TEV STRINGS AND ULTRAHIGH-ENERGY COSMIC RAYS*

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The origin and nature of ultrahigh-energy cosmic ray events, above the Greisen-Zatsepin-Kuzmin (GZK) cutoff energy, constitute a long-standing, unsolved mystery. Neutrinos are proposed candidates but their standard interactions with matter are too weak. In the context of a TeV-scale string theory, motivated by possible extra space dimensions, the neutrino-nucleon scattering is examined. Resonant string contributions increase substantially the standard model neutrino-nucleon cross section. Although they seem insufficient to explain the trans-GZK cosmic ray events, their effects might be detected in next experiments.

1. The mystery of ultrahigh-energy cosmic rays

Cosmic rays (CRs) were discovered in balloon experiments by V. Hess in 1911. They have always been a fruitful particle physics laboratory: positrons, muons and many other particles were first observed there. CRs have a broad spectrum, ranging from 1 GeV to $10^{20}$ eV, including energies far beyond the ones available at man-made accelerators. The flux of CRs decreases rapidly with the energy: e.g. 1 particle/m$^2$/s at $10^{11}$ eV and only 1 particle/km$^2$/year at about $10^{18}$ eV.

At high energies, a CR may hit an atmospheric nucleon producing a cascade of secondary particles, extensive air showers (EAS), that cover at ground an area with a diameter of tens of meters at $10^{14}$ eV to several kilometers at $10^{20}$ eV. Incidentally, it is easier to observe the EAS at ground level for energies where direct detection of CRs at satellite or balloon experiments becomes impractical. The primary particle (mass and energy) can only be inferred from the parameters of the shower, such as number of electrons, muons or hadrons, shapes of lateral distributions, etc.

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We will focus on the part of the spectrum above the Greisen-Zatsepin-Kuzmin cutoff energy \[ E_{\text{GZK}} \approx 5 \times 10^{19} \text{ eV} \] (see below). Unexpectedly, over seventy trans-GZK ultrahigh-energy cosmic ray events have been observed over the last forty years by five different experiments [2]. These events exhibit large-scale isotropy and small-scale anisotropy (a very unlikely pairing within the angular resolution occurs: three doublets and one triplet!). They constitute a puzzle in modern astrophysics. The main open questions are:

- **What is the primary particle?**

  Protons with energies above the reaction threshold \( E_{\text{GZK}} \) lose part of their energy by scattering with cosmic microwave background (CMB) photons:

  \[
p + \gamma_{2.7K} \rightarrow \Delta^+ \rightarrow p + \pi^0 (n + \pi^+)\.
  \]

  Above a distance of \( D_{\text{GZK}} \approx 50 \text{ Mpc} \) the energy of the proton gets below \( 10^{20} \text{ eV} \), regardless of its initial value, after a series of such collisions. This implies that the source cannot be too distant for trans-GZK events if they are originated by protons. Similar cutoffs exist at lower energies for other primaries: nuclei undergo photodisintegration in CMB and IR radiations; electrons lose energy very rapidly via synchrotron radiation; and photons have a relatively short absorption length.

- **What is the acceleration mechanism?**

  The conventional (Fermi) mechanism is the stochastic shock-wave acceleration in magnetized clouds. Powerful enough accelerators [3] are pulsars, AGN cores and jets or radio-loud quasars. Gamma ray bursters may accelerate protons up to \( 10^{20} \text{ eV} \) but they are too distant objects [4]. Alternatively, “top-down” scenarios [5] have been proposed, based on the decay or annihilation of super-heavy particles produced by topological defects [6] or supermassive cosmological relics [7]. Another possibility is the assumption of exotic physical laws, such as breakdowns of Lorentz invariance [8] or general relativity [9].

- **Where are the sources?**

  According to the conventional acceleration mechanism there are very few sources at sight within the GZK distance, insufficient to explain the number of observed trans-GZK events. Directional and temporal decorrelation with the source due to magnetic fields of the order of \( \mu G \) has been proposed as a possible explanation [10]. Distant sources like BL Lacertae blazers, outside the GZK volume, have received attention.
recently, since they may emit very high energy photons cascading into secondary ones still above the GZK cutoff while approaching us [11].

To evade most of the problems above, neutrinos are good candidates: they are neutral (unbent by magnetic fields, allowing pairing), stable and able to propagate unimpeded to earth from very distant sources: even if their cross section was of hadronic size, their mean free path would be of the order of 100 Gpc among galaxies and 1 Mpc within a galaxy, so they can only be stopped by the atmosphere. It is also suggestive that the flux of cosmogenic neutrinos matches well the flux of observed trans-GZK CRs, since the energy-degrading protons photoproduce pions decaying into neutrinos. The main problem is that neutrino cross sections are smaller than those of hadronic or electromagnetic interactions by six orders of magnitude. Two possible types of solutions have been proposed to the small cross section problem of neutrinos:

– “Z-bursts” [12]. Cosmogenic neutrinos are not the primaries but resonantly create a local flux of nucleons and photons (within the GZK distance) with $E \gtrsim E_{\text{GZK}}$ via scattering with the cosmic neutrino background. The resonance is a $Z$ boson. The necessary neutrino mass range fits well with the one inferred from oscillation experiments, but large neutrino fluxes are required. Similarly, “gravi-bursts” [13] may proceed via resonant Kaluza-Klein excitations of the graviton in theories with large compact extra dimensions or in localized gravity models.

– Enhance the neutrino cross section at high energies. Neutrinos are here the primary particles reaching the atmosphere. This will be our approach, in the context of a weakly-coupled string theory assuming extra space dimensions at the TeV scale.

2. The effects of extra dimensions

Extensions of the Standard Model (SM) with extra dimensions offer new ways to accommodate the hierarchies observed in particle physics [14]. A very attractive possibility would be to bring the scale of unification with gravity from $M_{\text{Planck}} \approx 10^{19}$ GeV down to the electroweak scale $M_{\text{EW}} \approx 1$ TeV. This could result if gravity propagates along a $(4+n)$–dimensional flat space with $n$ compact submillimeter dimensions (ADD model [15]) or along a $(4+1)$–dimensional slice of anti-deSitter space with a warp factor in the metric (RS model [16]). These higher dimensional field theories, however, must be considered effective low-energy limits only valid below the mass scale of a more fundamental theory. And nowadays, only string theory [17] provides a consistent framework for the unification of gravity with the standard model.
In string theory the massless graviton comes as the zero mode of a closed string, whereas the gauge bosons and matter fields are the lightest modes of open strings. The string vibration modes, with squared masses $M^2 = nM_S^2$ (integer $n > 1$), are the so-called string Regge (SR) excitations. The fundamental string scale is $M_S = \alpha'^{-1/2}$ where $\alpha'$ is the string tension.

At energies where the effects of a higher dimensional graviton are un-suppressed one expects the presence of its SR excitations giving an effect of the same size. This is necessary in order to avoid the pathologies of spin-2 field theories, such as the quantum field theory of gravity.

\[ \text{Fig. 1. String graphs for (a) } \mathcal{O}(g) \text{ vertex of three open strings, (b) } \mathcal{O}(\kappa) \text{ vertex of two open strings and one closed string, (c) } \mathcal{O}(g^2) \text{ tree-level 2-body scattering of open strings (no gravity), (d) } \mathcal{O}(\kappa^2) \text{ tree-level 2-body scattering with only gravity, }
\]

\[ \text{(e) } \mathcal{O}(g^4) \text{ loop-level 2-body scattering of open strings, topological equivalent to (d) and hence } \kappa^2 = \mathcal{O}(g^4) , \text{ meaning that (d) is } g^2 \text{ smaller than (c).} \]

Now, as emphasized in [18, 19], the exchange amplitude of a closed string has an order $g^2$ suppression versus the exchange of an open string, in string perturbation theory. This is illustrated in Fig. 1. In consequence, processes that receive sizeable contributions from SR and Kaluza-Klein excitations of the graviton and also from SR excitations of the gauge bosons will be dominated by the second ones, for $s \lesssim M_S^2/\alpha$. That is, gravity is subleading versus gauge interactions and hence the reactions are dominated at tree level by open-string diagrams. This is a generic feature in models of higher dimensional gravity embedded in a weakly-coupled string theory.

\[ ^1 \text{ When the amplitude } A_{\text{gauge}} \sim \alpha_s/M_S^2 \lesssim A_{\text{grav}} \sim \alpha^2 s^2/M_S^6 , \text{ namely for } \alpha_s \gtrsim M_S^2 , \text{ gravity dominates and non-perturbative effects become important.} \]
3. Phenomenology of TeV strings

Cullen, Perelstein and Peskin introduced a TeV-scale string model for QED in [19]. It contains electrons and photons at low energies and massive SR excitations above the string scale. Their results have been generalized in order to obtain string amplitudes for neutrino–quark elastic scattering [20], mediated in the SM by a Z boson in the $t$–channel.

The model results from a simple embedding of the SM interactions into Type IIB string theory. It is assumed that the 10–d space of the theory has 6 dimensions compactified on a torus with common periodicity $2\pi R$, and that $N$ coincident D3-branes (4–dimensional hypersurfaces where open strings may end) are stretched out in the 4 extended dimensions. Such configuration has N=4 supersymmetry and $U(N)$ gauge symmetry. One assumes that the extra symmetry of the massless string modes can be eliminated by an appropriate orbifold projection, resulting an acceptable model with (at least) the SM fields. The parameters of this theory would be the string scale $M_S = \alpha'^{-1/2} \approx 1$ TeV and the dimensionless gauge coupling constant $g$, unified at $M_S$. Proposals for splitting these couplings can be found in [21]. For more general D-brane models see [22] and references therein.

A tree-level amplitude of open string states on a D-brane is given [19, 23] as a sum of ordered amplitudes multiplied by Chan-Paton traces. For the processes under study we have

$$A(1, 2, 3, 4) = g^2 S(s, t) F^{1243}(s, t, u) \text{Tr}[t^1 t^2 t^3 + t^3 t^2 t^1] + g^2 S(s, u) F^{1234}(s, u, t) \text{Tr}[t^1 t^2 t^3 + t^3 t^2 t^1] + g^2 S(t, u) F^{1324}(t, u, s) \text{Tr}[t^1 t^2 t^3 + t^3 t^2 t^1].$$

In this expression,

$$S(s, t) = \frac{\Gamma(1 - \alpha' s) \Gamma(1 - \alpha' t)}{\Gamma(1 - \alpha' s - \alpha' t)}$$

is basically the Veneziano amplitude [24]; $(1, 2, 3, 4)$ label the external particles involved in the process: $(\nu_L^{in}, u_L^{in}, \nu_L^{out}, u_L^{out})$ for instance; the Chan-Paton factors $t^a$ are representation matrices of $U(N)$; and $F^{abcd}(s, t, u)$ is a factor depending on the vertex operators for the external states and their ordering. In our case all the vertex operators correspond to (massless) Weyl spinors of helicity (directed inward) + or −, giving

$$F^{-+++}(s, t, u) = -4 \frac{t}{s};$$

$$F^{+++-}(s, t, u) = -4 \frac{u^2}{st} = 4 \frac{u}{s} + \frac{u}{t};$$

$$F^{--++}(s, t, u) = -4 \frac{s}{t}.$$  

(1)
Therefore the amplitude for the process $\nu_L u_L \rightarrow \nu_L u_L$ reads

$$A = -4g^2 \left[ \frac{s}{t} S(s, t) T_{1243} + \frac{s}{u} S(s, u) T_{1234} + \frac{s^2}{tu} S(t, u) T_{1324} \right], \quad (4)$$

with $T_{abcd}$ the Chan-Paton traces. To understand the phenomenological consequences of this amplitude let us start with the limit $s, t \rightarrow 0$. Since $\Gamma(1) = 1$, we have all the Veneziano factors $S(0, 0) = 1$. The amplitude expresses then the exchange of massless vector modes in the $t$– and the $u$–channels. The former would correspond to the $Z$ gauge boson, whereas the field exchanged in the $u$–channel is in the $(3, 1)$ and/or the $(3, 3)$ representations of $SU(3)_C \times SU(2)_L$ and has electric charge $Q = -2/3$. We are interested, however, in models that reproduce the SM result at low energies, with no massless leptoquarks. We obtain this limit if the Chan-Paton factors assigned to $u_L$ and $\nu_L$ are such that $T_{1243} - T_{1324} = -1/10$ and $T_{1234} = T_{1324}$, where we have used $\sin^2 \theta_W = 3/8$, implied by gauge-coupling unification.

In terms of $T_{1234} \equiv -a/10$ the amplitude (4) becomes

$$A = 2g^2 \left[ \frac{s}{t} \left[ (1 + a) S(s, t) - a S(t, u) \right] + \frac{s}{u} a [S(s, u) - S(t, u)] \right]. \quad (5)$$

At low $s$ this amplitude is $A_0 \approx (2/5)g^2 s/t$ and corresponds to the exchange of a $Z$ boson in the $t$–channel. The $Z$ is then a massless SR mode that acquires its mass $M_Z$ only through the Higgs mechanism. We shall neglect the corrections of order $M_Z^2 / M_S^2$ that may affect the massive SR modes.

As the energy increases the Veneziano factor $S(s, t)$ gives a series of poles (at $1 - \alpha' s = 0, -1, -2, ...$) and zeroes (at $1 - \alpha' s - \alpha' t = 0, -1, -2, ...$). It can be expressed as

$$S(s, t) = \sum_{n=1}^{\infty} \frac{\alpha' t + \alpha' s - 1}{\alpha' t + n - 1} \frac{\prod_{k=0}^{n-1} (\alpha' t + k)}{(\alpha' s - n) (n - 1)!}. \quad (6)$$

At $s = nM_S^2$ the amplitude describes the exchange of a collection of resonances with the same mass and different spin (see below). Away from the poles the interference of resonances at different mass levels produces the usual soft (Regge) behavior of the string in the ultraviolet. Obviously, these resonances are not stable and at one loop will get an imaginary part in their propagator. When the total width of a resonance (which grows with its mass) is similar to the mass difference with the resonance in the next level one cannot see resonances and interference effects dominate also at $s = nM_S^2$. 
Let us first analyze the case with \( \alpha = 0 \) in Eq. (5). The amplitude is just \( \mathcal{A}(\nu_L u_L \to \nu_L u_L) = (2/5)g^2(s/t) \cdot S(s, t) \). Near the pole at \( s = nM_S^2 \),
\[
\mathcal{A}_n \approx \frac{2}{5}g^2 \frac{nM_S^4}{t} \frac{(t/M_S^2)}{(t/M_S^2 + 1) \cdots (t/M_S^2 + n - 1)} (n - 1)! (s - nM_S^2).
\]
This amplitude corresponds to the \( s \)-channel exchange of massive leptoquarks in the \((3, 3)\) representation of \( SU(3)_C \times SU(2)_L \) with electric charge \( Q = 2/3 \). At each pole we have contributions of resonances with a common mass \( \sqrt{nM_S} \) but different spin, going from zero to the order of the residue \( P_n(t) = \mathcal{A}_n \cdot (s - nM_S^2) \). In this case the maximum spin at the \( n \) level is \( J = n - 1 \). To separate these contributions we first write the residue in terms of the scattering angle \( \theta \), with \( t = -(nM_S^2/2)(1 - \cos \theta) \). Then we express \( P_n(\theta) \) as a linear combination of the \( d \)-functions (rotation matrix elements):
\[
P_n(\theta) = \frac{2}{5}g^2nM_S^2 \sum_{J=0}^{n-1} \alpha_n^J d_{0,0}^J(\theta).
\]
The coefficient \( \alpha_n^J \) gives the contribution to our amplitude of a leptoquark \( X_n^J \) of mass \( nM_S^2 \) and spin \( J \). For example, at the first SR level we find a scalar resonance with \( \alpha_1^0 = 1 \), at \( s = 2M_S^2 \) there is a single vector resonance with \( \alpha_1^2 = 1 \), whereas at \( s = 3M_S^2 \) there are modes of spin \( J = 2 \) (\( \alpha_3^J = 3/4 \)) and \( J = 0 \) (\( \alpha_3^0 = 1/4 \)).

The general case with \( \alpha \neq 0 \) is completely analogous, with resonant contributions from the terms proportional to \( S(s, t) \) and \( S(s, u) \). Taking \( u = -(nM_S^2/2)(1 + \cos \theta) \) and expressing again the residue in terms of \( d \)-functions we find the same type of resonances but with different \( \alpha_n^J \) coefficients: \( \alpha_1^0 = 1 + 2a \), \( \alpha_2^1 = 1 \), \( \alpha_3^3 = 3(1 + 2a)/4 \) and \( \alpha_3^0 = 1(1 + 2a)/4 \).

\section*{4. Application to the \( \nu \)-nucleon scattering}

From the resonant amplitude \( \nu_L u_L \to X_n^J \to \nu_L u_L \) we can now obtain the partial width \( \Gamma_n^J = \Gamma(X_n^J \to \nu_L u_L) \):
\[
\Gamma_n^J = \frac{g^2}{40\pi} \frac{\sqrt{nM_S} |\alpha_n^J|}{2J + 1}.
\]
Notice that for a given spin \( J \), the variation with \( n \) of \( \alpha_n^J \) gives the \textit{running} of the coupling with the energy. We obtain numerically that the coupling of heavier resonances decreases like the power law \( \alpha_n^J \approx 1/n \).

The partial width \( \Gamma_n^J \) can be used to obtain the cross section \( \sigma_n^J(\nu_L u_L) \equiv \sigma(\nu_L u_L \to X_n^J) \) in the narrow-width approximation (NWA):
\[
\sigma_n^J(\nu_L u_L) = \frac{4\pi^2 \Gamma_n^J}{\sqrt{nM_S}} (2J + 1) \delta(s - nM_S^2).
\]
At each mass level \( n \) there is a tower of resonances of integer spin \( J \) from 0 to \( n - 1 \). We find a sum rule for the production rate \( \sigma_n(\nu_L u_L) \equiv \sum_J \sigma_n^J(\nu_L u_L) \) of any of these resonances:

\[
\sigma_n(\nu_L u_L) = \begin{cases} 
\frac{2}{5} \frac{\pi g^2}{4} (1 + 2a) \delta(s - nM_S^2) & \text{for } n \text{ odd} \\
\frac{2}{5} \frac{\pi g^2}{4} \delta(s - nM_S^2) & \text{for } n \text{ even.}
\end{cases}
\]  

Eq. (11) is equivalent (for \( a = 0 \)) to the production rate of a single resonance of mass \( \sqrt{nM_S} \) and coupling \( (2/5)g^2 \) \[25\]. This is a very interesting result: the coupling of heavier SR modes decreases quadratically with the energy, but the number of modes (and the highest spin) at each mass level \( n \) grows also quadratically making \( \sum_J \alpha_n^J \) a constant independent of \( n \).

In the NWA \( \sigma(\nu_L u_L) \equiv \sum_{n,J} \sigma(\nu_L u_L \rightarrow X_n^J \rightarrow \text{anything}) \) is then \( \sigma(\nu_L u_L) = \sum_n \sigma_n(\nu_L u_L) \). In this limit the cross section is proportional to a collection of delta functions and thus all interference effects are ignored. This is a good approximation as far as the total width of a resonance is smaller than the mass difference with the next resonance of same spin. Although the coupling (and any partial width) decreases with the mass, the total width of heavier resonances grows due to the larger number of decay modes that are kinematically allowed. We estimate that contributions to \( \sigma(\nu_L u_L) \) from modes beyond \( n \cdot (g^2/4\pi) \approx 1 \) are a continuum. In this regime any cross section goes to zero exponentially at fixed angle \( (s \text{ large, } t/s \text{ fixed}) \) and like a power law at small angles \( (s \text{ large, } t \text{ fixed}) \) \[17\]. We neglect these contributions. We keep only resonant contributions from levels \( n < n_{\text{cut}} = 50 \). Our result depends very mildly on the actual value of \( n_{\text{cut}} \).

To evaluate the total \( \nu N \) cross section one also needs the elastic amplitudes \( \nu_L d_L, \nu_L u_R, \nu_L d_R \) and \( \nu_L \bar{q}_{L(R)} \). \( \mathcal{A}(\nu_L d_L \rightarrow \nu_L d_L) \) takes the same form as the amplitude in Eq. (5) with the changes \( (2/5, a) \rightarrow (-3/5, a') \). The massive resonances exchanged in the \( s \)-channel are now an admixture of an \( SU(2)_L \) singlet and a triplet. The singlet contribution is required in \( n\)-even levels, otherwise an \( SU(2)_L \) gauge transformation would relate the parameters \( \alpha_n^a \) obtained here with the ones deduced from Eq. (5). In \( n\)-odd mass levels gauge invariance could be obtained with no singlets for \( a' = -(2+a)/3 \). The cross section \( \sigma_n(\nu_L d_L) \) can be read from Eq. (11) just by changing \( (2/5, a) \rightarrow (3/5, a') \).

The calculation of amplitudes and cross sections for \( \nu_L \bar{q}_R \) are completely analogous. We obtain

\[
\sigma_n(\nu_L u_R) = \begin{cases} 
\frac{2}{5} \frac{\pi g^2}{2} b \delta(s - nM_S^2) & \text{for } n \text{ odd} \\
0 & \text{for } n \text{ even.}
\end{cases}
\]
The cross sections $\sigma_n(\nu_L d_R)$, $\sigma_n(\nu_L \bar{\nu}_R)$ and $\sigma_n(\nu_L \bar{d}_R)$ coincide with the expression in Eq. (12) with the changes $(2/5, b) \rightarrow (1/5, b')$, $(2/5, a) \rightarrow (2/5, a')$, respectively. For the left-handed antiquarks, $\sigma_n(\nu_L \bar{\nu}_L)$ and $\sigma_n(\nu_L \bar{d}_L)$ can be read from Eq. (11) by changing $(2/5, a) \rightarrow (2/5, b)$ and $(2/5, a) \rightarrow (1/5, b')$, respectively.

Now the total neutrino-nucleon cross section due to the exchange of SR excitations can be very easily evaluated. In terms of parton distribution functions $q(x, Q)$ ($q = q_{L,R}, \bar{q}_{L,R}$) in a nucleon ($N \equiv (n + p)/2$) and the fraction of longitudinal momentum $x$, it is

$$\sigma(\nu_L N) = \sum_{n=1}^{n_{\text{cut}}} \sum_q \frac{\tilde{\sigma}_n(\nu_L q)}{n M_S^2} x q(x, Q),$$

(13)

where $x = nM_S^2/s$, $s = 2M_N E_\nu$, $Q^2 = nM_S^2$ and $\tilde{\sigma}_n(\nu_L q)$ is the factor multiplying the delta function in the cross section $\sigma_n(\nu_L q)$.

Fig. 2. Neutrino-nucleon cross section versus the incident neutrino energy $E_\nu$. The SM contribution (solid) includes neutral and charged current interactions. The SR contribution is for $M_S = 0.5$ TeV (dashes) and $M_S = 2$ TeV (dots) for the cases (i) $a = a' = b = b' = 0$ and (ii) $a = a' = b = b' = 5$.

In Fig. 2 we plot the neutrino-nucleon cross section at energies from $10^2$ to $10^{13}$ GeV for $M_S = 0.5, 2$ TeV. We have used the CTEQ5 parton distributions in the DIS scheme [26] extended to $x < 10^{-5}$ with the methods in [27]. We show the SM cross section and the string corrections for $a =$
\( a' = b = b' \) equal 0 and 5 (notice that in the first case there are no \( s \)-channel resonances mediating the \( \nu_L q_R \) amplitude). The modes beyond \( n_{\text{cut}} = 50 \) are not included, since there we expect that the narrow width approximation is poor.

As a final comment, let us remark that the SM neutrino-nucleon interaction probes values of \( x \) much below the current HERA DIS data (\( x_{\text{HERA}}^{\text{min}} \sim 10^{-4} \)) for ultrahigh-energy neutrinos,

\[
\langle x \rangle \approx \frac{1}{\sigma_{\text{SM}}} \frac{G_F^2 M_W^2 M_W}{16 \pi} \approx 10 \left( \frac{E_\nu}{\text{GeV}} \right)^{0.8} \approx 10^{-8} \text{ for } E_\nu = 10^{11} \text{ GeV}.
\]

This is not the case for the process mediated by SR resonances, where the probed values of \( x \) are above

\[
x_{\text{min}} \approx 0.5 \times 10^6 \left( \frac{M_S}{\text{TeV}} \right)^2 \left( \frac{E_\nu}{\text{GeV}} \right)^{-1} \approx 0.5 \times 10^{-5} \text{ for } E_\nu = 10^{11} \text{ GeV},
\]

taking \( M_S = 1 \text{ TeV} \).

5. Conclusions

Cosmic rays hit the nucleons in the atmosphere with energies of up to \( 10^{11} \) GeV. If the string scale is in the TeV range, these cosmic rays have the energy required to explore the fundamental theory and its interactions. In particular, ultrahigh-energy neutrinos are interesting since they can travel long distances without losing a significant fraction of energy and are not deflected by magnetic fields. In addition, the SM interactions of a neutrino are much weaker than those of a quark or a charged lepton, which makes easier to see deviations due to new physics.

With this motivation we have analyzed the string \( \nu N \) cross section at energies much larger than the fundamental scale \( M_S \approx 1 \) TeV. In a weakly-coupled string theory the process is given at tree level by open-string graphs, whereas gravity effects appear as a one-loop correction.\(^2\) We have fixed the arbitrary parameters of the model imposing phenomenological constraints, namely, the massless SR modes must account for the standard model particles. Four Chan-Paton traces remain as free parameters. The massive SR modes include leptoquarks that resonantly mediate the process in the \( s \)-channel for energies above the string scale. The presence of massive leptoquarks is not a peculiarity of our toy model but a generic feature of any string model, and has to do with \((s,t)\) and/or \((t,u)\) duality of the open-string amplitudes (see \textit{e.g.} Eq. (4)).

\(^2\) Gravity effects in the context of weak-scale string theories at high energies have been considered in the Regge picture by [28].
A very simple sum rule for the production rate of all the s-channel leptoquarks, with different spins in the same mass level \( n \), makes possible the calculation of the total \( \nu N \) cross section in the narrow-width approximation. The effect of these leptoquarks is not just a correction of order \( M_Z^2/M_S^2 \) to the SM cross section, as one would expect on dimensional grounds. Such SR excitations give a contribution that can dominate for \( M_S \approx 1 \text{ TeV} \). However, for the expected flux of ultrahigh-energy neutrinos it seems unlikely that the cosmic ray events observed above the GZK limit correspond to the decay of string resonances produced in \( \nu N \) scattering. A similar conclusion has been recently drawn in [29] for graviton-mediated \( \nu N \) scattering and black hole production in TeV-gravity models.

Nevertheless, since neutrinos are very penetrating particles, the enhancement of the cross-section may make possible the detection of horizontal air showers produced by ultrahigh-energy CRs in upcoming experiments [30]. Furthermore, in second-generation neutrino telescopes, able to detect neutrinos in the TeV to PeV range, there is also a chance to probe this and other models of TeV-scale quantum gravity at more moderate energies [31].

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