Monte-carlo renormalization group study of gauged $RP^2$ spin models in two dimensions

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The 2D $RP^2$ gauge model is studied using the Monte-Carlo Renormalization Group (MCRG). We confirm the first-order transition reported in [1] ending in a critical point associated with vorticity. We find evidence for a new renormalized trajectory (RT) which is responsible for a cross-over from the vortex dominated regime to the $O(3)$ regime as the coupling is reduced. Near to the cross-over region a good signal for scaling will be observed in $RP^2$ but this is illusory and is due to the proximity of the RT. We suggest that this is the origin of the ‘pseudo’-scaling observed in [2]. We find that the continuum limit of $RP^2$ is controlled by the $O(3)$ fixed point.

1. Introduction

In [2] the mass gap measurement in the $SO(4)$ matrix model was compared with the Bethe-Ansatz prediction and found to disagree by a factor of four although for the covering group model there was good agreement. It was concluded that the signal for scaling in the $SO(4)$ simulation was only apparent and that a true continuum limit had not been achieved. It was conjectured that the deception was due to vortices, present in $SO(4)$ but absent in the covering group. Here we investigate whether vortices can cause a bogus signal for scaling in the 2D $RP^2$ gauge model which allows an interpolation between the pure $RP^2$ and the $O(3)$ spin models and which also contains vortices.

We find evidence for the $O(3)$ fixed point and for a scaling flow which indicates the presence of a new renormalized trajectory (RT). We suggest that this RT will cause an apparent scaling signal of the kind reported in [2] and that it is responsible for an observed cross-over effect.

It has also been conjectured [3] that in 2D the continuum limit in pure $RP^2$ is distinct from that in pure $O(3)$. Niedermayer et al. [4] and Hasenbusch [5] have suggested that this conjecture is incorrect and that the continuum limit in the $RP^2$ model is controlled by the $O(3)$ fixed point. The work we report here supports this conclusion.

The action used is

$$S = -\beta \left( \sum_{x,\mu} S_x \cdot S_{x+\mu} \sigma_{x,\mu} + \mu \sum_P P(\sigma) \right), \quad (1)$$

where $S_x$ is a unit length three-component vector at site $x$ and $\sigma_{x,\mu}$ is a gauge field on the link $x,\mu$ taking values in $[1, -1]$ . The plaquette of gauge fields is denoted by $P(\sigma)$ and vortices reside on plaquettes where $P(\sigma) = -1$. The pure $O(3)$ and $RP^2$ spin models correspond to $\mu \rightarrow \infty$ and $\mu = 0$ respectively.

A local update was used comprising a combination of heat-bath, microcanonical and demon schemes. Lattice sizes ranged from $64^2$ to $512^2$ with typically $\sim 5 \cdot 10^5$ configurations per run. The simulations were carried out on the
2. The MCRG scheme

To establish the topology of RG flows we study how the mean values of the spin-spin interaction, $A$, and of the plaquette, $P$, flow under blocking. For a given configuration these operators are given by

\[ A = \frac{1}{2V} \sum_{x, \mu} S_x \cdot S_{x+\mu} \sigma_{x, \mu} , \]

\[ P = \frac{1}{V} \sum_\nu P(\sigma) . \]  

(2)

where $\nu$ runs over nearest neighbour displacements from $x$, and $\alpha$ was chosen to be 0.0625. To block the links the gauge field products were computed for the three shortest Wilson paths $W_0, W_{+}, W_{-}$ joining the end points of the blocked link. The blocked gauge field was assigned the majority sign of the link. The blocked gauge field was assigned the $W$ computed for the three shortest Wilson paths

\[ S^{B}_{x, \mu} = \left( S_x + \alpha \sum_\nu S_{x+\nu} \sigma_{x, \nu} \right) / |\ldots| . \]

For given coupling constants $(\beta, \mu)$ and given lattice size $L^2$ each configuration was blocked by successive transformations until the blocked lattice was $8^2$. The operators $A_L(\beta, \mu)$ and $P_L(\beta, \mu)$ were then measured and averaged over all configurations. This was done for $L = 64, 128, 256, 512$ giving a flow segment in the $(\langle A \rangle, \langle P \rangle)$ plane with each point labelled by initial lattice size. The flow can be extended by tuning new couplings $(\beta', \mu')$ so that

\[ A_{L'}(\beta', \mu') = A_L(\beta, \mu) , \]

\[ P_{L'}(\beta', \mu') = P_L(\beta, \mu) , \]  

(3)

for some $L$ and $L'$. The segment for fixed $(\beta', \mu')$ can then be computed. These flows result from

\[ \langle A \rangle \text{ (dashed) and } \langle P \rangle \text{ (solid) versus } \mu \text{ for } \beta = 8.0 \text{ showing the first-order transition.} \]

3. Results and Conclusions

There is a line of first-order transitions, first reported in ref. 1, for $\mu \sim -0.3$ and $\beta \geq 7.5$. We find that this line of transitions ends at the “vorticity” critical point. In figure 1 we plot $\langle A \rangle$ and $\langle P \rangle$ versus $\mu$ for $\beta = 8.0$. The order parameter is the vorticity. The linear combination $U = 2A + \gamma P$ with $\gamma \sim -0.29$ shows no discontinuity in this region. $U$ is closely related to the action, $S$ (eqn. 1), and it is reasonable to interpret $U$ as the free energy. Also, $U$ is the correct operator to interpolate the vector state associated with the $O(3)$ critical surface where the corresponding correlation length will diverge but which remains finite at the vorticity critical point.

There is a set of neighbouring flows on which the observables scale. The example shown in figure 2 shows that there is a RT in the vicinity of the $O(3)$ critical surface which must be associated with a new fixed point. To see that observables scale each flow segment of four points was successively overlaid using eqn. 3 with $L' = L/2$. For the scaling flows the points of the overlaid flow segments coincide very
well within errors. The flow is consistent with an exponent $\nu = 4.75(15)$ with critical couplings $(\beta^*, \mu^*) \approx (7.5, -0.3)$. Of course, the new fixed point will not lie in the $(\beta, \mu)$ plane and its relationship with the vorticity critical point is unclear. Open questions are: can the continuum limit of the new RT be taken at the vorticity critical point and does there exist a continuum limit with a non-zero vorticity density? It is intriguing that the position of the vorticity critical point and the critical couplings deduced from the fit for $\nu$ are very similar. The nature of the new fixed points needs more investigation.

Scaling on the $O(3)$ RT ($\mu = \infty$) was verified and agreement with the 3-loop perturbative $\beta$–function was found after effects due to finite $L$ were accounted for using perturbation theory.

The pure $RP^2$ model ($\mu = 0$) does not intersect any critical surface except the one controlled by the $O(3)$ fixed point at $(\beta, \mu) = (\infty, \infty)$ (for $(\langle A \rangle, \langle P \rangle) = (1, 1)$). This confirms the conjectures of [4,5] that $RP^2$ and $O(3)$ have the same continuum limit. In figure 3 flow segments for the $RP^2$ are shown for various $\beta$ in the range 3.9 to 4.5. There is a clear cross-over in the blocked vorticity, $\mathcal{V}$, as $\beta$ increases through this range. Moreover, the flows to larger $\mathcal{V}$ renormalize close to the scaling flow, shown in figure 3 as a dotted line. Hence, for $\beta \sim 4.0$ we should expect to see a good signal for scaling induced by the associated new RT. However, this scaling is not a signal for a continuum limit in $RP^2$ but is due to the proximity of the cross-over region to the new fixed point to which it is due. As $\beta$ is increased through the cross-over region scaling will be violated and eventually reappear in association with the true continuum limit controlled by the $O(3)$ fixed point. We believe that this is the origin of the bogus scaling observed in [2].
In figure 4 we shown an artist’s impression of a possible topology for the RG flows.

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