Reputation in Majority Rule Model leading to democratic states

Leonardo S. Oliveira¹, Anderson C. Rodrigues² and Fabricio L. Forgerini¹,²

¹Institute of Humanities, Arts and Science, Campus Paulo Freire, Universidade Federal do Sul da Bahia
45988-058 Teixeira de Freitas - BA Brazil
²Campus Jorge Amado, Universidade Federal do Sul da Bahia
45613-204 Itabuna - BA Brazil
E-mail: fabricio.forgerini@ufsb.edu.br

Abstract. We study the Majority Rule (MR) model, a sociophysics model developed to describe how a group of agents with initial different opinions can reach consensus. At each instant of time, a group is selected at random and discuss among each other. After the discussion, all members of this discussion group follow the majority opinion. The number of agents in the discussion group is not fixed and it is selected each instant of time from a Gaussian distribution. The system dynamics stops when only one opinion survive. In this work we introduced in the MR model a 'reputation' for each agent, a weight to be considered in the system dynamics. Our results show that the introduction of reputation leads the system to a steady state in which not every agent on the system have exactly the same status, but a majority of them sharing the same opinion. In addition, our model with the inclusion of reputation do not show the critical point usually observed. Instead we have obtained the critical point \( p_c = 1 \), in contrast with the standard Majority Rule model.

1. Introduction
In the last decade much work have been done in the dynamics of opinion sharing and competing, attracted the attention from researchers from distinct areas such as physics and sociology. Different models have been proposed to investigate how competing opinions among agents evolve in populations [1, 2, 3, 4, 5]. In general, a dynamic opinion model is about how a group of people reaches an agreement. The dynamics of agreement and disagreement is treated in terms of the variation of the number of different opinion states in population, where each agent (i.e. each individual) can have a few opinions [6]. Usually the opinion of an agent is treated as a discrete variable (±1), but there is no restriction to the opinion change continuously [7].

The Majority Rule Model (MR) is a sociophysics model originally created to describe the human interactions regarding to how a group of N people reach consensus [8]. A random number of agents, \( g \), is selected from a given distribution (usually a Gaussian distribution) to debate. This group of agents is called “discussion group” and they interact among each other and, after the discussion, all of them follow the most popular opinion, the majority opinion of the group [7, 8].

The system initially starts with a fraction of agents with same opinion, \( p_0^+ \), favoring a slight
majority for one opinion, let’s say +1. If \( p^+_0 > p_c \), in which \( p_c \) is a critical value of initial density of agents with opinion +1, the dynamics is characterized by a threshold in \( p_c \) and, for a sufficient long time \( t \to \infty \), all agents will have the same opinion +1. This problem can be analytically solved by mean field analysis for the case where \( g \) is fixed [9]. Three fixed points are observed, \( p^+_0 = 0, p^+_0 = 1 \) and \( p^+_0 = 1/2 \), where the last one is an unstable fixed point. The time to reach consensus, called consensus time, scale with \( \ln N \) and for any \( p^+_0 \neq 1/2 \), at the end, all agents converge to the initial majority [10].

The two absorbing states at long run have all agents with opinions +1 or -1, describing the public opinion in a dictatorship and do not represent a common situation nowadays in a public debate or in a real opinion dynamics. Only a small number of studies have been focused on modifications of the MR Model, searching for a method that can limit the capacity of persuasion of the majorities [11, 12].

In this work we introduce into the MR Model the Reputation, a mechanism that limits the persuasion capacity of the agents, similar as used in others opinion models [2, 13]. Reputation, \( r \), is a score associated to each agent. It is realistic to believe that the individuals will change their opinions under the influence of highly respected persons, or in other words, under the influence of an agent with high reputation. We show that the inclusion of reputation in the MR model change the usual behavior of the model’s dynamics and the system reaches a state with majority of agents with same opinion, however not all agents, which corresponds to a democracy-like situation.

This paper is organized as follows. In Sec. 2 we present our model details and the dynamic rules of opinion interactions. In Sec. 3 we describe our main results and, finally, in Sec. 4, we summarize the results and present our conclusions.

2. The Model
As previously presented, by the introduction of the mechanism of reputation, an agent may not be convinced by others in discussion group. If the agent’s reputation is higher then the average reputation (calculated among the group holding the majority opinion), he will remain inflexible, as one can see in Fig. 1.

![Figure 1](image-url)

**Figure 1.** Representation of a discussion group. In (a) one can see the original MR model, in which all agents inside the discussion group follow the majority opinion. On the other hand, in (b), by the inclusion of reputation, one can see that an agent may not follow the most popular opinion if his reputation is higher than the average reputation.
In the case when an agent have a higher reputation than the average reputation of the most popular opinion inside the discussion group, this agent will keep his own opinion. As a result of the inclusion of reputation, a steady state is reached with the majority of same opinion, but not totality, as one can see in Fig. 2.

The dynamics of our model including reputation is ruled by the following microscopic rules:

(i) from $N$ agents, we selected at random the initial fraction of agents with up opinion, $p_0^+$;
(ii) all agents receive a score, the reputation $r$, taken from a Gaussian distribution, centered at origin and with $\sigma = 1$;
(iii) at each time step, $g$ agents are select at random. The size of $g$ is chosen from a Gaussian distribution centered at 3 with $\sigma = 1$ (only integer numbers);
(iv) we calculate from the discussion group the majority opinion and the average reputation, $\langle r_m \rangle$ of the agents with the majority opinion in $g$;
(v) all agents with reputation smaller than the average reputation of majority opinion ($r_i < \langle r_m \rangle$) flip and change their opinions;
(vi) the average opinion of the system is calculated as $\langle op \rangle = \frac{1}{N} \sum_{i=0}^{N} op_i = p_+ + p_-$ at each time step. One time step is defined as the time required to visit all the $N$ agents.

The consensus time is reached when average opinion, $\langle op \rangle$, does not change in time, only fluctuates around the mean value. One can describe the final average opinion in terms of the initial fraction of agents with same opinion, $p_0^+$. This general picture of the model is shown in Fig. 3, for three different values of $p_0^+$.

![Figure 2](image-url)  
**Figure 2.** Evolution of opinion for $p_0 = \pm 0.6$. The system reaches a situation in which a majority of agents s the same opinion, but not all of them.

![Figure 3](image-url)  
**Figure 3.** Opinion as function of time for three different values of the initial fraction of agents with same opinion, $p_0$, and for $N = 100000$.

For the MR model, the consensus time depends on $N^2$ [14] and, for our model, show dependence with the distribution of reputation, however, the choice of $\sigma$ does not affect our results.

3. **Numerical Results**
We performed several simulations with different configurations for the values of $p_0^+$ and $N$ in a square lattice with size $L$. Our calculations were realized taking averages over 100 samples
for each configuration as one can see in Fig. 3 for two values of $p_0^+$ and $N = 10000$ agents. An exception is the data in figure 2, in which the data presented is for a single run for each configuration.

In Fig. 4 we show our results for the average final opinion $\langle Op \rangle$ versus the initial fraction of up opinions, $p_0^+$, for different values of the system size $L$. We calculated $\langle Op \rangle$ by taking the final opinion of a system after it reached the steady state (about $100 \times 10^3$ time steps), and averaged over 100 different samples. Surprisingly, our results do not depend on the system size. From larger to smaller values of $N$, the initial fraction of up opinions lead the system to the same result.

![Figure 4](image_url)

**Figure 4.** Final Opinion versus the initial fraction of same opinion, $p_0^+$. The results are averaged over 100 different samples. The dashed lines highlight the two linear regimes for the final opinion.

One can see two different linear regimes in Fig. 4: a higher slope regime for small values of $p_0^+$ and, for $p_0^+ > p_0^+ \times$ a linear growth with smaller slope. We can define $p_0^+ \times$ as the initial fraction of agents with same opinion with characterize the transition of these two different regimes.

The dashed lines highlight the two linear regimes and we can find $p_0^+ \times = 0.0583$. Our results show that a consensus is reached after a long time, however, as one can see in Fig. 4, a full consensus is only obtained for the case when the initial fraction of same opinion is 1, which means that we have obtained the critical point $p_c = 1$.

By the introduction of the mechanism of reputation in the Majority Rule Model, the critical point usually observed was not obtained. We also show that the reputation in the MR Model produced results with democratic-like situations and also are independent of the systems size, which can be applied to a variety of real-world situations.

The spatial distribution of the agents’ opinion, allowing one to observe the evolution of the system’s opinion. It can be studied in a square lattice in which all agents have a fixed position and the discussion group $g$ select the agents at random. In other words, this mean that the agents may be connected to every one in the system to participate in a discussion group. The Figure 5 shows the agents opinions at three different instants of time steps ($t = 0$, $t = 2 \times 10^3$, $t = 100 \times 10^3$), starting with an initial configuration where $p_0^+ = 0.1$. As one can see in Figure 5 by the choice of agents selected at random to participate of the discussion group, we do not observe formation of clusters of agents with the same opinion.
Figure 5. Evolution of the agent’s opinions in a square lattice. Three different instants of time for the same initial configuration is shown. The positive opinion is represented by a black pixel, while the white represent the negative ones. From left panel to right, one can see the beginning of the dynamics, an intermediate instant of time and the steady state.

4. Conclusions
In this work we studied a sociophysic model, the majority rule model in which we introduced the reputation as a score for each agent. Our results show that the inclusion of the reputation in this specific model leads the system to a stationary state in which the consensus is reached but not with all agents with same opinion at long run.

We performed simulations for several different lattices sizes and different values of $p^+_0$. Our results shows that they are independent of the systems size and the critical value of $p_c^+ = 1$, which means that a full consensus obtained only for the case when all agents already had, initially, the same opinion. We believe that the inclusion of the reputation mechanism makes the majority rule model more realistic, leading to democratic states and, due to this fact, may be used for real voting situations.

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