Orthogonal Array Testing for Transmit Precoding based Codebooks in Space Shift Keying Systems

Mohammed Al-Ansi1,*, Syed Alwee Aljunid1, Essam Sourour2, Anuar Mat Safar1, and C B M Rashidi1
1School of Computer and Communication Engineering, University Malaysia Perlis (UniMAP), Perlis, Malaysia.
2Department of Electrical Engineering, College of Engineering, Prince Sattam Bin Abdulaziz University, 11991, Wadi Addwasir, Saudi Arabia.

E-mail: ma.ahmed@studentmail.unimap.edu.my

Abstract. In Space Shift Keying (SSK) systems, transmit precoding based codebook approaches have been proposed to improve the performance in limited feedback channels. The receiver performs an exhaustive search in a predefined Full-Combination (FC) codebook to select the optimal codeword that maximizes the Minimum Euclidean Distance (MED) between the received constellations. This research aims to reduce the codebook size with the purpose of minimizing the selection time and the number of feedback bits. Therefore, we propose to construct the codebooks based on Orthogonal Array Testing (OAT) methods due to their powerful inherent properties. These methods allow to acquire a short codebook where the codewords are sufficient to cover almost all the possible effects included in the FC codebook. Numerical results show the effectiveness of the proposed OAT codebooks in terms of the system performance and complexity.

1. Introduction
Spatial Modulation (SM) is a hybrid Multiple Input Multiple Output (MIMO) and digital modulation techniques proposed as a potential technology to satisfy both requirements of low complexity and Energy Efficiency (EE) in the next 5G communication systems [1, 2]. In each instant of SM transmission, the conventional Amplitude and Phase Modulation (APM) constellation symbol is transmitted through a single Transmit Antenna (TA) while the other TAs are idle where the index of active TA is specified randomly according to the user data. Therefore, the first part of the user data is mapped into the conventional APM constellation symbol while the second part is mapped to a spatial constellation. One special form of SM is the Space Shift Keying (SSK) where the user information bits are mapped only to the spatial constellation and unmodulated data, such as +1 symbol is transmitted through the active TA. This elimination of constellation symbol in SSK decreases the data rate but it reduces the detection complexity in the receiver side. SM/SSK methods are then generalized to increase the Spectrum Efficiency (SE) through activating more than one antenna at a time as in [3] and [4].

The novel idea of SM/SSK systems offers several benefits in comparison to the conventional MIMO. One of them is the low receiver complexity since the optimal Maximum Likelihood (ML) detector is applied at a single-stream decoding complexity. SM systems also consume less power because they require single Radio Frequency (RF) Chain which is one of the most consuming power in the transceiver.
Moreover, the inter channel interference is avoided and no need for antenna synchronization due to transmit from single TA at each transmission time [5].

One limitation of SM/SSK methods is the lack of diversity gain, thus gets low Bit Error Rate (BER) system performance due to activating a less number of TA’s. Transmit Precoding (TPC) techniques are recently proposed to weight the channel matrix and accordingly improve the system BER performance provided that the instantaneous Channel State Information (CSI) is known. However, the challenge is how to find the best precoder that maximize the Minimum Euclidean Distance (MED) between the received constellation points since the BER is predominated by the MED [6]. The precoder is employed to construct a TPC diagonal matrix which basically consists of two matrixes, real numbers represent the Power Allocation Precoding (PAP) matrix and a complex numbers represent the Phase Rotation Precoding (PRP) matrix. In PAP, the values are used to scale the magnitude of the channels while the phase can be scaled through the PRP matrix.

In the literature, the authors of [7, 8] proposed iterative algorithms with the aim of maximizing the MED through joint design of PAP and PRP matrixes and consequently get minimum BER. Other iterative schemes are introduced in [9, 10] based on designing a PAP matrix, while the TPC matrix constructed in [11, 12] is designed to change the phase of the TAs through PRP matrix. These iterative methods provide a global minimum of BER, since they designed to match the instantaneous CSI. However, they cannot be considered in practice due to their high complexity, especially for the high number of TAs. Further, full knowledge of CSI is essential to apply the algorithms and this necessity an infinite feedback channel.

On the other hand, most of the restrictions found in the iterative algorithms can be tackled when TPC based codebook precoding is applied. In codebook precoding, the CSI is known only to the receiver and a codebook is predefined to both sides of the transceiver. Given the instantaneous CSI, the receiver selects one of codewords in the codebook that satisfies maximizing the MED and then fed back the codeword index to the transmitter. In [13], the researchers apply the TPC based codebook precoding and noteworthy performance was achieved. However, the selected codeword is not exactly matching the instantaneous CSI and hence the MED is not globally minimized. Accordingly, the performance is less than what accomplished in the case of iterative schemes. One solution proposed for this problem is to design the codebook through matching the CSI on the average as in [14]. Though, this method requires infinite rate of feedback channel where the transmitter should be updated with the whole codebook whenever the CSI changes.

In this research, we consider constructing a Full-Combinations (FC) codebook with systematic structure based PRP precoding [15]. Given the CSI to the receiver, it tests all the combinations in FC codebook and selects the most desirable codeword that satisfy the selection criteria to maximize the MED. Then, the index of this codeword is fed back to the transmitter in order to weight the channel matrix. However, this exhaustive search increase the time of the whole precoding process and it becomes more challenging, particularly in the case of high number of TAs. Moreover, we noticed that there are several combinations (i.e. codewords) can satisfy the maximum MED criteria. Our initial objective of this work is to design a codebook contains only the most significant and concrete codewords which have the capacity to cover all the possible phase changes in order to precode the TAs. Thus, this research proposes to take advantage of the well-known Orthogonal Array Testing (OAT) methods for constructing the TPC codebooks. OAT properties allow designing codebooks with very short codebook length and consequently attain a considerable reduction of time during the selection process. Furthermore, these OAT codebooks have the capability to provide near to the high BER performance obtained by FC codebooks.

The remainder of this paper is organized as follows: in section 2, SSK system model and FC codebook precoding are introduced. The proposed OAT codebook is explained in section 3. Numerical results are presented in section 4. Finally, section 5 concludes the paper.
2. SSK System Model with Codebook Precoding

Consider SSK system with \( N_t \) antennas at the transmitter and \( N_r \) antennas at the receiver as shown in Figure 1. In each transmission instant, a length of \( \log_2(N_r) \) bits is taken from the user data bits and mapped to one of the spatial constellations to get the \((N_r \times 1)\) transmission vector \( x_k \) given by:

\[
x_k = [0, \ldots, 1, \ldots, 0]^T
\]

this indicates that only the \( k^{th} \) transmit antenna is active. By applying the TPC Codebook predefined for both sides of the transceiver, the \( N_c \times N_r \) codebook \( C \) is comprised of \( N_c \) codewords as

\[
C = [p_0, p_1, \ldots, p_b, \ldots, p_{N_c-1}]^T
\]

Each codeword \( p_b \) in the codebook contains \( N_c \) elements, where \( p_b = [p_{b,0}, p_{b,1}, \ldots, p_{b,N_t-1}]^T \), for \( 0 \leq b \leq N_c - 1 \).

In the receiver, the complex received signal can be expressed as:

\[
r = H P x_k + w
\]

where \( H = [h_0, h_1, \ldots, h_{N_r-1}] \) is the \((N_r \times N_t)\) complex channel matrix known to the receiver only with entries are assumed to be identical and independently distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. The precoding matrix \( P \) is formed in a diagonal structure to satisfy the SSK requirement of single TA active at a time as \( P = \text{diag}(p_b) \). The \((N_r \times 1)\) noise vector \( w \) is the white noise whose elements are i.i.d. complex Gaussian random variables with zero mean and variance \( 1/\gamma \), where \( \gamma \) is the Signal to Noise Ratio (SNR) per receive antenna.

Among all the codewords in the codebook \( C \), the codeword \( p_b \) is selected as the best one to maximize the MED between the received constellation points where the upper bound BER of SSK is dominated by the MED [6]. Therefore, the precoding codeword \( p_b \) is selected as follows:

\[
p_b = \arg\max_{p_b \in \mathbb{C}^{N_c \times 1}} \left( \min_{i \neq j} \left\| p_k, h_i - p_k, h_j \right\|^2 \right)
\]

then, the receiver feeds-back the index \( b \) of this codeword to the transmitter using \( N_b \) bits.

2.1. Full-Combinations (FC) Codebook

Full-Combination (FC) Codebook contains all the combinations cover the possible \( L \) phase changes for \( N_t \) antennas. Each combination represents one codeword and its elements can be obtained through \( p_{b,j} = \exp\left( j \frac{2\pi l_{b,j}}{L} \right) \). This allows the FC codebook to have a systematic structure. We assume the first column elements are equal to one (i.e. \( p_{b,0} = 1 \)) which means no phase change, so the weight of the first antenna is not scaled and becomes as a reference. Then, the other elements in each codeword
for the remaining antennas are obtained through the conversion of the codeword index \( b \) into a Base- \( L \) number with \( N_t - 1 \) digits. Hence, the codebook length is \( N_c = L^{(N_t - 1)} \) codewords, each with \( N_t \) elements.

All the \( N_t \) codewords go through the selection criteria (i.e. eq. (4)) at the receiver to find the best one provides the maximum MED. Then, the index of this codeword is fed back to the transmitter and the number of feedback bits is \( \lceil N_f b \rangle = (N_t - 1) \lfloor \log_L L \rfloor \). Therefore, FC codebook is considered as a reference since it covers all the \( L \) possible phase changes to weight the channel matrix and it can guarantee reaching the minimum BER performance achieved by any codebook based PRP. Even though, the exhaustive search among all combinations becomes more challenging when \( N_t \) is high. Moreover, we noticed from the numerical simulation that there are several combinations provide the same maximum MED and the first one tested is being selected. This motivates us to look for a low complexity method or TPC codebook maintain less BER with acceptable complexity.

### 3. Orthogonal Array Testing (OAT) Codebook

Orthogonal Array Testing matrix (OAT) is a statistical method mainly used for experimental testing in industrial engineering. It’s useful to avoid the difficulty of testing all possible combinations of experimental factors when each factor has various possible levels [16]. The Methodology of the OAT is based on the selection of a concise and effective set from the full-combinations such that this set is enough to consider all effects of each factor [35]. This selection guarantees to test all the pairwise combinations in the set. Typically, OAT is described with the notation \( O_{\text{Runs}} (\text{Levels}^{\text{Factors}}) \), where \( \text{Runs} \), \( \text{Levels} \), and \( \text{Factors} \) are integers. The parameter \( \text{Runs} \) is the total number of chosen combinations to test (number of rows in the matrix). The parameter \( \text{Factors} \) is the number of factors to test in each combination (number of columns in the matrix). Hence, the matrix size is \( (\text{Runs} \times \text{Factors}) \) with entries from 0 to \( \text{Levels} - 1 \), where the \( \text{Levels} \) is the number of levels to test for each factor. For efficient testing, the integer \( \text{Runs} \) typically satisfies \( \text{Runs} \ll \text{Levels}^{\text{Factors}} \). More details about the description of the properties and implementation of OAT are available in [16-19].

#### 3.1. OAT Codebook Design

While OAT is invented for experimental testing of an industrial design with sufficient test coverage, it found applications in communication systems [20]. In this paper, the concept of the OAT is utilized to design the precoding codebook where the integers \( \text{Factors}, \text{Levels}, \text{Runs} \), represent the number of TAs \( (N_t) \), the number of phase rotations \( (L) \), and the number of codewords in the codebook \( (N_c) \), respectively. Therefore, the OAT codebook dimension is \( (N_c \times N_t) \). The codebook entries are integers \( l_{b,j} \) which take values from 0 to \( L - 1 \). Similar to the FC codebook, these entries are converted into the \( L \) phasors \( p_{b,j} \).

As an illustration, the OAT codebook \( O_8 (2^4) \) is shown in Table 1 for \( N_t = 4 \) and \( L = 2 \). This codebook involves eight codewords to test the effect of four factors and two levels for each factor. However, the same situation requires 16 codewords in the FC codebook. Constructing OAT codebook for certain factors and levels can be done by several schemes, but the simplest way is to select the appropriate table from the available libraries [17, 21] and in the case if it is not available in the library, it can be designed with taking into consideration the rules and properties of the OATs.
Table 1. OAT Codebook $O_B(2^4)$

| Antenna1 | Antenna2 | Antenna3 | Antenna4 |
|----------|----------|----------|----------|
| -1       | -1       | -1       | -1       |
| -1       | -1       | -1       | 1        |
| -1       | 1        | 1        | -1       |
| -1       | 1        | 1        | 1        |
| 1        | -1       | 1        | -1       |
| 1        | -1       | 1        | 1        |
| 1        | 1        | -1       | -1       |
| 1        | 1        | 1        | 1        |

The size of OAT codebooks implemented in this paper is the typical size $N_c = L \times N_r$. Hence, the number of feedback bits is $N_{fb} = \log_2 L + \log_2 N_r$, which is much smaller than the FC codebook for large $N_r$. The construction of the OAT codebooks for $L = 2$ with $N_i = 8$ and $N_r = 16$ is based on the tables available in [22]. The codebooks for $L = 4$ with $N_i = 8$ and $N_r = 16$ are based on the tables available in [23] and [24] respectively.

4. Numerical Results

This section presents the BER performance of SSK system using the proposed OAT codebook precoding. Then, we compare between the OAT, FC codebook [15] and DFT codebook [13]. Perfect channel estimation is assumed at the receiver and finite feedback channel is applied in the system. Figure 2 shows the SSK BER performance with $N_i = 8$ and $N_r = 2$. It’s observed that almost 4 dB gain at BER of $10^{-3}$ is achieved applying OAT codebook with $L = 2$ phases compared to the conventional SSK without precoding. Additional of 1 dB gained when $L = 4$ phase changes are employed. Besides, this figure shows the capability of OAT codebooks to provide a close BER to the FC codebook [15]. From the system complexity side, the OAT codebook length $N_c = 16$ and $N_c = 32$ codewords for $L = 2$ and 4 phases, respectively. While the FC codebooks contain $N_c = 128$ and $N_c = 16384$ codewords for $L = 2$ and 4 phases, respectively.

![Figure 2. BER performance comparison between OAT and FC codebooks for SSK systems with $N_i = 8$ and $N_r = 2$. Solid lines indicate employing $L = 2$ phases, while dashed lines indicate $L = 4$ phases.](image-url)
Tables 2 and 3 illustrate the considerable gain accomplished by reducing the codebook size where the precoding time and the number of feedback bits are decreased when OAT codebook implemented. For example, OAT codebook contains 64 codewords and 6 bits are required to feed-back the index for $N_t = 16$ and $L = 4$ phases. In contrast, FC codebook comprises more than $10^9$ codewords and needs more than 29 feedback bits for the same number of TA and phases.

Table 2. Comparisons between the length of OAT and FC Codebooks for different cases of $N_t$ with $L = 2$ phases.

| Codebook Type | $N_t = 8$ | $N_t = 16$ | $N_t = 32$ |
|---------------|-----------|-----------|-----------|
| FC            | 128       | 32768     | 2.14E+9   |
| OAT           | 16        | 32        | 64        |

Table 3. Comparisons between the length of OAT and FC Codebooks for different cases of $N_t$ with $L = 4$ phases.

| Codebook Type | $N_t = 8$ | $N_t = 16$ | $N_t = 32$ |
|---------------|-----------|-----------|-----------|
| FC            | 16384     | 1.07E+9   | 4.6E+18   |
| OAT           | 32        | 64        | 128       |

Figure 3 compares the OAT codebook to the related method in the literature DFT codebook [13] with $N_t = 16$ and $N_r = 2$. It shows that applying the OAT codebook with $L = 4$ phases provides lower BER than the case of codebook generated using DFT matrix.

Figure 3. BER performance comparison between OAT and DFT codebook precoding for SSK systems with $N_t = 16$ and $N_r = 2$. Solid lines indicate employing $L = 2$ phases, while dashed lines indicate $L = 4$ phases.

5. Conclusions

This research proposed Orthogonal Array Testing (OAT) methods to construct a codebook based Phase Rotation Precoding (PRP) to weight the transmit channel vectors of SSK systems. In the Full-Combinations (FC) codebook, all possible combinations of the phase changes should be verified to catch the optimal codeword that maximizes the MED between the received constellation points. However, this extensive search becomes more challenging, particularly in the case of high number of transmit antennas. In contrast, the proposed OAT methods demonstrate that it has a robust inherent properties
allow to design a codebook contains less number of combinations. These combinations are effective and enough to cover all needed effects of the reference FC codebook. Thus, the numerical results show the great reduction achieved when OAT codebook applied. Accordingly, the implementation time is considerably minimized and consequently the number of feedback bits is reduced.

References
[1] E. Basar, Index modulation techniques for 5G wireless networks, IEEE Commun. Mag., 54, pp. 168-175, (2016).
[2] C. X. Wang, F. Haider, X. Gao, X. H. You, Y. Yang, D. Yuan, et al., Cellular architecture and key technologies for 5G wireless communication networks, IEEE Commun. Mag., 52, pp. 122-130, (2014).
[3] J. Jeganathan and A. Ghrayeb, Generalized space shift keying modulation for MIMO channels, presented at the Proc. IEEE 19th PIMRC’08, Cannes, (Sep. 2008).
[4] A. Younis, N. Serafimovski, R. Mesleh, and H. Haas, Generalised spatial modulation, in Proc. IEEE Signals Syst. Comput., Nov. 2010, pp. 1498-1502.
[5] R. Mesleh, H. Haas, S. Sinanovic, C. Wook Ahn, and S. Yun, Spatial modulation, IEEE Trans. Veh. Tech., 57, pp. 2228-2241, (Jul. 2008).
[6] J. Jeganathan, A. Ghrayeb, L. Szczecinski, and A. Ceron, Space shift keying modulation for MIMO channels, Trans. Wireless Commun., 8, pp. 3692-3703, (Jul. 2009).
[7] M. C. Lee, W. H. Chung, and T. S. Lee, Generalized precoder design formulation and iterative algorithm for spatial modulation in MIMO systems with CSIT, IEEE Trans. Commun., 63, pp. 1230-1244, (Apr. 2015).
[8] P. Yang, Y. L. Guan, Y. Xiao, M. D. Renzo, S. Li, and L. Hanzo, Transmit precoded spatial modulation: Maximizing the minimum Euclidean distance versus minimizing the bit error ratio, IEEE Trans. Wireless Commun., 15, pp. 2054-2068, (Mar. 2016).
[9] P. Yang, Y. Xiao, S. Li, and L. Hanzo, A Low-Complexity Power Allocation Algorithm for Multiple-Input Multiple-Output Spatial Modulation System, IEEE Trans. Veh. Technol., 65, pp. 1819-1825, (2016).
[10] P. Yang, Y. Xiao, B. Zhang, S. Li, M. El-Hajjar, and L. Hanzo, Power allocation-aided spatial modulation for limited-feedback MIMO systems, IEEE Trans. Veh. Techn., 64, pp. 2198-2204, (May 2015).
[11] C. Masouros, Improving the diversity of spatial modulation in MISO channels by phase alignment, IEEE Commun. Lett., 18, pp. 729-732, (May 2014).
[12] P. Yang, Y. Xiao, B. Zhang, M. El-Hajjar, S. Li, and L. Hanzo, Phase rotation-based precoding for spatial modulation systems, IET Commun., 9, pp. 1315-1323, (Feb. 2015).
[13] M. C. Lee, W. H. Chung, and T. S. Lee, Precoder design for space shift keying in MIMO systems with limited feedback, in Proc. IEEE 19th PIMRC’08, Sep. 2014, pp. 6-10.
[14] M. C. Lee, W. H. Chung, and T. S. Lee, Limited feedback precoder design for spatial modulation in MIMO systems, IEEE Commun. Lett., 19, pp. 1909-1912, (Nov. 2015).
[15] M. Al-Ansi, S. A. Aljunid, and E. Sourour, Phase Rotation Codebook Precoding for Space Shift Keying MIMO Systems, in Mobile and Wireless Technologies 2017: ICMWT 2017, K. J. Kim and N. Joukov, Eds., ed Singapore: Springer Singapore, (2018), pp. 94-101.
[16] A. Hedayat, N. Sloane, and J. Stufken, Orthogonal Arrays: Theory and Applications. New York, USA: Springer, (Jul. 1999).
[17] W. F. Kuhfeld. (2009). SAS Technical Support Notes. N. J. A. Sloane. (2005). A Library of Orthogonal Arrays.
[18] C. Weng, F. Yang, and A. Z. Elsherbeni, Linear antenna array synthesis using Taguchi's method: A novel optimization technique in electromagnetics, IEEE Trans. Ant. and Propag., 55, pp. 723-730, (Mar. 2007).
[19] C. Ying and L. Derong, Multiuser detection using the Taguchi method for DS-CDMA systems, IEEE Trans. Wireless Commun., 4, pp. 1594-1607, (July 2005).
[20] W. F. Kuhfeld. TS-723 Orthogonal Arrays.
[21] W. F. Kuhfeld. (2009). A library of Strength-Two Orthogonal arrays. Available: N. J. A. Sloane. (2005). A Library of Orthogonal Arrays with 64 Runs and 21 Factors.
[22] N. J. A. Sloane. (2005). A Library of Orthogonal Arrays with 32 Runs and 9 Factors.