Bidding Strategies with Gender Nondiscrimination Constraints for Online Ad Auctions

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ABSTRACT

Interactions between bids to show ads online can lead to an advertiser’s ad being shown to more men than women even when the advertiser does not target towards men. We design bidding strategies that advertisers can use to avoid such emergent discrimination without having to modify the auction mechanism. We mathematically analyze the strategies to determine the additional cost to the advertiser for avoiding discrimination, proving our strategies to be optimal in some settings. We use simulations to understand other settings.

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1 INTRODUCTION

Prior work found Google showing an ad for the Barrett Group, a career coaching service promoting the seeking of high paying jobs, more often to simulated men than women [5]. Later work enumerates possible causes of this disparity [6].

One possibility, raised by Google itself [17], is that the Barrett Group targeted both men and women equally, but other advertisers, on average, focused more on women, which would be in line with subsequent findings [13]. In this possibility, the Barrett Group found itself outbid for just women by the other advertisers who were willing to pay more than it was for reaching women but not for men. These other advertisers might be promoting products that many find acceptable to target toward women, such as makeup. Thus, it’s possible that each advertiser’s targeting appears reasonable in isolation but interacts to bring about emergent discrimination for a job-related ad.

For conscientious advertisers of products that should be broadcasted to women and men at equal rates, such an outcome is unacceptable but currently difficult to avoid. While Google offers the ability to skew ads toward men or toward women, it provides no way to ensure that both men and women see the ad an equal number of times. As discussed above, simply not targeting by gender is not enough to guarantee parity. Even running two ad campaigns of equal size is insufficient since the size is determined by budget and not the number of ads shown, which means that parity would only be achieved if women and men are equally expensive to reach.

In this work, we consider how advertisers can ensure approximate demographic parity for its ads without changing Google’s ad auction mechanism, which is based on a second-price auction [9]. Given that an advertiser wishes to maximize its utility by reaching the people most likely to respond to its ads, we model the advertiser’s utility function along with the parity goal as a constrained bidding problem. We consider both a very strict absolute parity constraint and a more relaxed relative constraint inspired by the US EEOC’s four-fifths rule on disparate impact [8]. While using a second-price auction suggests that the advertisers should bid their true value of showing an ad, a parity constraint and multiple rounds of the auction interact to make deviations from this truthful strategy optimal. Intuitively, as in multi-round second-price auctions with budget constraints [10], it is sometimes better to bid less to preserve the ability to participate in later auctions with a lower cost of winning. More interestingly, unlike with just budget constraints, it is also sometimes better to bid more to ensure an acceptable degree of parity, enabling participation in other auctions later.

Given these complexities, finding an optimal bidding strategy for such a constrained bidding problem is non-trivial. We do so by modeling them as a Markov Decision Processes (MDPs). Solving these MDPs using traditional methods, such as value iteration, is made difficult by the continuous space of possible bid values over which to optimize. To avoid this issue, we find recursive formulae for each type of constraint providing the optimal bid value and solve for their values instead. This approach allows us to solve the MDPs without needing to explicitly maximize over the possible actions as in value iteration.

We compare this optimal constrained bidding strategy to the optimal unconstrained strategy for both real and simulated data sets. The cost to the advertiser for ensuring parity varies by setting, but is manageable under the more realistic settings explored. In all cases, the revenue of the simulated Google remains roughly the same or goes up.

By not modifying the core auction algorithm used by Google and instead suggesting bidding strategies that could be deployed by the advertisers, we believe this work provides a practical path towards nondiscriminatory advertising.

2 RELATED WORK

The most closely related work, recently looked at enforcing parity constraints with auction mechanisms, whereas we do so with bidding strategies [4]. While both approaches have their use cases, we believe ours is easier to deploy since just the advertisers wanting the feature need to make changes to implement it. We further discuss tradeoffs...
between deployment approaches in Section 8. Our approach also differs by using strict constraints whereas theirs uses probabilistic constraints. Probabilistic constraints allow more utility but may be insufficient in cases where approximate parity is required, as when disparate impact is prohibited. At an algorithmic level, they differ by using gradient decent.

A similar alternative approach could use auction mechanisms with Guaranteed ad Delivery (GD) [15, 18]. An advertiser can act as two parties to the auction, one for each gender, and use GD to ensure an equal number of wins for each party. Unlike our bidding strategy, which an advertiser can unilaterally employ, this approach requires the ad exchange to change its auction mechanisms.

Prior works have looked at how to enforce (proportional) parity constraints on the classifications produced by ML algorithms [2, 3, 12, 20]. We instead look at auctions.

Prior works have used MDPs to model ad slot auctions. Li et al. [14] and Iyer et al. [11] have used them to find optimal bidding strategies when advertisers do not know the exact values of each type of ad slot and learn values by winning them. They showed advertisers should overbid to learn more information. Gummadi et al. [10] described the optimal bidding strategy for the second-price auction in which each advertiser has a limited budget, which leads to underbidding. Zhang et al. [21] derived optimal real-time bidding strategies when each ad slot have different properties.

3 ONLINE AD AUCTIONS

When a person visits a webpage, the webpage will often contain dynamically loaded ads at fixed locations on the page. These ads each occupy an ad slot, a location at a time (or page load) on the webpage. In some cases, the website selects which ads to show in which slots itself, such as with Facebook. In other cases, the website contracts with a third-party, to fill and charge for the slots with Guaranteed ad Delivery (GD) [15, 18]. An advertiser can act all bids are simply offers to pay for showing the ad.

The quality of the ad and its fit for the slot. For simplicity, the amount of bonuses it receives, or to avoid annoying visitors, might consider the quality of the ad and it’s fit for the slot. For simplicity, we will not consider these complications and instead presume that all bids are simply offers to pay for showing the ad. Second price auctions is a common mechanism for resolving such auctions, with Google using a variation of one [9], and we will presume the ad exchange uses one. In this auction mechanism, the exchange selects the highest bidder as the winner but only charges the bidder the price offered by the second highest bidder. Under certain circumstances, this mechanism ensures that each bidder’s optimal strategy is to bid the actual amount it values the slot at, making the mechanism truthful. Since ad exchanges sometimes sell more than one slot at time, such as for a webpage with multiple slots, they often use generalized second-price auctions, known as position auctions [7, 19].

We model the above economy as a sequential game of incomplete information, where in each round of the game a set of self-interested rational advertisers bids to win an ad slot through a second-price auction. We allow bids to vary over auctions and assume that each advertiser has a geometric lifespan. For simplicity, we make the total number of advertisers $a$ equal in all auctions by assuming that every time an advertiser dies a new advertiser joins.

At time $t$, each advertiser $i$ submits a bid $b_i$. Let $b_{i,j}$ be the bids of other the advertisers. The ad exchange platform runs a second-price auction where Advertiser $i$ wins the ad slot if its bid is higher than all other bids: $b_{i} > \max b_{i,j}$. For simplicity, we assume no ties, ensuring that such a winner exists. Let $a_i$ be 1 if the advertiser $i$ wins at round $t$ and be 0 otherwise. If the advertiser $i$ wins it will pay the second highest bid $d_i = \max b_{i,j}$. The cost of the auction $t$ is $c_i = c_i + d_i$ since the advertiser $i$ only pays if it wins.

The ad slot auctioned at $t$ has a value $v_i$ for the advertiser $i$. When an advertiser $i$ wins auction $t$, it gets an immediate reward, which is the value $v_i$ less its price $d_i$. Thus, the utility of advertiser $i$ gained at each round is $u_i = a_i v_i - c_i = a_i (v_i - d_i)$. Let the geometric parameter for the lifespan distribution for advertiser $i$ be $\delta_i$. The total utility for each advertiser is $U_i = \sum_{t=0}^{\infty} \delta_i a_i (v_i - d_i)$ where $\delta_i$ is exponentiation, not indexing like the others.

The advertiser $i$ should select its bids $b_i$ to maximize the expected value of $U_i$ where the expectation is over its value $v_i$ and the bids of other advertisers $b_{i,j}$. The advertiser can use market research, its prior experiences, and any information provided by the ad exchange to estimate these uncertain values. In the case of a pure second-price auction, the values of the other bids $b_{i,j}$ are irrelevant and the optimal strategy is to always set its bid $b_i$ equal to its estimation of its value $v_i$.

However, this result does not carry over to all second-price auctions with constraints, including the parity constraints we consider. In this case, the behavior the other advertisers matters, but estimating it for individual ad slots is difficult. Furthermore, the advertiser is unlikely to estimate the value of every ad slot individually even for...
a pure second-price auction. Rather, the advertiser will likely model 
ad slots as each having a type belonging to a set \( \Theta \) of reasonable 
size. The types will represent the most important information to the 
advertiser about the slot. For simplicity, we will typically assume that 
\( \Theta \) is equal to \( \Gamma \).

For each type \( \theta \), the advertiser will estimate the expected value \( V^i_\theta \) 
of a slot of type \( \theta \). For estimating the other bids, prior research [11] 
has shown it reasonable to model them as coming a stationary fixed 
distribution due to the large number of other advertisers. To simplify 
the future analysis, we denote the CDF of other bids for a slot of 
type \( \theta \) by \( g^\theta_i \). Finally, let \( p^\theta_i \) be the probability that the advertiser 
assigns to type \( \theta \).

With these estimations, we compute estimations of other key 
quantities. The probability of winning on auction \( t \) for a slot of type 
\( \theta \) with a bid of \( x \) is \( q(x; g^\theta_i) = \Pr(b^t_i \leq x) = g^\theta_i(x)^{\alpha(t)} \) where \( \alpha \) is the number of advertisers at each ad slot auction.

The expected value of the utility for the advertiser \( i \) for a single 
auction given the distribution of the other advertisers’ bids \( g^\theta_i \) and 
\( g^\theta_o \) is \( E[U^i_i] = \sum_{\theta_o} p^\theta_i q(V^i_\theta; g^\theta_i) \times (V^i_\theta - d^i_t) \). The expected value of 
the total utility for each advertiser is

\[
E[U_i] = \sum_t \delta^{t-1} \sum_\theta p^\theta_i q(V^i_\theta; g^\theta_i) (V^i_\theta - d^i_t) \tag{1}
\]

4 PARITY CONSTRAINTS

Advertisers may have concerns in addition to attempting to maximize 
the utility \( U_i \), such as complying with laws and social norms. In some 
cases, this will include ensuring that its ads reach various protected 
groups to the same degree. For example, an employer may desire that 
a job ad be shown to an equal number of women and men to 
comply with laws prohibiting gender discrimination in hiring [6]. 
Such advertisers would like to place their bids in a manner to ensure 
such demographic parity.

However, the above auction mechanism, as well Google’s actual 
mechanism as far as we can tell, does not offer any way of ensuring 
that a job ad is shown to an approximately equal number of women 
and men, as required by laws prohibiting gender discrimination in 
hiring [6]. Furthermore, ad exchanges may be unwilling to support 
such constraints given that only some advertisers have such concerns. 
Thus, our goal is to provide advertisers with a bidding strategy that 
dynamically adjusts bids to preserve the gender parity of the viewers, 
which advertisers can unilaterally use without needing changes to 
the auction mechanism of the ad exchange.

As an additional benefit of not modifying the ad auction 
mechanism, our bidding strategy can be used for any type of auction. 
However, we design and analyze them with only with second-price 
auctions in mind.

To state our goal more precisely, we have to distinguish between 
absolute (additive) and ratio (relative) parity. An advertiser has K-
strict absolute parity, or K-parity for short, if, after each auction, 
the maximum difference between the number of auctions that it wins 
for each gender is not more than \( K \). An advertiser has \( R \)-ratio parity, 
after each auction, if the maximum ratio of the number of auctions 
that it wins for each gender is not more than \( R \).

Our goal is to find the optimal bidding strategy for advertisers 
obeying either type of constraint. This is difficult since a con- 
strained advertiser must consider not just the immediate reward of 
winning a slot, but also how it may close or open the possibility of 
winning additional slots later. To see this, we will consider three 
examples involving a simplified setting in which an advertiser \( i \) is 
subject to 1-parity and knows exactly how long it will live. In each 
example, it values men and women both at \( 20 \) (no variance), but that 
other advertisers value women at an expected value of \( 21 \) and men 
at an expected value of \( 5 \). This setting reflects that advertisers are 
willing to pay more, on average, for women than men [13].

In the first example we consider, the advertiser knows that it will 
live for exactly one ad auction. In this case, the advertiser \( i \) will bid 
the value of the immediate reward \( 20 \) that it receives for winning 
an auction regardless of whether it is subject to a 1-parity constraint 
since winning the auction has no effect other than that immediate 
reward. It will win an auction for a man and lose an auction for a 
woman.

Next, consider the advertiser’s behavior for a series of two auc-
tions. The interesting case is two men in a row. In this case, advertiser 
\( i \) can only win one of the slots since it is subject to a 1-parity policy. 
Thus, the utility of the advertiser will be smaller from having \( i \) 
win the woman and pay the second price of \( 1 \) compared to winning 
the first woman. This is similar to how underbidding is optimal in 
some repeated second-price auctions with a constrained budget [10]. The degree 
of underbidding must balance the chance at getting a male slot at 
a discount with the risk of either losing both auctions or getting a 
female slot for the second auction.

The opposite, overbidding, can also occur. To see this, consider 
a series of three auctions with a woman followed by two men. In 
this case, the advertiser \( i \) can win both men, despite the 1-parity 
constraint, provided that it first wins the woman. Thus, winning 
the woman produces not just an immediate reward, but also a future 
reward by unlocking the ability to win more men. If we presume 
negligible variance in the other bids, the advertiser \( i \) will have to bid 
\( 22 \) to win the woman and pay the second price of \( 21 \), yielding an 
immediate reward of \( -1 \) by bidding \( 1 \) over the inherent value \( 20 \) of the 
female ad slot to the advertiser. However, since the immediate 
reward of a male slot is \( 20 - 5 = 15 \), being able to win the second 
man means a net positive gain of \( 15 - 1 \). (We ignore the effects of 
underbidding since we are now considering negligible variance in 
the other advertiser’s bids, which makes the effect go away.)

We find this distinction between the immediate reward and the 
future rewards coming from future flexibility useful for determining 
the optimal bidding strategy. However, doing so requires not only 
making the above intuitions quantitative, but also dealing with addi-
tional probabilistic factors, such as the genders of ad slots not being 
known in advance and the uncertain duration of the auction sequence. 
To overcome these difficulties, we switch to a more systematic model 
for each type of constraint.
5 ABSOLUTE PARITY CONSTRAINTS

An advertiser wants to show an ad to equal numbers of men and women. A particularly careful advertiser may desire that this parity constraint holds not only at the end of ad campaign but throughout. Such continuous parity ensures that the advertiser would pass an audit checking for this property at any point in time. It also ensures meeting the parity goal if the ad campaign must be cut short or if a sudden influx of competing advertisers prevents winning additional slots.

Meeting this strict goal is impossible since the first ad must go to either a man or woman, and not both. To account for this, we relax this goal by allowing a difference to arise. We use $\delta$ to denote the maximum allowed difference where $K = 1$ is the strictest constant compatible with showing any ads.

To make this precise, we let $\Gamma$ denote a set of groups. We are typically interested in the case where $\Gamma = \{m, w\}$ with $m$ denoting men and $w$ women. In this case, we use $p$ to denote the probability of a male ad slot (i.e., $p_{m}^{\Gamma}$). We use $n_{i}$ to denote the number ad slots for people in group $i$ won by the constrained advertiser.

DEFINITION 1 (K-PARITY). An advertiser obeys a $K$-absolute parity constraint, or $K$-parity for short, for a set of groups $\Gamma$ iff, after each auction, for all groups $i$ and $j$ in $\Gamma$, the number of auctions that it wins satisfies $n_{i} - n_{j} \leq K$.

We study approximating the optimal bidding strategy that an advertiser desiring to meet a $K$-parity constraint using $\Gamma$ can do so. In our analysis, we assume all of the advertisers have an unlimited budget. Thus, they can bid on all auctions in its lifespan, unless maintaining $K$-parity constraint precludes it.

5.1 Modeling

To find the optimal bidding strategy for the $K$-parity advertiser, we model the problem as a Markov Decision Problem (MDP). The obvious state space for such an MDP would have states of the form $(n_{m}, n_{w}, \theta)$, where $n_{m}$ and $n_{w}$ is the current number male and female viewers, respectively, and $\theta$ is the type of the ad slot currently being auctioned off, which we presume corresponds to a gender. ($\theta$ could be generalized to allow targeting toward certain men and women.) Observing that only $n_{m} - n_{w}$ matters, we instead use a smaller space of $[\theta] \times (2K + 1)$ states. We denote each state by a tuple $(k, \theta)$, where $k$ is the difference between male and female viewers. When the advertiser wins an ad slot for a male viewer, the advertiser goes from state $k$ to $k + 1$; for a female, it goes from $k$ to $k - 1$. The value of the $\theta$ is decided by a random process depending upon the value of $p$, where $p$ is probability of the viewer being male.

To find the optimal solution, we write the Bellman equation for the MDP in the steady state. Since we consider the steady state regime we also replace the value of each ad slot by its expected value (i.e., $v_{i}^{\theta}$). The value function for each state except for two states $(K, m)$ and $(-K, w)$ has two parts: a reward function $R$ that indicates the immediate reward of taking action $b_{i}$ and $N$ that is the future value the advertiser gets by doing that action. We write the value functions as follows:

$$V(k, \theta; g_{i}) = \max_{b_{i}} \left[ R^{\theta}(b_{i}; g_{i}^{\theta}) + \delta N^{\theta}(b_{i}; k; g_{i}) \right]$$

(2)

$$R^{\theta}(b_{i}; g_{i}^{\theta}) = q(b_{i}; g_{i}^{\theta})(v_{i}^{\theta} - d_{i}^{\theta})$$

(3)

$$N^{\theta}(b_{i}, k; g_{i}),$$ the future value that advertiser $i$ gets by bidding $b_{i}$ at state $(k, \theta)$, consists of two parts with the first part $N_{\text{win}}^{\theta}$ being the value that the advertiser gets if it wins and the second part $N_{\text{lose}}^{\theta}$ being the value when it loses. We treat $g_{i}$ as providing both $g_{i}^{\theta}$ and $g_{i}^{\theta'}$. $R^{\theta}(b_{i}; g_{i}^{\theta})$ and $R^{\theta}(b_{i}; g_{i}^{\theta'})$ show the reward value that advertiser $i$ will receive if it wins an ad slot auction viewed by female and male.

We have:

$$N^{\theta}(b_{i}, k; g_{i}) = q(b_{i}; g_{i}^{\theta}) * N_{\text{win}}^{\theta}(k; g_{i}) + (1 - q(b_{i}; g_{i}^{\theta})) * N_{\text{lose}}^{\theta}(k; g_{i})$$

with

$$N_{\text{win}}^{m}(k; g_{i}) = pV(k + 1, m; g_{i}) + (1 - p)V(k + 1, w; g_{i})$$

$$N_{\text{win}}^{w}(k; g_{i}) = pV(k - 1, m; g_{i}) + (1 - p)V(k - 1, w; g_{i})$$

$$N_{\text{lose}}^{\theta}(k; g_{i}) = pV(k, m; g_{i}) + (1 - p)V(k, w; g_{i})$$

As for the two edge cases, their values are solely determined by the values of their successor states since the advertiser cannot win the current auction:

$$V(K, m; g_{i}) = \delta * (pV(K, m; g_{i}) + (1 - p)V(K, w; g_{i}))$$

$$V(-K, w; g_{i}) = \delta * (pV(-K, m; g_{i}) + (1 - p)V(-K, w; g_{i}))$$

5.2 Computing Optimal Bidding Strategies

Computing $V$ with MDP solvers, such as value iteration, is complicated by the bid space being continuous. Computing $V$ for a discretization of this space will require a fine discretization to avoid rounding errors, which will mean slow convergence. Using numerical optimization methods is complicated by $V$ not being a linear function in $b_{i}$. To avoid these complexities, we instead rewrite $V$ in a form that can be solved without any optimization.

To identify the optimal bidding strategy, we observe that the two edge cases do not involve a decision and the strategy of bidding $0$ is forced for them. We also observe that for the remaining states the valuation function (2) includes many terms that do not change under various bidding strategies. We collect these constants into a term $\Lambda_{i}$, which we can ignore while optimizing the strategy. We replace $q(b_{i}; g_{i}^{\theta})$ by $c(b_{i}; g_{i}^{\theta})$ that indicates the estimated cost of each ad slot. The remainder of the valuation function provides the joint valuation function $\Phi^{\theta}_{i}$. In more detail,

$$V(k, \theta; g_{i}) = \max_{b_{i}} \left[ q(b_{i}; g_{i}^{\theta})\Phi^{\theta}_{i}(k; g_{i}) - c(b_{i}; g_{i}^{\theta}) + \Lambda_{i}(k; g_{i}) \right]$$

(4)

where

$$\Lambda_{i}(k; g_{i}^{\theta}) = \delta(pV(K, m; g_{i}) + (1 - p)V(K, w; g_{i}))$$

The joint valuation $\Phi$ represents the reward for winning, both immediate and long-term, which is why it is multiplied by the probability of winning $q(b_{i}; g_{i}^{\theta})$. The expected cost of winning $c(b_{i}; g_{i}^{\theta})$ is subtracted from this product. $\Phi$ breaks down along the lines of winning and losing cases, as $N$ did:

$$\Phi^{\theta}(k; g_{i}) = v_{i}^{\theta} + \delta(\Phi^{\theta}_{\text{win}}(k; g_{i}) - \Phi^{\theta}_{\text{lose}}(k; g_{i}))$$

(5)
where
\[ \Phi^0_{\text{win}}(k; g_i) = pV(k + 1; m; g_i) + (1 - p)V(k + 1; w; g_i) \]
\[ \Phi^0_{\text{lose}}(k; g_i) = pV(k - 1; m; g_i) + (1 - p)V(k - 1; w; g_i) \]

The term \( \Phi^0_{\text{win}} \) represents the immediate value of winning the ad slot. The reminder considers the gain that the advertiser gets from the future by winning (moving to a new state) or losing (staying put). The difference between future rewards for winning and those for losing corresponds to the amount of overbidding called for, which explains the subtraction in (5).

The following theorem shows the usefulness of this decomposition. It uses the following lemma:

**LEMMA 1 (IYER et al. 2011).** For any continues non-decreasing function \( q(x) \) on \([0, 1] \times [0, 1] \), function \( f(x, v) = q(x)(v - x) + \int_0^x q(u) \, du \) gains its maximum when \( x = v \).

**THEOREM 1.** For any given \( g_i \) and \( K \), the optimal bid at all states \((k, \theta)\) other than the edge cases \((K, m)\) and \((-K, w)\) is \( \Phi^0_{\text{i}}(k; g_i) \).

**PROOF.** Without loss of generality we assume all of the bids are between 0 and 1. The bidding strategy that maximize the equation (4) will be the optimal strategy. To maximize this equation, we can omit the \( \Lambda_k \) function since it is constant for each \( b_i \). Similar to [11], we rewrite the cost function \( c(b_i; g^0_\text{i}) \) as

\[ c(b_i; g^0_\text{i}) = q(b_i; g^0_\text{i})b_i - \int_0^{b_i} q(u; g^0_\text{i}) \, du \]

Now, we can rewrite the decision problem of the advertiser \( i \) as

\[ \max_{b_i} \left\{ q(b_i; g^0_\text{i})\Phi^0_{\text{i}}(k; g^0_\text{i}, g^0_\text{w}) - c(b_i; g^0_\text{i}) \right\} \]

\[ = \max_{b_i} \left\{ q(b_i; g^0_\text{i})\Phi^0_{\text{i}}(k; g_i) - \int_0^{b_i} q(u; g^0_\text{i}) \, du \right\} \]

\[ = \max_{b_i} \left\{ q(b_i; g^0_\text{i})(\Phi^0_{\text{i}}(k; g_i) - b_i) + \int_0^{b_i} q(u; g^0_\text{i}) \, du \right\} \]

We know \( q(b_i; g^0_\text{i}) \) is a continues non-decreasing function. Therefore, we can use Lemma 1 with \( q(x) = q(x; g^0_\text{i}) \) to conclude equation (4) is at its maximum when the bid is \( \Phi^0_{\text{i}}(k; g_i) \) for \( \theta = m \). Similarly we can show for equation (4) that the optimal bid is \( \Phi^0_{\text{i}}(k; g_i) \) where \( \theta = w \).

This theorem means that we do not need to search the space of possible bid values to find the optimal bid. Rather, we can just compute the optimal bid using \( \Phi \). While \( \Phi \) depends upon the value function \( V \), we can recursively make use of this fact to compute \( V \) without such a search either. In particular, the theorem implies that

\[ V(k, \theta; g_i) = R^\theta(\Phi^0_{\text{i}}(k; g_i); g^0_\text{i}) + \Delta N^\theta(\Phi^0_{\text{i}}(k; g_i), k; g_i) \]

However, this equation is still not a closed form solution. Thus, Algorithm 1 does this calculation iteratively to converge to the states’ values. Although, showing the convergence in general is an open problem, as discussed in Section 7, our experiments find convergence within a reasonable tolerance within a feasible number of iterations.

To use our approach, an advertiser (or DSP) runs Algorithm 1 to compute the value function \( V \) and stores it as a look-up table. Then, for each new ad auction, the advertiser first checks if it winning

**ALGORITHM 1** : Iterative approach to find \( V \)

**Input:** \( K, g_i, \alpha, v^m, v^w, \epsilon \)

**Initialize:** \( V[-K : K, m] \leftarrow \frac{v^m + v^w}{2} \), \( V[-K : K, w] \leftarrow \frac{v^m + v^w}{2} \)

**repeat**

\[ \Delta \leftarrow 0 \]

**for** \( k \) in \([-K, \ldots, K] \)**

**for** \( \theta \) in \([m, w] \)**

\[ V'[k, \theta] \leftarrow R^\theta(\Phi^0_{\text{i}}(k; g_i); g^0_\text{i}) + \Delta N^\theta(\Phi^0_{\text{i}}(k; g_i), k; g_i) \]

\[ \Delta \leftarrow \max(\Delta, |V'[k, \theta] - V[k, \theta]|) \]

**end**

**end**

\[ V \leftarrow V' \]

**until** \( \Delta < \epsilon \)

the auction would violate the parity constraint. If so, it will not participate in the auction (i.e., bids zero). Otherwise, The advertiser bids the value of \( \Phi^0_{\text{i}}(k) \), which can be easily computed from value functions.

### 6 RATIO CONSTRAINTS

While constraints on the difference between the number of ads shown to each gender are intuitive, the EEOC’s four-fifths rule found in US regulations against disparate impact in employment instead focuses on a ratio [8]. The ratio considered is not simply between the number of ads shown to each gender. Rather, it acknowledges that parity can be unrealistic due to having differing numbers of male and female applicants. It adjusts for that factor by comparing the fraction of female applicants receiving a job offer to the fraction male applicants receiving a job offer. It requires that this ratio of ratios be between 5/4 and 4/5. Similarly, our ratio constraint compares two ratios, checking whether the fraction of female ad slots won is within a factor of \( r \) to the fraction of male ad slots won.

Strictly enforcing this check creates problems when the number of slots seen so far is small since the fractions won may be very far apart even when the number of ads shown to each gender only differs by 1. To avoid this issue, we also allow an additive difference in the number of ads show to each gender. The resulting rule may be viewed as a hybrid between a pure ratio constraint and the absolute constraint we have already presented.

We use similar notation as in Section 5.1 to express this constraint in a manner that avoids division by zero.

**DEFINITION 2 **((r, K)-RATIO). An advertiser obeys a (r, K)-ratio constraint, for a set of groups \( \Gamma \) iff, after each auction, for all groups \( i \) and \( j \) in \( \Gamma \), the number of auctions that it wins satisfies \( p_i \alpha_j \leq p_j \alpha_i + K \) where \( p_i \) and \( p_j \) is the probability of seeing slots for groups \( i \) and \( j \), respectively.

#### 6.1 Modeling

Similar to the \( K \)-parity constraint, we limit ourselves to the case where \( \Gamma \) and \( \Theta \) only contain two types, which we treat as male and female. We use \( p \) as the probability of a male. We denote each state by a triplet \((\eta_m, \eta_w, \theta)\), where \( \eta_m \) and \( \eta_w \) is the current number male and female viewers, respectively.

While we reuse the immediate reward function \( R^\theta \) from (3), we rewrite the value function \( V \) and future value function \( N \). When
When a female may not be won since \( r(1-p)(n_m+K) > p n_w + K \) where \( n_m \) is the current number of males won, \( V(n_w, n_m, m; g_i) = \delta (p V(n_m, n_w, m; g_i) + (1-p) V(n_m, n_w, w; g_i)) \) 

When a female may not be won since \( r(n_w, n_m, m; g_i) = \delta (p V(n_m, n_w, m; g_i) + (1-p) V(n_m, n_w, w; g_i)) \) 

We use a similar approach as in Section 5.2 to find optimal strategies. As before, we force the strategy to bid zero when winning would violate the constraint, 

\[
V(n_w, n_m, \theta; g_i) = \max_{b_i} \left[ R^0(b_i; g_i^0) + \delta N^0(b_i, n_m, n_w; g_i) \right]
\]

Algorithm 2: Iterative approach to find \( V \)

**Input:** \( r, K, p, g_i, \alpha, v^m, v^w, \epsilon, \mu \)

**Initialize** \( V[0 : M, 0 : r(1-p)/\mu + K, m] \leftarrow v^{m+w} \)

repeat

\[ \Delta \leftarrow 0 \]

for \( n_w \) in \( \{0, \ldots, \mu\} \) do

for \( b_i \) in \( \{0, \ldots, \delta \} \) do

\[ V'[n_w, n_m, \theta] \leftarrow R^0(\Phi^0(n_m, n_w; g_i^0) + \delta N^0(\Phi^0(n_m, n_w), n_m, n_w; g_i)) \]

end

\[ V \leftarrow V' \]

until \( \Delta < \epsilon \)

We can extend this approach to recover if the advertiser underestimates \( \mu \). In this case, the advertiser can use a linear approximation to estimate the optimal bid. To do so, let \( \rho = \frac{m}{\mu}(\mu-1) \). If \( \rho \) is an integer value, then the advertiser bids \( \Phi^0(\rho, \mu-1) \). Otherwise, the advertiser bids \( \Phi^0(\lfloor \rho \rfloor, \mu-1) + (\rho - \lfloor \rho \rfloor) \Phi^0(\lfloor \rho \rfloor, n_m) - \Phi^0(\lfloor \rho \rfloor, n_m) \).

7 EXPERIMENTS

We simulate various scenarios to show the feasibility of our method and to measure the impact of our fairness constraints on utility. To do so, we implemented a second-price auction simulator in Python, where each advertiser gets the gender of the website viewer before selecting its bid and participating in the ad slot auction. To simulate the viewer, we draw their genders independent and identically from a binomial distribution with probability \( p \) where \( p \) is the probability of the viewer being male.

We focus on a single advertiser \( i \) and measure how its utility changes when it has either one of our fairness constraints or not. When having fairness constraints, it uses our bidding strategy, with \( \delta \) set to 0.999 (unless otherwise noted) and \( \epsilon \) set to 0.001. When not, it bids immediately value \( v^i \) for the ad slot \( t \), as is rational for an unrestricted second-price auction. We assume that the other advertisers are unrestricted, that they always bid their values. To obtain distributions over ad values, we used both a real dataset (The Yahoo! Ad Search Marketing Advertiser Bidding Dataset) and a simulated one. The Yahoo! Ad data does not have exact timestamp values, so we could not use it to estimate the number of advertisers (i.e., \( \alpha \)) for each ad auction. To estimate \( \alpha \), we visited top websites that have ads using header bidding method [16] for one month (June 2019) and collected how many advertisers bid on a specific ad slot. In our experiments we never saw more than 10 advertisers bid on an ad slot auction. In line with our observation, we assume that there are \( \alpha = 10 \) advertisers bidding for each ad slot.

7.1 Real Dataset

The Yahoo! Ad Search Marketing Advertiser Bidding Dataset contains anonymized bids of advertisers participating in Yahoo! Search

Algorithm 2: Iterative approach to find \( V \)

**Input:** \( r, K, p, g_i, \alpha, v^m, v^w, \epsilon, \mu \)

**Initialize** \( V[0 : M, 0 : r(1-p)/\mu + K, m] \leftarrow v^{m+w} \)

**repeat**

\[ \Delta \leftarrow 0 \]

**for** \( n_w \) in \( \{0, \ldots, \mu\} \) do

**for** \( b_i \) in \( \{0, \ldots, \delta \} \) do

\[ V'[n_w, n_m, \theta] \leftarrow R^0(\Phi^0(n_m, n_w; g_i^0) + \delta N^0(\Phi^0(n_m, n_w), n_m, n_w; g_i)) \]

\[ \Delta \leftarrow \max(\Delta, |V'[n_w, n_m, \theta] - V[n_m, n_w, \theta]|) \]

**end**

**end**

\[ V \leftarrow V' \]

**until** \( \Delta < \epsilon \)

We can extend this approach to recover if the advertiser underestimates \( \mu \). In this case, the advertiser can use a linear approximation to estimate the optimal bid. To do so, let \( \rho = \frac{m}{\mu}(\mu-1) \). If \( \rho \) is an integer value, then the advertiser bids \( \Phi^0(\rho, \mu-1) \). Otherwise, the advertiser bids \( \Phi^0(\lfloor \rho \rfloor, \mu-1) + (\rho - \lfloor \rho \rfloor) \Phi^0(\lfloor \rho \rfloor, n_m) - \Phi^0(\lfloor \rho \rfloor, n_m) \).
Marketing auctions for the top 1000 search queries from June 15, 2002, to June 14, 2003. The dataset includes 18 millions bids from more than 10,000 advertisers, but without the exact timestamps or information about the ad viewer. Each record in this dataset indicates a course timestamp with 15 minutes precision, the advertiser, the keyword, and the bid.

In our analysis, we assumed bids have stationary distribution. We evaluate this assumption on our dataset. We use a specific keyword (keyword number 2) and we gathered all of bids from different advertisers in four days period (starting 2/15/2003). Then, we compute the empirical distribution of the bids of the first two days and the second two days. Figure 2 presents the distribution of the bids for these periods, showing that the distributions are very similar in both periods, supporting our stationarity assumption. The figure also shows that the bids follow a log-normal distribution, in line with the findings of supporting our stationarity assumption. The figure also shows that the bids follow a log-normal distribution, in line with the findings of Balseiro et al. [1].

Each keyword in our dataset has a different bid value distribution and the restricted adviser can model each keyword separately. We use the similar approach in our simulations and for each simulation we compute the optimal bidding strategy for a specific keyword. We assume that restricted advertiser updates his model parameters every two days.

As mentioned, the Yahoo A1 dataset does not contain the exact timestamps. Therefore, we cannot exactly determine which advertisers participated in any single ad auction. We randomly select a set of advertisers’ bids from each 15 minutes interval for each of our ad auctions. Since the dataset does not include information about the viewers, we sample the bids for both female and male viewers from the same set of bid values, making their values equal.

Figures 3(a) and 3(c) show the total utility ratio of the K-parity and (r, K)-ratio versions to the unrestricted version of the advertiser i for various values of K, r, and p on Yahoo A1 bid dataset. Here, and in the other simulations, we compute this ratio by simulating restricted and unrestricted versions of the advertiser i, using the same draw of values across the two versions. We do this 100 times, computing the average of total utilities $U_i$ for each version. We then plot the ratio of these two averages. Since the value of ad slots for both female and male viewers are equal, the total utility of an unrestricted advertiser will not change for different values of p. On the other hand, a restricted advertiser will get different utilities based on the distribution of the men and women viewers. K-parity and (r, K)-ratio constraints are harder to achieve for extreme values of p. Turning to the effects of K, the results show that when K is large, the K-parity advertiser can reach the utility of the unrestricted advertiser. Also by relaxing r, r-ratio advertiser achieves higher utility. To show the benefit of our approach compared to simply bidding immediate values, we compare the utility ratio both approaches. Figure 3(d) shows that our bidding strategy allows the advertiser achieve a higher utility.

### 7.2 Synthetic Data

A major limitation of the real dataset for our purposes is that it does not show which ad slots are for men and which for women. Thus, we use a synthetic dataset to explore how changing their relative values affects the advertiser’s utility. We generate two synthetic datasets using a log-normal distribution to sample the advertisers bids. Table 1 shows the model parameter settings used for the two scenarios.

To show the effect of assigning different values to men and women, consider an advertiser that gives more value to female slots than to male ones, as shown in the Equal price - Female valuable parameter settings. Figure 4(c) shows the utility ratio for the K-parity and unrestricted versions of the advertiser in this scenario. The K-parity version has its maximum utility ratio when there are more male than female slots. This may seem counter-intuitive since the advertiser values females more, but the measured ratio reflects that an abundance of males means that the K-parity version will not have to operate much differently from the unrestricted one. This is due to their abundance making overbidding less needed, decreasing the K-parity version’s costs. Lambrecht et al. [13] empirically showed that young women are more expensive to show ads to. To simulate this setting, we considered a scenario in which the other advertisers prefer females (i.e., $g_j^w < g_j^m$ for all $j \neq i$). We used the Expensive female - Equal value parameter settings for this scenario. As in the first scenario, we have advertiser i value both types equally, at the average of the two different values used by the other advertisers. Figure 4(d) plots the total utility ratio as before (solid line). Note that as women become rare, the K-parity version struggles relative to the unrestricted one since the other advertisers snap up the few women leaving the constrained version unable to bid for men. The figure also shows the total utility ratio for a constrained version of

| Name                      | Others $\mu_i^m$ | $\mu_i^w$ | Advertiser i $\mu_i^m$ | $\mu_i^w$ |
|---------------------------|-----------------|----------|------------------------|----------|
| Equal price - Female valuable | -2.8            | -2.8     | -3.5                   | -2.4     |
| Expensive female - Equal value | -3.5            | -2.4     | -2.8                   | -2.8     |

### Table 1: Parameters for the log-normal distribution used in modeling the bids in the ad slot auctions. $\sigma^2$ is always 0.7.
the advertiser $i$ that uses the same simple bidding strategy as the unrestricted advertisers (dashed line). Note that ratio is lower than with our optimal bidding strategy, showing its value. This difference comes from our optimal bidding strategy overbidding for the female viewers, delaying the aforementioned effect. Figure 4(e) tells a similar story for the ratio constraint.

Figures 4(f) and 4(g) further explore overbidding using a variation on the Expensive female - Equal value scenario. Rather than keep the value that the advertiser $i$ assigns to males fixed at $\mu^m_i = -2.5$, we vary this value to see its effect on overbidding. Rather than plot $\mu^m_i$ itself, we plot the ratio of $\mu^m_i$ to the value assigned to males by the other advertisers. Figure 4(f) shows this value ratio by using various lines. For all such ratios above 1, as the rate $p$ of male viewers increases, the optimal $K$-parity advertiser will increase its overbidding on the female viewers since they are more scared. Figure 4(g) shows that as $\mu^m_i$ (and, thus, the male value ratio of advertiser $i$ to the other advertisers) increases, the overbidding for females increases. Figure 4(h) plots the utility ratio as the value of the rate $\delta$ at which the advertiser $i$ will leave the ad network changes. Rather than plot $\delta$ directly, it plots the expected lifespan of the advertiser computed from $\delta$. It shows that for short lived advertisers, $K$-parity has no effect since the advertiser is unlikely to reach $K$ wins for either gender. However, the constraint rapidly has an effect as the advertiser lives long enough to win this number of slots.

Ad Exchange Revenue. Also important is how our strategy impacts the revenue of the ad exchange. We explored the ratio of the ad exchange’s revenue when there is one restricted advertiser for each ad slot auction to the case where all advertisers are unrestricted for all of our scenarios (both real and synthetic dataset). In most cases the ad exchange revenue will not decrease at all. The worst case happens for (1.0, 1)-ratio constraint advertisers on Yahoo! A1, the ratio of revenues is 0.993. The ad exchange can have a lower bound on the $K$ and $r$ to make sure it does not lose any revenue. Therefore, implementing this feature will not significantly reduce the ad exchange’s revenue. Our observations show restricted advertisers are more likely to overbid which increases the ad exchange revenue. Figure 5 we compare the revenue of ad exchange’s for different number of restricted advertisers ($\rho$) for Yahoo! A1 dataset. As expected by increasing the number of restricted advertisers the revenue of ad exchange’s increases.

Performance. Algorithms 1 and 2, each of which only has to run once for each parameter setting, completed in under 2 minutes and under 10 minutes, respectively. Calculating bids during auctions, each took the 2 microseconds. We used a 2013 MacBook Pro with a 2.3 GHz Intel Core i7 and 16 GB of 1600 MHz DDR3 memory.

8 CONCLUSION AND DISCUSSION

Adding parity constraints results in a surprisingly complex bidding problem, exhibiting both over- and underbidding relative to the advertiser’s immediate value of an ad slot. Despite this complexity, we show a practical way of computing optimal bids, to within a small approximation factor $\epsilon$. This enables us to characterize how
the cost of parity depends upon not just its level of strictness $K$ or $R$, but also the base rate $p$ of types, their relative values to both the governed advertiser $i$ and to other advertisers, and the lifespan (or discounting factor) $\delta$, in sometimes counter-intuitive ways.

We envision two ways in which advertisers could use our bidding strategy. Firstly, ad exchanges might implement it for them as a feature in the ad buying interface. Such exchanges could use the data it has to determine the demographics of individuals viewing ad slots and adjust bids accordingly. While this would require a change to the ad exchange, it would not require modifying the core auction mechanism, making it a more straightforward feature to add.

Secondly, the strategy could be used either directly by the advertiser or offered to them by demand-side platforms as a feature. This approach has the advantage of not requiring any changes to the ad exchange. It has the disadvantage of only working for ad exchanges that support real-time bidding and programmatic advertising with rich enough data to infer the group membership of the people viewing ad slots. Additionally, such rich data can pose privacy concerns.

Figure 4: Experimental results for synthetic datasets
We believe that either of these approaches to deployment would be more straightforward than any way of deploying an auction mechanism that enforces parity constraints [4] or Guaranteed ad Delivery (GD) [15, 18]. Only the ad exchange would be able to implement such functionality. Presumably, ad exchanges have already selected the auction mechanism that they believe would be best for their business and would be reluctant to change it in a way that could have wide ranging effects. Given that Google uses a generalization of second-price auctions [9], it may believe that the theoretical result that second-price auctions are uniquely optimal in certain settings has some bearing on its setting. Thus, it may believe that any change to its auction mechanism is likely to reduce its profits, a strong disincentive. We believe that ad exchanges would be more willing to implement a change that instead only alters the bids of advertisers who opt in since it would be equivalent to one that advertisers could already implement unilaterally by altering their bids. Furthermore, since our approach changes just opted-in advertisers’ bids, there is a sense in which they pay for it.

Future work can explore more complex forms of nondiscrimination constraints, such as ones holding probabilistically or asymptotically. The use of bonuses for ad clicks and online tracking to assign different expected values to individual ad slots could be considered. Future work could accommodate constraints for non-binary sensitive attributes, such as location (a proxy for race, which is apparently not explicitly tracked by any ad exchange) or for multiple constraints simultaneously. Although our MDPs can straightforwardly be extended to such cases using a cross-product-like construction, the MDP size will be exponential in the number of constraints and their values, motivating even more significant future work.

The constraints we explore are very strict in that they must hold at all times, as opposed to holding with high probability or asymptotically, which might be acceptable in some settings. In related problems, parity may only be required at the end of certain checkpoints, such as at the end of a hiring season. Exploring such relaxations can be future work.

We used a simple model in which the expected value of each female slot is equal to the others.Advertisers can use online tracking, machine learning, and other techniques to compute more fine-grained estimations of slot values. Furthermore, our model of ad exchanges does not include that they are often paid more when the viewer clicks on the ad. Thus, their expected value for selling an slot to an advertiser depends upon not just the bid prices but also the fits of the ads for the slot, which also can be estimated with online tracking and machine learning. Such tracking and machine learning can be another route to discrimination [6].

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Figure 5: Ratio of the ad plot form revenue for Yahoo A1 dataset with (0.8,5)-ratio restricted advertiser for different number of restricted advertisers.

We used a simple model in which the expected value of each female slot is equal to the others, and the expected value of each male slot is equal to the others. Advertisers can use online tracking, machine learning, and other techniques to compute more fine-grained estimations of slot values. Furthermore, our model of ad exchanges does not include that they are often paid more when the viewer clicks on the ad. Thus, their expected value for selling an slot to an advertiser depends upon not just the bid prices but also the fits of the ads for the slot, which also can be estimated with online tracking and machine learning. Such tracking and machine learning can be another route to discrimination [6].
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