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Constraining Palatini–Horndeski theory with gravitational waves after GW170817

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Abstract In this paper, we investigate the possible parameter space of Palatini–Horndeski theory with gravitational waves in a spatially flat Universe. We develop a general method for obtaining the speed of gravitational waves in the Palatini formalism in the cosmological background and we find that if the theory satisfies the following condition: in any spatially flat cosmological background, the tensor gravitational wave speed is the speed of light $c$, then only $S = \int d^4x \sqrt{-g} \left[ K(\phi, X) - G_3(\phi, X) \Box \phi + G_4(\phi) R \right]$ is left as the possible action in Palatini–Horndeski theory. We also find that when $G_5(\phi, X) \neq 0$, the tensor part of the connection will propagate and there are two different tensor gravitational wave speeds.

1 Introduction

The successful detection of gravitational waves made up the last puzzle missing in the experimental verification of general relativity [1–5]. Therefore, general relativity has become the most successful theory of gravity so far.

However, there are still many theoretical problems that can not be explained by general relativity, such as how to explain the hierarchy between the Planck scale and the electroweak scale [6–8] and how to quantize gravity [9]. In addition, the phenomena observed in experiments, such as the accelerated expansion of the Universe [10] and the flat rotation curves of galaxies [11], can not be explained by general relativity. For these reasons, many modified theories of gravity were considered [6–8,12–16] in the hope of answering the problems that general relativity could not answer.

Adding additional scalar field is one way to modify gravity. Theories obtained in this way are called scalar–tensor theories. In Ref. [17], a tentative indication for scalar transverse gravitational waves was reported. If this is further confirmed in the future, it will strongly suggest that the gravity theory describing our world should have a scalar degree of freedom. In order to avoid the Ostrogradsky instability [18–21], we expect to give priority to those theories that can derive second-order field equations. In the metric formalism, the most general scalar–tensor theory that can derive second-order field equations is Horndeski theory [13].

However, Refs. [22–24] pointed out that the observation of the speed of tensor gravitational waves in the Universe by the gravitational wave event GW170817 together with the gamma ray burst GRB170817A would severely constrain the possible parameter space of metric Horndeski theory. Specifically, GW170817 and GRB170817A require the tensor gravitational wave speed $c_g$ to meet [25,26]

$$-3 \times 10^{-15} \leq \frac{c_g}{c} - 1 \leq 7 \times 10^{-16}. \quad (1)$$

This shows that in a very high precision, we can say that the tensor gravitational wave speed in the Universe is equal to the basic constant $c$ (speed of light). Considering that the cosmic background is also evolving during gravitational wave propagation, the most economic and natural assumption made by this observation result for the theory seems to be that: in any cosmological background, tensor gravitational waves always propagate at the speed of light. However, the possible subclasses of metric Horndeski theory satisfying this assumption only remain [22,27]

$$S = \int d^4x \sqrt{-g} \left[ K(\phi, X) - G_3(\phi, X) \Box \phi + G_4(\phi) R \right]. \quad (2)$$
This constraint limits the application of scalar–tensor theories. Therefore, we expect to find scalar–tensor theories beyond the metric Horndeski framework. Reference [28] studied the orbital evolution of eccentric binary systems in metric Horndeski gravity. There are also many studies using GW170817 to constrain modify gravity theories [29–40].

Further analysis shows that not all higher derivative theories have the Ostrogradsky instability. The higher derivative theory without the Ostrogradsky instability is required to satisfy the degeneracy condition [21, 41, 42]. In the metric formalism, the scalar–tensor theory with higher derivative but without the Ostrogradsky instability is called degenerate higher-order scalar–tensor (DHOST) theory [43–47]. In addition to DHOST theory, considering the teleparallel framework is another way to go beyond the metric Horndeski framework. In teleparallel Horndeski theory established by Bahamonde et al., metric Horndeski theory is included in the teleparallel framework as one of many subclasses [48, 49].

Considering the scalar–tensor theory in the Palatini formalism may be another way to go beyond metric Horndeski framework. There have been some works on scalar–tensor gravity in the Palatini formalism [50–67]. Cosmology in Palatini–Horndeski theory is different from that in metric Horndeski theory and their stability properties are different [60]. Different from metric Horndeski theory, under some parameter spaces, the connection of Palatini–Horndeski theory will introduce some new degrees of freedom [60]. In addition, the polarization modes of gravitational waves in Palatini–Horndeski theory are different from that in metric Horndeski theory [63]. Thus, it seems that Palatini–Horndeski theory may be different from metric Horndeski. However, it is necessary to further investigate the possible parameter space of Palatini–Horndeski theory.

In this paper, we will find possible subclasses of Palatini–Horndeski theory that satisfied the following condition: the speed of tensor gravitational waves is the speed of light in any spatially flat cosmological background. In Sect. 2, we review Palatini–Horndeski theory. In Sect. 3, we develop a general method in the Palatini formalism to obtain the speed of tensor gravitational waves in the spatially flat cosmological background and constrain the parameter space. The conclusion is given in Sect. 4.

We use the natural system of units in this paper. Greek alphabet indices \((\mu, \nu, \lambda, \rho)\) and Latin alphabet indices \((i, j, k, l)\) range over spacetime indices \((0, 1, 2, 3)\) and space indices \((1, 2, 3)\), respectively.

2 Palatini–Horndeski theory

In the Palatini formalism, the connection is independent of the metric. Therefore, it is necessary to take the variations of the action with respect to the metric and the connection independently. The Riemann tensor \(\tilde{R}^\mu_{\nu\rho\sigma}\) and Ricci tensor \(\tilde{R}_{\mu\nu}\) in the Palatini formalism are defined as

\[
\tilde{R}^\mu_{\nu\rho\sigma} = \partial_\nu \Gamma^\rho_{\mu\lambda} - \partial_\mu \Gamma^\rho_{\nu\lambda} + \Gamma^\mu_{\sigma\lambda} \Gamma^\sigma_{\nu\rho} - \Gamma^\mu_{\nu\sigma} \Gamma^\sigma_{\rho\lambda}.
\]

Furthermore, the action of Palatini–Horndeski theory is defined as follows:

\[
S = \int d^4x \sqrt{-\tilde{g}} \left( \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \right),
\]

where

\[
\mathcal{L}_2 = K(\phi, X),
\]

\[
\mathcal{L}_3 = -G_3(\phi, X) \tilde{\Box} \phi - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{\Box} \phi
\]

\[
\mathcal{L}_4 = G_4(\phi, X) \tilde{\Box} \phi
\]

\[
\mathcal{L}_5 = G_5(\phi, X) \left( \tilde{\Box} \phi \right)^2 - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{g}_{\rho\sigma} \tilde{\Box} \phi
\]

\[
\mathcal{L}_6 = G_6(\phi, X) \left( \tilde{\Box} \phi \right)^3 - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{g}_{\rho\sigma} \tilde{\Box} \phi
\]

Here, \(\tilde{\Box} = \tilde{\Box} \tilde{\Box} \tilde{\Box} \tilde{\Box} \phi, X = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi, \) and \(K, G_3, G_4\) and \(G_5\) are real analytic functions of the variables \(\phi, X\). To distinguish the quantities in the metric formalism, we add tilde to represent the corresponding quantities defined in the Palatini formalism. A comma in subscript means partial derivative, e.g., \(G_4, K \equiv \partial G_4/\partial X\).

In the Palatini formalism, the compatibility condition \(\tilde{\Box} \tilde{\Box} \tilde{\Box} \tilde{\Box} \phi, X = 0\) is generally no longer valid. Therefore, the definition of \(\tilde{\Box}_{\nu}, \tilde{\Box} \phi, \tilde{\Box} \tilde{\Box} \phi, \) and \(\tilde{\Box} \tilde{\Box} \tilde{\Box} \phi, X\) in the action (5) in the Palatini formalism is not unique [60, 62]. In this paper, we take the definition in Ref. [63]:

\[
\tilde{\Box} \phi = g_{\mu\nu} \tilde{\Box} \phi, \tilde{\Box} \phi, X = g_{\mu\nu} \tilde{\Box} \phi, X = g_{\mu\nu} \tilde{\Box} \phi, X
\]

3 The speed of tensor gravitational waves

In this section, we will calculate the speed of tensor gravitational waves propagating in a spatially flat cosmological background and find possible subclasses of Palatini–Horndeski theory that satisfy the following condition: the speed of tensor gravitational waves is the speed of light in any spatially flat cosmological background.

We first consider the background evolution of a spatially flat Universe in the Palatini–Horndeski theory. For a spatially
flat Universe, the metric $g_{\mu\nu}$ is the spatially flat Friedmann-Robertson-Walker (FRW) metric, and the connection $\Gamma^\lambda_{\mu\nu}$ and scalar field $\phi$ are only functions of time:

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \delta_{ij} dx^i dx^j,$$

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu}(t), \quad \phi = \phi(t). \quad (11)$$

We consider that the connection has spatial isotropy, that is, under the spatial rotation transformation, the components of the connection $\Gamma^\lambda_{\mu\nu}$ are invariant. This condition further limits the value of the connection. Specifically, under the spatial rotation transformation, the transformation law of the components of the connection is the same as that of the third-order tensor, which requires that the connection $\Gamma^\lambda_{\mu\nu}$ satisfies $[68,69]$

$$\Gamma^0_{0\alpha} = \Gamma^i_{0i0} = 0, \quad \Gamma^i_{ij} = \Gamma^0_{0ij}, \quad \Gamma^i_{0j} = \Gamma^i_{0i0}/g^i_j,$$

$$\Gamma^i_{jk} = \Gamma^3_{3i} \delta_{ijk} = \Gamma^1_{3i} \epsilon_{ijk} = 0. \quad (12)$$

Here, $\delta_{ij}$ is the Kronecker delta, and $\epsilon_{ijk}$ is the Levi-Civita tensor. It can be seen that for the components of the connection, only $\Gamma^0_{00}, \Gamma^0_{01} = \Gamma^0_{11} = \Gamma^0_{22} = \Gamma^0_{33}$ and $\Gamma^1_{01} = \Gamma^2_{02} = \Gamma^3_{03}$ may not be zero.

By substituting Eqs. (11) and (12) into the action (5), we obtain the action that describes the evolution of a spatially flat Universe:

$$S = \int dt \int d^3 x \times L \left( \dot{\phi}, \dot{\phi}, \dot{\phi}, \dot{N}, N, a, \Gamma^0_{00}, \Gamma^0_{01}, \Gamma^0_{11}, \Gamma^1_{01}, \Gamma^1_{01} \right), \quad (13)$$

where,

$$\frac{L}{a^3 \dot{N}} = K + \frac{3G_4 \Gamma^0_{00} \dot{\phi}}{a^2} - \frac{G_3}{N^2} \left( \Gamma^0_{00} \phi - \ddot{\phi} \right) + \frac{3G_4}{a^2} \left( \Gamma^0_{00} \Gamma^0_{11} + \Gamma^0_{11} \Gamma^0_{01} + \Gamma^0_{11} \right) + \frac{G_3}{N^2} \left( \Gamma^0_{00} \Gamma^0_{11} - \Gamma^0_{01} \Gamma^0_{11} \right) + 3a^2 \left( -\Gamma^0_{00} \Gamma^1_{01} + \Gamma^1_{01} \Gamma^1_{01} \right) + \frac{3G_5}{2a^2 N^4} \left( N^2 \left( \Gamma^0_{00} \Gamma^0_{11} + \Gamma^0_{11} \Gamma^0_{01} + \Gamma^0_{11} \right) + 3a^2 \right) \left( \Gamma^0_{00} \Gamma^1_{01} \Gamma^1_{01} \right) + \frac{G_4 X \dot{\phi}}{a^4 N^2} \left( 9N^2 \Gamma^0_{11} \dot{\phi} - 3a^2 N^3 \Gamma^0_{11} \left( 2\Gamma^0_{00} \phi + \Gamma^0_{11} \phi - 2\phi \right) + 2a^4 \left( N \Gamma^0_{00} - N \right) \left( \Gamma^0_{00} \phi - \phi \right) - \frac{G_5 X}{2a^6 N^4} \dot{\phi} \right). \quad (14)$$

Here and below, the dot on the letter represents the derivative of the corresponding quantity with respect to time, $K, G_3, G_4, G_5, G_4, X,$ and $G_5, X$ are functions of $(\phi, \dot{\phi}^2 / 2N^2)$.

One may want to set $N(t) = 1$ at the level of action, and then obtain the evolution equations by varying the variables $(a, \phi, \Gamma^0_{00}, \Gamma^0_{01}, \Gamma^1_{01})$. However, this will miss one equation [70]. Thus, in order to obtain complete evolution equations, $N$ should be kept in the action.

In addition to the gravitational field, the ideal fluid material field is also distributed in the spatially flat Universe. Therefore, in addition to the gravitational field action (13), we should also add an action $S_m$ that describes the ideal fluid into the total action

$$S_{tot} = S + S_m. \quad (15)$$

Here, $S$ is defined by (13). In the Palatini formalism, $S_m$ is only a function of the metric and the material field, and it is independent of the connection. Varying the action $S_m$ with respect to $g_{\mu\nu}$, we obtain

$$\delta S_m = -\frac{1}{2} \int d^4 x \sqrt{-g} \Gamma^{\mu\nu} \delta g_{\mu\nu}. \quad (16)$$

Here, $T^{\mu\nu}$ is the energy-momentum tensor of the ideal fluid:

$$T^{\mu\nu} = (P + \epsilon) u^{\mu} u^{\nu} + Pg^{\mu\nu}. \quad (17)$$

where $\epsilon$ is the matter density, $P$ is the matter pressure. The four-velocity $u^{\mu}$ satisfies $u^0 = 1, u^\mu = 0$.

By varying the action (15) with respect to $N, \phi, a, \Gamma^0_{00}, \Gamma^0_{11}$ and $\Gamma^1_{01}$, we obtain the background equations:

$$\frac{d}{dt} \frac{\partial L}{\partial N} - \frac{\partial L}{\partial N} = a^3 \epsilon = 0,$$

$$\frac{d^2}{dt^2} \frac{\partial L}{\partial \phi} - \frac{\partial L}{\partial \phi} = 0,$$

$$\frac{\partial L}{\partial a} - 3a^2 P = 0,$$

$$\frac{\partial L}{\partial \Gamma^0_{00}} - \frac{\partial L}{\partial \Gamma^0_{11}} = 0,$$

$$\frac{d}{dt} \frac{\partial L}{\partial \Gamma^1_{01}} - \frac{\partial L}{\partial \Gamma^1_{01}} = 0.$$

Here, $L$ is defined by (14). Because the specific expressions of the background equations (18) are very lengthy and easy to obtain, they will not be listed in this paper. In the following, we take $N(t) = 1$.\[\square\]
In order to study the tensor gravitational waves, we need to obtain the linear perturbation equations of the tensor perturbations on the spatially flat cosmological background.

Since the metric and connection are independent in the Palatini formalism, they should be perturbed independently:

$$g_{\mu \nu} \rightarrow g_{\mu \nu} + h_{\mu \nu}, \quad \Gamma^\lambda_{\mu \nu} \rightarrow \Gamma^\lambda_{\mu \nu} + \Sigma^\lambda_{\mu \nu}. \quad (19)$$

We take the part describing tensor gravitational waves in perturbations:

$$h_{00} = h_{0i} = 0, \quad h_{ij} = H_{ij}, \quad \Sigma^0_{00} = \Sigma^0_{0i} = \Sigma^0_{ij} = 0, \quad \Sigma^i_{0j} = \Sigma^j_{0i} = B^i_j, \quad \Sigma^i_j = \delta^i_j C^j_i + \delta^i_j D^j_i, \quad (20)$$

Here, $H_{ij}, A_{ij}, B_{ij}, C_{ij}$ and $D_{ij}$ are symmetric transverse traceless tensors. They satisfy

$$H_{ij} = H_{ji}, \quad A_{ij} = A_{ji}, \quad B_{ij} = B_{ji}, \quad C_{ij} = C_{ji}, \quad D_{ij} = D_{ji},$$

$$H^i_j = A^i_j = C^i_j = D^i_j = 0, \quad (21)$$

Only in this paragraph, we use $\delta^{ij}$ ($\delta_{ij}$) to raise and lower the index. In Appendix 1, we give the decomposition of the connection and explain why the perturbations describing the tensor gravitational waves are given by Eq. (20).

Without losing generality, we consider the propagation direction of gravitational waves as $+z$ direction. At this time, it can be seen from (20) that the components of the perturbations $h_{\mu \nu}$ and $\Sigma^\mu_{\mu \nu}$ that may not be zero are

$$h_{11} = -h_{22}, \quad h_{12}, \quad \Sigma^0_{11} = -\Sigma^0_{22}, \quad \Sigma^0_{12},$$

$$\Sigma^1_{01} = -\Sigma^2_{02}, \quad \Sigma^1_{02} = \Sigma^2_{01}, \quad \Sigma^1_{13} = -\Sigma^2_{23},$$

$$\Sigma^2_{13} = \Sigma^3_{12}, \quad \Sigma^3_{11} = -\Sigma^3_{22}, \quad \Sigma^3_{12}. \quad (22)$$

By expanding the second-order terms of the perturbations (22) in the action (15) and varying the action with respect to the perturbations, we can obtain the linear perturbation equations describing the tensor gravitational waves. These equations are very lengthy and easy to obtain, so they are not listed here.

Now we have obtained the linear perturbation equations describing the tensor gravitational waves. Next, we will use the equations to obtain the speed of the tensor gravitational waves.

Before that, we will take metric Horndeski theory as an example to demonstrate how to obtain the speed of tensor gravitational waves from the linear perturbation equation. In metric Horndeski theory, the linear perturbation equation describing the tensor gravitational waves is given by [27]

$$\ddot{h} + b(t) \dot{h} - \frac{c_t^2(t)}{a^2(t)} \Delta h = 0, \quad (23)$$

where $h$ is the component $h_{11}$ or $h_{12}$, $b$ and $c_t$ are functions of time, and $\Delta$ is the Laplace operator. For $h(t, z)$ propagating along the $+z$ direction, we make a Fourier transform:

$$h(t, z) = \int d^3k \, \tilde{h}(k) e^{-ikz}. \quad (24)$$

By substituting Eq. (24) into Eq. (23), and using the linearity of Eq. (23), we obtain the following equation:

$$\ddot{\tilde{h}}(k) + b(t) \dot{\tilde{h}}(k) + \frac{c_t^2(t)}{a^2(t)} \Delta \tilde{h}(k) = 0. \quad (25)$$

This allows us to consider only the case with a single spatial wave vector $k_3$:

$$h = f(t)e^{-ikz}, \quad (26)$$

where $f(t)$ can always be expressed as

$$f(t) = F(t) e^{ik_0(t)r}. \quad (27)$$

Here, $F$ is the norm of $f(t)$ and $k_0(t)r$ is the argument. Therefore, $F$ and $k_0$ are real numbers.

Considering that the gravitational wave is observed near time $t_0$ and the observation duration is $\Delta T$, that is, the observation time $t \in [t_0 - \Delta T/2, t_0 + \Delta T/2]$. The duration $\Delta T$ is about the same order of magnitude as the period of the gravitational wave, and during this time, the amplitude and phase of the gravitational wave change very little:

$$\Delta T \sim \frac{2\pi}{k_0} \sim \frac{1}{k_0}, \quad \Delta T \ll F, \quad k_0 \Delta T \ll k_0. \quad (28)$$

Thus, $h = F(t)e^{[k_0(t)r - k_3z]}$ can be approximated as a plane gravitational wave near $t_0$:

$$h = F(t_0)e^{[k_0(t)r - k_3z]}, \quad (29)$$

For the evolution of the cosmic background, the changes of $a, b$ and $c_t$ in Eq. (23) during this period are also small:

$$\dot{a} \Delta T \ll a, \quad \dot{b} \Delta T \ll b, \quad \dot{c_t} \Delta T \ll c_t. \quad (30)$$

So Eq. (23) near $t_0$ can be approximated as

$$\ddot{h} + b(t_0) \dot{h} - \frac{c_t^2(t_0)}{a^2(t_0)} \Delta h = 0. \quad (31)$$

By substituting Eq. (29) into Eq. (31), we can obtain

$$-k_0^2(t_0) + i b(t_0) k_0(t_0) + \frac{c_t^2(t_0)}{a^2(t_0)} k_3^2 = 0. \quad (32)$$

The gravitational waves we can observe have large $k_0$ and $k_3$, which makes the linear term of wave vector component (uniformly recorded as $k$) in the above equation negligible compared with the quadratic term of $k$. Thus, by Eq. (32), the relationship between $k_0$ and $k_3$ will satisfy

$$-k_0^2(t_0) + \frac{c_t^2(t_0)}{a^2(t_0)} k_3^2 = 0. \quad (33)$$

Just write (29) as

$$h = F(t_0)e^{ik_0(t_0)r} e^{-\left(\frac{k_3}{k_0}\right)^2} a(t_0) e^{-iz}, \quad (34)$$
and using Eq. (33), we can see that the tensor gravitational wave speed $c_{g}$ at time $t_{0}$ is

$$c_{g}(t_{0}) = \frac{a(t_{0}) k_{0}(t_{0})}{k_{3}} = \frac{a(t_{0})}{a(t_{0})} k_{3} = c_{t}(t_{0}). \quad (35)$$

The speed (35) obtained by this method is the same as that of Refs. [27,71].

Similar to the above analysis, for Palatini–Horneski theory near a certain time, we also approximate the coefficients of perturbations in the linear perturbation equations to constants which are independent of time. In addition, we also approximate the perturbations (22) to the form of plane gravitational waves:

$$h_{\mu\nu} = \bar{h}_{\mu\nu} e^{i(k_{0}t - k_{3}x)}, \quad \Sigma_{\mu\nu}^{\lambda} = \bar{\Sigma}_{\mu\nu}^{\lambda} e^{i(k_{0}t - k_{3}x)}. \quad (36)$$

Here, $\bar{h}_{\mu\nu}$ and $\bar{\Sigma}_{\mu\nu}^{\lambda}$ are amplitudes. Similar to the above example of metric Horndeski theory, by substituting (36) into the approximated linear perturbation equations, we can obtain the linear equations with amplitudes $\bar{h}_{\mu\nu}$ and $\bar{\Sigma}_{\mu\nu}^{\lambda}$ as the variable. This equations can be written in matrix form:

$$AX = 0, \quad (37)$$

where $A$ is a $10 \times 10$ matrix and it depends on variables $k_{0}$ and $k_{3}$. $X$ is a column vector composed of the components of the amplitudes $\bar{h}_{\mu\nu}$ and $\bar{\Sigma}_{\mu\nu}^{\lambda}$. The specific expression of $A$ is very lengthy and easy to obtain, so it is not listed in this paper.

Equation (37) has a gravitational wave solution if and only if

$$\det(A) = 0. \quad (38)$$

As in the above example, we consider $k_{0}$ and $k_{3}$ to be large. Thus, the lower power terms of $k$ in $\det(A)$ are ignored, and only the highest power terms of $k$ are retained. We mark the remaining quantity in $\det(A)$ as $A$. Therefore, according to the equation

$$A(k_{0}, k_{3}) = 0, \quad (39)$$

we can know the relationship between $k_{0}$ and $k_{3}$, so as to solve the speed of tensor gravitational waves.

Now, we will calculate the speed of tensor gravitational waves propagating in a spatially flat cosmological background.

We divide the parameter space of Palatini–Horneski theory into two classes.

**Class I**: $G_{5}(\phi, X) = 0$. In this class, by solving Eq. (39), we find that the tensor gravitational wave speed $c_{g}$ is given by

$$c_{g}^{2} = \frac{a^{2} k_{0}^{2}}{k_{3}^{2}} = \left(1 + \frac{1}{2} \frac{G_{4,\chi} \phi^{2}}{G_{4}}\right)^{2}. \quad (40)$$

Thus, the condition that the tensor gravitational wave speed is always the speed of light in any spatially flat cosmological background requires

$$G_{4,\chi} \phi^{2} = 0 \quad (41)$$

for any spatially flat cosmological background.

In this class, by solving the background equation (18), we find that $(\ddot{a}, \ddot{\phi}, \dot{P}_{00}, \dot{P}_{01}, \dot{P}_{11})$ can be expressed as functions of $(a, \dot{a}, \phi, \dot{\phi}, P, \epsilon)$. Further considering the equation of state and the energy conservation equation of the ideal fluid, we can also use $(P, a, \dot{a})$ to express $(\dot{P}, \dot{\epsilon})$. Therefore, as long as we know $(a, \dot{a}, \phi, \dot{\phi}, P, \epsilon)$, we can obtain $(\ddot{a}, \ddot{\phi}, \dot{P}_{00}, \dot{P}_{01}, \dot{P}_{11}, \dot{\epsilon})$. This determines the initial value condition of the background equation (18). The specific expressions of these variables are very lengthy and easy to obtain, so we have not listed them.

Considering that there are different equations of state for different types of matters, $P$ and $\epsilon$ can be considered as independent variables. Therefore, the condition that the tensor gravitational wave speed is the speed of light in any spatially flat cosmological background is equivalent to the following condition: at any values of the variables $(a, \dot{a}, \phi, \dot{\phi}, P, \epsilon)$, condition (41) is always true. This requires that $G_{4,\chi}$ is always vanishing.

In this way, we find that in Class I, only subclass

$$G_{5} = 0, \quad G_{4,\chi} = 0 \quad (42)$$

satisfies the condition that the tensor gravitational wave speed is the speed of light in any spatially flat cosmological background.

**Class II**: $G_{5}(\phi, X) \neq 0$. In this class, by solving the background equation (18), we can see that $(\ddot{a}, \ddot{\phi}, \dot{P}_{00}, \dot{P}_{01}, \dot{P}_{11})$ can be expressed as functions of $(a, \phi, \dot{\phi}, P_{00}, P_{01}, P_{11}, P, \epsilon)$. Further considering the state equation and the energy conservation equation of the ideal fluid, we can also use $(P, a, \dot{a})$ to express $(\dot{P}, \dot{\epsilon})$. Therefore, as long as we know $(a, \phi, \dot{\phi}, P_{00}, P_{01}, P_{11}, P, \epsilon)$, we can solve $(\ddot{a}, \ddot{\phi}, \dot{P}_{00}, \dot{P}_{01}, \dot{P}_{11}, \dot{\epsilon})$. This determines the initial value condition of the background equation (18). The specific expressions of these variables are very lengthy, so we do not list them. By substituting these expressions into Eq. (39) and solving it, we can obtain the tensor gravitational wave speed expressed by the variables $(a, \phi, \dot{\phi}, P_{00}, P_{01}, P_{11}, P, \epsilon)$.

In fact, the tensor gravitational wave speed we solved is not unique in this class, and it has two possible solutions $c_{g1}$ and $c_{g2}$. These two speeds are generally different. However, when the matter pressure $P = 0$, we have $c_{g1} = c_{g2}$. The specific expressions of $c_{g1}$ and $c_{g2}$ are very lengthy, so we do not list them.

The first speed $c_{g1}^{2}$ can be expressed as a fraction

$$c_{g1}^{2} = \frac{N}{D}. \quad (43)$$

It can be seen that $c_{g1}^{2} = 1$ is equivalent to the numerator part $N$ on the right side of Eq. (43) minus the denominator
Therefore, the above condition requires
\[ M \equiv N - D = 0. \] (44)

By expanding the brackets, \( M \) can be expressed as a polynomial about the variables \((a, \phi, \phi, \Gamma^{00}, \Gamma^{01}, P, \epsilon)\). If we require the tensor gravitational wave speed \(c_{\epsilon 1}\) to be the speed of light under any spatially flat cosmological background, then for any values of the variables \((a, \phi, \phi, \Gamma^{00}, \Gamma^{01}, P, \epsilon)\), this polynomial should be 0. We notice that in this polynomial, the term where \((\Gamma^{00})^4 \epsilon\) appears is
\[ 6a^{14} (G_5)^5 \phi^5 \left(5G_{5,X} \phi^2 - 2G_5\right)(\Gamma^{00}_0)^4 \epsilon. \] (45)

Therefore, the above condition requires
\[ G_5 = 0 \text{ or } G_5 = \frac{5}{2} \phi^2 G_{5,X}. \] (46)

If substituting the condition \(G_5 = \frac{5}{2} \phi^2 G_{5,X}\) into Eq. (44), we again notice that in this polynomial, the term where \((\Gamma^{01})^4 \epsilon\) appears is
\[ -\frac{1171875}{16}a^{14}(G_{5,X})^6 \phi^{17} (\Gamma^{01}_0)^4 \epsilon. \] (47)

Therefore, the above condition further requires \(G_{5,X} = 0\). Combining with the condition (46) we have \(G_5 = 0\). However, this is inconsistent with the assumption \(G_5 \neq 0\) in this class.

For the second solution \(c_{\epsilon 2}\), using the same analysis method as that used to analyze the first solution \(c_{\epsilon 1}\), we find that the condition of \(c_{\epsilon 2} = 1\) also requires \(G_5 = 0\).

To sum up, for Palatini–Horndeski theory, the parameter space satisfying the condition that the tensor gravitational wave speed is the speed of light under any spatially flat cosmological background is only \(G_{4,X} = 0\) and \(G_5 = 0\).

4 Conclusion

In this paper, we calculated the speed of tensor gravitational waves in the spatially flat cosmological background. Unlike the metric formalism, in the Palatini formalism, the background perturbation describing tensor gravitational waves is no longer just the tensor perturbation of metric \(h_{\mu\nu}^{TT}\), but there are also several tensor perturbations of connection (see Eq. (22)). Therefore, linear perturbation equations describing tensor gravitational waves in the Palatini formalism are often a system of equations. For such a system of equations, we developed a general method in the Palatini formalism to obtain the speed of tensor gravitational waves in the spatially flat cosmological background in Sect. 3. This method is generally applicable to any theory in the Palatini formalism.

It is worth noting that we found that there are two possible speeds of tensor gravitational waves in Class II. This is due to the additional degrees of freedom introduced by the tensor perturbations of the connection. It seems to imply that if we observe two tensor gravitational waves with different speeds in the future, the theory of gravitation describing our world may be described by the Palatini formalism.

However, if we further require the tensor gravitational wave speed to be the speed of light \(c\) in any spatially flat cosmological background, then only
\[ S(g, \Gamma, \phi) = \int d^4x \sqrt{-g} \left[ K(\phi, X) - G_3(\phi, X) \Box \phi + G_4(\phi) \tilde{R} \right]. \] (48)

is left as the possible action in the above two subclasses of Palatini–Horndeski theory. Reference [60] pointed out that the action (48) in the Palatini formalism is actually equivalent to the following action in the metric formalism:
\[ S(g, \phi) = \int d^4x \sqrt{-g} \left[ \tilde{K}(\phi, X) - G_3(\phi, X) \Box \phi + G_4(\phi) R \right]. \] (49)

Here,
\[ \tilde{K} = K + \left(-2G_3G_{4,\phi} + 3G_4^2 - \frac{2}{3}G_3^2 - \frac{X}{G_3} \right). \] (50)

It can be seen that the action (48) in the Palatini formalism actually still belongs to metric Horndeski theory. Therefore, Palatini–Horndeski theory described by action (48) does not have the Ostrogradsky instability. It should be noted that the action (49) is the only subclass of metric Horndeski theory that is compatible with the condition that the tensor gravitational wave speed is the speed of light \(c\) in any spatially flat cosmological background.

Despite some progress, the maximum parameter space without the Ostrogradsky instability in Palatini–Horndeski theory has not yet been fully found. Specifically, when \(G_3(\phi, X) \neq 0\), the connection will introduce additional degrees of freedom. It is completely unknown whether the theory has the Ostrogradsky instability in this case [60]. When only considering the case of the evolution of a spatially flat Universe, conducting the Hamiltonian analysis of the action (13), one can find that there is at least one constraint in the theory. It can be expected that by carefully analyzing the constraint, all parameter spaces without the Ostrogradsky instability can be found in the considered case. Providing such an analysis will help us deepen our understanding of the Ostrogradsky instability in the Palatini-Horndeski theory. This requires future research.

Finally, it should be pointed out that this does not mean that the scalar–tensor gravity in the Palatini formalism must not beyond the framework of metric Horndeski theory. This is because in this paper, we have taken the simplest and natural
assumption for the speed of tensor gravitational waves, that is, in any spatially flat cosmological background, the tensor gravitational wave speed is the speed of light c. By weakening this assumption, it is possible to find other parameter spaces of Palatini–Horndeski theory compatible with GW170817. In addition, Palatini–Horndeski theory considered in this paper is not the most general theory of scalar–tensor gravity in the Palatini formalism. A more general discussion needs to study more general action. These all need to be studied in future work.

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### Appendix A: Decomposition of connection

In this appendix, we use $\delta^{ij}$ ($\delta_{ij}$) to raise and lower the index. Therefore, it will not cause ambiguity if the upper and lower indices of a tensor are not distinguished.

We can decompose the perturbation of the connection $\Sigma_{\mu \nu}^{\lambda}$ into the following forms:

$$\Sigma_{00}^{00} = \Sigma_{0i}^{0i}, \quad \Sigma_{0j}^{0j} = \delta_i F + G_i, \quad \Sigma_{0i}^{0i} = \partial_i M + N_i. \quad \text{(A.1)}$$

$$\Sigma_i^{0j} = \delta_i B + 2 \delta_i C_j + 2 \delta_i E_j, \quad \Sigma_{ij} = \delta_{ij} B + \delta_{ij} C + \delta_{ij} E + U_{ij} + V_{ij}. \quad \text{(A.2)}$$

$$\Sigma_{i j}^{k} = B_{jk}^{i} + \partial_k C_{jk} + \partial_j D_{ik} + \partial_j E_{ik} + \partial_i \partial_j f_{ik} + \partial_i \partial_j \dot{g}_k + \partial_i \partial_j \dot{h}_k + h_l \delta_{jk} \delta_{ki} + q_{ij} \delta_{kim}. \quad \text{(A.3)}$$

$$\dot{\partial}_i G^i = \partial_i N^i = \partial_i C^i \equiv \dot{\partial}_i \bar{C}^i \equiv \dot{\partial}_i \bar{D}^i = \partial_i f^i = \partial_i \bar{g}^i = \partial_i \bar{h}^i = \partial_i \dot{q}^i = 0. \quad \text{(A.5)}$$

$$\dot{S}^i_0 = U^i_0 = V^i_0 = C^i_0 = 0, \quad \partial_i S^0_j = \partial_i U^0_j = \partial_i V^0_j = \partial_i C^0_j = \partial_i D^0_j = \partial_i E^0_j = 0, \quad S_{ij} = S_{ji}, \quad U_{ij} = U_{ji}, \quad C_{ij} = C_{ji}. \quad \text{(A.6)}$$

$$D_{ij} = D_{ji}, \quad V_{ij} = -V_{ji}, \quad E_{ij} = -E_{ji}. \quad \text{(A.7)}$$

The notations used in the appendix are not related to those in the text and should not be confused. The method to prove that the perturbation of the connection $\Sigma_{\mu \nu}^{\lambda}$ can always be decomposed into the above form is the same as the method for the perturbation of the metric $h_{\mu \nu}$.

Similarly, we can decompose the linear perturbation equations into several sets of coupled equations. The perturbation describing the tensor gravitational waves is the transverse traceless tensor part of the metric perturbation $h_{\mu \nu}^{TT}$. Consider that each term in the linear perturbation equations is a function of $\delta_{ij}$, $\dot{\delta}_{ij}$, a time dependent function and a perturbation, and since $h_{\mu \nu}^{TT}$ is a transverse traceless symmetric tensor, only $S_{ij}, U_{ij}, C_{ij}$ and $D_{ij}$ are coupled with $h_{\mu \nu}^{TT}$ in the perturbations of the connection (A1–A4). It is the reason why we take the perturbations (A1–A4) in the text.

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