The effect of the Landau–Zener transitions on nuclear fission dynamics

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Abstract

In the paper, it is studied the influence of Landau–Zener transitions between nuclear many-body states on the dissipative properties of nuclear large-amplitude collective motion. Within the cranking–like approach, we describe the time evolution of a nuclear many-body system as the self-consistent motion in a space of intrinsic excitations and in a space of a single collective (deformation) parameter. By that we measure how the spectral statistics of the nuclear energy levels affects the fission rate at quite large initial temperatures of heavy nuclei.
I. INTRODUCTION

It has been found that the perturbative dynamics of complex quantum systems has a diffusive behaviour in a space of the occupancies of adiabatic eigenstates of the system’s Hamilton operator. As a result of such a quantum mechanical diffusion, the complex quantum systems may absorb an energy of external perturbation. From this perspective, it will be interesting to measure how the quantum diffusion reveals itself on macroscopic level, i.e., when the time variations of macroscopic parameter are not arbitrary but provide a constancy of total energy of the system. Such a study is important to understand the quantum nature of fluctuations and dissipation appearing in dynamics of macroscopic (collective) coordinates in finite Fermi systems.

The main purpose of the present paper is to investigate different time regimes of the quantum mechanical diffusion and to find its effect on time evolution of the macroscopic coordinate. The plan of the paper is as follows. In Sect. II we start from the time–dependent Schrödinger equation and introduce adiabatic basis of the system’s Hamiltonian. In the weak–coupling limit, we get a closed set of equations for the time evolutions of the occupancies of adiabatic states. In Sect. IV we derive equation of motion for the macroscopic variable. In Sect. V we apply the obtained Langevin–like equation for the macroscopic variable to the problem of thermal overcoming of the nuclear fission barrier and calculate the fission rate over a model parabolic barrier. Conclusions and discussion of the main results of the paper are given in the Summary.

II. INTRINSIC EXCITATIONS DYNAMICS

We start our discussion of the time evolution of a nuclear many–body system from the Liouville equation for the density matrix operator \( \hat{\rho} \),

\[
\frac{\partial \hat{\rho}(t)}{\partial t} + i \hat{L}(t) \hat{\rho}(t) = 0. \tag{1}
\]

Here, \( \hat{L} \) is the Liouville operator defined in terms of the commutator

\[
\hat{L} \hat{\rho} = \frac{1}{\hbar} \left[ \hat{H}, \hat{\rho} \right], \tag{2}
\]

where \( \hat{H}(t) \equiv \hat{H}[q(t)] \) is a Hamilton operator of the nuclear system that parametrically depends on a single deformation (collective) variable \( q \).
In the sequel, we shall deal with dynamics of the density matrix whose diagonal part describes real transitions between states of the quantum system and which is important to understand microscopically the appearance of dissipation in macroscopic collective motion. With that purpose, we apply the Zwanzig’s projection technique \[12\] to Eq. (1), we introduce a projection operator $\hat{\mathcal{P}}$ and split the density matrix operator into the diagonal and non–diagonal parts,

$$\hat{\rho} = \hat{\rho}_d + \hat{\rho}_{od},$$  \hspace{1cm} (3)

where the diagonal part is defined as

$$\hat{\rho}_d = \hat{\mathcal{P}} \hat{\rho},$$ \hspace{1cm} (4)

and the non–diagonal part is given by

$$\hat{\rho}_{od} = (1 - \hat{\mathcal{P}}) \hat{\rho}.$$ \hspace{1cm} (5)

It is assumed that the projection operator $\hat{\mathcal{P}}$ is linear and time–independent. Acting on the Liouville equation (1) by the operators $\hat{\mathcal{P}}$ and $1 - \hat{\mathcal{P}}$ from the left, we obtain a system of equations for $\hat{\rho}_d$ and $\hat{\rho}_{od}$

$$\frac{\partial \hat{\rho}_d}{\partial t} + i \hat{\mathcal{P}} \hat{L}(\hat{\rho}_d + \hat{\rho}_{od}) = 0,$$ \hspace{1cm} (6)

$$\frac{\partial \hat{\rho}_{od}}{\partial t} + i (1 - \hat{\mathcal{P}}) \hat{L}(\hat{\rho}_d + \hat{\rho}_{od}) = 0.$$ \hspace{1cm} (7)

Formally, a solution to Eq. (7) can be written as

$$\hat{\rho}_{od}(t) = \hat{\rho}_{od}(t = 0) - i \int_0^t \exp \left\{ -i \int_0^{t-t'} (1 - \hat{\mathcal{P}}) \hat{L}(t'') dt'' \right\} (1 - \hat{\mathcal{P}}) \hat{L}(t') \hat{\rho}_d(t') dt'.$$ \hspace{1cm} (8)

Substituting solution (8) into Eq. (6), we obtain a closed kinetic equation for the diagonal part of the density matrix operator $\hat{\rho}_d$

$$\frac{\partial \hat{\rho}_d(t)}{\partial t} = -i \hat{\mathcal{P}} \hat{L}(t) \hat{\rho}_d(t) - i \hat{\mathcal{P}} \hat{L}(t) \hat{\rho}_{od}(t = 0)$$

$$+ \int_0^t \hat{\mathcal{P}} \hat{L}(t) \exp \left\{ -i \int_0^{t-t'} (1 - \hat{\mathcal{P}}) \hat{L} dt'' \right\} (1 - \hat{\mathcal{P}}) \hat{L}(t') \hat{\rho}_d(t') dt'.$$ \hspace{1cm} (9)

Let us write the basic kinetic equation (9) in matrix form. With that, we use an adiabatic basis of the Hamilton operator $\hat{H}(q)$,

$$\hat{H}(q) \Psi_n(q) = E_n(q) \Psi_n(q).$$ \hspace{1cm} (10)
This basis is determined by a set of static wave functions $\Psi_n$ and energies $E_n$ found at each fixed value of the macroscopic variable $q$. Using the adiabatic basis (10), the time–dependence of the matrix elements of the density matrix operator $\hat{\rho}$ are given by

$$\rho_{nm}(t) = \exp \left\{ -i \int_0^t \omega_{nn}(t') dt' \right\} \langle \Psi_n | \hat{\rho} | \Psi_m \rangle,$$

and the matrix elements of the Liouville operator $\hat{L}$ (2) are equal to

$$(\hat{L})_{nmn'm'} = \exp \left\{ -i \int_0^t \omega_{nn'}(t') dt' \right\} \langle \Psi_n | \frac{\partial}{\partial t} | \Psi_{n'} \rangle \delta_{mm'} - \exp \left\{ -i \int_0^t \omega_{m'n}(t') dt' \right\} \langle \Psi_{m'} | \frac{\partial}{\partial t} | \Psi_m \rangle \delta_{nn'},$$

where $\omega_{nn} = (E_n - E_m)/\hbar$.

The second term on the left–hand side of Eq. (9) with the choice of the projection operator $\hat{P}$,

$$(\hat{P})_{nmn'm'} = \delta_{nn'} \delta_{m'm} \delta_{mm'},$$

vanishes, since in this case

$$\hat{P} \hat{L} \hat{P} = 0.$$  

Therefore, from Eq. (9) we obtain

$$\frac{\partial \hat{\rho}_d(t)}{\partial t} = - \int_0^t \hat{P} \hat{L}(t) \exp \left\{ -i \int_0^{t-t'} (1 - \hat{P}) \hat{L} dt' \right\} (1 - \hat{P}) \hat{L}(t') \hat{\rho}_d(t') dt'.$$  

Writing down the last equation in the matrix notations, we obtain

$$\frac{\partial \rho_{nm}(t)}{\partial t} = \sum_{m \neq n} \int_0^t H_{nmnm}(t, t') [\rho_{mm}(t') - \rho_{nn}(t')] dt'.$$

Here, the integral kernel $H_{nmnm}$ is equal to

$$H_{nmnm}(t, t') = - \left( \hat{P} \hat{L}(t) \exp \left\{ -i \int_0^{t-t'} (1 - \hat{P}) \hat{L} dt' \right\} (1 - \hat{P}) \hat{L}(t') \right)_{nmnm},$$

where it was used the fact that

$$\sum_m H_{nmnm} = 0.$$  

We will proceed by considering the integral kernel $H_{nmnm}$ of our basic kinetic equation (16). Since the expression (12) contains the strongly oscillating exponential factors, we can approximately put the energy distances $E_n - E_m$ at the same time instant $t$. Thus, by using Eq. (10) one gets from (17)

$$H_{nmnm}(t, t') \approx \sum_{abcd} \frac{\dot{q}(t)\dot{q}(t')}{(E_a - E_b)(E_c - E_d)(q[t])} \left( \frac{\partial H}{\partial q} \right)_{ab} (q[t]) \left( \frac{\partial H}{\partial q} \right)_{cd}^{*} (q[t']) \times \exp(-i\omega_{ab}(q[t]) \cdot t) \exp(i\omega_{cd}(q[t']) \cdot t') G_{abcd}(t, t') (\delta_{mn} - \delta_{an})(\delta_{dm} - \delta_{cm}),$$

(19)
where star stands for the complex conjugation and

\[ G_{abcd}(t, t') = \left( \exp \left\{ -i(1 - \hat{P}) \int_0^{t-t'} \hat{L}(t'')dt'' \right\} \right)_{abcd}. \]  (20)

Factor \( G_{abcd} \) determines non–perturbative response of the quantum system (1)–(2) to the macroscopic parameter’s variations.

In this analysis, complexity of a quantum system is understood as the absence of any special symmetries in the system. Such system is expected to have some universal statistical properties which can be modelled by random matrix ensembles [13]. Within the random matrix approach [13], we average the right–hand side of Eq. (16) over suitably chosen statistics of the randomly distributed matrix elements \( h_{nm} \equiv (\partial \hat{H}/\partial q)_{nm} \) and the energy spacings \( E_n - E_m \). First, we perform the ensemble averaging over the matrix elements. They are treated as complex random numbers with the real and the imaginary parts which are independently Gaussian distributed. The correlation functions of the real and the imaginary parts of the matrix elements can be written in the following general form [9, 14]

\[ \frac{h_{nm}(q)h^*_{n'm'}(q')}{\delta_{nn'}\delta_{mm'}} = \frac{\sigma_0^2(q)}{\sqrt{\Omega(E_n)\Omega(E_m)}} f(|E_n - E_m|/\Gamma) Y(q - q'). \]  (21)

Here, we take into account both the energy and temporal correlations of the coupling matrix elements \( (\partial \hat{H}/\partial q)_{nm} \). In Eq. (21), \( \Omega \) is the average density of states at given excitation energy, \( f \) is the shape, \( \sigma_0^2 \) is the strength and \( \Gamma \) is the width of the energy distribution of the ensemble averaged matrix elements. The correlations of the matrix elements, existing at different values of the macroscopic parameter, \( q \) and \( q' \), are measured with the help of the correlation function \( Y \). Since the energy correlations between two different states \( n \) and \( m \) drop out with the rise of a distance between them, it is rather obvious that \( f \to 0 \) with \( |E_n - E_m|/\Gamma \to \infty \) and \( f \sim 1 \) at \( |E_n - E_m|/\Gamma << 1 \).

In the sequel, we will only study weak–coupling regime of the driven dynamics (16)–(20) when

\[ G_{abcd} = \delta_{ac}\delta_{bd}. \]  (22)

The applicability of such a regime will be discussed later on in next Section. Under the condition (22), we obtain (see Eqs. (16) and (21))

\[ \frac{\partial \rho(E_n, t)}{\partial t} = \frac{2\sigma_0^2(q(t))}{\sqrt{\Omega(E_n)\Gamma}} \int_0^t dt' \int_{E_{gs}}^{+\infty} dE_m \sqrt{\Omega(E_m)} f(|E_n - E_m|/\Gamma) \]

\[ \times Y(q[t] - q[t']) \cos\left( \frac{[E_n - E_m][t - t'/\hbar]}{(E_n - E_m)^2} \right) [\hat{\rho}(E_m, t') - \hat{\rho}(E_n, t')], \]  (23)
where $E_{gs}$ is the ground–state energy and the summation over all discrete states $m$ was replaced by the integration over the corresponding continuous energy variable $E_m$.

The energy spacings part of the ensemble averaging procedure is defined through the two–level correlation function, $R(\Omega|E_n - E_m|)$, that is the probability density to find the state $m$ with energy $E_m$ within the interval $[E_m, E_m + dE_m]$ at the average distance $|E_n - E_m|$ from the given state $n$ with energy $E_n$. We give an explicit form of the function $R$ for Gaussian Orthogonal Ensemble (GOE) [15]

$$R_{GOE}(x) = 1 - \left(\frac{\sin(\pi x)}{\pi x}\right)^2 + \left(\int_0^1 dy \frac{\sin(\pi xy)}{y} - \frac{\pi}{2}\right) \left(\frac{\cos(\pi x)}{\pi x} - \frac{\sin(\pi x)}{(\pi x)^2}\right),$$

(24)

Gaussian Unitary Ensemble (GUE)

$$R_{GUE}(x) = 1 - \left(\frac{\sin(\pi x)}{\pi x}\right)^2,$$

(25)

and Gaussian Symplectic Ensemble (GSE) of levels

$$R_{GSE}(x) = 1 - \left(\frac{\sin(2\pi x)}{2\pi x}\right)^2 + \int_0^1 dy \frac{\sin(2\pi xy)}{y} \left(\frac{\cos(2\pi x)}{2\pi x} - \frac{\sin(2\pi x)}{(2\pi x)^2}\right),$$

(26)

where $x \equiv |E_n - E_m|\Omega(E_n)$. The behaviour of the two–level correlation function $R(x)$ with the normalized level spacing $x$ for the different statistical ensembles (24), (25) and (26) is shown in Fig. 1. The main difference between the statistics, seen in Fig. 1, is the behaviour of $R(x)$ at small energy spacings $x$. For the GOE statistics one has the linear repulsion between levels, $R_{GOE} \sim x$, the GUE statistics implies the quadratic level repulsion, $R_{GUE} \sim x^2$, while in the GSE case we have $R_{GSE} \sim x^4$. On the other hand, $R_{GOE}$, $R_{GUE}$ and $R_{GSE}$ are similar at moderate spacings $x$, when the spectral correlations between levels consistently disappear.

Introducing the new energy variables,

$$E \equiv E_n, \quad e \equiv E_n - E_m,$$

(27)

we rewrite the dynamical equation (23) for the occupancies of the quantum adiabatic states within the random matrix approach as

$$\frac{\partial \rho(E, t)}{\partial t} = \frac{2\sigma_0^2 \dot{q}(t)}{\sqrt{\Omega(E)\Gamma}} \int_0^t dt' q(t') Y(q[t] - q[t']) \int_{-\infty}^{+\infty} de \sqrt{\Omega(E - e)R(\Omega|e|)f(|e|/\Gamma)} \times \frac{\cos(e|t - t'|/\hbar)}{e^2} \left[\tilde{\rho}(E - e, t') - \tilde{\rho}(E, t')\right].$$

(28)
The integration limits over the energy spacing \( e \) were extended to infinities since the time changes of the occupancy \( \bar{\rho}(E, t) \) of the given state with the energy \( E \) are mainly due to the direct interlevel transitions from the close–lying states located at the distances \( |e| << E \). The same assumptions enable us to truncate expansion to \( e^3 \)–order terms,

\[
\sqrt{\Omega(E - e)}[\bar{\rho}(E - e, t') - \bar{\rho}(E, t')] = -\sqrt{\Omega(E)}\frac{\partial \bar{\rho}(E, t')}{\partial E} e + \frac{1}{2\sqrt{\Omega(E)}} \frac{d\Omega(E)}{dE} \frac{\partial \bar{\rho}(E, t')}{\partial E} e^2 + \sqrt{\Omega(E)} \frac{\partial^2 \bar{\rho}(E, t')}{\partial E^2} e^2 + O(e^3). \tag{29}
\]

Substituting the expansion (29) into Eq. (28), the odd-\( e \) terms drop out and we obtain the diffusion–like equation of motion for the occupancy \( \bar{\rho}(E, t) \),

\[
\Omega(E) \frac{\partial \bar{\rho}(E, t)}{\partial t} \approx \sigma_0^2 \dot{q}(t) \int_0^t dt' K(t, t') \dot{q}(t') \frac{\partial}{\partial E} \left[ \Omega(E) \frac{\partial \bar{\rho}(E, t')}{\partial E} \right]. \tag{30}
\]

In Eq. (30), the memory kernel, \( K(t, t') \), is defined as

\[
K(t, t') = \frac{1}{\Gamma} Y(q - q') \int_{-\infty}^{+\infty} d\epsilon f(|\epsilon|/\Gamma) R(|\epsilon|\Omega(E)) \cos(\epsilon[t - t']/\hbar). \tag{31}
\]

One can treat dynamical process (30) as a quantum mechanical diffusion of energy in space of the occupancies of quantum adiabatic states, where \( \Omega(E)\bar{\rho}(E, t) \) gives a probability density to find the quantum system with energy lying in the interval \( [E, E + dE] \) at the moment of time \( t \). Due to our truncation of the expansion (29), the diffusion is Gaussian. Certainly, keeping the higher order terms in Eq. (29) would imply a non–Gaussian character of the diffusive process (30).

### III. DIFFERENT REGIMES OF THE QUANTUM MECHANICAL DIFFUSION

Features of the perturbed dynamics (30) are defined by the static characteristics of the quantum system, its average density of states \( \Omega \), the strength \( \sigma_0^2 \) and width \( \Gamma \) of the energy distribution (21) of the coupling matrix elements (21), as well as by the amplitude \( \Delta q \equiv q(t) - q(t = 0) \) and velocity \( \dot{q}(t) \) of the macroscopic coordinate’s variations. First, we shall investigate under what conditions the weak–coupling regime (22) of the driven dynamics is realized. With this purpose, we ensemble average the non–perturbative factor \( G_{abcd} \) (20).

One sees that \( G_{abcd} \) will only contain even powers of the Liouville operator \( \hat{L} \) and such an expansion in terms of \( \hat{L} \) is determined by a perturbation parameter

\[
\alpha \sim \left| \left( \int_0^t dt_1 \int_0^{t_1} dt_2 (1 - \mathcal{P})\hat{L}(t_1)(1 - \mathcal{P})\hat{L}(t_2) \right)_{abab} \right|. \tag{32}
\]
With the help of Eqs. (13) and (21), one can show that

\begin{align}
\alpha &= \frac{\sigma_0^2}{\Gamma} \left| \int_0^t dt_1 \dot{q}(t_1) \int_0^{t_1} dt_2 \dot{q}(t_2) \right| \left\{ \sum_c \frac{f(|E_c - E_a|/\Gamma)}{(E_c - E_a)^2 \sqrt{\Omega(E_c)\Omega(E_a)}} \cos\left(\frac{[E_c - E_a][t_1 - t_2]}{\hbar}\right) \right. \\
&\quad + \sum_d \frac{f(|E_d - E_b|/\Gamma)}{(E_d - E_b)^2 \sqrt{\Omega(E_d)\Omega(E_m)}} \cos\left(\frac{[E_d - E_b][t_1 - t_2]}{\hbar}\right) \\
&\quad - 2 \frac{f(|E_a - E_b|/\Gamma)}{(E_a - E_b)^2 \sqrt{\Omega(E_a)\Omega(E_b)}} \cos\left(\frac{[E_a - E_b][t_1 - t_2]}{\hbar}\right) \right|.
\end{align}

Making simplifying assumption on equidistant spectrum of the quantum system, we get a condition for the applicability of the weak–coupling regime (22):

\begin{equation}
\alpha \sim \frac{\sigma_0^2 \Omega^3 (\Delta q)^2}{\Omega \Gamma} \ll 1.
\end{equation}

Our next goal here is to discuss time scales defining the driven dynamics. Three different timescales enter the diffusive dynamics (30)–(31). The first timescale, \(\tau_{\text{cor}}\), is characteristic interval in time over which the coupling matrix elements \(\partial \hat{H}/\partial q\) \(nm(q[t])\) and \(\partial \hat{H}/\partial q\) \(nm(q[t'])\) are effectively correlate. Putting \(q[t'] \approx q[t] + \dot{q}t\) and by using perturbation theory with respect to small parameter \(|\dot{q}t/\Delta q|\), we can estimate the correlation time \(\tau_{\text{cor}}\) as

\begin{equation}
\tau_{\text{cor}} \sim \frac{1}{(\sigma_0/\sqrt{\Omega \Gamma})\dot{q}}.
\end{equation}

The second one is the characteristic timescale \(\hbar/\Gamma\) caused by the finite width \(\Gamma\) of the energy distribution of the ensemble averaged matrix elements (21). In the limit \(\Gamma >> 1/\Omega\) (when the features of the location of neighboring energy levels (24)–(25) are unsignificant), the memory kernel \(K(t, t')\) (31) is defined by the cosine–Fourier transform of the matrix elements’ energy distribution:

\begin{equation}
K(t, t') = 2\pi Y(q - q') F_{\cos} \left( f[(t - t')/(\hbar/\Gamma)] \right).
\end{equation}

And the third timescale is the characteristic time of the macroscopic coordinate’s variations \(\tau_{\text{macr}}\). In the paper, we do not investigate the effect of the time correlations of the coupling matrix elements by putting

\begin{equation}
Y(q - q') = 1.
\end{equation}

This is so when we neglect the time variations of the coupling matrix elements, \((\partial \hat{H}/\partial q)_{nm}(q[t]) \approx (\partial \hat{H}/\partial q)_{nm}(q[t = 0])\). On the other hand, the condition (37) can be
considered as a consequence of the weak–coupling limit since in this case, the correlation time $\tau_{cor}$ of the matrix elements is the largest timescale in the system,

$$\tau_{cor} \gg \tau_{macr} \gg \tau_{macr}.$$

(38)

At the end of this discussion, we would like to point out the following fact. As is shown in Refs. [8, 9], the existence of temporal correlations between the coupling matrix elements plays an important role at the non–perturbative regime of parametrically driven dynamics of complex quantum systems. Thus, taking into account of these correlations gives rise to significant reduction of the diffusion coefficient compared to the well–known one given by the Kubo formula.

In fact, the different dynamical regimes of the quantum mechanical diffusion is defined by the relationship between the characteristic time $\tau_{macr}$ of the macroscopic coordinate’s variations and the characteristic time scale of the driven dynamics $\hbar/\Gamma$:

(i) $\hbar/\Gamma << \tau_{macr}$. To understand this limiting situation, when the interaction between the complex quantum states almost the same over a large energy window $\Gamma$, we assume at the moment that initially only one fixed eigenstate $n_0$ of the system is occupied. Then, with a time run, the initial occupation peak will spread out over a huge number of neighboring states. One can also say that the quantum chaotic system (1)–(2) adopts almost instantaneously to the external perturbation $\dot{q}(t)$. In this case, the memory kernel is sharply peaked with respect to $t - t'$ and we get a normal diffusive regime of the quantum driven dynamics:

$$\Omega(E)\frac{\partial \rho(E,t)}{\partial t} = \frac{\hbar \sigma^2_0 \dot{q}^2(t)}{\Gamma} \frac{\partial}{\partial E} \left[ \Omega(E) \frac{\partial \rho(E,t)}{\partial E} \right].$$

(39)

Such situation corresponds to the usual Kubo formula regime, when the energy diffusion coefficient $D_E = \hbar \sigma^2_0 \dot{q}^2/\Gamma$ is proportional to the square of the external parameter’s velocity $\dot{q}^2$. Formally, at $\Gamma = \infty$, the energy diffusion disappears since all the eigenstates becomes equally occupied implying the absence of any energy flows in the system and therefore, any kind of diffusion.

(ii) $\hbar/\Gamma >> \tau_{macr}$. Now, each eigenstate is effectively coupled only to a few neighboring states and therefore, the initial occupation distribution will slightly disperse and remain almost unchanging for a quite long time of order $\hbar/\Gamma$. The memory kernel $K(t, t')$ can be well
approximated by constant at $\hbar/\Gamma \to \infty$ and the non–Markovian diffusion–like equation (30) for the occupancies becomes a wave equation of the following form:

$$\Omega(E) \frac{\partial^2 \bar{\rho}(E, t)}{\partial t^2} = \sigma_0^2 \dot{q}(t) \Delta q(t) \frac{\partial}{\partial E} \left[ \Omega(E) \frac{\partial \bar{\rho}(E, t)}{\partial E} \right]. \quad (40)$$

In fact, the transport of energy from the occupied states to unoccupied ones undergoes as a wave propagation with the speed $\sigma_0^2 \dot{q} \Delta q$. Here, we have a ballistic regime of the quantum driven dynamics (30).

(iii) At moderate values of the width $\Gamma$, the quantum driven dynamics (30) is essentially influenced by memory effects, when both the diffusive and ballistic regimes coexist. At the constant driven velocity, $\dot{q}(t) = q_0$, in the beginning the ballistic regime of the quantum mechanical diffusion (30) sets in

$$\text{var}_E \equiv \langle E^2 \rangle - \langle E \rangle^2 \approx \sigma_0^2 q_0^2 \cdot t^2, \quad 0 < t \leq \frac{\hbar}{\Gamma}, \quad (41)$$

where

$$\langle \ldots \rangle \equiv \int^\infty_{E_g} \ldots \bar{\rho}(E, t) \Omega(E) dE. \quad (42)$$

The ballistic energy transport is relieved by the the normal energy diffusion

$$\text{var}_E \approx \frac{\hbar \sigma_0^2 q_0^2}{\Gamma} \cdot t, \quad t \geq \frac{\hbar}{\Gamma}. \quad (43)$$

IV. CRANKING APPROACH

It is rather interesting to see how the dynamical regimes (39)–(40) of the quantum mechanical diffusion show up macroscopically. More specifically, would it be a microscopic source for damping of different types of collective excitations in complex many–body systems, when the macroscopic collective modes of motion are coupled to infinite bath of the intrinsic degrees of freedom? The appropriate approach for such investigation is a cranking model. Following the ideology of this approach, we define the time evolution of a macroscopic collective variable from the condition of the energy conservation.

To treat selfconsistently dynamics of the classical macroscopic coordinate and quantum system within the cranking approach, one has to clarify the following. The energy diffusion regime of the perturbed dynamics implies statistic interpretation. One can say that actually we have an ensemble of quantum chaotic system, characterizing by its own initial density
matrix operator $\hat{\rho}(t = 0)$ and energy path $E(t)$ \[30\]. Therefore, in order to provide the constancy of the total energy of the system, one has necessarily to introduce a bunch of macroscopic parameter’s trajectories. Or, we are able to claim that the time evolution of the macroscopic classical parameter should be random (non–deterministic). By that, we microscopically derived the fluctuations in the macroscopic collective dynamics.

To obtain an equation of motion for the driving parameter, we first find the average energy of the system, $\Sigma(t) = Tr[\hat{H}\{q(t)\}, \rho(t)]$. Calculating its time change, we get

$$\frac{d\Sigma}{dt} = \sum_i \dot{q}_i \frac{\partial E_{gs}}{\partial q_i} + \sum_i \dot{q}_i \sum_{n,m} \left( \frac{\partial \hat{H}}{\partial q_i} \right)_{mn} \rho_{nm} + \sum_n \dot{q}_i \sum_n \left( \frac{\partial \hat{H}}{\partial q_i} \right)_{nn} \rho_{nn} + \sum_n E_n \frac{\partial \rho_{nn}}{\partial t} + \sum_i \dot{q}_i \sum_{n,m} \left( \frac{\partial \hat{H}}{\partial q_i} \right)_{mn} \rho_{nm}. \tag{44}$$

The first term in the right–hand side of Eq. (44) describes a change of the macroscopic potential energy. The second contribution to the energy rate $d\Sigma/dt$ is defined by the non–diagonal components of the density matrix $\rho_{nm}(t)$. Its time evolution is caused by the virtual transitions among the adiabatic states. We believe that such a term is a microscopic source for the appearance of the macroscopic kinetic energy. To demonstrate that, we write it as

$$\left( \frac{d\Sigma}{dt} \right)^{virt} \equiv \dot{q} \sum_{nm} \left( \frac{\partial \hat{H}}{\partial q} \right)_{mn} \rho_{nm} = 2\dot{q} \sum_{nm} \int_0^t dt' V_{nm}(t, t') \dot{q}(t') [\rho_{mm}(t') - \rho_{nn}(t')], \tag{45}$$

where

$$V_{nm}(t, t') = h_{nm}(t) h_{mn}(t') \cos(\omega_{nm}[t - t']) / \omega_{nm}. \tag{46}$$

We formally extend the lower limit of the time integration in Eq. (45) to $-\infty$. In this way, we would like to study stationary dynamics of the complex quantum system, i.e., when the dynamics of the system does not depend on the choice of initial time. Thus, integrating by parts the time integral in the right–hand side of Eq. (45), one can show that

$$\int_{-\infty}^t dt' V_{nm}(t, t') \dot{q}(t') [\rho_{mm}(t') - \rho_{nn}(t')] \approx +\infty \sum_{l=0} \omega_{nm}^{-(2l+3)} \times \frac{d(2l+1)}{dt(2l+1)} (\dot{q} h_{nm} h_{mn} [\rho_{mm} - \rho_{nn}]). \tag{47}$$

In the weak–coupling limit (22), we obtain

$$\left( \frac{d\Sigma}{dt} \right)^{virt} \approx \dot{q} B(q) \ddot{q} + \frac{\partial B}{\partial q} \dot{q}^3, \tag{48}$$

where a term

$$M = \sum_{n,m} h_{nm} h_{mn} \omega_{nm}^{-3} [\rho_{mm} - \rho_{nn}] \tag{49}$$

can be associated with a macroscopic inertia coefficient.
The third term in the right–hand side of Eq. (44) is determined by the real transitions between the adiabatic states thus, defining how the energy of macroscopic motion is transferred into the energy of the intrinsic excitations of the quantum chaotic system:

\[
\left( \frac{d\Sigma}{dt} \right)_{\text{real}} = \sigma_0^2 \dot{q}(t) \int_0^t dt' K(t, t') \dot{q}(t') \int_{E_{gs}}^{+\infty} dE \Omega(E) E \frac{\partial}{\partial E} \left[ \Omega(E) \frac{\partial \bar{\rho}(E, t')}{\partial E} \right] + \dot{q}(t) \xi(t),
\]

(50)

where Eq. (30) was used. \( \xi(t) \) in Eq. (50) is a stochastic term whose ensemble averaged value is zero. By using Eq. (21), one can show that its correlation function is given by

\[
\langle \xi(t) \xi(t') \rangle = 4 \sum_{nk} h_{nk} h_{nk}^* \bar{\rho}_{nk}(t = 0)|^2 \cos[\omega_{nk}(t - t')].
\]

(51)

The fourth term in the r.h.s of Eq. (44) is given by the distribution of slopes of the adiabatic eigenstates \( E_{nn} \). Within the random matrix model (21)–(26), the negative and positive slopes of the adiabatic states are assumed to be equally distributed. Therefore, under the averaging over all random realizations of the random matrices, modeling the nuclear many body spectrum, one can neglect the contribution from the fourth term in the rhs of Eq. (44).

Thus, putting together different contributions (48) and (50) to the energy rate (44), we get transport description of the macroscopic coordinate’s dynamics

\[
M \ddot{q} = -\frac{\partial M}{\partial q} \dot{q}^2 - \frac{\partial E_{gs}}{\partial q} - \sigma_0^2 \int_0^t dt' K(t, t') \dot{q}(t') \int_{E_{gs}}^{+\infty} dE \Omega(E) E \frac{\partial}{\partial E} \left[ \Omega(E) \frac{\partial \bar{\rho}(E, t')}{\partial E} \right] - \xi(t).
\]

(52)

The transport equation (52) should be supplemented by an equation of motion for the occupancies of the quantum states (30). Importantly that the dynamics of the external macroscopic parameter is damped only when the average level–density of the quantum chaotic system \( \Omega(E) \) is a growing function of the intrinsic excitation \( E \). If the initial excitation \( E_0 \) of the quantum system is sufficiently large than the typical variations of the energy associated with the macroscopic parameter then, one can obtain approximately

\[
M \ddot{q} = -\frac{\partial M}{\partial q} \dot{q}^2 - \frac{\partial E_{gs}}{\partial q} - \sigma_0^2 \left. \frac{d\Omega(E)}{dE} \right|_{E_0} \int_0^t dt' K(t, t') \dot{q}(t') - \xi(t),
\]

(53)

where the normalization condition

\[
\int_{E_{gs}}^{+\infty} \bar{\rho}(E, t) \Omega(E) dE = 1
\]

(54)

was used.
V. NUCLEAR FISSION RATE

To measure the role of memory effects in collective dynamics of the nuclear system on the way from ground state to saddle point, we restrict ourselves by considering a one-dimensional collective motion $q(t)$ over a schematic parabolic barrier. The potential energy $E_{\text{pot}}$ presents a single-well barrier formed by a smoothing joining at $q = q^*$ of the potential minimum oscillator with the inverted oscillator

$$E_{\text{pot}} = \frac{1}{2} M \omega_A^2 (q - q_A)^2, \quad q \leq q^*, \tag{55}$$

$$= E_{\text{pot,B}} - \frac{1}{2} M \omega_B^2 (q - q_B)^2, \quad q > q^*.$$

We also use a constant value for the mass parameter $M = \frac{1}{5} AmR_0^2$, \tag{56}

where $A$ is the mass number of a nucleus, $m$ is the nucleon mass and $R_0$ is the radius of the nucleus. In numerical calculations, it was solved the generalized Langevin equation:

$$M \ddot{q} = -\frac{\partial E_{\text{pot}}}{\partial q} - \kappa_0 \int_0^t e^{-|t-t'|/\tau} \dot{q}(t') dt' - \xi(t), \tag{57}$$

where $\tau$ is a memory time and the stochastic force term $\xi(t)$ in the last equation is related to the memory-dependent friction force as

$$\langle \xi(t)\xi(t') \rangle = T \kappa_0 e^{-|t-t'|/\tau}. \tag{58}$$

Here,

$$T = \frac{d\Omega(E)/dE}{\Omega(E)} \bigg|_{E_0} \tag{59}$$

is understood as an initial temperature of the nucleus.

In Fig. 1, we plotted an escape rate $R_f$ over the parabolic potential barrier as a function of the initial temperature $T$ for quite small memory time $\tau = 2 \times 10^{-23}$ s (points line) and for fairly large memory time $\tau = 8 \times 10^{-23}$ s (solid line).

We see that with the increase of the initial temperature the role of the non-Markovian features of the macroscopic dynamics \tag{57} grows.
FIG. 1: The fission rate $R_f$ for the generalized Langevin dynamics as a function of the initial nuclear temperature $T$ at quite small memory time $\tau = 2 \times 10^{-23} \text{ s}$ (points line) and at fairly large memory time $\tau = 8 \times 10^{-23} \text{ s}$ (solid line).

VI. SUMMARY

In the paper, we have considered the response of a complex quantum system on time variations of a single macroscopic coordinate $q$. The driven dynamics has been studied in terms of the time evolution of the adiabatic occupancies of the system’s Hamilton operator $\hat{H}(q)$. In the limit of weak coupling of the macroscopic coordinate to the quantum system, we have obtained diffusion–like equations of motion for the adiabatic occupancies, see Eq. (30). Within the random matrix model (21)–(26), the time features of the quantum mechanical diffusion (30) have been investigated.

Thus, we have found the normal regime of the diffusion (39) with the diffusion coefficient quadratically proportional to the driving velocity $\dot{q}$. This regime is realized for sufficiently spread $\Gamma$ energy distribution of the coupling matrix elements $\partial \hat{H}/\partial q$ (21), when the characteristic time $\hbar/\Gamma$ is the shortest time scale in the system. In the opposite limit of quite small values of the spreading width $\Gamma$, we get fully ballistic regime (40) of the quantum driven dynamics (30). Here, the energy transport in a space of the adiabatic occupancies undergoes as a process of a wave propagation. At moderate values of $\Gamma$, the ballistic and diffusive regimes of the driven dynamics coexist.

Within the cranking approach (44), we have made an attempt to measure the macroscopic
manifestation of the quantum mechanical diffusion \cite{30}. For the first time, we have naturally include into the standard framework of the cranking approach fluctuative properties of the macroscopic classical coordinate $q$. We have also established memory effects in the motion of the macroscopic coordinate appearing due to the time features of the intrinsic diffusive dynamics.

We used the obtained equation for the macroscopic variable \cite{53} to the study of a nuclear escape over a model parabolic fission barrier. In practise, it was numerically solved the generalized Langevin equation of motion \cite{57} for the nuclear shape parameter $q$ and calculated the fission rate $R_f$ at different values of the memory time $\tau$. We found that the memory effects in the nuclear fission dynamics become stronger with the growth of the initial temperature of a nucleus (see Fig. 1).

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