Modelling the dynamics of superfluid neutron stars

1 Introduction

As the temperature of a system decreases towards absolute zero, matter either freezes to a solid or becomes superfluid. The first case is dominated by a structured lattice. In the second case, the system acts as a macroscopic quantum system. Regardless of the outcome, the rules that apply in an extremely cold system may be very different from those that describe the system at higher temperatures. This is, of course, a well-known fact and the different possible phases of matter have been studied in great detail. In particular, increasingly sophisticated laboratory experiments continue to provide insights into the details of superfluid/superconducting systems, ranging from the standard Bose-Einstein condensates to superfluid Helium (Donnelly 1991) and atomic systems exhibiting fermion Cooper pairing. This area of research is highly relevant for those that are fascinated by neutron star physics. In fact, mature neutron stars may provide the ultimate testing ground for theoretical physics. They are expected to contain an elastic nuclear crust permeated by superfluid neutrons. An outer core where superfluid neutrons coexist with superconducting protons transitions to the deeper core which may contain exotic like superfluid hyperons and deconfined quarks in a colour superconducting state. As if this was not enough, the presence of a “solid” core remains a possibility (even though the relevant lattice parameters are unknown).

Trying to understand the details of the various neutron star phases and their possible impact on astronomical observations is an exciting challenge. First of all, it requires a working knowledge of much of modern theoretical physics. Secondly, one will need some understanding of the observations. This includes decades of data for radio pulsars, in particular associated with the glitch events that provide the strongest evidence of the presence of (at least partially) decoupled superfluid components. X-ray observations of accreting neutron stars in binary systems, and isolated neutron stars closer to the Earth, are common and the data for the existence of magnetars is now very convincing. In fact, the recent observations of likely crustal oscillations following the giant flares in SGR1806-20 and SGR1900+14 [see the contributions by Israel, Watts and Samuelsson] may be the harbingers of neutron star asteroseismology. Future gravitational-wave observations should add another observational window, although it will likely require an advanced generation of detectors sensitive at high frequencies.

As the observations continue to improve we will have more precise quantitative opportunities to test our theoretical models. To meet these tests the current models have to be improved. In fact, many of our “favourite” models are far too rudimentary to pass any closer scrutiny. This is not surprising given that the relevant theory problems are difficult. Consider for example the possibility of superfluid vortex pinning in the neutron star crust. At the phenomenological level, the association between the observed spin-up of the crust in a glitch and the transfer of angular momentum from an initially faster spinning (superfluid) component is natural. To invoke vortex unpinning as the key agent that motivates the event is also natural. Yet it is clear from the effort that has gone into glitch modelling that it is difficult to describe the process in a truly quantitative way.
The dynamics of a superfluid neutron star is associated with a number of similar problems that need to be better understood. The main aim of the research described in this brief review is to arrive at such an understanding. Right now we are quite far away from this goal. In some cases, e.g. problems concerning basic shear viscosity, we may have a clear picture already, but in others, e.g. how the calculated shear viscosity coefficients are used in the case of a superfluid system, we are only beginning to be able to pose (what might be) the right questions.

This brief review is a summary of my presentation at the London neutron star meeting. It is in no sense a complete review of the various problems. Rather it is a self-centered view of my thinking at the time of writing. The interested reader will find references to the relevant literature in the cited papers.

Before I discuss our recent work, let me provide a list of problems that I find particularly interesting:

1. How do pulsar glitches really work? As already alluded to above, this remains a vexing issue. In my (somewhat pessimistic view) view, the present models are not truly quantitative. They likely contain the right elements, but the details of the underlying physics are not yet agreed upon. It is also worth noting that most models predict how the glitch proceeds and how the system relaxes back to quasi-equilibrium. Very few models provide a real mechanism to explain why the glitch happened in the first place.

2. How do we understand neutron star free precession? The observational indications that some neutron stars are wobbling now seem relatively strong. The obvious question is why this behaviour is so rare. After all, free precession is the most natural mode of motion for any rotating body. The answer will provide insight into the problem of why this mode of motion is so rare. Yet a multi-fluid system has extra degrees of freedom that we can apply the standard single fluid equations to. The answer will provide insight into the problem of why this mode of motion is so rare.

3. Do the superfluid degrees of freedom affect the oscillation properties of the star? This is an important question that impacts on future attempts to probe neutron star physics via asteroseismology. To provide a “useful” answer we need to study the problem using realistic models for the superfluid pairing gaps (which, of course, are not agreed upon by theorists) within general relativity. The latter is key in order to make the results accurate enough that it is meaningful to compare to observations. In principle, I don’t think the mode calculation would be very difficult. At least not for non-rotating stars, and as long as one is prepared to accept that the pairing gaps are likely to remain unknown up to perhaps a factor of a few. However, if one wants to account for the presence of the crust (which should be penetrated by superfluid neutrons) and the magnetic field, then the modelling becomes much more challenging. In the case of the crust, we have not (until very recently (Carter & Samuelsson 2006)) had any theoretical formulations that allow for the presence of a superfluid component, while in the magnetic field case we do not have a good understanding of the internal field structure.

Neutron stars may radiate detectable gravitational waves through a number of scenarios. One possibility is that the inertial r-modes are driven unstable as they radiate gravitationally. The basic mechanism behind this instability is well understood, and a large number of damping mechanisms that counteract the instability have been considered. Thus it has become clear that the key deciding factors relate to superfluidity. The presence of hyperons in the deep stellar core may prevent the instability from happening, but if the hyperons are superfluid then the chemical reactions that lead to bulk viscosity are suppressed and the effect on the instability may not be considerable. Another important mechanism is the so-called mutual friction, which in this context relates to the damping due to the scattering of electrons off of magnetic fields associated with the superfluid neutron vortices. So far all studies of mode-damping due to this effect have been based on straight vortices (as in a rigidly rotating star). But this may not be the appropriate model! If there is a significant oscillation in the star, which has some vorticity associated with it, then it seems likely that the vortices will get tangled up. The system would then be in a “turbulent” state, and from the analogous problem in superfluid Helium (Donnelly 1991) we know that the mutual friction force is then rather different.

Finally, I would like to emphasise the multi-fluid aspects of these problems. In most available studies, concerning for example viscosity, it is implicitly assumed that we can apply the standard single fluid equations of motion. Yet a multi-fluid system has extra degrees of freedom (the second sound in Helium and the analogous “superfluid modes” in an oscillating neutron star). These are unlikely to be “passive”. In fact, we know that the equations that are used to model simple superfluid neutron stars admit a so-called two-stream instability (Andersson & Comer 2004). One can speculate that this instability becomes relevant when the two components of the star rotate at different rates, as in the case of a spinning down neutron star with a pinned superfluid component. The instability mechanism is very simple and familiar from other physical systems, and it would be interesting to know whether it can operate in a neutron star as well.

2 Shear viscosity

We have recently investigated the effects of superfluidity on the shear viscosity in a neutron star core (Andersson, Comer & Glampedakis 2005). We were motivated to do...
this by a puzzling result from the literature. The available results suggest that the shear viscosity is stronger in a superfluid neutron star than it is when the star forms a normal-fluid system (Cutler & Lindblom 1987). This result contradicts our experience from other superfluid systems like Helium, and since the shear viscosity affects mode damping, free precession and relaxation after spin-up events in a crucial way, it is important that we understand it.

To address the problem we combined existing theoretical results for the viscosity coefficients with data for the various superfluid energy gaps into a “complete” description. This lead to a simple model for the electron viscosity which is relevant both when the protons form a normal fluid and when they become superconducting. This turns out to be the key distinguishing factor.

Below the neutron superfluid transition temperature the dominant contribution to the shear viscosity comes from the scattering of relativistic electrons. To model the viscosity we need to consider two contributions. Above the transition temperature at which the protons become superconducting, electron-proton scattering leads to a viscosity coefficient

\[ \eta_{\text{ep}} \approx 1.8 \times 10^{18} \left( \frac{x_p}{0.01} \right)^{13/6} \rho_{15}^{13/6} T_s^{-2} \text{ g/cm s} \quad (1) \]

where \( x_p \) is the proton (electron) fraction, \( T_s = T/10^8 \text{K} \) and \( \rho_{15} = \rho/10^{15} \text{g/cm}^3 \). Meanwhile, when the protons are superconducting the dominant effect is due to electrons scattering off of each other. Then we have

\[ \eta_{\text{ee}} \approx 4.4 \times 10^{19} \left( \frac{x_p}{0.01} \right)^{3/2} \rho_{15}^{3/2} T_s^{-2} \text{ g/cm s} \quad (2) \]

The protons play the key role since individual scattering processes add like “parallel resistors”. That is, we have

\[ \tau = \left[ \frac{1}{\tau_{\text{ee}}} + \frac{1}{\tau_{\text{ep}}} \right]^{-1} \quad \text{where} \quad \tau_{\text{ee}} \gg \tau_{\text{ep}} \quad (3) \]

and it is clear that the most important contribution comes from the most frequent scattering process.

When the protons are superconducting the electron-proton scattering will be suppressed, essentially because there will be fewer states available for the protons to scatter into. In order to allow for the transition to proton superconductivity, we can introduce a suppression factor \( R_p \) such that

\[ \tau_{\text{ep}} \rightarrow \frac{\tau_{\text{ep}}}{R_p} \quad (4) \]

Far below the critical transition temperature at which the protons become superconducting we should have \( R_p \rightarrow 0 \), and we see from (3) that the electron-electron scattering then dominates the shear viscosity.

Our results, which are illustrated in Figure 1, explain in a clear way why proton superconductivity leads to a significant strengthening of the shear viscosity. It should be noted that the superfluidity of the neutrons is not the factor which leads to electron-electron scattering becoming the main shear viscosity agent. Rather, it is the fact that the onset of superconductivity suppresses the electron-proton scattering. As can be seen from Figure 1, the electron-electron shear viscosity is not too different from the result for neutrons scattering off of each other. This means that, in the temperature range where shear viscosity dominates, the damping of neutron star oscillations will be quite similar (modulo multi-fluid effects) in the extreme cases when i) the neutrons and protons are both normal, and ii) when the neutrons and protons are superfluid/superconducting, respectively. The contrast with the case when the neutrons are superfluid and the protons normal (and viscosity is dominated by \( \eta_{\text{ep}} \)) is clear from Figure 1. This is an interesting observation because it shows that proton superconductivity (or rather absence thereof) could have a significant effect on the dynamics of a neutron star core.

Future work needs to i) implement the different degrees of freedom of a multifluid system, and ii) determine the required suppression factors (like \( R_p \)).

**3 Hyperon viscosity**

The presence of hyperons is expected to affect a neutron star in several important ways. First of all, the \( \Sigma^- \) carries negative charge which means that the lepton fractions drop significantly following its appearance. In fact,
in some models there are virtually no electrons present in the core of the star. This may have implications for the shear viscosity results that we discussed earlier. Secondly, the hyperons can act as an extremely efficient refrigerant. This is mainly because the hyperons may undergo direct URCA reactions essentially as soon as they appear, in clear contrast to the protons which must exceed a threshold value of \( x_B \sim 0.1 \). Studies of the effect of this enhanced cooling have shown that a neutron star with a sizeable hyperon core would cool extremely fast. In fact, it would cool so fast that it would be much colder than observational data suggests. This could be taken as evidence against an exotic core, but more likely it suggests that the hyperons are (at least partly) superfluid. This is a natural explanation since superfluidity leads to a reduction of the relevant reaction rates.

Analogous non-leptonic reactions have recently been invoked as a mechanism for producing a very strong bulk viscosity that may suppress the gravitational-wave driven instability of the r-modes (Lindblom & Owen 2002) [see also the contribution by Chatterjee & Bandyopadhyay]. This mechanism is particularly important because, in contrast to the standard bulk viscosity associated with \( \beta \)-reactions it is relevant at low temperatures (the coefficient scales as \( T^{-2} \) rather than \( T^6 \)). For a while it was argued that the presence of hyperons would make the r-mode instability completely irrelevant. This is now understood not to be the case (Nayyar & Owen 2006). Basically, the hyperons must be superfluid in order not to lead to conflicts with cooling data. Then the nuclear reactions that lead to the bulk viscosity must also be suppressed, and the effect on the r-mode instability may not be so great after all, see Figure 2. In fact, the most recent estimates suggest that the r-modes may be unstable in accreting neutron stars in LMXBs, possibly regulating the spin of these stars. Since the associated gravitational-wave signal may be detectable by advanced detectors, it is important that we understand this problem better.

Future work needs to i) improve our understanding of the hyperon pairing gaps and the associated suppression factors for bulk viscosity, and ii) develop a multi-fluid description of bulk viscosity that can be used below the relevant superfluid transition temperature.

### 4 Modelling multifluids

The equations that govern a general conservative multifluid system can be derived from a constrained variational principle (Prix 2004; Andersson & Comer 2006a). The fundamental variables in this framework are the number densities \( n_x \), where \( x \) is a “constituent” index that identifies the different particle species, and the associated transport velocities \( v_i^x \). To complete the model one must provide an energy functional which represents the equation of state. In the isotropic (meaning that there are no preferred directions) two-fluid problem this functional takes the form \( E(n_{n}, n_{p}, w^2) \) where \( w^2 = v_i^nx - v_i^y \) \( (x \neq y \text{ are constituent indices}) \). This immediately leads to

\[
dE = \sum_{x=n,p} \mu_x d n_x + \alpha dw^2 \tag{5}
\]

where \( w^2 = w_i^nxw_i^x \) and

\[
\tilde{\mu}_x = \mu_x \frac{1}{m_B} = \frac{\partial E}{\partial n_x} \quad \text{and} \quad \alpha = \frac{\partial E}{\partial w^2} \tag{6}
\]

(with \( m_B \) the baryon mass) defines the chemical potential per unit mass \( \tilde{\mu}_x \) and the entrainment parameter \( \alpha \).

The equations of motion for the system can be written

\[
\partial_t n_x + \nabla_i (n_x v_i^x) = 0 \tag{7}
\]

and

\[
n_x \left( \partial_t + L_{v_x} \right) p^n_x + n_x \nabla_i \left( \mu_x - \frac{1}{2} m_B v_x^2 \right) = 0 \tag{8}
\]

where \( L_{v_x} \) represents the Lie derivative along \( v_i^x \) (see Andersson & Comer (2006b) for an explanation why the
Lie derivative is natural in this context). In (8) the momentum (per particle) is given by $p_i^x = m_B (v_i^x + \varepsilon_{x} w_i^x)$, where $\varepsilon_{x} = 2\alpha/m_B n_x$. From this we can understand the nature of the entrainment effect. It is such that the velocity and momentum of each constituent in the multifluid system are no longer parallel. An alternative, perhaps more intuitive, description would represent this mechanism in terms of an effective mass $m^*_x$ (Andersson, Sidery & Comer 2006). In the case of the outer core of a neutron star, where superfluid neutrons coexist with superconducting protons the entrainment arises because of the strong interaction. Each proton/neutron is endowed with a cloud of particles of the opposite species, and when it flows it drags some of the other fluid along with it thus affecting its momentum.

The entrainment is known to play an important role in neutron star dynamics. In particular, it has been shown to affect the spectrum of stellar pulsation (Prix, Comer & Andersson 2004; Gusakov & Andersson 2006). Moreover, it is one of the key ingredients in the prescription for the vortex mediated mutual friction.

Future work needs to i) provide the entrainment parameters at finite temperatures (as in Gusakov & Haensel (2005)) (essentially doing equation of state calculations “out of equilibrium”), and ii) develop fluid models that allow for the presence of both superfluid condensates and the quasiparticle excitations that will be present when the star is no longer at $T = 0$.

### 5 Mutual friction

A superfluid mimics bulk rotation by forming an array of vortices. The dynamics of these vortices, and their interaction with the different particle species may significantly affect the evolution of the system. In a neutron star the presence of vortices leads to “mutual friction” between interpenetrating superfluids (neutrons and protons). This couples the two fluids on a relatively short timescale, which has implications for both glitch recovery estimates and the damping of pulsation modes. In fact, an estimate by Lindblom and Mendell indicates that the gravitational-wave driven instability of the fundamental $f$-modes will be prohibited by the mutual friction (Lindblom & Mendell 1995). The same effect is also relevant, albeit not quite in such a dramatic fashion, for the $r$-modes (Lindblom & Mendell 2000).

In a recent study, we have revisited the problem of the mutual friction force for neutron stars (Andersson, Sidery & Comer 2006). It is known that the entrainment plays a key role in determining the strength of the mutual friction, and to make progress and study various astrophysical scenarios we needed to develop a description of this mechanism within our multifluid framework. Our results provide useful checks on the classic work by Alpar, Langer & Sauls (1984) (who were the first to discuss the problem) and Mendell (1991) (whose mutual friction coefficients were used in the studies of mode-instabilities in superfluid neutron stars).

Briefly, the mutual friction that is expected to be the most important in the outer core of a mature neutron star originates as follows. The superfluid neutrons form vortices, which represent the quantisation of the momentum circulation. That is, we have

$$\kappa^i = \frac{\hbar}{2m_n} \hat{k}^i = e^{ijk} \nabla_j \rho^n_k$$  \hspace{1cm} (9)

where $\hat{k}^i$ is a normalised tangent vector to the vortex. Here it is important to understand that it is the circulation of momentum that is quantised, not that of the velocity. At first sight this distinction may seem to suggest that our model differs from the standard picture. However, this would be wrong. One must remember that in the orthodox description for superfluid Helium, the so-called superfluid “velocity” is in fact a rescaled momentum (Prix 2004). Hence, the two pictures are consistent, the only difference being that we make the true dynamical role of the variables clearer. Now, the flow of neutrons associated with the vortex induces flow in part of the protons because of the entrainment. This leads to the generation of a magnetic field, see Figure 3, the strength of which is estimated as

$$B \approx 2 \times 10^{14} G \left( \frac{x_p}{0.05} \right) \rho_{14} \left| \frac{m_p - m_n^*}{m_p^*} \right|$$  \hspace{1cm} (10)

where $\rho_{14} = \rho/10^{14} \text{g/cm}^3$. Finally, electrons scatter dissipatively off of this magnetic field, leading to a coupling of the two fluids in the system (the superfluid neutrons and a conglomerate of all the charged particles).

To arrive at the standard expression for the mutual friction force, we balance the Magnus “force”, which acts on a vortex as the superfluid flows past it, and a resistive force due to electrons scattering off of the vortex. It should be noted that the Magnus effect is already present in the smooth averaged equations of motion (8), which means that in practice we only add resistivity to

![Figure 3: A schematic illustration of the flow around a superfluid neutron vortex. Because of the entrainment effect, the vortex induces flow also in the protons. This leads to a magnetic field forming on the vortex, and the scattering of electrons off of this magnetic field leads to a dissipative mechanism known as mutual friction.](image-url)
the right-hand side of the equations. Anyway, the force-balance leads to the mutual friction force acting on the neutrons being

\[ f_{\text{mf}}^i = \mathcal{B} \rho n_v \epsilon^{ijk} \kappa_j \epsilon_{klm} \kappa^l w_{np}^m + \mathcal{B}' \rho n_v \epsilon^{ijk} \kappa_j w_{np}^k \]  

(11)

This force has been used to model a number of astrophysical scenarios. In particular, the coupling of the two fluids following a glitch event, see the following section, and the damping of oscillations in a superfluid neutron star. It is, however, important to understand that this is only a first approximation to the real mutual friction interaction. There are presently two alternatives that should be taken seriously. The first, which has been advocated by Sedrakian and collaborators for over 20 years [see Sedrakian & Sedrakian (1997) and references given therein], is based on the notion of vortex clusters. The idea is that each neutron vortex is associated with a large collection of proton fluxtubes. This increases the scattering cross section for the electrons, and as a result the mutual friction force is many orders of magnitude stronger than given above. The upshot of this is (somewhat counter-intuitively) that the coupling between the fluids take place on a much longer timescale (months rather than seconds). The other alternative accounts for the curvature of the vortices, and the possibility that they will get tangled up in a state that in many ways is reminiscent of turbulence. The lack of a preferred direction of the vortex array obviously affects the form of the mutual friction force. This is well-known from work on superfluid Helium (Donnelly 1991). Nevertheless, this alternative form for the mutual friction has only recently been discussed for neutron stars. Peralta et al (2005) are leading the way by carrying out numerical simulations of a turbulent two-fluid system. This work has a number of interesting implications, and it will be exciting to see where this will take us.

Future work needs to investigate the alternative manifestations for the mutual friction in detail. In my opinion the most pressing issue concerns the turbulence. Since it is known to be the key effect in experiments on superfluid Helium one has every reason to expect that it will be relevant for neutron stars as well.

6 Dynamical coupling

One of the most important problems that can be addressed once we know the form of the mutual friction force regards the relaxation timescale following a pulsar glitch. The idea is that in the initial state the vortices are pinned. This leads to a build-up of a rotational lag between the two fluids as the charged component (the crust and the core protons/electrons) spins down. At some critical relative rotation rate, the pinning force will no longer be able to hold the vortices in place. Global unpinning then leads to evolution described by the equations that we have derived (provided that the vortices remain straight during the evolution, a debatable assumption). All we need in order to be able to estimate the relaxation timescale following the glitch are the coefficients in (11).

As we have recently shown (Andersson, Sidery & Comer 2006), the dimensionless coefficients are \( B' = B^2 \) and

\[ B = \frac{\rho_n \kappa}{\rho_n} \approx \]

\[ \approx 4 \times 10^{-4} \left( \frac{\delta m_p}{m_p} \right)^2 \left( \frac{m_p}{m_p^*} \right)^{1/2} \left( \frac{x_p}{0.05} \right)^{7/6} \rho_{14}^{1/6} \]  

(12)

where \( \delta m_p = m_p - m_p^* \). Working out the dynamical coupling timescale we easily find

\[ n_v \partial_t p_p + \ldots = f_{\text{mf}}^i \]

\[ n_p \partial_t p_p^* + \ldots = -f_{\text{mf}}^i \]  

\[ \rightarrow \frac{m_p^*}{m_p} \partial_t w_{np}^p + \ldots \approx -\frac{\mathcal{B}_{Kn_v}}{x_p} w_{np}^{p} \]  

(13)

That is, the timescale on which the two fluids are (locally) coupled can be estimated as

\[ \tau_d \approx \frac{m_p^*}{m_p} \frac{x_p}{\mathcal{B}_{Kn_v}} \approx \]

\[ \approx 10 P(s) \left( \frac{m_p^*}{\delta m_p} \right)^2 \left( \frac{x_p}{0.05} \right)^{-1/6} \rho_{14}^{-1/6} \]  

(14)

This suggests that the two core fluids are coupled on a timescale of about 10 rotation periods (for typical parameters). This estimate should be compared to the classic result of Alpar & Sauls (1988) who suggest that the coupling timescale is \( 400 - 10^4 \) periods. This relatively rapid coupling is usually taken as evidence that glitches must originate in the crust superfluid, not in the core.

Our calculations, with a coupling that is about 1 order of magnitude faster than the old result, obviously support this conclusion. However, we need to understand why our estimate differs from that of Alpar and Sauls. The result may seem surprising since the parameters in the mutual friction force are similar in the two studies. The difference comes from the set-up of the coupling timescale problem. Our estimate followed immediately from the two-fluid equations, and represents the timescale on which mutual friction couples the two components in the system. In contrast, Alpar and Sauls consider the equation of motion for the vortices. Setting the system into relative rotation, they work out the timescale for the vortices to relax to a new equilibrium position. The key difference between the two estimates is that in the Alpar-Sauls scenario the rotational lag between the fluids is maintained while the vortices relax. This means that their model does not conserve angular momentum during the relaxation, which explains why the two results are different.

7 Assignments

At the end of my presentation, I gave the audience a number of “homework assignments”. These represent problems that I feel need more attention, and which I hope
to consider in the near future. It seems appropriate to conclude this brief write-up by outlining these problems. As we are beginning to understand the basic dynamics of the two-fluid model for neutron stars, we should look ahead and develop a framework that would allow us to make our models more realistic. Most importantly, we need to account for finite temperature effects and viscosity. This is a difficult problem area, but I feel that we cannot shirk away from it. Recently, Greg Comer and I have developed a flux-conservative formalism for multifluid systems, including both dissipation and entrainment (Andersson & Comer 2006a). Our formulation has much in common with work on the mechanics of mixtures and multiphase flows. Our first study of the problem is quite formal, and should be relevant for any multifluid situation in Newtonian physics. Of course, we want to apply the framework to neutron stars. To do this, we have considered two model systems. The results make it painfully clear that more realistic models will be significantly more complex than the systems we have considered so far. Our analysis shows that the simplest “reasonable” model for a neutron star core, starting with four fluids (neutrons, protons, electrons and entropy) and reducing to three degrees of freedom (superfluid neutrons, fluids (neutrons, protons, electrons and entropy) and releasing to three degrees of freedom (superfluid neutrons, electrons and entropy) and reducing to three degrees of freedom (superfluid neutrons, entropy and everything else), requires no less than 19 dissipation coefficients.

Fig. 4 An illustration of the different kinds of motion that lead to dissipation via particle scattering in a two-fluid problem (blue and red). The “red” fluid is assumed to be at rest, and the “blue” fluid i) flows linearly, ii) contracts and expands, and iii) undergoes shearing motion.

It will be a challenge to understand the role and values of these different coefficients. Yet it may not be too bad a problem, because the meaning of many of the new coefficients is quite clear. Consider for example dissipation due to particle scattering. In a single fluid problem scattering leads to viscosity due to shearing motion in the flow. In a situation where a relative flow is possible, there are more degrees of freedom that lead to particle scattering. Consider the schematic illustration in Figure 4, where the two fluids are represented by blue and red particles. There are three different ways that relative flow can induce scattering, and result in dissipation. Taking the “red” fluid to be at rest, the blue fluid can i) flow linearly, ii) contract and expand, or iii) undergo shearing motion. The viscosity coefficients associated with these different degrees of freedom are related and should only differ by “geometric factors”. They should certainly be calculable, and might actually be known from kinetic theory already.

As described in other contributions to these proceedings, it is likely that we now have the first observations of neutron star oscillations. This is fantastic news, but it provides us with a challenge that must be met in the near future. We need to develop accurate theoretical models to put contraints on e.g. the crust physics via these observations. However, if we want to do this we must consider fully relativistic models. This is the only way to obtain results that have the precision required to make a comparison with observations meaningful. In fact, we might even have to account for the presence of superfluid neutrons in the crust, a very difficult problem. Moreover, the observed systems are all magnetars and one might expect the magnetic field to be important. It seems likely (at least to me) that the magnetic field will couple any motion in the crust to the core, thus altering the nature of the pulsation modes and making any analysis of “pure crust oscillations” less relevant (Glampedakis, Samuelson & Andersson 2006). This would force us to study global oscillations of magnetised fully relativistic stars, certainly not a simple problem.

As I have already indicated, we need to worry about superfluid “turbulence”. I would not be at all surprised if studies of this problem lead to results that change our understanding of neutron star dynamics significantly. One advantage is that there has been a lot of work on the analogous problem for superfluid Helium (Donnelly 1991), and we can hope to benefit from these results. Most importantly we need to understand whether a turbulent description is relevant for neutron stars. If it is, how does it manifest itself? What is the effect on, for example, glitch relaxation and mutual friction damping of neutron star oscillations?

In addition to these problems, I can think of a number of issues concerning multifluid aspects of exotic phases like hyperons and deconfined quarks. Certainly the neutron star community is bustling with exciting ideas, but we have a lot to do before we truly understand the dynamics of superfluid neutron stars.

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