Cold dark matter and the cosmic phase transition

Bikash Sinha
Variable Energy Cyclotron Centre, 1/AF Bidhannagar
Kolkata 700064, India
E-mail: bikash@vecc.gov.in; bsinha1945@gmail.com

Abstract. It is entirely plausible that during the primordial quark-hadron phase transition in
the universe, microseconds after the Big Bang, supercooling takes place, accompanied by mini-
inflation. With $\mu/T \sim 1$ ($\mu$ is chemical potential), leading to a first order phase transition from
quarks to hadrons; there will be relics in the form of quark nuggets, and, that they consist of
Strange Quark Matter. The possibility that these SQM nuggets may well be the candidates of
cold dark matter is critically examined. A cursory comparison with the neutron star is presented
at the end.

1. Introduction
Over the years abundant evidence has been accumulated indicating the presence of large
quantities of unseen matter surrounding normal galaxies including our own [1,2,3]. The nature
of this dark matter, however, remains unknown except for that it cannot be normal, such as the
matter in stars, gas or dust.

The evidence of the existence of dark matter is primarily gravitational. The discrepancy
between the luminous mass and the gravitational mass clearly indicates the presence of a large
amount of unseen mass of the universe, usually referred to as dark matter. By now, it is
also established that dark matter accounts for approximately 27% of the energy budget of the
universe.

All kinds of speculations are in circulation. Axions, massive neutrinos, other kind of yet
unknown exotic particles, and even weakly interacting massive particles, referred to usually as
WIMPs have been proposed as candidates of dark matter, but none of which are experimentally
observed so far. Alcock et al. [4] and Aubourg et al. [5] (using gravitational microlensing
method suggested by Paczynski [6]) discovered the existence of this dark matter. Alcock et al.
[4] suggested that the dark matter can be explained by normal matter, known collectively as
Massive Compact Halo Objects (MACHOs) [4]. They went on to speculate that MACHOs might
be brown dwarfs, Jupiter like objects, neutron stars, or even black holes. It has been argued
by us [8,9,10,11] quite exhaustively, that a natural explanation will be that the MACHOs are
the relics from the putative cosmic phase transition from quark to hadrons about a microsecond
after the Big Bang. MACHOs, it is argued, can be the quark nuggets, which have survived from
that primordial epoch. It is entirely plausible that these relic quark nuggets are made of strange
matter, the true ground state of QCD, originally suggested by Witten [12].

This assertion has acquired further credibility from the recent experimental observations of
the Bose Institute group [13,14] engaged in studying cosmic strangelets at a mountain altitude.
Although heavy- ion collision is unlikely to produce strangelets, strange quark nuggets in the
form of strangelets hurtling down through the cosmos will tend to pick up mass \cite{13} from the atmospheric atoms, as they fall to the earth. The analysis of Banerjee et al. \cite{13} indicates that a strangelet with an initial mass $\sim 64$ amu and charge $\sim 2$ (typically for strangelet $Z/A << 1$) acquires mass as it passes through the atmosphere, and goes on to, eventually, evolve to a mass $\sim$ of 340 amu or so, at the end of its cosmic journey. This is at an altitude of $\sim 3.6$ km above sea level, typically the Himalayan mountain region in India such as Darjeeling. The collected data, just mentioned, suggest the interpretation of exotic cosmic ray events of very small $Z/A$, arising from the Strange Quark Matter droplets.

Recently, Harvey et al. \cite{15} have suggested that collisions between galaxy clusters provide a test of the non-gravitational forces acting on dark matter. Using the Chandra and Hubble Space Telescopes, they observe 72 collisions, including both “major” and “minor” mergers. Combining these measurements statistically, they find the existence of dark mass of $7.6 \sigma$ significance. The position of the dark mass has remained closely aligned, implying a self-interaction cross section $\sigma_{DM}/m < 0.47$ cm$^2$/g and disfavoring some proposed extensions to the standard model.

This observation \cite{15} is in general agreement, it seems with the identification of dark matter with strange quark nuggets, relics of the quark-hadron phase transition. It is interesting \cite{10} to observe that going over from a radiation dominated universe to gravity dominated universe these strange quark nuggets begin to clump around the temperature $\sim 1$MeV of the universe. It was found by us \cite{10} that after clumping is over, these objects feel only the influence of gravity as has been observed by Harvey et al. \cite{15}. Thus, SQNs are in some sense the WIMPs, massive but weakly interacting, by gravity only.

Armed with the recent observations \cite{13,14} of strangelets at mountain top and the very first observations of MACHO collaboration at Mount Stromlo by Alcock et al. \cite{4} and by Aubourg et al. \cite{5} of EROS collaboration at La Silla, Chile, it is proposed to go through a critical analysis of the origin of Strange Quark Matter (SQM) and it’s survival through the cosmological time scale, and, finally identifying SQMs as the candidates for cold dark matter (CDM) which may indeed close the universe \cite{10}.

There are two central issues: one the formation of quark nuggets immediately after the quark-hadron phase transition and second, their survival from that primordial epoch to now. Since the chemical potential of the universe at that point of time was close to zero and the temperature was around 200 MeV, the wisdom of lattice will lead the universe to cross over to the hadrons with no relic of QCD phase transition. It should be noted however that reliability of lattice calculation with bare quarks in an expanding universe is a non trivial issue.

In the next section, I analyze the circumstances precipitated essentially by some degree of supercooling accompanied by a “mini inflation.” With $\mu/T \sim 1$, the entry point of the universe to quarks is the world of hadrons is shifted along the phase boundary, and a first order phase transition takes place. In the following, we analyse this scenario critically.

2. Cosmic Phase Transition from Quark to Hadrons

It is conventionally assumed that the baryon asymmetry $\eta = (n_B - n_B) / \gamma$ at that primordial epoch of phase transition is the same as that of today’s universe $\eta \sim 10^{-10}$. There are however reasonably straightforward arguments \cite{12,16,17} that $\eta$ at that epoch is much higher and indeed of the order of $\eta \sim \mathcal{O}(1)$ unity. However, it is seen that after the phase transition $\eta$ goes back to $10^{-10}$, as it is today. The consequence of such a possibility is discussed in the following.

Witten \cite{12} and others \cite{8,11,16,17,18} have argued that a first order phase transition is plausible with a “small” supercooling. In a recent private communication Witten \cite{19} further asserted that if $n_B \approx n_B / \gamma$ remains $\sim 10^{-10}$ at the point of q-h phase transition, as it is in the current universe, then supercooling is implausible. However, he also points out \cite{19} that if the baryon to photon ratio is not small during the QCD phase transition and becomes small because of some phenomena at later times, then supercooling is plausible in principle. In the following,
we demonstrate that this is entirely possible.

This is the central issue, the relevance of baryon asymmetry at that primordial epoch.

One of the more compelling scenarios of baryogenesis is based on its generation from leptogenesis through topological sphaleron transitions occurring around the electroweak transition temperature. Leptogenesis occurs through out-of-equilibrium decays of heavy right-handed neutrinos which occur naturally within a seesaw mechanism, leading to Majorana masses for neutrinos, (as well as neutrino oscillation parameters) within observable ranges. Fermions with only Majorana masses and no Dirac masses (Majorana fermions) are charge self-conjugate spin-1/2 particles for any global U(1) charge. If this U(1) charge is associated with lepton number, then the charge self-conjugate property automatically implies that Majorana mass terms violate lepton number. Thus, it is this supposedly Majorana nature of neutrinos (even if they have a Dirac component as well) which lies at the heart of the incipient lepton number violation. The positive aspect of this mechanism of leptogenesis-induced baryogenesis is that one obtains a numerical result close to the observed baryon-photon ratio of \( \mathcal{O}(10^{-10} - 10^{-9}) \) without any fine tuning.

The resolution of the issue as to whether neutrinos are predominantly Majorana fermions, as happens to be the common prejudice currently, is to be decided by the currently ongoing experiments on neutrinoless double beta decay. If, contrary to extant belief, such experiments happen to yield null results, and neutrinos are confirmed to be Dirac fermions, this scenario of baryogenesis loses its prime attraction, entailing unsavoury fine tuning.

Given such a volatile situation, alternative scenarios of baryogenesis cannot be ruled out. Prominent among these is the nonthermal Affleck-Dine mechanism [20].

The Affleck-Dine mechanism [20] has the potential to produce a baryon asymmetry of \( \mathcal{O}(1) \) without requiring superhigh temperatures. However, the observed baryon asymmetry of \( \mathcal{O}(10^{-10}) \) at CMB temperatures needs to emerge naturally from such a scenario. This is what is achieved through a “little inflation” of about 7 e-folding occurring at a lower temperature, which may be identified with the QCD first order phase transition [16]. Such an inflation naturally dilutes the baryon photon ratio to the observed range, even though the baryon potential before the first order phase transition may have been high (of \( \mathcal{O}(1) \) in photon units). Comparing this “little inflation” with the more standard Guth’s inflationary model [21], one finds that the patterns of entropy variation in the two cases are very different. In the standard inflationary model [21] the entropy is conserved during exponential expansion, and increases, due to reheating when bubbles collide, at the end of the transition. However in Guth’s scenario, supercooling is there though very large; in the little inflation scenario for the case of quark-hadron phase transition, on the other hand, the entropy is constantly increasing during the quark-hadron phase transition.

The possibility and the criterion of a mini-inflationary epoch can be demonstrated in a simple way within the Friedman model of a spatially flat universe, which is homogeneous and isotropic along with an appropriate equation of state (EOS). Let the scale factor be \( R \) with an energy density \( \epsilon \), and then the Friedman equation reads

\[
\dot{R} - CR\sqrt{\epsilon} = 0 \tag{1}
\]

\[
\dot{\epsilon} - 3(\dot{R}/R)(\epsilon + P) = 0 \tag{2}
\]

with \( C = (8\pi/3)^{1/2}/M_p \) the Planck mass \( M_p = 1.2 \times 10^{9} \text{ GeV} \). The corresponding equation of state, relating energy density \( \epsilon \) and the pressure \( p \) using the bag model reads for QGP

\[
\epsilon_{qq} = (37\pi^2/90)T^4 + B \tag{3}
\]

\[
p_{qq} = (\epsilon_{qq} - 4B)/3 \tag{4}
\]
\[ p = p_{qg} + p_{bg}; \quad \epsilon = \epsilon_{qg} + \epsilon_{bg}; \quad \epsilon_{bg} = 3p_{bg} \]  

(5)

\[ p_{bg} = 14.25\pi^2T^4/90 \]  

(6)

The cosmic evolution will be an inflationary one if the expansion is accelerated, \( \ddot{R} \geq 0 \) which leads to using equations (1) and (2)

\[ \ddot{R} = -C^2 R(\epsilon + p)/2 \geq 0 \]  

(7)

and

\[ 3p + \epsilon < 0, \]  

(8)

with the solution

\[ \epsilon = B \coth^2\left[2C\sqrt{B}(t - t_c) + \text{arcth}\left(\sqrt{\epsilon_c/B}\right)\right], \]  

(9)

\[ R = \sinh^{1/2}\left[2C\sqrt{B}(t - t_c) + \text{arcth}\left(\sqrt{\epsilon_c/B}\right)\sinh^{-1/2}\left(\text{arcth}\sqrt{\epsilon_c/B}\right)\right] \]  

(10)

at \( t >> t_{\text{exp}} = (2C\sqrt{B})^{-1} \); the expansion proceeds exponentially as \( R \propto \exp(C\sqrt{B}t) \), with \( \epsilon_c = 5.5B \). It is worthwhile to note that in the present case, \( R \propto \exp(C\sqrt{B}t) \), where \( C = (8\pi/3)^{1/2}/M_p \); for Guth inflation [21] \( R \propto \exp(\chi t) \) and \( \chi = \sqrt{(8\pi/3)G\rho_0} \), \( \rho_0 \) is the initial energy density of the universe.

As can be seen Guth’s inflation involves the gravitational constant, whereas mini-inflation involves the Bag constant; supercooling in the standard inflation model of the universe is by 28 or more orders of magnitude, whereas here in the quark hadron phase transition its only 7 e-folds.

Thus in the cosmological context, phase transition seems to be intimately connected with supercooling.

Equations (3) to (8) are satisfied for \( T < T_i \) with \( T_i \cong 0.5B^{14} \).

For temperatures below \( T_0 = 0.65B^{1/4} \), the pressure becomes negative leading to acceleration of the universe. This is exactly what is achieved by the mini-inflation Fig. 1.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Sketch of a possible QCD phase diagram with the evolution path of the universe in the little inflation scenario, see Refs. [16], [18].
The temperature drops primarily as per the standard $t^{-1/2}$ law. Then it increases only slightly with the release of latent heat as quarks go to hadrons and the degrees of freedom are quenched. Finally $T$ again decreases as $t^{-1/2}$ again.

The scale factor $R$ evidently does not follow the standard $t^{1/2}$ law of the standard model. As shown in Fig. 2, the guiding equation for mini-inflation reads as $R \propto \exp(C\sqrt{BT})$; here the scale factor grows exponentially. In more general terms, the scale factor $R$ gets multiplied by $\sim 10^4$, where $T$ only decreases by only $\sim 70$ MeV and the entropy increases by $3 \times 10^9$ times (see Fig. 2). Thus in the “little inflation” scenario, the entropy is constantly increasing during the quark-hadron phase transition and the product $R(t)T(t)$ does not remain constant where $R$ is the scaling factor.

![Graphs showing temperature and scale factor as functions of time.](image)

**Figure 2.** (Upper panel) The temperature as a function of time, Standard Cosmology. (Lower panel) The Scale factor as a function of time at the time of mini inflation: $R \propto \exp(C\sqrt{BT})$, $C = \sqrt{8\pi/3}/M_p$.

The increase in entropy changes $n_q/n_\gamma$ radically with dramatic consequences. The quark number decreases as $R^{-3}$, and $n_\gamma \propto T^3$, and thus the ratio $n_q/n_\gamma \propto (RT)^{-3}$ is proportional to the inverse of the entropy. More specifically, the scale factor $R$ is multiplied by $\sim 10^4$ where $T$ only decreases from (say) 90 to 18 MeV. The product $RT$ is multiplied by more than $1.4 \times 10^3$, and thus the entropy increases by $3 \times 10^9$. 


In conclusion for this part of the argument, the entropy is multiplied by $10^9$ during the quark-hadron phase transition, while the baryon asymmetry is divided by the same factor. Thus we get back $n_q/n_\gamma \sim 10^{-9}$ at the end of the phase transition, as required for correct prediction of primordial nucleosynthesis, it also means, for $T > T_c$, the ratio $n_q/n_\gamma \sim 1$, and we get back our first order phase transition. This result contrasts sharply with normal adiabatic expansion in which baryon asymmetry does not change and remains as $n_q/n_\gamma \approx 10^{-9}$.

To recapitulate, the universe is assumed to begin with a large baryon chemical potential acquired through an Affleck-Dine [20] type of mechanism. It then undergoes a period of inflation, Fig. 1, crossing the QCD first order phase transition line, while remaining in a deconfined and a chirally symmetric phase. The universe is then trapped in a false metastable QCD vacuum state.

The delayed phase transition then releases the latent heat and produces concomitantly a large entropy density, which effectively reduces the baryon asymmetry to currently observable values. It then enters a reheating phase all the way up to the usual reheating temperature with no significant change in the baryon potential, and then the universe follows the standard path to lower temperatures.

Experimental observations of Alcock et al. [4], Aubourg et al. [5] and more recently by the Bose Institute group [13,14] are the clinching proof of such a scenario.

Finally, as suggested by Witten [19] stable quark lumps (nuggets) are extremely optimistic. It would be very lucky to be true. The relics as pointed out in this paper are the acid test of that luck. The recent observations [13,14] along with the old MACHO observations [4,5] seem to survive the acid test. Thus Cold Dark Matter is the relic of the cosmic phase transition.

3. Survivability of Cosmological Strange Quark Nuggets (SQN)

In Ref. [11] the present author presents a detailed discussion on the nature of the phase transition and about the survivability of QSNs. Tracing the history from Alcock and Farhi (see Ref. [11]) onwards to Madsen, Heiselberg and Riisager [22], and then to the detailed work of Bhattacharya et al. [8] using Chromo Electric Flux Tube (CEFT), we come to the conclusion that QNs with baryon number $\geq 10^{39} - 10^{40}$ will indeed be cosmologically stable. It is thus very relevant to ask what fraction of the dark matter could be accounted for by the surviving QNs. To put it yet another way what we wish to address in this paper is whether the cosmological dark matter, accounting for 90% or more of all the dark matter in the universe, can be made up entirely of QNs?

As per ref. [9,10,11], the universe is closed by the baryonic dark matter trapped in QNs, and we should have

$$N_B^H(t_p) = N_B^{QN} n_{QN} V^H(t_p),$$  \hspace{1cm} (11)

where $N_B^H(t_p)$ is the total number of baryons required to close the universe ($\Omega_B = 1$) at $t_p$, $N_B^{QN}$ is the total number of baryons contained in a single QN, and $V^H(t_p = (4\pi/3)(ct_p)^3$ is the horizon volume. With $v/c \approx 1/\sqrt{3}$

$$N_B^{QN} \leq 10^{-4.7} N_B^H(t_p)$$  \hspace{1cm} (12)

As per standard Big Bang Nucleosynthesis (SBBN) $\eta \equiv n_B/n_\gamma(10^{-9}, 10^{-10})$ for convenience (as an estimate), the baryon number is well within the horizon limit at the QCD epoch $\approx 10^{49}$.

These usual baryons constitute only $\sim 10\%$ of the closure density ($\Omega_B \sim 0.1$ from SBBN), a total number of $10^{50}$ within the horizon at a temperature of $\sim 100$ MeV would close the universe baryonically, provided these baryons do not take part in SBBN, and a criterion fulfilled by QNs. This would require $N_B^{QN} \leq 10^{45.3}$, clearly above the survivability limit of QNs.
4. Epilogue
The mini-bang between two nuclei, although mimicking the big bang of the universe, is somewhat different that this phenomenon. This is discussed in detail elsewhere [11].

It is clear however, that [23] the role played by the Newtonian constant of Gravity, G, in the big bang is somewhat analogous to the role played by the vacuum energy density often referred to as the bag constant, B. The Big Bang is a display of gravity, space, and time, whereas the little bang is essentially dealing with confinement and subsequently deconfinement in extreme conditions.

On the other extreme end of the phase diagram lies a domain of very high baryon density at rather low temperature, the scenario of neutron star matter, of compressed baryonic matter (CBM) at zero temperature. It will be of some interest to compare and contrast the “perfect fluid” property of the quark matter in the microsecond universe with the “perfect fluid” of the core of the neutron star.

It is interesting to note that for the early universe, we have depleting quark matter as hadronisation progresses and the universe expands in space and time. From the canonical value of $\eta/s \leq 1/4\pi$, with hadronisation, $\eta/s$ will go on increasing as pointed at Roy et al. [24]. Eventually, the SQNs will be floating in a dilute hadronic fluid, which is not so perfect, facing more viscous drag than its quark matter counter part.

In the case of the neutron star, however, the scenario is opposite; more hadrons will be transformed to quarks, so $\eta/s$ will decrease towards the canonical value $\eta/s \leq 1/4\pi$. For the neutron star, an approximate estimate of $\eta/s \sim T\lambda_F \sigma_s$ will indicate that (with very low value of $\lambda_F \equiv (\rho\sigma)^{-1}$, a very high $\rho$ and an extremely low temperature $\eta/s$) for the quark core of the star with $c_p \sim 1/\sqrt{3}$ (say) may well go down below the generic value $1/4\pi$, close to zero, making the core, a perfect fluid splashing on the membrane [12] of hybrid hadronic matter and quark core. It will be of great interest to explore this in future.

Thus, ‘G’ ensures gravitational attraction and ‘B’ ensures confinement [23]. Similarly, the Hawking radiation from a black hole, with its celebrated connection to entropy [25], has an analogous scenario to that of the quark nuggets that survive the cosmic phase transition. The strange quark nuggets radiate neutrons but remain dark and cold (non-relativistic). These so-called Strange Quark Nuggets are somewhat analogous to black holes, in the sense that they tend to absorb matter as they hurtle through the cosmos [14]. An attempt is being made here to find out a modified entropy “entropy equivalence” between black hole and SQNs.

For SQNs, the deciding role is played by the Bag Pressure, B, making the SQNs dark, veiled to an exterior observer. The natural length scale for SQNs, heuristically, can be argued as $L_B = \frac{M_{\odot}}{M_{\text{SQN}}} (B^{1/4})^{-1}$, so that the entropy of SQNs is $S_{\text{SQN}} = \frac{A}{4L_B^2} = \pi R^2 B^{1/2} (M_N/M_{\odot})^{-2}$, $M_N$ being the mass of the quark nugget and $M_{\odot}$ is the solar mass, whereas, for the black hole, $S_{\text{BH}} = \frac{A}{4\ell_p^2} = \frac{c^3 A}{4\ell_p^2}$ : $A = 16\pi (GM/C^2)^2$, where M is the mass of the black hole. At this preliminary stage, the above ansatz of SQNs’ entropy seem to broadly agree with other result (more will be reported later [26]).

5. Conclusion
Driven by the familiar standard model and introducing a “mini- inflation” at the cosmic quark-hadron phase transition, one can precipitate a first order phase transition from quarks to hadrons.

It has been argued in this paper that it does not introduce any non-standard cosmological scenario and indeed this mini-inflation comes in quite naturally, raising the value of $\eta = n_B/n_\gamma$ substantially. It is argued that the MACHOs will survive the cosmological time scale and beyond a critical baryon number window of $\sim 10^{40}$, are the candidates of cold dark matter observed some years ago [4,5] and rather more recently [13,14].

The interestingly satisfying thought lingers that we can accommodate all this in the
framework of the standard model without invoking yet unobserved exotic physics. Clearly, we should look for SQM nuggets, as candidates for cold dark matter, more vigorously.

5.1. Acknowledgment
First of all I like to thank Edward Witten for his insightful comments and Jürgen Schaffner-Bielich for explaining to me the purpose of mini inflation some time ago.

I like to thank my colleagues Partha Mazumdar, Debasis Mazumdar, Pijush Bhattacharya, and Sibaji Raha; Chiranjib Barman’s help is gratefully acknowledged. Finally, I would like to thank Victor Matveev for inviting me to the Dubna Conference on Strange Quark Matter, and the Indian National Science Academy for my emeritus position.

6. References
[1] Trimble V A 1987 Rev. Astr. Astrophys. 25 425
[2] Fich M, Tremaine S A 1991 Rev. Astr. Astrophys. 29 409
[3] Griest K 1991 Astrophys. J. 366 412
[4] Alcock C et al. 1993 Nature 365 621
[5] Aubourg E et al. 1993 Nature 365 623
[6] Paczynski B 1986 Astrophys. J. 304 1
[7] Berrett D et al. 1993 Ann. N. Y. Acad. Sci. 688 619
[8] Bhattacharya P et al. 1993 Phys. Rev. D 48 4630
[9] Bhattacharye A et al. 2000 Phys. Rev. D 61 083509
[10] Banerjee S et al. 2003 Mon. Not. R. Astron. Soc. 340 284
[11] Sinha B 2014 Int. JMP A 29 1432004
[12] Witten E 1994 Phys. Rev. D 30 272
[13] Banerjee S et al. 1999 Phys. Rev. Lett. 85 2000
[14] Basu B et al. 2015 Astro. Par. Phys. 61 88
[15] Harvey D et al. 2015 arXiv:1503.07675v2 [astro-ph.co]
[16] Boeckel T and Schaffner-Bielich J 2010 Phys. Rev. Lett. 105 041301
[17] Borghini N et al. 2000 J. Phys. G 26 771
[18] Boeckel T and Schaffner-Bielich J 2012 Phys. Rev. D 85 103506
[19] Witten E 2014 Private Communication
[20] Affleck I and Dine M 1985 Nucl. Phys. B 249 361
[21] Guth A H 1981 Phys. Rev. D 23 347
[22] Madsen J, Heiselberg H and Riisager K 1986 Phys. Rev. D 34 2947
[23] Shuryak E, Private Communication
[24] Lacey R A et al. 2007 Phys. Rev. Lett. 98 092301
[25] Hawking S W 1974 Nature 244 B 30; Hawking S W 1976 Phys. Rev. D 13 191
[26] Sinha B to be published