Computing Bayes Nash Equilibrium Strategies in Auction Games via Simultaneous Online Dual Averaging

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Numerous games studied in microeconomic theory, such as auctions and contests, are modeled as Bayesian games with continuous type and action spaces. However, explicit solutions in the form of Bayes-Nash equilibria for such games are only known under highly specific assumptions regarding the agents’ prior distributions or utility functions. Given the continuous nature of these games, existing equilibrium solvers cannot be straightforwardly applied and necessitate an additional discretization step.

This paper introduces algorithms to compute approximate Bayes-Nash equilibria in games with continuous types and actions. These algorithms are evaluated across a diverse range of games, including standard single-unit auctions, combinatorial auctions, and contests. Distinguished from other existing algorithms, we compute equilibria using distributional strategies on a discretized version of the game. Intuitively, distributional strategies are a generalization of mixed strategies to Bayesian games. Most other existing approaches use pure strategies, which often cause discontinuity issues in auctions. By using distributional strategies, we avoid these continuity problems and do not have to make any assumptions on the shape of the bid function. Importantly, this formulation renders our utility functions linear, and well-known methods and convergence results from online learning theory can be applied.

We provide compelling evidence through empirical analysis that our methods closely approximate the (pure) analytical equilibrium in various auction and contest games. In addition, we demonstrated the versatility of our method by applying it to settings where the equilibrium strategies are unknown. Notably, our algorithms demonstrate fast termination times, often within minutes or even seconds, even on standard laptop configurations. This is remarkable since online learning algorithms do not converge in many finite matrix games and can cycle or even behave chaotically. Our algorithm allows for interdependent valuations and different types of utility functions.

In addition to our experimental findings, we offer an approximation result that bounds the utility loss resulting from the discretization process for typical single-unit auctions. Also, while providing a general convergence proof for a larger class of games is a very challenging problem that is clearly out of the scope of our paper, we provide some heuristic explanations for the excellent performance of our algorithms when applied to auctions and contests.

A full version of this paper can be found at https://arxiv.org/abs/2208.02036.

CCS Concepts: • Theory of computation → Exact and approximate computation of equilibria; Convergence and learning in games.

Additional Key Words and Phrases: auctions, Bayes-Nash equilibrium, online convex optimization

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