Charm in Deep Inelastic Scattering†

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Abstract

We outline the existing descriptions of the charm component of the deep inelastic proton structure function $F_2$. We discuss recent approaches to include charm mass effects in the parton evolution equations and the coefficient functions.

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Both H1 and ZEUS have used $c \rightarrow D^*$ to measure charm production in the HERA domain \[1\]. They find $F_2^c/F_2 \sim 25\%$, as compared to the EMC measurement of $F_2^c/F_c \sim 1\%$ at lower fixed-target energies. Moreover, the precision of the measurements of $F_2^c(x,Q^2)$ will improve, particularly when vertex detectors become available. Such measurements should reveal important information on the gluon, which enters at LO through the photon-gluon fusion (PGF) process $\gamma g \rightarrow c\bar{c}$. Clearly it is important to see how best to include the charm mass effects in the analysis. At NLO we have

$$F_2^c(x,Q^2) = \int_x^1 dz \frac{x}{z} \left[ \frac{8}{9} C_{q=c}(z, Q^2, \mu^2) c\left(\frac{x}{z}, \mu^2\right) + \frac{4}{9} C_g(z, Q^2, \mu^2) g\left(\frac{x}{z}, \mu^2\right) \right], \quad (1)$$

where the coefficient functions $C_c = C_c^{(0)} + \alpha_s C_c^{(1)}$ and $C_g = \alpha_s C_g^{(1)}$.

(a) Massless charm evolution

The simplest approach is to assume that the charm distribution $c(x, \mu^2) = 0$ for $\mu^2 < \mu_c^2$, where $\mu_c \sim m_c$, and then to evolve assuming $c$ is a massless parton both in the splitting and coefficient functions. This is the approach used in the MRS and all but the latest (CTEQ4HQ \[2\]) of the CTEQ global analyses. MRS take $\mu_c^2 = 2.7\text{GeV}^2$ so as to obtain a satisfactory description of $F_2^c$ of EMC \[3\]. On the other hand CTEQ set $\mu_c = m_c$; this is a consequence of their choice to use the ACOT scheme \[4, 5\], explained below, to define the parton densities. In both cases, the number of quarks in the hard scattering coefficients is the number of “active” quarks.

Corresponding conditions are applied for the $b$ quark. This is a minor issue in the global fitting because of the small contribution of processes involving $b$ quarks.

Although phenomenologically successful, the massless model clearly is inadequate in the charm threshold region. For instance an on-shell $c\bar{c}$ pair can be created by photon-gluon fusion (PGF) provided

$$W^2 = Q^2 (1-x)/x \geq 4m_c^2$$

where $W$ is the $\gamma^* g \rightarrow c\bar{c}$ centre-of-mass energy. Thus at small $x$, $c\bar{c}$ production is not forbidden even for $Q^2 < \mu_c^2$ where the massless approach gives zero. The physical threshold for $c\bar{c}$ production, $W^2 = 4m_c^2$, is not the threshold that is provided by the massless model. Only to the extent that charm production is small in the threshold region does the massless model give a useful approximation to $F_2$.

(b) Photon-gluon fusion

In this approach charm is treated as a heavy quark and not a parton. That is we put $c = 0$ for all $Q^2$ in \[1\] and use the known $m_c \neq 0$ gluon coefficient function $C_g^{\text{PGF}}$ for $C_g$. This is called a fixed flavour number scheme (FFNS) with $n_f = 3$. In contrast to the situation for massless
quarks, there is no collinear divergence in $\gamma^* g \rightarrow c\bar{c}$ since the integral over the $c\bar{c}$ transverse momentum is regulated by $m_c$. However, this in turn means that at large $Q$ there are large logarithmic corrections in higher orders: $\ F_2 \sim [\alpha_S \ln(Q^2/m^2_c)]^n$ at $n^{th}$ order. In terms of the FFNS all these contributions should be absorbed in the coefficient function. However, these are just the large logarithms that are resummed by DGLAP evolution in the purely massless approach. This implies that at large $Q^2$ we should treat charm as a parton.

(c) Variable flavour number schemes: ACOT and MRRS

The aim is to obtain a universal charm distribution $c(x, Q^2)$ all the way down to the resolution threshold $Q^2 \sim m^2_c$ in order to make predictions for other processes. To do this we must include the nonzero mass of the charm quark in the calculation in a consistent way.

The ACOT method is to exploit the techniques of Collins, Wilczek and Zee. Below $\mu = m_c$, the parton densities are those of the FFNS with $n_f = 3$, and above $\mu = m_c$, they are pure $\overline{\text{MS}}$ distributions. The evolution coefficients are independent of mass in the $\overline{\text{MS}}$ scheme, so they are the same as in a purely massless calculation, with the number of active flavors changing from 3 to 4 at $\mu = m_c$. At leading order explicit calculation shows that the parton densities are continuous at $\mu = m_c$. The order $\alpha^2_S$ corrections to the matching conditions have been calculated but not yet implemented.

In the ACOT scheme, the remaining dependence on the nonzero charm-quark mass appears in the coefficient functions. In the threshold region, the charm density $c(x, \mu = Q)$ by itself does not accurately provide the charm contribution to the structure function $F^c_2$. The necessary correction, at the first non-trivial order is provided by the gluon coefficient in Eq. 1. Consistency with calculations in the FFNS is obtained at the appropriate level of accuracy.

Buza et al. have evaluated the coefficient functions for $Q^2 \gg m^2_c$ in the VFNS and the FFNS to NLO. Their definition of the parton densities is exactly that of ACOT. They find that for $Q^2 \sim 20 \text{GeV}^2$ the VFN and FFN schemes agree very closely. They also work out the matrix relation in the limit $Q^2 \gg m^2_c$ which allows a matching of all the parton four flavour densities above the scale $\mu^2 = m^2_c$, to the three flavour densities below that point. A direct calculation of the matching conditions from the partonic operator matrix elements confirms the matching conditions computed by Buza et al., for the subset of the cases that have so far been computed. This relation may be used to calculate other inclusive cross sections (e.g. large $E_T$ jets etc.) This particular calculation only applies for $Q^2 \gg m^2_c$, which does not solve the

*The NLO corrections to the PGF structure function are known and a leading twist analysis has been used to perform a resummation of the $[\alpha_S \ln(Q^2/m^2_c)]^n$ terms for $Q^2 \gg m^2_c$. Unfortunately such an approach is not applicable for $F_2$ in the threshold region $Q^2 \sim m^2_c$.

† Note that the definition includes the full unapproximated dependence of the parton densities on the charm mass, and that "$\overline{\text{MS}}$" refers to the ultra-violet renormalization of the parton densities. Also, the position of the change of definition need not be at $\mu = m_c$; any other value in the neighborhood would be suitable, provided that corresponding changes in the matching conditions are made.
problem of what to do at $Q^2 \sim m_c^2$. One of the authors (JCC) considers the principles of
the problem solved; a paper is in preparation [11]. Work is in progress to obtain the ACOT
coefficient functions at order $\alpha_S^2$ from the Buza et al. calculations; these coefficient functions
will apply at all values of $Q$ of order $m_c$ and larger. Another author (MGR) considers that there
has to be another matching condition which provides parton densities which are continuous in
the threshold region.

In common with the coefficient functions used in all the global analyses, those of ACOT are
for inclusive cross sections, $F_2$ for the case under discussion here. So a different, but related
calculation must be made if, for example, one wishes to compute the distribution of charm
in the final state. The reason why it is necessary to treat each process separately can be
illustrated by this example. The charm $p_T$ spectrum associated with the first $c\bar{c}$ pair produced
by evolution from a gluon is proportional to

$$\frac{dp_T^2}{p_T^2 + m_c^2} \left[ z^2 + (1 - z)^2 + \frac{2m_c^2 \frac{z(1-z)}{p_T^2 + m_c^2}}{z(1-z)} \right].$$

(2)

This same $p_T$ spectrum appears in the calculation of the ACOT matching conditions for the
parton densities (integrated over $p_T$). After application of the rules for computing an $\overline{\text{MS}}$
distribution, ACOT obtains the matching condition given previously. However only the first
term in eq. (2) is operative in giving the (mass-independent) evolution kernel of the ACOT
distributions. The second term is hidden in the matching conditions and coefficient functions.
The full contribution to $F_2^c$ at order $\alpha_S$ also needs the gluonic coefficient function in eq. (1).

The problem that worries MGR is that the second term in (2) is proportional to $\alpha_S(m_c^2)$. Thus it cannot be absorbed in a fixed order coefficient function, which necessarily depends on
$\alpha_S(Q^2)$. On the other hand the matching takes place at one fixed point $Q^2 = m_c^2$, where we
deal with $\alpha_S(m_c^2)$. It is possible to choose such a matching which gives the correct description
for $Q^2 \gg m_c^2$ but it looks impossible to correct the whole $p_T$ spectrum in the threshold region
by just changing the parton densities at one point.

A different approach to include $m_c \neq 0$ effects has been proposed by MRRS [12]. Their aim
is to formulate the evolution procedure with $m_c \neq 0$ so as to generate universal parton distributions
applicable to any inclusive or exclusive process with the (known) coefficient functions in
the conventional $\overline{\text{MS}}$ renormalisation scheme. In order to do this they analyse the leading log
contributions which come from the relevant (ladder type) Feynman diagrams. It turns out that
a remarkable simplification occurs. Recall that the leading logs come from the configuration
where the $p_T$ of the partons are strongly ordered along the ladder. At NLO accuracy it was

\[ A \text{ closely related case is that of the Drell-Yan cross-section, } d\sigma_{DY}/dQ^2dy \text{ integrated over transverse momentun } q_T. \] The physical cross section is analytic at $q_T = 0$, but the lowest-order, or parton-model, contribution
is proportional to $\delta(q_T)$. Higher order terms give a cross section $d\sigma/d^4q$ that, without resummation, diverges
as $q_T \to 0$ and that has delta-function with divergent coefficients. Integration over $q_T$ cancels the divergences and gives a correct estimate of $d\sigma/dQ^2dy$.\]
found to be sufficient to take into account the $m_c \neq 0$ effect for the charm parton with $p_T \sim m_c$.

In fact all the $m_c \neq 0$ effects at NLO occur only\[^\text{§}\] in $P_{cg}$

$$P_{cg} = \left[ z^2 + (1 - z)^2 + \frac{2m_c^2}{Q^2} z(1 - z) \right]$$

with $Q^2 = m_c^2 + p_T^2$. In this way the MRRS procedure automatically reproduces the correct $p_T$ spectrum of charm quark. It is straightforward to generalise the MRRS to higher orders \[^\text{[12]}\].

In a VFNS we have to take care to avoid double counting. The same Feynman graphs are generated by the evolution with zero-order coefficient function (corresponding to $\gamma^* c \to c$ with a spectator $\bar{c}$, or vice versa with $c \leftrightarrow \bar{c}$) and the first order coefficient function (which describes photon-gluon fusion $(g \to c\bar{c}) \otimes (\gamma^* c \to c)$). Thus we must subtract the contribution generated by evolution from the PGF contribution. This is done by both ACOT and MRRS.

The charm component $F_2^c$ of $F_2$ is totally determined by the gluon and light quark densities. The only parameter is the value of $m_c$. The MRRS predictions are shown in Fig. 1. They are in good agreement with both HERA and fixed-target EMC data, which together cover a wide range of $x$ and $Q^2$. Unfortunately in order to match on to conventional $\overline{\text{MS}}$ coefficient functions the evolution scale is required to be $Q^2 = m_c^2 + p_T^2$ leading to a rather low charm threshold at $Q^2 = m_c^2$. The value of $\alpha_s$ is not sufficiently small in this region to neglect NNLO corrections. One of the main effects is to move the threshold to $Q^2 \gtrsim 4m_c^2$ which, due to the Uncertainty Principle, is the virtuality of the photon required to resolve a charm quark within the $g \leftrightarrow c\bar{c}$ fluctuations. In the NLO predictions in Fig. 1 the NNLO modification has been imposed by hand in a simplified way. This is the origin of the artificial structure at $Q^2 = 4m_c^2$. Now that a complete NLLO calculation is available it may be used to smooth out this behaviour.

A corresponding plot from CTEQ, with the use of the ACOT scheme, is shown in Fig. 2. It compares the experimental data on the charm contribution to DIS with calculations with two sets of parton densities. One set, CTEQ4M, is obtained from a global fit which uses the massless scheme, and the other, CTEQ4HQ1, is obtained from a global fit \[^\text{[2]}\] that used the ACOT scheme for DIS.

\[^\text{§}\]Apart from the corresponding adjustment in the $\delta(1 - z)$ term in $P_{ag}$ \[^\text{[12]}\].
References

[1] H1 collaboration: C. Adloff et al., Z. Phys. C72 (1996) 593; ZEUS Collaboration; J.Breitweg et al., DESY 97-089.

[2] H.L. Lai and W.K. Tung, Z. Phys. C74 (1997) 463.

[3] A.D. Martin, R.G. Roberts and W.J. Stirling, Phys. Rev. D50 (1994) 6734.

[4] J.C. Collins and W.-K. Tung, Nucl. Phys. B278 (1986) 934; F.I. Olness and W.-K. Tung, Nucl. Phys. B308 (1988) 813; M.A.G. Aivazis, F.I. Olness and W.-K. Tung, Phys. Rev. Lett. 65 (1990) 2339.

[5] M.A.G. Aivazis, J.C. Collins, F.I. Olness and W.-K. Tung, Phys. Rev. D50 (1994) 3102.

[6] E. Laenen, S. Riemersma, J. Smith and W.L. van Neerven Nucl. Phys. B392 (1993) 162.

[7] M. Buza, Y. Matiounine, J. Smith and W.L. van Neerven, DESY report 96-258, Z. Phys. (in press).

[8] J.C. Collins, F. Wilczek, and A. Zee, Phys. Rev. D18, 242 (1978). See also Ch. 8 of J.C. Collins, “Renormalization” (Cambridge University Press, Cambridge, 1984).

[9] See [5]. The identity of the definitions of the parton densities between Buza et al. and ACOT has been confirmed in extensive discussions with J. Smith.

[10] J.C. Collins, unpublished.

[11] J.C. Collins, “Hard-scattering factorization with heavy quarks: A general treatment”, preprint in preparation.

[12] A.D. Martin, R.G. Roberts, M.G. Ryskin and W.J. Stirling, Durham preprint DTP/96/102, Z. Phys. (in press).

[13] EM collaboration: J.J. Aubert et al., Nucl. Phys. B213 (1983) 31.

[14] W.-K. Tung, private communication.
Fig. 1: The predictions of MRRS [12] for $F_2^c$ compared with the EMC [13] and HERA [1] measurements. The dotted, continuous and dashed lines correspond to $m_c = 1.2$, 1.35 and 1.5 GeV respectively. The starred data points in the HERA domain are obtained by interpolating ZEUS measurements, whereas the other HERA data correspond to the H1 measurements.
Fig. 2: The CTEQ predictions [14] for $F_2^e$ when the ACOT scheme is used. The dashed line gives the prediction from the CTEQ4M distributions, while the continuous line shows the prediction from the CTEQ4HQ1 distributions, which were obtained from a global fit [2] where the ACOT method was used. The data is from H1 [1].