MAJORANA NEUTRINO MIXING

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Abstract

The most natural see-saw explanation of the smallness of the neutrino masses is based on the assumption that total lepton number is violated at a large scale and neutrinos with definite masses are Majorana particles. In this review we consider in details difference between Dirac and Majorana neutrino mixing and possibilities of revealing Majorana nature of neutrinos with definite masses.

1 Introduction

Phenomenon of neutrino oscillations, discovered in the Super Kamiokande [1], SNO [2], KamLAND [3], K2K [4] and other neutrino experiments [5, 6, 7, 8] is one of the most important signature of a new, beyond the Standard Model physics.

Investigation of neutrino oscillations is based on the following assumptions:

1. Neutrino interaction is given by the standard CC and NC La-grangians

\[ \mathcal{L}_I^{CC} = -\frac{g}{2\sqrt{2}} j_{CC}^{\alpha} W_{\alpha} + \text{h.c.}; \quad \mathcal{L}_I^{NC} = -\frac{g}{2\cos\theta_W} j_{NC}^{\alpha} Z_{\alpha} \]  

Here \( g \) is the \( SU(2) \) gauge coupling, \( \theta_W \) is the weak angle and

\[ j_{CC}^{\alpha} = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_L \gamma_\alpha j_L; \quad j_{NC}^{\alpha} = \sum_{l=e,\mu,\tau} \bar{\nu}_L \gamma_\alpha \nu_L \]

are the leptonic charged current and the neutrino neutral current.

2. In the total Lagrangian a neutrino mass term, source of neutrino masses and mixing, enters.
For neutrinos, particles with equal to zero electric charges, two types of mass terms are possible (see reviews [9, 10]):

I. Dirac mass term

\[ \mathcal{L}^D = -\bar{\nu}_R M^D \nu_L + \text{h.c.} \]  

Here \( M^D \) is a non diagonal complex matrix and

\[ \nu_L = \begin{pmatrix} \nu_e^L \\ \nu^\mu_L \\ \nu^\tau_L \end{pmatrix}; \quad \nu_R = \begin{pmatrix} \nu_e^R \\ \nu^\mu_R \\ \nu^\tau_R \end{pmatrix}. \]

After the diagonalization of the matrix \( M^D \) the mass term (3) takes the standard form

\[ \mathcal{L}^D = -\sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i. \]

and flavor field \( \nu_{lL}(x) \) is given by

\[ \nu_{lL}(x) = \sum_{i=1}^{3} U_{li} \nu_{iL}(x). \]

Here \( \nu_i(x) \) is the field of neutrino with mass \( m_i \) and \( U \) is an unitary matrix. Thus, in general case of non diagonal matrix \( M^D \) flavor fields \( \nu_{lL}(x) \) in charged and neutral currents (2) are "mixed fields".

In the case of the Dirac mass term the total Lagrangian is invariant under global gauge transformations

\[ \nu_{lL}(x) \rightarrow e^{i \alpha} \nu_{lL}(x); \quad \nu_{lR}(x) \rightarrow e^{i \alpha} \nu_{lR}(x); \quad l(x) \rightarrow e^{i \alpha} l(x), \]

where \( \alpha \) is an arbitrary constant phase. This invariance means that the total lepton number \( L = L_e + L_\mu + L_\tau \) is conserved and \( \nu_i(x) \) are four-component fields of neutrinos (\( L = 1 \)) and antineutrinos (\( L = -1 \)).

II. Majorana mass term

\[ \mathcal{L}^M = -\frac{1}{2} (\nu'_L)^T M^M \nu'_L + \text{h.c.} \]

Here

\[ (\nu'_L)^c = C \bar{\nu}_L^T \quad (\nu'_L)^c = -\nu'_L^T C^{-1} \]
(C is the matrix of the charge conjugation), $M^M$ is a non diagonal, complex matrix and

$$
\nu'_i = \begin{pmatrix}
\nu_{eL} \\
\nu_{\mu L} \\
\nu_{\tau L} \\
\vdots
\end{pmatrix}.
$$

(10)

In the column $\nu'_L$ in addition to three flavor fields $\nu_{iL}$ ($l = e, \mu, \tau$) could also be other fields. These fields do not enter into the Lagrangian and corresponding particles have no standard electroweak interaction. Such additional fields are called sterile.

In the case of the Majorana mass term the total lepton number is violated and neutrinos with definite masses $\nu_i$ are Majorana particles. We will consider this case in some details.

The Fermi-Dirac statistics of neutrino fields requires that

$$
(M^M)^T = M^M.
$$

(11)

Symmetrical matrix can be diagonalized with the help of an unitary matrix:

$$
M^M = (U^\dagger m U^+)^T
$$

(12)

where $U^\dagger U = 1$ and $m_{ik} = m_i \delta_{ik}$; $m_i > 0$.

From (9) and (12) we find

$$
\mathcal{L}^M = -\frac{1}{2} \bar{\nu}^M m \nu^M = -\frac{1}{2} \sum_i m_i \bar{\nu}_i \nu_i.
$$

(13)

Here

$$
\nu^M = U^\dagger \nu'_L + (U^\dagger \nu'_L)^c = \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\vdots
\end{pmatrix}
$$

(14)

The field $\nu_i(x)$ is the field of neutrino with mass $m_i$. From (14) it follows that fields $\nu_i(x)$ satisfy Majorana condition

$$
\nu^c_i(x) = \nu_i(x),
$$

(15)

where $\nu^c_i(x) = C \bar{\nu}^T_i(x)$. 

3
From this condition it follows that
\[ \nu_i(x) = \int N_p \left( e^{-ipx} u^r(p) a_i^r(p) + e^{ipx} \nu^r(p) a_i^{r\dagger}(p) \right) d^3p \]  
(16)

Here \( N_p = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}} \) and \( a_i^r(p) ( a_i^{r\dagger}(p)) \) is the operator of absorption (creation) of a Majorana neutrino with momentum \( p \), helicity \( r \) and mass \( m_i \). Thus, Majorana neutrinos and antineutrinos are identical.

From (14) we find that flavor fields \( \nu_{1L}(x) \) are mixed fields
\[ \nu_{1L}(x) = \sum_i U_{1i} \nu_i L(x); \quad l = e, \mu, \tau, \]  
(17)

where \( \nu_i(x) \) is the field of the Majorana neutrino with mass \( m_i \).

If \( n_s \) sterile fields \( \nu_{sL} \) enter into the mass term, the number of the Majorana fields \( \nu_i(x) \) is equal to \( (3 + n_s) \), \( U \) is \( (3 + n_s) \times (3 + n_s) \) unitary matrix and in addition to (17) we have
\[ \nu_{sL}(x) = \sum_i U_{si} \nu_i L(x); \quad a = 1, \ldots, n_s. \]  
(18)

So called Majorana and Dirac mass term
\[ \mathcal{L}^{M+D} = -\frac{1}{2} (\bar{\nu}_L)^c M_{LM} \nu_L - \bar{\nu}_R M^{D} \nu_L - \frac{1}{2} \bar{\nu}_R M^{M}_R (\nu_R)^c + \text{h.c.} \]  
(19)

is of special interest. In (19) \( M_{LM,R}^{M} \) are \( 3 \times 3 \) Majorana symmetrical complex matrices and \( M^{D} \) is a \( 3 \times 3 \) Dirac complex matrix. The mass term \( \mathcal{L}^{M+D} \) can be presented in the form
\[ \mathcal{L}^{M+D} = -\frac{1}{2} (\bar{\nu'}_L)^c M^{M+D} \nu'_L + \text{h.c.}, \]  
(20)

where
\[ \nu'_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}; \quad M^{M+D} = \begin{pmatrix} M^{M}_{LM} & (M^{D})^T \\ M^{M}_{LR} & M^{M}_{RR} \end{pmatrix}. \]  
(21)

After the diagonalization of the mass term \( \mathcal{L}^{M+D} \) we will find
\[ \nu_{1L}(x) = \sum_{i=1}^{6} U_{1i} \nu_i L(x); \quad (\nu_{1R}(x))^c = \sum_{i=1}^{6} U_{1i} \nu_i L(x); \quad l = e, \mu, \tau. \]  
(22)

Existing neutrino oscillation data, with the exception of the LSND data \(^1\), are in a good agreement with the assumption that the

\(^1\) The LSND result is going to be checked by the MiniBooNE experiment at the Fermilab.
number of the massive neutrinos is equal to the number of the flavor neutrinos (three) and there are no light sterile neutrinos. The type of neutrino mass term and, correspondingly, the nature of the massive neutrinos $\nu_i$ (Majorana or Dirac?) are unknown. These problems will hopefully be solved in future experiments.

From neutrino oscillation data only neutrino mass-squared differences can be obtained. The mass of the lightest neutrino at present is unknown. From the data of the tritium $\beta$-decay experiments Mainz [13] and Troitsk [14] it was found \(^2\)

$$m_\beta \leq 2.3 \text{ eV.}$$

(23)

From cosmological data for the sum of neutrino masses the upper bounds in the range

$$\sum_i m_i \leq (0.4 - 1.7) \text{ eV.}$$

(24)

were inferred (see [15]).

These data together with neutrino oscillation data, which we will discuss in the next section, tell us that neutrino masses are different from zero and much smaller then masses of quarks and leptons. For example, for the particles of the third generation we have:

$$m_t \simeq 174.3 \text{ GeV}, m_b \simeq 4.6 \text{ GeV}, m_\tau \simeq 1.78 \text{ GeV}, m_3 \lesssim 2 \cdot 10^{-9} \text{ GeV}$$

Thus, within one generation masses of quarks and lepton differ by one-two orders of magnitude. Neutrino mass is about 9-11 orders of magnitude smaller than masses of quarks and lepton.

The most plausible mechanism of the generation of neutrino masses which are much smaller than the masses of quarks and leptons is the see-saw mechanism [16]. In order to explain an idea of this mechanism, let us consider the case of one generation (say, first). Assume that the Dirac mass term

$$\mathcal{L}^D = -m_D \bar{\nu}_e R \nu_e L + \text{h.c.}$$

(25)

is generated by the standard Higgs mechanism via spontaneous symmetry breaking. Masses of particles, generated by this mechanism, are

\(^2\)Small neutrino mass-squared differences can not be resolved in tritium experiments. Thus, $m_\beta \simeq m_0$, where $m_0$ is common neutrino mass.
proportional to the constant $v = (\sqrt{2} G_F)^{-\frac{1}{2}} \simeq 250$ GeV, which characterizes the scale of the electroweak symmetry breaking. It is natural to expect that $m^D$ is of the same order of magnitude as masses of $u$, $d$ quarks and electron. Experimental data tell us, however, that neutrino mass is much smaller than masses of quarks and leptons. Thus, an additional mechanism which leads to suppression of the neutrino mass is needed.

Let us assume that in addition to (25) a lepton number violating right-handed Majorana mass term

$$L^M = -\frac{1}{2} M_R(\nu_{eR})^c + h.c. \tag{26}$$

is generated by some mechanism. The total mass term is of the Majorana and Dirac type

$$L^{M+D} = -\frac{1}{2} (\nu'_L)^c M^{M+D} \nu'_L + h.c., \tag{27}$$

where

$$\nu'_L = \left( \begin{array}{c} \nu_{eL} \\ (\nu_{eR})^c \end{array} \right); \quad M^{M+D} = \left( \begin{array}{cc} 0 & m^D \\ m^D & M_R \end{array} \right). \tag{28}$$

After the standard diagonalization of the mass term (27) we find

$$L^{M+D} = -\frac{1}{2} \sum_{i=1,2} m_i \bar{\nu}_i \nu_i. \tag{29}$$

where $\nu_{1,2}$ are Majorana fields. Neutrino masses $m_i$ are given by the relation

$$m_{1,2} = \frac{1}{2} \left| M_R \mp \sqrt{M_R^2 + 4 m^2_D} \right| \tag{30}$$

For $\nu_{eL}$ and $(\nu_{eR})^c$ we find the mixing relations

$$\nu_{eL} = -i \cos \theta \nu_{1L} + \sin \theta \nu_{2L}$$

$$(\nu_{eR})^c = i \sin \theta \nu_{1L} + \cos \theta \nu_{2L}, \tag{31}$$

where mixing angle $\theta$ is given by

$$\tan 2 \theta = \frac{2 m^D}{M_R}. \tag{32}$$

\footnote{Let us stress that such mass term is allowed only for neutrinos. Because of the conservation of the electric charge, Majorana mass terms for quarks and leptons are not allowed.}
Let us assume now that the mass $M_R$ which characterize the scale of the violation of the lepton number is much larger than $m_D$. From (30) and (32) we obtain in this case

$$m_1 \simeq \frac{m_2^2}{M_R} \ll m_D; \quad m_2 \simeq M_R; \quad \theta \simeq \frac{m_D}{M_R} \ll 1. \quad (33)$$

The field of Majorana neutrino and the field of heavy Majorana particle are given by

$$\nu_1 \simeq i(\nu_{eL} - (\nu_{eL})^c); \quad \nu_2 \simeq (\nu_{eR} + (\nu_{eR})^c). \quad (34)$$

In the case of three generation the standard see-saw symmetrical mass matrix has the form

$$M^{M+D} = \begin{pmatrix} 0 & (M^D)^T \\ M^D & M_R \end{pmatrix}, \quad (35)$$

where $(M_R)^T = M_R$ and $M^D \ll M_R$. By analogy with the case of one generation we will choose the unitary matrix $U$ in Eq. (12) in the form

$$U = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} M_R^{-1} M^D & -(M^D)\dagger (M_R^{-1})\dagger \\ 1 \end{pmatrix}. \quad (36)$$

From (35) and (36) in linear approximation we have

$$U^T M^{M+D} U = \begin{pmatrix} (M^D)^T M_R^{-1} M_D & 0 \\ 0 & M_R \end{pmatrix}, \quad (37)$$

The $3 \times 3$ matrix

$$m_\nu = (M^D)^T M_R^{-1} M_D \quad (38)$$

is neutrino mass matrix.

Thus, if see-saw mechanism is the mechanism of the generation of neutrino masses, in this case:

- Neutrinos are Majorana particles with masses which are much smaller than masses of leptons and quarks.
- Heavy Majorana particles, see-saw partners of light Majorana neutrinos, must exist. CP-violating decays of these particles in the early Universe is considered as a probable source of the barion asymmetry of the Universe (see review [17]).
2 Briefly on the status of neutrino oscillations

From (6) and (17) for the state of flavor neutrino $\nu_l$ produced in a CC weak process together with lepton $l^+$ we have (see, for example, [9, 10])

$$|\nu_l\rangle = \sum_i U_{li}^* |\nu_i\rangle,$$

where $|\nu_i\rangle$ is the state of neutrino with mass $m_i$, momentum $\vec{p}$ and energy $E_i \simeq p + \frac{m_i^2}{2p}$.

In the case of the three-neutrino mixing for the probability of the transition $\nu_l \rightarrow \nu_{l'}$ in vacuum (during time $t$) we find the following expression

$$P(\nu_l \rightarrow \nu_{l'}) = |\sum_{i=1}^{3} U_{l'i} e^{-iE_i t} U_{li}|^2 = |\delta_{l'i} + \sum_{i=2,3} U_{l'i} (e^{-i\frac{\Delta m_{l'i}^2 L}{2E_i}} - 1) U_{li}^*|^2,$$

(40)

Here $\Delta m_{l'i}^2 = m_{l'i}^2 - m_{l}^2$ and $L \simeq t$ ($L$ is the distance between production and detection points).

In the general case the probability $P(\nu_l \rightarrow \nu_{l'})$ depends on six parameters: two neutrino mass-squared differences $\Delta m_{12}^2$ and $\Delta m_{23}^2$, mixing angles $\theta_{12}$, $\theta_{23}$, $\theta_{13}$ and CP violating phase $\delta$ (last four parameters characterize PMNS [18, 19] mixing matrix $U$). Existing data, however, are well described by simple two-neutrino expressions for transition probabilities, which can be obtained from (40) in the leading approximations (see [10]). This approximation is based on the smallness of two parameters

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \simeq 3.3 \cdot 10^{-2}; \quad \sin^2 \theta_{13} \leq 5 \cdot 10^{-2}.$$

(41)

The value of the parameter $\frac{\Delta m_{12}^2}{\Delta m_{23}^2}$ was obtained from analysis of all neutrino oscillation data. The inequality in (41) was obtained from analysis of the data of the reactor CHOOZ experiment [20], in which no indications in favor of neutrino oscillations were found.

If we neglect in the transition probability (10) small terms proportional to $\frac{\Delta m_{12}^2}{\Delta m_{23}^2}$ and $\sin^2 \theta_{13}$, we will find that in the region of $L/E$ in which $\Delta m_{23}^2 L/E \gtrsim 1$ (atmospheric and long baseline neutrino experiments) dominant transitions are $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$. For the
probability of $\nu_\mu$ ($\bar{\nu}_\mu$) to survive from (40) we obtain the standard two-neutrino expression

$$P(\nu_\mu \to \nu_\mu) = P(\bar{\nu}_\mu \to \bar{\nu}_\mu) = 1 - \frac{1}{2} \sin^2 2\theta_{23} (1 - \cos \Delta m^2_{23} \frac{L}{2E}).$$ \hspace{1cm} (42)

In the case of solar and KamLAND experiments for which small $\Delta m^2_{12}$ is relevant effect of the “large” $\Delta m^2_{23}$ is averaged. For $\nu_e$ ($\bar{\nu}_e$) survival probability in vacuum (or in matter) the following general expression can be obtained [21]:

$$P(\nu_e \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_e) = \sin^4 \theta_{13} + (1 - \sin^2 \theta_{13})^2 P^{(12)}(\nu_e \to \nu_e),$$ \hspace{1cm} (43)

where $P^{(12)}(\nu_e \to \nu_e)$ is the two-neutrino $\nu_e$ ($\bar{\nu}_e$) survival probability in vacuum (or in matter) which depends on $\Delta m^2_{12}$ and $\sin^2 \theta_{12}$. In the leading approximation the probability of reactor $\bar{\nu}_e$ to survive in vacuum is given by

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{12} (1 - \cos \Delta m^2_{12} \frac{L}{2E}),$$ \hspace{1cm} (44)

where the second term is the sum of (approximately equal) transition probabilities $P(\bar{\nu}_e \to \bar{\nu}_\mu)$ and $P(\bar{\nu}_e \to \bar{\nu}_\tau)$.

The probability of solar $\nu_e$ to survive in matter in the leading approximation is given by two-neutrino expression which depend on $\Delta m^2_{12}$, $\tan^2 \theta_{12}$ and electron number density $\rho_e(x)$.

From analysis of the Super Kamiokande atmospheric data for the parameters $\Delta m^2_{23}$ and $\sin^2 2\theta_{23}$ the following 90 % CL ranges were obtained [1]

$$1.5 \cdot 10^{-3} \leq \Delta m^2_{23} \leq 3.4 \cdot 10^{-3} \text{eV}^2; \quad \sin^2 2\theta_{23} > 0.92.$$ \hspace{1cm} (45)

From the global analysis of solar and KamLAND data it was found that [2]

$$\Delta m^2_{12} = 8.0^{+0.6}_{-0.4} \cdot 10^{-5} \text{eV}^2; \quad \tan^2 \theta_{12} = 0.45^{+0.09}_{-0.07}.$$ \hspace{1cm} (46)

### 3 Majorana mixing matrix

An unitary $n \times n$ matrix $U$ is characterized by $\frac{n(n-1)}{2}$ angles and $\frac{n(n+1)}{2}$ phases. The matrix $U$ can be presented in the form

$$U = S^T(\beta) \, U^0 \, S(\alpha),$$ \hspace{1cm} (47)
where $S(\alpha)$ and $S(\beta)$ are diagonal phase matrices:

$$S_{\mu l}(\beta) = e^{i\beta_l} \delta_{\mu l}; \quad S_{ik}(\alpha) = e^{i\alpha_k} \delta_{ik}.$$  \hspace{1cm} (48)

Because common phase is unobservable, one of $2n$ phases $\beta_l$ and $\alpha_i$ can be put equal to zero. We will choose $\alpha_n = 0$.

Let us consider first the case of the Dirac neutrinos. Phases of Dirac fields are arbitrary unphysical quantities. Thus, in the case of Dirac neutrinos phase factors $e^{i\beta_l}$ and $e^{i\alpha_i}$ can be absorbed, respectively, by the fields $l(x)$ and $\nu_i(x)$. Therefore the Dirac mixing matrix is given by

$$U^D = U^0.$$  \hspace{1cm} (49)

The Dirac mixing matrix is characterized by $\frac{n(n-1)}{2}$ angles and $\frac{n(n+1)}{2} - (2n - 1) = \frac{(n - 1)(n - 2)}{2}$ physical phases. In $n = 3$ case the mixing matrix is characterized by three angles and one phase. In the standard parametrization the matrix $U^0$ has the form

$$U^0 = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}. $$  \hspace{1cm} (50)

Here $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$.

In the case of the Majorana neutrinos only phase factors $e^{i\beta_l}$ can be included in the Dirac leptonic fields $l(x)$. Neutrino fields $\nu_i(x)$ satisfy Majorana condition (15) which fix phases of the fields $\nu_i(x)$. Thus, in the Majorana case phases $\alpha_i$ are physical quantities. Majorana mixing matrix has the form

$$U^M = U^0 S(\alpha).$$  \hspace{1cm} (51)

In the general $n \times n$ case it is characterized by $\frac{n(n-1)}{2}$ angles and

$$\frac{n(n + 1)}{2} - n = \frac{n(n - 1)}{2}$$

physical phases.

Let us notice that we can include $(n-1)$ Majorana phases $\alpha_i$ into Majorana fields and introduce new fields $\nu'_{i} = e^{i\alpha_i} \nu_{i}$. In this case for the mixing we have

$$\nu_{iL} = \sum_i U^M_{li} \nu_i = \sum_i U^0_{li} \nu'_i.$$  \hspace{1cm} (52)
The Majorana condition for the fields $\nu'_i$ takes the form

$$\nu'^c_i(x) = e^{-2i\alpha_i} \nu'_i(x).$$

(53)

Thus, there are two alternative ways of the inclusion of the Majorana phases into mixing relations:

- We can choose Majorana condition in the form (15). In this case Majorana mixing matrix is characterized by $\frac{n(n-1)}{2}$ phases and has the form Eq. (51).
- We can choose Majorana condition in the form (53). In this case Majorana mixing matrix is characterized by $\frac{(n-1)(n-2)}{2}$ phases and has the same form as Dirac mixing matrix. Other $n-1$ physical phases enter into Majorana conditions.

We will demonstrate later that both ways give the same physical results.

Up to now we have considered general neutrino mixing. Let us discuss now the case of the CP invariance in the lepton sector. In this case we have

$$V_{CP} L_{I}^{CC}(x) V_{CP}^{-1} = L_{I}^{CC}(x'),$$

(54)

where $V_{CP}$ is the operator of the CP conjugation and $x' = (x^0, -\vec{x})$.

In the case of the neutrino mixing the CC Lagrangian has the form

$$L_{I}^{CC} = -\frac{g}{\sqrt{2}} \sum_{i,l} U_{li}^* \bar{\nu}_L \gamma_\alpha l_L W^\alpha - \frac{g}{\sqrt{2}} \sum_{i,l} U_{li} \bar{l}_L \gamma_\alpha \nu_i L W^\alpha \dagger$$

(55)

We will consider first the case of the Dirac neutrino fields. The arbitrary CP phase factors of the Dirac fields can be chosen to be equal to one. For neutrino fields we have

$$V_{CP} \nu_{iL}(x) V_{CP}^{-1} = \gamma_0 C \nu^T_{iL}(x'); \quad V_{CP} \bar{\nu}_{iL}(x) V_{CP}^{-1} = -\nu^T_{iL}(x') C^{-1} \gamma_0$$

(56)

Taking into account that $C \gamma_\alpha C^{-1} = -\gamma^T_\alpha$ and $V_{CP} W^\alpha(x) V_{CP}^{-1} = \eta^\alpha (W^\alpha)\dagger(x')$ ($\eta^\alpha = (-1, 1, 1, 1)$) we find

$$V_{CP} \bar{\nu}_{iL}(x) \gamma_\alpha l_L(x) W^\alpha(x) V_{CP}^{-1} = \bar{l}_L(x') \gamma_\alpha \nu_{iL}(x') W^\alpha\dagger(x')$$

(57)

From (54), (55) and (56) we conclude that in the case of the CP invariance in the lepton sector the Dirac mixing matrix is real:

$$(U^D)^* = U^D$$

(58)
For Majorana field $\nu_i(x)$ we have
\begin{equation}
V_{CP}\nu_i(x)V_{CP}^{-1} = \eta_i^* \gamma_0 \nu_i(x'),
\end{equation}
where $\eta_i$ is the CP parity of the Majorana neutrino with mass $m_i$. This quantity can take the values $\eta_i = \pm i$. With the help of Eq. (59) in the Majorana case we obtain the following relation
\begin{equation}
V_{CP}\bar{\nu}_iL(x)\gamma_\alpha l_L(x)W_\alpha L(x) = \eta_i \bar{l}_L(x')\gamma_\alpha \nu_iL(x')W_\alpha^* L(x')
\end{equation}
From (54), (55) and (60) we find that in the case of the CP invariance in the lepton sector Majorana mixing matrix satisfies the relation
\begin{equation}
(U^M_{li})^* = \eta_i^* U^M_{li}
\end{equation}

4 Nature of neutrinos can not be revealed via investigation of neutrino oscillations

Neutrino oscillations is an interference phenomenon. Investigation of neutrino oscillations allow to measure such small values of the neutrino mass-squared differences which presumably are not reachable in other experiments. However, the study of neutrino oscillations in vacuum or in matter does not allow to reveal the nature of neutrinos with definite masses [23, 24, 25].

Let us consider neutrino oscillations in vacuum. The probability of the transition $\nu_l \rightarrow \nu_l'$ can be written in the form
\begin{equation}
P(\nu_l \rightarrow \nu_l') = \left| \sum_i U_{l'i} e^{-i\Delta m^2_{li} \frac{E}{2}} U^*_{li}\right|^2,
\end{equation}
The Majorana mixing matrix is connected with the Dirac mixing matrix by the relation (61). From Eq. (62) it is obvious that additional $n - 1$ Majorana phases $\alpha_i$ drop out from the expression for the transition probability. Thus, neutrino transition probability has the same form in the case of Dirac and Majorana neutrinos [23, 24]
\begin{equation}
P^M(\nu_l \rightarrow \nu_l') = P^D(\nu_l \rightarrow \nu_l')
\end{equation}
The investigation of neutrino transitions in matter also does not allow us to reveal the nature of massive neutrinos \[25\].

In fact, in the flavor representation the standard effective Hamiltonian of neutrino in matter is given by the expression \[26\]

\[
H^{m}_{\nu_{l}';\nu_{l}}(t) = \langle \nu_{l}'|H_{0}|\nu_{l}\rangle + \sqrt{2}G_{F}\rho_{e}(t)X_{\nu_{l}';\nu_{l}}.
\]

(64)

Here \(H_{0}\) is the free Hamiltonian, \(\rho_{e}\) is the electron number density and \(X_{\nu_{l}';\nu_{l}} = \delta_{\nu_{l}';\nu_{e}}\delta_{\nu_{l};\nu_{e}}\).

The Hamiltonian of the effective interaction of neutrino with matter (the second term of (64)) is determined by the amplitude of the elastic \(\nu_{e} - e\) scattering in the forward direction. This term does not depend on neutrino masses and mixing.

The neutrino masses and mixing enter only into the free Hamiltonian. The state of the flavor neutrino is given by the relation

\[
|\nu_{l}\rangle = \sum_{i} U^{*}_{li}|\nu_{i}\rangle, \quad (65)
\]

where \(|\nu_{i}\rangle\) is the eigenstate of the free Hamiltonian

\[
H_{0}|\nu_{i}\rangle = E_{i}|\nu_{i}\rangle; \quad E_{i} \simeq p + \frac{m_{i}^{2}}{2p}. \quad (66)
\]

From (65) and (66) (up to unessential unit matrix) we have

\[
\langle \nu_{l}'|H_{0}|\nu_{l}\rangle = \sum_{i} U_{l'i}^{*} \frac{m_{i}^{2}}{2E} U_{li} = \left(U \frac{m_{i}^{2}}{2E} U^{\dagger}\right)_{l'l}. \quad (67)
\]

Taking into account (51), we have

\[
U^{M} \frac{m_{i}^{2}}{2E} U^{M} = U^{D} \frac{m_{i}^{2}}{2E} U^{D\dagger}. \quad (68)
\]

Thus, additional Majorana CP phases \(\alpha_{i}\) do not enter into the Hamiltonian of neutrino in matter. In other words the Hamiltonian of neutrino in matter has the same form for Dirac and Majorana neutrinos and the nature of neutrinos can not be revealed through the investigation of transitions of neutrinos in matter.

5 On the equivalence of theories with massless Dirac and Majorana neutrinos

All existing data are in perfect agreement with the assumption that neutrino interaction is given by the standard Lagrangians (1) and (2).
For such an interaction theories with massless Dirac and Majorana neutrinos are equivalent \[27\]. This theorem is based on the fact that for massless neutrinos left-handed Dirac and Majorana fields are connected by the unitary transformation

\[
V \nu^D_{\nu L} V^{-1} = \nu^M_{\nu L}.
\]

(69)

The equivalence of theories with massless Dirac and Majorana neutrinos can be seen in the following way. From Majorana condition \[15\] it follows that for the Majorana field right-handed and left-handed components are connected by the relation

\[
\nu_{iR}(x) = (\nu_{iL}(x))^c
\]

(70)

In the Dirac case right-handed and left-handed components are independent. If \( m_i = 0 \), the right-handed fields do not enter into the standard Lagrangian. Hence, there is no way to distinguish Dirac and Majorana neutrinos in this case.

If neutrino masses are equal to zero \( \nu_{iL}(x) \) in \[11\] and \[12\] are quantum fields. In this case invariance of the Lagrangian under the global gauge transformations

\[
l'(x) = e^{i\Lambda_l} l(x); \quad \nu'_{iL}(x) = e^{i\Lambda_l} \nu_{iL}(x) \quad (l = e, \mu, \tau)
\]

(71)

takes place and flavor lepton numbers \( L_e, L_\mu \) and \( L_\tau \) are strictly conserved.

Thus, in the framework of the standard weak interaction observable effects of the massive Majorana neutrinos are proportional to neutrino mass. Because neutrino masses are very small, \textit{effects which allow to reveal the Majorana nature of neutrinos are strongly suppressed.}

\section{6 Violation of total lepton number in neutrino processes}

In the case of the neutrino mixing the standard Lagrangian of the CC interaction of lepton-neutrino pair with \( W \)-boson is given by \[55\]. Two types of neutrino (and antineutrino) processes can be induced by the interaction \[56\]. In the processes of the first type neutrino-production process is due to one term of the interaction \[56\] and
neutrino-detection process is induced by another term of the interaction. For example, the first term of (55) provides production of neutrino in the transition

$$W^+ \rightarrow l^+ + \nu_i$$  \hspace{1cm} (72)

Due to the second term of the Lagrangian (55) neutrino can be absorbed in the transition

$$\nu_i \rightarrow l'^- + W^+.$$  \hspace{1cm} (73)

In the physical processes, based on the transitions (72) and (73), total lepton number is conserved. Neutrino in transitions (72) and (73) can be Dirac or Majorana particles.

In the matrix elements of the processes of production and absorption of neutrino with momentum $p$ and mass $m_i$ enter the spinor $\frac{1-\gamma_5}{2} u^r(p)$. Taking into account linear in $m^2_i E$ terms, we have

$$\frac{1-\gamma_5}{2} u^r(p) = \frac{1-r}{2} u^r(p) + r \frac{m_i}{2E} \gamma_0 u^r(p).$$  \hspace{1cm} (74)

In neutrino experiments energies of neutrino are much larger than neutrino masses. Thus, in transitions (72) and (73) predominantly left-handed neutrinos are produced and absorbed.

Due to Heisenberg uncertainty relation production and absorption of different $\nu_i$ can not be resolved in neutrino experiments. As a result together with $l^+$ flavor neutrino $\nu_l$ is produced. Lepton $l'^-$ can be produced in a CC weak process by flavor neutrino $\nu_l'$. The state of flavor neutrino is given by the relation (39). All observed neutrino process are based on (72) and (73).

*If neutrinos $\nu_i$ are Majorana particles* the second type of neutrino processes are possible. For Majorana neutrinos operator $\nu_iL(x)$ is the sum of the operators of absorption and creation of neutrinos. Thus, neutrinos which are produced in transition (72) due to the first term of the Lagrangian (55) can be absorbed in the transition

$$\nu_i \rightarrow l'^+ + W^-$$  \hspace{1cm} (75)

due to the same term of the Lagrangian. It is obvious that in the chain of the processes induced by (72) and (75) the total lepton number is changed by two. In the matrix element of the absorption of neutrino
enter spinor \(\frac{1 - \gamma_5}{2} v^r(p)\) (\(v^r(p) = C (\bar{u}^r(p))^T\)). Taking into account linear in \(\frac{m_i}{2E}\) terms we have

\[
\frac{1 - \gamma_5}{2} v^r(p) = \frac{1 + r}{2} v^r(p) + r \frac{m_i}{2E} \gamma^0 v^r(p),
\]

(76)

The chain of the processes induced by the transitions (72) and (75) (for example, \(\pi^+ \rightarrow \mu^+ + \nu_i; \ \nu_i + N \rightarrow e^+ + X\)). is, however, strongly suppressed. In fact, from (74) it follows that in the neutrino-production process mainly left-handed neutrinos are produced. From (76) we see that in the cross section of the absorption of such neutrinos small factors \((\frac{m_i}{2E})^2\) enter. The probability of the production of the right-handed neutrinos, which have "large" weak absorption cross section, is suppressed by the factors \((\frac{m_i}{2E})^2\). Thus, the chain of the processes, induced by the first term of the Lagrangian (55), in which \(l^+\) and \(l'^+\) are produced, are suppressed with respect to usual neutrino processes, induced by the first and the second terms of the Lagrangian (55), by the helicity suppression factor not larger than \((\frac{m_3}{2E})^2\). \(m_3\) is the mass of the heaviest neutrino.\(^5\) Taking into account that neutrino energies in neutrino processes \(\gtrsim\) MeV and \(m_3 \lesssim 2\) eV we conclude that the suppression factor is extremely small

\[
\frac{m_3^2}{4E^2} \leq 10^{-12}.
\]

(77)

Thus, it is not possible in foreseeable future to reveal the neutrino nature in neutrino experiments via the observation of the violation of the total lepton number.\(^6\)

7 \(|\Delta L| = 2\) processes with virtual Majorana neutrinos

We will consider now \(|\Delta L| = 2\) processes with virtual Majorana neutrinos which are induced by the first (or the second) term of the Lagrangian (55). Examples of such processes are neutrinoless double

\(^5\)It is clear that the same arguments are applied to the second term of the Lagrangian (55). This term induces the chain of neutrino processes in which \(l^-\) and \(l'^-\) are produced.

\(^6\)In 1957 R. Davis [28] made an experiment in which he searched for \(^{37}\)Ar production in a process of the interaction of antineutrinos from reactor with \(^{37}\)Cl. He did not find \(^{37}\)Ar in the detector and obtain upper bound on the corresponding cross section. As we discussed before in the case of massive Majorana neutrinos such process in principle is allowed. It is suppressed, however, by the extremely small factor (77).
\( \beta \)-decay (0\( \nu \beta \beta \)-decay) of even-even nuclei

\[
(A, Z) \rightarrow (A, Z + 2) + e^- + e^-,
\]

(78)

the decays

\[
K^+ \rightarrow \pi^- + \mu^+ + \mu^+; \quad K^- \rightarrow \pi^+ + \mu^- + e^-,
\]

(79)

the process

\[
\mu^- + (A, Z) \rightarrow (A, Z - 2) + e^+,
\]

(80)

etc. The leptonic part of the operator which give contribution to matrix elements of the processes of the type (78) and (79) is given by (see \[9\])

\[
\sum_{i,k} T(\bar{L}_L(x_1) \gamma_\alpha \langle 0 | T(\nu_i L(x_1) \nu^T_{kL}(x_2)) | 0 \rangle \gamma_\beta \bar{L}_{kL}^T(x_2)) U_{li} U_{lk} \]

(81)

Let us consider neutrino propagator. From the Majorana condition (15) we have

\[
\nu^T_k = -\bar{\nu}_k C
\]

(82)

Taking into account this relation we find

\[
\langle 0 | T(\nu_i L(x_1) \nu^T_{kL}(x_2)) | 0 \rangle = -\delta_{ik} i \frac{1-\gamma_5}{(2\pi)^4} \int e^{-ip(x_1-x_2)} \frac{\gamma_p+m_i}{p^2-m_i^2} \frac{1-\gamma_5}{2} d^4 p \]

(83)

For small neutrino masses \( m_i^2 \ll p^2 \) and we have

\[
\langle 0 | T(\nu_i L(x_1) \nu^T_{kL}(x_2)) | 0 \rangle \simeq -\delta_{ik} m_i \frac{i}{(2\pi)^4} \int e^{-ip(x_1-x_2)} \frac{1}{p^2} d^4 p \frac{1-\gamma_5}{2} C \]

(84)

Thus, the propagator of the left-handed components of the neutrino fields is proportional to neutrino mass. In the limit \( m_i \rightarrow 0 \) processes of the type (77), (78), (79) are forbidden in accordance with the Dirac-Majorana equivalence theorem discussed in the section 5.

From (81) and (84) it follows that the matrix elements of the pro-
cesses of the type \( (78) \) and \( (79) \) are proportional to
\[
m_{\ell\ell'} = \sum_i U_{li} U_{l'i} m_i. \tag{85}
\]

Analogously, the matrix elements of the processes of the type \( (80) \) are proportional to \( m^*_{\ell\ell'} \). Taking into account the unitarity of the mixing matrix, we have
\[
|m_{\ell\ell'}| \leq m_3, \tag{86}
\]
where \( m_3 \) is the mass of the heaviest neutrino. If we take into account the data of the Mainz and Troitsk experiments \cite{13,14} we find
\[
|m_{\ell\ell'}| \leq 2.3 \text{ eV}. \tag{87}
\]

The probabilities of the \( |\Delta L| = 2 \) processes with virtual Majorana neutrinos are extremely small. First, they are the processes of the second order in the Fermi constant \( G_F \). And, second, they are helicity suppressed processes. In the probabilities of such processes enter very small helicity suppression factor \( \frac{m^2_3}{\langle Q^2 \rangle} \), where \( \langle Q^2 \rangle \) is an average momentum-transfer squared (typically \( \gtrsim 10 \text{ MeV}^2 \)).

The sensitivities of the experiments on the search for the processes \( (79), (80) \) and other similar processes are much worse than the upper bound \( (87) \). In the latest experiment \cite{29} on the search for the process \( \mu^- \text{Ti} \rightarrow e^+ \text{Ca} \) the following upper bound was obtained
\[
\frac{\Gamma(\mu^- \text{Ti} \rightarrow e^+ \text{Ca})}{\Gamma(\mu^- \text{Ti} \rightarrow \text{all})} \leq 1.7 \cdot 10^{-12}
\]

The best upper bound on the probability of the decay \( K^+ \rightarrow \pi^- \mu^+\mu^+ \) was reached in \cite{30}:
\[
\frac{\Gamma(K^+ \rightarrow \pi^- \mu^+\mu^+)}{\Gamma(K^+ \rightarrow \text{all})} \leq 3 \cdot 10^{-9}
\]

\(^7\)It is easy to show that the two ways of introduction of Majorana phases, discussed above, give the same result. In fact, we obtained \( (85) \) assuming that the Majorana condition has the form \( (15) \). If the Majorana condition has the form \( (53) \) we have \( \nu_k^T = -e^{2i\alpha} \bar{\nu}_k^T C \) and
\[
m_{\ell\ell'}' = \sum_i U_{li} U_{l'i} e^{2i\alpha} m_i = \sum_i U_{li} U_{l'i} m_i = m_{\ell\ell'}
\]
From these data it was obtained, correspondingly, the following bounds

\[ |m_{\mu e}| \leq 82 \text{ MeV}; \quad |m_{\mu\mu}| \leq 4 \cdot 10^4 \text{ MeV} \]

The exceptional process, sensitive to the expected Majorana neutrino masses, is neutrinoless double $\beta$-decay of some even-even nuclei. Possibilities to use large targets (in present-day experiments tens of kg, in future experiments about 1 ton and may be more), to reach small background and high energy resolution make experiments on the search for this decay an unique source of information about the nature of massive neutrinos $\nu_i$. In the next section we will consider this process in some details.

8 Neutrinoless double $\beta$-decay

8.1 Probability of the decay. Experimental data

The standard effective Hamiltonian of the weak interaction of electron-neutrino pair and hadrons is given by

\[ H_{T}^{CC} = \frac{G_F}{\sqrt{2}} 2 \bar{\nu}_e \gamma_\alpha \nu_e j^\alpha + \text{h.c.}, \]  

(88)

where $j^\alpha$ is the hadronic charged current. We will assume the Majorana neutrino mixing

\[ \nu_{eL} = \sum_i U_{ei} \nu_{iL}, \]  

(89)

where $\nu_i$ is the field of Majorana neutrino with mass $m_i$. The matrix element of $0\nu\beta\beta$-decay is given by the following expression

\[
    \langle f \mid S \mid i \rangle = -4 \left( \frac{G_F}{\sqrt{2}} \right)^2 \frac{i}{(2\pi)^4} N_{p_1} N_{p_2} m_{\beta\beta} \bar{u}(p_2) \gamma_\alpha \frac{1}{2} - \gamma_\beta \frac{1}{2} \bar{\nu}_e \gamma_\alpha \nu_\alpha \int e^{ip_1 x_1 + ip_2 x_2} e^{-ip(x_1-x_2)} \frac{1}{p^2} \langle \Psi_f | T(J^\alpha(x_1)J^\beta(x_2)) \Psi_i \rangle d^4x_1 d^4x_2 d^4p \]

(90)

Here

\[ m_{ee} \equiv m_{\beta\beta} = \sum_i U_{ei}^2 m_i. \]  

(91)

is effective Majorana mass, $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are vectors of the initial and final nuclei, $N_{p_i} = \frac{1}{(2\pi)^3 \sqrt{2p_i^0}}$ and $J^\alpha(x)$ is hadronic CC in
Heisenberg representation. From (90) for half-life of the $0\nu\beta\beta$-decay the following expression can be obtained (see reviews [32, 33])

$$\frac{1}{T_{1/2}(A, Z)} = |m_{\beta\beta}|^2 |M(A, Z)|^2 G^{0\nu}(E_0, Z).$$  \hspace{1cm} (92)

Here $M(A, Z)$ is nuclear matrix element (NME) and $G^{0\nu}(E_0, Z)$ is phase-space factor ($E_0$ is the energy release). Let us stress that NME is determined only by nuclear properties and strong interaction and does not depend on neutrino masses and mixing.\footnote{If $\nu_i$ are Majorana particles the $0\nu\beta\beta$-decay mechanism considered here definitely exists. Let us notice that there are different beyond the Standard Model mechanisms of violation of the total lepton number and $0\nu\beta\beta$-decay: the right-handed currents, SUSY with violation of $R$-parity, etc. (see [33])}

There exist at present data of many experiments on the search for $0\nu\beta\beta$-decay (see [34]). The most stringent lower bounds on the half-time of $0\nu\beta\beta$-decay were obtained in the recent CUORICINO [37] and in the Heidelberg-Moscow [38] experiments.\footnote{An indication in favor of the $0\nu\beta\beta$-decay of $^{76}$Ge, found in [35], is going to be checked by the GERDA experiment started at Gran Sasso [36].}

CUORICINO is a cryogenic experiment on the search for $0\nu\beta\beta$ decay of $^{130}$Te. An array of 62 TeO$_2$ crystals with total mass 40.7 kg was placed in a refrigerator at the temperature $T = 8$ mK. Heat capacity is proportional to $T^3$ and the increase of the temperature at tiny energy release can be recorded in the experiment. $^{130}$Te nuclei has high natural abundance (33.8\%) and relatively large $Q$-value ($Q = 2528.8 \pm 1.3$ keV). The background in $0\nu\beta\beta$ region in the CUORICINO experiment was 0.18 counts/keV kg year. No signal in the region of the $0\nu\beta\beta$ decay of $^{130}$Te was found. For the half-life the following lower bound was obtained in the CUORICINO experiment [37]

$$T_{1/2}^{0\nu}(^{130}\text{Te}) \geq 1.8 \cdot 10^{24} \text{ years}; \hspace{1cm} (90\%\text{CL}) \hspace{1cm} (93)$$

Taking into account different calculations of NME from [33] for the upper bound of the effective Majorana mass the following range of values was inferred

$$|m_{\beta\beta}| \leq (0.2 - 1.1) \text{ eV}. \hspace{1cm} \text{(CUORICINO)} \hspace{1cm} (94)$$

In the Heidelberg-Moscow experiment [38] the $0\nu\beta\beta$ decay of $^{76}$Ge was studied. 5 crystals of 86 \% enriched $^{76}$Ge with the total mass 11.5 kg was used. In the $0\nu\beta\beta$ range of energies the energy resolution
and background were 4.23 ± 0.14 keV and 0.163 counts /kg keV year. For the half-life of $^{76}$Ge the following lower bound was obtained in the experiment:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 1.55 \cdot 10^{25}\text{ years}$$  \hspace{1cm} (95)

Taking into account uncertainties in NME calculations from this result for the effective Majorana mass the following bounds can be obtained

$$|m_{\beta\beta}| \leq (0.3 - 1.2) \text{ eV}. \quad \text{(Heidelberg - Moscow)}$$  \hspace{1cm} (96)

Several future experiments on the search for $0\nu\beta\beta$-decay are in preparation at present. Detectors in these experiments will be much larger than in today’s experiments (about 1 ton and even more). All groups, preparing new experiments, plan to decrease significantly background and to improve energy resolution. The aim of the future experiments on the search for $0\nu\beta\beta$-decay is to reach sensitivity

$$|m_{\beta\beta}| \simeq \text{a few } 10^{-2} \text{ eV.}$$  \hspace{1cm} (97)

We will mention several proposals (for more detailed discussion see [39]).

The CUORE experiment [37] will be based on the same technique as the CUORICINO experiment. An array of 988 of TeO$_2$ crystals with total mass 741 kg will be used. Significant reduction of the background (up to $10^{-2} - 10^{-3}$ counts/keV kg year) and improvement of the energy resolution (≈ 5 keV) are planned to be reached. Expected sensitivity to the effective Majorana mass will be about $(2 - 3) \cdot 10^{-2}$ eV.

Majorana collaboration [40] plans to use 500 kg of 86 % enriched $^{76}$Ge. It is expected that the granularity of the detector and improved pulse-shape analysis will reduce background significantly. The anticipated sensitivity of the Majorana experiment is equal to

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \simeq 4 \cdot 10^{27}\text{ years}$$  \hspace{1cm} (98)

This corresponds to the following sensitivity to the effective Majorana mass

$$|m_{\beta\beta}| \simeq (2 - 7) \cdot 10^{-2} \text{ eV.}$$  \hspace{1cm} (99)

In the EXO experiment [41] $0\nu\beta\beta$-decay of $^{136}$Xe will be searched for. In this experiment not only the total energy of two electrons will be measured but also daughter nuclei will be detected. In the $0\nu\beta\beta$-decay of $^{136}$Xe ion $^{136}\text{Ba}^{++}$ is produced. This ion will be neutralized
to $^{136}\text{Ba}^+$ and localized. Then $^{136}\text{Ba}^+$ ion will be optically detected through the irradiation by photons from two lasers. It is expected that about $10^7$ photons/sec will be emitted by one ion. If the method of detection of $^{136}\text{Ba}^+$ will be realized the only background in the EXO experiment will come from $2\nu\beta\beta$-decay. Improvement of the energy resolution is crucial for the success of the experiment. With 10 ton enriched $^{136}\text{Xe}$ TPC detector and $^{136}\text{Ba}^+$ tagging the sensitivity

$$T_{1/2}^{0\nu}(^{136}\text{Xe}) \simeq 1 \cdot 10^{28} \text{ years}$$

is expected. This sensitivity corresponds to

$$|m_{\beta\beta}| \simeq (1.3 - 3.7) \cdot 10^{-2} \text{ eV}.$$ 

### 8.2 Effective Majorana mass and neutrino oscillation data

The effective Majorana mass $m_{\beta\beta}$ is determined by the values of neutrino masses $m_i$ and elements $U_{ei}^2$. In the standard parametrization we have

$$U_{e1}^2 = \cos^2 \theta_{13} \cos^2 \theta_{12} e^{2i\alpha_1}; \quad U_{e2}^2 = \cos^2 \theta_{13} \sin^2 \theta_{12} e^{2i\alpha_2};$$

$$U_{e3}^2 = \sin^2 \theta_{13} e^{2i\alpha_3}.$$ 

Information about the parameters $\tan^2 \theta_{12}$ and $\sin^2 \theta_{13}$ can be obtained from the data of solar-KamLAND and CHOOZ experiments (see (46) and (41)). The phases $\alpha_i$ are unknown.

The values of the neutrino masses are determined by the neutrino mass-squared differences $\Delta m_{23}^2$ and $\Delta m_{12}^2$, which are known from the data of the Super-Kamiokande and solar-KamLAND experiments (see (45) and (46)), the mass of the lightest neutrino and character of the neutrino mass spectrum.

For three neutrino masses two neutrino mass spectra are possible:

1. Normal spectrum

$$m_1 < m_2 < m_3; \quad \Delta m_{12}^2 \ll \Delta m_{23}^2$$
2. Inverted spectrum\textsuperscript{10}

\[ m_3 < m_1 < m_2; \; \Delta m_{12}^2 \ll |\Delta m_{13}^2| \]  

(104)

For neutrino masses in the case of the normal spectrum we have

\[ m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}, \; m_3 = \sqrt{m_1^2 + \Delta m_{12}^2 + \Delta m_{23}^2}. \]  

(105)

In the case of the inverted spectrum neutrino masses are given by

\[ m_1 = \sqrt{m_3^2 - \Delta m_{13}^2}, \; m_2 = \sqrt{m_3^2 - \Delta m_{13}^2 + \Delta m_{12}^2}. \]  

(106)

The lightest neutrino mass \( m_1(m_3) \) is at present unknown. Upper bound for the neutrino mass obtained from the data of the tritium experiments, is given in (23).

In the leading approximation neutrino transition probabilities have the same form for both types of neutrino mass spectra. Thus, in order to reveal the type of the neutrino mass spectrum it is necessary to study small effects beyond the leading approximation. The size of such effects and possibilities to measure them depend on the value of the parameter \( \sin^2 \theta_{13} \). We will see that the investigation of the neutrinoless double-\( \beta \)-decay will allow us to obtain an information about neutrino mass spectrum independently on the value of the small parameter \( \sin^2 \theta_{13} \).

We will consider three standard neutrino mass spectra (see [43])

I. Hierarchy of neutrino masses

\[ m_1 \ll m_2 \ll m_3 \]  

(107)

In this case neutrino masses \( m_{2,3} \) are determined by neutrino mass-squared differences:

\[ m_2 \simeq \sqrt{\Delta m_{12}^2} \simeq 8.9 \cdot 10^{-3} \text{eV}; m_3 \simeq \sqrt{\Delta m_{23}^2} \simeq 4.9 \cdot 10^{-2} \text{eV}. \]  

(108)

\textsuperscript{10}In order to keep for the solar-KamLAND neutrino mass-squared difference notation \( \Delta m_{12}^2 > 0 \), neutrino masses are usually labeled differently in the cases of normal and inverted neutrino spectra. In the case of the normal spectrum \( \Delta m_{23}^2 > 0 \) and in the case of the inverted spectrum \( \Delta m_{13}^2 < 0 \). Thus, with such notations of the neutrino masses the character of the neutrino mass spectrum is determined by the sign of atmospheric neutrino mass-squared difference. It clear, however, that the sign of of the atmospheric neutrino mass-squared difference has no physical meaning: it is a convention based on the notation \( \Delta m_{ik}^2 = m_k^2 - m_i^2 \). Notice that in both cases of neutrino mass spectra for mixing angles the same notations can be used.

\[23\]
The lightest neutrino mass satisfies inequality: $m_1 \ll \sqrt{\Delta m_{12}^2}$. Neglecting the contribution of $m_1$, for the effective Majorana mass we obtain the following expression

$$|m_{\beta\beta}| \simeq \left| \sin^2 \theta_{12} \sqrt{\Delta m_{12}^2} + e^{i \alpha_{23}} \sin^2 \theta_{13} \sqrt{\Delta m_{23}^2} \right|, \quad (109)$$

where $\alpha_{23} = \alpha_3 - \alpha_2$.

The first term in Eq. (109) is small because of the smallness of $\sqrt{\Delta m_{12}^2}$. Contribution of “large” $\sqrt{\Delta m_{23}^2}$ is suppressed by the small factor $\sin^2 \theta_{13}$. If we will use the CHOOZ bound (41) the modulus of both terms in (109) are approximately equal. Hence the terms in (109) could even cancel each other.\(^{11}\) From (45), (46) and (109) for the upper bound of the effective Majorana mass we find the value

$$|m_{\beta\beta}| \leq 6.6 \cdot 10^{-3} \text{ eV}. \quad (110)$$

Thus, in the case of the neutrino mass hierarchy upper bound of $|m_{\beta\beta}|$ is smaller that the best expected sensitivity of the future experiments on the search for $0\nu\beta\beta$-decay.

II. Inverted hierarchy of the neutrino masses

$$m_3 \ll m_1 < m_2. \quad (111)$$

For neutrino masses $m_{1,2}$ in the case of the inverted hierarchy we have

$$m_1 \simeq \sqrt{|\Delta m_{13}^2|}; \quad m_2 \simeq \sqrt{|\Delta m_{13}^2|} (1 + \frac{\Delta m_{12}^2}{2|\Delta m_{13}^2|}) \simeq \sqrt{|\Delta m_{13}^2|}. \quad (112)$$

The lightest neutrino mass is small: $m_3 \ll \sqrt{|\Delta m_{13}^2|}$.

Neglecting the contribution of the small term $m_3 \sin^2 \theta_{13}$, for the effective Majorana mass we obtain the following expression

$$|m_{\beta\beta}| \simeq \sqrt{|\Delta m_{13}^2|} (1 - \sin^2 2\theta_{12} \sin^2 \alpha_{12})^{\frac{1}{2}}, \quad (113)$$

where the only unknown parameter is $\sin^2 \alpha_{12}$. For the effective Majorana mass we obtain the following range of values of the effective Majorana mass

$$\cos 2\theta_{12} \sqrt{|\Delta m_{13}^2|} \leq |m_{\beta\beta}| \leq \sqrt{|\Delta m_{13}^2|} \quad (114)$$

\(^{11}\)For detailed calculations of the probability of the $0\nu\beta\beta$-decay of different nuclei in the case of the neutrino mass hierarchy see [42]
From analysis of the solar-KamLAND neutrino oscillation data it was found that $\theta_{12} < \pi/4$ (see (46)). Thus, the lower bound of the effective Majorana mass in the case of the inverted mass hierarchy is different from zero. From (45), (46) and (114) we find the following 90 % CL range

$$0.9 \cdot 10^{-2} \leq |m_{\beta\beta}| \leq 5.8 \cdot 10^{-2} \text{ eV} \quad (115)$$

The anticipated sensitivities to $|m_{\beta\beta}|$ of the future experiments on the search for $0\nu\beta\beta$ are in the range (115). Thus, next generation of the $0\nu\beta\beta$- experiments will probe the inverted hierarchy of the neutrino masses.

III. Quasi-degenerate neutrino mass spectrum

If the lightest neutrino mass satisfies inequality

$$m_1 \gg \sqrt{\Delta m_{23}^2} \quad (m_3 \gg \sqrt{|\Delta m_{13}^2|}) \quad (116)$$

neutrino mass spectrum is practically degenerate

$$m_1 \simeq m_2 \simeq m_3 \quad (117)$$

For the effective Majorana mass we have in this case

$$|m_{\beta\beta}| \simeq m_0 \left(1 - \sin^2 2\theta_{12} \sin^2 \alpha_{12}\right)^{\frac{1}{2}}, \quad (118)$$

where $m_0$ is the common neutrino mass. From this expression we have

$$\cos 2\theta_{12} m_0 \leq |m_{\beta\beta}| \leq m_0 \quad (119)$$

The common mass $m_0$ can be measured in tritium $\beta$-decay experiments. The expected sensitivity of the future KATRIN experiment [44] is

$$m_0 \simeq 0.2 \text{ eV}. \quad (120)$$

In the case of the quasi-degenerate neutrino mass spectrum an information about the value of $m_0$ can be also obtained from the $0\nu\beta\beta$-decay data. From (46) and (114) we find

$$|m_{\beta\beta}| \leq m_0 \leq 4.4 |m_{\beta\beta}| \quad (121)$$

The three neutrino mass spectra, we have considered, correspond to different mechanisms of neutrino mass generation (see [45]). Masses of quarks and charged leptons satisfy hierarchy of the type (107). Hierarchy of neutrino masses is a typical feature of GUT models (like
SO(10)) in which quarks and leptons are unified. Inverted spectrum and quasi-degenerate spectrum require specific symmetries of the neutrino mass matrix.

We will discuss now briefly possibilities to reveal CP violation in the case of the Majorana neutrinos. (for the detailed discussion see [46]) We will consider only inverted and quasi-degenerate neutrino mass spectra. If CP is conserved in the lepton sector, neutrino mixing matrix satisfies the condition (see section 3)

\[ U_{li} = \eta_i U_{li}^*, \]  

(122)

where \( \eta_i = \pm i \) is the CP parity of \( \nu_i \). Let us present \( \eta_i \) in the form

\[ \eta_i = e^{i \frac{\pi}{2} \rho_i}, \]  

(123)

where \( \rho_i = \pm 1 \). From (122) and (123) we find

\[ e^{2i \alpha_i} = e^{i \frac{\pi}{2} \rho_i} \]

(124)

In the quark sector violation of the CP invariance can be revealed

- through the observation of the processes which are forbidden if CP is conserved
- through the measurement of CP-odd asymmetries which are equal to zero in the case of CP conservation.

The observation in 1964 of the decay \( K_L \rightarrow \pi^+\pi^- \) [54], which is forbidden if CP is conserved, marked the discovery of the CP violation in the quark sector. The example of the second type of measurement was high precision measurement at the B-factories of the CP-asymmetry in the decays \( B^0(\bar{B}^0) \rightarrow J/\Psi K_S \) [55, 56].

The violation of CP invariance in the case of Majorana neutrinos can be revealed only through the measurement of the value of the effective Majorana mass. From (113) and (118) we find

\[ \sin^2 \alpha_{12} = \frac{1}{\sin^2 2\theta_{12}} \left( 1 - \frac{|m_{\beta\beta}|^2}{|\Delta m^2_{13}|} \right) \]  

(125)

and

\[ \sin^2 \alpha_{12} = \frac{1}{\sin^2 2\theta_{12}} \left( 1 - \frac{|m_{\beta\beta}|^2}{m_0^2} \right) \]  

(126)

correspondingly, for inverted hierarchy and quasi-degenerate spectrum.
In the case of the CP conservation from (124) we have
\[ \sin^2 \alpha_{12} = \sin^2 \frac{\pi}{4} (\rho_2 - \rho_1) \] (127)

Thus if CP is conserved parameter \( \sin^2 \alpha_{21} \) takes \textit{two values}
\[ \sin^2 \alpha_{21} = 0 \ (\rho_2 = \rho_1); \ \sin^2 \alpha_{21} = 1 \ (\rho_2 = \rho_1). \] (128)

These values correspond to the upper and lower bounds in (114) and (119).

In the case of inverted hierarchy the determination of the CP parameter \( \sin^2 \alpha_{21} \) requires only measurement of the effective Majorana mass.\(^{12}\) In the case of the quasi-degenerate spectrum common mass \( m_0 \) must be also known.

The major uncertainty in the determination of the effective Majorana mass \( |m_{\beta\beta}| \) is connected with nuclear matrix elements. We will discuss this problem in the next section. Even if the problem of NME will be solved, CP violation in the case of the Majorana mixing can be revealed only if half-lives of \( 0\nu\beta\beta \)-decay will be measured with high accuracy \(^{16}\). For illustration let us consider in the case of the inverted hierarchy of neutrino masses maximal CP violation \( (\alpha_{21} = \frac{\pi}{4}) \) and CP conservation. For effective Majorana mass we have, correspondingly.

\[ |m_{\beta\beta}|_{\text{max}} = 0.76 \sqrt{|\Delta m^2_{31}|} \]
\[ |m_{\beta\beta}|_{\text{CP1}} = \sqrt{|\Delta m^2_{31}|}; \ |m_{\beta\beta}|_{\text{CP2}} = 0.38 \sqrt{|\Delta m^2_{31}|} \] (129)

Apparently, it will be easier to exclude one of the CP values of the effective Majorana mass than to decide whether CP is violated or not.

### 8.3 On nuclear matrix elements

Effective Majorana mass is not directly measurable quantity. From experimental data only the product of the effective Majorana mass and nuclear matrix element can be obtained. In order to determine \( m_{\beta\beta} \) we must know nuclear matrix elements.

\(^{12}\)Notice that parameters \( |\Delta m^2_{13}| \) and \( \sin^2 2 \theta_{12} \) will be known from the data of future experiments with much better accuracy than today. In T2K experiment the parameter \( |\Delta m^2_{31}| \) will be measured with an accuracy 5\% \(^{17}\). In future solar neutrino experiments, in which \( pp \) neutrinos will be detected, an accuracy \( \simeq 5\% \) in the measurement of the parameter \( \sin^2 \theta_{12} \) is planned to be reached \(^{18}\).
The calculation of NME is a complicated nuclear problem (see reviews \cite{49}). NME is the matrix element of an integral which includes the T-product of two hadronic charged weak currents and neutrino propagator. Many intermediate nuclear states must be taken into account in calculations.

Two different approaches are used for the calculation of NME: Nuclear Shell Model (NSM) and Quasiparticle Random Phase Approximation (QRPA). In literature exist many QRPA-based models. Different calculations of NME for the same nuclear transition differ by factor 2-3 and more. In such a situation it is important to find a possibility to test NME calculations.

We will discuss here such a possibility \cite{50} which is based on the factorization property of the matrix element of $0\nu\beta\beta$-decay (see section 7).

As we have discussed in the beginning of this section in several future experiments on the search for $0\nu\beta\beta$-decay of different nuclei comparable sensitivities to $|m_{\beta\beta}|$ are expected. Thus, if $0\nu\beta\beta$-decay of one nuclei will be discovered in a future experiment it is quite probable that the decay will be observed also in other experiments with different nuclei.

A model of the calculation of NME is compatible with data only in the case if the value of the effective Majorana mass, determined from the results of experiments on the detection of $0\nu\beta\beta$-decay of different nuclei, is the same. From this requirement for a model $M$ we obtain the following relation

$$R_M(i;k) = \frac{T_{1/2}^{0\nu}(A_k,Z_k)}{T_{1/2}^{0\nu}(A_i,Z_i)}$$

where

$$R_M(i;k) = \left( \frac{|M^{0\nu}(A_i,Z_i)|^2}{|M^{0\nu}(A_k,Z_k)|^2} \right)_M \frac{G^{0\nu}(E^i_0,Z_i)}{G^{0\nu}(E^k_0,Z_k)}.$$  \hspace{1cm} (131)

For illustration we will consider three latest models of NME calculations:

- $(M_1)$ Shell Model \cite{51}
- $(M_2)$ QRPA model \cite{52} (QRPA parameter $g_{pp}$ is determined from the data of the experiments on the measurement of half-lives of the $2\nu\beta\beta$-decay.)
• $(M_3)$ QRPA model [53](QRPA parameters are determined from the \(\beta\)-decay data of nearby nuclei)

The results of the calculation of the ratios \(R(^{130}\text{Te}; ^{76}\text{Ge}), R(^{136}\text{Xe}; ^{76}\text{Ge})\)

and \(R(^{130}\text{Te}; ^{136}\text{Xe})\), are presented in the Table I.
Table I

The ratios $R_m(i; k)$, calculated in three recent NME models $M_1$ [51], $M_2$ [52], $M_3$ [53].

|                  | $M_1$ | $M_2$ | $M_3$ |
|------------------|-------|-------|-------|
| $R^{(130\text{Te}; 76\text{Ge})}$ | 4.08  | 2.65  | 7.76  |
| $R^{(76\text{Ge}; 136\text{Xe})}$ | 0.56  | 0.80  | 0.07  |
| $R^{(130\text{Te}; 136\text{Xe})}$ | 2.29  | 2.11  | 0.53  |

Because we use the factorization property of the matrix elements of the $0\nu\beta\beta$-decay we can compare with experimental data only ratios of NME. However, for the determination of the effective Majorana mass we need to know the value of NME. It could happen that for specific nuclei the ratios of NME calculated in different models are practically the same, in spite the values of NME being different. We see from the Table I that $R^{(130\text{Te}; 136\text{Xe})}$ for the models $M_1$ and $M_2$ differ less than 10%. However, the values of the effective Majorana mass which can be obtained with the help of these two models are quite different:

$$|m_{\beta\beta}|^2_{M_1} = 1.90 \ |m_{\beta\beta}|^2_{M_2} \quad (132)$$

It is evident from the Table I that the observation of the $0\nu\beta\beta$-decay of $^{130}\text{Te}$ and $^{76}\text{Ge}$ could easily allow to decide which of the three considered models is compatible with data (if any). Generally, we can conclude that the observation of $0\nu\beta\beta$-decay of three (or more) nuclei would be an important tool for the test of the models of NME calculation and for the determination of the value of the effective Majorana mass.

9 Conclusion

Discovery of neutrino oscillations driven by small neutrino masses and neutrino mixing took many years of enormous efforts of many physicists. From the point of view of the modern physics it is quite natural
that neutrinos have masses\textsuperscript{13}. The puzzling feature of the discovered phenomenon is extreme smallness of neutrino masses. The most natural see-saw explanation of the smallness of neutrino masses requires a violation of the total lepton number and Majorana neutrinos.

Neutrino oscillations are very sensitive to small neutrino mass-squared differences. However, neutrino oscillations and transition of neutrinos in matter are blind to the nature of $\nu_i$. The Majorana nature of $\nu_i$ can be established only through the observation of processes in which total lepton number is not conserved. It is a general feature of the standard weak interaction that the probabilities of such processes are very strongly suppressed. Investigation of the $0\nu\beta\beta$-decay of even-even nuclei is the most sensitive probe of the Majorana nature of neutrinos. Experiments on the search for $0\nu\beta\beta$-decay have very good chances to solve the fundamental problem of the nature of neutrinos with definite masses.

The observation of $0\nu\beta\beta$-decay would be a direct proof that $\nu_i$ are Majorana particles. From the study of this decay very important quantity, the effective Majorana mass $|m_{\beta\beta}|$, can be inferred. Determination of $|m_{\beta\beta}|$ would allow to obtain an information about the pattern of the neutrino mass spectrum and lightest neutrino mass. However, to determine $|m_{\beta\beta}|$ from experimental data we need to know nuclear matrix elements. The calculations of NME is a challenging problem for nuclear physics. At the moment there is no agreement between different calculations. Further progress is definitely needed. A possible test of NME calculations was discussed here.

If $\nu_i$ are Majorana particles neutrino mixing matrix contain additional (with respect to the Dirac case) CP phases (two for the three-neutrino mixing). Effect of these phases can be revealed through the study of $0\nu\beta\beta$-decay. However, this can be done only after the problem of NME will be solved. The high-precision measurement of the half-lives of $0\nu\beta\beta$- decay will also be required.

We can consider four possible scenarios assuming that the problem of NME will be solved and expected sensitivities of future experiments on the search for $0\nu\beta\beta$-decay and on the search for distortion of the end-point part of the $\beta$-spectrum of tritium will be reached.

1. In the KATRIN experiment effect of the neutrino mass is de-

\textsuperscript{13}However, at the sixties and at the seventies when first ideas of neutrino oscillations and mixing were proposed \cite{1S19} there was a common opinion, based on the success of the two-component neutrino theory, that neutrinos are massless particles.
ected, but $0\nu\beta\beta$-decay is not observed. This would mean that neutrino spectrum is quasi-degenerate and $\nu_i$ are Dirac particles.

2. Neutrinoless double $\beta$-decay with effective Majorana mass in the range $(119)$ is observed but in the KATRIN experiment effect of the neutrino mass is not detected. This would mean that $\nu_i$ are Majorana particles, spectrum is quasi-degenerate, but the sensitivity of the KATRIN experiment is not enough to see effect of neutrino mass.

3. Neutrinoless double $\beta$-decay of different nuclei is observed with $|m_{\beta\beta}|$ is in the range $(114)$. It would be a proof that $\nu_i$ are Majorana particles and inverted hierarchy of neutrino masses is realized.

4. Neutrinoless double $\beta$-decay is not observed in the future experiments and in the KATRIN experiment no distortion of $\beta$-spectrum of tritium is detected. This would mean that either $\nu_i$ are Dirac particles with masses smaller than the sensitivity of the KATRIN experiment or $\nu_i$ are Majorana particles but spectrum is hierarchical.

In some years we will apparently know which scenario was prepared for us by the Nature.

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