Numerical investigation on natural convection of hybrid nanofluid Al₂O₃ –MWCNT/water inside a vertical annulus

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Abstract. A numerical inspection on natural convection inside a vertical annulus interior of which water based (Al₂O₃ +MWCNT) (7:3) hybrid nanofluid has presently been carried out using CFD solver Ansys Fluent. Finite volume method (FVM) is used to perform the numerical simulations at constant heat flux of 3kW/m² and various volume fraction (0-0.04) of nanoparticle. The thermal conductivity and viscosity of hybrid nanofluid is computed by modified Maxwell model and Batchelor model respectively. The paper carried explicit analysis on effect of nanoparticle concentration, Nu number and heat transfer coefficient. The maximum enhancement in average heat transfer coefficient found to be 73.67% when gauged on volume fraction of 0.04.

Keywords: Natural Convection, Vertical Annulus, CFD, and Hybrid Nanofluid

1. Introduction

Natural convection occurs due to the density difference. Natural convection is generally preferred in power plants to avoid accidents by eliminating danger of pumps especially during shut off periods [1]. The more inputs are given through machines like pump and blower in case of the forced convection there for natural convection is a better choice when it comes to save the input energy. The design of the heat exchange devices has a significance part in increasing rate of heat transfer in natural convection. Annular geometries have wide application in various heat exchange devices. Various experimental and numerical studies are reported in literature concerning natural convection inside a vertical annulus. Husain et al. [2] performed mathematical inspection to analyse the effects of various geometrical parameters of annulus like radius ratio, aspect ratio and inclination on thermal and hydrodynamic performance. The numerical results extracted from study reveals that Reynolds number increases with inclination while Nusselt number decreases with power law. Husain and his team extended their work by conducting experiments on two phase flow of water inside vertical annulus and revealed that the length of the boiling zone increases with heat flux and observed that flow patterns are periodic in nature [3-5]. Mebarek -Oudina et al. [6] in his numerical study found that heat transfer increases as the length of the annulus decreases and further stated that shape and dimensions of gap inside annulus plays a significant role in increasing the heat transfer coefficient. Researchers have already made several attempt to enhance the coefficient of heat transfer in natural convection by altering the design of the heat exchangers. Kiwan and his co-workers [7] carried out numerical
investigation on natural convection using porous wall in closed vertical annuli and observed that buoyancy effect is dominant at stages away from the entrance.

Many researchers were made attempts to enhance heat transfer rate by dispersing solid nanoparticles to conventional fluids and thus new fluid formed were coined by [8] as nanofluid. Researchers came up with the idea of regular and hybrid nanofluid to enhance coefficient of heat transfer. Hybrid nanofluid is formed by mixing two different materials inside the base fluid. The Buoyancy induced flow of regular nanofluid (Al\textsubscript{2}O\textsubscript{3}/water) and hybrid nanofluid (Cu-Al\textsubscript{2}O\textsubscript{3}/water) inside eccentric horizontal were examine by Tayebi and Chamkha [9]. The research concluded that average Nusselt number enhances more effectively in the case of hybrid nanofluid rather than regular nanofluid. Khan et al. [10-11] performed numerical study on heat and fluid flow of nanofluid (Al\textsubscript{2}O\textsubscript{3}/water) inside a vertical annulus by considering different models of viscosity and thermal conductivity. The research infers that contradiction in results concerning buoyancy induced flow of nanofluids in various geometries arises because of the uncertainty in computation of thermal conductivity and viscosity of nanofluids due to adoption of discrete models.

It can be summarized from literature that experimental and numerical studies reported in literature concerning buoyancy induced flow of hybrid nanofluid in annular geometries are limited. Therefore a numerical investigation of buoyancy induced flow inside a vertical annulus interior of which water based (Al\textsubscript{2}O\textsubscript{3} +MWCNT) (7:3) hybrid nanofluid has presently been carried out using CFD solver Ansys Fluent.

2. **Formulation of the problem**

The annulus geometry (vertical) is selected as computational geometries shown in the figure 1. Geometry consist of two cylinder with different radius over one another on the same axis, the wall at outer is considered as non-heat transfer, while inner wall is fed with heat. The geometry has three parts lower, middle and upper. The outer diameter of the cylinder taken in geometry is 45mm and the inner diameter is 38 mm which makes the annular gap of 3.5 mm. Overall height of the annulus is 1232mm which makes aspect ratio as 352. The water based hybrid nanofluid (Al\textsubscript{2}O\textsubscript{3}+MWCNT) flows from bottom to the top covering the whole length of the geometry against the gravitational force. The heat is supplied via heat flux of 3kw/m\textsuperscript{2}in the middle region of geometry from \(y = y_1\) to \(y_2\).

![Figure 1. Computational geometry.](image-url)
3. CFD methodology

3.1 Governing equations for single phase model

Things taken in consideration for making the analysis:

- The geometry is taken as two dimensional.
- Cylinders of the annulus lie on the same axis i.e Axisymmetric.
- The circulating fluid in the interior of the annulus is considered as Newtonian and dense.
- The property of the fluid at the interior of the annulus is independent of temperature (Boussinesq Approximation).
- Single phase approach is used to model the flow of nanofluids.

Continuity equation controls the energy transfer and circulation of nanofluid interior of the vertical annulus, Navier-Stokes equation and the energy equation which in their elliptic form are shown below:

\[ \frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial v}{\partial y} = 0 \]  
(3.1)

\[ (u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial y}) = \frac{1}{\rho_h} \frac{\partial p}{\partial r} + \nabla \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \right] - \frac{\partial^2 u}{\partial y^2} \]  
(3.2)

\[ (u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial y}) = \frac{1}{\rho_h} \frac{\partial p}{\partial y} + \beta \rho_i (T - T_a) + \nu \frac{\partial^2 v}{\partial y^2} \]  
(3.3)

\[ (u \frac{\partial T}{\partial r} + v \frac{\partial T}{\partial y}) = h \frac{1}{\rho_i} \left( \frac{RT_T}{\rho_i} \right) + \frac{\partial^2 T}{\partial y^2} \]  
(3.4)

Below mentioned framework are used for non dimensionalization:

\[ R = \frac{r}{A}, \ Y = \frac{y}{A}, \ U = \frac{u}{A \lambda_{h_f}}, \ V = \frac{v}{A \lambda_{h_f}}, \ P = \frac{p}{1/\rho_{h_f} \left( \frac{A}{\lambda_{h_f}} \right)^2}, \ \theta = \frac{T - T_a}{k_{h_f}} \]

Aspect ratio, \( A_R = \frac{H}{A} \), Radius ratio, \( RR = \frac{r_o}{r_i} \), Annular gap \( A = r_o - r_i \)

Hence, the commanding equation in non dimensionalized state will be:

\[ \frac{1}{R} \frac{\partial (RU)}{\partial R} + \frac{\partial V}{\partial Y} = 0 \]  
(3.5)

\[ U \frac{\partial U}{\partial R} + V \frac{\partial U}{\partial Y} + Pr \left[ \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U}{\partial R} \right) \right] + \frac{\partial^2 U}{\partial Y^2} \]  
(3.6)

\[ U \frac{\partial V}{\partial R} + V \frac{\partial V}{\partial Y} = \frac{\partial P}{\partial Y} + Ra Pr \theta + Pr \left[ \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial V}{\partial R} \right) + \frac{\partial^2 V}{\partial Y^2} \right] \]  
(3.7)

\[ U \frac{\partial \theta}{\partial R} + V \frac{\partial \theta}{\partial Y} = \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \theta}{\partial R} \right) + \frac{\partial^2 \theta}{\partial Y^2} \]  
(3.8)

The set of commanding equations for buoyancy induced flow of water based hybrid nanofluid interior of vertical annulus must be solved synchronously. The non-dimensional state of length in axially is depicted as: lower adiabatic section is \( Y = 0 \) to \( Y = Y_1 = 21 \) and the middle heating zone is \( Y = Y_1 = 21 \) to \( Y = Y_2 = 300 \) and upper adiabatic zone \( Y = Y_2 = 300 \) to \( Y = A_R = 352 \). For radial direction \( R_1 = \frac{r_1}{A} \), and \( R_2 = \frac{r_2}{A} \)

**Boundary Conditions:**

(a) Inner wall (\( R = R_1 \)):

i) Lower unheated zone:

\[ U = V = 0 \text{ and } \frac{\partial \theta}{\partial R} = 0 \text{ for } 0 \leq Y < Y_1 \]
ii) Middle heated zone:

\[ U=V=0 \text{ and } \frac{\partial \theta}{\partial R} = -1 \text{ for } Y_1 < Y \leq Y_2 \]

(b) Outer wall (R = R_o):

\[ U=V=0 \text{ and } \frac{\partial \theta}{\partial R} = 0 \text{ for } 0 < Y \leq A_R \]

(c) Inflow (inlet) condition (Y = 0):

\[ \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial Y} = 0 \text{ and } \theta = 0 \text{ for } R_i < R < R_o \]

(d) Outflow (outlet) condition (Z = A_R):

\[ \frac{\partial U}{\partial Y} = \frac{\partial V}{\partial Y} = 0 \text{ and } \frac{\partial^2 \theta}{\partial Y^2} = 0 \text{ for } R_i < R < R_o \]

The heat transfer coefficient at any point can be numerated by:

\[ h = \frac{q_w}{(T_w - T_i)} \quad (3.9) \]

The bulk fluid temperature is numerated by:

\[ T_i = \frac{\int \rho c_p \bar{V} \Delta A}{\int \rho c_p \bar{V} \Delta A} \quad (3.10) \]

where \( \bar{V} \) is the resultant velocity.

Nu Number \( \text{Nu}_y = \frac{q_w A}{k(T_w - T_b) y} = \frac{hA}{k} \quad (3.11) \)

3.2 Solution Methodology

The Ansys Fluent computational fluid dynamics (CFD) solver is used for simulation, and the Ansys ICEM CFD grid tool is used in mesh development. Figure 2 demonstrates the distribution of the grid in the full computational domain. In the axial direction and non-uniform in the radial direction, the grid created has a uniform dissemination. To determine the governing equations, the two-dimensional, segregated laminar solver of Ansys is used. For discretized convective terms of the momentum and energy equations respectively, the third order MUSCL and second order upwind scheme are used. The Least Square Gradient system is used to overcome terminology for diffusion and PRESTO! For the discretization of pressure terms, formulating is used. The decoupling of pressure velocity is carried out using the semi-implicit method for pressure-linked equations.

![Figure 2. Representative Numerical grid.](image-url)
3.3 Thermo physical Properties of Hybrid nanofluids

The hybrid nanofluid is formed by mixing nano-sized particles (MWCNT and Al₂O₃) and water in an appropriate volume fraction as given in Table 1. Table 2 shows the thermo-physical properties of nanoparticles (MWCNT and Al₂O₃) and water.

Table 1. Volume fraction of nanoparticles in hybrid nanofluid.

| \( \alpha_{\text{MWCNT}} \) | \( \alpha_{\text{Al}_2\text{O}_3} \) in Al₂O₃/water | \( \alpha = \alpha_{\text{MWCNT}} + \alpha_{\text{Al}_2\text{O}_3} \) in MWCNT+ Al₂O₃/water |
|-----------------|-----------------|----------------------------------|
| 0.003           | 0.007           | 0.01                             |
| 0.006           | 0.014           | 0.02                             |
| 0.009           | 0.021           | 0.03                             |
| 0.012           | 0.028           | 0.04                             |

Table 2. Thermophysical properties of nanoparticles and water.

| Thermophysical properties | Base fluid (Water) | Nanoparticle (MWCNT) | Nanoparticle (Al₂O₃) |
|---------------------------|--------------------|----------------------|----------------------|
| Thermal conductivity (W/mK) | 0.613              | 3000                 | 40                   |
| Density (kg/m³)           | 997.1              | 2100                 | 3970                 |
| Specific Heat (J/kgK)     | 4179               | 600                  | 765                  |
| Volumetric expansion coefficient (1/K) | 0.00021 | -0.0000012 | 0.0000085 |
| Viscosity (Kg/m-s)        | 0.001123           | -                    | -                    |

The effective density of hybrid nanofluid

\[
(\rho)_f = (1-\alpha)(\rho)_b + \alpha(\rho)_p
\]

The heat capacitance of hybrid nanofluid as

\[
(C_p)_f = \frac{(1-\alpha)(\rho C_p)_b + \alpha(\rho C_p)_p}{(\rho)_f}
\]

Volumetric expansion coefficient:

\[
(\beta)_f = \frac{(1-\alpha)(\rho \beta)_b + \alpha(\rho \beta)_p}{(\rho)_f}
\]

Following models are used to evaluate viscosity and thermal conductivity of hybrid nanofluid

**Viscosity:** Viscosity of hybrid nanofluid is evaluated using Batchelor [12] model:

\[
\mu_f = (1+2.5\alpha+6.2\alpha^2)\mu_b
\]

**Thermal conductivity:** To evaluate the thermal conductivity of hybrid nanofluid, equation of Maxwell model [13] is rewrite in the following form:
4. Results and Discussion

The numerical investigation of buoyancy influenced flow of water based hybrid nanofluid (Al$_2$O$_3$+MWCNT) interior of a vertical annulus has been carried at constant heat flux of 3.5 kW/m$^2$. The thermal performance of hybrid nanofluids in vertical annulus is judge by temperatures, heat transfer coefficient and Nusselt number as presented and discussed in subsequent section.

4.1 Wall and Liquid Temperatures

The numerical outcome is displayed in form of wall and liquid temperature. The bulk liquid and wall temperatures increases along axial length of the annulus for both water and hybrid nanofluid as shown in figure 3. The wall temperature seems to increase at faster rate in the lower heated zone and then axially, it increases linearly up to reaches a maximum value. It then decreases in the upper non heated region of the annulus. The bulk temperature of water and hybrid nanofluid increases linearly in the heated region and almost become constant in the upper adiabatic region as shown in figure. The lower heated region of annulus describes the developing boundary layer where temperature of wall rises rapidly. The remaining heated zone signifies the evolved flow zone where temperature profile of wall and fluid becomes corresponding. The wall and bulk liquid temperature diminishes with volume concentration of nanoparticle which can be seen in the fig. This is because adding nanoparticles increases the thermal conductivity of the base fluid. Thus, higher thermal conductivity of hybrid nanofluid provides opportunity to extract more heat from heated wall.

![Figure 3. Contrast of wall and fluid temperature of water and hybrid nanofluid axially.](image-url)
4.2 Heat Transfer coefficient

Local heat transfer coefficient is evaluated using equation 3.9. Since the wall and fluid temperature varies axially and so is the heat transfer coefficient for both water and hybrid nanofluid as shown in figure 4. It is seen from figure, heat transfer coefficient drips abruptly in the lower entry zone signifies evolved flow and afterwards it seems to be constant representing fully developed flow. Further, it can be concluded through graph that heat transfer coefficient increases with volume fraction of nanoparticles. The effect of volume fraction of nanoparticles on average heat transfer coefficient is shown in figure 5. Average heat transfer coefficient is enhanced by 73.67% in case of hybrid nanofluid as compared to pure water on dispersion nanoparticles with volume fraction of 0.04.

![Figure 4. Contrast of local heat transfer coefficient axially.](image)

![Figure 5. Variation of average heat transfer coefficient with nanoparticle concentration](image)
4.3 Nusselt Number
The non-dimensional form of heat transfer coefficient is describes as Nu number that is calculated using equation 3.11. Contrast of Nu number axially is depicted in figure 6. The Nu number seems to drop abruptly in the lower entry zone signifies evolved flow region, afterwards it seems to be constant representing completely evolved flow. The Nu number estimated in case of hybrid nanofluid inside a vertical annulus is higher than pure water. However concentration of nanoparticles does not have significant effect on Nusselt number, because the commanding eqn. states that higher thermal conductivity of hybrid nanofluid suppresses the Nu number.

![Figure 6](image_url)

**Figure 6.** Contrast of Nu number axially.

5. Conclusion
The numerical inspection of buoyancy induced flow in vertical annulus interior of which water based hybrid nanofluid (Al₂O₃+MWCNT) has been carried out. The important points concluded from the present study:

- The lower heated region of annulus describes the developing boundary layer where temperature of wall rises rapidly. Whereas the remaining heated region signify the completely evolved region where temperature profile of wall and fluid becomes aligned.
- The local heat transfer coefficient increases with volume fraction of nanoparticles.
- Average heat transfer coefficient is enhanced by 73.67% in case of hybrid nanofluid as compared to pure water on dispersion nanoparticles with volume fraction of 0.04.
- The Nusselt number estimated in case of hybrid nanofluid inside a vertical annulus is higher than pure water.
- The concentration of nanoparticles does not have significant effect on Nusselt number of hybrid nanofluids.
## NOMENCLATURE

| Symbol | Description                                      | Greek            |
|--------|--------------------------------------------------|------------------|
| $A_f$  | Flow area                                        | $A$ Annular gap (m) |
| $c$    | Specific heat                                    | $\lambda$ Thermal diffusivity (m$^2$/s) |
| $d$    | Diameter of tube (m)                             | $\beta$ Expansion coefficient (K$^{-1}$) |
| $h$    | Heat transfer coefficient (W m$^{-2}$C$^{-1}$)    | $\rho$ Density (kg m$^{-3}$) |
| $k$    | Thermal conductivity (W m$^{-1}$C$^{-1}$)        | $\mu$ Dynamic viscosity (N s/m$^2$) |
| $L$    | Length (m)                                       | $\theta$ Non-dimensional Temperature |
| $g$    | Acceleration due to gravity (m s$^{-2}$)         | $\nu$ Kinematic Viscosity (m$^2$ s$^{-1}$) |
| $p$    | Pressure (Pascal)                                | Dimensionless parameters |
| $q_w$  | Heat flux (W/m$^2$)                              | $A_R$ Aspect ratio (L/$\delta$) |
| $r$    | Radial distance (m)                              | $Nu$ Nusselt Number ($\frac{hA}{k}$) |
| $R$    | Non-dimensional Radial distance                 | $Pr$ Prandtl Number ($\frac{v}{h}$) |
| $T$    | Temperature (°C)                                 | $Ra$ Rayleigh Number ($\frac{g\beta|q_w|A_f^4}{kv\lambda}$) |
| $Ri$   | Richardson Number                                | $RR$ Radius ratio ($\frac{Ra}{R_{i}}$) |
| $u$    | Radial velocity (m s$^{-1}$)                     | Subscripts and Superscripts |
| $U$    | Non-dimensional radial velocity                 | $a$ Ambient |
| $v$    | Axial velocity (m s$^{-1}$)                      | $h$ Heated |
| $V$    | Non-dimensional axial velocity                  | $i$ Inner |
| $y$    | Axial distance (m)                               | $o$ Outer |
| $Y$    | Non-dimensional axial distance                  | $bf$ Basefluid |
|        |                                                  | $hf$ Hybrid Nanofluid |
References

1. Mustafa, J., Husain, S., & Siddiqui, M. A. (2017). Experimental studies on natural convection of water in a closed-loop vertical annulus. *Experimental Heat Transfer*, 30(1), 25-45.

2. Husain, S., Siddiqui, M. A., & Khan, S. A. (2019). Effect of geometrical parameters on natural convection of water in a narrow annulus. *Progress in Nuclear Energy*, 112, 146-161.

3. Husain, S., Siddiqui, M. A., & Ahmad Khan, S. (2019). Wall boiling in a vertical annulus: Effect of inlet sub cooling and mass flow rate. *Numerical Heat Transfer, Part A: Applications*, 75(11), 776-793.

4. Husain, S., Siddiqui, M. A., & Khan, S. A. (2019). Visualization of flow patterns of water in open-ended vertical annulus during natural convection flow. *Journal of Flow Visualization and Image Processing*, 26(3), 209-222.

5. Husain, S., Khan, S. A., & Siddiqui, M. A. (2021). Wall boiling of Al2O3-water nanofluid: Effect of nanoparticle concentration. *Progress in Nuclear Energy*, 133, 103614.

6. Mebarek-Oudina, F. (2017). Numerical modeling of the hydrodynamic stability in vertical annulus with heat source of different lengths. *Engineering science and technology, an international journal*, 20(4), 1324-1333.

7. Kiwan, S., & Alzahrany, M. S. (2007). Effect of using porous inserts on natural convection heat transfer between two concentric vertical cylinders. *Numerical Heat Transfer, Part A: Applications*, 53(8), 870-889.

8. Choi, S. U., & Eastman, J. A. (1995). *Enhancing thermal conductivity of fluids with nanoparticles* (No. ANL/MSD/CP-84938; CONF-951135-29). Argonne National Lab., IL (United States).

9. Tayebi, T., & Chamkha, A. J. (2016). Free convection enhancement in an annulus between horizontal confocal elliptical cylinders using hybrid nanofluids. *Numerical Heat Transfer, Part A: Applications*, 70(10), 1141-1156.

10. Ahmad Khan, S., & Altamush Siddiqui, M. (2020). Numerical studies on heat and fluid flow of nanofluid in a partially heated vertical annulus. *Heat Transfer*, 49(3), 1458-1490.

11. Khan, S. A., Siddiqui, M. A., & Husain, S. (2020). Numerical Studies on Thermally Induced Flow of Nanofluid in a Vertical Annulus. In *Proceedings of International Conference in Mechanical and Energy Technology* (pp. 87-100). Springer, Singapore.

12. Batchelor, G. K. (1972). Sedimentation in a dilute dispersion of spheres. *Journal of fluid mechanics*, 52(2), 245-268.

13. Maxwell, J. C. (1873). A treatise on electricity and magnetism (Vol. 1). Oxford: Clarendon Press.