Linear active disturbance rejection control for the electro-hydraulic position servo system

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Abstract

Valve-controlled asymmetric cylinder is widely used in servo loading system. As a kind of typical electro-hydraulic servo system (EHSS), it inherently has the characteristics such as high order nonlinear, strong coupling, and uncertain, therefore, conventional control strategy is difficult to satisfy the requirements of high-performance control. In this paper, a novel linear active disturbance rejection control (LADRC) method was proposed, in which the internal and external disturbances were actively estimated by the third-order linear extended state observer (LESO) in real-time, and rejected by the control law of proportional integral control (PID) with acceleration feed-forward. The stability of the proposed method was proved, and the influence rules of the LADRC parameters on the control performance were revealed by simulation. Finally, comparative experiments between LADRC and PID control were carried out, results showed that the disturbances can be effectively compensated and the control goals can be successfully achieved with the proposed method.

Keywords

Valve-controlled asymmetric cylinder, electro-hydraulic servo system (EHSS), disturbance, linear extended state observer (LESO), linear active disturbance rejection control (LADRC)

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Introduction

As a kind of important actuator, valve-controlled asymmetric cylinder electro-hydraulic servo system (EHSS) has been widely used in servo loading system such as robotic arms,\(^1\) automotive suspension,\(^2\) and steel manufacturing equipment,\(^3\) etc. due to the characteristics such as high power-mass ration, fast response, high stiffness, and less space occupation. However, EHSS exhibits significant higher nonlinear dynamics, as for instance dead zone of the valve, flow, pressure, friction, and output saturation of the controller. Moreover, it is common that some of the system parameters are uncertain, especially the time-varying load and the structure of the asymmetry hydraulic cylinder. Therefore, in order to improve the tracking performance of EHSS, treatment of the uncertainties and development of high-performance closed-loop controllers have become great challenges in both academic and industrial fields over the last decade.

To achieve the high-performance tracking control of EHSS, model-based design method, known for its rigor, is highly focused in the modern control theory, such as adaptive control,\(^4\) robust control,\(^5\) variable structure control,\(^6-8\) H-infinity,\(^9-11\) fuzzy control,\(^12,13\) neural network control,\(^14,15\) feedback linearization control,\(^16-18\) internal model control (IMC),\(^19\) back-stepping control,\(^20\) and so forth. However, the influence of various uncertain factors on the control performance is rarely considered comprehensively, such as change of the load, the modeling error, the sensor noise, and the un-modeled dynamics. Adaptive control is mainly applicable to linear systems, and the uncertainty can be expressed in the form of unknown linear parameters. Unfortunately, the nonlinear uncertain often go far beyond the scope of the adaptive control theories in the EHSS, which will result in more complexity of the adaptive algorithm, and difficulty for the direct application in engineering practice. Robust control technique has also been utilized in hydraulic servo systems, to deal with parametric uncertainties and uncertain nonlinearities. However, typical nonlinear robust control usually requires that the uncertainties are bounded in some categories and have certain structural properties. In engineering practice, the value of the disturbance is too difficult to express precisely, especially the varying payload. To ensure the robustness of the control system, the upper bound of the system uncertainties is usually expressed in a conservative way, which will limit the application scope of the controller. Variable structure control (VSC) is a special kind of nonlinear control strategy. When the system state is operating in the sliding mode or the self-stabilizing domain, the system parameter variations and external disturbances are completely invariant, but the upper bound of the uncertain dynamic or disturbance still need to be provided in order to realize the sliding mode of the self-stabilizing domain threshold. At the same time, high frequency modes of the system is easy to be excited by the vibration caused by the switching control function of VSC, and the control performance will be degraded and even become unstable.

The feedback linearization method has some robustness with respect to both parametric uncertainties and uncertain nonlinearities. However, the accurate mathematical model and system states are also needed to design the controller. In the
back-stepping design process, explosive terms are easy to produce for the high order EHSS, and the designing control law is difficult to implement in engineering due to the complexity.

What is more, various disturbance estimation techniques have been proposed to compensate or suppress the effect of disturbance on the EHSS, such as the disturbance observer (DOB), \(^{21,22}\) the sliding mode observer (SMO), \(^{23}\) high gain state observer (HGSO), \(^{24}\) and the extended state observer (ESO) \(^{25}\) etc. In most estimators, such as DOB, SMO, and HGSO, the disturbance is reconstructed under assumption that the exact mathematical model is available. Its applicability is however often unsatisfied due to the lack of detailed plant model in practice, or the high cost for obtaining the model. Firstly, the structural and parametric uncertainties, especially the time-varying payload make it difficult to describe the dynamic model by a precise mathematical expression. Secondly, the system position, velocity, acceleration, and other system states are needed in the process of the controller design. Nevertheless, only the piston displacement can be directly measured in real engineering practice. Finally, the EHSS process can easily falls into unstable state if the control is designed solely based on a model because of the significant changes of the payload and the uncertain parameters.

Therefore, the strict model-based designs show their limitations, and the control design must be carried out with less model information. Hence, the PID control strategy always plays an important role in EHSS, thanks to the advantages of simple principle, convenient use, and strong adaptability. However, due to the hysteresis effect, overshoot, and oscillation is easily to form in PID control, and the high-performance of the control requirements is difficult to meet.

From the above analysis, it can be concluded that a control approach, suitable for EHSS with great disturbance rejection ability, strong robustness, easy to apply, and independent of an exact model is urgently needed for high performance displacement tracking control.

Based on the extended state observer (ESO), the active disturbance rejection controller (ADRC) is an efficient control method, to deal with the plants containing lumped uncertain dynamics and disturbances, and without depending on the exact model information. The key of ADRC is to estimate the total uncertainty (or total disturbance) online by the ESO, which lumps the internal uncertain dynamics and the external disturbances. Under the framework of ADRC, the total disturbance is treated as the extended state, and effectively estimated by the ESO and compensated in real time. The original nonlinear dynamic system will be forced to behave like a simple, disturbance-free plant.

It has attracted application of several industrial companies, such as Parker Hannifin extrusion plant and Texas instrument. Up to now, the idea of ADRC technique has been successfully applied in solving various kinds of engineering problems, such as chemical, \(^{26}\) electric system, \(^{27}\) robot, \(^{28}\) and other process industrial systems. \(^{29}\)

This research aims to develop a robust and easy-to-implement control approach, and provide a practical solution based on LADRC technology to achieve
high-performance tracking control of EHSS, even the EHSS model information is unknown. The closed-loop stability of the EHSS has been verified, and the feasibility and effectiveness of the proposed method has been demonstrated by simulation and experiments.

The rest of this paper is organized as follows. Section 2 describes the detailed nonlinear model equations with a single-rod electro-hydraulic system. Section 3 presents the combined structure of LADRC with acceleration feed-forward, stability analysis is given in section 4. Comparative simulation and experiments are carried out in section 5, and the influences of the observer bandwidth and controller bandwidth on control performance are also revealed. Then the conclusions are drawn in section 6.

**System modeling and problem formation**

It is known that EHSS is a complex multivariable coupled SISO (Single-Input Single-Output) nonlinear system. The schematic diagram of EHSS configuration is show in Figure 1, which mainly consists of the filter 1, motor 2, hydraulic pump 3, three-position four-way electro-hydraulic proportional valve 4, overflow valve 5, single-outlet hydraulic cylinder 6, load 7, controller 8, linear displacement sensor 9, analog-to-digital converter 10, and the digital-to-analog converter 11. The supply pressure $P_s$ is the invariable, and the return pressure $P_r$ is small since it is directly connected to the oil tank. The goal of the system is to obtain the elastic and inertial load tracking the expected smooth motion trajectory as close as possible, only with the position value $x$ collected by the linear displacement sensor with a $0.5 \mu m$
resolution, even under circumstances encountering nonlinear and time-varying uncertain disturbances.

In operation, the real-time load displacement $x$ is measured by the linear displacement sensor, and converted into a digital signal by the A/D converter, then compared with the given reference signal and taken as the input of the controller. The output of the controller is a digital signal DC voltage $u$ converted by the bipolar D/A converter. It is no more than 10 volts and is used to adjust the spool displacement of the three-position four-way electro-hydraulic proportional valve. When the spool moves, the orifices in the valve are opened. The fluid goes through one of the valve ports to the hydraulic cylinder, then flow back to the valve from the cylinder through another port. The flow that goes into and gets out of the cylinder has two different pressures $P_1$ and $P_2$, at the piston side and rod side, respectively. Both $P_1$ and $P_2$ act on the piston and the load is driven to move.

Under the control of the LADRC, the exact model of EHSS is not required. Nevertheless, the model information is particularly helpful for the design of the controller and enhance of the control quality. So, the dynamic mechanism model of the EHSS is constructed based on the nominal values of physical parameters.

Based on Newton’s second law, the dynamics of elastic and inertial load can be described by

$$
\ddot{x} = \frac{1}{M} \left( P_1 A_1 - P_2 A_2 - B_p \dot{x} - c x - d - f_u(x, \dot{x}, t) \right)
$$

where $M$ is the payload mass, $B_p$ and $c$ are the viscous damping coefficient of the actuator and the load stiffness, respectively, $d$ is the unknown external load and uncertainties of the mechanical dynamics, $f_u(x, \dot{x}, t)$ is the un-modeled dynamics, $P_1$ and $P_2$ denote pressures at the piston side and rod side, respectively, $A_1$ and $A_2$ are the annulus areas of the piston side and the rod side, $x$, $\dot{x}$, and $\ddot{x}$ are the piston position, velocity, and acceleration, respectively.

Ignoring the external leakage of the cylinder, the flow continuity equation of the two chambers can be described by

$$
\dot{P}_1 / \beta_e = V_1^{-1} (Q_1 - A_1 \dot{x} - C_t (P_1 - P_2))
$$

$$
\dot{P}_2 / \beta_e = V_2^{-1} (-Q_2 + A_2 \dot{x} + C_t (P_1 - P_2))
$$

where $V_1 = V_{10} + A_1 x$ and $V_2 = V_{20} - A_2 x$ are the total control volume of the piston chamber and rod chamber respectively, in which $V_{10}$ and $V_{20}$ are the initial values. $C_t$ is the internal leakage coefficient of the cylinder, $\beta_e$ is the effective bulk modulus of the oil, $Q_1$ and $Q_2$ are the supplied flow rate into the piston chamber and return flow rate from the rod chamber, respectively.

Define switching functions $S(\cdot)$ as

$$
S(\cdot) = \begin{cases} 
1 & (\cdot) \geq 0 \\
0 & (\cdot) < 0
\end{cases}
$$

then the flow rate modulated by the electro-hydraulic proportional valve can be described by
\[ Q_1 = k_q x_v \left( S(x_v) \sqrt{P_s - P_1} + S(-x_v) \sqrt{P_1 - P_r} \right) \]  (3a)

\[ Q_2 = k_q x_v \left( S(x_v) \sqrt{P_2 - P_r} + S(-x_v) \sqrt{P_s - P_2} \right) \]  (3b)

where \( k_q = C_d \omega \sqrt{2/\rho} \), \( C_d \), and \( \omega \) are the flow coefficient and area gradient of the electro-hydraulic proportional valve, \( \rho \) is the oil density, \( x_v \) is the spool displacement, \( P_s \) and \( P_r \) are the supply pressure and return pressure, respectively.

Considering that the dynamic response speed of the electro-hydraulic proportional valve used is much higher than the hydraulic cylinder operation frequency band, the linear model of the valve can be expressed by

\[ x_v = k_v u (|u| \leq 10) \]  (4)

where \( k_v \) is the gain of electro-hydraulic proportional valve, and \( u \) is the control output voltage.

Substituting equation (4) into equation (3), then \( Q_1 \) and \( Q_2 \) can be described by

\[ Q_1 = k_q k_v u \left( S(u) \sqrt{P_s - P_1} + S(-u) \sqrt{P_1 - P_r} \right) \]  (5a)

\[ Q_2 = k_q k_v u \left( S(u) \sqrt{P_2 - P_r} + S(-u) \sqrt{P_s - P_2} \right) \]  (5b)

For simplicity, define

\[ \gamma = k_q k_v \]  (6)

and

\[ \begin{cases} R_1 = S(u) \sqrt{P_s - P_1} + S(-u) \sqrt{P_1 - P_r} \\ R_2 = S(u) \sqrt{P_2 - P_r} + S(-u) \sqrt{P_s - P_2} \end{cases} \]  (7)

where \( R_i = 1, 2 \) are nonlinear time-varying function of the \( P_s, P_r, \) and \( P_i = 1, 2 \) respectively. Then equation (5) can be simplified as

\[ Q_1 = \gamma R_1 u \]  (8a)

\[ Q_2 = \gamma R_2 u \]  (8b)

Substituting equation (8) into equation (2), then

\[ \dot{P}_1 = \beta_e V_1^{-1} (\gamma R_1 u - A_1 \dot{x} - C_i (P_1 - P_2)) \]  (9a)

\[ \dot{P}_2 = \beta_e V_2^{-1} (-\gamma R_2 u + A_2 \dot{x} + C_i (P_1 - P_2)) \]  (9b)

Differentiate the equation (1) and substitute equation (9) into it, then the dynamics of elastic and inertial load can be expressed as
\[
\begin{align*}
\ddot{x} &= a_2 \dot{x} + a_1 \dot{x} + a_0 + b_1 u \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 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valve port by adjusting the control input voltage, to realize the adjustment of the two-chamber flow of the inflow and outflow cylinders, then the piston operating speed, and finally the piston displacement control can be realized.

Since

\[ Q_1 = A_1 \dot{x}, \quad Q_2 = A_2 \dot{x} \]

Considering time delay exists in the EHSS, from control input \( u \) to the output of the piston, displacement \( x \) can be described by

\[ e^{-\tau t} \]

where \( \tau \) is an unknown uncertain time constant \( \tau > 0 \).

Zhao and Gao\(^{33} \) proposed a modified ADRC design method to accommodate time delay. If the delay time is known, an equivalent delay is introduced to the control signal, which will synchronize the signals that go into the observer. Finally, the purpose of improving the state estimation accuracy, enhancing the anti-interference ability, and reducing the observer bandwidth are realized. However, the setting delay time mismatch may cause the degrade of control performance.

Generally, the time delay could be approximated by a first-order inertial link\(^{32} \) as

\[ e^{-\tau s} \approx \frac{1}{1 + \tau s} \]

Comprehensively consider equations (4), (8), (11), and (12), and perform Laplace transform. The relationship between the control input voltage \( u \) and the piston output displacement \( x \) can be approximated as second-order system.

\[ \frac{x(s)}{u(s)} = \kappa \frac{1}{s} \cdot \frac{1}{1 + \tau s} \]

Where \( \kappa \) is the time-varying constant, and \( \kappa > 0 \).

From equation (14), the EHSS can be considered as a second-order system for the next controller design.

Control gain estimation value \( b_0 \) is a critical parameter of the ADRC, which has an important effect on the system control performance. Based the equation (10), it has at least the following two forms, when described by a second-order system.

\[ \ddot{x}(t) = \frac{1}{a_2} (\ddot{x} - a_1 \dot{x} - a_0 - b_1 u + a_2 b_0 u + \Delta) + b_0 u, \quad (|u| \leq 10) \]

\[ \ddot{x}(t) = \int_{0}^{t} (a_2 \ddot{x} + a_1 \dot{x} + a_0 + b_1 u) dt - b_0 u + \Delta + b_0 u, \quad (|u| \leq 10) \]
The total disturbance $f(x, d, P_1, P_2, u)$, in equations (15) and (16), are identical in form, including model uncertainties, input disturbance and external disturbances, where $\Delta$ is the un-modeled dynamics.

In equation (15), the constant coefficient $b_0$ is the estimation value of the

$$b = -b_1/a_2 = \beta_ek_qk_v(R_1A_1/V_1 + R_2A_2/V_2)/B_p$$  \hspace{1cm} (17)

yet, in equation (15), it is the estimated value of the

$$b = \int_0^t bdt = \int_0^t \left(\beta_ek_qk_v(V_1 + A_2V_2)/M\right)dt$$  \hspace{1cm} (18)

Although the value of $\beta_ek_qk_v(A_1R_1/V_1 + A_2R_2/V_2)$ is exactly the same in both equations (17) and (18), and they are all molecular terms. Nevertheless, in equation (18), the effective payload $M$ has a significant time-varying uncertainty in practice, after integration, the control gain is difficult to estimate. In equation (17), the viscous damping coefficient $B_p$ is also time-varying, but the variation is relatively limited, the problem of the estimating control gain becomes relatively simple.

The time delay transfer function can be described as

$$G(s) = e^{-\tau s}$$  \hspace{1cm} (19)

and the frequency characteristics are

$$G(j\omega) = e^{-j\tau\omega} = \cos(-\tau\omega) + jsin(-\tau\omega) \hspace{2cm} A(\omega) = \sqrt{(\cos(\tau\omega))^2 + (-\sin(\tau\omega))^2} = 1 = \text{const} \hspace{2cm} \varphi(\omega) = -\tau\omega$$  \hspace{1cm} (20)

Equation (20) shows that the time delay only affects the phase and does not affect the amplitude.

Therefore, it is reasonable to use the second-order system of equation (15) to describe EHSS, which is beneficial to the design of the LADRC controller.

**Design of the LADRC control law**

Based on the extended state observer (ESO) and as an alternative solution, LADRC can actively estimates and compensates for the effects of discrepancies between the real process and the assumed mathematical model as well as the influence of any unknown external disturbances in real time, and the real process will forced to behave like a simple, disturbance-free plant. Unlike the tradition control methods, the controller draws the information needed to control the plant is not depended on the model of the plant but on the ESO. It will offer a new and inherent robust control method for EHSS. The dependence on explicit modeling will be greatly reduced, and it has the advantages of simplicity in engineering implementation and superior performance. Based on equation (15), the LADRC schematic
The diagram of the EHSS is shown in Figure 2, which includes the tracking differentiator (TD), state feedback controller, and linearly extended state observer (LESO).

In order to solve the contradiction between the fastness and stability of the control system, TD is used to arrange the transition process. $v$, $\dot{v}$, and $\ddot{v}$ are the outputs of TD, which are the desired track position, velocity, and acceleration of the EHSS, respectively.

The third-order linear TD has the following form:

$$
\begin{align*}
\dot{x}_1 &= v_1 \\
\dot{x}_2 &= v_2 \\
\dot{x}_3 &= v_3 \\
\end{align*}
$$

where $r$ is positive constants, and is used to regulate the tracking speed, $v$ is input signal to be tracked.

Define $x_1 = x$, $x_2 = \dot{x}$, and $x_3 = f$, assuming $f$ and its derivative $\eta$ are locally Lipchitz in their arguments and bounded within the domain of interest. In addition, the initial conditions are assumed as $f|_{t=0} = 0$, and $\eta|_{t=0} = 0$. Then, the plant in equation (14) can be described by

$$
\begin{align*}
\dot{x} &= Ax + Bu + E\eta \\
y &= cx
\end{align*}
$$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ b_0 \\ 0 \end{bmatrix}$, $E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $f$.

Based on equation (22), the third-order LESO such as
\[
\begin{align*}
\dot{x} &= Ax + Bu + L(y - \hat{y}) \\
\hat{y} &= C\hat{x}
\end{align*}
\]  
(23)

With the observer gain

\[
L = [l_1 \ l_2 \ l_3]^T
\]  
(24)

can be constructed, so that \(x\) can be accurately estimated by \(\hat{x}\). The observer gains are chosen such that the characteristic polynomial \(s^3 + l_1 s^2 + l_2 s + l_3\) is Hurwitz. For tuning simplicity, all the observer poles are placed at \(-\omega_o\). It results in the characteristic polynomial of (23) to be \(\lambda_o(s) = s^3 + l_1 s^2 + l_2 s + l_3 = (s + \omega_o)^3\), where \(\omega_o\) is the observer bandwidth and

\[
L = [l_1 \ l_2 \ l_3]^T = \begin{bmatrix} 3\omega_o & 3\omega_o^2 & \omega_o^3 \end{bmatrix}^T
\]  
(25)

with a well conducted observer, the states of the system \(x, \dot{x},\) and \(f\) can be closely tracked by the observer output of \(\hat{x}_1, \hat{x}_2,\) and \(\hat{x}_3\), respectively.

Employing the third-LESO of (22) in the form of (23), the state feedback control law with acceleration feed-forward is given as

\[
u = \frac{1}{b_0} [k_1 (v - \dot{x}_1) + k_2 (\dot{v} - \dot{x}_2) - \dot{x}_3 + \ddot{v}]
\]  
(26)

where \(k_1\) and \(k_2\) are the controller gains parameters.

Submitting equation (26) into equation (15), then the closed EHSS become

\[
\dot{y} = (f - \dot{x}_3) + k_1 (v_1 - \dot{x}_1) + k_2 (v_2 - \dot{x}_2) + v_3
\]  
(27)

With a well design LESO, the first term in the right hand side (RHS) of (27) is negligible and the rest of the terms in the RHS of (27) constitutes a generalized PD controller with an acceleration feed-forward.

For the convenience of parameter tuning of the state feedback controller, the parameterization technique proposed by Gao is adopted, and \(k_{i=1,2}\) are selected to make the characteristic polynomial of the closed loop system as

\[
s^2 + k_2 s + k_1 = (s + \omega_c)^2
\]  
(28)

Then \(k_1 = \omega_c^2, k_2 = 2\omega_c,\) and \(\omega_c\) is the controller bandwidth.

**Stability analysis**

To assess the reliability of the proposed method, it is very important to perform the stability analysis of the proposed method.

**Convergence of the third-order LESO**

Define the state estimation error vector of third-order LESO as
\[
\ddot{e} = \ddot{x} - \dot{x} = \begin{bmatrix} x_1 - \dot{x}_1 & x_2 - \dot{x}_2 & x_3 - \dot{x}_3 \end{bmatrix}^T
= \begin{bmatrix} \ddot{e}_1 & \ddot{e}_2 & \ddot{e}_3 \end{bmatrix}^T
\]  
(29)

Subtracting equation (22) from equation (23), the error dynamics of the LESO is as follows
\[
\dot{\tilde{e}} = (A - LC)\tilde{e} + E\eta
\]  
(30)

where \(A, L, C, E,\) and \(\eta\) are defined as that in equations (22) and (23), respectively.

For the purpose of parameterization and the stability analysis, the states transformation is define as
\[
\tilde{e}_i = 1, 2, 3 \quad (t) = \tilde{e}_i(t)/\omega_o^{i-1}
\]  
(31)

Then, equation (30) can be written as
\[
\dot{e}(t) = \omega_o A_\varepsilon e + E\frac{\eta}{\omega_o^2}
\]  
(32)

where \(e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}, \ A_\varepsilon = \begin{bmatrix} -3 & 0 & 1 \\ -3 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.\)

Solving equation (32), the solution of \(e(t)\) can be expressed as
\[
e(t) = e^{\omega_o A_\varepsilon t} e(0) + \int_0^t e^{\omega_o A_\varepsilon (t-\tau)} E \frac{\eta}{\omega_o^2} d\tau
\]  
(33)

Let \(p(t) = \int_0^t e^{\omega_o A_\varepsilon (t-\tau)} E \frac{\eta}{\omega_o^2} d\tau\), since \(\eta\) is bounded, that is \(|\eta| \leq \delta\) and \(\delta > 0\)
\[
|p_i(t)| \leq \frac{\int_0^t \left| e^{\omega_o A_\varepsilon (t-\tau)} E \right| \eta d\tau}{\omega_o^2} \leq \frac{\delta \int_0^t \left| e^{\omega_o A_\varepsilon (t-\tau)} E \right| d\tau}{\omega_o^2} = \frac{\delta}{\omega_o^2} \left\{ \left[ (A_\varepsilon^{-1} E) \right]_i \right\}
\]  
(34)

Since \(A_\varepsilon^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & -3 \\ 0 & 1 & -3 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\) hence
\[
\left| (A_\varepsilon^{-1} E)_{i = 1,2,3} \right| = \begin{bmatrix} -1 \\ -3 \\ -3 \end{bmatrix}
\]  
(35)

The matrix \(A_\varepsilon\) is Hurwitz, there exists a finite time \(T_1 > 0\) such that
Let $A_{i}^{-1} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}$ and $e^{\omega_i A_i} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$ then

$$\left| \left[ e^{\omega_i A_i} \right]_{ij} \right| \leq \frac{1}{\omega_i^3} \tag{36}$$

for all $t \geq T_1$, $i, j = 1, 2, 3$. Hence

$$\left| \left[ e^{\omega_i A_i} E \right]_{i = 1, 2, 3} \right| \leq \frac{1}{\omega_i^3} \tag{37}$$

for all $t \geq T_1$. Considering $T_1$ depends on $\omega_i A_i$.

Substitute equations (35) and (36) into the inequality (32), then

$$\left| \left( A_{i}^{-1} e^{\omega_i A_i} E \right)_{i = 1, 2, 3} \right| = |s_{1i} d_{13} + s_{12} d_{23} + s_{13} d_{33}|$$

$$\leq \left( \frac{1}{\omega_i^3} \right)_{i = 1} \frac{|s_{12} + s_{13}|}{\omega_i^3} = \left( \frac{1}{\omega_i^3} \right)_{i = 2, 3} \frac{|s_{12} + s_{13}|}{\omega_i^3} \tag{38}$$

Substitute equations (35) and (36) into the inequality (32), then

$$|p_i(t)|_{i = 1, 2, 3} \leq \frac{\delta}{\omega_i^3} \left( 3 + \frac{4}{\omega_i^3} \right) = \frac{3\delta}{\omega_i^3} + \frac{4\delta}{\omega_i^6} \tag{39}$$

for all $t \geq T_1$.

Note that $\left| \left[ e^{\omega_i A_i} \right]_{ij} \right| = |d_{ij}|_{i, j = 1, 2, 3} \leq \frac{1}{\omega_i^3}$.

Let $\varepsilon_{sum}(0) = |\varepsilon_1(0)| + |\varepsilon_2(0)| + |\varepsilon_3(0)|$, the following relationship will be held.

$$\left| \left[ e^{\omega_i A_i} \varepsilon \right]_{i = 1, 2, 3} \right| = |d_{i1} \varepsilon_1(0) + d_{i2} \varepsilon_2(0) + d_{i3} \varepsilon_3(0)|$$

$$\leq \frac{1}{\omega_i^3} |\varepsilon_1(0) + \varepsilon_2(0) + \varepsilon_3(0)| \leq \frac{1}{\omega_i^3} \frac{|\varepsilon_1(0)| + |\varepsilon_2(0)| + |\varepsilon_3(0)|}{\omega_i^3} = \frac{\varepsilon_{sum}(0)}{\omega_i^3} \tag{40}$$

for all $t \geq T_1$. From equation (33), the following will be held.

$$|\varepsilon_i(t)|_{i = 1, 2, 3} \leq \left| \left[ e^{\omega_i A_i} \varepsilon \right]_{i = 1, 2, 3} \right| + |p_i(t)| \tag{41}$$

Let $\tilde{\varepsilon}_{sum}(0) = |\tilde{\varepsilon}_1(0)| + |\tilde{\varepsilon}_2(0)| + |\tilde{\varepsilon}_3(0)|$.

According to $\tilde{\varepsilon}_i = 1, 2, 3 = \frac{\tilde{\varepsilon}_i}{\omega_i^3}$ and equations (39)–(41), the states estimation error of the third-order LESO have

$$|\tilde{\varepsilon}_i(t)|_{i = 1, 2, 3} \leq \frac{\varepsilon_{sum}(0)}{\omega_i^3} + \frac{\delta \nu}{\omega_i^{4-i}} + \frac{\delta \nu}{\omega_i^{7-i}} = \mu \tag{42}$$
for all $t \geq T_1$.

The equation (42) show that the higher of observer bandwidth $\omega_o$, the smaller of the absolute error for the states estimation $|\tilde{e}_i(t)|_{i=1,2,3}$.  

**Convergence of the LADRC**

Define the expected tracking state vector as
\[
\mathbf{v} = [v_1 \ v_2 \ v_3]^T = [v \ \dot{v} \ \ddot{v}]^T
\]
and the closed-loop tracking error vector as
\[
\mathbf{e} = \mathbf{v} - \mathbf{x} = [v_1 - x_1 \ v_2 - x_2]^T = [e_1 \ e_2]^T
\]
then the equations (43) and (44) will be hold
\[
\dot{e}_1 = \dot{v}_1 - \dot{x}_1 = v_2 - x_2 = e_2
\]
\[
\dot{e}_2 = \dot{v}_2 - \dot{x}_2 = v_3 - (x_3 + b_0u)
\]

Submitting equations (26) and (28) into equation (46), then
\[
\dot{e}_2 = v_3 - (x_3 - \dot{x}_3 + k_1(v - \dot{x}_1) + k_2(\dot{v} - \dot{x}_2) + \ddot{v})
\]
\[
= v_3 - x_3 + \dot{x}_3 - k_1[v - (x_1 - \tilde{e}_1)] - k_2[\dot{v} - (x_2 - \tilde{e}_2)] - \ddot{v}
\]
\[
= - k_1(e_1 + \tilde{e}_1) - k_2(e_2 + \tilde{e}_2) - \dot{x}_3
\]

The error dynamics of the EHSS feedback loop can be expressed by
\[
\dot{\mathbf{e}}(t) = A_e\mathbf{e}(t) + A_{\tilde{e}}\tilde{\mathbf{e}}(t)
\]
where
\[
A_e = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}; A_{\tilde{e}} = \begin{bmatrix} 0 & 0 & 0 \\ -k_1 & -k_2 & 1 \end{bmatrix}.
\]

Solving equation (47), the analytical expression is
\[
\mathbf{e}(t) = e^{A_e t}\mathbf{e}(0) + \int_0^t e^{A_{\tilde{e}}(t-\tau)}A_{\tilde{e}}\tilde{\mathbf{e}}d\tau
\]

Since
\[
\begin{align*}
A_e\tilde{e}_i |_{i=1} &= [0 \ 0 \ 0][\tilde{e}_1 \ \tilde{e}_2 \ \tilde{e}_3]^T = 0 \\
A_e\tilde{e}_i |_{i=2} &= [-k_1 \ -k_2 \ -1][\tilde{e}_1 \ \tilde{e}_2 \ \tilde{e}_3]^T \\
&= -k_1\tilde{e}_1 - k_2\tilde{e}_2 - \tilde{e}_3 \\
&\leq k_1|\tilde{e}_1| + k_1|\tilde{e}_2| + |\tilde{e}_3| \\
&\leq (1 + k_1 + k_2)\mu = \sigma
\end{align*}
\]

Let $\varphi(t) = \int_0^t e^{A_{\tilde{e}}(t-\tau)}A_{\tilde{e}}\tilde{\mathbf{e}}d\tau$ and defined
\[ \Psi = [0 \ \sigma]^T \]  

\[ \varphi_i(t) = \left[ \int_0^t e^{A_e(t-\tau)} A^c \bar{e}(\tau) d\tau \right]_i \]

\[ \leq \left[ \int_0^t e^{A_e(t-\tau)} \Psi d\tau \right]_i \]

\[ \leq \left| (A_e^{-1} \Psi)_i \right| + \left| (A_e^{-1} e^{A_e t} \Psi)_i \right| \]

Since

\[ A_e^{-1} = \begin{bmatrix}
-\frac{k_2}{k_1} & -\frac{1}{k_1} \\
1 & 0
\end{bmatrix} = \begin{bmatrix}
-\frac{2}{\omega_c} & -\frac{1}{\omega_c^2} \\
1 & 0
\end{bmatrix} \]

so

\[ \left| (A_e^{-1} \Psi)_{i=1,2} \right| = \begin{cases}
\frac{\sigma}{\omega_c^2} & i=1 \\
0 & i=2
\end{cases} \]

Considering the matrix \( A_e \) is Hurwitz, there exists a finite time \( T_2 > 0 \), such that

\[ \left| [e^{A_e t}]_{ij} \right| \leq \frac{1}{\omega_c^3} \]  

for all \( t \geq T_2, i,j = 1,2,3 \).

Note that \( T_2 \) depend on \( A_e \),

Let \( e^{A_e t} = \begin{bmatrix} o_{11} & o_{12} \\ o_{21} & o_{22} \end{bmatrix} \) and \( e_{sum}(0) = \sum_{i=1}^{i=2} |e_i(0)| \). Then, the following inequality will hold

\[ \left| [e^{A_e t} e(0)]_{i=1,2} \right| = |o_{11} e_1(0) + o_{12} e_2(0)| \leq \frac{e_{sum}(0)}{\omega_c^3} \]  

for all \( t \geq T_2, i = 1,2 \).

Let \( T_3 = \max\{T_1,T_2\} \), then

\[ \left| (e^{A_e t} \Psi)_i \right| \leq \frac{\sigma}{\omega_c^3} \]  

for all \( t \geq T_3, i = 1,2 \), and

\[ \left| (A_e^{-1} e^{A_e t} \Psi)_i \right|_{i=1,2} \leq \begin{cases}
\frac{(2\omega_c + 1)\sigma}{\omega_c^3} & i=1 \\
\frac{(2\omega_c + 1)\sigma}{\omega_c^3} & i=2
\end{cases} \]

for all \( t \geq T_3 \). From equations (52), (54), and (58), the following result can be hold
\[
|\varphi_{i=1,2}(t)| \leq |(A_c^{-1}\psi_i)| + |(A_c^{-1}e^{A_c t}\psi_i)| \leq \left\{ \frac{\sigma}{\omega_c^3} + \frac{\sigma^2}{\omega_c^2} + \frac{2\omega_c + 1}{\omega_c} \right\} i = 1 \tag{59}
\]

for all \( t \geq T_3 \).

From equations (49), (52), (56), and (59), the following result can be hold

\[
|e_{i=1,2}(t)| \leq \left\{ \frac{e_{sum}(0)}{\omega_c^3} + \frac{\sigma}{\omega_c^2} + \frac{\sigma(2\omega_c)}{\omega_c} \right\} i = 1
\]

\[
\frac{e_{sum}(0)}{\omega_c^2} + (\omega_c^2 + 2\omega_c + 1)\sigma i = 2
\]

\[
(\omega_c^2 + 2\omega_c + 1)(2\omega_c + 1)\mu i = 1
\]

\[
\frac{e_{sum}(0)}{\omega_c^2} + (\omega_c^2 + 2\omega_c + 1)\mu i = 2
\]

\[
= \rho_1
\]

\[
= \rho_2
\]

\[\leq \rho\]

for all \( t \geq T_3 \), where \( \rho = \max\{\rho_1, \rho_2\} \).

The equation (60) shows that the tracking error of the EHSS output is bounded and the upper bound of the error monotonously decreases with the increase of the observer bandwidth \( \omega_o \) and the controller bandwidth \( \omega_c \).

**Simulations and experimental verification**

To verify the feasibility and effectiveness of the proposed control strategy, simulations and experiments have been carried out for the valve controlled asymmetric cylinder electro-hydraulic servo system, including trapezoid instruction and sine closed-loop tracking control tests.

**Simulation analysis**

Based on the differential equations (1)–(5) obtained in section 2 and the actual working conditions, the dynamic model of EHSS was built using the S-function in Matlab/Simulink, and the model parameters were selected as in Table 1.

It means that in the following simulation, the influence of all disturbances on the performance of the control system is comprehensively considered, including structure uncertainty, parameter uncertainty and the time-varying payload etc. The simulation configurations were as follows: ode3 (Bogacki-Shampine) was selected as the solver. The fixed-step and sampling period were set to 0.001 s, which were the same as the real experiment system. Finally, the robustness of various LADRC
parameters ($b_0$, $\omega_o$, and $\omega_c$) were analyzed. PID controller was selected as the performance reference, and the control performance was comparatively analyzed.

**Case 1: Influence of the control gain estimate variations on the control performance.** Due to the structural asymmetry and the initial volume uncertainty of the cylinder, the control gain of the EHSS has a great uncertainty in the process during the piston moving back and forth. In order to verify the robustness of the proposed control method, time-varying control gain $b = \beta_k k_d k_v (A_1 R_1 / V_1 + A_2 R_2 / V_2)$ was replaced by its estimated value $b_0$. In the control law equation (25), the following values of $b_0$ were considered: $b_0 = [3.5 \ 4 \ 4.5 \ 5 \ 5.3 \ 5.5 \ 6] \times 10^5$. Each controller was assessed in closed-loop for the same plant, the bandwidth of the feedback controller and LESO were designed as $\omega_c = 30 \text{rad/s}$ and $\omega_o = 120 \text{rad/s}$, respectively. The output of the EHSS under the control of the LADRC is shown in Figure 3.

The simulation results showed that: If the values of $\omega_c$ and $\omega_o$ were configured properly and unchanged, the control gain estimate $b_0$, as a critical parameter of the proposed method, can be valued in a larger range. After approximate 1.5 ms, the piston was driven to the expected position, and the steady-state error was no more than 2 mm. This demonstrates the high tracking performance of the LADRC. Nevertheless, with the estimate value of $b$ gradually deviates from the actual value, the dynamic tracking error $e_1$ will increase gradually, and obvious overshoot and oscillation has been produced. In order to ensure the good dynamic and static performance of the system, the value of $b_0$ should satisfy the following constraint: $0.9b \leq b_0 \leq b$.

**Case 2: Robustness against the variations in the LESO bandwidth.** In order to verify the robustness of LESO with the bandwidth $\omega_o$ variations, the following values were considered: $\omega_o = [70, 80, 90, 100, 110, 120, 150] \text{rad/s}$. The control gain estimate and the state feedback controller bandwidth were designed as $b_0 = 5.3 \times 10^5$ and $\omega_c = 35 \text{rad/s}$, respectively. Each LESO was assessed in a closed-loop for the same plant. The simulation results were shown in Figures 4 and 5.

The simulation results showed that: the observer bandwidth $\omega_o$ can be valued in a large range, when the controller parameters $\omega_c$ and $b_0$ were kept constant. The steady-state of the system output arrived quickly in 1.5 s. As $\omega_o$ increased gradually from small to large (from 80 to 120), the system state estimation accuracy was also

### Table 1. Parameters of the EHSS.

| Parameter | Value       | Parameter | Value          |
|-----------|-------------|-----------|----------------|
| $k_v (m/V)$ | $1.575 \times 10^5$ | $V_{02} (m^3)$ | $1.418 \times 10^{-4}$ |
| $k_q$     | $2.863 \times 10^{-9}$ | $\beta_k (\text{Pa})$ | $1.05 \times 10^9$ |
| $C_v (m^3/s \cdot \text{pa})$ | $1.28 \times 10^{-12}$ | $K (\text{N.m}^{-1})$ | $7.5 \times 10^4 \ t \leq 5$ |
| $P_s (\text{Pa})$ | $1.5 \times 10^7$ | $M (\text{kg})$ | $100 \ t \leq 5$ |
| $A_1 (m^2)$ | $1.964 \times 10^{-3}$ | $B_p (N \cdot s/m)$ | $700 \ t \leq 5$ |
| $A_2 (m^2)$ | $9.73 \times 10^{-4}$ | $V_{01} (m^3)$ | $700 + \sin(2(t - 5))(\text{sign}(v)) \ t > 5$ |
improved, and the piston displacement tracking error decreased gradually. As $\omega_v$ increased continuously, the response speed of the system remained invariably, but the accuracy of the state estimation decreased, and the steady state error increased. This phenomenon can be interpreted as follows: under the LADRC control framework, the LESO is not only a state observer but also a high frequency filter. As long as the observer bandwidth is not exceeding the optimal value $120$, the high frequency measurement noise can be effectively suppressed, and the states of the system can be accurately estimated in real time. If $\omega_v$ continue to increase, high-frequency filtering performance will be obtained and the performance of LESO will be decreased. Also, because the displacement information is polluted by the high frequency measurement noise entering the forward control channel through

**Figure 3.** Tracking response curves of the EHSS under the control of LADRC with different control gain estimation values ($\omega_c = 30, \omega_v = 120$). (a) Displacement tracking response curve. (b) Displacement tracking error
Figure 4. State estimation errors of LESO under different observer bandwidths ($\omega_c = 35$, $b_0 = 3.5 \times 10^5$).

Figure 5. Displacement tracking curves of the EHSS under the control of LADRC with different observer bandwidths ($\omega_c = 35$, $b_0 = 5.3 \times 10^5$).
feedback, serious deviations of the state feedback controller output from the expectations and degradation of the control performance have been introduced. Therefore, the bandwidth of the observer needs to be tradeoff between the measurement noise and the accuracy of the state estimation.

**Case 3: Robustness against the variations of $\omega_c$.** In order to verify the robustness of the proposed control method under variations controller bandwidth $\omega_c$, the control laws formed by the combination of equations (22) and (24) were replaced by $k_1 = \omega_c^2$, $k_2 = 2\omega_c$. The following values of $\omega_c$ were considered: $\omega_c = [15, 20, 30, 33, 35, 40]$. The control gains and observer bandwidth were designed as $b_0 = 5.3 \times 10^5$ and $\omega_o = 120$, respectively. Thus, each controller was assessed in the closed-loop for the same plant. The simulation results are shown in Figure 6.

The simulation results showed that: when both values of $\omega_o$ and $b_0$ remained unchanged, the state feedback controller bandwidth $\omega_c$ was varied in a large range. As $\omega_c$ increasing, the absolute of the EHSS steady-state error decreased gradually from 5 mm to 1 mm, and the rise time was reduced from 1 s to 0.7 s. This means that the larger of $\omega_c$ the faster of the response speed and the smaller of the steady-state error. But high bandwidth means high cost of the control system implementation. Therefore, the controller bandwidth $\omega_c$ should be reduced as much as possible under the premise of the control requirements.

**Case 4: Analysis of the disturbance rejection performance.** In order to evaluate the disturbance rejection ability of the presented method, the generalized disturbances including the external disturbance, the unknown dynamics and the high frequency measurement noise were comprehensively considered into the research. The random white noise of the power, sampling period and the seed were selected as $1 \times 10^{-10}$, $1 \times 10^{-3}$, and 23,341, respectively, and imposed on the output terminal of the EHSS to simulate the measurement noise. To imitate the external disturbance, the alternating pulse width signal, after simulation for 5 s, was superimposed on the actuator with an amplitude $A_0 = 3 \times 10^3 \text{ N}$ and a period of $T = 6 \text{ s}$. The PID controller was selected as the performance reference, which was widely used in industrial areas, and the parameters were tuned as $k_p = 0.012$, $k_i = 0.0009$, and $k_d = 0.0001$, using the critical ratio method. The LADRC parameters were selected as $b_0 = 5.3 \times 10^5$, $\omega_c = 40$, and $\omega_o = 120$. Thus, both controllers were assessed in the same plant. The simulation results under the disturbances are shown in Figure 7.

Figure 7(a) shows that: LADRC controller has a strong robustness against the uncertain disturbance. When trapezoid instruction is tracked, the tracking error and the steady error are only 1.8 mm and 0.1 mm, both are only 30% and 10% of that in PID control, respectively. After 5 s, with the introduction of disturbances, the control performance of LADRC is almost unchanged, yet the PID control performance degraded dramatically. When different frequency sine signals such as $0.14 \sin (0.5t) \text{ m}$ and $0.14 \sin (2t) \text{ m}$ is tracked, respectively, there are obvious oscillations in the output of the system, and even the stability is lost with the increase of the tracking signal frequency, under the control of PID. In contrast, the developed
method exhibits excellent robustness to the disturbances. This phenomenon can be interpreted as follows: in PID control, the error between the reference and the output, the derived and integrated error are weighted to generate a control signal forcing the output to track the reference. However, the performance is limited by the constant gains. Firstly, the integral of the error was expected to be effective when the error was very small. When the error was very large, the integral of the error might lead to the saturation of the control signal, which was the same as that in the constant proportional gain. Secondly, the derivative error was needed to be active when the error was large in order to prevent overshoot. But when the error was small, normally in steady state, the derivative error only contributed high frequency

Figure 6. Displacement tracking curves of EHSS under the control of LADRC with different controller bandwidths ($b_0 = 5.3 \times 10^5, \omega_0 = 120$). (a) Displacement tracking response curve. (b) Displacement tracking error.
Figure 7. Displacement tracking curves of the EHSS under the control of LADRC ($b_0 = 5.3 \times 10^5$, $\omega_0 = 120$, and $\omega_c = 40$) and PID ($k_p = 0.012$, $k_i = 0.0009$, and $k_d = 0.0001$), respectively: (a) $r = 0.14/(m)$, (b) $r = 0.14 \sin(0.5t)/(m)$, (c) $r = 0.14 \sin(2t)/(m)$, and (d) $r = 0.14 \sin(\pi t)/(m)$.
measurement noise, which was introduced into the system through the feedback loop, and the control performance would be decreased. Thirdly, when different kinds of loads were driven by the hydraulic actuating cylinder, the dynamics of the system would be changed dramatically, the wide application would be limited by the constant gains. In LADRC control, the disturbance could be observed by the LESO in real time and compensated. At the same time, the high frequency measurement noise could be suppressed effectively by the LESO.

**The test bench and experiments**

Simulation results show that the developed method was simple, effective, easy to implement, and robust to the uncertain disturbances. The piston trajectory can be tracked quickly and accurately in the absence of a precise plant model. In this section, the developed method was carried out on the test bench of the EHSS to verify the effectiveness of the theory and simulation.

Figures 8 and 9 showed the system and schematic diagram of the experimental platform. It was mainly composed of two parts: the real-time control system and the test bench of the EHSS. The real-time control system, with a 1000 Hz sampling frequency, mainly composed of a host industrial computer, a slave digital processor 1103, the associated PCI-bus data acquisition cards including a 16-bit bipolar A/D converter and a 16-bit bipolar D/A converter. The ControlDesk software was employed to develop the human-computer interaction in the host computer, and the control algorithm was carried out in Matlab /Simulink with C language. The test bench is consist of the load mass, the Keyence magnetic levitation linear
displacement sensor (1800X10400), the three-position four-way electro-hydraulic proportional valve (Rexroth 4WREE10V-75-7X/6EG24N9K31/AV), constant pressure oil source and two single rod cylinders, the first cylinder (also called the main-cylinder) acted as the system to be controlled. In order to adjust the load force of the main-cylinder, the second cylinder was controlled by the proportional relief valve (Rexroth DBEE10/5X-315YG24K31K4M) in real time. The mass block geometric centerline was coincided with the axis of the two single cylinders, and directly contacted with the surface of test bench.

In order to verify the effectiveness of the proposed method, two sets of comparative experiments were carried out on the experimental platform, under the control of LADRC and PID, respectively. The PID controller parameters are set as $k_p = 3.3$, $k_i = 0.07$, and $k_d = 0.00025$, which were tuned by the critical ratio method. For LADRC, the control parameters were selected as $b_0 = 5.3 \times 10^5$, $\omega_c = 40$, and $\omega_p = 120$. All the parameters were remaining constant throughout the whole experiment process.

Figures 10 and 11 presented the comparative experiment results tracking the trapezoid instruction and the $60 \sin (0.5t)\text{mm}$, respectively.

Figure 10 showed that the proposed method can follow the reference trajectory so precisely that the two curves were almost indistinguishable. In the transient
Figure 10. Comparative experimental results tracking the trapezoid instruction: (a) LADRC and (b) PID.
Figure 11. Comparative experimental results tracking the sinusoidal signal: (a) LADRC and (b) PID.
region, the maximum tracking error was only 1.5 mm. Furthermore, there was no change in the steady state error under the external disturbance, and the absolute value of steady state error was no more than 0.5 mm. In contrast, under the control of PID, the maximum dynamic tracking error was about 3 mm, and the steady-state error of the system exceeded 1 mm, under the action of external disturbance.

Similarly, Figure 11 showed that, under the control of the proposed method, the tracking error was reduced instead, even the disturbance frequency increased significantly. The corresponding results have a good agreement with the theoretical analysis and simulation.

The results indicate that the proposed method can improve the tracking performance of EHSS, and achieve high-precision and smooth position control, due to the disturbances effective estimation and compensation. It is worth emphasizing that, the proposed control methods LADRC is independent of exact mathematical model, and the system information is only corresponded with the outputs of the displacement. Thus, the implementation of the developed algorithm in practice is rather convenient.

Conclusion

In this paper, a LADRC strategy with an acceleration feed-forward has been proposed and successfully implemented in EHSS to track the desired trajectory. The closed-loop system stability was proved. Through simulation, the influencing rule of the LADRC parameter variation on the control system performance was obtained.

Relationship between the LESO bandwidth and the state estimation accuracy was analyzed. The feasibility and effectiveness of the proposed method was verified by means of simulation and experiment. Compared with the general PID control, the nonlinear uncertainty disturbances in the EHSS can be effectively suppressed by the proposed method, and fast and accurate trajectory tracking can be realized. The following conclusions can be drawn based on the researches.

1. The proposed method can be easily used in engineering application and the structure of the controller is simple. In the proposed method, minimal priori information of the EHSS is required, the unknown dynamics and disturbances of the plant can be actively estimated and compensated by the third-order LESO.
2. The parameters such as the estimate value $b_0$ of EHSS, the bandwidth of the feedback controller $\omega_c$, and the LESO bandwidth $\omega_o$ of LADRC can be valued independent of each other in a wide range. The larger of $\omega_c$ the faster of the response speed. Generally, $\omega_o = (4\sim 10)\omega_c$, and the control gain estimation $b_0$ should satisfy the following constraint: $0.9b \leq b_0 \leq b$.
3. When the bandwidth of the observer increases, the estimated accuracy of the state observer will increase gradually, but when the bandwidth exceeds the critical value, the estimated accuracy of the state observer will decrease.
on the contrary because of the influence of the noise, and the bandwidth of the observer needs to be adjusted backwards correspondingly.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work is supported by the Open Research Fund of State Key Laboratory of High Performance Complex Manufacturing, Central South University (No. Kfkt2018-13), Shanxi Province Major Science and Technology Projects (No. 20191102009), and the Doctoral Scientific Research Foundation of Taiyuan University of Science and Technology (No. 20202043).

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**References**

1. Shao J, Ren D and Gao B. Recent advances on gait control strategies for hydraulic quadruped robot. *Recent Pat Mech Eng* 2018; 11: 15–23.
2. Huang Y, Na J, Wu X, et al. Approximation-free control for vehicle active suspensions with hydraulic actuator. *IEEE Trans Ind Electron* 2018; 65: 7258–7267.
3. Yan X. Machinery-electric-hydraulic coupling vibration control of hot continuous rolling mills. *J Mech Eng* 2011; 47(17): 61–65. (in Chinese).
4. Wos P and Dindorf R. Adaptive control of the electro-hydraulic servo-system with external disturbances. *Asian J Control* 2013; 15: 1065–1080.
5. Nguyen MN, Tran DT and Ahn KK. Robust position and vibration control of an electrohydraulic series elastic manipulator against disturbance generated by a variable stiffness actuator. *Mechatronics* 2018; 52: 22–35.
6. Bonchis A, Corke PI, Rye DC, et al. Variable structure methods in hydraulic servo systems control. *Automatica* 2001; 37: 589–595.
7. Fung R, Wang Y, Yang R, et al. A variable structure control with proportional and integral compensations for electrohydraulic position servo control system. *Mechatronics* 1997; 7: 67–81.
8. Gdoura EK, Feki M and Derbel N. Sliding mode control of a hydraulic servo system position using adaptive sliding surface and adaptive gain. *Int J Modell Identif Control* 2015; 23: 248.
9. Fales R and Kelkar A. Robust control design for a wheel loader using and feedback linearization based methods. *ISA Trans* 2009; 48: 312–320.
10. Milić V, Šitum Z and Essert M. Robust position control synthesis of an electro-hydraulic servo system. *ISA Trans* 2010; 49: 535–542.
11. Jian Y, Bing X and Huayong Y. Robust control of an electrohydraulic proportional speed control system with a single-rod hydraulic actuator. Chin J Mech Eng 2005; 18(4): 597–602.
12. Wang LP, Wang JZ, He YD, et al. A dual-fuzzy pressure compensation based symmetric control scheme of single-rod electro-hydraulic actuator. Adv Mater Res 2013; 816–817: 379–384.
13. Zhang YJ and Li HR. Study on control strategy for an electrohydraulic servo system with valve controlled asymmetric cylinder. China Mech Eng 1999; 10: 100–103.
14. Pedro J and Dahunsi O. Neural network based feedback linearization control of a servo-hydraulic vehicle suspension system. Int J Appl Math Comput Sci 2011; 21: 137–147.
15. Qi H, Liu Z and Lang Y. Symmetrical valve controlled asymmetrical cylinder based on wavelet neural network. Eng Comput 2017; 34: 2154–2167.
16. Seo J, Venugopal R and Kenné J. Feedback linearization based control of a rotational hydraulic drive. Control Eng Pract 2007; 15: 1495–1507.
17. Yang JH, Yin ZQ and Li SY. Nonlinear modeling and feedback linearization of valve-controlled asymmetrical cylinder. Chin J Mech Eng 2006; 42(5): 203–207.
18. Yu J, Zhuang J and Yu DH. Feedback linearization control for an electro-Hydraulic servo system using Lyapunov functions. J Xi’an Jiaotong Univ 2014; 48(7): 71–76. (in Chinese).
19. Zhang YJ and Fang X. The hydraulic system of intelligent asphalt disperser based on neural network internal model control. Appl Mech Mater 2014; 541–542: 1191–1197.
20. Chen S and Fu L. Observer-based backstepping control of a 6-dof parallel hydraulic manipulator. Control Eng Pract 2015; 36: 100–112.
21. Kim W, Shin D, Won D, et al. Disturbance-observer-based position tracking controller in the presence of biased sinusoidal disturbance for electrohydraulic actuators. IEEE Trans Control Syst Technol 2013; 21: 2290–2298.
22. Bahrami M, Naraghi M and Zareinejad M. Adaptive super-twisting observer for fault reconstruction in electro-hydraulic systems. ISA Trans 2018; 76: 235–245.
23. Chen Y. Backstepping controller design for electro-hydraulic servo system with sliding observer. In: Proceedings of the 29th Chinese control conference, Beijing, China, 29–31 July 2010, paper no.1, pp.391–394. Beijing: CCDC.
24. Won D, Kim W and Tomizuka M. High-gain-observer-based integral sliding mode control for position tracking of electrohydraulic servo systems. IEEE/ASME Trans Mechatron 2017; 22: 2695–2704.
25. Wang C, Quan L, Zhang S, et al. Reduced-order model based active disturbance rejection control of hydraulic servo system with singular value perturbation theory. ISA Trans 2017; 67: 455–465.
26. Chen Y, Chen ZQ, Sun MW, et al. Multivariable inverted decoupling active disturbance rejection control and its application to a distillation column process. Acta Autom Sin 2017; 43(6): 1080–1088. (in Chinese).
27. Sun L, Li D, Hu K, et al. On tuning and practical implementation of active disturbance rejection controller: a case study from a regenerative heater in a 1000 MW power plant. Ind Eng Chem Res 2016; 55: 6686–6695.
28. Martínez-Fonseca N, Castañeda LA, Uranga A, et al. Robust disturbance rejection control of a biped robotic system using high-order extended state observer. ISA Trans 2016; 62: 276–286.
29. Li S, Zhang K, Li J, et al. On the rejection of internal and external disturbances in a wind energy conversion system with direct-driven PMSG. *ISA Trans* 2016; 61: 95–103.

30. Huang Y, Xue W and Zhao C. Active disturbance rejection control: methodology and theoretical analysis. *J Syst Sci Math Sci* 2011; 31(9):1111–1129 (in Chinese).

31. Zhang C, Zhu J and Gao Y. Order and parameter selections for active disturbance rejection controller. *Control Theory Appl* 2014; 31(11): 1480–1485. (in Chinese).

32. Han J. *Active disturbance rejection control technique*. Beijing, China: National Defense Industry Press, 2008, pp.197–211 (in Chinese).

33. Zhao S and Gao Z. Modified active disturbance rejection control for time-delay systems. *ISA Trans* 2014; 53(4): 882–888.

34. Gao Z. Scaling and bandwidth-parameterization based controller tuning. In: *Proceedings of the American control conference*, 2003, pp.4989–4996. New York: IEEE.

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