Electron–photon correlations and the third moment of quantum noise

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Abstract. The radiation generated by a quantum conductor should be correlated with the electrons crossing it. We have measured the correlation between the fluctuations of the high-frequency electromagnetic power generated by a tunnel junction at very low temperature and the low-frequency voltage fluctuations across it. This quantity corresponds to a third-order correlator of the electromagnetic field. At low-voltage bias, i.e. when the junction emits no radiation into the impedance-matched vacuum connected to it, we observe that the voltage fluctuations of the electric signal generated by the junction has zero third moment. Yet we show by a careful analysis of environmental contributions that the intrinsic third moment of the high-frequency current fluctuations in the junction is given by $e^2 I$.

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1. Introduction

The existence of fluctuations of the electromagnetic field in a vacuum even at zero temperature is a hallmark of the quantum nature of light. The quantized electromagnetic field at frequency $f$ is described by a harmonic oscillator. The uncertainty in the position of the oscillator in its ground state implies fluctuations of the electromagnetic field and an associated energy $\frac{1}{2} hf$, the so-called half-photon of a vacuum [1]. In a conductor, electromagnetic fluctuations (usually referred to as noise) are associated with fluctuations of local charges and currents, i.e. with the motion of electrons. Although these are described by a complex Hamiltonian that includes, for example, the effects of disorder and electron–electron interactions, the zero-point motion of electrons leads to current fluctuations which, at equilibrium and zero temperature, have a variance given by $S_{I^2}(f) = G hf$, with $G$ the linear conductance of the sample [2, 3]. The factor $hf$ in this expression comes from the fact that high-frequency current fluctuations in a conductor at very low temperature are indeed a direct consequence of the quantum nature of electricity.

Theoretical predictions in quantum optics are extremely powerful thanks to the fact that photon field operators in different modes commute with each other, and that the theory of photodetection is well established [1, 4]. Quantum noise in mesoscopic conductors is a much younger field, and how to model the measurement process, which may involve macroscopic apparatus as well as other quantum conductors, is still a very active field of research. The main difficulty comes from the fact that current operators $\hat{I}$ (which belong to the Hilbert space of the electron gas) taken at different times or frequencies do not commute with each other. For example, the measurement of the classical current–current correlator $\langle I(f)I(-f) \rangle = |I(f)|^2$ (with $I(f)$ the Fourier component at frequency $f$ of the current), i.e. the average power at frequency $f$, must be related to the quantum correlators $\langle \hat{I}(f)\hat{I}(-f) \rangle$ and $\langle \hat{I}(-f)\hat{I}(f) \rangle$. Those two differ by the commutator $\langle [\hat{I}(f), \hat{I}(-f)] \rangle = 2G hf$ independent of temperature or voltage bias. To know which one is measured in general is not a trivial question [5]. These particular correlators of order two have, however, a clear physical picture in terms of photons: one corresponds to the absorption of photons while the other one corresponds to their emission [6–9]. Depending on the detector, one or the other can be measured, and the case of a classical detector is given by the symmetric combination $\langle \hat{I}(f)\hat{I}(-f) + \hat{I}(-f)\hat{I}(f) \rangle/2$ [10]. This gives, in equilibrium and at zero temperature, $S_{I^2}(f) = G\hbar|f|$, since no photon can be emitted by the sample.
In the case of higher-order correlators, no such interpretation exists, and if there are some models of idealized detectors (such as a single spin [11], a lossless LC circuit read out by an electrometer [6] or a one-dimensional wire [12]), these are far from real ones, diodes, mixers or digitizers. Thus, how to order operators to predict the outcome of a given experiment and how to design an experiment to measure a given combination of operators are still open theoretical problems [5].

Here we report an experiment that aims to deepen the understanding of quantum noise by the measurement of the correlator \( \langle P_f(\varepsilon) V(-\varepsilon) \rangle \). \( V(\varepsilon) \) denotes the voltage fluctuation at low frequency \( \varepsilon \) such that \( h\varepsilon \ll k_B T, eV \). \( V \) is the dc voltage across the sample and \( T \) the temperature. \( P_f(\varepsilon) \) is the Fourier component at frequency \( \varepsilon \) of the power fluctuations of the electromagnetic field oscillating at frequency \( f \). \( P_f(0) \) corresponds to the average power at frequency \( f \), therefore it is proportional to the noise spectral density \( S_{V^2} = \langle |V(f)^2| \rangle \). At finite frequency \( \varepsilon \) this generalizes to \( P_f(\varepsilon) \propto V(f) V(-f + \varepsilon) + V(-f) V(f + \varepsilon) \). Thus the correlator we measure is proportional to the third-moment of voltage fluctuations \( S_{V^3} = \langle V(\pm f) V(\mp f + \varepsilon) V(-\varepsilon) \rangle \), which describes the correlation between Fourier components of the voltage taken at three different frequencies.

This quantity is closely related to the intrinsic third moment \( S^\text{int}_{V^3}(f, \varepsilon) = \langle I(f) I(-f + \varepsilon) I(-\varepsilon) \rangle \) of the current fluctuations generated by the sample, i.e. to the statistics of the charge crossing the junction. \( S^\text{int}_{V^3} \) is usually the quantity that is addressed theoretically since it can be expressed as a third-order correlator of current operators \( \hat{I} \) (with the ordering problem discussed above). However its measurement requires a high-frequency ammeter, i.e. an amplifier with zero input impedance, which does not yet exist. Even in the simplest circuits the sample is connected to an electromagnetic environment with a finite impedance (in particular at high frequency), which modifies the statistics of the voltage or current that is measured [13–16]. The most relevant experimental case consists of a detector being matched to the sample, i.e. having the same impedance. This case corresponds to the strongest coupling between the sample and its environment since all the power radiated by the sample is absorbed by the environment and vice-versa. Our measurement is performed in this regime. Thus, to deduce \( S^\text{int}_{V^3} \) from \( S_{V^3} \) requires a very careful characterization of the environmental effects. Our experiment at the lowest temperatures corresponds to the quantum regime \( hf \gg k_B T \). For \( eV < hf \), no photon of frequency \( f \) is emitted, and the noise detected at frequency \( f \) is that of vacuum fluctuations. In the following, we show that the fluctuations of the electromagnetic field at high frequency have zero skewness, \( S_{V^3}(f, 0) = 0 \) as long as \( eV < hf \), while the intrinsic current fluctuations in the junction have a finite skewness \( S^\text{int}_{V^3}(f, 0) = e^2 I \). The dominant environmental contribution that leads to such a striking difference comes from the modulation of the sample’s noise by the high-frequency vacuum fluctuations of the environment. Our experimental result \( S^\text{int}_{V^3}(f, 0) = e^2 I \) is in agreement with theoretical predictions that use time-ordered current operators according to the so-called Keldysh technique [17–20].

2. Principle of the measurement

We have chosen to perform the measurement on the simplest system that exhibits well-understood shot noise, the tunnel junction. The sample is an Al/Al oxide/Al tunnel junction similar to that used for noise thermometry [21]. It is mounted on an rf sample holder placed on the mixing chamber of a dilution refrigerator, at a temperature that we vary between 4.2 K and 12 mK. We apply a 0.1 T perpendicular magnetic field to turn the Al normal. The principle
Figure 1. Experimental setup for the measurement of the third moment of voltage fluctuations $S_{V^3}(f, 0)$. The symbol $\otimes$ represents a multiplier, the output of which is the product of its two inputs. The diode symbol represents a square-law detector, the output of which is proportional to the low-frequency part of the square of its input. Right: all the measurements presented in these plots have been performed at $T = 20$ mK. The solid lines in insets D and E correspond to fits with equation (6). (D) (green triangles) Measured high-frequency noise $S_{V^2}(f)$, with $f = 6$ GHz, so that $hf/k_B T = 15$. It exhibits a crossover between quantum noise $|eV| < hf$ and shot noise $|eV| > hf$. (E) (orange triangles) Measured low-frequency noise $S_{V^2}(\epsilon) \simeq S_{V^2}(0)$, with $\epsilon < 400$ MHz, i.e. $\hbar \epsilon/k_B T < 0.25$. It exhibits a crossover between thermal noise $|eV| < k_B T$ and shot noise $|eV| > k_B T$. (F) (red triangle) Measured third moment of voltage fluctuations $S_{V^3}(f, \epsilon)$.

of the experimental setup, sketched in figure 1, is the following: a bias tee, sketched by an inductor and a capacitor, allows the dc current to separate, imposed by a current source, from the current fluctuations, which can be read-out and amplified at point A. To select the frequencies at which we want to observe $S_{V^3}$, we use a diplexer that separates voltage fluctuations emerging from the sample into two branches depending on frequency: frequencies below 1 GHz (the low
frequency (LF) branch, on the left in figure 1) and above 1 GHz (the high frequency (HF) branch, on the right in figure 1). Note that the diplexer is a non-dissipative element that does not add any noise. In the LF branch, the signal is low-pass filtered and amplified, to give a fluctuating voltage $v_B(t)$ which has Fourier components $v_B(\epsilon) \propto V(\epsilon)$ with $|\epsilon| < \Delta f_{LF} \approx 400$ MHz. In the HF branch, voltage fluctuations enter a circulator and a bandpass filter centered on the frequency $f = 6$ GHz before being amplified by a 4–8 GHz cryogenic amplifier. Thus the voltage $v_C(t)$ has Fourier components $v_C(\pm f + \epsilon') \propto V(\pm f + \epsilon')$ with $|\epsilon'| < \Delta f_{HF} \approx 1$ GHz. A fast diode takes the square of $v_C(t)$ and filters out the high frequencies (close to $2f$), so that the voltage $v_D$ at its output has Fourier components given by $v_D(\epsilon) \propto v_C(\pm f + \epsilon)v_C(\pm f - \epsilon)$, or, in terms of Fourier components: $\langle v_D \rangle \propto \int v_C(\pm f + \epsilon)v_C(\pm f - \epsilon) d\epsilon \propto S_{V^2}(f) \Delta f_{HF}$.

where $S_{V^2}(f) = \langle (V(f) - V(-f))^2 \rangle = \langle |V(f)|^2 \rangle$ is the time-averaged voltage noise (variance of the fluctuations) at frequency $f$. Experimental values of $\langle v_D \rangle$ are reported in figure 1(D). In the right hand of equation (1), we have neglected the frequency dependence of $S_{V^2}$ on the scale of $\Delta f_{HF}$. However, the solid line in figure 1(D) corresponds to a fit with (6) taking into account the frequency dependence of $S_{V^2}(f)$. Neglecting the frequency dependence just results in a slight overestimate of the temperature. Since the dc output of the diode $\langle v_D \rangle$ is the time-averaged noise power, its ac output $v_B(t) - \langle v_D \rangle$ has the physical meaning of fluctuations of the noise power. This signal is (after further room-temperature amplification, not shown), multiplied with the signal coming from the LF branch $v_B(t)$, using an analogue, active multiplier. The dc output of the multiplier is given by $\langle v_F \rangle \propto \langle v_B \times (v_D - \langle v_D \rangle) \rangle$, i.e. is proportional to $S_{V^3}(f, 0) \Delta f_{LF} \Delta f_{HF}$ (in the following we will replace $\epsilon$ by 0 since $h\epsilon \ll k_B T$). $\langle v_F \rangle$ is recorded as a function of the dc current in the sample and averaged for $\sim 24$ h for each temperature.

3. Results

The measurement of the low-frequency noise spectral density $S_{V^2}(0)$ shown in figure 1(E) allows us to extract the electron temperature $T$. Our lowest temperature is $T = 20$ mK. The curve of figure 1(D) shows the measurement of the high-frequency noise spectral density $S_{V^3}(f)$, as obtained at point D. We observe a very clear plateau at low voltage which separates the quantum regime $|eV| < hf$ from the classical regime [22]. The rounding of the curve at the threshold between the two regimes is due to the finite bandwidth $\Delta f_{HF}$ ($h\Delta f_{HF}/k_B = 50$ mK) as well as the finite temperature. In the quantum regime, the voltage fluctuations across the sample correspond to the vacuum fluctuations. To these adds the voltage noise of the HF amplifier, so that the noise temperature $R_0S_{V^2}/(2k_B) = 7.1$ K (measured by $\langle v_D \rangle$) at zero voltage is much higher than the equivalent noise temperature of the vacuum fluctuations, $hf/(2k_B) = 0.15$ K. Here $R_0 = 50 \Omega$ is the characteristic impedance of the microwave circuitry, as well as the input impedance of the amplifiers. The resistance of the sample $R = 50.4 \Omega$ is close enough to $R_0$ so that corrections due to impedance mismatch can be neglected.

Our experimental setup gives access to the third-order voltage correlator $S_{V^3}(f, 0)$. We show in figure 2, right axis, its dependence on the dc bias current at high temperature, in
Figure 2. Normalized third moment of current and voltage fluctuations in the classical regime $hf \ll k_B T$ measured at different temperatures. Right axis: the normalized third moment of voltage fluctuations $S_{V^3}(f, 0)$ across the sample, which contains the environmental contributions. The data set has been multiplied by 0.1 for clarity. Left axis: the normalized intrinsic third moment of current fluctuations $S_{I^3}(f, 0)$ obtained from $S_{V^3}(f, 0)$ after subtraction of the environmental contributions.

The link between the third moment of voltage fluctuations and that of the intrinsic current fluctuations generated by the sample is known to be strongly affected by the electromagnetic environment [13–16]. These could be avoided if we had ammeters (i.e. amplifiers with very low input impedance) working at frequencies of several GHz and being noiseless. Unfortunately, such devices do not yet exist, and the best available alternative is to have a well-controlled electromagnetic environment in order to be able to subtract environmental contributions with confidence. We have measured very carefully the contributions of the environment to the statistics of current fluctuations so that we can extract the intrinsic third moment of current fluctuations $S_{I^3}(f, 0)$. Hereafter we show the results while the next section is devoted to the determination of environmental contributions.

We show in figure 2, left axis, the results for $S_{I^3}(f, 0)$ in the classical regime (high temperature, $k_B T \gg hf$) for various temperatures. We clearly observe that the intrinsic third moment is temperature independent, given by $S_{I^3}^{int} = e^2 I$. Since environmental contributions depend on temperature, so do voltage fluctuations, as can be seen on the right axis of figure 2. This proves that environmental contributions are very well accounted for, and provides a strong
Figure 3. The normalized intrinsic third moment of current fluctuations $S_{fI^3}^{\text{int}}(f, 0)$ as a function of the dc current for seven temperatures between 20 mK and 4.2 K. The dashed black line corresponds to the theoretical expectation $S_{fI^3}^{\text{int}}(f, 0) = e^2I$. The shaded area corresponds to the quantum regime $eV < hf$.

confidence in their subtraction at the lowest temperatures, where the high-frequency third moment is not known. Our results for $S_{fI^3}^{\text{int}}$ in both the classical and quantum regime (shaded area) are shown in figure 3: one clearly observes $S_{fI^3}^{\text{int}}(f, 0) = e^2I$ over the whole temperature range, which goes from $k_B T/h f = 0.07$ to 14. Only data taken at high temperatures are used to determine parameters that characterize the environmental contributions. Yet, we obtain that $S_{fI^3}^{\text{int}}(f, 0) = e^2I$ even at the lowest temperature.

We now focus on the data at the lowest temperature $T = 20$ mK. $S_{fI^3}^{\text{int}}(f, 0)$ (blue circles) and $S_{fV^3}$ (red triangles) are shown together in figure 4 as a function of the reduced bias voltage $eV/h f$. There is no anomaly at $eV = hf$ for $S_{fI^3}^{\text{int}}$, in strong contrast with what happens for $S_{fV^2}$ or $S_{fI^2}$ (see figure 1(D)) and for $S_{fV^3}$. We observe $S_{fV^3}(f, 0) \simeq 0$ at low voltage $eV < hf$. As we show below, this result comes from the sample and the detection setup being impedance matched and the influence of the amplifier’s noise being weak enough. An environment with an impedance different from that of the sample leads to a different result [23].

4. Environmental contributions

Environmental effects correspond to the back-action of the measuring apparatus, which cannot measure current without inducing voltage variations across the sample. These, in turn, influence electron transport.
Figure 4. The measured normalized intrinsic third moment of current fluctuations $S_{I3}^{\text{int}}(f, 0)$ (blue circles, left axis) and the normalized third moment of voltage fluctuations $S_{V3}(f, 0)$ (red triangles, right axis) at the lowest temperature $T = 20$ mK, as a function of the reduced voltage $eV/(hf)$. The shaded area corresponds to the quantum regime $eV < hf$.

4.1. Theory

The influence of voltage fluctuations on the third moment of current fluctuations is captured by the correlator $\langle P_{f1}(f_2)V(-f_2) \rangle$. If the voltage across the sample does not fluctuate, i.e. $V(f) = 0$, this correlator vanishes. The response of the noise of the sample measured at frequency $f_1$ to a small excitation at frequency $f_2$, i.e. $P_{f1}(f_2)$, is given by $P_{f1}(f_2) = \chi_{f1}(f_2)V(f_2)$, where $\chi_{f1}(f_2)$ is the noise susceptibility, which has been both calculated [24, 25] and measured [26]. Thus, what needs to be determined is the variance of voltage fluctuations experienced by the sample $\langle |V(f_2)|^2 \rangle$. These are due to (a) the noise generated by the electromagnetic environment and (b) the current fluctuations from the sample itself across the impedance of the environment: $\langle |V(f_2)|^2 \rangle = |Z_{\parallel}(f_2)|^2[S_{I2}^{\text{int}}(f_2) + S_{I2}^{\text{ext}}(f_2)]$ where $Z_{\parallel} = ZZ_{\text{ext}}/(Z + Z_{\text{ext}})$ is the impedance equivalent to the sample of complex impedance $Z(f)$ in parallel with the environment of impedance $Z_{\text{ext}} \simeq 50 \Omega$; see figure 5. $S_{I2}$ is the noise spectral density of the current fluctuations generated by the sample itself (we omitted the ‘int’ superscript to simplify the notation). $S_{I2}^{\text{int}}$ is the noise spectral density of the current fluctuations generated by the external circuit. The frequencies $(f_1, f_2)$ relevant for our experiment are $(f, \epsilon)$, $(f, -f + \epsilon)$ and $(\epsilon, f)$, which physically correspond to both the modulation of the HF noise by LF voltage fluctuations and the modulation of the LF noise by HF voltage fluctuations.

Putting all the contributions together, we obtain

$$S_{V3}(f, 0) = R_{\text{eff}}^3 \left[ -S_{I3}^{\text{int}}(f, 0) + S_{I3}^{\text{fb}}(f, 0) + S_{I3}^{\text{ext}}(f, 0) \right],$$

(3)
Figure 5. Schematics of the detection setup equivalent to the high-frequency part of the circuit depicted in figure 1. For clarity, the dc biasing circuit has been removed. $Z$ and $Z_{\text{ext}}$ are the impedances of the sample and the external circuit, respectively. The current source $i$ represents the fluctuating current in the sample and $i_{\text{ext}}$ represents the current fluctuations emitted by the detection setup. The noise spectral density of the sample at frequency $f$ is $S_{I^2}(f) = \langle |i(f)|^2 \rangle$. That of the environment is $S_{I^2,\text{ext}}(f) = \langle |i_{\text{ext}}(f)|^2 \rangle$. $\Gamma_1 = (Z - Z_{\text{ext}})/(Z + Z_{\text{ext}})$ is the voltage reflection coefficient of the sample.

where $R_{\text{eff}}^3 = Z_{\|}(0) \left| Z_{\|}(f) \right|^2$. For $Z = Z_{\text{ext}} = 50 \Omega$, $R_{\text{eff}} = 25 \Omega$. $S_{I^2}(f, 0)$ refers to the correlations induced by the feedback of the environment:

$$S_{I^2}(f, 0) = Z_{\|}(0) S_{I^2}(0) \chi_0(f) + 2 \text{Re} \left( Z_{\|}(f) \right) S_{I^2}(f) \chi_f(f).$$

(4)

The first term corresponds to the modulation of the HF noise of the sample by its LF current fluctuations (of variance $S_{I^2}(0)$). The second term corresponds to the modulation of the LF noise by current fluctuations at frequencies $f$ and $-f$ (using $\chi_f(f) = \chi_f(0)$).

The second contribution to $S_{V^2}(f, 0)$ in equation (3), $S_{I^2,\text{ext}}(f, 0)$, represents the correlations induced by the noise generated by the environment

$$S_{I^2,\text{ext}}(f, 0) = 2Z_{\|}(0) \left| \Gamma(0) \right|^2 S_{I^2}(0) \chi_0(f) + 2 \text{Re} \left( 2Z_{\|}(f) \Gamma(-f) \right) S_{I^2}(f) \chi_f(f),$$

(5)

where $\Gamma = (Z - Z_{\text{ext}}) / (Z + Z_{\text{ext}})$ is the voltage reflection coefficient of the sample. The first term represents the modulation of the HF noise of the sample by the LF environmental voltage fluctuations; the second that of the LF noise by the HF environmental voltage fluctuations. Note that $S_{V^2}(f, 0)$ is only affected by the external noise at frequencies $\sim 0$ (here below 400 MHz) and $f$ (here $6 \pm 0.5$ GHz). All the frequencies which are not within the detection bandwidth are irrelevant.

Equations (3)–(5) are a simple extension of the results obtained at zero frequency [14, 15], which have been confirmed experimentally [13]. In the classical regime $hf \ll k_B T$, $S_{I^2}(f) \approx S_{I^2}(0)$, $\chi_f(f) \approx \chi_0(f)$ and we recover equation (1) of [14]. These contributions to the total signal we measure at the lowest temperature are shown in figure 6.
For the sake of completeness we recall the noise and noise susceptibilities of the tunnel junction as a function of temperature, voltage and frequency:

\[ S_I^f(f) = \text{Re} \left( \frac{k_B T}{2Z(f)} \right) \left\{ F' \left( \frac{eV + hf}{k_B T} \right) + F' \left( \frac{eV - hf}{k_B T} \right) \right\}, \]

\[ \chi_0(f) = \text{Re} \left( \frac{e}{2Z(f)} \right) \left\{ F' \left( \frac{eV + hf}{k_B T} \right) + F' \left( \frac{eV - hf}{k_B T} \right) \right\}, \]

\[ \chi_f(f) = \text{Re} \left( \frac{ek_B T}{2Z(f)hf} \right) \left\{ F \left( \frac{eV + hf}{k_B T} \right) - F \left( \frac{eV - hf}{k_B T} \right) \right\}, \]

with \( F(x) = x/\tanh(x/2) \) and \( F'(x) = dF/dx \). Note that \( \chi_0(f) = \frac{dS_I^a(f)}{dV} \) since it represents the adiabatic modulation of the noise measured at frequency \( f \). It is almost zero as long as \( eV < hf \), thus the low-frequency environmental noise (the first term in equation (5)) does not contribute to \( S_{V^3} \) in this limit. In contrast, \( \chi_f(f) \) increases linearly with \( V \) at low bias (as observed in [26]): the high-frequency voltage fluctuations of the environment influence the slope of \( S_{V^3} \) versus \( I \) for \( eV < hf \).

Figure 6. The normalized \( S_{V^3}(f, 0) \) at \( T = 20 \) mK (purple triangles) as a function of the dc current. \( S_{I^3}^{\text{int}}(f, 0) \) (blue circles) is obtained after subtraction of the environmental terms corresponding to the feedback of the environment \( S_{I^3}^{\text{fb}}(f, 0) \) (solid line) and the noise of the environment \( S_{I^3}^{\text{ext}}(f, 0) \) (dashed line), according to equations (3)–(6).
4.2. Experimental determination

We now describe the procedure for the determination of the environmental contributions to $S_{V^3}(f,0)$.

4.2.1. Impedance of the sample $Z$. We have measured $Z$ versus frequency with a network analyzer in the range 300 kHz–8 GHz. We observe that the tunnel junction can be well modeled by a resistor $R = 50.4 \, \Omega$ in parallel with a capacitor $C = 0.6 \, \text{pF}$, leading to a frequency cutoff of about 6 GHz. For now we suppose $Z_{\text{ext}} = 50 \, \Omega$ (deviations from this value are irrelevant here and will be discussed later), so $R_{\text{eff}} = 22.8 \, \Omega$.

4.2.2. Gain. To determine the gain of the setup, we use the fact that $S_{V^3}(f,0) \simeq \beta I$ in the limit $eV \gg hf, k_B T$ with a temperature-independent slope $\beta$, as shown in figure 7. $\beta$ is independent of $S_{\text{ext}}^{\text{I2}}$. From the measurements of $\beta$ (and $Z$, $Z_{\text{ext}}$), we determine the gain of the setup.

4.2.3. Environmental noise $S_{I^2}^{\text{ext}}$. The low-frequency noise $S_{I^2}^{\text{ext}}(0)$ is measured in situ by turning the junction superconducting (i.e. at zero magnetic field), so that the low-frequency noise of the environment is fully reflected by the sample and is measured at point E in figure 1 (the gain of the LF branch is calibrated by the measurement of the low-frequency noise of the sample $S_{I^2}(0)$ versus bias voltage). We obtain $S_{I^2}^{\text{ext}}(0) = 2k_B T_{\text{LF}}/R_0$ with $T_{\text{LF}} = 11.6 \pm 0.1 \, \text{K}$. The high-frequency noise is that of the load of the circulator at temperature $T$, plus the noise
Figure 8. $\alpha$ as a function of $eV/(k_B T)$ at different temperatures in the classical regime $k_B T \gg hf$, from 0.75 to 4.2 K, calculated according to (8) from the measurement of $S_{V^3}(f, 0)$. Dashed lines are expectations for a $Z_{ext} = R_0 = 50 \Omega$ environmental impedance. Solid lines correspond to $Z_{ext} = 48 \Omega$. Inset: temperature dependence of the high current value $\alpha_0$ of $\alpha$. The solid line is the expectation for $Z_{ext} = 48 \Omega$, given by (9).

Note that at 6 GHz, this noise is dominated by the vacuum fluctuations of the load of the circulator. Thus the associated environmental contribution to $S_{V^3}$ corresponds to the zero-point voltage fluctuations modulating the noise of the sample.

4.2.4. Low-frequency environmental impedance $Z_{ext}(0)$. The contribution of the external noise is proportional to the reflection coefficient times the noise of the environment; see equation (3). At low frequency $\Gamma(0)$ is very small but $S_{V^3}^{ext}(0)$ is large, whereas at $f = 6$ GHz, $\Gamma(f)$ is larger but $S_{V^3}^{ext}(f)$ tiny. Thus, the determination of the environmental effects relies on the precise value of $\Gamma(0)$, i.e. that of $Z_{ext}(0)$. This value is very difficult to measure directly with enough accuracy, therefore we adopted another strategy: we analyzed the low-voltage part of $S_{V^3}$, which is very sensitive to $\Gamma(0)$. For this we define

$$\alpha(I) = (S_{V^3} - \beta I) / (R_{eff}^3 e^2),$$

which is plotted in figure 8: dashed lines are the expectations for $\alpha(I)$ taking $Z_{ext} = 50 \Omega$. The fit is good at 4.2 K where the high-frequency environmental noise is strong, but not as good...
at lower temperatures. Taking $Z_{\text{ext}} = 48 \, \Omega$ gives a much better fit of $\alpha(I)$ at every temperature (solid lines in figure 8). To obtain this value we rely on the asymptotic value $\alpha_0$ of $\alpha$ at high current:

$$\alpha_0 = \alpha(eV \gg k_B T) = \gamma(0) S_{\text{int}}^0(0) + \gamma(f) S_{\text{int}}^f(f),$$

which is plotted as a function of temperature in the inset of figure 8. Here $\gamma(0)$ (resp. $\gamma(f)$) is a temperature-independent coefficient which depends on the impedances at frequency 0 (resp. $f$), according to equation (5). Thus the temperature dependence of $\alpha_0$ comes solely from that of $S_{\text{int}}^0(f)$ given by equation (7), while $\gamma(0)$ only adds a constant to $\alpha_0$ and can be determined precisely by the low-temperature limit of $\alpha_0$. As shown in the inset of figure 8, the full temperature dependence of $\alpha_0$ is very well fitted by taking $Z_{\text{ext}} = 48 \, \Omega$ (solid line), while taking $Z_{\text{ext}} = 50 \, \Omega$ leads to a shift in the curve (dashed line).

It is crucial to note that within our procedure of determination of the environmental contributions we did not make any assumption about the low-voltage behavior of the intrinsic third moment $S_{\text{int}}^f$. Only the fact that $S_{\text{int}}^f = e^2 I$ at large voltage (i.e. in the classical regime) has been used.

5. Discussion

Previous measurements have shown that the third moment of current fluctuations created by a tunnel junction measured at low frequency is temperature independent and is given by $S_{\text{int}}^0(0, 0) = e^2 I$ [13, 27]. At high frequency, a preliminary measurement suggested that $S_{\text{int}}^f(f, 0) = e^2 I$ regardless of $f$ [23]. The various correlators involved in the calculation of $S_{\text{int}}^f(f, f')$ depend on voltage, frequency and temperature [28]. Only certain combinations of those correlators give the result $S_{\text{int}}^f(f, f') = e^2 I$. At least, the so-called Keldysh ordering does [17–20]. The present experiment confirms this theoretical prediction without any ambiguity.

We have measured the voltage fluctuations across a sample embedded in a microwave circuit. From this measurement and a careful characterization of the circuit we have deduced the third moment of the intrinsic current fluctuations generated by the sample, which is what would be obtained with a perfect ammeter, noiseless and of zero input impedance. This procedure is usual in mesoscopic physics because the electromagnetic environment of a sample can be varied at will, as opposed to the case of an atom in the vacuum, or even in a cavity. It is, however, tempting to try to understand the physical meaning of our raw result for $S_{\text{int}}^f$ of figure 1(F): it represents the third moment of the fluctuations of the voltage (or the electric field) in the coax cable connected to the sample. In our setup, the sample is well matched to the coax cable, and the contribution from the amplifiers is small. Thus we have measured the third moment of voltage fluctuations of a tunnel junction connected to an impedance-matched vacuum, i.e. to a semi-infinite coax cable of the same impedance. This situation is probably the most relevant one when dealing with noise at high frequencies. Whereas current noise can be measured directly with an ammeter at low frequency, this quantity is not so relevant in experiments performed in the quantum regime $hf \gg k_B T$, because it is not what is measured in real experiments. A similar situation occurs in atomic physics: one measures the energy levels of the hydrogen atom in the presence of a vacuum of impedance 377 $\Omega$, not the energy levels of the bare atom. Our observation that $S_{\text{int}}^f = 0$ at low bias voltage means that the statistics of the electromagnetic field in the semi-infinite coax does not carry the skewness that the statistics of the electrons

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crossing the junction bear, which is characterized by $S^\text{int}_{I^3} = e^2 I$ at any frequency. The origin of this difference lies in the environmental vacuum fluctuations modulating the noise generated by the sample. The corresponding contribution to $S_{V^3}$ cancels exactly that of the intrinsic third moment $S^\text{int}_{I^3}$ for $eV < h\nu$. This cancelation occurs only if the sample and the environment have the same impedance.

6. Conclusion

Mesoscopic physics has started with the study of electron transport in quantum devices [29, 30], followed by that of the noise (i.e. photons) generated by such devices [22, 31]. Our experiment provides the measurement of the correlation between them, i.e. between the electron transport at low frequency and the photon field at high frequency, both when photons are emitted ($eV > h\nu$) and when the electromagnetic field is solely due to vacuum fluctuations. In this regime, we observe that the fluctuations of the electromagnetic field have zero skewness, $S_{V^3}(0, \nu) = 0$, i.e. the power of the high-frequency vacuum fluctuations are not correlated with the low-frequency voltage fluctuations. Yet, the electron current generated by the junction fluctuates with an intrinsic third moment having a spectral density $S^\text{int}_{I^3}(\nu, 0) = e2 I$, regardless of the frequency. Thus the electron shot noise in the junction does not imprint in a simple way its statistics into the electromagnetic field it generates in a matched environment.

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