Effects of $\rho$-meson width on pion distributions in heavy-ion collisions

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The influence of the finite width of $\rho$ meson on the pion momentum distribution is studied quantitatively in the framework of the S-matrix approach combined with a blast-wave model to describe particle emissions from an expanding fireball. We find that the proper treatment of resonances which accounts for their production dynamics encoded in data for partial wave scattering amplitudes can substantially modify spectra of daughter particles originating in their two body decays. In particular, it results in an enhancement of the low-$p_T$ pions from the decays of $\rho$ mesons which improves the quantitative description of the pion spectra in heavy ion collisions obtained by the ALICE collaboration at the LHC energy.

Recent measurements of the transverse momentum, $p_T$-distributions of identified particles produced in $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb collisions at CERN Large Hadron Collider (LHC) [1] revealed an excess of low-momentum ($p_T \lesssim 0.3$ GeV) pions over the conventional fluid-dynamical calculations [1,3].

It is well known that pions originating from decays of resonances have a steeper $p_T$-distribution than the thermal pions [4], and that they provide a dominant contribution to the spectrum at low transverse momentum. Thus, resonance decays require a particular attention when modeling spectra of particles originating from an expanding thermal fireball.

In fluid-dynamical calculations, the interacting hadrons are usually described by the hadron resonance gas (HRG), where the system is modeled as a gas of free hadrons with resonances considered as particles with vanishing widths. This approximation yields reasonable description of the bulk properties of the hadronic medium [5,8]. The HRG model also provides a very satisfactory description of particle yields measured in heavy ion collisions [9,17], as well as the hadronic equation of state and some fluctuation observables obtained in lattice QCD (LQCD) [18-22]. However, as we show in this letter, when $p_T$-differential observables are involved, a more refined approach may be necessary.

To properly address the dynamics of hadrons, the effect of resonance width must be included. A conventional approach is to impose a Breit-Wigner distribution on the resonance mass. Unfortunately, this approach proves to be too crude in many circumstances. For example, for a broad resonance like the $\sigma$ meson [23], or the (yet-to-be-confirmed) $\kappa$ meson [24], the Breit-Wigner approach can give misleading results on the resonance contribution to the thermodynamics.

We thus take a more fundamental approach to evaluate the properties of interacting hadrons based on the S-matrix formulation of Dashen, Ma and Bernstein [25]. For elastic scatterings, the interaction part of the partition function reduces to the Beth-Uhlenbeck form for the second virial coefficient, expressed in terms of the scattering phase shifts [26]. In the context of heavy-ion physics, this approach has been applied to evaluate the contribution of $\pi N$ [5,7,27], $\pi\pi$ [5,23], and $\pi K$ interactions [5,24] to the thermodynamics of hadronic matter, and to analyse the resonance production [28].

In this letter, to make the effects of resonance width on particle $p_T$-spectra more tractable, we concentrate on the $\pi\pi$ system. As shown in Refs. [5,23], the effects of the scalar-isoscalar and the scalar-isotensor channels largely cancel each other. This cancellation remains when the single particle distribution of pions is evaluated. Thus for our purposes it is sufficient to consider only the vector-isovector channel, i.e. the channel of the $\rho$ meson.

In the S-matrix formalism, the density of states per unit volume and unit invariant mass $M$, assuming thermal equilibrium at temperature $T$, is given by [5,7,26,28]

$$\frac{d n_{IJ}}{d M} = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2\pi} \mathcal{B}(M) f(E(M,p),T),$$

where $f$ is the Bose-Einstein or Fermi-Dirac distribution, and $\mathcal{B}(M)$ is an effective spectral weight,

$$\mathcal{B}(M) = 2 \frac{d^3 f_{IJ}}{d M},$$

derived from the scattering phase shift $\delta_{IJ}$, of the isospin $I$ and spin $J$ channel.

In the elastic region ($M \lesssim 1$ GeV), the empirical phase shift [29,31] of the ($I = 1, J = 1$) channel can be effectively described by a phenomenological formula, inspired by a one-loop perturbative calculation of the $\rho$ self-energy [32,33],

$$\delta_{11}(M) = \tan^{-1} \left( \frac{2}{3M} \frac{a_0}{\rho_{CM}^3} \frac{\rho_{CM}^3}{M^2 - m_0^2} \frac{1}{1 + c \rho_{CM}^2} \right),$$

where $\rho_{CM}(M) = \frac{1}{2} \sqrt{M^2 - 4m_\pi^2}$ is the center-of-mass momentum of the scattering pions, and $a_0 = 3.08$, $m_0 = 0.77$ GeV, and $c = 0.59$ GeV$^{-2}$ are the model parameters chosen to reproduce not only the phase-shift data, but
also the known value of the P-wave scattering length. The phase shift and the scattering length are related as
\[ a_1^1 = \frac{\delta_{11}}{p_{\text{CM}}} \bigg| _{p_{\text{CM}} \rightarrow 0}. \] (4)

We constrain the scattering length to \( a_1^1 = 0.038 m_{\pi}^{-3} \), matching the experimental value and chiral perturbation theory prediction \( a_1^1 = 0.038(2) m_{\pi}^{-3} \) [34] and 0.037(10) \( m_{\pi}^{-3} \) [35] [36], respectively. This requirement is essential for the correct description of the near-threshold behaviour of the density function, introduced in Eq. (2).

An important feature of the current approach is the use of the effective spectral weight \( B(M) \) instead of the standard spectral function. This effective weight includes contributions from both a pure \( \rho \) state and the correlated \( \pi\pi \) pair. The latter tends to shift the strength of the weight function towards the low invariant-mass region [2]. Such a shift can potentially translate into an enhancement of the low-\( p_T \) daughter pions from the decays of \( \rho \) mesons.

To quantify this expectation, we evaluate the distribution of \( \rho \)’s using the Cooper-Frye description [37], with the thermal distribution augmented by the effective spectral weight \( B \) in Eq. (2), as
\[ \frac{dN_\rho}{dy p_T dp_T d\phi} = \int dM_\rho \int d\sigma_\rho p_\rho^0 \frac{1}{2\pi} B(M_\rho) \times \frac{d_\rho}{(2\pi)^3} f_\rho(p \cdot u, T), \] (5)
where \( f_\rho, d_\rho \) are respectively the Bose-Einstein distribution and the spin degeneracy for \( \rho \), and \( u \) is the flow velocity. In the case of a static source, the integration over the surface, \( \int d\sigma_\rho p_\rho^0 \), becomes a simple multiplication by the volume of the system, \( V \), and by the energy of the particle, \( E \). The momentum spectrum of the decay pions can be evaluated by applying the conventional decay kinematics [31] [38] [39] to the distribution of \( \rho \)’s from Eq. (5). For a static source, one gets
\[ \frac{dN_\pi^{de}}{dy p_T dp_T d\phi} = V \int dM_\rho \frac{1}{2\pi} B(M_\rho) \times \frac{M_\rho}{2p_{\rho \text{CM}}} \int_{E^-_\rho}^{E^+_\rho} dE_\rho E_\rho d_\rho (2\pi)^3 f_\rho(E(M_\rho), T), \] (6)
where
\[ E^\pm_\rho = \frac{M_\rho}{2m_\pi} (E_\pi M_\rho \pm 2p_{\pi \text{CM}}). \] (7)

We evaluate the \( p_T \) distributions at \( T = 155 \text{ MeV} \), in the vicinity of the pseudocritical temperature obtained in the lattice formulation of QCD [40] [41].

In Fig. 1 Left we show the rapidity and azimuthal angle integrated transverse momentum spectra of \( \pi^+ \) originating from \( \rho \) decays. The \( \rho \)’s are treated as zero-width particles, particles with the standard Breit-Wigner width, or according to the S-matrix approach introduced in Eq. (6). The latter description leads to a substantial enhancement of the pion decay spectra. The effect is most prominent in the low-\( p_T \) region of the decay pions, where at \( p_T \approx 0 \) one observes a factor of two increase of the differential pion yield. Note that at larger values of the transverse...
momentum the spectrum of decay pions is practically unaffected by the width of \( \rho \). For future reference, we also present results on decay \( \pi^+ \) spectra from the system of \( \pi K \) interaction (sum of S- and P-wave) and from \( \pi N \) interaction in the \( \Delta \)-channel. In all the channels studied we find overall enhancement of low-\( p_T \) pions in the S-matrix approach compared to the zero-width results. Nevertheless, the difference is most noticeable in the \( \rho \) sector.

To illustrate the effect of leading resonances on the pion yield in the HRG, we show in Fig. 2 right the temperature dependence of the contributions from various sources to pion density after resonance decays. For this analysis, we have included the three-body decays of zero-width \( \eta \) and \( \omega \) (with branching ratios of 0.228 and 0.893, respectively). Furthermore, we have applied the S-matrix treatment to the \( \pi \pi \) (S-wave) and \( \pi K \) (S- and P-wave) systems and the \( \Delta(1232) \)-channel of \( \pi N \). At \( T = 155 \text{ MeV} \), when no heavier resonances are included, the relative abundance of \( \pi^+ \) from \( \rho \) decay is 25.1%, while the thermal pion yield remains dominant at 49.4%. Three-body decays considered constitute 12.2%, and the sum of the rest of two-body channels we treated give 13.3% of the total yield. The S-matrix treatment significantly affects the yield of pions from \( \rho \) decays, resulting in its increase by approximately 15%, whereas the effect is smaller for other channels considered. However, because of the contribution from all the other sources, the overall change in the final pion yield due to the S-matrix approach is only a few per cent.

In general, on the level of particle yields, and at higher temperatures \( T > 100 \text{ MeV} \), the zero-width treatment of resonances gives comparable results to the S-matrix approach [5] despite the fact that the phase shifts in most cases do not resemble a step function and the assumption of a zero (and at times even a narrow) width is strictly speaking not justified. However, as already seen in Fig. 1, essential differences can appear when \( p_T \)-differential observables of individual resonance channels are studied. Evidently, the more physical treatment by the S-matrix formulation is needed for precision calculations of particle spectra, as e.g. in modeling data in heavy-ion collisions.

In a realistic heavy-ion collision, however, the situation is further complicated by the expansion of the system, and the presence of all the other resonances. To gauge whether the S-matrix description of \( \rho \) mesons would affect the pion distributions observed in heavy-ion collisions, we describe the system using a blast-wave model [42]. There, the thermal source is assumed to be a boost-invariant [43] cylindrically symmetric transversely expanding tube of radius \( R \), from which particles are emitted at constant longitudinal proper time \( \tau \) with the radial flow velocity \( v(r) = v_{\text{max}}(r/R) \).

We calculate the distributions of all the resonances in the Particle Data Book up to the 2 GeV mass, apply the two- and three-body decay kinematics, and sum the contributions to the spectrum of thermal pions. We take advantage of the recent finding in the dynamical model calculations in heavy-ion collisions that the pion \( p_T \)-distribution changes only very little during the subsequent evolution in the hadronic phase [44]. Thus, we fix the freeze-out temperature at \( T = 155 \text{ MeV} \), which coincides with the chiral crossover in LQCD. The further parameters of the blast-wave model, \( \tau = 13.7 \text{ fm}, R = 10 \text{ fm} \), and the transverse flow velocity, \( v(\tau) = 0.2 \text{ for } \tau > 3 \text{ fm} \), are fixed at their default values.

The S-matrix and zero-width approaches are compared with the measured momentum distributions of positively charged pions, protons and \( \Omega \) baryons in 0-10% most central \( \sqrt{s_{\text{NN}}} = 2.76 \text{ TeV} \) Pb+Pb collisions as measured by the ALICE collaboration [1, 45], and fitted using a blast-wave model. In both panels the solid lines correspond to the S-matrix approach result and the dashed lines to the conventional zero-width approximation.
fm, and $v_{\text{max}} = 0.8$ were chosen to get the best description of spectra for positive pions in 0-10% most central $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb collisions as measured by the ALICE collaboration. The above freeze-out temperature and the resulting volume of the fireball, $V \approx 4300$ fm$^3$, are consistent with that obtained previously in the HRG model description of hadron production yields and some fluctuation observables in heavy-ion collisions at the LHC [10, 22].

The resulting pion distribution is shown in the left panel of Fig. 2. In this calculation the conventional zero-width treatment of $\rho$'s leads to a distribution which underestimates the data in the low-$p_T$ region ($p_T \lesssim 200$ MeV). When $\rho$ mesons are treated according to the S-matrix description, there is a clear, up to 7%, increase of the low-$p_T$ pions, which is sufficient to reach the data.

To check further the quality of the model parameterisation, we also show in the right panel of Fig. 2 the pion, proton and $\Omega$ baryon distributions in a broader $m_T$-range. As seen in this figure, the pion data are well described up to $m_T \approx 2$ GeV, and the model predictions are also consistent with the data for the $\Omega$ distribution. These results verify the chosen values for temperature and volume, and they are also consistent with the idea that $\Omega$ baryons hardly rescatter in the hadronic phase [16, 17], and thus their spectra are fixed at the phase boundary [17]. On the other hand, the proton distribution is steeper than the data, and the overall yield of protons is larger than the experimental value. The observed deviation on the level of proton yield is already discussed in the literature [10]. The deviations in the proton spectrum could be possibly due to their further rescattering during evolution in the hadronic phase [16, 48].

In conclusion, we have investigated how the explicit treatment of the $\rho$-meson width affects the pion yield and $p_T$ distribution in $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb collisions at LHC. We have used the S-matrix approach to describe $\rho$ mesons, and found that compared to the conventional zero-width treatment the pion yield increases, particularly at low values of transverse momentum. This indicates that the observed enhancement of low-$p_T$ pions may be possibly explained in fluid-dynamical calculations by a proper implementation of the width of resonances within the S-matrix approach. However, the S-matrix treatment of $\rho$'s alone may not be fully sufficient.

A natural extension of this work is to apply a more complete model for the fluid dynamical calculations [19-53], as well as, to account for a possible medium modification on the phase shifts. Essential in-medium effects for $\rho$ mesons are suggested by studies based on many-body Green’s function [34, 42, 58]. This, together with the S-matrix treatment of three-body decays, can presumably further increase the pion yields in the low-$p_T$ region. We leave this as a matter of future investigation. Nevertheless, even in their present level, our results demonstrate the importance of the proper treatment of resonances in modeling heavy-ion collisions, and the need to improve on the customary hadron resonance gas models for precision calculations of particle spectra at low values of transverse momentum. These studies are also important in hydrodynamics-cascade hybrid models [51-59] for particle production in heavy ion collisions when describing partialization of the fluid as an input to hadronic transport.

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