Solving the nucleon spin puzzle
based on the chiral quark soliton model

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An incomparable feature of the chiral quark soliton model as compared with many other effective models like the MIT bag model is that it can give reasonable predictions not only for quark distributions but also for antiquark distributions. This will be exemplified by the argument on the positivity constraint for $\bar{u}(x) + \bar{d}(x)$ as well as the Soffer inequality for quark and antiquark distributions. We also explain how the model can resolve the so-called nucleon spin puzzle without assuming large gluon polarization at the low energy scale.

1. Introduction

Undoubtedly, the EMC measurement in 1988 and the NMC measurement in 1991 are two of the most striking findings in the recent experimental studies of nucleon structure functions [1, 2]. A prominent feature of the chiral quark soliton model (CQSM) is that it can simultaneously explain the above two big discoveries in no need of artificial fine-tuning [3, 4]. What is the chiral quark soliton model, then? First of all, it is a relativistic field theoretical model effectively incorporating the idea of large $N_c$ QCD [5]. For large enough $N_c$, a nucleon is thought to be a composite of $N_v$ valence quarks and infinitely many Dirac sea quarks bound by the self-consistent pion field of hedgehog shape. After canonically quantizing the spontaneous rotational motion of the symmetry breaking mean field configuration, we can perform nonperturbative evaluation of any nucleon observables with full inclusion of valence and Dirac sea quarks [6]. It is this incomparable feature of the model that enables us to make a reasonable estimation not only of quark distributions but also of antiquark ones, as we shall show later. Finally, but most importantly, only 1 parameter of the model was already fixed by low energy phenomenology, which means that we can give parameter-free predictions for parton distributions function at the low renormalization scale.

2. CQSM and twist-2 PDF

For obtaining quark distribution functions, we need to evaluate nucleon matrix elements of quark bilinear operators with light-cone separation. By using the path integral formulation of the CQSM, such nonlocality effects in time as well as spatial coordinates can be treated in a consistent manner [6, 7].

The following novel $N_c$ dependencies follow from the theoretical structure of the model, i.e. the mean-field approximation and the subsequent perturbative treatment of collective

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1Talk given at 14th International Spin Physics Symposium (Spin2000), Osaka, Japan, 16-21 Oct. 2000.
rotational motion \[ u(x) + d(x) \sim N_c \left[ O(\Omega^0) + 0 \right] \sim O(N_c^1), \]
\[ u(x) - d(x) \sim N_c \left[ 0 + O(\Omega^1) \right] \sim O(N_c^0), \]
\[ \Delta u(x) + \Delta d(x) \sim N_c \left[ O(\Omega^0) + O(\Omega^1) \right] \sim O(N_c^1 + O(N_c^0)). \]

Because of the peculiar spin-isospin correlation embedded in the hedgehog mean field, there is no leading-order \( N_c \) contribution to the isovector unpolarized distribution as well as to the isoscalar longitudinally polarized one, in contrast to the other combinations. This especially means that the isoscalar or flavor-singlet axial charge is parametrically smaller than the isovector one, in conformity with the EMC observation.

3. Numerical results

In Fig. 1, we summarize our parameter-free predictions for the twist-2 PDF at the model energy scale. We emphasize that seeds of all the success of the model are already contained in these four figures. Here, the functions in the negative \( x \) region should be interpreted as antiquark distributions according to the rule \[ u(-x) = -[\bar{u}(x) + \bar{d}(x)] \quad (0 < x < 1), \]
\[ \Delta u(-x) = \Delta \bar{u}(x) + \Delta \bar{d}(x) \quad (0 < x < 1). \]

The long-dashed curves peaked around \( x \approx 1/3 \) are the contributions of \( N_c \) valence quarks, while the dash-dotted curves represent those of Dirac-sea quarks. The sum of these two contributions are denoted by solid curves.

![Figure 1](image_url)

Figure 1: The theoretical predictions of the CQSM for the unpolarized distributions \( u(x) + d(x) \) and \( u(x) - d(x) \) as well as for the longitudinally polarized distributions \( \Delta u(x) + \Delta d(x) \) and \( \Delta u(x) - \Delta d(x) \).

The crucial importance of the Dirac-sea contribution is most clearly seen in the isoscalar unpolarized distribution. Here, the “valence-quark-only” approximation leads
to positive $u(x) + d(x)$ in the negative $x$ region, thereby violating the positivity of the antiquark distribution [1]. On the other hand, if we include the vacuum polarization of Dirac-sea quarks, the positivity constraint for the antiquark distributions holds properly. The effect of Dirac-sea quarks is very important also for the isovector unpolarized distribution function [2, 10]. Especially interesting here is the fact that $u(x) - d(x) > 0$ in the negative $x$ region, which means that $\bar{u}(x) - \bar{d}(x) < 0$ for the physical value of $x$, just as required by the NMC measurement. In fact, after taking account of the scale-dependence by means of the DGLAP equation, the theory turns out to successfully explain the NMC data for the unpolarized nucleon structure functions $F_2^p(x)$ and $F_2^n(x)$ [11].

Turning to the longitudinally polarized distributions, one observes very different $x$ dependencies between the isoscalar and isovector ones. One interesting feature of the isoscalar distribution is its sign change in the small $x$ region. It has been shown that this sign change is just what is required by the recent experimental data for the longitudinally polarized structure functions of the deuteron [11]. Turning to the isovector distribution, we notice that the effect of Dirac-sea quarks has a peak of positive sign around $x \simeq 0$. What is remarkable here is the positivity in the negative $x$ region. It means that anti-quark distributions are isospin asymmetric also for the longitudinally polarized distributions [11]. It is interesting to point out that some support is already given to this unique prediction of the CQSM by several semi-phenomenological and/or semi-theoretical analyses [12].

![Figure 2: The theoretical check of Soffer inequality. The distributions in the negative $x$ region denote the antiquark distributions.](image)

To complete the list of twist-2 PDF, we need another distribution function $\delta q(x)$, usually called the transversity distribution. It is known that this distribution function must satisfy the so-called Soffer inequality [13]:

$$|\pm \delta q(x)| \leq \frac{1}{2} (\pm q(x) + \Delta q(x)) \quad (x > 0, \ x < 0).$$  \hspace{1cm} (7)

Now the question is whether the predictions of the CQSM fulfill this inequality or not. Fig.2 show that, if one includes the vacuum polarization effects properly, the Soffer inequality is well satisfied for both of $u$-quark and $d$-quark. On the other hand, if one ignores
the Dirac-sea contributions, the Soffer inequality is badly broken for the antiquark distributions. An important lesson learned from this observation is that the field theoretical nature of the model, that is, the proper inclusion of the vacuum polarization effects, plays essential roles in giving reasonable predictions for antiquark distributions. Another lesson is that the frequently-used saturation Ansatz of the Soffer inequality for estimating $\delta q(x)$ is not justified.

Also noteworthy is another consequence of the soliton picture of the nucleon. Shown in Fig.3 are the spin and the orbital angular momentum distribution functions at the model energy scale [14]. One notices that the Dirac-sea contribution to the orbital angular momentum distribution function is sizably large and peaked around $x \approx 0$. Among others, large support in the negative $x$ region suggests that sizable amount of orbital angular momentum is carried by antiquarks. After integration over $x$, one also finds that only about 35% of the total nucleon spin comes from the quark spin, while the remaining 65% is due to the orbital angular momentum of quark and antiquarks [14, 3]. It is interesting to see that the dominance of the orbital angular momentum part over the intrinsic spin one is also indicated by the recent lattice QCD simulation [15].

![Figure 3](image)

Figure 3: (a) The theoretical predictions of the CQSM for the quark and antiquark orbital angular momentum distribution functions $q_L(x)$ and (b) the isosinglet quark polarization $\Delta u(x) + \Delta d(x)$. The curves have the same meaning as in Fig.1.

The spin and orbital angular momentum contents of the nucleon are of course scale-dependent quantities. We recall that, at the NLO with the gauge-invariant factorization scheme, $\Delta \Sigma$ has a weak scale dependence mainly at low $Q^2$. The theoretical value $\Delta \Sigma = 0.31$ obtained at $Q^2 = 10 \text{ GeV}^2$ is qualitatively consistent with the recent SMC result, $\Delta \Sigma_{SMC}^{exp} = 0.22 \pm 0.17$ [16].

4. Conclusion

In summary, an incomparable feature of the CQSM as compared with many other effective models like the MIT bag model is that it can give reasonable predictions also for the antiquark distribution functions as exemplified by the argument on the posititivity constraint for $\bar{u}(x) + \bar{d}(x)$ and also on the Soffer inequality for antiquarks. It has been emphasized that parton distribution functions evaluated at the model energy scale contain all the seeds of the success of the model in explaining existing experimental data given at the high energy scale. It naturally explains the excess of $\bar{d}$ sea over the $\bar{u}$ sea in the proton. The most puzzling observation, i.e. unexpectedly small quark spin fraction of the nucleon can also be explained in no need of large gluon polarization at the low energy scale.
As a further unique prediction of the model, we pointed out the possibility of large isospin asymmetry of the spin-dependent sea-quark distributions, which seems to be a natural consequence of the large $N_c$-counting rule, but appears inconsistent with the naive “meson cloud convolution model”. Then, if this large asymmetry of the longitudinally polarized sea is experimentally established, it would offer a strong evidence in favor of nontrivial spin-isospin correlation imbedded in the “large $N_c$ chiral soliton picture” of the nucleon.

The talk is based on the collaborations with T. Watabe and T. Kubota.

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