Supplementary Materials for

**Diffuse reflection and reciprocity-protected transmission via a random-flip metasurface**

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**S1. Reciprocal transmission through stratified media and the flip configuration**

Reciprocity of transmission through stratified media (31) amounts to the statement that, for the geometry in Fig. S1B, which is reciprocal to geometry in Fig. S1A, the transmission coefficient \( t' \) should be equal to \( t \). Stratified media in Fig. S1A and B are the same while the incident beam in Fig. S1B is from the opposite direction of the transmission beam in Fig. S1A. In spite of the reciprocity-protected transmission, reflections of structures in those two cases are not reciprocity-protected. By virtue of unitarity and the equality of transmission, the lack of reciprocity in the reflection in non-absorbing stacks is a matter of phase change only. Moreover, those features hold for both transverse electric (TE) and transverse magnetic (TM) polarization.

**Fig. S1. Reciprocity-protected transmission of a stack and its flipped counterpart.**

Here, we study the transmission and reflection property of stratified media and verify the reciprocity of transmission by using the transfer matrix method. The characteristic matrix connects the complex amplitude of the electric and magnetic fields at two sides of a stack:

\[
\begin{bmatrix}
E_i + E_r \\
H_i + H_r
\end{bmatrix} = M \begin{bmatrix}
E_r \\
H_r
\end{bmatrix},
\]  

(S1)

where the subscript \( i, r, \) and \( t \) represent incident, reflected, and transmitted waves, respectively. The characteristic matrix of a uniform, isotropic layer is:

\[
M = \begin{bmatrix}
\cos(\delta) & -\frac{i}{\eta} \sin(\delta) \\
-\eta \sin(\delta) & \cos(\delta)
\end{bmatrix}.
\]  

(S2)

Here \( \eta \) is given by \( \eta = \sqrt{\epsilon_0/\epsilon, \mu_0/\mu} \cos(\theta) \) (\( \eta = \sqrt{\epsilon_0/\epsilon, \mu_0/\mu} \cos(\theta) \)) for TE (TM) polarization and \( \delta = -nk_0h \cos(\theta) \), where \( \epsilon, \mu, n, h, \) and \( \theta \) are separately the relative permittivity, relative permeability, refractive index and thickness of the layer and the refraction angle.
The transmission and reflection coefficients $t$ and $r$ of a stack whose combined character matrix is $M = \prod_{m} M_m$, where subscript $m$ depicts the $m$th layer from the incident side to the transmitted side, are given by

\[ r_{TE} = \frac{E_T}{E_i} = \frac{M[11] \eta_0 + M[12] \eta_0^2 - M[21] - M[22] \eta_0}{M[11] \eta_0 + M[12] \eta_0^2 + M[21] + M[22] \eta_0}, \quad (S3a) \]
\[ r_{TM} = \frac{H_T}{H_i} = \frac{M[11] \eta_0 + M[12] \eta_0^2 - M[21] - M[22] \eta_0}{M[11] \eta_0 + M[12] \eta_0^2 + M[21] + M[22] \eta_0} \quad (S3a) \]

and

\[ t_{TE} = \frac{E_T}{E_i} = \frac{2 \eta_0}{M[11] \eta_0 + M[12] \eta_0^2 + M[21] + M[22] \eta_0}, \quad (S4a) \]
\[ t_{TM} = \frac{H_T}{H_i} = \frac{2 \eta_0}{M[11] \eta_0 + M[12] \eta_0^2 + M[21] + M[22] \eta_0} \quad (S4b) \]

for TE and TM polarization, respectively, where $\eta_0$ is given by $\eta_0 = \sqrt{\varepsilon/\mu} \cos(\theta_0)$ ($\eta_0 = \sqrt{\varepsilon_0/\mu_0} \cos(\theta_0)$) for TE (TM) polarization and $M[jk]$ represents the element of $M$ in the $j$th row and the $k$th column.

The combined characteristic matrix for original and reciprocal geometries in Fig. S1A and B are separately $M_+ = M_1 M_2 \cdots M_m$ and $M_- = M_m M_{m-1} \cdots M_1$.

For both TE and TM polarized incidence with incident angle $\theta_0$, the transmission coefficient is given by

\[ t = \frac{2 \eta_0}{M_+[11] \eta_0 + M_+[12] \eta_0^2 + M_+[21] + M_+[22] \eta_0} \quad (S5) \]

and

\[ t' = \frac{2 \eta_0}{M_-[11] \eta_0 + M_-[12] \eta_0^2 + M_-[21] + M_-[22] \eta_0}. \quad (S6) \]

By inserting Eq. S2 into Eq. S5 and 6, we get $t = t'$ for both TE and TM polarizations. This identical relation is protected by optical reciprocity. While the reflection coefficients for cases in Fig. S1A and B can be different $r \neq r'$.

In random-flip metasurface design, asymmetric structures are flipped over to construct a pair of meta-atoms. Fig. S1C shows the flipped stack under the same incidence as that in Fig. S1A. One may find that under a simple in-plane rotation operation, the flipped case in Fig. S2C is identical to the reciprocal case in Fig. S1B and the transmissions in both cases equal, i.e., $t'' = t'$. Since $t = t'$, we get $t'' = t$, that is transmission in the flipped case equals to that in the original case. While the reflection in the flipped case $r''$ can be different from that of the original case $r$.

In practical applications, metasurfaces are usually fabricated on a substrate. Taking into consideration the substrate, an asymmetric background composed of two half-infinite media is considered in meta-atom design. As an example, we assume that the transmitted half-infinite space is filled with medium with relative permittivity $\varepsilon_s$ and permeability $\mu_s$. In such asymmetric background, it can be demonstrated that transmissions in original and reciprocal cases are still identical through a derivation similar to the above. However, the flipped case isn’t identical to the
reciprocal case because the asymmetric background isn’t flipped as the structure and hence the identical relation of transmissions in flipped and original cases is no longer valid. The difference between the transmission coefficients of original and flipped cases can be obtained as
\[ \Delta t = t - t'' = \frac{2(\eta_1 \eta_g (\eta_2 / \eta_1 - \eta_2 / \eta_1) \sin(\delta_1) \sin(\delta_2))}{\prod_i (M_i[11] \eta_0 + M_i[12] \eta_0 \eta_g + M_i[21] + M_i[22] \eta_g)} (\eta_0 - \eta_g). \] (S7)
where \( \eta_g = \sqrt{\varepsilon_g / \mu_g \cos(\theta_g)} \) (\( \eta_g = \sqrt{\varepsilon_g / \mu_g \cos(\theta_g)} \)) for TE (TM) polarization, \( \theta_g \) is the angle of refraction and \( t = \sqrt{\eta_1 / \eta_g} E_i / E_i \) (\( t = \sqrt{\eta_1 / \eta_g} H_i / H_i \)) for TE (TM) polarization. Here, transmission coefficients are defined so that their square amplitudes provide the associated incident and transmitted powers.

We note that a minor difference between optical properties of the two semi-infinite background media doesn’t destructively affect the reciprocity of transmission. In the main text, random-flip metasurface reported is fabricated on a silica substrate with \( \varepsilon_S = 2.25 \). For an interface between air and silica, the transmission coefficients of head and tail blocks corresponding to the geometries in Fig S1. A and C are still close to each other (Fig. 3C). Here, we consider a specific two-layer stack with \( \varepsilon_1 = 12, \varepsilon_2 = 6 \), \( d_1 = 125 \text{ nm} \) and \( d_2 = 300 \text{ nm} \). Fig. S2A shows the transmittance (upper panel) and transmission phase (lower panel) of this stack (black solid curves) and its flipped counterpart (red dashed curves) on an air-silica interface under normal incidence. In Fig. S2A, negligible deviation in transmissions of the original and flipped stack occurs because of the minor difference between \( \varepsilon_g (1) \) and \( \varepsilon_g (2.25) \). As shown in Fig. S2B and C, even for TE and TM polarized oblique incidence with \( \theta_i = 45^\circ \), transmissions of inverse two-layer stacks are still very close to each other.

**Fig. S2. Transmission of the original and flipped two-layer stack in asymmetric background.** Transmittance (upper panels) and transmission phase (lower panels) of a specific two-layer stack (black solid curves) and its flipped counterpart (red dashed curves) located on an air-silica interface under normal incidence (A) and TE (B) and TM (C) polarized oblique incidence with an incident angle of 45°. The thicknesses of the first and second layers are \( d_1 = 125 \text{ nm} \) and \( d_2 = 300 \text{ nm} \) and the relative permittivities are separately \( \varepsilon_1 = 12 \) and \( \varepsilon_2 = 6 \).
S2. Incident-angle-independent reciprocal transmission

Reciprocity and space inversion protected identical transmission and the distinct reflection of the flipped blocks are independent of the incident angle. We take the nano-rod metasurface blocks shown in Fig.2E as an example and calculated their transmission and reflection coefficients by the finite-difference time-domain (FDTD) method. Fig. S3 shows the calculated transmission (middle panels) and reflection (right panels) of a pair of flipped nano-rod blocks under the illumination of a TE-polarized oblique-incident plane wave (left panels) with incident angle \( \theta = 30^\circ \) (A) and 60° (B), respectively, verifying the incident-angle-independent broadband identical transmission and distinct reflection of flipped metasurface blocks.

![Fig. S3.](image)

**Fig. S3.** Broadband identical transmission (middle panels) and distinct reflection (right panels) of RFM blocks composed of a gold nano-rod embedded in silica layer.

S3. Several binary maps of head and tail blocks

For a general random-flip metasurface composed of head and tail blocks with lattice constant \( D \), each block can be treated as an individual radiation source with pattern function \( f_e(\theta, \varphi) \), scattering phase \( \varphi \), and scattering amplitude \( A \). Under the illumination of a normal incidence plane wave, the scattered far-field function of the metasurface can be derived as:

\[
f_f(\theta, \varphi) = \sum_{m=1}^{M} A_m f_{e,m}(\theta, \varphi) \exp\{-i(\varphi_m + k \sin(\theta) \cos(\varphi) x_m + k \sin(\theta) \sin(\varphi) y_m]\},
\]

where \( \theta \) and \( \varphi \) are the elevation and azimuth angles respectively. \( x \) and \( y \) are the coordinates. Subscript \( m \) depicts the \( m \)th block.

We assume a square RFM that contains \( 8 \times 8 \) equal-sized head or tail blocks with lattice constant \( D=0.86\lambda \), where \( \lambda \) is the wavelength in vacuum. And the pattern function of each block can be approached as \( f_e(\theta, \varphi) = \cos(\theta) \). The difference between scattering phases of head \( \varphi_h \) and tail blocks \( \varphi_t \) is assumed to be 180°, i.e., \( \varphi_t = \varphi_h + 180^\circ \). And the amplitude \( A \) is set as 1. Under those assumptions, we can perform optimization on the sequence of blocks in head or tail state by
taking the maximum value of the scattered far-field function \( \max(f_{r,m}(\theta, \varphi)) \) as the fitness function. Firstly, to get the best sequence in one direction e.g., x-direction, we employ the ergodic algorithm that is calculating all 28 cases of arrangement. The optimized sequence corresponding to the minimum fitness function is 01010011, where 1 depicts the head block and 0 depicts the tail block. Then we apply the optimized 1 dimensional (1D) sequence in both x and y directions by using a logistic XOR gate to form a 2 dimensional (2D) sequence. An XOR gate implements an exclusive or; that is, a true (1) output results if one, and only one, of the inputs to the gate is true. If both inputs are false (0) or both are true, the output is a falses. The obtained 2D sequence is shown in the upper panel of Fig. S4A and the calculated scattered far-field distributions are shown in the lower panel of Fig. S4A.

The fabricated RFM is designed by filling the binary phase maps in Fig. S4A with head and tail blocks, which are composed of 3×3 head or tail blocks shown in Fig. 3B. The side length of each block is 0.86\( \lambda \), which is coincident with the lattice constant in the optimization. What’s more, blocks composed of 3×3 blocks can efficiently keep the periodic of block and hence the interaction with incident beams.

Besides 01010011, there are also several 1D sequences contributing to a small maximum value of the scattered far-field function, e.g., 1110100, 11010100, etc. By combining these 1D sequences, we can get several 2D maps. Figure S4 B-F shows 5 cases of these 2D maps (upper panel) can the corresponding scattered far-field distributions (lower panel). From calculated scattered far-field distributions in Fig. S4, one may find that the radiation power is distributed into many directions, which is evidence of diffusion.
Fig. S4. Several 2D maps of random-flip units and corresponding scattered far-field distributions.

S4. Far-field radiation power patterns for a PTM

Figure S5 shows the Far-field radiation power patterns for a pure tail metasurface (PTM) under the illumination of a TE-polarized Gaussian beam at the incident angle (marked by white arrow) of $\theta_i = 0^\circ$, $10^\circ$, $20^\circ$, and $30^\circ$, respectively. Since the PTM is a simple flip of the PHM, and thus according to the reciprocity principle, main lobes corresponding to the distortion-free transmission and specular reflection of PTM observed in Fig. S5 are almost the same as that of PHM shown in top panels Fig. 3F.
Fig. S5. Far-field radiation power patterns for a PTM

S5. Far-field radiation power patterns for TM polarized incidence

In the main text, Fig. 3F shows the far-field radiation power patterns for a PHM composed of only head blocks and the designed RFM under the TE (with electric field along x-direction) polarized incidence. Here, in Fig. S6, we show the far-field radiation power pattern for the same PHM (upper panels) and RFM (lower panels) under the illumination of a TM (with magnetic field along y-direction) polarized Gaussian beam at the incident angles of \( \theta_i = 0°, 10°, 20°, \) and \( 30° \), respectively. The physical properties of gold adopted in simulations are from Ref. 32 and the refractive index of silica is set as 1.5. The white arrows depict incident beams in x-z plane. As can be found from Fig. S6, the main lobe in the transmission half space (z<0) representing transmitted beams dominates and are of outstanding directivity, indicating distortion-free transmission through both the PHM and RFM. While comparing the main lobe with outstanding directivity in the reflection half space (z>0) from the PHM, several small radiation lobes appear in RFM cases, which is the signature of diffuse reflection.

Fig. S6. Far-field radiation power patterns for a PHM and RFM under TM polarized incidence.
S6. Tolerance of phase/amplitude difference between the reflection from the head and tail structures

In the last section, we select several binary maps by assuming a maximum phase difference between head and tail blocks, that is 180°. In this section, we investigate the influence of phase difference and amplitude difference on the diffuse reflection effect. We take the binary map shown in Fig. S4A as an example and calculate the scattering far-field intensity for various phase differences by using Eq. S8. Figure S7 shows the calculated maximum value of scattering far-field intensity from the RFM, which is normalized by the PHM case. As can be found from Fig. S7, for phase difference within 90° -270°, the normalized maximum value of scattered far-field intensity is reduced by at least 50%. To investigate the influence of the amplitude difference between the reflection of the head and tail structures, we take the binary map shown in Fig. S4A as an example again and define \( \eta \) as the ratio of the reflection amplitude of the head structure to that of the tail structure. We then calculate the far-field scattering intensity in reflection as a function of \( \eta \) by using Eq. S8 and assuming that the phase difference between the head and tail structures is 180°. Figure S8 shows the maximum value of the far-field scattering intensity of the RFM as a function of \( \eta \), which is normalized by that of a PHM in reflection. As can be found in Fig. S8, for the cases of \( \eta \) within the wide range of (0.17-5.9), the normalized maximum value of the far-field scattering intensity is reduced by at least 50%. When \( \eta \) is within the range of (0.53-1.9), the normalized maximum intensity is reduced by over 90%.

![Fig. S7](image_url)

**Fig. S7.** The normalized maximum value of far-field scattering intensity as a function of the phase difference between head and tail blocks.

![Fig. S8](image_url)

**Fig. S8.** The normalized maximum value of far-field scattering intensity as a function of the reflectance ratio between head and tail structures.
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