Tunable band gaps of axially moving belt on periodic elastic foundation

Lei Lu, Fuyao Liu, and Jihui Wu

Abstract
The present paper investigates the band structure of an axially moving belt resting on a foundation with periodically varying stiffness. It is concluded that the band gaps appear when the divergence of the eigenvalue occurs and the veering phenomenon of mode shape begins. The bifurcation of eigenvalues and mode shape veering lead to wave attenuation. Hence, the boundary stiffness modulation can be designed to manipulate the band gap where the vibration is suppressed. The contribution of the system parameters to the band gaps has been obtained by applying the method of varying amplitudes. By tuning the stiffness, the desired band gap can be obtained and the vibration for specific parameters can be suppressed. The current study provides a technique to avoid vibration transmission of the axially moving material by designing the foundation stiffness.

Keywords
tunable band gap, axially moving belt, dispersion relation, periodic foundation, wave properties

1. Introduction
The axially moving belts are widely employed in various engineering applications, such as driving belts, transmission belts, magnetic belts, et al. The periodic structure is also a common device that can be found in modern industrial fields as the vibration isolator and mechanical filter. The dynamic properties of periodic structures, similar to phononic crystals, are attractive to engineers and researchers. The question then arises: Is that possible the vibration of axially moving materials can be suppressed by the periodic designed foundation instead of the constant foundation? The axially moving continuum have been studied for centuries (John Aitken, 1878). The transverse vibrations have been investigated widely (Ding and Chen, 2019; Ghayesh and Balar, 2010; Wickert and C. D. Mote, 1990; Yang and Zhang, 2014). The analysis and control of transverse vibration of axially moving string were studied systematically by Chen (Chen, 2005). Considering various constraints (Ding et al., 2018a, 2018b; Ghayesh, 2009), the axially moving continua were investigated in the available literature. Yang et al. (Yang et al., 2010) investigated the axially moving beam resting on the elastic foundation with constant stiffness and obtained the explicit expression of critical velocity. Bhat et al. (Bhat et al., 1982) investigated the dynamic response of the moving belt supported by the elastic foundation by numerical techniques. The researches of the foregoing literature are about uniform structures. When the systems involve periodic parameters, such as cross-section (Sorokin and Thomsen, 2015), stiffness, etc., the systems transform into periodic structures. One of the distinctive features of periodic structures is the appearance of band gaps. The wave attenuation occurs in the band gaps for systems composed of periodic units (Martínez-Sala et al., 1995). The fascinating characteristics of the band gaps of the periodic structures have drawn much attention for recent decades.

The properties of the wave propagation in periodic structures were investigated by Brillouin (Brillouin, 1953) systematically. Mead (Mead, 1970) reviewed a notion of the wave propagation constants in periodically supported infinite beams. Asfar and Nayfeh (Asfar and Nayfeh, 1983) applied the method of multiple scales to the periodic structures. The axially moving belt with periodic cross-section has been investigated by Sorokin (Sorokin and Thomsen, 2017) recently. He applied the method of varying amplitude to periodic structures for vibration suppression. Inspired by Sorokin (Sorokin, 2019), we consider a uniform axially moving belt with periodic varying foundation stiffness and stiffness modulation which composes a periodic structure. The band gap depends on the...
system parameters which would vary in different cases. The investigation leads to a tunable band gap that stems from the research of photonic crystal (Yablonovitch, 1987). Some investigations of the tunable band gap focused on the photonic crystal or phononic crystal (Fan et al., 2019; Li et al., 2020; Shkunov et al., 2002). Various wave guides were designed to control the wave propagation or tune the band gap (Airoldi and Ruzzene, 2011; Grinberg et al., 2018; Harrison et al., 2019; Zhou et al., 2020). Yang et al. (Yang et al., 2020) investigated a two-dimensional periodic lattice tuned by parametric excitations to control the wave propagation.

The moving belt and string supported on elastic foundation have been studied for decades (Bhat et al., 1982; Parker, 1999; Perkins, 1990). With the development of the metamaterial, especially the phononic crystal, the periodic structures become a hot topic and broaden the research of band gaps (Chen et al., 2013; Yang et al., 2019; Li et al., 2020; Shkunov et al., 2002). Various citations to control the wave propagation.

Assuming that the stiffness $S(X)$ varies harmoniously

$$S(X) = S_0(1 + \sigma \cos \varphi X)$$

where $S_0$ is a constant that stands for the stiffness of foundation without modulation, $\sigma$ represents the stiffness modulation, $\varphi = 2\pi/\Phi$ denotes the density of the stiffness varying with the length which represents one-dimensional reciprocal lattice (Brillouin, 1953).

To discuss the mechanism of the tunable band gap and to simplify the calculation, introducing the nondimensional variables and parameters

$$u = \frac{U}{\Phi}, \quad \varphi = \frac{2\pi}{\Phi}, \quad c = \sqrt{\frac{P}{\rho}}, \quad x = \varphi X, \quad t = \varphi c T,$$

$$v = \frac{V}{c}, \quad s = \frac{S_0}{\varphi^2 P}$$

Equation (2) is rewritten as

$$u_{tt} + 2vu_{tx} - (1 - v^2)u_{xx} + s(1 + \sigma \cos x)u = 0$$

The solution to equation (5) is assumed as

$$u(x,t) = A(x)e^{i\omega t}$$

By substituting equation (6) into equation (5), one obtains

$$(1 - v^2)A'' - 2i\omega vA' + [\omega^2 - s(1 + \sigma \cos x)]A = 0$$

When the foundation stiffness $S(X) = 0$, the system degenerates into the same form as Wickert and Mote (Wickert and C. D. Mote, 1990).

2. Governing equations

Consider an axially moving belt with velocity $V$ resting on a periodic elastic foundation as shown in Figure 1. The displacement in the transverse direction is denoted by $U(X, T)$, where $X$ is the spatial coordinate and $T$ is time. The linear density is $\rho$ and the tension is $P$. $S(X)$ denotes the varying stiffness of the elastic foundation with the period $\Phi$ at the point $X$, and $S(X) = S(X + \Phi)$.

The equations of motion for axially moving continua are derived in detail (Wickert and C. D. Mote, 1990).

$$\rho(U_{TT} + 2VU_{XT} + V^2U_{XX}) - PU_{XX} = 0$$

The model of the present paper is treated as the second-order model adding the constraining force $S(X)U$. The governing equation is derived instantly by Newton’s second law or Hamilton’s principle as

$$\rho(U_{TT} + 2VU_{XT} + V^2U_{XX}) - PU_{XX} + S(X)U = 0$$

3. Solution by the method of varying amplitudes

The method of varying amplitudes is based on the method of direct separation of motions (Sorokin, 2013). It is useful for studying dynamics of spatially periodic structures and avoiding the small parameters (Sorokin and Thomsen, 2015). By applying the method of varying amplitudes...
(Sorokin and Thomsen, 2017; Sorokin, 2019), we seek the solution to equation (7) in the form of an infinite series as

\[
A(x) = \sum_{m=-\infty}^{+\infty} a_m(x)e^{imx} \tag{8}
\]

Substituting equation (8) back into equation (7), one obtains

\[
\sum_{m=-\infty}^{+\infty} \left\{ (1-v^2)(a''_m + 2ima'_m - m^2a_m) - 2iov(a'_m + ima_m) + [o^2-s(1+\sigma\cos x)]a_m \right\} e^{imx} = 0, \ m \in \mathbb{Z} \tag{9}
\]

By transforming \cos x to the well-known Euler’s formula and balancing the coefficients of the harmonic involved, a set of equations is obtained as

\[
(1-v^2)(a''_m + 2ima'_m - m^2a_m) - 2iov(a'_m + ima_m) + (o^2-s)a_m - \frac{1}{2}\sigma s(a_{m-1} + a_{m+1}) = 0, \ m \in \mathbb{Z}. \tag{10}
\]

Assuming \( K = -ik \) is the eigenvalue of equation (10), where \( k \) is the wave number, and \( \mathbf{a} = (a_0, a_1, a_2, a_3, \ldots)^T \) is the associated eigenvector. The coefficients of the right-hand side of equation (8) can be rewritten in vector form as

\[
\mathbf{a}(x) = \begin{pmatrix} a_0(x) \\ a_1(x) \\ a_{-1}(x) \\ a_2(x) \\ a_{-2}(x) \\ \vdots \\ a_n(x) \\ \vdots \end{pmatrix} = \begin{pmatrix} a_0 \\ a_1 \\ a_{-1} \\ a_2 \\ a_{-2} \\ \vdots \\ a_n \\ \vdots \end{pmatrix} e^{-ikx} \tag{11}
\]

Then equation (10) turns to the matrix form

\[
\mathbf{Aa} = 0 \tag{12}
\]

where the matrix \( \mathbf{A} \) has the following elements

\[
a_{mn} = \begin{cases} (v(k-m) - \omega)^2 - (k-m)^2 - s, & m = n; \\ \frac{1}{2}\sigma s, & m = n \pm 1; \\ 0, & \text{otherwise} \end{cases} \tag{13}
\]

Here, \( a_{mn} \) denotes the element in the \( m \)-th row and the \( n \)-th column of the matrix \( \mathbf{A} \).

Combining equations (6), (8), and (11), the solution to equation (5) can be written as

\[
u(x,t) = \sum_{m=-\infty}^{+\infty} a_m e^{i[(m-k)x+s]} \tag{14}
\]

where the frequency is \( \omega \) and the compound wavenumber is \( m-k \). From equation (14), it follows that for a continuous periodic system, the corresponding compound wave comprises infinite components with the same frequency, but different wavenumbers.

4. Dispersion relation and band gaps

4.1 Dispersion relation

To obtain the non-trivial roots of equation (12), the determinant of the matrix \( \mathbf{A} \) should be equal to zero, which leads to the following eigenfunction

\[
f(\omega,k,v,s,\sigma) = \det(\mathbf{A}) = \prod_{m=-\infty}^{+\infty} \left( (v(k-m) - \omega)^2 - (k-m)^2 - s \right) - \frac{1}{2}\sigma s \prod_{m=-\infty}^{+\infty} \left( (v(k-m) - \omega)^2 - (k-m)^2 - s \right) = 0 \tag{15}
\]

where \( f \) is an implicit function with five parameters: frequency \( \omega \), wavenumber \( k \), axial velocity \( v \), foundation stiffness \( s \), and stiffness modulation \( \sigma \). The dispersion relation \( \omega(k) \) can be determined by the given parameters. Thus, equation (15) can be rewritten as an implicit function with respect to \( \omega \) and \( k \), that is, \( f(\omega,k) = 0 \). For any integer \( m \), \( f(\omega,k+m) = f(\omega,k) \), that is, \( f(\omega,k) = 0 \) is an implicit periodic function of wavenumber \( k \) with the period 1, where 1 is the least positive period of \( f \). Uncertainty in the wavenumber \( k \) is best avoided by restricting \( k \) to the interior of the first Brillouin zone, that is, \( k \in [-0.5, 0.5] \).

In order to investigate the dispersion relation, the frequency and the wavenumber should be drawn out from eigenfunction. Equation (15) is a function of \((s\sigma)^2m\) where \( m \) is an integer. If \( s < 1 \) and \( \sigma < 1 \), then \((s\sigma)^2m \ll 1\); thus, the result is reliable when the higher order terms of the function are neglected. As an illustration, we start on the first band gap. Assuming that the foundation stiffness \( s = 0 \), or \( s \ll 1 \), it is reasonable to take into account only one term of the eigenfunction, that is, let \( m = 0 \), in equation (15). Then, it follows

\[
f = (vk - \omega)^2 - k^2 - s = 0 \tag{16}
\]

According to the definition of the group velocity \( dv/dk \) (Achenbach, 1975), one obtains

\[
c_g = \frac{d\omega}{dk} = \pm \frac{k}{\sqrt{k^2 + s}} \tag{17}
\]
The direction of wave propagation can be detected by the sign of the group velocity, namely, the positive sign denotes forward travelling wave and the negative one denotes backward travelling wave. Thus, with regards to the axially moving belt, the corresponding compound waves are divided into two sets with different wavenumbers, but the same frequency. However, each frequency corresponds to one wavenumber in the first Brillouin zone.

As an illustration, Figure 2 demonstrates the dispersion relation of the belt with different parameters in the case of band gaps vanishing. If the foundation stiffness decreases toward zero, the band gap vanishes, which agrees well with the result of (Sorokin and Thomsen, 2017) as shown in Figure 2(a). Where \( m \) represents the \( m \)th order mode, \( c_g^+ \) and \( c_g^- \) denote the group velocities of forward travelling wave and backward wave, respectively. The compound wave of the system is comprised of different modes which can be divided into two opposite directions. The slope of the lines denotes the group velocity of the wave propagation. The positive one represents the forward travelling wave and the negative one represents the backward travelling wave. The speed of the forward travelling wave increases with the axial velocity while the backward travelling wave speed decreases with the axial velocity, as demonstrated in Figure 2(b). When the axial velocity limits to the critical velocity, the slope of the lines, that is, the speed of the backward travelling wave approaches zero as shown in Figure 2(c). The maximum speed of the forward travelling wave is \( v + 1 \) which can be derived from equation (17) instantly, where \( v \) is the axial velocity.

### 4.2 The existence of band gaps

The dependence of eigenvalues on system parameters can be illustrated by a family of loci (Pierre, 1988). With the varying of foundation stiffness, the frequency curves veer, and eigenvalues diverge. Analogously, the eigenfunction mode shapes veering in the transition zone (Leissa, 1974), we find that the mode shapes of the wave change dramatically in the band gap. As shown in Figure 3(a), when \( \omega = 0.28 \) and \( \omega = 0.4 \), mode shapes in the band pass are normal, when \( \omega = 0.27 \) and \( \omega = 0.45 \), mode shapes are irregular in the first band gap in Figure 3(b) and the second band gap in Figure 3(c), respectively. The mode shapes undergo abrupt changes when the frequency changes from band pass to band gap. If the length of the compound wave modes becomes infinite, the wavenumber or reciprocal wave length approaches zero, that is, the wave attenuates in these areas. The survey of wave propagation in the vicinity of band gap shows that: the modes of wave are normal when the frequency falls in the band pass, while the modes become abruptly and the bounded amplitude is broken when the frequency falls in the band gap, where the wave propagation is stopped.

Another interpretation of wave attenuation is in terms of wave velocity. When the speed of forward travelling wave is equal to the backward one, the superposition of wave velocity is zero, that is, the wave propagation is stopped. These points in the frequency-wavenumber plane are named veering points. The mode shapes veering and the

![Figure 2](dispersion_relation.png)

**Figure 2.** Dispersion relation \( \omega(k) \), for (a) \( v = 0, s = 0 \); (b) \( v = 0.5, s = 0 \) and (c) \( s = 0.1, \sigma = 0.5, v = 0.999 \) (the critical velocity is 1 (Wickert and C. D. Mote, 1990)).
eigenvalues changing occur instantaneously at these points. As demonstrated in Figure 4(a), the slopes of the curves denote the group velocities of the wave, and $c_g^+$ and $c_g^-$ represent the same meaning as the previous section.

Figure 3. Mode shapes change in the band gaps, for $v = 0.5, s = 0.1, \sigma = 0.5$. (a) Frequencies in the vicinity of two band gaps; (b) mode shapes change in the first band gap; (c) mode shapes change in the second band gap.

Figure 4. The band gaps are marked by gray shaded areas. (a) $s = 0.1, v = 0, \sigma = 0$, (b) $s = 0.1, v = 0.5, \sigma = 0.5$, and (c) $s = 0.5, v = 0.5, \sigma = 0.5$.

Usually, the dynamical system with the axially moving belt can show nonlinear properties. The nonlinear band gaps can result in the nonreciprocal transmission of elastic waves (Li et al., 2018, 2019), which is in accord with the wave attenuation of the present paper.

Actually, the number of the band gaps is dependent on the terms of the expansion in equation (15), and the maximum number of the band gaps is $2m + 1$. If $m = 0$, then
2m + 1 = 1, that is, the maximum number of the band gaps is one. If the expansion equation (15) is restricted in the first Brillouin zone, let \( \sigma = 0 \), equation (15) degenerates to the same case of \( m = 0 \). The dispersion relation is as shown in Figure 4(a) which has only one band gap. When \( m = 1 \), there will be three band gaps as shown in the remaining of Figure 4, but it is too tiny to be observed. If \( m \) is not restricted, there are also fourth band gap or higher band gaps. Sorokin and Thomsen (Sorokin and Thomsen, 2017) analyzed the band gap by employing the perturbation method, they found only one band gap because they truncated the perturbation parameter in second order, so they missed the higher frequency band gaps.

Magnifying the third band gap and fourth band gap of Figure 4(b) a thousand and five thousand times respectively yields the enlarged views as shown in Figure 5. Three different values of \( \sigma \) are given, which demonstrates that the

**Figure 5.** The partial enlarged views of the third band gap (a) and the fourth band gap (b) of the axially moving belt with parameters \( s = 0.1, \nu = 0.5 \) and the stiffness modulation \( \sigma = 0.1, \sigma = 0.3, \) and \( \sigma = 0.5 \), respectively.

**Figure 6.** The dependent surface of critical frequency on the axial velocity and the foundation stiffness in the first band gap.

**Figure 7.** Frequency versus foundation stiffness. The band gaps are blank areas. (a) \( \nu = 0.5, \sigma = 0.2 \); (b) \( \nu = 0.5, \sigma = 0.5 \).
stiffness modulation effects on the second or higher band gap. If $\sigma$ decreases toward zero, the width of the second or higher band gap approaches zero. Thus, the existence and the number of the band gap are dependent upon the periodic foundation stiffness and the stiffness modulation of the axially moving belt. The system, at least, has one band gap if the foundation stiffness exists. There will be more than two band gaps if the stiffness modulation appears with the foundation stiffness. The higher band gaps accompany with foundation stiffness and the stiffness modulation.

5. Tuning band gaps by varying parameters

The band gap can be manipulated by varying the parameters, such as the stiffness modulation and the foundation stiffness, which make the vibration frequency fall in the band gaps. Considering the influence of various parameters on the band gaps, the relations of the band gaps with the foundation stiffness and the stiffness modulation will be investigated in this section.

![Figure 8](image1.png)  
**Figure 8.** Frequency versus axial velocity. The band gaps are blank areas. (a) $s = 0.1$, $\sigma = 0.2$ and (b) $s = 0.1$, $\sigma = 0.5$.

![Figure 9](image2.png)  
**Figure 9.** Frequency versus stiffness modulation. The band gaps are blank areas. (a) $\nu = 0.5$, $s = 0.1$ and (b) $\nu = 0.5$, $s = 0.3$.

5.1 Varying the foundation stiffness

Considering the influence of the foundation stiffness on the band gaps, by controlling the frequency in the interval of the band gap, the wave propagation is stopped and the vibration is suppressed. For the stiffness modulation is small ($\sigma < 1$), the influence is neglectable in the first band gap. To calculate the first band gap in the first Brillouin zone, it is reasonable to take into account only one term of $f(m = 0)$, when $s < 1$, $\sigma < 1$. As discussed in Section 4.1, according to equations (16) and (17), when the foundation stiffness vanishes, the group velocities are $c_g = \nu \pm 1$, as shown in Figure 2(a). The wave propagations show in two opposite directions. When the group velocity $d\omega/dk$ vanishes, that is, the group velocity is zero at the cut-off frequencies, then the wave propagation is stopped (Achenbach, 1975). Thus, the wave attenuation occurs at the veering point

$$k = \nu \sqrt{\frac{s}{1 - \nu^2}}$$ (18)
The critical frequency with respect to stiffness and axial velocity is obtained explicitly
\[ \omega_c = \sqrt{s(1 - v^2)} \]
the first band gap is below the critical frequency \( \omega_c \). The width of the first band gap is presented as \( |\omega_c| \). As shown in Figure 6, the cut-off frequency of the first band gap decreases with the axial velocity but increases with the foundation stiffness.

An example of the vibration suppression by tuning the foundation stiffness is given. The initial state of the system with the frequency \( \omega_1 \), axial velocity \( v_1 \), and the mean foundation stiffness \( s_1 \) induces a band gap. When the axial velocity varies from \( v_1 \) to \( v_2 \), the corresponding frequency will change from \( \omega_1 \) to \( \omega_2 \). If we want to maintain the system still in the band gap for the new axial velocity \( v_2 \) without changing the value of the frequency, the stiffness can be tuned. As demonstrated in Figure 6, the critical frequency remains stationary in the first band gap, that is, when the axial velocity varies from \( v_1 \) to \( v_2 \), and the foundation stiffness varies from \( s_1 \) to \( s_2 \), the frequencies \( \omega_1 = \omega_2 \). With the aid of equation (16), the relation of the velocity and the stiffness can be computed as
\[ \Delta s = \frac{s_2^2 - s_1^2}{1 - v_2^2} \]
where \( \Delta s = s_2 - s_1 \). Thus, the vibration of the system caused by the velocity perturbation can be suppressed by varying the foundation stiffness, and vice versa, if the vibration originates from the foundation stiffness, the band gap can be tuned by axial velocity
\[ \Delta v = \frac{-\omega_1 s_2 - \sqrt{s_2^2 - (1 - v_1^2)s_1 s_2}}{s_2} \]
where \( \Delta v = v_2 - v_1 \).

Considering more than one band gap, the foundation stiffness has a significant influence on the first band gap as shown in Figure 7. The similar result was discussed by Mustafa et al. (Mustafa et al., 2018). The cut-off frequency and the width of the band gap both increase with the foundation stiffness. The band gap will vanish if the foundation stiffness vanishes which can be drawn by comparing Figures 2(a) and 4(a). According to equation (4), the dimensionless foundation stiffness \( s \) increases with the structure period \( \Phi \). If \( \Phi \) decreases toward zero, the foundation stiffness decreases toward zero which means that the band gap vanishes. Moreover, in contrast with the case of \( \Phi \) increasing, the foundation stiffness \( s \) increasing, which means that the first band gap increases with the structure period. The band gap increases with the foundation stiffness, but decreases with the axial velocity, as seen in Figure 8. Both the cut-off frequency and the band gap decrease with the velocity, but the width of second band gap is not sensitive to it. Note that, there are more than two band gaps in Figure 8, if one magnifies the figures, the tiny higher band gaps will emerge.

5.2 Varying the stiffness modulation

The band gap also can be manipulated by varying the stiffness modulation. To obtain the second band gap, we consider \( f \) with two terms \( m = 0 \) and 1, and it follows that
\[ f = (v k - \omega)^2 - k^2 - s \]
\[ - (k - 1)^2 - s - \frac{1}{4} s^2 a^2 = 0 \]
\[ \frac{a}{d f} = \frac{-d f/dk}{d f/d\omega} \]
\[ c_g = \frac{d \omega}{d k} = \frac{\omega}{d f/d\omega} \]

Similar to the previous subsection, the wave attenuation occurs at the veering point \( \omega_k \), when \( c_g = 0 \). The band gaps are obtained numerically as depicted in Figures 7–9.

The second band gap is more sensitive to the stiffness than the first one. The lower cut-off frequency of the second band gap decreases with the stiffness modulation while the upper cut-off frequency increases with it. Thus, the width of the second band gap increases with the stiffness modulation. Assuming the stiffness modulation \( \sigma < 1 \), the first band gap almost remains stationary when the stiffness modulation \( \sigma \) changes, as shown in Figure 9. In contrast to the case illustrated in Figure 7, the first band gap depends more on the foundation stiffness \( s \), while the width of the second one is almost independent of the axial velocity, as depicted in Figure 8. Note that, with the increasing of the stiffness modulation, the third band gap emerges, as shown in Figure 9(b).

Based on the analysis above, if the frequency need to be suppressed locates in the range of the second band gap, the effect of tuning the stiffness modulation is much better than tuning the foundation stiffness. Hence, by the fine design of the periodic foundation, the vibration of the axially moving belt can be suppressed for a specific frequency.

By making a comprehensive comparison of Figures 7–9, the foundation stiffness dominates the first band gap and the stiffness modulation dominates the second and higher band gaps. Both the band gaps and the cut-off frequency increase with the foundation stiffness but decrease with the axial velocity. It should be noted that the lower cut-off frequency of the second band gap decreases with the stiffness modulation and the upper cut-off frequency increases with the stiffness modulation, which means that the first band gap with the stiffness modulation.
6. Conclusions

In this paper, the relation of the band gap with the system parameters are obtained. The band gaps can be manipulated actively by tuning the system parameters, and the programmable structure can be designed according to the relations of the system parameters. Three main conclusions are as follows.

1. The eigenvalues divergence and the mode shapes veering at veer points are two different manifestations of the wave attenuation of the system which lead to the wave propagation stop.

2. The band gaps vanish when the foundation stiffness approaches zero, and the higher band gaps are more than the second further depend on the stiffness modulation. The foundation stiffness dominates the first band gap, and the stiffness modulation dominates the higher band gaps.

3. The width of band gaps increases with the foundation stiffness and the stiffness modulation but decreases with the axial velocity.

Based on the conclusions of band gaps, any frequency vibrations of the axially moving belts can be suppressed by the designed foundation with varying stiffness. The current study may trigger new ideas to lower the vibrations of moving structures.

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ORCID iD
Lei Lu @ https://orcid.org/0000-0002-1855-236X

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