Some remarks about pseudo gap behavior of nearly antiferromagnetic metals

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In the antiferromagnetically ordered phase of a metal, gaps open on parts of the Fermi surface if the Fermi volume is sufficiently large. We discuss simple qualitative and heuristic arguments under what conditions precursor effects, i.e. pseudo gaps, are expected in the paramagnetic phase of a metal close to an antiferromagnetic quantum phase transition. At least for weak interactions, we do not expect the formation of pseudo gaps in a three dimensional material. According to our arguments, the upper critical dimension $\nu$ for the formation of pseudo gaps is $\nu_{\text{c}} = 2$. However, at the present stage we cannot rule out a higher upper critical dimension, $2 \leq \nu_{\text{c}} \leq 3$. We also discuss briefly the role of statistical interactions in pseudo gap phases.

Experiments on metals close to an antiferromagnetic quantum critical point (QCP) show clearly that these systems cannot be described by standard Fermi liquid theory. This is not very surprising, as at the QCP magnetic fluctuations dominate and electronic quasi particles scatter from spin-fluctuations characterized by a diverging correlation length. Indeed, a theory of quantum critical fluctuations interacting weakly with Fermi liquid quasi particles[1] can explain a substantial part of the experiments at least if effects like weak impurity scattering are properly taken into account[2]. However, a number of experiments seems to contradict the standard spin-fluctuation scenario, presently the best studied example for this is probably CeCu$_6$-$_x$Au$_x$[3]. It has been speculated that this might be due to anomalous two-dimensional spin fluctuation[4] or a partial breakdown of the Kondo effect[5].

In this paper we discuss a different route which can lead to a breakdown of the theory of weakly interacting spin fluctuations, first proposed by Hertz[6]. The general idea[6] is the following: close to the QCP, the behavior of the system is dominated by large antiferromagnetic domains of size $\xi$, slowly fluctuating on the time scale $\tau_\xi \sim \xi^{\nu_{\text{op}}}$ where $\nu_{\text{op}}$ is the dynamical critical exponent of the order parameter. As $\xi$ is diverging when the QCP is approached, it is suggestive to assume that the electrons will adjust their wave functions adiabatically to the local antiferromagnetic background and will therefore show a similar behavior as in the antiferromagnetically ordered phase. If the Fermi surface is sufficiently large, the (staggered) order parameter of the antiferromagnetic phase induces gaps in parts of the Fermi surface with $\epsilon_k \approx \epsilon_{k+Q} \approx 0$, where $\epsilon_k$ is the dispersion of the quasi particles measured from the Fermi energy and $Q$ the ordering wave-vector of the antiferromagnet. Will precursors of this effect show up and induce pseudo gaps in the paramagnetic phase for sufficiently large $\xi$? Pseudo gaps play an important role in the physics of underdoped cuprates[7] and it has been speculated that they are indeed precursors of gaps in either superconducting, antiferromagnetic, flux or striped phases. In this report we want to investigate qualitatively on the basis of simple physical arguments under what generic conditions such pseudo gaps are expected to occur close to an antiferromagnetic QCP. We will consider only systems, where the ordered antiferromagnet is metallic, therefore our discussion might have less relevance for the high $T_c$ superconductors where the undoped antiferromagnet is an (Mott-) insulator.

![FIG. 1. Schematic plot of the Fermi surface. In the ordered phase of a metallic antiferromagnet gaps open at the boundaries of the magnetic Brillouin zone.](image)

To define the concept of a pseudo gap more precisely, we first analyze the ordered phase where in mean field theory the Hamilton of the electrons is of the form

$$H_\Delta = \sum_{\sigma,k} (c_{\sigma,k+Q}^\dagger c_{\sigma,k+Q}) \left( \frac{\epsilon_k}{\sigma} - \frac{\sigma \Delta}{\epsilon_{k+Q}} \right) \left( \frac{c_{\sigma,k}}{\epsilon_{k+Q}} \right).$$

(1)

$\Delta$ is proportional to the staggered order parameter (assumed to point in $z$ direction) and the $k$ sum extends over a magnetic Brillouin zone. Close to the “hot lines” on the Fermi surface (“hot points” in two dimensions) with $\epsilon_{k_h} = \epsilon_{k_h+Q} = 0$, a gap opens (see Fig. 1) and the band structure at $k = k_h + \delta k$ is approximately given by

$$\epsilon_{\delta k} \approx \frac{1}{2} \left( (v_1 + v_2)\delta k \pm \sqrt{(v_1 - v_2)^2 \delta k^2 + 4 \Delta^2} \right).$$

(2)

where $v_1 = v_{k_h}$ and $v_2 = v_{k_h+Q}$ are the Fermi velocities close to the hot points. The gap is e.g. visible if one integrates the spectral function $A_k(\omega)$ for $k$-vectors along a direction $\hat{n}$ in the $(v_1, v_2)$ plane perpendicular to $v_1 + v_2$.
antiferromagnetic order on length scales smaller than
of the quasi particles adjusts adiabatically to the local
usual Fermi liquid phase space arguments. From general
field theory no precursor of the gaps show up and
a dynamical critical exponent (see below). Within mean
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with
A
are quantitatively wrong. Interactions of quasi parti-
cles far away from
the fields Φ(→ ⟨Φ
k
h
with each other and with the
spin-fluctuations will actually induce some small weight
within these (renormalized) gaps but this does not invali-
date the mean field picture:˜
within these (renormalized) gaps but this does not invali-
date the mean field picture: A(ω) vanishes rapidly in the
limit ω → 0 in the ordered phase as it is obvious from the
usual Fermi liquid phase space arguments. From general
scaling arguments one expects in the paramagnetic phase
close to the QCP, that at T = 0

A(ω) ∼ ωα f (1/(ωξ2))

with f (x → 0) ∼ const. and f (x → ∞) ∼ xα where z is a
dynamical critical exponent (see below). Within mean
field theory no precursor of the gaps show up and α = 0.
However, one would expect α > 0 if the wave function
of the quasi particles adjusts adiabatically to the local
antiferromagnetic order on length scales smaller than ξ.

a) 1
\sqrt{A_0} + α
\sqrt{A_1} + α
\sqrt{A_2} + α
\sqrt{A_3} + ... 

b) 1
\sqrt{A_0} + α
\sqrt{A_1} + α
\sqrt{A_2} + α
\sqrt{A_3} + ... 

FIG. 2. a) Effective action S[Φ] according to Hertz for
the model defined in Eqn. 3 after the electrons have
been integrated out. The lines denote free Greens func-
tions G0(x − x′, τ − τ′) of the electrons, the wiggles are
the fields Φ(x, τ). b) Quadratic part of the effective action
Φ → (Φ) + δΦ in the ordered phase ( denotes the order
parameter (Φ)). Combinatorial prefactors are omitted both
in a) and b).

This paper focuses on the T = 0 behavior directly
at the QCP as we are mainly interested in the question
whether pseudogaps affect the quantum critical behav-
ior and therefore 3 with α > 0 serves as a definition
for a pseudogap. Note that pseudogap physics can be
considerably more pronounced in other regimes, e.g. for
nearly magnetic metals with Heisenberg or xy symmetry
in d = 2 for low, but finite temperatures in a parameter
regime, where the system is deep in the ordered phase
at T = 0. This regime has for example been investigated in
detail by Vlik, Tremblay et al. 12

For definiteness, we will consider a model of Fermions
jkn coupled linearly to a collective bosonic field Φq with
the following action in imaginary time 1

S = \int_0^β dτ \left[ \sum_{σ,k} f^*_q(∂_τ + ε_k) f_q + \sum_{q} \Phi^* q \frac{1}{|q|} \Phi_q 
+ \sum_{q,k,σ,σ'} \Phi^* q f^+_q \Phi^* k,σ f_q,σ' \right] + h.c. 

where σi are the Pauli matrices and β = 1/T the inverse
temperature. Integrating out the collective field induces
a spin-spin interaction J of the Fermions. For realistic
models one should also add charge-charge interactions,
which are, however, not expected to change the physics
close to a magnetic QCP qualitatively.

Many years ago, Hertz has proposed to describe the
QC metallic antiferromagnet in the spirit of a Ginzburg-
Landau-Wilson approach in terms of a fluctuating order
parameter Φ(x, τ) with an effective action

S = S_0 + S_{int}

S_0 = \frac{1}{β} \sum_{k,ω_n} \Phi^* k,ω_n (r + (k ± Q)^2 + γ[ω_n]) Φ k,ω_n

S_{int} = U \int_0^β dτ \int d^d r |Φ(x, τ)|^4

where ω_n = 2πν/β are bosonic Matsubara frequencies and
Φ k,ω_n is the Fourier transform of Φ(x, τ). The term
linear in ω_n is due to the scattering from quasi parti-
cles which induce the Landau damping of the spin-
fluctuations. As discussed in detail in the original paper
by Hertz, the action S describes only the leading terms
in an expansion which is derived by integrating out the
Fermions in Φ. The expansion is shown schematically
in Fig. 2b (due to time-reversal symmetry, cubic terms
vanish in the limit ω_n → 0). A simple scaling analysis
with k ~ 1/L, ω ~ 1/L^zop, Φ(x, τ) ~ L^{1−(d+zop)/2} with
zop = 2 shows that the interaction term S_{int} vanishes
~ 1/L^d+zop−4, i.e. U is (dangerously) irrelevant in di-
dimensions d > 4 − zop = 2. Furthermore, higher order
interactions and frequency and momentum dependencies
of the effective vertices are even more irrelevant. A pseu-
dogap as it is defined in 3 would certainly change the
critical exponent zop, as it would strongly reduce the
damping of the spin-fluctuations. As the scaling anal-
ysis sketched above does give no indications for such a
phenomenon for d ≥ 4 − zop, it strongly suggests that
a strong-coupling effect like a pseudo gap should never
occur in dimensions d > 4 − zop at least as long as the
(bare) interactions are not too strong.

This line of arguments (which would be completely
valid close to a classical phase transitions) is not reliable
in the case of a quantum phase transition in a metal.
This can be seen for example by considering the ordered
phase. An expansion of the Hertz action 3 around the
mean field Φ = ⟨Φ⟩ + δΦ suggests that the transverse
spin-fluctuations (assuming Heisenberg or xy symmetry)
are damped. However, the Goldstone theorem guaran-
tees that the spin-waves are not damped in the limit
The physical origin of this is essentially the same as in the previous discussion of pseudo-gap formation: the wave function of the electrons adjust to the slowly varying antiferromagnetic background. A simple RPA approximation based upon the mean field Hamiltonian [1] correctly describes this effect on a qualitative level. It is therefore instructive to investigate how the RPA contribution arise in the effective action $S[\Phi]$. In Fig. 2b, it is shown that spin-spin interactions $\Phi^n$ of arbitrarily high order $n$ are needed to recover the trivial RPA+mean-field result.

Two scenarios seem to be possible to resolve the apparent conflict that contributions which are irrelevant by power counting are important in the ordered phase. The first possibility is that all the higher interactions are indeed irrelevant in the sense that the physics of the formation of undamped spin fluctuations does not influence the quantum critical behavior on the paramagnetic side of the phase diagram in any qualitative manner – in technical terms, they are “dangerously irrelevant” and important only in the ordered phase. The analysis given below suggests that this situation is actually realized in three dimensions. The second possibility is that pseudo gap formation is important and that spin-spin interactions of arbitrarily high order have to be kept which implies that [3] does not describe the physics properly and the “true” critical theory cannot be formulated in terms of the order parameter alone but has to include fermionic modes. For example, the spin fluctuation theory of the cuprates as it is worked out by Abanov and Chubukov[2] suggests such a scenario in $d=2$. What can go wrong with the simple scaling arguments given above? Belitz, Kirkpatrick et al. have recently shown in their analysis of the dirty nearly ferromagnetic metal that scaling is indeed not reliable due to a very simple physical reason: The Hertz action implies that a domain of size $\xi$ fluctuates very slowly on the time scale $\tau_\xi \propto \xi z_{op}$ with $z_{op} = 2$. However, in a clean metal there is a much faster and more efficient way to propagate information from one side of a fluctuating domain to the other: ballistic electrons can traverse the domain in the time $\tau_F \propto \xi z_F$ with $z_F = 1$. This defines a second dynamical critical exponent $z_F$ (which can be renormalized due to scattering from spin fluctuations, see below). Power-counting is not reliable because two different dynamical exponents $z_{op}$ and $z_F$ exist simultaneously – while there is only one large length scale $\xi$, two rather different time scales exist. The question which of these scales is relevant for a given process generally requires a detailed analysis and is not at all obvious. This physics should therefore be investigated in a careful renormalization group calculations which includes both fermionic and bosonic degrees of freedom. We will not try such an analysis here but instead use a properly modified scaling argument to investigate the possibility of pseudo gap formation.

For our scaling analysis[3], we assume that the susceptibility at the QCP is of the form suggested by (3) $(d \geq 2)$

$$
\chi_{q \pm} Q(\omega) \sim \frac{1}{q^2 + (i\omega)^2 / z_{op}^*}.
$$

We are mainly interested in the case $z_{op} = 2$, smaller values for $z_{op}$ might be relevant if pseudo gap formation takes place[14], larger values have e.g. been used to fit experiments in CeCu$_{6-x}$Au$_x$ and have been claimed[15] to be relevant in $d = 2$. It is not difficult to generalize the following arguments for susceptibilities with other $q$ and $\omega$ dependencies.

The strategy of the following scaling analysis is to estimate the effective amplitude of the quasistatic collective field seen by the electrons. Obviously the answer will depend on which time- and length-scale the electrons probe the background magnetization. The main idea is, that a lower bound for the relevant time- and length scales can be obtained from Heisenberg’s uncertainty relation and the effective size of the gap. The main assumptions of the following arguments are discussed in detail in the second half of the paper: we assume that above the upper critical dimension for pseudogap formation, the nature of the electrons is not changed completely by the quantum critical fluctuations. According to the mean field result [2] a gap of size $\omega^* = \Delta$ opens in a $(d - 2)$ dimensional stripe in momentum space of width $k^* = \Delta / v_F$. Below, we will discuss the effect of interactions which can change this relation to $\omega^* \sim (k^*)^2 \sim \Delta z^2$ where $z_F = 1$ is the mean field exponent. Heisenberg’s uncertainty relation dictates that the electrons have to see a quasi-static antiferromagnetic background for a time $\tau^* \gtrsim 1/\omega^*$ on a length scale of order $\xi^* \gtrsim 1/k^*$ perpendicular to the direction of the hot lines to develop the pseudo gap. What is the effective size of the quasi-static antiferromagnetic order $\langle \Phi_{\xi^*,\tau^*} \rangle_{\xi^*,\tau^*}$ on these length and time scales? The following estimate should at least give an upper bound at the QCP

$$
\langle \Phi_{\xi^*,\tau^*} \rangle_{\xi^*,\tau^*}^2 \lesssim \int_0^\omega d\omega' \int q^2 d^d q \int_\infty^\infty d^{d-2} q_{\parallel} \text{Im} \chi_{q \pm} Q(\omega)
$$

$$
\sim (k^*)^{d + z_{op} - 2} + (k^*)^2 (w^*)^{2z_{op} - 4} / z_{op}^4
$$

$$
\sim \Delta (d + z_{op} - 4) \frac{v_F}{z_{op}^2} + 2
$$

where the anisotropic integration of $q$ takes into account that the momentum of the electrons parallel to the hot line can vary on the scale $k_F$. In [3] we assumed $z_F \leq 2 z_{op}$. For our scaling argument, it does not matter whether we use $\text{Im} \chi(\omega)$ or e.g. $\chi(-i\omega)$ in [3], the version given above is motivated by the estimate of the quasi-elastic weight obtained in a $T = 0$ neutron scattering experiment with limited resolution $\omega^*$ and $k^*$. If we assume furthermore that $\Delta$ is proportional to $\langle \Phi_{\xi^*,\tau^*} \rangle_{\xi^*,\tau^*}$ as suggested by the mean field analysis (which should be valid above the upper critical dimension), we obtain the inequality $\Delta^2 \lesssim \text{const} \cdot \Delta (d + z_{op} - 4) \frac{v_F}{z_{op}^2}$. This implies that, at least in a weak coupling situation, pseudo gaps should appear only if
Using (8) and (12) we obtain at the QCP through perturbation theory, the self energy of those electrons at the quasi-particles close to the hot lines. In leading order of the staggered magnetization and the respective role of amplitude and angular fluctuations, the second aspect concerns strong coupling effects due to the scattering from singular spin fluctuations. The scattering from spin fluctuations strongly modifies the behavior of electrons in such a situation is much less understood. To investigate the pseudo gap phase in this case, probably the most obvious theoretical route is to describe the adiabatic adjustment of the wave function of the electrons into the local direction of the slowly fluctuating order parameter. This approach has been used by a

\[ d + z_{op} \leq 4 \]  

which is the central result of this paper. We believe, that it is accidental that (11) coincides with the condition for the relevance of the $\Phi^4$ interaction in the Hertz model as is evident from the fact that $z_F$ enters the inequality. Within the approach of Hertz, $z_{op} = 2$ and the critical dimension for pseudo gap formation is therefore $d_c = 2$. From our scaling arguments we cannot say much about what will happen in $d = d_c = 2$ (or for $d < d_c$). Based on the observation, that the ordered phase is not well described by (3), we suspect that the Hertz description of a quantum critical antiferromagnet is not valid in $d = 2$ – this point of view agrees with the results of Abanov and Chubukov, who have analyzed the spin-fermion problem in $d = 2$ in a certain large $N$ expansion. In the pseudo gap phase we expect by comparison to the ordered phase that $1 \leq z_{op} < 2$. Therefore it seems to be possible that the critical dimension is not two but some $d > 2$ is larger than 2 in $d = 2$ depending on the number of hot spots). In three dimensions, pseudo-gap formation will probably not invalidate the Hertz approach, at least for weak coupling.

The derivation of (11) is far from being rigorous and based on a number of assumptions. In the following two of these, which are probably the most important ones, are discussed in more detail. First we consider non-Fermi liquid effects due to the scattering from singular spin fluctuations, the second aspect concerns strong coupling effect and the respective role of amplitude and angular fluctuations of the staggered magnetization.

The scattering from spin fluctuations strongly modifies the quasi-particles close to the hot lines. In leading order perturbation theory, the self energy of those electrons at $T = 0$ is given by

\[
\text{Im} \Sigma_k(\Omega) \approx g_S^2 \sum_{k' < k} \int_0^\Omega \text{d} \omega \text{Im} \chi_{k-k'}(\omega) \text{Im} g_k^{\text{op}}(\omega - \Omega),
\]  

where $g_S$ is the vertex of the coupling of electrons to spin-fluctuations (here, we assume the absence of pseudo gap formation and therefore $g_S$ is finite) and \( g_k^{\text{op}}(\omega) \approx 1/(\omega - \epsilon_k + i\delta) \) is the Greens function of the (free) fermions. Using (8) and (12) we obtain at the QCP

\[
\text{Im} \Sigma_{k_0+\delta k}(\Omega) \sim \Omega^{1 + \frac{d-3}{2}} f \left( \frac{(\delta k)^2}{\Omega^2 z_{op}} \right)
\]  

where $\delta k \sim \delta k \cdot v_{k_0+Q}$ is a distance from the hot line and $f$ is some scaling function with $f(x \rightarrow 0) \sim const.$ and $f(x \rightarrow \infty) \sim 1/x^{\frac{d-3}{2}}$. For $z_{op} = 2$ and far away from the hot lines, Fermi liquid behavior is recovered. Our previous arguments suggest that typical frequencies and momenta for the pseudo gap formation are $\delta k \sim \Delta$ and $\Omega \sim \Delta/F$, therefore the typical argument $\Delta^2 (1 - z_F/z_{op})$ of $f$ is small and the momentum dependence of $\text{Im} \Sigma$ can be neglected for $z_F < z_{op}$ and will not induce new effects for $z_F = z_{op}$ (this is the reason, why we used $k^* \sim \Delta$ in our scaling analysis). From this we obtain $\Sigma(\Omega_{\text{typical}}) \sim \Omega^{1 + \frac{d-3}{2}}$. Below three dimensions, the quasi particle picture breaks down close to the hot lines and therefore some of our perturbative arguments might fail. Ignoring this possibility, we conclude that typical energies $E_k$ of the (incoherent) fermionic excitations are determined from $E_k + c E_k^{1 + \frac{d-3}{2}} \sim v_F(k - k_F)$ (because we can neglect the $k$ dependence of $\Sigma$) and therefore

\[
z_F = \max \left[ 1, \frac{z_{op}}{d + z_{op} - 3} \right]
\]  

which is the value which should be used in our previous arguments for $d + z_{op} \geq 4$, i.e. in the absence of pseudo gap formation. An effect which we haven’t taken into account in our discussion, is that generically, close to the antiferromagnetic QCP, a superconducting phase is stabilized, however, at least in $d \geq 3$ the ordering temperature of the superconductor $T_c$ is usually much smaller than the typical scale $T^*$ below which the quantum critical behavior of the antiferromagnet dominates. In $d = 2$ the situation might be different with $T_c \sim c T^*$, where $c$ is a constant of order 1.

It is important to emphasize, that our estimate (11) of $\langle \Phi \rangle_{\text{eff}}$ and therefore our main result (1) is based on the assumption that amplitude fluctuations of the staggered order parameter are present and can be described by (8). Electrons adjust their wave functions much better to angular fluctuations of the direction of the staggered magnetization than to fluctuations of its size, because a rotation of the spin-quantization axis does not cost any energy in the long wave length limit (assuming weak spin-orbit coupling and/or a sufficient high symmetry of the underlying crystal). This adiabatic adjustment is not included in our estimates. Numerical results of Bartosch, Kopietz, Millis and Monien show that in $d = 1$ amplitude and phase fluctuations have a drastically different effect on pseudo gap formation. Nevertheless, our approach to focus on amplitude fluctuations in our previous discussion was valid as within the theory of Hertz (8), the interactions of spin-fluctuations are irrelevant and amplitude fluctuations exist for $d > 2$. If they are present they should be the dominating mechanism to destroy pseudo gap behavior. Below the upper critical dimensions, one expects that amplitude fluctuations are frozen out and only angular fluctuations dominate the critical regime. Even in dimensions larger than 2 such a picture might be appropriate in a strong coupling regime, e.g. if one considers a Heisenberg model with a large antiferromagnetic coupling $J_{AF}$ coupled to a metal. Unfortunately, the behavior of electrons in such a situation is much less understood.
number of authors interested in the pseudo gap phase of the cuprates. A natural model to discuss this type of physics consists of a non-linear $\sigma$-model coupled to the spin $S(r) = \frac{1}{2} f^{\sigma}(r) \sigma_{\alpha,\beta} f_{\beta}(r)$ of fermions $f$. The non-linear $\sigma$-model $S_\sigma$ describes the directional fluctuations of the staggered order parameter $\sigma$ in the absence of amplitude fluctuations. The action in terms of $n$ with $n^2 = 1$ and the Grassmann fields $f$ is given by

$$S = S_f + S_\sigma + S_{f\sigma}$$

$$S_f = \int_0^\beta d\tau \sum_{\sigma,k} f_{\sigma,k}(\partial_\tau + \epsilon_k)f_{\sigma,k}$$

$$S_\sigma = \frac{1}{g} \int_0^\beta d\tau \int d^d r (\partial_\tau n)^2 + (\nu \partial_\tau n)^2$$

$$S_{f\sigma} = \Delta \int_0^\beta d\tau \int d^d r \cos(Q r) n(\tau) S(\tau).$$

We have not written down the proper spin Berry phase which is essential to describe the Kondo lattice correctly. For simplicity, we focus in the following on a model with an $O(2)$ symmetry $n = (0, \sin \phi(r,t), \cos \phi(r,t))$ and comment below on the more difficult situation with $O(3)$ symmetry. To describe the pseudo fermions, we define new fields $c$ with a quantization axis rotated in the local direction of the order parameter $\tilde{\phi}$.

$$c_\uparrow(r, \tau) = \exp \left[ i \tilde{\phi}(r, \tau) \frac{\sigma_\tau}{2} \right] c_\uparrow(r, \tau)$$

The new fields $c$, which we call “pseudo fermions” in the following, do not transform under a global rotation around $x$-axis, this implies a separation of spin and charge degrees of freedom if the low energy excitations are well described by $c$ (see below). The advantage of the transformation is, that $S_{f\sigma}$ now describes the scattering of the pseudo fermions from a static order parameter pointing always in $z$ direction which can be treated non-perturbatively. The pseudo fermions are the natural degrees of freedom in a situation, where the single-particle wave function adjusts to the (collective) magnetic background. If one neglects the residual interactions with $n$, gaps open along the hot lines and the action of the pseudo fermions is given by

$$S_c = \int_0^\beta \sum_{\sigma,k} c_{\sigma,k}^* \partial_\tau c_{\sigma,k} + H_\Delta(c^*, c)$$

where $H_\Delta(c^*, c)$ is the mean field Hamiltonian.

The residual interaction of $n$ and $c$ arises from the Berry phase $f^* \partial_\tau f$ and kinetic energy of the electrons. The semi-classical contributions $S_{c\sigma}^{\text{sc}}$ is given by the minimal substitution which corresponds to the gauge transformation $[\delta \phi]$. Using $$(\partial_\mu \phi) \frac{\sigma_\tau}{2} = ((\partial_\mu n) \times n) \cdot \sigma_\tau$$, we obtain

$$S_{c\sigma}^{\text{sc}} = -i \int_0^\beta d\tau \int_{-\infty}^{\infty} d^d r \left[ (\partial_\tau + (\nu \partial_\tau n) n) \times n \right] \cdot S.$$
$f_{\mathbf{k}}$. In this case, we do not expect any pseudo gaps. As we are not aware of methods which can describe such a confinement transition, it is difficult to give an estimate under which conditions a pseudo gaps will occur in the model \( [5] \). We can only speculate that the formation of pseudo gaps might be controlled by the area-density of vortices, i.e. the number of vortices per area $n_A$ piercing through a given area in space time at the QCP, to be compared to $(\Delta/\varepsilon_F)^2$. Both $n_A$ and $\Delta$ are non-critical at the transition. If these are the relevant parameters, then pseudo gap behavior is expected only if the density of vortices at the QCP is small.

If the magnet has O(3) instead of xy symmetry, one can follow the same steps which have been discussed before and one faces again the problem, that statistical interactions are induced as soon as pseudo fermions are introduced. Kübert and Muramatsu have proposed in the context of a theory of a slightly doped $t$-$J$ model a convenient way to keep track of this statistical interaction with the help of a CP$^3$ representation of $n$ using two complex fields $z_1$ and $z_2$ with $|z_1|^2 + |z_2|^2 = 1$ and $n = z_1^* \sigma_{\alpha\beta} z_2$. In this language the pseudo fermions interact strongly with the CP$^3$ fields via a local $U(1)$ gauge theory. Again, confinement seems possible.

In this paper, we have investigated the possibility of pseudo gap behavior close to the QCP of a nearly antiferromagnetic metal. Based on heuristic scaling arguments we suggest that generically, amplitude fluctuations destroy pseudo gaps in dimensions $d > 2$. In three dimensions we expect that the Hertz theory is valid at least for not too strong coupling while in $d = 2$ it is probably modified due to pseudo gap formation and the strong interaction of spin-fluctuations and fermionic modes. These questions should be studied in a renormalization group treatment of both fermionic and bosonic modes. We were not able to derive any criteria for pseudo gap formation in a situation where amplitude fluctuations are completely frozen out and emphasized that the motion of the fermions on top of the spin background leads to strong statistical interactions of the fermionic modes with the excitations of the magnet.

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