CORRELATED RADIAL VELOCITY AND X-RAY VARIATIONS IN HD 154791/4U 1700+24

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ABSTRACT

We present evidence for approximately 400 day variations in the radial velocity of HD 154791 (V934 Her), the suggested optical counterpart of 4U 1700+24. The variations are correlated with the previously reported ≈400 day variations in the X-ray flux of 4U 1700+24, which supports the association of these two objects, as well as the identification of this system as the second known X-ray binary in which a neutron star accretes from the wind of a red giant. The HD 154791 radial velocity variations can be fitted with an eccentric orbit with period 404 ± 3 days, amplitude $K = 0.75 \pm 0.12$ km s$^{-1}$, and eccentricity $e = 0.26 \pm 0.15$. There are also indications of variations on longer timescales $\geq 2000$ days. We have reexamined all available All-Sky Monitor (ASM) data following an unusually large X-ray outburst in 1997–1998 and confirm that the 1 day averaged 2–10 keV X-ray flux from 4U 1700+24 is modulated with a period of 400 ± 20 days. The mean profile of the persistent X-ray variations was approximately sinusoidal, with an amplitude of 0.108 ± 0.012 ASM counts s$^{-1}$ (corresponding to 31% rms). The epoch of X-ray maximum was approximately 40 days after the time of periastron, according to the eccentric orbital fit. If the 400 day oscillations from HD 154791/4U 1700+24 are due to orbital motion, then the system parameters are probably close to those of the only other neutron star symbiotic-like binary, GX 1+4. We discuss the similarities and differences between these two systems.

Subject headings: binaries: symbiotic — stars: individual (HD 154791) — stars: neutron — X-rays: binaries

1. INTRODUCTION

The X-ray source 4U 1700+24 (also known as 2A 1704+241; $l = 45\degree 15, b = 32\degree 99$) was first discovered in Ariel 5 scans for high-latitude X-ray sources (Cooke et al. 1978) and by the Uhuru (SAS 1) X-ray observatory (Forman et al. 1978). It has a typical flux of (1–10) $\times 10^{-11}$ ergs cm$^{-2}$ s$^{-1}$ (2–10 keV), equivalent to 0.5–5 mcrab as measured by a range of X-ray missions; Masetti et al. (2002) and experiences occasional outbursts. During a 100 day X-ray high state in 1997–1998, 4U 1700+24 was observed by the Rossi X-Ray Timing Explorer (RXTE: Jahoda et al. 1996). This high state was the larger of two known outbursts of this system, and the X-ray flux reached a peak of 40 mcrab. The X-ray spectrum of 4U 1700+24 is similar to that of other accreting neutron stars and generally requires a blackbody with a temperature of 0.9–1.3 keV plus a hard component at higher energies for an acceptable fit. The X-ray spectral hardness and the lack of UV continuum imply that the compact object is almost certainly not a white dwarf (Garcia et al. 1983). A black hole companion is also unlikely since the thermal X-ray component in black hole candidates is typically an order of magnitude lower in temperature than that measured in 4U 1700+24. However, no rapid quasi-periodic or coherent X-ray variations have been confirmed (Masetti et al. 2002), which is rather unusual for an accreting neutron star.

Based on Einstein position measurements, Garcia et al. (1983) proposed the association of 4U 1700+24 with the 8th magnitude M2 III star HD 154791. In a recent reanalysis of ROSAT HRI data, however, Morgan & Garcia (2001) found only a 10% chance that the two sources are associated, but did not propose an alternative candidate. Previous optical spectroscopy ruled out radial velocity variations with amplitudes greater than 5 km s$^{-1}$ on timescales of 40 minutes to ~1 yr (Garcia et al. 1983), strongly suggesting that either the orbital period is significantly longer than 1 yr, or the system is viewed very close to face-on (inclination angle $i \approx 0$).

HD 154791 may be only the second symbiotic-like binary known to contain a neutron star, after the well-studied M giant/X-ray pulsar system V2116 Oph/GX 1+4. However, these systems are quite different in X-rays (Table 1). Furthermore, the optical spectrum of V2116 Oph exhibits extremely strong, variable emission lines, probably powered by UV photons originating from an accretion disc (Chakrabarty & Roche 1997). In contrast, HD 154791 has much weaker line emission. In fact, optically the spectrum is very close to that of an isolated M giant (e.g., Garcia et al. 1983).

Here we describe optical and X-ray observations that go some way toward resolving the uncertainties surrounding this system. We compare the properties of HD 154791/4U 1700+24 with those of V2116 Oph/GX 1+4 and discuss what may be inferred about the former in light of our results.

2. OBSERVATIONS

2.1. Radial Velocity Variations

Echelle spectra of HD 154791 were obtained using intensified Reticon detectors on the 1.5 m Tillinghast Reflector at the Whipple Observatory on Mount Hopkins, Arizona, and also on the 1.5 m Wyeth reflector at Oak
Ridge Observatory, Massachusetts. The observations were made between 1982 April 12 and 1996 October 26, with the 18 observations before 1992 October 7 taken at Mount Hopkins, and those since taken at Oak Ridge. The instruments are described in detail by Latham (1992). The spectra cover a 44 Å bandpass centered around 5200 Å, with a resolution of approximately 8.5 km s⁻¹/C0. Each observation consisted of a 10–20 minute integration of HD 154791, with 90 s Th-Ar comparison scans before and after. The positions of around 30 strong spectral lines in each calibration scan were fitted with a fifth-order polynomial, and the pre- and post-observation scans were then combined to calibrate the wavelength scale for each spectrum. Typical residuals to the polynomial fit were 0.5 pixel, corresponding to 0.01 Å or 0.5 wavelength scale for each spectrum. Typical deviations from circularity were significant at less than the 1 σ level.

The radial velocity measurements show significant variations of approximately 1 km s⁻¹ about the mean systemic velocity of −48.7 ± 0.1 km s⁻¹ (Fig. 1). We searched the data for periodic signals in the range 50–1000 days by calculating the Lomb-Scargle periodogram (e.g., Press et al. 1992) and computing the Lomb-normalized power 10.5. Estimating significance from bootstrap simulations, 3.3 σ deviations from circularity were significant at less than the 1 σ level. The best-fit eccentricity was 0.26, the deviations from circularity were significant at less than the 2 σ level.

There was also evidence for variations in the radial velocity on longer timescales. We calculated χ²/k for an eccentric orbit as a function of trial period for periods of up to 4000 days. While the 404 day signal gave rise to a χ²/k value close to 1, lower values were achieved at substantially longer periods of 1930 days (χ²/k = 1.04), 2370 days (1.02), and 3700 days (0.91). The irregular spacing of the measurements did not allow us to distinguish clearly between these periodicities; eccentric orbital solutions with any one of the three periods can account for the long-term radial velocity variations. The observations do not cover more than one full cycle for these longer period signals, so more observations are needed to determine whether they are true periodic variations or aperiodic trends related to irregular red giant variability.

2.2. RXTE/ASM Observations

Masetti et al. (2002) noted tentative evidence for an approximately 400 day periodicity in RXTE All-Sky Monitor (ASM) data. To investigate the 400 day periodicity, we obtained 1 day averaged ASM measurements of 4U 1700+24 between 1996 January 5 and 2002 April 18 (MJD 50,087–52,382). The mean quiescent ASM count rate was 0.2 counts s⁻¹. During the flare observed between 1997 September and December (MJD 50,700–50,800), the 1 day averaged rate reached 2.5 counts s⁻¹ (Fig. 2, top panel).

This information was taken from the ASM Web site at http://xte.mit.edu.

TABLE 1

| Parameter | 4U 1700+24 | GX 1+4 | References |
|-----------|-----------|--------|------------|
| Distance (kpc) | 0.42 ± 0.04 | 3–6/12–15 | 1, 2 |
| P_{rot} (s) | 1 | ≈100–137.7 | 3, 4 |
| P_{orb} (days) | 404 ± 37 | 304? | 5, 6, 7 |
| L_X (10^{35} erg s^{-1}) | (0.2–10) × 10^{-3} | ~10/100 | 1, 2 |
| M (10^{39} g s^{-1}) | (0.1–5) × 10^{-3} | 5/50 | 1, 2 |
| Companion spectra | M2 III | M3–6 III | 1, 2 |
| L_{opt}/L_X | 200 | 0.25 | 1 |

a Depending on the evolutionary status of the mass donor, first-ascent red giant/beginning initial ascent of the asymptotic giant branch.

b Depending on the distance.

c Assuming a neutron star with R_X = 10 km and M = 1.4 M_{⊙}.

References.—(1) Masetti et al. 2002; (2) Chakrabarty & Roche 1997; (3) Damle et al. 1988, Leahy 1989; (4) Jablonski et al. 2002; (5) This paper; (6) Cutler, Dennis, & Dolan 1986; (7) Pereira, Braga, & Jablonski 1999.

TABLE 2

| Parameter | Value |
|-----------|-------|
| Radial Velocity Amplitude K (km s⁻¹) | 0.75 ± 0.12 |
| Projected semimajor axis a sin i (10^{8} km) | 4.2 ± 0.7 |
| Orbital period P_{orb} (days) | 404 ± 3 |
| Epoch of periastron T_P (MJD) | 49.090 ± 80 |
| Eccentricity e | 0.26 ± 0.15 |
| Longitude of periastron ω (degrees) | 260 ± 40 |
| Mass function f_M (10^{-3} M_{⊙}) | 1.8 ± 0.9 |
| Model fit χ² | 95.96 (76 dof) |
To avoid a biased period measurement due to the presence of the 1997–1998 outburst, we initially calculated a Lomb-normalized periodogram over the interval from the end of the outburst (around 1998 March 28) to 2002 April 18. The result was a peak with Lomb-normalized power 14.05 (equivalent to 3.9 \sigma), at 383 ± 30 days. We estimated the period uncertainty by folding the ASM data on the detected period and calculating 1000 sets of statistically equivalent data sets with the same time sampling, average profile, and noise properties per bin as measured in the original data set. The variance in the detection periods of the simulated data sets is then a measure of the uncertainty in the actual detection.

To obtain a more precise estimate of the quiescent X-ray periodicity, we excluded those measurements with errors of more than 0.5 counts s\(^{-1}\). Around 13% of the 1 day averaged ASM measurements had errors above this threshold, possibly due to source variability on timescales of less than a day, but more likely arising from low-significance detections due to solar contamination or other suboptimal observing conditions. These low-significance measurements appeared to introduce a bias to the period measurements, and also tended to contribute disproportionately to the estimated uncertainty in the period. A Lomb-normalized periodogram calculated on this subset of data following the outburst resulted in a peak power of 24.04 (estimated significance \(4.2 \times 10^{-8}\), equivalent to 5.5 \sigma) at 392 ± 14 days. To confirm the above results, we used the phase-dispersion minimization technique of Stellingwerf (1978) and folded the ASM light curve on a grid of periods between 200 and 1000 days. We found that the X-ray flux was most likely modulated with a period of 404 ± 13 days. Combining the results from the latter two period-search techniques, we determine the period of the X-ray oscillation to be 400 ± 20 days with the epoch of maximum at MJD 51,140 ± 10 days. The mean profile of the post-outburst ASM measurements folded on the 400 day period is also shown in Figure 2 (inset, bottom panel). The profile was approximately sinusoidal, with an amplitude of 0.108 ± 0.012 ASM counts s\(^{-1}\), or 31% rms. With the periastron time from the orbital fit used as a reference phase for the fold, we find that, on average, the X-ray maximum occurs 0.1 in phase (~40 days) after periastron.

For comparison, we also calculated an oversampled Lomb-normalized periodogram of the entire ASM data set (including the 1997–1998 outburst), which revealed a significant periodicity at 383 days (identical to that measured following the outburst). The 1997–1998 outburst was unusual; no similar events were observed by the ASM in ~6.5 yr of monitoring. Thus, it may have occurred because of a mechanism distinct to some extent from the periodic behavior of the quiescent emission. Furthermore, there is evidence that the behavior of the persistent emission prior to the outburst was different than afterward. Although only a little more than one 400 day interval was covered before the outburst, the persistent flux was weakly anticorrelated, compared to the mean profile measured following the outburst. Applying the same error threshold (\(\sigma < 0.5\) counts s\(^{-1}\)) to the full data set resulted in an increase in both the peak power and period, to 51.1 (estimated significance 1.2 \times 10^{-20} or ~8 \sigma) at 385 days. The shorter measured periods using the full ASM data set may be attributed to aliasing resulting from a delay between the nearest phase of X-ray maximum for the persistent emission and the peak of the 1997–1998 outburst. If 4U 1700+24 is similar to wind-accreting neutron stars in high-mass binaries, we may expect large outbursts to correspond only approximately with periastron phase (e.g., Stella, White, & Rosner 1986). In fact, the peak of the 1997 outburst falls around MJD 50,780, approximately 40 days after the nearest epoch of maximum flux, according to the 400 day periodicity detected in the post-1997 ASM measurements, and approximately 70 days after periastron passage, according to the orbital solution discussed in § 2.1 (Fig. 2).2

2 While this article was in press, a second large outburst was detected by the ASM, beginning around 2002 June 15 (MJD 52,440) and peaking ~40 days later at ~2 rms counts s\(^{-1}\). The time of the outburst corresponds to phase 0.4 in the proposed 404 day orbit, relative to the epoch of periastron. The timing of this outburst is consistent with a trigger that is largely independent of the orbital cycle, as we have suggested. Furthermore, the delay between the previous outburst and this one (~1730 days) does not correspond closely to any of the possible longer periods in the radial velocity history.
3. DISCUSSION

We have found evidence for significant variation in the radial velocity measurements from HD 154791, which we fitted with an eccentric orbit with period 404 ± 3 days. We have also found evidence for a significant variation in the X-ray flux from 4U 1700+24. Our best estimate of the X-ray period is 400 ± 20 days. Hence, the periods measured from the radial velocity and ASM X-ray flux variations are consistent. It is compelling to suggest that these two periodicities arise from a common source. Naturally, it is also possible that the agreement between the two periodicities is merely coincidence and that HD 154791 and 4U 1700+24 are unrelated. Estimating the probability of such a coincidence depends on the origin of the periodicities in each star, as well as on the intrinsic distributions of these phenomena. In practice, it is not possible to make such an estimate with much confidence. However, it is clear that the likelihood of an unrelated red giant and X-ray source being at nearly the same measured position at high Galactic latitude, and also both having a periodicity that would be unusual for either type of source separately, is extremely small. The detection of a 400 day modulation from both stars therefore supports the original suggestion that HD 154791 is the optical counterpart of 4U 1700+24.

3.1. Cause of the Correlation

There are several plausible mechanisms for a correspondence between the radial velocity and the persistent X-ray flux. We consider two possibilities for the origin of the 404 day period in the HD 154791 radial velocity curve: orbital motion, and pulsational variation of the M giant. Precise optical photometric monitoring could distinguish between these two possibilities, since the maximum brightness occurring at any phase besides that of maximum recession would favor orbital motion.

If the radial velocity variations are due to orbital motion, the 400 day X-ray flux modulation may arise from changes in the accretion rate onto the neutron star as it follows an elliptical orbit around the companion and moves through different nebular densities. Such modulation is common in high-mass wind-accreting X-ray binaries (e.g., Stella et al. 1986). Moreover, the times of periastron passage in our best-fit orbital model are very well correlated with the times of increased X-ray flux. For a 1.4 $M_\odot$ neutron star and red giant mass $\lesssim 2.5 M_\odot$, the 404 day period and low amplitude of the radial velocity variations imply that the inclination is $i \lesssim 2^\circ$. Although the a priori probability of observing such a system is $\lesssim 7 \times 10^{-4}$, such a low inclination could explain the lack of X-ray pulsations or quasi-periodic oscillations (QPOs) from the source (see § 3.2). If the 400 day variation is in fact due to orbital motion, the longer term variations in the radial velocities may be related to irregular variations in the red giant (Percy, Wilson, & Henry 2001).

Alternatively, red giant pulsations with $P = 400$ days could modulate the stellar wind, and hence the accretion rate onto the neutron star. The IRAS fluxes of HD 154791 ($f_{12} = 2.77, f_{25} = 0.736, f_{60} < 0.4,$ and $f_{100} < 1.0$, where $f_{12}$ is the flux density $f_{\nu}$ at 12 $\mu$m in Jy) indicate that it is not a Mira variable (Kenyon, Fernández-Castro, & Stencel 1988). However, non-Mira M giants also pulsate. Although typical fundamental pulsation periods lie in the range 20–200 days, variations with a period up to an order of magnitude larger are also seen (Percy et al. 2001). If the 400 day variations are due to pulsation, one of the slower trends observed in the radial velocity variations could be related to orbital motion. In this case, the possible range of orbital inclinations becomes much more likely. An orbital period of 2370 days gives an inclination of $4^\circ–6^\circ$, while for a period of 3700 days the likely range is $5^\circ–7^\circ$. Also, the low accretion rate onto the neutron star would be more understandable if the orbital period is longer than 400 days. However, a longer orbital period alone cannot account for the discrepancy in luminosity between 4U 1700+24 and GX 1+4.

Finally, the shape of the 400 day radial velocity variation argues against a red giant pulsation. Pulsation light curves are typically nonsinusoidal (in the sense that more time is spent with brightness below the median value than above it), and similar behavior would be expected for the radial velocity variations.

3.2. Comparison with GX 1+4

If the 400 day periodicity in the HD 154791 radial velocities is due to orbital motion, then the orbital periods of HD 154791/4U 1700+24 and GX 1+4 may be quite similar (see Table 1). This similarity makes the contrast between their other observed properties puzzling. The X-ray luminosity of GX 1+4 appears to be 2–3 orders of magnitude greater than that of 4U 1700+24. In GX 1+4, there is clear evidence for an accretion disk (Chakrabarty & Roche 1997), whereas in 4U 1700+24 there is no measurable UV continuum (Garcia et al. 1983). GX 1+4 has a rich optical emission-line spectrum, and HD 154791 shows very little in the way of emission lines. Finally, GX 1+4 is a 2 minute X-ray pulsar, whereas no coherent or quasi-periodic oscillations have been convincingly detected from 4U 1700+24.

One explanation for the differing X-ray luminosities is a larger mass-loss rate from the late-type giant in GX 1+4. Chakrabarty & Roche (1997) find that the M6 giant in GX 1+4 is probably near the tip of the first-ascent red giant branch. HD 154791 has a spectral type M2, and so would be expected to be losing mass at a lower rate. The relative lack of optical/UV emission lines suggests that either the ionized nebula in HD 154791/4U 1700+24 is substantially smaller and/or less dense than in GX 1+4, or there are not enough UV photons to power nebular line emission in 4U 1700+24/HD 154791. Either of these conditions can plausibly arise if the mass loss from the companion is much smaller. Moreover, if the binary separation is smaller in GX 1+4 than in 4U 1700+24/HD 154791, a larger fraction of the red giant wind could be accreted. The higher X-ray luminosity in GX 1+4 could also illuminate the red giant and increase the mass loss from the red giant further, in a sense producing a feedback effect. The observed variability of the optical spectrum of V2116 Oph is evidence for the significant role played by X-ray heating. On the other hand, in 4U 1700+24/HD 154791 the X-ray luminosity is perhaps low enough that very little illumination of the red giant or enhanced mass loss is expected. Even during the 1997 X-ray outburst, Tomassella et al. (1997) note that the optical spectrum was unchanged.

Finally, the lack of pulsations or QPOs in 4U 1700+24 remains puzzling. Garcia et al. (1983) and Morgan & Garcia (2001) proposed a 900 s QPO, but this QPO was not confirmed in high-quality RXTE data (Maseetti et al. 2002). High-precision optical photometry has also revealed a lack of pulsations (down to a limit of 0.5% fractional variation).
or rapid variations of any kind (Sokoloski, Bildsten, & Ho 2001). This behavior is again in contrast to GX 1+4, where Jablonski et al. (1997) find an oscillation in the optical emission with an amplitude of from 0.4% to 4.5%, plus stochastic variations. Given that the optical light from GX 1+4 has a significant contribution from an accretion disk, the lack of rapid optical variations from HD 154791 provides evidence that any contribution from a disk in 4U 1700+24 is negligible in the optical regime as well as in the UV regime. The lack of X-ray pulsations is expected if the 400 day radial velocity variations are due to orbital motion. As discussed in § 3.1, the low amplitude of the radial velocity variations requires \( i \approx 0 \), in which case any magnetic hot spots on the neutron star will be continuously in view (assuming the NS spin axis is aligned with angular momentum axis of the binary). For higher and a priori more probable values of \( i \), the nondetection of pulsations may be explained if the accretion flow is not appreciably disrupted by the magnetic field of the neutron star above the surface (e.g., Ghosh & Lamb 1979a, 1979b) and spreads the accreting material more or less evenly over the neutron star surface. Given the extremely low luminosity (and hence accretion rate) of this source, standard calculations of ram and \( B \)-field pressure balance would require that the surface magnetic field strength of the neutron star be \( \leq 10^6 \) G, far below the canonical value for low-mass X-ray binaries or even old neutron stars in recycled millisecond pulsars. A more likely scenario consistent with larger \( i \) is that the neutron star spin is very slow.

In conclusion, all scenarios for 4U 1700+24/HD 154791 have some difficulties. We have presented some options, but more observational constraints are needed to select between them. Additional observations to determine whether the 400 day radial velocity variations are due to orbital motion or red giant pulsation would also enable one to better interpret the differences between 4U 1700+24/HD 154791 and GX 1+4.

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