Renormalization Group Improved Higgs Inflation with a Running Kinetic Term

Fuminobu Takahashi\textsuperscript{1,2} and Ryo Takahashi\textsuperscript{3}

\textsuperscript{1}Department of Physics, Tohoku University, Sendai 980-8578, Japan
\textsuperscript{2}Kavli Institute for the Physics and Mathematics of the Universe (WPI), TODIAS, University of Tokyo, Kashiwa 277-8583, Japan
\textsuperscript{3}Graduate School of Science, Tohoku University, Sendai 980-8578, Japan

Abstract

We study a Higgs inflation model with a running kinetic term, taking account of the renormalization group evolution of relevant coupling constants. Specifically we study two types of the running kinetic Higgs inflation, where the inflaton potential is given by the quadratic or linear term potential in a frame where the Higgs field is canonically normalized. We solve the renormalization group equations at two-loop level and calculate the scalar spectral index and the tensor-to-scalar ratio. We find that, even if the renormalization group effects are included, the quadratic inflation is ruled out by the CMB observations, while the linear one is still allowed.
1 Introduction

A Higgs boson was discovered at the LHC [1,2], which completed the last missing piece of the standard model (SM). While the properties of the Higgs boson are consistent with the expected ones for the SM Higgs boson within experimental uncertainties, a deeper understanding of the Higgs sector might be a key to unravel various phenomena which cannot be explained within the SM, such as dark matter, baryogenesis, etc. In fact, the Higgs field can play a role of the inflaton field responsible for cosmic acceleration in the early Universe [3–7]. There have been proposed a variety of Higgs inflation models (see e.g. Refs. [8–35] and references therein).

The successful Higgs inflation requires a rather flat potential. At large field values, the SM Higgs potential at tree-level is approximately given by a quartic potential with a coefficient of order $0.1$, which, however, is too steep to drive successful inflation and the density perturbations would be too large if we extrapolate the potential up to super-Planckian values. Therefore, the Higgs potential must be somehow modified at large field values for successful inflation. In this paper, we consider the Higgs inflation with a running kinetic (RK) term [16,24], where the kinetic term of the Higgs field is allowed to depend on the Higgs field itself. Such inflation models with a RK term as well as their phenomenological and cosmological implications were studied in detail in Refs. [36,37].

In the simplest realization, the kinetic term of the Higgs field is given by [16,24]

$$\mathcal{L}_K = \frac{1}{2} \left( 1 + \xi h^2 \right) (\partial h)^2,$$

where $\xi$ is a numerical coefficient and $h$ is the Higgs field. Thus, at sufficiently large field values, the quartic potential becomes a simple quadratic potential when expressed in terms of a canonically normalized field, $\hat{h} \propto h^2$. In contrast to the original Higgs inflation with a non-minimal coupling to gravity [9], the predicted tensor-to-scalar ratio, $r$, is relatively large, which is therefore tightly constrained by the recent Planck and BICEP2/Keck Array experiments [38,39].

It is known that the Higgs potential is modified by the running effects of coupling constants, and it is especially sensitive to the top Yukawa coupling. So far, the Higgs inflation model with a RK term was studied only at tree-level, and therefore, it was not clear if the simplest realization of the model is still allowed by observations, once one properly takes account of the running effects.

In this paper we take into account the running effects of relevant coupling constants under renormalization group equations (RGEs) at two-loop level with the experimentally observed value of the Higgs mass $M_h = 125.6$ GeV, and study the inflation dynamics of the Higgs inflation with a RK term. Then we calculate the scalar spectral index $n_s$ and the tensor-to-scalar ratio, and compare them with the observations. We will show that the recent Planck and BI-
CEP2/Keck Array results rule out the simplest realization where the inflaton potential is given by the quadratic term, even if one takes account running effects of coupling constants under RGEs. We also study another realization of the RK Higgs inflation where the inflaton potential is given by a linear term, which is shown to be consistent with the observations even if the running effects are taken into account.

The rest of this paper is organized as follows: In the next section, we investigate the RK Higgs inflation with a quadratic or linear term, taking account of the RGEs for the relevant coupling constants. The last section is devoted to discussion and conclusions. In the Appendix we give $\beta$-functions up to two-loop level for the RGEs of relevant coupling constant.

2 RGE improved RK Higgs inflation

In this section we investigate two types of the RK Higgs inflation model, including the running effects of relevant coupling constants by solving their RGEs. We first study the simplest realization where the inflaton potential is given by the quadratic term, and compare the predicted $n_s$ and $r$ with observations. We also study another realization where the inflaton potential is a linear term.

2.1 Quadratic model

The base model of the RK inflation is given by \cite{16, 18, 24, 36, 37}

$$\mathcal{L} = \frac{1}{2}(1 + \xi \phi^2)(\partial \phi)^2 - V(\phi),$$

(2)

where $\xi$ is a positive numerical coefficient much larger than unity, $\phi$ is the inflaton, and $V(\phi)$ is the inflaton potential. Here and in what follows we adopt the Planck units where the reduced Planck mass $M_{\text{Pl}}$ is set to be unity. The canonically normalized inflaton field is given by $\hat{\phi} \sim \sqrt{\xi} \phi^2$ at $\phi \gtrsim 1/\sqrt{\xi}$, because the kinetic term is dominated by the $\xi \phi^2$ term at large field values. Thus, the scalar potential changes its form above the critical value,

$$V(\phi) \rightarrow V(\hat{\phi}^{1/2}/\xi^{1/4}).$$

(3)

For instance, if the scalar potential in the original frame contains the quartic term $\phi^4$, it turns into the quadratic one, $\hat{\phi}^2/\xi$, at $\phi \gtrsim 1/\sqrt{\xi}$ when expressed in terms of the canonically normalized field. Thus, the inflaton potential becomes flatter at large field values due to the RK term.

In order to have successful large-field inflation, one has to keep a handle on the interactions of the inflaton, especially the large coupling in the kinetic term, at super-Planckian field values.
One possible way is to impose a shift symmetry on $\phi^2$,

$$\phi^2 \rightarrow \phi^2 + C, \quad (4)$$

where $C$ is a (real) transformation parameter. Then, the canonically normalized inflaton field $\hat{\phi}$ is necessarily proportional to $\phi^2$ at sufficiently large field values, as the form of the kinetic term is dictated by the symmetry. In this case, the largeness of $\xi$ can be understood because the ordinary kinetic term as well as the potential breaks the shift symmetry, and they should be accompanied by a small order parameter. Once one normalizes the inflaton field $\phi$ so that it is canonically normalized at $\phi = 0$, the small order parameter is translated to the large $\xi$.

We apply the above base model to the SM Higgs field $H$, and the relevant part of the Lagrangian is given by

$$L \supset \frac{1}{2}(1 + \xi h^2)(\partial h)^2 - \frac{\lambda}{4}(h^2 - v^2)^2, \quad (5)$$

where $h$ is the physical Higgs field. The largeness of $\xi$ in Eq. (2) can be explained by the smallness breaking of the shift symmetry,

$$|H|^2 \rightarrow |H|^2 + C, \quad (6)$$

which keeps the potential under control at large field values. For $h \gg 1/\sqrt{\xi}$, the relevant terms in Eq. (5) are given by

$$L \approx \frac{1}{2}(\partial \hat{h})^2 - \frac{\lambda}{\xi} \hat{h}^2, \quad (7)$$

with the canonically normalized field $\hat{h} \equiv \sqrt{\xi} h^2 / 2$. Thus, the quadratic chaotic inflation takes place if $\hat{h}$ is initially located at large field values.

The largeness of $\xi$ in Eq. (2) may raise doubts in the validity of the RK inflation. Namely, the high energy scattering amplitudes for $\phi\phi \rightarrow \phi\phi$ imply that the system enters into a strongly-coupled regime at energy scales much below the Planck scale. On the other hand, as we have seen above, the large $\xi$ arises from a small shift-symmetry breaking parameter, and clearly the inflation dynamics is described in a weakly-coupled regime. The apparent tension can be understood by noting that the perturbative unitarity bound is actually field-dependent \[40\], and that the kinetic term grows as the inflaton field increases in a controlled way thanks to the shift symmetry. First, the cutoff scale due to the non-minimal kinetic term is

$$\Lambda(h) \sim \begin{cases} M_{pl}/\sqrt{\xi} & \text{for } h \lesssim \frac{M_{pl}}{\sqrt{\xi}} \\ \sqrt{\xi h^2}/M_{pl} & \text{for } h \gtrsim \frac{M_{pl}}{\sqrt{\xi}} \end{cases}, \quad (8)$$

3
while the cutoff due to purely gravitational interactions is of order the Planck mass, and we explicitly show the dependence of the Planck mass for clarity. Therefore, during inflation when \( h > M_{\text{pl}}/\sqrt{\epsilon} \), the cutoff is of order the Planck mass, and there is no problem in using the effective field theory to describe the inflaton dynamics. The situation is quite analogous to the Higgs inflation with a non-minimal coupling to gravity. Indeed, as noted in Ref. [27], for a certain range of the Higgs field, the quadratic potential can also be obtained if one has the following Higgs-gravity coupling,

\[
S = \int d^4x \sqrt{-g} \left( \frac{M_{\text{pl}}^2}{2} f(h) R + \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h - V(h) \right)
\]

with

\[
f(h) = 1 + \frac{h^2}{\sqrt{6} \epsilon M_{\text{pl}}^2}.
\]

In the Einstein frame, the kinetic term of \( h \) is

\[
\mathcal{L}_K = \frac{1}{2} \left( \frac{1}{f} + \frac{3f'^2 M_{\text{pl}}^2}{2f^2} \right) \partial_\mu h \partial^\mu h,
\]

where the prime denotes the derivative with respect to \( h \). For \( \epsilon M_{\text{pl}} \lesssim h \lesssim \sqrt{\epsilon} M_{\text{pl}} \), the above kinetic term can be approximated by

\[
\mathcal{L}_K \simeq \frac{1}{2} \left( 1 + \epsilon^2 \frac{\tilde{h}^2}{M_{\text{pl}}^2} \right) \partial_\mu h \partial^\mu h \simeq \frac{1}{2} \partial_\mu \tilde{h} \partial^\mu \tilde{h}
\]

which is the same as Eq. (5) if \( \xi = 1/\epsilon^2 \). This implies that the RK Higgs inflation is equivalent to the Higgs inflation with the above Higgs-gravity coupling for \( \tilde{h} \lesssim M_{\text{pl}} \), and that the RGE effects of the relevant couplings can be similarly taken into account. At super-Planckian field values, the inflaton potential is still given by the quadratic potential in our model because of the shift symmetry. Secondly, the form of the kinetic term is determined by the shift symmetry, and therefore, it is robust against radiative corrections. In particular, the canonically normalized inflaton \( \tilde{h} \) is always proportional to \( h^2 \) at sufficiently large field values, and this relation is not spoiled by including radiative corrections. This implies that the inflaton potential can be well approximated by a quadratic mass term for a canonically normalized inflaton, as long as the scalar potential is dominated by a quartic term in Eq. (5). When one takes account of the running of coupling constants, there is a subtlety in the renormalization prescription, but this does not affect the validity of the RK inflation. In particular, our main result remains unchanged, and the reason for this will become clear shortly.

Now we study the above inflation model, including the running effects by solving the RGEs at two-loop level for relevant coupling constants, which enables precise comparison between
predictions and observations. The corresponding RGEs are given by

\[(4\pi)^2 \frac{dX}{dt} = \beta_X,\]  

(13)

where \(X\) collectively denotes the SM gauge coupling constants \(g_i\) \((i = 1, 2, 3)\), the top Yukawa coupling \(y_t\), and the Higgs quartic coupling \(\lambda\). \(t\) is defined by \(t \equiv \ln(\mu/1 \text{ GeV})\) where \(\mu\) is the renormalization scale. We numerically solve the RGEs within the renormalization scale of \(M_Z \leq \mu \leq m_{\text{pl}}\) where \(M_Z\) is the \(Z\) boson mass \(M_Z = 91.2\) GeV and \(m_{\text{pl}}\) is the Planck mass \(m_{\text{pl}} = 1.22 \times 10^{19}\) GeV. The \(\beta\)-functions for the coupling constants are given in the Appendix.

In the analysis, we take the Higgs mass to be \(M_h = 125.6\) GeV.

The metric perturbations are usually characterized by the scalar spectral index \(n_s\) and the tensor-to-scalar ratio \(r\). In our analysis we adopt the first-order expressions, \(n_s = 1 - 6\varepsilon + 2\eta\) and \(r = 16\varepsilon\), respectively. Here the slow roll parameters (in the Planck units) are defined as

\[\varepsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = \frac{V''}{V},\]  

(14)

where \(V' \equiv dV/d\hat{\phi}\) and \(V'' \equiv d^2V/d\hat{\phi}^2\). Here \(\hat{\phi}\) is the canonically normalized inflaton (Higgs) field. The inflation ends when either of \(\varepsilon\) or \(|\eta|\) exceeds the unity. The e-folding number \(N\) is given by

\[N = \int_{\hat{\phi}_0}^{\hat{\phi}_{\text{end}}} \frac{V}{V'} d\hat{\phi},\]  

(15)

where \(\hat{\phi}_0\) and \(\hat{\phi}_{\text{end}}\) are the initial and final field values during the inflation, respectively. We have evaluated \(n_s\) and \(r\) for the e-folding number \(N\) between 50 and 60, and obtained the following results,

\[0.960 \lesssim n_s \lesssim 0.967, \quad 0.132 \lesssim r \lesssim 0.159 \quad \text{for} \quad 50 \leq N \leq 60,\]  

(16)

where we take the scalar amplitude as \(A_s = (2.196^{+0.051}_{-0.060}) \times 10^{-9}\), and set the prior on the parameter \(\xi\) as \(\xi \gtrsim 1.36 \times 10^5\) and the top mass as \(M_t \leq 171.2\) GeV, for numerical stability. We have found that the values of \(n_s\) and \(r\) are rather robust against the renormalization-group effects, and their variations with respect to the top mass are very small, and approximately given by \(\delta n_s \simeq 2 \times 10^{-5}\) and \(\delta r \simeq 10^{-4}\) for a fixed e-folding number. Here we vary the top quark mass as \(168\) GeV \(< M_t < 171.2\) GeV. Thus, even if the running effects of coupling constants under RGEs are taken into account, the base RK Higgs inflation model with a quadratic potential is ruled out by the recent Planck [38] and BICEP2/Keck Array [39] results, which placed an upper bound on \(r\) as \(r < 0.07\) at 95% CL.
The reason for the robustness of $n_s$ and $r$ can be understood as follows. Even though the Higgs quartic coupling is sensitive to the top quark mass, as long as it is positive during inflation, the inflaton potential is still well approximated by the quadratic potential (plus small logarithmic corrections) except for the top quark mass $M_t \gtrsim 170$ GeV. The values of $n_s$ and $r$ are sensitive only to the shape of the potential, and they do not depend on the overall normalization of the potential, which is determined by the size of the quartic coupling at large field values, and that is why their values are robust against including the RGE effects. On the other hand, the value of $\xi$ is determined by the normalization of the curvature perturbations, and therefore, it is sensitive to the absolute value of $\lambda$ and $M_t$.

In Figure 1, we show the dependence of $\xi$ on $M_t$; the black line indicates the dependence in this quadratic potential model. As the top quark mass increases up to $M_t \simeq 170$ GeV, the value of $\lambda$ becomes smaller and smaller, and in order to generate the curvature perturbations of the right magnitude, the value of $\lambda$ and $\xi$ must satisfy

$$\frac{\lambda}{\xi} \simeq 2 \times 10^{-11},$$

at a scale relevant for inflation. This explains the behavior of $\xi$ in Figure 1.

For $M_t \gtrsim 170$ GeV, the quartic coupling $\lambda$ becomes even smaller, and at a certain point, the inflaton potential develops local maximum and minimum. In order for the inflaton to roll down to the electroweak vacuum, its initial position $h_{\text{ini}}$ must be smaller than $h_{\text{max}}$ at which the inflaton potential takes the local maximum. Thus, the quartic coupling $\lambda$ during inflation cannot be arbitrarily small, and it is bounded below [26],

$$\lambda(h) \gtrsim \lambda(h_{\text{max}}) \simeq 5 \times 10^{-6}. \quad (18)$$

Thus, in order to explain the observed density perturbations, $\xi$ is also bounded below as $\xi \gtrsim 3 \times 10^5$, which is consistent with Figure 1. As $M_t$ further increases, the location of the local maximum becomes smaller than $M_{\text{pl}}$. As a result, one of the slow-roll parameters, $|\eta|$, exceeds unity, and the slow-roll inflation becomes difficult to realize. This explains why the allowed region rapidly shrinks at $M_t \gtrsim 170$ GeV.

### 2.2 Linear model

Let us now consider a RK Higgs inflation with a linear potential. The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}(1 + \xi^3 \phi^6)(\partial \phi)^2 - \frac{\lambda}{4} \phi^4. \quad (19)$$

At large field values, $\phi \gtrsim 1/\sqrt{\xi}$, the canonically normalized inflaton field is $\hat{\phi} \sim \xi^{3/2} \phi^4$, and the effective inflaton potential becomes a linear potential. Thus, the inflaton potential becomes flatter compared to the quadratic one, and the tensor-to-scalar ratio is expected to be suppressed.
Figure 1: The dependence of $\xi$ on the top quark mass $M_t$ in the RGE improved Higgs inflation with RK term models. Black (top) and red (bottom) lines correspond to models with quadratic and linear potentials, respectively.

In order to apply the above model to the SM Higgs field $H$, we introduce a shift symmetry

$$|H|^4 \rightarrow |H|^4 + C,$$

and introduce the ordinary kinetic term and the Higgs potential as small explicit breaking of the symmetry. We consider the following Lagrangian,

$$\mathcal{L} \supset \frac{1}{2}(1 + \xi^3 h^6)(\partial h)^2 - \frac{\lambda}{4}(h^2 - v^2)^2,$$

where $h$ is the physical Higgs. At large field values as $h \gg \xi^{-3/4}$, the relevant terms in Eq. (5) are given by

$$\mathcal{L} \approx \frac{1}{2}(\partial \hat{h})^2 - \frac{\lambda}{\xi^{3/2}} \hat{h},$$

with the canonically normalized field $\hat{h} \equiv \xi^{3/2} h^4/4$. Thus, the linear inflation is realized.

We solve the inflaton dynamics, taking into account of the RGE evolution of the relevant couplings. Then we obtain the following results for $n_s$ and $r$, model leads

$$0.970 \lesssim n_s \lesssim 0.975, \quad 0.066 \lesssim r \lesssim 0.079 \quad \text{for} \quad 50 \leq N \leq 60,$$

(23)
where we take the scalar amplitude as $A_s = (2.196^{+0.051}_{-0.060}) \times 10^{-9}$, and set a prior on the parameter $\xi$ as $\xi \gtrsim 7.01 \times 10^8$, and the top mass as $M_t \leq 171.1$ GeV for numerical stability. We have found that, similar to the quadratic inflation, $n_s$ and $r$ are robust against including the RGE evolution of the relevant couplings, since they are sensitive to the inflaton potential shape only, which changes only logarithmically due to the RGE effects. Specifically, we have found the values of $n_s$ and $r$ vary with respect to the top quark mass as $\delta n_s \simeq 2 \times 10^{-5}$ and $\delta r \simeq 10^{-4}$ for a fixed e-folding number. Here we vary the top quark mass as $168 \text{GeV} < M_t < 171.1 \text{GeV}$. Thus, this model is consistent with the CMB observation [38,39]. The value of $\xi$, on the other hand, depends on the top quark mass through the RGE running of the Higgs quartic coupling. We show the dependence as red (bottom) lines in Fig. 1. As expected, the value of $\xi$ decreases as the Higgs quartic asymptotes to zero at $M_t \simeq 171$ GeV.

3 Discussion and Conclusions

In our analysis we set an upper bound of the top quark mass $M_t \lesssim 171$ GeV. This is because the Higgs quartic coupling must be positive at a scale relevant for inflation. The upper bound on the top quark mass, taken at a face value, is out of the experimentally measured value ($M_t = 173.34 \pm 0.76$ GeV [41]). This discrepancy, however, is not so problematic [29,42], because the experimental value is derived as an invariant mass for the final states of color singlets to fit data while $M_t$ in the above calculation of the RGEs is the pole mass. There might be a few GeV discrepancy between the singlet final state and the color octet $t \bar{t}$ pair, which is dominant at the hadron collider [43]. That said, it is also possible to have successful RK Higgs inflation for the top quark mass close to the experimental central value, by introducing other particles such as a singlet scalar dark matter with a TeV mass and heavy right-handed neutrinos. Since the new particles modify the $\beta$-functions of the coupling constants, the Higgs quartic coupling remains positive at a higher scale for a heavier top mass [28,30,41,43]. In such an extension, the model can also explain dark matter by the singlet scalar, and small active neutrino masses and baryon asymmetry of the Universe by heavy right-handed neutrinos, respectively. The extension for this model will be given in another publication.

The RK Higgs inflation with a quadratic potential predicted a too large tensor-to-scalar ratio. In this paper we have considered another form of the kinetic term so that the effective potential is a linear term. Another way to modify the inflaton potential is to add higher dimensional operators. Then the inflaton potential is given by a polynomial instead of a monomial one [40].

Our Universe has experienced an accelerating expansion in the early epoch, i.e. the so-called inflation. Yet it remains unknown what the inflaton is. The recently discovered Higgs particle is the unique elementary scalar particle among the known particles, and it might play a role of the
inflation. The idea of the Higgs inflation has attracted much attention, and studied extensively in the literatures. In this paper, we have focused on the so-called RK Higgs inflation, where the kinetic term is allowed to depend on the Higgs field itself and therefore the effective potential becomes flatter at large field values. So far, the RK Higgs inflation was studied only at tree level, and therefore, it was not clear if its simplest realization \[^1\] is still allowed by the CMB observations.

In this paper we have first considered the running effects of various coupling constants under the RGEs on two types of the RK Higgs inflation. One of the models has the quadratic potential and the other has a linear potential. We have shown that the quadratic potential model leads $0.960 \lesssim n_s \lesssim 0.967$ and $0.132 \lesssim r \lesssim 0.159$ for the e-folding number $N$ between 50 and 60, and that the values of $n_s$ and $r$ are quite robust against including the RGE effects. In fact, their variations are $\delta n_s \simeq 2 \times 10^{-5}$ and $\delta r \simeq 10^{-4}$ for a fixed e-folding number and the top quark mass $168 \text{ GeV} < M_t < 171.2 \text{ GeV}$. Thus, the RK Higgs inflation model with a quadratic potential is ruled out by the recent Planck and BICEP2/Keck Array ($r < 0.07$) \[^{38,39}\] result even if one takes account running effects of coupling constants under RGEs. Next, we have discussed the model with a linear potential. Similarly, we have shown that $n_s$ and $r$ are robust against including the running effects, and they are given by $0.970 \lesssim n_s \lesssim 0.975$ and $0.066 \lesssim r \lesssim 0.079$ with $\delta n_s \simeq 2 \times 10^{-5}$ and $\delta r \simeq 10^{-4}$, which are consistent with the current observations. While $n_s$ and $r$ are not significantly modified by the RGE effects, the coefficient $\xi$ in the kinetic term is sensitive to the running effects. This is because, while $n_s$ and $r$ are sensitive only to the inflaton potential shape, the magnitude of the curvature perturbation is also sensitive to the overall scale of the inflaton potential. As a result, $\xi$ varies with respect to the top quark mass, and we have shown that it decreases toward the critical value of the top quark mass at which the Higgs quartic coupling becomes extremely small at a scale relevant for inflation.

One of the virtues of the Higgs inflation with a RK term is that the predicted value of tensor-to-scalar ratio is larger than the Higgs inflation with a non-minimal coupling to gravity; the typical size of $r$ is of $\mathcal{O}(0.01 - 0.1)$. Future observations of the CMB B-mode polarization will refute or support the RK Higgs inflation with a linear term or even flatter or polynomial potential.

**Acknowledgments**

This work was supported by the Grant-in-Aid for Scientific Research on Scientific Research A (No.26247042 [FT]), Scientific Research B (No. 26287039 [FT]), Young Scientists B (No. 24740135 [FT]) and Innovative Areas (No. 23104008 and No.15H05889 [FT]). This work was supported by World Premier International Research Center Initiative (WPI Initiative), MEXT,
Appendix: $\beta$-functions

We give the $\beta$-functions of the relevant coupling constants at two-loop level. The $\beta$-functions for the gauge coupling constants are

$$
\beta_{g_1} = \frac{3}{5} \left( \frac{81 + s}{12} \right) g_1^3 + \frac{1}{16\pi^2} \frac{3}{5} g_1^3 \left[ \frac{199}{30} g_1^2 + \frac{9}{2} g_2^2 + \frac{44}{3} g_3^2 - \frac{17}{6} s y_t^2 \right],
$$

$$
\beta_{g_2} = -\frac{39 - s}{12} g_2^3 + \frac{1}{16\pi^2} g_2^3 \left[ \frac{9}{10} g_1^2 + \frac{35}{6} g_2^2 + 12 g_3^2 - \frac{3}{2} s y_t^2 \right],
$$

$$
\beta_{g_3} = -7 g_3^3 + \frac{1}{16\pi^2} g_3^3 \left[ \frac{11}{10} g_1^2 + \frac{9}{2} g_2^2 - 26 g_3^2 - 2 s y_t^2 \right].
$$

The $\beta$-functions for the top Yukawa and the Higgs are given by

$$
\beta_{y_t} = y_t \left[ \left( \frac{23}{6} + \frac{2}{3} s \right) y_t^2 - \left( \frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2 \right) \right]
+ \frac{1}{16\pi^2} \left[ \frac{1187}{600} g_1^4 - \frac{23}{4} g_2^4 - 108 g_3^4 - \frac{9}{20} g_1^2 g_2^2 + g_2 g_3^2 + \frac{19}{9} g_1^2 g_3^2 \right].
$$

$$
\beta_{\lambda} = 6(1 + 3 s^2) \lambda^2 - 6 y_t^4 + 12 \lambda y_t^2 - 3 \lambda \left( \frac{3}{5} g_1^2 + 3 g_2^2 \right) + \frac{3}{8} \left[ 2 g_4^2 + \left( \frac{3}{5} g_1^2 + g_2^2 \right)^2 \right]
+ \frac{1}{16\pi^2} \left[ - (48 + 288 s - 324 s^2 + 624 s^3 - 324 s^4) \lambda^3 \right]
+ \lambda^2 \left[ \frac{3}{5} (9 + 18 s + 9 s^2) g_1^2 + (27 + 54 s + 27 s^2) g_2^2 \right]
- \lambda \left[ - \frac{90 + 377 s + 162 s^2}{24} g_1^4 - \frac{3 - 18 s + 9 s^2}{4} \frac{3}{5} g_1^2 g_2^2 + \frac{181 + 54 s + 27 s^2}{8} g_4^2 \right]
+ \frac{912 + 3 s}{48} g_2^6 - \frac{290 - s}{48} g_2 g_4^2 - \frac{560 - 9}{48} g_1^4 g_2^2 - \frac{380 - s}{48} g_1^2 g_2^4 - \frac{193}{48} g_1^4 g_3^2 - \frac{21}{2} g_2^4
- \frac{9}{4} g_2^4 y_t^2 + \lambda y_t^2 \left( \frac{17}{2} g_1^2 + \frac{45}{2} g_2^2 + 80 g_3^2 \right) + \frac{3}{5} g_1^2 y_t^2 \left( - \frac{193}{4} g_1^2 + \frac{21}{2} g_2^2 \right)
- (36 + 108 s^2) \lambda^2 y_t^2 - (12 - 117 s + 108 s^2) \lambda y_t^4 + (38 - 8 s) y_t^6 \right],
$$

1 Another prescription to obtain the RGEs at one-loop level has been presented in [33]. In the work, the authors demand that the Higgs inflation model can be expanded by small parameters in large and mid field regime, and all loop corrections can be absorbed in counterterms at each loop level.
where we take the renormalization scale as $\mu = \phi$. We do not take into account the running effect of $\xi$ as we fix its value during inflation to generate density perturbations of the right magnitude, assuming its running effects on other coupling constants in SM are sufficiently small. Thus, the value of $\xi$ shown in our analysis should be understood as those evaluated during inflation. In addition, the cut-off in our model is field-dependent one and the model remains in a weak coupling regime during inflation like the ordinary Higgs inflation model (see e.g. Ref. [40]). Thus, the RGEs can be safely used from the low energy up to the inflationary scale.

$s$ is a factor which should be multiplied to all the loop-lines of scalar (Higgs) field $\phi$ ($h$). The factor $s$ is given by

$$ s = \left( \frac{d\phi}{d\phi} \right)^{-2} = \frac{1}{1 + \xi \phi^2}. $$

(29)

This procedure is similar to the case of Higgs inflation with non-minimal coupling [31] but note that the definition of $s$ is different from the Higgs inflation with non-minimal coupling. Lastly, we note that a non-zero non-minimal coupling is induced by the RGE effects (e.g., see [31,47,50] and references therein), but its value is considered to be small so that the inflationary prediction of $(n_s, r)$ is hardly modified.

References

[1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716 (2012) 1 [arXiv:1207.7214 [hep-ex]].

[2] S. Chatrchyan et al. [CMS Collaboration], JHEP 1306 (2013) 081 [arXiv:1303.4571 [hep-ex]].

[3] A. H. Guth, Phys. Rev. D 23 (1981) 347.

[4] A. A. Starobinsky, Phys. Lett. B 91 (1980) 99.

[5] K. Sato, Mon. Not. Roy. Astron. Soc. 195 (1981) 467.

[6] A. D. Linde, Phys. Lett. B 108 (1982) 389.

[7] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48 (1982) 1220.

[8] J. L. Cervantes-Cota and H. Dehnen, Nucl. Phys. B 442 (1995) 391 [astro-ph/9505069].

[9] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B 659 (2008) 703 [arXiv:0710.3755 [hep-th]].
[10] F. Bezrukov and M. Shaposhnikov, JHEP 0907 (2009) 089 [arXiv:0904.1537 [hep-ph]].

[11] M. B. Einhorn and D. R. T. Jones, JHEP 1003 (2010) 026 [arXiv:0912.2718 [hep-ph]].

[12] C. Germani and A. Kehagias, Phys. Rev. Lett. 105 (2010) 011302 [arXiv:1003.2635 [hep-ph]].

[13] S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, Phys. Rev. D 82 (2010) 045003 [arXiv:1004.0712 [hep-th]].

[14] H. M. Lee, JCAP 1008 (2010) 003 [arXiv:1005.2735 [hep-ph]].

[15] S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, Phys. Rev. D 83 (2011) 025008 [arXiv:1008.2942 [hep-th]].

[16] K. Nakayama and F. Takahashi, JCAP 1102 (2011) 010 [arXiv:1008.4457 [hep-ph]].

[17] K. Kamada, T. Kobayashi, M. Yamaguchi and J. Yokoyama, Phys. Rev. D 83 (2011) 083515 [arXiv:1012.4238 [astro-ph.CO]].

[18] M. P. Hertzberg, JCAP 1208, 008 (2012) [arXiv:1110.5650 [hep-ph]].

[19] I. Masina and A. Notari, Phys. Rev. D 85 (2012) 123506 [arXiv:1112.2659 [hep-ph]].

[20] I. Masina and A. Notari, Phys. Rev. Lett. 108 (2012) 191302 [arXiv:1112.5430 [hep-ph]].

[21] K. Kamada, T. Kobayashi, T. Takahashi, M. Yamaguchi and J. Yokoyama, Phys. Rev. D 86 (2012) 023504 [arXiv:1203.4059 [hep-ph]].

[22] K. Allison, JHEP 1402 (2014) 040 [arXiv:1306.6931 [hep-ph]].

[23] Y. Hamada, H. Kawai and K. y. Oda, PTEP 2014 (2014) 023B02 [arXiv:1308.6651 [hep-ph]].

[24] K. Nakayama and F. Takahashi, Phys. Lett. B 734 (2014) 96; [arXiv:1403.4132 [hep-ph]].

[25] J. L. Cook, L. M. Krauss, A. J. Long and S. Sabharwal, Phys. Rev. D 89 (2014) 10, 103525 [arXiv:1403.4971 [astro-ph.CO]].

[26] Y. Hamada, H. Kawai, K. y. Oda and S. C. Park, Phys. Rev. Lett. 112 (2014) 24, 241301 [arXiv:1403.5043 [hep-ph]].

[27] F. Bezrukov and M. Shaposhnikov, Phys. Lett. B 734 (2014) 249 [arXiv:1403.6078 [hep-ph]].
[28] N. Haba and R. Takahashi, Phys. Rev. D 89 (2014) 11, 115009 [Phys. Rev. D 90 (2014) 3, 039905] [arXiv:1404.4737 [hep-ph]].

[29] Y. Hamada, H. Kawai and K. y. Oda, JHEP 1407 (2014) 026 [arXiv:1404.6141 [hep-ph]].

[30] N. Haba, H. Ishida and R. Takahashi, PTEP 2015 (2015) 5, 053B01 [arXiv:1405.5738 [hep-ph]].

[31] H. J. He and Z. Z. Xianyu, JCAP 1410 (2014) 019 [arXiv:1405.7331 [hep-ph]].

[32] Y. Hamada, K. y. Oda and F. Takahashi, Phys. Rev. D 90 (2014) 9, 097301 [arXiv:1408.5556 [hep-ph]].

[33] D. P. George, S. Mooij and M. Postma, JCAP 1604 (2016) no.04, 006 [arXiv:1508.04660 [hep-th]].

[34] S. Di Vita and C. Germani, Phys. Rev. D 93 (2016) no.4, 045005 [arXiv:1508.04777 [hep-ph]].

[35] S. F. Ge, H. J. He, J. Ren and Z. Z. Xianyu, [arXiv:1602.01801 [hep-ph]].

[36] F. Takahashi, Phys. Lett. B 693 (2010) 140 [arXiv:1006.2801 [hep-ph]].

[37] K. Nakayama and F. Takahashi, JCAP 1011 (2010) 009 [arXiv:1008.2956 [hep-ph]]; JCAP 1011 (2010) 039 [arXiv:1009.3399 [hep-ph]].

[38] P. A. R. Ade et al. [Planck Collaboration], arXiv:1502.02114 [astro-ph.CO].

[39] P. A. R. Ade et al. [BICEP2 and Keck Array Collaborations], Phys. Rev. Lett. 116, 031302 (2016) [arXiv:1510.09217 [astro-ph.CO]].

[40] F. Bezrukov, D. Gorbunov and M. Shaposhnikov, JCAP 1110, 001 (2011) [arXiv:1106.5019 [hep-ph]].

[41] [ATLAS and CDF and CMS and D0 Collaborations], [arXiv:1403.4427 [hep-ex]].

[42] N. Haba, K. Kaneta, R. Takahashi and Y. Yamaguchi, Phys. Rev. D 91 (2015) 1, 016004 [arXiv:1408.5548 [hep-ph]].

[43] T. Horiguchi, A. Ishikawa, T. Suehara, K. Fujii, Y. Sumino, Y. Kiyo and H. Yamamoto, [arXiv:1310.0563 [hep-ex]].

[44] N. Haba, K. Kaneta and R. Takahashi, JHEP 1404 (2014) 029 [arXiv:1312.2089 [hep-ph]].
[45] N. Haba, H. Ishida, K. Kaneta and R. Takahashi, Phys. Rev. D 90 (2014) 036006 arXiv:1406.0158 [hep-ph].

[46] K. Nakayama, F. Takahashi and T. T. Yanagida, Phys. Lett. B 725, 111 (2013) arXiv:1303.7315 [hep-ph]; JCAP 1308, 038 (2013) arXiv:1305.5099 [hep-ph]; Phys. Lett. B 737, 151 (2014) arXiv:1407.7082 [hep-ph].

[47] E. Elizalde and S. D. Odintsov, Phys. Lett. B 321 (1994) 199 hep-th/9311087.

[48] E. Elizalde, S. D. Odintsov, E. O. Pozdeeva and S. Y. Vernov, Phys. Rev. D 90 (2014) no.8, 084001 arXiv:1408.1285 [hep-th].

[49] T. Inagaki, S. D. Odintsov and H. Sakamoto, Astrophys. Space Sci. 360 (2015) no.2, 67 arXiv:1509.03738 [hep-th].

[50] E. Elizalde, S. D. Odintsov, E. O. Pozdeeva and S. Y. Vernov, JCAP 1602 (2016) no.02, 025 arXiv:1509.08817 [gr-qc].