Abstract

For high energy cosmic neutrinos Athar, Ježabek, and Yasuda (AJY) have recently shown that the existing data on neutrino oscillations suggests that cosmic neutrino flux at the AGN/GRB source, $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 2 : 0$, oscillates to $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 1 : 1$. These results can be confirmed at AMANDA, Baikal, ANTARES and NESTOR, and other neutrino detectors with a good flavor resolution. Here, we re-derive the AJY result from quasi bi-maximal mixing, and show that observation of $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 1 : 1$ does not necessarily establish cosmic neutrino flux at the AGN/GRB source to be $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 2 : 0$. 

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1 Introduction

The solar neutrino anomaly, the LSND excess events, and the Super-Kamiokande data on atmospheric neutrinos, find their natural explanation in terms of oscillations of neutrinos from one flavor to another [1–3]. The only experiment so far that provides a direct evidence of oscillation from one flavor to another is the LSND experiment. However, the LSND result is still debated by the KARMEN collaboration [4]. It is expected to be settled by the dedicated Fermi Lab. experiments. Nevertheless, a strong tentative evidence for neutrino oscillations seems established.

In this Letter we shall neglect any possible CP violation in neutrino oscillations. We shall adopt the standard three-flavor neutrino scheme. In that framework one can accommodate any of the following two sets of data: (a) Data on the atmospheric neutrinos and solar neutrino anomaly, or (b) Data on atmospheric neutrinos and LSND excess events. In the quasi bi-maximal mixing, the angle $\theta$, see Eq. (7) below, can accommodate either the LSND results or the solar anomaly, but not both.

In the context of this experimental setting, and the stated theoretical framework, this Letter establishes the abstracted result. The origin for the abstracted result lies in the observation that the observed $L/E$ flatness of the electron-like event ratio in the Super-Kamiokande atmospheric neutrino data strongly favors [6,7] a quasi bi-maximal mixing matrix (and in fact this is what drives the AJY result). Here we show that quasi bi-maximal mixing transforms $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 2 - a$ to $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 1 : 1$. Note, the latter flux neither carries an $a$ dependence, nor is it affected by the angle $\theta$. This robustness has the consequence that by studying the departures from the $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 1 : 1$ for the observed cosmic high energy flux one may be able to explore new and interesting sources/physics of high energy cosmic neutrinos. The data, however, may also be used to study unitarity-preserving deformations of bi-maximality.

In the next section, we summarize the AJY result under study. Section 3 shows the quasi bi-maximal mixing as the physical origin of flux equalization for AGN/GRBs, it then presents the theorem advertised in the Abstract, and it ends by introducing a deformed bi-maximal mixing and its affect on the flux equalization. Section 4 is devoted to conclusion.
2 Brief review of AJY flux equalization

Without CP violation, the three-flavor neutrino oscillation framework carries five phenomenological parameters. These are the two mass-squared differences, $\Delta m^2_{32}$ and $\Delta m^2_{21}$, and the three mixing angles:

$$U(\theta, \beta, \psi) = \begin{pmatrix} e^{i\theta} & c_\theta c_\beta & s_\theta c_\beta \\ -c_\theta s_\beta s_\psi - s_\theta c_\psi & c_\theta c_\psi - s_\theta s_\beta s_\psi & c_\beta s_\psi \\ -c_\theta s_\beta c_\psi + s_\theta s_\psi & -s_\theta s_\beta c_\psi - c_\theta s_\psi & c_\beta c_\psi \end{pmatrix}$$ (1)

The columns of the mixing matrix $U$ are numbered by the mass eigenstates, $j = 1, 2, 3$, while the rows are enumerated by the flavors, $\ell = e, \mu, \tau$. Here, we have introduced the usual abbreviations: $c_x = \cos(x)$, and $s_x = \sin(x)$.

For a phenomenological study, the essential question is what are the parameters of the neutrino oscillations and what information may be extracted from them about particle physics, and astrophysical and cosmological processes/sources. New flavor-sensitive detectors with a collection area exceeding 1 km$^2$ shall provide us valuable information about the high-energy cosmic neutrino flux. This flux carries important information about the conventional processes of AGNs and GRBs, but it may also serve as a probe of certain quantum gravity effects and explore possible violations of the equivalence principle[8–15].

For high energy neutrinos, $E \gtrsim 10^6$ GeV, with sources in AGNs and GRBs, the source-detector distance far exceeds the kinematically induced oscillation lengths suggested by any of the solar, atmospheric, and the LSND data. Under these circumstances the AGN and GRB neutrino flux, $F^S$, at the source is roughly in the ratio:

$$F^e_e : F^S_\mu : F^S_\tau :: 1 : 2 : 0$$ (2)

The oscillated flux, $F^D_\ell$, measured at terrestrial detectors, becomes independent of the mass squared differences, and is given by [5]:

$$F^D_\ell = \sum_{\ell'} P_{\ell\ell'} F^S_{\ell'}$$ (3)

with

$$P_{\ell\ell'} = \sum_j |U_{\ell j}|^2 |U_{\ell' j}|^2$$ (4)
Using the solar, reactor, atmospheric, and the accelerator, neutrino data AJY have made a detailed numerical analysis. The result is [5]:

AJY’s numerical analysis: \( F_D^e : F_D^\mu : F_D^\tau :: 1 : 1 : 1 \) \hspace{1cm} (5)

Analytically [16], AJY show that the above result follows from the data-dictated assumptions:

\[ |U_{e3}|^2 \ll 1, \]
\[ |U_{\mu j}|^2 - |U_{\tau j}|^2 \ll 1, \quad j = 1, 2, 3. \] \hspace{1cm} (6)

3 Quasi Bi-maximal origin of flux equalization and an ambiguity theorem

We now show that this result is in fact a direct consequence of the quasi bi-maximal mixing inferred from the \( L/E \)-flatness of the electron-like event ratio observed in the Super-Kamiokande data on atmospheric neutrinos. Then, in the next section, we show that the flux equalization is not a unique signature of the source flux given by Eq. (2).

It was argued in Refs. [6,7] that the observed \( L/E \)-flatness of the electron-like event ratio in the Super-Kamiokande data on atmospheric neutrinos places severe analytical constraints on the mixing matrix. Without reference to the solar neutrino deficit, or the data on the LSND excess events, it was shown that these constraints yield a quasi bi-maximal mixing matrix.² This result is contained in Eq. (26) of Ref. [7], and reads:

\[
U = \begin{pmatrix}
c_\theta & s_\theta & 0 \\
-s_\theta/\sqrt{2} & c_\theta/\sqrt{2} & 1/\sqrt{2} \\
s_\theta/\sqrt{2} & -c_\theta/\sqrt{2} & 1/\sqrt{2}
\end{pmatrix}
\] \hspace{1cm} (7)

The mixing matrix (7), when coupled with Eq. (4), yields:

1 Also see, Ref. [16]
2 The quasi bi-maximal mixing reduces to the bi-maximal mixing for \( \theta = \pi/4 \). Apart from Refs. [6,7], other early references on bi-maximal mixing are [17–19].
3 An invertible quasi bi-maximal mixing matrix \( U \), Eq. (7), necessarily yields a \( P \) matrix that is non-invertible. This mathematical observation shall underlie the physical content of the theorem to be presented below.
\[
P = \begin{pmatrix}
    s_\theta^4 + c_\theta^4 & c_\theta^2 s_\theta^2 & c_\theta^2 s_\theta^2 \\
    c_\theta^2 s_\theta^2 & \frac{1}{4}(1 + s_\theta^4 + c_\theta^4) & \frac{1}{4}(1 + s_\theta^4 + c_\theta^4) \\
    c_\theta^2 s_\theta^2 & \frac{1}{4}(1 + s_\theta^4 + c_\theta^4) & \frac{1}{4}(1 + s_\theta^4 + c_\theta^4)
\end{pmatrix}
\]

Substituting the obtained \(P\) in Eq. (3) furnishes with the prediction:

Quasi Bi-maximal mixing: \(F_e^D : F_\mu^D : F_\tau^D :: 1 : 1 : 1\) (9)

This is precisely the result (5) which AJY obtained based on a detailed numerical analysis [5]. On the analytical side [16], the AJY constraints (6) are manifestly satisfied by the quasi bi-maximal mixing matrix (7).

Clearly, the AGN/GRB related \(\mathcal{F}^S\) satisfy this flux equalization criterion with \(a = 2\). For supernovae explosions, \(a \approx 1\). Once again, one obtains the flux equalization. The early results of Learned and Pakvasa [20], and Weiler et al. [21], are seen to follow as a special case associated with \(\theta = 0\) and \(a = 2\).

The result (9) is independent of the mixing angle \(\theta\) – the angle relevant for the solar, or LSND, data (see Refs. [6,7]). This implies that the high energy cosmic neutrino flux is robust in that it does not depend on the (vacuum) mixing angle obtained from the solar neutrino anomaly, or from the LSND data. This robustness can be exploited to systematically study other possible significant sources of neutrino flux, especially those which may arise from sources other than the decay of charged pions. The latter component of the neutrino flux may appear as a departure from the evenly proportioned flux of the three neutrino flavors discussed here. The departures may also serve as a probe of certain quantum gravity effects and possible violations of the equivalence principle[8–15]. However, we now emphasize that detecting a flux (9) does not necessarily imply the source flux to be (2).

In interpreting any deviations from the result (9), one must be careful to note the following ambiguity theorem. Let

\[
\mathcal{F}^S \equiv F_e^S : F_\mu^S : F_\tau^S :: 1 : a : 2 - a, \quad 0 \leq a \leq 2
\]

Then, under the already stated framework, the quasi bi-maximal mixing has the effect

\[
\mathcal{F}^S \rightarrow \mathcal{F}^D
\]

where

\[
\mathcal{F}^D \equiv F_e^D : F_\mu^D : F_\tau^D :: 1 : 1 : 1
\]
The proof is straightforward.

From an aesthetic point of view, a view which is also consistent with the existing data, the quasi bi-maximal mixing is a strong candidate to emerge as the unitary matrix behind the neutrino oscillations. The widely discussed bi-maximal mixing [6,7,17–19,22], as already noted, is a special case of the quasi bi-maximal mixing. In this special case one may introduce a unitarity-preserving deformation of the bi-maximality, and constrain it by the existing data as follows:

\[
U' = \begin{pmatrix}
\frac{c_\beta}{\sqrt{2}} & \frac{c_\beta}{\sqrt{2}} & s_\beta \\
-(1 + s_\beta)/2 & (1 - s_\beta)/2 & c_\beta/\sqrt{2} \\
(1 - s_\beta)/2 & -(1 + s_\beta)/2 & c_\beta/\sqrt{2}
\end{pmatrix}, \quad \beta \ll 1
\] (13)

This deformed bi-maximal mixing transforms \( F^S \) given by Eq. (10) into

\[
F'^D \equiv F'^D_e : F'^D_\mu : F'^D_\tau :: 1 : 1 + (a - 1)s_\beta^2 : 1 + (1 - a)s_\beta^2
\] (14)

and carries an essentially unique signature for the deformation parameter \( \beta \), and for the source flux parameter \( a \) (associated with the class of neutrino fluxes under consideration).

4 Conclusion

The observed \( L/E \) flatness of the electron-like event ratio in the Super-Kamiokande atmospheric data strongly favors a quasi bi-maximal mixing for neutrino oscillations. This quasi bi-maximal mixing contains one unconstrained mixing angle, \( \theta \). The angle \( \theta \) can either be used to accommodate the LSND excess events, or to explain to the long-standing solar neutrino anomaly. For high energy cosmic neutrinos, the Source-Detector distance far exceeds any of the relevant kinematically induced oscillations lengths. When this information is coupled with the Super-Kamiokande implied quasi bi-maximal mixing, characterized by the angle \( \theta \), we find that a whole range of neutrino fluxes, \( \mathcal{F}^S \), defined in Eq. (10), and characterized by \( a \), oscillate to equal fluxes of \( \nu_e, \nu_\mu, \) and \( \nu_\tau \). This result carries a remarkable robustness in its \( \theta \)- and \( a \)-independence.

Observation of equal \( \nu_e, \nu_\mu, \) and \( \nu_\tau \) fluxes from AGN/GRBs, and supernovae explosions, can be used to establish if they belong to flux class, \( \mathcal{F}^S \), defined above. Deviations of these fluxes, \( \mathcal{F}^D \), as observed in terrestrial detectors, from the ratio \( 1 : 1 : 1 \) can become a robust measure of the departure of the source flux ratio from \( 1 : a : 2 - a \). A detailed study of these departures
carries the seeds to discover new physics, and to characterize cosmic neutrino sources. In particular, it is to be emphasized that $a$ remains unmeasurable, if the mixing is quasi bi-maximal (of which, bi-maximal mixing is a special case). Furthermore, the angle $\theta$, that, e.g., can be adjusted to resolve the solar neutrino anomaly, does not influence the expected flux equalization. Because the high-energy cosmic neutrino flux as detected in terrestrial laboratories is insensitive to the underlying mass-squared differences, measurements on the flavor spectrum of the high-energy cosmic neutrino flux can be used to probe a whole range of parameters associated with neutrino oscillations. Since each of these parameters – from those related to the deformed bi-maximal mixing, to those that characterize a whole range of quantum-gravity effects (including those violating the principle of equivalence) – is likely to yield a different signature, high-energy cosmic neutrinos provide a powerful probe into new physics.

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