Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
Sustainable dynamic lot sizing models for cold products under carbon cap policy

Rami As’ad *, Moncer Hariga, Abdulrahim Shamayleh

Department of Industrial Engineering, College of Engineering, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates

ABSTRACT

Amid the ever growing interest in operational supply chain models that incorporate environmental aspects as an integral part of the decision making process, this paper addresses the dynamic lot sizing problem of a cold product while accounting for carbon emissions generated during temperature-controlled storage and transportation activities. We present two mixed integer programming models to tackle the two cases where the carbon cap is imposed over the whole planning horizon versus the more stringent version of a cap per period. For the first model, a Lagrangian relaxation approach is proposed which provides a mean for comparing the operational cost and carbon footprint performance of the carbon tax and the carbon cap policies. Subsequently, a Bisection based algorithm is developed to solve the relaxed model and generate the optimal ordering policy. The second model, however, is solved via a dynamic programming based algorithm while respecting two established lower and upper bounds on the periodic carbon cap. The results of the computational experiments for the first model display a stepwise increase (decrease) in the total carbon emissions (operational cost) as the preset cap value is increased. A similar behavior is also observed for the second model with the exception that paradoxical increases in the total emissions are sometimes realized with slightly tighter values of the periodic cap.

1. Introduction

Concerns over global warming and its detrimental consequences continue to rise, making carbon emissions reduction an increasingly critical matter. It is a global consensus that carbon emissions stand out as the prime cause leading to climate change and global warming (IPCC, 2014). The recent surge in carbon emissions and its negative impact is mainly attributed to the fast-paced industrial growth and the associated intensive energy consumption. The amount of carbon footprint generated evidently varies across different sectors of the economy, where the potential reduction of such emissions in the industrial, construction, and agricultural sectors was discussed by Huisingsh, Zhang, Moore, Qiao, and Li (2015). The authors stated that carbon emissions due to logistics operations in specific constitute a small percentage to possibly more than 10% of the global emissions depending on the characteristics of goods as well as the transportation mode adopted. As a matter of fact, in the year 2017, it is reported that 29% and 28% of greenhouse gas emissions in the United States were generated from transportation and electricity, respectively.1

On the notion of goods characteristics, cold supply chain has unique dynamics and peculiarities that sets it apart from other supply chains. Cold, or temperature-sensitive, products are typically of perishable nature and encompass a wide range of products such as fresh produce, poultry and dairy products, fishery items, pharmaceuticals, vaccines, among many others. Efficiently managing the logistics operations for such products has recently received an ever growing interest from both industrial practitioners and academic researchers alike, which may partially be attributed to two main reasons. First, the expanded market reach of these products and the unprecedented surge in their sales figures due to the advancement in the enabling cooling technologies coupled with the increase in the number of third party logistics service providers specialized in handling this type of products. For instance, it has been estimated that global cold chain market was valued at $189.92 Billion in 2017 and it is projected to reach $293.27 Billion by 2023.2 In the year 2018, the overall global capacity of cold storage reached around 616 million cubic meters.3 These numbers clearly illustrate the great

---

* Corresponding author.
E-mail address: rafif@aus.edu (R. As’ad).
1 https://www.epa.gov/ghgemissions/sources-greenhouse-gas-emissions.
2 https://www.marketsandmarkets.com/Market-Reports/cold-chains-frozen-food-market-811.html.
3 https://www.gcca.org/resources/global-cold-storage-capacity-report.

https://doi.org/10.1016/j.cie.2020.106800
Received 21 October 2019; Received in revised form 7 July 2020; Accepted 29 August 2020
Available online 4 September 2020
0360-8352/© 2020 Elsevier Ltd. All rights reserved.
cold products are typically chilled (0 ◦C). Such products along each step across the supply chain. Secondly, since the potential that exists towards optimizing the movement and storage of cold products ought to explicitly account for sustainability aspects with cold products. Therefore, supply chain practitioners dealing with cold products could have a devastating impact on the cost, quality, environment and customer service level. Furthermore, this extensive use of energy triggers elevated levels of carbon emissions. For instance, dairy products, fishery items and fresh food exhibit daily or weekly varying demand stressing the need for dynamic lot sizing models. Here, more specifically, cold or perishable products in general, typically exhibit time varying demand patterns. In case of daily demand patterns are more practical as they better capture real-life daily decision-making processes. As a matter of fact, different carbon regulatory policies have been considered internationally to help abate carbon footprint. Currently, there are four such policies in place which are carbon trading, carbon cap, cap-and-trade, and cap-and-offset (Benjaafar, Li, & Daskin, 2013, Konur & Schaefer, 2014). Following the carbon tax policy, a company is charged on a per unit basis (e.g., ton) for its generated carbon emissions in the form of taxes. In essence, the governing entity sets the price of carbon emissions and allows the market to determine the amount of emission reductions. On the contrary, it is not allowed to infringe the carbon emissions level under the cap policy, as the cap establishes a strict upper bound on a firm’s emissions. On the other hand, the cap-and-trade policy calls for allowing a firm to buy/sell emission credits if its emission level is higher/lower than the imposed cap. Lastly, the carbon cap-and-offset policy allows companies to invest in carbon offset projects towards increasing their allotted caps. A more elaborate discussion on the four regulatory policies, the efficiency and the potential benefits that each policy offers along with implementation practices of various policies across different countries is provided in Chelly, Nouira, Frein, and Hadj-Alouane (2019).

Several authors considered different demand patterns when addressing sustainable lot sizing problems for non-perishable and cold items such as deterministic (Battini, Calzavara, Isolan, Sgarbossa, & Zangaro, 2018; Lee, Yoo, & Cheong, 2017), dynamic or time varying demand (Absi, Dauzé-Pérès, Kedad-Sidhoum, Penz, & Rapine, 2013; Shuang, Diabat, & Liao, 2019), and stochastic (Mohammed, Selim, Hassan, & Syed, 2017; Muriana, 2016). However, the dynamic or stochastic demand patterns are more practical as they better capture real-life dynamics of the demand for many products such as the ones under study here. More specifically, cold or perishable products in general, typically exhibit time varying demand stressing the need for dynamic lot sizing modeling approach to optimize inventory and transportation related decisions on a periodic basis, and accordingly control costs and carbon emissions. For instance, dairy products, fishery items and fresh food exhibit daily or weekly varying demand patterns. In case of daily demand, this demand is much higher during the weekend than during weekdays as most consumers, living in the vicinity of the retail stores, frequently tend to purchase their needs of such items on weekends. The remaining consumers prefer to go to the retail store when the need arises during the weekdays. Similarly, the demand varies from week-to-week, where the demand in the first week of the month is especially higher during the weekdays. Similarly, the demand varies from week-to-week, where the demand in the first week of the month is especially higher. The intensive energy usage, and accordingly the cost, of cold products logistics operations coupled with their negative environmental impact renders the development of efficient and sustainable replenishment policies of such products an inevitable necessity. A bad selection for the number and capacity of temperature-controlled transport and storage facilities are needed to handle them making this industry highly energy intensive. In fact, Kayfeci, Kecegasus, and Gedik (2013) pointed out that energy consumption in cold chains amounts to 30% of the total world energy consumption. Furthermore, this extensive use of energy triggers elevated levels of carbon emissions, where it has been reported by James and James (2010) that the cold chain is held accountable for almost 1% of the world’s greenhouse gas emissions. The intensive energy usage, and accordingly the cost, of cold products logistics operations coupled with their negative environmental impact renders the development of efficient and sustainable replenishment policies of such products an inevitable necessity. A bad selection for the number and capacity of temperature-controlled transport and storage facilities are needed to handle them making this industry highly energy intensive. In fact, Kayfeci, Kecegasus, and Gedik (2013) pointed out that energy consumption in cold chains amounts to 30% of the total world energy consumption. Furthermore, this extensive use of energy triggers elevated levels of carbon emissions, where it has been reported by James and James (2010) that the cold chain is held accountable for almost 1% of

| Reference          | Supply chain configuration | Market demand | Planning horizon | No. of Items | Transp. cost structure | Limited Vehicle capacity | Carbon regulatory policy |
|--------------------|----------------------------|---------------|------------------|-------------|------------------------|--------------------------|--------------------------|
| Bonney and Jaber   | Single echelon             | C: Constant   | √                | √           | √                      | Simplified               | Emission cost            |
| Wahab et al.       | SV-SB                      | √             | √                | √           | Simplified              | Emission cost            |
| Hsu et al.         | Single echelon             | √             | √                | √           | None                   | Cap & trade              |
| Bouchery et al.    | Two echelon                | √             | √                | √           | None                   | Carbon tax               |
| Absi et al.        | Single echelon             | √             | √                | √           | None                   | Carbon cap               |
| Arslan and Turkay  | Single echelon             | √             | √                | √           | None                   | Four policies            |
| Benjaafar et al.   | Single echelon             | √             | √                | √           | None                   | Four policies            |
| Chen et al.        | Single echelon             | √             | √                | √           | None                   | Four policies            |
| Battini et al.     | Single echelon             | √             | √                | √           | Fixed Var.             | Emission cost            |
| Bazan et al.       | Single echelon             | √             | √                | √           | Fixed                  | Carbon tax               |
| Andriolo et al.    | Single echelon             | √             | √                | √           | Fixed Var.             | Emission cost            |
| Absi et al.        | Single echelon             | √             | √                | √           | Fixed Var.             | Carbon cap               |
| Cheng, Qi, Wang,   | Two echelon                | √             | √                | √           | Fixed Var.             | Four policies            |
| and Zhang          | Multi echelon              | √             | √                | √           | Simplified              | Four policies            |
| Mohammed et al.    | Multi echelon              | √             | √                | √           | Simplified              | Four policies            |
| Ghosh et al.       | SV-SB                      | √             | √                | √           | None                   | Carbon cap               |
| Li, Su, and Ma     | Two echelon                | √             | √                | √           | Simplified              | Carbon tax               |
| (2017)             |                            |               |                  |             |                       |                          |
| Lee et al.         | Single echelon             | √             | √                | √           | Fixed Var.             | Cap & trade              |
| Battini et al.     | Single echelon             | √             | √                | √           | Fixed Var.             | Cap & trade              |
| Shuang et al.      | Multi echelon              | √             | √                | √           | Fixed Var.             | Cap & trade              |
| Tao and Xu         | Single echelon             | √             | √                | √           | None                   | Carbon tax, Cap & trade  |

Table 1: A classification of the relevant sustainable inventory models for non-perishable products.
exhibiting a discrete time varying demand over a finite planning horizon. As modular temperature-controlled units are used to store and transport such products, the carbon footprint generated due to these activities is taken into account through the carbon cap regulatory policy. The cap is firstly set on the total emissions throughout the planning horizon and then a stricter version is considered with the cap being imposed on each period’s emissions. For each of the two cases, a mixed integer linear programming (MILP) model is developed along with a solution algorithm that yields the optimal ordering policy. To the authors’ best knowledge, this work is the first to tackle this problem in the context of cold products using temperature-controlled transportation trucks and warehousing facilities while accounting for the products limited shelf life aspect.

The remainder of this paper is organized as follows. Section 2 presents a review of the state-of-the-art literature pertaining to the problem at hand, and highlights the contributions of this work. Section 3 provides the mathematical models along with the proposed solution algorithms. The impact of the carbon cap (both total and periodic) as well as other key problem parameters on the lot sizing policy, the operational cost and the carbon footprint generated is assessed in Section 4 through sensitivity analysis. Concluding remarks in addition to suggestions for future research avenues are presented in Section 5. Lastly, the detailed derivation of the economic and environmental objective functions coefficients along with illustrative examples for the total and periodic cap cases are shown in the appendices.

2. Relevant literature

As the work presented herein tackles sustainable lot sizing problems for cold/perishable products with limited shelf life considerations, the state-of-the-art review of the relevant literature is divided into two parts: (1) Environmentally-conscious inventory models for non-perishable products, and (2) Inventory models for perishables with particular emphasis on those accounting for shelf life limitations.

2.1. Environmentally-conscious lot sizing models for non-perishables

While economic goals have long been the dominating performance measure for most inventory models, several research works have also incorporated environmental aspects to better align with stringent government regulations on carbon emissions and the elevated public awareness of sustainability related matters. Moreover, recent supply chain literature alongside emerging practices recognize that significant cost savings and carbon emissions reductions can be achieved via better-coordinated inventory and transportation related decisions. Therefore, different carbon emissions related constraints have been integrated with conventional operational constraints such as demand, supply, capacity, and inventory balance constraints. Similarly, the objective function has also been reformulated with the inclusion of total carbon emissions minimization (Chiranjit & Jharkharia, 2019). The aforementioned carbon regulatory policies may impose restrictions on an organization’s day-to-day operations, ultimately forcing such organizations to make adjustments to their operational strategy so that it better adheres to these regulations. In this section, we highlight the relevant inventory models incorporating carbon emissions, with a special emphasis on the regulatory policies of carbon tax and carbon cap. Table 1 provides a classification of the existing literature pertaining to sustainable inventory models based on several dimensions, which helps better situation the present work and highlight its contributions. The transportation cost structure column displays what structure is adopted (if any), where (fixed + var.) indicates that the authors explicitly accounted for the fixed cost per truck and the variable cost calculated on a per unit of the product and/or unit distance traveled. It can be noted from Table 1 that the majority of these models address single echelon environments and assume a single product setting.

Several research works have investigated the role that the inclusion of carbon emissions cost in the objective function plays towards curbing the carbon footprint levels under various operational settings. One of the earliest such works is that of Bonney and Jaber (2011) who devised a mathematical model that considers environmental aspects via minimizing the emissions social cost alongside other operational costs. The authors assumed a “simplified” structure for the transportation cost which solely depends on the number of orders/trips made and the distance traveled while overlooking relevant aspects such as trucks’ capacities, number of trucks utilized as well as the load and speed of these trucks. A similar structure for the transportation cost was also adopted by Wahab, Mamun, and Ongkunuruk (2011) but in the context of a single-vendor single-buyer (SV-SB) integrated inventory system. Bouchery, Ghaffari, Jemai, and Dallery (2012) presented a sustainable Economic Order Quantity (EOQ) model which jointly optimizes the cost as well as a set of sustainability criteria for the single echelon case. They later extended the work to address a two echelon serial system comprised of a warehouse and a retailer. Along the same lines, Battini, Persona, and Sgarbossa (2014) put forward a sustainable EOQ model that considers multiple transportation modes. In their work, the emissions generated due to truck transportation were assumed to be a function of the number of trucks utilized and the load factor of each truck. Through a multi-objective EOQ model, Andriolo, Battini, Persona, and Sgarbossa (2015) illustrated the economic and emissions reduction benefits realized when two buyers cooperate to share transportation paths and handling units. All of the above works have considered the cost of carbon emissions in the context of the EOQ model which assumes a deterministic and constant demand and an infinite planning horizon.

Other works have considered the strict carbon cap regulatory policy, which prevents firms from violating a predetermined level of carbon emissions. For instance, Chen, Benjaafar, and Elomri (2013) studied the EOQ model under the carbon cap policy and provided conditions under which the relative reduction in emissions is greater than the relative increase in cost. In another work, Hoen, Tan, and Fransoo (2014) studied the trade-off between lead time, inventory costs, and transportation costs while applying a cap to reduce carbon footprint. Konur (2014) analyzed an integrated inventory control and transportation problem while accounting for environmental aspects through a carbon cap constraint on the total emissions. He jointly addressed transportation and inventory control decisions to better capture per truck costs and truck capacities. Abi et al. (2013) considered a sustainable version of the dynamic lot sizing problem (DLSP) for a single item under the carbon cap scheme. In a follow-up work, Abi, Dauzère-Pérès, Kedad-Sidhoum, Penz, and Rapine (2016) extended their former work to account for different supplying modes with fixed carbon emissions associated with each mode. In a relevant work, Helmrich, Jans, van den Heuvel, and Wagelmans (2015) extended the work of Abi et al. (2013) where a more general function for the emissions associated with setups and inventory holding is adopted. They showed that the problem is NP-hard and subsequently developed a Lagrange relaxation based approach to obtain a lower bound to the problem. Ghosh, Jha, and Sarmah (2017) proposed a cost minimization model for a two-echelon supply chain under stochastic demand assumption with allowed partial backorders while incorporating a carbon emission constraint. They assessed the impact of the imposed carbon cap policy on the optimal order quantity, reorder point, and number of shipments.

Another stream of research works were devoted to comparing the lot sizing strategy and the supply chain performance under the four different policies or a subset of them (see carbon regulatory policy column in Table 1). For instance, He, Wang, and Wang (2012) assessed the effectiveness and efficiency of the cap-and-trade and carbon tax policies with respect to seven criteria, including the average emission price and the actual emission. Hua, Cheng, and Wang (2011) employed the EOQ model to examine the impacts of carbon cap and carbon price on the order size and the associated carbon emissions. Benjaafar et al. (2013) modeled the four regulatory policies to study the impact of carbon emissions on the dynamic lot sizing problem with the objective of...
| Reference | Supply chain configuration | Perishability nature | Market Demand | No. of items | Limited Shelf life | Transp. cost structure | Trucks capacity | Freezers capacity | Carbon regulatory policy |
|-----------|---------------------------|----------------------|---------------|--------------|-------------------|------------------------|----------------|-------------------|--------------------------|
| Hsu (2000) | Single echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Hsu (2003) | Single echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Kanchanasuntorn and Techanitisawad (2006) | Two echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Olsson and Tydén (2010) | Single echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Wu and Zhao (2014) | SV-SB                    | √                    |               |              |                   |                        |                |                   |                          |
| Soysal et al. (2014) | Multi echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Bozorgi et al. (2014) | Single echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Önal et al. (2015) | Single echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Bozorgi (2016) | Single echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Muriana (2016) | Single echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Önal (2016) | Two echelon              | √                    |               |              |                   |                        |                |                   |                          |
| Pauls-Worm et al. (2016) | Single echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Sarvar et al. (2016) | Single echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Sargut and Işık (2017) | Single echelon            | √                    |               |              |                   |                        |                |                   |                          |
| Hariga et al. (2017) | Multi echelon            | √                    |               |              |                   |                        |                |                   | Carbon tax              |
| Tiwari, Daryanto, and Wee (2018) | SV-SB                    | √                    |               |              |                   |                        |                |                   | Carbon tax              |
| Jansen et al. (2018) | Single echelon            | √                    |               |              |                   |                        |                |                   | Simplified               |
| Shamayleh et al. (2019) | Single echelon            | √                    |               |              |                   |                        |                |                   | Carbon tax              |
| Babagolzadeh et al. (2020) | SV-MB                    | √                    |               |              |                   |                        |                |                   | Carbon tax              |
| Al Theeb et al. (2020) | Two echelon              | √                    |               |              |                   |                        |                |                   | Carbon tax              |
| This paper | Single echelon            | √                    |               |              |                   |                        |                |                   | Carbon cap              |
minimizing the total costs of holding, ordering and backordering. They used the proposed models to study the extent to which carbon reduction requirements can be addressed by operational adjustments, as an alternative or a supplement to costly investments in carbon-reducing technologies. They also discussed the impact of collaboration on supply chain members’ costs and carbon emissions. Tüftel, Özli, and Konur (2014) analyzed a retailer’s joint decisions on inventory replenishment and carbon emission reduction. They extended the basic EOQ model to consider carbon emissions reduction investment availability under carbon cap, carbon tax, and cap-and-trade policies. Under the assumption of a constant demand, Hariga, Babekian, and Bahroun (2019) presented an integrated economic and environmental model for the two-stage SVSB supply chain under a vendor managed consignment inventory arrangement. They studied the impact of the carbon tax and the carbon cap regulations on the chain-wide total costs and carbon emissions. They concluded that the generated carbon footprint can be drastically reduced – without significantly increasing the operational costs – via adjusting the vendor’s production and buyer’s delivery quantities. Recently, Shuang et al. (2019) considered the carbon tax and the cap-and-trade regulations with transportation fleets while allowing for lost sales to take place. The objective of their non-linear mathematical model is to select the optimal carbon control policy with optimal production, inventory, and delivery quantities. Their results showed that the selected carbon policy may significantly affect the supply chain performance with carbon price being the key parameter. A more comprehensive discussion on sustainable inventory models incorporating carbon emission considerations are provided in the review works of Tang and Zhou (2012), Memari, Abdul Rahim, Ahmad, and Hassan (2016), Das and Jarkharia (2018), and Chelly et al. (2019).

While these works account for environmental concerns when tackling lot sizing and/or transportation decisions, they do so in the context of non-perishable products. As such, they overlook practical aspects governing the logistical operations of perishables, such as limited lifetime and the use of temperature-controlled transport and storage facilities. As noted in the review paper of Vrat, Gupta, Bhatnagar, Pathak, and Pulzele (2018), very few papers have considered sustainability aspects in the transportation of perishable products. This work enriches the existing literature where it jointly optimizes the lot-sizing and transportation decisions for cold products whilst incorporating relevant operational and environmental restrictions as an integral part of the decision making process.

2.2. Inventory models for cold/perishable products

Perishability is an important aspect characterizing a wide range of products. Due to their widespread applicability, an increasing body of literature has been devoted to modeling the logistical operations of these products while explicitly accounting for their perishable nature. As noted by Janssen, Claus, and Sauer (2016), the development of inventory models for perishable goods began in the 1960s with 17 review papers on the topic published by the end of 2015. However, when considering perishability, it is particularly important to point out that the literature distinguishes between two major categories, namely decay models and finite lifetime models, with the latter being further classified into fixed and random finite lifetime models (Kanchanasuntorn & Techanitisawad, 2006). Along the same lines, Pahl and Voß (2014) clearly highlighted the differences between decay/deterioration and perishability, and noted that these terms are used interchangeably in the literature. They pointed out that deteriorating items continuously lose their utility/value on a gradual basis over time, while perishable ones are considered no longer valid and lose all their utility at once after a certain point in time (shelf life). Table 2 provides a characterization of the relevant research works based on several dimensions where the products are classified according to their nature as perishable (those with limited shelf life consideration), deteriorating, and cold products (those requiring modular temperature-controlled transportation and storage units).

A stream of researchers addressed the lot sizing problem of perishable products under the assumption of dynamic demand, including Soysal, Bloemhof-Ruwaard, and van der Vorst (2014), Pires, Amorim, Martins, and Almada-Lobo (2015), Onal, Romeijn, Sapra, and van den Heuvel (2015), Onal (2016), Pauls-Worm, Hendrix, Alcoba, and Haijema (2016) and Sazvar, Mirzapour Al-e-hashem, Govindan, and Bahli (2016) as seen in Table 2. Soysal et al. (2014) put forward a sustainable optimization model for the network logistics problem of a beef supply chain, with cost and emission minimization as the two objectives. Pires et al. (2015) developed a mixed integer programming model for planning the production of multiple perishable products having a fixed shelf life while adopting a last-expired first-out (LEFO) consumption order. They further extended the model to account for the value of food freshness and age dependent demand. In another work, Onal et al. (2015) also considered the lot sizing problem of a perishable product with a deterministic expiration date. However, besides the LEFO consumption order, they also considered first-expired first-out (FEPO), first-in first-out (FIFO) and last-in first-out (LIFO) patterns. A dynamic programming (DP) based approach was developed to attain the optimal solutions for all consumption orders within polynomial solution time. While this latter work addressed single stage systems, Onal (2016) tackled the two-stage single-product lot sizing problem with fixed shelf life, proved the problem to be NP-hard, and presented special cases solvable within polynomial time. Following the FEPO consumption order, Sazvar et al. (2016) developed the replenishment strategy for multiple perishable products with each product having a specific expiry date while allowing shortages to take place. Pauls-Worm et al. (2016) studied the production planning problem of food items having a fixed shelf life, long production lead times and non-stationary stochastic demand. Under the FIFO consumption policy, a mixed integer programming model is developed to decide on the optimal lot sizing strategy while satisfying a fill rate constraint. Other works tackled lot sizing problems for perishable products of fixed shelf life under the assumption of a stochastic demand, including Kanchanasuntorn and Techanitisawad (2006), Olsson and Tydesjö (2010), Muriana (2016), Chua, Mokhlesi, and Sainathan (2017) and Janssen, Diabat, Sauer, and Herrmann (2018). For a more comprehensive synopsis of inventory models for perishable products, interested readers are referred to the review works of Pahl and Voß (2014) and Chaudhary, Kulshrestha, and Routroy (2018).

In case of cold supply chains, accounting for carbon emissions is even more critical, which is partly attributed to the associated high energy costs, negative environmental impacts and the stipulated stringent government regulations. For instance, Meneghetti and Monti (2015) pointed out that refrigeration accounts for a major portion of energy consumption and carbon emissions in fresh food supply chains. While cold products typically exhibit either a decay or perishability, the works listed hereafter differ from the ones mentioned above in the sense that they specifically incorporate the distinguishing peculiarities characterizing the logistical operations of cold products such as the use of limited-capacity temperature-monitored trucks and storage freezers, while mostly accounting for the generated carbon emissions. For instance, under the assumption of a constant demand, Bozorgi, Pazour, and Nazzal (2014) developed a single-period single-product model that jointly minimizes the operational cost as well as carbon footprint generated due to transportation and storage activities. A set of exact solution algorithms were proposed to find the optimal order quantity while assuming that the operational status of the freezers does not change throughout the period. Bozorgi (2016) extended the previous work to address the case of multiple cold products where products were grouped into families based on their compatibility. Following the carbon tax policy, Hariga, As’ad and Shamayleh (2017) assessed the impact of accounting for carbon emissions, resulting from transportation and storage activities, in the context of a three-stage cold supply chain under the assumptions of constant demand and infinite planning horizon. They
concluded that the incorporation of carbon-related costs, through carbon tax regulation, may call for adjusting the adopted operational policy resulting in a minor increase in the operational cost which is outweighed by the savings generated from carbon-related cost. In a related work, Shamayleh, Hariga, As’ad and Diabat (2019) proposed a model to optimize the replenishment strategy for a firm facing a discrete time-varying demand for a cold product under carbon tax policy. The sensitivity analysis results showed that higher cost savings and carbon reductions are realized for increasing values of the carbon tax rate. More recently, Babagolzadeh et al. (2020) extended the work of Hariga et al. (2017) to address a two-stage single-supplier multiple-retailers supply chain with the retailers facing a stochastic demand. The authors jointly optimize the inventory replenishment decisions alongside the routing of the vehicles among the retailers under the carbon tax policy. While overlooking the environmental perspective, Al Theeb, Smadi, Al-Hawari, and Aljarrah (2020) tackled the pure cost-minimizing inventory routing problem (IRP) of multi-stage multi-product cold supply chain where split deliveries were allowed. Further readings on works addressing various aspects of cold/perishable products logistics while accounting for environmental considerations are found in the review papers of Xu, Sunab, Zenga, and Liua (2015), Zhu et al. (2018) and Vrat et al. (2018).

2.3. Research contributions

For perishable products in general, accounting for shelf life aspect is of practical relevance as it directly affects the replenishment strategy and the associated carbon emissions as well as the potential waste generated and ultimately a firm’s profitability. Despite its paramount importance, it is noted in the review paper of Pahl and Vo (2014) that little research has been done in the modeling of lifetime restrictions to prevent wastage and disposal of perishable products, especially in a dynamic planning context. As cold products are usually of a perishable nature, this work contributes to such deficiency through addressing the lot-sizing policy of a cold product facing time varying demand while explicitly considering environmental and limited lifetime aspects. Stated more formally, we next highlight the three main contributions of this paper.

(1) A variant of the classic dynamic lot sizing problem for cold items is addressed while accounting for practical considerations, such as limited shelf life constraints and trucks and freezers capacity as well as the generated carbon emissions, through a carbon cap constraint (either on the total emissions or on the periodic emissions). Moreover, this work mimics real life practices in many retailing outlets whereby the cold storage area is partitioned into compartments (freezer units) and the number of such units to be switched on/off is optimized on a periodic basis depending on the amount of on-hand inventory. To the best of the authors’ knowledge, none of the existing research works have tackled this problem under the same settings in the context of cold products.

(2) It is known that Lagrangian relaxation approach provides lower bounds to complex minimization problems. As such, this approach is typically used to generate approximate (near-optimal) solutions. However, as shown in this paper, the novelty in the proposed hybrid Lagrangian relaxation and Bisection based solution procedure is that it generates the optimal solution to the optimization problem when a carbon cap is imposed on the total carbon emitted over the planning horizon. In addition, based on the optimal value of the Lagrangian multiplier, we present the policymakers with a novel idea to choose between the carbon tax and the total carbon cap policy. Unlike the work of Shuang et al. (2019) where the authors defined a binary variable to indicate the choice of a specific policy, we instead utilize the Lagrangian multiplier to assess the economic and environmental performance of these two policies and determine for what range of the tax rate one would outperform the other.

(3) Despite the existence of additional constraints that govern the logistics of cold products, such as the finite capacity of the transport and storage facilities and the limited shelf life, the work presented in this paper is the first to extend some of the established results for non-perishable items and illustrate their validity for cold products as well. More specifically, via making operational adjustments to the lot sizing strategy, it might be possible to substantially reduce carbon emissions without significantly compromising the operational cost (a similar result was reported by Chen et al. (2013) and Hariga et al. (2019) for non-perishables). Also, paradoxical increases in the total emissions are sometimes realized with slightly tighter values of the periodic cap (a similar result was obtained by Benjaafar et al. (2013) for non-perishables).

3. Mathematical models and solution procedures

Consider a cold product facing known time-varying demand over a discrete planning horizon of length $T$ periods. Due to its perishable nature, the cold product has a pre-determined shelf life and requires refrigerated trucks for its transportation and modular temperature-controlled storage units (freezers) for its storage at the retailer’s facility. As in the work of Muriana (2016), shelf life herein denotes the remaining portion of the product total lifetime as they enter the retailing stage. The planning horizon is discretized into equally spaced periods, where the demand is satisfied throughout the period and replenishments (if any) are to be received at the beginning of the period. Although a homogenous fleet of refrigerated trucks (reefers) is used to transport the cold product to the retailer, the calculations of the transportation cost presented herein, as well as the carbon emissions, take into account whether the truck is fully loaded to its maximum capacity $R$, which is better aligned with real life practices. In a similar fashion, the storage area at the retailer’s facility is partitioned into a number of identical-sized temperature-controlled compartments (freezers) each having a capacity of $U$ units. This ensures better energy utilization along with a reduction in the resulting carbon emissions through providing the retailer the flexibility to alter the operational status of those freezers on a periodic basis depending on the on-hand inventory levels. For each period, $t$, the optimization problem is then to determine the retailer’s ordering quantity, $Q_t$, the number of refrigerated trucks, $n_t$, and the number of freezers to be operated, $n_t$, that minimize the sum of ordering, transportation, holding, and freezers’ operational costs subject to the imposed restriction on the amount of carbon emissions due to storage and transportation activities. In this paper, we consider the two cases where the carbon cap is imposed on the amount of carbon emitted over the entire planning horizon and on the carbon generated during each period of the planning horizon.

Following are the assumptions underlying the mathematical models developed in this paper:

(1) Orders are delivered only at the beginning of the periods.

(2) Ideal transportation and storage conditions are always maintained which prohibits the deterioration of the cold product during those activities.

(3) The cold product is perishable in the sense that it loses its value/utility all at once upon exceeding the fixed shelf life (see Pahl and Vo (2014) for a more detailed discussion). Accordingly, a quantity delivered at the beginning of period $t$ can only be held in stock for a limited number of periods equal to its shelf life, $SL$.

(4) Demand fulfillment follows a first-in first-out (FIFO) stock outflow pattern. In this case, ending inventory left from an order delivered at the beginning of period $t$, is $g$ periods old at the end of period $(t + g - 1)$, where $g$ is less or equal to $SL$. 
(5) The lead time associated with shipment delivery from the supplier is negligible.

(6) Demand backordering is not permitted, and holding cost is charged against inventory at the end of each period.

(7) The operational status of a freezer unit does not change during the period, and a freezer is switched off only when it is completely emptied.

(8) There is no restriction on the number of reefer and freezers available to transport and store the cold product, respectively.

(9) The adopted carbon regulatory policy is carbon cap, which sets a limit on the amount of carbon footprint generated from the transportation and storage activities.

(10) Without loss of generality, the initial (ending) inventory at the beginning (end) of the planning horizon is set equal to zero.

The following main notations are adopted in the development of the mathematical models. More notations will be introduced as needed.

**Problem parameters:**
- \( h \): Unit storage cost of the cold product for one period, excluding energy consumption cost.
- \( A \): Fixed ordering cost.
- \( SL \): Shelf life of the cold product.
- \( U \): Freezer storage capacity.
- \( R \): Full truckload (FTL) capacity of one truck.
- \( \hat{D} \): Demand for the cold product in period \( t \).
- \( \hat{y} \): The smallest integer larger than or equal to \( x \).
- \( \hat{x} \): The largest integer smaller than or equal to \( x \).

**Decision variables:**
- \( b_t \): Inventory level at the beginning of period \( t \).
- \( e_t \): Inventory level at the end of period \( t \).
- \( Q_t \): Ordering quantity in period \( t \).
- \( m_t \): Number of refrigerated trucks needed to transport \( Q_t \) units of the cold product in period \( t \).
- \( n_t \): Number of freezers in the operating mode during period \( t \).
- \( y_t \): Binary variable, equals to one when an order is placed and received at the beginning of period \( t \) and zero otherwise.

Towards curbing carbon emissions, the carbon cap regulation has proven to be an effective regulatory policy that is gaining wider acceptance nowadays. Following such policy, a cap is set on the maximum carbon footprint generated by an organization where this cap may span the entire planning horizon (e.g., a year) or could simply be imposed on a shorter-range periodic amounts of carbon emissions (e.g., a month), with the latter being obviously a more strict version of the former. In this paper, we address both cases where the following subsection presents the mathematical formulation and the solution algorithm for the case of a cap being defined for the entire planning horizon whilst the next subsection tackles the shorter interval caps. For a better organization of the paper, the derivation details of the mathematical expressions of the coefficients for the total operational cost function and total carbon footprint constraints are delegated to Appendix A. Furthermore, illustrative examples on the two optimization problems, with total and periodic caps, are shown in Appendices B and C, respectively.

### 3.1 Imposed carbon cap on the emissions over the entire planning horizon

Using the expressions for the costs associated with the reefer and freezers operations derived in Appendix A, the operational cost incurred during period \( t \) is:

\[
OC_t = Ay_t + hE_t + CQ^*Q_t + TR^*m_t + FR^*n_t
\]

where the first two terms are the ordering and holding costs incurred during period \( t \), respectively. The sum of next two terms represents the transportation cost and the last term is the freezers’ operational cost. Accordingly, the total operational cost over the planning horizon (TOC) is given by:

\[
TOC = \sum_{t=1}^{T} [A^*y_t + h^*E_t + CQ^*Q_t + TR^*m_t + FR^*n_t]
\]

The total carbon footprint over the entire planning horizon (TCF) is directly attainable from Eq. (A7) and it is given by:

\[
TCF = \sum_{t=1}^{T} CF_t = \sum_{t=1}^{T} [\overline{CQ}Q_t + \overline{TR}m_t + \overline{FR}n_t]
\]

Therefore, the constraint stipulating that the allowable carbon cap over the planning horizon is not exceeded would be given by:

\[
\sum_{t=1}^{T} [\overline{CQ}Q_t + \overline{TR}m_t + \overline{FR}n_t] \leq C
\]

In the following, an ordering policy is defined by the vector \( Q = (Q_1, Q_2, \ldots, Q_T) \) in the sense that once \( Q \) is determined then the remaining decision variables can be found using:

\[
y_t = \begin{cases} 1 & \text{if } Q_t > 0 \\ 0 & \text{if } Q_t = 0 \end{cases} \quad t = 1, 2, \ldots, T
\]

\[
E_t = \sum_{j=1}^{T} (Q_i - D_i) \quad t = 1, 2, \ldots, T
\]

\[
m_t = \left[ \frac{Q_t}{R} \right] \quad t = 1, 2, \ldots, T
\]

\[
n_t = \left[ \frac{E_{t-1} + Q_t}{U} \right] \quad t = 1, 2, \ldots, T
\]

Hence, the operational cost optimization problem with carbon emission constraint is given by:

\[
\text{(CAP1)}: \quad Z(0) = \text{Minimize } TOC(Q) = \sum_{t=1}^{T} [A^*y_t + h^*E_t + CQ^*Q_t + TR^*m_t + FR^*n_t]
\]

Subject to:

\[
TCF(Q) = \sum_{t=1}^{T} [\overline{CQ}Q_t + \overline{TR}m_t + \overline{FR}n_t] \leq C
\]

\[
E_{t-1} + Q_t - E_t - D_t = 0, \quad t = 1, 2, \ldots, T
\]

\[
Q_t \leq m_t R \quad t = 1, 2, \ldots, T
\]

\[
E_{t-1} + Q_t \leq n_t U \quad t = 1, 2, \ldots, T
\]

\[
Q_t \leq \hat{y}_t \sum_{j=1}^{T} D_j \quad t = 1, 2, \ldots, T
\]

\[
E_{t-1} + Q_t \leq \sum_{j=t-SL}^{T} D_j \quad t = 1, 2, \ldots, T - SL
\]

\[
E_t \geq 0 \quad t = 1, 2, \ldots, T
\]

\[
y_t \in \{0, 1\} \quad t = 1, 2, \ldots, T
\]

\[
E_0 = E_T = 0
\]

\[
m_t, n_t \in N^+ \quad t = 1, 2, \ldots, T
\]

Eq. (3) represent the classical inventory balance constraints, while constraints (4) establish the number of trucks needed to transport the lot size of that period. The number of operational freezers in a particular period is dictated by the beginning inventory as seen in constraint set
are the shelf life constraints, which mandate that the inventory at the beginning of a period does not exceed the demand for SL periods including that own period’s demand. The remaining constraints represent the non-negativity, binary and integrality restrictions on the respective decision variables. We next provide some insights leading to the development of an optimal solution algorithm to problem (CAP$_T$).

Feasibility assumption: The carbon cap should satisfy the condition $C_{\text{min}} \leq C \leq C_{\text{max}}$.

The lower bound carbon cap, $C_{\text{cap}}$, is the minimum carbon footprint over the entire planning horizon obtained by solving the following carbon cap-unconstrained environmental optimization problem, since its objective function is to minimize the total carbon emitted over the planning horizon:

\[
\begin{align*}
\text{Cap}_{\text{min}}: & \\
\text{Min } & \text{TOC}(Q) \\
\text{Subject to constraints } & (3)-(11)
\end{align*}
\]

The upper bound, $C_{\text{max}}$, is the total carbon emissions of the optimal lot sizing policy of the following carbon cap-unconstrained operational cost optimization problem.

\[
\begin{align*}
\text{Cap}_{\text{max}}: & \\
\text{Min } & \text{TOC}(Q) \\
\text{Subject to constraints } & (3)-(11)
\end{align*}
\]

In order to solve the optimization problem (CAP$_T$), the carbon cap constraint (2) is relaxed by augmenting it to the objective function via the use of a Lagrangian multiplier ($\lambda > 0$). The Lagrangian relaxation problem is then:

\[
\begin{align*}
\text{Z}(\lambda) = \text{Minimize} & \{ \text{TOC}(Q) + \lambda [\text{TCF}(Q) - C] \} \\
\text{Subject to constraints } & (3)-(11)
\end{align*}
\]

For a given $\lambda$, the relaxed optimization problem (CAP$_T(\lambda)$) can be solved optimally using the dynamic programming based solution procedure proposed in Shamayleh et al. (2019). The objective function of the dual problem to (CAP$_T(\lambda)$) is then:

\[
\begin{align*}
\text{maxZ}(\lambda) & \quad \substack{\text{Subject to } \lambda \geq 0 \text{ and } Q \in S(\lambda))} \\
\end{align*}
\]

Given the integrality of the demand figures and the lot sizes, the set of feasible solutions to constraints (3)-(11) is finite. We denote this set of feasible $Q$’s by $S = (Q^1, Q^2, \ldots, Q^K)$, where $K$ is the number of feasible solutions. In this case, the objective function of the dual problem can be rewritten as:

\[
\begin{align*}
\text{maxZ}(\lambda) & = \max_{\lambda \geq 0} \min_{k=1,2,\ldots,K} \{ \text{TOC}(Q^k) + \lambda [\text{TCF}(Q^k) - C] \} \\
\end{align*}
\]

The objective function of (D) is, therefore, a piecewise linear concave function of $\lambda$. The dual function is shown in Fig. 1 using the data of the illustrative example presented in Appendix B. Note that the function $Z(\lambda)$ has a finite number of breakpoints where it is not differentiable, and between each two consecutive breakpoints the functions TOC(Q) and TCF(Q) remain constant.

We show in the following Lemma that TOC(Q) and TCF(Q) are non-decreasing and non-increasing functions of $\lambda$, respectively.

Lemma 1. (i) TOC(Q) is a non-decreasing function of $\lambda$.

(ii) TCF(Q) is a non-increasing function of $\lambda$.

Proof. Let Q(0) be the optimal ordering policy for the unconstrained carbon cap problem (Cap$_{\text{max}}$). Clearly, this unconstrained ordering policy has the least total operational cost, TOC(Q(0)) and the largest total carbon footprint, TCF(Q(0)). Therefore, for a given positive $\lambda$, the total operational costs, TOC(Q(\lambda)), of the optimal ordering policy Q(\lambda) to the Lagrangian relaxation problem (CAP$_T(\lambda)$) cannot be smaller that TOC(Q(0)). Similarly, it can be seen that TCF(Q(\lambda)) ≤ TCF(Q(0)). Next, consider the optimal solution Q(\lambda') to the Lagrangian relaxation problem (CAP$_T(\lambda')$), with $\lambda' = \lambda + \Delta \geq \lambda$. In its expanded form, the objective function of (CAP$_T(\lambda')$) is:

which can be rewritten as:
Lemma 2. Let \( t_e \) be the tax rate charged for each ton of carbon emitted. If \( t_e < \lambda^* (t_e > \lambda^*) \), then

a. Total carbon emitted under carbon tax policy is larger (smaller) than the one generated by the carbon cap policy.

b. The total operational cost of the carbon tax policy is smaller (larger) than the one of the carbon cap policy.

Proof. The proof is straightforward by referring to Fig. 1.

3.2. Imposed carbon cap on the emissions over each period of the planning horizon

In case the cap is imposed on the amount of carbon emitted over each period of the planning horizon, the carbon cap constraints (2) of the optimization problem (CAP_1) are replaced by:

\[
CF_t = \bar{FR}^* n_t + \bar{FR}^* Q_t + \bar{CQ}^* Q_t \leq CP \quad \text{for } t = 1, 2, \ldots, T
\]

where \( \bar{FR} \) and \( \bar{CQ} \) are given in equations (A9) and (A11).

The optimization problem to be solved is then:

(\text{CAP}_3)

Minimize \( TOC(Q) = \sum_{t=1}^{T} [A_{ij} + h_i Q_t + CQ^* Q_t + \bar{FR}^* n_t] \)

Subject to:

Constraints (3)–(11) and (15)

Feasibility assumption:

The carbon cap per period, \( CP \), should satisfy the condition:

\( CP_{\min} \leq CP \leq CP_{\max} \)

The periodic upper carbon cap is the maximum carbon emitted per period over the entire planning horizon when following the optimal lot sizing policy of the unconstrained carbon cap optimization problem, i.e., \( CP_{\max} = \max(CP_t, t = 1, 2, \ldots, T) \). On the other hand, the periodic lower carbon cap, \( CP_{\min} \), is the optimal objective function value of the following Min-Max optimization problem:

\( CP_{\min} \)

Min X

Subject to

\( X \geq \bar{FR}^* n_t + \bar{FR}^* Q_t + \bar{CQ}^* Q_t \quad \text{for } t = 1, 2, \ldots, T \)

Constraints (3)–(11)

In order to solve the optimization problem (CAP_{\min}), we propose the following dynamic programming based algorithm. We first rewrite the carbon footprint during period \( t \) as:

\[
CF_t = \bar{FR} \left[ \frac{Q_t}{R} + \bar{FR} \left( \frac{E_{t-1} + Q_t}{U} \right) + \bar{CQ} Q_t \right] \quad \text{for } t = 1, 2, \ldots, T
\]

Next, note that the ordering quantity in a period \( t \) depends only on the beginning inventory in the same period. For example, when \( t = T \), the ordering quantity is equal to \( D_T = E_{t-1} \), since the ending inventory at period \( T \) must be zero. For other periods, the beginning inventory, \( E_{t-1} + Q_{t-1} \), must satisfy:

\[
D_t \leq E_{t-1} + Q_{t-1} \leq \sum_{j=t}^{T-1} D_j \quad t = 1, 2, \ldots, T - SL
\]

Therefore, we define \( E_{t-1} \) as the state variable at period \( t \). We also let \( F_t(E_{t-1}) \) be the minimum of the maximum carbon footprint generated during periods \( t \) through \( T \) when the beginning inventory at period \( t \) is \( E_{t-1} \). The periodic lower carbon cap is then \( F_t(0) \).

For period \( T \), we have

\[
F_T(E_{T-1}) = \min \left[ \bar{FR} \left( \frac{D_T - E_{T-1}}{R} \right) + \bar{FR} \left( \frac{D_T}{U} \right) + \bar{CQ} (D_T - E_{T-1}) \right] \quad \text{for } E_{T-1} \leq 0, 1, \ldots, D_T
\]
On the other hand, for any period $t < T$, the beginning inventory at period $t$ should satisfy Eq. (17).

Note the right-hand-side inequality is due to the product shelf-life constraint. Under these two inequality constraints, the following recursive equation can be used to solve the $T$-period problem for Cap$_{min}$:

$$ F_t(E_{t-1}) = \min \left\{ \begin{array}{l} \max_{D_t \geq 0} \left( \frac{Q_t}{R} + \lambda \right) \\
\max_{0 \leq Q_t \leq \sum_{j=t}^{T-1} D_j} \left( \frac{Q_t}{R} + \lambda \right) \\
+ \tilde{C} \bar{Q} \tilde{F}_t(E_{t-1} + Q_t - D_t) \right\} \text{ for } 0 \leq E_{t-1} \leq \sum_{j=t}^{T-1} D_j \text{ and } 1 < t < T \\
\left( 19 \right) \\
\end{array} \right.$$

and

$$ F_0(0) = \min \left\{ \begin{array}{l} A + TR \left( \frac{Q_0}{R} \right) + FR \left( \frac{Q_0}{U} \right) + C \bar{Q} Q_0, \\
F_1(E_0) \right\} \text{ for } E_0 = E_{t-1} \text{ at the beginning of period } t. $$

$$ \left( 20 \right) $$

In order to solve the optimization problem (Cap$\text{$_t$}$), we can use Lagrangian relaxation method by relaxing the set of constraints in (15). For given Lagrangian multipliers $(\lambda_t : t = 1, 2, \ldots, T)$ associated with constraints (15), the dynamic programming based solution procedure proposed in Shamayleh et al. (2019) can be used to find the optimal lot sizing policy. However, the Bisection method cannot be used to solve the dual problem to (Cap$\text{$_t$}$) $(\lambda_t : t = 1, 2, \ldots, T)$ as we are dealing with more than one Lagrangian multiplier. Instead, other solution procedures, such as the sub-gradient method, have to be used which may not guarantee the attainment of an optimal lot sizing policy.

In the following, we propose a DP based algorithm similar to the one used to solve (Cap$\text{$_min$}$) with the same state variable $E_{t-1}$ to solve the optimization problem (Cap$\text{$_t$}$). Let $K_t(E_{t-1})$ be the minimum total operational costs satisfying the demands and carbon footprint limit during periods $t$ through $T$ when the state variable is $E_{t-1}$ at the beginning of period $t$. The minimum total operational costs over the planning horizon is then $K_0(0)$.

For a given $E_{T-1}$, the operational costs during period $T$ is:

$$ K_T(E_{T-1}) = A + TR \left( \frac{D_T - E_{T-1}}{R} \right) + FR \left( \frac{D_T}{U} \right) + C \bar{Q} \bar{F} (D_T - E_{T-1}) \text{ for } E_{T-1} = 0, 1, \ldots, D_T - 1 $$

$$ \left( 21 \right) $$

$$ K_t(E_{t-1}) = FR \left( \frac{D_t}{U} \right) \text{ for } E_{t-1} = D_t, $$

with $E_{T-1}$ satisfying

$$ TR \left( \frac{D_t - E_{t-1}}{R} \right) + FR \left( \frac{D_t}{U} \right) + C \bar{Q} \bar{F} (D_t - E_{t-1}) \leq CP $$

Next, for any period $t < T$, the beginning inventory should satisfy Eq. (19) and

$$ TR \left( \frac{Q_t}{R} \right) + FR \left( \frac{E_{t-1} + Q_t}{U} \right) + C \bar{Q} \bar{Q} \leq CP $$

Then, the recursive equation to be used is:

$$ K_t(E_{t-1}) = \min \left\{ A + TR \left( \frac{Q_t}{R} \right) + FR \left( \frac{Q_t}{U} \right) + C \bar{Q} \bar{Q} + h(E_{t-1} + Q_t - D_t) \right\} \text{ for } 0 \leq E_{t-1} \leq \sum_{j=t}^{T-1} D_j $$

$$ \left( 22 \right) $$

where $y_t = 1$ if $Q_t > 0$ and 0 otherwise.

### 4. Sensitivity analysis and managerial insights

In this section, we analyze the impact of the main problem parameters, such as the imposed carbon cap, inventory related cost (ordering and holding costs), as well as cooling infrastructure related parameters (driver’s wage, trucks capacity, fuel price and electricity price) on the resulting lot sizing policy for the two optimization problems (Cap$\text{$_t$}$) and (Cap$\text{$_min$}$).

#### 4.1. Impact of the imposed carbon cap

Being a key model parameter, we assess in this section the performance of the two mathematical models under various values of the imposed carbon cap (both total and periodic). The purpose is to draw analytic managerial insights pertaining to the impact of the carbon caps on the resulting lot sizing strategy and the associated operational cost and carbon emissions. The importance of conducting such analysis is that it aids the policymakers in setting appropriate values for these caps to effectively reduce the carbon emissions rather than doing so in a complete ad-hoc manner. From the individual corporations’ perspective, it greatly helps them capitalize on and maneuver within the caps imposed by the legislative entities through adopting the most-cost effective lot sizing and shipping policy that is in accordance with the allowed caps limits. It shall be noted that the analysis carried hereafter is based on the illustrative example presented in the appendix, wherein the chosen values for the carbon cap respect the established limits.

For the optimization problem (Cap$\text{$_t$}$), where the cap is imposed over the entire planning horizon, we examine 10 different values of the cap falling in the feasible range $1.573 \leq C \leq 1.715$ with a step size $\varepsilon = 0.0142$ tons (or 14.2 Kgs). The resulting operational strategy and the associated cost and total emissions are shown in Table 3 and Fig. 2, respectively. For the other optimization problem (Cap$\text{$_min$}$), the feasibility condition stipulates that the periodic cap falls in the range $0.1543 \leq CP \leq 0.2673$. In a similar fashion, we test 10 different values

| Total cap | Number of trucks | Number of freezers | $\varepsilon$ |
|-----------|------------------|-------------------|--------------|
| 1.574-1.64 | 11               | 46                | 794.04       |
| 1.641-1.7146 | 12         | 43                | 498.62       |
| 1.7147-1.715 | 13          | 41                | 0            |

| Table 3 | Results for the optimization problem (Cap$\text{$_t$}$) under different total cap values. |
for the periodic cap with a step size of $\varepsilon = 0.1543$. The obtained operational strategy along with the accompanying total cost and total emissions are reported in Table 4 and Fig. 3, respectively.

Closely examining the tables and figures in this section, one can draw the following insights pertaining to the impact of the imposed carbon caps.

**Insight 1:** The Lagrangian multiplier ($\lambda^*$) for the relaxed model of the optimization problem (CAP$_T$) provides the marginal value for the carbon cap constraint/resource or alternatively its dual price. As such, as the total cap is relaxed (i.e., assumes a higher value), this resource becomes more abundant and each additional ton of the emissions cap becomes less worthy. That is, it can be seen from Table 3 that as the total cap is increased, the optimal $\lambda^*$ value decreases until reaching the upper limit of the feasibility range, or the unconstrained carbon cap policy, at which point the optimal dual price value $\lambda^* = 0$. Besides the important role of the Lagrange multiplier in assessing the performance of the carbon tax and the carbon cap policy (as seen in Lemma 2), it also provides the policymaker with valuable information on the value of each additional ton of emissions cap helping ultimately with the setting of appropriate values for those caps.

**Insight 2:** As the total emission cap becomes less tight, more frequent shipments take place and/or more trucks are utilized in conjunction with fewer number of freezer units turned on (see Table 3). This pattern is justified as the model takes advantage of the increased permits on the...
cap and thus strives to minimize the operational cost, through minimizing the holding and freezers operational costs, at the expense of an increase in the ordering and transportation costs, where the savings in the former two cost components outweigh the increase in the latter two. As can be seen from Fig. 2, a stepwise decrease (increase) in the operational cost (total emissions) is realized for higher values of the emissions cap. Explicitly accounting for the limited trucks and freezers capacities induce such stepwise behavior where the Lagrange multiplier, and accordingly the lot sizing policy, remains the same for a range of the emission cap values.

Insight 3: Through making operational adjustments to the lot sizing strategy, it might be possible to substantially reduce carbon emissions without significantly compromising the operational cost. As can be seen from Table 3 and Fig. 2, when the emission cap is reduced from 1.7147 to 1.7146 tons, this induces a change in the operational strategy, and the order quantities, where one fewer truck and two more freezers are used. This in turn yields a reduction of 4.35% in the carbon emissions with a minor increase of 5.32% in the operational cost. This result is in line with those established in the literature under different problem settings (e.g. Hariga et al., 2017) for carbon tax policy as applied to cold products, and Benjaafar et al. (2013) for carbon cap policy for non-perishables). This clearly illustrates the efficiency of the carbon cap policy and further suggests the adjustment of the operational strategy as a viable and a possibly more economical alternative to curbing carbon emissions instead of investing in costly environmentally-friendly and more energy-efficient technology.

Insight 4: For the more stringent optimization problem with periodic caps (CAP_t), examining Fig. 3 reveals a general stepwise decreasing (increasing) trend in the operational cost (carbon emissions) with higher values of the cap. This is particularly the case for cap values smaller than or equal to 0.253 tons/week. For higher cap values, however, there are exceptions to this trend specifically for carbon emission where paradoxically higher total emissions may take place for tighter caps. For instance, when the periodic cap is set at 0.257 tons/week, this results in total emissions of 1.804 tons over the 12 weeks planning horizon versus 1.8189 tons of emissions for a tighter cap value of 0.246 tons/week. The same can also be noted when going from a cap of 0.2673 down to 0.258 tons/week. This surprising outcome takes place as imposing caps on a period-by-period basis may force a firm to order in bigger quantities (and use more trucks and freezers) in periods preceding those with high demand figures. This happens when ordering at the beginning of that future high demand period would entail an amount of carbon emissions that exceeds the allowed periodic caps. Note that this insight is also in line with Benjaafar et al. (2013) work where they addressed the case of non-perishable products. The authors provide another justification for this phenomenon where they point out that imposing periodic caps on a firm’s emissions could possibly deny the firm the chance to generate more emissions in one period if this allows significantly lower emissions
in future periods. Hence, the analysis conducted in this paper is the first to extend the validity of their results and illustrate their applicability to the case of cold products as well. This is also an important insight as the specifics of implementing the carbon cap policy in reality (total vs. periodic) could potentially lead to significant differences in the operational cost and the carbon footprint on the long run.

4.2. Impact of inventory and cooling infrastructure related parameters

We further assessed the impact of the changes in each of the inventory related cost (ordering and holding costs) as well as cooling infrastructure related parameters (driver’s wage, trucks capacity, fuel price, electricity price) on the lot sizing strategy and the associated operational cost and carbon emissions. To that end, we changed one parameter at a time while keeping the remaining parameters at their base values. In the following, we discuss the results of the sensitivity analysis when conducted for the case of a cap imposed on the emitted carbon over the entire planning horizon. We considered a total carbon cap of 1.7 tons over the 12-week planning horizon as the base value, where the solution of the base case is shown in the fourth column of Table B2. We did not present the results for the second case (periodic cap) as we observed almost the same effect for most of the parameters, with few exceptions, where the justification behind such exceptions is provided at the end of the previous section.

Surprisingly, the increase in the holding cost did not affect the total number of orders placed over the planning horizon. Typically, as the holding cost increases, one would expect the lot size to decrease at the expense of an increase in the number of orders. However, it should be noted here that the total cost represents a tradeoff between the holding and ordering cost on one end, and the trucks and freezers cost on the other end. For smaller lot sizes, there will be a need for more trucks due to more orders and, consequently, larger trucks related costs. Accordingly, in order to minimize the overall cost function, it is more economical to maintain the same number of orders for larger holding cost rates. Additionally, the increase in the holding cost did not have any impact on the total carbon emissions or the number of trucks and freezers used over the planning horizon. This can be explained by the same rational provided above. As can be noted from Fig. 4, increasing the holding cost resulted in an increase in the optimal Lagrangian multiplier value, which implies that the carbon tax policy will result in smaller (larger) total operational cost (carbon footprint) than the carbon cap policy over a wider range of tax values. Such implication is justifiable using the results of Lemma 2.

Turning to the ordering cost, as anticipated, the obtained results indicate that an increase in this cost would lead to a decrease in the number of orders with an accompanying increase in the number of operational freezers (see Fig. 5). Furthermore, increasing the ordering cost triggers a decrease in the optimal value of the Lagrangian

Fig. 6. Impact of the ordering cost on the Lagrange multiplier and the carbon footprint.

Fig. 7. Impact of the driver’s wage on the Lagrange multiplier value ($\lambda$).
multiplier, and an increase in the amount of carbon footprint generated (as seen in Fig. 6). As opposed to the holding cost, while the ordering cost is increased, it prevails over the rest of the cost components (trucks and freezers costs) leading the model to prefer a reduction in the number of orders while naturally utilizing more freezers due to the increase in the lot sizes. The heightened levels of carbon footprint is attributed to better utilization of the trucks capacities through FTL shipments, where the CO₂ emissions are related to the load as explained in Appendix A.

Lastly, the induced reduction in the Lagrangian multiplier results in a smaller range for the tax values for which the carbon tax policy outperforms that of the carbon cap in terms of the operational cost while yielding larger carbon footprint.

As for the driver’s wage, it turns out that increasing this fixed component of the transportation cost leads to smaller λ values (see Fig. 7), while the number of orders, trucks and freezers utilized as well as the associated carbon footprint remain unchanged. This is caused by the fact that the variable fuel based transportation cost outweighs the fixed transportation cost since the former is a key component that contributes to the carbon footprint, which has to be maintained within the imposed cap. Accordingly, all carbon related variables (mt, ni, and TCF(Q)) remain unchanged. For the obtained reduction in the λ value, the same rationale provided above for the ordering cost holds true.

Assessing the effect of varying the capacities of the trucks is particularly important as it aids the decision maker with the selection of the most suitable truck size, whether small, medium or large. It shall be noted here that by changing the truck capacity, we need also to change the gallon fuel consumption per mile and minimum and maximum carbon emissions over the planning horizon (Cmin and Cmax) as we are dealing with different types of trucks. The values of the fuel consumption per mile for an empty truck (GPM0) and a full truckload (GPM) for the different sized trucks are obtained from the work of Hariga et al. (2017). With larger truck capacity, larger lot sizes can be shipped and, therefore, less total number of orders are placed and fewer total number of trucks will be used. As can be noted from Table S, the medium size truck (R = 1125 units) yields the lowest operational cost and CO₂ emissions, and shall thus be selected.

Upon increasing the fuel price (pg), the observed outcome is a decrease in the total carbon footprint as well as the number of trucks and the number of orders whereas the number of freezers increased (see Table 6). Clearly, the reduction in CO₂ emissions is attributed to the use of fewer trucks where this reduction outweighs the additional emissions as a result of using more freezers. In order to minimize the fuel consumption related cost associated with larger values of the fuel price, the number of trucks (and orders) have to be decreased, which is what we observed. Obviously, decreasing the number of orders implies an increase in the lot size and the initial inventory in each period and consequently the number of freezers needed to accommodate them.

Finally, it is noted that varying the electricity price (EP) had no apparent impact on the lot sizing policy as it only affects the freezer related cost where this cost is overshadowed by the other cost components. In principle, as the electricity price increases, one may expect the number of freezers to decrease and accordingly smaller initial inventory and lot sizes are realized. However, such a decrease would imply an increase in the number of trucks and the number of orders, and consequently the trucks and ordering related costs. In order for this not to happen, the number of freezers, trucks, and orders remains the same.

### 5. Conclusion and future research directions

In adherence to external pressures from legislative entities and conscientious customers with regard to carbon emissions, companies are constantly facing the challenge to adopt environmentally friendly practices that aim at reducing their generated carbon footprint. To that end, this paper tackles the lot sizing problem of a temperature-sensitive product having a limited shelf life while accounting for environmental constraints via the carbon cap regulatory policy. We present mathematical models to address the two scenarios where the carbon cap is imposed on the emissions generated throughout the planning horizon or those generated per period. For each case, solution algorithms that yield the optimal ordering strategy are developed. For the first case, a hybrid Lagrangian relaxation and Bisection based solution approach is developed to obtain the optimal ordering policy, after establishing the upper and lower bounds on the total emissions. Through the use of the Lagrangian multiplier, we compare the performance of the carbon tax and the carbon cap policies from both environmental and economic perspectives, which ultimately aids the policymakers in choosing the most effective policy. For the second case, a dynamic programming based solution algorithm is devised which also guarantees the convergence to the optimal lot sizing policy.

Furthermore, a one-way sensitivity analysis is conducted on the key model parameters to assess their impact on the lot sizing policy, and the resulting operational cost and carbon emissions. The results indicate that operational adjustments to the lot sizing strategy may pose as a viable and a more affordable alternative towards reducing carbon emissions as compared to making substantial investments in costly energy-efficient technology. Also, it turned out that making operational adjustments, through modifying the order quantities and accordingly the number of trucks and freezers used, may significantly reduce the carbon footprint generated at the expense of a minor increase in the operational cost. Furthermore, it is noted from the computational analysis that the operational cost (carbon emission) is decreasing (increasing) with higher values of the carbon cap, where the later increase could possibly take place on an intermittent basis for the periodic carbon cap case. The work presented in this paper provides the policymakers or legislative entities with a decision making tool to aid them in setting appropriate values for the carbon caps towards effectively reducing the carbon emissions rather than doing so in a complete ad-hoc manner. From the corporate decision making perspective, once those caps have been set, individual corporations may make use of this work to establish the most cost-effective lot sizing and shipping policy that adheres to the imposed cap limits whilst attaining operational efficiency goals.

The work presented in this paper may be extended in several directions. One may opt to analyze similar problem settings to the one presented herein but for a more involved supply chain structure encompassing multi stages with one or more firm at each stage.

### Table 6: Sensitivity analysis results for the fuel price.

| Fuel price (pg) | Number of orders | Number of trucks | Number of freezers | Total carbon footprint |
|-----------------|------------------|------------------|--------------------|-----------------------|
| 3.5             | 10               | 12               | 43                 | 1.64                  |
| 8               | 9                | 11               | 46                 | 1.584                 |
| 12              | 9                | 11               | 46                 | 1.584                 |
| 20              | 9                | 11               | 46                 | 1.584                 |

### Table 5: Sensitivity analysis results for the truck capacity.

| Truck capacity (R) | GPM₀ | GPM | Cmin | Cmax | Number of orders | Total operational cost | Number of trucks | Number of freezers | Total carbon footprint |
|--------------------|------|-----|------|------|------------------|------------------------|-------------------|---------------------|------------------------|
| 750                | 0.0798 | 0.1008 | 1.573 | 1.715 | 11               | 697.78                | 13                | 41                  | 1.7146                 |
| 1125               | 0.101 | 0.1288 | 1.488 | 1.655 | 10               | 673                   | 10                | 41                  | 1.655                  |
| 1500               | 0.1302 | 0.2142 | 1.772 | 1.982 | 6                | 951.8                 | 6                 | 54                  | 1.772                  |
Similarly, this work tackled a single cold product situation while an interesting extension would be to explore the multi cold product case which further complicates the analysis and renders a more challenging problem to solve. The consideration of other regulatory policies, such as cap-and-trade, or situations wherein the demand is stochastic, rather than deterministic, pose as other promising future research avenues. Finally, our work could be extended to situations of unforeseen disruptions such as the recent COVID-19 outbreak through devising contingency plans that seek to prioritize logistics needs in terms of required storage and transportation capacity against the uncertainties pertaining to the availability and delivery of the items (see Ivanov & Dolgui, 2020). In such times of rising uncertainties, considering the resilience profiles of involved firms as well as the pre-booking of logistics capacity to minimize costs becomes the imperative rather than a choice. To that end, deploying simulation based approaches is of particular significance towards assessing the negative effects of such disruption and developing effective risk mitigation strategies (see Aldrighetti, Zemaro, Finco, & Battini, 2019).

Appendix A

In this appendix, we derive the coefficients of the economic and environmental objective functions. In particular, we show how to compute the transportation and cold storage costs as well as the amount of carbon footprint emitted due to transportation and storage activities. The following notations are adopted in the derivation of these equations:

- \( PG \): Fuel price per gallon
- \( EP \): Electricity price per kWh
- \( L_d \): Hourly driver wage
- \( GPM \): Gallons per mile for a full truckload
- \( GPM_0 \): Gallons per mile for an empty truckload
- \( L \): Distance traveled per truck
- \( w \): Truck driving speed
- \( CEGF \): Carbon emissions level per gallon of fuel
- \( TCFE \): Total carbon footprint of one kWh energy
- \( TECF \): Total energy consumption by one freezer operated for one period

A.1. Truck Transportation cost

We express the truck transportation cost as the sum of a fixed cost, which is independent of the fuel consumption, and a variable cost, which depends on the fuel consumption level as detailed next.

A.1.1. Fixed transportation cost

As in the work of Stellingwerf, Laporte, Cruissen, Kanellopoulos, and Bloemhof (2018), we assume that the driver wage is the major cost component of the truck fixed transportation cost when compared to other components such as the cost of ownership, maintenance and repairing cost, and insurance cost. The fixed transportation cost during period \( t \) can then be expressed as:

\[
L \cdot f_s \cdot \frac{L}{v} = \text{Fixed Transportation cost, } \forall t
\]  

(A1)

A.1.2. Variable transportation cost

In the transportation of cold products, the fuel is consumed for the purposes of generating motive power as well as for maintaining the temperature of the truckload at appropriate level (Stellingwerf et al., 2018). As observed by Tassou, De-Lille, and Ge (2009), the fuel consumption of the truck refrigeration equipment cannot be accurately estimated since it depends on several factors such as type of product (chilled vs. frozen), multiple vs single stops/drop-offs, surface area of the truck, difference in temperature between the inside and the outside of the truck, among many others. In addition, through a search in the relevant literature, we found different equations that are adopted in the calculation of the fuel used for temperature control (see for example Stellingwerf et al. (2018), and Wang and Wen (2020)). Based on the data of 17 temperature-controlled trucks, Tassou et al. (2009) reported that irrespective of the truck type, the average fuel consumption of the refrigeration equipment in the sample was between 15% and 25% of the engine fuel consumption. Accordingly, in this paper we mark up the available engine fuel consumption figures from the literature by a certain percentage to account for the additional consumption due to refrigeration. In the following, we assume the gallon per mile (GPM) figures include both fuel used for power motive and temperature control.

The total fuel consumption is a function of the number of trucks utilized, the distance traveled, the FTL capacity and the actual load in each truck. As the lot size in a specific period \( Q_t \) is transported to the retailer via a fleet of \( m \) identical trucks each having a capacity of \( R \) units, one may note that \( m_t - 1 \) trucks will be fully loaded while the load of the last truck is simply \( |Q_t - (m_t - 1)R| \). Assuming a linear relationship between the gallons of fuel...
consumed and the truckload, the total fuel consumption in period \( t \) for the full \((m_t - 1)\) trucks is \( L(m_t - 1)\text{GPM} \), and the fuel consumption for the last partially loaded truck is:

\[
L \left( \frac{Q_t - (m_t - 1)R}{R} \right) \left( \text{GPM} - \text{GPM}^0 \right) + \text{GPM}^0
\]

The total fuel consumption by all trucks during period \( t \) is then given by:

\[
L \left( (m_t - 1)\text{GPM} + \frac{Q_t - (m_t - 1)R}{R} \left( \text{GPM} - \text{GPM}^0 \right) + \text{GPM}^0 \right)
\]

The fuel consumption cost during period \( t \) is obtained by multiplying the last equation by the fuel price per gallon, \( pg \), which after some mathematical simplification can be written as

\[
(pg \times L \times \text{GPM}^0 R) Q_t + (pg \times L \times \text{GPM}^0) m_t
\]  
(A2)

The total transportation cost is calculated as the sum of the fixed transportation cost given in (A1) and variable transportation cost shown in (A2). It can then be expressed as:

\[
CQ \times Q_t + TR \times m_t
\]  
(A3)

where

\[
CQ = pg \times L \times \frac{\text{GPM} - \text{GPM}^0}{R} \quad \text{and} \quad TR = pg \times L \times \text{GPM}^0 + f_w \frac{L_v}{v}
\]  
(A4)

A.2. Cold storage cost

Similarly, the cold storage cost depends on the electricity consumed by the freezers, where the latter is a function of the number of operational freezers per period \( n_t \) and the periodic energy consumption by each freezer, \( TECF \). As such, the freezers’ operational cost during period \( t \) is given by:

\[
EP^*TECF^*n_t = FR^*n_t
\]  
(A5)

where

\[
FR = EP^*TECF
\]  
(A6)

Using Eqs. (A3) and (A5), the total operational costs due to transportation and storage activities of the cold product during period \( t \) is given by

\[
CQ^*Q_t + TR^*m_t + FR^*n_t
\]  
(A7)

Given that \( m_t = \left\lceil \frac{Q_t}{R} \right\rceil \) and \( n_t = \left\lceil \frac{Q_t}{U} \right\rceil \), where \( \left\lceil x \right\rceil \) is the smallest integer larger than or equal to \( x \), it is clear that the above function given in (A7) is a stepwise function of the ordering quantity with discontinuity that are multiples of the truck capacity and freezer capacity (see Fig. A1).

A.3. Carbon footprint

In this paper, carbon emissions are mainly generated by the transportation and storage activities of the cold products. The calculations of the carbon footprint for each activity are shown next.
A.3.1. Carbon footprint due to transportation

The carbon emissions in period \( t \) due to transportation activities is obtained through multiplying the carbon emissions level per gallon of fuel, \( CEGF \), by the total fuel consumption by all trucks during period \( t \). The carbon emitted because of transportation activities during period \( t \) is then:

\[
\overline{CQ}_t + \overline{TR}_t \]

or

\[
CEGF^*L^* \left( \frac{GPM - GPM^0}{R} Q_t + \frac{GPM^0}{R} m_t \right) ,
\]

\[ \text{or} \]

(A8)

where

\[
\overline{CQ} = CEGF^*L^* \frac{GPM - GPM^0}{R} \quad \text{and} \quad \overline{TR} = CEGF^*L^*GPM^0
\]

(A9)

A.3.2. Carbon footprint due to cold storage

The freezers at the retailer’s facility are held accountable for high levels of energy consumption and increased amount of CO$_2$ emissions. In particular, the carbon footprint generated per period due to cold storage activities is a function of the number of freezers operational during that period \( n_t \), the energy consumption by one freezer operated for one period, and the carbon footprint of one kWh energy. It is thus given by:

\[
TCFE^*TECF^*n_t, \quad \text{or} \quad FR^*n_t,
\]

\[ \text{or} \]

(A10)

where

\[
\overline{FR} = TCFE^*TECF
\]

Using Eqs. (A8) and (A10), the carbon footprint generated due to transportation and storage activities during period \( t \) is:

\[
CF_t = \overline{CQ}_t Q_t + \overline{TR}_t m_t + \overline{FR}_t n_t
\]

(A12)

Appendix B. Illustrative example for optimization problem (CAP$_T$)

In order to demonstrate the solution procedure for the optimization problem (CAP$_T$), we solve the following example in which some of the parameters values are adapted from the cold supply chain literature and are cited when used. For the sake of clarity, we provide detailed calculations of the objective function and carbon cap constraint coefficients.

A cold product, with a shelf life of \( SL = 3 \), is facing a discrete time-varying demand over a planning horizon of 12 weeks, as shown in Table B1. All administrative activities associated with the placement of an order is estimated to be \$5 per order. The distance between the supplier and the retailer is 100 miles (161 km). It is assumed here that the fuel type used is diesel, which costs \$3.25 per gallon. The cold product, in this case dairy products, are moved in unit load of crates with dimensions of 15 1/4’ long with internal dimensions of 17’10” x 7’5” x 7’5”. The total value of the dairy products held in each crate is worth \$50. The holding cost of \$0.1 per unit per week (0.1 * \$50/52) is given. The drivers wage is \$3.4 per hour after converting the monthly wage in local currency to dollar and assuming the driver is working for 160 h per month.

The fuel related parameters were estimated based on the works by Tassou et al. (2009), Madre, Andre, Leonardi, Ottmann, and Rizet (2010) and Hariga et al. (2017). Madre et al. (2010) and Hariga et al. (2017) provided a modified version of Madre et al. (2010) calculations with the fuel consumption figures shown in gallons per mile. In this work, we opted to use the gallon per mile as well since it is more useful towards determining the amount of gallons needed to drive a certain distance. In this paper, the fuel consumption per mile for the full and empty truck is assumed to be 0.072 gallons and 0.057 gallons, respectively for an average speed of 60 Km/h for a small truck (Madre et al., 2010, Hariga et al., 2017). The aforementioned fuel consumption figures are only valid for regular trucks. However, since we are dealing with refrigerated trucks, there will be an increase in energy consumption to preserve the truckload at the desired temperature. As mentioned in Appendix A, Tassou et al. (2009) pointed that the average fuel consumption of the refrigeration systems varies between 15% and 25% of the engine fuel consumption. This percentage is much higher for countries experiencing high ambient temperatures and humidity during most months of the year, such as Middle East countries. Accordingly, we adopt herein a 40% fuel consumption markup, which results in gallon per mile values of 0.1008 and 0.0798 for full and empty trucks, respectively. The Refrigerated truck used is 20’ long with internal dimensions of 17’10” x 7’5” x 7’5”. Given the truck and crate dimensions, this translates into a loading capacity of 750 units.

| Week $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|----------|---|---|---|---|---|---|---|---|---|----|----|----|
| $D_t$    | 400 | 600 | 400 | 300 | 400 | 1200 | 800 | 800 | 300 | 1000 | 300 | 1400 |
The amount of carbon emission generated per gallon of fuel consumed, \( CE \), is 0.0112 ton\(^4\). The freezer’s capacity is 9 m\(^3\) which is enough to store a maximum of \( U = \) 250 crates with the same dimensions as above. It is assumed that a single stage recuperating compressors and evaporative condensers is used for the refrigeration system with an energy consumption of 57.6 kWh/year/m\(^3\) (Bazan, Jaber, & El Saadany, 2015, James & James, 2010). Accordingly, the total energy consumption by one freezer, \( TECF \), is 518.4 (\( TECF = 57.6 * 9 \) kWh/year or equivalently 10 kWh/week. The price per kWh, \( EP \), depends on the consumption amount and the country electricity tariffs, which is set at $0.11/kWh.\(^4\) Finally, the total carbon footprint for 1 kWh of energy, \( TCFE \), is \( 7.44 \times 10^{-4} \) tons/kWh\(^5\). Using these parameters’ values, the cost coefficients of the objective function \( CQ \), \( TR \), and \( FR \) are calculated as follows:

\[
CQ = pgL \times \frac{GPM-GPM}{R} = 3.25 \times 100 \times (0.1008 - 0.0798)/750 = 0.0091/unit
\]

\[
TR = pgL \times \frac{GPM}{R} + f_{\frac{L}{R}} = 3.25 \times 100 \times 0.0798 + 3.4 \times 161/60 = 33.05/truck
\]

\[
FR = EP \times TECF = 0.11 \times 10 = $1.1/freezer
\]

The carbon cap constraint related coefficients \( \hat{CQ} \), \( \hat{TR} \), and \( \hat{FR} \) are computed as follows:

\[
\hat{CQ} = CE \times GPM / \frac{GPM}{R} = 0.0112 \times 100 \times (0.1008 - 0.0798)/750 = 0.0112 \times 10 = 0.0894 tons/unit
\]

\[
\hat{TR} = CE \times GPM / R = 0.0112 \times 100 \times 0.0798 = 0.8948 tons/truck
\]

\[
\hat{FR} = TCFE \times TECF = 0.000744 \times 10 = 0.00744 tons/freezer
\]

After solving the \( \text{(Cap}_{\text{min}}, 7) \) and \( \text{(Cap}_{\text{max}}, 7) \) optimization problems, we get \( C_{\text{min}} = 1.573 \) tons and \( C_{\text{max}} = 1.715 \) tons, respectively. Table B2 reports the obtained results for carbon cap values larger than 1.715 (unconstrained problem), as well as for the intermediate values of 1.7 and 1.6 tons. For a carbon cap of 1.7 tons, the optimal ordering policy resulted in a minimum total operational cost of $734.85 and a total carbon footprint of 1.64 tons over the 12 weeks planning horizon. This optimal ordering policy resulted in a reduction of 4.37% in the total carbon emissions when compared to the optimal solution of the unconstrained carbon cap policy. Furthermore, in an attempt to reduce the carbon emissions due to transportation activity, it is noted that less frequent orders are made for a cap value of 1.7 tons along with better utilization of the trucks’ capacity when compared to the unconstrained policy. On the other hand, a limit of 1.6 tons on the total carbon emitted over the planning horizon yields a reduction in carbon footprint by 8.28%. As it can be noticed from Table B2, the reduction in CO\(_2\) emissions is mainly due to the drop in the total number of trucks used over the entire planning horizon. This reduction in the carbon footprint due to the use of fewer trucks clearly outweighs the additional emissions induced by the use of more freezer units.

Appendix C. Illustrative example for optimization problem (CAP1)

Using the same illustrative example given in Appendix B, we first get \( CP_{\text{min}} = 0.1543 \) tons and \( CP_{\text{max}} = 0.2673 \) tons. The optimal ordering schedules along with the periodic carbon emissions under no carbon cap restriction and under maximum carbon emission of 0.25 tons and 0.20 tons per week are reported in Table C1.

When setting the periodic cap at a value larger than 0.2673 tons per week, it turned out that the trucks were fully utilized (FTL) in only 3 out of the 12 weeks planning horizon. A similar pattern with many less-than-truckload (LTL) replenishments over several weeks is also observed for a periodic cap of 0.25 tons per week. The dispatching of semi-loaded trucks\(^7\) is justified as the carbon emissions, as well as the transportation cost, increase with additional load rendering LTL shipments a more economical option. This also explains the positive slackness in the periodic carbon cap constraints where these constraints turn out to be non-binding across all 12 weeks.

As a tighter cap on periodic emissions is imposed, this induces an increase in the operational cost of the lot sizing policy where this increase is not necessarily proportional to how tighter the cap gets (i.e., 14.87% and 2.54% cost increases are realized when the periodic cap is reduced from 0.2673 to 0.25 and from 0.25 to 0.20, respectively). From a carbon emissions perspective, it can be seen that a tighter cap value of 0.25 tons/week had unexpectedly resulted in higher total emissions (1.819 tons) as compared to 1.715 tons for a cap value greater than or equal to 0.2673 tons/week. This

---

\(^4\) www.eia.gov.

\(^5\) www.epa.gov.
is due to the fact that 2 trucks are used in weeks 7 and 11 for an emission cap of 0.25 versus one truck in both weeks for the unconstrained case (cap > 0.2673). The use of more trucks for a tighter cap is justified as this prevents the use of more trucks in subsequent periods (see week 12 for example), where the emissions associated with future period(s) ship may exceed the imposed periodic cap (i.e. a shipment of 1400 units in week 12 results in 0.2673 tons of emissions which exceeds the 0.25 tons/week limit).

Appendix D. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.cie.2020.106800.

References

Abdi, N., Dauncey-Pérez, S., Kedad-Sidhoum, S., Penz, B., & Rapine, C. (2013). Lot sizing with carbon emission constraints. European Journal of Operational Research, 227(1), 55–61.

Abdi, N., Dauncey-Pérez, S., Kedad-Sidhoum, S., Penz, B., & Rapine, C. (2016). The single-item green lot-sizing problem with fixed carbon emissions. European Journal of Operational Research, 248(3), 849–855.

Aldighetti, R., Zennaro, I., Finco, S., & Battini, D. (2019). Healthcare supply chain simulation with disruption considerations: A case study from northern Italy. Global Journal of Flexible Systems Management, 20, 81–102.

Al Theeb, N., Smadi, H., Al-Hawari, T., & Aljarrah, M. (2020). Optimization of vehicle routing with inventory allocation problems in Cold Supply Chain Logistics. Computers & Industrial Engineering, 142, 106341.

Andriolo, A., Battini, D., Persona, A., & Sgarbossa, F. (2015). Haulage sharing approach to achieve sustainability in material purchasing: New method and numerical applications. International Journal of Production Economics, 164, 308–318.

Andan, M., & Turkay, M. (2013). EOQ revisited with sustainability considerations. Foundations of Computing and Decision Sciences, 38, 223–249.

Bagabolzadeh, M., Shrestha, A., Abbasi, B., Zhang, Y., Woodhead, A., & Zhang, A. (2020). Sustainable cold supply chain management under demand uncertainty and carbon tax regulation. Transportation Research Part D, 80, 102245.

Battini, D., Calzavara, M., Isolan, I., Sgarbossa, F., & Zangaro, F. (2018). Sustainability in material purchasing: A multi-objective economic order quantity model under carbon trading. Sustainability, 10, 4438.

Battini, D., Persona, A., & Sgarbossa, F. (2014). A sustainable EOQ model: Theoretical formulation and application. International Journal of Production Economics, 149, 145–153.

Bazan, E., Jaber, M. Y., & El Saadany, A. (2015). Carbon emissions and energy effects on manufacturing-remanufacturing inventory models. Computers and Industrial Engineering, 88, 307–316.

Benjaafar, S., Li, Y., & Daskin, M. (2013). Carbon footprint and the management of supply chains: Insights from simple models. IEEE Transactions on Automation Science and Engineering, 10, 99–116.

Bonney, M., & Jaber, M. Y. (2011). Environmentally responsible inventory models: Non-classical models for a non-classical era. International Journal of Production Economics, 133, 43–53.

Bouchery, Y., Ghaffari, A., Jemai, Z., & Dallery, Y. (2012). Including sustainability criteria into inventory models. European Journal of Operational Research, 222, 229–240.

Bozorgi, A. (2016). Multi-product inventory model for cold items with cost and emission consideration. International Journal of Production Economics, 170, 123–142.

Bozorgi, A., Pasour, J., & Nazal, D. (2014). A new inventory model for cold items that considers costs and emissions. International Journal of Production Economics, 155, 114–125.

Chaudhary, V., Kuhlreuther, R., & Routroy, S. (2018). State-of-the-art literature review on inventory models for perishable products. Journal of Advances in Management Research, 15(3), 306–346.

Chelly, A., Nouria, I., Frein, Y., & Hadj-Alouane, A. B. (2019). On the consideration of carbon emissions in modelling-based supply chain literature: The state of the art, relevant features and research gaps. International Journal of Production Research, 57, 4977–5004.

Chen, X., Benjaafar, S., & Elomri, A. (2013). The carbon constrained EOQ. Operations Research Letters, 41, 172–179.

Cheng, C., Qi, M., Wang, X., & Zhang, Y. (2016). Multi-period inventory routing problem under carbon emission regulations. International Journal of Production Economics, 162, 263–275.

Chiranjit, D., & Jharkharia, S. (2019). Low carbon supply chain: A state-of-the-art literature review. Journal of Manufacturing Technology Management, 29, 398–428.

Chua, G., Mokhlesi, R., & Sainathan, A. (2017). Optimal discounting and replenishment policies for perishable products. International Journal of Production Economics, 186, S20.

Das, C., & Barkharia, S. (2018). Low carbon supply chain: A state-of-the-art literature review. Journal of Manufacturing Technology Management, 29, 398–428.

Ghosh, A., Jha, J. K., & Sarmah, S. P. (2017). Optimal lot-sizing under strict carbon cap policy considering stochastic demand. Applied Mathematical Modelling, 44, 688–704.

Hagiza, M., Asad, R., & Shamyaleh, A. (2017). Integrated economic and environmental models for a multi-stage cold supply chain under carbon tax regulation. Journal of Cleaner Production, 166, 1357–1371.

Hagiza, M., Babekian, S., & Bahrouz, Z. (2019). Operational and environmental decisions for a two-stage supply chain under vendor managed consignment inventory partnership. International Journal of Production Research, 57, 3642–3662.

He, Y., Wang, L., & Wang, J. (2012). Cap-and-trade vs. carbon taxes: A quantitative comparison from a generation expansion planning perspective. Computers and Industrial Engineering, 63, 708–716.

Helmirch, M. J., Jans, R., van den Heuvel, W., & Wagelmans, A. (2015). The economic lot-sizing problem with an emission capacity constraint. European Journal of Operational Research, 241, 50–62.

Hoen, K. M. R., Tan, T., & Fransoo, J. (2014). Effect of carbon emission regulations on transport mode selection under stochastic demand. Flexible Services and Manufacturing Journal, 26, 170–195.

Hua, V. N. (2000). Dynamic economic lot size model with perishable inventory. Management Science, 46(8), 1159–1169.

Hua, V. N. (2003). An economic lot size model for perishable products with age-dependent inventory and backorder costs. IIE Transactions, 35, 775–780.

Hua, V. N., Cheng, T. C. E., & Wang, S. (2011). Managing carbon footprints in inventory management. International Journal of Production Economics, 132, 178–185.
Huisings, H., Zhang, Z., Moore, J., Qiao, Q., & Li, Q. (2015). Recent advances in carbon emissions reduction: Policies, technologies, monitoring, assessment and modeling. Journal of Cleaner Production, 102, 1-12.

Intergovernmental Panel on Climate Change (IPCC). (2014). Climate change 2014: Synthesis report. Geneva: Switzerland.

Ivanov, D., & Dolgui, A. (2020). Viability of intertwined supply networks: Extending the supply chain resilience angles towards survivability. A position paper motivated by COVID-19 outbreak. International Journal of Production Research, 58(10), 2904–2915.

James, S. J., & James, C. (2010). The food cold-chain and climate change. Food Research International, 43, 1944-1956.

Janssen, L., Claus, T., & Sauer, J. (2016). Literature review of deteriorating inventory models by key topics from 2012 to 2015. International Journal of Production Economics, 186, 86–112.

Janssen, L., Diabat, A., Sauer, J., & Herrmann, F. (2018). A stochastic micro-periodic age-based inventory replenishment policy for perishable goods. Transportation Research Part E, 118, 445–465.

Kanchanasuntorn, K., & Techanitisawad, A. (2006). An approximate periodic model for fixed-life perishable products in a two-echelon inventory distribution system. International Journal of Production Economics, 100, 101–115.

Kayfeci, M., Kecibas, A., & Gedik, E. (2013). Determination of optimum insulation thickness of external walls with two different methods in cooling applications. Applied Thermal Engineering, 50, 217-224.

Konur, D. (2014). Carbon constrained integrated inventory control and truckload transportation with heterogeneous freight trucks. International Journal of Production Economics, 153, 268–279.

Konur, D., & Schaefer, B. (2014). Integrated inventory control and transportation decisions under carbon emissions regulations: LTL vs. TL carriers. Transportation Research Part E: Logistics and Transportation Review, 68, 14–38.

Lee, S.-K., Yoo, S. H., & Cheong, T. (2017). Sustainable EOQ under lead-time uncertainty and multi-modal transport. Sustainability, 9, 476.

Li, J., Su, Q., & Ma, L. (2017). Production and transportation outsourcing decisions in the supply chain under single and multiple carbon policies. Journal of Cleaner Production, 141, 1109-1122.

Madre, J., Andre, M., Leonardi, J., Ottmann, P., & Rizer, C. (2010). Importance of the loading factor in transport CO\textsubscript{2} emissions. 12th WCTR. Lisbon, Portugal.

Memari, A., Abdul Rahim, A., Ahmad, R., & Hassan, A. (2016). A literature review on green supply chain modelling for optimizing CO\textsubscript{2} emission. International Journal of Operational Research, 26, 509–525.

Meneghetti, A., & Monti, L. (2015). Greening the food supply chain: An optimization model for sustainable design of refrigerated automated warehouses. International Journal of Production Research, 53, 6567–6587.

Mohammed, F., Selim, S., Hassan, A., & Syed, M. (2017). Multi-period planning of closed-loop supply chain with carbon policies under uncertainty. Transportation Research Part D, 51, 146–172.

Mursiana, C. (2016). An EOQ model for perishable products with fixed shelf life under stochastic demand conditions. European Journal of Operational Research, 255, 388–396.

Önal, M. (2016). The two-level economic lot sizing problem with perishable items. Operations Research Letters, 44(3), 403-408.

Önal, M., Romeijn, H. E., Sapra, A., & van den Heuvel, W. (2015). The economic lot-sizing problem with perishable items and consumption order preference. European Journal of Operational Research, 244(3), 881-891.

Olsson, F., & Tydén, P. (2010). Inventory problems with perishable items: Fixed lifetimes and backlogging. European Journal of Operational Research, 202(1), 131–137.

Pahl, J., & Voss, S. (2014). Integrating deterioration and lifetime constraints in production and supply chain planning: A survey. European Journal of Operational Research, 238, 654–674.

Pauls-Worm, K., Hendrix, E., Alcoba, A., & Hajjema, R. (2016). Order quantities for perishable inventory control with non-stationary demand and a fill rate constraint. International Journal of Production Economics, 181, 236-246.

Pires, M. J., Amorim, P., Martins, S., & Almada-Lobo, B. (2015). Production planning of perishable food products by mixed-integer programming. In J. Almeida, J. Oliveira, & A. Finta (Eds.), Operational Research. CIM Series in Mathematical Sciences (Vol. 4, pp. 331–352). Cham: Springer.

Sargut, F. Z., & Isik, G. (2017). Dynamic economic lot size model with perishable inventory and capacity constraints. Applied Mathematical Modelling, 48, 806–820.

Sazvar, Z., Mirzapour Al-e-habshem, S. M., Govindan, K., & Bahli, B. (2016). A novel mathematical model for a multi-period, multi-product optimal ordering problem considering expiry dates in a FEFO system. Transportation Research Part E, 93, 232–261.

Shamayleh, A., Hariga, M., Ar’ad, R., & Diabat, A. (2019). Economic and environmental models for cold products with time varying demand. Journal of Cleaner Production, 212, 847–863.

Shuang, Y., Diabat, A., & Liao, Y. (2019). A stochastic reverse logistics production routing model with emissions control policy selection. International Journal of Production Economics, 213, 201–216.

Soyal, M., Bloemhof-Ruwaard, J. M., & van der Vorst, J. G. (2014). Modelling food logistics networks with emission considerations: The case of an international beef supply chain. International Journal of Production Economics, 152, 57–70.

Stellingwerf, H. M., Laporte, G., Crijnsen, F., Kanellopoulos, A., & Bloemhof, J. M. (2018). Quantifying the environmental and economic benefits of cooperation: A case study in temperature-controlled food logistics. Transportation Research Part D, 65, 178–193.

Tang, C. S., & Zhou, S. (2012). Research advances in environmentally and socially sustainable operations. European Journal of Operational Research, 223, 585–594.

Tao, Z., & Xu, J. (2019). Carbon-regulated EOQ models with consumers’ low-carbon awareness. Sustainability, 11, 1094.

Tassou, S., De-Lille, G., & Ge, Y. T. (2009). Food transport refrigeration – Approaches to reduce energy consumption and environmental impacts of road transport. Applied Thermal Engineering, 29, 1467–1477.

Tiwari, S., Daryanto, Y., & Wee, H. M. (2018). Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission. Journal of Cleaner Production, 192, 281–292.

Toptal, A., Özli, H., & Konur, D. (2014). Joint decisions on inventory replenishment and emission reduction investment under different emission regulations. International Journal of Production Research, 52, 243–269.

Vrat, P., Gupta, R., Bhatnagar, A., Pathak, D. K., & Pulzlee, V. (2018). Literature review analytics (LRA) on sustainable cold-chain for perishable food products: Research trends and future directions. OPSSEARCH, 55, 601–627.

Wahab, M. I. M., Mamun, S. M. H., & Ongkunaruk, P. (2011). EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact. International Journal of Production Economics, 134, 151–158.

Wang, Z., & Wen, P. (2020). Optimization of a low-carbon two-echelon heterogeneous-fleet vehicle routing for cold chain logistics under mixed time window. Sustainability, 12, 1967.

Wu, C., & Zhao, Q. (2014). Supplier-buyer deterministic inventory coordination with trade credit and shelf-life constraint. International Journal of Systems Science: Operations & Logistics, 1(1), 36–46.

Xu, Z., Sunah, D., Zenga, X., & Liu, D. (2015). Research developments in methods to reduce the carbon footprint of the food system: A review. Critical Reviews in Food Science and Nutrition, 55, 1270–1286.

Zhu, Z., Chu, F., Dolgui, A., Chu, C., Zhou, W., & Piramuthu, S. (2018). Recent advances and opportunities in sustainable food supply chain: A model-oriented review. International Journal of Production Research, 56(17), 5700–5722.