Contrib. Plasma Phys. 41 (2001) no. 2–3, in press

**Strongly Coupled Coulomb and Nuclear Plasma in Inner Crusts of Neutron Stars**

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**Abstract**

Matter of subnuclear density in the inner crusts of neutron stars consists of neutron-rich atomic nuclei immersed in strongly degenerate relativistic gas of electrons and strongly non-ideal liquid of neutrons. Thermodynamic and kinetic properties of this matter are greatly affected by Coulomb and nuclear interactions and can be studied, in principle, from observations of thermal radiation of young (age \(\lesssim 100\) yr) neutron stars.

**1 Introduction**

Neutron stars (NSs) are the most fascinating stars in the Universe. Their masses are \(M \sim 1.4 M_\odot\), and their radii \(R \sim 10\) km. Accordingly, their mean mass density is about \((2–3) \rho_0\), where \(\rho_0 = 2.8 \times 10^{14} \text{ g cm}^{-3}\) is the mass density of matter in atomic nuclei. Thus, NSs contain matter of supranuclear density compressed by huge gravitational forces. Accordingly, they are treated as unique astrophysical laboratories of such matter. We show that young NSs are also laboratories of strongly coupled Coulomb–nuclear plasmas of subnuclear density.

A NS consists of a very thin atmosphere, outer crust, inner crust, outer core and inner core (e.g., refs. \([1, 2]\)). The outer crust is a few hundred meter thick extending to the neutron drip density \(\rho = \rho_d \approx 4 \times 10^{11} \text{ g cm}^{-3}\). It mainly consists of strongly degenerate, almost ideal electrons and fully ionized atoms (atomic nuclei). The nuclei form a strongly nonideal Coulomb plasma, liquid or solid, depending on \(\rho\) and temperature \(T\). The properties of this matter are discussed in a companion paper \([3]\). The inner crust (e.g., refs. \([4, 5, 6]\)), our main subject, extends from \(\rho = \rho_d\) to \(\rho = \rho_{cc} \approx \rho_0/2\), and is about 1 km thick. Its matter consists of electrons, neutron-rich nuclei, and Fermi liquid of neutrons dripped from the nuclei. The neutrons are likely superfluid due to the attractive part of nucleon-nucleon interaction. The critical temperature of superfluidity is model dependent; its typical values range from \(10^9\) to \(10^{10}\) K. The nuclei arrange in a Coulomb crystal; the nucleus shape and the parameters of neutron liquid are governed by strong interaction. The properties of this unique mixture of Coulomb and strong interactions in matter of subnuclear density are not well known from the theory. They are especially uncertain at the crust bottom, \(\rho \gtrsim 10^{14} \text{ g cm}^{-3}\), where the nuclei can be nonspherical and form clusters \([6]\).

Matter of the outer NS core consists of neutrons, with admixture of protons, electrons and possibly muons. This core may be about several km thick and extends to \(\rho \lesssim 2 \rho_0\). The inner core occupies deeper central layers creating the main mystery of NSs since its composition cannot be determined uniquely by the present theories. There are many theoretical models predicting appearance of hyperons, pion or kaon condensates, or quark matter.
2 Inner crust matter

Let us adopt the ground-state model of crustal matter \[4, 5\] and focus on the temperature range from \(\sim 10^8\) to \(\sim 10^9\) K of interest for NS cooling (Sect. 3). The nuclear composition does not depend on \(T\) for \(T \lesssim 3 \times 10^9\) K. The nuclear charge number is \(Z \sim 40–50\). The number of neutrons per a Wigner–Seitz cell can reach \(\sim 10^3\) at \(\rho \sim 10^{13}\) g cm\(^{-3}\); they mainly belong to the neutron liquid outside the nuclei. At \(\rho \sim 10^{14}\) g cm\(^{-3}\) the neutron and proton density distributions within the nucleus become smooth and the nuclear radius becomes \(\sim 0.5a\), where \(a\) is the radius of the Wigner–Seitz cell. For \(T = 10^9\) K at this \(\rho\) the Coulomb coupling parameter \(\Gamma = (Ze)^2/(aT)\) reaches \(\sim 10^3\), and \(T/T_p\) is \(\sim 0.3\), where \(T_p\) is the ion plasma temperature. Accordingly the Coulomb crystal is nearly classical, although it becomes quantum at lower \(T\).

The equation of state (EOS) in the inner crust is almost temperature independent. The pressure is mainly determined by electrons at \(\rho \sim \rho_d\) and by neutrons at \(\rho \sim \rho_{cc}\). The neutron drip greatly softens the EOS at \(\rho > \rho_d\), but the neutron liquid introduces considerable stiffness at \(\rho \gtrsim 10^{13}\) g cm\(^{-3}\).

If the neutrons were nonsuperfluid they would determine the heat capacity in the inner crust. The superfluidity can greatly reduce the neutron contribution, making the heat capacity of Coulomb crystal \[7\] dominant, for given temperatures.

The neutrino emissivity in the crust is produced by several mechanisms. The most important are neutrino pair bremsstrahlung due to scattering of electrons off nuclei (e.g. ref. \[8\]) and plasmon decay into neutrino pair.

Finally, the thermal conductivity in the inner crust is mainly provided by electrons which scatter off atomic nuclei \[9\]. It depends weakly on \(T\) for \(T = 10^8–10^9\) K as shown in fig. 1a where we use smooth–composition model \[8\] of spherical nuclei \[4, 5\] at \(\rho < 10^{14}\) g cm\(^{-3}\) and its extrapolation to higher \(\rho\) in the crust. It is important that at \(\rho \gtrsim \rho_{cc}/10\) the conductivity is sensitive to the size of the proton charge distribution within the nuclei. For instance, calculation of the conductivity for pointlike nuclei at \(\rho \sim 10^{14}\) g cm\(^{-3}\) underestimates the conductivity by a factor of 3–5. The conductivity in the core \[10\] is \(\sim 10^2\) times higher (fig. 1a) since there are no such efficient electron scatterers as atomic nuclei there.

3 Cooling of young neutron stars

NSs are born very hot in supernova explosions, with the internal temperature \(T \sim 10^{11}\), but gradually cool down. We have calculated a number of cooling curves, the surface temperatures \(T_s^\infty\) as detected by a distant observer versus stellar age \(t\). The detailed description of the results is given elsewhere \[11\].

We have used the code which calculates NS cooling by solving the equations of heat conduction within the NS taking into account neutrino energy losses from the NS interior and photon emission from the surface. The effects of General Relativity are included explicitly. We have adopted the same EOS in the NS core (composed of neutrons, protons and electrons) as in ref. \[11\]. The code includes all relevant sources of neutrino energy losses, heat capacity and thermal conductivity in the core and crust. The effects of neutron superfluidity in the core and crust and proton superfluidity in the core have been incorporated to test various theoretical models of superfluidity.

Although the inner crust is hidden deeply within the NS, the cooling is sensitive to its properties at the early cooling stage (\(t \lesssim 10–300\) yr) as long as the internal thermal relaxation is not established. The NS energy losses at this stage are mainly provided
by neutrino emission. The lower thermal conductivity in the crust delays the crustal thermal relaxation making it very pronounced in the cooling curves.

To test this statement we have run several models switching on and off various ingredients. Fig. 1b shows some cooling curves for a nonsuperfluid NS model with $M = 1.5M_\odot$ and $R = 11.38$ km. The central density $1.42 \times 10^{15}$ g cm$^{-3}$ is high enough to allow direct Urca process, the most powerful neutrino emission mechanism, to operate in a central NS kernel. This leads to fast neutrino cooling of the NS core (e.g., refs. [11, 12]). However the surface temperature remains independent of the core temperature at the initial nonrelaxed stage. The solid line is the real cooling curve. The end of the relaxation stage manifests itself in a spectacular drop of $T_s^\infty$ by about one order of magnitude. The relaxation time, defined [13] as the moment of the steepest fall of $T_s^\infty(t)$, is $t_r \approx 52$ yr, for the real cooling. Switching off all neutrino emission in the crust (the upper dotted line) would delay the relaxation to 258 yr keeping extremely high surface temperature, $T_s^\infty \sim 10^7$ K, at the nonrelaxed stage. Turning on the neutrino bremsstrahlung ($eZ$) alone would lead to $t_r = 54$ yr, close to the real value. Restoring full neutrino emission but switching off the heat capacity of neutrons ($C_n = 0$) in the crust (imitating thus the effect of strong neutron superfluidity) would speed up the relaxation to 15 yr. If we additionally turned off the heat capacity of nuclei the relaxation would speed up further to about 11 yr. On the other hand, if we restored the full heat capacity but ignored quantum suppression of the heat capacity of nuclei at $T \ll T_p$, the latter heat capacity would become important in older NSs, $t \gtrsim 10^4$ yr, strongly delaying the cooling. Finally, switching on the heat capacity and neutrino emission but assuming infinite thermal conductivity (lower dotted line) would remove the relaxation stage at all and lead to the fast drop of $T_s^\infty$ in a young NS. The relaxation time is very sensitive to the values of the thermal conductivity $\kappa$ in

![Fig. 1: (a) Electron thermal conductivity in NS crust and core at three values of $T$ for finite-size and pointlike nuclei in the crust. (b) Decrease of surface temperature $T_s^\infty$ of a young NS for several NS crust models.](image-url)
the crust at $\rho \sim 10^{14}$ g cm$^{-3}$. For instance, using the lower conductivity for pointlike nuclei (fig. 1a) would delay the relaxation to 130 yr.

Our calculations confirm the scaling relation \[ t_r = \alpha \tau, \]
where $\alpha = (\Delta R/1 \text{ km})^2(1 - r_g/R)^{-3/2}$ is the factor which depends on the crust thickness $\Delta R = R - R_{\text{cc}}$, NS mass and radius $r_g = 2GM/(c^2R)$ being the gravitational radius, while $\tau$ depends solely on physical properties of crustal matter. For the NS model in fig. 1b we have $\Delta R = 0.93$ km and $\alpha = 1.81$. The scaling enables one to calculate $t_r$ for other NS models.

It is well known that the NS cooling theory can be used for interpretation of observations of middle-aged NSs ($t \sim 10^4$–$10^6$ yrs) providing viable information on physical properties of matter in the NS cores (e.g., ref. \cite{12}). Now we see that cooling of young NSs, $t \lesssim 100$ yr, depends strongly on the thermal conductivity, heat capacity, and neutrino emissivity of the Coulomb–nuclear plasma in the inner NS crusts, at $\rho \sim 10^{14}$ g cm$^{-3}$. This gives potentially powerful method to test theoretical predictions of the properties of such plasma, particularly, sizes of highly unusual atomic nuclei and critical temperatures of the neutron superfluidity. Unfortunately, such young NSs have not been detected so far. Hopefully they will be observed in the near future in not too distant supernova explosions. This will enable one to realize the above method in practice.

Acknowledgements

The work was supported in part by RFBR (grant No. 99-02-18099) and INTAS (grant No. 96-0542). DGY and AYP are grateful to DFG for support provided to attend PNP10.

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Received 1 October 2000