Forecasting Daily Dynamic Hedge Ratios of Indian Index Future Contracts

Anuja Gupta, Manoj Jha, Namita Srivastava

Abstract: Forecasting plays a crucial role in determining the direction of future trends and in making necessary investment decisions. This research presents the forecasting performance of three multivariate GARCH models: SGARCH, EGARCH, and GJR-GARCH based on Gaussian and Student’s t-distribution. The forecasting ability of the models is evaluated on the basis of forecasting performance measures: MAE, SSE, MSE, and RMSE. This is done by examining the hedged portfolios of three indices of NSE: NIFTY50, BANKNIFTY, and NIFTYTAT. Daily data from Jan 2006 to Dec 2017 is taken and forecasts are conducted using out of sample data from Jan 2016-Dec 2017. Minimum mean square error (MMSE) forecasting method is used to generate conditional variance and covariance forecasts which in turn generate hedge ratios and corresponding hedged portfolio. Minimum variance hedge ratio framework of Ederington (1979) is used for hedging. The in-sample analysis shows that SGARCH with both the distribution performed better than the other models while out-of-sample analysis provides mixed results. EGARCH model assigns the lowest hedge ratio to NIFTY50 and BANKNIFTY while SGARCH model assigns the lowest hedge ratio to NIFTYTAT. Forecasting performance measures show the least value for SGARCH and EGARCH model. In future these models are able to reduce maximum risk from the spot market. The results of this research has important implications for financial decision and policy makers.

Index Terms: forecasting measures, hedge ratio, hedging effectiveness, multivariate GARCH models.

I. INTRODUCTION

Price discovery, hedging, gambling, and risk distribution are the main functions of future markets. Investors and hedgers use them to mitigate the risk of spot market by taking the equal and opposite position in future markets. Due to these different hedging theories have been proposed in the literature. The naïve hedging theory assumed an optimal hedge ratio of one for a spot position. But this theory failed to deliver due to the lack of coordination between spot and future price movements [1]. Working’s hedging theory (1953) is an improvement over naive hedging theory [2]. According to this theory, the hedger act as a speculator. This theory is based on the unrealistic assumption of maximizing returns by speculating on basis [1]. This theory is also criticized due to maximizing returns at any level of risk. Thereafter, portfolio approach to hedging given by Johnson (1960) and Stein (1961) came into being ([3],[4]). It assumed that the objective of hedger is neither minimization of risk nor maximization of returns. The main objective of hedger is the optimization of risk-return trade-off in the portfolio ([3], [4]). Ederington (1979) extended the work of Johnson and Stein and proposed the concept of MVHR and hedging effectiveness ([5], [6]). Portfolio hedging theory gained much importance as compared to other theories. It helps in estimating both static and dynamic hedge ratios [1].

Various studies estimated optimal hedge ratio from Ordinary Least Squares (OLS) method. Also hedging effectiveness has been measured using R-square. Hedge ratio obtained through OLS is criticized due to its biased nature. This biasedness occurs when the returns of cash and future markets are co-integrated. Another difficulty with this hedge ratio is the autocorrelation in OLS residuals. There is always present some heteroscedasticity in cash and future price [7]. Financial time series have fat tails, heteroscedasticity, leverage effects, autocorrelation and co-movements in volatilities. GARCH models can easily capture these features of financial time series [8]. Due to these difficulties GARCH models are proposed in the literature. Forecasting from any method depends on some reasonable assumptions. First, spot and future prices are properly correlated to obtain reliable hedge ratios. Second, time horizons should be as long as possible. Longer time horizon increases forecasting accuracy due to inclusion of more data points and provides more accurate results than shorter horizon. Third, the activities of competitors and decision makers also affect forecasting to a great extent [8]. Therefore, forecasting of hedge ratios is essential for traders and investors.

A. Research motivation

Though, there exists a plethora of research works in estimating hedge ratios and hedging effectiveness. There is a scarcity of literature in forecasting hedge ratio for Indian index future markets. Also, a very few research work available for estimating forecasting capacity and exactness of models [7]. This research tries to fill the above research gap. Forecasting enables an investor to understand the performance of future markets. It also assists portfolio managers in making ideal hedging and trading decisions [8]. It helps an investor in selecting and adjusting a suitable portfolio in dynamic hedging. This is done by investigating ex-post and ex-ante portfolio performance in spot and futures markets. On the basis of hedge ratios of portfolios, an investor select a portfolio with minimum risk.
B. Focus of the present study
This research investigates forecasting performance of SGARCH, EGARCH and GJR-GARCH models using daily data of CNXNIFTY50, BANKNIFTY, and NIFTYIT index futures. Forecasting is done using MMSE forecasting method in MATLAB software. To our knowledge these models and methods have never been considered for forecasting using the Indian index future contracts. MMSE is a versatile measure of forecasting which is easy to calculate and has not been considered before for forecasting hedge ratios. Forecasting performance is compared through MAE, MSE, SSE and RMSE which is of great help to investors and traders in selecting a particular hedging strategy.

C. Novelty of the proposed approach and research contribution
Forecasting done in this research offers many new dimensions when compared with the existing literature on forecasting using GARCH models. The data taken in this research is large enough to include all properties of time series data. The models used in this research are known to remove computational problems arising due to other GARCH models and the ordinary least square (OLS) model. Therefore, this research will provide substantial contribution to the literature and will help researchers and financial analysts in proper decision-making.

D. Organisation of the paper
Rest of the paper is structured as follows: next section contains the literature review. Section 3 describes some preliminaries and the models used in hedging. Section 4 describes numerical illustration with data description, methodology and results. The last section 5 concludes the paper.

II. LITERATURE REVIEW
Floros and Vougas evaluated out-of-sample hedging effectiveness of Greek stock index futures contracts using OLS, ECM, VECM and BGARCH (1,1) [9]. Hsu, Tseng, and Wang estimated hedge ratio and hedging effectiveness using copula-based GARCH models. Comparison is done with the conventional static, CCC-GARCH, and the DCC-GARCH models [10]. Bhaduri and Durai evaluated hedge ratios using OLS, VECM, VAR and DVEC-GARCH models [7]. Hedged and unhedged portfolios are compared using mean and variance for different hedging horizons. Kenourgios, Samitas, and Drosos forecasted hedge ratio and hedging effectiveness using OLS, ECM, GARCH, and EGARCH models [6]. Zhou and Wu forecasted hedge ratios and hedging effectiveness using OLS, naïve, and SGARCH hedging strategy [11]. Choudhry and Wu investigated forecasting ability of bivariate GARCH, BEKK GARCH, GJR-GARCH and Kalman filter [12]. Yang and Lai forecasted hedge ratios and hedging effectiveness using GJR-EC-GARCH and EC-OLS models [13]. Iqbal proposed an approach for forecasting conditional correlation and VaR using QMLE, LAD, and B-estimator of SGARCH and other multivariate GARCH models [14]. Ramasamy and Munisamy investigated forecasting ability of GARCH, GJR-GARCH, and EGARCH models [15]. Salvador and Arago estimated optimal hedge ratio using linear and non-linear GARCH models. Conclusion derived is that non-linear GARCH models provide better hedge ratios for in and out sample data [16]. Choudhary and Zhang forecasted hedge ratios using GARCH, BEKK GARCH, GARCH-X, BEKK-X, Q-GARCH, and GJR-GARCH. The estimation is done on the basis of normal and Student’s t-distribution [8]. Singh suggested an optimal hedge ratio by analyzing NIFTY, BANKNIFTY, and NIFTYIT using OLS, GARCH, EGARCH, TARCH, VAR, and VECM [1]. Lai forecasted hedge ratios using dynamic copula GARCH models [17].

From the literature review, it is concluded that majority of the investigations on index futures are associated with developed nations. There are a very few studies in the context of emerging nations like India. The results obtained through dynamic hedge ratios are more efficient than static hedge ratios [18]. In this research, the models used for forecasting and hedging are used in very few studies and the results obtained from them are more reliable. So an attempt is made to compare their hedging and forecasting performance by taking a sufficiently large hedging horizon.

III. PRELIMINARIES
The basic concepts used in this paper are briefly discussed in this section.

A. Hedge ratio
To hedge with future contracts, an investor should select an appropriate hedge ratio. It is determined by the number of future contracts required to reduce the risk of spot portfolio ([6]-[8]). Minimum value of hedge ratio should be taken into account. Higher hedge ratios require more future contracts which in turn demand higher investment. The hedge ratio which minimizes the variance of hedged portfolio is called optimal hedge ratio (OHR). It is also called the minimum variance hedge ratio (MVHR). Suppose the log difference of spot and future price generates spot and future portfolio returns. Let \( r_{s,t} \), \( r_{f,t} \) be the returns of spot and future portfolio at any time \( t \) based on the information up to time \( t-1 \).

According to Johnson (1960), Baillie, and Myers (1991) ([2], [19]), the minimum variance hedge ratio \( \beta ([6]-[8],[11],[16]) \) is given by:

\[
\beta = \frac{\text{cov}(r_{s,t},r_{f,t})}{\text{var}(r_{f,t})} \tag{1}
\]

Where numerator in (1) is the covariance between spot and future portfolio. Denominator is the variance of future portfolio.

The return \( r_h \) of hedged portfolio ([6]-[8]) can be calculated as:

\[
r_h = r_{s,t} - \beta r_{f,t} \tag{2}
\]

And the variance of the hedged portfolio ([6]-[8]) can be calculated as:

\[
\text{Var}(H) = \text{Var}(r_{s,t}) + \beta^2 \text{Var}(r_{f,t}) - 2\beta \text{cov}(r_{s,t},r_{f,t}) \tag{3}
\]

B. Hedging effectiveness
The performance and usefulness of hedging strategies can be examined by finding the hedging effectiveness of each strategy. The concept was given by Ederington in 1979.
It may be defined as the percentage decline in the variance of hedged portfolio relative to the unhedged portfolio [5]. Hedging effectiveness ([6-8]) is given by

$$ HE = \frac{\text{Var}(U) - \text{Var}(H)}{\text{Var}(U)} \quad (4) $$

Where \( \text{Var}(U) \) = variance of unhedged portfolio i.e., variance of spot portfolio
\( \text{Var}(H) \) = variance of hedged portfolio given by (3).

C. Models used in hedging

In this research, the data taken is large enough to cover all properties of time series data. According to central limit theorem large sample sizes tend towards normal distribution [20].

The residual term \( (e_t) \) in the mean equation of GARCH models is supposed to be conditionally normal. It has mean 0 and variance \( \sigma^2 \). For large sample sizes financial time series tends to be non-normal. It has leptokurtic fatter tail distribution. Non-normality, skewness and kurtosis are mostly found in the returns of financial time series. This makes the conditional normal distribution of the error term unfit with the observations. In this case Student’s t-distribution is more advantageous than normal distribution. This distribution is similar to normal distribution for large degrees of freedom. In this research, both the normal (Gaussian) and Student t-distribution are used for forecasting hedge ratios.

1. Multivariate GARCH Models

To model ARCH terms in return series, GARCH models are proposed in the literature. These models are applied to the residual of the return series. Let the daily spot and future prices are denoted by \( P_t \), \( t=1, 2, \ldots, T \) and their logarithmic returns at time \( t \) be

$$ r_t = \frac{\ln P_t - \ln P_{t-1}}{t} \quad [7] $$

Model 1: SGARCH (Simplified multivariate Generalised Auto Regressive Conditional Heteroscedastic) model: This model is given by Harris, Stoja and Tucker in 2007 [21]. In this model, conditional variance is estimated using GARCH (1, 1) model. Conditional variance of spot and future return series is estimated. Also the same is estimated for the sum and difference of both series. The covariance is estimated using GARCH models. In this model, the conditional variance is given by

$$ \sigma_{s,t}^2 = \alpha + \beta \sigma_{s,t-1}^2 + \gamma e_{s,t-1}^2 $$

$$ \sigma_{f,t}^2 = \alpha + \beta \sigma_{f,t-1}^2 + \gamma e_{f,t-1}^2 $$

$$ \sigma_{s,t}^2 = \alpha + \beta \sigma_{s,t-1}^2 + \gamma e_{s,t-1}^2 $$

Where \( \sigma_{s,t}^2 \) and \( \sigma_{f,t}^2 \) are the conditional variances of the spot and future returns, \( e_{s,t} \) and \( e_{f,t} \) are the residuals of the spot and future returns, \( \alpha, \beta, \gamma \) are the parameters of the model.

Model 2: EGARCH (Exponential GARCH): A modified GARCH model. Different impacts on conditional variance can be modeled through positive and negative innovations to returns GJR-GARCH approach. Asymmetric effect can be modeled by adding a dummy variable in the GARCH (1, 1) model [22]. EGARCH model ([15],[22]) is represented by the expression:

$$ \sigma_{s,t}^2 = \alpha + \beta \sigma_{s,t-1}^2 + \gamma e_{s,t-1}^2 + \epsilon_{s,t-1}^2 $$

$$ \sigma_{f,t}^2 = \alpha + \beta \sigma_{f,t-1}^2 + \gamma e_{f,t-1}^2 + \epsilon_{f,t-1}^2 $$

$$ \sigma_{s,f,t}^2 = \alpha + \beta \sigma_{s,f,t-1}^2 + \gamma (e_{s,t-1} e_{f,t-1}) $$

Where \( \epsilon_{s,t} \) and \( \epsilon_{f,t} \) are the innovations in the spot and future returns.

Model 3: GJR GARCH (Glosten-Jagannathan-Runkle GARCH): Apart from non-normality and leptokurtosis, one more feature exists in the return series called the ‘leverage effect’. It is defined as the equity’s volatility negatively correlated with equity’s returns. GARCH models are based on square of the past observations and past variances. These observations and variances estimate current variance. Thus the sign of return does not affect volatilities. Glosten, Jagannathan and Runkle (1993) introduced a modified GARCH model. Different impacts on conditional variance can be modeled through positive and negative innovations to returns GJR-GARCH approach. Asymmetric effect can be modeled by adding a dummy variable in the GARCH (1, 1) model [22]. GJR-GARCH model ([15],[22]) is represented by the expression:

$$ \sigma_{s,t}^2 = \alpha + \beta \sigma_{s,t-1}^2 + \gamma e_{s,t-1}^2 $$

$$ \sigma_{f,t}^2 = \alpha + \beta \sigma_{f,t-1}^2 + \gamma e_{f,t-1}^2 $$

$$ \sigma_{s,f,t}^2 = \alpha + \beta \sigma_{s,f,t-1}^2 + \gamma (e_{s,t-1} e_{f,t-1}) $$

Where \( \epsilon_{s,t} \) and \( \epsilon_{f,t} \) are the innovations in the spot and future returns.
The exponential nature also indicates that external shocks strongly influence the forecasted volatility. Here $\xi > 0$ indicates the asymmetric effect and $\xi < 0$ shows the presence of ‘leverage effect’. Parameters of above GARCH models are estimated using maximum likelihood estimation technique in MATLAB software. Both the Guassian and Student t-distribution (with 118 degrees of freedom) are used for analysis.

D. MMSE (Minimum Mean Square Error) forecasting method

If $\sigma^2_t, \sigma^2_{t+1}, ..., \sigma^2_{t+h}$ are the given conditional variances and $h$ is the forecast horizon, then this method generate the predictions for $\hat{\sigma}^2_{t+1}, \hat{\sigma}^2_{t+2}, ..., \hat{\sigma}^2_{t+h}$

Suppose at time $t=1$, $\hat{\sigma}^2_{t+1}$ be the forecasted conditional variance based on the history up to time $t$. The conditional expected square loss function is defined as

$$E[(\sigma^2_{t+1} - \hat{\sigma}^2_{t+1}) | H_t]$$

(12)

Minimizing this loss function yields the MMSE forecast [25]

$$\hat{\sigma}^2_{t+1} = E(\sigma^2_{t+1} | H_t) = E(\hat{\sigma}^2_{t+1} | H_t)$$

(13)

Where $H_t$ is the information set up to time $t$.

The forecasted conditional variance and covariance are then used to estimate hedge ratios using (9). The corresponding hedged portfolios are constructed using (2). Forecasting accuracy of GARCH models is examined on the basis of performance measures MAE, SSE, MSE and RMSE. If $y_t$ is the actual value of an observation and $y'_t$ is the forecasted value of that observation then the error

$$e_t = y_t - y'_t$$

(14)

If the length of out-of-sample data is $n$, then different forecasting measures based on the value of $e_t$ can be defined as:

(a) **MAE (mean absolute error)**

$$\text{MAE} = \frac{1}{n} \sum_{t=1}^{n} |e_t|$$

(b) **MSE (mean square error)**

$$\text{MSE} = \frac{1}{n} \sum_{t=1}^{n} e^2_t$$

(c) **SSE (sum of the square error)**

$$\text{SSE} = \sum_{t=1}^{n} e^2_t$$

(d) **RMSE (Root Mean Square Error)**

$$\text{RMSE} = \sqrt{\text{MSE}}$$

(15)-(18)

The value of these statistics should be as small as possible for a better forecast ([24], [25]).

IV. NUMERICAL ILLUSTRATION

To examine the efficiency of the forecasting models described in section 3, three main indices of NSE are selected.

A. Data description and methodology

In this research data of NIFTY50, BANKNIFTY, and NIFTYIT is used to estimate dynamic hedge ratios. Daily closing spot and future price of NIFTY50, BANKNIFTY and NIFTYIT is downloaded from nseindia.com from 1 Jan 2006 to 31 Dec 2017. The data is divided into two parts. Training data set from 1-Jan-2006 to 31-Dec-2015 for estimating the parameters of models. Testing data set from 1-Jan-2016 to 31-Dec-2017 for assessing the predictive performance of models. A total of 7441 observations for NIFTY50, 6418 for BANKNIFTY and 3910 for NIFTYIT index are used for estimating the models. All futures price indices are continuous series. (Fig 1-3, where x axis represents time and y-axis price).
In hedging, investments of cash and futures market are combined to form a new investment [11]. Also a hedger takes offsetting position in cash and futures market. Hedge ratio and hedging effectiveness are estimated statistically. Before doing any calculations, statistical properties of a time series under study should be considered. To test the stationarity of the data, ADF (Augmented Dickey Fuller) test is conducted in EViews software (Table 1-3). At a certain significance level, the null hypothesis for ADF test is the variables contain a unit root or are non-stationary. The daily spot and future price series is shown in Fig 1-3(a) and stationary return series are shown in Fig 1-3 (b). After that monthly average of daily closing spot and future price is calculated. The reason behind this is that stock prices fluctuate daily due to supply and demand, market psychological effect and individual investor needs. Also just one day does not give an accurate view of the stock’s price movement for investment decision. Spot and future returns are calculated using (5). Descriptive statistics of spot and future returns of the indices (table 4) show excess kurtosis. It indicates the presence of fat tails and high peak values. These values also indicates non-normality in data. In table 4, the value of skewness is negative. It shows the existence of longer tail on the left-hand side for all price series.

### Table 1: Stationarity test of NIFTY50 index

| Null Hypothesis: SPOTPRICESERIES has a unit root | Exogenous: Constant | Lag Length: 3 (Automatic - based on SIC, maxlag=35) |
|--------------------------------------------------|---------------------|--------------------------------------------------|
| Augmented Dickey-Fuller test statistic           | t-Statistic         | Prob. |
| Test critical values:                            | -1.261192           | 0.6497 |
| 1% level                                         | -3.431055           |       |
| 5% level                                         | -2.566916           |       |
| 10% level                                        | 0.00834             | 0.0084 |
| 5% level                                         | 0.18146             | 0.1825 |
| 10% level                                        | 0.0000              | 0.0000 |

*MacKinnon (1996) one-sided p-values.

### Table 2: Stationarity test of BANKNIFTY index

| Null Hypothesis: SPOTPRICESERIES has a unit root | Exogenous: Constant | Lag Length: 0 (Automatic - based on SIC, maxlag=30) |
|--------------------------------------------------|---------------------|--------------------------------------------------|
| Augmented Dickey-Fuller test statistic           | t-Statistic         | Prob. |
| Test critical values:                            | -1.078047           | 0.7266 |
| 1% level                                         | -3.431656           |       |
| 5% level                                         | -2.862002           |       |
| 10% level                                        | -2.567059           |       |

*MacKinnon (1996) one-sided p-values.

### Table 3: Stationarity test of NIFTYIT index

| Null Hypothesis: SPOTPRICESERIES has a unit root | Exogenous: Constant | Lag Length: 3 (Automatic - based on SIC, maxlag=30) |
|--------------------------------------------------|---------------------|--------------------------------------------------|
| Augmented Dickey-Fuller test statistic           | t-Statistic         | Prob. |
| Test critical values:                            | -0.401251           | 0.9087 |
| 1% level                                         | -3.431656           |       |
| 5% level                                         | -2.862002           |       |
| 10% level                                        | -2.567103           |       |

*MacKinnon (1996) one-sided p-values.

### Table 4: Basic statistics for spot and future price of NIFTY50, BANKNIFTY and NIFTYIT

| NIFTY50 | BANKNIFTY | NIFTYIT |
|---------|-----------|---------|
| Spot returns | Futures returns | Spot returns | Futures returns | Spot returns | Futures returns |
| Mean | 0.00034 | 0.00845 | 0.01092 | 0.01093 | 0.0084 | 0.0084 |
| Median | 0.01885 | 0.01866 | 0.00978 | 0.00762 | 0.0179 | 0.0138 |
| Std. Deviation | 0.0605 | 0.06191 | 0.08237 | 0.08325 | 0.0675 | 0.0670 |
| Kurtosis | 3.99344 | 3.54815 | 4.20782 | 4.15425 | 4.14798 | 4.29127 |
| Skewness | -0.85057 | -0.77476 | -0.2685 | -0.22262 | -1.0265 | -1.06344 |
| Minimum | -0.27033 | -0.26776 | -0.24001 | -0.23966 | -0.32242 | -0.32674 |
| Maximum | 0.38146 | 0.18809 | 0.26485 | 0.26222 | 0.15917 | 0.15732 |
| Sum sq. dev. | 0.43192 | 0.45221 | 0.80055 | 0.81779 | 0.53757 | 0.5404 |
| Count | 119 | 119 | 119 | 119 | 119 | 119 |
| Jarque-Bera | 84.74522 | 67.27746 | 8.91919 | 7.26292 | 98.7544 | 103.728 |
| Probability | 0.0000 | 0.0000 | 0.01758 | 0.02648 | 0.0000 | 0.0000 |

The Jarque-Bera (JB) test statistics shows the rejection of null hypothesis that the financial time series taken in this research is normal. This is because of large values or high heteroscedasticity in the sample. ARCH or heteroscedasticity test in performed in EVIEWs software using ARCH -LM test (table 5-7). The null hypothesis for ARCH test is that there are no ARCH effects present. For hedging, monthly hedging horizon is chosen and forecasting is done using MMSE forecasting technique for two years.
Table 5: NIFTY50 ARCH test

| parameter | SGARCH | EGARCH | GJR-GARCH |
|-----------|--------|--------|-----------|
| constant | | | |
| a1 | 0.0006 | 0.0006 | 0.0006 |
| a2 | 0.0006 | 0.0006 | 0.0006 |
| a3 | 0.0002 | 0.0002 | 0.0002 |
| a4 | 0 | 0 | 0 |
| ARCH | | | |
| g1 | 0.1484 | 0.1362 | 0.1362 |
| g2 | 0.1421 | 0.1320 | 0.1320 |
| g3 | 0.1449 | 0.1335 | 0.1335 |
| g4 | 0.1609 | 0.1651 | 0.1651 |
| GARCH | | | |
| g1 | 0.8378 | 0.8452 | 0.8452 |
| g2 | 0.8440 | 0.8513 | 0.8513 |
| g3 | 0.8415 | 0.8461 | 0.8461 |
| g4 | 0.8318 | 0.8764 | 0.8764 |

Table 6: BANKNIFTY ARCH test

| parameter | SGARCH | EGARCH | GJR-GARCH |
|-----------|--------|--------|-----------|
| constant | | | |
| a1 | 0.0006 | 0.0006 | 0.0006 |
| a2 | 0.0006 | 0.0006 | 0.0006 |
| a3 | 0.0002 | 0.0002 | 0.0002 |
| a4 | 0 | 0 | 0 |
| ARCH | | | |
| g1 | 0.1484 | 0.1362 | 0.1362 |
| g2 | 0.1421 | 0.1320 | 0.1320 |
| g3 | 0.1449 | 0.1335 | 0.1335 |
| g4 | 0.1609 | 0.1651 | 0.1651 |
| GARCH | | | |
| g1 | 0.8378 | 0.8452 | 0.8452 |
| g2 | 0.8440 | 0.8513 | 0.8513 |
| g3 | 0.8415 | 0.8461 | 0.8461 |
| g4 | 0.8318 | 0.8764 | 0.8764 |

Parameters of GARCH models discussed in section 3 are estimated using maximum likelihood estimation in MATLAB software (table 8-10). After estimating parameters, forecasting is done using MMSE forecasting method discussed in section 3 to obtain hedge ratio and hedging effectiveness of in and out of sample data of the indices (table 11-14). Effectiveness of hedge ratios is judged using mean and variance of hedged and unhedged portfolios.

Table 7: NIFTYIT ARCH test

| parameter | SGARCH | EGARCH | GJR-GARCH |
|-----------|--------|--------|-----------|
| constant | | | |
| a1 | 0.0006 | 0.0006 | 0.0006 |
| a2 | 0.0006 | 0.0006 | 0.0006 |
| a3 | 0.0002 | 0.0002 | 0.0002 |
| a4 | 0 | 0 | 0 |
| ARCH | | | |
| g1 | 0.1484 | 0.1362 | 0.1362 |
| g2 | 0.1421 | 0.1320 | 0.1320 |
| g3 | 0.1449 | 0.1335 | 0.1335 |
| g4 | 0.1609 | 0.1651 | 0.1651 |
| GARCH | | | |
| g1 | 0.8378 | 0.8452 | 0.8452 |
| g2 | 0.8440 | 0.8513 | 0.8513 |
| g3 | 0.8415 | 0.8461 | 0.8461 |
| g4 | 0.8318 | 0.8764 | 0.8764 |

Table 8: Parameter estimation of the three GARCH models for the index NIFTY50. The values in parenthesis denote the corresponding standard error.
Table 10: Parameter estimation results of the three GARCH models for the index NIFTYIT. The values in parenthesis denote

| parameter            | SGARCH | EGARCH | GJR-GARCH |
|----------------------|--------|--------|-----------|
|                      | Gaussian | t-dist. | Gaussian | t-dist. | Gaussian | t-dist. |
| Constant (α1)        | 0.00036 | (0.00046) | 0.00036 | (0.00045) | 0.00022 | (0.00025) |
|                       |         |        | -0.00856 | (0.01133) | 0.00020 | (0.00030) |
| α2                   | 0.00033 | (0.00044) | 0.00034 | (0.00046) | -0.00668 | (0.0050) |
|                       |         |        | -0.00882 | (0.01712) | 0.00211 | (0.00025) |
| α3                   | 0.00138 | (0.00177) | 0.00140 | (0.00180) | 0.00000 | (0.00000) |
|                       |         |        | 0.00000 | (0.00000) | 0.00000 | (0.00000) |
| α4                   | 0         | 0        | 0         | 0        | 0         | 0        |
| ARCH term (θ1)       | 0.151932 | (0.12761) | 0.14516 | (0.12580) | -0.16206 | (0.12521) |
|                       |         |        | -0.17392 | (0.12521) |         |         |
| λ2                   | 0.14663 | (0.12849) | 0.13890 | (0.12519) | -0.17531 | (0.05449) |
|                       |         |        | -0.12882 | (0.09647) |         |         |
| λ3                   | 0.14944 | (0.12723) | 0.14218 | (0.12564) | 0.13574 | (0.04848) |
|                       |         |        | -0.13574 | (0.04848) |         |         |
| λ4                   | 0.18254 | (0.13408) | 0.20800 | (0.14929) |         |         |
| GARCH term (ω1)      | 0.77283 | (0.20389) | 0.77319 | (0.20792) | 0.98551 | (0.00066) |
|                       |         |        | 0.98595 | (0.01015) | 0.82666 | (0.15942) |
|                       |         |        | 0.85605 | (0.15676) |         |         |
| α2                   | 0.78344 | (0.20276) | 0.78736 | (0.20870) | 0.98783 | (0.00833) |
|                       |         |        | 0.99780 | (0.01612) | 0.82875 | (0.16190) |
|                       |         |        | 0.86172 | (0.16170) |         |         |
| α3                   | 0.77799 | (0.20346) | 0.77833 | (0.20840) | 0.86609 | (0.14155) |
|                       |         |        | 0.63639 | (0.06332) | 0.68609 | (0.14135) |
|                       |         |        | 0.63639 | (0.06332) |         |         |
| α4                   | 0.66028 | (0.13172) | 0.65821 | (0.13961) |         |         |
| Leverage term        | --      | --      | -0.19130 | (0.04372) | -0.20105 | (0.05402) |
|                       |         |        | 0.23414 | (0.10781) | 0.16996 | (0.12832) |
|                      |         |        |         |         |         |         |

The values in parenthesis denote the corresponding standard error.

Table 11: In sample analysis of NIFTY50, BANKNIFTY and NIFTYIT using GARCH models with Gaussian distribution.

| Table 11: In sample analysis of NIFTY50, BANKNIFTY and NIFTYIT using GARCH models with Gaussian distribution. |
|---------------------------------------------------------------|
| In sample analysis (Gaussian distribution)                    |
| SGARCH | EGARCH | GJR-GARCH |
|---|---|---|
| Return (unhedged) | 0.00884 | 0.00884 | 0.00884 |
| Return (hedged) | 0.00007 | -0.00006 | -0.00002 |
| variance (unhedged) | 0.00363 | 0.00366 | 0.00366 |
| variance (hedged) | 0.00007 | 0.00012 | 0.00008 |
| hedge ratio | 0.97811 | 1.08983 | 1.02170 |
| hedging effectiveness | 99% | 97% | 98% |
| BANKNIFTY Return (unhedged) | 0.01092 | 0.01092 | 0.01092 |
| Return (hedged) | 0.00012 | -0.00175 | -0.00110 |
| variance (unhedged) | 0.00678 | 0.00678 | 0.00678 |
| variance (hedged) | 0.00012 | 0.00003 | 0.00022 |
| hedge ratio | 0.99890 | 1.16180 | 1.09799 |
| hedging effectiveness | 99% | 97% | 97% |
| NIFTY50 Return (unhedged) | 0.00864 | 0.00884 | 0.00884 |
| Return (hedged) | 0.00005 | -0.00009 | -0.00009 |
| variance (unhedged) | 0.00456 | 0.00456 | 0.00456 |
| variance (hedged) | 0.00008 | 0.00017 | 0.00015 |
| hedge ratio | 0.99277 | 1.59428 | 1.10641 |
| hedging effectiveness | 99% | 97% | 97% |
| BANKNIFTY Return (unhedged) | 0.00892 | 0.00884 | 0.00884 |
| Return (hedged) | 0.00005 | -0.00009 | -0.00009 |
| variance (unhedged) | 0.00456 | 0.00456 | 0.00456 |
| variance (hedged) | 0.00008 | 0.00017 | 0.00015 |
| hedge ratio | 0.99277 | 1.59428 | 1.10641 |
| hedging effectiveness | 99% | 97% | 97% |
| NIFTY50 Return (unhedged) | 0.00864 | 0.00884 | 0.00884 |
| Return (hedged) | 0.00005 | -0.00009 | -0.00009 |
| variance (unhedged) | 0.00456 | 0.00456 | 0.00456 |
| variance (hedged) | 0.00008 | 0.00017 | 0.00015 |
| hedge ratio | 0.99277 | 1.59428 | 1.10641 |
| hedging effectiveness | 99% | 97% | 97% |

Table 12: Out of sample analysis of NIFTY50, BANKNIFTY and NIFTYIT using GARCH models with Gaussian distribution.

| Table 12: Out of sample analysis of NIFTY50, BANKNIFTY and NIFTYIT using GARCH models with Gaussian distribution. |
|---------------------------------------------------------------|
| Out sample analysis (Gaussian distribution)                    |
| SGARCH | EGARCH | GJR-GARCH |
|---|---|---|
| NIFTY50 Return (unhedged) | 0.01166 | 0.01166 | 0.01166 |
| Return (hedged) | -0.00007 | 0.00553 | 0.00558 |
| variance (unhedged) | 0.00409 | 0.00409 | 0.00409 |
| variance (hedged) | 0.00012 | 0.00123 | 0.00399 |
| hedge ratio | 1.01025 | 0.54351 | 0.95435 |
| hedging effectiveness | 93% | 79% | 92% |
| BANKNIFTY Return (unhedged) | 0.00712 | 0.00712 | 0.00712 |
| Return (hedged) | -0.00008 | 0.00372 | 0.00381 |

Forecasting accuracy between the observed and the forecasted value is evaluated using forecasting performance measures discussed in section 3 (table 15). The best forecasting model is one which has the smallest value of these forecasting measures.

Table 15: Forecast accuracy test results NIFTY50 and BANKNIFTY
B. Results and discussion

Results from the proposed methodology are quite standard. Guassian and Student’s t-distribution are used for estimating GARCH models. All future price indices are continuous settlement prices. Returns are the log difference of spot (cash) and future price (fig 1-3). Results from ADF unit root test are given in Table 1-3. The null hypothesis of having unit root in cash and future price series is rejected. The stationary cash and future price series enables one to predict the returns significantly. Descriptive statistics of spot and future portfolio is given in table 4. Results from JB test statistics indicates non-normality in return series. Due to non-normality, returns are not equally distributed among buyers and sellers [1]. ARCH-LM test results indicates significant ARCH effects in the data (table 5-7).

The estimated parameters of the proposed models using both the distributions are provided in table 8-10. All the three models show significant ARCH and GARCH effects. EGARCH and GJR-GARCH models express considerable leverage effects. MMSE forecasting method is applied in MATLAB software to forecast conditional variances and covariance. This in turn generate hedge ratios and hedging effectiveness using (4) and (7) (table 11-14). The hedged portfolio is thus formed on the basis of forecasted hedge ratios. The mean and variance of hedged portfolio is estimated using (2) and (3). An in-sample analysis helps an investor to have an idea what the model performed in the past. An out-of-sample analysis assists in evaluating the predictive performance of strategies. Comparison between the forecasted and the actual returns thus provide relative accuracy of dynamic hedge ratios.

The results of forecasting performance measures for the GARCH models is given in table 15. For NIFTY50 and BANKNIFTY, EGARCH with both distributions has the least value for all the performance measures. While for NIFTYIT, SGARCH model has the best forecasting performance for NIFTY50 and BANKNIFTY using EGARCH model SGARCH model has the best forecasting performance for NIFTYIT. SGARCH model gives the minimum value of hedge ratio for NIFTYIT.

This research will help the financial decision makers in making proper investment decisions. The proposed methodology will help the investors to understand the role of future markets in trading and fund management. This will enable them to select the appropriate portfolio and make necessary adjustments in it.

C. Comparative analysis

Hedging effectiveness is an important component of hedging which helps in deciding a proper hedging strategy. Higher the value of hedging effectiveness, more effective will be the strategy. The proposed methodology provides better results as compared to the results of Bhaduri and Durai [7] and Singh [1]. The high value of hedging effectiveness in this research indicates removal of maximum risk of spot portfolio. The results will be beneficial for the market players to select the best hedging strategy.

V. CONCLUSION

This research attempts to estimate and forecast hedge ratios of NIFTY50, BANKNIFTY and NIFTYIT index futures. Proposed methodology is conducted using SGARCH, EGARCH and GJR-GARCH model with Guassian and Student t-distribution. The stationarity of cash and future market returns indicates proper distribution of information in both markets and so returns can be predicted significantly. Forecasting is conducted by means of MMSE forecasting method taking two year forecasting horizon. Performance measures are applied to test the forecasting ability of GARCH models. The outcome of performance measures illustrates mixed results. EGARCH model dominates other models for NIFTY50 and BANKNIFTY. Value of hedge ratio is the least for NIFTY50 and BANKNIFTY using EGARCH model SGARCH model has the best forecasting performance for NIFTYIT. SGARCH model gives the minimum value of hedge ratio for NIFTYIT.

This research will help the financial decision makers in making proper investment decisions. The proposed methodology will help the investors to understand the role of future markets in trading and fund management. This will enable them to select the appropriate portfolio and make necessary adjustments in it.

The proposed methodology with Student’s t-distribution offers better forecasting performance than that of Guassian distribution. There is no particular model to provide better forecasting performance for all index futures. Different models have different forecasting performance due to various distributions and set of data points. This research can be extended to include more complex dynamics of financial markets using various GARCH models and distributions.

REFERENCES

1. Singh, G., 2017. Estimating Optimal Hedge Ratio and Hedging Effectiveness in the NSE Index Futures. Jindal Journal of Business Research, 6(2), pp.108-131.
2. Working, H., 1976. Futures trading and hedging. In The Economics of Futures Trading (pp. 68-82). Palgrave Macmillan, London.
3. Johnson, L.L., 1960. The theory of hedging and speculation in commodity futures. The Review of Economic Studies, 27(3), pp.139-151.
4. Stein J.L., 1961. The simultaneous determination of spot and futures prices. American Economic Review, 51, pp.1012-1025.
5. Ederington, L. H., 1979. The hedging performance of the new futures markets. The Journal of Finance, XXXIV (1), 157-170.
6. Kenourgios, D., Samitas, A. and Drossos, P., 2008. Hedge ratio estimation and hedging effectiveness: the case of the S&P 500 stock index futures contract. Int. J. Risk Assessment and Management, Vol. 9.
7. Bhaduri, S.N. and Sethu Durai, S.R., 2008. Optimal hedge ratio and hedging effectiveness of stock index futures: evidence from India. Macroeconomics and Finance in Emerging Market Economies, 1(1), pp.121-134.
8. Zhang, Y. and Choudhry, T., 2015. Forecasting the daily dynamic hedge ratios by GARCH models: evidence from the agricultural futures markets. The European Journal of Finance, 21(4), pp.376-399.
9. Floros, C. and Vongas, D., 2006. Hedging effectiveness in Greek stock index futures market, 1990-2001. International Research Journal of Finance and Economics, 5, pp.7-18.
10. Hsu, C.C., Tseng, C.P. and Wang, Y.H., 2008. Dynamic hedging with futures: A copula-based GARCH model. Journal of Futures Markets: Futures, Options, and Other Derivative Products, 28(11), pp.1095-1116.
11. Zhou and Wu, 2009. Research on Real Futures Dynamic Hedging Based on S-GARCH Model. seeexploree
12. Choudhry, T. and Wu, H., 2009. Forecasting the weekly time-varying beta of UK firms: GARCH models vs. Kalman filter method. The European Journal of Finance, 15(4), pp.437-444.
13. Yang, M.J. and Lai, Y.C., 2009. An out-of-sample comparative analysis of hedging performance of stock index futures: dynamic versus static hedging. Applied Financial Economics, 19(13), pp.1059-1072
14. Farhat Igbal, 2013. Robust estimation of the simplified multivariate GARCH modelEmpirical Economics, Springer
15. Ramasamy, R. and Munisamy, S., 2012. Predictive accuracy of GARCH, GJR and EGARCH models select exchange rates application. Global Journal of Management and Business Research, 12(15).
16. Salvador, E. and Aragó, V., 2014. Measuring Hedging Effectiveness of Index Futures Contracts: Do Dynamic Models Outperform Static Models? A Regime-Switching Approach. Journal of Futures Markets, 34(4), pp.374-398.
17. Lai, Y.S., 2018. Dynamic hedging with futures: a copula-based GARCH model with high-frequency data. Review of Derivatives Research, pp.1-23.
18. Baille, R., and R. Myers. 1991. Bivariate GARCH Estimates of the Optimal Commodity Futures Hedge. Journal of Applied Econometrics 6 (2): 109–124
19. Bollerslev, T., R. Chou, and K. Kroner. 1992. “ARCH Modeling in Finance.” Journal of Econometrics 52 (1–2): 5–59.
20. Gupta, S.C. and Kapoor, V.K., 2019. Fundamentals of applied statistics. Suthan Chand & Sons.
21. R. D. F. Harris, E. Stoya, J. Tucker, 2007, A simplified approach to modelling the comovement of asset returns. Journal of Future Market 27:575–598.
22. Glosten, L., R. Jagannathan, and D. E. Runkle 1993. “On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks.” Journal of Finance 48 (5): 1779–1801.
23. Nelson, D.B., 1991, Conditional Heteroskedasticity in Asset Returns: A New Approach, Econometrica,Vol. 59, pp 347-370
24. Armstrong, J.S., 2001. Evaluating forecasting methods. In Principles of forecasting (pp. 443-472). Springer, Boston, MA.
25. Ma, J., Zhang, J. and Zhu, L., 2018. Study of the bullwhip effect under various forecasting methods in electronics supply chain with dual retailers considering market share. Complexity, 2018.

AUTHORS PROFILE

Ms. Anuja Gupta has done B.Sc. in 2006 (Barkatullah University, Bhopal), M.Sc. (Mathematics) in 2008 (Barkatullah University, Bhopal). Her research area includes Financial Mathematics and Portfolio Optimization. She has more than 7 years of teaching experience and is working as a Research Scholar in the Department of Mathematics and Computer Applications at Maulana Azad National Institute of Technology, Bhopal-462051 (MP) India.

Dr. Manoj Jha has done M.Sc. and M.Phil. in Mathematics. He has qualified State Level Eligibility Test (SLET). He has done his Ph.D from National Institute of Technology, Bhopal. His research area includes Financial Mathematics and Portfolio Optimization. He is having 20 years of teaching experience and is working as an Assistant Professor in the Department of Mathematics at Maulana Azad National Institute of Technology Bhopal, India. He has published more than 10 papers in National and International journals. He is a member of ISTE.

Dr. Namita Srivastava has done M.Sc. in Mathematics. She has done Ph.D in Financial Mathematics. Her research area includes Financial Mathematics and Portfolio Optimization. She is having 24 years of teaching experience and is working as a Professor and Dean FW in Maulana Azad National Institute of Technology. She has published 21 papers in International journal, 12 papers in National journals, 11 papers in National level conference, and 20 papers in International level conference. She is a member of ISTE and International Academy of Science Allahabad and Gwalior Academy of Mathematical Science. She guided more than 10 Ph.D scholars.