Nonfactorizable contributions to the decay mode
\[ D^0 \rightarrow K^0 \bar{K}^0 \]

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Abstract

We point out that the decay mode \( D^0 \rightarrow K^0 \bar{K}^0 \) has no factorizable contribution. In the chiral perturbation language, treating \( D^0 \) as heavy, the \( \mathcal{O}(p) \) contribution is zero. We calculate the nonfactorizable chiral loop contributions of order \( \mathcal{O}(p^3) \). Then, we use a heavy-light type chiral quark model to calculate nonfactorizable tree level terms, also of order \( \mathcal{O}(p^3) \), proportional to the gluon condensate. We find that both the chiral loops and the gluon condensate contributions are of the same order of magnitude as the experimental amplitude.

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1 Introduction

The decay mechanism of the weak nonleptonic \( D^0 \) decays has motivated numerous studies \cite{1}-.\cite{10}. For nonleptonic decays of \( D \) mesons, as well as for \( K \)'s and \( B \)'s, the so called factorization hypothesis has been commonly used. For nonleptonic decays, the effective Lagrangian at quark level has the form

\[ \mathcal{L}_W = \sum_i C_i Q_i, \]  

where the coefficients \( C_i \) contain all the short distance electroweak and QCD effects to a certain order in perturbation theory, and the \( Q_i \)'s are quark operators. Typically, these quark operators are products of (pseudo) scalar- or vector- currents: \( Q = j(1) j(2) \). Then, for a nonleptonic decay \( M \rightarrow M_1 + M_2 \), the factorization hypothesis (also called vacuum saturation approximation) gives prescriptions of the form

\[ \langle M_1 M_2 | Q | M \rangle \rightarrow \langle M_1 | j(1) | 0 \rangle \langle M_2 | j(2) | M \rangle. \]  

The factorization hypothesis are known to fail badly for nonleptonic \( K \) decays \cite{11}-.\cite{13}. On the other hand, there are certain heavy hadron weak decays where factorization might apply. Recently, the understanding of factorization for exclusive nonleptonic decays of \( B \) mesons in terms of QCD in the heavy quark limit has been considerably improved \cite{14}. In this paper we will discuss nonfactorizable terms for \( D \) decays, in particular for the decay mode \( D^0 \rightarrow K^0 \bar{K}^0 \).
Even though the factorization hypothesis might work reasonably well if one is interested in an order of magnitude estimate, it does not reproduce experimental data completely. For example, a naive application of factorization in charm decays leads to rates for the $D^0 \to \pi^0 \bar{K}^0$, $D^0 \to \pi^0 \pi^0$, $D^0 \to K^+ K^-$, $D^0 \to \pi^+ \pi^-$ decays which are too strongly suppressed. Moreover, and this is the important point of this paper: in $D^0 \to K^0 \bar{K}^0$, factorization misses completely, predicting a vanishing branching ratio, in contrast with the experimental situation.

To see this, note that at tree level the $D^0 \to K^0 \bar{K}^0$ decay might occur due to two annihilation diagrams [1] which could potentially create the $K^0 \bar{K}^0$ state. However, they cancel each other by the GIM mechanism. Moreover, in factorization limit, the amplitude is proportional to

$$
\langle K^0 \bar{K}^0|V_\mu|0\rangle \langle 0|A^\mu|D^0\rangle \approx (p_{K^0} - p_{\bar{K}^0})_\mu f_D p_D^\mu = 0.
$$

In many of the studies (e.g. [2, 3, 4, 5, 7]) this decay has been understood as a result of final state interactions (FSI). In the analysis of ref. [2] the rescattering mechanism included $K^+ K^-$ and $\pi^+ \pi^-$ states leading to a branching ratio $B(D^0 \to K^0 \bar{K}^0) = \frac{1}{2} B(D^0 \to K^+ K^-)$. Experimental data on the other hand are $B(D^0 \to K^0 \bar{K}^0) = (6.5 \pm 1.8) \times 10^{-4}$ and $B(D^0 \to K^+ K^-) = (4.25 \pm 0.16) \times 10^{-3}$. A recent investigation of the $D^0 \to K^0 \bar{K}^0$ decay mode performed in [3] has focused on the $s$ channel and the $t$ channel one particle exchange contributions. The $s$ channel contribution has been taken into account through the poorly known scalar meson $f_0(1710)$ and was found to be very small, while the one particle $t$-exchanges yielded higher contributions, with pion exchange being the highest. In the approach of [3] the $D^0 \to K^0 \bar{K}^0$ was realized through the scalar glueball or glue-rich scalar meson.

The second instructive case concerning the factorization hypothesis, is offered by the analyses of nonleptonic $K$ meson decays. Namely, it is well known that factorization does not work in nonleptonic $K$ decays. Among many approaches the Chiral Quark Model ($\chi$QM) [19] was shown to be able to accommodate the intriguing $\Delta I = 1/2$ rule in $K \to \pi \pi$ decays, as well as CP violating parameters, by systematic involvement of the soft gluon emission forming gluon condensates and chiral loops at $O(p^4)$ order [12]. In the $\chi$QM [17], the light quarks ($u, d, s$) couple to the would-be Goldstone octet mesons ($K, \pi, \eta$) in a chiral invariant way, such that all effects are in principle calculable in terms of physical quantities and a few model dependent parameters, namely the quark condensate, the gluon condensate and the constituent quark mass [12, 16, 18, 20, 21]. Also in “generalized factorization”, it was shown [13] that the inclusion of gluon condensates is important in order to understand the $\Delta I = 1/2$ rule for $K \to 2\pi \nu$ decays.

As the $\chi$QM approach successfully indicated the main mechanisms in $K \to \pi \pi$ decays, it seems worthwhile to investigate decays of charm mesons within a similar framework. However, in the case of $D$ meson decays one has to extend the ideas of the $\chi$QM to the sector involving a heavy quark (c) using the chiral symmetry of light degrees of freedom as well as heavy quark symmetry and Heavy Quark Effective Field Theory (HQEFT), Such ideas have already been presented in previous papers [19, 20, 21] and lead to the formulation of Heavy-Light Chiral Quark Models (HL$\chi$QM). In our formulation of the HL$\chi$QM Lagrangian, an unknown coupling constant appears in the term that couples the heavy meson to a heavy and a light quark. Our strategy is to relate expressions involving this coupling to physical quantities, as it is done within the $\chi$QM [12]. We perform the bosonization by integrating out the light and heavy quarks and obtain a heavy quark symmetric chiral Lagrangian involving light and heavy mesons [22, 23].

Because the $O(p)$ (factorizable) contribution is zero as seen in Eq. (3), we will try in this paper to approach to the $D^0 \to K^0 \bar{K}^0$ decay systematically to $O(p^3)$. We do this by including first the nonfactorizable contributions coming from the chiral loops. These are based on the weak Lagrangian corresponding to the factorizable $O(p)$ terms for $D^0 \to \pi^+ \pi^-$ and $D^0 \to K^+ K^-$. (Note that the
velocity \( v^\mu = p_D^\mu / m_D \) is considered to be \( \mathcal{O}(p^0) \) in the chiral counting). Second, we consider the gluon condensate contributions, also of \( \mathcal{O}(p^3) \) within the \( \chi \)QM and HL\( \chi \)QM framework. It should be noted that because the energy release in \( D \to K \bar{K} \) is of order one GeV (in contrast to 200 MeV for \( K \to 2\pi \)), the next to leading \( \mathcal{O}(p^5) \) terms might be almost of the same size numerically compared to our \( \mathcal{O}(p^3) \) terms. Still, the amplitude of \( D^0 \to K^0 \bar{K}^0 \) calculated within the framework of \( \mathcal{O}(p^3) \), has a reliable order of magnitude. Note that we have also omitted \( 1/m_Q \) terms in the framework of HQEFT.

Our paper is organized as follows: In Section 2 we write down the basic Lagrangians including the weak Lagrangian at quark level (with special emphasis on the terms giving rise to the nonfactorizable gluon condensate contributions) as well as the standard strong chiral Lagrangians for the light and heavy meson sectors. The chiral loop contributions to the decay amplitudes are presented in Section 3. The details of the Heavy - Light Chiral Quark Model (HL\( \chi \)QM) are presented in Section 4, while the bosonization of the weak quark currents is given in Section 5. The results are given in section 6. Appendix A contains some details from the chiral loop integrals, Appendix B some details about the \( D \) meson decay constant, while Appendix C contains some loop integrals within the HL\( \chi \)QM.

2 Basic Lagrangians

The effective weak Lagrangian at quark level relevant for \( D \to \pi \pi, K \bar{K} \) is

\[
\mathcal{L}_W = \tilde{G} \left[ c_A (Q_A - Q_C) + c_B (Q_B^{(s)} - Q_B^{(d)}) \right],
\]

where \( \tilde{G} = -2\sqrt{2} G_F V_{us} V_{cs}^* \), and

\[
Q_A = (\bar{s}L \gamma^\mu c_L) (\bar{u}_L \gamma_\mu s_L), \quad Q_C = (\bar{d}_L \gamma^\mu c_L) (\bar{u}_L \gamma_\mu d_L),
\]

\[
Q_B^{(q)} = (\bar{u}_L \gamma^\mu c_L) (\bar{q}_L \gamma_\mu q_L), \quad (q = s, d),
\]

are quark operators.

Using Fierz transformations and the following relation between the generators of \( SU(3)_c \) (\( i, j, l, n \) are color indices running from 1 to 3 and \( a \) is an index running over the eight gluon charges):

\[
\delta_{ij} = \frac{1}{N_c} \delta_{in} \delta_{lj} + 2 t^a \frac{t}{t^a} t^a ,
\]

one obtains

\[
Q_A = \frac{1}{N_c} Q_B^{(s)} + R_B^{(s)}, \quad Q_C = \frac{1}{N_c} Q_B^{(d)} + R_B^{(d)},
\]

\[
Q_B^{(s)} = \frac{1}{N_c} Q_A + R_A, \quad Q_B^{(d)} = \frac{1}{N_c} Q_C + R_C,
\]

where the \( R \)'s correspond to color exchange between two currents and is genuinely nonfactorizable:

\[
R_A = 2 (\bar{s}_L \gamma^\mu t^a c_L) (\bar{u}_L t^\mu s_L), \quad R_C = 2 (\bar{d}_L \gamma^\mu t^a c_L) (\bar{u}_L t^\mu \gamma_\mu d_L),
\]

\[
R_B^{(q)} = 2 (\bar{u}_L \gamma^\mu t^a c_L) (\bar{q}_L t^a q_L), \quad (q = s, d),
\]

The operators can be written in terms of currents, for instance:

\[
Q_B^{(s)} - Q_B^{(d)} = J_{Y,a}^X, \quad R_B^{(s)} - R_B^{(d)} = 2 J_{Y,a}^X,
\]
\[ J_Y^{\mu a} \equiv \bar{\sigma}_L \gamma^\mu t^a c_L ; \quad j_X^{\mu a} \equiv \bar{\sigma}_L t^a \gamma^\mu s_L - [s \to d]. \] (10)

The currents without color index are given by the corresponding expressions dropping the color matrix.

The factorization approach amounts to writing the currents \( J_Y^\mu \), \( j_X^\mu \) in terms of hadron (in our case meson) fields only, so that the operator \( Q_B^{(s)} - Q_B^{(d)} \) in the left equation of (11) is equal to the product of two meson currents. The color currents in (11) are then zero if hadronized (mesons are color singlet objects). There is also a replacement of the Wilson coefficients in the hadronized effective weak Lagrangian \( c_{A,B} \to c_{A,B}(1 + 1/N_c) \). Combining heavy quark symmetry and chiral symmetry of the light sector, we can obtain the weak chiral Lagrangian for nonleptonic \( D \) meson decays due to factorizable terms. Then we can first use this to calculate nonfactorizable contributions due to chiral loops. Second, we can calculate the color currents’ contribution using the gluon condensate within the framework of the HLχQM.

Treating the light pseudoscalar mesons as pseudo-Goldstone bosons one obtains the usual \( \mathcal{O}(p^2) \) chiral Lagrangian

\[ \mathcal{L}_{str}^{(2)} = \frac{f^2}{8} \text{tr}(\partial^\mu \Sigma \partial_\mu \Sigma^\dagger) + \frac{f^2 B_0}{4} \text{tr}(M_q \Sigma + M_q \Sigma^\dagger), \] (11)

where \( \Sigma = \exp(2i\Phi/f) \) with \( \Phi = \sum_j \lambda^j \pi^j \) containing the Goldstone bosons \( \pi, K, \eta \), while the trace \( \text{tr} \) runs over flavor indices and \( M_q = \text{diag}(m_u, m_d, m_s) \) is the current quark mass matrix. From this Lagrangian, we can deduce the light weak current to \( \mathcal{O}(p) \)

\[ j_X^\mu = -i \frac{f^2}{4} \text{tr}(\Sigma \partial_\mu \Sigma^\dagger \lambda^X), \] (12)

corresponding to the quark current \( j_X^\mu = \bar{q}_L \gamma_\mu \lambda^X q_L \). (\( \lambda^X \) is a SU(3) flavor matrix.)

In the heavy meson sector interacting with light mesons we have the following lowest order \( \mathcal{O}(p) \) chiral Lagrangian

\[ \mathcal{L}_{str}^{(1)} = -\text{Tr}(\bar{H}_{va} i v \cdot D_{ab} H_{vb}) - g \text{Tr}(\bar{H}_{va} H_{vb} \gamma_\mu A^\mu_{ab} \gamma_5), \] (13)

where \( iD^\mu_{ab} H_b = i \partial^\mu H_a - H_b V^\mu_{ba} \), the trace \( \text{Tr} \) runs over Dirac indices. Note that in (13) and the rest of this section \( a \) and \( b \) are flavor indices.

The vector and axial vector fields \( V_\mu \) and \( A_\mu \) in (13) are given by:

\[ V_\mu \equiv i \left( \xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right) ; \quad A_\mu \equiv i \left( \xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right), \] (14)

where \( \xi = \exp(i\Phi/f) \). The heavy meson field \( H_{va} \) contains a spin zero and spin one boson:

\[ H_{va} \equiv P_+(P_{ua} \gamma^\mu - iP_{5a} \gamma_5), \] (15)

\[ \overline{H}_{va} = \gamma^0 (H_{va})^\dagger \gamma^0 = \left[ P^\dagger_{ua} \gamma^\mu - iP^\dagger_{5a} \gamma_5 \right] P_+, \] (16)

with \( P_\pm = (1 \pm \gamma^\mu v_\mu)/2 \) being the the projection operators. The field \( P_5 (P_5^\dagger) \) annihilates (creates) a pseudoscalar meson with a heavy quark having velocity \( v \), and similar for spin one mesons.
For a decaying heavy quark, the weak current is given by
\[ J^\lambda_a = \bar{q}_a \gamma^\lambda L Q, \]  
(17)
where \( L = (1 - \gamma_5)/2 \) and \( Q \) is the heavy quark field in the full theory, in our case a \( c \)-quark, and \( q \) is the light quark field. (For flavor \( a = u \), this is the current \( J^\mu_Y \) in (9).)

From symmetry grounds, the heavy-light weak current is to \( \mathcal{O}(p^0) \) bosonized in the following way \[ J^\lambda_a = \alpha_H \text{Tr} [\gamma^\lambda L H_{ab} \xi_{ba}^\dagger] , \]  
(18)
where \( \alpha_H \) is related to the physical decay constant \( f_D \) through the well known matrix element
\[ \langle 0 | \pi \gamma^\lambda \gamma_5 | D^0 \rangle = -2 \langle 0 | J^\lambda_a | D^0 \rangle = im_D v^\lambda f_D . \]  
(19)
Note that the current (18) is \( \mathcal{O}(p^0) \) in the chiral counting.

### 3 Chiral loop contributions

In the factorization limit there are no contributions to \( D^0 \to K^0 \bar{K}^0 \) at tree level. The observation of a partial decay width \( B(D \to K^0 \bar{K}^0) = (6.5 \pm 1.8) \times 10^{-4} \) on the other hand implies that we can expect sizable contributions at the one loop level. Calculations to one loop in the framework of combined chiral perturbation theory and HQEFT involves a construction of the most general effective Lagrangian that has the correct symmetry properties in order to make the renormalization work. We discuss constructions of counterterms in the end of Sect. 5.

We work in the strict \( \overline{\text{MS}} \) renormalization scheme, where we put \( \overline{\Delta} = \frac{2}{\epsilon} - \gamma + \ln(4\pi) + 1 \) equal to one in the loop calculations. This choice, \( \overline{\Delta} = 1 \), determines the appropriate renormalization of couplings in the \( \mathcal{O}(p^3) \) effective Lagrangian and is the same as made by Stewart in [28], while it differs from the one used by authors of Ref. [22], who use \( \overline{\Delta} = 0 \). We consider only contributions coming from the \( c_A \) part of the weak Lagrangian as \( c_B \) is suppressed compared to \( c_A \) [25].

Writing down the most general one loop graphs with two outgoing Goldstone bosons (\( K^0 \) and \( \bar{K}^0 \)) one arrives at 26 Feynman diagrams. A number of these give zero contributions and are shown on Figures [1][2][3], while the graphs that do contribute to \( D^0 \to K^0 \bar{K}^0 \) decay are shown on Fig. 4. Note that factorizable loops which renormalize vertices are omitted (they do appear, however, in the loop determination of the \( \alpha_H \) coupling related to \( f_D \). See Appendix B.)

To shorten the notation, the common factors in the \( S \) matrix have been organized such that the amplitude is written
\[ \mathcal{M}(D^0 \to K^0 \bar{K}^0) = -\frac{G_F}{\sqrt{2}} c_A V_{us} V_{cs}^* \frac{F}{8\pi^2 \sqrt{m_D}} , \]  
(20)
where \( F = \sum F_n \) is the sum of the amplitudes corresponding to the graphs on Figs. 1, 2, 3, 4. The partial decay width for the decay \( D^0 \to K^0 \bar{K}^0 \) is then
\[ \Gamma_{D^0 \to K^0 \bar{K}^0} = \frac{1}{2\pi} \frac{G_F^2}{8m_D} c_A^2 |V_{us} V_{cs}^*|^2 \frac{|F|^2}{(8\pi^2)^2} p , \]  
(21)
where \( p \) is the \( K^0(\bar{K}^0) \) three-momentum in the \( D^0 \) rest frame
\[ p = \frac{1}{2} \sqrt{m_D^2 - 4m_K^2} . \]  
(22)
Figure 1: Diagrams that give zero contribution since the relevant vertices appearing in the heavy meson chiral Lagrangian (13) are zero. The double line represents heavy meson $D$ or $D^*$, while dashed lines denote pseudo-Goldstone bosons.

Figure 2: Diagrams that give zero contributions since the loop integrals are zero. The double line represents heavy meson $D$ or $D^*$, while dashed lines denote pseudo-Goldstone bosons.
Figure 3: Power suppressed diagrams (neglected in the calculation).

Figure 4: The nonzero diagrams in $D^0 \to K^0 \bar{K}^0$ decay.
The nonzero amplitudes corresponding to the graphs on Fig. 5 are

\[
F_1 + F_2 + F_3 = -g\alpha_H \frac{13}{4} \frac{m_D}{f^2} \left[ \Delta^*_q J_1(m_\pi, \Delta^*_d) - \Delta^*_s J_1(m_K, \Delta^*_s) \right],
\]

\[
F_4 = -\frac{\alpha_H m_D}{4 f^2} \left\{ \left[ m_D^2 - 2m_K^2 \right] \left[ N_0(m_\pi, m_D^2) - N_0(m_K, m_D^2) \right] + m_D^2 \left[ N_2(m_\pi, m_D^2) - N_2(m_K, m_D^2) \right] + \right. \\
\left. + \left[ N_3(m_\pi, m_D^2) - N_3(m_K, m_D^2) \right] - (m_\pi^2 - m_K^2) N_0(m_\pi, m_D^2) \right\},
\]

\[
F_5 + F_6 = \frac{\alpha_H m_D}{f^2} \frac{7}{24} \left[ I_1(m_\pi) - I_1(m_K) \right],
\]

\[
F_7 + F_8 = \frac{\alpha_H}{4 f^2} \left\{ \tilde{\Delta}_q \left( J_1(m_K, \tilde{\Delta}_d) + J_2(m_K, \tilde{\Delta}_d) \right) - \tilde{\Delta}_s \left( J_1(m_\pi, \tilde{\Delta}_s) + J_2(m_\pi, \tilde{\Delta}_s) \right) \\
+ m_D \frac{\Delta^*_d}{\Delta^*_d} I_2(m_K, \tilde{\Delta}_d) - m_D \frac{\Delta^*_s}{\Delta^*_s} I_2(m_\pi, \tilde{\Delta}_s) + \frac{m_D}{2 \Delta^*_d} I_1(m_K) - \frac{m_D}{2 \Delta^*_s} I_1(m_\pi) \right\},
\]

where \( \Delta^*_q = m_{D^0}^q - m_{D^0} \) and \( \tilde{\Delta}_q = m_{D^0}/2 + \Delta_q \) for \( q = d, s \). Note that \( \tilde{\Delta}_q \) are of the order \( m_{D^0}/2 \), a consequence of relatively high momenta flowing in the loops of graphs \( F_7, F_8 \). The functions \( I_1(m), I_2(m, \Delta), J_1(m, \Delta), J_2(m, \Delta), N_0(m, k^2), N_2(m, k^2), N_3(m, k^2) \) appearing in the amplitudes (24-27) can be found in Appendix A.

It should be noted that in eq. (23-26) all the expressions vanish in the exact \( SU(3) \) limit, where \( m_K \to m_\pi \) and \( \Delta_s \to \Delta_d, \Delta_s \to -\Delta_d \). This shows explicitly that the \( D^0 \to K^0 \bar{K}^0 \) decay mode is a manifestation of \( SU(3) \) breaking effects (as already noted by H. Lipkin [4], if \( U \) symmetry is exact, then \( \Gamma(D^0 \to K^0 \bar{K}^0) = 0 \)).

The amplitudes shown on Figs. 1, 2, 3, 4 are either exactly zero or are suppressed by powers of \( 1/m_D \) and \( g = 0.27 \). The amplitudes corresponding to diagrams on Figs. 5, 6, 7, 8 are zero due to symmetry reasons (because there are no such couplings in the heavy sector chiral Lagrangian [13]), or because of Lorentz covariance, while the amplitudes \( F_9, F_{10} \) and \( F_{11} \) shown on Fig. 3 are power suppressed. An analysis of the loop integrals leads to the conclusion that \( F_9 \sim g^3(m_{D^0}) F_4, F_{10} \sim g^3(m_{D^0}) F_4 \) and \( F_{11} \sim g^3(m_{D^0}) F_4 \), where \( \tilde{q} \) is a typical loop momentum less than \( m_{D^0}/2 \), so the suppression need not be substantial. However, a direct evaluation of the amplitude \( F_{10} \) shows that it is about 20 times smaller than \( F_4 \). Therefore, in our numerical calculation we neglect contributions of \( F_9, F_{10} \) and \( F_{11} \).
4 A Heavy-Light Chiral Quark Model (HLχQM)

The nonfactorizable contributions to $D^0 \rightarrow K^0\bar{K}^0$ coming from the chiral loop correction at the meson level obtained in the previous section are not the only contributions to $O(p^3)$. In the effective weak Lagrangian (4) there are, after Fierz transformations, terms that involve color currents (see (9),(10)). As mesons are color singlet objects, the product of color currents does not contribute at meson level in the factorization limit. However, at quark level they do contribute through the gluon condensate as will be shown in the next section. In order to estimate this contribution we have to establish the connection between the underlying quark-gluon dynamics and the meson level picture. This is done through the use of the Heavy-Light Chiral Quark Model (HLχQM).

Our starting point is the following Lagrangian containing both quark and meson fields:

$$L = L_{HQ} + L_{χQM} + L_{Int},$$

where

$$L_{HQ} = \bar{Q}_v i γ^μ D_μ Q_v + O(m^{-1})$$

is the Lagrangian for Heavy Quark Effective Field Theory (HQEFT). The heavy quark field $Q_v$ annihilates a heavy quark with velocity $v$ and mass $m_Q$. $D_μ$ is the covariant derivative containing the gluon field. The light quark sector is described by the Chiral Quark Model (χQM):

$$L_{χQM} = \bar{q}(iγ^μ D_μ - M_q)q - m_χ (\bar{q}_R Σ^† q_L + \bar{q}_L Σ q_R),$$

where $q = (u, d, s)$ are the light quark fields. The left and right-handed projections $q_L$ and $q_R$ are transforming under $SU(3)_L$ and $SU(3)_R$ respectively. $M_q$ is the current quark mass matrix, and $Σ$ is a 3 by 3 matrix containing the (would be) Goldstone octet ($π, K, η$), appearing already in (11).

The quantity $m_χ$ is interpreted as the (SU(3)-invariant) constituent quark mass for light quarks, supposed to appear due to chiral symmetry breakdown at a scale $Λ_χ \sim 1$ GeV.

The χQM has a “rotated version” with flavor rotated quark fields $χ$ given by:

$$χ_L = ξ^† q_L; \quad χ_R = ξ q_R; \quad ξ · ξ = Σ.$$  

In the rotated version, the chiral interactions are rotated into the kinetic term while the interaction term (proportional to $m_χ$ in (22) and responsible for the $π$-quark couplings) become a pure (constituent) mass term:

$$L_{χQM} = \bar{χ}[γ^μ(iD_μ + M_q)χ] - m_χ χ - \bar{χ}M_q χ,$$

and $M_q$ defines the rotated version of the current mass term:

$$\bar{M}_q ≡ ξ^† M_q ξ^† R + ξ M_q^† ξ L ≡ \bar{M}_q^R R + \bar{M}_q^L L ≡ \bar{M}_q^V + \bar{M}_q^A γ_5,$$

where $L = (1 - γ_5)/2$ is the left-handed projector in Dirac space, and $R$ is the corresponding right-handed projector. The Lagrangian (22) is manifestly invariant under the unbroken symmetry $SU(3)_V$ (if $M_q$ is formally chosen to transform as $Σ$). In the light sector, the various pieces of the strong chiral Lagrangian (22) can be obtained by integrating out the constituent quark fields $χ$. This is the bosonization to be discussed in more detail in the next section.

Similarly, a left handed current can be written ($λ^X$ is a SU(3) flavor matrix)

$$\bar{q}_L γ^μ λ^X q_L = \bar{χ}_L γ^μ Λ^X χ_L; \quad Λ^X ≡ ξ^† λ^X ξ.$$
By coupling the fields $A_\mu$, $\tilde{M}_q^{V,A}$, $\Lambda^X$ to quark loops, the chiral Lagrangians of the weak sector can be obtained.

In the heavy-light case, the generalization of the meson-quark interactions in the pure light sector $\chi Q M$ is given by the following $SU(3)_V$ invariant Lagrangian $\{13, 21, 24\}$:

\[ L_{\text{int}} = -G_H \left[ \bar{\chi} f H v_f + Q_v H_{vf} \chi_f \right], \tag{34} \]

where $G_H$ is a coupling constant which is related through bosonizations to physical quantities like $\alpha_H$ and $g$ appearing in (13) and (18), as well as $f_\pi$ and $m_\chi$. (See Appendix C).

Within HQEFT the heavy-light weak current in (17) will, below the renormalization scale $\mu = m_c$, be modified in the following way $\{31\}$:

\[ j^\lambda_a = C_{\gamma} (\mu) \bar{\chi} b \gamma^\dagger_{ba} \Lambda^\lambda L Q_v + C_v (\mu) \bar{\chi} b \gamma^\dagger_{ba} v^\lambda L Q_v, \tag{35} \]

where the coefficients $C_{\gamma,v}$ are determined by QCD renormalization for $\mu < m_c$. However, for $\mu \approx \Lambda^\chi$, $C_{\gamma} \approx 1$ and $C_v \approx 0$. The bosonization of (35) will lead to (18) by using (34).

5 Bosonization

The Lagrangian (27) from the previous section can now be used for bosonization, i.e. we integrate out the quark fields. This can be done in the path integral formalism, or as we do here, by expanding in terms of Feynman diagrams. For instance, the lowest order (kinetic) chiral Lagrangian (11) in the light sector (involving $\pi, K, \eta$’s) can be obtained by coupling two axial fields to a quark loop using the Lagrangian in Eq. (31):

\[ iL^{(2)}_{\text{str}}(\pi, K, \eta) = -N_c \int \frac{d^dp}{(2\pi)^d} Tr \left[ \left( \gamma_\sigma \gamma_5 A^\sigma \right) S(p) \left( \gamma_\rho \gamma_5 A^\rho \right) S(p) \right] \sim Tr \left[ A_\mu A^\mu \right], \tag{36} \]

where $S(p) = (\gamma \cdot p - m_\chi)^{-1}$, and the trace is both in flavor and Dirac spaces. This is the standard form of the lowest order chiral Lagrangian (11), which can easily be seen by using the relations

\[ A_\mu = -\frac{1}{2i} \xi (\partial_\mu \Sigma) \xi = \frac{1}{2i} \xi^\dagger (\partial_\mu \Sigma) \xi^\dagger. \tag{37} \]

Similarly one obtains the lowest order $O(p)$ strong chiral Lagrangian (13) in the heavy sector.

Let us now consider the bosonization of the pure light weak current. The lowest order term $O(p)$ is obtained when the vertex $\Lambda^X$ from (33) and axial vertex ($\sim A_\mu$) from (31) are combined with quark loops (see Fig. 3):

\[ j^X_\mu (A) = -iN_c \int \frac{d^dp}{(2\pi)^d} Tr \left[ \left( \gamma_\mu L \Lambda^X \right) S(p) \left( \gamma_\sigma \gamma_5 A^\sigma \right) S(p) \right] \sim Tr \left[ \Lambda^X A_\mu \right]. \tag{38} \]

This coincides with (37) when (37) is used.

To obtain a nonzero nonfactorizable contribution to $D^0 \rightarrow K^0 \bar{K}^0$ at tree level, we have to consider the color current $j^{X,a}_\mu$ to $O(p^3)$, involving insertions of the “mass fields” $\tilde{M}_q$ in (32). From Fig. 7, one obtains the contribution:

\[ j^{X,a}_\mu (G^b, A, \tilde{M}_q, \text{Fig. 7}) = i \int \frac{d^dp}{(2\pi)^d} Tr \left[ \left( \gamma_\mu L \Lambda^X \right) S(p) \left( \gamma_\sigma \gamma_5 A^\sigma \right) S(p) \tilde{M}_q S_1(p, G^b) \right], \tag{39} \]
Figure 6: Feynman diagram for bosonization of left-handed current to order $O(p)$

$$A_{\sigma} \quad \gamma_{\mu} L$$

Figure 7: Diagram for bosonization of the color current to $O(p^3)$

$$\gamma_{\mu} L a$$

where

$$S_1(p, G^b) = -\frac{g_s}{4} G^b_{\alpha \beta} t^b \left[ \sigma^{\alpha \beta} (\gamma \cdot p + m_\chi) + (\gamma \cdot p + m_\chi) \sigma^{\alpha \beta} \right] (p^2 - m_\chi^2)^{-2}$$

is the light quark propagator in a gluonic background (to first order in the gluon field) and $g_s$ is the strong coupling constant. Moreover, $a, b$ are color octet indices. Summing all six diagrams with permutated vertices compared to the one in Fig. 6 we obtain in total:

$$j^X_{\mu} (G^b, A, \overline{M_q}) = \frac{g_s}{12m_\chi} \frac{1}{16\pi^2} G^{a, \kappa \lambda} \left[ i \varepsilon_{\mu \rho \kappa \lambda} T^X_{\kappa \rho} + (\eta_{\mu \rho} \eta_{\rho \kappa} - \eta_{\mu \kappa} \eta_{\rho \kappa}) T^X_{\kappa \rho} \right],$$

where (We have used the analytical computer program FORM [30])

$$T^X_{\kappa \rho} = 4 S^K_{\rho} - 3 (S^L_{\rho} + S^R_{\rho})$$

$$T^X_{\kappa \rho} = \frac{1}{4i} \left[ \Lambda^X (D_\rho \Sigma) \Sigma^\dagger M_q^L + M_q^L (D_\rho \Sigma) \right]$$

$$T^X_{\kappa \rho} = \frac{1}{4i} \left[ \Lambda^X (D_\rho \Sigma) \Sigma^\dagger M_q^L + M_q^L (D_\rho \Sigma) \right]$$

The $S'$s are chiral Lagrangian terms:

$$S^L_{\rho} \equiv Tr \left[ \Lambda^X A_\rho \overline{M_q}^L \right] = \frac{1}{2i} \left[ \Lambda^X (D_\rho \Sigma) \Sigma^\dagger \right],$$

$$S^R_{\rho} \equiv Tr \left[ \Lambda^X \overline{M_q}^R A_\rho \right] = \frac{1}{2i} \left[ \Lambda^X M_q (D_\rho \Sigma^\dagger) \right],$$

$$S^K_{\rho} \equiv \frac{1}{2} Tr \left[ \Lambda^X \left( A_\rho \overline{M_q}^R + \overline{M_q}^L A_\rho \right) \right]$$

$$= \frac{1}{4i} \left[ \Lambda^X \left( (D_\rho \Sigma) \Sigma^\dagger M_q^L + M_q^L (D_\rho \Sigma) \right) \right]$$

Within the heavy-light sector, the weak current can be bosonized to lowest order ($O(p^0)$) by calculating the Feynman diagram shown in Fig. 8, left. The obtained result is Eq. (18) with $\alpha_H$ related to $G_H$ (see Appendix C).
Figure 8: Diagrams representing bosonization of heavy-light weak current. The boldface line represents the heavy quark, the solid line the light quark.

The bosonization of the color current given by (35) with an extra color matrix $t^a$ inserted and with an extra gluon emitted is given by the following loop integral (Fig. 8, right):

$$J^\sigma(H_v G^a)^f = - \int \frac{d^4k}{(2\pi)^d} Tr \left[ (-iG_H H_v \xi^\dagger)^f (iS_1(k, G^b)) (\gamma^\sigma L t^a)(i\Delta_v(k)) \right],$$  \hspace{1cm} (44)

where $\Delta_v(k) = P_+/k \cdot v$ is the heavy quark propagator. Notice that emission of a gluon from the heavy quark is suppressed by $1/m_Q$ and omitted. The result can be written

$$J^\sigma(H_v G^a)^f = G_H g_s G_{\alpha\beta} G^{a,\alpha\beta} [iB_\varepsilon \varepsilon_{\mu\alpha\beta} v^\sigma + B_\eta (\eta_{\mu\alpha} v_\beta - \eta_{\mu\beta} v_\alpha)] ,$$  \hspace{1cm} (45)

where $B_\varepsilon$ and $B_\eta$ are obtained from loop integrals given in Appendix C. Keeping only the pseudoscalar field $P_5$ representing $D_0$, we find

$$J^{Y,\alpha}_\mu(P_5, G^b) = \frac{g_s G_H}{16\pi^2} (P_5 \xi^\dagger)^Y G^{a,\alpha\beta} \left[ iB_\varepsilon \varepsilon_{\mu\alpha\beta} v^\sigma + B_\eta (\eta_{\mu\alpha} v_\beta - \eta_{\mu\beta} v_\alpha) \right],$$  \hspace{1cm} (46)

where $B_\varepsilon, \eta$ are obtained from loop integrals in (45). Then we find the nonfactorizable (gluon condensate) contribution:

$$\mathcal{L}_{eff}(D^0\text{decay})_{(G^2)} = 2G_{\varepsilon, \eta} \left( \frac{g_s G_H}{16\pi^2} \right) \left( \frac{g_s}{12m_\chi} \right) \frac{1}{16\pi^2} \langle G^2 \rangle \times v_\rho \left[ B_\varepsilon T_\varepsilon \xi^{X,\rho} + B_\eta T_\eta \xi^{X,\rho} \right] (P_5 \xi^\dagger)^Y ,$$  \hspace{1cm} (47)

where $\langle G^2 \rangle$ is the gluon condensate, obtained by the prescription

$$G^{a,\mu\nu} G^{a,\alpha\beta} \to \frac{1}{12} (\eta_{\mu\alpha} \eta_{\nu\beta} - \eta_{\mu\beta} \eta_{\nu\alpha}) \langle G^2 \rangle .$$  \hspace{1cm} (48)

In order to make predictions, we have to relate $G_H$ in (47) and the various loop integrals to physical quantities like $m_\chi$, $f_\pi$ and $\alpha_H \approx f_D \sqrt{m_D}$. It should be noted that there are apriori other terms than the one in (47). There is one possible term where the field $\widetilde{M}_q$ occurring in Fig. 7 may instead be attached to the light quark line in diagram in Fig. 8 (right). However, this term will not give contributions to $D^0 \to K^0 \bar{K}^0$. Moreover, there is apriori a term where the field $A_\sigma$ attached in Fig. 7 is instead attached to the light quark line in Fig. 8 (right). This term is identically zero.

In the language of chiral perturbation theory, the term (47) can be interpreted as a counterterm. To be more specific, the (divergent part of the) counterterm has the Lorentz and flavor structure of the second line of (47) and is multiplied with a (divergent) coefficient adjusted to cancel the loop divergences obtained in Sect. 3.
6 Results

In our numerical calculation we use the values of \( \alpha_H \), \( g \) and \( f \) obtained within the same framework in \([28, 27, 22, 32, 33]\). The coupling \( g \) is extracted from existing experimental data on \( D^* \to D\pi \) and \( D^* \to D\gamma \) decays. This analysis \([28]\) includes chiral corrections at one loop order and yields \( g = 0.27^{+0.04+0.05}_{-0.02-0.02} \), leaving the sign undetermined. The one loop chiral corrections reduce the pion decay constant from \( f_\pi = 0.132 \text{ GeV} \) to \( f = 0.120 \text{ GeV} \) \([28]\). In order to obtain the \( \alpha_H \) coupling, we use present experimental data on \( D_s \) leptonic decays. Namely, at the tree level there is a relation

\[
M_D = \frac{\alpha_H}{\sqrt{m_D}}.
\]

This relation receives \(10^{-20}\%\) chiral corrections \([22, 32]\). From the experimental branching ratio \( D_s \to \mu\nu \) and the \( D_s \) decay width \([15]\) one gets \( M_{D_s} = 0.23 \pm 0.05 \text{ GeV} \). Using the strict \( \overline{MS} \) prescription \( \Delta = 1 \) as in \([28]\). We put everywhere \( \mu = 1 \text{ GeV} \simeq \Lambda_{\chi} \).

For the Wilson coefficients \( c_{A,B} \) of \((4)\) we use \( c_A = 1.10 \pm 0.05 \) and \( c_B = -0.06 \pm 0.12 \) \([25]\), calculated at the scale \( \mu = 1 \text{ GeV} \) with the number of colors \( N_c = 3 \). Within the framework of “new” or “generalized” factorization, where nonfactorizable effects are taken into account in a phenomenological way, one uses the “effective values” \( c_{A,B}^{\text{eff}} = 1.26 \) and \( c_{B}^{\text{eff}} = -0.47 \). However, in this paper we calculate nonfactorizable effects in terms of chiral loops and gluon condensates, and therefore we use the values of \([25]\). Due to the suppression of \( c_B \) in comparison with \( c_A \), we do not include terms proportional to \( c_B \).

We present our numerical results for the nonzero amplitudes in Table 1.

\[
\begin{array}{|c|c|}
\hline
- & M_i \times 10^{-7} \text{ GeV} \\
\hline
M_1 & -0.42 \\
M_2 & -0.31 \\
M_3 & -0.62 \\
M_4 & 0.28 - 2.44i \\
M_5 & -0.81 \\
M_6 & -0.61 \\
M_7 & -0.99 \\
M_8 & 0.92 \\
\hline
\sum_i M_i & -2.56 - 2.44i \\
\hline
\end{array}
\]

Table 1: Table of the one chiral loop amplitudes (see Fig. 4), where \( M = \sum_n M \) is defined in \([24]\). In the last line the sum of all amplitudes is presented. It can be compared with the experimental result \( |M_{\text{Exp}}| = 3.80 \times 10^{-7} \text{ GeV} \).

The imaginary part of the amplitude comes from the \( F_4 \) graph, when the \( \pi \)'s or the \( K \)'s in the loops are on-shell. All other graphs contribute only to the real part of the amplitude. The imaginary part of the amplitude is scale and scheme independent within chiral perturbation theory. This amplitude is also obtained fromunitarity, and is valid beyond the chiral loop expansion. We also mention that the rescattering contribution, considered in \([2,10]\) is the same contribution as the one we calculate from graphs on Fig. 5.

In order to cancel divergences one has to construct counterterms. In our case, this is described at the end of section 5. Generally, one can do that by using the symmetry arguments, as it has

\footnote{Even if the “new factorization” values had been used, the \( c_B \) part of weak interaction would be suppressed by \( 1/3 \) compared to the \( c_A \) one.}
been done in [27, 28] for the semileptonic decays of heavy mesons and $D^*$ decays. In the case of $D^*$ [28] it was estimated that the contribution of counterterms is not substantial.

To obtain the $D^0 \to K^0 \bar{K}^0$ amplitude due to gluon condensate we have to know the coupling $G_H$. In addition, we have to find the tensors $S$ in (12, 13) (and thereby the $T$’s) for $K^0 \bar{K}^0$ in the final state. We find to lowest order for the parts of $T_{g,x}$:

$$S^L_\mu = - S^R_\mu = - \frac{1}{f^2} (m_s - m_d) (p + \bar{p})_\mu; \quad S^K_\mu = \frac{2}{f^2} (m_s + m_d) (p - \bar{p})_\mu,$$

where $p$ and $\bar{p}$ are the momenta of $K^0$ and $\bar{K}^0$ respectively. From (16) we see, that the momenta will be contracted with $v^\mu = p^\mu_D/M_D$; where $p_D = p + \bar{p}$. It is important that $S^L_\mu$ and $S^K_\mu$ have a different momentum structure than $\langle K^0 \bar{K}^0 | V_\mu | 0 \rangle$ in Eq. (3), and they will give a nonfactorizable contribution to $D^0 \to K^0 \bar{K}^0$ proportional to $\langle G^2 \rangle$, while $S^K_\mu$ does not. Note that $T_{g,x}$ is the next order contribution and $T_{g,x}^{\tilde{X},\rho}$ do not contribute. We find the gluon condensate contribution:

$$\mathcal{M}(D^0 \to K^0 \bar{K}^0)_{\langle G^2 \rangle} = c_A (\tilde{G} m^2_D) \frac{(m_s - m_d)}{m_\chi} \frac{\beta \delta_G}{6 N_c} B g f_D$$

where:

$$\delta_G \equiv N_c \frac{\alpha_s G^2/\pi}{8 \pi^2 f^4}, \quad G_H \equiv \beta \frac{G^2}{f^2}, \quad B_g = 16 i \pi^2 (I_{G1} - I_{G2}) = \frac{\pi}{4}.$$

When we take into account the various relations between the loop integrals ($I$’s) and $G_H$, we find that $\beta \simeq 1/4$. Using the values [12] $\langle \alpha_s G^2 \rangle \simeq (333 \text{ MeV})^4$, $m_\chi = 200$ MeV, and $m_s \simeq 150$ MeV, we obtain the numerical value:

$$\mathcal{M}(D^0 \to K^0 \bar{K}^0)_{\langle G^2 \rangle} \simeq 0.87 \times 10^{-7} \text{ GeV};$$

which is also of the same order of magnitude as the experimental value.

Adding both the chiral loops and the gluon condensate contributions, we obtain the total amplitude to $\mathcal{O}(p^3)$

$$\mathcal{M}_{\text{Th}} = (-1.7 - 2.4 i) \times 10^{-7} \text{ GeV},$$

or in terms of branching ratio

$$B(D^0 \to K^0 \bar{K}^0)_{\text{Th}} = (4.3 \pm 1.4) \times 10^{-4},$$

where the estimated uncertainties reflect the uncertainties in the input parameters, especially $\alpha_H$.

Around the charm mesons mass region there are many resonances. One might think that their contribution will appear in this decay mode, either as scalar resonance exchange like in [9] or as $K^*$ exchanges [8, 9, 11]. Within our framework they would appear as the next order contribution ($\mathcal{O}(p^3)$) in the chiral expansion. This is, however, beyond the present scope of our investigations. It is interesting to point out that the effects we calculate, both from chiral loops and from the gluon condensate, are results of the $SU(3)$ flavor symmetry breaking. In the limit of exact symmetry both contributions will disappear.

We can summarize that we indicate the leading $\mathcal{O}(p^3)$ nonfactorizable contributions to $D^0 \to K^0 \bar{K}^0$. The chiral loops and gluon condensates give the contributions of the same order of magnitude as the amplitude extracted from the experimental result.
A List of integrals from chiral loops

Here we list the dimensionally regularized integrals needed for evaluation of χPT and HQEFT one-loop graphs shown on Fig. 4:

\[ i\mu^c \int \frac{d^{4-\epsilon} q}{(2\pi)^{4-\epsilon}} \frac{1}{q^2 - m^2} = \frac{1}{16\pi^2} I_1(m), \]  
\[ i\mu^c \int \frac{d^{4-\epsilon} q}{(2\pi)^{4-\epsilon}} \frac{1}{(q^2 - m^2)(q \cdot v - \Delta)} = \frac{1}{16\pi^2} \frac{1}{\Delta} I_2(m, \Delta), \]

with

\[ I_1(m) = m^2 \ln \left( \frac{m^2}{\mu^2} \right) - m^2 \bar{\Delta}, \]  
\[ I_2(m, \Delta) = -2\Delta^2 \ln \left( \frac{m^2}{\mu^2} \right) - 4\Delta^2 F \left( \frac{m}{\Delta} \right) + 2\Delta^2 (1 + \bar{\Delta}), \]

where \( \bar{\Delta} = \frac{2}{\epsilon} - \gamma + \ln(4\pi) + 1 \) (in calculation \( \bar{\Delta} = 1 \)), while \( F(x) \) is the function calculated by Stewart in [28], valid for negative and positive values of the argument

\[ F \left( \frac{1}{x} \right) = \begin{cases} -\frac{\sqrt{1-x^2}}{x} \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right) \right] & |x| \leq 1 \\ \frac{\sqrt{x^2-1}}{x} \ln \left( x + \sqrt{x^2-1} \right) & |x| \geq 1 \end{cases} \]

The other integrals needed are

\[ i\mu^c \int \frac{d^{4-\epsilon} q}{(2\pi)^{4-\epsilon}} \frac{q^\mu}{(q^2 - m^2)(q \cdot v - \Delta)} = \frac{v^\mu}{16\pi^2} [I_2(m, \Delta) + I_1(m)], \]  
\[ i\mu^c \int \frac{d^{4-\epsilon} q}{(2\pi)^{4-\epsilon}} \frac{q^\mu q^{\nu}}{(q^2 - m^2)(q \cdot v - \Delta)} = \frac{1}{16\pi^2} \Delta \left[ J_1(m, \Delta) \eta^{\mu\nu} + J_2(m, \Delta) v^\mu v^\nu \right], \]

with

\[ J_1(m, \Delta) = (-m^2 + \frac{2}{3}\Delta^2) \ln \left( \frac{m^2}{\mu^2} \right) + \frac{4}{3}\Delta^2 - m^2) F \left( \frac{m}{\Delta} \right) \]  
\[ - \frac{2}{3}\Delta^2 (1 + \bar{\Delta}) + \frac{1}{3} m^2 (2 + 3\bar{\Delta}) + \frac{2}{3} m^2 - \frac{4}{9}\Delta^2, \]  
\[ J_2(m, \Delta) = (2m^2 - \frac{8}{3}\Delta^2) \ln \left( \frac{m^2}{\mu^2} \right) - \frac{4}{3} (4\Delta^2 - m^2) F \left( \frac{m}{\Delta} \right) \]  
\[ + \frac{8}{3}\Delta^2 (1 + \bar{\Delta}) - \frac{2}{3} m^2 (1 + 3\bar{\Delta}) - \frac{2}{3} m^2 + \frac{4}{9}\Delta^2, \]

The functions \( J_1(m, \Delta), J_2(m, \Delta) \) differ from the ones in Boyd - Grinstein list of integrals [27] by the last two terms in (62) that are of the order of \( O(m^2, \Delta^2) \). These additional finite terms originate from the fact that \( \eta^{\mu\nu} \) is \( 4 - \epsilon \) dimensional metric tensor.
The chiral loop integrals needed are

\[
\begin{align*}
\mu^\epsilon \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} \frac{1}{((q + k)^2 - m^2)(q^2 - m^2)} &= \frac{1}{16\pi^2} N_0(m, k^2), \\
\mu^\epsilon \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} \frac{q^\mu}{((q + k)^2 - m^2)(q^2 - m^2)} &= \frac{k^\mu}{16\pi^2} N_1(m, k^2) = -\frac{k^\mu}{2} \frac{1}{16\pi^2} N_0(m, k^2), \\
\mu^\epsilon \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} \frac{q^{\mu\nu} q^{\nu\nu}}{((q + k)^2 - m^2)(q^2 - m^2)} &= -\frac{k^\mu k^{\nu}}{16\pi^2} N_2(m, k^2) = -\frac{\eta^{\mu\nu}}{16\pi^2} N_3(m, k^2),
\end{align*}
\]

where

\[
\begin{align*}
N_0(m, k^2) &= -\bar{\Delta} + 1 - H\left(\frac{k^2}{m^2}\right) + \ln\left|\frac{m^2}{\mu^2}\right| - i\pi \Theta\left(-\frac{m^2}{\mu^2}\right) \text{sign}(\mu^2), \\
N_2(m, k^2) &= \frac{1}{3} \left[ \bar{\Delta} + \frac{7}{6} - 2 \frac{m^2}{k^2} + 2 \left(\frac{m^2}{k^2} - 1\right) \left(1 - \frac{1}{2} H\left(\frac{k^2}{m^2}\right)\right) \right. \\
&\left. - \ln\left(m^2/\mu^2\right) + i\pi \Theta\left(-\frac{m^2}{\mu^2}\right) \text{sign}(\mu^2) \right], \\
N_3(m, k^2) &= \frac{1}{2} \left(m^2 - \frac{k^2}{6}\right) \Delta - \frac{1}{2} \left(\frac{1}{3} (8m^2 + k^2) \left[1 - \frac{1}{2} H\left(\frac{k^2}{m^2}\right)\right] - \frac{8}{3} m^2 \right. \\
&\left. - \frac{5}{18} k^2 + (m^2 + k^2) \left(\ln\left|m^2/\mu^2\right| - i\pi \Theta\left(-\frac{m^2}{\mu^2}\right) \text{sign}(\mu^2)\right) \right],
\end{align*}
\]

and

\[
H(a) = \begin{cases} 
2 \left(1 - \sqrt{4/a - 1}\right) \arctan\left(\frac{1}{\sqrt{4/a - 1}}\right) & 0 < a < 4 \\
2 \left(1 - \frac{1}{2} \sqrt{1 - 4/a} \left[\ln\left|\frac{\sqrt{1 - 4/a} + 1}{\sqrt{1 - 4/a} - 1}\right| - i\pi \Theta(a - 4)\right]\right) & \text{otherwise}
\end{cases}
\]

while \(m^2\) is assumed to be positive.

**B** **D meson decay constant**

Here we list results for one-loop chiral corrections to \(D\) meson decay constants and use them to obtain coupling \(\alpha_H\) from experimental data. The one-loop chiral corrections have been calculated in \cite{23, 27} using \(\bar{\Delta} = 0\), while the leading logs have been obtained already in \cite{32, 33}.

\[
\begin{align*}
\frac{f_D}{\sqrt{m_D}} &= \frac{\alpha_H}{32\pi^2 f^2} \left[1 + \frac{3g^2}{32\pi^2 f^2} \left(\frac{3}{2} C(\Delta_{D^*D}, m_\pi) + C(\Delta_{D^*_1D}, m_K) + \frac{1}{6} C(\Delta_{D^*_2D}, m_\eta)\right) \right. \\
&\left. - \frac{1}{32\pi^2 f^2} \left(\frac{3}{2} I_1(m_\pi) + I_1(m_K) + \frac{1}{6} I_1(m_\eta)\right)\right], \\
\frac{f_{D_s}}{\sqrt{m_D}} &= \frac{\alpha_H}{32\pi^2 f^2} \left[1 + \frac{3g^2}{32\pi^2 f^2} \left(2 C(\Delta_{D^*_1D_*}, m_K) + \frac{2}{3} C(\Delta_{D^*_2D_*}, m_\eta)\right) \right. \\
&\left. - \frac{1}{32\pi^2 f^2} \left(2I_1(m_K) + \frac{2}{3} I_1(m_\eta)\right)\right].
\end{align*}
\]
where $C(\Delta, m) = J_1(m, \Delta) + \Delta \frac{\partial}{\partial \Delta} J_1(m, \Delta)$, while $J_1(m, \Delta)$ and $I_1(m)$ can be found in appendix A. Using $f = 120\text{MeV}$, $\mu = 1\text{GeV}$ and $\bar{\Delta} = 1$ one gets the numerical values

$$f_D = \frac{\alpha_H}{\sqrt{m_D}} (1 + 0.18 - 0.37 g^2) ,$$

$$f_{D_s} = \frac{\alpha_H}{\sqrt{m_D}} (1 + 0.35 + 0.38 g^2).$$

To obtain the $\alpha_H$ coupling we use experimental data on decays of $D$ mesons into leptons. From the experimental value for branching ratio $B(D_s \rightarrow \mu \nu \mu) = (4.6 \pm 1.9) \times 10^{-3}$ and the $D_s$ decay time $\tau_{D_s} = (0.496^{+0.010}_{-0.009}) \times 10^{-12} \text{s}$ one gets $f_{D_s} = 0.23 \pm 0.05 \text{GeV}$. Using this value and $g = 0.27$ in (71b) we get $\alpha_H = 0.23 \pm 0.04 \text{GeV}^{3/2}$.

Using $\alpha_H = 0.23 \pm 0.04 \text{GeV}^{3/2}$ in (71a) and $g = 0.27$ we can also calculate $f_D = 0.194 \pm 0.045 \text{GeV}$, where the uncertainties are due to the uncertainties in $\alpha_H$. The average value for the ratio $f_{D_s}/f_D = 1.19$ is in fair agreement with the recent lattice results [29].

### C Heavy-light quark loop integrals

The integrals entering heavy quark loops like the ones in Fig.8 are of the form:

$$R_{p,q} \equiv \int \frac{d^d k}{(2\pi)^d} \frac{1}{(v.k)^p} \frac{1}{(k^2 - m^2)^q}. \quad (72)$$

Performing a shift of momentum integration combined with Feynman parameterization, we obtain

$$R_{p,q} = 2^p \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} K(p+q,p-1), \quad (73)$$

where

$$K(n,r) \equiv \int_0^\infty d\lambda \int \frac{d^d l}{(2\pi)^d} \frac{\lambda^r}{(l^2 - m^2 - \lambda^2)^n}. \quad (74)$$

One should notice that to obtain the result in (13), we have to do the identification

$$8iN_c G_H^2 I_{HH} = 1, \quad (75)$$

where $I_{HH}$ is a logarithmically divergent loop integral given below. (There is also a similar relation for $g$.) One should notice that some authors use an extra factor $m_H$, the mass of the heavy meson, in front of the right hand side of (13). Choosing the normalization in (13), it means that a factor $\sqrt{m_H}$ is included in the heavy meson field $H_v$. For the left handed current in (18) and (19) we find that we have to identify:

$$\alpha_H = -4iN_c G_D I_{HW}, \quad (76)$$

where $I_{HW}$ is a quadratically divergent loop integral.

The regularization can be done in various ways (various cut-off prescriptions or by $\overline{MS}$) and each regularization correspond to slightly different versions of this type model [12, 18, 19, 20, 21, 24]. For instance, in the version we use, when soft gluon emission is included in (45) above, gluon condensate contributions should also be included in loop integrals $I_{HH}$ and $I_{HW}$, as it is for $f_\pi$ in the light
sector \([12, 13]\). However, we will not go into these details here. Still, as in the pure light quark case, one obtains numbers in the right ball park by parameterizing the quadratic divergent integral as \(\Lambda_\chi^2\), and the logarithmically divergent integral as \(\log(\Lambda_\chi^2/m^2)\). Anyway, using the expression for \(f_\pi\) obtained in the \(\chi\)QM, we obtain from (75) to leading order

\[
G_H \simeq \frac{2\sqrt{m_\chi}}{f_\pi} .
\]  

(77)

It can be seen from Ward identities for the loop diagrams for, say Fig. 8 (left) that the quadratic divergence in \(I_{HW}\) is related to the quark condensate of the light quark, which is also quadratically divergent. Then, similar to (77), we obtain from (76) to leading order

\[
G_H \simeq -2\frac{m_\chi}{f_\pi} \frac{\alpha_H}{\langle \bar{q}q \rangle} ,
\]  

(78)

Combining (77) and (78) we obtain

\[
\alpha_H \simeq -\frac{\langle \bar{q}q \rangle}{f_\pi \sqrt{m_\chi}} ,
\]  

(79)

which for the values \(m_\chi = 200\) MeV, \(f_\pi = 131\) MeV and \(\langle \bar{q}q \rangle = (-240\) MeV\(^3\) gives the value for \(\alpha_H\) cited in Appendix B. Furthermore, using (77) and (78) we obtain

\[
\beta \simeq -2\frac{m_\chi f_\pi^2}{\langle \bar{q}q \rangle} \simeq 0.25 ,
\]  

(80)

to be used in (50) and (51).

Within dimensional regularization, the expressions for some values of \(n\) and \(r\) are listed below:

\[
K(2, 0) = -4K(3, 2) \quad ; \quad m^2 K(3, 0) = (3 - d) K(3, 2) ,
\]  

(84)

Comparing with a cut-off regularization, we see that \(K(2, 1)\) is quadratically and \(K(3, 1)\) is logarithmically divergent. In a primitive cut-off regularization \(K(2, 0)\) and \(K(3, 2)\) appear as linearly divergent \([14]\), while they here appear as finite!

Note also that some of the integrals (74) can be obtained as the limits of integrals listed in appendix \(A\) if one lets \(\Delta \to 0\). Thus one has the relations

\[
K(2, 0) = -\frac{i}{32\pi^2} \lim_{\Delta \to 0} \frac{1}{\Delta} I_2(m, \Delta) ,
\]  

(85)

\[
K(2, 1) = \frac{i}{32\pi^2} \lim_{\Delta \to 0} \left[ I_2(m, \Delta) + I_1(m) \right] = \frac{i}{32\pi^2} I_1(m) ,
\]  

(86)

\[
K(2, 2) = -\frac{i}{32\pi^2} \lim_{\Delta \to 0} \Delta J_2(m, \Delta) .
\]  

(87)
The loop integral's (the $I$'s) are defined as:

\begin{align}
I_{HH} & \equiv mK(3,1) + K(3,2), \\
I_{HW} & \equiv K(2,1) + mK(2,0), \\
I_{G1} & \equiv K(3,1) + mK(3,0), \\
I_{G2} & \equiv K(3,1).
\end{align}

References

[1] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C 34, 103 (1987).
[2] X. Y. Pham, Phys. Lett. B 193, 331 (1987).
[3] Y. S. Dai et al., Phys. Rev. D 60, 014014 (1999).
[4] H. Lipkin, Phys. Rev. Lett. 44, 710 (1980).
[5] J.- M. Gerard, J. Pastieau, J. Weyers, Phys. Lett. B 436, 363 (1998).
[6] A. N. Kamal, A. B. Santra, T. Uppal, R. C. Verma, Phys. Rev. D 53, 2506 (1995).
[7] P. Zenczykowski, Phys. Lett. B 460, 390 (1999).
[8] K. Terasaki, Phys. Rev. D 59, 11401 (1999).
[9] A. J. Buras, J.- M. Gerard, R. Rückl, Nucl. Phys. B 268, 16 (1986).
[10] F. Buccella, M. Lusignoli, G. Miele, A. Pugliese and P. Santorelli, Phys. Rev. D 51, 3486 (1995), F. Buccella, M. Lusignoli and A. Pugliese, Phys. Lett. B 379, 249 (1996).
[11] See for example: A.J Buras, M. Jamin, and M. E. Lautenbacher, Nucl. Phys. B 408, 209 (1993), Phys. Lett. B 389, 749 (1996), S. Bertolini, M. Fabbrichesi, J.O. Eeg, Rev. Mod. Phys. 72 (2000) 65, and references therein.
[12] S. Bertolini, J.O. Eeg and M. Fabbrichesi, Nucl. Phys. B 449 (1995) 197, V. Antonelli, S. Bertolini, J.O. Eeg, M. Fabbrichesi and E.I. Lashin, Nucl. Phys. B 469 (1996) 143, S. Bertolini, J.O. Eeg, M. Fabbrichesi and E.I. Lashin, Nucl. Phys. B 514, 63 (1998), and ibid. B 514, 93 (1998).
[13] H. Cheng, epsilon'/epsilon in Chin. J. Phys. 38 (2000) 1044 [hep-ph/9911202].
[14] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett 83, 1914 (1999).
[15] Review of Particle Physics, D.E. Groom et al., Eur. Phys. J., C 15, 1 (2000).
[16] A. Pich and E. de Rafael, Nucl. Phys. B 358, 311 (1991).
[17] J. A. Cronin, Phys. Rev. 161, 1483 (1967), S. Weinberg, Physica, 96 A, 327 (1979), A. Manohar and H. Georgi, Nucl. Phys. B 234, 189 (1984), J. Bijnens, H. Sonoda and M.B. Wise, Can. J. Phys 64, 1 (1986), D. I. Diakonov, V. Yu. Petrov and P.V. Pobylitsa, Nucl. Phys. B 306, 809 (1988), D. Espriu, E. de Rafael and J. Taron, Nucl. Phys. B 345, 22 (1990).
[18] J. O. Eeg and I. Picek, Phys. Lett. B 301, 423 (1993), J. O. Eeg and I. Picek, Phys. Lett. B 323, 193 (1994) 193, A. E. Bergan and J.O. Eeg, Phys. Lett. 390, 420 (1997) 420.

[19] W. A. Bardeen and C. T. Hill, Phys. Rev. D 49, 409 (1994).

[20] D. Ebert, T. Feldmann, R. Friedrich and H. Reinhardt, Nucl. Phys. B 434, 619 (1995).

[21] A. Deandrea, N. Di Bartelomeo, R. Gatto, G. Nardulli, and A. D. Polosa, Phys. Rev. D 58, 034004 (1998), A. D. Polosa, hep-ph/0004183.

[22] R. Casalbuoni, A. Deandrea, N. Di Bartelomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. 281, 145 (1997).

[23] M. B. Wise, Phys. Rev. D 45, R2188 (1992).

[24] A. Hiorth and J. O. Eeg; in preparation

[25] A. J. Buras, Nucl. Phys. B 434, 606 (1995).

[26] G. Burdman and J. F. Donoghue, Phys. Lett. B280, 287 (1992).

[27] C. G. Boyd and B. Grinstein, Nucl. Phys. B 442, 205 (1995).

[28] I. W. Stewart, Nucl. Phys. B 529, 62 (1998).

[29] D. Becirevic, hep-lat/0011075

[30] J. A. M. Vermaseren: “Symbolic Manipulation with FORM”, Published by CAN, Kruislaan 413, 1098 SJ, Amsterdam, The Netherlands

[31] M. Neubert, Phys. Rep. 245, 259 (1994).

[32] B. Grinstein et al., Nucl. Phys. B 380, 369 (1992).

[33] A. F. Falk, B. Grinstein, Nucl. Phys. B 416, 771 (1994).

[34] J. L. Goity, Phys. Rev. D 46, 3929, (1992).