An updated comparison of the $M_\bullet$ vs $M_G\sigma^2$ relation with $M_\bullet$ vs $\sigma$ and the problem of the masses of galaxies

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Abstract We have studied, in a series of papers, the properties of the $M_\bullet$ versus $M_G\sigma^2$ relation and we have found that it is useful to describe the evolution of galaxies in the same way as the HR diagram does for stars and to predict the masses of Supermassive Black Holes that are difficult to be guessed using other scaling relations. In this paper, analyzing five samples of galaxies, we find that this relation has intrinsic scatter similar to the $M_\bullet - \sigma$, but follows the theoretical models much better than the $M_\bullet - \sigma$. Furthermore, we analyze the role of the bulge mass in the behavior of $M_\bullet$ versus $M_G\sigma^2$ relation because the difference with the $M_\bullet - \sigma$ is often determined by the choice of the right sample of galactic masses.

Keywords host galaxies · SMBHs · masses of galaxies

1 Introduction

An evidence of the last three decades of astrophysical observations is that almost each galaxy hosts a Supermassive Black Hole (SMBH). Another important discovery in this field of research is the existence of a correlation between the mass of SMBHs and the properties of the host galaxies, such as the velocity dispersion (Ferrarese and Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002), the bulge luminosity or mass (Kormendy and Richstone 1995; van der Marel 1999; Richstone et al. 1998; Magorrian et al. 1998; Marconi et al. 2001; Merritt and Ferrarese 2001; Laor 2001; Wandel 2002; Gebhardt et al. 2003; Marconi and Hunt 2003; Häring and Rix 2004; Gültekin et al. 2009), the galaxy light concentration (Graham et al. 2001), the effective radius (Marconi and Hunt 2003), the Sersic index (Graham and Driver 2005, 2007), the kinetic energy (Feoli and Mele 2005, 2007), the inner core radius (Lauer et al. 2007), the gravitational binding energy and gravitational potential (Aller and Richstone 2007), the momentum parameter (Soker and Meiron 2011), the number of globular clusters (Burkert and Tremaine 2010; Snyder et al. 2011), the spiral arm pitch angle (Seigar et al. 2008; Berrier et al. 2013).

Hence there is a co-evolution of galaxies and their central SMBHs that can be described and studied in the light of the scaling relations found. The aim of our paper is neither to compare all these relations to discover the best one, nor to study all their interesting applications or predictions. We focus our analysis only on one of them.

Fifteen years ago Feoli and Mele (2005) proposed a new correlation between the mass of a supermassive black hole and the kinetic energy of the host galaxy. The main results of this line of research that have been found during this period of time are summarizable in this way: 1) there is no doubt that the correlation exists. It has been tested with a lot of different samples and fitting methods (Benedetto et al. 2013; Feoli and Mele 2007; Feoli and Mancini 2009; Mancini and Feoli 2012); 2) there is no doubt that the relation is very competitive with all the others to fit the experimental data, in particular its intrinsic scatter is very low (Saglia et al. 2016) just like the more popular $M_\bullet - \sigma$ relation and the $M_\bullet \propto M_G^{1/2}\sigma^2$ proposed by Hopkins (2007a); 3) the relation can be very useful to understand the evolution of galaxies, just like the HR diagram is for the evolution of stars (Feoli and Mancini 2009) and allows good predictions of the masses of some black holes that do not follow the $M_\bullet - \sigma$ (Benedetto...
et al. 2013) as well as of the behavior of AGN (Mancini and Feoli 2012).

In this paper we want to study other two aspects of the relation that have not been deepened before:

1) the role of the masses of galaxies that makes the relation different from the $M_\bullet - \sigma$;
2) the correspondence of the scaling laws with possible theoretical models.

Since in recent papers there is a trend to reduce all the scaling laws to only one, considered as the most important (“the $M_\bullet - \sigma$ relation is the optimum universal relation...The Fundamental Plane and the $M_\bullet - \sigma$ relation together constitute a basis that can define other scaling relations applicable to galaxies of all types.” – van den Bosch 2016), we will measure the performance of our relation compared to the $M_\bullet - \sigma$ used as a reference standard.

2 Samples

For the relation $M_\bullet \propto M_G \sigma^2$, it is important to have a sample of the masses as homogeneous as possible, since it is precisely the mass of the galaxy that makes the difference with respect to the relation $M_\bullet - \sigma$. Therefore, we have identified three possible homogeneous databases of the masses of galaxies in the recent literature (Cappellari et al. 2013; Saglia et al. 2016; van den Bosch 2016) and used them to compose five samples of objects that form the starting point of our statistical analysis. First of all we underline that we have not taken the entire database of Saglia because we consider useless to repeat their so detailed analysis performed in Saglia et al. (2016), so we assumed it appropriate to consider their data starting from a subsample. We have avoided to choose directly which galaxies to consider and which to neglect and we have adopted the selection made by de Nicola et al. (2019), who have discarded 26 galaxies from the Saglia’s sample, retaining only 71. Furthermore, we know that van den Bosch (2016) does not report explicitly the masses of galaxies, but following Cappellari’s suggestion (Cappellari et al. 2006), we have calculated the dynamical masses in this way:

$$M_{dyn} = \frac{5R_e \sigma^2}{G}$$

where $R_e$ is the effective radius of host spheroidal component, $\sigma$ the velocity dispersion of the host galaxy and $G$ the gravitational constant.

After these two specifications, the samples we considered are as follows:

- **The 1st Sample** is composed of 47 early-type galaxies, obtained by intersecting the data of Cappellari et al. (2013) for masses of early-type galaxies and velocity dispersions, and van den Bosch (2016) for the corresponding masses of supermassive black holes. The only exception is for the mass of the black hole within the galaxy NGC4486, for which the most recent estimated value is considered (The Event Horizon Telescope Collaboration 2019). NGC 4429 was excluded from the intersection, as its morphological classification is doubtful.

- **The 2nd Sample** is obtained from van den Bosch (2016) considering, among his large collection of data, only the 181 galaxies with the mass of BH measured with a relative error on $\log(M_\bullet)$ not greater than or equal to 1 (Beltramonte et al. 2019). Furthermore, we have excluded from this subset NGC404, because its mass is too low for a SMBH and NGC4486b that “deviates strongly from every correlation involving its black hole mass” Saglia et al. (2016). In the end, we have preferred to exclude also NGC221, NGC1277, NGC1316, NGC5845 and UGC1841, reaching the final number of 174 galaxies for our 2nd Sample.

- **The 3rd Sample** is nothing else but the subsample of the previous one, obtained considering only the 108 early-type galaxies.

- **Our 4th Sample** consists of 71 elements, carried out from de Nicola et al. (2019), of which we consider the velocity dispersions and masses of SMBHs associated with galaxies, while the respective masses of galaxies $M_G$ are taken from Saglia et al. (2016), where they are denoted by the symbol $M_{Bu}$.

- **Finally, the 5th Sample** is composed by the same 71 galaxies of the previous one, but all the data were entirely extrapolated from Saglia et al. (2016).

The files used for the fit containing the data of the five samples are available at the link http://people.ding.unisannio.it/feoli/IF2020.zip

3 Two-parameter fit

We have considered the five samples described in the previous Section and we have looked for the best fit line using the linear regression routines LINMIX_ERR and MPFITEXY that work well when there are experimental errors on both variables. Then, we have also reported the ordinary least squares fit of Mathematica as a test in which we neglect the errors on the experimental data.

The linear regression routine LINMIX_ERR allows to determine the slope, the normalization, and the intrinsic scatter $\varepsilon_0$ (which is that part of the variance that cannot be attributed to specific causes – Novak et al. 2006) of the relation

$$\log(M_\bullet) = b + m \log(x) + \varepsilon_0.$$  \hspace{1cm} (2)

In our case the two relations that we want to study have $x = \sigma$ or $x = M_G \sigma^2/c^2$, where $c$ is the speed of light. LINMIX_ERR is a Bayesian fitting method already used by us.
and several authors in other papers, hence we avoid to describe it and invite to read the reference (Kelly 2007) for details.

The frequentist statistical approach can be followed starting from the routine FITEXY (Press et al. 1992), that minimizes the $\chi^2$:

$$\chi^2 = \sum_{i=1}^{N} \frac{(y_i - b - m x_i)^2}{\Delta y_i^2 + m^2 \Delta x_i^2}.$$  \hspace{1cm} (3)

To obtain the most efficient and unbiased estimate of the slope, it is necessary to introduce the intrinsic scatter $\epsilon_0$. It can be done using MPFITEXY, an evolution of FITEXY (Tremaine et al. 2002). When the reduced $\chi^2_{red} = \chi^2/(N - 2)$ of the fit is not equal to 1, MPFITEXY automatically normalizes $\chi^2_{red}$ including $\epsilon_0$ in the equation (3):

$$\chi^2_{red} = \frac{1}{N - 2} \sum_{i=1}^{N} \frac{(y_i - b - m x_i)^2}{\Delta y_i^2 + \epsilon_0^2 + m^2 \Delta x_i^2}.$$  \hspace{1cm} (4)

Finally, we also used a standard routine of Mathematica that performs an ordinary least squares fit, without considering the errors. We have analyzed the five samples of galaxies respectively, and we have obtained with MPFITEXY, LINMIX_ERR and with Mathematica the values shown in Tables 1a, 1b, 1c.

We have also included in the table the Pearson linear correlation coefficient calculated with the formula

$$R = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N} (x_i - \bar{x})^2 \sum_{i=1}^{N} (y_i - \bar{y})^2}}.$$  \hspace{1cm} (5)

From an inspection of the tables it is evident that the slope of $M_* - M_G \sigma^2$ relation is near the unity for the first three samples, while it drastically changes for the last two samples where it decreases to $m = 0.72$. On the other side the slope of $M_* - \sigma$ relation oscillates around $m = 5$ for all the five samples. Considering the values of the intrinsic scatter and of the correlation coefficient, we observe that the two relations are on the same level for the first sample, while for

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### Table 1a Two-parameter fits for Cappellari’s sample

|        | $m$  | $b$  | $\epsilon_0$ | $R$   | $m$  | $b$  | $\epsilon_0$ | $R$   |
|--------|------|------|--------------|-------|------|------|--------------|-------|
| LINMIX_ERR | 0.99 ± 0.10 | 3.78 ± 0.46 | 0.39 ± 0.06 | –   | 5.15 ± 0.51 | –   | 3.34 ± 1.15 | 0.38 ± 0.06 | –   |
| MPFITEXY  | 0.99 ± 0.09 | 3.76 ± 0.42 | 0.35   | –   | 5.20 ± 0.46 | –   | 3.44 ± 1.05 | 0.35   | –   |
| Mathematica | 0.96 ± 0.09 | 3.88 ± 0.41 | 0.413  | 0.845 | 4.88 ± 0.46 | –   | 2.75 ± 1.03 | 0.413  | 0.845 |

Note: $m$ and $b$ are the slope and the intercept of the linear relation respectively, $\epsilon_0$ is the intrinsic scatter of the relation and $R$ the linear correlation coefficient shown for $M_* = \frac{M_{Lum \sigma^2}}{c^2}$ and $M_* - \sigma$

### Table 1b Two-parameter fits for van den Bosch’s samples

|        | $m$  | $b$  | $\epsilon_0$ | $R$   | $m$  | $b$  | $\epsilon_0$ | $R$   |
|--------|------|------|--------------|-------|------|------|--------------|-------|
| LINMIX_ERR | 1.01 ± 0.06 | 3.27 ± 0.28 | 0.52 ± 0.04 | –   | 5.06 ± 0.26 | –   | 3.30 ± 0.60 | 0.49 ± 0.03 | –   |
| MPFITEXY  | 1.02 ± 0.05 | 3.21 ± 0.26 | 0.49   | –   | 5.10 ± 0.25 | –   | 3.39 ± 0.56 | 0.47   | –   |
| Mathematica | 0.95 ± 0.05 | 3.52 ± 0.26 | 0.569  | 0.807 | 4.89 ± 0.24 | –   | 2.91 ± 0.55 | 0.526  | 0.838 |

Note: $m$ and $b$ are the slope and the intercept of the linear relation respectively, $\epsilon_0$ is the intrinsic scatter of the relation and $R$ the linear correlation coefficient shown for $M_* = \frac{M_{Lum \sigma^2}}{c^2}$ and $M_* - \sigma$
Table 1c  Two-parameter fits for 4th and 5th samples

| 4th sample: de Nicola - Saglia | 5th sample: Saglia |
|--------------------------------|-------------------|
| $\log(M_\bullet) - \log(M_{\mathrm{MB}}\sigma^2)$ | $\log(M_\bullet) - \log(\sigma)$ |
| $m$   | $b$   | $\epsilon_0$ | $R$   | $m$   | $b$   | $\epsilon_0$ | $R$   |
| LINMIX_ERR       | 0.72 ± 0.04 | 5.19 ± 0.17 | 0.35 ± 0.04 | – | 4.98 ± 0.28 | –3.09 ± 0.64 | 0.37 ± 0.04 | – |
| MPFITEXY         | 0.72 ± 0.04 | 5.19 ± 0.17 | 0.34 | – | 4.99 ± 0.26 | –3.11 ± 0.61 | 0.35 | – |
| Mathematica      | 0.72 ± 0.04 | 5.19 ± 0.17 | 0.386 | 0.919 | 4.92 ± 0.27 | –2.94 ± 0.62 | 0.405 | 0.911 |

| 5th sample: Saglia |
|--------------------|
| $\log(M_\bullet) - \log(M_{\mathrm{MB}}\sigma^2)$ | $\log(M_\bullet) - \log(\sigma)$ |
| $m$   | $b$   | $\epsilon_0$ | $R$   | $m$   | $b$   | $\epsilon_0$ | $R$   |
| LINMIX_ERR       | 0.73 ± 0.04 | 5.16 ± 0.18 | 0.36 ± 0.04 | – | 5.04 ± 0.27 | –3.22 ± 0.64 | 0.37 ± 0.04 | – |
| MPFITEXY         | 0.73 ± 0.04 | 5.16 ± 0.17 | 0.34 | – | 5.05 ± 0.27 | –3.24 ± 0.62 | 0.36 | – |
| Mathematica      | 0.72 ± 0.04 | 5.17 ± 0.17 | 0.392 | 0.919 | 4.97 ± 0.27 | –3.05 ± 0.62 | 0.407 | 0.912 |

Note: $m$ and $b$ are the slope and the intercept of the linear relation respectively, $\epsilon_0$ is the intrinsic scatter of the relation and $R$ the linear correlation coefficient shown for $M_\bullet \sim M_{\mathrm{MB}}\sigma^2$ and $M_\bullet \sim \sigma$

Fig. 1 2nd sample is plotted with different colors for different morphological type. It is possible to see in blue the elliptical galaxies, in fuchsia the lenticular barred or with a weak bar, in yellow the spirals and in green the intermediate and barred spirals.

the third, fourth and fifth samples the $M_\bullet \sim M_G\sigma^2$ relation has slightly better values than the $M_\bullet \sim \sigma$ and the opposite occurs for the second sample.

Since van den Bosch’s sample has a sufficient number of spirals, we can plot in Fig. 1 the relation $M_\bullet \sim M_G\sigma^2$, highlighting the various morphological types with different colors. Lenticular galaxies are excluded from the plot, as they are uniformly scattered. The distribution of galaxies in the figure is such that there is a separation between the elliptical galaxies placed in the upper right part of the plot and the barred lenticulars, spirals and barred spirals placed in the lower left part, as already shown in previous papers on this subject (Benedetto et al. 2013; Feoli and Mancini 2009).

4 One-parameter fit

After the standard analysis contained in the previous section, we want to propose a different point of view that consists in the assumption for the two relations of a fixed slope that can be better explained by a theoretical model. We choose to test for the $M_\bullet \sim \sigma$ relation the slopes $m = 4$ and $m = 5$, while for the $M_\bullet \sim M_G\sigma^2$ relation the values $m = 0.75$ and $m = 1$ (Feoli 2014 and references therein). Furthermore, in recent papers, the statistical analysis of the scaling relations is done using more and more sophisticated machinery. Our different point of view consists also in coming back to a basic approach. We choose a very simple method for fitting the considered data sets that is the ordinary least squares.
comes down, for the one parameter case, to calculate, with the following exact formulas, the intercept \( b \) (Feoli and Mele 2007) of the best fit line:

\[
b = \frac{\sum_{i=1}^{N} \left( \frac{y_i - m x_i}{c^2} \right)}{\sum_{i=1}^{N} \left( \frac{1}{\left( m^2 + (\Delta x)^2 \right)^2} \right)}
\]

and its uncertainty

\[
(\Delta b)^2 = \frac{1}{\sum_{i=1}^{N} \left( \frac{1}{\left( m^2 + (\Delta x)^2 \right)^2} \right)}
\]

In one-parameter fit the reduced \( \chi^2 \) (4) must be divided by \( N - 1 \) and not by \( N - 2 \). For the uncertainty on the intercept we have used a slightly different formula from (7) that is used to calculate “the standard error of the weighted mean (scale corrected)”. Indeed, when the value of \( \chi^2_{\text{red}} \) is too large, the right uncertainty \((\Delta b)^2\) is obtained multiplying \((\Delta b)^2\) given by (7) for the \( \chi^2_{\text{red}} \). The corresponding results for the five samples are displayed in Table 2a, 2b, 2c.

In all the samples for the \( M_* - \sigma \) relation, the slope \( m = 5 \) is better than the \( m = 4 \), as we expected. For the \( M_* - M_G \sigma^2 \) relation, we have \( m = 1 \) for the first three samples and \( m = 0.75 \) for the last two samples, as we had already argued in the previous section. The difference is that for the first three samples the \( M_* - M_G \sigma^2 \) relation gives significantly better values of \( \chi^2_{\text{red}} \) with respect to the \( M_* - \sigma \) relation and the same occurs for the last two samples, comparing the result of \( m = 0.75 \) for \( M_* - M_G \sigma^2 \) with the other relation having \( m = 5 \).

### 5 The problem of the masses of galaxies

The estimation of the mass of a galaxy can be done by using different methods (see for example Appendix B of Feoli and Mancini 2009) and each author, according with the aim of his paper, can also choose to calculate the total mass of the galaxy or only one of its components: bulge, disk, dark matter halo. The consequence is that the different estimation
of galactic masses is reflected in the resulting slopes and intercepts of the scaling relations. Analyzing the various fits for the $M_* \propto M_G \sigma^2$ relation, it has been noticed that there was a strong difference between the best fit angular coefficient obtained for the fourth and fifth samples using the Saglia’s masses ($M_{Bu}$) and those predicted in all the other cases ($M_{JAM}$ and $M_{dyn}$). What we can deduce about the different slope is that:

- it does not depend on the estimate of errors. In fact we have used three different fit programs, in two of which the errors have been considered in the fits, while in the third Mathematica the errors have not been considered. The results were almost unchanged, so we can deduce that this is not what affects the final results.
- It does not depend on the choice of the method to fit, because for each sample the three routines find almost the same slope within the limits of uncertainty.
- It does not depend on the morphological type of galaxies, since, both analyzing the Cappellari’s sample, where we have only early-type galaxies, and analyzing the van den Bosch’s sample, where 65 spirals are present, we have still obtained similar values. So we can say that considering the spirals or not, the final result does not change.

For the considerations just explained, we are led to think that this difference depends on the estimation of the masses, i.e. masses estimated with different methods lead to a different slope. In order to confirm this point of view, we have made three graphics (Fig. 2) formed by taking only the values of the masses of galaxies that are in common between the two samples considered for each plot (Saglia $\rightarrow M_{Bu}$, Cappellari $\rightarrow M_{JAM}$ and van den Bosch $\rightarrow M_{dyn}$). What we noticed is that Cappellari-van den Bosch’s masses follow the bisector ($y = x$) of the graphic, while this does not happen in the other two plots, where the masses of the galaxies of Saglia are present. The best fit lines shown in the three plots were derived using Mathematica and lead to the following relations:

$$M_{dyn} \propto M_{Bu}^{(0.567)} \simeq (M_{Bu})^{1/2}$$  \hspace{1cm} (8)

$$M_{JAM} \propto M_{Bu}^{(0.582)} \simeq (M_{Bu})^{1/2}$$  \hspace{1cm} (9)

Fig. 2. The comparison of the masses of the galaxies extracted from different databases. Considering only the galaxies common to both samples, we have plotted: (a) Cappellari-Saglia, (b) van den Bosch-Saglia and (c) van den Bosch-Cappellari, where the solid line indicates the best fit, while the dashed line indicates what is expected if the masses of both samples have the same values.
\[ M_{\text{dyn}} \propto M_{\text{JAM}}^{1.079} \simeq M_{\text{JAM}} \] (10)

Thanks to (8) and (9), the Feoli and Mele’s relation \( M_\bullet \propto M_G \sigma^2 \) (2005) can be confused with the one proposed by Hopkins et al. (2007a, 2007b). \( M_\bullet \propto M_G^{1/2} \sigma^2 \), because \( M_\bullet \propto M_{\text{JAM}} \sigma^2 \) and \( M_\bullet \propto M_{\text{dyn}} \sigma^2 \) are equivalent to \( M_\bullet \propto M_{\text{JAM}}^{1/2} \sigma^2 \).

6 Conclusions

We have analyzed the behavior of the \( M_\bullet \propto M_G \sigma^2 \) relation using five samples of galaxies taken by different sources. The result is that the relation works as well as the \( M_\bullet \propto \sigma \). Furthermore, we have fixed the slope of the two relations to understand their concordance with possible theoretical models. In fact, relations of the kind \( M_\bullet \propto (M_G \sigma^2)^{0.79} \) or \( M_\bullet \propto \sigma^{4.63} \) have poor physical meaning. The one - parameter fit (at least with the samples used in this paper) shows that the matching of the \( M_\bullet \propto M_G \sigma^2 \) relation with the slopes \( m = 1 \) or \( m = 0.75 \) is significantly better than the matching of \( M_\bullet \propto \sigma \) with the slope \( m = 4 \) or \( m = 5 \). Finally, the difference in the slope found using different samples is often due to the estimation of bulge masses. We have found a not trivial difference for the mass of the same object in different databases and this fact can induce a confusion between the \( M_\bullet \propto M_G \sigma^2 \) relation and the \( M_\bullet \propto M_G^{1/2} \sigma^2 \) proposed by Hopkins et al. (2007a).

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