Masses and Strong Decay properties of Radially Excited Bottom states B(2S) and B(2P) with their Strange Partners B_s(2S) and B_s(2P)

Pallavi Gupta and A. Upadhyay
School of Physics and Materials Science, TIET, Patiala - 147004, Punjab, INDIA
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(1) INTRODUCTION

In the past years, the heavy-light mesons ($Qar{q}$) which contain heavy quark $Q$ and light anti-quark $\bar{q}$ have received considerable experimental and theoretical attention due to the existence of heavy quark and chiral symmetry. The study of heavy light mesons provide a better understanding of non-perturbative quantum chromodynamics (QCD). Many new discoveries have filled the gap in bottom meson spectroscopy. The ground state charm mesons such as $D(1S), D(1P), D_s(1S)$ and $D_s(1P)$ are well established and are listed in particle data group [1]. New candidates for higher radial and orbital excitations in the charm spectra include newly observed mesons like $D_0(2560), D_1^*(2680), D_2(2740), D_1^*(2760), D_J(3000), D_1^*(3000)$ and $D_J(3000)$ and strange states $D_{s1}(2860), D_{s1}(2860)$ and $D_{s1}(3040)$ [2]. Non-strange charm states $D_J^*(2460), D_J(2560), D_J^*(2680), D_J(2740), D_J^*(2760), D_J(3000)$ and $D_J(3000)$ are studied in our previous work [2] where their $J^P$'s are assigned as $1P_2^2, 2S_1^2, 2P_1, 1D_2^{-}, 1D_2^0, 2P_1, 1$ and $2P_1^0$ respectively. Observing the bottom spectroscopy, it is realized that unlike the success in charm sector, experimental information on higher excited bottom states is scare. Till now, only ground state bottom mesons for $B(1S)$ and excited $P$-wave mesons $B(1P)$ with $s_1 = 3/2$ along with their strange partners are experimentally available [1][2]. Recently in 2015, LHCb has observed new bottom mesons, which have diverted theorists interest towards the bottom sector. LHCb collaboration studied $B^+\pi^-$ and $B^0\pi^-$ mass distributions by analyzing the $p-p \rightarrow B(2S), B(2P)$ with their strange partners $B_s(2S)$ and $B_s(2P)$ states, which will be useful for both finding and understanding these excited bottom mesons in future.

In this paper, we analyzed the experimentally available radially excited charm mesons to predict the properties of experimentally missing radially excited bottom states $B(2S), B(2P)$ with their strange partners $B_s(2S)$ and $B_s(2P)$ states, which will be useful for both finding and understanding these excited bottom mesons in future.

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in 2013, the CDF collaboration studied the $B^0\pi^+$ and $B^+\pi^-$ mass distributions and have observed neutral bottom state $B_J(5970)$ with mass $M = 5978 \pm 5 \pm 12 \text{ MeV}$ and decay width $\Gamma = 70^{+30}_{-20} \pm 30 \text{ MeV}$ [14]. Since, this resonance decays in $B\pi$ final state, this state is supposed to have a natural spin parity state. The properties of $B_J(5970)$ state observed by CDF collaboration are consistent with $B_J(5960)$ state observed by LHCb collaboration, so they are assumed to be the same state. Theoretically, these states are studied by using various models like relativistic quark model [15], effective Lagrangian approach [16], quark model [17] etc. Being the fact that, $B(5960)$ decays to $B\pi$ final states and its mass being close to the mass of $2S_1$ state given in Ref’s [18][20], this state is considered to belong to $n=2$, with $J^P = 1^-$, S-wave state in the bottom spectroscopy. $B_1(5721)$ and $B_2^0(5747)$ are observed to belong to the bottom states $B(1^1P_1(1^+))$ and $B(1^1P_2(2^+))$ respectively. Beside these recently observed states, the information on the higher orbital and radial excited bottom states and their strange partners is still unknown.

In this paper, we aim on predicting the properties of experimentally missing radially excited bottom states $B(2S), B(2P)$ with their strange partners $B_s(2S)$ and $B_s(2P)$ states, which will be useful for both finding and understanding these excited bottom mesons in future. We will enlighten some of the properties like masses, strong decay widths, branching ratios, branching fractions, strong coupling constants for these radial excited bottom states. For this, we use Heavy Quark Effective Theory (HQET) as our framework that includes heavy quark spin-flavor symmetries making it invariant under SU(2)$\rightarrow$ transformations. In the heavy quark limit for the heavy meson doublets, we expect that the mass splittings among the different doublets and the partial decay widths are independent of the heavy quark flavor [22]. The flavor symmetry implies that the spin averaged...
mass splittings between the higher states and the ground state i.e. $\Delta_F$ and the mass splittings between the spin partners of the doublets i.e. $\Delta_F^{(b)} = \Delta_F^{(c)}$ and $\lambda_F^{(b)} = \lambda_F^{(c)}$. We apply this heavy quark symmetry on the experimental available data for n=2 charmon mesons to predict the properties of the corresponding bottom meson spectroscopy. We provide the mass spectra and strong decays for the bottom sector that will not only help in identifying the recent observed experimental bottom mesons, but will also help in validating the authenticity of the HQET model. This paper is organized as follows: Section 2 gives the brief review of the HQET framework where we define the heavy quark symmetry parameters and the possible QCD and $1/m_Q$ corrections to them. Section 3 represents the numerical analysis, where we calculate the masses based on the heavy quark symmetry and the corrections involved for the bottom states $B(1S)$, $B(1P)$, $B_s(1S)$ and $B_s(1P)$. Next, we use these calculated masses as an application, to predict the strong decay widths in terms of the couplings. Section 4 presents the conclusion of our work.

II. HEAVY QUARK EFFECTIVE THEOREY

In heavy quark effective theory, spin and parity of the heavy quark decouples from the light degrees of freedom as they interact through the exchange of soft gluons only. Heavy mesons are classified in doublets in relation to the total conserved angular momentum i.e. $s_1 = s\tau + l$, where $s\tau$ is the spin of the light anti-quark and $l$ is the orbital angular momentum of the light degree of freedom. For $l = 0$ (S-wave) the doublet is represented by $(P, P^s)$ with $J_{s_1}^P = (0^-, 1^-)_{1/2}$, which for $l = 1$ (P-wave), there are two doublets represented by $(P_0^1, P_1^1)$ and $(P_1^s, P_2^s)$ with $J_{s_1}^P = (0^+, 1^+)_{1/2}$ and $(1^+, 2^+)_{1/2}$ respectively. These doublets are described by the effective super-field $H_a$, $S_a$, $T_a$ [21,22], where the field $H_a$ describes the S-wave doublets, $S_a$ and $T_a$ fields represent the P-wave doublets for $J^P$ values $(0^+, 1^+)_{1/2}$ and $(1^+, 2^+)_{1/2}$ respectively. Radially excited states with radial quantum number n=2 are notated by $\tilde{P}$, $\tilde{P}^s$ and so on i.e. adding a $\sim$ symbol to the n=1 states. These fields are expressed as:

$$H_a = \frac{1 + \gamma_5}{2} \{ P_{a\mu} \gamma^\mu - P_a \gamma_5 \}$$

$$S_a = \frac{1 + \gamma_5}{2} \{ P_{a\mu} \gamma_\mu \gamma_5 - P_{a\mu} \gamma_5 \}$$

$$T_a^\mu = \frac{1 + \gamma_5}{2} \{ P_{2a\mu} \gamma_\nu - P_{1a\nu} \sqrt{3} \gamma_\mu \}$$

HQET is developed by expanding the QCD lagrangian in power of $1/m_Q$, in which heavy quark symmetry breaking terms are studied order by order. Applying finite heavy quark mass corrections, HQET lagrangian to order of $1/m_Q$ is

$$\mathcal{L} = \mathcal{L}_{\text{QCD}}(\mathbf{i}D_{\mu}D^\mu) + \frac{1}{m_Q} \mathcal{L}_{\text{HQET}}$$

Where, $D_{\mu}^a = \mathbf{i}D_{\mu} - v^\mu v_{\mu}$ is orthogonal to heavy quark velocity $v$, and $G^\mu_{\nu} = T_a G_{\mu\nu}^a = \frac{1}{g_s} [D_{\mu}, D_{\nu}]$ is the gluon field strength tensor. In the limit $m_Q \rightarrow \infty$, only first term $\mathcal{L}_{\text{QCD}}$ survives. The second term $D_{\mu}^a$ is arising from the off shell residual momentum of the heavy quark in the non relativistic model and it represents the heavy quark kinetic energy $\frac{p_a^2}{2m_Q}$ [23]. This term breaks the flavor symmetry because of the explicit dependence on $m_Q$, but does not break the spin symmetry of the HQET. The third term in the above equation i.e. $g\sigma_{\mu\nu}G_{\mu\nu}$ represents the magnetic moment interaction coupling of the heavy quark spin to the gluon field. This term breaks both the flavor and spin symmetry and is known as chromomagnetic term. The mass of the heavy-light hadron to the first order of $1/m_Q$ is represented as :

$$M_X = m_Q + \Lambda - \frac{\lambda_1}{2m_Q} - d_H \frac{\lambda_2}{2m_Q}$$

In this equation, $d_H = -4(S_Q \cdot S)$ is the Clebsch factor, with $d_H = -3$ or 1 for $J = 0$ or 1 respectively for $S_q = 1/2$ and $d_H = -5$ or 3 for $J = 1$ or 2 for $S_q = 3/2$. The $\lambda_1$, $\Lambda$ and $\lambda_2$ are the non-perturbative parameters, whose values are of the order of $\Lambda_{QCD}$ where

$$\Lambda = \frac{1}{2} \langle H^{(Q)}|H_0|H^{(Q)} \rangle$$

$$\lambda_1 = \frac{\langle H_Q|\overline{Q}(iD_{\perp})^2Q|H_Q \rangle}{2m_{H_Q}^3}$$

$$\lambda_2 = \frac{\langle H_Q|\overline{Q}\frac{1}{2}\sigma.GQ|H_Q \rangle}{2d_H m_{H_Q}^3}$$

The parameter $\Lambda$ is the energy of the light quark fields (i.e. brown muck), $\lambda_1$ term represents the kinetic energy of the heavy quark $Q$ and the term $\lambda_2$ gives the chromomagnetic interaction energy [23,24]. Since value of kinetic energy of the heavy quark is positive, the value of the parameter $\lambda_1$ should be negative. $\Lambda$ is the HQET parameter whose value is same for all the particles in a spin-flavor multiplet. $\Lambda$ does not depend on the light quark flavor if there is $SU(3)$ symmetry, but for the breaking of this symmetry $\Lambda$ is different for strange and non-strange heavy -light mesons and is denoted by $\overline{\Lambda}$ and $\overline{\Lambda}_{a,d}$ respectively.

In limit $m_Q \rightarrow \infty$, only the first term of the HQET lagrangian (equation 4) will have the effect of interaction. Based on various fields defined in equations 1-3, the kinetic terms of the heavy meson doublets and of the $\Sigma$...
field of light pseudo-scalar mesons, present in the effective Lagrangian \[21\] are as:

\[
L = iTr[H_\mu \partial_\mu H_a] + \frac{f_0^2}{8}Tr[\partial_\mu \Sigma \partial_\mu \Sigma']
+ Tr[S_b(i\gamma^\mu D_\mu - \delta_{ba} \Delta S)S_a]
+ Tr[T_b(i\gamma^\mu D_\mu - \delta_{ba} \Delta T)T_{a\alpha}]
\]

where the operator D is given as:

\[
D = -\delta_{ba} \partial^\mu + V_{\mu ba}
= -\delta_{ba} \partial^\mu + \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)ba
\]

The mass parameter \(\Delta_F\) describes the mass splitting between the higher mass doublets \((F)\) and the ground state H field doublet. This mass parameter \(\Delta_F\) \((F = S, T)\) can be written in terms of the spin average mass of these doublets as:

\[
\Delta_F = \overline{M}_F - \overline{M}_H, F = S, T
\]

where

\[
\overline{M}_H = (3m_{HQ}^{(Q)} + m_{HQ}^{(Q)})/4 \hspace{1cm} (12)
\]

\[
\overline{M}_S = (3m_{HQ}^{(Q)} + m_{HQ}^{(Q)})/4 \hspace{1cm} (13)
\]

\[
\overline{M}_T = (5m_{HQ}^{(Q)} + 3m_{HQ}^{(Q)})/8 \hspace{1cm} (14)
\]

The mass degeneracy between the members of the doublets breaks at the \(1/m_Q\) corrections to the heavy quark limit. This correction is of the form of

\[
\mathcal{L}_{1/m_q} = \frac{1}{2m_q} \lambda_H Tr[H_a \gamma_{\mu\nu} H_a \gamma_{\mu\nu}]
- \lambda_S Tr[S_a \gamma_{\mu\nu} S_a \gamma_{\mu\nu}]
+ \lambda_T Tr[T_a \gamma_{\mu\nu} T_a \gamma_{\mu\nu}]
\]

where \(\lambda_H, \lambda_S\) and \(\lambda_T\) are the hyperfine splittings between the spin partners in each doublet with

\[
\lambda_H = \frac{1}{8}(M_{P_H}^2 - M_{P_H}^2) \hspace{1cm} (16)
\]

\[
\lambda_S = \frac{1}{8}(M_{P_S}^2 - M_{P_S}^2) \hspace{1cm} (17)
\]

\[
\lambda_T = \frac{3}{16}(M_{P_T}^2 - M_{P_T}^2) \hspace{1cm} (18)
\]

Flavor symmetry implies

\[
\Delta_F^{(c)} = \Delta_F^{(b)} \hspace{1cm} (19)
\]

\[
\lambda_F^{(c)} = \lambda_F^{(b)} \hspace{1cm} (20)
\]

i.e. \(\Delta_F\) parameter which represents the mass splittings between the higher mass doublets and the ground state doublet and the \(\lambda_F\) parameter which represents the hyperfine splittings between spin partners, are same for both charm and bottom mesons.

This symmetry is broken by the higher order terms in the HQET lagrangian involving terms of factor \(1/m_Q\) and the parameters \(\Delta_F\) and \(\lambda_F\) are modified by extra term \(\delta \Delta_F\) and \(\delta \lambda_F\) \[25\].

The hyperfine splitting term \(\lambda_F\) which originates from the chromomagnetic interaction is dominated by the QCD corrections and the \(1/m_Q\) effect is neglected \[26\]. QCD corrections change the \(\lambda_F\) relation to

\[
\lambda_F^{(b)} = \lambda_F^{(c)} \left(\frac{\alpha_s(m_h)}{\alpha_s(m_c)}\right)^{9/25} \hspace{1cm} (21)
\]

Difference of the spin averaged masses at \(1/m_Q\) order is

\[
\overline{M}_S - \overline{M}_H = \overline{M}_S - \overline{M}_H - \frac{\Lambda_S}{2m_Q} + \frac{\Lambda_H}{2m_Q} \hspace{1cm} (22)
\]

which modifies the parameter \(\Delta_F\) by \(\delta \Delta_F\), i.e.

\[
\Delta_F^{(b)} = \Delta^{(c)} + \delta \Delta_F, \hspace{1cm} (23)
\]

Also in the heavy quark effective theory, the strange states possess the property, that the effect of the strange quark is to shift the mass of a given state by the same amount in the fundamental mode \(n=1\) and for \(n=2\) radial excitation doublet.

\[
M_{P_S} - M_P = \overline{M}_{P_S} - \overline{M}_P \hspace{1cm} (24)
\]

Masses of the heavy hadrons can be used to calculate other properties like strong decays, radiative decays, magnetic moments etc. Strong interactions are very important for the study of heavy hadrons containing one heavy and one light quark in the non-perturbative regime.

At the leading order approximation, using the lagrangians \(L_{HH}, L_{SH}, L_{TH}, L_{HY}, L_{ZH}\), the two body strong decays of \(Q\) heavy-light charm mesons are given as

\[
(0^+, 1^-) \rightarrow (0^-, 1^-) + M
\]

\[
\Gamma(1^- \rightarrow 1^-) = C_M \frac{g_H^2 M_f p_M^3}{8 \pi f_2^2 M_i} \hspace{1cm} (25)
\]

\[
\Gamma(1^- \rightarrow 0^-) = C_M \frac{g_H^2 M_f p_M^3}{6 \pi f_2^2 M_i} \hspace{1cm} (26)
\]

\[
\Gamma(0^- \rightarrow 1^-) = C_M \frac{g_H^2 M_f p_M^3}{2 \pi f_2^2 M_i} \hspace{1cm} (27)
\]

\[
(0^+, 1^+) \rightarrow (0^-, 1^-) + M
\]

\[
\Gamma(1^+ \rightarrow 1^-) = C_M \frac{g_H^2 M_f (p_M^2 + m_M^2)p_M}{2 \pi f_2^2 M_i} \hspace{1cm} (28)
\]

\[
\Gamma(0^+ \rightarrow 0^-) = C_M \frac{g_H^2 M_f (p_M^2 + m_M^2)p_M}{2 \pi f_2^2 M_i} \hspace{1cm} (29)
\]
In the above decay widths, $M_i$ and $M_f$ stands for initial and final meson mass, $p_M$ and $m_M$ are the final momentum and mass of the light pseudo-scalar meson respectively. The coefficient $C_{πφ}, C_{Kφ}, C_{Kφ^0}, C_{πφ^0} = 1$, $C_{πφ} = \frac{2}{3}$ and $C_{πφ} = \frac{2}{3}$ or $\frac{1}{3}$. Different values of $C_{πφ}$ corresponds to the initial state being $πφ, φπ$ or $φπ$ respectively. All hadronic coupling constants depends on the radial quantum number. For the decay within $n=1$ they are noted as $g_{HH}, g_{SH}$ etc, and the decay from $n=2$ to $n=1$ they are represented by $g_{HH}, g_{SH}$. Higher order corrections for spin and flavor violation of order $\frac{1}{m_Q^2}$ are excluded to avoid new unknown coupling constants.

III. NUMERICAL ANALYSIS

The masses, decay widths and $J^{P'}$s for the experimentally available radial excited charm mesons $D_0(2560), D_1^0(2680), D_J(3000), D_J^*(3000)$ have been analyzed with various theoretical approaches [7,28,29]. In particular the predicted $J^{P'}$ values for $(D_J(2560), D_J^*(2680))$ are given as $(0^+, 1^-)_\frac{1}{2}$ for $n=2$ and $L=0$ and $J^{P'}$ for states $(D_J^*(3000)), (D_J(3000))$ are given as $(0^+, 1^+)_1$ for $n=2$ and $L=1$.

In bottom sector, only one radially excited bottom state i.e. $B_J(5970)$, is experimentally known, whose $J^{P}$ is associated with $1^−_1$ for $B_J^*(2S)$ state. Other radially excited bottom states $(B(2S), B(2P), B_+(2S)$ and $B_+(2P))$ are still unavailable. In this paper, we aim to predict the masses, decay widths, branching ratios of these missing radially excited bottom states $(2S)$ and $B(2P)$ in the framework of HQET. To study the behavior of the heavy-light mesons for their spectroscopy, masses are the most important property to be studied so, we start our calculations by predicting the masses of these bottom meson states. To calculate these masses, we use the flavor symmetry property of the heavy quarks $λ_F^{(b)} = λ_F^{(c)}$ and $Δ_F^{(b)} = Δ_F^{(c)}$. This flavor symmetry parameters are defined in terms of the spin averaged mass splittings between the higher state doublets and the ground state doublet, represented by $Δ_F$ and $λ_F$ which is the mass splittings between the spin partners of the doublets. From the LHCb data [2], spin averaged mass splittings $Δ_F$ and the hyperfine splittings $λ_F$ for the recently observed $2S$ and $2P$ charm mesons states comes out to be:

$Δ_F^{(c)} = 660.33 \pm 3.8 MeV, \quad λ_F^{(c)} = (213.43 \pm 3.9 MeV)^2$

$\Delta_F^{(c)} = 1009.44 \pm 6.12 MeV, \quad λ_F^{(c)} = (164.72 MeV \pm 2.4)^2$

$Δ_F^{(c)} = 1034.19 \pm 1.2 MeV, \quad λ_F^{(c)} = (208.41 MeV \pm 1.4)^2$

for the $n=2$ odd parity, low lying even parity and for the excited even parity $c\bar{c}$ mesons. The charm mesons for $n=2$, $P$-wave with $j = 3/2$ are experimentally unavailable, so we have taken the theoretical masses for $(\bar{D}_1, \bar{D}_2)$ having values (2932.50, 3020.60)MeV [19,30,31,34]. In this, we have taken the SU(2) isospin symmetry for the non-strange charm mesons, i.e. $M(\bar{c}\bar{d}) = M(c\bar{d})$. The small statistical errors in $Δ_F$ and $λ_F$ for the non-strange radial excited charm mesons reflect the precision of the LHCb results [2].

Calculated masses obtained using these symmetries are listed in the 2nd column of Table I. Here, mass of bottom state $B_J$ for $n=2$ and $J^{P'} = 1^−$ comes out to be 5981.50 MeV, which is very close to experimentally observed mass 5978 MeV and 5969.20 MeV for bottom state $B_f(5970)$ observed by CDF and LHCb collaboration respectively. Closeness in the experimentally observed mass for bottom state $B_J$ and the mass obtained by HQET shows the authenticity of this heavy quark symmetry. Since the experimental information on radial excited bottom state is limited, the authenticity of this symmetry cannot be completely justified just on the basis on the one experimental available bottom state.

Based on these limitations, we have also compared the predicted masses for other $n=2$ bottom states, listed in Table I with some of the theoretically available data. The masses calculated using the heavy quark symmetry in our work are in agreement with the masses obtained by the potential model in Ref. [34]. Masses of the non-strange bottom field $H$ deviates by 0.4% and 0.5% when compared with the masses in Ref. [34] for $B_0$ and $\bar{B}_1$ states respectively. Similar pattern is observed for $P$-wave masses where the deviations are below 1%. In contrary to this, these bottom masses are deviating at most by 3.98% with the theoretical data in Ref. [31].

We have obtained a set of bottom spectra for $n=2$ using flavor symmetry which upto a very good approximation matches with other theoretical data as discussed above. Now, we would like to look into the QCD and $1/m_Q$ corrections in the HQET lagrangian. QCD and $1/m_Q$ corrections are applied to a scale of $\Lambda_{QCD}/m_Q$, where they can be an important input to decide the level of breaking of symmetry. The corrections to $Δ_F$ and $λ_F$ (where $F = H, \bar{S}, \bar{T}$) parameters are coming in the form of

$λ_F^{(b)} = λ_F^{(c)} + δλ_F^{(b)} \quad Δ_F^{(b)} = Δ_F^{(c)} + δΔ_F^{(b)}$ (33)

The $λ_F$ parameter originates from the chromomagnetic interaction, thus only QCD corrections are dominated and $1/m_Q$ effect is small. For applying the QCD corrections to the spin hyperfine splitting relation $λ_F$, the values of the parameter $α_s(m_b)$ and $α_s(m_c)$ are taken as 0.22 and 0.36 [25]. The leading QCD correction
ally excited charm mesons are deviating by 1 or 2 MeV except for the
mass of $J^P = (1^+)_s$ (n=2, L=0) for $D_{sJ}(2700)$ and $(1^+)_b$
(n=2, L=1) for $D_{sJ}(3040)$ state. The other strange 2S and 2P charm and bottom mesons are still unknown. Here, we have predicted the unavailable 2S and 2P charm masses using equation 24, and are listed in Table III and are matched with other theoretical data.

In the same way, strange bottom masses are calculated and are reported in Table III. The 2nd column of the Table gives the masses without any correction and the masses listed in column 3rd and 4th include the corrections to both the $\lambda_F$ and $\Delta_F$ relation respectively. This is followed by the masses in 5th column which gives the masses obtained by using corrections to the $\lambda_F$ and $\Delta_F$ relations simultaneously. In general, QCD correction to $\lambda_F$ relation changes the bottom masses by few MeV. Correction to $\Delta_F$ relation results in deviating the mass of S-wave bottom states by 0.22%. And the masses for P-wave get deviated by 0.77% and 0.58% respectively for $s_1 = 1/2^+$ and $s_1 = 3/2^+$.

Now we study the various other properties of bottom mesons like strong decay widths, branching ratios, strong coupling constants. We apply the effective Lagrangian approach discussed in Sec II to calculate the OZI allowed two body strong decay widths and the various branching ratios involved with the bottom states $B(2S)$, $B(2P)$, $B_s(2S)$ and $B_s(2P)$. The numerical value of the partial and total decay widths of 2S and 2P family are collected in Table IV, V, VI, VII where Table IV and V are for the n=2, S and P wave bottom meson with $s_1 = 1/2$, and the Table VI is for the other P wave bottom meson states having $s_1 = 3/2$.

For the radially ground state S-wave bottom states, Table IV reveals $B^*\pi^-$ mode to be the dominant decay mode both for $B^+_s$ and $B_0^-$ bottom states with branching fraction of 37.97% and 64.16% respectively. And for their strange partners, $B^+K^-$ is seen to be the leading decay mode with branching fraction 20.34% and 33.60% for $B_{s1}^*$ and $B_{s0}^-$ state respectively. Hence the decay modes $B^+\pi^-$ and $B^+K^-$ are suitable for the experimental search for the missing non-strange and strange 2S bottom meson states. Total decay width for bottom state $B_1^*$ is 64.32 MeV, which matches with its experimental value of 70 MeV [14] (observed by CDF Collaboration), where strong coupling constant $g_{HH}$ is used as 0.14 [23]. This coupling for the other S-wave bottom states gives decay width $\Gamma(B_0) = 49.48$ MeV, $\Gamma(B_{s1}^*) = 77.05$ MeV and $\Gamma(B_{s0}^*) = 55.14$ MeV. Similarly, for n=2 low lying P-wave bottom states(0^+, 1^+), Table V shows that the dominant decay modes for bottom state $B_0^*$ and $B_1^*$ are $B^+\pi^-$ and $B^+K^-$ respectively. These decay modes contribute 42.15% and 42.83% to the total decay widths of $B_0^*$ and $B_1$ state. And for the strange states $B_{s0}^*$ and $B_{s1}^*$, $B^-K^+$ and $B^-K^0$ decay modes emerges as the prominent modes for their experimental exploration in future. Using the coupling constant $g_{SH} = 0.12$, the total decay...
width for these \( s_l = 1/2 \) P-wave bottom states are obtained as: \( \Gamma(\bar{B}^*_1) = 242.70 \text{ MeV} \), \( \Gamma(\bar{B}^*_2) = 203.46 \text{ MeV} \), \( \Gamma(\bar{B}^*_{s0}) = 276.85 \text{ MeV} \) and \( \Gamma(\bar{B}^*_{s1}) = 243.66 \text{ MeV} \).

Apart from the mentioned partial decay widths, these bottom states also decay to D-wave bottom mesons. But these decays are suppressed in our calculations because of their small contribution.

Lastly, for the other P-wave bottom states having \( s_l = 3/2 \), Table IV.III points \( B^* \pi^\pm \) and \( B^* K^- \) decay modes to be the best suitable for the study of the missing non-strange states \( (\bar{B}_1, \bar{B}_2^*) \) and strange states \( (\bar{B}_{s1}, \bar{B}_{s2}^*) \) respectively. While observing the total decay width values from Table, we notice that lower values of coupling constant \( g_{TH} \) will give realistic decay width value for these states. Assuming that the coupling constant \( g_{NH} \) and \( g_{TH} \) will not vary much for higher excited states, total decay width corresponding to the coupling constant \( g_{TH} = 0.12 \) value are obtained as \( \Gamma(\bar{B}_1) = 188.69 \text{ MeV} \), \( \Gamma(\bar{B}_2) = 226.39 \text{ MeV} \), \( \Gamma(\bar{B}_{s1}) = 259.89 \text{ MeV} \) and \( \Gamma(\bar{B}_{s2}) = 313.98 \text{ MeV} \). Thus the states \( \bar{B}_2^* \) and \( \bar{B}_{s2}^* \) are obverse to be broader states as compared to their spin partner states \( \bar{B}_1 \) and \( \bar{B}_{s1} \) respectively. Here, we need to emphasize that the estimated total width of these states does not include the contribution from the decays to \( n=1 \) D-wave bottom mesons since the phase space is very small for these decay states.

**IV. CONCLUSION**

With so many newly available charm states, the data for the higher excited bottom states is limited as compared to the charm sector. In this work, we focuses on predicting the masses and the strong decay widths of the experimentally missing radially excited bottom states \( B(2S), B(2P), B_s(2S) \) and \( B_s(2P) \) using heavy quark effective theory.

- We apply the heavy quark symmetry property to the experimentally available radially excited charm mesons observed by LHCb and evaluate the similar spectra for the bottom sector. The predicted mass of bottom state \( \bar{B}^*_1 \) in our work is only 0.33% deviating from the experimentally measured mass of \( B_J(5970) \) state. The Closeness in the experimentally observed mass for bottom state \( \bar{B}^*_1 \) and the mass obtained in our work shows the authenticity of this heavy quark symmetry.

- These masses are then studied by taking the QCD and \( 1/m_Q \) corrections in the form of \( \delta \Delta_F \) and \( \delta \lambda_F \). Masses obtained using these corrections are tabulated in Table IV. While studying these correction, we realize that \( \delta \lambda_F \) shifts the bottom masses at most by 0.06% and has reduced the existing gap between the experimental and the observed mass of \( \bar{B}^*_1 \) state. While the correction \( \delta \Delta_F \) results in proportionate reduction in the bottom masses by the amount of \( |\delta \Delta_F| \).

- Evaluating the strange bottom masses in the similar manner, we discover that calculated masses are in good agreement with maximum 2.5% deviation from the other theoretical stranged bottom masses. Corrections results in deviating the S-wave masses by 0.22%. And the masses for P-wave get deviated by 0.77% and 0.58% respectively for \( s_l = 1/2^+ \) and \( s_l = 3/2^+ \). The calculated strange bottom masses in this work for \( n=2 \) are \( \sim 90 \text{ MeV} \) higher than the masses of the non-strange bottom masses except for the low lying S-wave states. These kind of corrections for \( n=2 \) bottom mesons are showing similar pattern of behavior but with a reduced effect, when compared with results for \( n=1 \) bottom states. This is something we expect when we study the properties of the heavier mesons for higher excited states.

- With these obtained masses, we further calculated the strong decay widths of these bottom states, which are collected in Table IV.IV. The predicted decay widths are in the terms of strong coupling constants \( g_{NH} \), \( g_{SH} \) and \( g_{TH} \). To avoid these unknown couplings, we have also computed the branching ratios and fractions for the possible OZI allowed decay channels.

- While analysing the decay widths in the Table IV.V, we observe \( B^- \pi^- (B^+ K^-) \) mode to be the dominant decay mode both for \( \bar{B}^*_1 (\bar{B}^*_{s1}) \) and \( \bar{B}_0 (\bar{B}_{s0}) \) bottom states. Similarly, for the low lying P-wave bottom mesons \( \bar{B}^*_0 \) and \( \bar{B}_0 \), \( B^* \pi^- \) and \( B^*+ \pi^- \) emerges as the dominant decay modes with contribution of 42.15% and 42.83% to their total decay widths. For their strange partners, \( B^- K^+ \) and \( B^* K^+ \) are seen as the prominent modes for \( \bar{B}_{s0}^* \) and \( \bar{B}_{s1}^* \) states respectively. Lastly, for the excited P-wave bottom mesons, \( B^* \pi^+ \) and \( B^* K^- \) are seen to be the best suitable decay modes for the study of the missing non-strange states \( (\bar{B}_1, \bar{B}_2^*) \) and strange states \( (\bar{B}_{s1}, \bar{B}_{s2}^*) \) respectively. With the obtained decay widths, we have further check their sensitivity to the QCD and \( 1/m_Q \) corrected masses. It is found that these corrections shifts the decay width values at most by 30 MeV (\( \sim 12\% \)).

Finally, we have predicted the masses and the strong decay widths of the experimentally not yet observed \( n=2 \) bottom mesons. These predicted bottom states has opened a window to investigate the higher excitations of bottom mesons and can be confronted with the future experimental data at the LHCb, D0, CDF.
[1] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016).
[2] R. Aaij et al.[LHCb Collaboration], JHEP 1309 145 (2013).
[3] P. del Amo Sanchez et al.[BaBar Collaboration], Phys. Rev. D 82, 111101 (2010).
[4] R. Aaij et al.[LHCb Collaboration], Phys. Rev. D 94, 072001 (2016).
[5] B. Aubert et al., [BaBar Collaboration], Phys. Rev. D80 (2009) 092003.
[6] R. Aaij, et al., [LHCb Collaboration], Phys. Rev. Lett. 113 (2014) 162001.
[7] P.Gupta and A. Upadhyay, Phys. Rev. D 97, 014015 (2018).
[8] S. Behrends et al. (CLEO), Phys. Rev. Lett. 50, 881 (1983).
[9] K. Han et al., Phys. Rev. Lett. 55, 36 (1985).
[10] T. Aaltonen et al., Phys. Rev. Lett. 102 (2009) 102003.
[11] T. A. Aaltonen et al., [CDF Collaboration], Phys. Rev. D90 (1) (2014)012013.
[12] J. Lee-Franzini et al., Phys. Rev. Lett. 65 (1990) 2947-2950.
[13] R. Aaij et al. [LHCb Collaboration],JHEP 1504, 024 (2015).
[14] T. A. Aaltonen et al. [CDF Collaboration],Phys. Rev. D 90, no. 1, 012013 (2014).
[15] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).
[16] Hao Xu, Xiang Liu, Takayuki Matsuki, Phys. Rev. D 89, 097502 (2014).
[17] L. Y. Xiao and X. H. Zhong, Phys. Rev. D 90, no. 7, 074029 (2014).
[18] T. Matsuki, T. Morii, and K. Sudoh, Eur. Phys. J. A 31, 701 (2007).
[19] D. Ebert, R. N. Faustov, and V. O. Galkin, Eur. Phys. J. C 66, 197 (2010).
[20] Y. Sun, Q.-T. Song, D.-Y. Chen, X. Liu, and S.-L. Zhu, Phys. Rev. D 89, 054026 (2014).
[21] P. Colangelo, F. De Fazio, F. Giannuzzi, S. Nicotri, Phys. Rev. D 86 054024 (2012).
[22] A. Falk and T. Mehen, Phys. Rev. D 53 231 (1996).
[23] Zhi-Gang Wang, Eur.Phys.J.Plus 129 (2014) 186.
[24] Cheunh C.Y, Hwang C.W., J. High Energ. Phys. (2014) 2014:177.
[25] Hai-Yang Cheng, Fu-Sheng Yu, Phys. Rev. D 89, 114017 (2014).
[26] G. Amoros, M. Beneke and M. Neubert, Phys. Lett. B 401, 81 (1997).
[27] A. Upadhyay et. al., Adv.High Energy Phys. 2014 (2014)619783.
[28] Zhi-Gang Wang, Phys. Rev. D 88, 114003 (2013).
[29] G. L. Yu, Z.-G. Wang, Z.-Y. Li, and G.-Q. Meng, Chin. Phys. C 39, 063101 (2015).
[30] M. Di Pierro and E. Eichten, Phys. Rev. D 64, 114004 (2001).
[31] T.A. Lahde, Nucl. Phys. A, 674, 141-167 (2000).
[32] Cheng H.Y., Yu F.S., Eur. Phys. J. C (2017) 77: 668.
[33] Stephen Godfrey, Kenn Moats, Phys. Rev. D 93 034035(2016).
[34] Virendrasinh Kher et.al., Chin.Phys. C41 (2017), 073101.
[35] Pietro Colangelo, Fulvia De Fazio, Phys. Rev. D 81 094001 (2010).
[36] B. Zhang, X. Liu, W.Z. Deng, S.L. Zhu, Eur. Phys. J. C50, 617 (2007), hep-ph/0609013,352.
[37] J. Segovia, D.R. Entem, F. Fernandez, Phys. Rev. D91, 094020 (2015).
TABLE I: Predicted values of the radially excited non-strange 2S and 2P bottom meson states. All the masses are in MeV units.

|                  | 0$^-$($2^1S_0$) | 1$^-$($2^3S_1$) | 0$^+$($2^3P_0$) | 1$^+$($2^1P_1$) | 1$^+$($2^3P_1$) | 2$^+$($2^3P_2$) |
|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Without corrections | 5950.96         | 5981.50         | 6335.83         | 6318.67         | 6336.30         | 6354.56         |
| Corrections in $\lambda_F$ | 5954.68         | 5980.26         | 6333.74         | 6319.37         | 6338.16         | 6353.45         |
| Corrections in $\Delta_F$ | 5928±13         | 5959±13         | 6291±37         | 6274±37         | 6306±9          | 6324±9          |
| Correction in both parameters $\lambda_F$ and $\Delta_F$ | 5932±13         | 5957±13         | 6289±37         | 6274±37         | 6308±9          | 6323±9          |

Ref.[30] 5985 6019 6264 6278 6308 6324
Ref.[19] 5976 5992 6318 6345 6359
Ref.[31] 5939 5966 6143 6160 6170
Ref.[34] 6003 6029 6367 6387 6382

TABLE II: Predicted values of the radially excited strange 2S and 2P charm meson states. All the masses are in MeV units.

| $J^p$($n^{2s+1}L_J$) | Our | Ref.[34] | Ref.[19] | Ref.[33] |
|----------------------|-----|----------|----------|----------|
| 0$^-$($2^1S_0$) | 2682.96 | 2680 | 2688 | 2673 |
| 1$^-$($2^3S_1$) | 2754.63 | 2719 | 2731 | 2732 |
| 0$^+$($2^3P_0$) | 3007.80 | 3022 | 3054 | 3005 |
| 1$^+$($2^1P_1$) | 3009.90 | 3081 | 3154 | 3018 |
| 1$^+$($2^3P_1$) | 3089.61 | 3092 | 3067 | 3038 |
| 2$^+$($2^3P_2$) | 3127.71 | 3109 | 3142 | 3048 |

TABLE III: Predicted values of the radially excited strange 2S and 2P bottom meson states. All the masses are in MeV units.

|                  | 0$^-$($2^1S_0$) | 1$^-$($2^3S_1$) | 0$^+$($2^3P_0$) | 1$^+$($2^1P_1$) | 1$^+$($2^3P_1$) | 2$^+$($2^3P_2$) |
|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Without corrections | 6039.67         | 6071.85         | 6335.72         | 6336.72         | 6429.02         | 6447.41         |
| Corrections in $\lambda_F$ | 6043 | 6070.54 | 6335.84 | 6336.68 | 6430.89 | 6446.29 |
| Corrections in $\Delta_F$ | 6025±6 | 6058±6 | 6286±8 | 6287±8 | 6391±20 | 6410±20 |
| Correction in both parameters $\lambda_F$ and $\Delta_F$ | 6029±6 | 6056±6 | 6286±8 | 6287±8 | 6393±20 | 6409±20 |

Ref.[30] 5886 5920 6163 6175 6194 6188
Ref.[19] 5890 5906 6221 6209 6281 6260
Ref.[31] 5822 5848 6010 6022 6028 6040
Ref.[34] 5926 5947 6297 6295 6311 6299
TABLE IV: Strong decay width of non-strange and strange n=2 S-wave bottom mesons $B(2^1S_1)$, $B(2^3S_0)$, $B_s(2^3S_1)$ and $B_s(2^1S_0)$. Ratio in 5th column represents the $\hat{\Gamma} = \frac{\Gamma}{\Gamma(B^{*+} \rightarrow B^{++} \pi^+)}$ for the non-strange mesons and $\hat{\Gamma} = \frac{\Gamma}{\Gamma(B_s^{*+} \rightarrow B_s^{++} K^+)}$ for the strange mesons. Fraction gives the percentage of the partial decay width with respect to the total decay width.

| State | nLsJ' | Decay channel | Decay Width(MeV) | Ratio | Fraction |
|-------|-------|----------------|------------------|-------|----------|
| $B_1^*(5981.50)$ | $2S_{1/2}^1^-$ | $B^+\pi^+$ | 1246.27$g_{HH}^2$ | 1 | 37.97 |
| | | $B^+\pi^0$ | 626.01$g_{HH}^2$ | 0.50 | 19.07 |
| | | $B^+\eta$ | 37.18$g_{HH}^2$ | 0.02 | 1.13 |
| | | $B^+K$ | 96.36$g_{HH}^2$ | 0.07 | 2.93 |
| | | $B^0\pi^0$ | 377.54$g_{HH}^2$ | 0.30 | 11.50 |
| | | $B^+\pi^-$ | 753.14$g_{HH}^2$ | 0.60 | 22.94 |
| | | $B^0\eta$ | 32.41$g_{HH}^2$ | 0.02 | 0.98 |
| | | $B_sK$ | 112.72$g_{HH}^2$ | 0.09 | 3.43 |
| | | Total | 3281.67$g_{HH}^2$ | | |
| $B_0(5950.96)$ | $2S_{1/2}^0^-$ | $B^+\pi^+$ | 1629.05$g_{HH}^2$ | 1 | 64.16 |
| | | $B^+\pi^0$ | 818.67$g_{HH}^2$ | 0.50 | 32.24 |
| | | $B^+\eta$ | 33.15$g_{HH}^2$ | 0.02 | 1.30 |
| | | $B_sK$ | 57.78$g_{HH}^2$ | 0.03 | 2.27 |
| | | Total | 2538.67$g_{HH}^2$ | | |
| $B_s^*(6071.85)$ | $2S_{1/2}^1^-$ | $B^0K^0$ | 520.79$g_{HH}^2$ | 0.65 | 13.24 |
| | | $B^+K^-$ | 529.94$g_{HH}^2$ | 0.66 | 13.47 |
| | | $B_s\pi^0$ | 384.18$g_{HH}^2$ | 0.48 | 9.77 |
| | | $B_s\eta$ | 134.70$g_{HH}^2$ | 0.16 | 3.42 |
| | | $B^0K^0$ | 784.86$g_{HH}^2$ | 0.98 | 19.96 |
| | | $B^{-}K^-$ | 799.72$g_{HH}^2$ | 1 | 20.34 |
| | | $B^0\pi^0$ | 628.19$g_{HH}^2$ | 0.78 | 15.97 |
| | | $B_s\eta$ | 148.94$g_{HH}^2$ | 0.18 | 3.78 |
| | | Total | 3931.35$g_{HH}^2$ | | |
| $B_{s0}(6039.67)$ | $2S_{1/2}^0^-$ | $B^0K^0$ | 924.75$g_{HH}^2$ | 0.97 | 32.86 |
| | | $B^{-}K^-$ | 945.50$g_{HH}^2$ | 1 | 33.60 |
| | | $B_s^+\pi^0$ | 815.12$g_{HH}^2$ | 0.86 | 28.97 |
| | | $B_s^+\eta$ | 128.26$g_{HH}^2$ | 0.13 | 4.55 |
| | | Total | 2813.64$g_{HH}^2$ | | |
TABLE V: Strong decay width of non-strange and strange n=2 P-wave with \( s_t = 1/2 \) bottom mesons \( B(2^3P_0) \), \( B(2^1P_1) \), \( B_s(2^3P_0) \) and \( B_s(2^1P_1) \). Ratio in 5th column represents the \( \frac{\Gamma}{\Gamma(B^* \rightarrow B^+ \pi^-)} \) for the non-strange mesons and \( \frac{\Gamma}{\Gamma(B^*_s \rightarrow B^{*-} K^+)} \) for the strange mesons. Fraction gives the percentage of the partial decay width with respect to the total decay width.

| State        | \( nLsJ^I \) | Decay channel  | Decay Width (MeV) | Ratio | Fraction |
|--------------|--------------|----------------|-------------------|-------|----------|
| \( B^*_0(6335.83) \) | 2\( P_{1/2}^0 \) | \( B^- \pi^+ \) | 7087.60\( g_{SH}^2 \) | 1     | 42.15    |
|              |              | \( B^0 \pi^0 \) | 3542.06\( g_{SH}^2 \) | 0.49  | 21.06    |
|              |              | \( B^0 \eta \)  | 1063.91\( g_{TH}^2 \) | 0.15  | 6.32     |
|              |              | \( B_s \)       | 5118.49\( g_{TH}^2 \) | 0.72  | 30.44    |
|              |              | Total           | 16812.10\( g_{SH}^2 \) |       |          |
| \( B^*_1(6318.67) \) | 2\( P_{1/2}^1 \) | \( B^* \pi^+ \) | 6052.72\( g_{SH}^2 \) | 1     | 42.83    |
|              |              | \( B^* \pi^0 \) | 3027.69\( g_{SH}^2 \) | 0.50  | 21.42    |
|              |              | \( B^* \eta \)  | 890.83\( g_{TH}^2 \)  | 0.14  | 6.30     |
|              |              | \( B^*_s \)     | 4158.15\( g_{TH}^2 \) | 0.68  | 29.42    |
|              |              | Total           | 14129.40\( g_{SH}^2 \) |       |          |
| \( B^*_{0s}(6335.72) \) | 2\( P_{1/2}^0 \) | \( B^s \pi^0 \) | 2839.71\( g_{SH}^2 \) | 0.43  | 14.77    |
|              |              | \( B^s \eta \)  | 3308.90\( g_{TH}^2 \) | 0.50  | 17.21    |
|              |              | \( B^s K^0 \)   | 6530.27\( g_{SH}^2 \) | 0.99  | 33.96    |
|              |              | \( B^s K^+ \)   | 6547.13\( g_{SH}^2 \) | 1     | 34.05    |
|              |              | Total           | 19226.00\( g_{SH}^2 \) |       |          |
| \( B^*_{1s}(6336.72) \) | 2\( P_{1/2}^1 \) | \( B^* s \pi^0 \) | 2490.50\( g_{SH}^2 \) | 0.42  | 14.71    |
|              |              | \( B^* s \eta \) | 2834.34\( g_{TH}^2 \) | 0.48  | 16.75    |
|              |              | \( B^{*+} K^0 \)| 5792.74\( g_{SH}^2 \) | 0.99  | 34.23    |
|              |              | \( B^{*+} K^+ \)| 5803.61\( g_{SH}^2 \) | 1     | 34.29    |
|              |              | Total           | 16921.20\( g_{SH}^2 \) |       |          |
TABLE VI: Strong decay width of non-strange and strange n=2 P-wave with $s_l = 3/2$ bottom mesons $B(2^1P_1)$, $B(2^3P_2)$, $B_s(2^1P_1)$ and $B_s(2^3P_2)$. Ratio in 5th column represents the $\Gamma = \frac{\Gamma}{\Gamma(B^0 \rightarrow B^+ \pi^-)}$ for the non-strange mesons and $\hat{\Gamma} = \frac{\Gamma}{\Gamma(B_s^0 \rightarrow B^{*-} K^+)}$ for the strange mesons. Fraction gives the percentage of the partial decay width with respect to the total decay width.

| State         | $nLs/J^P$ | Decay channel | Decay Width(MeV) | Ratio | Fraction |
|---------------|-----------|---------------|------------------|-------|----------|
| $B_1(6336.30)$ | $2P_{3/2}^{1+}$ | $B^0 \pi^+$   | 7021.89 $\tilde{g}_T^2$ | 1     | 53.58    |
|               |           | $B^0 \pi^0$   | 3522.08 $\tilde{g}_T^2$ | 0.50  | 26.87    |
|               |           | $B^0 \eta$    | 512.74 $\tilde{g}_T^2$  | 0.07  | 3.91     |
|               |           | $B_s K$       | 2046.74 $\tilde{g}_T^2$ | 0.29  | 15.61    |
|               |           | Total         | 13103.50 $\tilde{g}_T^2$|       |          |
| $B_1^*(6354.56)$ | $2P_{3/2}^{2+}$ | $B^+ \pi^+$   | 4571.90 $\tilde{g}_T^2$ | 1     | 29.08    |
|               |           | $B^+ \pi^0$   | 2292.95 $\tilde{g}_T^2$ | 0.50  | 14.58    |
|               |           | $B^+ \eta$    | 345.57 $\tilde{g}_T^2$  | 0.07  | 2.19     |
|               |           | $B_s K$       | 1392.05 $\tilde{g}_T^2$ | 0.30  | 8.85     |
|               |           | $B_s \pi^0$   | 1849.52 $\tilde{g}_T^2$ | 0.40  | 11.76    |
|               |           | $B_s \pi^- $  | 3694.17 $\tilde{g}_T^2$ | 0.80  | 23.49    |
|               |           | $B_s \eta$    | 300.86 $\tilde{g}_T^2$  | 0.06  | 1.91     |
|               |           | $B_s K$       | 1274.68 $\tilde{g}_T^2$ | 0.27  | 8.10     |
|               |           | Total         | 15721.70 $\tilde{g}_T^2$|       |          |
| $B_{1s}(6429.02)$ | $2P_{3/2}^{1+}$ | $B_s^0 \pi^0$ | 3587.85 $\tilde{g}_T^2$ | 0.57  | 19.87    |
|               |           | $B_s^{**} K^0$| 6149.63 $\tilde{g}_T^2$ | 0.98  | 34.07    |
|               |           | $B_s^{**} K^+$| 6212.01 $\tilde{g}_T^2$ | 1     | 34.41    |
|               |           | $B_s^0 \eta$  | 2098.43 $\tilde{g}_T^2$ | 0.33  | 11.62    |
|               |           | Total         | 18047.90 $\tilde{g}_T^2$|       |          |
| $B_{2s}(6447.41)$ | $2P_{3/2}^{2+}$ | $B_s^0 \pi^0$ | 2336.94 $\tilde{g}_T^2$ | 0.57  | 10.71    |
|               |           | $B_s^{**} K^0$| 4053.03 $\tilde{g}_T^2$ | 0.99  | 18.58    |
|               |           | $B_s^{**} K^+$| 4092.49 $\tilde{g}_T^2$ | 1     | 18.76    |
|               |           | $B_s^0 \eta$  | 1419.93 $\tilde{g}_T^2$ | 0.34  | 6.48     |
|               |           | $B_s^0 K^-$   | 3387.56 $\tilde{g}_T^2$ | 0.82  | 15.53    |
|               |           | $B_s K^0$     | 3352.34 $\tilde{g}_T^2$ | 0.81  | 15.37    |
|               |           | $B_s \pi^0$   | 1912.00 $\tilde{g}_T^2$ | 0.46  | 8.76     |
|               |           | $B_s \eta$    | 1255.43 $\tilde{g}_T^2$ | 0.30  | 18.58    |
|               |           | Total         | 21804.70 $\tilde{g}_T^2$|       |          |