ULTRA-HIGH-ENERGY COSMIC RAYS FROM A MAGNETIZED STRANGE STAR CENTRAL ENGINE FOR GAMMA-RAY BURSTS
O. Esquivel,1,2 and D. Page,3
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RESUMEN
Se ha tratado de asociar a los rayos cósmicos de ultra alta energía (RCUAE) a las fuentes más variadas y poderosas que se conocen en el universo. Los estallidos de rayos gamma (ERG) son candidatos naturales. En este trabajo sostienemos que tales rayos cósmicos pueden ser acelerados por ondas electromagnéticas de alta amplitud (OEAA), las cuales son creadas al momento en que la aproximación MHD del campo en el viento generado por el generador central del ERG deja de ser válida. En tal situación se considera como generador central a una estrella extraña con rotación diferencial la cual obtiene la estrella al momento en que una estrella de neutrones se transforma en una estrella extraña dentro de un sistema binario de rayos X de baja masa. Las OEAA generadas de esta manera aceleran los iones ligeros a las más altas energías, \( E = q\eta \Delta \Phi_{\text{max}} \), en donde la eficiencia \( \eta \sim 10^{-1} \) da cuenta de todas las posibles pérdidas energéticas. De manera alternativa, también consideramos la posibilidad de que una vez formadas, las OEAA se vuelquen entre sí, dando paso así a una turbulencia electromagnética relativisticamente poderosa. Bajo esta suposición, se estima un límite inferior para la eficiencia, \( \eta \sim 10^{-2.5} \). Debido a su edad, los sistemas binarios de rayos X de baja masa pueden localizarse en regiones de baja densidad de materia interestelar, p. ej., en cúmulos globulares o inclusive en el medio intergaláctico en el caso de los sistemas con un gran movimiento propio; en cuyo caso las pérdidas energéticas sufridas por los rayos cósmicos en el medio circundante son evitadas y dichas partículas pueden experimentar el voltaje máximo disponible, produciendo eventos que rebasarían inclusive las observaciones existentes.

ABSTRACT
Ultra-high-energy cosmic rays (UHECRs) have been tried to be related to the most varied and powerful sources known in the universe. Gamma-ray bursts (GRBs) are natural candidates. Here, we argue that cosmic rays can be accelerated by large amplitude electromagnetic waves (LAEMWs) when the MHD approximation of the field in the wind generated by the GRB’s magnetized central engine breaks down. The central engine considered here is a strange star born with differential rotation from the accretion induced conversion of a neutron star into a strange star in a low-mass X-ray binary

1Astronomisches Rechen-Institut am Zentrum für Astronomie der Universität Heidelberg, Heidelberg, Germany
2Fellow of IMPRS for Astronomy and Cosmic Physics, Heidelberg.
3Departamento de Astrofísica Teórica, Instituto de Astronomía, UNAM, México,
system. The LAEMWs generated this way accelerate light ions to the highest energies $E = q\eta \Delta \Phi_{\text{max}}$ with an efficiency $\eta \sim 10^{-1}$ that accounts for all plausible energy losses. Alternatively, we also consider the possibility that, once formed, the LAEMWs are unstable to creation of a relativistically strong electromagnetic turbulence due to an overturn instability. Under this assumption, a lower limit to the efficiency of acceleration is estimated to be about $\eta \sim 10^{-2.5}$. Due to their age, low mass X-ray binary systems can be located in regions of low interstellar medium density as, e.g., globular clusters or even intergalactic medium in case of high proper motion systems, and cosmic ray energy losses due to proton collisions with photons at the decelerating region are avoided, thus opening the possibility for particles to exploit the full voltage available, well beyond that currently observed.

Key Words: Cosmic Rays — Gamma-ray sources — gamma-ray bursts — Neutron stars

1. INTRODUCTION

A long standing puzzle in astrophysics remains the origin of the highest energy cosmic ray events with energies in excess of $10^{20}$ eV (Nagano & Watson 2000). The ultra-high-energy cosmic rays (UHECRs) observed so far follow a well known energy spectrum $N(E) \sim E^{-2.7}$ beyond the feature known as the 'ankle' located at $10^{18.5}$ eV (Axford 1994; Bird et al. 1994). The origin of the particles with energies greater than $\sim 10^{19}$ eV is usually attributed to extragalactic sources (Axford 1994). The distance an UHECR can travel is limited by the energy losses due to photo-pion production in collisions with cosmic microwave background photons. This sets a radius, $R_{\text{GZK}} \sim 50$ Mpc, beyond which the energy losses are copious and where the energy of a cosmic ray can be lowered even by a few orders of magnitude. This mechanism would generate a signature in the spectrum at $\sim 5 \times 10^{19}$ eV, known as the GZK cutoff (Greisen 1996; Zatsepin & Kuzmin 1966).

However, in recent years serious constraints on its existence prompted the attention to models containing new physics as the possible solution for the UHECR enigma (see Berezinsky 2008 for a review of the models).

A plethora of astrophysical objects have been considered as possible sources of UHECRs. Among them gamma ray bursts (GRBs) have received special attention (Vietri 1995; Milgrom & Usov 1995; Waxman 1995; Chen, et al. 2002; Vietri et al. 2003). In fact, the energetics associated to local GRBs is found to enable a GRB-UHECR association (Wick et al. 2004). Wick et al. (2004) considered that a good estimate for the local emissivity $\dot{\epsilon}_{\text{UHECR}}^{GRB}$ needed to power cosmic rays with energies $\geq 10^{20}$ should be $\dot{\epsilon}_{\text{UHECR}}^{GRB} \approx U/t_{\gamma\pi} \approx 10^{44}U_{21} \text{ ergs Mpc}^{-3} \text{ yr}^{-1}$, where $U = 10^{21}U_{21} \text{ ergs cm}^{-3}$ is the observed energy density in $> 10^{20}$ eV cosmic rays ($U_{21} \approx 0.5$ for the HiRes experiment and $U_{21} \approx 2$ for AGASA) and the timescale of a proton with energy $\sim 10^{20}$ eV for photo-pion production is $t_{\gamma\pi}$. For a local distribution of GRBs ($z \ll 1$), which is inside the GZK cutoff, pre-BeppoSAX estimates
of the luminosity density (Vietri 1995; Waxman 1995) argue for a value of \( \dot{\epsilon}_{\text{GZK}}^{(\text{GRB})} \approx 10^{44} \dot{\epsilon}_{44} \) ergs Mpc\(^{-3}\) yr\(^{-1}\), where \( \dot{\epsilon}_{44} \approx \) few. In a more recent analysis, Vietri et al. (2003) have arrived at a value of \( \dot{\epsilon}_{44} \approx 1.1 \). Even considering the uncertainties in these estimates, the similarity of the two luminosity densities \( \dot{\epsilon}_{\text{UHECR}}^{(\text{GZK})} \) and \( \dot{\epsilon}_{\text{GRB}}^{(\text{GZK})} \) make it reasonable to consider that UHECRs may be powered by the local GRBs.

The \( \sim 10^{50} – 10^{52} \) ergs (Frail 2004) required to fit the estimated GRBs' energetics, as well as the short observed GRB durations, naturally point toward gravitational energy associated to compact objects as the original energy source. One broad class of central engine (CE) models consider the merger of two neutron stars (Paczyński 1986), or a black-hole and a neutron star (Paczyński 1991), in a binary system or an isolated “failed” supernova (Woosley 1993) in which the released gravitational energy is temporarily stored in thermal energy and part of it, through neutrinos, is inefficiently converted into relativistic outflow and successive shocks required to fit the GRBs' light curves. Most of these models eventually involve a black-hole which also swallows a very large part of the energy. And in addition to that, the recent analysis of (Soderberg et al. 2006) shows that the supernova-GRB connection is not as strong as previously thought, inducing (Gal-Yam et al. 2006) to suggest a new explosive process to explain the lack of GRB-associated supernova detections. Another class of CE models takes advantage of the possibility of storing a significant part of the released gravitational energy into rotational kinetic energy of the newly formed compact object (Usov 1992), hence limiting the tremendous energy drain by thermal neutrinos and avoiding the black-hole gluttony. This kinetic energy can then be extracted on time scales of seconds using the torque exerted by the strong magnetic field of the compact star. In the “DROCO” (Differentially Rotating Compact Object) scenario (Kluźniak & Ruderman 1998) differential rotation of the compact object acts as a powerful dynamo and results in magnetic field strengths of the order of \( 10^{17} \) G or higher where subsequent buoyancy forces induce emergence of flux ropes at the stellar surface and produce successive sub-bursts.

2. DIFFERENTIALLY ROTATING COMPACT OBJECTS

Here, we explore the possibility that UHECRs are accelerated in the Poynting-dominated outflow of a GRB when the central engine is a new born strange star with a rotation period \( \Omega \sim 10^4 \) s\(^{-1}\) and a magnetic field of the order of \( 10^{17} \) G. Strange stars (Alcock et al. 1986; Haensel et al. 1986) are self-bound compact object made entirely of deconfined quark matter, comprising the three flavors of light quarks \( u, d \) and \( s \), which is assumed to be the true ground state of hadronic matter. Within this “strange matter hypothesis” (Bodmer 1971; Witten 1984) once a seed of quark matter appears within a compact star it will rapidly convert the whole star into a strange star in a time scale of the order of milliseconds up to a few seconds (Lugones et al. 1994; Harko et al. 2004). If this can happen in core collapse supernovae then all objects we call “neutron stars” may actually be strange stars; there
is however some evidence that this is not the case (see, e.g., (Weber 2005) for a discussion). We will consequently consider a more plausible scenario assuming that the critical density for appearance of quark matter is large enough that only the most massive neutron stars may convert into strange stars. Such a conversion is thus expected to occur in neutron stars which have accreted a significant fraction of a solar mass in a low-mass X-ray binary system (LXMB). The energetics, comparable to a typical core-collapse supernova, and the expectable frequency of such events were considered by Cheng & Dai (1996) who proposed them as candidates for GRBs. Given the accretion spin-up of the neutron star and its subsequent shrinking during its conversion to a strange star, the new-born strange star is plausibly rotating at sub-millisecond period while the differential rotation due to the very different density profile of a strange star compared to a neutron star can be expected to results in a DROCO as described in (Dai & Lu 1998), i.e., with a magnetic field of the order of $10^{17}$ G. An attractive aspect of the strange star scenario is that, most baryonic matter being converted into quark matter, it avoids the problem of baryon contamination of the outflow (Haensel et al. 1991). Nevertheless, the outer crust of the neutron star, with a mass of about $10^{-5} \, M_\odot$, is most probably not converted into strange matter and rather expelled by the neutrino flux and the highly super-Eddington thermal luminosity of the bare strange star (Usov 1998, 2001; Page & Usov 2002), as well as by the flux ropes that emerge at the stellar surface (Kluźniak & Ruderman 1998), providing the protons which can be accelerated at energies well above $10^{20}$ as we will describe later.

3. THE FLUX

The particle flux which one can expect from such event can be estimated by analogy with standard pulsar physics. In the magnetosphere of the CE a number of processes take place that enhance the multiplicity of pairs compared to the proton current (Usov 1992), besides the pairs produced by the strange star surface. For an estimate we consider the pulsar’s proton current as determined by the Goldreich-Julian current (Goldreich & Julian 1969)

$$\dot{N}_p = f \dot{N}_{GJ} \approx f \frac{\Omega^2 B_f R^3}{ec} \sim 10^{42} \Omega_4^2 f B_{17} \, s^{-1}$$

where $\Omega_4 = \Omega/10^4 \, s^{-1}$ and $B_{17} = B/10^{17} \, G$, and have introduced an enhancement factor $f$ which could be larger than 1. We can write the particle flux at the Earth, for energies $\geq 10^{20} \, eV$, as

$$I(\geq 10^{20} \, eV) \sim \frac{c}{4\pi} \dot{N}_p \Delta t \alpha \nu_{GRB} n_c T_{20}$$

where $\Delta t$ is the duration of acceleration activity of the source, $\nu_{GRB}$ the local GRB rate, a fraction $\alpha$ of them being of the type we consider, $n_c = 0.02 \, \text{Mpc}^{-3}$ the local density of galaxies and $T_{20}$ the lifetime of protons above $10^{20} \, eV$. For energies in excess of $10^{20} \, eV$, correspond propagation times
$T_{20} \leq 3 \times 10^8$ yr, however, the vast majority of the sources within the region limited by $R_{GZK}$ lie at $\sim 40 - 50$ Mpcs from us, thus it is appropriate to use $T_{20} \sim 3 \times 10^8$ yr (Aharonian & Cronin 1994). So far the true birthrate of local GRBs has not been established. The enhancement factor which accounts for the collimation of the luminosity has been calculated recently by a fitting procedure to both the observed differential peak flux and redshift distributions at the same time (Firmani et al. 2004), yielding a local GRB rate $\nu_{GRB} \sim 5 \times 10^{-5}$ yr$^{-1}$. A reasonable estimate (see below) for $\alpha$ is of the order of 10%.

This gives us, considering a production life-time $\Delta t \sim 10$ s,

$$I(\geq 10^{20}\text{eV}) \sim 2 \times 10^{-20}fB_{17}\Omega_{17}^2\Delta t_{10} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

which is safely comparable, and even above if $f > 1$, to the observed flux $I(\geq 10^{20}\text{eV}) \sim 2.6^{+2.5}_{-0.6} \times 10^{-20}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ (Nagano & Watson 2000). Notice that the energy in cosmic rays of $10^{20}$ eV, $\dot{N}_p(\Delta t)10^{20}\text{eV} \sim 10^{51}$ ergs, represents about 1% of the total initial kinetic energy of the star. Since not all the kinetic energy lost by the star will necessarily be detectable, in the GRB, this energy may be comparable, or even larger, than the GRB energy.

Acceleration of cosmic rays in GRBs has been considered, as in many other astrophysical objects, to be due to the Fermi acceleration process (Fermi 1949), this mechanism taking place either in the plasma outflow (Waxman 1995) or in the external shock of the outflow when the latter hits the interstellar medium at a large distance from the CE (Vietri 1995). Nonetheless, it has been shown by Gallant and Achterberg (Gallant & Achterberg 1999) that the energy gain by this mechanism in external shocks when starting from nonrelativistic energies is much lower than predicted previously, so in a more realistic situation the Fermi acceleration process could hardly reproduce the highest energy events so far observed. Chen, et al. (2002) outlined a different scenario in which Alfvén shocks traveling through the plasma can accelerate light nuclei in a similar fashion a terrestrial linear acceleration does, i.e., the acceleration is stochastic and particles are accelerated by the interplay of the accelerating and decelerating phases of the plasma wakefields generated by the Alfvén shocks. In this stochastic process, the energy losses associated with curvature and synchrotron radiation and with ultra-high-energy proton-plasma proton collisions are minimal, so the system can be regarded as one in which the acceleration is maximum.

Instead of considering Alfvén shocks, we consider here that large-amplitude electromagnetic waves (LAEMWs) are responsible not only of accelerating particles to produce a GRB event (Usov 1994) but also responsible for the acceleration of plasma protons to the highest energies. Poynting flux dominated models of GRBs have been explored recently (Zhang & Mészáros 2004) as an alternative to the fireball model of Rees & Mészáros (1992) because the efficiency of radiation is rather high and the peak photon energy distribution $E_p$ could be well reproduced, among other observational facts. LAEMWs are naturally generated when the MHD approximation of the magnetized outflow breaks down. This happens at a radius, $R_{MHD}$, from the CE where
the plasma density \( n_\pm \), which decreases like \( \sim r^{-2} \) (Rhoads 1999), drops below the density needed for applicability of the MHD approximation, the Goldreich-Julian density \( N_{GJ} \), which decreases like \( \sim r^{-1} \) (Goldreich & Julian 1969). The LAEMWs generated this way have a frequency \( \Omega \sim 10^4 \, \text{s}^{-1} \), change polarity on a length scale of \( 2\pi (c/\Omega) \sim 2 \times 10^7 \, \text{cm} \), and are very strong which can be seen if we consider the adimensional Lorentz-invariant parameter \( \nu \equiv eB/mc\Omega \sim 10^9 \) which is the ratio of the cyclotron frequency in the wave field \( \omega = eB/mc \) to the wave frequency (Gunn & Ostriker 1971). Such a high \( \nu \) value can be interpreted by saying that the particles move in a nearly static field. Particles \( \text{surf-ride} \) (Arons 2003) the LAEMWs and suffer a ponderomotive force in the radial direction due to the inductive electric field orthogonal to the magnetic field \( f_{\text{pond}} = (q/c)\langle \delta v \times \delta B \rangle = mc\Omega\nu \). Because the particles move relativistically, the radius of curvature associated to an UHE proton \( r_c = 2R_l\Gamma_w^2 \gg r \), where \( R_l = mc^2\gamma/eB \) is the Larmor radius and \( \Gamma_w \) is the wind’s Lorentz factor (Arons 2003), and hence excessive synchrotron and adiabatic losses are negligible. In principle, following Ruderman & Sutherland (1975), the maximum differences in the voltage of the wind outflow at large distances from the central engine are

\[
\Delta \Phi_{\text{max}} = \frac{\Omega^2 B_f R^3}{2e^2} \approx 1.1 \times 10^{24} \left( \frac{\Omega}{10^4 \, \text{s}^{-1}} \right)^2 \times \left( \frac{B_f}{2 \times 10^{17} \, \text{G}} \right) \left( \frac{R}{10^6 \, \text{cm}} \right)^3 \, \text{V.} \quad (4)
\]

so that we have an extractable energy well beyond the currently observed data. The work produced by the ponderomotive force can be approximated by

\[
W_{\text{pond}} \approx q\eta\Delta \Phi_{\text{max}} \quad (5)
\]

where \( \eta \) is the efficiency factor. From analysis of pulsar wind nebulae it is seen that about 10% of the total voltage is transferred to the ions (Arons 2003).

In the situation prevailing in a GRB outflow, an UHE proton can loose some of its energy due to collisions with ambient protons. This can be estimated in an easy way (Chen, et al. 2002) by integrating the collision probability, \( \sigma_{pp} n_p(r)/\gamma \), from \( R_{MHD} \), the transition radius, to \( R_{MHD} + R_{mfp} \) where \( R_{mfp} \) is the mean-free path radius and the collision cross section \( \sigma_{pp} \sim 30 \, \text{mb} \). If we want the system be collision-free, the threshold condition

\[
\frac{\sigma_{pp} N_{GJ} R_{MHD}}{\Gamma_w} \leq 1 \quad (6)
\]

should be satisfied, where we have taken \( n_p(r) = N_{GJ} \) for the plasma density at the transition region. It is easy to see that for appropriate values this condition is always satisfied.

However, as an alternative to this high efficiency acceleration mechanism, we have also considered the possibility that the interaction of LAEMWs with the plasma can lead to instabilities. Lyutikov & Blackman (2001) argued that
the interaction of LAEMWs with the plasma can lead to violent instabilities where the MHD approximation breaks down because an overturn instability of LAEMWs, creating a broad spectrum of random electromagnetic fields. The mixing of various parts of the wave destroys its oscillatory structure thus yielding a relativistically strong electromagnetic turbulence. When the correlation length, $l_c$, of the turbulent electric field is much shorter than the radiative length, $l_r$, there will be an effective root-mean-squared deficit in the coherence of the ion velocity and the accelerating electric field. If the turbulent energy is concentrated near the smallest scale $\sim$ skin depth, a lower limit to the efficiency of acceleration, $\eta$, is obtained, which in case of protons can be expressed in terms of the classical proton radius $r_p$ and the wave frequency $\Omega$, $\eta = (2r_p\Omega/c)^{1/7} \approx 10^{-2.5}$. We can see that even if this lower factor is taken into account, we have an extractable voltage sufficient to account for the highest energy events.

Whether the turbulence is considered or not, the principal energy losses for a high energy proton once the acceleration process is almost completed are due to collisions with the intense thermal photon field left after adiabatic expansion of the relativistic outflow. Each collision undermines $\sim 10\%$ of the UHE proton energy (Aharonian & Cronin 1994) although the characteristic time for this energy loss mechanism (Waxman 1995) $t_{p\gamma} \sim 10/n\gamma\sigma_{p\gamma}c$ is larger than the time of expansion of the outflow even when this is entering the decelerating regime. The collision cross section in this case is $\sigma_{p\gamma} \approx 10^{-29}$ cm$^2$. The photon number density $n_\gamma$ was calculated by using the photon luminosity in the decelerating region, $L_\gamma = 4\pi r_d^2 c\gamma n_\gamma \epsilon_\gamma$, where $\epsilon_\gamma$ is the observed photon energy, i.e., $\epsilon_\gamma \sim 1$ MeV.

Due to their age, LMXB systems are not associated with star formation regions and an interesting possibility arises if we consider the fraction of LMXB systems related to globular clusters, $\sim 10\%$ (Liu et al. 2001), in which the CE is surrounded by a very tenous ambient medium. We would thus expect in this case that an equal fraction of the GRBs discussed here would not produce an afterglow, or at least a very faint event (Castro-Tirado et al. 2005). This is another feature which avoids energy losses of UHECRs due to the photon field of the afterglow. However, we cannot exclude that the impact of the burst on the strange star’s companion will produce a detectable signal (Ramirez-Ruiz 2004), but with negligible effect on the bulk of UHECRs since it will be of small angular size, restricted to the direction of the companion, and, in addition, the zone where the acceleration takes place is far beyond the location of the companion.

Details of the expected UHECR spectrum are impossible to predict with confidence. In case of pure electromagnetic spindown of the central engine (Arons 2003) an $E^{-1}$ dependence is expected while if acceleration can be regarded as a quasi-stochastic process a spectrum $\sim E^{-2}$ would be more appropriate (Chen, et al. 2002). Propagation effects are usually argued to cause a steepening of the spectrum, and to that respect it has been argued that the consistancy between the data obtained by the HiRes-I-II, Yakutsk,
and Auger experiments proves the existence of a GZK cutoff in opposition to the results obtained by the Akeno-AGASA experiment, (Berezinsky 2008; Kampert 2008), but the lack of robust data makes impossible to reach a final conclusion. The model as presented here would agree with an extension of events beyond the highest energy values observed, $> 10^{20}$ eV.

Finally we mention that the above outlined scenario can be applied equally well to the accretion induced collapse of a white dwarf into a neutron star, considering a cataclismic variable as an initial system instead of a low mass X-ray binary. However, a young strange star with a bare quark surface will emit, for a possibly long period of time, super-Eddington luminosities through pair production at its surface (Usov 1998, 2001; Page & Usov 2002) and later through electron-electron bremsstrahlung from its electrosphere (Jaikumar et al. 2004), with unique spectral characteristics which would allow to identify it unambiguously. On the contrary, a neutron star born through accretion induced collapse of a white dwarf is difficult, if not impossible, to be differentiated from its siblings born through the core collapse of a massive star. Identification of strange stars in binary systems (see, e.g., for some possibilities (Page & Cumming 2005)) would give support to our scenario.

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Oscar Esquivel: Astronomisches Rechen-Institut am Zentrum für Astronomie der Universität Heidelberg, Mönchhofstraße 12-14, 69120 Heidelberg, Germany (esquivel@ari.uni-heidelberg.de).

Dany Page: Departamento de Astrofísica Teórica, Instituto de Astronomía, Universidad Nacional Autónoma de México, México D.F. 04510, México (page@astroscu.unam.mx).