Interpreting the Atmospheric Neutrino Anomaly

R.P. Thun and S. McKee

Abstract

We suggest that the atmospheric neutrino anomaly observed in the Super-Kamiokande (and other) experiments results from the combined effects of muon-neutrino to tau-neutrino oscillations with a $\Delta m^2$ value of approximately 0.4 $eV^2$ and oscillations between muon neutrinos and electron neutrinos (and vice-versa) with $0.0001 < \Delta m^2 < 0.001$ $eV^2$. With an appropriate choice of a three-neutrino mixing matrix, such a hypothesis is consistent with essentially all neutrino observations.

Key words: neutrino oscillations
1 Introduction

Statistically significant anomalies have been reported in observations of solar neutrinos [1–5], atmospherically-produced neutrinos [6–8], and accelerator-generated neutrinos [9]. These observations raise the question whether they can all be explained by a simple three-neutrino oscillation model [10–15]. So far, data on the atmospheric neutrino anomaly have been fitted only to a two-neutrino oscillation hypothesis. We believe that this may lead to an incorrect conclusion regarding the oscillation process and its associated parameters.

Several aspects of the Super-Kamiokande data motivate us to propose that muon-neutrino to tau-neutrino oscillations are observed with $\Delta m^2$ around 0.4 eV$^2$ and that, furthermore, oscillations between muon neutrinos and electron neutrinos (and vice-versa) are seen with $0.0001 < \Delta m^2 < 0.001 \text{ eV}^2$. First, low-energy ($E < 0.75 \text{ GeV}$) “overhead” (L~20 km) events show a highly significant deficit of muons, relative to electrons, indicating an oscillation process with $\Delta m^2 > 0.04 \text{ eV}^2$. This would be consistent with the positive oscillation result obtained in the LSND experiment which, when combined with negative results from other experiments, requires $\Delta m^2 \sim 0.4 \text{ eV}^2$. Second, Super-Kamiokande observes an additional, significant muon deficit for “upward” events for which the pathlength L~10,000 km. These events also show some enhancement of electron events, suggestive of muon-neutrino to electron-neutrino oscillations.

2 Assumed Model for Neutrino Oscillations

In what follows, we assume the simplest general case of three-neutrino mixing:

$$
\begin{pmatrix}
  v_e \\
v_\mu \\
v_\tau
\end{pmatrix} =
\begin{pmatrix}
  U_{11} & U_{12} & U_{13} \\
  U_{21} & U_{22} & U_{23} \\
  U_{31} & U_{32} & U_{33}
\end{pmatrix}
\begin{pmatrix}
v_1 \\
v_2 \\
v_3
\end{pmatrix}
$$

(1)

where $v_e, v_\mu, v_\tau$ are, respectively, the electron, muon, and tau neutrino states, and $v_1, v_2, v_3$ are the neutrino mass eigenstates with masses $m_1 < m_2 < m_3$. For simplicity, the unitary mixing matrix $U$ is assumed to be real and
expressed as:

\[
U = \begin{pmatrix}
  c_1 c_3 & s_1 c_3 & s_3 \\
  -s_1 c_2 - c_1 s_2 s_3 & c_1 c_2 - s_1 s_2 s_3 & s_2 c_3 \\
  s_1 s_2 - c_1 c_2 s_3 & -c_1 s_2 - s_1 c_2 s_3 & c_2 c_3
\end{pmatrix}
\] (2)

where c stands for cosine and s for sine of the appropriate three angles, \(\theta_1\), \(\theta_2\), and \(\theta_3\).

Oscillations of neutrinos of type \(\alpha\) into neutrinos of type \(\beta\) occur with a probability given by:

\[
P(\alpha \to \beta) = \delta_{\alpha\beta} - 4 \sum_{i=1}^{3} \sum_{j=1}^{3} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left[ \frac{(m_i^2 - m_j^2)L}{4E} \right]
\] (3)

where \(\delta_{\alpha\beta} = 1\) if \(\alpha = \beta\) and \(\delta_{\alpha\beta} = 0\) if \(\alpha \neq \beta\), \(L\) is the length of the neutrino flight path, and \(E\) is the neutrino energy. If \(m\) is given in eV, \(E\) in GeV, and \(L\) in km, then the factor of \(L/4E\) in the sine term becomes 1.27 \(L/E\).

We now discuss the interpretation of various observations in terms of this simple model. Given our hypothesis regarding the atmospheric neutrino data, we shall assume that \(m_3 \gg m_2 > m_1\). We define:

\[
m^2 \equiv m_2^2 - m_1^2 \\
M^2 \equiv m_3^2 - m_2^2 \\
\text{and } M^2 + m^2 = m_3^2 - m_1^2
\] (4)

with \(M^2 \gg m^2\).

For the mixing parameters \(U_{ij}\) obtained in our analysis, the value of \(M^2\) allowed by LSND is highly constrained by the results [16,17] of the Bugey reactor experiment \((M^2 > 0.2 \text{ eV}^2)\) and of the CDHS oscillation search \((M^2 < 0.4 \text{ eV}^2)\). We shall assume that \(M^2 = 0.4 \text{ eV}^2\) for reasons noted below. To produce a significant effect in Super-Kamiokande the value of \(m^2\) must be at the 0.0001 \(eV^2\) level or larger. The CHOOZ reactor experiment [19] has shown that \(\Delta m^2\) is smaller than 0.001 \(eV^2\). We shall therefore require that 0.0001 \(< m^2 < 0.001 \text{ eV}^2\).
3 LSND

The LSND probability to observe oscillations of muon neutrinos into electron neutrinos is given by:

\[
P(\text{LSND}) = -4 U_{23} U_{13} U_{21} U_{11} \sin^2 \left[ \frac{(M^2 + m^2)L}{4E} \right] \\
-4 U_{23} U_{13} U_{22} U_{12} \sin^2 \left( \frac{M^2 L}{4E} \right) \\
-4 U_{22} U_{12} U_{21} U_{11} \sin^2 \left( \frac{m^2 L}{4E} \right)
\]

(5)

In the LSND experiment [9] L=30 m and 36 < E < 60 MeV, so that for \( m^2 < 0.001 \text{ eV}^2 \) the third term in the above expression can be neglected. The first two terms can be combined by approximating \( M^2 + m^2 \approx M^2 \) and using the unitarity of the mixing matrix:

\[
P(\text{LSND}) = 4[U_{23} U_{13}]^2 \sin^2 \left( \frac{M^2 L}{4E} \right) = 4(s_{23} s_{33})^2 \sin^2 \left( \frac{M^2 L}{4E} \right)
\]

(6)

The measured value is \( P(\text{LSND})=0.0031 \pm 0.0011(\text{stat}) \pm 0.0005(\text{syst}) \). When other neutrino data are also considered, the smallness of \( P(\text{LSND}) \) arises in part from the smallness of \( s_3 \). For \( M^2 = 0.4 \text{ eV}^2 \), L=30 m, and a mean value of \( E=42 \text{ MeV} \), we obtain \( s_{23} s_3=0.0784 \).

4 Atmospheric Neutrinos

Super-Kamiokande will observe oscillations of muon neutrinos into tau neutrinos with a probability given by:

\[
P(\text{SK}; \nu_\mu \rightarrow \nu_\tau) = -4 U_{23} U_{33} U_{21} U_{31} \sin^2 \left[ \frac{(M^2 + m^2)L}{4E} \right] \\
-4 U_{23} U_{33} U_{22} U_{32} \sin^2 \left( \frac{M^2 L}{4E} \right) \\
-4 U_{22} U_{32} U_{21} U_{31} \sin^2 \left( \frac{m^2 L}{4E} \right)
\]

(7)

For the oscillation parameters considered in this paper, the third term can be
neglected and the first two terms combined to obtain:

\[
P(SK; v_\mu \rightarrow v_\tau) = 4[s_2 c_2 c_3^2]^2 \sin^2 \frac{M^2 L}{4E} = 2[s_2 c_2 c_3^2]^2
\]

(8)

where the variation in E and L allow us to average the sine-squared factor to a value of 0.5.

The smallness of the LSND effect and the failure to observe electron-neutrino disappearance in reactor experiments [16,19] indicate that the overall deficit of muons observed in the Super-Kamiokande atmospheric data for zenith angles satisfying \(\cos \theta_z > -0.6\) results primarily from oscillations of muon neutrinos into tau neutrinos. Using the result that \(P(SK)=0.30\) for such events [6], we obtain \(s_2 c_2 c_3^2 = 0.387\). Combining this with the LSND results that \(s_2 c_3 s_3 = 0.0784\), we find \(\theta_2 = 26.5^\circ\) and \(\theta_3 = 10.3^\circ\):

We quote results using central values for various parameters. Clearly, the uncertainty in an input such as \(P(\text{LSND})\) will produce uncertainties in a parameter such \(\theta_3\) (of order \(\pm 2^\circ\)). Our purpose is to show that these central values can account for essentially all observations.

Super-Kamiokande will also observe oscillations between muon neutrinos and electron neutrinos with a probability given by:

\[
P(SK; v_\mu \leftrightarrow v_e) = -4U_{23} U_{13} U_{21} U_{11} \sin^2 \left[\frac{(M^2 + m^2) L}{4E}\right]
- 4U_{23} U_{13} U_{22} U_{12} \sin^2 \left(\frac{M^2 L}{4E}\right)
- 4U_{22} U_{12} U_{21} U_{11} \sin^2 \left(\frac{m^2 L}{4E}\right)
\]

(9)

The sine-squared factors of the first two terms of this expression each average to a value of 0.5 and the two terms can be combined to yield:

\[
P(SK; v_\mu \leftrightarrow v_e) = 2[U_{23} U_{13}]^2 - 4U_{22} U_{12} U_{21} U_{11} \sin^2 \left(\frac{m^2 L}{4E}\right)
\]

(10)

For our values of \(\theta_2\) and \(\theta_3\), the first term is small (\(\sim 0.02\)), justifying our assumption that, for most of the zenith-angle range, the atmospheric muon anomaly is caused by muon-neutrino to tau-neutrino oscillations. However, the second term will produce observable effects for “upward” going events for which \(\frac{m^2 L}{4E}\) is assumed to be of order one or greater. We return to this point in the discussion below.
We have examined the question of matter (MSW) effects on oscillations between atmospheric electrons and muon neutrinos [18]. For the large mixing angles obtained for our model, resonance effects do not have a significant impact on our analysis. However, for $m^2$ values near 0.0001 $eV^2$, oscillations between electron and muon neutrinos are strongly damped for neutrino energies above 1 GeV. Such damping effects decrease rapidly with increasing values of $m^2$, and are insignificant at the upper end (0.001 $eV^2$) of the $m^2$ range considered here.

5 Solar Neutrinos

The MSW mechanism is not expected to have a significant impact on solar neutrinos for the large mass-squared differences considered in this paper, so that the probability for a solar electron neutrino to remain as such is:

$$P(solar; \nu_e \rightarrow \nu_e) = 1 - 4[U_{13} U_{11}]^2 \sin^2 \left[ \frac{(M^2 + m^2)L}{4E} \right] - 4[U_{13} U_{12}]^2 \sin^2 \left( \frac{M^2 L}{4E} \right) - 4[U_{12} U_{11}]^2 \sin^2 \left( \frac{m^2 L}{4E} \right)$$

(11)

For the E and L of solar neutrinos detected on Earth, the sine-squared terms in the above expression average each to 0.5 and:

$$P(solar; \nu_e \rightarrow \nu_e) = 1 - 2(s_3 c_3)^2 - 2(s_1 c_1 c_3^2)^2$$

(12)

Within the uncertainties of solar neutrino theory, essentially all solar neutrino observations (with the possible exception of those from the Homestake experiment) are consistent with $P(solar)=0.5$ [20]. Using this value and our previous result for angles $\theta_2$ and $\theta_3$, we obtain $\theta_1 = 37.6^\circ$.

6 Reactor Experiments

The general expression derived for solar neutrinos also applies to reactor experiments. Such experiments have not been sensitive to $m^2$ values less than 0.001 $eV^2$. However, oscillations with $M^2 = 0.4 eV^2$ are, in principle, observable. For values of E and L that allow an averaging of the oscillation factor,
we expect:

\[ P(\text{reactor}; v_e \rightarrow v_e) = 1 - 2(s_3 c_3)^2 \]  \hspace{1cm} (13)

With \( \theta_3 = 10.3^\circ \), we predict \( P(\text{reactor}) = 0.94 \) compared to measured values of \( 0.99 \pm 0.01 \text{(stat)} \pm 0.05 \text{(syst)} \) for Bugey [16] and \( 0.98 \pm 0.04 \text{(stat)} \pm 0.04 \text{(syst)} \) for CHOOZ [19]. Given the size of the errors, this is reasonable if not perfect agreement. In the Bugey experiment, the detectors were sufficiently close to the reactor to be sensitive, in principle, to modulations in the positron spectrum produced by the \( \sin^2 \frac{M^2 L}{4E} \) factor. We discuss this below.

7 Summary and Discussion

We have found that most features of oscillation-related neutrino data can be explained by the following mixing matrix, corresponding to the angles \( \theta_1 = 37.6^\circ, \theta_2 = 26.5^\circ, \) and \( \theta_3 = 10.3^\circ \).

\[ U = \begin{pmatrix} 0.78 & 0.60 & 0.18 \\ -0.61 & 0.66 & 0.44 \\ 0.15 & -0.45 & 0.88 \end{pmatrix} \]  \hspace{1cm} (14)

and by assuming that neutrino masses \( m_1, m_2, m_3 \) satisfy \( m_3 >> m_2 > m_1 \), that \( 0.0001 < (m_2^2 - m_1^2) < 0.001 \text{ eV}^2 \), and that \( M^2 = m_3^2 - m_2^2 = 0.4 \text{ eV}^2 \). This choice of parameters appears to be in reasonable accord with essentially all neutrino observations, including the zenith-angle behavior of the Super-Kamiokande atmospheric neutrino data for “upward” going \( (\cos \theta_z < -0.6) \) events. We demonstrate this by assuming that \( L \) is sufficiently large that all sine-squared factors involving mass-squared differences average to 0.5. We find that:

\[ P_{\mu\mu} = P(SK; v_\mu \rightarrow v_\mu) = U_{21}^4 + U_{22}^4 + U_{23}^4 \]

\[ P_{ee} = P(SK; v_e \rightarrow v_e) = U_{11}^4 + U_{12}^4 + U_{13}^4 \]

\[ P_{\mu e} = P_{\mu e} = P(SK; v_\mu \leftrightarrow v_e) = 2[U_{23} U_{13}]^2 - 2U_{22} U_{12} U_{21} U_{11} \]  \hspace{1cm} (15)

The double ratio \( R \) for these “upward” going events:

\[ R = \frac{(N_\mu/N_e) \text{ measured}}{(N_\mu/N_e) \text{ no oscillation}} \]  \hspace{1cm} (16)
can be estimated to be:

\[
R = \frac{(2P_{\mu\mu} + P_{e\mu})/(P_{ee} + 2P_{\mu e})}{2}
\]  

(17)

given the fact that atmospheric \(\nu_\mu\) are produced at approximately twice the rate of \(\nu_e\). We obtain the result that \(R=0.44\) compared to the measured value of \(0.41\pm0.04\) at the largest zenith angles.

We show in Fig. 1 the predicted ratio \(R\) as a function of \(L/E\) for three values of \(m^2 \equiv m_2^2 - m_1^2\). The curve of \(R\) vs. \(L/E\) has been smeared by a resolution function that is Gaussian in \(\ln(L/E)\) with a width that corresponds to a factor of three uncertainty in the \(L/E\) measured for any given event. In an experiment such as Super-Kamiokande the angle between the incoming neutrino and the direction of the detected lepton can be quite large [6], leading to a significant uncertainty in \(L\). The general shape of the curve of \(R\) vs. \(L/E\) in Fig. 1 depends only weakly on the assumed smearing, with factors of two or four giving qualitatively the same results as our assumed factor of three. We have also plotted in Fig. 1 the latest Super-Kamiokande results [6] for which a reasonably good fit is obtained for \(m^2\) values around \(0.0003 \text{ eV}^2\).

The most recent Super-Kamiokande paper [6] makes available the ratio of the number of measured to the number of expected events, assuming no oscillations, for electrons and muons separately. We show these data in Fig. 2, with the overall flux normalization reduced by 12\% to give a best fit to the three-neutrino mixing model presented in this paper. Such a normalization shift is well within the errors of the theoretical flux assumed by the Super-Kamiokande Collaboration. Our model does not give a very good fit to the individual electron and muon ratios. However, it is important to note that the double ratio \(R\) shown in Fig. 1, for which we obtain a good fit, is much less sensitive to systematic errors than are the individual electron and muon ratios. Until these systematic errors are reduced by improved measurements of atmospheric cosmic-ray particle production, one should keep an open mind about the significance of individual distributions in the atmospheric neutrino data.

As with previous attempts to explain all neutrino anomalies with a simple three-neutrino mixing model, the one outlined in this paper is subject to the resolution of various experimental inconsistencies and ambiguities. For example, we have assumed that the LSND result is an oscillation effect rather than unexplained background. It must be noted that the KARMEN experiment has not confirmed the LSND result. For our assumed parameters, recent KARMEN data [21] should have yielded approximately two signal events in addition to roughly three background events, whereas no oscillation candidate events were seen. However, the LSND collaboration has reported additional
Fig. 1. The predicted double ratio $R$ as a function of $L/E$ for the mixing matrix with $\theta_1 = 37.6^\circ$, $\theta_2 = 26.5^\circ$, $\theta_3 = 10.3^\circ$ and for three values of $m^2 \equiv m_2^2 - m_1^2$. The value of $M^2 \equiv m_3^2 - m_2^2 = 0.4 \text{ eV}^2$. The data are the most recent Super-Kamiokande results [6].

With respect to the atmospheric neutrino data of Super-Kamiokande, our model with $\theta_1 = 37.6^\circ$, $\theta_2 = 26.5^\circ$, $\theta_3 = 10.3^\circ$, $M^2 \equiv m_3^2 - m_2^2 = 0.4 \text{ eV}^2$ and $m^2 \equiv m_2^2 - m_1^2 = 0.0003 \text{ eV}^2$ gives a good overall fit to the data. To distinguish our model from one with a single oscillation process with a small $\Delta m^2 (\sim 0.0022 \text{ eV}^2)$ requires precise measurements of multi-GeV, “overhead” ($\cos \theta_z \sim 1$) events. Our model predicts $R \sim 0.72$ for such events whereas the small-$\Delta m^2$ model yields $R \sim 1$. Present Super-Kamiokande data are inconclusive in this low-$L/E$ region.
Our model requires that solar neutrino deficits observed on Earth should show no energy dependence and no MSW effects. The data from the gallium experiments (GALLEX and SAGE) and from Super-Kamiokande, which sample very different parts of the solar neutrino energy spectrum, are in general agreement with this prediction. However, the Homestake Cl-37 experiment has historically reported a substantially larger solar neutrino deficit [23] than is observed in these other experiments. We note, however, that the measured Homestake solar neutrino flux of $2.56 \pm 0.16 \text{(stat)} \pm 0.16 \text{(syst)}$ SNU represents $0.403 \pm 0.025 \pm 0.025$ of the expected flux computed by Turck-Chieze and Lopes [24], not far from the value 0.50 assumed in this paper. A final interpretation of all the data awaits a better understanding of both the theory and of possible
systematic errors in various experimental results.

Our model agrees within one standard deviation with the fluxes measured in reactor experiments. However, for $\theta_3 = 10.3^\circ$, the Bugey results [16] would be expected to show some modulation (at the level of several percent) of the shape of the measured positron spectrum. We note, however, that statistical and systematic uncertainties in the LSND result permit values of $\theta_3$ as small as 8$^\circ$, which would reduce such modulations below the level of detectability. Our choice of $M^2 = 0.4 \text{ eV}^2$, the maximum allowed by the CDHS oscillation search [17], is dictated by the requirement that $\theta_3$ be small.

We note that our model predicts large effects for terrestrial, long-baseline experiments such as K2K, MINOS, and those planned for Gran Sasso. Furthermore, if muon neutrinos oscillate into tau neutrinos with $M^2 = 0.4 \text{ eV}^2$, then observable effects are just beyond the reach of the present short-baseline experiments CHORUS and NOMAD but accessible to a new, improved experiment such as proposed by the TOSCA collaboration.

Finally, we remark that our model differs from previous versions of simple three-neutrino mixing [10–15] in its unique combination of a $M^2$ value that can explain the LSND results and a $m^2$ value significantly larger than is usually assumed in solar neutrino models. It is, of course, possible that eventually no simple three-neutrino mixing model will give a satisfactory account of past, present and future experimental results. In that case, more complicated models will have to be considered [25].

8 Acknowledgement

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At the Neutrino '98 Conference (Takayama, Japan) the following results were reported for solar neutrino results, where SSM stands for standard solar model prediction:

- Super-Kamiokande: data/SSM = 0.474
- GALLEX: data/SSM = 77.5/132 = 0.587
- SAGE: data/SSM = 66.6/137 = 0.486

with overall errors on data/SSM in the range of 10% to 15%.