PARAMETER DEGENERACY AND REACTOR EXPERIMENTS *

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Degeneracies of the neutrino oscillation parameters are explained using the sin²θ_{13}–s²_{23} plane. Measurements of sin²θ_{13} by reactor experiments are free from the parameter degeneracies which occur in accelerator appearance experiments, and reactor experiments play a role complementary to accelerator experiments. It is shown that the reactor measurement may be able to resolve the degeneracy in θ_{23} if sin²θ_{13} and cos²θ_{23} are relatively large.

1. Introduction

Thanks to the successful experiments on atmospheric and solar neutrinos and KamLAND, we now know approximately the values of the mixing angles and the mass squared differences of the atmospheric and solar neutrino oscillations: (sin²θ_{12}, |Δm_{21}²|) ≃ (0.8, 7 × 10⁻⁵ eV²) for the solar neutrino and (sin²2θ_{23}, |Δm_{31}²|) ≃ (1.0, 3 × 10⁻³ eV²) for the atmospheric neutrino. In the three flavor framework of neutrino oscillations, the quantities which are still unknown to date are the third mixing angle θ_{13}, the sign of the mass squared difference Δm_{31}² of the atmospheric neutrino oscillation, and the CP phase δ. Among these three quantities, the determination of θ_{13} is the next goal in the near future neutrino experiments.

In this talk I will first explain briefly the ambiguity due to the parameter degeneracies, which occur in the long baseline experiments, using the sin²2θ_{13}–s²_{23} plane, and then I will show that reactor experiments will play a role complementary to accelerator experiments, and it may resolve a certain degeneracy when combined with an accelerator experiment. This talk

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is based on the work\textsuperscript{1}, in which references on the subject can be found.

\section{Parameter degeneracies}

It has been realized that we cannot determine the oscillations parameters $\theta_{jk}$, $\Delta m_{jk}^2$, $\delta$ uniquely even if we know precisely the appearance probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ in a long baseline accelerator experiment with an approximately monoenergetic neutrino beam due to so-called parameter degeneracies. There are three kinds of the parameter degeneracies: the intrinsic ($\theta_{13}, \delta$) degeneracy, the degeneracy of $\Delta m_{31}^2 \leftrightarrow -\Delta m_{31}^2$, and the degeneracy of $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$. Each degeneracy gives a two-fold solution, so in total one has eight-fold solution if the degeneracies are exact. When these degeneracies are lifted, there are eight solutions for the oscillation parameters such as $\theta_{13}$ etc., and it will be important in future long baseline experiments to discriminate the real solution from fake ones.

To explain the parameter degeneracies, let me consider the contours which are given by $P \equiv P(\nu_\mu \rightarrow \nu_e)$ and $\bar{P} \equiv P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ at the same time in the $\sin^2 2\theta_{13} - s_{23}^2$ plane. If there were no matter effect $A \equiv \sqrt{2} G_F N_e = 0$, if the mass squared difference $\Delta m_{21}^2$ of the solar neutrino oscillation were exactly zero, and if $\theta_{23}$ is exactly $\pi/4$, then we would have one solution with 8-fold degeneracy as is shown in Fig.1(a). If we lift the conditions in the order ($A = 0, \Delta m_{21}^2 = 0, \theta_{23} - \pi/4 = 0$) $\rightarrow$ ($A = 0, \Delta m_{21}^2 = 0, \theta_{23} - \pi/4 \neq 0$) $\rightarrow$ ($A = 0, \Delta m_{21}^2 \neq 0, \theta_{23} - \pi/4 = 0$) $\rightarrow$ ($A \neq 0, \Delta m_{21}^2 \neq 0, \theta_{23} - \pi/4 \neq 0$), then the exact degeneracies are lifted as is depicted in Figs.1(a) (one solution with 8-fold degeneracy) $\rightarrow$ (b) (two solutions with 4-fold degeneracy) $\rightarrow$ (c) (four solutions with 2-fold degeneracy) $\rightarrow$ (d) (eight solutions without any degeneracy). Furthermore, if we assume the neutrino energy to be approximately monoenergetic and to satisfy the oscillation maximum condition $|\Delta m_{21}^2 L/4E| = \pi/2$ as is the case at the JHF experiment, then the contours look like Fig.1(e) and in this case only ambiguity which causes a problem is the $\theta_{23}$ degeneracy, since the intrinsic ($\theta_{13}, \delta$) degeneracy is exact and there is little ambiguity due to the $\text{sgn}(\Delta m_{31}^2)$ degeneracy because $|AL/2| \ll 1$.

Here let me explain briefly the behaviors of the contours in Fig.1(e), which is shown in Fig.2 in detail. Using the analytic approximate formulae\textsuperscript{3} in the case of the oscillation maximum,

\begin{align}
P &= x^2 f^2 - 2xyf g \sin \delta + y^2 g^2, \\
\bar{P} &= x^2 \bar{f}^2 + 2xy\bar{f} g \sin \delta + y^2 \bar{g}^2,
\end{align}
Figure 1. Contours which are given when both $P$ and $\bar{P}$ are known. The eight-fold parameter degeneracy is lifted as the small parameters are switched on: $(\cos^2 2\theta_{23}, |\Delta m_{21}^2/\Delta m_{31}^2|, AL)$ (a)(= 0, = 0, = 0), (b)(≠ 0, = 0, = 0), (c)(≠ 0, ≠ 0, = 0), (d)(≠ 0, ≠ 0, ≠ 0), (e)(≠ 0, ≠ 0, ≠ 0, at the first oscillation maximum). The solid (dashed) line stands for the case for $\Delta m_{21}^2 > 0$ ($\Delta m_{21}^2 < 0$), respectively, (a),(b),(c). $\bar{P} = P(\bar{P} > P)$ is assumed in (a), (b) and (c) ((d) and (e)), respectively.
where \( x \equiv s_{23}\sin 2\theta_{13}, \quad (f, \bar{f}) \equiv \cos(AL/2)/(1 \mp AL/\pi), \quad g \equiv \sin(AL/2)/(AL/\pi), \quad x \equiv s_{23}\sin 2\theta_{13}, \quad y \equiv \epsilon s_{23}\sin 2\theta_{12} \) and \( \epsilon \equiv |\Delta m_{21}^2/\Delta m_{31}^2| \), one can show for any given value of \( s_{23}^2 \)

\[
\sin^2 2\theta_{13}|_{\Delta m_{31}^2 > 0} - \sin^2 2\theta_{13}|_{\Delta m_{31}^2 < 0} = \frac{1}{s_{23}^2} \left( \frac{1}{f - \bar{f}} (P - \bar{P}) \right) \approx \frac{AL}{\pi s_{23}^2} (P - \bar{P}), \tag{2}
\]

if \( |AL/2| \ll 1 \). Therefore the thick solid and thick dashed lines are close to each other in Fig.2 because the matter effect and \( \epsilon \) are both small. Notice that \( |P - \bar{P}| \) could be large if \( \epsilon \) were large even if \( |AL/2| \ll 1 \). One can also show

\[
\max s_{23}^2|_{\Delta m_{31}^2 < 0} - \max s_{23}^2|_{\Delta m_{31}^2 > 0} = \frac{1}{\epsilon^2 \sin^2 2\theta_{12}} \frac{1}{g^2 f + \bar{f}} (P - \bar{P}) \approx \frac{1}{\epsilon^2} \frac{(2/\pi)^2 AL}{\pi} (P - \bar{P}) \tag{3}
\]

if \( |AL/2| \ll 1 \). (3) indicates that \( \max s_{23}^2|_{\Delta m_{31}^2 < 0} - \max s_{23}^2|_{\Delta m_{31}^2 > 0} \) is...
large in Fig.2 because a small quantity $AL(P - \bar{P})$ is enhanced by a large factor $1/\epsilon^2$. In the case of the JHF experiment, which has $L=295$km, $A \simeq 1/(1900$km), $\epsilon \simeq 1/35$, if $P=0.025$ and $\bar{P}=0.035$, then the right hand side of (2) $\simeq 5 \times 10^{-4}$, and the right hand side of (3) $\simeq 0.3$.

3. Reactor measurements of $\theta_{13}$

In the three flavor framework the disappearance probability of the reactor neutrinos with a baseline less than 10km is given by

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right),$$

(4)
to a good approximation. Hence reactor measurements are free from the ambiguity due to the matter effect, the CP phase $\delta$ and $\theta_{23}$. It has been shown$^{1,2}$ that a reactor measurement at the Kashiwazaki-Kariwa nuclear power plant potentially has sensitivity down to $\sin^2 2\theta_{13} \simeq 0.02$.

![Figure 3](image_url)

Figure 3. Situations of the long baseline accelerator experiment at the oscillation maximum ($|\Delta m_{31}^2 L/4E| = \pi/2$): (a) the case with $\theta_{23} \simeq \pi/4$, (b) the case with $\theta_{23} \neq \pi/4$.

4. Resolution of the $\theta_{23}$ ambiguity by the LBL and reactor experiments

Once we have the results from, say, the phase 2 of the JHF experiment on $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ and the reactor measurement of $P(\bar{\nu}_e \to \bar{\nu}_e)$, there are two possibilities. One is the case where $\sin^2 2\theta_{23}$ is close to 1, as is shown in Fig.3(a). In this case one could not resolve the ambiguity within
Figure 4. Resolution of the $\theta_{23}$ degeneracy by combining the results of the reactor and the long baseline accelerator experiments at the oscillation maximum: (a) the case with $\theta_{23} < \pi/4$, (b) the case with $\theta_{23} > \pi/4$. The shaded region stands for the error in $\sin^2 2\theta_{13}$ of the reactor experiment.
the experimental error nor would one have to worry about the ambiguity, as the difference is small. On the other hand, if \( \sin^2 2\theta_{23} \) turns out to be away from 1 as is depicted in Fig.3(b), then the reactor result may enable us to resolve the \( \theta_{23} \) ambiguity.

As is shown in Fig.4(a) and (b), if the ambiguity
\[
\delta_{de}(\sin^2 2\theta_{13}) \equiv |\sin^2 2\theta'_{13} - \sin^2 2\theta_{13}|
\]
due to the degeneracy, where \( \theta_{13} \) and \( \theta'_{13} \) stand for the values of \( \theta_{13} \) for the real and fake solutions, is larger than the error
\[
\delta_{re}(\sin^2 2\theta_{13})
\]
of the reactor measurement, then we can resolve the \( \theta_{23} \) ambiguity. \( \delta_{de}(\sin^2 2\theta_{13}) \) can be computed from the equation (26) in Ref.\(^4\), and we obtain
\[
\frac{\delta_{de}(\sin^2 2\theta_{13})}{\sin^2 2\theta_{13}} = |1 - \tan^2 \theta_{23}| \left[ 1 + \frac{\epsilon^2}{\sin^2 2\theta_{13}} \frac{\tan^2 (aL/2)}{(aL/\pi)^2} \right] |1 - (\frac{aL}{\pi})^2| \sin^2 2\theta_{12} \].
\]
Hence \( \delta_{de}(\sin^2 2\theta_{13})/\sin^2 2\theta_{13} \) can be approximated by \( |1 - \tan^2 \theta_{23}| \un-
less $\epsilon$ is large such as 0.1. As for the error of the reactor measurements, we have $\delta_{re}(\sin^2 2\theta_{13}) \approx 0.018$ for any value of $\sin^2 2\theta_{13}$ in the case with the systematic error 0.8% and 40t-yr. $\delta_{re}(\sin^2 2\theta_{13})/\sin^2 2\theta_{13}$ and $\delta_{de}(\sin^2 2\theta_{13})/\sin^2 2\theta_{13}$ are shown in Fig.5(a) and (b), and from these figures one can read off the region where the $\theta_{23}$ ambiguity is resolved. In general the $\theta_{23}$ ambiguity is resolved if $\sin^2 2\theta_{13}$ and $1 - \sin^2 2\theta_{23}$ are both large.

5. Summary

In this talk I have shown that the 8-fold parameter degeneracy in long baseline experiments can be visualized in the $\sin^2 2\theta_{13}-s^2_{23}$ plane. I have demonstrated that one may be able to resolve the $\theta_{23}$ ambiguity by combining the results of the JHF experiment at the oscillation maximum and a reactor experiment whose sensitivity for $\sin^2 2\theta_{13}$ is 0.02, if $\sin^2 2\theta_{13}$ and $\cos^2 2\theta_{23}$ are both relatively large. This scenario offers one of the strategies which are expected to resolve the ambiguities due to the parameter degeneracies.

References

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As other alternatives to resolve the ambiguities, Parke\textsuperscript{5} proposed to measure both $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ at JHF off the oscillation maximum and at NuMI at the oscillation maximum, whereas Donini\textsuperscript{6} discussed the measurements of the golden channel $P(\nu_e \to \nu_\mu)$ and the silver channel $P(\nu_e \to \nu_\tau)$ at a neutrino factory.