Soundness in Object-centric Workflow Petri Nets

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Abstract. Recently introduced Petri net-based formalisms advocate the importance of proper representation and management of case objects as well as their co-evolution. In this work we build on top of one of such formalisms and introduce the notion of soundness for it. We demonstrate that for nets with non-deterministic synchronization between case objects, the soundness problem is decidable.

1 Introduction

Traditional workflow nets often focus on a single case in isolation. However, in reality, the notion of a case is often more complex and may consist of multiple simultaneously “active” objects with complex inter-relations. This issue has been already recognized and addressed in various works. Among such, the seminal work on workflow patterns [6] with the related multiple instance patterns, works on proclists [2,3], in which the authors studied interactions between multiple workflows.

When it comes to analyzing such nets, it is often assumed that, even in the presence of multiple, simultaneously active process instances each of which has its own lifecycle, the global system state is composed of their individual (local) states that, in turn, are rarely addressed in isolation. This becomes evident when analyzing the process correctness using the concept of soundness. In the nutshell, the process is called sound if and only if any reachable process state, starting from its initial configuration, can properly terminate by reaching the final process configuration without leaving any intermediate process resources unused. Such soundness requires that the system model has clearly defined start and end points, and focuses only on the global termination. This also implies that the model must have a clearly defined “global” lifecycle, forcing all the lifecycles it embeds to synchronize on the aforementioned global start and end points.

This assumption is quite common in practice, but it cannot capture a more complex setting in which, instead of the global lifecycle, one models an environment or a protocol without clearly defined global start and end points, and with process instances whose own lifecycles are treated as “first class citizens” (and thus their local termination comes to the fore). An example of such setting can be easily found in higher education, where process models can represent the whole educational lifecycle of a student of more refined scenarios such as work on a project for some course. Notice that such models can include multiple, possibly interacting participants (that is, active cases) and one might be interested in the proper completion only of one of them. Like that the concept of the
global state becomes redundant as each single case can be treated in isolation. All this calls for a new notion of soundness that is not based on global, terminal states.

In this work, we address the aforementioned scenario by using the recently proposed formalism of object-centric Petri nets [4] — a weak variant of colored Petri nets enriched with a special kind of transfer arcs. Using such nets, one can easily represent lifecycles of multiple objects and define their local start and end points. By drawing inspiration from multi-agent systems, where each agent is represented with a separate workflow and its successful completion in isolation can happen irrespectively from other workflows, we introduce a new notion of soundness. There, we focus on local terminal states only, assuming that a system can have any number of participants being simultaneously active. We propose constructive techniques for isolating behaviors of interest without alternating the global process behavior and introduce a new notion of soundness that focuses on an individual object lifecycle, treating all other objects as its environment. Moreover, we conjecture that checking this new variant of soundness is decidable.

2 Motivating example

We start by discussing an example that illustrates issues arising when one wants to model an environment consisting of multiple lifecycles and touches upon main ideas that we develop later on in our proposal. The example in Figure 1 models a project work scenario in a university. There, all three workflow nets model behaviors of three different entities (or objects): a project team leader, a project team member and a project. Both project team leaders and members are students. Projects are performed in small teams each of which consists of three students: one team leader and two team members.

In Figure 1, separate entities are modeled using ordinary WF-nets. Figure 1a shows a workflow of a team member. Figure 1b illustrates how a team leader behaves. Project itself has its own prescribed sequence of key milestones. These make up a process workflow that is shown in Figure 1c. Let us consider behavior of separate entities in more detail.

We will begin from the project workflow shown in Figure 1c. Firstly, the project has to be created. It is done by a student who was chosen to be a team leader. Student can start working on a project if there are three members in their team. Thus, two team members have to join to as soon as the team was created. After this the students can start working on their project (this decision is taken by the team leader). From project point of view, the team has to submit part of the solution to supporting learning environment at least once. There can be multiple submissions as well. Then, the project finishes either by successful completion or by failure.

Let us consider now how student members behave. One may notice similarities between models in Figure 1a and Figure 1b. A team leader (see Figure 1b) creates the project, then starts it. After that each leader attends the initial meeting and, at any time, can discuss with other team members the project. Any submission of each team need to be approved by the team leader. The final point of each project is its presentation that can be attended by all the students participating to the project. The project is evaluated based on the presentation, which may be found unsatisfactory. In that case, the presentation can be repeated. Otherwise, the project is be evaluated as
completely failed or successfully completed. A regular team member (see Figure 1a) joins a team as first or second team member. Then, they participate to the same project-related activities as team leaders do.

Notice these models are basic workflow nets. In Figure 1a and Figure 1b black tokens in places represent a student’s state, whereas in Figure 1c tokens account for a project’s state.

These three models show behavior of separate entities participating to the same scenario. There are a lot of activities performed by several scenario participants together. For example, activity submit m. is performed in all three workflows: a student submits the work, a team leader approves this submission, a project changes its (progress) state. Three students (leader and two members) always need to perform the initial meeting, whereas discussions can be performed by two or more students.

The question is how to model the whole process with all interactions between its participants. This can be done using the formalism of object-centric Petri nets introduced in 4. Figure 2 shows how student and project workflows can be merged in the single object-centric workflow net. To combine separate nets we merge transitions with same labels from all workflow nets. It is important that we use one project net, one leader net, and two member nets. This leads to weights on some of arcs in the merged
OC WF-net. Moreover, discussions can be started and finished by an arbitrary number of students. We use special variable arcs to model this fact. Now we have a single model which represents the whole learning process with synchronous communications.

1 Upon transition firing, a variable arc transfers all the tokens from the input place it is connected to into the output place of the same type that is connected with another variable arc.

Fig. 2: A Petri net model representing the university project scenario
between process agents. Note that asynchronous communications can always be modeled via synchronous mechanisms.

In this paper, we address the problem of OC WF-net soundness. It is known how to define and to check soundness of basic WF-nets. For example, we can state that separate participants of our learning process from Figure 1 are modeled by sound WF models. But what can we say about soundness (correctness) of the OC WF-net that is shown in Figure 2? And how soundness of agent models can be related to soundness of the OC WF-net obtained by their merge?

3 Preliminaries

Multisets, Petri nets. Let us start with fixing some standard notions related to multisets. Given a finite set \( B \), a multiset \( m \) over \( B \) is the mapping of the form \( m : B \rightarrow \mathbb{N} \). Given an element \( b \in B \), \( m(b) \in \mathbb{N} \) denotes the number of times \( b \) appears in the multiset. We write \( b^n \) if \( m(b) = n \). Given two multisets \( m_1 \) and \( m_2 \) over \( B \):

(i) \( m_1 \subseteq m_2 \) (resp., \( m_1 < m_2 \)) iff \( m_1(b) \leq m_2(b) \) (resp., \( m_1(b) < m_2(b) \)) for each \( b \in B \);

(ii) \( (m_1 + m_2)(b) = m_1(b) + m_2(b) \);

(iii) \( m_1 \subseteq m_2, (m_2 - m_1)(b) = m_2(b) - m_1(b) \);

(iv) \( |m| = \sum_{b \in B} m(b) \).

We use \( B^\omega \) to denote the set of all finite multisets over \( B \).

Given two disjoint sets \( P \) and \( T \) of places, multiset \( F : (P \times T) \cup (T \times P) \rightarrow \mathbb{N} \) and function \( \ell : T \rightarrow A \cup \{\tau\} \), where \( A \) is a finite set of activity labels and \( \tau \) is a special symbol denoting silent transitions, we call \( N = (P,T,F,\ell,A) \) a labeled place/transition net (or a labeled P/T-net).

The notions of marking, transition enablement and firing for classical P/T nets are defined as usual. We do not give precise definitions here, since we will not use them explicitly in what follows.

Transition systems, bisimulation equivalence. Given a finite set of action labels \( A \) together with a special silent label \( \tau \), a (labeled) transition system (LTS) is a tuple \( \Gamma = (S,A,s_0,\rightarrow) \), where \( S \) is a set of states, \( s_0 \) is an initial state and \( \rightarrow \subset (S \times A \times S) \) is a transition relation. In what follows, we shall write \( s \overset{a}{\rightarrow} s' \) instead of \( (s,a,s') \in \rightarrow \).

Definition 1 (Strong bisimulation). Let \( \Gamma_1 = (S_1,A,s_{01},\rightarrow_1) \) and \( \Gamma_2 = (S_2,A,s_{02},\rightarrow_2) \) be two labeled transition systems. Then relation \( R \subset (S_1 \times S_2) \) is called a strong bisimulation between \( \Gamma_1 \) and \( \Gamma_2 \) iff for every pair \((p,q) \in R \) and \( a \in A \) the following holds:

(i) if \( p \overset{a}{\rightarrow_1} p' \), then there exists \( q' \in S_2 \) such that \( q \overset{a}{\rightarrow_2} q' \) and \((p',q') \in R \);

(ii) if \( q \overset{a}{\rightarrow_2} q' \), then there exists \( p' \in S_1 \) such that \( p \overset{a}{\rightarrow_1} p' \) and \((p',q') \in R \).

Given \( a \in A \), \( p \overset{a}{\rightarrow} q \) denotes a weak transition relation that is defined as follows:

- \( p \overset{a}{\rightarrow} q \) iff \( p(\overset{a}{\rightarrow})^* q \); \( a \overset{a}{\rightarrow} q_2(\overset{a}{\rightarrow})^* q \);

- \( p \overset{\tau}{\rightarrow} q \) iff \( p(\overset{\tau}{\rightarrow})^* q \).

Here, \((\overset{a}{\rightarrow})^* \) denotes the reflexive and transitive closure of \( \overset{a}{\rightarrow} \).
Definition 2 (Weak bisimulation). Let $\Gamma_1 = (S_1, A, s_{01}, \rightarrow_1)$ and $\Gamma_2 = (S_2, A, s_{02}, \rightarrow_2)$ be two labeled transition systems. Then relation $R \subseteq (S_1 \times S_2)$ is called a weak bisimulation between $\Gamma_1$ and $\Gamma_2$ iff for every pair $(p, q) \in R$ and $a \in A$ the following holds:

(i) if $p \xrightarrow{a}_{\Gamma_1} p'$, then there exists $q' \in S_2$ such that $q \xrightarrow{a}_{\Gamma_2} q'$ and $(p', q') \in R$;
(ii) if $q \xrightarrow{a}_{\Gamma_2} q'$, then there exists $p' \in S_1$ such that $p \xrightarrow{a}_{\Gamma_1} p'$ and $(p', q') \in R$.

We say that a state $p \in S_1$ is strongly (weakly) bisimilar to $q \in S_2$, written $p \sim q$ (correspondingly, $p \approx q$), if there exists a strong (weak) bisimulation $R$ between $\Gamma_1$ and $\Gamma_2$ such that $(p, q) \in R$. Finally, $\Gamma_1$ is said to be strongly (weakly) bisimilar to $\Gamma_2$, written $\Gamma_1 \sim \Gamma_2$ (correspondingly, $\Gamma_1 \approx \Gamma_2$), if $s_{01} \sim s_{02}$ ($s_{01} \approx s_{02}$).

A Workflow net is a labeled P/T net with two special places: $\text{in}$ and $\text{out}$. These places are used to mark the beginning and the ending of the workflow process.

Definition 3 (Workflow net). A labeled P/T net $N = (P, T, F, \ell, A)$ is called a workflow net (WF-net) iff:

1. There is one source place $\text{in} \in P$ and one sink place $\text{out} \in P$, s.t. $\text{in} = \text{out} = \emptyset$.
2. Every node from $P \cup T$ is on a path from $\text{in}$ to $\text{out}$.
3. The initial marking in $N$ contains the only token in its source place and is denoted by $[\text{in}]$.

The marking with the only token in the sink place $\text{out}$ is called final and is denoted by $[\text{out}]$.

4 Object-centric Petri nets

In this section, we formally define the class of object-aware workflow Petri nets, called Object-Centric Petri nets (OC-nets). Specifically, we partially adopt the formalism of object Petri nets presented in [4], which was introduced as a target model for multi-perspective process mining.

OC-nets can be thought of as a rather limited version of colored Petri nets (CPNs), where each color type contains exactly one color. Thus, the color types coincide with the colors, tokens in the place labeled with the color type are indistinguishable from each other, in which case we do not need variables in arc expressions, but only the natural numbers denoting the multiplicity of the arc, as in P/T-nets.

By analogy with [4], colors in OC-nets are called object types. We denote the set of object types as $O$, and reserve $E$ to denote a special “empty” type. As in [4], OC-nets also allow to use variable arcs — a special type of arcs for simultaneous transferring of a non-deterministic amount of tokens from one place to another.

Definition 4 (Object-centric net). An OC-net $ON$ is a tuple $(O, P, T, \ell, \text{type}, A)$, where:

1. $P$ and $T$ are finite sets of places and transitions, s.t. $P \cap T = \emptyset$;
2. $\text{type} : P \rightarrow O$ is a place typing function;
3. $F : E \rightarrow \mathbb{N} \cup \{\mu\}$ is a multiset of flow arcs, where $E = (P \times T) \cup (T \times P)$ and $\mu$ is used to indicate variable arcs;
4. for every \( p \in P \) and \( t \in T \), if \( F(t, p) = \mu \), then
   - there is exactly one \( p' \in P \) s.t. \( F(p', t) = \mu \) and \( \text{type}(p) = \text{type}(p') \), and
   - there is no \( p'' \in P \) s.t. \( \text{type}(p) = \text{type}(p'') \), \( F(p'', t) > 0 \) or \( F(t, p'') > 0 \);

5. \( A \) is a finite set of activity names, and \( \ell : T \to A \cup \{ \tau \} \) is a function which maps transitions to activities with \( \tau \) denoting a silent (invisible) activity.

The meaning of variable arcs in OC-nets remains in line with \cite{4}: they are meant to model the transfer of multiple objects from one place to another. The condition 4 in the above definition is similar to the consistent variability condition from \cite{4} stipulating that for any transition \( t \) and any object type \( d \) there can be no more than one pair of ingoing and outgoing for \( t \) variable arcs adjacent to places of type \( d \). For ease of notation, we denote with \( T_\mu \) the set of all transitions \( t \in T \) s.t. \( F(t, p) = \mu \), for some \( p \in P \).

Since the tokens residing in the same place cannot be distinguished, the OC-net marking is defined similarly to the marking for P/T-nets. More specifically, a marking of OC-net \( ON = (\mathcal{O}, P, T, F, \text{type}) \) is a function \( M : P \to \mathbb{N} \). When \( M(p) = n \) and \( n > 0 \) for some \( p \in P \), we say that there are \( n \) objects of type \( \text{type}(p) \) in state \( M \). We write \( \langle ON, M \rangle \) to denote OC-net \( ON \) marked with \( M \) and use symbol \( M_0 \) to define the initial marking of the net. For ease of notation, we also denote with \([p_1^n, \ldots, p_k^n]\) a concrete multiset representing a marking in which each place \( p_k \) contains \( k \) tokens. Finally, with \( M_P \) we denote the countably infinite set of all possible markings defined on top of \( P \).

Let us now specify the net dynamics. As customary, given \( x \in P \cup T \), we use \(*_x := \{ x \mid F(\_x, x) > 0 \text{ or } F(\_x, x) = \mu \} \) to denote the \textit{preset} of \( x \) and \( x^* := \{ x \mid F(x, \_x) > 0 \text{ or } F(x, \_x) = \mu \} \) to denote the \textit{postset} of \( x \).

Since a transition can have multiple incoming and outgoing variable arcs of different types, we would like to be able to explicitly identify how many tokens from each place adjacent to the variable arc are transferred by the transition firing. To this end, we introduce a function \( \alpha : \mathcal{O} \to \mathbb{N} \) specifying the transition’s \textit{transfer mode}. Note that by Definition \cite{4} a transition \( t \) in an OC-net cannot have two transitions \( p_1, p_2 \in *t \) s.t. \( F(p_1, t) = F(p_2, t) = \mu \). Thus, \( \alpha \) does not create ambiguities by relating to object types only.

We then say that transition \( t \) is \textit{enabled} in marking \( M \) with transfer mode \( \alpha \), written \( M[t, \alpha] \), iff, for every \( p \in *t \), the following conditions hold:

\( i) \ F(p, t) \leq M(p), \text{ if } F(p, t) \in \mathbb{N} \), and
\( ii) \ M(p) \geq \alpha(\text{type}(p)), \text{ if } F(p, t) = \mu. \)

When \( t \) is enabled in marking \( M \) with transfer mode \( \alpha \), it may \textit{fire}, yielding new marking \( M' \) that is defined for every \( p \in P \) as follows:

\[
M'(p) = \begin{cases} 
M(p) - \alpha(\text{type}(p)), & \text{if } F(p, t) = \mu \\
M(p) + \alpha(\text{type}(p)), & \text{if } F(t, p) = \mu \\
M(p) - F(p, t) + F(t, p), & \text{otherwise}
\end{cases}
\]

We denote this as \( M[t, \alpha, M'] \) and assume that the definition is inductively extended to sequences \( \sigma \in (T \times \mathcal{O} \to \mathbb{N})^* \) of transition firings. We say that \( M' \) is \textit{reachable} from
Algorithm 1 Variable arc elimination

Input: \((\text{ON}, M_0)\), where \(\text{ON} = (\emptyset, P, T, F, \text{type}, \ell, A)\), s.t. for some \(t \in T\) and \(p \in P\) it holds that \(F(t, p) = \mu\)

Output: \((\text{ON}', M'_0)\), where \(\text{ON}' = (\emptyset', P', T', F', \text{type}', \ell', A)\), s.t. for all \(t \in T'\) and \(p \in P'\) it holds that \(F'(t, p) \neq \mu\)

\[\begin{align*}
\emptyset' &= \emptyset \cup \{\emptyset\} \\
P' &= P \cup \{\text{lock}\} \\
T' &= T \\
\text{for all } t \in T_{\mu} \text{ do} \\
    P' &= P' \cup \{\text{lock}_t\} \\
    T' &= T' \cup \{\text{start}_t, \text{add}_t\} \\
\text{end for} \\
\text{for all } p, p' \in P', t \in T' \text{ do} \\
    k &= \text{if } F(p, t) = k \\
    1 &= \text{if } (p, t) = (\text{lock}, t) \\
    F'(p, t) &= \begin{cases} 
        k, & \text{if } F(p, t) = k \\
        1, & \text{if } (p, t) = (\text{lock}, t) \\
    \end{cases} \\
    \text{if } F(p, t) = F(t, p') = \mu \text{ and } (p, t) \in \{(p, \text{start}_t), (\text{lock}, \text{start}_t), (\text{lock}_t, t), (p', t)\} \\
    \text{end for}
\end{align*}\]

\[\begin{align*}
\text{type}'(p) &= \begin{cases} 
    \text{type}(p), & \text{if } p \in P \\
    \emptyset, & \text{otherwise} \\
\end{cases} \\
\ell'(t) &= \begin{cases} 
    \ell(t), & \text{if } t \in T \\
    \tau, & \text{otherwise} \\
\end{cases} \\
M'_0 &= \begin{cases} 
    M_0(p), & \text{if } p \in P \\
    1, & \text{if } p = \text{lock} \\
    0, & \text{otherwise} \\
\end{cases}
\end{align*}\]

\(M\) if there exists \(\sigma \in (T \times \emptyset \rightarrow \mathbb{N})^*\), s.t. \(M[\sigma]M'\). For an OC-net \(\text{ON}\), we write \(\mathcal{R}(\text{ON}, M_0)\) to denote the set of all markings reachable from its initial marking \(M_0\).

The execution semantics of an OC-net can be captured with a possibly infinite-state LTS that accounts for all possible executions starting from OC-net’s initial marking. Formally, an OC-net \(\text{ON} = (\emptyset, P, T, F, \text{type}, \ell, A)\) with initial marking \(M_0\) induces a labeled transition system \(\Gamma_{\text{ON}} = (S, A, s_0, \rightarrow)\), where:

- \(S = \mathcal{R}(\text{ON}, M_0)\) and \(s_0 = M_0\),
- for \(M, M' \in S\) it holds that: \(M \xrightarrow{a} M'\) iff \(M[t, \alpha]M'\), for some \(t \in T\) s.t. \(\ell(t) = a\) and some transfer mode \(\alpha\).

Now, after we have defined the syntax and semantics of OC-nets, we would like to observe that every OC-net \(\text{ON}\) can be represented with OC-net \(\text{ON}'\) that does not contain variable arcs. To this end, we provide a variable arc elimination algorithm and show that its output is always weakly bisimilar to its input. The weak bisimilarity is conditioned by the fact that in order to correctly represent the behavior of \(\text{ON}\), \(\text{ON}'\)
must include additional intermediate steps that are, however, invisible and therefore not relevant to the comparison of net behavior. That is why we need to use a form of bisimulation allowing to “skip” transitions irrelevant for the behavioral comparison [?]. Finally, we say that two marked OC-nets \( \langle ON, M_0 \rangle \) and \( \langle ON', M'_0 \rangle \) are weakly bisimilar (and denote it as \( \langle ON, M_0 \rangle \approx \langle ON', M'_0 \rangle \)) if for transition systems \( \Gamma_{ON} \) and \( \Gamma_{ON'} \) they respectively induce it holds that \( \Gamma_{ON} \approx \Gamma_{ON'} \).

Given a generic OC-net, Algorithm 1 allows to construct from it a behaviorally equivalent OC-net that has no variable arcs. For example, consider a simple net in Figure 3a. By applying the above algorithm to it, we obtain a new OC-net (see Figure 3b) that has no variable arcs. As it has been mentioned before, the algorithm produces a net that behaves exactly like the input one. Indeed, the net in Figure 3b models removed variable arcs with a sub-net representing a “lossy” transfer, working under the assumption that at least one token gets transferred from \( in \) to \( out \) (this assumption is in line with Definition 4). First, using \( start_t \), the net enters into a critical section that is guarded by the special lock place \( lock \). Notice that this lock is global to the whole net and is used to guard firing of every transition from the original net. When the net acquires the lock, it can “transfer” tokens from \( p_1 \) to \( p_2 \) using transition \( add_t \). As soon as at least one token has been generated in \( p_2 \), the transfer can be finalized by firing \( t_1 \) that releases the global lock. This example leads us to the following statement.

**Proposition 1.** Let \( \langle ON, M_0 \rangle \) be a marked OC-net, where \( ON = (O, P, T, F, type, \ell, A) \). Then we can effectively construct a marked OC-net \( \langle ON', M'_0 \rangle \), such that \( ON' \) has no variable arcs and \( \langle ON, M_0 \rangle \approx \langle ON', M'_0 \rangle \).  

Recall that OC-nets specify a class of nets that can be used to model the behavior of multiple, possibly interacting, objects of different types within one system. Notice that the formalism of OC-nets does not allow you to distinguish between objects of the same type. However, at the same time, one can focus on the lifecycle represented by one type of objects. Unlike the life-cycle of a single object, the lifecycle of an object type can encompass various possible behaviours for objects of that type. Next, we introduce a definition of a projection that supports retrieving a subnet based on a given object type.

Let \( ON = (O, P, T, F, type, \ell, A) \) be an OC-net, \( M \) be a marking in \( ON, d \in O \). In what follows, we will make use of the following notations for \( d \)-based restrictions of OC-net components: \( P|_d = \{ p \mid p \in P, \ type(p) = d \} \), \( T|_d = \{ t \mid t \in T, \ there \ is \ p \in \)
\*t \cup \*t \text{ s.t. } \text{type}(p) = d\}, \ F|_a = F \cap (P|_a \times T|_a) \cup (T|_a \times P|_a) \ 
\text{type}|_a = \text{type}|_{P|_a}, \\
\ell|_a = \ell|_{T|_a} \text{ and } M|_a = M|_{P|_a}.

**Definition 5 (Object type projection).** A d-typed projection of an OC-net \( ON = (\Box, P, T, F, \text{type}, \ell, A) \) is the OC-net \((\{d\}, P|_a, T|_a, F|_a, \text{type}|_a, \ell|_a, A)\). We denote the projection as \( ON|_d \).

Since all places in an object-type projection of an OC-net are labeled with the same and only type, this labeling can be omitted, and the object projection can be considered as a P/T-net. Then, abusing the notation, we denote both the d-typed projection of OC-net \( ON \) and the corresponding P/T-net by \( ON|_d \).

Finally, let \( d' \notin \Box \) be a ‘fresh’ name for a data type. By \( ON|_{d/d'} \) we denote the OC-net \( ON' = (\Box', P', T', F', \text{type}', \ell', A) \) such that:

- graphs \((P, T, F)\) and \((P', T', F')\) are isomorphic via some isomorphism function \( \iota \),
- \( \Box' = (\Box \setminus \{d\}) \cup \{d'\} \),
- for \( p \in P \), \( \text{type}'(\iota(p)) = d' \) if \( \text{type}(p) = d \), otherwise \( \text{type}'(\iota(p)) = \text{type}(p) \),
- for \( t \in T \), \( \ell'(\iota(t)) = \ell(t) \).

In other words, \( ON|_{d/d'} \) is a copy of \( ON \) in which \( d \) is replaced with \( d' \).

5 Object-centric Workflows

In this section, we show how object-centric nets can be used to represent workflows of multiple, possibly interacting, artifacts of different types.

A classical, untyped, workflow net is used to represent workflows or lifecycles of one single object (or case) in isolation. There are also generalized workflow nets [16], where the initial state contains several indistinguishable objects (black dots) in the source place, and the final state contains all these objects in the sink place, so that a workflow run terminates when all items initially residing in the source place are processed and located in the sink place.

Next, we will show how to extend the concept of a workflow net to cover multiple objects of different types. For this we use OC-nets, in which a typed place is a location for tokens of one type. In OC workflow nets, tokens of a specific type will be processed according to a specific workflow, but can interact with each other and with tokens of other types. Another important feature is that new objects can appear in source places at any moment. Thus, while it is assumed that the proper behavior of each object terminates, the system represented by the OC workflow net can run infinitely long.

**Definition 6.** An OC-net \( ON = (\Box, P, T, F, \text{type}, \ell, A) \) is called an object-centric workflow net (or OC WF-net for short), iff:

1. for every \( d \in \Box \), \( ON|_d \) is a WF-net;
2. for every transition \( t \in T \), \( \*t \cup \*t \neq \emptyset \), i.e. there are no isolated transitions in \( ON \).

Thus, a classical WF-net is the OC WF-net with exactly one object type, and OC WF-nets form true extension of WF-nets.
For OC WF-net \( ON = (\emptyset, P, T, F, \text{type}, \ell, A) \), we use the notation \( \text{in}_d \in P \) and \( \text{out}_d \in P \) for its (only) source and sink places of type \( d \), i.e. \( \text{type}(\text{in}_d) = \text{type}(\text{out}_d) = d \) and \( \bullet \text{in}_d = \text{out}_d^* = \emptyset \). By \( P_{\text{in}} \) and \( P_{\text{out}} \), we denote the sets of all source and sink places, respectively.

For OC WF-nets, we allow several tokens in each of its source places for an initial marking, not restricting the consideration to a workflow for just one case and just one initial marking. We also do not suppose that there are some fixed final markings for OC WF-nets and that reaching a final marking means termination of the system.

The synchronous composition of two nets \( N_1 \) and \( N_2 \) is a net obtained by pairwise merging of transitions in \( N_1 \) with transitions with the same labels in \( N_2 \). Fig. 4 gives an example of a synchronous composition of two nets. Note that though both components in Fig. 4a are simple and sound WF-nets, their synchronous composition is dead in its initial marking, provided the initial marking is the “canonical” one (that is, only input places have tokens assigned to them).

It is easy to note that every OC WF-net \( ON = (\emptyset, P, T, F, \text{type}, \ell, A) \) is a synchronous composition of all its \( d \)-projections \( N|_d \), where \( d \in \emptyset \), and \( d \)-projection \( N|_d \) represents the lifecycle for objects of type \( d \).

So far, we have been considering nets allowing to model co-evolution of multiple objects of various object types. The main limitation of the OC-net formalism is that the identity of such objects is always hidden: one can only see how many anonymous objects of the same type are stored in a place. Yet, in certain scenarios it might be still useful to track a single object. This tracking can be achieved by, first, isolating the behavior of the object and then carefully merging the extracted subnet with the original net. To this end, we first define a new, more restrictive type of projection that, instead of focusing on all possible behaviors of a single object type, captures behavior of one object.

**Definition 7 (1-safe projection).** Let \( ON = (\emptyset, P, T, F, \text{type}, \ell, A) \) be an OC-net, \( d \in \emptyset \). OC-net \( ON' \) is called 1-safe \( d \)-typed projection of OC-net \( ON \) (denoted \( ON|_d^{1} \)) iff \( ON' \) coincides with \( ON|_d^{1} \) except the arc multiplicity function, which takes the value 1 throughout its domain. Namely, \( ON|_d^{1} = (\{d\}, P|_d, T|_d, F|_d^{1}, \text{type}|_d, \ell|_d, A) \), where \( F|_d^{1} (x, y) = 1 \), if \( F|_d (x, y) \neq 0 \), and \( F|_d^{1} (x, y) = 0 \), otherwise. \( \triangleright \)
Now, using 1-safe projection, we can construct a net that can track a single object evolution as well as its potential interactions with other objects of the same or any other type in the net.

**Definition 8 (Tracking OC WF-net).** Let \( ON = (O, P, T, F, \text{type}, \ell, A) \) be a OC WF-net and \( ON' = (\{d\}', P', T', F', \text{type}', \ell', A) \), such that \( ON' = (ON|_{d'})_{d/d'} \). Then its \((d, d')\)-typed tracking extension, denoted as \( ON_{d+d'} \), is the OC WF-net \((\emptyset \cup \{d\}', P \cup P', T \cup T' \cup T'', F'', \text{type} \cup \text{type}', \ell'', A)\) where:

- \( T'' = \{t'' \mid t \in T_1 \cup T_2 \} \), where \( T_1 = (\bigcup_{p \in P \cap \ell} p) \setminus T' \) and \( T_2 = T' \cap T_p \):
  
  \[
  F'(x, y), \quad \text{if} \quad (x, y) \in (P \times T) \cup (T \times P) \\
  F'(x, y), \quad \text{if} \quad (x, y) \in (P' \times T') \cup (T' \times P') \\
  F(x, t) - 1, \quad \text{if} \quad (x, y) \in (P \times T'), t \in T \\
  F(t, x) - 1, \quad \text{if} \quad (x, y) \in (T' \times P), t \in T \\
  F(t, x), \quad \text{if} \quad (x, y) \in (P \times (T_1 \cup T_2)), t \in T \\
  F(t, x), \quad \text{if} \quad (x, y) \in ((T_1 \cup T_2) \times P), t \in T
  \]

For \((d, d')\)-typed tracking extension of OC WF-net \( ON \), we call \( ON \) the ground component, and \( ON' = (ON|_{d})_{d/d'} \) the tracking component.

**Proposition 2.** Let \( ON = (O, P, T, F, \text{type}, \ell, A) \) be a OC WF-net, \( M_{in} \) – its initial marking. Let also \( d \in O \) and \( d' \notin O \).

Then \( ON \) with the initial marking \( M_{in} \) is weakly bisimilar to its \((d, d')\)-typed tracking extension \( ON_{d+d'} \) with the initial marking \( M_{in'} \), s.t.

\[
M_{in'}(in_{d'}) = 1, \quad \text{if} \quad M_{in}(in_d) \geq 1, \quad M_{in'}(in_d) = 0, \quad \text{if} \quad M_{in}(in_d) = 0, \\
M_{in'}(in_d) = 0, \quad \text{if} \quad M_{in}(in_d) \geq 1, \quad M_{in'}(in_d) = 0, \quad \text{if} \quad M_{in}(in_d) = 0, \\
\text{for every } p \in P \setminus \{in_d\}, \quad M_{in'}(p) = M_{in}(p).
\]

Notice that the tracking extension itself can be used for monitoring concrete objects. The property below demonstrates that one can essentially monitor a finite number of objects at once.

**Proposition 3.** Let \( ON = (O, P, T, F, \text{type}, \ell, A) \) be a OC WF-net. Let also \( d_1, d_2 \in O, \ d_1', d_2' \notin O \), and \( d_1 \neq d_2, \ d_1' \neq d_2' \).

Then nets \((ON_{d_1+d_1'})_{d_2+d_2'} \) and \((ON_{d_2+d_2'})_{d_1+d_1'} \) are identical, i.e. the result of the successive application of multiple tracking extensions does not depend of their order.

**Example 1.** Let us consider an example of tracking extension. Figure 5 shows a OC WF-net \( N \) consisting of two nets related to two object types \( d_1 \) and \( d_2 \) that, for simplicity, are respectively depicted with green and yellow colors.

Let us consider tracking extensions for both object types of \( N \). By properly identifying the tracking component for \( d_1 \), we demonstrate \( N \)’s \((d_1, d_1')\)-tracking extension in Figure 5 (here, \( d_1' \) is represented with indigo color). Similarly to \( d_1 \), we present \( N \)’s \((d_2, d_2')\)-tracking extension in Figure 7 (\( d_2' \) is in red).

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2 Here, given two functions \( f : X \to A \) and \( g : Y \to B \), we write \( f \cup g : X \cup Y \to A \cup B \) iff \( f|_{X \cap Y} = g|_{X \cap Y} \).
Fig. 5: An OC WF-net with two object types $d_1$ (in yellow) and $d_2$ (in green)

Fig. 6: Tracking OC WF-net for an object of type $d_1$

Fig. 7: Tracking OC WF-net for an object of type $d_2$

6 Object-centric Soundness

The most crucial correctness criterion for workflow nets is soundness [5]. For classical WF-nets, a workflow case execution terminates properly, iff starting at the initial marking with a single token in its source place it terminates with a single token in its sink place, i.e. there are no “garbage” tokens after termination. A WF-net is called sound iff its execution can terminate properly, starting at any reachable marking.
The soundness definition from above always considers one object (or, more traditionally, one case) in isolation, or several objects of the same type executing a common task, so that they start and finish together (generalized soundness). At the same time, as Definition 6 suggests, OC WF-nets need to account for multiple objects of different types, when objects of the same type follow similar tasks, defined by a common for them workflow, and each object needs to terminate properly. The motivating example in Section 2 illustrates such a system. To this end, we extend the classical notion of soundness with object types as well as multiple typed source and sink places.

We start by recalling the classical notion of soundness used for WF-nets and then show how it can be extended to the object-centric case.

**Definition 9 (WF-net soundness).** Let $N = (P, T, F)$ be a WF-net, and let $in, out \in P$ be its source and sink places, respectively. The net is called sound iff

1. for every $M \in R(N, [in])$, it holds that $[out] \in R(N, M)$ (option to complete);
2. for every $M \in R(N, [in])$, if $[out] \subseteq M$, then $M = [out]$ (proper completion);
3. for every $t \in T$, there exists $M \in R(N, [in])$, s.t. $M \triangleright t$ (no dead transitions).

Now we come to defining object-oriented soundness for OC WF-nets.

**Definition 10 (Object soundness, system soundness).** Let $ON$ be a $O$-typed OC WF-net, $d \in O$ be one of its types, $d' \notin O$, and $ON_{d+d'} = (O', P', T', F', type', t', A)$ be the $(d, d')$-typed tracking extension of $ON$.

$ON$ is called object-sound for type $d$ (d-sound for short) iff for every input marking $M_{in} \in P'_{in}$ in $ON_{d+d'}$ s.t. $M_{in}(in_d) = 1$, it holds that for any marking $M \in R(ON, M_{in})$, there exist initial marking $M'_{in} \in (P'_{in})^\oplus$ and target marking $M' \in (P')^\oplus$ s.t.:

- $M' \in R(ON', M + M'_{in})$, i.e., target marking $M'$ can be reached from $M$, possibly by adding more tokens to source places;
- $M'(out_d) = 1$, i.e., the (tracked) object of type $d$ has reached its final state in target marking $M'$;
- for all $p \in P$ s.t. $type(p) = d$ and $p \notin in_d$, we have $M'(p) = 0$, i.e., the (tracked) object of type $d$ terminates properly (without leaving “garbage” behind, i.e. in the tracking component);
- there are no dead transitions in 1-safe $d$-typed projection $ON_{d}^{1}$ of OC-net $ON$ with the initial marking that contains the only token in source place $in_d$.

The same $ON$ is called object-centric sound (OC-sound, for short) iff it is d-sound for every $d \in O$.

As in classical WF-nets, this definition focuses on a “concrete” object and traces its lifecycle until the end. This is possible due to the relaxed treatment of object identifiers: the OC WF-net cannot inspect the content of its tokens, but can always check types of places that carry the tokens. Thus, it suffices to look into quantities of objects available in a net marking.

However, unlike in classical WF-nets, this single object is not considered in isolation. Our notion of OC-soundness allows to account for potentially multiple objects in the source places, but cares only for one single object when it comes to checking...
whether the process has been completed or not (that is, we want to check only whether
the “object of interest” has reached the source place). At the same time, we take no
notice of other objects that were potentially “helping” that very selected object to com-
plete the process. This means that the net may still contain tokens corresponding to
other objects in some intermediate places.

**Example 2.** Consider the scenario discussed in Section 2. There, a project can always
be completed regardless “concrete” team members and leaders that are currently present
in the net (i.e., we can always introduce into the net marking additional students to
complete “the selected” project). Likewise, each team member has an opportunity to
complete their task.

The following statements follow directly from the definition of OC-soundness.

**Proposition 4.** Let $N = (P, T, F)$ be a WF-net. If $N$ is sound, then it is also OC-sound
as OC WF-net $ON = (\{d\}, P, T, F, \text{type})$, where for all $p \in P$, $\text{type}(p) = d$.

Let us now come back to the projections. We now show how OC WF-nets and their
1-safe projections can be related.

**Proposition 5.** Let $ON = (\emptyset, P, T, F, \text{type}, \ell, A)$ be an OC WF-net. If $ON$ is OC-
sound, then for any $d \in O$, its 1-safe $d$-typed projection $ON \mid d$ is sound as a WF-net.

From this proposition it follows that the reachability set (set of all reachable mark-
ings) of a 1-safe projection $ON \mid d$ of sound OC WF-net $ON$ is finite, since the reacha-
bility set of sound (classical) WF-net is always finite.

**Proposition 6.** Each 1-safe projections of an OC-sound OC WF-net has finitely many
reachable markings.

For OC WF-nets we do not fix either input, or output markings. Sound OC WF-nets
have, generally speaking, an infinite number of reachable markings. When defining
OC-soundness, we take care of the possibility to terminate properly for one selected
object. For this object, all other objects play the role of resources required to complete
its task. Unlike to workflow nets with resources, in OC WF-nets, these resources are
dynamic and can be added as needed. A well-designed system should guarantee “equal
rights” to all participants, i.e. each object has an ability to terminate properly from
any reachable marking. OC-soundness defines this formally. The good news is that this
crucial property is decidable.

**Theorem 1.** The problem of checking OC-soundness for an OC WF-net is decidable.

### 7 Related Work

#### 7.1 Modeling Object-centric and Data-aware Processes using Petri nets

There are multiple Petri net-based approaches used for modeling data- and object-aware
processes. Here we briefly cover only the most recent ones.

The formalism of Petri nets with identifiers (PNIDs) was studied in [17]. In such
nets, tokens carry tuples of object identifiers (or references). Moreover, transitions can
produce globally new data values that are different from those in a current marking.
PNIDs have been also used as the process modeling language in the Information Sys-
tems Modeling Language (ISML) presented in [22]. In ISML, PNIDs are paired with
special CRUD operations defining operations over relevant data. Such data are structured according to an ORM model.

A viewpoint on multi-case and data-aware process models is covered by works on so-called Proclets. This concept was first proposed in [2]. Proclets are essentially quasi-workflows representing lifecycles of particular objects and allowing for different types of communication between such lifecycles via dedicated channels. The Proclet notation was applied to model processes with complex interactions between process objects [10] as well as artifact-centric processes [9]. Process artifacts here are documents, orders, deliveries, and other types of data objects which are manipulated within the process. Artifact-centric process models consider these objects as first-class citizens. Note that in such models process cases are often related to object types that leads to inter-case dependencies when more complicated communication patterns are involved.

In [12] the authors introduced catalog-nets (CLog-nets). Conceptually, the formalism marries read-only relational databases with a variant of colored Petri nets [18] which allows for generation of globally fresh data values upon transition firing. In CLog-nets, transitions are equipped with guards that simultaneously inspect the content of tokens and query facts stored in the database.

### 7.2 Soundness for Workflow Nets and their Extensions

The notion of soundness for classical WF-nets was initially discussed in [1] 3 Since then, many different variants of soundness have been proposed for WF-nets and their non-high-level extensions. An extensive study on such variants as well as related decidability results can be found in [5]. In the following, we briefly touch upon some of the soundness notions. Perhaps one of the most important generalizations of classical soundness are $k$-soundness and generalized soundness [16]. A WF-net is $k$-sound if, starting with $k$ tokens in its source place, it always properly terminates with $k$ tokens in its sink place. Hence this notion allows handling multiple individual processes (cases) in a WF-net. The classical soundness then coincides with 1-soundness. Generalized soundness is $k$-soundness for all $k$. It was proved in [16] that both $k$-soundness and generalized soundness are decidable.

One of the important aspects of workflow development concerns resource management. In classical Petri nets, one can distinguish places, corresponding to control flow states, and places, representing different kinds of resources. There are various works that study different resource-aware extensions of WF-nets together with related notions of soundness. In [7] the authors studied a specific class of WFR-nets for which soundness was shown to be decidable. In [15,23] a more general class of Resource-Constrained Workflow Nets (RCWF-nets) was defined. The constraints are imposed on resources and require that all resources that are initially available are present again after all cases terminate, and that for any reachable marking, the number of available resources does not override the number of initially available resources. In [15] it was proven that for RCWF-nets with a single resource type generalized soundness can be effectively checked in polynomial time. Decidability of generalized soundness for RCWF-nets with an arbitrary number of resource places was shown in [23].

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3 This notion coincides with the one presented in the beginning of Section 6.
All the above mentioned works assume resources to be permanent, i.e., they are acquired and released later at the end of a (sub-)process. Since resources are never created, nor destroyed, the process state space is explicitly bounded. To study a more general case of arbitrary resource transformations (that can arise, for example, in open and/or adaptive workflow systems), in [8] WF-nets with resources (RWF-nets) were defined. RWF-nets extended RCWF-nets from [15] in such a way that resources can be generated or consumed during a process execution without any restrictions. Unfortunately, even 1-sound RWF-nets are not bounded in general, hence existing soundness checking algorithms are no applicable in that case. The decidability of soundness for RFW-nets was shown for a restricted case, in which a RWF-net has a single unbounded resource place.

There are also other works that propose more high-level extensions of classical WF-nets as well as the related notion(s) of soundness. In [19,11,20], the authors investigated data-aware soundness for data Petri nets, in which a net is extended with guards manipulating a finite set of variables associated with the net. That type of soundness was shown to be decidable. In [21], the authors proposed both a workflow variant of $\nu$-Petri nets as well as its resource-aware extension. The authors also defined a suitable notion of soundness for such nets and demonstrated that it is decidable by reducing the soundness checking problem to a verification task over another first order logic-based formalism. [14,13] considered the soundness property for BPMN process models with data objects that can be related to multiple cases. The approach consists of several transformation steps: from BPMN to a colored Petri net and then to a resource-constrained workflow net. The authors then check k-soundness against the latter.

8 Conclusions

In this work, we investigated a notion of soundness for a workflow variant of the recently proposed formalism of object-centric Petri nets, in which each object type is associated with a dedicated workflow net. Given this structural characteristic, proposed soundness focuses only on local termination of a single instance of an object type and assumes that the entire net can have any number of tokens being assigned to other dedicated workflows. We also conjectured that checking this new variant of soundness is decidable.

References

1. van der Aalst, W.M.P.: The application of petri nets to workflow management. J. Circuits Syst. Comput. 8(1), 21–66 (1998). [https://doi.org/10.1142/S0218126698000043]
2. van der Aalst, W.M.P., Barthelmess, P., Ellis, C.A., Wainer, J.: Workflow modeling using proclets. In: Etzion, O., Scheuermann, P. (eds.) Cooperative Information Systems, 7th International Conference, CoopIS 2000, Eilat, Israel, September 6-8, 2000, Proceedings. Lecture Notes in Computer Science, vol. 1901, pp. 198–209. Springer (2000). [https://doi.org/10.1007/10722620_20]
3. van der Aalst, W.M.P., Barthelmess, P., Ellis, C.A., Wainer, J.: Proclets: A framework for lightweight interacting workflow processes. Int. J. Cooperative Inf. Syst. 10(4), 443–481 (2001). [https://doi.org/10.1142/S0218843001000412]
4. van der Aalst, W.M.P., Berti, A.: Discovering object-centric petri nets. Fundam. Informaticae 175(1-4), 1–40 (2020). [https://doi.org/10.3233/FI-2020-1946]
5. van der Aalst, W.M.P., van Hee, K.M., ter Hofstede, A.H.M., Sidorova, N., Verbeek, H.M.W., Voorhoeve, M., Wynn, M.T.: Soundness of workflow nets: classification, decidability, and analysis. Formal Asp. Comput. 23(3), 333–363 (2011)
6. van der Aalst, W.M.P., ter Hofstede, A.H.M., Kiepuszewski, B., Barros, A.P.: Workflow patterns. Distributed Parallel Databases 14(1), 5–51 (2003). [https://doi.org/10.1023/A:1022883727209]
7. Barkaoui, K., Benayed, R., Sbai, Z.: Workflow Soundness Verification Based on Structure Theory of Petri Nets. International Journal of Computing and Information Sciences (IJCIS) 5, 51–62 (2007)
8. Bashkin, V.A., Lomazova, I.A.: Decidability of k-Soundness for Workflow Nets with an Unbounded Resource, pp. 1–18. Springer (2014)
9. Fahland, D.: Artifact-centric process mining. In: Encyclopedia of Big Data Technologies. Springer (2019)
10. Fahland, D.: Describing behavior of processes with many-to-many interactions. In: Proc. of Petri nets 2019. pp. 3–24. Springer (2019)
11. Felli, P., de Leoni, M., Montali, M.: Soundness verification of decision-aware process models with variable-to-variable conditions. In: ACSD. pp. 82–91. IEEE (2019)
12. Ghilardi, S., Gianola, A., Montali, M., Rivkin, A.: Petri nets with parameterised data-modelling and verification. In: Fahland, D., Ghidini, C., Becker, J., Dumas, M. (eds.) BPM’20. Lecture Notes in Computer Science, vol. 12168, pp. 55–74. Springer (2020). [https://doi.org/10.1007/978-3-030-58666-9_4]
13. Haarmann, S., Montali, M., Weske, M.: Technical report: Refining case models using cardinality constraints. CoRR abs/2012.02245 (2020)
14. Haarmann, S., Weske, M.: Cross-case data objects in business processes: Semantics and analysis. In: BPM (Forum). Lecture Notes in Business Information Processing, vol. 392, pp. 3–17. Springer (2020)
15. van Hee, K.M., Serebrenik, A., Sidorova, N., Voorhoeve, M.: Soundness of resource-constrained workflow nets. In: ICATPN. Lecture Notes in Computer Science, vol. 3536, pp. 250–267. Springer (2005)
16. van Hee, K.M., Sidorova, N., Voorhoeve, M.: Generalised soundness of workflow nets is decidable. In: ICATPN. Lecture Notes in Computer Science, vol. 3099, pp. 197–215. Springer (2004)
17. van Hee, K.M., Sidorova, N., Voorhoeve, M., van der Werf, J.M.E.M.: Generation of database transactions with petri nets. Fundam. Inform. 93(1-3), 171–184 (2009)
18. Jensen, K., Kristensen, L.M.: Coloured Petri Nets - Modelling and Validation of Concurreny Systems. Springer (2009)
19. de Leoni, M., Felli, P., Montali, M.: A holistic approach for soundness verification of decision-aware process models. In: ER. Lecture Notes in Computer Science, vol. 11157, pp. 219–235. Springer (2018)
20. de Leoni, M., Felli, P., Montali, M.: Strategy synthesis for data-aware dynamic systems with multiple actors. In: KR. pp. 315–325 (2020)
21. Montali, M., Rivkin, A.: Model checking petri nets with names using data-centric dynamic systems. Formal Aspects Comput. 28(4), 615–641 (2016). [https://doi.org/10.1007/s00165-016-0370-6]
22. Polyvyanyy, A., van der Werf, J.M.E.M., Overbeek, S., Brouwers, R.: Information systems modeling: Language, verification, and tool support. In: Giorgini, P., Weber, B. (eds.) Advanced Information Systems Engineering - 31st International Conference, CAiSE 2019, Rome, Italy, June 3-7, 2019, Proceedings. Lecture Notes in Computer Science, vol. 11483, pp. 194–212. Springer (2019). [https://doi.org/10.1007/978-3-030-21290-2_13]
23. Sidorova, N., Stahl, C.: Soundness for resource-constrained workflow nets is decidable. IEEE Trans. Syst. Man Cybern. Syst. 43(3), 724–729 (2013)