Development and Research of Technological Equipment that Implements Dynamic Control of Process of Additive Fabrication of Parts of Complex Spatial Shapes Based on Mechanisms with a Hybrid Layout

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Abstract. The article is devoted to the study of the accuracy of fabrication of the surface layer of engineering products by additive methods. The analysis of the advantages and disadvantages of technology for layer-by-layer synthesis of products took place. It was revealed that with additive fabrication, the accuracy characteristics of the surface layer are significantly different from the characteristics of the surface layer of products obtained by traditional methods. The analysis of domestic and foreign works on the topic of research allowed to reveal that in order to increase the accuracy characteristics of products obtained by additive methods, it was necessary to provide dynamic control of the spatial orientation of the final link of the fabrication system of the additive installation in the process of fabrication. To control the spatial orientation of the working body of additive equipment, the use of mechanisms with a hybrid layout was proposed. The problem of parametric synthesis of a mechanism with a hybrid layout has been solved, which allows to create a space of design parameters that provide, at the early stages of design, the required formative capabilities of the mechanism for additive fabrication with a hybrid layout.

1. Introduction
Formulation of parts by additive methods is characterized by high values of shape error. This is due to the fact that the formation of the surface layer of a complex shape part occurs line by line (in layers), and the orientation of the final link of the fabrication system (extruder) of the additive installation is unchanged and independent of the curvature of the formed surface [1-3]. To reduce the error in the shape of the part, it is necessary, when formulation surface points, to ensure its orientation at which the normal at the point of the surface being formed will coincide with the axis of the final link of the formulation system (Figure 1) [17-23].
Figure 1. Fabrication of a point on the surface of a part: a) in the traditional way; b) - with the dynamic orientation of the part relative to the axis of the extruder

2. Designing of Equipment for Dynamic Process Control of Additive Formulation of a Part

To solve the problem of dynamic control of the process of additive fabrication, in paper [4] it was proposed to use a mechanism with parallel kinematics — a hexapod having six degrees of freedom, on which the part is formed owing to its moving part. When the additive installation extruder approaches the part surface, the hexapod changes its orientation, ensuring that the extruder axis coincides with the normal to the part surface at the point which is to be formed.

However, the use of mechanisms with parallel kinematics does not provide sufficient angles of rotation of the part relative to the vertical axis. To eliminate this drawback, it is proposed to create an installation using a mechanism with a hybrid layout (Figure 2).

Figure 2. Additive installation with a hybrid layout: 1 - molded part, 2 - extruder, 3 - stepper motors, 4 - rotating table, 5 - guides, 6 - hinges, 7 - lead screws

The designed equipment for additive fabrication is based on a parallel structure mechanism with constant-length rods. It is proposed to install a rotating table on a moving platform, which, if there are 5 degrees of freedom, will allow the angle of rotation of the part relative to the vertical axis in the range \( \pm \pi \). This will significantly improve access to the points on the surface of the formed part.

This installation has 5 degrees of freedom – movement along the axis \( X, Y \) and \( Z \); the inclination of the platform relative to the axis \( X \); table rotation relative to the axis \( Z \).

The main task of the design of technological equipment is the problem of parametric synthesis, the solution of which will allow to create a space of design parameters providing the required formative capabilities of the installation at the early stages of design, such as:
- the ability of the extruder to contact all points of the formed part (internal points of the part and points on its surface);
- the ability to set the normal to the surface of the part in the formed point for all points of the surface of the part along the axis of the extruder (Figure 1).

The solution of this problem is possible by compiling, on the basis of a generalized model, of the mechanisms with parallel kinematics [3], a system of six equations in accordance with the number of rods, as follows:

\[
\begin{align*}
L &= |\mathbf{R}_1(q_1) - A_{21}(u,v)\mathbf{S}_1|; \\
L &= |\mathbf{R}_2(q_1) - A_{21}(u,v)\mathbf{S}_2|; \\
L &= |\mathbf{R}_3(q_2) - A_{21}(u,v)\mathbf{S}_3|; \\
L &= |\mathbf{R}_4(q_3) - A_{21}(u,v)\mathbf{S}_4|; \\
L &= |\mathbf{R}_5(q_4) - A_{21}(u,v)\mathbf{S}_5|; \\
L &= |\mathbf{R}_6(q_4) - A_{21}(u,v)\mathbf{S}_6|;
\end{align*}
\]

where \( L \) – rod lengths; \( \mathbf{R}_i(q_1), \mathbf{R}_2(q_1), \ldots, \mathbf{R}_6(q_4) \) – vectors determining the position of the movable joints in the coordinate system of the installation base; \( q_1, q_2, q_3, q_4 \) – controlled coordinates of the installation, determining the position of the hinges along the axis \( Z \); \( A_{21}(u,v) \) – a matrix that determines the position of the movable platform at which the extruder will contact the point on the surface of the part with coordinates \( (u, v) \), and its axis will coincide with the normal to the surface of the part; \( \mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_6 \) – vectors that specify the position of the hinges of the moving platform in its own coordinate system.

Due to the redundancy of this system of equations for the designed installation, it can be reduced to the following one:

\[
\begin{align*}
L &= |\mathbf{R}_1(q_1) - A_{21}(u,v)\mathbf{S}_1|; \\
L &= |\mathbf{R}_3(q_2) - A_{21}(u,v)\mathbf{S}_3|; \\
L &= |\mathbf{R}_4(q_3) - A_{21}(u,v)\mathbf{S}_4|; \\
L &= |\mathbf{R}_6(q_4) - A_{21}(u,v)\mathbf{S}_6|.
\end{align*}
\]

The matrix \( A_{21}(u,v) \) will be determined by means of generation of the geometric closure equation (Figure 3) of the installation coordinate systems:

\[
A_{23} = A_{21}(u,v) \cdot A_{10}(q_5) \cdot A_{00}(u,v),
\]

where \( A_{23} = A^{[3]}(H - h_e) \) – matrix determining the position of the extruder in the base coordinate system; \( H \) – height of the installation; \( h_e \) – length of the extruder; \( A^{[3]} \) – axis \( Z \) transition matrix:

\[
A^{[3]}(z) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & H - h_e \\
0 & 0 & 0 & 1
\end{bmatrix};
\]

\( A_{00}(q_5) \) - matrix determining the rotation of the table with the formed part by the value \( q_5 \) relative to its initial position:
$A_{10}(q_5) = A^{(6)}(q_5) = \begin{bmatrix}
\cos q_5 & -\sin q_5 & 0 & 0 \\
\sin q_5 & \cos q_5 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$

$A_{ij}(u,v)$ – transition matrix from the coordinate system of the part to the coordinate system of a point of the surface being formed with coordinates $(u, v)$, the calculation method of which is given in [1];

$A_{23}(u,v) = A_{23} \cdot [A_{10}(q_5) \cdot A_{ij}(u,v)]^{-1}.$

Table rotation parameter $q_5$ and the corresponding matrix $A_{10}(q_5)$ can be found by the solution of the following vector equation:

$\vec{e}_1 \cdot \vec{e}_{3,j} = 0,$

where $\vec{e}_1$ – versor determining the positive direction of the axis $X_3$ of the extruder coordinate system $\vec{e}_1 = [1 \ 0 \ 0 \ 0]^T$; $\vec{e}_{3,j}$ – versor determining the positive direction of the axis $Z_j$ in the coordinate system of the point on the surface of the part, represented in the extruder coordinate system:

$\vec{e}_{3,j} = [A_{23}]^{-1} A_{10}(q_5) \cdot A_{ij}(u,v) \cdot \vec{e}_3,$

where $\vec{e}_3$ - versor determining the positive direction of the axis $Z_j$ in the coordinate system of the point on the surface of the part $\vec{e}_3 = [0 \ 0 \ 1 \ 0]^T$;

Using the system of equations (2), it becomes possible to determine the points on the space of design parameters at which this system will have a solution, which, therefore, will represent the range of their permissible values.

3. Results of Solving the Problem of Parametric Synthesis of a Device for Dynamic Control of the Process of Additive Product Fabrication

As an example, we will consider the finding of the range of permissible values for the lengths of the installation rods with constant dimensions of the base and the movable installation platform (Figure 4).
Thus, the vectors that determine the position of the movable hinges in the coordinate system of the installation base (see Fig. 4), will be set as follows:

$$\bar{R}_1 = \begin{bmatrix} h_{1R} & -\left(\frac{B_R}{2} - b_{1R}\right) & q_1 & 1 \end{bmatrix}^T;$$

$$\bar{R}_4 = \begin{bmatrix} h_{1R} & \left(\frac{B_R}{2} - b_{1R}\right) & q_2 & 1 \end{bmatrix}^T;$$

$$\bar{R}_5 = \begin{bmatrix} \frac{H_R}{2} - h_{2R} & b_{2R} & q_3 & 1 \end{bmatrix}^T;$$

$$\bar{R}_6 = \begin{bmatrix} \frac{H_R}{2} - h_{2R} & -b_{2R} & q_4 & 1 \end{bmatrix}^T.$$

The vectors that specify the position of the hinges of the moving platform in its own coordinate system will be defined as follows:

$$\bar{S}_1 = \begin{bmatrix} h_{1S} & -h_{1S} & 0 & 1 \end{bmatrix}^T;$$

$$\bar{S}_4 = \begin{bmatrix} h_{1S} & b_{1S} & 0 & 1 \end{bmatrix}^T;$$

$$\bar{S}_5 = \begin{bmatrix} h_{1S} + h_{2S} & b_{1S} & 0 & 1 \end{bmatrix}^T;$$

$$\bar{S}_6 = \begin{bmatrix} h_{1S} + h_{2S} & -b_{1S} & 0 & 1 \end{bmatrix}^T.$$

We will set the following device parameters for calculation:

- Device height: $H = 600 \text{ mm}$;
- Extruder length: $h_e = 100 \text{ mm}$;
- Hinges location options:
  - $H_R = 500 \text{ mm}$, $h_{1R} = 85 \text{ mm}$, $h_{2R} = 85 \text{ mm}$, $B_R = 650 \text{ mm}$, $b_{1R} = 25 \text{ mm}$, $b_{2R} = 65 \text{ mm}$, $h_{1S} = 85 \text{ mm}$, $h_{2S} = 20 \text{ mm}$, $b_{1S} = 65 \text{ mm}$; $h_e = 20 \text{ mm}$.

Table 1 shows the values of minimum $L_{\text{min}}$ and maximum $L_{\text{max}}$ rod lengths in the fabrication of parts such as a hemisphere described by the following equation:
\[
\rho_0(\theta; z) = \begin{bmatrix}
\sqrt{R^2 - z^2} \cdot \cos(\theta) \\
\sqrt{R^2 - z^2} \cdot \sin(\theta) \\
z \\
1
\end{bmatrix},
\]

where \( \theta, z \) – surface curvilinear coordinates; \( R \) – hemisphere radius \((R=50..100 \text{ mm})\).

### Table 1. Calculated values for permissible rod lengths

| \( z \), mm | \( R \), mm | \( L_{\text{min}} \) | \( L_{\text{max}} \) | \( L_{\text{min}} \) | \( L_{\text{max}} \) | \( L_{\text{min}} \) | \( L_{\text{max}} \) |
|----------------|-------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0              | 50    | 319            | 387            | 319            | 363            | 319            | 339            |
| 5              | 75    | 313            | 384            | 315            | 361            | 316            | 338            |
| 15             | 100   | 300            | 380            | 306            | 357            | 310            | 335            |
| 25             | 50    | 284            | 380            | 297            | 356            | 303            | 333            |
| 40             | 75    | 260            | 388            | 282            | 356            | 292            | 331            |
| 50             | 100   | 235            | 422            | 271            | 357            | 284            | 331            |
| 75             | 50    | -              | -              | 235            | 397            | 264            | 337            |
| 100            | 75    | -              | -              | -              | -              | -              | -              |

In Table 1, the values \( L_{\text{min}} \) are determined based on the solution of the equation system (2) for various hemisphere \( z \) parameter values, which are included into the equation (9), and the value \( L_{\text{max}} \) is determined by the limitations of the size of the working area imposed on the coordinates \( q_1, \ldots, q_4 \).

Figure 5 shows the range of permissible lengths of the rods of additive equipment for various values of the radius of the formed part in the form of a hemisphere.

![Figure 5. Range of allowable rod lengths L: a) – with the hemisphere radius R=50 mm; b) – with the hemisphere radius R=75 mm; c) – with the hemisphere radius R=100 mm](image-url)

Thus, with an increase in the radius of the formed surface in the form of a hemisphere, there is a decrease in the range of rod lengths that ensure during the fabrication process the following – the
extruder contacts all points of the formed part (internal points of the part and points on its surface); setting the normal to the surface of the part in the formed point for all points of the surface of the part along the axis of the extruder.

Thus, to ensure the hemisphere formulation conditions outlined above with \( R = 50 \text{ mm} \), the range of permissible rod lengths is in the range \( L = 319 \ldots 380 \text{ mm} \); \( R = 75 \text{ mm} \) - \( L = 319 \ldots 356 \text{ mm} \); \( R = 100 \text{ mm} \) - \( L = 319 \ldots 331 \text{ mm} \).

In a similar way, the range of permissible values for other parameters of the designed installation can be obtained.

### 4. Conclusions

Thus, based on the method described above, parametric synthesis of equipment for dynamic control of the process of additive formulation of products can be performed, as well as a range of values of the parameters of the formed surfaces at which the extruder will be in contact with all points of the formed surface (internal points of the part and points on its surface), as well as normal to the surface of the part will be established at the formed points of the surface along the axis of the extruder.

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### References

[1] Burns, M. 1993 Automated Fabrication: Improving Productivity in Manufacturing (Englewood Cliffs. N.J. USA: PTR Prentice Hall) 369 p.

[2] Sapyrkin, A.A. 2006 Increasing the Productivity of the Process of Selective Laser Sintering in the Manufacture of Prototypes Thesis of candidate of technical Sciences (Yurga: Tomsk Polytechnic University) 153 p.

[3] Pronikov, A., Averyanov, O.I. and Apollo, Yu. S. 1994 Designing of Metal-Cutting Machines and Machine Tools (Handbook-Textbook vol 3) (Moscow: N.E. Bauman MSTU: Mechanical Engineering) 444 p.

[4] Kuts, V.V., Razumov, M.S., Grechukhin, A.N. and Bychkova, N.A. 2016 Improving the Quality of Additive Methods for Formulation the Surfaces of Odd-Shaped Parts with the Application of Parallel Kinematics Mechanisms International Journal of Applied Engineering Research 11 P. 11832-11835

[5] Dobroskok, V.L., Abdurayimov, L.N. and Chernyshov, S.I. 2010 Rational Orientation of Products with their Layer-by-Layer Shaping on the Basis of the Original Triangulation 3d Model Scientific notes of the Crimean Engineering and Pedagogical University 24 P. 13-21

[6] Singhal, S.K., Pandey, A.P., Pandey, P.M. and Nagpal, A.K. 2005 Optimum Part Deposition Orientation in Stereolithography Computer-Aided Design & Applications 2 P. 319–328

[7] Hong, S., Byun Kwan H., Lee 2006 Determination of Optimal Build Direction in Rapid Prototyping with Variable Slicing Int. J. Adv. Manuf. Technol. 28 P. 307–313

[8] Hong, S., Byun Kwan H., Lee 2004 Optimal Part Orientation of Rapid Prototyping Using a Genetic Algorithm Computers & Industrial Engineering P. 426–431

[9] Hur, J., Lee, K. 1998 The Development of a CAD Environment to Determine the Preferred Build-up Direction for Layered Manufacturing Manuf. Technol 14 P. 247–254

[10] Kim, J.Y., Lee, K. and Park, J.C. 1994 Determination of Optimal Part Orientation in Stereolithographic Rapid Prototyping Technical Report. Department of Mechanical Design and Production Engineering (Seoul: Seoul National University) P. 356-366

[11] Lan, P.T., Chou, S., Chent, Y., Gemmill, L.D. 1997 Determining Fabrication Orientations for Rapid Prototyping with Stereolithography Apparatus Computer-Aided Design 29 P. 53–62

[12] Massod, S.H., Rattanawong, W., Iovenitti, P. 2003 A Generic Algorithm for Part Orientation System for Complex Parts in Rapid Prototyping J. Mater. Process. Technol 139 P. 110–116
[13] Masood, S.H., Rattanawong, W. 2002 A Generic Part Orientation System Based on Volumetric Error in Rapid Prototyping Int. J. Adv. Manuf. Technol. 19 P. 209–216
[14] Egorov, I.N. 2010 Position-Force Control of Robotic and Mechatronic Devices Vladimir State University Vladimir 243 p.
[15] Lashnev, S.I., Borisov, A.N., Emelyanov, S.G. 1997 Geometric Theory of Surface Formation by Cutting Tools (Kursk: Kursk State Tech. Un-t) 391 p.
[16] Emelyanov, S.G. 2001 Development of the Theory, Methods and Means of Surface Formation by Assembled Metal-Cutting Tools on the Basis of System Modeling of their Design Process Thesis of Doctor of Technical Sciences (Moscow) 407 p.
[17] Kuts, V.V. 2012 Methodology of Pre-Project Studies of Specialized Metal-Cutting Systems Thesis of Doctor of Technical Sciences (Kursk) 366 p.
[18] Grechukhin, A.N., Kuts, V.V., Razumov, M.S. and others 2018 Improving the Accuracy of Additive Formulation Methods Innovation, Quality and Service in Engineering and Technology P. 128-131
[19] Grechukhin, A.N., Kuts, V.V., Razumov, M.S. 2018 Control of Spatial Orientation of Robot Units in the Process of Additive Formulation of Products Bulletin of Voronezh State Technical University 4 P. 122-129
[20] Grechukhin, A.N., Kuts, V.V., Razumov, M.S. 2018 Experimental Determination of the Cross-Section Parameters of a Single Layer in the Additive Formulation Products News of Tula State University. Technical Science 10 P. 264-270
[21] Grechukhin, A.N., Anikutin, I.S., Byshkin, A.S. 2018 Management of Space Orientation of the End Effector of Generation of Geometry System Five-Axis Manufacturing Machinery for Additive Generation of Geometry MATEC Web of Conferences Vol 7 P. 128-136
[22] Grechukhin, A.N., Kuts, V.V., Razumov, M.S. 2018 Ways to Reduce the Error of Additive Methods of Formulation MATEC Web of Conferences Vol 7 P. 142-150
[23] Grechukhin, A.N., Razumov, M.S., Kudelina, D.V. and others 2018 Development of Information-Analytical System for Technological Requests Monitoring, Taking into Account Regional Specifics International Conference on Actual Issues of Mechanical Engineering Vol 157 P. 198-202
[24] Sokolov, U.A. 2018 Optimization of Additive Manufacturing Metalworking 5 P. 48-54.