Robust Beamforming and Power Minimization Design in MISO Health Monitoring System

Mingjun Pei¹, Xuemeng Jia¹, Rong Fu¹ and Zhanwei Jiao²,*
¹State Grid Henan Electric Power Company Xinan Power Supply Company, China
²Zhengzhou University, China
E-mail: *11892646@qq.com

Abstract. In this paper, we study the power splitting (PS) scheme for each user in a multiple-input single-output (MISO) health monitoring system, while considering simultaneous wireless information and power transmission (SWIPT). We construct an optimization problem with robust power minimization constrained by the signal-to-inference-plus-noise ratio (SINR) and energy harvesting (EH). For channel uncertainty, this problem is a non-convexity and is not easy to solve and it can be translated into semidefinite program (SDP) by utilizing S-Procedure. Then the above optimization problem is combined with different channel uncertainty models, and the Bernstein-type inequality and Gaussian error function are introduced to further simplify the optimization problem. Simulation results prove the performance of the proposed robust scheme and point out its effective help for the development of modern telemedicine systems.

1. Introduction
With the rapid development of technologies in 5-th generation wireless systems (5G), more and more patients is begining to use wireless information technology for real-time health monitoring. As a key scheme that can provide medical and health care for many patients, wearable or chip type wireless sensor implanted into human body can timely transmit information such as blood pressure, body temperature and heart rate to wireless medical platform for detection, and quickly receive corresponding medical advice and guidance [1-4]. However, the energy of wireless sensor equipment is usually limited which seriously affects the the normal operation of wireless sensor equipment. So it is necessary to solve this problem if we want to establish a longterm and reliable health monitoring system [5].

As a promising technology to solve the energy shortage of wireless communication in 5G, simultaneous wireless information and power transfer (SWIPT) provides an effective solution for the above problems in the medical field [6]. [7] describes SWIPT in detail, and introduces power division (PS) and time switching (TS) for decoding information and harvesting power. In the related research of SWIPT, many systems have studied and used the PS scheme, such as multiple-input-single-output (MISO) system [8], interference channel [9], and multicasting system [10]. In order to better balance the maximum traversal capacity and the maximum average harvest energy of information transmission, an optimal PS rule based on dynamic PS strategy is given [11].

Whether in the health care or other fields, most of the current research about SWIPT assumes that perfect channel state information (CSI) can be obtained. If we cannot obtain the CSI at the transmitter, it will be difficult for us to carry out in-depth optimization studies. Robust
techniques were considered to assimilate the channel uncertainties which can effectively avoid imperfections of the CSI in [12]-[15].

The main contributions of this paper are summarized as follows.

• Constructing a robust power minimization problem constrained by the signal-to-noise ratio (SINR) and EH which is a non-convex problem in terms of channel uncertainty, and it can be transformed into a semidefinite program (SDP) by using S-Procedure.

• In the above problem, we consider the uncertainty of two Gaussian channels. For the non-convex problem based on probability constraints, Bernstein-type inequality and Gaussian error function can transform the problem into a convex problem that is easy to solve.

The rest parts of this paper is organized as follows. Section 2 describes the System model. Section 3 and Section 4 proposes the robust optimization scheme for the uncertainty of two Gaussian channels. Section 5 presents the simulation results of proposed robust schemes. Section 6 summarizes this work.

2. System Model

We consider a MISO SWIPT health monitoring downlink system with one base station (BS) equipped with $M$ antennas and $K_m$ users, where each user only has one antenna. In this system, the user needs to complete two tasks of information decoding (ID) and EH which can be accomplished according to the PS scheme. The transmitted data and beamforming vector of $i$-th user can be donated as $x_i$ and $w_i$, respectively, where $i \in \{1, \ldots, K\}$ and $E\{|x_i|^2\} = 1$. The received signal at the $i$-th user can be donated as

$$y_i = h_i^H w_i x_i + h_i^H \sum_{l \neq i} w_l x_l + \varepsilon_i,$$

where $h_i$ is the channel coefficients of the $i$-th user and $\varepsilon_i$ denotes the additive Gaussian noise (AGN) which has zero mean and a variance of $\sigma^2_i$. $y_i$ can be divided into two parts with the help of PS ratio for ID and EH. We can let $\Gamma = \sum_{l=1}^{K_m} w_l x_l$ and two signal fractions are expressed as

$$y_i^{ID} = (h_i^H \Gamma + n_i)\sqrt{\gamma_i} + \varepsilon_i', \quad y_i^{EH} = (h_i^H \Gamma + \varepsilon_i)\sqrt{1 - \gamma_i},$$

where $\varepsilon_i'$ denotes AGN with zero mean and a variance of $\sigma^2_i'$ at the ID part of the $i$-th user. The SINR and harvested power can be written as

$$\text{SINR}_i = \frac{\gamma_i |h_i^H w_i|^2}{\gamma_i (\sum_{l \neq i} |h_l^H w_i|^2 + \sigma^2_i) + \sigma^2_i}, \quad E_i = \left(\sum_{i=1}^{K_m} |h_i^H w_i|^2 + \sigma^2_i\right)(1 - \gamma_i),$$

So the power minimization formula can be donated as

$$\min_{w_i, \gamma_i} \|w_i\|^2, \quad s.t. \text{SINR}_i \geq \delta_i, \quad E_i \geq \varphi_i, \quad i \in \{1, 2, \ldots, K_m\},$$

where $\delta_i$ and $\varphi_i$ are the predefined SINR and harvested power, respectively.

Our discussion above is based on obtaining the perfect CSI for all users at the transmitter, which can be converted to SDP by calculation. However, as mentioned before, channel estimation and quantization errors can affect how we obtain perfect CSI at the transmitter, and make this goal extremely difficult to achieve. In the later part of the article, we introduce several channel uncertainty models (CUMs) and carry out research on robust power minimization problems combined with these models, aiming at converting robust optimization problems into SDPs.
3. Robust Power minimization Problem Based on Statistical CUM I

The CUM I can be used in a system where the transmitter can estimate the user’s channel of the uplink, and the channels have correlations for each other, which can be specifically modeled as a joint Gaussian random variable. The channel uncertainty model of the $i$-th user can be expressed as $h_{i} = \hat{h}_{i} + \Delta_{i}$, where $\hat{h}_{i}$ is the estimated channel at the BS, and $\Delta_{i}$ is the error associated with corresponding channel with zero-mean and the error covariance matrix $\mathbf{T}_{i}$. If let $\mathbf{U}_{i} \sim \mathcal{CN}(0, \mathbf{I})$, we also have $\Delta_{i} = \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{U}_{i}$. This optimization problem combined with CUM I can be denoted as

$$\min_{\mathbf{w}_{i}, \gamma_{i}} \sum_{i=1}^{K_{m}} ||\mathbf{w}_{i}||^{2}$$  

s.t. $\Pr \left\{ \frac{\gamma_{i}||\hat{h}_{i} + \Delta_{i}||^{H} \mathbf{w}_{i}}{\left( \sum_{l \neq i} \gamma_{l}||\hat{h}_{i} + \Delta_{i}||^{H} \mathbf{w}_{l} ||^{2} + \sigma_{l}^{2} \right)} \geq \delta_{i} \right\} \geq 1 - P_{i}$,  

$$\Pr \left\{ \sum_{i=1}^{K_{m}} (\hat{h}_{i} + \Delta_{i})^{H} \mathbf{w}_{i} ||^{2} \geq \varphi_{i} \right\} \geq 1 - P_{i}'$$,  

(5)

where $\gamma_{i} \in [0, 1]$, $P_{i}$ and $P_{i}'$ are the outage probability of the SINR and EH constraints which be bounded in (0, 1), respectively. Due to the influence of $\mathbf{w}_{i}$, $P_{i}$ and $P_{i}'$, if we don’t convert the constraints into deterministic constraints, then (5) cannot be converted into conventional convex optimization problems to solve. We can achieve this transformation with the help of methods such as Bernstein-type inequality.

If we define some algebraic transformations and if we defined that $\mathbf{W}_{i} = \mathbb{E}\{\mathbf{w}_{i} \mathbf{w}_{i}^{H}\}$, $\mathbf{W} = \sum_{i=1}^{K_{m}} \mathbf{W}_{i}$, $\mathbf{V}_{k} = \mathbf{W}_{i} \gamma_{i}^{-1} - \sum_{l \neq i} \mathbf{W}_{l}$, (5) can be equivalently reformulated as

$$\min_{\mathbf{W}_{i} \geq 0, \gamma_{i}} \sum_{i=1}^{K_{m}} \text{Tr} (\mathbf{W}_{i})$$  

s.t. $\Pr \left\{ u_{i}^{H} \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{V}_{i} \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{U}_{i} + \hat{h}_{i}^{H} \mathbf{V}_{i} \hat{h}_{i} + 2 \Re \{ u_{i}^{H} \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{V}_{i} \hat{h}_{i} \} \geq \sigma_{i}^{2} + \frac{\sigma_{i}^{2}}{\gamma_{i}} \right\} \geq 1 - P_{i}$,  

$$\Pr \left\{ u_{i}^{H} \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{V}_{i} \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{U}_{i} + \hat{h}_{i}^{H} \mathbf{W}_{i} \hat{h}_{i} + 2 \Re \{ u_{i}^{H} \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{W}_{i} \hat{h}_{i} \} \geq \frac{\varphi_{i}}{1 - \gamma_{i}} - \sigma_{i}^{2} \right\} \geq 1 - P_{i}'$$,  

(6)

Then (6) can be reformulated as following by employing the Lemma in [16],

$$\min_{\mathbf{W}_{i} \geq 0, \Theta} \sum_{i=1}^{K_{m}} \text{Tr} (\mathbf{W}_{i})$$  

s.t. $\Theta, s_{i} \mathbf{I} + \mathbf{F}_{i} \geq 0$,  

$$\left\| \begin{bmatrix} \text{vec}(\mathbf{F}_{i}) \\ \sqrt{2} \mathbf{r}_{i} \end{bmatrix} \right\|_{2} \leq t_{i}, q_{i} \mathbf{I} + \mathbf{G}_{i} \geq 0, \left\| \begin{bmatrix} \text{vec}(\mathbf{G}_{i}) \\ \sqrt{2} \mathbf{r}_{i} \end{bmatrix} \right\|_{2} \leq r_{i},$$  

$$\text{Tr} (\mathbf{F}_{i}) - \sqrt{2} \ln (P_{i}) \mathbf{r}_{i} + \ln (P_{i}) s_{i} + \text{Tr} (\hat{h}_{i}^{H} \mathbf{V}_{i} \hat{h}_{i}) \geq \sigma_{i}^{2} + \frac{\sigma_{i}^{2}}{\gamma_{i}},$$  

$$\text{Tr} (\mathbf{G}_{i}) - \sqrt{2} \ln (p_{i}') s_{i} + \ln (p_{i}') q_{i} + \text{Tr} (\hat{h}_{i}^{H} \mathbf{W}_{i} \hat{h}_{i}) \geq \frac{\varphi_{i}}{1 - \gamma_{i}} - \sigma_{i}^{2},$$  

$$\{0 \leq \gamma_{i} \leq 1, t_{i} \geq 0, s_{i} \geq 0, r_{i} \geq 0, q_{i} \geq 0 \} \in \Theta, i = 1, \ldots, K_{m}.$$  

(7)

where $\mathbf{F}_{i} = \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{V}_{i} \mathbf{T}_{i}^{\frac{1}{2}}$, $\mathbf{f}_{i} = \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{V}_{i} \hat{h}_{i}$, $\mathbf{G}_{i} = \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{W} \mathbf{T}_{i}^{\frac{1}{2}}$, $\mathbf{g}_{i} = \mathbf{T}_{i}^{\frac{1}{2}} \mathbf{W} \hat{h}_{i}$. The problem (5) is finally transformed into a convex problem (7) and the interior-point method can be used to solve this problem efficiently.
4. Robust Power minimization Problem Based on Statistical CUM II

The channel uncertainty model (CUM) II can be used for systems with uplink and downlink reciprocity and the transmitter can estimate the user’s channel through the uplink, and the channels are not related at the same time, because the covariance matrix error can be considered as multiple independent error sources are generated, so specifically the elements in each channel error covariance matrix are modeled as independent Gaussian random variables with zero mean and variance $\Phi_{i}$. The above assumption can also be used in the case of $\Phi_{i}$'s statistically related entries with unequal variances. The CUM II can be represented as $\mathbf{H}_{i} = \mathbf{\hat{H}}_{i} + \Phi_{i}$, where $\mathbf{\hat{H}}_{i} = \mathbf{\hat{h}}_{i}\mathbf{\hat{h}}_{i}^{H}$ is the estimated channel covariance matrix at the BS, $\Phi_{i}$ represents an error covariance matrix that can describe the degree of mismatch between the estimated and the true channel covariance matrix, and it is also a Gaussian uncertainty matrix with real and complex elements, which can be assumed that the mean and variance of complex elements are zero and $\zeta_{i}^{2}$.

Based on the CUM II, the SINR $i$ and $E_{i}$ can be rewritten as follows:

$$E_{i} = \left(\sum_{l=1}^{K_{m}} w_{l}^{H}(\mathbf{\hat{H}}_{i} + \Phi_{i})w_{l} + \sigma_{\epsilon_{i}}^{2}\right)(1 - \gamma_{i}), \quad \text{SINR}_{i} = \frac{\gamma_{i} w_{l}^{H}(\mathbf{\hat{H}}_{i} + \Phi_{i})w_{l}}{\gamma_{i} \sum_{l \neq i} w_{l}^{H}(\mathbf{\hat{H}}_{i} + \Phi_{i})w_{l} + \sigma_{\epsilon_{i}}^{2}}$$  \hspace{1cm} (8)

For simplicity, we can let $\varrho_{i} = \sigma_{i}^{2} + \sigma_{\epsilon_{i}}^{2} \gamma_{i}^{-1}$, $\varrho_{i}^{*} = \varphi_{i}(1-\gamma_{i})^{-1} - \sigma_{i}^{2}$, and the robust power minimization problem combined with CUM II based constraints is donated as

$$\min_{\mathbf{W}_{i} \gamma_{i}} \sum_{i=1}^{K_{m}} \text{Tr}({\mathbf{W}_{i}})$$

$$\text{s.t.} \text{Pr}\left\{ \text{Tr}\left[ (\mathbf{\hat{H}}_{i} + \Phi_{i}) \mathbf{V}_{i} \right] \geq \varrho_{i} \right\} \geq 1 - P_{i}, \text{Pr}\left\{ \text{Tr}\left[ (\mathbf{\hat{H}}_{i} + \Phi_{i}) \mathbf{W} \right] \geq \varrho_{i}^{*} \right\} \geq 1 - P_{t}.$$ \hspace{1cm} (9)

Due to the existence of channel uncertainty, the above problem is still a non-convex problem. Therefore, we try to use the Lemma for further operations, and the two constraints in the previous formula can be donated as

$$\int_{-\infty}^{+\infty} \frac{1}{\sigma_{\epsilon_{i}^{2}}^{2} + \sigma_{\epsilon_{i}^{2}}^{2}} \frac{1}{2\pi \zeta_{i} \| \mathbf{V}_{i} \|} \exp \left( - \frac{(\alpha - \text{Tr}(\mathbf{\hat{H}}_{i} \mathbf{V}_{i}))^{2}}{2 \sigma_{\epsilon_{i}^{2}}^{2} \| \mathbf{V}_{i} \|^{2}} \right) d\alpha \geq 1 - P_{i}, \quad (10a)$$

$$\int_{-\infty}^{+\infty} \frac{1}{\sigma_{\epsilon_{i}^{2}}^{2} + \sigma_{\epsilon_{i}^{2}}^{2}} \frac{1}{2\pi \zeta_{i} \| \mathbf{W} \|} \exp \left( - \frac{(\beta - \text{Tr}(\mathbf{\hat{H}}_{i} \mathbf{W}))^{2}}{2 \sigma_{\epsilon_{i}^{2}}^{2} \| \mathbf{W} \|^{2}} \right) d\beta \geq 1 - P_{t}, \quad (10b)$$

where $\alpha_{i} \sim N_{R}(\text{Tr}(\mathbf{\hat{H}}_{i} \mathbf{V}_{i}), \gamma_{i}^{2} \text{Tr}(\mathbf{V}_{i} \mathbf{V}_{i}^{H}))$, $\beta_{i} \sim N_{R}(\text{Tr}(\mathbf{\hat{H}}_{i} \mathbf{W})), \gamma_{i}^{2} \text{Tr}(\mathbf{W} \mathbf{W}^{H}))$ and $N_{R}$ is the Gaussian distribution with real elements.

When considering the performance of common communication systems, the outage probability can usually be used as a reference index to measure the performance. As the promotion of performance, the outage probability is close to zero from less than 0. The Gaussian error function $\text{erf}(\cdot)$ will be considered and the robust power minimization problem can be written as

$$\min_{\mathbf{W}_{i} \gamma_{i}} \sum_{i=1}^{K_{m}} \text{Tr}({\mathbf{W}_{i}})$$

$$\text{s.t.} \text{Tr}(\mathbf{\hat{H}}_{i} \mathbf{V}_{i}) + \varrho_{i} \geq \sqrt{2} \gamma_{i} \text{erf}^{-1}[2(1-P_{i})-1] \| \mathbf{V}_{i} \|,$$

$$\text{Tr}(\mathbf{\hat{H}}_{i} \mathbf{W}) - \varrho_{i}^{*} \geq \sqrt{2} \gamma_{i} \text{erf}^{-1}[2(1-P_{t})-1] \| \mathbf{W} \|,$$  \hspace{1cm} (11)
The optimization problem is finally transformed into (11) which is a convex problem and the interior-point method can be used to solve this problem efficiently.

5. Simulation Results

In this paper, a health monitoring system with a transmitter equipped with $M = 5$ antennas and $K_m = 3$ users is simulated, and the number of antenna is one for each user. The E and SINR can be assumed as 10 dBm and 5 dB. The channel error variance can be assumed that $\zeta_i = \zeta_i \| \hat{h}_i \|_2^2$, and the channel error covariance is $T_i = \zeta_i^2 I$, where $\zeta^2 = 0.01$. $\sigma_i^2$ and $\sigma_i^2$ can be setted as $-60$ dBm and $-50$ dBm, respectively, and both $P_i$ and $P_i'$ are 0.05.

As we can see in Figure 1, the performance of the robust transmit power is shown when there are different SINRs and we compare robust power minimization problem combining deterministic and statistical channel uncertainties. It can be seen from the figure that the transmit power increases with a predefined SINR when the target harvest power is 10 dBm or 0 dBm. In addition, in terms of power consumption, the $S$-Procedure and Bernstein-type inequality consume more energy than a robust solution based on a Gaussian error function. In Figure 2, the transmit power with different Es when robust scheme have the perfect CSI can be seen. As the harvested power increasing when SINR is 5dB or 0 dB, it can be sure that the trend of transmit power has increasingly upward. Robust scheme which is based on Gaussian error function has the better performance than others.

Figure 3 gives the performance comparison of the average PS ratio which has different SINRs. As the increasing of SINR, the trend of average PS ratio is gradually rising, which also shows that the proportion of observation signals used for ID is gradually increasing. A robust scheme based on a random distribution requires more observations than robust schemes with a lower SINR for ID. When the value of SINR is large, the fraction is less than another robust scheme. Figure 4 illustrates that the PS ratio decreases gradually as the harvested power increases, which indicates that the observation fraction for EH becomes smaller. In addition, other schemes have bigger PS ratio of the observation for EH than the scheme which is based on random distribution.

6. Conclusion

The robust power minimization problem of SWIPT in the MISO health monitoring system combined with different CUMs is proposed in this paper. The robust power minimization problem has been formulated constraints with different CUMs which have SINR and EH constraints. But we cannot directly solve it because this problem is non-convex, so the robust scheme which is based on $S$-Procedure can be introduced to relax this problem for norm-bounded
channel uncertainties. Simulation results show that compared with other methods, our proposed method can significantly improve system performance.

Reference
[1] A. Pantelopoulos and N. G. Bourbakis, “A survey on wearable sensor based systems for health monitoring and prognosis,” IEEE Trans. Syst., Man, Cybern. C, Appl. Rev., vol. 40, no. 1, pp. 1–12, Jan. 2010.
[2] R. Zhang and C. Ho, “MIMO broadcasting for simultaneous wireless information and power transfer,” IEEE Trans. Wireless Commun., vol. 12, no. 5, pp. 1989–2001, May 2013
[3] E. Hossain, M. Rasti, H. Tabassum, and A. Abdelnasser, “Evolution toward 5G multi-tier cellular wireless networks: An interference management perspective,” IEEE, Wireless Commun., vol. 21, no. 3, pp. 118–127, Jun. 2014.
[4] V. Raghunathan, S. Ganeriwal, and M. Srivastava, “Emerging techniques for long lived wireless sensor networks,” IEEE, Commun. Mag., vol. 44, no. 4, pp. 108–114, Apr. 2006.
[5] L. Varshney, “Transporting information and energy simultaneously,” in Proc. 2008 IEEE Int. Symp. Inf. Theory, pp. 1612–1616, July, 2008.
[6] P. Grover and A. Sahai, “Shannon meets tesla: Wireless information and power transfer,” in Proc. 2010 IEEE Int. Symp. Inf. Theory, pp. 2363–2367, June, 2010.
[7] R. Zhang and C. K. Ho, “MIMO broadcasting for simultaneous wireless information and power transfer,” IEEE Trans. Wireless Commun., vol. 12, pp. 1989–2001, May 2013.
[8] Q. Shi, L. Liu, W. Xu, and R. Zhang, “Joint transmit beamforming and receive power splitting for MISO SWIPT systems,” IEEE Trans. Wireless Commun., vol. 13, no. 6, pp. 3269–3280, Jun 2014.
[9] S. Timotheou, I. Krikidis, G. Zheng, and B. Ottersten, “Beamforming for MISO interference channels with QoS and RF energy transfer,” IEEE Trans. Wireless Commun., vol. 13, no. 5, pp. 2646–2658, May 2014.
[10] M. Khandaker and K.-K. Wong, “Swipt in MISO multicasting systems,” IEEE Wireless Commun. Lett., vol. 3, no. 3, pp. 277–280, Jun. 2014.
[11] L. Liu, R. Zhang, and K.-C. Chua, “Wireless information and power transfer: A dynamic power splitting approach,” IEEE Trans. Commun., vol. 61, no. 9, pp. 3990–4001, Sept. 2013.
[12] K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, “Robust interference control techniques for multi-user cognitive radios using worst-case performance optimization,” in Proc. Asilomar Conf. Sign., Syst. and Comp., Pacific Grove, CA, pp. 378–382, Oct. 2008.
[13] K. Cumanan, Z. Ding, B. Sharif, G. Tian, and K. Leung, “Secrecy rate optimizations for a MIMO secrecy channel with a multiple-antenna eavesdropper,” IEEE Trans. Vehicular Technol., vol. 63, no. 4, pp. 1678–1690, May 2014.
[14] Q. Li, A.-C. So, and W.-K. Ma, “Distributionally robust chance-constrained transmit beamforming for multiuser MISO downlink,” in Proc. IEEE, ICASSP, pp. 3479–3483, May 2014.
[15] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge, UK: Cambridge University Press, 2004.
[16] K.-Y. Wang, A. M.-C. So, T.-H. Chang, W.-K. Ma, and C.-Y. Chi, “Outage constrained robust transmit optimization for multiuser MISO downlinks: Tractable approximations by conic optimization,” arXiv preprint arXiv:1108.0982, 2011.