Research on Availability Model of Two-dimensional Warranty Products Based on Incomplete Maintenance

Rongcai Wang¹, Zhonghua Cheng¹,* and Qian Wang²

¹ Shijiazhuang Campus, Army Engineering University, Shijiazhuang, China
² The No.32153nd Troop of PLA, Zhangjiakou, China

E-mail: asd3v36@163.com

Abstract. In view of the problem that the availability of two-dimensional warranty products is difficult to meet the actual needs of users during the warranty period, on the basis of incomplete preventive maintenance, this paper uses the accelerated failure time (AFT) model to describe the failure rate form of products, constructs the two-dimensional warranty product availability model and gives an effective solution. On this basis, taking the maximum availability of two-dimensional warranty products as the optimization objective, the optimal preventive maintenance interval and warranty period optimization scheme are calculated. Finally, the validity and accuracy of the model are verified by an example, which provides a scientific basis for manufacturers to formulate two-dimensional product warranty strategy.

1. Introduction

In modern society, with the rapid development of economy, the market competition is becoming more and more fierce. Therefore, in order to reduce costs and improve product market competitiveness, many product manufacturers have to find another way to seek better market competition strategy. This includes the two-dimensional warranty strategy of products, especially in the field of durable goods such as automobiles, the two-dimensional warranty has been widely used. Product warranty refers to the technical support activities provided by product manufacturers to users within a certain period of time after the products are sold, which is the main content of after-sales service provided by manufacturers. The reasonable warranty strategy can not only effectively reduce the cost of product warranty, but also effectively improve the availability of products within the warranty period. At present, the general meaning of warranty refers to the initial warranty, that is, warranty. Blischke and Murthy [1] have generally accepted the definition of warranty that "warranty is a kind of guarantee provided by product manufacturers for products or services purchased by consumers". Two-dimensional warranty refers to that the warranty period of a product is determined by two dimensions, as shown in Figure 1, which are calendar time and usage degree. When one of these two dimensions reaches the threshold value, the warranty service of the product is terminated [2]. For example, the two-dimensional warranty period of a certain brand of car is 4 years and 80000 kilometers, that is, when the car calendar time reaches 4 years or the driving mileage reaches 80000 kilometers, the warranty service will end.
At present, product manufacturers generally pay more attention to the problem of two-dimensional warranty cost. In two-dimensional warranty, it is generally believed that the failure rate of products is affected by two factors: calendar time and use degree within a certain period of time. Different users use different products, the improvement of product usage will lead to the increase of product failure rate. In order to prevent the high cost of warranty, the two-dimensional warranty strategy came into being. In the social production practice, many high-value products have very powerful functions, and the downtime loss caused by product function failure is also increasing, so users' requirements for the availability of related products are also increasing. In product warranty service, preventive maintenance is usually carried out to reduce the failure rate of the product during the warranty period, so as to improve the availability of the product and the satisfaction of users. At present, the research of preventive maintenance is mainly focused on one-dimensional warranty period, and the research results of preventive maintenance in two-dimensional warranty period are relatively few, and most of the research is based on the analysis of product manufacturers, with more research on warranty cost, especially the lack of research on product availability in two-dimensional warranty period that users pay attention to. At present, the research on preventive maintenance of products in warranty period is mostly on the basis that preventive maintenance is complete maintenance (repair as new), and the repair maintenance of products after functional failure is also mostly complete maintenance. But in fact, due to the limitation of maintenance cost or technical ability, the product can not reach the state of "repair as new" after maintenance, that is, the failure rate of the product after maintenance has declined but not fully recovered, so the application of incomplete maintenance [3] is more extensive and close to the reality. At present, scholars have carried out a lot of research on incomplete maintenance. Yu [4] established a two-dimensional product warranty cost model with incomplete maintenance improvement factor and time domain as variables to be optimized. Baneijee [5] models and analyzes the warranty cost under different warranty strategies by studying the incomplete and minimum maintenance strategies in two-dimensional warranty. In the two-dimensional warranty period, Shahanaghi [6] takes the minimum warranty cost as the goal, adopts the preventive maintenance strategy with incomplete cycle, establishes the warranty decision-making model, and optimizes the number and degree of preventive maintenance in the warranty period.

In the field of product two-dimensional warranty research, the foundation of product maintenance modelling is the construction of two-dimensional failure rate function. In the traditional one-dimensional warranty, the failure rate function of products is a function of time. This method will cause large errors in the two-dimensional warranty modelling, so it is necessary to construct the appropriate two-dimensional failure rate function expression. At present, there are three methods to construct two-dimensional failure rate function in the field of two-dimensional warranty, which are single factor variable method [7][8], two factor variable method [9][10], and composite scale method [11]. The three methods are applied in the field of two-dimensional product
warranty. Through the research and summary of relevant literature, the single factor variable method and the two factor variable method are most widely used. The single factor variable method assumes that the product utilization rate remains constant for a single user, which may not be completely consistent with the actual situation, but this assumption can effectively transform the two-dimensional problem into one-dimensional problem, which is convenient for the establishment and solution of the model; the two factor variable method is a binary distribution function, which is more practical in form, but the specific expression is not easy to determine, and the parameter estimation The calculation is also complex, and the two-dimensional warranty modeling and solution process of this method is very tedious, which requires a lot of approximate estimation, so the final result will have a large error. Based on the limitations of the existing two-dimensional failure rate function construction methods, lawless [12] uses the accelerated failure time (AFT) model and the proportional risk model to study the impact of product utilization on failure. The accelerated failure time model is a method to construct two-dimensional failure rules of products based on the idea of accelerated life test. This method has certain theoretical basis and is widely used for maturity. In the field of two-dimensional warranty availability, Cheng [13] establishes the model of product joint preventive warranty availability by distinguishing the recovery degree of product failure rate caused by preventive maintenance conducted jointly by manufacturers and users, which provides guidance for manufacturers and users to make warranty decisions. Han [14] used the virtual working age method to deal with incomplete maintenance, established a two-dimensional warranty availability model, and obtained the optimal interval of two-dimensional preventive maintenance.

2. Two-dimensional availability model of warranty products

2.1. Two-dimensional warranty failure modeling

Based on the above research, this paper uses the accelerated failure time (AFT) model to construct the failure rate function of two-dimensional warranty products. Suppose the cumulative fault distribution function of the product under the design utilization rate \( r_1 \) is \( F(t; \alpha_1, \beta) \), where, \( \alpha_1 \) and \( \beta \) are the scale parameters and shape parameters of the fault distribution respectively. When the actual utilization rate is \( r \) (hereinafter referred to as "utilization rate"), the cumulative failure distribution function of the product has the same form as that under the design utilization rate \( r_1 \), but the scale parameter becomes:

\[
\alpha(r) = \alpha_1 \left( \frac{r}{r_1} \right)^k
\]

Among them, acceleration factor \( k \geq 1 \). Therefore, when the utilization rate is \( r \), the cumulative failure distribution function of the product can be expressed as:

\[
F(t; \alpha(r), \beta) = F_1 \left( t; \alpha_1 \left( \frac{r}{r_1} \right)^k, \beta \right)
\]

(2)

It is found that the shape parameter does not change with the failure rate, so it is omitted in the following formula. Therefore, when the utilization rate is \( r \), the failure rate function of the product can be obtained as follows:

\[
\lambda(t; \alpha(r)) = \frac{f(t; \alpha(r))}{1 - F(t; \alpha(r))}
\]

(3)

The basis of incomplete maintenance modeling is how to express the influence of incomplete maintenance on product failure rate. The treatment methods for incomplete maintenance can be roughly divided into two categories: improvement factor method [15] and virtual length of service method [16]. In this paper, the virtual working age method proposed in document 16 is used to describe the effect of incomplete maintenance on product failure rate function. The starting time of incomplete preventive maintenance is set as 0, and the effect of this preventive maintenance is only
related to the state of the previous stage, so that $v_i$ represents the virtual working age of the product after the $i$-th incomplete preventive maintenance, of which $i \geq 1$. $\theta(x)$ is the improvement factor of incomplete preventive maintenance, which indicates the reduction degree of virtual working age. Its value range is $[0,1]$. It is a reduction function of maintenance level $x$. Therefore, when $x$ increases, $\theta(x)$ decreases, and virtual working age decreases, that is to say, maintenance effect is better. In particular, when $\theta(x) = 0$, the product reaches the state of "repair as new", that is, complete maintenance; when $\theta(x) = 1$, the product reaches the state of "repair as old", that is, minimum maintenance. Because of the incomplete preventive maintenance strategy adopted in this paper, the value range of improvement factor $\theta(x)$ should be $(0,1)$. Suppose that the two-dimensional preventive maintenance interval of the product is $(T_0, U_0)$ during the product warranty period, so for the given maintenance level $x$, when the product is under incomplete preventive maintenance with calendar time $t$, the change of the failure rate of the product is shown in Figure 2.

![Incompleter preventive maintenance](image)

**Figure 2.** Change of product failure rate under incomplete preventive maintenance.

When the incomplete preventive maintenance is carried out on calendar time $t$, the expression of failure rate function $\lambda(t; \alpha(r))$ when the maintenance level of the product is $x$ is:

$$\lambda(t; \alpha(r)) = \begin{cases} 
2(t + (i-1)T_0(\theta(x)-1); \alpha(r)) & (i-1)T_0 \leq t < iT_0, i = 1,2,\ldots,n \\
2(t + nT_0(\theta(x)-1); \alpha(r)) & nT_0 \leq t < T_w 
\end{cases} \quad (4)$$

If the product is under incomplete preventive maintenance with service level $u$, the expression of failure rate function $\lambda(t; \alpha(r))$ of the product at maintenance level $x$ only needs to replace $T_0$ in the above formula with $U_0/r$.

### 2.2. Model description and assumption

It is assumed that the warranty product is repairable, and the failure rate of the product increases with the increase of calendar time and use degree. There is a certain linear relationship between calendar time and use degree, i.e. $r = u/t$. For the same user, product utilization $r$ is a certain value, and different users have different utilization rates of products. For the same batch of products, product utilization $r$ is a random variable subject to a specific distribution. In order to improve the availability of the product in the two-dimensional warranty period $(T_0, U_0)$, especially for the warranty products with large downtime loss, and to ensure the user's use efficiency in the product warranty period, the manufacturer implements the two-dimensional periodic incomplete preventive maintenance strategy in the two-dimensional warranty period of the product. This strategy means...
that if one of the product calendar time and use degree reaches the preventive maintenance interval \( T_0 \) or \( U_0 \), incomplete preventive maintenance will be carried out for the product; when there is a failure in the preventive maintenance interval, the minimum maintenance will be carried out immediately, regardless of the response time of the product failure. In a certain period of time, the minimum maintenance can be decomposed into a limited number of independent non-homogeneous Poisson processes, and the expected minimum maintenance times of products are:

\[
E[N(t)] = \int_0^t \lambda(s)ds = -\ln(1 - F(t))
\]  

(5)

Among them, \( N(t) \) represents the failure times of products within \([0,t]\), \( \lambda(s) \) represents the failure rate function, and \( F(t) \) represents the cumulative failure distribution function.

2.3. Model notations

\((T_0, U_0)\): Two-dimensional incomplete preventive maintenance interval, \( T_0 \) represents calendar interval, \( U_0 \) represents usage interval.

\((T_w, U_w)\): The two-dimensional warranty period of the product, \( T_w \) represents the calendar time warranty period, \( U_w \) represents the usage warranty period.

\( T_f \): Time consuming of single repair maintenance.

\( T_p \): Time consuming for a single incomplete preventive maintenance.

\( T_R \): Total repair time during warranty period.

\( T_p \): Total preventive maintenance time during warranty period.

\( R, H(r), h(r) \): Utilization rate of products, distribution function and probability density function of utilization rate.

\( r_t \): The lowest and highest utilization rate of a batch of products respectively.

\( r_0 \): The ratio of preventive maintenance interval \( U_0 \) to \( T_0 \), two-dimensional warranty period \( U_w \) to \( T_w \).

\( \lambda(t; \alpha(r)) \): The failure rate function of the product when the utilization rate is \( r \).

\( \theta(x) \): Improvement factor of incomplete preventive maintenance, \( x \) represents maintenance level.

\( A_w(T_w, U_w) \): The average availability of the product within the two-dimensional warranty period \((T_w, U_w)\) when the preventive maintenance interval is \((T_0, U_0)\).

2.4. The model

In a two-dimensional warranty, product availability can be expressed as:

\[
A_w(T_w, U_w) = \frac{\text{Warranty period–Expected downtime}}{\text{Warranty period}}
\]  

(6)

As shown in Figure 3, when building a two-dimensional warranty availability model, it needs to be analyzed and discussed in two situations, namely \( r_0 \leq r_w \) and \( r_0 > r_w \).
Table 1 shows the times of incomplete preventive maintenance under different warranty periods in the availability model.

Table 1. Number of incomplete preventive maintenance under different warranty periods.

| Number of preventive maintenance | Warranty period | Numerical value |
|----------------------------------|-----------------|-----------------|
| $N_1$                            | $T_w$           | $\text{int} \left[ T_w / (T_0 + T_p) \right]$ |
| $N_2$                            | $T_w$           | $\text{int} \left[ (T_w r) / (U_0 + T_p r) \right]$ |
| $N_3$                            | $U_w$           | $\text{int} \left[ U_w / (U_0 + T_p r) \right]$ |
| $N_4$                            | $U_w$           | $\text{int} \left[ U_w / (T_0 + T_p r) \right]$ |

Note: “int” indicates rounding down the value.

Case 1. $r_0 \leq r_w$

In this case, it can be further divided into the following three situations: $r \leq r_0$, $r_0 < r \leq r_w$ and $r > r_w$.

1) $r \leq r_0$

The preventive maintenance interval of the product is $T_0$, and the number of preventive maintenance under the two-dimensional warranty period is $N_1$. At this time, the expected downtime of the product is $T_1$:

$$T_1 = T_{pp} + T_f$$

$$= T_p * N_1 + T_f * \int_{0}^{T_0} \lambda(t; \alpha(r))dt$$

$$+ T_f * \sum_{i=1}^{N} \int_{(i-1)(T_0 + T_p)}^{i(T_0 + T_p)} \lambda \left( t + i * (T_0 + T_p) (\theta(x) - 1) ; \alpha(r) \right) dt$$

$$+ T_f * \int_{N(T_0 + T_p)}^{T_0} \lambda \left( t + N_1 * (T_0 + T_p) (\theta(x) - 1) ; \alpha(r) \right) dt$$

(7)

2) $r_0 < r \leq r_w$
The preventive maintenance interval of the product is $U_0/r$, and the number of preventive maintenance under the two-dimensional warranty period is $N_2$. At this time, the expected downtime of the product is $T_2$:

$$T_2 = T_p + T_p$$

$$= T_p \cdot N_2 + T_f \cdot \int_0^{U_0} \lambda(t; \alpha(r))dt$$

$$+ T_f \cdot \sum_{i=1}^{N_1-1} \int_{U_i/r}^{U_{i+1}/r} \lambda \left(t + i \cdot \left(U_0/r + T_p\right) \left(\theta(x) - 1\right); \alpha(r)\right)dt$$

$$+ T_f \cdot \int_{U_N/r}^{T_f} \lambda \left(t + N_2 \cdot \left(U_0/r + T_p\right) \left(\theta(x) - 1\right); \alpha(r)\right)dt$$

(8)

3) $r > r_w$

The preventive maintenance interval of the product is $U_0/r$, and the number of preventive maintenance under the two-dimensional warranty period is $N_3$. At this time, the expected downtime of the product is $T_3$:

$$T_3 = T_p + T_p$$

$$= T_p \cdot N_3 + T_f \cdot \int_0^{U_0} \lambda(t; \alpha(r))dt$$

$$+ T_f \cdot \sum_{i=1}^{N_1-1} \int_{U_i/r}^{U_{i+1}/r} \lambda \left(t + i \cdot \left(U_0/r + T_p\right) \left(\theta(x) - 1\right); \alpha(r)\right)dt$$

$$+ T_f \cdot \int_{U_N/r}^{T_f} \lambda \left(t + N_3 \cdot \left(U_0/r + T_p\right) \left(\theta(x) - 1\right); \alpha(r)\right)dt$$

(9)

Therefore, in the case of two-dimensional warranty period $(T_w, U_w)$ and $r_0 \leq r_w$, the average availability of product two-dimensional warranty is:

$$A_w(T_w, U_w) = \int_0^{U_N} (U_w - T_i)/T_i dH(r) + \int_{U_N}^{U_f} (U_w - T_2)/T_2 dH(r)$$

$$+ \int_{U_f}^{U_N} (U_w/r - T_3)/(U_w/r) dH(r)$$

(10)

Case 2. $r_0 > r_w$

In this case, it can also be divided into three cases: $r \leq r_w$, $r_w < r < r_0$ and $r > r_0$.

1) $r \leq r_w$

The preventive maintenance interval of the product is $T_0$, and the number of preventive maintenance under the two-dimensional warranty period is $N_1$. At this time, the expected downtime of the product is $T_4$:

$$T_4 = T_p + T_p$$

$$= T_p \cdot N_1 + T_f \cdot \int_0^{T_f} \lambda(t; \alpha(r))dt$$

$$+ T_f \cdot \sum_{i=1}^{N_1-1} \int_{(T_i+T_p)/r}^{(T_{i+1}+T_p)/r} \lambda \left(t + i \cdot \left(T_0 + T_p\right) \left(\theta(x) - 1\right); \alpha(r)\right)dt$$

$$+ T_f \cdot \int_{(T_N+T_p)/r}^{T_f} \lambda \left(t + N_1 \cdot \left(T_0 + T_p\right) \left(\theta(x) - 1\right); \alpha(r)\right)dt$$

(11)
The preventive maintenance interval of the product is $T_0$, and the number of preventive maintenance under the two-dimensional warranty period is $N$. At this time, the expected downtime of the product is $T$. 

$$T = T_p + T_f$$

$$= T_p \cdot N_1 + T_f \cdot \int_0^{T_0} \lambda(t; \alpha(r)) dt$$

$$+ T_f \cdot \sum_{i=1}^{N-1} \int_{t+T_i}^{T_i+T} \lambda(t+i; (T_0+T_p)(\theta(x)-1); \alpha(r)) dt$$

$$+ T_f \cdot \int_{T_0}^{T_0+T} \lambda(t+N_3; (T_0+T_p)(\theta(x)-1); \alpha(r)) dt$$

To sum up, in the two-dimensional warranty period $(T, U)$, when the product utilization rate $r$ is subject to a certain distribution $H(r)$, under the regular two-dimensional incomplete preventive maintenance, the average availability of the product is:

$$A_u(T, U) = \int_0^T (T - T_i) / T_i dH(r) + \int_0^T (U_i / r - T_i) / (U_i / r) dH(r)$$

$$+ \int_0^T (U_i / r - T_i) / (U_i / r) dH(r)$$

Therefore, the average availability of product two-dimensional warranty is:

$$A_u(T, U) = \int_0^{T_f} (T_f - T_i) / T_i dH(r) + \int_0^{T_f} (U_i / r - T_i) / (U_i / r) dH(r)$$

$$+ \int_0^{T_f} (U_i / r - T_i) / (U_i / r) dH(r)$$

3. Numerical example

3.1. Problem description
In the social survey of a certain type of commercial household car, it is found that the use of this type of car has two dimensions: calendar time and use degree (mileage). In order to improve the availability and customer satisfaction of automobile products during the warranty period, the warranty contract signed by the manufacturer and the user stipulates that the automobile manufacturer shall provide the user with incomplete preventive maintenance service on a regular basis, and shall take the minimum maintenance for the faults occurring in the interval period of preventive maintenance, and the warranty cost shall be borne by the manufacturer.

Assuming that the initial two-dimensional warranty period of this model of vehicle is 3 years, 60000km, the relationship between maintenance level and incomplete preventive maintenance improvement factor [17] is as follows:

\[ \theta(x) = (1 + x)e^{-x} \]  
(16)

The cumulative fault distribution function of the product under the design utilization rate \( r_i \) obeys the Weibull distribution:

\[ F_i(t;\alpha_i) = 1 - \exp \left( - \frac{t}{\alpha_i} \right)^\beta_i \]  
(17)

When the product utilization rate is \( r \), the cumulative failure distribution function of the product is:

\[ F(t;\alpha(r)) = 1 - \exp \left( - \left( \frac{r}{r_i} \right)^{k_i} \left( \frac{t}{\alpha_i} \right)^\beta_i \right) \]  
(18)

Therefore, the product failure rate function is:

\[ \lambda(t;\alpha(r)) = \frac{f(t;\alpha(r))}{1 - F(t;\alpha(r))} = \frac{\beta_i \left( \frac{r}{r_i} \right)^{\beta_i} \left( \frac{t}{\alpha_i} \right)^{\beta_i}}{t^{\beta_i-1}} \]  
(19)

Through the statistical analysis of the same type of automobile products, we can get the approximate distribution of product utilization. In order to facilitate the analysis and calculation, it is assumed that the utilization rate of products is uniformly distributed:

\[ R \sim U[0, 200] \]  
(20)

Other parameter settings are shown in Table 2:

| Parameter | Value |
|-----------|-------|
| \( T_f \) | 4 days |
| \( T_p \) | 2 days |
| \( T_w \) | 3 years |
| \( U_w \) | 60000 km |
| \( x \) | 2 |
| \( \alpha_i \) | 0.5 |
| \( \beta_i \) | 0.8 |
| \( k \) | 1.05 |
| \( r_b \) | 30000 km/year |
| \( r_f \) | 1000 km/year |
| \( r_i \) | 20000 km/year |

3.2. Problem solving
Using software for calculation and numerical solution, according to the fault characteristics of this type
of vehicle, determine the appropriate value range of $T_0$ and $U_0$, let $T_0$ take the value in [0.01,1], the corresponding step is taken as 0.01 year, $U_0$ take the value in [0.02,2], the step is taken as 0.02 thousand kilometers, generate 10000 groups of $(T_0, U_0)$, a year is calculated as 365 days, and get the corresponding availability value of each group of $(T_0, U_0)$ through iterative calculation by MATLAB. Draw three-dimensional diagram of availability model under different two-dimensional preventive maintenance interval $(T_0, U_0)$, as shown in Figure 4.

![Availability model 3D](image)

**Figure 4.** Availability $A_w(T_w, U_w)$ for different $(T_0, U_0)$.

### 3.3. Result analysis

This section presents the analysis and judgment process of Figure 4.

1) When the product availability is the optimization objective, there is an optimal two-dimensional incomplete preventive maintenance interval. It can be seen from Figure 4 that there are extreme points in the three-dimensional image of product availability model, which shows that the optimal two-dimensional preventive maintenance interval can be obtained by implementing the two-dimensional incomplete preventive maintenance under the condition of availability as the constraint. Through calculation, after the implementation of incomplete preventive maintenance, the maximum product availability is 0.9336, the corresponding optimal two-dimensional incomplete preventive maintenance interval is $(0.5$ year, 19400 km) during the warranty period.

2) It can be seen from Figure 4 that the three-dimensional image of product availability is in a fluctuating state, and the calendar time and use degree jointly affect the product availability. When the utilization rate of products is uniformly distributed, the availability of calendar time and utilization degree is not the same under different combinations. When the fixed calendar time is used, the availability decreases first and then increases with the increase of the usage degree; when the fixed usage degree is used, the availability also decreases first and then increases with the calendar time. Under the joint action of the two variables, the maximum value is finally obtained at $(0.5$ year, 19400 km).

### 4. Conclusion

The author takes the availability of two-dimensional warranty products within warranty period as the research object, puts forward the warranty strategy of implementing two-dimensional incomplete preventive maintenance within warranty period, and establishes the corresponding product availability model. The example shows that the warranty strategy can improve the availability of two-dimensional warranty products, meet the user's demand for higher availability of
products, and provide the basis for manufacturers and users to make warranty decision-making scheme.

References
[1] Blischke W R and Murthy D N P 1996 Product Warranty Handbook (New York: Marcel Dekker)
[2] Murthy D N P 2006 Product Warranty and Reliability Annals of Operations Research 143 133-46
[3] Chan P K W and Downs T 1978 Two Criteria for Preventive Maintenance IEEE Transactions on Reliability 27 272-3
[4] Yu J Mao C M and Chen X Z 2010 Two-dimensional Product Guarantee Cost of Incomplete Maintenance Industrial engineering and management 15 57-61
[5] Banicic R and Bhattachaijee M C 2012 Analysis of a Two-dimensional Warranty Servicing Strategy with an Imperfect Repair Option Quality Technology & Quantitative Management 9 23-33
[6] Shahanaghi K and Noorossana R 2013 Failure Modeling and Optimizing Preventive Maintenance Strategy during Two-dimensional Extended Warranty Contracts Engineering Failure Analysis 28 90-102
[7] Tong P Song X and Liu Z 2014 Designing and Pricing of Two-dimensional Extended Warranty Contracts Based on Usage Rate International Journal of Production Research 52 6362-80
[8] Bouguerra S and Rezg N 2012 Opportunity Study on the Adoption of an Extended Two dimensional Warranty for a Randomly Failing Product 9th Int. Conf. on Modelling Optimization & Simulation
[9] Pal S and Murthy 2003 An Application of Gumbel’s Bivariate Exponential Distribution in Estimation of Warranty Cost of Motorcycles International Journal of Quality & Reliability Management 20 488-502
[10] Li X Y and Jia Y X 2015 Optimal Two-dimensional Preventive Maintenance Policy Based on Asymmetric Copula Function Telkommika 13 722-9
[11] Duchesne T and Lawless J F 2000 Alternative Time Scales and Failure Time Models Lifetime Data Analysis 6 157-79
[12] Lawless J Hu J and Cao J 1995 Methods for the Estimation of Failure Distributions and Rates from Automobile Warranty Data Lifetime Data Analysis 1 227-40
[13] Cheng Z H Wang Q Bai Y S Zhao R D and Guo S Q 2019 Optimization Model of Two dimensional Warranty Equipment Availability Journal of Ordnance Engineering College 33 34-9
[14] Han Y C. 2015 Research on Optimization Model of Two-dimensional Preventive Maintenance Interval of Equipment (Shijiazhuang: Ordnance Engineering College)
[15] Malik M A K 1979 Reliable Preventive Maintenance Policy AIIE Transactions 11 221-28
[16] Kijima M 1989 Some Results for Repairable Systems with General Repair Journal of Applied Probability 26 89-102
[17] Sahin I and Polatoglu H 1996 Maintenance Strategies Following the Expiration of Warranty IEEE Transactions on Reliability 45 220-8