Resonant relaxation near a massive black hole: the dependence on eccentricity

M. Atakan Gürkan1⋆ and Clovis Hopman2⋆

1Astronomical Institute ‘Anton Pannekoek’, University of Amsterdam, Kruislaan 403, 1098 SJ Amsterdam, the Netherlands
2Leiden Observatory, Leiden University, PO Box 9513, NL-2300 RA Leiden, the Netherlands

Accepted 2007 May 14. Received 2007 May 13; in original form 2007 April 20

Abstract

The orbits of stars close to a massive black hole (MBH) are nearly Keplerian ellipses. Such orbits exert long-term torques on each other, which lead to an enhanced angular momentum relaxation known as resonant relaxation. Under certain conditions, this process can modify the angular momentum distribution and affect the interaction rates of the stars with the MBH more efficiently than non-resonant relaxation. The torque on an orbit exerted by the cluster depends on the eccentricity of the orbit. In this paper, we calculate this dependence and determine the resonant relaxation time-scale as a function of eccentricity. In particular, we show that the component of the torque that changes the magnitude of the angular momentum is linearly proportional to eccentricity, so resonant relaxation is much more efficient on eccentric orbits than on circular orbits.

Key words: black hole physics – stellar dynamics – celestial mechanics – Galaxy: centre – Galaxy: kinematics and dynamics.

1 INTRODUCTION

It is now commonly accepted that many galaxies contain massive black holes (MBHs) at their centres (e.g. Gebhardt et al. 2003; Miller 2006), with masses $10^6 \gtrsim M_\bullet \gtrsim 10^9 M_\odot$. The proximity of our own Galactic Centre, at a distance of $7.62 \pm 0.32$ kpc (Eisenhauer et al. 2005), allows for the astrometric study of individual stellar orbits in the potential well of the MBH (e.g. Schödel et al. 2002; Ghez et al. 2003; Eisenhauer et al. 2005). These observations showed that the Galactic MBH mass is $M_\bullet = (3.61 \pm 0.32) \times 10^6 M_\odot$ (Eisenhauer et al. 2005). The presence of very dense stellar clusters near MBHs leads to a wide variety of interesting phenomena. For a comprehensive review of stellar phenomena in the Galactic Centre, see Alexander (2005). Many of these phenomena arise because the two-body relaxation time is shorter than the age of the systems, so that the orbits of stars are significantly redistributed during their lifetimes. For example, the relaxation time in the Galactic Centre is estimated to be several Gyr (Alexander 1999; Alexander & Hopman 2003).

A common assumption in stellar dynamics is that the mechanism through which stars exchange angular momentum and energy is dominated by uncorrelated two-body interactions (e.g. Chandrasekhar 1943). The orbits of the stars are largely determined by the potential of the smoothed density and the deviations from this potential caused by the individual stars lead to perturbations that evolve the orbits (Hénon 1973). Systems with MBHs at their centres cannot (yet) be studied by direct $N$-body integrations using a realistic number of particles. Hence, the methods used for studying the dynamics of these systems rely on the above assumptions. Such studies include Monte Carlo simulations (e.g. Shapiro & Marchant 1978; Freitag 2001; Freitag & Benz 2002; Freitag 2003; Freitag, Amaro-Seoane & Kalogera 2006) and Fokker–Planck methods (e.g. Bahcall & Wolf 1976, 1977; Murphy, Cohn & Durisen 1991; Hopman & Alexander 2006b).

Near an MBH, the potential is nearly Keplerian, which leads to closed elliptic orbits. There are two main reasons for deviations from Keplerian orbits: the contribution to the potential from the stars and general relativistic effects. However, for most orbits where the potential is dominated by the MBH, the time-scale for precession is large, and the orbits remain nearly stationary with respect to each other over many periods. Consequently, the assumption that the interactions between the stars are independent is not valid, as shown by Rauch & Tremaine (1996). They argued that a better description is given when the interactions are considered to be between different orbits, rather than between different point particles. Averaged over many periods, one can think of the mass of a star being smoothly distributed over the orbit, with the linear density in a small segment proportional to the time the star spends in this segment. The orbits then form massive ‘wires’. Rauch & Tremaine (1996) showed that the torques between the wires lead, under some circumstances, to very efficient angular momentum relaxation.

Part of the interest in efficient angular momentum relaxation is due to interesting phenomena which occur when stars have very

⋆E-mail: ato@science.uva.nl (MAG); clovis@strw.leidenuniv.nl (CH)
close interactions with MBHs when they are on highly eccentric orbits. Such phenomena include the tidal disruption of stars (e.g. Frank & Rees 1976; Lightman & Shapiro 1977; Rees 1988), and the emission of gravitational waves with detectable frequencies (e.g. Sigurdsson 1997; Freitag 2003; Hopman & Alexander 2005, 2006a).

Since resonant relaxation increases the rate of angular momentum scattering, stars reach highly eccentric orbits more rapidly. Rauch & Ingalls (1998) studied the consequences for the tidal disruption rate, while Hopman & Alexander (2006a) considered the enhancement of the rate at which gravitational wave sources spiral in. Resonant relaxation appears to be more important for the latter case, since it occurs closer to the MBH, where deviations from a 1/r potential are smaller.

Resonant relaxation affects both the magnitude and the direction of angular momenta. For processes that require orbits of high eccentricity, only the part which affects the overall magnitude of the angular momentum is of interest. Rauch & Tremaine (1996) estimate the torques of a stellar cluster on a test star, and deduce the relaxation rate from this. They do not consider the dependence of this process on the eccentricity. This is potentially important, as can be seen from the following example.

Consider a star in a circular orbit in the x-y plane, and a mass at a general point \( p = (x, y, z) \). In this case, the angular momentum of the star is in the z-direction, \( \mathbf{J} = J \hat{z} \). In the approximation that the orbit does not change, the torque \( \tau \) that the mass at \( p \) exerts on the star would have no z-component since the contributions from one-half of the orbit will cancel the contributions from the other half, \( \tau = (\tau_x, \tau_y, 0) \). Since \( \tau \cdot \mathbf{J} = 0 \), the mass at \( p \) can rotate the orbit, but never affect the magnitude of the angular momentum, and since \( p \) was a general point, this is true for all other points as well. We can therefore conclude that resonant relaxation can never modify the eccentricity of an \( e = 0 \) orbit.

For any orbit with \( e > 0 \), resonant relaxation will change the eccentricity. In this paper, we calculate how the efficiency of resonant relaxation depends on the eccentricity of the orbit of a given star.

2 RESONANT RELAXATION

In a Keplerian orbit there is a 1:1 resonance between the angular and radial frequency and hence the orientation of the orbit is fixed in space. Because of the general relativistic effects, all orbits around an MBH actually precess and are never exactly Keplerian; in addition, the contribution to the potential from the star cluster around the MBH will also lead to precession. However, for nearly Keplerian systems, the orientation of the orbit with respect to other orbits can be assumed to be fixed in space over some time \( t_{\text{RR}} \gg P \), where

\[
P(a) = 2\pi \left( \frac{a^3}{GM^*} \right)^{1/2}
\]

is the period of the star.

Over a time-scale \( P < t \ll t_{\text{RR}} \), the stars can be represented as massive wires (Rauch & Tremaine 1996) with the mass smeared out over their orbits. These wires exert mutual torques on each other. The magnitude of the torque exerted by a star of mass \( M^* \) and the semimajor axis \( a \) on another star with equal semimajor axis is estimated by

\[
\tau_1 \sim \frac{GM^*}{a}.
\]

For a large number \( N \) of stars in the region near\(^2\) \( a \), the sum of the torques will nearly average out to zero. However, because of statistical fluctuations, there will be an excess torque in an unknown direction of the order of \( \tau_{\text{ex}} \sim \sqrt{N} \tau_1 \). If the orbit of the test star lies in the x-y plane, so that its angular momentum is in the \( \hat{z} \)-direction, only the \( z \)-component of the torque can affect the magnitude of the angular momentum (or the eccentricity) of the star. The resulting angular momentum changes were called scalar resonant relaxation by Rauch & Tremaine (1996). If the eccentricity of a test star is \( e \), then the typical net torque in the \( \hat{z} \)-direction will be

\[
\tau_z(a, e) = \beta_1(e) \sqrt{N} \frac{GM^*}{a}.
\]

where \( \beta_1(e) \) is a dimensionless function of eccentricity.

In Rauch & Tremaine (1996) and later papers, the eccentricity dependence of \( \beta_1 \) was ignored. Rauch & Tremaine (1996) performed \( N \)-body simulations to find the eccentricity-averaged value of \( \beta_1(e) \), and found that for an isotropic cluster with eccentricity distribution \( N_{\text{iso}}(e) = 2e \text{de} \),

\[
\beta_1 = 0.53 \pm 0.06 \quad \text{(Rauch & Tremaine 1996, table 4d).}
\]

However, from the discussion in the Introduction section, it follows that \( \beta_1(0) = 0 \).

After a time \( t_{\text{RR}} \), the orientation of the test star with respect to the ambient cluster changes. This can happen either because its orbit has precessed, or because the orbits of most of the stars around it have precessed. Over this time, the angular momentum would change by

\[
\Delta J_a = J_{t_{\text{RR}}} = \beta_1(e) \sqrt{N} \frac{GM^*}{a} \tau_{\text{RR}}(a, e).
\]

For \( t > t_{\text{RR}} \), the torques on a particular star-wire become random, and the change in angular momentum grows as a random walk. The resonant relaxation time \( t_{\text{RR}} \) is defined as the time it takes for a star to have its angular momentum changed by an amount of the circular angular momentum

\[
J_a(a) = \sqrt{GM^*a}.
\]

Since it takes \( (J_a/\Delta J_a)^2 \) random steps to make this excursion in angular momentum space, and each step takes a time \( t_{\text{RR}} \), the resonant relaxation time is given by

\[
t_{\text{RR}}(a, e) = \frac{\left( \frac{J_a}{\Delta J_a} \right)^2}{t_{\text{RR}}} t_{\text{RR}} = \left[ \frac{1}{2\pi \beta_1(e)} \right]^2 \left( \frac{M^*}{M_\odot} \right)^2 \frac{1}{N(a)} t_{\text{RR}}(a, e).
\]

In the following section, we describe a method to find the torque on a wire which we use to determine the eccentricity dependence of \( \beta_1 \). We note that from equation (7) it can be seen that since \( \beta_1(e) \to 0 \) for \( e \to 0 \), scalar resonant relaxation becomes very ineffective for nearly circular orbits.

3 THE WIRE APPROXIMATION FOR TORQUE COMPUTATION

For our computations, we use a simple model that describes the Galactic Centre. We assume \( M_\odot = 3.6 \times 10^8 M_\odot \) and \( M^* = M_\odot \) for the mass of the MBH and the mass of each star, respectively. The radius of influence of the MBH, where the mass in stars is equal to

\(^2\) In Section 3, we show that the maximal distance at which stars still have a large contribution to the torque is 2a.
\( M_\star \), is \( r_0 = 2 \text{ pc} \), and within this distance from the MBH there is a cusp of stars,
\[
N_{\text{cusp}}(<a) = N_0 \left( \frac{a}{r_0} \right)^{3-\alpha},
\]
where \( N_0 = M_\star/M_\odot \) is the number of stars within the radius of influence \( r_0 \), and \( \alpha \) is the slope of the number density profile, for which we adopt the value \( \alpha = 1.4 \) (Alexander 1999; Genzel et al. 2003; Alexander 2005).

In order to compute the torque from a cluster of stars on a given test star efficiently, we make use of the ‘wire approximation’ suggested by Rauch & Tremaine (1996). We consider a test star in an orbit of some given initial eccentricity \( e \), and the semimajor axis \( a_0 = 0.01 \text{ pc} \) in the \( x\)-\( y \) plane, with the orbit’s peri-apse on the positive \( x \)-axis. It is surrounded by a cluster of field stars whose eccentricities are drawn randomly from an isotropic distribution \( N_{\text{iso}}(e) \) (see equation 4) and semimajor axes from the distribution given in equation (8).

We truncate the semimajor axes of the field stars at \( 5_a = 0.05 \text{ pc} \), giving \( N = 10\,000 \) stars. The orbits for the field stars start in a configuration similar to that of the test star and undergo a number of rotations: they are first rotated around the \( z \)-axis by an angle \( \phi \), drawn from a uniform distribution in \([0, 2\pi]\); then rotated around their latus rectum by an angle \( \theta \), cosine of which is drawn from a uniform distribution in \([-1, 1]\); and finally rotated around their major axis by an angle \( \gamma \), drawn from a uniform distribution in \([0, 2\pi]\). For a point starting from \((x, y, 0)\) the final coordinates \((x', y', z')\) are given by
\[
x' = -y \cos \phi \sin \theta \sin \gamma + y \sin \phi (2 \cos^2 \phi \cos \theta - \cos 2\phi) \cos \gamma + x \cos \phi (\cos 2 \phi \cos \theta + 2 \sin^2 \phi),
\]
\[
y' = -y \sin \phi \sin \theta \sin \gamma + y \cos \phi (2 \sin^2 \phi \cos \theta + \cos 2\phi) \cos \gamma + x \sin \phi (\cos 2 \phi \cos \theta - 2 \cos^2 \phi),
\]
\[
z' = -(\cos \theta \sin \gamma + 2 \cos \phi \sin \phi \sin \theta \cos \gamma) - x \cos 2 \phi \sin \theta.
\]

To calculate the torques, we represent the orbits by discrete points that are equidistantly spaced in the mean anomaly of the orbit. We start by using 64 points on each orbit to calculate the torque. We estimate the error in our calculation by recomputing the torque with the points which are in the middle (in mean anomaly) of the points just used. If the relative difference between the two torques calculated, \( \delta_t = |(\tau_1 - |\tau_2|)/(|\tau_1| + |\tau_2|)| \), is larger than 0.01, we double the number of points and repeat the computation. Once the desirable tolerance is reached, we use \( \tau = (\tau_1 + \tau_2)/2 \) for torque. To limit the time spent for computation, we use at most 65 536 points per orbit. When we quadrupled this value during the test runs, we obtained virtually identical results. Typically, a few thousand points per field star were required.

### 4 RESULTS

#### 4.1 The torque

To determine the eccentricity dependence of torque on an orbit, we made simulations with \( e_0 = 0, 0.2, 0.4, 0.6, 0.8, 0.99 \) and 0.999. For each value of \( e_0 \), we carried out 80 simulations with \( N = 10\,000 \) stars each and averaged over the results.

As a first result, we confirmed that the total torque is proportional to \( N^{1/2} \). However, stars very far from the test star will not exert any discernible torque. We therefore first determine beyond which point the contribution from stars becomes negligible. In Fig. 1, we plot the \( z \)-component of the torque on a star of the semimajor axis \( a_0 \) as a function of \( a_{\max} \), where \( a_{\max} \) is the cut-off for the semimajor axes of the field stars. This figure shows that stars with the semimajor axis larger than the test star’s apocentre distance \( r_{\text{apo}} = a_0(1 + e_0) \), contribute very little to the net torque on the test star. Motivated by this, we normalize the torque by
\[
\tilde{\tau} = \sqrt{N(<2a_0)} \frac{GM_\star}{a_0}.
\]

Fig. 1 already shows a strong dependence of the \( z \)-component of the torque on eccentricity. As expected, the torque vanishes for \( e \to 0 \), and has finite values for \( e > 0 \). We find that the result is consistent with a linear growth of the torque as a function of \( e \). The best linear fit gives
\[
\tau_e = \beta(e) \tilde{\tau} = 0.25e \tilde{\tau}.
\]
In Fig. 2, a cusp with \( \alpha = 1.4 \) was assumed. We have performed another set of calculations in which \( \alpha = 2 \), which also showed a linear eccentricity dependence of \( \tau_\epsilon \). It can thus be concluded that the result in equation (13) is not strongly dependent on the particular choice of \( \alpha \).

The component of the torque perpendicular to the angular momentum, \( \tau_\perp \equiv \sqrt{\tau_2^2 + \tau_1^2} \), changes its direction but not magnitude. The resulting relaxation process is called vector resonant relaxation by Rauch & Tremaine (1996). We plot this component as a function of eccentricity in Fig. 3. Here, the data are consistent with the torque being a quadratic function of eccentricity:

\[
\tau_\perp = \beta_\epsilon(e) \tau = 0.28 \left( e^2 + \frac{1}{2} \right) \tau. \tag{14}
\]

Note that the \( x \)-component of the torque vanishes for large \( e \), and \( x \)-and \( y \)-components become equal to each other for small \( e \).

### 4.2 The resonant relaxation time

For the translation of the torques into a relaxation time, the time-scale for the stars to change their orientation with respect to the host cluster \( t_\omega \) in equation (7) needs to be determined. Three relevant processes are: (1) precession of the test star due to general relativity; (2) precession of the test star due to the extended distribution of the host cluster; and (3) precession of the host cluster itself due to its own extended distribution.

Torques are assumed to be coherent when the orbit has precessed less than an angle \( \omega \), and to make a random walk for angles \( \gtrsim \omega \). The precise value of the coherence angle \( \omega \), which determines the steps of the random walk, is not clear. In particular, the coherence angle may itself depend on eccentricity, \( \omega = \omega(e) \). Our wire method, which does not include the evolution of the orbits, is not well suited to determine this dependence, and we do not consider this possibility here. By rotating a test wire in a cluster, we find that typical variations in the torque occur over angles of \( \sim \pi/2 \). Motivated by this, we use \( \omega = \pi/2 \), smaller than the value of \( \omega = \pi \) assumed by Rauch & Tremaine (1996). We note that a larger value for \( \omega \) leads to more effective resonant relaxation. We now discuss the three processes leading to reorientation of the test star’s orbit with respect to the cluster.

The general relativistic precession time is given by

\[
t_{GR}(a, e) = \frac{4}{3} \left( \frac{J}{J_{LSO}} \right)^2 P = \frac{a}{12r_s} (1 - e^2)^2 P(a), \tag{15}
\]

where

\[
J_{LSO} = \frac{4GM_*}{c} \tag{16}
\]

is the angular momentum of the last stable circular orbit, and \( r_s = 2GM_*/c^2 \) is the Schwarzschild radius of the BH.

Since the potential is not exclusively dominated by the MBH, but there is a contribution of the stellar cluster as well, the orbit of the test star precesses. The precession rate of a star in a cusp near an MBH was derived by Ivanov, Polnarev & Saha (2005). Here we briefly summarize the result for \( \alpha = 3/2 \). Let \( \delta\omega(a, e) \) be the change in the angle of the pericentre during one orbit. The time-scale for precession due to extended mass distribution is then given by

\[
t_\omega(a, e) = \frac{\pi}{2 \delta\omega(a, e)} P(a). \tag{17}
\]

Ivanov et al. (2005) showed that

\[
\delta\omega(a, e) = 4 \left( \frac{a}{r_a} \right)^{3/2} \sqrt{1 - e^2} \frac{dE}{de} F(e), \tag{18}
\]

where

\[
F(e) = \frac{2}{3} \sqrt{(1 + e)} \left[ 4E \left( \sqrt{\frac{2e}{1 + e}} \right) - (1 - e)K \left( \sqrt{\frac{2e}{1 + e}} \right) \right], \tag{19}
\]

and \( E \) and \( K \) are complete elliptic integrals. For this result, an \( \alpha = 3/2 \) power law was assumed, which simplifies the equations. For more general expressions see Ivanov et al. (2005).

Since general relativistic precession takes place in the opposite direction, the rate at which the star’s orbit precesses due to the combined effects of general relativity and the extended potential is (Hopman & Alexander 2006a):

\[
t_{GR}^e(a, e) = \frac{1}{t_{GR}(a, e)} - \frac{1}{t_\omega(a, e)}^{-1}. \tag{20}
\]

For some combinations of \( (a, e) \), the precession time \( t_{GR}^e(a, e) \) can become very large, implying that the star does not precess with respect to inertial space. However, for the efficiency of resonant relaxation, it is the orientation of the orbit with respect to the other stars that matters. If most of the other stars do precess, the torque on the test star will still fluctuate. We therefore define the precession time of the stellar cluster as

\[
t_{GR}^c(a, e) \equiv t_{GR}^e(a, e, = 0.7). \tag{21}
\]

The eccentricity \( e = 0.7 \) is the median eccentricity for an isothermal eccentricity distribution function; approximately half of the star precess more rapidly than \( t_{GR}^e(a, e = 0.7) \), and half of the star experience slower precession.

The limiting time-scale for resonant relaxation is then

\[
t_\omega(a, e) = \min \left[ t_{GR}^e(a, e), t_{GR}^c(a) \right]. \tag{22}
\]

Using equation (22) in equation (7) gives the resonant relaxation time as a function of \( a \) and \( e \).

### 4.3 Resonant relaxation for a simple model of a galactic nucleus

We apply our results to a simple model which may describe a galactic nucleus similar to our Galactic Centre. For masses, we assume that \( M_* = 3.6 \times 10^6 M_\odot \) and \( M_m = M_\odot \). The radius of influence of the MBH, where the mass in stars is equal to \( M_* \), is \( r_2 = 2 \) pc, and there is a cusp of stars, with \( \alpha = 3/2 \) (see equation 8).
The determination in this paper of $T_{RR}(a, e)$ as a function of the semimajor axis $a$ and eccentricity $e$ allows for implementation of resonant relaxation in Monte Carlo codes such as those presented in Freitag & Benz (2001, 2003). The fact that the torques depend on $e$ can be of considerable importance for these results. In a companion paper (Hopman & Gürkan, in preparation), we use the eccentricity dependence derived in this paper to find the steady-state angular momentum distribution of stars in presence of resonant relaxation, and address the consequences for the processes mentioned here.

5 SUMMARY AND DISCUSSION

In this paper, we have shown that the net torque of a cluster of stars on a test star of eccentricity $e$ is proportional to $e$ (equation 13). From this dependence, and the dependence of the precession time on eccentricity, we determine for the first time the resonant relaxation time as a function of $e$ and $a$ (equations 7 and 22).

Resonant relaxation may play an important role in several phenomena near MBHs. Rauch & Tremaine (1996) and Rauch & Ingalls (1998) estimated that resonant relaxation may increase the rate of tidal disruptions of stars by the MBH by a factor of $\sim 2$ due to the increased rate at which stars are driven towards the loss-cone. It has also been suggested that resonant relaxation has modified the distribution of the young star cluster known as the ‘S-stars’ in the Galactic Centre (Hopman & Alexander 2006a; Levin 2007; Perets, Hopman & Alexander 2007).

Resonant relaxation also plays a role in the formation of gravitational wave sources. Compact remnants that spiral into MBHs due to the emission of gravitational waves are an important potential source of gravitational waves for the Laser Interferometer Space Antenna (LISA). With the exception of the Galactic Centre (Hopman, Freitag & Larson 2007), such extreme mass ratio gravitational wave sources are not observable until they orbit on very tight orbits with periods less than an hour. Since such stars originate from orbits relatively close ($\sim 0.01$ pc) to the MBH (Hopman & Alexander 2005), resonant relaxation plays an important role in the event rate, and can lead to an increase of nearly an order of magnitude (Hopman & Alexander 2006a). With the exception of Freitag (2001, 2003), estimates of the event rate have relied on semi-analytical models which were not fully two dimensional in $(E, J)$ space. In particular, Hopman & Alexander (2006a) treated the resonant relaxation time as averaged over eccentricities.

ACKNOWLEDGMENTS

We thank Tal Alexander, Marc Freitag and Yuri Levin for helpful discussions, and Ann-Marie Madigan for comments on this manuscript. MAG was supported by a Marie Curie Intra-European fellowship under the sixth framework programme, and CH by a Veni scholarship from the Netherlands Organization for Scientific Research (NWO). CH thanks the University of Amsterdam, where most of the work was done, for their hospitality. The computations in this paper are done at the Lisa cluster at SARA supercomputing centre in Amsterdam.

REFERENCES

Alexander T., 1999, ApJ, 527, 835
Alexander T., 2005, Phys. Rep., 419, 65
Alexander T., Hopman C., 2003, ApJ, 590, L29
Bahcall J. N., Wolf R. A., 1976, ApJ, 209, 214
Bahcall J. N., Wolf R. A., 1977, ApJ, 216, 883
Chandrasekhar S., 1943, ApJ, 97, 255
Eisenhauer F. et al., 2005, ApJ, 628, 246
Frank J., Rees M. J., 1976, MNRAS, 176, 633
Freitag M., 2001, Class. Quantum Gravity, 18, 4033
Freitag M., 2003, ApJ, 583, L21
Freitag M., Amaro-Seoane P., Kalogera V., 2006, ApJ, 649, 91
Freitag M., Benz W., 2001, A&A, 375, 711
Freitag M., Benz W., 2002, A&A, 394, 345
Gehardy K. et al., 2003, ApJ, 583, 92
Genzel R. et al., 2003, ApJ, 594, 812
Ghez A. M., Becklin E., Duchjne G., Hornstein S., Morris M., Salim S., Tanner A., 2003, Astron. Nachr, Suppl. 1, 527
Hénon M., 1973, in Contopoulos G., Hénon M., Lynden-Bell D., eds, Dynamical Structure and Evolution of Stellar Systems. Geneva Observatory, Switzerland p. 183
Hopman C., Alexander T., 2005, ApJ, 629, 362
Hopman C., Alexander T., 2006a, ApJ, 645, 1152
Hopman C., Alexander T., 2006b, ApJ, 645, L133
Hopman C., Freitag M., Larson S. L., 2007, MNRAS, submitted (astro-ph/0612337)
Ivanov P. B., Polnarev A. G., Saha P., 2005, MNRAS, 358, 1361
Levin Y., 2007, MNRAS, 374, 515
Lightman A. P., Shapiro S. L., 1977, ApJ, 211, 244
Miller M. C., 2006, MNRAS, 367, L32
Murphy B. W., Cohn H. N., Durisen R. H., 1991, ApJ, 370, 60
Perets H. B., Hopman C., Alexander T., 2007, ApJ, 656, 709
Rauch K. P., Ingalls B., 1998, MNRAS, 299, 1231
Rauch K. P., Tremaine S., 1996, New Astron., 1, 149
Rees M. J., 1988, Nat, 333, 523
Schödel R. et al., 2002, Nat, 419, 694
Shapiro S. L., Marchant A. B., 1978, ApJ, 225, 603
Sigurdsson S., 1997, Class. Quantum Gravity, 14, 1425

This paper has been typeset from a \TeX file prepared by the author.