Algorithms of walking and stability for an anthropomorphic robot

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Abstract. Autonomous movement of an anthropomorphic robot is considered as a superposition of a set of typical elements of movement - so-called patterns, each of which can be considered as an agent of some multi-agent system [1]. To control the AP-601 robot, an information and communication infrastructure has been created that represents some multi-agent system that allows the development of algorithms for individual patterns of moving and run them in the system as a set of independently executed and interacting agents. The algorithms of lateral movement of the anthropomorphic robot AP-601 series with active stability due to the stability pattern are presented.

Introduction. The problem is considered to realize the lateral step of an anthropomorphic robot AR-601 series produced by "Android Technics" with ensuring its stability [1, 6]. To manage the robot, an information and communication infrastructure is developed, which is a multi-agent system that allows running algorithms for executing individual moving patterns as independently functioning and interacting agents. Agents are located on one or more controllers and interact with the main processor of the robot through an Ethernet type network. In this paper, algorithms for lateral moving of an anthropomorphic robot AP-601 series with active resistance are presented. In this case, the mathematical models and algorithms of the robot step patterns in the side are separately considered and stability of the robot in its longitudinal symmetry plane is ensured. When performing the work, the experience of modeling in related fields of science is taken into account [7, 8].

Algorithm of stability. Consider the robot stability pattern in the longitudinal plane. Figure 1 shows the construction of the foot of AP-601 robot. The foot is connected to the shin by means of an electric drive, which sets the angle between them. A sole of the foot is attached to the foot using a spring. There are force sensors, which allow to obtain the values of the moments of forces that arise in the articulation of the foot with the shin. Assumed that the body of the robot, hip and shin are fixed relative to each other, stability is achieved by changing the inclination of the shin relative to the foot.
Fig. 1 Scheme of the node shin robot AR-601

In Figure 1, $\alpha$ - is the angle of rotation the shin to the foot, $\beta$ - is the angle of tilt of the foot relative to the sole (floor).
Then $\gamma = \alpha + \beta$ – tilt of the foot relative to the floor.
Let $M_g = l \cdot m \cdot g \cdot \sin M_g = l m g \sin (\gamma)$ – the torque created by the weight of the robot, $M_\beta = K_\beta \cdot \beta$ – the torque created by springs in the soles of the robot, $l$ –the distance from the foot to the center of mass of the robot, $g$ – acceleration of gravity, $mg$ – Weight of the robot, $K_\beta$ – coefficient of elasticity of the sole.
The equation of dynamics tilt of the robot is described by the following differential equation:
$I_m \gamma' = l m g \sin (\gamma) - K_\beta \beta$ (1)

Assuming the value of $\gamma$ to be small, replace $\sin \gamma$ by $\gamma$:
$I_m \gamma' = l m g \gamma - K_\beta \beta$ (2)

It is required to ensure the stability of the $\gamma$ angle.
Let the required dynamics be described by the equation:
$I_m \gamma' = K_1 \gamma + K_2 \gamma'$ (3)

$K_1$ and $K_2$ Coefficients of elasticity of the left and right soles of the robot.

To reduce the equation to the following form:
$K_1 \gamma + K_2 \gamma' = l m g \gamma - K_\beta \beta$ (4)

We deduce $\beta$ by $\alpha$: $\beta = \gamma - \alpha$
Then:
$K_1 \gamma - l m g \gamma + K_\beta \gamma + K_2 \gamma' = K_\beta \alpha$ (5)

$\alpha = \frac{K_1 + K_\beta - l m g}{K_\beta} \gamma + \frac{K_2}{K_\beta} \gamma'$ (6)

If $\alpha = \alpha (t)$ is given by the formula (6), then the dynamics of the robot oscillations on the feet will correspond to the expression (3). $K_1$ and $K_2$ Are determined from the desired dynamics,$l$, $m$ and $K_\beta$ are known.
The algorithm for controlling feet to ensure stability: from the moment sensors in the feet of the robot, the indicators $u, \gamma(t_i)$ at the moment $t_i$ are taken and the values $\gamma(t_i), \dot{\gamma}(t_i)$ are calculated. Calculates $\alpha = \alpha(t)$ according to the formula (6) and is sent for execution in the foot drive.

**Step algorithm.**
We consider the step of the robot in the side as a typical element of walking [1]. It is assumed that the rotational movement of the robot around the vertical axis, which occurs when the free leg is moved forward, is compensated by rotations of the robot body. This makes it possible to divide the motion in the longitudinal (sagittal) plane from motion in the transverse plane. Therefore, here we consider the dynamics of the lateral (in the transverse plane) movement of the robot that occurs when walking.

Step is carried out in the following order. The robot crouches on one knee, then the knee is bent and the robot is given an impulse, which results in movement on one leg according to the law of motion of the pendulum. If the center of mass does not intersect the vertical plane passing through the point of support of the foot, then after a while the robot will return to the initial position with support on both legs. The time for which the robot pushed off and stood on both feet is the step time. The task is to develop an algorithm for straightening the jogging leg, which provides a given time for the step.

The time of the step is determined by the speed of the center of mass of the robot acquired at the moment of tearing off the leg, because force influence on the robot at the moment of detachment of the jogging leg ends [1]. We believe that we can provide a horizontal position of the robot's torso. Consider the equations for calculating the distance from the fulcrum to the center of mass of the robot [2] (Figure 2).

![Fig. 2 The position of the center of mass of the robot](image)

$L$ – distance from the point of support of the foot to the center of mass of the robot,
$l$ – current length of the leg, from the fulcrum to the attachment of the leg to the thigh
$h$ – length from the point of attachment of the leg to the thigh to the center of mass of the robot, const;
$\theta$ – an angle between the center of mass of the robot and the initial position of the support leg, const;
$\gamma$ – current angle formed after the transfer of the leg.
$\alpha$ - angle between the new position of the foot and the center of mass.
$\alpha = \theta + \gamma$;

By the cosine theorem [5], we calculate the length $L$:

$$L^2 = h^2 + l^2 + 2hl\cos\alpha.$$  \hspace{1cm} (7)

Consider robot movements on one leg according to the law of a physical pendulum, inverted 180 degrees:

$$J\ddot{\phi} = mgL \cdot \sin(\varphi)$$

$m$ - robot mass,
$\varphi$ – Angle of tilt vertically,
g – acceleration of gravity (9.81 m/s²),
L - length from the point of support of the foot to the center of mass of the robot,
J – moment of inertia about the axis passing through the point of support, where

\[ J = J_0 + mL^2 \]

\( J_0 \) – Moment of inertia relative to the center of mass of the robot.

Let the angle \( \varphi \) remain small all the time (less than 10 degrees), we can replace it \( \varphi = \sin(\varphi) \).

Then the equation takes the following form:

\[ J\ddot{\varphi} - mgL\varphi = 0 \]  \( (8) \)

The solution of the differential equation:

\[ \varphi(t) = C_1e^{S_1t} + C_2e^{S_2t} \]  \( (9) \)

\[ \dot{\varphi}(t) = S_1C_1e^{S_1t} + S_2C_2e^{S_2t} \]  \( (10) \)

where,

\[ S_{1,2} = \pm \sqrt{\frac{mgL}{J}} \]  \( (11) \)

Under the initial conditions \( t_0 = 0, \varphi_0 = C_1 + C_2 \)

\[ C_1 = \varphi_0 - C_2 \]  \( (12) \)

\[ C_2 = \frac{\varphi_0 - S_2\varphi_0}{S_1-S_2} \]  \( (13) \)

By specifying \( \varphi_0, \dot{\varphi}_0 \) we calculate the behavior of the function.

We calculate the necessary time for the pendulum to reach the upper point \( (t_{max}) \), at zero initial velocity

\( (\varphi_0 = 0, \text{that} \varphi_{max} = 0). \) (Figure 3)

![Fig. 3 Time to reach "zone" 0”](image)

\( \varphi_{max} \) – angle of reaching the top position of the pendulum, at. \( t_{max} \).

We substitute in (10) the initial condition \( (\varphi_0 = 0): \)

\[ \varphi_0(t_{max}) = S_1C_1e^{S_1t} + S_2C_2e^{S_2t} = 0 \]

\[ S_1C_1e^{S_1t} = -S_2C_2e^{S_2t} \]

Proceeding from (11) \( S_1 = -S_2 = S: \)
\[ e^{St} = \sqrt{\frac{C_1}{C_2}} \]

Substituting formulas (12) and (13)

\[ e^{St} = \frac{\varphi_0 S - \varphi_0}{\sqrt{\varphi_0 S + \varphi_0}} \] (14)

\[ t = t_{\text{max}} \]

\[ t_{\text{max}} = \frac{1}{2S} \left( \ln(\varphi_0 S - \varphi_0) - \ln(\varphi_0 S + \varphi_0) \right) \] (15)

The time step \( T = 2t_{\text{max}}, \) substituting (14) in (9)

\[ \varphi_{\text{max}} = \sqrt{\frac{\varphi_0^2 - \varphi_0^2}{S^2}} \] (16)

Calculating the time and angle at which the physical pendulum and the center of mass of the robot are in the "zone "0" of the vertical plane, using equations (15) and (16), we find the maximum permissible initial velocity \( \varphi_{\text{max}} \) so that the robot does not fall and is in the "zone "0".

"Zone" 0" is a short-term equilibrium position on the supporting leg, when the leg is lifted from the floor. In this case, the robot is in the vertical position when the angle of rotation of the foot of the support leg is in the zero position, or in a position close to 0.

Set \( \varphi_0 \), it is necessary to find \( \varphi_0' \), that \( \varphi_{\text{max}} = 0 \)

It follows from (17):

\[ \varphi_{\text{max}} = -S\varphi_0 \] (17)

Thus, to consider the behavior of the robot on one leg, we regard it as a physical pendulum rotated 180 degrees, where the initial angle of deviation from the vertical \( \varphi_0 \) is known, \( h, J, m \), is calculated \( L \). We set this initial velocity \( \varphi_0 \) so that it is not higher then \( \varphi_{\text{max}} \).

Consider the pattern of repelling the leg from the support surface. The range of permissible speed is known \( 0 \leq \varphi_0 \leq \varphi_{\text{max}}, \forall \varphi_0 \). It is necessary to bend the knee of the jogging leg by a certain angle \( \beta \) to ensure \( \varphi_0' \) that it is at the given angle \( \varphi_0 \).

Fig. 4.1 the position of the legs after the first step.

Fig. 4.2 squatting on the jogging leg.

Fig. 4.3 squat the knee. Side view

\( l_c \) – Length of leg with squat knee,

\( l_1 \) – Length of the leg and thigh of the robot's leg,
α – Angle of bend in the plane Oxy,
β – Squatting angle of the knee in the plane Oxy,
γ - Angle of bend in the plane Oxy.

At the moment of repulsion, when straightening there will be a sharp slowing of the speed, because the weight of the carried body will be equal to the total weight of the robot. At the moment of repulsion from the reference surface the initial velocity should be equal $\varphi_0$, it is necessary to sit down at angle $\beta_0$.

The algorithm was tested on a robot model in the Gazebo simulator, also tested on an anthropomorphic robot AR-601, whose control by ROS. [4]

**Conclusion:** When starting the pattern of active stability, we can calculate such a $\beta$ (knee squatting angle in the Oxy plane) and the knee straightening speed so that, given the initial conditions and the robot's position, it was possible to bring the robot to "zone 0", for further controlled “fall” of the robot and exercising a walking robot.

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