Microscopic collective inertial masses for nuclear reaction in the presence of nucleonic effective mass

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Collective inertial mass coefficients with respect to translational, relative, and rotational motions are microscopically calculated, along the collective reaction path self-consistently determined, based on the adiabatic self-consistent collective coordinate (ASCC) method. The impact of the time-odd component of the mean-field potential on the inertial masses are investigated. The results are compared with those calculated with the cranking formulae. The inertial masses based on the ASCC method reproduce the exact total nuclear mass for the translational motion as well as the exact reduced masses as the asymptotic values for the relative and rotational motions. In contrast, the cranking formulae fail to do so. This is due to the fact that the (local) Galilean invariance is properly restored in the ASCC method, but violated in the cranking formulae. A model Hamiltonian for low-energy nuclear reaction is constructed with the microscopically derived potentials and inertial masses. The astrophysical $S$-factors are calculated, which indicates the importance of microscopic calculation of proper inertial masses.

I. INTRODUCTION

Theoretical description of low-energy nuclear reaction with a solid microscopic foundation is still a challenging subject in nuclear physics. It may provide us with a deep insight into the reaction mechanism and the quantum dynamics of many-nucleon systems.

Starting from a microscopic many-body Hamiltonian (an energy density functional), the time-dependent Hartree-Fock (TDHF) method [1–3] has been established as a useful tool to study low-energy nuclear reactions. The TDHF real-time simulations have been intensively carried out with significant developments [2, 4–9], including the time-dependent Hartree-Fock-Bogoliubov (TDHFB) calculations [5, 10–17]. In the TDHF(B) simulations, the reaction path is given by a time-dependent generalized Slater determinant which is determined by the TDHF(B) equations as an initial-value problem. It is successful in describing the microscopic dynamics in major reaction channels on average. However, it is known that important quantum effects are missing in the TDHF(B). For instance the sub-barrier fusion reaction and spontaneous fission can not be properly described within the TDHF(B) framework. The reaction path is given by an energy density functional, the time-dependent Hartree-Fock (TDHF) method [1–3] has been established as a useful tool to study low-energy nuclear reactions. The TDHF real-time simulations have been intensively carried out with significant developments [2, 4–9], including the time-dependent Hartree-Fock-Bogoliubov (TDHFB) calculations [5, 10–17]. In the TDHF(B) simulations, the reaction path is given by a time-dependent generalized Slater determinant which is determined by the TDHF(B) equations as an initial-value problem. It is successful in describing the microscopic dynamics in major reaction channels on average. However, it is known that important quantum effects are missing in the TDHF(B). For instance the sub-barrier fusion reaction and spontaneous fission can not be properly described within the real-time TDHF(B) simulations [1, 5, 18].

To recover the quantum fluctuations of nuclear collective dynamics that is missing in TDHF(B), we aim at the requantization of TDHF(B) on the collective subspace [5]. First, we find a collective subspace spanned by a few selected collective canonical variables, which is well decoupled from the intrinsic excitations. The collective subspace and the collective variables can be extracted using the adiabatic self-consistent collective coordinate (ASCC) method [5, 19–22]. The ASCC is derived from the invariance principle of the TDHF(B) equation [23] under an assumption of adiabatic (slow) collective motion, namely, second-order expansion with respect to the collective momenta [19]. It also has a close connection with a theory of large amplitude collective motion developed in Refs. [24, 25]. The collective subspace in the adiabatic regime is given by a series of time-even Slater determinants and generators of collective coordinates and momenta locally defined at each state. In the present paper, we apply the method to the low-energy nuclear reaction, to identify the optimal reaction path and the canonical variables. The requantization is performed on this self-consistently determined collective reaction path.

One of the key ingredients of the requantization procedure is the inertial mass parameters with respect to the collective coordinates. This is essential for the construction of the collective Hamiltonian. For the nuclear reaction, the relative distance $R$ between two colliding nuclei should be a proper choice of the collective coordinate in the asymptotic region ($R \to \infty$). The corresponding inertial mass should be the reduced mass $\mu_{\text{red}} = A_PA_Tm/(A_P + A_T)$, where $A_P$ ($A_T$) is the mass number of the projectile (target) nucleus and $m$ is the bare nucleon mass. Therefore, a theory can be tested by examining its asymptotic limit. On the other hand, the inertial mass in the interior region where two nuclei touch each other is highly nontrivial. Thus, a microscopic theory to calculate the mass on the entire reaction path is necessary.

A commonly used method to calculate the collective mass coefficient is the cranking formula [26–28] which is derived based on the adiabatic perturbation theory. However, it is known that the cranking formula fails to re-
produce the total mass of a whole nucleus for the transla-
tional motion, when the mean-field potential has velocity
dependence [18]. This is due to violation of the Galilean
invariance in the mean-field level which must be corrected
by residual fields caused by the translational motion [29].
From this failure, we may expect that the cranking for-
ma cannot reproduce the reduced mass for the relative
motion of two nuclei at \( R \to \infty \). Since most of the real-
istic nuclear energy density functionals give effective mass,
\( m^*/m < 1 \), the mean-field potential indeed has the velo-
city dependence. Therefore, it is highly desirable to find
a proper method for the calculation of inertial masses.
In this paper, we show that the ASCC method, which is
capable of taking into account the residual effects miss-
ing in the cranking formula, provides a promising tool to
microscopically calculate the proper inertial masses.

In our previous works [30, 31], we calculated the ASCC
inertial masses for the relative motion between two nu-
clei, for the velocity-independent mean-field potential.
We also examined those of the cranking formula, which
turned out to be almost identical to the ASCC mass at
\( R \to \infty \). However, this is not true in general. In this pa-
per, we show that the asymptotic value in the cranking
formula for a velocity-dependent potential does not agree
with the reduced mass, while that in the ASCC method
exactly reproduces it.

In general, the non-local mean-field potential produces
the effective mass \( m^*/m \neq 1 \). For the energy density
functionals of the Skyrme type, the effective mass comes
from the \( t_1 \) and \( t_2 \) terms (momentum-dependent terms
of the Skyrme interaction). In order to guarantee the
Galilean invariance of the energy density functional, we
need densities that are time-odd with respect to the time-
reversal transformation. These time-odd densities vanish
in the ground state of even-even nuclei. Therefore they
do not contribute to the static mean-field potential. How-
ever, the time-odd mean fields may appear and play im-
portant roles in time-dependent dynamics, because the
time-odd densities are non-zero in general for the time-
dependent states. In this paper, we investigate the im-
 pact of the time-odd mean fields on the collective inertial
masses in the context of low-energy reaction dynamics of
light nuclei.

The paper is organized as follows: In Sec. II, we re-
capitulate the theoretical methods, mainly the ASCC
method and cranking formulae, for the calculation of in-
ertial masses. In Sec. III, in the presence of time-odd
mean-field potential, we apply different methods to cal-
culate the mass coefficients for the translational, relative
and rotational motions, the applications are carried out
for the reaction systems of \( \alpha + \alpha \to ^{8}\text{Be}, \alpha + ^{16}\text{O} \to ^{20}\text{Ne} \)
and \( ^{16}\text{O} + ^{16}\text{O} \to ^{32}\text{S} \). The impacts of inertial masses on
the \( S \)-factors of sub-barrier fusion are discussed. A sum-
mary and concluding remarks are given in Sec. IV.

II. THEORETICAL METHODS

It is customary to use a model Hamiltonian for studies
of low-energy nuclear reactions. The relative distance \( R \)
between the projectile and the target nuclei is assumed to
be a dynamical coordinate, then, the Schrödinger equa-
tion is given in a form,
\[
\left\{ -\frac{1}{2\mu_{\text{red}}} \frac{d^2}{dR^2} + \ell(\ell + 1) 2\mu_{\text{red}}R^2 + V(R) - E \right\} u_{\ell}(R) = 0, \tag{1}
\]
where \( \mu_{\text{red}} \) is the reduced mass, \( \mu_{\text{red}} = A_P A_T m/ (A_P + A_T) \).
We use the unit \( \hbar = 1 \) throughout this paper.
Here, in addition to the assumption of the coordinate \( R \),
the two nuclei are approximated as structureless “point
” particles, which leads to the trivial inertial masses, \( \mu_{\text{red}} \)
and \( \mu_{\text{red}}R^2 \). This approximation becomes exact in the
asymptotic limit \( (R \to \infty) \). However, at finite distance
\( R \), even if the two nuclei are well apart, it is not triv-
ial to justify the point-particle approximation. In fact,
as we show later, the rigid-body values of the moments of
inertia are significantly different from those of the point-particle approximation, \( J_{\text{rigid}} \neq \mu_{\text{red}}R^2 \), even at
\( R > R_P + R_T \) where \( R_P (R_T) \) indicates the radius of the projectile (target) nucleus. It should be noted that
the moments of inertia for deformed nuclei are given by
the rigid-body values in the harmonic-oscillator-potential
model [29]. Therefore, it is of great interest to examine
how far the point-particle approximation can be justified.
To reveal the coordinate \( \langle R \rangle \) dependence and dynamical
properties of the inertial masses, we apply the ASCC
method to an energy density functional. In this section
we recapitulate the ASCC method for the calculations
of inertial masses. For comparison, we also give a brief
description of the cranking formulæ.

A. ASCC inertial masses

The ASCC method provides a collective subspace
which describes nuclear reaction. The generators of the
collective variables \( (q,p) \) are locally given by \( (\hat{Q}(q),\hat{P}(q)) \)
and determined by the ASCC equations. In the asym-
ptotic region on the reaction path (collective subspace) at
\( p = 0 \), these should correspond to the relative distance
and its conjugate momentum. Thus, we expect that
\[
\hat{Q}(q)\Psi(q) \propto \hat{R}\Psi(q), \quad \hat{P}(q)\Psi(q) \propto \frac{\partial}{\partial \hat{R}}\Psi(q), \tag{2}
\]
where the proportional constants are arbitrary as far as
they satisfy the weak canonicality condition
\[
\langle \Psi(q) | [\hat{Q}(q),\hat{P}(q)] \Psi(q) \rangle = 1. \tag{3}
\]
In this paper, we study systems that preserve the axial
symmetry on the reaction path. The one-body operator
\( \hat{R} \) for the relative distance between projectile and target
nuclei can be defined in the following way. The symmetry
axis is chosen as the \( z \) axis, assuming the projectile on
the right \((z > 0)\) and target on the left \((z < 0)\). We introduce a separation plane at \(z = z_s\), so that

\[
\int d\mathbf{r} \, \theta(z - z_s) \rho(\mathbf{r}) = A_P,
\]

where \(\rho(\mathbf{r})\) is the nucleon density and \(\theta(x)\) is the step function. The operator \(\hat{R}\) is given by \(\hat{R} = \hat{Z}_P - \hat{Z}_T\), with

\[
\hat{Z}_{P(T)} = \sum_{s = \uparrow, \downarrow} \sum_{q = n_p, p} \int d\mathbf{r} \, \hat{\psi}_{sq}^\dagger(\mathbf{r}) \hat{\psi}_{sq}(\mathbf{r}) \frac{\theta(\pm(z - z_s))}{A_{P(T)}},
\]

where the upper sign is adopted for \(\hat{Z}_P\) and the lower for \(\hat{Z}_T\).

The operator \(\hat{R}\) is artificially defined as a convenient measure for the relative distance. Its meaning is clear and well defined in the asymptotic region, though it is no longer meaningful when the projectile and the target are merged into a single nucleus. Nevertheless, this choice of \(\hat{R}\) does not affect the final result as far as there is a one-to-one correspondence between \(q\) and \(R\), because the reaction path \(|\Psi(q)\rangle\) and its canonical variables \((q, p)\) are self-consistently determined without any assumption. After determining all the quantities in the collective Hamiltonian, we perform the coordinate transformation from \(q\) to \(R\), by using the relation \(R(q) = (\Psi(q)|\hat{R}|\Psi(q))\). This is merely a change in the scale (and the dimension) of the coordinate.

In the ASCC method, the reaction path \(|\Psi(q)\rangle\) at \(p = 0\) and the local generators \((\hat{Q}(q), \hat{P}(q))\) are determined by solving the ASCC equations

\[
\delta\langle\Psi(q)|\hat{H}_{mv}, \hat{Q}(q)\rangle = 0, \quad \delta\langle\Psi(q)|\hat{H}_{mv}, \frac{1}{i} \hat{P}(q) - \frac{\partial^2 V(q)}{\partial q^2} \hat{Q}(q)|\Psi(q)\rangle = 0, \quad \delta\langle\Psi(q)|\hat{H}_{mv}, \hat{J} \rangle = 0,
\]

where curvature terms are neglected for simplicity [19]. The Hamiltonian in the moving frame \(\hat{H}_{mv}\) and the potential \(V(q)\) are respectively defined as

\[
\hat{H}_{mv} \equiv \hat{H} - \frac{\partial V(q)}{\partial q} \hat{Q}(q), \quad V(q) \equiv \langle\Psi(q)|\hat{H}|\Psi(q)\rangle. \tag{9}
\]

Equation (6) which determines \(\hat{Q}(q)\) determines the reaction path \(|\Psi(q)\rangle\). The local generators, \(\hat{P}(q)\) and \(\hat{Q}(q)\), are given by a solution of Eqs. (7) and (8). However, since solutions of Eqs. (7) and (8) are not unique, we need to select the one corresponding to the reaction path we study. Normally, we choose a solution with the lowest frequency, \(\omega^2 = (\partial^2 V/\partial q^2) M^{-1}(q)\), except for the Nambu-Goldstone (NG) modes. The generators should satisfy the asymptotic condition of Eq. (2).

In Eq. (8) there appears the inverse mass parameter \(M(q)\) with respect to the coordinate \(q\). The magnitude of \(M(q)\) depends on the scale of the coordinate \(q\) which is arbitrary. We can choose to set the mass \(M(q)\) in Eq. (8) to be a constant, for instance \(M_q = 1\) MeV\(^{-1}\) \(|q|^{-2}\) without losing generality [30], where \(|q|\) represents the unit of \(q\).

In order to obtain an intuitive picture of the collective dynamics, we map the collective coordinate \(q\) to the relative distance \(R\) between projectile and target nuclei. The inertial mass should be transformed as

\[
M(R) = M(q) \left(\frac{dq}{dR}\right)^2 = M(q) \left(\frac{dR}{dq}\right)^{-2}, \tag{10}
\]

where the derivative \(dR/dq\) can be obtained as

\[
\frac{dR}{dq} = \frac{\langle\Psi(q)|\hat{R}|\Psi(q)\rangle}{\langle\Psi(q)|\frac{1}{i} \hat{P}(q)|\Psi(q)\rangle} = 2 \sum_{n \in p, j \in h} R_{nj}(q) P_{nj}(q), \tag{11}
\]

with the local generator \(\hat{P}(q)\). \(R_{nj}(q)\) and \(P_{nj}(q)\) are the particle-hole (ph) matrix elements of \(\hat{R}(q)\) and \(\hat{P}(q)\), obtained by solving ASCC equations (7) and (8) [30]. The calculated mass \(M(R)\) will replace the constant mass \(\mu_{red}\) in the first term of Eq. (1).

Apart from the the relative motion of our current interest, Eqs. (7) and (8) provide solutions for the NG modes, such as the translation and the rotation. These modes have natural and global generators, namely, the total linear momentum \(\hat{P}\) for the translation and the total angular momentum \(\hat{J}\) for the rotation. For example, the rotational motion around the \(x\) axis can be generated by \(\hat{J}_x\) without any cost of energy \((\omega^2 = 0)\), leading to the Thouless-Valatin equations [18, 32],

\[
\delta \langle\Psi(q)|\hat{H}_{mv}, \hat{J}_x\rangle |\Psi(q)\rangle = 0, \tag{12}
\]

\[
\delta \langle\Psi(q)|\hat{H}_{mv}, i\hat{\Theta}\rangle - \frac{1}{J(q)} \hat{J}_x |\Psi(q)\rangle = 0, \tag{13}
\]

\[
\sum_{i} \langle\Psi(q)|i\hat{J}_x, \hat{\Theta}(q)\rangle |\Psi(q)\rangle = 1, \tag{14}
\]

where \(\hat{\Theta}\) is the angle variable conjugate to \(\hat{J}_x\), and \(J(q)\) is the moment of inertia around \(x\) axis. Equations (12) and (13) indicate that \((\hat{Q}(q), \hat{P}(q)) = (\hat{\Theta}(q), \hat{J}_x)\) correspond to a solution of the ASCC equations (7) and (8) with \(\partial^2 V/\partial q^2 = 0\) and \(M(q) = J(q)\). The calculated rotational moments of inertia can be used to replace \(\mu_{red} R^2\) in the second term of Eq. (1).

To calculate the inertial masses for the NG modes, an efficient method was proposed in Ref. [33]. The inertial mass of the NG modes \(M(q)\) are given by the zero-frequency limit of the strength function in the linear response for the momentum operator \(\hat{P}\). We apply this method to the calculations of the translational and rotational inertial masses.
B. CHF+cranking method

A simple method based on the cranking formula to calculate the inertial mass is widely used for nuclear collective motion. The collective path (reaction path) is usually produced by the constrained-Hartree-Fock (CHF) calculation with a constraining operator \( \hat{C} \) given by hand. The CHF states are given by the condition,

\[ \langle \Psi_0(\lambda) | \hat{H}_s | \Psi_0(\lambda) \rangle = 0, \quad \hat{H}_s \equiv \hat{H} - \lambda \hat{\mathbf{C}}, \]  

(15)

where \( \lambda \) is the Lagrange multiplier. Upon the CHF states \( |\Psi_0(\lambda)\rangle \) with \( R = \langle \Psi_0(\lambda) | \hat{R} | \Psi_0(\lambda) \rangle \), the inertial mass with respect to the coordinate \( \hat{R} \) is calculated using the cranking formula \([34]\):

\[ M_{\text{cr}}^{np}(R) = 2 \sum_{m} \frac{\langle |\psi_{m}(R)\rangle |\partial / \partial R |\psi_{0}(R)\rangle |^2}{E_{m}(R) - E_{0}(R)} \]  

(16)

\[ = 2 \sum_{n, p, j \in h} \frac{\langle |\varphi_{n}(R)\rangle |\partial / \partial R |\varphi_{j}(R)\rangle |^2}{e_{n}(R) - e_{j}(R)}, \]  

(17)

where the single-particle states \( |\varphi_{\mu}(R)\rangle \) and energies \( e_{\mu}(R) \) are defined with respect to the CHF single-particle Hamiltonian, \( \hat{h}_{\lambda} = \hat{h}_{\text{HF}}[\rho] - \lambda \hat{\mathbf{C}} \), as

\[ \hat{h}_{\lambda} |\varphi_{\mu}(R)\rangle = e_{\mu}(R) |\varphi_{\mu}(R)\rangle, \quad \mu \in p, h. \]  

(18)

Note that the Lagrange multiplier \( \lambda \) is a function of \( R \) given by the condition, \( R = \langle \Psi_0(\lambda) | \hat{R} | \Psi_0(\lambda) \rangle \). The constraining operator \( \hat{C} \) can be in general different from the relative-distance operator \( \hat{R} \). In the following calculations in Sec. III, we use the mass quadrupole and octupole operators as \( \hat{C} \).

Another cranking formula, which is even more frequently used in many applications, can be derived by neglecting the rearranged fields induced by the change of \( \lambda \), namely, \( \delta \hat{h}_{\text{HF}} / \delta \rho \cdot \partial \rho / \partial \lambda = 0 \). Normally, one chooses the constraining operator as the collective coordinate, \( \hat{C} = \hat{R} \), for the CHF calculation. However, in the present calculation, since we need to find the separation plane \( z = z_s \) to determine the operator \( \hat{R} \), it is convenient to adopt a different one-body operator \( \hat{C} \). Let us derive the formula for this case. From Eq. (18) and the orthonormality condition \( \langle \varphi_{\mu}(R) | \varphi_{\nu}(R) \rangle = \delta_{\mu \nu} \), it is easy to find

\[ \langle \varphi_{\mu} | \partial / \partial R | \varphi_{\nu} \rangle = -\frac{\langle \varphi_{\mu} | \partial h_{\lambda} / \partial R | \varphi_{\nu} \rangle}{e_{\nu} - e_{\mu}} \approx \frac{\lambda H_{\lambda} |\varphi_{\nu}\rangle}{e_{\nu} - e_{\mu}}. \]  

(19)

Neglecting the rearrangement, \( \hat{h}_{\lambda + \Delta \lambda} \approx \hat{h}_{\lambda} - \Delta \lambda \hat{\mathbf{C}} \), the derivative \( dR / d\lambda \) is estimated with the first-order perturbation as \( dR / d\lambda = 2S_{k}(\hat{R}, \hat{C}) \), where

\[ S_{k}(\hat{Q}_1, \hat{Q}_2) = \frac{1}{2} \sum_{m > 0} \langle \Psi_{0}| \hat{Q}_1 |\Psi_{m}\rangle \langle \Psi_{m}| \hat{Q}_2 |\Psi_{0}\rangle + \text{c.c.} \]  

\[ = \frac{1}{2} \sum_{n \in p, j \in h} \langle \varphi_{j}| \hat{Q}_1 |\varphi_{n}\rangle \langle \varphi_{n}| \hat{Q}_2 |\varphi_{j}\rangle + \text{c.c.} \]  

\[ = S_{k}(\hat{Q}_2, \hat{Q}_1). \]  

(20)

Thus, the formula reads

\[ M_{\text{cr}}^{np}(R) = \frac{1}{2} S_{3}(\hat{C}, \hat{C}) \left\{ S_{1}(\hat{R}, \hat{C}) \right\}^{-2}. \]  

(21)

According to Ref. [34], we call the former one in Eq. (17) “non-perturbative” cranking inertia and the latter in Eq. (21) “perturbative” one. In contrast to the ASC mass, the cranking masses of Eqs. (17) and (21) have a drawback that they both neglect residual effect of the time-odd density fluctuation. As we shall see in Sec. III, when the nucleonic effective masses are present, the cranking formulae produce wrong masses for the translation and for the relative motion at \( R \rightarrow \infty \).

For the NG modes such as the translation and the rotation, we know the generator of the collective coordinate. For instance, in the translational case, the center-of-mass (COM) coordinates \( X_{k} \) \((k = x, y, z)\) are the trivial collective coordinates. Then, the non-perturbative cranking formula (17) can be transformed into

\[ M_{\text{cr}}^{np}(X_{k}) = 2 \sum_{n \in p, j \in h} \frac{\langle \varphi_{n}(X_{k}) |\partial / \partial X_{j} |\varphi_{j}(X_{k})\rangle |^2}{e_{n}(X_{k}) - e_{j}(X_{k})} \]  

(22)

\[ = 2 \sum_{n \in p, j \in h} \frac{\langle \varphi_{n} |\hat{p}_{k} |\varphi_{j}\rangle |^2}{e_{n} - e_{j}}, \]  

(23)

where \( \hat{p}_{k} \) is the single-particle linear momentum. In the second equation, we take advantage of the fact that the single-particle energies and the wave functions relative to the COM are invariant with respect to its position \( X_{k} \). For the rotation, we may replace \( X_{k} \) by an angle \( \phi_{k} \) and \( \hat{p}_{k} \) by the angular momentum \( \hat{j}_{k} \). This leads to nothing but the cranking formula for the moment of inertia, originally proposed by Inglis [35],

\[ (J_{\text{cr}}^{np})_{k} = 2 \sum_{n \in p, j \in h} \frac{\langle \varphi_{n} |\hat{j}_{k} |\varphi_{j}\rangle |^2}{e_{n} - e_{j}}. \]  

(24)

It is instructive to investigate properties of the cranking mass formula for the NG modes [18, 29, 32]. Here, let us examine the formula of Eq. (23) for the translation, following the argument given in Ref. [29]. The summation with respect to the index \( n \) in Eq. (23) is restricted to the particle states. This restriction can be removed, because the summation with respect to the hole-hole components, \( \sum_{n \in h} \sum_{j \in h} \), identically vanishes. When the mean-field (Hartree-Fock) Hamiltonian \( \hat{h}_{\text{HF}}[\rho] \) conserves the Galilean invariance,

\[ \left[ \hat{h}_{\text{HF}}, \hat{x}_{k} \right] = -\frac{i}{m} \hat{p}_{k}, \quad k = x, y, z, \]  

(25)

the translational mass becomes

\[ M_{\text{cr}}^{np} = 2 \sum_{\mu} \sum_{j \in h} \frac{\langle \varphi_{j} |\hat{p}_{k} |\varphi_{\mu}\rangle \langle \varphi_{\mu} |\hat{p}_{k} |\varphi_{j}\rangle}{e_{\mu} - e_{j}}. \]
Therefore, the Galilean invariance of the mean-field Hamiltonian guarantees that the cranking formula reproduces the total mass $Am$ for the translation. However, the Galilean invariance is violated by velocity dependence of the one-body mean-field potential in most of nuclear energy density functionals. This results in a wrong mass $M_{ct} \neq Am$ in the cranking formula. Indeed, the ASCC mass reproduces the exact total mass even when the mean-field potential violates the Galilean invariance, which will be shown in Sec. III.

### III. APPLICATION

In the following numerical calculations, in order to express the orbital wave functions, the grid representation is employed, discretizing the rectangular box into three-dimensional (3D) Cartesian mesh [4]. The model space is set to be $12 \times 12 \times 18$ fm$^3$ for the systems $\alpha + \alpha \rightarrow ^8$Be and $^{16}$O+$\alpha \rightarrow ^{20}$Ne. It is $12 \times 12 \times 24$ fm$^3$ for $^{16}$O+$^{16}$O $\rightarrow ^{32}$S. The mesh size is set to be 1.0 fm for the system $\alpha + \alpha \rightarrow ^8$Be and 1.1 fm for the other two systems.

For numerical calculations of the ASCC method, we use the finite amplitude method (FAM) [36–39]. The FAM provides us with high numerical efficiency with simple computer programs, because only the calculations of the mean-field (single-particle) Hamiltonian constructed with independent bra and ket states are required [36]. The matrix FAM (m-FAM) prescription [39] is adopted to solve the moving RPA equations (7) and (8). On the other hand, the iterative FAM (i-FAM) [36–38] is adopted to calculate the response functions for the NG modes. The moving mean-field equation (6) is solved by using the imaginary-time method [40].

#### A. Modified BKN energy density functional

In order to investigate the effect of this time-odd mean-field potential on the collective inertial masses, we adopt an energy density functional of the simplest choice, namely, the BKN energy density functional [41] with the minimum extension.

The original BKN functional assumes the spin-isospin symmetry without the spin-orbit interaction, which is a functional of the isoscalar kinetic and local densities, $\tau(r)$ and $\rho(r)$, only. The mean-field potential is local and has no velocity dependence. Thus, the nucleon’s effective mass is identical to the bare nucleon mass. However, in most of realistic energy density functionals, the effective mass is smaller than the bare mass, typically $m^*/m \sim 0.7$. In order to introduce the effective mass, we extend the energy density by adding terms $\rho \tau - j^2$ where $j(r)$ is the isoscalar current density. The appearance of the current density is necessary to preserve the Galilean invariance.

The modified BKN energy density functional reads,

\[
E[\rho] = \int \frac{1}{2m} \tau(r) \, dr + \int dr \left\{ \frac{3}{8} t_0 \rho^2(r) + \frac{1}{16} t_3 \rho^3(r) \right\} + \int dr' \rho(r) v(r-r') \rho(r') + B_3 \int dr \left\{ \rho(r) \tau(r) - j^2(r) \right\},
\]

where $\rho(r)$, $\tau(r)$, and $j(r)$ are calculated as

\[
\rho(r) = 4 \sum_{j=1}^{A/4} |\psi_j(r)|^2, \quad \tau(r) = 4 \sum_{j=1}^{A/4} \nabla \psi_j(r)|^2,
\]

\[
j(r) = 4 \sum_{j=1}^{A/4} \left\{ \psi_j^*(r) \nabla \psi_j(r) - \psi_j(r) \nabla \psi_j^*(r) \right\}.
\]

In Eq. (27), $v(r)$ is the sum of the Yukawa and the Coulomb potentials [41],

\[
v(r) = V_0 a e^{-r/a} + \frac{(e/2)^2}{r}.
\]

The new parameter $B_3$ controls the effective mass and the velocity dependence of the mean-field potential.

The variation of the total energy with respect to the density (or equivalently single-particle wave functions) defines the mean-field (Hartree-Fock) Hamiltonian,

\[
\hat{h}_{HF}(r) = -\nabla \frac{1}{2m^*(r)} \nabla + \frac{3}{4} t_0 \rho(r) + \frac{3}{16} t_3 \rho^2(r)
\]

\[
+ \int dr' v(r-r') \rho(r') + B_3 (\tau(r) + i \nabla \cdot j(r))
\]

\[
+ 2t_3 j(r) \cdot \nabla,
\]

where the effective mass is now deviated from bare nucleon mass,

\[
\frac{1}{2m^*(r)} = \frac{1}{2m} + B_3 \rho(r).
\]

For $B_3 \neq 0$, Eq. (28) indicates the velocity (momentum) dependence of the mean-field potential and the presence of the time-odd mean fields, $iB_3(\nabla \cdot j + 2j \cdot \nabla)$. For the time-even states, such as the ground state of even-even nuclei, the current density disappears, $j(r) = 0$. Nevertheless, as will be shown later, the terms associated with $j(r)$ play an important role in the collective inertial mass.

The parameters, $t_0$, $t_3$, $V_0$, and $a$ are adopted from Ref. [41], and we vary $B_3$ to change the effective mass and the time-odd mean fields.
However, at a field Hamiltonian $\hat{H}$ independent mean-field potential, for which the mean-mass reproduces the total mass, $M$ of the parameter $A_m$ produces the correct total mass with its $z$ component $Z_{cm}$ in the numerical calculation. In the present model, neutrons and protons have the identical mass coordinate $\rho_{cm}$ as shown in our former paper \cite{30} for $B_3 = 0$. With $B_3 \neq 0$, basic features of the reaction path is the same, although the ground state of $^8\text{Be}$ becomes more elongated, developing a prominent $2\alpha$ cluster structure. The curves in Fig. 2 show the inertial mass and total energy for the relative motion between two alpha particles as a function of the relative distance $R$. The ASCC mass is calculated based on the self-consistent ASCC collective path. For the cranking masses, the mass quadrupole operator $\hat{Q}_{20}$ is used as a constraint to construct the reaction path with the CHF calculation.

At large $R$ where the two alpha particles can be approximated as point particles, we expect that the inertial mass with respect to $R$ is identical to the reduced mass, $2m$. For the non-perturbative cranking mass, this is true at $B_3 = 0$, while it monotonically decreases as $B_3$ increases (Fig. 3). On the other hand, the ASCC mass reproduces the correct reduced mass at large $R$, irrespective of the value of $B_3$. The main difference between the ASCC and the cranking masses is the inclusion of the effect of time-odd residual fields. Therefore, we may conclude that the time-odd residual effect is essential to reproduce the reduced mass in the asymptotic region of $R \rightarrow \infty$.

The perturbative cranking mass seems to be unable to reproduce the correct reduced mass. It is larger than $\mu_{rel} = 2m$ even for $B_3 = 0$. As is seen in the first equation of (19), the non-perturbative cranking mass comes from the $R$-dependence of the Lagrange multiplier $\lambda$ and from that of the HF Hamiltonian (rearrangement) $\hat{h}_{HF}$. When we take the asymptotic limit of $R \rightarrow \infty$, since the interaction between two \(\alpha\) particles vanishes, we naturally expect $\lambda \rightarrow 0$. Then, we have $d\lambda/dR \rightarrow 0$, and $M_{\alpha\alpha}^{cp}$ solely comes from the rearrangement effect. In the perturbative treatment, we neglect the rearrangement in the last equality of Eq. (19). Instead, we replace $d\lambda/dR$ by the perturbative value $(2S_1(\vec{R}, \hat{C}))^{-1}$. It is thus difficult to justify the perturbative approximation in the

\[ (m^*/m)_{av} \equiv \frac{2m}{A} \int \frac{\rho(r)}{2m^*(r)} dr. \]
\textbf{FIG. 2.} (Color online) Inertial mass $M(R)$ for the reaction $\alpha + \alpha$ as a function of the relative distance $R$. The top panel shows the results of the ASCC method, while the middle panel shows those of the perturbative and non-perturbative cranking formulae $M^p_{\alpha\alpha}$ (thin lines) and $M^{np}_{\alpha\alpha}$ (thick lines). The bottom panel shows the potential energies along the ASCC collective paths. Solid (red) and dashed (blue) lines indicate those of $B_3 = 0$ and 75 MeV fm$^2$, respectively. In the upper panel, the dots correspond to the positions of the minimum energy in the bottom panel.

The cranking masses are shown in the middle panels in Figs. 4 and 5. The perturbative and non-perturbative cranking masses are calculated based on the CHF reaction paths that are constructed with the constraining operator $\hat{C} = \hat{Q}_{20}$ (mass octupole) for $\alpha + ^{16}\text{O} \rightarrow ^{20}\text{Ne}$, and with $\hat{C} = \hat{Q}_{20}$ (mass quadrupole) for $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S}$. If we adopt $\hat{C} = \hat{Q}_{20}$ for the former reaction system, we cannot obtain a continuous reaction path [31]. Again, for $B_3 \neq 0$, the cranking mass does not reproduce the correct reduced mass at $R \rightarrow \infty$, neither with the perturbative cranking formula nor with the non-perturbative one. Furthermore, the perturbative cranking mass is larger than the reduced mass, even for $B_3 = 0$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig2}
\caption{(Color online) Inertial mass $M(R)$ for the reaction $\alpha + \alpha$ as a function of the relative distance $R$. The top panel shows the results of the ASCC method, while the middle panel shows those of the perturbative and non-perturbative cranking formulae $M^p_{\alpha\alpha}$ (thin lines) and $M^{np}_{\alpha\alpha}$ (thick lines). The bottom panel shows the potential energies along the ASCC collective paths. Solid (red) and dashed (blue) lines indicate those of $B_3 = 0$ and 75 MeV fm$^2$, respectively. In the upper panel, the dots correspond to the positions of the minimum energy in the bottom panel.
}
\end{figure}

\textbf{FIG. 3.} (Color online) Non-perturbative cranking inertial masses calculated at $R = 8$ fm for the relative motion between two $\alpha$ particles divided by the reduced mass $\mu_{\text{red}} = 2m$, as a function of $B_3$. The blue dash-dotted curve indicates $(m^*/m)_{av}$ of Eq. (30).

2. $\alpha + ^{16}\text{O} \rightarrow ^{20}\text{Ne}$ and $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S}$

Next we show the inertial masses $M(R)$ of the relative motion for two reaction systems $\alpha + ^{16}\text{O} \rightarrow ^{20}\text{Ne}$ and $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S}$. The self-consistent reaction paths for these were presented in our former paper [31], but only for $B_3 = 0$. Similar to the $\alpha + \alpha \rightarrow ^{8}\text{Be}$ case, the increase of $B_3$ favors larger deformation. The upper panels in Figs. 4 and 5 show $M(R)$ for $\alpha + ^{16}\text{O} \rightarrow ^{20}\text{Ne}$ and $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S}$, respectively. The smaller effective mass enlarges the size of the nucleus, thus, the touching points between the two nuclei take place at larger $R$. The minimum energy position is also shifted to larger $R$ for larger $B_3$. The drastic increase in $M(R)$ in the interior region is due to the increase of $dq/dR$ in Eq. (10). In contrast to variations of $M(R)$ in the interior region, $M(R)$ is very stable and independent from the value of $B_3$ when two nuclei are separated. In the asymptotic region ($R \rightarrow \infty$), we always observe the correct limit, $M(R) \rightarrow \mu_{\text{red}}$. The reduced mass for $\alpha + ^{16}\text{O} \rightarrow ^{20}\text{Ne}$ is $\mu_{\text{red}} = 3.2m$, and $\mu_{\text{red}} = 8m$ for $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S}$.

Asymptotic limit.

Figure 3 clearly demonstrates that the cranking inertial mass at $B_3 \neq 0$ underestimates the exact reduced mass for the relative motion ($\mu_{\text{red}} = 2m$). Again, it well agrees with the average effective mass, $2(m^*/m)_{av}$. Thus, the failure of the cranking mass is due to the violation of the Galilean invariance.
FIG. 4. (Color online) Inertial mass $M(R)$ for the reaction path $\alpha+{^{16}}O$ as a function of $R$, in units of the nucleon mass. The top panel shows the results of the ASCC method, the middle panel shows those of the perturbative and non-perturbative cranking formulae $M_p^{cr}$ (thin lines) and $M_{np}^{cr}$ (thick lines), and the bottom panel shows the potential energies along the three ASCC collective paths. Solid (red) and dashed (blue) lines indicate those of $B_3 = 0$ and $75$ MeV fm$^5$. In the bottom panel, in order to show it in the given scale, the potential for $B_3 = 75$ MeV fm$^5$ (dashed line) is shifted downwards by 60 MeV.

D. Rotational moments of inertia

Next let us present results for the moment of inertia which is another important inertial mass for the collective Hamiltonian in Eq. (1). The rotational motion is a NG mode whose ASCC inertial mass can be calculated with the strength function for the angular momentum operator at zero frequency [33], instead of solving the Thouless-Valatin equation (13). For this purpose, we perform the i-FAM calculation [36–38] based on the reaction path self-consistently constructed by the ASCC method. For comparison, we also apply the cranking formula to the moments of inertia, Eq. (24), based on the reaction path calculated with the CHF method.

In Eq. (1), the point-particle approximation was adopted, leading to the moment of inertia, $\mu_{red}R^2$. There is another classical limit, namely, the rigid-body moment of inertia. In the present case, the system preserves the axial symmetry on the reaction path. Choosing the axis...
of the symmetry as the \( z \) axis, we have \( J_z = J_y \) whose rigid-body value is given by

\[
J_{\text{rig}} = m \int \left( y^2 + z^2 \right) \rho(r) dr = m \int \left( z^2 + x^2 \right) \rho(r) dr
\] (31)

The rigid-body moment of inertia about the \( z \) axis is non-zero, \( J_z \neq 0 \), which contradicts with the trivial quantum mechanical requirement that there exists no rotation around the symmetry axis. On the other hand, it is known that the cranking moments of inertia, \( J_x \) and \( J_y \), for axially deformed nuclei in the harmonic-oscillator-potential model are given by the rigid-body value at the equilibrium [29]. The cranking formula also satisfies the quantum mechanical condition, producing \( J_z = 0 \) in the axially symmetric case.

When two nuclei are far away, we expect the point-particle approximation is good. Therefore, we expect that the moments of inertia, \( J_x \) and \( J_y \), on the reaction path changes from the point-particle value \( J(R) = \mu_{\text{red}} R^2 \) at large \( R \) to the rigid-body value \( J(R) = J_{\text{rig}} \) near the equilibrium (ground) state. It is of significant interest and of importance to examine where and how this transition takes place during the reaction.

In Ref. [42], we have published the result for \( \alpha + \alpha \rightarrow ^{16}{\text{Be}} \). Thus, in this paper, we present the results for \( \alpha + ^{16}{\text{O}} \rightarrow ^{20}{\text{Ne}} \) and \( ^{16}{\text{O}} + ^{16}{\text{O}} \rightarrow ^{32}{\text{S}} \). First, let us show results for the velocity-independent potential with \( B_3 = 0 \). Figure 6 shows the moments of inertia calculated for the two reaction systems with \( B_3 = 0 \). The ground state of \( ^{20}{\text{Ne}} \) corresponds to \( R = 3.8 \text{ fm} \) in the upper panel of Fig. 6, and the superdeformed minimum of \( ^{32}{\text{S}} \) is located at \( R = 5 \text{ fm} \) in the lower panel. We can see that the ASCC and the (non-perturbative) cranking formulas give results very similar to each other. This is the consequence of the local Galilean invariance of the mean-field potential under the momentum transformation \( p \rightarrow p - m(\omega \times r) \) [29]. The calculated moments of inertia are close to their rigid-body values near these equilibrium states, \( J(R_e) \approx J_{\text{rig}}(R_e) \), where \( R_e \) represents the value of \( R \) at the potential equilibrium \( (dV/dR = 0) \). However, when the nucleus is more elongated along the reaction path, \( J(R) \) decreases as \( R \) increases. Since the rigid-body value is a monotonically increasing function of \( R \), they become smaller than the rigid-body values, \( J(R) < J_{\text{rig}}(R) \), then, quickly approach the point-particle values, \( \mu_{\text{red}} R^2 \). Beyond the scission point \( R = R_s \) where the system splits into two separated nuclei, we have \( J(R) \approx \mu_{\text{red}} R^2 \) \( (R \geq R_s) \). The calculated moments of inertia give parabolic convex lines as functions of \( R \), showing their minima located around the midpoint between \( R_e \) and \( R_s \).

It is surprising that the moments of inertia decrease as the deformation develops near the equilibrium. This contradicts with our naive expectation based on the classical model. For instance, the rigid-body moments of inertia linearly increases with the deformation \( \delta \), \( J_{\text{rig}} \approx J_0 (1 + \delta/3) \). Another classical model, irrotational fluid model suggests that it increases quadratically with the deformation as \( J_{\text{irrot}} \approx J_{\text{rig}} \delta^2 \) [29]. In any case, a single classical model cannot explain the reduction in \( J(R) \) as a function of \( R \). A hint to understand this behavior may come from the harmonic oscillator model in the quantum mechanics which reproduces both the rigid-body value and the irrotational-fluid value depending on the configuration of nucleus [29]. During the evolution of deformation as a function of \( R \), different configurations appear, which may lead to moments of inertia corresponding to different classical models. This suggests the importance of the quantum mechanical calculation.

Another striking feature in Fig. 6 is \( J(R) \approx \mu_{\text{red}} R^2 \) at \( R \geq R_s \), which is significantly smaller than \( J_{\text{rig}}(R) \). When the two nuclei are separated, the structure of the projectile and the target nuclei are almost invariant with respect to \( R \). Let us denote the rigid-body moments of inertia of the projectile and the target with respect to their own center-of-mass coordinates as \( J_P \) and \( J_T \), respectively. The total rigid-body moments of inertia can be written as

\[
J(R) = J_P + J_T + \mu_{\text{red}} R^2.
\] (32)

This equation is easy to prove in the rigid-body case. In the quantum mechanical treatment of the present cases, there is another trivial result, namely, \( J_P = J_T = 0 \). This is because both the projectile and the target nuclei (\( \alpha \) and \( ^{16}{\text{O}} \)) are spherical in the ground state, thus, the quantum mechanics requires the vanishing moments of inertia (See also Eq. (24)). Assuming that Eq. (32) is valid for the quantum mechanical systems, we find \( J(R) \approx \mu_{\text{red}} R^2 \) in the region where the projectile and the target become spherical. This is also a consequence of the quantum mechanics.

Next, let us show how the \( B_3 \) term influences the moments of inertia. Figure 7 shows the results. For \( B_3 \neq 0 \), the velocity dependence in the mean-field potential violates the local Galilean invariance. The calculated cranking moments of inertia are significantly smaller than the rigid-body value \( J_{\text{rig}} \) at \( R \approx R_e \). They are also smaller than the point-particle value \( \mu_{\text{red}} R^2 \) at \( R \geq R_e \). On the other hand, the ASCC calculation includes the residual effects of the time-odd mean fields which restore the local Galilean invariance, then, nicely reproduces both rigid-body and point-particle values at \( R \approx R_e \) and at \( R > R_e \), respectively. In fact, for these reaction systems, the ASCC moments of inertia are insensitive to the \( B_3 \) value over the entire region.

E. Impact on astrophysical S factors

In this section, we present the calculation of the astrophysical \( S \) factor for fusion reactions, using the potentials and the inertial masses obtained in Sec. III C and III D. The ASCC calculation provides us with the collective Hamiltonian with the optimal reaction paths for \( ^{16}{\text{O}} + \alpha \rightarrow ^{20}{\text{Ne}} \) and \( ^{16}{\text{O}} + ^{16}{\text{O}} \rightarrow ^{32}{\text{S}} \). The Hamiltonian for the reaction is given as that of Eq. (1) in which
the moment of inertia $\mu_{\text{red}}R^2$ is replaced by $\mathcal{J}(R)$, and the reduced mass $\mu_{\text{red}}$ by $M(R)$. We investigate the dependence of the sub-barrier fusion cross sections on these inertial masses.

The sub-barrier fusion cross section is calculated with the WKB approximation, following the procedure in Ref. [43]. The total sub-barrier fusion cross section is given by the sum of all the partial waves with angular moment $\ell$. Under the WKB approximation, the transmission coefficient for the partial wave $\ell$ at incident energy $E_{\text{c.m.}}$ is given by

$$T_\ell(E_{\text{c.m.}}) = [1 + \exp(2I_\ell)]^{-1},$$

with

$$I_\ell(E_{\text{c.m.}}) = \int_a^b dR \left\{ 2M(R) \times \left( V(R) + \frac{\ell(\ell + 1)}{2\mathcal{J}(R)} - E_{\text{c.m.}} \right) \right\}^{1/2}.$$  \tag{34}

where $a$ and $b$ are the classical turning points on the inner and outer sides of the barrier respectively. In addition to the potential $V(R)$, the coordinate-dependent inertial mass for the relative motion $M(R)$ and for the rotational moment of inertia $\mathcal{J}(R)$ are necessary for the calculation. The term $\ell(\ell + 1)/2\mathcal{J}(R)$ represents the centrifugal potential.

The fusion cross section is given by

$$\sigma(E_{\text{c.m.}}) = \frac{\pi}{2\mu_{\text{red}}E_{\text{c.m.}}} \sum_\ell (2\ell + 1)T_\ell(E_{\text{c.m.}}).$$ \tag{35}

For identical incident nuclei, $^{16}\text{O} + ^{16}\text{O}$, Eq. (35) must be modified according to the proper symmetrization. Only the partial wave with even $L$ contribute to the cross section as

$$\sigma(E_{\text{c.m.}}) = \frac{\pi}{2\mu_{\text{red}}E_{\text{c.m.}}} \sum_\ell [1 + (-)^L(2\ell + 1)T_\ell(E_{\text{c.m.}})].$$ \tag{36}

Instead of $\sigma(E_{\text{c.m.}})$, we show the astrophysical $S$ factor defined by

$$S(E_{\text{c.m.}}) = E_{\text{c.m.}}\sigma(E_{\text{c.m.}}) \exp[2\pi Z_1 Z_2 e^2 / \hbar v],$$ \tag{37}

where $v$ is the relative velocity at $R \to \infty$. The astrophysical $S$ factor is preferred for sub-barrier fusion because it removes the change by tens of orders of magnitude present in the cross section due to the penetration through the Coulomb barrier. The $S$ factor can reveal...
in a more transparent way the influence of the nuclear structure and dynamics.

Figure 8 shows the calculated $S$ factor with $B_3 = 0$ for the scattering of $\alpha + ^{16}O$ (upper panel) and $^{16}O + ^{16}O$ (lower panel), respectively. In order to clarify the effect of the inertial masses, we use the same potential $V(R)$ for all the curves in each panel of Fig. 8, which is the ASCC potential obtained for $B_3 = 0$ (solid lines in the bottom panels of Fig. 4 and 5). Different curves show the calculations with different inertia masses. For the case of $B_3 = 0$, the ASCC and the cranking inertial masses are similar to each other. Thus, the calculation with the cranking inertial masses produces the astrophysical $S$ factor similar to the ASCC result. Generally speaking, the larger the inertial mass $M(R)$ for the relative motion is, the smaller the astrophysical $S$ factor is. The opposite effect can be seen for the rotational moments of inertia $J(R)$. The larger moments of inertia give the larger $S$ factor. The replacement of the constant mass $\mu_{\text{red}}$ by ASCC $M(R)$ gives a significant effect for the fusion cross section. For the reaction system $^{16}O + ^{16}O$, we find a strong suppression of the fusion cross section at $E_{\text{c.m.}} < 4$ MeV compared to the calculation with $\mu_{\text{red}}$, which is an order of magnitude or even larger.

Figure 9 shows the same $S$ factors with inertial masses calculated with $B_3 = 25$ MeV fm$^3$ for the scattering of $\alpha + ^{16}O$ (upper panel) and $^{16}O + ^{16}O$ (lower panel), respectively. In order to see the effect of inertial masses, we use the potential $V(R)$ obtained for $B_3 = 0$. Thus, the differences between Figs. 8 and 9 come from the change in inertial masses. Again, the change from the reduced mass $\mu_{\text{red}}$ into $M(R)$ gives the largest impact on the $S$ factor. The suppression effect of $M(R)$ is even more significant than the $B_3 = 0$ case, because the $M(R)$ is larger for finite values of $B_3$, as we can find in Fig. 4 and 5.

Although the BKN density functional provides us only with qualitative results, these calculations reveal the important roles of the inertial masses, and suggest significant influence of the inertial masses on the sub-barrier fusion cross sections.
IV. SUMMARY

We calculated the ASCC inertial mass coefficients with respect to the translational, relative and rotational motions. The numerical calculations are performed using the FAM in the 3D coordinate space representation for the reaction systems $\alpha+\alpha \rightarrow ^4\text{Be}$, $\alpha+^{16}\text{O} \rightarrow ^{20}\text{Ne}$ and $^{16}\text{O}+^{16}\text{O} \rightarrow ^{32}\text{S}$. We investigated the impact of time-odd mean-field potentials on the collective inertial masses. In the presence of nucleonic effective mass, the cranking mass can neither reproduce the total mass $Am$ of translational motion nor the correct reduced mass $\mu_{red}$ for the relative motion. In addition, the cranking formula for the rotation produces neither the point-particle value $\mu_{red}R^2$ in the asymptotic region ($R \rightarrow \infty$), nor the rigid-body value $J_{rig}$ near the equilibrium states. In contrast, the ASCC masses is able to properly take into account the residual time-odd effects, producing the total mass $Am$ for translation, the reduced mass $\mu_{red}$ for relative motion, and the point-particle value $\mu_{red}R^2$ (the rigid-body value $J_{rig}$) after the scission (near the equilibrium). We also calculated the astrophysical $S$ factors with these microscopic inputs as the inertial masses of a reaction model. It turns out that replacement of the reduced mass $\mu_{red}$ by $M(R)$ obtained with ASCC gives the largest impact on the $S$ factors. It could lead to a strong suppression of the fusion cross section at low energy.

In the present study, a schematic BKN interaction plus time-odd terms are adopted for a qualitative investigation. It is highly desired to use realistic modern nuclear energy functionals. The paring correlation is also expected to play a critical role in low-energy collective dynamics. These issues are currently under investigation.

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