Oscillating bubbles in ultrasonic acoustic field

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Abstract. Behavior of oscillating bubbles is of fundamental study for acoustic cavitations, and is of great importance for medical field. For example, angiographic and diagnosis of cancer of liver. We focused on behavior of multiple air bubbles exposed to ultrasonic wave. The bubbles were injected into the static water from a vertical capillary tube, and then the ultrasonic wave of 20 kHz was applied from above toward the bubbles. Vibrating motion of the bubbles was captured by a high-speed camera at frame rates up to 45000 fps. Excitations of surface wave and shape oscillation with distinct mode number were realized. Correlation between the accelerated bubble behavior and the bubble-bubble distance.

1. INTRODUCTION
To understand a dynamic behavior of the bubble, especially its shape and volume oscillation, under periodic external force is of one of great importance in order to understand and control multiphase flows. Theoretical and experimental works on oscillation of bubble have been widely conducted as fundamental study for such as acoustic cavitations, cavitation noise, wave propagation in bubbly liquid, sonoluminescence production, and boiling heat transfer. A large number of researches have indicated the excitation of volume/shape oscillations and oscillation-mode-transition problems in a single bubble system under small-amplitude external force as reviewed by Plesset & Prosperetti and Feng & Leal. Little knowledge has been accumulated, however, on highly-nonlinear behavior of a single bubble nor interaction on multiple bubbles under oscillations with different mode. The present paper discusses dynamics of a single or multiple bubbles exposed to ultrasonic vibration of wide range of amplitude in water under isothermal condition.

2. EXPERIMENTS
Experimental apparatus is shown schematically in Fig. 1. In a series of experiments, a single or successive multiple bubbles were injected by use of micro-syringe into a test fluid in a rectangular tank. Various types of stainless micro-syringe tip of 0.15~2.50 mm in inner diameter were employed to form bubbles of different diameter from 0.3 to 4.0 mm. The test fluid was pure water or water with tiny amount of surfactant. Ultrasonic oscillator of 20 kHz which intensity of imposed ultrasonic field is 56.65(W/cm²) was immersed in the fluid from the top, and was triggered after the bubbles detached from the tip of the syringe; the bubbles were exposed to the acoustic pressure field from the top as the bubble rose in liquid. The acoustic pressure was measured with hydrophone. Figures 2 and 3 are time series of signal detected by the hydrophone in acoustic field without bubbles and its spectrum respectively. The acoustics pressure field consists of a single component of 20kHz. The bubble behaviors were captured by high-speed camera at frame rates up to 45,000 fps. The bubbles were visualized with continuous light source illuminated from backside.
3. RESULTS & DISCUSSIONS

In the case of a single bubble under a small-amplitude oscillation, surface wave with a small wavelength emerges over the free surface as shown in Fig. 4. Through experimental observations, the wavelength of the surface wave is about $1.1 \sim 1.5 \times 10^2 \mu m$, and almost independent of the bubble diameter under the present experimental conditions. This surface wave can be explained by considering Kelvin wavelength (Feng & Leal, 1997 & Eisenmenger, 1959) described as follows:

$$\lambda = \sqrt[3]{\frac{2\pi \sigma}{\rho f_c^2}} = \sqrt[3]{\frac{8\pi \sigma}{\rho f_c^2}}.$$

where $\sigma$ and $\rho$ are the surface tension and the density of solvent, respectively, $f_c$ the frequency of capillary wave, $f_c^e$ the frequency of ultrasonic vibration. The observed value is in good agreement with the prediction by this equation of $1.2 \times 10^2 \mu m$. In case of large amplitude of the ultrasonic vibration, the bubble exhibits a non-linear shape oscillation with longer wavelength accompanying with the surface wave at a later stage of the oscillation as shown in the last frame in Fig. 4.
Fig. 4 Time series of surface wave development on a single bubble under ultrasonic vibration of 20 kHz. Scale bar in the left frame corresponds to 1.0 mm.

Fig. 5 Shape oscillation of a single bubble with an oscillation mode \((n, m) = (15, 0)\) under large-amplitude ultrasonic vibration of 20 kHz (top), and predicted bubble shape with spherical harmonics of \((n, m) = (15, 0)\) (bottom). Scale bar in the top left frame corresponds to 1.0 mm.

Fig. 6 Non-axisymmetric shape oscillation with a mode \((n, m) = (14, 3)\); experimental result (left) and predicted shape by spherical harmonics (right).

With larger amplitude, axisymmetric shape oscillation with a distinct mode number appears at an early stage. Typical example of shape oscillation of \(n = 15\) and \(m = 0\) on 1.6-mm-diameter bubble is shown in Fig. 5. It is noted that radial oscillation emerges through the excitation of the oscillation first; irradiation of ultrasonic vibration directly leads the bubble toward shape oscillation of \((0, 0)\) mode, then shape oscillation with distinct mode number. This transition is discussed later in this section. This bubble behavior after the radial oscillation is perfectly described with spherical harmonics (bottom frames in Fig. 5) as follows;

\[
r = R(t) + a_n(t)Y_n^m(\theta, \phi)
\]  

(2)
Where $R$ is the instantaneous bubble mean radius, $a_n$ is the amplitude of the surface distortion, $Y^m_n$ is a spherical harmonics, $\theta$ and $\phi$ the polar and azimuthal angles, respectively.

\[
\omega_{nm}^2 = \frac{(n-1)(n+1)(n+2)\sigma}{\rho R^3}
\]

Where $\sigma$ and $\rho$ are the surface tension and the density, respectively. The evaluated value of the oscillating frequency for $(n, m) = (15, 0)$ from the experiments, which corresponds to the case as shown in Fig. 5 is $2.0 \times 10^3$ Hz, which is in quite good agreement with the predicted value by Eq. (3) of $2.14 \times 10^3$ Hz. The observed values under the present experimental conditions agree well with the predicted one.

This equation (4) is obtained from Rayleigh-Plesset equation and polytropic change equation.

\[
\omega_t = \frac{1}{4\pi^2 R} \sqrt{\frac{1}{\rho} \left( \frac{3\kappa P_g}{R} - \frac{2\sigma}{R} \right)}
\]

Where $P_g$ and $\kappa$ are the inner pressure of bubble before exposed to the ultrasonic vibration and the polytropic exponent. This equation ignores vapor pressure in the bubble; because that the vapor pressure in the bubble is ignored in this equation; its pressure is treated to have a small effect in the present scale of the bubble.

Correlation between the frequency of the shape oscillation $\omega_{nm}$ and the natural frequency of volume oscillation $\omega_t$

Equation 5 is predicted as follows (Feng & Leal, 1997.)

\[
\omega_v = 2\omega_{nm}
\]

Fig. 7: Preferable mode number $n$ of shape oscillation as a function of bubble diameter. Prediction by Rayleigh (1917)-Plesset (1949) & Feng, Z. C. & Leal, L. G. (1997) also indicated. Intensity of imposed ultrasonic field is about 57(W/cm$^2$) for all data.
One can observe non-axisymmetric shape oscillation as well. Typical example of this oscillation is shown in Fig. 6 in the case of \((n, m) = (14, 3)\). Through the experimental observation, the natural frequency of the non-axisymmetric shape oscillation is almost the same as the one in the case of axisymmetric oscillation if the oscillating bubble evolves the same mode number \(n\). This tendency is explained from Eq. (2), which is independent of the azimuthal mode number \(m\). Figure 7 presents mode number \(n\) of the shape oscillation emerged under the present experimental condition. The mode number ranges from 6 to 18. Theoretical value is made from Eq. (3), (4) and (5). In the case that the diameter of bubble is 3.5–4.5, the shape of the bubble near ellipse even under the effect of the surfactant. That’s why effective diameters change and the resultant mode number distribute in more scattered manner. On the other hand, in the case that a surface wave instead of the shape oscillation emerges over the free surface of the preceding bubble, surface wave appears on the following bubbles (see the second frame from the left in Fig. 9), and then the bubbles exhibit the shape oscillation with distinct mode (the last frame in Fig. 9). That is, if the energy of ultrasonic vibration is not fully absorbed by the preceding bubble, the acoustic field leads the preceding one to be covered with the surface wave, and still maintains enough energy to trigger a shape oscillation in the following bubbles. One can also observe that contacting bubbles under shape oscillation seldom coalesce.

![Image](image1.png)

*Fig. 8 Effect of the preceding bubble; in case that the preceding bubble exhibits shape oscillation*

The black object at the bottom of each frame is the tip of the syringe needle. Scale bar: 2mm

![Image](image2.png)

*Fig. 9 Effect of the preceding bubble; in case that surface wave emerges on the preceding bubble.*
It is of great importance to discuss the effect of the preceding bubble upon the following one in terms of the distance between them. The authors can qualitatively indicate that the following bubble exhibits its oscillation in almost the same manner as the preceding one if the distance $L$ is larger than about $10R_0$, where $R_0$ is the mean radius of the bubbles. That is, the following bubble shows a shape oscillation with the same distinct mode as the preceding one. In the case of moderate distance as $3R_0 < L < 10R_0$, it is quite seldom for the following bubble to oscillate in the same manner; almost no distinct shape oscillation is observed and highly non-linear oscillation accompanying with vigorous deformation. If the bubbles are located very closely before irradiation of the acoustic pressure, $L < 3R_0$, the bubbles oscillate in almost the same phase, and the following one is strongly sucked toward the preceding one.

In the case of successive multiple bubbles, the effect of preceding bubble upon the oscillation behavior of following bubble (and vise versa) becomes significant as shown in Fig.8 and .9. In case that the following bubble is rising at a distance between the preceding one in a certain range, the following bubble maintains almost smooth surface despite the preceding bubble vigorously oscillates as shown in the first two frames in Fig. 8. As the following bubble comes closer to the preceding one, the following bubble accelerates towards the preceding one, and deformation and shape oscillation emerge in the following bubble. After those bubbles almost contact, not coalesce, the oscillation in the preceding bubble propagates to the following one, and the both oscillate in quite similar manner. Any distinct shape oscillation does not emerge.

![Time series of volume variation of oscillating bubbles](image-url)
Here the authors focus upon variation of the bubble volume during the oscillation. The volume of the oscillating bubbles are evaluated by extracting the bubble edge (or perimeter) along a horizontal pixel line in the obtained images, and integrating the dish along the vertical axis under the assumption that the bubble oscillation are axisymmetric; this volume evaluation is conducted until the bubble seems to be beyond axisymmetric from the image. In addition, the depression part of the bubble is neglected in the evaluation process. The evaluated volume thus involves some error; the volume must be overestimated. The authors, however, estimate this error be small enough comparing to the major volume variation. Fig.10 indicates a time series of volume variation of the preceding & following bubbles in the case of \( L < 10R_0 \). The shape of later bubble As an early stage of the oscillation, both bubbles exhibit \((n, m) = (0, 0)\) oscillation perfectly in phase as shown in Fig. 11(a). After a while, the preceding bubble changes into oscillating manner into a combined mode of the surface wave and shape oscillation as shown in Fig.11(b). The following bubble, on the other hand, a shape oscillation with \((17, 0)\) mode dominates the bubble. It should be noted that the preceding & following bubbles
exhibit completely different behaviors at last, although they do oscillate almost perfectly in phase with almost the same amplitude.

4. CONCLUDING REMARKS
Behavior of bubble(s) and interaction among successively rising bubbles in water under ultrasonic vibration were discussed. In a single bubble case, excitation of the surface wave over the free surface or shape oscillation with high-order mode was indicated through the observation by use of the high-speed camera. The behavior depends upon the amplitude of the vibration. In the case of multiple bubbles successively rising in a row, the effect of the preceding bubble upon the behavior of the following bubbles was presented.

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6. REFERENCES
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7. NOMENCLATURE
\( a_n \) Amplitude of the surface distortion \([\text{mm}]\)
\( f_c \) Frequency of ultrasonic vibration \([\text{Hz}]\)
\( f_k \) Frequency of capillary wave \([\text{Hz}]\)
\( L \) Distance \([\text{mm}]\)
\( r \) Radius of the oscillating bubble \([\text{mm}]\)
\( R \) Radius of the spherical bubble \([\text{mm}]\)
\( R_0 \) Mean radius of the bubbles \([\text{mm}]\)
\( t \) Time \([\text{s}]\)
\( Y_n^m \) Spherical harmonics \([-]\)
\( \theta \) Polar angle \([\text{rad}]\)
\( \lambda \) Kelvin wavelength \([\text{mm}]\)
\( \rho \) Density \([\text{kg/m}^3]\)
\( \sigma \) Surface tension \([\text{N/m}]\)
\( \phi \) Azimuthal angle \([\text{rad}]\)
\( \omega_{n,m} \) Natural angular frequency of (n, m) mode oscillation \([\text{Hz}]\)
\( \omega_v \) Angular Frequency of volume oscillation \([\text{Hz}]\)
\( \kappa \) polytropic exponent \([-]\)