\(\partial\)-Explainer: Abductive Natural Language Inference via Differentiable Convex Optimization

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Abstract

Constrained optimization solvers with Integer Linear programming (ILP) have been the cornerstone for explainable natural language inference during its inception. ILP based approaches provide a way to encode explicit and controllable assumptions casting natural language inference as an abductive reasoning problem, where the solver constructs a plausible explanation for a given hypothesis. While constrained based solvers provide explanations, they are often limited by the use of explicit constraints and cannot be integrated as part of broader deep neural architectures. In contrast, state-of-the-art transformer-based models can learn from data and implicitly encode complex constraints. However, these models are intrinsically black boxes. This paper presents a novel framework named \(\partial\)-Explainer (Diff-Explainer) that combines the best of both worlds by casting the constrained optimization as part of a deep neural network via differentiable convex optimization and fine-tuning pre-trained transformers for downstream explainable NLP tasks. To demonstrate the efficacy of the framework, we transform the constraints presented by TupleILP and integrate them with sentence embedding transformers for the task of explainable science QA. Our experiments show up to \(\approx 10\%\) improvement over non-differentiable solver while still providing explanations for supporting its inference.

1 Introduction

Constrained optimization solvers based on Integer Linear Programming (ILP) have been proposed as a candidate method to address complex and knowledge-intensive natural language inference tasks (Thayaparan et al., 2020a; Khashabi et al., 2018; Khot et al., 2017; Khashabi et al., 2016). This family of approaches provides a viable mechanism to encode explicit and controllable assumptions, casting natural language inference as an abductive reasoning problem over semi-structured knowledge bases (Xie et al., 2020; Clark et al., 2018; Jansen et al., 2018). Abductive reasoning is a process known as inference to the best explanation (Lipton, 2003), which starts from complete/incomplete observations to find the hypothesis that best explain a given observation. As such, inference based on constrained optimization constitutes a cornerstone for explainability and transparency, as it can be used to construct structured explanations for the underlying inference process (Thayaparan et al., 2020b).

While delivering explainability, existing optimization solvers cannot be integrated as part of a deep neural network, and are often limited by the exclusive adoption of the constraints for inference. This shortcoming prevents these methods from being optimized end-to-end on annotated corpora and achieving performance and robustness comparable with deep learning counterparts.

State-of-the-art models for natural language inference, in particular, are almost exclusively represented by Transformers-based language models (Devlin et al., 2019), thanks to their ability to transfer linguistic and semantic information to downstream tasks. However, Transformers are typically regarded as black-box models, posing serious concerns about the interpretability and transparency of their predictions (Rudin, 2019; Guidotti et al., 2018).

In an attempt to leverage the best of both worlds and alleviate the aforementioned limitations, this paper proposes \(\partial\)-Explainer, the first hybrid framework for abductive natural language inference that combines constraint satisfaction layers with pre-trained neural representations, enabling end-to-end differentiability for optimization-based solvers.
Recent works have shown that certain convex optimization problems can be represented as individual layers in larger end-to-end differentiable networks (Agrawal et al., 2019b,a; Amos and Kolter, 2017), demonstrating that these layers can be adapted to encode constraints and dependencies between hidden states that are hard to capture via standard neural networks. In this paper, we build upon this work, demonstrating that convex optimization layers can be successfully integrated with Transformers to achieve explainability and robustness in complex natural language inference problems.

To demonstrate the impact of end-to-end differentiability, we approximate the constraints of an existing ILP system (i.e., TupleILP (Khot et al., 2017)). Specifically, we transform the constraints into differentiable convex optimization layers and subsequently integrate them with pre-trained sentence embeddings from Transformers (Reimers et al., 2019). Our experiments on complex, multi-hop question answering demonstrate that the performance of non-differentiable solvers can be improved by up to \( \approx 10\% \) when fine-tuned end-to-end, while still providing structured explanations in support of their inference.

In summary, the contributions of the paper are as follows:

1. A novel differentiable framework that incorporates constraints via convex optimization layers into broader transformers-based architectures. To the best of our knowledge, we are the first to apply this methodology for abductive natural language inference tasks such as multi-hop commonsense and science question answering.
2. Empirically demonstrates that the proposed framework allows end-to-end differentiability on downstream tasks for both explanation and answer selection, leading to a substantial improvement when compared to non-differentiable constraint-based solvers.
3. Demonstrate that \( \partial\)-Explainer is more robust to distractors when compared to Transformer-based models augmented with external evidence extractors and adopted as readers for downstream tasks.

2 Related Work

Constraint-Based NLI Solvers ILP has been employed to model structural and semantic constraints to perform abductive natural language reasoning. TableILP (Khashabi et al., 2016) is one of the earliest approaches to formulate the construction of explanations as an optimal sub-graph selection problem over a set of structured tables and evaluated on multiple-choice elementary science question answering. In contrast to TableILP, TupleILP (Khot et al., 2017) was able to perform inference over free-form text by building semi-structured representations using Open Information Extraction. SemanticILP (Khashabi et al., 2018) also comes from the same family of solvers that leveraged different semantic abstractions, including semantic role labelling, named entity recognition and lexical chunkers for inference.

Differentiable Convex Optimization Layers

Our work is in line with previous works that have attempted to incorporate optimization as a neural network layer. These works have introduced differentiable modules for quadratic problems (Donti et al., 2017; Amos and Kolter, 2017), satisfiability solvers (Wang et al., 2019) and submodular optimizations (Djolonga and Krause, 2017; Tschatschek et al., 2018). Recent works also offer differentiation through convex cone programs (Bussi et al., 2019; Agrawal et al., 2019c). Since every convex problem can be cast into a cone problem, these models can be used to define convex optimization problems. In this work, we use the differentiable convex optimization layers proposed by Agrawal et al. (2019b). These layers provide a way to abstract away from the conic form, letting users define convex optimization in natural syntax, providing a natural degree of freedom.

Semi-Definite Programming

Integer linear programming has been a key tool for subgraph-selection formulations of abductive natural language reasoning problems (Khashabi et al., 2018; Khot et al., 2017; Khashabi et al., 2016). However, these approaches are non-convex by nature and cannot be integrated into an end-to-end differentiable system that can learn from examples. *Semi-definite programming* (SDP) is often used as a convex approximation of traditional NP-hard combinatorial graph optimization problems (Lovász and Schrijver, 1991), such as the max-cut problem, the dense k-subgraph problem and the quadratic \( \{0, 1\} \) programming problem that implicitly underlies the problem formulation in Section 3. SDP relaxations of such problems
provide tighter bounds than linear relaxations, while still being solvable with polynomial-time algorithms (Helmberg, 2000). Furthermore, they are within the scope of the differentiable convex optimization layers in Agrawal et al. (2019b), crucially allowing optimization within SDP constraints to be integrated into larger end-to-end differentiable systems.

3 Abductive Natural Language Reasoning via Differentiable Convex Optimization

Abductive natural language inference is the task of combining two or more pieces of evidence from external knowledge sources to build a plausible explanation for a given reasoning task. Performing abductive inference with the constraint of providing a supporting natural language explanation has previously been formalized as a graph construction and subgraph selection problem on tasks such as multiple-choice commonsense and science question answering (Khashabi et al., 2018; Khot et al., 2017; Khashabi et al., 2016).

Given a graph with nodes defined by question texts, candidate answers and candidate supporting facts, an ideal system would score edge weights in such a way that combinatorial optimization strategies would select a reasonable path or subgraph that serves as an explanation for the underlying inference process. There are many design elements involved in the modelling of this the problem, and in previous works, it has been framed explicitly in terms of an Integer Linear Programming formulation, where linear constraints guide a sensible selection of the supporting subgraph. We follow the underlying problem formalization in (Khot et al., 2017), which can be described as follows:

As illustrated in Figure 1, Given a question \( Q \) and a candidate answer \( c_1 \), firstly solvers convert the question-answer pairs into a hypothesis \( h_1 \) and retrieve a set of relevant facts \( F = \{ f_1, f_2, f_3, \ldots, f_k \} \). Followed by construction an explanation graph \( G = (V, E) \) where the nodes \( V \) represent respective hypothesis and candidate explanations.

In this setting, the main challenge is in designing graph weights that capture some notion of semantic overlap and/or relevance, so that a combinatorial optimization strategy would produce a plausible reasoning path in justification of an answer choice. Part of the modelling problem in-
ling approaches for subgraph selection cannot be trained end-to-end and take advantage of the integration with neural representations (such as pre-trained language models (Devlin et al., 2019)) to improve performance and generalization.

2. Integer Programming is known to be NP-complete, with the special case of 0-1 integer linear programming being one of Karp’s 21 NP-complete problems (Karp, 1972). Therefore, as the size of the optimization problem increases, finding exact solutions becomes computationally intractable. This intractability is a strong limitation for abductive natural language inference in general since these systems typically operate on large knowledge bases and corpus.

3.2 Subgraph Selection via Semi-Definite Programming

To overcome the limitations discussed above, we propose an adaptation of the subgraph selection problem so that the edge-weighted representation of the graph \( G \) may also be optimized during the training of the end-to-end reasoning task. To this end, we turn to a semidefinite-programming relaxation of the NP-hard subgraph selection problem

\[
\max_{y \in \{0,1\}^n} y^T Wy. \tag{1}
\]

where \( W \) is the weighted adjacency matrix of \( G \) and \( y \in \{0,1\}^n \) is the node variable where 1 indicates the inclusion of a given node in the selected subgraph while 0 indicates that it is excluded.

We follow Helmberg (2000) in their reformulation and relaxation of this problem. Instead of vectors \( y \in \{0,1\}^n \), we optimize over the set of positive semidefinite matrices satisfying the SDP constraint in the following relaxed optimization problem:

\[
\maximize \langle W, E \rangle \tag{2}
\]

such that \( E - \text{diag}(E)\text{diag}(E)^T \succeq 0 \) \tag{3}

(\text{where } \langle W, E \rangle = \text{tr}(WE), E = yy^T, \text{diag}(E) = y \text{ and a matrix } M \text{ satisfies } M \succeq 0 \text{ if it is positive semi-definite}). This is a convex optimization problem within the scope of differentiable solution methods (Agrawal et al., 2019b,a; Amos and Kolter, 2017). Moreover, the semi-definite program relaxation can be solved adopting the interior-point method (De Klerk, 2006; Vandenberghe and Boyd, 1996) which has been proved to run in polynomial time (Karmarkar, 1984).

4 End-to-end Differentiable Architecture

In this paper, we hypothesize that convex optimization layers can be effectively integrated within broader transformers-based neural architecture to address abductive natural language inference in an end-to-end differentiable manner.

To validate this hypothesis, we propose \( \partial \)-Explainer, an abductive natural language inference system that adopts differentiable convex optimization for modelling the explanation subgraph selection problem. In order to empirically measure and demonstrate the impact of differentiability, \( \partial \)-Explainer reproduces the formalization introduced by previous ILP solvers. Specifically, we approximate and make differentiable the constraints presented in TupleILP (Khot et al., 2017), an integer linear programming solver that imposes structural and semantic constraints over a semi-structured knowledge base of OpenIE tuples.

The motivation behind the choice of TupleILP is two-fold:

1. TupleILP represents the current state-of-the-art when considering the class of structured and integer linear programming solvers, demonstrating good performance in complex natural language inference tasks that require the construction of multi-hop reasoning chains.

2. Compared to other ILP-based approaches, the constraints of TupleILP are relatively easier to reproduce, providing more control to validate the contribution deriving from the differentiability of the proposed approach.

To demonstrate the impact of integrating a convex optimization layer into a broader end-to-end neural architecture, \( \partial \)-Explainer employs a Transformer-based sentence embedding model. Specifically, we incorporate a differentiable convex optimization layer with Sentence-BERT (SBERT) (Reimers et al., 2019), a model that has demonstrated state-of-the-art performance on semantic sentence similarity benchmarks. Specifically, SBERT is adopted to estimate the similarity between two sentences during the construction of the explanation graph. We use SBERT
as a bi-encoder architecture to minimize the computational overload and operate on large sentence graphs.

Figure 2 describes the end-to-end architectural diagram of $\partial$-Explainer. For a given hypothesis, derived from the combination of a question and a candidate answer, $\partial$-Explainer adopts a sentence embedding model (SBERT) to encode the hypothesis and a set of candidate explanatory facts retrieved from an external knowledge base. SBERT is used to calculate the semantic relevance of a given fact via cosine similarity between the respective sentence embeddings.

The semantic relevance score is complemented with a lexical relevance score computed considering the shared terms between hypotheses and facts. Subsequently, semantic and lexical scores are weighted by a set of learnable $\theta$ parameters to construct an explanation graph $G = (V, E)$ supporting the candidate answer. The constructed graph is passed through a differentiable convex optimization (DCX) layer imposing structural and semantic constraints to select the optimal supporting graph $G^* = (V^*, E^*)$ including the selected hypothesis and explanatory facts.

Since the architecture is end-to-end differentiable, it is possible to compute a loss relative to the correct answer and target explanation, which can be subsequently back-propagated to jointly fine-tune the sentence representation model and the $\theta$ parameters.

### 4.1 Graph Construction

Given a question ($Q$) and a set of candidate answers $C = \{c_1, c_2, c_3, \ldots, c_n\}$ we convert them into hypotheses $H = \{h_1, h_2, h_3, \ldots, h_n\}$ using the approach proposed by Demszy et al. (2018). The aim of our approach is to select the correct candidate answer and the respective explanation subgraph.

For each hypothesis $h_i$, we adopt a fact retrieval model to return a list of candidate explanatory facts with their respective relevance score $F^{hi} = \{f_1^{hi} : s_1^{hi}, f_2^{hi} : s_2^{hi}, f_3^{hi} : s_3^{hi}, \ldots, f_k^{hi} : s_k^{hi}\}$ from a background knowledge base. We extract the facts with the highest relevance score $F = \{f_1, f_2, f_3, \ldots, f_k\}$ to construct the base knowledge graph $G$. We also extract SPO tuples $f_{il} = \{f_{il}^S, f_{il}^P, f_{il}^O\}$ for each fact $f_i$ using an Open Information Extraction model (Stanovsky et al., 2018). From the hypothesis $h_i$, we extract the set of unique terms $h_i^{ht} = \{t_1^{hi}, t_2^{hi}, t_3^{hi}, \ldots, t_i^{hi}\}$ excluding stopwords.

Given $n$ candidate hypotheses and $k$ candidate explanatory facts, $A$ represents an adjacency matrix of dimension $((n + k) \times (n + k))$ where the first $n$ columns and rows denote the candidate hypotheses, while the remaining rows and columns represent the candidate explanatory facts. The adjacency matrix denotes the lexical connections between hypotheses and facts in the graph. Specifically, each entry in the matrix $A[i, j]$ contains the following values:

$$A[i, j] = \begin{cases} 1, & i \leq n, j > n, |t(h_i) \cap t(f_{j-n})| > 0 \\ 1, & j \leq n, i > n, |t(h_i) \cap t(f_{i-n})| > 0 \\ 0, & \text{otherwise} \end{cases}$$ (4)

Here, the function $t$ extracts the set lemmatized unique terms from the given text.

We assign edge weights using following scores:

**Semantic Relevance ($S_{hi}$):** Given a hypothesis $h_i$, fact $f_j$ the sentence vectors is represented by $\vec{h}_i = \text{SBERT}(h_i)$ and $\vec{f}_j = \text{SBERT}(f_j)$. We
problems. DCP consists of functions (or atoms) and per-argument monotonicities. In addition to these, DPP also consists of Parameters which are symbolic constants with an unknown numerical value assigned during the solver run.

Given a hypothesis, the principal function is to select the correct hypothesis. We implement this via the following constraint:

$$\sum_{i=1, j=1}^{i=n, j=n} E[i, j] = 1 \quad \forall i = j$$

As defined in Section 3.2, each entry in the diagonal represents a value between 0 and 1, indicating whether the corresponding node in the graph should be included in the explanation subgraph. Similar to the adjacency matrix $A$, the first $n$ rows and columns present the hypothesis in the edge variable matrix. Given these premises, the above constraint implies that the solver must select only one hypothesis from the list of $n$ hypotheses.

In addition to the aforementioned constraints and semidefinite constraints specified in Equation 3, we adopt part of the constraints from TupleILP (Khot et al., 2017). The constraints in DPP format are described in Table 1.

### 4.3 Answer and Explanation Selection

The output from the DCX layer returns the solved edge adjacency matrix $E$ with values between 0 and 1. We interpret the value to be the probability of the specific node to be part of the explanation.

As discussed in Section 4.1, we encode the constraints in such a way that the first $n$ entries of the diagonal vector represents the probability of a given hypothesis to be the correct one (the sum of these values add up to 1). We consider the hypothesis with the highest probability to be the correct answer. In order to optimize our approach end-to-end, we compute the loss function considering the first $n$ output in the diagonal vector. We use the following cross-entropy loss function for answer selection.

The rest of the entries of the diagonal matrix corresponds to the candidate explanatory facts. To backpropagate loss on the selection of explanation, we adopt binary cross-entropy loss (BCELoss) using the annotated gold explanations (if available).
Support graph must have $\leq w_1$ active tuples

$$\sum_{i=1,j=1}^{n+k} E[i,j] \leq w_1 + 1 \quad \forall i = j$$

Active hypothesis term must have $\leq w_2$ edges

$$H_k[1, i] \cap E \leq w_2 \quad i = \{1, 2, 3, \ldots, k\}$$

$H_0$ is populated by hypothesis term matrix $H$ with dimension $((n + k) \times (n + k) \times l)$ and the values are given by:

$$H[1, i, k] = \begin{cases} 1 \quad &k \leq l, i \leq n, j > n, \\
1 \quad &k \leq l, i \leq n, j > n, \ 
\text{t}_k \in \text{t}(h_i), \text{t}_k \in \text{t}(f_j) \\
0 \quad &\text{otherwise} \end{cases}$$

Active tuple must have active subject

$$E \odot T^S > = E \odot A_0$$

As populated by adjacency matrix $A$, $T^S$ by subject tuple matrix $T^S$ with dimension $((n + k) \times (n + k))$ and the values are given by:

$$T^S[i, j] = \begin{cases} 1 \quad &i \leq n, j > n, \ 
|t(h_i) \cap t(f_j)| > 0 \\
1 \quad &i \leq n, j > n, \ 
|t(h_i) \cap t(f_j)| > 0 \\
0 \quad &\text{otherwise} \end{cases}$$

Active tuple must have $\geq w_3$ active fields

$$E \odot T^S + E \odot T^P + E \odot T^{Q'} \geq w_3(E \odot A_0)$$

$A_0$ populated by adjacency matrix $A$ and $T^S, T^P, T^{Q'}$ populated by subject, predicate and object matrices $T^S, T^P, T^{Q'}$ respectively. Predicate and object tuples are converted into $T^S, T^P$ matrices similar to $T^Q$.

Active tuple must have an edge to some hypothesis term

Implemented during graph construction by only considering tuples that have lexical overlap with a hypothesis

- Parameters

| Description | DPP Format | Parameters |
|-------------|------------|------------|
| $\sum_{i=1,j=1}^{n+k} E[i,j] \leq w_1 + 1 \quad \forall i = j$ | - | |
| $H_k[1, i] \cap E \leq w_2 \quad i = \{1, 2, 3, \ldots, k\}$ | $H_0$ populated by hypothesis term matrix $H$ with dimension $((n + k) \times (n + k) \times l)$ and the values are given by: | |
| $E \odot T^S > = E \odot A_0$ | As populated by adjacency matrix $A$, $T^S$ by subject tuple matrix $T^S$ with dimension $((n + k) \times (n + k))$ and the values are given by: | |
| $E \odot T^S + E \odot T^P + E \odot T^{Q'} \geq w_3(E \odot A_0)$ | $A_0$ populated by adjacency matrix $A$ and $T^S, T^P, T^{Q'}$ populated by subject, predicate and object matrices $T^S, T^P, T^{Q'}$ respectively. Predicate and object tuples are converted into $T^S, T^P$ matrices similar to $T^Q$. | |

Table 1: Adopting TupleILP constraints in DPP format. For this work we set $w_1=2$, $w_2=2$ and $w_3=1$

5 Empirical Evaluation

Question Sets: We use the following multiple-choice question sets to evaluate the $\partial$-Explainer.

1. **WorldTree Corpus** (Xie et al., 2020): The 2,290 question and explanations in the WorldTree corpus are split into three different subsets: train-set (987), dev-set (226) and test-set (1,077). We use the dev-set to assess the explainability performance since the explanations for test-set are not publicly available.

2. **ARC-Challenge Corpus** (Clark et al., 2018): ARC-Challenge is a multiple-choice question dataset which consists of question from science exams from grade 3 to grade 9. We only consider the Challenge set of questions. These questions have proven to be challenging to answer for other LP-based question answering and neural approaches.

Experimental Setup: We use *nli-bert-base-max-pooling* model as the Sentence BERT model for the sentence representation in $\partial$-Explainer. The motivation to chose this model is to use a pre-trained model on natural language inference and BERT Base is smaller, enabling us to encode a larger number of facts.\(^3\)

Similarly, for fact retrieval representation, we use *nli-bert-base-max-pooling* trained with gold explanations of WorldTree Corpus to achieve a MAP of 40.11 in the dev-set. We cache all the facts from the background knowledge using this model and retrieve the top $k$ facts using MIPS retrieval (Johnson et al., 2017).\(^4\)

Baselines. In order to assess the complexity of the task and the potential benefits of the convex optimization layers presented in our approach, we show the results for different baselines. We run all models with $k = \{1, \ldots, 10, 25, 50, 75, 100\}$ to find the optimal setting for each baseline and perform a fair comparison. For each question, the baselines take as input a set of hypotheses, where each hypothesis is associated with $k$ facts, ranked according to the fact retrieval model.

\(^3\)We fine-tune using a learning rate of $5e-5$, 10 epochs, with a batch size of 16 and a maximum sequence length of 32.

\(^4\)We use L2 distance for similarity search.
1. **IR Solver** (Clark et al., 2018): This approach attempts to answer the questions by computing the accumulated score from all $k$ obtained from summing up the retrieval scores. In this case, the retrieval scores are calculated using the cosine similarity of fact and hypothesis sentence vectors obtained from the SBERT model trained on gold explanations. The hypothesis associated with the highest score is selected as the one containing the correct answer.

2. **BERT**<sub>Base</sub> and **BERT**<sub>Large</sub> (Devlin et al., 2019): To use BERT for this task, we concatenate every hypothesis with $k$ retrieved facts, using the separator token `[SEP]`. We use the HuggingFace (Wolf et al., 2019) implementation of **BertForSequenceClassification**, taking the prediction with the highest probability for the positive class as the correct answer.

3. **Aprox-TupleILP** Removing all convex optimization layers from our approach provides an approximation of TupleILP. Here we consider the same weight for overlap and relevance scores ($\theta_r = \theta_o = 1$) and frozen S-BERT representations. This baseline is particularly useful as part of our ablation studies, analysing the impact of adding different learnable components.

5.1 **Answer Selection**

**WorldTree Corpus** Table 2 presents the answer selection performance on the WorldTree corpus in terms of accuracy, presenting the best results obtained for each model after testing for different values of $k$. To establish the impact of the differentiable convex optimization layers in our approach, we provide the results across two different settings: (i) Fine-tuning only with respect to parameters $\theta$, (ii) Fine-tuning for both $\theta$ and S-BERT. We also present the results for two different training goals, optimizing for only the answer and optimizing jointly for answer and explanation selection.

| Model                  | Acc  |
|------------------------|------|
| Baselines              |      |
| IR Solver              | 50.48|
| BERT<sub>Base</sub>    | 58.06|
| BERT<sub>Large</sub>   | 59.32|
| Aprox-TupleILP         | 49.91|

| $\partial$-Explainer  |      |
|------------------------|------|
| Optimized only on answer selection $\theta$ fine-tuning | 54.00|
| $\theta$ & S-BERT fine-tuning | 58.74|
| Optimized on answer and explanation selection $\theta$ fine-tuning | 54.33|
| $\theta$ & S-BERT fine-tuning | 60.10|

Table 2: Answer selection performance for the baselines and across different configurations of our approach on WorldTree Corpus.

Using our approach confirm that adding optimizable components to constrained-based approaches in a learning setting can provide a boost in performance. By learning only the parameters $\theta$, an increase of up to 4.42% accuracy can be achieved compared to the non fine-tuned model (Aprox-TupleILP). When adding the generalization abilities of fine-tuned pre-trained language models to the mixture, the improvement is even more significant with an increase of 10.19% in the score. Our best results are obtained in the setting optimized for explanation and answer selection; such results were expected since facts of higher quality will most likely lead to a better inference process. We also achieve an increase of 2.04% and 9.62% in performance over BERT<sub>Base</sub> as reader and solver while using a similar model for our inference.

**ARC Corpus** The addition of differentiable learnable components also reflects on a performance improvement when applied to different datasets, as shown in Table 3. This table presents ablation studies for our approach used to ARC Corpus with two different background knowledge bases: TupleInf, the same as used by TupleILP (Khot et al., 2017), and WorldTree. For this dataset, we use $k=50$, the same configuration as the best model applied to WorldTree, and fine-tune for only answer selection since ARC does not have gold explanations.

The results presented in Table 3 are consistent with our previous findings. Again, we observe that learning the parameters $\theta$ and S-BERT representations lead to better performance. While
Aprox-TupleILP (Khot et al., 2017)
TupleInf 23.83
Aprox-TupleILP
TupleInf 30.71
Aprox-TupleILP
WorldTree 32.50
θ fine-tuning
TupleInf 31.07
θ & S-BERT fine-tuning
TupleInf 33.36
θ fine-tuning
WorldTree 32.91
θ & S-BERT fine-tuning
WorldTree 37.77

Table 3: Answer Selection performance on ARC corpus with θ-Explainer fine-tuned on answer selection.

Aprox-TupleILP struggles in this multiple-choice scenario, using its constraints in a differentiable setting can enable this model to produce more substantial results. With our approach, we obtain an increase of 2.65% and 7.06% in performance, using TupleInf and WorldTree as KB, respectively. We also obtain an improvement of 9.53% over the original TupleILP model with the same background knowledge (TupleInf). The obtained results fully support our claims regarding the advantages of adding optimizable differentiable components to constrained-based approaches. However, the increase in performance observed in ARC is lower compared to that achieved in WorldTree question answering. We conjecture that this lack of improvement is due to the background knowledge base having a low knowledge coverage for question-answering for ARC.

5.2 Explanation Selection

As highlighted by our previous findings, optimizing for explanations can lead to better answer selection. The differentiable components also can impact the quality of generated explanations, as shown in Table 4. We evaluate our approach on the WorldTree KB, comparing our model’s selected facts with the gold explanations.

| Model          | Background KB | Accuracy |
|----------------|---------------|----------|
| Aprox-TupleLP  | TupleInf      | 31.07    |
| Aprox-TupleLP  | WorldTree     | 32.91    |

| Model          | Precision | Recall | F1 |
|----------------|-----------|--------|----|
| Aprox-TupleILP | 28.80     | 31.20  | 27.57 |
| Optimized only on answer selection |
| θ fine-tuning | 29.62     | 32.24  | 28.36 |
| θ & S-BERT fine-tuning | 29.52 | 32.84 | 28.57 |
| θ fine-tuning | 31.88     | 34.43  | 30.26 |
| θ & S-BERT fine-tuning | 31.88 | 34.43 | 30.26 |

Table 4: F1 score for explanation selection in WorldTree.

As we can see, the IR Solver decreases in performance as we add more facts, while the scores for transformer-based models start deteriorating for $k > 5$. Such results are counter-intuitive since it would be natural to expect that a model’s performance would increase as we add supporting facts. However, in practice, that does not apply as by adding more facts, there is an addition of distractors that such models may not filter out. We can prominently see this for BERT_large: it has a sudden drop in performance for $k = 10$, going from 56.61 to 30.26. Such drop is likely being caused...
**Question (1):** In a lake near a farm the growth of algae suddenly increased. This increase was most likely due to which of the following?

**Correct Answer:** Fertilizer runoff from the farm.

**Predicted Answer:** A decrease in air temperature.

**Predicted Explanations:**
(i) Fertilizers have a positive impact on algae growth
(ii) As the population of algae increases, the amount of available oxygen for fish will decrease.

**Predicted Answer:** Fertilizer runoff from the farm.

**Predicted Explanations:**
(i) Fertilizers have a positive impact on algae growth
(ii) Runoff contains fertilizer from cropland.

**Question (2):** What layer of Earth is located just below the crust?

**Correct Answer:** Mantle.

**Predicted Answer:** Inner core.

**Predicted Explanations:**
(i) The mantle is located just below the crust of the Earth
(ii) The mantle is located between the crust and the outer core.

**Predicted Answer:** Mantle.

**Predicted Explanations:**
(i) The mantle is located just below the crust of the Earth
(ii) The mantle is located between the crust and the outer core.

**Question (3):** Seafloor spreading provides evidence of which of the following Earth processes?

**Correct Answer:** Movement of crustal plates.

**Predicted Answer:** Formation of sedimentary rocks.

**Predicted Explanations:**
(i) Seafloor spreading causes cooled magma to form on the edges of tectonic plates on Earth
(ii) The formation of metamorphic rock is a stage in the formation process of rock cycle.

**Predicted Answer:** Movement of crustal plates.

**Predicted Explanations:**
(i) Seafloor spreading causes cooled magma to form on the edges of tectonic plates on Earth
(ii) The movement of crustal plates causes seafloor spreading.

Table 5: Example of predicted answers and explanations obtained from our model with different levels of fine-tuning.

by substantial overfitting; with the added noise, the model partially lost the ability for generalization. A softer version of this phenomena is also observed for BERT_{Base}.

In contrast, our model’s performance increases as we add more facts, reaching a stable point around $k = 50$. Such performance stems from our combination of overlap and relevance scores along with the structural and semantic constraints. The obtained results highlight our model’s robustness to distracting knowledge, allowing its use in data-rich scenarios, where one needs to use facts from extensive knowledge bases. On the other hand, for smaller values of $k$ our model is outperformed by transformer-based approaches, hinting that our model is more suitable for scenarios involving large knowledge bases such as the one presented in this work.

### 5.4 Qualitative Analysis

Different levels of learning can also lead to different quality of obtained answers and explanations, as shown in Table 5, where we present examples on what type of questions is the fully fine-tuned approach better at answering than the model only fine-tuned for $\theta$.

Overall, we found that fine-tuning S-BERT representation renders our approach less likely to be distracted by lexical overlap. As seen in question (1), both $\theta$ fine-tuned and S-BERT fine-tuned models likely decide on the answer based on the lexical overlap between question, answer and ex-
explanations. In contrast to $\theta$ which is distracted by the wrong lexical cues, the end-to-end fine-tuning of the S-BERT model aids in selecting based on correct term overlap.

Similarly, Question (2) contains multiple cases of lexical overlap between explanations and possible answers. Both models retrieve the same facts, but the S-BERT fine-tuned model seems to filter the relevant associations and reach the correct answer. Finally, Question (3) shows the effect of retrieving good explanations for obtaining the correct answer. While the $\theta$ fine-tuned model selects distracting explanations, the S-BERT model selects complementing facts that will not let the model drift away from the correct answer.

6 Conclusion

We presented a novel framework for encoding explicit and controllable abductive natural language inference assumptions as part of an end-to-end learning framework. We also empirically demonstrated how incorporating these constraints and learning end-to-end can improve natural language inference. While the work presented only adopts constraints from TupleILP, this work can be extended to encode different constraints with varying degrees of complexity. In summary, we believe that this work will open new research areas for incorporating optimizations leading to controllable, transparent and explainable natural language reasoning solvers.

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