Analysis of numerical simulations on triaxial compression tests using different constitutive models of the soil behaviour

M C Olarte¹, and J C Ruge¹
¹ Universidad Militar Nueva Granada, Bogotá, Colombia
E-mail: juan.ruge@unimilitar.edu.co

Abstract. To reproduce in an accurate way by artificial methods the real behaviour of the soil is extremely complicated, given the intrinsic heterogeneity of the soil and the innumerable external factors that affect its mechanical-hydraulic response. At present, there are several constitutive models, both traditional and modern, that attempt to simulate the possible stress paths that a geomaterial can undergoes. Within the practical engineering the most used model has been Mohr-Coulomb, which shows an elastic-perfectly plastic response. However, the soil is far from exhibiting a behaviour similar to that shown in this model. Modern constitutive laws can simulate in a more appropriate way the non-linear behaviour of the soil. However, they use several parameters that increase the complexity of the models. The objective of these constitutive models is none other than to extrapolate its mathematical formulation to the global behaviour of geotechnical structures. Before adjusting the parameters that should be used in the numerical modelling of real geotechnical problems, it is necessary to understand the behaviour at the elementary level (experimental) of the constitutive model. Normally this process is called calibration of parameters. In this work, the numerical results of triaxial compression tests for a typical normally consolidated soil are shown. Various constitutive models are used, which reveal different stress paths, for which the dispersion in the obtained results is considered.

1. Introduction
In previous times there was no possibility to implement numerically constitutive models to reproduce the behavior of the soil, so the theories used at the beginning for geomaterials were borrowed from other types of materials that do have a behavior with an elastic and plastic range effectively accented, which is not typical of a soil. For this reason, and to this day, traditional models for soils still keep characters of these models. Even today, whether by custom or traditionalism, geotechnical engineers still use in the first modelling the elastic law known as Hooke’s law, knowing that it is not the most adequate model to simulate the physical response of geomaterials. The simplest model to simulate the behavior of the soil continues to be Mohr-Coulomb, which shows an elastoplastic behavior often used to obtain a general behavior of the soil. This it is considered a first-order model [1]. Currently, there is a series of modern constitutive models capable of appropriately resemble soil behaviour, even with specific extensions depending on the nature of the soil to be simulated, i.e. peculiar characteristics such as cementation, partial saturation, expansivity, etc. Virtual simulations of laboratory tests using element-test programs have become important in the last decade due to its versatility to adjust the modeling to the 'real' behavior of the soil.

Current research covers a broad spectrum of constitutive models generally used in consulting and research activities in the geotechnical environment. Utilitarian software specialized in the artificial simulation of laboratory tests was used with the possibility of modifying and controlling different stages
during the test. Due to the nature itself of how the different models were conceived, there is a dispersion of the results obtained, which is analyzed in qualitative terms considering the primary factors that govern the mathematical formulation and physical meaning of each one of the constitutive parameters.

2. Background

The majority of constitutive models have been developed according to precepts already established from the state of the art of the subject, that is to say, practical terms, there is already a way taken in terms of the generation of models. In the mechanics of soils, it is essential to use realistic constitutive models that simulate the stress-strain behaviour correctly under various load conditions. These models must be generated through specialized laboratory tests and with the use of mathematical methodologies accompanied by consistent theoretical bases, such as elasticity and plasticity. If the model is capable of simulating soil response with these specialized tests, parameter calibration should not be necessary. However, access to this type of trial is limited. The quality of the constitutive model also depends on soil parameters that can be obtained from more affordable trials, such as monotonic triaxial or oedometric tests. According to [2], there are some aspects that have controlled the improvement of the models, among them: effects of virgin loading and loading-unloading, confining stress, stress-history, instability, anisotropy and rotation of stresses. Simpler models can describe the primary aspects, however advanced models are required to resemble more complex phenomena, such as those mentioned.

Since Hooke's law was devised, most materials have gone through a numerical evaluation, taking into account this elastic model. The main characteristic of this law is that the history of stresses is impossible to simulate under this model and that to know the current value of the effort, it is imperative to know the equally updated value of the deformation or in mathematical terms: the stress is a function of the strain. Another important aspect is that the elastic materials do not reveal recoverable deformations, that is if the load solicitation is removed, the deformation occurred associated with the imposed load is completely reversed, this is clearly observed in Figure 1. Regarding the non-linear elastic behavior, the main distinction against an elastoplastic response is the reversibility of the deformations. In a non-linear elastic material, its original shape would return after a charge-discharge cycle (Figure 1). Among these models, the one from [3] stands out.

![Figure 1. Elastic, non-linear elastic, Elastoplastic without hardening, with hardening and hypoplasticity.](image)

In contrast, an elastoplastic material would undergo permanent deformations and the stress-strain space would show a hysteretic and ratcheting behaviour, i.e. the gradual accumulation of permanent deformation of soils subject to load cycles, especially in granular soils [4] (Figure 1). Particularly, elastoplasticity (EP) presents a presumption that characterizes these models. It is the decomposition of the total tension tensor in an elastic tensioner and a plastic tensioner. The main aspects that are related to the mathematical formulation of EP (at least in the plastic range) is the failure criterion, that is, a boundary between admissible and non-admissible stress states, in terms of deviatoric stress, mean stress; and the plastic potential, which is defined through the plastic potential gradient and the hardening law (in some models), which represents a realistic approximation of soils that are subjected to compression paths and they are strengthened versus the load solicitation [5]. Within this category we can find the models CC and CCM [6-11].
Modern non-linear incremental models such as the hypoplasticity proposed by [12] are accepted today as quite promising soil behaviour laws. The first developments were focused on granular materials. However, in recent years, different extensions have been included to describe fine grain soils. Especially, the contributions related to the influence of barotropy, picnotropy and response to small deformations [13]. These models consider a logarithmic compression law [14] and, it is also centered on the theory of critical state, the basis of modern models existing in the state-of-the-art (Figure 1).

3. Methodology

3.1. Basic characterization of the material
For the numerical simulation, real triaxial compression tests will be used for a collapsible porous clay soil. The stratigraphic profile of the site was identified by in-situ tests with very low resistance to penetration according to the SPT and CPT. The samples for the tests were taken at 6 m. of depth. Due to infiltration laterization processes, this soil has a structure that is susceptible to collapse when it presents wetting in a simultaneous manner to a load loading in the material. Although the objective of the research is totally numerical, it is important to know the index parameters of the material. In Table 1 some of the properties of the material are shown.

| Sample | Depth (m) | Gs | LL (%) | LP (%) | IP (%) | w (%) |
|--------|-----------|----|--------|--------|--------|-------|
| 1      | 3.00 – 3.25 | 2.65 | 56.4   | 31.2   | 25.2   | 35    |
| 2      | 6.00 – 6.30 | 2.66 | 59.8   | 34.6   | 25.2   | 32    |
| 3      | 8.70 – 9.00 | 2.63 | 66.8   | 36.4   | 30.4   | 30    |

These lateritic soils, which present clay micro-concretions in the presence of silt and sand sizes, can present identification problems generating false negatives in the results of the particle size distribution. That is, they show a granulometric behavior that is not typical of this clay soil. This is due to ligations formed by cementing minerals that form packages or clusters. To avoid this, tropical soils such as the one analyzed must be characterized in their granulometry with the addition of a dispersing agent to the test, as shown in Figure 2.

![Figure 2](image)

Figure 2. (a) Particle size distribution and (b) Pore size distribution (PSD).

Nowadays it is not only important to analyze the soil samples in relation to the particle size distribution, it is also necessary to study it from the poral point of view. This is because part of the mechanical-hydraulic response depends in the same way on the water that is mobilized, both in micropores and in macropores. Porosimetry by mercury injection allows defining the pore size
distribution (PSD). The technique is based on the intrusion of mercury in the pores of the soil, in order to know the pressures necessary to fill the pores. The pore size is inversely proportional to the induced pressure. Although in Figure 2 only the derivative of the accumulated pore volume is shown, the directly obtained parameter is the volume injected into the pores. The reflected peaks of the inflection points reveal the predominance of micropores and macropores in the sample.

3.2. Drained triaxial tests
In Figure 3 the stress-strain curves show an expected behavior, in relation to a proportional increase in the deviating stress with the confining stress. The invariants of Cambridge p'-q stresses were estimated, observing the critical state line.

![Figure 3. Results of drained triaxial tests.](image)

3.3. Calibration of parameters
After having analysed in the literature review a panorama of constitutive models from several points of view, it was decided to use five of these models to simulate the behaviour of the soil chosen in a conventional drained triaxial test, Mohr-Coulomb (MC), modified cam clay (CCM), Hypoplasticity (HP), hardening soil (HS) and soft soil (SS). Basically, the calibration of parameters was carried out with a trial and error procedure, considering a criterion of which parameters affected certain aspects in the graph, i.e. elastic branch, critical state, peak state, etc. The MC model has five parameters in its most known version (E, µ, C, f, y). According to the procedure proposed by [15], the Young's modulus was estimated from the secant module for 50% of the peak deviatoric stress. The internal friction angle and the cohesion were calibrated with the intention of representing the peak state in the results of the drained triaxial test (Table 2). The curves q vs. e were simulated with the parameters obtained according to the previous procedure, as can be seen in Figure 4.

To determine the parameters of the CCM model, the information of consolidation tests has to be known, in order to estimate the slopes of the virgin and the discharge line. Using these two slopes, the values of λ, γ, κ can be obtained, more details in [16]. For the software used it is necessary to only insert three additional parameters, µ, Poisson's ratio to download and recharge; M, the slope of the critical state line, obtained from the triaxial tests and e₀, the initial void ratio of the consolidation test. The version of hypoplasticity (HP) used in the research is based on the proposal of [17]. The model uses five parameters: N, λ, κ, φ₀, r (Table 2). The first three are obtained in a similar way to the CCM model by means of a one-dimensional consolidation test, but its mathematical formulation of the model differs in values, that is, they are not necessarily equal. This model has a creep surface called the State Boundary Surface (SBS) and its shape is related to λ, κ, φ₀ [18].

The parameter r can be found directly as the relationship between the bulk volumetric module and the shear module. However, a parametric study is recommended to rule out dependence on other parameters of the model [19]. The angle of friction of the critical state φ₀ is revealed from the main stresses in the critical state of the soil, most modern models use this parameter, since it is the theoretical
basis essential in the generation of constitutive models. A linear regression of critical state points is required to correctly estimate $\phi_c$.

The Hardening Soil (HS) model uses seven parameters, where the formulation on plasticity predominates and elasticity is not considered with depth. The parameter $m$ is recommended to be set at 1 in the case of soft soils, this value represents the dependence between stresses and deformations. $E_{\text{ref}}^{50}$ is determined as the slope of the projection of the elastic branch for 50% of the value of rupture of the material, this was the same criterion to find the Young's modulus in the MC. $E'_{\text{ur}}$ is the slope of the primary compression in a consolidation test. However, there are also correlations with $E'_{\text{ur}}$. The modules $E_{\text{ur}}$ simulate the discharge and recharge behavior, that is, the Young's module corresponding to a reference stress of 100 kPa, for this case (Table 2). It is usually appropriate to set $E_{\text{ur}} = 3E_{\text{ref}}^{50}$. The peak state is simulated with the plasticity parameters of the MC model. The parameters of the SS model, like CCM and HP, use compression and expansion (stiffness) indexes, naturally obtained from consolidation tests, typical of soft soils. It also appropriates the MC model to reproduce the failure. In Table 2 can see the parameters for all the models used.

| Table 2. Parameters of the five constitutive models used. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Model MC        | Model CCM       | Model HP        | Model HS        | Model SS        |
| Parameter       | Value           | Parameter       | Value           | Parameter       | Value           | Parameter       | Value           |
| $E$             | 11500 kPa       | $\lambda$       | 0.072           | $\lambda$       | 25.2           | $E_{\text{ref}}^{50}$ | 11500 kPa       |
| $\mu$           | 0.39            | $\kappa$        | 0.015           | $\kappa$        | 25.2           | $E_{\text{ref}}^\text{oed}$ | 9000 kPa       |
| $C$             | 26 kPa          | $\mu_{ur}$      | 0.390           | $N$             | 30.4           | $E'_{\text{ur}}$ | 34000 kPa       |
| $\phi$          | 20°             | $\phi_c$        | 39°             | $m$             | 1              | $\phi'$        | 20°             |
| $\psi$          | 0°              | $e_0$           | 0.850           | $r$             | 0.09           | $C'$           | 26 kPa          |
| $\psi$          | 0°              | $e_0$           | 0.850           | $r$             | 0.09           | $C'$           | 26 kPa          |
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| $\psi$          | 0°              | $e_0$           | 0.850           | $r$             | 0.09           | $C'$           | 26 kPa          |
| $\psi$          | 0°              | $e_0$           | 0.850           | $r$             | 0.09           | $C'$           | 26 kPa          |

4. Results

4.1. Numerical simulations of the triaxial drained compression test

Figure 4 shows the numerical simulations of the triaxial drained compression test (TCD) for the clay normally consolidated under study. In models such as MC, HS and SS, their stiffness parameters are based on different conceptions of Young's modulus. The strength is simulated from the peak state due to the cohesion and friction angle. For this reason, in these models an equal value was set for these parameters in the three models. It is important to note that the break occurs practically in the same values. However, the elastic part corresponding to the stiffness presents variations that are perceived in Figure 4 and that are consistently explained in the mathematical formulation of each of these models.

In the MC model, only one Young module is used for the elastic branch, so it is not capable of simulating the non-linear behaviour of the soil from the first stress levels. The HS model uses up to three different values of stiffness, simulating in a better way this elastic range of the real test. In the SS model, its stiffness is simulated by the slopes of the virgin branches and discharge of a consolidation test. For this reason, it shows some similarity with CCM and HP models that use a similar theoretical base. As for the models based on the critical state of the soil (CCM and HP), they are the ones that best simulate the non-linear behavior of the soil, even from the first deformations levels. The stiffness that is determined in both models from $\lambda$ and $\kappa$, show a quite similar behavior, since both values are obtained from the consolidation test and have some variation related to the model's own conception.
Concerning the way in which the models address the critical state, they are practically the same. They only reveal some variation in the test with the highest confinement stress value. This residual strength part of the actual test is not adequately simulated by these two types of tests. Probably, one of the reasons is that the soil has certain peculiarities that can be seen reflected in its last states, in this case it is a soil prone to experience collapsibility. These specificities are difficult to simulate using numerical tools.

![Numerical modelling of various constitutive models](image)

**Figure 4.** Numerical modelling of various constitutive models.

5. Conclusions

Indeed, the software type element-test is a valuable numerical tool that allows to correctly understand the behavior of the soil in an artificial way knowing the details of the models used, since it is necessary to be able to insert the parameters indicated in the simulation.

The process of trial and error implemented to adjust the parameters as much as possible to the real test, ends up being didactic, in the sense that the parametric study that is done by setting a value and varying the others, allows to understand the physical meaning and the appropriate reproduction of the phases simulated by parameters of stiffness or rupture of the material. As expected, the models based on a very accentuated elastic and plastic branch end up presenting a high dispersion because they are not able to describe the non-linear response of the soil, these models are MC, HS and SS. The models based on the critical state of the soil and with stiffness parameters based on the virgin branches of compression and expansion simulate adequately the first levels of deformations.
As mentioned above, the dispersion in virtual simulations of tests is not as high as when talking about complex geotechnical problems with different levels of deformations and stresses in the geotechnical structure. At that time, the true quality of the model is measured, however, virtual simulation of tests is the first step to a successful numerical modelling since it establishes a compulsory nature for the user to understand the model and the physical meaning of its parameters.

It is important to emphasize that the use of constitutive models must be in accordance with the geotechnical problem addressed. Each model reproduces particular conditions of the material or an external factor that may affect it. Therefore, depending on the problem, an appropriate choice must be made of the constitutive law to be implemented.

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