Combined Task and Motion Planning as Classical AI Planning

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Abstract—Planning in robotics is often split into task and motion planning. The high-level, symbolic task planner decides what needs to be done, while the motion planner checks feasibility and fills up geometric detail. It is known however that such a decomposition is not effective in general as the symbolic and geometrical components are not independent. In this work, we show that it is possible to compile task and motion planning problems into classical AI planning problems; i.e., planning problems over finite and discrete state spaces with a known initial state, deterministic actions, and goal states to be reached. The compilation is sound, meaning that classical plans are valid robot plans, and probabilistically complete, meaning that valid robot plans are classical plans when a sufficient number of configurations is sampled. In this approach, motion planners and collision checkers are used for the compilation, but not at planning time. The key elements that make the approach effective are 1) expressive classical AI planning languages for representing the compiled problems in compact form, that unlike PDDL make use of functions and state constraints, and 2) general width-based search algorithms capable of finding plans over huge combinatorial spaces using weak heuristics only. Empirical results are presented for a PR2 robot manipulating tens of objects, for which long plans are required.

I. INTRODUCTION

Planning problems in robotics involve robots that move around, while manipulating objects and avoiding collisions. These problems are thought to be outside the scope of standard AI planners, and are normally addressed through a combination of two types of planners: task planners that handle the high-level, symbolic reasoning part, and motion planners that handle motion and geometrical constraints [1] [11] [23] [24] [14]. These two components, however, are not independent, and hence, by giving one of the two planners a secondary role in the search for plans, approaches based on task and motion decomposition tend to be ineffective and result in lots of backtracks [16].

In recent years, there have been proposals aimed at addressing this combinatorial problem by exploiting the efficiency of modern classical AI planners. In one case, the spatial constraints are taken into account as part of a goal-directed replanning process where optimistic assumptions about free space are incrementally refined until plans are obtained that can be executed in the real environment [31]. In another approach [7], geometrical information is used to update the heuristic used in the FF planner [13]. Other recent recent approaches appeal instead to SMT solvers suitable for addressing both task planning and the geometrical constraints [26] [3].

The work in this paper also aims at exploiting the efficiency of modern classical AI planning algorithms but departs from prior work in two ways. First, task and motion problems are fully compiled into classical planning problems so that the classical plans are valid robot plans. Motion planners and collision checkers [17] are used in the compilation but not in the solution of the classical problem. The compilation is thus sound, and probabilistically complete in the sense that robot plans map into classical plans provided that the number of sampled robot configurations is sufficient. In order to make the compiled problems compact, we move away from the standard PDDL planning language and appeal instead to Functional STRIPS [8], a planning language that is expressive enough to accommodate procedures and state constraints. State constraints are formulas that are forced to be true in every reachable state, and thus represent implicit action preconditions.

In the CTMP planning encoding, state constraints are used to rule out spatial overlaps. Procedures are used in turn for testing and updating robot and object configurations, and their planning-time execution is made efficient by precompiling suitable tables. The size and computation of these tables is also efficient, and allows us to deal with 3D scenarios involving tens of objects and a PR2 robot simulated in Gazebo [15].

The second departure from prior work is in the classical planning algorithm itself. Previous approaches have built upon classical planners such as FF and LAMA [13] [28], yet such planners cannot be used with expressive planning languages that feature functions and state constraints. The Functional STRIPS planner FS [5] handles functions and can derive and use heuristics, yet these heuristics are expensive to compute and not always cost-effective to deal with state constraints. For these reasons, we build instead on a different class of planning algorithm, called best-first width search (BFWS), that has been recently shown to produce state-of-the-art results over classical planning benchmarks [20]. An advantage of BFWS is that it relies primarily on exploratory novelty-based measures, extended with simple goal-directed heuristics. For this work, we adapt BFWS to work with Functional STRIPS with state constraints, replacing a Functional STRIPS heuristic that is
expensive and does not take state constraints into account by a fast and simple heuristic suited to pick and place tasks.

Given that classical AI planning is planning over finite and discrete state spaces with a known initial state, deterministic actions, and a goal state to be reached [9], it is not surprising that the combined task and motion planning can be fully compiled into a classical planning problem once the continuous configuration space is suitably discretized or sampled [17]. Moreover, modern classical planners scale up very well and like SAT or SMT solvers are largely unaffected by the size of the state space. If this approach has not been taken before, it is thus not due to the lack of efficiency of such planners but due to the limitations of the languages that they support [24]. Indeed, there is no way to compile non-overlap physical constraints into PDDL in compact form. We address this limitation by using a target language for the compilation that makes use of state constraints to rule out physical overlaps during motions, and procedures for testing and updating physical configurations. This additional expressive power prevents the use of standard heuristic search planning algorithms [13] [28] but is compatible with a more recent class of width-based planning methods that are competitive with state-of-the-art heuristic search approaches [21] [20].

The paper is organized as follows. We describe first the planning language and how the combined task and motion planning problem is modeled as a classical problem. We present then the preprocessing involved, the planning algorithm, and the empirical results. Videos displaying some of the problems and plans can be seen at [bitly/2fnXeAd](http://bitly/2fnXeAd)

## II. PLANNING LANGUAGE

For making a general use of functions and procedures in the planning encoding, we use Functional STRIPS, a logical extension of the STRIPS planning language [3]. Functional STRIPS is a general modeling language for classical planning that is based on the variable-free fragment of first-order-logic where action a have preconditions Pre(a) and effects f(t) := t', where the precondition Pre(a) and goals G, are variable-free, first-order formulas, and f(t) and t' are terms with f being a fluent symbol. Functional STRIPS assumes that fluent symbols, namely, those symbols whose denotation may change as a result of the actions, are all function symbols. Constant, functional and relational (predicate) symbols whose denotation does not change are called fixed symbols, and their denotation must be given either extensionally by enumeration, or intentionally by means of procedures as in [4] [25].

Terms, atoms, and formulas are defined from constant, function, and relational symbols in the standard first-order-logic way, except that in order for the representation of states to be finite and compact, the symbols, and hence the terms, are typed. A type is given by a finite set of fixed constant symbols. The terms f(c) where f is a fluent symbol and c is a tuple of fixed constant symbols are called state variables, as the state is just the assignment of values to such “variables”.

As an example, the action of moving a block b onto another block b' can be expressed by an action move(b, b') with precondition [clear(b) = true ∧ clear(b') = true], and effects loc(b) := b' and clear(loc(b)) := true. In this case, the terms clear(b) and loc(b) for block b stand for state variables. clear(loc(b)) is a valid term, but not a state variable, as loc(b) is not a fixed constant symbol. The denotation of the term clear(loc(b)) in a state is a function of the loc(b) and clear(b) state variables; whenever loc(b) = b' holds in a state, the value of clear(loc(b)) will be that of the state variable clear(b').

Formally, a state is an assignment of values to the state variables that determines a denotation (value) for every term and formula in the language. The denotation of a symbol or term t in the state s is written as t, while the denotation r* of terms made up of fixed symbols only and which does not depend on the state, is written as r*. By default, non-standard fixed constant symbols c, which usually stand for object names, are assumed to denote themselves, meaning that e* = e. The states s just encode the denotation f* of the functional fluent symbols, which as the types of their arguments are all finite, can be represented as the value [f(c)]* of a finite set of state variables. The denotation [f(t)]* of a term f(t) for an arbitrary tuple of terms t, is then given by the value [f(c)]* of the state variable f(c) where e* = t'. The denotation e* of all terms, atoms, and formulas e in the state s follows in the standard way.

An action a is applicable in a state s if [Pre(a)]* = true, and the state s0 that results from the action a in s satisfies the equation f*(t*) = w* for all the effects f(t) := w of a, and otherwise is equal to s. This means that the action a changes the value of the state variable f(c) to w* in the state s iff there is an effect f(t) := w of action a such that t* = c. For example, if X = 2 is true in s, the update X := X + 1 increases the value of X to 3 without affecting other state variables. Similarly, if loc(b) = b' is true in s, the update clear(loc(b)) := true in s is equivalent to clear(b') := true.

A problem is a tuple P = ⟨S, I, O, G, F⟩ where S includes the non-standard symbols (fixed and fluent) and their types, the atoms I and the procedures in F provide the initial denotation s0 of such symbols, O stands for the actions, and G is the goal. A plan for P is a sequence of applicable actions from O that maps the state s0 into a state s that satisfies G. It is assumed that standard symbols like “+”, 1, etc. have their standard denotation. Fixed functional symbols f whose denotation is given by means of procedures in F are written as #f. The denotation of the other functional symbols must be given extensionally in I.

### A. State Constraints

A. We will make use of a small fragment of Functional STRIPS, we will also need a convenient extension; namely, state constraints [18] [30]. State constraints are formulas that are forced to be true in all reachable states, something achieved by interpreting state constraints as implicit action preconditions. State constraints are not to be confused with state invariants that refer to formulas that are true in all reachable states without imposing extra constraints on actions. For example, in the blocks world, the formula describing that no block is on
the action

which may require moving obstructing objects as well. The configuration to a final configuration or set of configurations, the initial state.

for each such action

as (explicit) action preconditions but their semantics is different:
The syntax for these formulas is the same as for those encoding stands for a set of formulas expressing the state constraints.

tuple $P$ provides initial values for the state variables

procedures. All objects assumed to have the same shape. Initial situation pro-

CTMP Model Fragment in Functional STRIPS: Action and state

Fig. 1: CTMP Model Fragment in Functional STRIPS: Action and state

constraints schemas. Abbreviations used. Symbols preceded by "@" denote

procedures. All objects assumed to have the same shape. Initial situation pro-

vides initial values for the state variables $Base$, $Arm$ (resting), $Traj$ (dummy), and $Conf(o)$ for each object. Goals describe target object configurations. State constraints prevent collisions during arm motions. Motion planners and collision checkers used at compilation time, not at plan time, as detailed in the Preprocessing section.

two blocks at the same time is a state invariant. On the other hand, if we assert the formula $\neg [on(b_3, b_4) \land on(b_4, b_5)]$ as a state constraint, we are ruling out actions leading to states where the formula $[on(b_3, b_4) \land on(b_4, b_5)]$ holds.

A Functional STRIPS problem with state constraints is a tuple $P' = (S, I, O, G, C, F)$ where the new component $C$ stands for a set of formulas expressing the state constraints. The syntax for these formulas is the same as for those encoding (explicit) action preconditions but their semantics is different: an action $a$ is deemed applicable in a state $s$ when both $[Pre(a)]^s = true$ and the state $s_a$ that results from applying $a$ to $s$ is such that $c_{e^a} = true$ for every state constraint $c \in C$.

A plan for $P'$ is thus a sequence of actions from $O$ that maps the state $s_0$ into a state $s$ that satisfies $G$, and such that for each such action $a$, $Pre(a)$ is true in the state $s$ where the action $s$ is applied, and all constraints in $C$ are true in the resulting state. It is assumed that the state constraints hold in the initial state.

III. Modeling Pick-and-Place Problems

We consider CTMP problems involving a robot and a number of objects located on tables of the same height. The tasks involve moving some objects from some initial configuration to a final configuration or set of configurations, which may require moving obstructing objects as well. The model is tailored to a PR2 robot using a single arm, but can be generalized easily.

The main state variables $Base$, $Arm$, and $Hold$ denote the configuration of the robot base, the arm configuration, and the content of the gripper, if any. In addition, for each object $o$, the state variable $Conf(o)$ denotes the configuration of object $o$.

The configuration of the robot base represents the 2D position of the base and its orientation angle. The configuration of the robot arm represents the configuration of the end effector: its 3D position, pitch, roll, and yaw. Finally, object configurations are 3D positions, as for simplicity we consider object that are symmetric, and hence their orientation angle is not relevant. There is also a state variable $Traj$, encoding the last trajectory followed by the robot arm, which is needed for checking collisions during arm motions. All configurations and trajectories are obtained from a preprocessing stage, described in the next section, and are represented in the planning encoding by symbolic ids. When plans are executed, trajectory ids become motion plans; i.e. precompiled sequences of base and arm join vectors, not represented explicitly in the planning problem.

The encoding assumes two finite graphs: a base graph, where the nodes stand for robot base configurations and edges stand for trajectories among pairs of base configurations, and an arm graph, where nodes stand for end-effector configurations (relative to a fixed base), and edges stand for arm trajectories among pairs of such configurations. The details for how such graphs are generated are not relevant for the planning encoding and will be described below. As a reference, we will consider instances with tens of objects, and base and arm graphs with hundreds of configurations each, representing thousands of robot configurations.

A fragment of the planning encoding featuring all the actions and the state constraints is shown in Figure 1. Actions $MoveBase(e)$ take an edge $e$ from the base graph as an argument, and update the base configuration of the robot to the target configuration associated with the edge. The precondition is that the source configuration of the edge corresponds to the current base configuration, and that the arm is the resting configuration $ca0$. Actions $MoveArm(t)$ work in the same way, but the edges $t$ of the arm graph are used instead.

There are also actions $Grasp(o)$ and $Place(o)$ for grasping and placing objects $o$. The grasping action requires that the gripper is empty and that $@graspable(Base, Arm, Conf(o))$ is true, where the procedure denoted by the symbol $@graspable$ checks if the robot configuration, as determined by its base and (relative) arm configuration, is such that object $o$ in its current configuration can be grasped by just closing the gripper. Likewise, the atoms $Hold = o$ and $@placeable(Base, Arm, Conf(o))$ are preconditions of the action $Place(o)$.

The total number of ground actions is given by the sum of the number of edges in the two graphs and the number of objects. This small number of actions is made possible by the planning language where robot, arm, and object configurations do not appear as action arguments. The opposite would be true in a STRIPS encoding where action effects are determined solely by the action (add and delete lists) and do not depend
on the state. The number of state variables is also small, namely, one state variable for each object and four other state variables. Atoms whose predicate symbols denote procedures, like \( \text{@graspable(Base,Arm,Conf(o))} \), do not represent state variables or fluents, as the denotation of such predicates is fixed and constant. These procedures play a key role in the encoding, and in the next section we look at the preprocessing that converts such procedures into fast lookup operations.

The only subtle aspect of the encoding is in the state constraints used to prevent collisions. Collisions are to be avoided not just at beginning and end of actions, but also during action execution. For simplicity, we assume that robot-base moves do not cause collisions (with mobile objects), and hence that collisions result exclusively from arm motions. We enforce this by restricting the mobile objects to be on top of tables that are fixed, and by requiring the arm to be in a suitable resting configuration (ca0) when the robot base moves. There is one state constraint \( \text{@nonoverlap(Base,Traj,Conf(o),Hold)} \) for each object, where \( \text{Traj} \) is the state variable that keeps track of the last arm trajectory executed by the robot. The procedure denoted by the symbol \( \text{@nonoverlap} \) tests whether a collision occurs between object \( o \) in configuration \( \text{Conf(o)} \) when the robot arm moves along the trajectory \( \text{Traj} \) and the robot base configuration is \( \text{Base} \). The test depends also on whether the gripper is holding an object or not. As we will show in the next section, this procedure is also computed from two overlap tables that are precompiled by calling the MoveIt collision-checker \( [32] \) a number of times that is twice the number of edges (trajectories) in the arm graph.

IV. PREPROCESSING

The planning encoding shown in Fig. [1] assumes a crucial preprocessing stage where the base and arm graphs are computed, and suitable tables are stored for avoiding the use of motion planners and collision checkers during planning time. This preprocessing is efficient and does not depend on the number of objects, meaning it can be used for several problem variations without having to call collision checkers and motion planners again. Indeed, except for the overlap tables, the rest of the compilation is local and does not depend on the possible robot base configurations at all.

To achieve this, we consider the robot at a virtual base \( B_0 = (x, y, \theta) \) with \( x = y = \theta = 0 \) in front of a virtual table whose height is the height of the actual tables, and whose dimensions exceed the (local) space that the robot can reach without moving the base. By considering the robot acting in this local virtual space without moving from this virtual base \( B_0 \), we will obtain all the relevant information about object configurations and arm trajectories, that will carry to the real robot base configurations \( B \) through a simple linear transformations that depend on \( B \). The computation of the overlap tables is more subtle and will be considered later.

First of all, the \( x, y \) space of the virtual table is discretized regularly into \( D \) position pairs \( x_i, y_i \). If the height of the objects is \( h' \) and the height of the tables is \( h \), then the virtual object configurations are set to the triplets \( (x_i, y_i, z) \) where \( z = h + h'/2 \). Each virtual object configuration represents a possible center of mass for the objects when sitting at location \( x_i, y_i \) over the virtual table. For each such configuration \( C = (x_i, y_i, z) \), \( k \) grasping poses \( A_i^j \) are derived from which an object at \( (x_i, y_i, z) \) could be grasped, and a motion planner (MoveIt) is called to compute \( k' \) arm trajectories for reaching such each grasping pose \( A_i^j \) through \( k' \) different waypoints from a fixed resting pose and the robot base fixed at \( B_0 \). This means that up to \( k \times k' \) arm trajectories are computed for each virtual object configuration, resulting in up to \( D \times k \times k' \) arm trajectories in total and up to \( k \times D \) grasping poses. For each reachable grasping pose \( A_i^j \), we store the pair \( (A_i^j, C) \) in a hash table. The table captures the function \( \text{vplace} \) that maps grasping poses (called arm configurations here), into virtual object configurations. The meaning of \( \text{vplace}(A) = C \) is that when the robot base is at \( B_0 \) in front the virtual table and the arm configuration is \( A \), an object on the gripper will be placed at the virtual object configuration \( C \).

The arm graph has as nodes the arm configurations \( A \) that represent reachable grasping poses \( A = A_i^j \) in relation to some virtual object configuration \( C \), in addition to the resting arm configuration. The arm trajectories that connect the resting arm configuration \( A_0 \) with an arm trajectory \( A \) provide the edge in the arm graph between \( A_0 \) and \( A \). The graph contains also the inverse edges that correspond to the same trajectories reversed. Grasping configurations that are not reachable with any trajectory from the resting arm configuration are pruned and virtual object configurations all of whose grasping poses have been pruned, are pruned as well.

The base graph is computed by sampling a number of configurations \( N_B \) near the tables and calling the MoveIt motion planner to connect each such configuration with up to \( k_B \) of its closest neighbours. The number of robot configurations results from the product of the number of arm configurations \( k \times D \) and the number of base configurations \( N_B \). In the experiments we consider numbers that go from tens to a few hundred and which thus result into thousands of possible robot configurations. The computation of the base and arm graphs defines the procedures used in the \text{MoveBase} and \text{MoveArm} actions that access the source and target configuration of each graph edge.

The set of (real) object configurations are then defined and computed as follows. The virtual object configuration \( C = (x, y, z) \) represents the 3D position of the object before a pick up or after a place action, with the arm at configuration \( A \) and the robot base at the virtual base configuration \( B_0 = (0, 0, 0) \). As the robot moves from this “virtual” base to an arbitrary base \( B \) in the base graph, the point \( X \) is determined by the transformation \( T_B(C) \) of \( C \). The set of actual object configurations is then given by such triplets \( T_B(C) = (x', y', z) \) for which 1) \( B \) is a
node of the base graph, 2) \( C \) is a virtual object configuration, and 3) the 2D point \( x', y' \) falls within a table in the actual environment. That is, while the virtual object configurations live only in the virtual table with the base fixed at \( B_0 \), the actual object configurations depend on the virtual object configurations, the base configurations, and the real tables in the working space. We will write \( T_B(C) = \perp \) when \( C \) and \( B \) are such that for \( T_B(C) = (x', y', z') \), the 2D point \( x', y' \) does not fall within a table in the actual environment. In such a case, \( T_B(C) \) doesn’t denote an actual object configuration.

Given the linear transformation \( T_B \) and the function \( vpose(A) \) defined above, that maps an arm configuration into a virtual object configuration that is relative to the virtual base \( B_0 \), the procedures denoted by the symbols \( @graspable, @placeable \), and \( @pose \) in the planning encoding are defined as follows:

\[
@pose(B, A) = C' \quad \text{iff} \quad C' = T_B(vpose(A))
\]
\[
@graspable(B, A, C') = \text{true} \quad \text{iff} \quad C' = @pose(B, A)
\]
\[
@placeable(B, A) = \text{true} \quad \text{iff} \quad @pose(B, A) \neq \perp .
\]

We are left to specify the compilation of the tables required for computing the \( @nonoverlap \) procedure without calling a collision checker at planning time. This procedure is used in the state constraints \( @nonoverlap(B, Traj, Conf(o), Hold) \) for ruling out actions that move the arm along a trajectory \( Traj \) such that for the current base configuration \( B \) and content of the gripper \( Hold \), will cause a collision with some object \( o \) in its current configuration \( Conf(o) \). For doing these tests at planning time efficient, we precompile two additional tables, called the holding and non-holding overlap tables (HT, NT), which are made of pairs \( (Tr, C) \) where \( Tr \) is a trajectory in the arm graph, and \( C \) is what we will call a relative object configuration different than both the virtual and real object configuration. Indeed, the set of relative object configurations is defined as the set of configurations \( C^{-1}(C) \) for all bases \( B \) and all real object configurations \( C \), where \( C^{-1} \) is the inverse of the linear transformation \( T_B \) above. If \( C \) is a real 3D point obtained by mapping a point \( C' \) in the virtual table after the robot base changes from \( B_0 \) to \( B \), then \( C'' = T_B^{-1}(C) \) for \( B' = B \) is just \( C' \) but for \( B' \neq B \), it denotes a point in the “virtual” space relative to the base \( B_0 \) that may not correspond to a virtual object configuration, and may even fail to be in the space of the virtual table (the local space of the robot when fixed at base \( B_0 \)). Relative object configurations \( C'' \) that do not fall within the virtual table, are pruned. The holding overlap table (HT) contains then the pair \( (Tr, C) \) for a trajectory \( Tr \) and a relative object configuration, iff the robot arm moving along trajectory \( Tr \) will collide with an object in the virtual configuration \( C \) when the robot base is at \( B_0 \) and the gripper is carrying an object. Similarly, the pair \( (Tr, C) \) belongs to the non-holding overlap table (NT) iff the same condition arises when the gripper is empty. Interestingly, each of these two tables is compiled by calling a collision checker (MoveIt) a number of times that is given by the total number of arm trajectories. Indeed, for each trajectory \( T \), the collision checker tests in one single scan which relative configurations \( C \) are on the way.

The procedure \( @nonoverlap(B, Tr, Conf(o), Hold) \) checks whether trajectory \( Tr \) collides with object \( o \) in configuration \( Conf(o) \) when the robot base is \( B \). If \( Hold \) is None, this is checked by testing whether the pair \( (Tr, T_B^{-1}(Conf(o))) \) is in the NT table, and if \( Hold \) is not None, by testing whether the pair is in the HT table. These are lookup operations in the two (hash) tables NT and HT, whose size is determined by the number of trajectories and the number of relative object configurations. This last number is independent of the number of objects but higher than the number of virtual configurations. In the worst case, it is bounded by the product of the number \( N_B \) of robot bases and the number of real object configurations, which in turn is bounded by \( N_B \times N_C \), where \( N_C \) is the number of virtual object configurations. Usually, however, the number of entries in the overlap tables NT and HT is much less, as for most real object configurations \( C \) and base \( B \), the point \( T_B^{-1}(C) \) does not fall into the “virtual table” that defines the local space of the robot when fixed at \( B_0 \). The size of the hash table \( (Tr, C) \) precompiled for encoding the function \( vpose(Tr) \) above is smaller and given just by the number of arm trajectories \( Tr \), to the number of edges in the arm graph, which in turn is equal to \( 2 \times D \times k \times k' \), where \( D \) is the number of virtual object configurations, \( k \) is the number of grasping poses for each virtual object configuration, and \( k' \) in the number of trajectories for reaching each grasping pose.

V. PLANNING ALGORITHM

The compilation of task and motion planning problems is efficient and results in planning problems that are compact. Yet, on the one hand, standard planners like FF and LAMA do not handle functions and state constraints, while planners that do compute heuristics that in this setting are not cost-effective [5]. For these reasons, we build instead on a different class of planning algorithm, called best-first width search (BFWS), that combines some of the benefits of the goal-directed heuristic search with those of width-based search [19].

Pure width-based search algorithms are exploration algorithms and do not rely on goal directed heuristics. The simplest such algorithm is IW(1), which is a plain breadth-first search where newly generated states that do not make an atom \( X = x \) true for the first time in the search are pruned. The algorithm IW(2) is similar except that a state \( s \) is pruned when there are no atoms \( X = x \) and \( Y = y \) such that the pair of atoms \( (X = x, Y = y) \) is true in \( s \) and false in all the states generated before \( s \). More generally, the algorithm IW(k) is a normal breadth-first except that newly generated states \( s \) are pruned when their “novelty” is greater than \( k \), where the novelty of \( s \) is \( i \) iff there is a tuple \( t \) of \( i \) atoms such that \( s \) is the first state in the search that makes all the atoms in \( t \) true, with no tuple of smaller size having this property [19]. While simple, it has been shown that the procedure IW(k) manages to solve arbitrary instances of many of the standard benchmark domains in low polynomial time provided
that the goal is a single atom. Such domains can be shown indeed to have a small and bounded width $w$ that does not depend on the instance size, which implies that they can be solved (optimally) by running IW($k$). Moreover, IW($k$) runs in time and space that are exponential in $k$ and not in the number of problem variables. IW calls the procedures IW(1), IW(2), . . . sequentially until finding a solution. IW is complete but not effective in problem with multiple goal atoms. For this, Serialized IW (SIW) calls IW sequentially for achieving the goal atoms one at a time. While SIW is a blind search procedure that is incomplete (it can get trapped into dead-ends), it turns out to perform much better than a greedy best-first guided by the standard heuristics. Other variations of IW have been used for planning in the Atari games and those of the General Video-Game AI competition [22, 10, 29].

Width-based algorithms such as IW and SIW do not require PDDL-like planning models and can work directly with simulators, and thus unlike heuristic search planning algorithms, can be easily adapted to work with Functional STRIPS with state constraints. The problem is that by themselves, IW and SIW are not powerful enough for solving large CTMP problems. For such problems it is necessary to complement the effective exploration that comes from width-based search with the guidance that results from goal-directed heuristics. For this reason, we appeal to a combination of heuristic and width-based search called Best-First Width Search (BFWS), that has been shown recently to yield state-of-the-art results over the classical planning benchmarks [20]. BFWS is a standard best-first search with a sequence of evaluation functions $f = (h, h_1, \ldots, h_n)$ where the node that is selected for expansion from the OPEN list at each iteration is the node that minimizes $h$, using the other $h_i$ functions lexicographically for breaking ties. In the best performing variants of BFWS, the main function $h = w$ computes the “novelty” of the nodes, while the other functions $h_i$ take the goal into account.

For our compiled CTMP domain, we use BFWS with an evaluation function $f = (w, h_1, \ldots, h_n)$, where $w$ stands for a standard novelty measure, and $h_1, \ldots, h_n$ are simple heuristic counters defined for this particular domain. The novelty $w$ is defined as in [20]; namely, the novelty $w(s)$ of a newly generated state $s$ in the BFWS guided by the function $f = (w, h_1, \ldots, h_n)$ is $i$ if there is a tuple (conjunction) of $i$ atoms $X = x$, and no tuple of smaller size, that is true in $s$ but false in all the states $s'$ generated before $s$ with the same function values $h_1(s') = h_1(s)$, . . . , and $h_n(s') = h_n(s)$. According to this definition, for example, a new state $s$ has novelty 1 if there is an atom $X = x$ that is true in $s$ but false in all the states $s'$ generated before $s$ where $h_i(s') = h_i(s)$ for all $i$.

For the tie-breaking functions $h_i$ we consider three counter.

The first is the standard goal counter $\#g$ where $\#g(s)$ stands for the number of goal atoms that are not true in $s$. The second is an slightly richer goal counter $h_M$ that takes into account that each object that has to be moved to a goal destination has to involve two actions at least: one for picking up the object, and one for placing the object. Thus $h_M(s)$ stands for twice the number of objects that are not in their goal configurations in $s$, minus 1 in case that one such object is being held.

The last tie-breaker used corresponds to the counter $\#c(s)$ that tracks the number of objects that are in “obstructing configurations” in the state $s$. This measure is determined from a set $C$ of object configurations $C$ computed once from the initial problem state, as it is common in landmark heuristics. The count $\#c(s)$ is $i$ if there are $i$ objects $o$ for which the state variable $\text{Conf}(o)$ has a value in $s$ that is in $C$. The intuition is that a configuration is “obstructing” if it’s on the way of an arm trajectory that follows a suitable relaxed plan for achieving a goal atom. More precisely, we use a single IW(2) call at preprocessing for computing a plan for each goal atom in a problem relaxation that ignores state constraints (i.e., collisions). These relaxed problems are “easy” as they just involve robot motions to pick up the goal object followed by a pick up action, more robot motions, and a place action. The search tree constructed by IW(2) normally includes a plan for each goal atom in this relaxation, and often more than one plan. One such relaxed plan “collides” with an object $o$ if a MoveArm($t$) action in the plan leads to a state where a state constraint $\text{@nonoverlap(Base,Arm,Conf(o),Hold)}$ is violated (this is possible because of the relaxation). In the presence of multiple plans for an atomic goal in the relaxation, a plan is selected that collides with a minimum number of objects. For such an atomic goal, the “obstructing configurations” are the real object configurations $C$ such that a state constraint $\text{@nonoverlap(Base,Arm,C,Hold)}$ is violated in some state of the relaxed plan where $\text{Conf(o)} = C$ for some object $o$. We further consider as obstructing those configurations that in a similar manner obstruct the achievement of the goal of holding any object $o$ that is in an obstructing configuration in the initial state, recursively and up to a fixpoint. The set $C$ is then the union of the sets of “obstructing configurations” for each atomic goal, and $\#c(s)$ is the number of objects $o$ for which the value $C$ of the state variable $\text{Conf(o)}$ in $s$ belongs to $C$. Note that unlike the other two heuristics $\#g$ and $h_M$, which must have value zero in the goal, the $\#c(s)$ counter may be different than zero in the goal. Indeed, if a problem involves exchanging the configuration of two objects, $\#c(s)$ will be equal to 2 in the goal, as the two goal configurations are actually obstructing configurations as determined from the initial state. The set $C$ of obstructing configurations is computed once from the initial state in low polynomial time by calling the IW(2) procedure once. The resulting $\#c(s)$ count provides an heuristic estimate of the number of objects that need to be removed in order to achieve the goal, a version of the minimum constraint removal problem [12] mentioned in [7].

The counters $h_M$ and $\#c$ used in the BFWS algorithm for CTMP planning can be justified on domain-independent grounds. Indeed, $h_M$ corresponds roughly to the cost of a problem where both state constraints and preconditions involving procedures have been relaxed. So the plans for the relaxation are sequences of pickup and place actions involving the goal objects only. The counter $\#c$ is related to landmark heuristics under the assumption that the goals will be achieved.
We have a total of \( 480 \) feasible motion plans for the rest. The number of arm trajectories is \( 42 \). The total number of virtual grasping poses is \( 15 \). In turn, the arm is moved from the resting position to configurations where an object could be picked up or placed. This restriction reduces the average branching factor of the planning problem, in particular when the number of arm motions in the arm graph is large.

Finally, for the experimental results we have found useful to add an extra precondition to the action \( \text{MoveArm}(t) \). This precondition requires that \( \text{@target-a}(t) \) is the resting configuration \( ca0 \) or that \( \text{@placeable}(\text{Base}, \text{@target-a}(t)) \) is true. In other words, the arm is moved from the resting position to configurations where an object could be picked up or placed. This restriction reduces the average branching factor of the planning problem, in particular when the number of arm motions in the arm graph is large.

VI. EXPERIMENTAL EVALUATION

We test our model on two environments having one and three tables, the characteristics of which are shown in Table I. As explained above, the virtual space of the robot is discretized into \( D = 15 \) position pairs or virtual configurations, with \( k = 4 \) grasping poses per virtual configuration and \( k' = 4 \) arm trajectories for each of those grasping poses, obtained from Moveit. Thus, the maximum number of (virtual) grasping poses will be \( D \times k = 60 \), of which those for which no motion plan is found get pruned. In our benchmark environments, the total number of virtual grasping poses is \( 42 \). In turn, the maximum number of arm trajectories is \( D \times k \times k' = 240 \) in each direction, i.e. \( 480 \), while in both of our environments we have a total of \( 268 \) such trajectories, since again no feasible motion plans are found for the rest. The number of sampled bases is \( 124 \) for the one-table environment and \( 323 \) for the three-table environment, while each robot base in the base graph is connected to a maximum of \( 12 \) closest base configurations. Importantly, the output of the precompilation phase, which takes \( 5 \) min. (\( 13 \) min.) for the one-table (three-tables) environment, is valid for all instances with that number of tables, regardless of number of objects, initial robot and object configurations, and particular goals of the problem.

For each environment, we generate a number of semi-random instances with increasing number of objects, ranging from \( 10 \) to \( 40 \), and increasing number of goals, ranging from \( 2 \) to \( 8 \), where a problem with e.g. \( 4 \) goals might require that \( 4 \) different objects be placed in their respective, given target configurations. The initial and goal states of a sample problem instance are shown in Fig. 2 where the robot needs to place all blue objects in one table and all red objects in another. Tables IIa and IIb show the results of our BWFS planner on each generated instance, running with a maximum of 30 minutes and 8GB of memory on an AMD Opteron 6300@2.4Ghz. The planner uses ROS [27], Gazebo [15], and Moveit [32], in the preprocessing and in the simulations, but not at planning time. Videos showing the execution of the computed plans in the Gazebo simulator, for some selected instances, can be found in [bit.ly/2mXeAd]. The results show that our approach is competitive and scales well with the number of objects in the table. The length of the obtained plans ranges from \( 22 \) to \( 220 \) steps. Problems with up to \( 20 \) objects, both for one and three tables, for example, are solved in a few seconds and requiring only the expansion of a few thousands of nodes in the search tree. Problems with a up to \( 30 \) and even \( 40 \) objects are solved with relative ease in the environment with three tables, but as expected become much harder when we have one single table, because the objects clutter almost all available space, making it harder for the arm robot to move collision-free. Indeed, the results show that the key parameter for scalability is \( k' \), which in a sense indicates how cluttered the space is in the initial situation. When this number is not too high, as in the three-table environment, our approach scales...
up with relative ease with the number of different specified goals. Finally, preprocessing times scale up linearly with the number of objects, regardless of the number of goals, thanks to the low-polynomial cost of the IW(2) pass on which the preprocessing is based, as detailed above.

### VII. DISCUSSION

The presented work is closest to [6, 31]. What distinguishes our approach is that combined task and motion planning problems are fully mapped into classical AI planning problems encoded in an expressive planning language. Motion planners and collision checkers are used at compile time but not at planning time. The approach is sound (classical plans map into valid executable robot plans) and probabilistically complete (with a sufficient number of configurations sampled, robot plans have a corresponding classical plan). For the approach to be effective, three elements are essential. First, an expressive planning language that supports functions and state constraints. Second, a width-based planning algorithm that can plan effectively for models expressed in such a language without requiring the use of accurate but expensive heuristic estimators. Third, a preprocessing stage that computes the finite graphs of robot bases and arm configurations, the possible object configurations, and the tables that allow us to resolve procedural calls into efficient table lookups. We have shown that the compilation process is efficient and independent of the number of objects, that the compiled problems are compact, and that the planning algorithm can generate long plans effectively.

For the experiments, we have considered the type of pick and place problems that have been used in recent work [6, 31]. For these problems, it is sufficient to sample robot base configurations that are close to the physical tables, and arm trajectories that can pick up and place objects in the local space of the robot at a height that corresponds to the height of the tables. This part of the problem is not modeled explicitly in the Functional STRIPS planning encodings, which implicitly assume a finite graph of robot bases and one of robot arm configurations computed at preprocessing. In the future, we want to represent this information explicitly in the planning encoding so that the preprocessing stage can be fully general and automatic. This requires a general representation language for CTMP problems so that the compilation will be a mapping between one formal language and another. Unfortunately, there are no widely accepted and shared formal models and languages for CTMP, which makes it difficult to compare approaches empirically or to organize “CTMP competitions”, that in the case of AI planning or SAT solving.

### TABLE I: Compilation data for one and three tables

| #o | #g | #c | L | E | Prep | Search | Total |
|----|----|----|---|---|------|--------|-------|
| 10 | 1  | 4  | 38 | 700| 2.4  | 0.08   | 2.48  |
| 20 | 2  | 6  | 67 | 5.7k| 2.42 | 0.64   | 3.06  |
| 15 | 8  | 3  | 73 | 6.1k| 2.22 | 0.72   | 2.94  |
| 15 | 1  | 6  | 49 | 778| 3.4  | 0.1    | 3.5   |
| 15 | 2  | 8  | 81 | 9.8k| 3.76 | 1.27   | 5.03  |
| 15 | 3  | 10| 80 | 7.7k| 4.13 | 0.97   | 5.1   |
| 20 | 1  | 12| 86 | 39k | 5.44 | 4.46   | 9.9   |
| 20 | 2  | 14| 122| 63.3k| 5.85| 9.42   | 15.27 |
| 20 | 3  | 22| 159| 49.2k| 5.66| 7.26   | 12.92 |
| 25 | 1  | 4 | 22 | 206| 7.42 | 0.03   | 7.45  |
| 25 | 2  | 4 | 45 | 39.1k| 7.29| 5.54   | 12.83 |
| 25 | 3  | 18 | MO| - | - | - | - |
| 30 | 1  | 4 | 22 | 67.6k| 9.21| 10.16  | 19.37 |
| 30 | 2  | 38 | MO| - | - | - | - |
| 30 | 3  | 38 | TO| - | - | - | - |

(a) Manipulating objects, one single table.

### TABLE II: Per-instance results for one and three tables

| #o | #g | #c | L | E | Prep | Search | Total |
|----|----|----|---|---|------|--------|-------|
| 10 | 2  | 6  | 54 | 1.3k | 8.1 | 0.23 | 8.33 |
| 10 | 4  | 2  | 101| 3.9k | 8.1 | 0.8  | 8.9  |
| 10 | 6  | 2  | 121| 3.9k | 7.18| 0.6  | 7.78 |
| 10 | 8  | 2  | 150| 4.5k | 8.26| 0.91 | 9.17 |
| 20 | 2  | 4  | 65 | 6.2k | 19.19| 1.32 | 20.51 |
| 20 | 4  | 4  | 89 | 9.6k | 17.9 | 2.29 | 20.19 |
| 20 | 6  | 6  | 130| 3.1k | 17.66| 0.73 | 18.39 |
| 20 | 8  | 8  | 141| 5.9k | 18.42| 1.26 | 19.68 |
| 25 | 2  | 8  | 46 | 1.1k | 23.74| 0.23 | 23.97 |
| 25 | 4  | 8  | 80 | 2.3 | 24.44| 0.54 | 24.98 |
| 25 | 6  | 10 | 120| 3.5k | 27.04| 0.91 | 27.95 |
| 25 | 8  | 12 | 158| 3.4k | 23.74| 0.69 | 24.43 |
| 30 | 2  | 4  | MO| - | - | - | - |
| 30 | 4  | 2  | 74 | 1.6k | 30.37| 0.4  | 30.77 |
| 30 | 6  | 8  | 123| 2.6k | 30.09| 0.64 | 30.73 |
| 30 | 8  | 10 | 161| 3.5k | 32.22| 0.86 | 33.08 |
| 40 | 2  | 4  | 52 | 1.3k | 45.64| 0.33 | 45.97 |
| 40 | 4  | 14 | 114| 55.5k| 45.65| 13.12| 58.77 |
| 40 | 6  | 10 | 178| 166k| 47 | 41.36| 88.36 |
| 40 | 8  | 14 | 220| 201k| 46.46| 55.57| 102.03 |

(b) Manipulating objects, three tables.
have been an essential ingredient for progress. We believe that Functional STRIPS can actually serve both roles: as the basis for a general, integrated representation language for CTMP problems and as a convenient target language of the compilation representations. This work is a first step towards this goal where we have shown that the compilation is indeed feasible and effective both representationally and computationally.

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