Random field Ising model swept by propagating magnetic field wave: Athermal nonequilibrium phase diagram

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The dynamical steady state behaviour of the random field Ising ferromagnet swept by a propagating magnetic field wave is studied at zero temperature by Monte Carlo simulation in two dimensions. The distribution of the random field is bimodal type. For a fixed set of values of the frequency, wavelength and amplitude of propagating magnetic field wave and the strength of the random field, four distinct dynamical steady states or nonequilibrium phases were identified. These four nonequilibrium phases are characterised by different values of structure factors. State or phase of first kind, where all spins are parallel (up). This phase is a frozen or pinned where the propagating field has no effect. The second one is, the propagating type, where the sharp strips formed by parallel spins are found to move coherently. The third one is also propagating type, where the boundary of the strips of spins is not very sharp. The fourth kind, shows no propagation of strips of magnetic spins, forming a homogeneous distribution of up and down spins. This is disordered phase. The existence of these four dynamical phases or modes depends on the value of the amplitude of propagating magnetic field wave and the strength of random (static) field. A phase diagram has also been drawn, in the plane formed by the amplitude of propagating field and the strength of random field. It is also checked that, the existence of these dynamical phases is neither a finite size effect nor a transient phenomenon.

Keywords: Random field Ising model, Monte Carlo simulation

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I. Introduction:

The behaviour of Ising ferromagnet with randomly quenched magnetic field is an active field of modern research. The zero temperature ferro-para transition [1] by the random field, made it more interesting. A simple model was proposed to understand the hysteresis incorporating the return point memory [2] and Barkhausen noise [3]. Later, the statistics [4] and the dynamic critical behaviour [5] of Barkhausen avalanches were studied in random field Ising model (RFIM). The hysteresis loop was exactly determined in one dimensional RFIM [6]. The RFIM was also studied [7] in the Bethe lattice.

The dynamics of the domain wall in RFIM is another area of interest in modern research of statistical physics. The motion of domain wall shows very interesting depinning transition at zero temperature. Due to the energy barriers created by disorder (random field), the domain wall is pinned and the motion of the domain wall remains stopped up to a critical field [8, 9]. However, at any finite temperature the depinning transition is softened and the thermal fluctuation assists to overcome the energy barrier. Thus with the application of sufficiently small amount of field, the motion of domain wall can reach a nonzero mean velocity, which is known as creep motion [10, 11, 12]. Very recently, the creep motion of domain wall in the two dimensional RFIM was studied (by Monte Carlo simulations) with a driving field [13] and observed field-velocity relationship and estimated the creep exponent.

All the above mentioned studies are done with constant and uniform magnetic field applied to RFIM. The behaviour of RFIM in the presence of time dependent magnetic field is not yet studied widely. A study, with time dependent (but uniform over the space) magnetic field applied to RFIM, shows dynamical symmetry breaking and athermal nonequilibrium phase transition [14]. The dynamical phase boundary was also drawn in the plane formed by the width of the disorder (random field) and the amplitude of the oscillating magnetic field. This study with time dependent magnetic field, showing dynamic symmetry breaking transition, is unable to extract any information about the morphology of the system. It would be interesting, to study the dynamic structural behaviours of RFIM, one has to use propagating magnetic field. In this paper, the dynamic behaviours of spins of RFIM swept by propagating magnetic field are studied by Monte Carlo simulation.

The paper is organised as follows: the model and the simulation technique is discussed in section II, the numerical or simulational results are given in section III and the paper ends with a summary in section IV.
II. The model and simulation:

The time dependent Hamiltonian representing the random field Ising model swept by a propagating magnetic field wave may be written as

\[ H(t) = -J \sum s(x, y, t) s(x', y', t) - \sum h(x, y) s(x, y, t) - \sum h_p(y, t) s(x, y, t) \]  \hspace{1cm} (1)

The \( s(x, y, t) \) represents the Ising spin variable \((\pm 1)\) at lattice site \((x, y)\) at time \(t\) on a square lattice of linear size \(L\). \(J(>0)\) is the ferromagnetic (uniform) interaction strength. The first sum for Ising spin-spin interaction is carried over the nearest neighbours only. \(h(x, y)\) is the random field acting at \((x, y)\) lattice site. The distribution of the random field is bimodal type, and represented as

\[ P(h(x, y)) = \frac{1}{2} \delta(h(x, y) - w) + \frac{1}{2} \delta(h(x, y) + w) \]  \hspace{1cm} (2)

Here, \(w\) represents the width of random field. The \(h_p(y, t)\) is the value of the propagating (along the positive \(y\)-direction) magnetic field at time \(t\) acting at \((x, y)\) lattice site.

\[ h_p(y, t) = h_0 \cos(2\pi ft - 2\pi y/\lambda) \]  \hspace{1cm} (3)

The \(h_0\), \(f\) and \(\lambda\) represent the amplitude, frequency and the wavelength respectively of the propagating magnetic field wave. We have considered the initial \((t = 0)\) configuration as all spins are up \((s(x, y, t = 0) = +1\) for all \(x\) and \(y)\). The spins are updated randomly (a site \((x, y)\) is chosen at random) and spin flip occurs only when the energy decreases due to flipping (the zero temperature Metropolis algorithm). The boundary condition is periodic in both \((x\) and \(y)\) direction. Here, the value of magnetic field is measured in the unit of \(J\).

III. The Results:

The simulation was performed on a square lattice of linear size \(L = 100\) with periodic boundary condition. The lattice spacing is taken as unity. Initially all the spin variables assume the value +1. A lattice site \((x, y)\) is chosen randomly and the spin variable is updated/flipped only if the flipping reduces the energy. \(L^2\) such random updates of spin variables is defined as unit Monte Carlo Step. The time \(t\), in the problem is measured in this unit.
The frequency ($f$) and the wavelength ($\lambda$) of the propagating magnetic field wave are kept fixed ($f = 0.01$ and $\lambda = 20$) throughout the present study. The long time behaviour and the dynamical steady states are studied here. As the wave propagates through the system of random field Ising model, the dynamical steady states are observed to form in a variety of morphology depending upon the values of $h_0$ and $w$. One such variety of morphology (the spin configurations on the lattice) are depicted in Figure-1. Here, figure-1a, shows the strong ferromagnetic (ordered) phase (where all spins are up) for $h_0 = 3.5$ and $w = 0.4$. This (phase-1) is a non-propagating frozen or pinned phase. As the width ($w$) of random field increases to $w = 1.0$, the propagating dynamical mode is observed. Here, the groups of parallel spins forms strips and these strips of alternate down and up spins propagate along the direction of propagation of the magnetic field wave. The wavelength of this propagation is same as that of externally applied propagating magnetic field wave. This coherent propagation of strips of alternate down and up spins is shown in figure-1b. Here, the groups of down spins forms strips of sharp boundary. The strips formed by parallel spins are very compact in structure. No down spin can be observed in the strip formed by up spins. These strips propagates along the direction of propagation of the magnetic field wave. This coherent propagation of strips of down spins forms phase-2. After further increases of the width ($w = 4.0$) of random field, another dynamical phase appears. In this phase, the strips of down spins are formed but the boundary of these strips are not as sharp as that observed in phase-2. In this phase, the strips are not very compact. The down spin may be found in the strip formed by up spins or vice versa. Here also, the propagation of these noncompact strips is observed. This is phase-3 and shown in figure-1c. The fourth kind of dynamical phase, i.e., phase-4 appears for larger values of width ($w = 6.0$) of the random field. This is almost a homogeneous mixture of up and down spins and constitutes phase-4 or disordered phase. This is non-propagating in nature and shown in figure-1d for $w = 6.0$. It may be noted that, except in phase-1, in all other phases the total magnetisation vanishes.

The time dependent line magnetisation $m(y,t) = \frac{1}{L} \sum_x s(x,y,t)$ is calculated and plotted against $y$ for different times. These are shown in figure-2 for three different times ($t = 3000, 3030$ and $3060$). Here, $h_0 = 3.5$ and $w = 1.0$. The propagating nature of the strips is clear from the picture. To check the existence of propagating modes, for different sets of values of $h_0$
and \( w \), the line magnetisations \( m(y, t) \) are plotted against \( y \) for different times. These are depicted in figure-3. Figure-3a and figure-3b shows the variations of line magnetisation \( m(y, t) \) with \( y \) for different times, \( t = 3000 \) and \( t = 3030 \) respectively. Here, \( h_0 = 2.0 \) and \( w = 3.0 \). This indicates the propagation. Figure-3c and figure-3d shows the same (for different values of \( h_0(= 1.0) \) and \( w(= 5.0) \). This shows no propagation.

The four different dynamical phases are characterised by measuring the structure factor \( S(k, t) = \frac{1}{L} \int_0^L m(y, t)e^{iky}dy \). This integration is essentially a summation on the lattice. It is noticed that the magnitude of this structure factor, \( |S(K, t)| \) takes different values in different phases. This measurement is quite significant to get the spin configurations in the lattice. If one calculate the total magnetisation and tries to use this to characterise the different dynamical phases, this will not be successful. The total magnetisation will vanish for phase-2, phase-3 and phase-4. The structure factor, \( |S(k, t)| \) is a function of \( h_0 \) and \( w \). The phase-1 is characterise by \( |S(k, t)|=0. \) \( |S(k, t)| \) will assume a value above 0.5 in phase-2 and in phase-3 it will lie between 0.1 and 0.3. In phase-4, \( |S(k, t)| \) will be very close to zero (but not exactly equal to zero as was in phase-1). The structure factor was calculated after 6000 MCS and averaged over 10 different randomly disordered samples (for different random field configurations). The structure factor \( |S(k, t)| \) is plotted, against the width of the disorder (random field) for different values of \( h_0 \), in figure-4. For \( h_0 = 0.25 \), the system remains in phase-1 up to \( w = 3.7 \) and then it transits to phase-4. Now, for higher value of \( h_0 = 1.75 \), the system remains in phase-1 up to \( w = 2.1 \), then it enters into phase-3. Here the value of \( |S(k, T)| \) becomes very close to 0.3. The system remains in phase-3 up to \( w = 3.9 \) and finally it transits to phase-4 (characterised by \( |S(k, t)| \leq 0 \)). For this value of \( h_0 = 1.75 \), the phase-2 was not attained by the system. Increasing the value of \( h_0 \) to 3.75, the variation of \( |S(k, T)| \) with \( w \), shows that the system passes through all four nonequilibrium phases. In this case, transition from phase-1 to phase-2 occurs at \( w = 0.3 \), transition from phase-2 to phase-3 occurs at \( w = 3.9 \) and finally the transition from phase-3 to phase-4 occurs at \( w = 5.7 \). A comprehensive phase diagram is plotted and is shown in figure-5. This diagram shows the regions of four different phases in the \( w - h_0 \) plane.

The stability of the dynamical phases (mainly propagating, phase-2) in time is confirmed by a study for much longer period (\( t = 30000 \) MCS). The size dependence was also studied for larger (\( L = 200 \)) lattices. These are shown in figure-6. This shows that the existence of spin wave propagating
phase is neither a finite size effect (figure-6a) nor a transient phenomenon (figure-6b).

IV. The summary:

To summarize the results, the random (bimodal) field Ising ferromagnet swept by propagating magnetic field wave is studied by Monte Carlo simulations in two dimensions at zero temperature. For a fixed value of the frequency and wavelegth of the propagating magnetic field, the system resides in four distinct nonequilibrium phases, depending upon the values of amplitude of the propagating magnetic field and the width of the quenched random field. These four phases are: phase-1, strongly correlated phase where all spins are parallel (up), phase-2, coherent spin wave (with sharp strips formed by down spins) propagation, phase-3, coherent spin wave (the boundary of strips is not quit sharp) propagation and phase-4, the disordered phase, where a homogeneous distribution of up and down spins is observed. A phase diagram is drawn in the plane formed by the width of random field and the amplitude of propagating field. These four phases are not arising due to finite size of the system. Even, these are not transient states. These study is new and hope will show some interesting nonequilibrium effects.

Here, in this paper, the preliminary results are reported. There are some important works have to be done in this respect. They are (i) to study the velocity of spin wave propagation as a function of width of disorder, (ii) the existence of diverging length scale (if any) along the phase boundary[15] (iii) the detailed finite size analysis and the scaling behaviour. It is also important to check whether this type of propagating phases exist in the case of a Gaussian distribution of random field. This requires lot of computational task to get the phase diagram for these two types of distribution of random fields. However, it seems that the boundaries will be moved towards the right (that means transition will happen for higher values of the strength of the random field) in the phase diagram. Since, the Gaussian distribution will provide the major contribution of the random field in the central region the higher values of strength is required to destroy the ordering.

The single crystal of $Mn_{12} - ac$ and $LiHo_xY_{1-x}F_4$ are two physical examples of RFIM. The nonequilibrium phases discussed here may be observed in $Mn_{12} - ac$ experimentally by relaxation experiments using conventional SQUID magnetometer[16]. In the sample $LiHo_xY_{1-x}F_4$ the propagating mag-
netic field would act like time dependent perturbation (for very small am-
plitudes) and measuring the intensity variations, of the spectral lines of the
transitions, the nonequilibrium phases may be observed[17].

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Fig-1. Morphology of four different nonequilibrium steady states. Here, $L = 100$, $t = 3000$ MCS, $f = 0.01$, $\lambda = 20$, $h_0 = 3.5$. Clockwise from top-left, (a) phase of first kind (note that here all spins are up and only the down spins are indicated by dots), obtained for $w = 0.4$, (b) phase of second kind (propagating sharp strips) obtained for $w = 1.0$, (c) phase of third kind (propagating strips of rough boundary) obtained for $w = 4.0$ and (d) phase of fourth kind, obtained for $w = 6.0$. 
Fig-2. The propagation of spin waves. The value of $m(y, t)$ is plotted against $y$ for different time $t$. Here, $f = 0.01$, $\lambda = 20$, $h_0 = 3.5$, $w = 1.0$. Different snapshots are taken at different times (MCS). From top to bottom, (a) for $t = 3000$ MCS, (b) for $t = 3030$ MCS and (c) for $t = 3060$ MCS.
Fig-3. The line magnetisation $m(y, t)$ for different set of values of parameters. (a) $h_0 = 2.0$, $w = 3.0$ and $t = 3000$ MCSS (b) $h_0 = 2.0$, $w = 3.0$ and $t = 3030$ MCSS, (c) $h_0 = 1.0$, $w = 5.0$ and $t = 3000$ MCSS and (d) $h_0 = 1.0$, $w = 5.0$ and $t = 3030$ MCSS.
Fig-4. The structure factor ($|s(k)|$) plotted against the strength ($w$) of random field for different values of the amplitude ($h_0$) of the propagating magnetic field wave represented by different symbols. Here, $h_0 = 3.75(*)$, $h_0 = 1.75(\times)$ and $h_0 = 0.25(+)$. Continuous or dotted lines are guides to the eye. Here, $L = 100$, $f = 0.01$, $\lambda = 20$. 
Fig-5. The phase diagram in $w - h_0$ plane. Different phase regions are indicated and the transition boundaries are shown by different symbols. Solid or dotted lines are guides to the eye. Here, $L = 100$, $f = 0.01$, $\lambda = 20$. 
Fig-6. The steady state behaviour for different times. From top, (a) $t = 3000$ and (b) $t = 30000$. Here, $L = 200$, $f = 0.01$, $h_0 = 3.5$, $w = 1.0$. 

\[ \text{Fig-6. The steady state behaviour for different times. From top, (a) } t = 3000 \text{ and (b) } t = 30000. \text{ Here, } L = 200, f = 0.01, h_0 = 3.5, w = 1.0. \]