Quantum stirring in a one-dimensional Bose gas

Roberta Citro\textsuperscript{1,2}, Anna Minguzzi\textsuperscript{2}, Frank W. J. Hekking\textsuperscript{2}

\textsuperscript{1}Dipartimento di Fisica "E.R. Caianiello", Universit\`{a} di Salerno, and Unit\`{a} C.N.I.S.M., I-84081 Baronissi, Italy
\textsuperscript{2}Universit\`{e} Joseph Fourier, Laboratoire de Physique et Mod\`{e}lisation des Milieux Condens\`{e}s, C.N.R.S. B.P. 166, 38042 Grenoble, France
E-mail: citro@sa.infn.it

Abstract. We propose quantum stirring with a laser beam as a probe of superfluidity in a strongly interacting one-dimensional Bose gas confined to a ring. Within the Luttinger liquid theory framework, we calculate the fraction of stirred particles per period as a function of the stirring velocity, the interaction strength and the coupling between the stirring beam and the bosons. The fraction of stirred particles crosses over at a critical velocity from a constant at low velocities to a characteristic power law at high velocities. The critical velocity depends on the size of the ring and vanishes in the thermodynamic limit. Details of this cross-over depend on the interaction strength and on the coupling between the stirring beam and the bosons. Our results are relevant for ongoing experiments on ring-trapped Bose-Einstein condensates.

The remarkable experimental progress in the last few years in developing effective magnetic and optical trapping techniques for ultracold neutral atoms has enabled the realization of bosonic lattices with complex spatial structures. Recently, magnetic trapping schemes \cite{1, 2, 3} have been used to obtain the confinement of interacting bosons in a ring geometry. If the transverse confinement is tight enough, the motion of the particles is confined into a quasi-one-dimensional (1D) geometry, and the system becomes a realization of a Luttinger liquid \cite{4, 5, 6}. Together with this stimulating experimental work, numerous theoretical and experimental studies have focused on the manifestation of the superfluid behavior of the Bose-Einstein condensate (BEC) or on the observation of a persistent flow \cite{3}. For example, recent experiments have confirmed the superfluid behavior of a BEC by demonstrating a critical velocity below which a laser beam could be moved through the gas without causing excitations \cite{7, 8}, or by detecting an irrotational flow through the creation of vortices \cite{9} and vortex lattices \cite{10} in both rotating and non-rotating traps. While all previous studies were focused on 3D geometries, the analysis of superfluid properties of a one-dimensional system remains a challenge.

Recently, much theoretical effort has been devoted to investigate the possibility of inducing a persistent flow of particles by parametric pumping in various condensed matter systems. During adiabatic pumping \cite{11}, periodic (AC) perturbations of the system yield a DC current as long as the external perturbations are slow enough such that the system always remains in the ground state. The number of particles transferred in each cycle is then independent of the pumping period $T$ and the number of transferred particles per cycle is quantized. Up to now, spectacular precision of quantization of the pumped current has only been achieved in experiments with nano-electronics based devices \cite{12, 13, 14}.

In this paper we discuss an example of quantum stirring. Quantum stirring is accomplished by the adiabatic cyclical variation of one system parameter, while preserving the characteristics...
of a pump, i.e. the direction of particle flow is fixed. When considering the flow of a 1D Bose gas in the presence of a moving obstacle, the absence of drag is usually used as a criterion for superfluidity [15, 16]. As an alternative probe we propose quantum stirring with a laser beam in an interacting 1D Bose gas twisted into a ring. We suggest that the non-quantization of particle transport in the gas, originating from the absence of the creation of excitations, could be a striking demonstration of superfluid behavior.

To start with, we perform our analysis within the Luttinger liquid (LL) framework for a 1D interacting Bose system [4, 5, 6] whose Hamiltonian is given by:

$$H = \frac{\hbar}{2\pi} \int dx \left[ \frac{u_s}{K} (\nabla \phi(x))^2 + \frac{v_s K}{\hbar^2} (\pi \Pi(x))^2 \right].$$  (1)

This Hamiltonian accounts for the fluctuations of the phase $\phi(x)$ which represents the phonon mode associated with the density wave given by:

$$\rho(x) = [\rho_0 - \frac{1}{\pi} \nabla \phi(x)] \sum_{p=-\infty}^{\infty} e^{i2p(\pi \rho_0 x - \phi(x))},$$  (2)

where $\rho_0 = N/L$ is the average density of particles, $L$ being the system size; $\Pi(x) = \hbar \nabla \theta(x)$, is the conjugate variable to $\phi(x)$, $[\phi(x), \Pi(x')] = i\delta(x-x')$. In the case of a contact interaction between bosons $U_0 \delta(x)$, the sound velocity and the Luttinger parameter, $v_s$ and $K$, used in (1), are given [6] by: $v_s K = \frac{\pi h \rho_0}{m}$, as follows from galilean invariance, and $\frac{v_s}{K} = \frac{\rho_0}{m}$ in the weak coupling limit.

In addition, we consider an externally tunable potential $U(x, t)$ such that the Hamiltonian (1) acquires an explicit time-dependent term $\delta H(t) = \int dx U(x, t) \rho(x)$. In particular we consider a well localized blue-detuned laser beam stirred with a velocity $V$, i.e. $U(x, t) = V_0 \delta(x - Vt)$. By considering the bosonized expression for the density (2) and keeping only the most relevant harmonics, the time-dependent part of the Hamiltonian becomes:

$$\delta H(t) = V_0[\rho_0 - \frac{1}{\pi} \nabla \phi(Vt) + 2\rho_0 \cos(2\pi \rho_0 Vt - 2\phi(Vt))].$$  (3)

The term proportional to $\nabla \phi$ is analogous to a slowly varying chemical potential and can be absorbed in $H$ by a redefinition of the field $\phi, \phi \rightarrow \phi - (K/v_s) \int^\infty dx' U(x')$. An interpretation of the remaining terms can be inferred from the comparison with the bosonization procedure for the spinless fermions system which corresponds to the so-called Tonks-Girardeau regime, i.e. the hard-core limit. In this regime the corresponding expression applies: $2\pi \rho_0 \rightarrow 2k_F$, where $k_F$ is the Fermi wavevector. Thus the second term in (3) can be interpreted as a forward scattering of a fermion at $\pm k_F$ with small momentum change, while the third term represents a scattering by the impurity inducing a momentum change $\sim \pm 2k_F$, i.e. a backscattering process.

Since the backscattering term breaks the continuous chiral symmetry [17], the response to the motion of the delta barrier is given by the backscattering current, whose corresponding operator is given by $I^b = \frac{i}{\hbar} [N_L, \delta H] = -\frac{i}{\hbar} [N_R, \delta H]$, where $N_R(L) = \int dx \rho_R(L)(x)$ and $\rho_R(L)$ is the charge density of right(left) movers whose bosonized expression is $\rho_{R(L)} \approx \frac{\rho_0}{2} [\nabla \theta(x) \pm \nabla \phi(x)]$. The expression of the backscattering current is $I^b = i \gamma(t) \bar{b}(t) - \text{h.c.}$ where $\gamma(t) = V_0 e^{i 2 \pi \rho_0 Vt} = V_0 e^{i \omega_b t}$, with $\omega_b = 2\pi \rho_0 V$ the characteristic “backscattering” frequency, and $\bar{b}$ is the backscattering operator $\bar{b} \sim \rho_0 e^{i 2 \phi(Vt)}$.

Within the S-matrix expansion in the interaction representation for a weak barrier, the leading order contribution to the stirred particle current is given by:

$$I_b \approx i \int_{-\infty}^{t} dt' \langle [I_b(t), \delta H_{S\gamma}(t')] \rangle_{H_0} = \int_{-\infty}^{+\infty} dt' \text{Im} \left( \gamma(t) \gamma^*(t') \right) \frac{G^< (t-t') - G^> (t-t')}{i}$$  (4)
where \( G^{<}(t - t') \) is the analytic continuation of the Green’s function of the bosonized backscattering operator [18], \( G(\tau \to \pm it) \), whose long time behavior is determined by \( \sim (\frac{\alpha}{\alpha + \nu_s (t-t')})^{2K} \), \( \alpha \sim \rho_0^{-1} \) is the short distance cutoff. For an infinite system \( N, L \to \infty \) with \( \rho_0 = N/L \) constant, and in the limit of small stirring velocity \( V \), the expression for the backscattered current can be related to the Fourier transform of the Green’s function at the characteristic backscattering frequency:

\[
I_b \approx \frac{(2\pi)^{2K-2}}{\Gamma(2K)} \frac{V_0^2}{(\hbar \nu_s)^2} \left( \frac{V}{\nu_s} \right)^{2K-2} 2\pi \rho_0 V, \tag{5}
\]

with \( \Gamma \) being the Euler Gamma function. The number of stirred particles in one cycle is obtained from the backscattering current according to \( N_{\text{stir}}/N = I_b/\omega_b \). Note that as the Luttinger liquid theory is an effective low-energy model, it describes correctly the system at frequencies smaller than \( \nu_s/\alpha \), hence the expression (5) is valid only if \( V < \nu_s \), and cannot treat the supersonic regime. In the Tonks-Girardeau limit \( K \to 1 \) the fraction of particles stirred per cycle is given by \( N_{\text{stir}}/N \approx (V_0/\hbar \nu_s)^2 \). This result is independent of the frequency \( \omega_b \) and hence adiabatic [11]. In the small \( \omega_b \) limit this result is in agreement with a direct calculation of the fraction of stirred particles using the Bose-Fermi mapping and the Thouless expression [11].

The effects of the interactions and of a finite system size \( L \) can be captured using the renormalization group (RG) approach. The potential scales as \( V_0(l) = \left( \frac{l}{\alpha} \right)^{1-K} V_0 \) under the RG flow, where the length scale \( l \) can be inferred from (5) and is set by \( l = \alpha \nu_s / V \). The smallest value \( l \) can take is set by the short-distance cut-off \( \alpha \). This limits the stirring velocity to values smaller than the speed of sound, \( V < V_{\text{high}} \approx \nu_s \), as discussed above. On the other hand, the RG procedure must be stopped as soon as \( l \sim L \). This yields the lowest velocity \( V_{\text{low}} = \nu_s / N \) for which Eq.(5) holds for a finite ring. Indeed, \( m V_{\text{low}} \sim \pi \hbar / L \) is the momentum of the lowest bosonic mode that contributes to the renormalization of the barrier strength. Below this value, no excitations are possible because of discreteness of the spectrum and the stirred fraction is constant. The value of \( V_{\text{low}} \) found agrees with the one obtained by using a Gross-Pitaevskii type of approach [19]. As main result of this work we find that at the critical velocity \( V_{\text{low}} \) the fraction of stirred particles crosses from a power-law to a constant (adiabatic regime). Note also that the adiabatically stirred fraction decreases with decreasing interaction strength as \( 1/\Gamma(2K) \): when \( K \) grows, the system becomes more superfluid, hence the interaction with the external barrier decreases.

![Figure 1. Fraction of stirred particles per cycle as a function of the stirring velocity (in units of \( \nu_s \)) at decreasing barrier strengths \( V_0 \) from the upper to the lower curve. The vertical dotted lines at \( V^*/\nu_s \) indicate the crossover region from the weak link to the weak barrier limit. \( V/N \nu_s \) indicates the critical velocity below which the stirring remains adiabatic.](image-url)
The bosonized hamiltonian corresponding to the hopping across the weak link can be inferred from the duality representation [5] and the tunneling current can be calculated in the linear response theory as well. Its leading contribution is given by:

\[ I_t = \frac{(2\pi)^2}{(\hbar v_s)^2} \Gamma(1/2K) \left( \frac{V}{v_s} \right)^{2-2K} 2\pi \rho_0 V. \]  

(6)

The fraction of stirred particles in the presence of tunneling is given by

\[ N_{\text{stir}}/N = 1 - I_t/\omega_b. \]

In the hard-core limit \((K = 1)\) the stirred current will be again linear in the frequency of the stirring. We thus recover the adiabatic limit. As dictated by the RG flow, \(t(\frac{l}{\alpha}) = (\frac{l}{\alpha})^{1-K} t_0\), the perturbation is relevant for repulsively interacting bosons with contact interaction \((K > 1)\) and the stirring will be adiabatic as long as \(V < V_{\text{low}}\). Therefore, upon decreasing the stirring velocity the effective tunneling strength increases, thereby decreasing the stirred particle fraction. Perturbation theory breaks down when the effective tunneling strength reaches unity and the RG flow must be stopped at the \(V^*\) determined by \(t_0(\frac{l}{\alpha}) = 1\), i.e. \(V^* = v_s \rho_0^{K/(1-K)}\). Below \(V^*\) the stirred fraction of particles is governed by the weak barrier limit. The full dependence of the stirred fraction as a function of the stirring velocity is plotted for different barrier strengths and \(K > 1\) in Fig. 1.

On the experimental side, the persistent flow of Bose-condensed atoms in a toroidal trap has been recently observed [3]. It was found that when the local flow velocity exceeds the critical velocity, the flow will no longer be superfluid, resulting in the observed decay of the flow. A variant to such experiment by the addition of a cyclic moving plug beam in a quasi-1D ring, and the measure of the stirred fraction of particles could be a valuable realization of the present probe of superfluidity.

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