Numerical modeling of a cutting torch

B R Mancinelli¹, F O Minotti¹,² and H Kelly¹,²
¹ Grupo de Descargas Eléctricas, Departamento Ing. Electromecánica,
Universidad Tecnológica Nacional, Regional Venado Tuerto, Laprida 651,
Venado Tuerto (2600), Santa Fe, Argentina
² Instituto de Física del Plasma (CONICET), Departamento de Física, Facultad de
Ciencias Exactas y Naturales (UBA) Ciudad Universitaria Pab. I, (1428) Buenos
Aires, Argentina

E-mail: bmancinelli@arnet.com.ar

Abstract. A two-dimensional turbulent model of a low-current intensity (30 A) cutting
plasma torch was developed using the local thermodynamic equilibrium approximation.
A good agreement was found between measured and modelled results of plasma
temperature and velocity, the latter of which has not been previously reported in the
literature for high energy density torches. The cutting performance was also studied in
terms of the heat flux to the work-piece and the value of the force exerted by the
impinging plasma jet.

1. Introduction
Plasma hydrodynamic modeling by numerical simulation in cutting torches is a common tool to
predict the values of the fundamental physical quantities, namely the plasma temperature, the
particles concentration and the fluid velocity. These numerical codes are employed to understand
the relevant physical processes ruling the plasma behavior in order to interpret the experimental
results of several plasma diagnostics, and ultimately to obtain optimized designs of such devices
[1].

The purpose of this work is to present a turbulent two-dimensional local thermodynamic
equilibrium (LTE) [2] model of a 30 A oxygen high-energy density cutting torch. Two
configurations were used: the experimental one in which the arc is transferred to the rim of a
rotating anode 6 mm away, and the cutting one in which a solid work-piece, acting as the anode,
is 3 mm away from the nozzle exit. In the following section the plasma torch, model assumptions,
governing equations, boundary conditions and the physical details of the model are presented.
The calculated distributions of temperature and velocity and its comparison with the experimental
data together with the calculated values of the heat flux to the work-piece and the force exerted
by the impinging plasma jet are shown in section 3. The conclusion is summarized in section 4.

2. Mathematical model

2.1 Computational domain
The schematic of the modeled domain for the experimental configuration simulation is presented
in figure 1. A mass flow rate of 0.71 g s⁻¹ with a vortex injection that leads to a ratio of the
azimuthal to the axial inlet velocity of \( \tan(13^\circ) = 0.23 \) was used at the torch inlet CD. More details on the torch characteristics can be found elsewhere [3].

**Figure 1.** Cutting torch computational domain.

### 2.2 Model assumptions

(a) The plasma flow is two-dimensional and axisymmetric.
(b) The plasma is considered as a Newtonian fluid following the Navier-Stokes equation.
(c) The plasma gas is assumed to be pure oxygen in LTE, whose thermodynamic and transports coefficients were calculated by Murphy [4].
(d) Hall currents and gravitational effects are considered negligible.
(e) In the energy equation the viscous dissipation term is considered negligible.
(f) The anode was considered as a porous free boundary characterized by its electrostatic potential in the experimental configuration, while was considered as an extensive solid plate in the cutting one.

### 2.3 Governing equations

The fluid part of the thermal plasma model can be expressed as:

**Total mass conservation**

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0.
\]  

**Momentum conservation**

\[
\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + p \mathbf{I} - \mathbf{T}) - \mathbf{J} \times \mathbf{B} = 0.
\]

**Internal energy conservation**

\[
\frac{\partial (\rho e)}{\partial t} + \nabla \cdot (\rho e \mathbf{u} + \mathbf{q}) - \mathbf{J} \cdot \mathbf{E} + p \nabla \cdot \mathbf{u} + 4\pi \varepsilon N \frac{p}{P_{ATM}} = 0,
\]

where \( \rho \) represents the total mass density, \( \mathbf{u} \) the fluid velocity (having, axial \(-u_x\), radial \(-u_y\) and azimuthal \(-u_z\) components), \( p \) the pressure, \( \mathbf{I} \) the identity tensor, \( \mathbf{T} \) the stress tensor, \( \mathbf{J} \) the current density, \( \mathbf{B} \) the magnetic field (only the azimuthal component was considered), \( e \) the internal energy, \( \mathbf{q} \) the total heat flux, \( \mathbf{E} \) the electric field and \( \varepsilon N \) the plasma radiation net emission coefficient (NEC) [5].

Two further equations are required to describe the electromagnetic part of the plasma model. The first is the current continuity equation

\[
\nabla \cdot \mathbf{J} = 0,
\]

where
\[ J = -\sigma \nabla \phi, \]  

and the second is one of Maxwell’s equations

\[ \nabla \times B = \mu_0 J, \]

where \( \sigma \) is the electric conductivity, \( \phi \) is the electrostatic potential and \( \mu_0 \) the magnetic permeability of free space.

The total heat flux in (3) describes the heat transported by conduction and the enthalpy transport by mass diffusion, and is defined as

\[ \bar{q} = -\kappa_e \nabla T + \Gamma_e \rho_e, \]

where \( \kappa_e \) is the effective thermal conductivity and \( \Gamma_e \) is the electron mass diffusion that can be approximated by

\[ \Gamma_e = -\frac{m}{e} J, \]

where \( e' \) is the elementary electric charge and \( m \) is the electron mass. Equation (8) neglects the charge transported by ions. In (7) \( h_e = 5 k_B T / (2m) \) represents the specific electron enthalpy \( (k_B \) is the Boltzmann’s constant).

The effective viscosity is

\[ \mu_e = \mu_i + \mu_t, \]

and the effective thermal conductivity is

\[ \kappa_e = \kappa + \frac{\mu_i C_p}{P_r}, \]

where \( C_p \), \( \mu_i \) and \( \kappa \) are the plasma specific heat at constant pressure, viscosity and thermal conductivity, respectively. The turbulent Prandtl number was adopted as \( P_r = 1 \).

The turbulent viscosity \( \mu_t \) was calculated taking into account the effect of the vortex injection \[1\]

\[ \mu_t = \rho \lambda^2 \left\{ \left( \frac{\partial u_x}{\partial y} \right)^2 + \left( y \frac{\partial}{\partial y} \left( \frac{u_x}{y} \right) \right)^2 \right\}^{1/2}, \]

where the Prandtl mixing length was chosen as \[6\]

\[ l_m \equiv c \lambda, \]

here \( c \) is a parameter and \( \lambda \) is a local thermal radius which characterizes the boundary of the high velocity arc core, defined as the radial distance from the axis to the point at 2000 K.

The NEC in (4) was taken for one atmosphere and a plasma radius of 0.5 mm \[5\].

2.4 Boundary conditions

Table 1 summarizes the prescribed values of the physical quantities (or its spatial derivatives) on the boundaries shown in figure 1. At the hafnium insert AB the maximum value of the axial current density on the axis of the geometry was limited to \( \leq 170 \) A mm\(^{-2}\) \[1\]. Besides, at the interface between the plasma and the anode, in order to maintain the conservation of the energy flux and current intensity at this boundary, the following relations (neglecting radiation \[1\]) were used to calculate the local thermal and electric conductivities

\[ -\kappa \left( \frac{\partial T}{\partial x} \right)_{\text{anode}} = -\kappa \left( \frac{\partial T}{\partial x} \right)_{\text{plasma}} + J_x \left( \frac{5}{2} \frac{k_B}{e} (T - T_{\text{anode}}) + \phi_a + \phi_w \right), \]
here $\varphi_A$ and $\varphi_w$ are the anode voltage drop and the anode work function, respectively.

### Table 1. Boundary conditions.

| $p$   | $u$    | $T$    | $\varphi$ |
|-------|--------|--------|-----------|
| AB    | 0      | 3500 K | $-$       |
| BC    | 0      | 500 K  | $-$       |
| CD    | mass flow 0.71 g s$^{-1}$ | 300 K  | $\frac{\partial \varphi}{\partial x} = 0$ |
| DE    | 1 atm  | 500 K  | $-$       |
|      | $\frac{\partial u_x}{\partial y} = \frac{\partial u_y}{\partial y} = \frac{\partial u_z}{\partial y} = 0$ | $\frac{\partial \varphi}{\partial y} = 0$ |
| EF    | $\frac{\partial u_x}{\partial x} = \frac{\partial u_y}{\partial x} = \frac{\partial u_z}{\partial y} = 0$ | 300 K  | $\frac{\partial \varphi}{\partial x} = 0$ |
| FG (exp. conf.) | $\frac{\partial p}{\partial x} = 0$ | 1500 K | $\phi = 0$ |
| FG (cutting conf.) | $u_x = u_y = u_z = 0$ |     | $\phi = 0$ |

2.5 **Numerical aspects**

The unsteady form of the model equations was solved using a time-marching method [7,8]. The specific values used for the initial guesses did not impact the final converged results. The set of governing equation was discretized in time using a Taylor series first-order accuracy and in space using the finite volume method and solved with the given boundary conditions on a $81 \times 15$ non-uniform internal grid and $39 \times 47$ non-uniform external grid, by using the predictor-corrector algorithm [7,8]. The time-step used in the time-marching algorithm was chosen so that the CFL criterion was fulfilled [7,8]. The calculation was stopped when the relative variation of the plasma variables between two consecutive time iterations was $< 10^{-3}$. This convergence criterion was found to be sufficient. The accuracy of the calculations was tested by repeating them with a $38 \times 15$ internal grid and $19 \times 47$ external grid. The change in the plasma temperature was everywhere less than 15 %, while the changes in the axial velocity were less than 20 %. The finer grid was then used for generating the results to be presented in the following Section.

3. **Model Results**

Figures 1 a) and b) show the comparison among the theoretical profiles (corresponding to $c = 0.08$) and the experimentally derived temperature and velocity profiles, respectively. Calculations correspond to the same experimental configuration that was used in the experiments [9-12]. As it can be seen, the model results are in good agreement with both, temperature and velocity experimental data. Model validations for the high velocity values characteristic of the high energy density torches have not been reported previously.

The theoretical distribution of the heat flux to the work-piece (located at 3 mm from the nozzle exit) is given in figure 2. As it can be seen, the energy carried by the electron current is the dominant term at the centre of the distribution reaching about $10^8$ W m$^{-2}$, while the conduction term prevailing at larger radial values. The calculated power delivered by the plasma flow at the
work-piece was about 1800 W. The calculated pressure force due to the impinging high-velocity plasma jet was around 1 N.

Figure 1. Radial profiles of the plasma temperature (a) and velocity (b) predicted by the model at 3.5 mm from the nozzle exit together with experimental data.

Figure 2. Radial profile of the heat flux to the anode located at 3 mm from the nozzle exit.

4. Conclusions
A turbulent two-dimensional LTE model of a 30 A oxygen high-energy density cutting torch has been reported. The calculated distributions of temperature and velocity in the nozzle exit-anode gap have been validated using experimental data. Within the experimental uncertainties, it was found that the model reproduces both the experimental data of velocity and temperature. The cutting process performance was studied in terms of the calculated values of the heat flux to the work-piece and the force exerted by the impinging plasma jet.

Acknowledgements
This work was supported by grants from the Universidad de Buenos Aires (PID X108), CONICET (PIP 5378) and Universidad Tecnológica Nacional (PID Z 012). F. O. M. and H. K. are members of the CONICET.

References
[1] Gleizes A, Gonzalez J J and Freton P 2005 J. Phys. D: Appl. Phys. 38 R153.
[2] Boulos M, Fauchais P and Pfender E 1994 *Thermal plasmas fundamentals and applications* vol.1 (Plenum Press).
[3] Prevosto L, Kelly H and Mancinelli B 2008 IEEE Trans. Plasma Sci. **36** 263.
[4] Murphy A B and Arundell C J 1994 Plasma Chem. Plasma Process. **14** 451.
[5] Naghizadeh-Kashani Y, Cressault Y and Gleizes A 2002 Phys. D: Appl. Phys. **35** 2925.
[6] Yan J D, Nuttall K I and Fang M T C 1999 J. Phys. D: Appl. Phys. **32** 1401.
[7] Ferziger J H and Peric M 2002 *Computational Methods for Fluid Dynamics*, (Springer-Verlag).
[8] Fletcher C A J 1991 *Computational Techniques for Fluid Dynamics* vol 1, (Springer-Verlag).
[9] Prevosto L, Kelly H and Minotti F O 2008 IEEE Trans. Plasma Sci. **36** 271.
[10] Prevosto L, Kelly H and Mancinelli B 2009 IEEE Trans. Plasma Sci. **37** 1092.
[11] Prevosto L, Kelly H and Mancinelli B 2010 J. Appl. Phys. **107** 023304.
[12] Prevosto L, Kelly H and Mancinelli B 2009 J. Appl. Phys. **106** 053308.