Physics of Collisionless GRB Shocks and Their Radiation Properties

Mikhail V. Medvedev

CITA, University of Toronto, Toronto, Ontario, M5S 3H8, Canada
Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

Abstract.
We present a theory of ultrarelativistic collisionless shocks based on the relativistic kinetic two-stream instability. We demonstrate that the shock front is unstable to the generation of small-scale, randomly tangled magnetic fields. These fields are strong enough to scatter the energetic incoming (in the shock frame) protons and electrons over pitch angle and, therefore, to convert their kinetic energy of bulk motion into heat with very high efficiency. This validates the use of MHD approximation and the shock jump conditions in particular. The effective collisions are also necessary for the diffusive Fermi acceleration of electrons to operate and produce an observed power-law. Finally, these strong (sub-equipartition) magnetic fields are also required for the efficient synchrotron-type radiation emission from the shocks.

The predicted magnetic fields have an impact on polarization properties of the observed radiation (e.g., a linear polarization from a jet-like ejecta and polarization scintillations in radio for a spherical one) and on its spectrum. We present an analytical theory of jitter radiation, which is emitted when the magnetic field is correlated on scales smaller than the gyration (Larmor) radius of the accelerated electrons. The spectral power of jitter radiation is described by a sharply broken power-law: \( P(\nu) \propto \nu^1 \) for \( \nu < \nu_j \) and \( P(\nu) \propto \nu^{-(p-1)/2} \) for \( \nu > \nu_j \), where \( p \) is the electron power-law index and \( \nu_j \) is the jitter break, which is independent of the magnetic field strength but depends on the shock energetics and kinematics. Finally, we present a composite jitter+synchrotron model of GRB \( \gamma \)-ray emission from internal shocks which is capable of resolving many puzzles of GRB spectra, such as the violation of the “line of death”, sharp spectral breaks, and multiple spectral components seen in some bursts (good examples are GRB910503, GRB910402, etc.). We stress that simultaneous detection of both spectral components opens a way to a precise diagnostics of the conditions in GRB shocks. We also discuss the relation of our results to other systems, such as internal shocks in blazars, radio lobes, and supernova shocks.

THE STRUCTURE OF COLLISIONLESS SHOCKS

The conventional paradigm of GRBs assumes optically thin synchrotron radiation from ultra-relativistic shocks where the radiation is produced by Fermi-accelerated electrons moving in strong, nearly equipartition magnetic fields. This purely phe-
nomenological model contains several serious assumptions which require justifi-
cation: (i) standard hydrodynamic shock physics must be valid for these highly
collisionless shocks, i.e., one needs effective collisions; (ii) magnetic fields must be
generated in situ much faster than the dynamical time; (iii) acceleration of electrons
requires multiple scatterings (i.e., effective collisions) in the shock.

These problems have successfully been resolved [2,3]. It was shown that magnetic
fields are naturally produced via the relativistic two-stream instability operating
at the shock front. In essence, this work [2] makes the bridge between the theories
non-relativistic collisionless shocks [4] (those observed in the interplanetary space
were studied in situ in great details by many satellites, and their ultra-relativistic
counterparts. Here we briefly describe the main results.

- The two-stream instability operates in both internal and external shocks. The
  field is produced by both the electrons and protons.
- The generated magnetic field is randomly oriented in space, but always lies in
  the plane of the shock front.
- The characteristic e-folding time in the shock frame for the instability is \( \tau \sim \gamma_{sh}^{1/2}/\omega_p \) (where \( \gamma_{sh} \) is the shock Lorentz factor) which is \( \sim 10^{-7} \) s for internal
  shocks and \( 10^{-4} \) s for external shocks. This time is much shorter than the
dynamical time of GRB fireballs.
- The characteristic coherence scale of the generated magnetic field is of the
  order of the relativistic skin depth \( \lambda \sim c\sqrt{\gamma / \omega_p} \) (where \( \bar{\gamma} \) is the mean thermal
  Lorentz factor of particles), i.e. \( \sim 10^3 \) cm for internal shocks and \( \sim 10^5 \) cm
  for external shocks. This scale is much smaller than the spatial scale of the
  source.
- The instability converts a large fraction of the kinetic energy of particles into
  magnetic energy, hence \( [B^2/8\pi]/[mc^2n(\bar{\gamma} - 1)] = \eta \sim 10\% \). This agrees well
  with direct particle simulations.
- Random fields scatter particles over pitch-angle and, thus, provide effective
  collisions. Therefore MHD approximation works well for the shocks. The
  magnetic fields communicate the momentum and pressure of the outflowing
  fireball plasma to the ambient medium and define the shock boundary.
- The instability isotropizes and heats the electrons and protons. Moreover,
  effective collisions will diffusively further accelerate the electrons to higher
  energies.

**RADIATION FROM SHOCKS**

Since the geometry of magnetic fields is not entirely random, it affects the ob-
served properties of radiation. In particular, polarization of radiation will always be
radial. Therefore, one expects non-vanishing degree of polarization observed at any wavelength for a non-spherically symmetric explosion, e.g., a jetted geometry. An optical transient of GRB990510 shows linear polarization of the degree $\sim 1 - 2\%$. For a spherically symmetric case, radio scintillations may reveal the polarization map of the source (afterglow) [5].

The small-scale nature of magnetic fields affects the radiation process as well. In fact, it breaks the conventional paradigm of synchrotron nature of the radiation from shocks. The magnetic field produced in GRB shocks randomly fluctuates on a very small scale of roughly the relativistic skin depth, which is much smaller than the Larmor radius of the ultra-relativistic emitting electron. Therefore, the electron trajectories are not helical, as they would be in a homogeneous field, as in Figure 1. Thus, the theory of synchrotron radiation derived for homogeneous fields is not applicable and the spectrum of the emergent radiation is different. Such a situation has never been considered in the astrophysical literature.

If the magnetic field is randomly tangled and the correlation length is less then a Larmor radius of an emitting electron, then the electron experiences random deflections as it moves through the field. Its trajectory is, in general, stochastic. This is similar to a collisional motion of an electron in a medium. Bremsstrahlung quanta are emitted in every collision. Unlike the bremsstrahlung case, here “collisions” are due to small-scale inhomogeneities of the magnetic field rather than due to electrostatic fields of other charged particles. Since the Lorentz force depends on particle’s velocity, the emergent spectrum will be somewhat different from pure bremsstrahlung. There is also an alternative physical interpretation of the process. For an ultrarelativistic electron, the method of virtual quanta applies. In the rest frame of the electron, the magnetic field inhomogeneity with wavenumber $k \sim 1/\lambda$ is transformed into a transverse pulse of electromagnetic radiation with frequency $kc$. This radiation is then Compton scattered by the electron to produce observed radiation with frequency $\sim \gamma^2 kc$ in the lab frame.

Keeping this general physical picture in mind, we now analyze the problem in more details. Let’s consider a nonuniform random magnetic field with a typical

![Diagram](image)

**FIGURE 1.** Illustration of the process of radiation in homogeneous and inhomogeneous fields.
correlation scale $\lambda_B$, the Larmor radius of the electron, $\rho_e = \gamma m_e c^2/eB_\perp$ is less or comparable comparable to $\lambda_B$. The emerging spectrum depends on the relation between the particle’s deflection angle, $\alpha$, and the beaming angle, $\Delta \theta$. For ultrarelativistic particles and small deflection angles, the latter is estimated as follows. The particle’s momentum is $p \sim \gamma m_e c$. The change in the perpendicular momentum due to the Lorentz force acting on the particle during the transit time $t \sim \lambda_B/c$ is $p_\perp \sim eB_\perp \lambda_B/c$. The angle $\alpha$ is then $\alpha \sim p_\perp / p \sim eB_\perp \lambda_B / \gamma m_e c^2$. We now define the deflection-to-beaming ratio as follows,

$$\delta \equiv \frac{\gamma}{k_B \rho_e} \sim \frac{\gamma \lambda_B}{\rho_e} \sim \frac{\alpha}{\Delta \theta} \sim \frac{eB_\perp \lambda_B}{m_e c^2}. \tag{1}$$

It is interesting to note that this ratio is independent of particle’s energy (i.e., of $\gamma$) and is determined by $B$ and $\lambda_B$.

There are two limiting cases, as in Figure 2a,b. First, $\delta \sim \alpha / \Delta \theta \gg 1$; an observer sees radiation coming from short segments (“patches”) of the electron’s trajectory, the magnetic field is almost uniform but it varies from patch to patch. The radiation is pulsed with a typical duration $\tau_p \sim 1/\omega_c$ as for pure synchrotron. The ensemble-averaged spectrum completely identical to synchrotron radiation from large-scale weakly inhomogeneous magnetic fields. Second, $\delta \sim \alpha / \Delta \theta \ll 1$; the particle moves along the line of sight almost straight and experiences high-frequency jittering in the perpendicular direction due to the random Lorentz force. The emergent spectrum is determined by random accelerations of the particle.

Spectra of jitter radiation were calculated in Ref. [5]. They are well approximated by the sharply broken power-law, as is seen from Figure 2c. The jitter break frequency is

$$\omega_{jm} = 2^{7/4} \gamma_{sh} \gamma_{\text{int}} \gamma_{\text{min}}^{-1/2} \omega_{pe} \propto n_e^{1/2}, \tag{2}$$

where $\gamma_{sh}$, $\gamma_{\text{min}}$, $\gamma_{\text{int}}$, $\bar{\gamma}_e$ are the Lorentz factors of the ejecta, internal shock, electron power-law cutoff, and the mean thermal $\gamma$-factor of electron in front of the shock.

**FIGURE 2.** (a,b) The emission process in $\delta \sim \alpha / \Delta \theta \gg 1$ and $\delta \ll 1$ regimes. (c) Spectral power of jitter radiation for various $\delta$; synchrotron spectrum is shown by dashed curve.
Note, this break frequency is independent of the magnetic field strength; instead it directly “measures” the density of particles in the shock. Below and above the break, the photon spectra scale as ($p$ is the electron power-law index)

\[ F(\omega < \omega_{jm}) \propto \omega^0, \quad F(\omega > \omega_{jm}) \propto \omega^{-(p+1)/2}, \]

that is the spectrum is harder than synchrotron ($\propto \omega^{-2/3}$) at low frequencies.

As was mentioned earlier, the magnetic field is produced by both the electrons and the protons. However, only the electron-produced field is small-scale enough to produce jitter radiation, whereas for the proton-produced field $\delta > 1$ and synchrotron spectrum is expected. The ratio of the jitter and synchrotron break frequencies and peak fluxes completely determine both free parameter of the model: the small-to-large scale field strength ratio and the deflection-to-beaming ratio,

\[ \frac{\omega_{jm}}{\omega_{sm}} \simeq \frac{2 B_{SS}}{3 B_{LS}} \delta^{-1}, \quad \frac{F_{J,\text{max}}}{F_{S,\text{max}}} \simeq \delta^2. \]

This jitter+synchrotron model explains perfectly well the diversity of time-resolved GRB $\gamma$-ray spectra and resolves several long-standing puzzles, namely

- the violation of the constraint on the low-energy spectral index called the synchrotron “line of death” in about a third of BATSE and BSAX bursts [6];
- the sharp spectral break at the peak frequency seen in some bursts which is inconsistent with the broad synchrotron bump [7];
- the evidence for two spectral sub-components seen in some GRBs [8];
- possible existence of emission features (“GRB lines”) seen in few bursts [9].

All this strongly supports that (i) the proposed jitter radiation mechanism operates in astrophysical objects and (ii) the magnetic field is generated in shocks by the two-stream instability. In general, the detection of both spectral components in GRB spectra would be a powerful and precise tool to investigate the properties of cosmological fireballs.

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