A New and Fast Bias Correction and Segmentation Model with Application to Human Brain MR Images

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Abstract. Magnetic resonance (MR) images have been used in detecting human brain disease more and more in recent years. The most important problem in the diagnosis process is to find accurate regions, which makes image segmentation necessary and important. Though there are kinds of segmentation approaches for MR images, different limits and defects also appear in the practical application for bias fields and complex structures in MR images, such as inaccurate segmentation, over-correction and long iteration time. Considering those shortcomings, this paper presents a new level set model by expressing the image intensity in a continuous form and applying the split Bregman method. Thanks to the two improvements, this model can segment and correct MR images simultaneously and efficiently, and can achieve bias correction results without over-correction. Then the authors applied this model to a large quantity of MR images, and gained many good results. For better observation, quantitative and qualitative comparisons have been done between this model and the multiplicative intrinsic component optimization (MICO) model. The experimental results indicate that this model has performed well for challenging intensity inhomogeneity problems.

1. Introduction
With the frequent application of magnetic resonance (MR) images in diagnosing human brain disease in recent years, accurate segmentation and bias correction for MR images have been attracting a lot of attention. For MR images segmentation, there are already some classical methods[1,2,3,4,5], such as the Chan-Vese (CV) model [6], the region-scalable fitting energy (RSF) model [7] and other methods. However, those models are difficult to deal with serious intensity inhomogeneity. In MR images, intensity inhomogeneity is very serious because of the technique reason, and we can call it as bias field. Bias Field is a distortion existing in MR images, which can be an inhomogeneous influence spreading over MR images and prevent accurate segmentation for images. Typically, an image with bias is characterized by a phenomenon that pixels in different issues are not separable based on their intensities, which turns out to be the essential reason for wrong segmentation results [8,9]. Inefficient bias correction and over bias correction can cause inaccurate even wrong segmentation results, which is an important problem to be solved. To handle the problems above, the authors had proposed a variational level set method to segment and correct MR images moderately, and the proposed model approximates the image intensity by a group of basis functions. So the model can preserve the natural variation and weak regions in original images and work well in eliminating serious bias fields. The authors introduce the split Bregman method to the iteration process [10,11,12,13,14,15], which can sharply decrease the computing time and then improve the efficiency.
2. The proposed model

2.1. The energy formulation

Different from the assumption that the intensity of a true image is a piecewise constant function, this model believes that the intensity of a true MR image is a piecewise continuous function, which means in a distinct issue the intensity can slowly vary. This idea makes sense because all of the images in real world can impossibly be described as piecewise constant. Let $\Omega \subset \mathbb{R}^2$ be the image domain, and $u : \Omega \rightarrow \mathbb{R}$ denote the target image to be segmented and corrected. The image intensity continuous model can be expressed by:

$$E(\sigma, b, I) = \sum_{i=1}^{N} \int_{\Omega} |u(x) - bI_i|^2 A_i(\sigma(x)) \, dx + \nu |\nabla \sigma(\sigma)| + \mu \text{Pen}_\sigma(\sigma),$$

where $\sigma$ is the zero level set of the segmentation contour. And $A_i = H(\sigma)$, $A_2 = 1 - H(\sigma)$. $\nu$ and $\mu$ are two parameters. $b$ is the bias field, $I=(I_1, I_2, \cdots, I_N)$, where $I_i$ is the intensity which approximates the intensities in the ith region. $H$ represents the Heaviside function defined as ($\sigma$ is a positive constant parameter):

$$H_\sigma(x) = \frac{1}{2} [1 + \frac{2}{\pi} \arctan(\frac{x}{\sigma})].$$

In this model, the image intensity $I_i$ is approximated by a continuous function which is combined by a group of basis functions in the ith region, also the bias field $b$ is approximated by a continuous function, then we present the energy formulation as follows:

$$E(\sigma, a, q) = \sum_{i=1}^{N} \int_{\Omega} |u(x) - a(x)B(x)^T q_i(x)B(x)^T|^2 A_i(\sigma(x)) \, dx + \nu |\nabla \sigma(\sigma)| + \mu \text{Pen}_\sigma(\sigma),$$

where $b = a(x)B(x)^T$, $I_i = q_i(x)B(x)^T$, $a(x)$ and $q_i(x)$ are the coefficient values. $\text{Len}_\sigma(\sigma)$ is the length term and $\text{Pen}_\sigma(\sigma)$ is a regular term. The length term can be expressed as:

$$\text{Len}_\sigma(\sigma) = \int_{\Omega} \nabla H_\sigma(\sigma(x)) \, dx,$$

and the regular term is defined as:

$$\text{Pen}_\sigma(\sigma) = \int_{\Omega} \frac{1}{2} \left( \nabla \sigma(x) \right)^2 \, dx.$$ (5)

To apply the split Bregman method, some changes should be done in the energy formulation because the method only works for $L_1$ convex minimization problems. Firstly, the authors calculate the derivative of the energy formulation in terms of time $t$ and get:

$$\frac{e^{\sigma}}{\hat{t}} = \delta_\sigma(\sigma)(S_1 + S_2) + \nu \delta_\sigma(\sigma) \text{div} \left( \nabla \sigma \left( \left| \nabla \sigma \right| \right) \right) + \mu \left( \nabla^2 \sigma - \text{div} \left( \frac{\nabla \sigma}{\left| \nabla \sigma \right|} \right) \right),$$

where $S_1 = -\int_{\Omega} \mu(y) - a(x)B(x)^T q_1(x)B(x)^T \right|^2$, $S_2 = \int_{\Omega} \mu(y) - a(x)B(x)^T q_2(x)B(x)^T \right|^2$ and $\delta_\sigma(x)$ is the derivative of $H_\sigma(x)$.

Because the last term in equation (4) is a normalized term and just designed to improve the re-initialization problem in curve evolution, here the authors remove this term to satisfy the form of the split Bregman method. Then, the equation (4) is converted to a new one, for $\nu=1$, without changing the coincident stationary solution:

$$\frac{e^{\sigma}}{\hat{t}} = (S_1 + S_2) + \text{div} \left( \frac{\nabla \sigma}{\left| \nabla \sigma \right|} \right).$$

According to the simpler flow equation (5), the new energy functional can be defined as:
\[ E(\sigma) = \int t(|\nabla(u_0(x))|) |\nabla \sigma(x)| \, dx + \int \sigma(x) e(x) \, dx, \]

where \( t(\eta) = \frac{1}{1 + \beta |\eta|^p} \) is a boundary detection function, \( \beta \) is a constant parameter.

### 2.2. The minimization process based on the split Bregman method

After all the transformation and simplification, now the minimization can be expressed as:

\[ \min_{-2 \sigma \leq \sigma \leq 2} E(\sigma) = \min_{-2 \sigma \leq \sigma \leq 2} \left( |\nabla \sigma| + <\sigma, e> \right), \]

where \(|\nabla \sigma| = \int t(|\nabla u_0(x)|) |\nabla \sigma(x)| \, dx\), \( <\sigma, e> = \int \sigma(x) e(x) \, dx\). With application of the split Bregman method, the minimization process is completed. By applying the split Bregman method, the authors introduce an auxiliary variable \( \tilde{c} \) such that \( \tilde{c} = \nabla \sigma \) to equation (7) firstly, then the constrained problem is turned to an unconstrained one:

\[ (\sigma^*, \tilde{c}^*) = \arg \min \left( |\tilde{c}| + <\sigma, e> + \frac{\theta}{2} \| \tilde{c} - \nabla \sigma \|^2 \right), \]

where \( \theta \) is the parameter to control the last term. Secondly, the Bregman iteration is applied by introducing an auxiliary variable \( r \) :

\[ (\sigma^*, \tilde{c}^*) = \arg \min \left( |\tilde{c}| + <\sigma, e> + \frac{\theta}{2} \| \tilde{c} - \nabla \sigma - r \|^2 \right), \]

where the variable \( r \) is updated by:

\[ r^{n+1} = r^n + (\nabla \sigma^{n+1} - \tilde{c}^{n+1}). \]

As we can see, \( r \) will add the "calculation noise" back in the problem, thus the value of \( \theta \) can be fixed as a constant parameter.

Finally, for the new minimization problem, the alternating minimization algorithm is used to get solutions. The authors get the updating schemes for all the variables as follows:

\[
\begin{align*}
\alpha_{i,j} &= c_{i-1,j}^x - c_{i,j}^x + c_{i+1,j}^x - c_{i,j}^y - (r_{i-1,j}^x - r_{i,j}^x + r_{i+1,j}^x - r_{i,j}^y), \\
\nu_{i,j} &= \frac{1}{4} (\sigma_{i-1,j} + \sigma_{i+1,j} + \sigma_{i,j-1} + \sigma_{i,j+1} - e + \alpha_{i,j}), \\
\sigma_{i,j} &= \max\{\min\{\nu_{i,j}, 2\}, -2\}, \\
\tilde{c}^{n+1} &= \text{shrink}_b (r^n + \nabla \sigma^{n+1}, \frac{1}{\theta} = \text{shrink}_b (r^n + \nabla \sigma^{n+1}, \frac{1}{\theta}).
\end{align*}
\]

where the shrink operator is defined as:

\[ \text{shrink}_b(x, y) = \begin{cases} 
\frac{x}{|y|} \max(|x| - y, 0), & x \neq 0 \\
0, & x = 0.
\end{cases} \]

\( b \) and \( I_c \) is simply updated by minimizing equation (3) with respect to each variable.

### 3. Application to MR images

In this section this model has been applied to a large quantity of brain MR images. The authors also compare this model with the MICO model to demonstrate the accuracy and efficiency of this model. The initial level set functions \( \sigma \) is simply initialized as a binary step function taking 2 inside a region and -2 outside, \( \theta = 1 \times 10^{-5}, \beta = 100 \) and \( \sigma = 1 \) are used for this model.
Figure 1 shows the results of the proposed model and the MICO model for a brain MR image with bias field in the centre part. It is a typical brain MR image with bias field. The results of two models are presented in Row 1 and Row 2, respectively. Images in Column 1 are the original images with initial contours. Images in Column 2 show segmentation results of two models and images in Column 3 show the bias correction results of two models. As can be seen from the original image, the light bias field appearing in the centre part is corrected by the proposed model. However, the MICO model fails to deal with the problem.

In Figure 2, the human brain MR image is disturbed by an over-bright bias field in the white matter. Row 1 and Row 2 display the results of the proposed model and the MICO model, respectively. Original images, segmentation results and bias correction results are shown in Column 1, Column 2 and Column 3. Observing the results, we can find that the over-bright bias field is removed with the proposed model. But the MICO model cannot achieve a proper bias correction result. Therefore, the advantage of the proposed model is proved.

In Figure 3, the authors present results by this model for two MR images with serious bias field, one of which is an over-bright bias distortion and another is an over-dark distortion. In Row 1, the original image, the initial contour, the segmentation result and the bias correction results are displayed. The authors can see that a serious over-bright bias field is removed by this model, and in Column 4, the bias correction image is more clear so that an accurate segmentation result is presented in Column 2. Then in Row 2, an image disturbed by a serious over-dark bias field is tested by this model. In this circumstance, this model also works well by giving both good segmentation and correction results in Column 2 and Column 3. Through two images with apparent bias field, the authors have demonstrated the advantages of this model in correcting serious bias field, which makes sense in increasing the accuracy of segmentation relatively.

More objective and precise comparison of the segmentation and bias correction accuracy of the two methods can be performed by calculating the segmentation results using the coefficient of variation (CV) value, which is defined as a quotient of the standard deviation divided by the mean value of one selected tissue class and can be considered as a metric to evaluate homogeneous degree of an image. And a good algorithm for bias correction and segmentation should give low CV values for bias corrected intensities within each distinct region. The definition of the CV value is as follows:

\[ CV = \frac{SD}{MN}, \]

where SD represents the standard deviation of the image, MN is the mean value of the image. The smaller the CV value is, the better the correction is.

Table 1 displays the CV values of our model and the MICO model for four different images. It can be seen that the CV values after correction by our method are all lower than the MICO model, which indicates the bias correction images of our method are much more homogeneous than those of the MICO model. Compared with the MICO model, our method performs better in alleviating intensity in homogeneity and gives more surprising results.

Table 2 tells us the computing time of two methods, we can see that all the CPU time (in second) of our model are shorter than the MICO model, which clearly ensures that our model provides a more efficient approach for MR image segmentation and bias correction. Though an extra step is added to fit intensity in our model, the application of the split Bregman method maintains the efficiency stably.
Figure 1. Comparison results between the proposed model and the MICO model for a human brain MR image with bias fields in the centre part. (a) and (d): The original images. (b): The segmentation result of the proposed model. (c): The bias correction result of the proposed model. (e): The segmentation result of the MICO model. (f): The bias correction result of the MICO model.

Figure 2. Comparison results between the proposed model and the MICO model for a human brain MR image with bias fields in the white matter. (a) and (d): The original images. (b): The segmentation result of the proposed model. (c): The bias correction result of the proposed model. (e): The segmentation result of the MICO model. (f): The bias correction result of the MICO model.

Figure 3. Bias correction results of the proposed model for two human brain MR images. Row 1: Results of the proposed model for an image with an over-bright bias field in the centre. Row 2: Results of the proposed model for an image with an over-dark bias field. (a) and (d): The original images. (b) and (e): The segmentation results. (c) and (f): The correction results.

| WM  | GM  | WM  | GM  | WM  | GM  |
|-----|-----|-----|-----|-----|-----|
| Original image | 0.197 | 0.976 | 0.163 | 1.118 | 0.135 | 1.088 |
| The MICO model | 0.181 | 0.975 | 0.184 | 1.081 | 0.996 | 1.058 |
| The proposed model | 0.175 | 0.842 | 0.145 | 1.078 | 0.921 | 0.975 |

Table 1. Comparison of CV values between this model and the MICO model

| WM  | GM  | WM  | GM  | WM  | GM  |
|-----|-----|-----|-----|-----|-----|
| The MICO model | 2.09 | 1.79 | 1.93 | 1.97 | 1.95 | 1.92 |
| The proposed model | 0.41 | 0.26 | 0.35 | 0.28 | 0.21 | 0.33 |

Table 2. Comparison of CPU time between this model and the MICO model
4. Conclusion
In this paper, the authors proposed an improved model for MR image segmentation and bias correction. Considering that the intensity of an image can't be two absolute constants, the authors use a strategy in the proposed model by introducing as low variation function to approximate the true image intensity in different regions, which fits the real acquaintance much more. Then we applied the split Bregman method to enhance the efficiency and reduce the iterations steps. The new model is able to deal with images with different degree of intensity inhomogeneity, such as brain MR images. A fair quantity of brain MR images are tested by our model, and we give experimental results and also compare with a classical method called the MICO model. A tentative experiment of variable initial curves is implemented to analyse the robustness of the new model, which turns out to be stable extremely. The CV values of the segmentation and bias correction results are calculated to measure the correction precision quantitatively. From the numerical results, we can easily maintain the superiority of our model in dealing with intensity inhomogeneity when compared with the MICO model, especially in some dramatic circumstance which the MICO model fails to handle.

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