Effect of viscosity on one dimensional hydrodynamic flow and direct photons from 200 AGeV S+Au collisions at CERN SPS.

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We have analysed the direct photon data obtained by the WA80 collaboration in 200 A GeV S+Au collision at CERN SPS, in a one dimensional hydrodynamical model. Two scenario was considered: (i) formation of quark-gluon plasma and (ii) formation of hot hadronic gas. For both the scenario, ideal as well as extremely viscous fluid was considered. It was found that direct photon yield from QGP is not affected much whether the fluid is treated as ideal or extremely viscous. The yield however differ substantially if hadron gas is produced. Both the scenario do not give satisfactory description of the data.

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Recently WA80 collaboration has published their direct photon emission data for 200 A GeV S+Au collisions at CERN SPS [1]. The final version differ from their preliminary analysis [2] in several ways. The data was improved by increasing the detector coverage and also enlarging the data sample. The analysis technique was also improved upon. Most importantly, the data now gives the upper limit of the invariant yield for direct photon production at the 90% confidence limit, rather than the absolute yield. Much interest was aroused after the publication of the WA80 preliminary result of the direct photon data [3], as it was hoped that the direct photons can be a conclusive probe of the much debated quark-gluon plasma (QGP) expected to be produced in relativistic heavy ion collisions. The preliminary data were analysed by several authors. Xiong and Shuryak [4] analysed the data assuming a mixed phase formation and found excess photons. Srivastava and Sinha [5] analysed the data considering two possible scenarios after the collision, one with the phase transition to QGP, the other without it. It was claimed that the data were explained only in the phase transition scenario. We had also analysed the preliminary version of the WA80 direct photon data [6]. It was shown that formation of viscous hadron gas in the initial state, can explain the data.

In the present paper, we analyse the WA80 direct photon data, assuming formation of QGP or hot hadronic gas in the initial state. In cognizance of the fact that QGP or hot hadronic matter are not ideal, we consider two extreme cases of fluid flow, (i) ideal fluid flow and (ii) maximal viscous flow. Realistic fluid flow must be within this two extreme limits. Though, photon spectra thus calculated, will have uncertainties, as will be shown below, important conclusions can still be reached by comparing with experimental data.

Here we consider two possible scenarios that can arise after the collision: (i) the phase transition (PT) scenario, when the collision leads to formation of baryon free quark-gluon plasma (QGP) and (ii) the no phase transition scenario (NPT), when hot hadronic gas, again baryon free, is formed after the collision. In the PT scenario, it is assumed that after the collision, a baryon free QGP is formed at initial time $\tau_i$ and at temperature $T_i$. It expands and cools till the critical temperature $T_c$ is reached and the fluid enters into the mixed phase (we are assuming a 1st order phase transition). The matter remains in the mixed phase until all the QGP matter is converted adiabatically into the hadronic matter. The hadronic matter then cools further till the freeze-out temperature $T_F$ is reached at time $\tau_F$. In the NPT scenario it will be assumed that the matter is formed as (baryon free) hot hadronic gas at initial time $\tau_i$ at temperature $T_i$. It expands and cools till the freeze-out temperature $T_F$ is reached at $\tau_F$. In both the scenario, as mentioned earlier, we will consider the ideal and the maximal viscous fluid flow. We assume longitudinal boost-invariant hydrodynamic expansion of the system, without transverse expansion. We feel this to be justified at SPS energy as transverse expansion is small [6] and its effect on photon production is only marginal [4].

The space-time evolution of the fluid (QGP or hadronic gas) is governed by the energy-momentum conservation equation, which for a longitudinal similarity flow can be written as [7–9]:

$$\frac{d\varepsilon}{d\tau} = -(\varepsilon + p - \frac{4\eta}{3\tau} - \frac{\zeta}{\tau})/\tau$$

(1)

where, $\eta$ and $\zeta$ are the shear viscosity and the bulk viscosity coefficients. The other variables are having their usual meaning. In the ideal case, $\eta = \zeta = 0$. In the other extreme case with maximal viscous force, $(4\eta/3 + \zeta)/\tau = p$, as the total viscous force can not be larger than the pressure. For both the cases eqn. 1 can be solved analytically for a $p = 1/3\varepsilon$ type of equation of state. In the present work, for the QGP phase the bag equation of state $p_q = a_q T_q^4 - B$, with $a_q = 37\pi^2/90$ and for the hadronic phase the pion gas equation $p_h = a_h T^4$ with $a_h = 4.6\pi^2/90$ were used [10–12]. The mixed phase was described by the Maxwell construct $p_q(T_c) = p_h(T_c)$, which also gives the bag constant B.

For the ideal fluid, as the expansion is isentropic, the initial temperature $T_i$ of the fluid at an initial time $\tau_i$ can
be obtained by relating the entropy density with the observed pion multiplicity (assuming pion decoupling to be adiabatic) [10].

\[ T_i^{3} \tau_i = \frac{1}{\pi R_A^2} \frac{c}{4e\hbar} \frac{dN}{dy}(b = 0) \quad (2) \]

where \( c = 2\pi^4/45\zeta(3), \alpha_i = g_{a,b}\pi^2/90 \) and \( R_A \) is the transverse radius of the system. \( b = 0 \) corresponds to central collisions. However, for viscous flow, entropy is generated, the flow is no longer isentropic, the above equation can not be used. However, we can still assume that pion decoupling is adiabatic. Since bulk of the pions are produced at freeze-out, as argued in [1] we equate the final entropy density with the pion multiplicity.

\[ T_f^{3} \tau_f = \frac{1}{\pi R_A^2} \frac{c}{4e\hbar} \frac{dN}{dy}(b = 0) \quad (3) \]

to obtain the freeze-out time \( \tau_F \) for a given freeze-out temperature \( T_F \). The initial temperature of the fluid is then obtained from eqn.1 for a given \( \tau_i \). In both the PT and NPT scenarios, the initial time i.e. the proper time from which onward the hydrodynamical approach become applicable, is not known. It is presumed to be small and one generally uses the canonical value 1 fm. However, \( \tau_i \)'s for the QGP and the hadron gas need not be same. Elementary considerations suggests that \( \tau_i \) for the hadronic gases will be three times larger than that for the QGP [1]. In cognizance of the uncertainty in \( \tau_i \), we have used three different values: \( \tau_i=0.3,0.6 \) and 1 fm in the PT scenario and \( \tau_i=1,2 \) and 3 fm in the NPT scenario. In table 1, we have shown the initial temperature of the fluid in the PT and NPT scenario for a pion multiplicity \( dN/dy=225 \), which is appropriate for the central S+Au collisions [13].

In fig.1, we have shown the WA80 revised data (the solid bars) and direct photon yield in the PT scenario (the shaded region). The shaded region indicate the uncertainty in the direct photon production due to viscosity, the upper and lower limits of which were obtained by assuming ideal and maximal viscous flow respectively. The uncertainty in direct photon yield is not much, though the initial temperature of the ideal and the maximal viscous flow differ substantially. This can be understood. In the QGP scenario, both for the ideal and the viscous flow, the bulk of the photons are from the mixed and the hadron phase where the temperature variation is limited from 160-100 MeV. Then as viscosity is proportional to temperature, effect of viscosity is less in QGP. For all practical purposes, fluid can be considered ideal in the PT scenario. It is interesting to note that photon spectra are nearly identical whether \( \tau_i=0.3 \) fm or 1.0 fm, though the initial temperatures are substantially different. This indicates that contribution to direct photons from pure QGP sector is small compared to hadronic sector [4]. It is also evident that the PT scenario does not describe the WA80 data well. Insufficient number of large \( p_T \) photons are produced and the data are underpredicted by a factor of 10 or more at large \( p_T \).

In fig.2, we have shown the same results in the NPT scenario. Here again, the shaded region indicate the uncertainty due to viscosity. In contrast to the PT scenario, we find a substantial difference in the photon yield with \( \tau_i \). As mentioned earlier, in the PT scenario, initial QGP phase contribute little to the photon yield. On the otherhand, in the NPT scenario, contributions from the initial state are significant. Then as \( \tau_i \) is increased, photon yield at high \( p_T \) is reduced, as a result of reduced initial temperature. For the same reason, uncertainty in direct photon yield due to viscosity is substantial in this scenario. The uncertainty also grows with \( p_T \). This is also understood easily. Initially
temperatures of the ideal and extreme viscous fluid differ quite substantially, however, at later time this difference is reduced, ultimately, both the fluid freeze-out at the same temperature at the same time. Accordingly, uncertainty is large at high $p_T$ (early times) and lessened at low $p_T$ (later times). It is evident that if the collision leads to formation of ideal hadron gas, the data is not explained. More photons are produced than in experiment. For $\tau_i=1$ fm, even extreme viscosity can not lower the yield to agree with experiment. For $\tau_i=2$-3 fm, the large $p_T$ photons are within the shaded region, but the data are overpredicted in the intermediate (1-2 GeV) $p_T$ range. The NPT scenario also do not give satisfactory description of the WA80 direct photon data.

One of the deficiency of the present model is the neglect of pre-equilibrium photons emitted before $\tau_i$. Traxler and Thoma calculated pre-equilibrium photons and found them to be order of magnitude less than the equilibrium photons. Recently Roy et al also calculated pre-equilibrium photons. They used Fokker-Plank equations and found that pre-equilibrium photons are less or at best equal to the equilibrium photons. Even if the pre-equilibrium photons contribute as much as equilibrium photon, in the PT scenario, their inclusion will not raise the calculated yield sufficiently high to agree with experiment. In the NPT scenario, inclusion of pre-equilibrium photons can only worsen the discrepancy further. The present result that both the PT and NPT scenario do not give satisfactory description of the WA80 direct photon data will remain valid even if pre-equilibrium photons are included. Some unconventional scenario is then needed to explain the WA80 direct photon data. For example, in the PT scenario, as argued by Xiong and Shuryak, if the fluid expands slowly in the mixed phase, (very soft equation of state), the photon yield can be increased.

In conclusion, we have studied the effect of viscosity on direct photon production in PT and NPT scenario, currently in vogue. In the PT scenario, when QGP is assumed to be produced in the initial state, it was shown that the effect of viscosity on direct photon yield is minimum. Fluid in the PT scenario can be well approximated by a ideal fluid. This is not so in the NPT scenario. Viscosity can have a large effect there, and it is essential that one take into the account of viscosity in the NPT scenario. Though, the present calculations were done for direct photons, these conclusions are valid for other probes also. Viscosity coefficients of hot hadronic matter are then of primary importance, and efforts must be made to calculate them accurately. The present study also shows that the revised version of WA80 direct photon data could not be explained in the PT or the NPT scenario, irrespective of the viscosity.

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FIG. 1. The direct photon yield in the phase transition scenario for three different $\tau_i$’s. The solid bars are the WA80 direct photon data.

FIG. 2. The direct photon yield in the no phase transition scenario for three different $\tau_i$’s. The solid bars are the WA80 direct photon data.
TABLE I. The initial temperature of the ideal fluid $T_{i}^{\text{ideal}}$ and the extremely viscous fluid $T_{i}^{\text{viscous}}$ in the phase transition (QGP) and the no phase transition (Hadronic gas) scenario for different initial time $\tau_i$’s.

| $\tau_i$ (fm) | QGP $T_{i}^{\text{ideal}}$ (MeV) | QGP $T_{i}^{\text{viscous}}$ (MeV) | Hadronic Gas $T_{i}^{\text{ideal}}$ (MeV) | Hadronic Gas $T_{i}^{\text{viscous}}$ (MeV) |
|---------------|-------------------------------|-------------------------------|---------------------------------|---------------------------------|
| 0.3           | 304                           | 226                           | 1.0                             | 407                             |
| 0.6           | 241                           | 187                           | 2.0                             | 323                             |
| 1.0           | 203                           | 160*                          | 3.0                             | 283                             |
|               |                               |                               |                                 |                                 |

*For $\tau_i=1$fm, the multiplicity is not enough to produce the fluid in pure QGP state. We then assumed it to be formed in the mixed phase with proper fraction of QGP and hadronic fluids.
