Spatial Advertising Structure Analysis on Selected Examples

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Abstract. In the paper, the static and dynamic analysis of spatial advertising structures with many degrees of freedom is presented. The computations for numerical models with different height-to-width ratio of the screen and variable location of the pole in the system are conducted. An attempt to model the time-independent wind forces and time-dependent external loads on the screen and way of effectively applying them to the structure is made. The influence of assumed geometry of the advertising banner on the obtained results of natural frequencies, displacements and internal forces is discussed. The difference in system behaviour under static loads and under dynamic force distribution within and outside the resonance range is examined. Comparison of the results of individual analyses showed that the pole is the most crucial element in the considered structures. It experiences the largest displacements and values of bending moments in relation to other parts of the system. At the same time, the pole cross-section changes most affect the values of first natural frequencies of the entire system and have a large impact on the obtained results of the dynamic analysis. In fact, advertising banners are often located on the open space and are exposed to the dynamic forces, which are random loads that should be considered by using complex stochastic processes. However, obtained results from specific analyses allowed drawing general conclusions on the recommended geometry for the advertising structures to receive the high-capacity system and simultaneously to minimize the risk of negative effect of the resonance due to dynamic excitation.

1. Introduction

Nowadays, an increasingly faster development of different advertising elements designed to attract the attention of the customer can be observed. Structures with various shapes from planar systems to spatial forms are designed. Their common features are a large advertising screen in the central part of the scheme and the location of objects in the open space, which in turn makes them sensitive to the dynamic load caused by wind or motion of large vehicles.

Some references to advertising banners can be found in the codes for steel structures designing and for determining particular loads. Unfortunately, it is difficult to find the guidelines for this type of objects geometry selection in the sources available for the engineers. Commonly designed banners are those with rectangular shape of screen in which the width is greater than the height and with a symmetrical location of the pole. However, in many cases the different shape of screen like square or in the form of a vertically oriented rectangle can be found as well as banners with no symmetrical pole setting. Therefore, the question arises whether these differences result from design reasons or only aesthetic, and on the other hand, how do they affect the safety and stability of the entire system?

The described advertising structures may be located on different types of terrain. They can be found by the road in an open space or in a built-up area. For this reason, they are exposed to various
time-dependent loads like dynamic wind forces or violent gusts of air from the large dimension vehicles motions. Therefore, the statics cannot be the only method of analysis at the stage of design process. The biggest problem with the loads variable in time is the resonance phenomenon [1], [2], that occurs, when the frequency of dynamic excitation is close to the first natural frequency of the system. It results in overlapping vibrations and a significant increase in the displacement and internal force amplitudes, which can cause the damage of the system.

The most relevant issue is that the described dynamic wind forces [3] and vehicle traffic are stochastic loads in reality and we can determine their frequencies only with a limited probability [4], [5], [6]. At the stage of designing advertising banners, this part of calculations is omitted due to the complexity of stochastic processes and instead the so-called safety coefficients for the occurrence of dynamic loads are used. Therefore, the question arises if we are able to influence the reduction of the negative effects of the resonance phenomenon and create an object less sensitive to dynamic loads, by appropriate selection of the system geometry. The results presented in the paper are aimed at obtaining answers to the described problems.

2. Finite element method for the system with many degrees of freedom
In this chapter the summation notation is included, which means that in following equations, two repeated indices imply the sum. It results in greater transparency of presentation. Let us consider the complex structure with \( N \) degrees of freedom. Taking into account the external work, kinetic and potential energy of the system, using the Lagrange’s equation of the second type, the set of the equations of motion can be received in the form [7, 8]:

\[
M_{\alpha\beta}\ddot{q}_\beta + C_{\alpha\beta}\dot{q}_\beta + K_{\alpha\beta}q_\beta = Q_\alpha(t); \quad \alpha, \beta = 1, 2, \ldots, N
\]

where \( q_\beta, Q_\alpha(t) \) are the vectors of generalized coordinates and external loads while \( M_{\alpha\beta}, C_{\alpha\beta}, K_{\alpha\beta} \) are mass, damping and stiffness matrices respectively. Considering the free vibrations without damping (1) is reduced to the formula:

\[
M_{\alpha\beta}\ddot{q}_\beta + K_{\alpha\beta}q_\beta = 0; \quad \alpha, \beta = 1, 2, \ldots, N
\]  

(2)

Denoting \( N \) as the number of normalized mode shapes, the generalized coordinate vector during the computation can be expressed by the equation:

\[
q_\alpha = z_\beta \Phi_{\alpha\beta}; \quad \alpha, \beta = 1, 2, \ldots, N
\]

with \( z_\beta, \Phi_{\alpha\beta} \) being normal (modal) coordinate vector and the eigenvector matrix respectively. \( \Phi_{\alpha\beta} \) consist of sequentially ordered eigenvectors \( \phi(n)_{\beta} \) corresponding to particular eigenvalues \( \omega^2(n) \). It should be noted that the indices in round bracket as \( (n) \) are not subject to the summation convention.

The solution of Eigen problem can be given by the formula:

\[
(K_{\alpha\beta} - \Omega(\alpha\beta)M_{\alpha\beta})\Phi_{\beta} = 0
\]

(4)

where \( \Omega(\alpha\beta) \) is the diagonal matrix with entries being the squares of natural frequencies \( \omega^2(n) \). Every eigenvalue \( \omega^2(n) \) and corresponding it eigenvector \( \phi(n)_{\beta} \) satisfy the equation:

\[
(K_{\alpha\beta} - \omega^2(n)M_{\alpha\beta})\phi(n)_{\alpha} = 0
\]

(5)

It is commonly known, that Equation (5) has the solution if

\[
\det(K_{\alpha\beta} - \omega^2(n)M_{\alpha\beta}) = 0
\]

(6)
The solution of (5) and (6) brings $N$ Eigen pairs consisting of eigenvalue $\omega_2(n)$ and eigenvector $\phi(n)$. It allows to determine the $n$-th value of natural frequency and a corresponding mode shape included in $\phi(n)$.

To obtain the response of the dynamic system with many degrees of freedom in the form of node displacements or internal forces, the approximate methods can be applied. In the following paper, the harmonic analysis and Newmark method are used. These can be found in [7].

3. Numerical models
The geometry of the system in the form of a steel pole with the structural part of the screen presented in Figure 1. is adopted for the analysis. The pole with the horizontal and vertical bars of the banner are modelled as frame elements, while the bracing as truss elements. The bottom part is fixed in the ground. To transfer the external loads to the structural system of the banner, the cladding form is used. It results in obtaining the system with many degrees of freedom.

Figure 1. Assumed geometry of the advertising structure, dimensions in [m]

Numerical computations are made for three different screen dimensions: horizontally oriented rectangular – labelled as shape A (Figure 2a), square – shape B (Figure 2b) and vertically oriented rectangular – shape C (Figure 2c). Next, for every screen shape, three cases of location of the pole are considered: symmetrically or on a small and big eccentric about the center of the banner (according to Figure 3). Hence, nine numerical models are obtained, on which static and dynamic analysis is carried out. It is worth noticing that the dimensions of the screens have been selected so that areas are the same in all schemes.
Figure 2. The division of models according to the main dimensions of the screen, [cm]:
  a) Shape A; b) Shape B; c) Shape C.

Figure 3. The division of models depending on pole location:
  a) symmetrical; b) on small eccentric; c) on big eccentric.

4. Static Analysis
Static computations are made for three load cases [9]:
- dead load of the structural elements [10],
- advertisement load assumed as 20 kg/m$^2$,
- static wind load.

Static wind load is determined by using equation [11]:

$$ F_w = c_s c_d \cdot c_f \cdot q_p(z_e) \cdot A_{ref}, \quad (7) $$

where:
- $c_s c_d$ – structural factor,
- $c_f$ – force coefficient for the structure or structural element,
- $q_p(z_e)$ – peak velocity pressure at reference height [kN/m$^2$]
- $A_{ref}$ – reference area, [m$^2$].

Since main aim of the paper is the eigenproblem and dynamic analysis, the complex computations in the statics are omitted and for simplification, the coefficients $c_s c_d$ and $c_f$ equal 1.0 and 1.8 respectively are assumed. The reference screen area is 19.84 cm$^2$ for every model. By locating the banner in Poland in the first zone of wind load and the III category of terrain [11], the resultant force of the wind load equals 17.65 kN is calculated. In accordance to [11], obtained force should be applied
to the system on the level of the screen center of gravity but on the horizontal eccentric \( e = \pm 0.25B \), where \( B \) denotes the width of the screen. The point of force application for particular models – labelled as P, is presented in Figure 2 and Figure 3. In considered example the area of external surfaces perpendicular to the direction of the wind is significantly smaller than the area of parallel surfaces. Therefore, during the analysis the influence of friction forces acting parallel to the direction of the wind speed component is neglected.

Using the internal force and displacement results, the optimal steel cross-sections for individual element groups in the system are selected [12]: horizontal bars – C-section C140, vertical bars - L-shape 80x80x6 [mm], elements connecting the pole with the structural part of the screen – plate 40x40x10 [mm]. The pole turns out to be the crucial element in the whole system and because of the significant displacement and internal force values requires analysis that is more detailed. To find the optimal pole cross-section for all nine models the static computations for five different steel pipes with external diameter \( D \) and thickness \( t_p \) are made. Selected results obtained from model with A shape screen (Figure 2a) and symmetrical pole are presented in Table 1.

5. Eigen problem and dynamic analysis

The solution of Eigen problem brings the natural frequencies of the system and corresponding them eigenvectors. During the computations, the influence of the assumed pole cross-sectional area on the results of the first natural frequency and mode shape is examined.

As already mentioned, dynamic wind force and fluctuations in air velocity are random loads and should be considered by stochastic processes. However, the main goal of this paper is to determine the effect of resonance phenomena from loads variable over time on the values of displacements and internal forces in the system. It aims at finding the optimal geometry of the advertising banner, which will provide creation of the object less sensitive to dynamic loads. To simplify the computations and for a deeper understanding the described problem in the time-dependent analysis, two types of loads are considered: harmonic \( F_1(t) \) and periodical \( F_2(t) \) added to the system in point P with amplitude based on static wind action (compare Figure 4). For the same reason damping effect is omitted during calculations.

\[ F_1(t) = F_{w1}\sin(2\pi n_dt), \]  

where:
- \( F_{w1} \) – force amplitude, [kN]
- \( n_d \) – load frequency, [Hz].
- \( t \) – time, [s].

![Figure 4](image-url). Dynamic load cases: a) harmonic, b) periodical.

The first load case is determined by using the equation:

\[ F_1(t) = F_{w1}\sin(2\pi n_dt), \]  

where:
- \( F_{w1} \) – force amplitude, [kN]
- \( n_d \) – load frequency, [Hz].
- \( t \) – time, [s].
Since $F_1(t)$ and $F_2(t)$ are applied to the system in form of dynamic non-static loads the value 9.807 kN is assumed as the amplitude, which is equal the static wind load without taking into account the force coefficient. The excitation frequency equals $\nu=2.0$ Hz is adopted. The first case load computations are made by using harmonic analysis, while the second one by using the Newmark method with a time step equals 0.001 s. Subsequently, the results of maximum displacements and bending moments in the pole support point for different pipe cross-sections are compared.

6. Numerical results and discussions

The computations are carried out by using the program Autodesk Robot Structural Analysis (ARSA). Statics is conducted for a combination of three mentioned type of load including the load factors and from each case separately. Comparison of results shows that the highest value of bending moment is received in the fixed support of the pole as expected. For all models this value stayed at the same level $M_{comb.}=156$ kNm from the load combination and $M_{wind}=104$ kNm from the static wind load. The changes of steel pipe pole and the screen shape affect the cross-section strength and the value of the maximum displacement in the system. The decisive factor in the selection of the optimal pole cross-section becomes the displacement of its top point labelled as $u$. For the smallest pipe that fulfill the load capacity condition $u$ turns out bigger than the limit value equals to $u_{bound.}=\text{Length}/150$. All results obtained for model presented in Figure 2a. with symmetrical pole are given in Table 1, where $\sigma_{\text{max}}$ and $f_d$ means the maximum stress and computational strength of the material. For comparison, the boundary displacement for this scheme is equal 4.7 cm.

| Type of steel pipe pole | Cross-sectional area | $u$ [cm] | $\sigma_{\text{max}}/f_d$ |
|-------------------------|-----------------------|----------|--------------------------|
| 298.5x8.8               | 80.1                  | 9.0      | 0.91                     |
| 298.5x12.5              | 112.0                 | 6.6      | 0.65                     |
| 355.6x10                | 109.0                 | 7.0      | 0.55                     |
| 406.4x8                 | 100.0                 | 3.8      | 0.52                     |
| 406.4x8.8              | 110.0                 | 3.5      | 0.47                     |

Static calculations carried out on all models showed that the smallest value of $u$ is obtained for the model with A shape screen, while the biggest for the C shape screen. However, the bigger the width of the screen the higher the values of torsion moment from the wind load in the pole, which is certainly an unfavourable phenomenon. The same result can be obtained when the pole is located on eccentric.

The solution of the Eigen problem in the form of natural frequencies and corresponding eigenvectors shows that the first three mode shapes in models with a symmetrical pole location depend on its deformation. Only at higher vibration frequencies, the flexural and torsional deformations of the screen can be observed. Asymmetrical location of the pole in the model results in a greater share of flexural and torsional deformations of the screen in the second and third mode shape and reduces the first eigenvalue frequency. This allows us to conclude that the larger the eccentric of the pole, the system becomes more sensitive to dynamic loads associated with torsional screen vibrations.

The natural frequencies for various numerical models oscillate in the range of 2.0 to 3.5 Hz. Thorough analysis of received results demonstrate that the smaller the cross-sectional area of the pipe assumed for the pole, the lower the value of the system frequency. The smallest frequencies are obtained for the model with A shape screen, whose eigenvalue show simultaneously the greatest sensitivity to changing the location of the pole. Exemplary results are summarized in Table 2.
Table 2. First eigenvalues for banners with A shape screen

| Type of steel pipe pole $Dx_t$ [mm] | First eigenvalue frequency [Hz] |
|-------------------------------------|---------------------------------|
| Symmetric pole                      | Pole on small eccentric         |
| Pole on big eccentric               |                                 |
| 298.5x8.8                           | 2.19                            |
| 298.5x12.5                          | 2.49                            |
| 355.6x10                            | 2.96                            |
| 406.4x8                             | 3.29                            |
| 406.4x8.8                           | 3.42                            |

Dynamic computations are conducted for two different functions various in time — harmonic and periodical. The assumed excitation frequency in both cases is equal 2 Hz. Resonance phenomenon can be observed for models with natural frequency close to the values of the excitation frequency. It is characterized by a sudden increase in displacements of the pole and the value of the moment in the fixed support. Differences in the vibrations of the pole end point obtained for the model with A shape screen under periodical load in and out of the resonance area are presented in Figure 5.

However, an in-depth dynamic analysis of the numerical models shows that the assumed geometry of the banner plays a significant part in obtained values of displacements and internal forces in the resonance area. The smallest increase in displacements and bending moments is observed for the A shape screen and the symmetrical location of the pole. Enlarging the height of the screen in relation to the width causes greater sensitivity to the negative effects of the resonance (compare Figure 6).

![Figure 5](image)

**Figure 5.** Displacements for model with A screen shape and symmetric pole under periodical load

The most important result is determination of impact of the pole location on the structure behaviour when the system natural frequency is close to the excitation frequency. It turns out that placing a pole on the eccentric causes a significant increase in the displacement and moment values in all three screen shapes (compare Figure 7). Exemplary results for the model with A shape screen obtained from the harmonic load examined in time of 4 seconds are summarized in Table III.
Based on the received data, it can be concluded that the rapid increase in displacement in the case of asymmetric pole positioning is caused by the susceptibility of the obtained system to torsion and detaching of the board.

**Figure 6.** Displacements for model with symmetric pole under periodical load (pipe 298.5x8.8).

**Figure 7.** Displacements for model with A shape screen under periodical load (pole made of pipe 298.5x8.8)
Table 3. First eigenvalue results for banners with A shape screen

| Pole location            | Frequency of Eigen vibration [Hz] | $u_{\text{max}}$ [cm] | $M_{\text{max}}$ [kNm] |
|--------------------------|------------------------------------|------------------------|-------------------------|
| Symmetrical              | 2.19                               | 18.0                   | 206.85                  |
| On small eccentric       | 1.99                               | 42.2                   | 483.41                  |
| On big eccentric         | 1.95                               | 86.7                   | 992.59                  |

7. Conclusions
The static and dynamic analysis of the considered banner on models of different geometry allowed to draw important conclusions as to what principles should be followed at the design stage of the system, not only to create a safe and stable scheme, but to protect it against possible variable loads over time. In order to limit the negative effects of the resonance phenomenon that may occur from external loads in this type of structures, the shape of a rectangular screen with a width greater than the height and with a symmetrical pole location should be adopted. It allows reducing the torsion effect in the system from the applied forces, which can limit the susceptibility of the board detaching. This type of geometry minimizes all negative effects of dynamic loads in the resonance area.

In the case of advertising structures located in an area with high probability of sudden and intense gusts of wind, it is worth expanding the dynamic analysis of the model with additional computation preceded by field tests.

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