Semi-Analytical Correlation for Pool Boiling Heat Transfer Using Nanofluids Based on Fractal Theory

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Abstract. On the basis of fractal dispersion of active nucleation sites on boiling surface, a revised fractal model for nucleate pool boiling of nanofluids was established incorporating microlayer evaporation, nature convection and transient conduction. It has fewer empirical constants than that of traditional ones. Results of model using pure fluids and nanofluids conform to data from previous investigations. Thus, accuracy of model is verified. Effects of liquid properties and contact angle on pool boiling heat transfer coefficient (HTC) using nanofluids are discussed according to the present model. It is found that the HTC of nanofluid and the fractal dimension of nucleation sites decrease with the decreasing contact angle. The effect of liquid properties enhancement due to the presence of nanoparticles has ignorable influence on HTC of dilute nanofluids while the decreased contact angle plays a more important role. It is also found that the surface with lower contact angle had a higher natural convection component.

1. Introduction
Boiling heat transfer is broadly applied in nuclear reactors, electricity generating, and electronic component cooling. But the low heat conductivity of traditional thermal transfer fluids, which primarily limit the HTC and thermal performance of above-mentioned systems. Conventional method such as fins has reached its limit. Thus, to explore newly developed techniques to improve the thermal behavior of traditional heat transfer fluids, great endeavor has been made.

Since Choi [1] first proposed the concept of nanofluids in 1995, nanofluids is expected as a new method to enhance thermophysical performance of thermal fluids by dispersing nanoparticles in basic fluids have attracted many researchers' attention. In this way, the pool boiling thermal transfer properties of nanofluids in different conditions were investigated experimentally [2-15]. Some experiment results of pool boiling HTC of nanofluids are listed in table 1.

Conflicting experimental results in table 1 indicated either an enhancement [3, 4, 5, 6], or deterioration [7-14], or in some cases, no effect [2, 9, 15] on the HTC due to the presence of nanoparticles. Boiling of nanofluid refers to changes in liquid thermophysical performance as well as surface conditions in comparison with pure liquid [16]. Nanoparticle deposition on boiling surface that changes the contact angle during microlayer evaporation period, is thought to be the major reason for the changes of HTC [10].
Table 1. Summary of some investigations on nanofluid HTC.

| Author                        | Nanofluid     | HTC                        |
|-------------------------------|---------------|----------------------------|
| You et al. [2]                | Al₂O₃-water   | No effect                  |
| Das et al. [7]                | Al₂O₃-water   | Deterioration              |
| Vassallo et al. [15]          | SiO₂-water    | No effect                  |
| Bang and Chang [8]            | Al₂O₃-water   | Deterioration              |
| Suriyawong and Wongwises [3]  | TiO₂-water    | Enhancement or Deterioration|
| Kwark et al. [9]              | CuO-water     | No effect or Deterioration |
| Gerardi et al. [10]           | Diamond-water | Deterioration              |
| Kole and Dey [4]              | ZnO-EG        | Enhancement or Deterioration|
| Hegde et al. [11]             | CuO-water     | Deterioration              |
| Shahmoradi et al. [12]        | Al₂O₃-water   | Deterioration              |
| Jung et al. [13]              | Al₂O₃-water/LiBr | Deterioration           |
| Vazquez and Kumar [5]         | Silica-water  | Enhancement                |
| Tang et al. [6]               | Al₂O₃-R141b   | Enhancement (Surfactant)   |
| Sarafraz and Hormozi [14]     | Cu-water      | Enhancement (Surfactant) Deterioration (No Surfactant) |

Experimental research on pool boiling of nanofluids have been investigated extensively, however, theoretical understanding of the mechanism underlying the phenomena still remains unclear and requires further investigations. Therefore, analytical models suitable for predicting the pool boiling HTC of nanofluids are required.

To date, comparing with pure liquids, models for heat flux and nucleation site density of nanofluid boiling are very fewer. Rohsenow correlation [17] is widely used to represent the pool boiling since it was proposed in 1952, as shown

\[
q = \mu_i h_{fg} \sqrt{\frac{(\rho_l - \rho_s)}{\sigma}} \left( \frac{1}{C_{st}} \right)^{\frac{1}{2}} Pr^{\frac{1}{2}} \left( \frac{C_s \Delta T_{sat}}{h_{fg}} \right)^{\frac{1}{2}}
\]

(1)

\(C_{st}\) represents surface constant which depends on combination of surface and liquid performance, and \(s\) and \(R\) are the constants which depend on the combination of liquid surfaces. Taylor and Phelan [18] indicated nearly all experiment results of nanofluids boiling could be consistent with (1) while \(C_{st}\) changed between 0.0065 and 0.018. These phenomena indicated that since \(C_{st}\) is just related to surface status, surface modification due to nanoparticle deposition is the main factor determining the HTC of nanofluids.

Nevertheless, this does not mean that Eq. (1) can be easily used to calculate the pool boiling HTC of nanofluids because \(C_{st}\) must be determined through experimental. Rohsenow correlation is useful from the application point of view. Meanwhile, the reason for selecting the value of \(C_{st}\) is not given in the Rohsenow correlation, and \(C_{st}\) has a strong impact on the calculated results.

In theory, according to Bankoff’s [19] bubble nucleation mechanism, the correlation formula proposed by Yang and Kim’s [20] is considered to be a useful mechanism method to determine the density of active nucleation center.

\[
N_a = N \int_0^\theta f(\beta) d\beta \cdot \int_{D_{min}}^{D_{max}} f(D_c) dD_c
\]

(2)

in which \(N\) represents density of cavity number on heating surface, \(f(\beta)\) and \(f(D_c)\) represent the
probability density function of the cone angle and cavity mouth size separately. However, due to lack of basic data, accurate correlations for $f(\beta)$ and $f(D_c)$ have not been established.

Recently, according to the experiment results from Gerardi et al. [10], the model for active site density which considering influence of the contact angle has been established by Li et al. [21].

$$N_{a, sf} = 1.206 \times 10^4 (1 - \cos \theta') \Delta T_{sat}^{2.06}$$  \hspace{1cm} (3)

Following similar approach to Ganapathy and Sajith [22], model for active site density was obtained by Li and Liu [21].

$$N_a = \max \{612.5(1 - \cos \theta), 19.7\} \cdot \left(\frac{1}{\gamma} \Pr^3 (R_{sat})^{-0.4} \Delta T_{sat}^3\right)$$  \hspace{1cm} (4)

Unfortunately, the application scope of equation (3) is restricted by experimental conditions. There are few empirical constants with no clear physical meanings contained in each model.

As can be seen in the above-mentioned, the mechanism of nanofluid boiling still remains unclear. Thus, further theoretical research on pool boiling HTC using nanofluid has important value in both theory and practice. In present work, a revised fractal model incorporating the main thermal transfer theory in boiling process and fractal dispersion of nucleation sites on heating surfaces is obtained. The description of nucleation site density based on the fractal dispersion of nucleation sites provides a mean to investigate influence of surface properties on HTC for pool boiling of nanofluids. Contact angle is used as a parameter to define surface characteristics in present work.

2. Effective Properties of Nanofluid

Heat conductivity of nanofluids is obtained with equation as follows [23]:

$$k_{nf} = k_{bf} \frac{1 - \varphi_n + 2\varphi_n \frac{k_{np}}{k_{np} - k_{nf}}} {\ln \left(\frac{k_{np} + k_{nf}}{2k_{nf}}\right)}$$  \hspace{1cm} (5)

According to the mixture rule, the density of nanofluid is expressed as [24]:

$$\rho_{nf} = \varphi_n \rho_{np} + (1 - \varphi_n) \rho_{bf}$$  \hspace{1cm} (6)

Likewise, specific heat and heat expansion coefficient of nanofluid can be expressed [24] as:

$$C_{nf} = \frac{\varphi_n \rho_{np} C_{np} + (1 - \varphi_n) \rho_{bf} C_{bf}} {\rho_{nf}}$$  \hspace{1cm} (7)

$$\beta_{nf} = \frac{\varphi_n \rho_{np} \beta_{np} + (1 - \varphi_n) \rho_{bf} \beta_{bf}} {\rho_{nf}}$$  \hspace{1cm} (8)

The dynamic viscosity can be expressed as [24]:

$$\nu_{nf} = (1 + 2.5\varphi_n + 6.2\varphi_n^2) \nu_{bf}$$  \hspace{1cm} (9)

3. Model Development

3.1. Fractal Analysis of Active Sites on Boiling Surface

In the pool boiling process, carefully characterize the distribution of active sites is crucial. Because of the complex dispersion of active sites, actual values can be hard to be obtained by traditional method.

Fractal theory provide an important means to study the irregular and complicated phenomenon or objects in the nature, and has been widely employed to predict the transport properties and the thermal
properties of porous medium. Similar to pores in fractal porous media, Yu et al. [25] found that dimension dispersion of nucleation center on heating surface obey fractal power law. The number of active cavities with size higher than or equal to \( D_e \) can be expressed as

\[
N_e (D \geq D_e) = \left( \frac{D_{e,\text{max}}}{D_e} \right)^d \quad \text{with} \quad D_{e,\text{max}} \leq D_e \leq D_{e,\text{min}}
\]

(10)
in which \( d \) is the fractal dimension, \( D_{e,\text{max}} \) and \( D_{e,\text{min}} \) represents the highest and lowest diameters of nucleation sites, respectively.

By the differential forms of (10), the number of active sites with sizes of infinitesimal interval from \( D_e \) to \( D_e + dD_e \) can be obtained as

\[
-dN_e = dD_e^d D_e^{-d+1} dD_e
\]

(11)
The total number of active sites with diameter ranging from \( D_{e,\text{min}} \) to \( D_{e,\text{max}} \) is obtained as

\[
N_{e,\text{tot}}(D) = \left( \frac{D_{e,\text{max}}}{D_{e,\text{min}}} \right)^d
\]

(12)

Chu and Yu [26] proposed the expression for fractal size \( D_f \) of nucleation sites in pool boiling and is given as

\[
d_f = \ln \left[ \frac{1}{2} \left( \frac{D_{e,\text{max}}}{D_{e,\text{min}}} \right)^2 \right] / \ln \left( \frac{D_{e,\text{max}}}{D_{e,\text{min}}} \right)
\]

(13)

where \( \overline{D_{e,\text{max}}} \) represents mean value over all maximal active sites:

\[
\overline{D_{e,\text{max}}} = \frac{1}{m} \sum_{i=1}^{m} D_{e,\text{max}} \{ T_{\text{wall},i} \}
\]

(14)

Hsu et al. [27] suggested that the highest and lowest diameters of active cavity can be predicted by

\[
D_{e,\text{max}} = \delta_i (T_{\text{wall}} - T_{\infty}) \left[ 1 + \sqrt{\frac{8c_i(T_{\text{wall}} - T_{\infty}) \alpha T_{\infty}}{c_i(1 - \sin \theta)(1 - \sin \theta) T_{\text{sat}} \delta_i h_{\text{fg}} \rho_i}} \right]
\]

(15)

\[
D_{e,\text{min}} = \delta_i (T_{\text{wall}} - T_{\infty}) \left[ 1 - \sqrt{\frac{8c_i(T_{\text{wall}} - T_{\infty}) \alpha T_{\infty}}{c_i(1 - \sin \theta)(1 - \sin \theta) T_{\text{sat}} \delta_i h_{\text{fg}} \rho_i}} \right]
\]

(16)

where \( c_i = (1 + \cos \theta)/\sin \theta \), \( c = 1/\sin \theta \), \( T_{\infty} \) is the average temperature of liquid, \( T_{\text{wall}} \) and \( T_{\text{sat}} \) are, respectively, the wall temperature and saturation temperature of liquid, \( \theta \) is the liquid-solid contact angle. \( \delta_i \) is the thickness of thermal boundary layer

\[
\delta_i = \frac{k_{\text{wall}}}{h_{\text{nc}}}
\]

(17)

where \( h_{\text{nc}} \) is the average HTC of natural convection which can be calculated as [26]

\[
h_{\text{nc}} = 0.14 \rho_C \left( \frac{\beta g \Delta T^2 C_p}{v_i} \right)^{\frac{1}{2}}
\]

(18)

3.2. Fractal model for Boiling Heat Flux of Nanofluid

According to the analytical method proposed by Benjamin and Balakrishnan [28], same methodology has been employed in the current work. Combined with the fractal character of the active sites on boiling surface, a revised fractal model for boiling of nanofluids is proposed. The pool boiling heat flux is partitioned into 3 parts:

1. Latent heat supplied for microlayer evaporation \( (q_{\text{me}}) \);
(2) Heat consumed by regeneration of heat boundary layer after the bubble leaves \((q_{bc})\):

(3) natural convection of base fluid in the region of the surface uninfluenced by bubble cycle \((q_{nc})\):

Considering the period of bubble grow and departure, the total heat flow is obtained by time average expression

\[ q_{\text{tot}} = \frac{q_{bc} \tau_g + q_{nc} \tau_w}{\tau_g + \tau_w} \] (19)

where \(\tau_g\) represents time for bubble growth, and \(\tau_w\) represents waiting time for bubble regeneration.

### 3.2.1. Latent Heat Supplied for Bubble Growth

While the bubble grows, latent heat supplied for evaporating the microlayer under the bottom of the bubble through heat conduction. The expression for bubble growth is given as

\[ q_{\text{bg}} = \frac{0.135 l}{D} \theta \phi' \gamma \alpha \rho_h h_f \Delta T_{\text{sat}} \] (20)

where \(B\) equal to 1.55 for liquids like water, carbon tetrachloride and n-hexane, with density and boiling point are high. For liquids like n-pentane and acetone, \(B\) equal to 1/1.55, with low density and boiling point. The surface-liquid interaction parameter is \(\gamma = (k_s \cdot \rho_s \cdot C_{p,s}) / (k_l \cdot \rho_l \cdot C_{p,l})\), Jacob number is \(Ja = (\rho_l \cdot C_{p,l} \cdot \Delta T_{\text{sat}}) / (\rho_v \cdot h_{fg})\) and Archimedes number is \(Ar = (g/\nu^2) \left(\sigma / \rho_l \cdot g\right)^{1/3}\).

By deriving the volume of liquid evaporated in microlayer in growing process, expression for heat flux was expressed as \((21)_a\):

\[ \frac{0.135 l}{D} \theta \phi' \gamma \alpha \rho_h h_f \Delta T_{\text{sat}} = \int_0^{D_{\text{c,min}}} \int_0^{D_{\text{c}}/2} \frac{6 \sigma}{\Delta \rho g} \frac{3}{3} D_{\text{c}}^{1/3} \text{d}N_{\text{c}} \] (21a)

\[ \phi' = \cos^2 \theta, \quad \theta \leq 45^\circ \] (21b)

\[ \phi' = 0.5, \quad 45^\circ < \theta < 90^\circ \] (21c)

In equation (21), a unified distribution of size for every active nucleation site on boiling surface is assumed. Since the size distribution of active nucleation site is not uniform and conforms to the fractal power law, latent heat supplied for bubble growth by nucleation sites between \(D_{\text{c,min}}\) and \(D_{\text{c}} + dD_{\text{c}}\) is expressed as

\[ dq_{bc} = \int_0^{D_{\text{c,min}}} \int_0^{D_{\text{c}}/2} \frac{6 \sigma}{\Delta \rho g} \frac{3}{3} D_{\text{c}}^{1/3} \text{d}N_{\text{c}} \] (22)

The total heat flux of bubble growth for all nucleation sites within ranging from \(D_{\text{c,min}}\) to \(D_{\text{c,max}}\) is given as

\[ q_{\text{bg}} = \int dq_{bc} = \int_0^{D_{\text{c,max}}} \int_0^{D_{\text{c}}/2} \frac{6 \sigma}{\Delta \rho g} \frac{3}{3} D_{\text{c}}^{1/3} \text{d}N_{\text{c}} \] (23)

Equation (23) can be integrated if \(D_{\text{bd}}\) can be represented in \(D_{\text{c}}\). The diameter of bubble departure obtained by Kirichenko [29] is given as

\[ D_{\text{bd}} = \left[ \frac{6 \sigma}{\Delta \rho g} \right]^{1/3} \left[ \frac{1}{2} \right]^{1/3} \] (24)

Substituting equations (15) and (28) into (27), (27) can now be integrated to give as

\[ q_{\text{bg}} = \int_0^{D_{\text{c,max}}} \int_0^{D_{\text{c}}/2} \frac{6 \sigma}{\Delta \rho g} \frac{3}{3} D_{\text{c}}^{1/3} \text{d}N_{\text{c}} - \left( \frac{D_{\text{c,max}}}{D_{\text{c,min}}} \right)^{1/3} \] (25)

### 3.2.2. Transient Conduction After Bubble Departure

After the bubble departs, cooling liquid follows towards heating surface. Then, the thermal boundary layer reforms, which is named as waiting time, \(\tau_w\).
Thermal transfer from heating surface to liquid can be regarded as the transient conduction from the hot surface to the semi-infinite body (in this case, the liquid). Thus, heat flux caused by regeneration of thermal boundary layer can be expressed as

\[ q_{\text{tc}} = h_{\text{tc}} A_0 \Delta T_{\text{sat}} \]  \hspace{1cm} (26)

where \( h_{\text{tc}} = \frac{2k\rho C_p}{\pi \tau_w} \) represents transient heat transfer coefficient. \( A_i \) represents the influence area of bubble departure per unit area, assuming influence area do not interference, \( A_i \) is given as

\[ A_i = \frac{2b d_2}{4 \pi a_a \pi D} \]  \hspace{1cm} (27)

where \( K \) is the influence area factor, which is identified as ratio between the area of bubble influence to the projected area of bubble at departure time, which is suggested to be 4 by various researchers \[18, 24\]. Thus, the heat flux during the bubble during the waiting time is given as

\[ q_{\text{w}} = 2 \left[ \frac{k\rho C_p}{\pi \tau_w} \right] \left[ N_{\text{sat}} \pi D_{\text{sat}}^2 \right] \Delta T_{\text{sat}} \]  \hspace{1cm} (28)

Coupling the internal influence of bubble growth and departure, the growth time of bubble is obtained as

\[ \tau_g = \frac{D_{\text{sat}}^2}{B^2 A_r 0.27 J_a^{-1} \alpha_i} \]  \hspace{1cm} (29)

Van Stralen et al. \[30\] suggested that the waiting time is triple of growing time, as shown

\[ \tau_w = 3\tau_g = 3\left[ \frac{6\sigma}{\Delta \rho g} \right]^{1/2} B^{-2} A_r^{-0.27} J_a^{-1} \alpha_i \]  \hspace{1cm} (30)

Among the waiting process, the transient conduction heat transferred by all nucleation sites from the minimum site \( D_{\text{c,min}} \) to the maximum site \( D_{\text{c,max}} \) can be obtained as

\[ q_{\text{nc}} = 2 \left[ \frac{3\pi k\rho C_p}{3} \right] \Delta T_{\text{sat}} \left[ \frac{6\sigma}{\Delta \rho g} \right]^{1/2} B A_r^{-0.18} J_a^{0.5} \alpha_i^{0.5} \times \left[ \frac{3d,}{3d,} - 1 \right] D_{\text{c,max}}^{2/3} \left( N_{\text{sat}} - \frac{D_{\text{c,max}}}{D_{\text{c,min}}} \right) \]  \hspace{1cm} (31)

3.2.3. Natural Convection. Natural convection heat flux \( q_{\text{nc}} \) over a horizontal flat plate is given as

\[ q_{\text{nc}} = A_{\text{nc}} h_{\text{nc}} \Delta T_{\text{sat}} \]  \hspace{1cm} (32)

where \( A_{\text{nc}} \) is the area which is not affected by the bubble cycle per unit heater surface. According to the method of Benjamin and Balakrishnan \[28\], the area not influenced by the bubbles for flat plate surfaces should be written as

\[ A_{\text{nc}} = 1 - \frac{1}{4} N_{\text{sat}} \left( \frac{\pi D_{\text{sat}}^2}{4} \right) \left[ \frac{3}{4} N_{\text{sat}} \pi D_{\text{sat}}^2 \right] \]  \hspace{1cm} (33)

Yu et al. \[25\] obtained the volume (area) fraction of nucleation sites, given as

\[ \chi = \left( \frac{D_{\text{c,min}}}{D_{\text{c,max}}} \right)^{d-1} \]  \hspace{1cm} (34)

where \( \chi \) represents volumetric (or area) fraction, \( d = 2 \) indicated the 2D space for heated surfaces. Substituting (34) into (33), the fractal model for natural convection is expressed as

\[ q_{\text{nc}} = \left[ 1 - \frac{1}{16} \left( \frac{D_{\text{c,min}}}{D_{\text{c,max}}} \right)^{2-d} \right] \left( \frac{3}{4} \right) \frac{D_{\text{c,max}}}{D_{\text{c,max}}}^{2-d} h_{\text{nc}} \Delta T_{\text{sat}} \]  \hspace{1cm} (35)
Substituting equations (25), (31) and (35) into (19), a fractal model of total heat flux is obtained as

\[
q_{\text{tot}} = \frac{\sqrt{\pi B}}{10} \phi \gamma \alpha \rho \frac{h_l}{\Delta \rho g} \left( \frac{6 \sigma}{\Delta \rho g} \right)^{1/3} \left( \frac{3d_l}{3d_i - 1} \right) D_{c,\text{max}}^{0.5} \left( N_{c,\text{tot}} - \frac{D_{c,\text{min}}}{D_{c,\text{max}}} \right) + \frac{\sqrt{3 \pi k \rho c_p}}{2} \Delta T_w \left( \frac{6 \sigma}{\Delta \rho g} \right)^{1/3} B \left( \phi \alpha \rho \frac{h_l}{\Delta \rho g} \right)^{1/3} \left( \frac{3d_l}{3d_i - 1} \right) D_{c,\text{max}}^{0.5} \left( N_{c,\text{tot}} - \frac{D_{c,\text{min}}}{D_{c,\text{max}}} \right) + \left( 1 - \frac{1}{16} \frac{D_{c,\text{min}}}{D_{c,\text{max}}} \right)^{2-d_l} \frac{3}{4} \frac{D_{c,\text{min}}}{D_{c,\text{max}}}^{2-d_l} \gamma \phi \alpha \rho \frac{h_l}{\Delta \rho g} \Delta T_w \right)
\]

Equation (36) indicates that the total heat flux is a function of thermal physical properties of nanofluid, wall superheat, fractal dimension, the total number of nucleation sites, the smallest and largest sizes of active nucleation sites and contact angle.

4. Results and Discussion

A commonly feature in the boiling of nanofluids is nanoparticle deposition. To verify this present model, calculated results are in comparison with experimental results from different literatures [8, 10, 31, 32]. Details of experimental condition can be seen from table 2.

| Reference         | Nanofluid     | \(\Theta\) (clean surface) | \(\theta\) (nanocoated surface) | Pressure |
|-------------------|---------------|---------------------------|---------------------------------|----------|
| Bang et al. [8]   | 0.5vol.% Al₂O₃-water | 60°                       | 30°                             |          |
| Gerardi et al. [10] | 0.1vol.% Silica-water | 80°                       | 16°                             | 0.1 MPa  |
| Yang et al. [31]  | 0.36vol.% Silica-water | 60.7°                    | 23.5°                           |          |
| Shahmoradi et al. [17] | 0.02vol.% Al₂O₃-water | 45°                       | 20°                             |          |

As shown in figures 1 and 2, the predicted results with different contact angle and thermal physical properties conform well with the experimental data, and error is within ±20%. In addition, figures 1a and 1b show that HTC decreases with the decreasing contact angle, which also conforms well with the experimental results from Hegde et al. [11] and Li et al. [25]. The above phenomenon indicates that the present model is reasonable.

It is believed that the higher HTC of nanofluids is induced by the enhancement in thermophysical properties of liquid, by adding nanoparticle [1, 2, 7]. However, currently these hypotheses have been questioned and confuted for that the changes in the transport and thermal properties of nanofluid had nearly no effect on the boiling curve, especially for dilute nanofluid (<0.1vol %) [21]. To investigate influence of liquid properties on boiling thermal transfer for nanofluid, boiling curves of Al₂O₃-water nanofluids with varied volume concentration (0%, 0.02 vol.%, 0.05 vol.%, 0.1 vol.%, 0.5 vol.% and 1 vol%) which obtained from (40) and experimental data of Shahmoradi et al. [12] are plotted in figure 3. For convenience, assuming the contact angles of all particle concentration are equal to 45° under the condition that only consider the liquid properties.

As can be seen from figure 3 that the predicted HTC has nearly no significant enhancement when particle concentration increasing from 0% (pure water) to 0.1 vol.%. However, predication result by considering two factors of thermal physical performance and contact angle, still conforms well with results of experiment. This comparison in figure 3 shows that thermophysical performance of nanofluids sole, is still not enough to increase dilute nanofluid HTC. Therefore, it is evidence that the decrease of active nucleation is more obvious than properties changes, although the thermal conductivity increases by adding nanoparticles, thus the heat flux still decreases.
Figure 1. Comparison of predicated and experimental results for heat flux of pure water and nanofluid.

Figure 2. The errors between the experimental and model results.

Figure 3. Effect of liquid properties on boiling curve for nanofluids.

Figure 4 depicts that fractal size of active nucleation sites rises with superheat increasing, while reduces as the contact angle going down from 80° to 30°. According to equations (16)-(20), as the contact angle decreases, more cavities on boiling surface are flooded by liquids. Thus, the decrease in nucleation sites results in decreasing of the fractal dimension of active nucleation sites. It is indicated that $d_f$ is a function of the wall overheat and contact angle, which is proportion to nucleation sites density and HTC.

It is shown in figure 5 that the relative contribution of transient conduction flux, microlayer evaporation flux and natural convection flux to the total heat flux. The microlayer evaporation component increases with wall superheat increasing and account for 77.66-79.39% as heat flux closes to CHF. From figure 5, we can see that component of natural convection, which dominates the total heat flux at lower superheats, gradually decreases with increasing wall superheat and reach 9.4-11.38% when approaching the CHF. Nonetheless, the natural convection term should not be ignored even near CHF. It is important to note that surface with smaller contact angle has a higher natural convection component. This can be explained that the lower contact angle, fewer active nucleation sites present on boiling surfaces due to bubble cycle phenomena; thus, a bigger size of surface can be used for natural convection. Similar trend was found by Ganapathy and Sajith [22].
5. Conclusion
A fractal model to predict the pool boiling HTC using nanofluids is established according to fractal theory as well as heat partition method in this paper. Model data conforms well with the available experiment results. Conclusions in this study are listed as follows:

(1) HTC of nanofluid decreases with the decreasing contact angle. For dilute nanofluids, Thermal conductivity and other thermal properties are heightened even caused by presence of nanoparticles, boiling performance of nanofluids was more dominantly influenced by the reduction of active nucleation sites that induced by nanoparticle deposition.

(2) Fractal size of active nucleation sites are associated with wall overheating and contact angle. It is also proportion to nucleation sites density and wall superheat while reduces as the contact angle reduces.

(3) Thermal transfer contribution caused by microlayer evaporation grows with wall overheat. Transient conduction component reached a peak, then slightly went down. Component of natural convection gradually decreases with increasing wall superheat and reach 9.41-11.38% at high heat flux.

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