Dust and gas distribution in molecular clouds: an observational approach.

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Abstract. The interstellar medium (ISM), gas and dust, appears to be arranged in clouds, whose dimensions, masses and densities span a large range of scales: from giant molecular clouds to small isolated globules. The structure of these objects show a high degree of complexity appearing, in the range of the observed scales, as a non-homogeneous (“clumpy”) distribution of matter. The arrangement of the ISM is clearly relevant for the study of the fragmentation of the clouds and then of the star formation processes. To quantify observationally the ISM structure, many methods have been developed and our study is focused on some of them, exploiting multiwavelength observations of IS objects. The investigations presented here have been carried out by considering both the dust absorption (in optical and near IR wavelengths) and the gas emission (in the submm-radio spectral range). We present the maps obtained from the reduction of raw data and a first tentative analysis by means of methods as the structure function, the autocorrelation, and the Δ-variance. These are appropriate tools to highlight the complex structure of the ISM with reference to the paradigm given by the supersonic turbulence. Three observational cases are briefly discussed. In order to analyse the structure of objects characterized by different sizes, we applied the above-mentioned algorithms to the extinction map of the dark globule CB 107 and to the CO(J=1-0) integrated intensity map of Vela Molecular Ridge, D Cloud. Finally we compare the results obtained with synthetic fractal maps known as “fractional Brownian motion” fBm images.

1. Introduction
Molecular clouds are the sites for active star formation both in our Galaxy and in the external galaxies. Because the conditions favoring the star formation mechanism are related with the physical status of the molecular cloud, many observational and theoretical efforts have been done in the recent years with the aim to infer on the possible relationships between the physics of the clouds and the mechanisms activating the star formation. One of the most important properties characterizing a molecular cloud is its own structure; in this respect two main approaches have been adopted to describe the mass distribution of IS clouds. One is the hierarchical approach that considers the clouds formed by clumps that, in turn, contain cores, with an increase in density as the mass and size decrease, until the protostellar scale is reached [1]. The other one is the fractal approach that considers molecular clouds as fractal objects.

Recently, thanks to the improved observing capabilities and the progress in the simulations, many observational aspects of the molecular clouds have been interpreted in the hypothesis that
the gas in these objects is in a turbulent regime; in this respect the fractal point of view seems to be the most appropriate approach to describe the IS clouds. In this work we illustrate a structural study based on the effects due to the dust component observed in optical ([2]) and NIR ([3]) imaging of the dark globule CB 107, and the emission of the gas component by means of sub-mm mapping of Vela Molecular Ridge (VMR), D Cloud. Then a structural analysis of these objects is proposed by applying some methods discussed in the context of the fractal objects, namely the structure function, the autocorrelation and the $\Delta$-variance. Here, for reason of space we present in detail only the results obtained analyzing CB 107, comparing these with those obtained for synthetic $fBm$ images.

2. Observations and data reduction
CB 107 is a small molecular cloud located at 180 pc ([4]) and projected toward a rich stellar background, with galactic coordinates $\ell = 2^{\circ}.8528, b = -2^{\circ}.7538$. The observations of CB 107 were carried out at the ESO-NTT telescope with the EMMI camera in 1999, for the optical images ($B$, $V$, $I$) and the SOFI camera in 2000, for the NIR images ($J$, $H$, $K_s$). For calibration purposes we also acquired a set of images of standard star fields, selected from the list of [5] and [6], for the optical and NIR range. As usual, bias and dark frames as well as flat-field images were also taken for the purpose of correcting our images for systematics.

The raw observational data were treated by following the procedure usually applied to CCD images, obtaining three final images for each spectral region; in Figure 1 (left panel) we show the final image obtained in the $B$ band. The photometry of the stellar objects detected in our images has been done by means of the DAOPHOT package, and the conversion of the instrumental magnitudes in a calibrated system was obtained by using the standard stars observed.

The method adopted to map the mass distribution in a molecular cloud, in the optical and NIR range, is essentially based on the effects produced by dust grains on the light of the background stars. The observable we determine is the extinction of the stellar light which can be quantified by means of images of the stellar field taken at different wavelengths. Because the extinction efficiency of the IS dust decreases with increasing wavelength, the optical images are better suited to analyze the edge of a cloud, with relatively low column densities. On the other hand the NIR images are more suited for mapping the innermost dark regions of the cloud.

Two methods are commonly used to obtain an extinction map. The first one uses the counting of the stars revealed in each box of a superimposed spatial grid, and consists in comparing the stellar densities observed in the obscured regions with a reference density determined in a nearby, presumably unobscured, comparison field ([7]). Using the knowledge of the luminosity function of the field stars, the ratio of the stellar densities on and off the cloud can be converted in extinction. Another valuable method for obtaining the extinction is based on the determination of the stellar colors which are reddened by differential extinction. This reddening is proportional to the projected column density of dust and is quantified by the color excess: $E(\lambda_1 - \lambda_2) = A_{\lambda_1} - A_{\lambda_2}$.

The photometric information obtained on the stars detected in the $B$, $V$, and $I$ images, and $J$, $H$, and $K$ images of CB 107 allows us to obtain both $[B - V]$, $[V - I]$ and $[J - H]$, $[H - K]$ colors for the stars detected in all the $B$, $V$, and $I$, as well as in the $J$, $H$, and $K$ images. Then the color excess of a given star can be determined by comparing its position in the $[B - V]$ vs $[V - I]$ or $[J - H]$ vs $[H - K]$ diagram, with the locus of the unreddened stars [2],[3]. Once the extinction law is known, the color excesses, determined in this way, can be used to obtain the extinction map. In the case of the $J$, $H$ and $K$ bands, assuming a normal IS reddening ([8]), the visual extinction is related to the excesses by: $A_V = 9.35 \ E[J - H]$ or $A_V = 15.87 \ E[H - K]$. In Figure 1 (right panel) we show the extinction map obtained.

The other object, VMR, is a cloud complex extending in galactic longitudes $\ell = 260^\circ - 273^\circ$, and confined to latitudes $b = \pm 2^\circ$ ([9]). It is composed of four molecular clouds ([10]); the cloud D (VMR-D) is located at a distance $d = 700$ pc ([11]). The CO($J = 1 - 0$) line emission (2.6 mm,
Figure 1. **Left panel**: final image of CB 107 in band $B$. The field size is $8.5' \times 8.5'$. The image contains several bright saturated stars whose effect is to degrade locally the image. Because of this, the regions affected by saturation effects have been masked before proceeding with further analyses. **Right panel**: contour levels of iso-extinction superimposed to the extinction map as determined in [3]. Each pixel corresponds to a square box of $L \sim 25''$ on the sky plane. The contour levels are drawn in steps of 1 mag. The blank pixels around the position (9,25) correspond to boxes containing less than 5 stars.

115 GHz) observations of VMR-D were carried out at SEST (ESO, La Silla, Chile) in 1999 and 2003. The spectral resolution was $\Delta \nu \sim 41.7$ kHz. We moved the antenna on the sky in steps of 50", finally obtaining, after the data reduction, a $53 \times 73$ pixels map of CO integrated intensity ([12]). For the purpose of comparison we also generated a $80 \times 80$ pixels synthetic $fBm$ image. The $fBm$ images are fractal sets characterized in the Fourier domain by both power-law power spectrum and random phases. Here, we chose a value of the power spectrum slope $\beta = 2.8$.

3. Application of statistical tools
The interest for studying the structure of the IS clouds is grown in the recent years, because of the important connections expected between the dominating physical processes and the internal structure of the IS clouds, and also because of the improved observational capabilities that make possible detailed tests for the model predictions. A more detailed discussion is given in [13].

The so called $\Delta$-variance method, originally proposed by [14], is a 2-D generalization of the Allan-variance method which has been traditionally used in the stability and drift analysis of instrumentation and electronic devices. If $s(x,y)$ is a 2-dimensional scalar function the $\Delta$-variance is defined as the variance of the convolved function

$$\sigma_\Delta^2(L) = \frac{1}{2\pi} < (s \ast L)^2 >_{x,y}$$

where

$$L(r) = \begin{cases} \frac{1}{\pi(L/2)^2} & (r \leq L/2) \\ -\frac{1}{8\pi(L/2)^2} & (L/2 < r \leq 3L/2) \\ 0 & (r > 3L/2) \end{cases}$$

is an axially symmetric filter function of scale $L$ defined as the *down-up-down cylinder function* with $r = (x^2 + y^2)^{1/2}$. This method aims to the detection of the drift behavior of the 2-D signal.
contained in the cloud images that can be related to the possible fractal structure underlying the mass distribution of the IS clouds. In fact, it can determine the slope of the power spectrum of the cloud image. This approach is particularly attractive, especially for relatively small sized maps, because the $\Delta$-variance method does not imply image transformations in the Fourier domain, avoiding in this way the drawbacks related to the finiteness and sampling of the image. A further advantage is also given by the ability of the method to easily separate the small scale noise from the structure signal ([15]). In this line, the analysis of the extinction map of CB 107, of the integrated intensity map of VMR-D, and of the $fBm$ test image has been done by convolving the corresponding images by sequentially scanning with different filter sizes and computing the variance of the obtained signal, related to the power spectrum of the images. The advantages of this method are partially counterbalanced by the fact that it works accurately only for the $fBm$ images because it allows to compute the exponent $\beta$ of the power-law power spectrum directly in the spatial domain ([14]) by the equation $\sigma(L)^2 \propto L^{2-\beta}$. In this respect, because some CO and HI maps of interstellar clouds seem to be similar to $fBm$ images ([14]) we decided to use this method.

Translating the results of the $\Delta$-variance analysis in terms of fractal dimension for the cloud structure we obtain $D = (3E + 2 - \beta)/2 \sim 2.6$ ([14]), where $E = 2$ represents the Euclidean dimension of the image. Note that this value is derived under the reasonable hypothesis that we are dealing with isotropic fractals. To estimate the uncertainty we repeated the same analysis for a set of simulated $fBm$ cloud images, with the same image size and fractal dimension. In this way we estimated the statistical error of our procedure that amounts to $\Delta D = 0.1$ ([3]).

The results obtained for the extinction map of CB 107 (left panel) and for the $fBm$ set (right panel) are presented in the Figure 2 that shows the drift behaviour of the $\Delta$-variance vs size.

The structure function of order $p$ can be defined and calculated for a mapped observable $A$ to quantify its scaling and fluctuations in the map:

$$S_p(\Delta r) = \langle |A(r) - A(r + \Delta r)|^p \rangle,$$

where $r$ identifies a position in the map, $\Delta r$ is an increment, and $\Delta r = |\Delta r|$ its module; the average $\langle \rangle$ is extended to all map positions and all directions of $\Delta r$. For a $fBm$ structure, it is
found that

\[ S_p(\Delta r) \propto \Delta r^{p_H} \equiv \Delta r^{\zeta(p)} . \tag{4} \]

For a turbulent velocity field, [16] found the relation \( \zeta(p)/\zeta(3) = p/3 \) in the case of incompressibility. Two refined and more complex models have been proposed by [17] and [18] by considering an improvement of the incompressible case, for which \( \zeta(p)/\zeta(3) = p/9 + 2[1 – (2/3)^{p/3}] \), and the supersonic compressible case, for which \( \zeta(p)/\zeta(3) = p/9 + 1 – (1/3)^{p/3} \) respectively. We calculated the structure functions of order \( p = 1 \div 15 \) for the extinction map of CB 107 (as in [19]) and of the \( fBm \) image (shown in the Figure 3), and of the VMR-D integrated intensity map (as in [20]). By fitting the linear region of the log-log diagram we calculated the slope \( \zeta \) at each value of \( p \). Finally, the fit of the linear region of this \( \zeta(p) \) curve (normalized by the \( \zeta(3) \) exponent), makes possible, in principle, to determine the exponent \( \beta \) and the fractal dimension \( D \) of the image.

The autocorrelation function (ACF) is the expected value of the product of the pixel values of a map with a shifted version of itself:

\[ C(\Delta r) = \langle A(r) A(r + \Delta r) \rangle . \tag{5} \]

For a \( fBm \) image the relation with the power spectrum is given by [14]:

\[ \log \left( 1 - \frac{C(\Delta r)}{\sigma^2} \right) = (\beta - 2) \log \Delta r + c , \tag{6} \]

where \( \sigma^2 = \langle (A(r) - \langle A(r) \rangle)^2 \rangle \), and \( c \) is a constant. It is then possible to derive the \( \beta \) exponent from Equation 6. In the Figure 4 we show the results of the best-fit procedure performed for CB 107 and for the \( fBm \) test image.

4. Discussion

Examining the results obtained by means the three adopted statistical tools (see Table 1), we can make several considerations. The investigated clouds exhibit a fractal behavior, as suggested by the fractal dimension of the 3-D structures, that is found significantly greater than 2. We can say that the results of the \( \Delta \)-variance and the autocorrelation appears to be consistent. Moreover, these two methods well reproduce the fractal characteristics of the reference \( fBm \) image. On the
Figure 4. ACF and determination of the power-law power spectrum slope $\beta$ calculated for the extinction map of CB 107 (left) and for the $fBm$ test image (right).

Table 1. An overview of the $\beta$ values obtained with different statistical tools.

|                | CB 107 | VMR-D | $fBm$ |
|----------------|--------|-------|-------|
| $\Delta$-variance | 2.77   | 2.59  | 2.78  |
| Structure function | 2.22   | 2.33  | 2.70  |
| Autocorrelation   | 2.60   | 2.63  | 2.80  |

contrary, the structure function method provides values for $\beta$ that appear underestimated, and then requires a more extended application to synthetic images, in order to increase the statistic of the investigated cases. These results can be the first step toward a better knowledge of the physical processes governing the dynamics of the ISM.

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