All-order Finiteness in $N = 1$ SYM Theories: Criteria and Applications

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Abstract

As a motivation, we first recall the possible connection of electric-magnetic duality to finiteness in $N = 1$ super-Yang-Mills theories (SYM). Then, we present the criterion for all-order finiteness (i.e., vanishing of the $\beta$-functions at all orders) in $N = 1$ SYM. Finally, we apply this finiteness criterion to an $SU(5)$ SGUT. The latter turns out to be all-order finite if one imposes additional symmetries.

1 Introduction

Our aim is to present a criterion for all-order finiteness in $N = 1$ SYM theories, to outline its derivation, and to exhibit an application yielding an all-order finite supersymmetric GUT. As a motivation, we would first like to attempt at situating the discussion of all-order finiteness within the larger and exciting context of (electric-magnetic) duality in supersymmetric gauge theories. Holomorphy and duality in minimal and extended supersymmetry [1] form indeed an extremely active domain of research, to which the present conference is devoted.

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We start, following [2, 3], by recalling basic ideas about duality and perturbative finiteness. The electric-magnetic duality of Maxwell’s equations in the vacuum is a symmetry under the exchange of the electric and magnetic fields. In the presence of sources, one is led to introduce magnetic monopoles with magnetic charges $q_m$, in addition to the electric charges $q_e$, which obey Dirac’s charge quantization condition $q_e q_m = 2\pi n$, $n \in \mathbb{N}$. For the elementary charges ($n = 1$), duality therefore exchanges $q_e$ with its inverse (up to the $2\pi$ factor) $2\pi/q_e = q_m$.

The electric-magnetic duality of Maxwell’s theory motivates one to search for an analogous symmetry in the realm of quantum field theory, e.g., QED. The latter, supporting no magnetic monopoles, has to be rejected, and one therefore turns to spontaneously broken Yang-Mills theories. In that framework, due to the running of the couplings, duality can only be established if it can be made to hold at any scale $\mu$, that is, the symmetry under the exchange of $\lambda(\mu)$ with the dual coupling $\sim 1/\lambda(\mu)$ has to be scale-independent. The latter can be achieved provided the couplings do not run, i.e., provided their $\beta$-functions vanish exactly. This is the case in $N = 4$ supersymmetric Yang-Mills theories, for which Montonen and Olive [4] have conjectured that electric-magnetic duality might be an exact symmetry. Similarly, $N = 2$ SYM theories can be made finite by choosing appropriate combinations of gauge group and matter fields which lead to vanishing one-loop $\beta$-functions. A non-renormalization theorem then guarantees that the $\beta$-functions vanish above one-loop [5]. The discussion of perturbative finiteness in $N = 2, 4$ SYM is hence seen to be closely related to that of electric-magnetic duality.

There is an obvious interest in discussing the case of $N = 1$ SYM theories as well. In contrast to $N = 4$ or $N = 2$, the $N = 1$ case is relevant to the supersymmetric GUT’s of low-energy phenomenology, as those yielding the minimal supersymmetric standard model (MSSM). Therefore, in the same spirit of relating perturbative finiteness to electric-magnetic duality, we focus in the present paper on the case of minimal, $N = 1$ supersymmetry. We present the criterion for all-order finiteness in $N = 1$ SYM of [6, 7], give an outline of the proof and produce an example of application.

The paper is structured as follows. In Section 2, after reviewing the criteria for one- and two-loop finiteness in $N = 1$ SYM theories [9], we show how lower-orders finiteness can be extended to all orders [4, 7], by imposing, as a consistency condition for higher orders, that the gauge and Yukawa couplings obey reduction in the sense of Oehme and Zimmermann [10]. All-order scale invariant theories hence possess a single independent coupling, which does not run. It should be noted, in the context of electric-magnetic duality, that the proof of this finiteness criterion for $N = 1$ SYM is a rigorous extension of a formal argument proposed for the finiteness of $N = 4$ [3].

In Section 3, we present the criterion for all-order vanishing $\beta$-functions in $N = 1$ SYM of [7], an exact result which is based on hypotheses operating exclusively at the one-loop level. We start by recalling the structure of the supercurrent multiplet anomaly [11, 12, 13], which yields an important relation [6] connecting the conformal anomalies, i.e., the $\beta$-functions and the anomalous dimensions, to the axial and $R$-axial anomalies. The latter axial and $R$-axial
anomalies being non-renormalized, they are given by their one-loop values. Vanishing of the
latters is among the hypotheses of the finiteness criterion. The consistency requirement of
couplings reduction translates into a further hypothesis on the unicity of the solution to the
conditions of vanishing one-loop Yukawa $\beta$-functions.

It turns out that fulfilling the criterion generally means imposing global, Lie-type or
discrete symmetries that restrict the superpotential. As an illustration we present, in Section 4,
among the possible all-order finite $N = 1$ SYM models, a realistic supersymmetric $SU(5)$ gauge
theory with discrete symmetries \cite{14}. The latter finite model can be tested phenomenologically,
based on its prediction for the top quark mass, which is in agreement with experimental data.

To conclude this Introduction, let us mention that there exists related approaches to all-
order finiteness in $N = 1$ SYM, as those in refs. \cite{15, 16, 17}. The work of Intrilligator, Leigh
and Strassler \cite{17} is of special relevance here due to its implications for duality symmetry.

2 Finite $N = 1$ Supersymmetric Gauge Theories

We consider a chiral, anomaly free, $N = 1$ globally supersymmetric Yang-Mills theory based
on a group $G$ with gauge coupling $g$, and with the superpotential

$$W(\phi^i) = \frac{1}{2} m_{ij} \phi^i \phi^j + \frac{1}{6} \lambda_{ijk} \phi^i \phi^j \phi^k,$$  \hspace{1cm} (2.1)\]

where $m_{ij}$ and $\lambda_{ijk}$ are gauge invariant tensors and the matter fields $\phi^i$ transform according
to the irreducible representation $R_i$ of $G$. The renormalization constants associated with the
superpotential (2.1), assuming that supersymmetry is preserved, are

$$\phi^{0i} = (Z_j^i)^{1/2} \phi^j, \quad m^{0ij} = Z_{ij}^{j'} m_{j'j'}, \quad \lambda^{0ijk} = Z_{ij}^{j'k'} \lambda_{j'k'}. \hspace{1cm} (2.2)$$

The non-renormalization theorem for $N = 1$ SYM \cite{18} ensures the absence of mass and cubic
infinities, therefore

$$Z_{ij}^{j'k'} (Z_{j'}^{j''})^{1/2} (Z_{k''}^{k''})^{1/2} = \delta_{ij}^{j''} \delta_{jk'}^{k''}, \quad Z_{ij}^{j'} (Z_{j'}^{j''})^{1/2} = \delta_{ij}^{j''}.$$  \hspace{1cm} (2.3)\]

As a result the only surviving possible infinities are those associated to the wave-function
renormalization constants $Z_i^j$, i.e., one infinity for each field. The one-loop gauge $\beta$-function is
given by \cite{3, 19}

$$\beta_g^{(1)} = \frac{dg}{d \ln \mu} = \frac{g^3}{16\pi^2} \left[ \sum_i T(R_i) - 3 C_2(G) \right], \hspace{1cm} (2.4)\]

where $\mu$ is the renormalization scale, $T(R_i)$ is the Dynkin index of $R_i$ and $C_2(G)$ is the quadratic
Casimir of $G$. The one-loop $\beta$-functions $\beta^{(1)}_{ijk}$ of the Yukawa couplings $\lambda_{ijk}$ are related to the
matrix $\gamma^{i (1)}$ of one-loop anomalous dimensions of the matter fields $\phi^i$ as

$$\beta_{ijk}^{(1)} = \frac{d \lambda_{ijk}}{d \ln \mu} = \lambda_{ij} \gamma_k^{(1)} + \lambda_{ik} \gamma_j^{(1)} + \lambda_{jk} \gamma_i^{(1)}, \hspace{1cm} (2.5)$$
with \[1, 9\]

\[
\gamma_i^{(1)} = \frac{1}{32\pi^2} \left[ \lambda^{ikl} \lambda_{jkl} - 2 g^2 C_2(R_i) \delta_i^j \right],
\]

where \(C_2(R_i)\) is the quadratic Casimir of the irrep. \(R_i\), and \(\lambda^{ijk} = \lambda^{*ijk}\).

Necessary and sufficient conditions for one-loop finiteness result from demanding that the one-loop gauge \(\beta\)-function \((2.4)\), respectively the one-loop matter fields anomalous dimensions \((2.6)\), vanish, i.e.,

\[
\sum_i T(R_i) = 3 C_2(G),
\]

\[
\lambda^{ikl} \lambda_{jkl} = 2 g^2 C_2(R_i) \delta_i^j.
\]

These one-loop finiteness conditions are known to be necessary and sufficient for finiteness at the two-loop level \([9]\). In case supersymmetry is broken by soft terms, one-loop finiteness of the soft sector imposes further constraints on it \([20]\). In addition, the same set of conditions that are sufficient for one-loop finiteness of the soft breaking terms renders the soft sector of the theory two-loop finite \([21]\).

The one- and two-loop finiteness conditions \((2.7)-(2.8)\) restrict considerably the possible choices of the irreps. \(R_i\) for a given group \(G\) as well as the Yukawa couplings in the superpotential \((2.1)\). Note in particular that the finiteness conditions cannot be applied to the supersymmetric standard model (SSM), since the presence of a \(U(1)\) gauge group is incompatible with the condition \((2.7)\), due to \(C_2[U(1)] = 0\). This naturally leads to the expectation that finiteness should be attained at the grand unified level only, the SSM being just the corresponding low-energy, effective theory.

Another important consequence of one- and two-loop finiteness is that supersymmetry (most probably) can only be broken by soft breaking terms. Indeed, due to the unacceptability of gauge singlets, F-type \([22]\) spontaneous supersymmetry breaking terms are incompatible with finiteness, as well as D-type \([23]\) spontaneous breaking which requires the existence of a \(U(1)\) gauge group.

A natural question to ask is what happens at higher loop orders. The answer is contained in a theorem \([7]\) which states the necessary and sufficient conditions to achieve finiteness at all orders. Before we formulate the theorem let us make some introductory remarks. The one and two-loop finiteness conditions \((2.7)-(2.8)\) restrict the possible choices of matter representations and impose relations between the gauge and Yukawa couplings [see \((2.8)\)]. To require such relations which render the couplings mutually dependent, at a given renormalization point, is trivial. What is not trivial is to guarantee that relations leading to a reduction of the couplings hold at any renormalization point\(^4\). The necessary, but also sufficient, condition for this to happen is to require that such relations are solutions to the reduction equations of Oehme and

\(^4\) A recent paper \([24]\) deals with the possibility of extending at all orders the fixed points of \(\lambda_{ijk}/g\) (a simple form of reduction of the couplings), by discussing lower-order results.
Zimmermann [10],

\[ \beta_g \frac{d \lambda_{ijk}}{dg} = \beta_{ijk}, \]  

(2.9)

and hold at all orders. It is remarkable that the existence of all-order solutions to (2.9) can be decided at the one-loop level [10]. We shall come back to this point in Section 3.

Attempts at reducing the couplings can be found in refs. [25, 26, 27, 28]. These models, in particular the ones in ref. [28], as well as the finite model that will be described in Section 4, predict among other things a window for the top quark mass [29, 28]. More accurate measurements of the latter will decide on their validity.

We now return to the all-order finiteness theorem of [7]. It is based on (a) the structure of the supercurrent in \( N = 1 \) SYM [11, 12, 13], on (b) the non-renormalization properties of \( N = 1 \) chiral anomalies [6, 7], and finally on (c) the requirement of reduction of the couplings, in the sense discussed above. The theorem states that for an \( N = 1 \) supersymmetric gauge theory based on a simple gauge group (with representations that are free of gauge anomalies), the necessary and sufficient conditions for \( \beta_g \) and \( \beta_{ijk} \) to vanish at all orders are the following: (i) the one-loop finiteness conditions (2.7)-(2.8) hold; (ii) the reduction equations (2.9) admit a formal power series solution; (iii) the latter, in its lowest order, is also a solution of the condition (2.8). Since, as mentioned above, the existence of all-order solutions to the reduction equations can be decided at the one-loop level, the theorem can be recast in the form of a criterion for all-order finiteness [7], which is based on exclusively one-loop hypotheses. We shall develop that formulation in the next section.

3 Criterion for All-order Vanishing \( \beta \)-Functions

We start by describing formally, within the “algebraic renormalization” approach [30], i.e., through renormalized Ward identities, the action for \( N = 1 \) SYM (with a simple gauge group \( G \)) and its symmetries. In order to avoid problems with regularization, the theory is assumed to be renormalized according to the superspace renormalization scheme of [13].

The real gauge and chiral matter superfields are resp. denoted by \( V \) and \( \phi^i \). The gauge-invariant superfield action reads (in a notation that differs from the one used in the original literature, references [4, 3]):

\[
S_{\text{invariant}} = -\frac{1}{128g^2} \text{Tr} \left( \int d^4x d^2 \theta W^\alpha W_\alpha + \text{h.c.} \right) + \frac{1}{16} \int d^4x d^4 \theta \phi_i^j (e^V)^i_j \phi^j + \frac{1}{6} \left( \int d^4x d^2 \theta \lambda_{ijk} \phi^i j \phi^j + \text{h.c.} \right),
\]

(3.1)

where we have omitted the supersymmetric mass terms of the superpotential (2.1). In other words, we consider all fields to be massive, but we treat the theory only asymptotically at large Euclidean momenta. We therefore avoid all infrared problems associated with the dimension zero of the vector superfield \( V \). [The construction of \( N = 1 \) SYM theories with supersymmetry
breaking masses, using the algebraic renormalization approach (in the Wess-Zumino gauge) developed in [31], has been addressed recently [32].

The detailed gauge-fixing of the action (3.1) [13] is beyond our purpose. Following the BRS quantization procedure, one usually constructs the generating functional of one-particle irreducible Green’s functions (the classical action) as

\[ S_{cl} = S_{\text{invariant}} + S_{\text{gauge fixing}} + S_{\text{Faddeev-Popov}} + S_{\text{external sources}}, \] (3.2)

and defines the quantum theory generating functional

\[ \Gamma = S_{cl} + \mathcal{O}(\hbar^n) \] (3.3)

to be the most general solution of a set of renormalized constraints given by the gauge condition, the equations of motion, the rigid and BRS symmetries, etc. A subset of these constraints is relevant to the present context:

1. **R-symmetry.** On a generic superfield \( \varphi = \phi^i, \phi_i^\dagger, V, \) ghosts, antighosts, etc., \( R \)-transformations act infinitesimally as

\[ \delta_R \varphi = i \left( n_\varphi + \theta^a \partial_{\theta^a} - \bar{\theta}^{\dot{a}} \partial_{\bar{\theta}^{\dot{a}}} \right) \varphi, \] (3.4)

with \( R \)-weights \( n_\varphi \) given by \( n_{\varphi^i} = -n_{\phi_i^\dagger} = -\frac{2}{3}, \) and all other \( n_\varphi = 0. \) The functional \( R \)-Ward identity reads

\[ \mathcal{W}_R \Gamma \equiv -i \sum_\varphi \int \delta_R \varphi \frac{\delta \Gamma}{\delta \varphi} \simeq 0, \] (3.5)

where the symbol \( \simeq \) denotes equality up to soft breakings (i.e., breaking terms that vanish in the deep Euclidean region) of \( R \)-symmetry induced by supersymmetric masses, and the convenient superspace integration measure is subsumed in \( f. \)

2. **Supersymmetry,** expressed through the usual (unbroken) Ward identities

\[ \mathcal{W}_a \Gamma = 0, \quad \bar{\mathcal{W}}_a \Gamma = 0. \] (3.6)

3. **BRS invariance,** acting as, e.g.,

\[ s e^V = e^V c_+ - \bar{c}_+ e^V, \quad s \phi^i = -c_+ a(T^a_R)^i_j \phi^j, \quad s c_+ = -\frac{1}{2} \{c_+; c_+\}, \] (3.7)

where \( c_+, \bar{c}_+ \) denote Faddeev-Popov ghosts and antighosts. BRS invariance is encoded in a (non-linear) Ward identity, the Slavnov identity

\[ S(\Gamma) \equiv -i \int \left[ \sum_\varphi |s\varphi| \text{non-linear} \frac{\delta \Gamma}{\delta Y_\varphi} \frac{\delta \Gamma}{\delta \varphi} + \sum_\varphi |s\varphi| \text{linear} s \varphi \frac{\delta \Gamma}{\delta \varphi} \right] \simeq 0, \] (3.8)

where the \( Y_\varphi \)’s are external sources coupled to the non-linear BRS variations \( s \varphi, \) e.g., those displayed in (3.7). The corresponding \( \int s \varphi Y_\varphi \) terms in \( S_{\text{external sources}} \) make it possible to define in the quantum theory the composite BRS variations \( s \varphi \) as functional derivatives of \( \Gamma \) w.r.t.
the sources $Y_\varphi$. The Slavnov identity (3.8) is satisfied provided there is no gauge anomaly [13, 33]. The symbol $\simeq$ means that (3.8), similarly to (3.3), holds up to soft breakings of BRS invariance induced by supersymmetric masses.

4. A possible set of rigid chiral symmetries acting solely on the matter superfields,

$$\delta_a \phi^i = i e_a^i \phi^i, \quad \delta_a \bar{\phi}^i = -i \bar{e}_a^i \bar{\phi}^i, \quad \delta_a V = \delta_a c_+ = \delta_a \bar{c}_+ = \ldots = 0,$$

(3.9)

generated by Hermitean charges $e_a = \bar{e}_a^i$. The “chiral” Ward identity

$$\mathcal{W}_a \Gamma \equiv -i \int \delta \phi \frac{\delta \Gamma}{\delta \phi} = \sum_i e_a^i \left[ \int d^2 \theta \frac{\delta}{\delta \bar{\phi}^i} \Gamma - \text{h.c.} \right] \simeq 0$$

(3.10)

is satisfied, up to soft breakings of chiral symmetry due to supersymmetric masses [see also (3.3) and (3.8)], provided $\lambda_{ijl} e_a^l k + \lambda_{jkl} e_a^l i + \lambda_{klj} e_a^l j = 0$.

The Ward operators for supersymmetry, translations and $R$-transformations close under supersymmetry. As a consequence, a superfield Ward operator $\hat{W}$ can be constructed out of the Ward operators for $R$-transformations $\mathcal{W}_R$, supersymmetry $\mathcal{W}_\alpha$, $\mathcal{W}_{\bar{\alpha}}$, and translations $\mathcal{W}_T^\mu$, as

$$\hat{W} = \mathcal{W}_R - i \theta^\alpha \mathcal{W}_\alpha + i \bar{\theta}^\dot{\alpha} \mathcal{W}_{\bar{\alpha}} - 2 \theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^\dot{\alpha} \mathcal{W}_T^\mu;$$

(3.11)

the Ward identities (3.5), (3.6) are thus comprised in

$$\hat{W} \Gamma \simeq 0.$$  

(3.12)

The component Noether currents associated to the symmetries in (3.11), i.e., the $R$-current $R_\mu$, the supersymmetry currents $Q_{\mu \alpha}$, $\bar{Q}_{\mu \dot{\alpha}}$, and the energy-momentum tensor $T_{\mu \nu}$, form a supermultiplet – the supercurrent [11, 12] $V_\mu$

$$V_\mu(x, \theta, \bar{\theta}) = R_\mu(x) - i \theta^\alpha Q_{\mu \alpha}(x) + i \bar{\theta}^\dot{\alpha} \bar{Q}_{\mu \dot{\alpha}}(x) - 2(\theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^\dot{\alpha}) T_{\mu \nu}(x) + \ldots,$$

(3.13)

with $V_{\alpha \dot{\alpha}} = \frac{1}{2} \sigma_{\alpha \dot{\alpha}} \mu \nu \gamma^\mu$ obeying the supertrace identity [12], which we write schematically as

$$\bar{D}^\dot{\alpha} V_{\alpha \dot{\alpha}} \sim 2 D_\alpha S + \text{c.t.}. $$

(3.14)

“c.t.” denotes the relevant contact terms. Applying $D^\alpha = \partial_{\alpha} - i \sigma^\alpha_{\mu \dot{\alpha}} \bar{\theta}^\dot{\alpha} \partial^\mu$ to (3.14) and $\bar{D}^\dot{\alpha} = -\partial_{\dot{\alpha}} + i \theta^\gamma \sigma^\gamma_{\alpha \dot{\alpha}} \partial^\mu$ to the conjugate equation, and combining, one gets the conservation condition for the supercurrent

$$\partial^\mu V_\mu \sim i (DD S - \bar{D} \bar{D} \bar{S}) + \text{c.t.}. $$

(3.15)

In (3.14) and (3.13), $S$ is a chiral superfield insertion of dimension 3 and $R$-weight $-2$ known as the supercurrent anomaly [12]. It has the structure of a supermultiplet containing the anomalies of the supercurrent components, i.e., the anomalous divergence of the $R$-current, the $(\gamma)$-traces of the supersymmetry currents, and the trace of the energy-momentum tensor. Indeed, in the lowest component, (3.15) yields an Abelian chiral anomaly breaking the $R$-current divergence

$$\partial_\mu R^\mu \sim i (DD S - \bar{D} \bar{D} \bar{S}) + \text{c.t.}. $$

(3.16)
It is remarkable that the supertrace identity (3.14) also yields dilation anomalies in the energy-momentum trace
\[ T_\mu \sim \omega_D \Gamma - \frac{3}{2} (D D S + \bar{D} \bar{D} \bar{S}), \tag{3.17} \]
where \( \omega_D \) is just, upon integration, the Ward operator of dilatations
\[ \int \omega_D \equiv W_D \equiv -i \sum \delta_D \varphi \frac{\delta}{\delta \varphi}, \tag{3.18} \]
with the infinitesimal transformation law
\[ \delta_D \varphi = (d_\varphi + x^\mu \partial_\mu + \frac{1}{2} \theta^a \partial_{\theta^a} + \frac{1}{2} \bar{\theta}^\dot{a} \partial_{\bar{\theta}^{\dot{a}}}) \varphi. \tag{3.19} \]

Our task is now to relate the Abelian chiral anomaly in the \( R \)-current divergence (3.16) and the dilation anomalies in the energy-momentum trace (3.17) to the Abelian anomalies associated to the (possible) chiral symmetries \( W_a \) (3.10). The natural setting for deriving such a relation is provided by the Callan-Symanzik equation. We shall arrive at its formulation by expanding the supercurrent anomaly \( S \) in a basis of dimension 3, BRS-invariant, chiral insertions \( \{ L_n \} \) as
\[ S = \beta_g L_g + \sum_{i,j,k} \beta_{ijk} L_{ijk} - \sum_{i,j} \gamma^i_j N^i_j + \ldots, \tag{3.20} \]
where the dots stand for insertions which are not essential in the present context. The \( L_n \)’s (the choice of which shall be justified by the Callan-Symanzik equation) are defined in terms of derivatives of the action functional w.r.t. the gauge and Yukawa couplings,
\[ \partial_g \Gamma \equiv \int d^4 x \, d^2 \theta \, L_g + \text{h.c.}, \quad \partial_{\lambda_{ijk}} \Gamma \equiv \int d^4 x \, d^2 \theta \, L_{ijk} + \text{h.c.}, \tag{3.21} \]
and in terms of the “counting operators” \( N^i_j \), defined as
\[ N^i_j \Gamma \equiv \left[ \int d^4 x \, d^2 \theta \, \phi^i \frac{\delta}{\delta \phi^j} + \text{h.c.} \right] \Gamma \equiv \int d^4 x \, d^2 \theta \, L^i_j + \text{h.c.}. \tag{3.22} \]
Let us replace into the energy-momentum trace (3.17) the expansion for \( S \) (3.20) and the definitions of the \( L_n \)’s (3.21), (3.22). Then, relating the (broken) Ward identity of dilatations to the scaling operator through the dimensional analysis identity \( W_D \Gamma = \mu \partial_\mu \), one arrives at the Callan-Symanzik equation
\[ \left[ \mu \partial_\mu + \beta_g \partial_g + \sum_{i,j,k} \beta_{ijk} \partial_{\lambda_{ijk}} - \sum_{i,j} \gamma^i_j N^i_j + \ldots \right] \Gamma = 0. \tag{3.23} \]
The latter describes how dilation invariance is broken by the \( \beta \)-functions \( \beta_g, \beta_{ijk} \) associated to the renormalization of the gauge, resp. Yukawa couplings, and by the anomalous dimensions \( \gamma^i_j \). Eq. (3.23) justifies \textit{a posteriori} the choices of the coefficients in (3.20), and of the definitions (3.21), (3.22).
We now perform a change of basis for the counting operators $N^i_j$ (3.22),

$$\{N^i_j\} \rightarrow \{N_{0a} \equiv e_{0a}^i, N^i_j\} \oplus \{N_{1k}\}, \tag{3.24}$$

where the $e_{0a}^i$ are charge matrices corresponding to the center of the algebra $\{W_a\}$ formed by the chiral symmetries (3.10), i.e.,

$$W_{0a} \Gamma \equiv \sum_{i,j} e_{0a}^i [\int d^4x d^2\theta \phi^i \delta \phi^j - \text{h.c.}] \Gamma \simeq 0 \tag{3.25}$$

with

$$[W_{0a}; W_b] = 0, \quad \forall b. \tag{3.26}$$

The new counting operators $N_{0a} \equiv e_{0a}^j N^i_j$ (3.24) annihilate the superpotential in the (asymptotic) action (3.1), in the sense that

$$N_{0a} \left[ \int d^4x d^2\theta \lambda_{ijk} \phi^i \phi^j \phi^k + \text{h.c.} \right] = 0. \tag{3.27}$$

One can show that the supercurrent anomaly $S$, as well as each of the insertions of its expansion in the new basis $\{N_{0a}, N_{1k}\}$ (3.24), omitting the unessential terms $N_{1k}$,

$$S = \beta_g L_g + \sum_{\lambda_{ijk}} \beta_{ijk} L_{ijk} - \sum_a \gamma_0a L_{0a} + \ldots, \tag{3.28}$$

can be expressed as the $K_3^0$-dependent terms,

$$S = \bar{D} D (r K_3^0 + \ldots), \tag{3.29}$$

and

$$L_g = \bar{D} D \left( \frac{1}{128 g^3} + r_g \right) K_3^0 + \ldots, \quad L_{ijk} = \bar{D} D r_{ijk} K_3^0 + \ldots, \quad L_{0a} = \bar{D} D r_{0a} K_3^0 + \ldots, \tag{3.30}$$

where $K_3^0$ is the ghost number zero, supersymmetric Chern-Simons term, and the dots stand for unessential terms. Replacing these expressions into (3.28), and identifying the coefficients of the $K_3^0$-dependent terms, yields an important relation among the coefficients of the anomaly expansion (3.28) and those of (3.29), (3.30):

$$r = \beta_g \left( \frac{1}{128 g^3} + r_g \right) + \sum_{\lambda_{ijk}} \beta_{ijk} r_{ijk} - \sum_a \gamma_0a r_{0a}, \tag{3.31}$$

where $r_g$, $r_{ijk}$ are of order $\hbar$ at least, whereas $r$ and $r_{0a}$ are strictly of order $\hbar$.

Indeed, specializing to the case under consideration, the non-renormalization theorem for chiral anomalies in $N = 1$ SYM (see [4, 7]) tells us that $r$ and $r_{0a}$ in (3.31) are non-renormalized, i.e., they are of order one in $\hbar$. $r$ is the coefficient of the Abelian anomaly of the $R$-axial current,
and the $r_{0a}$’s are the coefficients of the Abelian anomalies of the axial currents associated to the chiral $\mathcal{W}_{0a}$-symmetries (3.23). $r$ and $r_{0a}$ are proportional to their one-loop values \[ r \sim \beta_{g}^{(1)} \sim \sum T(R_i) - 3 C_2(G) \] \[ r_{0a} \sim \sum e_{a}^{j} T(R_i) \] (3.32)

Note that at the order one in $\bar{h}$, (3.31) reduces to $\beta_{g}^{(1)} = 128 g^3 r$, which just corresponds to the first “∼” in eqs. (3.32), up to the coefficient.

The proof of the non-renormalization theorem \[ \bar{3} \bar{4} \] uses the fact that the supersymmetric Chern-Simons three-form $K_0^3$ is related through the supersymmetric descent equations to the zero-form $K_0^3 = \frac{1}{3} \text{Tr} c_1^3$, the cubed ghost field insertion. The non-renormalization theorem for chiral vertices \[ \bar{3} \bar{4} \bar{4} \] guarantees the finiteness of the latter insertion. (Another derivation of the finiteness of $\text{Tr} c_1^3$, which is based on the supersymmetric antighost equation, has been given in $\bar{3} \bar{5}$).

We now state the criterion for all-order vanishing $\beta$-functions, in the form announced at the end of Section $\bar{2}$.

**Criterion for all-order vanishing $\beta$-functions:**

Consider an N=1 super-Yang-Mills theory with simple gauge group. If

(i) there is no gauge anomaly,

(ii) the gauge $\beta$-function vanishes at one loop [eq. (2.7)],

\[ \beta_{g}^{(1)} = 0 \] \[ (3.33) \]

(iii) there exist solutions of the form $\lambda_{ijk} = \rho_{ijk} g$, $\rho_{ijk} \in \mathfrak{C}$, to the conditions of vanishing one-loop matter fields anomalous dimensions [eqs. (2.8)]

\[ \gamma_{j}^{(1)} = 0 \] \[ (3.34) \]

and (iv) these solutions are isolated and non-degenerate when considered as solutions of the conditions of vanishing one-loop Yukawa $\beta$-functions [see eq. (2.5)]

\[ \beta_{ijk}^{(1)} = 0 \] \[ (3.35) \]

then each of the solutions $\lambda_{ijk} = \rho_{ijk} g$ can be uniquely extended to formal power series in $g$, and the associated SYM models depend on a single coupling constant (e.g., the gauge coupling $g$) with a $\beta$-function which vanishes at all orders.

Some comments are in order. By “isolated”, we mean that the solutions cannot be multiple zeroes, whereas by “non-degenerate” we forbid parametric families of solutions. Indeed, the solutions of $\gamma_{j}^{(1)} = 0$ are generally multiple zeroes or come in one-parameter families. To obtain a SYM model with one isolated and non-degenerate solution (i.e., a unique solution for that model), one generally needs to restrict the superpotential by imposing global, chiral or discrete, symmetries. One solution of $\gamma_{j}^{(1)} = 0$ which is unique when regarded as a solution of $\beta_{ijk}^{(1)} = 0$
therefore corresponds, if it exists, to a given global symmetry of the superpotential. This is the meaning of the finiteness criterion: there can be different, arbitrarily multiple or degenerate solutions to \( \gamma_j^{(1)} = 0 \). Each of them may yield a finite SYM model with global symmetries, assuming that such symmetries exist. If more than one finite model can be constructed for a given unconstrained \( N = 1 \) SYM theory, then each of these models corresponds to the original theory with an additional global symmetry specific to that model.

Note that, due to the fact that the couplings are complex, there generally are undetermined phases left in the solution of \( \beta_{ijk}^{(1)} = 0 \), hence the latter is a parametric family. These phases \( \varphi \) can be set to zero by hand provided the corresponding \( \beta \)-functions, schematically \( \beta_\varphi = \text{Im}(\beta_\lambda/\lambda) \), vanish. \( \beta_\varphi = 0 \) holds if one uses a renormalization scheme that preserves at all orders the one-loop relation \( (2.3) \) between the \( \beta \)- and \( \gamma \)-functions. For details, see [7].

The conditions \( \beta_g^{(1)} = \gamma_j^{(1)} = 0 \) have been known for some time to guarantee one- and two-loop vanishing of the \( \beta \)-functions \( 1 \) (see Section 2). Models which fulfill these conditions are tabulated in \( 36 \), for the most popular (simple) gauge groups. Conditions (iii) and (iv) represent therefore consistency requirements that are necessary in order to extend the vanishing of the \( \beta \)-functions at all orders.

Some models satisfying the all-order finiteness criterion are known. An all-order finite \( SU(6) \) SYM theory has been presented in \( 7 \). Other attempts at finding all-order finite models have resulted in constraining the initial theory by imposing discrete (orbifold-type) symmetries \( 14 \); an example is presented in Section 4 below.

Let us now sketch the proof of the finiteness criterion. With the expressions for \( r \) and \( r_{0a} \) \( (3.32) \), it follows from (ii) and (iii) that

\[
0 = \beta_g \left( \frac{1}{128 g^2} + r_g \right) + \sum_{\lambda_{ijk}} \beta_{ijk} r_{ijk} .
\]

That the Yukawa couplings \( \lambda_{ijk} \) are proportional to \( g \) in the one-loop approximation as a consequence of (iii) is clear from \( 2.9 \). At higher orders, \( \lambda_{ijk} = \lambda_{ijk}(g) \) are formal power series in \( g \), and one needs to impose for consistency that these functions satisfy the reduction equations \( 2.9 \).

A power series solution to the reduction equations exists at all orders if there is a lowest-order solution which is unique. We now look at this point in more details, following \( 7, 10 \). At one-loop, the reduction equations \( 2.9 \) reduce to

\[
\beta_{ijk}^{(1)}(\lambda_{ijk}, g) = 0 , \quad \forall \lambda_{ijk} ,
\]
which is hypothesis (iv). Separating the complex $\lambda_{ijk}$’s into their real and imaginary parts, we assume all Yukawa couplings to be real and denote them by $\lambda_i$. The reduction equations \(2.9\) now read
\[
\beta_g \frac{d\lambda_i}{dg} = \beta_i ,
\]
where the $\beta$-functions have the forms
\[
\beta_i = \sum_{n=1}^{\infty} \sum_{a=0}^{n} \sum_{k} C_i^{(n)} k_{2n+1} g^{2n-2a} \lambda_{k_{2n+1}} \lambda_{k_{2n+1}} = C_i^{(1)} k g^2 \lambda_k + C_i^{(1) klm} \lambda_k \lambda_l \lambda_m + \mathcal{O}(\hbar^2) \quad (3.40)
\]
with $n$ denoting the loop order, and
\[
\beta_g = g^3 \sum_{n=2}^{\infty} \sum_{a=0}^{n-1} \sum_{k} B(n) k_{2a} g^{2n-2-2a} \lambda_{k_{2a}} = \mathcal{O}(\hbar^2) . \quad (3.41)
\]
Having assumed hypothesis (ii), i.e., $\beta_g^{(1)} = 0$, we look for a solution of (3.39) of the form
\[
\lambda_i(g) = \sum_{n=0}^{\infty} \rho_i^{(n)} g^{2n+1} . \quad (3.42)
\]
Inserting (3.41), (3.41) and (3.42) into (3.39), one finds that $\rho_i^{(0)}$ must be a solution of the equations
\[
F_i[\rho^{(0)}] \equiv C_i^{(1)} k \rho_k^{(0)} + C_i^{(1) klm} \rho_k^{(0)} \rho_l^{(0)} \rho_m^{(0)} = 0 , \quad (3.43)
\]
which are just eqs. (3.38), i.e., hypothesis (iv). At higher orders, one gets the recurrence conditions
\[
M_i^k \rho_k^{(n)} = f_i , \quad n \geq 1 , \quad (3.44)
\]
of which the right side depends only on $\rho^{(p)}$, $p < n$. The matrix $M$ depends on $\rho^{(0)}$ exclusively,
\[
M_i^k = \frac{\partial F_i[\rho^{(0)}]}{\partial \rho_k^{(0)}} . \quad (3.45)
\]
If this matrix is non-singular, i.e., iff the solution $\rho^{(0)}$ of (3.43) is unique, then (3.44) determines the higher order coefficients of (3.42) in terms of $\rho^{(0)}$.

Having done this, we replace the reduction equations [in their form (2.9)] into (3.37) and get
\[
0 = \beta_g \left[ \frac{1}{128 g^3} + r_g + \sum_{\lambda_{ijk}} \frac{\partial \lambda_{ijk}}{\partial g} r_{ijk} \right] , \quad (3.46)
\]
which is of the form $0 = \beta_g [1/128 g^3 + \mathcal{O}(\hbar)]$, i.e., the bracket in (3.46) is perturbatively invertible. It follows that $\beta_g = 0$ at all orders, for the unique remaining (independent) coupling of the theory, e.g., the gauge coupling $g$. 

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4 A Realistic Finite SGUT Based on $SU(5)$

From the classification of theories with $\beta_g^{(1)} = 0$ [36], one can see that, using $SU(5)$ as gauge group, there exist only two candidate models which can accommodate three fermion generations. These models contain the chiral supermultiplets $5, \bar{5}, 10, \bar{10}, 24$ with the multiplicities $(6, 9, 4, 1, 0)$ and $(4, 7, 3, 0, 1)$, respectively.

Only the second model contains three families and can describe in a self-consistent way (i.e., without reference to a larger model) the spontaneous symmetry breaking (SSB) of $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$. We therefore concentrate here on that model, for which the most general $SU(5)$ invariant, $N = 1$ supersymmetric, cubic superpotential is

$$W = \frac{1}{2} g_{i ja} \bar{10}_i \bar{10}_j H_a + \bar{g}_{i ja} 10_i \bar{5}_j H_a + \frac{1}{2} g'_{ijk} 10_i \bar{5}_j \bar{5}_k + \frac{1}{2} g_{iab} 10_i \bar{H}_a \bar{H}_b$$

$$+ f_{ab} \bar{H}_a 24 H_b + h_{iab} \bar{5}_i 24 H_a + p 24^5,$$

where $i, j, k = 1, 2, 3$ and $a, b = 1, 2, 3, 4$; we have suppressed the $SU(5)$ indices. The $10_i$'s and $\bar{5}_i$'s are the usual three generations, and the $24$ contains the scalar superfield. The four ($5 + \bar{5}$) Higgses are denoted by $H_a, \bar{H}_a$.

Given the superpotential, the $\gamma^{(1)}$'s can be easily computed [$\beta_g^{(1)}$ vanishes of course]. Eq. (2.8) imposes the following relations among the Yukawa and gauge couplings

$$\begin{align*}
H &: \quad 4 \bar{g}_{i ja} \bar{g}^{i jb} + \frac{24}{5} f_{ac} f^{bc} + 4 q_{iac} q^{ibc} = \frac{24}{5} g^2 \delta^b_a, \\
\bar{H} &: \quad 3 g_{i ja} g^{i jb} + \frac{24}{5} f_{ca} f^{cb} + \frac{24}{5} h_{iab} h^{ib} = \frac{24}{5} g^2 \delta^b_a, \\
\bar{5} &: \quad 4 \bar{g}_{kia} \bar{g}^{k ja} + \frac{24}{5} h_{iab} h^{ja} + 4 g'_{ikl} g^{ijkl} = \frac{24}{5} g^2 \delta^i_j, \\
10 &: \quad 2 \bar{g}_{ika} \bar{g}^{ika} + 3 q_{ika} q^{ika} + q_{iab} q^{jka} + g'_{kli} g^{jkl} = \frac{36}{5} g^2 \delta^i_j, \\
24 &: \quad f_{ab} f^{ab} + \frac{21}{5} p p^* + h_{iab} h^{ia} = 10 g^2.
\end{align*}$$

To realize finiteness at all orders, we must find a unique solution of $\beta_{ijk} = 0$ [eq. (3.33)], that is consistent with the vanishing of the $\gamma^{(1)}$'s [eqs. (4.2)]. Such a search contrasts with most of the previous studies of the present model [37, 38], where no attempt has been made to find isolated and non-degenerate solutions. These studies have rather pursued an opposite goal. They have used the freedom offered by the fact that the solutions are not isolated in order to make specific Ansätze that could lead to phenomenologically acceptable predictions.

Following [14], we concentrate on finding a model (1) that is phenomenologically interesting, e.g., the $SU(5)$ SGUT based on the (unconstrained) superpotential (4.1), and (2) which yields a solution to (4.2) that is unique as a solution of (3.33) in order to realize finiteness. As a first approximation to the Yukawa matrices, a diagonal solution (that is, one without intergenerational mixing) may be considered. It turns out that this can be achieved by imposing the $Z_7 \times Z_3$ discrete symmetry presented in Table 1, plus a multiplicative $Q$-parity on the superpotential $W$. Under the latter $Q$-parity, the $10_i$'s and $\bar{5}_i$'s describing the fermion multiplets are odd, while all other superfields are even. These symmetries allow only $g_{iia}, g_{iiia}, f_{aa}$ and $p$ to be non-vanishing. Furthermore, looking at this problem from the point of view of
Table 1: The charges of the $Z_7 \times Z_3$ symmetry

|     | $10_1$ | $10_2$ | $10_3$ | $5_1$ | $5_2$ | $5_3$ | $H_1$ | $H_2$ | $H_3$ | $H_4$ |
|-----|--------|--------|--------|-------|-------|-------|-------|-------|-------|-------|
| $Z_7$ | 1      | 2      | 4      | 4     | 1     | 2     | 5     | 3     | 6     | 0     |
| $Z_3$ | 1      | 2      | 0      | 0     | 0     | 1     | 2     | 0     | 0     |       |

the first formulation of the finiteness criterion (see Section 2), we have found that there indeed exists a unique power series solution of the reduction equations (2.9) that satisfies the finiteness conditions (3.34), (3.35). Defining

\[ \alpha_{ija} = \frac{|g_{ija}|^2}{4\pi}, \quad \bar{\alpha}_{ija} = \frac{|\bar{g}_{ija}|^2}{4\pi}, \quad \alpha_{fab} = \frac{|f_{fab}|^2}{4\pi}, \quad \alpha_p = \frac{|p|^2}{4\pi}, \quad \text{and} \quad \alpha_{\text{GUT}} = \frac{|g|^2}{4\pi}, \quad (4.3) \]

this unique solution is given by [14]

\[ \alpha_{iii} = \frac{8}{5}\alpha_{\text{GUT}} + \mathcal{O}(\alpha_{\text{GUT}}^2), \quad \bar{\alpha}_{iii} = \frac{6}{5}\alpha_{\text{GUT}} + \mathcal{O}(\alpha_{\text{GUT}}^2), \quad \alpha_{f44} = \alpha_{\text{GUT}} + \mathcal{O}(\alpha_{\text{GUT}}^2), \quad \alpha_p = \frac{15}{7}\alpha_{\text{GUT}} + \mathcal{O}(\alpha_{\text{GUT}}^2). \quad (4.4) \]

The $\mathcal{O}(\alpha_{\text{GUT}}^2)$-terms are power series in $\alpha_{\text{GUT}}$ that can be uniquely computed to any finite order if the $\beta$-functions of the unreduced model are known to the corresponding order. The reduced model in which gauge and Yukawa couplings are unified has $\beta$-functions that identically vanish to that order.

In the above model, we have found a diagonal solution for the Yukawa couplings, with each family coupled to a different Higgs. However, we may use the fact that mass terms do not influence the $\beta$-functions in a certain class of renormalization schemes, and introduce appropriate mass terms that allow to rotate in the Higgs sector so that only one pair of Higgs doublets, coupled to the third family, remains light and acquires a non-vanishing VEV (this is similar to [38]). Note that the effective coupling of the Higgs doublet to the first family is very small, hence avoiding a potential problem with proton lifetime [39]. Thus, effectively, we have at low energies the minimal supersymmetric standard model (MSSM) with only one pair of Higgs doublets.

Supersymmetry breaking can be achieved through soft breaking terms, which do not influence the $\beta$-functions beyond $M_{\text{GUT}}$. It is worth mentioning that renormalization group invariant relations in the soft supersymmetry breaking sector (which hold up to two loops) have been revived [40]. When these conditions are applied to (a generalization of) the present $SU(5)$ model, the finite case emerges as the only possibility.

Since the $SU(5)$ symmetry is spontaneously broken below $M_{\text{GUT}}$, the finiteness conditions obviously do not restrict the renormalization property at low energies. All one gets at such regimes is a boundary condition on the gauge and Yukawa couplings, which have to be so chosen that they satisfy (1.4) at $M_{\text{GUT}}$. So we examine the evolution of the gauge and Yukawa couplings according to their renormalization group equations at two-loops, taking into account all the
boundary conditions at $M_{\text{GUT}}$ \cite{14}. A recent analysis \cite{29} based upon updated experimental data on Standard Model parameters yields as a prediction for the top quark mass in the present model

$$m_t = (185 \pm 5) \text{ GeV}.$$  

(4.5)

5 Conclusions

Electric-magnetic duality can be implemented in supersymmetric Yang-Mills theories if the relations exchanging the weak and strong coupling regimes can be made to hold at any renormalization scale, \textit{i.e.}, if the couplings do not run. This illustrates the connection between electric-magnetic duality and exact vanishing of the $\beta$-functions, \textit{i.e.}, all-order perturbative finiteness. We did not attempt here at giving a formal description of that connection, which we have used mainly as a motivation for discussing all-order finiteness. Hoping to contribute to the exciting discussion on duality symmetries, we have concentrated on the case of all-order finite $N = 1$ supersymmetric gauge theories.

The criterion we have presented at the end of Section 3 for all-order vanishing $\beta$-functions is attractive due to the fact that it does involve only one-loop hypotheses. The condition that the solutions be isolated and non-degenerate [hypothesis (iv)] is generally not met by an unconstrained model. This does however not mean that SYM theories cannot be made finite; finiteness can be achieved by enforcing the unicity of such solutions through additional symmetry requirements on the superpotential.

In general, the procedure of constructing an all-order finite SYM theory involves two or more steps. One first reduces the number of independent Yukawa couplings by means of global symmetries. Then one checks if the solution of $\gamma_j^{(1)} = 0$ considered as a solution of $\beta_{ijk}^{(1)} = 0$ is isolated and non-degenerate. If not, the process has to be restarted, imposing an enlarged global symmetry to the superpotential. The process stops successfully if unicity of the solution of $\beta_{ijk}^{(1)} = 0$ is attained.

One course, a more practical point of view may be adopted. Starting from an unconstrained SYM model, one imposes the global symmetries that are motivated by phenomenology (as, \textit{e.g.}, family symmetry), and then checks if finiteness is realized. One may hope that the global symmetries which are necessary for finiteness turn out to be physically relevant and to carry predictive power.

We have presented an application of the finiteness criterion to a $SU(5)$ SGUT. The latter is shown to be all-order finite provided one imposes discrete symmetries of the type $Z_7 \times Z_3$, plus a multiplicative $Q$-parity. The same model yields a prediction for the top quark mass, $m_t = (185 \pm 5) \text{ GeV}$, which is in agreement with the present experimental data.

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