Combined Electroweak Analysis

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Abstract.
Recent developments in the measurement of precision electroweak measurements are summarised, notably new results on the mass of the top quark and mass and width of the W boson. Predictions of the Standard Model are compared to the experimental results which are used to constrain the input parameters of the Standard Model, in particular the mass of the Higgs boson. The agreement between measurements and expectations from theory is discussed.

1. Introduction
On the level of realistic observables such as measured cross sections, ratios and asymmetries, the set of electroweak precision data consists of over thousand measurements with partially correlated statistical and systematic uncertainties. This large set of results is reduced to a more manageable set of 17 precision results, so-called pseudo observables, in a largely model-independent procedure, by the LEP and Tevatron Electroweak Working Groups. The pseudo observables updated for this conference are summarised. Using in addition external constraints on the hadronic vacuum polarisation at the Z pole and “constants” such as the Fermi constant $G_F$, analyses within the framework of the Standard Model (SM) are performed [1].

2. Measurements
More than 3/4 of all pseudo observables arise from measurements performed in electron-positron collisions at the Z resonance, by the SLD experiment and the LEP experiments ALEPH, DELPHI, L3 and OPAL. The Z-pole observables are: 5 observables describing the Z lineshape and leptonic forward-backward asymmetries, 2 observables describing polarised leptonic asymmetries measured by SLD with polarised beams and at LEP exploiting tau polarisation, 6 observables describing $b$- and $c$-quark production at the Z pole, and finally the inclusive hadronic charge asymmetry. The Z-pole results and their combinations are final and published since last year [2]. The remaining pseudo observables are: the mass and total width of the W boson measured by CDF and DØ at the Tevatron and by the four LEP-II experiments, and the top-quark mass measured by the Tevatron experiments only.

The heavy-flavour results at the Z-pole were the last precision electroweak Z-pole results to become final; details on the various measurements are given in [2]. The combination of these measurements has a rather low $\chi^2$ of 53 for (105 – 14) degrees of freedom: all forward-backward asymmetries are very consistent, and their combination is still statistics limited. The combined values for $A_{tb}^{0,b}$ and $A_{tb}^{0,c}$ are compared to the SM expectation in Figure 1 (left), showing that they agree well with the SM expectation for a medium Higgs-boson mass of a few hundred GeV.
Assuming the SM structure of the effective coupling constants, the measurements of the various asymmetries are compared in terms of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ in Figure 1 (right). The average of all $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ determinations is $\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23153 \pm 0.00016$, with a $\chi^2$/dof of 11.8/5, corresponding to a probability of 3.7%. The enlarged $\chi^2$/dof is solely driven by the two most precise determinations of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$, namely those derived from the measurements of $A_G$ by SLD, dominated by the left-right asymmetry result, and of $A_0^{\text{b}}$ at LEP, preferring a low and high Higgs-boson mass, respectively. The two measurements differ by 3.2 standard deviations.

In 1995 the Tevatron experiments CDF and D0 discovered the top quark in proton-antiproton collisions at 1.8 TeV centre-of-mass energy, by observing the reaction $p\bar{p} \rightarrow t\bar{t} \, X$, $t\bar{t} \rightarrow b\bar{b}W^+W^-$. The results on the mass of the top quark presented at this conference [3], based on data collected during Run-I (1992-1996) and the ongoing Run-II (since 2001) are combined by the Tevatron Electroweak Working [4]: $M_t = 170.9\pm 1.1 \, (\text{stat.}) \pm 1.5 \, (\text{syst.}) \, \text{GeV}$, corresponding to an overall precision of 1.1%.

Final results on the mass and width of the W boson from CDF and D0 are available for the complete Run-I data set. First results based on the Run-II data set are available for $\Gamma_W$ from D0 and, recently, for $M_W$ from CDF, with combined results of $M_W = 80.429 \pm 0.039 \, \text{GeV}$ and $\Gamma_W = 2.078 \pm 0.087 \, \text{GeV}$ [5]. The results on $M_W$ and $\Gamma_W$ from the LEP-2 experiments ALEPH, DELPHI, L3 and OPAL are all final. However, the LEP combined estimation of colour-reconnection effects based on dedicated studies, used in limiting that uncertainty in the LEP $M_W$ combination, is still preliminary, and thus is the LEP combination of $M_W$ and $\Gamma_W$: $M_W = 80.376 \pm 0.033 \, \text{GeV}$ and $\Gamma_W = 2.196 \pm 0.083 \, \text{GeV}$ [6]. The LEP and Tevatron results are in good agreement; the new preliminary world-average values are: $M_W = 80.398 \pm 0.025 \, \text{GeV}$ and $\Gamma_W = 2.140 \pm 0.060 \, \text{GeV}$. Within the SM, these $M_W$ results points to a low Higgs-boson mass, as shown in Figure 2 (left), in contrast to $A_0^{\text{b}}$.

3. Combined Electroweak Analysis

Within the framework of the SM, each pseudo observable is calculated as a function of five relevant input parameters: the running electromagnetic and strong coupling constants evaluated at the Z pole, $\alpha_{\text{em}}$ and $\alpha_s$, and the masses of Z boson, top quark and Higgs boson, $M_Z$, $M_t$, $M_H$. Using the Fermi constant $G_F$ allows to calculate the mass of the W boson. The running electromagnetic coupling is represented by the hadronic vacuum polarisation $\Delta\alpha^{(5)}_{\text{had}}$; as it is this contribution which has the largest uncertainty, $\Delta\alpha^{(5)}_{\text{had}} = 0.02758 \pm 0.00035$ [7]. The dependence on $M_t$ and $M_H$ enters through electroweak loop corrections. The predictions are calculated with the computer programs TOPAZ0 and ZFITTER, which incorporate state-of-the-art calculations of radiative corrections [8].

Using the Z-pole measurements of SLD and LEP-I, electroweak radiative corrections are evaluated allowing to predict the masses of top quark and W boson. The resulting 68% C.L. contour curve in the $(M_t, M_W)$ plane is shown in Figure 2 (right). Also shown is the contour curve corresponding to the direct measurements of both quantities at the Tevatron and at LEP-II. The two contours overlap, successfully testing the SM at the level of electroweak radiative corrections. The diagonal band in Figure 2 (right) shows the constraint between the two masses within the SM, which depends on the unknown mass of the Higgs boson, and to a small extent also on the hadronic vacuum polarisation (small arrow labeled $\Delta\alpha$). Both the direct and the indirect contour curves prefer a low value for the mass of the SM Higgs boson.

The best constraint on $M_H$ is obtained by analysing all data. This global fit has a $\chi^2$ of 18.2 for 13 degrees of freedom, corresponding to a probability of 15.1%. The pulls of the 18 measurements fitted are shown in Figure 3 (left). The single largest contribution to the $\chi^2$ arises from the $A_0^{\text{b}}$ measurement discussed above, with a pull of 2.9. The fit yields $M_H = 76_{-29}^{+34} \, \text{GeV}$, a 37% constraint on $M_H$, which corresponds to a one-sided 95% C.L. upper limit on $M_H$ of
144 GeV including the theory uncertainty, as shown in Figure 3 (right). This limit increases to 182 GeV when including the LEP-2 direct-search limit of 114.4 GeV [9] in the analysis.

The fitted $M_H$ is strongly correlated with the fitted hadronic vacuum polarisation (correlation of $-0.54$) and the fitted top-quark mass ($+0.39$). The strong correlation with $M_t$ implies a shift of 15% in $M_H$ if the measured $M_t$ changes by 2 GeV. Thus a precise measurement of $M_t$ is very important. Also shown are the $\chi^2$ curves obtained with the more precise but theory-driven evaluation of $\Delta \alpha^{(5)}_{\text{had}}$ [10], yielding a correlation of only $-0.2$ with $M_H$, or including the results obtained in low-$Q^2$ interactions: atomic parity violation [11], Moller scattering [12], and NuTeV’s measurement of deep-inelastic lepton-nucleon scattering [13]; with the two former measurements in agreement with the expectations but the latter differing by 3 standard deviations. Both analyses yield nearly the same upper limits on $M_H$.

The theoretical uncertainty on the SM calculations of the observables is shown as the thickness of the blue band. It is dominated by the uncertainty in the calculation of the effective electroweak mixing angle, where a completed two-loop calculation is needed. The shaded part in Figure 3 (right) shows the $M_H$ range up to 114.4 GeV excluded by the direct search for the Higgs boson at 95% confidence level [9]. Even though the minimum of the $\chi^2$ curve lies in the excluded region, the uncertainties on the fitted Higgs mass are such as that the results are well compatible.

4. Conclusions and Outlook

Over the last 2 decades many experiments have performed a wealth of measurements with unprecedented precision in high-energy particle physics. These measurements test all aspects of the SM of particle physics, and many of them show large sensitivity to electroweak radiative corrections, and point to a light SM Higgs boson. Most measurements agree well with the expectations as calculated within the framework of the SM, successfully testing the SM at Born and loop level. There are two “3 standard deviations effects”, namely the spread in the various determinations of the effective electroweak mixing angle, within the SM analysis disfavouring the measurement of $A^{\text{on-shell}}_{\text{fb}}$, and NuTeV’s result, most pronounced when interpreted in terms of the on-shell electroweak mixing angle. For the future, precise theoretical calculations including theoretical uncertainties are needed, in particular a completed two-loop calculation for the effective electroweak mixing angle and a NLO reanalysis of the NuTeV measurement. Experimentally, the next few years will bring further improvements in the measurements of $W$-boson and top-quark masses, allowing to constrain $M_H$ to 28% (Tevatron/LHC) and even 16% (ILC/GigaZ). Of course, the discovery of the Higgs boson is eagerly awaited, with a measurement of its mass to sub-GeV precision and of other properties.

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Figure 1. Left: Contour curves in the \((A_{fb}^0, A_{fc}^0)\) plane. The SM expectations are shown as the arrows for \(M_t = 170.9 \pm 1.8\) GeV, \(M_H = 300^{+700}_{-186}\) GeV and \(\Delta\alpha_{\text{had}}^{(5)} = 0.02758 \pm 0.0035\). Right: The effective electroweak mixing angle from asymmetry measurements.

Figure 2. Left: Contour curves of 68\% C.L. in the \((M_W, \Gamma_W)\) plane. Right: Contour curves of 68\% C.L. in the \((M_t, M_W)\) plane for the direct measurements and the indirect determinations. The band shows the correlation between \(M_W\) and \(M_t\) expected in the SM.

Figure 3. Left: Pulls of the measurements used in the global SM analysis. Right: \(\Delta \chi^2\) curve as a function of \(M_H\). Also shown are the curves using a theory-driven evaluation of \(\Delta\alpha_{\text{had}}^{(5)}\) or including the low-\(Q^2\) measurements in the analysis.