Controllable finite ultra-narrow quality-factor peak in a perturbed Dirac-cone band structure of a photonic crystal slab

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We show that by using a perturbed photonic Dirac-cone, one can realize ultra-narrow and finite $Q$-factor peak in the wavevector space, with both the peak value and the width separately tunable. We also discuss a lower bound in the minimal viable width given a peak $Q$-value while maintaining sufficient $Q$ differentiation among modes. The strong angular and frequency $Q$-selection finds applications in optical devices where strong angle- and frequency-selection is needed.

Engineering the photonic band structure has led to significant advances in the developments of optoelectronic devices [1–3]. Most band-structure engineering focuses on the eigenfrequency of the modes as a function of the wavevector. Recently on the other hand, engineering the quality-factor ($Q$-factor) as a function of the wavevector has drawn increasing interest due to a number of applications such as light-trapping, two-dimensional lasing, and meta optics [4–12]. One of the desired features in such engineering is to obtain a fully-controllable finite and narrow high-$Q$ peak. This allows the devices to interact with the electromagnetic waves only at a certain angle and frequency, which is important in angle-selective optical devices such as absorbers [13–24] and thermal emitters [25–38]. This feature is also useful in achieving single-mode large-area lasing for high-power applications. Such devices have always been challenging since as the device-size scales, the spacing between the modes in the wavevector space reduces, leading to a diminishing $Q$- and hence threshold-difference [39–43].

By exploring a perturbed photonic Dirac-cone [44–46], in this Letter we demonstrate a ultra-narrow finite $Q$ peak in a photonic-crystal slab. Both the peak $Q$-factor and its width in the wavevector space are independently controllable. Our approach exploits the strong mixing between the Dirac-cone bands to the linear order in the wavevector. Such mixing leads to a drastic $Q$ reduction away from the Brillouin zone center $\Gamma$. The peak $Q$ can be tuned by the strength of the perturbation, i.e. the size of the additional small holes in the photonic-crystal slab. The width of the peak can be tuned by the thickness of the slab. We also derive a trade-off relation between the peak $Q$ value and the width, which gives a lower bound of the latter given the former. Our construction can be used to fabricate perfect absorbers that are both direction- and frequency-selective. The design of surface-emitting lasers can also benefit from this result, as a controllable outstanding high-$Q$ mode at $\Gamma$ promotes single-mode lasing in a scaled device.

MAIN RESULT

To motivate our results, we consider generically how a peaked $Q$ can be obtained. In the following, we examine optical resonators with light incident or output in the out-of-plane direction, with $k = (k_x, k_y)$ representing the 2D in-plane wavevector, as is illustrated in Fig. 1. Our goal is to obtain a peaked $Q$ as a function of the angles. We start with a uniform dielectric slab, which can be found in vertical-cavity surface-emitting lasers (VCSELs). There are Fabre-Pérot resonances in the vertical direction in the slab. Their $Q$-factors as a function of the wave incident angle is nearly constant near the nor-

![Figure 1. Comparison of the angular $Q$ variation in different structures. (a) A Fabry-Pérot band at the frequency of 0.5c/d in a uniform slab of thickness d and permittivity of 12, where $c$ is the light speed. (b) A guided resonance mode at the frequency of 0.36c/l in a photonic-crystal slab, where $l$ is the periodicity of the photonic-crystal. The slab has a thickness of 0.8l. The side length of the isosceles right triangle holes is 0.5l. (c) The quadrupole band for the perturbed photonic-crystal slab structure shown in Fig. 2.](image-url)
The photonic-crystal slab has a square lattice with period \( b \) by the approach described above and perturb it such that the system thus does not have the desired singly degenerate state has an infinite Q-factor. However, in all previous works, due to the symmetry of the structure used, the singly degenerate state [44, 45, 47]. However, in all previous works, due to the symmetry of the structure used, the singly degenerate state has an infinite \( Q \). The resulting Hamiltonian

\[
\hat{h}(k) = \begin{pmatrix}
  i\gamma_d & iv_g k \\
  -iv_g k & i\gamma_q
\end{pmatrix},
\]

along a direction \( k \) in the Brillouin zone. Here \( k = |k| \) is the magnitude of the in-plane wavevector, \( v_g \) is the group velocity of the bands, \( \gamma_d \) is the radiation constant of the low-Q mode at \( \Gamma \), and \( \gamma_q \) is that of the high-Q mode at \( \Gamma \). The resulting \( Q(k) \) function for the band containing mode \( |q⟩ \) is plotted in Fig. 1c, assuming the parameters of \( \gamma_q = 6.7 \times 10^{-7} \), \( \gamma_d = 3.8 \times 10^{-4} \), and \( v_g = 0.11 \, c/2\pi \). Due to the strong mixing that is linear in \( k \), the \( Q \) factor reduces drastically away from \( \Gamma \), leading to a finite and narrow \( Q \) peak.

The remaining task is to design a structure that gives the effective Hamiltonian in Eq. (1). The linear mixing in Eq. (1) corresponds to a Dirac-cone band structure [44, 45]. In a photonic crystal slab, one can form a Dirac cone by creating an accidental degeneracy at the \( \Gamma \) point between a pair of two-fold degenerate dipole states and a singly degenerate state [44, 45, 47]. However, in all previous works, due to the symmetry of the structure used, the singly degenerate state has an infinite \( Q \). The resulting system thus does not have the desired \( Q(k) \) dependency [46]. Our approach is to start with a Dirac-cone formed by the approach described above and perturb it such that the infinite-Q mode becomes radiative.

To implement our approach, we consider the photonic-crystal slab structure as is shown in the inset of Fig. 2b. The photonic-crystal slab has a square lattice with period \( l \) in both \( x \) and \( y \) directions. We normalize all the other geometrical parameters in units of \( l \). The slab has a thickness of \( t = 0.8 \, l \), and a permittivity of 12 as is typical for common semiconductors in the infrared wavelength range. In each unit-cell, there is a large square-shaped hole at the center with a side length of \( a_1 \) and two smaller holes with a side length of \( a_2 = 0.05 \, l \) located at \((0.4 \, l, 0)\) and \((0, 0.4 \, l)\), respectively. The large hole is through the slab, while the smaller holes have a depth of \( t_2 = 0.05 \, l \). Without the smaller holes, the PhC slab has a \( D_{4h} \) point-group symmetry [48]. The symmetry reduces to \( C_{2v} \) with the smaller holes added.

The band structure and the \( Q(k) \) functions can be directly calculated by monitoring the poles of the scattering matrix on the complex frequency plane [49] [50]. The results are plotted as blue circles in Fig. 2a and b, respectively. In the absence of the smaller holes, and with a side length of \( a_1 = 0.69 \, l \), three modes are tuned to degeneracy at \( \Gamma \). In the resulting band structure in Fig. 2, both of the bands are dispersive, while the third forms a flat band. Both the band structure in Fig. 2a and the \( Q(k) \) functions in Fig. 2b show two phases separated by exceptional points [45]. At \( \Gamma \), the infinite-Q mode belongs to the \( B_{1g} \) representation of the \( D_{4h} \) point group and is of quadrupole nature. The low-Q mode belong to the \( E_u \) representation, which is of dipolar nature and strongly couples to free-space radiation. The resulting \( Q(k) \) is sharply peaked for one of the bands. However, the mode at \( \Gamma \) in this band has a infinite \( Q \)-factor and is a bound state in the continuum (BIC) [5], which can not couple to free space to perform light detection, absorption, or emission.

To create a sharply peaked \( Q(k) \) where \( Q \) is finite at the peak, we perturb the structure by introducing two

![Figure 2. Band structure and the \( Q(k) \) function of the photonic Dirac-cone. Circles represent numerical calculation results, while solid curves represent results of the effective model in Eq. (1). (a) band structure of the unperturbed structure. Two dispersive (blue) bands and the flat-band (gray) are plotted. Inset: schematic of the photonic-crystal slab. The frequency is offset to the Dirac-cone frequency at 0.496 \( c/l \). (b) \( Q(k) \) functions of the unperturbed structure (blue) and the perturbed (red). The flat-band is plotted in Grey circles. Black circles represent the \( Q(k) \) function of the photonic-crystal slab with triangular holes. Inset: diagram of the first Brillouin zone.](image-url)
smaller holes as mentioned above to break the $D_{4h}$ symmetry, such that the quadrupole mode is allowed to radiate. With the perturbation, the real part of the band structure is virtually identical to the one without in Fig. 2a, hence is not plotted. The $Q(k)$ functions with the perturbation are shown as red circles in Fig. 2b. We observe that the $Q(k)$ functions for both the high-$Q$ and the low-$Q$ bands are very close to those without perturbation, except near the Brillouin zone center $\Gamma$. At $\Gamma$, the high-$Q$ mode now has a finite $Q$-factor rather than infinite. Away from $\Gamma$, the $Q$-factor quickly reduces, thus forming a sharp peak at $\Gamma$. As a reference, the $Q(k)$ function of the band at the frequency of 0.36 $c/l$ in a photonic-crystal slab with a square lattice and triangular holes is also plotted in the black curve in Fig. 2b [8, 49]. This is a singly degenerate band with no other bands nearby in frequency, hence does not form a Dirac-cone. A schematic of this structure can be found in Fig. 1b. The peak $Q$-factor is at a level of $10^5$, similar to our design here. However, the $Q(k)$ function is much more slowly varying. The $Q$ peak width is reduced by an order of magnitude in our work through exploiting the strong linear-order mixing.

The band structure and the $Q(k)$ function of the two dispersive bands are described by the effective Hamiltonian in Eq. (1) [45]. Here, $\gamma_d$ and $\gamma_q$ can be read out from the numerical calculations at the Brillouin zone center $\Gamma$. $v_g$ can be obtained by fitting to the slope of the bands outside of the exceptional points. The complex eigenfrequencies of the Dirac-cone bands are then obtained by solving Eq. (1). The resulting band structure and the $Q(k)$ functions are plotted as solid curves in Fig. 2. The results very accurately reproduce the numerical simulations. In existing studies of the photonic Dirac-cones, $\gamma_q$ is zero, leading to an infinite $Q$-factor. In our work, $\gamma_q$ becomes finite due to the introduction of the smaller holes. The perturbation can also cause small corrections to other parameters in Eq. 1, which are omitted in the lowest order. From Eq. (1) it is clear that the off-diagonal terms that is linear in $k$ causes a strong mixing between the dipolar and the quadrupole modes, resulting in a sharp $Q$ peak. It is important to note that the entire band structure and the $Q(k)$ functions are controlled by only three parameters, i.e. $\gamma_d$, $\gamma_q$, and $v_g$.

INDEPENDENT ENGINEERING OF THE PEAK Q AND ITS WIDTH

In our approach to engineer the $Q(k)$ function, both the peak $Q$ and its width can be tuned. The peak $Q$ value can be straightforwardly engineered by controlling the size of the small holes. For example, in Fig. 3(a) we plot the $Q(k)$ function of the same structure as above, but with the small holes having a reduced side length and depth of $a_2 = t_2 = 0.04l$. The resulting $Q$-factor at $\Gamma$ now increases by an order of magnitude while the width of the peak remains largely unchanged.

To tune the width of the $Q$ peak without changing the peak value, we can change the $Q$-factor of the dipolar mode at $\Gamma$. As is discussed above, the $Q(k)$ functions of both bands are controlled by only three parameters, i.e. $\gamma_q$, $\gamma_d$, and $v_g$. By increasing the $Q$-factor of the dipolar mode, the region inside the exceptional points shrinks, leading to a reduced peak width. A demonstration is shown in Fig. 3(b), in which we compare the $Q(k)$ function of the design above that corresponds to the red curve in Fig. 3a and one with a reduced slab thickness of $t = 0.775l$ (purple). The side length of the large hole is adjusted to 0.695 $l$ to maintain the degeneracy of the real part of the bands. We also tune the smaller holes to have a side length and depth of $a_2 = t_2 = 0.035l$, to maintain the peak $Q$-value at $\Gamma$. The resulting $Q(k)$ functions of both numerical calculation (circles) and the analytical model (solid curves) are shown. The $Q$-factor of the dipolar mode increased by four times in the thinner photonic-crystal slab. As a result, the high-$Q$ bandwidth reduces by a factor of 2 while the peak $Q$ value is unchanged, as is observed in Fig. 3(b).

PEAK-WIDTH RELATION

The above discussion of the tuning of the high-$Q$ width gives a hint of a relation between the minimal viable width and the peak $Q$. In real devices, it is often desirable to design a certain $Q$-factor at the zone center $\Gamma$ and at the same time a strong $Q$-differentiation against other modes away from $\Gamma$. For simplicity, We define the width of the high-$Q$ region by the width of the wavevector region inside the exceptional points. The location of the exceptional point can be calculated from Eq. (1) as $k_c = (\gamma_d - \gamma_q)/2v_g$. It is clear that a higher $Q_d$, i.e. a smaller $\gamma_d$, leads to a smaller $k_c$. By the requirement of
In summary, in this Letter we presented an approach to obtain a finite and ultra-narrow $Q$ peak, with the peak value and width independently tunable. This is realized by employing a perturbed photonic Dirac-cone. Utilizing the linear mixing between modes in the Dirac-cone, the modes at $\Gamma$ strongly remix away from $\Gamma$, leading to a rapidly decreasing $Q(k)$ function for a high-$Q$ band. The narrow $Q$ peak is useful in scenarios where angular selection is needed, such as directional absorber or emitter. The feature can be useful in surface-emitting lasers as well, since a large $Q$ differentiation promotes single-mode lasing. Although we focus on 2D PhC structure, this approach applies to other dimensions such as 1D distributed Bragg reflectors (DBRs).

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CONCLUSION

Figure 4. The absorption spectra of the angle- and frequency-selective absorber. The frequency is offset to the Dirac-cone point at $0.485\,\text{(c/}\lambda)$. The incident wave is polarized such that the in-plane projection of the electric field is pointed in the $45^\circ$ direction.

In summary, in this Letter we presented an approach to obtain a finite and ultra-narrow $Q$ peak, with the peak value and width independently tunable. This is realized by employing a perturbed photonic Dirac-cone. Utilizing the linear mixing between modes in the Dirac-cone, the modes at $\Gamma$ strongly remix away from $\Gamma$, leading to a rapidly decreasing $Q(k)$ function for a high-$Q$ band. The narrow $Q$ peak is useful in scenarios where angular selection is needed, such as directional absorber or emitter. The feature can be useful in surface-emitting lasers as well, since a large $Q$ differentiation promotes single-mode lasing. Although we focus on 2D PhC structure, this approach applies to other dimensions such as 1D distributed Bragg reflectors (DBRs).

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