Kohn condition and exotic Newton-Hooke symmetry in the non-commutative Landau problem

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Abstract

$N$ “exotic” [alias non-commutative] particles with masses $m_a$, charges $e_a$ and non-commutative parameters $\theta_a$, moving in a uniform magnetic field $B$, separate into center-of-mass and internal motions if Kohn’s condition $e_a/m_a = \text{const}$ is supplemented with $e_a \theta_a = \text{const}$. Then the center-of-mass behaves as a single exotic particle carrying the total mass and charge of the system, $M$ and $e$, and a suitably defined non-commutative parameter $\Theta$. For vanishing electric field off the critical case $e \Theta B \neq 1$, the particles perform the usual cyclotronic motion with modified but equal frequency. The system is symmetric under suitable time-dependent translations which span a $(4 + 2)$-parameter centrally extended subgroup of the “exotic” [i.e., two-parameter centrally extended] Newton-Hooke group. In the critical case $B = B_c = (e \Theta)^{-1}$ the system is frozen into a static “crystal” configuration. Adding a constant electric field, all particles perform, collectively, a cyclotronic motion combined with a drift perpendicular to the electric field when $e \Theta B \neq 1$. For $B = B_c$ the cyclotronic motion is eliminated and all particles move, collectively, following the Hall law. Our time-dependent symmetries are reduced to the $(2+1)$-parameter Heisenberg group of centrally-extended translations.

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I. INTRODUCTION

Kohn’s theorem [1] says that a system of charged particles in a uniform magnetic field can be decomposed into center-of-mass and relative coordinates if the charge-to-mass ratios are the same for all particles,

\[ \frac{e_a}{m_a} = \text{const} = \frac{e}{M}, \]  

where \( e = \sum_a e_a \) and \( M = \sum_a m_a \) are the total charge and mass, respectively. Renewed interest in the question [2, 3] comes from its relation to Newton-Hooke symmetry [4–6].

For an isolated system, the possibility of having a center-of-mass decomposition relies on the non-trivial cohomology of the Galilei group [7]. In \( d \geq 3 \) space dimensions the latter is the same as the one which generates the central extension by the mass. In the plane the Galilei group also admits an “exotic” central extension [8, 9], though, highlighted by the non-commutation of boosts. “Exotic” Galilean symmetry is realized by non-commutative particles in the plane; such a particle can be coupled to an electromagnetic field, leading to the non-commutative Landau problem [10]. In the critical case \( B = B_c = (e\Theta)^{-1} \) the system becomes singular, and all motions follow a generalized Hall law [9–11].

In this Note we combine and extend these results to a system of \( N \) exotic particles in the plane. First we briefly review some aspects of the Landau problem for \( N \) ordinary particles.

Our new results are presented in Sections III and IV: we first generalize Kohn’s theorem to exotic particles, and then we establish their symmetry under suitable time-dependent translations. Off the critical case i.e. for \( e\Theta B \neq 1 \) our conserved quantities realize the two-fold “exotic” extension of the Newton-Hooke group [3, 12], further extended, as in the Galilean case [3, 7], by internal rotations and time translations.

In Section IV we show how the Hall effect arises. In the critical case \( B = B_c \) all particles move, collectively, as dictated by the Hall law. The dimension of the phase space of the system drops from \( 4N \) to \( 2N \), and our “time-dependent translation” – symmetry reduces to the Heisenberg group with \( -(\Theta)^{-1} = -eB_c \) as central parameter.

We find it worth noting that, in both the regular and singular cases, the motion is fully determined by the respective conserved quantities.
II. KOHN’S THEOREM AND NEWTON-HOOKE SYMMETRY OF THE LANDAU PROBLEM

A simplest proof of Kohn’s theorem is obtained by using the equations of motion; alternatively, the standard Lagrangian can be decomposed as \( L = L_{\text{com}} + L_{\text{int}} \), where \( L_{\text{com}} \) and \( L_{\text{int}} \) only depend on the center-of-mass, \( \mathbf{X} = \sum_a m_a x_a / M \) and the relative coordinates, \( x_{ab} = x_a - x_b \), respectively [2]. Recent interest in Kohn’s theorem arose [2, 3] when it was noticed that when (1) holds, the “time-dependent translation” (or “boost”),

\[
x_a \rightarrow x_a + b(t)
\]

is a symmetry of the equations of motion whenever \( b \) satisfies

\[
M \ddot{b} = e \dot{b} \times \mathbf{B}.
\]

(2) plainly leaves the relative coordinates invariant and only act on the center-of-mass. The Lagrangian changes, moreover, as

\[
L \rightarrow L - \left( \mathbf{X} + \frac{1}{2} \mathbf{b} \right) \cdot \left\{ M \ddot{\mathbf{b}} - e \dot{\mathbf{b}} \times \mathbf{B} \right\} + \frac{d}{dt} \left\{ \left( \mathbf{X} + \frac{1}{2} \mathbf{b} \right) \cdot (M \dot{\mathbf{b}} + \frac{e}{2} \dot{\mathbf{b}} \times \mathbf{B}) \right\},
\]

confirming that (2) is indeed a symmetry when (3) holds. Working in the plane, this equation is solved by \( b(t) = R(-\omega t) \mathbf{a} + \mathbf{c} \), where \( \mathbf{a} \) and \( \mathbf{c} \) are constant vectors, \( \omega = eB/M \), and \( R \) denotes a planar rotation. For \( \mathbf{a} = 0 \) we get an ordinary translation and for \( \mathbf{c} = 0 \) a “boost” [17]. Adding rotations and time translations provides us with Newton-Hooke transformations [4, 5, 12]. Then Noether’s theorem provides us with conserved “magnetic momentum” and “magnetic center-of-mass”,

\[
\Pi_i = M \left( \dot{X}_i - \omega \varepsilon_{ij} X_j \right), \quad \mathbf{K} = M R(\omega t) \dot{\mathbf{X}},
\]

respectively. Eliminating \( \dot{\mathbf{X}} \) yields the usual cyclotronic motion,

\[
X^i(t) = \frac{\varepsilon_{ij}}{eB} \left( \Pi^j - (R(-\omega t) \mathbf{K})^j \right).
\]

The value of \( \Pi \) determines the center around which the vector \( \mathbf{K} \) rotates with common frequency \( \omega \), shared by all particles.

Moreover, the usual one-particle commutation relations \( \{ p_a^i, p_b^j \} = e_a B \varepsilon^{ij} \delta_{ab}, \{ x_a^i, p_b^j \} = \delta^{ij} \delta_{ab}, \{ x_a^i, x_b^j \} = 0 \), imply that

\[
\{ P^i, P^j \} = eB \varepsilon^{ij}, \quad \{ X^i, X^j \} = 0, \quad \{ X^i, P^j \} = \delta^{ij},
\]

(7)
where \( \mathbf{P} = \sum_a \mathbf{p}_a \). The conserved quantities (5) satisfy therefore
\[
\{ \Pi^i, \Pi^j \} = -M\omega \varepsilon^{ij}, \quad \{ K^i, K^j \} = M\omega \varepsilon^{ij}, \quad \{ \Pi^i, K^j \} = 0.
\] (8)

These relations are consistent with those of the mass-centrally-extended Newton-Hooke group [2–5] [18]. We just mention that the equations of motion [omitted here] are Hamilton’s equations for
\[
\mathbf{h} = \sum \mathbf{p}_a^2/2m_a; \quad \mathbf{h}_{\text{int}} = \sum \frac{m_a m_b}{M_3} \mathbf{p}_{ab}, \quad \mathbf{p}_{ab} = \frac{M}{m_a m_b} (m_b \mathbf{p}_a - m_a \mathbf{p}_b)
\] (9)
shows that \( \mathbf{h} \) alone would yield the center-of-mass equations, owing to \( \{ \mathbf{X}^i, \mathbf{h}_{\text{int}} \} = \{ \mathbf{P}^i, \mathbf{h}_{\text{int}} \} = 0 \).

III. THE LANDAU PROBLEM FOR EXOTIC PARTICLES

Let us now consider \( N \) “exotic” particles endowed with masses, charges and non-commutative parameters \( m_a, e_a \) and \( \theta_a \), respectively, moving in a planar electromagnetic field. Although our theory works for any \( B \) and \( E \), we assume, for simplicity, that both fields are constant. Generalizing the 1-particle equations in [9], we describe our system by
\[
m_a^* \dot{x}_a^i = \dot{p}_a^i - m_a e_a \theta_a \varepsilon^{ij} E^j, \quad \dot{p}_a^i = e_a B \varepsilon^{ij} \dot{x}_a^j + e_a E^i,
\] (10)
where \( m_a^* = m_a(1 - e_a \theta_a B) \) is the effective mass of the particle labeled by \( a = 1, \ldots, N \). Note, in the first relations, also the “anomalous velocity terms” perpendicular to the electric field. Summing over all particles, we find that when \( e_a/m_a \) and \( e_a \theta_a \) are both constants i.e. when the generalized Kohn conditions
\[
\frac{e_a}{m_a} = \frac{e}{M}, \quad e_a \theta_a = e \Theta, \quad \Theta = \sum_a \frac{m_a^2 \theta_a}{M^2}
\] (11)
hold (where \( e = \sum_a e_a \) is the total charge), then the center-of-mass splits off,
\[
M^* \dot{X}^i = \dot{P}^i = Me \Theta \varepsilon^{ij} E^j, \quad \dot{P}^i = eB \varepsilon^{ij} \dot{X}^j + eE^i, \quad M^* = M(1 - e\Theta B).
\] (12)
The center-of-mass behaves hence as a single “exotic” particle carrying the total mass, charge and non-commutative parameter, \( M, e \) and \( \Theta \), respectively.

We first consider the purely magnetic case \( E = 0 \), postponing that of \( E \neq 0 \) to Sect. [IV]. Then the center-of-mass performs the usual cyclotronic motion but with modified frequency,
\[
\mathbf{X}(t) = R(-\omega^* t) \mathbf{A} + \mathbf{B}, \quad \omega^* = \frac{eB}{M^*} = \frac{\omega}{1 - e\Theta B},
\] (13)
where \( A \) and \( B \) are constant vectors.

Now we generalize the symmetry (2). Ignoring the relative coordinates, an easy calculation shows that a time-dependent boost (2), implemented as \( X \rightarrow X + b, \ P \rightarrow P + M^* \dot{b} \), is a symmetry for (12) whenever \( b \) satisfies

\[
M^* \dot{b}^i - eB \varepsilon^{ij} \dot{b}^j = 0. \tag{14}
\]

cf. (3). Alternatively, we note that the \( N \)-body exotic equations (10) derive from the first-order phase space Lagrangian [9],

\[
L_{exo} = \sum_a \left\{ (p_a + eA_a) \cdot \dot{x}_a - \frac{p_a^2}{2m_a} + \frac{\theta_a}{2} p_a \times \dot{p}_a \right\}. \tag{15}
\]

When the generalized Kohn conditions (11) hold, our Lagrangian can be decomposed into the sum of center-of-mass and internal parts, \( L_{exo} = L_{com} + L_{int} \), with [19],

\[
L_{com} = (P + eA) \cdot \dot{X} - \frac{P^2}{2M} + \frac{\Theta}{2} P \times \dot{P}, \tag{16}
\]

\[
L_{int} = \frac{1}{2} \sum_{a \neq b} \frac{m_a m_b}{M^2} (p_{ab} + eA_{ab}) \cdot \dot{x}_{ab} - h_{int} + \frac{\Theta}{4} \sum_{a \neq b} \left( \frac{m_a m_b}{M} \right)^2 p_{ab} \times \dot{p}_{ab}, \tag{17}
\]

where \( A_{ab} = A_a - A_b \).

Let us now consider a time-dependent translation \( b(t) \). Implementing it on \( p_a \) according to \( p_a \rightarrow p_a + m_a^* \dot{b} \), we see that it leaves invariant the internal part, and only acts (consistently with our earlier implementation) on the center-of-mass. Then a tedious calculation shows that our Lagrangian changes as

\[
\delta L_{exo} = \frac{d}{dt} \left[ \frac{1}{2} \Theta M^* \varepsilon^{ij} P^j \dot{b}^i + \frac{1}{2} M^* \dot{b}^i \dot{b}^i + M^* \dot{b}^i X^i + \frac{1}{2} eB \varepsilon^{ij} \dot{b}^j X^j \right]
+ \left( \frac{\Theta}{2} P^i \varepsilon^{ij} - \frac{1}{2} \dot{b}^i - X^i \right) \left( M^* \dot{b}^j - eB \varepsilon^{jk} \dot{b}^k \right), \tag{18}
\]

which is a surface term when (14) is satisfied.

Assume first that we are off the critical case, \( M^* \neq 0 \), i.e., \( e \Theta B \neq 1 \). Then (14) is solved as in the ordinary case but with frequency \( \omega \rightarrow \omega^* \),

\[
b(t) = R(-\omega^* t) a + c, \tag{19}
\]

where \( a \) and \( c \) are constant vectors. The associated conserved quantities are therefore

\[
P^i = M^* (\dot{X}^i - \omega^* \varepsilon^{ij} X^j), \tag{20}
\]

\[
K = \frac{M^*}{M} R(\omega^* t) P = \frac{M^*^2}{M} R(\omega^* t) \dot{X}. \tag{21}
\]
Eliminating $\dot{X}$ yields the classical trajectories cf. (13),

$$X^i(t) = \frac{\varepsilon_{ij}}{eB} \left( P^j - \frac{[R(-\omega^*t)K]^j}{1 - e\Theta B} \right). \quad (22)$$

The commutation relations of our system of exotic particles [9, 10],

$$\begin{align*}
\{p_a^i, p_b^j\} &= \frac{e_aB\varepsilon^{ij}}{1 - e_a\theta_aB} \delta_{ab}, \\
\{x_a^i, p_b^j\} &= \frac{\delta^{ij}}{1 - e_a\theta_aB} \delta_{ab}, \\
\{x_a^i, x_b^j\} &= \frac{\theta_a\varepsilon^{ij}}{1 - e_a\theta_aB} \delta_{ab},
\end{align*} \quad (23)$$

imply the center-of-mass Poisson brackets,

$$\begin{align*}
\{P^i, P^j\} &= \frac{eB\varepsilon^{ij}}{1 - e\Theta B}, \\
\{X^i, P^j\} &= \frac{\delta^{ij}}{1 - e\Theta B}, \\
\{X^i, X^j\} &= \frac{\Theta\varepsilon^{ij}}{1 - e\Theta B}.
\end{align*} \quad (24)$$

These Hamiltonian structures would allow us to recover both exotic equations of motion (10) and (12). Moreover, the commutation relations of the exotic conserved quantities (21) are,

$$\begin{align*}
\{P_i^a, P_j^b\} &= -M^*\omega^*\varepsilon^{ij}, \\
\{K_i^a, K_j^b\} &= (1 - e\Theta B)M^*\omega^*\varepsilon^{ij}, \\
\{P_i^a, K_j^b\} &= 0.
\end{align*} \quad (25)$$

Our new formulae can be compared to those in the ordinary i.e. case $\theta_a = 0$ in eqn. (8). Remembering that $M^*\omega^* = M\omega = eB$ we see that the commutation relations of the conserved magnetic momenta $P^i$ are the same as in the commutative Landau problem. The boost-with-boost relation are also similar to those in (8) to which they reduce when $\Theta = 0$. For $\Theta \neq 0$, however, they pick up the characteristic factor $1 - e\Theta B$. The new relations (25) correspond in fact to the two-parameter centrally-extended “exotic” Newton-Hooke symmetry $\tilde{NH}$ [5, 12]. We only mention here that, like in the Galilean case [9, 10], the non-commutativity contributes to the c-o-m angular momentum,

$$J = X \times (P + eA(X)) + \frac{\Theta}{2}P^2, \quad (26)$$

where the “exotic” contribution, $(\Theta/2)P^2 = M\Theta H$, comes from the manifestly rotation-invariant exotic term in (16). Moreover, if the interaction potential between the particles is itself radial, $V = V(|x_a - x_b|)$, then the system will be also invariant w.r.t. internal rotation around the center-of-mass, inducing a second, conserved, “internal angular momentum”

$$j_{\text{int}} = \frac{1}{2} \sum_{a,b} \frac{m_am_b}{M^2} (x_{ab}) \times (p_{ab} + eA_a) + M\Theta h_{\text{int}}. \quad (27)$$
The “exotic” contribution here is again proportional to the internal energy, $h_{\text{int}}$ in (9). All this is understood by observing that, for $V = 0$, the equations of internal motions,

$$M^* \dot{x}_{ab} = p_{ab}, \quad \dot{p}_{ab}^i = eB \varepsilon^{ij} \dot{x}_{ab}^j,$$

have indeed the same form as those of the center of motion, (12). The internal motions are therefore once again cyclotronic with common frequency $\omega^* = eB/M^*$.

The two terms in the total angular momentum, $J = J + j_{\text{int}}$, are separately conserved— as are $H$ and $h_{\text{int}}$ in (9). The latter are in fact associated with “internal rotations” and “internal time translations” which act as symmetries, independently of those acting on the center of mass. In conclusion, for $N \geq 2$, the full symmetry of the system is

$$\overline{N}H \otimes (\text{SO}(2) \times \mathbb{R}),$$

in analogy with the Galilean, and extending the commutative cases [3, 7].

**IV. THE HALL EFFECT IN NON-COMMUTATIVE MECHANICS**

The most interesting physical application of the exotic model is to the Hall effect [9–11, 16] that we now extend to $N$ exotic particles. We first assume that the electric field is turned off. Then, in the critical case,

$$M^* = 0 \quad \text{i.e.} \quad B = B_c = \frac{1}{e\Theta},$$

the only way to satisfy the equations of motion (12) is $P = \dot{X} = 0$ and thus $X(t) = X_0$: the center-of-mass becomes fixed. Note that while the frequency diverges, $\omega^* \to \infty$, the radius of the circle, $\left| (eB(1 - e\Theta B))^{-1} \mathcal{K}/1 \right|$, shrinks to zero as the critical value is approached. In fact, the whole system gets “frozen” into a static “crystal” configuration: all effective masses vanish, $m_a^* = 0$ when $B = B_c$; all individual positions are therefore fixed.

Eqn. (14) only allows for $b = \text{const}$: the 4-parameter symmetry algebra reduces to mere translations [20]. The associated conserved quantities behave as $\mathcal{P}^i \to -eB_c \varepsilon^{ij} X_0^j$ and $\mathcal{K} \to 0$ as $M^* \to 0$ i.e. $B \to B_c = (e\Theta)^{-1}$.

Let us now restore the electric field, $E$, assumed constant for simplicity. This amounts to adding $\sum_a e_a E \cdot x_a$ to the Lagrangian (15). But the Kohn condition (11) allows us to infer that this is simply $eE \cdot X$; the electric field only effects therefore the center-of-mass, but not the internal motion.
Let us now consider the center-of-mass equations, (12). We show that the constant electric field can be eliminated as in the commutative case by a suitable Galilean boost. Consider indeed $x_a \to \tilde{x}_a = x_a + ut$, $p_a \to \tilde{p}_a = p_a + m_a u$, where $u$ is a constant vector. It acts on the center-of-mass as $X \to \tilde{X} = X + ut$, $P \to \tilde{P} = P + Mu$. This is not a symmetry of the system, though, but yields rather

$$M \dot{\tilde{X}}^i - \dot{\tilde{P}}^i = eM(\Theta Bu^i + \varepsilon^{ij}E^j) = e\left(-B\varepsilon^{ij}u^j + E^i\right).$$

Choosing $u^i = -\varepsilon^{ij}E^j/B$ the electric term is therefore eliminated, leaving us with the pure magnetic problem we just solved. The motion in the original frame is thus obtained by combining the purely-magnetic motions with the constant-speed drift $-u$, perpendicular to the electric field. Off the critical case, $e\Theta B \neq 1$, we get, both for individual particles and their center-of-mass, collectively drifted cyclotronic motions with common frequency $\omega^*$ plus drift velocity $\varepsilon^{ij}E^j/B$.

In the critical case, $B = B_c$ and for $E = 0$ instead, all “motions” are mere fixed points: as we have seen, our particles are frozen into static equilibrium. Boosting backwards allows us to conclude that, when (30) holds, all particles, and hence also their center-of-mass, drift, collectively, with common Hall velocity

$$\dot{x}_a^i = \varepsilon^{ij}E_j^i \quad \text{for all} \quad a = 1, \ldots, N.$$  
(31)

By (28), the internal motions are still frozen.

It is convenient to introduce the guiding centers of the particles, $q_a = x_a - (m_a/e_a B_c^2)E$ which move, together with their center of mass, $Q = (\sum_a m_a q_a)/M = X - eM\Theta^2 E$, following the Hall law, (31). Their equations of motion can be derived from the reduced Lagrangian [obtained from (15) by Faddeev-Jackiw reduction [9, 10]], which splits once again into c-o-m plus internal parts,

$$L^{red} = \sum_a \frac{1}{2\Theta_a} \varepsilon^{ij} q^i_a \dot{q}^j_a - \sum_a e_a E \cdot q_a$$
(32)

$$= \left\{ \frac{1}{2\Theta} \varepsilon^{ij} Q^i \dot{Q}^j - eE^i Q^i \right\} + \sum_{a,b} \frac{1}{4\Theta} \frac{m_a m_b}{M^2} \varepsilon^{ij} (q^i_a - q^i_b) (\dot{q}^j_a - \dot{q}^j_b).$$

As noticed by Landau-Lifshitz in the commutative case, [14], the guiding center coordinates are non-commuting,

$$\{q^i_a, q^j_b\} = -\Theta_a \delta_{ab} \varepsilon^{ij} = -\frac{1}{e_a B_c} \delta_{ab} \varepsilon^{ij}, \quad \{Q^i, Q^j\} = -\Theta \varepsilon^{ij} = \frac{1}{e B_c} \varepsilon^{ij}. \quad (33)$$
The Hamiltonian has no kinetic term and reduces the potential alone [15]. As
\[ \delta L^{\text{red}} = \frac{d}{dt} \left( \sum_a \frac{1}{2 \theta_a} \varepsilon^{ij} q_a^i - e_a E^i t \right) b^i = \frac{d}{dt} \left( \frac{1}{2 \Theta} \varepsilon^{ij} Q^j - e E^i t \right) b_i \] (34)
under a translation \( b \), the associated conserved quantities are non-commuting,
\[ \mathcal{P}_{\text{red}}^i = -\frac{1}{\Theta} \varepsilon^{ij} \left( Q^j - \varepsilon^{jk} E^k \frac{B_c}{t} \right), \quad \{ \mathcal{P}_{\text{red}}^i, \mathcal{P}_{\text{red}}^j \} = -\frac{1}{\Theta} \varepsilon^{ij} = -e B_c \varepsilon^{ij}, \] (35)
showing that our residual symmetry is the *Heisenberg group* with central extension parameter
\(-\Theta^{-1} = -e B_c\). These relations are consistent with letting \( B \rightarrow B_c \) in (21) and (25).
Amusingly, the [Hall] motions can, once again, be recovered from the conserved quantity,
\[ Q^i(t) = \Theta \varepsilon^{ij} \mathcal{P}_{\text{red}}^j + \varepsilon^{ij} E^j \frac{B_c}{t}. \] (36)

V. CONCLUSION

The intuitive meaning of the Kohn condition is to guarantee a collective behavior: all particles rotate with the same frequency, shared also by their center-of-mass. The additional condition \( e_a \theta_a = \text{const} \) implies that the typical factors \((1 - e_a \theta_a B)\) are the same for all particles, namely \((1 - e \Theta B)\), allowing us to extend Kohn’s theorem to exotic particles.

Our second result is to prove the *two-parameter centrally-extended “exotic” Newton-Hooke symmetry* for our a system of \( N \) exotic particles [21]. As in the Galilean case, the commutation relations only differ from the ordinary (1-parameter) case in the boost-boost relation, which now also involves the non-commutative parameter \( \Theta \), and is supplemented by internal rotations and time translations.

Off the critical case \( e \Theta B \neq 1 \), the motions are analogous to those in the ordinary Landau problem but with modified frequency. In the critical case \( B = B_c = (e \Theta)^{-1} \), however, all particles are frozen into a static “crystal” configuration when the electric field vanishes, and drift perpendicularly to the electric field with Hall velocity when \( E = \text{const} \neq 0 \).

It is worth saying that our description for the reduced system presented in Sect. IV is in fact that of *pointlike vortices in the plane* [10], consistently with Laughlin’s suggestion, who explains the Hall effect by the motion of charged vortices [16].

All our investigations have been purely classical. It is not difficult to quantize our system, though, as in the ordinary case [2].
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[17] They are in fact “imported Galilean translations and boosts” [3, 6].

[18] The same symmetry was found earlier for Chern-Simons vortices in a constant electromagnetic field, see Ref. [6].

[19] An interaction potential $V(\mathbf{x}_a - \mathbf{x}_b)$ would modify the equations of motion (10) by contributing to the electric fields felt by each particle. The extra terms would, however, drop out under
summing, leaving the center-of mass equations unchanged and affecting the internal motion only. Since our main interest lies in the center-of-mass motion, we only consider $V = 0$ in what follows.

[20] All diffeomorphisms of the plane become instead symmetries.

[21] Owing to the equivalence of the uniform-B–field and oscillator problems, the latter plainly shares the same exotic Newton-Hooke symmetry.