Muons spectra of Quasi-Elastic and 1-pion production events at the KEK LBL
neutrino oscillation experiment

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I. INTRODUCTION

Recently the KEK to Kamioka long-baseline neutrino oscillation experiment (K2K) has published its first result [1], which confirms the existence of \( \nu_e \) oscillation as seen in the Super-Kamiokande (SK) atmospheric neutrino data [2]. The observed oscillation parameters from K2K agree well with the neutrino mass and mixing angles deduced from the atmospheric neutrino oscillation data [2].

\[
\sin^2 2\theta \simeq 1 \quad \text{and} \quad \Delta m^2 \simeq 3 \times 10^{-3}\text{eV}^2 .
\]

As is well known, in a two flavor scenario, the probability for a muon neutrino with energy \( E_\nu \) to remain a muon neutrino after propagating the distance \( L \) is given by the following expression

\[
P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right) . \tag{1}
\]

Basically, the standard approach to measure the oscillation parameters is to determine the oscillation probability in Eq. (1) in dependence of \( E_\nu \). At the position of the minimum \( \Delta m^2 \) can be determined from the condition \( \Delta m^2 L = \frac{\pi}{2} \) and \( \sin^2 2\theta \) from \( P_{\mu\mu}(E_\nu_{\min}) = 1 - \sin^2 2\theta \). The neutrino energy is not directly measurable but can be reconstructed from the simple kinematics of quasi-elastic (QE) scattering events. Measuring the energy \( E_\mu \) and the polar angle \( \cos \theta_\mu \) of the produced muon allows to reconstruct \( E_\nu \) with help of the following relation (even if the scattered proton is not observed)

\[
E_\nu = E_\nu[E_\mu, \cos \theta_\mu] = \frac{2M E_\mu - m_\mu^2 / 2}{M - E_\mu + \left| \vec{k}_\mu \right| \cos \theta_\mu} . \tag{2}
\]

Here \( M \) denotes the proton mass, \( m_\mu \) the muon mass and \( \vec{k}_\mu \) is the three-momentum of the muon in the laboratory system.

However, in practice there are some difficulties. First of all, the experimental one-ring muon events (1R\( \mu \)) are not pure QE event samples. About 30% of the 1R\( \mu \) events are 1-pion production events with unidentified or absorbed pions. For the 1-pion events Eq. (2) would systematically underestimate the true neutrino energy [3]. Secondly, the reconstruction of \( E_\nu \) gets more complicated including binding energy \( \epsilon_B \) and Fermi motion of the target nucleons

\[
E_\nu = E_\nu[E_\mu, \cos \theta_\mu, \vec{p}, \epsilon_B] = \frac{(E_p + \epsilon_B)E_\mu - (2E_p\epsilon_B + \epsilon_B^2 + m_\mu^2)/2 - \vec{p} \cdot \vec{k}_\mu}{E_p + \epsilon_B - E_\mu + \left| \vec{k}_\mu \right| \cos \theta_\mu - \left| \vec{p} \right| \cos \theta_p} ,
\]

where \( \vec{p} \) is the three momentum and \( E_p = \sqrt{M^2 + \vec{p}^2} \) the energy of the initial nucleon. Further, \( \theta_p \) is the polar angle of the target nucleon w.r.t. the direction of the incoming neutrino. Neglecting \( \epsilon_B \) and the momentum \( \vec{p} \) Eq. (2) is recovered. Since the momentum \( \vec{p} \) is unknown, \( 0 \leq \left| \vec{p} \right| \leq p_F \) where \( p_F \) is the Fermi momentum, this will lead to an uncertainty of the reconstructed neutrino energy at given values \( E_\mu \), \( \cos \theta_\mu \), and \( \epsilon_B \) of about -9% to +6% for a single event.

Hence we can see no reliable way for reconstructing the neutrino energy for the 1R\( \mu \) sample on an event by event basis. On the other hand the muon energy is a directly measurable quantity for each event. Therefore it seems to us to be a better variable for testing the spectral distortion phenomenon compared to the reconstructed neutrino energy.

In this talk we summarize the basic ideas and the main results in [4] where we have used kinematic considerations to predict the muon energy spectra of the QE and 1-pion resonance production events which constitute the bulk of the charged-current \( \nu_e \) scattering events in the K2K experiment. These predictions can be checked with the observed muon energy spectra from the nearby detector. We also present the distortions of these muon spectra due...
to $\nu_\mu$ oscillation, which one expects to see at the SK detector. Comparison of the predicted muon spectra with those of the observed QE and 1-pion events at the SK detector will be very useful in determining the oscillation parameters.

## II. FLUX AVERAGED MUON ENERGY SPECTRA

The flux averaged muon energy spectra for QE and 1-pion events are given by

\[
\langle \frac{d\sigma^R}{dE_\mu} \rangle \equiv \int f(E_\nu) \frac{d\sigma^R}{dE_\mu} dE_\nu \tag{4}
\]

where \(f(E_\nu)\) is the neutrino flux at K2K for the nearby detector (ND) and 'R' denotes QE and $\Delta$ resonance contributions to 1-pion production, which dominates the latter. Simple kinematic considerations lead to the following approximation for the flux averaged muon energy spectra, both, for QE and 1-pion production [3]

\[
\langle \frac{d\sigma^R}{dE_\mu} \rangle \simeq \sigma^R_{tot}(E_R) f(E_R) , \tag{5}
\]

with

\[
E_R = E_\mu + \Delta E^R = E_\mu + \begin{cases} 
0.15 \text{ GeV for QE} \\
0.4 \text{ GeV for 1-pion} 
\end{cases} \tag{6}
\]

Furthermore, it is well known that the total cross sections for QE and $\Delta$ production tend to constant values for neutrino energies of about 1 GeV and 1.4 GeV, respectively: \(\sigma^R_{tot}(E_\nu) \to N^R\). Hence, for muon energies larger than about 1.2 GeV, Eq. (5) can be further simplified by replacing \(\sigma^R_{tot}\) by its constant asymptotic value \(N^R\):

\[
\langle \frac{d\sigma^R}{dE_\mu} \rangle \simeq N^R \times f(E_R) \quad \text{for} \quad E_\mu \gtrsim 1.2 \text{ GeV} , \tag{7}
\]

with

\[
N^R = \begin{cases} 
4.5 \text{ fb for QE} \\
5.5 \text{ fb for 1-pion} 
\end{cases} \tag{8}
\]

The normalizations correspond to the average cross-section per nucleon for a H$_2$O target [3].

Thus we conclude that at large muon energies \(E_\mu \gtrsim 1\) GeV the flux averaged muon energy cross section \(\langle \frac{d\sigma^R}{dE_\mu} \rangle\) is directly proportional to the neutrino flux shifted in energy. The normalizations \(N^R\) and energy shifts \(\Delta E^R\) are predictions of our theoretical calculation [4] which can be verified experimentally with the muon energy spectra of the QE and 1-pion events observed at the ND. Furthermore, Eqs. (4), (5), and (7) also apply to the far detector (FD) if one replaces the flux at the ND by the flux at the FD which is distorted by the neutrino oscillation probability \(P_{\mu\mu}(E_\nu)\):

\[
f(E_\nu) \to f_{SK}(E_\nu) = f(E_\nu) \times P_{\mu\mu}(E_\nu) . \tag{9}
\]

Comparing these predictions with the observed muon energy spectra of the QE and 1-pion events of the SK detector will test the spectral distortion due to $\nu_\mu$ oscillation and determine the oscillation parameters. Particularly, on the higher energy side of the peak the relative size of the SK to the ND cross-sections provides a direct measure of the spectral distortion and hence the underlying oscillation parameters:

\[
\frac{\langle \frac{d\sigma^R}{dE_\mu} \rangle_{FD}}{\langle \frac{d\sigma^R}{dE_\mu} \rangle_{ND}} \simeq P_{\mu\mu}(E_R) \quad \text{for} \quad E_\mu \gtrsim 1.2 \text{ GeV} . \tag{10}
\]

In Sec. III we present exact calculations of the QE [6] and 1-pion production cross sections. The dominant contribution to 1-pion production is from $\Delta$ resonance production for which we take the formalism of Refs. [7, 8, 9]. For completeness we have also included the $P_{11}(1440)$ and $S_{11}(1535)$ resonance contributions for which we used the parameterization and the form factors of Ref. [10]. We estimate the contribution from the still higher resonances along with the non-resonant background to be no more than $5 - 10\%$ of the 1-pion production cross section at these low energies (see Fig. 1). Therefore the accuracy of our prediction should be as good as that of the K2K experiment.

We compare our exact calculations with the approximation in Eqs. (7) and (10) from which the range of validity can be inferred. A comparison with the approximation in Eq. (5) can be found in Ref. [3] (see Figs. 2 and 3).

## III. NUMERICAL RESULTS

In this section we present our numerical results. Fig. 1 shows the K2K neutrino energy spectrum. In addition, we show an approximated spectrum by a Lorentzian given by

\[
f_{1L}(E_\nu) = \frac{N}{\pi} \frac{\Gamma}{(E_\nu - E_0)^2 + \gamma^2} , \tag{11}
\]

where \(N\) is an appropriate normalization factor.

Fig. 2 shows the predicted muon energy spectra for the QE (solid line) and 1-pion production (dashed line) processes. Clearly one can see that the peak at \(E_0 = 1.2\) GeV is shifted left by $\Delta E \simeq 0.15\text{ GeV}$ for the QE and $\Delta E \simeq 0.4 - 0.5\text{ GeV}$ for the $\Delta$ resonance production which dominates the 1-pion production process. The steepness of the muon energy spectra at the low energy reflects the threshold rise of $\sigma^R$ and the steep neutrino flux. On the other hand the muon energy spectra closely
FIG. 1: K2K neutrino energy spectrum. The solid line is the exact spectrum normalized to unit area. The dashed line shows the approximated spectrum by the Lorentzian in Eq. (11). We also show for comparison as dotted line the same Lorentzian, but with normalization 1.4 instead of 1.25.

follow the shape of the neutrino energy spectrum on the right side of the peak. The predicted exact muon energy spectra of Fig. 2 agree reasonably well with the corresponding spectra of the K2K ND [1] for both the QE and the non-QE sample. In particular one can compare the predicted QE spectrum with their simulated QE spectrum shown in Fig. 1 of Ref. [1]. Their Figs. 1a and c show separately the QE muon momentum distribution for the 1-Ring muon (1Rµ) sample of the 1KT and the QE enhanced sample of the FGD respectively. The two play complementary roles in covering the complete muon energy range, as the 1KT and the FGD have high efficiencies at $E_\mu < 1$ GeV and $E_\mu \gtrsim 1$ GeV, respectively [1]. One can not compare our predicted muon energy spectra with these figures quantitatively without folding in these efficiency factors, which are not available to us. But there is good qualitative agreement between the predicted QE spectrum of our Fig. 2a with their Fig. 1c at $E_\mu \gtrsim 1$ GeV and Fig. 1a at $E_\mu < 1$ GeV. While the former shows the position of the peak and the shape of the spectrum to the right, the latter shows the broadening of spectrum down to $E_\mu \simeq 0.4$ GeV. Similarly one sees good agreement between the predicted muon energy spectrum of our Fig. 2a for 1-pion events with the non-QE spectra of their Fig. 1c,d at $E_\mu \gtrsim 1$ GeV and Fig. 1a at $E_\mu < 1$ GeV. Thus one has a simple and robust prediction for the shape of the muon energy spectrum in terms of the neutrino spectrum not only for the QE events but also for the 1-pion production events, which dominate the inelastic events.

Fig. 2: Exact predictions of the muon energy spectra for the (a) Nearby and (b) SK detectors of the K2K experiment. The QE (solid line) and the 1-pion production (dashed line) cross-sections are shown along with the $\Delta$ resonance contribution (dotted line).

For the estimation of the pion detection efficiency it is necessary to consider nuclear effects, i.e., Pauli blocking, nuclear absorption and charge exchange of the produced pions which can rescatter several times in the nucleus. Therefore we have included these effects following the prescription of Refs. [5, 11, 12]. Since the dominant contribution to 1-pion production processes comes from $\Delta$ resonance production on oxygen, we have evaluated the effects of nuclear absorption and rescattering on the produced pions for this case. The relevant charged current subprocesses are $\nu p \to \mu^- p\pi^+$, $\nu n \to \mu^- n\pi^+$ and $\nu n \to \mu^- p\pi^0$ with relative cross-sections 9 : 1 : 2 for the dominant contribution from $\Delta$ resonance.

Fig. 3 shows the effects of nuclear corrections on the produced $\pi^+$ and $\pi^0$ spectra from these processes for the nearby detector averaged over the neutrino spectrum. The results are very similar for the SK detector. Nuclear
rescattering effects result in enhancing the $\pi^0$ events at the cost of the dominant $\pi^+$ component. But taken together we see a nearly 20% drop in the rate of 1-pion events due to nuclear absorption of the produced pion. Moreover about 10% of the remaining events corresponds to the pion momentum being less than the Cerenkov threshold of 100 MeV. Therefore one expects about 70% of the $\Delta$ events to give a detectable pion ring at the SK detector while the remaining 30% appears like a QE event. Adding the latter to the 35% of genuine QE events would imply that about 50% of the CC events will appear QE-like at the SK detector. Alternatively the observed muon energy spectrum of the sum of 1-Ring and 2-Ring muon events could be compared with the predicted spectrum of the sum of QE and 1-Pion events. Although a part of the 2-Ring events may come from multi-pion production, the resulting error may be small since multi-pion events at the ND constitute only $\sim 15\%$ of CC events.

**FIG. 3:** The momentum distribution of the decay pion for charged current resonance production in oxygen with (solid line) and without nuclear correction (dotted line). The dashed line takes only into account the effect of Pauli blocking.

**FIG. 4:** The predicted exact muon energy spectra (solid lines) and the approximation (dashed lines) according to Eq. 4 for QE and 1-pion production for the (a) Nearby detector and (b) SK detector of the K2K experiment.

Next, we turn in Fig. 4 to a comparison of the exact calculation for the muon energy spectra (solid lines) with the approximation (dashed lines) made on the right hand side of Eq. (7). Fig. 4a shows the results for the ND and Fig. 4b for the FD obtained with the neutrino flux specified in Eq. (9). One can see that the approximation is in perfect agreement with the exact calculation for muon energies $E_\mu \geq 1.2$ GeV for single pion production and $E_\mu \geq 1$ GeV for QE scattering. Hence, in this region the muon energy spectra are directly proportional to the neutrino flux shifted in energy.

**FIG. 5a** and **5b** show the far-near-ratio of the muon energy spectra for QE and 1-pion production, respectively. In this case, the dashed lines depict the exact result and the solid lines the oscillation probabilities $P_{\mu\mu}$ given in Eq. (1) which have been evaluated at the shifted energies $E_R$ given in Eq. (6). Again, at $E_\mu \geq 1$ GeV (QE) and $E_\mu \geq 1.2$ GeV (1-pion production) the approximation in Eqs. 7 and 10 works well
and a measurement of the far-near-ratio of such pure QE or 1-pion event samples in this kinematic region would give direct access to the oscillation probability. However, as has been discussed above, the observable 1-Ring and 2-Ring muon events are superpositions of QE, 1-pion, and multi-pion events making the analysis more complicated.

IV. CONCLUSIONS

The muon energy spectra of QE and 1-pion events provide a complementary approach to experimental extrac-
tions of the atmospheric muon oscillation parameters at K2K. The results are based on quite general kinematic considerations and will also be applicable to other future long baseline experiments like J2K, MINOS and the CERN-Gran Sasso experiments which plan to use low energy $\nu_\mu$ beams\[13,14\]. Therefore it will be very useful to extend this analysis for the beam energy spectra and the target nuclei of these experiments.

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