Thermodynamical behaviour of the Variable Chaplygin gas

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Abstract

The thermodynamical behaviour of the Variable Chaplygin gas (VCG) model is studied, using an equation of state like \( P = -\frac{B}{\rho} \), where \( B = B_0 V^{-\frac{n}{3}} \). Here \( B_0 \) is a positive universal constant, \( n \) is also a constant and \( V \) is the volume of the fluid. From the consideration of thermodynamic stability, it is seen that only if the values of \( n \) are allowed to be negative, then \( (\frac{\partial P}{\partial V})_S < 0 \) throughout the evolution. Again thermal capacity at constant volume \( c_V \) shows positive expression. Using the best fit value of \( n = -3.4 \) as previously found by Guo et al.\(^1\) gives that the fluid is thermodynamically stable throughout the evolution. The effective equation of state for the special case of, \( n = 0 \) goes to \( \Lambda \)CDM model. Again for \( n < 0 \) it favours phantom-like cosmology which is in agreement with the current SNe Ia constraints like VCG model. The deceleration parameter is also studied in the context of thermodynamics and the analysis shows that the flip occurs for the value of \( n < 4 \). Finally the thermal equation of state is discussed which is an explicit function of temperature only. It is also observed that the third law of thermodynamics is satisfied in this model. As expected the volume increases as temperature falls during adiabatic expansions. In this case, for \( T \to 0 \), the thermal equation of state reduces to \( (-1 + \frac{n}{6}) \) which is identical with the equation of state for the case of large volume.

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1. Introduction

Recent observational evidences suggest that the present universe is accelerating.\(^2\) A Chaplygin type of gas cosmology \(^3\) is one of the plausible explanations of recent phenomena, which is a new matter field to simulate dark energy. This type of equation of state (EOS) is not applicable in the case of primordial universe. This was discussed in the several articles.\(^4\)\(^5\)\(^6\)\(^7\)\(^8\). Such equation of state leads to a component which behaves as dust at early stage and as cosmological constant (\( \Lambda \)) at later stage. The form of the equation of state (EoS) of matter is the following,

\[
P = -\frac{B}{\rho}
\]

Here \( P \) corresponds to the pressure of the fluid and \( \rho \) is the energy density of that fluid and \( B \) is a constant. Recently a variable Chaplygin gas (VCG) model was

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proposed and constrained using SNeIa gold data \cite{9,10}, assuming the $B$ to depend on scale factor of our metric chosen. Now we have taken the above relation as $B = B_0 V^{\frac{n}{3}}$ where $V$ is the volume of the fluid. For $n = 0$ the VCG equation of state reduces to the original Chaplygin gas equation of state. The value of $n$ may be positive or negative. Guo et al \cite{9} showed that the best fit value of $n = -3.4$ using gold sample of 157 SNe Ia data. Later in another article \cite{10} they constrained on VCG and determine the best fit value of $n = 0.5^{+1.0}_{-1.1}$ using gold sample of 157 SNe Ia data and X-ray gas mass fraction in 26 galaxy cluster \cite{11}. This result favors a phantom-like Chaplygin gas model which allows for the possibility of the dark energy density increasing with time. Relevant to mention that recently there are some indications that a strongly negative equation of state, $w \leq -1$, may give a good fit \cite{12,14} with observations. But in another work \cite{15}, we have seen that the value of $n$ lie in the interval ($-1.3, 2.6$) $[\text{WMAP 1st Peak + SNe Ia(3$\sigma$)}]$ and ($-0.2, 2.8$) $[\text{WMAP 3rd Peak + SNe Ia(3$\sigma$)}]$. Recently Santos et al \cite{16} have studied the thermodynamical stability in generalised Chaplygin gas model. In the present work we investigate thermodynamical behaviour of the variable Chaplygin gas (VCG) by introducing the integrability condition equation (3) and the temperature of equation (16). All thermal quantities are derived as functions of temperature and volume. In this case, we show that the third law of thermodynamics is satisfied with the Chaplygin gas. Furthermore, we find a new general equation of state, describing the Chaplygin gas as function of either volume or temperature explicitly. For the variable Chaplygin gas we expect to have similar behaviours as the Chaplygin gas did show. Consequently, we confirm that Chaplygin gas could show a unified picture of dark matter and energy which cools down through the universe expansion. Returning to the stability criterion of the Chaplygin gas we find that the value of $n$ should be negative. Interestingly Guo et al \cite{9} showed that the best fit value of $n = -3.4$ from probability contour. As mentioned earlier \cite{15} the best fit value of $n$ may be positive or negative. From the thermodynamical stability considerations we can constrain the value of $n$ and found that $n$ should always have negative value. The paper is organised as follows: in section 2 we build up the thermodynamical formalism of the VCG model and discuss the thermodynamical behaviour of this model. Finally in section 3 we give a brief discussion.

2. Formalism

Before proceeding further we define the uniform density of the fluid filling the universe as

$$\rho = \frac{U}{V}$$  \hspace{1cm} (2)

where $U$ and $V$ are the internal energy and volume filled by the fluid respectively. Now the energy $U$ and pressure $P$ of Variable Chaplygin gas may be taken as a function of its entropy $S$ and volume $V$. From general thermodynamics \cite{17}, one
has the following relationship

\[
\left( \frac{\partial U}{\partial V} \right)_S = -P \tag{3}
\]

With the help of equations (1) - (3) we get

\[
\left( \frac{\partial U}{\partial V} \right)_S = B_0 V^{-\frac{4}{3}} \frac{V}{U} \tag{4}
\]

Integrating we get

\[
U = \left[ \frac{6B_0 V^{\frac{2-n}{6}}}{6-n} + c \right]^{\frac{1}{2}} = \left( \frac{2B_0 V^{-\frac{4}{3}}}{N} \right)^{\frac{1}{2}} V \left\{ 1 + \left( \frac{\epsilon}{V} \right)^N \right\}^{\frac{1}{2}} \tag{5}
\]

the parameter \( c \) is the integration constant which may be a universal constant or a function of entropy \( S \) only; \( c = c(S) \) and \( B_0 = B_0(S) \). The term \( N = \frac{6-n}{3} \) and \( \epsilon = \left[ \frac{Ne}{2B_0} \right]^{\frac{1}{N}} \) which has the dimension of volume. Now the energy density \( \rho \) of the VCG reduces to the following form

\[
\rho = V^{-\frac{n}{6}} \left[ \frac{6B_0}{6-n} + cV^{-\frac{6-n}{3}} \right]^{\frac{1}{2}} = \left( \frac{2B_0 V^{-\frac{4}{3}}}{N} \right)^{\frac{1}{2}} \left\{ 1 + \left( \frac{\epsilon}{V} \right)^N \right\}^{\frac{1}{2}} \tag{6}
\]

Now we want to discuss the thermodynamical behaviour of this model.

(a) Pressure :

The pressure \( P \) of the VCG is also determined as a function of entropy \( S \) and volume \( V \) in the following form

\[
P = -\frac{B_0 V^{-\frac{4}{3}}}{\left[ \frac{6B_0}{6-n} + cV^{-\frac{6-n}{3}} \right]^{\frac{1}{2}}} = -\left[ \frac{NB_0 V^{-\frac{4}{3}}}{2} \right]^{\frac{1}{2}} \left\{ 1 + \left( \frac{\epsilon}{V} \right)^N \right\}^{-\frac{1}{2}} \tag{7}
\]

For \( n = 0 \), the above results reduce to CG model [3] and its thermodynamical behaviour was discussed earlier by Santos et al [16].

It is seen from the fig-1 that for \( n \leq 0 \), the pressure goes more and more negative with volume. We get \( P = 0 \) at \( V = 0 \) for any value of \( n \). It also follows from the fig-1 that as \( n \) becomes more and more negative the pressure falls sharply.

(b) Equation of state:

Now from the equations (6) and (7) we get the effective equation of state as

\[
\mathcal{W} = \frac{P}{\rho} = -\frac{N}{2} \frac{1}{1 + \left( \frac{\epsilon}{V} \right)^N} \tag{8}
\]
Figure 1: The nature of variations of $P$ and $V$ for different values of $n$ are shown. This figure shows that $P$ goes more and more negative with $V$. (Taking $B_0 = 1, c = 1$).

Figure 2: The variations of $W$ and $V$ for different values of $n$ are shown. This figure shows that a quiescence phenomenon for $n = 0$ and phantom-like phenomenon for $n < 0$. (Taking $B_0 = 1, c = 1$).

i) For small volume, $V \ll \epsilon$, i.e., $\frac{v}{V} \gg 1$
$W \rightarrow 0$, therefore $P \approx 0$. we get dust dominated universe and the EoS is independent of $n$.

ii) For large volume, $V \gg \epsilon$, i.e., $\frac{v}{V} \ll 1$

$W \approx -1 + \frac{n}{6}$ (9)

it follows from equation (9) that if $n < 6$, $W$ is always greater than $-1$. So this is not $\Lambda$CDM, but for $n = 0$, this will be $\Lambda$CDM. Influence of $n$ is prominent in this case. From equation (9) it follows that for positive values of $n$, the value of $W$ will be $0 > W > -1$. So we get a quiescence phenomenon and the big rip is avoided. However, in what follows we shall presently see that to preserve the thermodynamic stability of VCG $n$ should be negative. For $n < 0$ we get $W < -1$, the phantom-like model. It is seen from the fig-2 that $W$ is more negative for $n = -3.4$. In an earlier work [18] the present author studied modified Chaplygin gas in higher dimensional space time and showed that in the presence of extra dimension the model became phantom like, but when the extra dimension is absent our results seem to be
ΛCDM. One may see that the experimental results favour like VCG model \[9,10,15]\.

(c) Deceleration parameter:

Now we calculate the deceleration constant.

\[
q = \frac{1}{2} + \frac{3 P}{2 \rho} = \frac{1}{2} - \frac{3N}{4} \frac{1}{1 + \left(\frac{\epsilon}{V}\right)^N} \tag{10}
\]

Figure 3: The variations of \(q\) and \(V\) for different values of \(n\) are shown. Early flip is shown for \(n = 0\). (Taking \(B_0 = 1, c = 1\)).

i) For small volume, \(V \ll \epsilon\), i.e., \(\frac{\epsilon}{V} \gg 1\) which gives \(q \approx \frac{1}{2}\) i.e., \(q\) is positive, universe decelerates for small \(V\).

ii) For large volume, \(V \gg \epsilon\), i.e., \(\frac{\epsilon}{V} \ll 1\) which gives \(q \approx -1 + \frac{n}{4}\)

Thus we see that initially, i.e., when volume is very small there is no effect of \(n\) on \(q\). \(q\) is positive, universe decelerates. From fig-3 it follows that as volume increases \(q\) goes to zero first and then universe accelerates. For flip to occur the flip volume \((V_f)\) is in the following form

\[
V_f = \epsilon \left[\frac{2}{4 - n}\right]^{\frac{1}{n}} \tag{11}
\]

A little analysis of the equation (11) shows that for \(V_f\) to have real value \(n < 4\). Otherwise there will be no flip. This is in accord with the findings of the observational result \[9\].

(d) Stability:

To verify the thermodynamic stability conditions of a fluid along its evolution, it is necessary (i) to determine if the pressure reduces through an adiabatic expansion \(\left(\frac{\partial P}{\partial V}\right)_S < 0\) and (ii) to examine if the thermal capacity at constant volume, \(c_V > 0\) \[17\]. Using equations (1) and (7) we get
\[
\left( \frac{\partial P}{\partial V} \right)_S = \frac{P}{6V} \left[ (6 - n) \left\{ 1 - \frac{1}{1 + \left( \frac{\Theta}{\lambda} \right)^N} \right\} - n \right]
\]  

(12)

Figure 4: The variations of \( \frac{\partial P}{\partial V} \) and V for different values of n are shown. The nature of evolution of graphs are quite different for \( n = 0 \) and \( n = -3.4 \) but they give \( \frac{\partial P}{\partial V} < 0 \) throughout the evolution. (Taking \( B_0 = 1, c = 1 \)).

In an earlier work Sethi et al. [15] showed that the range of n lies in the interval (−1.3, 2.6) [WMAP 1st Peak + SNe Ia(3σ)] and (−0.2, 2.8) [WMAP 3rd Peak + SNe Ia(3σ)]. But from equation (12) we see that for \( n \leq 0 \), \( \left( \frac{\partial P}{\partial V} \right)_S < 0 \) throughout the evolution. Fig-4 gives similar type conclusion. So the positive value of n is not compatible in VCG model. It may be concluded that to get thermodynamical stable evolution the positive value of n should be discarded. One may mention that the nature of evolution of graphs are quite different initially for \( n = 0 \) and \( n = -3.4 \) but they both give \( \frac{\partial P}{\partial V} < 0 \) throughout the evolution. This is due to the influence of n.

Now we should also verify if the thermal capacity at constant volume \( c_V \) is always positive. First, we determine the temperature \( T \) of the Variable Chaplygin gas as a function of its volume \( V \) and its entropy \( S \). The temperature \( T \) of this fluid is determined from the relation \( T = \left( \frac{\partial U}{\partial S} \right)_V \). Using this relation of the temperature and with the help of equation (5) we get the expression of \( T \) as follows

\[
T = \frac{1}{2} \left[ \frac{V^N dB_0}{N dS} + \frac{dcV}{dS} \right] \left[ \frac{2B_0V^N}{N} + c \right]^{-\frac{1}{2}}
\]  

(13)

If \( c \) and \( B_0 \) are also assumed to be universal constants, then \( \frac{dc}{dS} = 0 \) and \( \frac{dB_0}{dS} = 0 \), the fluid, in such condition, remains at zero temperature for any value of its volume and pressure. Therefore, to check the thermodynamic stability of the variable Chaplygin gas whose temperature varies during its expansion, it is necessary to assume that the derivatives in equation (13) are not simultaneously zero. We have no \textit{apriori} knowledge of the functional dependance of \( B_0 \) and \( c \) on \( S \). From physical considerations, however, we know that this function must be such as to give positive temperature and cooling along an adiabatic expansion, and we choose that \( \left( \frac{dc}{dS} \right) > 0 \).
Now from dimensional analysis, we observe that \( [c]^{\frac{1}{2}} = [U] \) which implies \([c]^{\frac{1}{2}} = [U] = [T][S] \). Thus

\[
c = \frac{1}{\beta^2} S^2 \tag{14}
\]

Here \( \beta^{-1} \) is a universal constant with the dimension of the inverse of the temperature: \( \beta^{-1} = \tau \). Differentiating equation (14) we get

\[
\frac{dc}{dS} = 2\tau^2 S \tag{15}
\]

In order to have positive temperatures and cooling along an adiabatic expansion, we must impose for mathematical simplicity \( \frac{dB_0}{dS} = 0 \), which makes the constant \( B_0 \) a universal constant. In that case equation (13) reduces to

\[
T \approx \frac{1}{2} \frac{dc}{dS} \left( \frac{2B_0 V N}{N} + c \right)^{-\frac{1}{2}} = \tau^2 S \left( \frac{2B_0 V N}{N} + \tau^2 S^2 \right)^{-\frac{1}{2}} \tag{16}
\]

After straightforward calculations we get the expression of entropy as

\[
S = \left( \frac{2B_0}{N} \right)^{\frac{1}{2}} V^\frac{n}{6} T^\frac{n}{2} \left( 1 - \frac{T^2}{\tau^2} \right)^{-\frac{1}{2}} \tag{17}
\]

For positive and finite entropy \( 0 < T < \tau \). Evidently at \( T = 0 \), \( S = 0 \) implying that the third law of thermodynamics is satisfied.

The thermal capacity at constant volume can be written as

\[
c_V = T \left( \frac{\partial S}{\partial T} \right)_V = \left( \frac{2B_0}{N} \right)^{\frac{1}{2}} V^\frac{n}{6} T^\frac{n}{2} \frac{1}{(1 - \frac{T^2}{\tau^2})^{\frac{1}{2}}} \tag{18}
\]

Since, \( 0 < T < \tau \), \( c_V > 0 \) is always satisfied irrespective of the value of \( n \).

**(e) Thermal equation of State:**

Since \( P = P(T, V) \), using equations (5), (14) and (17) we get the internal energy as a function of both \( V \) and \( T \) as follows

\[
U = \left( \frac{2B_0}{N} \right)^{\frac{1}{2}} V^\frac{n}{6} \left( 1 - \frac{T^2}{\tau^2} \right)^{-\frac{1}{2}} \tag{19}
\]

Now with the help of equations (1), (2) and (19) the pressure will be

\[
P = - \left( \frac{B_0 N}{2} \right)^{\frac{1}{2}} V^{-\frac{n}{6}} \left( 1 - \frac{T^2}{\tau^2} \right)^{\frac{1}{2}} \tag{20}
\]

which is also a function of both \( V \) and \( T \). For \( T = \tau \), \( P = 0 \), the universe behaves like a dust-like or a pressureless universe, as the Chaplygin gas equation of state can not explain the primordial universe. Unlike the work of Santos et al. [16] we do not get de Sitter like universe due to the presence of the term \( V^{-\frac{n}{6}} \) in equation (20) for the case of \( T \to 0 \). Again we have seen that the isobaric curve for the VCG do
not coincide with its isotherms in the diagram of thermodynamic states. Now using equation (2) and (19) we further get,

$$\rho = \left( \frac{2B_0}{N} \right)^{\frac{1}{2}} V^{-\frac{n}{6}} \left( 1 - \frac{T^2}{\tau^2} \right)^{-\frac{1}{2}}$$

(21)

We find exactly similar expressions of $\rho$ with the help of equations (1) and (20). From equations (20) and (21) we get the thermal equation of state parameter

$$\omega = \frac{P}{\rho} = \left( -1 + \frac{n}{6} \right) \left( 1 - \frac{T^2}{\tau^2} \right)$$

(22)

This thermal equation of state parameter is an explicit function of temperature only and it is also depends on $n$. As volume increases temperature falls during adiabatic expansions. In our case, for $T \to 0$, the equation (22) yields $\omega = -1 + \frac{n}{6}$ which is identical with the equation (9) as it is the case of large volume. Again as $T \to \tau$ (the maximum temperature), $\omega \to 0$ which is indicating dust dominated universe as expected.

Now we have to examine the well known thermodynamical relation as

$$\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial P}{\partial T} \right)_V - P$$

(23)

Using equations (19) and (20) we find the relation (23) is also satisfied.

We can also express the maximum temperature $\tau$ as a function of the initial conditions of the expansion. If we consider that the initial conditions at $V = V_0$ are $\rho = \rho_0$, $P = P_0$ and $T = T_0$, then we can get from equation (5) as

$$c = \left( \frac{\rho^2}{\rho_0^2} - \frac{2B_0}{N} V_0^{-\frac{n}{3}} \right) V_0^2$$

(24)

With the help of equations (6), (7) and (24), we obtained the energy density $\rho$ and the pressure $P$ as a function of the volume $V$ as

$$\rho = V^{-\frac{4}{3}} \rho_0 \left[ \frac{2B_0}{N \rho_0^2} + \left( 1 - \frac{2B_0}{N \rho_0^2} V^{-\frac{n}{6}} \right) \left( \frac{V_0}{V} \right)^2 V^\frac{n}{3} \right]^{\frac{1}{2}}$$

(25)

and

$$P = -\frac{B_0^{\frac{1}{2}} \left( \frac{B_0}{\rho_0^2} \right)^{\frac{1}{2}} V^{-\frac{n}{6}}}{\left[ \frac{2B_0}{N \rho_0^2} + \left( 1 - \frac{2B_0}{N \rho_0^2} V^{-\frac{n}{6}} \right) \left( \frac{V_0}{V} \right)^2 V^\frac{n}{3} \right]^{\frac{1}{2}}}$$

(26)

Now the equations (20), (25) and (26) can be written as function of the reduced parameters $\varepsilon$, $v$, $p$, $\kappa$, and $t$ such that

$$\varepsilon = \frac{\rho}{\rho_0}, \quad v = \frac{V}{V_0}, \quad p = \frac{P}{B_0^{\frac{1}{2}}},$$

$$\kappa = \frac{2B_0}{N \rho_0^2}, \quad t = \frac{T}{T_0}, \quad \tau^* = \frac{\tau}{T_0}$$

(27)
the equations (20), (25) and (26) can be written in the reduced units respectively as

\[ p = -\left(\frac{N}{2}\right)^{\frac{1}{2}} V^{-\frac{n}{6}} \left(1 - \frac{t^2}{\tau^*^2}\right)^{\frac{1}{2}} \quad (28) \]

\[ \varepsilon = V^{-\frac{n}{6}} \left[\kappa + (1 - \kappa V^{-\frac{n}{3}}) \frac{V^{\frac{n}{3}}}{v^2}\right]^{\frac{1}{2}} \quad (29) \]

\[ p = -\frac{\kappa^{\frac{1}{2}} \left(\frac{N}{2}V^{-\frac{n}{3}}\right)^{\frac{1}{2}}}{\left[\kappa + (1 - \kappa V^{-\frac{n}{3}}) \frac{V^{\frac{n}{3}}}{v^2}\right]^{\frac{1}{2}}} \quad (30) \]

At \( P = P_0, V = V_0 \) and \( T = T_0 \), we have \( t = 1 \) and \( v = 1 \), and we get from equations (28) and (30)

\[ p_0 = -\kappa^{\frac{1}{2}} \left(\frac{N}{2}\right)^{\frac{1}{2}} V_0^{-\frac{n}{3}} = -\left(\frac{N}{2}\right)^{\frac{1}{2}} V_0^{-\frac{n}{6}} \left(1 - \frac{1}{\tau^*^2}\right)^{\frac{1}{2}} \quad (31) \]

hence \( \kappa \) and \( \tau^* \) can be determine as follows

\[ \kappa = V_0^{\frac{n}{3}} \left(1 - \frac{1}{\tau^*^2}\right) \quad (32) \]

and

\[ \tau^* = \frac{1}{\left(1 - \kappa V_0^{-\frac{n}{3}}\right)} \quad (33) \]

Interestingly, we have seen that \( \tau^* \) is depend on both \( \kappa, V \) and \( n \) also. For \( n = 0 \), all the above equations reduce to the equations of Santos et al. \[16\]. At present epoch, \( \kappa = \frac{2B_0}{N\rho_0} \), therefore, \( \rho_0 = \left(\frac{2B_0}{N\kappa}\right)^{\frac{1}{2}} \). If we considered that the temperature \( \tau = 10^{32}\text{K} \) (temperature of Planck era) and \( T_0 = 2.7\text{K} \) (the temperature of the present epoch), the ratio, \( \tau^* = \frac{\tau}{T_0} = 3.7 \times 10^{31} \). So the ratio \( \kappa \) will be,

\[ \kappa = V_0^{\frac{n}{3}} \left[1 - \frac{1}{(3.7 \times 10^{31})^2}\right] \approx V_0^{\frac{n}{3}} \quad (34) \]

Again from equation (21), for the case of present epoch when temperature \( T \) is very small ( i.e., \( T \rightarrow 0 \),

\[ \rho_0 \approx \left(\frac{2B_0}{NV_0^{\frac{n}{3}}}\right)^{\frac{1}{2}} \approx \left(\frac{2B_0}{N\kappa}\right)^{\frac{1}{2}} \quad (35) \]

The same result can be obtained from equation (6) for large volume.

Thus, consideration from equation (14), at the present epoch, the energy density \( \rho \) of the universe filled with the VCG must be very close to \( \left(\frac{2B_0}{N\kappa}\right)^{\frac{1}{2}} \).
3. Discussion

We have studied thermodynamical behaviour of VCG model. We consider the value of $n = -3.4$ which was found by Guo et al. [9]. In an earlier literature we have seen that the value of $n$ may be negative or positive [15]. For large volume, when $n > 0$ the effective equation of state results in a quiescence type whereas $n = 0$ goes to $\Lambda$CDM model. Again for $n < 0$ it favours phantom-like cosmology. Some important results are given below:

i) As we have considered $n = -3.4$, the pressure goes more and more negative as volume increases (fig-1).

ii) The effective equation of state is shown in equation (8). At large volume we have seen that for $n = 0$, gives $\Lambda$CDM model. Influence of $n$ is prominent for sufficiently large volume. For $n < 0$, $W < 0$ leads to phantom like cosmology which is in favour of the current SNe Ia constraints like VCG model. The above phenomena is shown in fig-2.

(iii) The deceleration parameter is studied in the context of thermodynamics and is shown in fig-3. Our analysis shows that for the flip to occur the value of $n < 4$. This is in accord with the findings of the observational results [9].

(iv) The most important area of our concern is the question of the thermodynamic stability of the gas chosen. Firstly, we have to determine whether $\left(\frac{\partial P}{\partial V}\right)_S < 0$. The analysis shows that only for the negative value of $n$, $\left(\frac{\partial P}{\partial V}\right)_S < 0$ throughout the evolution. So one important conclusion done here using equation (12) is that the value of $n$ must be negative. Interestingly, this result is in agreement with the observational result found earlier by Guo et al. [9]. Due to this reason we have taken $n = -3.4$ to study the whole work done in this article. In this context the thermal capacity at constant volume $c_V$ is also determined and it is seen that $c_V$ is always positive irrespective of the value of $n$. So both the conditions of thermodynamic stability of the fluid are studied which shows that the fluid is thermodynamically stable through out the evolution process.

(v) The expression of entropy is derived and shown in equation (17). In this equation it is seen that at $T = 0$, $S = 0$ implying that the third law of thermodynamics is satisfied.

(vi) Finally the thermal equation of state is discussed in this work where it is seen that the volume is not explicitly present in the expression (22). This thermal equation of state parameter is an explicit function of temperature only. As volume increases temperature falls during adiabatic expansions. In this case, for $T \to 0$, the equation (22) yields $\omega = -1 + \frac{n}{6}$ which is identical with the equation (9) as it is the case of large volume. Again as $T \to \tau$, $\omega \to 0$ pointing to a dust dominated universe. As this type of equation of state can not explain the primordial universe. Here the maximum temperature $\tau$ is expressed as a function of the initial conditions.
of the expression.

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