The Varying Speed of Light in Eddington-inspired Born-Infield Gravity with Rainbow Metric

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(Dated: September 19, 2018)

Abstract

In this paper, we consider the auxiliary metric $g_{\mu\nu}$ in Eddington-inspired Born-Infield gravity as rainbow metric. When we study FRW universe, we find the energy of test particle will increase with rising energy density of universe. When considering the energy of test particle is proportional to the temperature of cosmic microwave background radiation, we find rainbow gravity would place constraints on Eddington-inspired Born-Infield gravity. We also research the varying speed of light for three different kinds of rainbow functions, two results show that the speed of light is decreasing while the energy density of universe is rising. The other is constant speed of light.

PACS numbers: 98.80.-k, 04.50.Kd

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I. INTRODUCTION

Since General Relativity was found by Einstein, some phenomena were perfectly explained. It not only predicted and proved gravitational red shift and light deflection, but also explained the expansion of universe by the Friedmann equation. But the explanation about accelerating expansion of early universe is lack of a satisfied solution. For looking for the answer of the problem, many reformed gravity theories were put forward. Based on Eddington theory [1] and non-linear electrodynamics of Born and Infeld [2], Eddington-inspired Born-Infeld (EiBI) gravity was proposed by Máximo Bañados and Pedro G.Ferreira [3]. Different from the metric Born-Infeld-Einstein theory [4] and the purely affine Eddinton theory[1], EiBI gravity is a Palatini theory of gravity. This theory [3] supposes the metric $g_{\mu\nu}$ and the connection $\Gamma^\alpha_{\beta\gamma}$ are varied independently and can get a no-singularity universe [5] or bounding universe [3].

Based on Special Relativity and modified energy-momentum dispersion relation, the rainbow gravity was developed by João Magueijo and Lee Smolin [6–8]. In this theory, the geometry of spacetime depends on the energy of the test particle and observers with different energy would see different geometries of spacetime. Hence, a family of energy-dependent metrics named as rainbow metrics will describe the geometry of spacetime, which is different from general gravity theory. Some researches show that a rainbow gravity can also get no-singularity universe [9, 10] or bounding universe [11, 12] in cosmic background. Through modified energy-momentum dispersion relation, the rainbow gravity might lead to the varying speed of light (VSL) scenario [13, 14], which depends on the form of rainbow function. In this paper, we relate EiBI gravity with rainbow gravity.

In Sec.II, we review the general ideas about Eddington-inspired Born-Infeld gravity and rainbow gravity. In Sec.III, we identify the auxiliary FRW metric in EiBI
gravity with rainbow FRW metric and find some contacts between the energy density of FRW universe and the energy of test particle using three diverse rainbow functions. Sec.IV gives a summarization.

II. THE BACKGROUND ABOUT EDDINGTON-INSPIRED BORN-INFELD GRAVITY AND RAINBOW GRAVITY

A. Eddington-inspired Born-Infeld gravity

The EiBI action was proposed as $[3, 15]:$

$$S_{EiBI}[g, \Gamma, \Psi] = \frac{1}{8\pi G\kappa} \int d^4x [\sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}|} - \lambda \sqrt{|g|}] + S_M[g, \Gamma, \Psi],$$

(1)

where, $R_{\mu\nu}$ only depends on connection $\Gamma^\sigma_{\mu\nu}$. The equations of motion for this theory are in the following. By varying with respect to the metric $g_{\mu\nu}$, one can get

$$\frac{\sqrt{|g + \kappa R|}}{\sqrt{|g|}} [(g + \kappa R)^{-1}]^{\mu\nu} - \lambda g^{\mu\nu} = -\kappa T^{\mu\nu}. \quad (2)$$

By introducing an auxiliary metric $q_{\mu\nu}$ compatible with $\Gamma^\lambda_{\mu\nu}$, the variation with respect to connection $\Gamma^\lambda_{\mu\nu}$ can be simplified as

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}, \quad (3)$$

and $\Gamma^\mu_{\alpha\beta} = \frac{1}{2} q^{\mu\sigma} (q_{\sigma\alpha,\beta} + q_{\sigma\beta,\alpha} + q_{\alpha\beta,\sigma}).$ Combining the two equations, one derives the equation

$$\sqrt{q} q^{\mu\nu} = \lambda g^{\mu\nu} - \kappa T^{\mu\nu}. \quad (4)$$

Now we focus on cosmology. Assume that the FRW metric corresponding to $g_{\mu\nu}$ can be shown as

$$ds^2 = -dt^2 + a(t)^2 d\vec{x} \cdot d\vec{x}, \quad (5)$$

3
which describes a homogeneous and isotropic flat universe. The metric couples with ideal fluid \( T^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu} \). The components of the other metric \( q_{\mu\nu} \) can also be assumed as \( q_{00} = -U \) and \( q_{ij} = a^2 V \delta_{ij} \), so we have

\[
d s_q^2 = -U dt^2 + a(t)^2 V d\vec{x} \cdot d\vec{x}.
\] (6)

The calculation indicates\(^3\)

\[
U = \frac{D}{1 + \kappa \rho T}, \quad V = \frac{D}{1 - \kappa P_T},
\] (7)

where \( D = \sqrt{(1 + \kappa \rho T)(1 - \kappa P_T)^3} \), \( \rho_T = \rho + \Lambda \), and \( P_T = p - \Lambda \).

According to Ref.\(^{16}\), the particular “bigravity” theory with metrics \([g_{\mu\nu}, q_{\mu\nu}]\) was studied with the action

\[
S = \frac{1}{16\pi G} \int \sqrt{-g}(K - 2\Lambda) + \sqrt{-q}(R - 2\Upsilon) + \frac{1}{l^2} \sqrt{-q}[\pm q^{\alpha\beta} g_{\beta\gamma}] + S_M[g, q, \Psi],
\] (8)

which was studied in \([16][25]\). Here \( \Lambda \) and \( \Upsilon \) are cosmological constants for each sector of the theory. \( K \) and \( R \) are the Ricci scalars for metric \( g_{\mu\nu} \) and \( q_{\mu\nu} \). Varying with respect to \( q_{\mu\nu} \), the equation can be algebraically solved for this field

\[
q_{\mu\nu} = \frac{1}{\Upsilon}(R_{\mu\nu} \pm \frac{1}{l^2} g_{\mu\nu}).
\] (9)

Supposed a special condition \( K = 0 \), we find the Eq.(9) continues to be kept. As a consequence, EiBI gravity would be seen as a kind of bigravity. When we choose plus sign in Eqs.(8) and (9), we can get

\[
S = \frac{1}{16\pi G} \int \sqrt{-g}(-2\Lambda) + \sqrt{-q}(2\Upsilon) + S_M[g, \Gamma, \Psi],
\] (10)

when \( \frac{1}{\Upsilon} = \kappa \), \( \Upsilon l^2 = 1 \) and \( \Lambda = \frac{\Lambda}{\kappa} \), the Eq.(10) is the same with Eq.(1), \( S = S_{EiBI} \).
B. rainbow gravity

It is generally believed that the geometry of spacetime is fundamentally described by a quantum theory and the Planck energy plays an important role of threshold separating the classical description from the quantum description. There is a choice considering Planck energy\[6\]

\[ E^2 \cdot f^2(E/E_P) - p^2 \cdot g^2(E/E_P) = m^2, \]  
(11)

which is a modified energy-momentum dispersion relation. This can be realized by the action of a non-linear map from momentum space to itself, denoted as \( U : P \rightarrow P \), given by \[7\]

\[ U \cdot (E, p_i) = (U_0, U_i) = (f(E/E_P)E, g(E/E_P)p_i), \]  
(12)

which implies that momentum space has a non-linear norm, given by

\[ \|p\|^2 = \eta^{ab}U_a(p)U_b(p). \]  
(13)

This norm is preserved by a non-linear realization of the Lorentz group, given by

\[ \tilde{L}_a^b = U^{-1} \cdot L_a^b \cdot U, \]  
(14)

where \( L_a^b \) are the usual generators.

An approach developed in Ref.\[26\] shows that free field theories in flat space-time have plane wave solutions, even through the 4-momentum satisfies deformed dispersion relations. For this to be possible the contraction between position and momentum must remain linear. This is

\[ dx^a p_a = dx^0 p_0 + dx^i p_i. \]  
(15)

If momentum transforms non-linearly (from a non-linear action derived from the generators Eq.\((14)\)), this requires that the \( dx^a \) transformation is energy-dependent,
as explained in [26]. Based on a map $U$ in [12], it is not difficult to show that the spacetime dual leads to [6]

$$ds^2 = -\frac{1}{f^2}dx_0^2 + \frac{1}{g^2}d\vec{x} \cdot d\vec{x}.$$  \hfill (16)

When the method was extended to curved spacetime, the deformed FRW metric can be shown [6]

$$ds^2 = -\frac{1}{f^2}dt^2 + \frac{1}{g^2}a(t)^2d\vec{x} \cdot d\vec{x},$$  \hfill (17)

here, $f$ and $g$ are energy-dependent rainbow functions.

### III. THE RELATIONSHIP BETWEEN EDDINGTON-INSPIRED BORN-INFELD GRAVITY AND RAINBOW GRAVITY

Generally, based on Eq.(10), the EiBI gravity can be explained as bigravity with some constraint conditions, however, it is still a question what the physical explanations about the two metrics is. Rosen had proposed that at each point of spacetime, in addition to the Riemannian metric tensor which describes the geometry of spacetime and the gravitational field, there is an Euclidean metric tensor which refers to the flat spacetime and describes the inertial force [27, 28]. In massive gravity, except for general Riemannian metric, a reference metric, corresponding to the background metric around which fluctuations take the Fierz-Pauli form, was introduced in order to construct the interaction term [29–31].

The similarity between the Eqs.(6) and (17) inspires us to think that the auxiliary metric $q_{\mu\nu}$ may be the rainbow metric, so the two metrics in EiBI gravity can be respectively explained as Riemannian metric and rainbow metric. Based on this hypothesis, we will proceed to following our work.

Currently, through some researches of physicists, three rainbow functions were put forward [32, 33]. In order to test the feasibility for our hypothesis, the three rainbow
functions will be respectively studied. Firstly, originated from loop quantum gravity and noncommutative spacetime\cite{34, 35}, the rainbow functions are

\[ f(E/E_P) = 1, \quad g(E/E_P) = \sqrt{1 - \eta E/E_P}, \quad (18) \]

where $\eta$ is a model parameter. Based on the hypothesis above, $U = 1$ and $V = \frac{1}{g(E/E_P)^2}$ can be given, so the relation between the energy of test particle and energy density of universe can be shown as

\[ E = \frac{E_P}{\eta} \left( 1 - \frac{1}{(1 + \kappa \rho_T)^{2/3}} \right), \quad (19) \]

which is described in Fig.1. Eq.(15) indicates when $(\kappa \rho_T)^{2/3}$ tends to zero the energy of test particle tends to zero.

![Figure 1. $E - \rho_T$ diagram. It corresponds to $E_P = 1$ and $\eta = 1$. From top to bottom, the $\kappa$ takes respectively 1, 0.1, 0.05.](image)

Generally, the density parameter $\Omega$ was showed as

\[ \Omega = \frac{8\pi G \rho}{3H^2}. \quad (20) \]
Here, the present Hubble parameter $H_0 = (73.52 \pm 1.62) \, km \cdot s^{-1} \cdot Mpc^{-1} = (2.38 \pm 0.05) \times 10^{-18} s^{-1}$ [36]. In general, $G = 6.67 \times 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2}$ and $k = 1.38 \times 10^{-23} J \cdot K^{-1}$ and Plank energy $E_P = 1.959 \times 10^9 J$. So the critical energy density $\rho_C \approx (1.01 \pm 0.04) \times 10^{-26} kg/m^3$. Suppose that the energy of test particle is proportional to cosmic microwave background radiation (CMBR) temperature $E = kT$. Because $\rho_T$ includes cosmological constant, so we suppose $\Omega = \rho_T/\rho_c = 1$ and $\rho_T = \rho_c$. Using taylor expansion for Eq.(19) when $E/E_P$ or $\kappa \rho_T$ is smaller than 1

$$kT \approx \frac{E_P}{\eta} \frac{2}{3} \kappa \rho_T. \quad (21)$$

The constraint between $\kappa$ and parameter $\eta$ can be given

$$\frac{\kappa}{\eta} \approx \frac{4\pi G kT}{\Omega E_P H^2} = 2.8345 \times 10^{-6} m^3/kg, \quad (22)$$

here, we suppose the energy of test particle is proportional to the temperature of CMBR $T \cong 2.72548 \pm 0.00057 K$ [37]. Due to $E \leq E_P$, so we can get $\eta \geq 1$ with $\kappa \leq 2.8345 \times 10^{-6} m^3/kg$. As a result, $\kappa \rho_T \leq 2.8628 \times 10^{-32}$ for current universe.

Generally, the VSL scenario [13, 14] is an interesting alternative to cosmological inflation. It was found in [26, 38, 39] that some deformed dispersion relations might lead to a realization of VSL. According to Eq.(11), one makes $f_3 = \frac{\theta(E)}{f(E)}$ and can find (here, the mass of photon $m = 0$)

$$c = \frac{dE}{dp} = \frac{f_3}{1 - \frac{E_P f_3}{f_3}}, \quad (23)$$

where, $f_3' = \frac{df_3}{dE}$. When $E \to 0$ or $f_3 = 1$, one can find the speed of light is a constant which is $c_0$. From rainbow function of Eq.(18) we have

$$c = \frac{2}{(1 + \kappa \rho_T) + (1 + \kappa \rho_T)^{1/3}}, \quad (24)$$

For current universe, we can find $c \approx (1 - 1.9085 \times 10^{-32})c_0$. 8
Secondly, in order to explain the hard spectra of gamma-ray bursts at cosmological distances, the rainbow functions are proposed to be

\[
\begin{align*}
  f(E/E_P) &= \frac{\exp(\alpha E/E_P) - 1}{\alpha E/E_P}, \\
  g(E/E_P) &= 1,
\end{align*}
\]

(25)

where \( \alpha \) is a model parameter. Similar to above discussion, \( U = \frac{1}{f(E/E_P)^2} \) and \( V = 1 \) can lead to

\[
\kappa \rho_T = \frac{\exp(\alpha E/E_P) - 1}{\alpha E/E_P} - 1,
\]

(26)

which is described in Fig. 2. Through Eq. (26), we can find that \( E \) will also increase monotonically with rising \( \rho_T \). In particular, when \( \kappa \rho_T \) tends to zero, the energy \( E \) tends to zero. Using taylor expansion for Eq. (26) when \( E/E_P \) or \( \kappa \rho_T \) is smaller than 1, we can get

\[
kT \approx \frac{E_P}{\alpha} 2\kappa \rho_T.
\]

(27)

Figure 2. \( E - \rho_T \) diagram. It corresponds to \( E_P = 1 \) and \( \kappa = 1 \). From top to bottom, the \( \alpha \) takes respectively 2, 1, 0.5.
The constraint between $\kappa$ and parameter $\alpha$ can be given

$$\frac{\kappa}{\alpha} \approx \frac{4\pi GkT}{3\Omega_E H^2} = 0.9448 \times 10^{-6} m^3/kg. \quad (28)$$

Suppose that $\alpha \geq 1$, we can obtain $\kappa \leq 0.9448 \times 10^{-6} m^3/kg$. As a result, $\kappa \rho_T \leq 0.9542 \times 10^{-32}$ for present universe. For the rainbow function of Eq.(25), the speed of light can be expressed as

$$c = \exp \left(-\frac{\alpha E}{E_P}\right) \approx \exp \left(-2\kappa \rho_T\right), \quad (29)$$

so we can find for present universe $c \approx (1 - 1.9084 \times 10^{-32})c_0$ and $c_0 \approx c$.

Thirdly, providing a constant speed of light and a solution to the horizon problem, the rainbow functions are proposed to be \[41\]

$$f\left(\frac{E}{E_P}\right) = g\left(\frac{E}{E_P}\right) = \frac{1}{1 - \lambda_1 \frac{E}{E_P}}, \quad (30)$$

where $\lambda_1$ is a model parameter and may have either sign\[7\]. When $U = V = \frac{1}{f\left(\frac{E}{E_P}\right)^2} = \frac{1}{g\left(\frac{E}{E_P}\right)^2}$, we can get

$$E = \frac{E_P}{\lambda_1} \left(1 - \sqrt{1 + \kappa \rho_T}\right), \quad (31)$$

which indicates $E \leq 0$ with positive $\kappa \rho_T$ and $\lambda_1$. In order to make $E > 0$, we take $\lambda = -\lambda_1 > 0$. It is described in Fig.(3). Using taylor expansion for Eq.(31) when $E/E_P$ or $\kappa \rho_T$ is smaller than 1

$$kT \approx \frac{E_P}{\lambda} \frac{1}{2} \kappa \rho_T. \quad (32)$$

The constraint between $\kappa$ and $\lambda$ can be given

$$\frac{\kappa}{\lambda} \approx \frac{16\pi GkT}{3\Omega_E H^2} = 3.7793 \times 10^{-6} m^3/kg. \quad (33)$$

Suppose that $\lambda \geq 1$, we can also obtain $\kappa \leq 3.7793 \times 10^{-6} m^3/kg$. As a result, $\kappa \rho_T \leq 3.8171 \times 10^{-32}$ for present universe. Due to $f_3 = 1$ in Eq.(30), the speed of light keeps constant.
IV. CONCLUSION AND DISCUSSIONS

In the paper, we review EiBI gravity and rainbow gravity. Because EiBI gravity can be seen as a special bigravity, the physical explanation may be a focusing question about the two metrics. The similarity between auxiliary FRW metric in EiBI gravity and rainbow FRW metric inspires us to think that the auxiliary metric is in agreement with FRW rainbow metric. Based on above hypothesis, the three forms of rainbow function have been respectively studied.

Firstly, based on three rainbow functions, we get three relations between the energy of test particle $E$ and the energy density of universe $\rho_T$ and find that the energy of test particle is increasing with rising energy density for all of them. When $\rho_T \to 0$, $E \to 0$ and $\rho_T \to \infty$, $E \to \frac{E_P}{\eta}, E \to \frac{E_P}{\alpha}, E \to \frac{E_P}{\lambda}$. These results indicate the energy density $\rho_T$ may be sizable in early universe and may be extremely small in current universe.

Secondly, because some features of evolution of universe are reflected by the tem-
perature of CMBR, we suppose that the energy of test particle $E = kT$ (here, the temperature $T$ equals to the temperature of CMBR), which shows three constraints between $\kappa$ and other parameters ($\eta$, $\alpha$ and $\lambda$). In this paper, due to smaller energy density in current universe, we get three approximate expression about them. The restriction would give a way to test the validity of EiBI gravity.

Thirdly, based on three rainbow functions, we research whether the speed of light would be influenced by them. For two conditions of them (Eqs. (18) (25)), we find those speeds of light are increasing with the evolution of universe and only depend on cosmic time. In early universe, the speed of light is smaller than current state, but in current universe the speed of light approaches to $c_0$. In particular, the rate of change about the speed of light is negligible in current universe. The solutions allow us to imagine that the speed of light is approximate constant in current universe. For another rainbow function Eq. (30), the speed of light keeps constant. Based on above researches, we find our assumption might be a right choice to study EiBI gravity.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

ACKNOWLEDGMENTS

We would like to thank the National Natural Science Foundation of China (Grant No.11571342) for supporting us on this work.
REFERENCES

[1] RD Carmichael et al. As eddington, the mathematical theory of relativity. *Bulletin of the American Mathematical Society*, 31(9-10):563–563, 1925.

[2] Max Born and Leopold Infeld. Foundations of the new field theory. *Proceedings of the Royal Society of London*, 144(852):425–451, 1934.

[3] Maximo Banados and Pedro G Ferreira. Eddingtons theory of gravity and its progeny. *Physical review letters*, 105(1):011101, 2010.

[4] Stanley Deser and GW Gibbons. Born-infeld-einstein actions? *Classical and Quantum Gravity*, 15(5):L35, 1998.

[5] Jose Beltrán Jiménez, Lavinia Heisenberg, Gonzalo J Olmo, and Diego Rubiera-Garcia. On gravitational waves in born-infeld inspired non-singular cosmologies. *Journal of Cosmology and Astroparticle Physics*, 2017(10):029, 2017.

[6] Joao Magueijo and Lee Smolin. Gravity’s rainbow. *Classical and Quantum Gravity*, 21(7):1725, 2004.

[7] Joao Magueijo and Lee Smolin. Generalized lorentz invariance with an invariant energy scale. *Physical Review D*, 67(4):044017, 2003.

[8] Joao Magueijo and Lee Smolin. String theories with deformed energy-momentum relations, and a possible nontachyonic bosonic string. *Physical Review D*, 71(2):026010, 2005.

[9] Yi Ling. Rainbow universe. *Journal of Cosmology and Astroparticle Physics*, 2007(08):017, 2007.

[10] Adel Awad, Ahmed Farag Ali, and Barun Majumder. Nonsingular rainbow universes. *Journal of Cosmology and Astroparticle Physics*, 2013(10):052, 2013.
[11] Yi Ling and Qingzhang Wu. The big bounce in rainbow universe. *Physics Letters B*, 687(2-3):103–109, 2010.

[12] Wen-Jian Pan and Yong-Chang Huang. Bouncing universe with modified dispersion relation. *General Relativity and Gravitation*, 48(11):144, 2016.

[13] Andreas Albrecht and João Magueijo. Time varying speed of light as a solution to cosmological puzzles. *Phys. Rev. D*, 59:043516, Jan 1999.

[14] João Magueijo. New varying speed of light theories. *Rept. Prog. Phys.*, 66:2025, 2003.

[15] Shao-Wen Wei, Ke Yang, and Yu-Xiao Liu. Black hole solution and strong gravitational lensing in Eddington-inspired Born-Infeld gravity. *Eur. Phys. J.*, C75:253, 2015. [Erratum: Eur. Phys. J.C75,331(2015)].

[16] Maximo Banados, Andres Gomberoff, Davi C Rodrigues, and Constantinos Skordis. Note on bigravity and dark matter. *Physical Review D*, 79(6):063515, 2009.

[17] CJ Isham, Abdus Salam, and J Strathdee. F-dominance of gravity. *Physical Review D*, 3(4):867, 1971.

[18] CJ Isham and D. Storey. Exact spherically symmetric classical solutions for the f-g theory of gravity. *Phys. Rev. D*, 18:1047, 1978.

[19] Thibault Damour. T. damour, ii kogan, and a. papazoglou. *Phys. Rev. D*, 66:104025, 2002.

[20] Thibault Damour and Ian I Kogan. Effective lagrangians and universality classes of nonlinear bigravity. *Physical Review D*, 66(10):104024, 2002.

[21] H. Georgi Arkani-Hamed Nima and MD Schwartz. Effective field theory for massive gravitons and gravity in theory space. *Ann. Phys.(NY)*, 305:96, 2003.

[22] Diego Blas, Cedric Deffayet, and Jaume Garriga. Causal structure of bigravity solutions. *Classical and Quantum Gravity*, 23(5):1697, 2006.

[23] Z Berezhiani, D Comelli, F Nesti, and L Pilo. Spontaneous lorentz breaking and massive gravity. *Physical review letters*, 99(13):131101, 2007.
[24] Z Berezhiani, D Comelli, F Nesti, and Luigi Pilo. Exact spherically symmetric solutions in massive gravity. *Journal of High Energy Physics*, 2008(07):130, 2008.

[25] Cedric Deffayet. Spherically symmetric solutions of massive gravity. *Classical and Quantum Gravity*, 25(15):154007, 2008.

[26] Dagny Kimberly, João Magueijo, and João Medeiros. Nonlinear relativity in position space. *Phys. Rev.*, D70:084007, 2004.

[27] N. Rosen. General relativity and flat space. i. *Phys. Rev.*, 57:147–150, Jan 1940.

[28] N. Rosen. General relativity and flat space. ii. *Phys. Rev.*, 57:150–153, Jan 1940.

[29] Claudia de Rham and Gregory Gabadadze. Generalization of the fierz-pauli action. *Phys. Rev. D*, 82:044020, Aug 2010.

[30] Claudia de Rham, Gregory Gabadadze, and Andrew J. Tolley. Resummation of massive gravity. *Phys. Rev. Lett.*, 106:231101, Jun 2011.

[31] S. F. Hassan, Rachel A. Rosen, and Anngis Schmidt-May. Ghost-free Massive Gravity with a General Reference Metric. *JHEP*, 02:026, 2012.

[32] Seyed Hossein Hendi and Mir Faizal. Black holes in gauss-bonnet gravity’s rainbow. *Physical Review D*, 92(4):044027, 2015.

[33] Xue-Mei Deng and Yi Xie. Gravitational time advancement under gravity’s rainbow. *Physics Letters B*, 772:152–158, 2017.

[34] Giovanni Amelino-Camelia. Quantum-spacetime phenomenology. *Living Reviews in Relativity*, 16(1):5, 2013.

[35] Uri Jacob, Flavio Mercati, Giovanni Amelino-Camelia, and Tsvi Piran. Modifications to lorentz invariant dispersion in relatively boosted frames. *Physical Review D*, 82(8):084021, 2010.

[36] Adam G. Riess et al. Milky Way Cepheid Standards for Measuring Cosmic Distances and Application to Gaia DR2: Implications for the Hubble Constant. 2018.

15
[37] D. J. Fixsen. The temperature of the cosmic microwave background. *The Astrophysical Journal*, 707(2):916, 2009.

[38] Stephon Alexander and Joao Magueijo. Noncommutative geometry as a realization of varying speed of light cosmology. In *Proceedings, XIIIth Rencontres de Blois on Frontiers of the Universe: Blois, France, June 17-23, 2001*, pages 281–297, 2004.

[39] Stephon Alexander, Robert Brandenberger, and Joao Magueijo. Noncommutative inflation. *Phys. Rev.*, D67:081301, 2003.

[40] Giovanni Amelino-Camelia, John Ellis, NE Mavromatos, Dimitri V Nanopoulos, and Subir Sarkar. Tests of quantum gravity from observations of γ-ray bursts. *Nature*, 393(6687):763, 1998.

[41] Joao Magueijo and Lee Smolin. Lorentz invariance with an invariant energy scale. *Physical Review Letters*, 88(19):190403, 2002.