Magnetic field inversion symmetry in quantum pumps with discrete symmetries

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We investigate the magnetic field inversion symmetry of the pumped currents in quantum pumps with various discrete symmetries using Floquet scattering matrix approach. We found the pumped currents can have symmetries \( I(B, \phi) = -I(-B, -\phi) \) and \( I(B, \phi) = I(-B, -\phi) \), where \( \phi \) is the phase difference of two time dependent perturbations, depending on the discrete symmetry considered. The results in the adiabatic limit for each discrete symmetry are compared with those of Brouwer’s formula.

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I. INTRODUCTION

The quantum pump is a device that generates a dc current at zero bias potential through cyclic change of system parameters\(^{1,2,3}\). Recently, adiabatic charge pumping in open quantum dots not only has attracted considerable theoretical attention\(^{4-17}\), but was also experimentally realized by Switkes et al.\(^{18}\). After a cycle of the adiabatic shape change one returns to the initial configuration, but the wavefunction may have its phase changed with respect to the initial wavefunction. This is the geometric or Berry’s phase\(^{19}\). The additional phase is equivalent to pumped charges that pass through the quantum dot\(^{2}\). From another point of view, the quantum pump is a time dependent system driven by (at least) two different time periodic perturbations with the same angular frequency and a phase difference \( \phi \). One can deal with this problem under the adiabatic approximation, or by using the more general Floquet approach\(^{20,21,22}\). Recently, it has been shown that in the adiabatic limit with small strength of the oscillation potentials the Floquet and the adiabatic approach give exactly equivalent results\(^{21,22}\).

The puzzle unsolved in the experiment performed by Switkes et al.\(^{15}\) is the magnetic field inversion symmetry (MIS) in pumped currents. It was suggested theoretically\(^{15}\) that the pumped current \( I \) is invariant upon magnetic field reversal

\[
I(B) = I(-B). \tag{1}
\]

The subsequent experiment on open quantum dots appears to be in a good agreement with Eq.\(^{11,15}\). It became immediately clear, however, that such symmetry is not valid in theory\(^{8}\). Recently, Brouwer proposed that the rectified displacement current could account for the MIS\(^{11}\), which means the current observed in the experiment is not attributed to the adiabatic quantum pumping.

Even though in general quantum pumps do not fulfill the MIS, additional discrete symmetries of quantum dots can lead to such MIS. The effect of discrete symmetries on the MIS has been studied by Aleiner, Altshuler, and Kamenev\(^{3}\) in the adiabatic limit. They found the reflection symmetries give rise to relations

\[
I(B) = I(-B) \quad \text{or} \quad I(B) = -I(-B) \tag{2}
\]

both in the adiabatic and in the non-adiabatic cases. On the other hand, dots with UD symmetry obey

\[
I_{UD}(B, \phi) = I_{UD}(-B, \phi) \tag{3}
\]

both in the adiabatic and in the non-adiabatic cases, as well. IV symmetry does not have any relevant symmetry for the magnetic field inversion. From the Floquet scattering approach, however, the pumped current is shown to be vanishingly small in the adiabatic limit. It is emphasized that Eqs.\(^{2}\) and\(^{3}\) do not correspond to

\[
I_{LR}(B) = I_{LR}(-B) \quad \text{and} \quad I_{UD}(B) = -I_{UD}(-B), \tag{2}
\]

respectively. In the adiabatic limit it is always true that \( I(B, \phi) = -I(B, -\phi) \), consequently \( I(B, \phi = 0) = 0 \), while it is no longer available in the non-adiabatic case.

In Sec. II Floquet scattering matrix formalism is presented for the case without magnetic field. In Sec. III we discuss magnetic field inversion operation in a quantum pump, and investigate the MIS for three discrete symmetries, i.e. LR, UD, and IV symmetry. In Sec. IV we compare the results obtained in Sec. III with those from Brouwer’s formula, which shows they are consistent with each other in the adiabatic limit. Finally, we conclude in Sec. V.
I. FLOQUET SCATTERING MATRIX FORMALISM FOR A QUANTUM PUMP

First, we introduce Floquet scattering matrix formulation without magnetic field for simplicity. Consider the one-dimensional time-dependent Schrödinger equation $i\hbar(\partial/\partial t)\psi = H(t)\psi$ for a non-interacting electron with mass $\mu$ and $H(t) = -\hbar^2\nabla^2/2\mu + U(x,t)$, where $U(x,t + T) = U(x,t)$ and $U(x,t) = 0$ at $x \to \pm \infty$. Due to the periodicity in time, a solution can be written as

$$\Psi_\epsilon(x,t) = e^{-i\epsilon t/\hbar} \sum_{n=-\infty}^{\infty} \chi_n(x)e^{-in\omega t},$$

where $\epsilon$ is the Floquet energy which takes a continuous value in the interval $[0,\hbar\omega]$. Since the potential is zero at $x \to \pm \infty$, $\chi_n(x)$ is given by the following form

$$\chi_n(x) = \begin{cases} A_n e^{ik_n x} + B_n e^{-ik_n x}, & x \to -\infty \\ C_n e^{ik_n x} + D_n e^{-ik_n x}, & x \to +\infty, \end{cases}$$

where $k_n = \sqrt{2\mu(\epsilon + n\hbar\omega)}/\hbar$. By wave matching, the incoming and the outgoing waves can be connected by matrix $M$

$$\begin{pmatrix} \bar{B} \\ \bar{C} \end{pmatrix} = M \begin{pmatrix} \bar{A} \\ \bar{D} \end{pmatrix}. \quad (6)$$

If we keep only the propagating modes ($k_n$ is real), we can obtain the unitary Floquet scattering matrix $S$ [23], which can be expressed in the following form [24, 25, 26]

$$S = \begin{pmatrix} r_{00} & r_{01} & \cdots & r'_{00} & r'_{01} & \cdots \\ r_{10} & r_{11} & \cdots & r'_{10} & r'_{11} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ t_{00} & t_{01} & \cdots & t'_{00} & t'_{01} & \cdots \\ t_{10} & t_{11} & \cdots & t'_{10} & t'_{11} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix},$$

where $r_{nm}$ and $t_{nm}$ are the reflection and the transmission amplitudes respectively, for modes incident from the left with energy $\epsilon + m\hbar\omega$ and outgoing to modes with energy $\epsilon + n\hbar\omega$; $r'_{nm}$ and $t'_{nm}$ are similar quantities for modes incident from the right.

The total transmission coefficient from the left to the right as a function of an energy of an incident electron is given by

$$T^{\rightarrow}(E) = \sum_{n=0}^{\infty} |t_{nm}(\epsilon)|^2,$$

where $E = \epsilon + m\hbar\omega$. The total transmission from the right to the left $T^{\leftarrow}(E)$ can also be determined in a similar way. Then, the pumped current to the right $I_r$ is given by the difference between two currents having the opposite directions [21, 22].

$$I_r = I^{\rightarrow} - I^{\leftarrow},$$

where

$$I^{\rightarrow(\leftarrow)} = \frac{2e}{\hbar} \int_0^\infty dEf(E)T^{\rightarrow(\leftarrow)}(E). \quad (10)$$

Here, $f(E)$ represents the Fermi-Dirac distribution. Without a bias $f(E)$ has the same form for all reservoirs. The pumped current to the left $I_l$ is equal to $-I_r$. Without specification $f$ denotes $I_r$ below. To derive Eq. (10), we need to assume that the reservoirs are always at equilibrium even under external time-dependent perturbation [27]. This is guaranteed if the relaxation time of an electron in the reservoirs is much faster than a pumping cycle $T$.

II. MAGNETIC FIELD INVERSION IN A QUANTUM PUMP

Let us now consider the two-dimensional quantum dot driven by time-periodic perturbations under a static...
magnetic field, which is attached to two leads (see Fig. 2), whose Hamiltonian is given by

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ \frac{i\hbar \nabla + eA}{2\mu} + V(r,t) \right] \psi,$$

where $A$ is the vector potential. The potential $V(r,t)$ is given by

$$V(r,t,\phi) = V_0(r) + V_1(r) \cos(\omega t - \phi/2) + V_2(r) \cos(\omega t + \phi/2),$$

where $V_0$ represents the confined potential of a quantum dot, and $V_1$ and $V_2$ are the spatial dependence of two time dependent perturbations. $\phi$ is the initial phase difference of two time-dependent perturbations, and $r$ denotes $(x,y)$. Note that $V(r,-t,\phi) = V(r,t,-\phi)$. When $\phi = n\pi$ without magnetic field, the quantum pump is microscopically reversible (or time reversible), i.e. $t_{nm}^{\alpha\beta} = t_{nm}^{\alpha\beta}$, where $t_{nm}^{\alpha\beta}$ represents the transmission amplitude from the Floquet side band $m$ of the channel $\alpha$ in the left lead to the side band $n$ of the channel $\beta$ in the right lead, and $t_{nm}^{\alpha\beta}$ is a similar quantity for the opposite direction. In a quantum pump the two time dependent perturbations with a finite $\phi$ ($\neq n\pi$) as well as the magnetic field break the time reversal symmetry.

If we take the complex conjugate of Eq. (11) and at the same time, reverse the vector potential ($A \to -A$) and time ($t \to -t$), we obtain

$$i\hbar \frac{\partial \psi^*}{\partial t} = \left[ \frac{i\hbar \nabla + eA}{2\mu} + V(r,-t) \right] \psi^*.$$

Compared with Eq. (11) one can see that $\psi_{-B,\phi}(r,t) = \psi_{B,-\phi}^*(r,-t)$. The complex conjugation combined with the time inversion of the solution in Eqs. (11) and (12) simply corresponds to the reversal of the momentum direction of the incoming or the outgoing plane waves and simultaneously the replacement of absorption by emission, and vice versa. This simply implies that $S \to S^T$ ($S^T$ is the transpose of $S$), i.e. $S_{nm}^{\alpha\beta} \to S_{mn}^{\alpha\beta}$. For completion of the magnetic field inversion operation, $\phi \to -\phi$ also has to be considered.

The total transmission coefficient to the right reservoir after the magnetic field inversion is given by

$$T_{-B,\phi}^+(E) = \sum_{\alpha\beta} \sum_{n=0}^{\infty} |t_{nm}^{\alpha\beta}(\epsilon, -B, \phi)|^2$$

and

$$T_{-B,\phi}^-(E) = \sum_{\alpha\beta} \sum_{n=0}^{\infty} |r_{nm}^{\alpha\beta}(\epsilon, B, -\phi)|^2.$$

where $E = \epsilon + m\hbar \omega$. It must be noted that Eq. (14) is not equivalent to $T_{B,-\phi}^-(E)$. Figures 1(a) and (b) represent the usual setup of side bands for calculating the total transmission coefficients, e.g. $T_{B,\phi}^{-+}(E)$, and the setup for calculating $T_{B,\phi}^{-\pm}(E)$, respectively. In general, they are different from each other, so are $I(B)$ and $I(-B)$. The MIS is not valid in Floquet formalism, as well.

A. LR symmetry

If we assume LR symmetry [$V_{0}(x,y) = V_{0}(-x,y)$ and $V_{1}(x,y) = V_{2}(-x,-y)$], the following relations are obtained [see Fig. 1(a)]

$$t_{nm}^{\alpha\beta}(\epsilon, B, \phi) = t_{nm}^{\alpha\beta}(\epsilon, -B, -\phi),$$

$$r_{nm}^{\alpha\beta}(\epsilon, B, \phi) = r_{nm}^{\alpha\beta}(\epsilon, -B, -\phi).$$

Then, Eq. (14) can be rewritten as

$$T_{-B,\phi}^+(E) = \sum_{\alpha\beta} \sum_{n=0}^{\infty} |t_{nm}^{\alpha\beta}(\epsilon, B, -\phi)|^2 = T_{B,-\phi}^-(E).$$

From Eq. (10) one reaches $I(B, \phi) = -I(-B, -\phi)$, i.e. Eq. (2). Since such relation results from the Floquet approach, it is available for both the adiabatic and the non-adiabatic cases. Note that when $\phi = 0$ the result obtained in Ref. 9 is recovered, and its validity is now extended even to the non-adiabatic case. At zero magnetic field one finds $I(\phi) = -I(-\phi)$, consequently $I(\phi = 0) = 0$ even for the non-adiabatic case.

B. UD symmetry

For UD symmetry [$V_{0}(x,y) = V_{0}(x,-y)$ and $V_{1}(x,y) = V_{2}(x,-y)$] one finds

$$t_{nm}^{\alpha\beta}(\epsilon, B, \phi) = t_{nm}^{\alpha\beta}(\epsilon, -B, -\phi),$$

$$r_{nm}^{\alpha\beta}(\epsilon, B, \phi) = r_{nm}^{\alpha\beta}(\epsilon, -B, -\phi).$$

FIG. 2: Schematic diagrams for describing the configuration of side bands with given channels $\alpha$ and $\beta$ to calculate the total transmission coefficients through the time-periodic perturbation $V(t)$ for (a) $T_{B,\phi}^+(E)$ and (b) the case given in Eq. (15). (c) The setup (a) (the solid arrows) superimposed by the setup (b) reversed by using IV symmetry given in Eq. (20) (the dashed arrows).
[see Fig. 1(b)]. Then, Eq. (14) can be rewritten as

$$T^{-\rightarrow}_{B,\phi}(E) = \sum_{\alpha\beta} \sum_{n=0}^{\infty} |t^{\beta\alpha}_{nm}(\epsilon, B, -\phi)|^2$$

$$= T^{-\rightarrow}_{B,\phi}(E),$$

(19)

which leads to $I(B, \phi) = I(-B, -\phi)$, i.e., Eq. (19). Like the LR symmetry, such symmetry of the quantum pump is correct for both the adiabatic and the non-adiabatic cases. Note also that when $\phi = 0$ the result obtained in Ref. 3 is recovered not only for the adiabatic limit but also for the non-adiabatic case. At zero magnetic field one finds $I(\phi) = I(-\phi)$, which can be understood from the fact that the exchange of $V_1$ and $V_2$, i.e., $\phi \rightarrow -\phi$, does not make any difference in Fig. 1(b).

C. IV symmetry

Finally, we consider IV symmetry $[V_0(x,y) = V_0(-x,-y)$ and $V_1(x,y) = V_2(-x,-y)]$. If one assume IV symmetry, one obtains [see Fig. 1(c)]

$$t^{\beta\alpha}_{nm}(\epsilon, B, \phi) = t^{\beta\alpha}_{nm}(\epsilon, B, -\phi),$$

$$r^{\beta\alpha}_{nm}(\epsilon, B, \phi) = r^{\beta\alpha}_{nm}(\epsilon, B, -\phi).$$

(20)

Eq. (15) is reexpressed as

$$T^{-\rightarrow}_{B,\phi}(E) = \sum_{\alpha\beta} \sum_{n=0}^{\infty} |r^{\beta\alpha}_{nm}(\epsilon, B, \phi)|^2,$$

(21)

which is different from $T_{B,\phi}(E)$ since the summation is taken for $n$ as shown in Fig. 2(c). In general, there is no relevant MIS for dots with IV symmetry. When $\phi = 0$, however, Eq. (20) leads to $T^{-\rightarrow}_{B}(E) = T^{-\rightarrow}_{B}(E)$, consequently $I = 0$. The result in Ref. 4 is also recovered for both the adiabatic and non-adiabatic case.

IV. ADIABATIC LIMIT

The adiabatic condition in the quantum pump implies that any time scale of the problem considered must be much smaller than the period of the oscillation of an external pumping field. We can then define the instantaneous scattering matrix with time dependent parameters, namely $X_n(t)$,

$$\mathcal{S}^{ad}(E, t) = \mathcal{S}^{ad}(E, \{X_n(t)\}) = \begin{pmatrix} \hat{t}^{\alpha}_{ad} & \hat{t}^{\alpha}_{ad} \\ \hat{r}^{\alpha}_{ad} & \hat{r}^{\alpha}_{ad} \end{pmatrix}$$

(22)

Due to the time periodicity of $X_n$’s, using a Fourier transform one can obtain the amplitudes of side bands for particles traversing the adiabatically oscillating scatterer with incident energy $E$ as following

$$\mathcal{S}^{ad}(E, \{X_n(t)\}) = \sum_n \mathcal{S}^{ad,n}(E) e^{-i\text{ang}t},$$

(23)

where

$$\mathcal{S}^{ad,n}(E) = \frac{1}{T} \int_0^T dt e^{i\text{ang}t} \mathcal{S}^{ad}(E, \{X_n(t)\}).$$

(24)

Thus we can construct the adiabatic Floquet scattering matrix as follows

$$S(E_n, E) \approx S(E, E-n) \equiv \mathcal{S}^{ad,n}(E),$$

(25)

where $E_n = E + n\hbar\omega$. By exploiting Eqs. (24), (23) and (25), the MIS’s of adiabatic scattering matrices can be obtained directly from those of Floquet scattering matrices for the three symmetry classes.

Once the adiabatic scattering matrix is found, by using Brouwer’s formula, the pumped charge $Q$ per one cycle can be expressed as

$$Q(B) = g[i^{ad}(B), i^{ad}(B)] = \int_0^T dt \{f[i^{ad}(B)] + f[\hat{i}^{ad}(B)]\},$$

(26)

where

$$f(s) = \frac{e}{2\pi} \sum_{k=1}^2 \sum_{\alpha\beta} \text{Im} \frac{\partial S_{\alpha\beta}}{\partial X_k} a_{\alpha\beta} \frac{dX_k}{dt}.$$  

(27)

Then, the pumped current $I$ is $Q/T$. Below, we will use Eq. (20) for the current.

A. LR symmetry

The relation $\mathcal{L}$ implies $\hat{i}^{\beta\alpha}_{ad,n}(E; B, \phi) = \hat{i}^{\beta\alpha}_{ad,n}(E; -B, -\phi)$ in the adiabatic limit, which, by using Eq. (22), immediately leads to $\hat{i}^{ad}(E, t; B, \phi) = \hat{i}^{ad}(E, t; -B, -\phi)$. From Eq. (20) one finds

$$I_r(-B, \phi) = g[i(-B, \phi), i(-B, \phi)] = g[i^{ad}(B, \phi), i^{ad}(B, \phi)]$$

$$= I^{ad}(B, -\phi) = -I_r(B, -\phi).$$

(28)

This exactly corresponds to Eq. (2). Note that if the relation $I(\phi) = -I(-\phi)$, which is available only in the adiabatic limit, is considered, one reaches simpler form $I(B) = I(-B)$.

B. UD symmetry

Like the previous subsection the relation $\mathcal{L}$ implies $\hat{i}^{\beta\alpha}_{ad,n}(E; B, \phi) = \hat{i}^{\beta\alpha}_{ad,n}(E; -B, -\phi)$ in the adiabatic limit, which, by using Eq. (23), leads to $\hat{i}^{ad}(E, t; B, \phi) = \hat{i}^{ad}(E, t; -B, -\phi)$. From Eq. (20) one obtains

$$I_r(-B, \phi) = g[i(-B, \phi), i(-B, \phi)]$$

$$= g[i^{ad}(B, -\phi), i^{ad}(B, -\phi)]$$

$$= I_r(B, -\phi).$$

(29)
This exactly corresponds to Eq. (3). Note also that if the relation \( I(\phi) = -I(-\phi) \) is considered, one reaches simpler form \( I(B) = -I(-B) \).

C. IV symmetry

The relation \( \hat{t}_{ad,ad}^a(E;B,\phi) = \hat{t}_{ad,n,n}^a(E;B,-\phi) \) in the adiabatic limit, which, by using Eq. (28), leads to \( \hat{t}_{ad}(E,t;B,\phi) = \hat{t}_{ad}(E,t;B,-\phi) \).

Considering the magnetic field inversion operation, one can obtain

\[
\hat{t}_{ad}(E,t;B,\phi) = \hat{t}_{ad}(E,t;-B,\phi), \\
\hat{r}_{ad}(E,t;B,\phi) = \hat{r}_{ad}(E,t;-B,\phi).
\]

From Eq. (29) one finds

\[
I_r(-B,\phi) = g[\hat{t}_{ad}(-B,\phi),\hat{r}_{ad}(-B,\phi)] = g[\hat{t}_{ad}^T(B,\phi),\hat{r}_{ad}^T(B,\phi)].
\]

There is no relevant symmetry in the adiabatic limit, again.

Eq. (29) implies that \( t_{m+n,m} \approx t_{n,m-n} \) in the adiabatic limit. After summing up all the side band contributions, the two configurations shown in Fig. 1(c) (the solid and the dashed arrows) give the approximately equivalent total transmission coefficients, i.e. \( T^\pm_B(E) \approx T^\mp_B(E) \). By applying similar procedure to the reflection coefficient one can also find \( R^\pm_B(E) \approx R^\mp_B(E) \). (make sure that the arrow is now reversed). It is also satisfied that \( T^\pm_B(E) = T_{-B}^\mp(E) \) since the unitarity of Floquet scattering matrix requires \( T^{+(-)} + R^{+(-)} = 1 \). From these two relations the pumped current is shown to be vanishingly small, \( I(B) = \int dE [T^\pm_B(E) - T^\mp_B(E)] \approx 0 \) since \( T^\pm_B(E) \approx T_{-B}^\mp(E) \approx T^\mp_B(E) \). In Table I, we summarize the MIS of the pumped currents with various situations for the three symmetries.

V. SUMMARY AND DISCUSSION

In summary, we have investigated magnetic field inversion symmetry in a quantum pump with various discrete symmetries using Floquet scattering matrix approach. We found the pumped currents can possess symmetries \( I(B,\phi) = -I(-B,-\phi) \) for LR symmetry, and \( I(B,\phi) = I(-B,-\phi) \) for UD symmetry. For IV symmetry there is no relevant symmetry. The adiabatic limit for each discrete symmetry has been considered by using Brouwer’s formula.

One possibility to realize the experimentally observed MIS, i.e. \( I(B) = I(-B) \), is that the quantum dot would obey LR symmetry. Then, the MIS is recovered only in the adiabatic limit, while it will be broken in nonadiabatic case. In general, however, it is difficult that a chaotic quantum dot possesses any discrete symmetry like the LR symmetry.

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In order to construct the scattering matrix we multiply an identity to both sides, \( K^{-1}K \equiv M^{-1}K^{-1}K^{-1}K \), where \( K_{nm} = \sqrt{k_n} \delta_{nm} \). Then we have \( \hat{J}_{in} = \hat{J}_{in} \), where \( \hat{J} \) represents the amplitude of probability flux and \( \hat{M} \equiv KMK^{-1} \). Eq. (4) is obtained from \( \hat{M} \).
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TABLE I: The symmetries of the pumped currents for the magnetic field inversion. TRS denotes time reversal symmetry by two time dependent perturbations, i.e. TRS for $\phi = n\pi$ and no TRS for $\phi \neq n\pi$. $\times$ represents that there is no relevant symmetry.

|       | adiabatic | non-adiabatic |
|-------|-----------|---------------|
|       | TRS       | no TRS        |
| LR    | $I = 0$   | $I(B) = I(-B)$| $I(B) = -I(-B)$ | $I(B, \phi) = -I(-B, -\phi)$ |
| UD    | $I = 0$   | $I(B) = -I(-B)$| $I(B) = I(-B)$ | $I(B, \phi) = I(-B, -\phi)$ |
| IV    | $I = 0$   | $I \approx 0$ | $I = 0$         | $\times$ |

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