Synchrony-optimized power grids

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(Dated: August 29, 2014)

We investigate synchronization in power grids, which we assume to be modeled by a network of Kuramoto oscillators with inertia. More specifically, we study the optimization of the power grid topology to favor the network synchronization. We introduce a rewiring algorithm which consists basically in a \textit{hill climb} scheme where the edges of the network are swapped in order enhance the main measures of synchronization. As a by-product of the optimization algorithm, we typically have also the anticipation of the synchronization onset for the optimized network. We perform several robustness tests for the synchrony-optimized power grids, including the impact of consumption peaks. In our analyses, we investigate synthetic random networks, which we consider as hypothetical decentralized power generation situations, and also a network based in the actual power grid of Spain, which corresponds to the current paradigm of centralized power grids. The synchrony-optimized power grids obtained by our algorithm have some interesting generic properties and patterns. Typically, they have the majority of edges connecting only consumers to generators in the decentralized case, whereas synchrony-optimized centralized power grids have a minimal number of vertices with just one or two neighbors, known generically as dead ends and which have been recently identified as extremely vulnerable and responsible for cascade faults. Despite the extreme simplifications adopted in our model, our results, among others recently obtained in the literature, can provide interesting principles to guide future growth and development of real power grids.

PACS numbers: 89.75.Fb, 05.45.Xt, 89.75.-k

I. INTRODUCTION

Since Thomas Edison’s Pearl Street Station in Manhattan started operating in 1882, the power grids have continued to grow and are today probably the largest machines ever built \cite{1,2}. Furthermore, their growth is still far away from being complete, as the pursuit of renewable sources of energy and new technologies drive the integration of different power grids into intercontinental machines. The widespread use of alternating current (AC) creates the necessity to keep the whole power grid synchronized. Every generator in the power plants and all devices connected to the other side of the transmission lines need to operate with the same frequency (typically, 50 Hz in Europe and 60 Hz in most of the Americas) to keep the grid working correctly. A disruption in this synchronization causes malfunctioning, leading to power outages with possibly catastrophic proportions in real scenarios.

The phenomenon of synchronization, a key factor for the perfect operation of a power grid, has been the object of study for a long time \cite{3-6} in different areas of knowledge. It is present in a myriad of situations, arising naturally in many areas of biology, physics, social sciences, etc. It has been, however, only recently that a complex system approach has been devised to study the synchronization of power grids \cite{7}. Typically, a single power plant is a complicated machine, with a lot of tunable parameters necessary to its correct functioning. Although power grids can be, and surely are, analyzed and studied in all their finer details taking into account hundreds of power plants, substations, transmission lines, and many other devices, the idea of \cite{7} is to focus on the complexity of the underlying network of connections \cite{8,9} and its role on the overall synchronization process. In order to achieve such a goal, one treats the power plants as simple generators and the loads on the other side of transmission lines as passive machines.

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Energy conservation in this context yields a set of equations similar to the well know Kuramoto Model \cite{6}. This similarity, in its turn, enables us to study power grid synchronization with the vast set of tools and ideas already developed in the recent literature. Indeed, we have witnessed recently many works devoted to the analysis of power grids in the context of complex system as, for instance, the analysis of the European power grid \cite{10}, the effects of decentralization of energy production in the British power grid \cite{11}, the identification of parameters in individual vertices that turn the synchronous state more stable \cite{12}, the existence of Braess’s paradox \cite{13,14}, the role of the network topology on synchronization \cite{15} and a stability analysis of blackouts using basin-stability measures \cite{16}.

In the present paper, we study the optimization of the power grid topology in order to favor synchronization. More specifically, we adapt a previous algorithm proposed in \cite{17,18} to optimize the synchrony of a network built from usual Kuramoto oscillators to the case of power grids. The application of this algorithm produces power grids with many interesting properties besides the overall enhancing of the synchronization measures as, for instance, an anticipated onset of synchronization. We study the topological properties of these optimized networks and their robustness for edges (transmission lines) removal. We also discuss the impact of consumption peaks. Our results show that the optimized power grids exhibit, for considerably large increase of power consumption and for a wide range of parameters, a stable state where the
whole system synchronizes with a frequency smaller than the expected one. In contrast, for the non-optimal networks such a state of synchronization typically exists only for very small increases in consumption.

The paper is organized as follows. In Sec. II, we review briefly the power grid model of [1] and discuss some of its properties with relevance to our analysis. Sec. III is devoted to the introduction of our optimization algorithm. In Sec. IV, we show the numerical results obtained for both synthetic random network models and the Spanish power grid. The last section is left to some concluding remarks.

II. POWER GRIDS

For the sake of completeness, we will briefly present the derivation of the equations of motion for the power grid model of Ref. [1], and also derive some simple results concerning the synchronization of power grids which are important to our analysis. A power grid will be represented here by a network composed of $N$ vertices corresponding to two types of machines: $N_G$ generators and $N_C$ consumers (motors). The numbers of generators and consumers do not need to be equal. The power transmission lines correspond to $m$ edges connecting the vertices. The connectivity pattern is described by the usual symmetric adjacency matrix $A$, with elements $a_{ij}$ such that $a_{ij} = 1$ if vertices $i$ and $j$ are connected, and $a_{ij} = 0$ otherwise.

Each individual element $i$ of the network is a synchronous machine, generator or consumer, characterized by a power $P_i$, which is positive for generators and negative for the consumers. For each individual node of the network, simple energy conservation implies that this power must be equal to the sum of three contributions: the rate of change of the machine kinetic energy

$$P_{i \text{kin}} = I_i \dot{\theta}_i,$$

where $I_i$ and $\theta_i$ stand for, respectively, the moment of inertia and the phase of the $i$-th generator/consumer; the rate that energy is dissipated through friction

$$P_{i \text{diss}} = \gamma_i \dot{\theta}_i^2,$$

where $\gamma_i$ is dissipation constant associated to the machine at vertex $i$; and the total power transmitted to other vertices. In particular, the power transmitted from vertex $i$ to $j$ is given by

$$P_{i \text{trans}} = P_{ij}^{\text{trans}} = P_{ij}^{\text{max}} \sin(\theta_j - \theta_i),$$

where $P_{ij}^{\text{max}}$ represent the maximum power that can be transmitted along the transmission line connecting $i$ and $j$ vertices. Summing all the terms, one has

$$P_i = I_i \dot{\theta}_i + \gamma_i \dot{\theta}_i^2 - \sum_{j=1}^{N_{\text{trans}}} P_{ij}^{\text{max}} \sin(\theta_j - \theta_i).$$

From now on, we restrict ourselves to the idealization often assumed for power grids, namely that all elements in the grid have the same moment of inertia $I$, the same dissipation constant $\gamma$ and all transmission lines have the same capacity of transmission $P_{ij}^{\text{max}}$. Relaxing this hypothesis does not, apparently, lead to new interesting dynamical behavior, but makes all the analysis much more intricate. For the proper functioning of the power grid, all of the elements must operate with the same frequency $\Omega$ (for instance, 50 or 60 Hz for real power grids). In order to take into account small fluctuations around this value, we write the element phases as

$$\theta_i(t) = \Omega t + \phi_i(t),$$

with $\phi_i \ll \Omega$. Taking into account the above simplifications and keeping only linear terms in the perturbation $\phi_i(t)$ in (4), one has the so-called Kuramoto equation with inertia

$$\frac{d^2 \phi_i}{dt^2} = P_i - \alpha \frac{d\phi_i}{dt} + K \sum_{j=1}^{N} a_{ij} \sin(\phi_j - \phi_i),$$

where $P_i = (P_i - \gamma \Omega^2)/\Omega$, $\alpha = 2\gamma/I$, $K = P_{ij}^{\text{max}}/\Omega$, and $a_{ij}$ is the usual adjacency matrix for the underlying network. Some useful information can be obtained even before solving equations (6). The first point to take notice is the existence of a synchronized stationary state with the grid frequency $\Omega$. If such a state exists, it must obey $\dot{\phi}_i = \phi_i = 0$, which means that the stationary phases $\phi_i$ of the oscillators satisfy the equation

$$P_i + K \sum_{j=1}^{N} a_{ij} \sin(\phi_j - \phi_i) = 0,$$

where, interestingly, the dissipation parameter $\alpha$ takes no role. Summing both sides of equation (6) with respect to the index $i$, since the sine function is odd, we have that a necessary condition for the existence of this stationary state is that the powers $P_i$ must sum to zero, what obviously is nothing else than a statement of energy conservation for the whole network (neglecting, of course, transmission losses).

In order to calculate the network average frequency perturbation $\langle \phi \rangle = \frac{1}{N} \sum_{i=1}^{N} \phi_i$, we sum both sides of equation (6), resulting in

$$\frac{d}{dt} \langle \phi \rangle = -\alpha \langle \phi \rangle$$

which can be trivially solved and leads to

$$\langle \phi \rangle = \langle \phi_0 \rangle e^{-\alpha t}.$$

The average frequency perturbation vanishes in the limit $t \to \infty$, but this does not imply that each frequency $\phi_i$ converges to zero, since they can also attain a state with symmetric distribution of positive and negative frequencies with null mean. However, if we have synchronization, they do vanish individually. Equations (9) confirm an earlier numerical result that was verified in [13]: the time scale to reach asymptotically stationary states is proportional to $\alpha^{-1}$. The average phase can be easily evaluated from $\langle \phi \rangle = \frac{d}{dt} \langle \phi \rangle$,

$$\langle \phi \rangle = \langle \phi_0 \rangle + \frac{1}{\alpha} \langle \phi_0 \rangle \left( 1 - e^{-\alpha t} \right).$$
Notice also that, by virtue of equation (10), the asymptotic value of the average phase depends on the initial conditions and also on the parameter $\alpha$.

In order to analyze the synchronization process in our networks, we will use the order parameter $z(t)$ introduced originally by Kuramoto,

\[ z(t) = \rho(t)e^{i\phi(t)} = \frac{1}{N} \sum_{j=1}^{N} e^{i\phi_j(t)}, \tag{11} \]

which corresponds to the centroid of the phases if they are viewed as a swarm of points moving around the unit circle. For incoherent motion, the phases are scattered on the circle homogeneously and $\rho \approx N^{-1/2}$ for large $N$ as a consequence of the central limit theorem, while for a synchronized state the points should move in a single lump and, consequently, $\rho \approx 1$. As our equations are of second order, we will also use the average squared frequency,

\[ \langle \dot{\phi}_j^2 \rangle = \frac{1}{N} \sum_{j=1}^{N} \dot{\phi}_j^2(t). \tag{12} \]

Synchronization for the power grid requires, of course, that all elements have the common frequency $\Omega$, which implies that $\dot{\phi}_j(t) = 0$. In our calculations, we will use extensively the averages

\[ r = \left| \frac{1}{\delta T} \int_{T_i}^{T_i+\delta T} z(t) dt \right|, \tag{13} \]

and

\[ \nu^2 = \frac{1}{\delta T} \int_{T_i}^{T_i+\delta T} \dot{\phi}_j(t)^2 dt. \tag{14} \]

The value of $T_i$ must be long enough to ensure that a stationary state has been reached, and $\delta T$ cannot be too small to assure good statistics.

III. THE OPTIMIZATION ALGORITHM

The optimization algorithm employed here is adapted from that one introduced in [17] and [18] for the original Kuramoto model, which corresponds to the model of the last section without inertia ($I = 0$). Here, by optimization, we mean a rewiring of the edges such that the network has higher values of the order parameter $r$. As we will see, as a byproduct of the optimization, we typically have an earlier onset of synchronization, i.e., the power grid synchronizes with smaller values of the transmission line capacity $K$. As it occurs typically in synchronization problems, there is a phase transition from a non synchronized state to a synchronized one occurring at a critical value of the capacity $K_c$, i.e., for $K > K_c$, the order parameter $r$ is an increasing function of $K$, while for $K < K_c$ we have typically $r \approx 0$ (no synchronization).

The strategy of the algorithm is roughly the following. We choose a value of $K^* > K_c$ such that $r(K^*)$ is reasonably high. Then, one considers a rewiring: a randomly selected edge connecting two vertices is removed if it does not disconnect the network, and two randomly chosen disconnected vertices are connected; and the new value of $r(K^*)$ is evaluated. If the rewiring results in a higher value for $r(K^*)$, one keeps the modification or, otherwise, one discharges the rewiring and returns the network to its previous configuration. This procedure is repeated until $r(K^*)$ attains a maximum value. In practice, our algorithm limits the maximum number of iterations up to $10^4$ and also stop after a certain number of consecutive iterations failed to achieve a higher value of $r$ (we used $4m$ iterations, with $m$ the number of edges). These edge swaps preserve the average degree of the initial network (as the number of edges is kept the same), but not the degree distribution (this is not a problem a priori, since we do not know any reason to keep a particular degree distribution for power grids).

As for the original Kuramoto model [17][18], we found that this kind of hill climb algorithm produces networks that generally present the desired properties of higher values of the order parameter $r$ and smaller values of $\nu^2$ for all the values of $K$, and not just for $K = K^*$, the value for which the optimization was performed. Typically, a considerably earlier onset of synchronization is also observed for the optimal network. Moreover, the properties of the optimal network are independent from the initial conditions and also from the precise value of the capacity $K = K^*$ for which the rewiring is done, as long as it is chosen to satisfy $K^* > K_c$, i.e., it corresponds to a synchronization regime. For small networks, typically with the number of vertices up to $N = 20$, the optimized network seems to be unique, since all runs return the very same optimal network. However, for larger networks, different runs of our algorithm can lead to different optimized networks, but their properties are essentially the same, showing just some small fluctuations over the average values. In this way, strictly speaking, we cannot guarantee that the results found with the algorithm are global maxima, and this is particularly important for large networks. Nevertheless, the returned optimized networks always show a substantial improvement of the synchronization properties when compared to the original ones.

IV. NUMERICAL RESULTS

In this section, we will show some of the results obtained using the optimization algorithm discussed in the last section. We analyze two situations which we call decentralized and centralized energy production. The first case corresponds to a network with $N$ vertices in which $N/2$ are generators with $P_i = 1$ and the other half are consumers with $P_i = -1$. This situation tries to mimic future development in power grids where many (perhaps small) power plants, with different energy sources, will be connected to the grid. The second case, on the other hand, represent the current situation, with energy production confined to a small number (compared to the number of consumers) of large power plants. Our numerical computations were done by using the SciPy package for python [19]. The system of ordinary differential equations (6), in
particular, is solved with SciPy odeint routine, which is indeed a implementation of lsoda from the FORTRAN library odepack. We have also made extensive use of the NetworkX package\textsuperscript{[20]} for calculating network properties and characteristic parameters and for creating the network graphs.

### A. Decentralized power grid

As our first example, we applied the hill climb algorithm to the power grid depicted in Figure\textsuperscript{[1]}, which corresponds to a network built starting from the Erdos-Renyi (ER) model\textsuperscript{[8]} with $n = 100$ vertices and average degree $\langle k \rangle = 4$. In ER networks, edges are added with probability $p = \langle k \rangle / n$, independent from any other edge already connecting the vertices. Half of the vertices were randomly selected as generators with $P_1 = 1$, and the other half as consumers, with $P_2 = -1$. We assume $\sigma = 1.0$. Figure\textsuperscript{[1]} shows the optimized network obtained by applying our algorithm. Figures\textsuperscript{[2]} and\textsuperscript{[3]} show the evolution of the order parameter $r$, given by equation\textsuperscript{[11]}, for the non-optimized and optimized networks, respectively. Figures\textsuperscript{[2]} and\textsuperscript{[3]} show the average squared velocity $v^2$ given by equation\textsuperscript{[12]}. In this case, 10 different runs of the algorithm were computed and the synchronization diagrams for each one are depicted in the graphics.

The synchronization diagrams of Figure\textsuperscript{[2]} were calculated starting from $K = 0$ and integrating the equations of motion\textsuperscript{[6]} for a time interval $T_r = 300$ and then averaging the corresponding values of $r$ and $v^2$ in the next time interval of length $\delta T = 100$. We repeated these steps, using the outcome of the last run as initial conditions of the next step, with increments of $\delta K = 0.02$ until $K$ reached the value $K = 1.5$. From this point, we reversed the step direction and started to decrease the value of $K$, again with steps of size $\delta K$. By means of this procedure, we have two synchronization diagrams, called the forward and backward continuations\textsuperscript{[21]}, respectively, and the hysteresis loop appears when they do not coincide.

For the non-optimized networks, both the forward and backward continuations are the same, except for some small fluctuations, indicating a sort of second order phase transition, without any hysteresis behavior. For the optimized network, however, interesting new behaviors arise. First, we have a significant earlier onset of synchronization (smaller values of $K_r$). Second, the values of the order parameter $r$ attain much higher values, indicating that the phases, although not all equal, have a much narrower distribution than for the non-optimized case. Third, and very interesting, the type of the phase transition seems to change from second to first order with a hysteresis behavior, as the forward and backward curves no longer match each other. The existence of a first order phase transition with hysteresis behavior was already studied for the model\textsuperscript{[6]} in the case of an all-to-all topology,\textsuperscript{[22]}. Our results indicate that the topology plays a fundamental role in the synchronization properties of a network, since a rewiring of the network, keeping everything else unchanged, is capable of modifying the kind of synchronization transition. This kind of first-order phase transition for the synchronization is the key dynamical point of the so-called explosive synchronization behavior, which is now under intensive investigation, see\textsuperscript{[21]} and\textsuperscript{[23]} for instance.

Figure\textsuperscript{[3]} shows the evolution of some network characteristic parameters which are typically used to describe the topology\textsuperscript{[8,9]} along the optimization process labeled by the average order parameter value $r$, which is, by construction, strictly increasing in the optimization process. The panels depict the results for each of the 10 runs of the optimization process for the network in Figure\textsuperscript{[1]}. The standard deviation for the degree distribution, $\sigma_k$, depicted in Fig.\textsuperscript{[3]a}, is decreasing for each case, indicating that the whole grid changes to a more homogeneous situation (the average degree is invariant along the optimization process). Note that in this case, differently from\textsuperscript{[17]}, we do not observe a sudden increase in $\sigma_k$ when $r$ approaches 1. This is due to the fact that we do not have perfect phase synchronization in the power grid model, otherwise this

![Figure 1](image1.png)

**FIG. 1.** An example of optimized network (figure b) obtained with the hill climb algorithm discussed in the text. We used an Erdos-Renyi network (figure a) with $n = 100$ vertices and $m = 202$ edges as initial condition. There are 50 blue squares and 50 red circles representing consumers and generators, with $P_2 = -1.0$ and $P_1 = 1.0$, respectively. Vertex’s size is proportional to its degree.

![Figure 2](image2.png)

**FIG. 2.** Synchronization diagrams for the networks depicted in figure\textsuperscript{[1]} Both the forward (red) and backward (blue) continuations in $K$, with $\delta K = 0.02$, are shown. Panels (a) and (b) show, respectively, the order parameter $r$ and the average squared velocity $v^2$ for the non-optimized network, suggesting a second order phase transition, since the forward and backward continuations roughly coincide. In turn, panels (c) and (d) show the results of 10 runs of the optimization algorithm. In this case, the phase transition appears to be of first order with a hysteresis behavior. The arrow in panel (a) shows where the value of $K^*$ for which the optimization process was performed.

![Figure 3](image3.png)

**FIG. 3.** The evolution of some network characteristic parameters which are typically used to describe the topology\textsuperscript{[8,9]} along the optimization process labeled by the average order parameter value $r$, which is, by construction, strictly increasing in the optimization process.
would imply that no energy would be transmitted, see equation (7) and Fig. 5. The average shortest path length $\langle l \rangle$, Fig. 3p, increases through the optimization, as it was expect from the homogeneity of the degree distribution we have just seen and the fact that the clustering coefficient $C$ (Fig. 3) fluctuates around the same small value for all the runs. Finally, as it was also observed in [17], the fraction $p_-$ of edges connecting consumers and generators increases monotonically to values close to 1, see Fig. 3.

![Figure 3](image3.png)

**FIG. 3.** The evolution of the main network characteristic parameters through the optimization algorithm steps. The lines are the results for each one of the 10 runs of the optimization algorithm applied in the Erdos-Renyi network depicted in figure 1 with $n = 100$ vertices and average degree $\langle k \rangle = 4.0$. The panels depict: (a) the standard deviation for the degree distribution $\sigma_k$, (b) the average shortest path length $\langle l \rangle$, (c) the clustering coefficient $C$, and (d) the fraction $p_-$ of edges connecting consumers to generators.

For the decentralized cases studied here, the best way to build the power grid is to guarantee that transmission lines connect only consumers to generators. Knowing this optimal pattern of connections, we could use the algorithm proposed in [24] for optimizing decentralized power grids. Since it is an algorithm to maximize $p_-$ for the network, it should provide synchrony-optimized decentralized power grids as well. The advantage of the algorithm of [24] is that it is, typically, much faster than ours. Another interesting point is the dynamics of the generators and consumers. As we have seen earlier, the time scale to reach the stationary state is determined only by the dissipation parameter $\alpha$, see equation (9). Nevertheless, the way that the individual phases distribute themselves is determined by the topology of the power grid, see equation (7). Figure 4 shows the evolutions of the phases for the networks in Figure 1. Interestingly, for the same value of the capacity $K$, we found that for the optimized power grids the phases have a much narrower distribution compared to the results of the non-optimized network. Figure 5 depicts the phases of Figure 4 around the unitary circle, at $t = 20$ (the stationary regime), and the corresponding histogram of the absolute values of the sine of the phase difference along the edges connecting vertices $i$ and $j$. As we see, the optimized grids have typically much narrower phase differences $|\phi_i - \phi_j|$, and hence smaller transmitted power per edge. This has important implications for the power grid functioning. The phase difference between connected vertices is obviously related to the transmitted power, but also to the losses in the transmission lines [23]. The ability of keeping the phases closer to each other with the same value of the capacity $K$ has an important role in the search for efficiency.

![Figure 4](image4.png)

**FIG. 4.** The phases of the generators (red lines) and consumers (blue lines) as a function of the time for the networks in figure 1. Top and bottom panels show the phases for the non-optimized and optimized networks, respectively. The initial phases were randomly draw from the uniform distribution in $(0, 2\pi)$ and the velocities from the uniform distribution in $(0, 1)$. The parameters are $\alpha = 1.0$ and $K = 2.0$.

![Figure 5](image5.png)

**FIG. 5.** The phases of the generators (red circles) and consumers (blue circles) depicted around the unit circle, at $t = 20$ (stationary regime), for the networks of Figure 4. Inserted, the histograms of the absolute values of the sine of the phase difference along the edge connecting vertices $i$ and $j$. The optimized-grid has a narrower distribution $\sin(\phi_i - \phi_j)$ and consequently smaller transmitted power per edge in the network.
it is possible to include in the optimization process a measure of robustness, as the ones proposed in \cite{26,27}. For example, it is rather simple to change the algorithm in such a way that an edge swap is accepted only if it increases both the synchronization order parameter and the robustness measure adopted. However, we can assure that our optimization procedure does not weaken the robustness of the original network against edge removals.

In order to test if the properties of the optimized network are independent of the initial topology, we also applied the hill-climb algorithm to networks generated from the Barabasi-Albert model \cite{23} constructed by using a preferential attachment mechanism which results in power law degree distributions. We found that the optimized networks properties are still fairly the same.

\section{Centralized power grid}

After having analyzed the decentralized energy production scenario, we move on to the centralized case, which is perhaps better suited for describing the current status of power grids. However, for these cases, instead of synthetic networks we now use a real power grid, the Spanish grid \cite{29}. Grossly, it is a network of 192 vertices and 287 edges, a grid of intermediary size which is convenient for our analysis having a reasonable compromise between results and CPU time. In the Spanish network, 64 vertices are generators and the others 128 are consumers. We set the power of each consumers $P_i = -1$ and in order to supply the necessary demand, each generator has a power $P = 2$. Again, we use $\alpha = 1.0$. For this case, we use $T_r = 300$ and $\delta T = 100$

Figure \ref{fig:7} shows the original grid in the left handed side and the optimized network is in the right one, with the corresponding degree distributions. An important question in the optimization of real grids is related to the costs of rewiring since it physically means changing transmission lines, and this is not an easy task. For the case of figure \ref{fig:7}, we found that the optimized network shares around 37% of edges with the original Spanish grid. A by-product of the optimization algorithm in the case of centralized power grids is the decrease of the number of vertices with just one or two neighbors, see Figure \ref{fig:7}, known as dead ends. Ref. \cite{16} studies the stability of the power grid in the case of a short circuit that disconnects a generator to the rest of the grid. In this situation, the generator is unable to transmit electrical energy, leading to an increase in its rotational frequency. When the transmission lines returns to its proper functioning, the generator is out of its synchronized state and this can cause a cascade of disconnections. In particular, the vertices in these dead ends are extremely vulnerable in these situations. By reducing the dead ends, the optimization algorithm employed here also helps to avoid such cascade faults. Our exhaustive numerical experiments show that the algorithm performs extremely well in optimizing the networks, with a robust improvement in the synchronization properties as figure \ref{fig:8} shows. Even keeping in mind that we are employing here a very simplified model that ignores many details, the result is interesting as it shows that there is plenty of room for optimizing real grids.

Along the optimization process, the topological properties of the network show in some cases a similar behavior to the decentralized case discussed in the previous section. Figure \ref{fig:9} depicts the evolution of the main network characteristic parameters along the optimization. The standard deviation $\sigma_k$ for the degree distribution also decreases, thus indicating a much more regular topology. The average shortest path length $\langle l \rangle$ decreases from a value of around 6 to oscillate around 5. Interesting, the clustering coefficient decreases from a considerably high value to almost 0. The fraction $p_c$ of edges connecting consumers to generators is also increasing. However, due to physical limitations since there are much more consumers than generators, $p_c$ can only saturates at a value considerably smaller than 1. With respect to edge removal, the situation is similar to that one we have found earlier for the decentralized case, for the three removal rules studied, the behavior of both the non-optimized and the optimized networks

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6}
\caption{The size of the giant component $S(m)$ as a function of the number $m$ of edges removed for the networks in figure \ref{fig:1} for different removal strategies. Solid blue lines correspond to random removal of edges (averaged over 100 runs), red dashed lines to degree product rule and green dash-dot lines to degree betweenness rule. Top and bottom panels show the giant components for the non-optimized and optimized networks, respectively. No appreciable differences were detected.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7}
\caption{An example of an optimized network with centralized power generation. The initial network topology is the real Spanish grid (left handed side) with 128 blue squares and 64 red circles representing consumers and generators, with $P_i = -1.0$ and $P = 2.0$, respectively. The optimized network is shown in the right handed side. For both cases, the vertex’s size is proportional to its degree. The corresponding degree distribution for the original and optimized networks are also shown in the panels.}
\end{figure}
are very similar, see Figure 10. Again, we can assure that our optimization procedure does not weaken the robustness of the original network against edge removals.

We will also study how the networks depicted in 7 respond to a sudden increase in the consume of energy. This means that the power $P_i$ consumed by each machine experiences suddenly an increase such that we no longer have $\sum_{i=1}^{N} P_i = 0$ during a typically short time period. This kind of consumption peak situation has been a recurrent topic in many recent works, see 7 [11] [15], for instance. Here, however, we will analyze what happens to the power grid during the transient interval of increased consumption. Notice that due to the unbalance $\sum_{i=1}^{N} P_i \neq 0$, the results of Section I for the synchronized state are not valid anymore. Figure 11 shows such situation for both the non-optimized and optimized networks. From $t = 0$ to $t = 250$ the system is balanced, $\sum_{i=1}^{N} P_i = 0$, and the power grid reaches a stationary state of synchronization with frequency $\Omega$ since $\nu^2 = 0$. Then, during the period of time from $t = 250$ to $t = 280$, each consumer increases its power to $P = -6$. After that, they return to their previous values. For the non-optimized network, the system desynchronizes during the consumption peak; most of the generators increase their frequencies whereas the consumers decrease. For the optimized networks, however, both generators and consumers decrease their frequencies at the same pace in such a way that they reach a new synchronized state with a lower frequency, which implies a decreasing in the rate of energy dissipation (that is proportional to the squared frequency), allowing the grid to supply the extra demanded power.

This new synchronization state during a consumption peak corresponds to the solution $\phi(t) = \omega t$. Plugging this form of $\phi_i$ in equation 4, we have that the frequency shift $\omega$ should satisfy

$$\omega = \frac{1}{\alpha N} \sum_{i=1}^{N} P_i. \quad (15)$$

When the sum $\sum_{i=1}^{N} P_i < 0$, the frequency shift becomes negative, as is the case of figure 11. The value of $\omega$ calculated from 15, shown as the black lines in 11, matches perfectly the numerical values. We stress that this synchronized state with lower frequency is asymptotically stable for all the parameters used here, the dynamics of the power grid tend to it spontaneously during episodes of consumption peaks. Notice that 15 is also valid when $\sum_{i=1}^{N} P_i > 0$, which would correspond to a decrease in consumption, but keeping the same power pumped into the grid. We would have, for the optimized network, a new synchronized state with a larger frequency, allowing in this way the network to dissipate the power pumped in excess. From the dynamical point of view, episodes of consumption peaks and decreases are equivalent, respectively, to generation shortages and increases.

In order to study the range of parameters for which this new synchronized state is indeed stable, we consider the difference between the average frequency of the generators and consumers, $\langle \phi \rangle_g - \langle \phi \rangle_c$, during the period of time (that we took to be 30) in which each consumer has a power increase of $\delta P$.  

![FIG. 8. Synchronization diagrams for the networks depicted in figure 7. Both the forward (red) and backward (blue) continuations in $K$, with $\delta K = 0.02$, are shown. Panels (a) and (b) show the order parameter $r$, equation 11, and the average squared velocity $\nu^2$, equation 12, for the non-optimized network. For the optimized network, panels (c) and (d) show an expressive improvement in the synchronization properties. The arrow in panel (a) shows where the optimization process was done. Notice the characteristic hysteresis behavior in the synchrony-optimized network.](image1)

![FIG. 9. The evolution of some network properties through the optimization algorithm steps. The blue lines are the results for the optimization of the Spanish grid in figure 7. The different panels show (a) the standard deviation $\sigma_i$ of the degree distribution, (b) the average shortest path length $l$, (c) the clustering coefficient $C$ and (d) the fraction $p_e$ of edges connecting consumers to generators.](image2)

![FIG. 10. The size of the giant component $S(m)$ as a function of the number $m$ of edges removed for the network in figure 7 for different removal strategies. Solid blue lines correspond to random removal of edges (averaged over 100 runs), red dashed lines to degree product rule and green dash-dot lines to edge-betweenness rule. Top and bottom panels show the giant components for the non-optimized and optimized networks, respectively. Again, no appreciable differences were detected.](image3)
This situation is depicted in Figure 12, where we see that for the non-optimized network, the synchronization state with frequency \( \Omega + \omega \) lasts only to a value of \( \delta P \approx 3 \). On the other hand, for the optimized network, the average frequencies are the same to a much larger range, until \( \delta P \approx 6 \). Moreover, the same qualitative result holds for different values of \( \alpha \).

FIG. 12. Difference in average frequency, \( \langle \dot{\phi} \rangle_g - \langle \dot{\phi} \rangle_c \), of the generators and consumers for the non-optimized, top panel, and optimized network, bottom panel, during the period where each consumers increases their power consume to \( \delta P \), whereas the generators keep the same power. We used \( K = 4 \) and different values of \( \alpha \).

V. FINAL REMARKS

In this paper, we have studied how to optimize a network topology for enhancing the synchronization properties of power grids. By applying a simple hill-climb algorithm that swaps edges, we show it is possible to enhance the network synchronization measures and also to expressively reduce the synchronization threshold \( K_c \). We applied our proposed algorithm to synthetic random networks and also to network inspired in a real world power grid, namely the Spanish grid. Having a smaller value of the threshold \( K_c \) means that the power grid could work in a stable state with transmission lines with smaller transmission capacities, reducing the associated costs. This is achieved, in the optimized grids obtained by our algorithm, through a better distribution of the electric flow, typically reducing the burden on the transmission lines. The improvement of the phase synchronization, with higher values of the order parameter \( r \) and, consequently, smaller phase difference between the vertices, also means smaller losses in the transmission.

The synchrony-optimized power grids obtained by our algorithm have some interesting generic properties besides optimal synchronization patterns. For the decentralized generation scenarios, which try to mimic future development in power grids where many small power plants, probably with different energy sources providing the demanded power to the grid, the optimized network typically have the majority of edges connecting only consumers to generators. On the other hand, for the cases corresponding to the contemporary paradigm of centralized generation power grids, the synchrony-optimized power grids have a minimal number of vertices with just one or two neighbors, known generally as dead ends and which have been recently identified as extremely vulnerable and responsible for cascade faults [16]. Also, during power supply unbalances, the synchrony-optimized power grids exhibit a synchronized state with frequency \( \Omega + \omega \), with \( \omega \) given by equation (15) for much larger range of parameters than the original power grids. Obviously, the models and operations considered here are extreme simplifications of the real world power grids. For instance, implementing a rewiring actually means rebuilding transmission lines through perhaps hundreds of kilometers in places without a favorable geography. Nevertheless, our results, together with the previous on obtained in [10–16], show that there is plenty of room for optimizing real power grids and even provide interesting simple principles to guide the future growth and developments.

ACKNOWLEDGMENTS

The authors thank CNPq, CAPES and FAPESP (grant 2013/09357-9) for the financial support. AS thanks Prof. Leon Brenig for several discussions and for the warm hospitality at the Free University of Brussels, where the initial part of this work was done.

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