Some conceptual problems in the CCU 2016 draft of the New SI

Franco Pavese
formerly Consiglio Nazionale delle Ricerche, Istituto di Metrologia, Torino, Italy

Abstract. After a short introduction to the present SI (Système International des Unités), this poster presentation illustrates a digest of the basics of the proposed New SI and of its difference with respect to the present SI. This involves a review of some unresolved problems in the CCU 2016 draft, concerning: the role of the constants of physics in it and their role in the conceptual construction of this international standard; the implications for science of the New SI implementation. Consequences and new duties for the users are illustrated, involving a possible hierarchy between countries that would be installed by the new definition.

1. Introduction
The SI is a system evolving in time, as shown in [1]. The original definition of the SI can be found on the website of the BIPM, under “Metre Convention” [2] and under “SI Brochure” [3]:

“The Convention was signed in Paris in 1875 by representatives of seventeen nations. … the Metre Convention established a permanent organizational structure for member governments to act in common accord on all matters relating to units of measurement. The Convention, … remains the basis of international agreement on units of measurement. The BIPM now has fifty-seven Member States, including all the major industrialized countries.” [2].

“This SI consists of a set of base units, prefixes and derived units, as described in these pages. The SI base units are a choice of seven well-defined units which by convention are regarded as dimensionally independent: the metre, the kilogram, the second, the ampere, the kelvin, the mole, and the candela. Derived units are formed by combining the base units according to the algebraic relations linking the corresponding quantities.” [3].

“Les unités choisies doivent être accessibles à tous, supposées constantes dans le temps et l’espace, et faciles à réaliser avec une exactitude élevée” (from the official French text) [3].

2. Mandatory features of a modern system of units, and the present SI
A sound system of units of modern conception must fulfil specific requirements, some explicitly indicated in the above citations from [2, 3]. The units should be:

1. constants in time and space
2. organised in a rational system
3. easy to realise with high precision
4. accessible to everybody.

The present SI is built according to the above principles. The definition of the system consists in the definition of seven base units, in which a numerical indication of the magnitude of each unit is specified. In the specific jargon, it is of the type called “Explicit Unit Definition” (EUD). Each of the base unit is individually defined. The present definitions are of different types:

(a) Make use of an artefact: e.g., for mass “The kilogram is the unit of mass; it is equal to the mass of...
the international prototype of the kilogram”;
(b) Make use of a physical state or condition: e.g., for temperature “The kelvin, unit of thermodynamic
temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water”.
Similar approach for time, amount of substance and luminous intensity;
(c) Define a realisation method of the unit, e.g. for length: “The metre is the length of the path
travelled by light in vacuum during a time interval of 1/299 792 45 8 of a second”. Similar approach
for the electrical current.
(Actually, also the type (a) and (b) base units define indirectly a so-called “realisation method”, to be
implemented with a procedure—not to be confused with the “mise en pratique” that is a (set of)
approximations of the definition).

At present, four base units violate the indication of the SI Brochure that “les grandeurs de base sont,
par convention, considérées comme indépendantes”, because, using for convenience also some derived
units, the definition of: the unit of length involves the second; the unit of amount of substance involves
the kilogram; the unit of electric current involves the newton and the metre; the unit of luminous
intensity involves the hertz, the watt and the steradian.

The paper schematically illustrates the four above requirements, in particular in respect to the
conceptual structure proposed in [4].

3. A summary of the New SI structure
The proposed change of definition, normally named “New SI”, is present planned to be promulgated in
autumn 2018. The use of some ‘constants’ has been proposed—with stipulated numerical values.

3.1 Definition of the New SI
The following “Definition of the SI” is reported in the last available draft [5]:
“The International System of Units, the SI, is the system of units in which:
• the unperturbed ground state hyperfine splitting frequency of the caesium 133 atom ΔνCs is 9 192 631 770 Hz,
• the speed of light in vacuum c is 299 792 458 m/s,
• the Planck constant h is 6.626 070 040 ×10−34 J s,
• the elementary charge e is 1.602 176 620 8 ×10−19 C,
• the Boltzmann constant k is 1.380 648 52 ×10−23 J/K,
• the Avogadro constant NA is 6.022 140 857 ×1023 mol−1,
• the luminous efficacy Kcd of monochromatic radiation of frequency 540 ×1012 hertz is 683 lm/W.
The numerical values of the seven defining constants have no uncertainty”.

The constants (no more labelled “fundamental”) are of five different types:
– for time a specific electronic transition ΔνCs of a specific substance, 133Cs—the same used at present;
– for luminous intensity, a technical constant Kcd, of physiological origin;
– for electrical current, the elementary charge e, via the fine-structure constant α = e²/(2cε₀h);
– for temperature and amount of substance, the Boltzmann constant kB and the Avogadro constant Nₐ,
respectively, considered two ‘conversion factors’;
– for length and mass, the speed of light in vacuum c₀—the same as presently—and the Planck
constant h, respectively, the only two real “fundamental” constants.

This can be called the ‘group-definition”—in jargon, it is of the type called “Explicit Constant
Definition” (ECD) or “Global Constant Definition” (GCD), since these constants should be considered
and treated as a group.
It is indicated as the sufficient new SI definition. Table 1 in [5] indicates that these constants define the
following units: Hz = s⁻¹, m s⁻¹, J s = kg m² s⁻¹, C = A s, J K⁻¹, mol⁻¹, lm W⁻¹ = cd sr W⁻¹.

3.2 Definitions of the New SI units: Base units
Additionally to the group definition of the constants, in [5] there is also stated that the “description in
terms of base and derived units is maintained in the present definition of the SI, but has been
reformulated as a consequence of adoption of the defining constants”. A reason may be found in the other statement: “Preserving continuity is an essential feature of any changes to the International System of Units”. Then “the definitions of the traditional base units of the SI ... follow from the definition of the seven defining constants” (emphasis added). Therefore, also the definition for each single base unit is provided. At present, the above statement is implemented in [5] for each of the seven base units as follows:

- “The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency ΔνCs, the unperturbed ground-state hyperfine splitting frequency of the caesium 133 atom, to be 9 192 631 770 when expressed in the unit Hz, which is equal to s⁻¹ for periodic phenomena.”
- “The mole, symbol mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles. It is defined by taking the fixed numerical value of the Avogadro constant NA to be 6.022 140 857 ×10²³ when expressed in the unit mol⁻¹.”
- “The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299 792 458 when expressed in the unit m s⁻¹, where the second is defined in terms of the caesium frequency ΔνCs.”
- “The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be 6.626 070 040 ×10⁻³⁴ when expressed in the unit J s, which is equal to kg m² s⁻¹, where the metre and the second are defined in terms of c and ΔνCs.”
- “The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be 1 602 176 620 8 ×10⁻¹⁹ when expressed in the unit C, which is equal to A s, where the second is defined in terms of ΔνCs.”
- “The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380 648 52 ×10⁻²³ when expressed in the unit J K⁻¹, which is equal to kg m² s⁻² K⁻¹, where the kilogram, metre and second are defined in terms of h, c and ΔνCs.”
- “The candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540 ×10¹² Hz, Kcd, to be 683 when expressed in the unit lm W⁻¹, which is equal to cd sr W⁻¹, or kg⁻¹ m⁻² s⁻³ cd sr, where the kilogram, metre and second are defined in terms of h, c and ΔνCs.”

The above definitions imply that the present base units—second, metre, kilogram, ampere, kelvin, mole, and candela—remain in the New SI, however with a totally differently-based definition and function: they are not anymore the definition of the SI. They are maintained in order to fulfil the “continuity condition” and are indicated to “follow” from the SI definition—the ‘group’ one of the constants. However, some units remain not defined even in the above definitions: Hz, J, lm, W, sr: they are defined as derived only later in the conceptual roadmap.

4. Basic features of the New SI

The proposal of the New SI, to anchor the magnitude of the base units to constants included in basic laws of physics [8], entails some basic new features that are the pillars of the new definition and can be considered an interesting advancement with respect to the present SI.

Internal consistency of the base-unit system. Only a degree of consistency can be checked, because four of the seven constants (c₀, h, e, kₐ) are multi-dimensional, so an arbitrary choice of the unit magnitudes in not allowed. However, for stipulation, the present values of the seven constants were obtained by taking advantage of the CODATA studies, which fix the (minimum) degree of consistency that is acceptable for the SI from now on. This is an important asset of the proposal.

Continuity (constancy) of the unit magnitude. The continuity requirement (also called constancy in time of the unit magnitudes) is stressed in the proposal as a basic need. Because of this requirement, the set of base units is taken un-altered from the present SI, but only as to the base quantities and unit denomination, while the definition of the base units is now founded on the same constants of the group definition. In addition, since the stipulated values of the constants come from the present-SI, the magnitudes of those constants are assumed to remain un-altered.

Actually, only the use of three of the seven constants is new: h, kₐ and e. In fact, ΔνCs, c₀, Nₐ and Kcd
are already included in the present-SI definitions, though in an Explicit Unit Definition (EUD) type.

**Lack of definitional methods.** In the New SI there is not anymore a definitional method specified for each base unit. This is maybe the most important practical change in the features of the SI of interest of the users. It means that “the realizations are separated conceptually from the definitions” [5].

In addition, to be at the top step of the hierarchical ladder, an institute should now demonstrate that its standards comply with the new condition set by New SI: that, by using them, they are able to obtain the stipulated value of each relevant constant—in the past they had to demonstrate that they own consistent realizations of the relevant “definitional methods”.

**Lack of reference to any specific substance: uniqueness.** The preference of using of constants in strict sense—though of different types—was born from the fact that they are not a property of any specific substance. Only for the time, whose definition is formulated differently from the present one but is basically the same, the atomic transition is one of $^{133}$Cs. The advantage is that a unique property of the constants: their lack of definitional non-uniqueness. Consequently, the “value of a constant” is the only relevant property here, which is intrinsically invariant in time and space. Its numerical value, instead, is contingently assigned as the ‘best’ one available at the time of the SI change in definition.

5. Main criticalities in the CCU 2016 Draft of the New SI

5.1 Concerning unit definitions

Only the main issue will be illustrated, concerning the conceptual frame of the new definition: it is based on an analysis that can be found in [6].

The SI-2018, differently from the present one, is structured into two conceptual ‘frames’, which will be called here A (constants) and B (base units), which can be found in the text of the CCU 2016 Draft [5] in the following order:

- **General principles**
  - A1) Some present advantages considered achieved in the present SI “led to the decision to define all units with the help of defining constants”;
  - B2) The “description in terms of base and derived units is maintained in the present definition of the SI, but has been reformulated as a consequence of adoption of the defining constants” (emphasis added);
- **“Definition of the SI”**
  - A3) The ‘group-definition’. For each constant, it is of the type: “the [constant] [symbol] is [numerical value] [present-SI units]”, where the indicated units, single or algebraic combination, can be formed by either present base units or derived units. For example, “the Planck constant $h$ is $6.626 \times 10^{-34}$ J s”;
  - A4) Then, Table 1 in [5] shows the caption “The seven defining constants of the SI, and the seven corresponding units they define” (emphasis added), where the name, symbol, numerical value and “Unit” are reported, for example for $h$ is $J s = kg \ m^2 \ s^{-1}$, where the initial units are then spelled out in terms of only the present base units, except for the luminous efficacy $K_{cd}$, which is $lm \ W^{-1} = cd \ sr \ W^{-1}$, where a derived unit and a special name unit are indicated;
- **Definitions of the base units**
  - A5-B5) “Preserving continuity is an essential feature of any changes to the International System of Units, which has always been assured in all changes to the definitions. The numerical values of the defining constants have been chosen to be consistent with the earlier definitions insofar as advances in science and knowledge allow”;
  - B6) “The definitions of the traditional base units of the SI, as listed in Table 2 [of [5]], follow from the definition of the seven defining constants” (emphases added);
  - B7) The new definition of each of the seven present base units is provided. They are reported here above. For each base unit, it is of the type: “[The base unit] is defined by taking the fixed numerical value of [a constant, one and different for each base unit], to be [numerical value] when expressed in
the [present-SI units], where the [other present-SI base units indicated] is [or are] defined in terms of the [other relevant constant indicated in the group definition]”. For example, “the kilogram … is defined by taking the fixed numerical value of the Planck constant \( h \) to be \( 6.626 \, 070 \, 040 \times 10^{-34} \) when expressed in the unit J s, which is equal to kg m\(^2\) s\(^{-1}\), where the metre and the second are defined in terms of \( c \) and \( \Theta / \Theta_{\text{Cs}} \)."

 Definitions of the derived units

B8) Table 4 [5] “The [derived] 22 SI units with special names and symbols”: “Derived units are defined as products of powers of the base units”, namely the joule (J), hertz (Hz), watt (W), coulomb (C), lumen (lm) and steradian (sr).

The splitting of the New SI structure into two frames entails several difficulties in its correct implementation, many still unresolved.

 Group definition (frame A). There is an obvious mismatch in the conceptual sequence. In fact, at step A3 concerning the definition of the SI, the new definitions of the base and derived units were not yet provided: it is incorrect in a definition to refer to a prior knowledge of the reader about the existence of certain previous units—statement B2 in itself is not sufficient to alleviate the problem.

It is not possible to disentangle the kind of definition A3 and resolve the mismatch, because the reason is the fact that the numerical value of the constant is indicated, entraining the need to specify the units in which it is expressed. The reason for the choice cannot be found within frame A, but is due to frame B according to step B5.

Actually, the choice of the constants does not depend on frame B, except for the fact they must be seven according to the proposers as the present base units—according to principle B5—and involving all the seven present ones, but without one-to-one correspondence with respect to the base units. However, considering that four of the seven are multidimensional, in principle, one could select instead less than seven constants, provided that they include all the dimensions of the base units.

From a conceptual viewpoint, the group definition does not need—and cannot—anticipate any numerical value. It should be considered as the “SI-fundamental” definition [4], and it would be sufficient to say:

“The SI-fundamental is the coherent system of units in which \( C_1 \) is the fundamental-unit of frequency, \( C_2 \) is the fundamental-unit of speed and \( C_3 \) is the fundamental-unit of action, …” [4], where the \( C_i \) are the chosen constants. “In this way, according to this definition, for example the Planck constant has a value 1 fundamental unit of action”. [4]

Actually, the seven constants do not define the (previous) units indicated at step A4, but, as indicated in [7], frequency, speed, action, electrical charge, heat capacity, reciprocal amount of substance and luminous intensity, which are the true new “base” units of the SI-fundamental system.

However, due to the principle B5, the base units cannot be changed. That principle is not arising from the underlying physical principles governing the choice of the SI-fundamental, but from a practice that is considered highly desirable in metrology (and in measurement science): the practice issues are what can be labelled “SI-conventional” [4], here frame B, which is the only frame of the present-SI.

Then the numerical values come, originating from the present base units.

 Base units (frame B). The need to maintain the use of base units is not justified in B1, but in B5, where it is also clear that the magnitude of each base unit should remain unchanged through the change of definitions. This is the (implicit in the 2016 Draft [5]) reason for the use of numerical values for the constants that are simply the ‘best available’ at the moment of the change, as said in the second sentence in B5.

However, in B6 there is a logical inversion of this justification, where it is said that their definitions, as implemented in B7, “… follow from the definition of the seven defining constants” (emphasis added). In fact, in B7 the reported numerical value is obviously the one before the change of the SI definition, retained in the group definition: again a consequence of the inversion of the conceptual order illustrated in the previous subsection.
This inconsistency also reflects in B7, the text of the new base unit definitions, when they refer to the group definition for the definition of the other base units appearing in the definition of the relevant one: e.g., in the definition of the kilogram, which is based on the Planck constant only, unit “kg m² s⁻¹, where the metre and the second are defined in terms of c and c₀” (emphasis added). Here a circular reasoning occurs, because the group-definition is obliged to refer to the present-SI units while reporting the numerical values, but the latter refers now to the group-definition.

Thus, the use of a separate “SI-conventional” indicated above [4], logically coming after frame A, the SI-fundamental, would also resolve the problem in the base unit definitions [4]:

“The SI-conventional is the coherent system of units in which
(A) the unit of time (duration) is the second, symbol s, the unit of length is the metre, symbol m, the unit of mass is the kilogram, symbol kg, …;
(B) the frequency f is k₁ s⁻¹, the speed c is k₂ m s⁻¹, the action h is k₃ kg m² s⁻², …;
(C) the numerical values are k₁ = [num. value]×10⁻¹⁰, k₂ = [num. value]×10⁻⁹, k₃ = [num. value]×10⁻³⁴, …”,

where the numerical values are stipulated according to the ‘best’ values at the moment when the new definition comes into effect. Notice that also the base units should be seen as a group.

In practice, the numerical values spelled out in the CCU group-definition are the conversion factors from the “SI-fundamental” to the “SI-conventional”, the two distinct frames.

What is intended for units in the SI:2018. The Introduction in [5] states: “The definition of the SI units is established in terms of a set of seven defining constants. From the units of these defining constants the complete system of units can be derived. These seven defining constants are the most fundamental feature of the definition of the entire system of units” (emphases added). The expression “the units of these defining constants” is ambiguous: does it refer to the present SI units (e.g., present-SI “velocity” for c₀), or to the present set of base units (e.g., present -SI “length” and “time” for c₀), or to a New SI unit for c₀?

In addition, the third sentence is a statement about the prevalence of the definition of the group of constants (called the group-definition hereinafter) with respect to the ‘base units’. In fact, the period then states: “The specific constants have been identified as the best choice reflecting the previous definition of the SI based on seven base units and the progress in science” (emphasis added), putting the base units as the alternative way “reflecting” the previous SI definition.

According to this approach, Section 1.1 “Motivation for the use of defining constants to define the SI” [5] states: “the realizations are separated conceptually from the definitions” (emphasis added), a basic statement implicitly meaning that the definition does not indicate any ‘definitional method’ [5] like the previous SI, where instead in practice “the definition and the realization” were “equivalent”. The lack of ‘definitional methods’, indicated as the effect of using constants in the definition, looks one of the few—important—practical reasons for the latter choice. However, the implementation problems are actually moved, from the realization by definitional methods to the relationship between definition and realizations, and to their link to the previous SI.

6. Final remarks
With respect to the CCU 2013 Draft, the 2016 draft is improved because: the constants are not labelled as “fundamental” anymore; the term “mise en pratique” is replaced by the term “realization”—though considered equivalent; the “continuity condition” is introduced explicitly. However, other conceptual issues still need further consideration. In particular one can notice that many units are defined by using the reciprocal of a constant. This is not without consequences, because, as the reciprocal of a rational number is normally a real number, so the reciprocal of a stipulated number is also a real number, not a stipulated one unless a decision is taken to truncate the latter. This issue may reflect an inversion of the reasoning with respect to the correct conceptual flow.
References

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