Production of Terahertz Radiations by Short Pulse Lasers

Sandeep Kumar¹, Niti Kant¹, Shivani Vij², Alka Mehta¹ and Vishal Thakur¹*

¹. Department of Physics, Lovely Professional University, G.T. Road, Phagwara - 144411, Punjab, India
². Department of Applied Sciences, DAV Institute of Engineering and Technology, Jalandhar, 144008, India

E-mail: vishal20india@yahoo.co.in

Abstract

In this paper, we develop an analytical formalism for THz generation from Laser filaments in the presence of static electric field in the magnetized collisional plasma. Two femtosecond laser pulses with different frequencies undergo filamentation in magnetized collisional plasma to have non-linear coupling in the presence of transverse static electric field. This results in balancing action of static ponderomotive force with pressure gradient force and forms transverse density ripple and non-linear ponderomotive force couple with density ripple to provide strong non-linear transverse current which results in excitation of THz Radiations at resonance. This coupling is further enhanced by electric static field.

Keywords: THz Radiations, magnetized collisional plasma, non-linear ponderomotive force

I. Introduction

In modern era THz radiation has become an important research tool due to its scientific and commercial applications such as Topography, remote sensing [1], medical imaging [2], communication [3], explosives detection [4], spectroscopy [5]. More over THz radiations can also be used to produce second harmonic generation and third harmonic generation[6], which further can be
used in various applications like particle accelerators [7,8]. Out of various schemes proposed in the literature, THz pulse energy achieved from laser filament is very high [9].

Filamentation denotes a peculiar phenomenon related to propagation of a beam of light through a medium without apparent diffraction. Counteracting the natural spreading of the beam is possible with intense laser pulses owing to optical Kerr effect, which causes a change of refractive index in the medium proportional to the beam intensity. The core of beam is more intense than the wings results in self focusing of beam. One of the main feature of filaments is their ability to generate plasmas in the wave of propagating pulse which in turn modifies narrow band pulse laser into broadband pulse. [10,11] Houard et al. [12] studied and observed 3 order of magnitude enhancement of THz energy due to filamentation of femtosecond laser pulse in air in presence of a static Transverse electric field approximate10kv/cm [13,14]. The Laser intensity was approximate 9x10^13 W/cm². Wu et al. [15-17] studied significant enhancement in the efficiency of THz generation, where Transverse magnetic field is applied.

In this paper we provide Theoretical treatment for THz generation from Laser filaments in the presence of static (D.C. biased) electric field in magnetized collisional plasma.

Consider Two Transversely amplitude modulated laser beams with electric fields propagating along z-axis, polarized along y-axis and amplitude modulated along x-axis. To produce magnetized plasma, mag. field is applied along y-axis D.C. electric field is applied across x-axis, which can provide d.c drift to electrons.

The laser exerts a Ponderomotive force $F_{qw}$ as well as ponderomotive static force $F_{pq}$ on electron. In the steady state, the static Ponderomotive force is balanced by pressure gradient force. It results in Transverse density ripple at zero frequency and wave vector that is $k_{0q}$. The beat frequency Ponderomotive force is responsible for velocity and density oscillations.
The density oscillations beats with D.C. drift to produce a Transverse current $J_{\omega,k}$ which is responsible for THz generation. In section II we obtain the expressions for non linear velocity perturbation at THz frequency $\omega = (\omega_1 - \omega_2)$ and wave vectors $k = k_1 - k_2$, $k + q$ and non linear density perturbation at THz frequency and wave vectors $k$, $k + q$ by choosing non linear coupling of suitable factors an expression for non linear current density at frequency $\omega$ and wave vectors $k$ is obtained. In section III we determine THz wave generation and determine normalized amplitude. In section IV there is a discussion of results. Most of the researchers have not considered the collisions between electrons and neutrals in plasmas but in reality collisions are present in every non linear system. In this paper we will also discuss the resonance condition and the effect of collisions on it.

II. Production of Non-Linear Current

Consider a magnetized collisional plasma of electrons density $n_0$ having a D.C. electric field applied along X-axis. Due to the $E_{d.c.}$ electrons will acquire drift velocity

$$\vec{V}_{d.c} = \frac{-e\vec{E}_{d.c}}{mv_e}$$

(1)

Where $-e$ = charge of electron
$m$ = mass of electron
$\nu_e$ = collisional frequency of electron

We incident two transversely amplitude modulated (by filamentation) lasers in to the plasma which is magnetized collisional plasma. The electric fields of lasers are

$$\vec{E}_j = \hat{A} A_{j0} \left[ 1 + \mu_j \cos(qx) \right] e^{-(\omega_j t - k_j x)} \text{where } j = 1,2$$

(2)

$\mu_j$ = index of modulation.
Frequency difference of lasers $\omega = \omega_1 - \omega_2$ lies in THz range.

The laser filaments import oscillatory velocities to plasma electrons.

$$\vec{v}_j = \frac{-e\vec{E}_j}{m(\omega_j - v)}$$

(3)
They also exert static Ponderomotive force

$$F_{pq} = e \bar{V} \phi_{pq}$$  and beat frequency ponderomotive force

$$F_{p\omega} = e \bar{V}_{p\omega}$$

$$\varphi_{p\omega} = \frac{e A_{10} A_{20}}{2 m (\omega_1 - v_e)(\omega_2 - v_e)} \left\{ 1 + \left( \frac{\mu_1 + \mu_2}{2} \right) e^{iqx} \right\} e^{-i(\omega_1 - k_2x)}$$

$$\varphi_{p\omega} = \frac{e}{4mT_e} \left\{ \frac{A_{10}^2 \mu_1}{(\omega_1 - v_e)^2} + \frac{A_{20}^2 \mu_2}{(\omega_2 - v_e)^2} \right\} e^{iqx}$$

$$T_e$$, the equilibrium temperature of electrons

The beat frequency ponderomotive force, collisional force and magnetic field force will provide oscillatory velocity to electrons.

By using equations of motion,
\[
\vec{V}_{p}^{NL} = \frac{\vec{r}_{p}^{NL} \omega_{c}}{m(v-\omega)} - \frac{\vec{r}_{p}^{NL \omega_{c}}}{m(v-\omega)^2} - \frac{\omega_{c}^2}{(v-i\omega)^2} \vec{V}_{p}^{NL}
\]

\(\bar{\omega}_{c} = e B_{y}/m\) is cyclotron frequency of electrons

\[
\vec{V}_{x}^{NL} = \frac{(v-i\omega)(F_{x} \hat{x} + F_{z} \hat{z})}{[m(v-\omega)^2 + \omega_{c}^2]} + \frac{\omega_{c} F_{x} \hat{x}}{[m(v-i\omega)^2 + \omega_{c}^2]}
\]

The components of velocity along X and Z axis

\[
\vec{V}_{x}^{NL} = \frac{(v-i\omega)F_{x}}{m(v-\omega)^2 + \omega_{c}^2} + \frac{\omega_{c}}{m(v-\omega)^2 + \omega_{c}^2} F_{x}
\]

And

\[
\vec{V}_{z}^{NL} = \frac{(v-i\omega)F_{z}}{m(v-\omega)^2 + \omega_{c}^2} - \frac{\omega_{c}}{m(v-\omega)^2 + \omega_{c}^2} F_{z}
\]

Put [(v - \omega)^2 + \omega_c^2] = \omega_d^2

\[
\vec{V}_{x}^{NL} = \frac{(v-i\omega)F_{x}}{ma \omega_d^2} + \frac{\omega_{c}}{ma \omega_d^2} F_{x}
\]

\[
\vec{V}_{z}^{NL} = \frac{(v-i\omega)F_{z}}{ma \omega_d^2} + \frac{\omega_{c}}{ma \omega_d^2} F_{x}
\]

\[
\vec{V}_{x}^{NL} = \frac{(v-i\omega)}{ma \omega_d^2} \left( \frac{e^2 A_{10} A_{20} (\mu_1 + \mu_2)}{4m(\omega_1 - v_e)(\omega_2 + v_e)} e^{-i(\omega t - kx - qx)} (i q) \hat{x} \right.
\]

\[
+ \frac{\omega_{c}}{ma \omega_d^2} \left( \frac{e^2 A_{10} A_{20}}{2m(\omega_1 - v_e)(\omega_2 + v_e)} e^{-i(\omega t - kx)} (i q) \hat{x} \right)
\]

\[
+ \frac{\omega_{c}}{ma \omega_d^2} \left( \frac{e^2 A_{10} A_{20} (\mu_1 + \mu_2)}{4m(\omega_1 - v_e)(\omega_2 + v_e)} e^{-i(\omega t - kx - qx)} (i q) \hat{z} \right)
\]

\[
(13)
\]
\[
\mathbf{\tilde{V}}_{2}^{NL} = \frac{(v - i\omega)}{m\omega_a^2} \left( \frac{e^2 A_{1\theta} A_{20}(ik\mathbf{\hat{z}})}{2m(\omega_1 - \nu_e)(\omega_2 + \nu_e)} e^{-(i\omega t - kz - qx)} \right.
\]
\[
+ \frac{i(v - i\omega)k \mathbf{\hat{z}} - \omega_c q \mathbf{\hat{x}}}{2m\omega_a^2(\omega_1 - \nu_e)(\omega_2 + \nu_e)} e^{-(i\omega t - kz - qx)}
\]
\[
\tilde{v}_{\mathbf{\hat{z}}} = \frac{(v - i\omega)}{m\omega_a^2} \left( \frac{e^2 A_{1\theta} A_{20}(\mu_1 + \mu_2)(ik)}{4m(\omega_1 - \nu_e)(\omega_2 + \nu_e)} e^{-(i\omega t - kz - qx)} \right)
\]
\[
- \frac{\omega_c}{m\omega_a^2} \left( \frac{e^2 A_{1\theta} A_{20}(\mu_1 + \mu_2)(iq)}{4m(\omega_1 - \nu_e)(\omega_2 + \nu_e)} e^{-(i\omega t - kz - qx)} \right) \mathbf{\hat{x}}
\]

(14)

Along X-axis
\[
\mathbf{\tilde{n}}_{w,k} = \frac{n_0^0 e^2 A_{1\theta} A_{20} [ik^2] \omega_c}{2m^2 \omega_a^2(\omega_1 - \nu_e)(\omega_2 + \nu_e)} e^{-(i\omega t - kz)}
\]
\[
\mathbf{\tilde{n}}_{w,k+q} = \frac{n_0^0 e^2 (A_{1\theta} A_{20})(\mu_1 + \mu_2)}{4m^2(\omega_1 - \nu_e)(\omega_2 + \nu_e)} - \frac{i[(v - i\omega)^2 q^2 + \omega_c k^2]}{\omega \omega_a^2} e^{-(i\omega t - kz - qx)}
\]

(15)

Along Z-axis
\[
\mathbf{\tilde{n}}_{w,k} = \frac{n_0^0 e^2 A_{1\theta} A_{20} [ik^2] (v - i\omega)}{2m^2 \omega_a^2(\omega_1 - \nu)(\omega_2 + \nu)} e^{-(i\omega t - kz)}
\]
\[
+ \frac{n_0^0 e^2 (A_{1\theta} A_{20})(\mu_1 + \mu_2)}{4m^2(\omega_1 - \nu)(\omega_2 + \nu)} - \frac{i[(v - i\omega)^2 k^2 + \omega_c q^2]}{\omega \omega_a^2} e^{-(i\omega t - kz - qx)}
\]

(16)
Non linear current density at $\omega, k$ erases due to coupling between

(i) Equilibrium plasma density $n_0^*$ and non linear velocity $\vec{v}_{NL, w,k}$

(ii) Non linear density perturbation $\vec{v}_{NL, w,k}$ and dc electrons velocity $\vec{V}_{dc}$

(iii) Zero frequency transverse density ripple $n_{0,q}$ and non linear velocity $\vec{v}_{NL,\omega, w,k}$

\[ \vec{J}_{\omega,k}^{NL} = -n_{0,q} e \vec{v}_{NL,\omega,k+q}^* - n_0^* e \vec{v}_{w,k} - \vec{v}_{NL,\omega,k} e \vec{V}_{dc} \]

\[ = -n_0^* e^3 \left( \frac{A_{10}^2 \mu_1}{(\omega_1 - v_e)^2} + \frac{A_{20}^2 \mu_2}{(\omega_2 + v_e)^2} \right) e^{-iqx} \left( e^{2A_{10}A_{20}(\mu_1 + \mu_2)} \right) \frac{i[(v - \omega)q \hat{x} + \omega_c k \hat{z}]}{4m^2 \omega_e^2(\omega_1 - v_e)(\omega_2 + v_e)} e^{-i(\omega t - k_x - qx)} \]

\[ = -n_0^* e^3 \left( \frac{A_{10}^2 \mu_1}{(\omega_1 - v_e)^2} + \frac{A_{20}^2 \mu_2}{(\omega_2 + v_e)^2} \right) e^{-iqx} \left( e^{2A_{10}A_{20}(\mu_1 + \mu_2)} \right) \frac{i[(v - \omega)k \hat{z} + \omega_c q \hat{x}]}{4m^2 \omega_e^2(\omega_1 - v_e)(\omega_2 + v_e)} e^{-i(\omega t - k_x - qx)} \]

\[ = -n_0^* e^3 \left( \frac{A_{10}^2 \mu_1}{(\omega_1 - v_e)^2} + \frac{A_{20}^2 \mu_2}{(\omega_2 + v_e)^2} \right) e^{-iqx} \left( e^{2A_{10}A_{20}(\mu_1 + \mu_2)} \right) \frac{i[(v - \omega - \omega_c)q \hat{x}]}{4m^2 \omega_e^2(\omega_1 - v_e)(\omega_2 + v_e)} e^{-i(\omega t - k_x)} \]

\[ -n_{0,k} e \vec{V}_{dc} \]

\[ = -n_0^* e^3 A_{10}A_{20}[(\omega_1 - v_e)(\omega_2 + v_e)] \left( -\frac{e\vec{E}_{dc}}{mv_e} \right) e^{-i(\omega t - k_x)} \]

\[ = -n_0^* e^3 A_{10}A_{20}[(\omega_1 - v_e)(\omega_2 + v_e)] \left( -\frac{e\vec{E}_{dc}}{mv_e} \right) e^{-i(\omega t - k_x)} \]

\[ = -n_0^* e^4 A_{10}A_{20} \vec{E}_{dc} \left( k^2[\omega_c + v - \omega_x] e^{-i(\omega t - k_x)} \right) \]

\[ \vec{J}_{\omega,k}^{NL} = \frac{-n_0^* e^4 A_{10}A_{20}[(\omega_1 - v_e)(\omega_2 + v_e)]}{2m^2 \omega_e^2} \left( K^2 \frac{\vec{E}_{dc}}{\omega v_e} \right) \left( \frac{A_{10}^2 \mu_1}{(\omega_1 - v_e)^2} + \frac{A_{20}^2 \mu_2}{(\omega_1 + v_e)^2} \right) (v - \omega - \omega_c) e^{-i(\omega t - k_x)} \]

\[ = \frac{-n_0^* e^4 A_{10}A_{20}[(\omega_1 - v_e)(\omega_2 + v_e)]}{2m^2 \omega_e^2} \left( \frac{K^2 \vec{E}_{dc}}{\omega v_e} \right) \left( \frac{A_{10}^2 \mu_1}{(\omega_1 - v_e)^2} + \frac{A_{20}^2 \mu_2}{(\omega_1 + v_e)^2} \right) (v - \omega - \omega_c) e^{-i(\omega t - k_x)} \]

\[ (17) \]
\[ T_{\omega,k}^{NL} = \frac{-n_0^2 e^4 A_1 A_2 |l|}{2m^2 \omega_n^2 (\omega_1 - v_c)(\omega_2 + v_c)} \left\{ \frac{ek(\mu_1 - \mu_2)}{8T_e} \left( \frac{A^2_1 \mu_1}{(\omega_1 - v_c)^2} + \frac{A^2_2 \mu_2}{(\omega_1 + v_c)^2} \right) (v - i\omega + \omega_c) \\
+ \frac{mk [(v - \omega) + \omega_e]}{e} \right\} e^{i(\omega t - kx)} \]

\( (18) \)

**III. Generation of THz radiations**

The wave equation governing the propagation of THz waves can be written as:

\[ -\nabla^2 \vec{E} + \nabla \cdot (\nabla \cdot \vec{E}) = \frac{4\pi \omega}{c^2} j^{NL} + \frac{\omega^2}{c^2} \vec{E} \]

In the presence of magnetic field along Y-axis in collisional plasma, dielectric constant assumes the form of anisotropic Tensor \( \varepsilon \)

X and Z components of THz field are

\[ -\frac{\omega^2}{c^2} \varepsilon_{zz} E_z - \frac{\omega^2}{c^2} \varepsilon_{xx} E_x = \frac{4\pi \omega}{c^2} j_z^{NL} \]

And

\[ -2ik' \frac{\partial E_x}{\partial z} + k'^2 E_x - \frac{\omega^2}{c^2} \varepsilon_{xx} E_x - \frac{\omega^2}{c^2} \varepsilon_{xx} E_x = \frac{4\pi \omega}{c^2} j_x^{NL} \]

By using phase matching condition

\[ k'^2 = \frac{\omega^2}{c^2} \left( \varepsilon_{xx} + \frac{\varepsilon_{xx}^2}{\varepsilon_{zz}} \right) \]

\[ k'^2 = \frac{\omega^2}{c^2} \left\{ \frac{\omega_p^2 (v - i\omega)}{i\omega \omega_A} + \frac{(-\omega_p^2 \omega_A^2)}{\omega_p^2 \omega_A^2} \right\} \]

\[ k_1 - k_2 + q = \frac{\omega}{c} \left( 1 - \frac{\omega_p^2}{i\omega \omega_A^2} \left( \frac{v - i\omega}{i\omega \omega_A^2 - \omega_p^2 (v - i\omega)} \right) \right)^{1/2} \]
By Using above Phase matching condition electric field component along the X-axis

\[
E_x = \frac{-2\pi \omega}{k' c^2} \left\{ \frac{j^{NL}}{j_{\omega k}^{NL}} + \frac{\epsilon_{xx}}{j_{\omega k}^{NL}} \right\} z
\]

\[
E_x = \frac{-2\pi \omega}{k' c^2} \left( \frac{-v_0^4 A_{1o} A_{2o} [1]}{2m^2 \omega_0^4 (\omega_1 - \omega_2 + \omega_e) (\omega_1 + \omega_e)} \right) \left( \frac{K^2 E_{d,c}(v - \omega + \omega_e)}{\omega v_e} - \frac{e q (\mu_1 + \mu_2)}{8 T_e} \frac{A_{1o}^2 \mu_1}{(\omega_1 - \omega_e)^2} + \frac{A_{2o}^2 \mu_2}{(\omega_2 + \omega_e)^2} \right) (v - \omega + \omega_e)
\]

\[
- \omega + \omega_c + \frac{\omega_c \omega_0^2}{\omega_0 \omega_0^2 - \omega_0^2 (v - \omega) (v - \omega)} \left( \frac{\omega_0^2}{(\omega_1 - \omega_e)^2} + \frac{A_{1o}^2 \mu_1}{(\omega_1 - \omega_e)^2} + \frac{A_{2o}^2 \mu_2}{(\omega_2 + \omega_e)^2} \right) (v - \omega + \omega_e)
\]

\[
+ \frac{mK}{e} (v - \omega + \omega_e)
\]

(19)

The normalized THz amplitude

\[
\frac{e E_x}{\omega p c} = \frac{\omega z' V_{10}' V_{20}' i}{4 k' \omega_0 v_0^5 (\omega_0 p_1 - v_e') (\omega_0 p_2 + v_e')} \left( \frac{K_{1d,c}^2}{\omega v_e'} \right) \left( \frac{K_{1d,c}}{\omega v_e'} \right) \left( \frac{K_{2d,c}}{\omega v_e'} \right) \left( \frac{K_{3d,c}}{\omega v_e'} \right) \frac{V_{10}' \mu_1}{(\omega_0 p_1 - v_e')^2} + \frac{V_{20}' \mu_2}{(\omega_0 p_2 + v_e')^2} (v_e' - \omega' - \omega_c')
\]

(20)

Where \( V_{10}' = e A10/\omega p c \), \( V_{20}' = e A20/\omega p c \), \( \omega' = \omega/\omega p \), \( \omega p = \omega 1/\omega 2 \), \( \omega p = \omega 2/\omega p \), \( v_e' = \omega c/\omega p \), \( E_{d,c}' = e E_{d,c}/\omega p c \), \( K_0 = C K/\omega p \), \( K' = C K'/\omega p \), \( \omega_c = \omega c/\omega p \), \( q' = C q/\omega p \), \( V_{th}' = V_{th}/C \)

\( \omega_1 = 2.4 \times 10^{14} \text{ rad/s} \), \( \omega_2 = 2.1 \times 10^{14} \text{ rad/s} \), \( E_{d,c}' = 0.053 \), \( \omega_p = 2.0 \times 10^{13} \text{ rad/s} \), \( \mu_1 - \mu_2 = 0.3 \), \( q^* = 0.3 \)

\( \omega' = 2.0 \rightarrow 5.0 \), \( V_{10}' = V_{20}' = 0.005 \rightarrow 0.01 \), \( \omega p_1 = 12 \), \( \omega p_2 = 10.5 \), \( v_e = 1.5 \times 10^{13} \text{ rad/s} \), \( B = 5 \text{ T} \)

\( \omega_c = 0.879 \times 10^{12} \text{ rad/s} \), \( \omega_c' = 0.04 \), \( v_e' = 0.75 \), \( K_1 = 0.8 \times 10^6 \text{ m} \), \( K_2 = 0.7 \times 10^6 \text{ m} \), \( K_0 = 1.5 \)

\( q^* = 0.3 \), \( q = 0.2 \times 10^5 \text{ m} \), \( K' = 1.2 \times 10^5 \), \( K'' = 1.8 \), \( Z' = 100 \), \( V_{th}' = 0.0067 \)
IV. Result and Conclusion

The plot between normalized THz amplitude and normalized THz frequency is as shown above, when \( \omega' \) lies between 2.5 to 5.0 normalized THz amplitude increases

![Variation of normalized amplitude of THz radiation with \( \omega'/\omega_p \)](Figure: 2)

Coupling is enhanced in magnetized collisional plasma in the presence of D.C. electric field, which helps in THz radiation amplitude with frequency with the help of External magnetic field we can also tune up the frequency of THz radiations.

Field amplitude and efficiency of THz is reduced due to electron neutral collisions in the magnetized collisional plasma.

References

1. B. Ferguson and X.C. Zhang, Not. Mater. 1,26 (2002)
2. P.Y. Han, G.C.ChO and X.C. Zhang, opt. Letter,25,242(2000)
3. J. Federic and L. Moeller, J.Appl.Phys.107,111101(2010)
4. M. Tonouchi, Nature Photon 1, 97(2007)
5. M.C. Beard, G.M. Turner and C.A. Schmattenmor, “Terahertz spectroscopy”, J.Phys. Cheme B 106,7146(2002)
6. Jyoti Rajput, Niti kant , Harjit Singh, Vikas Nanda, “Resonant third harmonic generation of a short pulse laser in plasma by applying a wiggler magnetic field.” Optics Communications Volume 282, issue 23, 1 December 2009, Pages-4614-4617.

7. Jyoti Rajput, Niti kant and Arvinder Singh, "Electron acceleration due to a circularly polarized laser pulse on a downward plasma density ramp in the presence of an azimuthal magnetic field." AIP Conference Proceedings Volume 2006, Issue 1 10.1063/1.5051281.

8. Niti Kant, Jyoti Rajput, Arvinder Singh. "Electron acceleration from rest to GeV energy by chirped axicon Gaussian laser pulse in vaccum in the presence of wiggler magnetic field." Volume 26, March 2018, Pages 16-22

9. L. Bhasin, V.K. Tripath, "THz generation from Laser filaments in the presence of static electric field in plasma", Phys. Plasma 18,123106.

10. Prateek Varohnety, Vivek saijal, Prashant chuhan, Ravinder Kumar and Navneet K. Sharma, "Effects of Transverse static by heating of two Transversely modulated Gaussian Laser beams in plasma". Laser and particle beams (2014) 32, 375-381

11. S. Tzortakis, M. Franco, Y.B. Andre, A. Chiron, B. Lenourouz, B. Prade and A. Mysyrowicz, 1989 Phys. Rev. E 63 F 3505

12. A. Couairon. 2003 Phys. Rev. A 68015

13. R. MeLaughlin, A. Corohia, M.B. Johnston Q. Chen, C.M. Clesla, D.D. Araone, G.A.C. Jones, E.H. Linfield, A.G. d and M. Papper 2000, Appl. Phys. Letter 76 2038

14. A. Hourd, Y.Liu, B. Prade, V.T. Tikhanchuk and A. Myoyzovie Phys. Rev Lett. 100, 255006(2008)

15. H.C. Wu, Z.M. Sheng, Q.L. Dose M. Ku and J. Zhang 2001, Physq., Rev. E 75016407

16. Niti Kant, Jyoti Rajput, Pankaj Giri, Arvinder Singh. "Effect of axial magnetic field on axicon laser-induced acceleration." Volume 18, March 2016, Pages 20-25.
17. Arvinder Singh, Jyoti Rajput and Niti Kant. “Combined influence of azimuthal and axial magnetic fields on resonant electron acceleration in plasma.” Published 26 October 2017 Laser Physics, Volume 27, Number 11.