Three-dimensional foam flow resolved by fast X-ray tomographic microscopy

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received on 17 June 2015; accepted by B. Andreotti on 27 July 2015
published online 24 August 2015

PACS 83.80.Iz – Emulsions and foams

Abstract – Adapting fast tomographic microscopy, we managed to capture the evolution of the local structure of the bubble network of a 3D foam flowing around a sphere. As for the 2D foam flow around a circular obstacle, we observed an axisymmetric velocity field with a recirculation zone, and indications of a negative wake downstream the obstacle. The bubble deformations, quantified by a shape tensor, are smaller than in 2D, due to a purely 3D feature: the azimuthal bubble shape variation. Moreover, we were able to detect plastic rearrangements, characterized by the neighbor swapping of four bubbles. Their spatial structure suggests that rearrangements are triggered when films faces get smaller than a characteristic area.

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Foam rheology is an active research topic [1–4], motivated by applications in ore flotation, enhanced oil recovery, food or cosmetics [5]. Because foams are opaque, imaging their flow in bulk at the bubble scale is challenging. To bypass this difficulty, 2D flows of foams confined as a bubble monolayer, whose structure is easy to visualize, have been studied. However, the friction induced by the confining plates may lead to specific effects [6], irrelevant for bulk rheology. In 3D, diffusive-wave spectroscopy has been used to detect plastic rearrangements [7,8]. These events, called T1s, characterized in 2D by the neighbor swapping of four bubbles in contact, are of key importance for flow rheology, since their combination leads to the plastic flow of foams. Magnetic resonance imaging has also been used to measure the velocity field in 3D [9]. However, both these techniques resolve neither the bubble shape nor the network of liquid channels (Plateau borders, PBs) within a foam. In contrast, X-ray tomography renders well its local structure. However, the long acquisition time of a tomogram, over a minute until very recently, constituted its main limitation, allowing to study only slow coarsening processes [10,11].

Here, we report the first quantitative study of a 3D foam flow around an obstacle. Such challenge was tackled thanks to a dedicated ultra-fast and high-resolution imaging set-up, recently developed at the TOMCAT beam line of the Swiss Light Source [12]. High-resolution tomogram covering a volume of $4.8 \times 4.8 \times 5.6 \text{mm}^3$ with a voxel edge length of $5.3 \mu\text{m}$ could be acquired in around $0.5 \text{s}$, allowing to follow the evolving structure of the bubbles and PB network. Our image analysis shows that the 3D foam flow around a sphere is qualitatively similar to the 2D flow around a circular obstacle: we reveal an axisymmetric velocity field, with a recirculation zone around the sphere in the frame of the foam, and a negative wake downstream the obstacle. Bubble deformations are smaller (in the diametral plane along the mean direction of the flow $z$) than for a 2D flow, thanks to the extra degree of freedom allowing an azimuthal deformation: bubbles appear oblate before, and prolate after, the obstacle. Finally, we were able to detect plastic rearrangements, characterized by the neighbor swapping of four bubbles and the exchange of two four-sided faces. Our observations suggest that those events are triggered when the bubble faces get smaller than a characteristic size around $R_c^2$, given by a cutoff length of the PB $R_c \simeq 130 \mu\text{m}$ in the case of our foam.

Experimental set-up: We prepared a foaming solution following the protocol of [13]: we mixed 6.6% of sodium
A typical X-ray projection image is shown on the right. The acquired tomograms cover the central region with a volume of $4.8 \times 4.8 \times 5.6$ mm$^3$. A typical X-ray projection image is shown on the right.

The experiments were performed at the TOMCAT beamline of the Swiss Light Source. Filtered polychromatic X-rays with mean energy of 30 keV were incident on a custom-made flow cell (fig. 1) attached to the tomography stage with three translational and one rotational degrees of freedom. The X-rays passing through the foam in the chamber were converted to visible light by a 100 $\mu$m thick LuAG:Ce and detected by a 12 bit CMOS camera. Typically 550 radiographic projections acquired with 1 ms exposure time at equidistant angular positions of the sample were reconstructed into a 3D volume of $4.8 \times 4.8 \times 5.6$ mm$^3$ with isotropic voxel edge length of $ps = 5.3$ $\mu$m. Such a 3D snapshot of the flowing foam is acquired in $t_{\text{scan}} = 0.55$ s, ensuring that motion artifacts are absent since $t_{\text{scan}} < ps/v_{\text{flow}}$. In order to follow the structural changes of the foam during its flow around the obstacle, we recorded a tomogram every 35 seconds for approximately 20 minutes (resulting in around 36 tomograms). The tomograms quality is enhanced using not only the X-rays attenuation by the sample, but also the phase shift of the partially coherent X-ray beam as it interacts with the foaming solution in the PBs and senses the electron density variation in the sample [12]. This phase shift was retrieved using a single-phase object approximation [14].

**Image analysis**: The tomograms are then segmented, separating the PBs and vertices from air. Figure 2 shows two successive time steps of the 3D snapshots of the reconstructed PBs network during the foam flow around the sphere. We measured the liquid fraction from the segmented images, by computing the relative surface occupied by the PBs and vertices on individual horizontal slices. We measured an averaged liquid fraction of 4% over a tomogram, which did not evolve significantly during our experiments.

Then, we reconstructed and identified individual bubbles of the flowing foam, following the procedure we recently developed and validated on static foam samples, imaged at the same acquisition rate and spatial resolution [15]. We did not observe any evolution of the size distribution of the polydisperse foam studied here, with an average volume $V = 0.36 \pm 0.13$ mm$^3$, hence coarsening remains negligible. Typically, 160 bubbles are tracked between two successive 3D snapshots, leading to statistics over 5600 bubbles. Bubbles smaller than 0.01 mm$^3$ cannot be discriminated from labeling artifacts [15], and thus, are discarded.

**Velocity field**: From the bubble tracking, we could measure their velocity $\vec{V}$ around the obstacle. Statistics are performed in the diametral ($p$z) plane of the cylindrical coordinates ($p$ $\phi$ $z$). We have checked that our results do not depend significantly on the angular coordinate $\phi$. A typical liquid fraction of 4% only the X-rays attenuation by the sample, but also
can be used. The unit vectors ($\hat{\rho}, \hat{z}$) or ($r, \theta$) are plotted for $r = 1.5$ mm in the plane. The normalized velocity field obtained by subtracting the mean flow velocity is shown in (b). The gray half-disc represents the obstacle (diameter 1.5 mm). The red arrow centered on the semi-obstacle gives the velocity scale of 8 μm/s. Blue arrows show the negative wake effect.

Fig. 3: (Color online) Velocity fields in the ($\rho z$) plane, (a) in the lab frame. Both the ($\rho z$) and the ($r \theta$) polar coordinates can be used. The unit vectors ($\hat{\rho}, \hat{z}$) or ($r, \theta$) are plotted for $r = 1.5$ mm in the plane. The normalized velocity field obtained by subtracting the mean flow velocity is shown in (b). The gray half-disc represents the obstacle (diameter 1.5 mm). The red arrow centered on the semi-obstacle gives the velocity scale of 8 μm/s. Blue arrows show the negative wake effect.

Fig. 4: (Color online) Velocity components measured at a distance $r = 1.5$ mm from the obstacle center as a function of the polar angle $\theta$: $V_r$ (blue circles) and $V_\theta$ (green squares) in the ($r \theta$) frame. The lines hold for a potential flow model.

should be improved in the future. The strong fore-aft asymmetry of the flow evidenced by this negative wake confirms that the foam cannot be modelled as a viscoelastic fluid, which gives a fore-aft-symmetric flow [23]: it is intrinsically viscoelastoplastic.

To further quantify the velocity field, its components $V_r$ and $V_\theta$ at a distance $r = 1.5$ mm (one obstacle diameter) from the obstacle center are plotted as a function of $\theta$ in fig. 4. We have checked that choosing another distance (e.g. $r = 2$ mm) does not change the qualitative features of the velocity field. The component $V_\theta$ is negative because $\theta$ is directed upstream. $|V_\theta|$ is maximum at $\theta = \pi/2$, and $V_\theta$ is almost fore-aft symmetric (i.e. symmetric with respect to the axis $\theta = \pi/2$). The component $V_r$ is positive for $\theta < \pi/2$, and negative for $\theta > \pi/2$. Contrary to $V_\theta$, $V_r$ is fore-aft asymmetric. The absolute value of $V_r$ monotonously grows on both sides away from $\pi/2$ (albeit with noise near 0), it reaches a local extremum near $3\pi/4$, then decreases as $\theta$ increases towards $\pi$. To further reveal this asymmetry, a comparison is made with a potential flow model, whose velocity field reads [24] $V_r = U(1 - r^3/a^3) \cos \theta$, and $V_\theta = -U(1 + r^3/2a^3) \sin \theta$, where $U$ is the uniform velocity far from a spherical obstacle of diameter $2a$. We proceed as follows: first, we fit $V_\theta$ with $U$ as the sole free fitting parameter. This procedure gives the dotted line in fig. 4, with $U = 8.2 \mu$m/s. We then use this value of $U$ in the potential flow formula for $V_r$, and we plot it as a dashed line in fig. 4. While $V_\theta$ is very similar to the potential flow case (which is expected, since this only tests its fore-aft symmetry), $V_r$ exhibits deviations from the potential flow close to $\theta = 0$ and $\pi$. In particular, $V_r$ reaches a value significantly larger than $U$ close to $\theta = 0$, which is a signature of the negative wake. The deviation close to $\theta = \pi$ is more difficult to interpret, and might be due to the presence of the capillary holding the obstacle.
Bubble deformation: Given the set of coordinates \( \{ r \} \) of the voxels inside a bubble, we define its inertia tensor \( I = (r - r) \otimes (r - r) \), and its shape tensor as \( S = I^{1/2} \). This operation is valid because \( I \) (and hence \( S \)) is a symmetric and definite tensor. Alike the velocity field, averages are performed inside boxes to obtain a shape field (fig. 5). The bubble deformation is quantified by the eigenvectors/values of the shape tensor. In good approximation, two of them \((S_{\rho\rho}^+ \) and \( S_{\rho\rho}^- \)) are found inside the \((\rho z)\) plane, the other corresponds to the projection of the tensor along the azimuthal direction \((S_{\phi\phi})\). An effective radius is defined by \( R_{\text{eff}} = (S_{\rho\rho}^+ - S_{\rho\rho}^- S_{\phi\phi})^{1/3} \). The bubbles deformation in the \((\rho z)\) plane is represented by ellipses of semi-axes \( S_{\rho\rho}^+ \) and \( S_{\rho\rho}^- \). The direction of the largest one, \( S_{\rho\rho}^+ \), is emphasized by a line across the ellipse (fig. 5). Deformation in the azimuthal direction is quantified by \((S_{\rho\rho} - R_{\text{eff}})/R_{\text{eff}} \) in colormap. The orientation of the ellipses in the \((\rho z)\) plane exhibits a clear trend comparable to the 2D case [17, 25]. They are elongated streamwise on the obstacle side and at the trailing edge. In between, the ellipses rotate 180° to connect these two regions. We noticed that the deformation of the bubbles is much smaller than for a 2D foam with the same liquid fraction [17, 25]. The deformation in the azimuthal direction exhibits dilation/compression up to 5% only. The quantity \((S_{\phi\phi} - R_{\text{eff}})/R_{\text{eff}} \) is positive upstream (oblate shape) or negative downstream (prolate shape) to favor the passage around the obstacle (fig. 5). The third dimension tends therefore to reduce the bubble deformation in the \((\rho z)\) plane by increasing the deformation in the azimuthal direction. This effect is opposite downstream, right after the obstacle, where the bubbles are prolate.

These features are further quantified by plotting the normal components of the shape tensor \( S_{\rho\rho} \), \( S_{\theta\theta} \) and \( S_{\phi\phi} \) as a function of \( \theta \) at \( r = 1.5 \text{ mm} \), in fig. 6. This graph shows that these normal components remain within a narrow range, between 0.19 mm and 0.22 mm, confirming that the bubbles are weakly deformed. These values correspond to the typical bubble size. For \( \theta > \pi/2 \) (i.e. upstream the obstacle), \( S_{\rho\rho} \) is lower than \( S_{\theta\theta} \) and \( S_{\phi\phi} \), which are approximately equal: hence, the bubbles are squashed against the obstacle. Conversely, for \( \theta < 1.2 \text{ rad} \), \( S_{\rho\rho} \) is larger than \( S_{\theta\theta} \) and \( S_{\phi\phi} \): the bubbles are stretched away from the obstacle. Hence, close to the axis \( \theta = 0 \), the bubbles are elongated streamwise more than spanwise. The origin of the negative wake then becomes clear: by elastically relaxing this deformation, the bubbles “push” the streamlines away from the axis \( \theta = 0 \). Hence, the velocity has to decrease towards its limiting value \( U \) as the bubbles are advected away from the obstacle.

Plastic rearrangements: Automated tracking of bubble rearrangements was hindered by the high sensitivity of such procedure to small defects in the reconstruction of the bubble topology. Description of the contact between bubbles requires to rebuild precisely the faces between bubbles, which would require a finer analysis [26]. Nevertheless, we managed to detect manually four individual events, corresponding to the rearrangements of neighboring bubbles. We provide below a detailed description of one typical example (fig. 7); the features of the three other ones were found to be the same. Those rearrangements consist of the swapping of four neighboring bubbles, with an exchange of four-sided faces, called T1s or quadrilateral-quadrilateral (QQ) transitions by Reinelt and Kraynik [27, 28]. We did not observe three-sided faces during a T1 as reported by [29]. These are likely highly unstable, transient states which are too short-lived to be captured by tomography. The QQ transitions observed involve two bubbles losing one face and two bubbles gaining one face. As can be seen on the projection plane across
these four bubbles (fig. 7(a) and (d)), this is analogous to T1s in 2D, which always involve four bubbles, two losing one side and two gaining one side. The distance between these four bubbles in contact decreases of 150 μm, from 1.10 mm before the T1 to 0.95 mm after, while the distance between the two bubbles losing contact increases of 200 μm, from 1.03 mm before the T1 to 1.23 mm after. We checked that the distances between the other bubbles around this T1 change much less. This corroborates the vision of a T1 quite similar as in 2D, acting as a quadrupole in displacement, with most effect on the bubbles in the plane. On the other hand, the variation of shape anisotropy of the bubbles involved in the T1 did not show significant trends.

We went further on in the characterization of the spatial structure of those rearrangements. Bubble faces comprise a thin film surrounded by a thick network of PBs and vertices. We have observed that the thin film part is usually very small for faces that are about to disappear, or that have just appeared, during a rearrangement. However, due to the finite radius of the PBs and of the finite size of the vertices, the “skeleton” of these faces is not arbitrarily small. Quantitatively, we measured on the images a PB radius \( R_c = 130 \) μm. We also measured the area of the skeleton of the faces on 2D projections along the plane of the faces (we did not observe significantly non-planar faces). We always found skeleton areas larger than 3.4 \( \times 10^4 \) μm², which is of the order of \( R_c^2 \). This suggests an interesting analogy with the cut-off edge length in 2D foams expected in theory [30,31], and measured in both simulations [32] and experiments [16]. In 2D foams and emulsions, when the distance between two approaching vertices reaches a certain length, a rearrangement occurs. This happens usually when the two PBs decorating the two neighboring vertices start to merge; hence, the order of magnitude of the cut-off length is \( R_c \). For 3D foams, our observations suggest that there is a cut-off area of the order of \( R_c^2 \) below which a face becomes unstable, triggering a rearrangement.

In summary, we have provided the first experimental measurement of a 3D time- and space-resolved foam flow measured directly from individual bubble tracking, with novel results on all the essential features of liquid foam mechanics: elasticity, plasticity and flow, through descriptions of shape field, T1 events, and velocity field. Such experimental results could be achieved thanks to the recent advances of both high-resolution and fast X-ray tomography and quantitative analysis tools. We discovered differences between 2D and 3D flows in that the range of influence of the obstacle on the flow field is smaller in the 3D case. The same is true for the deformation of the bubbles which is much smaller in the 3D case. Perspectives include further refinements of the analysis tools [15,26], to fully automatize the detection of rearrangements, to increase statistics and to study various geometries. Imaging the 3D flow at the bubble scale may shed new light on pending issues on shear localization [9] and nonlocal rheology [33].

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