Semi-hard scattering of partons at SPS and RHIC: A study in contrast

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We analyze the contribution of pQCD based semi-hard parton (re)scattering to the reaction dynamics of relativistic heavy-ion collisions at SPS and RHIC. While such processes are able to account for the measured yield of high momentum direct photons at SPS energies, the conditions necessary for jet-quenching are not fulfilled. The situation changes dramatically at RHIC energies.

The detection of a quark gluon plasma would confirm one of the important predictions of QCD, namely the de-confinement of quarks and gluons when nuclear matter is heated to a temperature exceeding \( T_c \approx 170 \text{ MeV} \). Several of the proposed signatures of the formation of a quark-gluon plasma \([1]\) in relativistic heavy ion collisions have been observed in collisions of lead nuclei at the CERN SPS. Even though explanations in terms of solely hadronic interactions for many of these observations have been proposed from time to time, the simplest and most consistent interpretation of the observations is in terms of the formation of a de-confined state \([2]\). Microscopic calculations confirm that the energy density reached in these collisions is, indeed, high enough for quark de-confinement to occur \([3]\).

The success \([4]\) of perturbative QCD in describing the transverse momentum spectrum of high-\( p_T \) pions measured in Pb+Pb collisions at the SPS \([5]\) suggests that the perturbative scattering of nuclear partons contributes to the energy deposition in these reactions. On the other hand, the apparent absence of a visible energy loss by these partons, in stark contrast to the observations at RHIC \([6]\), indicates a dearth of multiple scattering that could lead to local equilibration of the partons before hadrons are formed.

Prompt photons are a good probe of these phenomena, since they escape from the strongly interacting matter without major distortion by final state interactions. We have recently shown that the measured photon yield can be related to the number of semi-hard scatterings among the partons, because a substantial fraction of the photons are emitted in secondary interactions of scattered partons \([7]\).

In the present study we report the results of a parton cascade model calculation of photon emission in central collision of Pb nuclei at CERN-SPS energies. We address the following questions:

- How large is the contribution of pQCD based parton scattering to the reaction dynamics of nuclear collisions at the SPS?
- Can this mechanism account for the direct photon yield as measured by the WA98 experiment \([8]\)?
- Does the perturbative scattering of partons create conditions necessary for jet quenching, and how do these conditions compare to those predicted by the same model for collisions at RHIC energies?

The parton cascade model (PCM) provides a detailed space-time description of nuclear collisions at high energy, from the onset of hard interactions among the partons of the colliding nuclei up to the moment of hadronization \([9]\). It is based on the relativistic Boltzmann equation for the time evolution of the parton density due to perturbative QCD interactions:

\[
p^\mu \frac{\partial}{\partial x^\mu} F_i(x,p) = C_i[F].
\]  

The PCM collision term \( C_i \) is a nonlinear functional of the phase-space distribution function \( F(x,p) \), containing the matrix elements which account for the following processes:

\[
\begin{align*}
    gg &\rightarrow gg, & gg \rightarrow q\bar{q}, & gg \rightarrow qg, \\
    qq' &\rightarrow qq', & qq \rightarrow qg, & q\bar{q} \rightarrow q'\bar{q}', \\
    q\bar{q} &\rightarrow q\bar{q}, & q\bar{q} \rightarrow gg, & qg \rightarrow q\gamma \\
    gg &\rightarrow q\gamma, & q\bar{q} \rightarrow g\gamma, & q\bar{q} \rightarrow g\gamma 
\end{align*}
\]  

with \( q \) and \( q' \) indicating different quark flavors. The corresponding scattering cross sections are expressed in terms of spin- and color-averaged amplitudes \( |\mathcal{M}|^2 \) \([10]\):

\[
\left( \frac{d\hat{\sigma}}{dQ^2} \right)_{ab\rightarrow cd} = \frac{1}{16\pi s^2} (|\mathcal{M}|^2)
\]  

The total cross section, necessary for the transport calculations is obtained from \([8]\):

\[
\hat{\sigma}_{ab}(\hat{s}) = \sum_{c,d} \int_{(p_T^\text{min})^2}^{\hat{s}} \left( \frac{d\hat{\sigma}}{dQ^2} \right)_{ab\rightarrow cd} dQ^2 .
\]  

The low momentum-transfer cut-off \( p_T^\text{min} \) regularizes the infrared divergence of the parton-parton cross section. We have used a value of \( Q_0^2 \approx (p_T^\text{min})^2 \). We choose the GRV-HO parameterization \([11]\) and sample the distribution functions at the initialization scale \( Q_0^2 \).
| SPS | RHIC |
|-----|------|
| cut-off $p_T^{\text{min}}$ (GeV) | 0.7 | 1.0 |
| # of hard collisions/event | 255 | 3618 |
| # of fragmentations/event | 17 | 2229 |
| average momentum transfer $\langle Q^2 \rangle$ (GeV$^2$) | 0.8 | 1.7 |
| average c.m. energy $\langle \sqrt{s} \rangle$ (GeV) | 2.6 | 4.7 |

TABLE I: Comparison between the total number of semi-hard partonic collisions and fragmentations in Pb+Pb collisions at SPS and Au+Au collisions at RHIC.

Additionally, we include the branchings $q \rightarrow qg$, $g \rightarrow gg$ and $g \rightarrow q\gamma$ [13]. The soft and collinear singularities in the showers are avoided by terminating the branchings when the virtuality of the time-like partons drops below $\mu_0 = 1$ GeV. Some of these aspects were discussed in [14] and explored within framework of the PCM implementation VNI [15]. The present work is based on our thoroughly revised, corrected, and extensively tested implementation of the parton cascade model, called VNI/BMS [16].

The starting point of our investigation is the comparison of the number and average properties of perturbative parton-parton collisions between nuclear reactions at the SPS and RHIC, displayed in table 1. Following [17] we choose a low-momentum cut-off of 0.7 GeV for our SPS calculation and 1.0 GeV for the RHIC calculation. The number of perturbative parton-parton scatterings at SPS is on average 225 per event – clearly too small to provide a significant effect on the dynamics and characteristics of the bulk of the matter. The number of perturbative scatterings increases by a factor of 14 from SPS to RHIC – at RHIC these interactions are as important for the collision dynamics as the soft, non-perturbative processes. The number of fragmentations increases likewise by more than two orders of magnitude – at SPS it is virtually negligible and has no influence on the parton collision cascade. The final entries of table 1 refer to the average momentum transfer $\langle Q^2 \rangle$ in semi-hard collisions and their average center of mass energy $\sqrt{s}$. For the SPS, both values are clearly cut-off dominated. The lack of fragmentations at the SPS and the small values for the average c.m. energy and momentum transfer are a clear indication that the conditions for jet-quenching to occur are not fulfilled at this energy.

A more detailed comparison between the regimes at SPS and RHIC can be seen in Fig. 1. The upper frame shows the $\sqrt{s}$ distribution for binary parton-parton collisions at nuclear reactions at the SPS and RHIC. Obviously the collision regime at SPS is much softer than at RHIC reflecting the strong suppression in the number of semi-hard scatterings observed at the SPS. The two distributions differ strongly not only in magnitude, but also in shape – the distribution at RHIC cannot be obtained by simply rescaling the distribution for the SPS energy and exhibits strong enhancement towards higher values of $\sqrt{s}$. The bottom frame of Fig. 1 shows a comparison of the collision number distributions for SPS and RHIC. Although multiple parton-parton scatterings occur at SPS energy, such re-scattering is strongly suppressed compared to RHIC. E.g., the probability for a parton to scatter five times is more than two orders of magnitude larger at RHIC than at the SPS, and the difference between the two curves grows with the number of multiple scatterings.

In the following, we shall focus on photon production at the SPS, since high-$k_T$ photons constitute a direct probe of semi-hard parton scattering [8]. As a first step, we discuss invariant transverse momentum spectra of photons obtained when performing the PCM calculation within the eikonal approximation, as well as with the restriction to primary interactions. These should correspond to a lowest order pQCD estimate (see upper frame of Fig. 1). We consider near central collision of Pb nuclei with impact parameters $0 < b < 4.5$ fm, corresponding
FIG. 2: Upper panel: Transverse momentum distribution of photons from central collision of lead nuclei at SPS in lowest order pQCD (solid line) in the eikonal approximation (diamonds), and for primary collisions only (squares), generated with VNI/BMS. Lower panel: Photon transverse momentum spectra for multiple scatterings only (diamonds) and for multiple scatterings and fragmentations (× symbols). The solid line shows the predictions of lowest-order perturbative QCD.

to the centrality covered in the WA98 experiment. All the results are for the region of the central rapidity. The close agreement between these three different calculations confirms their insensitivity to the assumptions as well as the accuracy of our numerical implementations. Deviations from these results would then provide evidence for multiple scattering.

The lower frame of Fig. 2 shows calculations when (a) only multiple scattering among the partons, and (b) multiple scatterings as well as fragmentations of the partons are included. Multiple scattering of partons leads to a broadening of the $k_T$ distribution for intermediate $k_T$ in the range $1.0 \text{GeV} \leq k_T \leq 2.5 \text{GeV}$. We find that at SPS the fragmentations do not result in an increasing number of collisions. The increase in photon production, all due to the fragmentation $q \rightarrow q\gamma$, is of the of the order of 30%. This result contrasts sharply with the results obtained for collisions at RHIC where fragmentation processes enhance multiple scatterings by a factor of almost 2.4 (in the absence of LPM suppression).

The effect of higher order corrections via an effective $K$-factor or inclusion of an intrinsic $k_T$ in the momenta of the partons is investigated in Fig. 3. The intrinsic $k_T$ is sampled from a distribution $\sim \exp(-k_T/p_0)$, with $p_0 = 0.44 \text{ GeV}$ [15]. While the inclusion of $K$-factor increases the yield of photons roughly in proportion (here $K = 2$), the inclusion of intrinsic $k_T$ affects the results in a more complex manner: it reduces the yield of photons with $p_T < 1.2 \text{ GeV}$, but increases the yield of photons with larger $p_T$ by up to a factor of 2 (we do not consider the results below $p_T^{\text{min}}$ which are strongly affected by the cut-off). A similar observation has been made in [18], where the importance of nuclear broadening effects for the understanding of the WA98 photon data at the SPS.
FIG. 4: A comparison of single photon production in the parton cascade model, including multiple scatterings and fragmentations of partons along with an intrinsic partonic transverse momentum, characterized by \( (k_T) \approx 0.44 \) GeV, with the WA98 data.

was emphasized.

Figure 4 shows the comparison of our most complete scenario (parton re-scattering, fragmentation and intrinsic \( k_T \)) with data from the WA98 experiment [8]. The PCM is able to account for the measured direct photon yield for \( k_T \geq 2.7 \) GeV. For smaller \( k_T \) the PCM falls short of the data, which is not surprising since hadronic processes not included in the PCM dominate the photon yield in this range [9].

In summary, our calculations are able to describe the observed photon yield at \( k_T \geq 2.7 \) GeV measured by WA98 if higher-order corrections in the form of intrinsic \( k_T \) are taken into account. Only a small increase in the photon yield is seen when the scattered partons are allowed to fragment (mostly due to \( q \rightarrow q\gamma \) reactions), since the number of secondary collisions does not change as the fragmentations are rather few. We conclude that the parton medium is dilute and does not form a dense parton plasma, at SPS. This is in stark contrast to the results obtained at RHIC energies where the multiple scatterings and fragmentations increase the production of photons manifold over the prompt photon yield. Our results suggest that the observed absence of jet quenching at SPS is a consequence of the modest amount of cascading and fragmentation of the scattered partons, while these processes occur in abundance at RHIC energies, resulting in a significant suppression of high-momentum hadrons due to multiple scattering.

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