On the Reconstruction of the Convection Pattern Below an Active Region of Solar Corona.

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Abstract. In order to better understand magneto-convective patterns and flux emergence, we use the Nudging Back and Forth, a data assimilation method with an anelastic convection model to reconstruct the convection zone below a solar active region from observed solar surface magnetograms. To mimic photosphere, vector magnetograms are computed using force free hypothesis. We find that the observed arcade system of AR9077-20000714 (“the slinky”) of magnetic lines is actually formed by $\Omega$ and $U$ loops generated in the convection zone. We generate temperature maps at top of the convective zone and find that high magnetic fields on either sides of the neutral line produce a local cooling by impeding the overturning motions.

1. Introduction
Convection in the Sun is at the origin of magnetic dynamo, and thus, ultimately of coronal activity. The theoretical problem of turbulent compressible convection has been studied by many authors (e.g. Brummell et al., 2006; Bushby et al., 2008) including radiation effects (Nordlund, 2008; 2009; Nordlund and Stein, 2007). In fact, since the convective envelope is strongly stratified and of relatively low Mach number, the flow is anelastic. The advantage here is that the equations do not take acoustic waves into account. There are indeed acoustic waves in the Sun. Helioseismology is using surface signature to attempt the reconstruction of the interior of the Sun and in particular the convective envelope (Hanasoge et al., 2006; Christensen-Dalsgaard, 2002). But there is also a numerical advantage: time steps (CFL) do not need to be small enough to follow acoustic waves. Anelastic approximation for convection has been studied in details for a long time (e.g. Gough, 1969) and improved recently for astrophysical convective envelopes with variable physical properties (Lantz and Fan, 1999). If one wants to predict the evolution of coronal activity, and in particular a solar flare, it would be necessary to simultaneously compute the MHD flows inside the Corona and also the convective layer below. This has been attempted (Abbett and Fisher, 2002; Hurlburt and De Rosa, 2004) for realistic but for purely theoretical flows. To simulate realistic convection including granulation is possible, but to reproduce observed magnetograms and temperatures in a particular area of about $175,000 km \times 175,000 km$ using simulation of convection remains a challenge.

In this study, we try to reconstruct the convective layer below a particular active region AR9077-20000714. For this purpose, we need to know the actual temperature, velocity and magnetic field at top of convection layer (photosphere) continuously in time. Temperature and magnetic fields are observed by satellite and ground observatories and publicly accessible in the...
form of heliocentered 2D pixel maps (Fig.1). However, many problems arise from the datasets.

Firstly, magnetograms are usually not given at the same times and cadence as temperature maps: typical SOHO-MDI magnetograms are given at a 96 minute cadence. Also velocities are not easily observed and must be computed by correlation techniques (e.g. Rieutord et al., 2000; Longcope, 2004). To surmount this, we use Nudging Back and Forth (NBF) (Auroux et al., 2008), a data assimilation technique (Daley, 1991; Lewis et al., 2006; Bouttier and Courtier, 2003), akin to Quasi-Linear Inverse 3D (Kalnay et al., 1999). This is needed to solve the state of the convection zone that matches available observations taken as upper boundary conditions.

Our manuscript is organized as follows. In next section, we set the anelastic equations for solar convection and we explain the data assimilation technique for an active region. Then, in section 3, we assimilate observed vector magnetograms to reconstruct the convection pattern below the active region.

2. Anelastic convection model and NBF data assimilation

We are using the Back and Forth Nudging algorithm (NBF) (Auroux et al., 2008) to reconstruct the convective zone below the photosphere given a pair of observed magnetograms.

2.1. Anelastic convection below an active region.

The convective layer of the Sun is strongly stratified in density and mostly we can regard the Mach number as being small. In this regime, magnetohydrodynamic anelastic approximation allowing for depth dependent parameters is valid (Lantz and Fan, 1999). The advantage is that we do not need to deal with acoustic waves that would impose a very small time-step. A realistic set of anelastic equations for convection in the sun is given in Fan et al. (1999). Computed quantities are fluctuations around given mean vertical profiles (Fig. 2) denoted by a $z$ subscript, i.e. it can only vary with depth (Sect. 2.2). The system of conservation equations is in non dimensional units (Lantz and Fan, 1999):
\[ \nabla \cdot (\rho z V) = 0 \quad (1) \]
\[ \rho z \left[ \frac{\partial V}{\partial t} + (V \cdot \nabla)V \right] = -\nabla p - \rho g e_z - \frac{1}{R_o} \rho z (\Omega \times V) + (\nabla \times B) \times B + \frac{1}{R_e} \nabla \cdot \vec{\tau} \quad (2) \]
\[ \rho z T_z \left[ \frac{\partial s}{\partial t} + (V \cdot \nabla)(s_z + s) \right] = \frac{1}{P_e} \nabla \cdot (\kappa \rho z T_z \nabla s) + \frac{1}{R_m} \eta (\nabla \times B)^2 + \frac{1}{R_e} (\vec{\tau} \cdot \nabla) \cdot V \quad (3) \]
\[ \nabla \cdot B = 0 \quad (4) \]
\[ \frac{\partial B}{\partial t} = \nabla \times (V \times B) - \frac{1}{R_m} \nabla \times (\eta \nabla \times B) \quad (5) \]
\[ \frac{\rho}{\rho z} = \frac{p}{P_z} - \frac{T}{T_z} \quad (6) \]
\[ \frac{s}{c_p} = \frac{T}{T_z} - \frac{\gamma - 1}{\gamma} \frac{p}{P_z} \quad (7) \]

Here the gas is a polytrope with adiabatic parameter \( \gamma \), \( s \) specific entropy, \( \kappa \) is the diffusivity of small scales entropy fluctuations, \( \eta \) is magnetic diffusivity, \( c_p \) is specific heat at constant pressure and \( \vec{\tau} = \mu \left[ \frac{\partial V}{\partial x_j} + \frac{\partial V}{\partial x_i} - \frac{2}{3} \delta_{ij} \nabla \cdot V \right] \) is the viscous rate of strain tensor.

Boundary conditions are non-penetrating stress-free top and bottom and periodic in the horizontal directions (Fan et al., 1999). Non dimensional numbers are set in our simulations as listed in table 1. Corresponding dimensional physical parameters \( g^*, \eta^*, \kappa^*, \mu^*, c_p^*, \Omega^* \) have been computed and listed in table 2.

| \( R_e \) | \( R_m \) | \( P_e \) | \( R_o \) |
|---------|---------|---------|---------|
| 750     | 1000    | 1       | 1       |

Table 1. Non dimensional numbers used in our simulations.

| \( g^* (m.s^{-2}) \) | \( \eta^* (m^2.s^{-1}) \) | \( \kappa^* (m^2.s^{-1}) \) | \( \mu^* (kg.m^2.s^{-1}) \) | \( c_p^* (J.kg^{-1}.K^{-1}) \) | \( \Omega^* (s^{-1}) \) |
|---------------------|----------------------|---------------------|---------------------|---------------------|---------------------|
| 260                 | \( 10^9 \)           | \( 1.3 \times 10^9 \) | \( 1.3 \times 10^{10} \) | \( 3 \times 10^4 \)   | \( 1.6 \times 10^{-3} \) |

Table 2. Reference values for physical parameters in our simulations.

Since Rossby number is 1, corresponding \( \Omega \) is three orders of magnitude higher than it should. We do not have radiation model and we do not simulate the photosphere. As a time scale, we use the Free Fall Time (FFT) defined by: \( t = \frac{H_0}{v_r} = \sqrt{\frac{H_0}{\delta_r g}} \approx 7.10^2 \text{s} \) where \( \delta_r = 0.4 \) is the superadiabaticity (Stix, 2002) and \( H_0 \) is the pressure height scale.
2.2. Mean profiles (convection):
Mean vertical polytropic profiles for pressure, entropy, density and temperature $P_z$, $s_z$, $\rho_z$, $T_z$ (Fig.2) are given as follows:

\[ P_z(z) = P_o \left[ 1 - \frac{z}{(m+1)H_o} \right]^{m+1} \quad (8) \]

\[ \rho_z(z) = \rho_o \left[ 1 - \frac{z}{(m+1)H_o} \right]^m \quad (9) \]

\[ s_z(z) = (m+1) \log \left[ 1 - \frac{z}{(m+1)H_o} \right] \quad (10) \]

\[ T_z(z) = T_o \left[ 1 - \frac{z}{(m+1)H_o} \right] \quad (11) \]

The temperature gradient $\beta$ in hydrostatic equilibrium, polytropic index $m$ and pressure height scale $H_o$ are related by:

\[ \beta = -\frac{1}{(m+1)H_o} \quad (12) \]

In our problem, a reasonable value is: $m = \frac{1}{\gamma - 1} \approx \frac{1}{5/3 - 1} \approx 1.5$, where $\gamma = \frac{5}{3}$ for an atomic ideal gas and isentropic process (Fan et al., 1999). It can differ for an ionized gas (e.g., Lantz, 1992). Here gravity $g$ and pressure scale height $H_o$ are taken at the middle of the convective zone. Mean profiles are shown Fig.2. Due to our large numerical aspect ratio, we could not take exact profiles in our direct numerical simulation.

**Figure 2.** Density, pressure, temperature and entropy mean profiles from our simulation (——) and from a standard solar model (Bahcall et al., 2004) (- - - -). Pressure scale is about 55 Mm at top of the convective zone. Theoretical profiles are slightly deeper than computed.
The code has been parallelized using Open-MP and, with 8 processors, the speed-up is increased by almost a factor 3 (Fig. 3). The software we used is “AN-MHD” written by Solar Muri team (Fan et al., 1999; Abbett et al., 2001; http://solarmuri.ssl.berkeley.edu/public/software/Anmhd/). The machine we used is a cluster of 1024 cores (128 nodes 2-processors Intel Xeon E5462 quad-cores 3.0 GHz, 16 GB memory-node) at RQCHP (Réseau Québécois de Calcul Haute-Performance). We did also used extensively a 160 nodes cluster at GRPS (Groupe de Recherche en Physique Solaire de l’Université de Montréal).

Figure 3. Computational performance of the code. Wall clock time is almost decreased by a factor 3 when the number of cores is increased from 1 to 8. It takes about a few hours to simulate a free fall time.

Numerical resolution is $256 \times 256 \times 64$ grid points corresponding to $175 \times 175 \times 175$ $Mm$ with an 4:1 numerical aspect ratio. We use a periodic boundary condition. With this resolution, we can reach $Re = 750$, $Pr = 1$, $Ro = 1$ and $Rm = 1000$: In solar interior Reynolds number is about $10^6$ and magnetic Reynolds number is about $10^9$, Rossby number is about $2 \times 10^2$. No current simulations from literature approach real solar Reynolds number (Stein, 2011).

2.3. Recreating the convective zone below an active region.
We assimilate the first magnetograms with the NBF algorithm at times when they are available (01:39 and 03:15 on 14/07/2000). This enables to reconstruct a 3-D initial condition in $T$, $P$, $\rho$, $\mathbf{V}$ and $\mathbf{B}$. To do this, we add a term in the equation for magnetic induction conservation (eq. 5):

$$
\frac{\partial \mathbf{B}}{\partial \ell} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \frac{1}{Rm} \nabla \times (\eta \nabla \times \mathbf{B}) + K \times \delta(z - z(Lz - 2\delta z)) \left\{ (B^{obs}_x - B^{cal}_x)\mathbf{e}_x + (B^{obs}_y - B^{cal}_y)\mathbf{e}_y + (B^{obs}_z - B^{cal}_z)\mathbf{e}_z \right\}
$$

(13)

where $\delta(z - z(Lz - 2\delta z))$ is a Dirac function and $\delta z$ is the vertical grid-step. $K$ is the Nudging parameter, possibly a matrix, equivalent to a Lagrange multiplier (Auroux et al., 2008). We take $K = 0.6$ (free fall time units)$^{-1}$ and $Lz = 64$. Thus, the NBF is a Newtonian relaxation technique. $B^{obs}_{x,y,z}$ and $B^{cal}_{x,y,z}$ are observed and computed vectorial magnetograms. We did also assimilate the two other components $B_x$ and $B_y$ of the vector magnetograms reconstructed from $B_z$ using Linear Force Free approximation.
2.4. Simulation of a single buoyant flux tube, a test for validation.

To validate, the whole procedure can be done in three steps with a purely numerical test. First, by mean of a direct simulation of the anelastic code, we run a simple but realistic case of U-shaped turbulent flux tube. The resulting data are regarded as “the true state” or observations. We then extract top layers of this true state at various times and these are our observed “magnetograms”. Third, we run the same code and same parameters but this time in NBF mode and with a random thus different initial state. To compare computed and “observed data”, we use a linear Pearson correlation coefficient

\[
C(B_{z,\text{cal}}, B_{z,\text{obs}}) = \frac{\sigma_{(\text{cal,obs})}}{\sigma_{\text{cal}}\sigma_{\text{obs}}} = \frac{\sum_{x,y} (B_{z,\text{cal}} - \bar{B}_{z,\text{cal}})(B_{z,\text{obs}} - \bar{B}_{z,\text{obs}})}{\sqrt{\sum_{x,y} (B_{z,\text{cal}} - \bar{B}_{z,\text{cal}})^2} \sqrt{\sum_{x,y} (B_{z,\text{obs}} - \bar{B}_{z,\text{obs}})^2}} \tag{14}
\]

where \(B_{z,\text{cal}}\) and \(B_{z,\text{obs}}\) are computed and observed magnetograms (top layer). A result close to one indicates that the two magnetograms are strongly correlated whereas a zero value indicates uncorrelated magnetograms. In this experiment, we found a good agreement between predictions and “observations” over a few Free Fall Times (\(FFT \approx 11\min\)). This is shown Fig.4 where \(B_{z,\text{cal}}(x_0, y_0, t)\) and \(B_{z,\text{obs}}(x_0, y_0, t)\) are plotted. Here we take \(x_0 = \frac{L_x}{5}\) and \(y_0 = \frac{L_y}{5}\).

![Figure 4](image)

**Figure 4.** Local predictability using the NBF algorithm. Time evolution of the vertical component of magnetic field \(B_z\) at one place \((x = \frac{L_x}{5}, y = \frac{L_y}{5}\) of observed (dashed line) and computed (full line) magnetograms. Observed and computed \(B_z(x_0, y_0, t)\) remain close each other up to a few Free Fall Times.

How far in time can we predict magnetograms could be estimated using the RMS error function \(<\sqrt{(B_{z,\text{cal}} - B_{z,\text{obs}})^2}(z)\) at top \((z = L_z - 4\delta z)\) that is directly related to the Nudging forcing term in eq.13 but here we used correlation function defined above (eq. 14) and averaged over the entire magnetogram (Fig.5). Simulation is restarted at \(t=0\) with initial condition obtained after data assimilation over \(t=0,5\ FFT\) (the assimilation window). From \(t=0,12\ FFT\), computed and observed magnetograms remain correlated within 8 Free Fall Times. This includes the data assimilation window and a part of the prediction window.
Figure 5. Test of validity. Decreasing of Pearson linear correlation coefficient between computed and observed vertical magnetograms with time. $C(B_{z}^{\text{obs}}, B_{z}^{\text{cal}})$ at end of NBF was optimal at 0.4 in this case. We restarted from $t=0$.

This shows that an anelastic model for solar convective zone combined with the NBF can be used for a prediction purpose over at least a few Free Fall Time (here about 30 minutes).

2.5. Unicity of solution using regularization

Since this problem is under-determined, we would not get a unique solution. This is the case for stiff problems like inversion problems and we have to use here an additional constraint called a regularization function (see Appendix)

$$
\phi(x, y, z) = \frac{\int_{3D} \|
abla B(x, y, z)\|^2 dV}{\int_{3D} dV}.
$$

Practically, we add to eq.13:

$$
B_{z}^{\text{cal}}(x, y, z)^{(t+\delta t)} = B_{z}^{\text{cal}}(x, y, z)^{(t)} + \lambda \times \left[ B_{z}^{\text{cal}}(x + \delta x, y, z) + B_{z}^{\text{cal}}(x - \delta x, y, z) + B_{z}^{\text{cal}}(x, y + \delta y, z) + B_{z}^{\text{cal}}(x, y - \delta y, z) + B_{z}^{\text{cal}}(x, y, z + \delta z) + B_{z}^{\text{cal}}(x, y, z - \delta z) - 6 \times B_{z}^{\text{cal}}(x, y, z) \right]^{(t)}
$$

The solution is then unique but the regularization functional that we used (Tikhonov and Goncharsky, 1987) arbitrarily maximizes correlation between neighbouring points. It could be seen as a minimum entropy method. It is highly reasonable to do this because direct simulations of convection in sun-like conditions show a high degree of coherent structures. Not only we get a solution but also it is smoother than without regularization. By trial and error, we found that an optimal value for the Lagrange multiplier is $\lambda = 10^{-6}$. A largest value would distort magnetic patterns.
2.6. Continuous top boundary condition using time interpolation

During each run, we assimilate two vector magnetograms. The first corresponding to initial time and the second to final time. At each time step $t$, we interpolate linearly to get a continuous variation of magnetograms (here $B_z$).

$$B_z^{\text{interp}}(t) = \left( 1 - \frac{t - t_0}{t_1 - t_0} \right) \times B_z^{\text{obs}}(t_0) + \left( \frac{t - t_0}{t_1 - t_0} \right) \times B_z^{\text{obs}}(t_1)$$  \hspace{1cm} (17)

where $t_0$ and $t_1$ are initial and final times. There is an approximation here. Linear order is efficient to interpolate in time the largest scales but not the smallest scales that de-correlate much faster ($<10\text{min}$) than about $1H30$, the time between two SOHO magnetograms.

However, in the tests with ascending flux tubes, we noted that there is no large distortion between the simulated continuous time varying sequence of “observed magnetograms” and the sequence of interpolated magnetograms. The anelastic set of equations (eqs. 1-7) is not equivalent to a linear interpolator but we found that if the time interval between two magnetograms is small enough ($\approx 15\text{min}$ or a free fall time) the differences are not large.

The final test of validity would be to compare real observations of magnetograms and temperatures with predicted $B$ and $T$ computed with our code but we do not do this here.

3. Results.

3.1. Magnetic induction.

We have reconstructed magnetic patterns below active region AR9077-20000714. Application of a local Nudging term has modified the whole convective zone and not only a thin layer at surface. Fig.6 shows a comparison between a snapshot of $B_z$ component of magnetic field at top of our simulation (left) and the corresponding assimilated magnetogram (right). To extract only the largest scales on the graphics of Figs. 9,10,11 we have used a second order low pass filter (Roberts and Roberts, 1978).

**Figure 6.** (Left) Computed $B_z$ component of magnetic field at $z = L_z - 2\delta_z$. (Right) Assimilated data from \textit{SOHO}/MDI. Time is a fraction of a free fall time. We see that largest scales are well reproduced.

Our results (Fig.7, bottom) show plume like structures commonly observed in simulations of magneto-convection of the quiet sun (Stein and Nordlund, 2006; Vögler \textit{et al.}, 2005; Abbett \textit{et...
al., 2004) but also an intense concentration of magnetic flux in the upflows (Fig. 7, top). This is often observed in magneto-convective simulations of active regions (Stein, 2011). The maximal magnetic flux intensity is located near the neutral line, where gradients of magnetic field are higher.

**Figure 7.** Data assimilated magnetic intensity: $|\mathbf{B}(x, y, z)|$. View from top. Time is a fraction of a free fall time. Magnetic induction at top corresponds to Figure 6 (left). Application of a local Nudging term has modified the whole convective zone and not only a thin layer at surface. At bottom of the figure, magnetic field is concentrated in downflows, showing plumes-like structure observed in magneto-convective simulations of the quiet sun. On the opposite, at top of the figure, magnetic field is highly concentrated in the upflows, forming an active region.

3.2. Temperature fluctuations.
Temperature fluctuations at top of the convective zone shows cool and hot places (Fig. 8 left). Coldest areas correspond to very high magnetic field amplitude (Fig. 8 right). High amplitude magnetic field should decrease convective heat transfer from inside layers and produce a local cooling (Stein, 2011). We observe that plasma located at the neutral line is hotter and indeed there are high electric currents there (Fig. 9) taking the form of a sheet at top of the convective zone. This is because the horizontal magnetograms $B_x, B_y$ we use have been reconstructed from the vertical magnetogram $B_z$ using a linear force free algorithm. As a result, electric currents are parallel to the magnetic field. We did this on purpose because we have no photosphere in our anelastic equations. Additional heating of the neutral line may also be due to viscous friction as significant velocity shears develop there (Fig. 10, right).

Current lines are displayed Fig. 9. High currents are indicated by a lighter color. We see a direct correspondence between this high current line and the magnetic arcade system around it (Fig. 10, right:top).

3.3. Magnetic field lines throughout the entire simulated region.
Although we do not simulate the corona or the photosphere, magnetic field lines from our simulations show often typically $\Omega/U$ loop structures (Fig. 10, right). Magnetic lines do reconnect but outside of the computational domain, in the corona, to form $\Omega$ loops. Magnetic field lines have been computed (VAPOR; Clyne et al., 2005, 2007) assuming horizontal periodicity. Simulations of flux tubes emergence in the quiet sun and in active regions is known
Figure 8. Left: Simultaneous display of temperature fluctuations and contour of the vertical magnetic field showing the polarity inversion line (neutral line). Temperature is lower where magnetic field is high. Right: Simultaneous display of magnetic field intensity and contour of the vertical magnetic field. Strongest magnetic fields are located on either sides of neutral line where magnetic gradients are maximal.

Figure 9. Comparison between magnetic field lines (red/yellow) and plasma current lines (blue). Intensity of the current increases from blue to white. Current intensity is high inside the $\Omega/U$ loop and maximal around the “polarity inversion line” where magnetic gradients are higher (Fan and Gibson, 2004).

to produce this kind of structures (Cheung et al., 2010; Abbett et al., 2004). Slow diverging upflows raise the flux and make magnetic lines emerging from the convective zone to form $\Omega$ loop, while fast downflows pin it down to produce $U$ loops (Fig. 10 left). Simultaneous upward and downward flows result in a vertical velocity shear.

Observations of the active region AR9077-20000714 have shown an arcade system in the corona “the slinky” (SOHO/EIT). Our results show that velocity shears due to convection are at the origin of this pattern (Figs. 10, 11).
Figure 10. Left: Comparison between magnetic field lines and vertical component’s velocity. Magnetic field lines go from red to yellow. Blue areas indicate that the plasma is moving outward (upflows) whereas red indicates an inward movement (downflows). Magnetic field lines emerge from the convective zone in upflows. A low pass filter has been used to generate this graphics. Right: Vertical slice from left panel showing \( \Omega/U \) loop structure for magnetic field above an active region. Magnetic field lines show an \( \Omega \) loop structure above the computational domain and a \( U \) loop structure in it due to vertical velocity shearing.

Figure 11. Simultaneous display of magnetic field lines and vertical components of magnetic field. Blue indicates that magnetic field is pointing outward whereas red indicates an inward vertical component. Arcade system of AR9077-20000714 of magnetic lines is actually formed by the \( \Omega \) and \( U \) loops generated in the convection zone below.

4. Conclusion and future work
We have reconstructed the interior of solar convection zone below active region AR9077-20000714 by using the Nudging Back and Forth algorithm (Auroux et al., 2008). To get a unique solution we added a regularization function that is also acting as a smoothing function. This simple data assimilation technique allows us to get the velocity and magnetic 3D fields as well as
pressure, density, entropy and temperature inside the computed volume by assimilating vector magnetograms at top of convective zone. One could add more observations like temperature maps for instance. This is realistic because photospheric temperature is also observed by satellite. It would be included in a future work.

In our simulations, we see coherent magnetic patterns as well as places where magnetic fields are very weak. High magnetic intensity at top of the convective zone corresponds to low temperature. We see local Ohmic heating, heat diffusivity from underlying layers as well as viscous friction. We found that velocity shears between slow diverging upflows and fast turbulent downflows produce \( \Omega \) and U-shaped magnetic field loops. This proves that that the coronal arcade system of AR9077-20000714 has its origin inside the convective zone below.

Our long-term goal is to find a possible 3D convective pattern that can be used as an initial condition to predict in real time, the evolution of temperature, magnetic field and velocity at photospheric levels based on magnetohydrodynamical modeling. Such reconstruction would be an important step toward flare prediction.

4.1. Appendix: Regularization

This problem is made stiff because of the discontinuous nature of the top boundary (Fig.6). One way to get smoother 3-D fields is to add a regularization. This step is under way and is not shown here. The idea is to minimize a functional \( \Phi \) called the deviation function which quantifies the deviation of the calculated unknown quantities from the observed ones:

\[
\Phi((B, T)_{3D}) = \int_{B,T} \left[ R(B, T)_{obs} - R(B, T)_{cal} \right]^2 dS + \lambda R(B, T)
\]

where \((B, T)\) is the unknown 3D field, \(R\) is a projection \((B, T)_{3D}\) at top of the convective layer. \(\lambda\) is a numerical Lagrange multiplier (small enough) and the second part of \(\Phi\) (the deviation function) is a regularization functional \(R(B, T)\) because we have here a stiff problem. We use Tikhonov regularization functional that maximizes correlation between neighboring points (Tikhonov and Goncharsky, 1987):

\[
R(B, T) = \frac{\int_{3D} ||\nabla(B, T)||^2 dV}{\int_{3D} dV}
\]

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