Manipulation of light in a generalized coupled Nonlinear Schrödinger equation

R.Radha*,1 P.S.Vinayagam1

1 Centre for Nonlinear Science (CeNSc), PG and Research Department of Physics, Government College for Women (Autonomous), Kumbakonam 612001, India

K. Porsezian†,2

2 Department of Physics, Pondicherry University, Puducherry-605014, India.

Abstract

We investigate a generalized coupled nonlinear Schrödinger (GCNLS) equation containing Self-Phase Modulation (SPM), Cross-Phase Modulation (XPM) and Four Wave Mixing (FWM) describing the propagation of electromagnetic radiation through an optical fibre and generate the associated Lax-pair. We then construct bright solitons employing gauge transformation approach. The collisional dynamics of bright solitons indicates that it is not only possible to manipulate intensity (energy) between the two modes (optical beams), but also within a given mode unlike the Manakov model which does not have the same freedom. The freedom to manipulate intensity (energy) in a given mode or between two modes arises due to a suitable combination of SPM, XPM and FWM. While SPM and XPM are controlled by an arbitrary real parameter each, FWM is governed by two arbitrary complex parameters. The above model may have wider ramifications in nonlinear optics and Bose-Einstein Condensates (BECs).

Key words: Coupled Nonlinear Schrödinger system, Bright Soliton, Gauge transformation, Lax pair
2000 MSC: 37K40, 35Q51, 35Q55

* Corresponding author.

Email addresses: radha_ramaswamy@yahoo.com,
Telephone: (91)-0435-2403119, Fax: (91)-0435-2403119 (R.Radha*),
ponzsol@yahoo.com† (K. Porsezian†).
1 Introduction

The potential of solitons to carry information through optical fibres governed by the nonlinear Schrödinger (NLS) equation \[1,2,3,4\] and the freedom to switch energy between two laser beams in a fibre described by the celebrated Manakov model has made a dramatic turnaround in the field of optical communications. The concept of shape changing collisional dynamics of solitons in the Manakov model is governed by the coupled NLS equation of the following form

\[iq_1 t + q_{1xx} + 2(g_{11}|q_1|^2 + g_{12}|q_2|^2)q_1 = 0, \quad (1a)\]
\[iq_2 t + q_{2xx} + 2(g_{21}|q_1|^2 + g_{22}|q_2|^2)q_2 = 0, \quad (1b)\]

where \(q_i(x,t)(i=1,2)\) corresponds to the envelope of the electromagnetic radiation passing through the optical fibre and \(i\) is the imaginary unit. In the above equation, \(g_{11}\) and \(g_{22}\) correspond to the Self Phase Modulation (SPM) while \(g_{12}\) and \(g_{21}\) represent the Cross Phase Modulation (XPM). Equations \((1)\) have been shown to be integrable and admit Painlevé property \[5\] if either (i) \(g_{11} = g_{12} = g_{21} = g_{22}\) or (ii) \(g_{11} = g_{21} = -g_{12} = -g_{22}\). The first choice corresponds to the celebrated Manakov model \[6,7\] while the second choice represents the modified Manakov model \[8,9\] and it has been observed that both admit shape changing collisional dynamics of bright solitons \[10\]. Recently, it was shown \[11\] that one can rotate the trajectories of the bright solitons by varying the system parameters, namely SPM and XPM without violating the integrability of the Manakov (or modified Manakov) model.

It should be mentioned that the inelastic collision of bright solitons which is concerned with redistribution of energy between two modes (optical beams) is brought about by varying the parameters associated with the phase of bright solitons combined with a suitable combination of coupling coefficients. Can one manipulate the intensity (or energy) in a given mode (electromagnetic radiation) or in a given bound state of the electromagnetic radiation ?. Can one manipulate optical pulses by varying its interaction with the medium rather than changing the parameters associated with the phase of solitons ?. The answer to these questions assumes tremendous significance as one will have the flexibility of desirably energizing a given mode or a given bound state of the optical pulse as two laser beams propagate through optical fibres. This situation is reminiscent of manipulating binary interaction through Feshbach resonance \[12\] in Bose-Einstein Condensates \[13\].

In addition to Manakov (or modified Manakov) model, a generalized coupled NLS equation (GCNLS) by including Four Wave Mixing (FWM) with SPM and XPM has been investigated by Park and Shin \[14\] which subsequently led to the identification of four different classes of integrable models dealing with the propagation of optical beams through birefringent fibres. Even though
the variants of the GCNLS equation were investigated recently by Wang and Agalar et al.\cite{15,16} respectively, with three arbitrary parameters (two real parameters corresponding to SPM and XPM and one complex parameter for FWM), the impact of FWM when reinforced with SPM and XPM on the collisional dynamics of solitons has not yet been clearly spelt out. In addition, the impact of the freedom associated with four arbitrary parameters in a GCNLS has not been probed yet.

In this paper, we investigate a GCNLS equation involving four system parameters. The two real arbitrary parameters are associated with SPM and XPM while the two arbitrary complex parameters are associated with Four Wave Mixing (FWM). We then construct the Lax-pair of the GCNLS equation and generate bright solitons. We then show that one can not only manipulate the intensity (or energy) between two laser beams, but also manoeuvre the energy distribution among the bound states of a given laser beam. The freedom to manipulate intensity arises from a suitable combination of nonlinear interaction parameters associated with the system.

2 Mathematical model and Lax-Pair

We know that the coupling between co-propagating optical beams in a nonlinear medium determines the application of optical fibres. Considering the propagation of optical pulses through a nonlinear birefringent fibre, the dynamics is governed by the generalized coupled NLS (GCNLS) equation of the following form

\begin{align}
  i\psi_{1t} + \psi_{1xx} + 2(a|\psi_1|^2 + c|\psi_2|^2 + b\psi_1\psi_2^* + d\psi_2\psi_1^*)\psi_1 &= 0, \quad (2a) \\
  i\psi_{2t} + \psi_{2xx} + 2(a|\psi_1|^2 + c|\psi_2|^2 + b\psi_1\psi_2^* + d\psi_2\psi_1^*)\psi_2 &= 0, \quad (2b)
\end{align}

In equations\cite{2}, $\psi_1$ and $\psi_2$ represent the strengths of electromagnetic beams. The nonlinear coefficients $a$ and $c$ which are real account for the SPM and XPM respectively while the arbitrary real parameters $b$ and $d$ correspond to FWM. It should be mentioned that even though one can allow FWM parameters $b$ and $d$ to be complex, we have retained them to be real in the present model. When FWM effects ($b$ and $d$) are equal to zero and the coefficients $a = c$, then the above model reduces to the celebrated Manakov or modified Manakov\cite{6,8} model. Equation\cite{2} has also been investigated recently for $d = b^*$\cite{15} and the impact of FWM on the collisional dynamics of solitons has been investigated. The above equation\cite{2} admits the following linear eigenvalue problem of the following form,
\[ \Phi_x + U\Phi = 0, \quad (3) \]
\[ \Phi_t + V\Phi = 0, \quad (4) \]

where \( \Phi = (\phi_1, \phi_2, \phi_3)^T \) and

\[ U = \begin{pmatrix} i\zeta_1 & \psi_1 & \psi_2 \\ -R_1 - i\zeta_1 & 0 \\ -R_2 & 0 & -i\zeta_1 \end{pmatrix}, \quad (5) \]

\[ V = \begin{pmatrix} -i\zeta_1^2 + i\frac{1}{2}\psi_1 R_1 + i\frac{1}{2}\psi_2 R_2 - \zeta_1 \psi_1 - \frac{i}{2}\psi_1 x - \zeta_1 \psi_2 + i\frac{1}{2}\psi_2 x \\ \zeta_1 R_1 + i\frac{1}{2} R_{1x} \psi_1 - i\zeta_1^2 - i\frac{1}{2}\psi_1 R_1 - i\frac{1}{2}\psi_2 R_1 \\ \zeta_1 R_2 + i\frac{1}{2} R_{2x} - i\frac{1}{2}\psi_1 R_2 - i\zeta_1^2 - i\frac{1}{2}\psi_2 R_2 \end{pmatrix}, \quad (6) \]

and

\[ R_1 = -a\psi_1(x,t)^* - b\psi_2(x,t)^*, \]
\[ R_2 = -d\psi_1(x,t)^* - c\psi_2(x,t)^*. \]

In the above equation, the spectral parameter \( \zeta_1 \) is isospectral. It is obvious that the compatibility condition \( (\Phi_x)_t = (\Phi_t)_x \) leads to the zero curvature equation \( U_t - V_x + [U, V] = 0 \) which yields the integrable generalized coupled NLS equation \( (2) \).

Recently Agalarov et al.,\[16\] investigated the GCNLS equation \( (2) \) for \( d = b^* \) and transformed it to Manakov model. It should be mentioned that equation \( (2) \) with four independent arbitrary parameters \( a, c, b \) and \( d \) can also be mapped onto the celebrated Manakov model only for the parametric choice \( b = d \) and \( ac - bd = \sigma = \pm 1 \) under the following transformation

\[ \psi_1 = q_1 - dq_2, \quad \psi_2 = aq_2. \quad (7) \]

so that we obtain equation \( (1) \) (after suitable algebraic manipulation) with \( g_{11} = g_{21} = a, \ g_{12} = g_{22} = a\sigma \).

We again emphasize that GCNLS equation \( (2) \) offers the freedom to choose arbitrary \( a, b, c \) and \( d \) and the Manakov and modified Manakov model only arises as a special case of equation \( (2) \). We would like to emphasize that any conversion of GCNLS equation \( (2) \) to either Manakov (or modified Manakov) model deprives us the freedom to choose \( a, b, c \) and \( d \) arbitrarily.
Fig. 1. Elastic collision of solitons in the celebrated Manakov model for $a \equiv c = 1$ and $b \equiv d = 0$, $\alpha_{10} = 0.15$, $\alpha_{20} = 0.15$, $\beta_{10} = 0.25$, $\beta_{20} = 0.25$, $\chi_1 = 2$, $\chi_2 = 3$, $\delta_1 = 4$, $\delta_2 = 5$, $\epsilon^{(1)}_1 = 0.5$, $\epsilon^{(2)}_1 = 0.5$

Fig. 2. FWM induced rotation and enhancement of intensities of solitons for $a \equiv c = 1$ and $b = d = 0.5$ with the other parameters as in fig. (1)

3 Bright Solitons and Collisional Dynamics

It is worth pointing out at here that the above type of equation such as Davey-Stewartson (DS) equation has been investigated by different methods like first integral method [17], variational iteration method [18] and decomposition method [19], but gauge transformation approach [20] which is employed to investigate the model equations. [2] is more effective and handy to generate multi soliton solution. Employing gauge transformation approach one obtains
Fig. 3. Enhancement of the intensity $I_1$ by manipulating $d = 3.5$ keeping the other parameters as in fig. (2) except $\varepsilon^{(1)}_1 = 0.8$ and $\varepsilon^{(2)}_1 = 0.5$

\begin{equation}
\psi^{(1)}_1 = 2\varepsilon^{(1)}_1 \beta_1 \text{sech}(\theta_1) e^{i(-\xi_1)},
\end{equation}

\begin{equation}
\psi^{(1)}_2 = 2\varepsilon^{(1)}_2 \beta_1 \text{sech}(\theta_1) e^{i(-\xi_1)},
\end{equation}

where

\begin{align}
\theta_1 &= 2x\beta_1 - 4\int (\alpha_1 \beta_1) dt + 2\delta_1, \\
\xi_1 &= 2x\alpha_1 - 2\int (\alpha_1^2 - \beta_1^2) dt - 2\chi_1.
\end{align}

with $\alpha_1 = \alpha_{10}(a\tau_1^2 + b\tau_1 \tau_2 + d\tau_1 \tau_2 + c\tau_2^2)$, $\beta_1 = \beta_{10}(a\tau_1^2 + b\tau_1 \tau_2 + d\tau_1 \tau_2 + c\tau_2^2)$ while $\delta_1$, $\chi_1$, $\tau_1$ and $\tau_2$ are arbitrary parameters and $\varepsilon^{(1)}_1, \varepsilon^{(2)}_1$ are coupling parameters, subject to the constraint $|\varepsilon^{(1)}_1|^2 + |\varepsilon^{(2)}_1|^2 = 1$.

It is obvious from the above that the amplitude of the bright solitons depends not only on the SPM and XPM parameters $a$ and $c$, but also on FWM parameters $b$ and $d$. This freedom in the system parameters can be manipulated
to switch energy between two light pulses or between two bound states of a
given light pulse. The gauge transformation approach [20] can be extended to
generate multisoliton solution. For example, the two-soliton solution
given light pulse. The gauge transformation approach [20] can be extended to
generate multisoliton solution. For example, the two-soliton solution $\psi^{(2)}_{1,2}$ for
the two modes can be expressed as

$$
\psi^{(2)}_1 = 2I \frac{A_1}{B},
$$

(12a)

$$
\psi^{(2)}_2 = 2I \frac{A_2}{B},
$$

(12b)

where

$A_1 = M_{121} M_{222} (\zeta_2 - \zeta_1) (\zeta_1 - \bar{\zeta}_1) (\zeta_2 - \bar{\zeta}_2) + M_{122} M_{221} (\zeta_2 - \zeta_1) (\bar{\zeta}_2 - \zeta_1) (\bar{\zeta}_2 - \bar{\zeta}_1) + M_{112} M_{221} (\zeta_2 - \zeta_1) (\zeta_2 - \bar{\zeta}_1) (\bar{\zeta}_2 - \bar{\zeta}_1)$,

$A_2 = M_{121} M_{211} (\zeta_2 - \zeta_1) (\zeta_1 - \bar{\zeta}_1) (\zeta_2 - \bar{\zeta}_2) + M_{112} M_{212} (\zeta_2 - \zeta_1) (\zeta_2 - \bar{\zeta}_1) (\zeta_2 - \bar{\zeta}_2) + M_{212} M_{221} (\zeta_2 - \zeta_1) (\zeta_2 - \bar{\zeta}_1) (\zeta_2 - \zeta_1) (\zeta_2 - \zeta_1) (\zeta_2 - \zeta_1)$,

$B = (M_{122} M_{211} + M_{121} M_{212}) (\zeta_1 - \zeta_1) (\zeta_2 - \zeta_2) + (M_{112} M_{221} + M_{111} M_{222}) (\zeta_2 - \zeta_1) (\zeta_2 - \zeta_1)$,

$\zeta_2 = \bar{\zeta}_2^* = \alpha_2 + i\beta_2$,

$$
M_{11j} = e^{-\theta_j} \sqrt{2}; \quad M_{12j} = e^{-i\xi_j} \varepsilon_1^{(j)}; \quad M_{13j} = e^{-i\xi_j} \varepsilon_2^{(j)};
$$

$$
M_{21j} = e^{i\xi_j} \varepsilon_1^{*(j)}; \quad M_{22j} = e^{\theta_j} / \sqrt{2}; \quad M_{23j} = 0;
$$

$$
M_{31j} = e^{i\xi_j} \varepsilon_2^{*(j)}; \quad M_{32j} = 0; \quad M_{33j} = e^{\theta_j} / \sqrt{2},
$$

where

$$
\theta_j = 2\beta_j x - 4 \int (\alpha_j \beta_j) dt + 2\delta_j,
$$

(13)

$$
\xi_j = 2\alpha_j x - 2 \int (\alpha_j^2 - \beta_j^2) dt - 2\chi_j,
$$

(14)

and $j = 1, 2$

The two soliton solution given by equations [12 14] can be rewritten asymptotically (i.e) at $t = \pm \infty$ in the following form [15]
Before collision:

\[
\psi_{(1)}^{(2-)} = A_1^{(2-)} \xi_1^{(1)} \left[ \frac{C_1 + C_2 + C_3 + C_4}{B_1 + B_2} \right] e^{i(\xi_1)},
\]
\[
\psi_{(2)}^{(2-)} = A_2^{(2-)} \xi_1^{(1)} \left[ \frac{C_1 + C_2 + C_3 + C_4}{B_1 + B_2} \right] e^{i(\xi_2)},
\]

(15)

After collision:

\[
\psi_{(1)}^{(2+)} = A_1^{(2+)} \xi_1^{(2)} \left[ \frac{C_1 + C_2 + C_3 + C_4}{B_1 + B_2} \right] e^{i(\xi_1 - \xi_2)},
\]
\[
\psi_{(2)}^{(2+)} = A_2^{(2+)} \xi_1^{(2)} \left[ \frac{C_1 + C_2 + C_3 + C_4}{B_1 + B_2} \right] e^{i(\xi_2 - \xi_1)},
\]

(16)

In the above expression, the (-) and (+) sign indicates before and after collision and the subscript and superscript depicts the component (mode) and soliton respectively, with,

\[
A_1^{(2-)} = \alpha_1 \left[ \frac{\zeta_1 - \zeta_2}{\zeta_1 - \zeta_2} \right], \quad A_2^{(2-)} = \beta_1 \left[ \frac{\zeta_1 - \zeta_2}{\zeta_1 - \zeta_2} \right],
\]
\[
A_1^{(2+)} = \alpha_2 \left[ \frac{\zeta_2 - \xi_1}{\zeta_2 - \xi_1} \right], \quad A_2^{(2+)} = \beta_2 \left[ \frac{\zeta_2 - \xi_1}{\zeta_2 - \xi_1} \right].
\]

(17)

where

\[
C_1 = \{-2\beta_2[(\alpha_2 - \alpha_1)^2 - (\beta_1^2 - \beta_2^2)] - 4i\beta_1\beta_2(\alpha_2 - \alpha_1)\} e^{(\theta_1^2 + \theta_1^2 + \xi_2^2)},
\]
\[
C_2 = \{-2\beta_2[(\alpha_2 - \alpha_1)^2 + (\beta_1^2 + \beta_2^2)]\} e^{(-\theta_1^2 + \xi_2^2)},
\]
\[
C_3 = \{-2\beta_1[(\alpha_2 - \alpha_1)^2 + (\beta_1^2 - \beta_2^2)] + 4i\beta_1\beta_2(\alpha_2 - \alpha_1)\} e^{(i\xi_1^2 + \theta_1^2 + \theta_2^2)},
\]
\[
C_4 = -4i\beta_1\beta_2[(\alpha_2 - \alpha_1) - i(\beta_1 - \beta_2)] e^{(i\xi_1^2 + \theta_1^2 - \theta_2^2)},
\]
\[
B_1 = -4\beta_1\beta_2[\sinh(\xi_1) \sinh(\xi_2) + \cos(\xi_1 - \xi_2)],
\]
\[
B_2 = 2 \cosh(\xi_1) \cosh(\xi_2) [(\alpha_2 - \alpha_1)^2 + (\beta_1^2 + \beta_2^2)],
\]

with \( \alpha_j = \alpha_{j0}(a\tau_1^2 + b\tau_2 + d\tau_1\tau_2 + c\tau_2^2), \beta_j = \beta_{j0}(a\tau_1^2 + b\tau_1\tau_2 + d\tau_1\tau_2 + c\tau_2^2) \) and the notation \( \theta_j - \theta_j^* = 2i\beta_j x - 4(\alpha_j^2 - \beta_j^2) t \) and \( \theta_j^* + \theta_j = 2\beta_j x - 4\alpha_j \beta_j t \) where \( j = 1, 2 \).

If we choose

\[
\frac{\alpha_{10}}{\alpha_{20}} = \frac{\beta_{10}}{\beta_{20}},
\]

(18)

with \( \xi_1^{(1)} = \xi_1^{(2)} \) (or) \( \xi_1^{(1)} \neq \xi_1^{(2)} \) keeping the condition \( |\epsilon_1^{(j)}|^2 + |\epsilon_2^{(j)}|^2 = 1, \)

\( j = 1, 2 \), one observes elastic collision of bright solitons. Any violation of the
above condition given by equation. \([18]\) results in inelastic collision of solitons leading to the exchange of energy between two optical beams (modes).

3.1 Intramodal collision of bright solitons

We first consider the elastic collision of solitons in the Manakov model shown in figure. \([1]\) under the parametric choice given by equation. \([18]\). The corresponding amplitudes of two soliton solution before and after collision can be rewritten as

\[
A_j^{(2-)} = \left( \frac{\alpha_1}{\beta_1} \right) (a \tau_{1}^2 + c \tau_{2}^2) \varepsilon_{1}^{(1)} \cdot \frac{\zeta_1 - \zeta_2^*}{\zeta_1 - \zeta_2},
\]

\[
A_j^{(2+)} = \left( \frac{\alpha_1}{\beta_1} \right) (a \tau_{1}^2 + c \tau_{2}^2) \varepsilon_{1}^{(2)} \cdot \frac{\zeta_2 - \zeta_1^*}{\zeta_2 - \zeta_1}, \quad j = 1, 2
\]

(19)

When we introduce FWM, the amplitudes of two solitons solution asymptotically take the following form
Fig. 6. Rotation of bright solitons and switching of energy for $b = 0.5$ and $d = 3.5$ keeping the other parameters the same as in fig. 5 except $\varepsilon_1^{(1)} = 0.8$ and $\varepsilon_1^{(2)} = 0.5$

$$A_j^{(2-)} = \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} \left( a \tau_1^2 + (b + d) \tau_1 \tau_2 + c \tau_2^2 \right) \varepsilon_1^{(1)} \frac{\zeta_1 - \zeta_2^*}{\zeta_1 - \zeta_2},$$

$$A_j^{(2+)} = \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} \left( a \tau_1^2 + (b + d) \tau_1 \tau_2 + c \tau_2^2 \right) \varepsilon_1^{(2)} \frac{\zeta_2 - \zeta_1^*}{\zeta_2 - \zeta_1^*}, \quad j = 1, 2 \quad (20)$$

From the above, it is obvious that the introduction of FWM parameters $b$ and $d$ contributes to the enhancement of intensities as shown in figure. 2. In addition, one observes the rotation of the trajectories of solitons.

Now, to enhance the intensity of a given bound state in a given mode (optical pulse), we manipulate the FWM parameter say $d$ and choose unequal coupling parameter $\varepsilon_1^{(1)} = 0.8$ & $\varepsilon_1^{(2)} = 0.5$. The corresponding density profile shown in figure. 3 enhances the intensity of one bound state ($I_1$) at the expense of the other ($I_2$) in each mode (optical pulse). By manipulating the real parameter $b$ instead of $d$ and keeping $\varepsilon_1^{(1)} = 0.5$ & $\varepsilon_1^{(2)} = 0.8$, the density profile is shown in figure. 4, where one observes the enhancement of $I_2$ at the expense of $I_1$ in each mode (optical beam).

3.2 Intermodal collision of bright solitons

To manipulate the intensity of a given optical beam (mode), we now begin with the celebrated inelastic collision of solitons in the Manakov model [21] shown in figure. 5 in the absence of FWM. The asymptotic form of two solitons can be written as
When we introduce unequal FWM now and reverse the choice of coupling parameters (i.e.,) \( \varepsilon^{(1)}_1 = 0.8 \) & \( \varepsilon^{(2)}_1 = 0.5 \), the intensity redistribution shown in figure (6) shows that one can manipulate energy in a given bound state of the beam desirably.

Interchanging the values of the FWM & coupling parameters results in the reversal of intensity distribution of the solitons as shown in fig.7.

The above results indicate that one can not only manoeuvre the intensity distribution between the light beams, but also manipulate the intensity distribution of the given bound state in a given mode (optical pulse) and this freedom arises due to a suitable combination of FWM, SPM and XPM. It is also worth pointing out that the introduction of unequal SPM and XPM alongwith FWM in the above interaction of solitons contributes to a marginal increase of intensities besides rotating the trajectories of solitons. From the above, we also observe that the intensity redistribution among the bound states of a given optical beam (mode) or between two optical beams (modes) through manipulation of FWM parameters \( b \) & \( d \) is always accompanied by rotation to sustain the stability of solitons.

When we neglect SPM and XPM in equation (2) (a=c=0), we obtain the
following coupled NLS equation

\[
\begin{align*}
  i\psi_{1t} + \psi_{1xx} + 2(b\psi_1^*\psi_2 + d\psi_2^*\psi_1^*)\psi_1 &= 0, \\
  i\psi_{2t} + \psi_{2xx} + 2(b\psi_1^*\psi_2 + d\psi_2^*\psi_1^*)\psi_2 &= 0.
\end{align*}
\] (22)

It should be mentioned that equation \((22)\) arises only as a special case of equation \((2)\) and the phenomenon of soliton reflection and noninteraction of solitons \([15]\) can also be obtained as a special case. It should also be mentioned that under suitable transformation, the model governed by GCNLS equation \([2]\) can be mapped onto its counterpart in Gross-Pitaevskii (GP) equation which means that one can switch matter wave intensities desirably and this mechanism can be employed for matter wave switching.

4 Discussion

In this paper, we have derived a generalized coupled NLS (CGNLS) equation containing four arbitrary real parameters with two real parameters corresponding to SPM and XPM and the other two real parameters accounting for FWM. The collisional dynamics of bright solitons shows that one can have the luxury of sustaining desirable intensity in a given bound state of the optical beam or in a given optical pulse and the celebrated Manakov does not have the same freedom. It should be emphasized that the manipulation of light intensities in the above GCNLS equation \((2)\) explicitly depends on the interaction of light with the medium unlike the Manakov model where the intensity redistribution occurs by changing the parameters associated with the phase of solitons. Our investigation may open the floodgates for optical and matter wave switching in nonlinear optics and BECs. It would be interesting to study the ramifications of complex FWM parameters on the dynamics of bright solitons.

5 Acknowledgements

Authors would like to acknowledge Dr. Telman Gadzhimuradov in sharing his perspective in improving the contents of the paper. PSV wishes to thank Department of Science and Technology (DST) for the financial support. RR wishes to acknowledge the financial assistance received from DST (Ref.No:SR /S2/HEP-26/2012), UGC (Ref.No:F.No 40-420/2011(SR), Department of Atomic Energy -National Board for Higher Mathematics (DAE-NBHM) (Ref.No: NBHM / R.P.16/2014/Fresh dated 22.10.2014) and Council of Scientific and Industrial Research (CSIR) (Ref.No: No.03(1323)/14/EMR-II dated 03.11.2014) dated 4.July.2011) for the financial support in the form Major Research Projects.
KP thanks the DST, NBHM, IFCPAR, DST-FCT and CSIR, Government of India, for the financial support through major projects.

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