Due to the lack of historical data, the inability to carry out a large number of reliability tests on the machine tool, the lack of reliability information collected, and the change of the working environment of the machine tool, there are a lot of uncertainties in the machine tool. In the existing research, only unilateral uncertainty factors are usually considered, while in practical problems, random uncertainty and cognitive uncertainty exist at the same time. Therefore, in this study, the random uncertainty and cognitive uncertainty in the system are characterized by random variables, fuzzy variables, probability box variables, and interval variables. For the structural function with more variables, the dimension is reduced first, and then the reliability model of the universal generating function (UGF) is established. Taking the heavy CNC machine tool as an example, the structural reliability analysis model of the fatigue strength of the milling shaft based on the UGF is constructed. The local sensitivity analysis and global sensitivity analysis of the variables affecting the fatigue strength of the milling shaft are carried out, and the factors that have the greatest impact on the fatigue strength of the milling shaft are obtained. The case study shows the effectiveness of the method proposed in this study.

1. Introduction

CNC machine tools are widely used in modern manufacturing industry, known as the “machine tool” of modern manufacturing industry, and its performance level also directly affects the rapid development of manufacturing industry in the whole country [1]. Heavy CNC machine tool has the characteristics of complex system structure, difficult process, long R&D cycle, high manufacturing cost, large driving load, and complex working condition stress. Due to the high R&D cost, long development cycle, insufficient experimental conditions, small batch customization, and other factors of Heavy CNC machine tools, it is impossible to obtain sufficient test data and field use data, which makes the reliability data include random uncertainty and cognitive uncertainty. How to deal with uncertainty is the key and difficulty of reliability evaluation. Uncertainty includes random uncertainty and cognitive uncertainty. Random uncertainty comes from the randomness and volatility of the system itself, which can be quantified by probability theory; cognitive uncertainty is due to cognitive bias, incomplete information, and other reasons. The subjective information given by experts or engineers often has cognitive uncertainty, which can be gradually reduced with the deepening of understanding and the increase of information [2]. At present, the main theories dealing with cognitive uncertainty include fuzzy mathematics, possibility theory, interval analysis, evidence theory, and Bayesian method. These theories still have certain limitations in dealing with cognitive uncertainty, and these theories need to make various assumptions about the probability distribution of component parameters or the independence of components [3], the
reliability evaluation method based on various assumptions has a certain degree of unreliability. In addition, none of the above theories can effectively quantify random uncertainty and cognitive uncertainty at the same time. How to establish a unified model to quantify random uncertainty and cognitive uncertainty is the main problem of complex system reliability evaluation.

Xiahou et al. [4] considered the uncertainty of component degradation parameters in the system, quantitatively characterized the component degradation parameters by evidence theory, and established the system reliability evaluation model based on evidence theory. Yang [5] combined evidence theory with common cause failure and proposed a reliability analysis method of complex systems under the coexistence of cognitive uncertainty and common cause failure. Lu [7] proposed a random fuzzy uncertainty under the coexistence of cognitive uncertainty and common cause failure, and combined evidence theory with common cause failure and the traditional fault tree with cognitive uncertainty. Mi [6] proposed a reliability evaluation model based on evidence theory. Yang [5] characterized the component degradation parameters by the traditional reliability discretization analysis method of uncertain variables, studies the random and cognitive uncertainty quantification based on the UGF, and establishes the reliability evaluation model. The feasibility and reliability of the two methods are fully verified by Monte Carlo analysis under the condition of small uncertainty.

2. Mathematical Description of UGF

The UGF represents discrete variables in the form of polynomials, defines the calculation rules and combination operators of discrete variables, and obtains the general form of system construction through recursive calculation.

Let all the possibilities of the discrete variable X be \((x_1, x_2, \ldots, x_k)\), and the corresponding probabilities are \((p_1, p_2, \ldots, p_k)\) respectively. Then the mapping of \(x_i \rightarrow p_i\) is called the probability quality function and satisfies \(\sum_{i=1}^{k} p_i = 1\). The UGF of a discrete variable X is defined as

\[
m(t) = E(e^x) = \sum_{i=1}^{k} e^{x_i} p_i.
\]

Let Z represent \(e^x\), then the UGF of X is expressed as

\[
u(z) = \sum_{i=1}^{k} p_i \cdot z^{x_i}.
\]

For two universal generating functions, the algorithm of [21] \(u_1(z) = \sum_{i=1}^{k} p_i \cdot z^{x_i}\), \(u_2(z) = \sum_{i=1}^{k} p_i \cdot z^{x_i}\):

\[
u_1(z) + u_2(z) = \sum_{i=1}^{k} \sum_{j=1}^{k} p_i p_j z^{x_i + x_j},
\]

\[
u_1(z) - u_2(z) = \sum_{i=1}^{k} \sum_{j=1}^{k} p_i p_j z^{x_i - x_j},
\]

\[
u_1(z) \times u_2(z) = \sum_{i=1}^{k} \sum_{j=1}^{k} p_i p_j z^{x_i x_j},
\]

\[
u_1(z) \otimes u_2(z) = \sum_{i=1}^{k} \sum_{j=1}^{k} p_i p_j z^{x_i^* x_j^*}.
\]

According to the algorithm of the UGF, considering “n” independent discrete variables \(X = (X_1, X_2, \ldots, X_n)\), the UGF \(u_g(z)\) of the function \(M = g(X)\) can be expressed as

\[
u_g(z) = u_{x_1}(z) \otimes u_{x_2}(z) \otimes \ldots \otimes u_{x_n}(z) = \sum_{j_1=1}^{k_1} \sum_{j_2=1}^{k_2} \ldots \sum_{j_n=1}^{k_n} \prod_{i=1}^{n} P_{ij_i} z^{g(X_{ij_1}, \ldots, X_{ij_n})},
\]

where, \(\otimes\) is a compound operator, \(g(X_{ij_1}, \ldots, X_{ij_n})\) is calculated according to the four algorithms of \(M = g(X)\). It can be seen from equation (4) that the number of items in the equation is \(\prod_{i=1}^{n} k_i\). For simple calculation, equation (4) can be rewritten as

\[
u(z) = \sum_{i=1}^{k} p_i z^x,
\]

where \(K = \prod_{i=1}^{n} k_i\).
2.1. UGF for Different Types of Variables

2.1.1. UGF Representation of Random Variables. Let the probability density function of a random variable \( X \) be \( f(x) \), where the distribution of samples is known and bounded, and it is recorded as \([a_0, a_n]\). Divide \([a_0, a_n]\) into \( n \) sub-intervals, as shown in Figure 1. The corresponding mean and probability of each subinterval are, respectively:

\[
\mu_i = \left( \frac{a_0 + a_1}{2}, \ldots, \frac{a_{n-1} + a_n}{2} \right),
\]

\[
P_i = \int_{a_{i-1}}^{a_i} f(x)dx, \quad i = 1, 2, \ldots, n.
\]

The universal generating function of random variables from equation (2) can be expressed as

\[
u(z) = \sum_{i=1}^{n} p_i z^{\mu_i}.
\]

2.1.2. UGF Representation of Probability Box Variables. Let the probability density function of the probability box variable \( X \) be \( f(\bar{x}) \) and the distribution interval be \([a, \bar{a}]\). To determine the mean value and probability of each interval of \( X \), the interval \([a, \bar{a}]\) can be divided into \( l \) cells on average, which can be expressed as

\[
\left\{ [a_1, \bar{a}_1], [a_2, \bar{a}_2], \ldots, [a_l, \bar{a}_l] \right\}.
\]

The mean value of each interval is

\[
\mu_i = (\mu_1, \mu_2, \ldots, \mu_l) = \left( \frac{a_1 + \bar{a}_1}{2}, \frac{a_2 + \bar{a}_2}{2}, \ldots, \frac{a_l + \bar{a}_l}{2} \right).
\]

The probability value corresponding to the mean value of each interval is

\[
p_i = (p_1, p_2, \ldots, p_l).
\]

2.1.3. UGF Representation of Fuzzy Variables. Fuzzy information is also a common type of information in reliability engineering. Fuzzy information is described by the membership function. Triangular fuzzy number is a common membership function, which is represented by \( \bar{X} \) represents the fuzzy variable, and the triangular fuzzy membership function is shown in Figure 2:

\[
\mu(\bar{X}) = \begin{cases}
\frac{(\bar{X} - a)}{(b - a)}, & a \leq \bar{X} < b, \\
1, & \bar{X} = b, \\
\frac{(\bar{X} - c)}{(b - c)}, & b < \bar{X} \leq c, \\
0, & \text{others}.
\end{cases}
\]

For the fuzziness problems contained in variables, the existing theoretical methods often use the form of cut sets to transform the membership function into the form of intervals. In this paper, the method of entropy equivalence will be used to transform the fuzziness problems in variables into random problems.

Shannon defines the probability entropy of random variables under uncertainty as:

\[
PE(x) = -\int_{-\infty}^{\infty} f(x) \ln f(x)dx.
\]
2.2. Structural Reliability Modeling with Mixed Uncertainty.

Variables describing mixed uncertainty usually include: random variable \( x \), fuzzy variable \( \tilde{x} \), interval variable \( y \), probability box variable \( X \), if \((X, \tilde{X}, Y, X)\) is a variable that affects the state and nature of the structure, then the functional function of the structure can be expressed as

\[
Z = g(X, \tilde{X}, Y, X). \tag{23}
\]

According to the stress strength interference model, when \( Z < 0 \) is, the structure fails; when \( Z > 0 \) the structure is in a safe state; when \( Z = 0 \), the structure reaches the limit state. In Figure 3, three limit states of the structural function in the case of two-dimensional variables.

In general, the failure rate of the structure is obtained by multiple integration of the joint probability density function of each variable in the region of \( Z < 0 \). When the structure contains different types of variables and the number of variables is large, it is very difficult to solve it by multiple integrals. The UGF simplifies the calculation to a certain extent. The structural failure probability under various uncertainties based on the UGF can be expressed as

\[
F = P_f \{g(X, \tilde{X}, Y, X) < 0\}. \tag{24}
\]

When the number of variables affecting reliability is small, the calculation using equation (24) is relatively simple, but when the number of variables affecting reliability is large, it will lead to the problem of large amount of calculation. Therefore, the approximate dimension reduction method is used to reduce the dimension of the multivariable computing function, obtain the approximate function of the multivariable computing function, and then build a UGF model to solve the failure efficiency.

\[
2.2.1. \text{Dimensionality Reduction of Multivariable Function.} \tag{23}
\]

Rahman and Xu [23] proposed an approximate dimensionality reduction method for multivariable functions. The main idea of the multivariable dimensionality reduction method is to decompose the original multivariable function into the sum of multiple single variables. Multivariable dimensionality reduction methods are mainly divided into two categories: Multivariable dimensionality reduction method based on mean point expansion and multivariable dimensionality reduction method based on the maximum possible failure point. This paper mainly adopts the multivariable dimensionality reduction method based on mean point expansion. Let the mean value of random variables in \( n \) intervals \( \mu = (\mu_1, \mu_2, \ldots, \mu_n) \), then the dimension reduction approximation model of the function \( g(X) \) of the mean value is as follows:

\[
g(X) \equiv \tilde{g}(X) = \sum_{i=1}^{k} g(\mu_x, \mu_{x_2}, \ldots, \mu_{x_k})
- (n-1)g(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_k}) \tag{25}\]

For variables with different types, the mean values of \((X, \tilde{X}, Y, X)\) are, respectively, \( \mu_x = (\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_k}), \mu_{\tilde{x}} = (\mu_{\tilde{x}_1}, \mu_{\tilde{x}_2}, \ldots, \mu_{\tilde{x}_k}), \mu_Y = (\mu_{Y_1}, \mu_{Y_2}, \ldots, \mu_{Y_k}), \mu_\tilde{X} = (\mu_{\tilde{X}_1}, \mu_{\tilde{X}_2}, \ldots, \mu_{\tilde{X}_k}) \).
Reliable domain $Z > 0$
Failure domain: $Z < 0$
Limit state: $Z = 0$

\[ f(z) = \exp\left(\lambda_0 + \sum_{i=1}^{s} \lambda_i z^i\right), \]  \hspace{1cm} (32)

\[ \lambda_0, \lambda_i, i = (1, 2, \ldots, s) \] is a Lagrange multiplier which can be obtained by the nonlinear programming method. The probability density function expression of multiple variables with the maximum entropy distribution satisfying the constraint conditions in equation (31):

\[ f(z) = \exp\left(\lambda_0 + \lambda_1 z + \lambda_2 z^2 + \lambda_3 z^3 + \cdots + \lambda_s z^s\right). \]  \hspace{1cm} (33)

Then it is transformed into the form of a universal generating function according to equation (2).

Since the function $g_2(Y)$ is an interval variable, the minimum and maximum values are determined through the linear programming optimization model, that is,

\[
\begin{aligned}
\frac{g_L^2}{g_U^2} &= \min \frac{g_L}{g_U} \\
&\text{subject to} \\
Y_1^L &\leq Y_1 \leq Y_1^U, \\
&\vdots \\
Y_r^L &\leq Y_r \leq Y_r^U.
\end{aligned}
\]  \hspace{1cm} (34)

According to equations (26) and (34), the maximum and minimum values of the functional functions $g(X, \bar{X}, Y, X)$ in the interval variables are, respectively.

\[
\begin{aligned}
g(X, \bar{X}, Y, X) &= g_{\min} = g(X) + g_4(\bar{X}) + g_3^L, \\
g(X, \bar{X}, Y, X) &= g_{\max} = g(X) + g_4(\bar{X}) + g_3^L.
\end{aligned}
\]  \hspace{1cm} (35)

The minimum and maximum failure probability values can be obtained by using the calculation method of UGF as shown in Figure 4.

3. Fatigue Strength Reliability Analysis of Milling Shaft Based on UGF

3.1. Reliability Analysis of Fatigue Strength of Sub Milling Shaft. For the heavy CNC boring and milling machine, the milling shaft is the core component of the machine tool. When the machine tool is working, the motor drives the motor.
milling shaft to rotate through the gear transmission mechanism, and the milling shaft bears the meshing force from the external gear in the process of rotation; at the same time, the milling shaft bears the cutting force, which is transmitted to the milling shaft by the milling cutter. There is little difference between the maximum outer diameter and the minimum inner diameter of the milling shaft, so the milling shaft can be simplified as a hollow shaft with an inner diameter \( d_2 \) of 130 mm and an outer diameter \( d_1 \) of 221 mm. Under the combination of bending and torsion, the stress diagram of the milling shaft is shown in Figure 5:

\[ F_c \] is the external load (kN); \( M_1 \) and \( M_2 \) are the bending moment on the dangerously section (kN·m); \( F_r \) is the radial force at the meshing part of the gear, and \( F_t \) is the circumferential force at the meshing part of the gear; \( a, b, \) and \( c \) are the dimensions of the milling shaft (m).

The functional function of the structure under the fatigue strength failure mode of the milling shaft is as follows:

\[
F = \Pr \left( \frac{\sigma - 1}{K} - \frac{\sqrt{M_H^2 + M_V^2 + (aM_T)^2}}{W} < 0 \right)
\]  

(36)

where \( M_T \)—torque of milling shaft (kN·m); \( M_H \)—bending moment on the horizontal plane of milling shaft (kN·m); \( M_V \)—bending moment on the vertical plane of milling shaft (kNm); \( W \)—bending section coefficient (m²); \( K \)—the correction coefficient is related to the surface quality and size of the milling shaft; \( \sigma - 1 \)—fatigue strength of standard smooth specimen (MPa);

The statistical table of fatigue strength related parameters of machine tool milling shaft is shown in Table 1:

For random variables, parameter 1 is the mean of normal distribution and parameter 2 is the variance of normal distribution; for the probability box variable, parameter 1 is the mean interval of normal distribution, and parameter 2 is the variance of normal distribution; for interval variables, parameter 1 is the interval lower limit, marked as \( a \), and parameter 2 is the interval upper limit, marked as \( b \).

There are many dependent variables. First, approximate dimensionality reduction of \( g(\sigma - 1, M_T, M_H, M_V, W, a, K) \) at the mean value:

\[
g(\sigma - 1, M_T, M_H, M_V, W, a, K) = \frac{\sigma - 1}{1.4} - \frac{0.42 + (0.55 \times M_T)^2}{0.0022} + 1649.35
\]

(37)
For the random variable $\sigma_{z}$, $M_T$, let $z = \sigma_{z}/1.4 - (0.42 + (0.55 \times M_T)^{1/2}) + 1649.35$, the first four order origin moments of $Z$ are obtained by (30), where the first four order origin moments are shown in Table 2:

The probability density function of $Z$ obtained from (33) is

$$f_z = \exp\left(\begin{array}{l}
-1001.257597 + 0.749403 \times z + 1.142073 \times 10^{-7} \times z^2 \\
+ 3.000513 \times 10^{-10} \times z^3 - 3.77152 \times 10^{-11} \times z^4
\end{array}\right).$$

(38)

Divide $Z$ into 5 cells, and the mean value and probability of each interval are

\[
\begin{align*}
\bar{Z} &= (1715, 1745, 1775, 1805, 1835), \\
p_z &= (0.060868, 0.260884, 0.404255, 0.221053, 0.041135).
\end{align*}
\]

(39)

For interval variables $K$, $W$, $\alpha$, let $z_1 = 1000/K - 1.3/W - (0.42 + (2.05\alpha)^{1/2})/0.0022$ the maximum and minimum value of $z_1$ can be obtained through optimization.

$$\begin{align*}
\min (\max)z_1 &= \frac{1000}{K} - \frac{1.3}{W} - \frac{(0.42 + (2.05\alpha)^{1/2})}{0.0022}, \\
1.3 &\leq K \leq 1.5, \\
0.0021 &\leq W \leq 0.0023, \\
0.5 &\leq \alpha \leq 0.6.
\end{align*}$$

(40)

According to the $3\sigma$ principle, the distribution interval $[\mu - 3\sigma, \mu + 3\sigma]$ of the probability box variable $M_H$ is divided into 7 cells, and the mean value and probability of each cell of $M_H$ are obtained:

\[
\begin{align*}
\bar{M}_H &= (0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8), \\
p_{M_H} &= (p_{M_H}^1, p_{M_H}^2, \ldots, p_{M_H}^7), \\
p_{M_H}^1 + p_{M_H}^2 + \ldots + p_{M_H}^7 &= 1, \\
p_{M_H}^i &\in \left[0.0013, 0.0214\right], \left[0.0214, 0.1359\right], \left[0.1359, 0.3413\right], \left[0.3413, 0.3413\right].
\end{align*}
\]

(41)

Similarly, $M_V$ is divided into 5 cells, and the mean value and probability of each cell of $M_V$ are obtained:

\[
\begin{align*}
\bar{M}_V &= (0.162, 0.286, 0.410, 0.534, 0.658), \\
p_{M_V} &= (p_{M_V}^1, p_{M_V}^2, \ldots, p_{M_V}^5), \\
p_{M_V}^1 + p_{M_V}^2 + \ldots + p_{M_V}^5 &= 1, \\
p_{M_V}^i &\in \left[0.0243, 0.0379\right], \left[0.2108, 0.2623\right], \left[0.4627, 0.4647\right].
\end{align*}
\]

(42)
According to equation (24), the calculation program is compiled with MATLAB software to obtain the minimum and maximum values of the fatigue strength and failure probability of the milling shaft. The calculation results with Monte Carlo method are shown in Table 3:

The fatigue strength of the milling shaft is calculated by using the multivariable dimension reduction method and the UGF. Compared with the results calculated by the Monte Carlo method, the error of the two methods is small, and the method proposed in this paper does not need a large number of data points and has high efficiency.

3.2. Sensitivity Analysis of Milling Shaft. If the reliability is sensitive to the changes of some factors, attention should be paid to the design, manufacturing, and processing of the structure to strictly ensure its accuracy. On the contrary, if the uncertainty of variables has no obvious impact on the structural reliability, it can be treated as a fixed value in the analysis, to reduce the difficulty of analysis and improve the calculation efficiency. Reliability sensitivity analysis can provide useful guidance for reliability design, construction, and maintenance of structures. The local sensitivity method and global sensitivity analysis method are used to analyze the fatigue strength of the milling shaft, and then the effects of several important variables that have the greatest impact on the fatigue strength of the milling shaft on the functional value will increase the failure probability.

3.2.1. Local Sensitivity Analysis Based on Monte Carlo. Local sensitivity is the value of the partial derivative of the input variable to the failure probability at a given parameter value. When the Monte Carlo method is used for analysis, the failure probability is expressed as

$$P_f = \frac{1}{N} \sum_{i=1}^{N} I(g < 0). \quad (43)$$

$I(\cdot)$ is a decision function, and its value can be expressed as follows:

$$\begin{cases} g < 0, & I(\cdot) = 1, \\ g \geq 0, & I(\cdot) = 0. \end{cases} \quad (44)$$

According to the definition of sensitivity, the expression is

$$\frac{\partial P_f}{\partial \theta_x} = \int \ldots \int \frac{\partial f(x)}{\partial \theta_x} dx. \quad (45)$$

$\theta_x$ is the characteristic parameter of the random variable distribution, and $f(x)$ is the probability density function of the basic random variable.

By transforming the integral in equation (45) into the expression of mathematical expectation, the Monte Carlo method can be used to calculate the structural failure sensitivity as follows:

$$\begin{align*}
\frac{\partial P_f}{\partial \theta_x} &= \int \ldots \int \frac{\partial f(x)}{\partial \theta_x} \frac{1}{f(x)} f(x) dx \\
&= \int \ldots \int I(\cdot) \frac{\partial f(x)}{\partial \theta_x} \frac{1}{f(x)} f(x) dx \\
&= E\left( \frac{I(\cdot)}{f(x)} \frac{\partial f(x)}{\partial \theta_x} \right) = \frac{1}{N} \sum_{i=1}^{N} I(\cdot) \frac{\partial f(x)}{\partial \theta_x} \quad (46)
\end{align*}$$

The sensitivity analysis of the fatigue strength limit state equation of the milling shaft is carried out in combination with the Monte Carlo method. Taking $\{d_{a1}, M_T, M_V, M_H\}$ as random variables and $\{K, W, \alpha\}$ as interval variables, the local sensitivity analysis results are shown in Table 4:

If the sensitivity is positive, increasing $\theta_x$ value will reduce the failure probability, negative sensitivity and increase $\theta_x$ will increase the failure probability.

3.2.2. Global Sensitivity Analysis Based on Variance. The local sensitivity analysis method is used to test the influence of the change of a single parameter at the nominal value point on the model results, while the global sensitivity analysis extends the change range of the parameters to the whole definition domain. In this paper, the variance sensitivity method is mainly used to analyze the global sensitivity.

Variance sensitivity analysis uses variance to describe the uncertainty of input variables. It can allocate the variance of output variables to each input variable and the interaction between different input variables, and decompose the response function $Y = g(X)$ into

$$Y = g(X) = g_0 + \sum_{i=1}^{n} g_i(X_i) + \sum_{1 \leq i \neq j \leq n} g_{ij}(X_i, X_j) + L + g_{1,2\ldots,n}(X_1, \ldots, X_n). \quad (47)$$

In this formula, $g_0 = E(Y)$, $\hat{g}_i = E(Y|X_i) - g_0$, $g_{ij} = E(Y|X_i, X_j) - g_0 - g_i - g_j$. The variance of the output variable $y$ can be decomposed into

$$V(Y) = \sum_{i=1}^{n} V_i + \sum_{1 \leq i \neq j \leq n} V_{ij} + \cdots + V_{1,2\ldots,n}. \quad (48)$$

### Table 2: First four order origin moments of random variable Z.

| First order moment | Second order moment | Third order moment | Fourth order moment |
|--------------------|---------------------|--------------------|--------------------|
| $1.7724 \times 10^3$ | $3.1421 \times 10^6$ | $5.5717 \times 10^9$ | $9.8825 \times 10^{12}$ |

### Table 3: Calculation results of failure probability calculated by different methods.

| Failure probability | Paper method | Monte Carlo method |
|---------------------|--------------|--------------------|
| $P_f^i$             | $0.3586 \times 10^{-4}$ | $0.43 \times 10^{-4}$ |
| $P_f^i$             | $0.0456$     | $0.03272$          |
Table 4: Local sensitivity analysis results of fatigue strength of milling shaft.

| Parameter | Local sensitivity |
|-----------|------------------|
| $\sigma_{-1}$ | $3.32 \times 10^{-8}$ | $-1.42 \times 10^{-8}$ |
| $M_T$ | $-1.61 \times 10^{-5}$ | $1.59 \times 10^{-5}$ |
| $M_H$ | $-1.05 \times 10^{-5}$ | $1.00 \times 10^{-6}$ |
| $M_V$ | $-4.72 \times 10^{-6}$ | $7.77 \times 10^{-6}$ |
| $W$ | $-0.005$ | $0.005$ |
| $\alpha$ | $1 \times 10^{-5}$ | $-1 \times 10^{-5}$ |
| $K$ | $-3.33 \times 10^{-6}$ | $3.33 \times 10^{-6}$ |

Table 5: Variance and partial variance.

| Basic variable | Variance $V_{p_i}$ | Variance $V_{\sigma_{-1}}$ | Variance $V_{M_T}$ | Variance $V_{M_H}$ | Variance $V_{M_V}$ | Variance $V_W$ | Variance $V_{\alpha}$ | Variance $V_K$ |
|----------------|--------------------|--------------------------|-------------------|-------------------|-------------------|----------------|-------------------|----------------|
| $V_{p_i}$      | 4438.82            | 342.95                   | 453.41            | 355.99            | 209.37            | 249.62         | 525.64           | 2297.91        |

Table 6: Global sensitivity analysis results based on main effect.

| Basic variable | Main effect sensitivity $S_i$ |
|----------------|-------------------------------|
| $\sigma_{-1}$  | 0.077                         |
| $M_T$          | 0.102                         |
| $M_H$          | 0.080                         |
| $M_V$          | 0.047                         |
| $W$            | 0.056                         |
| $\alpha$       | 0.118                         |
| $K$            | 0.517                         |

In this formula, $V_{p_i} = V(g_i) = V(E(Y|X_i))$, $V_{ij} = V(g_{ij}) = V(E(Y|X_i, X_j)) - V_i - V_j$ etc.

Sobol defines the sensitivity index as

$$S_i = \frac{V_{i}}{V(Y)} = \frac{V(E(Y|X_i))}{V(Y)} \quad (49)$$

$S_i$ indicate the influence degree of input variable $X_i$ on the output variable $Y$.

The reliability sensitivity analysis of Heavy CNC is carried out in combination with the variance sensitivity analysis method, and the fatigue strength limit state equation of milling shaft is as follows:

$$g(\sigma_{-1}, M_T, M_V, M_H, K, W, \alpha) = \frac{\sigma_{-1}}{K} \sqrt{\frac{M^2_H + M^2_V + (\alpha M_T)^2}{W}} \quad (50)$$

The fatigue strength variance of the milling shaft is calculated by using equations (47) and (48), as shown in Table 5.

Apply equation (48) to obtain the main effect sensitivity of each variable, as shown in Table 6.

It can be seen from the table that the uncertainty of the correction coefficient $K$ has the greatest impact on the fatigue strength of the milling shaft, and the others are $\alpha$, $M_T$, $M_H$, $\sigma_{-1}$, $W$, and $M_V$ in turn.

4. Conclusions

A reliability model based on UGF is established. An example shows that the method is effective.

The results of local sensitivity analysis are positive and negative. Positive sensitivity shows that the impact on reliability is positive, and negative sensitivity has a negative impact on reliability. Variable $M_T$, $M_V$, $M_H$ mean sensitivity is negative and variance sensitivity is positive, indicating that increasing the mean will increase the failure probability, and increasing the variance will reduce the failure probability. Increasing the mean value of variable $\sigma_{-1}$ decreases the failure probability, and increasing the variance of variable $\sigma_{-1}$ increases the failure probability. For interval variables, the value of $K$ and $a$ shall be increased and the value of $W$ shall be reduced to reduce the failure probability.

In the global sensitivity analysis results, the main effect sensitivity index value of the comprehensive correction coefficient $K$ is the largest, followed by the stress correction coefficient $\alpha$, the bending moment $M_H$ on the horizontal plane, the torque $M_T$ on the milling axis and the fatigue strength of the standard smooth specimen $\sigma_{-1}$ are the second, and the bending moment $M_V$ and bending section coefficient $W$ on the vertical plane are the smallest. Therefore, the uncertainty of the correction coefficient $K$ has a great impact on the uncertainty of the fatigue strength of the milling shaft. Increasing the correction coefficient $K$ can reduce the failure probability of the fatigue strength of the milling shaft the most.

Data Availability

The formula parameter data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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