We present polarization spectra near maximum light for the strongly subluminous Type Ia supernova SN 1999by that show that the supernova is intrinsically polarized. SN 1999by has an observed, overall level of polarization of ≈0.3%–0.8%, a rise of the polarization P redward of 6500 Å, and a change in polarization across the Si ii λ6150 feature of about 0.4%. The presentation of the polarization at different wavelengths in the Q-U plane is shown to be a powerful tool to determine the overall geometry and the interstellar component. The distribution of points with wavelength using this empirical Q-U plane method reveals that SN 1999by has a well-defined axis of symmetry and suggests an interstellar polarization (ISP) vector with P_{ISP} = 0.3% and position angle Θ = 150° with an error circle in the Q-U plane of radius about 0.1%. Synthetic non-LTE spectra for axisymmetric configurations based on delayed-detonation models have been computed assuming ellipsoidal geometry. The input ejecta structure and composition are based on a Chandrasekhar mass delayed-detonation model. The parameters of the explosion are chosen to reproduce the time evolution of IR spectra of SN 1999by without further adjustments. Spherical models are then mapped onto ellipsoidal geometries and the axis ratio, viewing angle, and ISP adjusted to provide the best agreement with the polarization spectra. Both flux and polarization spectra can be reasonably well reproduced by models with an asphericity of ≈20% observed equator-on. The general properties of the polarization can be understood as a consequence of the structure of subluminous models. Best fits are obtained for the theoretical models with P_{ISP} = 0.25% and Θ = 140°, consistent with the empirical method. We discuss our results for this subluminous Type Ia in the context of “normally bright” Type Ia supernovae. For normally bright Type Ia, the photosphere is near the inner iron-rich layers at maximum light and the ubiquitous iron lines give a rapid variation to the model polarization spectra. In subluminous models, the photosphere near maximum is in the silicon layers with fewer lines and a smoother overall polarization spectrum, as observed for SN 1999by. Though data are sparse, the low upper limits for polarization determined for many normal events in contrast to the high polarization in SN 1999by may suggest a relation between the asymmetry we observed and the mechanism that produces a subluminous Type Ia. Among various mechanisms, rapid rotation of the progenitor white dwarf and/or an explosion during a binary white dwarf merger process are likely candidates to explain the asphericity in SN 1999by.
Woosley & Weaver 1986; HK96). Only the first two scenarios seem to be in agreement with current observations. Delayed-detonation (DD) models (Khokhlov 1991; Yamaoka et al. 1992; Woosley & Weaver 1994) of Chandrasekhar mass CO WDs and their variations can account for the spectral and light-curve evolution of “normally bright” and subluminous SNe Ia in the optical and IR (Höflich 1995a; Höflich, Khokhlov, & Wheeler 1995a, hereafter HKW95; Wheeler et al. 1998; Lentz et al. 1999; Höflich et al. 2000). In this model, a burning front starts as a subsonic deflagration and then undergoes a transition to a supersonic detonation. Pure deflagration models like W7 (Nomoto, Thielemann, & Yokoi 1984) can also account for optical spectra of normal SNe (e.g., Nugent et al. 1997). The merging scenario remains an interesting alternative that to a great extent has only been parameterized in theoretical studies but may produce results in agreement with some SNe (HK96). The sub-Chandrasekhar model triggered by edge-lit helium ignition is disfavored on the basis of predictions of delayed detonations and different explosion scenarios, and discuss the implications for the use of SNe Ia as standard candles.

2. OBSERVATIONS AND DATA REDUCTION

2.1. The Imaging Grism Polarimeter

All observations were taken with the dual-beam Imaging Grism Polarimeter (IGP) at the Cassegrain focus of the McDonald Observatory 2.1 m telescope. IGP is the Imaging Grism Instrument (IGI) combined with spectropolarimetry optics (Goodrich 1991). IGI is a simple focal reducer with a grism that can be moved into the collimated beam. Off-the-shelf optics are used that have been anti-reflection coated for λ > 4000 Å. Polarization capability is provided by a modified Glan-Taylor polarizing beam splitter and a rotatable wave plate. The Glan-Taylor prism is made up of two calcite blocks. The ordinary ray (o-ray) is totally internally reflected, while the extraordinary ray (e-ray) passes through, as a result of different indices of refraction. The wave plate acts as a half-wave retarder and rotates the plane of polarization of incoming light. The half-wave plate is “superachromatic” with a retardance of 180° ± 2° from 3200 to 11,000 Å. The intrinsic polarization of IGP has been previously determined to be low, P < 0.1 (see Wang, Wheeler, & Höflich 1997a, hereafter WWH97; Wang et al. 2000b). This was confirmed with the observation of null polarization standards. More extensive information about the data reduction techniques, standard stars, and observational setup is available in Howell (2000).

2.2. Setup of the Instrument

A 2° slit was used with the 6000 Å grism, giving a resolution of about 14 Å. For all observations, the 85 mm lens was used in conjunction with the TK4 1024 × 1024 CCD, yielding a plate scale of 0′.5 (~3.9 Å) pixel−1. The spectra were wavelength calibrated with an argon lamp. Polarization standards were observed at least once per night. The observing log is shown in Table 1.

2.3. Data Reduction

The data were reduced in IRAF, using tasks written by the authors. The reduction methods followed were those presented in Miller, Robinson, & Goodrich (1988), Trammell (1994), and Howell (2000). For SN 1999by, two sets of data were taken each night with each “set” consisting of four 20–30 minute integrations at wave plate position angles of 0°, 45°, 22.5°, and 67.5°. Polarization and flux standards were observed each night. Since spectropolarimetry measurements require a high signal-to-noise ratio, the SN was observed over the course of three nights, and the data were combined. The data presented here have been corrected for redshift, though this correction is negligible (z = 0.00213).

2.4. SN 1999by

SN 1999by was discovered independently by R. Arbour, South Wonston, Hampshire, England (Arbour et al. 1999), and by the Lick Observatory Supernova Search (LOSS; see Treffers et al. 1997; Li et al. 1999). The SN was found on the outskirts (100° west and 91° north of the nucleus) of the well-known Sb galaxy NGC 2841. According to the NASA/IPAC Extragalactic Database² (NED), the host galaxy is a LINER and has a heliocentric radial velocity of 638 ± 3 km s−1. According to the Lyon/Meudon Extragalactic

² http://nedwww.ipac.caltech.edu/index.html.
produced SN 1912A (type uncertain) and the "peculiar Type I modulus of 30.48 mag. This prodigious galaxy has also pro-

\[ H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}, \] this puts the distance to NGC 2841 at 12.49 Mpc, with a distance modulus of 30.48 mag. This prodigious galaxy has also produced SN 1912A (type uncertain) and the "peculiar Type I supernovae" SN 1957A and SN 1972R.

Database\(^3\) (LEDA), the velocity corrected for Virgo infall is 811 km s\(^{-1}\). Using \( H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}, \) this puts the distance to NGC 2841 at 12.49 Mpc, with a distance modulus of 30.48 mag. This prodigious galaxy has also produced SN 1912A (type uncertain) and the "peculiar Type I supernovae" SN 1957A and SN 1972R.

According to Bonanos et al. (1999), SN 1999by reached a maximum light of \( B = 13.80 \pm 0.02 \) on UT 1999 May 10.5 and a maximum in the \( V \) band of \( V = 13.36 \pm 0.02 \) (date not given). According to W. Li (2000, private communication), the maxima were May 10.3 (\( B \)) and May 12.3 (\( V \)). Using a distance modulus of 30.48 and the photometry of Bonanos et al. (1999), the absolute magnitudes at peak are \( M_B = -16.68 \) and \( M_V = -17.12 \). Thus, SN 1999by is underluminous by roughly 2.5 mag compared to a

\(^3\) http://www.obs.univ-lyon1.fr/leda/home_leda.html.
typical SN Ia. SN 1999by also had a large $\Delta m_{15}(B)$ of 1.87. This makes SN 1999by nearly as steeply declining as SN 1991bg, which had $\Delta m_{15}(B) = 1.93$. SN 1999by is thus one of the most underluminous and rapidly declining SNe Ia known. SN 1999by is similar to SN 1991bg in other respects as well. The Si II line at 5800 Å is strong relative to the line at 6150 Å, another signature of the subluminous subclass. According to Bonanos et al. (1999), the flux near 4000 Å was depressed as in other subluminous SNe Ia, a feature thought to be due to Ti II absorption. The O I feature at 7500 Å is also strong in both SN 1991bg and SN 1999by.

3. RESULTS

3.1. Flux Spectra

Flux spectra of SN 1999by can be seen in Figure 1. Maximum light in $B$ occurred on May 10; therefore, the spectra shown are 2 days before, 1 day before, and at maximum light. Several features are apparent upon inspection (Fig. 1). Most obvious is the fact that SN 1999by is unquestionably a subluminous SN Ia. It is similar to the prototype, SN 1991bg.

Beginning at the blue side of the spectrum, we see that the spectrum starts near 4300 Å in a huge absorption feature. This is thought to be due to Ti II, a feature only observed in subluminous events (Filippenko et al. 1992, hereafter F92). The fact that this ionization state of Ti is seen indicates a relatively low temperature in the ejecta, since Ti II is ionized in typical SNe Ia (Nugent et al. 1995, hereafter N95). Ti II is also responsible for absorption features in the 4500–5000 Å region (F92). Moving redward, another signature of subluminous events is seen in the pair of Si II lines on either side of 5400 Å. As one moves down the sequence from overluminous to subluminous SNe, the blue line is overtaken in strength by the red one (N95). Here the bluer line is scarcely seen at all, consistent with our finding that SN 1999by is a very subluminous SN Ia. From the blueshift of the absorption features of Si, one can measure an approximate velocity of this element (and thus its placement in the ejecta, since $v \propto r$). For SN 1999by we measure $v(\text{Si}) \sim 10,000$ km s$^{-1}$, comparable to speeds of 10,000 and 10,600 km s$^{-1}$ reported for SN 1991bg by F92 and Leibundgut et al. (1993), respectively. This is slightly lower than the average Si velocity in a normally bright SN Ia, 11,000–13,000 km s$^{-1}$, though velocities derived from absorption minima of broad doublets as seen in the Si II 6150 feature are only accurate to 1000 km s$^{-1}$.

On the far red end of the spectrum, the absorption at 7500 Å is yet another hallmark of subluminosity. This line is due largely to a blend of O I λ 7774 and Mg II on the red wing, separated here by narrow atmospheric absorption. This feature is not seen in the luminous SN 1991T but increases in strength as one goes down the luminosity sequence to subluminous SNe Ia (N95). This O I feature is also absent in SN 1998de, a fast-declining SN Ia that was peculiar in several ways (Modjaz et al. 2001).

3.2. Polarization

The dominant sources of opacity in an SN Ia, line scattering and electron scattering, can both change the polarization of incident flux. Electron scattering polarization is independent of wavelength. In the case of axisymmetric scattering, the polarization position angle can take on only one of two values: 0° or 90° with respect to the symmetry axis (see, e.g., Cropper et al. 1988; J91; H91). Line scattering need not lead to complete angular redistribution. The degree to which line scattering can polarize depends on the lower and upper level total angular momenta of that line (J91). Despite the fact that lines can polarize, most line scattering is less polarizing than electron scattering by an order of magnitude. As a result, electrons are the chief polarization mechanism, while the lines generally depolarize the flux previously polarized by electron scattering. Lines can redirect photons by decay in the same transition, but they can also redistribute photons to both higher and lower frequencies by further excitation or ionization from the level at which the photon is absorbed or by decay into multiple discrete lower levels (J91; Höflich 1995a).

Dust can polarize by scattering through dichroic extinction and through emission of polarized photons. Dust scattering is usually wavelength dependent, though the results vary with scattering angle. For dust typical of the interstellar medium (ISM), the polarized flux of optical photons undergoing a single scattering can decrease with wavelength for small scattering angles, increase with wavelength for large scattering angles, or remain constant for $\Theta_{ac} \approx 90^\circ$ (White 1979; Webb et al. 1993). Non spherical dust grains with aligned rotation axes (from, e.g., a magnetic field) can be a source of polarization through dichroic extinction. In this case, the polarization position vector is perpendicular to the grain alignment vector.

Table 2 gives all the SNe Ia for which some polarization data are available. Figure 2 shows the normalized Stokes parameters $Q$ and $U$ and the flux spectrum of SN 1999by. The dashed line is the unsmoothed data. A darker colored line on the $Q$ and $U$ plots shows the data smoothed with a running boxcar smoothing function of 17 pixels (65 Å). Features from 4800 to 5100 Å are blue, from 5100 to 5270 Å light blue. The Si II features from 5270 to 5600 Å are green. The Si II features are marked by two different colors. The 5800 Å feature is yellow, while the 6150 Å line is magenta. Black represents the area between the Si features and the

![Fig. 1.—Flux spectra of SN 1999by at −2 days, −1 day, and $B$ maximum. The data are in arbitrary units. Earth symbols denote features not intrinsic to the SN.](image-url)
area slightly redward. Finally, the large, broad polarization feature to the red of 6500 Å is colored red.

In Figure 3 we present \( P = (Q^2 + U^2)^{1/2} \) for the unsmoothed data, but we note that this indicator is biased high for low signal-to-noise data. Therefore, we generate the smoothed version, \( P_s \) (dark solid line), from the smoothed \( Q \) and \( U \) values. In other words,

\[
P_s = \sqrt{Q_s^2 + U_s^2},
\]

where the subscript \( s \) denotes smoothed values. Similarly, the smoothed polarization position angle is

\[
\Theta_s = 0.5 \arctan \left( \frac{U_s}{Q_s} \right).
\]

In addition to \( P \), Figure 3 also shows \( \Theta \) and the associated errors \( \sigma(P) \) and \( \sigma(\Theta) \).

Figure 4 shows the smoothed values for \( Q \) and \( U \) in the \( Q-U \) plane for SN 1999by. The colors are identical to the color scheme used in Figure 2. In such a plot, each point defines a vector from the origin. The length of this vector is \( P_s \), and the orientation is \( 2\Theta_s \). Figure 4 and its implications are discussed further in § 4.

As is often the case in spectropolarimetry measurements, the data for any single set of observations have a low signal-to-noise ratio. Thus, we have combined the data for all three nights to produce Figures 2 and 3. These figures show the sum of all observations and consequently have the best signal-to-noise ratio, with \( \sigma(P) \sim 0.1\% \) and \( \sigma(P_s) \sim 0.025\% \). Note that despite this low formal statistical error, systematic errors prevent us from actually discriminating small-scale changes in \( P_s \) of order 0.05%–0.1%. We use this “lowest noise” case for most subsequent analysis, but we also look to the individual data sets for confirmation that features are real. Figure 5 shows the results of combining the data in different ways. The top panel shows \( P \) derived from all observations. The second panel shows \( P \) constructed from only the best four data sets. The third panel shows \( P \) from the best single data set, taken on May 10 when the SN was at maximum light. Finally, the bottom panel of Figure 5 shows the sum of the best data from May 8 and 9 combined with the second best data set from May 10. These are the next three best data sets after the best set from May 10. The dashed lines in Figure 5 are 1 \( \sigma \) errors. Our indicator \( P_s \) falls beneath the midpoint of the unsmoothed data because \( P \) is biased high as an indicator, though \( P_s \) is less biased. The degree of the bias to \( P \) increases with low signal-to-noise data. We also tested Wang’s indicator (WWH97) and the rotated Stokes parameter (e.g., Tran 1995), and both agree with \( P_s \) to within 0.02% across all wavelengths. It is important to remember that there is no perfect indicator for the total degree of polarization (Leonard et al. 2001; Simmons & Stewart 1985). Three features stand out in every data set. These are (1) the overall level of \( P \) \sim 0.25\%, (2) the change in \( \Theta \) across the Si II λ6150 feature, and (3) the rise in \( P \) to 0.6% at 6900 Å. We discuss each in turn.

The level of \( P \).—The first of these significant features is that throughout most of the spectrum \( P \sim 0.25\% \), with some excursions to lower values. Not every bump in the smoothed data in Figure 5 is significant because smoothing can introduce artifacts into low signal-to-noise data; however, the overall level of \( P_s \approx 0.25\% \) shown in Figure 3 is significant. The statistical error for \( P \) is \( \sigma(P) \sim 0.1\% \) in the region of interest, and the statistical error for the smoothed spectrum is \( \sigma(P_s) \sim 0.025\% \). We should also keep in mind that the fact that we have smoothed the data means that we cannot distinguish features narrower than 65 Å.

Si II λ6150.—An overall level of polarization is interesting but could be caused by interstellar or circumstellar sources. Perhaps the most striking feature of the spectropolarimetry of SN 1999by is the change in position angle, \( \Theta \), of the polarization through the Si II λ6150 feature as seen in Figure 3. The position angle jumps from 100° to 150° over this region, peaking at 6200 Å. As shown in the second panel of Figure 3, \( \sigma(\Theta) \sim 12° \) without smoothing. After smoothing, \( \sigma(\Theta) \sim 3° \). Even without smoothing, the change in \( \Theta \) across Si is a 4 \( \sigma \) result. This is clear evidence that the SN is intrinsically polarized.

**Table 2**

| SN     | Galaxy | Spec | Sub | Epoch | \( P \) (%) | Detection | Reference |
|--------|--------|------|-----|-------|-------------|-----------|-----------|
| 1972E  | NGC 5253 | b    | n   | 30    | 0.35 ± 0.2 | Undetermined | 1         |
| 1975N  | NGC 7723 | b    | n   | 0; 34: | 1.5         | ISP       | 2         |
| 1981B  | NGC 4536 | b    | n   | 56    | 0.41 ± 0.14 | Undetermined | 3         |
| 1983G  | NGC 4753 | s    | n   | −2:   | 2.0         | Upper limit | 4         |
| 1986G  | NGC 5128 | b    | s   | −9, −8 | 5.2         | ISP       | 5         |
| 1992A  | NGC 1308 | s    | n   | 15; 100: | 0.3 ± 0.3 | Undetermined | 6         |
| 1994D  | NGC 4526 | b    | n   | −10   | 0.35 ± 0.2 | Undetermined | 7         |
| 1994ae | NGC 3370 | b    | n   | ≥30   | 0.3         | Upper limit | 7         |
| 1995D  | NGC 2962 | b    | n   | 14, 41 | 0.2         | Upper limit | 7         |
| 1996X  | NGC 5061 | s    | n   | −7, 30 | 0.3 ± 0.3 | Maybe?     | 8         |
| 1997dt | NGC 7448 | s    | n   | ≥2; 0: | ?           | Yes?      | 9         |
| 1998bu | M 96    | b    | n   | −4, −3, −2 | 2.1 ± 0.1 | ISP       | 10        |
| 1999by | NGC 2841 | s    | s   | −2, −1, 0 | 0.8 ± 0.1 | Yes       | 11        |

* Type of measurement: broadband (b) or spectropolarimetric (s).
* Strength of SN: normal (n) or subluminous (s).
* Approximate number of days past maximum light observations were taken.
* Whether or not this is a detection of polarization intrinsic to the SN.

**References**—(1) Wolstenholme & Kemp 1972. (2) Shokhovskoi 1976. (3) Shapiro & Sutherland 1982. (4) McCullough et al. 1984a. (5) Hough et al. 1987. (6) Spyromilio & Bailey 1993. (7) Wang et al. 1996. (8) Wang, Wheeler, & Höflich 1997b. (9) Leonard et al. 2000. (10) Hernandez et al. 2000. (11) This paper.
It is also remarkable that the polarization change in Si is roughly equal to the polarization change across the entire wavelength region. At 4800 Å, Θ ≈ 150°, but by 7100 Å, Θ has dropped to ≈ 90°. This is significant because we will later show that there is a single choice of ISP that, when subtracted from the data, reduces the position angle across all wavelengths to nearly a single value.

The 6900 Å rise.—The rise in polarization toward the red peaks at P ≈ 0.6% at 6900 Å. This feature is wider (1000 Å) than any particular line, and it occurs in a region normally blanketed by Fe peak lines. Such a rise in polarization in this wavelength region was predicted theoretically by WWH97. We will revisit this feature when we discuss the theoretical interpretation of the data in § 5.

4. DATA ANALYSIS USING THE Q-U PLANE

The Q-U plane is a powerful tool for interpreting SN polarimetry data with respect to both testing the overall geometry and determining the interstellar component of the polarization.

4.1. Intrinsic Properties of SN 1999by

The Q-U plot for the smoothed data of SN 1999by is given in Figure 4. Each point corresponds to 1 pixel. The points are color coded to reflect their distribution in wavelength. The color coding is from Figure 2, which shows the smoothed Q, U, and flux spectra.

Polarization arising from axisymmetric scattering will show up as a straight line on the Q-U plot [see, e.g., Cropper et al. 1988; J91]. Uncorrected for ISP, the data for SN 1999by clearly define an axis of symmetry in Figure 4. The scatter is ≈0.1%, comparable to the scatter expected from photon statistics and systematic effects.

The distribution of the Si II λ6150 feature is particularly interesting, as noted in the previous section. Here the Si feature is colored magenta. It was previously noted that the
variation in position angle across the Si feature matches the variation in position angle seen across the entire wavelength region of study. Stated another way, Si
variation in position angle seen across the entire wavelength variation in position angle across the Si feature matches the one spot on the Q-U diagram but is distributed throughout the “line” demarcated by the rest of the points in the plot, except the points from the 6900 Å region, which are colored red. Physically, this means that Si does not have a special geometry—it shares the same axis of symmetry as the rest of the SN.

There is a slight deviation from linearity in the red points on Figure 4. If we take this jog to be real, noting that it is nearly perpendicular to the rest of the points, then the jog corresponds to a switching from an oblate to a prolate geometry in the corresponding wavelength region or vice versa. This seems to be an unnecessary complication of the model. The discrepant points can be explained by noise. If a point deviates from the line by 3σ, smoothing can introduce correlated errors, causing several neighboring points to appear to deviate from the line as well. On the basis of these few points, it is not possible to conclude that there are deviations from the global geometry in this wavelength region.

4.2. Interstellar Polarization

ISP due to dust in the ISM of both the host galaxy and the Milky Way can change the observed polarization spectrum of an object. The difficult question of how much of the polarization signature measured for SN 1999by is intrinsic and how much is interstellar plagues polarization measurements of all objects. The issue is particularly difficult in the case of extragalactic objects, which can have two sources of ISP: one contribution from our Galaxy and one from the host galaxy. Fortunately, we can estimate the ISP for SN 1999by via two essentially independent methods, the empirical Q-U plane method and theoretical considerations. We will discuss these specific techniques below, but in this section we provide general constraints on ISP.

The maximum ISP allowable, corresponding to the most favorable alignment of dust grains along the line of sight, is 9% × [E(B-V)] (Serkowski, Mathewson, & Ford 1975). The Galactic longitude of SN 1999by is 166°.91 and latitude 44°12 (NED). The Galactic extinction toward NGC 2841 is estimated to be A_V = 0.070 mag (Schlegel, Finkbeiner, & Davis 1998) though Burstein & Heiles (1982) give A_V = 0.000 mag. Schlegel et al. (1998) also estimate E(B-V) = 0.016 mag. Using Schlegel’s estimate of E(B-V) = 0.016 mag, the maximum Galactic component of polarization is 0.144%. This is indicated by a dashed circle in the Q-U plane diagram of Figure 4.

Some limited information about the Galactic component of ISP can be gleaned from observations of stars near the line of sight to SN 1999by. There are only three stars with polarization measurements in the literature within ±5° of SN 1999by, and they are presented in Table 3. They are also plotted as light blue asterisks on the Q-U plane in Figure 4. The closest star to SN 1999by is HD 82328. It is 2":16 away from the SN on the sky. While HD 82328 shows a low level of polarization, P = 0.01%, it is also the closest star at 19 pc, so it does not sample much of the ISP between us and the SN. The next closest star, HD 82621, is 2":7 away with P = 0.05%, but it is only 40 pc away. If we go out to HD

### TABLE 3

| Name          | Glon (deg) | Glat (deg) | Pol (%) | GPA (deg) | EPA (deg) | V (mag) | SpT | A_V (mag) | μ (mag) |
|---------------|------------|------------|---------|-----------|-----------|---------|-----|-----------|---------|
| 82621         | 164.82     | 45.87      | 0.05    | 81.3      | 6.0       | 4.6     | A2V | 0.0       | 3.0     |
| 82328         | 165.42     | 45.68      | 0.01    | 11.1      | 115.0     | 3.2     | F6IV| 0.0       | 1.4     |
| 77770         | 169.27     | 41.90      | 0.37    | 79.0      | 175.0     | 7.5     | B2IV| 0.0       | 10.6    |
| SN 1999by     | 166.91     | 44.12      |         |           |           |         |     |           |         |

Note.—Data are from Mathewson et al. 1978, available at http://tarantella.gsfc.nasa.gov/cgi-bin/viewer/specific.pl?file=catalog.dat&catalog=2034A.

* HD name.
* Galactic longitude.
* Galactic latitude.
* Degree of polarization.
* Position angle of the E-vector in galactic coordinates.
* Position angle of the E-vector in equatorial coordinates.
* Visual magnitude.
* Spectral type.
* Visual absorption.
* Distance modulus [m − M = 5 log (r/10)].
77770, we can look through 1.3 kpc of the Galaxy to see \( P = 0.37\% \), but now we are looking 3'25 away from the line of sight to the SN. This is probably the reason that this star is outside the allowed Galactic ISP region of Figure 4. The three stars also have different polarization position angles, and it is likely that any Galactic polarization toward SN 1999by would have still a different position angle. In addition, if there is a contribution from ISP in the spectrum of SN 1999by, it could arise from the host galaxy. These three stars alone do not allow us to place meaningful limits on the ISP. Fortunately, the \( Q-U \) plane does suggest a choice of ISP.

4.3. ISP Subtraction

ISP is a smoothly varying function of wavelength, described by the Serkowski law:

\[
\frac{P}{P_{\text{max}}} = \exp \left(-K \ln^2 \left( \frac{\lambda_{\text{max}}}{\lambda} \right) \right),
\]

where \( p \) is the percent polarization at wavelength \( \lambda \), \( P_{\text{max}} \) is the maximum polarization at wavelength \( \lambda_{\text{max}} \), and \( K = 0.01 + 1.66\lambda_{\text{max}} \) (\( \lambda \) is in \( \mu m \); Whittet et al. 1992).

To remove ISP, we subtract a Serkowski law from the data. This is essentially a vector subtraction in the \( Q-U \) plane. Correcting for ISP can either increase or decrease the derived intrinsic polarization of the object, and it can turn absorption features into emission features and vice versa. This is easy to understand if we remember that we are “undiluting” the light from the SN by removing contaminating ISP. Mathematically, this is because the combination of the intrinsic polarization and ISP is a vector addition.

To remove the interstellar signature from our data, we must first determine \( P_{\text{ISP}} \). As remarked above, the data points fall on a line in the \( Q-U \) plane of Figure 4. We can assume for SNe Ia that some parts of the spectra are depolarized (WWH97). Taken together, this allows two possible choices of ISP (see Wang et al. 2000a). If we place the ISP at
either end of the line delineated by the data points and subtract it, then the data will fall along a radial line in $Q-U$ space with essentially one constant value of $\Theta$ for the system (note that $\Theta$ is one-half the angle of a vector on the $Q-U$ plane). It is unlikely that the ISP point could be placed well within the spread of unsubtracted SN 1999by points. In this case, after subtraction the position angle would change abruptly where there was no particular feature in the polarization spectrum. We argue that this behavior is not likely to be physical.

The two choices for ISP are the light blue circles marked as A and B in Figure 4. The diameters of the circles correspond to the approximate uncertainty in the placement of the ISP. ISP A has a value of $P = 0.65\%$ and $\Theta = 82.5^\circ$. ISP B has a value of $P = 0.3\%$ and $\Theta = 150^\circ$. Both of the choices for ISP lie outside of that allowed for the Galaxy along the line of sight to SN 1999by. Assuming that the $E(B-V)$ of Schegel et al. (1998) is correct, this implies that some of the ISP arose in the host galaxy. Note that we could place the ISP any place on the line defined by the data in the $Q-U$ plane that was not well within the spread of data points. The farther the ISP from the data, the more tightly, but arbitrarily, the ISP position angle would be defined. We cannot rule out more extreme choices but argue that the ISP is most plausibly at either A or B so that some portions of the data represent low polarization in the SN. We think that this argument applies to SNe Ia, but note that the data on the Type II SN 1999em show that at some phases there is finite polarization at all observed wavelengths (Wang et al. 2000a).

**ISP choice A.**—First we consider ISP A. Figure 6 presents the resulting $Q$, $U$, $P$, and $\Theta$ with this choice of $P_{\text{isp}}$ subtracted from the data. Note that in the bottom panel, the effect of removing the ISP is that the position angle becomes nearly constant across the entire wavelength region. There is some deviation at longer wavelengths. This is due to the fact that when the ISP-corrected points are relocated on the $Q-U$ plane, the long-wavelength points are closest to the origin, and deviations in position angle are magnified. ISP A is disfavored for three reasons: it produces a decrease in polarization at 7000 Å, it produces an increase in polarization in the Si II $\lambda 6150$ feature, and it is less likely because of reddening considerations. We consider each in turn.

Subtracting ISP A produces a high $P \sim 0.8\%$ at short wavelengths and a decrease in polarization at wavelengths greater than 6500 Å. This is contrary to theoretical expectations. Numerical calculations presented in WWH97 predicted a rise in polarization longward of 6500 Å as a result of decreased line blanketing. This is discussed in further detail in §5.

The removal of ISP A from the data also causes the polarization to be enhanced in the Si II $\lambda 6150$ feature. While this increase cannot be ruled out completely, from a theoretical perspective this feature is easier to understand if it is a depolarization. P Cygni lines are generated above the photosphere, so polarization produced by electron scattering at the photosphere is expected to be depolarized by absorption and reemission in the layers above it (see Appendix A). Note that by using this qualitative theoretical prediction to discriminate between the two choices of ISP, the “empirical method” is not completely independent of theory. However, the approach is significantly different from the determination of ISP from a quantitative comparison to a specific theoretical model as done in §5.

Another consideration that slightly disfavors the choice of ISP A is the observational constraint that the SN does not appear to be reddened. As mentioned above, the maximum ISP allowable, corresponding to the most favorable alignment of dust grains along the line of sight, is

![Image](https://via.placeholder.com/150)

**Fig. 5.** Different data sets of $P$ for SN 1999by. Top panel: All observations combined. Second panel: Four best sets of observations. Third panel: Best single data set, the first data set taken on the night of maximum light, May 10. Bottom panel: Similar to the “four best” case, but without the best spectrum (May 10[1]). This is the sum of the data sets May 8(1), May 9(1), and May 10(2), the second through fourth best sets.

![Image](https://via.placeholder.com/150)

**Fig. 6.** All observations: $Q$, $U$, $P$, and $\Theta$ for SN 1999by after subtraction of $P_{\text{isp}}$ A. The resulting polarization spectrum argues against this choice on theoretical grounds.
9% \times [E(B-V)]. We can use this to place a lower limit on the reddening expected from our choice of ISP. Inverting, we have

\[ E(B-V) \geq \frac{P_{\text{ISP}}}{9\%}. \] (3)

Thus, for model A, \( E(B-V) \geq 0.072 \), and for a standard extinction law (Savage & Mathis 1979), \( A_V \geq 0.22 \). Given that SN 1999by is already bluer than SN 1991bg, which was thought to have low extinction along the line of sight, it is likely that SN 1999by is not significantly reddened. SN 1999by has a pseudocolor of \( B_{\text{max}} - V_{\text{max}} = 0.44 - 0.47 \) (Bonninos 1999; W. Li 2000, private communication). This compares to a value of \( (B-V)_{\text{max}} = 0.74 \) for SN 1991bg (Mazzali et al. 1997). Note that \( B_{\text{max}} - V_{\text{max}} \) and \( (B-V)_{\text{max}} = 0.74 \) are not, strictly speaking, the same quantity, but this is the closest comparison we can make until the photometry data are published. Both SN 1991bg and SN 1999by are redder than typical SNe Ia, but this is thought to be intrinsic to subluminous SNe Ia (F92; Leibundgut et al. 1993; HKW95; Turatto et al. 1996; Mazzali et al. 1997). Since neither the extinction toward SN 1991bg nor that toward SN 1999by is known with certainty, and we can only place a lower limit on the reddening expected from our model, this is not a hard limit.

**ISP choice B.**—The subtraction of ISP B from the data, like ISP A, causes the SN data to lie essentially on a straight radial line in the \( Q-U \) plane as can be seen in Figure 7. For this choice of ISP, the polarization deduced for the SN has a constant position angle of \( \Theta \approx 80^\circ \) across the entire wavelength region. In the bottom panel of Figure 8, the ISP-subtracted data are compared to the position angle from the non-ISP-corrected data. This figure also shows smoothed and unsmoothed versions of \( P, Q, \) and \( U \) after ISP correction. ISP B is slightly favored because of reddening considerations. For this model \( E(B-V) \geq 0.033 \), and \( A_V \geq 0.10 \), less than half that of ISP A. From a theoretical perspective, this choice also gives the desired depolarization of the Si feature. ISP B also gives a rise in polarization at red wavelengths as theoretically predicted. For these reasons it is a better choice than ISP A.

With ISP B subtracted, we find \( P \sim 0.4\% \) from 5600 to 6600 Å with the exception of the Si feature. The difference between the polarization maximum at 6900 Å and the overall level (e.g., at 5800 Å) is 0.4%. This relative difference does not depend on the choice of ISP given the assumption that intrinsic polarization defines a radial line on the \( Q-U \) plot. The maximum degree of polarization after subtraction of ISP B is \( P = 0.8\% \). As noted above, subtraction of ISP A gives the same maximum value, but for ISP B the maximum polarization occurs at wavelengths \( \geq 6500 \) Å. This is where the maximum polarization is expected from theory (WWH97). The subtraction of ISP B produces relative depolarization around 4800–5600 Å, where the level is 0.2% or lower. This is where line blanketing opacity dominates electron scattering opacity and thus is where depolarization is expected from theory (Höflich, Müller, & Khokhlov 1993, hereafter HMK93; WWH97).

We have estimated the degree of uncertainty associated with this choice of ISP by choosing different values of \( P_{\text{ISP}} \) close to ISP B and observing the degree to which they cause the axis ratio of the system to deviate from a single value across all wavelengths. These are shown in Howell (2000) but are omitted here for brevity. After consideration of the uncertainties, our best choice of ISP, case B, is \( P = 0.3\% \pm 0.05\% \) at \( \Theta = 150^\circ \pm 5^\circ \).

Regardless of whether we choose ISP A or B, the intrinsic polarization of SN 1999by, given by the total length of the distribution of points in the \( Q-U \) plane, is \( P = 0.8\% \). As
mentioned earlier, Si is distributed throughout most of the line in the $Q-U$ plane and hence samples this total intrinsic polarization. One implication is that the Si $\lambda6150$ feature will always show a minimum or maximum in the polarization after subtraction of a reasonable choice of ISP lying at one or the other extreme of the data in the $Q-U$ plane.

5. COMPARISON WITH THEORETICAL MODELS

The chemical and density structures for the initial models are based on calculations of the stellar evolution from the main sequence to WD formation and the subsequent phase of accretion onto the WD (Umeda et al. 1999; Höflich et al. 2000). Spherical dynamical explosions and corresponding light curves are first calculated. Aspherical structures are then constructed by mapping the spherical structures in the homologous expansion phase onto ellipsoidal density structures with an axis ratio $a/b$. Here $a$ is the equatorial major axis and $b$ is the polar major axis. Subsequently, detailed non-LTE spectra are calculated for the flux and polarization. Details are given in Appendix A.

DD models and pulsating DD models have been found to reproduce the optical and infrared light curves and spectra of SNe Ia reasonably well (Hoźńich 1995b; HKW95; HK96; Nugent et al. 1997; Wheeler et al. 1998; Lentz et al. 1999; Höflich et al. 2000; C. L. Gerardy et al. 2001, in preparation). In particular, the DD models with a variation in the parameter $\rho_{\text{tr}}$, at which the transition is made from deflagration to detonation give a range of ejecta structures and mass of $^{56}\text{Ni}$. Models with smaller transition density give less nickel and hence both lower peak luminosity and lower temperatures. The latter gives lower opacities and hence a steeper decline in the light curve. The DD models thus give a natural and physically well-motivated origin of the brightness-decline relation of SNe Ia within the paradigm of thermonuclear combustion of Chandrasekhar mass CO WDs (HKW95; Höflich et al. 1996b).

Here we present the polarization spectra of a DD model with parameters that have been adjusted to provide a fit to the time evolution of the infrared spectra observed for SN 1999 by (C. L. Gerardy et al. 2001, in preparation) without further tuning. In the DD scenario, the transition density, $\rho_{\text{tr}}$, is the dominant factor that determines the amount of $^{56}\text{Ni}$ produced and hence distinguishes the normally bright from the subluminous models (HKW95; Höflich 1995a; Iwamoto et al. 1999). We contrast the ejecta structure of a normally bright SN Ia model with that required to reproduce a subluminous event like SN 1999by in Figure 9. The same progenitor structure and central density at the time of the explosion have been taken as the normally bright SNe Ia studied in Höflich et al. (2000). We assume a WD with a Chandrasekhar mass and solar metallicity that originates from a main-sequence star of 7 $M_{\odot}$ (Umeda et al. 1999). At the time of the explosion, the central density is $\rho_c = 2 \times 10^9$ g cm$^{-3}$. To produce a subluminous SN consistent with the IR spectra of SN 1999by, $\rho_{\text{tr}}$ has been chosen to be $1 \times 10^7$ g cm$^{-3}$ compared to $\rho_{\text{tr}} = 2.5 \times 10^7$ g cm$^{-3}$ for the normally bright model. The subluminous model produced 0.103 $M_{\odot}$ of $^{56}\text{Ni}$ compared to 0.701 $M_{\odot}$ for the normally bright model.

![Diagram of chemical structures](image)

**Fig. 9.** — Final chemical structures of models are given for a strongly subluminous (top) and a normally bright (bottom) SN during the phase of homologous expansion. Identical progenitors and prescription for the deflagration and detonation fronts have been used, but the deflagration–detonation transition densities, $\rho_{\text{tr}}$, of $1 \times 10^7$ and $2.5 \times 10^7$ g cm$^{-3}$ have been used to produce the subluminous and normally bright models, respectively. Note that in the subluminous model, $^{56}\text{Ni}$ is confined to a lower region of velocity space, beneath the optical photosphere at maximum light.
bright model. A typical feature of subluminous DD models is the greater production of O, S, and Si (0.251, 0.431, and 0.221 \(M_\odot\), respectively) compared to normally bright SNe Ia (0.065, 0.215, and 0.123 \(M_\odot\), respectively). Another feature of subluminous SN Ia models is that the \(^{56}\)Ni is constrained to the inner layers with low expansion velocities that become visible a few weeks after maximum light. In contrast, for normally bright SN Ia models, the outer edge of the \(^{56}\)Ni layers extends to about 10,000–13,000 km s\(^{-1}\) so that the Ni is already visible at maximum light (Fig. 9).

For more details on the explosion models, light curves, and spectral evolution, see Höflich et al. (2000) and C. L. Gerardy et al. (2001, in preparation).

For this paper we calculated the polarization spectrum at day 15 after the explosion, which corresponds to maximum light in \(B\). At this epoch the model gives \(M_V = -17.52\) and \(B-V = 0.46\). The temperature structures for the aspherical models were based on the spherical calculations. The optical spectrum is formed in layers with expansion velocities between \(\approx 8000\) and 14,000 km s\(^{-1}\), which corresponds to about 1 \(M_\odot\) in mass coordinate, i.e., in layers dominated by intermediate-mass elements (Fig. 9).

Currently, three-dimensional models are not a suitable tool to adjust parameters such as the main-sequence mass, accretion rates, metallicity, rotation, and combustion physics to find best fits. Therefore, we restrict our discussion of the fluxes to the main spectral features.

In our model, the spectrum at day 15 is formed at the outer edge of the Si-rich layers, i.e., well above the region containing \(^{56}\)Ni (Fig. 9). The top panel of Figure 10 shows reasonable agreement between theoretical and observed features and thus confirms the validity of this model in terms of Doppler shifts of the lines, their strengths, the ionization stages, and the overall slope of the spectrum. Most of the strong features can be attributed to the intermediate-mass elements O I, S II, and Si II. Most of the weaker lines are due to iron group elements that are singly ionized as

![Figure 10](image-url)

Fig. 10.—Flux spectra (top panel) and polarization spectra (second panel) at day 15 after the explosion for a subluminous DD model (see text). The observations are not corrected for reddening, since it is uncertain, though they are corrected for the negligible redshift (\(z = 0.00213\)). The model SN has been mapped into an oblate ellipsoid with axis ratio 1.17 shown for various inclinations \(i\) and compared to the observations of SN 1999by at about maximum light in \(B\) (see § 2). For comparison with the models, an interstellar component with \(P = 0.25\%\) and \(\Theta = 140^\circ\) has been subtracted from the observations. The observed flux spectrum is the solid black line. Model flux spectra are shown by solid magenta, dotted green, and dashed blue lines for inclination angles, \(i\), of 9.1, 28.3, and 52.1, respectively. In the second panel, the raw polarization data are shown by a light blue line, while the smoothed data (with \(\Delta \lambda = 45\) \(\AA\)) are shown by the black line. The model polarization spectra are the same as in the first panel. The third panel presents the statistical error in \(P\) for the raw data (blue line), smoothed data (dotted blue line), and theoretical model (red line; binned to \(\Delta \lambda = 12.5\) \(\AA\)). The model seen nearly equator-on (\(i = 9.1^\circ\)) does the best job of reproducing the general features of the polarization spectrum: depolarization from 4900 to 5500 \(\AA\), moderate polarization from 5500 to 6100 \(\AA\), a depolarization in the Si \(\lambda 6150\) feature, and a rising polarization to the red. In the bottom panel, the observed polarization angle \(\Theta\) is shown as a function of wavelength. The unbinned data are given by dashed lines and the binned data by solid lines. The data are shown before and after correcting for the ISP contribution determined from the models (see text). After correction for ISP it is clear that the position angle is essentially constant across all wavelengths. There is an 80° shift across the Si II trough feature, though this is an artifact of low polarization close to the origin giving an uncertain position angle. Compare this choice of ISP to the empirically determined choice, Fig. 8.
expected for subluminous SNe Ia with a relatively cool photosphere (Höflich et al. 1995b). The O I feature at about 7450 Å is too weak in the models. This is not due to a lack of oxygen (Fig. 9), but the strength of the O I line depends sensitively on the excitation. The discrepancy may be an artifact of the approximations imposed by the numerical treatment, or it may hint at the need for a slightly higher excitation in the outer layers, e.g., due to nonthermal electrons or an increased γ-ray flux. We note that some fast-declining (subluminous) SNe Ia do not show strong O I in their spectra (Mojjaz et al. 2001).

The flux spectrum depends on the inclination, i (here taken to be the angle between the equatorial plane and the line of sight). For oblate geometries, the flux increases with i because of the increased escape probability of photons toward polar directions (H91). Although the asphericity is small in our example, the absolute flux varies by about 30% or 0.3 m from equator to pole. With increasing i, weak absorption features (e.g., due to Fe II, S II) tend to be smeared out, and the overall spectrum becomes smoother as shown in the top panel of Figure 10. These absorption features are formed in a narrow region close to the origin of the quasi-continuum that is produced by electron scattering, bound-free processes, and a large number of very weak lines (Höflich 1995a). Because \( v \propto r \), for an oblate geometry the photosphere seen by an observer spans a wider velocity range as i increases. In addition, the probability of multiple scattering of photons tangential to the photosphere into the “absorption” feature increases with i in the case of oblate geometries, thus diluting the depolarization of the line.

Flux and polarization spectra of a model with 17% asymmetry observed at various inclination angles are shown in Figure 10 in comparison with observations. The model polarization spectra have been rebinned with \( \Delta \lambda = 12.5 \) Å, to provide a statistical error comparable to the observations that, for this section, have been rebinned with \( \Delta \lambda = 45 \) Å. The model polarization spectra for various choices of the inclination, i, and the raw and binned data are given in the second panel of Figure 10. The statistical errors in the model and observations and the polarization angle before and after correcting for the model-determined ISP are given in the third and fourth panels of Figure 10, respectively.

In the models, the polarization is produced by electron scattering. As discussed in Appendix A, we assume complete redistribution for lines, which causes depolarization in lines. As expected from both analytical and numerical studies, the polarization \( P \propto \cos^2 i \) (e.g., van de Hulst 1957; H91). The frequency dependence of the polarization is governed by the absorption probability in lines relative to electron scattering. Most prominent is the strong depolarization by the Si II line at 6150 Å and the O I, Mg II, and Si II features at 7600 Å (Fig. 10, second panel). At shorter wavelengths, a
large number of overlapping lines due to iron group ele-
ments are responsible for the low polarization. The wave-
lengh region above 5400 Å shows an increasing degree of
polarization due to the decreasing importance of line opa-
cities. Shortward of the Si II line at 6150 Å, the polarization
level in the models is generally lower and shows modula-
tions due to moderately strong Si II, S II, and Fe II lines, as
expected from the opacity pattern for Si-rich layers at tem-
peratures between 5000 and 10,000 K (see Fig. 1 in
HMK93).

We note that the frequency pattern of the model polariza-
tion spectra for subluminous SNe is rather different from
normally bright SNe Ia at the same phase (e.g., SN 1996X;
WWH97). In the latter, the polarization spectra near maxi-
mum light are dominated by iron group elements because
the photosphere is between 10,000 and 12,000 km
s$^{-1}$, i.e., right at the interface between complete and in-
complete Si burning (Fig. 9). In addition, the photospheric tem-
peratures are higher than in subluminous events by several
thousand degrees, and thus iron group elements are present
in both the second and third ionization stages. The presence
of these ubiquitous iron lines gives a more pronounced
rapid frequency dependence of the model polarization
spectra for the normally bright SNe Ia near maximum as
contrasted with the relatively smooth variation predicted
for the polarization spectra of the subluminous events.

We have iterated the component of the ISP to optimize
the fits with respect to the line width and depth of the strong
lines, in particular the Si II feature at 6150 Å. The best
agreement is for $P_{\text{ISP}} = 0.25\%$ and $\Theta = 140^\circ$. Small changes
of the ISP vector result in slightly broader components of
the depolarization features. With increasing deviations from
the optimized values, local maxima appear in the line
centers that eventually dominate the spectra. The vector for
the ISP derived in this way is consistent with the contribu-
tion of the ISP found with the empirical method described
in § 4.3. The agreement supports the soundness and reli-
ability of both the empirical and theoretical approaches.

As shown in Figure 10 (second panel), the observed polar-
ization spectrum can be reproduced by an ellipsoid with an
axis ratio of 1.17 seen equator-on ($i \approx 0^\circ$). The overall level
of polarization in different wavelength ranges and the velo-
city shift and strength of features produced by strong lines
are consistent.

The theoretical polarization spectrum shows a distinct
physical pattern between 5400 and 6000 Å at the 0.1% level
that cannot be discriminated in the data. These small-scale
fluctuations may be valuable tools to analyze small-scale
structures caused by inhomogeneous mixing if better data
become available in the future.

An axis ratio of 1.17 is a lower limit to the amount of
asphericity that is required, though we can also estimate an
upper limit. As an alternative to a model with small aspheri-
city seen equator-on, models with larger asphericity may be
able to reproduce the observations if seen from larger incli-
nations, $i$. To test this possibility, we have calculated a set of
models with larger axis ratios as presented in Figure 11. To
reproduce the overall level of polarization for $i$ of 28° and
52°, the axis ratio $a/b$ must be boosted to 1.25 and 1.5,
respectively. Strong discrepancies at the 0.3% level are
present for $i = 52^\circ$, which are well beyond the level of uncer-
tainty. Most noticeable are the strong peaks in $P$ at about
6000 and 6400 Å, as well as the change of 0.6% between
5800 and 6000 Å. Such variations cannot be eliminated by a
different choice for ISP. For $i = 28^\circ$ and $a/b = 1.25$, the
agreement with the observation of $P$ is better, but problems
remain on the level of 0.2%, and there are some problems
with the location of the Si II minima. The model with
$i = 28^\circ$ may be marginally consistent with the observations.
We can conclude that SN 1999by has intrinsic asymmetries
of the order of $\approx 20\%$ and that it is seen almost equator-on.

6. DISCUSSION AND CONCLUSIONS

Whereas core collapse SNe are commonly found to be
polarized at 1% or greater, the degree of polarization is
much smaller in SNe Ia (Wang et al. 1996, 2000b). Spectro-
polarimetric and broadband measurements of polarization
in SNe Ia are still exceedingly rare. For most measurements,
the interstellar component cannot be determined, making
any intrinsic component impossible to determine. Table 2
shows all SNe Ia for which polarization data are available.
McCall et al. (1984b) measured no significant polarization
intrinsic to SN 1983G but did place (large) upper limits on
spectropolarimetric signatures. The first slightly sub-
luminous SN Ia with a polarization measurement was SN
1986G. Broadband $UBVRIJH$ polarimetry with a
maximum polarization of 5.2% is well fitted by a Serkowski
law (Hough et al. 1987). Broadband polarimetry of SN
1998bu is presented in Hernandez et al. (2000). The polar-
ization, ranging from 1% to 2%, again follows the Ser-
kowski law. Spyromilio & Bailey (1993) placed limits on the
spectropolarimetry of SN 1992A, which was observed
about 15 days and 100 days past maximum light. The
overall level of $P$ is roughly 0.3%, comparable to the noise
level. While Spyromilio & Bailey (1993) make no claim for
intrinsic polarization of the SN, polarization signatures of
the type suggested for SN 1996X (WWH97) cannot be ruled
out from the published data. Leonard, Filippenko, &
Matheson (2000) presented spectropolarimetry of SN
1997dt, showing apparent changes in $P$ across spectral fea-
tures. The level of intrinsic polarization versus ISP is not
known.

WWH97 observed the normally bright supernova SN
1996X about 1 week before maximum light in $V$. The data
showed a small average polarization with modulations on
the level of 0.2%. WWH97 concluded that SN 1996X could
have only a very small, if any, global asymmetry in the
geometry, given the lack of detectable mean polarization,
but that the modulations were consistent with inhomoge-
neities in distributions of the elements within the ejecta.

Figure 12 shows the data for SN 1996X and SN 1999by in
the $Q-U$ plane. The lack of any preferred orientation to the
data and hence a global asymmetry for SN 1996X contrasts
distinctly with the linear distribution of points for SN
1999by. The finer scale variations in the polarization
spectra of SN 1996X are qualitatively similar to the expecta-
tions for the behavior of a photosphere significantly con-
taminated with iron peak elements as a normally bright SN
Ia should be (Fig. 9; WWH97). A comparison with detailed
model calculations for a DD model confirmed that the data
are consistent with little or no global asymmetry in the
density structure of SN 1996X. The continuum polarization
in the model is modulated by ubiquitous weak lines of
Fe II–III, Co II–III, and Ni II–III that serve to depolarize on
scales of $\approx 100$ Å. These lines are formed at the interface
region between complete and incomplete Si burning.
WWH97 suggest that there is an asymmetry in the chemical
structure that may be a relic of the chemical plumes rising
during the deflagration phase. Strong depolarization over the Si II 6150 feature is predicted by the model, but not observed. As noted in WW97, another problem with the comparison to theory of SN 1996X is that the model shows a rise in the polarization longward of 6500 Å that is not apparent in the data. SN 1999by shows both the depolarization at the Si II feature and the rise in the polarization level to the red.

One of the open questions in SN Ia research is the nature of the very subluminous subclass. The prototypical example of this subclass is SN 1991bg. Light curves and spectra of SN 1991bg are presented in F92, Leibundgut et al. (1993), and Turatto et al. (1996). Other subluminous SNe include SN 1992K (Hamuy et al. 1994), SN 1997cn (Turatto et al. 1998), and SN 1998de (Modjaz et al. 2001). Subluminous SNe Ia may be rare, but some analyses indicate that they could make up 16% (Li et al. 2001) or more (Schaefer 1996) of all local SNe Ia. Some defining characteristics of the subclass are rapidly declining light curves \( \Delta m_{14}(B) \approx 1.9 \), fainter than normal peak magnitudes by 2–3 mag, redder colors at maximum \( [(B-V)_{\text{max}} \approx 0.4–0.74] \), a nonexistent or weak secondary maximum in \( R \) and \( I \), and an earlier transition to the nebular phase. Moreover, the \(^{56}\)Ni region is limited to expansion velocities below 4000–4500 km s\(^{-1}\), i.e., about a factor of 2–3 times smaller than in normally bright SNe Ia. By comparison, the expansion velocity implied by Si II indicates a wider range of the Si-dominated layers in subluminous SNe Ia \( (\approx 5000–12,000 \text{ km s}^{-1}) \) compared to normally bright SNe Ia \( (\approx 9000–13,000 \text{ km s}^{-1}) \); Turatto et al. 1996; C. L. Gerardy et al. 2001, in preparation). Subluminous events also show strong absorption features at 4200 (Ti II), 5800 (Si II), 7500 (O I and Mg II) and 8300 Å (Ca II). Theoretical interpretations of subluminous SNe Ia given in Ruiz-Lapuente et al. (1993), Woosley & Weaver (1994), HKW95, N95, Mazazzi et al. (1997), and Milne, The, & Leising (1999) include all three types of explosion mechanisms outlined in § 1.

SN 1999 by is the first subluminous SN Ia to show definitive evidence for intrinsic polarization. Before ISP correction, the SN shows an overall level of polarization, a rise in \( P \) redward of 6500 Å, and a change in polarization position angle across the Si II 6150 feature. The strong wavelength dependence and the individual features are not expected from ISP and show the need for a component intrinsic to the SN.

We employ a new method (see Wang et al. 2001b) for analyzing the polarization data of SNe by using the \( Q-U \) plane. This representation provides important constraints on the overall geometry of the configuration, and it allows strong constraints to be placed on the ISP if the data points are aligned. The small spread of the data points in SN 1999by shows about a line in the \( Q-U \) plane reveals that the SN has a well-defined axis of symmetry. Though the spread around this line is consistent with rotational symmetry, we cannot exclude small additional off-axis components. In the classical approach to determine the polarization component of the ISM, the object needs to be observed at several epochs from which the polarization vector can be deduced by the reasonable assumption that it is time invariant. In the \( Q-U \) plane and for reasonable assumptions for the polarization mechanism, we have shown that the component of the ISM can be determined by the vector between the origin of the \( Q-U \) plane and the endpoint of the line of the polarization data points. Obviously, with this choice, the polarization position angle should not vary over lines. The residuals can be used to judge the quality of the data and the assumptions. For SN 1999by, this method suggests that \( P_{\text{ISP}} = 0.3\% \) with \( \Theta = 150^\circ \), which is consistent with the best fits for the ISM based on detailed models \( (P_{\text{ISP}} = 0.25\%, \Theta = 140^\circ) \).

Based on a DD model, synthetic non-LTE (NLTE) spectra have been computed and compared to the observations. The free parameters have been adopted from a spherical DD model that reproduces the infrared spectrum of SN 1999by between 1 and 2 \( \mu m \) and its evolution with time (C. L. Gerardy et al. 2001, in preparation). No further tuning of the initial model is done to fit the polarization data, although three new parameters must be introduced: the axis ratio, the angle of the line of sight to an observer, and the ISP. The ISP could be taken from the observations, but the theoretical analysis independently confirms the empirical value. The axis ratio and inclination angle are constrained by two aspects of the observations: the overall level of polarization and the shapes of spectral features. The initial spherical model is remapped to an ellipsoidal geometry with an axis ratio of \( a/b \) and a change in polarization position angle across this line is consistent with rotational symmetry, the SN shows an overall level of polarization, a rise in \( P \) redward of 6500 Å, and a change in polarization position angle across the Si II 6150 feature. The strong wavelength dependence and the individual features are not expected from ISP and show the need for a component intrinsic to the SN.

The relatively high polarization we observe for SN 1999by may be a significant clue to the nature of SNe Ia. Existing limits on polarization of other SNe Ia are very sparse (seven objects at the time of this paper; see Table 2). The polarization due to asphericity decreases with inclination, further reducing the significance of any statistics. Within these limitations, we would have expected to see more polarized SNe Ia if \( P \approx 0.6\%–0.8\% \) is common. The
lack of such detections supports the notion that the intrinsic polarization of the subluminous SN 1999by is unusually large. Another argument is that, although small, the size of the asymmetry we derive implies a change of the observed luminosity of about 0.3" from equator to pole. This spread is larger by a factor of \( \approx 1.5 \) than the mean dispersion in the brightness-decline relation \( M(\Delta m_{15}(B)) \) (Hamuy et al. 1995; Rieiss et al. 1999). The dispersion in \( M(\Delta m_{15}(B)) \) is dominated by the normally bright SNe Ia, and the orientation of the symmetry axis of a given SN is arbitrary. Taken together, these facts again suggest that the asphericity is unusually large for SN 1999by compared to most normally bright SNe Ia. The low mean polarization observed for the normal SN 1996X (WWH97) is consistent with this. If future observations confirm that the large asphericity of SN 1999by is characteristic of subluminous SNe Ia, this may provide important clues to the physical reasons for both normal and less than normal peak luminosity.

Our quantitative analysis of the data of SN 1999by was based on DD models with imposed asphericity without addressing the question of the physical mechanism that produces aspherical envelopes. Within the DD model, possible mechanisms to induce asymmetry include the following: (1) instabilities in the nuclear burning front during the deflagration phase, (2) rapid rotation of the progenitor WD, (3) DD transition at a point rather than simultaneously on a sphere, and (4) impact of the SN ejecta on the secondary star; aspherical configurations may also be the result of (5) the merging process of two degenerate WDs and (6) in principle, edge-lit helium detonations in sub-Chandrasekhar mass WDs.

Studies of the deflagration phase of DD models reveal large plumes of burning material (e.g., Khokhlov 1995, 2001; Niemeyer & Hillebrandt 1995). These plumes may leave their imprint on the chemical structure of the ejecta. The plumes do not significantly perturb the density distribution and hence will not explain global asymmetries in the overall density structure (Khokhlov 2001), but they could leave an imprint at composition interfaces. WWH97 suggested this mechanism as a possible explanation for the polarization signatures seen in SN 1996X, but these plumes are an unlikely explanation for the global deviations from sphericity with a well-defined axis of symmetry as in SN 1999by.

Livne (1999) pointed out that we cannot expect a deflagration–detonation transition on a sphere as implicitly assumed in spherical models but that the transition may begin at one point. Detailed two-dimensional calculations showed large-scale asymmetries, but Livne only followed the explosion process. He did not follow the calculation into the homologous expansion, which may or may not destroy any initial asphericities by the time of maximum light when the observations of SN 1999by were taken. It is also not clear why such a mechanism should apply to subluminous SNe Ia and not, perhaps, to most SNe Ia.

In any accretion model, we expect an impact of the SN ejecta with the secondary star. Marietta, Burrows, & Fryxell (2000) computed two-dimensional numerical simulations of the impact of an SN Ia explosion with hydrogen-rich main-sequence, subgiant, and red giant companions. They find that the blast strips main-sequence and subgiant companions of 15% of their mass while red giants lose \( \sim 97\% \) of their envelopes \( (0.5 \, M_\odot) \). The impact of the ejecta with the secondary star creates a hole in the debris of angular size \( \sim 30^\circ–40^\circ \), corresponding to \( \sim 10\% \) of the ejecta surface. The result was similar for all cases considered because the companion was close enough to be in Roche lobe overflow. This effect may be consistent with the polarization observed in SN 1999by, but we would expect the same effect in both normal and subluminous SNe Ia, which may be in conflict with the low polarization observed in normally bright SNe Ia.

Mahaffey & Hansen (1975) calculated a rotating detonation model, but no detailed models have been calculated for rapidly rotating, deflagrating WDs. Rapid rotation will produce a global distortion of the density structure of the pre-SN WD (Müller & Eriguchi 1985) and may also affect the propagation of nuclear burning fronts in the SN, resulting in small asphericities. We regard this possibility as very attractive since the combined effect of rotational distortion and modification of the nuclear deflagration front has the potential to link the subluminosity of SN 1999by to the large, global asymmetry. A comparison with model 7 of Müller & Eriguchi (1985) suggests that even with solid body rotation (which produces the minimum distortion for a given angular momentum), sufficient rotation to distort the structure in the silicon layers of our subluminous model would need to have a rotation energy only about 5% of the gravitational energy. Thus, a significant, but not extreme, rotation may be sufficient to explain the degree of polarization we observe in SN 1999by. Rotation could thus serve as a single parameter that distinguishes normal SNe Ia arising in slowly rotating WD progenitors from subluminous SNe Ia that occur in more rapidly rotating WDs. One possible problem with the scenario of rotation as a single parameter is that SN properties appear to show a correlation with environment and, by implication, progenitor age (Howell 2000, 2001).

In the merging scenario, two CO WDs merge to produce an SN Ia (Webbink 1984; Iben & Tutukov 1984). This would provide a natural explanation for the lack of hydrogen seen in SN Ia spectra. There is evidence that such systems exist (Saffer, Livio, & Yungelson 1998), Khokhlov, Müller, & Höflich (1993) and HK96 parameterized merger events as spherically symmetric CO WDs with thick envelopes. They produced light curves that were in reasonable agreement with some SNe Ia. Doubts about the merger models have been raised. Three-dimensional hydrodynamical simulations of the process show that (unless the system ignites during the merger process) the less massive star is disrupted into a hot envelope and an accretion disk around the primary (Benz et al. 1990; Rasio & Shapiro 1995; Mochkovitch, Guerrero, & Segretain 1997). In this case, off-center carbon ignition produces a flame that propagates inward, converting the star to an O + Ne + Mg WD (Nomoto & Iben 1985; Saio & Nomoto 1985, 1998; Kawai, Saio, & Nomoto 1987). Magnesium-24 then undergoes electron capture, inducing accretion-induced collapse, to produce a neutron star (Saio & Nomoto 1985; Mochkovitch & Livio 1990; Nomoto & Kondo 1991). Alternatively, it may be possible to ignite the system as a result of tidal heating during the merger process (Iben, Tutukov, & Fedorova 1998), which could produce a highly aspherical explosion. This scenario has not been demonstrated in realistic simulations but deserves further study.

We have to stress the limits of this study, which must be seen only as a first step. Clearly, more high-quality data must be obtained to provide a statistical sample and to
make use of small-scale features to evaluate details of the
graphy and, ultimately, to probe details of the explosion
mechanism (e.g., the burning properties) and the scenario
(e.g., rotation of a single rapidly rotating WD vs. merging of
two WDs). Future data should have a noise level well below
0.1% to address questions of deviations from axial sym-
metry and to utilize the small-scale variations seen in the
models. This is a very feasible goal with the current and
upcoming generation of 8 m class telescopes. Our models
are based on aspherical envelopes that are distorted artifi-
cially. Although this is a reasonable approach to estimate
the size of asphericity and general properties, it hardly pro-
vides the desired link to the physical mechanisms
consistent hydrodynamic calculations of rotating WDs, def-
lagration fronts, and the multidimensionality of the
deflagration–detonation transition are feasible. Although
any successful model must reproduce the overall pattern in
the chemical composition, alternative scenarios with full
multidimensional hydrodynamics may do this job as well.
In particular, the merging scenario needs more attention.

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APPENDIX A

COMPUTATIONAL METHODS

A1. RADIATION HYDRODYNAMICS AND LIGHT CURVES IN SPHERICAL GEOMETRY

Explosion models are calculated using a one-dimensional radiation hydrodynamics code (HK96) that solves the hydrody-
namical equations explicitly by the piecewise parabolic method (Colella & Woodward 1984). Nuclear burning is taken into
account using an extended network of 216 isotopes (Thielemann, Nomoto, & Hashimoto 1996 and references therein).
The propagation of the nuclear burning front is given by the velocity of sound behind the burning front in the case of a detonation
wave and in a parameterized form during the deflagration phase based on detailed three-dimensional calculations (e.g.,
Khokhlov 1995, 2001; Niemeyer & Hillebrandt 1995). We use the parameterization as described in Dominguez & Höflich
(2000). We assume that \( v_{\text{burn}} = \max (v_i, v_f) \), where \( v_i \) and \( v_f \) are the laminar and turbulent velocities with

\[
v_i = C_1 \sqrt{\frac{\alpha_T g L_f}{\rho}},
\]

(A1)

with \( C_1 = 0.15 \), where

\[
\alpha_T = \frac{\alpha - 1}{\alpha + 1},
\]

(A2)

and with

\[
\alpha = \frac{\rho^- (r_{\text{burn}})}{\rho^+ (r_{\text{burn}})}.
\]

(A3)

Here \( \alpha_T \) is the Atwood number, \( L_f \) is the characteristic length scale, and \( \rho^+ \) and \( \rho^- \) are the densities in front of and behind
the burning front, respectively. The quantities \( \alpha \) and \( L_f \) are directly taken from the hydrodynamics at the location of the burning
front, and for this choice of \( C_1 \), we take \( L_f = r_{\text{burn}} \). The transition density is treated as a free parameter. The description of
the deflagration front does not significantly influence the final structure of the explosion. The amount of matter consumed
during burning (and the total \( ^{56}\text{Ni} \) production) is governed by the preexpansion of the WD and, consequently, is determined
by the transition density \( \rho_{\text{tr}} \), at which the burning front switches from the deflagration to the detonation mode (Höflich
1995a). The value \( \rho_{\text{tr}} \) can be adjusted to produce a given amount of \( ^{56}\text{Ni} \) that determines the brightness (Höflich 1995a) and,
through the temperature dependence of the opacity, the decline rate (HKW95; Höflich et al. 1996b).

The code also simultaneously solves for the energy and radiation transport with variable Eddington factors. The radiation
transfer portion of the code consists of (1) an LTE radiation transfer scheme based on the time-dependent moment equations,
which are solved implicitly; (2) a frequency-dependent radiation transport to determine the Eddington factors and the
frequency averaged opacities; (3) a detailed equation of state with an elaborate treatment of the ionization balance and the
ionization energies; (4) time-dependent expansion opacities that take into account the composition structure of the explosion
model; (5) photon scattering and thermalization calibrated by NLTE calculations; and (6) a Monte Carlo \( \gamma \)-ray deposition
scheme that takes into account all relevant \( \gamma \)-ray transitions and interaction processes. For more details, see Höflich, Wang, &
Wheeler (1999) and references therein.

A2. DENSITY AND CHEMICAL STRUCTURE OF THE THREE-DIMENSIONAL ENVELOPE

Aspherical density structures are constructed from the spherical density distribution by imposing a homologous expansion
function that depends on the angle \( \Theta \) from the equatorial plane and that conserves the total energy and the mass fraction per
The homology scaling constant, $C(\Theta)$, is determined to produce the desired axis ratio for an ellipsoid (Höflch et al. 1999), and the total explosion energy is normalized to that of the spherical model. Here the asphericities are small and the temperature profiles in SNe Ia are shallow (HMK93; Höflch 1995a). Therefore, we assume identical isodensity and isotemperature.

$$C(\Theta) = \frac{r(\Theta)}{R t},$$  \hspace{1cm} (A6)

where $t$ is the time since the explosion and $r(\Theta)$ is the distance of the mass element after time $t$.

Because little is known about the general geometry of the ejecta, we construct ellipsoidal isodensity contours with an axis ratio $E = a/b$ at the photosphere where $a$ is the distance of the photosphere in the $x$-$y$ (symmetry) plane and $b$ is the distance in the $(\text{axial}) z$-direction. This contour is given by

$$r(\Theta) = r(\Theta = 0) \sqrt{\cos^2(\Theta) + E^2 \sin^2(\Theta)}.$$

The homology scaling constant, $C(\Theta)$, is determined to produce the desired axis ratio for an ellipsoid (Höflch et al. 1999), and the total explosion energy is normalized to that of the spherical model. Here the asphericities are small and the temperature profiles in SNe Ia are shallow (HMK93; Höflch 1995a). Therefore, we assume identical isodensity and isotemperature contours.

### A3. SPECTRA FOR ASYMMETRIC CONFIGURATIONS

Our radiation transport code works both for spherical geometry and in three dimensions using different modules for the radiation transport. For details and references to the atomic database, see Höflch (1995a) and references therein. The density and abundance structure is taken from the hydrodynamic calculations. Excitation due to $\gamma$-rays is included via the Monte Carlo code. Based on the list of atomic data of Kurucz (1995), we have constructed detailed atomic models for a couple of ions similar to those used in WWH97. The use of three-dimensional geometries caused some restrictions on the complexity of the atomic models. Multilevel atoms have been restricted to the main ionization species of $\text{C} \, \text{I} \,(23/57), \text{O} \, \text{I} \,(28/75), \text{Mg} \, \text{II} \,(25/85), \text{Si} \, \text{II} \,(28/83),$ and $\text{Ca} \, \text{II} \,(26/90)$ of the intermediate-mass elements. Here the numbers in the parentheses denote the number of levels and of line transitions, respectively. For those ions, the statistical equations are solved consistently including bound-bound and bound-free transitions. The method of accelerated lambda iteration is applied to remove the global dependence of the level populations and the radiation field. In order to include the line blocking effect, about $10^8$ additional lines are taken into account under the assumption of LTE for the population number but including scattering terms calibrated by the NLTE elements. The electron temperature structure is based on the depth-dependent luminosity from the monochromatic light-curve calculation at a certain time. The complete system of equations is given by the time-independent radiation transport and the statistical equations, i.e., the rate equation, the particle conservation equation, and the charge conservation equation. All allowed bound-bound and bound-free transitions between the NLTE levels are taken into account. Complete redistribution over each individual line both in frequency and in angle is assumed (Mihalas 1978) in the comoving frame (but not the observer’s frame). This means that the populations of sublevels of the upper and lower transition are described by a Maxwell-Boltzman distribution and that the light becomes unpolarized.

#### A3.1. Radiation Transport in Three-Dimensional NLTE Spectra

The current calculations use a modified version of the Monte Carlo code previously applied to calculate the continuum in SN 1987A (H91) and line polarization in SN 1993J and SN 1996X (Höflch et al. 1995b; WWH97). The code is capable of handling arbitrary three-dimensional geometries, for both the density and the distribution of the sources. Polarization and flux spectra for rapidly expanding envelopes can thus be computed. Polarization is treated within the Stokes formalism (see, e.g., van de Hulst 1957). We include electron scattering as a polarization process but omit scattering at dust particles.

The modifications for the present work to the previous versions of our Monte Carlo scheme (H91) are mainly related to the inclusion of NLTE effects. In scattering-dominated atmospheres, deviations from LTE are also relevant at large optical depths; however, at large optical depths, Monte Carlo methods become very costly and/or inaccurate (Höflch et al. 1995b). Therefore, we use a hybrid scheme of Monte Carlo and nonequilibrium diffusion methods. In the latter case, we implicitly solve the time-dependent radiation transport equation in a nonequilibrium, diffusion approximation for three-dimensional geometry including the scattering and thermalization terms for the source functions and include the frequency derivatives in the formulation for the opacities and emissivities (Lucy & Solomon 1970; Karp et al. 1977). At large optical depths, this provides the solution for the full radiation transport problem. We note that the use of a nonequilibrium diffusion does not imply that the mean intensity $J$ is given by the blackbody field. To obtain the correction solution for the radiation transport equation at small optical depths, the difference between the solutions of the diffusion and full radiation transport equations is calculated by a Monte Carlo method. We calculate the difference between the solutions for computational accuracy and efficiency. Consistency between the solution at the outer and inner region is obtained iteratively. The same Monte Carlo solver is used that has been applied to compute $\gamma$-ray and polarization spectra (e.g., Höflch et al. 1995b). The Monte Carlo method is appropriate for this problem because of its flexibility with respect to the geometrical and velocity structures.
A3.2. Coupling of the Radiation Transport and Rate Equations

The solution of the radiation transport and the statistical equations for the level populations are coupled. A perturbation method is used to obtain consistency between the solutions (Höflich 1995a). In the equations for the microphysical quantities we express the actual mean intensity by the following equation:

\[
J^{(m)} = \Lambda^{(m)} S^{(m)} \approx \Lambda^{(m-1)} S^{(m-1)} + \Lambda^{(m-1)} [S^{(m)} - S^{(m-1)}].
\]  

(A8)

Here the indices in parentheses denote the iteration step and \(S\) and \(\Lambda\) are the source function and the radiation transport matrices, respectively. The matrix \(\Lambda^*\) is a band matrix with elements corresponding to the diagonal and first off-diagonal elements of the complete matrix \(\Lambda\). The elements of \(\Lambda\) are computed in the narrow line limit (Sobolev 1957; Höflich 1995a).

The use of a Monte Carlo scheme introduces an additional complication due to the “photon” noise. To achieve numerical stability for the solution in the rate equations, the deviation from the solution for \(J\) obtained by the Monte Carlo calculations and the nonequilibrium diffusion are set to zero if they are less than the statistical noise. This avoids instabilities during the Monte Carlo calculation from previous iterations, \(n\), taken into account with individual weight factors. Currently, we use a 1/(\(m-n\)+1) weight, where \(m\) is the total number of model iterations. Clearly, this weight function needs further fine-tuning. Tests for realistic structures of the envelope show that the resulting error in the population numbers inherent to our procedure is comparable to those introduced by discretization errors if we solve the radiation transport equation in a different scheme.
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