Analyzing the upper secondary school students’
difficulties in the rotation of rigid body

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Abstract. Previous studies have shown that students have difficulties in understanding some concepts related to circular motion of particles. However, there are few studies on rigid body rotational dynamics. In this study, we have studied difficulties that Upper secondary school students have about the rotation of a rigid body. We have designed an 8 questions open-ended questionnaire. In designing the questions, we took into account the key concepts that students have to know in order to understand the rotation of rigid body in a proper way. We conducted a study with 85 Upper secondary school students after their Mechanics course and the analysis has been performed using phenomenography, which is focused on the explanations rather than in the correctness of the answers. The results suggest that students have several difficulties in understanding some concepts of rigid body rotation and that there are significant alternative ideas among them.

1. Introduction
The literature has shown that Upper secondary school (16-18 year old students) and university students present some difficulties in understanding particles’ rotational movement. The research reveals that a significant amount of students think that a particle describing a uniform circular motion does not have acceleration, that no force is needed to produce circular motion and they consider centrifugal forces as “real” even for an inertial frame of reference [1]. Students show difficulties distinguishing between linear and angular acceleration and between linear and angular velocity. Moreover, they also show difficulties distinguishing between centripetal force and acceleration [2,3]. Taking into account all those difficulties, we expect that the gap in the comprehension of the concepts will grow up when the particle is substituted by a rigid body.

1.1. Difficulties in understanding rigid body rotation

The research on University students’ understanding of rigid body rotation has shown many difficulties. Frequently, students mix the concept of torque with the concept of force. In the same way, they do not establish any relation between the torque and the angular acceleration, they think that a constant torque results on a constant angular velocity and they show difficulties when taking into account the line of action of a force besides its point of application [4,5].

In relation to the moment of inertia of a rigid body, literature reports that many university students do not establish the dependence of the moment of inertia regarding the mass or its distribution around the axis of body rotation in a proper way. Others consider that the moment of inertia is proportional to angular acceleration of the body [2,6]. The research about university students’ difficulties regarding the angular momentum reveal that students do not recognize its
vector nature and they think that it is only applicable in cases where there is rotation. Moreover, students show difficulties in arguing about the conservation of angular momentum [2,4,7,8]. However, we have not found any research about Upper school students’ comprehension of rigid body rotation. The knowledge of the level of understanding that students have when they begin to study the topic in introductory physics course at University, constitutes a relevant contribution when designing the teaching material to be implemented into the classroom [9]. Research is required to show students’ ideas and difficulties in topics that have already been studied in Upper secondary schools, which are later studied in greater depth in introductory physics courses at University.

1.2. Problem statement
The objective of our study is to analyze how Upper secondary school students recognize and apply the concepts of moment of inertia, torque and angular momentum for a rotating rigid body. The aim is to relate the ability of students to apply these concepts with their understanding and to detect students’ comprehension difficulties when learning the concepts of moment of inertia, torque and angular momentum at Upper secondary level. We conducted a study to address these objectives. In section 2, we report the data collection process and data analysis strategy. In section 3, we report the results of our investigation and discuss relevant insights. Finally, in section 4 we conclude the study highlighting some implications in the teaching and learning the concepts of moment of inertia, torque and angular momentum.

2. Experimental design
We conducted a study with students from Upper secondary schools of the Basque Country (85, 16-18 year old students). After they had studied the rigid body rotation in class, we gave them an 8 questions open-ended questionnaire, where rotation was considered to be around a fixed axis in space. All students were finishing their semester of classical mechanics where the lectures and problem-solving sessions were focused on teaching rotational kinematics, rotational dynamics and moment of inertia, Newton’s second law for rotational motion, torque and angular momentum, as well as the conservation of angular momentum. In this study, we focus on the concepts of moment of inertia, torque and angular momentum.

Regarding the validation of questionnaire, once the questionnaire had been prepared, three experienced teachers expressed their opinions rewarding to the coherence between questions and their objectives. They agreed that the difficulty of the issues was adequate and that the questions responded to the objectives proposed by the researchers. Moreover, we carried out a draft test with 25 Upper secondary school students. The first analysis of questions showed that students understood the questions and were able to answer. Moreover, three teachers who were fully agreed with the relation of the aims and the formulation of the questions validated the aims of each question.

3. Methodology
This study, for analysing the results, uses a phenomenographic analysis to investigate “the qualitatively different ways in which people experience, conceptualize, perceive, and understand various aspects of, and phenomena in, the world around them” [10]. Phenomenography deals with how different ways of perceiving and understanding reality (concepts and associated ways of reasoning) can be considered as categories describing reality. These categories can be observed among a great number of individuals, and therefore all these representations together indicate a type of collective intellect.

According to Marton and Booth [11] creating categories must follow a specific set of criteria such as: a) Each category should be clearly related to research phenomena, so that each one tells us something distinct about a particular way of experiencing the phenomenon. b) Categories must be hierarchical, or in other words, they must progress from simple to complex relations. c) The categorization system should be parsimonious, meaning that, as few categories as is reasonably
possible should be explained. If the developed system of categories meets the above criteria, it will be theoretically and pedagogically useful.

3.1. Questions
The questions used on the questionnaire are quite familiar to students in the academic context and are usually mentioned in textbooks as examples of rigid body rotation. We present here three of the questions, figure 1, figure 2 and figure 3.

Q1. Two identical balls joined by a thin rod rotate with respect to an axis located in the middle of the rod (consider the rod as massless). The following three cases are presented:

Sort the cases according to the system’s moment of inertia with respect to the axis of rotation from BIGGEST ___ ___ ___ to SMALLEST. Argue your answer.

Figure 1. To inquire about moment of inertia, a rotational motion of two balls joined by a rod is proposed to students. They need to analyze the moment of inertia of the system in three different cases and sort those cases according to their moment of inertia.

Question Q1 aims to investigate whether students understand the meaning of the moment of inertia in a situation with rotation. According to the theoretical framework of classical physics, the moment of inertia measures the resistance that a body that rotates around an axis opposes to the change of rotational motion and is defined as: $I = \sum m_i r_i^2$. The moment of inertia with respect to an axis has an analogous role to that of mass in translational motion.

The three different cases in the question Q1 differ the distance from the balls to the axis of rotation and the balls’ mass; so that, students can make a qualitative and quantitative analysis when justifying their choice of ordering the cases according to the values of moment of inertia and its meaning.

Q2. Helen wants to drop a screw from the sink with a wrench. She has two possibilities. In the first one, she does a force of 3F module with an angle of 30°. In the second one, the force is of 2F but with an angle of 90°.

On which possibility does she obtain a major torque about the point B? Justify your answer.

Figure 2. To inquire about torque in relation to linear magnitudes, a rotation of a sink in two different ways is proposed to students. They need to analyze both cases and choose the way in which a major torque is obtained about the point B.
Torque is a vector which measures the tendency of a force to rotate an object about an axis. Torque in relation to linear magnitudes about a point is defined as: \( \vec{\tau} = \vec{r} \times \vec{F} = r F \sin \alpha \). In question Q2, students can argue using that definition that torque does not only depend on the force module but also on angle between the force vector and the position vector.

**Q3.** An autonomous car had an electronic fault and it started wheeling in the clockwise direction. John (the driver) does not know how to stop the wheel which can be touch only at the point 1 (see image). The following 3 modes are given:

![Diagram of three modes](image)

Which is the best mode? Justify your answer and indicate why you have chosen that option and not the others.

**Figure 3.** To inquire about torque and the importance of the point of application of the resultant force and its line of action and direction, a rotational motion of a wheel is proposed to students. They need to analyze the torque of the system in three different cases and choose the way in which a major torque is obtained in order to stop the wheel.

According to the definition of the torque in relation to linear magnitudes, the line of action of the resultant force and the direction of it play an important role. In question Q3, students need to consider that torque is a vector quantity; so that, the line of action of the resultant force is vital so as to obtain a major torque and also the direction of the vector needs to be taken into account.

3.2. Data analysis

The questionnaire was given to 85 Upper secondary students. We analyzed data with a phenomenographic qualitative method, considering the categories that emerge based on the students’ approach to each question. The categorization process was validated by three researchers, and the classification of students’ answers into categories was supported by Cohen’s kappa (\( \kappa = 0.92 \)).

4. Results and discussion

In this analysis, we first present the categories that emerged regarding moment of inertia, torque and angular momentum. Then, we show a summary of the results, highlighting the main findings and discuss their implications.

4.1. Difficulties in understanding moment of inertia

4.1.1. Question 1

Category A includes explanations that take into account both variables that influence the moment of inertia, the mass and the distance to the rotation axis, in a proper way according to the definition of moment of inertia (\( I = \sum m_i r_i^2 \)). See below examples of such responses:

“The greater mass an object has and the further away it is, the greater its moment of inertia. The mass and the moment are directly proportional but it is proportional to the square of the radius. That is why, C's moment is bigger than B's and as it has less mass A's moment is smaller than B's.” (Student No. 23)
Category B includes the responses that indicate either that only one of the two variables (mass or distance from the axis of rotation) needs to be taken into account, or indicate that both mass and distance influence the moment of inertia but they incorrectly establish the relation among the variables. The students reduce or confuse implicated variables. For example:

“The greater the distance they have from the axis in the center, the greater the moment of inertia, but as in cases A and B the distance is the same, A has a greater moment of inertia since it has less weight.” (Student No. 8)

Almost 20% of the answers (see Table I) that relate the moment of inertia of the system with the velocity of rotation of the system, but do not provide arguments that justifies this statement. For example:

“Case B will have the greater moment of inertia because having greater mass it rotates faster. In the same way, case A will have a greater moment of inertia than case C because it has less distance and it will take less time to turn.” (Student No. 51)

Table I. Categorization of question 1. Name and description of each category are shown in left and middle columns respectively. Right column shows the percentages of the answers classified in each category.

| Category | Explanation                                                      | %   |
|----------|------------------------------------------------------------------|-----|
| A        | Takes into account both, mass and distance from the axis of rotation in a proper way | 22.7 |
| B        | Takes into account either only one variable or both but with incorrectly related | 46.5 |
| C        | Based on the rotation velocity of the system                     | 19.3 |
| D        | Meaningless answer                                               | 4.5 |
| E        | No answer or answer with no explanation                          | 6.8 |

4.2. Difficulties in understanding torque in relation to linear magnitudes

4.2.1. Question 2.
Category A includes explanations that take into account the angle and the force’s module that influence the torque in a proper way according to the definition of torque in relation to linear magnitudes ($\vec{\tau} = r \vec{F} \sin \alpha = rF \sin \alpha$). See below an example of such responses:

“The second possibility. Torque is defined as: $\vec{\tau} = rF \sin \alpha$; so that, maximum torque will be when $\sin \alpha = 1$ what means: $\alpha = 90^\circ$. In the other possibility, $\sin 30^\circ$ is 0.5 so the force’s module should be double compared to the other possibility. In other words: $rF \sin 30^\circ = rF \sin 90^\circ$.” (Student No. 66)

Category B includes the responses that indicate the two variables (angle and module), however, they incorrectly establish the relation among the variables. For example:

“The first possibility. It is a greater force and a shorter angle.” (Student No. 43)
Almost 20% of the answers (see Table II) take into account just one variable, as we have already seen in question 1 reducing the variables, category C.

“The first possibility. It is a shorter angle so he reaches a major torque” (Student No. 58)

Table II. Categorization of question 2. Name and description of each category are shown in left and middle columns respectively. Right column shows the percentages of the answers classified in each category.

| Category | Explanation | %  |
|----------|-------------|----|
| A        | Takes into account both magnitudes (angle and module) in a proper way. | 63.5 |
| B        | Takes into account both magnitudes but with an incorrect relation. | 3.5 |
| C        | Takes into account only one magnitude | 17.7 |
| D        | Meaningless answer | 3.5 |
| E        | No answer or answer with no explanation | 11.8 |

4.2.2. Question 3.
Category A includes explanations that take into account the line of action and direction of the resultant torque with a strong argumentation. See below an example of such responses:

“The best mode is B. In the mode A the net torque is 0 and in the mode C the angular velocity would increase and not decrease,” (Student No. 71)

Category B includes the responses that indicate good relation between the force and torque but without a strong argumentation. For example:

“The best mode is B. The wheel turns in the clockwise direction, so the force needs to be done in the other direction.” (Student No. 2)

Category B includes the responses that take into account just the module of the force or the direction. For example:

“The best mode is A because it has the highest force and it is useful in order to stop the wheel. The mode B also is useful but it has a lower force.” (Student No. 63)

Table III. Categorization of question 3. Name and description of each category are shown in left and middle columns respectively. Right column shows the percentages of the answers classified in each category.

| Category | Explanation                                           | %  |
|----------|-------------------------------------------------------|----|
| A        | Takes into account the line of action and direction of the resultant torque with strong argumentation | 22.4 |
| B        | Takes into account the line of action and direction of the resultant torque without a strong argumentation | 54.1 |
| C        | Takes into account take into account just the module of the force or the direction | 5.9 |
| D        | Meaningless answer | 9.4 |
| E        | No answer or answer with no explanation | 8.4 |
4.3. Findings and discussion
This research has shown that Upper secondary school students have several difficulties in solid body rotation (see Figure 4).

In relation to the students' difficulties in understanding the concept of moment of inertia (question Q1, see Figure 1) most students recognize both variables that influence the moment of inertia (mass and distance to the axis of rotation). However, almost half of them established an incorrect dependence of the variables or only take into account one of them.

We note that a significant number of students (almost 20%, see Table I) intuitively relate the moment of inertia of the system with its velocity of rotation, but they do not offer an argument that relates both magnitudes.

In relation to the understanding of torque in relation to linear magnitudes, a vast majority of students correctly relates and interprets them (see Table II and Table III). However, it is important to consider that they have good intuitive responses but they have several difficulties in the argumentations.

![Discussion](image)

**Figure 4.** Histogram showing the absolute percentages of analyzed magnitudes. The analysis is classified in 1: Moment of inertia, 2: Torque in relation to linear magnitudes (the angle and module), 3: Torque in relation to linear magnitudes (the line of action and the direction). Correct answers are included in category A.

5. Conclusion
In this study, we observe that most of the students in Upper secondary school show significant difficulties in understanding the basic concepts of rigid body rotation. Data reveals that almost 20% of students do not distinguish any connection between the moment of inertia and the mass of the system or they think that the influence of the mass and distance from the axis of rotation are the same. This kind of misunderstanding can arise from their long learning process of translational dynamics.

So as to torque in relation to linear magnitudes data reveals that 20% of the students confused torque and force concepts and they have several difficulties in the argumentations.

Obtained data indicate that Upper secondary school students achieve little understanding of the magnitudes of the rigid body rotation, not only for the questions shown but also for the whole of the questionnaire. Therefore, the specific difficulties detected should be taken into account when planning the introductory physics course of the current University education.

Some of the difficulties we have observed are similar to those found in previous studies at University level, which indicates the importance and persistence of these difficulties. For example, the incorrect relation between the moment of inertia and the angular velocity [6,7].
In future research, we propose to pass the questionnaire to students of introductory physics courses at University to contrast the learning progression achieved and to establish improvements in instructional materials.

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