Study on speed transfer characteristics of double-deck ball bearing based on generalized four-terminal parameter method

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Abstract The speed transfer ratio is one of the most important parameters of the double-deck ball bearing. For the calculation of speed ratio, most of the theoretical estimation formulas related to the structure are used, but under some measured working conditions, the speed ratio is not only related to structural parameters. To explore the speed transmission mechanism of double-deck ball bearing, a new method for calculating the speed transmission characteristics of double-deck ball bearing is proposed in this paper. The generalized four-terminal parameter method is applied to the speed analysis of double-deck ball bearing for the first time. The speed transmission analysis model of double-deck ball bearing is established by combining the three theories of quasi-statics theory, elastohydrodynamic lubrication theory, and generalized four-terminal parameter theory. A variety of factors are taken into account to solve the research status of rotational speed transfer ratio only considering a single factor. The speed transmission characteristics of the double-deck ball bearing after the system is accelerated to the steady state are studied, and the correctness of the results is verified by experiments, which provides a new idea for the analysis of the speed transmission characteristics of the double-deck ball bearing and provides a theoretical basis for the exploration and design of the potential application value of the double-deck ball bearing.

Keywords Double-deck ball bearing · Speed transfer · Quasi-statics · Elastohydrodynamic lubrication · Generalized four-terminal parameter method

1 Introduction

As the main supporting component of rotating machinery, rolling bearing is widely used in mechanical industries such as aerospace [1], shipbuilding [2], and the automobile industry [3]. As the bearing area of rolling bearings is small, it is easy to cause serious mechanical damage when running at high speed. According to the requirements of application occasions, more and more bearings need to work at high speed [4], high temperature [5], and alternating load [6]. The high-speed operation of bearing is the premise of the high-
To improve the limited speed of bearing, the double-deck ball bearing structure was proposed, and many scholars have carried out relevant research on it. Har Prashad has studied the speed distribution theory, stiffness and damping characteristics, life, temperature rise and friction torque characteristics, and other aspects of double-deck ball bearings. Zhu et al. established a mechanical analysis model at high speed and studied the influence of structural parameters on mechanical properties. Jin et al. proposed the method of mapping network division of rolling element and local encryption of contact area grid and analyzed the contact stress and elastic deformation of rolling element and ferrule channel of double-deck ball bearing under certain load. Based on Hertz’s basic theory and safe contact angle theory, Yu et al. analyzed the mechanical properties of “Z” type double-deck ball bearing. Zhu et al. proposed a new “I” type double-deck ball bearing based on the prototype double-deck ball bearing. According to the principle of quasi-statics, the mechanical model of the new type “I” double-deck ball bearing is established, the corresponding calculation model is established on the MATLAB platform, the load test-bed is built, and the simulation results are verified. Zhu et al. proposed a speed control system using face-to-face inner ball bearings and conducted an experimental analysis on the influence of torque. Compared with traditional CB (catcher bearings), the use of new double-deck ball bearings helps to reduce the impact after rotor drop events. Based on the quasi-statics theory, Hu et al. established a double-layer algorithm solution model for the quasi-static solution of double-deck ball bearings and studied the numerical solution of double-deck ball bearings. Hu et al. optimized the design of a double-deck ball bearing based on the response surface method and optimized the problem of the mass increase caused by the double-deck structure.

Compared with traditional single-layer bearings, there are relatively few studies on double-deck ball bearings. In the above studies, most of them focus on the related research on stiffness, damping, temperature rise, plastic deformation, and so on, and there is little research on speed and transmission characteristics. The speed transfer ratio is one of the most important characteristics of double-deck ball bearings. The research on the speed transfer ratio is the basis of exploring the limit speed of double-deck ball bearings. However, in the current research, most of the theoretical estimation formulas of speed ratio related to structural parameters are used for the calculation of speed. However, under some complex and harsh working conditions of high temperature and high speed, the speed ratio is not a constant value, with the speed, stress, and temperature. The research on the speed transmission characteristics of double-deck ball bearings is of great significance to the high-speed performance design and to improve the limit speed of double-deck ball bearings.

The main contribution of this paper is:

1. A new method for calculating the speed transmission characteristics of double-deck ball bearings is proposed in this paper. The speed transmission model of the double-deck ball bearing is established, and the generalized four-terminal parameter method is applied to the speed transmission analysis of the double-deck ball bearing for the first time.
2. A variety of factors are taken into account to solve the research status of rotational speed transfer ratio only considering a single factor.
3. Combined with the two theories of quasi-statics and elastohydrodynamic lubrication of the bearing, the influence laws of different structural parameters and lubrication parameters on the speed ratio are analyzed, which provides a new idea for the speed transmission performance of double-deck ball bearing and provides a theoretical basis for the high-speed design of double-deck ball bearing.

The rest of this paper is organized as follows. Section 2 analyzes the mechanical characteristics and elastohydrodynamic lubrication analysis of double-deck ball bearing, which provides the basis for the following contents. In Sect. 3, a new method for calculating the speed transfer characteristics of
Study on speed transfer characteristics of double-deck ball bearings is proposed. The generalized four-terminal parameter method is applied to the speed analysis of double-deck ball bearings for the first time. The quasi-static theory, elastohydrodynamic lubrication theory, and generalized four-terminal parameter theory are combined to establish the speed transfer analysis model of double-deck ball bearings and analyze the effects of different working conditions, structural parameters, and lubrication parameters on the speed transfer ratio. In Sect. 4, build a testbed to test the speed transmission characteristics of double-deck ball bearings, and compare with the theoretical results to verify the correctness of the method proposed in this paper. Section 5 concludes the paper.

### 2 Mechanical characteristics and elastohydrodynamic lubrication analysis of double-deck ball bearing

#### 2.1 Mechanical analysis of double-deck ball bearing

##### 2.1.1 Mechanical model

The mechanical model of the double-deck ball bearing is shown in Fig. 1. The double-deck ball bearing is composed of primary and secondary ball bearings. Assuming that the bearing is subjected to radial force and axial force, the force balance equations are shown in formula (1).

\[
\begin{align*}
Q_{1m} \cos \alpha_{1m} - Q_{2m} \sin \alpha_{2m} - M_{im} \frac{\dot{z}_{1m}}{D_{ib}} (\dot{\alpha}_{1m} \cos \alpha_{1m} - \dot{\alpha}_{2m} \cos \alpha_{2m}) &= 0 \\
Q_{1m} \sin \alpha_{1m} + F_{im} - Q_{2m} \cos \alpha_{2m} + M_{im} \frac{\dot{z}_{1m}}{D_{ib}} (\dot{\alpha}_{1m} \sin \alpha_{1m} - \dot{\alpha}_{2m} \sin \alpha_{2m}) &= 0 \\
Q_{o1n} \sin \alpha_{01n} - Q_{o2n} \sin \alpha_{02n} - M_{on} \frac{\dot{z}_{o1n}}{D_{ob}} (\dot{\alpha}_{o1n} \cos \alpha_{o1n} - \dot{\alpha}_{o2n} \cos \alpha_{o2n}) &= 0 \\
Q_{o1n} \cos \alpha_{01n} + F_{on} - Q_{o2n} \cos \alpha_{o2n} + M_{on} \frac{\dot{z}_{o1n}}{D_{ob}} (\dot{\alpha}_{o1n} \sin \alpha_{o1n} - \dot{\alpha}_{o2n} \sin \alpha_{o2n}) &= 0
\end{align*}
\]

\[
\begin{align*}
\sum_{m=1}^{\infty} \left[ Q_{1m} \cos \alpha_{1m} + \frac{\dot{z}_{1m} M_{im}}{D_{ib}} \sin \alpha_{1m} \right] \cos \varphi_{im} &= F_r \\
\sum_{m=1}^{\infty} \left[ Q_{1m} \sin \alpha_{1m} - \frac{\dot{z}_{1m} M_{im}}{D_{ib}} \cos \alpha_{1m} \right] &= F_a \\
\sum_{n=1}^{\infty} \left[ Q_{o1n} \cos \alpha_{o1n} + \frac{\dot{z}_{o1n} M_{on}}{D_{ob}} \sin \alpha_{o1n} \right] \cos \varphi_{on} &= 0 \\
= &\sum_{m=1}^{\infty} \left[ Q_{2m} \cos \alpha_{2m} + \frac{\dot{z}_{2m} M_{im}}{D_{ib}} \sin \alpha_{2m} \right] \cos \varphi_{im} \\
\sum_{n=1}^{\infty} \left[ Q_{o1n} \sin \alpha_{o1n} - \frac{\dot{z}_{o1n} M_{on}}{D_{ob}} \cos \alpha_{o1n} \right] &= 0 \\
= &\sum_{m=1}^{\infty} \left[ Q_{2m} \sin \alpha_{2m} - \frac{\dot{z}_{2m} M_{im}}{D_{ib}} \cos \alpha_{2m} \right]
\end{align*}
\]
where $F_{im}$ and $F_{nm}$ are the centrifugal force of the rolling elements of the primary and secondary bearings; $M_{im}$ and $M_{nm}$ are the gyro moment of the rolling elements of the primary and secondary bearings; $Q_{1m}, Q_{2m}, Q_{1n}, Q_{2n}$ are the contact force between the rolling elements of the primary and secondary bearings and the inner and outer rings; $a_{i0}$ and $a_{o0}$ are the initial contact angle of primary and secondary bearings; $x_{i1m}, x_{i2m}, x_{i1n}, x_{i2n}$ are the contact angle between the rolling elements of primary and secondary bearings and the inner and outer rings; $\varphi_{im}$ and $\varphi_{on}$ are the position angle of primary and secondary bearing rolling elements; $\Delta_{ir}, \Delta_{io}, \Delta_{or}, \Delta_{on}$ are the radial and axial clearance of primary and secondary bearing.

2.1.2 Geometric model

The deformation coordination relationship of the double-deck ball bearing is shown in Fig. 2. The figure shows the schematic diagram of the relative position relationship between the inner and outer raceway curvature center and the rolling element center at the $m$ and $n$ rolling elements before and after the first and second stage bearings being loaded under radial load and axial load. After being loaded, the outer ring of the second stage bearing is fixed on the bearing seat, so it can be approximately considered that the central position $O_{o2n}$ of the raceway curvature of the first stage
bearing does not change before and after being loaded, and the central position of the inner race curvature of the inner race changes from $O_{o1n}$ to $O'_{o1n'}$. Because the double-deck ball bearing has an intermediate race and one more layer of bearings than the traditional single-layer bearing, the central position of the rolling element of the second stage bearing changes from $O_{o2m}$ to $O'_{o2m'}$, and the central position of the outer race curvature of the first stage bearing changes from $O_{o2m}$ to $O'_{o2m'}$. The curvature center of the inner race of the first stage bearing changes from $O_{i1m}$ to $O'_{i1m'}$, and the center position of the rolling element of the first stage bearing changes from $O_{ibm}$ to $O'_{ibm}$, which deviates from the connecting line of the curvature center of the inner and outer rings, resulting in the contact angle between the inner ball and the inner and outer rings no longer being equal. The deformation coordination relationship is shown in Formula (2).

\[
\begin{align*}
\lambda_{i2m} &= \sqrt{(A_{i2m} - \lambda_{o1a} + \Delta_{o1})^2 + (B_{i2m} - (\lambda_{o1} - \Delta_{o1a}) \cos \phi_{i1m})^2} - (f_{i2} - 0.5)D_{ib} \\
\lambda_{i1m} &= \sqrt{(A_{i1m} - A_{i1n})^2 + (B_{i1m} - B_{i1m})^2} - (f_{i1} - 0.5)D_{ib} \\
\lambda_{o2n} &= \sqrt{A_{o2n}^2 + B_{o2n}^2} - (f_{o2} - 0.5)D_{ob} \\
\lambda_{o1n} &= \sqrt{(A_{o1n} - A_{o1n})^2 + (B_{o1n} - B_{o1n})^2} - (f_{o1} - 0.5)D_{ob}
\end{align*}
\]

where $f_{i1}, f_{i2}, f_{o1}, f_{o2}$ are the radius of curvature coefficient of inner and outer ring grooves of primary and secondary bearings; $\lambda_{i1m}, \lambda_{i2m}, \lambda_{o1m}, \lambda_{o2n}$ are the contact deformation of primary and secondary bearings; $\Delta_{o1}, \Delta_{o1}, \Delta_{o1a}$ are the radial and axial clearance of primary and secondary bearings; $\lambda_{i1}, \lambda_{i2}, \lambda_{o1}, \lambda_{o2n}$ are the radial and axial displacement of primary and secondary bearings; $A_{i1m}, B_{i1m}, A_{o1n}, B_{o1n}$ are the axial projection and radial projection between the curvature center of the outer ring of the primary and secondary bearings and the ball center; $A_{i2m}, B_{i2m}, A_{o2n}, B_{o2n}$ are the axial projection and radial projection between the curvature centers of the inner ring and outer ring of the primary and secondary bearings.

2.2 Elasto-hydrodynamic lubrication analysis of double-deck ball bearing

Solving the elasto-hydrodynamic lubrication model requires simultaneous Reynolds equation, oil film thickness equation, lubricant density, viscosity and load-bearing equation, load balance equation, and energy equation. The calculation equation is shown in formula (3).

\[
\begin{align*}
\frac{\partial}{\partial x} \left[ \left( \frac{\rho}{\eta} \right) \frac{h^3}{\partial} \frac{p}{\partial} \right] + \frac{\partial}{\partial y} \left[ \left( \frac{\rho}{\eta} \right) \frac{h^3}{\partial} \frac{p}{\partial} \right] &= 12U_c \frac{\partial}{\partial x} \left( \rho e \right) \\
h(x, y, t) &= h_0(t) + \frac{x^2 + y^2}{2R} + \frac{y^2}{2R_y} \\
\rho &= \rho_0 \left[ 1 + C_1 p/(1 + C_2 p) - C_3 (T_r - T_0) \right] \\
\eta &= \eta_0 \exp \left( \left( \ln \eta_0 + 9.67 \right) \right) \\
\int \int \rho(x, y) \frac{p(x, y) dx dy}{\Omega} &= w \\
\int \int \frac{\partial T_r}{\partial x} \frac{\partial p}{\partial x} + \frac{\partial T_r}{\partial y} \frac{\partial p}{\partial y} - \left( \frac{\partial}{\partial x} \int_0^z \rho udz + \frac{\partial}{\partial y} \int_0^z \rho vdz \right) \right] = k \\
\frac{\partial^2 T_r}{\partial x^2} + \frac{\partial^2 T_r}{\partial y^2} &= \frac{\partial}{\partial x} \left( \frac{\rho e}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho e}{\partial y} \right) \\
\rho e &= \frac{\rho e}{\partial z} + \frac{\partial^2 T_r}{\partial z^2}
\end{align*}
\]

where $E'$ is the equivalent elastic modulus of the inner ring and rolling element; $h_0(t)$ is the film thickness at the center of a rigid body; $\rho_0$ is the density of the lubricant at room temperature; $C_1$ and $C_2$ are the compaction coefficient; $T_0$ is the inlet temperature; $T_r$ is the oil film temperature between the bearing inner ring and the rolling element; $z_0$ is the viscosity pressure coefficient; $\eta_0$ is the viscosity-temperature coefficient; $\eta$ is the viscosity-temperature coefficient at room temperature; $\beta$ is the viscosity-temperature coefficient at room temperature; $w$ is the radial load borne by the ball; The boundary condition of the energy equation is satisfied at the inlet boundary of the calculation domain $T(x_{in}, y, z) = T_0$; $c$ is the specific heat; $u, v$ is the lubricating oil and the velocity of the lubricating oil along the direction of x and y; $k$ is the thermal conductivity of lubricating oil, solid energy equation references [20–23].

Oil film stiffness calculation [24]

\[
K_{oil} = \frac{dw}{dh}
\]

Oil film damping calculation [25]

\[
C_f = \frac{\Delta w}{U_e} = \frac{1}{U_e} \int_A p(x, y) dx dy
\]
3 Speed transfer model and calculation of double-deck ball bearing

3.1 Transmission mechanism analysis of double-deck ball bearing

The double-deck ball bearing is composed of two-stage bearing. Assuming that the bearing is subjected to radial and axial forces, the force and speed are transmitted to the rolling body of the primary ball bearing through the inner ring of the primary ball bearing, and the rolling body of the primary ball bearing is transmitted to the outer ring of the primary ball bearing, the outer ring of the primary ball bearing is transmitted to the adapter ring, the adapter ring is transmitted to the inner ring of the secondary ball bearing and continues to be transmitted to the rolling body of the secondary ball bearing, and the rolling element of the secondary ball bearing is transmitted to the outer ring of the bearing to complete the transmission of a process. The transmission diagram is shown in Fig. 3.

3.2 Establishment of speed transfer model of double-deck ball bearing

Generally, a dynamic system can be simplified as a combination of three linear elements: mass element, spring element, and damping element. To sum up, the double-deck ball bearing is equivalent to the transfer concentration model and expressed by the mass element, spring element, and damping element. The inner ring of the first stage bearing is equivalent to the mass element, the rolling element of the primary ball bearing is equivalent to the spring and damping element, and the outer ring of the primary ball bearing is equivalent to the mass element. Because the material of the adapter ring can be replaced, treating the adapter ring as a mass element will bring errors to the study of the transmission characteristics of the whole system. To accurately analyze a complete transmission process, the adapter ring is equivalent to a spring and damping element. The inner ring of the second bearing is equivalent to the mass element, the rolling element of the secondary ball bearing is equivalent to the spring and damping element, and the outer ring of the secondary ball bearing is regarded as a rigid body because it is fixed on the bearing seat. The speed transmission equivalent analysis model which applies to the working condition under axial force and radial force is established, as shown in Fig. 4.

3.3 Analysis method of transmission characteristics of double-deck ball bearing

The modeling methods of mechanical structure dynamic systems include finite element method, multi stiffness method, four-terminal parameter method, impedance/admittance synthesis method, modal impedance method, dynamic condensation method, transfer wave method, etc. Because some methods need to establish differential equations of motion, the establishment of differential equations of motion for double-deck ball bearings becomes very cumbersome and difficult to solve. The mechanical impedance and admittance of mass, spring and damping elements can be written, so the system can be simulated and expressed by the electrical system. The four-terminal parameter method is the concept of network theory in the electrical system. The differential equation form of the four-terminal parameter method is consistent with that of the differential equation of motion in a mathematical expression. The four-terminal parameter method has many advantages: ①The system can be analyzed separately. The calculation of the four-terminal parameters of the system is related to its dynamic characteristics and has nothing to do with the front and rear structure of the system; ②The calculation is in the form of the matrix, which is relatively simple to solve the differential equation; ③The influence of nonrigidity of substructure...
can be considered in the calculation. Therefore, the four-terminal parameter method is used to analyze the transfer model.

### 3.3.1 The four-terminal parameter method

The four-terminal parameter method can be expressed as a single input and single output system as shown in Fig. 5. Point 1 is the input of the system and point 2 is the output of the system. It is the input force of the system, the input speed of the system, the output force of the system, and the output speed of the system. The expression form of the four-terminal parameters of the system is shown in formula (6):

\[
\begin{bmatrix}
F_{in}(i\omega) \\
V_{in}(i\omega)
\end{bmatrix} = \begin{bmatrix}
z_{ii}(i\omega) & z_{io}(i\omega) \\
z_{oi}(i\omega) & z_{oo}(i\omega)
\end{bmatrix} \begin{bmatrix}
F_{out}(i\omega) \\
V_{out}(i\omega)
\end{bmatrix}
\]  

(6)

where \(z\) is the four-terminal parameter.

The four-terminal parameter expression of the mass element is:

\[
\begin{bmatrix}
z_{ii}(i\omega) & z_{io}(i\omega) \\
z_{oi}(i\omega) & z_{oo}(i\omega)
\end{bmatrix} = \begin{bmatrix} 1 & i\omega M_r \\ 0 & 1 \end{bmatrix}
\]  

(7)

where \(M_r\) is the mass of the rigid body.

The four end-parameter expression of the spring element is:

\[
\begin{bmatrix}
z_{ii}(i\omega) & z_{io}(i\omega) \\
z_{oi}(i\omega) & z_{oo}(i\omega)
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ i\omega/K_s & 1 \end{bmatrix}
\]  

(8)

where \(K_s\) is the stiffness of the spring.

The four-terminal parameter expression of the damping element is:

\[
\begin{bmatrix}
z_{ii}(i\omega) & z_{io}(i\omega) \\
z_{oi}(i\omega) & z_{oo}(i\omega)
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/C_d & 1 \end{bmatrix}
\]  

(9)

where \(C_d\) is the damping coefficient.

The schematic diagram of the subsystem in series is shown in Fig. 6.

\[
\begin{bmatrix}
F_{in} \\
V_{in}
\end{bmatrix} = \begin{bmatrix} \alpha_{ii} & \alpha_{io} \\ \alpha_{oi} & \alpha_{oo} \end{bmatrix} \begin{bmatrix} F_{mid} \\
V_{mid}
\end{bmatrix}
\]  

(10)

After the system is connected in series, the four-terminal parameters are expressed as:

\[
\begin{bmatrix}
F_{in} \\
V_{in}
\end{bmatrix} = \begin{bmatrix} \gamma_{ii} & \gamma_{io} \\ \gamma_{oi} & \gamma_{oo} \end{bmatrix} \begin{bmatrix} F_{out} \\
V_{out}
\end{bmatrix}
\]  

(12)
The expression of the parallel subsystem is shown in Fig. 7:

According to Kirchhoff’s law in the circuit, when applied to the mechanical system:

\[
\begin{align*}
V_{in} &= V_{mid_{-1i}} = V_{mid_{-2i}} \\
V_{out} &= V_{mid_{-1o}} = V_{mid_{-2o}} \\
F_{in} &= F_{mid_{-1i}} + F_{mid_{-2i}} \\
F_{out} &= F_{mid_{-1o}} + F_{mid_{-2o}}
\end{align*}
\]  

\[ (13) \]

The four-terminal parameter expression of a single subsystem can refer to the formula (6). The four-terminal parameters of a parallel system can be obtained:

\[
\begin{bmatrix}
\gamma_{ii} & \gamma_{io} \\
\gamma_{oi} & \gamma_{oo}
\end{bmatrix} = \begin{bmatrix}
\frac{(\alpha_{oi}\beta_{oi} + \alpha_{oi}\beta_{oi})}{(\alpha_{oi} + \beta_{oi})} & \frac{(\alpha_{oi}\beta_{oi} + \alpha_{oi}\beta_{oi})(\alpha_{oi}\beta_{oi} + \alpha_{oi}\beta_{oi})}{(\alpha_{oi} + \beta_{oi})} \\
\frac{(\alpha_{oi}\beta_{oi})}{(\alpha_{oi} + \beta_{oi})} & \frac{(\alpha_{oi}\beta_{oi} + \alpha_{oi}\beta_{oi})}{(\alpha_{oi} + \beta_{oi})}
\end{bmatrix}
\]

\[ (14) \]

3.3.2 The generalized four-terminal parameter method

From the above description of the four-terminal parameter method, it can be seen that the four-terminal parameter method applies to the calculation of a single direction. However, in practical engineering calculation, when the input of the application system on many occasions is more than one direction, the error of the traditional four-terminal parameter method is relatively large, and even wrong results appear. Therefore, the generalized four-terminal parameter method is used to solve the double-deck ball bearing. The generalized
The four-terminal parameter method is an extension of the four-terminal parameter method. Its basis is still based on the four-terminal parameter theory. It only considers the input force and direction in multiple directions.

The series form of multi-input and the multi-output system is shown in Fig. 8. The total generalized four-terminal parameter expression after the series has the same form as the four-terminal parameters. The total series system is equal to the continuous multiplication of each subsystem. The generalized four-terminal parameter expression after the series connection is shown in formula (15):

$$\begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} = \begin{bmatrix} a_{ii} & a_{io} \\ a_{oi} & a_{oo} \end{bmatrix} \begin{bmatrix} \beta_{ii} & \beta_{io} \\ \beta_{oi} & \beta_{oo} \end{bmatrix} \begin{bmatrix} F_{out} \\ V_{out} \end{bmatrix}$$

(15)

The parallel form of the multi-input and multi-output system is shown in Fig. 9, and the generalized four-terminal parameter expression of each subsystem after parallel is shown in formula (16):

$$\begin{bmatrix} \gamma_{ii} \\ \gamma_{io} \\ \gamma_{oi} \\ \gamma_{oo} \end{bmatrix} = \begin{bmatrix} \alpha_{ii} (\beta_{oi}^{-1} + 1)^{-1} + \beta_{ii} (\alpha_{oi}^{-1} + 1)^{-1} + (\beta_{ii} - \alpha_{ii})(\alpha_{oi} - \beta_{oi})^{-1}(\alpha_{oo} + \beta_{oo} + \alpha_{oo} + \beta_{oo}) \\ \alpha_{oi} (\beta_{oi}^{-1} + 1)^{-1} + \beta_{oi} (\alpha_{oi}^{-1} + 1)^{-1} + (\beta_{ii} - \alpha_{ii})(\alpha_{oi} - \beta_{oi})^{-1}(\alpha_{oi} + \beta_{oi} + \alpha_{oi} + \beta_{oi}) \\ \alpha_{oi} (\beta_{ii}^{-1} + 1)^{-1} + \beta_{ii} (\alpha_{oi}^{-1} + 1)^{-1} + (\beta_{ii} - \alpha_{ii})(\alpha_{oi} - \beta_{oi})^{-1}(\alpha_{oi} - \beta_{oi} + \alpha_{ii} + \beta_{ii}) \\ \alpha_{oo} (\beta_{oo}^{-1} + 1)^{-1} + \beta_{oo} (\alpha_{oo}^{-1} + 1)^{-1} + (\beta_{ii} - \alpha_{ii})(\alpha_{oi} - \beta_{oi})^{-1}(\alpha_{oi} + \beta_{oi} + \alpha_{oi} + \beta_{oi}) \end{bmatrix}$$

(16)
As mentioned above, the generalized four-terminal parameter model of double-deck ball bearing is established, as shown in Fig. 10.

According to the transmission model of the system, the transmission relationship between the following system parameters is established, as shown in formulas (17–22). It should be noted that the transmission speed calculated by the system is linear, while the bearing input speed is rotational speed, which needs to be transformed into the formula for the corresponding calculation.

\[
\begin{bmatrix}
F_{in,y} \\
F_{in,z} \\
V_{in,y} \\
V_{in,z}
\end{bmatrix} =
\begin{bmatrix}
1 & i\omega M_{in} & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
F_{in1,y} \\
F_{in1,z} \\
V_{in1,y} \\
V_{in1,z}
\end{bmatrix}
\]

(17)

\[
\begin{bmatrix}
F_{in2,y} \\
F_{in2,z} \\
V_{in2,y} \\
V_{in2,z}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & i\omega M_{mid_1} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
F_{in3,y} \\
F_{in3,z} \\
V_{in3,y} \\
V_{in3,z}
\end{bmatrix}
\]

(19)

\[
\begin{bmatrix}
F_{in3,y} \\
F_{in3,z} \\
V_{in3,y} \\
V_{in3,z}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
F_{in4,y} \\
F_{in4,z} \\
V_{in4,y} \\
V_{in4,z}
\end{bmatrix}
\]

(20)

\[
\begin{bmatrix}
F_{in4,y} \\
F_{in4,z} \\
V_{in4,y} \\
V_{in4,z}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & i\omega M_{mid_2} & 0 \\
0 & 1 & 0 & i\omega M_{mid_2} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
F_{in5,y} \\
F_{in5,z} \\
V_{in5,y} \\
V_{in5,z}
\end{bmatrix}
\]

(21)

\[
\begin{bmatrix}
F_{in5,y} \\
F_{in5,z} \\
V_{in5,y} \\
V_{in5,z}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
F_{in6,y} \\
F_{in6,z} \\
V_{in6,y} \\
V_{in6,z}
\end{bmatrix}
\]

(22)

where \(F_{in,y}\) and \(V_{in,y}\) are the input radial force and radial velocity; \(F_{in,z}\) and \(V_{in,z}\) are the input axial force and axial speed; \(F_{in1,y}, F_{in2,y}, F_{in3,y}, F_{in4,y}, F_{in5,y}, F_{in6,y}\) and \(F_{in7,y}\) are the radial forces transmitted in the process; \(F_{in1,z}, F_{in2,z}, F_{in3,z}, F_{in4,z}, F_{in5,z}\) and \(F_{in6,z}\) are the axial force transmitted in the process; \(V_{in1,y}, V_{in2,y}, V_{in3,y}, V_{in4,y}, V_{in5,y}\) and \(V_{in6,y}\) are the radial velocity transmitted in the process; \(V_{in1,z}, V_{in2,z}, V_{in3,z}, V_{in4,z}, V_{in5,z}\) and \(V_{in6,z}\) are the axial velocity transmitted in the process; \(K_{ly}\) and \(K_{lz}\) are the radial and axial stiffness of the primary bearing rolling element; \(C_{ly}\) and \(C_{lz}\) are the radial damping and axial damping of the primary bearing rolling element; \(K_{2y}\) and \(K_{2z}\) are the radial and axial stiffness of the adapter ring, and \(C_{2y}\) is the radial damping and axial damping.
of the adapter ring, $K_{3z}$ is the radial and axial stiffness of the rolling element of the secondary bearing, and $C_{3z}$ is the radial damping and axial damping of the rolling element of the secondary ball bearing.

From Sect. 2, the radial and axial equivalent stiffness and equivalent damping of the primary and secondary bearings can be obtained, i.e., $K_{1y}$, $K_{1z}$, $C_{1y}$, $C_{1z}$, $K_{3y}$, $K_{3z}$, $C_{3y}$, and $C_{3z}$ are known. The stiffness and damping data of the adapter ring material are obtained from the following experiments.

3.4 Measurement and calculation of stiffness and damping of adapter ring of double-deck ball bearing

The stiffness and damping of the adapter ring are calculated by the test. The electronic universal testing machine is used to test the performance of the sample. Figure 11 shows the sample under test. The samples are tested in two directions, respectively. The measured stress–strain curves of the three adapter rings are...
shown in Fig. 12. The stress and strain from point a to point B meet Hooke’s law, and the test pieces are unloaded at point C. The test pieces are loaded and unloaded for one cycle and get the stress–strain relationship loop of adapter rings of different materials. The area surrounded by the loop can calculate the strain energy consumed by the test sample per unit volume in the process of loading and unloading. In the process of one cycle, the maximum energy stored is 

$$H = \frac{r_0^2}{2},$$

in which $r_0$ is the stress amplitude and $r_0$ is the strain amplitude.

The damping ratio is calculated according to the formula $J = H/\Theta$, and the damping factor of the material is calculated according to the formula $C_{\text{mid}} = J/2\pi$.

The stiffness and damping of the adapter ring calculated by the above method are shown in Table 1.

### Table 1  Material test data of adapter ring

| Material quality   | Radial stiffness (N/m) | Axial stiffness (N/m) | Radial damping (N·s/m) | Axial damping (N·s/m) |
|-------------------|------------------------|-----------------------|------------------------|-----------------------|
| 45# steel         | 4.60E + 05             | 1.71E + 07            | 0.926                  | 0.026                 |
| 7075 Aluminum alloy | 2.24E + 05             | 1.66E + 07            | 0.121                  | 0.01187               |
| TC4 Titanium alloy | 7.60E + 05             | 3.24E + 07            | 0.06                   | 0.011                 |

3.5 Numerical calculation of speed transfer model of double-deck ball bearing

The contact stiffness of double-deck ball bearing is calculated by quasi-static theory, and the oil film stiffness calculated by elastohydrodynamic lubrication theory is calculated in series and parallel. The calculation formulas are shown in (23, 24, 25, 26, 27, 28, 29 and 30).

The equivalent stiffness of the contact between the primary bearing rolling element and the inner and outer rings is calculated as follows:

$$K_{i1m} = \frac{(K_n)_{1m} \times (K_i)_{1m}}{(K_n)_{1m} + (K_i)_{1m}}$$  \hspace{1cm} (23)

$$K_{i2m} = \frac{(K_n)_{2m} \times (K_i)_{2m}}{(K_n)_{2m} + (K_i)_{2m}}$$  \hspace{1cm} (24)

(a) Axial test of 45# steel; (b) Axial test of 7075 aluminum alloy; (c) Radial test of TC4 titanium alloy

Fig. 12 Stress–strain curve of adapter ring specimen
where $K_{1m}$ and $K_{2m}$ are the equivalent stiffness of the contact between the $m$ rolling body and outer ring of the primary bearing; $(K_n)_{1m}$ and $(K_n)_{2m}$ are the contact stiffness of the contact between the $m$ rolling body and the outer ring; $(K_f)_{1m}$ and $(K_f)_{2m}$ are the oil film stiffness of the contact between the $m$ rolling body and the outer ring; the values of contact stiffness and oil film stiffness are calculated in Sect. 2.

The equivalent stiffness of the contact between the second bearing rolling element and the inner and outer rings is calculated as follows:
The high-speed performance of the bearing is evaluated by the $D_m n_i$ value (the product of bearing pitch diameter $D_m$ and speed $n_i$, mm·r/min). To verify the applicability of the method proposed in this paper for high-speed and low-speed conditions, this paper selects the calculation condition of speed as 10,000–40000r/min, spanning low-speed to high-speed conditions. In the same state, the smaller the ball diameter is, the higher the limit speed of the bearing is. If the large ball diameter bearing is selected for the test, the requirements of the working condition of high speed cannot be met. Therefore, the small ball diameter bearing is selected to meet the requirements of calculation and test in this paper. The parameters of double-decker ball bearings are shown in Table 2.

4.2 Method verification and transmission characteristics of the speed transmission model of the double-deck ball bearing

4.2.1 Analysis of the influence of working condition parameters on speed transmission characteristics

As shown in Fig. 14, the variation in speed transfer ratio with speed is shown. It can be seen from the figure that the transmission rate decreases with the increase in rotating speed. This is because, on the one hand, as the speed increases, the contact angle of the inner contact of the first and second stage bearings of the double-deck ball bearing continues to increase, while the contact angle of the outer contact continues to decrease. The axial contact stiffness is consistent with the sinusoidal change trend of the contact angle, increasing the axial stiffness of the inner contact of the first and second stage bearings, the decrease in the axial stiffness of the outer contact, and the decrease in the axial stiffness under the combined action. On the other hand, with the increase in speed, the oil film thickness increases, and the oil film stiffness is the derivative of the load to the film.
Therefore, with the increase in speed, the film thickness increases, and the oil film stiffness decreases. Damping is calculated by dividing the damping force by the speed, and the resistance is defined as the change of oil film load caused by the speed. Therefore, as the speed increases, the damping decreases. The combined effect of axial contact stiffness, oil film stiffness, and damping reduces the speed transfer ratio. This also explains the phenomenon that the transmission rate decreases with the increase in rotating speed in literary theory. In the figure, the transmission rate calculated by the Har Prashad method is 6.11% [8], and the transmission rate calculated by Yu [26] method is 5.76%, while the transmission rate of the method proposed in this paper is between 2.5 and 6.5%, and the transmission law of theoretical calculation is consistent with the law of experiment in literature, which can verify the correctness of this method. Next, according to the method proposed in this paper, the law of speed transfer ratio under the influence of multiple factors is studied.

**Change with axial force** As shown in Fig. 15, the changing trend of speed transfer ratio with axial force is shown. It can be seen from the figure that under different speeds, the speed transfer ratio increases with the increase in axial force. This is because, on the one hand, with the increase in axial force, the axial stiffness of the primary and secondary bearings changes in direct proportion, increasing axial stiffness. On the other hand, at the same speed, with the increase in axial load, the pressure and temperature of oil film increase, and the film thickness decreases, increasing oil film stiffness and damping. Under the comprehensive function, the speed transfer ratio increases, and the changing trend increases with the increase in speed.

**4.2.2 Analysis of the influence of structural parameters on speed transmission characteristics**

**Change with the rolling element material** Fig. 16 shows the relationship between the speed transfer ratio and the rolling element material. It can be seen from the figure that the transfer ratio of the primary and secondary rolling elements made of ceramic is the largest, the first level is made of ceramic, the second level made of steel is similar to that of all-ceramic, the
The first level is made of steel, and the second level made of ceramic is similar to that of all steel. This is because, on the one hand, the elastic modulus of ceramics is larger than that of steel, and the contact stiffness of the combination of steel for the primary rolling element and ceramic for the second rolling element is greater than that of the combination of all steel rolling elements. The primary bearing bears the main working speed in the two-stage series bearing, while the speed transmitted to the second bearing is relatively small, so the contact stiffness performance of ceramic steel combination for the primary and secondary rolling elements is better than that of steel ceramic combination. The contact stiffness of the two-stage rolling elements made of ceramic materials is the largest. On the other hand, the density of ceramic material is smaller than that of steel. At the same speed, the centrifugal force generated is smaller than that of steel, and the film thickness increases. When the rolling element of secondary bearing is made of steel, and the rolling element of primary bearing is made of ceramic, its oil film stiffness is smaller than that of steel, and the damping is smaller than that of steel; when the rolling element of the primary bearing is made of steel, and the second bearing is made of ceramic, the oil film stiffness is less than that of steel, and its damping is also less than that of steel. The first and second stages adopt ceramic combinations, with the largest contact stiffness and the smallest oil film stiffness and damping, while the value of contact stiffness is greater than the oil film stiffness. Under the comprehensive action, the speed transfer ratio of ceramic and ceramic combination is the largest, and the material of the second stage bearing has little effect on the speed transfer ratio. Similarly, the changing trend caused by different speeds is greater than that of axial force. In the actual design process, if tend to increase the speed transfer ratio, the rolling element of the primary bearing can be made of ceramic.

Change with groove radius of curvature Figs. 17 and 18 show the relationship between the speed transfer ratio and the groove curvature radius. With the increase in the groove curvature radius of the primary bearing, the speed transfer ratio decreases. This is because, on the one hand, the contact deformation increases with the increase in the radius of curvature of the groove, so the radial contact stiffness and axial contact stiffness decrease. On the other hand, with the increase in groove curvature radius, the comprehensive curvature radius decreases, the contact angle decreases, the load increases, and the contact area decreases, so that the contact stress increases, the oil film pressure increases, the film thickness decreases, the lubrication performance becomes worse, and the oil film stiffness and damping increase. Under the combined action, the equivalent stiffness decreases and the speed transfer ratio decreases. With the increase in the speed, the gap between the speed transfer ratio becomes larger and larger and with the increase in the axial force. The difference in speed transfer ratio changes little. With the change of the
curvature of the secondary bearing groove, the speed transfer ratio is unchanged and can be ignored.

Change with initial contact angle Figs. 19 and 20 show the relationship between the speed transfer ratio and the initial contact angle. With the increase in the initial contact angle, the axial stiffness increases, the oil film stiffness decreases, and the damping decreases. This is because, on the one hand, with the increase in the initial contact angle, the contact force between the rolling element and the inner and outer rings to balance the axial load performance of the bearing is better, resulting in the reduction in axial deformation and the increase in radial deformation, resulting in the reduction in the radial stiffness of the primary and secondary bearings and the increase in the axial stiffness. On the other hand, as the initial contact angle increases, the contact load decreases, the minimum film thickness of the oil film increases, the lubrication performance becomes better, and the maximum pressure decreases. Under the combined action, the equivalent stiffness increases, and the speed transfer ratio increases. With the increase in the speed, the gap of the speed transfer ratio becomes larger and larger and with the increase in the axial force. The difference in speed transfer ratio changes little. The change of the contact angle of the second
bearing has little effect on the speed transfer ratio, which can be ignored.

**Change with the number of rolling elements** Fig. 21 shows the relationship between the speed transfer ratio and the number of primary bearing rolling elements. It can be seen from the figure that with the increase in the number of primary bearing rolling elements, the speed transfer ratio decreases. This is because, on the one hand, with the increase in the number of rolling elements, the contact load and contact angle of the inner ring decrease, the radial stiffness of the internal and external contact of the primary and secondary bearings increases, and the axial stiffness decreases but changes little. On the other hand, when the number of rolling elements increases, the load on a single rolling element decreases, the pressure between the rolling element and the inner and outer rings decreases, the maximum contact stress decreases, and the oil film thickness increases, resulting in the reduction in oil film stiffness and damping. According to the calculation of generalized four end parameters, the transfer ratio decreases and the change rate of the transfer ratio increases with the increase in speed. With the increase in axial force, the change rate is not
obvious. When the number of rolling elements changes, the speed has a greater impact on the speed transfer ratio than the axial force.

Figure 22 shows the relationship between the speed transfer ratio and the number of secondary bearing rolling elements. It can be seen from the figure that the speed transfer ratio remains unchanged with the change in the number of secondary bearing rolling elements, which shows that changing the number of secondary bearing rolling elements has little effect on the speed transfer ratio.

Change with rolling element diameter Figs. 23 and Fig. 24 show the relationship between the speed transfer ratio and the rolling element material. With the increase in rolling element diameter, the speed transfer ratio decreases. This is because, on the one hand, the increase in the diameter of the rolling elements reduces the number of rolling elements, and the contact load borne by each rolling element increases, which increases the contact deformation of the rolling elements on the inner and outer rings and reduces the contact stiffness. However, the larger the
diameter of the rolling elements, the larger the area of the contact point on the inner and outer rings, which is conducive to improving the contact stiffness inside the bearing. Under the combined influence of the two factors, the axial stiffness decreases. On the other hand, with the increase in rolling element diameter, the equivalent radius of elastohydrodynamic lubrication calculation decreases, the angular velocity remains unchanged, the linear velocity decreases, and the oil film thickness decreases, increasing oil film stiffness and damping. Due to the structural characteristics of speed transmission, the oil film stiffness and damping of the second bearing are larger than those of the primary bearing. Under the combined action, the equivalent stiffness decreases, and the speed transfer ratio decreases.

Change with adapter ring material Fig. 25 shows the changing trend of the speed transfer ratio with the material of the adapter ring. The adapter ring is made of 45# steel, 7075 aluminum alloy, and TC4 titanium alloy. It can be seen from the figure that the speed transfer ratio of 7075 aluminum alloy is the largest and
that of TC4 titanium alloy is the smallest. To accurately calculate the transfer characteristics, different material characteristics of the adapter ring should be fully considered; when designing a double-deck ball bearing, it cannot be easily regarded as a definite parameter. The influence trend of rotating speed on the adapter ring is greater than that caused by different axial forces.

### 4.2.3 Analysis of the influence of lubrication parameters on speed transmission characteristics

As shown in Fig. 26, the speed transfer ratio changes with the viscosity of the lubricating oil of the primary bearing. For a concise description, the viscosity of the primary bearing is abbreviated as \( P \)-viscosity, and the viscosity of the second bearing is abbreviated as \( S \)-viscosity. It can be seen from the figure that the speed transfer ratio decreases with the increase in the viscosity of the lubricating oil. This is because the increase in viscosity will lead to an increase in film thickness.

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**Fig. 26** Change of speed transfer ratio with the viscosity of primary bearing lubricating oil

**Fig. 27** Change of speed transfer ratio with the viscosity of lubricating oil of secondary bearing
especially at high speed, the entrainment speed is relatively large, even if the lubricant with small viscosity is selected, a certain oil film can be formed, and the stiffness of the oil film decreases, and the damping slowly increases. Therefore, the speed transfer ratio is reduced.

As shown in Fig. 27, the viscosity of the secondary lubricating oil has little effect on the transmission mode of the bearing under the speed ratio.

4.3 Test verification of speed transmission characteristics of double-deck ball bearing

4.3.1 Test verification and result analysis of the influence of working condition parameters on speed transmission

The speed test bench of the double-deck ball bearing is shown in Fig. 28, which is used to verify the transmission characteristics. The testbed is composed of a driving system, loading device, test part, base, and protection device. It is assumed that the oil supply is sufficient in the
experiment, that is, intermittent oil supply is carried out during the experiment without considering the influence of lack of oil. To avoid the influence of indoor temperature on the viscosity of lubricating oil, the indoor temperature is controlled at about 23°C.

For the double-deck ball bearing to be tested, the angular contact ball bearings with models of 71901c and 71905c in the theoretical analysis part are selected, and the adapter ring is made of 45# steel, 7075 aluminum alloy, and TC4 titanium alloy. The three kinds of adapter ring samples processed are shown in Fig. 29. The assembled double-deck ball bearings made of three different adapter ring materials are shown in Fig. 30.

(1) Test and result analysis with axial force

In the test of changing with the axial force, the radial force is loaded to 200N, and four groups of tests are carried out under the working conditions of the same speed and different axial forces. To avoid some coincidence of the test data under the specific speed, seven groups of speed data are selected for the speed, and the test conditions are shown in Table 3. To reduce the experimental error, three tests are carried out for each group of working condition tests. When each group of tests reaches the required working condition and operates stably, read and record the speed data of the inner ring and intermediate race of the double-deck ball bearing, and bring it into the existing speed transfer ratio calculation formula to calculate the speed transfer ratio of each group of data. Because the results of the three tests are relatively close and stable, and there is no large deviation, it can be considered that the three groups of data can describe the test data of this working condition. Take the average of the three data to obtain the speed transfer ratio of the final test under this working condition, and obtain the value of the speed transfer ratio under each working condition. To show clearly and concisely in the figure, “B-M” stands for “By proposed method,” and “B-T” stands for “By test,” as shown in Fig. 31.

Figure 31 shows the comparison between the change of speed transfer ratio and the theoretical value with the change of axial force in the test. It can be seen from the figure that the trends of the experimental and theoretical models are highly similar, and there are some errors between the theoretical calculation and the measured speed transfer ratio in terms of numerical value. This is because in the actual test, due to a series of reasons such as assembly, processing, and environmental factors, the whole test system will have a certain eccentricity, which needs to offset part of the load compared with the theoretical calculation. Therefore, the test results deviate from the theoretical calculation results. The error between the measured speed transfer ratio and the theoretical calculation in reference [8] is within 26.09 ~ 30.43%, the error between the theoretical model and the measured value is within 27%, the value is within the acceptable range, and the transfer law is consistent with the theoretical calculation, which can verify the correctness of the theoretical model calculation in this paper.

(2) Test and result analysis of varying with rotating speed

![Double-decker Ball Bearing](image-url)
In the test varying with the speed, the test is carried out at the speeds of 10,000, 20,000, 30,000, and 40,000 r/min. Similarly, to avoid some coincidence of test data under specific stress, 13 groups of stress conditions are selected for the test, and the test conditions are shown in Table 4. Test each group of working conditions three times, record, and calculate the change of speed transfer ratio with speed under each group of working conditions, as shown in Fig. 32.
Figure 32 shows the changing trend of speed transfer ratio with speed. Whether the test results are acceptable can be evaluated from two aspects: ① The test trend is consistent with the theoretical trend; ② The error between the test data and the theory is within an acceptable range. Figure 32 shows that the changing trend of speed transfer ratio with speed is roughly consistent with the theoretical calculation. At the speed of 10000 r/min, the change of theoretical value is relatively small, but the theoretical data are an upward trend, which is in line with the test trend. In terms of test error, in the existing literature [8], the error of the existing method speed transfer ratio and the literature test is between 26.09 and 30.43%, while the test value of the speed transfer ratio calculated in this paper varies with the speed and the error of this method is within 27%, indicating that the error calculated in this paper is acceptable. The main reason for the error lies in that when the speed increases, the forward sliding phenomenon occurs. The sliding causes the speed of the middle ring to decrease, so the speed transmitted by the middle ring decreases, which is not calculated in the theoretical calculation, so there is a deviation between the theoretical calculation and the measured value.

4.3.2 Test verification and result analysis of the influence of structural parameters on speed transmission

(1) Test and result analysis of material change with rolling element

The single-layer bearings with the same model, but different materials are selected for combination, and the adapter ring is made of 45# steel, which is combined into four kinds of double-deck ball-bearing structures. The influence of the change of speed and axial force on the speed of the inner ring and middle ring of double-deck ball bearing is tested, respectively. Similarly, each working condition is tested three times, and the influence of the change of rolling element material on the speed transfer ratio is recorded and calculated. The results are shown in Fig. 33. It can be seen from the figure that the maximum error of the
(a)-1 $N_{bi}=14,N_{bo}=21$; (a)-2 $N_{bi}=15,N_{bo}=21$; (a)-3 $N_{bi}=16,N_{bo}=21$; (a)-4 $N_{bi}=17,N_{bo}=21$

(a) The change of speed transfer ratio with the number of primary rolling elements at different speeds

(b)-1 $N_{bi}=14,N_{bo}=21$; (b)-2 $N_{bi}=15,N_{bo}=21$; (b)-3 $N_{bi}=16,N_{bo}=21$; (b)-4 $N_{bi}=17,N_{bo}=21$

(b) The change of speed transfer ratio with the number of primary rolling elements under different axial forces

(c)-1 $N_{bi}=17,N_{bo}=18$; (c)-2 $N_{bi}=17,N_{bo}=19$; (c)-3 $N_{bi}=17,N_{bo}=20$; (c)-4 $N_{bi}=17,N_{bo}=21$

(c) The change of speed transfer ratio with the number of secondary rolling elements at different speeds

(d)-1 $N_{bi}=17,N_{bo}=18$; (d)-2 $N_{bi}=17,N_{bo}=19$; (d)-3 $N_{bi}=17,N_{bo}=20$; (d)-4 $N_{bi}=17,N_{bo}=21$

(d) The change of speed transfer ratio with the number of secondary rolling elements under different axial forces

Fig. 34 Comparison of measured speed transfer ratio with the number of rolling elements and theoretical value
measured data varying with speed is 24.4%. In the measured data varying with axial force, the maximum error is 18.7%, and the measured trend is consistent with the theoretical trend.

(2) Test and result analysis with the number of rolling elements

In practice, the structure is not easy to process due to the change in rolling element diameter and groove curvature radius, and the increase in rolling element diameter will cause the change of rolling element number, and the change of groove curvature radius will also cause the change of rolling element diameter and then cause the change of rolling element number. According to the theoretical calculation of speed transmission of double-deck ball bearing in Sect. 3, the changing trend of these three factors is the same. Therefore, in the experiment of structural parameters, the change of rolling element number, rolling element diameter, and groove curvature radius is analyzed and discussed together. The 14, 15, 16, and 17 rolling elements were selected for the primary bearing, and 18, 19, 20, and 21 rolling elements were selected for the secondary bearing. Under the working condition, the radial force is 200 N, the axial force is 200, 400, 600, and 800 N, and the rotating speed is 10000, 20000, 30000, and 40000 r/min. The experimental results are shown in Fig. 34.

As shown in Fig. 34, the influence of the number of measured rolling elements on the speed transfer ratio is shown. It can be seen from the figure that under coaxial load, the speed transfer ratio tends to decrease with the increase in the number of primary bearing rolling elements. At the same speed, with the increase in the number of primary bearing rolling elements, the speed transfer ratio also decreases. The trend and value of the measured value is similar to the
theoretical value. In the variation diagram with axial force, the maximum error is 18.7%, and in the variation diagram with rotating speed, the maximum error is 21%. In the numerical comparison, there is a certain error, which is because the effect of the cage is not considered in the theoretical calculation, but it is affected by the cage in the process of measurement, so there is a certain measurement error. The number of secondary bearing rolling elements has little effect on the speed transfer ratio, which is consistent with the theoretical calculation.

(3) Test and result analysis with the change of adapter ring material

The adapter ring is made of 45# steel, 7075 aluminum alloy, and TC4 titanium alloy. In the test conditions varying with the speed, the radial force and axial force are 200 N, and the speed is 10000r/min, 20000r/min, 30000r/min, and 40000r/min. In the test conditions that vary with the axial force, the speed is 10000r/min and the axial force is 200, 400, 600, and 800 N. Test each working condition three times, and record and calculate the speed transfer ratio data, as shown in Fig. 35.

As shown in Fig. 35, the influence law of the material of the adapter ring on the speed transfer ratio is measured. It can be seen from the figure that the speed transfer ratio of 7075 aluminum alloy is the largest, which is the same as the theoretical calculation trend. This is because, on the one hand, the adapter ring made of 7075 aluminum alloy has a relatively small density and is lightweight, so it is easier to drive, so the speed transfer ratio is better than 45# steel and TC4 titanium alloy. On the other hand, 7075 aluminum alloy has relatively large heat conductivity and better heat dissipation performance, and the viscosity of lubricating oil has less impact on 7075 aluminum alloy. Therefore, the speed transfer ratio of the 7075
aluminum-alloy adapter ring is the largest. There is a certain error between the measured value and the theoretical value, which is caused by the processing mode and assembly of the adapter ring, but the trend is in good agreement with the theoretical calculation.

4.3.3 Test verification and result analysis of the influence of lubrication parameters on speed transmission

In the test conditions varying with the speed, the radial force and axial force are 200 N, and the speed is 10000r/min, 20000r/min, 30000r/min, and 40000r/min. In the test conditions that vary with the axial force, the speed is 10000r/min and the axial force is 200, 400, 600, and 800 N. In the experiment, 4109, 4106, and 4011 aviation lubricating oils are selected, and their normal temperature dynamic viscosities are 0.033 Pa·s, 0.055 Pa·s, and 0.041 Pa·s, respectively. They are measured three times under each working condition. The calculated data for the speed transfer ratio are shown in Fig. 34.

As shown in Fig. 36, the influence of the viscosity change of the measured lubricating oil on the speed transfer ratio is shown. It can be seen from the figure that under coaxial load, the speed transfer ratio tends to decrease with the increase in the viscosity of the bearing lubricating oil. At the same speed, the speed transfer ratio also decreases. In the variation diagram with axial force, the maximum error is 18.99%, and in the variation diagram with speed, the maximum error is 18.74%. The trend of the measured value is close to the theoretical value. In the numerical comparison, there is a certain error, which is because the theoretical calculation is in the ideal environment, while in the measured data, there is a certain measurement error due to the influence of the test environment and temperature.

5 Conclusion

In this paper, the speed transfer analysis model of double-deck ball bearing is established, and the generalized four-terminal parameter method is applied to the speed transfer characteristics analysis of double-deck ball bearing for the first time. By combining the mechanical characteristics, elastohydrodynamic lubrication, and generalized four-terminal parameters of double-deck ball bearings, a new method for calculating the speed transmission characteristics is proposed. Different from the existing method, this new method proposes the speed of the transmission mechanism is visualized by considering multiple factors, which improves the current research situation of considering only one factor. The effects of different structural parameters, working conditions, and lubrication parameters on the transmission characteristics are analyzed. The speed transfer calculation method of double-deck ball bearing proposed in this paper provides a theoretical basis for the accurate calculation of speed transfer characteristics and the design of double-deck ball bearing. In future work, the influence of friction and wear factors, grease lubrication, and other factors on the speed transfer ratio will be considered. The above-proposed method can provide a theoretical basis for subsequent research work.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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