneity of the gas composition was ensured by pumping few millilitres of sample gas inside the micro gas chromatography unit before each measurement. The cheese packs were not kept after the analyses because of the too low amount of headspace volume. Therefore, new cheese packs produced from the same batch were used for each analysis in order to follow the kinetics of headspace composition during ripening. The gas chromatography unit was previously calibrated with gas bottles of known compositions. At least two measurements were carried out per each sample.

2.5 Description of the model

Evolution of CO₂ partial pressures in the foil packed blind cheese during ripening relies on the interplay of four mechanisms: (1) CO₂ production due to the activity of microorganisms (PAB), (2) gas transport within the cheese paste, (3) gas transfer at the cheese rind/headspace interface and (4) gas transmission through the packaging film. The four described mechanisms are illustrated in figure 1.
2.5.1 Model assumptions

The following assumptions were made in the present mono-directional modelling study:

- The composition of the atmosphere surrounding the packed cheese during ripening is constant and equals to 78.1% for $N_2$, 20.9% for $O_2$ and 0.03% for $CO_2$ (Widory and Javoy, 2003).

- Temperature in the packed cheese system and surrounding atmosphere is constant, without gradients.

- Total pressure of the system is constant and it equals atmospheric pressure (101325 Pa).

- The solubilization of $N_2$ inside the cheese was considered negligible compared to $CO_2$, because of its lower solubility in water (about 50 times less soluble than $CO_2$ in water at 20°C) (Dean, 1999). $O_2$ solubility and diffusivity in the cheese were set to constant values of $2.5 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$ and 1.3 mmol kg$^{-1}$ atm$^{-1}$ respectively and which correspond to solubility and diffusivity of the gas in water at 20°C (Chaix et al., 2014).

- The mechanical resistance to deformation of the cheese packaging is not taken into account in the model. Thus, volume changes according to moles content inside the pack and total pressure of headspace remains equal to external atmospheric pressure. Measuring volume changes during experiment has revealed that they are negligible and that the flexible pouch could easily compensate the small volume change due to its property of elongation.
- For the cheese with salt gradient, the initial NaCl gradient was measured in cheeses produced within the same production day and it was considered to remain constant during the experiment. Same assumption was used to quantify the proteolytic indicator (PTATN).

- Gradients in chemical composition in the cheese from its upper rind to its core are symmetric to the gradients from lower rind to the core.

2.5.2 Gas balance in headspace

The variation of concentration of the gas species $j$ in the packaging headspace $HS$ depends on the mass flow of gas permeating the packaging film $f$ from the surrounding atmosphere $\varphi_{j_f}$ (kg/s) and the mass flow of the gas species occurring at the interface between the cheese and the headspace $\varphi_{j_1}$.

Assuming that the packaging film does not oppose any mechanical resistance to deformation, when the amount of gas species in the headspace varies, the headspace volume $V_{HS}$ (m$^3$) changes, whereas the total pressure in the headspace $P_T$ remains constant and equal to atmospheric pressure (101325 Pa). The global mass balance of species $j$ in the headspace can hence be written as follows:

$$V_{HS} \frac{dC_{jHS}}{dt} + C_{jHS} \frac{dV_{HS}}{dt} = \varphi_{j_f} + \varphi_{j_1} \quad \text{(Eq. 1)}$$

where $C_{jHS}$ (kg/m$^3$) is the concentration of the gas in the headspace and $t$ is time (s).

Assuming that the gas mixture in the headspace obeys the ideal gas law, $\frac{dV_{HS}}{dt}$ can be calculated as a pondered sum of the different aforementioned mass flow $\varphi_{j_f'}$, $\varphi_{j_1}$ as follows:

$$\frac{dV_{HS}}{dt} = \frac{RT}{P_T} \left( \frac{\varphi_{CO_2 f} + \varphi_{CO_2_1}}{M_{CO_2}} + \frac{\varphi_{O_2 f} + \varphi_{O_2_1}}{M_{O_2}} + \frac{\varphi_{N_2 f}}{M_{N_2}} \right) \quad \text{(Eq. 2)}$$
where $M_f$ is the gas molar mass (kg/mol), $R$ the universal gas constant (J mol$^{-1}$ K$^{-1}$) and $T$ the temperature in Kelvin (K).

The concentration of the gas in the headspace $c_{j,HS}$ can be calculated according to (Eq. 3)

$$RT \sum_{j=O_2,CO_2,N_2}^{} \frac{c_{j,HS}}{M_j} = P_f = 10^5 Pa$$  (Eq. 3)

and similarly, for the partial pressure of the gas in the headspace $p_{j,HS}$ (Eq. 4),

$$p_{j,HS} = \frac{c_{j,HS}RT}{M_j}$$  (Eq. 4)

2.5.3 Gas permeation through the maturation foil

The gas permeation through the packaging film (maturation foil), assuming that the partial pressures in the atmosphere surrounding the cheese are constant, could be conveniently described by the Fick’s first law (steady state regime). Therefore, the mass flow $\phi_f$ of gases $j$ (N$_2$, O$_2$, CO$_2$) through the packaging film was calculated according to first Fick’s law (Fick, 1855) as stated in Eq. 5.

$$\phi_f = M_j Pe_j \frac{A_f}{l_f} (p_{j,\infty} - p_{j,HS})$$  (Eq. 5)

where $Pe_j$ is the permeability coefficient of the gas $j$ through the film (mol m$^{-1}$ s$^{-1}$ Pa$^{-1}$), $l_f$ is the thickness of the film (m), $A_f$ is the film surface (m$^2$) and $p_{j,\infty}$ and $p_{j,HS}$ are the partial pressure of the gas $j$ in the surrounding atmosphere (denoted by symbol $\infty$) and the packaging headspace, denoted by $HS$.

2.5.4 Mass flow at the cheese rind/gaseous interface

The mass flow at the cheese/rind gaseous interface was calculated as follows (Eq. 6):

$$\phi_j = k_f(\frac{M_jA_f}{RT})(p_{j,HS,i} - p_{j,HS})$$  (Eq. 6)

Where $k$(m/s) is the mass transfer coefficient at the interface between the cheese rind and the headspace and $A_f$ and $p_{j,HS,i}$ are the surface at the interface and the partial pressure at the
immediate vicinity of the cheese surface. The mass transfer coefficient $k$ is calculated according to Eq. 7

$$k = \frac{Bi \times D_j}{L}$$

(Eq. 7)

Where $Bi$ is the non-dimensional Biot number, assumed to be equal to $10^5$ (ratio between diffusivity of CO$_2$ in air and cheese) (Laridon, 2014), $D_j$ is the diffusivity of CO$_2$ in cheese (about $10^{-10}$ m$^2$ s$^{-1}$) and $L$ is the characteristic length of the considered material (length of the cheese equalled about 0.008m).

Assuming that the gaseous species in headspace are in thermodynamic equilibrium with the dissolved species at the cheese surface, the concentration of dissolved gas species $j$ at the cheese surface $C_{j, ch, x=0}$ (kg/m$^3$) relates to $p_{j, HS, i}$ according to Henry’s law:

$$p_{j, HS, i} = \frac{C_{j, ch, x=0}}{M_j S_j}$$

(Eq. 8)

Where $S_j$ is the solubility coefficient of the considered gas $j$ (mol m$^{-3}$ Pa$^{-1}$).

2.5.5 Gas diffusion inside the cheese paste and CO$_2$ production

The CO$_2$ diffusion within the cheese paste was represented by making use of Fick’s second law of diffusion for the transient state of diffusion and it was coupled to CO$_2$ production by adding production term $\nu$ (kg m$^{-3}$ s$^{-1}$) to Fick’s second law, leading to Eq. 9.

$$\frac{\partial C_{CO_2, ch}}{\partial t} = D_{CO_2} \frac{\partial^2 C_{CO_2, ch}}{\partial x^2} + \nu_{CO_2}$$

(Eq. 9)

Where $\frac{\partial C_{CO_2, ch}}{\partial t}$ is the partial derivative of the concentration of CO$_2$ in the cheese as regard to time, in the direction $x$ (m). $x$ represents the distance from the interface of the cheese rind to headspace and the considered position inside the cheese. $x$ varies from 0 to 8 (cheese thickness in cm). $D_{CO_2}$ is the effective diffusivity coefficient of CO$_2$ in cheese (m$^2$/s).
The same equation without the production term was used to model O$_2$ diffusion within the cheese paste.

### 2.5.6 Boundary conditions

Equality of flows was considered at the interface between the cheese rind and the headspace:

$$D_j \frac{\partial c_{ij}}{\partial x} = \frac{\varphi_i}{A_i} = k \left( \frac{M_j}{RT} \right) (p_{jHS} - p_{jHS})$$  at  $x = 0$  

(Eq. 10)

where $j$ stands for either CO$_2$ or O$_2$.

Since the cheese is pressed on continuous impermeable material during ripening due to gravity, we assumed no diffusion at the cheese bottom rind ($b_r$):  

$$D_j \frac{\partial c_{ij}}{\partial x} = 0$$  at  $x = b_r$  

(Eq. 11)

Assuming that the food sample was initially in equilibrium with a gas of fixed partial pressure $p_{j_0}$ (Pa), the initial conditions take the form:

$$c_{ij}(x, 0) = M_j \varphi_j (x, 0)$$  for  $t = 0$  

(Eq. 12)

### 2.5.7 Prediction of model input parameters for different ripening conditions

The following empirical equation was used in the model for predicting the changes in CO$_2$ production rate, $v_{CO2}$ (mmol kg$^{-1}$ d$^{-1}$) as a function of salt to moisture ratio, SM and temperature, $T$ (Acerbi et al., 2016a):

$$v_{CO2} = 8.527 + 0.0862 \ T^2 - 1.621 \ T + 5.367 \ SM - 0.415 \ SM \ T$$  

(Eq. 13)

SM (g/100g) is calculated as the percentage of the mass ratio between sodium chloride and moisture.

The following empirical equation was used in the model for predicting the value of $D_{CO2}$ at different temperatures, $T$, salt to moisture ratio, $SM$ and $PTATN$ (ratio of phosphotungstic acid soluble nitrogen on total nitrogen). The latter coefficient is a ripening indicator related to the...
proteolytic phenomena (especially amount of free amino acids) which relates to structure changes in the cheese during ripening (Acerbi et al., 2016d).

\[
D_{CO_2} = (-0.0428 T^2 + 1.3689 T - 0.6188 SM + 12.2421 PTATN - 9.6910) \times 10^{-10}
\]

(Eq. 14)

The value of \( S_{CO_2} \) (in mmol kg\(^{-1}\) atm\(^{-1}\)) was calculated by using a modified version of the equation given by Acerbi et al. (2016b):

\[
S_{CO_2} = 37.35879 - 0.83414 SM^2 + 4.4619 SM - 0.34768 T
\]

(Eq. 15)

The modification consisted in the use of a quadratic term for the salt effect instead of a linear term in the view of more recent results (figure 2).

**Figure 2**

The value of permeability \((KP)\) of the gas \( j \) (O\(_2\), CO\(_2\), N\(_2\)) was estimated at different ripening temperatures by interpolating the results of permeability coefficients \( KP \) given by Acerbi et al. (2016c) in units cm\(^3\) µm\(^{-2}\) d\(^{-1}\) bar\(^{-1}\). The \( KP \) values were converted into \( Pe \) values (mol m\(^{-1}\) s\(^{-1}\) Pa\(^{-1}\)) to be used in the model (Eq. 5).

\[
KP_{CO_2} = -108.03 T^2 + 4551.5 T - 3242.6
\]

(Eq. 16)

\[
KP_{O_2} = 386.53 T + 6313.6
\]

(Eq. 17)

\[
KP_{N_2} = 188.78 T + 3422
\]

(Eq. 18)

**2.5.8 Numerical solving and identification procedure**

This system of equations governing the coupled diffusion and production of gas in the cheese and the gas transfer and permeation in the cheese packaging headspace was solved using a dedicated algorithm “ode15s” developed in Matlab® computing software (The Mathworks Inc., Natick,
Mass, USA) and adapted to stiff systems where each of unknown variables may exhibit radically different variation kinetics. This algorithm adjusted automatically the size of the time step used for numerical integration of the equations. To evaluate goodness of fit between experimental and predicted data, the coefficient of variation of the root mean square deviation (CVRMSD) was calculated as follows:

$$\text{CVRMSD} = \left( \frac{\sqrt{\frac{\sum(y_{\text{pred}}-y_{\text{exp}})^2}{N^2}}}{{\mu_{\text{exp}}}} \right) \times 100$$  
(Eq. 19)

Where $y_{\text{pred}}$ and $y_{\text{exp}}$ are respectively the experimental and the simulated residual value, $N$ is the number of experimental measurements and $\mu_{\text{exp}}$ is the average of the experimental values. CVRMSD enables to compare the quality of the model with other models present in literature. A good quantitative model is considered to have $\text{CVRMSD} \leq 10\%$.

### 2.6 Model validation

The experimental validation was made in three steps of increasing complexity which are summarized in Table 1.

1. Firstly, the mono-directional CO$_2$ diffusion within the cheese and transfer at the cheese rind interface was validated on an old ripened high salted cheese (namely “old cheese”) with no salt gradient and without CO$_2$ production (no CO$_2$ producing bacteria). The cheese was high salted via brining (about 5% NaCl/dm, salt content on dry matter) in order to avoid possible gas production from unwanted microorganisms in such old cheese. The cheese, apart for its upper rind, was fully covered with a gas impermeable membrane (see Supplementary Material A) and it was housed in a controlled temperature cabinet (set at 19°C) with a continuous flow of wet CO$_2$ (gas supposed to be equilibrated at 100% of relative humidity after bubbling in a flask containing pure water at the temperature of the experiment). After 3 days, the CO$_2$ dissolved in different positions (e.g. the sample is sliced parallel to the interface) from the upper rind ($x=0$) to the
bottom rind (x=8) was assessed with the CO₂ determination method described above. Two repetitions were made in order to validate the diffusion phenomenon. For two other repetitions, after 3 days of CO₂ sorption, the cheese samples were then kept 3 days in a continuous flow of wet (100% RH) N₂ (desorption step). The gradient of dissolved CO₂ into the paste after these 3 days of desorption was then assayed as previously described for the sorption step (2 repetitions).

Secondly the coupling of CO₂ diffusion and production was experimentally assessed by using a cheese with PAB and average salt concentration (about 2.5% NaCl/dm, salt on dry matter, in the cheese, namely “young cheese”). This cheese also included a salt gradient due to its younger age (15 days from renneting) when salt is still slowly diffusing from rind to core of the cheese (brined cheese). The same experimental procedure previously described was used, apart for the following: the cheese stayed 1 day at 19°C, before the experiment with the contact of wet CO₂ from upper rind (x=0) started and it lasted for 4.4 days.

The initial CO₂ concentration gradient in the cheese was measured just before the experiment started (i.e. about 9.5 weeks from renneting for the first validation step and 15 days from renneting for the second more complex validation step including CO₂ production). This initial CO₂ gradient is indispensable to parametrize initial conditions in the numerical algorithm.

Thirdly, in order to compare the output of gas concentration in the headspace given by the model including permeation phenomenon, gas headspace composition was measured in the headspace of packed cheeses (2 replications). The headspace gas analyses were repeated on different cheeses during ripening (about at 10, 14, 16 ripening days at 19°C).

In each case of these aforementioned steps, the model was adapted accordingly to experimental conditions: system of Eq. (6) to (15) for the first and second steps - without the production term in Eq. (9) and related Eq. (13) – and system of Eq. (1) to Eq. (18) for the third, full case.
2.7. Statistics

Statistical tests were performed by using R software for statistical computing (R, 2014). Comparisons between the chemical compositions were performed by pairwise comparisons using $t$ test with pooled standard deviation. The Holm method for the multiplicity of error was used to adjust the p-value of multiple pairwise comparisons. Different letters were used for denoting significant difference between data sets (level of significance $\alpha = 0.05$, unless stated).

3. Results and discussion

Our model takes into account experimentally assessed parameters for describing CO$_2$ production and overall transfer in and out a cheese/packaging system during foil ripening. The focus was on the exhaustive description of the gas transport properties in the continuous cheese paste and at its interface with the surrounding atmosphere. We therefore used a model blind cheese (without eyes). We experimentally assessed the following parameters: transport of CO$_2$ in the cheese paste (effective diffusivity), transfer of CO$_2$ at the interface between cheese and headspace (solubility), oxygen, nitrogen and carbon dioxide transmission rate through the maturation foil (permeability) and CO$_2$ production rate from the bacteria. The cited parameters were previously assessed in function of the main ripening variables (different temperatures, ripening time, salt content amongst others) (Acerbi et al., 2016a, 2016b, 2016c, 2016d).

3.1 Compositional analyses

The chemical composition of the cheese without PAB (old cheese – data not shown) was not significantly different ($\alpha=0.05$) as a function of the position in the cheese, whereas a relevant gradient in PTATN (ratio of phosphotungstic acid soluble nitrogen on total nitrogen) and SM

17
(salt in moisture ratio) was confirmed from analyses made on cheese with PAB analysed at 15
and 20 days from renneting (figure 3).

Although the transfer properties of nitrogen components (PTATN) and salt were not
experimentally assessed within this study and not included in the model, the initial heterogeneous
composition was considered and a linear interpolation was used for predicting the initial
composition in all positions of the cheese including PAB. The composition in the position 4 to 8
cm was considered symmetrical to the position 0 to 4 cm.

Figure 3

A relevant gradient in organic acid concentration was found in the cheese with PAB both at 15
and 20 days from renneting (figure 4). No propionate and acetate was found in the outer rind (0
cm) at 15 days from renneting. The observed delay in PAB fermentation in the outer part of the
cheese was probably due to the negative effect of high salt to moisture ratio on the bacteria cells
(Guinee, 2004) and it was already demonstrated by Huc et al. (2014) in similar cheeses. The
molar ratio between propionate and acetate in the core of the cheese equalled about 2. The ratio
of 2 is obtained if PAB convert lactate exclusively via the Fitz pathway (Fröhlich-Wyder and
Bachmann, 2004; Piveteau, 1999; Fedio et al., 1994). We can therefore confirm that, in the
studied cheese, PAB fermented lactate primarily via the Fitz pathway and that probably no other
metabolic activity (producing acetate) occurred.

Figure 4
The pH of the cheese was measured because pH changes may affect both the solubility of carbon dioxide dissolved in the water phase of the cheese (Chaix et al., 2014) and its diffusivity coefficient due to a different organisation of the casein matrix structure (Lawrence et al., 1987). The pH in the studied cheese did not relevantly vary in the considered experimental time (figure 5a).

The enumeration of PAB cells confirmed the higher metabolic activity observed in the core of the cheese (figure 5b).

**Figure 5**

### 3.2 Initial CO\(_2\) concentration

The initial CO\(_2\) concentration in different positions (from upper to bottom rind) of the cheese was measured in 2 to 3 cheeses produced within the same production batch. Whereas the CO\(_2\) distribution was low and rather homogeneous for the old ripened cheese with no intentionally added CO\(_2\) producing bacteria (figure 6a, 3 repetitions), the initial CO\(_2\) gradient was relevant in the cheese with PAB (figure 6b, 2 repetitions). The low and homogeneous initial CO\(_2\) concentration observed in the older cheese was expected because this cheese did not contain intentionally added CO\(_2\) producing bacteria, while the CO\(_2\) gradient observed in the cheese with PAB at 15 days from renneting (14 days at 13°C and 1 days at 19°C) reflected the effect of salt gradient on PAB. There is an inhibition of PAB growth (Figure 5b) by the higher salt content close to the rinds with a consequently lower production of CO\(_2\) and propionic acid close to the rinds compare to the core of the product leading to this “bell like curve” for initial CO\(_2\) gradient in salted cheese with PAB. This evolution of salt content that is spatially-dependant, due to diffusion of salt from the periphery towards the centre of cheeses, has been found in previous
studies (Huc et al., 2014; Guinee, 2004; Hollywood & Doelle, 1984) and before by Mocquot (1979), Geurts et al. (1974, 1980) and Pauchard et al. (1980). Therefore, the median value for initial CO₂ concentration was used (4 mmol/kg) for the model validated for the cheese without PAB and the real gradient was used as input vector for initial CO₂ concentration for the cheese with PAB.

3.3. Investigating effect of salt content on CO₂ solubility

Prior model validation, additional points of solubility of CO₂ within the cheese paste were measured to complete the work of Acerbi et al. (2016b) and obtain more precise $S_{CO2}$ prediction (eq. 15). According to these new values (Figure 2), $S_{CO2}$ increased with light NaCl content, while it decreased at very high NaCl content, following a similar behaviour compared to salting in and salting out effect of protein. This phenomenon was ascribed to the possibly higher sorption of CO₂ in highly hydrated protein during salting in phase (Guinee, 2004), while the overall $S_{CO2}$ would decrease due to lower level of protein hydration at high salt concentration (salting out).

Guinee (2004) suggested that salting out effect of protein in cheese (Mozzarella) may take place for salt to moisture ratio (S/M) higher than 6.3% w/w, which is close to the highest salt level investigated in Figure 2 (about 6.7% S/M).

3.4 Model validation

The simplest model form describing CO₂ diffusion inside the cheese paste (without PAB and CO₂ production) and the transfer at the cheese rind/gaseous interface (100% CO₂) was successfully validated because the difference between predicted and experimental data was below 10% (figure...
7a). Same good validation with a 100% N\textsubscript{2} flux at the cheese rind surface was obtained (CVRMSD < 8%) (see Supplementary Material B).

When increasing the complexity of the model, including CO\textsubscript{2} production rate and considering a cheese with salt gradient, the observed error between experimental and predicted data was found higher (30%) (figure 7b). In the latter more complex model, the predicted line was generally underestimating the experimental gradient, leading to a high CVRMSD. The lack of fit of the more complex model could be due to (1) the adoption of a mono-directional model which may not be fully appropriate for diffusion in cheeses with gas production and/or (2) underestimations in either the prediction of CO\textsubscript{2} production rate and/or in the initial CO\textsubscript{2} concentration gradient.

Underestimation of the CO\textsubscript{2} production rate may be due to the linear approximation used by Acerbi et al. (2016a) to describe the effect of salt content. Indeed, lower salt contents were investigated in the previous work, which provided the predictive equation for CO\textsubscript{2} production rate, compared to the current study. Therefore, effect of high SM ratio is probably not well predicted by Eq.13 which was never validated for such SM ratio. This underestimation may be overcome by carrying out more experimental measurements of CO\textsubscript{2} production rate at higher salt content. A possible underestimation of the initial CO\textsubscript{2} concentration gradient in the cheese may be ascribed to natural deviations in metabolic activity of the PAB in different cheeses produced from the same batch. Both hypotheses were confirmed by simulating a 1.7 folds higher CO\textsubscript{2} production rate (figure 8) and then, a 2 folds higher initial CO\textsubscript{2} concentration (figure 9). In both cases, the predicted curve fit very well the experimental data.

Figure 7

Figure 8
Then, the full model considering permeation through ripening foil in addition to solubilisation/diffusion and production of CO\textsubscript{2} within the food was then used to predict the CO\textsubscript{2} partial pressure in headspace. A third set of experiments conducted on cheeses packed in ripening foils were conducted in the objective to validate it. Experimental difficulties arose because gas headspace analysis could not be assessed continuously in the pack (because of too high gas volume injected) therefore only a few experimental points have been collected (See supplementary material C). However, it has been noted that the model tended to underestimate the headspace partial pressure when CO\textsubscript{2} production occurs (in cheese with PAB). But, once again, considering a 1.7 fold higher CO\textsubscript{2} production rate, the prediction fitted better the experimental data. A better fit may also be obtained by reducing the value for CO\textsubscript{2} permeability of the packaging or increasing the initial CO\textsubscript{2} concentration gradient (results not shown). But this hypothesis does not sound well founded and therefore was not applied. The permeability values were especially assessed in the conditions encountered in the present work and were therefore considered relevant.

### 3.5 Exploratory analysis of model simulations

Simulations were carried by considering the min and max values of 4 parameters in their range of variation (table 2): CO\textsubscript{2} solubility (figure 10a), diffusivity (figure 10b), permeability (figure 10c) and production coefficients (figure 10d). During the simulation, only one of the mentioned input parameters was varied from the lowest to the highest value described in Table 2. The boundary...
conditions used during these simulations included that the packed cheese was in contact with the atmosphere (20.9% O₂, 0.03% CO₂, 78.1% N₂) from one rind (upper rind) and in contact with a non-permeable support from the below rind (shelf). The range of values for solubility and diffusivity was chosen in agreement with the min and max observed in the experimental campaign described by Acerbi et al. (2016b, 2016d). The values for permeability were decided to vary of factor 10 of the predicted value of Equation 16, 17, 18. The value of production rate was decided to vary of 4 units compared to the predicted value of equation 13, because it was considered a realistic variation for the different salt contents observed in different positions of the studied cheese at 19°C.

Table 2

The other input parameters were kept fixed (medium value) as respect of their position. This simplified sensitivity analysis had the goal of highlighting which input parameter had the strongest effect on the output (CO₂ gradients formed in cheese). Logically, changing S_{CO₂} led to light differences in CO₂ concentration only close to the gaseous interface (figure 10a) because this parameter only intervenes in the boundary condition at cheese/headspace interface: the lower the solubility, the lower the CO₂ content at the interface. Changing D_{CO₂} slightly affected the shape of the CO₂ gradient close to the cheese rinds, probably because these positions were characterized by lower v_{CO₂} due to the higher salt content (figure 10b) and it affected the overall shape of the curve. v_{CO₂} showed the highest effect on the CO₂ gradients, from -10 to +20 mmol/kg of difference for the lowest and highest v_{CO₂} respectively compared to the median value in the core of the cheese. Concerning the effect of different permeability, the lower the gas
permeability of the packaging, the higher was the CO₂ concentration at the cheese rind, but the overall change in CO₂ gradient due to different permeability was negligible.

Figure 10

This paper presents an unprecedented modelling approach that successfully describes the mechanisms of CO₂ diffusion, solubilisation and production by Swiss-type cheese with intensive PAB-based CO₂ production and also CO₂ permeation through the ripening foil. All mechanisms were dynamically coupled and experimentally validated permitting to achieve the initial objective of this paper which was to predict evolution with time of CO₂ gradients into the packed cheese. The developed model is the most complete one compared to precedent similar approach of the literature. For instance, the one of Jakobsen and Risbo, (2009), developed for prediction, among others, of the changes in solubilised carbon dioxide in semi-hard cheese packed in modified atmosphere packaging, neglected all mechanisms of CO₂ diffusion and production in their approach.

This model could be further used to predict CO₂ gradients into blind cheese or, once coupled with a mechanical model of bubble growth such as the one proposed by Laridon et al., (2014), could be used to predict eye growth in Swiss type cheese and then pilot the ripening step.

4. Conclusion

We presented the first experimentally validated model which couples the phenomena of CO₂ production, solubilisation/diffusion and permeation in a packed cheese system for predicting the CO₂ gradients formed in the cheese during ripening. A three steps validation procedure enabled to build a robust model for a quantitative description of CO₂ gradient formed in a cheese with or
without CO$_2$ production by PAB and including the phenomena of CO$_2$ diffusion in the cheese paste and transfer at the gaseous interface. The model was able to describe the shape of the CO$_2$ gradient formed in the cheese with, nevertheless, less precision when CO$_2$ production happens. This term was probably less accurately characterized for the conditions used in this paper and induces less performant prediction. A simplified sensitivity analysis highlighted CO$_2$ production as the most important input parameter affecting the CO$_2$ gradients formed in cheese during ripening. Results presented in this paper represent a solid basis for the description of the most important phenomena affecting the quality of cheese with intense CO$_2$ production during ripening.
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Figure 1: Scheme of the simplified food / packaging system with the four phenomena considered (mono-directional transfer through the headspace/cheese interface, \( br = \)cheese bottom-rind)

Figure 2. \( \text{CO}_2 \) solubility in semi-hard Swiss-type cheese as a function of salt content at 13°C. The point at intermediate NaCl content (1.5% w/w) is added to the ones reported by Acerbi et al. (2016b) and it was measured with same protocol on the same cheese. Different letters a, b, c, …denote significant difference for \( \alpha = 0.05 \).

Figure 3. (a) PTATN - ratio of phosphotungstic acid soluble nitrogen on total nitrogen and (b) SM - salt in moisture ratio- in different positions of the of the semi-hard Swiss-type cheese with PAB, from rind (0 cm) to core (4 cm) at 15 and 20 days from renneting, representing starting time and ending time of the validation experiment.

Figure 4. Propionate (a) and acetate (b) concentrations in different positions of the semi-hard Swiss-type cheese with PAB at 15 and 20 days from renneting (young cheese).

Figure 5. pH and PAB count in different positions of the semi-hard Swiss-type cheese with PAB at 15 and 20 days from renneting.

Figure 6. Median values for initial \( \text{CO}_2 \) concentration measured in 9.5 weeks after renneting for old semi-hard Swiss-type cheese without PAB (a) and in 2 weeks after renneting for young semi-hard Swiss-type cheese with PAB (b). Horizontal and vertical error bars represent cheese position used and min and max values for assessed \( \text{CO}_2 \) respectively.

Figure 7. Experimental (red dots) and predicted (solid line) \( \text{CO}_2 \) concentration in the semi-hard cheese without PAB (a) and with PAB (b) \( \text{CO}_2 \) production after about 4 ripening days at 19°C. Root mean squared error (CVRMSD) is indicated on each figure. Error bars in experimental data represent standard deviations.

Figure 8. Predicted \( \text{CO}_2 \) gradients in cheese (solid lines) after 1, 2, 3, 4 and 10 days of contact with 100% \( \text{CO}_2 \), calculated considering 1.7 folds higher \( \text{CO}_2 \) production rate (19°C). Red
solid line and red error bars correspond to predicted and experimental CO₂ gradient after 4 days of contact.

Figure 9. Predicted CO₂ gradients in cheese (solid or dotted lines) after 1, 2, 3, 4 and 10 days of contact with 100% CO₂, calculated considering 2 folds higher initial CO₂ gradient (19°C). Red solid line and red error bars correspond to predicted and experimental CO₂ gradient after 4 days of contact.

Figure 10. Effect of the intensity (low, medium and high, as stated in Table 2, in black, blue and red respectively) of the input parameter CO₂ solubility (a), diffusivity (b) permeability (c) and production rate (d) on the predicted CO₂ gradients in cheese ripened for 4 days at 19°C (age at beginning of ripening equalled 14 days from renneting).
Table 1. Target chemical composition of “old” and “young” cheeses and steps of model validation

| Type of cheese | Time after renneting | Target moisture % w/w | Target fat absolute % d.m. | Target salt content % NaCl/d.m. | Target pH | PAB | Validation step |
|----------------|----------------------|-----------------------|----------------------------|---------------------------------|-----------|-----|-----------------|
| “Old cheese”   | 15 days              | 42                    | 40                         | 5 %                             | 5.45      | No  | ① CO₂ diffusion only |
| “Young cheese” | 9.5 weeks            | 42                    | 8                          | 2.5 %                           | 5.45      | Yes*| ② CO₂ diffusion + production by PAB ③ CO₂ diffusion + production by PAB + CO₂ permeation through ripening foil |

*10⁶ CFU ml⁻¹ milk
Table 2. Range of values used in the simulations

| Parameter/level                        | low    | medium | high   |
|----------------------------------------|--------|--------|--------|
| Solubility (mmol kg\(^{-1}\) atm\(^{-1}\)) | 25     | 36     | 40     |
| Diffusivity (10\(^{-10}\) m\(^2\) s\(^{-1}\)) | 1      | 4      | 8      |
| Permeability (cm\(^3\) µm m\(^{-2}\) d\(^{-1}\) bar\(^{-1}\)) | 4000   | 44000  | 440 000|
| Production rate (mmol kg\(^{-1}\) d\(^{-1}\)) | 2      | 4      | 8      |
Figure 1: Scheme of the simplified food / packaging system with the four phenomena considered (mono-directional transfer through the headspace/cheese interface, \( br \) = cheese bottom-rind)
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Highlights

- We proposed the first validated model for the prediction of CO₂ gradient in cheese.
- CO₂ production is the most important parameter affecting CO₂ gradients in cheese.
- A variation of a factor 10 of CO₂ permeability of the packaging did not relevantly affect CO₂ gradients in cheese.
- CO₂ permeability of the packaging did not relevantly affect CO₂ gradients in cheese.