Probing deviations from tri-bimaximal mixing through ultra high energy neutrino signals

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Abstract

We investigate deviation from the tri-bimaximal mixing in the case of ultra high energy neutrino using ICECUBE detector. We consider the ratio of number of muon tracks to the shower generated due to electrons and hadrons. Our analysis shows that for tri-bimaximal mixing the ratio comes out around 4.05. Keeping $\theta_{12}$ and $\theta_{23}$ fixed at tri-bimaximal value, we have varied the angle $\theta_{13} = 3^\circ, 6^\circ, 9^\circ$ and the value of the ratio gradually decreases. The variation of ratio lies within 8% to 18% from the tri-bimaximal mixing value and it is very difficult to detect such small variation by the ICECUBE detector.

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1 Introduction

Various experiments for solar and atmospheric neutrinos provide a range for the values of solar mixing angle $\theta_{\odot} = \theta_{12}$ (the 1-2 mixing angle) [1] that corresponds to solar neutrino oscillations and also a range for atmospheric mixing angle $\theta_{\text{atm}} = \theta_{23}$ (the 2-3 mixing angle) [1] around their best fit values. The tri-bimaximal mixing condition of neutrinos are given by $\sin \theta_{12} = \frac{1}{\sqrt{3}}$, $\sin \theta_{23} = \frac{1}{\sqrt{2}}$ and $\sin \theta_{13} = 0$ [2]. Possible deviations from tri-bimaximal mixing can be obtained by probing the ranges of $\theta_{12}$ and $\theta_{23}$ given by the experiments. Also the exact 13 mixing angle $\theta_{13}$ is not known except that the CHOOZ [3] gives an upper limit for $\theta_{13} (< 9^\circ)$. Probing the deviations of $\theta_{12}$ and $\theta_{23}$ for different values for $\theta_{13}$ is significant not only to understand the neutrino flavour oscillations in general but also for the purpose of model building for neutrino mass matrices.

In this work we explore the possibility for ultra high energy (UHE) neutrinos from distant Gamma Ray Bursts (GRBs) for probing the signatures of these deviations of the values of the mixing angles from tri-bimaximal mixing as discussed above. One such proposition of using UHE neutrinos is described in a recent work by Xing [4]. Gamma Ray Bursts are short lived but intense burst of gamma rays. During its occurrence it outshines all other luminous objects in the sky. Although the exact mechanism of GRBs could not be ascertained so far but the general wisdom is that it is powered by a central engine provided by a failed star or supernova that possibly turned into a black hole, accretes mass at its surroundings. This infalling mass due to gravity bounces back from the surface of black hole much the same way as the supernova explosion mechanism and a shock is generated that flows radially outwards with enormous amount of energies ($\sim 10^{53}$ ergs). This highly energetic shock wave drives the mass outwards, in the form of a “fireball” that carries in it, protons, $\gamma$ etc. The pions are produced when the accelerated protons inside the fireball interacts with $\gamma$ through a cosmic beam dump process. UHE neutrinos are produced by the decay of these
pions. Thus a generic cosmic accelerator accelerates the protons into very high energies which then beam dump on $\gamma$ in the “fireball” as also at the cosmic microwave background (CMB) and ultra high energy neutrinos are produced.

The GRB neutrinos, due to their origin at astronomical distances from earth, provide a very long baseline for the earth bound detectors for UHE neutrinos such as ICECUBE [5]. The oscillatory part of the neutrino flavour oscillation probabilities ($\sin^2(\Delta m^2[L/4E])$) averages out to 1/2 because of this very long baseline $L$ ($\sim$ hundreds of Mpc) and the $\Delta m^2$ (mass square difference of two neutrinos) range obtained from solar and atmospheric neutrino experiments are $\Delta m^2_{21} \sim 10^{-4}$ eV$^2$ and $\Delta m^2_{32} \sim 10^{-3}$ eV$^2$ respectively ($L/\Delta m^2 >> 1$). Thus for neutrino flavour oscillation, in this case, the effect of $\Delta m^2$ is washed out and governed only by the three mixing angles namely $\theta_{12} = \theta_{\odot}$, $\theta_{23} = \theta_{\text{atm}}$ and $\theta_{13}$. The purpose of the present work is to probe whether or not the possible variations of $\theta_{12}$ and $\theta_{23}$ from their best fit values can be ascertained by UHE from distant GRBs.

The GRB neutrinos, on arriving the earth, undergo charged current (CC) and neutral current (NC) interactions with the earth rock and the detector material. The CC interactions of $\nu_{\mu}$ produce secondary muons and the same for electrons produce electromagnetic shower ($\nu_{\mu} + N \rightarrow \mu + X$ and $\nu_e + N \rightarrow e + X$). The former will produce secondary muon tracks and can be detected by track-signal produced by the Cerenkov light emitted by these muons during their passage through a large underground water/ice Cerenkov detectors like ICECUBE. The ICECUBE is a 1km$^3$ detector in south pole ice and can be considered to be immersed in the target material for the UHE neutrinos where the neutrino interactions are initiated. In case of $\nu_e$, the electrons from the $\nu_e N$ CC interactions, shower quickly and can also be detected by such ICECUBE detector. The case of $\nu_\tau$ is somewhat complicated. The first CC interaction of $\nu_\tau$ ($\nu_\tau + N \rightarrow \tau + X$) produces a shower (“first bang”) along with a $\tau$ track. But the $\nu_\tau$ is regenerated (with diminished energy) by the decay of $\tau$ and in the process produces another hadronic or
electromagnetic shower ("second bang"). The whole process is called double bang event. In case the first bang could not be detected, then by possible detection of second bang (with showers) the $\tau$ track can be reconstructed or identified and this scenario (the $\tau$ track and the second bang) is called the lollipop events. An inverted lollipop event is one where only the first bang ($\nu_\tau + N \rightarrow \tau + X$) is detected and the subsequent $\tau$ track is detected or reconstructed. As mentioned in Ref. [6], the detection of $\nu_\tau$ from their CC interaction mentioned above is not every efficient by a 1km$^3$ detector since the double bang events can possibly be detected only for the $\nu_\tau$ energies between 1 PeV to 20 PeV beyond which the tau decay length is longer than the width of such detector and at still higher energies the flux is too small for such detectors for their detection. Hence, in the present work we do not consider the events initiated by $\nu_\tau N$ CC interactions. However, for $\nu_\tau$ we consider the process that may yield events higher than the “double bang” events. We consider the decay channel of $\tau$ lepton [7], obtained from charged current interactions of $\nu_\tau$, where muons are produced ($\nu_\tau \rightarrow \tau \rightarrow \bar{\nu}_\mu \mu \nu_\tau$) which can then be detected as muon tracks [8] in ICECUBE detector. The neutral current (NC) interactions of all flavours however will produce the shower events at ICECUBE and they are considered in this investigation.

This paper is organised as follows. In Section 2 we describe the formalism for neutrino fluxes of the three species while reaching the earth. The nature of the GRB flux taken for present calculations is also discussed. The flux suffers flavour oscillations while traversing from GRB site to the earth. The oscillation probabilities are also calculated and the oscillated flux obtained on reaching the earth is determined. They are given in Section 2.1. We also describe in this section the analytical expressions for the yield of secondary muons and shower events at the ice Cerenkov kilometre square detector like ICECUBE. This is given in Section 2.2. The actual calculations and results are discussed in Section 3. Finally, in Section 4, some discussions and summary are given.
2 Formalism

2.1 GRB Neutrinos Fluxes

The neutrino production in GRB is initiated through the process of cosmological beam dump by which a highly accelerated protons from GRB interacts with $\gamma$ to produce pions which in turn decays to produce $\nu_\mu$ ($\bar{\nu}_\mu$) and $\nu_e$ ($\bar{\nu}_e$) much the same ways as atmospheric neutrinos are produced. They are produced in the proportion $2\nu_\mu : 2\bar{\nu}_\mu : 1\nu_e : 1\bar{\nu}_e$ \cite{9}.

For the present calculation we consider the isotropic flux \cite{10} resulting from the summation over the sources and as given in Gandhi et al \cite{11}. The isotropic GRB flux for $\nu_\mu + \bar{\nu}_\mu$ is given as

$$F(E_\nu) = \frac{dN_{\nu_\mu+\bar{\nu}_\mu}}{dE_\nu} = \mathcal{N} \left( \frac{E_\nu}{1\text{GeV}} \right)^{-n} \text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{GeV}^{-1}$$  \hspace{1cm} (1)

In the above,

$$\mathcal{N} = 4.0 \times 10^{-13}, \quad n = 1, \quad \text{for} \quad E_\nu < 10^5 \text{ GeV}$$

$$\mathcal{N} = 4.0 \times 10^{-8}, \quad n = 2, \quad \text{for} \quad E_\nu > 10^5 \text{ GeV}$$

Thus,

$$\frac{dN_{\nu_\mu}}{dE_\nu} = \phi_\nu_\mu = \frac{dN_{\bar{\nu}_\mu}}{dE_\nu} = \phi_{\bar{\nu}_\mu} = 0.5 F(E_\nu)$$  

$$\frac{dN_{\nu_e}}{dE_\nu} = \phi_{\nu_e} = \frac{dN_{\bar{\nu}_e}}{dE_\nu} = \phi_{\bar{\nu}_e} = 0.25 F(E_\nu)$$  \hspace{1cm} (2)

The neutrinos undergo flavour oscillation during their passage from the GRB to the earth. Under three flavour oscillation, the $\nu_e$ and $\nu_\mu$ originally created at GRB will be oscillated to $\nu_\tau$. Thus after flavour oscillations, the $\nu_e$ fluxes ($F_{\nu_e}$), $\nu_\mu$ fluxes ($F_{\nu_\mu}$), $\nu_\tau$ fluxes ($F_{\nu_\tau}$) become

$$F_{\nu_e} = P_{\nu_e \rightarrow \nu_e} \phi_{\nu_e} + P_{\nu_\mu \rightarrow \nu_e} \phi_{\nu_\mu}$$

$$F_{\nu_\mu} = P_{\nu_\mu \rightarrow \nu_\mu} \phi_{\nu_\mu} + P_{\nu_e \rightarrow \nu_\mu} \phi_{\nu_e}$$

$$F_{\nu_\tau} = P_{\nu_e \rightarrow \nu_\tau} \phi_{\nu_e} + P_{\nu_\mu \rightarrow \nu_\tau} \phi_{\nu_\mu}$$  \hspace{1cm} (3)
The transition probability of a neutrino of flavour $\alpha$ to a flavour $\beta$ is given by,

$$ P_{\nu_\alpha \rightarrow \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \sin^2 \left( \frac{\pi L}{\lambda_{ij}} \right) $$

(4)

In the above oscillation length $\lambda_{ij}$ is given by

$$ \lambda_{ij} = 2.47 \text{ Km} \left( \frac{E}{\text{GeV}} \right) \left( \frac{\text{eV}^2}{\Delta m^2} \right) $$

(5)

Because of astronomical baseline $\Delta m^2 L/E \gg 1$, the oscillatory part becomes averaged to half. Thus,

$$ \left\langle \sin^2 \left( \frac{\pi L}{\lambda_{ij}} \right) \right\rangle = \frac{1}{2} $$

(6)

Therefore

$$ P_{\nu_\alpha \rightarrow \nu_\beta} = \delta_{\alpha\beta} - 2 \sum_{j>i} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} $$

$$ = \delta_{\alpha\beta} - \sum_i U_{\alpha i} U_{\beta i} \left[ \sum_{j \neq i} U_{\alpha j} U_{\beta j} \right] $$

$$ = \sum_j |U_{\alpha j}|^2 |U_{\beta j}|^2 $$

(7)

where use has been made of the condition $\sum_i U_{\alpha i} U_{\beta i} = \delta_{\alpha\beta}$.

With Eq. (7), Eq. (3) can be rewritten in matrix form

$$
\begin{pmatrix}
F_{\nu_e} \\
F_{\nu_\mu} \\
F_{\nu_\tau}
\end{pmatrix} =
\begin{pmatrix}
U_{e_1}^2 & U_{e_2}^2 & U_{e_3}^2 \\
U_{\mu_1}^2 & U_{\mu_2}^2 & U_{\mu_3}^2 \\
U_{\tau_1}^2 & U_{\tau_2}^2 & U_{\tau_3}^2
\end{pmatrix}
\begin{pmatrix}
U_{\mu_1}^2 & U_{\mu_2}^2 & U_{\mu_3}^2 \\
U_{e_1}^2 & U_{e_2}^2 & U_{e_3}^2 \\
U_{\tau_1}^2 & U_{\tau_2}^2 & U_{\tau_3}^2
\end{pmatrix}
\begin{pmatrix}
F_{\nu_e} \\
F_{\nu_\mu} \\
F_{\nu_\tau}
\end{pmatrix}
\begin{pmatrix}
\phi_{\nu_e} \\
\phi_{\nu_\mu} \\
\phi_{\nu_\tau}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
U_{e_1}^2 & U_{e_2}^2 & U_{e_3}^2 \\
U_{\mu_1}^2 & U_{\mu_2}^2 & U_{\mu_3}^2 \\
U_{\tau_1}^2 & U_{\tau_2}^2 & U_{\tau_3}^2
\end{pmatrix}
\begin{pmatrix}
U_{e_1}^2 & U_{e_2}^2 & U_{e_3}^2 \\
U_{\mu_1}^2 & U_{\mu_2}^2 & U_{\mu_3}^2 \\
U_{\tau_1}^2 & U_{\tau_2}^2 & U_{\tau_3}^2
\end{pmatrix}
\begin{pmatrix}
1 \\
2 \\
0
\end{pmatrix}
\phi_{\nu_e}
$$

(8)
In Eq. (8) above, we have used the initial flux ratio from GRB to be $\phi_{\nu_e} : \phi_{\nu_\mu} = 1 : 2 : 0$. From Eq. (8) it then follows that,

$$F_{\nu_e} = \left\{ U^2_{e1}[1 + (U^2_{\mu1} - U^2_{\tau1})] + U^2_{e2}[1 + (U^2_{\mu2} - U^2_{\tau2})] + U^2_{e3}[1 + (U^2_{\mu3} - U^2_{\tau3})] \right\} \phi_{\nu_e}$$

$$F_{\nu_\mu} = \left\{ U^2_{\mu1}[1 + (U^2_{\mu1} - U^2_{\tau1})] + U^2_{\mu2}[1 + (U^2_{\mu2} - U^2_{\tau2})] + U^2_{\mu3}[1 + (U^2_{\mu3} - U^2_{\tau3})] \right\} \phi_{\nu_e}$$

$$F_{\nu_\tau} = \left\{ U^2_{\tau1}[1 + (U^2_{\mu1} - U^2_{\tau1})] + U^2_{\tau2}[1 + (U^2_{\mu2} - U^2_{\tau2})] + U^2_{\tau3}[1 + (U^2_{\mu3} - U^2_{\tau3})] \right\} \phi_{\nu_e}$$

(9)

The MNS mixing matrix $U$ for 3-flavour case is given as

$$U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13} \\
    -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\
    s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13}
\end{pmatrix}$$

(10)

We are not considering any CP violation here. Hence Eqs. (3) - (9) above also hold for antineutrinos.

\subsection*{2.2 Detection of GRB neutrinos}

The $\nu_\mu$'s from a GRB can be detected from the tracks of the secondary muons produced through the $\nu_\mu$ CC interactions.

The total number of secondary muons induced by GRB neutrinos at a detector of unit area is given by (following [12, 9, 13])

$$S = \int_{E_{\text{thr}}}^{E_{\text{max}}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\text{surv}}(E_\nu) P_\mu(E_\nu, E_{\text{thr}})$$

(11)

In the above, $P_{\text{surv}}$ is the probability that a neutrino reaches the detector without being absorbed by the earth. This is a function of the neutrino-nucleon interaction length in the earth and the effective path length $X(\theta_z)$
(gm cm⁻²) for incident neutrino zenith angle θ_z (θ_z = 0 for vertically downward entry with respect to the detector). This attenuation of neutrinos due to passage through the earth is referred to as shadow factor. For an isotropic distribution of flux, this shadow factor (for upward going neutrinos) is given by

\[ P_{\text{surv}}(E_\nu) = \frac{1}{2\pi} \int_{-1}^{0} d\cos \theta \int d\phi \exp[-X(\theta_z)/L_{\text{int}}]. \] (12)

where interaction length \( L_{\text{int}} \) is given by

\[ L_{\text{int}} = \frac{1}{\sigma_{\text{tot}}(E_\nu)N_A} \] (13)

In the above \( N_A(= 6.022 \times 10^{23}\text{gm}^{-1}) \) is the Avogadro number and \( \sigma_{\text{tot}}(= \sigma_{\text{CC}} + \sigma_{\text{NC}}) \) is the total cross section. The effective path length \( X(\theta_z) \) is calculated as

\[ X(\theta_z) = \int \rho(r(\theta_z, \ell))d\ell. \] (14)

In Eq. (9), \( \rho(r(\theta_z, \ell)) \) is the matter density inside the earth at a distance \( r \) from the centre of the earth for neutrino path length \( \ell \) entering into the earth with a zenith angle \( \theta_z \). The quantity \( P_\mu(E_\nu, E_{\text{thr}}) \) in Eq. (6) is the probability that a secondary muon is produced by CC interaction of \( \nu_\mu \) and reach the detector above the threshold energy \( E_{\text{thr}} \). This is then a function of \( \nu_\mu N \) (N represents nucleon) - CC interaction cross section \( \sigma_{\text{CC}} \) and the range of the muon inside the rock.

\[ P_\mu(E_\nu, E_{\text{thr}}) = N_A\sigma_{\text{CC}} \langle R(E_\nu; E_{\text{thr}}) \rangle \] (15)

In the above \( \langle R(E_\nu; E_{\text{thr}}) \rangle \) is the average muon range given by

\[ \langle R(E_\nu; E_{\text{thr}}) \rangle = \frac{1}{\sigma_{\text{CC}}} \int_0^{1-E_{\text{thr}}/E_\nu} dy R(E_\nu(1-y), E_{\text{thr}}) \frac{d\sigma_{\text{CC}}(E_\nu, y)}{dy} \] (16)

where \( y = (E_\nu - E_\mu)/E_\nu \) is the fraction of energy loss by a neutrino of energy \( E_\nu \) in the charged current production of a secondary muon of energy \( E_\mu \).

Needless to say that a muon thus produced from a neutrino with energy
$E_{\nu}$ can have the detectable energy range between $E_{\text{thr}}$ and $E_{\nu}$. The range $R(E_{\mu}, E_{\text{thr}})$ for a muon of energy $E_{\mu}$ is given as

$$R(E_{\mu}, E_{\text{thr}}) = \int_{E_{\text{thr}}}^{E_{\mu}} \frac{dE_{\mu}}{dX} \langle \frac{dE_{\mu}}{dX} \rangle \simeq \frac{1}{\beta} \ln \left( \frac{\alpha + \beta E_{\mu}}{\alpha + \beta E_{\text{thr}}} \right).$$

(17)

The average lepton energy loss with energy $E_{\mu}$ per unit distance travelled is given by [12]

$$\langle \frac{dE_{\mu}}{dX} \rangle = -\alpha - \beta E_{\mu}$$

(18)

The values of $\alpha$ and $\beta$ used in the present calculations are

$$\alpha = \{2.033 + 0.077 \ln[E_{\mu}(\text{GeV})]\} \times 10^{-3}\text{GeVcm}^2\text{gm}^{-1}$$

$$\beta = \{2.033 + 0.077 \ln[E_{\mu}(\text{GeV})]\} \times 10^{-6}\text{cm}^2\text{gm}^{-1}$$

(19)

for $E_{\mu} \lesssim 10^6$ GeV [14] and

$$\alpha = 2.033 \times 10^{-3}\text{GeVcm}^2\text{gm}^{-1}$$

$$\beta = 3.9 \times 10^{-6}\text{cm}^2\text{gm}^{-1}$$

(20)

otherwise [15]. For muon events obtained from $\nu_{\mu}$ CC interactions, $\frac{dN_{\nu}}{dE_{\nu}}$ in Eq. (11) will be replaced by $F_{\nu_{\mu}}$ (Eq. 9).

As discussed earlier, the events due to $\nu_{\tau}$ CC interactions is considered only for the process where the decay of secondary $\tau$ lepton produces muon which then detected by the muon track. The probability of production of muons in the decay channel $\tau \rightarrow \bar{\nu}_{\mu}\mu\nu_{\tau}$ is 0.18 [7, 8]. The generated muon carries a fraction 0.3 of energy of original $\nu_{\tau}$ (a fraction 0.75 of the energy of the $\nu_{\tau}$ is carried by secondary $\tau$ lepton and a fraction of 0.4 of $\tau$ lepton energy is carried by the muon [7, 12, 8]). For the detection of such muons, the Eqs. (10 - 16) is applicable with properly incorporating the muon energy described above. Needless to say, in this case, $\frac{dN_{\nu}}{dE_{\nu}}$ in Eq. (11) is to be replaced by $F_{\nu_{\tau}}$ (Eq. 9).
For the case of showers, we do not have the advantage of a specific track and then the whole detector volume is to be considered. The event rate for the shower case is given by

\[ N_{\text{sh}} = \int dE_\nu \frac{dN_{\nu}}{dE_\nu} P_{\text{surv}}(E_\nu) \times \int \frac{1}{\sigma^j} dy P_{\text{int}}(E_\nu, y). \]  

(21)

In the above, \( \sigma^j = \sigma^{\text{CC}} \) (for electromagnetic shower from \( \nu_e \) charged current interactions) or \( \sigma^{\text{NC}} \) as the case may be. In the above \( P_{\text{int}} \) is the probability that a shower produced by the neutrino interactions will be detected and is given by

\[ P_{\text{int}} = \rho N_A \sigma^j L \]  

(22)

where \( \rho \) is the density of the detector material and \( L \) is the length of the detector (\( L = 1 \) Km for ICECUBE).

For each case of shower events, \( \frac{dN_{\nu}}{dE_\nu} \) in Eq. (21) is to be replaced by \( F_{\nu_\mu} \) or \( F_{\nu_\mu} \) or \( F_{\nu_\tau} \) as the case may be.

### 3 Calculations and Results

The secondary muon yield at a kilometre scale detector such as ICECUBE is calculated using Eqs. (6 - 20). The earth matter density in Eq. (9) is taken from [9] following the Preliminary Earth Reference Model (PREM). The interaction cross-sections - both charged current and total - used in these equations are taken from the tabulated values (and the analytical form) given in Ref. [11]. In the present calculations \( E_{\nu_{\text{max}}} = 10^{11} \) GeV and threshold energy \( E_{\text{thr}} = 1 \) TeV are considered.

For our investigations, we first define a ratio \( R \) of the muon events (both from \( \nu_\mu \) (and \( \bar{\nu}_\mu \)) and \( \nu_\tau \) (and \( \bar{\nu}_\tau \))) and the shower events. As described in the previous sections, the muon events are from \( \nu_\mu \) (and \( \bar{\nu}_\mu \)) and \( \nu_\tau \) (and \( \bar{\nu}_\tau \)), whereas the shower events include electromagnetic shower initiated by CC
interaction of $\nu_e$ and NC interactions of neutrinos of all flavours. Therefore,

$$\mathcal{R} = \frac{T_\mu}{T_{sh}} \quad (23)$$

where,

$$T_\mu = S(\text{for } \nu_\mu) + S(\text{for } \nu_\tau)$$

$$T_{sh} = N_{sh}(\text{for } \nu_e \text{ CC interaction})$$
$$+ N_{sh}(\text{for } \nu_e \text{ NC interaction})$$
$$+ N_{sh}(\text{for } \nu_\mu \text{ NC interaction})$$
$$+ N_{sh}(\text{for } \nu_\tau \text{ NC interaction}) \quad (24)$$

The purpose of this work is to explore whether UHE neutrinos from GRB will be able to distinguish any variation of $\theta_{12}$ and $\theta_{23}$ from their best fit values. The tri-bimaximal mixing condition is denoted by the best fit values of $\theta_{12}$ and $\theta_{23}$ for $\theta_{13} = 0^\circ$. The best fit value of $\theta_{12} = 35.2^\circ$ and that of $\theta_{23} = 45^\circ$. We first vary $\theta_{12}$ in the limit $30^\circ \leq \theta_{12} \leq 38^\circ$ and vary $\theta_{23}$ in the limit $38^\circ \leq \theta_{12} \leq 54^\circ$ with $\theta_{13} = 0$ and for each case calculate the ratio $\mathcal{R}$ using Eqs. (1 - 24). We find that $\mathcal{R}$ varies from 3.14 to 4.25. One readily sees that the variation in muon to shower ratio is not very significant. The flux and other uncertainties of the detector may wash away this small variations. $\mathcal{R}$ obtained from tri-bimaximal condition given above is 4.05.

The same operation is repeated for three different values of $\theta_{13}$, namely $\theta_{13} = 3^\circ, 6^\circ$ and $9^\circ$ with similar results. The results are tabulated below.

| $\theta_{13}$ | $\mathcal{R}_{\text{Max}}$ | $\mathcal{R}_{\text{Min}}$ | $\mathcal{R}$ at $\theta_{12} = 35.2^\circ, \theta_{23} = 45^\circ$ |
|---------------|----------------|----------------|--------------------------------------------------|
| $0^\circ$     | 4.78           | 3.80           | 4.05                                             |
| $3^\circ$     | 4.75           | 3.77           | 4.01                                             |
| $6^\circ$     | 4.72           | 3.75           | 3.98                                             |
| $9^\circ$     | 4.69           | 3.73           | 3.96                                             |
Table 1. Maximum and minimum values of ratio $R$ for different values of mixing angles

We have also plotted the variation of $R$ with $\theta_{12}$ and $\theta_{23}$ for four fixed values of $\theta_{13}$ as given in Table 1. These are shown in Figs 1a - 1d for $\theta_{13} = 0^o$, $3^o$, $6^o$ and $9^o$ respectively.

As is evident from Table 1 and Fig. 1, the variation of muon tracks to shower ratio is not very significant with the deviation from the best fit values of the mixing angles. The ratio $R$ varies upto only $\sim 18\%$. We have also calculated the muon track signal for 1 year of ICECUBE run. For $\theta_{13} = 0$, this varies from $\sim 99$ to $\sim 115$, whereas the muon yield obtained for tri-bimaximal mixing is 103. So the variation for deviation from tri-bimaximal mixing condition is between $4\% - 11\%$. This variation is also not significant given the sources of uncertainty in the flux and the sensitivity of the ICECUBE detector. Firstly, the flux itself can be uncertain by several factors. This can induce errors in calculation of muon yield and shower rate. If the flux uncertainties are energy-dependent, even the ratio $R$ can also be affected. Also the simulation results for ICECUBE detector by Ahrens et al [16] shows the cosmic neutrino signal is well below the atmospheric neutrino background for one year data sample after applying suitable cuts (for the source flux $E^2 \nu \times dN_\nu/dE_\nu = 10^{-7}\text{cm}^2\text{s}^{-1}\text{sr}^{-1}\text{GeV}$). The diffuse flux needed for a $5\sigma$ significance detection after 1 year is well below the experimental limits [16, 17]. There can also be systematic uncertainty arises out of optical module (OM) sensitivity which is affected by the refrozen ice around OM, optical properties of the surrounding ice, trapped air bubbles in the OM neighbourhood etc. An estimation of these uncertainties for a $E^{-2}$ signal is calculated to be around 20\% [16]. Taking into account these uncertainties and sensitivity limit, it is difficult by a detector like ICECUBE to detect the deviation ($\lesssim 18\%$), if any, from tri-bimaximal mixing through the detection of UHE neutrinos from a GRB.
4 Summary and Discussions

In summary, we investigate the deviation from the well known tri-bimaximal mixing in the case of Ultra High Energy neutrinos from a Gamma Ray Burst detected in a kilometer scale detector such as ICECUBE. We have calculated the ratio $R$ of the muon track events and shower events (electromagnetic shower from charged current interactions of $\nu_e$ and hadronic showers from neutral current interactions of neutrinos of all flavours) for tri-bimaximal mixing condition given by $\theta_{12} = 35.2^\circ$, $\theta_{23} = 45.0^\circ$, $\theta_{13} = 0^\circ$. We then investigate the possible variation of $R$ from tri-bimaximal mixing condition by varying $\theta_{12}$ and $\theta_{23}$ within their experimentally obtained range for four different values of $\theta_{13}$ namely $0^\circ$, $3^\circ$, $6^\circ$ and $9^\circ$.

The isotropic flux of GRB neutrinos are obtained following Waxman-Bahcall [10] type parametrization of the flux and summation over the sources. The initial parametrization of neutrino flux can be written as

$$\frac{dN_\nu}{dE_\nu} = \begin{cases} \frac{A}{E_\nu E_b} & , E_\nu < E_b^b \\ \frac{A}{E_\nu^2} & , E_\nu > E_b^b \end{cases}$$

(25)

where $E_b^b$ is the spectral break energy ($\sim 10^5$ GeV) and is related to photon spectral break energy, Lorentz factor etc.

The GRB neutrinos after reaching the earth has to pass through the earth rock (for upward going events) to reach the detector to produce muon tracks or shower. In the calculation therefore, the attenuation of neutrinos through the earth (shadow factor) is estimated. The muons produced out of charged current interactions of neutrinos should also survive to enter the detector and produce tracks. Therefore, to estimate the muon track events, the energy loss of muons through the rock is also estimated. The average lepton energy loss rate (with lepton energy $E_\mu$) due to ionisation and the losses due to Bremsstrahlung, pair-production, hadron production etc. (catastrophic
losses) is parametrized as
\[
\left\langle \frac{dE_\mu}{dX} \right\rangle = -\alpha - \beta E_\mu
\]
where $\beta$ describes the catastrophic loss which dominate over the ionisation loss above a certain critical energy $\zeta = \alpha/\beta$. This induces a logarithmic dependence of the lepton energy loss.

The calculated ratio $R$ varies between $\sim 8\%$ to $\sim 18\%$ for variation from tri-bimaximal mixing scenario and for different values of $\theta_{13}$. Given the sensitivity of the ICExCUBE detector in terms of detecting GRB neutrino flux and considering other uncertainties like that in estimating the flux itself, the atmospheric background, low signal yield and the systematic uncertainties of the detector, it appears that ICExCUBE with its present sensitivity will not be able to detect significantly such a small variation due to deviations from tri-bimaximal mixing. Hence to detect such small deviation, very precise measurement is called for. This requires more data (more years of run) and larger detector size for more statistics. The increase in detector size will not widen the deviation of the ratio significantly as the total area factor of the detector cancels out in the ratio (Eq. 23) although the total number of both muon tracks and total shower yield increase significantly. For the case of shower, the whole detector volume is to be considered and from Eq. (22), there is indeed an $L$ dependence. This makes the deviation of the ratio wider although very marginally as we increase the detector dimension.

It is difficult to predict the detector dimension and/or the time of exposure that will be suitable for such a precision measurement discussed above. Detailed simulation studies taking into account factors like atmospheric neutrino background, photomultiplier tube efficiency and other possible uncertainties like the one carried out in Ref. [16] is required for being able to comment on the detector parameters for such precise measurements.

We also want to mention in passing that we have repeated the same calculation for single GRBs with fixed red shift ($z$) values with similar results.
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Figure Caption

Fig. 1 Variation of $\mathcal{R}$ with $\theta_{12}$ and $\theta_{23}$ for (a) $\theta_{13} = 0^o$, (b) $\theta_{13} = 3^o$, (c) $\theta_{13} = 6^o$ and (d) $\theta_{13} = 9^o$. See text for details.
Fig. 1a
Fig. 1b
Fig. 1c
Fig. 1d