A NONCOMMUTATIVE BISHOP PEAK INTERPOLATION-SET THEOREM

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Abstract. We prove a noncommutative version of Bishop’s peak interpolation-set theorem.

1. Introduction

For us, an operator algebra is a norm closed algebra of operators on a Hilbert space, or equivalently a closed subalgebra $A$ of a $C^*$-algebra $B$. In this paper we shall assume for simplicity that $A$ and $B$ share a common identity element. In ‘noncommutative peak interpolation’ as surveyed briefly in [3] say, one generalizes the classical peak interpolation theory to the setting of operator algebras, using Akemann’s noncommutative topology (see [1] and e.g. other references in [3]). In classical peak interpolation the setting is a subalgebra $A$ of $B = C(K)$, the continuous scalar functions on a compact Hausdorff space $K$, and one tries to build functions in $A$ which have prescribed values or behaviour on a fixed closed subset $E \subset K$ (or on several disjoint subsets). The sets that ‘work’ for this are the $p$-sets, namely the intersections of peak sets, where the latter term means a set of form $f^{-1}(\{1\})$ for $f \in A, \|f\| = 1$ (in the separable case, they are just the $p$-sets). A typical ‘peak interpolation result’, says that if $f \in C(K)$ is strictly positive, and $E$ is a $p$-set, then the continuous functions $g$ on $E$ which are restrictions of functions in $A$, and which are dominated in modulus by the ‘control function’ $f$ on $E$, have extensions $h$ in $A$ satisfying $|h| \leq f$ on all of $K$ (see e.g. II.12.5 in [16]; there are nice pictures illustrating this result in [3]). We shall call this the ‘Gamelin-Bishop theorem’ below. The special case of this where $g = 1$ in fact characterizes $p$-sets among the closed subsets of $K$ (e.g. see [15]). Combining this with a result of Glicksberg [16, Theorem II.12.7]), one obtains that the $p$-sets are the closed sets $E$ in $K$ with $\mu_E \in A^\perp$ for all measures $\mu \in A^\perp$. Equivalently, if and only if the characteristic function $\chi_E$ is in $A^{1\perp}$.

An interpolation-set for $A$ is a closed set $E \subset K$ such that $A|_E = C(E)$. A peak interpolation-set (resp. $p$-interpolation set) is a peak (resp. $p$-) set which is also an interpolation-set. In the light of the above it is clear and obvious that the $p$-interpolation sets may be characterized by the appropriate variant of the Gamelin-Bishop theorem above: if $f \in C(K)$ is strictly positive, then the continuous functions $g$ on $E$ which are dominated in modulus by the ‘control function’ $f$ on $E$, have extensions $h$ in $A$ satisfying $|h| \leq f$ on all of $K$. There are other characterizations of $p$-interpolation sets, e.g. as the closed sets in $K$ with $\mu_E = 0$ for all measures $\mu \in A^\perp$. By basic measure theory, the latter is equivalent to:

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1When we use the term ‘peak interpolation’ we mean in this sense. Others sometimes use this term to refer to what we call peak interpolation-sets below.
|µ|(E) = 0 for all such µ. By the Bishop peak interpolation-set theorem we shall mean the result that Bishop proved in [2]: namely that if µE = 0 for all measures µ ∈ A⊥ then the extension theorem stated a few lines back involving f, g, h, holds.

A special case of interest is when f above is 1, and when this case is applied to the disk algebra, together with the F & M Riesz theorem, one obtains the famous Rudin-Carleson theorem (see e.g. II.12.6 in [16]).

The noncommutative analogue of p-sets are the p-projections. Analogously to the classical case above they have been characterized as the infima (‘meet’) of peak projections, or as the closed projections in B∗∗ that lies in A⊥⊥, if A is a unital subalgebra of a C*-algebra B (see the start of Section 2 for references). See [3] for a survey of part of the ‘noncommutative peak set’ theory, and see also references therein for dozens of our other results. More recently Davidson and Clouâtre and Hartz and others, have studied forms of noncommutative peak interpolation-sets in specific classes of operator algebras (see e.g. [14, 10, 11, 12, 13] and references therein). Their aim is often to generalize to such classes the classical theory of interpolation sets for the ball algebra (see [21, Chapter 10]), the Rudin-Carleson theorem, etc. This work has strong connections to our general ‘noncommutative interpolation’ theory (from [3] and references therein), or in the present paper), indeed one of the main theorems in [14] follows quickly from our more general theory as we mention at the end of this Introduction. Exploring these connections raises many interesting questions and should lead to further important progress.

The following is the unital case of a very general noncommutative variant of the Gamelin-Bishop theorem [3, Theorem 3.4]:

**Theorem 1.1**. [3] Suppose that A is a subalgebra of a unital C*-algebra B, with 1B ∈ A. Suppose that q is a closed projection in B∗∗ that lies in A⊥⊥. If b ∈ A with bq = qb, and qb*qb ≤ qd for an invertible positive d ∈ B which commutes with q, then there exists an element a ∈ A with qa = qa = bq, and a*a ≤ d.

This result may fail without the ‘commuting hypothesis’ bq = qb, and even in the case d = 1. See [3, Corollary 2.4] for an example of a peak projection q and b ∈ A with ∥bq∥ ≤ 1, but there is no a ∈ Ball(A) with qa = bq. Thus as stated after that result, “Clearly the way to proceed from this point, in noncommutative peak interpolation, is to insist on a commutativity assumption of [this] type”. Let us apply this principle in an attempt to find a noncommutative version of the classic Bishop peak interpolation-set theorem above. We first note that [19, Propositions 3.2 and 3.4] may be regarded as such, however there is an ε parameter there that one does not see in Bishop’s result. Our next observation is that simple noncommutative versions of the Bishop peak interpolation-set theorem follow immediately from Theorem [14] and [19, Proposition 3.4]. Indeed right at the end of [3] we alluded to this: “Finally, we remark that simple Tietze theorems of the flavour of the Rudin-Carleson theorem mentioned on the first page, follow from our interpolation theorems by adding a hypothesis of the kind in Proposition 3.4 of” [19]. For example suppose that q is in the center of B∗∗. If A⊥ ⊆ (qB)⊥, and b ∈ B then [19, Proposition 3.4] provides a ∈ A with qa = bq = qa and qb*bq = qa*aq. Thus we may apply Theorem [14] to obtain g ∈ A with gg = gg = bq = aq, and g*g ≤ d as desired. However the assumption that q is central in B∗∗ is very strong. If q is not central but bq = qb then one may modify the argument above, but we were unable to see at the time of [3] how to get the desired conclusion without adding a strong or unappealing hypothesis. For example if A is commutative (which is the case in
many results discussed in e.g. [10, 11, 14], and of course it does not imply that $B$ is also commutative) then this modified argument works (see the proof of Corollary (5) below).

In any case, the desired noncommutative Bishop peak interpolation-set theorem suggested by the above would be a characterization of the closed projections $q$ in $A^⊥⊥$ with the following property: If $b ∈ B$ with $bq = qb$, and $qb^*bq ≤ qd$ for an invertible positive $d ∈ B$ which commutes with $q$, then there exists an element $a ∈ A$ with $aq = qa = bq$, and $a^*a ≤ d$. In the present paper we supply such a theorem.

Turning to notation, the reader is referred for example to [5, 4, 7, 3] for more details on some of the topics below if needed. We will use silently the fact from basic analysis that $X^⊥⊥$ is the weak* closure in $Y^{∗∗}$ of a subspace $X ⊂ Y$, and is isometric to $X^{∗∗}$. For us a projection is always an orthogonal projection. If $A$ is a unital operator algebra then its second dual $A^{∗∗}$ is a unital operator algebra with its (unique) Arens product, this is also the product inherited from the von Neumann algebra $B^{∗∗}$ if $A$ is a subalgebra of a $C^{∗}$-algebra $B$. Via semicontinuity, it is natural to declare a projection $q ∈ B^{∗∗}$ to be open if it is a increasing (weak*) limit of positive elements in $B$, and closed if its ‘perp’ $1 − q$ is open. We write $χ_E$ for the characteristic function of $E$. In the case that $B = C(K)$, and $E$ is an open or closed set in $K$, the projection $q = χ_E$ may be viewed as an element of $C(K)^{∗∗}$ in a natural way since $C(K)^{∗}$ is a certain space of measures on $K$. Thus if $B = C(K)$ the open or closed projections are precisely the characteristic functions of open or closed sets.

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2. On the ‘Bishop peak interpolation-set’ result

In this paper $A$ is a subalgebra of a unital $C^{∗}$-algebra $B$ with $1_B ∈ A$. Noncommutative peak sets for $A$ were introduced in the thesis of Damon Hay [19]. A projection $p ∈ A^{∗∗}$ is called open with respect to $A$ or $A$-open if there exists a net $x_t ∈ A ∩ pA^{∗}p$ with $x_t → p$ weak* (see [4, Section 2]). It is $A$-closed if $1 − p$ is $A$-open. These coincide with the projections in $B^{∗∗}$ that are open or closed in the $C^{∗}$-algebra sense, that also lie in $A^⊥⊥$ [4, Theorem 2.4]. They also coincide with
the \(p\)-projections for \(A\) (see e.g. [7] Theorem 1.2 and [6] Theorem 3.4; these results have also been generalized to Jordan operator algebras by the author and Neal).

**Lemma 2.1.** Suppose that \(A\) is a unital operator algebra, a subalgebra of a unital \(C^\ast\)-algebra \(B\). Let \(q\) be a closed projection in \(A^\ast\ast\). If \(A_0\) is the subalgebra \(\{a \in A : aq = qa\}\) then \(q\) is a closed projection in \(A_0^\ast\ast\) (and in \(B_0^\ast\ast\)). Moreover, \(qB_0\) and \(qA_0\) are norm closed.

**Proof.** Indeed suppose that \(x_t \in A \cap (1-q)A^\ast(1-q)\) with \(x_t \to 1-q\) weak*. Then \(qx_t = x_t q = 0\) so that \(1-x_t \in A_0\) and \(1-x_t \to q\) weak*. So \(q\) and \(1-q\) are in \(A_0^\perp\) and \(1-q\) is an open projection for \(A_0\) by the latter mentioned definition.

It follows from [19, Proposition 3.1] with \(X = A_0\) that \(qA_0\) is closed. Indeed if \(\varphi \in A_0^\perp\) then since \(qA_0 \subset A_0^\perp\) it follows that \(\varphi \in (qA_0)_\perp\). So \(qA_0\) is closed by [19, Proposition 3.1]. Hence, or similarly, \(q \in B_0^{\perp\perp}\) and is a closed projection there, and \(qB_0\) is closed.

**Remark.** Note that \(B_0^{\perp\perp} \subset B^{\ast\ast} \cap \{q\}'\) clearly, however one can show that these sets differ in general. Similarly, \(qB_0^{\perp\perp} \neq qB^{\ast\ast} \cap \{q\}' = qB^{\ast\ast}q\).

The following contains the desired characterization of ‘peak interpolation-sets’, as discussed in the second paragraph after Theorem 1.1. Indeed (iii) is precisely the class of projections discussed there, namely the projections corresponding to the (noncommutative version of the) extension property in the Bishop peak interpolation-set theorem. Item (vi) is a weaker version of this peak interpolation-set extension property, while (v) is saying that \(q\) is a noncommutative ‘interpolation-set’.

Items (i), (ii), and (iv) are noncommutative reformulations of the classical condition \(\mu_E = 0\) (or equivalently \(\chi_E d\mu = 0\)) for all measures \(\mu \in A^\perp\) in Bishop’s original theorem. We remark that the ‘noncommutative null sets’ in (iv) were first considered by Clouâtre and Timko in [12].

**Theorem 2.2.** (Noncommutative ‘Bishop peak interpolation-set’ result) Suppose that \(A\) is a unital operator algebra, a subalgebra of a unital \(C^\ast\)-algebra \(B\). Suppose that \(q\) is a closed projection in \(B^{\ast\ast}\), and that \(B_0 = \{b \in B : bq = qb\}\), and \(A_0 = A \cap B_0\). The following are equivalent:

(i) \(qB_0 \subset A_0^{\perp\perp}\).

(ii) \(A_0^\perp \subset (qB_0)_\perp\) (or equivalently \(A_0^\perp \subset qB^{\ast\ast}\)).

(iii) If \(b \in B_0\) and \(qb^*bq \leq qd\) for an invertible positive \(d \in B\) which commutes with \(q\), then there exists an element \(a \in A\) with \(aq = qa = bq\), and \(a^*a \leq d\).

(iv) \(q\) is \(A_0\)-null (that is, if \(\varphi \in A_0^\perp\) then \(|\varphi|(q) = 0\)).

If the above all holds, then \(q\) is a \(p\)-projection for \(A\) (or equivalently \(q \in A^{\perp\perp}\), or equivalently \(q\) is \(A\)-closed), \(qA_0\) is a \(C^\ast\)-algebra, and we also have

(v) \(qB_0 = qA_0\).

(vi) If \(b \in B_0\) and \(\|bq\| \leq 1\) then there exists an element \(a \in \text{Ball}(A)\) with \(aq = qa = bq\).

Item (vi) implies (v). If \(q\) is a \(p\)-projection for \(A\) then items (i)–(vi) are all equivalent.

**Proof.** Note that \(qB_0 \subset A_0^{\perp\perp}\) if and only if \(qB_0^{\ast\ast} = qB_0^{\perp\perp} \subset A_0^{\perp\perp}\); and if and only if \(A_0^\perp \subset (qB_0^{\perp\perp})_\perp\). The equivalence of (i) and (ii) follow from the bipolar theorem, or by taking upper and lower \(\perp\)’s. Note that since 1 is in \(B_0\) (or \(B_0^{\perp\perp}\), (i) and (ii)
force \( q \in A^{\perp\perp} \). At the start of this section we discussed the equivalences between \( B \)-closed projections in \( A^{\perp\perp} \), \( A \)-closed projections, and \( p \)-projections for \( A \).

By [19] Proposition 3.4], (ii) implies (v). Conversely, suppose that \( q \in A^{\perp\perp} \). Then \( q \) is \( A \)-closed as we just said, and \( q \in A_0^{\perp\perp} \) by Lemma [2.1] Then (v) implies that

\[
qB_0 = qA_0 \subset A_0^{\perp\perp} A_0 \subset A_0^{\perp\perp}.
\]

Thus (i) holds. Suppose in addition that \( b \in B_0 \) and \( qb^*bq \leq gd \) for an invertible positive \( d \in B \) which commutes with \( q \). Then there exists an element \( a_0 \in A_0 \) with \( a_0q = bq \), so that \( qa_0^*a_0q \leq gd \). By Theorem [1.1] there exists an element \( a \in A_0 \) with \( aq = a_0q = bq \), and \( a^*a \leq d \). We have shown that (v) implies (iii) if \( q \in A^{\perp\perp} \), and hence also that (ii) implies (iii) (since (ii) implies (v) and \( q \in A^{\perp\perp} \)).

That (iii) implies that \( q \in A^{\perp\perp} \) follows essentially from the principle that the Gamelin-Bishop theorem characterizes \( p \)-sets, or that in the noncommutative case the condition in the last sentence of Theorem 1.1 with \( b = 1 \) characterizes \( p \)-projections (see e.g. [19] Theorem 5.10) for an even better result). So by the above, (iii) implies (i). Clearly (iii) with \( d = 1 \) implies (vi). Conversely, (vi) implies (v) by scaling. Note that \( qB_0 \) is a \( C^* \)-algebra since it is the range of the map of multiplication by \( q \), which is a \( * \)-homomorphism from \( B_0 \) into its bidual.

The equivalences involving (iv) may be deduced from Clôatré and Timko’s proof of [12] Theorem 6.2, and follow from properties of the polar decomposition of a linear functional as may be found in basic \( C^* \)-algebra texts (e.g. 3.6.7 in [20] applied to the bidual). We include the proofs for completeness: Suppose that \( q \) is \( A_0 \)-null and \( \varphi \in A_0^\perp \) with polar decomposition \( \varphi = u|\varphi| \) with \( u \in B_0^* \). By Cauchy-Schwartz we have

\[
|\varphi(qb)|^2 = |\varphi|(qbu)^2 \leq C|\varphi|(q) = 0, \quad b \in B_0.
\]

So (ii) holds. Conversely if (ii) holds then, as we said above, we have (i) and (v) and also \( q \in A_0^{\perp\perp} \). Suppose that \( \varphi \in A_0^\perp \) with polar decomposition \( \varphi = u|\varphi| \) as above. We have \( qu^* \in qB_0^* \subset A_0^{\perp\perp} \). Thus \( |\varphi|(q) = \varphi(qu^*) = 0 \). So (iv) holds. \( \square \)

Note that \( A_0q \) is actually a \( C^* \)-algebra in the setting above, just as in the commutative case where \( q = \chi_E \) for a closed set \( E \subset K \), and \( A_0q = C(E) \). Unfortunately \( Aq \) is not generally a \( C^* \)-algebra, which seems to be further evidence for the consideration of \( A_0 \) in place of \( A \) in certain such results.

**Corollary 2.3.** (Alternative noncommutative ‘Bishop peak interpolation-set’ result in special cases) Suppose that \( A \) is a unital operator algebra, a subalgebra of a unital \( C^* \)-algebra \( B \). Suppose that \( q \) is a closed projection in \( B^* \). As in the previous result (see also [12] Theorem 6.2), \( qB \subset A^{\perp\perp} \) if and only if \( A^\perp \subset (qB)_{\perp} = (qB^*)_{\perp} \), and if and only if \( q \) is \( A \)-null that is, if \( \varphi \in A^\perp \) then \( |\varphi|(q) = 0 \). Moreover these conditions are equivalent to: \( qB = qA \) with \( q \in A^{\perp\perp} \). If these conditions all hold then so does the ‘Bishop peak interpolation-set’ result in (iii) of the previous theorem, under any one of the following extra hypotheses:

1. \( q \in B \).
2. \( A + B_0 \) is norm closed (or equivalently: every functional in \( A_0^\perp \) extends to a functional in \( A^\perp \)).
3. \( q \) is central in \( B^* \).
4. \( q \) is a minimal projection in \( B^* \).
5. \( A \) is commutative.
Proof. The first two ‘if and only ifs’ follow almost exactly as in the previous proof. Note that \(qB \subset A^{\perp\perp}\) if and only if \(qB^{**} = qB^{**} \subset A^{\perp\perp}\), and if and only if \(A^{\perp} \subset (qB^{**})_{\perp}\). The equivalence with \(q \in A^{\perp\perp}\) and \(qB = qA\) is noted in [12, Theorem 6.2]. Indeed one direction of this is obvious and as in the last proof: these conditions imply \(qB = qA \subset A^{\perp\perp} A \subset A^{\perp\perp}\). The other direction is immediate from [19, Proposition 3.4]. (There was a mistake in the statement and proof of this equivalence in the ArXiV version of [12], which we pointed out to Clouatre and has been corrected in the published version.) Finally we show that each of conditions (1)–(5) imply that the conditions in the last theorem are satisfied.

(2) If \(A + B_0\) is norm closed then by a principle in functional analysis, \(A^{\perp\perp} \cap B_0^{\perp\perp} = (A \cap B_0)^{\perp\perp} = A_0^{\perp\perp}\) (this follows easily for example from [18, Lemma I.1.14(a)]). Hence \(qB_0 \subset A^{\perp\perp}\) or \(qB \subset A^{\perp\perp}\) implies
\[
qB_0 \subset A^{\perp\perp} \cap B_0^{\perp\perp} = A^{\perp\perp}_0.
\]
So the conditions in the last theorem are satisfied. The other equivalence here is no doubt also a principle in functional analysis known in some quarters, which we now explain. Indeed if the restriction map from \(B^*\) to \(B_0^*\) maps \(A^\perp\) onto \(A_0^\perp\), then it is the dual of a bicontinuous injection \((A_0^\perp)_* = B_0/A_0\) into \((A^\perp)_* = B/A\). This injection is the canonical map \(b_0 + A_0 \mapsto b_0 + A\), since the dual of the latter is the restriction map. This argument is reversible: if the canonical map \(b_0 + A_0 \mapsto b_0 + A\) is bicontinuous then the restriction map from \(B^*\) to \(B_0^*\) maps \(A^\perp\) onto \(A_0^\perp\). By the closed range theorem applied to the canonical map \(A + B_0 \mapsto (A + B_0)/A \subset B/A\), if \(A + B_0\) is closed then \((B_0 + A)/A\) is isomorphic to \(B_0/A_0\) (see [18, Lemma I.1.14] if needed). Conversely, if the canonical map \(b_0 + A_0 \mapsto b_0 + A\) is bicontinuous then \((B_0 + A)/A\) is closed in \(B/A\), so that \(A + B_0\) is closed.

(1) If \(q \in B\) and \(qB \subset A^{\perp\perp}\) then \(q \in B \cap A^{\perp\perp} = A\). Then
\[
qB_0 \subset qA^{\perp\perp} - q = (qAq)^{\perp\perp} = \{a \in A : a = qa = aq\}^{\perp\perp} \subset A_0^{\perp\perp}.
\]
So again the conditions in the theorem are satisfied. Alternatively, we can apply [19, Proposition 3.4] and [3, Theorem 3.4] to \(B_0\) and \(qAq\).

(3) This is obvious e.g. from Theorem 2.2.

(4) We have \(q \in A^{\perp\perp}\) and so \(qB_0 = C q \subset A_0^{\perp\perp}\) by the lemma.

(5) Suppose that \(A\) is commutative and \(A^{\perp} \subset (qB)_{\perp}\). Then \(q \in A^{\perp\perp}\) as above and the latter is also commutative. So \(A = A_0\) and \(A_0^{\perp} \subset (qB)_{\perp}\subset (qB_0)_{\perp}\). This does it. Alternatively, by the argument towards the end of the paragraph after Theorem 2.2 there exists \(a \in A\) with \(aq = qa = bq\) and we may conclude as in that argument.

Remarks. 1) In Bishop’s result \(A\) need not be an algebra. One may therefore hope to extend some of our results above to the unital operator space setting. Note that [19, Proposition 3.4], an ingredient above, does not need \(A\) to be an algebra.

2) One approach to replacing \(A_0\) by \(A\), is to try to find conditions under which \(\varphi \in A_0^{\perp}\) implies that there is an extension \(\tilde{\varphi} \in A^{\perp}\). If this holds and if \(A^{\perp} \subset (qB)_{\perp}\), then it is easy to check condition (ii) in Theorem 2.2 directly. Indeed suppose that \(\varphi \in A_0^{\perp}\). If it has an extension \(\tilde{\varphi} \in A^{\perp}\) then by hypothesis \(\tilde{\varphi} \in (qB)_{\perp}\), so that \(\varphi \in (qB_0)_{\perp}\) (using also the lemma above).

We thank Ken Davidson for suggesting to us in the discussions mentioned earlier that perhaps there always exists an extension of functionals in \(A_0^{\perp}\) to \(A^{\perp}\), if and only if \(A + B_0\) is closed.
3) We do not know if \( qB \subset A_{\perp\perp} \) implies the conditions in Theorem 2.2 in all cases, nor do we have a counterexample at this time. By symmetry if \( q \) is \( A \)-null then we have both \( qB \subset A_{\perp\perp} \) and \( Bq \subset A_{\perp\perp} \). A main difficulty that arises is that by the argument towards the end of the paragraph after Theorem 2.2 one may obtain ‘left interpolating’ elements \( a_1 \) and ‘right interpolating’ elements \( a_2 \). We have \( a_1q = qa_2 \), but one really needs \( a_1 = a_2 \) and we have not been able to spot the trick to ensure this (except under strong hypotheses).

4) It seems possible that the ideas in [8, Corollary 5.4] (taking \( b \) there in \( B \)) might give another noncommutative variation of Bishop’s peak interpolation-set theorem.

5) In many cases discussed in [10–14] the algebra \( A \) is commutative so that (5) in the last theorem applies.

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