The methodology of economic recovery of commercial companies in crisis conditions

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Abstract. Even though commercial companies play a really important role in the economic functioning and development, now, in crisis conditions, the future for some of them is unknown. As a result of the release of crisis caused by „Coronavirus severe acute respiratory syndrome 2, abbreviated SARS-CoV-2“, these companies are facing novel negative situations, not being able to manifest temporarily or normally the activity. From a systematic point of view, this crisis has the role of a really powerful disturbing factor. The creditors and investors are in a similar fragile situation. This paper identifies some scientific methods which can decrease the negative appeared problems, building models that take into account the limited resources that companies face and that they can lead to improving their economic situation. As an application, part of the essay presents the synthesis of the main assets and malfunctions of a company established in Romania. The elaboration of future strategies will be achieved through the systematic research of the admissible bases, aiming at maximizing the objective function. The type of equilibrium by reverse induction will be established, thus obtaining the set of the best cost variants. Linear programming will generate the optimal solution capable of exploiting the opportunities offered by the market with a minimal impact of disruptive factors.

1. Introduction
At the moment, organizations face intense national and international competition, with quick changes, caused by the emergence and evolution of the phenomena caused by COVID-19. Because of the pandemic, the legislative framework has frequently changed and the business environment has been in an extremely vulnerable situation. Social protection, the prohibition of some activities or their continuation under a modified form, the provision of some measures with impact on the business partners lead to the need for quick and impactful decisions, taken by the management of organizations. In all situations, in order to adapt to market conditions, the management decisions carry high risks for organizations. The paper aims to present some mathematical models related to technical, managerial and economic phenomena. These models can be useful tools that can be used permanently within organizations. Given the volatility that characterizes the economic situation due to COVID-19, both at local, European or global level, in the current context, these models become useful tools in finding new solutions.

2. Background to the study
As it is presented in literature, operational research gives scientific methods and methodologies that leads to optimal decisions in management or the best solution in other domains. It makes up a dynamic domain of scientific research, in this moment are many complex optimization procedures and ways to be selected from, all of them having its own attributes and practical aim. It is uncomplicated because
the unsuitable decision of optimization methods or starting values cannot lead to a global optimal solution [1]. Even if a challenger suggests similar approaches, he should not be capable of implementing them or should not get a considerable amount of cost cutting on development compared to the company which owns the same capabilities [2]. Operational research provides scientific methods and methodologies that lead to optimal management decisions or to the best solutions in other areas. It is a dynamic field of scientific research, both theoretical and applicative, which studies optimization problems resulting from the mathematical modelling of some phenomena and processes. Operational research involves the application of scientific methods, by a multidisciplinary team, the aim is to obtain results that best serve the interests of the organization as a whole [3].

The research uses operational research techniques and linear programming. Both belong to various disciplines, they make use of algorithms, computer techniques, mathematics, economics concepts, and obviously statistics [4, 5]. The most important problem is then reducing the lack of methodologies and case studies which makes use of operational research, linear programming, incorporating both artificial intelligence techniques, multicriteria decision theory [6], and uncertainty modeling [7 - 9].

Operational research is mainly the basic instrument for decision theory, prediction and improvement [10, 11]. Limiting time and costs will give the organization a competitive advantage over the competition. The company can establish an improved capacity in advance to the competitor replication, advantage in the competition can be sustained if it is known in do time [12, 13].

Essentially, an optimization problem is a problem of choice. It assumes as given a collection of entities generically called permissible solutions (or variants, scenarios). Solutions can be compared with each other and classified by means of an assessment criterion (performance). In this context, the question which arises is of finding the most appreciated solution, also called the optimal solution of the problem [3]. In the domain of business, operational research can bring an improvement in all sectors (resources, organization structure, demands), their analysis can lead to an important competition progress [14]. Currently, operational research training have started to incorporated computer strategies in automated learning by machine and using artificial intelligence, in this way can be improvingly applied in domains such as medicine [15] forecast and improvement [16]. Work by Daniel Kahneman and Amos Tversky presents a theory of how people make decisions in conditions of uncertainty called the theory of perspective [17].

Game theory is a method of examining economics that is based on the rational choice of variants when the results depend on the unknown strategies chosen by others. It is a mathematical discipline, which defines types of solutions built on methods and algorithms for solving. An example of game theory that analyzes the study of two-person play is provided by Antoine Cournot [18]. The mathematician Emile Borel suggested a formal game theory [19], he published several papers on game theory, the problem being approached in order to find out the opponent's movement in a game with imperfect information. He designed game theory for use in economic and military applications, and the ultimate goal was to determine if there was a "best" strategy for a particular game and to find that strategy. However, the development and popularization of game theory belongs to John Von Neumann, who published his first work on game theory in 1928 [20], later developing it with economist Oskar Morgenstern [21].

Over time it was acknowledged as a strong instrument in different domains (biology, logic, economics, psychology, political science). It focuses on their attractiveness and refinement, thus proposing a measure to improve the game, grounded on information about uncertainty in the results of the game [22]. Chess and Mah Jong variants are also the subject of the application of a logistic model [23]. Another model was built on analyzing the development of game strategies and was applied to time-limited games [24]. Merrill Flood conducted one of the best-known early experiments to investigate this question [25]. This paper introduced the convict’s questioning considered to be the most exciting issues in the theory of games. Ken Binmore studied the role of game theory in the social sciences, while another work in this field is developed by two Nobel Prize winners (2002) D. Kahneman and A. Tversky [26]. They presented a theory of how people take decisions in conditions of uncertainty called perspective theory [27].
Complexity theory deals with the study of things that can be calculated when the resources we have at our disposal are limited. The fields of both theory of complexity and of proper algorithms refer to issues from opposite directions. A correct algorithm should be applied to find a solution to a problem and is at the same time an evidence of the correctness of the problem. Instead the theory of complexity the aim is to show that important issues are not to be resolved with limited means. Algorithms theory and theory of complexity try to estimate this. Therefore, the two fields of computer science, namely the theory of complexity, the design of correct algorithms and their analysis pass the boundaries between the choice of algorithm correctness implemented with accurate demands [28]. The complexity theory has identified some of the most fundamental and profound problems related to computing, has developed a powerful methodology for solving them and is generally considered one of the most challenging mathematical frontiers [36], deals with the study of calculable phenomena when resources are limited. From a certain point of view, the complexity theory is the opposite of the theory of algorithms. If this provides a solution to the problem within the limits imposed by resources, the complexity theory identifies the moment when resources are insufficient to solve the problem. Complexity is defined as a fabric of inseparable heterogeneous constituents [37].

The solving of algorithm issue is a basic topic in IT and a main part in programming [29]. Solving the problem and producing correct and optimal results constitute the main aims in computer training. There are many difficulties in solving algorithmic problems, mainly in using abstract actions in addressing complex problems in giving options for correct outcome and for introducing it in programs [30 - 32]. As a result, obtaining good solutions is not common. Thus understanding the steps in solving a problem is necessary. If the probabilistic approach must first construct a statistic pattern and only after give its coordinates on figures a data string, in the domain of algorithms the modal is directly defined by these measurement. The data table can be organized as a set of ordinary and aleatory pieces [33]. The algorithmic method in search of the shortest program will generate the obtained sequence [34]. One such method is to describe the set of symbols "1" in a binary word in the programs prefix and to describe in a suffix a number of binary words with such a number of "1" [33, 35].

3. Research organization
The modelling of an economic process for the purpose of optimization involves two types of values: constant (they might change from one period to another, but over a period of time, they can be considered to be invariable) and variable (which will be defined in such a way as to satisfy a certain performance criterion, taking into account the carrying out, under limitative conditions, of the economic process). The correct identification of restrictions will have implications for the accuracy of the model and hence the value of the optimal solution. In developing the mathematical model for a practical optimization problem it is recommended to go through the following steps [3].

- Identifying variable values, they will become the decision variables of the model;
- Identification of limitative conditions that characterize the modelled process and their formalization in mathematical relationships-inequalities and / or equalities - called restrictions;
- Specifying the performance criterion and formalizing it in a function of the choice variants, called the function of the objective;
- Specifying the explicit conditions imposed on variable sizes, such as: to take only real non-negative values or only integer values, etc.

After finalizing the model, solving the optimization problem will lead to the determination of the values of the decision variables that will satisfy the restrictions and conditions imposed on the variables and will give the objective function the maximum or minimum value. The permissible solutions will be identified with those sets of numerical values granted to decision variables that satisfy the explicit restrictions and conditions, and the optimal solution will be that permissible solution that gives the objective function the highest or lowest value, as the case may be [3].

Here is an example of the management of company that assembles closing systems for logistics center, warehouses, shops or production halls that have an intense flow and require an optimal level of
isolation. The management wants to maximize the profit of the company in relation to the interim management balances and to select the best option of costs in order to minimize them.

To maximize profit, the problem will be approached using operational research [38] and the solutions will be revealed by linear programming using Maple 2019 programs and C++ code from the CodeBlocks application. Then the game theory will be used to select the optimal consumption variant (figure 1).

![Diagram of resolution steps](image)

**Figure 1.** Scheme of resolution steps.

3.1. *Data provided by organization*

The management of the organization seeks to achieve a profit share of 20 percent of the added value, the study of the indicator in dynamics revealed the following structure:

\[ AV = 5 \times CM + 3 \times PY - C \]

where: 
- \( AV \) - added value
- \( CM \) - commercial margin
- \( PY \) - production of the financial year
- \( C \) - Third party consumption

The following variables will be used to perform the modelling: \( x_1 \) (commercial margin), \( x_2 \) (production of the financial year), \( x_3 \) (third party consumption) and \( f_P \) function \( (x_1, x_2, x_3) \), profit of the organization dependent on the commercial margin, production of the financial year and consumption. The maximum of the function is obtained in relation (1), the limitative conditions (2) being the following:

\[
\begin{align*}
\text{max } f_B &= \frac{22 \times (5 \times x_1 + 3 \times x_2 - 2 \times x_3)}{100} \\
5 \times x_1 + 3 \times x_2 &\geq 8295 \\
5 \times x_1 - 2 \times x_3 &\leq 2752 \\
3 \times x_2 - 2 \times x_3 &\leq -4986 \\
x_1 + x_2 + x_3 &= 10300 \\
x_1 &\geq 0, x_2 &\geq 0, x_3 &\geq 0
\end{align*}
\]
3.2. Finding the interval

By entering the data into the Maple2019 application the program generates the following array of solutions (3):

```maple
> with(SolveTools[Inequality]):
> SolveTools[Inequality]:-LinearMultivariateSystem
({5*x1+3*x2+8 295, 5*x1-2*x3 <= 2 752, 3*x2-2*x3 <= -4 986, x1+x2 + x3 =10 300, x1 >=0, x2 >=0, x3 >=0,},
[x1, x2, x3])
```

\[
\begin{align*}
\{x_1 &= 0, x_2 = \frac{15.614}{5}, x_3 = \frac{35.886}{5}, \\
&= 1659, x_2 = \frac{15.614}{5}, x_3 = \frac{35.886}{5}, \\
&= (1659, 3336), x_2 = 11676, x_3 = -1 376 + \frac{5x_1}{2}, \\
&= (0, 3336), x_2 = 0, 11676, x_3 = 10 300 - x_1 - x_2 \}
\end{align*}
\]

3.3. Identifying the solutions

In this way, the intervals that comply with all conditions were found. To find out the unknowns that lead to the maximum value, enter the data in a program such as C++ through the application CodeBlocks.

To start with, we will define 4 variables which will contain the maximum value of the maximized function (4) which is calculated by function B, which will be described below, and the 3 parameters (5) that led to this value.

```c
int ma = 0
int x1, x2, x3
```

The defined function called B calculates the value of the maximized function using the three given parameters which represent a set of values that is included in the array of solutions (6).

```c
int B (int x1, int x2, int x3)
{
    return (22*(5*x1+3*x2-2*x3)/100);
}
```

The returned value will be stocked in a new variable (7) and compared with variable (4) and, if the value is greater in variable (7), then the used parameters in function B (6) will be stocked in the 3 variables described before (5). To illustrate, here is an example:

```c
int p=B(0, 15614/5, 35886/5.0);
if(ma<p)
{ 
    ma=p;
    x1=0;
    x2=15614/5;
    x3=35886/5;
}
```

The same procedure will be applied for all intervals and unique solutions from the array of solutions which was given by Maple 2019. The maximum value of the function is 1 954 (m.u.), for \(x_1 = 2 752\) (m.u.), \(x_2 = 2 044\) (m.u.) and \(x_3 = 5 504\) (m.u.)
3.4. Balance type
This section aims to identify the Stackelberg balance type in order to minimize consumption. The company has three variants with the following cost matrix imposed by the demand curve. Game theory [39] consider a dyadic game of the three 3 variants, presented in the following matrix (8):

\[
\begin{bmatrix}
52.33 & 52.80 \\
52.21 & 52.15 \\
52.40 & 52.10 \\
52.56 & 52.44 \\
\end{bmatrix}
\begin{bmatrix}
52.33 & 52.10 \\
52.80 & 52.56 \\
51.72 & 51.72 \\
51.82 & 52.21 \\
\end{bmatrix}
\begin{bmatrix}
51.70 & 51.75 \\
51.76 & 51.66 \\
51.76 & 51.76 \\
51.66 & 51.82 \\
\end{bmatrix}
\] (8)

V1, choosing strategy 1, chooses the first matrix line. V2, knowing that V1 chose line 1 of the matrix, chooses either the first strategy, or the second strategy. We assume it will choose strategy 1. In the end, V3 chooses its strategy, that is, the first or the second column. Suppose it chooses strategy 1. As a result, the next evolution of the game is obtained in the following matrix (9):

\[
\begin{bmatrix}
52.33 & 52.80 \\
52.21 & 52.15 \\
52.33 & 52.80 \\
52.33 & 52.33 \\
\end{bmatrix}
\begin{bmatrix}
52.33 & 52.10 \\
52.80 & 52.56 \\
52.33 & 52.33 \\
52.33 & 52.33 \\
\end{bmatrix}
\begin{bmatrix}
51.70 & 51.75 \\
51.76 & 51.66 \\
51.70 & 51.70 \\
51.70 & 51.70 \\
\end{bmatrix}
\] (9)

It highlights how to find the Stackelberg balance by reverse induction: V3 chooses minimum in each line from the matrices in column 3. Inductively V2, knowing how V3 will choose its strategy, chooses from the highlighted items the lowest. V1 chooses, knowing the choices V2 and V3. In the Stackelberg game V1 chooses the strategy and communicates it to V2, this knowing the choice of V1, selects its strategy and communicates to V3 both the strategy chosen and the strategy chosen by V1. V3 chooses its strategy knowing the strategies chosen by V1 and V2. Finally, after all the strategies have been chosen, each variant obtains the value corresponding to its cost function. So, the game has a single uncertain Stackelberg balance (2; 1; 2) with costs (52,10; 51,76; 51,72). Profile (2; 1; 2) is not Nash balance.

3.5. Consumption minimization
The cost vector of size \( A = [52,33; 52,80; 52,21; 52,40; 52,10; 52,56; 52,44; 52,33; 52,10; 52,80; 52,56; 51,76; 51,76; 51,82; 52,21; 51,70; 51,75; 51,76; 51,66; 51,76; 51,76; 51,66; 51,82] \) with distinct positive integer elements and a total cost: \( TC \) will be considered. Costs can be selected n times, and the program will generate the solution closest to the established criteria.

C++ code from the CodeBlocks app:

```c++
#include <iostream>
#include <fstream>
using namespace std;

int main()
{
    double n;
    double a[50];
    fin>>n ;
    for(int i=0 ; i<24 ; i++)
        fin>>a[i];
    for(int i=0 ; i<23 ; i++)
        for(int j=i+1 ; j<24 ; j++)
```

```c++
    
```
```cpp
if(a[i]<a[j])
    swap(a[i],a[j]);
int i=0;
cout<<"For obtain total cost "<<n<<", we use:\n";
while(n>0 && i<24)
{
    int ap=0;
    while(n>a[i])
    {
        n-=a[i];
        ap++;
    }
    if(ap>0)
        cout<<"Cost "<<a[i]" de "<<ap<<" ori\n";
    i++;
}
if(n>0)
    cout<<"Cost "<<a[23]" de 1 time\n";
return 0;
}

The solution that ensures a selection of a minimum number of costs is given by choosing the cost 52,80 by 208 times and the cost 51,66 once.

4. Results interpretation
It is important, especially at a time when societies are in a precarious balance due to the phenomena caused by COVID-19, to understand the methods by which organizations can grow. There are many studies that identify this possibility, but how they can grow and what are the factors that influence their evolution is a very wide field. Determining the best infrastructure of the organization is a crucial task. To this end, different approaches are applied which may lead to the achievement of the proposed objective. The study revealed a maximum profit of 1 954 (m.u.), in the case of a commercial margin of 2 752 (m.u.). production of the financial year in the amount of 2 044 (m.u.) and 5 504 (m.u.) consumptions. An uncertain Stackelberg balance was found, minimizing consumption can be achieved by selecting a minimum number of costs.

5. Conclusion
Based on the income statement, a series of value indicators can be determined regarding the volume and profitability of the organizations' activity, this fact being reflected in the analysis of the intermediate management balances. The added value is the surplus achieved by the company over the material consumption within the current activity. The increase of this indicator can be achieved taking into account the balance between the commercial margin and the production of the year on the one hand and the consumptions from third parties on the other hand. Each indicator is reflected in the final result, their analysis together or separately can lead to a strategy suitable for each organization. In the paper, we can notice the manner of solving the problem of operational research in Maple2019 and CodeBlocks applications, C++ code, accessing several optimization programs not being necessary, thus increasing the efficiency of solving the case. In most cases the answer to a simple question leads to a complex system of solutions. Game theory can predict how variants cost could be selected helping to design the economic system, but certainly research can continue in order to fully understand the purpose, applicability and usefulness of these theories.
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