Landscape of the island of inversion studied by AMD

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Abstract. Deformation and many particle and many hole configurations in the island of inversion are investigated with Antisymmetrized Molecular Dynamics (AMD). The result suggests the coexistence of many particle and many hole configurations with different deformation at small excitation energy. In the case of $^{33}\text{Mg}$, we suggest the negative-parity assignment for the ground state. Other particle hole configurations are predicted as the excited states.

1. Introduction
The breaking of neutron magic number $N = 20$ that was firstly pointed out from the observation of the anomalous binding energy and spin of $^{31}\text{Na}$ [1, 2] has been one of the great interests in nuclear physics. Based on the theoretical calculations, neighboring nuclei around $^{31}\text{Na}$ was named “island of inversion” [3, 4] because they are dominated by the intruder configuration in which neutrons are promoted into $pf$-shell across $N = 20$ shell gap due to strong deformation [5, 6]. Since then, numerous experimental and theoretical studies have been devoted to the island. One of the recent experimental finding is that there are coexistence of “spherical and deformed shape” or “normal and intruder configurations” in many cases [7, 8, 9, 10]. It is expected that the exploration of the coexistence will bring us further understanding of the shell structure in neutron rich $N \sim 20$ region.

The onset of deformation in the island of inversion implies that there should be the coexistence of spherical and deformed shapes and many-particle and many-hole ($mpmh$) configurations associated with these deformations. Therefore the systematic investigation of the spectrum in the island of inversion is of importance to understand the shape coexistence phenomena.

The spectroscopy of odd-neutron nuclei is of importance and interest, because their spin and parity are related to the last neutron particle’s or hole’s orbit. For example, we have predicted the coexistence of many-particle and many-hole configurations at small excitation energy in $^{31}\text{Mg}$ ($N = 19$) [11], and some of these excited states are identified by the the proton knock-out reaction from $^{32}\text{Al}$ [12]. The spin-parity of the excited states are associated with the neutron orbits $[N, n_z, l_z, j_z] = [2, 0, 2, 3/2], [3,3,0,1/2], [2,0,0,1/2]$ and $[3,2,1,3/2]$ in terms of the Nilsson model.

In this talk, we report our study of island of inversion based on AMD. We first demonstrate the coexistence of the $mpmh$ configurations in the island and discuss the spectrum of $N = 21$ system ($^{33}\text{Mg}$).
2. Framework of AMD

For the detail of AMD framework, readers are directed to the references [13, 14]. The Hamiltonian used in this study is given as,

\[ H = \sum_i t_i + \sum_{i<j} v_N(ij) + \sum_{i<j} v_C(ij) - T_{\text{c.m.}}. \] (1)

As an effective nuclear force \( v_N \), we have used Gogny D1S force, and the Coulomb force is approximated by the sum of seven Gaussians. \( T_{\text{c.m.}} \) takes care of the center-of-mass energy.

In the AMD framework, the wave function of the center-of-mass motion is separable from the internal wave function and the calculation is free from the spurious center-of-mass energy. We employ the parity projected wave function as the variational wave function,

\[ \Phi^\pm = \hat{P}^\pm \Phi_{\text{int}} = \frac{(1 \pm \hat{P}_z)}{2} \Phi_{\text{int}}, \] (2)

and the intrinsic wave function is represented by a Slater determinant,

\[ \Phi_{\text{int}} = A\{\varphi_1, \varphi_2, \ldots, \varphi_A\}, \quad \varphi_i(r) = \phi_i(r)\chi_i\xi_i, \] (3)

where \( \varphi_i \) is the \( i \)-th single particle wave packet consisting of the spatial \( \phi_i \), spin \( \chi_i \) and isospin \( \xi_i \) parts. \( \phi_i \) is described by a triaxially deformed Gaussian,

\[ \phi_i(r) = \exp\left\{-\sum_{\sigma=x,y,z} \nu_\sigma \left(r_\sigma - \frac{Z_\sigma}{\sqrt{\nu_\sigma}}\right)^2\right\}, \quad \chi_i = \alpha_i\chi_\uparrow + \beta_i\chi_\downarrow, \quad \xi_i = p \text{ or } n. \] (4)

The centroids of wave packets (\( Z_i \)) and direction of spins (\( \alpha_i, \beta_i \)) are optimized to minimize the total energy under the constraint on the matter quadrupole deformation \( \beta \).

After the variation, we project out the eigenstate of the total angular momentum, \( \Phi^{J^+}_{MK}(\beta) = \hat{P}^{J^+}_{MK} \Phi^\pm(\beta) \). Finally, the wave functions \( \Phi^{J^+}_{MK}(\beta) \) which have the same parity and angular momentum but have different \( K \) and deformation \( \beta \) are superposed (GCM). Then the wave function of the system is written as

\[ \Phi^{J^+}_n = c_n \Phi^{J^+}_{MK}(\beta) + c'_n \Phi^{J^+}_{MK'}(\beta') + \cdots. \] (5)

The coefficients \( c_n, c'_n, \ldots \) are determined by the Hill-Wheeler equation.

3. Onset of the deformation in the island of inversion

Fig. 1 shows the energy curves of Ne, Mg and Al isotopes with \( N = 18, 20 \) and 22 calculated with AMD. Here Ne and Mg isotopes with \( N = 20 \) and 22 are in the island and Al isotopes are out of or on the border. In the case of \( ^{32}\text{Mg} \), there are three minima in the positive-parity energy curve and two in the negative-parity. These minima have different single-particle configurations. It is confirmed by the neutron single particle energies as function of quadrupole deformation shown in Fig. 2. In the spherical region, the last two neutrons occupy the orbital originates in \( sd \)-shell, and the system has a \( 0p0h \) configuration (Hereafter, we denote \( mphp \) configurations with respect to \( N = 20 \) shell closure). Around \( \beta = 0.3 \) two neutrons occupy an orbital protruding from \( pf \)-shell and the system has a \( 2p2h \) configuration. Further increase of deformation causes the intrusion of another orbital from \( pf \)-shell (\( 4p4h \) configuration). Thus, the minima on the positive-parity energy curve are understood to have \( 0p0h \), \( 2p2h \) and \( 4p4h \) configurations in ascending order of deformation. In the same way, the minima on the negative-parity energy curve have \( 1p1h \) and \( 3p3h \) configurations.
Figure 1. Energy curves of Ne, Mg and Al isotopes as function of quadrupole deformation. Open circles (open boxes) show the positive-parity (negative-parity) states before the angular momentum projection for $^{32}$Mg.

Figure 2. Single-particle levels for last 12 neutrons in $^{32}$Mg. Filled circles (open boxes) show the orbitals originates in the $p$- or $sd$-shell (the intruder orbital from $pf$-shell).

These are several striking features in the energy curves of Ne, Mg and Al isotopes, (i) there are several minima on the surface, (ii) number of minima depends on neutron number but not...
on proton number, (iii) relative energy between minima strongly depends on proton number. The particle-hole configuration of each minimum shown in Fig.1 tells that up to 4 particle or 4 hole configurations (from $np^4h$ to $4pn^h$) appear at small excitation energy. It is consistent with the behavior of neutron single-particle orbital in which two orbitals originate in the $pf$-shell and two from $sd$-shell cross as deformation becomes larger. Since the states on the energy curve are dominated by the neutron particle-hole configurations, (i) minima with different neutron configurations appear, and (ii) the single-particle configuration of each minimum depends on the neutron number. As proton number decreases, the binding energy of the spherical (or small deformed) states become smaller. It reduces the relative energy between different particle hole configurations and, in the island of inversion like $^{32}$Mg, strongly deformed configuration becomes the ground state.

Fig. 3 shows the spectrum of $^{32}$Mg after the GCM calculation compared with the observation. Note that AMD calculation predicts the coexistence of many bands with different particle-hole configurations as well as the strongly deformed ground band.

4. Low-lying level structure of $^{33}$Mg

The system with odd number neutrons may be the best example to show the coexistence of shapes and $mpmh$ configurations. Fig. 4 shows the calculated spectrum and the magnetic moment of $^{33}$Mg, and compares them with observations and the shell model calculation. There are conflicting assignments for the ground state spin-parity based on several experiments. From the observation of $\beta$ decay from $^{33}$Na [15] and to $^{33}$Al [16] the positive-parity assignment is suggested, while the negative-parity is suggested from the measurement of the magnetic moment [17] and $1n$ removal cross section [18].

Looking at the AMD result, we can see that $1p0h$, $2p1h$, $3p2h$ and $4p3h$ configurations coexist within very small excitation energy. The spin parity of these band head states are basically consistent with the last neutron single particle orbitals. It’s behavior is quite similar to that of $^{32}$Mg shown in Fig.2. Namely, at small deformation, the last neutron occupies $[3,3,0,1/2]$ orbital ($1p0h$) with $J^\pi = 7/2^-$. As deformation becomes larger, the last neutron occupies $[2,0,2,3/2]$ ($2p1h$), $[3,2,1,3/2]$ ($3p2h$) and $[2,0,0,1/2]$ ($4p3h$) orbitals that respectively generate $K^\pi = 3/2^+$, $3/2^-$ and $1/2^+$ bands. Among these bands, only the $J^\pi=3/2^-$ state has the negative-sign of the magnetic moment and consistent with the observation of the magnetic moment. Note that AMD calculation predicts the $mpmh$ configurations with different
deformation at small excitation energy and further experimental studies are needed.

5. Summary
We have investigated deformation and $mpmh$ configurations in the island of inversion based on AMD. The energy curves show the coexistence of $mpmh$ configurations with different deformation. In the case of $^{33}$Mg, these $mpmh$ configurations coexist at very small excitation energy. AMD calculation supports the negative-parity assignment for the ground state suggested by the observation of the magnetic moment [17] and the SD-PF shell model calculation [17].

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