Performance Analysis and Comparison Between Robust Adaptive Beamforming

M. Diao, H.Y. Song, J. Shi, B.S. Liu and C.Y. Yang

ABSTRACT

With the ever-growing needs of a high quality of wireless communication services, the field of multiple-antenna systems, which is often called Multiple-Input Multiple-Output (MIMO) systems, has evolved rapidly. In principle, multiple-antenna techniques can fully exploit the spatial domain information to enhance wireless communication quality, so have constituted the key technologies for modern wireless communications. Adaptive beamforming, which can be interpreted as a processor in conjunction with an array of antennas to provide an adaptive form of spatial filtering, is effectively utilized to improve the Signal Interference Noise Ratio (SINR) or suppress spatial noise and interference in a multiuser scenario. However, due to the array steering vector errors, small-sample errors and so on, its performance will suffer from a substantial degradation in practical engineering applications. In this paper, Norm Constraint Robust Capon Beamforming (NCRB) and Worst-Case Performance Optimization Robust Beamforming (WCRB) are respectively formulated as a standard SOCP form, and their performances are compared and analyzed in detail. Finally, computer simulations show the algorithms’ excellent performance for Signal Of Interest (SOI) power estimation and output SINR (Signal Interference Noise Ratio) as compared with the standard adaptive beamforming via a number of numerical examples.

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INTRODUCTION

With the ever-growing needs of a high quality of wireless communication services, Multiple-Input Multiple-Output (MIMO) systems have evolved rapidly (Bellofiore & Balanis 2002). Multiple-antenna techniques have constituted the key technologies for modern wireless communications. This is achieved by means of adaptive beamforming, which can be interpreted as a processor in conjunction with an array of sensors to provide an adaptive form of spatial filtering (Mietzner & Schober 2009).

Adaptive beamforming can also be called Standard Capon Beamformer (SCB) or statistically optimum beamforming, whose weights are chosen based on the statistics of the data received at the array. There are several different criteria for choosing statistically optimum beamformer weights, such as the Multiple Sidelobe Canceller (MSC), Maximization of Signal to Noise Ratio (MSNR), and so on (Van Veen & Buckley 1988). However, as implied by the terminology, adaptive beamforming relies on the availability of a precise mathematical description of the array steering vector. When the mathematical model fails to reflect the physical phenomena with a sufficient accuracy, the performance of adaptive beamforming will suffer from a substantial degradation (Krim & Viberg 1996). Therefore, robust approaches to adaptive beamforming appear to be of primary importance in practical engineering application. Until now, there have been several efficient approaches to the design of robust adaptive beamformers, such as the so-called Linearly Constrained Minimum Variance (LCMV) beamformer, Diagonal Loading (DL), subspace-based adaptive beamforming methods, signal blocking-based algorithms, Bayesian beamformer and so on (Oh & Kim 2005).

Recently, another kind of promising robust approaches based on a convex optimization using Second-Order Cone Programming (SOCP) has been studied in detail. The typical SOCP-based algorithms include Worst-Case Performance Optimization Robust Beamforming (WCRB), Norm Constraint Robust Capon Beamforming (NCRB), and so on. In principle, they do not provide any closed-form solution, but can be solved by several efficient convex optimization software tools, such as SEDUMI, CVX, and so on (Shahbazpanahi & Gershman 2003).

Although the literature on robust SOCP-based adaptive beamforming is quite extensive, prior research seldom consider the performance analysis and comparison between each other. In this paper, we first show how to efficiently formulate WCRB and NCRB as the standard SOCP form, and then analyze and compare the performance between them in detail. Numerical examples illustrate that under different SNRs or snapshots, both WCRB and NCRB can significantly improve the robustness of SCB.

CONVEX OPTIMIZATION AND SOCP THEORY

Convex optimization is a kind of special mathematic optimization problems, and is more prevalent in practice than was previously thought. There are many
advantages to recognizing or formulating a problem as a convex optimization problem. The most basic advantage is that the reliable and efficient optimal solution can be obtained through the interior point method or other special methods for convex optimization (Boyd & Vandenberghe 2004).

The second order cone program (SOCP) can be represented as:

\[
\begin{aligned}
\min_x & \quad f^T x \\
\text{s.t.} & \quad \|A_i x + b_i\|_2 \leq c_i^T x + d_i, \quad i = 1, \ldots, m \\
& \quad F x = g
\end{aligned}
\]

where \( x \in \mathbb{R}^n \) is the optimization variable, \( A_i \in \mathbb{R}^{n \times n} \), \( F \in \mathbb{R}^{p \times n} \). We call a constraint of the form \( \|A_i x + b_i\|_2 \leq c_i^T x + d_i \), \( A_i \in \mathbb{R}^{n \times n} \), a second order cone constraint.

**SOCP FORM AND SOLUTION TO ROBUST ADAPTIVE BEAMFORMING**

**Norm Constraint Robust Capon Beamformer**

Norm Constraint Robust Capon Beamforming (NCRB) imposes an additional constraint on the Euclidean norm of the weight vector to improve the robustness of the Capon beamformer (Li & Stoica 2004). So it can be formulated as follows:

\[
\begin{aligned}
\min_y & \quad \|y\|_1 \\
\text{s.t.} & \quad w^H a(\theta) = 1, \quad \|U w\| \leq y_1, \quad \|w\| \leq \sigma
\end{aligned}
\]

Through the zero cone and second order cone transformation, we can obtain:

\[
\begin{aligned}
\begin{bmatrix}
y_1 \\ U w
\end{bmatrix} &= \begin{bmatrix} 0 \\ \frac{1}{\sigma} \end{bmatrix} \begin{bmatrix} a(\theta)^H y \phantom{y} y \end{bmatrix} = c_1 - A_1^T y \in \mathbb{R}^{1 \times N} \\
\begin{bmatrix} \sigma \end{bmatrix} &= \begin{bmatrix} \sigma \\ \frac{1}{\sigma} \end{bmatrix} \begin{bmatrix} 0 \\ -I \end{bmatrix} = c_2 - A_2^T y \in \mathbb{R}^{1 \times N + 1}
\end{aligned}
\]

Let \( e = [c_1, c_2, c_5]^T \), \( A = [A_1, A_2, A_3]^T \), so

\[
\begin{aligned}
A^T &= \begin{bmatrix} 0 & a^H(\theta) \\ -1 & \frac{1}{\sigma} \end{bmatrix} \in \mathbb{R}^{(2N+4)(N+1)} \\
b &= [-1, \frac{1}{\sigma}]^T \in \mathbb{R}^{(N+1) \times 1} \\
c &= [1, 0, \sigma, 0]^T \in \mathbb{R}^{(2N+4) \times 1}
\end{aligned}
\]

Hence the optimization weight vector \( w \) which is obtained by SOCP satisfies the following equation (Vincent & Besson 2004, Zhang & Song, 2015):

\[
\begin{aligned}
\max_y & \quad b^T y \\
\text{subject to} & \quad c - A^T y \in \{0\} \times SOC_1^{1 \times N} \times SOC_2^{1 \times N}
\end{aligned}
\]

Then, apply the software tool SEDUMI to Equ.(9), we can obtain the optimization weight vector \( w_{\text{NCRB}} \).

**Worst-Case Performance Optimization Robust Beamforming**

Worst-Case Performance Optimization Robust Beamforming (WCRB), which is based on the optimization of worst-case performance, is a new powerful approach
to robust adaptive beamforming in the presence of an arbitrary unknown steering vector mismatch (Vorobyov & Gershman 2003). The algorithm can be represented as the following optimization problem:

\[
\begin{align*}
\min_{\tau, \mathbf{w}} & \quad \tau \\
\text{s.t.} & \quad \varepsilon \|\mathbf{w}\| \leq \mathbf{w}^H \mathbf{a} - 1, \quad \text{Im}\{\mathbf{w}^H \mathbf{a}\} = 0, \quad \|U\mathbf{w}\| \leq \tau
\end{align*}
\]  

(10)

Further let

\[
\begin{align*}
\mathbf{d} &= [1, \ 0^T] \in \mathbb{R}^{(2M+1)\times 1} \\
\mathbf{y} &= [\tau, \ \mathbf{w}^T] \in \mathbb{R}^{(2M+1)\times 1} \\
\mathbf{f} &= [\mathbf{0}^T, \ -\mathbf{1}, \ \mathbf{0}^T] \in \mathbb{R}^{(4M+3)\times 1} \\
\mathbf{F}^T &= \begin{bmatrix}
1 & \mathbf{0}^T \\
0 & \mathbf{U} \\
0 & \mathbf{a}^T \\
0 & \mathbf{e} \mathbf{I} \\
0 & \mathbf{a}^T
\end{bmatrix} \in \mathbb{R}^{(4M+3)\times (2M+1)}
\end{align*}
\]

(11)

(12)

(13)

(14)

And Equ.(10) can be represented as:

\[
\begin{align*}
\min_{\gamma} & \quad \mathbf{d}^T \mathbf{y} \\
\text{s.t.} & \quad \mathbf{f} + \mathbf{F}^T \mathbf{y} \in \text{SOC}_1^{2M+1} \times \text{SOC}_2^{2M+1} \times \{0\}
\end{align*}
\]

(15)

So the optimization weight vector is:

\[
\mathbf{w}_{\text{NCRB}} = \begin{bmatrix} \mathbf{\bar{w}}_1, \ldots, \mathbf{\bar{w}}_M \end{bmatrix}^T + \mathbf{j} \begin{bmatrix} \mathbf{\bar{w}}_{M+1}, \ldots, \mathbf{\bar{w}}_{2M} \end{bmatrix}^T
\]

(16)

So far, we have converted Equ.(10) into Equ.(15), which is SOCP and can be solved by software tool Sedumi conveniently.

In summary, both NCRB and WCRB can be converted into the SOCP form, and solved by SEDUMI (Sturm 1999).

**SIMULATION ANALYSIS**

**Spatial Spectrum Comparison**

Simulation conditions: the signal frequency is 1kHz, the number of snapshots is 200, the incidence angle is 20°, the number of the array elements is 16, element spacing is \(\lambda/2\) (\(\lambda\) is the wavelength). Suppose the \(\text{SNR} = 10\text{dB}\), the steering vector errors are \(-5\text{dB}\), the constraint parameters for NCRB and WCRB are respectively 0.3 and 2.0. Compare and analyze the spatial spectrum estimation results of NCRB, WCRB, SCB, and Conventional Beamforming (CBF).

![Figure 1. Spatial spectrum estimation results.](image-url)
As shown in Fig.1, CBF has wide main lobe and serious background fluctuation. Due to the influence of the array steering vector errors, noises and other factors, the performance of SCB, such as the suppression ability to the noise and interference, is severely degraded. Not only the main lobe becomes wider, but also the side lobe level is only about -7dB. However, under the conditions of the given constraint parameters, both WCRB and NCRB have a more acute peak and the greater background suppression ability. For example, the side lobe level can reach below -20dB. So both WCRB and NCRB can effectively improve the robustness of adaptive beamforming.

Output SINRs and the Signal Power Estimation under Different SNRs

Simulation conditions: suppose there exist a signal and two interferences. The incidence angle of the signal is $0^\circ$, the frequency is 2kHz. The incidence angles of two interferences are respectively $20^\circ$ and $30^\circ$, the frequencies are respectively 3kHz and 5kHz, the INRs are both 30dB. Consider a uniform linear array with 16 elements, and the array element spacing is half wavelength of the signal. The steering vector errors are caused mainly by the Direction Of Arrival (DOA) estimation bias $\Delta \theta = \Gamma^\circ$. The constraint parameters for NCRB and WCRB are respectively 0.3 and 1.0. The number of snapshots is 20, and the SNR ranges from -20dB to 30dB. The simulation results are shown in Figure 2.

Figure 2(a) are output SINRs for NCRB, WCRB and SCB under different SNRs. It can be seen that, with the increase of the SNRs, the output SINRs of NCRB and WCRB also gradually increase, and both are significantly higher than SCB. For example, when the input SNR is 5dB, the output SINRs of NCRB and WCRB are respectively about 11dB and 7dB. But SCB has poor robustness, and its output SINR is less than -10dB. Figure 2(b) are the signal power estimation for NCRB, WCRB and SCB under different SNRs. We can see that NCRB and WCRB can estimate the signal power nicely, except under the lower SNRs (about below -10dB), where there will be a slight deviation. But due to the influences of the noise, interferences and the steering vector errors, the robustness of SCB is significantly degraded. So there will appear greater power estimation errors, and we can no longer effectively estimate the actual signal power level. From the above analysis, under different SNRs, NCRB and WCRB can significantly improve the robustness of SCB.
Output SINRs and the Signal power Estimation under Different Snapshots

Simulation conditions: the simulation conditions are similar to simulation 3. SNR = 10dB, the number of snapshots ranges from 10 to 500. The constraint parameters for NCRB and WCRB are respectively 0.3 and 1.3. The simulation results are shown in Figure 3.

It can be seen from Figure 3 (a) that with the increase of snapshots, the output SINRs of WCRB and NCRB also gradually increased. However when the snapshots grow more than 150, both the output SINRs are stabilized up to 18dB. Due to the influence of the DOA estimation bias, the performance of SCB is seriously degraded, its output SINR is only about -10dB, which is well below the estimation results of WCRB and NCRB. Figure 3(b) shows that, when the number of snapshots is less than 20, the signal power estimated by SCB declines significantly, and obviously departs from the actual signal power. But WCRB and NCRB are more robust against the small snapshots, and the signal powers estimated by them are almost the same. When the snapshots grow more than 20, the signal power estimated by SCB, WCRB and NCRB are all stabilized. However both WCRB and NCRB can obtain the accurate estimation, while SCB has the deviation about 10dB. Thus, under different snapshots, both NCRB and WCRB have significantly superior robustness to SCB.

CONCLUSIONS

Adaptive beamforming constitutes the key technology for modern wireless communications, especially in the aspect of improving the Signal Interference Noise Ratio (SINR) or suppressing spatial noise and interference in a multiuser scenario. This paper converts NCRB and WCRB, which are both non-convex problems, respectively into SOCP form, and then makes use of the mathematical toolbox SEDUMI to solve the SOCP problems, and thus obtains the robust optimization weight vectors. Computer simulations show the algorithms’ excellent performance for SOI power estimation and output SINR as compared with the standard adaptive beamforming via a number of numerical examples.
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