Measurement problem: from De Broglie to theory of classical random fields interacting with threshold detectors

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Abstract. The quantum measurement problem as was formulated by von Neumann in 1933 can be solved by going beyond the operational quantum formalism. In our “prequantum model” quantum systems are symbolic representations of classical random fields. The Schrödinger’s dynamics is a special form of the linear dynamics of classical fields. Measurements are described as interactions of classical fields with detectors. Discontinuity, the “collapse of the wave function”, has the trivial origin: usage of threshold type detectors. The von Neumann projection postulate can be interpreted as the formal mathematical encoding of the absence of coincidence detection in measurement on a single quantum system, e.g., photon’s polarization measurement. Our model, prequantum classical statistical field theory (PCSFT), in combination with measurements by threshold detectors satisfies the quantum restriction on coincidence detections: the second order coherence is less than one (opposite to all known semiclassical and classical field models). The basic rule of quantum probability, the Born’s rule, is derived from properties of prequantum random fields interacting with threshold type detectors. Comparison with De Broglie’s views to quantum mechanics as theory of physical waves with singularities is presented.

1. Introduction

The measurement problem in quantum mechanics [1] is the unresolved problem of how (or if) wave function collapse occurs. The inability to observe this process directly has given rise to different interpretations of quantum mechanics and poses a key set of questions that each interpretation must answer. (See [2]-[7] for hot debates on the “right interpretation”.) The wave function in quantum mechanics evolves according to the Schrödinger equation which preserves linear superposition of different states, but actual measurements always find the physical system in a definite state. Any future evolution is based on the state the system was discovered to be in when the measurement was made, meaning that the measurement “did something” to the process under examination. Whatever that “something” may be does not appear to be explained by the basic conventional quantum theory.

During the last 12 years quantum foundations were discussed at the series of conferences which took place in Växjö (South-East Sweden), see, e.g., [2]-[7]. And the most exciting spectacle started each time when the question of interpretations of the wave function attracted the attention. Finally, it became clear that the number of different interpretations is in the...
best case equal to the number of participants. If you meet two people who say that they are advocates of, e.g., the Copenhagen interpretation of QM, ask them about the details. You will see immediately that their views on what is the Copenhagen interpretation can differ very much. The same is true for other interpretations. If two scientists tell that they are followers of Albert Einstein’s ensemble interpretation, ask them about the details. At one of the round tables (after two hours of debates with opinions for and against completeness of QM) we had decided to vote on this problem. Incompleteness advocates have won, but only because a few advocates of completeness voted for incompleteness. The situation is really disappointing: the basic notion of QM has not yet been properly interpreted (after 100 years of exciting, but not very productive debates).

We can also mention the Växjö interpretation of QM. This interpretation evolved essentially from the Växjö interpretation-2000 [8] which was based on “naive Einsteinian realism” (the values of physical observables can be assigned to a quantum system before measurement) to the Växjö interpretation 2009 in which measurement context played a fundamental role [9] (in particular, the values of physical observables cannot be assigned to a quantum system before measurement). The latter combines the views of Einstein on incompleteness of QM with the views of Bohr on contextual structure of quantum measurements. By the Växjö interpretation subquantum reality exists. However, for a moment we cannot approach it by means of available observables. The presently used observables, “quantum observables”, are not “elements of subquantum reality”: for example, the components of the electric and magnetic fields of photon or the density of electron’s charge, see W. Hofer [10] for the latter. In our model the presently used “quantum observables” are contextual: their values cannot be assigned to subquantum systems in advance, before measurement. The results of quantum measurements are determined by measurement contexts. This is really surprising, because typically contextuality was considered as a quantum feature. The basic classical physical model, classical statistical mechanics, is not contextual, the values of physical observables are considered as the values of an object. However, as the reader will see in the present paper, already classical wave models combined with measurement theory based on usage of threshold type detectors can exhibit contextual features.

This is well known (starting with the work of S. Gudder [12], see also K. Svozil [13] and the author’s book [9]) that contextual models can violate Bell’s inequality [14]. Therefore this is not surprising that PCSFT in combination with measurements of the threshold type can peacefully coexist with Bell’s no-go theorem. (This problem will not be considered in the present paper; see, however, [15] for the detailed presentation.)

There is a plenty of approaches to solve the measurement problem by using the formalism of quantum mechanics, e.g., [19], see also [20] for the detailed review. However, we speculate that it seems to be impossible to find the “real solution of this problem” in the standard quantum framework, since the quantum formalism (as was pointed by N. Bohr on many occasions, e.g., [21], [22]) is a formal operational formalism describing measurements for microsystems and not physical processes in the microworld [22]: “Strictly speaking, the mathematical formalism of quantum mechanics and electrodynamics merely offers rules of calculation for the deduction of expectations pertaining to observations obtained under well-defined experimental conditions specified by classical physical concepts”, cf. [23], [24], [25]. By using the operator representation

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1 This is a good place to point to a general scientific methodology which was advertised during many years by Atmanspacher and Primas [11]. Any scientific theory is based on two levels of description of reality: ontic (reality as it is) and epistemic (the image of reality obtained with the aid of a special class of observables). The QM-formalism is an example of an epistemic model. However, existence of an epistemic model does not prevent scientists to go beyond it to approach the ontic level.

2 See [16], [17], [18] for recently constructed prequantum models which do not contradict to the Bell’s argument. We do not comment these models in the present paper.
of observables we escape a detailed description of the process of measurement and such a description could not recovered in the quantum formalism. To solve the measurement problem of quantum theory, one has to go beyond the operational quantum formalism, cf. [19], [20], [26].

In this paper, we “solve the quantum measurement problem”\(^3\) by interpreting quantum systems as formal operational representations of classical prequantum fields (waves) and by describing the measurement process as the process of interaction of a classical wave with a threshold type detector. In section 2 we analyze the process of measurement of a classical wave by a detector which selects one of eigenfunctions from the coherent superposition representing the wave, see (3). After such a measurement this superposition is “collapsed”. This consideration is motivated by chapter 5 of L. De Broglie’s book [27], pp. 51-54; see also [28]. The “collapse” of a sound wave (emitted e.g., by a vibrating string) which interacts with the turning fork having one of the basic string’s frequencies can be operationally described in the same way as the collapse of the wave function of a photon (emitted in the state of superposition of a few frequencies) which interacts with a photo-detector. However, opposite to the quantum wave function, the classical wave is collapsed not to a single eigenfunction of the corresponding stationary equation, but to the decoherent mixture of such eigenfunctions. Thus the formal mathematical description is similar to the description of quantum measurement without selection of the fixed value of the observable. In the later case we also obtain not a single eigenvector, but a mixture, described by the density operator.

Roughly speaking in our approach the main difference between classical and quantum collapses is degeneration of the former, the impossibility to select a single output component of the signal (“quantum particle”). In quantum theory this problem is known as the problem of coincidence detection. It was intensively studied experimentally to reject (semi)classical models pretending to reproduce quantum predictions, see Grangier et al. [42], [41], also [43]. The quantum prediction that coincidence detections should occur relatively rarely was supported by experiments. It is commonly accepted that these experiments demonstrated that classical or semiclassical models without coincidence detection do not exist.

However, the field model elaborated in [29]-[36], prequantum classical statistical field theory (PCSFT), in combination with usage of the threshold type and properly calibrated detectors solves the coincidence detection problem, opposite to known (semi)-classical models. In [37] we found that quantum statistics can be obtained in a simple way: by combination of stationary random field describing spatial or internal degrees of freedom with the Brownian motion (Wiener process) describing temporal fluctuations, cf. with [15], where a more complicated stochastic processes were in use. In this paper we essentially improved the rigorosity of the presentation of the threshold detection scheme for random waves by coupling this problem with the well studied problem of probability theory, namely, the first hitting time problem. Unfortunately, probabilistic studies on hitting times were restricted to real valued stochastic processes. (These studies were essentially stimulated by applications to finances and here the real valued processes are in usage.) Development of theory of hitting times for complex valued stochastic processes is a complicated mathematical problem and we hope that our paper will stimulate research in this direction. For a moment, we can apply only results on hitting times for real processes and therefore we restrict our consideration to the case of real Hilbert space. In this paper we reproduce quantum statistics for density matrices with real elements. This is merely a question of the mathematical justification: in [37] we proceeded in the general case of complex Hilbert space, but not in the completely rigorous mathematical framework.

We remark that the aforementioned classical-quantum analogy between measurements of classical and quantum waves is valid only for disturbative classical measurements. It is commonly postulated that in classical physics it is possible to perform measurements with an arbitrary

\(^3\) Of course, one staying at the Copenhagen position would not consider our quantum solution of the measurement problem as the real solution of this problem.
precision; in particular, it is possible to determine the form of the wave without to destroy coherent superposition. In this paper, we do not criticize this postulate\(^4\). We just remark that quantum observables form only a subclass of observables for prequantum fields. These are coarse and disturbative measurements. PCSFT’s class of observables is essentially larger and it contains measurements of fields components, e.g., the electric and magnetic fields components of a photon. The modern experimental technology is still far from realization of such measurements, cf., however, with recent experimental results presented by W. Hofer (Conference “Quantum Mechanics as Emergent Phenomenon”, Vienna, November 2011) on a possibility to violate Heisenberg’s uncertainty relation.

At the very end of section 2, we compare the De Broglie’s Double Solution theory \([28],[27]\) with PCSFT (combined with threshold-type measurements) and with Bohmian mechanics. There is a rather common opinion that the De Broglie’s Double Solution theory is simply an early version of a more advanced theory, namely, Bohmian mechanics. However, the careful study of works of De Broglie shows that this viewpoint to the inter-relation between the De Broglie’s Double Solution theory and Bohmian mechanics is rather primitive, see section 2.

2. Classical fields: superposition, linear dynamics, measurement

In this section we consider classical waves with linear dynamics (in vacuum or some media). The process of measurement induces effects\(^5\) which formally can be described by the von Neumann projection postulate: linear superposition “collapses”. We claim that the essence of the “collapse” of superposition is the transition from one linear dynamics in the absence of measurement to another dynamics (nonlinear or even linear) of interaction with a measurement device.

A possibility to form linear superposition of waves (fields) and to split a wave into superposing summands is the basic feature of the classical wave theory. A crucial point is the existence of dynamics which preserve linear superpositions in the process of evolution. We can mention the dynamics of the string or the classical electromagnetic field (Maxwell’s equations)\(^6\).

Suppose that in the absence of interaction with measurement devices the field dynamics is described by the differential equation:

\[
\frac{\gamma}{\gamma} \frac{d\phi(t,x)}{dt} = L\phi(t,x),
\]

where \(L\) is a linear partial differential operator and \(\gamma\) is a constant. Since, at each instant of time \(t\), the field’s energy is finite, \(\int_{\mathbb{R}^3} |\phi(t,x)|^2 dx < \infty\), the equation (1) can be considered as a linear (ordinary) differential equation in the \(L^2\) space. This space has the Hilbert space structure. Denote it by the symbol \(H\). For all basic physical processes, we can assume that the operator \(L\) is self-adjoint in \(H\).

Sometimes it is convenient to proceed with complex vector valued fields, e.g., for the electromagnetic field, we can consider the Riemann-Silberstein representation, \(\phi(t,x) = E(t,x) + iB(t,x)\), where \(E\) and \(B\) are the electric and magnetic components, respectively. We remark that the system of the Maxwell equations can be written in such a form with \(\gamma = i\) \([30]\), i.e., in the same form as the Schrödinger equation \([38],[30]\). (We also mention the work of Strocchi \([39]\) who demonstrated that the Schrödinger equation can be written as the system of linear

\(^4\) One of the main objections is the presence of noise; for prequantum classical fields their coupling with the background field, vacuum fluctuations, is irreducible \([29]–[36]\). For a moment, we ignore this problem.
\(^5\) The interaction between a measurement device and the input wave can be linear; so nonlinearity is not crucial for collapse.
\(^6\) We remark that all these linear dynamics are approximate. Although these are very good approximations, one should not overestimate the role of linear dynamics. At the fundamental level the majority of processes are nonlinear, cf. De Broglie \([28]\).
Hamiltonian equations, see also [30].) In general, we work with real valued vector fields. The wave equation and Klein-Gordon equation cannot be written in the complex form, here fields are real and $\gamma = 1$. Solution of a linear dynamical equation with self-adjoint operator $L$ can be represented in the form of superposition of solutions $\phi_k = \phi_k(x)$ of the stationary equation:

$$L\phi_k(t, x) = \omega_k \phi_k(t, x);$$  

(2) here

$$\phi(t, x) = \sum_k c_k(t) \phi_k(t, x), c_k(t) = e^{\frac{\omega_k t}{\gamma}} c_{k0},$$  

(3) where

$$\phi(0, x) = \sum_k c_{k0} \phi_k(x)$$  

(4) is the expansion of the initial wave. The main statement is that this form of expansion, (4), is preserved in the process of evolution. If, for some $k$, the term with $\phi_k(x)$ was present in (4) at $t = 0$, then it will never disappear from (3), for any instant of time $t > 0$. (If the operator $L$ has continuous spectrum, then the sign of sum is changed to the sign of integral.)

As was pointed out by L. De Broglie [27], the expansion (3) is a formal mathematical representation. The field $\phi(t, x)$ cannot be imagined as “physical superposition” of fields $\phi_k(x)$. He stressed that the contributions of summands $\phi_k$ in (3) can be extracted from the field $\phi$ only through interaction with measurement devices, cf. Roychoudhuri [40].

In [27] the example of vibrating string was presented. The waves $\phi_k$ correspond to the basic frequencies $\omega_k$ of the vibrating string (for some type of boundary conditions). However, until we start measurement the waves $\phi_k$ do not present in the integral wave $\phi$ so to say physically. In this case measurement can be done with the aid of a turning fork. Put a turning fork nearby the vibrating string and by adjustment of turning fork\(^7\) we can find one of the basic frequencies, say $\omega_{k0}$. The sound emitted by turning fork at the frequency $\omega_{k0}$ corresponds to extracting from the integral wave $\phi$ its fixed component $\phi_{k0}$. Formally, this process can be described as the orthogonal projection $P_{k0}$ (in Hilbert space $H$) of the integral wave $\phi$ onto the one dimensional subspace corresponding to the wave $\phi_{k0}$. One can call this process the collapse of the classical wave $\phi$ or more precisely the collapse of the linear superposition (3). Of course, there is nothing mysterious in this collapse: in particular, it is completely clear that it is not instantaneous, the process of interaction with the turning fork has a finite duration; neither mystery is destruction of superposition (3): the dynamics of interaction with the turning fork is different from the dynamics (1) preceding interaction and hence the original superposition need not be preserved and it can be destroyed, collapsed. (The interaction dynamics need not be nonlinear. It can be linear as the original dynamics (1), but with another linear operator, say $L_1$. Dynamics with $L$ and $L_1$ can be unified through linear dynamics with time dependent generator.)

In fact, the situation is more complicated. Here the string plays the role of a source of sound waves (cf. with a source of quantum systems). Hence, the turning fork interacts not directly with the string, but with the emitted sound wave. To proceed rigorously, we have to use a different notation for this (sound) wave, say $\varphi(t, x)$ and the corresponding solutions of the stationary equation $\varphi_k(x)$. Before measurement, the dynamics of the sound wave $\varphi(t, x)$ can be well described by the linear partial differential equation, the wave equation. Hence, as well as (3), the representation

$$\varphi(t, x) = \sum_k c_k(t) \varphi_k(x),$$  

(5)\(^7\) We can use either a turning fork which frequency can be changed or what is may be even easier a collection of turning forks representing scale of frequencies.
can be used for mathematical calculations. Here we stress again that the physical (sound) wave
\( \varphi(t, x) \) is not composed of physical stationary (sound) waves \( \varphi_k(x) \).

To simplify consideration, we shall proceed with original vibrations of the string (by having
in mind that, in fact, the measurement device turning fork destroys the coherent superposition
(5) in the sound wave). We state again that one has to be very careful with terminology.
The expression “destroys coherent superposition” is related to the “mathematical wave” and
not the physical one. (Hence, “collapse” takes place in mathematical space.) In the presented
considerations we stressed the analogy between “collapses” of the classical wave and the quantum
wave function. The similarity of operational descriptions of “collapses”, i.e., by using projection
operators in Hilbert space, is especially important. However, although classical superposition
(3) collapsed as the result of interaction with the turning fork which is used for measurement
and in the operational formalism the extraction of the component \( \phi_{k_0}(x) \) can be described by
the projection operator \( b \), the reader experienced in quantum theory would point out that
there is a crucial difference between the collapse of the quantum wave function and the classical
wave. In the classical case, although the component \( \phi_{k_0}(x) \) is extracted from the signal, the
signal is not completely reduced to this component. If we put two turning forks, we can select
two basic frequencies, \( \omega_j, j = 1, 2 \). The selection of each component \( \phi_j \) can be operationally
represented by the corresponding projector \( P_j \) in \( H \). However, opposite to the quantum case (we
consider the case of nondegenerate spectrum, \( \omega_i \neq \omega_j, \, i \neq j \) the output signal is not reduced to
one of the selected components. And if we put many (and even infinitely many) turning forks,
it is possible to select corresponding components of the wave \( \phi \).

By the conventional interpretation of quantum mechanics, in the process of interaction with
a detector, a quantum system exhibits particle properties, hence, it could not be detected
simultaneously by two different detectors. In fact, the starting point was the analysis of the
two slit experiment by N. Bohr. This analysis played a fundamental role in elaboration of the
complementarity principle. In the two slit experiment by placing detectors directly behind slits
one does not observe coincidence detections, detectors never click simultaneously.

Of course, this is the idealization of the real physical situation. The first basic assumption
is that the used source is really a single-photon source. At least in 1920th such sources did not
exist; even nowadays one can only approximately assume that a source is of the one-photon
type (with sufficiently high approximation). Another problem is noise. In any event, there
are coincidence detections and their number is not negligibly small. Nevertheless, it has to be
relatively small.

The corresponding experiments have been done. The first experiment was done by Grangier
[41], [42], see also [43] for a review, in the framework of quantum optics: detection of the outputs
of two channels of the polarization beam splitter (PBS). \(^8\) By quantum mechanics a single photon
passing PBS could not be split and coincidence detection for two detectors placed in the two
outputs of PBS is impossible. Grangier demonstrated that the relative probability of coincidence
detection, the second order coherence:

\[
g^{(2)}(0) = \frac{P_{12}}{P_1 P_2} \tag{6}
\]

is less than 1. At the same time all known (semi)classical field models predicted that this
coefficient exceeds 1, see [43] for review.

In fact, the von Neumann projection postulate (for observables with nondegenerate spectrum)
is the formal Hilbert space description of the absence of coincidence detections. As the
result of quantum measurement only one detector can click. (We state again that we

\(^8\) Up to author’s knowledge the experiment on the coincidence detection in the real two slit experiment has never
been done, at least with high precision.
consider only noncompound systems.) This physical process is represented in the operational quantum formalism by projection of the state $\phi$ onto one fixed state $\phi_{k_0}$. The quantum collapse of superposition differs from the “classical collapse” of the waves superposition by its nondegeneration, the output is only a single state $\phi_{k_0}$. Hence, in quantum theory only a single result of measurement, say $\omega_{k_0}$, can be obtained. The state $\psi$ of the quantum system is “collapsed” to the eigenstate $\phi_{k_0}$. (Opposite to classical wave theory, all states are assumed to be normalized.) In the mathematical formalism this process is described by the von Neumann projection postulate:

$$\psi_{k_0} = \frac{P_{k_0} \psi}{||P_{k_0} \psi||}$$  \hspace{1cm} (7)

The probability to get the result $\omega_{k_0}$ is given by Born’s rule

$$P(\omega_{k_0}) = |\langle \psi | \psi_{k_0} \rangle|^2.$$ \hspace{1cm} (8)

We remark that in quantum theory this rule is a postulate, i.e., it is not derived from other natural physical assumptions. And there is no satisfactory derivation of this rule from other (“naturally justified principles”), in spite of numerous attempts to do this.\footnote{“The conclusion seems to be that no generally accepted derivation of the Born rule has been given to date, but this does not imply that such a derivation is impossible in principle”. See [44]; cf. ‘t Hooft [45], [46] and Hofer [10].}

In classical wave theory, in principle, it is possible to measure simultaneously all values $\omega_k$. (We stress the role of simultaneous measurement. We shall come back to this crucial point later.) Measurements induce extractions of the components $\Phi_k$ which are proportional to eigenwaves $\phi_k$ from the coherent superposition $\phi$.

$$\Phi_k = P_k \phi.$$ \hspace{1cm} (9)

Opposite to the quantum case this components are not normalized; the quantity

$$E_k = ||\Phi_k||^2 = |\langle \phi | \Phi_k \rangle|^2$$ \hspace{1cm} (10)

is the energy of the $k$th output wave, corresponding to the result of measurement $\omega = \omega_k$.

The common between the quantum and classical cases is that in any event the coherent superposition is destroyed. In the quantum case, it is transformed into one of eigenstates, in the classical case into decoherent mixture of eigenwaves $\phi_k$. We remark that in the quantum case if the result of the fixed measurement, $\omega = \omega_k$, is not selected, then we also get decoherent mixture of eigenstates:

$$\rho = \sum_k P(\omega_k) P_k.$$ \hspace{1cm} (11)

We point out that in the classical framework the Born rule for quantum probabilities can be formally written by using relative energies of the classical output waves:

$$\frac{E_k}{E_\phi}$$ \hspace{1cm} (12)

where $E_\phi$ is the total energy of the field. We state again that in the classical wave $\phi$ is not normalized. If now we use the normalized (by its total energy) wave, i.e.,

$$\phi \rightarrow \psi = \frac{\phi}{||\phi||}.$$ \hspace{1cm} (13)
we see that the formal mathematical expression for classical relative energy of the \( k \)th output channel coincides with (8), the Born’s rule.\(^\text{10}\) Consider the classical wave normalized by its total energy, i.e., the wave \( \psi \). In this case the output wave is the mixture of normalized waves. It can be written in the same way as (11) by using relative energies, instead of probabilities:

\[
\rho = \sum_k \frac{\mathcal{E}_k}{\mathcal{E}_\phi} P_k.
\] (14)

Heuristically it is clear that the number of detector’s clicks in \( k \)th channel is proportional to the energy distributed to this channel. However, it is not a trivial task to find a class of classical waves which produce quantum statistics of clicks given by the rule:

\[
P(\omega_k) = \frac{\mathcal{E}_k}{\mathcal{E}_\phi}.
\] (15)

It is evident that waves have to be random and detectors have to be (similar to quantum detectors) of the threshold type – to produce individual clicks and not continuous output signals. Another crucial constraint is that there should be no coincidence detections. Hence, the temporal structure of the random field has to be selected in such a way that, although the field is continuous and so to say it is present everywhere – in any detector, simultaneous clicks (corresponding to random energy spikes) would occur relatively rarely.

This is a good place to compare our model, PCSFT, with the De Broglie’s *Theory of the Double Solution*. By the latter quantum mechanics can be considered as an operational formalism for a deeper theory, theory of classical physical waves containing singularities. The singularities in waves correspond to quantum particles. In PCSFT, particles do not present at all. Singularities appear only at the level of measurements, as clicks of the threshold type detectors. However, the common point is that PCSFT-fields are also singular; these are stochastic processes valued in the \( L_2 \)-space. So, the prequantum fields are nonsmooth and discontinuous for almost all values of a random parameter.

This is also a good place to make a remark about nonlocality. The Theory of the Double Solution is closely related to Bohmian mechanics. The latter is definitely nonlocal, since the quantum potential is nonlocal. On the other hand, PCSFT is a local classical field theory, see \[29\]–\[36\]. In PCSFT a sort of classical nonlocality can be assigned to the background random field. This field is not considered in the present paper, it started to play a crucial role in classical wave modeling of compound systems which we do not study in this paper, see \[33\]–\[35\]. However, such a background field is a classical field, as in stochastic electrodynamics \[48\], and has nothing to do with nonlocality of the Bohmian type. There is a rather common opinion that creation of Bohmian mechanics was the improvement of the Theory of the Double Solution, so nonlocality was also firmly incorporated in De Broglie’s views. However, the real inter-relation between views of De Broglie and Bohm is more complicated. In particular, by reading the De Broglie’s book \[27\] I was not able to find any trace of nonlocality. In the description of a compound quantum system, De Broglie \[27\], p. 74-76, of course, used the nonlocal quantum potential induced by the wave function of a compound system. However, the wave function by itself and hence the potential corresponding to it were treated as related to the formal mathematical description. The propagations of real physical waves corresponding to a compound system are also coupled, but through random fluctuations of the field of random media, a kind of the background field. In this sense De Broglie’s views are close to the views of the author of this

\(^\text{10}\) We suspect that our consideration is similar to Born’s motivation of the rule for probabilities (8); the motivation which he did not present in his paper \[47\]. This is a pity that Born did not present any motivations for the rule (8). This formal postulation of the basic probabilistic rule of quantum theory makes the impression of something completely new and even mysterious.
paper. I also stress that both for De Broglie and for me the starting point of model’s creation was Einstein’s attempt to create a purely field model of physical reality, see, especially, Einstein and Infeld [49]; cf. De Broglie [27], p. 43: “My first attempts to interpret wave mechanics in terms of the Theory of the Double Solution in 1926-1927, were undoubtedly suggested to me by Einstein’s work on general relativity. Einstein believed that the physical world should be described wholly by means of fields, well defined at every point of space-time and obeying well-defined equations of a non-random nature.” As we shall see in section 3, the later statement is not valid for PCSFT, we shall operate with random fields. However, “late De Broglie” also rejected “naive Einsteinian determinism” and stressed the role of the random background field, see [27], p. viii.

3. Threshold detection of classical random fields

We consider a threshold type detector with the threshold $E_d$. It interacts with a random field $\phi(s, \omega)$, where $s$ is time and $\omega$ is a chance parameter describing randomness. For a moment, we consider the $\mathbb{C}$-valued random field (complex stochastic process). Later we shall consider random fields valued in finite and infinite dimensional complex Hilbert spaces. The finite dimensional case corresponds to detection of internal degrees of freedom such as, e.g., polarization. We stress that the real physical situation corresponds to random fields with infinite-dimensional state space, e.g., $H = L_2(\mathbb{R}^3)$; the space of complex valued fields $\phi : \mathbb{R}^3 \to \mathbb{C}$ (or $\mathbb{C}_k$). The energy of the field is given by $\mathcal{E}(s; \omega) = |\phi(s; \omega)|^2$ (hence, the random field has the physical dimension $\sim \sqrt{\text{energy}}$). A threshold detector clicks at the first moment of time $\tau = \tau(\omega)$, when field’s energy $\mathcal{E}$ exceeds the threshold:

$$\mathcal{E}(\tau(\omega), \omega) \geq E_d. \quad (16)$$

In the mathematical model the detection moment is defined as the first hitting time [50]

$$\tau(\omega) = \inf\{s \geq 0 : \mathcal{E}(\tau(\omega), \omega) \geq E_d\}. \quad (17)$$

We consider the following detection scheme. After arriving to a threshold type detector a classical random field behaves inside this detector as the Brownian motion in the space of (complex) fields. Thus $\phi(s, \omega)$ is the Wiener process: the Gaussian process having zero average at any moment of time $E\phi(s, \omega) = 0$; and the covariance function

$$E\phi(s_1, \omega)\phi(s_2, \omega) = \min(s_1, s_2)\sigma^2; \quad (18)$$

in particular, we can find average of its energy

$$E\mathcal{E}(s, \omega) = \sigma^2 s. \quad (19)$$

From this equation, we see that the coefficient $\sigma^2 = \frac{E\mathcal{E}(s, \omega)}{s^2}$ has the physical dimension of power. We are interested in average of the moments of the $E_d$-threshold detection for the energy of the Brownian motion. Since moments of detection are defined formally as hitting times, we can apply theory of hitting times for the Wiener process, see [50]:

$$\tilde{\tau} = E\tau = \frac{E_d}{\sigma^2} \quad (20)$$

or

$$\frac{1}{\tilde{\tau}} = \frac{\sigma^2}{E_d}. \quad (21)$$
Hence, during a long period of time $T$ such a detector clicks $N_\sigma$-times, where

$$N_\sigma \approx \frac{T}{\tau} = \frac{\sigma^2 T}{\mathcal{E}_d}. \quad (22)$$

Consider now a random $\phi(s, \omega)$ valued in the $m$-dimensional complex Hilbert space $H$, where $m$ can be equal to infinity. Let $(e_j)$ be an orthonormal basis in $H$. The vector-valued $\phi(s, \omega)$ can be expanded with respect to this basis

$$\phi(s, \omega) = \sum_j \phi_j(s, \omega)e_j, \quad (23)$$

where $\phi_j(s, \omega) = \langle \phi(s, \omega) | e_j \rangle$. This mathematical operation is physically realized as splitting of the field $\phi(s, \omega)$ into components $\phi_j(s, \omega)$ These components can be processed through mutually disjoint channels, $j = 1, 2, \ldots, m$.\(^{11}\) We now assume that there is a threshold detector in each channel, $D_1, \ldots, D_m$. We also assume that all detectors have the same threshold $\mathcal{E}_d > 0$.

Suppose now that $\phi(s, \omega)$ is the Wiener process valued in $H$. This process is determined by the covariance operator $B : H \rightarrow H$. Any covariance operator is Hermitian, positive, and trace-class and vice versa. The complex Wiener process is characterized by Hermitian covariance operator. We have, for $y \in H$, $E\{y, \phi(s, \omega)\} = 0$, and, for $y_j \in H, j = 1, 2$, $E\{y_1, \phi(s, \omega)\}E\{y_2, \phi(s, \omega)\} = \min(s_1, s_2)\langle By_1, y_2 \rangle$. The latter is the covariance function of the stochastic process; in the operator form: $B(s_1, s_2) = \min(s_1, s_2)B$. We note that the dispersion of the $H$-valued Wiener process (at the instant of time $s$) is given by $\Sigma^2(s) = E\|\phi(s, \omega)\|^2 = s\text{Tr}B$. The quantity $\mathcal{E}(s, \omega) = \|\phi(s, \omega)\|^2$ is the total energy of the Brownian motion signal at the instant of time $s$. Hence, the quantity

$$\Sigma^2 \equiv \frac{\Sigma^2(s)}{s} = \text{Tr}B \quad (24)$$

is the average power of this random signal. We stress that the average power is time-independent.

We also remark that by normalization of the covariance function for the fixed $s$ by the dispersion we obtain the operator,

$$\rho = B(s, s)/\Sigma^2(s) = B/\text{Tr}B, \quad (25)$$

which formally has all properties of the density operator used in quantum theory to represent quantum states. Its matrix elements have the form $\rho_{ij} = b_{ij}/\Sigma^2$. These are dimensionless quantities. The relation (25) plays a fundamental role in our approach: each classical random process generates a quantum state (in general mixed) which is given by the normalized covariance operator of the process. One can proceed the other way around as well: each density operator determines a class of classical random processes.

Consider components $\phi_j(s, \omega)$ of the vector valued signal $\phi(s, \omega)$. Then

$$E\phi_i(s, \omega)\phi_j(s, \omega) = \min(s_1, s_2)\langle Be_i, e_j \rangle = b_{ij}.$$  

In particular, $\sigma^2_j(s) \equiv E\phi_j(s, \omega)^2 \equiv E|\phi_j(s, \omega)|^2 = sb_{jj}$. This is the average energy of the $j$th component at the instant of time $s$. We also consider the average powers of components

$$\sigma^2_j = \frac{\sigma^2_j(s)}{s} = b_{jj}. \quad (26)$$

\(^{11}\) We consider separation of a signal into disjoint channels as a part of the measurement procedure. These channels correspond to separate detectors that are looking at the input state through some kind of beam splitter. And it is assume that an input state is a single mode photon state. (Hence, aforementioned channels do not correspond to different photon modes.)
We remark that the average power of the total signal is equal to the sum of the average powers of its components.

$$\Sigma^2 = \sum_j \sigma_j^2.$$  \hfill (27)

Consider now a run of experiment of the duration $T$. The average number of clicks for the $j$th detector can be approximately expressed as

$$N_j \equiv N_{\sigma_j} \approx \frac{\sigma_j^2 T}{E_d}.$$  \hfill (28)

The total number of clicks is (again approximately) given by $N = \sum_j N_j \approx \frac{\Sigma_j^2 T}{E_d}$. Hence, for the detector $D_j$, the probability of detection can be expressed as

$$P_j \approx \frac{N_j}{N} \approx \frac{\sigma_j^2}{\Sigma_j^2} = \rho_{jj}.$$  \hfill (29)

This is, in fact, the Born’s rule for the quantum state $\rho$ and the projection operator $\hat{C}_j = |e_j\rangle\langle e_j|$ on the vector $e_j$. For the detector $D_j$, the probability of detection can be expressed as

$$P_j = \text{Tr} \rho \hat{C}_j.$$  \hfill (30)

4. Coincidence counts

The relative number of coincidence counts is given by second order coherence:

$$g^{(2)}(0) = \frac{P_{12}}{P_1 P_2}$$  \hfill (31)

Probabilities $P_j, j = 1, 2$, for singles were found in (30); they coincide with corresponding quantum probabilities. For a single photon source, the $P_{12}$, the probability of clicks in both channels after a beam splitter, equals to zero. But, of course, nobody has yet seen really single photon sources. Therefore in reality $P_{12}$ differs from zero. Our threshold detection model definitely contradicts to quantum mechanics as a theory: the coincidence probability is positive (in any case for $P_1 = P_2 = 1/2$). However, we are not disappointed by this situation. We are sure that quantum mechanics is just an approximate model to describe probabilistic data. Moreover, this model cannot take into account experimental technicalities, such as e.g. the size of the discrimination threshold. Therefore its theoretical prediction $g^{(2)}(0) = 0$, for “single photon sources”, is far from the real experimental data. Nevertheless, even with the aid of this rough prediction it was possible to discard the semiclassical model. The latter predicts that $g^{(2)}(0) \geq 1$. And by getting $g^{(2)}(0)$ sufficiently small, experiments claim that their data match with the predictions of quantum mechanics and mismatch with predictions of the semiclassical model. By taking into account the above discussion, we have to test our model contra experiment and not contra the theoretical quantum formalism.

We were not able to obtain a formula for $P_{12}$ in the framework of classical Brownian model from the prequantum field. We were able only to obtain an estimate from above, see [37]. This estimate shows that in some range of variation of the detection threshold $E_d$ the second order coherence $g^{(2)}(0)$ decreases with the increase of $E_d$. Hence, $g^{(2)}(0) < 1$ for sufficiently large $E_d$. Thus our model does not say that there are no coincidence counts, neither that their number is relatively small irrelatively to magnitudes of experimental parameters. However, by our model these parameters can always be selected in such a way that detected data would be described (with a good approximation) by the quantum probabilistic formalism. Thus Devil is really in detectors and experimental technicalities.

Unfortunately, it seems that Grangier’s type experiments with detailed monitoring of dependence of the coincidence probability on the value of the threshold have never been done.
5. Conclusion
By following L. De Broglie we stressed the formal analogy between the operational descriptions of the processes of measurement of classical waves and quantum systems, including the wave collapse. Our analysis showed that the main experimental difference between measurements of classical and quantum waves is the absence of coincidence detections in the latter. Then we presented the classical wave model which reproduces the quantum detection probabilities (described by the Born’s rule) and at the same time the number of coincidence detections is relatively small (for sufficiently large detection threshold).

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[1] von Neumann J 1955 Mathematical Foundations of Quantum Mechanics (Princeton: Princeton Univ. Press)
[2] Khrennikov A (ed) 2001 Foundations of Probability and Physics. Quantum Probability and White Noise Analysis vol 13 (Singapore: WSP)
[3] Khrennikov A (ed) 2002 Quantum Theory: Reconsideration of Foundations. Ser. Math. Modelling vol 2 (Växjö: Växjö Univ. Press)
[4] Khrennikov A (ed) 2003 Quantum Theory: Reconsideration of Foundations-2. Ser. Math. Modelling vol 10 (Växjö: Växjö Univ. Press)
[5] Khrennikov A (ed) 2005 Foundations of Probability and Physics-3, vol 750
[6] Jaeger G, Khrennikov A, Schlosshauer M, Weihs G (eds) 2011 Advances in Quantum Theory vol 1327 (Melville, NY: AIP)
[7] Khrennikov A (ed) 2010 Quantum Theory: Reconsideration of Foundations-5 vol 1232 (Melville, NY: AIP)
[8] Khrennikov A 2002 On foundations of quantum theory. Quantum Theory: Reconsideration of Foundations. Ser. Math. Modelling vol 2 (Växjö: Växjö Univ. Press) pp 163-196
[9] Khrennikov A 2009 Contextual Approach to Quantum Formalism (Berlin-Heidelberg-New York: Springer)
[10] Hofer W 2011 Foundations of Physics 41 754
[11] Atmanspacher H and Primas H 2005 Epistemic and ontic quantum realities Foundations of Probability and Physics-3 vol 750 (Melville, NY: AIP) pp 49-62
[12] Gudder S 1970 J. Math. Physics 11 431
[13] Svozil K 2005 On counterfactuals and contextuality. Foundations of Probability and Physics-3 vol 750 (Melville, NY: AIP) pp 351-360
[14] Bell J 1987 Speakable and Unspeakable in Quantum Mechanics (Cambridge: Cambridge Univ. Press)
[15] Khrennikov A 2011 Quantum probabilities and violation of CHSH-inequality from classical random signals and threshold type properly calibrated detectors Preprint arxiv.org/abs/1111.1907
[16] Accardi L 2005 Some loopholes to save quantum nonlocality. Foundations of Probability and Physics-3 vol 750 (Melville, NY: AIP) pp 1-20
[17] Adenier G. 2008 Am. J. Phys. 76 147
[18] Hess K, Michielsen K and De Raedt H 2009 EPL 87 60007
[19] Allahverdyan A E, Balian R, and Nieuwenhuizen Th M 2005 The quantum measurement process in an exactly solvable model Foundations of Probability and Physics-3 vol 750 (Melville, NY: AIP) pp 16-24
[20] Allahverdyan A E, Balian R and Nieuwenhuizen Th M 2011 Understanding quantum measurement from the solution of dynamical models Preprint arxiv.org/abs/1107.2138
[21] Bohr N 1935 Phys. Rev. 48 696
[22] Bohr N 1987 The Philosophical Writings of Niels Bohr, 3 vols (Woodbridge, Conn.: Ox Bow Press)
[23] De Muynck W M 2002 Foundations of Quantum Mechanics, an Empiricists Approach (Dordrecht: Kluwer)
[24] D’Ariano G M and Tosini A 2010 Quant. Inf. Proc. 9 95
[25] D’Ariano G M 2010 Probabilistic theories: What is special about quantum mechanics? Philosophy of Quantum Information and Entanglement (Cambridge: Cambridge Univ. Press) pp 85-126
[26] Mehmami B and Nieuwenhuizen Th M 2010 An overview on single apparatus quantum measurements Preprint arxiv.org/abs/1007.3635
[27] de Broglie L and Andrade E Silva J (the author of a chapter) 1964 The Current Interpretation of Wave Mechanics. A Critical Study (Amsterdam-London-New York: Elsevier)
[28] de Broglie L 1960 Non-linear Wave Mechanics: A Causal Interpretation. (Amsterdam: Elsevier)
[29] Khrennikov A 2005 J. Phys. A 38 9051
[30] Khrennikov A 2006 Phys. Lett. A 357 171
[31] Khrennikov A 2008 Phys. Lett. A 372 6588
[32] Khrennikov A, Ohya M, and Watanabe N 2010 J. Russian Laser Research 31 462
[33] Khrennikov A 2010 J. Math. Phys. 51 082106
[34] Khrennikov A 2010 EPL 90 40004
[35] Khrennikov A 2010 Found. Phys. 40 1051
[36] Khrennikov A 2010 Physica E 42 287
[37] Khrennikov A, Nilsson B, and Nordebo S 2011 Classical signal model reproducing quantum probabilities for single and coincidence detections Preprint arxiv.org/abs/1112.5591
[38] Białynicki-Birula I 1994 Acta Phys. Polonica A 86 97
[39] Strocchi F 1996 Rev. Mod. Phys. 38 36
[40] Roychoudhuri Ch 2007 Shall we climb on the shoulders of the giants to extend the reality horizon of physics? Quantum Theory: Reconsideration of Foundations-4 vol 962 (Melville, NY: AIP) 195-205
[41] Grangier P 1986 Etude expérimentale de propriétés non-classiques de la lumière: interférence à un seul photon (Centre D’Orsay: Université de Paris-Sud)
[42] Grangier P, Roger G and Aspect A 1986 EPL 1 173
[43] Beck M (2007) J. Opt. Soc. Am. B 24 2972
[44] Landsman N P 2008 Compendium of Quantum Physics. Weinert F, Hentschel K, Greenberger D and Falkenburg B (eds) (Berlin: Springer)
[45] ’t Hooft G 2006 The mathematical basis for deterministic quantum mechanics (Preprint arXiv: quant-ph/0604008); do. 2007 J. Phys.: Conf. Series 67 012015
[46] ’t Hooft G 2011 Herald of Russian Acad. Sc. 81 907
[47] Born M 1926 Zeitschrift für Physik 37 863
[48] De la Pena L and Cetto A M 1996 The Quantum Dice: An Introduction to Stochastic Electrodynamics (Dordrecht: Kluwer)
[49] Einstein A and Infeld L 1961 Evolution of Physics: The Growth of Ideas from Early Concepts to Relativity and Quanta (New-York: Simon and Schuster)
[50] Mansuy R and Yor M 2006 Random Times and Enlargements of Filtrations in a Brownian Setting (Berlin: Springer)