Scaling properties of localization length in 1D paired correlated binary alloys of finite size

Felix M. Izrailev\textsuperscript{1,2,*}, Tsampikos Kottos\textsuperscript{1} and G. P. Tsironis\textsuperscript{1}

\textsuperscript{1} Department of Physics, University of Crete
and Research Center of Crete, P.O. Box 1527
71110 Heraklion-Crete, Greece

\textsuperscript{2} Budker Institute of Nuclear Physics, Novosibirsk 630090, Russia

October 9, 2018

Abstract

We study scaling properties of the localized eigenstates of the random dimer model in which pairs of local site energies are assigned at random in a one dimensional disordered tight-binding model. We use both the transfer matrix method and the direct diagonalization of the Hamiltonian in order to find how the localization length of a finite sample scales to the localization length of the infinite system. We derive the scaling law for the localization length and show it to be related to scaling behavior typical of uncorrelated Band Random Matrix, Anderson and Lloyd models.

\*email addresses: izrailev@vxinpb.inp.nsk.su; izrailev@physics.spa.umn.edu
1 Introduction

It is well known that in one-dimensional (1D) disordered models even small amount of disorder leads to an exponential localization of all eigenstates \([1, 2]\). On the other hand, recent studies of quasi-1D polymers have shown that short-range correlations embedded in a random sequence can lead to appearance of fully transparent states \([3, 4]\). In ref. \([4]\) in particular, various organic disordered systems were quoted with electrical properties. The prototypical case is that of the Random Dimer Model (RDM) \([3, 4]\) where (in the context of a tight-binding Hamiltonian) pairs of adjacent energy levels are assigned at random, leading to two-site correlations in an otherwise random model.

Since for infinite samples fully delocalized states appear only for specific energy values, there is no Anderson transition in the usual sense (see also \([5]\)). However, the number of transparent states for finite samples was found to be proportional to square root of the length of the sample \([3, 4]\). This fact is related to the divergence of the localization length in infinite samples when the energy approaches some critical values \([3, 4, 5]\). Therefore, these states may be important for conducting properties of finite samples \([3, 10]\).

In infinite samples the Anderson transition can be characterized in terms of the localization length; the latter is commonly defined from the decay of amplitude of eigenstates in the limit \(|n| \to \infty\), where \(n\) is the site label in the tight-binding picture. Contrary to what happens in infinite samples, the global properties of eigenstates of finite samples cannot be characterized in the same way; one needs to use other quantities (such as the inverse participation ratio), that are valid both for finite and infinite samples. Then, through the use of scaling conjectures, one can link the properties of eigenstates in infinite samples to those in finite samples. In the theory of disordered solids, the scaling approach proved to be extremely useful in describing the conductance and its fluctuations (see e.g. \([11, 12]\)). A similar approach has been recently
used in the theory of quantum chaos when describing localized eigenstates random on a finite scale \[13, 14\]. Such eigenstates also arise in the quasi-1D models with random potential. Extensive numerical and analytical studies (see e.g. \[13, 16\] and references therein) have revealed remarkable scaling properties of eigenstates, which seem to be of quite generic nature.

In this paper we study the RDM of finite size from the point of view of scaling properties of its eigenstates. The question of the relevance of the above mentioned results to models with correlated disorder is far from being trivial since short range correlations may cause significant difference in the structure of eigenstates, when compared with those for random potentials. In the next Section 2 we briefly describe the RDM and discuss different definitions of localization length, which are used in our numerical simulations. In Section 3 we present numerical data on scaling properties of eigenstates in the center of energy bands. In this case, the localization length in infinite samples has been obtained by the transfer matrix method. In Section 4, we study the energy region near the critical values \( E_{cr} \) by making use both numerical and analytical treatment of the localization length. Finally, in Section 5 we give a short summary of our investigation.

2 Finite size scaling approach to random dimer model

Our starting point is the 1D Schrödinger equation in the tight-binding approximation,

\[
i \frac{d c_n(t)}{d t} = \epsilon_n c_n(t) + c_{n+1}(t) + c_{n-1}(t),
\]

(1)

where \( c_n(t) \) is the probability amplitude for an electron to be at site \( n \) and \( \epsilon_n \) is the local site energy. By making the transformation \( c_n(t) = \exp(-i E t) x_n \) one can obtain the equation

\[
E \varphi_n = \varphi_{n+1} + \epsilon_n \varphi_n + \varphi_{n-1},
\]

(2)

for the eigenvalue \( E \) and the corresponding eigenstate \( \varphi_n(E) \). In what follows, we consider the RDM \[3, 8, 17\] which implies short range correlations in the sequence of
random $\epsilon_n$. In this model there are only two values of $\epsilon_n$ viz. $\epsilon_A$ or $\epsilon_B$ that appear in pairs in the sequence $\epsilon_n$'s. In other words, in order to create the dimer chain the pairs $AA$ and $BB$ each with energies $\epsilon_A$, $\epsilon_B$ respectively are distributed at random. We take for simplicity the probabilities, of occurrence of the paired to be equal, i.e. $P_{AA} = P_{BB} = 1/2$.

The RDM has been well studied for an infinite chain (see, e.g. [3, 5, 6, 7, 8, 17]). Its basic feature is that for two values of the energy $E_{cr} = \epsilon_A$ or $\epsilon_B$ its eigenstates are extended. In the vicinity of these values and for $\epsilon_{A,B} = |\epsilon_A - \epsilon_B|$ which is less than the critical value $\epsilon_{A,B}^{cr} = 2$, the localization length $l_\infty$ defined through the exponential decay of the amplitude of eigenfunction diverges as $l_\infty(E) \sim 1/E^2$ (for the specific value $\epsilon_{A,B} = 2$, the singularity law $l_\infty(E) \sim 1/E$ holds instead, see details in [3, 4, 8]). In spite of the fact that for other values of $E$ inside the spectrum the localization length is finite, the influence of nearly-transparent states on the electronic properties of finite samples is strong (see e.g. [9, 10]). This is related to the fact that the number of eigenstates with localization length larger than the size $N$ of the sample is proportional to $\sqrt{N}$.

Our main interest is in the structure of eigenstates for finite samples, both in the center of energy band and in the vicinity of the critical energies $E_{cr}$ where localization length in infinite sample diverges. Unlike the more simple case of infinite samples, the meaning of localization length for finite samples is not clear. Below, we follow the approach developed in the theory of quasi-1D disordered solids which is based on the evaluation of multifractal localization lengths (see, e.g. [15]). One of the commonly used quantities in this approach is the so-called entropic localization length, defined through to the information entropy $\mathcal{H}$ of eigenstates,

$$\mathcal{H} = - \sum_{n=1}^{N} w_n \ln w_n; \quad w_n = |\varphi_n|^2 \quad (3)$$

where $\varphi_n$ is the $n-$th component of an eigenstate in a given finite basis. For eigenstates normalized as $\sum_n w_n = 1$, the simplest case of $\varphi_n = N^{-1/2}$ results to the entropy equal to the maximum value, viz. $\mathcal{H} = \ln(N)$. We define therefore the localization
length $l_N$ as the number of basis states occupied by the eigenstate $\varphi_n$; the latter is equal to $\exp(\mathcal{H}_N)$ . Similar definitions have been used for the first time in ref. \cite{13} where different characteristics of eigenstates have been discussed in solid state applications. One can see that in the other limit case of an exponentially localized state with $\varphi_n = l_{\infty}^{-1/2} \exp(-|n - n_0|/l_{\infty})$ , the quantity $l_N$ is proportional to $l_{\infty}$ , viz. $l_N \approx e l_{\infty}$ (assuming $l_{\infty} \ll N$ ). One should note that, generically, the amplitudes $\varphi_n$ fluctuates strongly with $n$ and thus the coefficient of proportionality between $l_N$ and $l_{\infty}$ depends on the type of fluctuations.

To study the properties of chaotic states, localized on some scale in the finite basis in \cite{13,14} it was suggested to normalize the localization length $l_N$ in such a way that in the extreme case of fully extended states the quantity $l_N$ is equal to the size of the basis $N$ . In such an approach, the entropic localization length $l_N^{(1)}$ is defined as

$$l_N^{(1)} = N \exp(<\mathcal{H}_N > - \mathcal{H}_{ref})$$  

(4)

In Eq. (4) the ensemble average $<...>$ is introduced over the number of eigenstates of the same structure. The normalization factor $\mathcal{H}_{ref}$ has the meaning of an average entropy of the completely extended random eigenstates in the finite basis, therefore, it can be easily found analytically \cite{13},

$$\mathcal{H}_{ref} = \psi\left(\frac{N}{2} + 1\right) - \psi\left(\frac{3}{2}\right) \approx \ln\left(\frac{N}{2.07}\right)$$  

(5)

where $\psi$ is the digamma function and the distribution of components $\varphi_n$ is assumed to correspond to the Gaussian Orthogonal Ensemble (GOE). From (5) one can see that for $N \gg 1$ the entropic localization length of random eigenstates, defined simply as $\exp(\mathcal{H}_{ref})$, is approximately 2.07 times less than $N$ ; this result is due to gaussian fluctuations in the components $\varphi_n$ .

Analogously, the whole set of localization lengths $l_N^{(q)}$ can be defined in the following way \cite{13,19}:

$$l_N^{(q)} = N \left(\frac{<P_{q}>_{\text{ref}}}{P_{q}}\right)^{1/q}; \quad q \geq 2$$  

(6)
where
\[ P_q = \sum_{n=1}^{N} (w_n)^q \]
and \( P_{ref}^{(q)} \) is the average value of \( P_q \) for the reference ensemble of completely extended states. One should note that for the particular case \( q = 2 \) the quantity \( P_2 \) is known as the participation ratio; it is widely used in solid state physics. In the limit case of the GOE, one can find that 
\[ P_{ref}^{(2)} = 3/N, \]
therefore, the inverse participation ratio 
\[ (P_2)^{-1}, \]
which is commonly taken as the definition of localization length, for random eigenstates is 3 times as less as the “actual” length \( N \).

In fact, the above expressions for the localization lengths \( l_N^{(q)} \) is defined through the \( 2q \)-th moments of a distribution of components \( \varphi_n \) of eigenstates; non-normalized to \( P_{ref}^{(q)} \) quantities are well known in the multifractal analysis of wave functions. Such normalization turns out to be extremely important when establishing scaling properties of eigenfunctions. Indeed, by normalizing the localization lengths \( l_N^{(q)} \) to the size \( N \) of the sample,
\[ \beta_q = \frac{l_N^{(q)}}{N} \]
one can expect, in the spirit of renormalization theory, that the set of dimensionless parameters \( \beta_q \) is the proper quantity to characterize generic properties of eigenstates for finite samples. According to the scaling conjecture in the modern theory of disordered solids, it was assumed \[20\] that for quasi-1D disordered models described by Band Random Matrices the quantity \( \beta_q \) depends on the scaling parameter \( \lambda \) only, which is the ratio of the localization length \( l_\infty \) for the infinite sample, to the size \( N \) of the sample itself. Therefore, the scaling relation can be written as
\[ \beta_q = f_q(\lambda); \quad \lambda = \frac{l_\infty}{N} \]
Detailed studies, both numerical and analytical, have confirmed this conjecture for different models like the Kicked Rotator Model and Band Random Matrices (see, e.g. \[14, 15, 16\] and references therein). Moreover, the scaling function \( f_q(\lambda) \) has also been found.
Our main question is whether the relation of the type (9) is also valid for our dimer model with the correlated disorder. The first nontrivial question arises about the reference ensemble for the computation of the average entropy $H_{\text{ref}}$. Indeed, in application to 1D Anderson type models (see details in [21]) the reference ensemble cannot be chosen as an ensemble of full random matrices, like the GOE. This point is related to the fact that in the Anderson case fully extended states are not gaussian random functions but just plane waves which arise for zero disorder. In the dimer model, the situation is even more complicated due to strong dependence of the localization length on the energy. However, and this is our expectation, in spite of the presence of the extended states at the critical energies, scaling properties of the eigenstates in the dimer model of finite size $N$ are of generic type discovered for 1D and quasi-1D disordered models.

For this reason and in the spirit of Ref. [21, 22], we define the normalization factors $H_{\text{ref}}$ and $P^{(q)}_{\text{ref}}$ from the solution of Eq. (2) for the zero disorder, $\epsilon_n = 0$,

\begin{align*}
E^k &= 2\cos\frac{k\pi}{N+1}, \\
\varphi^k_n &= \sqrt{\frac{2}{N+1}}\sin\frac{nk\pi}{N+1},
\end{align*}

with $k, n = 1, \ldots, N$. The entropy $H_{\text{ref}}$ of the above eigenfunctions in the large $N$ limit has the same value for every eigenvalue $E^k$, i.e.,

$$H_{\text{ref}} = \ln(2N) - 1,$$

and correspondingly,

$$P^{(2)}_{\text{ref}} = \frac{3}{2N}.$$ 

3 Scaling properties of localization lengths in the center of energy bands

Since all results depend on the difference $\epsilon_{A,B} = |\epsilon_A - \epsilon_B|$ but not on the actual values $\epsilon_A$ and $\epsilon_B$ separately, we can set $\epsilon_A = 0$ for simplicity. One should stress that both
localization lengths $l_\infty$ and $l^{(q)}_N$ are functions of the energy $E$. For this reason, in our numerical experiments we consider ensembles of states specified by the values of the energy $E$ in a small window $\Delta E$ and by different realizations of random on-site energies $\epsilon_n$. We choose the size of the energy window in such a way that for every chosen value of $\epsilon_B$ the localization length $l_\infty$ is approximately constant inside this window (in all the cases $\frac{\Delta E}{l_\infty} \leq 0.06$).

In order to study scaling properties of the localized eigenstates we have used the transfer matrix method for infinite chains as well as the direct diagonalization of the Hamiltonians that one associated with Eq. (1), for finite chains of size $N$. To find the localization length $l_\infty$ we have studied the asymptotic behavior of the random matrix product $\prod M_n$, where $M_n$ is defined through the relation

$$\xi_{n+1} = M_n \xi_n; \quad M_n = \begin{pmatrix} v_n & -1 \\ 1 & 0 \end{pmatrix}; \quad v_n = E - \epsilon_n$$

(14)

for the vector $\xi_n = (x_n, x_{n-1})$ with the matrix $M_n$ known as the transfer matrix. Then the localization length $l_\infty$ is the inverse Lyapunov exponent $\gamma$; the latter is evaluated as the exponential decay rate of an initial vector $\xi_1$,

$$l^{-1}_\infty = \gamma = \lim_{N \to \infty} \frac{1}{N} \ln \prod_{n=1}^{N} \frac{|M_n \xi_n|}{|\xi_1|}.$$ 

(15)

Although the Lyapunov exponent $\gamma$ for finite $N$ depends on a particular realization of the disorder, for $N \to \infty$ it converges to its mean value. For the above calculations we have used samples of length $5 \times 10^5$ for relatively large values of $\epsilon_B$ and up to $4 \times 10^6$ for small values of $\epsilon_B$.

To reveal scaling properties of localization length for finite samples, we have computed two localization lengths $l^{(1)}_N$ and $l^{(2)}_N$ according to the relations discussed in the previous Section, with the normalization factors $H_{\text{ref}}$ and $P_{\text{ref}}^{(2)}$ in the forms (12) and (13). In the computations of these lengths, the energy window was taken in the center of the spectrum, around the value $E = \epsilon_B/2$ for $\epsilon_B$ equals 2, 1.8, 1.6, 1.2, 1, 0.8, 0.6, 0.4, 0.35 and for the fixed value of $N$. The width of the windows has
been numerically chosen to provide a small change of localization length inside any of windows. The values of $\beta_1$ and $\beta_2$ are obtained by the averaging over an ensemble of random samples of size $N = 100 \div 800$ for the values of $\epsilon_B$ cited previously. As a result, the total numbers of eigenstates in the energy windows were more than 1000.

All the data have been fitted to the scaling function $\beta_q$ found for quasi-1D disordered models [15]:

$$\beta_q = \frac{c_q \lambda}{1 + c_q \lambda}$$

(16)

In fact, this scaling relation is exact only for $q = 2$ , however, for other cases of small values $q$ , including $q = 1$ , it is very close to the correct one (see details in [15]).

Numerical data reported in Fig.1 give clear evidence of a scaling of the type (16). The fitting parameters $c_q$ are equal to $c_1 = 2.80$ and $c_2 = 1.55$ . From this figure one can see that the behaviour of $\beta_q$ is very different in the two limits of very localized ($\beta_q \ll 1$) and extended ($\beta_q \approx 1$) eigenstates. The dependence (16) has the remarkable property which can be seen in other variables,

$$Y_q = \ln\left(\frac{\beta_q}{1 - \beta_q}\right); \quad X = \ln\left(\frac{l_\infty}{N}\right)$$

(17)

which are more convenient when considering the whole region of both very localized and extended states. Indeed, in these variables the scaling has extremely simple form,

$$Y_q = a_q + b_q X$$

(18)

with $b_q = 1$ and $a_q = \ln(c_q)$ . The data for the scaling in variables $Y, X$ are presented in Fig.2. The fitting parameters $b_{1,2}$ are found to be quite close to 1 i.e. $b_1 = 1.02$ and $b_2 = 0.98$, for this reason in the Fig.2 we put $b_1 = b_2 = 1$. The remarkable result is that the above simple scaling relation holds in a very large region of the scaling parameter $\lambda = l_\infty/N$ . According to the fit to the dependence (18), the values $a_{1,2}$ are: $a_1 = 1.05$ and $a_2 = 0.45$, which gives $\Delta a_{1,2} = a_1 - a_2 = 0.6$. It is very interesting that these values of $a_{1,2}$ are the same as for common Anderson model [22] in the center of the
energy band. This fact is very important in establishing the link between the RDM and Anderson models of finite size.

It is of special interest to relate the entropy localization length $l_N^{(1)}$ and the localization length $l_N^{(2)}$ associated with the inverse participation ratio. Their interdependence is shown in Fig.3. We see that they are approximately equal for very localized and very extended states. It is also clear that $\beta_2$ is always less than $\beta_1$ since $P_N^{(q)} < P_N^{(q+1)}$, due to the definition of Eq. (7). Using the definition of Eq. (16) one can find the relation between $\beta_1$ and $\beta_2$:

$$\beta_2 = \frac{c\beta_1}{1 + (c-1)\beta_1}; \quad c = \frac{c_2}{c_1}$$

(19)

4 Scaling of localization lengths near the critical energy.

In the previous section we have shown that the scaling law (16), found for fully disordered 1D and quasi-1D models, also holds in our dimer model of finite size when considering eigenstates in the center of energy bands. In a sense, this property may be expected since far from critical energies where the localization length diverges, the eigenstates are assumed to be similar to that known for disordered models. The important question is whether this scaling holds for all energies inside the band, in particular, near the critical energies $E_{cr} = \epsilon_A, \epsilon_B$. Direct numerical computation of the localization length $l_\infty$ through the transfer matrix method is very difficult in this energy region due to very weak convergence of Lyapunov exponents. For this reason, we have used the analytical expression which was derived for $l_\infty$ near the critical energies in an approach developed in [8]:

$$l_\infty(E) \approx \frac{2\sin^2 \mu_0}{Q\delta^2 \cos^2 \mu_0}; \quad 2\cos \mu_0 = E$$

(20)

Here, the factor $Q$ stands for the probability for the pair $\epsilon_n = \epsilon_{n+1} = \epsilon_B$ to appear, and $\delta$ is defined by the relation $E = \epsilon_B - \delta \approx \epsilon_B$ [23]. We remind that in our case $Q = 1/2$ and $\epsilon_A = 0$ has been assumed for the simplicity. From the above expression
one can find that if the value of $\epsilon_B$ is far from the stability border $E_B = 2$, and the distance $\Delta = 2 - \epsilon_B$ is large compared to $\delta = \epsilon_B - E$, the localization length diverges as

$$l_\infty \approx \frac{2\Delta}{Q\delta^2}, \quad \delta \ll \Delta \ll 1$$

(21)

In the other limit case of $\epsilon_b = 2$ we have

$$l_\infty \approx \frac{2}{Q\delta}, \quad \delta \ll 1, \quad \Delta = 0$$

(22)

It is interesting to note that the same expressions (21) and (22) are obtained in Ref. [17] by assuming that localization length $l_\infty$ is determined by the reflection coefficient from a single pair $\epsilon_n = \epsilon_{n+1}$, embedded in a perfect lead. It is of interest to check how accurate are estimates found in [8] and [17] if to apply them for any energy inside the band.

To find the localization lengths $l_N^{(1)}$ and $l_N^{(2)}$ for finite samples of the size $N$, we have used the same approach described in the previous section, by examining the eigenstates with energies in a small energy window $\Delta E \leq 10^{-2}E_{cr}$ near the critical energy $E_{cr} = \epsilon_B$. Yet, since in the region of critical energies $l_\infty(E)$ and thus the localization properties of eigenstates, depend from the energy in a singular way, (see Eq. (21) and Eq. (22)) we took from the energy window only the eigenvector with the corresponding eigenstate which is closer to $E_{cr}$ (but always different from it, $E \neq E_{cr}$).

This was a natural choice in order to study statistical properties of eigenstates with the similar localization properties (i.e. the eigenstates just near the totally extended one).

The average values of $l_N^{(q)}$ have been obtained by statistical averaging over an ensemble of more than 3000 samples of the size $N = 100 \div 800$ with different pair-correlated disorder. The results are reported in Fig. 4 together with the fit to the expression (16).

One can see a quite good scaling of the form (16), in spite of fluctuations which are much larger in this energy region compared to that in the center of bands. The fitting coefficients $c_1 = 2.20$ and $c_2 = 1.06$ are slightly less than those in the band center.

This fact may be explained by an approximate character of the analytical expression
(20) (one should also note that for the values of $\beta_q$ very close to the limit $\beta_q = 1$ the computational errors are very large).

In Fig. 5 the same data are represented in the variables (17), with the fit correspondent to the dependence (18). It is interesting that in spite of a slight difference for the coefficients $a_1 = 0.75$ and $a_2 = 0.08$ in comparison for those found in the center of bands, the shift $\Delta a_{1,2} = a_1 - a_2 = 0.67$ remains almost the same (compared to 0.6).

5 Summary

We have studied a 1D tight-binding model with binary on-site disorder that is randomly assigned in every two sites. For such a model we know that there exist two special energies $E_{cr}$ at which transparent states appear [3, 6, 7, 8]. For other energies, but close to critical ones, the localization length is very large, leading to nearly-transparent states that are of great importance in the conducting properties of finite samples. This property is quite different from genuine disordered models of Anderson type.

Our numerical study of random dimer models of finite size deals with the scaling properties of the eigenstates. This study was motivated by the remarkable scaling law that has been found for different 1D and quasi-1D models, both dynamical (Kicked Rotator on a torus [13, 14]) and disordered (1D Anderson and Lloyd models [21, 22] and quasi-1D models [15, 20]). These latter results indicate that eigenstates in finite samples with disorder have generic properties, regardless of the details of the disorder.

The main result of our computations is that scaling properties of eigenstates of finite dimers are of the same type as for the disordered models mentioned above in spite of the existence of nearly-transparent states. In particular, both entropy localization length and localization length from inverse participation ratio normalized in the proper way, follow the universal scaling law of Eq. (16).

The scaling relation of Eq. (16) can be also represented in a very intriguing form
\[ \frac{1}{l_N^{(q)}(\epsilon, E)} = \frac{1}{l_\infty^{(q)}(\epsilon, E)} + \frac{1}{l_N^{(q)}(0, E)} \] (23)

which still has no physical ground. In Eq. (23), \( l_N^{(q)}(\epsilon, E) \) represents the localization length for a finite sample and finite disorder, \( l_\infty^{(q)}(\epsilon, E) \) is the localization length for an infinite sample with the same disorder and \( l_N^{(q)}(0, E) \) is the localization length for a finite sample with zero disorder. One should stress that all three localization lengths are defined in the same way given through expressions of Eq. (6) and (7). One can see that the form Eq. (23) is parameter independent; the same form holds also for the 1D Anderson and Lloyd models (see [21, 22]).

In our numerical study the energy window has been chosen in the middle of the spectrum as well as close to the critical energy, giving the same scaling form (16). The slight difference in the coefficients \( c_q \) for these two energy regions seems to indicate that the analytical expression (24) needs some correction related to an additional dependence on the energy when the latter is not close enough to the critical one. Our results indicate that the same scaling is expected to hold for other values of the energy inside the band. One should note that the scaling (16) (or, the same, (23)) can be used to check the accuracy of expressions for the localization length \( l_\infty \) in dependence of the parameters \( E \) and \( \epsilon_b \), if for some values of these parameters the scaling function \( \beta_q \) is found with a high accuracy.

It is of quite interest to check the scaling behaviour of localization lengths corresponding to the higher moments \( q \geq 3 \) in (3). Analytical treatment [13] for disordered models have shown that the scaling law (19) approximately holds also for higher moments. The correct expression for \( \beta_q(\lambda) \) is known only in the limit case of very localized (\( \lambda \ll 1 \)) and extended (\( \lambda \gg 1 \)) states. It has the same form (18) with \( b_q = 1 \) but with different values of \( a_q \) in these limits (see details in [13]). On the base of our results for \( q = 1, 2 \), it is quite natural to expect that for the dimer model the correspondence to the analytical predictions [13] also hold for higher moments, however, this question remains open.
Finally, we should comment that the results obtained in the present work can be generalized to cases with correlation blocks larger than dimers, viz. $m$-blocks with $m = 3, 4, 5, \ldots$. In these more general cases the following surprising result holds: Given an arbitrary distribution of correlated blocks with even length, i.e. an arbitrary distribution of dimers, quatromers, sextomers, octamers, etc, with the same energy $\epsilon$, that populate a lattice with sites that have some other energy value, there is always a resonant energy $E_{cr} = \epsilon$ that corresponds to a delocalized state. This result can be easily deduced from the general expressions of ref. [8]. On physical grounds, we expect the localization properties of the eigenstates of this system to follow similar scaling laws to the ones derived in the present work.

6 Acknowledgements

One of the authors (F. M. I.) wishes to acknowledge the support of Grant ERBCHRXCT 930331 Human Capital and Mobility Network of the European Community and also the support of Grant No RB7000 from the International Science Foundation. We thank Paolo Grigolini Bruce J. West and C. M. Soukoulis for discussions.
References

[1] P. W. Anderson, Phys. Rev. 109, (1958) 1492.

[2] E. N. Economou, Green's Functions in Quantum Physics, Springer Series in Solid State Physics, Vol. 7 (Springer-Verlag, Berlin, 1979).

[3] D. Dunlap, H.-L. Wu and P. Phillips, Phys. Rev. Let. 65, (1990) 88.

[4] P. Phillips and H.-L. Wu, Science 252, (1991) 1805.

[5] S. Gangopadhyay and A. K. Sen, J. Phys. Cond. Matt. 4, (1992) 9939.

[6] J. C. Flores, J. Phys. Cond. Matt. 1, (1989) 8471.

[7] A. Bovier, J. Phys. A 25, (1992) 1021.

[8] F. M. Izrailev, Tsampikos Kottos, G. P. Tsironis, Phys. Rev. B 52 (1995) 3274.

[9] P.K. Datta, D. Giri and K. Kundu, Phys.Rev. B 47 (1993) 10727.

[10] P.K. Datta, D. Giri and K. Kundu, Phys.Rev. B 48 (1993) 16347.

[11] E. Abrahams, P. W. Anderson, D. C. Licciardello and T. V. Ramakrishnan, Phys. Rev. Lett. 42 (1979) 673.

[12] J. L. Pichard, J. Phys. C : Solid State 19 (1986) 1519.

[13] G. Casati, I. Guarneri, F. M. Izrailev and R. Scharf, Phys.Rev.Lett. 64 (1990) 5.

[14] F. M. Izrailev, Phys.Rep. 196 (1990) 299.

[15] Y.V.Fyodorov and A.D.Mirlin, Int.J.Mod.Phys. 8 (1994) 3795.

[16] F.M.Izrailev, Chaos, Solitons & Fractals 5 (1995) 1219; Y.V.Fyodorov and A.D.Mirlin, Int.J.Mod.Phys. 8 (1994) 3795.

[17] S. N. Evangelou and A. Z. Wang, Phys. Rev. B 47 (1993) 13126.
[18] C. Papatriantafillou and E. N. Economou 13 (1976) 920.

[19] S. N. Evangelou and E. N. Economou, Phys. Lett. 151A (1991) 345.

[20] G. Casati, L. Molinari and F. M. Izrailev, Phys. Rev. Lett. 64 (1990) 1851.

[21] G. Casati, I. Guarneri, F. Izrailev, S. Fishman, L. Molinari, J. Phys. Cond. Matt. 4, (1992) 149.

[22] L. Molinari J. Phys. Cond. Matt. 5, (1993) L319.

[23] We have corrected the coefficient in Eq. (20) by the factor 2 comparing to that in Ref. [8].

[24] Y. V. Fyodorov and A. D. Mirlin, Phys. Rev. Lett. 69 (1992) 1093.
Figure Captions

Fig.1 Scaling of $\beta_q$ as a function of the localization ratio $\lambda = l_\infty/N$ for the RDM with $Q = 0.5$ and $\epsilon_A = 0$. The energies $E$ are taken in an energy window centered at $E = \epsilon_B/2$ for values of $\epsilon_B = 2.0; 1.8; 1.6; 1.2; 1.0; 0.8; 0.6; 0.4; 0.35$. Smooth curves correspond to the dependence (16) with $c_q$ as a fitting parameter.

(a) Scaling for $\beta_1$ with $c_1 = 2.80$;
(b) Scaling for $\beta_2$ with $c_2 = 1.55$.

Fig.2 The scaling of $\beta_1, \beta_2$ as a function of $\lambda = l_\infty/N$ in the variables $Y_{1,2}$ and $X$ (see (17)) for the same values of the parameters as in Fig. 1. Straight lines (1) and (2) correspond to the expression (18) with $a_1 = 1.05; b_1 = 1$ and $a_2 = 0.45; b_2 = 1$ respectively.

Fig.3 A plot of $\beta_2$ as a function of $\beta_1$. It is interesting to note that the fitting curve has the same form (19) as the ones for the case of $\beta_{1,2}$ plotted in Fig.1 as a function of $\lambda = l_\infty/N$.

Fig.4 The same as in Fig. 1 for the energies close to the critical one ($E_{cr} = \epsilon_B$), for $q = 1$ (Fig.4a) and $q = 2$ (Fig.4b). The values of $\epsilon_B$ are taken as $\epsilon_B = 1.8; 1.6; 1.4; 1.2; 1.0$.

Fig.5 The same as in Fig.2 for the parameters of Fig.4. Straight lines (1) and (2) correspond to the expression (18) with $a_1 = 0.75; b_1 = 1$ and $a_2 = 0.08; b_2 = 1$ respectively.
Fig. 3
Fig. 4b
