Analytical model of the laminar traffic phase states and its use for calculating the level of service indicators for road network

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Abstract. The article is dedicated to the description and discussion of theoretical models that may be used for calculating the level of service indicators for road network. Authors developed the approach for calculating the level of service indicators for urban road network sections with signalized intersections. The core of the proposed models of assessing the level of service is based on the method, developed by American experts for main roads and adapted by the authors for the city roads. The approach to assessing the level of service is based on the advanced analytical model of the fundamental diagram, describing various phases of traffic flow. Herein authors introduce the concept of a laminar traffic flow in the sections of the road network, which is implemented for the level of service calculating.

1. Introduction
Modern society needs high-quality transport services. At the same time, the demands for transportation of goods and passengers are constantly growing. The growing demand for passenger transport has led to an increase in the level of motorization and to the aggravation of related problems, including a high level of accidents on the roads, the occurrence of chronic congestion and environmental pollution.

In Russia the most typical example is Moscow. In Moscow every day occur about 650 traffic jam situations, each of which is idle an average of 500–600 cars. The average speed within central part of the City is about 18 km/h [1]. Similar conditions are observed in many other megacities of the world [2–5].

The most effective means of dealing with the consequences of uncontrolled motorization are intelligent transport systems (ITS). The ITS project should include three basic complex subsystems [6, 7]:
1) Subsystems that solve the problem of optimizing the transport system performance, focused on management of transport flows.
2) Subsystems of dispatching management of transport (public, municipal, special, freight) united by a single automated system of information for analytical support of Executive bodies.
3) Subsystems, providing safety and security of transport processes.

Exploitation of ITS is very expensive affair for a city administration. Therefore, an important task is a comprehensive assessment of the ITS functioning quality. From the point of view of the road users, the ability to move through the urban streets and city roads network is considered as a service.
In this case, assessment of situation must be based on evaluation of actual parameters of traffic flow on sections of the road network [5, 8].

A scientific basis for the assessment of traffic flow parameters (speed, density, intensity) was laid in the works of American scientists. In the second half of the last century they had formulated a number of indicators under the title "Level of service". They determined six levels of service designated A, B, C, D, E, F, where the “A” level indicates the lightest condition of traffic flow. Indicators for assessing these levels are set out in the Highway Capacity Manual (HCM), fundamental American document, which has been repeatedly supplemented and republished [9]. Each level of service depends on relation of average traffic flow speed and free flow speed for the highway. Similar approach was realized in Russia road branch methodical document ODM 218.2.020-2012 "Guidelines on the assessment of road capacity" [10].

However, the analysis of limit values of indicators in both these documents indicates that they are applicable only for highways and not designed to assess the level of service on urban roads. The reason for this conclusion is that the characteristics of traffic flow on city roads are strongly influenced by speed limits, prescribed by city authorities. These limits are significantly different in value from the speed limits for highways. This, in turn, significantly affects the boundary values of indicators assessing level of service. This article deals with the evaluation of the indicator "level of service" using the fundamental diagram [11, 12], which is considered as an analytical dependence of the average speed on the average density at different phase states of the traffic flow, and provides a critical analysis of an alternative approach to the description of the fundamental diagram based on the use of a single approximating analytic formula. When assessing the indicator "level of service" we take into account the speed limit, prescribed for the road sector.

2. Two types of car following models, used for describing traffic flow parameters

2.1. Continuous model of Van Aerde

The mathematical theory of traffic flow, started with fundamental diagram, was developed mainly by mathematicians and physicists. There were two branches in the development of the mathematical theory of traffic flow, using “car following model”. The first one is approximation theory, which try to construct mathematical model, using statistical data of science experiments. The well-known model was continuous model of Van Aerde [13]. This model described the gap (h) between two vehicles: leader and follower, as function of current velocity (v) and velocity of free flow state (v_0) as follows:

\[ h = c_1 + \frac{c_2}{v_0 - v} + c_3 v, \]

where \( c_1, c_2, c_3 \) – constants.

The inverse value is density (\( \rho \)). Therefore, we have:

\[ \rho = \frac{1}{c_1 + \frac{c_2}{v_0 - v} + c_3 v}, \]

The graph of the Van Aerde continuous “Speed – Density” model represented in fig. 1.

The approximation approach has its own advantages and disadvantages. Advantages include simplicity of the analytical model and good approximation to real situation, gained by choice of coefficients values. The analytical model of the "speed-density" shows that dependence is non-linear and decreasing function of the traffic flow density. When the traffic flow density changes from zero to a maximum value, the curve is changed so that in total it represent some different sections, which correspond to different phase states of the traffic flow.

Disadvantages are:

1) the model doesn’t explain cause of traffic flow phase state changes;
2) the character of curve for numeric interval of small values of density is in contradiction with real situation. Indeed, the curve slows down immediately after starting point of density value, equal to zero. It means that average speed of traffic flow drops at initial section of the curve, corresponding to situation of almost empty road. This is contradiction with the real situation on the road and drivers behavior. Really, within some interval near the zero density value, average speed of the traffic flow doesn’t drop down and stay constant.

![Figure 1. The graph of the Van Aerde continuous model](image)

2.2. Methods of engineering psychology, used to derive the model of traffic flow

In the models of the follow-the-leader system "driver-vehicle" treated as a closed system with feedback, methods of engineering psychology are used to identify the mechanism by which the driver selects a certain position in relation to other cars and the road in order to describe the processes of information processing and decision making in the management of the vehicle [14].

![Figure 2. Position of the leader and the car following the leader](image)

Engineering-psychological approach is used to examine the unique sensory and motor characteristics of the driver and restrictions: on this basis, strive to minimize the number of situations in which the driver cannot properly manage vehicle, as well as to regulate the absolute and relative speed [15–17]. The scheme of movement of the car following the "leader" is shown in fig. 2. Here “x” is the current position of the leader, and “x_{n+1}” is the current position of the car following the leader.

The advantage of the leader models, in comparison with the continuous model of Van Aerde, is a
detailed account of the human factor, in this case – a detailed account of driver behavior and its impact on phase state of traffic flow. In accordance with this approach, the traffic flow on the roads can be considered as a complex closed system "driver – vehicle – traffic conditions – means of regulation". In this system, a control element influencing the speed of movement is the driver. He, relative to the speed of movement, keeps in mind states of other components of the system. The person (driver) is the main link in the system "driver – vehicle–road–environment".

3. Four phase states of the “laminar” traffic flow

3.1. Explanation of “laminar traffic flow” term and reason to use it.
When we analyze the situation with traffic flow which occurs on long section of town’s arterial roads with few or without intersections, we can note that state of the flow changes slightly during the day time, without hops. In that case the average speed of the traffic flow doesn’t fall down to zero at any time and at any traffic conditions if there are no obstacles. So we may use an analogy with the term "laminar", which is taken from hydrodynamics theory and means the way in which the liquid layers move without agitation and pulsation (that is, without messy rapid changes of velocity and pressure). The meaning of the term "laminar" is consistent with the cases of traffic flow when traffic is moving relatively freely, the number of “bottle necks” is small and the changes of flow parameters is slow.

The main reason for the term "laminar traffic flow" is that in this state it behavior precisely described with “follow the leader” models. Therefore we shall formulate our definition of this term.

For definition construction we use analogy with the term "laminar", which is taken from hydrodynamics theory and means the way in which the liquid layers move without agitation and pulsation (that is, without messy rapid changes of velocity and pressure). The meaning of the term "laminar" is consistent with the cases of traffic flow when traffic is moving relatively freely, the number of overtaking is small and the change of flow parameters is slow.

Strictly speaking, in the case of free flow, when the cars move freely from one lane to another, the analogy with laminar flow is not quite correct. The cars move with different velocities, moving freely within the drive way, from right lane to left and back. However, in this case, we believe the main condition is the absence of obstacles to the flow. That is the main condition for laminar traffic flow occur (at any value of flow density). The second important condition for the existence of the laminar traffic flow is sustained flow condition close to the stationary, characterized by the absence of sharp changes. The third important condition, it is possible to consider the traffic flow in this state without interruption, even when the value of the flow density, close to the maximum density value. Though, the concept of “maximum density” is rather arbitrary, since it is impossible to tell the exact meaning of this value but only a certain range of these values, as mentioned above.

Definition: laminar traffic flow is the traffic flow which meets no artificial obstacles and bottlenecks in its path, not experiencing the sharp fluctuations of its velocity along the path and moving without stopping at any value of its average density [18–19].

Thus, we assume the traffic flow is laminar at any phase state if the following conditions take place:
1) Absence of obstacles to the flow of the vehicles.
2) Stability condition, expressed in the absence of sharp fluctuations of velocity and flow density.

When these conditions are met, the term laminar traffic flow is applicable for all its phase states.

Thus, we can assume laminar traffic flow under the above conditions arising in the areas of urban road network with uninterrupted motion.

Violation of the stability condition occurs when the transport streams convergence or split, when traffic streams have to go around a parked car or accident scene, at signalized intersection.

In conclusion, we can say that laminar traffic flow condition is important because it guaranties correctness of “follow the leader” models implementation for describing traffic flow behavior.

3.2. Analytical model of congested flow
Let us analyze the literature mathematical description of the motion of stationary traffic flow for different parts of the range of variation of traffic flow density. For this aim we can see analytical expressions of the function "speed-density".

As a basis we have taken a model of "follow the leader" for "congested flow" [15, 16]. Basic assumptions upon which the analytical model of a congested flow stream designed, when the density of the flow, tending to the limiting values \( \rho = \rho_{\text{max}} \), are as follows:

- the cars move on the lane, not overtaking each other;
- the movement of each car following its leading vehicle (leader), described by certain differential equations, which reflects the increased sensitivity of the driver at small changes of distance between the cars;
- the derivation of the analytical dependence of the average values of flow "speed - density" assumes the existence of an equilibrium state of the stream;
- when traffic flow density reaches the maximum value average flow speed reaches the minimum value, but different from zero.

It is proved that the behavior of cars in heavy traffic on a separate strip adequately described by the model of follow the leader, which is represented by the differential equation:

\[
x'(t)_{n+1} = Cx(t)_{n} \left( x(t)_{n} - x'(t)_{n+1} \right),
\]

where

- \( x(t)_{n} \) = current leader position in time \( t \);
- \( x(t)_{n+1} \) = current position of car, following the leader in time \( t \);
- \( x''(t)_{n+1} \) = acceleration of the car, following the leader in time \( t \);
- \( x'(t)_{n+1} \) = velocity of car, following the leader in time \( t \);
- \( x'(t)_{n} \) = velocity of leader car in time \( t \);
- \( C \) = the proportionality coefficient.

It has been shown (Haight, 1963) that the equation (3) resolved and shows the equation for average speed stationary traffic flow \( (V_{\text{flow}}) \) in the form:

\[
V_{\text{flow}} = C_0 e^{-C_1 \rho},
\]

where

- \( C_0, C_1 \) = coefficients;
- \( \rho \) = average traffic density.

The equation (4) shows that when the maximum density of a laminar traffic flow is reached, the flow velocity is minimal, but not equal to zero. This corresponds to the fact that cars in real traffic flow at a maximum density are not stop in the state of "bumper to bumper" corresponding to maximum density, as was suggested by the author of the first analytical model of traffic flow (Greenshields, 1934), and traffic flow continue to move at a low average speed keeping the smallest possible secure interval between cars. This corresponds to the real situation on the arterials and highway section when there are no obstacles for the traffic. We need to note that in this case “maximum density” term is idealization of the real situation.

3.3. Analytical model of “synchronized flow”

When density of traffic flow became less then it correspond to “congested” flow the average speed rise up to approximately 30 km/h, traffic flow phase state changes for "synchronized flow" and equation (4) is not adequate for this speed range.

The relation between the basic parameters of traffic flow in phase of it “synchronized motion” can be described by a rule formulated by the competent authorities of the State of California Department of Transport: "The minimum distance between two moving one behind the other cars, must be
proportional to the velocity of the slave vehicle” [16].
To formulate analytical description of this state, let us denote $x'_{n+1}(t)$ – current speed of the vehicle, following the leader at time $t$. Then this rule can be written in analytical form as follows:

$$x(t)_{n+1} = c (x_n - x_{n+1}) + k$$  \hspace{1cm} (5)

where

$x(t)_n$ = current leader position in time $t$;
$x(t)_{n+1}$ = current following the leader car position in time $t$;
$c$, $k$ = coefficients.

As noted above, the task of obtaining the desired ratios of the average values of speed and density of flow is solved in the assumption of the existence of the equilibrium state of the flow. That is, you must accept the assumption that there is a very stable equilibrium state in each phase. Note that this condition is just performed when “laminar traffic flow” conditions occur.

For the state of traffic flow equilibrium, described by model (5) we have the average distance between vehicles, which is inversely proportional to the average traffic flow density. Therefore, for average traffic flow speed ($V_{flow}$) we obtain the expression:

$$V_{flow} = \frac{c}{(\rho - 1)} + k$$  \hspace{1cm} (6)

where

$c$, $k$, $l$ = constants.

We can see that in the “synchronized flow” phase state may be described by a hyperbolic function. The coefficient $l$ in equation (6) is a value of flow density ($\rho_0$), which corresponds to the vertical asymptote of the hyperbola whose equation is $\rho = \rho_0$. With this in mind, equation (6) can be rewritten in the form:

$$V_{flow} = \frac{c}{(\rho - \rho_0)} + k$$  \hspace{1cm} (7)

3.4. Analytical model of free flow state
When conditions of free flow occur each driver chooses the most convenient speed depending on the speed limit prescribed for the road section. In free flow conditions the increase in traffic flow density within certain limits does not reduce the average traffic flow velocity ($V_{flow}$) within some narrow range of density near the zero value [18, 19]. Hence, the equation linking average speed of free and density of free-flowing traffic flow looks like this:

$$V_{flow} = V_{max} \quad 0 < \rho < \rho_0,$$  \hspace{1cm} (8)

where $V_{max}$ = constant; maximum value of average speed of traffic flow when free flow conditions occur.

The Graph of the function "speed-density" for "free flow" phase state (when traffic flow density changes within the range of $0 < \rho < \rho_0$) has the form of a straight line parallel to the $\rho$-axis.

3.5. Proof of the existence of the traffic flow fourth phase
When the density of the traffic flow exceed value of $\rho = \rho_0$, the traffic “free flow” phase state transform into another phase state.

Here we outline theoretical justification for the existence of a fourth phase, – the phase, which is transitioning traffic flow state from the state of "free flow" to the state of "synchronized flow”.

The formal proof of existence of “transition state” phase is based on the following considerations.
Since the traffic flow is orderly movement of motor vehicles of large mass, according to Newton’s laws of movement, "speed-density" function for different phase states should be smooth. This means that each value of argument (density) should not be a value of discontinuity of the function itself and its first derivative. As we can see above, different parts of "speed-density" function are described with different equation. The conditions of smoothness at the points of conjugation of the function different part could analytically be written as follows:

\[
\lim_{\rho \to \rho_0^-} V_i = \lim_{\rho \to \rho_0^+} V_i
\]

(9)

where

\(V_i, V_j\) – the value of the function "speed-density" to the left and to the right of the conjugation point;

\(\rho_{ij}\) = the conjugation point of the \(i\)-th and \(j\)-th sections of this function.

Similarly, for the first derivation of the function we have:

\[
\lim_{\rho \to \rho_0^-} V'_{ij} = \lim_{\rho \to \rho_0^+} V'_{ij}
\]

(10)

where \(V'_i, V'_j\) – the value of the first derivative of the function "speed-density" to the left and right of the conjugation point.

As shown above, the influence of density absents and average speed remains constant within some interval \([0, \rho_0]\) for the function section (8) Therefore, the value of the first derivative at any point of interval \([0, \rho_0]\) equal to zero.

From other side, synchronized traffic flow is represented by hyperbola, which is decreasing function. Therefore, the value of the first derivative at any value of density for this section is less than zero. If after “free flow” phase state, the next state is “synchronized flow” phase state, then condition (10) could not be fulfilled. That is impossible. Conclusion—there must be a “transition phase”, smoothly connecting “free flow” phase state and “synchronized flow” phase state. So, formally, the need for the existence of “transition phase” is justified. It means that laminar traffic flow has distinct four phase states.

3.6. Formal derivation of analytical expression for “transition phase” of laminar traffic flow

We now describe the analytical expressions derivation of the "speed – density" function for “transition phase”. To do this, we must note that this phase state appears when the vehicle moves at the high speed and the driver began to understand the dangerous situation, created by close distance from leader. Therefore, we perform the construction of differential equations, reflecting the behavior of the driver, following the leader in the “transition phase”, on the basis of the theoretical analysis of the behavior of vehicle drivers in emergency situations. We considered a theoretical analysis developed by the Russian researchers A. Utkin and V. Chvanov. Driving a car at the maximum allowable speed was regarded by them as an emergency situation.

A. Utkin detailed the features of the driver behavior and visual perception of the situation in case of dangerous situations. He took the time interval to possible collision as the measure of transport risk. He developed a model of driver behavior in the transport stream based on the description of the interaction of two or three cars moving sequentially one behind the other [20].

V. Chvanov performed analysis of the main patterns of drivers’ behavior, using the theory of engineering psychology concerning the activities of operators in the special conditions, to which Chvanov was assigned the driving of vehicles at maximum allowed speed [21].

We can conclude that the A. Utkin and V. Chvanov results adequately describe the behavior of the driver during the “transition phase” driving. Than we chose Chvanov’s verbal model "Danger - speed" as the basis for analytical model construction. The main content of this model is as follows: "A sense of the danger of the driver depends on the selected speed. The growth of danger sense can be
compensated by lowering car speed, which allows driver to maintain the emotional stress at acceptable level” [21].

We comment on this model as follows. When driving the car in the free flow condition, the main danger source is the high kinetic energy of the car, which moving at very high speed regarding to driving conditions. This factor creates the driver a feeling of great danger. So, minimum distance to the leader is estimated by the driver on the basis of the analysis of, distance, road conditions, vehicle brake system capacity and other factors. As a result, driver should change the car speed if the distance reduced so as to avoid a collision if a sudden and sharp deterioration of the situation will occur. Simultaneously, he must take in mind the car limited maneuvering and braking capabilities at high speed.

This implies that driver behavior should be based on the possibility of reducing the vehicle speed when approaching a leader to maintain high emotional stress at an acceptable level. In this case the negative acceleration during convergence with the leader should be gradually increased, not exceeding a certain value, in order to sustain the high speed movement of the car on the road. These terms in a differential equation describing the negative acceleration of the driven vehicle $(x''(t))_{n+1}$ can be represented as follows:

$$x(t)_{n+1} = \frac{C_1}{(x_n - x_{n+1})^n} + C_0,$$

where

$C_1 = \text{constant};$

$(x_n - x_{n+1}) = \text{distance value, which forced driver, following the leader, to react during convergence.}$

The expression (11) defines the value of negative acceleration when approaching to the leader during the “transition phase”. After integration equation (11) for following the leader car speed $x'(t)_{n+1}$ we have the equation:

$$x(t)_{n+1} = \frac{C_1}{(x_n - x_{n+1})^n} + C_0,$$

where

$C_0, C_1 = \text{constants}.$

For the equilibrium state of the flow in the “transition phase”, given the fact that the average distance between vehicles is inversely proportional to the average flow density and $C_0 = V_{\text{max}}$, we get from (12) for the average flow speed $(V_{\text{flow}})$:

$$V_{\text{flow}} = V_{\text{max}} - C \left( \rho - \rho_0 \right)^2, \quad \rho > \rho_0$$

$V_{\text{max}} = \text{maximum value of average flow speed during “free flow” phase; } \rho_0 = \text{maximum value of average flow density, when fare flow state occurs; }$

$C_1 = \text{constant}.$

Analysis of equation (13) shows that analytical expression of transition phase state of laminar traffic flow is one branch of parabola, which has maximum value $V_{\text{max}}$ when density equal to $\rho_0$. The conditions (9) and (10) are fulfilled in this point.

Analytical expressions of phase states of laminar traffic flow are shown in Table 1.
Table 1. Analytical expressions to calculate the speed and intensity of traffic flow in different parts of the curve "speed-density" density change of the above thread

| Curve section | Traffic flow phase | Traffic flow density (vehicles/km) interval | “Speed - density” equation for the curve section |
|---------------|-------------------|------------------------------------------|---------------------------------------------|
| Section 1     | Free flow phase   | 0                                        | \( V_{flow} = V_{max} \)                   |
| Section 2     | Transition phase  | \( \rho_0 \) to \( \rho_1 \)             | \( V_{flow} = V_{max} C (\rho - \rho_0)^2 \)|
| Section 3     | Synchronized flow phase | \( \rho_1 \) to \( \rho_2 \)         | \( V_{flow} = \frac{C_2}{(\rho - \rho_1) + k} \) |
| Section 4     | Congested flow phase | \( \rho_2 \) to \( \rho_{max} \)      | \( V_{flow} = V_o e^{-cp} \)               |

4. The results of experimental studies conducted to confirm the adequacy of the analytical models of the laminar flow phase states

4.1. Objects of experimental studies and technology for gathering initial data

Objects of experimental studies were:

- gathering of measured speed, capacity data of traffic flow in the different phase states;
- initial calculation of gathered data and estimating of analytical expressions parameters for “speed-density” functions for different phase state of traffic flow and fitting different part of the function to fulfill conditions (9), (10).

The data was gathered for street and arterials sections of Moscow road network with different speed limits (110 km/h, 80 km/h and so on). Each session lasted 15 or 30 minutes.

For measurements, a road section with a dry, flat surface was selected. Measurements were carried out in the years 2008–2011 [18]. A total of about 20 sessions were held in order to obtain baseline data. However, part of the data had to be discarded due to inappropriate conditions in which they were collected. Each session lasted 15 or 30 minutes, during which the transit time of a pre-selected and measured section of each vehicle in the selected lane was measured. The total number of vehicles passing through the lane during the session was calculated.

Processing the data collected was to calculate the traffic flow average speed capacity and density per lane. Then parameters of analytical expressions were calculated.

In Table 2, as an example, given the parametric model of those functions with numeric parameter values and ranges of flow density for different phase states on the main road with an established speed limit of 110 km/h [18]. Density concept refers to the flow, which is equivalent to the flow of homogenous flow of passenger’s cars.

Table 2. Parametric "speed-density" function model for various phase states of traffic flow on the left lane of the main road with limited speed of 110 km/h

| Number of state | Traffic flow phase   | “Speed - density” equation for the curve section | Flow density (vehicles/km) interval |
|-----------------|----------------------|-----------------------------------------------|-----------------------------------|
| 1.              | Free flow phase      | \( V_1(\rho) = 110 \)                        | From 0 to 9,1                     |
| 2.              | Transition phase     | \( V_2(\rho) = 110 - 0,17 (\rho - 9,1)^2 \)  | From 9,1 to 22,2                  |
| 3.              | Synchronized flow    | \( V_3(\rho) = 5 + \frac{799}{0,7\rho - 5} \) | From 22,2 to 49,0                 |
| 4.              | Congested flow       | \( V_4 = 86e^{-0,02} \)                      | From 49,0 to \sim 160            |

Figure 3 shows the graph of the stage “Speed-density” function, representing analytical equations of traffic phase states, represented in table 2.
Legend: Coordinates for transfer points for speed limit \( V=110 \text{ Km/h} \):
1. Transfer “Free flow” phase state to “Transition” phase state \((q=1001 \text{ Veh/h}, \rho=9.1 \text{ Veh/km})\)
2. Transfer “Transition” phase state to “Synchronized flow” phase state \((q=1794 \text{ Veh/h}, \rho=22.2 \text{ Veh/km})\)
3. Transfer “Synchronized flow” phase state to “Congested flow” phase state \((q=1583 \text{ Veh/h}, \rho=49.0 \text{ Veh/km})\)
4. Point 3 is point of indistinguishability for each “Capacity-Density” function. Their values become statistically indistinguishable when average density of traffic flow exceed density value, corresponding to point 3.

Figure 3. The graph for the stage “Speed-density” function, representing analytical equations of traffic phase states, represented in table 2

5. Calculating the level of service indicators for road network, keeping in mind speed limits for urban arterials and streets segments

The figure 3 represents 5 different graphs of “Capacity-Density” relationship corresponding to different traffic speed limits for urban arterials and streets segments.

We suppose that if average speed of traffic flow is close to established speed limit for a streets segments, then conditions correspond to “Free flow” phase state. We can see that initial parts of graphs are represented by straight-line segment, which corresponds to different constant average speed of traffic flow for “Free flow” phase state. Different slope of each initial part of graphs reflects different speed limits. Therefore transfer point coordinates inside “Capacity-Density” phase space for “Free flow” phase state depend on speed limit established for street or arterial segment. Further we can see that after some density value “Capacity-Density” relationship is statistically indistinguishable for each street or arterial section, and not depends on initial speed limit for a street or arterial segment. Then we can see that for small speed limit established, traffic flow has only three phase states. These conclusions give us opportunity to recalculate parameters of phase state of traffic flow. The results of this recalculation are represented in table 3.
Table 3. Phase states boundaries for laminar traffic flow on freeway flow sections with different values of speed limits

| Value of speed limit, km/h | Traffic flow phase state          | Density variation interval, vehicles/km |
|---------------------------|----------------------------------|----------------------------------------|
| $V_{\text{max}} = 60$ km/h| Free flow state                  | $0 < \rho \leq 24,5$                   |
|                           | Synchronized flow state          | $24,5 < \rho \leq 60$                  |
|                           | Congested flow state             | $60 < \rho \leq 140$                   |
| $V_{\text{max}} = 80$ km/h| Free flow state                  | $0 < \rho \leq 16,8$                   |
|                           | Transition state                 | $16,8 < \rho \leq 25,6$                |
|                           | Synchronized flow state          | $25,6 < \rho \leq 60,0$                |
|                           | Congested flow state             | $60 < \rho \leq 140$                   |
| $V_{\text{max}} = 90$ km/h| Free flow state                  | $0 < \rho \leq 13,9$                   |
|                           | Transition state                 | $13,9 < \rho \leq 25,6$                |
|                           | Synchronized flow state          | $25,6 < \rho \leq 60,0$                |
|                           | Congested flow state             | $60 < \rho \leq 140$                   |
| $V_{\text{max}} = 110$ km/h| Free flow state                  | $0 < \rho \leq 9,1$                    |
|                           | Transition state                 | $9,1 < \rho \leq 22,2$                 |
|                           | Synchronized flow state          | $22,2 < \rho \leq 60$                  |
|                           | Congested flow state             | $60 < \rho \leq 140$                   |

6. Conclusions
Analysis of traffic flow shows that there are some conditions when specific traffic behavior arises. This occurs when the traffic flow meets no artificial obstacles and bottlenecks in its path, not experiencing the sharp fluctuations of velocity along the path and moving without stopping at any value of its average density. This traffic behavior was named “laminar traffic flow”. We assume the traffic flow is laminar at any phase state if the conditions, mentioned above, take place.

Theoretical investigations results prove four phase stage of traffic flow and each phase stage can be described with different “Capacity-Density” relationship analytical expression. When “Free flow” phase stage arises average flow speed remains constant inside some density value interval.

A speed limits for traffic, established by local authorities for streets and arterials sections of urban roads network influences phase state of traffic, particularly changes density values interval for free flow state. This, in turn, influences estimations of traffic flow phase state.

When some average density value of traffic flow is exceeded, “Capacity-Density” relationship is statistically indistinguishable for each street or arterial section, and does not depend on initial speed limit of traffic flow for the sections.

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