Stock market volatility: An approach based on Tsallis entropy

Sónia R. Bentes\textsuperscript{1}, Rui Menezes\textsuperscript{2}, Diana A. Mendes\textsuperscript{2}  
\textsuperscript{1}ISCAL, Av. Miguel Bombarda, 20, 1069-035 Lisboa, Portugal  
\textsuperscript{2}ISCTE, Av. Forcas Armadas, 1649-025 Lisboa, Portugal.

Abstract

One of the major issues studied in finance that has always intrigued, both scholars and practitioners, and to which no unified theory has yet been discovered, is the reason why prices move over time. Since there are several well-known traditional techniques in the literature to measure stock market volatility, a central point in this debate that constitutes the actual scope of this paper is to compare this common approach in which we discuss such popular techniques as the standard deviation and an innovative methodology based on Econophysics. In our study, we use the concept of Tsallis entropy to capture the nature of volatility. More precisely, what we want to find out is if Tsallis entropy is able to detect volatility in stock market indexes and to compare its values with the ones obtained from the standard deviation. Also, we shall mention that one of the advantages of this new methodology is its ability to capture nonlinear dynamics. For our purpose, we shall basically focus on the behaviour of stock market indexes and consider the CAC 40, MIB 30, NIKKEI 225, PSI 20, IBEX 35, FTSE 100 and SP 500 for a comparative analysis between the approaches mentioned above.

PACS (2008): 87.23.Ge, 89.65.Gh, 89.70.Cf, 89.90.+n

Keywords: Stock market volatility; standard deviation; nonlinear dynamics; Tsallis entropy; econophysics

\textsuperscript{*}E-mail: soniabentes@clix.pt
Introduction

In the last few years there has been an increasing debate on the subject of stock market volatility. In spite of its present relevance, this is not an entirely new issue and has emerged in a systematic way when Shiller [1] first argued that the observed stock market volatility was inconsistent with the predictions of the present value models, quite popular in the past. Moreover, Grossman and Shiller [2] found out that the intemporal variation appeared to be inexplicably high and could not be rationalized even in models with a stochastic discount factor. Even though some authors questioned the conclusion of excessive volatility, like Flavin [3] or Kleidon [4], latter tests accounting for dividend nonstationarity and small sample bias continued to lend support to Shiller’s initial claim (see Refs. [5], [6], [7], [8], [9]). A new insight into this was brought by Schwert [10], who asked the seminal question “Why does stock market volatility change over time?”, having reached the conclusion that only a small amount of fluctuations could be explained by models of stock valuation. In this light, many other studies have appeared with the aim of studying every single aspect of stock market volatility, giving rise to an intense debate on the theme. Recognizing its relevance, Daly [11] summarizes some of the major reasons pointed out for its study: (i) Firstly, when market exhibits an excess volatility, investors may find it difficult to explain it based only upon the information about the fundamental economic factors. As a result an erosion of confidence and a reduced flow of capital into equity markets may occur. (ii) Secondly, for firms individually considered, volatility is an important factor in determining the probability of bankruptcy. The higher the volatility for a given capital structure, the higher the probability of default. (iii) Thirdly, volatility is also an important factor in determining the bid-ask spread. So, the higher the volatility of the stock the wider will be the spread between bid and ask prices, thus affecting the market liquidity. (iv) Fourthly, hedging techniques such as portfolio insurance are affected by the volatility level, with the prices of insurance increasing with volatility. (v) Fifthly, if consumers are risk averse, as the financial theory suggests, an increase in volatility will therefore imply a reduction in economic activity with adverse consequences for investment. (vi) Finally, increased volatility over time may induce regulatory agencies and providers of capital to force firms to allocate a larger percentage of available capital to cash equivalent investments, to the potential detriment of allocation efficiency.

In this brief overview we have tried to shed some light on the theme and to unfold some of its major implications. Nevertheless, given the impracticability of analyzing volatility as a whole we focus on its particular aspect of measurement. Here, however we face an obstacle: since volatility is not observed, there has been no agreement on how to measure it, thus emerging a plethora of techniques. Another conclusion that appeared to have arisen is that volatility
is volatile.

The main contribution of this paper is to compare two different approaches: one based on the statistical measure of the standard deviation or variance and another centred on the concept of entropy. In this regard, we particularly focus on the concept of Tsallis entropy, which constitutes a generalization of the Boltzmann-Gibbs or Shannon entropy. These measures were both generated in the domain of physics, although the latter is also attributed to the Information Theory, and their application to financial phenomena falls in the domain of the so-called econophysics. In an analogy with terms like biophysics, geophysics and astrophysics this word was originally introduced by Stanley et al. [12] in an attempt to legitimize the study of economics by physicists. One argument is that some regularities were found between these two areas. Another argument points out the benefits of the experimental method commonly used in physics, which departs from the observed data without imposing any previous model. Also, it is worthy to note the evidence of common research interests between these two areas. As Mantegna and Stanley [13] pointed out, an active domain of research in physics is the characterization of prices changes, i.e., volatility. In our particular research we apply the concept of entropy to capture the presence of nonlinear dynamics in seven international stock market indexes since the standard deviation can only detect linear relationships. The empirical analysis is conducted with data from Datastream in order to perform a comparative research.

The remainder of the paper is organized as follows: Section 1 describes the most commonly used measure of volatility - the standard deviation - and compares it with two different measures of entropy: the Shannon entropy and its generalization - the Tsallis entropy. Section 2 exhibits the empirical findings, and Section 3 draws the conclusions.

1 Volatility and Entropy Measures: Some Concepts

In this Section we define various measures of volatility. We begin with the standard deviation and then analyze the Tsallis entropy and its special case: the Shannon entropy. Before proceeding further on we shall first clarify the term volatility. According to a wide range of research, volatility can be broadly defined as the changeableness of the variable under consideration (see [1] and [14] for some references). As a result, the more this variable fluctuates over time, the more volatile that variable is said to be. Usually, this term is popular as a synonymous of risk and uncertainty; though its meaning is not quite the same. Yet, they are related concepts. Knight [15] established the difference between both of them in the following sense: while in a situation of risk we are not certain about the results of a given action but know exactly its probability
distribution function in uncertainty the p.d.f. is always unknown.

Another view was introduced by Hwang and Satchell [16], who considered that volatility could be regarded as a combination of two components: transitory noise and permanent fundamental volatility. While the former is temporary and caused by the trading noise, the latter is generated by the arrival of information. This is in accordance with the work of Ross [17], who has already pointed out the role of information in this context.

Based on the fact that volatility could be not constant over time, i.e., "volatility is volatile", some authors have divided the various techniques in two different categories: time invariant (or independent) and time variant (or dependent) measures. In the first group we include the techniques studied in this paper, since they are time independent. The other one clearly exceeds the scope of our research and is related to, for example, the ARCH (Autoregressive Conditionally Heteroskedastic) models, and their subsequent derivations.

1.1 A traditional measure of volatility

A popular way of measuring volatility is to compute the returns $R_t$ of the asset under consideration

$$R_t = \ln P_t - \ln P_{t-1},$$

where $P_t$ and $P_{t-1}$ denote the prices at time $t$ and $t-1$, respectively, and then estimate the corresponding standard deviation over some historical period $T$.

$$\sigma = \sqrt{\frac{\sum_{t=1}^{T} (R_t - \overline{R})^2}{T - 1}},$$

with $\overline{R}$ representing the sample average return, $\overline{R} = \sum R_t/T$.

Although this measure has some advantages since it is simple to estimate and has the ability to capture the probability of occurring extreme events, it also shows some drawbacks. One is that it could lead to an abrupt change in volatility once shocks fall out of the measurement sample. And, if shocks are still included in a relatively long measurement sample period, then an abnormally large observation will imply that the forecast will remain in an artificial high level even though the market is subsequently tranquil. Secondly, it assumes that recent and more distant events are equally weighted. However, the most likely situation is that the more recent ones have a stronger effect on volatility than the older ones. Finally, it only captures linear relationships,
ignoring all kinds of nonlinear dynamics among data. In this light, some more sophisticated measures have emerged aiming to improve the understanding of volatility. With regard to this, a measure that appears to be particularly relevant is the concept of entropy, which constitutes our major aim in this study.

Nonetheless, it is worthy to note that, in spite of all the flaws that have been recognized by a wide body of research, the standard deviation is still the most popular measure of volatility being used as a benchmark for comparing the forecast ability of more complex models.

1.2 Entropy as a measure of volatility

An alternative way to study stock market volatility is by applying concepts of physics which significant literature has already proven to be helpful in describing financial and economic phenomena. One measure that can be applied to describe the nonlinear dynamics of volatility is the concept of entropy. This concept was originally introduced in 1865 by Clausius to explain the tendency of temperature, pressure, density and chemical gradients to flatten out and gradually disappear over time. Based on this, Clausius developed the Second Law of Thermodynamics which postulates that the entropy of an isolated system tends to increase continuously until it reaches its equilibrium state. Although there are many different understandings of this concept, the most commonly used in literature is as a measure of ignorance, disorder, uncertainty or even lack of information (see [18]). Later, in a subsequent investigation, Shannon [19] provided a new insight into this matter showing that entropy was not only restricted to thermodynamics but could instead be applied in any context where probabilities can be defined. In fact, thermodynamic entropy can be viewed as a special case of the Shannon entropy since it measures probabilities in the full state space. Based on the Hartley’s [20] formula, Shannon derived his entropy measure and established the foundations of information theory.

For the probability distribution \( p_i \equiv p(X = i), \ (i = 1, ..., n) \) of a given random variable \( X \), Shannon (Boltzmann-Gibbs) entropy \( S(X) \) for the discrete case, can be defined as

\[
S(X) = -\sum_{i=1}^{n} p_i \ln p_i, \tag{3}
\]

with the conventions \( 0 \ln (0/z) = 0 \) for \( z \geq 0 \) and \( z \ln (z/0) = \infty \).

As a measure of uncertainty the properties of entropy are well established in literature (see [21]). For the non-trivial case where the probability of an event is less than one, the logarithm is negative and the entropy has a positive sign.
If the system only generates one event, there is no uncertainty and the entropy is equal to zero. By the same token, as the number of likely events duplicates the entropy increases one unit. Similarly, it attains its maximum value when all likely events have the same probability of occurrence. On the other hand, the entropy of a continuous random variable may be negative. The scale of measurements sets an arbitrary zero corresponding to a uniform distribution over a unit volume. A distribution which is more confined than this has less entropy and will be negative.

Shannon entropy has been most successful in the treatment of equilibrium systems in which short/space/temporal interactions dominate. However, there are many anomalous systems in nature that just do not verify the simplifying assumption of ergodicity and independence. Some examples are: metaequilibrium states in large systems involving long range forces between particles; metaequilibrium states in small systems \((100 - 200\) particles); glassy systems; some classes of dissipative systems, mesoscopic systems with nonmarkovian memory. With the aim of studying this kind of systems, Tsallis [22] derived a generalized form of entropy, known as Tsallis entropy. Although this measure was first introduced by Havrda and Charvát [23] in cybernetics and later improved by Daróczy [24], it was Tsallis [22] who really developed it in the context of physical statistics and, therefore, it is also known as Havrda-Charvát-Daróczy-Tsallis entropy.

For any nonnegative real number \(q\) and considering the probability distribution \(p_i \equiv p(X = i), i = 1, ..., n\) of a given random variable \(X\), Tsallis entropy denoted by \(S_q(X)\) for the discrete case, is defined as

\[
S_q(X) = \frac{1 - \sum_{i=1}^{n} p_i^q}{q - 1}
\]  

(4)

where the \(q\)-\textit{exponential} function is defined by

\[
y = \left[1 + (1 - q) x\right]^{1/q} \equiv e_q^x \quad (e_1^x = e^x)
\]

(5)

whose inverse is the \(q\)-\textit{logarithm} function

\[
\ln_q x \equiv \frac{x^{1-q} - 1}{1 - q} \quad (\ln_1 x = \ln x).
\]

(6)

The entropic index \(q\) characterizes the statistics we are dealing with; as \(q \to 1\), \(S_q(X)\) recovers \(S(X)\) since the \(q\)-logarithm uniformly converges to a natural logarithm as \(q \to 1\). This index may be regarded as a biasing parameter since \(q < 1\) privileges rare events and \(q > 1\) privileges common events (see [25]). A concrete consequence of this is that while the Shannon entropy yields exponenti-
tial equilibrium distributions, Tsallis entropy yields power-law distributions. As Tatsuaki and Takeshi [26] have already pointed out, the index $q$ plays a similar role as the light velocity $c$ in special relativity or Planck’s constant $\hbar$ in quantum mechanics in the sense of a one-parameter extension of classical mechanics, but unlike $c$ or $\hbar$, $q$ does not seem to be a universal constant. Furthermore, we shall mention that for applications of finite variance $q$ must lie within the range $1 < q < 5/3$. Additionally, in the case of financial series Tsallis, Anteneodo, Borland and Osorio [25] have proven that $q \approx 1.4 - 1.5$.

Tsallis entropy exhibits a series of notable properties described as follows (see, for example, Refs. [27], [28], [29], [22]):

i) Non-negativity: $S_q (X) \geq 0$ for any arbitrary set $\{p_i\}$. The equality holds for $q > 0$ and certainty (all probabilities equal zero excepting one which equals unity).

ii) Equiprobability: If $p_i = 1/W, \forall i$ (microcanonical ensemble) we obtain, $\forall q$, the following extreme value:

$$S_q (X) = \frac{W^{1-q} - 1}{1-q}.$$  \(7\)

iii) Pseudo-additivity: If $A$ and $B$ are two independent systems (i.e., $p_{ij}^{A+B} = p_i^A + p_j^B$), we verify that

$$\frac{S_q (A+B)}{k} = \frac{S_q (A)}{k} + \frac{S_q (B)}{k} + (1-q) \frac{S_q (A)}{k} \frac{S_q (B)}{k},$$  \(8\)

since in all cases $S_q (X) \geq 0$, $q < 1$, $q = 1$ and $q > 1$ respectively correspond to superadditivity (supreextensivity - $S_q (A+B) > S_q (A) + S_q (B)$), additivity (extensivity - $S_q (A+B) = S_q (A) + S_q (B)$) and subadditivity (subextensivity - $S_q (A+B) < S_q (A) + S_q (B)$).

iv) Additivity: It has been recently shown that $S_q (X)$ is also extensive, i.e.,

$$S_q (A_1 + A_2 + ... + A_N) \approx \sum_{i=1}^{N} S_q (A_i),$$  \(9\)

for special kinds of correlated systems, more precisely when the phase-space is occupied in a scale-invariant form (see, [30], [31], for some references). By being extensive for an appropriate value of $q$, $S_q (X)$ complies with Clausius’ concept of macroscopic entropy and with thermodynamics.

v) Reaction under bias: The Shannon entropy can be rewritten as
\[ S(X) = - \left[ \frac{d}{dx} \sum_{i=1}^{W} p_i^x \right]_{x=1} \]  

(10)

This can be seen as a reaction to a translation of the bias \( x \) in the same way as differentiation can be seen as a reaction of a function under a (small) translation of the abscissa. Along the same line, \( S_q(X) \) can be rewritten as

\[ S_q(X) = - \left[ D_q \sum_{i=1}^{W} p_i^x \right]_{x=1} \]  

(11)

where

\[ D_q h(X) \equiv \frac{h(qx) - h(x)}{qx - x} \quad \left( D_1 h(x) = \frac{dh(x)}{dx} \right) \]  

(12)

is Jackson’s 1909 generalized derivative, which can be seen as a reaction of a function under *dilatation* of the abscissa (or under a *finite* increment of the abscissa).

vi) Concavity: If we consider two probability distributions \( \{p_i\} \) and \( \{p'_i\} \) for a given system \( (i = 1, ..., W) \), we can define the convex sum of the two probability distributions as

\[ p''_i \equiv \mu p_i + (1 - \mu) p'_i \quad \quad (0 < \mu < 1). \]  

(13)

An entropic functional \( S(\{p_i\}) \) is said *concave* if and only if for all \( \mu \) and for all \( \{p_i\} \) and \( \{p'_i\} \)

\[ S(\{p''_i\}) \geq \mu S(\{p_i\}) + (1 - \mu) S(\{p'_i\}). \]  

(14)

By concavity we mean the same property where \( \geq \) is replaced by \( \leq \). It can be shown that the entropy \( S_q(X) \) is concave (convex) for every \( \{p_i\} \) and every \( q > 0 \) \((q < 0)\). It is important to stress that this property implies, in the framework of statistical mechanics, thermodynamic stability, *i.e.*, stability of the system with regard to energetic perturbations.

vii) Stability or experimental robustness: An entropic functional \( S(\{p_i\}) \) is said to be *stable* or *experimentally robust* if and only if, for any given \( \varepsilon > 0 \), exists \( \delta_\varepsilon > 0 \) such that, independently from \( W \),

\[ \sum_{i=1}^{W} |p_i - p'_i| \leq \delta_\varepsilon \implies \left| \frac{S(\{p_i\}) - S(\{p'_i\})}{S_{\text{max}}} \right| < \varepsilon. \]  

(15)
This implies that

\[
\lim_{\varepsilon \to 0} \lim_{W \to \infty} \left| \frac{S(\{p_i\}) - S(\{p'_i\})}{S_{\text{max}}} \right| = \lim_{W \to \infty} \lim_{\varepsilon \to 0} \left| \frac{S(\{p_i\}) - S(\{p'_i\})}{S_{\text{max}}} \right| = 0. \tag{16}
\]

Lesche [32] has argued that the experimental robustness is a necessary requisite for an entropic functional to be a physical quantity because it essentially assures that, under arbitrary small variations of the probabilities, the relative variation of entropy remains small.

Since its proposal, Tsallis entropy has been the source of most empirical research devoted not exclusively to physics but also comprising other scientific areas such as biology, chemistry, geophysics, medicine, economics and finance. It is our aim in this study to especially address the latter and find out whether Tsallis entropy is useful to measure stock market volatility.

2 Empirical Results

This Section explores the empirical relevance of the theoretical results obtained by both perspectives. To do so we have gathered data from several countries in order to detect whether some similarities can be found among them. This is especially relevant in the context of the globalization we are living in, which also constitutes another area of research interest.

2.1 Data

In our empirical research the data set compounds the daily returns of the CAC 40 (France), MIB 30 (Italy), NIKKEI 225 (Japan), PSI 20 (Portugal), IBEX 35 (Spain), FTSE 100 (U.K) and SP 500 (U.S.A.) extending from 8 January 1990 to 7 April 2006. Each index contains 4240 observations, which is large enough to make our analysis meaningful. These data were collected on a daily basis without considering the re-investment of dividends and were computed in accordance with Eq. 1 where the closing prices were the inputs. Fig. 1 plots the results.
As a preliminary analysis we may say that all indexes show evidence of changing volatility. However, a more in-depth analysis is required to draw consistent conclusions to this regard, as performed in the next subsections. Table 1 presents some descriptive statistics.

Table 1 Summary statistics of the daily returns

| Statistics    | CAC 40 | MIB 30 | NIKKEI 225 | PSI 20 | IBEX 35 | FTSE 100 | SP 500 |
|---------------|--------|--------|------------|--------|---------|----------|--------|
| Mean          | 0.000287 | 0.000215 | 7.67E−05   | 0.000199 | 0.000352 | 0.000246 | 0.000341 |
| Median        | 0.000220 | 0.000000 | 0.000000   | 0.000000 | 0.000308 | 0.000235 | 0.000257 |
| Maximum       | 0.061670 | 0.069017 | 0.093935   | 0.062732 | 0.063311 | 0.054147 | 0.053666 |
| Minimum       | −0.073584| −0.077873| −0.072108  | −0.080149| −0.082164| −0.053676| −0.070264|
| Skewness      | −0.177881| −0.182288| 0.058954   | −0.405571| −0.299988| −0.158930| −0.126998|
| Kurtosis      | 6.413817 | 6.078641 | 6.857375   | 11.48285 | 6.853787 | 6.536921 | 7.295682 |
| Jarque-Bera   | 2081.259 | 1967.934 | 2631.14    | 12828.95 | 2687.391 | 2227.915 | 3271.407 |
| Probability   | 0.000000 | 0.000000 | 0.000000   | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
From a statistical point of view there is evidence of weak negative asymmetry in all the returns considered, excluding the NIKKEI 225, which presents a weak positive asymmetry (together with a negative mean). In addition, all indexes exhibit excess kurtosis. As a consequence, unconditional normality is significantly rejected as the Jarque-Bera test $p$-value is less than 0.01 in all cases. In this light, there is strong evidence of fat-tails for all series, as expected.

### 2.2 Standard Deviation Results

We now proceed to the analysis of the standard deviation results as depicted in Fig. 2.

![Fig. 2 Relative standard deviation of the stock indexes returns](image_url)

The standard deviation is a measure of dispersion of a probability distribution that depends on the value of the underlying mean. Therefore, a more convenient representation of the volatility is based on the coefficient of variation which is a normalized measure of dispersion, and, thus, it is a dimensionless number. This measure is particularly useful for variables that are always positive and have a positive mean so that an appropriate alternative is the Relative Standard Deviation (RSD). The Relative Standard Deviation is just the absolute value of the ratio of the standard deviation to the mean multiplied
by 100. It provides a good picture of the overall linear dispersion underlying the data. Our results show that the NIKKEI 225 presents, by far, the highest linear volatility value among the seven indexes under consideration. This is obviously not surprising since the Japanese stock market was subjected to a severe instability over the period analyzed, showing a non-increasing long-run trend in the raw price series and quite sharp oscillations over time. This was thus transmitted to the returns series and translates into abnormally large oscillations or high volatility, as observed. Next, but by far lower than in the previous case, the MIB 30 shows the second highest value of linear volatility, followed by Portuguese PSI 20 and the French CAC 40. The Spanish IBEX 35 and the north-American SP 500 exhibit the lowest values of linear volatility as measured by the Relative Standard Deviation.

In order to have an idea of the relative discrepancy of the linear volatility coefficient across the seven markets under analysis, and taking the north-American SP 500 as our basis, we can observe that the Spanish IBEX 35 coefficient is 12% higher, whereas the British FTSE 100, the French CAC 40 and the Portuguese PSI 20 are, respectively, 28%, 37% and 49% higher than the SP 500. The MIB 30 and the NIKKEI 225 multiply by two and five, respectively, the SP 500 coefficient. Stock market volatility, therefore, appears to have a quite different pattern of behaviour around the world when measured in a linear way. Is this just a systematically linear behaviour or volatility also shows signs of nonlinear dynamics across markets? This is what we shall analyze in the next section.

### 2.3 Entropy Results

In the domain of the econophysics approach we have computed the Tsallis and Shannon entropies, which are depicted in Table 2.

| Statistics | Index (q) | CAC 40 | MIB 30 | NIKKEI 225 | PSI 20 | IBEX 35 | FTSE 100 | SP 500 |
|------------|-----------|--------|--------|------------|--------|---------|----------|--------|
| Shannon    | 1         | 3.0655 | 3.073  | 2.9163     | 2.6515 | 2.8951  | 3.0644   | 2.989  |
|            | 1.4       | 1.7229 | 1.7255 | 1.6718     | 1.5708 | 1.6977  | 1.7229   | 1.6948 |
| Tsallis    | 1.45      | 1.6216 | 1.6238 | 1.5766     | 1.4869 | 1.5997  | 1.6217   | 1.5967 |
|            | 1.5       | 1.5295 | 1.5313 | 1.4898     | 1.41  | 1.5104  | 1.5297   | 1.5074 |

All entropies were estimated with histograms based on equidistant cells. For the calculation of Tsallis entropy we have set values at 1.4, 1.45 and 1.5 for the
index $q$, which is consistent with the finding that when considering financial data their values lie within the range $q \simeq 1.4 - 1.5$ (see [25]). Since all entropies are positive we shall conclude that the data show nonlinearities. This phenomenon is more obvious for the MIB 30, CAC 40 and FTSE 100, and a little less so for the SP 500, NIKKEI 225 and PSI 20. When we look at the relative discrepancies of the data taking as our basis the SP 500, the overall difference across the seven markets is not very marked. For the Shannon entropy there is, in most cases, a relative change of $2.5\% - 3\%$, positive or negative. The exception is the PSI 20 that exhibits a negative change of $11\%$ relative to the SP 500. On the other hand, for the Tsallis entropy (using, for example, the results for $q = 1.45$), the relative change is even smaller: around $7\%$ less for the PSI 20 and $1\%$ for all other indexes.

Since the entropy is designed to capture the overall linear and nonlinear dispersion (or volatility) observed in the data, our results point to the conclusion that volatility appears to show a relatively homogeneous pattern across international stock markets. Curiously, the “less” volatile market, the PSI 20, is also the smallest and the most dependent of the seven markets analyzed. Globally, the Portuguese stock market appears to be $7\% - 11\%$ less volatile than the north-American one. However, in terms of linear dispersion, the former is $49\%$ more volatile than the latter. That is, the proportion of linear volatility on the overall volatility is higher in the Portuguese market than in the north-American one. This appears to reveal that the volatility in the Portuguese stock market is more linearly predictable than the volatility in the north-American market.

A similar situation occurs in the case of the Japanese stock market relatively to the north-American one, taken as a benchmark. Here, however, the proportion of the overall volatility explained by linear dependencies appears to be higher than in the previous case. For all other markets, the overall and the linear volatility figures are higher than the north-American benchmark so that it is not possible to make any final conclusion about the relative weight of the linear and nonlinear dispersion relatively to the US. Apparently, however, they are all more linearly predictable than the north-American stock market.

3 Conclusions

In this paper we have investigated the volatility of seven indexes: CAC 40, MIB 30, NIKKEI 225, PSI 20, IBEX 35, FTSE 100 and SP 500. Our major goal was to compare two different perspectives: one based on the standard deviation and another supported by the concept of entropy. For our purpose two variants of this notion were regarded: the Tsallis and Shannon statistics.
In particular, the results from both entropies have shown nonlinear dynamics in the volatility of all indexes and must be understood in complementarity. The results, however, must be compared with the relative standard deviation ones in order to have a full picture of the overall phenomenon. This is especially relevant for the decision making process in which all the information is regarded as necessary and useful. Nonetheless, in spite of all the divergences encountered, there is an apparent common behaviour in most European Markets.

In this study we especially address the concept of entropy as an alternative to the standard deviation since it can capture the uncertainty and disorder in a time series without imposing any constraints on the theoretical probability distribution, which constitutes its major advantage.

References

[1] R.J. Shiller, American Economic Review 71, 421 (1981).
[2] S.J. Grossman, R.J. Shiller, American Economic Review 71, 222 (1981).
[3] M.A. Flavin, Journal of Political Economy 91, 929 (1983).
[4] A.W. Kleidon, Journal of Political Economy 94, 953 (1986).
[5] N.G. Mankiw, D. Romer, M.D. Shapiro, Journal of Finance 40, 677 (1985).
[6] N.G. Mankiw, D. Romer, M.D. Shapiro, Review of Economic Studies 58, 455 (1991).
[7] K.D. West, Econometrica 56, 37 (1988).
[8] M. Zhong, A.F. Darrat, D.C. Anderson, Journal of Banking and Finance 27, 673 (2003).
[9] J. Coakley, A.-M. Fuertes, Journal of Banking and Finance 30, 2325 (2006).
[10] G.W. Schwert, Journal of Finance 44, 1115 (1989).
[11] K. Daly, Physica A 387, 2377 (2008).
[12] H.E. Stanley et al., Physica A 224, 302 (1996).
[13] R.N. Mantegna, H.E. Stanley, An Introduction to Econophysics: Correlations and Complexity in Finance (Cambridge University Press, Cambridge, 2004).
[14] S.D.H. Hsu, B.M. Murray, Physica A 380, 366 (2007).
[15] F Knight, Risk, Uncertainty and Profit, (The Riverside Press, Cambridge, 1921).
[16] S. Hwang, S.E. Satchell, Journal of Banking and Finance 24, 759 (2000).
[17] S.A. Ross, Journal of Finance 54, 1 (1989).
[18] A. Golan, Journal of Econometrics 107, 1 (2002).
[19] C.E. Shannon, The Bell System Technical Journal 27, 379 (1948).
[20] R.V.L. Hartley, Bell System Technical Journal 7, 535 (1928).
[21] C.E. Shannon, W. Weaver, The Mathematical Theory of Communication (The University of Illinois Press, Urbana, 1964).
[22] C. Tsallis, Journal of Statistical Physics 52, 479 (1988).
[23] J. Havdra, F. Chárvat, Kybernetica 3, 50 (1967).
[24] D.Z. Daróczy, Information and Control 16, 36 (1970).
[25] C. Tsallis, C. Anteneodo, L. Borland, R. Osorio, Physica A 324, 89 (2003).
[26] W. Tatsuaki, S. Takeshi, Physica A 301, 284 (2001).
[27] C. Tsallis, E. Brigatti, Continuum Mech. Thermodyn.16, 223 (2004).
[28] C. Tsallis, F. Baldovin, R. Cerbino, P. Pierobon, arXiv:cond-mat/0309093v1.
[29] E.M.F. Curado, C. Tsallis, J. Phys. A: Math. Gen. 24, L69 (1991).
[30] C. Tsallis, Milan Journal of Mathematics 73, 145 (2005).
[31] C. Tsallis, M. Gell-Mann, Y. Sato, Europhysics News 36, 186 (2005).
[32] B. Lesche, J. Stat. Phys. 27, 419 (1982).