CPR Analysis of a SD 7003 Airfoil with a Local Correlation Based Transition and Turbulence Model

Mazharul Islam\textsuperscript{1a}, Felix Langfeldt\textsuperscript{2b}, Jiří Fürst\textsuperscript{3c}, David H Wood\textsuperscript{1d}

\textsuperscript{1} Department of Mechanical Engineering, Kulliyah of Engineering, International Islamic University Malaysia, P.O. Box 10, 50728 Kuala Lumpur, Malaysia
\textsuperscript{2} Department of Automotive and Aeronautical Engineering, Hamburg University of Applied Sciences, Berliner Tor 9, 20099 Hamburg, Germany
\textsuperscript{3} Department of Technical Mathematics, Faculty of Mechanical Engineering, Czech Technical University in Prague, Karlovo nam. 13, 121 35 Praha, Czech Republic
\textsuperscript{4} Department of Mechanical and Manufacturing Engineering, Schulich School of Engineering, University of Calgary, 2500 University Dr NW, Calgary T2N1N4, Canada

E-mail: \textsuperscript{a} mazharul@iium.edu.my, \textsuperscript{b} felix.langfeldt@gmail.com, \textsuperscript{c} Jiri.Furst@fs.cvut.cz, \textsuperscript{d} dhwood@ucalgary.ca

Abstract.

The SD 7003 has been investigated quite extensively in the past at different Reynolds numbers. At moderate Reynolds numbers and low angles of attack, its upper surface is dominated by a large laminar separation bubble which provides a demanding test case for RANS-based transition models. In this article, a widely used open-source Computational Fluid Dynamics (CFD) toolbox has been used mainly to investigate the (i) aerodynamics coefficients and (ii) boundary layer transitional behaviors of a SD 7003 airfoil using a local correlation-based $\gamma - \text{Re}_{\theta_t}$ SST \cite{1} transition and turbulence model. Several researchers have proposed different correlations which can be used in the $\gamma - \text{Re}_{\theta_t}$ SST. In this article, five such correlations have been investigated for the SD 7003 at low Reynolds number which is particularly challenging for CFD analysis.

1. Introduction

Airfoils are used in different engineering applications like wind turbines, airplanes etc. In the operating conditions, airfoils typically encounter transitions from laminar to turbulent flow in the boundary layer (BL). The SD 7003 airfoil has been investigated quite extensively in the past at different Reynolds numbers through diversified computation techniques and experimental investigations. It has been found that at moderate Reynolds numbers and low angles of attack, the upper surface of SD 7003 is dominated by a large laminar separation bubble (LSB). According to \cite{2} - "At these low Reynolds numbers, the flow may remain laminar over a significant portion of the airfoil rendering it susceptible to separation from even mild adverse pressure gradients. For moderate incidence, separation leads to the formation of a closed laminar separation bubble (LSB) which reattaches through transition into turbulence."

CFD analysis of airfoils at low Reynolds number that is associated with LSB is a challenging task for computationally inexpensive RANS-based transition models. Diversified approaches can be used to model different types of BL transitions. Two popular transition and turbulence models in Computational Fluid Dynamics (CFD) analysis are:
(i) The local correlation-based transitional model called $\gamma - Re_\theta$ which was originally proposed by Menter et al. [1]; and
(ii) The phenomenological $k - k_L - \omega$ model [3].

Over the years, different researchers [4–8] have proposed different correlations for calculating the two undisclosed proprietary parameters required by the original $\gamma - Re_\theta$ model. In the present work, CFD analysis of SD 7003 airfoil has been performed at a low Reynolds number of 60,000. To tackle the transition modeling, $\gamma - Re_\theta$ model is used in OpenFOAM [9] using five different correlations, proposed by [4–8]. It should be noted that the present work is an extension of the earlier research activities with $k - k_L - \omega$ SST model which was presented in [10]. Selected final results from the CFD analysis have been compared with other computational (DNS & LES) and experimental datasets for verification and validation.

2. The OpenFOAM Case
To perform the CFD analysis, a widely used open-source CFD toolbox called OpenFOAM has been used in the present work. To verify the results, mesh sensitivity analysis has been conducted with 3 mesh sizes (termed as Coarse, Medium and Fine). In Table 1, the salient features of these three mesh sizes are presented. It should be noted that the same three mesh sizes were previously investigated in the previous research [10] using the $k - k_L - \omega$ model. It should be noted that the SD 7003 has sharp trailing edges for which the C-type mesh is preferred and thus used in the present work. More discussions about the computational domain and the mesh were included in [10].

In Figure 1, the mesh around SD 7003 is illustrated. Whereas, the mesh distribution around the leading and trailing edges of the airfoil are depicted in Figures 2 and 3 respectively. For the present analysis, a steady-state solver called "simpleFoam", which is available in OpenFOAM and employs the SIMPLE algorithm to solve the incompressible Navier-Stokes equations, was used. The Reynolds Number (Re) for all the cases is about 60,000 and all the CFD analyses were conducted at 4° angle of attack.

Figure 1: Mesh Around the SD 7003 Airfoil
Figure 2: Mesh Around the Leading Edge of the SD 7003 Airfoil

Figure 3: Mesh Around the Trailing Edges of the SD 7003 Airfoil

Table 1: Features of 3 Meshes

| Parameter                                      | Coarse   | Medium   | Fine     |
|------------------------------------------------|----------|----------|----------|
| Number of Cells                                | 223571   | 396506   | 519707   |
| Number of points over the airfoil surface      | 200      | 300      | 450      |
| Number of points in the direction normal to the airfoil surface | 80       | 120      | 180      |
| Maximum $y^+$ for the final iteration          | 0.9118   | 0.9126   | 0.9132   |

3. Results and Discussions

3.1. Mesh Sensitivity Analysis

To verify the CFD results, the variation of the pressure coefficients ($C_p$) around the surface of SD 7003 airfoil has been investigated using the $\gamma - Re_{th} SST$ [1] model using the correlation proposed by Tomac et al. [8], which is shown in Figure 4. The $C_p$ distributions for the three mesh sizes are quite close to each other for the entire surface. These results have been compared with the ILES results obtained by [2] and it can be seen that there are noticeable differences around certain locations. The variation of aerodynamic coefficients for 3 mesh sizes are presented in Table 2, which shows that further refinement of mesh sizes is not needed.

Table 2: Moment, Drag and Lift Coefficients for 3 Mesh Sizes

| Mesh                                           | $C_m$   | $C_d$   | $C_l$   |
|------------------------------------------------|---------|---------|---------|
| $\gamma - Re_{th} SST$ with Tomac et. al. - Coarse | -0.0329 | 0.0203  | 0.4772  |
| $\gamma - Re_{th} SST$ with Tomac et. al. - Medium | -0.0328 | 0.0202  | 0.4773  |
| $\gamma - Re_{th} SST$ with Tomac et. al. - Fine  | -0.0328 | 0.0202  | 0.4773  |

3.2. Validation of Aerodynamic Coefficients

In Table 3, aerodynamic coefficients ($C_l, C_d \& C_m$) obtained from this research work with five correlations are compared with - (1) CFD results of $k - \omega SST$, (2) $k - k_L - \omega$ (Falkner-Skan) - Fine [11] (3) ILES results from [12], (4) Experimental results from [13] \(^1\). It can be seen from

\(^1\) Interpolated for $\alpha = 4^\circ$ from the results at Re=61,400
Figure 4: Variation of Pressure Coefficient ($C_p$) around the SD 7003 Airfoil

Table 3: Comparison of Moment, Drag and Lift Coefficients

|                          | $C_m$   | $C_d$   | $C_l$   |
|--------------------------|---------|---------|---------|
| $k - k_L - \omega$       | -0.0387 | 0.0188  | 0.5150  |
| (Falkner-Skan) - Fine    |         |         |         |
| $\gamma - Re_{eq} SST$ with Tomac et. al - Fine | -0.0328 | 0.0202  | 0.4753  |
| $\gamma - Re_{eq} SST$ with Langtry and Menter - Fine | -0.0401 | 0.0255  | 0.4782  |
| $\gamma - Re_{eq} SST$ with Malan et. al - Fine | -0.0353 | 0.0215  | 0.4801  |
| $\gamma - Re_{eq} SST$ with Sorensen - Fine | -0.0346 | 0.0208  | 0.4780  |
| $\gamma - Re_{eq} SST$ with Sukusma et. al - Fine | -0.0381 | 0.0225  | 0.4807  |
| XFOIL [Nerit - 9]        | -0.0355 | 0.0199  | 0.4253  |
| ILES [GalbraithAndVisbal2010] | -       | 0.021   | 0.59    |
| Experimental [Selig et. al. 1995] [Interpolated for $\alpha = 4^\circ$ from the results at Re=61,400] | -0.0166 | 0.0038  |

3.3. Skin Friction Coefficients
In Figure 5, the variations of skin friction coefficients ($C_f$) around the SD 7003 airfoil that are obtained from the CFD analysis using the five correlations [4–8] with the $\gamma - Re_{eq} SST$ [1] model are shown. These results are compared with Implicit Large Eddy Simulation (ILES) [2] and XFOIL [14].
Both the transition from laminar to turbulent flow and the laminar separation bubble are situated between the separation point $x_{\text{separation}}$ (where the $C_f$ becomes negative near $x/c \approx 0.23$ for the ILES result) and reattachment point $x_{\text{reattachment}}$ (where the $C_f$ value becomes positive again near $x/c \approx 0.65$ for the ILES result). So, the distance between $x_{\text{separation}}$ and $x_{\text{reattachment}}$ is indicative of the length of the bubble size. The two major observations from this figure are:

- All the RANS-based correlations are over-predicting the reattachment points significantly when compared with the ILES result.
- Among the five correlations [4–8], the the distance between $x_{\text{separation}}$ and $x_{\text{reattachment}}$ is maximum for [7] and minimum for [8].

![Figure 5: Variation of Skin Friction Coefficient ($C_f$) around the SD 7003 Airfoil](image)

**4. Concluding Remarks**

Five different correlations for $\gamma - Re_\theta$ Semicond. Sci. Technol. model [1], proposed by [4–8], have been investigated through CFD analysis of SD 7003 airfoil at low Reynolds number in the present study. It has been found that all the correlations have under-predicted the $C_l$ values and over-predicted the $C_d$ values. However, aerodynamic coefficients ($C_m$, $C_l$, and $C_d$) obtained from the $k - k_L - \omega$ model [3] using the Falkner-Skan based modification [10] are closer to the experimental results than Tomac et al. [8]. Also, the correlation by Tomac et al [8] is predicting the separation and reattachment points, for the flow around SD 7003 at low Reynolds number, better than the other four correlations [4–7].
5. References

[1] Menter F R, Langtry R B, Likki S R, Suzen Y B, Huang P G and Völker S 2004 *Journal of Turbomachinery* **128** 413–422 ISSN 0889-504X URL http://dx.doi.org/10.1115/1.2184352

[2] Galbraith M C 2009 *Implicit Large Eddy Simulation of Low-Reynolds-Number Transitional Flow Past the SD7003 Airfoil* Master’s thesis University of Cincinnati URL http://rave.ohiolink.edu/etdc/view?acc_num=ucin1242939414

[3] Walters D K and Cokljat D 2008 A Three-Equation Eddy-Viscosity Model for Reynolds-Averaged Navier–Stokes Simulations of Transitional Flow

[4] Sørensen N N 2009 *Wind Energy* **12** 715–733 ISSN 1099-1824 URL http://dx.doi.org/10.1002/we.325

[5] Malan P, Suluksna K and Juntaaro E 2008 A Three-Equation Eddy-Viscosity Model for Reynolds-Averaged Navier–Stokes Simulations of Transitional Flow

[6] Galbraith M and Visbal M 2010 *Implicit Large Eddy Simulation of Low-Reynolds-Number Transitional Flow Past the SD7003 Airfoil* 40th Fluid Dynamics Conference and Exhibit Fluid Dynamics and Co-located Conferences (American Institute of Aeronautics and Astronautics) URL http://dx.doi.org/10.2514/6.2010-4737

[7] Selig M S, Guglielmo J J, Broeren A P and Giguère P 1995 *Summary of low speed airfoil data* vol 1 (SoarTech Publications) ISBN 0964674718

[8] Drela M 1989 *Lecture Notes in Engineering* 1–12 ISSN 01765035