BCS-BEC crossover and liquid-gas phase transition in nuclear matter

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Abstract. The effect of nucleon-nucleon correlations in symmetric nuclear matter at finite temperature is studied beyond BCS theory. We calculate the critical temperature for a BEC superfluid of deuterons, of a BCS superfluid of nucleons, and in the crossover between these limits. The effect of the correlations on the liquid-gas phase transition is discussed. Our results show that nucleon-nucleon correlations beyond BCS play an important role for the properties of nuclear matter, especially in the low-density region.

1. Introduction

Pairing and nucleon-nucleon correlations are important properties of interacting nuclear systems. For example, in the weak-coupling limit, i.e., at high density, the nucleons form Cooper pairs, and below a certain critical temperature $T_c$, the system is in a superfluid phase as described by the Bardeen-Cooper-Schrieffer (BCS) theory. In the strong-coupling limit, i.e., at low density, neutrons and protons form deuteron bound states which will condense if the temperature is below the critical temperature for the corresponding Bose-Einstein condensation (BEC). It was believed that there is a smooth crossover which connecting the BCS and BEC limits. Qualitatively, especially at zero temperature, these features can be studied within the BCS (mean field) approximation. Quantitatively, however, the critical temperature obtained in this way is too high because the BCS theory does not include the existence of non-condensed pairs at finite temperature. In order to go beyond mean field, one has to consider pair correlations already above the critical temperature, as in the Nozières-Schmitt-Rink (NSR) theory [1].

It is well known that there exists a liquid-gas phase transition in nuclear matter. Within mean-field theory, we know that the BCS-BEC crossover is completely covered by the instability region of the liquid-gas phase transition. In the case of the ultracold atomic Fermi gases, because the pair correlations stabilize the gas [1] such that the system does not collapse into its solid ground state but it remains in its metastable gas state. By analogy, one expects that pair correlations will stabilize low-density nuclear matter and thus reduce the liquid-gas coexistence region. One of our subjects of investigation is to evaluate quantitatively the strength of this effect of nucleon-nucleon correlations on the liquid-gas phase transition.
\[ G = G_{HF} + \Sigma (p, \omega) + \Sigma (p, \xi_p) \]

\[ \Sigma = T + \text{Exchange Term} \]

\[ T = + + + \ldots \]

**Figure 1.** The Feynman diagrams for the Green’s function (top), for the self-energy (middle), and for the T-matrix in ladder approximation (bottom).

### 2. Formalism

Based on the HF single particle Green’s function 
\[ G_{HF}(p, \omega) = \frac{1}{\omega - \xi_p + i\delta} \]
with the quasiparticle energy \( \xi_p = \frac{p^2}{2m} - \Sigma_{HF}(p) - \mu \) and the HF self-energy \( \Sigma_{HF} \), which will be calculated using the D1 Gogny force \[2\], we calculate the nucleon Green’s function including the nucleon-nucleon correlation from the T-matrix contribution,

\[ G^{-1}(p, \omega) = G_{HF}^{-1}(p, \omega) - \tilde{\Sigma}(p, \omega), \quad (1) \]

Since the Gogny effective interaction on HF is supposed to account most of the correlation effect, we need to do the substraction and define the self-energy as follows,

\[ \tilde{\Sigma}(p, \omega) = \Sigma(p, \omega) - \text{Re} \Sigma(p, \xi_p). \quad (2) \]

The whole process can be described by the Feynman diagram in fig.1. In principle, with the Green’s function (1) one can get all the properties of the system at this level of approximation. This can be done numerically with a selfconsistent Green’s function method\[3\]. But here we make the assumption that the correlations can be treated as a small correction to the Gogny HF self-energy. This allows us to use the HF Green’s function \( G_{HF} \) in the calculation of the T-matrix and of the self-energy. Then, for consistency, one should also keep only the first-order term of eq.(1),

\[ G(p, \omega) = G_{HF}(p, \omega) + G_{HF}^2(p, \omega) \tilde{\Sigma}(p, \omega). \quad (3) \]

In order to get a simple expression for the T-matrix, we use the separable Yamaguchi potential\[4\], \( V_\alpha(k, k') = -\lambda_\alpha v(k)v(k') \), \( v(k) = \frac{1}{\sqrt{k^2 + \beta^2}} \). Then the T-matrix has the following analytical form,

\[ T_\alpha(k, k', K, \omega) = \frac{V_\alpha(k, k')}{1 - J_\alpha(K, \omega)}, \quad (4) \]

where \( k \) and \( k' \) are the incoming and outgoing momenta in the center of mass frame, \( K \) is the total momentum, and

\[ J_\alpha(K, \omega) = \int \frac{d^3k}{(2\pi)^3} V_\alpha(k, k) \frac{1 - f(\xi_{K/2+k}) - f(\xi_{K/2+k})}{\omega - \xi_{K/2+k} - \xi_{K/2-k} + i\delta}. \quad (5) \]

In terms of the T-matrix, the self-energy in eq.(2) can be written as,

\[ \Sigma(p, i\omega_n) = \frac{3}{2} \sum_{\alpha=S_1, S_0} T \sum_{n'} \int \frac{d^3p'}{(2\pi)^3} G_{HF}(p', i\omega_n') T_\alpha(k, k, i\omega_n + i\omega_n') \quad (6) \]
Figure 2. The solid line is the full calculation, while the long dashes is the BCS result. The short dashes show the critical temperature of Bose-Einstein condensation of a deuteron gas.

With the Green’s function, the density of the system can be calculated with the following formula,

\[
n(T, \mu) = -4T \sum_n \int \frac{d^3k}{(2\pi)^3} G(k, i\omega_n). \tag{7}
\]

After a lengthy derivation (see [5]), one finds the formulas for the density of the nuclear system initially given in Refs. [6]: \( n = n_{HF} + n_{corr} \). Where the \( n_{HF} \) is the normal HF density[7] and the correlation contribution reads

\[
n_{corr} = 6 \int_{K > K_{Mott}} \frac{d^3K}{(2\pi)^3} g(\omega_b(K)) - 6 \int_{K > K_{Mott}} \frac{d^3K}{(2\pi)^3} g(\omega_0(K)) - 6 \sum_{\alpha=3S_1,1S_0} \int_{K=0} \frac{d^3K}{(2\pi)^3} \int_{\omega_0(K)}^{\infty} \frac{d\omega}{\pi} \left( \frac{d}{d\omega} g(\omega) \right) \left( \delta_\alpha - \frac{1}{2} \sin 2\delta_\alpha \right). \tag{8}
\]

In the upper formula, \( g(\omega) = 1/(e^{\omega/T} - 1) \) is the Bose function, \( \delta_\alpha \) is the nucleon-nucleon phase shift in the \( \alpha \) channel, and \( \omega_b(K) = K^2/4m - 2\mu^*, \omega_0(K) = E_b(K) - \omega_0(K) \) with the effective nucleon mass and effective chemical potential defined in [5].

3. Some results
In the above calculation, when the temperature is below some critical value, we get a divergence in the T-matrix. This pole corresponds to the formation of Cooper pairs at high density and to Bose-Einstein condensation of deuterons at low density. One can determine the critical temperature of the superfluid transition as the temperature where the T-matrix develops a pole at zero total momentum (\( K = 0 \)) and at zero energy (\( \omega = 0 \)). This is the well-known Thouless criterion[8] for the onset of superfluidity, coinciding with the BCS gap equation when the gap goes to zero,

\[
1 - J_\alpha(K = 0, \omega = 0; T = T_c) = 0. \tag{9}
\]

The critical temperature as a function of density is shown by fig.2. One can see that the phase boundary coincides with the BCS curve (long dashed line) in the high density region. This means the nucleon-nucleon correlation effect vanishes here. At very low density and temperature, the main contribution to the density comes from the deuteron bound state. Therefore the superfluid
critical temperature at low density coincides with the critical temperature for Bose-Einstein condensation of a deuteron gas which is shown by the short-dashed line.

Furthermore, one can get the pressure as a function of temperature and chemical potential, \( P(T, \mu) \), from the number density \( n(T, \mu) \) by integration over the chemical potential \( \mu \)\cite{5}. With the pressure, one can determine the coexistence region of the liquid and gas phases of nuclear matter from the conditions: \( P(T, n_1) = P(T, n_2) \) and \( \mu(T, n_1) = \mu(T, n_2) \). At the same time, we can determine the spinodal curve from the zeros of \( \partial P/\partial n \). In the region under the spinodal curve, the system cannot exist in a homogeneous phase. In the region between the phase boundary curve and the spinodal curve, the gas phase or the liquid phase can exist as a metastable state.

Combining the normal-superfluid phase transition and the liquid-gas phase transition, we get the phase structure for the nuclear matter at low temperature region shown by the fig.3. The results show that the coexistence curve crosses the superfluid curve at \( n \sim n_0 \), which means the homogeneous nuclear matter with pairing is stable above this density. The spinodal curve (dashed line) can be calculated until it reaches the superfluid region. Note that on the low-density side, the density region where the gas phase is metastable is strongly increased by the correlations, which indicates the correlations have a stabilizing effect. However, the BEC-BCS crossover lies still in the unstable region of the liquid-gas phase transition.

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