Analysis of water quality characteristic for water distribution systems

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ABSTRACT

Since governments all over the world are paying more attention to water quality in water distribution systems (WDS), a method based on mass balance and first-order chlorine decay model was proposed to assess the efficiency of WDS involving water quality (represented by residual chlorine). The concepts of surplus chlorine factor (S) for nodes in individual pipes and comprehensive surplus chlorine factor (CS) for nodes in WDS were put forward to represent the water quality characteristic of nodes in WDS based on the assumption that the structure of the pipe network and quantity of chlorine dose are definite. The proposed method was applied to two examples of WDS and sensitivity analysis regarding chlorine decay coefficient ($k_0$) was discussed. The results indicated that values of CS for nodes in WDS are affected by the inflow of nodes, which is determined by water demand and pipe length from water sources to nodes. In addition, the value of CS increases with $k_0$ when the inflow of the node is larger than the optimized inflow. The results verified that the deduction of S for a single pipe can be generalized to WDS, and can measure the water quality characteristics for nodes in WDS easily.

Key words | comprehensive surplus chlorine factor (CS), residual chlorine, surplus chlorine factor (S), water distribution system (WDS)

NOMENCLATURE

WDS Water distribution system

DOM Dissolved organic matter

$m$ Chlorine decay quantity

$q$ Flow within the pipe

$Q_{in}$ Pipe inflow

$Q_{out}$ Pipe outflow

$C_{in}$ Residual chlorine concentration at the inlet

$C_{out}$ Residual chlorine concentration at the outlet

$m_{in}$ Chlorine entering the pipe at the inlet

$m_{out}$ Chlorine leaving the pipe at the outlet

$k_0$ First-order kinetic decay coefficient

$L$ Pipe length

$D$ Pipe diameter

$(C_{out})_{max}$ Maximum chlorine concentration at the outlet

$k$ Chlorine transfer efficiency

$(Q_{in})_{opt}$ Optimized pipe inflow

$S$ Surplus chlorine factor

CS Comprehensive surplus chlorine factor

INTRODUCTION

Water quality in water distribution systems (WDS) is currently of widespread concern, which is influenced by a...
number of factors including the water age, water storage facility, and disinfectant methods (Mau et al. 1996). Chlorine is the most widely used disinfectant for preventing finished water from regrowth of microbial pathogens (Kim et al. 2015). Most of the chlorine dosed is consumed in reactions with other substances remaining in the water after treatment, particularly dissolved organic matter (DOM) (Fisher et al. 2012). Therefore, the concentration of residual chlorine is required to be kept at a certain level, especially in the extremities of WDS (Li et al. 2013; Blokker et al. 2014). The most widely used chlorine decay model in WDS is the first-order decay model, expressed by Equation (1) as follows:

\[
\frac{dC}{dt} = -k_0 C
\]

(1)

where \(C\) is the concentration of chlorine, \(t\) refers to time, and \(k_0\) stands for the chlorine decay coefficient (Hallam et al. 2002; Al-Jasser 2007; Fisher et al. 2011; Kim et al. 2014). In this paper, water quality characteristics of nodes in WDS based on the first-order chlorine decay equation mentioned above were analyzed.

The hydraulic characteristic and mechanical reliability of WDS have been much studied in the literature (Vaabel et al. 2006; Tanyimboh & Templeman 2007; Wu et al. 2011; D’Ercole et al. 2008). Mechanical reliability was defined as the probability that a component (new or repaired) experiences no structural failures (Kansal & Kumar 1995). The hydraulic reliability refers to the probability that a water distribution pipe can meet a required water flow level at a required pressure at each nodal demand (Ostfeld 2001). In order to increase the hydraulic capacity of a network and overcome sudden failures in WDS, the concept of hydraulic power capacity with consideration of both flow and pressure was identified (Vaabel et al. 2006). The hydraulic characteristic of nodes represented by the concept of surplus power factor based on hydraulic power was defined and applied to measure network resilience (Wu et al. 2011). However, both flow and water quality are of equal importance for WDS. The capacity concerning water quality for WDS has not been researched sufficiently in past studies (Gupta et al. 2012). Models have been developed of the chlorine concentrations at nodes in WDS with consideration of chlorine decay (Boulos et al. 1995; Mau et al. 1996; Hallam et al. 2002). The fraction of delivered quality (FDQ) was expressed to influence the water quality reliability, which is the ratio of simulation runs of supplied concentration below the threshold concentration to all the simulation runs (Ostfeld et al. 2002). Similarly, the ratio of days that the residual chlorine fulfills the residual chlorine standards to simulated days was proposed to represent water quality reliability (Zhao et al. 2010). The concept of node chlorine availability was proposed to define the water quality reliability of WDS (Li et al. 2013). The optimal operations of booster stations were proposed with the objective of minimizing the chlorine injection quantity (Tryby et al. 2002; Ostfeld & Salomons 2006; Kang & Lansey 2010). However, from the aspect of water quality management, the chlorine injection to WDS often remains fixed with no relationship to water demand. Under such circumstances, the water quality characteristics of nodes are affected by many factors, such as distance from water source to nodes, diameters and flows of pipes connecting with nodes, water demand of nodes, and chlorine decay coefficient, etc. The contribution of this paper is to research the degree of effect of various factors on water quality characteristics of nodes in WDS.

In this paper, first, based on the concepts of surplus chlorine factor (S) for nodes in single pipes deduced from a mass balance and first-order chlorine decay model, the concept of comprehensive surplus chlorine factor (CS) for nodes in WDS is proposed to analyze water quality characteristic of nodes in WDS. Second, CS was applied to measure the water quality characteristics in two examples of WDS based on an EPANET hydraulic and water quality extended simulation, and a sensitivity analysis of \(k_0\) to CS is presented. Finally, values of CS of nodes in WDS were measured and compared, and the factors affecting CS are indicated and discussed.

**METHODOLOGY**

**Surplus chlorine factor (S) for individual pipe**

The ideal parameter for assessing water quality reliability should have clear physical meaning, be able to distinguish nodes in WDS, and be easy to calculate. Based on mass balance and the chlorine-decay model, we can obtain
Equations (2) and (3) from the individual pipe shown in Figure 1. The quantity of decayed chlorine is termed $m$, and the flow within the pipe is termed $q$.

$$m_\text{in} = QinC_\text{in}$$  \hspace{1cm} (2)

$$m_\text{out} = QoutC_\text{out}$$  \hspace{1cm} (3)

where $Q_\text{in}$ is the inflow of the pipe (L/s), $Q_\text{out}$ is the outflow of the pipe (L/s), $C_\text{in}$ is the concentration of residual chlorine at the inlet of the pipe (mg/L), $C_\text{out}$ is the concentration of residual chlorine at the outlet of the pipe (mg/L), $m_\text{in}$ is the quantity of chlorine entering the pipe per second at the inlet (mg/s), and $m_\text{out}$ is the quantity of chlorine leaving the pipe per second at the outlet (mg/s).

Suppose residual chlorine decay follows the first-order kinetic reaction equation, the concentration of residual chlorine at the outlet is expressed by Equation (4) as follows:

$$C_\text{out} = C_\text{in} \exp (-k_0 \pi) = \frac{m_\text{in}}{Q_\text{in}} \exp \left( -\frac{k_0 L \pi D^2}{4Q_\text{in}} \right)$$  \hspace{1cm} (4)

where $k_0$ is the first-order kinetic decay coefficient (s$^{-1}$), $L$ and $D$ are the length (m) and diameter (m) of the single pipe, respectively.

Obviously, we can obtain Equations (5) and (6) for an individual pipe (shown in Figure 1), expressed as follows:

$$Q_\text{in} = Q_\text{out} = q$$  \hspace{1cm} (5)

$$m = m_\text{in} - m_\text{out}$$  \hspace{1cm} (6)

In Equation (4), $L$, $D$, $k_0$, and $m_\text{in}$ are usually known; however, $Q_\text{in}$ varies depending on the event. Therefore, $C_\text{out}$ in Equation (4) can be expressed as a function of $Q_\text{in}$ expressed by $C_\text{out}(Q_\text{in})$. The aim of this paper is to study the variation of $C_\text{out}$, and try to find the variation pattern of $C_\text{out}$ with $Q_\text{in}$ under the condition that the injection quantity of chlorine is definite. The optimized value of $Q_\text{in}$ is to make the outlet chlorine concentration $C_\text{out}$ reach the maximum value so as to get the water quality at the best operational condition. Therefore, according to Equation (4), with the assumption that $\frac{\partial C_\text{out}}{\partial Q_\text{in}} = 0$, we can deduce that $C_\text{out}$ gets its maximum value when $Q_\text{in}$ can satisfy the condition expressed by Equation (7), as follows:

$$\left( Q_\text{in} \right)_\text{opt} = \frac{k_0 L \pi D^2}{4} \hspace{1cm} (7)$$

where $(Q_\text{in})_\text{opt}$ is the optimized value of $Q_\text{in}$.

Accordingly, $C_\text{out}$ reaches the maximum value, which can be expressed by Equation (8) as follows:

$$\left( C_\text{out} \right)_\text{max} = m_\text{in} \frac{e}{e(Q_\text{in})_\text{opt}} = \frac{4m_\text{in} e \kappa_0 L \pi D^2}{e(Q_\text{in})_\text{opt}} \hspace{1cm} (8)$$

where $(C_\text{out})_\text{max}$ is the available maximum value of $C_\text{out}$, corresponding to $(Q_\text{in})_\text{opt}$. From Equation (8) we can find out that $(C_\text{out})_\text{max}$ has relationships with the pipe characteristics (length $L$ and diameter $D$), chlorine decay constant $k_0$, and the input chlorine quantity $m_\text{in}$.

Therefore, $(C_\text{out})_\text{max}$ is a definite value for a given pipe. Usually the actual flow in a pipe is different from $(Q_\text{in})_\text{opt}$, thus the actual chlorine concentration $C_\text{out}$ at the outlet is often different from $(C_\text{out})_\text{max}$. Therefore, to assess the chlorine transfer efficiency, we can obtain Equation (9), expressed as follows:

$$k = \frac{C_\text{out}}{(C_\text{out})_\text{max}} \hspace{1cm} (9)$$

where $k$ refers to the chlorine transfer efficiency.

By combining Equations (4) and (7), the coefficient $k$ can also be expressed by Equation (10) as follows:

$$k = \exp \left( -\frac{k_0 L \pi D^2}{4Q_\text{in}} \right) \frac{e(k_0 L \pi D^2)}{4Q_\text{in}} \hspace{1cm} (10)$$

In order to simplify Equation (10), parameter $\eta$ was defined by Equation (11) as follows:

$$\eta = \frac{k_0 L \pi D^2}{4Q_\text{in}} \hspace{1cm} (11)$$

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Figure 1 | Flows, concentration and chlorine loss for individual pipe.
In addition, \( \eta \) can also be expressed by Equation (12) as follows:

\[
\eta = \frac{(Q_{in})_{opt}}{Q_{in}} \tag{12}
\]

Therefore, Equation (10) was transformed into Equation (13) as follows:

\[
k = \frac{e\eta}{\exp(\eta)} \tag{13}
\]

The relationship curve between \( k \) and \( \frac{Q_{in}}{(Q_{in})_{opt}} \) (i.e. \( \frac{1}{\eta} \)) is shown in Figure 2, where \( k \) is presented as a function of \( \frac{Q_{in}}{(Q_{in})_{opt}} \) or \( \eta \). The scope of \( k \) is between 0 and 1.0. In effect, the water quality characteristics of WDS get better when the chlorine concentration at the outlet is higher. If \( k = 1 \), \( C_{out} \) reaches the maximum value \( (C_{out})_{max} \), which means that the pipe works at the best condition from the viewpoint of water quality corresponding to value of \( (Q_{in})_{opt} \) being 1.0 (also shown in Figure 2).

Since the coefficient \( k \) characterized the potentiality of chlorine concentration of the node, the concept of surplus chlorine factor \( (S) \) was proposed to represent the water quality characteristic of the node, expressed by Equation (14) as follows:

\[
S = 1 - k = 1 - \frac{e\eta}{\exp(\eta)} \tag{14}
\]

where \( S \) refers to surplus chlorine factor \( (S) \), which is applied to assess the characteristic of the node in view of water quality. The scope of \( S \) is between 0 and 1, which is similar to \( k \). If \( S \) decreases, water quality reliability of WDS will be improved. When \( S \) decreases to 0, \( C_{out} \) reaches \( (C_{out})_{max} \), which means WDS work with maximum chlorine concentration at the outlet.

From Figure 2, we can also find that when \( Q_{in} \) decreases or increases from \( (Q_{in})_{opt} \), \( C_{out} \) always decreases. Combining Equations (13) and (14), we can also conclude that if \( \eta \) or \( \frac{Q_{in}}{(Q_{in})_{opt}} \) is fixed, \( k \) and \( S \) remain unchangeable. When \( \frac{Q_{in}}{(Q_{in})_{opt}} \) varies in the range from 0 to 1.0, with the rise of \( \frac{Q_{in}}{(Q_{in})_{opt}} \), \( k \) increases and \( S \) decreases. On the contrary, when \( \frac{Q_{in}}{(Q_{in})_{opt}} \) varies in the range from 1.0 to \( \infty \), with the rise of \( \frac{Q_{in}}{(Q_{in})_{opt}} \), \( k \) decreases and \( S \) increases. The reason can be explained as follows. From Equation (14), we can deduce Equation (15), expressed as follows:

\[
\frac{dS}{d\eta} = -(1 - \eta) \exp(1 - \eta) \tag{15}
\]

From Equation (15), we can find that when \( \eta \) is bigger than 1.0, i.e. \( \frac{Q_{in}}{(Q_{in})_{opt}} \) is in the scope of \((0, 1.0)\), \( \frac{dS}{d\eta} \) is greater than 0, which means that \( S \) decreases with the decrease of \( \eta \) or the rise of \( \frac{Q_{in}}{(Q_{in})_{opt}} \). Similarly, when \( \eta \) is smaller than 1.0, i.e. \( \frac{Q_{in}}{(Q_{in})_{opt}} \) is in the scope of \((1.0, \infty)\), \( \frac{dS}{d\eta} \) is smaller than 0, which means that \( S \) increases with the decrease of \( \eta \) or the rise of \( \frac{Q_{in}}{(Q_{in})_{opt}} \).

**Relationship between surplus chlorine factor \( (S) \) and \( Q_{in} \) under different chlorine decay constant \( k_0 \)**

From Equations (10), (11), and (14), we can find parameters \( k \) and \( S \) vary with the decay coefficient \( k_0 \) for the individual pipe. The chlorine decay coefficient \( k_0 \) varies with flow...
velocity and temperature (Blokker et al. 2014). From Equation (8) we can draw the conclusion that $(C_{out})_{max}$ always decreases with the increase of $k_0$ when the other factors remain the same, which means that chlorine decay coefficient $k_0$ affects the reachable maximum chlorine concentration negatively. In addition, with the increase of $k_0$, the value of $(Qm)_{opt}$ increases according to Equation (7), which means that water flow in the pipe will increase to meet the chlorine concentration of $(C_{out})_{max}$ at the outlet. With the increased chlorine decay coefficient of $k_0$, the chlorine concentration at the outlet $C_{out}$ decreases. Therefore, the variation of $k$ and $S$ with $k_0$ depends on the relative increased degrees of $C_{out}$ and $(C_{out})_{max}$. The relationship curves between $S$ and $(Qin)$ under different values of $k_0$ are shown in Figure 3. The values of $k_0$ ranged from $+20\%$ to $-20\%$.

With the increase of $Q_{in}$, $S$ decreases from 1.0 to 0 initially, then increases with the rise of $Q_{in}$. When the value of $S$ is 0, $Q_{in}$ reaches the best value of $(Q_{in})_{opt}$. For a specific curve with certain $k_0$, when $Q_{in}$ is less than $(Q_{in})_{opt}$, $S$ decreases with the rise of $Q_{in}$. On the contrary, if $Q_{in}$ is greater than $(Q_{in})_{opt}$ for certain $k_0$, $S$ increases with the rise of $Q_{in}$. The values of $(Q_{in})_{opt}$ vary with $k_0$. When $k_0$ increases, $(Q_{in})_{opt}$ also increases, which is in accordance with Equation (7). Among all the values of $(Q_{in})_{opt}$, $(Q_{in})_{opt1}$ is the smallest, corresponding to 0.8$k_0$, and $(Q_{in})_{opt2}$ is the biggest, corresponding to 1.2$k_0$. From Figure 3 we can also find that when $Q_{in}$ is less than $(Q_{in})_{opt1}$, the higher the chlorine decay coefficient $k_0$ is, and the greater $S$ is for the same $Q_{in}$. However, when $Q_{in}$ is greater than $(Q_{in})_{opt2}$, the higher the chlorine decay coefficient $k_0$ is, and the smaller $S$ is for the same $Q_{in}$. The reason can be explained by the fact that when $Q_{in}$ is less than $(Q_{in})_{opt1}$, $\eta$ is always bigger than 1.0. From Equation (15), we can obtain that $\frac{dS}{d\eta}$ is greater than 0. Therefore, if $k_0$ increases, then $\eta$ increases correspondingly according to Equation (11), which leads to the rise of $S$ that coincides with the curve in Figure 3. Moreover, when $Q_{in}$ is greater than $(Q_{in})_{opt2}$, then $\eta$ is always smaller than 1.0, which leads to $\frac{dS}{d\eta}$ being less than 0. Therefore, if $k_0$ increases, then $\eta$ increases correspondingly according to Equation (11), which leads to the decline of $S$ that also coincides with the curve in Figure 3.

**Comprehensive surplus chlorine factor (CS) for water distribution network**

The consideration for the individual pipe can also be generalized for WDS. For a pipe net consisting of two pipes, shown in Figure 4, node $n$ was connected with two single pipes.

From Figure 4, we can obtain Equations (16)–(18), expressed as follows:

$$Q_{in1} + Q_{in2} = Q_{out}$$

(16)

$$Q_{in1} = q_1$$

(17)

$$Q_{in2} = q_2$$

(18)

where $Q_{out}$ is the flow of node $n$, $Q_{in1}$ is the flow in pipe $\oplus$ connecting with node $n$, $Q_{in2}$ is the flow in pipe $\oplus$ connecting with node $n$.

The value of CS for node $n$ is calculated by Equation (19) as follows:

$$CS = \sum_{i=1}^{k} w_i S_i = \sum_{i=1}^{k} \left( \frac{Q_i}{\sum Q_j} \right) S_i$$

(19)

![Figure 3](image-url) | Relationship curves between surplus chlorine factors ($S$) and $Q_{in}$ under different $k_0$.

![Figure 4](image-url) | Flows, concentration, and chlorine loss for a pipe net.
where CS is the comprehensive surplus chlorine factor for node \( n \), \( S_i \) \((i = 1, 2, \ldots, K)\) is the surplus chlorine factor for node \( n \) through the \( i^{th} \) pipe, \( Q_i \) \((i = 1, 2, \ldots, K)\) is the flow of the \( i^{th} \) pipe connecting with node \( n \), and \( K \) is the number of pipes connected with node \( n \).

For a given WDS that consists of multiple pipes, water sources and nodes, CS for nodes varies with the inflow to nodes, length of pipes from water sources to nodes, diameters of pipes connecting with nodes, water demand of nodes, and chlorine decay coefficient \( k_0 \) etc. For a certain node in WDS, the length of pipes from water sources to nodes and the diameters of pipes connecting with nodes are determined beforehand. However, the inflow \( Q_{in} \) of a node is variable depending on water demand by users, including residents and industrial enterprises. The chlorine decay coefficient \( k_0 \) varies with the water quality, water temperature, and the pipe material. Therefore, values of CS for nodes in a given WDS usually vary with water demand and chlorine decay coefficient \( k_0 \). For nodes in a WDS, the length of pipes from water sources to nodes and the diameters of pipes connecting with nodes are different. The values of CS for nodes vary with the differences in pipe lengths and pipe diameters.

**RESULTS AND DISCUSSION**

**Example 1**

**Base run**

The proposed methodology was demonstrated for a small illustrative system shown in Figure 5 (Rossman 1994). The initial residual chlorine of the water works was supposed to be 0.5 mg/L. During the hydraulic simulation progress of EPANET2, the Hazen–Williams function was used as the hydraulic simulation model, and the roughness coefficient was assumed to be 100. During the water quality simulation progress, the chlorine decay coefficient \( k_0 \) was set to be \(-1.55/\text{day}\) (Rossman 1994).

The system consists of 12 pipes, a water source, a pump, and an elevated storage tank. The system is subject to a 24-h representative demand pattern shown in Table 1.

The pump is fed by a source that is at a constant water level of 243.8 m. The water is delivered to a storage elevated tank at node 2 (at a ground level of 259.1 m), and to eight consumers located at nodes 11, 12, 13, 21, 22, 23, 31, and 32. The pump has a shutoff head value of 101.3 m and a maximum flow rate of 189.3 L/s. The tank is cylindrical with a diameter of 15.4 m. Its minimum, initial, and maximum levels above ground are 30.5 m, 36.6 m, and 45.7 m, respectively. The results of the base run for the case study are shown in Figure 6 and Table 2.

**Table 1** | 24-h demand pattern characteristics for example (Rossman 1994)

| Time of day | 1–2 | 3–4 | 5–6 | 7–8 | 9–10 | 11–12 | 13–14 | 15–16 | 17–18 | 19–20 | 21–22 | 23–24 |
|-------------|-----|-----|-----|-----|-----|-------|-------|-------|-------|-------|-------|-------|
| Multiplier of base demand | 1.0 | 1.2 | 1.4 | 1.6 | 1.4 | 1.2 | 1.0 | 0.8 | 0.6 | 0.4 | 0.6 | 0.8 |

Figure 5 | Example 1 (Rossman 1994).
The variation curves of the time multiplier and comprehensive surplus chlorine factors (CS) for all nodes in 24 h calculated by Equation (19) are shown in Figure 6. We can also find that for almost all nodes besides node 13, values of CS were the smallest at the 19th–20th hour when water demand is also the least, which means that values of CS for nodes vary with characteristics of the demand pattern for these nodes. It can be explained that when water demand decreases, the flow Qin decreases accordingly, which leads to direct increase of Cin under the condition that chlorine injection m_{in} is fixed. The outlet concentration Cout increases with Cin, which leads to the increase of k and decrease of S. The condition is consistent with the case for a single pipe, shown in Figure 3, when Qin is greater than (Q_{in})_{opt}. For comparing water quality characteristics of nodes in WDS, the average values of CS for nodes in 24 h are shown in Table 2.

From Table 2, we can find that the average value of CS for node 23 is significantly less than other nodes. The reason for this is that values of Qin for all nodes are greater than (Q_{in})_{opt} obtained by Equation (7), and \eta is always less than 1.0, but Qin for node 23 is closer to (Q_{in})_{opt}, since node 23 is the terminal node of the pipe network. Therefore, \eta for node 23 is closer to 1.0 than other nodes according to Equation (11), which leads to a smaller value of CS. The conclusion is in accordance with the case shown in Figure 3 for a single pipe; that is, the closer the node inflow is to (Q_{in})_{opt}, the smaller CS is for the node.

### Sensitivity analysis

Since chlorine decay coefficient k_0 is of significant importance in WDS, the sensitivity analysis of k_0 to CS is performed. The decay coefficient k_0 is set to vary from −20% to +20% compared with k_0 in the base run keeping other data the same as that in the base run. The results of average values of CS for all nodes are shown in Table 3.

| Nodes | Chlorine decay coefficient |
|-------|-----------------------------|
|       | 0.8k_0 | 0.9k_0 | k_0 | 1.1k_0 | 1.2k_0 |
| 11    | 0.64   | 0.61   | 0.55 | 0.54   | 0.51   |
| 12    | 0.79   | 0.76   | 0.71 | 0.72   | 0.70   |
| 13    | 0.59   | 0.55   | 0.49 | 0.48   | 0.45   |
| 21    | 0.59   | 0.77   | 0.71 | 0.72   | 0.70   |
| 22    | 0.79   | 0.60   | 0.55 | 0.55   | 0.52   |
| 23    | 0.54   | 0.30   | 0.25 | 0.23   | 0.20   |
| 31    | 0.68   | 0.64   | 0.59 | 0.58   | 0.55   |
| 32    | 0.57   | 0.53   | 0.47 | 0.46   | 0.45   |
We can observe that values of CS for nodes 11, 13, 22, 23, 31, and 32 decrease with the increase of $k_0$. Although node 12 and node 21 do not comply with the conclusion completely, the variation trends of CS with $k_0$ are the same as other nodes. Based on Figure 3, the relationship between $S$ and $k_0$ depends on $Q_{in}$ for individual pipes. According to the analysis above, that values of $Q_{in}$ for all nodes are greater than $(Q_{in})_{opt}$, $S$ decreases with the rise of $k_0$. Although a water distribution network is more complex than an individual pipe, the calculations of CS are on the basis of $S$ of individual pipes. Therefore, the variation trend of average values of CS for all nodes with chlorine decay coefficient $k_0$ can be explained by the conclusions resulting from individual pipes.

In addition, the hourly variations of CS for node 21 and node 23 are shown in Figure 7(a) and 7(b).

We can find from Figure 7(a) that although node 21 does not comply with the conclusion that average values of CS decrease with the increase of $k_0$, the hourly variation of CS corresponding to $k_0$ is the same as other nodes; that is, CS decreases with the increase of $k_0$, which is in accordance with the case shown in Figure 3 when $Q_{in}$ is greater than $(Q_{in})_{opt}$.

We can also find from Figure 7(b) that although average values of CS for node 23 follow the conclusion that average values of CS decrease with the increase of $k_0$, the hourly variation of CS does not follow the same conclusion for the valley hour of 18 h–19 h. The reason is that the inflow of node 23 is smaller than other nodes, and when the water demand is lower, the inflow of node 23 becomes lower than $(Q_{in})_{opt}$, which leads to the increase of CS with $k_0$. The conclusion is also in accordance with the case shown in Figure 3, when $Q_{in}$ is less than $(Q_{in})_{opt}$; that is, the higher the chlorine decay coefficient $k_0$, the bigger the water quality characteristic CS for the node.

Comparing results in Figure 7(a) and 7(b), hourly values of CS for node 21 are greater than hourly values of CS for node 23. The reason is that $Q_{in}$ at node 23 is smaller than $Q_{in}$ at node 21, and is closer to $(Q_{in})_{opt}$, which leads to the values of CS for node 23 being less than the values of CS for node 21, which means that the water quality characteristic of node 23 is better than node 21.

The variations of CS for all nodes under various $k_0$ corresponding to peak hour (8th hour) and valley hour (19th hour) were analyzed and are shown in Figure 8(a) and 8(b).

From Figure 8(a), we can conclude that at peak hour (8th hour), values of CS for all nodes decrease with the increase of $k_0$, which is in accordance with the results above. Moreover, under various $k_0$, values of CS decrease in the order of node 21 > node 12 > node 31 > node 11 > node 13 > node 22 > node 32 > node 23, which means that at peak hour the water quality characteristic is best at node 23, and worst at node 21. However, from Figure 8(b) at the valley hour (19th hour), we can find that the variation trend of CS for almost all nodes decreases with the rise of $k_0$ except for node 23, which is similar to the peak hour (8th hour). For node 23 at the valley hour (19th hour), the values of CS increased with the rise of $k_0$, which means that the water quality characteristic of node 25 became worse with the increase of $k_0$. The values of CS decreased in the order of node 21 > node 12 > node 22 > node 31 > node 11 > node 32 > node 15 > node 23.

Comparing results in Figure 8(a) and 8(b), values of CS at the peak hour (8th hour) are greater than at the valley hour.
hour (19th hour), which is in accordance with results obtained from Figure 6. When water demands at the valley hour are lower than water demands at the peak hour, the water quality characteristics of nodes are improved.

Example 2

In this case, the concept of $CS$ for the node was applied to a real-life network shown in Figure 9 (Example 3 of EPANET software (Rossman 1994)). The system consists of two sources, one elevated tank, 117 pipes, 97 demand nodes, and two pumps (the complete data used were exactly as that of Example 3, in Rossman (1994), and thus are not repeated here). The coefficient of chlorine decay was set to be $-1.55/\text{day}$.

The results of $CS$ for typical nodes in Example 2 are shown in Table 4.

For nodes at the extremities of the WDS in Example 2, the average values of $CS$ were lower than other nodes, which are
0.662, 0.221, 0.698, and 0.52 for node 30, 40, 70, and 73, respectively. The reason is that the inflows of nodes 30, 40, 70, and 73 are 6.953, 0.219, 3.488, and 1.391 L/s, respectively, lower than other nodes, which leads to the decrease of \( CS \), and improvement of the water quality characteristic of nodes. Moreover, the smaller inflow of nodes leads to a lower value of \( CS \). For example, the inflows of the four nodes increase in the order of node 40 < node 73 < node 30 < node 70. Accordingly, the values of \( CS \) for the four nodes increase in the same order. Although the inflow of node 27 is less than node 11, the pipe diameter connecting to node 27 is smaller than node 11, which leads to the value of \( CS \) at node 27 being larger than the value of \( CS \) at node 11. The reason for this is that \( \eta \) is affected by pipe diameter, pipe length from water sources to node, and inflow of nodes together, which is shown in Equation (11). The value of \( \eta \) determines the water quality characteristic of nodes expressed by \( CS \).

### CONCLUSION

In this paper, the concept of surplus chlorine factor (\( S \)) for a node in a single pipe based on the mass balance and first-order chlorine decay model was deduced. The relationship between \( S \) and \( \frac{Q_{in}}{(Q_{in})_{opt}} \), and the variation under different \( k_0 \) for a single pipe were revealed. In addition, comprehensive surplus chlorine factor (\( CS \)) for nodes in WDS based on \( S \) was put forward, and applied to two examples of WDS during 24 h in a day (shown in Figures 5 and 9). The results indicated that the value of \( CS \) decreases with the decrease of inflow at the node, which is caused by lower water demand and longer pipe length from water sources to node. In addition, average values of \( CS \) decrease with the increase of chlorine decay coefficient \( k_0 \). The conclusions are in accordance with the results obtained from a single pipe when \( Q_{in} \) is greater than \((Q_{in})_{opt}\). When the inflow at the node which is further from water sources becomes lower than \((Q_{in})_{opt} \) at the valley hour, the value of \( CS \) increases with the increase of chlorine decay coefficient \( k_0 \). In addition, the diameters of pipes connecting with nodes also affect the water quality characteristics of nodes; that is, the larger the pipe diameter, the better the water quality characteristics of nodes. The proposed method was proved to be efficient and easy to use for analyzing the water quality characteristic of nodes in WDS.

### Table 4 | Average comprehensive surplus chlorine factor (\( CS \)) for nodes in Example 2

| Nodes | Water demand of nodes (L/s) | Inflow of nodes (L/s) | Outflow of nodes (L/s) | Diameters of pipes connected with nodes (mm) | Average comprehensive surplus chlorine factor (\( CS \)) |
|-------|-----------------------------|----------------------|------------------------|---------------------------------------------|---------------------------------------------|
| 11    | 11.245                      | 11.245               | 0.000                  | 400, 400                                    | 0.848                                       |
| 15    | 11.983                      | 34.687               | 22.704                 | 300, 300, 300, 300                          | 0.964                                       |
| 19    | 14.869                      | 470.546              | 455.677                | 750, 750, 300, 300                          | 0.969                                       |
| 27    | 0.497                       | 8.890                | 8.393                  | 200, 200                                    | 0.873                                       |
| 30    | 2.332                       | 6.953                | 4.621                  | 300, 300                                    | 0.662                                       |
| 35    | 4.372                       | 384.523              | 380.151                | 750, 750                                    | 0.957                                       |
| 38    | 0.795                       | 359.491              | 358.696                | 750, 750, 350                               | 0.990                                       |
| 40    | 0.219                       | 0.219                | 0.000                  | 350                                         | 0.211                                       |
| 44    | 0.000                       | 75.843               | 75.843                 | 750, 600                                    | 0.998                                       |
| 49    | 0.000                       | 5.069                | 5.069                  | 200, 300                                    | 0.998                                       |
| 64    | 0.000                       | 45.475               | 45.475                 | 300, 300                                    | 0.996                                       |
| 70    | 3.488                       | 3.488                | 0.000                  | 350                                         | 0.698                                       |
| 73    | 1.391                       | 1.391                | 0.000                  | 300                                         | 0.520                                       |
| 86    | 0.000                       | 36.040               | 36.040                 | 300, 200, 300                               | 0.976                                       |
| 91    | 0.000                       | 31.084               | 31.084                 | 300, 200, 300                               | 0.972                                       |
| 92    | 0.000                       | 30.837               | 30.837                 | 300, 200, 300                               | 0.996                                       |
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