ABSORPTION VS DECAY OF BLACK HOLES IN
STRING THEORY AND T-SYMMETRY

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ABSTRACT

Classically a black hole can absorb but not emit energy. We discuss how this T-asymmetric property of black holes arises in the recently proposed (T-symmetric) microscopic models of black holes based on bound states of D-branes. In these string theory based models, the nonvanishing classical absorption is made possible essentially by the exponentially increasing degeneracy of quantum states with mass of the black hole. The classical limit of the absorption crosssection computed in the microscopic model agrees with the result obtained from a classical analysis of a wave propagating in the background metric of the corresponding black hole (upto a numerical factor).
0 Introduction

Recent rapid developments in string theory have opened up the exciting possibility of a microscopic derivation of the physics of black holes. Based on progress in understanding bound states of D-branes, microscopic models of black holes have been constructed in 4+1 and 3+1 dimensions. Perhaps the most promising feature of these models is that a counting of microscopic states correctly reproduces black hole degeneracy as required by the Bekenstein-Hawking formula \( S = \frac{1}{4} A \). There are, of course, many other aspects of black hole physics which one would like to derive from these simple microscopic models. The very existence of an event horizon is one such aspect. This implies that classically a black hole can absorb but not emit energy. Can one possibly understand this apparent lack of time-reversal symmetry in terms of the proposed microscopic models which are based on a manifestly time-reversal symmetric microscopic theory?

A priori it would seem that this question must remain unanswered at present. This is because while the microscopic models of black holes as bound states of D-branes have been constructed in the weakly coupled string theory regime, the semiclassical picture of the black hole is expected to be valid in the strong coupling regime, and exploring this regime is at present beyond our technical abilities. It is, therefore, surprising that the simple microscopic model of the Hawking emission proposed in \( \text{[8, 18]} \) yields an expression for the decay rate that agrees with the standard formula in all its essential details! Even though we don’t quite understand why this works, we are encouraged enough by this agreement to address the question about the classical absorption by a black hole as a first step towards exploring the horizon physics in the microscopic models. In this paper we will consider the microscopic model of \( \text{[8, 18]} \) and show that the time-reversed process of the Hawking emission leads to absorption by the black hole which is indeed nonzero in the classical limit, unlike the Hawking emission which vanishes in this limit. This leads to classical absorption but not decay by this microscopic model of the black hole.

This paper is organized as follows. In Section 1 we review the model

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\footnote{The fact that a black hole horizon leads to the abovementioned time-asymmetry has long been recognized in the literature. See \( \text{[16, 17]} \) for a rather detailed discussion and review.}
of [8, 18] for Hawking emission. In Section 2 we consider the time-reversed process and calculate the absorption coefficient. At the microscopic level the magnitude of the matrix element leading to decay is identical to that leading to absorption. Nevertheless, the classical limit of absorption by the macroscopic black hole is nonzero while the decay vanishes. In Section 4 we discuss the classical propagation of a massless scalar particle in the geometry of the black hole under discussion. We calculate the absorption crosssection and compare with the result of Section 3. In Section 5 we discuss possible strong coupling effects. We argue that the essential details of the result obtained here are expected to survive even in the strong coupling limit.

1 Hawking Decay

In this section we will review the model of Hawking decay of the 4+1-dimensional charged black hole considered in [8, 18]. We will use a notation that will be useful later for discussing absorption in the next section.

In the microscopic model the decay of a black hole is interpreted as the annihilation of two massless open string excitations on a D-brane, each with energy $\omega/2$, into a massless closed string quantum of energy $\omega$. As discussed in [8], these open string degrees of freedom live in the 2-dimensional space-time whose space part is the $S^1$ of radius $R$ along $x^5$ that is common to D-onebranes (which wrap around the compact coordinate $x^5$ in the 10-dim. space-time) and D-fivebranes (which wrap around the compact 5-dim. space $S^1 \times T^4$ labelled by $x^5, x^6, x^7, x^8$ and $x^9$). There are $N_B$ species of bosonic (and as many fermionic) open string levels for each value of momentum in the $x^5$ direction. $N_B = Q_1 Q_5$ for the extremal holes where $Q_1$ is the number of D-onebranes and $Q_5$ is the number of D-fivebranes. As discussed in [18], however, there are problems in applying this naive picture to realistic black holes, the so-called “fat” black holes. These problems are resolved in a modified model proposed in [18], which uses an observation made in [19]. In this modified model the D-onebranes and D-fivebranes arrange themselves inside the bound state in such a way that effectively the number of bosonic species $N_B = 1$, while the effective radius of the $S^1$ along the $x^5$ direction in which these bosons live is $L = R Q_1 Q_5$. Since the charged black holes considered in [8] and under discussion here, especially in sections 3 and 4 where we discuss the classical limit, are of the “fat” type, we will henceforth
work within the effective model of [18].

We may write a low-energy effective action for the interaction of a closed massless (in the 4+1 noncompact space-time, \( \vec{x} = (x^1, x^2, x^3, x^4, t) \)) string with two massless (in the 2-dim. space-time \( x^5 \)) open strings:

\[
S_{\text{int}} \propto g_{st} \int dt \int_0^{2\pi L} dx^5 \partial_a \phi(t, x^5) \partial^a \phi(t, x^5) h(t, \vec{x} = 0), \quad a = (t, x^5)
\]

Such an interaction can be inferred from the analyses of [20, 21, 22, 23]. In the above, dimensional reduction has been applied to the directions \( x^6, x^7, x^8, x^9 \) and hence the fields are independent of these coordinates. The field \( \phi(t, x^5) \) describes a massless open string excitation moving along \( x^5 \). The field \( h(t, \vec{x}) \) is a massless (scalar) closed string — its masslessness in the \((4+1)\)-dimensional noncompact space-time \((\vec{x}, t)\) implies that it is independent of \( x^5 \). Finally, \( g_{st} \) is the string coupling in 10-dimensions and \( L = RQ_1Q_5 \) is the effective radius of the \( S^1 \) along \( x^5 \).

The normal mode expansion of the field \( \phi(t, x^5) \) is

\[
\phi(t, x^5) = \sum_{h=-\infty}^{+\infty} \frac{1}{\sqrt{2\omega L}} \left( a_n \exp\left[\frac{in x^5}{L} - i\omega t\right] + c.c. \right), \quad \omega = \frac{|n|}{L}
\]

**Matrix element for decay**

Let us now consider an initial state \( |i\rangle \) with total left-moving (along \( x^5 \)) momentum \( \vec{N}_L \) and total right moving momentum \( \vec{N}_R \). There are, of course, many states \( |i\rangle \) corresponding to a given choice of \( \vec{N}_L \) and \( \vec{N}_R \). These are characterized microscopically by the set of number \( \{ \vec{N}_L(n), \ n = 1, 2, \cdots, \infty \} \) and \( \{ \vec{N}_R(n), \ n = 1, 2, \cdots, \infty \} \) where \( \vec{N}_L(n) = a^+_n a_n, \vec{N}_R(n) = a^+_{-n} a_{-n}, \ n = 1, 2, \cdots, \infty \). In other words,

\[
|i\rangle = \prod_{n=1}^{\infty} \left( \vec{N}_L(n)! \vec{N}_R(n)! \right)^{-\frac{1}{2}} \left( a^+_n \right)^{\vec{N}_L(n)} \left( a^+_{-n} \right)^{\vec{N}_R(n)} |0\rangle
\]

This field is assumed to be bosonic. In principle one should also consider fermionic open string excitations. In this work we have considered emission and absorption of bosonic closed strings only. Since the contribution of fermionic open string excitations in this case is much smaller than the contribution of bosonic open string excitations at the energies of interest to us, we have ignored them. However, the fermionic excitations do contribute to black hole degeneracy and we have included their contribution to the counting of microstates.
Clearly, $\tilde{N}_{L,R} = \sum_{n=1}^{\infty} n \tilde{N}_{L,R}(n)$.

It is a matter of simple one-dimensional thermodynamics to compute the number of microstates $|i\rangle$ corresponding to a given choice of $\tilde{N}_L$ and $\tilde{N}_R$. Including contribution from fermionic open string excitations also, we get

$$\Omega = e^S, \quad S = 2\pi \left( \sqrt{\tilde{N}_L} + \sqrt{\tilde{N}_R} \right). \quad (4)$$

This is related to the expression for degeneracy of nonextremal states given in [9, 8] by the relation [18]

$$\tilde{N}_{L,R} = Q_1 Q_5 N_{L,R}. \quad (5)$$

The final state $|f\rangle$ that we are interested in is obtained from the initial state $|i\rangle$ by the annihilation of a left-moving open string of momentum $\frac{m_L}{L} = \frac{\omega}{2}$ with a right-moving open string of momentum $\frac{m_R}{L} = \frac{\omega}{2}$ into a massless closed string quantum of energy $\omega$. The remaining gas of open string excitations in the final state $|f\rangle$ is, therefore, characterized by $\tilde{N}'_L = \tilde{N}_L - m_L$, $\tilde{N}'_R = \tilde{N}_R - m_R$:

$$|f\rangle = h_\omega^+ \otimes \prod_{n=1}^{\infty} (\tilde{N}'_L(n)!)^{\frac{1}{2}} (a_n^+) (\tilde{N}'_R(n)!)^{\frac{1}{2}} (a_n^-) \tilde{N}_{L,R}^{\tilde{N}'_L(n)} |0\rangle \quad (6)$$

Since we will be interested in closed strong quanta with zero momentum parallel to the branes, for us $m_L = m_R = m$.

As in (4), (5) we can easily compute the number of microstates $|f\rangle$ corresponding to a given choice of $\tilde{N}'_L$ and $\tilde{N}'_R$:

$$\Omega' = e^{S'}, \quad S' = 2\pi \left( \sqrt{\tilde{N}'_L} + \sqrt{\tilde{N}'_R} \right) \quad (7)$$

The $S$-matrix element for decay from the initial state $|i\rangle$ to the final state $|f\rangle$, to 1st order in string coupling $g_{st}$, can be computed using (1) following standard perturbation theory rules and is given by

$$\langle f|S|i\rangle \propto \frac{g_{st}}{\sqrt{m_L} \sqrt{m_R} \sqrt{\omega} V_4 \frac{m_L}{L} \frac{m_R}{L} L \delta_{m_L, m_R} \delta (\omega - \frac{m_L}{L} - \frac{m_R}{L}) (\tilde{N}_L(m_L) \tilde{N}_R(m_R))^{1/2} \quad (8)$$

where $m_L/L = \omega/2 = m_R/L$ and $V_4$ is the volume of the 4-dimensional noncompact space (box normalization).
Now, recall that the nonextremal black holes under discussion are characterized by six parameters which label the corresponding D-brane bound states. These are denoted $N_1, N_{\bar{1}}, N_5, N_{\bar{5}}, N_L$ and $N_R$ and stand for respectively the number of D-onebranes, anti-D-onebranes, D-fivebranes, anti-D-fivebranes, total left moving momentum and total right moving momentum. (Note that $Q_1 \equiv N_1 - N_{\bar{1}}$ and $Q_5 \equiv N_5 - N_{\bar{5}}$.) All microscopic states $|i\rangle$ which have a common value of these parameters refer to the ‘same’ macroscopic black hole (‘no-hair’). Therefore, the microscopic model of the black hole is a density matrix

$$\rho = \frac{1}{\Omega} \sum_{\{i\}} |i\rangle \langle i|$$

where the sum $\{i\}$ is over all possible distributions $\{\tilde{N}_L(n)\}$ and $\{\tilde{N}_R(n)\}$ keeping $N_L$ and $N_R$ fixed. It is this formula that leads to the entropy $S = -Tr \rho \ln \rho = -\sum_{\{i\}} \frac{1}{\Omega} \ln \frac{1}{\Omega} = \ln \Omega$.

Density matrices like the one in (9) are not unfamiliar in particle physics. They arise, e.g. in calculating the decay rate of an unpolarized particle into unpolarized products. As there, in the present case also, the total “unpolarized” transition probability is given by

$$P_{\text{decay}}(i \rightarrow f) = \frac{1}{\Omega} \sum_{\{i\},\{f\}} |\langle f|S|i\rangle|^2$$

The division by $\Omega$ represents averaging over initial states, while the final states are simply summed over. The passage to the decay rate $d\Gamma$ is usual and one gets

$$d\Gamma \propto d^4 \vec{k} G_N m \langle \tilde{N}_L(m) \rangle \langle \tilde{N}_R(m) \rangle$$

where $G_N$ is Newton’s constant in 4-dimensional noncompact space, $\omega = \frac{2m}{L}$ is the energy of the emitted massless closed string, $\vec{k}$ is its momentum in 4-dimensional space ($|\vec{k}| = \omega$) and $\langle \tilde{N}_{L,R}(m) \rangle$ is the average distribution in the initial state (with fixed total momenta $\tilde{N}_L$ and $\tilde{N}_R$). For large values of $\tilde{N}_L$ and $\tilde{N}_R$ one can compute these average distributions by approximating

\footnote{For an explicit demonstration that ‘microscopic’ quantum numbers do not show up for scattering off large black holes, see the analysis of elementary BPS black holes in heterotic string theory by \cite{24} who also showed that hair appears in subleading terms of $S$ matrix down by inverse powers of mass. For a more general analysis of hair in string models of black holes see \cite{27}.}
the microcanonical ensemble by a canonical ensemble (in the 1-dimensional thermodynamics that gave rise to (1) and (3)). One gets the standard Bose-Einstein distributions

\[ \langle \tilde{N}_{L,R}(m) \rangle = \left( e^{\beta_{L,R} \omega/2} - 1 \right)^{-1} \]

where

\[ \beta_{L,R} = \frac{\pi L}{\sqrt{\tilde{N}_{L,R}}} = \frac{\pi L}{\sqrt{Q_1 Q_5 N_{L,R}}} \]

and we have used that \( m = \omega L/2 \).

2 Absorption

Consider now the absorption of a massless closed string quantum by the black hole. The elementary process here is just the reverse of the decay process of the previous section. In fact, let us consider the absorption of a massless quantum of energy \( \omega = 2m/L \) by the initial state \(|i'\rangle\) labelled by the total momentum \( \tilde{N}'_L \) and \( \tilde{N}'_R \) (as in the final state \(|f\rangle\) of the previous section). The final state \(|f')\rangle\) of the black hole in this case, then, contains an additional left (right) moving open string mode of momentum \( m_L/L = \omega/2 \) (\( m_R/L = \omega/2 \)) (just like in the initial state \(|i\rangle\) of the previous section). Thus, in this absorption process the initial and final states of the previous section just get interchanged. Furthermore, it is trivial to see from (1) (or by using perturbative unitarity of string theory) that to first order is \( g_{st} \) perturbation theory, \( \langle f'|S|i'\rangle = \langle i|S|f\rangle = -\langle f|S|i\rangle^* \). It follows, therefore, that for the macroscopic black hole the absorption probability is

\[ P_{\text{abs}}(i' \rightarrow f') = \frac{1}{\Omega'} \sum_{\{i'\},\{f'\}} |\langle f'|S|i'\rangle|^2 = \frac{1}{\Omega'} \sum_{\{i\},\{f\}} |\langle f|S|i\rangle|^2 \]

The division by \( \Omega' \) signifies averaging over the microscopic initial states \(|i'\rangle\) whose degeneracy is the same as that of the state \(|f\rangle\) and is given by \( \Omega' \) in (7). From (10) and (14) we see that at the macroscopic level the absorption probability is related to the decay probability by the equation

\[ P_{\text{abs}}(i' \rightarrow f') = \frac{\Omega}{\Omega'} P_{\text{decay}}(i \rightarrow f) \]
Thus the absorption probability is larger than the decay probability by the factor $\Omega/\Omega'$ (recall that $\Omega$ increases exponentially with the mass of the black hole and that $\Omega'$ refers to the black hole with mass smaller than that to which $\Omega$ refers). As we shall see it is this enhancement factor that is responsible for a nonzero classical absorption by the black hole.

The passage from the absorption probability to the absorption cross-section $\sigma_A$ is usual. We get

$$\sigma_A \propto \frac{\Omega}{\Omega'} G_N \frac{\omega L}{2} \langle \tilde{N}_L(m) \rangle \langle \tilde{N}_R(m) \rangle$$

where $m = \omega L/2$ and $\langle \tilde{N}_{L,R}(m) \rangle$ are given in (12), (13).

3 Classical Limit

We would now like to discuss the results of the previous two sections in the classical limit. This limit is taken by letting the mass of the black hole become very large, i.e., $M \gg 1$ (in Planck units). Actually, the classical limit is more subtle in the case of charged black holes [26, 27, 28]. One also needs to ensure that the black hole is not too close to extremality. More quantitatively, for the 5-dimensional charged black holes under discussion, the appropriate conditions are

$$M \gg \Delta M \gg \frac{1}{M^2}, \quad M \gg 1$$

The second part of the first condition ensures consistency of thermal description ($\Delta M$ is mass deviation from the extremal limit) [26], while the first part ensures that deviations from extremality are small in the macroscopic sense.

Now, let us assume that the nonextremal charged black holes under discussion are obtained by perturbing $N_R$ away from its extremal value of zero. Let us take the classical limit by scaling $Q_1$, $Q_5$ and $N(\equiv N_L - N_R)$ as follows

$$Q_1 \to \lambda Q_1, \quad Q_5 \to \lambda Q_5, \quad N \to \lambda N, \quad \lambda \gg 1$$

keeping the ratio $N_R/N_L$ fixed and small. Such a scaling is natural for Reissner-Nordstrom black holes which satisfy the condition $Q_1 R/g_{st} = Q_5 R V/g_{st} = N/R$. These scalings have the following effect on the mass $M$ of the black hole, $\Delta M$ and the effective radius $L$ of the $S^1$ in the $x^5$ direction:

$$M \to \lambda M, \quad \Delta M \to \lambda \Delta M, \quad L \to \lambda^2 L.$$
The conditions in (17) are then automatically satisfied for $\lambda \gg 1$ in the case of absorption, while in the case of decay they are satisfied at least in the early stages of decay. Thus, a consistent way of taking the classical limit is to do the scalings (18), and let $\lambda$ become large.

Now, under the scalings (18) and (19), we see from (13) that $\beta_{L,R}$ scale as

$$\beta_L \to \sqrt{\lambda} \beta_L, \quad \beta_R \to \sqrt{\lambda} \beta_R \quad (20)$$

Note that it follows from (13) that $\beta_R$ is always much larger than $\beta_L$ (because of the condition $N_R \ll N_L$), and remains so under the scalings (20).

**Vanishing classical decay rate:**

We are now ready to discuss the classical limits of (11) and (16). Let us consider the decay first. Because of (20) the decay rate peaks at $\omega \sim 1/\beta_R$ in the classical limit. In (11) we may, therefore, expand $\langle \tilde{N}_L(m) \rangle$ and retain only the first term:

$$\langle \tilde{N}_L(m) \rangle \sim \frac{1}{\beta_L \omega} \quad (21)$$

Thus, the decay rate becomes [8]

$$d\Gamma \propto d^4 \vec{k} A_h (e^{\beta_R \omega/2} - 1)^{-1} \quad (22)$$

where $A_h \sim G_N \sqrt{Q_1 Q_5 N_L}$ is the area of the horizon of the black hole. Now, using (18) and the second of (20), we find that the decay rate vanishes exponentially in the classical limit:

$$d\Gamma \sim \lambda^{\lambda/2} e^{-\sqrt{\lambda}} \quad (23)$$

where in the last equation we have displayed only the $\lambda$-dependence.

**Nonvanishing classical absorption:**

To see what happens to the absorption crosssection, (16), in this limit, we also need to compute the enhancement factor $\frac{\Omega}{\Omega'}$. Using that $\tilde{N}_{L,R}' = \tilde{N}_{L,R} - m$ and $\omega = \frac{2m}{L}$, we get

$$\frac{\Omega}{\Omega'} = e^{[\frac{m}{2} \beta_L + o(\omega^2)]} e^{[\frac{m}{2} \beta_R + o(\omega^2)]} \quad (24)$$
The coefficient of the $\omega^2$ term in the first exponent is the derivative of $\beta_L \sim \sqrt{L/M}$ with respect to $M$, and under the scalings in (19) vanishes as $\lambda^{-1/2}$ for large $\lambda$. Similarly the corresponding coefficient in the second exponent involves the derivative of $\beta_R \sim \sqrt{L/\Delta M}$ with respect to $\Delta M$, which also vanishes as $\lambda^{-1/2}$ for large $\lambda$. The coefficients of higher powers of $\omega$ in both the exponents vanish even faster as $\lambda$ becomes large.\footnote{Equation (24) is actually valid under a more general scaling of $\Delta M$ than that given in (19), namely $\Delta M \rightarrow \lambda^\alpha \Delta M$ where $1 \geq \alpha > 2/3$.}

Using (24) and (12) in (16), we get

$$\sigma_A \propto G_N \frac{\omega L}{2} \left(1 - e^{-\beta_L \omega/2}\right)^{-1} \left(1 - e^{-\beta_R \omega/2}\right)^{-1}$$

which is clearly nonvanishing in the classical limit.

We will now restrict the above formula to frequencies $\omega$ satisfying $\omega \ll \beta_L^{-1}$. This is done for the following reason. In the classical calculation of the absorption crosssection in the next section, we have restricted ourselves to small values of $\omega$. The corrections are order $\omega r_0$, where $r_0 \sim (G_N M)^{-1/2} \sim \beta_L$ is the radius of the horizon. The corrections are, therefore, higher order in $g_{st}$. To include this consistently one must, therefore, also include higher order $g_{st}$ corrections in the microscopic model, which we have not done here.

Now under the condition $\omega \ll \beta_L^{-1}$, we may expand the first factor in brackets in (25) in powers of $\omega \beta_L$. Retaining only the first term, we get

$$\sigma_A \propto A_h \left(1 - e^{-\beta_R \omega/2}\right)^{-1}$$

Now we let $\lambda$ become large after doing the appropriate scalings given in (24). For any given fixed $\omega \ll \beta_L^{-1}$, $\beta_R \omega$ will eventually become very large as $\lambda$ becomes large.\footnote{The condition $\omega \ll \beta_L^{-1}$ actually puts a restriction on how large a value of $\lambda$ can be taken. However this does not affect our conclusion since the maximum value of $\beta_R \omega$ allowed by this condition, namely $\beta_R / \beta_L$, is very large.} Therefore in this limit (26) gives

$$\sigma_A \propto A_h$$

Thus, the enhancement factor $\Omega / \Omega'$ has ensured that the absorption coefficient remains nonzero in the classical limit! As we shall see in the next section, the result we have obtained above in (27) from a microscopic calculation matches in all its essential details with that obtained from a classical calculation of wave propagation in the appropriate black hole geometry.
4 Classical Wave Analysis and Absorption

In this section we consider classical propagation of a massless field in the geometry of the 4+1 dimensional black hole. We take the massless field to be one of the scalar moduli which has a simple propagation equation

\[ D_\mu \partial^\mu \phi = 0 \]  

Here the metric defining the Laplacian is

\[ ds^2 = -f^{-2/3}(r)g(r)dt^2 + f^{1/3}(r)[g(r)]^{-1}dr^2 + f^{1/3}r^2[d\chi^2 + \sin^2 \chi d\theta^2 + \sin^2 \chi \sin^2 \theta d\phi^2] \]

\[ g(r) = (1 - r_0^2/r^2) \]

\[ f(r) = (1 + \frac{r_0^2}{r^2}\sinh^2 \alpha)(1 + \frac{r_0^2}{r^2}\sinh^2 \gamma)(1 + \frac{r_0^2}{r^2}\sinh^2 \sigma) \]

The parameters \( r_0, \alpha, \gamma, \sigma \) appearing in the metric can be related to various parameters of the microscopic model by the relations

\[ Q_1 = \frac{Vr_0^2}{2g_{st}} \sinh 2\alpha, \quad Q_5 = \frac{r_0^2}{2g_{st}} \sinh 2\gamma, \quad N = \frac{R^2Vr_0^2}{2g_{st}^2} \sinh 2\sigma, \]

\[ M = \frac{RVr_0^2}{2g_{st}^2}(\cosh 2\alpha + \cosh 2\gamma + \cosh 2\sigma). \]

In order to calculate the absorption coefficient from (28), we will follow a procedure similar to the one used in [30] for the 3+1 dimensional Schwarzschild black hole.

Equation (28) admits the following separation of variables:

\[ \phi(r, t, \chi, \theta, \phi) = e^{-i\omega t}R_{\omega l}(r)Z_l(\chi) \]

We will be interested in the low frequency behaviour. It is then enough for us to concentrate on the s wave. The corresponding radial function \( R_{\omega l} \equiv R_{\omega l}|_{l=0} \) satisfies the differential equation

\[ \left[ \frac{g}{r^3} \frac{d}{dr}gr^3 \frac{d}{dr} + \omega^2 f \right]R_{\omega} = 0 \]
This equation can be alternatively written, in terms of $\psi \equiv r^{3/2} R_\omega$, as

\[
\left[ -\frac{d^2}{dr_*^2} + V_\omega(r_*) \right] \psi = 0
\]

\[
V_\omega(r_*) \equiv -\omega^2 f + \frac{3}{4} \frac{r_0}{2} (1 - \frac{r_0^2}{r^2}) (1 + 3 \frac{r_0^2}{r^2})
\]

where $r_* \equiv \int \frac{dr}{1 - \frac{r_0^2}{r^2}} = r + (1/2) r_0 \ln |(r - r_0)/(r + r_0)|$ is the “tortoise coordinate”.

**Solution in the Far Region** $(r \gg r_0, (r_0/\omega^2)^{1/3})$

In this region we can keep terms only up to $\frac{1}{r}$ in (34). The solution, given in terms of Coulomb wave functions, has the following asymptotic expansions:

(i) $\omega r \gg 1$

\[
R_\omega \sim r^{-3/2} (-2i\omega)^{-a} \left( e^{-i\omega} \frac{\Gamma(2a)}{\Gamma(a)} A + e^{i\omega} \left[ e^{-i\pi a} \frac{\Gamma(2a)}{\Gamma(a)} + B \right] \right) [1 + o(\omega r)^{-1}]
\]

(ii) $\omega r \ll 1$

\[
R_\omega \sim r^{a-3/2} e^{i\omega r} \left[ (A + B \frac{\Gamma(1 - 2a)}{\Gamma(1 - a)}) - (-2i\omega r)^{1-2a} B \frac{\Gamma(1 - 2a)}{\Gamma(1 - a)} \right] (1 + o(\omega r))
\]

Here

\[
a = \frac{1}{2} + \sqrt{1 - (\omega r_0)^2 (2 + s_1)}, \quad s_1 = \sinh^2 \alpha + \sinh^2 \gamma + \sinh^2 \sigma
\]

**Solution in the Near Region** $(r \to r_0)$

Here (33) reduces to

\[
(g \frac{d}{dr} g \frac{d}{dr} + \omega^2 f_0) R_\omega = 0 \Rightarrow \left[ (\frac{d}{dr_*})^2 + \omega^2 f_0 \right] R_\omega = 0
\]

where

\[
f_0 = f \big|_{r=r_0} = \cosh^2 \alpha \cosh^2 \gamma \cosh^2 \sigma
\]
The solution to (38), using the boundary condition that there is no outgoing exponential at the event horizon, is

\[ R_\omega \sim A_0 \exp[-i(\omega \sqrt{f_0} r + \delta)] \]  

(40)

Besides these solutions, it is also easy to derive the following exact \( \omega = 0 \) solution (at any \( r \))

\[ R_{\omega=0} = A_1 + \frac{B_1}{2} \ln |1 - \frac{r_0^2}{r^2}| \]  

(41)

Matching (36) and (40) with the \( r_0/r \to 0 \) and \( r_0/r \to 1 \) limits of (41) (cf. [30]) we get the following relations between various coefficients at low frequency (\( \omega r_0 \ll 1 \)):

\[ B = \beta A_0 \]

\[ A = [1 - \beta \frac{\Gamma(1-2a)}{\Gamma(1-a)}] A_0 \]  

\[ \beta \equiv 2i(\omega r_0)^3 \sqrt{f_0} \frac{\Gamma(a)\Gamma(2-2a)}{\Gamma(2a)\Gamma(1-2a)} \]  

(42)

Choosing \( A \) to be such that the coefficient of \( \frac{e^{i\omega r}}{r^{3/2}} \) in (35) is 1, we get

\[ R_\omega \simeq e^{-i\omega \frac{r}{r_0^{3/2}}} + R e^{i\omega \frac{r}{r_0^{3/2}}} \]  

(43)

The absorption coefficient is, therefore, to leading order in \( \omega r_0 \)

\[ |A|^2 \equiv 1 - |R|^2 = \frac{\pi}{2} (\omega r_0)^3 \sqrt{f_0} = \frac{1}{4\pi} \omega^3 A_h \]  

(44)

where we have used (39) and the expression for the area of the event horizon

\[ A_h = 2\pi^2 r_0^3 \cosh \alpha \cosh \gamma \cosh \sigma \]  

(45)

It is easy to show that the \( s \) wave absorption crosssection \( \sigma_A \) is related to the absorption coefficient by

\[ \sigma_A = \frac{4\pi}{\omega^3} |A|^2 \]  

(46)

which in this case, therefore, is

\[ \sigma_A = A_h \]  

(47)

as claimed in the previous section. Like (44), (47) is also calculated to the leading order in \( \omega r_0 \).
5 Concluding Remarks

In summary, in this letter we have computed the absorption crosssection of massless quanta by a near extremal 4+1 dimensional charged black hole within the context of the string theory based microscopic model proposed in [7, 8, 18]. The authors of [8] have correctly reproduced the Hawking radiation formula for a near extremal black hole (modulo a numerical coefficient). Our microscopic computation of the absorption crosssection agrees (modulo a numerical coefficient) with the classical calculation from the analysis of a massless wave propagating in the background metric of the appropriate black hole.

The basic reason why we get a nonzero absorption crosssection is the presence of the enhancement factor $\Omega/\Omega'$ in this calculation relative to the Hawking decay, which vanishes in the classical limit. The factor $\Omega/\Omega'$ depends only on the counting of the microscopic quantum states of a near extremal black hole and does not depend on the details of the matrix element calculation. This is just like the factors $\langle \tilde{N}_L(m) \rangle$ and $\langle \tilde{N}_R(m) \rangle$, which also depend only on the counting of states and in the decay calculation and give rise to the universal black body nature of the Hawking decay formula. The precise cancellation of these factors with the enhancement factor $\Omega/\Omega'$, in the classical limit, is then what gives a nonzero result for the classical absorption crosssection, as opposed to the Hawking decay, which vanishes in the classical limit.

We believe that it is reasonable to expect that the above feature of our calculation will not be modified by taking strong coupling effects into account, at least for near extremal black holes. On the other hand, one may, a priori, not have expected to get a detailed agreement of the classical limit of the microscopic calculation with classical absorption crosssection calculation. That the former agrees with the latter in all its essential details is, therefore, a surprise. The magic here is the same as the one that gives the Hawking decay coefficient proportional to the area of the horizon in the calculation of [8]. This is because the magnitude of the microscopic matrix element that is responsible for absorption is the same as the one that gives the decay, at this order of string coupling. It is possible that with a better understanding of the microscopic models of black holes we might understand why certain physical situations are insensitive to the strong coupling effects of the “dense horizon soup” [15]. In this context, it would be very interesting to compute
the numerical coefficient in front of the decay rate in (22) and the absorption coefficient in (27) and to see whether they agree with their expected values.

One of the essential features of the existence of a horizon is that classically it acts as a one way valve for particles and energy. It seems to us that the microscopic models which incorporate this feature must “know” about the existence of a horizon in the strong coupling regime. There are, of course, many other aspects of the physics of horizon that need to be explored. Hopefully, further study will provide a better and more detailed understanding of this and other aspects of black hole physics within the context of the microscopic models.

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