Methods for determining the maximum energy area for wind power turbine

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Abstract. The present paper analyses two control methods for wind power systems to achieve optimum performance in terms of energy. The load control methods at wind turbines (WT) to achieve optimal operation, is based on the fact that the energy captured by the wind energy depends significantly on the mechanical angular velocity (MAV). In order to perform a function in the maximum power point (MPP) area, the load on the electric generator (EG) is changed and the power given by the wind turbine estimated. The first method, known as the small perturbation method, is widely used, becoming classic. The second original method consists in determining the optimal angular mechanical velocity $\omega_{\text{OPTIM}}$, using a low power auxiliary wind turbine that operates without load, at maximum mechanical angular velocity $\omega_{\text{MAX}}$. This value of MAV takes into account the wind speed evolution in time. The method bases on the particularity of the $\omega_{\text{OPTIM}}/\omega_{\text{MAX}}$ report that does not depend on the time variation of wind speed values and has a constant value for a given WT.

1. Introduction

The operation problem in the maximum energy area of wind turbine’s (WT) is extensively treated in the literature at constant wind speeds over time, using different simplified mathematical models (MM - WT), different from those encountered in operation. At time-varying wind speeds, due to rapid wind speed changes and significant mechanical inertia the operation in the maximum power point (MPP) presents a complex problem. [1, 2]

The small perturbation method to reach the MPP area is considered to be the main used computing procedure, a method which does not provide an MPP operation and introduce power swing in the generator.

The dependence between the mechanical angular velocity and the power of a WT, ie the $P_{\text{WT}}(\omega)$, register, for a certain constant wind speed, a maximum at $\omega_{\text{OPTIM}}$. The power characteristic has some important points: the maximum power point (MPP) to which corresponds $\omega_{\text{OPTIM}}$ and the zero power point with $\omega_{\text{MAX}}$, the maximum angular velocity. The operation in the MPP zone is provided by the right determination, in operating conditions, of those points.

The values for $\omega_{\text{OPTIM}}$, the optimum angular velocity, are deduced based on simulations using the classic mathematical models of the Permanent Magnet Synchronous Generator (PMSG) and those of the wind turbines.
2. The small perturbation method

Considering the form of the power characteristic curve $P_{WT}(\omega)$, three distinct zones can be identified from the point of view of the sign of the power derivation $P_{WT}(\omega)/d\omega$: positive, negative and with the value of the derivative tending to zero. The optimal energy area, that contains the MPP, is the zone with the power derivative value pointing to zero. Depending on the wind speed, the simulation analyzes are done in two cases: at constant and at time variable wind speed.

2.1 Simulation analysis at constant wind speed

The power characteristics have the form:

$$P_{WT}(\omega,v)=1191.5 \cdot (\frac{v}{\omega} - 0.02)e^{-98.02(\frac{v}{\omega})} \cdot v^3$$  \hspace{1cm} (1)

For the wind speed of $v=11\,[m/s]$, $P_{WT}(\omega,11)$, the value for $\omega_{OPTIM}$ is obtained cancelling the derivative power,

$$\frac{dP_{WT}(\omega,v)}{d\omega} = 0$$  \hspace{1cm} (2)

In the wind power characteristics, according to (1), Figure 1, we can distinguish three areas: OA, with $dP_{WT}(\omega,v)/d\omega > 0$, AB, with $dP_{WT}(\omega,v)/d\omega \approx 0$ and BC with $dP_{WT}(\omega,v)/d\omega < 0$.

![Figure 1. Power characteristic of the wind turbine.](image)

In the area $\omega < \omega_{OPTIM}$ with $dP_{WT}(\omega,v)/d\omega > 0$, the load at the electric generator it's shrinking or becomes null and so the operation point is moving to the MPP area.

In the area $\omega > \omega_{OPTIM}$ with $dP_{WT}(\omega,v)/d\omega < 0$ the load at the electric generator is increasing and so the operation point is moving to the MPP area.

Considering that in point $P_1$ of OA area, initially the turbine operates, without load, at $\omega_i=300[rad/s]<\omega_{OPTIM}$, the derivative power has the value of $dP_{WT}(\omega,v)/d\omega = \Delta P_{WT}(\omega,v)/\Delta \omega$, were the value for $\Delta \omega$ was considered a value of 2% from $\omega_{OPTIM}$: $\Delta \omega=2\% \omega_{OPTIM}=7.28\,[rad/s]$.

Determining the power WT at a certain mechanical angular velocity (MAV) $\omega_k$ is done based on the measurement of the generated electricity and MAV, computing thus the captured energy of the wind turbine for a given time interval, $\Delta t$, as demonstrated in the following, starting from the movement equation [7, 8]:

$$J \frac{d\omega}{dt} = \tau_{WT} - \tau_{MPSG}$$  \hspace{1cm} (3)

with $\tau_{MPSG}$ – permanent magnet synchronous generator torque, $\tau_{WT}$ – wind turbine torque and $J$ – equivalent inertia moment.

Integrating relation (3) on the sampling interval, $\Delta t=t_i-t_{i-1}$, result:

$$J \cdot \left(\omega^2 - \omega_{i-1}^2\right) / 2 = \int_{t_{i-1}}^{t_i} P_{WT} \, dt - \int_{t_{i-1}}^{t_i} P_{MPSG} \, dt$$  \hspace{1cm} (4)
and computing the captured energy of the wind turbine from (4), we obtain the equivalent power of the WT:

\[
P_{WT-average} = \left( J \cdot (\omega_t^2 - \omega_{t-1}^2) / 2 + \int_{t-1}^{t} P_{MPG} \, dt \right) / \Delta t
\]

(5)

Considering the initial MAV \( \omega(0) = \omega_A(0) = 300 \) rad/s, a wind speed of 11 m/s and an time interval of \( \Delta t = 20 \) s, from the movement equation

\[
45 \frac{d\omega}{dt} = 1191.5 \left( \frac{11}{\omega} - 0.02 \right) e^{-0.06(11/\omega)} \cdot 11^3
\]

\[
\omega(0) = 300
\]

we obtain an MAV of \( \omega = \omega(20) = 301.8 \) rad/s for which the average power registers the value \( P_{WT, initial}(300) = \frac{1}{2} \left( J \cdot (\omega_{t-1}^2 - \omega_t^2) \right) / \Delta t = 730.31 \) W. In calculating the power derivative considering \( \Delta \omega = 2 \% \omega_{OPTIM} = 7.29 \) rad/s, we get \( \omega = \omega(0) + \Delta \omega = 307.29 \) rad/s. Based on this procedure, the value of 307.27 rad/s is considered again an initial value, which substituted in (6), leads to \( P_{WT, final}(307.29) = 748.03 \) W. With these results, in the area of P1, the derivative power has the value \( \Delta P_{WT}(\omega, v) / \Delta \omega = 2.437 \).

With this results, the optimum value for \( \omega_{OPTIM} \) of 364.26 rad/s, can be reached

AB area, with \( dP_{WT}(\omega, v) / d\omega \approx 0 \) and \( \omega = \omega_{OPTIM} \). Considering that the system function initial in point A at MAV of \( \omega_A = 360 \) rad/s, substituting in the movement equation (6), results \( \omega = \omega(20) = 361.03 \) rad/s and \( \Delta \omega = \omega_A - \omega = 361.03 - 0.03 = 360 \) rad/s, obtaining the average power at the beginning of the interval \( P_{WT, initial}(360) = \frac{1}{2} \left( J \cdot (\omega_{t-1}^2 - \omega_t^2) \right) / \Delta t = 835.49 \) W. At the end of the interval, MAV has the value \( \omega_A(35) = 301.67 \) rad/s. Substituting in the movement equation (6), \( \omega \) results \( \omega = \omega(20) = 368.3 \) rad/s and the average power at the end of the interval computed from the variation of the kinetic energies becomes \( P_{WT, final}(367.29) = 835.81 \) W. The derivative power in the AB area has the value \( \Delta P_{WT}(\omega, v) / \Delta \omega = 4.38 \cdot 10^{-2} \).

BC area, with \( dP_{WT}(\omega, v) / d\omega < 0 \) and \( \omega \approx \omega_{OPTIM} \). Considering that the system function initial in point P2 at MAV of \( \omega_A = 440 \) rad/s, we obtain, substituting in the movement equation (6), \( \omega = \omega(20) = 440.69 \) rad/s and \( \Delta \omega = \omega_A - \omega = 440.69 - 0.69 = 440 \) rad/s, obtaining the average power of \( P_{WT, initial}(440) = \frac{1}{2} \left( J \cdot (\omega_{t-1}^2 - \omega_t^2) \right) / \Delta t = 835.49 \) W. Finally, MAV has the value \( \omega_A = \omega(0) + \Delta \omega = 447.94 \) rad/s. Substituting in the movement equation (6), \( \omega \) results \( \omega = \omega(20) = 447.29 \) rad/s and the average power becomes \( P_{WT, final}(367.29) = 654.64 \) W. The derivative power in the BC area is \( \Delta P_{WT}(\omega, v) / \Delta \omega = -3.97 \).

In the MPP area the power derivation tends of zero. Based on this remark, an optimal control can be achieved. The necessary time of MPP localization is in the order of minutes, what under constant wind speed conditions does not create problems in the control system.

2.2 Simulation analysis at variable wind speed

For a time variable wind speed \( v(t) = 15 - 7 \cdot sin0.17943t \) as in Figure 2 [9], the obtained optimal values are \( P_{WT, MAX} = 2690 \) W and \( \omega_{OPTIM} = 579.7 \) rad/s [5].

Considering the equivalent power characteristics, Figure 3, we can also distinguish for this analysis three areas: OA with \( dP_{WT}(\omega, v) / d\omega > 0 \); AB with \( dP_{WT}(\omega, v) / d\omega \approx 0 \); BC with \( dP_{WT}(\omega, v) / d\omega < 0 \).

OA area, with \( dP_{WT}(\omega, v) / d\omega > 0 \) and \( \omega < \omega_{OPTIM} \). Considering the initial MAV of \( \omega(0) = 300 \) rad/s and a time interval \( \Delta t = 35s \), substituting in the time movement equation (6) we obtain \( \omega = \omega(35) = 301.67 \) rad/s. The MAV variation in the given interval, \( \Delta \omega = 1.67 \) rad/s, meet an average power of \( P_{WT, initial}(300) = 645.94 \) W. Considering \( \Delta \omega = 2 \% \omega_{OPTIM} = 11.594 \) rad/s for computing the power derivative, we obtain \( \omega = \omega(0) + \Delta \omega = 311.59 \) rad/s. Computing the movement equation for this new value, we achieved
\( \omega(35) = 313.4 \text{ rad/s} \) and \( P_{WT-Final}(311.59) = 772.22 \text{ W} \). With those results, the power derivative in the OA area becomes \( \frac{\Delta P_{WT}(\omega, v)}{\Delta \omega} = 7 \).

To observe better the AB area, with \( \frac{dP_{WT}(\omega, v)}{d \omega} \approx 0 \) and \( \omega \approx \omega_{OPTIM} \), we will analyse two cases: \( \alpha(0) = 500 \text{ rad/s} \) and \( \alpha(0) = 570 \text{ rad/s} \). Considering the analyse starting with \( \omega_0 = 500 \text{ rad/s} \), passing the same steps as for the OA area, the power derivative is 3.83 while for \( \omega_0 = 570 \text{ rad/s} \) the result power derivative is -0.28. It is noticeable that for this second case the value of the power derivative tends to zero, being much less than the first case this because the initial MAV in the second case is close to the optimum, \( \omega_{OPTIM} \).

Working without load in the OA area, the WT can be brought near the optimum energy area, Ab area, knowing the power derivative value. Near the AB area the power derivative value decrease and the electric generator is coupled to the load.

The energy taken over by the GSMP, can be computed by solving the movement equation,

\[
P_G = 845 \alpha^2 \left( 5R + 8 \right) \frac{4\omega^2 + 625R^2 + 2000R + 1600}{1250R^2 + 4000R + 3200 + 7\omega^2}
\]  \( (7) \)

For a load resistance of \( R_0 = 260.49 \Omega \) [5], the movement and energy equations, starting from the initial conditions \( \alpha(0) = 571 \text{ rad/s} \) and \( W_G(0) = 0 \), at times intervals of \( \Delta t = 35s \), we obtain the MAV and energies \( \alpha(35) = 571.01 \text{ rad/s} \) and \( W_G(35) = 71759J \) up to \( \alpha(5040) = 571.48 \text{ rad/s} \) and \( W_G(5040) = 10345 \cdot 10^7 \text{ J} \).

This value of the power is maxim for the analysed wind speed, Figure 2. To demonstrate this, we modify the load resistance with 10%, \( R_1 = 234.49 \Omega \) and resume the simulation analysis.

As it can be seen from Figure 4, MAV decreases as a result of load increasing at the GE. In the MPP area, the power derivative tends to zero.

BC area, with \( (dP_{WT}/d\omega) < 0 \) and \( \omega > \omega_{OPTIM} \). Considering that the system functions at MAV of \( \omega_0 = 666 \text{ rad/s} \), from the movement equation results \( \omega = \alpha(35) = 668.11 \text{ rad/s} \), for which the average power registers the value \( P_{WT-initial}(440) = 1809.6 \text{ W} \) and \( P_{WT-final}(677.59) = 1744.9 \text{ W} \), the power derivative in this area has the value \( \Delta P_{WT}/\Delta \omega = -5.58 \).
2.3 Power fluctuations at the generator

Considering the initial value for the load resistance $R=260.49\ \Omega$ and $\omega=571\text{rad/s}$, the generator's power is obtained computing in (7), $P_G=2055.5\text{W}$ and changing the load resistance with 10% to $R_1=234.49\ \Omega$, results $P_G=2270\text{W}$, an difference of $\Delta P_{GSMP}=224.5\text{W} \ (9.5\%)$. Therefore, changing the GSMP load creates power oscillations which accumulate with those generated by the wind speed variation over time.

The power oscillations caused by wind speed variation over time can be estimated; for the analysed $R_0$, wind speed variation and initial MAV $\omega(0)=571\ \text{rad/s}$, solving the movement equation, results the form for MAV variation in time, Figure 5.

This time variation of the MAV can be approximated by

$$\omega(t)=570.25+0.75\sin(0.179\cdot t) \ (8)$$

Based on this, the time variation of the generators power (7), is represented in Figure 6.

Based on the above analyse, the electrical power variations are due changes in the generators load and the wind speed time variation. The power oscillation decrease due to increased generator load, can be achieved by lowering the load shock, ie another load resistance for example a 5% decrease in load resistance.

Determining the optimal energy zone, with the operating point in the maximum power, MPP, by modifying the load at the PMSG, introduces power oscillation. The oscillation is, however, much lower than the power oscillations due to the variation of the wind over time.
Due to the time variation of the wind speed, MAV vary over time too, as it can be seen in Figure 7. In the time intervals \(\Delta t_1\) and \(\Delta t_3\), \(\omega\) has almost constant values, so \(d\omega/dt=0\), and in the time interval \(\Delta t_2\) and \(\Delta t_4\), \(\omega\) changes significantly, so \(d\omega/dt\neq0\). Therefore it is very important to choose the value of the time intervals between which the MAV is measured.

3. Method based on the report \(\omega_{\text{OPTIM}}/\omega_{\text{MAX}}\)

Determining \(\omega_{\text{OPTIM}}\) can be done quickly and with very small errors by using an auxiliary wind turbine. Equations should be centred and should be numbered with the number on the right-hand side. This has, based on the connection relationship of the \(\omega_{\text{OPTIM}}\), the variation of the kinetic energy, \(\Delta W_{\text{kinetic}}\) is computed as

\[
\Delta W_{\text{kinetic}} = J \left( \omega_{k-1} \omega_{k} \right) / 2
\]

where: \(J\) - equivalent inertia moment, \(\omega_k\) – MAV at \(t_k\), \(\omega_{k-1}\) – MAV at \(t_{k-1}\).

To illustrate the independence of the \(\omega_{\text{OPTIM}}/\omega_{\text{MAX}}\) ratio, we consider two possible situations with different wind speeds. For \(v(t)=15-7 \cdot \sin 0.17943t\), has been achieved \(\omega_{\text{OPTIM}}=579.7\) rad/s. At the operation without load, \(\omega_{\text{MAX}}=852\) rad/s, the ratio \(\omega_{\text{OPTIM}}/\omega_{\text{MAX}}\) being equal with 0.68. For \(v(t)=10-7 \cdot \sin 0.17943t\), we obtain \(\omega_{\text{OPTIM}}=453.25\) rad/s, the ratio \(\omega_{\text{OPTIM}}/\omega_{\text{MAX}}= 453.25/ 663.92 = 0.68269\).

Finally it can be concludes that the value of the \(\omega_{\text{OPTIM}}/\omega_{\text{MAX}}\) ratio is the same at any wind speed, \(\omega_{\text{OPTIM}}/\omega_{\text{MAX}}=0.68\).

Based on the link between \(\omega_{\text{OPTIM}}\) and \(\omega_{\text{MAX}}\) and computing the kinetic energy variations, we propose and simple and efficient control algorithm, Figure 8. The main steps of the control algorithm are:

- Step \(k\) at \(t_k\) moment: computing \(\omega_{\text{OPTIM}}\) from the operation without load of the auxiliary turbine, \(\omega_{\text{MAX}-ak}\), based on the connection relationship of the \(\omega_{\text{OPTIM}}=0.66 \cdot \omega_{\text{MAX}-ak}\);
The values of MAV $\omega_k$ and the over taken energy of the generator $W_G(\Delta t)$ are obtained based on measurements at the time moment $t_k$:

- Variation of the kinetic energies computed with $\omega_k$ and $\omega_{OPTIM-k}$, $\Delta W_k=J\cdot(\omega_k^2-\omega_{OPTIM-k}^2)/2$;
- Generator over taken energy in the conditions the system would have reached $\omega_{OPTIM-k}$: $W_k-\text{OPTIM}=W_G(\Delta t)+\Delta W_k$;
- The prescribed power of the generator to reach the optimal MAV, $\omega_{OPTIM-k}$, $P_{G,K-OPTIM}= W_k-\text{OPTIM}/\Delta t$

Through the proposed control algorithm, which is based only on experimental data: MAV at the generator and the auxiliary turbine, generator energy and power, an operation is in the optimal energy area is performed.

4. Conclusions
For the exploitation of the wind potential to become as less dependent on subsidence [12], the control methods that allow an efficiency increasing of this renewable technology, needs to be improved. By analysing several cases, it was possible to determine the basic sizes leading to an optimal operation, from energy point of view, and maintaining the wind turbine in the MPP area through speed and energy measuring at the generator. Essential is the correct determination of the maximum energy area, the energy captured by the wind turbine having maximum values in the MPP area. Through the simulations analysis, the PMSG loads could be identified, so that the Generator system will reach the optimal energy area in the briefest possible time. The second method is based on determination of the optimal MAV with an auxiliary WT, without load, on the direct proportionality of the optimum $\omega_{OPTIM}$ MAV to $\omega_{MAX}$. This prevents the use of mathematical models in the control process, which are only valid under certain conditions [13].

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