High Energy Astrophysical Neutrinos: the Upper Bound is Robust

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We elucidate the physical basis for the upper bound on high energy neutrino fluxes implied by the observed cosmic ray flux. We stress that the bound is valid for neutrinos produced either by \( p, \gamma \) reactions or by \( p−p(n) \) reactions in sources which are optically thin for high energy protons to photo-meson and nucleon-meson interactions. We show that the upper bound is robust and conservative. The Waxman-Bahcall bound overestimates the most likely neutrino flux by a factor \( \sim 5/\tau \), for small optical depths \( \tau \). The upper limit cannot be plausibly evaded by invoking magnetic fields, optically thick AGNs, or large hidden fluxes of extragalactic protons. We describe the implications of the bound for future experiments including the AMANDA, ANTARES, Auger, ICECUBE, NESTOR, and OWL/AIRWATCH detectors.

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1. INTRODUCTION

The observed cosmic ray flux at high energies implies an upper bound on the high-energy neutrino flux produced in astronomical sources that are, like gamma ray bursts and the observed jets of active galactic nuclei, optically thin to photo-meson interactions on protons \([1]\). The upper bound also applies to \( p−p \) or \( p−n \) collisions that create neutrinos via pion production in sources that are optically thin to nucleon-meson interactions. For simplicity, we shall refer in the following only to \( p−p \) collisions when we really mean either \( p−p \) or \( p−n \) collisions.

The high energy protons that produce neutrinos by \( p−\gamma \) or by \( p−p \) interactions contribute to the observed cosmic ray flux after they leave the site where the neutrinos are created. Since they have maintained a high energy in spite of the possibility of interactions with the cosmic microwave background, protons that are detected at earth with energies greater than \( 10^{18} \) eV must originate at redshifts \( z < 1 \). Therefore, the observed flux of high-energy cosmic rays determines the rate at which particles of those energies are being created in the relatively local universe. One can obtain an upper limit to the cosmic ray production rate in the whole universe by assuming that the rate increases with redshift like the fastest known population of astronomical sources, quasi-stellar sources (QSOs). Knowing an upper limit to the universal proton production rate, one can readily compute, by standard particle physics techniques, an upper limit to the rate of production of neutrinos by the same protons.

The conservative upper bound derived in this way has come to be known in the literature as the Waxman-Bahcall upper bound \([1]\).

The upper bound exceeds what one can reasonably expect to measure. As we shall see in Sec. [11], the limit is derived numerically by assuming that all of the energy of the high energy protons produced in the astronomical sources is transferred to pions by photo-pion or nucleon-proton interactions. Each time that a \( \pi^+ \) is produced by photo-pion interactions it receives only about 20% of the initial proton energy \([1]\). For \( p−p \) collisions at very high energies, the explicit calculation of the fraction of energy that is transferred to pions has not yet been done but is likely to be even less than for \( p−p \) interactions. Thus, at the very least, the Waxman-Bahcall upper bound is conservative by a factor of five, and the most likely neutrino flux is about \( 5/\tau \) smaller than the Waxman-Bahcall limit for small optical depths.

The Waxman-Bahcall bound is consistent with our prediction of the expected flux of high energy neutrinos from gamma-ray bursts (GRBs) \([1,2]\). Naturally, the flux expected from GRBs is less than the maximum allowed by the bound. For the GRB calculation, we estimated that \( \sim 20\% \) of the total proton energy is transferred by photo-meson interactions to pions (proton-nucleon interactions are less efficient \([3]\)). As described in the previous paragraph, we assumed, in deriving our upper bound, that 100% of the proton energy is transferred to pions.

The upper bound we set from the observed cosmic ray flux is two orders of magnitude lower than the intensity predicted in some previously published models for the production of neutrinos in AGN jets, implying that a km² neutrino detector would record at most 1 neutrino from AGN jets per year. The same argument also rules out models in which most or all of the gamma-ray background is produced by photo-meson interactions in AGN jets.

Can the upper bound on the high energy neutrino flux be evaded? This is an important question since very large neutrino detectors are being designed for installation in the ocean or a deep lake (see, e.g., Refs. [4][5]), under Antarctic ice (see, e.g., Ref. [6]), in space (see, e.g., Ref. [8]), and large area ground arrays (see, e.g.,
The design characteristics for these neutrino detectors are being determined in part by the best available theoretical models. It is therefore necessary to examine carefully the justifications for different predictions of high energy neutrino fluxes. We do so in this paper.

The goal of this paper is to clarify how the Waxman-Bahcall upper bound follows from the measured cosmic ray flux and, in the process of the clarification, to demonstrate that the upper bound is robust and conservative. We emphasize the implications of the bound for future neutrino and gamma-ray observations.

There are two special types of sources for which the Waxman-Bahcall bound does not apply; these sources could in principle produce a neutrino flux exceeding the Waxman-Bahcall limit. The first special type of source is one in which neutrinos are produced by processes other than photo-meson or proton-nucleon interactions; the second type of special source is one for which the photo-meson (or proton-nucleon) optical depth is high. We begin by summarizing in Sec. III various speculations regarding the existence of such sources, for which we have no observational evidence to date. This section is particularly relevant for future neutrino observational programs. We then turn to a detailed discussion of the derivation of the bound and of its validity and robustness.

In Sec. II and Fig. 1, we summarize the existing data on the cosmic energy spectrum, including the limited information at very high energies that is derived from the Fly’s Eye [1], AGASA [2], and Yakutsk [3] experiments. We show in this section that cosmic-ray observations imply a robust upper limit on neutrino fluxes in the energy range of $\sim 10^{16}$ eV to $\sim 10^{20}$ eV. What is expected at very high energies, $> 10^{20}$ eV, and at relatively low energies, $< 10^{16}$ eV? We argue in Sec. V that it is plausible to extrapolate the Waxman-Bahcall bound to neutrino energies that correspond to protons more energetic than the expected Greisen-Zatsepin-Kuzmin cutoff at $5 \times 10^{19}$ eV. In Sec. V we summarize the observational evidence for the conventional viewpoint that most cosmic rays below $10^{18}$ eV are heavier particles of Galactic origin. Thus, the proton flux that is to be used in deriving the upper limit to the neutrino flux is much less than the total observed cosmic ray flux at energies below $10^{18}$ eV. We discuss the important question of whether sources that are not included in the Waxman-Bahcall bound derivation may exist in this energy range.

It has been common practice for some time to motivate proposed observatories to search for high energy neutrinos by considering the predictions of AGN models for neutrino production for photo-meson interactions that also produce the gamma-ray background. We show in Sec. VIII that the large set of previously published AGN models for neutrino production exceed the Waxman-Bahcall bound by typically two orders of magnitude and therefore are not appropriate theoretical models upon which to base proposed observatories. We present estimates for the upper limit neutrino event rates photo-meson and nucleon-meson interactions that may be observed in the AMANDA, ANTARES, Auger, ICECUBE, NESTOR, and OWL detectors. The previously published AGN models based upon photo-meson interactions are also inconsistent with gamma-ray observations that show rapid time variability. We also discuss in this section the question of whether some AGN’s models may have a high photo-meson optical depth.

Can one increase the rate at which high energy protons are generated beyond the value contemplated in the Waxman-Bahcall bound? If so, one could obviously raise the experimentally important bound on the neutrino flux. The only way of exceeding the bound is by hypothesizing the existence of high energy proton accelerators which exist in environments that do not allow protons to escape. Can one fine-tune the conditions in which luminous astronomical sources are imagined to exist so that neutrinos and photons escape, but protons do not leave the system? Strong magnetic fields obviously provide a potential way of confining protons, but not neutrons, and we discussed this possibility briefly in Ref. [1]. In Sec. VII, we consider some specific scenarios for avoiding the Waxman-Bahcall limit by invoking strong magnetic fields and show by example that magnetic field confinement is not a plausible way of avoiding the limit.

In Sec. [11] we show that it is a good approximation for GRB’s to assume that the high energy neutrino flux is dominated by photo-pion interactions that proceed through the $\Delta$ resonance. The contributions from non-resonant interactions are shown to be small, confirming our previous estimates [4], according to which $\sim 20\%$ of the proton energy is converted in GRBs to pions. We summarize our results in Sec. [IX].

II. EXCEED, NOT VIOLATE, THE WB BOUND

Does the Waxman-Bahcall bound imply that neutrino fluxes from all conceivable astronomical sources must lie below this limit? Are all possible neutrino sources that will be searched for with very large area neutrino telescopes subject to this bound?

As we discussed in the Introduction, there are two (speculative) ways to exceed the limit without violating the constraint imposed by the observed cosmic ray background. The WB bound refers, see Sec. [II], only to sources that are optically thin to proton photo-meson and

*We point out in Ref. [1] that one can imagine "neutrino only" sources that are optically thick to proton photo-meson interactions and from which protons cannot escape. Sources of this kind could in principle contribute a flux in excess of the Waxman-Bahcall bound, but there is, by construction, no observational evidence (from baryons or high energy photons) for their existence.
proton-nucleon interactions and from which protons can escape. Thus, the first ‘way out’ which was also discussed in our original paper [14], is to assume the existence of sources which are optically thick to proton photo-meson or proton-nucleon interactions. We referred in [1] to such sources as “neutrino only factories” or “hidden core models” since they are not motivated by measurements of the cosmic-ray flux or by any electromagnetic observations.

Examples of optically thick scenarios include neutrinos produced in the cores of AGNs (rather than in the jets) by photo-meson interactions [14], or via $p - p$ collisions in a collapsing galactic nucleus [15] or in a cacooned black hole [16]. The optimistic predictions of the AGN core model [14] have already been ruled out by the AMANDA experiment [17] (see Fig. 3).

The second type of source that may exceed the WB bound is one in which neutrinos are produced by some mechanism other than photo-meson or proton-nucleon interactions. We list below some of the hypothesized sources of this type.

- Decays of supermassive, dark matter particles [26,27] might produce high energy neutrinos without violating the cosmic ray bound.
- Topological defects (see, e.g., Refs. [20–25]) might produce particles which decay, among other channels, to neutrinos with only a small associated cosmic ray proton flux.
- Superheavy relic neutrinos [26,27] which interact with the cosmic neutrino background have been proposed as one way of producing ultra-high energy cosmic rays.
- Ultrahigh-energy photons at large redshifts [28] could in principle produce a flux of ultrahigh-energy neutrinos.

III. COSMIC RAY OBSERVATIONS AND THE WB BOUND

In this section, we summarize the observations and calculations that lead directly to the upper bound on high energy neutrino fluxes.

Figure 1 shows the cosmic ray fluxes measured by the Fly’s Eye [11], AGASA [12], and Yakutsk [13] experiments. The smooth curve shown in Fig. 1 was used by Waxman and Bahcall [1] to compute the upper bound on high energy astrophysical neutrino sources from $p - \gamma$ interactions.

It is possible that the energy generation rate given locally by Eq. (1) increases with redshift. In particular, we have explained in Ref. [1] how cosmic rays observed at earth to have energies in excess of $10^{18}$ eV must have originated at small redshifts because of the large energy loss rate at these high energies. In order to establish a conservative upper limit, we assumed that the local rate given in Eq. (1) evolves with redshift at the maximum evolutionary rate exhibited by the quasars [29–31]. We also included the adiabatic energy loss due to the expansion of the universe.

Figure 1† shows that the smooth curve which we have used to estimate the cosmic ray flux above $10^{18}$ eV is a conservative (i.e., high) estimate of the observed rate.

What is the neutrino bound that results from the observed cosmic ray flux? Figure 3 shows the numerical limit that is implied by the cosmic ray observations. The upper horizontal curve is computed by assuming that the cosmic ray sources evolve as rapidly as the most rapidly evolving known astronomical sources. This very conser-
The Waxman-Bahcall (WB) upper bound on muon neutrino intensities ($\nu_\mu + \bar{\nu}_\mu$). The numerical value of the bound assumes that 100% of the energy of protons is lost to $\pi^+$ and $\pi^0$, and that the $\pi^+$ all decay to muons that also produce neutrinos. The WB upper bound exceeds the most likely neutrino flux by a factor of $5/\tau$ for small optical depths $\tau$. The upper solid line gives the upper bound corrected for neutrino energy loss due to redshift and for the maximum known redshift evolution (QSO evolution, see text). In what follows, we will refer to this conservative upper curve as the “Waxman-Bahcall bound.” The lower solid line is obtained assuming no evolution. The dotted curve is the maximum contribution due to possible extra-galactic component of lower-energy, < $10^{17}$ eV, protons as first discussed in [1] (see Sec. V for details). The dash-dot curve shows the experimental upper bound on diffuse neutrino flux recently established by the AMANDA experiment [17]. The dashed curves show the predictions of the GRB fireball model [14, 22].

**IV. BEYOND $10^{20}$ EV**

The theoretical curve in Fig. 1 shows the predicted decrease above $5 \times 10^{19}$ eV in the observed cosmic ray flux due to interactions with the cosmic microwave background [36]. The existing observations do not clearly demonstrate this decrease and therefore many authors have speculated that there may be a new source of ultra-high energy neutrinos beyond $10^{20}$ eV.

In our original discussion (see Fig. 1 of Ref. [1] or Fig. 2 of the present paper), we did not extend the upper bound to energies beyond $10^{20}$ eV because we judged that too little is presently known about the cosmic ray flux in that region. However, as a useful guide to designing experiments, it is a reasonable expectation that the production rate will not exceed by a large factor the flux predicted by an extension of the $E^{-2}$ production rate used in setting the limit at observed proton energies of $10^{19}$ eV.

In our view, it is reasonable to extend the Waxman-Bahcall limit beyond $10^{20}$ eV by simply extrapolating the horizontal line in Fig. 2 to higher energies. The validity of this extrapolation will be tested by future measurements of the spectrum of ultra-high energy cosmic rays, measurements that will determine observationally whether the spectrum exhibits the predicted Greisen-Zatsepin-Kuzmin (GZK) cutoff at high energies.

We have suggested elsewhere [34] that the clustering of sources of ultra-high energy ($>10^{20}$) cosmic rays may give rise to more events beyond the GZK cutoff than expected on the basis of a homogeneous distribution of sources. Other authors (see, for example, the references cited in Sec. V) have suggested exotic sources of ultra-high energy cosmic rays that are not described by the $E^{-2}$ proton generation rate. If any of these scenarios is correct, then the upper bound shown in Fig. 2 will have to be raised above the simple extrapolation we currently advocate as a useful guide. However, the existing cosmic ray data suggest that the factor by which the WB upper bound is conservative, $5/\tau$, will more than compensate for any increase in cosmic ray flux beyond the GZK limit.

**V. LOWER ENERGY: $E < 10^{18}$ EV**

Figure 1 shows that our smooth curve for the extragalactic proton cosmic ray flux falls below the total observed flux for energies less than $10^{19}$ eV. We have assumed (see Eq. 1) that the generation rate of extragalactic protons is proportional to $E^{-2}$, where $E$ is the proton energy. This $E^{-2}$ dependence is produced generically by the Fermi mechanism for accelerating high energy cosmic rays in shocks [35]. If the lower energy cosmic rays were protons from extragalactic sources, then (as we noted earlier [1]) one could raise the upper bound for astrophysical neutrinos at these lower energies.

Could the lower energy cosmic rays be primarily extragalactic protons? Unfortunately, the answer is no [2]. The available observational evidence suggests that only a small fraction of the cosmic-ray flux in the energy range of $10^{14}$ eV to $10^{17}$ eV is composed of protons. Direct (balloon) composition measurements at $10^{14}$ eV [32] show

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1In the original version of their paper, astro-ph/9812398 v1, Mannheim, Protheroe, and Rachen assumed that 100% of the lower energy cosmic rays could be extragalactic protons. In the third version of their paper, v3, they accepted the argument presented in the first version of our paper hep-ph/9902383 v1, and repeated here, that only a small fraction of these lower energy cosmic rays are extragalactic protons.
that the fraction of cosmic-ray flux composed of protons at this energy is \( \sim 20\% \). Air-shower and cosmic-ray tracking detectors measurements indicate \( \sim \) that the proton fraction decreases in the energy range of \( 10^{14} \) eV to \( 10^{16} \) eV. In fact, the Fly’s Eye and AGASA experiments support \( \sim \) a composition strongly dominated by (consistent with 100\%) heavy nuclei at \( 10^{17} \) eV. Moreover, both the Fly’s Eye \( \sim \) and the AGASA \( \sim \) collaborations have reported, for energies less than \( 3 \times 10^{18} \) eV, a small but statistically significant enhancement of the cosmic ray flux near the Galactic plane. This enhancement is expected for a Galactic, but not an extragalactic, origin for the cosmic rays in this energy domain.

In summary, the observational evidence is that most of the observed cosmic rays in the energy range \( 10^{14} \) eV to \( 10^{17} \) eV are not protons and therefore the total flux of cosmic rays cannot be used to raise the upper bound for neutrinos in the energy range \( 10^{14} \) eV to \( 10^{17} \) eV. The NASA satellite ACCESS is expected to provide accurate measurements of the composition of the cosmic rays at energies up to \( \sim 10^{15} \) eV sometime in the next decade.

Assuming, conservatively, that \( \sim 10\% \) of the cosmic rays in this energy region are protons, the neutrino bound may be raised at energies \( E_{\nu} < 10^{16} \) eV (Since the cosmological model we have used, see Eq. 1 and Fig. 3, accounts for \( \sim 10\% \) of the total cosmic ray flux at \( 10^{17} \) eV, and a progressively larger fraction at higher energies, one cannot increase the upper bound on neutrino fluxes at energies \( \geq 10^{18} \) eV). Since the energy density in cosmic rays is approximately proportional to \( E^{-1} \) for energies less than \( 10^{18} \) eV, the actual extragalactic neutrino flux could exceed the most stringent Waxman-Bahcall limit by a factor of \( (10^{16} \text{ eV}/E_{\nu}) \) for neutrino energies \( E_{\nu} < 10^{16} \) eV.

Figure 2 illustrates with a curved dotted line the maximum contribution from unrecognized extragalactic sources of protons, i.e., sources of extragalactic protons that do not contribute significantly to the observed cosmic ray flux at \( 10^{19} \) eV. The ‘unknown source’ contribution is obtained by multiplying the proton generation rate that is proportional to \( E^{-2} \) by an analytic approximation to the maximum additional contribution allowed at lower energies by the existing experiments. Thus

\[
\text{max. proton rate} \propto E^{-2} \left[ 1 + 0.1 \left( (10^{17} \text{eV})/E \right) \right], \quad (2)
\]

where \( E \) is the proton energy. The recent experimental bound established by the AMANDA experiment already rules out a contribution from unrecognized extragalactic sources of protons at the maximum level allowed by present cosmic-ray observations.

We note that including a contribution to the neutrino flux due to neutrino production by photo-meson interactions on heavy nuclei will not affect the neutrino bound. For heavy elements, the cross section for photodissociation is higher than, and its energy threshold is lower than, for photo-meson production. Thus, heavy elements will dissociate before losing a significant fraction of their energy to neutrino production.

### VI. AGN MODELS

What predictions have been made for the neutrino fluxes from AGN jets? Are the fluxes large enough to be measurable in a practical detector?

There are two classes of models that have been considered as explanations for the emission of radiation from AGN jets, ones in which radiation is due to electromagnetic processes \( \sim \) and others in which emission of radiation involves the acceleration of very high energy protons \( \sim \). For the first class of models, no significant flux of high energy neutrinos is expected. The second class of models has been normalized by postulating that the jets produce the observed gamma-ray background via photo-meson processes on high energy protons, which produce \( \pi^0 \)'s that decay into gamma rays. The photo-meson process, if it produces the gamma-ray background, would also produce a large flux of high energy neutrinos via charged meson decay. Therefore, the second class of models, involving photo-meson interactions on high energy protons, has received a lot of attention, especially from particle experimentalists.

The previously published AGN models that are discussed in the following subsection have been used to estimate the expected event rates in the AMANDA, ANTARES, Auger, ICECUBE, NESTOR, and OWL detectors \( \sim \). However, all of these models exceed by more than an order of magnitude the maximum neutrino flux permitted by the WB bound (see Fig. 3).

#### A. Previously published models

Figure 3 compares the Waxman-Bahcall upper bound to predictions by various proton acceleration models for AGN jets. The previously published models illustrated in Fig. 3 are inconsistent with the cosmic ray limit; they typically predict fluxes one to two orders of magnitude above the Waxman-Bahcall upper bound.

The expected detection rate in a \( \text{km}^2 \) detector (e.g., ANTARES, ICECUBE, or NESTOR) is less than one event per year from AGNs with spectral neutrino energy shapes like the models P97, HZ97, and M95B shown in Fig. 3 if a neutrino flux consistent with the Waxman-Bahcall upper bound.

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The neutrino event rate in the Auger detector\( \sim \) is less than or of order 1 event per year above a neutrino energy of \( 10^{19} \) eV for optically thin sources, i.e., sources that satisfy the WB bound. The rate may be two orders of magnitude larger if the OWL/AIRWATCH detector is built according to the preliminary specifications \( \sim \).

\[ \text{eq:neutrino} \]
The AGN models were normalized so that the calculated gamma-ray flux from the earlier papers of Mannheim [42] (marked M95B in the figure), Protheroe [43] (P97), and Halzen and Zas [44] (HZ97). The AGN models were normalized so that the calculated gamma-ray flux from \( \pi^0 \) decay fits the observed gamma-ray background. The AGN hidden-core conjecture (S92), to which the WB upper bound does not apply due to high background. The AGN hidden-core conjecture (S92), to which the WB upper bound does not apply due to high photo-meson optical depth of the source, is taken from [44]. Note, that this conjecture is already ruled out by the AMANDA upper bound [13].

Since \( p - \gamma \) reactions give rise to high energy gamma rays (via \( \pi^0 \) decay) as well as to high energy neutrinos (via charged pion decay), the flux of neutrinos is proportional to the flux of gamma rays. Since the models with high neutrino fluxes that are shown in Fig. 3 were all chosen by their authors so that they could account for the observed gamma ray flux, it is clear that the still allowed AGN models (with the 30 times lower neutrino flux) can only account for a few percent of the gamma ray background.

B. Are AGNs optically thick to photo-meson interactions?

The WB bound applies only to sources which are optically thin to photo-meson interactions. As mentioned above, previously published AGN jet models satisfy this requirement. One may argue [46], however, that new models may be constructed to have large optical depth and nevertheless fit the available data.

We discuss this possibility below. We first explain why published AGN models predict small optical depth, and then discuss whether new, high optical depth models can be constructed.

The optical depth to photo-meson interactions for protons of energy \( \epsilon_p \) can be related to the optical depth for pair production of photons of much lower energy, \( \epsilon_\gamma \).

The threshold relation for photo-meson interactions is \( \epsilon_\gamma \epsilon_\gamma \approx m_\pi m_p \) and the threshold relation for pair production is \( \epsilon_\gamma \epsilon_\gamma = 2m_e^2 \). Thus, as first pointed out in [1], a photon of energy \( \epsilon_\gamma \) that causes a photon-meson interaction with a proton of energy \( \epsilon_p \) could also pair-produce with a photon of energy \( 2m_e^2/(m_\pi m_p) \epsilon_p = 4 \times 10^{-6} \epsilon_p \).

Taking account of the ratio between photo-meson and pair production cross sections, one finds that

\[
\tau_{\gamma p}(\epsilon_p) \approx 5 \times 10^{-4} \tau_{\gamma \gamma}(\epsilon_\gamma = 4 \times 10^{-6} \epsilon_p).
\]

(3)

Observed AGN photon energy distributions typically follow a power-law, \( dn/d\epsilon_\gamma \propto \epsilon_\gamma^{-2} \). For such photon spectra, one can easily show [1] that \( \tau_{\gamma \gamma} \propto \epsilon_\gamma \), and hence that

\[
\tau_{\gamma p}(\epsilon_p) \approx 2 \tau_{\gamma \gamma}(\epsilon_\gamma = 10 \text{GeV}) \left( \frac{\epsilon_p}{10^{19} \text{eV}} \right).
\]

(4)

Emission of \( \sim 1 \) TeV photons from “blazars,” AGN jets nearly aligned with our line of sight, is now well established [48], and there is evidence that the high-energy photon spectrum extends as a power-law at least to \( \sim 10 \) TeV [49]. The high energy photon emission is the main argument used [50] in support of the hypothesis that high-energy emission from blazars is due to pion decay rather than inverse Compton scattering. However, the observation of \( > 1 \) TeV photons implies, as shown by Eq. (3), that the jet optical depth to photon-meson interaction is very small. Indeed, one can readily see from Fig. 4 of Ref. [51] that high-energy photon data of northern hemisphere blazars requires small pair-production optical depth at photon energy \( < 0.1 \) TeV. All photons blazar models shown in the figure have \( \tau_{\gamma \gamma} < 1 \) at this energy, i.e. \( \tau_{\gamma p} < 1 \) for \( \epsilon_p < 10^{19} \) eV.

TeV emission is observed from the nearest blazars, which are hence relatively low-luminosity blazars. It had recently been argued by Mannheim, Protheroe and Rachen (MPR, [10]), that the emission of high energy, \( \sim 1 \) TeV, photons may be suppressed in high luminosity blazars, for which models with high optical depth may therefore be constructed. The most intense high-energy gamma-ray source is 3C 279. Figure 6 of Ref. [13], which is reproduced here as Fig. 4, shows that the measured optical depth for pair production is at most \( \tau_{\gamma \gamma} \sim 1 \) at \( \epsilon_\gamma = 10 \) GeV. Combining this observation with Eq. (3) we see that \( \tau_{\gamma p}(\epsilon_p) < 1 \) for \( \epsilon_p < 10^{19} \) eV. Making the ad hoc assumption, that the pair-production optical depth for high luminosity blazars like 3C 279 exceeds unity for photons above 10 GeV, i.e. just above the highest energy

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Note: The photo-meson optical depth of high-energy protons in GRBs is discussed in [12]. It is shown there that the observation of high energy, \( \sim 100 \) MeV, photons in GRBs implies that the sources are not optically thick to photo-meson interactions. In the context of Fireball models for GRBs, the optical depth to nucleon-meson interactions is also small [33].
for which data are available (see Fig. 1), MPR concluded that the AGN neutrino flux may slightly exceed the WB bound at $\sim 10^{18}$ eV (see dotted curve in their Fig. 5a).

![FIG. 4. The emission spectrum of 3C 279 over 15 decades in frequency, adapted from Ref. [47]. The measured fluxes are plotted as small squares in the figure. The flare in June 1991 was studied by many different groups observing in a large number of wavelength domains. Out to the last measured point at about 10 GeV, the emission spectrum shows no evidence for turning over due to a large optical depth. The solid curves are the predictions for emission from a uniform relativistic moving sphere; more complex models are discussed in [47].](image)

In order to avoid the constraint given in Eq. (4), and construct AGN models with significant optical depth to photo-meson interaction, one must argue that the spectral distribution of the background photons (for photo-meson and pair-production interactions) deviates from a $d\nu_{\gamma}/d\nu_{\gamma} \propto \nu^{-2}$ power-law. Since the observed spectral distribution does follow an $\nu^{-2}$ power-law, one must argue that the background radiation with which high energy photons and protons may interact is not the radiation associated with the observed photon flux. This may indeed be the case for jets which expand with high Lorentz factor $\Gamma$. In this case, an ambient isotropic radiation field, $u_{\text{amb.}}$, through which the jet expands, will dominate pair-production and photo-meson interactions as long as its energy density exceeds a fraction $\Gamma^{-4}$ of the energy density associated with the observed radiation, $u_{\text{obs.}}$, which is produced within the jet. The presence of such radiation field can not be ruled out. However, the energy density of such ambient radiation is limited to values not much exceeding $u_{\text{amb.}}/u_{\text{obs.}} = \Gamma^{-4}$, since for $u_{\text{amb.}}/u_{\text{obs.}} > \Gamma^{-4}$ inverse Compton scattering emission of jet electrons would dominate over the synchrotron emission which produces the observed radiation in the radio to UV bands.

In order to obtain the maximum possible optical depth, MPR have therefore adopted the following ad hoc assumptions: (1) The pair production optical depth in luminous blazars, e.g. 3C 279, exceeds unity for photons above 10 GeV; (2) The optical depth is due to ambient radiation (which is not detected); (3) The spectrum of the ambient radiation deviates from a $\nu^{-2}$ power law in such a way that its energy density increases by an order of magnitude at photon energies just below those corresponding to the pair-production threshold of 10 GeV photons. Adopting these implausible assumptions, MPR concluded that the AGN neutrino flux may exceed the WB bound by a factor $\sim 2$ at $\sim 10^{18}$ eV (see thick dashed curve in their Fig. 5a).

VII. CAN ONE EVADE THE UPPER BOUND WITH THE AID OF MAGNETIC FIELDS?

In Sec. III of Ref. [1] we considered various ways that one might try to avoid the upper limit on the flux of high energy neutrinos by invoking magnetic fields. In particular, we discussed the possibility that the cosmic-ray density observed at Earth is lower than the average universe cosmic-ray density, due to confinement of protons by magnetic fields surrounding the cosmic-ray sources, or due to large-scale structure fields. We have shown that, given observational constraints on such magnetic fields, magnetic proton confinement can not affect the high-energy cosmic-ray flux, and therefore can not affect the neutrino bound.

In Sec. IIIc of Ref. [1] we have discussed in detail the possible effects of large-scale structure magnetic fields, and demonstrated that even if such fields are built to equipartition levels, protons of energy exceeding $\sim 10^{16.5}$ eV can not be confined to large-scale structures over a Hubble time. Large-scale structure magnetic fields can not therefore affect the WB bound at energy $\gtrsim 10^{15}$ eV. It should be pointed out here, that had proton confinement by large scale structure magnetic fields been possible, which might be the case for low energy protons, it would have most likely implied that the neutrino upper bound should be lower, rather than higher at $\nu_r < 10^{15}$ eV. This is due to the fact that confining high-energy cosmic-rays to high density regions would imply that the local cosmic-ray density is higher than the universe average. Since, on the other hand, neutrinos can not be confined and their density is uniform, their flux would be bound to a lower value than implied by our assumption of a uniform cosmic-ray density.

Recently, it has been argued by Mannheim, Protheroe and Rachen (MPR, [46]), that while high energy protons can not be confined to the vicinity of their sources
by magnetic fields, adiabatic losses of protons escaping magnetic halos surrounding AGN jets may lead to significant decrease in proton energy for $\epsilon_p < 10^{18}$ eV, which may lead to modification of the bound for $\epsilon_p < 10^{17}$ eV. Clearly, even if this claim is correct, proton AGN models are still in contradiction with the Waxman-Bahcall bound, since the bound is not affected at $\epsilon_p > 10^{17}$ eV (see Fig. 2). Moreover, the key arguments made by MPR in deriving the claimed adiabatic energy loss are flawed. First, the magnetic field model adopted by MPR in deriving the claimed adiabatic energy loss are flawed. First, the magnetic field model inferred from observations is high, tens of $\mu$G (e.g. [54]), depolarization measurements [53] imply small halo magnetic field, 0.1–1 $\mu$G at the central halo region, $r \sim 10$ kpc, and much smaller at large distances, $r \sim 100$ kpc. MPR assume a spherically symmetric field structure with amplitude decreasing as $r^{-2}$ from $30 \mu$G $r = 10$ kpc to 0.3 $\mu$G at $r = 100$ kpc. Second, MPR assume that high energy protons are produced in the central region of the jet, where the magnetic field is high, and then are confined to jet plasma which expands adiabatically to the halo plasma conditions of low magnetic field. This is an implausible description of the plasma evolution and of proton confinement. Acceleration of protons to high energy is typically expected to occur at the outer edge of the jet at the strong shock. Moreover, since the jet is narrow, typical opening angles are 10 degrees, the typical jet structures are $\sim 1$ kpc in scale, and neutrons produced by photo-meson interactions leading to $\nu$ production escape the jet before decaying.

VIII. CONTRIBUTIONS FROM NON-RESONANT INTERACTIONS

The high energy-neutrino flux from GRBs was calculated by Waxman & Bahcall [21] using the “$\Delta$-approximation,” i.e. assuming that photo-meson production is dominated by interaction of protons with photons of energy close to that corresponding to the $\Delta$-resonance. It has recently been argued by Mücke et al. [54] that for characteristic GRB photon spectrum, there is an additional contribution to photo-meson production from interaction of protons with photons of energy much higher than that corresponding to the resonance, leading to significant deviation from results derived based on the $\Delta$-approximation. We point out here, that the contribution to neutrino production of non-resonant interactions is small, leading to only negligible modification of results obtained using the $\Delta$-approximation.

The GRB photon spectrum is well fitted in the BATSE detector range, 30 keV–3 MeV, by a combination of two power-laws, $dn_{\gamma}/d\epsilon_{\gamma} \propto \epsilon_{\gamma}^{-\beta}$ with different values of $\beta$ at low and high energy [53]. The break energy (where $\beta$ changes) in the observer frame is typically $\epsilon_{b,\gamma}^{ob} \sim 1$MeV, with $\beta \simeq 1$ at energies below the break and $\beta \simeq 2$ above the break (the plasma emitting the GRB radiation expands with Lorentz factor $\Gamma \simeq 300$, and particle energies in the plasma frame are smaller by a factor $\Gamma^{-1}$ compared to observed particle energies). For protons of observed energy $\epsilon_p \lesssim 10^{16}$ eV, the (observed) photon threshold energy for photo-meson interaction is higher than the break energy $\epsilon_{b,\gamma}^{ob} \sim 1$MeV [2]. Such protons therefore interact only with the steep part of the photon spectrum, $dn_{\gamma}/d\epsilon_{\gamma} \propto \epsilon_{\gamma}^{-2}$. Thus, for $\epsilon_p \lesssim 10^{16}$ eV the contribution from interaction with high energy protons, i.e. photons of energy well above the threshold and therefore well above the energy corresponding to the $\Delta$-resonance, is negligible simply because there are very few photons of such high energy.

Protons of energy $\epsilon_p > 10^{16}$ eV may interact with photons well below the break, where the photon spectrum is flatter, $dn_{\gamma}/d\epsilon_{\gamma} \propto \epsilon_{\gamma}^{-1}$. In this case there may be significant contribution to photo-meson production from interaction of protons with photons of energy well above the threshold, and therefore well above the energy corresponding to the $\Delta$-resonance, since the photon spectrum is flat over significant energy range, from the threshold energy to $\epsilon_{b,\gamma}^{ob} \sim 1$MeV. However, contribution from non-resonant interaction becomes significant only at very high energy: for protons of energy $\epsilon_p > 10^{18}$ eV photo-meson production of pions is increased by only a factor of $\sim 2$ compared to the production rate based on the $\Delta$-approximation. Moreover, this increase has no effect on the neutrino production discussed by Waxman & Bahcall [21], since the high energy pions produced in interaction of protons with $\epsilon_p > 10^{18}$ eV lose their energy through synchrotron emission before decaying, and therefore do not contribute to the neutrino flux, which is strongly suppressed at observed neutrino energy $> 10^{16}$ eV [2,56].

IX. DISCUSSION

In this paper, we have shown that the Waxman-Bahcall upper limit on neutrino fluxes is robust for neutrino energies above $10^{16}$ eV. Cosmic ray experiments set a firm upper limit on the flux of extragalactic protons, and hence on the flux of extragalactic neutrinos, that are produced by $p+\gamma$ or $p-p$ interactions in astronomical sources that are optically thin to photo-meson and nucleon-meson interactions on high energy protons. The upper solid curve in Fig. 2 which is labeled WB bound, is a conservative representation of the limit imposed by the cosmic ray observations and includes the maximum plausible redshift evolution of the sources of high-energy cosmic rays. The WB bound exceeds the most likely neutrino flux from $p+\gamma$ or $p-p$ interactions by a factor of $5/\tau$, for small optical depths, $\tau$, to photo-meson and nucleon-meson interactions.

The Waxman-Bahcall upper bound is about two orders of magnitude less than published predictions [12,44] of neutrino fluxes expected from models for AGN jets that explain the gamma-ray background by photo-meson interactions on high energy protons (see Fig. 3). Given the
neutrino upper limit, photo-mesonic interactions in optically thin sources like AGN jets can at most account for a few percent of the flux of the gamma-ray background [7].

The Waxman-Bahcall bound implies an upper limit to the expected event rate from optically thin sources that may be observed in the AMANDA, ANTARES, Auger, ICECUBE, NESTOR, and OWL/AIRWATCH detectors [4–7]. In km² detectors like ANTARES, ICECUBE, or NESTOR, one expects to detect less than or of order 1 neutrino event per year from optically thin AGNs with neutrino spectral energy shapes like the AGN models P97, HZ97, and M95B shown in Fig. [3]. For the Auger detector, one expects less than or of order 1 neutrino event per year above 10^{19} eV from all sources satisfying the WB bound. The event rate could be ~ two orders of magnitude larger for the OWL detector if it performs according to the preliminary specifications. These estimates are upper limits and do not include the factor of 5/τ by which, for small optical depths τ, the WB bound exceeds the plausible expected neutrino flux. The rate estimates given in this paragraph are much less than the estimates given in the published papers describing the capabilities of the neutrinos detectors, since in those papers AGN models like those shown in Fig. [3] were assumed in the calculations.

At neutrino energies below 10^{16} eV, the flux of extragalactic neutrinos from optically thin sources may reach as high as the dotted curve in Fig. [3]. In order for this curve, labeled ‘max. extra-galactic p’ in Fig. [3] to apply, a large fraction of the cosmic rays with energies below 10^{17} eV must be protons of extra-galactic origin (cf. Eq. [2]). For cosmic rays more energetic than 10^{20} eV, there is not yet an accurate determination of the cosmic ray flux. Hence there is no way of rigorously extending the WB bound for neutrino energies > 10^{19} eV. We propose that horizontal line in Fig. [3] simply be extrapolated to higher energies as a plausible bound if the source spectrum continues to fall off at ultrahigh-energies as E^{-2}.

The large area neutrino experiments [4–7] that are currently being constructed are designed to detect neutrinos with energies < 10^{15} eV, corresponding to protons with energies less than or of the order of 10^{16} eV. The WB limit is constructed by normalizing to the observed flux of cosmic ray protons at 10^{19} eV and extrapolating to lower energies using the E^{-2} source spectrum. The extrapolation is plausible but not rigorous for the neutrino energy domain of the ANTARES, BAIKAL, ICECUBE, and NESTOR experiments.

There are two hypothetical ways of exceeding the Waxman-Bahcall limit without violating the constraint imposed by cosmic ray observations. First, the neutrinos could be produced in sources that are optically thick to photo-nucleon or nucleon-nucleon interactions. Second, the neutrinos could be produced by processes that do not give rise to high energy cosmic rays, such as the decay of dark matter particles, topological defects, superheavy relic neutrinos, or ultrahigh-energy photons. So far, there is no obervational evidence supporting any of these possibilities.

We have investigated a number of ad hoc scenarios invented to try to find ways of violating the Waxman-Bahcall bound. None of the suggested scenarios, including various ideas involving magnetic fields, provide a physically self-consistent mechanism for raising the upper limit to the neutrino flux implied by a straightforward interpretation of the cosmic ray observations.

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