Molybdenum disulfide and magnesium oxide nanoparticle performance on micropolar Cattaneo-Christov heat flux model

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Abstract This article intends to illustrate the Darcy flow and melting heat transmission in micropolar liquid. The major advantage of micropolar fluid is the liquid particle rotation through an independent kinematic vector named the microrotation vector. The novel aspects of the Cattaneo-Christov (C-C) heat flux and Joule heating are incorporated in the energy transport expression. Two different nanoparticles, namely, MoS₂ and MgO, are suspended into the base-fluid. The governing partial differential equations (PDEs) of the prevailing problem are slackening into ordinary differential expressions (ODEs) via similarity transformations. The resulting mathematical phenomenon is illustrated by the implication of fourth-fifth order Runge-Kutta-Fehlberg (RKF) scheme. The fluid velocity and temperature distributions are deliberated by using graphical phenomena for multiple values of physical constraints. The results are displayed for both molybdenum disulphide and magnesium oxide nanoparticles. A comparative benchmark in the limiting approach is reported for the validation of the present technique. It is revealed that the incrementing material constraint results in a higher fluid velocity for both molybdenum disulphide and magnesium oxide nanoparticle situations.

Key words micropolar fluid, Cattaneo-Christov (C-C) heat flux, molybdenum disulphide nanoparticle, melting effect, numerical solution

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1 Introduction

Heat transport in distinct fluids has a wide range of implications in modern industrial and engineering developmental areas. It is necessary to save energy and time to overcome the limitations. Nowadays, many researchers and scholars are working on the fluids in the presence of nanoparticles called nanofluids. Choi and Eastman[1] first reported such work in this direction. They used combinations of different metals and metal oxides to obtain a
nano substance. By this research, it came to know that the heat transport performance of nanofluids is better than that of pure fluids such as water and oil. The idea of nanofluids made a revolutionary change in the field of research. Gholinia et al. [2] used silver and copper nanoparticles to illustrate the flow over a circular cylinder along with the electric conductivity. Ashjaae et al. [3] scrutinized the magnetohydrodynamic (MHD) ferrofluid flow to examine the pressure and heat drop in the system. Patel et al. [4] used the semi-analytic method to solve the micropolar fluid model with ferro-nanoparticles. Venkateswarlu et al. [5] illustrated the fluid temperature between MgO and MoS2 water based nanofluids by considering the MHD and Cattaneo-Christov (C-C) heat flux over a sheet. Mohammad et al. [6] inspected a hybrid nanofluid over a porous medium with the effects of magnetic field and pressure gradient. Turkyilmazoglu [7] addressed both single and double phase nanofluid models in concentric annuli under slip mechanism. Mehryan et al. [8] described the magneto-hybrid nanofluid flow by considering the viscosity dependent on the magnetic field.

MHD flow has a wide range of applications in engineering, geophysics, astrophysics, study of earthquakes, aerospace engineering, and biological fields. The equations are the combination of Navier-Stokes and Maxwell equations. Turkyilmazoglu [9] described the magnetic field phenomena for mixed convected flows generated by a nonlinear deforming surface. Zeeshan et al. [10] scrutinized the MHD Couette-Poiseuille nanofluid flow by considering the activation energy along with the chemical reaction, and obtained the relation between concentration of the fluid and chemical reaction. The stagnation flow and the effect of Newtonian heating in the magnetic nanofluid over a surface were inspected by El-Halimi et al. [11]. Lu et al. [12] scrutinized the MHD Carreau nanofluid flow by applying a zero mass-flux condition. Khan et al. [13] considered the three-dimensional (3D) flow over two-directional stretching sheets, and used the Carreau rheological model. In the research of the magnetic effect, heat generation/absorption was taken into account. They revealed that the velocity is dependent on the Hartmann number. Reddy and Sandeep [14] explained the phenomena of MHD flow of Carreau nanofluid in the occupation of different effects.

Cattaneo [15] was the first person to change the Fourier heat conduction model by considering the time factor in heat transfer. Later, Christov [16] proposed the Maxwell-Cattaneo law which helped in eliminating heat flux and obtaining the single temperature equation. Straughan [17] continued the research on the C-C heat flux and explained the importance of higher Cattaneo numbers. Hayat et al. [18] illustrated a 3D flow under the C-C heat flux and used the optimal homotopy analysis method (OHAM) to obtain a solution. They showed that the temperature and concentration functions are inversely proportional to the relaxation parameter. Upadhya et al. [19] looked over the study of MHD fluid flow by considering dust and nanoparticles with the chemical reaction and C-C heat flux. Reddy [20] used the Oldroyd-B fluid to explain the significance and features of MHD flow over two different geometries with the C-C heat flux. The Runge-Kutta-Fehlberg (RKF) scheme was implemented to find the numerical solution. Mustafa [21] illustrated the C-C heat flux in a non-Newtonian fluid flow over a rotating system. Waqas et al. [22] illustrated the Burgers fluid flow with the C-C heat flux over a sheet. Hayat et al. [23] explained the C-C heat flux in the Maxwell fluid flow induced by a moving sheet.

Recently, Bakier [24] considered two types of flow to illustrate the melting effect in a porous medium over a plate. Cheng and Lin [25] scrutinized the mixed convection heat flux over a porous medium by considering the melting effect. Dinh et al. [26] considered the Lorentz force and melting effect on the MHD flow in a rotating system. Muhammad et al. [27] illustrated the heat transfer of a squeezing flow of hybrid nanofluids and nanofluids with viscous dissipation and melting effect, and revealed that the entropy generation is directly proportional to the squeezing, magnetic parameter, and Eckert number. Hayat et al. [28] illustrated the mixed convectional nanofluid flow in MHD over a stretchable plate with the melting effect. Kok [29] scrutinized heat transfer with or without fin in the presence of melting effect. Mehryan et al. [30] inspected the heat flow between vertical isothermal cylinders with the melting effect by using
the Galerkin finite element approach.

The literature review revealed that not much attention has been devoted to the magnetized micropolar fluid behaviors with suspension of molybdenum disulphide and magnesium nanoparticles induced by a melting sheet. Hence, the thermo-physical characteristics of the micro-polar non-Newtonian nanofluid over a melting sheet is expressed. The melting condition is developed for the execution of energy transmission of nanoparticles. The numerical scheme is adopted to visualize the results of modeled physical problem. The tabular and graphical descriptions are illustrated to express the behaviors of the problem. The results are also computed in a limiting approach to validate the methodology.

2 Modeling

The magnetized micropolar fluid flow induced by a melting surface with heat transmission is addressed. The melting sheet is stretched through a velocity \( U_w(x) = ax \). It is supposed that the melting sheet temperature is \( T_m \), and the temperature at the free-stream is \( T_\infty \) such that \( T_\infty > T_m \). The wall temperature is assumed to be a constant, i.e., \( T = T_\infty \). Further, the induced magnetic force is ignored due to a weaker magnetic Reynolds number. The physical illustration of the problem is shown in Fig. 1. The thermophysical values of nanofluids are demonstrated in Table 1.

![Physical model of problem (color online)](image)

**Table 1** Thermo-physical properties of nanofluid

| Property               | Nanofluid                        |
|------------------------|----------------------------------|
| Density                | \( \rho_{nf} = \phi \rho_s + \rho_f (1 - \phi) \) |
| Viscosity              | \( \mu_{nf} = \frac{\mu_f (1 - \phi)}{(1 - (1 - \phi))} \) |
| Heat capacity          | \( (\rho c_p)_{nf} = \phi (\rho c_p)_s + (\rho c_p)_f (1 - \phi) \) |
| Thermal conductivity   | \( \frac{k_{nf}}{k_f} = \frac{2(k_f k_s - 2(k_f - k_s)\phi)}{2k_f + k_s + 2(k_f - k_s)\phi} \) |
| Heat conductivity      | \( 1 + \frac{(2\phi - 1)^2}{(2\phi + 2)^2 - (2\phi - 1)^2} \phi \) |

The formulated boundary layer equations of momentum and energy transfer expressions are written as\(^5–6, 19–20\)

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)
\]
K in which \( \rho \) is the density, \( \mu_\text{nf} \) is the dynamic viscosity, \( \kappa \) is the vortex viscosity, \( \nu \) is the micro rotation density, \( c_p \) is the specific heat capacity, and \( \lambda_1 \) is the parameter of thermal relaxation.

The transformed conditions are

\[
\begin{align*}
\left\{ 
\begin{array}{l}
  u = U_m(x), \\ v = 0, \\ N = -\delta^*_{\nu} \frac{\partial u}{\partial y} \\
\end{array}
\right.
\begin{array}{l}
  T = T_m, \\
\end{array}
\right. \\
(5a)
\end{align*}
\]

where \( \delta^* \) is the melting constant, and its value lies between \( 0 \leq \delta^* \leq 1 \).

Introduce the following similarity transformations:

\[
\begin{align*}
u &= ax f'(\eta), \\ v &= -\sqrt{\alpha_v} f(\eta), \\ \eta &= \sqrt{\frac{a}{\nu_1} y}, \\ N &= ax \sqrt{\frac{a}{\nu_1} g(\eta)}, \\ \theta &= \frac{T - T_m}{T_\infty - T_m}. \\
\end{align*}
\]

Then, Eq. (1) is satisfied automatically, while Eqs. (2)–(4) can be reduced as follows:

\[
\begin{align*}
\left( \frac{1}{(1 - \phi)^2} + K \right) f'''' + \left( 1 - \phi + \frac{\rho_y}{\rho_l} \right) f f'''' - M \left( 1 + \frac{3(\frac{\sigma_y}{\sigma_l} - 1)\phi}{(\frac{\sigma_y}{\sigma_l} + 2)(\frac{\sigma_y}{\sigma_l} - 1)\phi} \right) f' \\
- \left( 1 - \phi + \frac{\rho_y}{\rho_l} \right) f'^2 + \lambda_3 \theta = 0, \\
\left( \frac{1}{(1 - \phi)^2} + \frac{K}{2} \right) g''' + \left( 1 - \phi + \frac{\rho_y}{\rho_l} \right) f g'' - \left( 1 - \phi + \frac{\rho_y}{\rho_l} \right) f'g - K(2g + f'') = 0, \\
\frac{1}{Pr} \left( k_s + 2k_l - 2\phi(k_l - k_s) \right) \theta'' + \left( 1 - \phi + \frac{\rho_c}{(\rho_c)_{\nu}} \right) \phi \left( f f'\theta' + f \theta'' \right) \\
\quad \cdot \left( 1 + \frac{3(\frac{\sigma_y}{\sigma_l} - 1)\phi}{(\frac{\sigma_y}{\sigma_l} + 2)(\frac{\sigma_y}{\sigma_l} - 1)\phi} \right) \phi = 0. \\
\end{align*}
\]

The transformed conditions are

\[
\begin{align*}
\left\{ 
\begin{array}{l}
  f'(0) = 1, \\ f(0) = 0, \\ g(0) = -\delta^* f''(0), \\
\end{array}
\right. \\
\left\{ 
\begin{array}{l}
  k_{\nu} M \theta'(0) + \left( 1 - \phi + \frac{\rho_\nu}{\rho_l} \phi \right) Pr F(0) = 0, \\
  \theta(0) = 0 \quad \text{at} \quad \eta = 0, \\
  f'(\infty) \to 0, \\
  g(\infty) \to 0, \\
  \theta(\infty) \to 1 \quad \text{as} \quad \eta \to \infty,
\end{array}
\right. \\
(10a)
\end{align*}
\]

in which \( K = \frac{\sigma_y}{\mu_l} \) is the material parameter, \( j = \frac{\mu_y}{\mu_l} \) is the micro rotation density, \( M = \frac{\sigma_\nu}{(\rho_c)_{\nu}} \) is the magnetic parameter, \( Pr = \frac{(\rho_c)_{\nu}}{k_l} \) is the Prandtl number, \( Ec = \frac{u^2}{\gamma_y(T_\infty - T_m)} \) is the Eckert
number, \( \lambda_2 = \frac{Gr}{Re_x} \) is the thermal buoyancy parameter, \( Gr_x = \frac{2\beta_{m}^2(T_\infty - T_m)x^2}{\nu} \) is the local thermal Grashof number, \( Re_x = \frac{U_x x}{\nu} \) is the local Reynolds number, \( M_1 = \frac{T_\infty - T_m}{c_p(\lambda + C_1(T_x - T_m))} \) is the melting parameter with \( C_1 \) being the heat capacity of the solid surface and \( T_x \) being the solid temperature, and \( \delta_1 = \lambda_1 a \) is the non-dimensional Deborah number. Note that the present flow analysis constrains to the hydrodynamic case by setting \( M = 0 \).

The dimensionless forms of the drag force and the heat transmission rate are addressed as

\[
C_{fx} = \frac{\tau_w}{\rho U_{w}^2}, \quad Nu_x = \frac{xq_w}{k_T(T_\infty - T_m)},
\]

where \( \tau_w \) and \( q_w \) are defined as

\[
\tau_w = (\mu_{nf} + \kappa) \frac{\partial u}{\partial y} + \kappa N \bigg|_{y=0},
\]

\[
q_w = -k_{nf} \frac{\partial T}{\partial y} \bigg|_{y=0} + (q_l)_w,
\]

in which \( C_{fx} Re_x^{1/2} \) and \( Nu_x Re_x^{-1/2} \) are defined as

\[
C_{fx} Re_x^{1/2} = \left( \frac{1}{(1 - \phi)^{2.5}} + K(1 - \delta^*) \right) f''(0),
\]

\[
Nu_x Re_x^{-1/2} = -\frac{k_{nf}}{k_T} \theta'(0).
\]

3 Solution procedure

In this part, the nonlinear ordinary differential expressions (ODEs) (7)-(9) with the prescribed condition (10) are calculated numerically by engaging the RKF technique using MATLAB. The major advantage of this method is its efficiency and easy implementation. This method is self-starting and stable unlike other multi-step techniques. In this context, the boundary value is developed into the initial value.

\[
\begin{aligned}
f &= y_1, \quad f' = y_2, \quad f'' = y_3, \\
f''' &= \frac{(1 - \phi)^{2.5}}{1 + K(1 - \phi)^{2.5}} \left( -\left( 1 + \frac{\rho_s}{\rho_l} \phi - \phi \right)y_1 y_3 + \left( 1 + \frac{\rho_s}{\rho_l} \phi - \phi \right)y_2^2 ight) \\
&\quad + M \left( 1 + \frac{\sigma_u}{\sigma_f} - (\frac{\sigma_u}{\sigma_f} - 1) \phi \right) y_2 - \lambda_2 y_6, \\
g &= y_4, \quad g' = y_5, \\
g'' &= \frac{2(1 - \phi)^{2.5}}{2 + K(1 - \phi)^{2.5}} \left( -\left( 1 + \frac{\rho_s}{\rho_l} \phi - \phi \right)y_1 y_5 + \left( 1 + \frac{\rho_s}{\rho_l} \phi - \phi \right)y_2 y_4 + K(2y_4 + y_3) \right), \\
\theta &= y_6, \quad \theta' = y_7, \\
\theta'' &= \frac{Pr(k_s + 2k_l + \phi(k_l - k_s))}{k_s + 2k_l + \delta_1 y_1 (k_s + 2k_l + \phi(k_l - k_s) - 2\phi(k_l - k_s))} \\
&\quad + \left( 1 + \phi \frac{\rho_s}{\rho_l} \right) y_1 y_7 + \left( 1 + \frac{\sigma_u}{\sigma_f} - (\frac{\sigma_u}{\sigma_f} - 1) \phi \right) M Ec y_2^2 + \delta_1 y_1 y_2 y_7.
\end{aligned}
\]
The transformed boundary conditions are

\[
\begin{align*}
&y_2 = 1, \quad y_1 = 0, \quad y_4 = -\delta^* y_3, \\
&\frac{k_n \alpha}{k_f} M_1 y_7 + \left(1 - \phi + \frac{\rho_n \alpha}{\rho_f} \phi \right) P r y_1 = 0, \quad y_6 = 0 \quad \text{at} \quad \eta = 0, \\
y_2 \to 0, \quad y_4 \to 0, \quad y_6 \to 1 \quad \text{as} \quad \eta \to \infty.
\end{align*}
\] (14a)

To solve Eqs. (11)–(13), we adopt \(y_3, y_5, y_7\) which are not given at the prior conditions. Later, the prior stages are declared by using the RKF method with the subsequent looping step of length 0.01.

4 Graphical analysis

The steady flow and heat transport of micropolar fluid over a melting sheet are investigated. Two types of oxides, namely, molybdenum disulphide and magnesium oxides, are suspended into the base-fluid water. The mixed convection heat transmission with the melting effect is described. The numerical mechanism, namely, the RKF fourth-fifth order together with the shooting scheme, is adopted to exploit the dimensionless modeled expressions of the current problem. The effects of various governing parameters on \(f'(\eta), h(\eta),\) and \(\theta(\eta)\) are demonstrated graphically. The present study is also compared with the available results of Venkateswarlu et al.\[5\] and Reddy and Sandeep\[14\] (see Table 2). The limiting results are in good match with each other.

Table 2  Comparison data of skin friction coefficient with Venkateswarlu et al.\[5\] and Reddy and Sandeep\[14\]

| \(M\) | Venkateswarlu et al.\[5\] \(-f''(0)\) | Reddy and Sandeep\[14\] \(-f''(0)\) | Present \(-f''(0)\) |
|---|---|---|---|
| 1 | -1.41421 | 1.41421 | -1.41123 |
| 5 | -2.44949 | 2.44980 | -2.44987 |
| 10 | -3.31662 | 3.31662 | -3.31676 |
| 50 | -7.14143 | 7.14142 | -7.13452 |

4.1 Fluid velocity \(f'(\eta)\)

Figures 2–6 execute the behaviors of the fluid velocity profiles against different physical parametric values. Figure 2 represents the nature of \(M\) on the velocity \(f'(\eta)\) profile. Here, it is achieved that higher values of \(M\) augment the fluid velocity. The compactness of the corresponding momentum layer rises for the improving trend of \(M\). Physically, a larger \(M\) implies stronger Lorentz forces. The improvement in the Lorentz force generates more drag to resist the liquid flow. The influence of the material parameter on the velocity profile \(f'(\eta)\) for both MoS\(_2\) and MgO nanoparticle cases is depicted in Fig. 3. Here, the incrementing nature of \(K\) booms the velocity \(f'(\eta)\) and compactness of momentum layer also cultivate for growing values of \(K\). Furthermore, the enhancement of fluid flow is higher in MgO nanoparticle situation than that in MoS\(_2\) nanoparticle situation. The connections of the buoyancy parameter \(\lambda_2\) on \(f'(\eta)\) for both MoS\(_2\) and MgO nanoparticle cases are visualized in Fig. 4. Here, it is observed that the uplifting values of \(\lambda_2\) scale back the fluid flow in both nanoparticle cases. Additionally, the thicker momentum boundary layer is achieved for larger \(\lambda_2\). Further, the fluid velocity is more dropped in MgO nanoparticles when compared with MoS\(_2\) nanoparticle case. Figure 5 exhibits the nanoparticle volume fraction \(\phi\) behavior on \(f'(\eta)\). It is perceived that the increment in \(\phi\) improves the fluid velocity for both cases. The influence of the melting parameter on the velocity profile \(f'(\eta)\) for both MoS\(_2\) and MgO nanoparticle cases is illustrated through Fig. 6. It is mentioned that the larger integrities of \(M_1\) rise the velocity of the fluid.
4.2 Angular velocity $g(\eta)$

In Fig. 7, the angular velocity distribution $g(\eta)$ is presented for different values of the material parameter $K$ for both MoS$_2$ and MgO nanoparticle cases. One may observe from this figure that the angular fluid velocity enhances for improving values of $K$. The momentum layer of both MoS$_2$ and MgO nanoparticle cases enhances for booming values of the material parameter. Additionally, the enhancement in the angular fluid flow is higher for MoS$_2$ nanoparticle case as compared to MgO nanoparticle case. The microrotation parameter $\delta^*$ aspects on the
angular fluid velocity \( g(\eta) \) distribution are observed in Fig. 8. A decrement in microrotation for both MoS\(_2\) and MgO nanoparticle cases is achieved for enlarging \( \delta^* \). Figure 9 exhibits the nanoparticle volume fraction \( \phi \) on the angular velocity profile \( g(\eta) \). The expanding values of \( \phi \) boost the angular fluid velocity and the corresponding boundary layer for both nanoparticle cases.

![Fig. 8](image)

**Fig. 8** Behaviors of \( \delta^* \) on \( g(\eta) \) (color online)

**Fig. 9** Behaviors of \( \phi \) on \( g(\eta) \) (color online)

### 4.3 Temperature profile \( \theta(\eta) \)

Figure 10 reports the temperature profiles \( \theta(\eta) \) for both MoS\(_2\) and MgO nanoparticle cases against varying Prandtl numbers \( Pr \). The temperature profile and thermal layer decay for incrementing \( Pr \). Physically, the melting aspect for advanced values of \( Pr \) has lower thermal diffusivity. Finally, larger values of \( Pr \) scale back the fluid temperature. The domination of nanoparticle volume fractions on \( \theta(\eta) \) for MoS\(_2\) and MgO nanoparticle cases is distinguished through Fig. 11. It is mentioned that the larger \( \phi \) rises the temperature of the fluid. Physically, the increment in \( \phi \) results in stronger thermal conductance of the material which returns to boost the fluid temperature. Figure 12 demonstrates the variations of \( \theta(\eta) \) against higher melting parameters. The larger values of \( M_1 \) lead to a reduction in the nanofluid temperature for both MoS\(_2\) and MgO nanoparticle cases. Indeed, more advanced \( M_1 \) values escort to more convected flow from the melting sheet towards the hot fluid. It follows that the fluid temperature is dropped. Figure 13 demonstrates the changes in \( \theta(\eta) \) for the Eckert number \( Ec \). As conventional, hotness of the fluid and compactness of thermal layer dwindle for both MoS\(_2\) and MgO nanoparticle cases for higher values of \( Ec \). In Fig. 14, the temperature distribution \( \theta(\eta) \) is presented for different values of \( \delta_1 \) for both MoS\(_2\) and MgO nanoparticle cases.

![Fig. 10](image)

**Fig. 10** Behaviors of \( Pr \) on \( \theta(\eta) \) (color online)

![Fig. 11](image)

**Fig. 11** Behaviors of \( \phi \) on \( \theta(\eta) \) (color online)
incrementing trend of $\delta_1$ values leads to weaker fluid temperature for both MoS$_2$ and MgO nanoparticle cases.

4.4 Friction factor and Nusselt number

Figure 15 describes the variations in $C_{f x} Re_x^{1/2}$ for varying magnetic and material parameters. Here, the improving magnetic constraint enhances $C_{f x} Re_x^{1/2}$, and a similar trend is observed for enhancing the material parameter values. The role of $M_1$ and $Pr$ on $Nu_x Re_x^{-1/2}$ is demonstrated through Fig. 16. This figure illustrates that higher values of $M_1$ and $Pr$ increase $Nu_x Re_x^{-1/2}$ for both MoS$_2$ and MgO nanoparticle cases.
The phenomenon of diversity of flow constraints on $Re_{x}^{1/2}C_{f}$ and $Re_{x}^{-1/2}Nu_{x}$ is illustrated in Table 3. The friction coefficient enhances by increasing $M, \lambda_{2}, Pr$, and $\delta_{1}$, but a reverse tendency is observed for magnifying $K, \phi, \delta^{*}$, and $M_{1}$. Furthermore, the heat transmission rate values are exaggerated by accumulating the values of $Pr, M_{1}, K$, and $\phi$, but a reverse trend is seen for $\lambda_{2}, \delta_{1}$, and $Ec$.

### Table 3  
**Numerical values of $Re_{x}^{1/2}C_{f}$ and $Re_{x}^{-1/2}Nu_{x}$ for different values of $K, M, M_{1}, \lambda_{2}, Pr, \delta^{*}, Ec, \delta_{1}$, and $\phi$**

| $K$ | $M$ | $\lambda_{2}$ | $Pr$ | $\delta_{1}$ | $Ec$ | $\phi$ | $\delta^{*}$ | $M_{1}$ | Friction factor coefficient | Nusselt number |
|-----|-----|--------------|-----|-----------|-----|------|--------|-------|---------------------------|--------------|
| 0.2 | 0.98324 | 0.30991 |
| 0.3 | 0.97765 | 0.31133 |
| 0.4 | 0.96852 | 0.32234 |
| 0.5 | 1.21323 | 0.26978 |
| 1.0 | 1.22314 | 0.31126 |
| 1.5 | 1.24122 | 0.32341 |
| 0.3 | 1.15674 | 0.52345 |
| 0.5 | 1.18902 | 0.52109 |
| 0.7 | 1.20045 | 0.51123 |
| 2   | 1.00092 | 0.32943 |
| 3   | 1.01321 | 0.37865 |
| 4   | 1.12094 | 0.40188 |
| 0.4 | 1.21322 | 0.53421 |
| 0.6 | 1.22134 | 0.52334 |
| 0.8 | 1.34210 | 0.51232 |
| 1.1 | 0.95543 | 0.48331 |
| 1.2 | 0.95342 | 0.45672 |
| 1.3 | 0.94432 | 0.44578 |
| 0.1 | 0.89900 | 0.73321 |
| 0.2 | 0.71121 | 0.74123 |
| 0.3 | 0.64321 | 0.74456 |
| 0.2 | 1.09821 | 0.71120 |
| 0.4 | 0.00123 | 0.81254 |
| 0.6 | 0.00009 | 0.98234 |
| 0.7 | 0.88921 | 0.62210 |
| 0.9 | 0.87651 | 0.71109 |
| 1.1 | 0.86698 | 0.89890 |

## 5 Conclusions

The molybdenum disulphide and magnesium oxide nanoparticle performance through C-C heat flux and melting process in magneto-micro nanofluid is studied. The outcomes of the velocity and temperature profiles for MoS$_{2}$ and MgO nanoparticle cases against various non-dimensional parametric values are presented graphically. Based on the tabulated and graphical results, we extract the following conclusions.

(i) The velocity profile rises up with $\phi$, $M_{1}$, and $K$, but a decay trend is noticed against improving values of $M$ and $\lambda_{2}$.

(ii) The temperature profile increases with mounting values of $Ec$ while decreases with augmented $Pr$, $M_{1}$, and $\delta_{1}$.

(iii) The skin friction coefficient rises up against higher values of $M$, $\lambda_{2}$, $Pr$, and $\delta_{1}$ while decreases for larger values of $K$, $\phi$, $\delta^{*}$, and $M_{1}$.

(iv) The Nusselt number is magnified with boosting values of $Pr$, $M_{1}$, $K$, and $\phi$ while decreases for increasing values of $\lambda_{2}$, $\delta_{1}$, and $Ec$.

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