Abstract—An efficient multilayer machine learning-assisted optimization (ML-MLAO)-based robust design method is proposed for antenna and array applications. Machine learning methods are introduced into multiple layers of the robust design process, including worst-case analysis (WCA), maximum input tolerance hypervolume (MITH) searching, and robust optimization, considerably accelerating the whole robust design process. First, based on a surrogate model mapping between the design parameters and the performance, the WCA is performed using a genetic algorithm to ensure reliability. MITH searching is then carried out using an MLAO-based framework to find the MITH of the given design point. Next, based on the training set obtained using MITH searching, correlations between the design parameters and the MITH are learned. The robust design is carried out using surrogate models for both the performance and the MITH, and these models are updated online following the ML-MLAO scheme. Finally, two examples, including an array synthesis problem and an antenna design problem, are used to verify the proposed ML-MLAO method. The numerical results and computation time are discussed to demonstrate the effectiveness of the proposed method.

Index Terms—Antennas, arrays, robust design, machine-learning assisted optimization (MLAO).

I. INTRODUCTION

Robust design is one of the most crucial aspects in the design of modern antennas and arrays, which aims to learn the correlation between the input and output tolerance, thereby finding a balance between the robustness and performance of the final design and providing guidelines for the integration and manufacturing process [1]–[5]. With the rapid development of commercial full-wave electromagnetic (EM) simulation tools, robust design independent of the computational EM process has been widely investigated with optimization schemes [3], [4], [6] and surrogate-assisted modeling approaches [2], [7]. A lot of methods such as worst-case analysis (WCA) [4], [8] were introduced to find the output tolerance when the input tolerance is known. Based on the reliable analysis of worst-case performance (WCP), various methods were developed to find the input tolerance when the output tolerance is known. A popular approach is maximum input tolerance hypervolume (MITH) searching, which aims to find the MITH that satisfies the predefined output tolerance. Some advanced algorithms ranging from global search algorithm combined with iterative input tolerance hypervolume (ITH) shrinking [4], to sampling strategy combined with surrogate modeling [2] were proposed to achieve efficient MITH searching and robust optimization for antenna designs. Nonetheless, in [4], the ratios between different design parameters must be decided in advance and cannot be changed during the iterative process. Although able to achieve the quasi-optimal shape of the MITH efficiently, the fidelity of the MITH in [2] is dependent on the sampling points in every iteration, and there is no guarantee that the real WCPs of the obtained MITH are found during the process, which leads to possible bias in the calculated MITH.

Recently, surrogate models built using different strategies have been introduced to deal with EM optimization and design problems [9]–[17]. As one of the data-driven modeling strategies, machine learning methods such as artificial neural networks [14] and Gaussian process regression (GPR) [9], [13] have been widely utilized to build surrogate models and then applied to machine learning-assisted optimization (MLAO) schemes for EM component designs. Combined with different searching algorithms, the surrogate models can be introduced to find the WCP [7] or applied in MITH searching [2], therefore accelerate the robust design process for EM components. The heavy computational burden due to the cost of building surrogate models with high predictive ability can be alleviated by applying multistage collaborative machine learning method based on multifidelity data sets [15]. However, the surrogate models have been used to replace only full-wave EM simulations; consequently, the large number of function calls required in the robust design process will still result in high time consumption.

We presented an efficient MITH searching process by learning the correlations between the design parameters and the WCP for a simple patch antenna application in [17]. This approach is investigated in detail and then extended to more general situations with arbitrary dimension problems in this study. Moreover, it is used to build a reliable MITH database under present prediction for responses in each iteration, and more importantly, integrated into an efficient and reliable robust design scheme named the multilayer MLAO (ML-MLAO) method. The utilization scope of the MLAO algorithm is expanded to the prediction and optimization of not only antenna and array responses and WCPs, but also MITHs and robust designs. The proposed algorithm greatly improves the reliability of the evaluated MITH and the efficiency when compared with conventional optimization-based robust design algorithms.

The rest of this communication is organized as follows. The MITH searching strategy based on MLAO is introduced in Section II. Section III integrates the proposed MLAO-based MITH searching method into the proposed ML-MLAO algorithm to achieve a robust design for antennas and arrays. Array synthesis and antenna design problems are used to verify the proposed method in Section IV. Finally, Section V concludes this work.

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II. WCA AND MITH SEARCHING

A. Mathematical Formulation of the Problems

Consider a set of \( Q \) nonlinear differentiable functions, each with \( P \) variables: \( y_q(x) = y_q(x_1, x_2, \ldots, x_P) \), \( q = 1, 2, \ldots, Q \). The uncertainties of the input parameters are defined by the input tolerances on the variables: \( \delta = [\delta_1, \delta_2, \ldots, \delta_P]^T \), \( \delta_p \geq 0 \). Therefore, the input tolerance interval is represented as

\[
\omega_{x, \delta} = [t | x - \delta \leq t \leq x + \delta] \tag{1}
\]

where \( t \) represents possible parameter values. The WCP is defined as the performance of the specific parameter \( \hat{x} \) that causes the maximum deviation; that is,

\[
F_q(\hat{x}) = \max_{\hat{x} \in \omega_{x, \delta}} f_q(\hat{x}) \tag{2}
\]

For \( Q > 1 \), there may exist multiple WCP points within the design space. The collection of WCPs is defined as \( F(x) \). The methods employed to search for the WCP represent the infrastructure of robust antenna design. Within a given input tolerance region (ITR), a global optimization method is normally required over the tolerance interval to search for reliable WCP points [8].

The uncertainties in the design parameters of a nominal point \( x \), known as the ITR, can be modeled as a hyperrectangle defined by \( \delta(x) \). The ITH can be defined to evaluate the size of the ITR, which can be represented as the product of the uncertainties

\[
T_{ITH}(x) = \prod_{p=1}^{P} \delta_p(x) \tag{3}
\]

A more important issue in robust antenna design is to search for the maximum \( T_{ITH} \) if the output tolerance region (OTR) is known, which can be regarded as an optimization problem if WCA can be performed with high accuracy. Similar to the ITR, the OTR can be defined as \( \Delta \triangleq [\Delta_1, \Delta_2, \ldots, \Delta_Q]^T \), \( \Delta_q \geq 0 \). The output tolerance interval can be represented as

\[
\Omega_{s, \Delta} \triangleq \{s | y - \Delta \leq s \leq y + \Delta\} \tag{4}
\]

where \( s = [s_1, s_2, \ldots, s_Q]^T \) represents possible function values. The MITH searching can be numerically represented as

\[
\max T_{ITH}(x), \quad s.t. \quad F(x) \in \Omega_{s, \Delta} \tag{5}
\]

B. MITH Searching

As the infrastructure of the robust design, the WCP searching process plays an important role by evaluating the robustness of the given design point and ITR. In [4], a global optimization algorithm is introduced to locate the WCP within the ITR for the given design point, and a novel tolerance region shrinking strategy is proposed to achieve 100% reliability robust optimization. Due to the high computational cost introduced by global optimization required by the WCP searching, the ratios between the design parameters are decided before MITH searching, therefore, ease the whole computation burden of the robust design. Instead of viewing the WCP searching as the fundamental unit of the MITH searching process, a novel sampling strategy is proposed in [2], in which an integrated process is introduced to achieve a direct solution to search for the quasi-optimal shape of the MITH. However, the independent WCP searching process leads to the reliability issue of the founded MITH. Here, the MLAO-based MITH searching method is proposed to solve the above-mentioned issues, and to avoid one-sidedly emphasize the efficiency, degree of design freedom, and reliability in the conventional algorithms.

Fig. 1. Flow diagram of the MLAO-based algorithm for MITH searching.

Similar to the approach in [7], the MLAO is able to achieve efficient WCP searching under given ITR. The optimization is operated using the low-fidelity surrogate models learned by machine learning methods. In antenna designs, different from the typical MLAO scheme [16], the validation and surrogate model update process are operated in outer iterations, rather than during each iteration of the WCP searching. For cases such as array synthesis, the high-fidelity models are utilized directly for optimization procedures instead of using MLAO, due to the negligible time cost required by the high-fidelity responses. Therefore, the computation time for one WCP searching is controlled within several seconds for both antenna and array designs, which is acceptable for future data set establishment. The workflow of the MLAO-based MITH searching algorithm is illustrated in Fig. 1. Detailed steps, technical considerations, and benchmark examples are shown as follows.

**Step 1 (Initialize):** For given design point, OTR and model, the design space should be defined to ensure that all possible fabrication error and practical bias are considered.

**Step 2 (Sample ITR Vectors):** Using both random and uniform sampling strategies, a number \( N_t + N_u \) of ITR vectors are sampled within the predefined design space. Other sampling strategies can also be introduced based on practical usage scenes.

**Step 3 (Calculate WCPs With ITRs):** The corresponding WCPs are calculated following the above-mentioned WCP searching methods for the sampled ITRs. The data sets are established containing ITRs and WCPs as input and output data, respectively. For multiobjective designs, there may exist multiple design parameter vectors related to WCPs of one or multiple design objectives.

**Step 4 (Update Surrogate Model):** Surrogate models are constructed using machine learning methods such as the single-output or multioutput GPR algorithms [15], based on the data sets including ITR vectors \( w_i \) and WCP responses \( w_i \) obtained in step 3. The surrogate models are then utilized to predict the values of WCP for any given ITR.

**Step 5 (Optimize Surrogate Model for MITH):** Here, the fitness function \( f_{MITH} \) is defined as follows:

\[
f_{MITH} = D(F(x), \Omega_{s, \Delta}, F(x) \in \Omega_{s, \Delta}) - T_{ITH}(x) \tag{6}
\]

where \( D(F(x), \Omega_{s, \Delta}) \) represents the distance between the WCP and the OTR

\[
D(F(x), \Omega_{s, \Delta}) = \sum_{q=1}^{Q} \max_{f_q(t) \in [\Omega_{s, \Delta}]} \left\{ \left| f_q(t) - (y_q + \Delta_q) \right| - \left| f_q(t) - (y_q - \Delta_q) \right| \right\} \tag{7}
\]
For cases in which \( F(x) \notin \Omega_{x,A} \), \( f_{\text{MITH}} \) is positive and decreases with the predicted WCP moving toward the OTR. For cases in which \( F(x) \in \Omega_{x,A} \), \( f_{\text{MITH}} \) is negative and decreases when \( T_{\text{MITH}} \) increases. The proposed fitness function offers a continuous optimization objective space and can find the MITH efficiently.

**Step 6 (Validate Optimized ITR):** The optimized ITR is then validated based on WCP searching, and then checked by termination criterion, such as the maximum number of iterations \( N_{\text{iter}} \) or the maximum number of available results \( N_{\text{num}} \). If no criterion is met, the surrogate models for WCP are updated, and then, step 4 is repeated.

The MLAO-based MITH searching method is a nested loop structure that uses WCP searching as the high-fidelity simulation for the MITH search process. In [2], a novel MITH search method called sampling strategy is introduced, in which \( N_i = 7500 \) sampling points in each iteration, and \( N_{\text{iter}} = 6 \) iterations are suggested for practical antenna applications. Two issues are worth further consideration: 1) for practical problems with large design spaces and no prior knowledge about the MITH dimensions, the MITH may not be found if the number of sampling points is relatively small and 2) the evaluated MITH may not be accurate because the WCP may not be located within the sampling set. Here, the reliability and degree of design freedom are ensured by using a global optimization process for MITH searching. And the consequent large number of function calls is alleviated by introducing the MLAO again and viewing the MITH searching as a constraint optimization problem.

A \( K \)-dimensional, nonconvex, nonseparable benchmark function Ackley is evaluated to verify the proposed MLAO-based MITH searching, which is defined by

\[
g(x) = \sum_{k=1}^{K-1} \left( e^{-0.2 \sqrt{x_k^2 + x_{k+1}^2}} + 3 (\cos (2 \pi x_k) + \sin (2 \pi x_{k+1})) \right) + 0.1 .
\]

The design point is set to \( x_k = 1, k = 1, \ldots, K \), with the OTR of \([-1, 1]\) around \( g(x) \). A performance comparison between the sampling strategy and the proposed MLAO-based method is given in Fig. 2 for different input dimensions and sampling point numbers \( N_i \) of sampling strategy. After calculating the MITHs using the two methods, the GA is introduced to find the WCPs, which are then compared with the given OTR to validate the MITHs. The deviations are calculated by comparing the WCPs with the upper and lower bounds and are shown using different colors and point sizes.

Fig. 2 reveals several issues. First, due to the limitation of the sampling point number, the sampling strategy may fail in finding the MITH. Second, in many cases, the WCP of the MITH found by sampling strategy is outside the OTR, which means that the true

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**Fig. 2.** Performance comparison between sampling strategy and the proposed MLAO-based MITH searching method.

WCPs have not been found. Third, with increasing input parameter numbers, the ability of the conventional sampling strategy to find the reliable MITH is limited. Compared with the sampling strategy, the proposed MLAO-based method can obtain MITH results with their WCPs within the OTR but close to the OTR bounds. Considering both efficiency and reliability, the proposed MLAO-based method can be very helpful for designing practical antennas and arrays.

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**III. ROBUST OPTIMIZATION**

Based on the WCA and MITH search algorithms proposed above, robust antenna design can be accomplished using robust optimization for different applications, including array synthesis and antenna design. The ML-MLAO method is proposed based on the MLAO scheme by applying MLAO-based MITH searching as the high-fidelity simulation procedure. The workflow of the ML-MLAO algorithm for robust antenna design is given in Fig. 3. The detailed steps are summarized as follows.

**Step 1 (Initialize):** The robust design procedure can be implemented after finishing the optimization procedure, from which the number \( N_{\text{ini}} \) of initial design points can be obtained. In addition, the design space and constraints should also be defined, as should the OTR.

**Step 2 (Sample Design Points):** One practical antenna problem is to find the most robust antenna design based on several optimal design vectors [4]. In this case, the robust design process is implemented based on assumptions that the most robust designs are located around the initial design points obtained by the optimization procedure. Therefore, \( k_d \) design points around each design point are randomly sampled using strategies such as Latin hypercube sampling.

**Step 3 (Calculate MITHs):** The MITHs corresponding to every sampled design point, in addition to the initial design points, are calculated using the MLAO-based MITH searching method.

**Step 4 (Update Surrogate Model):** Surrogate models are established to build relationships between the locations of the sampling points and the corresponding MITHs. For antenna designs with \( N \) parameters, \( N \) surrogate models are established to make predictions about the MITHs at new points. Here, the sampled design points \( x_i \) are regarded as input variables \( X \), and the corresponding MITH responses \( w_i \) are regarded as output variables \( Y \).

**Step 5 Optimize Surrogate Model:** Here, different optimization methods and different fitness functions can be implemented for different design applications. Different design objectives, such as antenna size, robustness, and performance, can be optimized simultaneously using multiobjective
optimization methods. Single-objective optimization can also be utilized if robust designs are needed only around the initial design points. 

*Step 6 (Validate Optimized Designs):* The predicted optimization results of the MITHs are verified using the proposed ML-MLAO-based MITH search method, and the predicted antenna performance is verified using EM simulations. The termination criterion, such as the maximum number of iterations $N_{\text{ML}}$ or the maximum number of unchanged iterations, is then checked. If one of the termination conditions is met, the process is stopped; otherwise, the database is updated, and step 4 is repeated.

The accuracy and robustness of the results are ensured by the accuracy of ML-MLAO-based MITH searching. Fig. 4 depicts the operating principle of the proposed ML-MLAO method. Based on the established surrogate model $R_{s1}$ between the design parameters and antenna responses, the WCP values of different design points are calculated and utilized to build a surrogate model $R_{s2}$ between the design parameters and the WCP. By using the two surrogate models established above, a training set consisting of antenna parameters and MITHs is constructed to build a surrogate model $R_{s3}$. Here, single-output GPR is introduced to build surrogate models in the ML-MLAO algorithm. The detailed information including input and output data sets of the utilized three GP surrogate models in ML-MLAO is given in Table I. Here, optimization has been introduced to optimize for best covariance functions among the squared exponential, the exponential, the Matern class, and the rational quadratic kernels, and also optimize for other parameters such as the noise standard deviation. By varying these parameters, the optimization aims to minimize the cross-validation loss and find the best parameters to construct the surrogate model. The ML-MLAO algorithm offers an efficient solution to build surrogate models, make predictions of new design points and update surrogate models online, thereby efficiently searching for robust design points or Pareto fronts consisting of robust information.

**IV. VERIFICATION EXAMPLES**

In this section, an array synthesis problem and an antenna design problem are used for verification.

**TABLE I**

| Models | Input                        | Output          |
|--------|------------------------------|-----------------|
| $R_{s1}$ | Ant. parameters              | Ant. responses  |
| $R_{s2}$ | Ant. parameters, responses and ITR | WCPs           |
| $R_{s3}$ | Ant. parameters and OTR     | MITH            |

A symmetric linear uniformly spaced antenna array consisting of 2N4i isotropic elements is considered. The element spacing is $d = 0.5\lambda$, and the magnitudes of the elements are uniformly excited. The phases of the elements are optimized to achieve the lowest SLL. Two design results are preliminarily optimized using MLAO optimization methods [18] and then examined and modulated using the ML-MLAO method to obtain robust design points around them. The search range for each design point is $\pm 20^\circ$ within the initial designs. The OTR is set as the SLL of $-12$ dB. The element phases, MITHs and SLLs before and after the robust design process are given in Table II.

Two designs give similar SLL initially, while design 1 suffers a much lower MITH than design 2, which means that the latter is more robust than the former before the robust design. Upon applying ML-MLAO, the two designs are both modulated to achieve better MITH performance. Although design 1 has a worse SLL performance than design 2 after the robust design, it has a higher MITH value of approximately 5705.8, which means that design 1 is more robust than design 2 after the robust design process.

**B. Antenna Design**

In contrast to the array synthesis problem discussed above, the solution of the antenna design problem relies on the prediction accuracy of the surrogate models trained with the datasets established by full-wave EM simulations. For antennas with a large design parameter range, acquiring accurate surrogate models for the entire design space is difficult if time is limited. Therefore, local surrogate models surrounding the possible optimal design points should be established and updated online during the robust design process rather than surrogate models suitable for the entire design space. The robust antenna design algorithm is divided into two phases: the optimization phase and the robust optimization phase.

In the optimization phase, the MLAO in [18] is applied to calculate the optimal designs under the constraint of forbidden areas by previously optimized design points [4]. By implementing forbidden areas, multiple local optima are obtained with predefined numbers. The acquired data sets are then utilized to build surrogate models for the robust optimization phase, which enables the surrogate models to achieve a balance between local prediction accuracy around the Pareto front and global prediction ability within the searching area.

In the robust optimization phase, the ML-MLAO algorithm is implemented. Particularly, for multiobjective robust design...
considering both antenna performance and robustness, the Pareto front $P_{pre,i,j}$ and the corresponding worst case $w_{i,j}$ are predicted based on the learned correlations, where $i$ and $j$ respectively represent the indexes of the outer loop and inner loop and the iteration number of the outer loop is represented by $I$. In the inner loop, ML-MLAO is implemented to achieve an accurate Pareto front based on the available dataset for antenna responses. In the outer loop, news design points are sampled around the calculated Pareto front, and the surrogate models for antenna responses are updated online. For each performance objective, single-objective optimization is carried out to optimize each individual objective using the surrogate models; the optimized results are then used to seed the multiobjective optimization procedure, thereby enhancing the performance [19].

In the inner loop, the predicted antenna performance responses $P_{pre,i,j}$ are verified using EM simulations. The surrogate models for the antenna performance are then updated. The corresponding MITH of $P_{pre,i,j}$ is also updated, with the obtained Pareto front named $P_{val1,i,j}$. The termination criterion can be set as the maximum number of iterations of the inner loop $J$ or a limit on the RMSE. If the termination criterion is fulfilled, then the algorithm shifts to the outer loop, in which a number of design points $N_{sam}$ are randomly sampled around every design point in the Pareto front with verified antenna responses $P_{val1,i,j}$ within circular regions of radii $k_{sam} \times D_{sam}$ and at the midpoint between each design point and the corresponding WCP point, where $D_{sam}$ is the distance between these two points and $k_{sam}$ is the coefficient. Local surrogate models are updated, so improved accuracy is achieved when predicting the antenna responses around $P_{val1,i}$. Surrogate models for the parameter tolerances are then updated based on the updated local surrogate models for the antenna responses using MITH searching and WCA. The parameter tolerances for the obtained Pareto front with verified antenna responses are then recalculated based on the updated surrogate models. The Pareto front obtained here is $P_{val2,i}$, in which the parameter tolerance can be regarded as a verified MITH based on the accurate local surrogate models. By choosing $k_{sam}$ slightly larger than 1, the surrogate models tend to model not only closely around the predicted Pareto front, but also the responses within and around the boundary of the ITR of the predicted MITH, which leads to the enhancement of the prediction ability of the MITH after each iteration.

Based on the verified results for both the antenna responses and the parameter tolerances, Pareto front $P_{upd,j}$ is then obtained based on the entire dataset, which is regarded as the final high-fidelity results in one outer loop. Whereas $P_{val2,i}$ gives a relatively accurate Pareto front prediction, the final Pareto front should be further updated due to the presence of inevitable prediction bias. The Pareto front obtained here is $P_{upd,i}$, which can be utilized for tradeoff purposes in the final design. The termination criterion for the outer loop can be set as the maximum number of iterations of the outer loop $I$. By introducing a two-level nested loop, the surrogate models for the design points on and around the predicted Pareto front are built and updated within the robust design framework. The Pareto front is verified and updated to achieve better accuracy. One antenna design is shown here to verify the proposed ML-MLAO algorithm.

The series-fed microstrip antenna array (SMAA) shown in Fig. 5 is analyzed based on the proposed optimization and robust design method. The SMAA is designed on the ground plane with a Rogers 5880 substrate having dimensions of $400 \times 30 \times 1.5$ mm$^3$ and a relative permittivity of $\varepsilon_r = 2.3$. The antenna is designed for 5.8 GHz with ten series-fed microstrip antenna elements of symmetric structure and shortened at the end. The distance between different elements, the width of the antenna elements and the width of the microstrip line are constant at $g_0 = 17$ mm, $m_0 = 2$ mm and $w_1 = 16.4$ mm. The output tolerance is set with a maximum $|S_{11}|$ better than $-14$ dB and an SLL better than $-18$ dB. The design vector is $x = [l_1, l_2, l_3, l_4, l_5]^T$, and the design space is defined by the center vector $x^0 = [20.5, 20, 18, 11.5, 9]^T$ mm with a variable range of $x^0 \pm \delta$, where $\delta = [1.5, 2, 3, 2.5, 3]^T$ mm. The WCA design space should be larger than the design space for design cases in which the WCP points appear near the boundaries; hence, the WCA design space is set as $x^0 \pm \delta_w$, where $\delta_w = [2, 2.5, 3, 3, 3.5]^T$ mm. The optimization and SA phases are performed in turn with $J = 1$, $I = 4$, $k_{sam} = 1.2$, $N_{sam} = 30$, $N_q = 5$, $N_l = 5$ and $N_{iter} = 5$. The design objectives are the reflection coefficient $|S_{11}|$ and SLL at the designated frequency point and the MITH. The Pareto front for the designed SMAA after four iterations is shown in Fig. 6, with $P_{pre,4}$, $P_{val2,4}$ and $P_{upd,4}$ presented. The predicted and verified antenna responses and tolerances are shown in Fig. 7. The algorithm gives a close prediction for both antenna responses and the MITH.

Three design points on the obtained final Pareto front are highlighted, and their corresponding dimensions and performance values are given in Table III. A clear tradeoff between the antenna
performance and robustness can be ascertained. The data in Table III demonstrate that while Design 3 has the best $|S_{11}|$ performance, Design 2 has comparable $|S_{11}|$ performance and much better SLL performance. Compared with Design 2, Design 1 suffers from worse $|S_{11}|$ and SLL performance but has much better parameter tolerances, which is more suitable when robustness is considered in the final fabrication process. A summary of the computation time for the SA process of the SMAA design is listed in Table IV. The computation time has been largely reduced by using surrogate models for not only the antenna responses $|S_{11}|$ and SLL but also the WCP points and MITHs. A total of 420 MITH searches are conducted, where each implementation takes approximately 10 min of computation time, and the corresponding MITHs are predicted 50,600 times, which means that the overall computation time increases to approximately 8,857 h if no surrogate models for the MITHs are used. A total of 14,678 WCAIs are conducted, where each implementation takes approximately 10.0 s of computation time, and approximately 65 million predictions are made for the WCP, with 154,857 predictions for the WCP in one MITH search process. The entire computation time can reach approximately 21.8 million hours if no surrogate models for the WCP and MITH are used. A computation time of 139.9 s is needed to conduct the high-fidelity simulation of the SMAA. In one WCA process, approximately 5,445 function calls for the $|S_{11}|$ and SLL performance are needed. If no surrogate models are used for the WCP, MITH, $|S_{11}|$ and SLL, then the computation time will increase to approximately 189 million years. It is worth noting that all computation times listed above, except the time using all surrogate models, are roughly estimated based on the evaluation times of the surrogate models; these estimates are calculated only to show how the application of surrogate models for different design objectives can help largely reduce the entire computation time.

### V. CONCLUSION

An ML-MLAO method has been proposed to achieve a reliable and efficient robust design for antenna and array applications. By introducing MLAO algorithms to different layers of the robust design process, rapid and trustworthy WCA, MITH searching, and robust optimization have been achieved. The heavy computational loads for global searching, EM simulation, and tolerance analysis have been greatly reduced by exploiting prior knowledge regarding the correlations between the antenna parameters and responses, WCP and MITH. This increased efficiency has been utilized to make predictions in different design layers, largely accelerating the whole robust design process. Moreover, the surrogate models for the aforementioned design objectives are updated online following the ML-MLAO algorithm. Array synthesis and antenna design have been described to verify the efficiency and reliability of the proposed method.

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