Coronal Loop Scaling Laws for Various Forms of Parallel Heat Conduction

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Abstract

The solar atmosphere is dominated by loops of magnetic fluxes that connect the multi-million degree corona to the much cooler chromosphere. The temperature and density structure of quasi-static loops are determined by the continuous flow of energy from the hot corona to the lower solar atmosphere. Loop scaling laws provide relationships between global properties of the loop (such as the peak temperature, pressure, and length); they follow from the physical variable dependencies of various terms in the energy equation, and, hence, the form of the loop scaling law provides insight into the key physics that control the loop structure. Traditionally, scaling laws have been derived under the assumption of collision-dominated thermal conduction. Here, we examine the impact of different regimes of thermal conduction—collision-dominated, turbulence-dominated, and free-streaming—on the form of the scaling laws relating the loop temperature and heating rate to its pressure and half-length. We show that the scaling laws for turbulence-dominated conduction are fundamentally different than those for collision-dominated and free-streaming conduction, inasmuch as the form of the scaling laws now depend primarily on conditions at the low-temperature, rather than high-temperature, part of the loop. We also establish regimes in the temperature and density space in which each of the applicable scaling laws prevail.

Key words: Sun: corona – Sun: flares

1. Introduction

In a landmark paper based on Skylab extreme ultraviolet observations, Rosner et al. (1978) showed that the solar corona is not a plane-parallel structure but rather is dominated by approximately isobaric loop-like structures with shapes controlled by the structure of the ambient magnetic field. The plasma confined by the magnetic loops is heated to multi-million degree temperatures in the corona (by a mechanism that remains largely as an open question), while the energy balance in the cooler lower atmosphere is dominated by radiative losses that dissipate the energy carried downward by thermal conduction. Hence, in quasi-static equilibrium, the plasma properties of coronal loops are strongly dependent on the conductive flux, which is usually taken to be proportional to the field-aligned temperature gradient (see Aschwanden 2004; Reale 2014, for reviews).

Rosner et al. (1978) studied the energy balance between heating, radiation, and field-aligned thermal conduction in quasi-static loops to deduce the now well-known “scaling laws” relating the maximum temperature, $T_M$ (K), and heating rate, $E_H$ (erg cm$^{-3}$ s$^{-1}$), in the loop to its half-length, $L$ (cm), and pressure, $p$ (dyne cm$^{-2}$), viz.:

$$T_M \approx 1.4 \times 10^3 (pL)^{1/3}$$

(1)

(their Equation (4.3)) and

$$E_H \approx 9.8 \times 10^4 p^{7/6} L^{-5/6}$$

(2)

(their Equation (4.4)). Kano & Tsuneta (1995) found a different coefficient and index for the temperature scaling law ($3.8 \times 10^4$ and $1/(5.1 \pm 0.5)$, respectively), based on observational measurements and line-fitting, and they discuss possible reasons for these discrepancies. The modeling in Rosner et al. (1978) assumed uniform volumetric heating (a condition later relaxed by Serio et al. 1981; Martens 2010), optically thin radiative losses, and heat conduction dominated by collisional transport of electrons, in which the heat flux is proportional to the local temperature gradient, with a coefficient that is temperature-dependent. However, in certain situations (that we discuss in Section 3), the heat flux can become saturated (and, hence, a function of only local density and temperature conditions rather than the temperature gradient) and reaches its free-streaming limit (Manheimer & Klein 1975; Cowie & McKee 1977; Campbell 1984), or the mean free path used in calculating the thermal conduction coefficient may be limited by some form of turbulence.

Observations of coronal loop-top hard X-ray sources in solar flares require that the bremsstrahlung-producing electrons are confined to the corona (e.g., Masuda et al. 1994; doschek et al. 1995; mariska et al. 1996; Tsuneta et al. 1997; veronig & Brown 2004; jin & ding 2008; Krucker et al. 2008; guo et al. 2012; Simões & Kontar 2013). Various authors have considered mechanisms that could be responsible for more effective confinement of accelerated electrons in the corona and, in particular, the possibility that turbulence enhances the angular scattering rate and, therefore, suppresses the rate of escape of nonthermal electrons from the coronal acceleration region (e.g., Kontar et al. 2014; bian et al. 2017). Of course, the presence of such turbulence will also act to suppress energy transport by thermal electrons. Further, it is likely that some form of small-scale turbulence exists in active region loops, particularly if coronal heating is due to small-scale processes, such as the flux-braiding and reconnection mechanism described by Parker (1988) or the interaction of counter-propagating Alfvén waves (van Ballegooijen et al. 2011). It is, therefore, of interest to consider the effect of small-scale turbulence on the thermal conductive flux and, hence, on the form of the loop scaling laws. At the other extreme, if the scattering length (either collisional or turbulent) is large compared to the loop half-length, $L$, then the heat flux can become saturated.
In this paper, we consider how modifications to the collision-dominated physics of thermal conduction, either toward suppression of conductive flux by turbulence or toward the free-streaming limit, affect the form of the scaling laws appropriate to active region loops. Our expressions modify those of Rosner et al. (1978) in not only a quantitative but, as we shall see, a qualitative way, in which conditions in the transition region of the loop can play a role comparable to, or even more important than, conditions in the hot regions near the coronal apex. We shall also explore the physical conditions in which each of these regimes dominates the form of the conductive flux and, hence, the form of the pertinent scaling law.

We begin Section 2 with a review of the fundamental energy balance equation in a static coronal loop structure, and we consider the form of this equation in regimes for which energy transport by thermal conduction takes place by a combination of collisional, turbulent, or free-streaming processes. Using some plausible assumptions, we derive approximate analytic solutions to this energy equation and, thus, deduce the corresponding loop scaling laws for both the peak temperature and, in general, antiparallel to the local temperature gradient and, therefore, in the direction of the magnetic field. We delineate the loop using the 1D coordinate $x$, measured from the loop base toward the loop apex, i.e., in the direction of positive temperature gradient and, in general, antiparallel to the direction of the magnetic field. The quasi-static energy balance is thus given by

$$\frac{dF}{ds} + E_R = E_H,$$  

(3)

where $E_H$ (erg cm$^{-3}$ s$^{-1}$) is the heat input, and $E_R = n^2 \Lambda(T) = \frac{p^2}{4k_B^2} \Lambda(T)$ is the (optically thin) radiative loss (erg cm$^{-3}$ s$^{-1}$) and $F$ (erg cm$^{-2}$ s$^{-1}$) is the conductive flux (aligned in a direction antiparallel to the temperature gradient). For thermal transport dominated by Coulomb collisions (denoted below by the notation [C]), there is the well-known Spitzer (1962) result:

$$F_C = -\kappa_o \frac{T^5}{T} \frac{dT}{ds}, \quad [C]$$  

(5)

In the above equations, $n$ (cm$^{-3}$) is the density, $T$ (K) is the electron temperature, $p = 2nk_BT$ (erg cm$^{-3}$) is the gas pressure, and

$$\kappa_o = \frac{k_B (2k_B)^{3/2}}{\pi m_e^{1/2} e^4 \ln \Lambda} \simeq 1.7 \times 10^{-6} \text{ erg cm}^{-1} \text{s}^{-1} \text{K}^{-7/2}$$  

(6)

is the Spitzer (1962) coefficient of thermal conductivity. $\Lambda(T)$ (erg cm$^{-3}$ s$^{-1}$) is the optically thin radiative loss function that, in the temperature range of interest ($10^5 \text{K} \leq T < 10^7 \text{K}$), is well approximated by the power-law form

$$\Lambda(T) = \chi T^{-1/2},$$  

(7)

where $\chi \simeq 1.6 \times 10^{-19}$ erg cm$^3$ s$^{-1}$ K$^{1/2}$.

As discussed by Bian et al. (2016), the presence of (for example) a spectrum of magnetic field fluctuations within the loop gives rise to an additional source of angular scattering for electrons, hereafter referred to as “turbulent scattering,” with an associated (velocity-independent) mean free path, $\lambda_T$ (cm). For example, for turbulent scattering associated with local inhomogeneities, $\delta B \ll$, in a background magnetic field, $B_0$,

$$\lambda_T = \lambda_B \left( \frac{\delta B}{B_0} \right)^{-2},$$  

(8)

where $\lambda_B$ is the magnetic correlation length. Although it is possible that the turbulent heat conductivity also depends on quantities, such as the magnetic energy release rate (via the fluctuation energy, $\delta B^2/8\pi$), we here, for simplicity, take $\lambda_T$ to be a constant parameter. When the heat flux is controlled by turbulent scattering, the expression for the heat flux becomes

$$F_T = -\frac{\kappa_o}{R} \frac{T^5}{T} \frac{dT}{ds}, \quad [T]$$  

(9)

where we introduced the notation $[T]$ and the turbulent heat flux correction factor, $R$ (Bian et al. 2018), reflects the ratio of the collisional to turbulent mean free paths:

$$R = \frac{\lambda_C}{\lambda_T} = \frac{(2k_BT)^3}{2\pi e^4 n \ln \Lambda \lambda_T} = \frac{(2k_BT)^3}{2\pi e^4 \ln \Lambda \lambda_T p} = c_R \left( \frac{T^3}{\lambda_T p} \right),$$  

(10)

where

$$c_R = \frac{4k_B^3}{\pi e^4 \ln \Lambda} \simeq 3.15 \times 10^{-12} \text{ erg cm}^{-2} \text{K}^{-3}.$$  

(11)

Substituting Equation (10) in Equation (9) gives

$$F_T = -\frac{\kappa_o}{c_R} \left( \frac{\lambda_T p}{c_R} \right) \frac{T^{-1/2}}{T} \frac{dT}{ds}, \quad [T]$$  

(12)

When the mean free path (either collisional or turbulent) becomes larger than the characteristic scale of the loop (e.g., its half-length, $L$), the thermal conductive flux is no longer inhibited by scattering processes and, therefore, approaches its free-streaming value. Since the electrons at a given point now originate from a wide range of positions (and, therefore, temperatures) within the loop, the value of the thermal conductive flux at a given point is in general a non-local quantity formed by the convolution of the expression for the local conductive flux as a function of temperature $T$ with the temperature profile of the loop (Emslie & Bian 2018). Since we are here interested in zero-dimensional (0D) global scaling laws, rather than 1D variations in quantities with the position, we will neglect this non-local factor, which by construction averages to zero over the loop. An upper limit to the heat flux is, therefore, set by the free-streaming limit in which particles move in the direction antiparallel to the local temperature gradient at the local thermal speed (see discussion in Bradshaw & Cargill 2006):

$$F_{\text{max}} = -E_{\text{th}} v_{\text{th}}.$$  

(13)
where \( E_{th} = (3/2) n k_B T \) is the electron thermal energy density and \( v_e = (k_B T / m_e) \) is the thermal speed. A correcting factor is usually also employed and, generally, based on Fokker–Planck simulations (e.g., Klimchuk et al. 2008), is taken to be one-sixth. The maximum electron heat flux (denoted by \( [S] \)) is, therefore,

\[
F_S \simeq \frac{1}{6} \times \frac{3}{2} n k_B T \sqrt{\frac{k_B T}{m_e}} \frac{n(2k_B T)^{3/2}}{m_e^{1/2}}. \quad [S]
\]

We now proceed to derive the loop scaling laws that follow from the various expressions in Equations (5), (9), and (14) for the conductive flux. For the collision-dominated and turbulence-limited cases, we start by recasting the energy equations in the form (see Equation (3.11) of Rosner et al. 1978):

\[
\frac{F_C}{\kappa_o T^{5/2}} \frac{dT}{dT} = \frac{p^2}{4k_B} \frac{N(T)}{T} - E_H; \quad \text{[C]}
\]

\[
\frac{c_R T^{1/2} F_T}{\kappa_o \lambda_T p} \frac{dT}{dT} = \frac{p^2}{4k_B} \frac{N(T)}{T} - E_H. \quad \text{[T]}
\]

Using Equation (7), Equation (15) may be written

\[
F_C dF_C = \frac{\chi_p}{4k_B} \frac{dT}{dT} - E_H T^{5/2} \frac{dT}{dT}; \quad \text{[C]}
\]

\[
F_T dF_T = \frac{\chi_p}{4k_B} \frac{dT}{dT} - E_H T^{5/2} \frac{dT}{dT}. \quad \text{[T]}
\]

which can both be straightforwardly integrated from \( T = T_0 \) (the base of the transition region) to \( T \) to give

\[
F_C^2(T) - F_C^2(T_0) = \frac{\chi_p}{2k_B} \frac{dT}{dT} \left[ (T - T_0) \right] T^{5/2}; \quad \text{[C]}
\]

\[
F_T^2(T) - F_T^2(T_0) = \frac{\chi_p}{4k_B} \frac{dT}{dT} \left[ \frac{1}{T_0} - \frac{1}{T} \right] T^{5/2}; \quad \text{[T]}
\]

(17)

(18)

(19)

(20)

(21)

These can be integrated again to yield an expression for \( s(T) \):

\[
s(T) - s(T_0) = \kappa_o^{1/2} \int_{T_0}^T \left[ \frac{\chi_p}{2k_B} \left( T - T_0 \right) \right. \left. - 4E_H \left( T^{7/2} - T_0^{7/2} \right) \right]^{1/2} T^{5/2} \frac{dT}{dT}; \quad \text{[C]}
\]

\[
s(T) - s(T_0) = \left( \kappa_o \lambda_T p \frac{\chi_p}{4k_B} \left[ \frac{1}{T_0^2} - \frac{1}{T^2} \right] \right)^{1/2} \left( \frac{\chi_p}{4k_B} \left( T^{7/2} - T_0^{7/2} \right) \right)^{1/2} T^{5/2} \frac{dT}{dT}; \quad \text{[T]}
\]

At this juncture, Rosner et al. (1978) neglect the second term in the square brackets in the first of these equations, arguing that near the base of the loop, the primary energy balance is between the heat flux and transition region radiative losses. We shall proceed somewhat differently here. First, we note from Equation (18) that since the temperature gradient vanishes at the loop apex \( (T = T_M) \),

\[
\frac{\chi_p}{4k_B} \left( \frac{1}{T_0^2} - \frac{1}{T_M^2} \right) = \frac{4E_H}{T} \left( T_M^{7/2} - T_0^{7/2} \right) \quad \text{[T]}
\]

Since \( T_M \gg T_0 \), Equation (20) gives, to a high degree of accuracy,

\[
T_M^{5/2} \simeq \frac{7}{8} \frac{\chi_p}{16k_B} E_H \quad \text{[C]}
\]

(21)

It is important to note that in the case of collision-dominated conduction, only positive powers of the temperature appear in Equation (20), and, therefore, the base electron temperature, \( T_0 \), does not play a significant role in determining the maximum temperature, \( T_M \). Physically, this is because the temperature gradient \( dT/ds \propto F_C/T^{7/2} \), thus, the low temperatures at the loop base mean that the temperature gradient is very large (compared to that in the corona) in order to support the incident heat flux in the presence of a much lower thermal conduction coefficient. Quantitatively, the thickness, \( \ell \), of a layer corresponding to a temperature range of \( T_1 < T < T_1 + \Delta T \) is given by \( \Delta T \propto (dT/ds)^{-1} \Delta T \propto T^{7/2} \), which is much smaller at the loop base \( (T \simeq T_0) \) than in the corona \( (T \simeq T_M) \). Thus, the amount of radiation emitted within such a layer is negligible, and the loop energetics are controlled principally by a balance between heat conduction and radiation in the corona.

On the other hand, for turbulence-limited conduction the heat flux is reduced by a temperature-dependent factor of \( R \propto T^3 \) (Equation (10)), so that the conductive coefficient in the expression for \( F_T \) is inversely proportional to \( T \), \( F_T \propto T^{-1/2} dT/ds \) (Equation (12)). Thus, the temperature gradient is \( dT/ds \propto T^{1/2} \) (see Equations (9) and (10)) and the thickness of a layer corresponding to a temperature difference is \( \Delta T \) is \( \Delta T \propto T^{1/2}/dT \), which is a quantity that is now larger at the loop base than in the corona (the temperature gradient need no longer steepen to support the incident heat flux because it is turbulence-limited). As a result of this fundamentally different scaling of the
half-length, \( T \) becomes more significant.\) This effect is sufficiently strong so that the integral in Equation (19) becomes dominated by the low-temperature material near the base of the loop. Therefore, conditions at the loop base control the scaling laws for the loop.

Using Equation (20) in Equation (19), we find that the loop half-length, \( L = s(T_M) - s(T_0) \), is given by

\[
L = \left( \frac{7 \kappa_0}{4 E_H} \right)^{1/2} \int_{T_0}^{T_M} \left[ \left( \frac{T_M^2 - T_0^2}{T_M - T_0} \right)(T - T_0) - \left( T^{7/2} - T_0^{7/2} \right) \right]^{1/2} dT
\]

Also, substituting values for \( \chi, \kappa_o, \) and \( k_B \). The first line in each of the results in Equations (23) and (24) are the Rosner et al. (1978) results expressed in Equations (1) and (2) above, with slightly different coefficients because of the difference of the Coulomb logarithm \( \ln \Lambda \) and, hence, \( \kappa_0 \) used.

The expressions for the case of the free-streaming heat flux are developed somewhat differently (see also Ciaravella et al., 1993, who first applied thermal conduction in the free-streaming limit to coronal loop models). Using expressions (4) and (14) in the basic energy Equation (3), we find that

\[
E_H = \frac{p^2}{4k_B T^2} \chi T^{-1/2}, \tag{25}
\]

from which

\[
dT/ds = \frac{2}{3} \frac{m_{e1/2}}{n (2k_B)^{3/2}} \frac{1}{T^{1/2}} \left[ \frac{p^2 \chi}{4k_B T^{5/2}} - E_H \right]. \tag{26}
\]

Setting \( dT/ds = 0 \) at the apex (\( T = T_M \)) gives

\[
E_H = \frac{p^2 \chi}{4k_B T_M^{5/2}}. \tag{27}
\]

Substituting this in Equation (26), using the relation of \( n = p/2k_B T \), inverting to get an expression for \( ds/dT \), and integrating this expression from the base temperature, \( T_0 \), to the apex temperature, \( T_M \), gives

\[
pL = \frac{3}{2} \left( \frac{29/2 m_e^{1/2}}{k_B^{5/2}} \chi \right) \int_{T_0}^{T_M} T^{2} \frac{dT}{1 - (T/T_M)^{5/2}} \simeq \frac{(2k_B)^{5/2}}{29/2 m_e^{1/2}} \chi T_M^{3/2}. \tag{28}
\]

This gives the scaling law

\[
T_M = \left( \frac{4m_e^{1/2} \chi}{k_B^{5/2}} \right)^{1/3} (pL)^{1/3} \simeq 4.4 \times 10^2 (pL)^{1/3}. \tag{29}
\]

and using this in Equation (27) gives the additional scaling law

\[
E_H = \frac{\chi}{4k_B} \left( \frac{k_B^{5/2}}{4m_e^{1/2} \chi} \right)^{5/6} p^{7/6} L^{-5/6} \simeq 5.1 \times 10^2 p^{7/6} L^{-5/6}. \tag{30}
\]

To summarize, for electron-dominated conduction, we have the following scaling laws in the three cases (collisional,
This differs from the more exact scaling law in Equation (31) by the more substantial factor of $(2/7) \times (25/28) \times [(7/8) \times (32/25)]^{1/2} \approx 1.02$. However, a similar simple exercise fails to determine the correct scaling laws for the case of turbulence-dominated conduction because it neglects the important role that radiation from the lower transition region plays in determining the maximum loop temperature and, hence, the conductive flux and the required heating rate to balance it.

\[ T_M = \left( \frac{25 \chi}{32 \kappa_o k_B^2} \right)^{1/6} (pL)^{1/3} \approx 1.3 \times 10^3 (pL)^{1/3}; \quad [\text{C}] \]

where we used Equation (34). This differs from the more exact scaling law in Equation (31) by the more substantial factor of $(2/7) \times (25/28) \times [(7/8) \times (32/25)]^{1/2} \approx 1.02$.

Similarly, equating the loop-averaged divergence of the saturated conductive flux (Equation (14)) with the radiative loss term at the peak (Equation (7)) gives

\[ \frac{1}{2^{2/7} \gamma_p} \left( \frac{2k_B}{m_e} \right)^{1/2} \frac{p^{1/2}}{L} = \frac{p^2}{4k_B} \chi \quad T_M^{-5/2}. \quad (36) \]

From this we find

\[ T_M = \left( \frac{2m_e^{1/2} \chi}{k_B^{5/2}} \right)^{1/3} (pL)^{1/3}, \quad [\text{S}] \]

which differs from the more exact result (Equation (31)) by a factor of only $(1/2)^{1/3} \approx 0.79$. Then, equating the heating and conduction terms gives

\begin{align*}
E_H &= \frac{1}{2^{2/7} \gamma_p} \left( \frac{2k_B}{m_e} \right)^{1/2} \frac{p^{1/2}}{L} \\
&= \frac{\chi}{4k_B} \left( \frac{k_B^{5/2}}{2m_e^{1/2} \chi} \right)^{5/6} p^{7/6} L^{-5/6}, \quad [\text{S}] \quad (38)
\end{align*}

which differs from the more exact result (Equation (32)) by a factor of $25/6 \approx 1.8$.

\[ 3. \text{ Regimes where Each Set of Scaling Laws Applies} \]

Using Equations (5), (6), (9, (10), and (14), the ratios of the magnitudes of the heat fluxes in the various limits (turbulent, collisional, and free-streaming) are

\[ F_T : F_C : F_S = \frac{n(2k_B)^{3/2}}{2 \pi e^4 \ln \Lambda m_e^{1/2}} \quad \frac{T^{1/2}}{T^{3/2}} \quad \frac{dT}{ds} \quad \frac{1}{2^{7/2}} \quad \frac{n(2k_B T)^{3/2}}{m_e^{1/2}} \quad (39) \]

where $T^{1/2} / T^{3/2} \approx \frac{2}{7} \frac{2 \pi e^4 \ln \Lambda m_e^{1/2}}{2^{7/2}}$.

\[ \approx \frac{1}{2^{7/2}} \frac{n(2k_B)^{3/2}}{m_e^{1/2}} \frac{T^{3/2}}{L} \quad \frac{T^{7/2}}{L} \quad (40) \]

where $\ln \Lambda$ is the Coulomb logarithm. Since the conductive heat flux is proportional to the mean free path or length scale, the ratio of the heat fluxes is simply the ratio of the corresponding scale lengths:

\[ F_T : F_C : F_{\text{max}} = \frac{\lambda_T}{\lambda_C} = \frac{3}{7} \frac{\lambda_C}{2^{9/2} \lambda_C} \quad (41) \]

where $\lambda_C$ is the collisional mean free path (Equation (10)).

In general, for a given set of physical conditions (temperature and density), the lowest of the three heat flux values (collision-
Temperatures, turbulence, weak turbulence, density, $n$, and $T$ are discussed. The dashed horizontal lines represent the quantities $\lambda_T$ and $(3/2^{9/2})L$, respectively (see Equation (41)). The left figure corresponds to a turbulence scale of $\lambda_T = 10^{3.5}$ cm (see Bian et al. 2016) and a loop half-length of $L = 10^{6.5}$ cm (3L/2$^{9/2}$ = 10$^{6.5}$ cm); the right figure corresponds to a very long turbulence scale of $\lambda_T = 10^{10}$ cm (and therefore weak turbulence) and the same loop half-length, $L$. In both panels, the relative values of these three length scales are highlighted at two values of the plasma temperature, $T$ (K), and density, $n$ (cm$^{-3}$): ($T = 10^7$, $n = 10^{10.5}$) and ($T = 10^8$, $n = 10^{10}$). For the latter set of ($n,T$) values, the blue dot marks where the vertical line crosses the horizontal surfaces denoting the values of $\lambda_T$ and $(3/2^{9/2})L$, respectively.

![Figure 1](image1.png)

Figure 1. Values of pertinent spatial scales for different conduction regimes. The slanted surface represents the quantity $(3/7)\lambda_T$, while the red and green horizontal surfaces represent the quantities $\lambda_T$ and $(3/2^{9/2})L$, respectively (see Equation (41)). The left figure corresponds to a turbulence scale of $\lambda_T = 10^{3.5}$ cm (see Bian et al. 2016) and a loop half-length of $L = 10^{6.5}$ cm (3L/2$^{9/2}$ = 10$^{6.5}$ cm); the right figure corresponds to a very long turbulence scale of $\lambda_T = 10^{10}$ cm (and therefore weak turbulence) and the same loop half-length, $L$. In both panels, the relative values of these three length scales are highlighted at two values of the plasma temperature, $T$ (K), and density, $n$ (cm$^{-3}$): ($T = 10^7$, $n = 10^{10.5}$) and ($T = 10^8$, $n = 10^{10}$). For the latter set of ($n,T$) values, the blue dot marks where the vertical line crosses the horizontal surfaces denoting the values of $\lambda_T$ and $(3/2^{9/2})L$, respectively.

![Figure 2](image2.png)

Figure 2. Values of pertinent spatial scales in different conduction regimes for $L = 10^{6.5}$ cm and $T = 10^7$ K. The diagonal line represents the quantity $(3/7)\lambda_T$, while the dashed horizontal lines represent the quantities $\lambda_T$ and $(3/2^{9/2})L$, respectively. The dotted–dashed vertical lines correspond to $n = 10^{8.5}$ cm$^{-3}$ and $n = 10^{10.5}$ cm$^{-3}$. The circles show where each of these vertical lines meet the lowest pertinent scale and, hence, the relevant conduction and scaling-law regime. Thus, for $\lambda_T = 10^{3.5}$ cm, conduction is controlled by turbulence if $n = 10^{10}$ cm$^{-3}$ and by collisions if $n = 10^{10.5}$ cm$^{-3}$; whereas for $\lambda_T = 10^{10}$ cm, conduction is controlled by free-streaming if $n = 10^{8.5}$ cm$^{-3}$ and by collisions if $n = 10^{10.5}$ cm$^{-3}$.

The results are as follows:

1. $[\lambda_T = 10^{3.5}$ cm, $L = 10^{9.5}$ cm] In this case, for the first set of parameters ($T = 10^{6.5}$ K, $n = 10^{10.5}$ cm$^{-3}$), the collision-related scale is the lowest and, hence, determines the rate of heat loss by conduction; in such a case, the pertinent scaling laws are the Rosner et al. (1978) scaling laws—denoted by [C] in Equations (31) and (32). For the second set of parameters ($T = 10^{7.5}$ K, $n = 10^{9.0}$ cm$^{-3}$), the lowest scale length is the turbulence scale length $\lambda_T = 10^7$ cm (the blue dot on the red horizontal surface in the left panel of Figure 1); in such a

dominated, turbulence-dominated, and free-streaming) controls the flow of the heat and, hence, determines the pertinent conductive flux regime and the associated scaling law. Since the heat flux is proportional to the corresponding length scale (Equation (41)), the issue of selecting the pertinent conductive regime thus reduces to selecting which of the three length scales in Equation (41) is the smallest.

Figure 1 compares the values of the three length scales in Equation (41) for two cases: ($\lambda_T$, $L$) = ($10^{7.5}$, $10^{9.5}$) cm (left panel) and ($\lambda_T$, $L$) = ($10^{10}$, $10^{10.5}$) cm (right panel). Figure 2 shows a slice through each of the 3D plots in Figure 1 at $T = 10^7$ K by way of an example.
the pertinent scaling laws are those denoted by $[T]$ in Equations (31) and (32).

2. $[\lambda_T=10^{10}\text{ cm}, L=10^{3.5}\text{ cm}]$ For the first set of parameters $(T=10^{6.2}\text{ K}, n=10^{10.5}\text{ cm}^{-3})$, the collision-relation scale is still the lowest; the pertinent scaling laws are still the Rosner et al. 1978 scaling laws—denoted by $[C]$ in Equations (31) and (32). For the second set of parameters $(T=10^{7.2}\text{ K}, n=10^{9.0}\text{ cm}^{-3})$, the lowest scale length is now that related to the loop half-length $L$ (the blue dot on the green horizontal surface in the right panel of Figure 1); in such a case, the heat conduction is controlled by free-streaming, and the pertinent scaling laws are now those denoted by $[S]$ in Equations (31) and (32).

4. Discussion and Conclusions

This paper has extended the work of Rosner et al. (1978) to include situations where the thermal conductive flux that redistributes heat within a coronal loop is governed by processes other than Coulomb collisions, specifically turbulent scattering and free-streaming. Equations (31) and (32) provide the pertinent scaling laws for the peak temperature, $T_{\text{peak}}$, and the volumetric heating rate, $E_{\text{HM}}$, respectively, in terms of the loop pressure, $p$, and the half-length, $L$. It is notable that because of the much weaker dependence of the thermal conduction coefficient, $\kappa$, on the temperature for the case of turbulent scattering by magnetic fluctuations, the characteristics of the loop in such a regime are governed not by the highest temperatures in the loop (as they are for both the collisional and free-streaming cases) but by conditions at the low-temperature (transition region) part of the loop.

Which of these scaling laws is appropriate in a particular environment depends on the ratios of the turbulent scale length, $\lambda_T$, to the collisional mean free path, $\lambda_c$, to the loop half-length, $L$ (Equation (41)). Figure 1 illustrates examples where each process, and, hence, scaling law, dominates.

Given the likely role of turbulence in active region loops, particularly those associated with flaring activity (Bian et al. 2018, and references therein), and modern observations of the faint, hot component of emission in non-flaring active regions, which are considered a signature of impulsive heating (e.g., Reale et al. 2009a, 2009b; Schmelz et al. 2009a, 2009b; Testa et al. 2011; Miceli et al. 2012; Brosius et al. 2014; Petralia et al. 2014; Marsh et al. 2018) that allow more precise estimation of loop temperatures and densities (and of the presence of turbulence; Kontar et al. 2017), we encourage the comparison of observed loop parameters with these extended scaling laws as a possible diagnostic of the physical conditions in active region coronal loops and, hence, of the energy required to create and sustain them.

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