Field Surveys and Numerical Modeling of Pumiceous Debris Flows in Amalfi Coast (Italy)

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The first objective of the work is to test a cost-effective tool for the collection of debris flows (DF) field data such as volumes, peak flow depths and deposit depths. Secondly, we show how these data can be used for the calibration of a depth-averaged propagation model. The case study is a DF of pumiceous sediments, occurred in the Amalfi Coast (Southern Italy) in October 2013. The DF path is a steep channel, ending in a small debris fan delimited by a gabion wall. The risk is high because DFs, having a return period of just few years, overtop the wall and hit a busy road. Both terrestrial laser scanner (TLS) and photogrammetric techniques were employed to survey the topography, before and after the event under study. The images of the channel were taken from an unmanned aerial vehicle (UAV). Digital terrain models (DTM) were obtained pre and post event while the traces left by the DF along the channel banks allowed the estimation of the peak flow depths. A finite volume two-dimensional numerical code (FLATModel), based on shallow-water equations, was used for modelling the propagation and deposition of the DF under study. Both Voellmy and pure Coulomb friction resistance laws were tested. The numerically predicted deposit was compared to the post event DTM. Such comparisons showed a good agreement in terms of both depths and shape of deposit. The calibrated model could be used to predict the DFs run-out distances in similar contexts.

Key words: terrestrial laser scanner, photogrammetry, FLATModel, pyroclastic debris flows

1. INTRODUCTION

Geophysical granular flows, such as debris flows (DF) or rock avalanches, represent dangerous phenomena for humankind and infrastructures in mountainous areas. Though in the last decades big efforts have been made for improving the mathematical description of the propagation stages of these flows [e.g., Laigle and Coussot, 1997; Iverson and Denlinger, 2001; Fracarollo and Papa, 2000; Sarno et al., 2011; Sarno et al., 2014], the dynamics of these complex phenomena is still not completely understood and the mathematical models have often a small predictive effectiveness if they are calibrated only on laboratory data.

One of the limiting factors for the progress in the research on DF phenomena is the poor database of field measurements [e.g., Jakob et al., 2005; Takahashi, 2007]. In order to overcome these difficulties, it is necessary to develop cost-effective tools, capable of extensively measuring the different features of real debris flows.

The present work shows the suitability of modern survey techniques, such as terrestrial laser scanner (TLS) and photogrammetry, for calibrating and validating depth-averaged mathematical models.

Campanian Apennine mountains (Italy) and, in particular, Lattari Mountains in Amalfi Coast are often concerned by debris flows [e.g., Cascini et al., 2008; Martino and Papa, 2008; Papa et al., 2011] involving the rapid flow of pyroclastic material originated from the ancient eruptions of Vesuvius volcano. The related risk is huge, due to the high frequency of these phenomena and the dense urbanization of the involved areas. The peculiar nature of the pyroclastic material [Basile et al., 2003] and the often important amount of lightweight pumices significantly influence the propagation mechanisms of Campanian debris flows and, thus, require particular attention. The case study concerns a small basin in Amalfi Coast, often affected by pumiceous debris flows. The high frequency of DF events and the good accessibility of the basin makes it an ideal site to systematically monitor such phenomena in a natural setting.

The work is composed of three parts. Firstly,
the chosen site under study and the debris flow occurred in October 2013 are described. In the second part, the employed survey techniques are described together with the DTMs derivation. In the third part, the outcomes of the surveys are employed to simulate the DF event by the means of a two-dimensional depth-averaged model, FLATModel [Medina et al., 2008].

2. CASE STUDY

The site under study (Fig. 1) is a small basin located in the municipality of Tramonti in Amalfi Coast, often concerned by pumiceous debris flows. The catchment intersects the Provincial Road SP1, approximately at coordinates 40.690910° N, 14.613087° E. A gabion wall that caused the formation of a deposit zone upstream of it delimits the road. Nevertheless, since the deposit area is very small and is rarely emptied, some debris flows hit the road in recent years. A picture of the deposit area, taken from the road soon after a big event occurred in September 2014, is reported in Fig. 2. This event is currently under study and, therefore, is not dealt with in the present paper.

The basin has an overall extent of about 5 ha, with an average altitude of 780 m a.s.l. The DFs run along a steep gully having an average slope of about 36°.

The soils covering the carbonate bedrock of the Amalfi Coast basins are made of ashes, pumice and scoriae, coming from the eruptive activity of the Somma-Vesuvius volcano (eruptions immediately before, and after the 79 AD, until the last one in 1944). Layers of pumice with thicknesses up to a few meters are often present.

The heavy rainfalls of autumn and spring periodically mobilize the pyroclastic soils, and particularly the lightweight pumice layers, causing shallow landslides that typically develop into DFs [Papa et al., 2013].

On the 10th October 2013, a cumulative rainfall of about 60 mm in 4 hours with a maximum intensity of about 50 mm/h (rain data were obtained from the closest rain gauge located in the town centre of Tramonti) provoked the failure of about 150 m³ of pumices. Such a failure gave place to a channelized DF that finally deposited upstream the gabion wall. In this event, the gabion wall was not overtopped by the flow. The surveys conducted before and after the event of October 2013 allowed for a numerical reconstruction of the DF, as showed in the following sections.

3. SURVEYS

In this section, we report the details of the surveys carried out in the site under study.

In particular, two TLS surveys of the debris fan were conducted: the first one before the studied event on 27/06/2013, the second one after the event on 18/10/2013. A picture of the gabion wall and the fan, taken on 18/10/2013 during the second survey, is reported in Fig. 3. Moreover, an extra photogrammetry survey of the whole catchment, including the channel upstream the debris fan, was carried out on 20/12/2013 by using an unmanned aerial vehicle (UAV), in order to obtain the digital terrain model (DTM) of the whole basin.

3.1 TLS survey

The instrument used for the TLS surveys is the TOPCON GLS-1500, provided by the company 3D TARGET. The survey project took into account the technical features of the instrumentation, the complexity of the site and the hindrances due to the instability of the pumiceous sediments. During the first survey, performed on 27/06/2013, three scan stations were employed: two of them were at the lateral extremities of the debris fan, while the third one was at the opposite part of the roadway, in order to suitably acquire the upper part of the
debris fan. In the second survey, a fourth scan station was added above the gabion wall. The scan stations are reported in Fig. 4. In both cases, in the post-elaboration phase, the scan resolution was set to 8 mm (at the distance of 10 m), to satisfy the precision requirements of the particular application. In order to export the data, the free software ReCap of Autodesk, which is a tool for the 3D elaboration from laser scanner data, was used.

The raw data were subsequently processed by using the reverse engineering software Geomagic Studio 2012, that allows managing point clouds from 3D scans, to align various scans and to generate the DTM. For our purposes, some operations, both manual and automatic, were required. In particular, we point out the elimination of the points characterized by excessive noise, due to the presence of vegetation. A further reduction of the points was performed in the overlapping zones. The latter task led to a more homogeneous point cloud and allowed for a remarkable data reduction.

Then, we performed a registration-alignment procedure for the final DTM creation. It consists in setting the reference system of a given scan as reference for the other scans. In order to achieve this goal, a manual registration function was implemented in two steps: selecting the scan no. 3 (Fig. 4) as fixed reference scan and, then, identifying at least 3 pairs of homologous points for obtaining the desired alignment. At the end of the above-mentioned operations, we obtained a DTM of about 5.000.000 triangles for each survey.

After obtaining the pre and post event DTMs of the fan through the TLS surveys, it was possible to align the two DTMs with respect to a same reference system (Fig. 5). Such alignment, which is necessary to assess the volume of mobilized material [Barba and Fiorillo, 2012], is performed by taking into account only natural targets (emerging rocks, gabion wall elements, etc.).

3.2 UAV photogrammetry survey

In addition to the surveys of the debris fan, we also performed a survey of the whole area, affected by the DF event (from the detachment area to the accumulation zone), through using a UAV equipped with Sony CX 260 camera of 9 Mpixel, supplied by the company TEDIDRONE (Fig. 6). It was also performed a terrestrial photogrammetric survey with the acquisition of 72 photos from the retention dam but, given the high instability of pumiceous sediments in the catchment, the employment of a drone has been preferred to a terrestrial survey. For the success of the photogrammetric project, a key role was played by the overlap of the camera shots at least by 60% [Barba, 2008; Cardone, 2013]. The overall project consists of 166 images, obtained in time-lapse mode with a one-second-interval between two subsequent camera shots. The acquired images
Fig. 6 The UAV equipment supplied by the company TEDIDRONE

were processed by the software Agisoft Photoscan (in the last licenced version, 1.0.4), that enables to obtain high-resolution orthophotos and texturized DTM [Fiorillo et al., 2012; Guidi et al., 2010]. The software employs an image-based 3D modelling algorithm capable of determining the camera position for each shot and, therefore, of aligning pictures and generating DTM without resorting to calibrated lens or particular shot conditions. The generated DTM, together with the spatial reconstruction of the camera positions and the related position of the frame and axis, is shown in Fig. 7. As one can see from Fig. 7, during the acquisition, the axis of the camera was kept horizontal, as far as possible, by using a gimbal connecting the camera to the UAV. The survey allowed for documenting the shape of the cross sections in the channel upstream the debris fan and the local slopes. This information has been used to estimate the peak flow of the hydrograph, used as input condition of the numerical simulations described in the next section.

In order to validate the output results of this approach, the photogrammetric DTM was overlapped on the fan mesh model obtained through the TLS survey of 18/10/2014. Then the deviation of the spatial coordinates of homologous points on the two surfaces was evaluated. The analysis of the propagation of the errors returned data with good accuracy, with a maximum value of the deviation less than 10 cm.

4. NUMERICAL MODELLING

The information obtained from the TLS surveys was fruitfully exploited to numerically reconstruct the DF event of October 2013 with the two-dimensional single-phase depth-averaged mathematical model, FLATModel [Medina et al., 2008]. Firstly, the pre- and post-event DTM of the accumulation area (debris fan), obtained as explained in Sect. 3, were converted into a structured rectangular mesh. In order to do so, a linear interpolation of the original unstructured mesh was carried out. The volume of deposit, estimated by simply comparing the elevation of the pre- and post-event DTM, is found to be equal to 146 m$^3$. In some small areas upon the gabion wall, the noise due to vegetation causes some errors in the estimation of the deposit, which are found altogether smaller than of 3 m$^3$. Therefore, the overall accuracy of the estimated volume of deposit is expected to be of the same order.

Let $(x, y, z)$ be a Cartesian frame of reference with the plane $xy$ being horizontal and $z$ pointing upwards. The model equations of FLATModel, which are derived from depth-averaging the three-dimensional conservation laws of a monophasic fluid [Medina et al., 2008; Ciervo et al., 2014], can be written in the following conservative
\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \left( h u + g \frac{h^2}{2} \right) + \frac{\partial}{\partial y} \left( h v + g \frac{h^2}{2} \right) = 0
\]

\[
h \left( g \tan \alpha_s - S_p \right)
\]

form where \( h \) represents the vertical flow depth, \( u \) and \( v \) are the \( x \)- and \( y \)-component of the flow velocity vector respectively, \( \alpha_s \) and \( \alpha_c \) are the inclination angles of the bottom surface in \( x \) and \( y \) direction, \( S_P \) and \( S_P \) are the basal resistances in the \( x \) and \( y \) directions, respectively. \( g_p = g \cos \theta + V^2/r_c \) is a corrected gravity acceleration, where \( g \) is the natural gravity acceleration, \( \theta \) is the angle between the flow velocity vector and the horizontal, \( V \) is the modulus of the flow velocity vector and \( r_c \) is the local curvature radius of the basal topography, calculated along the velocity direction.

Such a gravity correction takes into account slope and curvature variations of the topography. FLATModel incorporates the most popular basal resistance laws (e.g. Bingham, Herschel-Bulkley, frictional and Voellmy models) for describing mud and debris flows. The partial differential equation system, (1), is numerically integrated in FLATModel through a finite-volume numerical scheme, based on the Harten-Lax-Leer-Contact approximate Riemann solver [Toro et al., 1994]. The spatial domain of the numerical problem is chosen to completely contain the debris fan, the gabion wall and some part of the road facing the gabions. All the numerical simulations were performed by using the pre-event DTM as basal topography. The mesh size is set equal to \( \Delta x = \Delta y = 0.1 \) m for all runs.

The deposit distributions predicted by the numerical simulations are, then, compared with the field measurements resulting from post-event DTM.

### 4.1 Boundary conditions and input hydrograph

A no-flow boundary condition is set along the whole boundary of the spatial domain, with the exception of a narrow line approximately 2 m-long in the channelized area of the upstream boundary. Along this line, the upstream boundary condition is set as an in-flow condition, where a triangular hydrograph is assumed. For the sake of simplicity, we assumed that the DF event is composed of only one surge. The volume of the input hydrograph is set equal to the measured volume of deposit, 146 m³. The peak flow rate of the hydrograph is estimated by assuming that the critical state occurs in the cross section, located in the upstream channel, where it was possible to estimate the peak flow depth thanks to the post-event survey. Such a flow depth was measured equal to 85 cm with an uncertainty of roughly ±0.15 m.

Thanks to the geometric information provided by the DTM, obtained through the UAV survey, it was possible to reconstruct the geometry of the cross section there and, consequently, to calculate the corresponding critical flow rate, which is found to be equal to 5.15 m³/s. The time length of the hydrograph turns out to be 56.7s. Hereinafter, a sensitivity analysis on the critical flow depth, let vary within its accuracy range, is provided to support the robustness of our approach.

### 4.2 Choice of the resistance law

Since the grain size of the involved pumiceous material is very coarse, we excluded that a non-Newtonian rheology, such as Bingham and Herschel-Bulkley models, could properly describe the event under study. In fact, these rheological models are notorious for not being capable to properly capture the steep slopes of deposit in granular flows.

Differently, because the main dissipation mechanisms are due to friction and collisions among grains in a typical granular flow behaviour such as the flow under study, we chose the Voellmy model [Voellmy, 1955; Bartelt et al., 1999] to calculate the basal resistances. Several back-analyses on granular debris flows and granular avalanches showed that this model is often suitable to describe such events [e.g., Hungr and Evans, 1996; Rickenmann and Koch, 1997; Hürlimann et al., 2008].

The Voellmy model assumes that the basal shear stress, \( \tau \), can be regarded as the sum of two terms, a rate-independent friction term and a collisional turbulent-like term,

\[
\tau = \rho g h \cos \alpha \tan \phi + \frac{Dg}{C_s^2} V^2
\]

Where \( \alpha \) is the inclination angle of the basal surface, \( \phi \) the friction angle of the granular mixture, \( \rho \) is the bulk density and \( C_s \) is a Chezy-like conveyance coefficient of dimensions \([L^{1/2}/T]\). For \( C_s \to \infty \) the Voellmy formula, Eq. (2), simply degenerates into a Coulomb frictional resistance law. Nonetheless, using the PDE system, (1), amounts to assume an hydrostatic normal pressure distribution along the flowing pile, differently from the Savage-Hutter type.
models which employ a lateral pressure coefficient to better describe the Coulomb frictional behaviour of the flowing mass [e.g., Savage and Hutter, 1989; Hungr, 1995; Iverson and Denlinger, 2001; Sarno et al., 2013].

The angle of repose of the granular material, which can be regarded as an estimate of its static friction angle, was found to be around 35.5° by means of laboratory tests, performed at the LIDAM (Environmental and Maritime Hydraulics Laboratory) and consisting in the direct measurement of the steepest angle of descent of stable piles of material, quasi-statically amassed on an horizontal surface. The dynamic angle of friction of the mixture of pumiceous material and water, to be used in Eq. (2), is expected to be rather smaller than the angle of static friction. Therefore, the optimal value of $\phi$, together with the optimal value of the conveyance coefficient, $C_z$, is found through a back-analysis of the measured shape of deposit in the event under study. Wide ranges of friction angles, [25°, 35.5°], and of the coefficient, $C_z$, [10,100] m$^{1/3}$/s$^{-1}$, were systematically investigated. The deposit, numerically predicted by each simulation, is compared with the measured deposit from the post-event DTM.

### 4.3 Comparisons with field data

The following $L_1$-type error index [Sarno et al., 2013] is employed

$$E = \frac{\sum_{i=1}^{N} |h_{\text{sim},i} - h_{\text{measured},i}|}{\sum_{i=1}^{N} h_{\text{measured},i}}$$  \(3\)

where the summations are over the total number of cells $N$ in the domain, $h_{\text{sim},i}$ and $h_{\text{measured},i}$ represent the predicted and measured depth of deposit at the $i$-th cell, respectively.

The error index, $E$, is used to quantitatively assess the degree of agreement between the numerical simulation and the field measurements. The optimal agreement, corresponding to $E = 0.236$, is found by using $\phi = 28.5^\circ$ and $C_z = 30$ m$^{1/3}$/s. The distribution of flow depths at the deposit, predicted by the numerical code is reported in Fig. 8, together with the distribution of the absolute error,

$$\mu_i = |h_{\text{sim},i} - h_{\text{measured},i}|.$$  \(4\)

As one can see from Fig. 8, the agreement is very good. The average absolute error is 0.079 m and the maximum error is equal to 1.00 m. It is interesting to note that the numerical model nicely describes the observed shape of deposit and correctly predicts that almost no amount of granular material overtops the gabion wall.

As shown in Fig. 8, the main discrepancies between the numerical simulation and the field measurements lie on the right side of the computational domain close to the gabion wall. This phenomenon might be due to the no-flow boundary condition there, which might have induced higher unrealistic deposit depths. In a further study, the boundary condition there could be improved or a broader basal topography should be used. The discrepancies on the debris fan are, conversely, typically smaller than 0.50 m.

The reconstruction of the event was performed also under the hypothesis of a Coulomb frictional resistance law. In this case, a back-analysis was performed on the sole parameter, $\phi$. Several values of $\phi$, below the limiting value of the static angle of
friction, were investigated. The optimal agreement between the numerical simulation and the field data is obtained by using \( \phi = 32^\circ \) with an error index \( E = 0.299 \) and an average absolute error equal to 1.10 m. It is interesting to note that the optimal value of \( \phi \) is only a few degrees below the static angle of friction, as expected for the dynamic angle of friction [e.g., Hungr and Morgenstern, 1984]. The numerical predicted deposit, together with the absolute error, \( \mu \), is reported in Fig. 9.

Fig. 9 (a) depths of the deposit predicted by the frictional model (\( \phi = 32^\circ \)) on the pre-event DTM (background); (b) absolute errors between the numerical simulation and the measured deposit depths

| ID | \( h \) [m] | \( Q_{\text{max}} \) [m\(^3\)/s] | \( E \) | \( \mu \) [m] |
|----|------------|----------------|---|--------|
| 1  | 0.70       | 3.46           | 0.257 | 0.086 |
| 2  | 1.00       | 7.21           | 0.235 | 0.079 |

4.4 Sensitivity analysis on the input hydrograph

As reported, some uncertainties affect the measurement of the peak flow depth in the transportation channel. Such uncertainties affect also the estimation of the peak flow rate and, thus, of the input hydrograph. In order to assess the effects of such errors on the numerical simulations, we performed a sensitivity analysis on this quantity. In particular, we run different simulations by varying the peak flow depth, \( h \), within its uncertainty range, [0.70 m, 1.00 m]. The peak flow rate of the input hydrograph, \( Q_{\text{max}} \), calculated by imposing the occurrence of critical conditions, is found to be within the following range [3.46 m\(^3\)/s, 7.21 m\(^3\)/s]. Analogously to the previous numerical simulations, the time length of the hydrograph is calculated so that the total input volume is equal to the measured one. Such numerical simulations were carried out by using the Voellmy basal resistance law with the parameters found through the previous back-analysis: \( \phi = 28.5^\circ \) and \( C_z = 30 \text{ m}^{1/2}/\text{s} \). The error index, \( E \), and the averaged absolute error, \( \mu \), at the deposit, for the limiting cases investigated are reported in Table 1.

Although the flow dynamics of these simulations is obviously different due to different flow rates and time lengths of the hydrographs, it is noteworthy that the distribution of depths at the deposit are very similar and all of them exhibit very small discrepancies with respect to the measured deposit profile.

5. CONCLUSIONS

The three-dimensional surveys conducted before and after a pumice debris flow in Amalfi Coast provided relevant data, useful for the numerical simulation of the event and the calibration of the model parameters. Both terrestrial laser scanner (TLS) and photogrammetric techniques, based on the processing of images acquired by an ordinary digital camera, were employed to survey the topography of the debris flow channel and the debris fan. It resulted that the resolution of the photogrammetry, though it is noticeably lower than
the one of TLS, is enough for the purposes of debris flow modelling. Given also the reduced costs, the photogrammetry can be efficiently used for the survey of debris flow sites.

The digital terrain models of the site, before and after the event, were derived and used for assessing the basal topography of a numerical model, for the estimation of the amount of deposited volume and for the comparison of the numerically simulated deposit with the observed one.

The numerical simulations were performed by employing a single-phase depth-averaged model (FLATModel) based on the shallow-water equations and with a special treatment of slope and curvature variations in mountainous topographies. A back-analysis of the event under study showed that the pumiceous debris flows seem to exhibit a typical granular flow behaviour and, thus, can be successfully described by using a frictional-collisional resistance law of the Voellmy type or, even more minimalistically, a Coulomb type frictional resistance law. The results showed the good capacity of the model to describe the phenomenon and the geometry of the deposit. The depths of deposit are nicely reproduced with an average error lower than 10 cm. A further validation of the model will be performed by simulating other surveyed DF events in similar basins. The proposed model could be used, with pre-calibrated values of the input parameters, to make predictions of debris flow events involving similar mixtures.

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