Optimization of the technological process and equipment of complex profiled parts

C I Malea and E L Nitu
University of Pitesti, Faculty of Mechanics and Technology, Manufacturing and Industrial Management Department, Targul din Vale Street No.1, Romania
E-mail: malea.claudiu@gmail.com or claudiu.malea@upit.ro

Abstract: The modernization of the manufacturing industry requires the development of the manufacturing process and technological equipment, in general, but also for the realization of the complex parts. These components with complex geometries and multiple functional roles, for the most part, are safety parts manufactured from materials with superior physicochemical characteristics that have demanding technical quotas and conditions, leading to difficulties in establishing the processing methods and technological equipment used. For those considerations, it is proposed a three-stage design-optimization algorithm and an additional step, which represents a balanced and rational approach to technological process issues and in the manufacturing system. The first stage shall determine the optimal number and succession of operations based on a technological graph and the associated mathematical model, in which the objective function and the restrictions are defined concerning the purpose pursued. The second stage establishes, for each technological operation, the optimal orientation and fixation scheme (O-OFS) of the semi-finished part, which ensures the precision of machining and the technological process productivity, applying elements, of the sets theory and the global utility method to the method of optimizing of orientation and fixation of the parts. In the third step, based on the optimal orientation and fixation scheme, a mathematical model is formulated for the adapted technology chart to describe the construction variants of the orientation supports, and to provide the optimal device construction solution to the analysed operation. In situations where a reconfiguration of the technological system is desired, the fourth step of equipment optimization of the entire technological process is proposed.

1. Introduction
Technology, considered as the science of methods and means of transforming materials or in all manufacturing processes used to produce a product, has evolved in recent times thanks to approaches aimed at systematizing scientific knowledge, the development of rigorously reasoned methods capable of generating rational solutions for the construction of modern technological systems.

In all industrial sectors, the technological preparation of the manufacturing industry is forced to solve in a short period many problems and complications, a situation that often requires summary solutions in the design activity, which only aim at execution with obviously unfavourable results.

Overcoming technological limits implies the development of research activities in the field of manufacturing methods and equipment, for the complex optimization of technological systems, which is, after all, an important technical-economic particular problem.

In many publications [1-14] there is a trend to deepen the theoretical problems in the field of technological processes concerning new methods of operation, and their productivity in the field of
technological devices are concerns related to the optimization of the orientation and fixation of semi-finished products, construction, configuration, actuation, mechanization and modularization, and in the field of machine-tools, studies aim at their precision and automation.

The complex optimization of technological processes requires a combined approach to manufacturing processes and equipment problems. This article proposes a successive-simultaneous optimization method, which analyses and determines the most favourable solutions for the processes and methods to be handled according to the capacity of the technological system and, at the same time, allows modifying or redefining the characteristics and performance of technological equipment, in order to achieve technological processes with higher performance than existing equipment.

2. Successive-simultaneous method of optimizing technological processes and equipment

The modernisation of manufacturing processes and technological equipment implies the development and evolution of methods and means of processing, assembly or control, whatever the size of the production series.

The main objectives of the theoretical and experimental research and industrial applications developed in recent years [8, 13-18] are the development of mathematical models, optimisation algorithms, new processing methods and the realisation of special or modular structures, for the construction of complex processing devices, equipment and systems.

In this context, a combined method for optimizing the design of the technological process and equipment has been developed consisting of three mandatory steps and one additional step, the outline of which is shown in figure 1.

![Figure 1. Method for optimizing combined processes and technological equipment.](image)

The three mandatory stages are:
- **E01**: Determining the optimal number and sequence of technological operations
- **E02**: Determination of the optimal orientation and fixing scheme for the semi-finished product
- **E03**: Determining the optimal design variant of the device

And the additional stage is:
- **E04**: Establishing the optimum design variant of the process equipment in terms of technological processes

2.1. Establishing the optimal number and sequence of technological operations

The constructive-functional analysis of the drawing and of the overall drawing of which the workpiece is a part of is the starting point to evaluate the complexity and difficulty of making the piece, in comparison with the known manufacturing processes.

The content establishment and type of technological operations and their succession is influenced by the complexity of the workpiece and the surfaces that make it up, the degree of accuracy of the quotas and conditions imposed, the facilities and equipment performance, but also the experience of specialists working in the design activity.

In the paper [1], it is mentioned that the method of determining the optimal sequence of technological operations belongs to the category of problems that are solved by making objective decisions in a binary manner, with regard to the acceptance or rejection of processes, taking into account a particular optimization criterion, but also the conditions and requirements related to the precision of the part or the existing possibilities in the manufacturing process.

In order to achieve the mathematical model associated with the problem, it is considered that each processing operation can be carried out by one or more known or new processing operations that can be developed at that time.
Identifying optimal processing procedures requires the selection from a specified set of activities of those that best satisfy certain conditions. The X-column vector from equation (1) describes the set of operations:

$$X=[x_1 \ x_2 \ x_3 \ ... \ x_j \ ... \ x_n]^T$$

(1)

The components of the X-vector are bivalent variables associated with the processing operations:

$$x_j = 1 \text{ or } 0, \ j \in \{1,2,3,...,n\}$$

(2)

Is considered $x_j=1$, when the process is optimal and $x_j=0$ when the process is not optimal. Note the C line vector whose components represent the processing costs, in equation (3), for the procedures coded with $x_1$, $x_2$, $x_3$, ..., $x_n$:

$$C=[c_1 \ c_2 \ c_3 \ ... \ c_j \ ... \ c_n]$$

(3)

The mathematical model associated with the problem is the one in the equation (4):

$$\min(C \cdot X) = \min \left( \sum_{j=1}^{n} c_j \ x_j \right), \ j \in \{1,2,3,...,n\}$$

$$F(X) \ Rel \ O, \ i \in L=\{1,2,3,...,l\}$$

(4)

where:
- $C \cdot X$ is the objective function of the problem;
- $F(X)$ represents the l conditions (restrictions) of the problem;
- $Rel \ O$ is the relational operator of the form: $<, =, >, \leq, \geq$.

For the technological process analysed the set of coded variants, can be expressed by the primary technological graph from figure 2, in which: $x_1$, $x_2$, $x_3$, ..., $x_n$ are the technological operations analysed. The points (nodes) of the graph represent the moments of transformation of the geometry of the workpiece, $M= \{1, 2, 3, 4, ..., m\}$, and the procedures codified by $x_j$ are the graph arcs.

For a better highlighting of the technological variants, the secondary technological graph with shorter circuits is rewritten, as in figure 3.

![Figure 2. Primary technological graph of the analysed variants.](image1)

![Figure 3. Secondary technological graph of the analysed operations.](image2)

The coded processing procedures have the following meaning:

- $x_1 = x_1$, $x_2 = x_2$, $x_3 = x_3$, $x_4 = x_4$, $x_{n+1} = x_{n+1}$.

The column vector of the operations encoded in the secondary graph is:

$$X'=[x'_1 \ x'_2 \ x'_3 \ x'_4 \ x'_{n+1}]^T$$

(5)

To formulate the mathematical model [1]:
- Place the incidence matrix associated with the arcs of the graph, denoted by $A$, in clockwise browsing;
- The positive side of the incidence matrix \((A^+)^+\) is identified, which takes into account only the positive elements (starting from the node) denoted by +, the negative ones (-1) become 0;
- Enter the square matrices \(T_c, R_c\) of the tolerances and roughnesses capable, but also the line matrices \(T_p, R_p\) of the tolerances and roughnesses prescribed;
- Specify the column matrix \(U\) with \(n\) elements equal to 1.

The mathematical model is of the form:

\[
\begin{align*}
\text{min}(C \cdot X) \\
A^+ \cdot X &= U \\
A^+ \cdot (T_c \cdot X) &\leq T_p \\
A^+ \cdot (R_c \cdot X) &\leq R_p \\
x_j &= 1 \vee 0
\end{align*}
\]  

(6)

After replacing the values and solving the mathematical model, the optimal variant of the technological process is of the form:

\[
V_{Op} = [x_0, x_1, x_2, ..., x_{n+1}]
\]  

(7)

2.2. Method for optimizing the orientation and fixing scheme of the semi-finished product

In the operation plan, the optimum orientation and fixing scheme (O-OFS) is specified on the operation outlines using the graphic signs of the information symbolization. This is the starting point for the design of processing, assembly or control devices.

In classical technology, the orientation and fixation of the semi-finished products in the device have been conditioned almost exclusively by the experience of the technology and device designers and less by optimization methods. In modern technology, the literature [2, 6, 8, 19, 20] presents several optimization methods, from the simplified but less precise methodology to the more complex, rigorous but more laborious SEFA methodology.

In all these cases, the optimization method must be applied separately for each processing area, depending on the technical conditions and the type of operations, but obviously with less precision and productivity. For complex profile parts, the technological process \(x_n\) in the optimal version of the technological process (OV TP) requires the simultaneous implementation of O-OFS for two or more different processing zones of the semi-finished product, which will be carried out in the same operation.

For these reasons, in step E02 of the design-optimization presented in this paper, a method for the simultaneous determination of SOF-O is proposed, combining elements of sets theory and the global utility methodology with the SEFA optimization methodology [2], the analysis of the different process areas, which have specific quotas and conditions to be achieved.

2.2.1. Development of possible technical orientation systems (PT-OS)

A01. Operation sketch

- The processing areas specific to the operation are specified, areas that includes one or more surfaces;
- The outline of the operation must be carried out, in particular: the part in work station, with the numbering of the surfaces, the association of the XYZ axis system, the marking of the zones processed before the operation and the highlighting of those to be processed in the analysed operation, the quotas and conditions to be carried out in the analysed operation; movements of the machine with the semi-finished part and/or the tools necessary to generate the surfaces.

A02. Highlighting the technical conditions

Table 1 specifies the quotas and conditions imposed on the operation under review, the processing areas, the dimension bases, the areas that determine them, and the amount of deviations for linear quotas or other conditions.
Table 1. Quotas and conditions imposed, processing areas and dimension bases for the operation under review.

| No. cond. | Technical conditions imposed | Processing area | The processed surface | Dimension basis | Surfaces that determine them | Maximum deviation | Deviations according to
|-----------|-----------------------------|-----------------|-----------------------|-----------------|-----------------------------|------------------|-------------------------|
| C1        | \( \Phi_{s}^{a_{i}} \)     | Surface 19      | The axis of the surface 3 | Surface 3       | \( a_{i} \)           | SR EN 227668-1:1995  |
| C2        | \( L_{s}^{a_{i}} \)        | Surface 1       | The axis of the surface 8 | Surface 8       | \( a_{i} \)           | SR EN 227668-1:1995  |
| C3        | \( L_{s}^{a_{i}} \)        | ...             | ...                    | ...             | ...                       | ...               |
| ...       | ...                         | ...             | ...                    | ...             | ...                       | ...               |
| C9        | \( \bigcirc \)             | S17, S19, S16   | The axis of the surface 3 | Surface 3       | 0.3                       | SR EN 227668-1:1995  |
| ...       | ...                         | ...             | ...                    | ...             | ...                       | ...               |
| ...       | ...                         | ...             | ...                    | ...             | ...                       | ...               |
| Cn-1      | \( L_{n}^{a_{i}} \)        | Surface 24      | The axis of the surface 22 | Surface 22      | 0.1                       | SR EN 227668-1:1995  |
| Cn        | \( \perp \)                | ...             | ...                    | ...             | ...                       | ...               |

The notations are those of general character, but also others presented in the associated case study from the paper [21].

For the analysis of several groups of conditions, corresponding to the number of different areas to be carried out in the same operation, it is proposed to formulate additional connection conditions between them, which will generate a new method of surface machined, and a new device more precise in accuracy and with high productivity.

**A03. Selection of technical conditions**

- Note with \( M_1 \) and formulate the set of technical conditions required for this operation, for the first processing area:
  \[
  M_1 = \{c_1, c_2, c_3, ..., c_{j}, ..., c_n\} \quad \text{or} \quad M_1 = \{c_i | x \in c_i, x \leq c_j\} \quad (8)
  \]

- Note with \( M_2 \) and formulate the set of technical conditions required for this operation, for another processing area:
  \[
  M_2 = \{c_{j+1}, c_{j+2}, c_{j+3}, ..., c_{j}\} \quad \text{or} \quad M_2 = \{c_j | x \in c_j, x < c_{j+1}\} \quad (9)
  \]

- Specify the state of connection between the processing areas, which may be an explained or implicit condition of those entered in the table of technical conditions or may be an additional condition introduced for this purpose.

- **It is proposed as a selection criterion**, from the set of conditions highlighted, **that which relates to the orientation of the part** on the bases of the device and obviously ensures greater precision.

Applying this condition for the two sets \( M_1 \) and \( M_2 \) is obtained:

\[
M_{1-O} = \{c_2, ..., c_{j}, ..., c_{j+1}\} \quad (10)
\]

\[
M_{2-O} = \{c_{j+1}, ..., c_{j+4}, ..., c_{c_2}\} \quad (11)
\]

- The combination of the two sets \( M_{1-O} \) and \( M_{2-O} \) is a set made up of all the elements of the two sets which relate to the orientation of the workpiece in the device:

\[
M_{1-O} \cup M_{2-O} = \{c_{j} \in M_{1-O} \text{ sau } c_{j} \in M_{2-O}\} \quad (12)
\]

\[
M_{1-O} \cup M_{2-O} = \{c_2, ..., c_{j}, ..., c_{j+1}, ..., c_{j+4}, ..., c_{c_2}\} \quad (13)
\]
The set of conditions \( \{c_2, ..., c_9, ..., c_{j+1}, ..., c_{c,2}\} \) is the \textit{multitude of determinant conditions}, that are important, necessary and obligatory, which will be assured with certainty by the orientation of the part on the supports of the device. Other conditions are not related to the orientation of the workpiece in the device, but will be carried out by the construction of the tool, the device, the adjustment or obtained in other technological operations.

\textbf{A04. Geometrization of determining conditions}

The determining technical conditions must be rewritten as an equation between the geometric elements of surface type (S), line (D) or point (P), the areas to be processed and those of the quotation bases.

\[
C_{2} \quad S_{j} \leftarrow \begin{array}{c} L_{2}^{i} \end{array} \rightarrow D_{x_{j}} \cdot x_{i}
C_{j+2} \quad S_{16} \leftarrow \begin{array}{c} L_{2}^{i} \end{array} \rightarrow S_{v}

C_{9} \quad S_{16}, S_{17}, S_{18} \leftarrow \begin{array}{c} L_{2}^{i} \end{array} \rightarrow D_{z_{j}} \cdot z_{i}
C_{r-2} \quad \ldots \ldots \ldots \ldots

\textbf{A05. Extreme conditions selection}

S_{1}, ..., S_{16}, S_{17}, S_{18}, ..., they are the \textit{extreme directors}, they depend on the tool and they are reference elements.

D_{x_{j}} \cdot x_{i}, ..., D_{z_{j}} \cdot z_{i}, ..., P_{k}, they are \textit{extremely dependent}, because they depend on dimensions or conditions on the extreme ends and they ensure the orientation of the semi-finished product in the device.

\textbf{A06. Explanation of the dependent extremes}

The dependent extremes must be expressed by real and palpable elements, which correspond to surfaces, straight lines or points on the workpiece, in the form of:

\[
C_{2} \quad S_{j} \leftarrow \begin{array}{c} L_{2}^{i} \end{array} \rightarrow D_{x_{j}} \cdot x_{i}
C_{j+2} \quad S_{16} \leftarrow \begin{array}{c} L_{2}^{i} \end{array} \rightarrow D_{x_{j}} \cdot x_{i} \cdot P_{k}

C_{9} \quad S_{16}, S_{17}, S_{18} \leftarrow \begin{array}{c} L_{2}^{i} \end{array} \rightarrow D_{z_{j}} \cdot z_{i}
C_{r-2} \quad \ldots \ldots \ldots \ldots

\textbf{A07. Ordering the dependent extremes}

According to the frequency criterion or the degrees of freedom cancelled, the order is:

\[
E_{1} \quad D_{x_{j}} \cdot x_{i} ; \quad E_{II} \quad D_{z_{j}} \cdot z_{i} ; \quad E_{III} \quad P_{k}
\]

\textbf{A08. Symbolization of the materialization of the ordered dependent extremes}

The graphic signs of the information symbolisation accompanied by a number of brackets should be placed on the surfaces of the workpiece, acting as a basis for orientation and fixation, which also suggests the form, construction and functions of the device’s supports. For the semi-fabricated part of the case study presented in the following article [21], these are:

\[
E_{1} \quad D_{x_{j}} \cdot x_{i} ; \quad E_{II} \quad D_{z_{j}} \cdot z_{i} ; \quad E_{III} \quad P_{k}
\]

\textbf{A09. Symbolization of auxiliary supports}

It is only realized if necessary.

\textbf{A10. Symbolization of the additional supports}
If, for the semi-manufactured part, during the operation under analysis, the cutting or fastening forces can produce deformations or vibrations, they must be arranged in the points of maximum risk, preferably with self-setting and locking, to prevent deformation of the part and the occurrence of errors in the technological operation.

### 4.1. Establishing possible technical guidance schemes (PT-OS)

The centralizing table 2 must be constructed, in which the dependent extremes have the order EI, EII, EIII, with the numbers assigned to the symbols, the determined conditions identified with the calculated permissible orientation errors ($\varepsilon_{oa}$).

Orientation variants of PT-OS are combinations of symbols for supports elements, which must meet a mandatory kinematic condition of not exceeding the six degrees of freedom of full orientation.

| No. | PT-OS | Dx, Dy, Dz | EII | EIII | R.s. | C2 | C9 | C16 | TA-OS |
|-----|--------|------------|------|------|------|----|----|----|-------|
| 1   | X      | X          | X    | 0.867| 0    | ...| ...| ...| ...   |
| 2   | X      | X          | X    | 0.867| 0    | ...| ...| ...| ...   |
| 3   | X      | X          | X    | 0.867| 0    | ...| ...| ...| ...   |
| 4   | X      | X          | X    | 0.867| 0    | ...| ...| ...| ...   |
| 5   | X      | X          | X    | 0.867| 0    | ...| ...| ...| ...   |
| 6   | X      | X          | X    | 0.867| 0    | ...| ...| ...| ...   |
| 7   | X      | X          | X    | 0.867| 0    | ...| ...| ...| ...   |
| 8   | X      | X          | X    | 0.867| 0    | ...| ...| ...| ...   |

The symbols 6* and 7* are the same supports 6 and 7, which, after blocking, cancel a degree of freedom (Rx), for the extreme Pk.

### 2.2.2. Technical selection of orientation schemes

- The admissible orientation errors are calculated ($\varepsilon_{oa}$) and go to the table. The equation is (14):

$$\varepsilon_{oa} = \left( \frac{1}{2} \cdot \frac{1}{3} \right) \cdot T$$

where: T- is the tolerance of the piece to the analysed condition.

- The actual orientation errors are calculated ($\varepsilon_{or}$), determined by clearances or non-compliance of the dimensioning bases (DB) with the guide bases (GB) and are placed in the table. The equations in paper [9] are:

- for clearances:

$$\varepsilon_{or} = \sqrt{\left( \frac{uL}{uL} \right)^2 \cdot i^2 + \left( \frac{uL}{uL} \right)^2 \cdot j^2}$$

(15)

- for non-compliance of the dimensioning bases with the guide bases:

$$\varepsilon_{or} = \sqrt{\left( \frac{uL}{uL} \right)^2 \cdot T^2_{ij} + \left( \frac{uL}{uL} \right)^2 \cdot T^2_{ij}}$$

(16)

- The actual orientation errors are compared with the admissible ones and the variants that fulfil the condition $\varepsilon_{or} < \varepsilon_{oa}$ because all the errors analysed, they become technical acceptable orientation schemes (TA-OS) that will ensure with certainty the accuracy of the processing of the device.

### 2.2.3. Economic selection of technically acceptable orientation schemes

Technically acceptable orientation systems (TA-OS) are evaluated using the global utility method, assigning values for coefficients of importance (Ci) based on how well they respond to the economic criteria associated with [9] the types of supports, then they are written in the table.
If 0 ≤ Ci ≤ 1, or for another range of values, when the structure of the analysed supports is favourably related to the analysed criteria, the Ci values will be towards maximum values, and in the case of a less favourable response, the values will be towards minimum. For the case study presented in the article [21], Ci values are presented in table 3.

Once the importance coefficients have been entered, the symbol or combination of symbols with the maximum value is a local optimum for the extreme being analysed.

\[
SO-O = \text{Optimum } E_I + \text{Optimum } E_{II} + \text{Optimum } E_{III}
\]  

For the case study considered from the paper [21], the optimal orientation scheme is:

\[
\text{SO-O} = \begin{cases} 
(8) & (9) & (3) & (7) 
\end{cases}
\]

(17)

In a similar study, the optimal fixation scheme (SF-O) preceded by the symbol (11) is also established. The result of the optimization method applied in the study case:

\[
\text{O-OFS} = \text{SO-O} + \text{SF-O} = \begin{cases} 
(8) & (9) & (3) & (7) & (11) 
\end{cases}
\]

(18)

(19)

| No. TA-OS | Symbolization of information associated with dependent extremes | Criteria used (C-U) | Coefficients of importance (Ci) (Max.) | Decision |
|-----------|---------------------------------------------------------------|---------------------|----------------------------------------|----------|
| 1         | E_I                                                          | CE1 CE2 CE3 CE4 CE5 CE6 | 0.9 0.9 0.9 0.8 0.8 5.2                 | Optimum E_I |
| 2         | E_{II}                                                        |                      | 0.3 0.3 0.3 0.5 0.8 0.8 3                |          |
| 3         | E_{III}                                                       |                      | 0.9 0.8 0.6 0.6 0.8 0.8 4.5              |          |
| 4         | E_{III}                                                       |                      | 0.9 0.9 0.6 0.8 0.8 0.8 4.8 Optimum E_{II} |          |
| 5         | E_{III}                                                       |                      | 0.9 0.8 0.6 0.6 0.8 0.8 4.5              |          |
| 6         | E_{III}                                                       |                      | 0.9 0.9 0.6 0.8 0.8 0.8 4.8 Optimum E_{III} |          |

Table 3. Centralizing the criteria for economic optimization.

3. The design-optimization algorithm of the device construction

In phase E03, the optimal variant of device construction (VOC_D) is obtained after the design-optimization algorithm presented in [8], in the following sequence:

I. Identification of constructive options for O-OFS;
II. Determining the graphic of the associated design options O-OFS;
III. Formulate the mathematical model based on the graph with an indication of the objective function, restrictions, or other equations;
IV. Definition of the algorithm for solving mathematical models; 
V. Solving the mathematical model.

It is well known from the literature [2, 8, 20, 22] and from industrial practice that the field of devices constructions is distinguished by the large number of different types and constructional options. For this reason, for each symbol in O-OFS, several options for the construction of the supports will be analysed.

I. For a specific application, the construction options of the orientation support in O-OFS, with the graphic signs in the informational symbolization (SI1, SI2, SI3):

1. SI1, the graphic sign of the first support type, has for variants structures:
   \[ x_1, x_2, x_3, \ldots, x_n \]  
   (20)

2. SI2, the graphic sign of the second support type, with the structures variants:
   \[ x_1, x_2, x_3, \ldots, x_n \]  
   (21)

3. SI3, the graphic sign, for support three, with the structures variants:
   \[ x_1, x_2, x_3, \ldots, x_n \]  
   (22)

Similarly, the construction options of the fixing system, which do not appear in this evaluation, are analysed to simplify the presentation of the optimization method.

II. The construction options of the supports will be expressed by the graph in figure 4, where the peaks (nodes) of the graph are the orientation symbols of the O-OFS, and the arcs of the graph are the construction options of the supports that make up the whole device:

![Figure 4. Graph of support construction options.](image_url)

Mathematical model for the graph associated with all the construction options of the supports, with the objective function, the cost of the device (C), and as restrictions: precision (ε), productivity (p), flexibility (f), manufacturing preparation time (t) and operating behaviour (e), as follows:

\[
\begin{align*}
\text{Min} \quad C &= c_1 x_1 + c_2 x_2 + \ldots + c_n x_n \\
e_1 x_1 + e_2 x_2 + \ldots + e_n x_n &\leq e_{\max} \\
p_1 x_1 + p_2 x_2 + \ldots + p_n x_n &\geq p_{\min} \\
f_1 x_1 + f_2 x_2 + \ldots + f_n x_n &\leq f_{\min} \\
t_1 x_1 + t_2 x_2 + \ldots + t_n x_n &\leq t_{\min} \\
e_1 x_1 + e_2 x_2 + \ldots + e_n x_n &\geq e_{\min} \\
\sum_{j=1}^{j} x_j &= 1; x_j \geq 0; j \in \{1, 2, 3, \ldots, h\} \\
\sum_{j=1}^{j} x_j &= 1; x_j \geq 0; j \in \{1, 2, 3, \ldots, g\}
\end{align*}
\]  
(23)
The values for the restrictions $\varepsilon$, $p$, $f$, $t$, $e$, and $c$, from the objective function, it is determined with the calculation equations in the papers [2, 8, 9, 23] and are inserted into the inequation system of the mathematical model. For resolving, the mathematical model is written in matrix form, as follows:

\[
\begin{align*}
\mathbf{A}^+\mathbf{X} &= \mathbf{U} \\
\mathbf{A}^+(\varepsilon\mathbf{X}) &\leq \varepsilon_{\text{max}} \\
\mathbf{A}^+(p\mathbf{X}) &\geq p_{\text{min}} \\
\mathbf{A}^+(f\mathbf{X}) &\geq f_{\text{min}} \\
\mathbf{A}^+(t\mathbf{X}) &\leq t_{\text{max}} \\
\mathbf{A}^+(e\mathbf{X}) &\geq e_{\text{min}} \\
\text{Min}(C\mathbf{X})
\end{align*}
\]  

(24)

where:
- $\mathbf{A}$ is the incidence matrix associated with the graph;
- $\mathbf{A}^+$ is the positive side of the incidence matrix;
- $\mathbf{X}$ is the column vector of the construction variants;
- $\mathbf{U}$ is the column matrix with $n$ elements equal to 1.

After solving the mathematical model, the optimal version of the device construction $\text{VOC}_{D}$ is composed of the following structures:

\[
\text{VOC}_{D} = (x_{1}, x_{2}, x_{3})
\]

(25)

4. Establishing the optimum design variant of the process equipment in terms of technological processes

When a technological process does not provide the best conditions for an established production program or when other objectives are imposed at a given time, a redefinition of the manufacturing system is necessary.

For these reasons, it is often necessary to re-evaluate the processing, assembly or control equipment throughout the technological process, which is why it is proposed, in stage E04, to develop new structures in the field of devices, equipment construction and the modernization of machine-tools.

To this end, well-known solutions are identified and new technological equipment structures are developed, described using a technological graph, similar to the one presented in the construction of the devices, which will allow the analysis of the equipment based on specific technical and economic criteria.

After the formulation and the solving of the mathematical model, a combination of structures of the manufacturing equipment is also obtained, for each operation of the technological process, with characteristics, functions, and performances superior to the existing structures.

This type of approach, depending on the objectives pursued and the rigorous analysis of performance criteria, often leads to technical solutions with elements of obvious novelty and to the modernization of the manufacturing system.

One of these solutions, shown in figure 5, is a milling and centring head [16], which provides the optimum cutting speed and the different speed to the two tools, which can perform these operations on several types of machines.

The rotational movement of an electric motor or a drive device (DD) is taken by the flange input shaft 15 and is transmitted through special bolts 20 to the disc 11, and through wedge 21 is transmitted to the main shaft 3, supported by bearings 7, 9 and 10 from the main body 8, which is solidified with the milling disc 6. The satellites 13, of the planetary transmission on axes 16, which run on the ring gear 12, which also has the role of centring the cases 8 and 14, multiply the speed by the pinion 18 and
the wedge 17 transmits it to the inner axis 2, supported on the bearings 4, 5 and 19, which drive the centring drill 1, which rotates five times faster than the milling disc.

This drive system with compact tools at different speeds has enabled the construction of a flexible milling-centring machine consisting of modular structures in the rhythm of a normal lathe, which holds three patents [15-17] and operates successive- simultaneously the two processing phases. The modular milling and centring machine, with this level of technical innovation, is an industrial application in which the optimization of the process and technological equipment has generated a new method of processing, the development of new structures for the construction modernization of machine tools and technological system. The superior features and performance of such equipment ensure a high level of accuracy, productivity, and flexibility, as well as many reduced costs of production and operations.

Figure 5. Milling and centring head [16].

5. Conclusions
The successive optimization method with the three stages E01, E02 and E03, but also the method of simultaneous analysis and optimization of the structures for the development and reconfiguration of the technological system, proposed in stage E04, are part of the research concerns and the effort to achieve the complex optimization of manufacturing processes and equipment used in modern industry.

Calculation methods, mathematical models and design optimization algorithms enable objective decisions and ensure the evolution of manufacturing processes and equipment with certainty.

This simultaneous approach, as a method of optimization, allows the selection of the best solutions as a processing procedure for certain characteristics of the technological equipment, but also offers the possibility of manufacturing new equipment with superior characteristics and performance to the existing ones.

6. References
[1] Ivan N V, Paunescu T, Udroiu R, Ivan M C, Gavrus C and Pescaru R 2010 Machine Building Technology: Theory and innovative approaches (Brasov: Transilvania University of Brasov)
[2] Bragaru A, Picos C and Ivan N V 1996 Optimization of processes and technological equipment
[3] Zhu S W, Ding G F, Ma S W, Yan K Y and Qin S F 2013 Workpiece locating error prediction and compensation in fixtures Int. J. Adv. Manuf. Tech. 67 1423–1432

[4] Paunescu T A 2005 Monte Carlo approach of modular fixture systems locating accuracy Part 2 The Monte Carlo simulation 2nd Int. Conf. Mechanics and Machine Elements (Sofia, Bulgaria)

[5] Kang Y and Rong Y 2003 Computer-Aided Fixture Design Verification. Part 1 The framework and Modelling Int J Adv Manuf Technol 21(10-11) 827-835

[6] Paunescu T A, Bulea H and Paunescu R 2008 Modular devices. Mathematical models 2 (Brasov: University of Transylvania publishing)

[7] Qin G H and Zhang W H 2006 A mathematical approach to analysis and optimal design of a fixture locating scheme Int J Adv Manuf Tech 29 349-359

[8] Costea A and Rachiera N 2005 Flexibility and performance of processing equipment. Optimize device design (Bucharest: BREN Publishing)

[9] Iordache D M, Costea A and Baba A 2016 Calculation methods and mathematical models for design optimization devices (Pitesti: University of Pitesti Publishing)

[10] Zhang F P, Wu D, Zhang T H, Yan Y and Butt S I 2018 Knowledge component-based intelligent method for fixture design Int J Adv Manuf Technol 94 4139–4157

[11] Chu Y 1999 Workpiece Localization: Theory, Algorithms and Implementation (The Hong Kong University of Science and Technology)

[12] Vlase A 1996 Tehnologia constructiilor de masini (Bucharest: Technical Publishing)

[13] Nitu E, Baba A, Iordache M, Costea A and Iacomi D 2013 Mathematical model and algorithm to optimize the construction of adjustable multi-axis heads Applied Mechanics and Materials 371 617-621 (Switzerland: Trans Tech Publications)

[14] Nitu E, Costea A, Iordache M, Rizea A and Baba A 2017 Optimizing the construction of devices to control inaccessible surfaces - case study 11th International Congress of Automotive and Transport Engineering - Mobility Engineering and Environment (CAR) 252 (Pitesti)

[15] Costea A and Susa V 2005 Dispozitiv de antrenare a sculelor Patent RO no. 119072 B1, Int.Cl. B23 B5/24

[16] Costea A and Prunoiu M 2005 Dispozitiv de frezare – centruire Patent RO no. 119073 B1, Int.Cl. B23 B39/04

[17] Costea, A, Miñana M and Prunoiu M 1997 Masina de frezare – centruire Patent RO no. 110688 C, Int.Cl. B23 B 49/04

[18] Nitu E L, Costea A, Iordache M, Iacomi D T and Baba A 2017 Cap multiax reglablit modularizat Patent RO no. 130221 B1

[19] Iordache D M and Ungureanu I 2010 Technological devices (Pitesti: University of Pitesti Publishing)

[20] Vasii R S, Gogineschi N, Andronic C, Selariu M and Gherghel I 1982 Proiectarea dispozitivelor (Bucharest: Didactic and Pedagogic Publishing Bucharest)

[21] Malea C I and Iordache M D 2020 New methods and equipment for the machining of cross-axes parts- Case study ModTech2020 (in press)

[22] Paunescu T, Bulea H and Paunescu R 2006 Dispozitive modulare 1 Constructie, exploatare (Brasov: University of Transylvania publishing)

[23] Abrudan I 1996 Flexible Manufacturing Systems. Design and management concepts (Cluj-Napoca: Dacia Publishing)