Realistic Gamow shell model for resonance and continuum in atomic nuclei

F.R. Xu, Z.H. Sun, Q. Wu, B.S. Hu, and S.J. Dai
School of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
E-mail: frxu@pku.edu.cn

Abstract. The Gamow shell model can describe resonance and continuum for atomic nuclei. The model is established in the complex-moment (complex-k) plane of the Berggren coordinates in which bound, resonant and continuum states are treated on equal footing self-consistently. In the present work, the realistic nuclear force, CD Bonn, has been used. We have developed the full Q-box folded-diagram method to derive the realistic effective interaction in the model space which is nondegenerate and contains resonance and continuum channels. The CD-Bonn potential is renormalized using the V_{low-k} method. With choosing ^16O as the inert core, we have applied the Gamow shell model to oxygen isotopes.

1. Introduction

Advances in theory and experiment make it possible to study the properties of exotic nuclei far from the stability. However, the description of weakly-bound nuclei is still a challenge to theoretical nuclear physics. Exotic nuclei have low particle thresholds, and cannot be described properly within the standard shell model because of strong coupling to continuum. The weakly-bound property makes dripline nuclei a complex open quantum system (OQS), and therefore the interplay between scattering and bound states should be taken into account explicitly.

The traditional shell model which utilizes the harmonic oscillator (HO) basis does not always provide a good description of loosely-bound nuclei as the HO basis wave functions are well bound and localized. Different from the situation of the HO basis, OQS has small separation energy and large spatial spread, therefore continuum states play an important role. In many cases, the HO basis gives poor convergence for weakly-bound nuclei [1]. Even though super computers have been available for large-scale HO-shell-model calculations, most nuclei around driplines are still unreachable due to huge model dimensions. Methods which unify bound and continuum states are eager.

The importance of continuum has been recognized for a long time. Even in stable nuclei, the continuum can have effects. Tremendous efforts have been devoted to the treatment of the continuum in the framework of multi-configuration interactions, e.g., the continuum shell model (CSM) [2-4], the continuum coupled cluster [5] and the continuum-coupled shell model [6]. The configuration-interaction many-body Gamow shell model (GSM) has also been developed recently [7-10], which can be considered as a generalization of the HO-basis shell model by utilizing the Berggren basis in a complex-energy plane. The Berggren coordinates [11] describes
bound and unbound states in the same framework. With phenomenological interactions, the
GSM has been successfully applied to nuclear structure calculations [7-10].

Recently, we are pursuing a GSM calculation based on realistic nuclear forces [12]. The ab-initio no-core Gamow shell model (NCGSM) has been proposed [13]. However, the NCGSM is limited to light nuclei due to too large model dimension, e.g., only to light helium isotopes in Ref. [13]. The shell model by taking an inert core can dramatically increase the capability of numerical computations. The core GSM (CGSM) based on realistic forces has been successful [14, 15]. It is a challenge to derive the realistic effective interaction of the model space from a realistic force in the complex-\(k\) Berggren basis. Usually, the many-body perturbation theory is adopted to build the interaction, e.g., the \(\hat{Q}\)-box-plus-folded-diagram method. However, due to the existence of the continuum, the usual degenerate \(\hat{Q}\)-box approach is inappropriate for GSM. One needs to extend the \(\hat{Q}\)-box approach to the nondegenerate case. In the recent work [12], we have developed the full \(\hat{Q}\)-box folded-diagram method which is appropriate for the nondegenerate complex-\(k\) space.

2. The core Gamow shell model
In the present CGSM, we choose a doubly magic nucleus as the inert core. For the \(sd\)-shell nuclei, the \(^{16}\)O is chosen as the core. We take the Woods-Saxon (WS) potential with inclusion of the spin-orbit coupling [16] to provide single-particle basis states in the Berggren complex-\(k\) plane. The radial wave functions of the Berggren single-particle states are determined by the time-independent Schrödinger equation,

\[
\frac{d^2 u(k,r)}{dr^2} = \left[ \frac{l(l+1)}{r^2} + \frac{2m}{\hbar^2} U(r) - k^2 \right] u(k,r), \tag{1}
\]

where \(l\) is the orbital angular momentum of the nucleon motion. The momentum \(k\) and wave function \(u(k,r)\) can be complex numbers (or function). \(U(r)\) is the spherical WS potential generated by the core. We are investigating oxygen isotopes, therefore there is no Coulomb interaction for active valence neutrons. The piecewise perturbation method [17] has been used to solve the Berggren single-particle eigenvalue equation (1). For a resonant state, the energy eigenvalue is a complex number, \(\hat{\epsilon}_n = \epsilon_n - i\gamma_n / 2\), where \(\gamma_n\) stands for the resonance width.

Fig. 1 shows the Berggren complex-\(k\) plane. The Berggren single-particle states form a complete set of basis states with discrete bound and resonant states and continuum scattering.

**Figure 1.** Schematic Berggren complex-\(k\) plane.
states [11]. The wave functions of resonant states are not square integrable. The exponential increase and an infinitely oscillating behavior make that the resonant wave function cannot be normalized with conventional techniques. A regularization technique was introduced by Zel’dovich [18] by multiplying a convergence factor $e^{-\epsilon r^2}$ in normalization and taking the limit of $\epsilon \to 0$. In the present work, we use the exterior complex scaling method [19] for the normalizations of resonant states. The scattering states are normalized to Dirac delta [10].

We use the universal WS parameters but with a strength $|V_0|$ reduced by 2.3 MeV to obtain a reasonable $0d_{3/2}$ resonance width compared with data. The WS potential gives the bound $0d_{5/2}$ and $1s_{1/2}$ orbits at the energies of $-5.31$ and $-3.22$ MeV, respectively, and a resonant $0d_{3/2}$ at $\tilde{\epsilon} = 1.06 - 0.09i$ MeV. The higher orbits $f_{7/2}, iP_{3/2,1/2}$ ($i \geq 1$), $g_{9/2,7/2}$, $id_{5/2,3/2}$ ($i \geq 1$) and $is_{1/2}$ ($i \geq 2$) are continuum states. We limit the $l \leq 4$ orbits for the $sd$ shell-model calculations. $0d_{5/2}, 1s_{1/2}$ and $id_{3/2}$ ($i = 0, 1, 2 \cdots$) are taken as our model space. Contributions from other continuum channels as the intermediate states of scatterings are taken into account by performing the $Q$-box folded-diagram process. In practical computations, the continuum states on the contour $L^+$ need to be discretized. We use the Gauss-Legendre quadrature method [10, 14, 20] for the discretization.

The intrinsic $A$-body Hamiltonian has the following form,

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j}^{A} v_{ij} - \frac{P^2}{2Am}, \quad (2)$$

where $v_{ij}$ is the nucleon-nucleon interaction. $p_i$ is the nucleon momentum in the laboratory coordinate, while $P = \sum_{i=1}^{A} p_i$ is the center-of-mass (CoM) momentum of the system. In shell-model calculations, usually a one-body potential $U$ is written into the Hamiltonian, which makes calculations more convenient,

$$H = \sum_{i=1}^{A} \left( \frac{p_i^2}{2m} + U \right) + \sum_{i<j}^{A} \left( v_{ij} - U - \frac{p_i^2}{2Am} - \frac{p_j^2}{2Am} \right)$$

$$= H_0 + V, \quad (3)$$

with $H_0 = \sum_{i=1}^{A} \left( \frac{p_i^2}{2m} + U \right)$ having a one-body form. In the present calculations, $U$ takes the WS potential of the $^{16}$O core. $V$ is the residual two-body interaction with corrections from the CoM motion. The final goal is to solve the Schrödinger equation $H|\psi_\nu\rangle = E_\nu|\psi_\nu\rangle$, which is done in the framework of the Gamow shell model.

The Hamiltonian has been intrinsic, but the CoM effect in wave functions has not been removed. The CoM motion can bring spurious states. It is difficult to write wave functions in a relative coordinates for many-body systems, though the few-body Jacobi coordinates and the cluster-orbital shell model have been employed to remove the CoM effects. In the present paper, we calculate the low-lying excited states, therefore the CoM effect should be negligible.

3. Calculations and discussions

The CD Bonn potential [21] is considered to derive the realistic effective interaction for the model space of the $sd$ shell with the $^{16}$O core. While $0d_{3/2}$ is a resonant orbit, the remaining part of the $d_{3/2}$ channel is continuum. A resonant basis state plays a special role in the CGSM calculation. Therefore, the channel containing resonant state(s) should be included in the model space. In the present calculations, we choose $\{1s_{1/2}, 0d_{5/2}, 0d_{3/2}\}$-resonance, $d_{3/2}$-continuum as the model space of the CGSM. The bare CD Bonn potential is softened firstly by using the $V_{\text{low-k}}$ renormalization [22]. We have exploited the full $Q$-box folded-diagram method to the Berggren complex-$k$ space [12]. The $\bar{Q}$-box perturbation is to include effects from the core polarization.
Figure 2. Valence-linked Goldstone diagrams up to the second order of the $\hat{Q}$-box evaluation.

Figure 3. Convergence of spectroscopic calculations against the point number of discretizing the $L^+$ contour in $^{22}$O.

and the excluded space. Fig. 2 displays all the valence-linked diagrams up to the second order in the $\hat{Q}$-box evaluation. The folded-diagram process (i.e., the iteration is used in the extended Krenciglowa-Kuo (EKK) technique [23]) is to consider high-order contributions by summing up the subsets of diagrams to infinite order [24]. The present CGSM space is not degenerate, which includes bound, resonant and continuum orbits. The degenerate $\hat{Q}$-box folded-diagram method should be no longer valid for the nondegenerate Berggren space. At least the continuum channel must cross shells. In the present work, we exploit the EKK method [23] to the complex-$k$ space. The detail can be found in the previous publication [12]. We emphasize that effects from all continuum states are included either by being included as a part of the model space or through the $\hat{Q}$-box folded-diagram process of the effective interaction.
Figure 4. Calculated spectra of $^{20-23}$O, compared with data [30-33]. Resonant states are indicated by shading. $\lambda = 2.6$ fm$^{-1}$ is taken in the $V_{\text{low-}}k$ renormalization.

In practical computations, one has to discretize the continuum scattering states which lie either on the $L^+$ contour or on the real-$k$ axis, see Fig. 1. We use the Gauss-Legendre method to discretize the contour. An equal number of discretization points is set for each segment of the $L^+$ contour. Fig. 3 shows the convergence of the CGSM $^{20}$O calculations with changing the number of discretization points. We see that 18 discretization points have given a converged result. The convergences as a function of the cutoff $\Lambda$ of the $V_{\text{low-}}k$ technique and the starting energy of the $Q$-box folded diagram have been well tested in our previous calculations [12].

In Fig. 4, the calculated $^{20-23}$O energy spectra are reported, that agree well with experiments. In $^{23}$O, the calculations predict two resonant states, $3/2^+$ and $5/2^+$, at the excitation energies of $\sim 3.5$ and $\sim 6.9$ MeV, respectively. The two states are unstable toward neutron emissions with the resonance widths of $\sim 100 - 200$ keV. Oxygen isotopes near the neutron dripline and beyond have been intensely discussed [12]. The loosely-bound properties are well described [12]. However, we should admit that the three-nucleon force (3NF) is missing in the present calculations. Though, from the present calculations, it seems that the 3NF effect on spectra is small and ignorable (at least for the oxygen isotopes), it has been known that 3NF has important effect on binding energy calculations and plays a crucial role in the determination of the dripline [5, 25-29]. In the previous work [12], we have discussed the binding energy calculations and gave improved calculations by taking the experimental single-particle energies. However, the calculations without the inclusion of 3NF cannot give correctly the dripline position of the oxygen chain [12].

4. Summary
In this paper, we present the core Gamow shell model calculations for oxygen isotopes. Nuclear resonance and continuum are treated in a unified framework, named the Berggren complex-$k$ coordinates which describes bound, resonant and scattering states on equal footing. The calculations are based on a realistic nuclear force, the CD-Bonn potential. Due to the
nondegenerate property of the Gamow basis, the traditional degenerate \( Q \)-box folded diagram has to be extended to the nondegenerate complex-\( k \) space. The bare CD-Bonn potential is renormalized by using \( V_{\text{low-k}} \) to soften the hard short-range repulsive core of the strong interaction. \( ^{16}\text{O} \) is chosen as the inert core for the spectroscopic calculations of oxygen isotopes. The \( ^{23}\text{O} \) is predicted to have two low-lying resonant states which may be observed in future experiments. The 3NF is not considered in the present GSM calculations, even if we acknowledge that 3NF plays an important role to reproduce correctly for binding energies.

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