Large Leptonic Flavor Mixing and the Mass Spectrum of Leptons

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Abstract

Implications of a simple model for the mass generation of leptons are studied, in particular for the upcoming long-baseline neutrino experiments. The flavor mixing angles are large (nearly maximal). The probability for the long-baseline $\nu_\mu \leftrightarrow \nu_e$ oscillation is predicted to be about 1%.
Recently the Super-Kamiokande Collaboration has reported new and stronger evidence for the existence of the atmospheric neutrino anomaly. The data particularly favor an interpretation of the observed muon-neutrino deficit by a $\nu_\mu \leftrightarrow \nu_\tau$ oscillation with the mass-squared difference $\Delta m^2_{\text{atm}} \approx (0.5 \ldots 6) \times 10^{-3}$ eV$^2$ and the mixing factor $\sin^2 2\theta_{\text{atm}} > 0.82$ at the 90% confidence level [1]. The long-standing solar neutrino deficit has also been confirmed in the Super-Kamiokande experiment. Analyses of the energy shape and day-night spectra of solar neutrinos favor the mechanism of a long-wavelength vacuum oscillation with $\Delta m^2_{\text{sun}} \approx 10^{-10}$ eV$^2$ and $\sin^2 2\theta_{\text{sun}} \approx 1$ [2]. If these large mixing angles $\theta_{\text{atm}}$ and $\theta_{\text{sun}}$ are finally confirmed, one would have an indication that the physics responsible for neutrino masses and leptonic flavor mixing might be qualitatively different from that for the quark sector.

In 1996 a pattern of lepton mass matrices, based on the approximate flavor democracy for charged leptons and the near mass degeneracy for neutrinos, was proposed by the present authors [3]. In this approach the flavor mixing matrix in the symmetry limit is identical to the mixing matrix that one obtains in QCD for the light pseudoscalar mesons in the limit of chiral symmetry. Note that the mixing between the mass eigenstates $|\pi^0\rangle$, $|\eta\rangle$, $|\eta'\rangle$ and the QCD eigenstates $|\bar uu\rangle$, $|\bar dd\rangle$, $|\bar ss\rangle$ is caused by the gluon anomaly, which at the same time leads to a strong mass hierarchy: $M^2_{\pi^0}, M^2_{\eta} \ll M^2_{\eta'}$. Analogously the mixing of lepton flavors arises from a strong mismatch between the charged lepton and neutrino mass matrices. To leading order we find a constant flavor mixing matrix, independent of the lepton masses, linking the neutrino mass eigenstates to the neutrino flavor eigenstates:

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U_0
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
$$

where

$$
U_0 = \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix}.
$$

Therefore one obtains $\sin^2 2\theta_{\text{atm}} = 8/9$ and $\sin^2 2\theta_{\text{sun}} = 1$. Both angles are in good agreement with current data on atmospheric and solar neutrinos. As discussed in Ref. [3], this constant algebraic mixing matrix will receive small corrections, once the muon and electron masses are taken into account. We emphasize that in our approach the leptonic flavor mixing angles do not depend on the neutrino masses.

The present note aims at displaying the symmetry-breaking patterns of the lepton mass matrices more clearly and discussing their next-to-leading-order consequences on flavor mixing.
angles as well as specific predictions for the upcoming neutrino experiments. In particular it will be shown that a small correction to $U_{0}$, which makes its (1,3) element deviate slightly from zero, may lead to observable effects in the future long-baseline neutrino experiments.

Let us start with the symmetry limits of the charged lepton and neutrino mass matrices. In the flavor space in which charged leptons have the exact flavor democracy and neutrino masses are fully degenerate, the mass matrices can be written as

$$
M_{0_{l}} = \frac{c_{l}}{3} \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}, \\
M_{0_{\nu}} = c_{\nu} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

(3)

where $c_{l} = m_{\tau}$ and $c_{\nu} = m_{0}$ measure the mass scales of charged leptons and neutrinos, respectively. Note that the special case $c_{\nu} = m_{0} = 0$ is not excluded. The matrices given in Eq. (3) exhibit the underlying $S(3)_{L} \times S(3)_{R}$ symmetry for the charged leptons and the $S(3)$ symmetry for the neutrinos (see also Ref. [5]). We leave it open at the moment whether the neutrino masses are of Fermi-Dirac or Majorana type. Of course there is no flavor mixing in these symmetry limits, in which $m_{e} = m_{\mu} = 0$ and $m_{1} = m_{2} = m_{3} = m_{0}$ hold. A simple diagonal breaking of the flavor democracy for $M_{0l}$ and the mass degeneracy for $M_{0\nu}$, as introduced in Ref. [3], may lead to phenomenologically instructive predictions for neutrino oscillations. Below we proceed with two different symmetry-breaking steps.

(i) A small perturbation to the (3,3) elements of $M_{0l}$ and $M_{0\nu}$ is introduced [6]. The resultant mass matrices read

$$
M_{1l} = \frac{c_{l}}{3} \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 + \varepsilon_{l}
\end{pmatrix}, \\
M_{1\nu} = c_{\nu} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 + \varepsilon_{\nu}
\end{pmatrix},
$$

(4)

where $|\varepsilon_{l}| \ll 1$ and $|\varepsilon_{\nu}| \ll 1$. Now the charged lepton mass matrix ceases to be of rank one, and the muon becomes massive ($m_{\mu} = 2|\varepsilon_{l}|m_{\tau}/9$ to the leading order of $\varepsilon_{l}$). The neutrino mass matrix

\[M_{0\nu} = c_{\nu} \text{Diag}\{\eta_{1}, \eta_{2}, \eta_{3}\}\]

with $\eta_{i} = \pm 1$ and $|m_{i}| = m_{0}$ [4].
mass $m_3$ is no more degenerate with $m_1$ and $m_2$ (i.e., $|m_3 - m_0| = m_0|\varepsilon_\nu|$). It is easy to see, after the diagonalization of $M_{1l}$ and $M_{1\nu}$, that the second and third lepton families have a definite flavor mixing angle $\theta$. We obtain $\tan \theta = -\sqrt{2}$ if the small correction of $O(m_\mu/m_\tau)$ is neglected. Then neutrino oscillations at the atmospheric scale arise in $\nu_\mu \leftrightarrow \nu_\tau$ transitions with the mass-squared difference $\Delta m^2_{32} = \Delta m^2_{31} \approx 2m_0|\varepsilon_\nu|$, where $\Delta m^2_{ij} \equiv |m_i^2 - m_j^2|$. The corresponding mixing factor is in good agreement with current data ($\sin^2 2\theta \approx 8/9$).

(ii) A small perturbation to the (2,2) or (1,1) elements of $M_{1l}$ and $M_{1\nu}$ is introduced, in order to generate the electron mass and to lift the degeneracy between $m_1$ and $m_2$. It has been argued in Refs. [3, 5], in analogy to the quark case, that at this step a simple and instructive perturbation to $M_{1l}$ should be of the form that its (1,1) and (2,2) elements simultaneously receive small corrections of the same magnitude and of the opposite sign. The analogous correction can be introduced to $M_{1\nu}$. Then the mass matrices become

$$
M_{2l} = \frac{c_l}{3} \left( \begin{array}{ccc}
1 - \delta_l & 1 & 1 \\
1 & 1 + \delta_l & 1 \\
1 & 1 & 1 + \varepsilon_l
\end{array} \right),
$$

$$
M_{2\nu} = c_\nu \left( \begin{array}{ccc}
1 - \delta_\nu & 0 & 0 \\
0 & 1 + \delta_\nu & 0 \\
0 & 0 & 1 + \varepsilon_\nu
\end{array} \right),
$$

where $|\delta_l| \ll 1$ and $|\delta_\nu| \ll 1$. One finds $m_e = |\delta_l|^2 m_\tau^2/(27m_\mu)$ to the leading order as well as $m_1 = m_0(1 - \delta_\nu)$ and $m_2 = m_0(1 + \delta_\nu)$. The diagonalization of $M_{2l}$ and $M_{2\nu}$ leads to a full $3 \times 3$ flavor mixing matrix, given as $U_0$ in Eq. (2) if small corrections of $O(\sqrt{m_e/m_\mu})$ and $O(m_\mu/m_\tau)$ are neglected. Then the solar neutrino deficit can be interpreted by $\nu_e \leftrightarrow \nu_\mu$ oscillations with the mass-squared difference $\Delta m^2_{21} \approx 4m_0|\delta_\nu|$ and the maximal oscillation amplitude [3, 7].

If the corrections from nonvanishing muon and electron masses are taken into account, the leptonic flavor mixing matrix will in general read as $V = O_l U_0$, where $O_l$ is an orthogonal matrix. Three rotation angles of $O_l$ are functions of the mass ratios $m_e/m_\mu$ and $m_\mu/m_\tau$. Due to the strong hierarchy of the charged lepton mass spectrum [8], i.e.,

$$
\alpha \equiv \sqrt{\frac{m_e}{m_\mu}} \approx 0.0695 ,
$$

$$
\beta \equiv \frac{m_\mu}{m_\tau} \approx 0.0594 ,
$$

$O_l$ is expected not to deviate much from the unity matrix. In our specific symmetry-breaking
case discussed above, we obtain

\[
O_l = \begin{pmatrix}
1 - \frac{1}{2} \alpha^2 & \alpha & \sqrt{2} \alpha \beta \\
-\alpha & 1 - \frac{1}{2} \alpha^2 - \frac{1}{4} \beta^2 & -\frac{1}{\sqrt{2}} \beta \\
-\frac{3}{\sqrt{2}} \alpha \beta & \frac{1}{\sqrt{2}} \beta & 1 - \frac{1}{4} \beta^2
\end{pmatrix}
\]  
(7)

to the next-to-leading order. Note that there is another solution for \(O_l\) and it can directly be obtained from Eq. (7) with the replacements \(\alpha \rightarrow -\alpha\) and \(\beta \rightarrow -\beta\). The leptonic flavor mixing matrix turns out to be

\[
V_{(\pm)} = U_0 \pm (\alpha A - \beta B) - \left(\alpha^2 C - \alpha \beta D + \beta^2 E\right),
\]  
(8)
in which the constant matrices \(A \ldots E\) read as

\[
A = \begin{pmatrix}
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

\[
B = \begin{pmatrix}
0 & 0 & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\
-\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix},
\]

\[
C = \begin{pmatrix}
\frac{1}{2\sqrt{2}} & \frac{-2}{\sqrt{2}} & 0 \\
\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{-1}{\sqrt{6}} \\
0 & 0 & 0
\end{pmatrix},
\]

\[
D = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\
0 & 0 & 0 \\
\frac{-3}{2} & \frac{3}{2} & 0
\end{pmatrix},
\]

\[
E = \begin{pmatrix}
\frac{1}{4\sqrt{6}} & \frac{1}{4\sqrt{6}} & \frac{-1}{2\sqrt{6}} \\
\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}}
\end{pmatrix}.
\]  
(9)

The effects of \(O(\alpha^2)\), \(O(\alpha \beta)\) and \(O(\beta^2)\) on neutrino oscillations will be discussed subsequently.

In general a \(3 \times 3\) flavor mixing matrix \(V\) can be parametrized, in terms of three Euler angles and one \(CP\)-violating phase, as follows \[^3\]:

\[
V = \begin{pmatrix}
s_t s_\nu c + c_t c_\nu e^{-i\phi} & s_t c_\nu c - c_t s_\nu e^{-i\phi} & s_t s \\
c_t s_\nu c - s_t c_\nu e^{-i\phi} & c_t c_\nu c + s_t s_\nu e^{-i\phi} & c_t c \\
-s_\nu s & -c_\nu s & c
\end{pmatrix},
\]  
(10)

where \(s_t \equiv \sin \theta_t\), \(c_\nu \equiv \cos \theta_\nu\), \(s \equiv \sin \theta\), etc. Possible tiny \(CP\)-violating effects will not be discussed here, and we take \(\phi = 0\). We then obtain \(\tan \theta_t = 0\), \(\tan \theta_\nu = 1\) and \(\tan \theta = -\sqrt{2}\) in
Table 1: Numerical results for mixing angles $\theta_l$, $\theta_\nu$, $\theta$ and $\sin^2 2\theta_{\text{sun}}$, $\sin^2 2\theta_{\text{atm}}$ to the next-to-leading order.

| Case  | $\theta_l$ | $\theta_\nu$ | $\theta$ | $\sin^2 2\theta_{\text{sun}}$ | $\sin^2 2\theta_{\text{atm}}$ |
|-------|------------|--------------|----------|-----------------------------|-----------------------------|
| "$V(+)"$ | 3.6° | 44.4° | -57.0° | 0.99 | 0.84 |
| "$V(-)"$ | -4.3° | 44.4° | -52.2° | 0.99 | 0.94 |

the limit where terms of $O(\alpha)$ and $O(\beta)$ are neglected. Taking small corrections of $O(\alpha)$ and $O(\beta)$ into account, we arrive at $\tan \theta_l = \pm \alpha$, $\tan \theta_\nu = 1$ and $\tan \theta = -\sqrt{2} \left(1 \pm \frac{3\beta}{2}\right)$, where the "±" signs correspond to $V(\pm)$ in Eq. (8). The full next-to-leading-order results for three mixing angles are found to be

\[
\begin{align*}
\tan \theta_l &= \pm \alpha \left(1 \pm \frac{3\beta}{2}\right), \\
\tan \theta_\nu &= 1 - 3\sqrt{3} \alpha \beta, \\
\tan \theta &= -\sqrt{2} \left(1 \pm \frac{3\beta}{2}\right). 
\end{align*}
\]

(11)

One can see that the rotation angle $\theta_\nu$, which primarily describes the mixing between the first and second neutrino families, only receives a tiny correction from the charged lepton sector.

Following Ref. [3] we take $\Delta m_{\text{sun}}^2 = \Delta m_{21}^2$ and $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2 \approx \Delta m_{31}^2$ to accommodate current data on solar and atmospheric neutrino oscillations. Calculating the survival probability $P(\nu_e \to \nu_e)$ and the transition probability $P(\nu_\mu \to \nu_\tau)$ to the next-to-leading order, we arrive at

\[
\begin{align*}
\sin^2 2\theta_{\text{sun}} &= 1 - \frac{8}{3} \alpha^2, \\
\sin^2 2\theta_{\text{atm}} &= \frac{8}{9} \left(1 \pm \beta\right). 
\end{align*}
\]

(12)

The numerical results for mixing angles in Eqs. (11) and (12) are listed in Table 1. One can see that the flavor mixing patterns "$V(+)$" and "$V(-)$" are both consistent with the present experiments.

The near degeneracy of three neutrino masses assumed in the phenomenological scenario...
under discussion leads to
\[ \left| \frac{m_2 - m_1}{m_3 - m_2} \right| \approx \frac{\Delta m_{\text{sun}}^2}{\Delta m_{\text{atm}}^2} \sim 10^{-7}, \tag{13} \]
or \[ |\delta|/|\varepsilon| \approx \frac{\Delta m_{\text{sun}}^2}{2\Delta m_{\text{atm}}^2} \sim 10^{-7}. \] This kind of neutrino mass spectrum can account for the hot dark matter of the universe, if \( m_i \approx 2 \text{ eV} \) (for \( i = 1, 2, 3 \)). The relatively large gap between \( \Delta m_{21}^2 \) and \( \Delta m_{32}^2 \) (or \( \Delta m_{31}^2 \)) has some implications on the forthcoming long-baseline experiments, as we shall see later on.

Now we consider the effect of nonvanishing \( \theta_l \) on the survival probability of electron neutrinos in a long-baseline (LB) experiment, in which the oscillation associated with the mass-squared difference \( \Delta m_{21}^2 \) can be safely neglected due to \( \Delta m_{12}^2 \ll \Delta m_{32}^2 \approx \Delta m_{31}^2 \). It is easy to find
\[ P(\nu_e \rightarrow \nu_e)_{LB} = 1 - \frac{8}{3} \alpha^2 (1 \mp 2\beta) \sin^2 \left( 1.27 \frac{\Delta m_{32}^2 L}{|P|} \right), \tag{14} \]
where \( P \) denotes the momentum of the neutrino beam (in unit of GeV), and \( L \) is the distance between the neutrino production and detection points (in unit of km). The oscillation amplitude amounts to 1.1% (the “\( V_+ \)” case) or 1.4% (the “\( V_- \)” case) and might be detectable. Note that the CHOOZ experiment, in which the survival probability of anti-electron neutrinos is measured, indicates the oscillation amplitude \( \sin^2 2\theta_{\text{CH}} < 0.18 \) for \( \Delta m_{\text{CH}}^2 \geq 9 \times 10^{-4} \text{ eV}^2 \).\[ \]In the three-flavor scheme under consideration, it is appropriate to set \( \Delta m_{\text{CH}}^2 = \Delta m_{32}^2 \approx \Delta m_{31}^2 \), which essentially has no conflict with the Super-Kamiokande data. One can see that the small mixing obtained in Eq. (14) lies well within the allowed region of \( \sin^2 2\theta_{\text{CH}} \).

The transition probability of \( \nu_\mu \) to \( \nu_e \) in such a long-baseline neutrino experiment reads
\[ P(\nu_\mu \rightarrow \nu_e)_{LB} = \frac{16}{9} \alpha^2 (1 \mp \beta) \sin^2 \left( 1.27 \frac{\Delta m_{32}^2 L}{|P|} \right). \tag{15} \]
Here the mixing factor is about 0.8% (the “\( V_+ \)” case) or 0.9% (the “\( V_- \)” case). The proposed K2K experiment is expected to have a sensitivity of \( \sin^2 2\theta > 10\% \) for \( \nu_e \leftrightarrow \nu_\mu \) oscillations, while the MINOS experiment could probe values of the mixing as low as \( \sin^2 2\theta = 1\% \).\[ \]Thus a test of or a constraint on the prediction obtained in Eq. (15) would be available in such experiments. On the other hand, the probability for \( \nu_\mu \rightarrow \nu_\tau \) transitions in the assumed long-baseline experiment is essentially the same as that for the atmospheric neutrino oscillations, i.e.,
\[ P(\nu_\mu \rightarrow \nu_\tau)_{LB} = \frac{8}{9} (1 \mp \beta) \sin^2 \left( 1.27 \frac{\Delta m_{32}^2 L}{|P|} \right). \tag{16} \]
The mixing factor, corresponding to two different perturbative corrections of the magnitude \( \beta \sim 6\% \), takes the value 0.84 or 0.94 (see Table 1). It is also worth mentioning that the
transition probability of $\nu_e \to \nu_\tau$, which satisfies the sum rule

$$P(\nu_e \to \nu_e)_{LB} + P(\nu_e \to \nu_\mu)_{LB} + P(\nu_e \to \nu_\tau)_{LB} = 1,$$  \hspace{1em} (17)

is smaller (with the mixing factor $8\alpha^2/9 \approx 0.4\%$) and more difficult to detect.

As pointed out in Ref. [3], the three-flavor scenario with near degenerate neutrino masses and near-maximal mixing angles has no conflict with current data on the neutrinoless $\beta\beta$-decay [13], if neutrinos are of the Majorana type. However, it is not compatible with the result of the LSND experiment [14]. The new analysis from the KARMEN experiment [15] seems to be in contradiction with the LSND evidence for $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ oscillations, and a further examination of the latter will be available in the coming years.

Finally let us give some comments on the bi-maximal mixing scenario of three neutrinos, which is recently proposed by Barger et al [16]. The relevant flavor mixing matrix, similar to $U_0$ in Eq. (2), reads as follows:

$$U' = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$  \hspace{1em} (18)

Such a flavor mixing pattern is independent of any lepton mass and leads exactly to $\sin^2 2\theta_{\text{atm}} = \sin^2 2\theta_{\text{sun}} = 1$ for neutrino oscillations. We find that $U'$ can be derived from the following charged lepton and neutrino mass matrices:

$$\begin{align*}
M'_l &= \frac{c'_l}{2} \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \begin{pmatrix} \delta'_l & 0 & 0 \\ 0 & \varepsilon'_l & 0 \\ 0 & \varepsilon'_l & 0 \end{pmatrix} \right], \\
M'_\nu &= \frac{c'_\nu}{2} \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} \delta'_\nu & 0 & 0 \\ 0 & \varepsilon'_\nu & 0 \\ 0 & \varepsilon'_\nu & 0 \end{pmatrix} \right],
\end{align*}$$

where $|\delta'_{l,\nu}| \ll 1$ and $|\varepsilon'_{l,\nu}| \ll 1$. In comparison with the democratic mass matrix $M_0$ given in Eq. (3), which is invariant under the $S(3)_L \times S(3)_R$ transformation, the matrix $M'_l$ in the limit $\delta'_l = \varepsilon'_l = 0$ only has the $S(2)_L \times S(2)_R$ symmetry. However $M'_\nu$ in the limit $\delta'_\nu = \varepsilon'_\nu = 0$ takes the same form as $M_0$, which displays the $S(3)$ symmetry. The off-diagonal perturbation of $M'_l$ allows the masses of three charged leptons to be hierarchical:

$$\{m_e, m_\mu, m_\tau\} = \frac{c'_l}{2} \left\{ |\delta'_l|, |\varepsilon'_l|, 2 + \varepsilon'_l \right\}.$$  \hspace{1em} (20)
We get \( c'_l = m_\mu + m_\tau \approx 1.88 \text{ GeV}, \left| \varepsilon'_l \right| = 2m_\mu/(m_\mu + m_\tau) \approx 0.11 \) and \( |\delta'_l| = 2m_e/(m_\mu + m_\tau) \approx 5.4 \times 10^{-4} \). The off-diagonal perturbation of \( M'_\nu \) makes three neutrino masses non-degenerate:

\[
\{m_1, m_2, m_3\} = c'_\nu \{1 + \varepsilon'_\nu, 1 - \varepsilon'_\nu, 1 + \delta'_\nu\}.
\]  

(21)

Taking \( \Delta m^2_{\text{sun}} = \Delta m^2_{21} \) and \( \Delta m^2_{\text{atm}} = \Delta m^2_{32} \approx \Delta m^2_{31} \) for solar and atmospheric neutrino oscillations, respectively, we then arrive at \( |\varepsilon'_\nu|/|\delta'_\nu| \approx \Delta m^2_{\text{sun}}/(2\Delta m^2_{\text{atm}}) \sim 10^{-7} \), a result similar to that obtained above. The diagonalization of \( M'_l \) and \( M'_\nu \) leads straightforwardly to the flavor mixing matrix \( U' \). In Ref. [16] a different neutrino mass matrix has reversely been derived from the given \( U' \) in a flavor basis that the charged lepton mass matrix is diagonal. The emergence of the bi-maximal flavor mixing pattern from \( M'_l \) and \( M'_\nu \) in Eq. (19) is, in our point of view, similar to that of the near-maximal flavor mixing pattern from \( M_{2l} \) and \( M_{2\nu} \) in Eq. (5).

In summary, we have displayed a simple and phenomenologically instructive symmetry-breaking pattern for the charged lepton mass matrix with flavor democracy and the neutrino mass matrix with mass degeneracy. Large (near-maximal) leptonic flavor mixing angles, which are favored by recent Super-Kamiokande data on atmospheric and solar neutrino oscillations, emerge naturally from our scenario and have been evaluated to the next-to-leading order. The oscillation amplitudes of \( \nu_e \leftrightarrow \nu_e \) and \( \nu_e \leftrightarrow \nu_\mu \) transitions are predicted to be about 1% for the upcoming long-baseline neutrino experiments. We expect that further results from the Super-Kamiokande and other neutrino experiments could finally clarify if the solar neutrino deficit is attributed to the long-wavelength vacuum oscillation and provide stringent tests of the model discussed here.

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