WSN Topology Control algorithm for High Speed Railway Monitoring

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Abstract. In order to solve the safety problems caused by the continuous improvement of train speed and the continuous growth of railway scale, it is inevitable to apply wireless sensor network (WSN) to train operation environment monitoring. In order to prolong the survival time of wireless monitoring network, the game theory is used to construct a distributed topology model. In order to solve the problem of network lifetime shortening caused by uneven energy consumption of nodes, the Theil index is introduced to improve the energy consumption balance of nodes, and a topology control algorithm for energy consumption balance (EBTCA) is proposed. Experiments show that the algorithm has good results in terms of network lifetime and energy balance.

Keywords: Railway environment monitoring, wireless sensor networks, topology control.

1. Introduction
With the increase of train speed, the safety problem of high-speed trains has become increasingly prominent. For example, the deformation of viaduct bridge structure, the landslide of mountain along the railway, and the bad weather of wind, rain and snow all threaten the safety of high-speed train operation. Wireless sensor networks (WSN) consist of a large number of energy-constrained sensor nodes, which form a data-centric data acquisition and transmission network in a multi-hop self-organizing manner. WSN has the characteristics of wide coverage, low power consumption, low cost and self-organization. The application of WSN in the monitoring of the running environment of high-speed trains makes the monitoring network more flexible and effective, and at the same time, it will promote the development of the railway industry to the direction of information, high-speed and safety. Topology control is an important technology in WSN, which can effectively improve energy efficiency and prolong network life without affecting network connectivity and throughput [1]. Traditional topology control algorithms are based on geometric structure algorithm [2, 3], sleep scheduling algorithm [4], etc. With the expansion of the network scale and the sharp increase of the amount of data, the traditional topology control algorithm takes too long and needs to seek more efficient solutions. On the other hand, in the distributed topology control algorithm, nodes show selfishness because they cannot obtain global information. For selfish nodes, game theory is a powerful tool to describe competition and collaboration among decision makers. The MLPT algorithm based on link power consumption is designed in reference [5]. Its main idea is to run the minimum hop routing algorithm with maximum
power, that is, each node takes turns to execute the game and select the link with the optimal response strategy. Although the algorithm reduces the number of hops, the transmission power of the node is higher and the energy consumption is higher. In reference [6], the algebraic connectivity of graphs is used to measure the redundancy of networks, and a TCLE algorithm is proposed. The network topology constructed by this algorithm not only saves energy but also has certain fault tolerance. In reference [7], EFTCG-1 algorithm and EFTCG-2 algorithm are proposed for single-connected and double-connected networks, respectively. Although some of the above-mentioned algorithms based on game theory can improve network performance, they do not take into account the energy balance and energy efficiency of nodes. In order to improve the balance of energy consumption of nodes, the Theil index in economics is used to measure the balance of residual energy of nodes, and a topology control algorithm (EBTCA) is proposed, which comprehensively considers the balance of energy consumption and energy efficiency.

The rest of this article is organized as follows. In Section 2, we introduce related theoretical knowledge and establish a topology game model based on game theory. In Section 3, the EBTCA algorithm is proposed based on the model established in Section 2. In Section 4, we conduct experiments on EBTCA algorithm, EFTCG algorithm and MLPT algorithm and analyse the experimental results. Section 5 concludes this paper.

2. Topology Control Model based on Game Theory

2.1. Model Establishment

If the node is regarded as a participant in the game, the optional power set of the node is regarded as a strategy set of the game, and the utility function is designed according to the optimization objective, WSN can be expressed as a strategy game \( \Gamma = (N, S, \{u_i\}) \) [8]. Among them, \( N \) represents \( n \) participants in the game. \( S_i = \{s_{i,1}, s_{i,2}, ..., s_{i,k}\} \) is the strategy set of participant \( i \), and \( S = \prod_{i=1}^{n} S_i \) represents the strategy space of the strategy game. Generally, \( s = (s_i, s_{-i}) \) represents a strategy choice of a strategy game, where \( s_i \) represents the strategy chosen by participant \( i \), and \( s_{-i} \) represents the strategy chosen by the remaining \( n-1 \) participants. \( \{u_i\} \) is the income collection of participants.

**Definition 1**: Nash equilibrium means that each participant chooses the strategy to maximize his utility function. For \( \forall i \in N \) and \( \forall s_i \in S_i \) has

\[
u_i(s^*) \geq u_i(s_i, s_{-i})
\]

Then the strategy combination \( s^* = (s_i^*, s_{-i}^*) \) is a Nash equilibrium point of the strategy game \( \Gamma = (N, S, \{u_i\}) \).

The number of Nash equilibrium points in ordinary strategy games is uncertain, while the ordinal potential game is a special kind of strategy game, which has at least one Nash equilibrium [9].

**Theorem 1**: If there is a function \( V \), for \( \forall i \in N \), \( \forall s_{-i} \in S_{-i} \) and \( \forall a_i, a_i \in S_i \) has

\[
V(a_i, s_{-i}) - V(b_i, s_{-i}) > 0 \iff u_i(a_i, s_{-i}) - u_i(b_i, s_{-i}) > 0
\]

Then the strategy game is an ordinal potential game, and the function \( V \) is its ordinal potential function.

**Theorem 2**: If the strategy game \( \Gamma = (N, S, \{u_i\}) \) is an ordinal potential game and the function \( V \) is its ordinal potential function, then the strategy set that maximizes the function \( V \) is a Nash equilibrium point of the strategy game.

In order to optimize network performance more comprehensively and effectively, the following aspects should be considered when designing the utility function.

1) Network connectivity
Maintaining network connectivity is one of the basic requirements of topology control. If the network is disconnected, the information collected by the wireless node cannot be transmitted to the base station.

2) Transmission power of nodes

The transmission power of a node is an important factor affecting network energy consumption. Only when the node forwards the data packets with lower transmission power can it work longer under the condition of limited battery capacity.

3) Residual energy of nodes

In order to prolong the survival time of nodes with less residual energy as much as possible, nodes with more residual energy should be preferred in the construction of network topology.

4) Degree of nodes

The degree of nodes reflects the amount of data forwarding and the number of network links of nodes to a certain extent. In order to reduce the energy consumption of nodes and avoid the generation of loops, the degree of nodes should be limited.

5) Energy balance

If the residual energy of nodes is inconsistent, the survival time of nodes will also be inconsistent. Therefore, the residual energy of nodes should be consistent as much as possible.

In order to better reflect the residual energy balance degree of nodes, this paper introduces the Theil index to express it. The Theil index is an important indicator in economics that uses entropy to measure the income gap between individuals or regions [10]. The Theil index between $n$ individuals is calculated as follows

\[
T = \sum_{i=1}^{n} \frac{x_i}{\sum_{j=1}^{n} x_j} \cdot \ln \frac{x_i}{\bar{x}} \tag{3}
\]

Where $n$ is the total number of individuals, $x_i$ is the income of individual $i$, and $\bar{x}$ is the average income of $n$ individuals.

If the node is regarded as an individual and the residual energy of the node is regarded as the income of the individual, the residual energy balance degree of the node is calculated as follows

\[
T = \sum_{i=1}^{n} \frac{E_r(i)}{\sum_{j=1}^{n} E_r(j)} \cdot \ln \frac{E_r(i)}{E_r} \tag{4}
\]

Based on the above analysis, the utility function is defined as

\[
u_i(p, \{p_{-i}\}) = f_{p_i} \left( 4p_i^{\text{max}} - k_{p_i}p_i \right) + \frac{E_r(i)}{E_0(i) - E_r(i)} + \frac{E_r}{E_0} \left( p_i \right) + \frac{1}{T+1} \tag{5}
\]

Here, $f_{p_i}$ depends on the connectivity of the network. If the network is connected, then $f_{p_i} = 1$, otherwise $f_{p_i} = 0$. $k_{p_i}$ represents the degree of node $i$. $E_0(i)$ and $E_r(i)$ represent the initial energy and residual energy of node $i$, respectively. $\bar{E}_r(p_i) = \frac{1}{m} \sum_{j=1}^{m} \frac{E_r}{E_0}$ is the average ratio of residual energy to initial energy of neighbor nodes of node $i$ under the power $p_i$. $T$ is the Theil index that reflects the residual energy balance degree of nodes.

2.2. Model Analysis

**Theorem 3:** The strategy game $\Gamma = \langle N, S, \{u_i\} \rangle$ is an ordinal potential game, and the ordinal potential functions is
\[ V(p_i, p_{-i}) = \sum_{i \in N} \left[ f_{p_i} \left( 4p_i^{\text{max}} - k_p p_i \right) + \frac{E_r(i)}{E_0(i) - E_r(i)} + \bar{E}_i(p_i) + \frac{1}{T+1} \right] \]  \quad (6)

**Proof:** \( p_i \) and \( q_i \) are the two optional powers of node \( i \). Assuming that \( p_i > q_i \), then \( f_{p_i} \geq f_{q_i} \). The income difference when node \( i \) selects power \( p_i \) and \( q_i \) is calculated as follows

\[ \Delta u_i = u_i(p_i, p_{-i}) - u_i(q_i, p_{-i}) \]

\[ = \left[ f_{p_i} \left( 4p_i^{\text{max}} - k_p p_i \right) + \frac{E_r(i)}{E_0(i) - E_r(i)} + \bar{E}_i(p_i) + \frac{1}{T+1} \right] - \left[ f_{q_i} \left( 4p_i^{\text{max}} - k_q q_i \right) + \frac{E_r(i)}{E_0(i) - E_r(i)} + \bar{E}_i(q_i) + \frac{1}{T+1} \right] \]

\[ = f_{p_i} \left( 4p_i^{\text{max}} - k_p p_i \right) - f_{q_i} \left( 4p_i^{\text{max}} - k_q q_i \right) \]  \quad (7)

Similarly,

\[ \Delta V = V(p_i, p_{-i}) - V(q_i, p_{-i}) \]

\[ = \sum_{i \in N} \left[ f_{p_i} \left( 4p_i^{\text{max}} - k_p p_i \right) + \frac{E_r(i)}{E_0(i) - E_r(i)} + \bar{E}_i(p_i) + \frac{1}{T+1} \right] - \left[ f_{q_i} \left( 4p_i^{\text{max}} - k_q q_i \right) + \frac{E_r(i)}{E_0(i) - E_r(i)} + \bar{E}_i(q_i) + \frac{1}{T+1} \right] \]

\[ = \Delta u + \sum_{j \in N, j \neq i} \left[ (f_{p_j} - f_{q_j}) \left( 4p_j^{\text{max}} - k_p p_j \right) \right] \]  \quad (8)

Let \( \lambda_j = (f_{p_j} - f_{q_j}) \left( 4p_j^{\text{max}} - k_p p_j \right) \), then

\[ \Delta V = \begin{cases} 
\Delta u, & f_{p_i} = f_{q_i} \quad (a) \\
\Delta u + \sum_{j \in N, j \neq i} \lambda_j, & f_{p_i} \neq f_{q_i} \quad (b) 
\end{cases} \]  \quad (9)

For case (a), it is obvious that \( \Delta V > 0 \Leftrightarrow \Delta u > 0 \). For case (b), \( f_{p_i} = 1 \) and \( f_{q_i} = 0 \), so \( \Delta u > 0 \) and \( \Delta V > 0 \). Then, for both case (a) and case (b), \( \Delta V > 0 \Leftrightarrow \Delta u > 0 \). According to Theorem 1, the strategy game is an ordinal potential game.

3. **EBTCA Algorithm**

Based on the proposed topology model, a topology control algorithm (EBTCA) is proposed, which comprehensively considers the balance of energy consumption and energy efficiency. EBTCA is divided into three stages: initial stage, game stage and adjustment stage.

3.1. **Initial Stage**

The purpose of initial stage is to obtain the information of neighbour nodes and the optional power set of each node. The initialization process of node \( i \) is as follows,
1) Node $i$ broadcasts a "HELLO" message with the maximum transmission power $p_i^{\text{max}}$, which contains ID, remaining energy, transmission power and its position.

2) After node $j$ receives the "HELLO" message from node $i$, it immediately sends an "ACK" message to node $i$ with the maximum transmission power $p_j^{\text{max}}$. The message contains node $j$'s ID, remaining energy, transmission power and its position.

3) If node $i$ receives the "ACK" message from node $j$, it adds node $j$ to its neighbor list and calculates the minimum transmission power $p_{ij}$ to maintain the link $l_{ij}$.

4) $p_i^{\text{max}}$, 0 and the minimum transmission power between node $i$ and all neighbours constitute the optional power set of node $i$.

3.2. Game Stage

In the game stage, each node adjusts power in turn according to the node ID, and selects the optimal transmit power according to the topology control model. The game process of node $i$ is as follows,

1) The optional power set of node $i$ is arranged in descending and marked as $P_i = \{p_1, p_2, ..., p_{k+2}\}$. Among them, $k$ is the number of neighbour nodes of node $i$, $p_1 = p_i^{\text{max}}$, $p_{k+2} = 0$.

2) Set the initial power of node $i$ to $p_1$, and calculate the utility function value according to the utility function. That is $p_i^* = p_1$, $u_i^* = u_i(p_1, p_{-i})$.

3) Calculate the utility function value from $p_2$ to $p_{k+2}$ in turn, and compare it with $u_i^*$. If $u_i(p_j, p_{-i}) > u_i^*$, then let $p_i^* = p_j$, $u_i^* = u_i(p_j, p_{-i})$.

4) After selecting the optimal transmission power $p_i^*$, node $i$ broadcasts the transmission power $p_i^*$ with the maximum transmission power $p_i^{\text{max}}$.

3.3. Adjustment Stage

When the network runs for a period of time, the remaining energy of the node will change, so the network topology should be dynamically adjusted. In order to unify the time for nodes to adjust the topology, it is agreed to replay the game stage every minute.

3.4. Time complexity analysis of EBTCA algorithm

Suppose that the number of nodes in the network is $n$, and the average number of neighbor nodes is $m$. In the initial stage, each node broadcasts a "HELLO" message and receives an "ACK" message. The time complexity of the initial stage is about $O(n)$. The main time cost in the game stage is the node's calculation of the utility function value at each power. Since $m$ is a very small number, the time of the game stage complexity is about $O(n)$. The time complexity for a node to broadcast its own selected power is $O(n)$. The time required for sorting optional power sets is negligible. Therefore, the time complexity of the entire algorithm is $O(n)$.

4. Results and Analysis

Through experiments, the EBTCA algorithm is compared with MLPT algorithm [5] and EFTCG algorithm [7]. Assuming that the base station is located in the center of the monitoring area, the nodes in the network are randomly distributed and their positions are fixed, and the initial energy and maximum transmission power of each node are the same. During the experiment, each node sends a 1024 bytes packet to other nodes at the transmission rate of 106 bits/s. In order for the data packet sent by node $i$ to be received by node $j$, there is a minimum value for the transmit power of node $i$. The minimum value is calculated as follows

$$\omega_j = \frac{p^{\text{th}}(4\pi)^2 L}{G_i G_j \lambda^2}$$  \hspace{1cm} (10)
Where $p_{th}$ is the signal capture threshold of the node, $d_{ij}$ is the distance between node $i$ and node $j$, $L$ is the system loss, $G_t$ is the transmit antenna gain, $G_r$ is the receive antenna gain, and $\lambda$ is the wavelength. The specific parameters are shown in Table 1.

| Parameter                        | Value       |
|----------------------------------|-------------|
| monitoring area                  | 300m×300m   |
| initial energy of nodes          | 50J         |
| maximum transmission distance    | 100m        |
| accept signal threshold          | $7 \times 10^{-10}$ $\text{W}$ |
| wavelength                       | 0.1224m     |
| transmit antenna gain            | 1           |
| receive antenna gain             | 10          |
| system loss                      | 1           |

Fig. 1 shows the average transmission power of the three algorithms when the number of nodes is 50 to 100. It can be seen that the average transmission power of nodes decreases with the increase of node density, and the average transmission power of EBTCA algorithm is always lower than that of MLPT and EFTCG. The lower transmission power can reduce the energy consumption and prolong the lifetime of nodes.

![Figure 1. Average transmit power](image)

Fig. 2 shows the residual energy standard deviation of the three algorithms when the number of nodes is 100. Due to the use of the Theil index, the residual energy standard deviation of the EBTCA algorithm increases the slowest. This means that the TFTG algorithm is superior to the other two algorithms in terms of energy balance.

![Figure 2. Residual energy standard deviation](image)
Figure 2. Standard deviation of residual energy

If the failure time of the first node is taken as the network lifetime, Fig. 3 is a comparison of the network lifetime with 50 to 100 nodes. Network lifetime is an important index to evaluate topology control algorithms. Because the EBTCA algorithm reduces the transmission power of the node and improves the energy balance by introducing the Theil index, the network survival time of the Thiel algorithm is higher than that of the other two algorithms.

Figure 3. Network lifetime

5. Conclusions
In order to ensure the safety of train operation, it is a general trend to apply WSN to train environment monitoring. The energy of nodes in WSN is limited, and the uneven energy consumption can easily shorten the network lifetime. In this paper, a utility function considering energy efficiency and energy balance is designed by combining game theory and Theil index. On this basis, a topological game algorithm EBTCA for energy consumption equilibrium is proposed. From the simulation results, it can be concluded that the EBTCA algorithm can effectively improve the energy balance and prolong the network lifetime.

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