Minijet production and transverse energy are important not only to understand hadronic collisions, but also for the interpretation of nucleus collisions at RHIC and LHC, where it determines the “initial conditions” for the flow in a hadronic soup or a plasma. For high collision energies and small $q_\perp$ (minijets) we enter the BFKL region. This implies that we must take into account off-shell parton cross sections and non-integrated structure functions ($k_\perp$-factorization). It is also essential to avoid double counting, as one emitted parton is a participant in two different subcollisions. The LDC model, developed in Lund to describe DIS, provides a very convenient formalism to handle these problems. The result is a dynamical suppression of minijets for small $q_\perp$. The resulting $E_\perp$-flow is similar to the result from a “naive” calculation based on integrated structure functions with a $q_\perp$ cut-off around 2 GeV.

1 Introduction

In hadronic collisions, jets with large $q_\perp$ can be described by the expression

$$\frac{d\sigma}{dq_\perp^2} \sim F(x_a, q_\perp^2)F(x_b, q_\perp^2)\frac{\alpha_s^2(q_\perp^2)}{q_\perp^4}.$$  (1)

The structure functions $F(x, q_\perp^2)$ can here be described by DGLAP evolution; the initial parton cascades are ordered in $q_\perp$ towards the hard subcollision. The cross section for quasireal partons is approximately $\sim \alpha_s^2(q_\perp^2)/q_\perp^4$. For smaller $q_\perp$, minijets, we enter the BFKL domain, with important contributions from non-ordered parton chains. The expression in eq. (1) blows up for small $q_\perp$, and in phenomenological applications a cutoff around 2 GeV is needed to reproduce experimental data. This phenomenological cutoff appears to grow with energy, which makes it difficult to make predictions for very high energies, e.g. at the LHC. Thus we have two essential problems:

1. For non-ordered chains we get contributions from scattering of off-shell partons with $k_{\perp a}$ and/or $k_{\perp b} > q_\perp$, for which the cross section has to be modified. From $k_\perp$-factorization, this problem can be handled by non-integrated structure functions $F(x, k^2_\perp)$ and off-shell parton-parton cross sections.

2. In each chain as in fig. 1 there are many final state partons (minijets). Each final parton is connected to two links, which implies that it is a component in two different subcollisions. Therefore, we must be cautious to avoid double counting.
Both these problems can be conveniently treated within the Linked Dipole Chain (LDC) model. This model interpolates between DGLAP and BFKL. In a single formalism it describes different types of reactions in DIS: “Normal DIS”, boson-gluon fusion, and hard resolved $\gamma p$ scattering. When applied to $hh$ or $AA$ collisions it implies that the inclusive jet cross section is reduced for smaller $q_\perp$ cf. to the expression in eq. (1). The total $E_\perp$ flow is similar to the “naive” result if this has a low $q_\perp$ cut-off $\sim 2$ GeV.

2 DIS

When $Q^2$ is large and $x$ not too small (in the DGLAP region) the dominant contribution to $F_2$ is given by gluon chains, which are ordered in $x$ (or $q_L$) and in $k_\perp$. Each chain as in fig. 1 gives a contribution

$$\prod_i^n \frac{dx_i}{x_i} \frac{dq_{\perp,i}^2}{q_{\perp,i}^2}$$

where $\alpha \equiv 3\alpha_s \equiv \frac{\alpha_0}{\pi \ln(q_{\perp}^2)}$.

(2)

Summing over different values of $n$ gives the double leading log result $F \sim \exp(2\sqrt{\alpha_0 \ln 1/x \ln Q^2})$. The gluons in such a chain constitute the initial state radiation. To obtain the properties of the complete final state, we must add final state radiation within angular ordered regions.

When $Q^2$ is moderate and $x$ small (in the BFKL region), the $k_\perp$-ordered region is small, and non-ordered contributions are important although suppressed. The result is a power-like increase of $F$ for small $x$, $F \sim x^{-\lambda}$.

In the interpolation region we must calculate suppressed contributions from non-ordered chains. It is then necessary to specify the separation between initial state radiation and final state radiation. This is not given by Nature; it has to be defined by the calculation scheme. A particular scheme was chosen by Ciafaloni, Catani, Fiorani, and Marchesini (the CCFM model). Here the initial state radiation is ordered in angle (or rapidity) and energy (or $q_+ = q_0 + q_L$). The contribution from each chain is then determined by specific non-Sudakov form factors.

2.1 The Linked Dipole Chain model

The Linked Dipole Chain (LDC) model is a reformulation and generalization of the CCFM result in a scheme, where more gluons are treated as final state radiation. The initial chain is ordered in $q_+ = q_0 + q_L$ and $q_- = q_0 - q_L$, and $q_\perp$ satisfies $q_\perp > \min(k_{\perp,i},k_{\perp,i-1})$. This implies that there are fewer chains. One LDC chain corresponds to a set of CCFM chains. It then turns out that all the corresponding non-Sudakov form factors add up to just unity.
Figure 1. Fan diagrams in hh collisions and in DIS. In the LDC model the initial state emissions \( q_i \) form a chain in the \((y, \kappa = \ln(k_\perp^2))\)-plane. Final state radiation is allowed in the region below the horizontal lines.

Thus the contribution from each such chain is given by an expression identical to eq. (2). We note in particular that this expression is totally left-right symmetric, meaning that we get the same result if we start the chain in the photon end, instead of in the proton end.

We can express this result in the link momenta \( k_i \), instead of the final state momenta \( q_i \). Using the relations \( d^2q_\perp^2,i \sim d^2k_\perp^2,i \) and \( q_\perp^2,i \approx \max(k_\perp^2,i, k_\perp^2,i-1) \) we find the following weights

\[
\frac{d^2q_\perp^2,i}{q_\perp^2,i} \sim \frac{d^2k_\perp^2,i}{k_\perp^2,i} \quad \text{for} \quad k_\perp^2,i > k_\perp^2,i-1; \quad \frac{d^2q_\perp^2,i}{q_\perp^2,i} \sim \frac{d^2k_\perp^2,i}{k_\perp^2,i} \cdot \frac{k_\perp^2,i}{k_\perp^2,i-1} \quad \text{for} \quad k_\perp^2,i < k_\perp^2,i-1.
\]

(3)

Thus, for a step down in \( k_\perp \) we have an extra suppression factor \( k_\perp^2,i/k_\perp^2,i-1 \). This implies that if the chain goes up to \( k_{\perp,\text{max}} \) and then down to \( k_{\perp,\text{final}} \) we obtain a factor \( 1/k_{\perp,\text{max}}^4 \), which corresponds to the cross section for a hard parton-parton subcollision. An important feature is that different types of reactions can be described in the same formalism. Thus “normal” DIS corresponds to the case when \( k_{\perp,\text{final}}^2 < Q^2 \), boson-gluon fusion to \( k_{\perp,\text{final}}^2 > Q^2 \), while resolved \( \gamma p \) scattering is obtained when \( k_{\perp,\text{max}}^2 > k_{\perp,\text{final}}^2(> Q^2) \).

MC programs have been developed for both the CCFM model and the LDC model and compared to HERA data. Programs for the two models can fit \( F_2 \) and essential properties of the final states. A still puzzling feature is
that both programs can fit the forward jets only provided the nonsingular terms in the splitting functions are suppressed.

A running coupling \( \alpha_s \) favours small \( k_{\perp} \) in the chain. This implies that the structure function factorizes for small \( x \)-values. For the results in the next section we use the expression \( F \sim x^{-0.3} (\ln k_{\perp})^2 \) which is motivated by the MC fit to HERA data.

3 Jet Cross Section in Hadronic Collisions

The results presented in this section are obtained in collaboration with Gabriela Miu. For a resolved \( \gamma p \) scattering the symmetry of the formalism implies that it can be interpreted as evolution from both ends toward a central hard scattering. This obviously works also for pp or AA scattering. Study a single link with notation as in fig. 1. There are three different possibilities:

1. \( k_{\perp} > k_{\perp,a}, k_{\perp,b} \Rightarrow q_{\perp,a} \approx q_{\perp,b} \approx k_{\perp} \) which has a weight \( k_{\perp}^{-4} \).

2. \( k_{\perp,a} > k_{\perp}, k_{\perp,b} \Rightarrow q_{\perp,a} = k_{\perp,a}, q_{\perp,b} = k_{\perp} \), with weight \( k_{\perp}^{-2} \cdot k_{\perp}^{-2} \).

3. \( k_{\perp} < k_{\perp,a}, k_{\perp,b} \Rightarrow q_{\perp,a} = k_{\perp,a}, q_{\perp,b} = k_{\perp,b} \), with weight \( k_{\perp}^{-2} \cdot k_{\perp}^{-2} \).

In each case we have also a factor \( \alpha_s(q_{\perp,a}^2) \cdot \alpha_s(q_{\perp,b}^2) \). We note that there is a suppression when the virtuality of the colliding particles \( (k_{\perp,a}^2 \) and \( k_{\perp,b}^2 \) are larger than the momentum transfer \( k_{\perp}^2 \). This implies that \( \sigma_{\text{incl}} \) does not blow up for small \( q_{\perp} \)-values. The inclusive cross section can be expressed in terms of the non-integrated structure functions \( F(x, k_{\perp}^2) \).

\[
\frac{d\sigma_{\text{incl}}}{dq_{\perp}^2 dy} \sim \int F(x_a, k_{\perp,a}^2) \cdot F(x_b, k_{\perp,b}^2) \cdot \frac{1}{2} \cdot \frac{d\tilde{\sigma}}{dq_{\perp}^2}(q_{\perp}^2, k_{\perp,a}^2, k_{\perp,b}^2, \hat{s}),
\]

where \( \hat{s} = x_a x_b s \) and \( y = \frac{1}{2} \ln(x_a/x_b) \). After integration over \( k_{\perp,a}, k_{\perp,b}, x_a, \) and \( x_b \), the result can be written in the form (the factor \( 1/q_{\perp}^{2\lambda} \) originates from the \( x \)-dependence of \( F \sim x^{-\lambda} \))

\[
\frac{d\sigma_{\text{incl}}}{dq_{\perp}^2 dy} \propto \frac{s^\lambda}{q_{\perp}^{4+2\lambda}} \cdot \alpha_s^2(q_{\perp}^2) \cdot h(q_{\perp}^2).
\]

Each outgoing parton is connected to two links, and therefore counted in two subcollisions in fig. 1. Therefore, to avoid double counting, we must include an extra factor \( 1/2 \) in eq. (4). We can compare with the result of the “naive” approach in eq. (1), and define a corresponding function \( h_{\text{naive}} \). (We note that the functions \( h(q_{\perp}^2) \) are defined in such a way that scale independent structure functions \( F(x) \) in eq. (1) would correspond to \( h_{\text{naive}} = \text{constant} \).

As discussed in more detail in the expression in eq. (4) corresponds to two jets for each link in the chain. This double counting does, however, not give
Figure 2. A parton chain in the \((y, \ln q^2_\perp)\) plane. The emitted partons are marked by dots. The “naive” approach corresponds to an outgoing parton for each circle.

Figure 3. The function \(h(q^2_\perp)\) in eq. (5) (arbitrary scale), and the ratio \(h/h_{\text{naive}}\).

Figure 4. The \(E_\perp\) distribution, \(dE_\perp/dy dq^2_\perp\) (solid), compared to the “naive” estimate in eq. (1) (dashed). For comparison we also show the result obtained for scale-independent structure functions (dotted). Also shown is the integrated \(E_\perp\) distribution, \(\int q^2_\perp dq^2_\perp dE_\perp/dy dq^2_\perp\) (arbitrary scales).
a factor of 2 in the jet cross section, because the jets are expected to have transverse momenta given by the $k_\perp$ of the link, also when this is smaller than the $k_\perp a$ or $k_\perp b$ of the colliding partons. Therefore the distribution of hard jets becomes approximately correct, but the distribution of softer jets becomes strongly overestimated. This is illustrated in fig. 3.

As seen in fig. 3, $h(q_\perp^2)$ is much reduced cf. to $h_{\text{naive}}$ for small $q_\perp$. This is also seen in the $E_\perp$-flow presented in fig. 4. From the integrated $E_\perp$-distribution we see that the difference in total $E_\perp$ between our and the “naive” result corresponds roughly to the $E_\perp$ flow below 2 GeV in the “naive” approach. Consequently the total $E_\perp$ flow in our approach corresponds to the “naive” result, if the latter had a low $q_\perp$ cut-off around 2 GeV.

4 Summary

The Linked Dipole Chain model, developed for high energy DIS, is also applicable to hard subcollisions in hh, hA or AA collisions. The inclusive jet cross section is expressed in terms of non-integrated structure functions ($k_\perp$-factorization). The formalism also automatically avoids double counting for long parton chains. The result gives a dynamical suppression for small $q_\perp$, which corresponds to an effective cut-off around 2 GeV in a “naive” approach in terms of integrated structure functions.

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