Reputation blackboard systems

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Blackboard systems are motivated by the popular view of task forces as brainstorming groups in which specialists write promising ideas to solve a problem in a central blackboard. Here we study a minimal model of blackboard system designed to solve cryptarithmetic puzzles, where hints are posted anonymously on a public display (standard blackboard) or are posted together with information about the reputations of the agents that posted them (reputation blackboard). We find that the reputation blackboard always outperforms the standard blackboard, which, in turn, always outperforms the independent search. The asymptotic distribution of the computational cost of the search, which is proportional to the total number of agent updates required to find the solution of the puzzle, is an exponential distribution for those three search heuristics. Only for the reputation blackboard we find a nontrivial dependence of the mean computational cost on the system size and, in that case, the optimal performance is achieved by a single agent working alone, indicating that, though the blackboard organization can produce impressive performance gains when compared with the independent search, it is not very supportive of cooperative work.

I. INTRODUCTION

Understanding the conditions that improve the efficacy of cooperative work is a critical issue for the economy of developed countries, given the central role played by task force problem-solving (e.g., drug design, robotics engineering, software development, etc.) on producing technological innovations [1]. For instance, the study of the impact of imposed communication patterns (i.e., who can communicate with whom) on group performance dates back to the 1950s [2–4] (see [5, 6] for more recent contributions) and aims at offering scientific guidance for matching organizational structures to task complexity [7].

A classical team organization that allows relevant information (hints) to diffuse quickly among members of the group is the so-called blackboard system [8], which was introduced in the Artificial Intelligence domain in the 1980s to tackle the uncertainties inherent to speech understanding [9] and is now part of the AI problem-solving toolkit [10, 11]. In this organization, team members (or agents) simply read and write hints to a central blackboard that can be accessed by all members.

In this contribution we revisit and build on a seminal study of the efficacy of a minimal model of blackboard system to solve cryptarithmetic problems [12] (see also [13]). That study suggested that the blackboard organization could produce a superlinear speedup in the time to find the solution with respect to the number of group members. We find that although this organization indeed entails a considerable quantitative gain in comparison with the situation where the agents solve the problem independently of each other using a simple random trial-and-test search, it does not produce any qualitative change in the performance measures. In particular, the expected value of shortest time $T_M$ to find the solution decreases linearly with the number of agents $M$ and the asymptotic distribution of $T_M$ is an exponential distribution as in the case of the independent search. In addition, when the performance of the system is measured by the computational cost $C \propto M T_M$, which essentially yields the total number of agent updates until the solution is found, it becomes practically independent of the system size, as in the independent search again.

Here we propose a slightly different and, perhaps, more realistic blackboard organization, the so-called reputation blackboard system, in which the value of a hint (i.e., the probability of the hint being selected from the blackboard) is determined by the reputation of the agent that posted it. The reputation of an agent is a measure of the quality of its partial solutions to the problem. We find that the reputation blackboard always outperforms the standard blackboard, and that the probability distribution of the computational costs are still well fitted by an exponential distribution. However, in contrast with the results for the independent search and for the standard blackboard, the mean computational cost exhibits a complex dependence on the system size $M$ and, somewhat surprising, the minimum cost is achieved for $M = 1$, i.e., individual work is more effective than group work for this organization. Our findings suggests that the blackboard organization is not very supportive of cooperative work.

The rest of the paper is organized as follows. In Section II we present the cryptarithmetic problem used in our study and explain how the solutions are encoded in integer strings. In that section we also introduce the concept of hint and define the cost function of a string, which is used to assign a reputation to the agents. In Section III we present the minimal model for the standard blackboard organization [12] and study its performance on solving the cryptarithmetic puzzle. Then in Section IV we introduce the reputation blackboard organization and compare its performance with that of the standard blackboard. Finally, Section V is reserved to our concluding remarks.
II. THE CRYPTOARITHMETIC PUZZLE

Cryptarithmetic problems such as

\[ DONALD + GERALD = ROBERT \]  \hspace{1cm} (1)

are constraint satisfaction problems in which the task is to find unique digit-to-letter assignments so that the integer numbers represented by the words add up correctly \[ 1447603 \]. The cost value defined in eq. (3) applies to and \( F \) is the first operand \((DONALD)\) and \( S \) is the second operand \((GERALD)\). For the string \((0,2,9,4,8,1,7,6,3,5)\) we have \( R = 362435 \), \( F = 967019 \) and \( S = 843019 \) so that the cost associated is \( c = 1447603 \). The cost value defined in eq. (3) applies to all strings except those for which \( i_3 = 0 \) corresponding to the assignment \( D = 0 \), \( i_5 = 0 \) corresponding to the assignment \( G = 0 \) and \( i_9 = 0 \) corresponding to the assignment \( R = 0 \). Those are invalid strings because they violate the rule of the cryptarithmetic puzzles that an integer number should not have the digit 0 in its leftmost position. Hence for those strings we assign an arbitrary large cost value, namely, \( c = 10^9 \), so that now they become valid strings but they have the highest cost among all strings. If the cost of a string is \( c = 0 \) then the digit-to-letter assignment coded by that string is the solution to the cryptarithmetic problem.

An advantage of traditional blackboard systems is that they do not need the introduction of a cost function to weight the quality of the strings. Rather, those systems use hints that the agents read and write to a blackboard that is accessed by all agents. In the context of cryptarithmetic puzzles, hints are letter-digit assignments that add up correctly modulo 10 for at least one column. For example, considering the third column (from left to right) of the problem \((1)\) we have the hints \((N = 3, R = 2, B = 5), (N = 7, R = 8, B = 5), \) etc. Although this is the definition of hints used in Refs. \([12, 13]\), it is actually not very useful to solve the puzzle \((1)\) since the correct solution uses only 2 of those hints, namely, \((N = 6, R = 7, B = 3)\) and \((D = 5, D = 5, T = 0)\), whereas the other 4 columns only add up correctly if one considers the 1 that is carried from the adjacent column, e.g., \((L = 8, L = 8, R = 7), (A = 4, A = 4, E = 9), \) etc. This observation suggests that we extend the list of hints to include also the cases that the two letter-digit assignments plus the carry 1 add up correctly modulo 10 for each column, except for the leftmost one \((D + D = T)\), of course. In this scenario, there are 351 distinct hints and 6 of them correspond to the solution of the puzzle \((1)\). Here we will consider only the extended list of hints.

As we will see in the next section, the hints are discovered by the agents during their random exploration of the state space and displayed in the central blackboard for use by the other agents.

III. THE STANDARD BLACKBOARD SYSTEM

We consider a system composed of \( M \) agents and a central blackboard where the agents can read and write hints. However, hints cannot be erased from the blackboard. Each agent is represented by a string of 10 different digits representing a particular digit-to-letter assignment for the puzzle \((1)\) and so henceforth we will use the terms agent and string interchangeably. At time \( t = 1 \) all agents are initialized as random strings selected with equal probability from the pool of the 10! valid digit strings. The agents check for all possible hints from their strings and post them to the blackboard, unless they are already displayed on the board. Figure 1 shows the mean number of hints \( H \) displayed on the board at time \( t = 1 \) for different system sizes \( M \). For instance, for \( M = 1 \) we
have \( H \approx 1.1 \), for \( M = 100 \), \( H \approx 80 \) and for \( M > 5000 \) the board will almost certainly display all 351 hints already at the outset of the search.

Once the initial states of the agents and of the blackboard are set up, the search procedure develops as follows. It begins with a randomly chosen agent – the target agent – picking a hint at random from the blackboard. In the case that there are no hints (i.e., the blackboard is empty), or that the target agent is already using the chosen hint, the agent selects randomly a letter-to-digit assignment from the pool of valid digit strings. The incorporation of a hint into the digit string of the target agent involves the relocation of at most six digits of that string. For example, consider the assimilation of the hint \( (N = 1, R = 4, B = 5) \) into the target string \((0, 2, 9, 4, 8, 1, 7, 6, 3, 5)\). First, the assignment \( N = 1 \) is assimilated, yielding \((0, 2, 9, 4, 8, 7, 1, 6, 3, 5)\), then \( R = 4 \), yielding \((0, 2, 9, 3, 8, 7, 1, 6, 4, 5)\) and finally \( B = 5 \) resulting in the string \((0, 5, 9, 3, 8, 7, 1, 6, 4, 2)\) which contains the desired hint. In both events – incorporation of a hint into the target string or replacement of the target string by a random string – the agent checks for all possible hints from its new string and posts them to the blackboard.

After the target agent is updated, we increment the time \( t \) by the quantity \( \Delta t = 1/M \). Then another target agent is selected at random and the procedure described before is repeated. Note that during the increment from \( t \) to \( t + 1 \) exactly \( M \), not necessarily distinct, agents are updated. Let us denote by \( t_i^* = 1, 2, \ldots \) the length of time that agent \( i \) takes to find the solution of the cryptarithmic problem. AlthoughRefs. \[12, 13\] have focused on the distribution of \( t_i^* \), we think that a more suitable measure of the efficiency of the blackboard system is

\[
T_M = \min\{t_1^*, \ldots, t_M^*\},
\]

which is interpreted as the first time that the solution of the puzzle is found by one of the agents.

In the case that the agents explore the state space independently of each other, the \( t_i^* \)s, with \( i = 1, \ldots, M \), are identically distributed independent random variables distributed by the geometric distribution

\[
f(t_i^*) = p(1-p)^{t_i^*-1},
\]

where \( p = 1/10! \) is the success probability. (We recall that the puzzle \([1]\) has a unique solution.) The probability distribution of the minimum time \( T_M \) is also a geometric distribution \([18, 19]\) with success probability \( 1 - (1-p)^M \).

\[
\tilde{P}(T_M) = \left[ 1 - (1-p)^M \right] (1-p)^{M(T_M-1)} \approx M p \exp(-M pt_M),
\]

where in the last step we have assumed that the system size is much smaller than the size of the state space, i.e., \( Mp \ll 1 \). A useful measure of the performance of a problem-solving system is the computational cost of the search defined as

\[
C = M p T_M,
\]

which yields the total number of agent updates necessary to find the solution scaled by the effective size of the state space. Since \( P(C) = \tilde{P}(T_M)/M p \) we have \( P(C) = \exp(-C) \) for the independent search with \( M p \ll 1 \). The advantage of using \( C \) is that the dependence of its mean on the number of agents can be used to distinguish between truly cooperative from non-cooperative systems, for which \( \langle C \rangle \) is approximately constant.

Figure 2 shows the distribution of probability \( P(C) \) of the computational cost for the independent search and for the standard blackboard system in the realistic situation that \( M p \ll 1 \). As expected, the results for the independent search are independent of the system size \( M \). We find the same (qualitative) results for the standard blackboard system: although this organization reduces the time to solve the puzzle by a factor of 10, the mean computational cost \( \langle C \rangle \approx 0.1 \) is practically unaffected by the value of \( M \), provided it is not too large (see Fig. 3), as in the case of the independent search. This is not a surprise since the results shown in Fig. 1 indicate that the blackboard will display all hints after a very short time. In particular, in the case of a single agent \( (M = 1) \), it takes on the average only \( t/10! = 0.0035 \) updates to fill out the blackboard. Hence the cooperation (i.e., writing on the board) ceases at the very beginning of the search and from then on the \( M \) agents will explore the state space and the changeless blackboard independently of each other. We estimated that the replacement of the target string by a random string occurs with probability 0.0068, so most of the time the target agents are picking hints from the blackboard.
Our results indicate that although the standard blackboard organization can produce a tenfold speedup on the mean time to find the solution of the cryptarithmetic puzzle, it does not change the nature of the search, which maintains all characteristics of the independent search, contrary to the suggestion of Refs. [12, 13] that the exponential distribution of eq. (7) is replaced by a lognormal distribution for the blackboard organization. The main point, however, is that such organization is not really a cooperative problem-solving system, since once the blackboard is filled out, which happens in a very short time, the agents will pick hints on the board and explore the state space independently of each other.

IV. THE REPUTATION BLACKBOARD SYSTEM

A straightforward way to guarantee that the agents keep updating the blackboard with useful information even after all hints are already on display is to associate an indicator of quality to each hint. This indicator is the reputation of the target agent who wrote the hint, and is measured by the reciprocal of the cost function \( \lambda \). (The fact that the cost is zero for the solution of the puzzle is not a problem because the search is halted when the solution is found as we are interested in the statistics of \( T_M \) only.) Rather than picking hints from the board at random with equal probability as done in the standard blackboard scenario of the previous section, now the probability of selecting a hint is proportional to the reputation of the hint, so that hints posted by high-reputation agents are more influential than those posted by low-reputation agents, a situation that resembles the Ortega hypothesis about the scientists contributions to scientific progress [20].

The search procedure for the reputation blackboard is almost the same as for the standard blackboard, except that the hints are displayed on the board together with the reputation of the agents who last wrote them, and that the probability of selecting a hint is directly proportional to the reputation of the agent that posted it, as already mentioned. In particular, we assume that the target agent overwrites the hints on the blackboard so they will be displayed with its reputation until another agent overwrites them again. Of course, after the short time necessary to fill out the board with the 351 hints, only the reputation labels will change in the rest of the search.

Figure 2 shows the distribution of the computational cost for reputation blackboard systems of different sizes. In contrast to the results for the independent search and for the standard blackboard (see Fig. 2), these results show a marked dependence on the system size \( M \) that points to the collective nature of the search procedure. The distribution \( P(C) \) in the regime of not too small costs is well fitted by an exponential, which is a robust marker of the essentially random trial-and-error character of the search. However, for small computational costs we find significant deviations from the exponential fitting as shown in Fig. 2. This is probably due to the fact that the blackboard does not yet exhibit all the 351 hints in the short time region depicted in the figure. We find a similar but much less pronounced effect in the case of the standard blackboard as well. We note that the higher odds of finding the solution for low cost searches when compared with the predictions of the exponential fitting has only a minor effect on the mean computational cost,

![FIG. 2. (Color online) Probability distribution of the computational cost \( C = MT_M/10! \) to find the solution of the puzzle (1) for the independent search and the standard blackboard system as indicated. The system sizes are \( M = 1 \) (\( \Delta \)) and \( M = 100 \) (\( \circ \)). The curve fitting the data of the independent search is \( P(C) = \exp(-C) \), whereas the data of the blackboard system is fitted by \( P(C) = 10 \exp(-10C) \).

![FIG. 3. (Color online) Probability distribution of the computational cost \( C = MT_M/10! \) to find the solution of the puzzle (1) for reputation blackboards of sizes \( M = 1 \) (\( \Delta \)), \( M = 10 \) (\( \triangledown \)) and \( M = 100 \) (\( \circ \)) as indicated. Here the solid lines are the fittings using the exponential distribution \( P(C) = \exp(-C/\lambda_M)/\lambda_M \) with \( \lambda_1 = 0.0046 \), \( \lambda_{10} = 0.026 \) and \( \lambda_{100} = 0.053 \).]
A more revealing comparison between the standard and reputation blackboards is offered in Fig. 5 that shows the mean computational cost as function of the system size. Whereas the mean cost of the standard blackboard system remains constant within a vast range of system sizes and begins to increase noticeably only for $M > 10^4$ due to duplication of work (i.e., different agents searching the same region of the state space), the mean cost of the reputation blackboard system exhibits a somewhat complex behavior. In particular, the increase of $\langle C \rangle$ for small $M$ is probably the effect of hints posted by high-reputation agents trapped in local minima of the cost function (3). When the population is large enough this effect is attenuated by the presence of hints posted by agents close to the global minimum (solution), but then the duplication of work ends up increasing the computational cost again. These phenomena were observed in the study of imitative learning, for which the local optima play a major role [19, 21]. The novelty here is that the best performance is achieved by a single individual ($M_{opt} = 1$), whereas in the imitative learning case the optimal performance is achieved by a group size that is on the order of the logarithm of the size of the state space (i.e, $M_{opt} \approx \ln 10! \approx 15$) [17].

For instance, consider the popular puzzle
\[ WOW + HOT = TEA, \] 
whose state space is comprised of the $10! / 4! = 151200$ sequences of 6 distinct digits. This problem has 66 different solutions (we have generated and tested all possible sequences) so that the probability of finding the solution by picking a single sequence at random is $p = 66/151200$. Inserting this value of $p$ in eq. (8) allows us to compare the performances of the search heuristics on different puzzles since the effective sizes of their state spaces (i.e., $1/p$) are accounted for in the definition of the computational cost. Figure 6, which shows the mean computational cost for puzzle (9), confirms the general validity of our conclu-
sions. We recall that for the independent search one has 
\( C = M p / \left[ 1 - (1 - p)^{M} \right] \) so that \( C \approx 1 \) for \( M p \ll 1 \).
Interestingly, if we estimate the difficulty of a puzzle by the computational cost to find its solutions, then, from the perspective of the blackboard systems, puzzle \( (9) \) is more difficult than puzzle \( (1) \).

V. DISCUSSION

The appeal of the blackboard organization probably owes to the common view of working groups as teams of specialists exchanging ideas on possible approaches to solve a problem and writing the promising lines of investigation in a blackboard [11]. A minimal model of blackboard systems [12][13], which we refer to as the standard blackboard organization, assumes that the agents post hints anonymously on the blackboard, and that those hints have equal probability of selection or, equivalently, have equal value. The performance of the system is measured by the computational cost of the search, which is proportional to the total number of agent updates required to find the solution of the problem posed to the group (see eqs. (4) and (5)).

Our findings show that the standard blackboard organization produces a tenfold decrease on the mean computational cost to find the solution of the cryptarithmetic puzzle \( (1) \) when compared with the case that the agents work independently of each other (see Fig. 2). However, this cost is practically unaffected by changes on the number of agents \( M \) for not too large \( M \), as in the case of the independent search. This result reveals that this organization is not very effective on promoting cooperative work. In fact, once all hints are displayed on the blackboard, which happens in a very short time span compared with the total length of the search, the agents carry out the search independently of each other.

In this contribution we propose a perhaps more realistic model of blackboard organization, referred to as the reputation blackboard, in which the value of a hint (i.e., the probability of the hint being selected from the blackboard) is associated to the reputation of the agent that posted it. The reputation of, say, agent \( i \) is defined as the reciprocal of the cost function \( c(i) \) given in eq. (5).

We find that the computational cost of the reputation blackboard is always lower than the cost of the standard blackboard, and it exhibits a nontrivial dependence on the number of agents in the system (see Fig. 5), which reveals the cooperative nature of the search. However, for both blackboard organizations the probability distribution of the computational cost is an exponential distribution as in the case of the independent search. This observation is at variance with earlier findings [12][13], which predicted a lognormal distribution for the standard blackboard organization.

A most unexpected and, perhaps, instructive outcome of our analysis of the reputation blackboard is that the best performance is achieved for a single agent (i.e., \( M = 1 \)), which suggests that individual work is more efficient than group work in that case (see Figs. 3 and 5). The reason is that in our blackboard scenario all agents have the potential to solve the problem by themselves and in that case the optimal strategy is to employ a single agent that uses the blackboard as a notebook to keep track of its past failures and successes and decide its future actions, thus carrying out a sort of individual brainstorming.

In a real-world scenario, it is likely that none of the team members are able to solve the problem working alone, otherwise hiring a single specialist would obviously be the cheapest way to carry out the task, as we have shown here. It is easy to modify the minimal model of blackboard organization to guarantee that the agents cannot solve the problem by themselves (e.g., by allowing the agents to search only limited regions of the state space) and so to make cooperation mandatory for solving the problem. But then we would not be able to make a fair comparison between the performances of the cooperative system and the independent search, which provides a valuable quantitative appraisal of the efficiency of the blackboard system. Nevertheless, it would be instructive to find out what ingredients one should add to our reputation blackboard scenario in order to make group work more efficient than individual work.

From a more general perspective, we note that finding a cooperative organization that produces a superlinear speedup in the time to find the solution with respect to the number of agents remains a challenging issue for distributed cooperative problem-solving systems [22].

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