Mathematical modelling of formation process of a reinforced hole in a composite structure

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Abstract. Research of strained state of installation process of a reinforcing bushing in a hole of a part made of polymer composite material has been carried out using finite-element model. To determine forces and work of installation process and influence of design factors on bushing strained state along the contour of the reinforced hole, the mathematical model has been developed.

1. Introduction
Using of composite materials (CM) in aircraft structures [1, 2] and ground equipment allowed to develop structural components having predesigned properties, to decrease labor intensiveness when treating, and to reduce their mass on 15-30 \%, in some cases up to 50 \%. Prospectivity of composite materials is evident, and that is why constant nomenclature increase of complex engineering systems (aircraft, helicopters, rockets) and their components (fuselage, wing, stabilizers, rudders, rotors, rocket motor cases) [2] takes place in Russia and abroad. Among the most important factors which efficiency of CM in aircraft engineering depends on is formation of joints in composite material structures [3,4].

2. Formulation of the problem
When treating composite material products, some technological difficulties appear. Essential differences of physical-mechanical properties of composite materials and conventional ones result from the fact that these materials transmit loads worse than metals that transmit loads well (especially concentrated forces) from one element to another. This disparity between of loads transmission mechanisms in joints and the mechanism of involving in work separate layers of composition elements cause difficulties when designing mechanical point (rivet and bolt) joints of structures made of fiber reinforces materials. That is why most structural failures of composite material structures (60-85\%) happened in the joint zones. One of the ways to increase bearing capacity of joints is to install intermediate reinforcing bushing that allows to redistribute stresses and to decrease concentration of stress at a hole boundary [4].

The author has developed method of installation of a bushing [5]. The reinforcing element is glued in and plastically compressed in axial direction with formation of variable field of a radial compressing stresses at the joint boundary. This allows to reduce considerably radial tensile stresses at the boundary 'reinforcing element - sheet billet' under operating loads and thus to keep a glue interlayer between the hole wall in the sheet billet and the reinforcing bushing.

In article [5] the research were carried out on influence of strained state of a reinforcing element on the stressed state arising in the sheet billet after installation of the reinforcing element in the hole and
under tensile stresses. But during operation of products with reinforced holes it is necessary to know form and strain distribution in the reinforcing element, value of consumed work and a force necessary for axial compression of the element, height of plastic strain propagation.

Applying finite-element modelling, experimental research to determine geometrics of a reinforcing element and analysis of its strained state has been carried out. Carbon-filled plastic KMY-7T with thickness 4.0 mm was used as a sheet billet material, aluminium and titanium alloys were used as materials for the bushing. The bushings with dimensions shown in Figure 1 were used during the experimental research.

3. Formulation of the mathematical model

To determine forces necessary to press fit the bushing with segments energy method [3] was used. Work of external forces $A\sigma$ perpendicular to the segment butt is:

$$A\sigma = \int_F \sigma \cdot UdS = UP,$$

where $U$ – value of deforming motion.

From the condition of equality of works of external forces $A\sigma$ and deformation forces of the bushing $A_D$ we get:

$$P = A_D / U.\quad (2)$$

To determine $A_D$, boundary conditions were given at the outer contour of the bushing. Frictional force act at the upper and lower (under the segment) butts of the bushing and over its external surface. Shearing forces arise at the contact plane of the zone $A$ and the rigid zone $B$. Back pressure forces act from the sheet billet side, which are external forces in relation to the bushing, but the work of these forces can be considered as negative work. The bushing is considered as a plastic body with nonlinear hardening, and the sheet billet as an anisotropic medium complying with generalized Hooke law.

Total work of deformation $A_D$ is formed by the work of internal forces $A_{int}$ providing change of the bushing shape, work of frictional forces $A_{FP}$ at the segment butts, work of shearing forces $A_{CP}$ between plastic and hard zones, and work of negative forces of the sheet billet $A_{comp}$. 

Figure 1. Geometrics of reinforcing bushings used in the experimental research.
Figure 2. Strained condition of a reinforcing bushing with double-sided lugs.

\[ A_D = A_{st} + A_{TP} + A_{CP} + A_{comp}. \]  

Taking into account summable forces (3), total deformation work of the reinforcing bushing is the following:

\[ A_D = \frac{\sigma_0 U^{1+n} R}{(1-\varepsilon_i)\varepsilon^m_t} \left[ \frac{RD_1}{h^n(1+h)} + \frac{2\nu R^2 D_2}{\sqrt{3}h} + \frac{h^{1-n}D_3}{\sqrt{3}} \right] + \]

\[ U^2 R \frac{k-v_{12}+\frac{N}{2}(k+1)+\frac{N}{\pi}(k-1)}{4g} \left[ \frac{RD_4}{h} + \Psi_2 D_1 \right] \]

\[ D_1 = \left( \frac{\pi}{t} \tan \frac{\pi}{t} \right); \]

\[ D_2 = \frac{\pi}{3} + \frac{\sin \frac{\pi}{t}}{3\cos^2 \frac{\pi}{t}} + \frac{1}{3} \ln \left| tg \left( \frac{\pi}{4} + \frac{\pi}{2k} \right) - \tan \frac{\pi}{k} \right|; \]

\[ D_3 = \frac{\pi}{t} - \ln \left| tg \left( \frac{\pi}{4} + \frac{\pi}{2k} \right) \right|; \]

\[ D_4 = \frac{\pi}{t} - 2tg \frac{\pi}{t} + \left( \frac{tg \frac{\pi}{t}}{3} + \frac{\pi}{t} \right); \]

\[ U = \frac{1}{2} \varepsilon_s \cdot \left( 1 - \frac{r^2}{r^2 \cos^2 \varphi} \right); \]
\[\varepsilon_z = \frac{(h - h_0)}{h}; \]
\[n = \frac{\varepsilon_t}{(1 - \varepsilon_t)}; \]
\[\Psi = \mu + \frac{1}{8} \frac{\sigma}{h_0} (1 - \mu) \sqrt{\mu}; \]
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where \( \sigma \) - breaking point of the bushing material; \( h \) - initial height of the bushing with segments; \( h_0 \) - bushing height after deformation (without lugs); \( \varepsilon_t \) - reduction of area contributing to formation of neck stability; \( R \) - external bushing radius; \( r \) - internal bushing radius; \( \Psi \) - empirical coefficient taking into account state of frictional surfaces and the shape of deformation zone; \( \mu \) - friction coefficient; \( \sigma_r \) - radial stresses at the hole contour in the billet [39]; \( E_1 \) and \( E_2 \) - respectively modulus of elasticity of the sheet billet material in orthogonally related directions; \( \nu_{12} \) and \( \nu_{21} \) - Poisson ratio; \( G \) - shear modulus; \( \mu_2 \) - friction coefficient in the contact between the bushing and the hole wall of the sheet billet.

Knowing work of external forces \( A_D \), and using formula (2) value of an upset force of the reinforcing bushing is determined.

4. Conclusion
Mathematical model allowing to determine work and upset force of the bushing knowing design parameters of the process of installation of a reinforcing element in a hole has been developed. It also allows to determine strained state of bushing material along the outer contour. The model is adapted for applying in a standard finite-element software ANSYS Mechanical. It allows to choose manufacturing equipment necessary to perform technological process of development of reinforced holes in the products made of polymer composite materials, and to reduce complexity of results analysis when developing technological process of joint formation.

5. References
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