Filling factors and ionized masses in planetary nebulae

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Abstract. We calculate filling factors (ε) and ionized masses (Mᵢ) for a total of 84 galactic and extragalactic planetary nebulae (PNe) at known distances. To do these calculations, from the equation of energy propagation within a gaseous nebula, we have chosen forbidden line electron densities, observed angular diameters, and Hβ fluxes, from the most recent measurements available in the literature. Statistical analysis on the distributions of ε and Mᵢ show that (1) the ranges of values of these parameters is wider than what was previously found; (2) the mean value of the filling factor is between 0.3 and 0.4, for the different sets; (3) the mean value of the ionized mass is between 0.1 and 0.25 M☉, 0.2 M☉ representing an upper limit to the ionized mass for galactic disk PNe, and a typical value for galactic bulge and extragalactic PNe; (4) a clear correlation between ε and the dimensions of the PNe was not found when distance-independent sets of PNe were used; (5) for extragalactic PNe, where distance errors are not a factor, the filling factors and the ionized masses anticorrelate tightly with the electron densities. The results indicate that the modified Shklovsky distance method is correct.

Key words: Galaxy (the): the Bulge of – Galaxies : Magellanic Clouds – Planetary nebulae : general

1. Introduction

Statistical studies of planetary nebulae (PNe) have been largely developed in the recent years, giving results of great significance toward the knowledge of the final stages of stellar evolution. In particular, the analysis of samples of PNe at known distances, such as galactic bulge and Magellanic Cloud PNe, have been used to derive an increasingly complete picture of the overall nebular properties (e. g. Stasińska et al. 1991, heretofore STAS91, Dopita 1992). Nonetheless, in many of these statistical studies the important aspect of the filling factor (ε) has often been overlooked, generally because of the impossibility of estimating the exact value of this parameter for each PN of a large set.

The filling factor is a fundamental parameter, as it labels each individual PN by telling which fraction of the nebular volume is filled by ionized gas. It has been introduced by Osterbrock & Flather (1959) to explain the existing discrepancies between observational data and theoretical models in the case of the Orion nebula. Usually, in statistical studies, the filling factor is arbitrarily assumed to be constant for all PNe (for galactic disk PNe, a value equal to 0.65 has been adopted (e. g. Kaler 1970) and a value equal to 0.7 or 1 is generally taken for the extragalactic PNe (Wood et al. 1986; Dopita et al. 1988).

We do believe that this cannot be a satisfactory approximation. In fact, PNe do come from progenitors having a mass comprised in a wide range of values (0.8 ≤ Mₓ ≤ 8 M☉), Iben & Renzini (1983) and are affected by mass loss processes occurring at different rates according to their mass (Renzini 1989). PNe show different morphological structures and different degrees of ionization. Only the direct calculation of ε may provide a more realistic study of PNe. For the same reasons assuming a constant value for the ionized mass of a set of PNe could be severely misleading.

Pioneer work on the subject of the filling factor has been performed by Seaton (1966), who obtained the filling factor of fourteen planetary nebulae from photographic plates and drawings (average ε = 0.63). Later Webster (1969) found an average value of 0.8 for forty–nine PNe by using details of surface brightness and PNe dimensions. Similar values were obtained by O'Dell (1962). On the other hand, Torres–Peimbert & Peimbert (1977) calculated much smaller values of ε. More recently, Mallik & Peimbert (1988) have calculated the filling factor of 35 galactic PNe at known distance (independent from statistical methods) and found that ε, calculated to be between 0.001 and 1, anticorrelates with the nebular dimensions. The average value of ε found by Mallik & Peimbert (1988) was 0.28. Very recently Kingsburgh & Barlow (1992) and Kingsburgh & English (1992) calculated filling factors for different types of galactic nebulae, finding an average ε of of about 0.35 when excluding peculiar objects with ε > 1.

On these grounds, we propose to extend the calculation of ε and Mᵢ to several sets of galactic and extragalactic PNe whose distances are known independently from statistical methods. The main goal of this paper is to set constraints to ε and Mᵢ, to define the mean and most probable value for both parameters, and to examine possible differences among the distributions of these parameters in different stellar populations. In §2 we set the theoretical formulation to calculate ε and Mᵢ. In §3 we present the input physical parameters of our four sets of PNe, including galactic PNe, galactic bulge PNe. Large Magellanic Cloud PNe and Small Magellanic Cloud PNe. Results and correlations are reported in §4. In §5 we draw conclusions and future perspective of this study.
2. Theoretical formulation

In order to set the stage for our calculations let us consider a hydrogen–rich nebula (we chose $y=N(\text{He})/N(\text{H})=0.1$) which gets ionized by a star. The nebula is in thermal balance, and each ionization is balanced by a recombination. The equation that describes the propagation of energy within this gaseous nebula can be written as:

$$F_{H\beta} = \frac{(\alpha_{\beta} h \nu_{\beta})}{(4\pi d^2)} \int_0^{R_N} \varepsilon \cdot N_e(r)N_p(r) \cdot 4\pi r^2 dr \text{ erg cm}^{-2}\text{s}^{-1}, \quad (1)$$

where $F_{H\beta}$ is the Balmer $\lambda4861$ Å flux corrected for extinction, $\alpha_{\beta}$ is the hydrogen recombination coefficient, $\nu_{\beta}$ is the frequency of the $H\beta$ line, $d$ is the distance to the PN, $N_e(r)$ is the electron density, $N_p(r)$ the ion density, and $\varepsilon$ the filling factor, that represents which fraction of the nebular volume is occupied by ionized gas. The integration is performed over the whole volume of the nebula, $R_N$ being the nebular radius. The recombination coefficient can be written as:

$$\alpha_{\beta} = 9.69 \cdot 10^{-11} \cdot T_e^{-0.88} \text{ cm}^3 \text{s}^{-1}, \quad (2)$$

which represents an interpolation from values found in the literature (Brocklehurst 1971, Table V).

In the approximation that the density is constant throughout the nebula, from Eq. (1) we write:

$$\varepsilon \approx 2.47 \cdot 10^6 \cdot F_{H\beta} \left( \frac{1}{\alpha_{\beta} \theta^2 N_e d} \right). \quad (3)$$

In Eq. (3) the approximation of fine angles holds (i.e. the nebular radius $R_N \approx 6d$, where the angular radius $\theta$ is expressed in arcsec and the distance in kpc) and it has been assumed that

$$k = \frac{N_e}{N_p} \approx 1.15, \quad (4)$$

as to keep into account partial double–ionization of helium atoms.

To calculate the ionized masses we assume simple spherical geometry,

$$M_i = \frac{4\pi R_N^3}{3} \cdot \varepsilon N_e m_p \left( 1 + \frac{4y}{k} \right) \text{ g}, \quad (5)$$

where $y=0.1$ and $k=1.15$. If we use Eq. (3) for $\varepsilon$, $M_i$ turns out to be independent on $\theta$:

$$M_i \approx 3.45 \cdot 10^{-2} \cdot \left( \frac{F_{H\beta} d^2}{\alpha_{\beta} N_e} \right) M_\odot. \quad (6)$$

We then use Eqs. (3) and (6) through our calculation.

3. Input Database

We calculate $\varepsilon$ and $M_i$ for 84 PNe: twenty-nine belonging to the Galaxy, twelve to the galactic bulge, nineteen to the Small Magellanic Cloud, and twenty-four to the Large Magellanic Cloud. Our PNe database includes only known distance objects, that have been selected from the literature among those with electron density calculated by forbidden line analysis, and whose $H\beta$ fluxes and angular radii are reliably known. Following, we examine the parameter space for each set of PNe.

3.1. Galactic Planetary Nebulae

In Table 1 we list our choice of galactic PNe. Column (1) gives the PN name. In columns (2) and (3) the $H\beta$ flux (in units of erg cm$^{-2}$s$^{-1}$) and the extinction logarithmic factor, for which the flux needs to be corrected, are reported. Fluxes are already in the modern photometric scale as suggested in Shaw & Kaler (1982) and as explained in Cahn et al. (1992, hereafter CKS92).

The electron densities quoted in column (4) of Tab. 1 are, for the most part, obtained by averaging the density values obtained from the different ions, as calculated by Stanghellini & Kaler (1989, hereafter SK89). When these densities were not available, we have used other sources, as given in the footnotes to the Table. Densities from SK89 are preferable since they have been calculated by using the most recent atomic parameters. All SK89 densities listed in column (4) belong to the low error domain of the intensity ratios–densities curves. There are two galactic PNe for which the calculation of $\varepsilon$ and $M_i$ has been performed, but their densities are within the higher error domain, thus they have not been reported in the table nor counted for the statistics.

In column (5) we give the electron temperature from Kaler (1986), or from other sources listed in the footnotes. We calculated a weighted average of the [N II] and [O III] electron temperatures according to the electron densities that have been used to get the mean electron density in column (4), and by taking into account the fact that temperatures derived from the intensity of [N II] lines are well representative of regions of low ionization where oxygen and sulfur are easily ionized, and that those derived from the intensity of [O III] are instead well representative of regions of high ionization, where argon and chlorine are preferentially ionized (Torres–Peimbert & Peimbert 1977). In the case that no temperatures have been found in the literature, we adopt a standard value of 10000 K. This value is very close to the real value of these low density environments, and represents the typical average value guaranteed in the actual case of thermal balance (Osterbrock 1989).

In column (6) we give angular radii, and in column (7) the distances to the nebula which are for the most part reddening distances, the exceptions being some non–LTE model-dependent distances and two cluster memberships (see the footnotes to the Table; reference to CKS92 are relative to the distances there reported on Table 3). Heretofore, we refer to Set 1 as the group of galactic PNe with model-independent distances, and to Set 2 to these galactic PNe whose distances have been calculated by Mendez et al. (1988, heretofore MEA88). In columns (8), (9) and (10) we list the linear radii, and the calculated filling factors and ionized masses.

3.2. Galactic Bulge Planetary Nebulae

We have chosen radio selected objects from the catalogs by Gathier et al. (1983), and Pottasch & Acker (1989), since the radio observation helps to reduce the severe extinction effects present in the case of objects in (or in the direction of) the galactic bulge (Habing et al. 1989). We have taken all the PNe from these two catalogs with no repetition. There is good confidence that the objects of the two catalogs are true PNe (Pottasch 1983; Gathier et al. 1983; Pottasch & Acker 1989).

Table 2 is the analogous to Table 1 for galactic bulge PNe. The electron densities listed in column (4) are the means computed in the same way than for galactic PNe. Electron temperatures, given in column (5), are from Acker et al. (1989b).
3.3. Magellanic Cloud Planetary Nebulae

There are two reasons why we selected Magellanic Clouds PNe. First planetary nebulae of these two external galaxies have been extensively studied in the last ten years, and a wealth of information is ready for us to use. Furthermore, as Dopita & Meatheringham (1990) underline, these objects are at known distance and evolve in a low reddening environment. Our database consist of 19 SMC and 24 LMC PNe, collected from the catalog by Sanduleak et al. (1978) (hereafter SMP78). In fact, these were the only Magellanic Cloud PNe whose necessary physical parameters for the calculation of $\varepsilon$ and $M_i$ were available in the literature.

Tables 3 and 4, structured in the same way as Tab 2, give the required input data and the calculated parameters for Small and Large Magellanic Cloud PNe. Columns (2) and (3) of Tabs. 3 and 4 give H$\beta$ fluxes and logarithmic extinction correction factors; each flux value is an average value of all the values found in literature from Meatheringham et al. 1988, Wood et al. 1987 (hereafter WMMDM), Barlow 1987, Webster 1969 &1983, Aller 1983, and Osmer 1976. The WMDM fluxes have been rejected in the case of SMP 2, 11 and 17 in SMC and of SMP 40 in LMC since they are not in good agreement with the corresponding Meatheringham et al. (1988) quantities (see for further comments the latter paper). All the foretold fluxes are not corrected for the interstellar extinction and still need to be brought into the modern photometric scale, by subtracting 0.02 (Shaw & Kaler 1982).

The logarithmic extinction are from Kaler & Jacoby (1990) and Meatheringham & Dopita (1991). Thanks to the fact that in the Magellanic Clouds the extinction is very low, we have assumed a constant logarithmic extinction factor for those PNe whose extinction have not been measured. We then adopt respectively $c=0.12$ for SMC PNe and $c=0.21$ for LMC PNe, as explained in detail in Kaler & Jacoby (1990). We test the validity of the assumed average values of the extinction factor by estimating $c$ through its definition, though using the color excess of each Cloud calculated in Jacoby et al. (1990). From the definition of $c$,

$$c = \log \left( \frac{F_{H\beta}^{\text{theor}}}{F_{H\beta}^{\text{obs}}} \right) = \frac{A_\lambda (B-V)}{2.5},$$

where $F_{H\beta}^{\text{theor}}/F_{H\beta}^{\text{obs}}$ is the ratio of the corrected and observed H$\beta$ fluxes, $E(B-V)$ is the color excess, and $A_\lambda$ is a function of the wavelength. In the case of SMC we obtain $c=0.09$, in good agreement with the adopted value, and for LMC $c=0.14$.

The electron densities are averages over all possible values reported in the literature. For LMC and SMC PNe, the plasma diagnostics has been performed only with the [O II] $\lambda\lambda 3726 – 3729$ and the [S II] $\lambda\lambda 6717 – 6731$ intensity ratios, therefore the mean densities in columns (4) of tabs. 3 and 4 are calculated from the values from these ions, only as listed in Dopita et al. (1988), Monk et al. (1988), Meatheringham & Dopita (1991), and Barlow (1987). The average values of $N_e$ from Dopita et al. (1988) exclude those values from the highest velocity component as to avoid peculiarities.

The electron temperatures are means of all possible values found in the literature (Monk et al. 1988, and Meatheringham & Dopita 1991). We have taken temperatures calculated from the [N II] intensity ratios since all electron densities come from the [O II] and [S II] lines; missing these temperatures, we have taken the ones determined from the [O III] intensity ratios. A temperature of 10000 K has been assumed when no other reference was found.

In columns (6) of tabs 3 and 4 we list the angular radii. Only a few Magellanic Cloud PNe have well measured radii, given the difficulty to achieve a good resolution of these PNe from ground observations. We have taken all angular diameters of bright and compact objects ($r \leq 0.13$ pc) from Wood et al. (1986) speckle interferometry data, while those of faint and large PNe (for angular diameters larger than 0.7 arcsec) from WMDM ’s direct imaging. Some diameters come from Jacoby et al. (1990) and have been derived with the method used in WMDM. The mean angular diameters of SMP 47, 62 and 78 in LMC are uncertain as they have been calculated from limits with discordant signs.

Physical radii, filling factors and ionized masses are listed in the last three columns. The SMC distance modulus is $m-M=18.8$, equivalent to a distance of 57.5 kpc, while for LMC we have chosen $m-M=18.5$, corresponding to a distance of 50.1 kpc (Kaler & Jacoby 1990).

4. Results

4.1. Statistical analysis of the results

In Figures 1 and 2 we show cumulative distributions of $\varepsilon$ and $M_i$ for our four sets of PNe, where we restrict to values of the filling factor between 0 and 1. In this chapter we always refer to averages and correlations by excluding those objects with $\varepsilon > 1$. We see the off–limit cases in the next section. Let us analyse in detail one set of PNe at the time starting with galactic PNe.

Of the fourteen galactic PNe of Set 1, eleven have $\varepsilon \leq 1$ and for them the mean $\varepsilon$ is equal to 0.37 and the mean $M_i$ is 0.11 $M_\odot$ (averages of $\varepsilon$ and $M_i$ for all sets of PNe are on Table 5).
The derived values of $\varepsilon$ for galactic PNe are much smaller than the ones usually adopted (see §1), but very close to the ones calculated by Mallik & Peimbert (1988), and by Kingsburgh and collaborators. From the cumulative histogram of Fig. 1 we see that up to a value of about 0.35 the distribution of $\varepsilon$ in galactic PNe is systematically lower than those of galactic bulge and extragalactic PNe. This effect could be explained since late, low filling factor nebulae are easily seen at larger distance, due to perspective effects.

In Figure 2 we note a wide spread of the ionized masses, and that all bin intervals are well represented. A large percentage (~90%) of galactic PNe have ionized mass between 0 and 0.2 $M_\odot$, which means that the value of 0.2 $M_\odot$ well represents the upper limit to the mass of a typical disk PN, while does not mean that all PNe can be well modelled by this value of the mass.

In Figures 3 and 4 we show how $\varepsilon$ and $M_i$ correlate to nebular dimensions in galactic PNe. In these Figures, circles refer to Set 1 and squares to Set 2 (see §3). Some nebulae have been represented by two symbols connected with a vertical line. In these cases, the filled symbols always refer to have taken into account electron densities within the higher reliability interval, while open symbols include in the means the offset values (see SK89). The correlations found and drawn in the Figures only refer to the filled circles. In both graphs two objects (NGC 5315 and He 2-131) are plotted with an open circle only: for them the only electron densities available in the literature fall outside the higher reliability interval. One object, NGC 4361, of Set 2, is not plotted in the two graphs for scaling reasons and it considerably scatters from the general trend.

By analyzing Fig. 3, we found (for the PNe in Set 1, excluding those with $\varepsilon$ greater than 1)

$$\log \varepsilon = -1.29 - 0.69 \log R,$$

with correlation coefficient r=-0.4. Our correlation is weaker than what found by Mallik & Peimbert (1988). We feel that the use of more recent data have just disclosed that the relation is not real, but it is due to data scatter, in agreement with what found by Kingsburgh & Barlow (1992). Further support to this conclusion is given by the analysis of galactic bulge PNe, later on this paper.

We then go on and check if there exists any relation between the thickness of the nebulae and the filling factor. We find that most optically thin planetary nebulae (i.e. those with an optical thickness parameter $T > 3.13$, CKS92) but one (NGC 6565) fall in the same region of the $\log \varepsilon - \log R$ plane. In particular we found that $\varepsilon > 0.1$ for optically thick nebulae.

Figure 4 shows that ionized masses and nebular dimensions do correlate, as

$$\log M_i = 0.188 + 1.329 \log R,$$

with regression coefficient r=0.72 (the calculation was performed for Set 1 PNe with $\varepsilon \leq 1$ only). All objects provided...
with model dependent distances (filled and open squares) have systematically higher values of $M_i$, as they preferentially fall in the upper part of the graph where $M_i \geq 0.15 \, M_\odot$. This result adds suspicion on the MEA88 distances, and explains why we do not include these data on averages and correlations.

By comparing the results shown in Fig. 4 with a similar, theoretical plot by Schmidt–Voigt & Köppen (1987b, Fig. 8) we could think that our galactic PNe trace the 2W model the best. Nevertheless, by looking at the different mass loss rates on the AGB assumed by Schmidt–Voigt & Köppen (1987a&b), we conclude that the observed data do not follow either relation (2W nor 3W) in such a fashion to allow us to decide for one or the other.

We now pass on galactic bulge PNe. For these PNe, the average filling factor is equal to 0.39, very close to what found for galactic PNe, and the mean value of $M_i=0.22 \, M_\odot$ is slightly larger than that of galactic PNe. From stellar evolutionary models, we would expect that galactic bulge PNe, supposedly evolving from Population II progenitors, would have ionized masses quite smaller than the masses of disk PNe (Iben & Renzini 1983). The high values of ionized masses that we found could be due to different reasons: first, we can think to some selection effect that allows a more probable detection of brighter PNe, which means on average more massive PNe in direction of the galactic bulge (see also Pottasch 1983); second, massive objects, descended from progenitors of a younger population than expected, do exist (Rich 1991); or, the ionized masses of the galactic bulge PNe have been overestimated in the calculation.

In particular the distance may have played a major role in this sense, since $M_i$ goes as the distance squared, and an overestimate in the galactic bulge distance by 20% propagates into lowering the mean $M_i$ of about 0.1 $M_\odot$. Another possibility could be that the difference is produced by the error propagation from all observed parameters. In §4.2 we see that this uncertainty can produce a variation in the mass of $\sim 0.1 \, M_\odot$, which could explain the general trend in the differences.

![Fig. 5](image.png)

**Fig. 5.** $\log \varepsilon$ vs. $\log R$ for galactic bulge PNe.

In Figure 5 we show that the correlation between $\log \varepsilon$ and $\log R$ is almost lost when using galactic bulge PNe. We conclude that, the data available up-to-date do not indicate correlation between the filling factor and the nebular dimensions. Further analysis with distance independent PNe samples is in order. In this paper, we could not go further in this direction, since the determination of angular diameters for PNe of the Magellanic Clouds is still very uncertain; we propose to investigate this important aspect in the future, with the new available data on Magellanic Cloud PNe especially from space observation.

At last, we go on to Magellanic Cloud PNe. The Magellanic Clouds mainly contain massive stars and young star clusters. PNe not only may have descended from an old star progeny, but also from massive progenitors that have undergone a huge mass loss in AGB as to be left with a low remnant mass well below the Chandrasekhar limit. From Figure 2 we note that the mass spread in these galaxies is even larger than for galactic PNe. The masses of SMC PNe are in a range $0 \div 0.8 \, M_\odot$, with 47% of the objects with $M_i \leq 0.2 \, M_\odot$, and the average mass value is 0.24 $M_\odot$. Of the 19 objects of the SMC, 15 have filling factor smaller than 1, the mean value of these being 0.29. On the other hand, LMC PNe show an average mass value of 0.21 $M_\odot$ and a mean $\varepsilon$ of 0.32. Again the spread in mass values is large, going from 0 to 0.96 $M_\odot$.

![Fig. 6](image.png)

**Fig. 6.** $\log \varepsilon$ vs. $\log N_e$ for Small Magellanic Cloud PNe; The solid line represents the correlation found (see text).

![Fig. 7](image.png)

**Fig. 7.** $\log M_i$ vs. $\log N_e$ in Small Magellanic Cloud PNe; the solid line represents the correlation found (see text).
Figure 6 shows that the trend between filling factor and electron density in the SMC PNe:

$$\log \varepsilon = 3.41 - 1.14 \log N_e,$$  \hspace{2cm} (10)

with correlation coefficient \( r = -0.66 \), is weaker than expected from functional relation (Eq. 3). This trend, that was only hinted in galactic PNe, suggests that, as the expansion of the nebula proceeds, the nebular material gets actually diluted. In fact as the nebula expands the volume increases while the electron density decreases. The ionized material per unit volume decreases being partially dispersed in the interstellar medium.

In Figure 7 we plot the relation between \( M_i \) and \( N_e \) for SMC PNe. The logarithms of the two quantities seem to correlate well, and we found

$$\log M_i = 1.88 - 0.70 \log N_e$$  \hspace{2cm} (11)

with correlation coefficient \( r = -0.78 \). If we suppose that most SMC PNe are optically thick, we can test on this Figure the validity of Shklovsky’s assumption that the luminosity in the light of \( H_\alpha \) is constant for all PNe. If that were the case, \( M_i \propto N_e^{-1} \). Our relation is weaker than this, as \( M_i \propto N_e^{-0.7} \). On the other hand, for our data \( M_i \) is constant only for very low electron densities. If we compare the last two Equations, we can infer that \( M_i \propto \varepsilon^{0.6} \), which is very close to the assumption under Shklovsky’s modified method \( (M_i \propto \varepsilon^{0.5}) \), as used in CKS92, giving more strength to the CKS92 distance scale.

We now examine the possible correlation between the ionized masses and the masses of the parent stars. For galactic PNe of Set 1 we use the central star parameters as given in Stanghellini et al. (1993) and we locate the central stars on the log\( F_{\text{bol}} \) – log\( L/L_\odot \) plane to infer their masses. Given the large uncertainties on the distances and temperatures of these stars, we could not single out any correlation between the two quantities.

We then go on and test the same correlation in Magellanic Cloud PNe. In Figure 8 we show ionized versus core masses (in solar units) for SMC and LMC PNe. Open symbols refer to those nebulae whose filling factor is larger than unity, those whose physical parameters have the larger errors (see Sect. 4.2). The ionized masses have been taken from Tabs. 3 and 4, and the core masses from Kaler & Jacoby (1990, 1991). Although the scatter in Figure 8 is large, we can individuate two major trends, especially when we consider only the filled symbols: one group of PNe show a large spread in \( M_i \) on a relatively narrow \( M_c \) range, the other group has smaller \( M_c \) , on average, on a wider \( M_c \) domain. Before trying to interpret these results, let us analyze the uncertainties in the parameters.

The ionized mass approximates reasonably well the total nebular mass for optically thin nebulae, while \( M_i \) is an overestimate of the true value (see Kaler & Jacoby 1990, 1991). For optically thick PNe, on the other hand, \( M_i \) represents a lower limit to the PNe mass, while \( M_c \) is well determined. This means that the points in Figure 8 could be mistaken as they have to be shifted upward or leftward, depending on nebular thickness. From Kaler & Jacoby’s (1990) criterion for optical thickness \(( F(A3797)/F(H_\alpha) > 0.8 \) and 0.35 for LMC and SMC PNe respectively), and from the flux data listed in the latter paper, we found that most of the nebulae in our sample are thick, thus the core masses in Figure 8 are sound values. We chose three PNe for which the filling factor is less than unity, and for which the thickness criterion by Kaler & Jacoby (1990) is largely achieved to indicate the typical errorbars (the PNe with errorbars are SMP 21 and 40 in LMC and SMP 5 in SMC, for discussion on \( M_c \) errors see Kaler & Jacoby 1990).

From our error analysis we conclude that the two different trends are real, thus a correlation between the core and the planetary nebula masses can not be the same for all types of PNe. For more insight, we plot the relation between core and nebular mass from Renzini & Völi (1981, RV81); furthermore, from the calculation of Vassiliadis & Wood (1992, VW92), we derive another \( M_i – M_c \) relation by considering the amount of mass that has been ejected during the last thermal pulse (or group of pulses in the cases where the mass loss is there continuous, meaning that in case of a multiple shell PNe we only consider to be observing the innermost shell). Renzini & Völi (1981) use high mass loss during AGB evolution and earlier, while Vassiliadis & Wood produce their models by using mass loss rates derived from empirical period–mass loss rate relations of Miras and OH/IR stars. We conclude that the different theoretical predictions encompass all observed PNe, while no systematic discrepancy was found between the PNe that occupy different parts of the \( M_i – M_c \) plane.

### 4.2. Detailed analysis and peculiar objects

In several cases the filling factor is calculated to be larger than unity. Among galactic PNe, NGC 6565, NGC 6567, and NGC 6741 of Set 1, plus NGC 6629 and IC 418 of Set 2 have \( \varepsilon > 1 \). Except for the last nebula, where \( \varepsilon = 0.89 \) if one takes into account electron densities outside the low-error range, there are not evident peculiarities to produce the high values of \( \varepsilon \). For NGC 6629, Mendez et al. (1992) also found \( \varepsilon > 1 \), even using different distance and electron density.

Among galactic bulge PNe, Ha 1-31, M 2-6, M 2-30 and M 3-29 have a filling factor greater than unity. Ha 1-31 is the only one whose electron density is not reliable as it falls outside the low-error range. The reason for high values of the filling factors should reside in the observational errors.

We are aware that the available measurements of angular diameters might not represent well the complex morphologies of the objects considered, and large uncertainties affect the...
The fact that the PN is not uniformly thin or thick to the star may be balanced by the spatial density fluctuations. The fact that we can overestimate (or underestimate) the nebular radius may be balanced by the spatial density fluctuations. We expected many off-range values. In particular, we found filling factors much larger than unity when using angular diameters from Speckle-interferometry, which are highly unreliable (Wood 1992, private communication). The errors or wrong determinations of \( \theta \) do not propagate into errors in \( M_i \), which are then very reliable for Magellanic Cloud PNe.

We estimate the general errors on \( M_i \) and \( \varepsilon \) by looking at the errors quoted in the papers where the physical parameters were derived: for galactic PNe, the error estimates on fluxes are taken from CKS92, those on electron densities from SK89, the ones on distances from Gathier et al. (1986) and the ones on angular diameters from the references cited in CKS92; the error on the galactic center distance is of the order of 20%. Errors on distances and on fluxes of extragalactic PNe come from Kaler & Jacoby (1990), the ones on angular diameters from Wood et al. (1986). Angular radii affect the calculation of \( \varepsilon \) the most: a 20% error on the angular radius means a variation of \( 0.3 \div 0.6 \) in the estimate of the filling factor. An error on angular dimensions do not influence the ionized mass. The maximum uncertainty on \( M_i \) determined by the contributions of all observed parameters is 0.1 \( M_\odot \) for galactic PNe, and about 0.05 \( M_\odot \) in the Magellanic Clouds.

5. Conclusions and future perspectives

The present investigation underlines the complex nature of PNe and the various aspects that need to be taken into account when attempting statistical studies on these objects. Among 4 sets of galactic and extragalactic PNe at known distances, we found that the filling factors and ionized masses spread over wide ranges, suggesting that many different PNe types do exist.

The average value of the filling factor is between 0.3 and 0.4 for those nebulae whose observed parameters have errors within 20% of their values. We also found that the filling factors anticorrelate with nebular dimensions for galactic disk PNe, while for the other PNe this value is an appropriate upper limit to the ionized mass for our disk PNe, while for the other PNe this value represents the average ionized mass.

For Magellanic Cloud PNe, where the distance is not a factor, we found tight correlations between the filling factors and the ionized masses versus the electron densities, as derived from forbidden line analysis. From the analytical relations found, we derive that the assumptions under Sklowy’s modified distance scale, as used in CKS92, are reliable within the errors in the data.

Ionized masses for Magellanic Cloud PNe spread versus the core masses in the domain predicted by the different theoretical studies; a single, sharp relation between nebular and core mass could not be found, while the data seem to indicate that two different trends do exist.

We plan to further expand the calculation of filling factors and ionized masses to other samples of PNe, especially in extragalactic environments, as more spectroscopic data will become available, to give more statistical significance to the results found here.

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