Strong Couplings of Three Mesons with Charm(ing) Involvement

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Abstract. We determine the strong couplings of three mesons that involve, at least, one \( \eta_c \) or \( J/\psi \) meson, within the framework of a constituent-quark model by means of relativistic dispersion formulations. For strong couplings of \( J/\psi \) mesons to two charmed mesons, our approach leads to predictions roughly twice as large as those arising from QCD sum rules.

1 Three-meson strong coupling from meson–meson transition amplitudes

We determine the strong couplings of three mesons at least one of which is one of the charmonia \( \eta_c \) and \( J/\psi \), generically called \( g_{P P V} \) and \( g_{P V V} \) for pseudoscalar mesons \( P \) of mass \( M_P \) and vector mesons \( V \) of mass \( M_V \) and polarization vector \( \varepsilon_\mu \) and defined, for momentum transfer \( q \equiv p_1 - p_2 \), by the amplitudes

\[
\langle P'(p_2) V(q) | P(p_1) \rangle = -\frac{g_{P P V}}{2} (p_1 + p_2)^\mu \varepsilon_\mu(q),
\]

\[
\langle V'(p_2) V(q) | P(p_1) \rangle = -g_{P V V} \varepsilon_\mu(p_1) \varepsilon_\nu(p_2) p_1^\mu p_2^\nu,
\]

from the residues of \textit{poles} situated at the masses \( M_{P_R} \) and \( M_{V_R} \) of (appropriate) pseudoscalar and vector resonances \( P_R \) and \( V_R \) and contributing to \textit{transition form factors} \( F_+^{P P V}(q^2) \), \( V^{P V V}(q^2) \) and \( A_0^{P V V}(q^2) \), in terms of vector quark currents \( j_\mu \equiv \bar{q}_1 \gamma_\mu q_2 \) and axial-vector quark currents \( j_\mu^A \equiv \bar{q}_1 \gamma_\rho \gamma_5 q_2 \) defined by

\[
\langle P'(p_2) | j_\mu(p_1) \rangle = F_+^{P P V}(q^2) (p_1 + p_2)_\mu + \cdots,
\]

\[
F_+^{P P V}(q^2) = \frac{g_{P P V} f_{V_R}}{2 M_{V_R} (1 - q^2/M_{V_R}^2)};
\]

\[
\langle V(p_2) | j_\mu(p_1) \rangle = \frac{2 V^{P V V}(q^2)}{M_P + M_V} \varepsilon_\mu(p_1) \varepsilon_\nu(p_2) p_1^\rho p_2^\nu,
\]

\[
V^{P V V}(q^2) = \frac{(M_V + M_P) g_{P V V} f_{V_R}}{2 M_{V_R} (1 - q^2/M_{V_R}^2)};
\]

\[
\langle V(p_2) | j_\mu^A(p_1) \rangle = i q_\mu (\varepsilon^\nu(p_2) p_1) \frac{2 M_V}{q^2} A_0^{P V V}(q^2) + \cdots,
\]

\[
A_0^{P V V}(q^2) = \frac{g_{P V V} f_{P_R}}{2 M_V (1 - q^2/M_{P_R}^2)};
\]

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where the $P$ or $V$ decay constants $f_{P,V}$ parametrize the matrix elements of the interpolating currents $j^{(5)}_{\mu}$

\[ \langle 0| j^{(5)}_{\mu}| P(q) \rangle = i f_P q_\mu , \quad \langle 0| j^{(5)}_{\mu}| V(q) \rangle = f_V M_V \epsilon_\mu(q) . \]

Such strong-coupling results may prove to be useful for studies of long-distance QCD effects in hadron decays involving charmed mesons or charmonia in the final state of a kind similar to the one in Ref. [1].

## 2 Quark-model-underpinned dispersion analysis of transition form factors

We describe the relevant properties of the involved strongly coupling mesons by means of a relativistic constituent-quark model [2–4]. Of course, this requires us to match the QCD currents $j_{\mu}^{(5)}$ to associated constituent-quark currents, which is, for heavy quarks, easily effected by introducing form factors $g_{V,A}$,

\[ j_{\mu} = g_V \bar{Q} \gamma_\mu Q + \text{other Lorentz structures} , \quad j_{\mu}^{(5)} = g_A \bar{Q} \gamma_\mu \gamma_5 Q + \text{other Lorentz structures} , \]

for which we choose $g_V = g_A = 1$ [5] but, for light quarks, rendered rather involved [6, 7], for instance, if embedding partial axial-current conservation. For the radial meson wave functions, Gaussian shapes

\[ w_{P,V}(k^2) \propto \exp \left( -\frac{k^2}{2\beta_{P,V}^2} \right) , \quad \int d^2 k^2 w_{P,V}(k^2) = 1 , \]

with slopes $\beta_{P,V}$ given, together with all relevant mesonic features, in Table 1 [8–13], turn out to suffice for our purposes. Table 2 lists the numerical values adopted for the masses of the constituent quarks $q$.

Within the framework of a relativistic dispersion formalism (reviewed, e.g., in Ref. [14]), we represent for our purposes. Table 2 lists the numerical values adopted for the masses of the constituent quarks $q$.

\[ F(q^2) = \int d s_1 d s_2 \phi_1(s_1) \phi_2(s_2) \Delta_F(s_1, s_2, q^2) , \quad f_{P,V} = \int_{(m_1+m)^2}^{\infty} ds \phi_{P,V}(s) \rho_{P,V}(s) , \]

Table 1. Relevant parameters of the mesons: numerical values of mass $M$, leptonic decay constant $f$ and slope $\beta$.

| Meson | $M$ (GeV) | $f$ (MeV) | $\beta$ (GeV) |
|-------|-----------|-----------|--------------|
| $D$   | 1.87      | 206 ± 8   | 0.475        |
| $D^*$ | 2.010     | 260 ± 10  | 0.48         |
| $D_s$ | 1.97      | 248 ± 2.5 | 0.545        |
| $D_s^*$ | 2.11   | 311 ± 9   | 0.54         |
| $\eta_c$ | 2.980 | 394.7 ± 2.4 | 0.77        |
| $J/\psi$ | 3.097 | 405 ± 7   | 0.68         |

Table 2. Constituent mass of each quark flavour $Q = u, d, s, c$ [5] involved in charm(ing) three-meson couplings.

| Quark flavour | Quark mass $m$ (GeV) |
|--------------|---------------------|
| $u$          | 0.23                |
| $d$          | 0.23                |
| $s$          | 0.35                |
| $c$          | 1.45                |
and of the wave functions of all mesons entering the corresponding one- or two-meson matrix elements

$$\phi_{P,V}(s) = \frac{\pi}{s^{3/4}} \sqrt{\frac{s^2 - (m_1^2 - m_2^2)^2}{2(s - (m_1 - m_2)^2)}} \omega_{P,V} \left( \frac{(s - m_1^2 - m_2^2)^2 - 4m_1^2m_2}{4s} \right).$$

3 Three-meson strong coupling: determination from transition amplitudes

We fix the slopes $\beta_{P,V}$ such that the decay constants $f_{P,V}$ are reproduced by their spectral representation. Equipped with these $\beta_{P,V}$ values, we deduce all strong couplings from the spectral representation of the relevant form factors $F(q^2)$ derived sufficiently off the resonances at $M_R = (P_R, V_R)$, by interpolating pointwise given momentum dependences of $F(q^2)$ by three-parameter $(\sigma_1, \sigma_2, F(0))$ ansätze of the form

$$F(q^2) = \frac{F(0)}{1 - \sigma_1 q^2/M_R^2 + \sigma_2 q^2/M_R^4} \left( 1 - \frac{q^2}{M_R^2} \right), \quad \text{Res } F(M_R^2) = \frac{F(0)}{1 - \sigma_1 + \sigma_2}, \quad R = P_R, V_R,$$

and extrapolating $F(q^2)$ to the poles at $q^2 = M_R^2$, where the strong couplings emerge from the residues. Using $\sigma_{1,2}, F(0)$, and $M_R$ as fit parameters, all arising masses $M_R$ come close to the known resonances. Quite generally, a given strong coupling may show up in and therefore can be extracted from more than one meson–meson transition form factor, for example, $g_{\eta_c, J/\psi}$ from $F_{+}^{\eta_c, J/\psi}$ or $A_{0}^{D, J/\psi}$ (see Fig. 2) [15–17], $g_{DD}\phi$ from $F_{+}^{D,D, \phi}$ or $A_{0}^{D, \phi}$ (see Fig. 3(a)) [15–17] and $g_{DD}\eta_c$ from $F_{+}^{\eta_c, D}$, $A_{0}^{\eta_c, D}$ or $A_{0}^{D, \eta_c}$ (see Fig. 3(b)) [15–17]; for further examples of such multiple involvements, consult Tables III, V, and VI of Ref. [15].

![Figure 2](https://example.com/figure2.png)

Figure 2. Behaviour of the off-shell $\eta_c$-$\eta_c$-$J/\psi$ strong coupling $g_{\eta_c, \eta_c, J/\psi}$ with increasing resonance-mass-normalized momentum transfer $x \equiv q^2/M_R^2$ for the transition of $\eta_c$ to the $\eta_c$ (solid red line) or the $J/\psi$ meson (dotted blue line).
Figure 3. Behaviour of the “off-shell” $D-D-J/\psi$ and $D-D^*-\eta_c$ strong couplings $g_{D\psi\eta_c}$ and $g_{D\psi^*\eta_c}$, respectively, with increasing “resonance-mass-normalized” momentum transfer $\chi = q^2/M_R^2$: (a) $g_{D\psi\eta_c}(x) = 2 M_0 (1-x) A_0^{\psi\eta_c}(q^2)/f_D$ (blue dotted line and squares □) and $g_{D\psi}(x) = 2 M_0 (1-x) F_{3}^{\psi\eta_c}(q^2)/f_\phi$ (red solid line and triangles ▲), relying on interpolation (blue or red lines) or not (squares □, triangles ▲); (b) $g_{D\psi\eta_c}(x)$ (solid red line), $g_{D\psi^*\eta_c}(x)$ (dotted blue line) and $g_{D\psi^*\eta_c}(x)$ (dashed green line). For each transition, the relevant resonance, $R$, is identified by a circumflex.

4 Strong coupling predictions from relativistic constituent-quark approach

We collect our emerging strong-coupling findings — extracted, in the case of multipresence of one and the same three-meson coupling in more than one meson–meson transition amplitude, by a combined fit — in Table 3: Strange quark content instead of a down quark implies a reduction of the involved strong couplings, by roughly 10%. Confronting, in Table 4, our $D_{(s)}-D_{(s)}^*-J/\psi$ predictions with QCD sum-rule outcomes [18–20], the QCD sum-rule estimates prove to be lower than ours [15–17] by a factor of two.

**Table 3.** Charm(ing) three-meson strong couplings: quark-model-based dispersion-approach outcomes [15–17].

| $PP'V$ Coupling | Strong coupling $g_{PP'V}$ |
|-----------------|---------------------------|
| $\eta_c-\eta_c-J/\psi$ | 25.8 ± 1.7 |
| $D-D-J/\psi$ | 26.04 ± 1.43 |
| $D-D^*-\eta_c$ | 15.51 ± 0.45 |
| $D_s-D_s-J/\psi$ | 23.83 ± 0.78 |
| $D_s-D_s^*-\eta_c$ | 14.15 ± 0.52 |

| $PV'V$ coupling | Strong coupling $g_{PV'V}$ (GeV$^{-1}$) |
|-----------------|-------------------|
| $\eta_c-J/\psi-J/\psi$ | 10.6 ± 1.5 |
| $D-D^*-J/\psi$ | 10.7 ± 0.4 |
| $D^*-D^*-\eta_c$ | 9.76 ± 0.32 |
| $D_s-D_s^*-J/\psi$ | 9.6 ± 0.8 |
| $D_s^*-D_s^*-\eta_c$ | 8.27 ± 0.37 |

**Table 4.** Strong couplings of the $J/\psi$ meson to two charmed mesons: relativistic quark model vs. QCD sum rule.

| Coupling | Approach | Quark model [15–17] | QCD sum rules | References |
|----------|----------|---------------------|---------------|------------|
| $D-D-J/\psi$ | | 26.04 ± 1.43 | 11.6 ± 1.8 | [18] |
| $D-D^*-J/\psi$ | (10.7 ± 0.4) GeV$^{-1}$ | (4.0 ± 0.6) GeV$^{-1}$ | [18] |
| $D_s-D_s-J/\psi$ | 23.83 ± 0.78 | 11.96±1.134 | 19] |
| $D_s-D_s^*-J/\psi$ | 9.6 ± 0.8 GeV$^{-1}$ | (4.30±1.22) GeV$^{-1}$ | [20] |
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