Abstract
The physical meaning of the particularly simple non-degenerate supermetric, introduced in the previous part by the authors, is elucidated and the possible connection with processes of topological origin in high energy physics is analyzed and discussed. New possible mechanism of the localization of the fields in a particular sector of the supermanifold is proposed and the similarity and differences with a 5-dimensional warped model are shown. The relation with gauge theories of supergravity based in the $OSP(1/4)$ group is explicitly given and the possible original action is presented. We also show that in this non-degenerate super-model the physic states, in contrast with the basic states, are observables and can be interpreted as tomographic projections or generalized representations of operators belonging to the metaplectic group $Mp(2)$. The advantage of geometrical formulations based on non-degenerate super-manifolds over degenerate ones is pointed out and the description and the analysis of some interesting aspects of the simplest Riemannian superspaces are presented from the point of view of the possible vacuum solutions.
I. INTRODUCTION AND MOTIVATION

The study of the symmetries plays a fundamental role in modern physics. The geometrical interpretation of the physical phenomena takes as basic object the action, where all the dynamics of the theory is derived. The idea of to associate an underlying geometrical structure to these physical phenomena coming from of a fundamental idea of unification of all the interactions into the natural world and not from an heuristic thought. The interrelation between physical and mathematical definitions and concepts (i.e. geometry, groups, topology↔space-time, internal structure, fields) turns more and more concrete and basic in the physics of the XX and XXI centuries. If well there are elegant formulations of the
physical problems of interest from the mathematical point of view, there exists a lack of uniqueness in the geometrical definition of the Lagrangian density.

The great difficulties appear (or almost are evidently explicit) at the quantum level where the geometrical objects playing the role of Lagrangian or Hamiltonian pass to play the role of (super) operators. These troubles carry inexorably to the utilization of diverse methods or prescriptions that change the original form of the action (or Hamiltonian). This distortion of the original form of these fundamental operators at the classical level generally does not produce changes into the dynamical equations of the theory but quantically introduces several changes, because the spectrum of physical states is closely related with the form of the Hamiltonian. This fact was pointed out by the authors in the previous paper [10]. Clearly, in order to construct the Lagrangian and other fundamental invariants of the theory, the introduction of a manifold as the important ingredient is the relevant thing. In particular can be very interesting the introduction of a super-manifold (in the sense of [10] and references therein) in order to include the fermionic fields in a natural manner.

Several attempts have been made by various groups to construct the theory of supergravity as the geometry of a superspace possessing non-zero curvature and torsion tensors without undesirable higher spin states[1,2]. Only few years after those works, the consistent construction of the superfield supergravity was formulated in the pioneering papers independently by V.I. Ogievetsky and E. Sokatchev [3] and S.J. Gates and W. Siegel [4]. From these times in several areas of the theoretical physics the description of different systems was given in the context of the geometry of supermanifolds and superfields[5]: supergravity and d-branes models with warped supersymmetry[6], super-Landau systems[7], superbrane actions from nonlinearly realized supersymmetries[8], etc.

It is therefore of interest to study the geometry not only of the simplest superspaces, but also the more unusual or non-standard ones and elucidate all the gauge degrees of freedom that they possess. This fact will clarify and expand the possibilities to construct more realistic physical models and new mathematically consistent theories of supergravity. On the other hand, the appearance of supergroups must draw attention to the study of the geometries of the homogeneous superspaces whose groups of motions they are. Another motivation of the study of these Riemannian superspaces is in order to establish some degree of uniqueness in the obtained supersymmetric solutions.

Motivated by the above, we complete our previous paper [10] studying and analyzing
from the point of view of the possible vacuum solutions, the simplest non trivial supermetric
given by Volkov and Pashnev in [9] that was the “starting point” toy model of the first part
of this work

\[ ds^2 = \omega^{\mu} \omega_{\mu} + a \omega^\alpha \omega_{\alpha} - a^* \omega^\alpha \omega_{\alpha} \]  \hspace{1cm} (1)

This particular non-degenerate supermetric contains the complex parameters \( a \) and \( a^* \) that
make it different of other more standard supermetrics. As we shown in [10,11], the degenerate
supermetrics are not consistent into a well theoretically formulated supergeometry. Then,
our main task is to find the meaning and the role played by these complex parameters from
the geometrical and physical point of view. To this end, we compare the solution of ref.[10]
that was computed in the N=1 four dimensional superspace proposed in [9,11], compactified
to one dimension and restricted to the pure time-dependent case with:

i) the well known solution described in references[12,13] that was formulated in a superspace \((1 | 2)\).

ii) a multidimensional warped model described in [14], in this case also considering for
the proposed superspace the possible dependence of the solution on \( n \)–additional bosonic
coordinates \((d=n+4)\).

Our goal is to show that, from the point of view of the obtained solutions, the complex
parameters \( a \) localize the fields in a specific region of the bosonic part of this special
superspace, they explicitly breakdown the chiral symmetry when some conditions are required
and all these very important properties remain although the supersymmetry of the model
was completely broken. Also, besides all these highlights, we also show that the obtained
vacuum states from the extended supermetric are very well defined in any Hilbert space.

About the geometrical origin of this particularly special metric, we demonstrate that
it can be naturally derived from a theory of supergravity based in a \( OSP(1/4) \)-valuated
connection \( A \). When the symmetry of the \( OSP(1/4) \) model (super-\( SO(4,1) \)) presented
here is explicitly written as a function of its reductive components, a part as (1) appear plus
a first order (Dirac-like) fermionic term.

And finally we show that the physical states derived of the geometrical Lagrangian con-
structed with this particular supermetric are nothing more that the tomographic projections
(in the sense given by the authors of [34]) of the Heisenberg-Weyl and \( su(1,1) \) fundamental
operators (previously defined in [10]) $L_{ab} = \begin{pmatrix} a \\ a^+ \end{pmatrix}_{ab}$ and $\mathbb{L}_{ab} = \begin{pmatrix} a^2 \\ a^{+2} \end{pmatrix}_{ab}$ with respect to the basic coherent states (CS) fundamental solutions of the square root of the interval (1). These physical states have the following spin content: $\lambda = 1/2, 1, 3/2$ and 2 and the representations of these fundamental operators are in diagonal (Sudarshan-like) and in an asymmetric-(non diagonal) form, both representations forming part of a more general class of representations for operators recently proposed by Klauder and Skagerstam in ref.[35].

The plan of the paper is as follows: in Section 2 we give a brief review, based in a previous work of the author [10], about the $N = 1$ non-degenerate four dimensional superspace proposed by Volkov and Pashnev and its solution. Section 3 is devoted to analyze the relation of the supermetric under consideration with the superspace $(1 | 2)$ given explicitly under which conditions one is reduced to the other one from the point of view of the obtained vacuum solutions. The geometrical derivation of the supermetric from a gauge theory of supergravity based in the $OSP(1/4)$ group (super-$SO(4,1)$), a new superparticle model and the link between the complex parameters $a$ and $a^*$ and the cosmological constant $\Lambda$ are given in Section 4. In Section 5 a surprising connection between the extended supermetric and multidimensional warped gravity model solutions is shown and some hints of a possible new mechanism of the field localization and the idea of confinement is proposed. In Section 6 and 7 the interpretation of the physic states of the theory as tomographic representations of operators of the metaplectic group $Mp(2)$ are discussed, the spin content of these states is analyzed and the Gram-Schmidt operator is explicitly given. Finally in Section 8 we resume the main results and concluding remarks.

II. THE PARTICULAR FOUR DIMENSIONAL $N = 1$ SUPERSPACE

The superspace $(1, 3 | 1)$ has four bosonic coordinates $x^\mu$ and one Majorana bispinor: $(t, x^i, \theta^\alpha, \overline{\theta}^\dot{\alpha})$. Two possible realizations for this superspace are:

\[
\begin{cases}
osp(2,2) \rightarrow \text{Bosonic – Fermionic} \\
osp(1/2, \mathbb{R}) \rightarrow \text{Bosonic Fermionic}
\end{cases}
\]

with the following group structure for the bosonic-fermionic realization

\[
\begin{pmatrix}
SU(1,1) & Q \\
Q & SU(1,1)
\end{pmatrix}
\]
We will concentrate our analysis to the superspace \((1, 3 \mid 1)\) with extended line element as in \([9,11]\)

\[ ds^2 = \omega^\mu \omega_\mu + a \omega^\alpha \omega_\alpha - a^* \omega^\alpha \omega_\alpha \]

invariant to the following supersymmetric transformations

\[ x'_\mu = x_\mu + i \left( \theta^\beta (\sigma_\mu)_{\alpha\beta} \bar{\xi}^\beta - \xi^\beta (\sigma_\mu)_{\alpha\beta} \bar{\theta}^\beta \right) ; \quad \theta'^\alpha = \theta^\alpha + \xi^\alpha \; ; \quad \bar{\theta}'^\alpha = \bar{\theta}^\alpha + \bar{\xi}^\alpha \]

where the Cartan forms of the group of the supersymmetry are

\[ \omega_\mu = dx_\mu - i \left( d\theta \sigma_\mu \bar{\theta} - \theta \sigma_\mu d\bar{\theta} \right) ; \quad \omega^\alpha = d\theta^\alpha ; \quad \omega^\alpha = d\bar{\theta}^\alpha \]

The spinorial indices are related as follows (the dotted indices are similarly related, as usual):

\[ \theta^\alpha = \varepsilon^{\alpha\beta} \theta_\beta \; ; \quad \theta_\alpha = \theta^\beta \varepsilon_{\beta\alpha} \; ; \quad \varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha} \; ; \quad \varepsilon^{\alpha\beta} = -\varepsilon_{\beta\alpha} \; ; \quad \varepsilon_{12} = \varepsilon^{12} = 1 \]

The complex constants \(a\) and \(a^*\) in the extended line element are arbitrary. This arbitrariness for the choice of \(a\) and \(a^*\) are constrained by the invariance and reality of the interval (1). The solution for the metric in the time dependent case with 3 spatial dimensions compactified (i.e. \(\mathbb{R}^1 \otimes S^3\), ref.[15]) takes the form [10]

\[ g_{ab}(t) = e^{A(t)+\xi(t)} g_{ab}(0) \]

with the following superfield solution

\[ \varrho(t) = \phi_\alpha + \bar{\chi}_\alpha \]

(i.e. chiral plus anti-chiral parts). The system of equations for \(A(t)\) and \(\varrho(t)\) that we are looking for was given in [10], and is the following

\[ |a|^2 \ddot{A} + m^2 = 0 \]

\[ \ddot{\chi}_\alpha - i \frac{\dot{\varrho}}{2} (\sigma^0)^\alpha_\alpha \phi_\alpha = 0 \]

\[ -\ddot{\phi}_\alpha + i \frac{\dot{\varrho}}{2} (\sigma^0)^\beta_\alpha \bar{\chi}_\beta = 0 \]

The above system can be solved exactly given us the following result

\[ A = -\frac{1}{2} \left( \frac{m}{|a|} \right)^2 t^2 + c_1 t + c_2 \; ; \quad c_1, c_2 \in \mathbb{C} \]

and

\[ \phi_\alpha = \phi_\alpha \left( \alpha e^{i\omega t/2} + \beta e^{-i\omega t/2} \right) + \frac{2i}{\omega} (\sigma^0)^\beta_\alpha \bar{\chi}_\beta \]
\[ \mathbf{X}_\alpha = \left( \sigma^0 \right)^\alpha_{\dot{\alpha}} \hat{\phi}_\alpha \left( \alpha e^{i\omega t/2} - \beta e^{-i\omega t/2} \right) + \frac{2i}{\omega} \left( \sigma^0 \right)^\alpha_{\dot{\alpha}} Z_\alpha \]  

(6)

where \( \hat{\phi}_\alpha \), \( Z_\alpha \) and \( \overline{Z}_\beta \) are constant spinors and the frequency goes as: \( \omega^2 \sim \frac{4}{\left| a \right|^2} \). The superfield solution for the fields (see the "square states" of ref.[10,11]) that we are looking for, have the following form

\[ g_{ab} (t) = e^{-\frac{1}{2} \left( \frac{m}{\hbar} \right)^2 t^2 + c_1 t + c_2} e^{\xi \phi (t)} g_{ab} (0) \]  

(7)

with

\[ \phi (t) = \hat{\phi}_\alpha \left[ \left( \alpha e^{i\omega t/2} + \beta e^{-i\omega t/2} \right) - \left( \sigma^0 \right)^\alpha_{\dot{\alpha}} \left( \alpha e^{i\omega t/2} - \beta e^{-i\omega t/2} \right) \right] + \frac{2i}{\omega} \left[ \left( \sigma^0 \right)^\beta_{\dot{\beta}} \overline{Z}_\beta + \left( \sigma^0 \right)^\alpha_{\dot{\alpha}} Z_\alpha \right] \]  

(8)

and

\[ g_{ab} (0) = \langle \Psi (0) | L_{ab} | \Psi (0) \rangle \]  

(9)

that is nothing more that the "square" of the state \( \Psi [1] \) \( (L_{ab} = \begin{pmatrix} a & \cr a^+ & \end{pmatrix}_{ab}) \) with \( a \) and \( a^+ \) the standard creation and annihilation operators). The meaning of the expression (9) was given by the authors in ref.[10] and can be resumed as:

i) it can be interpreted as the "square" of the state \( \Psi \) and it is the fundamental solution of the square root of the interval (1), precisely describing a trajectory in the superspace[9,10,11];

ii) for these states \( \Psi \) the zero component of the current is not positively definite given explicitly by

\[ j_0 (x) = 2E \Psi^\dagger \Psi \]

but for the states \( g_{ab} \)

\[ j_0 (x) = 2E^2 g^{ab} g_{ab} \]

then, \( j_0 (x) \) for the states \( g_{ab} \) is positively definite (e.g. the energy \( E \) appears squared);

iii) from ii), such states \( \Psi \) are related with ordinary physical observables only through they "square" \( g_{ab} \) in the sense of expressions as (9), and this fact is very important in order to explain the reason why these fractional spin states are not easily observed or detected in the nature under ordinary conditions [10];

[1] This particular realization was initially introduced in ref.[28]) between the fundamental states \( | \Psi \rangle \) in the initial time, where the subalgebra is the Heisenberg-Weyl algebra (with generators \( a, a^+ \) and \( (n + \frac{1}{2}) \))
iv) and fundamentally we will take under consideration here only the particular case of spin 2 because for this state the Hilbert space is dense and these states lead a thermal spectrum\textsuperscript{[2]} \cite{10,16} (\(g_{ab}\) in the expression (9) has \(s=2\): each state \(\Psi\) contributes with a spin weight equal to one, see detailed explanation in Section 6). Other interesting possibilities given by these type of coherent states solutions and they physical meaning, that can give some theoretical framework for more degrees of freedom for the graviton in the sense of \cite{29-31}, will be analyzed with details in a separate paper\cite{16}(see also Section 6).

The \(g_{ab}\) at time \(t\) is given by the following expression\cite{10, Appendix}

\[
g_{ab}(t) = e^{-\left(\frac{m}{\hbar}\right)^2 t^2 + c_1 t + c_2} e^{\xi(t)} |f(\xi)|^2 \left( \begin{array}{c} \alpha \\ \alpha^* \end{array} \right)_{ab} \tag{10}
\]

where \(\alpha\) and \(\alpha^*\) are the respective eigenvalues of the creation-annihilation operators \(a\) and \(a^+\). And the dynamics for \(\Psi\) becomes now to

\[
\Psi_\lambda(t) = e^{-\frac{1}{2} \left[(\frac{m}{\hbar})^2 t^2 + c_1 t + c_2\right]} e^{\frac{\xi(t)}{2}} |f(\xi)| \left( \begin{array}{c} \alpha^{1/2} \\ \alpha^{*1/2} \end{array} \right)_\lambda \tag{11}
\]

It’s useful to remark here that there exist some misleadings and wrong asseverations about the non-degenerate supermetrics as (1) in some references (see e.g.:\cite{38}). The reason of these wrong claims coming from the misunderstanding that metrics as (1) in appearance don’t give a Dirac’s type (1\textsuperscript{st} order) equations of motion for fermions. In Section 5, when we discuss the origin of this type of metrics, this fact will be completely clear.

III. RELATION WITH THE (1 | 2) SUPERSPACE

We pass now to the description of the superspaces under consideration from the uniqueness of the possible solutions for the metric components, the supergroup structures defined by the possible group of motions and the possible physical interpretation of these results. The superspace (1, 2) has one bosonic coordinate \(t\) and two majorana spinors: \(x^\mu \equiv (t, \theta^1, \theta^2)\) and is the simplest low dimensional superspace of interest (we use similar notation as in refs.\cite{12,13}). The big group in which this superspace is contained is \(OSP(3, 2)\), schematically as

\textsuperscript{[2]} The other possibilities are squeezed states (non-thermal spectrum).
The solution for the metric in this case is given by [12,13]

$$g_{ab} = g_{ab} e^{2\sigma(t,\theta)}$$ (12)

where the following superfield was introduced

$$\sigma(t, \theta) = A(t) + \theta^\beta B_{\beta} + \theta^\alpha \theta_\alpha F(t)$$

From the Einstein equations for the (1 | 2) superspace we obtain the following set

\[
\begin{align*}
\dot{B}_\alpha + b_{\alpha\beta} B_{\beta} - AB_\alpha &= 0 \\
\ddot{A} - \frac{1}{2} A^2 + \frac{1}{2} B^\gamma B_\gamma &= \frac{4}{e^{2A}} - 1
\end{align*}
\] (13)

where \(b_{\alpha\beta} = b_{\beta\alpha}\) is an arbitrary symmetric matrix. Making a suitable transformation in the first of above equations the explicit form of the \(B_\gamma\) field that we are looking for is

$$B^\gamma B_\gamma = 2 \nu^\alpha \nu_\alpha e^{2A}$$ (14)

\(\nu_\alpha\) is a constant spinor and \(\sqrt{b}\) was associated in the ref.[13] with the mass. Inserting (14) in the second equation of the system (13) it leads the following new equation

$$\ddot{A}' - \frac{1}{2} A'^2 = \frac{\lambda}{4} \left( e^{2A'} - 1 \right)$$ (15)

where the transformation \(A' = A - \nu^\alpha \nu_\alpha\) was used. Notice that in the ref.[13] the derivation of the solution of the equation (15) was not explicitly explained, but however it is easy to see that can be reduced to the following expression

\[
\left( W \right)^2 = \frac{\lambda}{4} \left( W^2 + \frac{1}{2W^2} \right) + C
\] (16)

with \(W = e^{-\frac{A'}{\lambda}}\) and \(C\) is an arbitrary constant. When \(C = 0\) eq.(16) is the equation of motion for a supersymmetric oscillator in the potential of the form \(k \left( X^2 + \frac{1}{X^2} \right)\), for which the group \(O(3)\) is a dynamic symmetry group. Notice that from the point of view of a potential it is possible redefine it in order that \(C\) disappears, but the conservation of \(C\) is crucial for the determination of the families of solutions of the problem (is not possible to know completely this type of problems only inspectioning the potential). This type of
equations of motion for an oscillator with conformal symmetry was considered earlier in the non-supersymmetric case in [17]. The solutions for the possible values of the constant $C$ are

$$C = 0 \rightarrow e^{-A} = \frac{\sqrt{2}}{2} \text{Sinh} \left( \sqrt{\lambda} t + \varphi_0 \right), \quad \varphi_0 = \sqrt{\lambda} t_0$$

$$\frac{8C^2}{\lambda^2} < 1 \rightarrow e^{-A} = \frac{\sqrt{2}}{2} \left[ \text{Sinh} \left( \sqrt{\lambda} t + \varphi_0 \right) \sqrt{1 - \kappa^2} - \kappa \right], \quad \kappa = \frac{2\sqrt{2}C}{\lambda}$$

$$\frac{8C^2}{\lambda^2} = 1 \rightarrow e^{-A} = \frac{\sqrt{2}}{2} \left[ \frac{e^{(\sqrt{\lambda} t + \varphi_0)}}{\sqrt{2}} - \kappa \right]$$

$$\sigma(t, \theta) = A(t) + \theta^\alpha B_\alpha$$ (17)

Notice that $\lambda$ takes the place of the cosmological constant and is related with $b$ by $b = -\frac{\lambda}{2}$. The superfield solution (17) is N=2 (chiral or antichiral two components spinors), has conformal symmetry in the case $C = 0$ and is not unique: as was pointed out in the references [12,13,18] there exist a larger class of vacuum solutions. The dynamics of the solution is very simple as is easy to see from eqs.(17), that is not the case in the superspace (1, 3 | 1) as we showed in the previous Section.

With the description of both superspaces above, we pass now to compare them in order to establish if a one to one mapping exists between these superspaces. By simple inspection we can see that the fermionic part of the superspace solutions (2) and (17) is mapped one to one, explicitly (for the (1 | 2) superspace indexes 1 and 2 for $\alpha$ and $\beta$ are understood).

$$\nu_\alpha = -2\beta_{\bar{\phi}}^\alpha$$

$$2\sqrt{b} = \omega$$

$$\theta^1 \leftrightarrow \bar{\theta}^\alpha, \quad \theta^2 \leftrightarrow \theta^\alpha$$

$$\left(\sigma^0\right)_\alpha^\alpha \leftrightarrow \beta_1^2$$

if the following conditions over the four dimensional solution hold

$$\alpha = \beta, \quad Z_\alpha = \overline{Z}_\beta = 0$$

For the bosonic part of the superfield solutions (17) and (4) no direct relation exists between them. Only taking the limit of the constants $|a| \rightarrow \infty$ of the non-degenerate superspace (1, 3 | 1) (i.e. going to the standard (1, 3 | 1) superspace) the Gaussian solution (7) goes to the same type that the described in (17) for the (1 | 2) superspace, with $c_1 \approx \sqrt{\lambda}$.
and $c_2 \approx \varphi_0$. And this fact is non-trivial: this happens because the chirality is explicitly restored in this limit as we can easily seen from equations (3) when $|a| \to \infty, \omega^2 \to 0$. It is clear that the solution coming from four dimensional non-degenerate superspace is the physical one because represents a semiclassical (Gaussian) state of the Husimi’s type [10,19]: and this is a direct consequence of the non-degenerate characteristic of the supermetric. The important role played by the constants $a$ and $a^*$ in the extended line element (1) is localize the physical state in a precise region of the space-time, as is easily seen from expression (7). This fact can give some hints in order to explain and to treat from the mathematical point of view the mechanism of confinement, spontaneous compactification and other problems in high energy particle physics that can have a topological origin [16].

IV. SUPERGRAVITY AS A GAUGE THEORY AND THE ORIGIN OF THE SUPERMETRIC

Now, we will give some answers and hints about the origin of the non-degenerate supermetric under consideration and the structure of the equations of motion derived the respective super-Lagrangian constructed from it. The starting point is the $OSP(1/4)$ (some times called super-$SO(4, 1)$) superalgebra

\[
[M_{AB}, M_{CD}] = \epsilon_C (A M_B)_D + \epsilon_D (A M_B)_C \\
[M_{AB}, Q_C] = \epsilon_C (A Q_B) , \quad \{Q_A, Q_B\} = 2M_{AB} \tag{18}
\]

here the indices $A, B, C...$ stay for $\alpha, \beta, \gamma... (\hat{\alpha}, \hat{\beta}, \hat{\gamma}...) $ spinorial indices: $\alpha, \beta (\hat{\alpha}, \hat{\beta}) = 1, 2 (1, 2)$. We define the superconnection $A$ due the following ”gauging”

\[
A^p T_p \equiv \omega^{\alpha \beta} M_{\alpha \beta} + \omega^{\alpha \beta} M_{\alpha \beta} + \omega^{\alpha \beta} M_{\alpha \beta} + \omega^\alpha Q_\alpha - \omega^\alpha \overline{Q}_\alpha \tag{19}
\]

where $(\omega M)$ define a ten dimensional bosonic manifold[3] and $p \equiv $ multi-index, as usual. Analogically the super-curvature is defined by $F \equiv F^p T_p$ with the following detailed structure

\[
F (M)^{AB} = d\omega^{AB} + \omega^A_C \land \omega^{CB} + \omega^A \land \omega^B \tag{20}
\]

\[
F (Q)^A = d\omega^A + \omega^A_C \land \omega^C \tag{21}
\]

[3] Corresponding to the number of generators of $SO (4, 1)$ or $SO (3, 2)$ that define the group manifold.
From (20-21) is not difficult to see, that there are a bosonic part and a fermionic part associated with the even and odd generators of the superalgebra. Our proposal for the action is

$$S = \int F^p \wedge \mu_p$$  \hspace{1cm} (22)

where the tensor $\mu_p$ (that play the role of a $OSP(1/4)$ diagonal metric) is defined as

$$\mu_{\alpha\beta} = \zeta_{\alpha} \wedge \bar{\zeta}_{\beta} \quad \mu_{\alpha\beta} = \zeta_{\alpha} \wedge \zeta_{\beta} \quad \mu_{\alpha} = \nu \zeta_{\alpha} \text{ etc.}$$  \hspace{1cm} (23)

with $\zeta_{\alpha} \left( \bar{\zeta}_{\beta} \right)$ anticommuting spinors (suitable basis\cite{4}) and $\nu$ the parameter of the breaking of $OSP(1/4)$ to $SP(4) \sim SO(4,1)$ symmetry of $\mu_p$.

In order to obtain the dynamical equations of the theory, we proceed to perform the variation of the proposed action (22)

$$\delta S = \int \delta F^p \wedge \mu_p + F^p \wedge \delta \mu_p$$  \hspace{1cm} (24)

$$= \int d_A \mu_p \wedge \delta A^p + F^p \wedge \delta \mu_p$$

where $d_A$ is the exterior derivative with respect to the $OSP(1/4)$ connection and: $\delta F = d_A \delta A$ have been used. Then, as a result, the dynamics are described by

$$d_A \mu = 0 \quad , \quad F = 0$$  \hspace{1cm} (25)

The first equation said us that $\mu$ is covariantly constant with respect to the $OSP(1/4)$ connection: this fact will be very important when the $OSP(1/4)$ symmetry breaks down to $SP(4) \sim SO(4,1)$ because $d_A \mu = d_A \mu_{AB} + d_A \mu_A = 0$, a soldering form will appear. The second equation give the condition for a super Cartan connection $A = \omega^{AB} + \omega^A$ to be flat, as is easily to see from the reductive components of above expressions

$$F (M)^{AB} = R^{AB} + \omega^A \wedge \omega^B = 0$$  \hspace{1cm} (26)

$$F (Q)^{A} = d\omega^A + \omega^C_A \wedge \omega^B = d_\omega \omega^A = 0$$

where now $d_\omega$ is the exterior derivative with respect to the $SP(4) \sim SO(4,1)$ connection and $R^{AB} \equiv d\omega^{AB} + \omega^C_A \wedge \omega^{CB}$ is the $SP(4) \sim SO(4,1)$ curvature. Then

$$F = 0 \Leftrightarrow R^{AB} + \omega^A \wedge \omega^B = 0 \text{ and } d_\omega \omega^A = 0$$  \hspace{1cm} (27)

\cite{4} In general this tensor has the same structure that the Cartan-Killing metric of the group under consideration.
the second condition says that the $SP(4) \sim SO(4,1)$ connection is super-torsion free. The first says not that the $SP(4) \sim SO(4,1)$ connection is flat but that it is homogeneous with a cosmological constant related to the explicit structure of the Cartan forms $\omega^A$, as we will see when the $OSP(1/4)$ action is reduced to the Volkov-Pashnev model.

A. The geometrical reduction: origin of the extended supermetric

The supermetric under consideration, proposed by Volkov and Pashnev in [9], can be obtained from the $OSP(1/4)$ action via the following procedure:

i) the Inönu-Wigner contraction [32] in order to pass from $OSP(1/4)$ to the super-Poincare algebra (corresponding to the original symmetry of the model of refs.[10,9]), then, the even part of the curvature is splitted into a $R^{3,1}$ part $R^{\alpha\beta}$ and a $SO(3,1)$ part $R^{\alpha\beta}(R^{\alpha\beta})$ associated with the remaining six generators of the original $SP(4)$ group. This fact is easily realized knowing that the underlying geometry is reductive: $SP(4) \sim SO(4,1) \rightarrow SO(3,1) + R^{3,1}$, and rewriting the superalgebra (18) as

\begin{align}
[M, M] &\sim M \quad [M, \Pi] \sim \Pi \quad [\Pi, \Pi] \sim M \\
[M, S] &\sim S \quad [\Pi, S] \sim S \quad \{S, S\} \sim M + \Pi
\end{align}

(with $\Pi \sim M_{\alpha\beta}$ and $M \sim M_{\alpha\beta}(M_{\alpha\beta})$) and rescales $m^2\Pi = P$ and $mS = Q$, in the limit $m \rightarrow 0$ one recovers the super Poincare algebra. Notice that one does not rescale $M$ since one want to keep $[M, M] \sim M$ Lorentz algebra (that also is symmetry of (1))

ii) the spontaneous break down of the $OSP(1/4)$ to the $SP(4) \sim SO(4,1)$ symmetry of $\mu_p(e.g:\nu \rightarrow 0$ in $\mu_p)$ of such manner that the even part of the $OSP(1/4)$ action $F(M)^{AB}$ remains. (Notice the important fact that the super-action under consideration carry naturally the fermionic ”Dirac type” part that disappears with the particular breakdown of the symmetry of $\mu_p$, this fact was not having account in several refs. as in [38])

After these processes have been explicitly realized, the even part of the original $OSP(1/4)$ action (now super-Poincare invariant ) can be related with the original metric (1) as follows:

$$ R(M) + R(P) + \omega^\alpha\omega_\alpha - \omega^\alpha\omega^*_\alpha \rightarrow \omega^\mu\omega_\mu + a\omega^\alpha\omega_\alpha - a^*\omega^\alpha\omega^*_\alpha |_{VP} $$

Notice that there is mapping $R(M) + R(P) \rightarrow \omega^\mu\omega_\mu |_{VP}$ that is well defined and can be realized of different forms, and the map of interest here $\omega^\alpha\omega_\alpha - \omega^\alpha\omega^*_\alpha \rightarrow a\omega^\alpha\omega_\alpha - a^*\omega^\alpha\omega^*_\alpha |_{VP}$

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that associate the Cartan forms of the original $OSP(1/4)$ action (22) with the Cartan forms of the Volkov-Pashnev supermodel: $\omega^\alpha = (a)^{1/2} \omega_v^\alpha \mid_{VP}$; $\omega^\alpha = (a^*)^{1/2} \omega_v^\alpha \mid_{VP}$. Then, the origin of the coefficients $a$ and $a^*$ becomes clear from the geometrical point of view.

What about physics? From the first condition in (27) and the association (29) it is not difficult to see that, as in the case of the spacetime cosmological constant $\Lambda : R = \frac{\Lambda}{3} e \wedge e$ ($e \equiv space – time$ tetrad), there is a cosmological term from the superspace related to the complex parameters $a$ and $a^* : R = - (a_\omega^\alpha \omega_\alpha - a^* \omega^\alpha \omega_\alpha)$ and is easily to see from the minus sign in above expression, why for the standard supersymmetric (supergravity) models is more natural to use $SO(3, 2)$ instead $SO(4, 1)$.

On the associated spinorial action in the action (22), notice that the role of this part is constrained by the nature of $\nu_\alpha$ in $\mu_p$:

i) If they are of the same nature of the $\omega^\alpha$ this term is a total derivative, has not influence into the equations of motion, then the action proposed by Volkov and Pashnev in [9,10] has the correct fermionic form.

ii) If they are not with the same $SP(4) \sim SO(4, 1)$ invariance that the $\omega^\alpha$, the symmetry of the original model has been modified. In this direction a relativistic supersymmetric model for particles was proposed in ref. [33] considering an N-extended Minkowski superspace and introducing central charges to the superalgebra. Hence the underlying rigid symmetry gets enlarged to N-extended super-Poincare algebra. Considering for our case similar superextension that in ref.[33] we can introduce the following new action

$$S = -m \int_{\tau_1}^{\tau_2} d\tau \sqrt{\omega^\mu \omega_\mu + a^\alpha \theta_\alpha - a^\alpha \bar{\theta}_\alpha + i(\theta^{\alpha i} A_{ij} \theta^j_\alpha - \bar{\theta}^{\alpha i} A_{ij} \bar{\theta}^j_\alpha)} = \int_{\tau_1}^{\tau_2} d\tau L (x, \theta, \bar{\theta})$$

(30)

that is the N-extended version of the superparticle model proposed in [9], with the first order fermionic part. The matrix tensor $A_{ij}$ introduce the simplectic structure of such manner that now $\zeta_{\alpha i} \sim A_{ij} \theta^j_\alpha$ is not covariantly constant under $d_\omega$. Notice that the "Dirac-like" fermionic part is obviously inside the square root because it is part of the full curvature (the geometry of a N-superspace), fact that was not advertised by the authors in [33] that, specifically in they work, they not take account on the geometrical origin of the action. The interesting point is perform the same quantization that in the first part of this research [10] in order to obtain and compare the spectrum of physical states with the obtained in ref.[33]. This issue will be presented elsewhere [16].
In resume, we explicitly show here that the action under consideration constructed with the non-degenerate supermetric, can be derived from a $OSP(1/4)$ action and the structure of the dynamical equations for the fields of the theory depends on the coefficients of the tensor $\mu_p$ because they are responsible of the manner that the $OSP(1/4)$ symmetry of the model is breakdown, and a new N-extended version of the supermodel of [9] is presented.

V. "WARPED" GRAVITY MODELS, CONFINEMENT AND THE SUPERMETRIC

It is well known that large extra dimensions offer an opportunity for a new solution to the hierarchy problem [20]. Field theoretical localization mechanisms for scalar and fermions [21] as well as for gauge bosons [22] were found. The crucial ingredient of this scenario is a brane on which standard model particles are localized. In string theory, fields can naturally be localized on D-branes due to the open strings ending on them[23]. Up until recently, extra dimensions had to be compactified, since the localization mechanism for gravity was not known. It was suggested in ref.[24] that gravitational interactions between particles on a brane in uncompactified five dimensional space could have the correct four dimensional Newtonian behaviour, provided that the bulk cosmological constant and the brane tension are related. Recently, it was found by Randall and Sundrum that gravitons can be localized on a brane which separates two patches of AdS$_5$ space-time [25]. The necessary requirement for the four-dimensional brane Universe to be static is that the tension of the brane is fine-tuned to the bulk cosmological constant [24,25]. By the other hand, recent papers present an interesting model in which the extra dimensions are used only as a mathematical tool taking advantage of the AdS/CFT correspondence that claims that the 5D warped dimension is related with a strongly coupled 4D theory [26].

A remarkable property of the solution given by the expression (7) is that the physical state $g_{ab}(x)$ is localized in a particular position of the space-time. The supermetric coefficients $a$ and $a^*$ play the important role of localize the fields in the bosonic part of the superspace in similar and suggestive form as the well known "warp factors" in multidimensional gravity[14] for a positive (or negative) tension brane. But the essential difference is, because the $C$-constants $a$ and $a^*$ coming from the $B_{L,0}$(even) fermionic part of the superspace under consideration, not additional and/or topological structures that break the symmetries of the
model (i.e. reflection $Z_2$-symmetry) are required: the natural structure of the superspace produces this effect.

Also it is interesting to remark here that the Gaussian type solution (7) is very well defined physical state in a Hilbert space[10,19] from the mathematical point of view, contrarily to the case $u(y) = ce^{-H\lvert y\rvert}$ given in [14] that, although were possible to find a manner to include it in any Hilbert space, is strongly needed to take special mathematical and physical particular assumptions whose meaning is obscure. The comparison with the case of 5-dimensional gravity plus cosmological constant[14] is given in the following table:

| Spacetime       | (5 − D) gravity + $\Lambda$ | Superspace (1, d | 1) |
|-----------------|-------------------------------|------------------|
| Interval        | $ds^2 = A(y) dx_{3+1}^2 + dy$ | $ds^2 = \omega^\mu \omega_\mu + a \omega^\alpha \omega_\alpha - a^* \omega^\alpha \omega_\alpha$ |
| Equation        | $[-\partial_y^2 - m^2 e^{H\lvert y\rvert} + H^2 - 2H \delta(y)] u(y) = 0$ | $[(a^2) (\partial_0^2 - \partial_1^2) + \frac{1}{4} (\partial_\mu - \partial_\nu + i \partial_\mu (\sigma^{\mu}) \xi)^2 - \frac{1}{4} (\partial_\mu + \partial_\nu + i \partial_\mu (\sigma^{\mu}) \xi)^2 + m^2]_{cd} g_{ab} = 0$ |
| Solution        | $u(y) = ce^{-H\lvert y\rvert}, \ H \equiv \sqrt{-\frac{2\Lambda}{3}} = \frac{|T|}{M^3}$ | $g_{ab}(x) = e^{-\left(\frac{m}{\Lambda}\right)^2 x^2 + c_1 x + c_2 e^{\xi(x)} |f(\xi)|^2} \left( \begin{array}{c} \alpha \\ \alpha^* \end{array} \right)_{ab}$ |

Here, in order to make our comparison consistent, the proposed superspace has $d = n + 4$ bosonic coordinates and the extended superspace solution for $n = 0$ can depend, in principle, on any or all the 4-dimensional coordinates: $x \equiv (t, \overline{x})$, $c_1 x \equiv c_1^{\mu} x^\mu$ and $c_2$ scalar (e.g.: the $t$ coordinate in expression (7)); for $n \neq 0$ it depends on the $n$-additional coordinates.

Notice the following important observations:

i) that for that the solution in the 5-dimensional gravity plus $\Lambda$ case, the explicit presence of the cosmological term is necessary for the consistency of the model: the "fine-tuning" $H \equiv \sqrt{-\frac{2\Lambda}{3}} = \frac{|T|}{M^3}$, where $T$ is the tension of the brane and $M^3$ is the constant of the Einstein-Hilbert +$\Lambda$ action.

ii) about the localization of the fields given by the particular superspace treated here, the $Z_2$ symmetry is non-compatible with the solution that clearly is not chiral or antichiral. This fact is consistent with the analysis given for a similar superspace that the considered here in ref.[10,27] where the solutions are superprojected in a sector of the physical states that is not chiral or antichiral.

iii) because for $n = 0$ our solution (7) is attached on the 3+1 space-time but the localization occurs on the time coordinate (on in any of the remane 3 space coordinates) the physics seems to be very different with respect to the warped gravity model where the field
equation in final form for the 5-dimensional gravity depends on the extra dimension\textsuperscript{[5]}. This $n = 0$ case can give some hints for the theoretical treatment of the confinement mechanism with natural breaking of the chiral symmetry in high energy physics (e.g.: instanton liquid models, etc);

iv) for $n = 1$, our model with the solution depending on the extra coordinate, the situation changes favorably: the localization of the field is in the additional bosonic coordinate (as the graviton in the RS type model) but with all the good properties of the solution (7) already mentioned in the beginning of this paragraph.

From the points discussed above and the ”state of the art ” of the problem, we seen the importance of to propose new mechanisms and alternative models that can help us to understand and to handle the problem. Also is clearly important that the supermetric (1), cornerstone of this simple supermodel, is non-degenerate in order to solve in a simultaneous manner the localization-confinement of the fields involved and the breaking of the chiral symmetry. Then, it is not difficult to think to promote the particular supermetric under study towards to build a strongly coupled 4D model, using this particular N=1 toy superspace. We will treat this issue with great detail in a further work\textsuperscript{[16]}.

VI. GENERALIZED PHASE-SPACE REPRESENTATIONS: ”TOMOGRAPHIC” INTERPRETATION OF THE PHYSIC STATES

In this section we will treat to elucidate the meaning of the basic (non-observable) states and the physical ones of Section 2: $\Psi$ and $g_{ab}$ respectively. To this end, before to enter in more technical questions, some important points of the toy model presented here need to be reminded from the previous Sections and from reference [10] (mainly in Section 5):

\begin{itemize}
  \item For simplicity, only the bosonic part of the superfield wave solution will be analyzed in order to discuss conveniently the meaning of the states involved: the fermionic part, e.g.: $e^{\xi_g(\alpha+\alpha^*)}|f(\xi)|^2$ will be not discussed here.

  \item The detailed mathematical structure of the basic states $\Psi_{3/4}(t,\xi,q)$ and $\Psi_{1/4}(t,\xi,q)$ will be not considered (they will be studied elsewhere [16]).

  \item There are two spaces, that we denote $\mathcal{H}$ and $\overline{\mathcal{H}}$ (dotted and undotted indices, as usual
\end{itemize}

\textsuperscript{[5]} e.g.: in the Randall-Sundrum model the graviton is localized in the extra dimension
for different helicity states, each one being the direct sum of two irreducible subspaces \( \mathcal{H} = \mathcal{H}_{1/4} \oplus \mathcal{H}_{3/4} \) \((\mathcal{H} = \overline{\mathcal{H}}_{1/4} \oplus \overline{\mathcal{H}}_{3/4})\)

- Each irreducible subspace \( \mathcal{H}_{1/4} \) (with spin 1/4), \( \mathcal{H}_{3/4} \) (with spin 3/4) are spanned by even and odd states \(|2n\rangle\) and \(|2n+1\rangle\) respectively. They are eigenstates of \( N = a^{+}a \sim K_{0} \), that in more standard form:

\[
|n\rangle = \left| \frac{1}{4}, \frac{1}{2} \left( n + \frac{1}{2} \right) \right\rangle \quad \text{for } n \text{ even}
\]

\[
|n\rangle = \left| \frac{3}{4}, \frac{1}{2} \left( n + \frac{1}{2} \right) \right\rangle \quad \text{for } n \text{ odd}
\]

- The specific basic solutions naturally obtained from the non-degenerate super-space (see Section 2) are coherent states \( \Psi_{1/4}(t, \xi, q) \) and \( \Psi_{3/4}(t, \xi, q) \), eigenstates of the operator \( K_{-} = \frac{1}{2}aa \), spanning respectively the irreducible subspaces \( \mathcal{H}_{1/4} \) and \( \mathcal{H}_{3/4} \) (also for the ”dotted” case \( \overline{\mathcal{H}}_{1/4} \) and \( \overline{\mathcal{H}}_{3/4} \)).

- \( \Psi_{1/4}(t, \xi, q) \) and \( \Psi_{3/4}(t, \xi, q) \) are mutually orthogonal \( \langle \Psi_{3/4} | \Psi_{1/4} \rangle = \langle \Psi_{1/4} | \Psi_{3/4} \rangle = 0 \).

- One general state of any spin can be expanded in the \(|n\rangle\) or in the CS \(|\Psi\rangle\) basis, due the well known properties of such states.

- From the previous points and the explicit solution of the (super) relativistic wave equation, notice that there are four (non-trivial) representations for the group decomposition of the bispinor solution, as follows: \((1/4, 0) \oplus (0, 3/4), (3/4, 0) \oplus (0, 1/4), (1/4, 0) \oplus (0, 1/4)\) and \((3/4, 0) \oplus (0, 3/4)\) (Section 5 ref.[10]).

Is very well known, the quality of the CS of being "canonical quantizers" [37]. The CS quantization (BKT, Berezin-Klauder-Toeplitz) consists in associating with any classical observable \( f \) (function of the phase space variables \( q, p \) or equivalently \( z, \overline{z} \)) the operator valued integral

\[
\frac{1}{\pi} \int_{\mathbb{C}} f(z, \overline{z}) |z\rangle \langle z|dz^{2} = A_{f}
\]

The resulting operator (if it exists, almost in a weak sense), acts on the Hilbert space, spanned by the (over) complete set of CS \(|z\rangle\). In the weak sense we mean that the integral

\[
\frac{1}{\pi} \int_{\mathbb{C}} f(z, \overline{z}) |\langle \varphi |z\rangle|^{2}dz^{2} = \langle \varphi |A_{f}| \varphi \rangle
\]

should be finite by any \(|\varphi\rangle \in \mathcal{H}\) (or \(\in\) to some dense subset in \(\mathcal{H}\)). On immediately notices that if \(\varphi\) is normalized then the previous equation represents the mean value of the
function with respect to the \(\varphi\)-dependent probability distribution \(|\langle \varphi | z \rangle|^2\) on the phase space. Because this can be thought as a de-quantization, if we take account on one of the fundamental features of the CS that is the resolution of the unity in any Hilbert space \(\mathcal{H}\), the bridge between the classical and the quantum world can be established. Then, the fact that the obtained basic states of the superfield solution presented here are CS states is clearly important from the classical and quantum point of view.

We will see soon that there exist some type of operators, \(L_{ab}\) for example, where the integral as in the above equation involves non-diagonal projectors. That means that is necessary an extension of the set of ”acceptable classical observables” to those CS distributions \(T \in \mathcal{D}'(\mathbb{R}^2)\) (space of distributions) such that the product \(e^{-\eta z^2} T \in \mathcal{S}'(\mathbb{R}^2)\), e.g. tempered distributions in \(\mathcal{D}'(\mathbb{R}^2)\) that belong to the Schwartz space \(\mathcal{S}'(\mathbb{R}^2)\). As was pointed out in ref.[35], an increase in the family of representations of various systems offers new ways to study such systems. Representations of Hilbert space operators in the manner of the Weyl representation may be carried out for a great variety of groups, asymmetric representations of various forms (analogous to those presented in [35] for the Weyl group) can be introduced for other groups. In our case, the big group involved is the metaplectic group \(Mp(2)\) (the covering group of \(SL(2C)\)). This important group \(Mp(2)\) have been studied with some detail in several references [36] and is closely related with the para-bose coherent and squeezed states (CS and SS).

The starting point for our analysis is the following CS reconstructing Kernel for any operator \(A\) (not necessarily bounded[37,19])

\[
K_{\hat{A}}(\alpha, \alpha'; g) = e^{[|\alpha|^2 - |\alpha'|^2]} \langle \alpha | A | \alpha' \rangle
\]

where \(\alpha\) and \(\alpha'\) are complex variables that characterize a respective CS (in principle can depend on time, see expressions (32) below) and \(g\) is an element of \(Mp(2)\). From the first paper of this work and the previous sections, the possible basic states (CS not physically observables, regard the discussion in Section 2) are classified as

\[
\Psi_{1/4}(t, \xi, q) = f(\xi) |\alpha_+(t)\rangle
\]

\[
\Psi_{3/4}(t, \xi, q) = f(\xi) |\alpha_-(t)\rangle
\]

(32a)
with the following symmetric and antisymmetric non-equivalent combinations

\[
|\Psi^S\rangle = \frac{f(\xi)}{\sqrt{2}} (|\alpha_{+}\rangle + |\alpha_{-}\rangle) = f(\xi) |\alpha^S(t)\rangle
\]

\[
|\Psi^A\rangle = \frac{f(\xi)}{\sqrt{2}} (|\alpha_{+}\rangle - |\alpha_{-}\rangle) = f(\xi) |\alpha^A(t)\rangle
\]

and the important fact in order to evaluate the kernels (31) was the action of \(a\) and \(a^2\) over the states previously defined

\[
a|\Psi_{1/4}\rangle = \alpha |\Psi_{3/4}\rangle ; \ a |\Psi_{3/4}\rangle = \alpha |\Psi_{1/4}\rangle ; \ a |\Psi^S\rangle = \alpha |\Psi^S\rangle ; \ a |\Psi^A\rangle = -\alpha |\Psi^A\rangle
\]

\[
a^2 |\Psi_{1/4}\rangle = \alpha^2 |\Psi_{3/4}\rangle ; \ a^2 |\Psi_{3/4}\rangle = \alpha^2 |\Psi_{1/4}\rangle ; \ a^2 |\Psi^S\rangle = \alpha^2 |\Psi^S\rangle ; \ a^2 |\Psi^A\rangle = \alpha^2 |\Psi^A\rangle
\]

(similarly for states \(\overline{\Psi}\)). We have that the physical states are particular representations of the operators \(L_{ab}\) and \(L_{ab} (\in M p(2))\) in spinorial form in the sense of quasiprobabilities (tomograms in the \(\Psi_s\) plane) or as mean values with respect to the basic CS (32):

\(\langle \Psi_\lambda \rangle (\lambda = 1/4, 1/2, 3/4, 1)\). The possible generalized kernels(31) are the following

\[
g_{ab}(t, 2, \alpha)_{h w} = \langle \Psi^S(t) | L_{ab} | \Psi^S(t) \rangle = e^{-\left(\frac{m}{\sqrt{2}a}\right)^2 [(\alpha+\alpha^*)-B]^2+D e^{\xi e(\alpha+\alpha^*)} | f(\xi) |^2} \begin{pmatrix} \alpha \\ \alpha^* \end{pmatrix}_{(2)ab}
\]

\[
g_{ab}(t, 1, -\alpha)_{h w} = \langle \Psi^A(t) | L_{ab} | \Psi^A(t) \rangle = e^{-\left(\frac{m}{\sqrt{2}a}\right)^2 [(\alpha+\alpha^*)-B]^2+D e^{\xi e(\alpha+\alpha^*)} | f(\xi) |^2} \begin{pmatrix} -\alpha \\ -\alpha^* \end{pmatrix}_{(1)ab}
\]

\[
g_{ab}(t, 2, \alpha^2)_{s u(1,1)} = \langle \Psi^S(t) | L_{ab} | \Psi^S(t) \rangle = e^{-\left(\frac{m}{\sqrt{2}a}\right)^2 [(\alpha+\alpha^*)-B]^2+D e^{\xi e(\alpha+\alpha^*)} | f(\xi) |^2} \begin{pmatrix} \alpha^2 \\ \alpha^* \end{pmatrix}_{(2)ab}
\]

\[
g_{ab}(t, 1, \alpha^2)_{s u(1,1)} = \langle \Psi^A(t) | L_{ab} | \Psi^A(t) \rangle = e^{-\left(\frac{m}{\sqrt{2}a}\right)^2 [(\alpha+\alpha^*)-B]^2+D e^{\xi e(\alpha+\alpha^*)} | f(\xi) |^2} \begin{pmatrix} \alpha^2 \\ \alpha^* \end{pmatrix}_{(1)ab}
\]

\[
g_{ab}(t, 3/2, \alpha^2)_{s u(1,1)} = \langle \Psi_{3/4}(t) | L_{ab} | \Psi_{3/4}(t) \rangle = e^{-\left(\frac{m}{\sqrt{2}a}\right)^2 [(\alpha+\alpha^*)-B]^2+D e^{\xi e(\alpha+\alpha^*)} | f(\xi) |^2} \begin{pmatrix} \alpha^2 \\ \alpha^* \end{pmatrix}_{(3/2)ab}
\]

\[
g_{ab}(t, 1/2, \alpha^2)_{s u(1,1)} = \langle \Psi_{1/4}(t) | L_{ab} | \Psi_{1/4}(t) \rangle = e^{-\left(\frac{m}{\sqrt{2}a}\right)^2 [(\alpha+\alpha^*)-B]^2+D e^{\xi e(\alpha+\alpha^*)} | f(\xi) |^2} \begin{pmatrix} \alpha^2 \\ \alpha^* \end{pmatrix}_{(1/2)ab}
\]

where the constants \(D\) and \(B\) are related to the original constants \(c'_1\) and \(c'_2\) of the first part of this work as: \(D = \left(\frac{|a|c'_1}{\sqrt{2}m}\right)^2 + c'_2\) and \(B = \left(\frac{|a|}{m}\right)^2 c'_1\). The expressions (33) are in the
called Sudarshan’s diagonal-representation that lead, as important consequence, the \textit{physical states} with spin content $\lambda = 1/2, 1, 3/2, 2$. Precisely, the CS generate a map that relates the solution of the wave equation $g_{ab}$ to the specific subspace of the full Hilbert space where these CS live (see the reconstruction of the operators $L$ and $L$ eqs. (35-37) below).

However, there exists for operators $\in Mp(2)$ an asymmetric -kernel leading for our case the following $\lambda = 1$ state

$$g_{ab}(t, 1, \alpha)|_{hw} = \langle \Psi_{3/4}(t) \mid L_{ab} \mid \Psi_{1/4}(t) \rangle =$$

$$= \langle \Psi_{1/4}(t) \mid L_{ab} \mid \Psi_{3/4}(t) \rangle = e^{-\left(\frac{m}{\sqrt{2}|a|}\right)^2[(\alpha+\alpha^*)-B]^2+D} e^{\xi\phi(\alpha+\alpha^*)} |f(\xi)|^2 \begin{pmatrix} \alpha \\ \alpha^* \end{pmatrix} (1)_{ab}$$

because the non-diagonal projector involved in the reconstruction formula of $L_{ab}$ is formed with $\Psi_{1/4}$ and $\Psi_{3/4}$ spanning all the Hilbert space.

Observation 1: Due the unobservability of isolated basic states, the spin zero physical states appears as bounded states $g\mathbf{G}$, where $g_{ab}(t, s, w)$ and $\mathbf{G}_{ab}(t, s, w)$ are given by the bilinear expressions (33).

Observation 2: Each kernel represents a \textit{physical} state composed by fundamental ones, that are basic and unobservable.

Notice that the spectrum of the physic states are labeled not only by they spin content $\lambda$, but also for the ”eigenspinors”\cite{6} $\begin{pmatrix} \alpha \\ \alpha^* \end{pmatrix} (\lambda)_{ab}$ corresponding to the tomographic representations of $L_{ab}$ (map over a region of $\mathcal{H}$) ; and $\begin{pmatrix} \alpha^2 \\ \alpha^{*2} \end{pmatrix} (\lambda)_{ab}$ corresponding to the tomographic representations of $L_{ab}$.

The previous representations form part of a more general class of representations for operators recently proposed by Klauder and Skagerstam in ref.[35]. However, the operators can be reconstructed due the (over) completeness of the basic states $\mid \Psi_{1/4}(t) \rangle$ in $\mathcal{H}_{1/4}$, $\mid \Psi_{3/4}(t) \rangle$ in $\mathcal{H}_{3/4}$ and $\mid \Psi^S(t) \rangle (\mid \Psi^A(t) \rangle)$in the full Hilbert space $\mathcal{H} = \mathcal{H}_{1/4} \oplus \mathcal{H}_{3/4}$ through

\[\text{[6] This term was introduced here by us.}\]

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the following reconstruction formulas (analogically for the states $\Psi$):

\[
L_{ab} = \int \frac{d^2 \alpha}{\pi} \left[ \int \frac{d^2 \omega}{\pi} \int \frac{d^2 \alpha'}{\pi} e^{-\left(\frac{m}{\sqrt{2} \pi}\right)^2 \left[(\alpha + \alpha^* - B)^2 + D \xi \phi |\xi|^2 \right]} \left( \frac{\alpha^2}{\alpha'^2} \right)^{(2)ab} \times \right.

\times e^{\frac{|w|^2}{4} e^2 \left[(\alpha - \alpha^*) w^* + (\alpha^* - \alpha^*) w \right]} \left| \Psi^S (\alpha) \right\rangle \left\langle \Psi^S (\alpha) \right| \tag{35a}
\]

\[
L_{ab} = \int \frac{d^2 \alpha}{\pi} \left[ \int \frac{d^2 \omega}{\pi} \int \frac{d^2 \alpha'}{\pi} e^{-\left(\frac{m}{\sqrt{2} \pi}\right)^2 \left[(\alpha + \alpha^* - B)^2 + D \xi \phi |\xi|^2 \right]} \left( \frac{\alpha^2}{\alpha'^2} \right)^{(1)ab} \times \right.

\times e^{\frac{|w|^2}{4} e^2 \left[(\alpha - \alpha^*) w^* + (\alpha^* - \alpha^*) w \right]} \left| \Psi^A (\alpha) \right\rangle \left\langle \Psi^A (\alpha) \right| \tag{35b}
\]

\[
L_{ab} = \int \frac{d^2 \alpha}{\pi} \left[ \int \frac{d^2 \omega}{\pi} \int \frac{d^2 \alpha'}{\pi} e^{-\left(\frac{m}{\sqrt{2} \pi}\right)^2 \left[(\alpha + \alpha^* - B)^2 + D \xi \phi |\xi|^2 \right]} \left( \frac{\alpha^2}{\alpha'^2} \right)^{(3/2)ab} \times \right.

\times e^{\frac{|w|^2}{4} e^2 \left[(\alpha - \alpha^*) w^* + (\alpha^* - \alpha^*) w \right]} \left| \Psi_{3/4} (\alpha) \right\rangle \left\langle \Psi_{3/4} (\alpha) \right| \tag{35c}
\]

\[
L_{ab} = \int \frac{d^2 \alpha}{\pi} \left[ \int \frac{d^2 \omega}{\pi} \int \frac{d^2 \alpha'}{\pi} e^{-\left(\frac{m}{\sqrt{2} \pi}\right)^2 \left[(\alpha + \alpha^* - B)^2 + D \xi \phi |\xi|^2 \right]} \left( \frac{\alpha^2}{\alpha'^2} \right)^{(1/2)ab} \times \right.

\times e^{\frac{|w|^2}{4} e^2 \left[(\alpha - \alpha^*) w^* + (\alpha^* - \alpha^*) w \right]} \left| \Psi_{1/4} (\alpha) \right\rangle \left\langle \Psi_{1/4} (\alpha) \right| \tag{35d}
\]

\[
L_{ab} = \int \frac{d^2 \alpha}{\pi} \left[ \int \frac{d^2 \omega}{\pi} \int \frac{d^2 \alpha'}{\pi} e^{-\left(\frac{m}{\sqrt{2} \pi}\right)^2 \left[(\alpha + \alpha^* - B)^2 + D \xi \phi |\xi|^2 \right]} \left( \frac{\alpha'^2}{\alpha^*^2} \right)^{(2)ab} \times \right.

\times e^{\frac{|w|^2}{4} e^2 \left[(\alpha - \alpha^*) w^* + (\alpha^* - \alpha^*) w \right]} \left| \Psi^S (\alpha) \right\rangle \left\langle \Psi^S (\alpha) \right| \tag{36a}
\]
\[ L_{ab} = \int \frac{d^2 \alpha}{\pi} \left[ \int \frac{d^2 w}{\pi} \int \frac{d^2 \alpha'}{\pi} e^{-\left(\frac{w}{\sqrt{2|a|}}\right)^2 |(\alpha + \alpha^*) - B|^2 + D e^{\xi g(\alpha' + \alpha^*)}} |f(\xi)|^2 \begin{pmatrix} -\alpha' \\ -\alpha'^* \end{pmatrix} \right] \times \]

\times e^{\frac{|w|^2}{4} e^{\frac{i}{2} [(\alpha - \alpha')w^* + (\alpha^* - \alpha^*)w]}} \left| \Psi^A (\alpha) \right〉 \left〈 \Psi^A (\alpha) \right| \]  

and in the asymmetric case (34)

\[ L_{ab} = \int \frac{d^2 \alpha}{\pi} \left[ \int \frac{d^2 w}{\pi} \int \frac{d^2 \alpha'}{\pi} e^{-\left(\frac{w}{\sqrt{2|a|}}\right)^2 |(\alpha + \alpha^*) - B|^2 + D e^{\xi g(\alpha' + \alpha^*)}} |f(\xi)|^2 \begin{pmatrix} \alpha' \\ \alpha'^* \end{pmatrix} \right] \times \]

\times e^{\frac{|w|^2}{4} e^{\frac{i}{2} [(\alpha - \alpha')w^* + (\alpha^* - \alpha^*)w]}} \left| \Psi_m (\alpha) \right〉 \left〈 \Psi_n (\alpha') \right| \]  

with \( m, n = 1/4, 3/4 \) (\( m \neq n \)). The important point to remark here is that there exist two different reconstructions for the operator \( L_{ab} \) in the full Hilbert space \( \mathcal{H} \) with the same \( \lambda \): a diagonal representation and an asymmetric one. However, both representations are not equivalent: for the asymmetric one the basic states involved are one half than in the diagonal case and the eigenspinor \( \begin{pmatrix} \alpha' \\ \alpha'^* \end{pmatrix} \) has plus sign (compare expressions (37) and (36b)).

Then, as was pointed out for the authors of reference [35], in this case the asymmetric representation is absolutely necessary to describe completely the physical system (and also to reconstruct conveniently the operators).

Finally from the above expressions, the Gram-Schmidt operators can be easily constructed (\( \lambda = 1/2, 1, 3/2, 2 \)):

\[ G = \int \frac{d^2 \alpha}{\pi} \left[ \int \frac{d^2 w}{\pi} e^{\frac{|w|^2}{4} e^{\frac{i}{2} [(\alpha - \alpha')w^* + (\alpha^* - \alpha^*)w]}} \left| \Psi_{\lambda/2} (\alpha) \right〉 \left〈 \Psi_{\lambda/2} (\alpha) \right| \right] \]

\[ G = \int \frac{d^2 \alpha}{\pi} \left[ \int \frac{d^2 w}{\pi} e^{\frac{|w|^2}{4} e^{\frac{i}{2} [(\alpha - \alpha')w^* + (\alpha^* - \alpha^*)w]}} \left| \Psi_{\lambda/2} (\alpha) \right〉 \left〈 \Psi_{\lambda/2} (\alpha) \right| \right] \]

and in the asymmetric case (34)

\[ G = \int \frac{d^2 \alpha}{\pi} \left[ \int \frac{d^2 w}{\pi} e^{\frac{|w|^2}{4} e^{\frac{i}{2} [(\alpha - \alpha')w^* + (\alpha^* - \alpha^*)w]}} \left| \Psi_m (\alpha) \right〉 \left〈 \Psi_n (\alpha') \right| \right] \]

where we solve naturally the identity in each (sub) Hilbert space, as is required by, also in the non-diagonal case.
Summarizing explicitly the main results of this Section,

i) the $\text{Mp}(2)$ is the primary group that acts unitarily on $\mathcal{H}$ (and the same for the representation $\overline{\mathcal{H}}$)

i) In this model, the basic CS are the fundamental solutions generated by the most simplest non-degenerate supermetric where the supercoordinates are the fields of the theory.

ii) These basic states have spin $\lambda=1/4$ and $\lambda=3/4$ and are not physically observables.

ii) The basic CS generate a map (i.e. $g_{ab}$) that relates the operators $L_{ab}$ and $\mathbb{L}_{ab} \in \text{Mp}(2)$ to the specific subspace of the full Hilbert space where these CS live.

iii) The physical states are nothing more that tomographic representations or quasiprobabilities in the sense that are the mean of the operators $L_{ab}$ and $\mathbb{L}_{ab} \in \text{Mp}(2)$ (that forms, with the dotted representation, the double covering of $SL(2\mathbb{C})$) with respect to the basic CS solution of the superwave equation (given by expressions (32));

ii) The representations for the operators $L_{ab}$ and $\mathbb{L}_{ab} \in \text{Mp}(2)$ (expressions (35-37)) are particular cases of a more general kind of representations for operators recently proposed by Klauder and Skagerstam in [35];

iii) the set of physical states are labeled by the total spin $\lambda$ and the associated “eigen-spinors” as described in expressions (33-34);

iv) in the best reconstruction formulas for $L_{ab}$ and $\mathbb{L}_{ab}$ (reliable in the sense given in [34]) the basic CS involved in such formulas span all the Hilbert space $\mathcal{H} = \mathcal{H}_{1/4} \oplus \mathcal{H}_{3/4}$;

v) there exist two types of non-equivalent reconstruction formulas for $L_{ab}$ and $\mathbb{L}_{ab}$: a diagonal representation and an asymmetric one.

vi) from the previous point we conclude (in full agreement with the claim in [35]) that the asymmetric representation is absolutely necessary in order to describe completely the physical system (physic states).

VII. DISCUSSION

The proposal for the choice of model with a underlying basic structure starts from the very early. Today, the large effective group given by the standard model (multiplicity in the representations and the different coupling constants) stimulates from time ago the search of such models. As we saw in the first part of this work and other references, is that starting from the most simplest non-degenerate supermetric where the supercoordinates are the
fields of the theory and retaining the original form of the fundamental geometrical operators (namely Lagrangian or Hamiltonian) the physical states obtained are constructed from the the basic ones by mean operators that characterize the most fundamental symmetries of the spacetime.

The situation is more or less clear: although the supersymmetry is broken, the physical states are localized in the "even" part of the manifold due the metric coefficients of a non-degenerate supermetric. The physical states are composed by most fundamental (non-observable) basic states. Operators belonging to the metaplectic group (the most fundamental covering group of the SL(2C)) lead, due a map produced by the basic CS, the observable spectrum of physical states. This fact is clearly important as the "cornerstone" of a new realistic composite model of particles based in coherent states where the spacetime symmetry is directly connected with the physical spectrum.

VIII. CONCLUDING REMARKS

In the present paper we have analyzed from the point of view of the symmetries and the obtained vacuum solutions the superspace N=1 non-degenerate metric proposed by Volkov and Pashnev in [9]. This particular model, although its high simplicity, present a much richer structure than the others degenerate standard superspaces because it contains the complex parameters \( \alpha \) and \( \alpha^* \) that make it different. The important role played by the complex parameters \( \alpha \) and \( \alpha^* \) can be resumed as follows:

i) the \( \mathbb{C} \)-parameters \( \alpha \) and \( \alpha^* \) fix the field in a specific sector of the even part \( (B_{L,0}) \) of the supermanifold;

ii) these parameters, that are responsible of the non trivial part of the model, break the chiral symmetry of the field solution. The chiral symmetry is restored when the metric in question becomes degenerate in the limit \( |\alpha| \to \infty \) (with all other parameters of the model fixed);

iii) the fields remain attached in a specific region of the spacetime when the supersymmetry of the model is completely broken, even if all the fermions are switched off;

iv) we have analyzed and compared from the point of view of the obtained solutions the superspace \((1 \mid 2)\) with the particular superspace \((1, 3 \mid 1)\) proposed by Volkov and Pashnev [9,11], compactified to one dimension and restricted to the pure time-dependent case. The
possibility that the non-degenerate superspace $(1, 3 \mid 1)$ with extended line element is reduced to the superspace $(1 \mid 2)$ is subject to the condition $|a| \to \infty$. The fermionic part of both superspaces is mapped one to one by mean of a suitable definition of the fermionic variables and coefficients.

From the geometrical and group theoretical point of view the results are the following:

v) the supermetric can be derived from a gauge theory of supergravity based in the $OSP(1/4)$ group;

vi) the complex parameters $a$ and $a^*$ play similar role that the cosmological constant $\Lambda$ in the ordinary spacetime models. Then, add a $\Lambda$ constant by hand is not necessary in this supersymmetric model;

vii) a new generalization of the Volkov-Pashnev superparticle is presented for $N>1$ supersymmetry where the first order fermionic term appear explicitly in the action.

In comparison with the 5-dimensional gravity plus cosmological constant of refs.[14], the simple supersymmetric model under analysis here (now with $n$–extra bosonic coordinates) has the following advantages:

viii) for $n = 0$ the model, although is a very good candidate for a confinement mechanism with natural breaking of the chiral symmetry in high energy physics (e.g.:instanton liquid models, etc), cannot be compared directly with the Randall-Sundrum model because the localization of the fields are not in the bosonic extra-dimension (the physic are different in both cases).

For $n = 1$

ix) the mechanism of localization of the fields in the bosonic 4-dimensional part of the supermanifold does not depends on the cosmological constant;

x) the fields attached are Gaussian type solutions (7) very well defined physical state in a Hilbert space from the mathematical point of view, contrarily to the case $u(y) = ce^{-H|y|}$ given in [16];

xi) not additional and/or topological structures that break the symmetries of the model (i.e. reflection $Z_2$-symmetry) are required to attach the fields: the natural structure of the superspace produces this effect through the $C$-parameters $a$ and $a^*$.

From the point of view of the obtained spectrum of physic states we explicitly shown that

i) the $Mp(2)$ is the primary group that acts unitarily on $\mathcal{H}$ (and the same for the representation $\overline{\mathcal{H}}$)
i) In this model, the *basic* CS are the fundamental solutions generated by the most simplest non-degenerate supermetric where the supercoordinates are the fields of the theory.

ii) These basic states have spin $\lambda=1/4$ and $\lambda=3/4$ and are not physically observables.

iii) The basic CS generate a map (i.e. $g_{ab}$) that relates the operators $L_{ab}$ and $\mathbb{L}_{ab} \in Mp(2)$ to the specific subspace of the full Hilbert space where these CS live.

iii) The *physical states* are nothing more that tomographic representations or quasiprobabilities in the sense that are the mean of the operators $L_{ab}$ and $\mathbb{L}_{ab} \in Mp(2)$ (that forms, with the dotted representation, the double covering of $SL(2C)$) with respect to the basic CS solution of the superwave equation (given by expressions (32));

ii) The representations for the operators $L_{ab}$ and $\mathbb{L}_{ab} \in Mp(2)$ (expressions (35-37)) are particular cases of a more general kind of representations for operators recently proposed by Klauder and Skagerstam in [35];

iii) the set of physical states are labeled by the total spin $\lambda$ and the associated “eigen-spinors” as described in expressions (33-34);

iv) in the best reconstruction formulas for $L_{ab}$ and $\mathbb{L}_{ab}$ (reliable in the sense given in [34]) the basic CS involved in such formulas span all the Hilbert space $\mathcal{H} = \mathcal{H}_{1/4} \oplus \mathcal{H}_{3/4}$;

v) there exist two types of *non-equivalent* reconstruction formulas for $L_{ab}$ and $\mathbb{L}_{ab}$: a diagonal representation and an asymmetric one.

vi) the asymmetric representation is absolutely necessary in order to describe completely the physical system (physic states).

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X. APPENDIX

The dynamics of the $|\Psi\rangle$ fields, in the representation that we are interested in, can be simplified considering these fields as coherent states in the sense that are eigenstates of $a^2$ 

$$|\Psi_{1/4} (0, \xi, q)\rangle = +\infty \sum_{k=0} f_{2k} (0, \xi) |2k\rangle = +\infty \sum_{k=0} f_{2k} (0, \xi) \frac{(a\dagger)^{2k}}{\sqrt{(2k)!}} |0\rangle$$

$$|\Psi_{3/4} (0, \xi, q)\rangle = +\infty \sum_{k=0} f_{2k+1} (0, \xi) |2k+1\rangle = +\infty \sum_{k=0} f_{2k+1} (0, \xi) \frac{(a\dagger)^{2k+1}}{\sqrt{(2k + 1)!}} |0\rangle$$

From a technical point of view these states are a one mode squeezed states constructed by the action of the generators of the SU(1,1) group over the vacuum. For simplicity, we will take all normalization and fermionic dependence or possible CS fermionic realization, into the functions $f (\xi)$. Explicitly at $t=0$

$$|\Psi_{1/4} (0, \xi, q)\rangle = f (\xi) |\alpha_+\rangle$$

$$|\Psi_{3/4} (0, \xi, q)\rangle = f (\xi) |\alpha_-\rangle$$

where $|\alpha_{\pm}\rangle$ are the CS basic states in the subspaces $\lambda = \frac{1}{4}$ and $\lambda = \frac{3}{4}$ of the full Hilbert space. In the case of the physical state that we are interested in, we used the HW realization for the states $\Psi$

$$|\Psi\rangle = \frac{f (\xi)}{2} (|\alpha_+\rangle + |\alpha_-\rangle) = f (\xi) |\alpha\rangle$$

where, however, the linear combination of the states $|\alpha_+\rangle$ and $|\alpha_-\rangle$ span now the full Hilbert space (dense) being the correspond $\lambda$ to the CS basis. The ”square” state at $t=0$ are

$$g_{ab} (0) = \langle \Psi (0)| L_{ab} |\Psi (0)\rangle = \langle \Psi (0)\left( \begin{array}{c} a \\ a^\dagger \end{array} \right)_{ab} |\Psi (0)\rangle$$

$$= f^* (\xi) f (\xi) \left( \begin{array}{c} \alpha \\ \alpha^* \end{array} \right)_{ab}$$

The algebra (topological information of the group manifold) is ”mapped” over the spinors solutions through the eigenvalues $\alpha$ and $\alpha^*$. Notice that the constants $c_1^* c_2$ in the exponential functions in expressions (10) and (11) can be easily determined as functions of the frequency $\omega$ as in ref.[19] for the Schrödinger equation.
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