Rolling spinners on the water surface

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Angular momentum of spinning bodies leads to their remarkable interactions with fields, waves, fluids, and solids. Orbiting celestial bodies, balls in sports, liquid droplets above a hot plate, nanoparticles in optical fields, and spinning quantum particles exhibit nontrivial rotational dynamics. Here, we report self-guided propulsion of magnetic fast-spinning particles on a liquid surface in the presence of a solid boundary. Above some critical spinning frequency, such particles generate localized 3D vortices and form composite “spinner-vortex” quasiparticles with nontrivial, yet robust dynamics. Such spinner-vortices are attracted and dynamically trapped near the boundaries, propagating along the wall of any shape similarly to “liquid wheels.” The propulsion velocity and the distance to the wall are controlled by the angular velocity of the spinner via the balance between the Magnus and wall repulsion forces. Our results offer a new type of surface vehicles and provide a powerful tool to manipulate spinning objects in fluids.

INTRODUCTION
The dynamics of rotating matter carrying angular momentum is crucial in numerous problems spanning from microscopic quantum to giant astrophysical scales. These problems involve rotating or spinning particles, from quantum elementary particles (1) to planets and stars (2), and a variety of vortices, in classical (3) and quantum (4, 5) fluids as well as in classical (6, 7) and quantum (8) wave fields. Quite often, the angular momenta of spinning particles and vortices become closely related and mutually coupled (9–11). Not surprisingly, the dynamics of spinning particles and vortices share fundamental similarities, such as the Magnus and various Hall effects, i.e., the transverse angular momentum–induced transport (12–15).

Spinners in classical fluids naturally appear under optical manipulation with vortex or circularly polarized fields (9, 10), in externally imposed magnetic fields (16–18), and in sports ballistics (19, 20). Furthermore, liquid droplets on hot solid surfaces also behave as spinning liquid particles or “Leidenfrost wheels” (21–23). At microscale, i.e., at the Reynolds numbers of Re < 1, colloidal building blocks rotating using external magnetic field can be directed along user-defined paths, thus acting as colloidal microwheels (24). Recently, it was shown that water surface waves generating circular flows and polarizations (25) can efficiently interact with subwavelength spinners, which allows efficient manipulation of particles on the water surface (26). Therefore, understanding the dynamics of spinning particles in fluids constitutes a fundamental and applied problem important for physics and engineering.

Here, we report a previously unknown behavior in a known classical system: a spinning particle (spinner) floating on the water surface. We show that above some threshold angular velocity (i.e., above critical Reynolds number), the spinner generates a well-localized three-dimensional (3D) vortex around it, and this provides an effective coupled spinner-vortex “quasiparticle.” Such spinner-vortices become attracted and trapped near solid boundaries (where the wall-normal repulsion and Magnus forces balance each other) and roll with constant translational velocities along the boundaries of arbitrary shapes. This provides robust “liquid wheels,” which can be controlled via the angular velocity of the spinners and transported along any desired boundary without touching it. We find simple and robust dependencies between the main parameters of such self-guided propulsion: the angular velocity, the linear velocity, and the distance to the boundary. Our results provide a novel type of the spinner-vortex coupling in classical hydrodynamics and suggest a new type of robust self-navigated transport that can be used in liquid surface vehicles.

RESULTS
The main parameters and phenomenon
We study the motion of fast-spinning particles floating on the water surface in the absence of externally imposed flows but in the presence of fixed solid boundaries. Particles studied here are magnetized discs with radii in the range of a = (0.5–2.5) mm and a thickness of h = 1 mm. The spinning motion of the particles is generated and supported by an external rotating magnetic field (see the Supplementary Materials) such that the spinning frequency can be varied in the range of f = ω/2π = (1 – 50) Hz. The corresponding rotational Reynolds numbers are Reθ = 2a²ω/v = (20 – 200), where v = 10⁻⁶ m²/s is the kinematic viscosity of water. In these conditions, inertial effects must play an important role for the mechanisms of the spinner’s propulsion and its interaction with a solid boundary.

We observed that fast-spinning particles, which float on the water surface, are attracted to solid boundaries and roll along these boundaries of practically any shape with a fixed velocity V at a fixed distance δ from the boundary. Figure 1 shows examples of the spinner motion around triangular, star-shaped, and irregular “Australia-shaped” boundaries (movies S1 to S3).

Spinner-vortex quasiparticle
Spinners interact with other objects in a fluid by creating vortices around themselves. Therefore, we first look at the flow created by a spinning disc away from the wall. At low rotation frequency ω (or in a viscous fluid), the spinner creates a vortex around its axis of rotation. The fluid’s azimuthal velocity in the vortex at the distance ρ from the center of rotation is given by vₚ(ρ) = a²ω/ρ² (18). We
observe such a vortex with purely azimuthal flow at low rotational Reynolds numbers $Re_\omega < 20$ as shown in Fig. 2 (A and B). This experiment is performed on the surface of a glycerol solution, whose viscosity is 50 times higher than that of water. In this regime, the spinner remains motionless, and no propulsion is observed.

As the frequency is increased above some critical value $\omega > \omega_0$, which corresponds to the rotational Reynolds numbers $Re_\omega > Re_{\omega 0} = (22 \pm 2)$, the vortex around the spinner changes qualitatively. Namely, fluid particles are expelled from the vicinity of the spinner, forming a finite-size depletion zone whose radius $R$ is independent of the angular velocity $\omega$ for a given spinner size (Fig. 2C). Such a depletion zone was previously reported with regard to spinners on the liquid surface in (16), where it was referred to as the high-

$\Delta t = 1 \text{s}$. Note the reversed angular and linear velocities in the motion along the Australia-shaped boundary. See also movies S1 to S3.

![Fig. 1. Fast-spinning floating particles move along complex-shape boundaries.](image)

The spinner radius is $a = 1 \text{mm}$, and the spinning frequency is $\omega/2\pi = 12 \text{Hz}$. Time between consecutive frames is $\Delta t = 1 \text{s}$. Note the reversed angular and linear velocities in the motion along the Australia-shaped boundary. See also movies S1 to S3.

![Fig. 2. Generation of the “spinner-vortex” quasiparticle.](image)

(A) Flow around a spinner with $a = 0.5 \text{mm}$ and angular velocity $\omega < \omega_0$. (Re$_\omega = 6$ here). Here, the spinner floats on the surface of a glycerol-water solution. (B) Radial dependence of the azimuthal velocity in the vortex flow from (A). (C and D) Same as in (A) and (B) but for the spinner with $\omega > \omega_0$ on a water surface ($Re_\omega = 79$ here). A localized 3D vortex characterized by a depletion zone with the radius $R = 2.5 \text{mm}$ develops.

Dynamical instability and attraction to the boundary

The motionless state of the spinner-vortex with $\omega > \omega_0$ becomes unstable. It undergoes small random excursions from the initial position and then starts moving with exponentially growing linear velocity toward the wall (movie S4). When approaching the wall, the spinner-vortex becomes trapped at some distance $\delta$ and its velocity $V$ becomes stabilized and directed along the wall. Figure 3A shows an example of such evolution for the spinner originally located in the center of a circular pool and eventually trapped near its boundary. Thus, the only stable regime for such spinner-vortex is the “liquid-wheel rolling” along the wall. Note that this motion is in agreement with an intuitive picture of a “rolling wheel”: the linear velocity is directed $V \propto \omega \times \mathbf{n}$, where $\omega$ is the angular velocity vector, while $\mathbf{n}$ is the normal to the boundary directed toward the spinner. Reversing $\omega$ or placing the spinner on the opposite side from the wall reverses the direction of its linear motion.

When the spinner becomes unstable and starts moving within such a circular pool, its trajectory is well described by the parametric fit

$$r(\theta) = r_\omega \frac{\exp(b\theta)}{C + \exp(b\theta)}$$

where $(r, \theta)$ are the polar coordinates with the origin in the center of a circular boundary and $(b, r_\omega, C \gg 1)$ are constant parameters. The measured trajectory of a spinner, together with the parametric fit Eq. 1, are shown in Fig. 3 (B and C). At short times, or small $\theta$, the radius grows exponentially, $r \propto \exp(b\theta)$, which corresponds to a logarithmic spiral. Later, this growth saturates at the stationary radius along the wall-guided orbit at $r = r_\omega$. The temporal evolution of the spinner velocity, $V(t)$, and azimuthal angle, $\theta(t)$, are shown in Fig. 3 (D and E). One can see that the spinner travels with a constant angular velocity $\Omega = d\theta/dt$ along the spiral part of the trajectory (which is a characteristic feature of the logarithmic spiral) and then travels with another constant angular velocity along the circular wall. The way the spinner-vortex approaches the wall is very robust: The ratio of the spinner velocity $V$ along its trajectory to its terminal velocity toward the wall, $V_\omega$, is independent of its frequency $\omega$ (Fig. 3F).

To get some intuition about dynamical laws underlying this motion, we assume that the spinner-vortex moves with a constant angular velocity $\Omega, \theta \approx \Omega t$, and that the azimuthal velocity component makes the main contribution to the velocity $V$ during the spiral motion: $V \approx r \Omega$. Then, the evolution of the velocity can be written as

$$V(t) = V_\omega \frac{e^{\beta t}}{C + e^{\beta t}} \quad V(s) = V_\omega \left(1 - e^{-\beta s/V_\omega}\right)$$

where $V_\omega = \Omega r_\omega$, $\beta = \Omega b$. We introduced the coordinate along the spinner trajectory, $s = \int V dt$, and set the initial condition $V(s = 0) = 0$. 

$\Delta t = 1 \text{s}$. Note the reversed angular and linear velocities in the motion along the Australia-shaped boundary. See also movies S1 to S3.
torque into the propulsion force (the rolling wheel mechanism).

To quantify the relations between the main parameters of the rolling wheel motion, \( \omega, V, \) and \( \delta \), we performed a series of measurements to determine these parameters for spinners with different radii \( a \) and angular velocities \( \omega \). The results shown in Fig. 4 (C and D) reveal rather simple and robust dependencies between these quantities:

\[
V = aR(\omega - \omega_0), \quad V = V_0 \exp\left(-\frac{\delta - R}{L}\right)
\]
Here, \( \alpha, V_0, \) and \( L \) are parameters of no dimension, velocity dimension, and length dimension, respectively, and all of these are independent of the spinner size \( a \). The linear dependence \( V \approx (\omega - \omega_0) \) also follows from Eq. 3, where \( \alpha = A/B \). In our experiments, we found that \( \alpha \approx 0.05 \) (this is probably related to the critical rotational Reynolds number: \( \alpha \approx 1/Re_{\omega_0} \), \( V_0 \approx 59 \text{ mm/s} \), and \( L \approx 2.9 \text{ mm} \). Notably, the linear proportionality of the translational velocity to the angular velocity, the first Eq. 4, agrees with the intuitive picture of the rolling wheel. Because the effective angular velocity of the rolling motion is \( (\omega - \omega_0) \) rather than \( \omega \), one can expect that the Magnus force should also be modified as \( F_M = \bar{m}(\omega - \omega_0) V \). In turn, the second Eq. 4 reflects the fact that larger angular and linear velocities produce smaller distances \( \delta \) (as the Magnus force increases). The particular exponential dependence is apparently related to the form of the repulsion force. From Eq. 4, and the form of the Magnus force, we can write an effective form of the repulsion force: \( F_R(\delta) = F_0(\delta) \exp(-2(\delta - \tilde{R})/L) \), where \( F_0 = \bar{m}V_0^2/\alpha R \). Note that \( (\delta - \tilde{R}) \) is the distance between the spinner-vortex boundary and the wall.

A rigorous theoretical calculation of the repulsion force is a rather challenging problem depending on the details of the interaction between a moving localized 3D vortex and the wall. We leave it for further studies and demonstrate the presence of this repulsion force experimentally. For this purpose, we performed a series of measurements using instantaneous switching off the spinning motion, i.e., making \( \omega = 0 \) at some instant of time \( t = t_0 \). This switches off the Magnus force, and the spinner starts moving away from the wall under the action of the repulsion force (movie S5). Figure 5 shows that the repulsion force pushes the particle away from the border such that its trajectory has the radius of curvature approximately equal to \( L \approx 2.9 \text{ mm} \), independently of the spinner size \( a \) and its velocity \( V \) at \( t = t_0 \). The repulsion force can be estimated as \( m_0V_0^2/L \), where \( m_0 \) is the spinner mass. By equating it to the Magnus force at \( t < t_0 \), \( F_M = \bar{m}(\omega - \omega_0) V = \bar{m}V^2/\alpha R \), we find that \( \bar{m} = m_0aL/R \). This can be used only as a rough estimate because switching off the spinner destroys the localized vortex around it so that the parameters of this object (mass, size, etc.) can change considerably.

**DISCUSSION**

To summarize, we have reported a deterministic behavior of fast-spinning particles on the liquid surface. Above some critical rotational Reynolds number, the spinner is dynamically trapped and propels at a fixed distance from a boundary following its arbitrary shape. This phenomenon is accompanied by the generation of a localized 3D vortex around the spinner, and described by the balance of the Magnus and wall repulsion forces. The effect reported here is very robust and can be controlled by a single parameter: the angular velocity of the spinner. Note also that it does not rely on the surface-specific effects: The buoyancy and the surface tension just balance the gravitational force acting on a spinner. Thus, forces discussed here should also act on any spinning object in the presence of walls.

Our results offer exciting possibilities of developing self-navigating water surface vehicles, with various applications in laboratory and marine robotics, manipulation and sorting of spinning or magnetic particles, mixing of chemical or biological substances, etc. In this work, we have presented detailed experimental observations and measurements, together with their phenomenological analysis. An accurate theoretical description of the observed phenomena is a rather challenging problem for future studies. Even a toy mechanical model of the spinner-vortex behavior must describe a
nonconservative system with gain (via external torque) and loss (via friction) of energy, as well as with coupling between rotational and translational degrees of freedom. Hydrodynamic analysis of the problem is even more challenging because the flows around spinning bodies have been explored so far only in numerical simulations and only in a limited range of Reynolds numbers (18, 27, 28). Further understanding of the phenomenon of rolling spinners will require experimental and theoretical studies of the 3D fluid motion within the spinner–vortex structure, which seems to be the key to the observed effect. It would also be interesting to investigate in the future the motion of spinners along modulated walls (e.g., sinusoidal), as well as collective wall-guided motion of multiple spinners.

MATERIALS AND METHODS

Rotating magnetic field

The electric current in the coil pairs is driven by two wave amplifiers (Accel Instruments TS250-0), which create a rotating horizontal magnetic field at the surface of the liquid (fig. S1). Opposite coils are connected in series, and the current in the pairs is phase-shifted by $\pi/2$. The resulting magnetic field rotates at the frequency $\omega/2\pi$. The rotation direction can be reversed by changing the phase between the magnetic coil pairs from $\pi/2$ to $-\pi/2$. The horizontal magnetic field at the liquid surface is approximately constant: $\Delta B/B < 0.02$. This has been tested using a 2D scanning Hall probe at the coils horizontal mid-plane, corresponding to the liquid level.

Spinners and boundaries

Ferromagnetic spinners of different diameters are manufactured by 3D printing a template (PLAflex) consisting of a pattern of cylindrical holes. The holes are loaded with a mixture of polymers (Elite 3D printing a template containing the spinners is exposed to a horizontal uniform magnetic field at the liquid surface. The density of the spinners is adjusted by adding magnetic particles to the polymer. In addition, the top surface of the spinner is Teflon-coated to avoid wetting. This is done to eliminate the Cheerios effect (29), which could add to the spinner–wall interaction. When placed on the water surface, the torque on the spinner is imposed by the external rotating magnetic field. The spinner rotation frequency is verified using a high-speed camera.

Boundaries and liquid containers are made of polylactic acid thermoplastic using a 3D Ultimaker 2+ printer. To avoid menisci at the container boundary, a step at the level of 15 mm from the bottom of the container is made and a liquid is filled to that level. In experiments summarized in Figs. 3 and 4, the diameter of the circular container is 60 mm.

SUPPLEMENTARY MATERIALS

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/7/16/eabd4632/DC1

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