Thermocapillary rivulete structure in a flowing liquid film under conditions of constant wall temperature

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Abstract. A stationary thermocapillary rivulet structure formed in a film of a liquid falling over a semi-infinite heater is theoretically and experimentally investigated. Nonlinear three-dimensional rivulets on the surface of a liquid film under constant wall temperature conditions are simulated numerically. It is shown that the developed rivulet structure varies very little downstream. The results of calculations for small and moderate Reynolds numbers are in good agreement with experimental data.

1. Introduction

Non-isothermal liquid films determine the regime of heat transfer in irrigation cooling towers, absorbers, scrubbers, distillation columns, evaporators, condensers, chemical technology devices, during the movement of vapor-liquid mixtures in pipes. Theoretical studies of the wave dynamics of a heated film are based on the long-wave approximation, but two different approaches are used. In the first approach, based on the lubricant approximation, one evolution equation is derived for the film thickness of the Benney equation type [1], but taking into account the thermocapillary force on the film surface. This approach is applicable only for small values of the Reynolds number. According to the second approach in which the Reynolds number is not supposed small, the dynamics of a heated film is described by a system of the evolutionary equations with respect to the film thickness, the flow rate and temperature of the liquid in the film. Theoretical models [2-4] based on the second approach are derived directly from the Navier–Stokes equations using some assumptions about the velocity profile and the temperature profile in the film. In the case of film flow of a liquid along a heated surface, in addition to hydrodynamic instability, a thermocapillary instability mechanism also takes place, as a result of which the stationary rivulet-like structures are formed on the film surface, which significantly affect the dynamics and heat transfer. In the first experiments [5, 6], where thermocapillary rivulet structures were observed for the first time, the heater had a constant wall temperature. At the upper edge of such a heater, a horizontal bump was formed due to a large temperature gradient. Immediately behind the bump, the film disintegrated into a system of stationary rivulets with approximately the same distance between them. In [7, 8], on the basis of the developed model, the nonlinear development of instability, which leads to the formation of a rivulet structure in a uniformly heated liquid film, was studied by a numerical method. In this work, a rivulet thermocapillary structure in a liquid film formed on the upper edge of a semi-infinite heater is simulated by a numerical method. The calculation results are compared with experimental data.
2. Theoretical model

Let us consider a film of liquid flowing down a semi-infinite vertical heater. The surface of the liquid is in contact with a fixed gas, heat exchange between the liquid and the gas is described by means of a given heat transfer coefficient $\alpha$. All properties of a fluid are considered constant, with the exception of surface tension, which is linearly dependent on temperature: $\sigma = \sigma_0 - \gamma(T - T_0)$. We introduce a Cartesian coordinate system $Oxyz$ with the $Ox$ axis in the direction of gravity and the $Oy$ axis directed along the normal to the plate. Perturbations in the film are considered to be long-wave (i.e. the film thickness $h$ is much smaller than the characteristic size of the thermocapillary structure $L$). Taking the unperturbed film thickness $h_m$ as the distance scale, we introduce the velocity scale $u_m = gh_m^2/3\nu$, the time scale $t_m = h_m/u_m$, the flow rate scale $q_m = u_m h_m$, and the temperature scale $T_m = T_w - T_g$. In dimensionless variables $x/h_m, z/h_m, q/q_m, m/q_m, T/T_m, (T-T_g)/T_m$ the flow is described by the system of equations derived in [7] for the thickness films $h(x,z,t)$, rates $q(x,z,t)$, $m(x,z,t)$

$$
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h \frac{\partial T}{\partial x}) + \frac{\partial}{\partial z}(h^2 \frac{\partial T}{\partial z}) = \frac{3}{Re} \left( h \frac{\partial T}{\partial x} - \frac{q}{2h^2} \right) + \text{We} \frac{\partial h}{\partial x},
$$

$$
\frac{\partial m}{\partial t} + \frac{\partial}{\partial x}(m \frac{\partial T}{\partial x}) + \frac{\partial}{\partial z}(m \frac{\partial T}{\partial z}) = \frac{3}{Re} \left( m \frac{\partial T}{\partial x} + \frac{m}{h^2} \right) + \text{We} \frac{\partial h}{\partial x},
$$

$$
\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} + \frac{\partial m}{\partial z} = 0.
$$

Here $J_1 = \frac{6q^2}{5h} - \frac{Ma}{20} \frac{\partial T}{\partial x} + \frac{h^2}{120} \left( \frac{Ma}{20} \frac{\partial T}{\partial z} \right)^2$, $J_2 = \frac{6m^2}{5h} - \frac{Ma}{20} \frac{\partial T}{\partial x} + \frac{h^2}{120} \left( \frac{Ma}{20} \frac{\partial T}{\partial z} \right)^2$, $J_{1,2} = \frac{6mq}{5h} \frac{Ma}{40} \left( \frac{q}{\partial x} + \frac{m}{\partial z} \right) + \frac{h^2}{120} \left( \frac{Ma}{20} \frac{\partial T}{\partial x} \right)^2 + \frac{h^3}{120} \frac{Ma^2}{20} \frac{\partial T}{\partial x} \frac{\partial T}{\partial z}, \Delta h = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2}, T_m(x,z) = \frac{T_w}{1 + Bi h}$.

The flow of the heated film is determined by the following dimensionless criteria: $We = (3Fi/Re)^{1/3}$ is Weber number, $Fi = \sigma^2/\rho g^2 v^4$ is Kapitsa number, $Ma = \frac{\rho T_m}{\mu u_m}$ is Marangoni number, $Bi = \alpha h_m/\lambda$ is Biot number, $Re = gh_m^2/3\nu^2$ is Reynolds number. $h_m^{-} \sim Re^{1/3}$, $u_m^{-} \sim Re^{2/3}$, then $Bi = Bi^{-} Re^{1/3}$, $Ma = Ma^{-} Re^{2/3}$. Here, dimensionless complexes $Bi^* = 3^{1/3} \alpha \nu^{2/3} / \lambda g^{1/3}$ and $Ma^* = 3^{1/3} (T_m - T_g) / \rho g^{1/3} v^{4/3}$ are determined only by the properties of the liquid and the heating condition. Unperturbed flow corresponds to a trivial solution $h_0 = 1$, $q_0 = 1$, $m_0 = 0$, $T_{x,0} = (1 + Bi)^{-1}$.

3. Results of calculations of stationary rivulets

The stationary 3D rivulet structure with a spatial period $L_e$ was simulated by solving the non-stationary equations (1) by the finite-difference method. All calculations were made for water ($Fi^{1/3} = 3272$). In the calculations, the edge of the semi-infinite heater was located at a distance $x_H$ from the upper boundary of the computational region, and the dimensionless wall temperature was set in the form:

$$
T_w = 0 \text{ at } x < x_H - b; \ T_w = 1 \text{ at } x \geq x_H; \ T_w = \frac{1}{2} \left( 1 - \cos \left( \pi \frac{x + b - x_H}{b} \right) \right) \text{ at } x_H - b < x < x_H.
$$

Dependence (2), schematically shown in figure 1, takes into account the partial heating of the wall in the region of length $b$ above the heater edge due to the finite thermal conductivity of the substrate material into which the heater was embedded in experiments [9]. It follows from (2) that in the transition zone of length $b$, the dimensionless wall temperature increases monotonically from zero to unity. Near the edge of the heater, where the gradient of the surface temperature of the film reaches a
high value, a stationary horizontal bump is formed. The shape of the bump, studied in many papers (for example, in [10, 11]), was determined from the equations of the 2D flow:

\[
\frac{\partial q}{\partial t} + \frac{\partial J}{\partial x} = \frac{3}{\text{Re}} \left( h \frac{\partial^2 T}{\partial x^2} - \frac{q}{h^2} \right) + \text{We} h \frac{\partial^3 h}{\partial x^3},
\]

Unsteady equations (3) were solved by the establishment method, and the unperturbed flow of a cold film was set as the initial condition. In the calculations, the edge of the heater was located at a distance of \( x_0^0 = 1.5 \) cm from the origin of coordinates, the length of the transition zone was \( b = 0.5 \) cm.

Figure 1 shows the results of the calculation of the horizontal bump with \( \text{Re} = 2 \) for various values of parameter \( \text{Ma}^* \). The bump is located entirely in front of the heater edge in the transition zone, where a sharp increase in wall temperature occurs. It can be seen from the figure that with the increase in \( \text{Ma}^* \) the height of the bump increases, but its shape does not change.

Stationary 3D rivulets with a spatial period \( L_z \) were simulated by means of a perturbation, periodic in the \( z \) coordinate, superimposed on the stationary 2D flow previously calculated from (3). The position of the edge of the heater was set as \( x_\mu = x_0^0 + A \cos(\xi k) \). Here \( A \) is the small amplitude of the “roughness” of the heater edge, \( k = 2\pi / L_z \) is the wavenumber which corresponds to the maximum of spatial growth rate calculated according to the linear theory [8]. The portion of the film that impinges onto the protrusion of the edge of the heater is heated before those impinging into the cavity. Thus, the “roughness” of the heater edge sets a perturbation of the temperature of the fluid in the transverse direction. The corresponding thermocapillary force initiates a transverse perturbation of the flow, which develops downstream, and leads to the formation of a 3D rivulet structure.

The size of the computational region in the \( z \) coordinate was one period, i.e. the computational area was a rectangle \( 0 \leq x \leq X_{end} \), \( 0 \leq z \leq L_z \). On the lateral boundaries of the computational region \( z = 0 \) and \( z = L_z \) the conditions for periodicity \( h(x,0) = h(x, L_z) \), \( q(x,0) = q(x, L_z) \), \( m(x,0) = m(x, L_z) = 0 \) were set. Taking into account the symmetry conditions with respect to the middle of the interval \( h(-\xi, t) = h(\xi, t), \quad q(-\xi, t) = q(\xi, t), \quad m(-\xi, t) = -m(\xi, t) \) (here \( \xi = z - L_z / 2 \) ) allows one to carry out calculations on a half-period, i.e. for \( 0 \leq \xi \leq L_z / 2 \). During evolution, three-dimensional waves, running away downstream and leaving the computational region, were formed from the initial state in

![Figure 1](image1.png)  
**Figure 1.** Horizontal bump on the heater edge at \( \text{Re} = 2 \) for different values of \( \text{Ma}^* \): 80 (1), 100 (2), 125 (3). Open circles are dimensionless wall temperature.

![Figure 2](image2.png)  
**Figure 2.** Developed 3D rivulet structure with wavelength \( L_z = 8.7 \) mm at \( \text{Re} = 2, \text{Ma}^* = 150, \text{Bi}^* = 0.2 \).
the film. The computation was over, when in the entire computational domain there was a three-dimensional rivulet structure not changing over time.

Figure 2 shows a rivulet structure with a period of $L_z = 8.7$ mm for $Re = 2$, $Ma^* = 150$, $Bi^* = 0.2$. It can be seen from the figure that immediately after the horizontal bump the thickness of the film is modulated in the transverse direction. Downstream, the amplitude of transverse modulation of thickness increases with distance approximately exponentially (region of linear growth). Then, in a short section, non-linear developed rivulets, separated by a relatively thin film, are formed. The height of the hump increases abruptly, and the thickness of the film between the humps decreases abruptly to value $h \approx 0.1$. An additional rivulet of much smaller height $h \approx 0.3$ is formed between the main humps. Such a small amplitude rivulet located between the main rivulets (corresponding to the $L_z$ period) is clearly visible in experimental photos [12]. The developed rivulet structure varies very slightly downstream. The film thickness in the bridges between rivulets decreases gradually due to the flow of liquid in the direction of the humps under the action of thermocapillary forces, but this cross-flow is opposed by the gradient of capillary pressure due to the curvature of the film surface. The almost balanced effect of these two opposite factors (with some predominance of thermocapillary force) leads to a slight change in the developed rivulet structure downstream.

Figure 3 explains how “roughness” of the heater edge initiates perturbation of the flow in the transverse direction (the parameters are the same as in figure 2). The figure shows how the temperature of the film surface ($a$) and the longitudinal flow ($b$) varies along the path through the protrusion of the edge of the heater and along the path through the cavity of the edge of the heater. It can be seen from figure 3a that the liquid in the path $z = L_z/2$ is heated before the liquid along the path $z = 0$. The small difference in the coordinates of the start of heating for these two trajectories ($2A = 0.2$ mm) leads to a small difference in temperature at the beginning of the transition zone. However, this small inhomogeneity of temperature initiates a transverse thermocapillary force, which in the area of the bump redistributes the flow rate $q$ in the transverse direction. It can be seen from figure 3b that the flow rate decreases significantly with distance along the trajectory $z = L_z/2$ and increases significantly along the trajectory $z = 0$. Due to the redistribution of the flow, the film thickness at the end of the transition zone on the trajectory $z = 0$ is larger than on the trajectory $z = L_z/2$, as a result of which the difference in temperature for these trajectories increases significantly (see figure 3a). The transverse perturbation in the film, formed in the region of the horizontal bump, increases downstream under conditions of constant wall temperature.

![Figure 3](image1.png)

**Figure 3.** Temperature of film surface ($a$) and flow rate ($b$) depending on coordinate $x$ along the trajectory $z = L_z/2$ (curves $1$) and along the trajectory $z = 0$ (curves $2$).

Figure 4a shows the temperature distribution on the film surface of a 60% glycerol solution, calculated for the experimental conditions [13] at $Re = 0.1$. An experimental thermogram of the film surface in the region near the edge of the heater is also shown for comparison (figure 4b). The calculation was made for a rivulet structure with a period of $L_z = 8.6$ mm, which is clearly visible in
the experimental thermogram. The period of the structure agrees well with the wavelength of the maximum growing perturbation calculated by the linear theory, equal to 8.73 mm. It can be seen from the figure that the calculation agrees qualitatively and quantitatively with experimental data. The temperature rises in a transition zone by length of 0.5 cm in front of the heater. At the edge of the heater (i.e. at $x = 1.5$ cm), the temperature reaches a plateau, where the ravines (ditches), corresponding to the formed rivulets, are visible. On the plateau between the ravines there is a smaller notch (depression), which corresponds to an additional low-amplitude rivulet.

**Figure 4.** Temperature of film thickness of 60% water solution of glycerol at Re = 0.1. Experimental thermogram of film surface (a) and calculation (b) for experimental conditions of [13].

Figure 5 shows the rivulet structure calculated for the conditions in figure 4. It can be seen from the figure that for such a small Reynolds number Re = 0.1, the region of the linear growth of the perturbation is practically absent, and the developed rivulets appear immediately behind the horizontal bump.

**Figure 5.** Developed rivulet structure corresponding to the conditions in figure 4.
Conclusion

Based on the equations derived in the long-wave approximation, the formation of 3D thermocapillary rivulets in a liquid film flowing along a locally heated wall was investigated. The used model [7,8] based on modified IBL-model has wide application because it correctly describes the characteristics of nonlinear waves at moderate Re < 20 away from the instability threshold. This fact is confirmed by many studies and, in particular, by our work [14], comparing the stability calculations by the modified IBL-model and by the Orr-Sommerfeld equation. Analysis of the experimental data for various liquids and different Reynolds numbers showed that the distance between the rivulets corresponds well to the wavelength of the most amplified perturbation, calculated in accordance with the theory of stability. Three-dimensional rivulets were simulated by spanwise perturbation imposed on a pre-calculated stationary 2D flow. The specified spatial period of the perturbation \( L_z \) corresponded to the wavelength of the most amplified perturbation. The calculations have revealed the influence of dimensionless parameters on the characteristic spatial scale of the rivulet structure. Calculations have shown that the Re number increase in the range of 1–10 does not significantly change the shape and amplitude of the developed rivulets. The results of calculations of the rivulet structure are in good agreement with the experimental data.

References

[1] Benney D J 1966 *J. Math. Phys.* **45** 150–5
[2] Trevelyan M J, Scheid B, Ruyer-Quil C and Kalliadasis S 2007 *J. Fluid Mech.* **592** 295–334
[3] Skotheim J M, Thiele U and Scheid B 2003 *J. Fluid Mech.* **475** 1–19
[4] Scheid B, Kalliadasis S, Ruyer-Quil C and Colinet P 2008 *Phys. Rev. E* **78** 066311
[5] Kabov O A, Marchuk I V and Chupin V M 1996 *Russ. J. Engin. Thermophys.* **6** 104–38
[6] Kabov O A and Chinnov E A 1997 *Russ. J. Engin. Thermophys.* **7** 1–34
[7] Aktershev S P and Alekseenko S V 2017 *J. of Physics: Conference Series* **891** 012023
[8] Aktershev S P and Alekseenko S V 2019 *Int. J. Multiphase Flow* **114** 115–27
[9] Chinnov E A and Kabov O A 2004 *High Temperature* **42** 267–77
[10] Aktershev S P 2004 *Thermophys. Aeromech.* **11**, 287–98
[11] Gatapova E Ya, Kabov O A and Marchuk I V 2004 *Technical Physics Letters* **30** 46–52
[12] Kabov O A and Chinnov E A 2007 *Microgravity Science and Technology* **19** 18–22
[13] Chinnov E A and Shatskiy E N 2014 *Technical Physics Letters* **40** 7–9
[14] Aktershev S P and Alekseyenko S V 1996 *Russ. J. Engin. Thermophys.* **6** 307–20