How to build institutionalization on students: a pilot experiment on a didactical design of addition and subtraction involving negative integers

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Abstract. This study focuses on the design of a didactical situation in addition and subtraction involving negative integers at the pilot experiment phase. As we know, negative numbers become an obstacle for students in solving problems related to them. This study aims to create a didactical design that can assist students in understanding the addition and subtraction. Another expected result in this way is that students are introduced to the characteristics of addition and subtraction of integers. The design was implemented on 32 seventh grade students in one of the classes in a junior secondary school as the pilot experiment. Learning activities were observed thoroughly including the students’ responses that emerged during the learning activities. The written documentation of the students was also used to support the analysis in the learning activities. The results of the analysis showed that this method could help the students perform a large number of integer operations that could not be done with a number line. The teacher’s support as a didactical potential contract was still needed to encourage institutionalization processes. The results of the design analysis used as the basis of the revision are expected to be implemented by the teacher in the teaching experiment.

1. Introduction
In Mathematics, a negative number is a real number that is less than zero. The negative integer is part of the set Z of integers, the set N = { 0, 1 ,2, ... } of non-negative integers (or natural numbers), and the set N* = { 1 ,2, ... } of positive integers and the elementary properties of addition, multiplication, and order. For each a∈Z there exists -a∈Z such that a + (-a) = 0 (additive inverse). It can be written as a – b for a + (-b) [1]. Negative numbers with this abstract concept become an obstacle to students who previously knew the number as a real object. Counting-down operations involving negative numbers alter the structure of concepts they understand so far, that is, additions produce larger numbers while subtractions produce smaller numbers. When they find that 5 + (-3) = 2 or 7 - (-3) = 10, there is often a cognitive conflict in the students’ thinking.

The number line is the most commonly used model to represent numbers in practical learning in the classroom [2]. The number line can be used with arrows where the direction of the arrow indicates whether the number is positive (pointing to the right) or negative (pointing to the left), and the length of the arrow indicates how far away from zero the integer is located [2]. The number line is symmetrical...
based on the position of the numbers on the left and right of zero. Musser stated that this symmetry produces a useful concept of the relationship of positive and negative numbers, that is, the opposite concept of a number. The opposite of 4 is denoted as \(-4\), as the opposite of \(-4\) is to be 4. This number line is also used as a tool to determine the results of addition and subtraction operations of integers as found in many Mathematics textbooks [3]. This way is usually used for arithmetic operations on simple numbers. For example \(-4 + 3\) is solved as follows: From 0, move 4 units to the left, then continue 3 units to the right so as to obtain the end point, i.e., \(-1\), which is the result of \(-4 + 3\) [4].

Obstacles begin to appear when the students are dealing with arithmetic operations with larger numbers or multi-digit numbers, for example, \(-23 + 56\) or \(-17 - (-29)\). Several studies have shown that negative numbers are a major difficulty in arithmetic operations [5,6,7]. Regarding negative numbers, [8] it is stated that the students have similar difficulties, such as conceptualizing numbers less than zero, creating a negative number as a mathematical object, and formalizing rules for integer operations, especially the meanings for the opposite of negative numbers into positive numbers. By modeling the addition and subtraction problems and then generalizing the situation, the students can understand the addition and subtraction operations [9].

The instructional designers search for a way to support students organizing their thoughts that can be modeled or inscribed in the form of physical tools and symbols [8]. Didactical designs need to be of attention by the teacher in designing learning to anticipate all possible students' responses in a didactic situation [10]. The design of didactical situations created by the teacher in classroom learning activities is expected to develop students' potential, which can build their own knowledge to be achieved through a series of abstraction processes. Actions and feedbacks through strategies will enable the formation of new knowledge. This action consists of four main situations in a didactic situation, namely, situations of action, formulation, validation, and institutionalization [11,12]. Situations of institutionalization are those by which the cognitive status of knowledge or a piece of knowledge is fixed conventionally and explicitly [12]. It is a process that allows students to transform their previous knowledge into new knowledge through empowerment by the teacher who gives them the value of truth and makes it possible to use the new knowledge to solve the next problem.

Based on the descriptions that have been rendered, the researchers consider it necessary to design a didactical design that supports learning through appropriate models and media or tools in addition and subtraction involving negative numbers by using context appropriate to the students' thinking level. Learning is done through didactical situations with media and techniques that are easy for the students and in line with their level of thinking. The purpose of this study, in particular, is to describe how to build the process of students' institutionalization on addition and subtraction of integers involving negative numbers, while in general to contribute to the teacher in creating a design of learning through appropriate media.

2. Methods

2.1. Data collection and analysis

The didactical design of addition and subtraction of negative integers is part of the Hypothetical Learning Trajectory (HLT) that was designed by researchers in Negative Integer materials. The implementation of the design was carried out in 32 students in one of the 7th grades in one of the junior secondary schools in Palembang as the research subject. The researchers assumed a role as a participant observer that allowed the researchers to act as piloting teachers directly. The Mathematics teacher in the subject class took the task of being an observer to observe the learning activities with the intention of providing inputs and suggestions related to the learning path. Two cameras with two operators were used to record all learning activities. The students were divided into 4 groups based on the seating row. All data were analyzed descriptively qualitatively to get a comprehensive picture of the learning process, the teacher’s way of managing the learning, and the process of students’ finding new ways in addition and subtraction operations involving negative integers.
2.2. Learning design

The instructional design was organized based on the aspects of the 7th grade Mathematics curriculum, the concept of addition and subtraction, and the character of the students' thinking based on the preliminary study and literature study that have been studied by the researchers. In a set model, chips could be used to represent integers [2]. However, two colours of chips had to be used, one colour to represent positive integers (black) and the other to represent negative integers (red). One black chip represented a debit of one. Thus one black chip and one red chip cancelled each other, or "make a zero" so that they were called a zero pair. In this concept, each integer could be represented by chips in many different ways. White circles were also used to represent positive integers and black circles to represent negative integers with different signs [3]. The other way was using the jar as a model, which contained the symbol "+" as a negative number and the "-" symbol as a negative number [13]. A jar containing + and − was said to be zero or empty. Just as addition was represented by a "joining" action, subtraction was represented by a "take away" action. To subtract one integer from another, represent the first integer (the minuend) with a jar of charges; then remove from this jar a collection of charges representing the second integer (the subtrahend). The new charge on the first jar is the difference.

![Figure 1. Jar models for (a) addition and (b) subtraction of integer [13]](image)

The researchers then modified the above way (Figure 1) to perform the addition and subtraction operations by using two-coloured candies media. The learning phase was implemented based on situations in the theory of didactical situations: (1) action: Each student was faced with a problem on how to add and subtract integers and interact with other students, the teacher, and the milieu; (2) formulation: Each student had an opportunity to create their own models implicitly to reveal their strategies; the teacher could be involved in this process; (3) validation: Each student was given some problems related to addition and subtraction operations and they gave a clear and complete explanation of the theory in any way that had been used to solve the problems; (4) institutionalization: Each student no longer used the candies as a tool in reality but could abstract arithmetic operations with multi-digit numbers; the teacher supported as a facilitator who referred to new knowledge with the one that had been understood by the students.

3. Results and Discussion

The learning activity began by reminding a simple integer operation and then asking the students how to add or subtract integers involving negative numbers, for example, how to add positive with negative numbers or negative with positive numbers. Since the students studied the concept of integers with number lines previously, they argued that integer counting operations can be performed using a number line. Then the teacher asked the question on how to find the operating result from -25 - (-32). The students did not give definitive answers. The teacher conveyed that the lesson material to be studied was integer arithmetic operations.
3.1. Addition and subtraction with a jar and candies
The first stage in a didactical situation was situation of action. Every student was faced with a problem. Students interacted with other students, the teacher, and the milieu. The milieu is everything that may affect students [11]. In this study, each group got one set of jar containing 25 blue and red candies, a blue candy was a positive integer representation whereas a red candy was a negative integer representation. The rule about the meaning of zero was also put forward in the beginning by using the analogy of debt: If someone owed one item, the debt had to be paid out with one item too, so it was said to be empty or zero \((a + (-a) = 0)\). Actually this debt context should not be used in integer learning because this context cannot be considered a quantity that indicates direction [14]. The debt context in this case was used by the researchers to show the inverse only and it should not be used in the counting operation.

The teacher demonstrated the operation of \(8 + (-3)\), this inserted into a jar of 8 blue candies and 3 red candies. The students had to look for the paired candies, which were 3 blue candies and 3 red candies; then they removed these candies because the value was equal to zero. The remaining candies left were 5 blue candies or positive 5. So \(8 + (-3) = 5\). For the subtraction, the teacher gave \(-5 - 3\) as an example. Subtraction operations were interpreted as "take away" as the students had known so far. The teacher demonstrated the operation of \(-5 - 3\) by entering 5 red candies then adding 3 blue candies and 3 red candies (3 blue and 3 red candies equal to zero). Because -3 meant “take away 3”, then 3 blue candies were removed from the jar. The eight candies were left in the jar. So they got \(-5 - 3 = 8\).

Then, the students solved the operation of \(-5 - (-12)\). One group performed the operations in the following stages.

![Figure 2](image_url)  
**Figure 2.** Procedures performed by one group on the operation of \(-5 - (-12)\)

This situation was called situation of formulation, where the students were given the opportunity to create their own models implicitly to express the strategy with words that other students could understand, discuss, and argue that made other students accept the explanation [15]. The activity performed by the students showed how they expressed their own way and explained it to the other students in their group. The most common obstacles were operations involving negative numbers. Many studies have shown how some students experience some difficulties understanding the concept of negative numbers. Based on students’ difficulties, the minus sign plays a major role in the development of understanding and using negative numbers [16]. In the theory of didactical situations, to improve students' autonomy, the teacher should give more adidactical situations, that is, situations giving students an opportunity to be more autonomous learners by having a supportive milieu of independent learning without direct intervention [11,12,15,17,18]. Adidactical situations are designed to minimize the involvement of the teacher in the learning process [19]. However, in the learning practice that occurs, the teacher’s assistance is needed as anticipation of the students’ responses that develop during the
learning process; this can be in the form of providing questions that can lead the students to an understanding.

3.2. A subsection
For the problem of addition, as far as the researchers’ observation, there was no significant constraint. It only took some practice given to the students. The emphasis on reducing operations involving negative numbers needed to be given more attention, so that the students had an understanding that \( a-b=a+(-b) \), so that \( a-(-b)=a+b \).

In this case the students were expected to perform addition and subtraction operations through mathematical abstraction. Interventions from the teacher were still needed to help support institutionalization through the explanation of rules and procedures in the operation. Each positive number was denoted by “+” while the negative number was denoted by “-“. Each pair “+” and “-“ was equal to zero. To support this, the teacher gave the opening problem with simple numbers first, for example 8 + (-3). The procedure of operation 8 + (-3) can be seen in figure 3.

![Figure 3. Operation procedure for 8 + (-3)](image)

The next step was to encourage the students to perform the counting of negative numbers with larger numbers. The teacher gave -25- (-32) operation problem and asked the students to think about how to get the result by having a discussion in their group. Some students answered with -57 and 7. A student came in front of the class and wrote down the answer (see Figure 3). The student made 25 negatives signs, then 32 negatives signs below. Then he encircled 25 negative signs and 25 negative signs below it. Now 7 negative signs were left that caused the result to be negative 7. The answer showed the progress that students had achieved even though the answer was still not right.

The students’ response showed that the students had not been able to perform the count operation on the multi-digit numbers and still relied heavily on the tool, so the teacher gave re-explanation through the assisting questions as an anticipation on it. In situations of validation, the students convey their ideas, and here the teacher plays a role to bridge their knowledge to achieve the intended knowledge [12]. The process of filing a problem to construct new knowledge (which will later be used to solve a problem) sometimes does not guarantee a didactical situation under a didactical contract happens entirely [20]. In the ordinary learning process, an adidactical situation occurs rarely, but some situations have the potential to occur (adidactical potential). It is said to be ‘potential’ because the teacher can get involved managing the situation, evaluating the students’ answers without waiting for the students to react on the feedback from the milieu.

The teacher provided an explanation of the meaning "take away" of subtraction operations. In a didactical situation, where the students are given the opportunity to solve problems without the teacher’s intervention, the teacher’s assistance is still needed as part of a didactical contract. Conflict is experienced by the teacher; negotiation and the search for a new contract will continue the didactic relationship through a new didactical situation [12]. This new situation is then built by the teacher when the students have not been able to perform the calculation operations through the correct concept, so that the teacher needs to redirect the operating algorithm to be done by the students.

Different interactions introduce characteristics and illustrate the adidactical potential practice in a potential adidactical contract [21]. This type of contract offers new responsibilities to the students, an important responsibility to identify. This form of responsibility requires the students to explain their suggestions and contribute to rebuilding the cognitive path that leads them to the learning objectives through the questioning that builds the intended knowledge to the students. They must also develop the
skills to argue, properly communicating ideas and knowing about learning in other ways. One student’s answer showed how this process had happened to him figure 6.

Figure 4. (a) An answer of one student for operation -25- (-32), (b) a student answered for the operation of -25-(-32), (c) an answer of a student for -121 - (230)

This student's answer illustrates his understanding that a negative number with a negative number produces a positive number. The analogy used by this student is the multiplication where negative numbers multiplied by negative numbers produce positive numbers. Then 25 were subtracted by 32 and the result was 7. The reason that he gave did not seem to provide sufficient explanation of what he knew because he did not explicitly write it down. However, after being excavated through interviews, it was known that he had understood the nature that: 1) -(-a) = a, so 2) -a - (-b) = -a + b, and 3) -a + b = b + (-a) = b – a. It can be said that he correctly understood how the characteristics of addition applies to negative integers. The teacher was fully aware that the students might be caught in the procedure without understanding the concept. Therefore, the explanation given by the teacher was not only the operation procedure but the concept contained in it. When the teacher gave a problem -84 – (-31), a student came in front of the class to complete it. The procedure was correct, using the concept that subtraction is meaningful as a "take away" and the inverse concept of addition, but was still wrong in the calculation (see Figure 5). However this error could be corrected by giving a briefing and reminding of the inverse concept that -84 could be elaborated to -31 and -53; other students responded with negative 53 quickly.

Figure 5. One student’s procedure on the operation of -84 – (-31)

Better work was shown in the next question: -121 - (-230). One student gave the answer correctly (see Figure 5). In this case, the student had undergone a process of abstraction in his thinking. These students had actually known that: 1) a + (-a) = 0, 3) -a - (-b) = -a + b, and 3) -a + b = b + (-a). By this time the student had reached the institutionalization stage where he reached new knowledge and used it to solve problems, as Brousseau described. According to Piaget, the formation of the notion of an abstract object comes from one's empirical experience called empirical abstraction [22]. The new
concepts formed based on experimental concepts through previously established experiences and stored in one's thinking through a process of abstraction are called theoretical abstractions. The state and mental activity of a person is part of the theoretical abstraction.

4. Conclusion

In the process of learning activities, the students can develop their abilities through the use of appropriate context that they are well acquainted with. The intended mathematical concepts can be understood by the students with the delivery of learning in accordance with their level of thinking. Through the help of this candy as a tool, the students can perform addition and subtraction operations easily and then can move on to multi-digit numbers without using tools. This understanding will greatly assist the students in identifying the properties of addition and subtraction operations. The students’ responses that arise during learning are also important to the teacher creating new situations through the questions that build the intended knowledge. The application of didactical situations through the design of didactical situations created by the teacher in the learning activities taking place in the classroom is expected to develop the students’ potential, meaning that the students can construct their own knowledge to be achieved through a series of abstraction processes.

5. References

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