String and membrane dynamics may be unified into a theory of 2+2 dimensional self-dual world-volumes living in a 10+2 dimensional target space. Some of the vacua of this M-theory are described by the N=(2,1) heterotic string, whose target space theory describes the world-volume dynamics of 2+2 dimensional ‘M-branes’. All classes of string and membrane theories are realized as particular vacua of the N=(2,1) string: Type IIA/B strings and supermembranes arise in the standard moduli space of toroidal compactifications, while type I’ and heterotic strings arise from a $\mathbb{Z}_2$ orbifold of the N=2 algebra. Yet another vacuum describes M-theory on a $T^5/\mathbb{Z}_2$ orientifold, the type I string on $T^4$, and the six-dimensional self-dual string. We find that open membranes carry ‘Chan-Paton fields’ on their boundaries, providing a common origin for gauge symmetries in M-theory. The world-volume interactions of M-brane fluctuations agree with those of Born-Infeld effective dynamics of the Dirichlet two-brane in the presence of a non-vanishing electromagnetic field on the brane.
1. Introduction

Strong/weak coupling duality relations among the diverse supergravity theories, motivated by our current understanding of string dynamics, have begun to merge into a rather complete picture of a single, unifying entity – M-theory – which reduces to these various effective theories in different limits. These limits include the various critical string theories – type I, type IIA&B, and heterotic – as well as eleven-dimensional supergravity. It is clear from recent developments that there should exist a formalism flexible enough to describe strings, membranes, and perhaps higher p-branes in a unified way. Such a formalism has been lacking until now, and thus so has a basic definition of M-theory; nevertheless, the duality hypothesis has been used to infer a number of its features (see for instance [1] for a recent survey, and [2-13] for further results.).

Recently, two of us [14] showed that a route to the construction of M-theory is provided by the N=(2,1) heterotic string, which describes the dynamics of M-theory in particular backgrounds; this leads for the first time to a unified treatment of strings and membranes that goes beyond their effective low-energy descriptions. The central idea is that the target space theory of the N=(2,1) heterotic string generates the world volume dynamics of the various critical string and membrane theories, and hence serves as a sort of Rosetta stone to deciphering the language of M-theory. This target space theory is fundamentally 2+2 dimensional; however, in the N=(2,1) string the worldsheet gauge algebra includes a null current, which restricts the kinematics to 1+1 or 2+1 dimensions. This construction motivates a definition of M-theory as a theory of 2+2 dimensional membranes (we shall call them M-branes, following [6]) embedded in 10+2 dimensions with a null reduction, whose dynamics is that of self-dual dilaton gravity coupled to self-dual matter. The choice of the null vector which defines the gauged current determines whether one obtains strings or membranes: Strings if the null vector is entirely within the M-brane, membranes if it lies partly in the M-brane world-volume and partly along the transverse directions of the matter field-space.

The N=(2,1) string provides one with a tool to probe the 2+2 dimensional M-brane world-volume theory. Ultimately, one will want to study directly this 2+2 dimensional theory. However, before attempting that, it seems useful to get as much information as possible about aspects of the theory described by N=(2,1) strings. That is the goal of this paper.

The right-movers of the N=(2,1) string live in 2+2 dimensions, the left-movers in 10+2 dimensions. A common setting for the various backgrounds we will consider is
achieved by compactifying all spatial coordinates of the target. One finds a Narain moduli space of vacua \( \mathcal{M} \). At generic points in \( \mathcal{M} \), the target space dynamics is essentially 1+2 dimensional, describing the world-volume of a membrane. A special alignment of the moduli gives the 1+1 dimensional dynamics of strings. Certain solitonic sectors of the theory appear to describe \( p \)-branes.

The picture we find is consistent with string duality. We discuss three main classes of M-theory vacua. Each class contains several seemingly different theories; in our construction, they are continuously connected in the moduli space of vacua \( \mathcal{M} \). The three classes are:

1) Type IIA/B strings and the eleven-dimensional supermembrane. For generic values of the moduli \( m \in \mathcal{M} \), one finds a stretched supermembrane; double dimensional reduction yields the type IIA string, while aligning the vector that defines the null reduction with the membrane world-volume yields the type IIB string. This common setting for these three theories agrees with the fact that type IIA/B strings are related by T-duality, while the eleven-dimensional supermembrane describes the strong coupling limit of the type IIA theory.

2) The heterotic and type I strings, and M-theory on an \( S^1/\mathbb{Z}_2 \) orientifold. The generic point in moduli space describes an open supermembrane; different limits describe (a) the type \( I' \) string with gauge group \( SO(16) \times SO(16) \), and (b) the heterotic string with gauge group \( E_8 \times E_8 \). Aligning the null reduction as above yields the \( SO(32) \) heterotic string. Gauge groups for these theories appear from a common source – twisted sectors of M-theory orientifolds; our construction gives a new derivation of Chan-Paton factors, and clarifies the relation between the sources of gauge groups for type I and heterotic strings. The common setting for the above four theories agrees with the fact that (i) \( SO(32) \) and \( E_8 \times E_8 \) strings, and separately type I and type \( I' \) strings, are related by T-duality; (ii) the heterotic and type I theories are related by strong/weak coupling duality; and (iii) M-theory on \( S^1/\mathbb{Z}_2 \) describes the strong coupling limit of the \( E_8 \times E_8 \) heterotic string.

3) M-theory on a \( T^5/\mathbb{Z}_2 \) orientifold, type IIA strings on a \( T^5/\mathbb{Z}_2 \) orientifold, type IIB strings on \( K3 \), type I strings on \( T^5 \), and the six-dimensional self-dual string \[15\]. The construction describes a membrane whose ends are trapped on five-branes; different limits describe (a) a type IIA two-brane stretched between Dirichlet four-branes, (b) the type IIA string on a \( T^5/\mathbb{Z}_2 \) orientifold, which is equivalent to the type I string on \( T^5 \), and (c) the self-dual string, which carries a ‘Chan-Paton’ current algebra.
This common setting for the different theories agrees with the fact that (i) type IIA theory on a $T^5/Z_2$ orientifold is T-dual to the type I theory on $T^5$; (ii) M-theory on a $T^5/Z_2$ orientifold describes certain strong-coupling limits of type IIB strings on $K3$; and (iii) the type IIB theory on $K3$ with an $A_1$ singularity is equivalent to the eleven-dimensional M-theory with coincident five-branes, both of which lead to a description of the six-dimensional self-dual string.

Let us now describe the plan of the present article. In section two, we review the relevant aspects of the $N=2$ right-moving and $N=1$ left-moving gauge algebras of the $N=(2,1)$ string. A new ingredient is the twisted $N=2$ algebra [16,17] which is needed for some of the orbifolds in section four. In section three, we describe the construction of the type IIB string, and the eleven-dimensional/type IIA supermembrane (item (1) above); here, the right-moving $N=2$ gauge algebra is untwisted.

Section four explores orbifolds/orientifolds of M-theory. We begin with a construction of the type II string/eleven-dimensional supermembrane on $K3$. Then we consider the twisted $N=2$ algebra for the right-movers. Using this algebra, we present a unified treatment of type I and heterotic theories (item (2) above). We also discuss the M-theory orientifold $T^5/Z_2$ (item (3) above).

In section five we resolve a few apparent puzzles regarding our construction of the membrane; the main observation is that there is naturally a non-zero electromagnetic field on the world-volume of the membrane. We summarize our results and discuss directions for future research in section six.

2. Review of $N=(2,1)$ strings, and a twist

The $N=(2,1)$ heterotic string unites chiral $N=2$ local supersymmetry for the right-moving degrees of freedom with antichiral $N=1$ local supersymmetry for the left-moving degrees of freedom. Consistency also requires the introduction of a left-moving anomaly-free $U(1)$ super-current algebra [19]. The right-moving gauge algebra is generated by the $N=2$ superconformal currents $\bar{T}$, $\bar{G}_\pm$, $\bar{J}$. There are two classes of boundary conditions allowed for these currents [16].

First, since the supersymmetry currents $\bar{G}_\pm$ are charged under the $U(1)$ generated by $\bar{J}$, one may twist the former by a Wilson line of the latter. This allows for a continuous

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1 For more on N=2 strings and references to earlier work see e.g. [18,19]. For more background on our construction see [14].
family of boundary conditions that interpolate between the standard Ramond (R) and Neveu-Schwarz (NS) boundary conditions on the $\bar{G}^{\pm}$. Since the $U(1)$ symmetry generated by $\bar{J}$ is gauged, the sectors corresponding to different boundary conditions are identified.

Second, one can impose antiperiodic boundary conditions on the $U(1)$ current $\bar{J}$, together with a $\mathbb{Z}_2$ twist interchanging $\bar{G}^+$ with $\bar{G}^-$; this is usually called the twisted N=2 algebra \[16\]. Such boundary conditions arise when one gauges the above $\mathbb{Z}_2$ automorphism of the N=2 superconformal algebra, and lead to new sectors of the Hilbert space.

In this section, we review these two classes of N=2 algebra, together with the set of ghosts required for construction of the BRST charge and corresponding representatives of physical states for the right-movers \[20\], followed by a review of the relevant aspects of the N=1 algebra of the left-movers. In the next two sections we will combine these two sectors, first for vacua of the N=(2,1) string using the untwisted N=2 algebra, and then for vacua using the twisted N=2 algebra.

2.1. The right-moving N=2 algebra.

The inequivalent representations of the N=2 algebra are characterized by the boundary conditions on the currents. One possibility is to twist by $U(1)$ spectral flow

\[
\begin{align*}
\bar{T}(2\pi) &= \bar{T}(0) \\
\bar{G}^+(2\pi) &= -e^{2\pi i u} \bar{G}^+(0) \\
\bar{G}^-(2\pi) &= -e^{-2\pi i u} \bar{G}^-(0) \\
\bar{J}(2\pi) &= \bar{J}(0).
\end{align*}
\]

(2.1)

For instance, what one usually calls the Neveu-Schwarz (NS) sector has $u = 0$, and the Ramond (R) sector has $u = \frac{1}{2}$. In a theory with local $N = 2$ supersymmetry the $U(1)$ current is gauged, and all such sectors are equivalent, corresponding to different Wilson lines of the gauge field. To construct physical states in such a theory one needs the ghosts for the gauge algebra generated by $\bar{T}$, $\bar{G}_\pm$, $\bar{J}$; these will be denoted by $(\bar{b}, \bar{c})$, $(\bar{\beta}_\pm, \bar{\gamma}_\pm)$, and $(\bar{b}, \bar{c})$, respectively. In particular, vertex operators depend on the fields $\bar{\phi}_\pm$ arising from the bosonization of $\bar{\beta}_\pm, \bar{\gamma}_\pm$ in the usual way \[20\]: $\bar{\beta}_\pm \bar{\gamma}_\pm = \bar{\partial} \bar{\phi}_\pm$; also $\bar{b} \bar{c} = \bar{\partial} \bar{\phi}$.

One may gauge in addition the $\mathbb{Z}_2$ symmetry taking $\bar{J} \rightarrow -\bar{J}$ and exchanging $\bar{G}^\pm$. This gives rise to a sector in which the $N = 2$ algebra is twisted:

\[
\begin{align*}
\bar{T}(2\pi) &= \bar{T}(0) \\
\bar{G}_\nu(2\pi) &= \frac{1}{2}(\bar{G}^+ + \bar{G}^-)(2\pi) = -\bar{G}_\nu(0) \\
\bar{G}_a(2\pi) &= \frac{1}{2}(\bar{G}^+ - \bar{G}^-)(2\pi) = +\bar{G}_a(0) \\
\bar{J}(2\pi) &= -\bar{J}(0).
\end{align*}
\]

(2.2)
In this case it is convenient to use the corresponding combinations of the ghosts which diagonalize the $\mathbb{Z}_2$ twist: $\gamma_v, \beta_v$ and $\gamma_a, \beta_a$; the corresponding bosonizations are $\beta_v \gamma_v = \partial \bar{\phi}_v$, $\beta_a \gamma_a = \partial \bar{\phi}_a$.

In flat spacetime $\mathbb{R}^{2,2}$, the right movers of the $N=(2,1)$ heterotic string are four real scalar fields $x^\mu$, $\mu = 0, 1, 2, 3$ with signature $(-, -, +, +)$, and their superpartners under the $N = 2$ superconformal algebra, $\bar{\psi}^\mu$. The matter $N = 2$ superconformal generators can be written as:

\[
T = -\frac{1}{2} \partial \bar{x} \partial x - \frac{1}{2} \bar{\psi} \partial \bar{\psi} \\
\bar{G}^\pm = (\eta_{\mu \nu} \pm \mathcal{I}_{\mu \nu}) \bar{\psi}^\mu \bar{x}^\nu \\
\bar{J} = \mathcal{I}_{\mu \nu} \bar{\psi}^\mu \bar{x}^\nu
\]

$\eta_{\mu \nu}$ is a metric on $\mathbb{R}^{2,2}$, while the antisymmetric tensor $\mathcal{I}_{\mu \nu}$ corresponds to a non vanishing background field on the world-volume of the brane. The magnitude of this field is determined by the right-moving $N=2$ superconformal algebra; we will discuss its role further in section 5. Throughout this paper we will use $\mathcal{I}_{03} = \mathcal{I}_{12} = 1$.

The superconformal currents (2.2) that diagonalize the $\mathbb{Z}_2$ of the twisted algebra are

\[
\bar{J} = \mathcal{I}_{\mu \nu} \bar{\psi}^\mu \bar{x}^\nu \\
\bar{G}^v = \eta_{\mu \nu} \bar{\psi}^\mu \partial x^\nu \\
\bar{G}^a = \mathcal{I}_{\mu \nu} \bar{\psi}^\mu \partial x^\nu
\]

Standard vertex operators for the untwisted algebra have the form:

\[
V_{-1}(k) = e^{-\bar{\phi} - \bar{\phi}} e^{i k \cdot x} \\
V_{0}(k) = \bar{G}^+ - \frac{1}{2} \bar{G}^- e^{i k \cdot x}
\]

with the first line describing the vertex operator in the $-1$ picture, and the second, in the $0$ picture. To implement the twisted $N = 2$ algebra (2.2) one may for example mod out by the $\mathbb{Z}_2$ symmetry $(x_1, x_3) \rightarrow -(x_1, x_3)$, $(\bar{\psi}_1, \bar{\psi}_3) \rightarrow - (\bar{\psi}_1, \bar{\psi}_3)$, taking $\bar{J} \rightarrow - \bar{J}$, $\bar{G}^v \rightarrow \bar{G}^v$, $\bar{G}^a \rightarrow - \bar{G}^a$ (2.4). Standard vertex operators in the twisted sector have the form:

\[
V_{NS}^{\text{twisted}} = e^{-\bar{\phi} - \bar{\phi}} \Sigma_{13} S_{13} e^{\frac{1}{2} \bar{\phi} e^{i k \cdot x}} \\
V_{R}^{\text{twisted}} = e^{-\frac{1}{2} \bar{\phi} - \bar{\phi}} \Sigma_{13} S_{02} e^{\frac{1}{2} \bar{\phi} e^{i k \cdot x}}
\]

where $\Sigma_{13}$ and $S_{13}, S_{02}$ are the appropriate twist and spin fields that implement (2.2). The two operators $V_{NS}, V_{R}$ (2.6) are identified by the gauged spectral flow.
2.2. The left-moving $N = 1$ sector.

To describe the left-moving sector we need to find a $\hat{c} = 10$ $N = 1$ superconformal field theory (SCFT), to compensate the central charge of the $N=1$ superconformal ghosts. The $2 + 2$ dimensional spacetime coordinates $x^\mu$ (which are shared by the left- and right-movers), and their superpartners $\psi^\mu$ form a $\hat{c} = 4$ SCFT. Naively, anomaly cancellation requires an additional $\hat{c} = 6$, but there is a subtlety \[19\]. The $x^\mu$ live in $2+2$ signature, so the $N=1$ superconformal constraints are insufficient to guarantee elimination of negative-norm states; we need to gauge a null $U(1)$ supercurrent on the left side as well. The ghosts corresponding to this $U(1)$ carry $\hat{c} = -2$ so overall we are looking for an internal $\hat{c} = 8$ ($c = 12$) SCFT. A convenient representation is provided by eight left-moving superfields $y^a + \theta \lambda^a$, $a = 1, \cdots, 8$. The $y^a$ are eight left-moving scalar fields living on the $E_8$ torus for modular invariance (more general possibilities exist and will be discussed below), while $\lambda^a$ are eight Majorana – Weyl fermions. The total $N = 1$ superconformal algebra for the left movers can be taken to be:

$$
T = -\frac{1}{2} \partial x \partial x - \psi \partial \psi - \frac{1}{2} \partial y \partial y - \lambda \partial \lambda
$$

$$
G = \psi \partial x + \lambda \partial y.
$$

As for the $N = 2$ case, there are ghosts $b, c, \beta, \gamma$ needed to gauge (2.7) as well as ghosts of the gauged $U(1)$ current algebra, $\tilde{b}, \tilde{c}$ and their superpartners $\tilde{\beta}, \tilde{\gamma}$. Some vertex operators involve the bosonized ghost currents $\beta \gamma = \partial \phi$, $\tilde{b} \tilde{c} = \partial \tilde{\phi}$, $\tilde{\beta} \tilde{\gamma} = \partial \rho$.

A typical example of the left-moving part of a physical NS vertex operator is

$$
V_{-1}^a = e^{-\phi} \lambda^a e^{ik \cdot x}$$

$$
V_0^a = (\partial y^a + ik \cdot \psi \lambda^a) e^{ik \cdot x}
$$

Note that the 0 picture vertex involves the translation operator in the $y^a$ direction, $\partial y^a$. It is natural to assume that the scalar field $V^a$ lives on the same torus as $y^a$. In particular, if we compactify $(x^i, y^a)$, the target space theory lives on a spatial torus as well. Orbifolds of the (2,1) string that twist $x^i, y^a$ should correspond to spacetime orbifolds. We will use this interpretation in sections three and four.

The statistics of target space fields of the $N=(2,1)$ string appears to be determined solely by the NSR parity of the $N=1$ gauge algebra of the left-movers. In constructions using only the untwisted $N=2$ algebra, one integrates over Wilson line expectation values of a gauged $U(1)$ that interpolate between NS and R states; hence target fermion parity
cannot depend on right-moving fermion boundary conditions. Even in the twisted constructions, instanton operators generate a discrete remnant of spectral flow which flips this parity arbitrarily. Thus we will assign target space statistics according to the NSR parity of the left-movers alone. It would be useful to construct the target space torus partition function in order to verify that the appropriate signs arise from the different sectors.

3. Vacua with maximal supersymmetry.

3.1. The general structure.

To discuss different vacua of the N=(2,1) string in a unified way, it is convenient to compactify the spatial membrane world-volume coordinates \((x^2, x^3)\). We find ten spacelike right-moving scalars \(z^i, i = 2, \cdots, 11\), two left-moving ones \(\bar{z}^i, \bar{i} = 2, 3\), and their superpartners, \(\chi^i, \bar{\chi}^i\):

\[
z^i = (x^2, x^3, y^1, \cdots, y^8); \quad \bar{z}^\bar{i} = (\bar{x}^2, \bar{x}^3)
\]
\[
\chi^i = (\psi^2, \psi^3, \lambda^1, \cdots, \lambda^8); \quad \bar{\chi}^\bar{i} = (\bar{\psi}^2, \bar{\psi}^3)
\]

(3.1)

Including the (non-compact) timelike superfields \((x^\alpha, \psi^\alpha), \alpha = 0, 1\) which may be denoted by \((z^\alpha, \chi^\alpha)\), we have twelve left-moving fields \(z^\mu, \mu = 0, \cdots, 11\), and four right-moving ones \(\bar{z}^{\bar{\mu}}, \bar{\mu} = 0, \cdots, 3\). The left-moving superconformal algebra \((2.7)\) is generated by:

\[
T = -\frac{1}{2} \partial z^\mu \partial z^\mu - \chi^\mu \partial \chi^\mu; \quad G = \chi^\mu \partial z^\mu; \quad \mu = 0, \cdots 11
\]

(3.2)

The \(U(1)\) supercurrent we gauge has the general form:

\[
J = \alpha \cdot \partial z; \quad J|\text{phys}\rangle = 0
\]
\[
j = \alpha \cdot \chi; \quad j|\text{phys}\rangle = 0
\]

(3.3)

\(\alpha\) is an arbitrary null vector whose direction changes under \(SO(2,10)\) Lorentz transformations. For example, we may take \(\alpha\) to point in the \((x^1, x^3)\) direction, so that \(J = \partial x^1 + \partial x^3\), \(j = \psi^1 + \psi^3\).

The momenta of the compact scalars \((z^i, \bar{z}^{\bar{i}}), i = 2, \cdots, 11, \bar{i} = 2, 3\), live on an even self-dual Lorentzian lattice \(\Gamma_{10,2}\). The moduli space of vacua, \(\mathcal{M}\) is locally \(\mathcal{M} \simeq SO(10, 2)/SO(10) \times SO(2)\). Here we want to study some of its properties.

\(\text{\footnote{A word of caution: one should not confuse the two Lorentzian spaces with signature (10,2) discussed here. The left movers of the N=(2,1) string live in a 2+10 dimensional target space parametrized by \(z^\mu\); separately, if we compactify all ten spatial dimensions \((z^i, \bar{z}^{\bar{i}})\) \([3.1]\), their left- and right-moving momenta live on a \((10,2)\) dimensional Narain lattice \(\Gamma_{10,2}\).}}\)

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The theory is supersymmetric in target space. The supercharges form a 16 of \(SO(1, 9)\). They can be constructed in the standard fashion \([20]\) out of the \(SO(2, 10)\) spin fields \(S_\alpha\) for the 12 fermions \(\chi^\mu\), and the bosonized ghost current \(\phi\). Since one is twisting the fermionic current \(j\) \((3.3)\), the supercharges contain the dimension \(-1/8\) twist field \(\exp(\rho/2)\) for the bosonic ghosts \(\tilde{\beta}, \tilde{\gamma}\):

\[
Q_\alpha = \oint dz e^{-\frac{\phi}{2} + \frac{\rho}{2}} S_\alpha.
\]  
(3.4)

The \(SO(2, 10)\) spin field \(S_\alpha\) has 32 components, but only 16 of those satisfy the second gauge condition in \((3.3)\). These are supercharges satisfying

\[
\gamma^1 \gamma^3 Q = Q
\]  
(3.5)

which, taken together with the twelve-dimensional chirality projection

\[
\left( \prod_{\mu=0}^{11} \gamma^\mu \right) Q = Q ,
\]  
(3.6)

implies that the physical supercharges form a Majorana-Weyl spinor of the \(SO(1, 9)\) acting on \((x^0, x^2, y^1, \cdots, y^8)\).

The physical states assemble naturally into a gauge field in 2 + 10 dimensions:

\[
V = e^{-\phi} \chi^\mu e^{-\tilde{\phi}} e^{ik \cdot z + i\tilde{k} \cdot \bar{z}} .
\]  
(3.7)

The polarization and momentum vectors satisfy the familiar physical state conditions:

\[
k^2 = \bar{k}^2 = 0
\]  
\[
k \cdot \xi = 0; \quad \xi \sim \xi + \epsilon k.
\]  
(3.8)

The first line states that \(V\) is massless, while the second is a consequence of gauge invariance in 2 + 10 dimensions.

In addition to \((3.8)\), the gauged supercurrent \((3.3)\) leads to the following constraints on the physical states \((3.7)\):

\[
k^3 + k^1 = 0; \quad \xi^3 + \xi^1 = 0.
\]  
(3.9)

As discussed in \([14]\), this gauge invariance effectively sets \(k^3 = k^1 = \xi^3 = \xi^1 = 0\), removing two target space dimensions. Thus, to count physical degrees of freedom contained in \((3.7)\), we start with twelve polarizations of the 2 + 10 dimensional gauge field \(V\); the null
gauging reduces it to a gauge field in $1+9$ dimensions; gauge invariance \((3.8)\) implies that there are eight physical degrees of freedom.

Supersymmetry \((3.4)\) pairs these eight scalars with eight physical fermionic degrees of freedom:

$$F = u^\alpha(k) e^{-\frac{\phi}{2} + \frac{\bar{\phi}}{2}} S_\alpha e^{-\bar{\phi} + \phi} e^{i k \cdot z + i \bar{k} \cdot \bar{z}}.$$ \(\text{(3.10)}\)

The polarization vector $u$ satisfies a Dirac equation, and a super-$U(1)$ constraint analogous to \((3.5)\):

$$k_\mu \gamma^\mu u = 0; \quad \gamma^1 \gamma^3 u = u.$$ \(\text{(3.11)}\)

The two together describe eight propagating degrees of freedom, in agreement with supersymmetry.

The physical states assemble naturally into a $1+9$ dimensional vector superfield (after imposing the null reduction), but the theory does not in general live in $1+9$ dimensions. The spectrum of operators \((3.7)\), \((3.10)\), and supersymmetry structure \((3.4)\) is identical to that arising in the D-brane construction of \([21]\). By analogy, one would expect to find different sectors of the theory in which the momenta of $V, F$ are $1+p$ dimensional, and their interactions describe $p$-brane world-volume dynamics. In the $\text{N}=(2,1)$ construction, strings ($p=1$) and membranes ($p=2$) are singled out by the decompactification limits of $\Gamma^{10,2}$; if we go to the boundary of moduli space $\mathcal{M}$ where the theory describes two non-compact scalar fields and eight left-moving scalars living on the $E_8$ torus, one finds \([14]\) that the target space of the $\text{N}=(2,1)$ string is either $1+1$ or $1+2$ dimensional, depending on the orientation of the null vector \((3.3)\). We will describe here the string and membrane constructions, noting only that higher $p$-branes originate in mixed momentum/winding sectors of $\Gamma^{10,2}$; we leave a detailed analysis of these higher $p$-branes to future work.

### 3.2. Type II strings and the supermembrane.

An arbitrary Lorentzian even self-dual lattice $\Gamma^{10,2}$ can be brought by an $SO(10,2)$ Lorentz transformation to a standard form

$$\Gamma^{10,2} \simeq \Gamma^{1,1} \times \Gamma^{1,1} \times \Gamma^8$$ \(\text{(3.12)}\)

with the two $\Gamma^{1,1}$ factors describing two compact scalar fields, say $x^2, x^3$ and $\Gamma^8$ describing the remaining left-moving scalars, $y^1, \ldots, y^8$ living on the $E_8$ torus. Different vacua of the $\text{N}=(2,1)$ string can be studied by fixing the $\Gamma^{10,2}$ \((3.12)\), and varying the direction of the null vector $\alpha$ \((3.3)\) with respect to it.
We can choose the timelike component of $\alpha$ to point in the $x^1$ direction; this defines $x^1$. There are three logical possibilities for the spacelike component of $\alpha$ which we consider in turn: it could (a) lie purely in the $\Gamma^{1,1} \times \Gamma^{1,1}$ part of (3.12), (b) lie purely in $\Gamma^8$, or (c) have components in both.

In the first case, the null gauging effectively removes one of the two circles, and we are left with momenta lying in $\Gamma^{9,1} \simeq \Gamma^{1,1} \times \Gamma^8$. For example, we may take $J = \partial x^1 + \partial x^3$, in which case the $\Gamma^{1,1}$ momenta $(k_2, \bar{k}_2)$ describe the compact scalar field $x^2$, and the momentum in the $\bar{y}$ direction $\bar{p}$ belongs to the $E_8$ root lattice, $\Gamma^8$. We will focus on momentum modes of the compact scalar $x^2$. As discussed in [14], winding modes describe another copy of the structure (in this case a “winding string”), while mixed momentum – winding sectors give rise to higher $p$-branes, and will not be discussed here in any detail.

The zero winding number sector on $\Gamma^{1,1}$ together with the timelike non-compact dimension $x^0$ describes a 1+1 dimensional target space $(x^0, x^2)$. The physical spectrum includes a ten-dimensional $U(1)$ gauge superfield $V$, $F$ (3.7), (3.10), dimensionally reduced to two dimensions. This gives rise to an $(8, 8)$ supersymmetric theory of eight scalars and fermions (the transverse components of the gauge field). In [14] this target space theory has been interpreted as the world sheet theory describing a type IIB string in static gauge.

The transverse dimensions of that string are compactified on an eight-torus. The spatial world sheet coordinate $x^2$ is identified with the longitudinal direction on the string, hence the latter is compact as well. Thus we naturally find the type IIB string compactified on a spatial torus $T^0$, with only time, which is identified with the world sheet time coordinate $x^0$ by the static gauge condition, remaining non-compact.

Consider next the case where the null vector $\alpha$ has a spatial component purely in $\Gamma^8$, say $J = \partial x^1 + \partial y^1$. The zero winding sector in the two spatial circles describes this time a 1+2 dimensional target space parametrized by $(x^0, x^2, x^3)$, describing [14] the world-volume of a supermembrane. Dimensional reduction of (3.7), (3.10) gives rise now to seven scalars and a gauge field on the world-volume, as well as eight fermionic states, related by $N=8$ Green-Schwarz supersymmetry on the world-volume.

Type IIA strings appear in this formalism by double dimensional reduction of the membrane construction. It is well known that dimensional reduction of eleven-dimensional

\[ 3 \quad \text{The fact that this is the type IIB string (as opposed to a type IIA one) follows from the structure of the supersymmetry algebra, and in particular from the fact that left- and right-moving fermions } F \text{ (3.10) belong to different spinor representations of Spin}(8). \]
supergravity gives the non chiral type IIA supergravity in ten dimensions. Thus, the type II\(A\) string can be obtained by taking the radius of (say) \(x^3\) to zero in the second construction described above.\(^4\)

Note that we always find sixteen supercharges on the world-volume of the brane, half of the number of supercharges in a covariant description of the theory. The reduction is due to the imposition of static gauge, which allows only supercharges that close on translations along the world-volume of the brane.

The case when the spatial part of the null vector \(\alpha\) has components both in \(\Gamma^8\) and in \(\Gamma^{1,1} \times \Gamma^{1,1}\) is also interesting, as it allows one to continuously go from the membrane to the string vacuum of M-theory. Indeed, suppose:

\[
J = \partial x^1 + a \partial x^3 + b \partial y^1
\] (3.13)

with \(0 \leq a, b \leq 1\), and \(a^2 + b^2 = 1\). As we saw before, for \(a = 0\) (3.13) describes a supermembrane world-volume, while for \(b = 0\) it describes a type IIB string world sheet. If \((x^2, x^3)\) live on circles of radii \((R_2, R_3)\), for general \(a, b\) (3.13) the target space is 1+2 dimensional with the spatial dimensions effectively living on circles of radii \((R_2, bR_3)\); thus, indeed in the limit \(b \rightarrow 0\) the width of the membrane goes to zero and it turns into a string.

To summarize, in this section we have shown that the moduli space of the simplest \(N=(2,1)\) string vacua describes in target space the world-volume dynamics of type IIA/B strings and the eleven-dimensional supermembrane. All theories naturally arise compactified on a spatial torus. Which description is most natural changes as we vary the moduli.

The fact that these vacua of M-theory are closely related is known [22,23]. Compactified type IIA and IIB strings are related by T-duality, while the eleven-dimensional supermembrane and IIA theories are related by strong-weak coupling “duality” (the former is the strong coupling limit of the latter).

4. **Orbifolds/orientifolds of M-theory.**

In section 3 we have studied some vacua of M-theory with the largest possible supersymmetry. This excludes many interesting classes of vacua, such as type I and heterotic

\(^4\) Actually the limit \(R_3 \rightarrow 0\) yields a type IIB string propagating in a spacetime one of whose spatial dimensions is a circle with vanishing radius. T-duality relates that to a type IIA string on a large circle.
theories on tori, as well as type II theories and supermembranes on K3 manifolds. In this section we will examine some such vacua; the discussion is not intended to be exhaustive – the purpose is to demonstrate that many vacua of M-theory with reduced supersymmetry are realized by the N=(2,1) string. Another motivation is to compare the structures emerging from our version of M-theory to predictions of string duality that exist in the literature. We find qualitative agreement with those predictions.

4.1. Type II strings and M-theory on K3.

In section 3 we have discussed a vacuum of the N=(2,1) string describing the worldsheet of a type IIB string compactified on a nine-torus. It is not difficult to describe a type IIB string propagating on $K^3 \times T^5 \times \mathbb{R}$. Recall that the type IIB theory arose when we gauged $J = \partial x^1 + \partial x^3$; the target space of the N=(2,1) string (identified with the world sheet of a type IIB string) was the 1+1 dimensional space parametrized by $(x^0, x^2)$. To study the theory on a $K^3$ orbifold $T^4/\mathbb{Z}_2$ we can orbifold the N=(2,1) string by the discrete symmetry

$$
(y^5, y^6, y^7, y^8) \rightarrow -(y^5, y^6, y^7, y^8),
$$

$$
(\lambda^5, \lambda^6, \lambda^7, \lambda^8) \rightarrow -(\lambda^5, \lambda^6, \lambda^7, \lambda^8),
$$

(4.1)

inverting four of the left-moving spatial coordinates. As explained in section 2, the world sheet fields $y^a$ are essentially identified with the spacetime coordinates, so this orbifold can in fact be thought of as acting in the 1+9 dimensional spacetime in which the type IIB string lives.

Gauging the symmetry (4.1) breaks the $SO(1,9)$ Lorentz symmetry of the vacuum to $SO(1,3) \times SO(6)$. The spacetime supercharges (3.4) constructed in section 3 decompose as $(2, 4) \oplus (\bar{2}, \bar{4})$ under the unbroken Lorentz group. Half of them, say the $(2, 4)$, survive the projection (4.1). The unbroken supercharges form a (2,0) supersymmetry algebra in the six dimensions orthogonal to $K^3$. This chiral supersymmetry is precisely what one expects for a type IIB string on $K^3$ in static gauge.

To find physical states in the untwisted sector one needs to impose invariance under the $\mathbb{Z}_2$ symmetry (4.1) on the states (3.7), (3.10) discussed in section 3. This eliminates four

5 Indeed, the physical vertex operators (3.7) $V^a = e^{-\phi} \lambda^a e^{-\bar{\phi}_+ - \bar{\phi}_- e^i k \cdot x}$ with $a = 5, 6, 7, 8$ are inverted under the action of (4.1) as appropriate for a spacetime orbifold.
of the eight scalars $V$ and their superpartners; the invariant states are the ones describing translations in the $y^1, \cdots, y^4$ directions,

$$V^a = e^{-\phi} \lambda^a e^{-\bar{\phi}+\tilde{\phi}} e^{ik \cdot x}; \quad a = 1, \cdots, 4.$$  \hspace{1cm} (4.2)

In addition, there are four massless scalars (and their superpartners) coming from the twisted sector:

$$W^{i \alpha} = e^{-\phi} \Sigma^i S^\alpha e^{-\bar{\phi}+\tilde{\phi}} e^{ik \cdot x}.$$  \hspace{1cm} (4.3)

Here $\Sigma^i$ are twist fields for the chiral scalars $y^5, \cdots, y^8$; $S^\alpha$ are spin fields constructed out of $\lambda^1, \cdots, \lambda^4$. On a $T^4/\mathbb{Z}_2$ there are sixteen fixed points, but the $y^a$ are chiral scalars, so there are four twist fields $\Sigma^i$ (which can be described as spin fields after fermionization of $y^a$). The spin fields $S^\alpha$ belong to a 2 of $SO(4)$; their chirality is fixed by the GSO projection (or equivalently by mutual locality with the unbroken supercharges). Finally, out of the four chiral twist fields $\Sigma^i$ only two survive the GSO projection. Thus, (4.3) describes four scalar fields in the two-dimensional target space $(x^0, x^2)$; they can be thought of as Nambu-Goldstone modes for the translation invariance broken by the orbifold (4.1). The spectrum is in agreement with what one expects; the scalars $V^a, W^{i \alpha}$ are the transverse coordinates of the type IIB string, living in $T^4, T^4/\mathbb{Z}_2$ respectively.

One can repeat the analysis for the eleven-dimensional vacuum of M-theory. Gauging, as in section 3, $J = \partial x^1 + \partial y^1$, we find in the target space of the $N=(2,1)$ string a membrane world-volume $(x^0, x^2, x^3)$. Twisting by (4.1) leads to the eleven-dimensional theory on $K3$. The supersymmetry and spectrum are similar to those discussed above; the only difference is that one of the scalar fields (4.2), $V^1$ is eliminated by the gauge conditions and is replaced by a three-dimensional gauge field (which in three dimensions is equivalent to a scalar ). Double dimensional reduction of the supermembrane theory on $K3$ leads to a type IIA string, as before.

According to string duality, type IIB strings on $K3$ have a large duality group $SO(21, 5; \mathbb{Z})$. This symmetry would only become apparent in our formalism after second quantization of the $N=(2,1)$ string, and it is not surprising that we find a unique theory in the restricted class of vacua described by the $N=(2,1)$ string. It has also been argued \cite{22,8} that a compactification of the eleven-dimensional M-theory on $T^5/\mathbb{Z}_2$ is equivalent to the type IIB string on $K3$, and so in the next subsection we turn to a set of constructions that will allow us to study that equivalence.

\footnote{\hspace{1cm} Indeed, they transform as $2 \times \bar{2} = 4$ under the $SO(4)$ acting on $T^4/\mathbb{Z}_2$.}
String duality also suggests that the eleven-dimensional theory and the type IIA string compactified on $K3$ are equivalent to the heterotic string compactified on a three- and four-torus, respectively. That duality plays an important role in analyzing the strong coupling dynamics of heterotic and type II strings in dimensions four through seven. The IIA – heterotic duality will make an appearance in the constructions of the next subsection.

4.2. Twisted $N=(2,1)$ strings and open membranes.

In this subsection we will discuss a class of vacua of M-theory in which, in addition to $N=(2,1)$ chiral world-sheet supergravity, we gauge the $Z_2$ automorphism \((2.2)\) of the $N=2$ superconformal algebra discussed in section 2. This leads to M-theory on various orientifolds. Orientifolds arise rather generally in our construction since the $N=(2,1)$ string target space describes (stretched) strings and membranes in static gauge. Thus, some coordinates play a double role as both M-brane world-volume and spacetime coordinates. Twisting these coordinates leads to combined world-sheet/spacetime orbifolds or orientifolds (as well as $p$-branes).

We start with a description of M-theory on an $S^1/Z_2$ orientifold, and relate it to heterotic and type I string theories. Then we move on to a more intricate example: M-theory on a $T^5/Z_2$ orientifold, which is expected to describe certain strong coupling limits of type IIB strings on $K3$, as well as type I (and heterotic) strings on $T^4$. In all cases, the relations between the different vacua we find agree with expectations from string duality.

a) Type I and heterotic strings.

Type I and heterotic strings with gauge group $SO(32)$ are believed to be related by strong-weak coupling duality \([2.24]\). In fewer than ten non-compact dimensions they are furthermore related to the $E_8 \times E_8$ heterotic string by heterotic T-duality. The latter is in turn described at strong coupling by a certain $Z_2$ orientifold of the eleven-dimensional version of M-theory \([3]\). In this subsection we will construct a class of $N=(2,1)$ string vacua which makes these relations manifest. In different regions of the moduli space of $N=(2,1)$ string vacua the theory we will construct looks like:

1. M-theory on $S^1/Z_2$ (or equivalently the $E_8 \times E_8$ heterotic string).
2. The $SO(32)$ heterotic string (with gauge group possibly broken to $SO(16) \times SO(16)$ by Wilson lines).
3. The type I’ theory with gauge group $SO(16) \times SO(16)$ (which is T-dual to a type I theory with the same gauge group).
In the spirit of the discussion of section 3, we compactify the spatial coordinates \((z^i, \bar{z}^i)\) (3.1) on \(\Gamma^{1,1} \times \Gamma^{1,1} \times \Gamma^8\) (3.12). Then twist by:

\[
\begin{align*}
(x^1, x^3, y^1, \ldots, y^8) & \to -(x^1, x^3, y^1, \ldots, y^8) ; \\
(\psi^1, \psi^3, \lambda^1, \ldots, \lambda^8) & \to -(\psi^1, \psi^3, \lambda^1, \ldots, \lambda^8) ; \\
(\bar{\psi}^1, \bar{\psi}^3) & \to -(\bar{\psi}^1, \bar{\psi}^3) ;
\end{align*}
\]

(4.4)

Note that the \(Z_2\) symmetry (4.4) twists the left- and right-moving gauge algebras in the manner described in section 2:

\[
\begin{align*}
\bar{G}^+ & \leftrightarrow \bar{G}^- ; \\
\bar{J} & \to -\bar{J} \\
G & \to G ; \\
J & \to -J
\end{align*}
\]

(4.5)

Thus, in the twisted sector of the orbifold the \(N = 2\) algebra will be twisted. Also, (4.4) describes an asymmetric orbifold. The fields \(y^1, \cdots, y^8\) are chiral, while \(x^1, x^3\) have both left- and right-moving components. Correspondingly, the orbifold (4.4) is chiral for the \(y^i\) and non-chiral for \(x^1, x^3\).

The timelike component of the gauged current \(J\) (3.3) can be chosen to be \(\partial x^1\), without loss of generality. To specify the vacuum, one needs to fix the orientation of the spatial component of \(J\). Some general properties of the resulting vacua are independent of that choice and we will discuss these first.

Half of the supercharges (3.4) are broken by the orbifold (4.4). The surviving supercharges satisfy:

\[
\gamma^0 \gamma^2 Q = Q.
\]

(4.6)

The physical states fall into two classes. In the untwisted sector we have the eight scalar modes (3.7), and their superpartners under the unbroken supersymmetries (4.6). The twisted sector gives rise to sixteen fermionic states from each of the two fixed points of the (non-chiral) \(x^3\) orbifold:

\[
F^i = e^{-\frac{1}{2} \phi} \sigma \Sigma^i e^{-\bar{\phi}} e^{ik \cdot x}
\]

(4.7)

where \(\Sigma^i, i = 1, \cdots, 16\) are the chiral twist fields for \((y^1, \cdots, y^8)\), and \(\sigma\) contains the necessary twist and spin fields for \(x^\mu, \psi^\mu, \bar{\psi}^\mu\) and the \(U(1)\) ghosts:

\[
\sigma = \sigma_{13} S_{02} \bar{S}_{13} e^{\frac{1}{2} \bar{\phi} + \frac{1}{2} \bar{\phi}}
\]

(4.8)

The detailed structure of the theory depends on the particular construction. We next describe the different possibilities. As for other constructions encountered in this paper,
one can pass between the various theories we find by changing the Narain moduli of the \( N=(2,1) \) string.

To get M-theory on \( S^1/Z_2 \) we choose the gauged \( U(1) \) current to be:

\[
J = \partial x^1 + \partial y^1.
\] (4.9)

Before gauging (4.4) this gives rise to the eleven-dimensional supermembrane theory with world-volume \( (x^0, x^2, x^3) \) described in section 3. The orbifold introduces two boundaries on the world-volume, at \( x^3 = 0, \pi R_3 \), and correspondingly two boundaries in spacetime in the direction identified with \( x^3 \) by the static gauge condition.

To analyze the spectrum we are instructed to first find the states in the untwisted sector (3.7), (3.10) that are invariant under (4.4). The physical polarizations of the bosons \( V \) (3.7) that survive the projection are:

\[
V^a = e^{-\phi} \lambda^a e^{-\bar{\phi}_a} e^{ik \cdot x} \cos k_3 x^3; \quad a = 2, \ldots, 8 \] (4.10)

and the three-dimensional gauge field

\[
\xi_\mu A^\mu = e^{-\phi} \bar{\phi}_a e^{ik \cdot x} \left( \xi_0 \psi^0 \sin k_3 x^3 + \xi_2 \psi^2 \sin k_3 x^3 + \xi_3 \psi^3 \cos k_3 x^3 \right). \] (4.11)

In the above two equations \( k \) denotes the momentum in the \( (x^0, x^2) \) directions; \( \xi \) is the polarization of the gauge field. Dualizing the gauge field \( A^\mu \) via \( \epsilon_{\mu\nu\lambda} F^{\mu\nu} = \partial_\lambda \Phi \) we find (4.10), (4.11) that both \( V^a \) and \( \Phi \) satisfy Neumann boundary conditions at \( x^3 = 0, \pi R_3 \). Thus the theory contains eight fluctuating fields both in the bulk of the membrane and on the boundary\(^7\). It describes a membrane of the eleven-dimensional theory stretched between two nine-branes with world-volumes parametrized by \( \{x^0, x^2, \Phi, V^a\} \); in the original presentation, one has what would be described in type IIA language as a Dirichlet two-brane stretched between Dirichlet eight-branes (see figure 1). Note, however that there is a basic distinction between our description of the membrane and that of type IIA theory\(^7\).

In that theory the IIA string is taken to be fundamental, and the two-brane is treated as a soliton; our theory treats the two-brane as fundamental. The unbroken supersymmetry (4.6) pairs the scalars with eight fermions \( F \) (3.10). It is not obvious at this level what space the bosonic physical excitations \( (V^a, \Phi) \) live on. We believe that they live on an eight-torus, but a definite answer must await a more detailed analysis.

\(^7\) In particular, (4.4) describes an \( S^1/Z_2 \) orientifold and not a \( T^9/Z_2 \) one as one might have thought.
Next we turn to the twisted sectors of the orbifold. On each of the two-dimensional boundaries of the world-volume, which are labeled by \((x^0, x^2)\), we find sixteen chiral fermions (4.7) that are inert under the unbroken supersymmetry. In the limit \(R_3 \rightarrow 0\), the N=(2,1) string target space becomes two-dimensional; the fields living on the world-volume are eight massless scalars \(V^a, \Phi\) parametrizing \(T^8\), their eight right-moving superpartners \(F^\alpha\), and thirty-two left-moving fermions \(F^i\), sixteen from each of the fixed points. The charges (4.6) form an \((8,0)\) supersymmetry algebra, \(\{Q_\alpha, Q_\beta\} = \delta_{\alpha\beta}(p^0 + p^2)\). This is the right structure for a heterotic string compactified on \(T^8\). For finite \(R_3\) we find an open membrane with finite extent. This is precisely the structure expected for the \(E_8 \times E_8\) string at finite coupling [3]. Our analysis provides new evidence for the \(E_8\) current algebras conjectured in [3] to live on the boundaries of the membrane world-volume.

![Diagram](image)

**Figure 1.** A Dirichlet two-brane ends on a stack of Dirichlet eight-branes; 2-8 strings generate ‘Chan-Paton’ dynamics on the two-brane boundary. These degrees of freedom are the universal source of fundamental gauge symmetry in M-theory.

The \(SO(32)\) heterotic string vacuum of the theory is obtained by gauging

\[
J = \partial x^1 + \partial x^3. \tag{4.12}
\]

This gives a theory living in a 1+1 dimensional target space parametrized by \((x^0, x^2)\). The spectrum and supersymmetry algebra (4.6), (4.7) are again those of the heterotic string. The fermions \(F^i\) still split into two groups of sixteen coming from each of the two fixed points of the \(x^3\) orbifold, but now because of the null gauging (4.12) there is nothing in
the theory that can sense the separation in the \( x^3 \) direction. Thus, the thirty two fermions \( F \) appear symmetrically, just as one would expect for an \( SO(32) \) heterotic string. It is nevertheless possible that a non-perturbative analysis of the \( N=(2,1) \) string would reveal that \( SO(32) \) is broken by a Wilson line to \( SO(16) \times SO(16) \). We leave this question to future work.

The type I\( \prime \) string is in our formulation a close relative of the \( E_8 \times E_8 \) heterotic string, corresponding to a different dimensional reduction of the membrane discussed above. Gauging (4.9) again we get a membrane with world-volume \((x^0, x^2, x^3)\). To get the weakly coupled \( E_8 \times E_8 \) heterotic string we took the radius \( R_3 \) to zero. If instead we consider the limit \( R_2 \to 0 \) we find a theory with target space \((x^0, x^3)\); \( x^3 \) lives on a line segment. The target space theory describes a type II\( A \) string stretched between two eight-branes/orientifolds – a type I\( \prime \) world-sheet. The coordinate \( x^3 \) that is being twisted (4.4) plays, due to the static gauge condition, a dual role – it is both a world-sheet and a spacetime coordinate. Theories of this sort have been considered in the past \cite{24} and play an important role in type I T-duality as well as in type I – heterotic string duality \cite{2}.

The structure of the theory is compatible with the \( SO(16) \times SO(16) \) type I\( \prime \) string. Half of the supersymmetries are broken by the presence of the boundaries on the world sheet. The states that live in the bulk of the world-sheet are eight scalars and their eight superpartners. On the boundaries of the world-sheet, at \( x^3 = 0, \pi R_3 \) (which describe two orientifold planes), one finds sixteen fermionic states \( F^i \). In this case they are quantum mechanical degrees of freedom; the mass shell condition sets the momentum \( k \) in (4.7) to zero. Quantizing the theory, the \( F^i \) provide discrete anticommuting labels for states living on the boundaries of the world-sheet – they are the \( SO(16) \times SO(16) \) Chan-Paton labels.

It is satisfying to see that the gauge groups in type I and heterotic string theories, whose origins seemed quite different in the past, actually stem from the same source. In both cases the gauge group originates in the fermionic states \( F^i \) in the twisted sector (4.7) which, depending on the choice of the null current \( J \), give rise either to a current algebra on the world-sheet of a heterotic string or to Chan Paton labels living on the boundary of a type I world-sheet.

In hindsight, this common source of internal symmetry is not at all surprising. One can understand the appearance of the degrees of freedom \( F^i \) on the boundary of the open membrane by passing to the type IIA string description of figure 1. Our construction naturally gives us fundamental two-branes attached to a stack of eight-branes. Type IIA string theory also describes these objects, but as solitons. In that description, one has
‘2 − 8’ open strings in the theory [24], having one end on the two-brane and the other on an eight-brane. These strings are bound to the boundary of the two-brane where it attaches to the eight-brane. Thus one expects the one-dimensional boundaries of the open membrane to carry fields with group indices transforming under $U(n_2) \times U(n_8)$ [25]. The orientation projection reduces this symmetry to a combination of orthogonal/symplectic groups. These fields are the $F^i$ of (4.7). To see that these degrees of freedom should be identified as the source of heterotic gauge symmetry, perform a T-duality transformation on the membrane coordinate $x^3$ transverse to the boundary. This converts the two-brane to a one-brane (with worldsheet $(x^0, x^2$)), and turns the eight-brane into a nine-brane (again in the type IIA string effective description). The group indices are now carried by ‘1 − 9’ strings; we find the solitonic description of the heterotic string in the type I theory [2].

b) M-theory on $T^5/Z_2$.

In recent developments in string duality, an important role was played by type II theories compactified on $K3$ surfaces [22]. Type IIA string theory on $K3$ is dual to the heterotic string on $T^4$, which in turn is equivalent to type I on $T^4$. When the $K3$ CFT approaches certain singular points in its moduli space, the type II string exhibits interesting non-perturbative effects. For example, in the region of moduli space where a two-cycle on the $K3$ shrinks to zero size (i.e. an $A_1$ singularity develops) and certain Kalb-Ramond fields are suitably adjusted [26], the type IIA theory has additional light gauge fields, enhancing the gauge group to $U(2)$ (from $U(1) \times U(1)$), while a type IIB string near an $A_1$ singularity has a self-dual string with tension that goes to zero as one approaches the singularity. The states becoming light in these two cases can be thought of as arising from two- and three-branes, respectively, wrapping around the vanishing cycle. M-theory on a $T^5/Z_2$ orientifold provides a useful picture of all these phenomena, and we will discuss it in this subsection. We start with a brief explanation of the role of open membranes in type IIB theory on $K3$ which will prove useful for interpreting the results.

It has been shown recently [27] that a type II string theory on $\mathbb{R}^4/Z_2$, with a certain $Z_2$ discrete torsion turned on, describes a vacuum with an orientifold plane and an NS-NS five-brane sitting at the fixed point of the orbifold. That result (as well as the related work of [28,29]) allows one to study the behavior of type II theories on $K3$ in a simple manner.

Consider first the type IIA theory on a $K3$ of the form $(S^1)^4/Z_2$. T-duality on one of the circles turns this into a type IIB string and furthermore turns on the requisite
discrete torsion \cite{27}. The type IIB string thus lives on a four-torus with sixteen orientifold planes located at the fixed points of the orbifold, and sixteen NS-NS five-branes that are initially located at the orientifold planes as well\textsuperscript{8}. After the T-duality transformation, the blowing-up modes of the original orbifold that allow one to move in the moduli space of type IIA on $K^3$ turn into translational modes that move the five-branes away from the orientifold planes (the discrete torsion is essential to this interpretation, turning degrees of freedom that transform as $3 \oplus 1$ of the tangent space $SO(4)$ into the 4 required for Nambu-Goldstone modes). Note that this picture provides a global description of the $K^3$ moduli space.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Relation between $T^4/Z_2$ K3 orbifold and equivalent configuration of five-branes and orientifold five-planes. A vanishing two-cycle on the K3 is equivalent to pair of five-branes approaching one another.}
\end{figure}

The generic singular limit of the $K^3$ CFT corresponds in the T-dual language to a limit in which two of the five-branes approach one another (see figure 2). When the five-branes are well separated, states corresponding to strings with one end on each of the five-branes\textsuperscript{9} are massive. In the limit of coinciding five-branes their mass goes to zero; these open strings give rise to the additional light gauge fields that are responsible for the singularity.

\textsuperscript{8} One can use the $SL(2,\mathbb{Z})$ symmetry of type IIB string theory to turn this type IIB string into one living on a D-manifold with sixteen orientifolds and sixteen Dirichlet five-branes attached to them.

\textsuperscript{9} These are D-string states if the five-branes are NS-NS and fundamental string states after the $SL(2,\mathbb{Z})$ transformation of the previous footnote.
in the CFT (at least this is the interpretation of the singularity if one approaches it keeping $g_{\text{string}}$ fixed).

A similar story occurs for the type IIB string on $(S^1)^4/Z_2$ \([15,13]\). T-duality relates it to a type IIA string on a four-torus with sixteen orientifold planes corresponding to the fixed points of the orbifold and sixteen NS-NS five-branes. Again, the CFT becomes singular when two or more of the five-branes approach each other. This time the objects that can end on the NS-NS five-branes are Dirichlet two-branes. These objects become light in the limit where the two five-branes coincide, transforming into tensionless self-dual strings embedded in the five-brane. One of our purposes in this section is to study these strings.

An eleven-dimensional interpretation is also available \([7,8]\), and is closely related to the above ten-dimensional discussion. Compactification of the eleven-dimensional theory on $T^5/Z_2$ should be equivalent to type IIB theory on a $T^4/Z_2$ $K3$ orbifold when one of the twisted $S^1$’s shrinks away. Anomaly cancellation requires 16 five-branes to be distributed among the 32 fixed points of the orbifold \([8]\). Again, tensionless strings appear in the limit when two of these five-branes approach one another \([5,6]\).

In section 3 we discussed the eleven-dimensional supermembrane vacuum of M-theory by gauging the null vector $J = \partial x^1 + \partial y^1$. The membrane world-volume was parametrized by $(x^0, x^2, x^3)$. Here we will construct the state in M-theory on $T^5/Z_2$ which corresponds to a stretched open membrane with ends restricted to five-branes. To do that we split the 2+10 dimensions of spacetime into two groups of 1+5, and twist one of the two groups. For example, we can orbifold by the $Z_2$ twist:

\[
(x^1, x^3, y^1, y^2, y^3, y^4) \rightarrow -(x^1, x^3, y^1, y^2, y^3, y^4); \\
(\psi^1, \psi^3, \lambda^1, \lambda^2, \lambda^3, \lambda^4) \rightarrow -(\psi^1, \psi^3, \lambda^1, \lambda^2, \lambda^3, \lambda^4); \\
(\bar{\psi}^1, \bar{\psi}^3) \rightarrow -(\bar{\psi}^1, \bar{\psi}^3);
\]

Note that while the fields $y^1, \cdots, y^4$ are chiral, $x^1, x^3$ are non-chiral; consequently, the orbifold (4.13) is chiral for the $y^a$ but is a standard, left-right symmetric one for $x^1$ and $x^3$.

The twist (4.13) breaks half of the supersymmetries, just as in the $K3$ compactifications of type II and eleven-dimensional theories described above. The orbifold also introduces boundaries both on the membrane world-volume and in spacetime. To see their role we now turn to the physical excitation spectrum.
The analysis of the physical spectrum proceeds again in two stages. In the untwisted sector of the orbifold we find (compare to (4.10), (4.11)) eight scalars living in the bulk of the membrane. Half of them, \((V^2, V^3, V^4, \Phi)\), satisfy Neumann boundary conditions on the ends of the open membrane; their vertex operators have the form (4.10). The other half \((V^5, \cdots, V^8)\) satisfy Dirichlet boundary conditions, and so are fixed at the boundaries; their vertex operators also have the form (4.10), but with \(\cos(k_3x^3)\) replaced by \(\sin(k_3x^3)\) in order to survive the twist (4.13). Note that just as in the previous subsection, the orientifold \(T^5/Z_2\) is not directly related to the fields that appear in (4.13). Rather, the \(T^5/Z_2\) is parametrized by \((V^5, \cdots, V^8)\) and \(x^3\) (which serves as both a membrane world-volume and spacetime coordinate in static gauge). The world-volume of the five-brane is parametrized by \((x^0, x^2, V^2, V^3, V^4, \Phi)\).

The structure in the untwisted sector is similar to that described in [5,6]. The membrane with transverse coordinates \(V^a, \Phi\) has two ends stuck on five-branes. The boundary dynamics is described by the six-dimensional self-dual string of [15]. As we bring the two five-branes closer together, by taking \(R_3 \to 0\), the string on the boundary becomes tensionless.

The twisted sector of the orientifold doesn’t seem to have been addressed previously. It would describe states that live on the boundary of the membrane, which is a string, and could potentially modify the boundary dynamics. We thus turn next to the twisted sector.

The physical spectrum in the twisted sector includes the following bosonic states

\[
W^{i\alpha} = e^{-\phi} \sigma^{i} S^{\alpha} e^{-\bar{\phi}_e - \frac{i}{2} \bar{\phi}_o} e^{ik \cdot x}
\]

where \(\sigma\) is a combination of a twist field for \((x^1, x^3)\), spin fields for \((\psi^1, \psi^3)\) and \((\bar{\psi}^1, \bar{\psi}^3)\), twist fields for the left- and right-moving \(U(1)\) ghosts, and a twist field for the left-moving super \(U(1)\) ghosts:

\[
\sigma = \sigma_{13} S_{13} \bar{S}_{13} e^{\frac{i}{2} \phi + \frac{i}{2} \phi_0 + \frac{i}{2} \rho}
\]

\(\Sigma^i\) and \(S^\alpha\) are chiral twist fields for \((y^1, y^2, y^3, y^4)\) and spin fields for the corresponding fermions. The 1+2 dimensional membrane world-volume \((x^0, x^2, x^3)\) has two 1+1 dimensional boundaries corresponding to the two fixed points of the \(x^3\) orbifold (4.13); four scalar fields (4.14) (and their four superpartners under the unbroken supersymmetry) live on each boundary (see the discussion after equation (4.13) for the counting of states).

What are we to make of the operators \(W^{i\alpha}\)? They are analogs of the sixteen fermionic operators \(F^i\) (4.7) that we found in our \(E_8 \times E_8\) heterotic construction. There, they
described the $E_8$ current algebra degrees of freedom living on the boundaries of the open membrane. We found that the existence of these degrees of freedom could be deduced by considering (in type IIA language) $2 - 8$ strings that are bound to the boundary of the open membrane. In the current situation, the boundaries of the open membranes are described in type IIA language as residing on Dirichlet four-branes; the $2 - 4$ strings that pass between the two-brane and four-brane are bound to their intersection. These degrees of freedom generate the ‘Chan-Paton fields’ $W^{i\alpha}$. The presence of these fields on the boundaries of the open membrane clearly affects the dynamics of the six-dimensional self-dual string (in particular the latter secretly lives in more than six dimensions). We will leave a more detailed description of the self-dual string to future work, and conclude this subsection with a few comments on other theories described by the $T^5/Z_2$ orientifold of M-theory, and an observation about our construction of self-dual strings.

A T-duality transformation on $(x^3, V^5, \cdots, V^8)$ relates the type IIA theory on a $T^5/Z_2$ orientifold to the type I string on $T^5$; the Dirichlet four-branes become the nine-branes of the D-brane interpretation of type I string theory, while the two-brane is converted into a five-brane. Alternatively, double-dimensional reduction along $x^2$ of the eleven-dimensional description of the membrane turns it into an open string, and the five-branes on which it ends into four-branes. The theory again describes a type IIA string on a $T^5/Z_2$ orientifold, and again is equivalent by T-duality to the type I string on $T^5$. Dimensional reduction in one of the directions transverse to the fivebrane (e.g. $V^8$) leads to a type IIA string state with a Dirichlet two-brane ending on NS-NS fivebranes. As discussed in the beginning of this subsection this is a T-dual description of the sector of the type IIB theory that becomes massless as the theory approaches an $A_1$ singularity. Lastly, consider T-duality on the membrane coordinate $x^3$ transverse to the IIA four-brane; this operation turns the four-brane into a Dirichlet five-brane, and the membrane into a D-string, which is embedded in the five-brane. In this way, one can make contact with the ideas of [28].

In this subsection, we have described the open membrane of M-theory through an orientifold which twists half of the 10+2 spacetime coordinates. The boundary of the membrane is a one-dimensional object living in a five-brane sitting on the orientifold plane. These boundaries are the non-critical strings of [15]. This self-dual string appears as the boundary dynamics of an open membrane whose boundary lies in a five-brane [5,6]. The coincident five-brane limit isolates the boundary dynamics by freezing the bulk oscillations of the membrane. One might imagine that another way of isolating the boundary
5. Comparison of M-brane and D-brane fluctuations

When the target world-volume theory of the N=2 string is 2+1 dimensional, its field content and supersymmetries are compatible with those of the Dirichlet two-brane of ten-dimensional type IIA strings: seven scalars $V^a$, a vector $A$, and their eight fermion superpartners. However, as noted in [14], in the N=2 string there is a three-point interaction between two $V$’s and an $A$:

$$\langle V^a(1)V^b(2)A(3) \rangle = \delta^{ab}(k_1 \cdot \bar{k}_2 - k_2 \cdot \bar{k}_1) \frac{1}{2} \xi \cdot (k_1 - k_2), \quad (5.1)$$

where $\xi$ is the polarization of the gauge field. Such a coupling does not appear in the effective action of the two-brane in trivial backgrounds (it is not even Lorentz invariant in 2+1 dimensions), so one might wonder if the proposed identification is correct.

In this section we explore the precise relationship between the 2+1 dimensional version of the M-brane and the type IIA Dirichlet two-brane. Our main result is that (5.1) also
arises in the Dirichlet two-brane effective action in the presence of a constant background world-volume electromagnetic field (actually the two-brane couples to the gauge-invariant combination $\mathcal{F} = dA - B$, where $B$ is the universal antisymmetric tensor gauge potential of string theory). The strength and polarization of this background $\mathcal{F}$ field are related to the N=2 U(1) current $\mathcal{J} = \mathcal{I}_{\mu\nu} \bar{\psi}^\mu \psi^\nu$ by $\mathcal{I}_{\mu\nu} \propto \mathcal{F}_{\mu\nu}$; $\mu, \nu = 0, 2, 3$ (maintaining our convention that the left-moving null current is $J = \partial x^1 + \alpha_i \partial z^i$).

The effective dynamics of the type IIA two-brane is governed by the Born-Infeld action

$$S = \int d^3x \left[ e^{-\phi} \sqrt{\det(G + \mathcal{F})} \right].$$

(5.2)

Here $G$ is the induced metric $G_{\mu\nu} = \partial_\mu V^a \partial_\nu V^a$. Expand about the static gauge solution $V^a = \delta^a_\mu x^\mu + v^a$ in the presence of a constant background electromagnetic field $\mathcal{F}_{\mu\nu} = F_{\mu\nu} + \partial_{[\mu} a_{\nu]} = F_{\mu\nu} + f_{\mu\nu}$ and dilaton $e^{-\phi} = 1/\lambda$:

$$\det^{1/2}[G + \mathcal{F}] = \det^{1/2}[G] \left[ 1 + \frac{1}{2} G^{\mu\nu} G^\rho_\sigma \mathcal{F}_{\mu\rho} \mathcal{F}\nu_\sigma \right]^{1/2}$$

(5.3)

The background field $F$ breaks 2+1 dimensional Lorentz symmetry down to the one-dimensional subgroup leaving $F$ fixed. For instance, if $F$ is a magnetic field, only spatial rotations remain. In such a situation, one generically does not expect a Lorentz-invariant effective action to result upon expanding (5.3) in powers of the fluctuations. Remarkably, for a background magnetic field $F_{\mu\nu} = F_{\epsilon_0\mu\nu}$ with $\Lambda = [1 + \frac{1}{2} F_{\mu\nu} F^\mu_\nu]^{1/2}$, the rescaling $x^0 \to \Lambda x^0, a_{2,3} \to \Lambda a_{2,3}$ restores Lorentz invariance of the kinetic terms for both $v^a$ and $a_\mu$.

A similar modification works for other orientations of the background electromagnetic field. This rescaling simultaneously produces a Lorentz covariant form for the cubic interaction term (5.1). To cubic order we have

$$S = \frac{1}{\lambda} \int d^3x \left[ \Lambda^2 + \frac{1}{2} \partial v^a \partial v^a + \frac{1}{4} f^{\mu\nu} f_{\mu\nu}ight.$$

$$\left. - \Lambda^{-1} \partial^\mu v^a \partial^\nu v^a (F_{\mu\rho} f^{\rho}_{\nu} - \frac{1}{4} \eta_{\mu\nu} F_{\lambda\rho} f^{\lambda\rho}) + \ldots \right],$$

(5.4)

in perfect agreement with the N=2 string result (5.1) provided $F_{\mu\nu}/(1 + \frac{1}{2} F^2)^{1/2} = \mathcal{I}_{\mu\nu}$. This relation fixes the background electromagnetic field to a value determined directly by the choice of N=2 U(1) current. In other words, the background electromagnetic field breaks $O(1,2)$ symmetry on the D-brane down to $O(2)$ or $O(1,1)$ in precisely the same way that the N=2 U(1) current breaks $O(2,2)$ symmetry on the M-brane down to $U(1,1)$ or $GL(2, \mathbb{R})$. Since the normalization of the U(1) current is fixed by the central charge of the N=2 algebra, we see once again that the N=(2,1) string produces a very specific background of M-theory.
6. Discussion

6.1. Summary.

Using the definition of M-theory proposed in [14], we have initiated the detailed analysis of its vacua. We have discussed three broad classes of vacua. In section three, we showed that type IIA/B string and eleven-dimensional supermembrane theories are different limits within a single moduli space. In section four, we presented two additional classes, corresponding to M-theory on $S^1/Z_2$ and $T^5/Z_2$ orientifolds. The first of these describes type I and heterotic string theories; the second yields type II strings on K3, type I strings on $T^4$, and the six-dimensional self-dual string.

These constructions provide valuable insight into the nature of M-theory. Among the important lessons are:

i) Our results give strong new evidence for the validity of many conjectures regarding the properties of M-theory, its relation to different string vacua, and the various (S-, T-, and U-) dualities that relate them. Conversely, this evidence strongly supports the proposal of [14] for the dynamical structure underlying M-theory.

ii) The unified picture of string and membrane world-volume theories treats them as different reductions of 2+2 dimensional self-dual dynamics. This self-duality structure is imposed by the right-moving N=2 dynamics of the (2,1) string. The spacetime in which these 2+2 ‘M-brane’ world-volumes is embedded is 10+2 dimensional. This spacetime information is carried by the left-moving N=1 part of the (2,1) string, which typically has eight physical massless bosonic excitations together with fermionic partners related by Green-Schwarz type supersymmetry. The 2+2 dimensions of the world-volume together with these eight bosonic Nambu-Goldstone modes show that the world-volume is embedded in 10+2 dimensions.

iii) Our formalism treats the modes on the M-brane in a unified fashion as the dimensional reduction of a twelve-dimensional gauge field. Reminiscent of the D-brane construction [21], the M-brane splits these into a gauge field on the brane (the Kahler vector potential) and a set of scalars (the transverse fluctuations of the brane). Higher-dimensional branes make their appearance in mixed momentum/winding sectors of the N=2 string, when all spatial directions are compactified.

iv) Fundamental gauge symmetry in M-theory has a single source – boundaries of open membranes. Until now, the gauge group of the type I theory and that of the heterotic
v) String worldsheet dynamics is governed by conformal field theory; a beautiful generalization of this structure is emerging. The 2+2 dimensional M-brane dynamics typically describes membrane world-volumes; string worldsheets arise in particular limits. We have seen that many structures of conformal field theory, such as world-sheet/spacetime orbifolds, carry over to this new setting.

Thus our results provide an explicit construction of strings and membranes in a unified fashion, and demonstrate conclusively that the appropriate setting for M-theory is twelve-dimensional. Many conjectures about M-theory are placed on a solid footing. We have presented a small portion of the rich variety of constructions that are available within our framework. We expect that further elaborations will shed yet more light on the structure of M-theory.

6.2. Directions for future work.

Our results strongly suggest that a 2+2 dimensional world-volume theory of membranes is the appropriate setting for M-theory. They furthermore suggest that this theory is quantizable. There exist other formulations of membrane dynamics – the Born-Infeld and Howe-Tucker [30] actions – that do not lead to a quantizable theory. In this sense, the 2+2 dimensional theory should play in M-theory the role that the Polyakov formulation does in string dynamics, as compared to the Nambu-Goto formulation[10].

A direct understanding of the 2+2 dimensional world-volume theory would be desirable. To achieve that, we propose to extract information about such a theory from the N=(2,1) string, whose target space is this world-volume. The N=(2,1) sigma-model was analyzed in [14] precisely to begin addressing this issue. In any string theory, the sigma model tells us about the target space geometry, and gives off-shell information (for instance about gauge invariances and field content). Thus the N=(2,1) sigma-model geometry is precisely the classical geometry of the 2+2 dimensional M-brane world-volume theory. The structure that emerged from this analysis was a theory of self-dual dilaton gravity coupled to self-dual matter (e.g. self-dual \([U(1)]^{12}\) or \([U(1)]^{28}\) gauge theory). The symmetries of

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10 Indeed, we have seen in section 5 that (to the order we have checked) this self-dual dynamics is classically equivalent to that of the Born-Infeld action for membranes.
self-dual geometry play the same role here that conformal invariance does in string theory. Moreover, these symmetries lead to the integrability of the classical field equations.

The existence of (2,1) supersymmetry imposes constraints on the sigma-model background. The N=2 right-moving supersymmetry requires the geometry to be self-dual. The N=1 left-moving supersymmetry solders four of the fiber directions to the tangent bundle of the M-brane (i.e. embeds the spin connection in the gauge group). Indirectly, this then describes the embedding of the M-brane into spacetime. In addition, the left-moving null current algebra of the N=(2,1) string construction of the M-brane requires the existence of a covariantly constant null Killing vector along the fibers, compatible with the constraints that lead to (2,1) supersymmetry. The orientation of this null Killing vector relative to the embedded M-brane tangent bundle determines whether one finds a string or a membrane.

In other situations with analogous structures of symmetries and integrability, one is able to use Ward identities and/or geometric ideas to quantize the theory. While we have no proof, we strongly suspect that 2+2 dimensional self-dual dilaton gravity coupled to matter exists as a quantum theory. The theory should be strongly constrained by the Ward identities arising from the world-volume current algebra. Quantization of four-dimensional self-dual matter in this spirit has been discussed in [31].

We conclude from this discussion that it is important to understand the quantization of self-dual dilaton gravity coupled to matter in 2+2 dimensions; to construct its algebra of Ward identities along the lines of [32]; and to understand the null reduction that gives strings and membranes.

Finally, there are (at least) two possible attitudes as to what basic objects underly M-theory. First, one might suppose that there is a lowest-dimensional object – a string or membrane – which plays a fundamental role, such that the other p-branes appear as collective excitations. A second viewpoint posits that all p-branes are of equal importance [33]. A striking feature of the N=(2,1) string construction is the unification of all the dilaton-gravity-matter fields on the M-brane into what looks like a twelve-dimensional gauge field (since they all come from the left-moving polarization states of the N=1 superstring). As in the D-brane construction [21], the M-brane splits these into a gauge field on the brane and a set of scalars; but the spatial dimensions of the M-brane are at most two. This leads one to wonder whether there might be a generalization of the M-brane world-volume theory that leads to a consistent quantization of all p-branes, perhaps involving 10+2 dimensional Yang-Mills theory. Hints of this are found in the mixed momentum/winding sectors of the (2,1) string when all spatial dimensions are compact. Could the restriction
to strings and membranes be yet another artificial restriction of the $N=(2,1)$ string that disappears in the final formulation of M-theory?

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References

[1] C. Hull, [hep-th/9512181].
[2] J. Polchinski and E. Witten, Nucl. Phys. B460 (1996) 525; [hep-th/9510169].
[3] P. Horava and E. Witten, Nucl. Phys. B460 (1996) 506; [hep-th/9510209].
[4] E. Witten, Nucl. Phys. B460 (1996) 541; [hep-th/9511030].
[5] A. Strominger, [hep-th/9512059].
[6] P. Townsend, [hep-th/9512062].
[7] K. Dasgupta and S. Mukhi, [hep-th/9512196].
[8] E. Witten, [hep-th/9512219].
[9] C. Schmidhuber, [hep-th/9601003].
[10] M. Duff, R. Minasian, and E. Witten, [hep-th/9601036].
[11] E. Gimon and J. Polchinski, [hep-th/9601038].
[12] O. Ganor and A. Hanany, [hep-th/9602120].
[13] N. Seiberg and E. Witten, [hep-th/9603003].
[14] D. Kutasov and E. Martinec, [hep-th/9602049].
[15] E. Witten, contribution to STRINGS ’95; [hep-th/9507121].
[16] W. Boucher, D. Friedan, and A. Kent, Phys. Lett. 172B (1986) 316; A. Schwimmer
   and N. Seiberg, Phys. Lett. 184B (1987) 191.
[17] S. Ketov, O. Lechtenfeld, and A. Parkes, Phys. Rev. D51 (1995) 2872, [hep-th/9312150].
   J. Bischoff, S. Ketov, and O. Lechtenfeld, Nucl. Phys. B438 (1995) 373, [hep-th/9406101].
[18] N. Marcus, talk at the Rome String Theory Workshop (1992); [hep-th/9211059].
[19] H. Ooguri and C. Vafa, Nucl. Phys. B367 (1991) 83.
[20] D. Friedan, E. Martinec and S. Shenker, Nucl. Phys. B271 (1986) 93.
[21] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724; [hep-th/9510017].
[22] E. Witten, Nucl. Phys. B443 (1995) 85; [hep-th/9503124].
[23] P. Aspinwall, [hep-th/9508154].
[24] J. Polchinski, S. Chaudhuri and C. Johnson, [hep-th/9602052].
[25] M. Bershadsky, V. Sadov, and C. Vafa, [hep-th/9510225].
[26] P. Aspinwall, [hep-th/9507012].
[27] D. Kutasov, [hep-th/9512145] (revised version, to appear).
[28] M. Douglas, [hep-th/9512077].
[29] E. Witten, [hep-th/9207094].
[30] P. Howe and R. Tucker, J. Phys. A10 (1977) L155.
[31] A. Losev, G. Moore, N. Nekrasov, and S. Shatashvili, [hep-th/9509151].
[32] A.A. Belavin, A.M. Polyakov, and A.B. Zamolodchikov, Nucl. Phys. B241 (1984) 333.
[33] P. Townsend, [hep-th/9507048].

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