Anti-$PT$-Symmetry-Broken Nonreciprocity in a Linear Resonator

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Abstract

Entirely real eigenvalues emerge for non-Hermitian systems if their Hamiltonians are symmetric or antisymmetric under the parity-time ($PT$) operations. This fact has led to many unconventional discoveries such as loss-induced lasing, sensor response enhancement and topological energy transfer. One merit of anti-$PT$ systems is free of any gain, but nonlinear medium is still required in recent efforts on making and utilizing anti-$PT$ devices. Here, counterintuitively, we propose to achieve a linear anti-$PT$ device by combining dissipative optical coupling and the Sagnac frequency shifts of counterpropagating lights in a spinning resonator. Anti-$PT$ symmetry and its spontaneous breaking can be well controlled by tuning the spinning velocity of the resonator. Compared with its Hermitian counterpart, 6 times enhancement of optical isolation and 2 orders enhancement of sensing response can be achieved by breaking the anti-$PT$ symmetry. Configured as a new platform to study enhanced light-matter interactions, anti-$PT$ resonators can also be used in such a wide range of fields as anti-$PT$ lasers, anti-$PT$ gyroscopes and anti-$PT$ topological or chiral photonics.

Introduction.—Parity-time ($PT$) symmetry provides a natural and attractive way to relax the conventional condition of Hermiticity in quantum physics to ensure the real eigenvalues of systems [1, 2]. This discovery has been widely confirmed in experiments using various gain-loss-balanced systems [3–18]. As an emerging frontier, $PT$ physics has led to many unconventional phenomena, such as $PT$-enhanced lasing or perfect absorption [2, 7, 8], robust wireless power transfer [15], enhanced sensor response [17–20], thresholdless phonon amplifications [21, 22] and topological energy transfer [23].

Recently, a tremendous effort has also been witnessed in harnessing the power of anti-$PT$ symmetry [24]. One merit of anti-$PT$ systems is free of any gain medium, which is attractive for both fundamental studies on non-Hermitian quantum noises and various applications [25–27]. In experiments, anti-$PT$ symmetry has been firstly demonstrated in dissipatively coupled atomic beams [28], and then also in cold atoms [29], electrical circuits [30], diffusive thermal materials [31], and optical systems [32–35]. Unique effects associated with anti-$PT$ symmetry, such as refractionless light [28], energy-difference conserving dynamics [30] and chiral mode switching [32], have been observed as well.

In a very recent advance which is particularly related to our present work, a nonlinear anti-$PT$ optical resonator was demonstrated based on strong optical nonlinearity of the materials (i.e., the nonlinear Brillouin scattering process) [36]. Here, counterintuitively, we propose how to achieve anti-$PT$ symmetry by simply spinning a linear resonator, without the need of any gain or nonlinearity. We note that in recent experiments, by playing with the freedom of rotation, novel effects have been shown such as unidirectional light flow [37], sound circulation [38], and enhanced torque sensing [39]. Here, for the first time, we show that anti-$PT$ symmetry and its spontaneous breaking can emerge by spinning a linear resonator. As a result, light transmission becomes highly nonreciprocal in the anti-$PT$-broken regime featuring separated real eigenenergies. This is drastically different from optical nonreciprocity observed in $PT$-symmetric resonators [5, 6], which requires coalesced eigenenergies with both gain and nonlinearity. Compared with a Hermitian resonator with the same spinning velocity [37], the anti-$PT$ resonator can realize 6 times enhancement of optical isolation and 2 orders enhancement of sensing response. Our work provides a new versatile tool to study cavity photonics and, by combining external rotation and internal nonlinearity, more fascinating effects can be explored such as anti-$PT$ lasers or gyroscopes [40, 41], nonreciprocal quantum correlations [42, 43], and anti-$PT$ topological photonics [32].

Realization of anti-$PT$ symmetry.—Anti-$PT$ systems with two coupled optical modes can be described at the simplest level as [24]

$$H_{APT} = \begin{pmatrix} \omega & J \\ -J^* & -\omega^* \end{pmatrix},$$

where $\omega$ is the complex frequency and $J$ is the complex coupling strength between the two optical modes. It indicates two conditions are required for realizing anti-$PT$ symmetry: (i) two excited modes with opposite frequency detunings and same loss or gain; (ii) anti-Hermitian coupling between the two modes. In previous
experiments, anti-$PT$ symmetry has been realized in optics based on, e.g., nonlinear processes [36, 36] or complex spatial structures [28, 32].

As shown in Fig. 1(a), we consider a linear optical resonator driven by two lasers with frequency $\omega_d$ from the left and right, which can excite the clockwise (CW) and the counter-clockwise (CCW) travelling modes. When the resonator is spinning at an angular velocity $\Omega$ [37], the rotation-induced SagnacFizeau shift $\omega_c \rightarrow \omega_c \pm \Delta_{\text{sag}}$ is given by [44]

$$\Delta_{\text{sag}} = \frac{n R \omega_c}{c} \left(1 - \frac{1}{n^2} - \frac{\lambda}{n} \frac{dn}{d\lambda} \right),$$

where $\omega_c$ is the resonant frequency of a nonspinning resonator, $c$ ($\lambda$) is the speed (wavelength) of light, $n$ and $R$ are the refractive index and radius of the resonator, respectively. The dispersion term $dn/d\lambda$, characterizing the relativistic origin of the Sagnac effect, is relatively small in typical materials ($\sim \mathcal{E}()$) [37]. We fix the CCW rotation of the resonator; hence $+\Delta_{\text{sag}}$ ($-\Delta_{\text{sag}}$) corresponds to the CW (CCW) travelling mode, as shown in Fig. 1(b). Note that taper scattering induced dissipative backscattering will lead to the dissipative coupling $i\kappa$ between the countercirculating modes [40], which is anti-Hermitian, i.e., $-(i\kappa)^* = i\kappa$. Obviously, this Sagnac resonator naturally fulfills the conditions of the realization of anti-$PT$ symmetry.

In a frame rotating at driving frequency $\omega_d$, the effective Hamiltonian of this system can be written at the simplest level as

$$H_0 = \begin{pmatrix} \Delta_+ - i\gamma_c & i\kappa \\ i\kappa & -\Delta_- + i\gamma_c \end{pmatrix},$$

where $\Delta_\pm = \Delta_c \pm \Delta_{\text{sag}}$ are the optical detunings in spinning case with $\Delta_c = \omega_c - \omega_d$, and $\gamma_c = (\gamma_0 + \gamma_{\text{ex}}) / 2$ is the total optical loss including the intrinsic loss of the resonator $\gamma_0 \equiv \omega_c / Q$ with the quality factor $Q$ and the loss due to the coupling of the resonator with the fiber taper $\gamma_{\text{ex}}$. For the resonance case $\Delta_c = 0$, anti-$PT$ symmetry can be realized without any gain or nonlinear process, which is different from the scheme based on nonlinear Brillouin scattering to provide optical gain in a single optical resonator [36]. Unlike $PT$ symmetry, anti-$PT$ symmetry is independent on spatially separated gain-loss balanced structure, as shown in Fig. 1(c), which opens up a new route to explore non-Hermitian quantum effects.

The eigenfrequencies of this linear anti-$PT$-symmetric system are

$$\omega_\pm = -i\gamma_c \pm \sqrt{\Delta_{\text{sag}}^2 - \kappa^2},$$

which indicates an intriguing phase transition as $\Omega$ varies. As shown in Fig. 1(d), for small $\Omega$ ($\Delta_{\text{sag}} < \kappa$), the

![Fig. 1](image-url)

FIG. 1. Anti-$PT$ symmetry in a linear spinning resonator. (a) The resonator is driven by two lasers with the same frequency $\omega_d$ from the left and right. Anti-$PT$ symmetry is realized with the opposite frequency shifts induced by the mechanical rotation with angular speed $\Omega$ and the dissipative coupling $\kappa$ induced by the taper scattering. The output spectra in the anti-$PT$ symmetric (anti-PTS) and symmetry broken (anti-PTB) phases can be observed by tuning $\Omega$ for $\Omega < \Omega_{\text{EP}}$ and $\Omega > \Omega_{\text{EP}}$, respectively, where $\Omega_{\text{EP}}$ is the angular speed at exceptional point. (b) The schematics of Sagnac effect and dissipative coupling show the origin for realizing anti-$PT$ symmetry. (c) Differences between $PT$ symmetry and anti-$PT$ symmetry. (d) The real and imaginary parts of eigenfrequencies versus $\kappa$ and $\Omega$ reveal the spectral properties of anti-$PT$ symmetry. The white solid curves correspond to the case of $\kappa = 8$ kHz. For the other parameter values, see the main text.
two eigenmodes preserve anti-$PT$ symmetry with the same resonance frequency but different linewidths. The symmetry breaking occurs at EP ($\Delta_{\text{sag}} = \kappa$) where the two eigenstates coalesce. For large $\Omega$ ($\Delta_{\text{sag}} > \kappa$), the system enters symmetry-broken phase with the two eigenmodes bifurcate. Specifically, by taking the dissipative coupling strength as $\kappa = 8$ kHz, we can obtain the critical value of rotation speed at EP: $\Omega_{EP} = 333$ Hz (see Fig. 2).

Here, we take the experimentally accessible parameters [45–47]: $\lambda = 1550$ nm, $Q \approx 1 \times 10^{11}$, $\gamma_{\text{ex}} = \gamma_0/2$, $n = 1.44$, and $R = 50$ $\mu$m. A spinning ring resonator mounted on a turbine with $\Omega = 6.6$ kHz has been realized in a recent experiment [37], where a stable coupling between the resonator and the fiber taper is maintained by self-adjustment process. In addition, the dissipative coupling originating from taper scattering has been experimentally demonstrated in a spinning ring cavity coupled with a fiber taper [40]. For more details of experimental feasibility, including the situation with nanoparticles, see the Supplementary Information [48].

Symmetry-broken nonreciprocity.—When a probe light is incident from the left (right) side and using the input-output relation [48], the transmission rate $T_{cw}^A (T_{ccw}^A)$ can be obtained analytically as

$$T_{cw,ccw}^A = \left| 1 + \frac{i\gamma_{\text{ex}}(\delta_p \mp \Delta_{\text{sag}})}{(\delta_p + \Delta_{\text{sag}})(\delta_p - \Delta_{\text{sag}}) + \kappa^2} \right|^2,$$

where $\delta_p = \Delta_p - i\gamma_c$ and $\Delta_p = \omega_c - \omega_p$. The term of $\mp\Delta_{\text{sag}}$ describing the difference between $T_{cw}^A$ and $T_{ccw}^A$ is the origin of nonreciprocal implementations of light transmission. In anti-$PT$-symmetric regime, $\Delta_{\text{sag}} < \kappa$ limits the difference of the two transmission rates. However, symmetry-broken phase with $\Delta_{\text{sag}} > \kappa$ will enlarge the difference to enable better one-way light transmission. To confirm this picture, we study the isolation ratio of this anti-$PT$-symmetric system, i.e.,

$$I^A = 10 \log_{10} \left( T_{cw}^A / T_{ccw}^A \right),$$

and we find that the isolations in symmetry-broken phase are much larger than those in anti-$PT$ symmetry phase, as shown in Figs. 2(b) and 2(c).

For comparison, the isolation of Hermitian spinning resonator (HSR) $I^H$ [37] and the maxima of the isolation $\eta \equiv \max[I]$ for $\Delta_p \in [-15, 15]$ kHz with respect to $\Omega$ are revealed in Figs. 2(d) and 2(e), respectively. In HSR, the isolation becomes larger by increasing $\Omega$ due to the splitting of the counterpropagating modes induced by the Sagnac frequency shift; but it will be limited to 9.5 dB because of the fixed linewidths of the two modes [37]. In contrast, the dissipative coupling combined with the Sagnac frequency shift can alter the linewidths and extrema of the transmission spectrum, thus the isolation rate can be improved to 61.8 dB in symmetry-broken regime. This anti-$PT$-symmetric nonreciprocity, with up to 6 times enhancement isolation, is related to the interplay between the linear synthetic angular momentum and dissipative
backscattering, which is fundamentally different from the nonreciprocity in PT-symmetric system originating from the symmetry-breaking-enhanced nonlinearity [5, 6]. This linear non-Hermitian one-way device has a wide range of applications, including on-chip optical circulator [5, 6], invisible sensing [51, 52], and quantum optical computation [53].

**Anti-PT sensor.**—Taking the perturbation of a nanoparticle into account, ultrasensitive sensing can be realized by measuring the variation of the transmission spectrum. For a single nanoparticle falling onto the surface of the resonator and staying on the resonator [54], the Hamiltonian of this anti-PT-symmetric system is modified as [49, 54]

\[
H_1 = \left( \Delta'_g + i\gamma_c + J \right) \frac{i\kappa + J}{i\kappa + J} \Delta'_\omega - i\gamma_c + J, \tag{6}
\]

where \( \Delta'_\omega = \Delta_\omega + \Delta_{\text{sag}} \), and \( J = g_1 - i\gamma_1 \) is the complex perturbation introduced by the nanoparticle. As shown in Fig. 3(a), the eigenfrequency structure reveals how this system reacts on a sufficiently small perturbation. The sensitivity can be defined as the difference between the two eigenfrequencies, i.e.,

\[
\Delta_{\omega_1} = 2\sqrt{\Delta_{\text{sag}}}^2 - (\kappa - iJ)^2. \tag{7}
\]

Figure 3(b) shows the logarithmic behavior of Re\( [\Delta_{\omega_1}] \), and highlights the sensitivity enhancement of anti-PT sensor, compared to that of HSR sensor [37, 54]. With the same amount of perturbation, HSR sensor performs closely to anti-PT sensor operating in symmetry-broken phase, where the mode splitting does not exhibit a strong dependence on the perturbation. In symmetry-unbroken phase, we find the frequency splitting is smaller than that in symmetry-broken phase or at EP, which indicates a reduced sensitivity to the perturbation. However, at EP, \( \Delta_{\text{sag}} = \kappa \), the slope of the response is \( 1/2 \), thus confirming that perturbations experience an enhancement of the form \( J^{1/2} \). We can explain this behavior using perturbation theory. When \( J \) is much smaller than \( \kappa \), the complex frequency splitting is expected to approximately follow

\[
\Delta_{\omega_1} = 2\sqrt{2i\kappa J^{1/2}}. \tag{8}
\]

For larger \( J \), the slope is slightly larger than \( 1/2 \) because in this case Eq. (7) cannot be simplified to Eq. (8) (see more details in the Supplementary Information [48]). In experiments [19, 49], the sensitivity defined by frequency splitting can be assessed by monitoring the separation of the eigenfrequencies spectral lines in transmission spectrum.

Alternatively, measuring transmittance variation can also characterize the response of the perturbation [54]. Choosing one of the counterpropagating modes, e.g., the CW mode, the transmittance variation is defined as \( \mathcal{V}_s \equiv T_s - T_0 \), where \( s \) denotes the sensor of anti-PT symmetry (A) or HSR (H), \( T_s \) and \( T_0 \) are the transmissions with and without the perturbation, respectively. Compared with HSR sensor, we introduce the enhancement factor

\[
\chi = \mathcal{V}_A / \mathcal{V}_H, \tag{9}
\]

to show the sensitivity enhancement of anti-PT sensor. We find the enhancement factor is larger (smaller) than 1 in symmetry-broken (symmetry-unbroken) regime, as shown in Fig. 3(c). In particular, we find that for \( \Omega \approx 386 \text{ Hz} \), i.e., when the optical isolation reaches its maximum, 2 orders of magnitude enhanced sensitivity is achieved in anti-PT sensor. This symmetry-breaking enhanced sensing can also be understood from the interplay between Sagnac frequency shift and dissipative backscattering. Our results offer a new path to engineer ultrasensitive nanoparticle sensors, which are
crucial elements in medical diagnosis and environmental monitoring [55, 56].

Conclusions.—We have revealed anti-PT symmetry in a single spinning resonator without any sophisticated gain-loss balance structure and nonlinearity. The opposite frequency shifts induced by the Sagnac effect and the dissipative coupling induced by the taper scattering enable the Hamiltonian to be antisymmetric under the combined PT operations. Our results advance non-Hermitian physics by bridging to the field of synthetic dimension [57], where rich phenomena and applications in quantum optics aided by anti-PT symmetry could be anticipated. Moreover, symmetry-breaking-induced one-way light transmission and ultrasensitive particle sensing can be also realized in this linear anti-PT-symmetric system, which play a key role in quantum information processing [58] and sensitive sensing [17, 18].

The spinning anti-PT resonator is highly feasible in experiments and can be used to study a wide range of phenomena from chiral optical effects [59] to non-Hermitian topological or quantum effects [23, 26].

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Note added—In the preparation of this manuscript we became aware of an experiment achieving anti-PT symmetry in a nonlinear optical microcavity [36]. Here, in contrast, we propose to realize anti-PT symmetry in a spinning resonator, without the need of any nonlinearity.

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Supplementary Information for “Anti-\(\mathcal{PT}^{\ast}\)-Symmetry-Broken Nonreciprocity in a Linear Resonator”

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Here, we present technical details on anti-\(\mathcal{PT}^{\ast}\) symmetry-broken nonreciprocity in a linear resonator. In Sec. S1, we discuss the experimental feasibility of our scheme. In Sec. S2, we provide the detailed derivation of the anti-parity-time (\(\mathcal{PT}\))-symmetric Hamiltonian. In Sec. S3, we show the extended results about nonreciprocal light transmission and ultrasensitive nanoparticle sensing.

S1. EXPERIMENTAL FEASIBILITY

A. Stable coupling between the tapered fiber and the spinning resonator

To realize anti-\(\mathcal{PT}^{\ast}\) symmetry, one indispensable condition is two excited modes with opposite frequency detunings. Inspired by a recent experiment [S1], we find due to the Sagnac effect, counterpropagating modes with opposite frequency shifts naturally exist in a spinning resonator, which is pumped bidirectionally. Mounting a silica microtoroid resonator on a turbine and positioning it near a tapered region of a single-mode telecommunication fiber, the light can be coupled into or out of the cavity evanescently.

1. Self-adjustment process and critical coupling

According to the experiment [S1], the aerodynamic process plays a key role in stable resonator-fiber coupling. When the resonator rotates at an angular velocity \(\Omega\), a boundary layer of air will be dragged into the region between the taper and the resonator. Hence, the taper will fly above the resonator with a separation of several nanometers. If some perturbations cause the taper to rise higher than the stable-equilibrium height, it floats back to its original position, which is called “self-adjustment”.

Using similar methods in Ref. [S1], we consider the local deformation of the taper [see Fig. S1(a)], break it into a set of infinitesimal cylinders and focus on the outermost one. The air pressure on this infinitesimal cylinder, leading to a tiny displacement \(d\), is written as \(\Delta T_{\text{air}} = (\rho \Delta \theta) T_{\text{air}} / L\), where \(\rho\) (\(\theta\)) represents the radius (angle) of the winding shape for the deformed region of the fiber. Total pressure on the taper from a boundary layer of air (the “air bearing” surface), \(T_{\text{air}}\), can be estimated analytically from [S1]

\[
T_{\text{air}} = 6.19 \mu R^{5/2} \Omega \int_0^r \left( h - \sqrt{r^2 - x^2} + r \right)^{-3/2} \, dx, \tag{S1}
\]

where \(\mu\) is the viscosity of air, \(r\) (\(R\)) is the radius of the taper (resonator), \(h = h_0 + d\) is the taper-resonator separation with \(h_0\) being the gap between the taper and the surface of the non-spinning resonator. Note that in our device, the curvature of the microtoroid is so small compared with the width of the film-lubricated region of interest, thus its surface can be considered as a flat plane, which is similar to the case of spinning sphere in Ref. [S1].

The tension on this infinitesimal cylinder induced by the deformation can be calculated by \(\Delta T_{\text{ela}} = 2F \sin (\Delta \theta / 2) \approx F \Delta \theta\), where the elastic force on the taper \(F\) obeys Hooke’s law:

\[
\sigma = E \epsilon. \tag{S2}
\]

Here, \(E\) is the Young’s modulus of silica, \(\sigma = F / (\pi r^2)\) represents the uniaxial stress, \(\epsilon = \delta L / L\) is the strain, \(L\) stands for the original length of the deformation region of the taper, and \(\delta L = L' - L\) denotes the change in length, which
can be derived with the aid of $L' = \rho \theta$, $(L/2)^2 + (\rho - d)^2 = \rho^2$, and $\sin(\theta/2) = L/(2\rho)$. Therefore, in the case of stable equilibrium ($\Delta T_{\text{air}} = \Delta T_{\text{ela}}$), we can describe $T_{\text{air}}$ in another form:

$$T_{\text{air}} = 2\pi r^2 E [\arcsin (\phi) - \phi] \approx \pi r^2 \phi^3 E/3,$$

(S3)

with $\phi = 4Ld/(L^2 + 4d^2)$ being much smaller than 1 under the approximation of $d/L \ll 1$. The arcus function can be perturbatively expanded using a Taylor series: $\arcsin (\phi) = \phi + \phi^3/6 + \cdots$ for $|\phi| \ll 1$; then, the displacement $d$ caused by air pressure can be analytically estimated as

$$d = L \left( \tau - \sqrt{\tau^2 - 1} \right)/2,$$

(S4)

where $\tau = [\pi r^2 E/(3T_{\text{air}})]^{1/3}$. Accordingly, the strain of the taper can be rewritten as

$$\epsilon = \arcsin (\phi)/\phi - 1 \approx \phi^2/6.$$

(S5)

From this equation, we find the strain, i.e., the elastic force, is positively associated with the distance between the fiber and the surface of the rotating resonator:

$$\frac{\partial F}{\partial h} = \pi r^2 E \left( \frac{\partial \epsilon}{\partial d} \right) = \frac{16\pi r^2 E L^2 d (L^2 - 4d^2)}{3(L^2 + 4d^2)^3} > 0.$$

(S6)

Therefore, if any perturbation causes the gap to be larger than the stable-equilibrium distance, the elastic force will be stronger.

The “self-adjustment” behavior can be understood from the responses of air pressure and elastic force to the variation of the gap induced by a perturbation. As shown in Fig. S1(b), the air pressure is reduced dramatically when the fiber is far away from the resonator. Meanwhile, the elastic force becomes larger due to the stronger deformation, which indicates that the fiber can be dragged back to the original position; thus we can maintain the separation between the spinning devices and the couplers, which is essential for critical coupling [S1].

Moreover, to realize the fast spinning resonator experimentally, it requires the microtoroid cavity to be perfectly circular to maintain a stable taper-resonator coupling. However, the experimental toolbox and the fabrication methods of microresonators, in principle silica based whispering gallery mode microresonators in the form of toroid or sphere are so mature that such resonators can be fabricated with almost atomically small surface roughness and close-to perfect shape via surface tension or polishing techniques. Thus, the impact of the shape of the resonator is negligible and the coupling with the waveguide can be maintained stably.

Herein, we choose experimentally accessible parameters [S1, S2]: $E = 75$ GPa, $r = 544$ nm and $L = 5$ $\mu$m. As presented in Fig. S1(c), the deformation is extremely small ($d/L < 0.37\%$ and $\epsilon < 3.625 \times 10^{-5}$), thereby our approximation is physically reasonable.
2. Intermolecular forces

The intermolecular forces between the taper and the spinning resonator, including Casimir and van der Waals forces, can be described as [S1]:

$$T_{\text{int}} = rR \left( -\frac{A}{6\pi h^3} + \frac{B}{45\pi h^3} - \frac{\pi^2 c}{240h^4} \right),$$  
(S7)

where $B$ is a constant [S3], and $A$ is the Hamaker constant, which can be calculated from the following equation [S4]:

$$A = \frac{3\varepsilon_{-}^{(1)}\varepsilon_{+}^{(2)}k_{B}T}{4\varepsilon_{+}^{(1)}\varepsilon_{+}^{(2)}} + \frac{\nu\left[n_{-}^{(1)}n_{-}^{(2)}\right]^2}{n_{+}^{(1)}n_{+}^{(2)}\left[n^{(1)}_{+} + n^{(2)}_{+}\right]},$$  
(S8)

with $\nu = 3\sqrt{2h
u_{e}/16}$, $\varepsilon_{\pm}^{(j)} = \varepsilon_{j} \pm \varepsilon_{0}$, $n_{\pm}^{(j)} = \sqrt{n_{j}^2 \pm n_{0}^2}$, and $j = 1, 2$. Moreover, $\varepsilon_{0}$ ($n_{0}$), $\varepsilon_{1}$ ($n_{1}$) and $\varepsilon_{2}$ ($n_{2}$) represent the dielectric constant (the refractive index) of air, taper and spinning resonator, respectively; $\nu_{e}$ is a constant, $k_{B}$ is the Boltzmann constant, and $T$ is the temperature of the system [S4].

In previous studies, it has been shown that the intermolecular forces begin to attract the flyer towards the rotor when the gap between them is reduced to less than 10 nm, and to strongly repel them when the gap is narrowed further, normally to below 300 fm [S5]. In our system, the fiber-resonator separation is set to be 20 nm, thus we can safely omit the effects of Casimir and van der Waals forces. The experiment [S1] also shows some other factors, such as lubricant compressibility, tapered-fiber stiffness, and wrap angle of the fiber, may affect resonatorwaveguide coupling. However, the effects induced by these factors are confirmed to be negligible in the experiment.

3. Air friction

The air drag torque on the rotational resonator is given by [S6]:

$$M_{\text{air}} = 1.336\Omega P_{0}R^{4} \sqrt{\frac{2\pi m_{0}}{k_{B}T_{0}}} \propto \Omega,$$  
(S9)

where $P_{0}$ is the air pressure, $m_{0} = 4.6 \times 10^{-26}$ kg is the mass of the air molecule, and $T_{0}$ is the temperature of the surrounding air molecules. We note that Eq. (S9) is originally applied to a single spinning sphere. However, we can use it here since the surface of the microtoroid resonator can also be regarded as a flat plane within the film-lubricated region, as presented in Sec. S1 A 1. Moreover, the rotation speed used in our numerical simulations is below kHz level, which is comparatively small. For this reason, we can neglect rotation-induced heating on the air molecules, and assume $T_{0} = T = 300$ K and $P_{0} = 1.013$ bar. In our approximations, the maximum value of air drag torque is $M_{\text{air}} = 4.947 \times 10^{-12}$ N·m, which indicates air friction can be safely ignored in our system.

4. Stability analysis

The condition of $\tau \geq 1$ in Eq. (S4) yields the first limit of angular velocity:

$$\Omega_{0} = \frac{\varrho \pi r^{2}E}{18.57\mu R^{5/2}},$$  
(S10)

where

$$\varrho = \left[\int_{0}^{r} \left(h - \sqrt{r^{2} - x^{2}} + r\right)^{-3/2} \, dx \right]^{-1}. $$  
(S11)

Also, the approximation condition $d \ll L$ gives another limit ($d/L = 1\%$):

$$\Omega_{1} = \frac{\varrho \pi r^{2}E}{\nu \mu R^{5/2}} < \Omega_{0},$$  
(S12)
where \( \nu = 2.905 \times 10^5 \). Furthermore, the tiny displacement \( d \) should be smaller than the taper-resonator separation \( h \), thus provides the third limit:

\[
\Omega_2 = \frac{\sigma \rho \pi r^2 E}{18.57 \mu R^{5/2}},
\]  

(S13)

with \( \sigma = [4Lh/(L^2 + 4h^2)]^3 \). Finally, we consider the elastic limit of the material for the taper (\( \sigma = \Upsilon \)):

\[
\Omega_3 = \frac{\rho \pi r^2 \Upsilon}{3.095 \mu R^{5/2}} \sqrt{6 \Upsilon E},
\]  

(S14)

where \( \Upsilon \) is typically 9 GPa for silica [S7]. From the analyses made above, the mechanical limit of the spinning frequency can be given by:

\[
\Omega_{\text{max}} = \min \{ \Omega_1, \Omega_2, \Omega_3 \}.
\]  

(S15)

When operating at taper-resonator separations near 20 nm [S1], we find \( \Omega_1 = 13.9 \text{ kHz}, \Omega_2 = 893.6 \text{ Hz}, \) and \( \Omega_3 = 133.3 \text{ MHz} \), thereby the maximum value of angular velocity can be up to 893.6 Hz. This indicates that the local deformation of the taper dominates the stability of our system and it is reasonable to set \( \Omega = 700 \text{ Hz} \) in the main text.

### B. Dissipative coupling between the counterpropagating modes

The other condition for realizing anti-\( \mathcal{PT} \) symmetry is the dissipative coupling between the clockwise (CW) and counterclockwise (CCW) modes. According to a recent experiment on optical gyroscope [S8], the dissipative coupling can originate from the dissipative scattering induced by the fiber or any other dissipative scattering elements in the resonator. As given in Ref. [S8], in a standing-wave basis, the loss induced by the coupling of the fiber and the resonator can be expressed as

\[
H = \rho \pi r^2 (a_{\text{cw}}^{\dagger} a_{\text{cw}} - a_{\text{ccw}}^{\dagger} a_{\text{ccw}}) - \gamma (a_{\text{cw}}^{\dagger} a_{\text{cw}} + a_{\text{ccw}}^{\dagger} a_{\text{ccw}}),
\]  

(S16)

where \( \gamma_{1,2} \) are the losses induced by the fiber for the two optical modes. Transforming this Hamiltonian in the traveling-mode basis with \( a_{1,2} = (a_{\text{cw}} \pm a_{\text{ccw}})/\sqrt{2} \), we can get

\[
H = -i (\gamma_{1a} + \gamma_{2a}) a_{\text{cw}}^{\dagger} a_{\text{cw}}/2 - i (\gamma_{1a} + \gamma_{2a}) a_{\text{ccw}}^{\dagger} a_{\text{ccw}}/2
+ i (\gamma_{2a} - \gamma_{1a}) a_{\text{cw}}^{\dagger} a_{\text{ccw}}/2 + i (\gamma_{2a} - \gamma_{1a}) a_{\text{ccw}}^{\dagger} a_{\text{cw}}/2.
\]  

(S17)

Rewrite it in a matrix form

\[
H = \begin{pmatrix} a_{\text{cw}}^{\dagger} a_{\text{cw}} & a_{\text{ccw}}^{\dagger} a_{\text{ccw}} \\ a_{\text{ccw}}^{\dagger} a_{\text{ccw}} & a_{\text{cw}}^{\dagger} a_{\text{cw}} \end{pmatrix} \begin{pmatrix} -i\gamma & i\kappa \\ i\kappa & -i\gamma \end{pmatrix} \begin{pmatrix} a_{\text{cw}} \\ a_{\text{ccw}} \end{pmatrix},
\]  

(S18)

where \( \gamma = (\gamma_{1a} + \gamma_{2a})/2, \kappa = (\gamma_{2a} - \gamma_{1a})/2 \), and \( \kappa \) is the effective dissipative coupling strength induced by the fiber. This dissipative coupling plays a crucial role in the realization of anti-\( \mathcal{PT} \) symmetry in our scheme.

### C. Nanoparticle sensing process

In previous experiments [S9, S10], nanoparticles falling onto the stationary resonator randomly were detected and counted. These nanoparticles can be deposited on the surface of the resonator via a nozzle [S9]. The complex optical mode coupling induced by a single particle, related to the overlap between the particle and the mode volume of the resonator, can be expressed as [S9]

\[
\gamma_s = \frac{2\pi^2 \alpha^2 f^2(r) \omega_c}{3\lambda^3 V_c}, \quad g_s = -\frac{\alpha f^2(r) \omega_c}{2V_c},
\]  

(S19)

where \( r \) is the particle position, \( V_c \) is the mode volume of the resonator, \( f(r) \) is the normalized mode distribution function, and \( \lambda \) is the wavelength of the light. Here, \( \alpha \) is the particle polarizability, which depends on the size
and refractive index of the particle. This complex modal coupling could induce mode splitting in the transmission spectrum which can be utilized to estimate the size of the nanoparticle in the detection process.

In our scheme based on a spinning resonator, we consider that the scatterers fall onto and stay on the surface of the resonator, rotating with it, similar to the particles on a stationary ring cavity [S9, S10]. A nanoparticle can be detected in such a system only when the splitting is resolved in the transmission spectrum, which requires $\frac{\gamma_s}{g_s} < 1$ [S9]. While in our scheme, the locations of the scatterers may be changed with the fast spinning of the resonator, thus affect the mode volume and the overlap of the particles, leading to the change in the values of $\gamma_s$ and $g_s$ according to Eq. (S19). But the ratio $\gamma_s/g_s = -4\pi^2\alpha/(3\lambda^3)$ will not be changed. Thus, we take the fixed value of $\gamma_s/g_s \sim 0.05$ in the simulation of the perturbated system as in the experiment [S9, S11].

In this work, we compare in detail the different mode-splitting features for these two cases: the case with the scatter falling onto the anti-\PT resonator and the other case with the scatterer falling onto the Hermitian spinning resonator (HSR) [S1, S12]. We define the signal enhancement factor to evaluate the performance of these two sensors and show that the sensitivity is always enhanced by the anti-\PT sensor (see Sec. S3). For both of the two sensors, if the particle is not stationary on the mode volume (particle diffuses on the surface) then we will observe that mode splitting will change. A particle located in a high intensity field in the mode volume will lead to a larger mode splitting than the same particle located in a low intensity field. If the particle is detached from the resonator due to spinning or any other reasons, then mode splitting will return back to the case when there is no particle. However, Sagnac effect and dissipative coupling themselves are not affected by the particle position. Thus the symmetry-broken enhancement in sensitivity originating from the interplay between Sagnac effect and dissipative coupling will not be affected by the position of the particle. We can conclude that the location of the particle will not change the fact that anti-\PT sensor performs better than HSR sensor. In addition, HSR sensor performs better than a stationary-resonator sensor, i.e., diabolic point sensor, which has been revealed in Ref. [S12].

S2. DERIVATION OF THE EFFECTIVE ANTI-\PT-SYMMETRIC HAMILTONIAN

We will compare the anti-\PT-symmetric Hamiltonian with the \PT-symmetric Hamiltonian in this section, and present the detailed derivation of the anti-\PT-symmetric Hamiltonian. As proposed in Ref. [S13], non-Hermitian Hamiltonian can have entirely real eigenvalues if it is symmetric under combined \PT operations. As its counterpart, the anti-\PT-symmetric Hamiltonian, which was first proposed in Ref. [S14], follows \{PT, H\} = 0 mathematically.

We express the Hamiltonian possessing \PT symmetry and anti-\PT symmetry in the matrix form as

$$H = \begin{pmatrix} a_1^\dagger & a_2^\dagger \end{pmatrix} M \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}. \quad (S20)$$

A \PT-symmetric Hamiltonian is usually in a form of

$$M_{\PT} = \begin{pmatrix} \Delta + i\gamma & J \\ J & \Delta - i\gamma \end{pmatrix}, \quad (S21)$$

where $\Delta$ is the detuning of the optical modes, $\gamma$ represents the optical loss and the equal gain, and $J$ is the coupling strength between the two optical modes. Under the combined \PT operations, it is invariant, i.e., $[\PT, H] = 0$. In a general form, it can be expressed as

$$M_{\PT} = \begin{pmatrix} \omega & \kappa \\ \kappa^* & \omega^* \end{pmatrix}, \quad (S22)$$

where $\omega$ is the complex frequency and $\kappa$ is the complex coupling strength between the two optical modes.

An anti-\PT-symmetric Hamiltonian in the form of

$$M_{\APT} = \begin{pmatrix} \Delta - i\gamma & iJ \\ iJ & -\Delta - i\gamma \end{pmatrix}, \quad (S23)$$

will be mapped to its opposite under the combined \PT operations, i.e., $[\PT, H] = 0$. In a general form, it can be expressed as

$$M_{\APT} = \begin{pmatrix} \omega & \kappa \\ -\kappa^* & -\omega^* \end{pmatrix}. \quad (S24)$$
We can see by multiplying \( \mathcal{P}\mathcal{T} \)-symmetric Hamiltonian can become anti-\( \mathcal{P}\mathcal{T} \) symmetric mathematically.

In Tab. I, taking a two-mode optical system as an example, we summarize the conditions that need to be satisfied in \( \mathcal{P}\mathcal{T} \)-symmetric and anti-\( \mathcal{P}\mathcal{T} \)-symmetric systems. For \( \mathcal{P}\mathcal{T} \)-symmetric systems, one of the optical modes should be active with gain while the other mode has to be passive with equal loss. For the anti-\( \mathcal{P}\mathcal{T} \)-symmetric Hamiltonian, the opposite detunings and the antisymmetric coupling should be met at the same time. Different from \( \mathcal{P}\mathcal{T} \) symmetry, the realization of anti-\( \mathcal{P}\mathcal{T} \) symmetry is independent on sophisticated gain-loss balance structure [S15–S23]. The anti-\( \mathcal{P}\mathcal{T} \) symmetric system can be passive (with loss) [S15] or active (with gain) [S16].

In the following part, we will show how to construct the Hamiltonian in the form as shown in Eq. (S23). Inspired by the recent experiment on nonreciprocal light transmission with a spinning resonator [S1], we propose a scheme to realize anti-\( \mathcal{P}\mathcal{T} \) symmetry via a single linear optical resonator, which can support two counterpropagating modes, i.e., the CW and CCW modes. The Hamiltonian for such a system can be expressed as (\( \hbar = 1 \)):

\[
H_0 = (\omega_c - i\gamma_c) \left( a_{cw}^{\dagger} a_c + a_{ccw}^{\dagger} a_{ccw} \right) + i\kappa \left( a_{ccw}^{\dagger} a_{ccw} + a_{cw}^{\dagger} a_{cw} \right),
\]

where \( \omega_c = c/\lambda \) is the resonant frequency of the optical mode with \( \lambda \) (\( c \)) being the wavelength (speed) of light, \( \gamma_c = (\gamma_0 + \gamma_{ex})/2 \) is the total cavity loss, \( \gamma_0 \equiv \omega_c/Q \) is the intrinsic loss of the cavity mode, \( \gamma_{ex} \) is the loss induced by the coupling between the fiber and the resonator, \( \kappa \) is the dissipative coupling strength between the CW and the CCW modes, and \( a_{cw}(a_{ccw}) \) and \( a_{cw}^{\dagger}(a_{ccw}^{\dagger}) \) represent the annihilation and creation operators of the CW (CCW) cavity mode. This resonator is bidirectionally driven by two pumps with the same frequency \( \omega_d \). The driving terms can be expressed as

\[
H_{dr} = i\varepsilon_d \left( a_{cw}^{\dagger} e^{-i\omega dt} - a_{cw} e^{i\omega dt} \right) + i\varepsilon_d \left( a_{ccw}^{\dagger} e^{-i\omega dt} - a_{ccw} e^{i\omega dt} \right),
\]

where \( \varepsilon_d = \sqrt{\gamma_{ex} P_d/\hbar \omega_d} \) is the driving amplitude and \( P_d \) denotes the input power. Rewriting the Hamiltonian in the rotating frame with respect to

\[
U = \exp \left[ -i\omega_d \left( a_{cw}^{\dagger} a_c + a_{ccw}^{\dagger} a_{ccw} \right) t \right],
\]

we can derive the transformed Hamiltonian as

\[
H' = i\frac{dU^\dagger}{dt} U + U^\dagger H U,
\]

where

\[
H = H_0 + H_{dr}.
\]

Then we get the transformed Hamiltonian as

\[
H' = (\Delta_c - i\gamma_c) \left( a_{cw}^{\dagger} a_c + a_{ccw}^{\dagger} a_{ccw} \right) + i\kappa \left( a_{ccw}^{\dagger} a_{ccw} + a_{cw}^{\dagger} a_{cw} \right) + i\varepsilon_d \left( a_{cw}^{\dagger} - a_{cw} \right) + i\varepsilon_d \left( a_{ccw}^{\dagger} - a_{ccw} \right),
\]

with \( \Delta_c = \omega_c - \omega_d \).

If this resonator rotates at a speed of \( \Omega \) (in unit of Hz) [S1], the Sagnac effect will result in two opposite frequency shifts in the CW and CCW modes [S29], i.e.,

\[
\Delta_{sag} = \pm \frac{n R \Omega \omega_c}{c} \left( 1 - \frac{1}{n^2} \frac{\lambda}{dn} \right).
\]

### TABLE I. The summary of the coupled two-mode \( \mathcal{P}\mathcal{T} \)-symmetric and anti-\( \mathcal{P}\mathcal{T} \)-symmetric system. \( \Delta_1 \) and \( \Delta_2 \) are the detunings of the two optical modes. \( \gamma_1 \) and \( \gamma_2 \) are the optical loss or gain. \( \kappa_1 \) and \( \kappa_2 \) are the coupling strengths between the optical modes.

| Symmetry | Detuning | Coupling Strength | Gain/Loss |
|----------|----------|------------------|-----------|
| \( \mathcal{P}\mathcal{T} \): | \( \Delta_1 = \Delta_2 \) | \( \kappa_1^* = \kappa_2 \) | \( \gamma_1 = -\gamma_2 \) |
| Anti-\( \mathcal{P}\mathcal{T} \): | \( \Delta_1 = -\Delta_2 \) | \( \kappa_1^* = -\kappa_2 \) | \( \gamma_1 = \gamma_2 \) |
Hereafter, we replace \( n_2 \) with \( n \) to denote the refractive index of the resonator just for convenience. The dispersion term \( dn/d\lambda \), characterizing the relativistic origin of the Sagnac effect, is relatively small in typical materials (\( \sim 1\% \)) \[S1\]. Then the Hamiltonian becomes:

\[
H' = (\Delta_+ - i \gamma_c) a_{cw}^\dagger a_{cw} + (\Delta_- - i \gamma_c) a_{ccw}^\dagger a_{ccw} + i \kappa (a_{cw}^\dagger a_{ccw} + a_{ccw}^\dagger a_{cw}) \\
+ i \varepsilon_d (a_{cw}^\dagger - a_{cw}) + i \varepsilon_d (a_{ccw}^\dagger - a_{ccw}),
\]

with the detunings \( \Delta_\pm = \delta_c \pm \delta_{sag} \).

If we set \( \Delta_c = 0 \), the matrix form of the Hamiltonian without the driving terms becomes:

\[
H_0 = \begin{pmatrix} a_{cw}^\dagger & a_{ccw}^\dagger \end{pmatrix} M \begin{pmatrix} a_{cw} \\ a_{ccw} \end{pmatrix},
\]

where

\[
M = \begin{pmatrix} \delta_{sag} - i \gamma_c & i \kappa \\
- i \kappa & - \delta_{sag} - i \gamma_c \end{pmatrix}.
\]

Obviously, this Hamiltonian is anti-\( \mathcal{PT} \)-symmetric under the combined \( \mathcal{PT} \) operations:

\[
\{ \mathcal{PT}, H_0 \} = 0.
\]

We have constructed the anti-\( \mathcal{PT} \)-symmetric Hamiltonian using the opposite frequency shifts induced by rotation and the dissipative coupling induced by the dissipative elements in the resonator. We will examine the non-Hermitian degeneracy known as exceptional point (EP) in this system by solving the eigenfrequencies. From the equation (\( \mathbb{I}_2 \) is the identity 2 \( \times \) 2 matrix)

\[
|M - \omega \mathbb{I}_2| = 0,
\]

i.e.,

\[
\left| \begin{array}{cc} \delta_{sag} - i \gamma_c - \omega & i \kappa \\
- i \kappa & - \delta_{sag} - i \gamma_c - \omega \end{array} \right| = 0,
\]

TABLE II. Experimentally feasible parameters used for numerical simulations \[S1, S8, S24–S28\]. Here \( \lambda \) is the wavelength of the light, \( Q \) is the quality factor of the resonator, \( n \) is the refractive index of the resonator, \( R \) is the radius of the resonator, \( \kappa \) is the dissipative coupling, \( P_d \) is the pump power, and \( \gamma_{ex} \) is the coupling loss.

| \( \lambda \) | \( Q \) | \( n \) | \( R \) | \( \kappa \) | \( P_d \) | \( \gamma_{ex} \) |
|---|---|---|---|---|---|---|
| 1550 nm | \( \sim 1 \times 10^{11} \) | 1.44 | 50 \( \mu \)m | 8 kHz | 5 \( \mu \)W | \( \gamma_0/2 \) |
we obtain the eigenfrequencies of anti-$\mathcal{PT}$-symmetric systems:

$$\omega_{\pm} = -i\gamma_c \pm \sqrt{\Delta_{\text{sag}}^2 - \kappa^2}. \quad (\text{S38})$$

The experimentally accessible parameters are listed in Tab. II. We note that $Q$ has been improved to $10^{12}$ in experiment [S27], and the rotation speed $\Omega$, which is lower than 1 kHz in numerical simulations, is also experimentally feasible [S1].

As shown in Fig. S2(a), the term under the square root in Eq. (S38) is negative when $\Delta_{\text{sag}} < \kappa$, which leads to the difference of linewidth and enables the system to enter the anti-$\mathcal{PT}$ symmetry unbroken phase. When increasing the rotation speed until $\Delta_{\text{sag}} > \kappa$, the term under the square root becomes positive, thus frequency difference occurs and the system is within anti-$\mathcal{PT}$ symmetry broken phase. When $\Delta_{\text{sag}} = \kappa$, the two eigenfrequencies coalesce and the system is exactly at the EP. In Fig. S2(b), we plot the eigenfrequencies as a function of the rotation speed $\Omega$. Based on Eq. (S38), we fix the coupling strength at $\kappa = 8$ kHz, and obtain the rotation rate at EP with the aid of Eq. (S31): $\Omega_{\text{EP}} \approx 333$ Hz. In Fig. S2(b), when the rotation speed is below a critical value, i.e., $\Omega \leq \Omega_{\text{EP}}$, anti-$\mathcal{PT}$-symmetric phase appears with purely imaginary splitting. When $\Omega = \Omega_{\text{EP}}$, the coalescence occurs, and we can observe the broken phase of anti-$\mathcal{PT}$ symmetry with purely real splitting at $\Omega > \Omega_{\text{EP}}$.

S3. NONRECIPROCAL LIGHT TRANSMISSION AND ULTRASENSITIVE NANOPARTICLE SENSING

A. Symmetry-broken nonreciprocity

We consider the probe light to be incident from the left. The Hamiltonian can be expressed as

$$H' = (\Delta_+ - i\gamma_c) a^+_\text{cw} a_{\text{cw}} + (\Delta_- - i\gamma_c) a^+_{\text{ccw}} a_{\text{ccw}} + i\kappa (a^+_\text{cw} a_{\text{ccw}} + a^+_\text{ccw} a_{\text{cw}}) + i\epsilon_p (a^+_\text{cw} - a_{\text{cw}}), \quad (\text{S39})$$

where $\Delta'_\pm = \Delta_\pm \pm \Delta_{\text{sag}}$ with $\Delta_p = \omega_c - \omega_p$ being the probe detuning, $P_p$ denotes the probe power, and $\epsilon_p = \sqrt{\gamma_{\text{ex}} P_p / \hbar \omega_p}$ represents the amplitude of the probe light whose probe frequency is $\omega_p$. The equations of motion can be derived as

$$\dot{a}_{\text{cw}} = -i (\delta_\text{p} + \Delta_{\text{sag}}) a_{\text{cw}} + \kappa a_{\text{ccw}} + \epsilon_p,$$

$$\dot{a}_{\text{ccw}} = -i (\delta_\text{p} - \Delta_{\text{sag}}) a_{\text{ccw}} + \kappa a_{\text{cw}}, \quad (\text{S40})$$

with $\delta_\text{p} = \Delta_p - i\gamma_c$. In the strong-driving regime, we can get the steady-state solutions by making $\dot{a}_{\text{cw}} = 0$ and $\dot{a}_{\text{ccw}} = 0$:

$$a_{\text{cw}} = \frac{-i (\delta_\text{p} - \Delta_{\text{sag}}) \epsilon_p}{(\delta_\text{p} + \Delta_{\text{sag}}) (\delta_\text{p} - \Delta_{\text{sag}}) + \kappa^2}. \quad (\text{S41})$$

Applying the input-output relation [S30]

$$a^\text{out}_{\text{cw}} = a^\text{in}_{\text{cw}} - \sqrt{\gamma_{\text{ex}}} a_{\text{cw}}, \quad (\text{S42})$$

we obtain the transmission rate:

$$T^A_{\text{cw}} = \left| 1 - \frac{\gamma_{\text{ex}} a_{\text{cw}}}{\epsilon_p} \right|^2 = \left| 1 + \frac{i\gamma_{\text{ex}} (\delta_\text{p} - \Delta_{\text{sag}})}{(\delta_\text{p} + \Delta_{\text{sag}}) (\delta_\text{p} - \Delta_{\text{sag}}) + \kappa^2} \right|^2, \quad (\text{S43})$$

where the superscript A denotes anti-$\mathcal{PT}$-symmetric system. Similarly, when the probe is incident from the right, the solutions can be derived as

$$a_{\text{ccw}} = \frac{-i (\delta_\text{p} + \Delta_{\text{sag}}) \epsilon_p}{(\delta_\text{p} + \Delta_{\text{sag}}) (\delta_\text{p} - \Delta_{\text{sag}}) + \kappa^2}. \quad (\text{S44})$$

According to the input-output relation

$$a^\text{out}_{\text{ccw}} = a^\text{in}_{\text{ccw}} - \sqrt{\gamma_{\text{ex}}} a_{\text{ccw}}, \quad (\text{S45})$$
FIG. S3. (a) Normalized transmission rate of the HSR system. (b) Normalized transmission rate of the anti-$\mathcal{PT}$-symmetric system. The blue curves represent the transmission in the CW direction while the orange curves stand for the transmission in the CCW direction. The parameters used here are the same as those used in Fig. S2.

The transmission rate can be given by

$$ T_{ccw}^A = \left| 1 - \frac{\gamma_{ex} a_{ccw}}{\epsilon_p} \right|^2 = \left| 1 + \frac{i \gamma_{ex} (\delta_p + \Delta_{sag})}{(\delta_p + \Delta_{sag})(\delta_p - \Delta_{sag}) + \kappa^2} \right|^2. \quad (S46) $$

For comparison, we consider the light transmission in an HSR system with single mode [S1]. The steady-state solutions with the probe light incident from the CW and CCW directions are

$$ a_{cw} = \frac{-i \epsilon_p}{\delta_p + \Delta_{sag}}, \quad a_{ccw} = \frac{-i \epsilon_p}{\delta_p - \Delta_{sag}}, \quad (S47) $$

respectively. Likewise, the corresponding transmissions are

$$ T_{cw}^H = \left| 1 + \frac{i \gamma_{ex}}{\delta_p + \Delta_{sag}} \right|^2, \quad T_{ccw}^H = \left| 1 + \frac{i \gamma_{ex}}{\delta_p - \Delta_{sag}} \right|^2. \quad (S48) $$

where superscript H denotes the HSR system.

In Fig. S3, the normalized transmission rate $T_{cw,ccw}^s / \max[T_{cw,ccw}^s]$, is plotted as a function of the probe detuning $\Delta_p$ at different rotation speed, where $s$ denotes the anti-$\mathcal{PT}$ system (A) or the HSR system (H). For the HSR system, starting from the stationary case ($\Omega = 0$), the countercirculating modes overlap, owing to their expected degeneracy. As predicted by Eq. (S31), increasing the mechanical rotation frequency $\Omega$ results in a linear opposing frequency shift for the countercirculating modes, i.e., nonreciprocal light transmission [S1]. For the anti-$\mathcal{PT}$-symmetric system, as shown in Fig. S3(b), there is no splitting in the transmission spectrum in the symmetry unbroken phase ($\Omega < \Omega_{EP}$), and nonreciprocal light transmission is unclear compared with the HSR system, although the rotation speed has been increased over 200 Hz. This is owing to the fact that the rotation-induced Sagnac shift is smaller than the dissipative coupling and the transmission in two opposite directions is not greatly separated from each other [see Eqs. (S43) and (S46)]. Meanwhile, there is no dissipative coupling in HSR system, thus the nonreciprocal transmission is only dependent on the Sagnac shift, as shown in Eq. (S48). The mode splittings for CW and CCW modes appear in the symmetry broken phase ($\Omega > \Omega_{EP}$) by increasing the rotation speed, and the nonreciprocity becomes stronger than that in HSR system. Different from the nonreciprocity reported in the $\mathcal{PT}$-symmetric systems [S31, S32], which relies on the nonlinear process, the nonreciprocal light transmission in our system is free of nonlinearity or gain-loss balanced structure.
B. Anti-$\mathcal{PT}$ sensor

Apart from one-way control of transmission rates, in the following, we would like to present the potential of the anti-$\mathcal{PT}$-symmetric system for detection of nanoparticles [S9, S11, S12], which is highly desirable for widespread applications in, e.g., medical diagnosis and environmental monitoring [S9, S33, S34].

The Hamiltonian of our anti-$\mathcal{PT}$-symmetric system modified by nanoparticles can be expressed as a matrix form [S9, S12, S35]:

$$M_N = \left( \begin{array}{cc} \Delta_{\text{sag}} + \delta_N & i\kappa + C_N^- \\ i\kappa + C_N^+ & -\Delta_{\text{sag}} + \delta_N \end{array} \right),$$  \hspace{1cm} (S49)

with

$$g_N = \sum_{i=1}^{N} g_{s,i}, \quad \gamma_N = \sum_{i=1}^{N} \gamma_{s,i}, \quad C_N^\pm = \sum_{i=1}^{N} (g_{s,i} - i\gamma_{s,i}) \exp(\pm 2m\beta_i), \quad (i = 1, 2, 3, ..., N),$$  \hspace{1cm} (S50)

where $\delta_N = g_N - i(\gamma_c + \gamma_N)$, $N$ is the total particle number, $m$ is the azimuthal mode number, $\beta_i$ is the angular position of the $i$-th nanoparticle, and $g_{s,i}$ ($\gamma_{s,i}$) is the coupling strength (the loss rate) induced by the $i$-th nanoparticle. According to $|M_N - \omega L_2| = 0$, the eigenfrequencies for this perturbed system can be given by:

$$\omega_N^\pm = \delta_N \pm \sqrt{\Delta_{\text{sag}}^2 + (i\kappa + C_N^-)(i\kappa + C_N^+)}. $$  \hspace{1cm} (S51)

The corresponding frequency difference is

$$\Delta\omega_N^A = \omega_N^+ - \omega_N^- = 2\sqrt{\Delta_{\text{sag}}^2 + (i\kappa + C_N^-)(i\kappa + C_N^+)}.$$  \hspace{1cm} (S52)

To explore EP assisted sensitivity enhancement, we compare our EP sensor with HSR sensor [S12]. In the presence of perturbations, the eigenfrequencies for HSR sensor can be written as

$$\omega_N^\pm = \delta_N \pm \sqrt{\Delta_{\text{sag}}^2 + C_N^- C_N^+},$$  \hspace{1cm} (S53)

thus yields the frequency difference as follows:

$$\Delta\omega_N^H = 2\sqrt{\Delta_{\text{sag}}^2 + C_N^- C_N^+}.$$  \hspace{1cm} (S54)

It should be noted that the coupling between CW and CCW modes in HSR sensor is only induced by nanoparticles. By setting $N = 1$, we first consider the simplest case that only a single nanoparticle is deposited on the resonator with $g_1 = g_{s,1}$ and $\gamma_1 = \gamma_{s,1}$. In this case, angular position $\beta_1$ is set to be 0, leading to $C_N^+ = C_N^- = g_{s,1} - i\gamma_{s,1} = g_1 - i\gamma_1$. Here we have restricted our discussion to $J = g_1 - i\gamma_1$ for convenience, then the perturbed eigenfrequencies can be described as

$$\omega_1^\pm = J - i\gamma_c \pm \sqrt{\Delta_{\text{sag}}^2 - (\kappa - iJ)^2},$$  \hspace{1cm} (S55)

with the corresponding frequency difference:

$$\Delta\omega_1^A = \omega_1^+ - \omega_1^- = 2\sqrt{\Delta_{\text{sag}}^2 - (\kappa - iJ)^2}.$$  \hspace{1cm} (S56)

Likewise, we can write perturbed frequencies and their differences for HSR sensor as follows:

$$\omega_1^\pm = J - i\gamma_c \pm \sqrt{\Delta_{\text{sag}}^2 + J^2}, \quad \Delta\omega_1^H = 2\sqrt{\Delta_{\text{sag}}^2 + J^2}.$$  \hspace{1cm} (S57)

In Fig. S4(a), the logarithmic behaviour of the real part of the complex frequency splitting is shown to highlight the sensitivity enhancement of anti-$\mathcal{PT}$ sensor. For the same minuscule perturbation, anti-$\mathcal{PT}$ sensor at EP performs better than HSR sensor, which shows no strong dependence on perturbation. However, their behaviours are similar for large disturbance. This can be explained with the perturbation theory. The complex frequency splitting at EP ($\Delta_{\text{sag}} = \kappa$) is expected to approximately follow

$$\Delta\omega_1^A = 2\sqrt{2i\kappa J^{1/2}},$$  \hspace{1cm} (S58)
FIG. S4. Anti-$\mathcal{PT}$-symmetric ultrasensitive nanoparticle sensing. (a) The real part of the complex frequency splitting $\text{Re}[\Delta \omega_1]$ versus the perturbation $g_1$ where $s$ denotes the anti-$\mathcal{PT}$ sensor (A) or HSR sensor (H). (b) The signal enhancement factor as a function of the perturbation $g_1$. (c) The frequency splitting versus the total particle number $N$ to show performance of these sensors for multiparticle detection. The rotation speed of these two sensors is fixed at $\Omega_{\text{EP}} \approx 333$ Hz. $\gamma_1/g_1$ is set to be 0.05 as in the experiment [S10, S11]. The other parameters used here are the same as those in Fig. S2.

when $J$ is much smaller than $\kappa$, which indicates that perturbations experience an enhancement of the form $J^{1/2}$. For larger $J$, the slope of the splitting will be slightly larger than $1/2$, because in this case Eq. (S56) cannot be simplified to Eq. (S58), and the higher order terms should be taken into consideration:

$$\Delta \omega_1^A = 2\sqrt{2iK}J^{1/2} - \frac{i\sqrt{2iK}}{2\kappa}J^{3/2} + \cdots.$$  \hspace{1cm} (S59)

For HSR sensor, the complex frequency splitting

$$\Delta \omega_1^H = 2\Delta_{\text{sag}} + \frac{J^2}{\Delta_{\text{sag}}} - \frac{J^4}{4\Delta_{\text{sag}}} + \cdots,$$  \hspace{1cm} (S60)

is proportional to the square of the perturbation $J^2$ at least. Thus, HSR sensor will not show strong dependence on small perturbation.

A signal enhancement factor defined as

$$\xi = \left| \frac{\partial \Delta \omega_1^A}{\partial g_1} \right| = \left| \frac{2(\kappa - \gamma_1 - ig_1)(\gamma_1/g_1 + i)}{\sqrt{\Delta_{\text{sag}}^2 - (\kappa - \gamma_1 - ig_1)^2}} \right|,$$  \hspace{1cm} (S61)

is introduced to intuitively evaluate the performance of anti-$\mathcal{PT}$ sensor. Similarly, the signal enhancement factor for HSR sensor can be given by

$$\xi = \left| \frac{\partial \Delta \omega_1^H}{\partial g_1} \right| = \left| \frac{2(-\gamma_1 - ig_1)(\gamma_1/g_1 + i)}{\sqrt{\Delta_{\text{sag}}^2 + (-i\gamma_1 + g_1)^2}} \right|.$$  \hspace{1cm} (S62)

The dependence of signal enhancement factor $\xi$ on perturbations is shown in Fig. S4(b), from which we can see in the vicinity of EP, larger sensitivity can be obtained for smaller perturbations in anti-$\mathcal{PT}$ sensor. While in HSR sensor, the signal enhancement factor is small for minuscule perturbations. When the perturbation $J$ is much larger than $\kappa$, as aforementioned, the signal enhancement factor of these two sensors will be roughly equal ($\sim 2$, as illustrated in the figure) for the fact that the higher order terms in Eqs. (S59) and (S60) cannot be neglected.

We finally extend the discussions above to a more practical case for multiparticle detection. The real parts of complex frequency splittings versus total particle number $N$ are shown in Fig. S4(c) when continuously depositing particles on the surface of the resonator, with the orange circles denoting the splitting of anti-$\mathcal{PT}$ sensor and the blue diamonds denoting that of HSR sensor. The values of $g_{s,i}$, $\beta_i$, and $\gamma_{s,i}$ are random in this figure. The azimuthal mode number is set to be $m = 4$ [S10]. Similar to the case of single nanoparticle detection, the performance of this anti-$\mathcal{PT}$ sensor is superior to the HSR sensor even for multiparticle sensing. One of the potential applications of this anti-$\mathcal{PT}$-symmetric system, i.e., anti-$\mathcal{PT}$ based sensing and its enhancement to the sensitivity are carefully confirmed, which could open a new path towards engineering compacted ultrasensitive sensors for detections of nanoscale objects.
Finally, we get

\[ H_N = (\Delta'_+ + \delta_N) a^\dagger_{cw} a_{cw} + (\Delta'_- + \delta_N) a^\dagger_{ccw} a_{ccw} + (i\kappa + C_N^-) a^\dagger_{ccw} a_{cw} + (i\kappa + C_N^+) a^\dagger_{cw} a_{cw} + \epsilon_p (a^\dagger_{cw} - a_{cw}). \]  

(S63)

The equations of motion can be derived as

\[
\begin{align*}
\dot{a}_{cw} &= -i \left( \Delta'_+ + \delta_N \right) a_{cw} - i \left( i\kappa + C_N^- \right) a_{ccw} + \epsilon_p, \\
\dot{a}_{ccw} &= -i \left( \Delta'_- + \delta_N \right) a_{ccw} - i \left( i\kappa + C_N^+ \right) a_{cw}.
\end{align*}
\]

(S64)

The steady-state solutions can be obtained by making \( \dot{a}_{cw} = 0 \) and \( \dot{a}_{ccw} = 0 \), i.e.,

\[
\begin{align*}
0 &= -i \left( \Delta'_+ + \delta_N \right) a_{cw} - i \left( i\kappa + C_N^- \right) a_{ccw} + \epsilon_p, \\
0 &= -i \left( \Delta'_- + \delta_N \right) a_{ccw} - i \left( i\kappa + C_N^+ \right) a_{cw}.
\end{align*}
\]

(S65)

Finally, we get

\[
a_{cw} = \frac{-i \left( \Delta'_+ + \delta_N \right) \epsilon_p}{\left( \Delta'_+ + \delta_N \right) (\Delta'_- + \delta_N) - (i\kappa + C_N^-)(i\kappa + C_N^+)}. 
\]

(S66)

Here we consider only one single nanoparticle is deposited on the resonator \( (N = 1) \), then the steady-state solution becomes

\[
a_{cw} = \frac{-i \left( \Delta'_+ \right) \epsilon_p}{\left( \Delta'_+ + \delta_1 \right) (\Delta'_+ + \delta_1) - (g_1 - i\gamma_1)^2}. 
\]

(S67)

Applying the input-output relation in Eq. (S42), we can get the transmission rate as

\[
T_i^A = \left| 1 - \frac{\gamma_{ex} a_{cw}}{\epsilon_p} \right|^2. 
\]

(S68)

For HSR sensor, the steady-state solution is

\[
a_{cw} = \frac{-i \left( \Delta'_- \right) \epsilon_p}{\left( \Delta'_+ + \delta_1 \right) (\Delta'_- + \delta_1) - (g_1 - i\gamma_1)^2}. 
\]

(S69)

The corresponding transmission is

\[
T_i^H = \left| 1 - \frac{\gamma_{ex} a_{cw}}{\epsilon_p} \right|^2. 
\]

(S70)

To compare the performance of the these two sensors, we introduce an enhancement factor

\[ \chi = \mathcal{V}_A / \mathcal{V}_H, \]

(S71)

with the relative variation ratio defined as \( \mathcal{V}_A = T_i^A / T_0^A \) and \( \mathcal{V}_H = T_i^H / T_0^H \). Here \( T_0^A \) (\( T_0^H \)) is the transmission for anti-\( \mathcal{P}\mathcal{T} \) (HSR) sensor without nanoparticle (by making \( g_1 = 0 \) and \( \gamma_1 = 0 \)). The probe detuning is chosen to be \( \Delta_p = \text{Re} [\omega_-] \).

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