Life prediction of electric spindles with multiple degradation processes

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Abstract. In order to realize product life prediction without burst failure information, a method combining performance degradation information correlation analysis to construct a degradation process model is proposed. On the basis of obtaining the degradation information and establishing the single performance indicator degradation process model, the covariance is used to analyze the correlation of the multi-performance index degradation process, and the Copula connection function is introduced to establish the system reliability model under multi-degradation process. Achieve the reliability curve drawing and reliable life prediction. Finally, the effectiveness of the method is verified by taking the degradation information of a series of milling type electric spindles as an example.

Key words: Electric spindles; multi-degenerate process; reliability modelling; life prediction

1. Introduction

At present, it is difficult for domestically produced machine tools to form effective competitiveness. One of the reasons is that the reliability of the "heart component" electric spindle is not high [1]. By predicting the life of the electric spindle, it is possible to predict the failure time in advance, take measures to reduce the number of motor spindle shutdowns, and reduce downtime and maintenance costs during the fault repair process. Therefore, it is of great significance to carry out research on reliable life prediction of electric spindles.

At present, the life prediction methods are mainly divided into the following three categories: statistical-based methods, artificial intelligence-based methods, and model-based methods [2]. Based on the statistical principle, the life prediction needs to obtain sufficient product life information, and the reasonable statistical life model is obtained through model assumptions and parameter estimation. The model accuracy is related to the data volume. The artificial intelligence method needs to combine multi-source information fusion technology with artificial neural network, and the analysis process is relatively complicated [3]. Life prediction using models is more common in engineering practice [4].

The electric spindle is a long-life, high-reliability product, and it cannot obtain sufficient failure information in a short time. Therefore, the traditional life prediction method based on statistical principle is not applicable. The performance of the product is always degraded during use, so it can be combined with its degradation information for reliability modelling and life prediction research [5-6]. The multi-
degradation process coexists during the use of the electric spindle, and there is mutual influence between the degradation processes. Therefore, based on the multi-performance degradation process and correlation analysis of the electric spindle, this paper constructs a multi-performance degradation process model for the life prediction of the electric spindle.

2. Reliability modelling based on multiple degradation processes with mutual influence

2.1. Single degradation process modeling

The performance indicator degradation amount is calculated based on the performance measurement information. It is assumed that the degradation process obeys a common distribution function, and the Weibull distribution, the normal distribution, and the lognormal distribution are generally considered. The model parameters are estimated by the maximum likelihood estimation method. The d-test method (Kolmogorov-Smirnov test, or K-S test) is used to test the distribution model, and the error distribution is used to optimize the model distribution parameters. Then the performance index degradation data distribution function will be obtained.

2.1.1. Parameter estimation

Let \( X = \{X_1, X_2, \ldots, X_n\} \) be the overall degradation value of a certain performance indicator. Set \( f(x; \theta_1, \theta_2, \ldots, \theta_m) \) are parameters to be estimated with the amount of \( m \) as the population distribution density of the random variable \( X \). Construct a likelihood function from \( n \) observations of random variable \( X \).

\[
L(\theta_1, \theta_2, \ldots, \theta_m) = \prod_{i=1}^{n} f(x_i; \theta_1, \theta_2, \ldots, \theta_m) \tag{1}
\]

According to the principle of maximum likelihood method, the estimated value of \( \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_m \) can be obtained by solving the likelihood equation (2).

\[
L(\theta) = L(x; \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_m) = \max_{\theta \in \Theta} L(x; \theta_1, \theta_2, \ldots, \theta_m) \tag{2}
\]

2.1.2. Model checking

Commonly used model checking methods are the \( \chi^2 \)-test method and the d-test method. The \( \chi^2 \)-test method is mainly applicable to large samples. The d-test (Kolmogorov-Smirnov test, or K-S test) is finer than the \( \chi^2 \)-test. Besides, it is applicable to small samples. Therefore, this paper chooses the d-test method for the distribution model test.

The d-test is to arrange \( n \) test data in order of small to large. According to the hypothetical distribution, calculate the corresponding \( F_n(x_i) \) of each data and compared these with the empirical distribution function \( F_n(x_i) \). The maximum absolute value of the difference is the observation value of the test statistic \( D_n \). It will be compared to the threshold \( D_{n, \alpha} \) to determine whether to accept the null hypothesis.

\[
D_n = \sup_{-\infty < x < \infty} |F_n(x_i) - F_0(x_i)| = \max_{i=1}^n |d_i| \leq D_{n, \alpha} \tag{3}
\]

Where \( F_0(x_i) \) is null hypothesis distribution; \( F_n(x_i) \) is empirical distribution function with sample size \( n \).

2.1.3. Distribution model parameter optimization

The model parameters are actually the amount of change with time, further improve the accuracy of the model, and perform time series fitting on the model parameters, considering several commonly used time series fitting functions, such as linear functions, exponential functions, power functions, and logarithmic functions.

Using the sum of squared errors \(^7\) to reflect the excellent condition of the time series function fitting model parameters, the representation is as follows
\[ N = \sum_{i=1}^{n} (g_i - \hat{g}_i)^2 \]  

(4)

Where \( g_i \) indicates the actual value, \( \hat{g}_i \) indicates the fitted value.

The closer the sum of squared errors is to zero, the smaller the deviation between the actual data of the statistic and the fitted curve, indicating that the degree of fit of the function is higher.

2.2. Correlation analysis of multiple degradation processes

Covariance was used to test the correlation of multiple degradation processes. The covariance formula is:

\[ \text{cov}(x(t), y(t)) = E[(x(t) - \bar{x})(y(t) - \bar{y})] \]  

(5)

Where \( x(t) \) and \( y(t) \) are the degraded data of the two performance indicators of the product at time \( t \), \( \bar{x} \) and \( \bar{y} \) are the corresponding mean.

If the covariance is equal to zero or closer to zero, it means that the degradation processes of the two performance indicators are independent of each other. Besides, there is a correlation. The covariance value reflects the close relationship of the performance degradation process.

2.3. Multi-degradation process modeling based on Copula

The single degenerate process model is used as the edge distribution. The Copula connection function is introduced to construct the joint distribution model [8]. The maximum likelihood estimation is used to calculate the model parameters. The AIC criterion is used to select the most suitable Copula connection function to complete the reliability model construction.

The more typical Copula function in the commonly used Archimedes type distribution family is selected as the connection function. Its expression is shown in Table 1.

### Table 1. Copula function expression.

| Copula function     | Parameter range       | Function expression for binary |
|---------------------|-----------------------|--------------------------------|
| Gumbel Copula       | \( \alpha \in [0,1] \) | \( C(F_x, F_y; \alpha) = \exp\left[-\left(-\ln F_x\right)^\alpha + \left(-\ln F_y\right)^\alpha\right] \) |
| Clayton Copula      | \( \theta \in (0,\infty) \) | \( C(F_x, F_y; \theta) = \left(F_x\ln \theta + F_y\ln \theta - 1\right)^{-1/\theta} \) |
| Frank Copula        | \( \beta \in (-\infty,0) \cup (0, \infty) \) | \( C(F_x, F_y; \beta) = -\frac{1}{\beta} \ln \left[\frac{1 + (1-e^{-\beta F_x})(1-e^{-\beta F_y})}{e^{\beta}}\right] \) |

The AIC formula is as follows. The smaller the AIC criterion value, the better the model fits.

\[ \text{AIC} = -2\ln L + 2k \]  

(6)

Where \( L \) is the maximum likelihood function of the model, and \( k \) is the number of parameters of the model.

3. Examples

The electric spindle is a highly reliable and long-life product. It takes a long time to expose the motor spindle to defects or potential problems. Considering this situation, the test time is reduced by the constant stress accelerated life test when the test time is determined.

The relationship between the life \( t \) in the actual case and the corresponding life \( T \) under the acceleration test can be expressed as:

\[ T = K^{-1}t \]  

(7)

Where the acceleration factor \( K \) is taken 4.5.

According to the statistical analysis of the fault under actual use conditions, get \( t = 2339.6h, \) the completion probability is 45.60\%, and the test time for determining the accelerated life test was approximately 528h.

The eddy current sensor and the three-way acceleration sensor are used to collect the signal of the axial end runout and spindle vibration. The test data are shown in Table 2. According to the specification
of the electric spindle, when the axial end runout of the electric spindle reaches 1.6 times of the initial value, or when the spindle vibration of the electric spindle reaches 2 times the initial value, it is considered that the electric spindle has failed.

**Table 2.** Deterioration data of motor spindle performance index.

| time/h | Axial end runout (µm) | Spindle vibration (µm/s) | Performance index | time/h | Axial end runout (µm) | Spindle vibration (µm/s) |
|--------|------------------------|--------------------------|-------------------|--------|------------------------|--------------------------|
| 0      | 7.1                    | 123.2                    |                   | 288    | 8.9                    | 138.9                    |
| 24     | 7.8                    | 127.3                    |                   | 312    | 8.9                    | 140.9                    |
| 48     | 7.8                    | 127.6                    |                   | 336    | 8.9                    | 139.8                    |
| 72     | 7.8                    | 130.5                    |                   | 360    | 9.2                    | 141.8                    |
| 96     | 8.1                    | 130.9                    |                   | 384    | 9.4                    | 143.8                    |
| 120    | 8.2                    | 132.2                    |                   | 408    | 9.1                    | 146                      |
| 144    | 8.1                    | 130.2                    |                   | 432    | 9.6                    | 144.2                    |
| 168    | 8.4                    | 132.7                    |                   | 456    | 9.5                    | 149                      |
| 192    | 8.2                    | 134.1                    |                   | 480    | 9.5                    | 151                      |
| 216    | 8.4                    | 134.1                    |                   | 504    | 9.6                    | 149.3                    |
| 240    | 8.5                    | 136.9                    |                   | 528    | 9.9                    | 151.3                    |
| 264    | 8.8                    | 135.7                    |                   |        |                       |                           |

The maximum likelihood method is used to estimate the performance index degradation data, and the K-S test is performed. The results are shown in Table 3. According to the results of the K-S test, the lognormal distribution is selected as the optimal distribution of the axial end runout $D_x$ and the spindle vibration amount $D_y$ degradation data.

**Table 3.** K-S test result.

| distribution          | Performing index | Axial end runout $D_x$ | Spindle vibration $D_y$ |
|-----------------------|-------------------|------------------------|------------------------|
|                       | H                  | P                      | H                      | P                      |
| Normal distribution   | 0                  | 0.6436                 | 0                      | 0.6526                 |
| Lognormal distribution| 0                  | 0.6838                 | 0                      | 0.7056                 |
| Weibull distribution  | 1                  | 0.0148                 | 0                      | 0.4266                 |

The parameters of the distribution function are optimized, and are fitted by time series, and the function is selected by the sum of squared errors. The results are shown in Table 4.

According to the fitting situation, the corresponding function with the smallest sum of squared errors is selected, and the result of parameter fitting is obtained as follows:

1. Axial end runout: $\mu_x(t) = 5.002 \times 10^{-4}t + 2.028 \quad \sigma_x(t) = 3.043 \times 10^{-4}t + 1.855$

2. Spindle vibration: $\mu_y(t) = 4.833 \exp\left(7.002 \times 10^{-5}t\right) \quad \sigma_y(t) = 3.096 \exp\left(3.627 \times 10^{-5}t\right)$

The degradation process model of performance indicators is:
(1) Axial end runout: 
\[ F_x(t) = \Phi \left( \frac{\ln x(t) - \mu_x(t)}{\sigma_x(t)} \right) \]

(2) Spindle vibration: 
\[ F_y(t) = \Phi \left( \frac{\ln y(t) - \mu_y(t)}{\sigma_y(t)} \right) \]

The relationship between the two performance index degradation processes of the electric spindle is calculated as follows: 
\[ \text{cov} \left( x(t), y(t) \right) = 4.346 \]

There is a correlation between the two degradation processes. The reliability model is established by the Copula function. List the Copula function parameter values at some time, as shown in Table 5.

**Table 4.** Sum of square error.

| Function      | Axial end runout | Spindle vibration |
|---------------|------------------|-------------------|
|               | \( N_{\mu_x} \) | \( N_{\mu_y} \) | \( N_{\sigma_x} \) | \( N_{\sigma_y} \) |
| Linear function | 0.0098           | 0.00365           | 0.0074           | 0.000793           |
| Exponential function | 0.0100           | 0.00372           | 0.0073           | 0.000787           |
| Power function    | 0.0479           | 0.01753           | 0.0350           | 0.003793           |
| Logarithmic function | 0.0517           | 0.01847           | 0.0356           | 0.003825           |

Gumbel Copula's function parameter values are not within the specified limits and are not considered. The selection of the best model among the remaining two Copula functions is done by the AIC criterion. The calculation results of the AIC are shown in Table 6.

According to the calculation results of AIC, Clayton Copula>Frank Copula can be found, therefore, the Frank Copula model is selected.

Perform a linear time series fitting on the parameters \( \beta \) in the Frank Copula function \([9]\), and the parameter expression as: 
\[ \beta = 0.1081t + 0.0515 \]

Using the Frank Copula model, the reliability model of the multi-degradation process of the electric spindle is completed, and reliability model is obtained:
\[ R(t) = \frac{1}{\beta} \left[ 1 + \left( \frac{1-e^{-\beta t}}{e^{-\beta t-1}} \right) \right] \]

According to the reliability model of the electric spindle, the reliability curve of the electric spindle is drawn. The reliability curve of the electric spindle is shown in Figure 1.

**Table 5.** Copula function parameter values at some times.

| \( t \) | Gumbel Copula | Clayton Copula | Frank Copula |
|--------|---------------|----------------|--------------|
| 24     | 1.0473        | 0.9872         | 1.4253       |
| 264    | 0.3724        | 14.3267        | 26.4521      |
| 528    | -0.5896       | 30.9974        | 57.8054      |

**Table 6.** Copula function AIC calculation value at a certain moment.

| \( t \) | Clayton Copula | Frank Copula |
|--------|----------------|--------------|
| 24     | -9.3057        | -10.4209     |
| 264    | -8.4781        | -9.7896      |
| 528    | -7.9131        | -8.915       |

When a given value \( R \) of a certain reliability of a product is given, the corresponding time \( t_R \) of the electric spindle, and this value reflects the probability that the product can operate normally at time \( t \). The formula for the reliable life of an electric spindle can be expressed as:
\[ R(t_R) = R \]

After accelerated factor conversion, get median life: \( t_{0.5} = 2301.75h \), characteristic life: \( t_{e^{-1}} = 2617.37h \).

When \( R = 0.456 \) , \( t = 2407.51h \), compared the actual life time of the electric spindle under the same reliability of 2339.6h, the gap of life prediction result and the actual is 2.9%. Considering the existence of
the test error, the prediction result is acceptable, basically in line with the actual situation of the project.

The average time before failure is $MTTF = \int_{0}^{\infty} R(t) \, dt$, the MTTF is 1885.36h, which reflects the average normal working time of the product.

4. Conclusion

(1) Assume several commonly used distribution functions, estimate the parameters by the maximum likelihood method, use the d-test to select the distribution function of the performance degradation data, further optimize the model parameters, and determine the final single degradation process model.

(2) Using the covariance to judge the relationship between the multi-performance indicators of the electric spindle, and the performance indicators are related.

(3) Based on the correlation between multi-performance indicators, the maximum likelihood method is used to estimate the parameters of Copula function, the AIC criterion is used to complete the selection of Copula function, and the reliability modeling of the electric spindle is completed to achieve a reliable life prediction.

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