Well test mathematical model for fractures network in tight oil reservoirs

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**Abstract.** Well test, especially build-up test, has been applied widely in the development of tight oil reservoirs, since it is the only available low cost way to directly quantify flow ability and formation heterogeneity parameters. However, because of the fractures network near wellbore, generated from artificial fracturing linking up natural fractures, traditional infinite and finite conductivity fracture models usually result in significantly deviation in field application. In this work, considering the random distribution of natural fractures, physical model of fractures network is proposed, and it shows a composite model feature in the large scale. Consequently, a nonhomogeneous composite mathematical model is established with threshold pressure gradient. To solve this model semi-analytically, we proposed a solution approach including Laplace transform and virtual argument Bessel function, and this method is verified by comparing with existing analytical solution. The matching data of typical type curves generated from semi-analytical solution indicates that the proposed physical and mathematical model can describe the type curves characteristic in typical tight oil reservoirs, which have up warping in late-term rather than parallel lines with slope 1/2 or 1/4. It means the composite model could be used into pressure interpretation of artificial fracturing wells in tight oil reservoir.

1. Introduction

After long history development in theoretical research and field applications both, well test has been one of the most conventional dynamic reservoir monitoring technologies. Well test technology in high permeability reservoirs, has a relatively mature mathematical models and interpretation methods. However, there are still some confusion questions about the technology applications in tight oil reservoirs, especially on proper physical and mathematical well test model, because most of the production wells are artificial fractured.
A theoretical treatment for the behavior of pattern-waterflooding injection wells is established [1], which considered two cases: oil and water have same properties and different properties. This method could be used whether the surface pressure persists or not in shut-in period. Buckley-Leverett displacement theory also is used to establish a numerical model for pressure falloff test [2]. The effects of saturation gradient on pressure falloff data are examined with different factors, like relative permeability curves, injection rate, injection volume and distances to the external boundary. Then, the multi-phase well test theories is improved leading to non-linear convolution as well [3], which is solved by proposed rapid calculation method, based on non-linear estimation, to rated arbitrary flow. Considering threshold pressure gradient in tight oil reservoirs, a mathematical model is derived for single-layer heterogeneous model which ignore skin factor and wellbore storage effect and has analytical solution in Laplace space [4]. Meanwhile, considering characteristic of deformable of low permeability media, there also are differential equations for unsteady flow [5] and composite model [6].

The well test results in Changqing oilfield and Yanchang oilfield, which are two typical tight oil reservoirs in northwest China, show that bottom hole pressure (BHP) changes in shut-in period as little as 0.5~1 MPa. Furthermore, most of the type curves do not have linear or bilinear flow characteristics in log-log diagnosis plot. The traditional infinite and finite conductive fracture model is improper to describe flow in tight oil reservoirs [7-8].

In this paper, the characteristic of fractures network in large scale is analyzed, and the corresponding composite heterogeneous mathematical model is established as well with threshold pressure gradient. Based on Laplace transform and virtual argument Bessel function, the proposed nonhomogeneous model is solved semi-analytically. Lastly, field pressure data in typical tight oil reservoirs is interpreted by the proposed model.

2. Physical and mathematical model

2.1. Physical model
As we all know, there is usually heterogeneity resulted from natural fractures and ultra-low permeability in tight oil reservoirs. Generally, the wells need to be fractured before production. Since the random intersect of fractures nearby well holes in artificial fracturing process [9-12], a fractures network is formed around the hole by linking up natural fractures, as Figure 1 shows. Therefore, because of the distribution of fractures, the permeability near well bore is higher and decrease with distance from the well. It means there is a higher permeability zone, i.e. inner zone, and a lower permeability zone, i.e. outer zone. It can conclude that fractures network generates a composite model in large scale, as shown in Figure 2.

![Figure 1. Fractures network distribution.](image1)

![Figure 2. Fractures network physical model.](image2)

2.2. Mathematical model
In accordance with the well pressure buildup test in tight oil reservoirs, the following conditions are assumed: (1)slight compressible single-phase liquid has a plane radial flow in the formation; (2) the formation fluid flow is isothermal; (3) the well radius is \( r_w \); (4) the wellbore storage and skin effect are
considered; (5) the initial pressure \( p_i \) is homogeneously distributed; (6) the effects of gravity and capillary are ignored; (7) fluid flows as low velocity non-linear Darcy flow considering threshold pressure gradient \( \lambda_b \); (8) the production rate is \( q \); (9) the formation rock is slight compressible.

Based on the above assumptions, the nonhomogeneous mathematical model of the composite model is established (Eq. 1).

\[
\begin{align*}
\frac{\partial^2 p_1}{\partial r^2} + \frac{1}{r} \frac{\partial p_1}{\partial r} + \frac{1}{r^2} \frac{\partial p_1}{\partial \lambda_b} = \frac{\phi \mu C_1}{k_1} \frac{\partial p_t}{\partial t} \\
\frac{\partial^2 p_2}{\partial r^2} + \frac{1}{r} \frac{\partial p_2}{\partial r} + \frac{1}{r^2} \frac{\partial p_2}{\partial \lambda_b} = \frac{\phi \mu C_2}{k_2} \frac{\partial p_t}{\partial t} \\
p_1(r,t) \bigg|_{r=0, r=R} = p_i \\
p_2(r,t) \bigg|_{r=0, r=R} = p_t \\
p_r = p_i(r_w, t) - r_S \left( \frac{\partial p_t}{\partial r} + \lambda_b \right) \\
q_B = 2 \pi r_h \frac{k_2}{\mu_2} \left( \frac{\partial p_t}{\partial r} + \lambda_b \right) - C \frac{dp_{w}}{dt} \\
\lim_{r \to r_w} p_2(r,t) = p_t \\
2 \pi r_h \frac{k_2}{\mu_2} \left( \frac{\partial p_t}{\partial r} + \lambda_b \right) \bigg|_{r=R} = 2 \pi r_h \frac{k_2}{\mu_2} \left( \frac{\partial p_t}{\partial r} + \lambda_b \right) \bigg|_{r=R} \\
\end{align*}
\]

Where \( p_1, p_2 \) are formation pressure in inner and outer zone respectively, atm; \( r \), distance, cm; \( R \), inner zone radius, cm; \( \lambda_{b1}, \lambda_{b2} \) are threshold pressure gradient in inner and outer zone respectively, atm/cm; \( \phi \) is formation porosity; \( \mu_1, \mu_2 \) are viscosity in inner and outer zone respectively, mPa.s; \( C_1, C_2 \) are comprehensive compressibility coefficient, atm\(^{-1}\); \( k_1, k_2 \) are formation permeability in inner and outer zone respectively, D; \( t \), time, s; \( p_i \), initial formation pressure, atm; \( p_{w} \), well-bottom hole pressure, atm; \( S \), skin factor; \( q \), production or injection surface rate, cm\(^3\)/s; \( B \), volume factor; \( h \), formation thickness, cm; \( C \), wellbore storage coefficient, cm\(^3\)/atm.

3. Semi-analytical solution approach

3.1. Mathematical solution

By introducing dimensionless and Laplace transform equations (Eq. 2-3), the dimensionless mathematical model is obtained (Eq. 4).

\[
\begin{align*}
p_{D} = \frac{2 \pi k_h h (p_{i} - p_{f})}{qB \mu_i} \\
p_{D} = \frac{2 \pi k_h h (p_{i} - p_{f})}{qB \mu_i} \\
t_D = \frac{k_t}{\phi \mu C_i r_{w}^2} \\
r_D = \frac{r}{r_{w}} \\
C_D = \frac{C}{2 \pi k_h C_i r_{w}^2} \\
\lambda_{D1} = \frac{2 \pi k_h h r_{w} \lambda_{b1}}{qB \mu_i} \\
\lambda_{D2} = \frac{2 \pi k_h h r_{w} \lambda_{b2}}{qB \mu_2} \\
\overline{p_D} = \int_{0}^{\infty} p_D e^{-u/C_D} d (t_D / C_D)
\end{align*}
\]

Where \( u \) is complex variable in Laplace transform; subscript \( D \) means dimensionless.
\[
\begin{align*}
\frac{\partial^2 p_{1D}}{\partial r^2} + \frac{1}{r} \frac{\partial p_{1D}}{\partial r} + \frac{\lambda_{1D}}{r} = & \frac{u}{C_p} p_{10} \\
\frac{\partial^2 p_{2D}}{\partial r^2} + \frac{1}{r} \frac{\partial p_{2D}}{\partial r} + \frac{\lambda_{2D}}{r} = & \frac{u k_2 \mu_2 C_2}{C_p \mu_1 C_1} p_{20}
\end{align*}
\]

\[\bar{p}_{1D}(r_0) \bigg|_{r_0^N} = 0\]

Since the existence of the threshold pressure gradient, the mathematical model includes a non-homogeneous governing equation. In order to get semi-analytical solution, it is deduced of the general solution with virtual argument Bessel function based on Green function method as follows:

\[y = A I_0(x) + B K_0(x) + \int_0^\infty G(x, \xi) f(\xi) d\xi\]

Where \(y\) is the general solution; \(I_0(x), K_0(x)\) are the first and second kind modified Bessel function respectively.

Then, the analytical BHP in Laplace space can be obtained (Eq. 6) and the corresponding coefficients are shown in appendix A. The semi-analytical solution in Laplace space is then inverted numerically into real space with the Stehfest inversion algorithm [13].

\[\bar{p}_{1D} = A I_0(\beta_i) + B K_0(\beta_i) + \frac{M}{\beta_i} I_0(\beta_i) \int_0^\infty K_0(\xi) d\xi\]

3.2. Solution verification

In order to verify the analytical solution approach, as well as the accuracy of numerical inversion, the numerical inversion results are compared with analytical solutions (Eq. 7), when ignoring threshold pressure gradient, well storage and skin effect. The used reservoir parameters values are shown in Table 1.

\[p(r, t) = p_i - \frac{q \mu}{4 \pi \kappa h} \left[ -Ei\left( -\frac{r^2}{4 \eta t} \right) \right] \]

| Table 1. Reservoir Parameters in solution verification. |
|-------------------------------------------------------|
| Initial Formation Pressure, MPa | 15 | Steady Production, m³/d | 10 |
| Reservoir Permeability, mD | 10 | Wellbore Radius, m | 0.1 |
| Reservoir Porosity, f | 0.2 | Total Compressibility, 1/MPa | 0.002 |
| Effective Thickness, m | 10 | Starting Pressure Gradient, MPa/m | 0 |
| Fluid Viscosity, mPa.s | 2 | | |
Figure 3. Comparison of BHP between analytical and numerical inversion solution.

Figure 4. Comparison between analytical and numerical inversion solution at r=10m.

(AS-analytical solution; NI-numerical inversion)

Figure 3 and 4 show that there are exactly matching between analytical and numerical inversion solution, indicating that the semi-analytical solution approach is correct and numerical inversion calculation is accuracy enough.

4. Pilot field application

Generally, it is thought that artificial fracturing results in major fractures and linear or bilinear flow in tight oil reservoirs, which the curves in log-log coordinates are parallel and have a slope of $\frac{1}{2}$ or $\frac{1}{4}$. And the corresponding model is the infinite and finite conductivity fracture model.

However, most of the field type curves in typical tight oil reservoirs, like Changqing and Yanchang oilfield, show characteristics of composite model rather than linear or bilinear flow. The field type curves of well W1 are up warping and getting closer gradually, indicating the decrease of permeability along with distance to well, as shown in Figure 5. The parameters interpreted by proposed model shows that permeability in inner and outer zones are 39.6 and 0.22 mD respectively. As physical model in Figure 2 illustrates, the initial permeability of this tight oil reservoir is 0.22 mD, where fractures network generated from artificial fracturing improves formation permeability almost 200 times.

Figure 5. Type curves of well W1 and pressure interpretation results by proposed model.
Furthermore, trying to evaluate the infinite and finite conductivity fracture model, 1282 field type curves are matched by composite and fracture model respectively. Table 2 lists the matching results, which are divided into 3 classes:

Can’t be matched: type curves cannot be matched in all flow stages;
Matched nearly: trends and later-term curves are matched basically;
Matched Completely: type curves are matched in all flow stages.

Apparently, composite model has the best matching results to type curves in tight oil reservoir. Almost 58% of wells can be matched completely, whereas the ratio of matched completely by infinite and finite conductivity fracture models is less than 25%.

| Well Name       | Composite Infinite Conductivity model | Finite Conductivity Fracture Model |
|-----------------|---------------------------------------|------------------------------------|
| Matched completely | 748                                   | 321                                |
| Matched nearly   | 427                                   | 534                                |
| Can’t be matched | 107                                   | 748                                |

5. Conclusions

(1) Artificial fracturing may induce fractures network rather than major fractures, since natural fractures in tight oil reservoirs could be linked up in artificial fracturing process.

(2) As fractures network could improve formation permeability of wellbore zone, it usually illustrates composite characteristics in large scale.

(3) The proposed mathematical model of fractures network is non-homogeneous as considering threshold pressure gradient and its general solution can be obtained semi-analytically by virtual argument Bessel function based on Green function method.

(4) The proposed mathematical model is more applicable to well test in tight oil reservoirs than traditional infinite and finite conductivity fracture models.

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References

[1] Hazebrook P, Rainbow H, Matthews C S 1958 Pressure fall-off in water injection wells 32nd Annual Fall Meeting of Society of Petroleum Engineering in Dallas Tex 10 6-9

[2] Sosa A, Raghavan R, Limon T J 1981 Effect of relative permeability and mobility ratio on pressure falloff behavior Journal of Petroleum Technology 33(06) 1125-1135

[3] Ramakrishnan T S, Kuchuk F J 1994 Testing injection wells with rate and pressure data SPE Formation Evaluation 9(03) 228-236

[4] Feng W, Ge J 1985 Non-stationary and non-linear low speed filtrate problem in single medium and dual medium Petroleum Exploration and Development 12(1) 56-62

[5] Song F, Liu C 2000 Analysis of pressure and production in deformable porous media Petroleum Exploration and Development 27(1) 57-58

[6] Yang L, Lin H 2006 Stress-sensitive hypotonic composite reservoir well testing model West-China Exploration Engineering 18(2) 73-74

[7] Liu X, 2017 Shale-gas well test analysis and evaluation after hydraulic fracturing by stimulated reservoir volume (SRV) Natural Gas Industry B 3 577-584

[8] Sun H, Chawathe A, Zhou D, Maclvor K, Hoteit H 2015 Integrated Haynesville Production Analysis Unconventional Resources Technology Conference (URTEC)
Appendix A- Coefficients in Solution

\[ A_i = \frac{\beta_iK_i(\beta_iR)}{E} \left[ uK_i(\beta_i) + (uS + 1)\beta_iK_i(\beta_i) \right] \]

\[ B_i = \frac{D_i}{\left[ uK_i(\beta_i) + (uS + 1)\beta_iK_i(\beta_i) \right]} \frac{\beta_iK_i(\beta_iR)}{E} \left[ uI_i(\beta_i) - (uS + 1)\beta_iI_i(\beta_i) \right] \]

\[ M_1 = \frac{\lambda_{1d}}{u}, \quad \beta_1 = \sqrt{\frac{u}{C_{lo}}} \]

\[ M_2 = \frac{\lambda_{2d}}{u}, \quad \beta_2 = \frac{uK_2\mu_{k_2}C_{oi}}{\sqrt{\mu_{k_2}\mu_{k_2}C_{oi}}} \]

\[ D_1 = -\frac{M_1}{\beta_i} I_o(\xi)d\xi - \frac{\beta_i}{\beta_i} I_o(\beta_iR) \int_{\beta_i}^{\infty} K_o(\xi)d\xi \]

\[ + \frac{\mu_{k_2}M_1}{\beta_2} K_2(\beta_2R) \int_{\beta_2}^{\infty} I_o(\xi)d\xi + \frac{\mu_{k_2}M_2}{\beta_2} I_o(\beta_2R) \int_{\beta_2}^{\infty} K_2(\xi)d\xi \]

\[ D_2 = M_1K_1(\beta_1R) \int_{\beta_1}^{\infty} I_o(\xi)d\xi - \frac{\lambda_{1d}}{u} \]

\[ - M_1K_2(\beta_2R) \int_{\beta_2}^{\infty} I_o(\xi)d\xi + M_2I_o(\beta_2R) \int_{\beta_2}^{\infty} K_2(\xi)d\xi \]

\[ D_3 = \frac{1+\lambda_{3d} + uS_{3d}}{u} \left( I_o(\beta_i) \int_{\beta_i}^{\infty} K_o(\xi)d\xi + (uS + 1)M_1I_o(\beta_i) \right) \]

\[ E = \frac{\lambda_{1d}}{\beta_i} I_o(\beta_i) \int_{\beta_i}^{\infty} K_o(\xi)d\xi + (uS + 1)\beta_iK_i(\beta_i) \]

\[ - \beta_iK_i(\beta_iR)I_o(\beta_iR) \left[ uK_i(\beta_i) + (uS + 1)\beta_iK_i(\beta_i) \right] \]

\[ + \frac{\mu_{k_2}K_2(\beta_2R)}{\beta_2} K_2(\beta_2R) \left[ uK_o(\beta_i) - (uS + 1)\beta_iI_o(\beta_i) \right] \]

\[ + \frac{\mu_{k_2}K_2(\beta_2R)}{\beta_2} K_2(\beta_2R) \left[ uI_o(\beta_i) - (uS + 1)\beta_iI_o(\beta_i) \right] \]

\[ + \beta_1K_1(\beta_1R) \]