Avoiding an Empty Universe in RS I Models and Large-$\mathcal{N}$ Gauge Theories

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**Abstract**

Many proposed solutions to the hierarchy problem rely on dimensional transmutation in asymptotically free gauge theories, and these theories often have dual descriptions in terms of a warped extra dimension. Gravitational calculations show that the confining phase transition in Randall-Sundrum models is first-order and parametrically slower than the rate expected in large-$\mathcal{N}$ gauge theories. This is dangerous because it leads to an empty universe problem. We argue that this rate suppression arises from approximate conformal symmetry. Though this empty universe problem cannot be solved by using the radion for low-scale inflation, we argue that if the radion potential is asymptotically free, another instanton for the RS phase transition can proceed as $e^{-\mathcal{N}^2}$. We also discuss the existence of light magnetic monopoles ($\sim 100$ TeV) as a possible signature of such a phase transition.
1 Introduction

New confining gauge theories play a central role in many models of new physics, such as dynamical supersymmetry breaking, technicolor, and a composite higgs. If the universe is heated above the confining temperature, these theories undergo confining phase transitions as the universe cools. The vacuum energy before the phase transition is of order $T_c^4$, so that the universe in the deconfined phase expands with $H = T_c^2/M_{pl}$. If the phase transition rate per unit volume is smaller than $H^4$, bubbles of the new phase never collide, and there is an empty universe problem \[1\]. From the entropic argument of Section 2.1 the transition rate per unit volume is $\sim T_c^4 e^{-N^2}$ for a gauge group of rank $N$. Thus to avoid the empty universe problem, we require

$$N \lesssim 2 \sqrt{\log \left( \frac{M_{pl}}{T_c} \right)}.$$  \hspace{1cm} (1)

For $T_c \sim 1$ TeV, $N \lesssim 12$ is required for the phase transition to complete.

The RS-I model \[2\] also undergoes a phase transition, first discussed by Creminelli et al, in which the TeV brane nucleates from behind an AdS-Schwarzchild horizon \[3, 4\]. In fact, this is also a confining phase transition in the CFT dual description, which has rank $N = 4\pi(M_5L_{AdS})^{3/2}$ \[5\]. By studying the transition in the weakly coupled gravitational description, Creminelli et al argue that the transition is strongly first order and, if the model is weakly coupled, too slow to complete. If the universe was ever hotter than the weak scale, this presents a serious problem for weak scale physics that is well described by an RSI type model.

The analysis of \[3\] employed a generalized Goldberger-Wise radion stabilization with positive mass squared for the Goldberger-Wise field. In the dual picture, this corresponds to a confinement scale set by competition between a weakly coupled marginal operator and a slightly irrelevant operator. The radion contribution to the tunneling action depends on the mass of the Goldberger-Wise field and its boundary condition on the TeV brane; when these parameters are small, the radion action is calculable and dominates over the gravitational contributions of order $\sim N^2$.

The origin of this scaling behavior is somewhat mysterious in the gravitational treatment, and it is unclear whether the enhanced action is peculiar to non-interacting Goldberger-Wise-like stabilization, or endemic to a more general class of theories. We note that because the confining scale in Goldberger-Wise stabilization is determined by the cancellation of two weakly coupled operators, the theory has an approximate conformal symmetry at the confining scale, which is spontaneously broken by confinement. The weakly coupled radion is
a pseudo-Goldstone boson of this broken conformal symmetry, and the enhancement of the
brane-nucleation action is nothing more than the usual $S \propto 1/\lambda$ enhancement from the weak
radion coupling $\lambda$. This strongly suggests that the slow transition of $\lambda$ is not specific to the
Goldberger-Wise stabilization mechanism, but a result of approximate conformal symmetry.

This observation suggests a possible instanton for a faster transition in models, discussed
in Section 4 where the radion stabilization arises from cancellation of a marginal operator and
a slightly relevant one. Though the theory is approximately conformal at the confining scale
$\mu_{\text{TeV}}$, it is far from conformal at the lower scale $\Lambda_S$ at which the slightly relevant operator
becomes strongly coupled. Therefore, if the transition discussed above does not complete,
the hot deconfined theory enters a supercooling phase for $\Lambda_S \lesssim T \lesssim \mu_{\text{TeV}}$. At $T \sim \Lambda_S$,
the radion can tunnel to $\Lambda_S$ with a rate unsuppressed by weak couplings. A short period of
inflation follows as the radion rolls classically to $\mu_{\text{TeV}}$, but barring severe fine-tuning, this
cannot produce many $e$-foldings of weak-scale inflation. Even in this optimistic scenario, the
constraint (i) requires $N \lesssim 10 - 15$ and hence $L_{\text{AdS}} \lesssim 1/M_5$.

In Section 5, we discuss one additional phenomenological signature of the Randall-
Sundrum picture, namely the production of TeV-scale magnetic monopoles during the phase
transition. If the Standard Model gauge fields are composite at a scale $\mu_{\text{TeV}}$, then there exist
monopoles of mass $\mu_{\text{TeV}}/\alpha$. In the warped picture, these monopoles are brane black holes
that carry magnetic charge. We give several estimates of monopole production rates, under
various assumptions. The phenomenology of such light monopoles is quite different from
that of heavier monopoles, and surprisingly unconstrained. The Parker bound derived from
galactic magnetic fields is the tightest constraint on such monopoles. Most current searches
for monopoles in the earth or monopole flux are insensitive because these monopoles are so
light, but sensitive searches for stopped monopoles could be performed.

2 The Confinement and Brane-Nucleation Transitions

We begin by reviewing several well-known properties of confining phase transitions in large-
$N$ gauge theories. We argue that (i) the phase transition is first-order, (ii) the confining
temperature $T_c \sim \Lambda$, and (iii) the transition rate scales as $e^{-S}$, with action $S \sim N^2$.

The AdS/CFT correspondence relates Randall-Sundrum models with a weakly stabilized
radion to confining gauge theories in which explicit breaking of conformal symmetry is weak
at the confining scale. These models serve as a perturbative, calculable check of the large-$N$
scalings reviewed in 2.1 and those derived in Section 3. In 2.2 we review several properties
of these models, the correspondence, and the brane nucleation phase transition of RS I discussed in [3].

2.1 Confining Phase Transitions at Large $N$

Lattice studies [6] show that for $N \gtrsim 3$ the confining phase transition is first-order, growing more strongly first order as $N \to \infty$, and that the critical temperature is of order $\Lambda$. Indeed, $\Lambda$ is the only dimensionful scale of the theory. The approximate $N$-independence of $T_c/\Lambda$ is less obvious. In the deconfined phase, the $N^2$ gluons are independent massless degrees of freedom, leading to a free energy density $F_{\text{dec}} \sim -N^2 T^4$. The confined phase has $O(1)$ light degrees of freedom and hence low entropy, but as adjoint bilinears take VEVs of order $\Lambda$ and scale as $N^2$, the potential energy density is lowered by $E_{\text{con}} \sim -N^2 \Lambda^4$. The free energies of both phases are equal at the critical temperature $T_c \sim \Lambda$.

Alternatively, the $N$-independence can be seen by consideration of confining flux tubes [7]. A flux tube of length $r$ and tension $\Lambda$ has energy $\Lambda^2 r$, and there are $e^{\alpha \Lambda r}$ configurations for such a tube. The color of the two charges uniquely determines the $SU(N)$ representation of the flux tube, so $\alpha$ is $N$-independent. Thus the partition function for the flux tubes is

$$Z = \sum_r e^{\Lambda r (\alpha - \Lambda/T)} \quad (2)$$

and we see that $T_c \sim \Lambda$.

Finally, there is an argument that $T_c/\Lambda$ is $N$-independent [8] based on the bag model of hadrons. The transition temperature is the temperature at which the thermal density of hadron bags is space-filling, and this occurs when $n(T)V_{\text{bag}} \sim 1$, where $V_{\text{bag}} \sim \frac{1}{\Lambda^3}$ is the typical size of hadron bags (with no $N$-dependence) and $n(T)$ is the number density of hadrons. As long as there is a resonance with mass $\lesssim \Lambda$, we can estimate the parametric dependence using $n(T) \sim T^3$, yielding $T_c \sim \Lambda$.

The phase transition rate is controlled by the entropy difference between the two phases in a region the size of a critical bubble. The difference in entropy densities between the two phases at $T_c \sim \Lambda$ goes as $N^2 \Lambda^3$, and the critical bubble size is $\Lambda^{-3}$, so that the phase transition rate per unit volume is $\Gamma \sim \Lambda^4 e^{-cN^2}$ with $c \sim O(1)$. Our universe, in the confined phase, is very nearly flat space. The vacuum energy difference between the phases, and hence the cosmological constant of the deconfined phase, is $V_0 \sim N^2 \Lambda^4$, so that for $T \lesssim \Lambda$ it undergoes inflation with $H \sim \sqrt{V_0/M_{\text{pl}}} \sim N \Lambda^2/M_{\text{pl}}$. If $\Gamma \lesssim H^4$, bubbles of the confining
phase never merge. To avoid this empty universe problem, we require
\[ N^2 \lesssim \frac{4}{c} \ln \left( \frac{M_{pl}}{\Lambda} \right) \] (3)
whenever a gauge group of rank \( N \) confines at a scale \( \Lambda \) below the inflationary reheating temperature. Numerically, this requires \( N < 12 \) for \( \Lambda \) at the weak scale and \( c = 1 \).

### 2.2 AdS/CFT and Confinement

The RS I model provides an explicit AdS/CFT dual description of the analysis above. The model \([2]\) consists of a 'slice' of \( AdS_5 \) bounded by two \( 3 + 1 \)-dimensional branes. The bulk metric is given by
\[ ds^2 = e^{-2kr} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2, \] (4)
where \( k \) is the \( AdS_5 \) curvature and \( M \) is the five dimensional Planck scale. The UV brane is located on the hypersurface \( r = 0 \), while the TeV brane lies at \( r = r_c \). The warped metric means that the effective UV cutoff for physics on a slice at \( r \) is given by \( Me^{-2kr} \). If the Higgs is localized on the TeV brane, the natural scale for its mass is \( Me^{-\pi kr} \), while the effective four-dimensional Planck scale \( M_{pl} \) seen by observers on the TeV brane is given by
\[ M_{pl}^2 = \frac{M_{pl}^3}{k} (1 - e^{-2kr_c}) \] (5)
By tuning the bulk cosmological constant and brane tensions, \( r_c \) (and hence the radion \( \mu = ke^{-\pi kr_c} \)) can be made a flat direction. For a realistic model that explains the origin of the hierarchy, this flat direction must be lifted. This stabilization can be achieved by the presence of bulk fields that create an \( r_c \) dependent 'casimir energy', as in the Goldberger-Wise mechanism \([9]\).

The AdS/CFT correspondence clarifies many properties of the RS I model through a partial dictionary between the conventional description in bounded AdS space and a four-dimensional, softly broken conformal field theory coupled to gravity \([10]\). The correspondence is incomplete in that it is not known precisely which CFT corresponds to the Randall-Sundrum model with a given set of bulk and brane fields. It is useful nonetheless, because so many properties of the gauge theory are determined by conformal invariance and large-N scaling. Fields localized to the TeV brane correspond to emergent composite states of the CFT, and KK modes of bulk fields correspond to resonances. The bulk fields that stabilize \( r_c \) correspond to slowly running couplings or sources in the CFT.

Above the critical temperature \( T_c \) gauge theories deconfine, and composite states and resonances are no longer the relevant degrees of freedom. In the five-dimensional picture,
this hot CFT phase corresponds to an AdS-Schwarzschild geometry, with metric

\[ ds^2 = \frac{\rho^2}{L^2} \frac{Z(\rho)}{Z(\rho_{Pl})} d\tau^2 + \frac{d\rho^2}{\rho^2} + \frac{\rho^2}{L^2} \sum_i dx_i^2, \]

where \( Z(\rho) = 1 - \rho^4/\rho_h^4 \), and \( \rho_h = \pi L^2 T_h \) is the coordinate of the black brane. These coordinates are chosen so that the induced metric at the Planck brane boundary is identical to that of pure AdS with the same \( L \). When \( T = T_h \), the Hawking radiation from the black hole horizon is in stable thermal equilibrium with the bulk. The confining/deconfining phase transition is between the AdS-Schwarzschild and RS geometries. As the hot, deconfined, expanding universe cools below \( T_c \), the confined phase becomes energetically favorable.

Before turning to more detailed computations, a few comments are in order. In the AdS/CFT correspondence, \( N^2 = 16\pi^2 (ML)^3 \), so that large \( N \) corresponds to an AdS curvature that is much smaller than the 5-d Planck scale. This is a requirement for a sensible gravitational description. In the bulk description, the radion \( \mu(x) = ke^{−\pi kr_c(x)} \) is the position of the TeV brane, and consequently it is a gravitational degree of freedom. Thus its kinetic term comes from the Einstein Hilbert action, so it is accompanied by a factor of \((ML)^3 \propto N^2 \). We will argue below that in the CFT description the radion is a glueball state, so that the large-\( N \) scaling is consistent between the two descriptions.

On the CFT side, the black brane can be interpreted as a space-filling plasma of deconfined CFT matter. A ball of the confined phase corresponds to a bubble of the TeV brane protruding through the horizon. The wall of the bubble interpolates between the AdS-S horizon and the TeV brane, but the two meet at \( \mu = 0 \) and \( T_h = 0 \), which is pure AdS space. If the bubble is big enough, the radion potential takes over and the bubble expands at the speed of light, corresponding to the radion/glueball operator condensing out of the vacuum. As we will discuss below, the effective field theory breaks down when the temperature on the TeV brane exceeds the local red-shifted Planck scale, so we only have control over the regime \( \mu > T/(ML) \).

From the perspective of the bulk RS I description it might seem that the phase transition involves unknown UV physics – after all, the transition involves a topology changing GR instanton. However, these strong gravitational and stringy effects are merely mocking up complicated, low energy CFT behavior, and they are not sensitive to very high energy physics, such as new degrees of freedom at energies above \( T_c \). Unknown UV behavior does not limit our understanding of the transition.

Many quantities of interest can be calculated with both the CFT and RS descriptions. For the most part we will focus on the CFT side, so that the level of generality will be
obvious. Detailed RS model estimates were made in [3], [13]; we will consider them in turn and show how they fit into the general picture of confining/deconfining transitions.

Figure 1: An instanton for the TeV-brane nucleation transition in a Randall-Sundrum Model

3 Confinement with Approximate Conformal Symmetry

A confining gauge theory is characterized by its gauge group (which we take to be $SU(N)$ for convenience), its matter content, a confining scale $\Lambda$, and a strong coupling scale $\Lambda_S$. In typical asymptotically free theories, $\Lambda \approx \Lambda_S$. When confinement occurs at the scale of strong coupling, as in QCD, the approximate conformal symmetry of the high energy theory is badly broken, so that it plays no role in the phase transition. It is possible, however, for confinement to be induced by a combination of weakly coupled operators perturbing a CFT. The confining scale in such a theory, which we call $\mu_{\text{TeV}}$, is parametrically separated from the strong-coupling scale $\Lambda_S$ ($\Lambda_S \lesssim \mu_{\text{TeV}}$) generated by dimensional transmutation. Though this situation is not generic, it is of interest both as a calculable regime and because the AdS/CFT duals of Randall-Sundrum models are of this sort. At the confining scale $\mu_{\text{TeV}}$, an approximate conformal symmetry governs the dynamics. In this section we will show that conformal symmetry leads to a reduction in the confining phase transition temperature and an enhancement of the instanton action by powers of the conformal symmetry breaking spurion. To do this we must understand the free energies and relevant degrees of freedom of the two phases.
The approximate conformal symmetry at the scale $\mu_{TeV}$ is spontaneously broken. In light of the AdS/CFT duality between RS-I models and gauge theories with approximate conformal symmetries, we will refer to the pseudo-goldstone boson $[14]$ of conformal symmetry breaking as the “radion”. Because the radion is necessarily a singlet under both the $SU(N)$ gauge symmetry and all other symmetries, it is reasonable to assume that it is a glueball (or, more generally, an 'adjoint-representation-ball'), and this determines its large $N$ scaling properties. This is borne out in the specific case of $N = 4$ SYM theory, where the radion sets the overall scale of all moduli $[10]$.

In the confined phase there is a radion-glueball condensate at scale $\mu_{TeV}$, whereas in the deconfined phase at low temperatures we expect $\langle \mu \rangle \sim T \ll \mu_{TcV}^4$. The radion is an order parameter for the breaking of the approximate conformal symmetry, and since confinement can only occur after conformal symmetry is broken, it serves indirectly as an order parameter for confinement. For $\mu > \Lambda_S$, the radion is a good degree of freedom. In this regime, we can calculate its potential.

By large-$N$ counting, the radion Lagrangian scales with an overall factor of $N^2$ $[15]$, and we can write

$$V(\mu) = N^2 g(\mu) \mu^4, \quad (7)$$

$$\tilde{L}(\mu) = N^2 (\partial \mu)^2 - V(\mu). \quad (8)$$

As the scale-dependence $\beta$ of the coupling $g(\mu)$ explicitly breaks conformal invariance, we take it to be small at the minimum of $V(\mu)$, the confining scale $\mu_{TcV}$. It is important to note that this implies that $g(\mu_{TcV})$ must be small because

$$V'(\mu_{TcV}) = 0 \quad \Rightarrow \quad g(\mu_{TcV}) = -\frac{1}{4} \beta \ll 1, \quad (9)$$

which may be surprising since a $\mu^4$ term with constant coefficient is conformal. The entropy of the confined phase is small, so that to a good approximation

$$F_{conf}(T, \mu) = V_{conf}(\mu); \quad (10)$$

corrections are considered in Appendix $[13]$

This description is valid only for $\mu \gtrsim \Lambda_S$, for we do not know what the relevant degrees of freedom are below the strong coupling scale. Furthermore, when $T \gg \mu$, many resonances are thermally excited, and our description may break down. For $T \gtrsim N^{2/3} \mu$, the glueball

$^{4}$Since the radion is an operator that creates a glueball, $\langle \mu \rangle$ can in principle be calculated in the deconfined phase. This phase can persist to low temperatures because the phase transition rate is slow.
scattering rate per glueball volume ($\sim \mu^{-3}$) becomes $O(1)$ and the effective field theory at scale $\mu$ is highly suspect. The equivalent limit in the dual 5-d gravitational picture, $\mu > T/(ML)$, follows from demanding that the temperature seen by a TeV brane observer when the brane is at $\mu$ never exceeds the 5-d Planck scale (the ‘local Planck scale’ is $Me^{-\pi kr} = \mu(ML)$). In fact, the local string scale determined by $M_s \sim g_s^{1/4}M_{pl}$ is an even smaller cutoff for the effective theory.

To study the phase transition, we must calculate the free energy density as a function of the local plasma temperature $T_h$ in the deconfined phase. The free energy density is minimized when the local temperature $T_h(x)$ is equal to the average temperature $T$. Since $T_{\mu\nu}$ is traceless, the hot CFT matter has the same equation of state as radiation, and we expect $E \propto N^2T_h^4$ and $S \propto N^2T_h^3$. These two constraints specify the form of the free energy density up to an overall factor, and this is in agreement with the answer derived from the AdS-S geometry in appendix B [3],

$$F_{\text{dec}}(T_h) = E - TS = \frac{N^2\pi^2}{8}(3T_h^4 - 4TT_h^3).$$

In equilibrium, $F_{\text{dec}} = -\pi^2N^2T^4/8$. Figure 2 is a cartoon of the free energy for the system in both confined and deconfined phases. We have used conformal invariance in obtaining this expression for $F_{\text{dec}}$; conformal symmetry breaking corrections must be proportional to the spurious $g(T_h)$ or $g(T)$, which are assumed to be small. These correction were calculated in [3] for a Goldberger-Wise field in AdS-S, the case relevant to the RS-I model.

The transition temperature and phase transition rate depend on the relative normalizations and zero-points of $F_{\text{con}}$ and $F_{\text{dec}}$. In an RS-I model, the relative normalization is known because both free energies are calculated from the same gravitational action, but in the CFT description they are related by an unknown $O(1)$ factor. The zero-point energies are matched at $\mu = T_h = 0$, but as [7] is strongly coupled for $\mu \lesssim \Lambda_S$, we can say only that

$$F(T_h = T) - F(\mu_{\text{TeV}}) = -\pi^2N^2T^4/8 - g(\mu_{\text{TeV}})N^2\mu_{\text{TeV}}^4 + O(N^2\Lambda_S^4),$$

where $g(\mu_{\text{TeV}}) < 0$. The two phases have the same free energy at the critical temperature

$$T_c \propto [g(\mu_{\text{TeV}})]^{1/4}\mu_{\text{TeV}}.$$

The critical temperature is suppressed by $g^{1/4} \ll 1$ compared with the strongly coupled result, as claimed. Below $T_c$, tunneling between the deconfined and confined phases is allowed. The action for the instanton between them involves uncalculable contributions, which we expect to be $O(N^2)$ as in the conventional confining phase transition. But the phase transition also involves tunneling of the radion from near zero to $\mu_{\text{TeV}}$ in the potential
Figure 2: Radion potential for the confining phase transition in an approximately conformal field theory. The region to the left of the Y-axis represents the free energy of the deconfined phase as a function of $T_h$, while the region to the right represents the free energy of the confined phase as a function of $\mu$.

When $\mu$ is rescaled to have a canonical kinetic term, this has a weak coupling $\lambda = |g(\mu_{TeV})|/N^2$ and the tunneling occurs over a distance $\mu^* = N\mu_{TeV}$ in field space. In the thin-walled approximation [16], this part of the transition contributes an action

$$S_4 \sim 1/\lambda$$

(14)

to the $O(4)$-invariant instanton, and

$$S_3/T \sim \frac{1}{\sqrt{\lambda}} \frac{\mu^*}{T} \left(1 - \left(\frac{T}{T_c}\right)^4\right)^{-2}$$

(15)

to the finite-temperature action for $T \approx T_c$. These estimates are accurate to the extent that $\lambda \ll 1$.

The Goldberger-Wise stabilization considered by Creminelli et. al. has $g(\mu_{TeV}) \approx -\epsilon^{3/2}v_1^2/N^2$, where $\epsilon = m_{GW}^2 L_{AdS}^2/4$ is the anomalous dimension of one of the Goldberger-Wise operators (taken to be slightly irrelevant), and $v_1$ is the boundary condition for the Goldberger-Wise field on the TeV brane [3]. The actions they obtain are indeed of the parametric form expected from our one-parameter description, underscoring that the effect enhancing the thin-wall action for RS brane nucleation is precisely approximate conformal invariance at the scale $\mu_{TeV}$. 

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4 A Faster Transition from Strong Coupling in the IR

In the gauge theories considered above including the Randall-Sundrum model, the confining phase transition is very slow because the action for tunneling to $\mu = \mu_{\text{TeV}}$ is enhanced by inverse powers of the small conformal symmetry breaking spurion. (In the limit of exact conformal symmetry, confinement does not occur at any finite temperature [11]). If this spurionic operator is marginally irrelevant, then the breaking of conformal symmetry only gets weaker in the IR and the results of [3] apply. If the deformation is marginally relevant, conformal invariance becomes strongly broken at an IR scale $\Lambda_S$. In Randall-Sundrum models, one usually requires $\Lambda_S < \mu_{\text{TeV}}$, so that explicit breaking of conformal invariance is weak at the TeV scale.

Near the IR scale $\Lambda_S$, conformal symmetry plays no role in the dynamics and there is no parametrically light radion. Thus tunneling to $\mu \sim \Lambda_S$ is unsuppressed by small couplings, as in a generic asymptotically free theory, though it is only allowed for $T \lesssim \Lambda_S$. The essential idea is that there are two phase transitions – a confining transition in a gauge theory, and the spontaneous breakdown of conformal symmetry in a CFT. With the instanton of section 3 both transitions occur simultaneously, but in general it should be possible to allow the universe to cool until the confining transition proceeds in a strongly coupled regime, after which point the radion rolls to its conformal breaking minimum without any phase transition at all. In what follows we consider whether this process leads to an acceptable spectrum of density perturbations, and we briefly consider other possibilities for the phase transition.

The confining phase transition will occur through the nucleation of bubbles of size $\sim \frac{1}{\Lambda_S}$ with a rate per unit volume of $\Gamma \sim \Lambda_S^4 e^{-cN^2}$, where $c \sim O(1)$. If the transition is first order, which we expect for large-N gauge theories [6], then a latent heat of order $\Lambda_S^4$ is released during the nucleation process. It is energetically favorable for the radion $\mu$ to relax to $\mu_{\text{TeV}} >> \Lambda_S$, which is in the approximately conformal regime, where $g(\mu)$ is small and runs slowly. Though in principle the rolling radion could lead to many e-foldings of weak-scale inflation, this would require fine-tuning of the slow-roll parameter $\eta$. One might then worry that the late-stage inflation before tunneling destroys primordial density perturbations. We will see that this is not a danger.

It will be useful to introduce a series expansion of the potential for $\mu$, which we can write quite generally as

$$V(\mu) = \mu^4 N^2 g(\mu),$$

where $g(\mu)$ is the slowly running coupling of the conformal symmetry breaking operator [17].
In the regime where $g$ runs slowly, we can expand $g$ in $V(\mu)$ so that
\begin{equation}
V(\mu) = \mu^4 N^2 \left( g_0 + g_1 \left( \frac{\mu}{\mu_0} \right)^\epsilon + \ldots \right),
\end{equation}
where $\mu_0$ is the cutoff, the coefficients $g_i$ depend on $g(\mu_0)$, and $\frac{d\ln g}{d\ln \mu} = \epsilon$. There is a local extremum at $\mu_{\text{TeV}} \approx \mu_0 \left( \frac{g_0}{g_1} \right)^{\frac{1}{2}}$, exponentially below the cutoff $\mu_0$, at which the weak scale is stabilized. This scale $\mu$ can still be much larger than the IR scale $\Lambda_S$ at which $g$ gets strong.

It is easy to see that the radion should not be used as a weak scale inflaton. The flatness of the radion potential over an interval $\sim \mu_{\text{TeV}}$ is protected by the approximate conformal symmetry above the scale $\Lambda_S$. However, the conformal coupling $\mu^2 R$ between the radion and the Ricci scalar generates a mass term for $\mu$ of order $H^2 \sim \frac{V(\mu_{\text{TeV}})}{M_{\text{Pl}}^2}$, so that in an inflationary phase $\eta \equiv \left| \frac{V''(\mu)}{V(\mu)} \right| \sim \frac{H^2 m_{\text{Pl}}^2}{V(\mu_{\text{TeV}})} \sim 1$ and slow-roll conditions are violated. Tuning of the dynamics near $\mu \sim \Lambda_S$ is required to make $\eta$ small, and 60 $e$-foldings of inflation with nearly scale-invariant density perturbations after the phase transition seems untenable.

As density perturbations are not generated after the phase transition, they must not be destroyed during the transition. Therefore, it is important that the vacuum-energy-dominated supercooling phase between $T \sim \mu_{\text{TeV}}$ and $T \sim \Lambda_S$ does not blow out primordial density perturbations. In the supercooling phase below $\mu_{\text{TeV}}$, the temperature drops to $T \sim \Lambda_S$ over $\sim \ln \left( \frac{\mu_{\text{TeV}}}{\Lambda_S} \right)$ $e$-foldings of super-cooling. After nucleation of confining bubbles near the scale $\Lambda_S$, the confined phase continues to supercool as the radion degree of freedom $\mu$ relaxes to its weakly coupled minimum. Because $\eta \sim 1$, we don’t expect more than a few $e$-foldings of inflation after the transition.

It was briefly suggested in [3] that deSitter fluctuations could drive the phase transition. This means that we allow the universe to supercool in the deconfined phase until $T \sim H \sim \mu_{\text{TeV}}^2 / M_{\text{Pl}}$, and then hope that the transition proceeds quickly. This is not well defined in theories with a strong coupling scale $\Lambda_S > H$, where we do not know the relevant degrees of freedom in the deep IR. In theories where such a low temperature regime is well-defined, there is no reason to expect that corrections from deSitter space are qualitatively different from the thermal corrections considered in appendix B. Since we have cooled to $T \sim H$, to avoid the empty universe problem the transition must proceed very quickly: $T^4 e^{-S} \sim H^4$ implies $S \sim 0$. We still have a confining transition in a large N gauge theory, and so we do not expect such a tiny instanton action.

Before moving on, we should review what has been gained. We have shown in this section that it is plausible for an RS model to have a weakly coupled, approximately conformal description near the TeV scale without having a confining phase transition that is paramet-
rically slower than the transition in a generic, large N gauge theory. Unfortunately, since we need $N < 12$ for gauge theories that become strongly coupled at the TeV scale, this limits $(ML) < 1$ in RS models, so that even in the most optimistic scenarios, the extra-dimensional description is barely under control.

5 Monopoles From Composite Gauge Fields

A number of studies have searched for monopoles produced in accelerators and stopped in the detectors, obtaining bounds on the pair production cross-section for monopole masses up to 800 GeV (e.g. [18]). If the compositeness scale $\mu_{\text{TeV}}$ is $\sim 1$ TeV or even several TeV, the monopole mass scale is 100 times greater, well beyond the reach of these experiments. Moreover, even if an accelerator were to reach the monopole mass scale, the production cross-section is exponentially small. This suppression follows from the finite size expected for the monopoles—$1/\mu_{\text{TeV}}$, which is $1/\alpha$ times their Compton wavelength. The two monopoles must be produced at a spacelike separation $d$ exceeding their size, which implies a suppression of the production rate by $e^{-m d} \sim e^{1/\alpha}$. Therefore, the most feasible path to discovering light magnetic monopoles is not by producing them today, but by detecting primordial monopoles.

The gauge fields in RS models can be either in the bulk (elementary) or on the brane (composite). If the electroweak $SU(2) \times U(1)$ gauge fields are composite at a scale $\mu_{\text{TeV}}$, then there is a magnetic monopole field configuration of mass $\sim \mu_{\text{TeV}}/\alpha$ (this mass scale is simply the magnetic self-energy for a Dirac monopole solution cut off by new physics at the length scale $\mu_{\text{TeV}}^{-1}$ with magnetic coupling $1/\alpha$). In the dual gravitational description, the monopoles are magnetically charged 4-dimensional black holes at the TeV scale.

One might worry that these monopoles are produced so readily in the first-order phase transition that they overclose the universe. This fate is trivially avoided if the gauge fields are elementary. But even when the Standard Model gauge fields are composite, the abundance of monopoles is low enough that there is no significant constraint. They may, however, be produced in an abundance close to the Parker limit. We present two estimates of their density, and discuss constraints and possible searches for TeV-scale monopoles.

5.1 Monopole Formation in the Phase Transition

A lower bound on the monopole density (the Kibble density) is obtained by noting that gauge fields are not coherent between Hubble patches, so that at least one monopole should
exist per Hubble volume at the transition temperature, and hence

\[(n/s)_{\text{Kibble}} \sim \frac{H^3}{s} \sim \frac{g_s T^6 / M_{Pl}^3}{g_s S T^3} \sim 10 \left( \frac{T_c}{M_{Pl}} \right)^3 \sim 10^{-47}, \tag{18}\]

for \(T_c \sim 1\) TeV. We will see that, for a TeV-scale phase transition, this density is far too low to be observed.

The monopole density may be enhanced by thermal production in the super-heated regions where bubble walls collide. The temperature in these regions is naively \(T_{\text{wall}} \sim (R/\delta) T_c\), where \(R\) is the typical bubble size upon collision and \(\delta\) their thickness. Assuming the resulting production rate is still less than Hubble, so that the monopoles are born frozen out, the resulting monopole density is roughly

\[(n/s)_{\text{superheating}} \sim \frac{\delta}{R} e^{-2M/T_{\text{wall}}} \sim \frac{\delta}{R} e^{-2\delta/(R\alpha)}. \tag{19}\]

This can be much larger than the Kibble density—indeed, this mechanism or an alternative must be at work for monopoles to be visible.

### 5.2 Monopole phenomenology

The thermal velocity dispersion of light monopoles is quite small, so that their typical velocity relative to galaxies is dominated by the typical galactic velocity of order \(10^{-3}c\). The
Parker bound of \([19, 20]\) is applicable, and constrains their flux. This bound comes from demanding that the galactic magnetic field is regenerated by the dynamo effect faster than monopoles traveling through the galaxy deplete it. The galactic magnetic field is \(\sim 3\mu G\). Light monopoles of charge \(h\) that enter the galaxy at non-relativistic velocities typically gain energy \(\Delta E_1 = hB\ell \sim 10^{11}\) GeV passing through a region of size \(\ell \sim 10^{21}\) cm in which the magnetic field is coherent and of magnitude \(B\). If a monopole passes through \(\sim R/\ell\) uncorrelated regions before escaping the galactic field, where \(R \sim 10^{23}\) cm is the radius of the galaxy, then the energy gain of each monopole is \(E_M \sim \sqrt{R/\ell}\Delta E_1 \sim 10^{12}\) GeV. Assuming that the dynamo effect regenerates the galactic magnetic field on a timescale \(\tau \sim 10^8\) years, the flux of magnetic monopoles must not be sufficient to deplete the field on the same timescale. To avoid depleting the galactic magnetic field, then, the monopole flux must be \(\lesssim 10^{-15} cm^{-2} sr^{-1} s^{-1}\), for \(n \lesssim 10^{-23} cm^{-3}\) or \(Y \equiv n/s \lesssim 10^{-26}\).

Monopoles reaching the Earth have passed through at least one patch of coherent galactic magnetic field, from which they gain energy \(\sim 10^{11} - 10^{12}\) GeV. The electromagnetic energy loss rate of monopoles of mass \(M = 100\) TeV is discussed in \([21]\) (see also \([23]\)). When \(E \gg M^2/m_p \sim 10^{10}\) GeV, electromagnetic monopole-nucleus scattering transfers an order-1 fraction of the monopole energy to the scattered nucleus, which causes showering. Below this energy, the monopole still initiates showering, but does not lose energy so rapidly, with an energy loss of only \(\sim 2m_p E/M\) per collision. The resulting atmospheric showers are visible to cosmic ray detectors, and it has been suggested that relativistic monopoles could account for super-GZK cosmic rays \([21, 21, 25]\), though the angular and energy profile of observed super-GZK events may be inconsistent with this proposal \([26]\). At depths of several kilometers (e.g. the depth of the MACRO detector), monopoles typically still have \(\gamma \sim 10^5\), and they stop at depths of order \(1/10\) the Earth’s radius.

The sensitivity of monopole flux limits to 100 TeV monopoles is unclear. Because they are very relativistic and have mass \(\gg m_e\), the scattering of these monopoles off electrons is similar to that of ultra-relativistic cosmic ray muons. They are not energetic enough to traverse the Earth, so they contribute only a downward-going flux. Sensitivity is then limited by the muon background or the ability to distinguish monopoles from muons.

Though the underwater Cerenkov detectors Baikal and Amanda have measured the flux of downward-going relativistic charged particles, they have not reported resulting monopole flux bounds \([27, 28, 29, 30]\). Two analyses from the MACRO experiment (the combined scintillator-streamer tube analysis and the track-etch detector) are sensitive to relativistic monopoles\([31]\). The combined analysis, however, is only sensitive to \(\gamma \lesssim 10\) because more relativistic monopoles can shower in the detector and may be eliminated by the selection...
criteria designed to filter out cosmic-ray muons\textsuperscript{32}. The track-etch detector flux limit of \(1.5 \times 10^{-16} \text{cm}^{-2} \text{sr}^{-1} \text{s}^{-1}\) for monopoles with \(\beta \sim 1\) from \textsuperscript{31} may also be insensitive to ultra-relativistic monopoles because of the increasing mean free path and because the scattering of such monopoles frequently produces showers which are less readily distinguished from muons. Further analysis of the MACRO data would be necessary to determine its sensitivity to such light monopoles.

Monopoles that are trapped in the primordial Earth and through-going monopoles that stop in the vicinity of the crust could both contribute to a density of monopoles near the surface. Therefore, limits on the monopole density per atom of rock (e.g. \textsuperscript{33}, \textsuperscript{34} in lunar rock) imply bounds on both flux and primordial monopole density, though the translation of density limits is imprecise. A monopole that crosses through the Earth obliquely and exits the Earth with kinetic energy below \(\lesssim 10\ \text{TeV}\) can lose this kinetic energy through ionization in traversing the atmosphere\textsuperscript{21}, after which it is slowed to thermal velocities, and pulled by the Earth’s magnetic field and Brownian motion it can be bound in magnetized rock\textsuperscript{22}. Only one in \(\sim 10^8\) monopoles impinging on the Earth do so at an angle that leaves \(\lesssim 10\ \text{TeV}\) of residual energy, but the resulting monopole density could still be as high as \(\sim 10^{-8}\text{cm}^{-2}\). We note also that the earth’s magnetic field \(\sim .3\ \text{G}\) exerts a force \(\sim 10^{-3}\ \text{eV/nm}\) on a monopole of unit Dirac charge, whereas the force of gravity on a monopole of mass \(10^5\ \text{GeV}\) is only \(\sim 10^{-11}\ \text{eV/nm}\). Thus, it is possible that the stopped light monopole density is enhanced near the Earth’s magnetic poles. The poles may be interesting places to search for light monopoles.

6 Conclusions

Models in which our vacuum emerges from a slow first-order phase transition can be deadly to cosmology. If the nucleation rate \(\Gamma\) of bubbles of true vacuum is slow compared to \(H^4\) at the transition temperature, then bubbles of true vacuum never collide, and their interiors are cold and empty. Such models can only be viable if the universe never re-heated above the critical temperature, or if there is second epoch of inflation after the phase transition.

The brane nucleation phase transition in a Randall-Sundrum type-I model is parametrically slow. The AdS/CFT correspondence relates this transition to a confining phase transition in a particular limit of large-\(N\) gauge theory. This motivates studying the dynamics of the confining transition in more general large-\(N\) theories. The RS-I instanton is suppressed by two effects. The instanton action \(S\) is proportional to \(N^2\), an effect that is endemic to all large-\(N\) gauge theories, bounding \(N\) in viable weak-scale models to be \(\lesssim 10 - 14\), with

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stricter bounds for larger confinement scales. Furthermore, the calculable part of the instanton of Creminelli, Nicolis, and Rattazzi in RS-I has a larger action. This enhancement can be seen to arise from the approximate conformal symmetry at the scale of confinement. Indeed, when the radion potential is asymptotically free, the action for tunneling to the Landau-pole scale is only $\sim N^2$, as conformal breaking is not a small parameter on this scale.

Monopoles of mass $\Lambda_S/\alpha$ are always present in theories in which the electroweak gauge bosons are composite at a scale $\Lambda_S$. These monopoles are not produced in the phase transition at a high enough rate to cause a problem, but they could be produced at number densities comparable to the Parker limit. Their phenomenology is quite different from that of GUT-scale monopoles, and largely uncharted. If observed, they could offer an intriguing window on the phase transition.

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A The AdS-S Free Energy

For completeness, we follow [3] to derive the free energy of the AdS-S phase from the gravitational action.

At finite temperature $T$, the AdS-Schwarzchild metric with a black brane at the Hawking temperature $T_h = T$ solves Einstein’s equations. The black brane has entropy $\propto (ML)^3 \gg 1$, so that at high temperatures we expect this phase to have lower free energy than the RS phase. At very low temperatures, the AdS-S solution approaches pure Anti-deSitter space, and radion stabilization makes the RS solution energetically preferred.

We write the AdS-Schwarzchild metric as

$$ds^2 = \frac{\rho^2}{L^2} \frac{Z(\rho)}{Z(\rho_{Pl})} d\tau^2 + \frac{d\rho^2}{Z(\rho) L^2} \sum_i dx_i^2,$$

where $Z(\rho) = 1 - \rho^4 / \rho_h^4$, and $\rho_h = \pi L^2 T_h$ is the coordinate of the black brane. In these coordinates, the induced metric at the Planck brane boundary is identical to that of pure AdS with the same $L$. The thermal compactification of $\tau$ on the interval $(0, 1/T)$ is independent of $\rho_h$. 

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As in [3], we construct the free energy of the AdS-S space for all values of $T$, not limited to $T = T_h$. This will have two pieces—the bulk contribution and a contribution at $T \neq T_h$ from a conical singularity at the horizon, corresponding to its lack of thermal equilibrium with the surrounding space. Though the Einstein action has a contribution from the Planck brane, it must vanish because the boundary geometries have been identified. Physically this makes sense—UV details should not be important for the TeV scale free energy.

In the stationary-phase approximation, the free energy can be approximated as

$$ F = -T \log \left[ \int Dg_{\mu \nu} \exp \left( -\int_0^{1/T} d\tau \int d^4x \mathcal{L}_E(g_{\mu \nu}) \right) \right] $$

$$ \approx T \int_0^{1/T} d\tau \int d^4x \mathcal{L}_E(g_{\mu \nu}), $$

with Lagrangian density

$$ \mathcal{L} = 2M_5^3 \sqrt{-g}\left[ R - 12k^2 - 12k\delta(\rho - \rho_{Pl}) \right]. $$

Since $R = -20k^2$, the Lagrangian is simply proportional to the volume of the space, and

$$ F_{AdS-S} - F_{AdS} = 16TM^3k^5 \int_0^{1/T} \left( \int_{\rho_h}^{\rho_{Pl}} \sqrt{-g_{AdS-S}} - \int_{\rho_h}^{\rho_{Pl}} \sqrt{-g_{AdS}} \right) $$

$$ = 16M^3k^5 \int_{\rho_h}^{\rho_{Pl}} \rho^3 Z(\rho_{Pl})^{-1/2} d\rho - \int_{\rho_h}^{\rho_{Pl}} \rho^3 d\rho $$

$$ = -2\pi^4 (ML)^3 T_h^4 + \mathcal{O} \left( \frac{1}{\rho_{Pl}} \right). $$

If $T \neq T_h$, we can expand the near-horizon metric in $(\rho - \rho_h)/\rho_h = y^2/L^2$. Keeping only the leading terms in $y$ near the horizon and suppressing the three spatial dimensions, we find a metric

$$ ds^2 = \frac{4\rho_h^2 y^2}{Z(\rho_{Pl})L^4} dt^2 + dy^2. $$

If $\tau$ is compactified at a radius other than $1/T_h$, this metric has a conical singularity. As in [CITE], we regularize this singularity with a spherical cap of radius $r$, area $2\pi r^2(1 - T_h/T)$, and constant curvature $2/r^2$. The contribution of this cap to the free energy is $r$-independent and given by

$$ F_{horizon} = 8\pi^4 (ML)^3 T_h^4 \left( 1 - \frac{T}{T_h} \right). $$

Physically, the conical singularity reflects the non-equilibrium between the black hole and the bulk, which are at different temperatures, and is proportional to the temperature difference.
Thus, the AdS-S free energy (relative to pure AdS) is given by

\[ F_{\text{AdS-S}}(T_h) = 6\pi^4 (ML)^3 T_h^4 - 8\pi^4 (ML)^3 T T_h^3. \quad (29) \]

As expected, the minimum of \( F \) is at \( T_h = T \).

### B Thermal Effects and Additional Degrees of Freedom

If there are additional elementary degrees of freedom besides the CFT states, these will contribute to the energy and entropy of both phases. However, if these fields do not mix significantly with the CFT, then their properties do not change significantly across the phase transition, and their effects will be negligible. An analogous statement about QCD is that the electron has a very small effect on the QCD phase transition. In the RS I model, additional elementary degrees of freedom must reside in the bulk of AdS\(_5\) or on the UV brane (fields on the TeV brane are composite states in the CFT description). Bulk and UV brane fields are present as KK modes in both the AdS-S and RS phases, and they have similar energy and entropy densities in both phases, so their effects will tend to cancel.

We assumed above that the number of states in the confining phase was \( O(1) \), which is tiny in comparison to the \( O(N^2) \) states in the deconfined plasma. This is our expectation from the gauge theory perspective; in general we expect the number of degrees of freedom to decrease from the UV to the IR \cite{35}, and a parametrically large number of confined states would be very unusual. However, in RS I models it is possible to add a large number of confined states by hand by adding fields to the TeV brane. From the CFT perspective this seems rather pathological, but we consider it anyway for completeness.

Let \( g_* \) be the number of confined CFT states, or equivalently, the number of fields added to the TeV brane. These states must be light in order to make a sizable contribution to the entropy. In the limit that they are actually massless,

\[ \Delta F_{\text{RS}} \approx -\frac{\pi^2 g_*}{90} T^4 \quad (30) \]

Clearly for arbitrarily large \( g_* \) these thermal effects could alter our analysis, but in an RS model we would need \( g_* \sim 180\pi^2 (ML)^3 > 1700 \), where the limit comes from the fact that the 5-d Planck scale must be greater than the AdS curvature. It seems very unlikely that there are this many light composite degrees of freedom in our universe.

We can also consider the renormalization of the radion potential due to thermal effects
The radion couples to low-energy standard model degrees of freedom through

\[ L_{\text{int}} = \frac{\mu^2}{\langle \mu^2 \rangle} T^\nu_\nu \]  

(31)

where \( T^\nu_\nu \) is the trace of the energy momentum tensor of the light degrees of freedom. This coupling can be derived explicitly in RS I models, but it also follows directly from the fact that the radion non-linearly realizes conformal invariance. For fields such as massless gauge bosons that are approximately conformal, this contribution is very small, although it could be significant for massive fields, for which \( \langle T^\nu_\nu \rangle \propto m^2T^2 \) (ignoring terms proportional to small couplings). For standard model fields, \( m \propto \langle \mu \rangle \) through the higgs vacuum expectation value, so we find

\[ \Delta V(\mu) \sim \sum_i y_i \mu^2 T^2 = Y \mu^2 T^2 \]  

(32)

where the \( y_i \) are effective couplings such as the ratio of the Higgs mass to \( \mu_{\text{TeV}} \). For positive \( Y \) this effect will tend to push the TeV brane away from the Planck brane, making the phase transition slower. In the regime where conformal invariance is softly broken and the transition is under quantitative control, \( T_c < \mu_{\text{TeV}} \) and this contribution to the radion potential will be suppressed compared with the zeroth order \( V(\mu) \) coming from stabilization.

One might still hope that for \( Y \) negative, there may be regime where it is possible to tunnel to some \( \mu_T \ll T < T_c \), where the finite temperature correction dominates over \( V(\mu) \propto \mu^4 \). Unfortunately this is very unlikely – in order for the transition to be allowed, we require

\[ F_{\text{RS}}(\mu_T, T) \approx -YT^2\mu_T^2 \leq -2\pi^4(ML)^3T^4 = F_{\text{AdS-}}(T) \]  

(33)

Since \( Y \) must originate as some radion coupling we require \( |Y| < 2\pi^4(ML)^3 \) for consistency of the effective theory, and this forces \( \mu_T > T \). Thus we see that thermal effects cannot drive the transition or rescue the model.

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