Hadron Formation Time and Dilepton Mass Spectra in Heavy Ion Collisions

Peter Filip and Ján Pišút

Department of Theoretical Physics, Comenius University Bratislava,
SK-842 15 Bratislava, Slovak Republic

Abstract

We point out that formation time of pions produced in heavy ion collisions modifies the mass spectrum of dileptons produced via \( \pi^+\pi^- \rightarrow e^+e^- \). Increasing formation time enhances the production of dileptons with lower masses. The effect offers an explanation of a part of the enhanced production of dileptons below the \( \rho \)-meson mass as observed by the CERES and HELIOS Collaborations at the CERN SPS.

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I. INTRODUCTION

Dilepton production in heavy ion collisions is considered as a relatively clear signature of the nature of matter produced in heavy-ion collisions. This is one of reasons why the enhanced production of dileptons as observed recently in heavy ion collisions by CERES [1–3], HELIOS [4,5] and NA-38 [6,7] has received so much of attention.

An attractive explanation of the peculiar dilepton mass shape is based on the Brown-Rho [8] conjecture of decreasing $\rho$-meson mass with increasing density and temperature of hadronic matter, related to partial restoration of chiral symmetry.

When included into the transport codes the decreasing $\rho$-meson mass together with effects due to higher nucleon and $\Delta$ resonancies can lead to a qualitative agreement with data [4]. Good description of dilepton mass spectra in S-Au and S-W interactions has been obtained under the same assumption of the behaviour of $m_\rho$ by Li,Ko and Brown [10].

The purpose of the present note is to point out that the formation time of pions produced in heavy-ion collisions leads also to modifications of the shape of dilepton mass spectrum enhancing the production of lighter dileptons with respect to heavier ones.

The concept of the formation time has appeared a long time ago in connection with the problem of how fast is the electromagnetic field accompanying a charged particle reconstructed after an abrupt change of the momentum of the particle. Early literature on the question can be traced back e.g. from Ref. [11]. In connection with production of secondary particles in hadronic collisions the concept has been introduced by Stodolsky [12]. In analysis of data [13,14] on multiparticle production in proton - nucleus interactions it has turned out [13–16] that the cascading of secondaries is considerably lower than expected under the assumption that a secondary pion is able to interact immediately after it has been produced in a nucleon - nucleon collision and this has been ascribed to the formation time of secondary particles.

The term ”formation time” has somewhat different meaning in different models. In what follows we shall understand this term as the time interval between the time of the
collision in which the secondary hadron has been produced and the time when this hadron is able to interact strongly (with the full cross-section) with other hadrons. For the dilepton production in $\pi^+\pi^-$ interaction the formation time is relevant, since the process goes via the intermediate $\rho$-meson stage $\pi^+\pi^- \to \rho \to e^+e^-$ and the first part of the process is strong.

We also assume that the formation time of a pion is characterized by $\tau_f$ which denotes the formation time of the pion in its rest frame and in other frames it is Lorentz dilated.

To see in a very qualitative way the effect of formation time on dilepton spectra consider a set of pions produced at $t = 0$ in a volume of typical dimension $L$. A pion with four-momentum $(E, \vec{p})$ is unable to interact before time $t = \tau_f \cdot E/m_\pi$ where $\gamma = E/m_\pi$ is the Lorentz time-dilation factor. Within this time the pion traverses the distance $d = vt = \tau_f |\vec{p}|/m_\pi$. Faster pions traverse longer distances before interacting. As a consequence thereof, interactions of slower pions are preferred and this leads to softening of dilepton spectra produced via $\pi^+\pi^- \longrightarrow e^+e^-$. This mechanism will be important in situations when each of pions undergoes only a few collisions. When the number of collisions, even for fast pions is large, the system thermalizes and the spectrum of dileptons corresponds to interactions of fully formed pions in such a thermalized system.

II. SCHEME OF CALCULATION.

Suppose that the system of pions produced in a heavy ion collision is described by a distribution function $f(k, x)$ in phase space, where $x = (\vec{x}, t)$. The standard expression for the rate of dilepton production is (see e.g. [15]):

$$\frac{dN_{ee}}{dM^2d^4x} = \int \frac{d^3k_1}{(2\pi)^3} f(k_1, x) \int \frac{d^3k_2}{(2\pi)^3} f(k_2, x) [v, \sigma_e(M^2) \cdot \delta(M^2 - (k_1 + k_2)^2)]$$

(2.1)

where $\sigma_e(M)$ is the cross-section for $\pi^+\pi^- \longrightarrow e^+e^-$ at given $\sqrt{s} = M$, $s = (k_1 + k_2)^2$, and $v, \sigma_e(M^2)$ are the relative velocity. Distribution functions $f(k, x)$ depend on the cross-section $\sigma_\pi$ for $\pi^+\pi^- \longrightarrow \pi^+\pi^-$ which regulates scattering of pions, and on the value of the formation
time. Multiplying and dividing the right-hand side of Eq.(2.1) by $\sigma_\pi$ and integrating over $d^4x$ we obtain

$$\frac{dN_{ee}}{dM^2} = R \cdot \frac{\sigma_e(M^2)}{\sigma_\pi(M^2)} \int d^4x \frac{d^3k_1}{(2\pi)^3} f(k_1,x) \frac{d^3k_2}{(2\pi)^3} f(k_2,x) \left[ \delta(M^2 - (k_1 + k_2)^2) \cdot v_r \sigma_\pi(M^2) \right]$$

(2.2)

which can be rewritten as

$$\frac{dN_{ee}}{dM^2} = R \cdot \frac{\sigma_e(M^2)}{\sigma_\pi(M^2)} \cdot \frac{dN_\pi}{dM^2}$$

(2.3)

where $\frac{dN_\pi}{dM^2}$ is simply distribution of the number of $\pi\pi$ collisions as a function of the mass of the $\pi\pi$ system and $R$ denotes the ratio of the number of $\pi^+\pi^-$ collisions to all $\pi\pi$ interactions. The expression $\frac{dN_\pi}{dM^2}$ is calculated by Monte-Carlo method in a given cascade code. The term $\sigma_\pi(M^2)$ is the total $\pi\pi$ cross-section including $s$– and $p$– waves plus possibly higher ones. The term $\sigma_e(M^2)$ is the $\pi^+\pi^- \rightarrow e^+e^-$ cross-section which can be taken in the simplest approximation as

$$\sigma_e = \frac{4\pi \alpha^2}{3} \frac{m_e^2}{s} \left[ 1 - \frac{4m_e^2}{s} \right]^{1/2} \left[ 1 - \frac{4m_e^2}{s} \right]^{1/2} \left[ 1 + 2\frac{m_e^2}{s} \right] |F_\pi|^2$$

(2.4)

This result follows directly from a calculation of the Feynman diagram. Neglecting the electron mass $m_e$ with respect to $\sqrt{s} = M$ we obtain

$$\sigma_e = \frac{4\pi \alpha^2}{3} \frac{m_e^2}{s} \left[ 1 - \frac{4m_e^2}{s} \right]^{1/2} |F_\pi|^2$$

(2.5)

The form-factor squared $|F_\pi|^2$ can be simply parametrized in the Bright-Wigner approximation as

$$|F_\pi|^2 = \frac{m_\rho^4 + m_\rho^2 \Gamma_\rho^2}{(M^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2}$$

(2.6)

In practical calculations we have used a multiple scattering model inspired by that used by Humanic, see also Ref. [21]. Since $\pi\pi$ interactions are most likely responsible for the enhanced production of dileptons with masses below 1GeV/$c^2$ we have studied only a simple version of the multiple scattering model in which all secondary hadrons are pions. The initial state for the evolution of the cascade is given by set of momenta and positions
in which pions are created in nucleon - nucleon sub-collisions. This initial state has been
generated by the Monte-Carlo model of Závada [16]. We have used isospin averaged $\pi\pi$
cross section (see e.g. Appendix in Ref. [22]):

$$\frac{d\sigma_{\pi}(s, \theta)}{d\Omega} = \frac{4}{q^2} \left( \frac{1}{9} \sin^2 \delta_0^0 + \frac{5}{9} \sin^2 \delta_0^2 + \frac{27}{9} \sin^2 \delta_1^1 \cos^2 \theta \right)$$

(2.7)

The total cross-section is obtained by integrating over a half of the angular space (due
to identity of particles)

$$\sigma_{\pi}(s) = \frac{8\pi}{q^2} \left( \frac{1}{9} \sin^2 \delta_0^0 + \frac{5}{9} \sin^2 \delta_0^2 + \sin^2 \delta_1^1 \right)$$

(2.8)

Here $\delta_l^T$ is the phase shift in the partial wave with isospin $T$ and orbital momentum $l$, $\theta$
is the CMS scattering angle, $s$ is the CMS energy squared and $q = (1/2)\sqrt{s - 4m_\pi^2}$ denotes
the CMS momentum.

Rescattering program follows trajectory of each pion in short time steps and when the
distance between two pions is smaller than $\sqrt{\sigma_{\pi}(s)/\pi}$ the pions scatter according the cross-
section given by Eq.(2.7). The scattering angle is determined in CMS frame of two pions
being scattered and then momenta of pions are transformed back to the global frame of the
simulation. The scattering is assumed [19,20] to take time interval $t_i$ of the order of 1fm/c,
which roughly corresponds to $\hbar/\Gamma_\rho$. During this time interval the participating pions are
unable to interact again.

In contradistinction to other Monte-Carlo cascade codes, like e.g. [21] we take initial
conditions directly from a realistic model [16] of pion production in nuclear collisions and
introduce the formation time $\tau_f$ in the way discussed in Ref. [24]. The formation time
characterizes the formation of pion in its rest frame.

For times lower than Lorentz dilated formation time $(E_\pi/m_\pi)\tau_f$ a pion created in the
initial state is unable to interact. The formation time is Lorentz dilated, whereas the time $t_i$
is fixed in the global frame of simulation which is CMS of Pb+Pb system. Rate of collisions
per pion varies from 3.7 for $\tau_f = 0.2fm$ to 1.0 for $\tau = 1.0fm$ in our simulation. We guess
that effects of non-locality (see e.g. [23]) present also in our rescattering simulation do not
influence substantially results for dilepton mass spectrum.
III. RESULTS OF THE SIMULATION

We have run the rescattering program for three different values of formation time parameter $\tau_f$ using initial state of pion gas generated by cascading generator \cite{L} for Pb+Pb 160 GeV/n b=7fm collisions. Mass spectrum of dileptons was calculated according to the equation (2.3) from the mass spectrum of $\pi\pi$ collisions obtained from the simulation. Cross sections given by (2.4) and (2.8) were properly normalized and resulting distribution of dileptons was rescaled to the 100 MeV bin size, charged multiplicity $dN_{ch}/d\eta = 300$ and yield of dileptons per unit of rapidity. In Fig.1 we present the shape and magnitude of dilepton mass spectrum as a function of the formation time $\tau_f$. The interpretation of this dependence is simple. Dilepton yields for low masses are almost independent on $\tau_f$ since slower pions are formed rather fast. On the other hand dileptons with masses in the $\rho$-meson region are suppressed for larger values of the formation time. This is due to the Lorentz dilation factor ($E_\pi/m_\pi$) which permits faster pions to escape from the interaction region with little or no rescattering or annihilation.

![Fig.1 Dilepton invariant mass distributions obtained by rescattering simulation for three different values of formation time parameter.](image)

In Fig.2 we compare the excess of dileptons as observed by CERES collaboration \cite{2} with our calculations of dilepton production due to $\pi\pi$ annihilation as given in Fig.1. Data points
and the line which gives the expected dilepton production as extrapolated from proton-nucleus interactions are taken from [26]. Our results presented in Fig.1 are added to the extrapolated continuous line.

Fig.2 Excess of dileptons from $\pi\pi$ annihilation in hadron gas. Data points and background are taken from [26].

The comparison shows that a rough and qualitative agreement could be reached for formation times $\tau_f \approx 0.5 \text{ fm}$. This comparison (Fig.2) can only serve as a very qualitative argument, since the extrapolation procedure usually used to get the background line in Fig.2 is based on assumption that $\rho$-meson dilepton decays and other contributions to dilepton production can be simply rescaled from proton-nucleus to nucleus-nucleus collisions. This may be quite a rough approximation since $\rho$-mesons and other resonancies rescatter in Pb-Pb collisions with much larger probability than in proton-nucleon case. We think that quantitative analysis can be based only on a detailed Monte-Carlo code containing $\rho$-mesons and other resonancies in the initial state and taking their rescattering into account.

**IV. COMMENTS AND CONCLUSIONS**

We have shown above that the value of formation time $\tau_f$ strongly influences the shape and magnitude of the mass spectrum of dileptons produced in $\pi\pi$ annihilation. This concerns in particular the ratio of low mass dileptons to those in the $\rho$-meson region.
We have also shown that the shape and order of magnitude of the dilepton production
due to $\pi^+\pi^-$ annihilations roughly corresponds to the excess of dileptons as observed by the
CERES and HELIOS collaborations at the CERN SPS, both in the low mass and $\rho$-meson
region. Contribution around $m_{ee} \approx 500 - 600$ MeV/c$^2$ may still be missing.

The excess of dileptons as observed at CERN-SPS is of a similar shape as that observed
by DLS collaboration [27]. We think that the question whether formation time effects in
calculations like [28,29] can bring models closer to data deserves a detailed study.

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