Dark galaxies, spin bias and gravitational lenses

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ABSTRACT
Gravitational lensing studies suggest that the Universe may contain a population of dark galaxies; we investigate this intriguing possibility and propose a mechanism to explain their nature. In this mechanism a dark galaxy is formed with a low density disk in a dark halo of high spin parameter; such galaxies can have surface densities below the critical Toomre value for instabilities to develop, and following Kennicutt’s work we expect these galaxies to have low star formation rates. The only stellar component of the galaxies is a halo system, formed during the collapse of the proto-galactic cloud. We compute synthetic stellar population models and show that, at a redshift $z = 0.5$, such galaxies have apparent magnitudes $B \approx 28$, $R \approx 26$ and $I \approx 25$, and could be unveiled by deep searches with the Hubble Space Telescope. Dark galaxies have an initial short blue phase and then become essentially invisible, therefore they may account for the blue population of galaxies at high redshift. We find a strong mass-dependence in the fraction of dark galaxies, and predict that spiral galaxies will not be found in halos with masses less than about $10^{9} \, M_{\odot}$, if $\Omega = 1$. Above about $10^{12} \, M_{\odot}$, all halos can produce luminous disks. The mass-dependence of the galaxy-formation efficiency introduces the possibility of ‘spin bias’ – luminous galaxies being associated preferentially with strongly-clustered high-mass halos. A further prediction is that the slope of the faint-end luminosity function for galaxies will be flatter than the associated halo mass function.

Key words: cosmology: theory — cosmology: dark matter — galaxies: formation — galaxies: evolution

1 INTRODUCTION
The first gravitational lens was discovered by Walsh, Carr-nell & Weymann (1979), and comprised a pair of quasar images split by a clearly visible massive galaxy. Since then, many manifestations of gravitational lensing have been observed, including systems of two or four images, or a ring. Although in some cases a lensing galaxy can be seen, over the last few years extensive evidence has been accumulating that multiple and complex quasar images are being gravitationally lensed by dark objects of galaxy mass, and the phrase ‘dark galaxy’ has frequently been used to characterise such bodies (Weedman et al., 1982, Djorgovski & Spinrad 1984, Meylan & Djorgovski 1989, Hewett et al. 189, Wisotski et al. 1993, Hawkins et al. 1997).

Each configuration of quasar images presents its own particular problems in interpretation, and it is probably fair to say that any particular system can be dismissed on the basis of an improbable conjunction of phenomena. In the case of double quasars, in spite of arguments based on similarity of spectra, colour and redshift, there is still some residual doubt that they may be chance coincidences. In these systems, under the lensing hypothesis it is usually possible to put tight limits on the mass-to-light ratio of the lensing galaxy, typically several hundred $M_{\odot}/L_{\odot}$. For quadruple systems there is usually no doubt that they are indeed lensed systems, but the close proximity of the four images can make it hard to put useful limits on the magnitude of the lensing galaxy. Potentially the most conclusive case for dark galaxies comes from radio rings. Here a compact radio source is lensed to produce a ring. There are several examples already known where no lensing galaxy is clearly detectable, but optical emission from the radio source coupled with the small diameter of the rings tends to complicate the picture. Although in individual cases it is often possible to explain away the necessity for a dark galaxy, if one looks at the population of lensed quasars as a whole, the case for unseen lenses is hard to avoid. For example, a sample of double quasars has recently been analysed by Hawkins (1997),
and a strong statistical case is presented that dark objects of galactic mass must be present in most systems.

Rather surprisingly, in spite of the conclusion in a number of papers that lensing is probably being caused by dark galaxies or dark matter halos, little attempt has been made in the literature to explain the nature of these invisible gravitational lenses. Indeed, regardless of the lensing evidence, the question of whether dark galaxies can exist is an intriguing one ( Dekel & Silk 1983, Silk 1986).

In this letter we propose a mechanism to explain the nature of dark galaxies. While it seems difficult to prevent massive galaxies from producing stars on initial collapse (Tegmark et al. 1997; Padoan, Jimenez & Jones 1997), we suggest that sometimes only form halo stellar components, while their disks remain gaseous and quiescent if the surface density is low enough and they do not undergo (further) mergers. These galaxies are extreme cases of Low Surface Brightness (LSB) galaxies, and we propose to name them Low Density Galaxies (LDG).

We argue that the angular momentum of the galactic halo may play an important role in determining whether galaxies become LDGs and therefore dark. Scaling computations show that the possibilities for dark galaxies are higher for lower-mass halos, and we predict a minimum mass of halos with bright disks of around $10^7 M_\odot$ in an Einstein-de Sitter Universe. Dark lenses of high mass $\sim 10^{12} M_\odot$ would be the largest predicted by this theory. Nevertheless, there are important general implications of this work. Apart from the minimum spiral mass already mentioned, the predicted increasing fraction of dark galaxies towards lower masses modifies the luminosity function so it has a flatter faint slope than the associated halo mass function; the initial burst of star formation may provide a population of 'disappearing dwarfs' proposed for interpreting the faint galaxy counts ( cf Babul & Rees 1992, Metcalfe et al. 1996). Furthermore, for intermediate mass galaxies, the correlation between bright disks and halo mass provides a mechanism for biasing the luminous galaxy population towards high-density regions as it is known that more massive halos are expected to inhabit preferentially high-density regions ( Cole & Kaiser 1989, Mo & White 1996).

2 LOW DENSITY GALAXIES AND THE SPIN PARAMETER

It is known that some galaxies transform virtually all their gas into stars and others do not. The case of LSB galaxies is a good example where star formation in the disk has been inhibited, due to the low surface density of the disk ( van der Hulst et al. 1993). These galaxies form most of their stars in a first burst and then evolve into a rather quiescent phase ( Padoan, Jimenez & Antonuccio-Delogu 1997). On the other hand, High Surface Brightness (HSB) galaxies continually form stars, because of their higher surface density $\Sigma$.

The surface density in a galaxy disk and star formation activity can be related using the Kennicutt (1989) formalism, based on the Toomre (1964) stability criterion. If the Toomre parameter $Q = \Sigma_c/\Sigma$, where $\Sigma_c$ is a critical surface density, exceeds about unity, very little star formation takes place.

We assume that disks settle until they are rotationally-supported ( cf Fall & Efstathiou 1980, White & Rees 1978), so the final surface density is related to the initial spin parameter of the halo. If the spin parameter of the halo is large enough, $Q > 1$ and it is assumed that no star formation takes place in the disk.

The spin parameter of dark matter halos has been studied analytically and numerically by a number of authors ( e.g. Heavens & Peacock 1988, Barnes & Efstathiou 1987, Warren et al. 1992, Catelan & Theuns 1996), and is expected to anti-correlate only weakly with peak height ( and hence environment), and the range of spin parameters is broad $0.01 \lesssim \lambda \lesssim 0.1$.

In this section, we calculate the settling radius and final surface density of the gas, and compare with the Toomre (1964) criterion for instability of the disk. It is difficult to make the argument rigorously quantitative, because the effects of sub-clumping and interaction between the baryonic component and dark matter may not be trivial. However, the general picture may be illustrated by a model in which the gas settles in a ( given) dark matter potential, until it is rotationally-supported. For small baryon fractions, the dynamics are still dominated by the halo ( see below and Dalcanton et al. 1997), especially for the extended disk systems which are of most interest in this paper.

We assume that the gas component has the same specific angular momentum as the dark matter initially, both arising because of tidal torques ( see Navarro & Steinmetz for an alternative view). We take the universal dark matter profile found in simulations by Navarro, Frenk & White (1996):

$$\rho(r) = \frac{\delta r_s^3}{r(r + r_s)^2}$$

where $r_s \sim 5$ kpc is the ‘core’ radius of the halo, containing typically around 10% of the mass. $\delta \sim 5 \times 10^2 \Omega (1 + z_f)^3$ is the characteristic density in units of the background critical density ( White 1997). The bounding radius of the virialised halo, $r_{200}$ is defined as the radius within which the average density is 200 times the critical density, and is typically about $10 r_s$.

The spin parameter is $\lambda = J/|E|^{1/2}/GM^{5/2}$, where $J$ is the angular momentum of the halo and $E$ its energy. We assume that the dark matter is the dominant contributor to the potential, and compute the settling radius of baryonic matter of initial projected radius $r_i$. Equilibrium of the gas in the dark matter halo is achieved at a radius $r$ given by the solution to the equation $r M(< r) = a^2 \lambda^2 \Omega r_i M(< r_i)$, where $a \sim 2.5$, defined such that the square of the halo rotation speed at $r_i$ is $a^2 \lambda^2 GM(< r_i)/r_i$, and the value of $a$ corresponds to $\lambda \sim 0.4$ for rotational support ( White 1997). The surface density of the baryonic disk is given by $r \Sigma(r) = \Sigma(r_i) r_i dr_i/\delta r$, where the initial baryon surface density follows the dark matter profile, but scaled by the ratio $\Omega B/\Omega$. We take the baryon contribution to the density parameter $\Omega$ to be $\Omega_B = 0.035$, consistent with primordial nucleosynthesis if $H_0 = 65$ km s$^{-1}$ Mpc$^{-1}$ ( Walker et al. .

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so the self-gravity of the disk makes a small correction to the settling radius. The final surface density profile is shown for two halo masses in Fig. 1. Note that 8 percent of the halo mass resides within a radius of 0.2\(r_s\), so the dark matter still dominates there.

The mean value of the spin parameter, \(\simeq 0.05\) and its dispersion \(\sim 0.03\) are only weakly dependent on the power spectrum (Heavens & Peacock 1988, Barnes & Efstathiou 1987), so conclusions we draw from Fig. 1 are not model-sensitive. Also, the trends shown in Fig. 1 – larger gas distributions and lower densities for larger halo spin parameters, are clearly general features and not specific to the assumptions made here.

We turn now to the quantitative question of whether the resulting surface density is above or below the critical value apparently required for star formation. The critical surface density is \(\Sigma_c = \alpha\sigma_{\text{disp}}/(3.36G)\) (Kennicutt 1989), where \(\alpha \simeq 1,\) \(\sigma_{\text{disp}}\) is the velocity dispersion of the disk, and the epicyclic frequency, written in terms of the rotation speed, is \(\kappa = 1.41(V/r)\sqrt{1 + \ln V/d \ln r}\). The Toomre stability parameter \(Q \equiv \Sigma_c/\Sigma\) may then be written in terms of \(s \equiv r/r_s\), exploiting the scalings in the problem (ignoring a weak mass-dependence of \(\delta_r\)). (White 1997):

\[
Q(s) = \frac{\alpha}{3.36} \sqrt{\frac{\pi}{\Omega}} \frac{\sigma_{\text{disp}}}{r_s \Omega} \times \frac{f}{s^{3/2} I(s/f)} \frac{df}{d\lambda} \sqrt{M(s) + \frac{s^2}{(1 + s)^2}}. \tag{2}
\]

where \(f(s, \lambda) = r/r_i\) is the solution to \(sM(s) = \alpha^2 \lambda^2 s^2 M(s)\); the enclosed mass is related to \(M(s) \equiv \ln(1 + s) - s/(1 + s)\), and the initial surface density is related to \(I(s) = \sqrt{s^2 - 1}/2\). In addition to the dark lensing possibility, there are examples of halos with dark rotationally-supported disks. (e.g. Christodoulou et al. 1995), but note that the fastest-rotating \((\lambda \geq \Omega_B)\) disks should be stable (Mo et al. 1997).

In Fig. 1 and 2 we show the \(Q\) parameter for two halos of different mass, \(M = 10^9\,M_\odot\) and \(M = 10^{12}\,M_\odot\). The halos have formation redshift 1, but the curves are not very sensitive to these parameters. Following Bahcall and Casertano (1984), we take \(c_{\text{disp}} = 6\) and 15 km s\(^{-1}\) respectively, since we have no particular reason to believe that the velocity dispersion should depend sensitively on the star formation rate. Eleven curves are shown, corresponding to halos with \(\lambda\) in the expected range between 0.01 and 0.1. As expected, the higher spin halos are more stable, and the low-mass halo is stable for more-or-less all reasonable spin parameters. The high-mass halos, on the other hand, all produce stars somewhere in the disk. We do not consider here bar instabilities (e.g. Christodoulou et al. 1995), but note that the fastest-rotating \((\lambda \geq \Omega_B)\) disks should be stable (Mo et al. 1997).

In Fig. 3 and 4, we summarize the results as follows. For each halo mass and formation redshift, we calculate the maximum spin parameter which will produce stars somewhere in the disk (\(Q < 1\)). Fig. 3 shows results for \(c_{\text{disp}} = 10\,\text{km}\,\text{s}^{-1}\); for other values scale the mass by the cube of \(c_{\text{disp}}/10\). We see that the dependence on formation redshift is weak, and that higher mass halos have a progressively larger critical \(\lambda\). In Fig. 4, we show the fraction of halos giving rise to luminous disks in the model, assuming a gaussian distribution for \(\ln(\lambda)\) with mean \(\ln(0.05)\) and dispersion 0.5 (Mo et al. 1997). The solid curve is for \(c_{\text{disp}} = 10\,\text{km}\,\text{s}^{-1}\), the dashed curve allows a smooth mass-dependence of \(c_{\text{disp}}\) from 5 to 15 km s\(^{-1}\) as the mass changes from \(10^9\) to \(10^{12}\,M_\odot\). The fraction of luminous halos increases from 5\% at \(2 \times 10^9\,M_\odot\) to 95\% at just over \(10^{12}\,M_\odot\).

Dark gravitational lenses of high mass \(\sim 10^{12}\,M_\odot\), apparently required to explain the missing lens double quasar population (Hawkins 1997), are possible as the most massive examples of halos with dark rotationally-supported disks. These halos always have the possibility to cause splittings of images; the singular density profile assures this, and splittings of up to about 4 arcsec are possible from \(10^{12}\,M_\odot\) halos at \(z \sim 0.5\).

In addition to the dark lensing possibility, there are
The maximum halo spin parameter which will produce stars somewhere in the disk, according to the Toomre stability criterion, for halos of different total mass and formation redshift. The expected range of spin parameters for halos is $0.01 - 0.1$. The assumed disk velocity dispersion here is $10\,\text{km}\,\text{s}^{-1}$. Scaling solutions imply that the masses here scale as $c_{\text{disp}}^3$.

The fraction of halos producing luminous disks as a function of total halo mass. The solid curve assumes a disk velocity dispersion of $10\,\text{km}\,\text{s}^{-1}$, the dashed curve assumes the dispersion increases linearly with $\log(M)$ as indicated in the text. Many other interesting consequences of this model: the mass-dependence of the bright fraction will alter the luminosity function – it will no longer reflect the underlying halo mass function, which may be useful since most currently popular galaxy formation models over-predict the low-mass population; perhaps most interestingly, there should be no spiral galaxies with total masses below about $10^{10}M_\odot$ (cf. compilation by Roberts & Heynes 1994); low-mass or high-spin halos should lead to short-lived bright galaxies, affecting the interpretation of the faint blue number counts; luminous galaxies should be found in high-density environments. The last two points are explored in the next two sections.

It should be noted that the possibilities for dark galaxies are much lower if non-baryonic dark matter contributes much less than the critical density. In this case, the minimum spiral mass drops substantially, to around $10^7M_\odot$ if $\Omega_0 = 0.2$, but the disk self-gravity cannot be ignored in this case.

3 STAR FORMATION IN LOW DENSITY GALAXIES

LDGs will not be entirely dark, as it seems impossible to prevent some fragmentation of the initial halo (Tegmark et al. 1997; Padoan, Jimenez & Jones 1997). Without ongoing star formation in the disk, the galaxy will rapidly fade, producing a short-lived bright population of ‘disappearing dwarfs’. The details of the process are not crucial for the arguments in this paper, but for illustration we calculate the brightness of a halo of mass $10^{10}M_\odot$, using the star formation model of Padoan et al. 1997, with formation redshifts of 0.5 and 1 (Fig. 5). We also show the colour evolution in Fig. 6. The assumptions here are 2% star formation efficiency (a prediction of the model, but the results may be scaled if desired), and an initial metallicity of $Z=0.002$, $Y=0.24$. The graphs use the latest version of our synthetic stellar population code (Jimenez et al. 1997). These results evidently have consequences for the number counts of galaxies since the initial bright phase of LDGs may account for the population of blue galaxies found at high redshifts (cf Metcalfe et al 1997 and references therein). In the most extreme cases (Tyson et al. 1986) the observed limiting magnitudes for dark lenses are: $I > 24.5$, $R > 26$ and $B > 25$. And for the surface brightness $R > 27.9\,\text{mag}\,\text{arcsec}^{-2}$. Therefore, even at a redshift of 0.5, a LDG has a magnitude below the limits found by (Tyson et al. 1986), already 2-3 Gyr after the formation of its halo, and its mass-to-light ratio is about $1000M_\odot/L_\odot$, as required.

4 SPIN BIAS

The possible existence of dark galaxies has implications for bias. A bias in the luminous galaxy population has been invoked as a way to reconcile an Einstein-de Sitter universe with observations of low mass-to-light ratios in galaxy clusters, provided that luminous galaxies are found preferentially in clusters. On larger scales, a bias $b$ has important implications for determining the density parameter from peculiar velocity studies (e.g. Willick et al. 1997) or redshift-space distortions (e.g. Heavens & Taylor 1995), since linear studies determine $\Omega$ only in the combination $\beta \equiv \Omega^{0.6}/b$. Results from these studies are inconclusive at present, with values of $\beta$ ranging between about 0.5 and 1 (see e.g. Strauss & Willick 1995).

Spin bias will act in a similar way to high peak biasing, or natural biasing (Kaiser 1984, Davis et al. 1985, Bardeen et al. 1986, Cole & Kaiser 1989, Mo & White 1996), where lower-mass halos are anti-correlated with respect to higher-mass halos. The link with observation then requires
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5 SUMMARY

In this letter we have proposed a new scenario to explain the nature of dark lenses based on Low Density Galaxies, that we define as galaxies with a stellar halo component, and a rotationally supported gaseous disk unable to form stars because of its low surface density, which is mainly a consequence of the large spin parameter of the galactic halo.

The model with which we have chosen to demonstrate the ideas assumes a particular form of dark matter halo, and a specific star formation model to obtain the brightness of the galaxy, but these ingredients are not central to the argument. The general feature of high-spin systems with little initial star formation producing rotationally-supported spirals with large radii is robust. If one accepts the Kennicutt criterion for star formation, the possibility of dark galaxies then emerges quite naturally. Support for the relevance of the Kennicutt criterion comes from LSB galaxies, which show very little current star formation activity. Our analytic calculations can, of course, be regarded only as an approximation in hierarchical models of galaxy formation, and future gas-dynamic modelling will provide a more robust test of the hypothesis.

A firm prediction of our model is that LDGs should be seen in deep HI surveys in voids. The column density of LDGs can be high ($\sim 10^{21}$ cm$^{-2}$ but mass-dependent), but there is the possibility of ionization of the hydrogen, which would prevent their appearance in HI surveys. In any case, due to the way HI surveys are performed, there could be many undetected massive HI objects within a distance of 250$h^{-1}$ Mpc (Briggs 1990), but LDGs would be revealed by next generation HI surveys (e.g. Parkes).

The main conclusions of this work are:

- The spin parameter of the halo may play a critical role in determining whether galaxies quickly become dark.
- Spiral galaxies should have masses in excess of about $10^9 M_\odot$, and there is an upper limit for dark galaxies of around $10^{12} M_\odot$, if $\Omega_0 = 1$.
- LDGs convert about 2% of their baryonic mass into stars in the first couple of Gyr of their formation process. They consist of a stellar halo and a gaseous, quiescent disk. LDGs have apparent magnitudes $B \simeq 28$ mag at redshift $z=0.5$ and $B \simeq 32$ mag at $z=1.0$; they are therefore invisible. The colours of LDGs are very similar to the colours of LSB galaxies.
- A large fraction of the total number of galactic halos might contain an invisible LDG, with a higher probability in low mass halos, thus providing a mechanism, ‘spin bias’, for biasing the luminous galaxy distribution.
- The faint-end luminosity function of spiral galaxies should be flatter than the corresponding halo mass function, and there should be a population of ‘disappearing dwarfs’.

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† After this paper was completed, our attention was drawn to a paper by Meurer et al. (1996), in which they report the discovery of a very dark galaxy, with $M/L = 79$, a large HI disk, and star formation confined to the inner 0.8 kpc. This fits well with our predictions, being a transitional case of a galaxy which is unstable only in the central parts. The authors suggest that the star formation is controlled principally by the column density $n_{HI} > 10^{21}$ cm$^{-2}$, rather than $Q$, but the qualitative effect is the same, as seen in Fig. 1 and 2.
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