**Abstract.** The energy of a Gamma-Ray Burst is one of the most interesting factors that can help determining the origin of these mysterious explosions. After the discovery that GRBs are cosmological it was thought, for a while that they are standard candles releasing $\sim 10^{51}$ ergs. Redshift measurements that followed the discovery of GRB afterglow revealed that GRBs are (i) further than what was initially believed and (ii) have a very wide luminosity function. Some bursts revealed an enormous energy release with a record estimate of $1.4 \times 10^{54}$ ergs for GRB990123. The energy budget was stretched even further when it was suggested that the conversion efficiency of producing gamma-rays is low. The realization that GRBs are beamed changed this perspective and brought the energy budget of GRBs down back to a "modest" $\sim 5 \times 10^{50}$ ergs. I discuss here various estimates showing that GRB energy is narrowly distributed and discuss the implications of this conclusion to GRB models.

1. Introduction

The energy release in a Gamma-ray burst is one of the best clues on the nature of these objects. However, this simple and basic quantity is rather hard to get. The observed flux was readily available even for the first bursts. But the lack of a reliable distance estimate was a series obstacle. When redshift measurements became available it turned out that even this knowledge was not sufficient. Other complications arose. On one hand theoretical considerations showed that the efficiency of conversion of the energy to $\gamma$-rays could not be very high and estimates based on $\gamma$-rays alone were too low. On the other hand, it turned out that GRBs are beamed and estimates that assumed isotropic emission were too high. In this lecture I discuss the current understanding of the energetics of GRBs and the implication to models of the "inner engines" that power GRBs.

In the first GRB revolution BATSE (Meegan et al., 1992) demonstrated that GRBs are cosmological. This has changed the distance scale by six orders of magnitude and the energy scale by twelve orders of magnitude. BATSE’s count distribution for long bursts ($T_{90} > 2$ sec) is consistent with a cosmological standard candle distribution (Cohen & Piran, 1995) with $E \approx 10^{51}$ ergs. This has led to the believe that (at least long) GRBs are standard candles and that the bursts are observed by BATSE up to $z \approx 2$.

The second GRB revolution took place in 1997 with the discovery of GRB afterglow (Costa et al., 1997; van Paradijs et al., 1997). The Italian-Dutch
satellite BeppoSAX provided angular position of several dozen long bursts to within about 3 arc-minutes which enabled follow up observations in the x-ray (see Piro, 2000), optical, milli-meter and radio frequencies which has provided a wealth of information on these explosions. In several cases the redshift of the afterglow or the host galaxy could be measured. This provided a new and direct estimate of the distance and of the energy involved. The first redshift, $z = 0.835$ was obtained for GRB970508. The corresponding energy, $5.4 \times 10^{51}$ ergs, was more or less in line with the standard candles estimates!

It quickly became apparent that GRBs are not standard candles. With a redshift 3.4 and energy $2 \times 10^{53}$ ergs GRB971214 was too far and too energetic for the standard candle picture. GRB990123 was even more energetic. It turned out that the GRB energy distribution is very broad and that their rate follows the star formation rate (Totani, 1997; Sahu et al, 1997; Wijers et al, 1998). The consistency of a standard candle cosmological model with BATSE’s peak counts distribution remains an inexplicable coincidence.

The observations of extremely large energies (at least in some bursts) suggested that some GRBs involve energy release of stellar mass or more, ruling out several of the leading models at the time. The situation was even more worrisome when one considered the issue of efficiency. Most models and in particular the internal shocks model, cannot produce $\gamma$-rays at 100% efficiency. There was a concern that the efficiency of the internal shocks model is rather low (at the level of a few percent (Kobayashi et al, 1997; Daigne, & Mochkovitch, 1998). This increased further the energy requirement for GRBs, ruling out most models and leading to a GRB energy crisis (Kumar, 1999).

On the other hand GRBs could be beamed. The energy budget would then drop down significantly, by a factor $\theta^2/2$, where $\theta$ is the opening angle of the jet. Rhoads (1999) pointed out that an expanding relativistic jet would exhibit spherical like evolution as long as $\Gamma > \theta^{-1}$. It will expand rapidly sideways afterwards. This behaviour will produce a break in the afterglow light curve. Using this break one could determine the opening angles and estimate the true energy budget. Already in 1999 Sari, Piran & Halpern pointed out that the two most energetic GRBs (at that time) exhibit jet-like behaviour. This suggested that beaming is a strong factor in the energetics of GRB.

I summarize here several different estimates to the energy of GRBs. I argue that there are good indications that different GRBs emit relatively constant energy. GRBs are standard candles after all. However, differences in beaming factors lead to a variation of three orders of magnitude in the isotropic equivalent energies and to a very wide apparent luminosity function.

2. The Overall Picture and the Fireball Model

Following the discovery of GRB afterglow we have learned during the last four years a great deal about long duration gamma-ray bursts (GRBs). These observations are described well by the relativistic fireball model (see e.g. Piran, 1999). According to this model the energy from the central source is deposited in material that moves very close to the speed of light. The kinetic energy of this material is converted to the observed $\gamma$-rays as a result of collisions between fast moving material that catches up with slower moving ejecta. The afterglow
Figure 1. The internal-external shocks fireball model: The GRB is produced by internal shocks within the flow. The afterglow is produced by shock heated circum-burst matter.

Our goal is to find the best estimate for
• $E_{\text{tot}}$: the total energy emitted by the source.

However, it is practically impossible to determine this energy. It is not known what are all forms of energy release by the source. For example, the fireball model is based the conversion of the kinetic energy of relativistic flow (particles or Poynting flux) to $\gamma$-rays. It is impossible to detect non-relativistic particles that may also be emitted by the source. A second, more accessible quantity is

• $E_{\text{rel}}$: the energy of the relativistic flow emitted by the source.

A fraction, $\epsilon$, of $E_{\text{rel}}$ is converted via the internal shocks to the observed prompt $\gamma$-ray emission. The rest, $(1 - \epsilon)E_{\text{rel}}$, is dissipated later via external shocks on the circum-burst matter producing the afterglow. An unknown fraction of this energy is dissipated in the radiative phase (which has not been observed yet) during the first half hour of the afterglow. The remaining energy is:

• $E_K$: the kinetic energy during the adiabatic afterglow phase.

$E_K$ is dissipated gradually over a period of months or even years producing the observed afterglow. Clearly, we have: $E_K < (1 - \epsilon)E_{\text{rel}} \leq E_{\text{tot}}$. Observations
of long time tails of GRBs suggest that this losses are small (Burenin et al, 1999, Giblin et al, 1999; Tkachenko et al., 2000). Thus, $E_K \approx (1 - \epsilon)E_{rel}$. The efficiency, $\epsilon$, cannot be too large otherwise there won’t be afterglow. It cannot be too small either, otherwise we will reach a GRB energy crisis. Combining these facts we expect, therefore, that $E_K \approx E_{rel}$ to within say a factor of a few.

### 2.2. Energy - Observational Considerations

Given the observed $\gamma$-ray fluence and the redshift one can easily estimate

- $E_{\gamma,iso}$: the energy emitted in $\gamma$-rays assuming that the emission is isotropic.

$E_{\gamma,iso}$ can also be estimated from the BATSE catalogue by fitting the flux distribution to theoretical models (Cohen & Piran, 1995; Schmidt, 2001). As afterglow observations proceeded, alarmingly large values (Kulkarni et al. 1999) $(3.4 \times 10^{54}$ergs for GRB990123) were measured for $E_{\gamma,iso}$. As we discuss shortly (Rhoads, 1999; Sari Piran & Halpern, 1999) GRBs are beamed and $E_{\gamma,iso}$ is far larger than the actual energy emitted in $\gamma$-rays. We define instead:

- $E_{\gamma} \equiv (\theta^2/2)E_{\gamma,iso}$.

Here $\theta$ is the effective angle of $\gamma$-ray emission.

One would expect that $E_{\gamma}$ is a good estimate to $E_{rel}$ as: $E_{\gamma} = \epsilon E_{rel}$. However there are several problems. First $\epsilon$ is unknown. Moreover one would expect it to vary significantly from one burst to another. Second, the large Lorentz factor during the $\gamma$-ray emission phase, makes the observed $E_{\gamma}$ rather sensitive to angular inhomogeneities of the relativistic ejecta (Kumar & Piran, 2000). During the GRB phase the relativistic Lorentz factor is at least a few hundred (See e.g. Lithwick,& Sari, 2001 for a summary of the arguments concerning pair production opacity). Thus, the observed $\gamma$-rays come from a region whose angular size is $\gamma^{-1} \leq 10^{-2}$. This is narrower by a factor of ten than the angular width of the narrowest observed jets. Thus the observed $\gamma$-rays span only a small fraction of the actual emitting region. The estimated energy based on this data alone could be misleading.

### 2.3. The Efficiency of Internal Shocks

The conversion efficiency of kinetic energy to $\gamma$-rays, $\epsilon$, depends on two factors: the conversion efficiency of bulk kinetic energy to energy of accelerated electrons and the efficiency of radiating this energy to $\gamma$-rays. It is usually assumed that this second factor is close to unity. The first factor depends on the strength of the relevant shocks and in turn on the relative Lorentz factors between the different shells (Kobayashi, Piran & Sari, 1997). There have been numerous attempts to estimate this efficiency (Kobayashi, Piran & Sari, 1997; Daigne, & Mochkovitch, 1998; Kumar, 99; Spada et al, 2000; Beloborodov, 2000; Kobayashi, & Sari, 2001; Guetta et al, 2001) and the results range from a few percent to near unity. The conversion can be efficient (close to unity) if the distribution of Lorentz factors is very wide (Beloborodov, 2000, Kobayashi, & Sari, 2001). In those cases in which GRB afterglow is observed this efficiency could not be too large. Otherwise there would be no energy left to produce the afterglow.
2.4. Jets and Beaming

The original fireball model assumed spherical symmetry. As always this was partially because of simplicity. However, the spherical approximation is valid as long as $\Gamma > \Delta \theta^{-1}$, where $\Delta \theta$ is the scale of angular inhomogeneity. Because of time dilation sideways propagation is too slow and the information on the inhomogeneity could not propagate (Piran, 1995). As the blast wave is slowed down by the circum burst matter the Lorentz factor decreases and when $\Gamma \sim \theta^{-1}$ it begins to expand sideways (Rhoads, 1999). This produces a break at the light curve, which is accompanied by a change in the spectral index (Sari Piran & Halpern, 1999). Various models (usually analytic or semi-analytic) have been used to describe the hydrodynamic evolution of this expanding phase (Rhoads 1999; Sari Piran & Halpern, 1999; Panaitescu & Mészáros 1999; Moderski, Sikora & Bulik 2000; Kumar & Panaitescu 2000). Different assumptions have lead to different relations between the opening angle and the $t_b$, the time of the break in the light curve. Following Sari Piran & Halpern (1999) I adopt here:

$$\theta = 0.12(n/E_{51})^{1/8} t_{b,\text{days}}^{3/8} = 0.052(n/E_{K,51})^{1/6} t_{b,\text{days}}^{1/6},$$

where $E_{K,51}$ is the adiabatic kinetic energy in units of $10^{51}$ ergs, $E_{51} \equiv 2E_{K,51}/\theta^2$ is the isotropic-equivalent kinetic energy and $t_{b,\text{days}}$ is the break time in the afterglow light curve expressed in days.

Detailed numerical simulation (Granot et al, 2001) show that the sideline expansion is more complicated than what was assumed in the simple analytic models (Fig. 2 depicts the density and the velocity field from a jet long after the jet break. The sideline expansion is not as prominent as expected.) Still, somewhat surprisingly, a jet break arises more or less at $t_b$ according to Eq. 1.

3. Energy Estimates

The simplest and most direct estimates are of $E_{\gamma,\text{iso}}$. Schmidt (1999) have fitted the $\langle V/V_{\text{max}} \rangle$ distribution of BATSE’s GRBs a cosmological model that
follows the star formation rate. He finds a break energy of \(1.2 \times 10^{53}\) ergs in the 10-1000 keV band but as the higher slope is steeper the average energy is \(2.4 \times 10^{52}\) ergs. This is of course the isotropic energy. The energy estimate depends on the specific cosmological model (Schmidt mentions variation by a factor of 2.5 between different models), on the star formation rate and on the assumptions on the average spectral indices of the GRB.

For GRBs with known redshifts one can estimate \(E_{\gamma,iso}\). Here care should be taken to obtain an exact estimate of the spectrum (see Jimenez et al., 2001) and to add a proper K correction (Bloom et al., 2001). Fig 3 depicts the isotropic energy and the redshift for 17 bursts with known afterglow from Bloom et al., (2001). One sees a large spread (as seen also in Schmidt’s (1999) luminosity function). Jimenez et al (2001) consider 8 BATSE bursts with a spectroscopic redshift estimate and four additional bursts whose redshift has been estimated on the basis of the host galaxy R magnitude. They find an average \(E_{\gamma,iso}\) of \(1.3 \times 10^{53}\) ergs. This agrees with the average of Bloom’s sample but is higher by a factor of 5 than Schmidt’s average for the full BATSE sample.

The next step would be to correct \(E_{\gamma,iso}\) for beaming. Frail et al. (2001, hereafter F1) estimated \(E_\gamma\) for 18 bursts with redshift. They find a very narrow distribution with typical values around \(5 \times 10^{50}\) ergs (see Fig 3) and FWHM of a factor of 5. This energy estimate should be taken with care as it depends critically on the estimated jet opening angles \(\theta_j\). Frail et al (2001, hereafter F01) estimate the opening angles using Eq. 1. Panaitescu & Kumar (2001, hereafter PK01) perform a detailed modeling of the afterglow and obtain different estimates for \(\theta_i\). A comparison of the opening angles obtained by F01 and by PK01 for the same bursts shows that while many of the estimates agree in two cases out of 6 the angles differ by a factor of \(\sim 3\). Furthermore one has to worry about the possible angular inhomogeneity discussed earlier. Given these facts the narrowness of the \(E_\gamma\) distribution is remarkable!

PK01 have modeled the afterglow emission over a wide range of frequency and time for 8 well studied bursts. The model is fitted to the data and the fit yields several burst parameters, including the adiabatic energy, \(E_K\), and the jet opening angle. The results are also shown on Fig 3. Using their estimate of the opening angles \(\theta_i\) PK01 also estimate \(E_\gamma\) (see Fig. 3). The PK01 estimates for \(E_\gamma\) differ from the F01 estimates for the same bursts by up to a factor of 8!

4. The Width of the Energy Distribution

Both the F01 and PK01 results show that the energy distributions are rather narrow. F01 find that the FWHM of the \(E_\gamma\) distribution is only a factor of 5. PK01 \(E_K\) estimates are very narrow, the width is only a factor of 3. The \(E_\gamma\) estimates of PK01 are wider, but still narrow with a width of 11. These values should be compared with the \(E_{iso}\) distribution that spans a factor of \(10^4\). This motivates us to search for different limits on the width of the energy distribution.

Following Piran et al (2001a) I turn now to determine the spread of \(E_K\) using the x-ray afterglow flux. The key factor in this method is the observation that the x-ray luminosity at a fixed time after the burst depends almost linearly on the kinetic energy of the flow at that time and very weakly if at all on other parameters (Kumar 2000; Freedman & Waxman, 2001).
Consider a fireball with $\Gamma > \theta^{-1}$, so that the spherical approximation holds. The standard synchrotron fireball model implies that the isotropic equivalent luminosity at frequency $\nu$ above the cooling frequency, at a fixed elapsed time since the explosion, is given by (Kumar 2000; Freeland & Waxman, 2001):

$$L_x = \eta_p \left[ \frac{dE_K}{d\Omega} \right]^{(p+2)/4} e^{\epsilon_e - \epsilon_B (p-2)/4},$$

(2)

where $dE_K/d\Omega$ is the kinetic energy per unit solid angle, and $\eta_p$ is a constant. $L_x$ depends on the energy per unit solid angle in the explosion, and on the fractional energy taken up by electrons, $\epsilon_e$. $p$ here is the spectral index of the electron's energy distribution. Since typically $p \approx 2$ we have an almost linear dependence of $L_x$ on $E_K$. $L_x$ should be measured several hours after the burst before any jet break takes place.

One can attempt to estimate $E_K$ from measurements of $L_x$. However a simpler and more robust calculation will be to estimate the width of the energy distribution $\sigma_{E_K}$ which in turn is determined by the width of the x-ray luminosity distribution. Remarkably, the width of the x-ray luminosity distribution can be estimated from the observed x-ray flux distribution. The x-ray afterglow fluxes from GRBs have a power law dependence on $\nu$ and on the observed time $t$ (Piro, 2000): $f_\nu(t) \propto \nu^{-\beta} t^{-\alpha}$ with $\alpha \approx 1.4$ and $\beta \approx 0.9$. The observed x-ray flux per unit frequency, $f_x$, is related, therefore, to, $L_x$, the isotropic luminosity of the source at redshift, $z$ by:

$$L_x(t) = 4\pi d_L^2 f_x(t)(1+z)^{\beta-\alpha} \equiv f_x(t)Z(z),$$

(3)

where $Z(z)$ is a weakly varying function of $z$. For bursts with $0.5 < z < 4$ and with $\beta - \alpha \approx -0.5$ we find $\sigma_Z \approx 0.31$ (for a cosmology with $\Omega_m = 0.3$ and
Figure 4. From Piran et al., (2001) **Left:** The distribution of X-ray fluxes (2-10 keV) at t=11 hours after the GRB in 21 afterglows observed by BeppoSAX. The arrow marks two bursts for which there is only an upper limit. **Right:** Likelihood contour lines (corresponding to 99%, 90% and 69% confidence levels) in the log($f_x$), $\sigma_{f_x}$ plane for the X-ray flux distribution as inferred from 21 GRBs detected by BeppoSAX. The maximal likelihood is at log($f_x$) = $-12.2\pm 0.2$ and $\sigma_{f_x} = 0.43^{+1.2}_{-1.1}$.

$\Omega_\Lambda = 0.7$). Here and thereafter we denote by $\sigma_X$ the standard deviation of the log($X$), unless noted otherwise.

Piran et al (2001) use 21 BeppoSAX bursts (Piro, 2000) to determine log($f_x$) and $\sigma_{f_x} = 0.43^{+1.2}_{-1.1}$ for the observed x-ray flux in the 2–10 kev band at 11 hr after the GRB (For two of the 21 bursts there is only upper limit of $2 \times 10^{-13}$ergs/cm$^2$/sec to the x-ray flux. This is consistent with a predicted number of 3.5 burst with x-ray afterglows below this limit.).

If there is no correlation between the microscopic variables $\epsilon_e$, $\epsilon_B$, $p$ and $dE_K/d\Omega$ and if the x-ray flux does not depend on the redshift we have:

$$\sigma_{E_K}^2 + 4\sigma_\theta^2 = \sigma_{dE_K/d\Omega}^2 < \sigma_{L_z}^2 = \sigma_{f_x}^2 + \sigma_\theta^2 \approx \sigma_{f_x}^2 = (0.43^{+1.2}_{-1.1})^2.$$ (4)

PK01 and F01 have estimated the jet opening angles obtaining $\sigma_\theta(PK01) \approx 0.31 \pm 0.06$ and $\sigma_\theta(F01) \approx 0.28 \pm 0.05$ for 8 and 17 bursts respectively. If these values are representative for the whole GRB population we find a marginally viable solution within two $\sigma$ errors of $\sigma_{E_K} < 0.2$ (for the PK01 result) and $\sigma_{E_K} < 0.27$ (for the F01 data); to get a viable solution we had to take both the values of $\sigma_{L_z}$, one $\sigma$ above the mean and the value of $\sigma_\theta$ one $\sigma$ below the mean. This result suggests that there is a narrow energy distribution; the FWHM of $E_K$ being less than a factor of 5.

We have argued before that $E_K$, discussed here, is a rather good estimate to $E_{rel}$ the total energy emitted by the “inner engine”. The constancy of $E_K$ is another indication for it being a good measure of $E_{rel}$. The constancy of $E_K$ is also an indication that the assumptions that have lead to Eq. 2 are justified. Otherwise it would have been remarkable if starting from different levels of initial energy and having different amounts of energy losses the final kinetic energy of the afterglow would converge to a constant value.
5. Discussion

There are two striking results: the narrowness of the $E_{\gamma}$ and $E_K$ distributions and the fact that $\bar{E}_{\gamma} \approx 3\bar{E}_K$ (see Fig. 3). There seem to be an intrinsic inconsistency between the two results. A narrow $E_{\gamma}$ distribution with $\bar{E}_{\gamma} > \bar{E}_K$ implies (with no fine tuning) that $E_{rel}$ is narrowly distributed and $E_{\gamma} \approx E_{rel}$ (rather than $E_{rel} \approx E_K$). The fact that $E_K < \bar{E}_{\gamma}$ is also narrowly distributed implies that $\epsilon$, the conversion efficiency of relativistic kinetic energy to $\gamma$-rays, is close to unity and moreover, $\epsilon$ itself should be rather narrowly distributed (between 70-80%). This last conclusion would be astonishing considering the dependence of $\epsilon$ on the the distribution of energies and Lorentz factors of the different shells. The narrow distribution of $E_{\gamma}$ also implies that the emitting jets are rather homogeneous (otherwise we would expect significant variations in $E_{\gamma}$ even for the same burst when observed from slightly different direction). The results are possible but they require a (currently) inexplicable fine tuning.

There is no simple way out. It is of course possible that we have been misled by small number statistics and we have to wait for a larger data set in which we expect to find $E_K \geq E_{\gamma}$ and a larger spread in $E_{\gamma}$.

The current results would have been easier to understand if, for some reason, $E_K$ have been underestimated by a factor of 3. In this case we would have $E_{rel} \approx \bar{E}_{\gamma} \approx E_K$ with $\epsilon \approx 0.5$ and with a reasonable spread in $\epsilon$. Note that at least in the PK01 results we already have $\sigma_{E_{\gamma}} > \sigma_{E_K}$.

A second possibility is that in fact $E_K \geq \bar{E}_{\gamma}$ (where $\bar{E}_{\gamma}$ is the $\gamma$-ray energy averaged over the whole GRB jet). However, in our sample angular inhomogeneities resulting in bright spots have increased, according to the patchy shell model (Kumar & Piran, 2000), the observed $E_{\gamma}$ (in the afterglow sample) above the real (average) $\bar{E}_{\gamma}$ value. This would results in $\sigma_{E_{\gamma}} > \sigma_{E_K}$, as observed indeed in the PK01 results. However, the same angular inhomogeneities would result at times in a decrease in the observed flux. We should observe also bursts with $E_{\gamma} < E_K$. Such bursts are currently missing in the PK01 sample! Again more bursts may resolve this problem.

In any case these results indicate that one way or another GRB "inner engines" are standard candles releasing a rather constant energy. The wide distribution of directly and indirectly determined $E_{\gamma,iso}$ results from the wide distribution of beaming angles. The fact that GRB engines are "standard" engines in terms of their energy output provide a very severe constraint on the nature of these enigmatic explosions. For instance, in the collapsar model for GRBs the central engine is composed of a black hole (BH) and an accretion disk around it (Woosley 1993; Paczynski, 1998; MacFadyen & Woosley, 1999). This model has two energy reservoirs which can be tapped to launch a relativistic jet: the BH rotation energy and the gravitaional energy of the disk. Our result of nearly constant energy in GRBs implies that the mass accretion on to the BH plus the possible conversion of rotational energy of the BH to kinetic energy of the jet does not vary much from one burst to another in spite of the fact that both the disk mass and the BH spin are expected to vary widely in the collapse of massive stars.

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