Students’ Errors in Geometry Viewed from Spatial Intelligence

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Abstract. Geometry is one of the difficult materials because students must have ability to visualize, describe images, draw shapes, and know the kind of shapes. This study aim is to describe student error based on Newmans’ Error Analysis in solving geometry problems viewed from spatial intelligence. This research uses descriptive qualitative method by using purposive sampling technique. The datas in this research are the result of geometri material test and interview by the 8th graders of Junior High School in Indonesia. The results of this study show that in each category of spatial intelligence has a different type of error in solving the problem on the material geometry. Errors are mostly made by students with low spatial intelligence because they have deficiencies in visual abilities. Analysis of student error viewed from spatial intelligence is expected to help students do reflection in solving the problem of geometry.

1. Introduction
Mathematics is one of the important lessons in education. The study of mathematics is considered to be a very important basic education because mathematical calculation skills are used in every way of life [1]. Mathematics can train students to think systematically, logically, critically, creatively, and consistently. The purpose of mathematics education is to produce students who have skill in solving problems and fostering a high interest and motivation in mathematics [2]. Mathematical ability can be used to solve everyday problems, so problem solving has an important role in mathematical learning. Mathematics are studied at all levels of education. Mathematics studied at Junior High level includes arithmetic, algebra, geometry, trigonometry, opportunities and statistics. The mathematical aspect in geometry relates to the study of different forms [3]. Geometry is not only develops students' cognitive abilities, but also forms concrete thinking to abstract. Geometry helps students to analyze and interpret the world, and also equip them with tools that can be applied in other mathematical fields [4]. Geometry is one of the hard materials for students. Students must have the ability to visualize, describe images, draw shapes, and get to know shapes. Learning geometry is not only studies the definition but also analyzes the geometric properties of two dimensions (2D) and three dimensions (3D), and also develops geometric relationships to determinate location, apply transformation, use symmetry, visualization, spatial reasoning, and geometric modeling to solve problems [2]. In fact, students’ achievement in geometrical materials is low, especially student achievement in Indonesia. Based on the report of Trends in International Mathematics and Science Study in 2011 indicated that Indonesia obtained an average mathematical score of 377 on geometry material [5]. Indonesia is
ranked the bottom 3 compared to other countries. Other reports based on the results of the national junior examination in Indonesia showed that the mathematical value of the geometry material had decreased in the last 3 years. Geometry value in Indonesia reaches 60.58% in 2014, 52.04% in 2015, and 47.19% in 2016 [6-8]. These results indicated that students make many errors in solving geometry problems. The problems related with geometry in schools were caused to the high degree of geometry abstraction and the abstract ability of students' abstract object visualization [9]. The ability of object visualization is supported by the students' spatial intelligence. Spatial intelligence is the ability to perceive the spatial world accurately and transform spatial perception in various forms [10]. Spatial intelligence supports students to achieve learning outcomes and understanding of matter optimally. Mathematics and spatial thinking have a positive correlation in school-aged children, both in low or high spatial abilities [11]. Spatial intelligence is measured by using indicators including the ability to determine the vertical and horizontal direction of an object (spatial perception), the ability to see motion or displacement on the part of a configuration (visualization), the ability to determine the results of two- and three-dimensional rotation (mental rotation) (spatial relation), and the ability to guess the image of an object from a certain angle (spatial orientation) [12].

Identification of student error needs to be done to know the students' understanding in solving the problem. Students will more easily understand the concept of geometry after knowing the errors made when solving the problem. The most students' errors in solving the story problem were errors in understanding the meaning of a problem (comprehension), transformation, and carelessness [13]. The most common error for structured questions is misunderstanding, whereas the error in multiple choice questions was a transformation error [14]. However, student errors in solving geometric problems can occur due to the influence of students' spatial abilities. In this research, spatial intelligence was used to review student errors.

Student errors in solving geometric problems are described by using Newman's Error Analysis. Newman's procedure is a step in understanding and analyzing the way students solve problems. Students experienced obstacles when answering problems, namely reading problems, comprehension, transformation, process skills, and encoding [15]. Identification of student errors is required as a reference in selecting appropriate learning models and media based on students' spatial intelligence on geometric material. Students are not aware of the mistakes made. Students do not know the location of the mistake, so students cannot do reflection to fix errors that have been done. Therefore, it is necessary to conduct research to describe students' mistakes in solving geometric problems in terms of students' spatial intelligence.

2. Experimental Method

Research design in this research uses qualitative approach. The purpose of this research is to analyze student error in solving geometry problem by using Newman's Error Analysis viewed from spatial intelligence. The procedures in this study include analysis of the results of geometry solid test and interview. The sampling technique used is purposive sampling. The test instrument in this study is a test for identifying student error based on Newman's Error Analysis which is classified into five types of errors include reading, comprehension, transformation, process skills and encoding. Spatial intelligence of students is derived from the results of spatial intelligence tests. Spatial intelligence indicators used include spatial perception, visualization, mental rotation, spatial relations, and spatial orientation.

This research was conducted in 8th grade of SMPN 2 Ngawi Ngawi Regency, Indonesia in April 2017. This research was conducted in Ngawi regency because the mathematics value on the geometry material based on the national exam result during the last 3 years decreased. Geometry value in Ngawi Regency reaches 49.90% in 2014, 39.97% in 2015, and 35.47% in 2016 [6-8]. Subjects in this study were 35 students covering 11 students with high spatial intelligence, 17 students with medium spatial intelligence, and 7 students with low spatial intelligence.
3. Result and Discussion

Errors are always encountered in every lesson, including mathematics learning. The mean of mathematical error is deviations from the real solution. This study discusses students' error on the solid geometry (cubes, cuboids, prisms, and pyramids). Errors are analyzed based on student error solving problems and answering interview questions. Test results and interviews on solid geometry materials (cubes, cuboids, prisms, and pyramids) show that there are several types of mistakes made by students. The first error to be identified is the reading error. Reading errors include students' errors in reading and finding keywords or symbols in question [13]. In the aspect of reading error, no errors are performed by students with high, medium, or low spatial intelligence. Each category of intelligence is able to read and understand every known matter.

Another error to be identified is the comprehension error. Comprehension errors include errors that occur when students are able to read all the words on the question but they do not understand the meaning of the term used, so the students can not continue to solve the problem correctly [13]. In this study every category of spatial intelligence performs comprehension error. The most error made by students with low spatial intelligence category reached 9.0%. Students with medium spatial intelligence are doing 4.5% comprehension error while students with high spatial intelligence perform 3.0% comprehension error. Comprehension errors occur because students do not have the ability to interpret questions and strategies to manipulate questions. Students can not explain the information that is known and asked questions fully. Most misunderstandings are made in selecting information and students have difficulty in distinguishing between relevant and irrelevant information in task [16].

One of examples of comprehension error when selecting the information in this study is shown in Figure 1. Students are required to determine the surface area of the prism if known to the length of the base and the height of the prism. Students make a mistake because it considers the height of the triangle is 13. In fact, 13 is the side of the triangle. Students should find the height of the triangle using the phytagoras theorem. It shows students' answers that do comprehension errors.

![Figure 1. Example of Students' Comprehension Error](image)

Transformation error is the next type of error identified. Transformation error occurs when the student have understood the intent of the problem but the student can not identify the sequence of operations needed to solve the problem [13]. Students use the wrong concept in solving the problem, so they don’t perform the next operation properly. Most of the transformation errors are made due to incorrect math concepts [16]. In this study the greatest transformation error performed by students with low spatial intelligence category reached 14.9%. Students with spatial intelligence are performing a transformation error of 6.0% while students with high spatial intelligence perform a transformation error of 3.0%. One example of transformation error while using mathematical concepts in this study is shown in Figure 2. Students are required to determine the height of the cuboid if the length, width, and surface area of cuboid are known, Figure 2 shows that students do not use the correct formula to determine the height of the cuboid. Students must write the formula of the surface area of the cuboid correctly to get the height of the cuboid. This indicates that the student performed a transformation error.
The next type of error to be identified is the process skills error. Process skills error occurs when the student is able to identify the sequence of operations needed to solve the problem but does not use the procedure correctly [13]. Most errors occur because students make mistakes in the calculation procedure. Student’s skill in less mathematical calculation causes students to make mistakes in process skills error type. In this study, the process skills error is at most 14.9% performed by students with low spatial intelligence. Students with spatial intelligence are making a 9.0% error while students with high spatial intelligence make a mistake of 6.0%. One example of process skill error when not using mathematical procedures correctly in this study is shown in Figure 3. Students are required to determine the height of the beam if the length, width, and surface area of the beam are known. Figure 3 shows that students do not use the correct procedure to solve the problem. The student writes the surface area of the cuboid but uses the wrong calculation procedure. It only adds \(12t\) with \(9t\). Students should add \(12t\) with \(9t\) then multiplied by 2. This indicates that the student made a mistake on the type of process skills error.

Encoding error is the last type of error to be identified. Encoding errors include errors expressing problem solving in written form [13]. Students do not conclude the final answer correctly. In this study encoding error is mostly done by students with low spatial intelligence of 14.9%. Students with spatial intelligence are encoding error of 9.0% while students with high spatial intelligence perform encoding error of 6.0%. One example of student error for not concluding the answer correctly in this study is shown in Figure 4. Students are required to determine the surface area of the prism if known to the length of the base and the height of the prism. Figure 4 shows that the student does not write the final answer correctly because it is wrong in calculating the final result.
Most errors made by students from high spatial intelligence category were in process skills errors, and encoding errors. This indicates that students have problems in calculating and writing the correct answer. Most errors made by students from medium spatial intelligence category were in process skills errors, and encoding errors. This indicates that students have problems in calculating and writing the correct answer. Most errors made by students from low spatial intelligence category were in transformation errors, process skills errors, and encoding errors. This indicates that students have problems in using correct concept, calculating and writing the correct answer. Students with high spatial intelligence solving the geometry questions better than students with medium and low spatial intelligence. Students with high spatial intelligence had ability in solving mathematic problems [17]. They could imagine transformations of the orientation of objects. Spatial intelligence was an ability to see from different angles [18]. So, most errors in this research made by students with low spatial intelligence because they couldn’t imagine and solve the problem in geometry question well.

4. Conclusion
Students need to know the errors made when solving geometry problems so the students can do reflection to improve their errors. The results of this study indicate that there is a correlation between the spatial intelligence that students have with students’ errors in solving geometry problems. The student’s error in understanding and solving geometry problems in each category of spatial intelligence is different. Teachers should pay special attention to students with low spatial intelligence categories. Students with low spatial intelligence make more mistakes than students with moderate or high spatial intelligence. Low spatial ability causes students to make more errors in solving geometry problems, especially the type of transformation error, process skill error, and encoding error.

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