A note on correlation functions in AdS$_5$/SYM$_4$
correspondence on the Coulomb branch

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Abstract

We compute certain two-point functions in $D=4$, $\mathcal{N}=4$, SU($N$) SYM theory
on the Coulomb branch using SUGRA/SYM duality and find an infinite set of first
order poles at masses of order $(\text{Higgs scale})/(g_{YM}\sqrt{N})$. 
1. Apart from being a universal way of looking at the gravity/gauge theory relation, Maldacena’s supergravity (SUGRA)/super Yang-Mills (SYM) duality \[1, 2\] provides a useful tool for the study of strongly coupled gauge theories. The precise rules for relating SUGRA and SYM observables in the context of AdS$_5$/CFT$_4$ duality of ref.\[1\] were given in refs.\[4, 5\]. They consist of (A) the dictionary between gauge-invariant SYM operators and SUGRA fields, and (B) the precise relation between the generating functional of correlators of $\mathcal{N} = 4$ CFT$_4$ and the Type IIB SUGRA action $S(\phi^i)$ evaluated on the solutions $\phi^i$ of the classical equations satisfying the boundary conditions $\phi^i|_{\partial AdS} \sim \phi^i_0$, where $\phi^i_0$ are the sources for the operators of CFT$_4$.

\[
\left\langle \exp \int_{S^4} \phi^i_0 \mathcal{O}_i \right\rangle_{CFT} = \exp(-S(\phi^i)).
\] (1)

Rules A and B were used for the calculation of two, three and four point functions of strongly coupled conformal $\mathcal{N} = 4$ SYM in refs.[4–11]. Given that in the conformal case one can use SUGRA/SYM duality to calculate correlation functions of SYM$_4$, one naturally wonders if rules A and B can be modified to incorporate SYM$_4$ on the Coulomb branch. The AdS$_5$/SYM$_4$ correspondence on the Coulomb branch was first discussed in ref.\[1\], where the relevance of the multi-center D3-brane SUGRA solution to SUGRA/SYM$_4$ duality was first mentioned. Various aspects of this correspondence have since been discussed in ref.\[12\], but it stood on much weaker grounds compared to the correspondence in the conformal case. Recently, Klebanov and Witten \[13\] gave arguments, partly based on an earlier work ref.\[14\], in favor of AdS/SYM correspondence on the Coulomb branch.

Let us recall a generalization of rule B proposed in ref.\[14\]. There it was pointed out that $\delta\left(-S(\phi)\right) = \left\langle \mathcal{O}(x) \right\rangle_{\phi_0}$ is the expectation value of the operator $\mathcal{O}(x)$ in the presence of the boundary source $\phi_0$. In the context of Eq. \[1\], the bulk solution $\phi$ to the classical field equations is completely determined by the boundary value $\phi_0$ and the requirement of regularity in the bulk. This uniqueness fails if one admits singular fields corresponding to sources in the bulk. The generalization of Eq. \[1\] proposed in ref.\[14\] consists in introducing sources in the bulk. The SYM one-point function $\left\langle \text{tr}F^2(x) \right\rangle$ in the instanton background was computed in ref.\[14\] by considering the response of bulk SUGRA action in the D-instanton background to the change in boundary data. In the same sense as the relevant SUGRA background to consider in the case of SYM in the instanton background is AdS D-instanton background, multi-center D3-brane background is relevant in the case of SYM on the Coulomb branch \[13\].

2. In the present work we extend rules A and B to the case of SYM$_4$ with the Higgs vev $\bar{X}$ turned on, and apply the modified rules to the specific case of spherically symmetric distribution of eigenvalues of $\bar{X}$. Let $\mathcal{O}_i = \mathcal{O}_i[F, X]$ be gauge invariant SYM quantum

\[1\] For a recent review of SUGRA/SYM duality see ref.\[3\].
operators. We are interested in the connected Green’s functions

$$\langle O_1(x_1) \ldots O_n(x_n) \rangle_{\bar{X}} = \frac{1}{Z} \int DADXD\psi e^{-S[A,X+\bar{X},\psi]}O_1(x_1) \ldots O_n(x_n) \bigg|_{\text{conn.}} \ .$$ (2)

Note that $$\langle O_i \rangle_{\bar{X}} = 0.$$ 

Consider the 10D Type IIB SUGRA action $$S_{10}(\Phi_i)$$ in the multi-center D3-brane background:

$$ds^2 = H^{-1/2}(dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + H^{1/2} \sum_{j=1}^{6} dy_j^2$$

$$H(\vec{y}) = Q \int d^6y' \rho(\vec{y}) \frac{1}{|\vec{y} - \vec{y}'|^4} ,$$ (3)

where $$\rho$$ is the distribution function of D3-branes normalized as $$\int d^6y \rho(\vec{y}) = N$$ and Q is the charge of a single D3-brane. Now substitute the expansions of the fields $$\Phi_i$$ in spherical harmonics

$$\Phi_i(x, \vec{y}) = \sum \phi^I_i(x, r) Y^I(\Omega_5) , \quad r \equiv |\vec{y}|$$ (4)

into $$S_{10}(\Phi_i)$$ and integrate over the sphere $$S^5$$. We end up with a five dimensional action $$S_5 = S_5(\phi^I_i)$$ in some effective background which is asymptotically AdS_5. In the single-center D3-brane case $$S_5$$ coincides with the five dimensional gauged SUGRA action on AdS_5 background with the infinite tower of Kaluza-Klein fields included. In the conformal case rule A was formulated by matching the spectrum of SUGRA fields on the AdS_5 x S^5 background with the conformal dimensions of SYM operators \([15]\). A natural extension of rule A to our case consists in matching the spectrum of SUGRA fields in the infrared with the dimensions of the SYM operators in the ultraviolet. The modification of rule B reads as follows

$$\langle \exp \int_{\mathbb{R}^4} \phi^I_i O_i \rangle_{\bar{X}} = \exp(-S_5(\phi^I_i)) .$$ (5)

Since asymptotically the geometry is AdS_5, we impose the same boundary conditions for the bulk SUGRA fields $$\phi^I_i$$ as in the conformal case.

3. Let us apply the modified rules to the computation of the two-point functions of the operators tr($$F^2 X^I$$), where $$X^I = X^{i_1}X^{i_2} \ldots X^{i_l}C^I_{i_1i_2\ldots i_l}$$ \([10]\). These operators correspond to the Kaluza-Klein harmonics $$\phi^I$$ of the 10D dilaton. Consider the dilaton kinetic term

\[ H(\vec{y}) = Q \int d^6y' \rho(\vec{y}) \frac{1}{|\vec{y} - \vec{y}'|^4} , \]

where $$\rho$$ is the distribution function of D3-branes normalized as $$\int d^6y \rho(\vec{y}) = N$$ and Q is the charge of a single D3-brane. Now substitute the expansions of the fields $$\Phi_i$$ in spherical harmonics

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In general there will be no consistent truncation of $$S_5$$ to lowest KK modes $$\phi^I = 0$$. There is actually no fundamental reason for the existence of such a truncation in the generic case. The large N properties of SYM_4 are encoded in 10D Type IIB SUGRA on appropriate backgrounds and not in 5D truncations thereof.
in $S_{10}$ and Kaluza-Klein reduce it to 5D as described earlier.

$$\int d^{10}x (H\Phi \nabla^2_\| \Phi + \Phi \nabla^2 \Phi) \rightarrow$$

$$\rightarrow \int dx^4 drr^5 \left[ \phi \left( \int d\Omega_5 H \right) \nabla^2_\| + V_5 \frac{1}{r^5} \partial_r r^5 \partial_r \phi \right.$$

$$\left. + \left( \int d\Omega_5 HY^I \right) (\phi \nabla^2_\| \phi^I + \phi^I \nabla^2_\| \phi) + \left( \int d\Omega_5 HY^I Y^J \right) \phi^I \nabla^2_\| \phi^J \right.$$  

$$\left. + \left( \int d\Omega_5 Y^I Y^J \right) \phi^I \frac{1}{r^5} \partial_r r^5 \partial_r \phi^J + \frac{1}{r^2} \left( \int d\Omega_5 Y^I \nabla^2_5 Y^J \right) \phi^I \phi^J \right]. \quad (6)$$

where $\phi = \phi^0 = 0$ and $V_5 = \int d\Omega_5$. The form of the action Eq. (6) suggests that the correlation function $\langle \text{tr} F^2 \text{tr} (F^2 X^I) \rangle$ is non-vanishing for generic Higgs vev $\bar{X}$. However, in the spherically symmetric case $\rho = \rho(|\vec{y}|)$ the coupling between $\phi$ and $\phi^I$ vanishes, implying that this correlator vanishes.

Let us show how this happens in SYM. The distribution of D3-branes in SUGRA corresponds to the distribution of Higgs vev $\bar{X}^i$ in SYM. The general form of the correlator $\langle \text{tr} (F^2 X^I) (x) \text{tr} (F^2 X^J) (y) \rangle \bar{X}$ compatible with gauge and R symmetries is

$$f_1 \left( |x - y|, \text{tr} (\bar{X}^I \bar{X}_J), \text{tr} \bar{X}^I \text{tr} \bar{X}_J \right) \text{tr} (\bar{X}^I \bar{X}^J)$$

$$+ f_2 \left( |x - y|, \text{tr} (\bar{X}^I \bar{X}_J), \text{tr} \bar{X}^I \text{tr} \bar{X}_J \right) \text{tr} \bar{X}^I \text{tr} \bar{X}^J, \quad (7)$$

where $f_1$ and $f_2$ are some arbitrary functions. In the spherically symmetric case only the first term survives and it is proportional to $\delta^{IJ}$, in agreement with SUGRA.

Now consider the most general finite, spherically symmetric distribution of D3-branes. From Eq. (8) we see that the harmonic function becomes

$$H(\vec{y}) = \begin{cases} f(|y|) \quad & |y| \leq r_0 \\ \frac{NQ}{|y|^2} \quad & r_0 \leq |y| \end{cases} \quad (8)$$

As we argued in Eq. (9), in this case the KK harmonics are decoupled. In consequence, we can study them separately. The equations of motion for an arbitrary mode $\phi^I$ in this background are:

$$\frac{1}{r^5} \partial_r (r^5 \partial_r \phi^I) + \frac{q(I)}{r^2} \phi^I - k^2 f(r) \phi^I = 0 \quad r \leq r_0$$

$$\frac{1}{r^5} \partial_r (r^5 \partial_r \phi^I) + \frac{q(I)}{r^2} \phi^I - k^2 \frac{NQ}{r^4} \phi^I = 0 \quad r_0 \leq r, \quad (9)$$

where $q(I) = -l(l + 4)$ is the eigenvalue associated to spherical harmonics $Y^I$. Let $\chi$ and $\Psi$ be the two solutions of the equation for $r \leq r_0$ and assume that $\chi$ is well behaved at zero while $\Psi$ is well behaved at infinity. Then, matching the solutions of Eq. (6) at $r = r_0$ we have for the solutions in the interval $r \in [0, \infty)$:

- Solution well behaved at $r = \infty$ ($\xi = \frac{r_0}{r} = 0$):

$$\psi^I_0 (\xi) = \begin{cases} \Psi(\xi, \alpha) + \gamma^I(\kappa) \chi(\xi, \alpha) \quad & r \leq r_0 \\ \xi^2 I_{l+2} (\kappa \xi) \quad & r > r_0 \end{cases}. \quad (10)$$
Here $\kappa^2 = \frac{K N Q}{r_0^3}$.

- Solution well behaved at $r = 0$ ($\xi = \frac{r}{r_0} = \infty$):

$$\psi^J_2(\xi) = \begin{cases} 
\chi(\xi, I) & r \leq r_0 \\
\xi^2 K_{I+2}(\kappa \xi) + \beta^I(\kappa) \xi^2 I_{I+2}(\kappa \xi) & r > r_0
\end{cases}.$$  \hspace{1cm} (11)

Let us give two examples. For the case of the spherical shell distribution

$$\rho(|y|) = \frac{N}{V_3 r_0^6} \delta(r - r_0)$$  \hspace{1cm} (12)

we have $f(|y|) = \frac{Q N}{r_0^3}$ and

$$\chi(\xi, I) = \xi^2 I_{\nu} \left(\frac{\kappa}{\xi}\right), \quad \nu = 2 + l.$$  \hspace{1cm} (13)

For the case of uniform distribution

$$\rho(|y|) = \begin{cases} 
\frac{6N}{V_3 r_0^6} & |y| \leq r_0 \\
0 & r_0 \leq |y|
\end{cases}.$$  \hspace{1cm} (14)

we have $f(|y|) = \frac{Q N}{r_0^3}(3r_0^2 - 2r^2)$ and

$$\chi(\xi, I) = \xi^{-l} e^{\frac{i\pi}{2} + \frac{l}{2}} F_1 \left[\frac{3}{2} + l + \frac{3\sqrt{2}\kappa}{8}, l + 3, -\sqrt{2}i\kappa \xi^{-2}\right].$$  \hspace{1cm} (15)

Since we are interested in the limit $r \to \infty$ ($\xi \to 0$), only $\beta(\kappa)$ is relevant. It reads

$$\beta^I(\kappa) = \frac{\chi(\xi, I) \partial_\xi (\xi^2 K_{I+2}(\kappa \xi))) - \partial_\xi \chi(\xi, I) (\xi^2 K_{I+2}(\kappa \xi))|_{\xi = 1}}{\chi(\xi, I) \partial_\xi (\xi^2 I_{I+2}(\kappa \xi)) - \partial_\xi \chi(\xi, I) (\xi^2 I_{I+2}(\kappa \xi))|_{\xi = 1}}.$$  \hspace{1cm} (16)

With this solution the scalar Green’s function has the following expression

$$G^I_0(x, y) = G^I_0(x, y) + \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} \psi^I_2(x_0) \psi^I_2(y_0) \psi^I_1(\epsilon) \psi^I_2(\epsilon),$$  \hspace{1cm} (17)

where

$$G^I_0(x, y) = -\int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} \left\{ \begin{array}{ll}
\psi^I_1(x_0) \psi^I_2(y_0) & x_0 < y_0 \\
\psi^I_1(y_0) \psi^I_2(x_0) & x_0 > y_0
\end{array} \right.$$  \hspace{1cm} (18)

Using the fact that the action on this solution is

$$S[\phi] = -\frac{1}{2} \lim_{\epsilon \to 0} \int d^4 x e^{-3} \phi^I \partial_0 \phi^I \bigg|_{x_0 = \epsilon}$$  \hspace{1cm} (19)

with

$$\phi^I(x, x_0) = \int d^4 y \left( y_0^{-3} \frac{\partial^I}{\partial y_0} G^I_0(x, y) \right) \bigg|_{y_0 = \epsilon} \phi^I_0(y) e^{-l}$$

$$= \int d^4 y \phi^I_0(y) e^{-l} \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} \frac{\psi^I_2(x_0)}{\psi^I_2(\epsilon)}.$$  \hspace{1cm} (20)
we get the two-point function
\[ \langle O^I(x)O^J(y) \rangle = -\delta^{IJ} \int \frac{d^4k}{(2\pi)^4} e^{-i\vec{k}(x-y)} \left( \frac{2^{1-2\nu} \Gamma(1-\nu)}{\Gamma(\nu)} (k \sqrt{QN})^{2\nu} - \frac{2^{1-2\nu}}{\Gamma(\nu)\Gamma(\nu+1)} \beta^I(\kappa)(k \sqrt{QN})^{2\nu} \right), \quad (21) \]
where \( \nu = 2 + l \), and after performing the \( k \) integral we find
\[ \frac{\langle O^I(x)O^J(y) \rangle}{(QN)^\nu} = \delta^{IJ} \frac{2\nu^2(1+\nu)}{\pi^2|x-y|^{4+2\nu}} \left( 1 + \frac{2^{-2(\nu+1)}}{\Gamma(\nu+1)\Gamma(\nu+2)} \int_0^\infty dk \kappa^6 J_1(k) \beta^I \left( k \frac{\sqrt{QN}}{r_0|x-y|} \right) \right) \quad (22) \]
In the extreme UV limit, \( |x-y|r_0/\sqrt{QN} \ll 1 \), we have \( \beta^I \to 0 \) and we recover CFT correlators. Note that CFT behaviour is valid down to a much lower energy scale \( \frac{\mu}{\sqrt{YM/N}} \) compared to the Higgs scale \( r_0 \).

4. Performing the Wick rotation \( k \to -i\kappa \) to Minkowski space one opens the possibility of studying at least part of the spectrum of SYM theory on the Coulomb branch. Recalling the transformations of Bessel functions \( I_\nu(-iz) = e^{-iz}J_\nu(z) \), \( K_\nu(-iz) = \frac{\pi}{2}e^{iz}J_\nu(z) \) we find
\[ \beta^I(\kappa) \to (-1)^I \frac{i\pi}{2} + (-1)^I \frac{\pi}{2} \left| \frac{\partial \tilde{\chi}(\xi, I) \partial \xi (\xi^2 J_{l+2}(\kappa \xi)) - \partial \xi \tilde{\chi}(\xi, I) (\xi^2 J_{l+2}(\kappa \xi))}{\partial \xi \tilde{\chi}(\xi, I) (\xi^2 J_{l+2}(\kappa \xi))} \right|_{\xi=1}, \quad (23) \]
where \( \tilde{\chi} \) is the Wick rotated solution of the wave equation for \( r \leq r_0 \) which is well-behaved at \( r = 0 \). The first term in Eq. (23) cancels against a similar term coming from the conformal piece of the two-point function (first term in Eq. (21)). At this point we can read of the singularity structure of the two-point function (see figures 1 and 2). The poles are given by the solutions of the equation
\[ \tilde{\chi}(\xi, I) \partial \xi (\xi^2 J_{l+2}(\kappa \xi)) - \partial \xi \tilde{\chi}(\xi, I) (\xi^2 J_{l+2}(\kappa \xi)) \bigg|_{\xi=1} = 0. \quad (24) \]
The solutions of Eq. (24) which are first order zeroes correspond to states in SYM which are color singlets, have the right quantum numbers to couple to \( O^I = \text{tr}(F^2 X^I) \) and have masses of order \( \mu/\sqrt{YM/N} \). In the large \( N \) limit these states are stable. However, for finite \( N \) they become unstable against decay into photons. The strength of the coupling can be read from the two-point function.

The same set of states can be obtained in the approach of ref.[16]. One starts with the wave equation for the mode \( \phi^I \) and solves it as an eigenvalue equation subject to suitable boundary conditions required by normalizability. In our case, the solutions are those given in Eq. (11) and Eq. (11). Imposing normalizability of the solution at \( r = 0 \) singles out the solution in Eq. (11), while normalizability at infinity implies that the coefficient of
\[ \xi^2 K_{l+2}(\kappa \xi) \text{ (the denominator of } \beta^l(\kappa)) \text{ is zero. This is the Wick-rotated form of Eq. (24)} \]
giving therefore the same spectrum.

For the uniform distribution discussed before, a plot of the Wick-rotated function \( \beta \) for \( l = 0 \) looks as in figure 2.

\[ \text{Fig. 2: Plot of } \beta^{l=0}(\kappa) \text{ for the distribution Eq. (14).} \]

Using the explicit solution of the wave equation and/or numerics, one can show that all the poles are simple poles and thus correspond to physical states. Numerical analysis shows that the poles move towards higher masses as one increases \( I \). Similar behaviour arises for the spherical shell distribution.

5. One may wonder whether the poles of the two-point function we found are artifacts of continuous distribution of D3 branes. It would be very interesting to compute some two-point functions for the case of the SYM with the gauge group \( U(2N) \) broken to \( U(N) \times U(N) \) using the two-center D3 brane solution. For this purpose one has to solve 10D wave equation following from the 10D action Eq. (6) on the two-center D3 brane background. We expect that the two-point functions will have poles in this case as well.

\[ \text{Fig. 1.} \]
Acknowledgments

We are grateful to H. Nastase and M. Roček for useful discussions. We would also like to thank I. Klebanov, J. Maldacena, P. van Nieuwenhuizen, H. Ooguri, L. Rastelli, D. Vaman and T.T. Wu for stimulating discussions.

Note Added

As this work was being completed, there appeared refs. [17, 18] which discuss some other distributions of D3 branes and give conceptually different interpretation of AdS/SYM correspondence on the Coulomb branch. In ref. [17] the extreme smallness of masses at the poles of the two-point functions was interpreted as an artifact of SUGRA approximation. The argument was based on the fact that the curvatures of the geometries which were considered become large close to the brane distribution and therefore, at low energies, supergravity is not reliable. Our uniformly distributed branes solution does not share this feature, but we still find masses of the same order of magnitude, suggesting that the unnatural smallness of masses is not an artifact of SUGRA approximation.

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