Secure Quantum Passwords

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Abstract

We propose a quantum authentication protocol that is robust against the theft of secret keys. In the protocol, disposable quantum passwords prevent impersonation attacks with stolen secret keys. The protocol also prevents the leakage of secret information of a certification agent.
1 Introduction

Secure authentication plays an important role in modern society, supporting various types of transactions. Recently, there has been a surge in crimes involving skimming passwords in smart cards. In addition, computer viruses and criminals have led to authentication systems leaking customer passwords from certification agents which should be kept secret under normal circumstances. These issues expose the fragility of classical authentication methods. Classical authentication is performed essentially by cross-checking classical secret information composed of alphanumeric characters and shared by two parties to identify each other. Although improved methods have been developed which apply biometrics including fingerprints and iris codes, these cannot stop skimming at a fundamental level. The root of the problem is that there exists no principle to prohibit the cloning of classical information.

Meanwhile, quantum authentication is a possible way to assure such safety by employing fundamental physics principles. Under the no-cloning theorem [1] of quantum information, there exists no physical process to perfectly copy quantum states non-orthogonal to each other. Hence, by encoding secret information into non-orthogonal quantum states, it is possible to prevent perfect skimming. It is also known from the uncertainty principle [2] that measurements for eavesdropping by an adversary can be detected by checking the change of quantum states.

Several quantum protocols for identification have been proposed. Those methods can be classified into two basic classes of method. In the first method, common secret keys are generated as classical information composed of alphanumeric characters [3]–[9]. In identification processes, shared classical information is converted into quantum information and sent from $A$, who is a user, to $B$, who is a certification agent. In the second method [10]–[12], secret keys are shared as quantum information from the start. For example, $A$ stores the information in a specific portable device, a quantum smart card, which is slotted into the quantum authentication machines of $B$. Though these quantum protocols have advantages over classical methods, several unsatisfactory aspects remain. For example, in the first method, clone leakage of classical secret keys from $B$ cannot be prohibited in principle. For the second method, impersonation cannot be prohibited when $A$’s quantum smart card is stolen.
In this paper, we propose a quantum protocol in which $A$ can require $B$ to identify herself under high security, overcoming the above disadvantages by using Bell states and one-time-pad passwords together. $A$’s qubits of shared Bell pairs are stored in a quantum smart card. The password that $A$ determines is memorized only by $A$ and is not known by $B$ or others. The advantages of our protocol are the following. (I) Even if $A$’s quantum smart card is stolen by an adversary $E$, $E$ cannot impersonate $A$ without knowing her password. (II) No available information of the password is contained in the stolen card. (III) $B$ does not keep any information about $A$’s passwords, thereby avoiding the risk of clone-leakage from $B$’s storage by $E$. (IV) $E$ cannot make perfect copies of the passwords by eavesdropping on quantum channels. Moreover, by hiding quantum-mechanically encoded passwords behind many decoy qubits when sending to $B$, a high rate of eavesdropping detection can be achieved. (V) In identification tests of $A$ by $B$, entanglement between qubits of shared Bell pairs and qubits of passwords is not generated. Therefore, $B$ can use $A$’s quantum-password qubits once and then throw them away. Hence, $B$ must resend to $A$ only half of the qubits of the shared Bell pairs.

This paper is organized as follows. In section 2, typical protocols of quantum authentication proposed to date are briefly reviewed. In section 3, we propose a simplified protocol with quantum passwords by which we will explain the basic ideas of our full protocol. In section 4, security analysis is given for the simplified protocol. In section 5, an improved protocol with high security is proposed by extending the simplified protocol in section 3.

## 2 Protocols proposed to date

In this section, we characterize authentication by the following. (a) There exist two parties $A$ and $B$. (b) The purpose of authentication is that $B$ identifies $A$ with high success probability. (c) $A$ and $B$ have common secret keys that may be classical or quantum. (d) Authentication protocol are composed of the following phases. (d,i) Generation of secret keys. (d,ii) $A$ and $B$ work upon those secret keys by local operations and individually store them. (d,iii) In authentication, $A$ and $B$ are able to send their secret keys and other information using classical and quantum communication. (d,iv) $B$ can check whether a user communicating with $B$ is a legitimate person,
that is A, using locally accessible information. If something is wrong with this check, the process stops. (d,v) After B recognizes A, they are able to perform local operations and exchange information by classical and quantum communication. After that, the setup of (d,ii) is reproduced.

If A also wants to identify B, the above protocol applies with exchanged roles. The protocol is called quantum when it requires quantum information and quantum media. In the following, we briefly review some proposed protocols.

**Classical authentication:** In (c) above, secret keys can be composed of bit numbers. In (d,i), A and B meet and share a sequence of numbers. In (d,ii), they individually store the numbers without revealing them to any third party. For example, A memorizes the secret key and B stores the key in an electronic database. In (d,iii), A encodes the memorized information by classical ciphers and sends it to B. B decodes the information sent from A. In (d,iv), B compares the decoded result with her sequence of secret numbers. If the decoded information is consistent with the numbers, B recognizes A. If not, B stops the process. In this protocol, there is no need for phase (d,v) because B can discard the information sent from A. It is well known that this protocol cannot eliminate the danger of undetected leakage by cloning the classical information.

**Barnum 1:** Barnum [10] proposed two quantum protocols. The first method uses a sequence of qubit pairs in a fixed Bell state as secret keys of phase (c). In (d,i), A and B each share half of the Bell pairs. In (d,ii), A stores her qubit pairs in a quantum smart card, while B keeps hers in a quantum storage device. In (d,iii), A sends the quantum states stored in the smart card to B through a quantum channel. In (d,iv), B performs a Bell measurement of the qubits sent from A and the qubits stored in B’s storage device. From the measurement results, B verifies whether the two qubit states in the sequence are an original Bell state. In general, entanglement states such as Bell states exhibit purity properties only when all the entangled subsystems are gathered. A lack of some entangled subsystems leads to mixed states. If A is a legitimate person, the two qubits measured by B should be in a pure Bell state to give an acceptable result. If an adversary E attempts to impersonate A, qubits sent from E cannot reproduce pure states with the qubits stored in B’s storage device. Thus the Bell measurement by B detects inevitable errors for E’s qubits. In (d,v), after B recognizes A correctly, B resends half of the qubits of the Bell pairs in the right state to A. A then stores them again in her smart card. In this protocol, there is no danger
of clone leakage of A’s secret information from B, because B’s contracted quantum state of qubits is the maximal entropy state. This is a remarkable advantage over classical protocols. However, if A’s smart card is stolen by E, E is able to impersonate A easily and the protocol becomes insecure.

**Barnum 2:** The second protocol proposed by Barnum [10] uses catalyst states. First, A and B share an entangled state $|\phi_1\rangle$ of two qubits. We consider another state $|\phi_2\rangle$ of two qubits and assume that $|\phi_2\rangle$ is a state which is never converted from $|\phi_1\rangle$ by local operations and classical communication (LOCC). However, it is assumed to be possible that if A and B also share a certain entangled state $|\chi\rangle$, called a catalyst state, A and B are able to transform $|\phi_1\rangle$ into $|\phi_2\rangle$ by LOCC. Under the transformation, $|\chi\rangle$ remains unchanged. An explicit setting of $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\chi\rangle$ can be seen in a standard textbook [13]. Using a notion of catalyst states, Barnum proposed a quantum authentication protocol as follows. In (d,i), A and B share a sequence of qubit pairs in a catalyst state $|\chi\rangle$. In (d,ii), A stores half of the qubits in a quantum smart card and B keeps the other half in a quantum storage device. In (d,iii), B generates a sequence of qubit pairs in $|\phi_1\rangle$ and sends half of the pairs to A. In this step, A and B share a sequence of four-qubit composite systems in a state $|\phi_1\rangle \otimes |\chi\rangle$. They transform $|\phi_1\rangle \otimes |\chi\rangle$ into $|\phi_2\rangle \otimes |\chi\rangle$ by LOCC. Then, A resends half of the qubit pairs in $|\phi_2\rangle$ to B. In (d,iv), B performs a measurement which verifies whether the composite systems of qubits sent from A and the qubits stored by B generated first in $|\phi_1\rangle$ are really in $|\phi_2\rangle$. If B gets a positive result, B recognizes A. If not, B stops the process. When E personates A, the transformation from $|\phi_1\rangle$ to $|\phi_2\rangle$ cannot be achieved by E due to a lack of $|\chi\rangle$. The advantage of this protocol is that there is no need for A to send secret qubits in the catalyst state $|\chi\rangle$ to B. This reduces the risk that the secret key could be stolen in the transfer from A to B. However, as for Barnum 1, if A’s smart card, which contains the catalyst-state qubits, is stolen by E, E can impersonate A easily.

**Guo et al.** In addition to the protocol of Barnum [10], a method of quantum authentication by a different use of Bell states was proposed by Guo et al. [9]. In (d,i), A and B determine a classical password as a sequence of numbers composed of 1, 2, 3, 4. Each number is assigned to one of four orthogonal Bell states of two qubits. This generates a sequence of Bell states along the order of the classical password. In (d,ii), A stores half of the Bell pairs in a quantum smart card. B keeps the other half in a quantum storage device and also stores the classical password in an electronic database. In (d,iii), A sends the qubits stored in the smart card to B. In (d,iv), B performs
a Bell measurement of the qubits sent from $A$ and the qubits stored by $B$ so as to read out the classical password. $B$ checks whether the obtained results agree with the password stored in the database. If the results are correct, $B$ recognizes $A$. If not, $B$ stops the process. In (d,v), $B$ resends half the Bell pairs to $A$. In this protocol, similar to Barnum 1, the smart card does not contain any information about the classical password because the contracted quantum states of the qubits in the smart card are the maximal entropy state. However, this protocol is insecure against card theft. Moreover, there is a risk that copies of the classical password from $B$’s database could be leaked.

In the next section, we propose a secure protocol, which is robust against card theft. The protocol also prevents information leak from the verifier $B$.

3 Basic Protocol

In this section, we explain a basic protocol in order to outline the essence of the idea behind our full, more secure protocol, which we will be detail in section 5. The structure of the basic protocol is composed as follows. In (c) of the previous section, the secret keys are a sequence of qubit pairs in a Bell state. In (d,i), $A$ and $B$ meet and generate a sequence of qubit pairs in a Bell state $|+\rangle$. In (d,ii), $B$ puts half of the Bell pairs into a quantum storage device, while $A$ stores the other half in a quantum smart card. $A$ also generates a classical password composed of bit values 0 and 1 with the same length of the above sequence of Bell pairs. The password is not revealed to anybody, including $B$. $A$ performs a unitary transformation on each qubit in the smart card, dependent on the bit value of $A$’s password. When the bit value is 0, the unitary transformation is the identity transformation $I$. When the bit value is 1, the unitary transformation is $R$, which is not the identity transformation. The action of $R$ changes $|+\rangle$ into another Bell state $|\xi\rangle$. The state $|\xi\rangle$ is not orthogonal to $|+\rangle$. In (d,iii), $A$ encodes the classical password by using two non-orthogonal quantum states $|0\rangle$ and $|\alpha\rangle$ of a qubit. The part of the password with bit values 0 is replaced by the state $|0\rangle$. The other part with bit values 1 is replaced by $|\alpha\rangle$. We call the sequence of these qubits the quantum password. $A$ sends to $B$ both her quantum password and the qubits stored in her smart card. In (d,iv), $B$ combines the qubits sent by $A$ and the qubits that $B$ keeps. The system becomes a sequence of
composite three qubits which contains a qubit of A’s quantum password and two qubits of Bell pairs as secret keys. B performs a unitary transformation \( U \) on each three-qubit system. As seen below, if the input of \( U \) is legitimate, all the states of the Bell-pair part become \( |+\rangle \) independent of the bit values of A’s password. If not, other Bell components orthogonal to \( |+\rangle \) appear in the output of \( U \) with nonzero probability and give an error. B performs a measurement to check whether the state is really \( |+\rangle \). If the result is positive, B recognizes A. If not, B stops the process. In (d,v), B performs the inverse transformation \( U^{-1} \) to each set of three qubits in the sequence and resends only half of the Bell-pair part to A. The other half is entered again into B’s storage device. The qubits of A’s quantum password are discarded by B so that the information cannot be leaked.

In the following, we explain the protocol in more detail. First of all, we specify the security levels of the environment. The region of A is assumed to be secure against any attack by an adversary \( E \). Meanwhile, \( E \) is allowed to take classical information from the region of B, though \( E \) cannot get any quantum media in or out the B’s region. This assumption about B’s region implies for example that \( E \) can steal the data of the measurement results of B, but is not able to bring entangled qubits into B’s region for teleportation or to remove any quantum state of B. We also assume that a public channel of classical communication is available between A and B in which radiowaves with signals of A spread widely in open space towards B and no adversary stops the communication. The channel is used for announcements to B of the start of A’s protocol.

We now give a detailed explanation using a password example. The protocol is composed of nine steps, as follows.

(1) A and B meet and generate \( N \) qubit pairs in a Bell state \( |+\rangle \). For example, let us consider a case with \( N = 4 \). The state of the system is then given by \( |+\rangle|+\rangle|+\rangle|+\rangle \). A stores half of the qubits \( Q_A \) of the Bell pairs in a quantum smart card. B keeps the other half \( Q_B \) of the Bell pairs in a quantum storage device. The process is depicted in Fig. 1. Box A represents A’s smart card and box B represents the quantum storage device. The circles connected by wavy lines represent entangled qubits. (2) A generates an \( N \)-bit classical password \( K \) composed of 0s and 1s, keeping it secret from B and others. For instance, let us consider \( K = (0101) \). A performs a unitary transformation \( R \) on the qubits of \( Q_A \) corresponding to the bit values 1 of \( K \). The action of \( R \) changes \( |+\rangle \) into another Bell state \( |\xi\rangle \). In the example, the
state of $Q_A$ and $Q_B$ is transformed into $|+\rangle|\xi\rangle|+\rangle|\xi\rangle$. The process of (2) is depicted in Fig. 2. (3) In authentication, $A$ generates a quantum password $Q_K$. The bit values 0 of $K$ is encoded into a qubit state $|0\rangle$ and 1 of $K$ is encoded into a state $|\alpha\rangle$ non-orthogonal to $|0\rangle$. In the example, the quantum password is generated as $Q_K = |0\rangle|\alpha\rangle|0\rangle|\alpha\rangle$. (4) $A$ sends $Q_A$ and $Q_K$ to $B$ through a quantum channel. (5) $B$ unlocks $Q_A$ and $Q_B$ using a quantum device $UL$ with the quantum password $Q_K$. The device $UL$ operates so as to perform a unitary transformation $U$ on the composite system of $Q_K$, $Q_A$, and $Q_B$. $U$ does not change the input state $|0\rangle|+\rangle$ with bit values 0 of $K$ and transforms the input state $|\alpha\rangle|\xi\rangle$ with bit values 1 of $K$ into $|c\rangle|+\rangle$, where $|c\rangle$ is a quantum state of a qubit of $Q_K$. Fig. 3 depicts the input state for the $N = 4$ example and Fig. 4 shows the output state. (6) $B$ checks whether the output state of $Q_A$ and $Q_B$ is $|+\rangle^\otimes N$. This is done by a Bell measurement of $|+\rangle$. If a positive result is obtained, $B$ recognizes $A$. If not, $B$ stops the process. (7) $B$ locks $Q_A$ and $Q_B$ by a quantum device $L$ with $Q_K$. The action of $L$ is the inverse transformation of $U$. In the example, the output state of $L$ for $Q_A$ and $Q_B$ is given by $|+\rangle|\xi\rangle|+\rangle|\xi\rangle$. The output state of $L$ for $Q_K$ is given by $|0\rangle|\alpha\rangle|0\rangle|\alpha\rangle$. (8) $B$ breaks off $Q_K$ and erases the information so it cannot be stolen by others. (9) $B$ returns $Q_A$ to $A$ and $A$ restores $Q_A$ to the smart card.

We note that $Q_A$ and $Q_B$ cannot be decoded correctly by $B$ without the information of $K$ or $Q_K$. Therefore, even if the smart card is stolen by $E$, $E$ cannot impersonate $A$ without $K$. Thus, property (I) in section 1 is achieved. Moreover, the contracted states of the qubits of $Q_A$ become the maximal entropy state:

$$\rho_{\text{max}} = \frac{I}{2}.$$ 

Thus property (II) in section 1 is verified. Similarly, the contracted states of $Q_B$ also become the maximal entropy state. Therefore, the storage device of $B$ does not contain any information of $K$. It is also stressed that the only information that $B$ holds is $Q_B$. Hence $E$ cannot steal useful information about $K$ from $B$’s storage device. This guarantees property (III). In step (4), the information of $K$ is encoded by two non-orthogonal states. Thus, $E$ cannot perfectly obtain $Q_K$ by eavesdropping. By using an extension which will be proposed in section 5, rapid detection of eavesdropping also becomes possible and achieves property (IV). It is notable that there exists no
entanglement between $A$’s quantum password $Q_K$ and the composite system of $Q_A$ and $Q_B$. Therefore, $Q_K$ is disposable in each round of the protocol. Therefore property (V) is attained. $B$ cannot get perfect information of $K$ in steps (5) and (6) because the quantum qubits accessible by $B$ are all non-orthogonal to each other.

In the following, we give explicit forms of the unitary transformation $R$ and $U$. Let us define four Bell states orthogonal to each other as follows:

$$|\pm\rangle = \frac{1}{\sqrt{2}} [ |0\rangle |0\rangle \pm |1\rangle |1\rangle ],$$

$$|B_{\pm}\rangle = \frac{1}{\sqrt{2}} [ |0\rangle |1\rangle \pm |1\rangle |0\rangle ],$$

where $|0\rangle$ and $|1\rangle$ are two orthonormal states of a qubit. The first qubit is stored by $A$ and the second by $B$. The state $|\alpha\rangle$, which is used when $A$’s quantum password is generated, is given by

$$|\alpha\rangle = \alpha |0\rangle + \beta |1\rangle,$$

where $\alpha$ and $\beta$ are real constants such that $0 \leq \alpha < 1$ and $\beta = \sqrt{1 - \alpha^2}$. The unitary transformation $R$ is defined as follows:

$$R|0\rangle = e^{i\delta}|0\rangle,$$

$$R|1\rangle = e^{-i\delta}|1\rangle,$$

where $\delta$ is a real parameter. For later convenience, we introduce two real parameters $\xi$ and $\eta$ such that $e^{i\delta} = \xi + i\eta$, $0 \leq \xi < 1$ and $\eta = \sqrt{1 - \xi^2}$. Acting on $A$’s qubit in $|+\rangle$ with $R$ yields a new Bell state $|\xi\rangle$:

$$R \otimes I |+\rangle = |\xi\rangle,$$

where $|\xi\rangle$ is given by

$$|\xi\rangle = \xi |+\rangle + i\eta |-\rangle.$$

It is easy to check explicitly that $|\xi\rangle$ is a Bell state because the following relations hold:

$$\rho_A = \text{Tr}_B [ |\xi\rangle \langle \xi | ] = \frac{I}{2},$$

$$\rho_B = \text{Tr}_A [ |\xi\rangle \langle \xi | ] = \frac{I}{2}. $$
Because $|+\rangle$ is also a Bell state, the following relations are also satisfied:

$$\rho_A = \text{Tr}_B [|+\rangle\langle+|] = \frac{I}{2},$$
$$\rho_B = \text{Tr}_A [|+\rangle\langle+|] = \frac{I}{2}.$$

Consequently no information of $K$ can be extracted from only $Q_A$ or $Q_B$.

Here it should be noted that a similar idea of imprinting information into Bell states by a local operation has been proposed in [11]. However, the authors treat only orthogonal Bell states. In contrast, non-orthogonal Bell states play a crucial role in our protocol. Because $|+\rangle$ and $|\xi\rangle$ are not orthogonal to each other, $B$ cannot decode $K$ perfectly by a measurement of $Q_A$ and $Q_B$.

The unitary transformation $U$ is defined such that the following relations are satisfied:

$$U|0\rangle|+\rangle = |0\rangle|+\rangle,$$
$$U|1\rangle|+\rangle = \frac{\beta}{d}|1\rangle|+\rangle + u_{11}|0\rangle|\rangle + u_{21}|1\rangle|\rangle,$$

$$U|0\rangle|\rangle = -\frac{i\alpha\eta}{d}|1\rangle|+\rangle + u_{12}|0\rangle|\rangle + u_{22}|1\rangle|\rangle,$$
$$U|1\rangle|\rangle = -i\frac{\beta\eta}{d}|1\rangle|+\rangle + u_{13}|0\rangle|\rangle + u_{23}|1\rangle|\rangle,$$
$$U|b\rangle|B_{\pm}\rangle = |b\rangle|B_{\pm}\rangle,$$

where $d$ is a real parameter given by $d = \sqrt{1 - \alpha^2\xi^2}$ and $u_{ij}$ are complex numbers satisfying uniary relations given by

$$|u_{11}|^2 + |u_{21}|^2 = 1 - \frac{\beta^2\xi^2}{d^2},$$
$$|u_{12}|^2 + |u_{22}|^2 = 1 - \frac{\alpha^2\eta^2}{d^2},$$
$$|u_{13}|^2 + |u_{23}|^2 = 1 - \frac{\beta^2\eta^2}{d^2}.$$
\[ u_{11}^* u_{13} + u_{21}^* u_{23} = -\frac{i \beta \xi \eta}{d^2}, \]
\[ u_{12}^* u_{13} + u_{22}^* u_{23} = -\frac{\alpha \beta \eta^2}{d^2}, \]
\[ u_{11}^* u_{12} + u_{21}^* u_{22} = -i \frac{\alpha \beta \xi \eta}{d^2}. \]

\( U \) has the following properties. For bit values 0 of \( K \), the input state \(|0\rangle + \rangle\) does not change at all under \( U \) operation as seen in Eq. (2). For bit values 1 of \( K \), the input state \(|\alpha\rangle |\xi\rangle \) is transformed into

\[ U |\alpha\rangle |\xi\rangle = |c\rangle + \rangle, \quad (3) \]

where \(|c\rangle \) is a state defined by

\[ |c\rangle = \alpha |0\rangle + d |1\rangle. \quad (4) \]

Eq. (3) can be directly verified from Eq. (4) and the inverse relation \( U^{-1} |c\rangle + \rangle = |\alpha\rangle |\xi\rangle \), which is derived from unitary relations such that

\[ U^{-1} |0\rangle + \rangle = |0\rangle + \rangle, \]
\[ U^{-1} |1\rangle + \rangle = \frac{\beta \xi}{d} |1\rangle + \rangle + i \frac{\alpha \eta}{d} |0\rangle - \rangle + i \frac{\beta \eta}{d} |1\rangle - \rangle. \]

From Eq. (2) and Eq. (3), it is verified that entanglement between \( Q_K \) and \( Q_A + Q_B \) is not generated before and after the operation of \( U \) and \( U^{-1} \). Therefore, purity of the state for \( Q_A + Q_B \) is preserved even if \( Q_K \) is discarded by \( B \) after the authentication. This fact allows us to repeat the use of \( Q_A \) stored in the smart card.

4 Security Analysis

In this section, we present security analysis of the above protocol. First, we assume that \( E \) does not have \( A \)'s smart card and her password \( K \). The success probability \( p_s \) of \( E \) per qubit to pass the authentication test by \( B \) is evaluated as follows. Without access to \( K \), \( E \) has to prepare a universal
optimal state $|\Psi_E\rangle^\otimes N$ of $Q_K + Q_A$ as a forged quantum password and a forged smart card. Without loss of generality, $|\Psi_E\rangle$ is written as

$$|\Psi_E\rangle = \Psi_{00}|0\rangle|0\rangle + \Psi_{01}|0\rangle|1\rangle + \Psi_{10}|1\rangle|0\rangle + \Psi_{11}|1\rangle|1\rangle,$$

where $\Psi_{bb'}$ are complex coefficients satisfying the normalization condition of the state. The first qubit corresponds to $Q_K$ and the second to $Q_A$. Because the forged input of $E$ is not at all entangled with $Q_B$, the input state of $Q_B$ of $U$ is given by $I_B/2$, independent of the bit values of $K$. Hence $p_s$ is written as

$$p_s = \frac{1}{4} |\Psi_{00}|^2 + \frac{1}{4} |\Psi_{01}|^2 - \frac{1}{4} \left| \frac{\beta}{d} e^{-i\delta} \Psi_{10} - i \frac{\alpha \eta}{d} \Psi_{00} \right|^2 - \frac{1}{4} \left| \frac{\beta}{d} e^{i\delta} \Psi_{11} + i \frac{\alpha \eta}{d} \Psi_{01} \right|^2,$$

by use of the following unitary relations.

$$\langle 0|\langle +|U = \langle 0|\langle +|,$$

$$\langle 1|\langle +|U = \frac{\beta \xi}{d} \langle 1|\langle +| - i \frac{\alpha \eta}{d} \langle 0|\langle -| - i \frac{\beta \eta}{d} \langle 1|\langle -|.$$
We can show that $p_s$ is bounded above by applying Schwartz inequalities as follows:

$$p_s \leq \frac{1}{4} |\Psi_{00}|^2 + \frac{1}{4} |\Psi_{10}|^2$$

$$+ \frac{1}{4} (|\Psi_{10}|^2 + |\Psi_{00}|^2)$$

$$+ \frac{1}{4} (|\Psi_{11}|^2 + |\Psi_{01}|^2)$$

$$= \frac{1}{4} (|\Psi_{00}|^2 + |\Psi_{10}|^2) + \frac{1}{4}$$

$$\leq \frac{1}{2}.$$

Consequently, we get a lower bound of detection probability of $E$’s impersonation as

$$p_E^{(N)} \geq 1 - \left(\frac{1}{2}\right)^N.$$

Therefore, $B$ is able to detect $E$ with a high probability for a large $N$.

Next, we consider a case where $E$ succeeds in stealing $A$’s card. However, we assume that $E$ does not know $K$. In this case, $E$ must make a forged quantum password. The optimal state for each qubit is denoted by $\rho_E = |K_E\rangle\langle K_E|$. Assuming $K$ is randomly generated, the appearance probability of each bit value of $K$ is $1/2$. For bit values 0 of $K$, the state of $Q_A + Q_B$ is $|+\rangle$. Thus, the detection probability of $E$ per qubit is given by

$$p_{E0} = \text{Tr} \left[ U (\rho_E \otimes |+\rangle\langle +|) U^\dagger (I - |+\rangle\langle +|) \right].$$

For bit values 1, the state is $|\xi\rangle$. Hence, the detection probability is written as

$$p_{E1} = \text{Tr} \left[ U (\rho_E \otimes |\xi\rangle\langle \xi|) U^\dagger (I - |+\rangle\langle +|) \right].$$

The average detection probability per qubit is given by

$$\Delta_E = \frac{1}{2} p_{E0} + \frac{1}{2} p_{E1}.$$

We parametrize $\rho_E$ as
\[ \rho_E = r|0\rangle\langle0| + (1-r)|1\rangle\langle1| + (x + iy)|0\rangle\langle1| + (x - iy)|1\rangle\langle0|, \]

where

\[ 0 \leq r \leq 1, x^2 + y^2 = r(1-r). \]

The detection probability is then evaluated as

\[ \Delta_E = \frac{1}{2} \frac{1 - \xi^2}{1 - \alpha^2 \xi^2} \left[ 1 + \alpha^2 - 2\alpha^2 r - 2\alpha \beta x \right]. \]

The minimum value of \( \Delta_E \) in terms of \( r \) and \( x \) is easily obtained as

\[ p_n = \min \Delta_E = \frac{1}{2} \frac{(1 - \xi^2)}{1 - \alpha^2 \xi^2} (1 - \alpha). \]

For instance, taking typical values of \( \alpha \) and \( \xi \) as \( \alpha = \xi = 1/2 \), the value of \( p_n \) is evaluated as \( p_n = 1/5 \). It is noted that the total detection probability of \( E \) is given by

\[ p_n^{(N)} = 1 - (1 - p_n)^N. \]

Consequently, \( E \) can be detected with a high rate for a large \( N \).

A comment should be made about man-in-the-middle attacks. If \( E \) is able to secretly occupy classical and quantum channels between \( A \) and \( B \) and perform any attack allowed by physics laws, \( E \) is able to steal all the quantum states of \( A \) in the transfer through the channels. In order to impersonate \( A \) after this round of the protocol, \( E \) must keep the stolen qubits. However, we have assumed that \( E \) cannot prevent \( B \) from knowing the start of \( A \)'s authentication protocol through a public channel. In order to avoid \( B \) quickly noticing the impersonation, \( E \) has to send some forged qubits as \( Q_A \) and \( Q_K \) to \( B \). Then the identification test by \( B \) yields a wrong output and the man-in-the-middle attack is easily noticed.

Though our protocol has many advantages, as detailed above, some subtle loopholes exist. One of them may occur in step (4). Because of the non-orthogonality of \( |0\rangle \) and \( |\alpha\rangle \), perfect cloning of \( Q_K \) in the channel is prohibited. However, it is possible for \( E \) to attempt an approximate cloning of \( Q_K \). Even though the cloning leaves a disturbance in the states of \( Q_K \) received by \( B \), the detection rate of eavesdropping is not large. If \( B \) fails to detect \( E \), \( E \) may next try to steal \( A \)'s smart card. Let us assume that \( E \)
succeeds in obtaining the card. This card-steal attack with an approximate clone of $Q_K$ strongly decreases $B$’s probability of detecting impersonation. Another loophole may occur in step (8). $B$ discards $A$’s quantum password $Q_K$ after the authentication. If $E$ infiltrates $B$’s region and secretly measures $Q_K$ and accumulates the results, $E$ can estimate $K$ with high precision after several rounds by $A$ and $B$. This may lead to possible abuse of the information by $E$. In the next section, we give an improved protocol robust against these attacks, without loss of the advantages of the basic protocol.

5 Extended protocol

In this section, we present an extended protocol, which increases the detection probability of eavesdropping in quantum channels and decreases the amount of knowledge of $K$ leaked in the discarding step of $Q_K$. Before $A$ sends $Q_K$ to $B$, $A$ generates a long sequence of two non-orthogonal states of qubits like the BB84 quantum key distribution [14] and uses them as decoys to detect eavesdropping. In order to suppress the leakage of information about $K$, a one-time-pad method is adopted when $A$ generates a quantum password. The extended protocol is composed of 14 steps as follows.

1. $A$ and $B$ meet and generate $N$ qubit pairs in a Bell state $|+⟩$. $A$ stores half of the Bell pairs $Q_A$ in a quantum smart card. $B$ keeps the other half $Q_B$ in a quantum storage device.

2. $A$ generates an $N$-bit classical password $K$ composed of 0s and 1s and keeps it secret from $B$ and others. $A$ performs $R$ for $Q_A$ qubits corresponding to bit values 1 of $K$.

3. In authentication, the lock process is reversed using $K$. $A$ performs $R^{-1}$ for $Q_A$ qubits corresponding to bit values 1 of $K$.

4. $A$ generates $N$-bit pseudo-random numbers $\tilde{K}$. $\tilde{K}$ is used as a one-time-pad password in the transfer of $A$’s qubits. $A$ performs $R$ for $Q_A$ corresponding to bit values 1 of $\tilde{K}$.

5. $A$ generates a quantum password $Q_{\tilde{K}}$. The bit values 0 of $\tilde{K}$ are encoded into a qubit state $|0⟩$ and bit values 1 of $\tilde{K}$ into $|\alpha⟩$.

6. $A$ generates a sequence of $N_D$ pseudo-random numbers $K_D$ composed of 2, 3, 4, 5. $K_D$ is quantum mechanically encoded using four quantum states $|0⟩, |1⟩$ and ...
\[ |0_x \rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle), \]
\[ |1_x \rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle). \]

The number 2 in \( K_D \) is replaced by \( |0\rangle \), 3 by \( |1\rangle \), 4 by \( |0_x \rangle \) and 5 by \( |1_x \rangle \).

We call the sequence of qubits \( Q_D \). (7) \( A \) makes \( Q_{\tilde{K}} \) randomly slip into \( Q_D \) and sends \( Q_A \) and \( Q_{\tilde{K}} + Q_D \) to \( B \). (8) After they are received by \( B \), \( A \) announces to \( B \) the positions of the qubits of \( Q_D \) and the value of \( K_D \). \( B \) then separates \( Q_D \) from \( Q_{\tilde{K}} \). \( B \) measures \( Q_D \) in the basis of \{\( |0\rangle, |1\rangle \}\) if the value of \( K_D \) is 2 or 3 and in the basis of \{\( |0_x\rangle, |1_x\rangle \}\) if the value of \( K_D \) is 4 or 5. If the results are consistent with \( K_D \), \( B \) makes a judgement that there is no eavesdropping. If it is judged that eavesdropping may have occurred, \( B \) stops the process. (9) \( B \) unlocks \( Q_A \) and \( Q_B \) by the unitary transformation \( U \) with the password \( Q_{\tilde{K}} \). \( U \) does not change the input state \( |0\rangle\langle+| \) with bit value 0 of \( K \) and transforms the input state \( |\alpha\rangle\langle\xi| \) with bit value 1 of \( K \) into \( |c\rangle\langle+| \). (10) \( B \) checks whether the output state of \( Q_A + Q_B \) is \( |+\rangle^{\otimes N} \). If the result is positive, \( B \) recognizes \( A \). If not, \( B \) stops the process. (11) \( B \) locks \( Q_A \) and \( Q_B \) by \( U^{-1} \) with \( Q_{\tilde{K}} \). (12) \( B \) discards \( Q_{\tilde{K}} \). (13) \( B \) returns \( Q_A \) to \( A \) and \( A \) restores \( Q_A \) to the smart card. (14) \( A \) performs \( R^{-1} \) to the qubits of \( Q_A \) corresponding to bit values 1 of \( K \) to make the state of \( Q_A + Q_B \) to be \( |+\rangle^{\otimes N} \). Then, \( A \) locks \( Q_A + Q_B \) by the original password \( K \). \( A \) performs \( R \) on the qubits of \( Q_A \) corresponding to bit values 1 of \( K \).

The one-time-pad password method in step (4) prevents \( E \) from stealing the information of the original password \( K \) when the quantum password is discarded by \( B \). Step (8) of the protocol also prevents approximate cloning attacks by \( E \) eavesdropping by taking a large \( N_D \). A detailed security analysis will be reported elsewhere.

It is expected that the protocol will protect the basic infrastructure of the information-based society if a quantum smart card is devised which can store quantum information for a long period.

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Figure Captions

Fig. 1: In the basic protocol, $A$ and $B$ meet and share Bell pairs. The case with $K = (0101)$ is depicted. The state of the system is given by $|+\rangle|+\rangle|+\rangle|+\rangle$. $A$ stores half of the Bell pairs $Q_A$ in a quantum smart card. $B$ keeps the other half $Q_B$ in a quantum storage device. Box $A$ indicates the smart card and box $B$ indicates the quantum storage device. The circles connected by wavy lines represent entangled qubits.

Fig. 2: $A$ generates a classical password $K$ composed of 0s and 1s and keeps it secret, even from $B$. The case with $K = (0101)$ is depicted. $A$ performs a unitary transformation $R$ on the qubits of $Q_A$ corresponding to bit values 1 of $K$. The action of $R$ changes $|+\rangle$ into another Bell state $|\xi\rangle$. In this example, the state of $Q_A$ and $Q_B$ is given by $|+\rangle|\xi\rangle|+\rangle|\xi\rangle$.

Fig. 3: $B$ unlocks $Q_A$ and $Q_B$ using a quantum device $UL$ with the password $Q_K$. The device $UL$ operates so as to perform a unitary transformation $U$ on the composite system of $Q_K, Q_A$ and $Q_B$. The input state is depicted for the example.

Fig. 4: The output state of $UL$ is depicted.
Fig. 1
Fig. 2
Fig. 3
