There is increasing interest in a possible nonet of light scalar mesons (all of mass \( < 1 \) GeV). In addition to the well established \( f_0(980) \) and \( a_0(980) \) evidence of both experimental and theoretical nature for a very broad \( \sigma \) (\( \approx 560 \)) and a very broad \( \kappa \) (\( \approx 900 \)) has been presented [1]. The latter two resonances are difficult to identify cleanly because they appear to be of non Breit-Wigner type, signaling strong interference with the non-resonant background.

Such a nonet would most likely represent meson states more complicated than quark-anti quark type and hence would be of great importance for a full understanding of QCD in its non-perturbative low energy regime.

Clearly it is important to study the properties of the \( f_0(980) \) and \( a_0(980) \) from the point of view of how they fit into a putative nonet family. In particular, the reactions \( \phi \to f_0(980) \) and \( \phi \to a_0(980) \) have recently been observed [2] with good accuracy and are considered as useful probes of scalar properties. The theoretical analysis was initiated by Achasov and Ivanenko [3] and followed up by many others [4]. The models employed are essentially variants of the single \( K \) meson loop diagram to which a \( \phi \)-type vector meson, a photon and two pseudoscalars or a scalar are attached.

In the present note we introduce a complementary approach which emphasizes the "family" or symmetry aspects of the analysis. This enables us to study the correlations among a fairly large number of related radiative amplitudes in terms of a few parameters, without making a commitment to a particular quark structure for the scalars.

Our framework is that of a standard non-linear chiral Lagrangian containing, in addition to the pseudoscalar nonet matrix field \( \phi \), the vector meson nonet matrix \( \rho \mu \) and a scalar nonet matrix \( \eta \). Under chiral unitary transformations of the three light quarks; \( q_{L,R} \to U_{L,R} \cdot q_{L,R}, \) the chiral matrix \( U = \exp(2i\phi/F_\pi) \), where \( F_\pi \approx 0.131 \) GeV, transforms as \( U \to U \cdot U_R \cdot U \). The convenient matrix \( K(U_L, U_R, \phi) \) is defined by the following transformation property of \( \xi (U = \xi^2) \): \( \xi \to U_L \cdot \xi \cdot U_R^\dagger = K \cdot \xi \cdot U_R^\dagger \), and specifies the transformations of "constituent-type" objects. The fields we need transform as

\[
\begin{align*}
N &\to K \cdot N \cdot K^\dagger, \\
\rho \mu &\to K \cdot \rho \mu \cdot K^\dagger + (iK \cdot \partial \rho \mu K^\dagger, \\
F_{\mu\nu}(\rho) &= \partial \rho \nu - \partial \rho \mu - i\tilde{g} [\rho \mu, \rho \nu] \to K \cdot F_{\mu\nu} \cdot K^\dagger \end{align*}
\]

where the coupling constant \( \tilde{g} \) is about 4.04. One may refer to Ref. [5] for our treatment of the pseudoscalar-vector Lagrangian and to Ref. [7] for the scalar addition. The entire Lagrangian is chiral invariant (modulo the quark mass term induced symmetry breaking pieces) and, when electromagnetism is added, gauge invariant.

It should be remarked that the effect of adding vectors to the chiral Lagrangian of pseudoscalars only is to replace the photon coupling to the charged pseudoscalars as,

\[
i eA_\mu \text{Tr} \left( Q \phi \partial^\mu \phi \right) \to
\]

\[
i eA_\mu \left[ kF_\pi^2 \text{Tr}(Q\rho \mu) + i \left( 1 - k/2 \right) \text{Tr}(Q\phi \partial^\mu \phi) \right] + \cdots ,
\]

where \( A_\mu \) is the photon field, \( Q = \text{diag}(2/3, -1/3, -1/3) \) and \( k = \left( \frac{m_\phi}{2F_\pi} \right)^2 \) with \( m_\phi \approx 0.76 \) GeV. The ellipses stand for symmetry breaking corrections. We see that in this model, Sakurai’s vector meson dominance simply amounts to the statement that \( k = 2 \) (the KSRF relation [8]). This is a reasonable numerical approximation which is essentially stable to the addition of symmetry breakers [9]. We employ it here by neglecting the last term in Eq. (2). Although vector meson dominance must be somewhat modified in cases where the axial anomaly plays a role [10], it generally works quite well for processes such as those we study here.

**Vector Meson Dominance Model for Radiative Decays Involving Light Scalar Mesons**

Deirdre Black\(^{(a)}\),†, Masayasu Harada\(^{(b)}\),‡, and Joseph Schechter\(^{(c)}\),‡

\(^{(a)}\) Theory Group, Jefferson Lab, 12000 Jefferson Ave., Newport News, VA 23606.

\(^{(b)}\) Department of Physics and Astronomy, SUNY at Stony Brook, Stony Brook, NY 11794 and

\(^{(c)}\) Department of Physics, Syracuse University Syracuse, NY 13244-1130

We study a vector dominance model which predicts a fairly large number of currently interesting decay amplitudes of the types \( S \to \gamma \gamma, V \to S \gamma \) and \( S \to V \gamma \), where \( S \) and \( V \) denote scalar and vector mesons, in terms of three parameters. As an application, the model makes it easy to study in detail a recent proposal to boost the ratio \( \Gamma(\phi \to f_0(980))/\Gamma(\phi \to a_0(980)) \) by including the isospin violating \( a_0^0-f_0 \) mixing. However we find that this effect is actually small in our model.
The new feature of the present work is the inclusion of strong trilinear scalar-vector-vector terms in the effective Lagrangian:

$$ \mathcal{L}_{SVV} = \beta_A \epsilon_{abc} \epsilon^{a'b'd'} \left[ F_{\mu\nu}(p) \sigma_{\mu\nu} \right] \left[ F_{\mu\nu}(p) \sigma_{\mu\nu} \right] N^c $$

$$ + \beta_B \text{Tr} \left[ N \text{Tr} \left[ F_{\mu\nu}(p)F_{\mu\nu}(p) \right] \right] $$

$$ + \beta_C \text{Tr} \left[ N \text{Tr} \left[ F_{\mu\nu}(p)F_{\mu\nu}(p) \right] \right] $$

$$ + \beta_D \text{Tr} \left[ N \text{Tr} \left[ F_{\mu\nu}(p) \right] \right] \text{Tr} \left[ F_{\mu\nu}(p) \right] . $$

(3)

Chiral invariance is evident from (3) and the four flavor-invariants are needed for generality. (A term \( \sim \text{Tr} (FF\nu) \) is linearly dependent on the four shown). Actually the \( \beta_D \) term will not contribute in our model so there are only three relevant parameters \( \beta_A, \beta_B \) and \( \beta_C \). Equation (3) is analogous to the PVV interaction which was originally introduced as a \( \pi\rho\omega \) coupling a long time ago [12]. It is intended to be a leading point-like [3] description of the production mechanism. With (3) one can now compute the amplitudes for \( S \to \gamma\gamma \) and \( V \to S\gamma \) according to the diagrams of Fig. 1.

![Fig. 1: Feynman diagrams for (a) \( S \to \gamma\gamma \) and (b) \( V \to S\gamma \).](image)

The decay matrix element for \( S \to \gamma\gamma \) is written as \( (e^2/g^2)X_S \times (k_1 \cdot k_2 \epsilon_1 \cdot \epsilon_2 - k_1 \cdot \epsilon_2 k_2 \cdot \epsilon_1) \) where \( \epsilon_\mu \) stands for the photon polarization vector. It is related to the width by

$$ \Gamma (S \to \gamma\gamma) = \alpha \frac{2 \pi}{4} \frac{m_S}{g^2} \left| X_S \right|^2 , $$

(4)

and \( X_S \) takes on the specific forms:

$$ X_\sigma = \frac{4}{9} \beta_A \left( \sqrt{2}s - 4c \right) + \frac{8}{3} \beta_B \left( c - \sqrt{2}s \right) , $$

$$ X_{f_0} = \frac{4}{9} \beta_A \left( \sqrt{2}s + 4c \right) + \frac{8}{3} \beta_B \left( \sqrt{2}s + c \right) , $$

$$ X_{a_0} = \frac{4\sqrt{2}}{3} \beta_A . $$

(5)

Here \( \alpha = e^2/(4\pi), s = \sin \theta_S \) and \( c = \cos \theta_S \) where the scalar mixing angle, \( \theta_S \) is defined from

$$ \left( \begin{array}{ccc} \sigma \\ f_0 \end{array} \right) = \left( \begin{array}{ccc} c & -s \\ s & c \end{array} \right) \left( \begin{array}{ccc} N_3^2 \\ N_1 + N_2^2 \end{array} \right) / \sqrt{2} . $$

(6)

Furthermore ideal mixing for the vectors, with \( \rho^0 = (\rho_1 - \rho_2^2)/\sqrt{2}, \omega = (\rho_1^2 + \rho_2^2)/\sqrt{2}, \phi = \rho_3^2, \) was assumed for simplicity.

Similarly, the decay matrix element for \( V \to S\gamma \) is written as \( (e/g)C_S^V \times [p \cdot k\epsilon_V - c \cdot p \cdot \epsilon \cdot \epsilon_V] \). It is related to the width by

$$ \Gamma (V \to S\gamma) = \frac{\alpha}{3} \left| k_V^S \right|^3 \left| \frac{C_S^V}{g} \right|^2 , $$

(7)

where \( k_V^S = (m_V^2 - m_S^2)/(2m_V) \) is the photon momentum in the V rest frame. For the energetically allowed \( V \to S\gamma \) processes we have

$$ C_{f_0} = \frac{2\sqrt{2}}{3} \beta_A c - \frac{4}{3} \beta_B \left( \sqrt{2}c + s \right) $$

$$ + \frac{\sqrt{2}}{3} \beta_C \left( c - \sqrt{2}s \right) , $$

$$ C_{f_0}^\sigma = -\frac{2\sqrt{2}}{3} \beta_A s - \frac{4}{3} \beta_B \left( c - \sqrt{2}s \right) $$

$$ - \frac{2}{3} \beta_C \left( c + \frac{1}{\sqrt{2}}s \right) , $$

$$ C_{\rho^0} = \sqrt{2} \beta_C - 2\beta_A , $$

$$ C_{\rho^0}^\sigma = \frac{2\sqrt{2}}{3} \beta_A \left( c + \sqrt{2}s \right) + \frac{2\sqrt{2}}{3} \beta_B \left( c - \sqrt{2}s \right) $$

$$ - \frac{2}{3} \beta_C \left( \sqrt{2}c + s \right) , $$

$$ C_{\phi} = -2\sqrt{2} \beta_A c + 2\sqrt{2} \beta_B \left( c - \sqrt{2}s \right) . $$

(8)

In addition, the same model predicts amplitudes for the energetically allowed \( S \to V\gamma \) processes: \( f_0 \to \rho^0\gamma, f_0 \to \rho^0\gamma, a_0 \to \rho^0\gamma, a_0 \to \rho^0\gamma \) and, if \( \kappa^0 \) is sufficiently heavy \( \kappa^0 \to K^{\ast0}\gamma \). The corresponding width is

$$ \Gamma (S \to V\gamma) = \alpha \left| k_V^S \right|^3 \left| \frac{D_V^S}{g} \right|^2 , $$

(9)

where \( k_V^S = (m_S^2 - m_V^2)/(2m_S) \) and

$$ D_{f_0}^\sigma = \frac{2}{3} \beta_A \left( -2c + \sqrt{2}s \right) + \frac{2}{3} \beta_B \left( 2c + \sqrt{2}s \right) $$

$$ + \frac{2}{3} \beta_C \left( c - \sqrt{2}s \right) , $$

$$ D_{f_0}^\rho = -2\sqrt{2} \beta_A s + 2\beta_B \left( 2c + \sqrt{2}s \right) , $$

$$ D_{a_0}^\rho = 2\beta_C , $$

$$ D_{a_0}^\sigma = \frac{4}{3} \beta_A , $$

$$ D_{\kappa^0}^\sigma = -\frac{8}{3} \beta_A . $$

(10)

All the different decay amplitudes are described by the parameters \( \beta_A, \beta_B \) and \( \beta_C \). The reason \( \beta_D \) does not appear at all and \( \beta_C \) does not appear for \( S \to \gamma\gamma \) is that, noting Eq. (3), the Tr(\( F_{\mu\nu} \)) factor is seen to give zero when coupled to an external photon line. Because the \( \varphi \) and \( \kappa \) are so broad, the simple two body final state approximation in decays like \( \omega, \phi \to \sigma\gamma \to \pi^0\pi^0\gamma \) is not
accurate. It is better to consider these decays as having three body final states with the terms in Eq. (3) giving the vertices and to take into account large width corrections in the scalar propagators as well as non resonant background.

These formulas can be used for different choices of the quark structure of the scalar nonet $N_Q$ (e.g. the usual $q\bar{q}q\bar{q}$ scenario or the “dual” scenario $Q_aQ_b^\dagger$ where $Q_a \sim c_{abc}\bar{q}\bar{q}^\dagger$). The characteristic mixing angle $\theta_S$ is expected to differ, depending on the scheme. In the literature, besides conventional $q\bar{q}$ models, $qq\bar{q}\bar{q}$ models [4] meson-meson “molecule” models [5] and unitarized meson-meson [6] models have been investigated. Recently models featuring mixing between a $qq\bar{q}\bar{q}$ nonet and a heavier $q\bar{q}$ nonet have been proposed [7]; in this case two sets of interactions like Eq. (3) should be included.

Now we shall illustrate the procedure for the model of a single putative scalar nonet $\phi$ with a mixing angle, $\theta_S \approx -20^\circ$ (characteristic of $qq\bar{q}\bar{q}$ type scalars).

The parameters $\beta_A$ and $\beta_B$ may be estimated from the $S \rightarrow \gamma\gamma$ processes. Substituting $\Gamma_{exp}(a_0 \rightarrow \gamma\gamma) = (0.28 \pm 0.09)$ keV (obtained using [9] $B(a_0 \rightarrow \eta\gamma) = 0.177 \pm 0.024$) into Eqs. (4) and (5) yields $\beta_A = (0.72 \pm 0.12)$ GeV (assumed positive in sign). Of course, this value is independent of the value of $\theta_S$.

Next consider the $\phi$ radiative decays. Assuming $\phi \rightarrow \eta \pi^0 \gamma$ is dominated by $\phi \rightarrow a_0\gamma$, $\Gamma_{exp}(\phi \rightarrow a_0\gamma) = (0.47 \pm 0.07)$ keV and Eq. (3) determines $\beta_C$ as either $(7.7 \pm 0.5)$ GeV$^{-1}$ or $(-4.8 \pm 0.5)$ GeV$^{-1}$. Note that $|\beta_A|$ and $|\beta_B|$ are almost an order of magnitude smaller than $|\beta_C|$. Thus, the $\phi$ radiative decay rates are mainly determined by $|\beta_C|$. Knowing $\beta_A$, $\beta_B$ and $\beta_C$ we can predict $\Gamma(\phi \rightarrow f_0\gamma)$ using Eq. (3). There are four possibilities due to the two possibilities each for $\beta_B$ and $\beta_C$.

The largest number, $\Gamma(\phi \rightarrow f_0\gamma) = (0.21 \pm 0.03)$ keV corresponds [8] to the choice $\beta_B = (-0.62 \pm 0.10)$ GeV$^{-1}$ and $\beta_C = (7.7 \pm 0.5)$ GeV$^{-1}$.

Unfortunately this is still considerably smaller than the listed value [8]: $\Gamma_{exp}(\phi \rightarrow f_0\gamma) = (1.51 \pm 0.41)$ keV [9]. Recently Close and Kirk [8] proposed that the ratio $\Gamma(\phi \rightarrow f_0\gamma)/\Gamma(\phi \rightarrow a_0\gamma)$ could be boosted by considering the effects of the isospin violating $a_0^\pm(980)-f_0(980)$ mixing. We will now see that these effects are small in our model. One may simply introduce the mixing by a term in the effective Lagrangian: $\mathcal{L}_{af} = A_{af}a_0^\pm f_0$. A recent calculation [22] for the purpose of finding the effect of the scalar mesons in the $\eta \rightarrow 3\pi$ process obtained the value $A_{af} = -4.66 \times 10^{-3}$ GeV$^2$. It is convenient to treat this term as a perturbation. Then the amplitude for $\phi \rightarrow f_0\gamma$ includes a correction term consisting of the $\phi \rightarrow a_0^\pm\gamma$ amplitude given in Eq. (3) multiplied by $A_{af}$ and by the $a_0$ propagator. The $\phi \rightarrow a_0^\pm\gamma$ amplitude has a similar correction. In terms of the amplitudes in Eqs. (3), the desired ratio is then,

$$\frac{amp(\phi \rightarrow f_0\gamma)}{amp(\phi \rightarrow a_0^\pm\gamma)} = \frac{C_{\phi}^f + A_{af}C_{\phi}^a/D_a(m_f^2)}{C_{\phi}^a + A_{af}C_{\phi}^f/D_f(m_f^2)}.$$  

where $D_a(m_f^2) = -m_f^2 + m_a^2 - i m_a \Gamma_a$ and $D_f(m_f^2) = -m_f^2 + m_f^2 - i m_f \Gamma_f$. In this approach the propagators are diagonal in the isospin basis. The numerical values of these resonance widths and masses are, according to the Review of Particle Physics [23] $m_{a_0} = (984.7 \pm 1.3)$ MeV, $\Gamma_{a_0} = 50-100$ MeV, $m_{f_0} = 980 \pm 10$ MeV and $\Gamma_{f_0} = 40-100$ MeV. For definiteness, from column 1 of Table II in Ref. [23] we take $m_{a_0} = 987$ MeV and $\Gamma_{f_0} = 65$ MeV while in Eq. (4.2) of Ref. [24] we take $\Gamma_{a_0} = 70$ MeV. In fact the main conclusion does not depend on these precise values. It is easy to see that the mixing factors are approximately given by

$$\frac{A_{af}}{D_a(m_f^2)} \approx \frac{A_{af}}{D_f(m_f^2)} \approx \frac{iA_{af}}{m_a \Gamma_a} \approx -0.07i.$$  

Noting that $C_{\phi}^f/C_{\phi}^a \approx 0.75$ in the present model, the ratio in (11) is roughly $(0.75 - 0.07i)/(1 - 0.05i)$. Clearly, the correction to $\Gamma(\phi \rightarrow f_0\gamma)/\Gamma(\phi \rightarrow a_0\gamma)$ due to $a_0 f_0^\pm$ mixing only amounts to a few per cent, nowhere near the huge effect suggested in [23]. It may be remarked that Eq. (11) is practically accurate to all orders in $A_{af}$, corresponding to iterating any number of $a_0-f_0$ transitions. Then, after summing a geometric series, the numerator picks up a correction factor $[1 - A_{af}^2/(D_a(m_f^2)D_f(m_f^2))]^{-1}$ and the denominator, the similar factor $[1 - A_{af}^2/(D_a(m_a^2)D_f(m_f^2))]^{-1}$.

Vector meson dominance, together with the assumptions of SU(3) flavor symmetry and a single nonet of scalar mesons makes many more predictions. These are listed in Table I for two of the allowed ($\beta_A, \beta_B, \beta_C$) parameter sets, neglecting $a_0-f_0$ mixing. It will be interesting to see if future experiments confirm the pattern of predicted widths.

We have given a leading order correlation of many radiative decays involving scalars, based on flavor symmetry and vector meson dominance. Clearly further improvements can be made. Elsewhere, we will study flavor symmetry breaking effects, higher derivativ interaction terms, treatment of the $S\gamma$ final states as $PP\gamma$, and the case of mixed $q\bar{q}$ and $qq\bar{q}\bar{q}$ scalar nonets.

We are happy to thank N. N. Achasov for a suggested correction to the first version of this note and with A. Abdel-Rahim and A. H. Fariborz for very helpful discussions. D.B. wishes to acknowledge support from the Thomas Jefferson National Accelerator Facility operated by the Southeastern Universities Research Association (SURA) under DOE contract number DE-AC05-84ER40150. The work of M.H. is supported in part by
TABLE I: Fitted values of $\beta_A$, $\beta_B$ and $\beta_C$ together with the predicted values of the ratio $\Gamma(\phi \rightarrow f_\omega \gamma)/\Gamma(\phi \rightarrow a_0 \gamma)$ and the decay widths of $V \rightarrow S + \gamma$ and $S \rightarrow V + \gamma$. Units of $\beta_A$, $\beta_B$ and $\beta_C$ are GeV$^{-1}$ and those of the decay widths are keV.

| Parameter | Value |
|-----------|-------|
| $\beta_A$ | $0.72 \pm 0.12$ |
| $\beta_B$ | $0.61 \pm 0.10$ |
| $\beta_C$ | $7.7 \pm 0.52$ |
| $f_0/a_0$ ratio | $0.26 \pm 0.06$ |
| $\Gamma(\sigma \rightarrow \gamma \gamma)$ | $0.024 \pm 0.023$ |
| $\Gamma(\phi \rightarrow \sigma \gamma)$ | $137 \pm 19$ |
| $\Gamma(\omega \rightarrow \sigma \gamma)$ | $16 \pm 3$ |
| $\Gamma(\rho \rightarrow \sigma \gamma)$ | $0.23 \pm 0.47$ |
| $\Gamma(f_0 \rightarrow \omega \gamma)$ | $126 \pm 20$ |
| $\Gamma(f_0 \rightarrow \rho \gamma)$ | $19 \pm 5$ |
| $\Gamma(a_0 \rightarrow \omega \gamma)$ | $641 \pm 87$ |
| $\Gamma(a_0 \rightarrow \rho \gamma)$ | $3.0 \pm 1.0$ |

Grant-in-Aid for Scientific Research (A) #12740144 and USDOE Grant Number DE-FG02-88ER40388. The work of J.S. is supported in part by DOE contract DE-FG-02-85ER40231.

* e-mail: dblack@jlab.org
† e-mail: harada@tonic.physics.sunysb.edu
‡ e-mail: schechter@physics.syr.edu

[1] See the conference proceedings, S. Ishida et al. "Possible existence of the sigma meson and its implication to hadron physics," KEK Proceedings 2000-4, Soryuushiron Kenkyu 102, No. 5, 2001.

[2] N. N. Achasov et al. [SND Collaboration], Phys. Lett. B 479, 53 (2000); R. R. Akhmetshin et al. [CMD-2 Collaboration], Phys. Lett. B 462, 380 (1999); A. Aloisio et al. [KLOE Collaboration], arXiv:hep-ex/0107021.

[3] N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. B 315, 465 (1989).

[4] F. E. Close, N. Isgur and S. Kumano, Nucl. Phys. B 389, 513 (1993); N. N. Achasov and V. V. Gubin, Phys. Rev. D 56, 4084 (1997); Phys. Rev. D 57, 1807 (1998); N. N. Achasov, V. V. Gubin and V. I. Shevchenko, Phys. Rev. D 56, 203 (1997); J. L. Lucio Martinez and M. Napsuciale, Phys. Lett. B 454, 365 (1999); arXiv:hep-

[5] C. G. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177, 2247 (1969).

[6] M. Harada and J. Schechter, Phys. Rev. D 54, 3394 (1996).

[7] D. Black, A. H. Fariborz, F. Sannino and J. Schechter, Phys. Rev. D 59, 074026 (1999).

[8] J.J. Sakurai, Currents and mesons (Chicago U.P., Chicago, 1969).

[9] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966).

[10] In M. Harada and K. Yamawaki, Phys. Rev. Lett. 87, 152001 (2001), it was shown that this is also satisfied even at the quantum level in three flavor QCD.

[11] M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. 164, 217 (1988). See also M. Harada and K. Yamawaki, to appear in Phys. Rept.

[12] M. Gell-Mann, D. Sharp and W. G. Wagner, Phys. Rev. Lett. 8, 261 (1962). A general formulation of the PVV interaction in a gauge invariant manner was given in: T. Fujiwara, T. Kugo, H. Terao, S. Uehara and K. Yamawaki, Prog. Theor. Phys. 73, 926 (1985); O. Kaymak-

[13] N. N. Achasov, hep-ph/0201206, it is shown that some non pointlike effects are required to fit the experimental spectrum shape for $\phi \rightarrow \pi^0 \pi^\pm \gamma$. These may be introduced straightforwardly as higher derivative terms in our effective Lagrangian.

[14] R. L. Jaffe, Phys. Rev. D 15, 267 (1977).

[15] J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659 (1982).

[16] N. A. Tornqvist, Z. Phys. C 68, 647 (1995); E. van Beve-

[17] D. Black, A. H. Fariborz and J. Schechter, Phys. Rev. D 61, 074001 (2000); D. Black, A. H. Fariborz, S. Moussa, S. Nasri and J. Schechter, Phys. Rev. D 64, 014031 (2001); T. Teshima, I. Kitamura and N. Morisita, arXiv:hep-ph/0105107.

[18] We took $M_{f_0} = 987$ MeV here; taking 980 MeV would almost double the result.

[19] D. E. Groom et al. [Particle Data Group Collaboration], Eur. Phys. J. C 15, 1 (2000) and 2001 partial update for edition 2002 (URL: http://pdg.lbl.gov).

[20] This value is obtained by assuming that $\phi \rightarrow \pi^0 \pi^0 \gamma$ is dominated by $\phi \rightarrow f_0 \gamma \rightarrow \pi^0 \pi^0 \gamma$. In Ref. [19] it was noted that a very small contribution to decays like $\phi \rightarrow \pi^0 \pi^0 \gamma$ is obtained using two VVP vertices and vector meson dominance via $\phi \rightarrow \pi^0 \rho \rightarrow \pi^0 \pi^0 \omega \rightarrow \pi^0 \pi^0 \gamma$. This is usually called the “vector meson dominance contribution” in the literature. Of course it exists in our model too but we shall neglect it here; we assume that our effective VVS vertex describes the leading contribution.

[21] F. E. Close and A. Kirk, Phys. Lett. B 515, 13 (2001). The $a_0$-$f_0$ mixing was earlier studied in N. N. Achasov, S. A. Devyanyi and G. N. Shetstakov, Phys. Lett. B 88, 367 (1979). Very recently, the Close-Kirk model itself has been shown to lead to small $a_0$-$f_0$ mixing effects by N.N. Achasov and A. V. Kiselev; see hep-ph/0203042.

[22] A. Abdel-Rahiem, D. Black, A. H. Fariborz and J. Schechter, in preparation. The $a_0$-$f_0$ mixing is obtained from Eq. (2.10) of [9] in which the quark mass spurion is replaced by $M = \text{diag}(1 + y, 1 - y, x)$. The isospin violating parameter was taken to be $y = -0.202$ from Table III of [9]. Conservatively $y$ has about a 20 per cent uncertainty.

[23] M. Harada, F. Sannino and J. Schechter, Phys. Rev. D 54, 1991 (1996).

[24] A. H. Fariborz and J. Schechter, Phys. Rev. D 60, 034002 (1999).