Nonclassicality of local bipartite correlations

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Simulating quantum nonlocality and steering requires augmenting pre-shared randomness with non-vanishing communication cost. This prompts the question of how one may provide such an operational characterization for the quantumness of correlations due even to unentangled states. Here we show that for a certain class of states, such quantumness can be pointed out by superlocality, the requirement for a larger dimension of the pre-shared randomness to simulate the correlations than that of the quantum state that generates them. This provides an approach to define the nonclassicality of local multipartite correlations in convex operational theories.

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I. INTRODUCTION

Local measurements on a spatially separated quantum system can lead to a nonclassical box (set of correlations) which cannot be explained by shared classical randomness [1, 2]. This feature of quantum correlations is called nonlocality and is witnessed by the violation of a Bell inequality, which must be satisfied by the correlations that admit a local hidden variable (LHV) model [1]. The fact that the nonlocality of quantum theory is limited [3] led Popescu and Rohrlich to propose nonsignaling correlations which are more nonlocal than quantum correlations [4, 5]. One of the goals of studying generalized nonsignaling probability theories is to find out what physical principles limit quantum nonlocality [6].

Concepts like quantum discord [7–9] and local broadcasting [10] indicate the existence of quantumness even in separable states, and can be associated with the non-commutativity of measurements [11]. It is known that the observation of nonlocality or Einstein-Podolsky-Rosen (EPR) steering also implies the presence of incompatibility of measurements [12]. From an operational perspective, nonlocal or steerable states require augmenting pre-shared randomness with non-zero communication cost [13, 14].

Here we are concerned with the question of how to give such an operational characterization to the quantumness of local correlations arising from non-commuting measurements performed on separable states. By definition, such a box clearly requires zero communication cost.

We partially answer this question by providing evidence that for some such states in the bipartite two-input-two-output Bell scenario, the dimension of the pre-shared randomness required to simulate the box exceeds the dimension of the quantum system generating it. This is a specific case of superlocality [15]. The idea that superlocality occurs even in separable states implicitly finds mention in [16] (cf. in particular, Fig. 3 there). A detailed characterization of superlocality for the bipartite single-input-multiple-output scenario appears in Sec. 4.1 of [17]. Bounds on the quantum dimension required to reproduce Bell correlations in the bipartite multiple-input-multiple-output scenario are discussed in [18].

II. THE POLYTOPE OF NONSIGNALING BOXES

In the formalism of generalized no-signaling theory, bipartite correlations are treated as “boxes” shared between two parties. Let us denote the input variables on Alice’s and Bob’s sides x and y, respectively, and the outputs a and b. We restrict ourselves to the state space in which the boxes have two binary inputs and two binary outputs, i.e., x, y, a, b ∈ {0, 1}. In this case, the state of every box is given by the set of 16 conditional probability distributions $P(ab|A_xB_y)$.

Barrett et. al. [19] showed that the set $\mathcal{N}$ of two-input-two-output non-signaling (NS) boxes forms an 8 dimensional convex polytope with 24 extremal boxes, the 8 Popescu-Rohrlich (PR) boxes:

$$P_{PR}^{\alpha\beta\gamma\epsilon}(ab|A_xB_y) = \begin{cases} \frac{1}{2}, & a \oplus b = x \cdot y \oplus \alpha x \oplus \beta y \oplus \gamma \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

and 16 local-deterministic boxes

$$P_{D}^{\alpha\beta\gamma\epsilon}(ab|A_xB_y) = \begin{cases} 1, & a = \alpha x \oplus \beta \\ b = \gamma y \oplus \epsilon \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

Here, $\alpha, \beta, \gamma, \epsilon \in \{0, 1\}$ and $\oplus$ denotes addition modulo 2. All the deterministic boxes can be written as the product of marginals corresponding to Alice and Bob, $P_D(ab|A_xB_y) = P_D(a|A_x)P_D(b|B_y)$, whereas the 8 PR-boxes cannot be written in product form or even a convex combination over such product boxes. The marginals of the PR boxes are maximally mixed, i.e. $P(a|A_x) = \frac{1}{2} = P(b|B_y)$ for all x, y, a, b.

The polytope $\mathcal{N}$ can be divided into two disjoint sets: the local polytope, which is the convex hull of the 16

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local deterministic boxes (3), and its nonlocal complement. The extremal boxes in a given set are equivalent under “local reversible operations” (LRO). LRO is given by Alice changing her input $x \rightarrow x \oplus 1$, and changing her output conditioned on the input: $a \rightarrow a \oplus \alpha x \oplus \beta$. Bob can perform similar operations.

Fine [20] showed that a box has a LHV model iff it can be written in the above form. A local box satisfies the complete set of Bell-type inequalities [21]. The Bell-CHSH inequality [22] and its symmetries which are given by,

$$B_{αβγ} := (-1)^{γ} (A_0B_0) + (-1)^{β} (A_0B_1) + (-1)^{α} (A_1B_0) + (-1)^{αβγ}(A_1B_1) \leq 2,$$  

form the complete set, where $⟨A_xB_y⟩ = \sum_{xy}(-1)^{αβγ}P(ab|A_xB_y)$. All these tight Bell-type inequalities form the nontrivial facets for the local polytope. All nonlocal boxes, which lie outside the local polytope, violate a Bell-CHSH inequality.

Quantum boxes which belong to the Bell-CHSH scenario [22] are obtained obtained by two dichotomic measurements on bipartite quantum states described by the density matrix $\rho_{AB} ∈ B(ℋ_A ⊗ ℋ_B)$, the set of bounded operators in the joint Hilbert space of the two particles. The Born’s rule predicts the behavior of the quantum operators in the joint Hilbert space of the two particles. The Born’s rule predicts the behavior of the quantum operators in the joint Hilbert space of the two particles.

The noisy PR-box [24] is a mixture of a PR-box and white noise,

$$P = p_{PR}P_{PR} + (1 - p_{PR})P_N,$$  

where $p_{PR}$ is a real number such that $0 \leq p_{PR} \leq 1$, $P_{PR}$ denotes the PR-box $P_{PR}^0$, and $P_N$ is the maximally mixed box, i.e., $P_N(\alpha\beta|A_xB_y) = 1/4$ for all $x,y,a,b$. The noisy PR-box violates the Bell-CHSH inequality, i.e., $E_{000} = 4p_{PR} > 2$ iff $p_{PR} > 1/2$. If $p_{PR} > 1/2$, then $E_{000} > 2√2/2$ in violation of the Tsirelson bound for quantum correlations, and hence is physically prohibited.

We discuss two methods by which the box (6) can be generated quantum mechanically. In both cases, we use the noncommuting measurements $A_0 = \sigma_x$, $A_1 = \sigma_y$, $B_0 = \frac{1}{√2}(σ_x - σ_y)$ and $B_1 = \frac{1}{√2}(σ_x + σ_y)$.

In the first method, the above measurements are applied to the family of pure entangled states:

$$|ψ(θ)⟩ = cos θ |00⟩ + sin θ |11⟩; \quad 0 ≤ θ ≤ π/4,$$  

which produces the noisy CHSH box:

$$P_{C,CHSH} = 2 + (1)^{αβγ}√2C.$$  

Here $C = sin 2θ$, the concurrence [25] of state (7). The above statistics can be written in the noisy PR-box form with $p_{PR} = C/√2$. So, $p_{PR} > 0$ if and only if the state is entangled ($θ > 0$). This holds even when $P$ becomes local ($θ < \frac{π}{4}$).

In the second method to generate the box (6), consider the two-qubit Werner states,

$$ρ_{W} = W|ψ^+⟩⟨ψ^+| + (1 - W)\frac{I}{4},$$  

where $|ψ^+⟩ = \frac{1}{√2}(|00⟩ + |11⟩)$. The above states are entangled iff $W > \frac{1}{2}$ [26] and nonlocal iff $W > \frac{1}{2}\sqrt{2}$. It is known that the Werner states have nonzero quantumness (as quantified by discord) for any $W > 0$ [7]. For the noncommuting measurements that we used above, the Werner state (9) gives rise to the noisy PR-box (6) with $p_{PR} = \frac{W}{2√2}$. Even in the local range $0 < p_{PR} \leq \frac{1}{2}$, the box (6) cannot be reproduced by a pre-sharing just one bit each of classical random correlation. To see this, note that the noisy PR box corresponds to the following correlations:

$$⟨A_0⟩ = ⟨A_1⟩ = ⟨B_0⟩ = ⟨B_1⟩ = 0.$$  

$$⟨A_0B_0⟩ = ⟨A_0B_1⟩ = ⟨A_1B_0⟩ = –⟨A_1B_1⟩ = p_{PR}.\quad (10b)$$  

Quite generally, suppose that the pre-shared bit $λ$ determines the following indeterministic strategy. Alice outputs $a$ conditioned on input $x$ and pre-shared value $λ$ with probability $P_{A,λ}(a|x)$, and similarly Bob outputs $b$ conditioned on input $y$ and $λ$ with probability $P_{B,λ}(b|y)$. The value of $λ ∈ \{0,1\}$ is distributed according to the probability distribution $P_λ(λ)$.
Eq. (10a) implies:

\[
P_{A,0}(0|0)P_{A}(0) + P_{A,1}(0|0)P_{A}(1) = \frac{1}{2} \]

\[
P_{A,0}(0|1)P_{A}(0) + P_{A,1}(0|1)P_{A}(1) = \frac{1}{2} \]

\[
P_{B,0}(0|0)P_{A}(0) + P_{B,1}(0|0)P_{A}(1) = \frac{1}{2} \]

\[
P_{B,0}(0|1)P_{A}(0) + P_{B,1}(0|1)P_{A}(1) = \frac{1}{2} .
\]

Eq. (10b) implies:

\[
P_{A,0}(0|0)P_{B,0}(0|0)P_{A}(0) + P_{A,1}(0|0)P_{B,1}(0|0)P_{A}(1) = \frac{1 + p}{4} \]

\[
P_{A,0}(0|1)P_{B,0}(0|0)P_{A}(0) + P_{A,1}(0|1)P_{B,1}(0|0)P_{A}(1) = \frac{1 + p}{4} \]

\[
P_{A,0}(0|0)P_{B,0}(0|1)P_{A}(0) + P_{A,1}(0|0)P_{B,1}(0|1)P_{A}(1) = \frac{1 + p}{4} \]

\[
P_{A,0}(0|1)P_{B,0}(0|1)P_{A}(0) + P_{A,1}(0|1)P_{B,1}(0|1)P_{A}(1) = \frac{1}{4} .
\]

where we make use of the normalization for the relevant conditioned probabilities.

Now, subtracting the first two equations of (11), we find:

\[
(P_{A,0}(0|0) - P_{A,0}(0|1))P_{A}(0) + (P_{A,1}(0|0) - P_{A,1}(0|1))P_{A}(1) = 0
\]

while subtracting the first two equations of (12), we find:

\[
(P_{A,0}(0|0) - P_{A,0}(0|1))P_{B,0}(0|0)P_{A}(0) + \\
(P_{A,1}(0|0) - P_{A,1}(0|1))P_{B,1}(0|0)P_{A}(1) = 0.
\]

From Eqs. (13) and (14), we determine that \(P_{B,0}(0|0) = P_{B,1}(0|0)\). Plugging this in the first equation of (11), we find \(P_{B,0}(0|0) = P_{B,1}(0|0) = \frac{1}{2}\).

Proceeding thus for other conditional probabilities, \(P_{A,\lambda}(a|x)\) and \(P_{B,\lambda}(b|y)\), we derive their measurement independence on the underlying pre-shared variable, and their value to be \(\frac{1}{2}\). Substituting these values for the conditional probabilities in the first equation of Eq. (12), we find \(p_{\lambda}(0) + p_{\lambda}(1) = 1 + p\), a contradiction for any \(p > 0\). This entails that the dimension of the classical system simulating the noisy PR box must exceed the dimension (two) of the qubit, and is an instance of superlocality [15].

This observation prompts us to operationally identify the nonclassicality of the box (6) with superlocality. Since this characterization of nonclassicality depends only on the box and not how it is generated, our approach gives a general way to approach nonclassicality in local correlations in an arbitrary operational theory. For noisy PR boxes, we identify \(p_{PR}\) as a nonclassicality measure.

IV. ENTROPIC SUPERLOCALITY

In the Bell-CHSH scenario, the noisy local CHSH box (8) can be decomposed as:

\[
P_{\text{CHSH}}^{C=1/\sqrt{2}} = \frac{1}{8} \sum_{\alpha \beta \gamma} P_{D}^{\beta \gamma (\alpha \gamma \beta \gamma)}(ab|xy) \]

\[
= \frac{1}{4} \left( P_{D}^{0000} + P_{D}^{1100} \right) + \frac{1}{4} \left( P_{D}^{0010} + P_{D}^{1110} \right) \]

\[
+ \frac{1}{4} \left( P_{D}^{0101} + P_{D}^{1011} \right) + \frac{1}{4} \left( P_{D}^{0111} + P_{D}^{1011} \right) \]

\[
\equiv \frac{1}{4} \left( \Delta_{1} + \Delta_{2} + \Delta_{3} + \Delta_{4} \right)
\]

\[
= \frac{1}{2} P_{PR}^{00} + \frac{1}{2} P_{N}.
\]

Accordingly, for \(p_{PR} \leq 1/2\), the noisy PR box can be decomposed as follows:

\[
p_{PR} P_{PR}^{00} + (1 - p_{PR}) P_{N} = \frac{2 p_{PR}}{8} \sum_{\alpha \beta \gamma} P_{D}^{\beta \gamma (\alpha \gamma \beta \gamma)}(ab|xy) + (1 - 2 p_{PR}) P_{N},
\]

\[
= \frac{2 p_{PR}}{4} \sum_{j=1}^{4} \Delta_{j}(ab|xy) + (1 - 2 p_{PR}) P_{N}.
\]

\[
= \frac{1}{4} \sum_{\lambda=1}^{4} P_{\lambda}(a|x) P_{\lambda}(b|y).
\]

The expression (19) determines a classical simulation protocol with dimension four, which is known to suffice for local polytope in the Bell-CHSH scenario [15]. We can use (18) to define a classical communication protocol that bounds from above the average pre-shared information required to simulate an noisy PR box.

Assume that we wish to simulate an experiment with \(n\) trials, with sufficiently large \(n\). Alice and Bob pre-share a five-symbol string, say with symbols \(\sigma = 0, 1, 2, 3, 4\), such that the 0’s will appear with probability \(1 - 2p_{PR}\) and determine coordinates where each of them independently outputs unbiased random bits, when given either input. The remaining \(\sigma\) values, uniformly distributed, will determine when they will use one of the above local probabilistic strategies \(\Delta_{j}\).

In other words, we require a Shannon encoding for a source with five symbols determined by the probability distribution \((1 - 2p_{PR}, \frac{p_{PR}}{2}, \frac{p_{PR}}{2}, \frac{p_{PR}}{2}, \frac{p_{PR}}{2})\), where \(p_{PR} \leq \frac{1}{2}\). Therefore, on average, per trial Alice and Bob must pre-share \(l(p_{PR})\) bits, where

\[
l(x) \equiv (1 - 2x) \log(1 - 2x) + 2x \log \left( \frac{x}{2} \right).\]
We find that $l(x) \geq 1$ for $x \gtrsim 0.085$. Therefore, the noisy PR box may be considered, on average, entropically superlocal, and thus nonclassical, in the range $0.085 \lesssim p_{PR} \leq 0.5$.

V. TOWARDS QUANTIFYING SUPERLOCALITY

We now propose a measure to quantify superlocality, which is constructed to work for noisy PR boxes. Later we will discuss its limitations when applied to other boxes. Essentially, we require a quantification of the PR box fraction in a noisy PR box that would be independent of the particular PR box. We call this measure “Bell strength”, because it employs the Bell correlator.

Define the absolute Bell functions

$$B_{2\alpha+\beta} = |\langle A_0 B_0 \rangle + (-1)^\beta \langle A_0 B_1 \rangle + (-1)^\alpha \langle A_1 B_0 \rangle + (-1)^{\alpha+\beta} \langle A_1 B_1 \rangle|.$$ 

We construct the following triad of quantities

$$\Gamma_1 := \tau(B_0, B_1, B_2, B_3)$$
$$\Gamma_2 := \tau(B_0, B_2, B_1, B_3)$$
$$\Gamma_3 := \tau(B_0, B_3, B_1, B_2),$$

and so on. Combinatorially there are 24 possibilities for function $\tau$, but because of the three two-fold symmetries $\tau(a,b,c,d) = \tau(b,a,c,d) = \tau(a,b,d,c) = \tau(c,d,a,b)$, there is an 8-fold redundancy, so that only three terms $\Gamma_j$, as given in Eq. (21), are independent.

Here $\Gamma_j$ are constructed such that it satisfies the following axiomatic properties: (a) $\Gamma_j \geq 0$; (b) $\Gamma_j = 0$ for any $P_D^{\alpha\beta\gamma\epsilon}$; (c) the $\Gamma_j$ attains its maximum (of 4) on PR-boxes.

We define Bell strength as:

$$\Gamma := \min_i \Gamma_i.$$ 

The quantity $\Gamma$ is manifestly LRO invariant. Further, $\Gamma(P) = 4p_{PR}$ for the noisy PR box (6).

Any noisy PR box, $P = p\rho_{PR} + (1-p)\rho_N$ with $p > 0$ has the property that only one of the Bell functions is nonzero. This follows from the fact that $B_j(P_N) = 0$ for any $j$, and the property of monoandry, described below.

Given a no-signaling correlation shared by three particles, monogamy [27] refers to a bound on the sum of the absolute Bell function values for two different pairs of particles, with respect to any fixed Bell operator (say $B_0$). In contrast, monoandry refers to a bound on the sum of the absolute values for two different Bell operators (say $B_0$ and $B_3$), with respect to the same pair of particles.

Any given PR box has a tight association with the Bell functions, in that it takes the value 4 on precisely one of the four absolute Bell functions, and vanishes for the rest. For each of the 16 local deterministic boxes, the absolute Bell function takes the value 2. This leads to the following monoandry relation:

$$B_j(P) + B_k(P) \leq 4,$$ 

where $j \neq k$ and $j,k \in \{0,1,2,3\}$. To prove this, let $B_j^*$ denote $B_j$ without taking the absolute value. Consider the decomposition $P = \sum_{j=0}^3 G_j P_{PR}^{j\pm} + (1-G) L_{Bell}$, where $G = \sum_j G_j$, and $P_{PR}^{j\pm}$ is precisely one of the PR box/antibox pair $(P_{PR}^+, P_{PR}^-)$ such that $B_j^* (P_{PR}^{j\pm}) = \pm 4$. This decomposition always holds, since an equal mixture a PR box and its antibox is the maximally mixed state $P_N$, which can be included in $L_{Bell}$, the local box.

Consider any two distinct $B_j$’s. Without loss of generality, let these correspond to $j = 0, 3$. We then have:

$$B_0^*(P) = \pm 4G_0 \pm (1-G)B_0^*(L_{Bell}),$$

which implies that

$$B_0(P) \leq 4G_0 + 2(1-G).$$

Similarly,

$$B_3(P) \leq 4G_3 + 2(1-G),$$

from which it follows that

$$B_0(P) + B_3(P) \leq 4(G_0 + G_3) + 4(1-G) \leq 4,$$

since $G_0 + G_3 \leq G$. Clearly, this holds for any distinct pair $j, k$ in Eq. (24).

A tighter version of monoandry as applicable to quantum boxes is reported in Ref. [28], but the bound (24) is applicable to any two-input-two-output nonsignaling boxes, and is suitable for our purpose. Since $0 \leq B_j \leq 4$, monoandry (24) implies that $-4 \leq B_j(P) - B_k(P) \leq 4$, from which it follows that $|B_j(P) - B_k(P)| \leq 4$. Therefore, $0 \leq \Gamma \leq 4$.

If a box $P$ is nonlocal, then there is a $j$ such that $B_j(P) > 2$. By monoandry, $B_k(P) < 2$ for all $k \neq j$, so that none of the $\Gamma_j$ will vanish in Eq. (21). Therefore, $\Gamma(P) > 0$, entailing that boxes with vanishing $\Gamma$ are necessarily local. But not all local boxes satisfy $\Gamma = 0$. This arises from the fact that the set of boxes characterized by the property $\Gamma = 0$ is not convex, unlike the local polytope. In particular, the convex property

$$\Gamma \left( \sum_j p_j P_j \right) \leq \sum_j p_j \Gamma(P_j),$$

fails when the correlations $P_j$ in Eq. (29) correspond to the local-deterministic boxes. For these, $\Gamma = 0$, but then even local boxes arising from noncommuting measurements have nonvanishing $\Gamma$, according to the following result.
Theorem 1. Locally commuting projective measurements entail that $\Gamma = 0$ for any two-qubit state.

Proof. Any two-qubit state, up to local unitary equivalence, can be represented as [29]

$$\rho_{AB} = \frac{1}{4}(\mathbb{1}_A \otimes \mathbb{1}_B + \vec{r} \cdot \vec{\sigma} \otimes \mathbb{1}_B + \mathbb{1}_A \otimes \vec{s} \cdot \vec{\sigma}$$

$$+ \sum_{i=1}^{3} c_i \sigma_i \otimes \sigma_i)$$

(30)

where the coefficients $c_i = \text{Tr}(\rho_{AB} \sigma_i \otimes \sigma_i)$, $i = x, y, z$, form a diagonal matrix denoted by $C$. Here, $|\vec{r}|^2 + |\vec{s}|^2 + ||C||^2 \leq 3$ with equality holding for the pure states. The expectation value of the above states is given by,

$$(A_x B_y) = \hat{a}_x \cdot C \hat{b}_y.$$  

(31)

Let us calculate $\Gamma$ for the states as given in Eq. (30) for commuting measurements on Alice’s side. Suppose we choose measurement directions as $\hat{a}_0 = \hat{a}_1 = \hat{a}$, then the measurement observables commute, i.e., $[A_0, A_1] = 0$. For this choice of commuting measurements on Alice’s side, the state in Eq. (30) has $B_0 = B_1 = 2\hat{a}_0 \cdot C \hat{b}_0$, and, $B_2 = B_3 = 2\hat{a}_0 \cdot C \hat{b}_1$. These values imply that $\Gamma = 0$ for any choice of commuting measurements on Alice’s side and any choice of commuting/non-commuting measurements on Bob’s side. \hfill \Box

The above result means that there exist product boxes (characterized by vanishing pre-shared dimension) that have $\Gamma > 0$. Therefore, if one goes beyond noisy PR boxes then nonvanishing $\Gamma$ does not entail superlocality. Such nonzero $\Gamma$ product boxes do have quantumness due to noncommuting measurements, which leads to local randomness, but this nonclassicality is not pointed out by superlocality.

VI. CONCLUSIONS AND DISCUSSION

Nonlocality or steerability in the given correlations (or, box) in quantum mechanics or in an arbitrary convex operational theory can be characterized in terms of the communication cost that must supplement pre-shared randomness in order to simulate it. The question of an analogous characterization of nonclassicality arising from separable states is addressed here, and associated with superlocality.

However, it should be pointed out that the quantumness indicated by superlocality does not detect all quantum discord states. In particular, consider classical-quantum or quantum-classical states [10], which have the form:

$$\rho_{AB} = \sum_{j=0,1} p_j |j\rangle \langle j| \otimes \rho_j.$$  

(32)

In the Bell-CHSH scenario, for Alice measuring in basis $\{|j\rangle\}$, it is clear that the resulting box can be simulating by a probabilistic strategy using dimension 2. This observation holds even when Alice measures in any other basis (except that her random number generator will be possibly be more randomized).

It follows that zero-discord states, i.e. classical-classical correlations (corresponding to orthogonal $\rho_j$, in Eq. (32)), are also non-superlocal. Therefore, superlocal states are a subset of states with quantum-quantum correlations, and thus a strict subset of discordant states. This suggests that superlocality does not encompass all of the nonclassicality in local quantum states.

As our approach applies to boxes rather than specifically to quantum states, it leads in a natural way to nonclassicality in bipartite states in an arbitrary convex operational theory. These consideration can be extended to tripartite [30] and multipartite boxes. In this context, Refs. [31, 32] associate nonclassicality with the nonsimpliciality of the state space $\Sigma$ of such boxes in a probability theory. Here our criterion for nonclassicality, as indicated by superlocality, applies to individual boxes, rather than $\Sigma$.

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