Physics in 2006

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Abstract. Any consideration of future physics facilities must be made in the context of the Tevatron and the LHC. I discuss some examples of physics results which could emerge from these machines and the resulting questions which would remain for a high energy $e^+e^-$ collider. Particular attention is paid to the electroweak symmetry breaking sector. If a light Higgs boson exists, it will be observed at the LHC and the role of any later accelerator will be to map out the Higgs’s boson mass and couplings and then determine the space of possible models. If there is no light Higgs boson then some effects of a strongly interacting electroweak symmetry breaking sector will be observed at the LHC and I discuss the role of a high energy linear collider in exploring this scenario.

INTRODUCTION

Physicists are presently faced with a quandry. Plans and designs for the next generation of accelerators need to be formulated in the near future, since the construction of these machines spans many years. The “best” machine to build for the post-LHC era will only become clear, however, when we see what surprises the LHC holds. Faced with our imperfect knowledge, we must examine possible scenarios for LHC physics and determine the machine most likely to answer the physics questions remaining in the years following completion of the LHC and the fulfillment of its physics promise.

Here, I will consider the physics of electroweak symmetry breaking at a high energy $e^+e^-$ collider. This discussion must be made in the context of potential discoveries at the Tevatron and the LHC. I begin by reviewing the current experimental status of electroweak symmetry breaking, both from direct Higgs boson searches at LEP2 and from precision electroweak measurements. Theoretical

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1) Supported by the U.S. Department of Energy under Contract No. DE-AC02-76CH00016.
2) For values of the Higgs boson mass near and slightly above 100 GeV, a $\mu^+\nu^-\nu^+$ collider operating at the Higgs resonance can make extremely precise measurements of the Higgs mass and couplings. I will not discuss the physics potential of a muon collider here. [1]
expectations for the Higgs boson mass are then reviewed, with emphasis on the implications for physics at higher mass scales.

Next, I review the discovery prospects for the Higgs boson at both the Tevatron and the LHC. The working hypothesis is that a weakly interacting Higgs boson will be discovered, if it exists, at the Tevatron or the LHC, and so the role of the next generation of accelerators will be to study the properties of a Higgs boson. Precision measurements of the mass, decay widths, and production rates will all be necessary in order to verify that a particle is the Higgs boson of the Standard Model.

Aside from the couplings to fermions and gauge bosons, we would also like to know that the Higgs boson self-interactions result from the spontaneously broken scalar potential of the Standard Model. In order to do this, the three- and four-point self-couplings of the Higgs boson must be measured. These couplings can only be probed by multi-Higgs production, which has extremely small rates, both at the LHC and at a high energy $e^+e^-$ collider.

The focus in this note is on verifying the properties of the Standard Model Higgs boson. In order to do this, it is helpful to compare with the predictions of a supersymmetric model since these predictions may be quite different from those of the Standard Model. Distinguishing between the Standard Model and a supersymmetric model is an important test of our understanding of the electroweak sector.

If a Higgs boson is not found at the Tevatron or the LHC, the electroweak symmetry breaking sector must be strongly interacting. I end with a brief discussion of strong electroweak symmetry breaking and a view towards the future.

**INFERENCES FROM THE STANDARD MODEL**

The Standard Model of electroweak interactions has been verified to the .1% level through precision measurements at LEP and SLD. \[2\] In fact, the mechanism of electroweak symmetry breaking remains the only unconfirmed area of the Standard Model. The Standard Model predicts the existence of a physical scalar particle, termed the Higgs boson. The search for this particle is therefore a fundamental goal of all current and future accelerators since its discovery is needed to complete our knowledge of the electroweak sector. The mass is a free parameter of the theory and so the Higgs boson must be systematically sought in all mass regions.

The couplings of the scalar Higgs boson, however, are completely specified in terms of the Higgs vacuum expectation value, $v = 246 \text{ GeV}$. Hence branching ratios and production rates can be computed unambiguously in terms of the mass. Measurements of ratios of branching rates can then be used to test the validity of the model.

Since the Higgs boson contributes to electroweak radiative corrections at one loop, precision measurements from LEP and SLD can be used to infer a preferred value for the Higgs mass. The contribution of the Higgs boson to electroweak
observables is logarithmic and so the limit on the Higgs mass is not nearly as precise as the indirect limit on the top quark mass from precision measurements. The current 95% confidence level limit is, \[ M_h < 230 \text{ GeV}, \] Precision Measurements. \[ (1) \]

It is important to understand that this limit assumes the validity of the Standard Model. Quantum loops containing new particles can change this limit, as can new operators beyond those of the Standard Model. If there is new physics at the TeV scale, the limit of Eq. 1 can be evaded. \[ (3) \]

The Higgs boson mass is the only free parameter of the electroweak theory. Although we cannot compute its mass, there are certain theoretical restrictions following from the consistency of the theory. The scalar potential for an \( SU(2) \) scalar doublet \( \Phi \) is,

\[ V = -\mu^2 |\Phi|^2 + \lambda(|\Phi|^2)^2. \] \[ (2) \]

After the electroweak symmetry breaking has occurred, there remains the physical scalar Higgs boson \( h \). The quartic coupling, \( \lambda \), is related to the Higgs boson mass,

\[ \lambda = \frac{M_h^2}{2v^2}. \] \[ (3) \]

\[ \begin{array}{c}
\text{FIGURE 1.} \text{ Theoretical expectations for a Standard Model Higgs boson, as a function of the scale } \Lambda \text{ above which the Standard Model is no longer valid. The region above the solid curve has } \\
\lambda(\Lambda) \rightarrow \infty, \text{ while the region below the dotted line has } \lambda(\Lambda) < 0. \text{ The allowed region is the region between the curves.}
\end{array} \]
FIGURE 2. Discovery potential for a Standard Model Higgs boson at the Tevatron. The vertical axis shows the required luminosity to observe or exclude a given Higgs boson mass. At the lower values of the Higgs mass two curves are shown; the lower in each set is a neural net analysis, the upper a standard analysis with cuts on the signal and background. From Ref. 11.

Now $\lambda$ is not a fixed parameter, but scales with the relevant energy, $Q$, and so Eq.
2 is the potential at the electroweak scale. If $\lambda$ is large, (corresponding to a heavy Higgs boson), then at a scale $Q$, [4]

$$Q \frac{d\lambda}{dQ} = \frac{3}{4\pi^2} \lambda,$$

(4)

which can be solved to obtain,

$$\frac{1}{\lambda(\Lambda)} = \frac{1}{\lambda(M_h)} - \frac{3}{4\pi^2} \log\left(\frac{\Lambda^2}{M_h^2}\right).$$

(5)

A sensible theory will have $\lambda(\Lambda)$ finite at all scales, ($\lambda(\Lambda) \to \infty$ is termed the Landau Pole), or correspondingly $\frac{1}{\lambda(\Lambda)} > 0$. This yields an upper bound on $\lambda$ and hence on $M_h^2$,

$$M_h^2 < \frac{8\pi^2 v^2}{3 \log(\Lambda^2/M_h^2)}.$$  

(6)

If the Standard Model is valid to the GUT scale, $\Lambda \sim 10^{16} \text{ GeV}$, then we have an approximate upper bound on the Higgs mass, [4,5]
\[ M_h < 170 \text{ GeV} \]  \hspace{1cm} (7)

For any given value of \( \Lambda \), there is a corresponding upper bound on \( M_h \). \( \Lambda \) is often termed the “scale of new physics” since above this scale, the Standard Model interactions are not valid. This bound is the upper curve on Figure 1.

There is also a theoretical lower bound on \( M_h \). If \( \lambda \) is small (light \( M_h \)), then
\[
Q \frac{d\lambda}{dQ} \sim \frac{1}{16\pi^2} (B - 12g_t^4),
\]  \hspace{1cm} (8)

where \( g_t \) is the Higgs-top quark Yukawa coupling, \( g_t = -M_t/v \), and \( B \) is a function of the gauge coupling constants, \( B = \frac{3}{16}(2g^4 + (g^2 + g'^2)^2) \). We see that the large top quark mass tends to drive \( \lambda \) negative. In order for electroweak symmetry breaking to occur, the potential must remain bounded from below, and \( \lambda \) positive. Solving Eq. 8,
\[
\lambda(\Lambda) = \lambda(M_h) + \frac{B - 12g_t^4}{16\pi^2} \log\left( \frac{\Lambda}{M_h} \right).
\]  \hspace{1cm} (9)

Requiring \( \lambda(\Lambda) > 0 \) gives the lower bound on \( M_h \),
\[
\frac{M_h^2}{2v^2} > \frac{B - 12g_t^4}{16\pi^2} \log\left( \frac{\Lambda}{M_h} \right).
\]  \hspace{1cm} (10)

For large \( M_t \), this relation changes sign and the two-loop renormalization group corrections are important to obtain the numerical bound. Requiring that the Standard Model be valid to the GUT scale, \( \Lambda = 10^{16} \text{ GeV} \), gives the restriction on the Higgs boson mass [6]
\[ M_h > 130 \text{ GeV} \]  \hspace{1cm} (11)

This is shown as the lower curve in Figure 1. There have been many theoretical improvements to the naive bounds presented above, but the bottom line is the same: If the Standard Model is valid to the GUT scale, then
\[ 130 \text{ GeV} < M_h < 180 \text{ GeV}, \quad \Lambda \sim 10^{16} \text{ GeV}. \]  \hspace{1cm} (12)

A Higgs boson outside this mass region would be a signal for new physics at the corresponding scale \( \Lambda \). The mass region of Eq. 12 is particularly interesting since it could potentially be probed at the Tevatron with an upgraded luminosity.

There are also absolute bounds on the Higgs boson mass which are independent of the scale, \( \Lambda \). Unitarity of the WW elastic scattering amplitudes requires \( M_h < 800 \text{ GeV} \), while lattice calculations obtain a similar bound, \( M_h < 700 \text{ GeV} \). [7] All of the theoretical bounds of this section predict a Higgs boson comfortably within the discovery range of the LHC and so if the Standard Model is correct, a Higgs boson discovery should be just around the corner.
PROSPECTS FOR DISCOVERY

The current 95% confidence level limit on the Higgs boson mass from direct searches at LEP2 using data from \( \sqrt{s} = 189 - 202 \text{ GeV} \) is [8]

\[
M_h > 106 \text{ GeV}, \quad \text{LEP2.} \tag{13}
\]

This limit is not expected to improve substantially with further running at LEP2.

The minimal supersymmetric model has two neutral Higgs bosons, \( h^{SUSY} \) and \( H^{SUSY} \), a charged Higgs, \( H^\pm \), and a pseudoscalar, \( A \). The structure of the supersymmetric potential dictates that at lowest order all the couplings can be expressed in terms of two parameters, which are typically taken to be the pseudoscalar mass, \( M_A \), and the ratio of Higgs vacuum expectation values, \( \tan \beta \). All masses can then be expressed in terms of these two parameters. [9]

The experimental limit on the Higgs boson mass in a supersymmetric theory typically depends on \( \tan \beta \). If we require that the limit be valid for all \( \tan \beta \), there is a slightly lower 95% confidence level limit than for the Standard Model Higgs boson, [8]

\[
M_h^{SUSY} > 90 \text{ GeV} \quad \text{LEP2.} \tag{14}
\]

The minimal supersymmetric theory has the remarkable feature that there is an upper bound on the lightest Higgs boson resulting from the structure of the scalar potential. This bound is roughly

\[
M_h^{SUSY} < 110 - 130 \text{ GeV}, \tag{15}
\]

where the exact value depends on assumptions about the parameters of the theory. [10] This is tantalizingly close to the experimental limit of Eq. 14. We see that there is no overlap between the expected mass of the lightest Higgs boson of a supersymmetric model and the Standard Model Higgs boson when \( \Lambda \sim M_{GUT} \). Hence an observation of the Higgs boson with even an imprecise value for its mass will help to distinguish between the Standard Model and its minimal supersymmetric extension.

A Standard Model Higgs boson should be discovered at the Tevatron or the LHC. Due to the small rate, the Higgs boson will be extraordinarily difficult to observe at the Tevatron. The signal with the best signature is associated production with a \( W^\pm \). For \( M_h \sim 120 \text{ GeV} \), the cross section at \( \sqrt{s} = 2 \text{ TeV} \) is \( \sigma(p\overline{p} \rightarrow W^+h) \sim .3 \text{ pb} \). Even with \( 10 \text{ fb}^{-1} \), the \( 5\sigma \) discovery level is only \( M_h \sim 100 \text{ GeV} \), below the current LEP2 limit. This underscores the need for the highest possible luminosity.

Figure 2 illustrates the discovery potential for a Standard Model Higgs boson at the Tevatron. [11] For \( M_h < 140 \text{ GeV} \), the dominant signal results from \( p\overline{p} \rightarrow Wh, h \rightarrow b\overline{b} \), while at higher Higgs masses, the decay \( h \rightarrow WW^* \) becomes the most important. The discovery reach plot combines small signals from many different channels. In fact, the maximum \( S/\sqrt{B} \) in any channel is .9 for \( L = 1 \text{ fb}^{-1} \). A
Standard Model Higgs discovery at the Tevatron will almost certainly require the full 25–30 fb\(^{-1}\) of upgraded luminosity. The LHC, on the other hand, should discover a Standard Model Higgs boson in any mass region below 1 TeV, as illustrated in Figure 3, even with only 30 fb\(^{-1}\). From \(M_h \sim 120 \text{ GeV}\) all the way up to \(M_h \sim 700 \text{ GeV}\), the Higgs boson can be observed through the decay \(h \rightarrow ZZ \rightarrow 4l\). The discovery reach can be extended up to \(M_h \sim 1 \text{ TeV}\) through the channels \(h \rightarrow ZZ \rightarrow l^+l^-\nu\bar{\nu}\) and \(h \rightarrow W^+W^- \rightarrow l\nu\text{jetjet}\). With the full luminosity of 100 fb\(^{-1}\), the LHC will see a Higgs signal in multiple channels for all possible masses. The observation in multiple channels will allow preliminary measurements of the Higgs coupling constants, as discussed in the next section.

The Standard Model points to a Higgs boson in the 100–200 GeV mass range, while its minimal supersymmetric extension suggests that the lightest Higgs boson is just above the current experimental limit. In either case, such a light Higgs boson would be kinematically accessible through the process \(e^+e^- \rightarrow hZ\) at an \(e^+e^-\) collider with \(\sqrt{s} \sim 350 – 500 \text{ GeV}\). The rates for Higgs production at an \(e^+e^-\) collider are shown in Fig. 4. For an \(e^+e^-\) collider with \(\sqrt{s} \sim 500 \text{ GeV}\), the dominant production mechanism is \(e^+e^- \rightarrow Zh\) for \(M_h \sim 200 \text{ GeV}\). At higher energy, say \(\sqrt{s} \sim 1 \text{ TeV}\), the largest rate is from \(e^+e^- \rightarrow \nu\bar{\nu}h\). In the next section, we examine the capabilities and the required luminosities for linear colliders to measure the Higgs properties and contrast these potential future measurements with what we will know from the LHC.

**TABLE 1.** Indirect Measurements of \(M_h\)

| Collider    | \(\Delta M_W\)  | \(\Delta M_t\)  | \(\delta M_h/M_h\) |
|-------------|------------------|------------------|--------------------|
| LEPII, TeV  | 30 MeV           | 4 GeV            | 57 %               |
| LHC         | 15 MeV           | 2 GeV            | 26 %               |
| 500 GeV \(e^+e^-\) | 15 MeV          | 200 MeV          | 17 %               |

**PRECISION MEASUREMENTS OF MASS, COUPLINGS, AND BRANCHING RATIOS**

**Higgs Mass Measurements**

There are two complementary approaches to measuring the Higgs boson mass. The first is through the direct observation of the Higgs boson. For most values of \(M_h\), with an integrated luminosity of \(\int \mathcal{L} = 300 \text{ fb}^{-1}\), the LHC will measure \(\delta M_h/M_h \sim 10^{-3}\), as shown in Fig. 5. Even at \(M_h \sim 800 \text{ GeV}\), the expected precision is \(\delta M_h/M_h \sim 10^{-2}\).

At a high energy \(e^+e^-\) collider, the cross section for \(e^+e^- \rightarrow Zh\) is a sensitive function of the Higgs boson mass and we could hope to obtain an extremely
precise measurement of the mass. By measuring the rate as a function of $\sqrt{s}$, a measurement of order $[13]$

$$\delta M_h \sim 60 \text{ MeV} \sqrt{\frac{\mathcal{L}}{100 \text{ fb}^{-1}}}$$  \hspace{1cm} (16)$$

could be obtained for a Higgs boson in the 100 GeV region. An alternate method is to measure the recoil spectrum in the process $e^+e^- \rightarrow Zh \rightarrow he^+e^-, h\mu^+\mu^-$. This would yield a precision of,

$$\delta M_h \sim 300 \text{ MeV} \sqrt{\frac{\mathcal{L}}{100 \text{ fb}^{-1}}} ,$$  \hspace{1cm} (17)$$

again for a Higgs boson in the 100 GeV region. With 1000 $fb^{-1}$ the precision on $\delta M_h$ for a light Higgs boson at an $e^+e^-$ collider could be considerably better than at the LHC, using either the excitation spectrum or the recoil spectrum of the $e^+e^- \rightarrow Zh$ process.

Precise measurements of $M_W$ and $M_t$ at future colliders will allow a value of $M_h$ to be inferred [13], as shown in Table 1. (The $e^+e^-$ numbers in this table assume $\int \mathcal{L} = 1000$ $fb^{-1}$.) Since the Higgs boson contributes only logarithmically to electroweak observables, the precision is significantly less than the direct measurement. Consistency between the direct and the indirect measurements will provide an important check of the theory at the quantum level, however.

**Figure 3.** Discovery potential for a Standard Model Higgs boson at the LHC, using the ATLAS detector, with 30 $fb^{-1}$. From Ref. 12.
Measurements of Higgs Couplings

The measurement of the Higgs boson couplings is important to differentiate between the Standard Model and other possibilities. In a supersymmetric model, the Higgs couplings to both fermions and gauge bosons can be quite different from those of the Standard Model, as illustrated in Fig. 6 for an arbitrary choice of input parameters. The total decay width can differ by more than an order of magnitude between the Standard Model and a supersymmetric model.

The total Higgs boson width can be measured from the reconstructed Higgs peak at the LHC. This direct measurement is only possible for \( M_h > 200 \text{ GeV} \). Below this mass, the width of the resonance is narrower than the experimental resolution. For \( M_h > 200 \text{ GeV} \), the Higgs can be observed through the decay \( h \to ZZ \to 4l \) and the resulting measurement of the total width is shown in Fig. 7. With \( \mathcal{L} = 300 \text{ fb}^{-1} \), the LHC can measure \( \Delta \Gamma_h / \Gamma_h < 10^{-1} \) for \( 300 \text{ GeV} < M_h < 800 \text{ GeV} \).

Measurements of specific branching ratios are probably the most useful quantities for distinguishing between the Standard Model and other models. As an example, I discuss the coupling of the Higgs boson to the top quark. In the Standard Model the Yukawa coupling is given by,

\[ g_t = -\frac{M_t}{v}, \]  

while in the minimal supersymmetric model the coupling is modified by the factor \( C_{tth} \).
For some values of $\tan \beta$ and $M_A$, $C_{tth}$ can be quite different from 1, as shown in Fig. 8. Fig. 8 also shows the coupling of the heavier neutral Higgs boson of a supersymmetric theory, $H^{SU/SY}$, to the top quark. Again, the coupling can be far from the Standard Model coupling. Note that for $M_A \to \infty$, $C_{tth} \to 1$, $C_{ttH} \to 0$ and the Standard Model coupling is recovered.

At the LHC, the $t\bar{t}h$ coupling can be measured to roughly 20% through the process $pp \to t\bar{t}h$ in the mass region $M_h \sim 120$ GeV. [12] (For higher Higgs masses, the cross section becomes quite small.) A similar mass region can be probed at an $e^+e^-$ collider. The signal decays predominantly to $W^+W^-b\bar{b}b\bar{b}$ and so will be spectacular. A study of the signal and background showed that the signal could be extracted from the background using both the semi-leptonic and the hadronic decays of the $W$’s and a measurement of $g_t$ obtained. [14,15] Table 2 shows the expected precision for the measurement of $g_t$ at $\sqrt{s} = 500$ GeV and 1 TeV. [15] The message is clear. A precision measurement of $C_{tth}$ requires high energy and high luminosity ($L = 1000$ $fb^{-1}$) in order to improve on the LHC’s measurement.

The total rate for Higgs production in the process $e^+e^- \to Zh$ can be found by measuring the recoil mass of the lepton pair, $M_{l\bar{l}}$, from the decay $Z \to l^+l^-$. This measurement is independent of the Higgs boson decay mode. Once the total rate is known, the Higgs branching ratios can be measured by flavor tagging of the Higgs...
FIGURE 6. Total decay width for Standard Model Higgs boson and the Higgs bosons of a supersymmetric model with $\tan \beta = 2$, $A_t = M_{SUSY} = 1\ TeV$, and $\mu = 100\ GeV$.

TABLE 2. $\delta g_\ell$ in $e^+e^-$ interactions With 1000 $fb^{-1}$. From Ref. 15.

| $M_h\ (GeV)$ | $\sqrt{s} = 500\ GeV$ | $\sqrt{s} = 1\ TeV$ |
|--------------|-------------------------|-----------------------|
| 100          | .08                     | .06                   |
| 110          | .12                     | .06                   |
| 120          | .21                     | .07                   |
| 130          | .44                     | .08                   |

decay final states.

The measurements of the Higgs couplings to the lighter quarks can be done with a precision of $5 - 10\%$ with 500 $fb^{-1}$ at an $e^+e^-$ collider. Ref. [16] found roughly equivalent results for $\sqrt{s} = 350\ GeV$ and $\sqrt{s} = 500\ GeV$. The error on the measurements of the Higgs Yukawa couplings of Ref. [16] is dominated by theoretical uncertainty due to the measured input values of $\alpha_s$, $m_c$, and $m_b$, not by systematic or statistical errors.

Armed with measurements of the Higgs boson branching ratios, we can ask over what region of parameter space the minimal supersymmetric model can be distinguished from the Standard Model. The answer is shown in Figure 9, taken from Ref. [16]. First, the 95% confidence level value of the branching ratio for the Standard Model was computed. Ref. [16] then scans over the parameter space of the minimal supersymmetric model, taking $\tan \beta < 60$ and the mass parameters to be
FIGURE 7. Measurement of the total Higgs boson width at the LHC with 300 $fb^{-1}$, using the ATLAS detector. From Ref. 12.

less than $1 - 1.5 \, TeV$. For a given set of parameters, the Higgs branching ratio was then computed. In Fig. 9, the region to the right of the curves (going from left to right on the figure) has more than 68, 90 or 95% of the supersymmetric model solutions outside of the Standard Model 95% confidence level region. With 500 $fb^{-1}$, an $e^+e^-$ collider can distinguish between the Standard Model and the minimal supersymmetric model up to $M_A \sim 550 \, GeV$, while with 1000 $fb^{-1}$, the sensitivity is increased to $M_A \sim 730 \, GeV$. [16] This is remarkable given the decoupling of the Higgs sector of the minimal supersymmetric model for large $M_A$.

At the LHC, measurements of the Higgs couplings are less clearcut than at an $e^+e^-$ collider. At the LHC, measurements involving the Higgs boson typically involve combinations of Higgs couplings. For example, a measurement of the ratio of the $h \rightarrow \gamma\gamma$ and $h \rightarrow ZZ \rightarrow 4l$ rates would give the ratio of the $h \rightarrow \gamma\gamma$ and $h \rightarrow ZZ$ branching ratios, but not the absolute couplings. A study of the combinations of Higgs couplings which can be measured at the LHC is given in Ref. [21].

**VERIFYING THE STRUCTURE OF THE HIGGS POTENTIAL**

Once a Higgs particle is found, it will be necessary to investigate its self-couplings in order to reconstruct the Higgs potential and to verify that the observed particle is
indeed the Standard Model Higgs boson which results from spontaneous symmetry breaking. A first step in this direction is the measurement of the trilinear self-couplings of the Higgs boson which are uniquely specified by the scalar potential of Eq. 2.

After the symmetry breaking, the self-couplings of the Higgs boson are uniquely determined by $M_h$,

$$V = \frac{M_h^2}{2} h^2 + \frac{M_h^2}{2v} h^3 + \frac{M_h^2}{8v^2} h^4. \quad (20)$$

In extensions of the Standard Model, such as models with an extended scalar sector, with composite particles or with supersymmetric partners, the self-couplings of the Higgs boson may be significantly different from the Standard Model predictions.

In order to probe the three- and four-point Higgs couplings, it is necessary to measure multi-Higgs production. Higgs boson pairs can be produced by several mechanisms at hadron colliders:

- Higgs-strahlung $W^*/Z^* \rightarrow hhW/Z$,
- vector-boson fusion $WW, ZZ \rightarrow hh$,
- Higgs radiation off top and bottom quarks $gg, q\bar{q} \rightarrow Q\bar{Q}hh$,
- gluon-gluon collisions $gg \rightarrow hh$.

**FIGURE 8.** Couplings of the neutral Higgs bosons, $h^{SUSY}$ and $H^{SUSY}$, of a supersymmetric model to the top quark, in units of the Standard Model Higgs-top quark Yukawa coupling.
At the LHC, gluon fusion is the dominant source of Higgs-boson pairs in the Standard Model and arises from quark loops, with the dominant contribution coming from top quark loops. The rate, even at the LHC, is quite small as can be seen in Fig. 10. Although the rate is sensitive to the tri-linear coupling, the variation is probably too small to be observed. [17] A detailed study of the signal and background gives the results shown in Fig. 11. [18] This study computed the minimum rate necessary for a 5σ discovery of $hh$ production. It is clear that in the Standard Model, this physics will have to wait for the next generation of accelerators.

In a supersymmetric model, the $b$-quark contribution to $hh$ production will be enhanced for large $\tan\beta$. Even so, in the absence of large squark loop contributions, with $25 \text{ fb}^{-1}$ the Tevatron can only exclude a small region of parameter space with $M_A < 150 \text{ GeV}$ and $\tan\beta > 80$. The LHC will be able to exclude an even larger region of $M_A$ and $\tan\beta$ space. [18] However, the situation changes dramatically for light squarks and with the parameters chosen to maximize the squark tri-linear couplings. In this case, it is possible to obtain a significant enhancement of the rate, largely due to resonance effects. This is shown in Fig. 12 for the Tevatron. [18] In this very special situation, even the Tevatron will be extremely sensitive to double Higgs production.

At a high energy $e^+e^-$ collider, Higgs pairs are produced through similar mech-
FIGURE 10. Double Higgs production, \( pp \rightarrow hh \), at the LHC, \( \sqrt{s} = 14 \) TeV. The solid line is the Standard Model rate, while the dotted and dashed lines have the tri-linear Higgs couplings modified.

anisms as in hadronic collisions. At intermediate energies, \( \sqrt{s} \sim 500 \) GeV, the dominant mechanism is \( e^+e^- \rightarrow Zhh \), while at TeV scale energies, the process \( e^+e^- \rightarrow \nu\bar{\nu}hh \) is dominant. Just above the kinematic threshold, the sensitivity to the trilinear coupling is maximal in the \( e^+e^- \rightarrow Zhh \) process. With 2000 \( fb^{-1} \), the tri-linear coupling can be measured to \( \sim 15\% \) [19]. The cross sections for all sources of double Higgs production in an \( e^+e^- \) collider are small, on the order of a few femtobarn or less for \( M_h < 200 \) GeV. [19] This is clearly a measurement which requires the highest possible luminosity in order to isolate the signal from the background and make a measurement of the tri-linear Higgs coupling.

At present, it does not appear possible to measure the Higgs boson four-point coupling. In principle, it could be measured in triple Higgs production, but the rate is miniscule.

**STRONGLY INTERACTING SYMMETRY BREAKING**

If a Higgs boson is not found at the LHC, then the electroweak symmetry breaking is strongly interacting. Without the addition of some new type of physics, \( WW \) scattering will violate unitarity at an energy scale somewhere below 3 TeV. There are two classes of effects which could potentially be observed in this scenario.

The first possibility is that whatever new physics unitarizes the \( WW \) scattering
is at too high an energy scale to be observed at either the LHC or an $e^+e^-$ collider with $\sqrt{s} \sim 500 \text{ GeV} - 1 \text{ TeV}$. In this case the only effects which can be observed are small deviations in absolute rates. The Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{f_i}{\Lambda^2} \mathcal{O}_i,$$  \hspace{1cm} (21)

where $\mathcal{L}_{SM}$ is the Lagrangian of the Standard Model with the Higgs boson removed. Without the Higgs boson, the Lagrangian can be written in terms of an expansion in powers of $\frac{s}{\Lambda^2}$, where $\Lambda$ is the scale of new physics. The $f_i$ are dimensionless coefficients of the new operators, $\mathcal{O}_i$. A complete set of operators at order $s/\Lambda^2$ can be found in Ref. [20]. The goal of the LHC or a high energy $e^+e^-$ collider in this scenario would be to measure the $f_i$ and attempt to distinguish between models. At the LHC, there will be a very small number of events [12] and it is doubtful if it will be possible to tell the difference between the various possible models. An $e^+e^-$ collider with $\sqrt{s} \sim 1.5 \text{ TeV}$ could measure some of the $f_i$ to $\mathcal{O}(10^{-3})$, but a complete set of measurements will take still higher energy.

In the second case, the new physics which unitarizes the $WW$ scattering amplitudes produces resonances which can be observed. Numerous studies have found that an $e^+e^-$ collider with $\sqrt{s} \sim 1.5 \text{ TeV}$ has roughly the same sensitivity to $TeV$ scale resonances as does the LHC. [21] Both machines will be sensitive to resonances on the order of $1.5 \text{ TeV}$.
**FIGURE 12.** Double Higgs production at the Tevatron in a supersymmetric model with enhanced squark contributions. From Ref. 18.

**CONCLUSION**

Even after the LHC has successfully run for a few years, there will still be unanswered physics questions. If a weakly interacting Higgs boson exists, either from a supersymmetric model or the Standard Model, it will be observed at the LHC. The LHC will make preliminary measurements of the Higgs boson mass and couplings, but a high energy $e^+e^-$ collider with high luminosity will significantly improve on the precision. Precise measurements of the Higgs width are particularly important for differentiating between models. Measurements of double Higgs production and strong symmetry breaking in particular will require the highest possible energy and luminosity.

This note has considered only electroweak symmetry breaking. There will of course be many exciting questions to be answered in other areas of particle physics such as supersymmetry, QCD, CP violation, etc. Interesting times await us!

**REFERENCES**

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