On extrapolations below the Planck scale in models with Lorentz symmetry violation (I)

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Abstract
Most current models of Lorentz symmetry violation (LSV) at the Planck scale involve power-like extrapolations of the Lorentz-beaking term down to accelerator and even much lower energies. It is therefore assumed that no intermediate energy scale alters this behaviour. But this is not the only possible scenario: a more sophisticated energy-dependence is possible, and would even be natural, involving significant effective thresholds at intermediate energies. Such thresholds may exist between the Planck scale and the highest cosmic-ray energies, or between ultra-high cosmic-ray energies and the TeV scale, leading to interesting scenarios. In many cases, experimental predictions of LSV patterns can be dramatically modified and space experiments become necessary irrespective of AUGER results. By combining both kinds of experiments, future results of cosmic-ray observations will hopefully be able to test, for a large family of models involving various patterns of Planck-scale physics, the possible existence of an absolute local rest frame in the real world.

1 Introduction
By now, preliminary AUGER data [1] seem not to exclude a possible absence of the Greisen-Zatsepin-Kuzmin (GZK) cutoff [2, 3]. In this case, Lorentz symmetry violation at the Planck scale [4] would be a natural candidate to explain such an observation, provided it is assumed that a privileged rest frame exists (the vacuum rest frame). But it may also happen that future data with better statistics show the existence of such a cutoff. This would not rule out all LSV patterns.

The model we proposed in 1997 in [4], and also in [5, 6] and in other papers of the same period (see arXiv.org), is indeed able to explain a possible absence of the GZK cutoff, contrary to that considered previously by Kirzhnits and Chechin [7] which was shown [8, 9] not to be able to account for such an effect. The reason is that the Kirzhnits-Chechin model is a form of STRONG doubly special relativity (SDSR), where it is required that the laws of Physics be exactly the same in all inertial frames [10, 11]. This approach assumes that the action is invariant under space-time diffeomorphisms, forbidding the appearance of a preferred reference frame. It turns out to imply a substantial violation of energy-momentum conservation in the physical inertial frames and precludes the effect announced by the authors of [7]. If the GZK cutoff turns out not to exist in the real world, our LSV pattern can account for such an effect, but not SDSR.

Models like that proposed in [4] can fit into a larger family of doubly special relativity patterns (WEAK doubly special relativity, WDSR), where it is still required that the laws of physics be described by two universal constants, $c$ and the fundamental length $a$, but the equations of motion are not identical in all inertial frames: they are identical only in the limit $a \to 0$. The vacuum rest frame can then, in the examples we consider, be characterized by the isotropy of the laws of physics. The universality of these laws remains up to the boost between the inertial frame considered and the vacuum rest frame. Such a boost is a linear energy-momentum transformation, preserving the additiveness of these four observables. Similar patterns are assumed for more involved laws of
Physics. An example of WDSR can be provided by a dispersion relation of the kind (1) - (3) presented below, assumed to be valid in the vacuum rest frame, and its Lorentz-transformed sets of equations in the other inertial frames. This is a form of deformed relativistic kinematics (DRK) in WDSR.

Expressions like (1) - (3) involve a power-like extrapolation from the Planck scale down to all scales above the highest mass scale of the particles considered. Actually, this is only the simplest possible scenario and other forms of energy-dependence are possible. The WDSR models proposed in our papers since 1997 are not field-theoretical in the sense considered by [12]. They therefore escape the criticism formulated by these authors. Furthermore, because of the collapse of the final-state phase space at ultra-high energy [13], the implementation of unitarity at these energies can undergo important changes in WDSR as compared to standard scenarios. This important issue be discussed in a forthcoming paper. Also, the general requirement that the deformation term of WDSR be negative in order to prevent spontaneous decays of ultra-high energy (UHE) particles was first emphasized in our 1997 papers (f.i. [14]), where possible violations of the equivalence principle were explicitly discussed and the question of the universality of the coefficient of the deformation term was dealt with [5, 6, 15, 16]. It was made clear that this universality is not possible for large bodies, and that a different law is needed where the deformation coefficient depends on the mass of the body considered [5, 17]. For elementary particles, the universality of the deformation appeared as a natural but not a priori compulsory hypothesis, often imposed in practice to a good approximation by phenomenological considerations. This analysis was further developed in [18] to obtain bounds on standard LDRK parameters (see Section 3) and led in this case [18, 19] to the bound $E_{QG} \gtrsim 10^{26}$ GeV where $E_{QG}$ is the effective quantum gravity scale or any other relevant fundamental scale playing a similar role.

In what follows, the word "deformation" stands for a set of special-relativity violating terms which tend to zero (possibly up to very small constants) in the infrared limit, faster than the conventional squared momentum term of relativistic kinematics. This may correspond to the existence of a preferred inertial reference system as in WDSR, or to a pattern without a vacuum rest frame (SDSR).

Usually, like in (1) - (3), the deformation is power-like down to the particle mass scales and involves no intermediate energy thresholds. However, such thresholds may exist and play a significant role, as will be discussed in the present paper.

## 2 QDRK with a vacuum rest frame and energy-momentum conservation

The role of possible LSV in astrophysical processes at very high energy has been discussed and updated in [18, 20], and later in [21, 22]. Assuming energy and momentum conservation in the vacuum rest frame, ultra-high energy cosmic rays (UHECR) [24] provide a laboratory to test LSV.

As we suggested in 1997, a simple LSV pattern with an absolute local rest frame and a fundamental length scale $a$ (e.g. the Planck scale) where new physics is expected to occur [4] is given by a quadratically deformed relativistic kinematics (QDRK) of the form [4, 5]:

$$E = (2\pi)^{-1} \hbar c a^{-1} e (k a)$$  \hspace{1cm} (1)

where $\hbar$ is the Planck constant, $c$ the speed of light, $k$ the wave vector, and $[e (k a)]^2$ is a convex function of $(k a)^2$ obtained from vacuum dynamics. Expanding equation (1) for $k a \ll 1$, we can
write in the absence of other distance and energy scales [5]:

\[ e (k a) \simeq [(k a)^2 - \alpha (k a)^4 + (2\pi a)^2 \hbar^{-2} m^2 c^2)]^{1/2} \]  

(2)

\( \alpha \) being a model-dependent constant, possibly in the range \( 0.1 - 0.01 \) for full-strength violation of Lorentz symmetry at the fundamental length scale, and \( m \) the mass of the particle. For momentum \( p \gg mc \), we get:

\[ E \simeq p c + m^2 c^3 (2 p)^{-1} - p c \alpha (k a)^2 / 2 \]  

(3)

It is assumed that the earth moves slowly with respect to the absolute rest frame. The ”deformation” approximated by \( \Delta E = - p c \alpha (k a)^2 / 2 \) in the right-hand side of (3) implies a Lorentz symmetry violation in the ratio \( E p^{-1} \) varying like \( \Gamma (k) \simeq \Gamma_0 k^2 \) where \( \Gamma_0 = - \alpha a^2 / 2 \). If \( c \) is a universal parameter for all particles, the QDRK defined by (1) - (3) preserves Lorentz symmetry in the limit \( k \to 0 \), contrary to the standard \( THe\mu \) model [24]. QDRK can lead to several dramatic observable effects at phenomenologically reasonable energy scales [18, 19, 20], as reminded in subsections 2.1 and 2.2. It seems to be, to date, the best-suited LSV model for phenomenology.

2.1 Transition energy effects

At energies above \( E_{trans} \approx \pi^{-1/2} h^{1/2} (2 \alpha)^{-1/4} a^{-1/2} m^{1/2} c^{3/2} \), the deformation \( \Delta E \) dominates over the mass term \( m^2 c^3 (2 p)^{-1} \) in (3) and modifies all kinematical balances: physics gets closer to Planck scale than to electroweak scale and ultra-high energy cosmic rays (UHECR) become potentially an efficient probe of Planck-scale physics. The standard parton model (in any version) does no longer hold, and similarly for standard formulae on Lorentz contraction and time dilation [15]. See, however, [8, 9] on the possible role of (formal) extra dimensions.

Because of the negative value of \( \Delta E \) [14], it costs more and more energy, as \( E \) increases, to split the incoming longitudinal momentum in the laboratory rest frame. As the ratio \( m^2 c^3 (2 p \Delta E)^{-1} \) varies like \( \sim E^{-4} \), the transition at \( E_{trans} \) is very sharp. Using these simple power-like laws, QDRK can lead [18, 20] to important observable phenomena. In particular:

- In astrophysical processes at very high energy, similar mechanisms can inhibit [15, 20] radiation under external forces (e.g. synchrotron-like, where the interactions occur with virtual photons), photodisintegration of nuclei, momentum loss through collisions (e.g. with a photon wind in reverse shocks), production of lower-energy secondaries...

- Unstable particles with at least two stable particles in the final states of all their decay channels become stable at very high energy [4]. Above \( E_{trans} \), the lifetimes of all unstable particles (e.g. the \( \pi^0 \) in cascades) become much longer than predicted by relativistic kinematics. The neutron or even the \( \Delta^{++} \) can be candidates for the primaries of the highest-energy cosmic ray events. If \( c \) and \( \alpha \) are not exactly universal, many different scenarios are possible [14].

2.2 Limit energy effects

\( E_{trans} \) is not the only phenomenological energy scale naturally generated by DRK, and other effects are also present:

- The allowed final-state phase space of two-body collisions is strongly reduced at very high energy, [14], with a sharp fall of partial and total cross-sections for cosmic-ray energies above \( E_{lim} \approx (2 \pi)^{-2/3} (E_T a^{-2} \alpha^{-1} \hbar^2 c^2)^{1/3} \), where \( E_T \) is the target energy. Using the previous figures for LSV parameters, above some energy \( E_{lim} \) between \( 10^{22} \) and \( 10^{24} \) eV a cosmic ray will not deposit most of its energy in the atmosphere and can possibly fake an exotic event with much less energy [14]. Contrary to \( E_{trans} \), \( E_{lim} \) depends on the target energy \( E_T \).
- For $\alpha a^2 > 10^{-72} \text{ cm}^2$, and assuming universal values of $\alpha$ and $c$, there is no GZK cutoff for the particles under consideration.

- Requiring simultaneously the absence of GZK cutoff in the region $E \approx 10^{30} \text{ eV}$, and that cosmic rays with $E$ below $\approx 3.10^{20} \text{ eV}$ deposit most of their energy in the atmosphere, leads to the constraint $10^{-72} \text{ cm}^2 < \alpha a^2 < 10^{-61} \text{ cm}^2$, equivalent to $10^{-20} < \alpha < 10^{-9}$ for $a \approx 10^{-26} \text{ cm}$ ($\approx 10^{21} \text{ GeV}$ scale). Assuming full-strength LSV forces $a$ to be in the range $10^{-36} \text{ cm} < a < 10^{-30} \text{ cm}$, but a $\approx 10^{-6}$ LSV at Planck scale can still explain the data. Thus, the simplest version of QDRK naturally fits with the expected potential role of Planck-scale dynamics if ultra-high energy cosmic rays (UHECR) are the right probe.

2.3 QDRK with intermediate energy thresholds

However, the simple power-like extrapolation used above for $\Delta E$ over at least 19 orders of magnitude is not the only possible behaviour of the deformation at energies below Planck scale. A simple modification would be to write:

$$\Delta E = - p c \alpha (k^2 a^2)^{3/2}$$

where $k' = (k^2 + k_0^2)^{1/2} - k_0$ and $k_0$ is a new wavevector scale of dynamical origin associated to the energy scale $E_0 = (2\pi)^{-1} h c k_0$. For $k \ll k_0$, one has: $k' \approx k^2 (2k_0)^{-1}$ and:

$$\Delta E = - p c \alpha k^4 a^2 k_0^{-2}/8$$

so that the deformation becomes much smaller below the $\approx k_0$ scale whereas, for $k \gg k_0$, $\Delta E$ has the same form as previously up to non-leading terms. Again, the transition between the two regimes is rather sharp. The new effective threshold scale $k_0$ ($E_0$) is to be related to some intermediate scale where new physics becomes apparent, and the parameterization used for the new deformation is just an illustrative example. The $E_0$ scale can be chosen to be below $E_{\text{trans}}$, between $E_{\text{trans}}$ and $E_{\lim}$ or above $E_{\lim}$, leading to various phenomenological predictions.

In particular, if $k_0$ is chosen to be above the expected GZK cutoff scale, it is possible to build scenarios where the cutoff is present at the energies predicted in [2, 3] but disappears at a higher energy scale where the QDRK effects described above manifest themselves. Satellite experiments seem to be the natural way to explore the possible existence of such a form of QDRK, irrespective of the future results of the AUGER experiment.

More generally, it would be interesting to explore scenarios where:

$$\Delta E = - p c \alpha (k' a)^\sigma (k a)^\tau /2$$

with $\sigma + \tau = 2$, $\sigma$ and $\tau$ being real and positive. In this way, it is possible to further regulate the effect of the new energy scale.

3 LDRK with a vacuum rest frame
and energy-momentum conservation

LDRK, linearly deformed relativistic kinematics, was discarded in our 1997 and subsequent papers for phenomenological reasons [13], but has been proposed by several authors (see e.g. [25], for
cosmic-ray and gravitational-wave phenomenology, and various versions of the pattern have been considered more recently [26, 10]. The function $e(k a)$ is then a function of $k a$ and, for $k a \ll 1$:

$$e(k a) \simeq [(k a)^2 - \beta (k a)^3 + (2\pi a)^2 h^{-2} m^2 c^2]^{1/2}$$

(7)

$\beta$ being a model-dependent constant. For momentum $p \gg mc$:

$$E \simeq p c + m^2 c^2 (2 p)^{-1} - p c \beta (k a)/2$$

(8)

the deformation $\Delta E = - p c \beta (k a)/2$ being now driven by an expression linear in $k a$. LDRK can be generated by introducing a background gravitational field in the propagation equations of free particles [27]. If existing bounds on LSV from nuclear magnetic resonance experiments are to be interpreted as setting a bound of $\approx 10^{-21}$ on relative LSV at the momentum scale $p \sim 100 M eV$, this implies $\beta a < 10^{-34} cm$. But LDRK seems to lead to inconsistencies with cosmic-ray experiments unless $\beta a$ is much smaller [18, 19]. Concepts and formulae presented for QDRK can be readily extended to LDRK, and we get now:

$$E_{\text{trans}} \approx \pi^{-1/3} h^{1/3} (2 \beta)^{-1/3} a^{-1/3} m^2 c^5 / 3$$

$$E_{\text{lim}} \approx (2 \pi)^{-1/2} (E_T a^{-1} \beta^{-1} h c)^{1/2}$$

(9)

(10)

For a high-energy photon, LDRK is usually parameterized [27] as:

$$E \simeq p c - p c \beta (k a)/2 = p c - p^2 M^{-1}$$

(11)

where $M$ is an effective mass scale. Tests of this model through gamma-ray bursts, measuring the delays in the arrival time of photons of different energies, have been considered in [28, 11, 29] for the Gamma-ray Large Area Space Telescope (GLAST), and more generally in [27] and in subsequent papers by several authors (see the references in [10]). But, from the same considerations developed in our 1997-99 papers and more systematically in [18, 19] taking QDRK as an example, stringent bounds on LDRK can be derived. Assume that LDRK applies only to photons, and not to charged particles, so that at high energy we can write for a charged particle, $ch$, the dispersion relation:

$$E_{\text{ch}} \simeq p_{\text{ch}} c + m_{\text{ch}}^2 c^3 (2 p_{\text{ch}})^{-1}$$

(12)

where the $ch$ subscript stands for the charged particle under consideration. Then, it can be readily checked that the decay $ch \rightarrow ch + \gamma$ would be allowed for $p$ above $\simeq (2 m_{\text{ch}}^2 M^3 c^3)^{1/3}$, i.e:

- for electrons, above $E \approx 2 TeV$ if $M = 10^{16} GeV$, and above $\approx 20 TeV$ if $M = 10^{19} GeV$;
- for muon and charged pions, above $E \approx 80 TeV$ if $M = 10^{16} GeV$, and above $\approx 800 TeV$ if $M = 10^{19} GeV$;
- for protons above $E \approx 240 TeV$ if $M = 10^{16} GeV$, and above $\approx 2.4 PeV$ if $M = 10^{19} GeV$;
- for $\tau$ leptons, above $E \approx 400 TeV$ if $M = 10^{16} GeV$, and above $\approx 4 PeV$ if $M = 10^{19} GeV$; so that none of these particles would be observed above such energies, apart from very short paths. Such decays seem to be in contradiction with cosmic ray data, but avoiding them forces the charged particles to have the same kind of propagators as the photon, with the same effective value of $M$ up to small differences. Similar conditions are readily derived for all "elementary" particles, leading for all of them, up to small deviations, to a LDRK given by the universal dispersion relation:

$$E \simeq p c + m^2 c^3 (2 p)^{-1} - p^2 M^{-1}$$

(13)

For instance, $\pi^0$ production would otherwise be inhibited. But if, as it seems compulsory, the $\pi^0$ kinematics follows a similar law, then the decay time for $\pi^0 \rightarrow \gamma \gamma$ will become much longer than
predicted by special relativity at energies above $\approx 50\,\text{TeV}$ if $M = 10^{16}\,\text{GeV}$ and $\approx 500\,\text{TeV}$ if $M = 10^{19}\,\text{GeV}$. Again, this seems to be in contradiction with cosmic-ray data. Requiring that the $\pi^0$ lifetime agrees with special relativity at $E \approx 10^{17}\,\text{eV}$ would force $M$ to be above $\approx 10^{26}\,\text{GeV}$, far away from the values to be tested at GLAST. Another bound is obtained from the condition that there are $3.10^{20}\,\text{eV}$ cosmic-ray events. Setting $E_{\text{lim}}$ to this value, and taking oxygen to be the target, yields $M \approx 3.10^{21}\,\text{GeV}$. It therefore appears very difficult to make LDRK, with $M$ reasonably close to Planck scale, compatible with experimental data.

It often said that high-threshold ($\sim 10^{19}\,\text{eV}$) experiments like EUSO can test ”TeV gravity”. By ”TeV gravity” it is meant LDRK models where the effective fundamental scale is somewhere between $10^{16}$ and $10^{19}\,\text{GeV}$. As emphasized in our previous papers [18, 20] and reminded above, present data can already be used to exclude all versions of this LSV pattern able to lead to observable effects in the $\text{TeV}$ region (but the situation may be different for SDSR versions of LDRK). It has also been claimed that, from an experimental point of view, the test of ”TeV gravity” will be possible only after having studied ultra-high energy neutrinos. Obviously, the study of UHE neutrinos will provide crucial information, but there is no physical reason for such a restriction. The condition that a UHE proton does not spontaneously decay by emitting a photon involves only the dispersion relations of these two particles in the physical vacuum and does not depend on any property of neutrino physics. Therefore, ”TeV gravity” based on (WDSR) LDRK patterns is ruled out by global phenomenological considerations, independently of future neutrino results.

Similarly, the discussion [13] of the sharp fall of multiparticle phase space for a UHE particle interacting with the atmosphere involves only a balance between the deformation of hadronic kinematics and the target energy which, in standard relativistic models, is expected to provide the multiparticle transverse energy. As a nonrelativistic target is accelerated to a relativistic speed by the UHE collision, its rest mass turns into a much smaller mass term ($\approx m^2 p^{-1}_{OT}$ where $p_{OT}$ stands for the outgoing target momentum) and the released energy usually provides the transverse energy of the event as well as the multiparticle mass terms. However, the presence of a negative deformation term in (1) - (3) or (7), (8) growing like a power of $p$ in the kinematics of the incoming UHE particle alters standard kinematical balances as studied in our 1997-2000 papers. Above $\approx E_{\text{lim}}$, there is less and less energy available to provide mass terms and form the transverse multiparticle phase space, so that atmospheric showers cannot be generated for and the conventional UHECR event does no longer occur. It is on these and similar grounds that we excluded LDRK models long ago.

### 3.1 LDRK and the bound from the Crab nebula synchrotron radiation

Our claim that LDRK, in its standard WDSR power-like form, cannot be made consistent with experiment, has also been confirmed by the analysis astrophysical of synchrotron radiation. In [30], Jacobson, Liberati and Mattingly considered the LDRK dispersion relation:

$$E^2(p) = m^2 + p^2 + \eta M^{-1}$$

(14)

is considered, $M$ being the Planck mass, $\eta$ a negative constant and $E_{\text{QG}} = M |\eta|^{-1}$ the effective quantum gravity scale, to discuss data on synchrotron radiation from the Crab nebula. They obtained a lower bound $E_{\text{QG}} \gtrsim 10^{26}\,\text{GeV}$. These authors refer to [20] as having first pointed out the existence of a cutoff in synchrotron radiation in the presence of Lorentz symmetry violation. In [20], we had made explicit the calculations for a conclusion already stated in our 1997 papers. It should also be noticed, as reminded above, that the bound $E_{\text{QG}} \gtrsim 10^{26}\,\text{GeV}$ had first been obtained in [19] from a very reasonable requirement on $\pi^0$ lifetime.

In [20], we explicitly pointed out that the transition at $E_{\text{trans}}$ introduces an essential modification of the energy absorption required for radiation synchrotron emission and must lead to a sharp cutoff.
for this emission. Assuming $\alpha$ to be positive in (3), as otherwise LSV would lead to spontaneous decays at ultra-high energy \[5, 14\], and using the fact that it costs more and more energy to split the incoming longitudinal momentum as energy increases above $E_{\text{trans}}$, we gave a schematic illustration of the effect of DRK on synchrotron radiation using the same QDRK model as in Section 2. If relativistic kinematics applies, a UHE proton with energy-momentum \(\simeq [p c + m^2 c^3 (2p)^{-1}, p]\) can emit in the longitudinal direction a photon with energy \((\epsilon, \epsilon c^{-1})\) if, for instance, it absorbs an energy-momentum $\delta E \simeq m^2 c^2 p^{-2} \epsilon/2$. This expression for $\delta E$ falls quadratically with the incoming energy. With QDRK and above $E_{\text{trans}}$, we get instead $\delta E \simeq 3 \alpha \epsilon (k a)^2/2$, quadratically rising with proton energy. At high enough energy, the proton can no longer emit synchrotron radiation apart from (comparatively) very small energy losses. We therefore expect protons to be accelerated to higher energies in the presence of Lorentz symmetry violation. Similar considerations obviously apply to LDRK as well, where the expression $E_{\text{trans}} \approx \pi^{1/3} h^{1/3} (2 \beta)^{-1/3} a^{-1/3} m^{2/3} c^{5/3}$ is to be compared with the cutoff $E_{\text{max}}$ obtained in \[30\] which differs only by a factor 0.93 from $E_{\text{trans}}$ and corresponds, up to this factor, to the same analytic expression. This is the actual origin of the cutoff rediscovered in \[30\]. A similar result for $E_{\text{max}}$, again identical to our definition of $E_{\text{trans}}$, was also obtained in \[26\], where a detailed calculation is performed using explicitly a Liouville string model.

On the grounds of specific Liouville string models, where quantum gravity corrections amount to introducing defects in space-time with vacuum quantum numbers \[31\], it was claimed \[26\] that charged particles may follow a different kinematics from that of the photon, yet not spontaneously decaying through photon emission (although kinematically allowed) as they would not see the quantum-gravitational medium and could not emit Cherenkov radiation. But, although violations of the equivalence principle by Lorentz-violating terms are obviously possible and were already considered in our 1997 papers, it does not appear that the claim of \[26\] has actually been demonstrated and the validity of the argument is not obvious. Quantum electrodynamics implies that a physical charged particle is made of the bare charged particle surrounded by a cloud of virtual photons and these photons do see the D-particle medium. The basic question is whether a virtual photon can spontaneously materialize, and the answer seems to be that this is kinematically allowed in the models considered in \[26\]. The virtual photon is indeed real if it has the required on-shell kinematical properties, and can then escape from the bare electron. The fact that the charged particle does not see the space-time foam is not enough to contradict our assertion. The authors of \[26\] compare the situation with the Cherenkov effect, but precisely in this case the decisive ingredient is how photons see the medium and whether their critical speed becomes lower than that of the electron.

In the application of Liouville strings considered here, particles have energies well below Planck scale. Then, the virtual photon is comparatively long-lived and will be emitted in the quantum-gravitational medium even if the bare charged particle does not see such a medium. Therefore, it does not seem that spontaneous electron decay may be inhibited if it is kinematically allowed. Furthermore, the charged particle can always absorb a virtual photon from an external electromagnetic field and in this case photon emission becomes a scattering which is obviously not forbidden.

### 3.2 LDRK with intermediate energy thresholds

But, as for QDRK, it is possible to explore scenarios where $\Delta E$ is not power-like between the Planck scale and the particle mass scales. Again, we can write: $\Delta E = - p c \beta (k a)/2$ where $k'$ has the same definition as before. We then get for $k \ll k_0$:

$$\Delta E = - p c \beta k^2 a k_0^{-1}/4$$  \[15\]
At scales below $k_0$, one has a form of QDRK whose exact predictions will depend on the value of $\alpha_L = \beta (k_0 a)^{-1}$. If $k_0$ corresponds to an energy scale above the $10^{20}$ eV region, then $\alpha_L \approx 10^{-6}$ or larger can account for the possible suppression of the GZK cutoff. If $a$ is the Planck length and $\beta \approx 1$, a value of $\alpha_L$ between $10^{-6}$ and 1 implies the $E_0$ scale to lie between $\approx 10^{22}$ eV and the Planck scale. This interval clearly includes the grand unification scale. Above $E_0$, we recover the LDRK expression (8) up to non-leading terms.

In this way, it may be possible to make QDRK phenomenology compatible with an initial LDRK pattern generated at the Planck scale.

As for QDRK, it is also possible to explore LDRK models of the form:

$$\Delta E = - p c \beta (k a)^{\sigma'} (k a)^{\tau'} / 2$$

with $\sigma' + \tau' = 1$, $\sigma'$ and $\tau'$ being real and positive. Then, below $E_0$, one could explore hybrid scenarios between QDRK and LDRK.

4 Strong Doubly Special Relativity (SDSR)

As previously quoted, other authors [10, 11] require that the laws of physics be exactly the same in all inertial frames (SDSR).

We follow here the papers [32] by Amelino Camelia et al., and refer to the papers quoted in this article and in [10, 11]. These authors use the following deformed dispersion relation ($\lambda$ being the deformation parameter) in a two-dimensional space-time:

$$0 = \frac{2}{\lambda^2} [\cosh(\lambda E) - \cosh(\lambda m)] - p^2 e^{\lambda E} \simeq E^2 - p^2 - m^2 - \lambda E p^2$$

which can indeed be valid in all inertial frames at the cost of a $\lambda$-dependent deformation of the boost transformations (SDSR), but can also be interpreted as being valid only in a preferred reference system (WDSR). Amelino-Camelia et al. emphasize that this formulation of SDSR is not compatible with standard energy-momentum conservation. To show this incompatibility, they use the dependence of energy-momentum on the rapidity parameter $\xi$:

$$\cosh(\xi) = \frac{e^{\lambda E} - \cosh(\lambda m)}{\sinh(\lambda m)}, \quad \sinh(\xi) = \frac{\lambda p e^{\lambda E}}{\sinh(\lambda m)},$$

where $\xi$ here is the amount of rapidity needed to take a particle from its rest frame ($E = m$, $p = 0$) to a frame in which its energy is $E$ and its momentum is $p(E)$ from the dispersion relation (17). For $\lambda \to 0$, one gets the standard special relativistic relations:

$$\cosh(\xi) = \frac{E}{m}, \quad \sinh(\xi) = \frac{p}{m}.$$  

Amelino-Camelia et al. check that, if one is to enforce the standard additive law of energy-momentum conservation in a framework where (17) and (18) hold in all inertial frames, such a law can only hold in one inertial frame. They conclude that this cannot be the SDSR law of energy-momentum conservation and, referring to previous authors, propose a conservation law whose form to first order in $\lambda$ is:

$$E_a + E_b - \lambda p_a p_b - E_c - E_d + \lambda p_c p_d = 0,$$
\[ p_a + p_b - \lambda(E_a p_b + E_b p_a) - p_c - p_d + \lambda(E_c p_d + E_d p_c) = 0 \]  

(21)

Such a conservation law means in particular that energy and momentum are not additive. Therefore, we are not dealing with free particles strictly speaking, as the deformation generates an effective interaction between the "free particle" and the other particles in the Universe. Referring to [33], Smolin [11] describes this situation by saying that "while it is the covariant energy and momentum which are observed, it is the contravariant 4-vectors which are conserved linearly".

The fact that the particles under consideration are not really free raises the question of whether they all see the same space-time properties as they propagate in a SDSR frame. Furthermore, how do we define the particle velocity? It has been recognized [34] that there is a spectator problem in such an approach, as any particle in the Universe interacts with all the other particles and the Hamiltonian involves kinematical terms relating each single particle to the rest of existing matter. But then, velocity should be defined in terms of the global Hamiltonian and not of the formal Hamiltonian of an isolated particle which has no real physical meaning. To date, there seems to be no clear solution to this possible source of fundamental inconsistency.

Actually, to consistently define the velocity of a single free particle, one must be able to separate its individual Hamiltonian from that of the rest of the world. This does not seem to be possible in SDSR. By willing to preserve the strict universality of the laws of Nature in all inertial frames, an old fundamental principle is abandoned: that of the separability of a free particle from the rest of matter. It thus seems impossible to determine the velocity of a single "free" particle without knowing the existence and motion of all matter in the Universe. The matter motion and distribution observed will in principle depend on the inertial frame, so that SDSR may naturally generate its own breaking in the real universe. Perhaps this is a strong indication that, in the real world, no consistent departure from the standard Lorentz symmetry can afford itself working without a preferred inertial frame.

It is well known that there is in SDSR another possible conservation law, obtained defining the physical energy and momentum \( E' \) and \( p' \) such that:

\[
\frac{E'}{m} = \frac{e^{\lambda E} - \cosh (\lambda m)}{\sinh (\lambda m)}, \quad \frac{p'}{m} = \frac{\lambda p e^{\lambda E}}{\sinh (\lambda m)},
\]

(22)

so that \( E' \) and \( p' \) are additive and conserved. With these two variables and standard definitions of space and time, we readily recover special relativity. This may raise a conceptual puzzle for SDSR time of travel tests. If each inertial SDSR reference frame can be related to a (non physical) standard inertial frame of special relativity (the SR frame), two photons of different energies emitted simultaneously and at the same position in the SR frame will follow identical paths in a relevant system of local SR frames and arrive simultaneously to the detector in its local SR rest frame. Then, a macroscopic difference in arrival time cannot be generated by the local transformation between the SR and the SDSR frame at the detector position.

The requirement that the laws of physics be exactly the same in all inertial frames naturally implies that most usually suggested tests of LSV are not suitable for the "strict" interpretation of DSR advocated by Amelino-Camelia et al. [32] and other authors. According to their study, the possible tests of SDSR through observable effects seem to be essentially those based on time-of-travel experiments (but there may be a nontrivial problem in determining the speed of a "free" particle). This result is to be compared with our previous analysis [8, 9], where we pointed out that actually the Kirzhnits-Chechin ansatz (KCh) was not able to reproduce the GZK cutoff. As previously stressed, the KCh model is nothing but a form of SDSR, where it is required that the Finsler space law replaces special relativity in all inertial frames. Thus, more recent references generalize our 2002 result.
4.1 SDSR with intermediate energy thresholds

No basic principle prevents patterns of the SDSR type from presenting the same kind of intermediate energy thresholds considered in subsections 2.1 and 3.1. Then, the absence of a measurable effect in time of travel experiments would not necessarily provide a clear way to falsify SDSR. But if the effect is found and turns out to have no natural explanation, it may provide a serious evidence for SDSR, as the WDSR pattern would lead to many unwanted phenomena in this case.

5 On models involving superluminal particles

The idea that conventional Lorentz symmetry could be only an approximate property of equations describing a sector of matter, and that it would be broken at very high energy and short distance, was already put forward in our 1995-96 papers on superluminal particles [35]. This kind of Lorentz symmetry breaking, due to the mixing with superluminal sectors, had then to be a general property of the equations of the "ordinary" sector of matter, including propagators and dispersion relations deformed by the Lorentz breaking mixing.

In [35] it was also pointed out that superluminal particles with positive mass and energy (super-bradyons) must necessarily emit "Cherenkov" radiation, i.e. particles with lower critical speed in vacuum. This was the basic property used later in [36] to claim bounds on models of the $TH\epsilon\mu$ type with non-universal critical speed in vacuum. Similarly, in [35] we already suggested scenarios of Lorentz symmetry Violation, with superluminal particles, allowing to escape the GZK cutoff.

In [37], we also considered for the first time: a) models breaking simultaneously Lorentz symmetry and the symmetry between particles and antiparticles, as well as the possibility that this mechanism explains the difference between matter and antimatter in the Universe; b) specific DRK patterns generated by the mixing with superluminal particles. More recent papers are [38].

As an example, the scale corresponding to the rest energy of a superluminal particle (or a family of them) can naturally set an intermediate energy scale for the energy dependence of the deformation term in DRK patterns. More involved mechanisms can also be considered.

6 Conclusion

The AUGER experiment alone cannot completely settle the most crucial issues of UHECR phenomenology, and must be completed by UHECR space experiments. These experiments should not only be sensitive to the highest possible cosmic-ray energies but must have at the same time an energy threshold as low as possible. This second requirement reflects the need to understand as well as possible the first cosmic-ray interactions in the atmosphere, as well as the beginning of cascade development. The $\pi^0$ lifetime at UHE, for instance, is a crucial parameter.

The question of whether a vacuum rest frame exists remains to be answered by experiment. The absence of the GZK cutoff would be a clear indication against SDSR and in favour of WDRS with a vacuum rest frame, unless a more conventional explanation could be found.

If the GZK cutoff is found, this will not rule out all possible WDSR patterns, and models with an intermediate energy scale above $\approx 10^{20}$ eV will have to be explored and checked.
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