Precision computation of the leptonic $D_s$-meson decay constant in quenched QCD

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We summarize a computation of the leptonic decay constant $F_{D_s}$ of the $D_s$-meson in quenched QCD on the lattice. We perform a direct simulation at the masses of the strange and the charm quarks at four different lattice spacings from approximately 0.1 fm to 0.05 fm. Fully non perturbative $O(a)$-improvement is employed. After taking the continuum limit we arrive at a value of $F_{D_s} = 252(9)$ MeV, when setting the scale with the Kaon decay constant $F_K = 160$ MeV. Setting the scale with the nucleon mass instead leads to a decrease of about 20 MeV of $F_{D_s}$.

1 Introduction

To get reliable estimates of weak decay constants like $F_B$ lattice QCD has often to be supplemented by chiral extrapolations and/or heavy quark effective theory. These introduce substantial systematic errors [1]. In addition, the usual lattice errors like statistical errors, discretization errors, finite volume effects, contamination from excited states, perturbative renormalization and quenching [2] have to be understood or eliminated.

The $D_s$-meson however is special in this context. It consists of a strange and a charm quark. Both can be implemented precisely directly on the lattice as has been done recently by the ALPHA collaboration [3,4]. Thus neither a chiral extrapolation nor an extrapolation to heavy quarks (or similar strategies) have to be used. Still we expect the $D_s$-meson to be similar to other heavy light systems. Therefore it can be studied to understand all the other error sources mentioned above.

The goal of our work, which has been recently published in [5], is a precision computation of $F_{D_s}$ in quenched QCD. We aim at a combined error of three percent. To eliminate the discretization errors, we perform fully non perturbative $O(a)$-improvement and simulate at four different lattice spacings. This allows us to take a reliable continuum limit. Finite volume effects have been shown to be negligible in [3]. At our masses they are expected to be even smaller. We define our plateaus such that the contamination by excited states stays below five per mille. Finally all the uncertainty due to perturbative renormalization has been eliminated by using the non perturbative renormalization techniques of the ALPHA collaboration [6,7]. The only systematic uncertainty we cannot deal with at present is the quenching error.

A precise value of $F_{D_s}$ in quenched QCD is desirable since together with a computation of the decay constant in the static approximation it will enable us to see how far the heavy quark effective theory can be applied safely. In unquenched simulations we can then rely on this experience in the computation of $F_B$.

The $D_s$-meson is stable in QCD. It decays weakly by an emission of a $W$-boson into a lepton and a neutrino. The branching ratios can be measured experimentally. They are summarized in [8].

Recent experimental data are shown in table 1 [9]. The status of lattice computations was reviewed in [10].

| Experiment | $F_{D_s}$ [MeV] |
|------------|-----------------|
| ALEPH      | 285 ± 19 ± 40   |
| DELPHI     | 330 ± 95        |
| L3         | 309 ± 58 ± 50   |
| CLEO       | 280 ± 17 ± 42   |
| BEATRICE   | 323 ± 44 ± 36   |
| E653       | 194 ± 35 ± 24   |

Table 1. Experimental data for $F_{D_s}$.
For unexplained notation we refer to [20]. We define conditions [18, 19] on a lattice QCD using Schrödinger functional boundary theorem [15]. Here boundary, respectively. From these we compute the physical conditions are constant to a sufficient precision.

| Table 2. Lattice results for $F_{D_s}$ in quenched QCD. |
|---------------------------------------------|
| $F_{D_s}[\text{MeV}]$                      |
|-----------------------------|
| UKQCD [11]                  | 229(3) + 23 − 12               |
| APE [12]                    | 234(9) + 5 − 0                 |
| MILC [13]                   | 223(5) + 19 − 17               |

2 Strategy

The decay constant $F_{D_s}$ is defined by the QCD matrix element

$$\langle 0| A_\mu(0)| D_s(p) \rangle = i p_\mu F_{D_s}$$  \hspace{1cm} (2)

of the axial current $A_\mu = \bar{s}\gamma_\mu\gamma_5 c$. To formulate this problem on the lattice we eliminate the bare parameters of the QCD Lagragian in favour of physical observables in one chosen hadronic scheme. Our strategy [14, 3, 4] is to use the kaon decay constant $F_K$ to set the scale, that means to compute the lattice spacing $a$ in physical units as a function of the bare coupling $g_0$. The bare strange quark mass and the bare charm quark mass are eliminated by the masses $m_K$ and $m_D$, of the kaon and the $D_s$-meson, respectively. We neglect isospin breaking and take the quark with flavour indices $i$.

The decay constant

$$\langle O \rangle = \frac{1}{Z_A (t)} A_\mu(x) A_\nu(x) \langle 0 \rangle$$

is defined by the QCD matrix elements. The contribution

$$f_A(x) = \frac{1}{2} \langle O A_0(x) \rangle$$

is expected to exhibit a plateau at intermediate times when the contribution $\eta_A^0 e^{-x_0 \Delta}$ of the first excited state and the contribution $\eta_A^0 e^{-(T-x_0) m_G}$ from the $O^+$ glueball both are small. A plateau average can then be performed to increase the signal and is understood in [21]. Further explanations for equation (7) and details can be found in [21].

4 Parameters

We discretize the space-time cylinder using four different lattice spacings $a$ but keep $L$ and $T = 2L$ approximately constant in physical units. To this end we use the same bare couplings that have been used in the determinations of the strange and the charm quark masses in [3, 4] by the ALPHA collaboration. From this work we also take the hopping parameters for the quarks. Our choice of parameters is shown in table 3. In table 4 we show that with this choice indeed the correlation functions

$$f_1(x_0) = -\frac{1}{2} \langle O A_0(x) \rangle$$

and

$$f_1 = -\frac{1}{3 L^5} \langle O' O \rangle.$$  \hspace{1cm} (6)

Here $A_\mu(x)$ denotes the improved axial current. It receives a scale independent multiplicative renormalization $Z_A$ on the lattice. In terms of these correlation functions $F_{D_s}$ can be written as

$$F_{D_s} = -2Z_A (1 + b_A (am_{q,i} + am_{q,j})) / 2$$

$$\times \frac{f_A^I (x_0)}{\sqrt{f_1}} (m_{D_s} L^3)^{-1/2} e^{-x_0 - T / 2} m_{D_s}$$

and

$$\times \left\{ 1 - \eta_A^1 e^{-x_0 \Delta} - \eta_A^0 e^{-(T-x_0) m_G} \right\}$$

$$+ O(a^2).$$  \hspace{1cm} (7)

Here the factor $(m_{D_s} L^3)^{-1/2}$ takes into account the normalization of one particle states. The contribution $f_1^{-1/2}$ cancels out the dependence on the meson sources. Because of the exponential decay of the correlation function $f_A^I$ the product in (7) is expected to exhibit a plateau at intermediate times when the contribution $\eta_A^0 e^{-x_0 \Delta}$ of the first excited state and the contribution $\eta_A^0 e^{-(T-x_0) m_G}$ from the $O^+$ glueball both are small. A plateau average can then be performed to increase the signal and is understood in [21]. Further explanations for equation (7) and details can be found in [21].
5 Computation of the decay constant

To compute the decay constant $F_{D_s}$ we use the combination of correlation functions. For all parameter choices we find plateaus as functions of $x_0$. These plateaus are shown in figure 1. Their extent is roughly from $4r_0$ to $5r_0$. At small respectively large times we fit to the functions $F_{D_s}(x_0)$ the expected contributions of the first excited state and the glueball. For details see [5]. We define the plateau such that their sum stays below 5 per mille.

Since we deal with heavy quark propagators on APE1000 in single precision we have to check that the rounding errors are small enough. Our check against runs with a double precision code [22] reveals that the rounding errors are smaller than one per mille.

After averaging $F_{D_s}$ over the plateaus defined above we get the values shown in table 5. These data can be extrapolated to the continuum limit. Here we leave out the coarsest lattice. The extrapolation can be performed in $(a/r_0)^2$ since we employ non perturbative $O(a)$ improvement. Here we take $b_A$ from the Los Alamos group [23]. Since this involves an extrapolation of their data we have also used 1-loop perturbation theory [24]. Then we get $r_0F_{D_s} = 0.631(24)$, which is in perfect agreement with our main result, $r_0F_{D_s} = 0.638(24)$, or, using $r_0 = 0.5$ fm, $F_{D_s} = 252(9)$ MeV. The continuum extrapolation is shown in figure 2.

Table 4. Demonstration of constant physical conditions.

| $\beta$ | $L/r_0$ | $r_0m_{D_s}$ |
|---------|---------|--------------|
| 6.0     | 2.98    | 4.972(22)    |
| 6.1     | 3.79    | 4.981(23)    |
| 6.2     | 3.26    | 5.000(25)    |
| 6.45    | 3.06    | 5.042(29)    |

Table 5. Simulations results and continuum limit for $F_{D_s}$.

| $\beta$ | $r_0F_{D_s}$ |
|---------|--------------|
| 6.0     | 0.540(14)    |
| 6.1     | 0.570(13)    |
| 6.2     | 0.598(16)    |
| 6.45    | 0.614(15)    |
| c.l.    | 0.638(24)    |

Figure 1. Behaviour of $aF_{D_s}^{\text{bare}}$. The full symbols denote the plateau range.

Figure 2. Continuum extrapolation of $F_{D_s}$.

6 The quenched scale ambiguity

To estimate the quenched scale ambiguity of $F_{D_s}$ under a scale shift of 10 percent, which is typical for the quenched approximation, we consider $r_0F_{D_s} = f(z)$ as a function of the meson mass $z = r_0m_{D_s}$. We expand $f(z)$ around the physical value $z_0 = 4.988$ up to first order. A 10 percent increase of $r_0$ corresponds to $z - z_0 = 0.5$. With an estimate of $f'(z_0)$ from a linear fit of $r_0F_{D_s}$ around $m_{D_s}$ we get $f(z) - f(z_0) \approx 0.008(3)$. Converting back to physical units, now using $r_0 = 0.55$ fm we find that $F_{D_s}$ decreases by 20 MeV, corresponding to eight percent. The estimate of $f'(z_0)$ is possible since in addition to the hopping parameters already discussed we have also performed simulations around the charm quark mass value.

7 Conclusion

The leptonic $D_s$-meson decays can be studied on the lattice without chiral extrapolations or heavy quark
effective theory. This has enabled us to perform a computation of $F_{Ds}$ with a precision that matches the precision goals at future experiments, for example, CLEO. The precise value of $F_{Ds}$ in quenched QCD, together with new precise data in the static approximation, will show how far heavy quark effective theory can be applied safely. This is of importance for the unquenched computation of $F_B$ in the future.

We will supplement this analysis with more data around the charm mass. Part of this data has already been used to estimate the quenched scale ambiguity of $F_{Ds}$ under a scale shift of 10 percent.

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