Effective fermion couplings in warped 5D Higgsless theories

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We consider a five dimensional $SU(2)$ gauge theory with fermions in the bulk and with additional $SU(2)$ and $U(1)$ kinetic terms on the branes. The electroweak breaking is obtained by boundary conditions. After deconstruction, fermions in the bulk are eliminated by using their equations of motion. In this way Standard Model fermion mass terms and direct couplings to the internal gauge bosons of the moose are generated. The presence of these new couplings gives a new contribution to the $\epsilon_3$ parameter in addition to the gauge boson term. This allows the possibility of a cancellation between the two contributions, which can be local (site by site) or global. Going back to the continuum, we show that the implementation of local cancellation in any generic warped metric leaves massless fermions. This is due to the presence of one horizon on the infrared brane. However we can require a global cancellation of the new physics contributions to the $\epsilon_3$ parameter. This fixes relations among the warp factor and the parameters of the fermion and gauge sectors.

I. INTRODUCTION

Higgsless models [1, 2, 3, 4, 5, 6, 7, 8, 9], which have been proposed as gauge theories in extra dimensions as an alternative to the standard electroweak symmetry breaking mechanism, can also be understood as four dimensional deconstructed theories in the context of linear moose models [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. Deconstruction is a useful tool, both for computing and also for finding renormalizable extensions [24].

One of the interesting features of the Higgsless models is the possibility to delay the unitarity violation scale via the exchange of massive KK modes [1, 25, 26, 27, 28, 29, 30, 31]. However, in the simplest versions of these models, it is difficult to reconcile a delayed unitarity with the electroweak constraints: in fact the $\epsilon_3$ parameter tends to get a large contribution. A recent solution to the $\epsilon_3$ problem, which does not spoil the unitarity requirement at low scales, has been found by delocalizing the fermions along the fifth dimension [23, 32, 33].
This solution has also a deconstructed correspondence, which is obtained by introducing direct couplings between ordinary left-handed fermions and the gauge vector bosons along the moose string \[22\]. The direct coupling is realized in terms of a product of non-linear $\sigma$-model scalar fields that in the continuum limit becomes a Wilson line. Also the fermion mass term in the deconstructed model is obtained by a Wilson line connecting the left end of the chain with the right end. The solution to the $\epsilon_3$ problem is found by fine tuning the contribution from the gauge fields with the one from the fermions. This cancellation may be "local" that means site by site \[22, 34\] or global. The possibility of this cancellation was already noticed in \[35\] within a strongly interacting electroweak framework which can be reduced to a linear moose model. In the continuum limit this solution is obtained by allowing the left-handed fermion fields to have some finite extent in the extra dimensions \[32, 33\]. A different solution, suggested by holographic QCD, has been recently proposed \[36\], where different metrics are felt by axial and vector states.

Aim of this paper is to show how the direct couplings of the fermions to the bulk gauge bosons defined in \[22\] can be effectively obtained by integrating over the bulk fermions. This result is obtained in the deconstructed theory. By performing the continuum limit we get an extension of the analysis of \[23, 33\] to a generic warped case.

Our paper is organized as follows: starting with a 5 dimensional gauge theory in a generic warped metric we write down the equivalent deconstructed moose. Then we study the new physics effects in the low energy limit by eliminating the fermions living on the internal sites with the solutions of their equations of motion in the limit in which the kinetic terms are negligible. In this way effective couplings of the standard fermions to the gauge fields along the moose are generated. They give a contribution to $\epsilon_3$ which can cancel "locally" or "globally" the gauge sector contribution. In addition, with the same mechanism, fermion masses are generated. Going back to the continuum limit, we have shown that the implementation of local cancellation in any generic warped metric leaves massless fermions. This is due to the presence of one horizon on the infrared (IR) brane. However we can require a global cancellation of the new physics contributions to the $\epsilon_3$ parameter. This fixes relations among the warp factor and the parameters of the fermion and gauge sectors. As in the flat case \[23, 33\], in the range of parameters allowed by the electroweak constraints and compatible with the unitarity requirement, it is possible to achieve fermion mass values up to the charm or to the bottom quark. In order to give
mass to the top quark, a possible solution, following \cite{23, 32}, is to break five dimensional Lorentz invariance in the fermion sector. This is done by considering the fermionic action over a fifth dimension of inverse length \((\pi R)^{-1}\) and curvature \(k\) rescaled by a factor \(\kappa\). This procedure does not spoil the flavour universality avoiding problems with FCNC and preserves the results for \(\epsilon_3^N\) cancellation (with \(\epsilon_3^N\) we indicate the new physics contribution to \(\epsilon_3\)). We show that the overall results are quite independent on the warping of the space.

In section \textbf{II} we consider a continuum model describing a \(SU(2)\) gauge theory in five dimensions with \(SU(2)\) symmetry broken to \(U(1)\) by boundary conditions on the branes. Following \cite{23} we add kinetic terms on the branes both for bosons and fermions in a suitable way to recover, at low energy, the standard model (SM). After discretization we write down the equivalent moose. In section \textbf{III} we consider the fermion action and we deconstruct it. In section \textbf{IV} we study the new physics effects in the low energy limit by decoupling the heavy fermions. This is done by eliminating the non-standard model fields with their equations of motion in the limit in which the kinetic terms are negligible. In section \textbf{V} we study the possible cancellations in the \(\epsilon_3^N\) parameter going back to the continuum. Conclusions can be found in section \textbf{VI}.

\section{II. REVIEW OF THE MODEL: GAUGE SECTOR}

Let us consider a theory with one extra dimension with arbitrary curvature which preserves the Poincarè invariance in the usual four dimensions:

\[
d s^2 = e^{-2\phi(y)} d x^\mu d x^\nu \eta_{\mu \nu} - d y^2
\]

The function \(e^{-2\phi(y)}\) is called the warp-factor. It is always possible to perform a coordinate transformation \(z = z(y)\) to get a conformally flat metric. This is achieved by requiring the following relation between \(d y\) and \(d z\)

\[
d y = e^{-A(z)} d z
\]

such that

\[
d s^2 = e^{-2A(z)} (\eta_{\mu \nu} d x^\mu d x^\nu - d z^2)
\]

with \(A(z) = \phi(y)\).
Let us consider a $SU(2)$ gauge theory in the 5-dimensional bulk, with additional brane kinetic terms to lower the mass values of the lightest gauge bosons, and with boundary conditions which break the gauge group to the electromagnetic $U(1)$. Let us generalize the action in [33] on a curved background, with a compact fifth dimension varying on a segment of length $\pi R$. The gauge action is:

$$S_{gauge} = -\frac{1}{4} \int d^4x dz \sqrt{|g(z)|} \left[ G_{a}^{a} g^{MP} g^{NQ} \right] - \frac{1}{4} \int d^4x dz \sqrt{|g(z)|} \left[ G_{a}^{a} g^{\mu\nu} g^{\rho\sigma} \right] - \frac{1}{4} \int d^4x dz \sqrt{|g(z)|} \left[ G_{3}^{3} g^{\mu\nu} g^{\rho\sigma} \right]$$

with

$$G_{MN} = \partial_{M} V_{N} - \partial_{N} V_{M} - i[V_{M}, V_{N}]$$

where $V_{M} = V_{M} a^{a}/2$, $M = (5, \mu)$, and $a = 1, 2, 3$ is the $SU(2)$ index. We have included a $z$-dependence in the gauge coupling constant. We will comment on that later on. We impose the Dirichlet boundary conditions $V_{\mu}^{1,2} | z = \pi R = 0$, and the Neumann boundary conditions $\partial_{z} V_{\mu}^{a} | z = 0 = 0$ [33].

By substituting $\sqrt{|g(z)|} = e^{-A(z)}$ we get:

$$S_{gauge} = -\frac{1}{4} \int d^4x dz \frac{1}{g_{5}^{a}(z)} e^{-A(z)} \left[ G_{a}^{a} g^{\mu\nu} - 2 G_{a}^{a} G^{\mu55} \right] - \frac{1}{4} \int d^4x dz \frac{1}{g_{5}^{a}(z)} e^{-A(z)} G_{a}^{a} g^{\mu\nu}$$

$$- \frac{1}{4} \int d^4x dz \frac{\delta(\pi R - z)}{g_{5}^{a}(z)} e^{-A(z)} G_{a}^{a} g^{\mu\nu}$$

Let us now review the deconstruction procedure of a gauge theory in 5 dimensions [10, 11, 12, 13, 37]. In order to discretize the fifth dimension and to write the moose action which, in the continuum limit corresponds to eq. [33], let us divide the $z$-segment in $K + 1$ intervals of size $a$. In general we can consider different $a_{j} = z_{j+1} - z_{j}$ lattice spacings near site $j$, but, since we are working with a general warped metric with the only requirement of Poincaré invariance (flat branes), we can safely consider equal spacing $a_{j} = a$ without loosing generality. The continuum limit is obtained by taking $a \to 0$ and $K \to \infty$. Through discretization of the fifth dimension we get a finite set of 4-dimensional gauge theories, each of them acting at a particular lattice site. In this way the discretized version of the original 5-dimensional gauge theory is substituted by a collection of four-dimensional gauge theories synthetically described by a moose diagram (see Fig. 1).
$S_{\text{lattice gauge}} = -\frac{1}{2} \int d^4x \left[ \sum_{j=1}^{K} \frac{a}{g_{5j}} \text{Tr}(G_{j}^{5})^2 e^{-A_{j}} - 2 \sum_{j=1}^{K+1} \frac{a}{g_{5j}} e^{-A_{j}} \text{Tr}(C_{\mu 5}^{j})^2 \right]$

$$-\frac{1}{2} \int d^4x \frac{1}{g^2} e^{-A_{0}} \text{Tr}(G_{0}^{0})^2 - \frac{1}{2} \int d^4x \frac{1}{g^2} e^{-A_{K+1}} \text{Tr}(G_{K+1}^{K+1})^2$$

(7)

where, for sake of simplicity, we have omitted the gauge index; $G_{j}^{5}$ are the 4-dimensional field strengths on the sites and

$$G_{j}^{\mu 5} = \partial_{\mu} V_{5}^{j} - \frac{1}{a} (V_{\mu}^{j} - V_{\mu}^{j-1}) - i[V_{\mu}^{j}, V_{5}^{j}] \quad j = 1, \ldots, K + 1$$

(8)

Here $V_{\mu}^{j} = V_{\mu}^{j a \tau a}/2$ and $g_{5j}$ are the gauge fields and gauge coupling constants associated to the groups $G_{j}$, $j = 1, \cdots, K$, and $V_{\mu}^{0} = \tilde{W}_{\mu}^{a \tau a}/2$, $V_{\mu}^{K+1} = \tilde{Y}_{\mu}^{3 \tau 3}/2$, are the gauge fields associated to $SU(2)_L$ and $U(1)_Y$ respectively.

![FIG. 1: The linear moose model.](image)

Let us introduce the link variables and their covariant derivatives for the discretized extra dimension:

$$\Sigma_{j} = e^{-ia V_{5}^{j}} \quad j = 1, \ldots, K + 1$$

$$D_{\mu} \Sigma_{j} = \partial_{\mu} \Sigma_{j} - i V_{\mu}^{j-1} \Sigma_{j} + i \Sigma_{j} V_{\mu}^{j}$$

(9)

For small lattice spacing,

$$D_{\mu} \Sigma_{j} \sim -i (a \partial_{\mu} V_{5}^{j} - V_{\mu}^{j-1} + V_{\mu}^{j} - ia [V_{\mu}^{j-1}, V_{5}^{j}]) \sim -ia G_{\mu 5}^{j}$$

(10)

and so:

$$(G_{\mu 5}^{j})^2 \sim \frac{(D_{\mu} \Sigma_{j})^\dagger (D^{\mu} \Sigma_{j})}{a^2}$$

(11)

from which the action for the discretized theory, in the small spacing limit, follows

$$S_{\text{lattice gauge}} = -\frac{1}{2} \int d^4x \left[ \sum_{j=1}^{K} \frac{a}{g_{5j}} e^{-A_{j}} \text{Tr}(G_{j}^{5})^2 - 2 \sum_{j=1}^{K+1} \frac{a}{g_{5j}} e^{-A_{j}} \text{Tr}(D_{\mu} \Sigma_{j})^\dagger (D^{\mu} \Sigma_{j}) \right]$$

$$-\frac{1}{2} \int d^4x \frac{1}{g^2} e^{-A_{0}} \text{Tr}(G_{0}^{0})^2 - \frac{1}{2} \int d^4x \frac{1}{g^2} e^{-A_{K+1}} \text{Tr}(G_{K+1}^{K+1})^2$$

(12)
This action can be related to a linear moose action, based on the $SU(2)$ symmetry and written in terms of $K + 1$ non-linear $\sigma$-model scalar fields $\Sigma_i$, $i = 1, \cdots, K + 1$, $K$ gauge groups, $G_i$, $i = 1, \cdots, K$, a global symmetry $SU(2)_L \otimes SU(2)_R$, in which the standard electroweak gauge group $SU(2)_L \times U(1)_Y$ is obtained by gauging a subgroup of $SU(2)_L \otimes SU(2)_R$. The moose action has the following form:

$$S_{\text{moose gauge}} = \int d^4x \left[ \sum_{j=1}^{K+1} \frac{f_j^2}{g_j^2} \text{Tr}[D_\mu \Sigma_j D^\mu \Sigma_j] - \frac{1}{2} \sum_{j=0}^{K+1} \frac{1}{g_j^2} \text{Tr}(G_j^{\mu \nu})^2 \right]$$  (13)

Therefore, by comparing eq. (12) with eq. (13), we have a matching between the 5D parameters of the discretized theory (the gauge coupling constants $g_{5j}$, the lattice spacing $a$, the warp factors $A_j$, the gauge couplings $\bar{g}$ and $\bar{g}'$ on the branes) and the parameters of the 4D deconstructed theory (the gauge couplings along the chain $g_j$, the link couplings $f_j$, the gauge couplings at the left and right ends of the chain $g_0$ and $g_{K+1}$). Namely

$$\begin{align*}
\frac{ae^{-A_j}}{g_{5j}} & \leftrightarrow \frac{1}{g_j} \quad j = 1, \ldots, K \\
\frac{e^{-A_j}}{ag_{5j}} & \leftrightarrow \frac{f_j^2}{g_j^2} \quad j = 1, \ldots, K + 1 \\
\frac{e^{-A_0}}{\bar{g}^2} & \leftrightarrow \frac{1}{g_0^2} \\
\frac{e^{-A_{K+1}}}{\bar{g}'^2} & \leftrightarrow \frac{1}{g_{K+1}^2}
\end{align*}$$  (14)

In order to compare with the moose lagrangian given in ref. [22] we only have to replace $V_j^\mu \rightarrow g_j V_j^\mu$ and $f_j \rightarrow f_j g_j$. Let us perform these rescalings for all $j = 0, \ldots, K + 1$. Notice that, since the mass dimension of the gauge couplings is $[g_{5j}^2] = -1$, then $[V_j] = 1$, that is $V_j$ have the canonical 4D mass dimension, and $g_j$ are dimensionless.

From the first two relations in eq. (14), we obtain that, for all the sites

$$a^2 = \frac{1}{f_j^2 g_j^2}$$  (15)

this means that, for generic $f_j$, it must be $g_j = \frac{1}{a f_j}$ and $g_{5j}^2 = \frac{e^{-A_j}}{a f_j}$. In the following, we will consider two possibilities:

- $f_j = \bar{f}$ and $g_j = \bar{g}$ do not depend on $j$. In that case the gauge coupling constants depend on the site through: $g_{5j}^2 = \bar{g}^2 ae^{-A_j}$. The flat case corresponds to $e^{-A_j} = 1$;

- $g_{5j} = g_5$ does not depend on $j$, then $f_j e$ and $g_j$ are not constant; in particular $g_j^2 = \frac{g_5^2 e^{A_j}}{a}$ and $f_j^2 = \frac{e^{-A_j}}{ag_5^2}$. 

III. REVIEW OF THE MODEL: FERMION SECTOR

Proceeding as for the gauge sector, let us consider fermions propagating in the warped 5-dimensional space with additional kinetic terms localized on the boundaries of the fifth extra dimension:

\[
S_{\text{ferm.}} = \int d^4x dz \sqrt{|g|} \left[ \left( \frac{i}{2} \bar{\psi} e^M_a \Gamma^a \tilde{D}_M \psi + \text{h.c.} \right) - M \bar{\psi} \psi \right.
\]

\[
+ \frac{\delta(z)}{t_L^2} \bar{\psi} e^L_a \Gamma^a \tilde{D}_L \psi + \delta(\pi R - z) i \bar{\psi} \left( \frac{1}{t_R^2} \right) e^R_a \Gamma^a \tilde{D}_R \psi \right] \tag{16}
\]

where \(e^M_a\) are the inverse fünfbein defined by \(g^{MN} = e^M_a e^N_b \eta^{ab}\). In the conformally flat metric defined by eq. (3) we have \(e^M_a = e^{A(z)} \delta^M_a\). The five dimensional Dirac matrices are defined in terms of the 4D ones by \(\Gamma^a = (\gamma^\mu, -i \gamma^5)\). This is a generalization in warped space of the fermion sector in [33].

The 5D fermions are equivalent to 4D Dirac fermions, \(\psi = (\psi_L, \psi_R)\), where \(\psi_L\) and \(\psi_R\) are \(SU(2)\) left and right-handed doublets for each family. The boundary conditions we impose for the bulk fermions are: \(\psi_R|_0 = 0\), \(\psi_L|_{\pi R} = 0\). A sum over the flavors is implicit and the couplings \(t_L, t_R\) can in general be different for each flavor. Following [33] we assume an universal \(t_L\), while \(t_R\) will be fixed for each flavor in order to reproduce the fermion mass spectrum. In eq. (16), \((1/t_R^2)\) is to be understood as a \(2 \times 2\) diagonal matrix with different entries for up and down fermions for each family:

\[
\left( \frac{1}{t_R^2} \right) = \begin{pmatrix} (1/t_R^2)^u & 0 \\ 0 & (1/t_R^2)^d \end{pmatrix} \tag{17}
\]

The parameters \(t_{L,R}\) set the weight of the brane kinetic terms with respect to the bulk one. They parameterize the amount of extra dimension which is felt by the fermions on the two branes. That is the delocalization in the bulk of the brane fermions. If we do not include brane kinetic terms, the boundary conditions considered, imply the absence of a zero mode for fermions [33, 38], and the mass of the KK excitations is set to the only mass scale in the bulk, \(1/R\). On the contrary, in the limit \(t_{L,R} \to 0\) the connection through the bulk kinetic terms is negligible and the model describes massless left-handed fermions gauged under a \(SU(2) \otimes U(1)\) gauge group living on the left brane, and massless right-handed fermions gauged under a \(U(1)\) living on the right brane. As we will see, the masses
of the SM fermions depend on these two parameters. In fact the bulk fermions make the communication between the light states possible, generating their masses.

In eq. (16), $\tilde{D}_M$ is the covariant derivative $\tilde{D}_M = D_M + \frac{1}{2} w^b_M \sigma_{bc}$ where $w^b_M$ is the spin connection

$$w^a_{bc} = \frac{1}{2} g^{RP} e^a_R \partial_{[M} e_{P]} \sigma + \frac{1}{4} g^{RP} g^{TS} e^a_R e^b_T \partial_{[P} e^c_R e^d_M \eta_{cd}$$

(18)

with $\sigma_{ab} = \frac{1}{4} \gamma_{a} \gamma_{b}$, and

$$D_M \psi = (\partial_M - iT^a V^a_M(z) - iY_L V^3_M(\pi R))\psi$$

(19)

where $T^a = \frac{\tau^a}{2}$ and $Y_L$ is the left hypercharge. Due to the additional kinetic terms, the 4D part of the covariant derivatives on the left and right branes are:

$$D_\mu \psi_L |_{z=0} = (\partial_\mu - iT^a V^a_\mu(0) - iY_L V^3_\mu(\pi R))\psi_L$$

$$D_\mu \psi_R |_{z=\pi R} = (\partial_\mu - iY_R V^3_\mu(\pi R))\psi_R$$

(20)

where $Y_R = T^3 + Y_L$ is the right hypercharge.

It is straightforward to show that, when the background geometry is conformally flat with a conformal factor which depends only on the fifth coordinate $z$, as given by eq. (3), the spin connection contributions cancel each other. We get

$$S_{\text{ferm.}} = \int d^4x dz e^{-4A(z)} \left[ \left( \frac{i}{2} \bar{\psi} \Gamma^M D_M \psi + h.c. \right) - M(z) \bar{\psi} \psi \right]$$

$$+ \int d^4x dz e^{-4A(0)} \delta(z) \frac{\delta(z)}{\ell_L^2} \bar{\psi}_L \gamma^\mu D_\mu \psi_L + \int d^4x dz e^{-4A(\pi R)} \delta(\pi R - z) i\bar{\psi}_R \left( \frac{1}{\ell_R^2} \right)^\mu D_\mu \psi_R$$

(21)

where we have defined

$$M(z) = Me^{-A(z)}$$

(22)

As for the gauge sector, let us discretize the fifth dimension.

We will include in the action, following [23], a $\kappa$ parameter in front of the $D_5$ term to be able to get the top quark mass. As we will see, the fermion masses arise from the $\partial_5$ terms which mix left-handed and right-handed fermions. The factor $\kappa$ will enhance the value of the fermion masses. We get (in the unitary gauge, $V_5 = 0$):
where \( \psi_j, j = 0, \ldots, K + 1 \) are the fermions at the \( j \)-site, and we have introduced the notation \( (\psi_0)_L = L_0, (\psi_{K+1})_R = R_{K+1} \); as before \( a \) is the lattice spacing. In eq. (23), the 4D covariant derivatives acting on the \( j \)-site fermion have the form:

\[
D_\mu \psi_j = (\partial_\mu - i g_j T^a V^{a}_\mu - i \bar{g}' Y_L V^{K+1}_\mu) \psi_j \\
D_\mu \psi_0 = (\partial_\mu - i \bar{g}' V^{0}_\mu - i \bar{g}' Y_L V^{K+1}_\mu) \psi_0 \\
D_\mu \psi_{K+1} = (\partial_\mu - i \bar{g}' Y_R V^{K+1}_\mu) \psi_{K+1}
\]

(24)

Introducing the shorthand notation: \( L_j = \psi^j_L \), \( R_j = \psi^j_R \), and using the property that, in
the continuum limit for the case of a continuous metric, $e^{-4A_j} \simeq e^{-4A_{j+1}}$, we get

$$S_{\text{lattice ferm.}} = \int d^4x i \sum_{j=1}^{K} a e^{-4A_j} \bar{\psi}_j \gamma^\mu D_\mu \psi_j + \kappa \sum_{j=0}^{K} e^{-4A_j} (\bar{L}_j R_{j+1} + h.c.)$$

$$- \sum_{j=1}^{K} e^{-4A_j} (aM_j + \kappa) (\bar{L}_j R_j + h.c.) + \frac{i}{t_L} e^{-4A_0} \bar{L}_0 \gamma^\mu D_\mu L_0$$

$$+ ie^{-4A_{K+1}} \bar{R}_{K+1}(\frac{1}{t_R^2})\gamma^\mu D_\mu R_{K+1}$$

(27)

It is convenient to rescale the fermion fields in order to make them canonical in 4 dimensions. We define

$$\hat{\psi}_j = \sqrt{ae^{-2A_j}} \psi_j, \quad j = 1, \ldots, K$$

$$\hat{L}_0 = \frac{e^{-2A_0}}{t_L} L_0$$

$$\hat{R}_{K+1}^{u,d} = \frac{e^{-2A_{K+1}}}{t_R^{u,d}} R_{K+1}^{u,d}$$

(28)

where we have used the notation $\psi = \begin{pmatrix} \psi^u \\ \psi^d \end{pmatrix}$ for each family and $t_R^{u,d}$ are the eigenvalues of the $t_R$ matrix (see eq. (17)).

With the redefinitions of eq. (28) we get:

$$S_{\text{lattice ferm.}} = \int d^4x \left[ \sum_{j=1}^{K} i \bar{\hat{\psi}}_j \gamma^\mu D_\mu \hat{\psi}_j + i \bar{\hat{L}}_0 \gamma^\mu D_\mu \hat{L}_0 + i \bar{\hat{R}}_{K+1} \gamma^\mu D_\mu \hat{R}_{K+1} + \frac{1}{a} \sum_{j=1}^{K} (aM_j + \kappa) (\bar{\hat{L}}_j \hat{R}_j + h.c.) + \frac{\kappa}{a} \sum_{j=1}^{K-1} (\bar{\hat{L}}_j \hat{R}_{j+1} + h.c.)$$

$$+ \frac{t_L}{\sqrt{a}} \kappa (\bar{\hat{L}}_0 \hat{R}_1 + h.c.) + \sum_{f=u,d} \frac{t_f}{\sqrt{a}} \kappa (\bar{\hat{L}}_f \hat{R}_f^{K+1} + h.c.) \right]$$

(29)

This action describes fermions on the $j$-sites with $j = 0, \ldots, K+1$, with a mass term

$$m_j = (aM_j + \kappa)/a, \quad j = 1, \ldots, K$$

(30)

which ”hop” from one site to the near one. These ”hopping” terms come from the derivative term along the fifth dimension after discretization. The ”hopping” strengths are:

$$\alpha_0 = \kappa t_L/\sqrt{a}$$

(31)
which parameterizes the probability for the fermions on the left end of the moose to hop to the \( j = 1 \) site;

\[
\alpha^u_d = \kappa \frac{t^u_d}{\sqrt{a}} \tag{32}
\]

parameterizing the probability for the fermions on the right end of the moose to hop to the \( j = K \) site, and

\[
\alpha_j = \kappa / a \tag{33}
\]

parameterizing the probability for the fermions on the \((j-1)\)-site to hop to the \( j \)-site. Notice that all the \( \alpha_j \) are equal for \( j = 1, \ldots, K - 1 \).

IV. DECOUPLING THE HEAVY FERMIONS

Let us study the effects of the \( \psi_j \ (i = 1, \ldots, K) \) fermions in the low-energy limit, that is for kinetic terms negligible with respect to mass terms.

This can be done by eliminating the \( \psi_j \) fields with the solutions of their equations of motion. Actually we want to derive the trilinear effective interactions between the light fermions living on the left and right ends of the moose with the gauge bosons on the \( j \)-sites. These effective vertices are provided by the mixing between the light fermions and the heavy ones living on the \( j \)-sites. This means that the contributions to the effective interactions could come only from the quadratic interactions among fermions described in eq. \[29\]. For this reason, in solving the equations of motions, it is enough to consider the quadratic part of the fermionic action.

Let us solve the equations of motion for the fields \( \hat{L}_j \), \( (j = 1, \ldots, K) \) and \( \hat{R}_j \), \( (j = 1, \ldots, K) \) in terms of \( \hat{L}_0 \) and \( \hat{R}_{K+1} \) which respectively are the left and right components of the SM fermions. The equations of motion, from the quadratic part of the action, and neglecting the kinetic term contributions are:

\[
\alpha_j \hat{L}_j - m_{j+1} \hat{L}_{j+1} = 0, \quad j = 0, \ldots, K - 1
\]

\[
\alpha_j \hat{R}_{j+1} - m_j \hat{R}_j = 0, \quad j = 1, \ldots, K \tag{34}
\]
The solutions are:

\[ \hat{L}_1 = \frac{\alpha_0}{m_1} \hat{L}_0; \]
\[ \hat{L}_j = \left( \frac{\alpha_0}{m_j} \prod_{i=1}^{j-1} \frac{\alpha_i}{m_i} \right) \hat{L}_0; \quad j = 2, \ldots, K \]

\[ \hat{R}^u_{K+1} = \left( \frac{\alpha_K}{m_K} \prod_{i=j}^{K-1} \frac{\alpha_i}{m_i} \right) \hat{R}^{u,d}_{K+1} \quad j = 1, \ldots, K-1 \]

\[ \hat{R}^{u,d}_{K} = \frac{\alpha_K}{m_K} \hat{R}^{u,d}_{K+1} \quad (35) \]

By substituting in eq. (29), we get:

\[ S_{\text{ferm.}}^{\text{lat.}} = \int d^4 x [i \tilde{L}_0 \gamma^\mu D_\mu \hat{L}_0 + \sum_{j=1}^{K} b^L_j \ i \tilde{L}_0 \gamma^\mu (\partial_\mu - ig^a T^a V_\mu - i\tilde{g}' Y L V_\mu^{K+1}) \hat{L}_0 \]
\[ + i \tilde{R}_{K+1} \gamma^\mu D_\mu \hat{R}_{K+1} + \sum_{j=1}^{K} \sum_{f=u,d} b^{R\mu}_j \ i \tilde{R}_{K+1} \gamma^\mu (\partial_\mu - ig^a T^a V_\mu - i\tilde{g}' Y L V_\mu^{K+1}) \hat{R}_f^{K+1} \]
\[ + \sum_{j=1}^{K} \sqrt{b^{Ru}_j} \sqrt{b^{Rd}_j} \frac{g_j}{\sqrt{2}} (\tilde{R}^u_{K+1} \gamma^\mu V_\mu^{+j} \hat{R}_f^{K+1} + \text{h.c.}) - \sum_{f=u,d} \tilde{m}_f (\tilde{L}_0 \hat{R}_f^{K+1} + \text{h.c.})] \quad (36) \]

where \( D_\mu \hat{L}_0 \) and \( D_\mu \hat{R}_{K+1} \) are given in eq. (24), \( V_\mu^{+j} = (V^{1j}_\mu - iV^{2j}_\mu)/\sqrt{2} \), and

\[ b^L_j = \left( \frac{\alpha_0}{m_j} \prod_{i=1}^{j-1} \frac{\alpha_i}{m_i} \right)^2 \]
\[ b^{Ru}_j = \left( \frac{\alpha_K}{m_K} \prod_{i=j}^{K-1} \frac{\alpha_i}{m_i} \right)^2 \quad b^{Rd}_j = \left( \frac{\alpha_K}{m_K} \prod_{i=j}^{K-1} \frac{\alpha_i}{m_i} \right)^2 \]
\[ \tilde{m}^{u,d} = \alpha_0 \frac{\alpha_K}{m_K} \prod_{i=1}^{K-1} \frac{\alpha_i}{m_i} \quad (37) \]

Notice that the following relations for the up and down-type fermion masses hold for all \( j = 1, \ldots, K \):

\[ \tilde{m}^u = \sqrt{b^L_j} \sqrt{b^{Ru}_j} m_j, \quad \tilde{m}^d = \sqrt{b^L_j} \sqrt{b^{Rd}_j} m_j \quad (38) \]

The canonical kinetic terms for the standard fermions are obtained by the following redefinitions:

\[ \hat{L}_0 \rightarrow \frac{1}{\sqrt{1 + \sum_{i=1}^{K} b^L_i}} \hat{L}_0 \]
\[ \hat{R}_f^{K+1} \rightarrow \frac{1}{\sqrt{1 + \sum_{i=1}^{K} b^{R\mu}_i}} \hat{R}_f^{K+1} \quad f = u, d \]

\[ (39) \]
so that the mass generated for the SM fermions is, \( f = u, d \):

\[
m_f' = m_j \sqrt{\frac{b_i^L}{1 + \sum_{i=1}^K b_i^L}} \sqrt{\frac{b_i^{Rf}}{1 + \sum_{i=1}^K b_i^{Rf}}} \quad \forall j = 1, \ldots, K
\]  

(40)

The mass difference between, for example, the top and the bottom quark, can be obtained by choosing \( b_j^{Rt} \gg b_j^{Rb} \). We will discuss this issue in more detail in Section \[\text{V}\].

The action in eq. (36) can be directly compared with the moose model one given in ref. \[\text{[22]}\], where only standard model fermions were considered, coupled to the SM gauge fields at the ends of the chain. Direct couplings of the left-handed fermions to the fields \( V_{\mu}^j \) were introduced by generalizing the procedure suggested in the BESS model \[\text{[39, 40]}\]. For each \( \psi_L \), the following \( SU(2) \) doublets were constructed

\[
\chi_j^L = \Sigma_j^\dagger \Sigma_{j-1}^\dagger \cdots \Sigma_1^\dagger \psi_L, \quad j = 1, \ldots, K
\]

(41)

Therefore a term containing direct left-handed fermion couplings to \( V_{\mu}^j \), invariant under the symmetry transformation of the model, could be added. In the unitary gauge \( \Sigma_j = I \), therefore additional couplings between left-handed fermions and the gauge bosons on the \( j \)-sites were generated. They are exactly the ones in eq. (36) with strength \( b_j^L \). In ref. \[\text{[22]}\] these additional couplings were described by free parameters; in the present derivation, \( b_j^L \) are generated by the presence of heavy fermions in the bulk. Notice that the sign of \( b_j^L \) is positive definite and, as we will show, this is the right sign to compensate for the contribution of the gauge bosons to the parameter \( \epsilon_3^N \).

With the same mechanism, additional couplings \( b_j^R \) of the right-handed standard fermions to the \( V_{\mu}^j \) gauge bosons are generated. According to our choice of different values of \( t_R \) for up and down-type fermions, the couplings of the right-handed fermions to the heavy gauge bosons along the moose are different for up and down components and for the various fermion families.

The effects of the \( V_{\mu}^j \) (\( j = 1, \ldots, K \)) particles in the low-energy limit can be considered by eliminating the \( V_{\mu}^j \) fields with the solution of their equations of motion for large gauge coupling constants \( g_j \). This limit corresponds to heavy masses for the \( V^j \) fields. In fact in this limit the kinetic terms of the new resonances are negligible. The corresponding effective theory will be considered up to order \( (1/g_j)^2 \).
By separating charged and neutral components, we get

\[ V_j^\pm = \frac{1}{g_j} (\tilde{g} \tilde{W}_j^\pm z_j) \]  

(42)

\[ V_j^3 = \frac{1}{g_j} (\tilde{g}' \tilde{Y}_j y_j + \tilde{g} \tilde{W}_j^3 z_j) \]  

(43)

Here we are neglecting the fermion current contributions which give current-current interactions. These terms turn out to be quadratic in the \( b^{L,R}_j \) parameters.

Coming back to the standard notation: \( \hat{L}_0 = \frac{1 - \gamma_5}{2} \hat{\psi} \) and \( \hat{R}_{K+1} = \frac{1 + \gamma_5}{2} \hat{\psi} \), we get the following effective interaction lagrangians:

\[
\mathcal{L}^{charged}_{\text{eff}} = \frac{\tilde{e}}{\sqrt{2} s_\theta} \left\{ (1 - \frac{b^L}{2}) \tilde{\psi}_d \gamma^\mu \frac{1 - \gamma_5}{2} \tilde{\psi}_u \tilde{W}^- \mu \\
+ \frac{1}{\sqrt{2}} b^{Ru}_j b^{Rd}_j \tilde{\psi}_d \gamma^\mu \frac{1 + \gamma_5}{2} \tilde{\psi}_u \tilde{W}^- \mu \right\} + \text{h.c.}
\]  

(44)

\[
\mathcal{L}^{neutral}_{\text{eff}} = -\frac{\tilde{e}}{s_\theta c_\theta} (1 - \frac{b^L}{2}) \tilde{\psi}_\gamma \gamma^\mu T^3 \frac{1 - \gamma_5}{2} \tilde{\psi}_u \tilde{Z}_\mu \\
- \frac{\tilde{e}}{s_\theta c_\theta} \sum_{f=u,d} \frac{b^{Rf}}{2} \tilde{\psi}_\gamma \gamma^\mu T^3 \frac{1 + \gamma_5}{2} \tilde{\psi}_u \tilde{Z}_\mu \\
+ \frac{\tilde{e}}{c_\theta} s_\theta \tilde{\psi}_\gamma \gamma^\mu Q \tilde{\psi}_u \tilde{Z}_\mu - \tilde{e} \tilde{\psi}_\gamma \gamma^\mu Q \tilde{\psi}_d \tilde{A}_\mu,
\]  

(45)

with \( \tilde{e} = \tilde{g} s_\theta = \tilde{g}' c_\theta \) and

\[
b^L = 2 \frac{\sum_{i=1}^K b^L_i y_i}{1 + \sum_{i=1}^K b^L_i}, \quad b^{Rf} = 2 \frac{\sum_{i=1}^K b^{Rf}_i z_i}{1 + \sum_{i=1}^K b^{Rf}_i}, \quad f = u, d
\]  

(46)

Here fields and couplings are "tilded" because they need renormalization (see [22]). From eqs. (44), (45) we see that the \( b^{Rf}_j \) parameters give rise to charged and neutral right-handed currents coupled to the SM gauge bosons. There are strong phenomenological constraints on the \( b^{Rf} \) parameters, coming for example, from right-handed fermion coupling to charged \( W \) contribution to the \( b \rightarrow s \gamma \) process \[41\] and to the \( \mu \) decay \[42\]. Nevertheless, as it is clear from eq. (37), in order to generate a mass term for the SM fermions, we need all the \( \alpha_j \neq 0 \) for \( j = 0, \ldots, K \).

However we will make the assumption, \( \alpha_j \gg \alpha_0 \gg \alpha^K_f \) for \( j = 1, \ldots, K - 1 \); this means \( b^L_j \gg b^{Rf}_j \). (An exception will be done for the top quark which will require \( \alpha^K_t \sim \alpha_0 \), that
is $t^i_R \sim t^i_L$ in order to obtain the physical value for $m_t$ (see Section V). This choice will not spoil the results obtained since the top does not enter in the new physics contribution to the $\epsilon_3$ parameter).

In this approximation we can proceed exactly as in ref. [22] concerning the low-energy limit of the model, the fields and couplings renormalization and the calculation of the electroweak $\epsilon^N_1$, $\epsilon^N_2$ and $\epsilon^N_3$ parameters.

Following the same lines of ref. [22], one can expand up to the first order in $b^L_j$ and neglect terms $O(b^L_j/g^2_j)$. Analogously we neglect corrections coming from $b^R_f$. With this approximation, we can also neglect the contribution from the effective four-fermion couplings.

Finally, the new physics contribution to the $\epsilon$ parameters is:

\begin{align*}
\epsilon^N_1 & \approx 0, \\
\epsilon^N_2 & \approx 0, \\
\epsilon^N_3 & \approx \sum_{i=1}^{K} y_i \left( \frac{e^2}{s^2_{\theta} g^2_i} z_i - b^L_i \right) \tag{47}
\end{align*}

where $e$ is the electric charge, $s_\theta$ is the sine of Weinberg angle defined by $G_F = \sqrt{2}e^2/(8s^2_\theta c^2_\theta M^2_Z)$, and

\begin{align*}
z_i = \sum_{j=i+1}^{K+1} x_j, \quad x_i = f_i^2, \quad \frac{1}{f^2_i} = \sum_{i=1}^{K+1} \frac{1}{f^2_i}, \quad \sum_{i=1}^{K+1} x_i = 1, \quad y_i = 1 - z_i \tag{48}
\end{align*}

with $f_i$ given in eq. (14) and rescaled according to $f^2_i \rightarrow f^2_i g^2_i$.

The expression for $\epsilon^N_3$ suggests that the additional fermion couplings to $V_i$ proportional to $b^L_i$ can compensate site by site for the contribution of the tower of gauge vectors with the choice

\begin{align*}
b^L_i = \frac{e^2}{s^2_{\theta} g^2_i} z_i, \quad \forall i = 1, \cdots K \tag{49}
\end{align*}

or the whole contribution from the gauge sector can cancel with the fermion one.

In order to explore in detail the various possible way to realize this compensation, let us perform the continuum limit.

V. THE CONTINUUM LIMIT

In this section we consider the continuum limit of the deconstructed model, obtained for $a \rightarrow 0$, $K \rightarrow \infty$ with the condition $(K+1)a = \pi R$, which is length of the segment in the
fifth dimension. Let us define:

\[
\lim_{a \to 0} \frac{b_{L,R}^j}{a} = b_{L,R}^j(z), \quad \lim_{a \to 0} \alpha f_j^2 = f_j^2(z), \quad \lim_{a \to 0} \alpha g_j^2 = g_j^2(z)e^{A(z)}
\]  

(50)

Using the relations between the moose parameters and the ones of the 5D theory given in eq. (14), we obtain

\[
\frac{1}{f^2} = \int_0^{\pi R} \frac{dz}{f^2(z)} = \int_0^{\pi R} dz \ g_5^2(z)e^{A(z)}
\]

(51)

From eq. (37), using eqs. (30),(31),(33), we get the continuum limit for the additional fermionic couplings, generated by the decoupling of the heavy fermionic modes in the bulk:

\[
b_L^j(z) = \lim_{a \to 0} \frac{\kappa^2 t_j^2}{(aM_j + \kappa)^2} \exp[\sum_{i=1}^{j-1} \log \left( \frac{\kappa^2}{(aM_j + \kappa)^2} \right)] = t_L^j e^{-\frac{2}{\kappa} \int_0^z dt M(t)}
\]

(52)

\[
b_R^j(z) = (t_R^f)^2 e^{-\frac{2}{\kappa} \int_0^{\pi R} dt M(t)}
\]

(53)

Finally, using eqs. (40), (37), (30), we get the following expression for the fermion masses (neglecting terms \(O(b^2)\)):

\[
m_f = \kappa \sqrt{b_L^j(z)b_R^j(z)} = \sqrt{t_L^j T_R^f} e^{-\frac{1}{\kappa} \int_0^{\pi R} M(z)dz} = \kappa \frac{T_R^f}{\sqrt{\pi R}} \sqrt{b_L^j(\pi R)}
\]

(54)

where we have introduced the dimensionless parameters \(T_L = t_L \sqrt{\pi R}, \ T_R^f = t_R^f \sqrt{\pi R}\) with \(f = u, d\).

In order to have a suppressed \(\epsilon_3^N\) parameter, we can require the contribution from the gauge sector to cancel with the one from the fermion sector. This cancellation may be local (for each value of the fifth coordinate) or global, namely we can require that the two contributions, integrated over the \(z\)-coordinate, do cancel.

### A. Local cancellation in \(\epsilon_3^N\)

Let us start investigating the possibility of a local cancellation. We need the continuum limit of the following expression, appearing in eq. (17),

\[
\lim_{a \to 0} \frac{e^2}{a s_b g_5^2} = e^2 e^{-A(z)} \int_0^{\pi R} dt g_5^2(t)e^{A(t)} = \frac{e^2}{s_b g_5^2} f^2(z) \int_z^{\pi R} dt f^2(t)
\]

(55)
Local cancellation requires:

\[ b^L(z) = \frac{T^2_L}{\pi R} e^{\frac{2}{\kappa} \int_0^z dt M(t)} = \frac{e^2}{s_y^2} \int_0^z dt \frac{1}{f^2(t)} \] (56)

This equality must hold for each value of the fifth coordinate. The quantity on the right-hand side vanishes in \( z = \pi R \) (at least for continuous functions \( f(z) \)). As a consequence \( b_L(\pi R) = 0 \), and, according to eq. (54), the fermion mass is zero: if we impose the local cancellation of the \( \epsilon_3^N \) parameter, we can’t give mass to fermions.

As an example, let us consider the first choice of parameters described at the end of Section II. This choice, \( (f_j = \bar{f} \text{ and } g_j = \bar{g}) \), leads to a \( z \)-dependent five dimensional gauge coupling constant, with \( g_5^2(z)e^{A(z)} \) independent on \( z \) due to eq. (50). Local cancellation requires:

\[ b^L(z) = \frac{T^2_L}{\pi R} e^{\frac{2}{\kappa} \int_0^z dt M(t)} = \frac{e^2}{s_y^2} \int_0^\pi R dt \frac{1}{f^2(t)} \] (57)

where we have normalized \( e^{-A(0)} = 1 \). In order to satisfy eq. (57), we have to take:

\[ e^{A(z)} = 1 - \frac{z}{\pi R} \]
\[ M = \frac{\kappa}{2\pi R}, \quad T^2_L = \frac{T^2_L}{\pi R} = \frac{e^2}{s_y^2 g_5^2(0)} \] (58)

With this choice for the metric, which turns out to be singular on the right brane, it is not possible to give mass to fermions. This follows from the fact that in our model the right handed fermions are on one horizon of the metric and are causally disconnected from the left handed fermions.

This conclusion is general: even if one can find a metric such to obtain a local cancellation of the gauge and fermion contributions to the \( \epsilon_3^N \) parameter, this metric has a singularity on the right end brane which prevents the fermions to acquire a mass.

**B. Global cancellation in \( \epsilon_3^N \)**

The other possibility is to require that the whole contribution to \( \epsilon_3^N \) from the gauge sector cancels with the one from fermions. That means

\[ \epsilon_3^N = \int_0^{\pi R} dz \frac{z}{(\pi R)^2} \left[ \frac{e^2}{s_y^2 g_4^2(z)} (1 - \frac{z}{\pi R}) - T^2_L e^{\frac{2}{\kappa} \int_0^z dt M(t)} \right] = 0 \] (59)
\[ g_2^2(z) = g_2^2(z)/(\pi R) \] (60)

For a flat metric, corresponding to the choice \( e^{-A(z)} = 1 \), leading to \( M(z) = M = \hat{M}/(\pi R) \) and \( g_4(z) = g_4 \), we get (this analysis has already been performed by [33])

\[ \epsilon_3^N = \frac{\lambda^2}{6} - T_L^2 \hat{A} \] (61)

where

\[ \lambda = \frac{e}{s_\theta g_4}, \quad \hat{A} = \frac{\kappa^2}{4M^2} - \frac{\kappa}{2\hat{M}}(1 + \frac{\kappa}{2\hat{M}})e^{-2\hat{M}/\kappa} \] (62)

The requirement \( \epsilon_3^N = 0 \) links the parameter of the gauge sector \( \lambda \), with the fermion parameters \( t_L \) and \( M \) as found in [33].

We are now in the position to explore how these results can change by considering warped-metrics. Let us, for example, specialize to a Randall-Sundrum metric:

\[ ds^2 = \frac{1}{(1 + k z)^2} (dx^2 - dz^2) \] (63)

with \( k \) the curvature along the \( z \)-coordinate.

The fermion mass, turns out to be:

\[ m^f = \kappa \frac{T_L T_R^f}{\pi R} (1 + k \pi R)^{-\frac{M}{\kappa}} \] (64)

Again, let us analyze the two cases described at the end of Section III

- \( f_j = \bar{f} \) and \( g_j = \bar{g} \) do not depend on \( j \); \( g_2^2(z) = g_2^2(0)/(1 + k z) \).

\[ \epsilon_3^N = \frac{\lambda^2(0)}{6} - \frac{T_L^2}{k \pi R} \left[ \frac{(1 + k \pi R)^{-2 M/\kappa + 1}}{(-2 M/\kappa + 1)} + \frac{1 - (1 + k \pi R)^{-2 \hat{M}/\kappa + 2}}{k \pi R (-2 M/\kappa + 1) (-2 \hat{M}/\kappa + 2)} \right] \] (65)

where we have used \( g_2^2(z)(1 + k z) = g_2^2(0) \) and introduced \( \lambda(0) = e/(s_\theta g_4(0)) \).

Obviously the contribution from the gauge sector is the same of the flat case, in fact we have chosen the \( z \) dependence of \( g_2(z) \) in a way to compensate the warp factor.

Following the analysis in [22], we can derive the mass of the lightest charged gauge boson, the \( W \), which, after fields and couplings renormalization, in the limit of small \( \lambda(0) \) is given by:

\[ M_W \sim \frac{\lambda(0)}{\pi R} (1 - \frac{\lambda^2(0)}{6}) \] (66)
To have an estimate of the contribution coming from the fermion sector, we can consider the case of large curvature: \( k >> (\pi R)^{-1} \), \( M \). In this limit we get:

\[
\epsilon_3^N \sim \frac{\lambda^2(0)}{6} - \frac{T_L^2}{2}
\]

(67)

which is the same result obtained for flat metric and \( M \to 0 \). This was expected since, in this limit, all the mass scales are negligible with respect to the curvature \( k \). The warping factor is multiplied by \( M \), so its effect is weakened for \( M \to 0 \). The global cancellation requires

\[
T_L \sim \frac{1}{\sqrt{3}} \pi R M_W
\]

(68)

This implies that the fermion mass is:

\[
m_f \sim \kappa \frac{T_R^f}{\sqrt{3}} M_W
\]

(69)

- \( g_5(z) = g_5 \).

The expression for \( \epsilon_3^N \) in this case is quite complicated. For large curvature: \( k >> (\pi R)^{-1} \), \( M \) we get

\[
\epsilon_3^N \sim \frac{1}{4k\pi R} \lambda^2 - \frac{T_L^2}{3}
\]

(70)

and, to the first order in \( \lambda \),

\[
M_W \sim \frac{\lambda}{\pi R} \sqrt{\frac{2}{k\pi R}}
\]

(71)

Both the results given in eq. (70) and eq. (71) contain the factor \( \lambda_{\text{eff}} = \lambda/\sqrt{k\pi R} \). This is a consequence of the following observation. The bulk gauge eigenstates are localized near the right brane of our model, which corresponds to the IR brane in the Randall-Sundrum metric given in eq. (63). The bulk fields that connect the \( SU(2)_L \) and the \( U(1)_Y \) theories transmit the breaking down to \( U(1)_{\text{em}} \). Then the phenomena responsible for the electroweak symmetry breaking happen mainly near the IR brane. Since \( g_5 \) is a dimensional parameter, it gets shifted by the warp factor \( g_5^2 \to g_5^2/(1 + k\pi R) \). In the limit of large curvature this rescaling is responsible for the extra factor in \( \lambda_{\text{eff}}^2 \).

In this case the global cancellation requires

\[
T_L \sim \pi R \sqrt{\frac{3}{8}} M_W
\]

(72)
a result which is very close to the flat case. Analogously, for the fermion masses, we get
\[ m_f \sim \kappa \sqrt{\frac{3}{8}} T_R f M_W \]

Therefore, in both cases the numerical value of \( T_L \), necessary to satisfy the electroweak constraints, is determined once \( R \) is given. For instance choosing \( R^{-1} \sim 1 \text{ TeV} \), we have \( T_L \sim 0.15 \). A smaller \( T_L \) requires a higher \( R^{-1} \) which implies a partial wave unitarity violation at a lower scale.

From eq. (73) we notice that the ratio \( T^t_R / T^b_R = m_t / m_b \) does not depend on the parameters \( M, k, \kappa \) and \( R \). In addition the product of \( T^t_R T^b_R \) is constrained by the limit on right-handed charged currents.

In this model a right-handed coupling \( tbW \) is generated by eliminating the fermions in the bulk. In eq. (44), in the low energy limit, we can read the effective coupling of the standard \( W \) boson to the right-handed fermion current.

\[ L^{tbW} = \frac{\bar{g}}{\sqrt{2}} \kappa_{CC} \left( \bar{\psi}_b \gamma^\mu \frac{1 + \gamma_5}{2} \psi_t \bar{W}_\mu + h.c. \right) \]

with
\[ \kappa_{CC} = \frac{1}{2} \sqrt{b^t_R b^b_R} = \sum_{j=1}^{K} \sqrt{b_{ij}^t} \sqrt{b_{ij}^b} z_j \]

The tilded variables differ from the physical ones by corrections which are of the second order in \( 1/g_j^2 \) and first order in \( b_j^{L,R} \). Since the \( tbW \) coupling is already of the first order in \( b_j^R \), we can neglect these additional corrections.

By performing the continuum limit, we get
\[ \kappa_{CC} = \frac{T^t_R T^b_R}{\pi R} \int_0^{\pi R} dz \exp \left[ -\frac{2}{\kappa} \int_z^{\pi R} M(t) dt \right] \frac{\int_z^{\pi R} g_5(t) e^{A(t)} dt}{\int_0^{\pi R} g_5^2(t) e^{A(t)} dt} \]

Therefore assuming \( M(t) > 0 \), we have \( \kappa_{CC} \leq T^t_R T^b_R \).

We will make use of the 2\( \sigma \) experimental bound for \( \kappa_{CC} \), \( |\kappa_{CC}| \leq 4 \times 10^{-3} \) [41], by saturating the strongest constraint \( T^t_R T^b_R \sim 4 \times 10^{-3} \). In this way we determine \( T^t_R = 0.35 \) and \( T^b_R = 0.01 \). With this choice the top mass value can be obtained, assuming \( R^{-1} \sim 1 \text{ TeV} \) and \( \kappa \sim 10 \), for both cases. Therefore the KK fermion excitations are approximately ten times heavier than the corresponding gauge excitations, thus enforcing our derivation of the effective theory by integrating out the bulk fermions. The value of \( T^b_R \) is consistent with the
experimental bounds on the corrections to the right-handed coupling $g_R^b$ of the bottom to $Z$. In our model $\delta g_R^b \sim (T_R^b)^2 \sim 10^{-4}$, whereas the experimental bound is $\delta g_R^b \leq 0.0219$. In a particular case of the BESS model considered in [44], similar conclusions are reached.

From our analysis it turns out that, in the large curvature limit, the warping does not substantially modify the discussion and the conclusions of the flat case once the global cancellation in $\epsilon_N^3$ is required.

VI. CONCLUSIONS

Five dimensional gauge models with flat or warped metric offers an attractive alternative to the problem of the electroweak breaking. A general feature of these scenarios is however, like for technicolor models, a large contribution to the $\epsilon_3$ parameter. In this paper we have investigated the possibility of a cancellation between the gauge contribution and the fermion one in the $\epsilon_3$ parameter once fermions are delocalized. This cancellation can be local (in the fifth dimension) or global. Starting from the deconstructed version of a five dimensional $SU(2)$ gauge theory with bulk fermions in a generic warped metric, we have studied the effects of the new physics in the low energy limit by eliminating the bulk fermions. In this way effective couplings of the standard model fermions to the bulk bosons and also mass terms for the fermions are generated. We have shown that the requirement of local cancellation in $\epsilon_N^3$ necessarily implies the vanishing of the fermion masses. This is due to the presence of one horizon on the right (IR) brane and it holds true in any generic warped metric. Finally we have considered the global cancellation both in the flat and in the warped case.

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