Influence of rising the corners of a simply supported reinforced concrete slab on its reinforcement by numerical methods

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Abstract. The article presents a method for calculating a simply supported slab with a supple support contour using a numerical method in a computational software package. The support contour of a simply supported slab is formed by groups of two nodal finite elements, which allow modeling a linear supple bond in the vertical direction. They take into account the possibility of changes in the design scheme due to subsidence of the walls. The suppleness of the contour of prefabricated elements is 1 mm without taking into account the bends of the corners of a simply supported slab. The forces and reinforcement of a simply supported square slab were calculated according to the proposed methodology with a calculation where the joint s of the contour are set to move in the vertical direction. As a result, the bending moments in the middle of the span increase, and the torques in the corners of the slab decrease. The reinforcement of the upper area of the slab is not required. The reinforcement of the lower area of the slab is increased by the amount of released reinforcement from the upper area of the slab.

1. Introduction

Renovation of urban agglomerations, caused by historical changes in urban areas, leads to a constant increase in the demolition of buildings and structures around the world [1]. Destructive demolition technologies are associated with the dustiness of adjacent territories and lead to the formation of a huge amount of construction waste, which makes up from 50 to 70% of the volume of household waste dumps, which increases the environmental problems of municipal solid waste dumps in large cities [2, 3]. In Russia, The Department of Building Structures and Foundation Engineering named after Professor Yu.M. Borisov of Voronezh State Technical University developed and implemented technologies for the separate demolition of buildings and structures with the repeated use of building materials and products and structures in the construction of low-rise buildings and structures [4-6], which have proved their high efficiency. The development of low-rise construction technologies with repeated use of bricks is inevitably associated with the design of monolithic floor slabs, based on the supple foundation of freshly laid brick walls.

When constructing prefabricated buildings and structures, there is always the task of constructing a reinforced concrete floor slab, simply supported along a contour on brick or similar walls. With the advent of computational software systems, solving a number of design problems becomes faster and more accurate [7-11]. However, in conditions of fierce competition, the designer must calculate any tasks in a short time. Such conditions sometimes lead to errors. For example, the choice of the fixing
boundary conditions for the floor slab, simply supported along the contour, affects the final result of the calculation.

The support of simply supported beams or slabs in the computational software complex is defined by the bonds of the support joints for vertical movements and one of the joints in horizontal directions. Let us consider a square monolithic slab with a span of l=6 m. The recommended thickness of a flat slab for civil buildings is h=160 mm [12]. We set the design load on the slab p=10 kN/sq.m, which takes into account the dead weight of the slab, floor weight, and useful load. We take the concrete class B30 and a protective layer of 30 mm, longitudinal reinforcement of class A400. The calculation is performed in the Lira-SAPR software package. Bending moments are concentrated in the span of the slab (figure 1.a). However, the floor slab has an upper reinforcement in the corner zones (figure 1.c). This effect is due to the large torques \( M_{xy} \) (figure 1.b) with respect to the bending moments of \( M_x \) and \( M_y \) at the corners of the slab. As the plate thickness increases, the reinforcement of the corner zones will decrease. Due to the ratio of these moments, the reinforcement in the corners of the slab is intense, and in the span, it is underestimated.

In fact, the corners of a simply supported slab lift when bearing the load. A simply supported slab is usually placed on a wall with a support area of 120 mm wide. Such support provides load transfer from the ceiling to the wall, but the support itself is considered to be hinged. Such a contour of the slab cannot be considered fixed in the horizontal plane. For example, for the building roof slab, the parapet of the masonry does not press it at all.

Brick masonry does not have sufficient rigidity comparable to that of reinforced concrete. After the construction of a building, the brickwork gives sediment for two or more years [13]. Thus, the contour of the slab can be considered supple relative to the horizontal design plane of the slab, and it is incorrect to impose bonds on the joints along vertical displacements. If the slab is reinforced according to the scheme (figure1.c, figure1.d), then with relatively large spans or its small thickness the design scheme will change due to deflections, bends of the corners.

The aim of the study is to determine the methodology for calculating a simply supported slab with a supple support contour, modeled in a software package, with the subsequent calculation of forces and reinforcement.

![Stress contour plot for Mx](image1.png)

![Stress contour plot for My](image2.png)

![Stress contour plot for Mxy](image3.png)

![Stress contour plot for Mxy](image4.png)
2. Modeling supple support along the contour of a simply supported square slab in the software package

We established above that the contour is supple and has displacements along the vertical axis. In fact, the intensity of the load distribution from the slab depends on the stiffness of the masonry or support beam. The pressure distribution is also influenced by the loading diagram of the wall or beam, the strength of the mortar and the degree of its hardening, the height of the masonry, the presence and placement of openings, etc. It is not possible to find the desired value of the displacements of the contour joints taking into account the above factors. However, we can set a limit value for these displacements.

Based on the norms for designing stone structures [13], the difference in the free deformations of adjacent sections of the same wall is shown in the table of Appendix E. These values can be considered empirical since before they get into the design standards they were repeatedly checked at the construction site. Based on the analysis of this table, we find that for one floor with a wall height of 3 m, the difference in free deformations will be 1 mm. Then the permissible maximum displacements of the contour joints for the slab are 1 mm from wall deformations.
Let us check the movements of the upper nodes of the conditional wall of 3 m high and 6 m long, on which the floor slab described above will rest. We take the wall thickness of 380 mm and the minimum strength values for masonry [13], for example, for brick grade M35 and mortar grade M4, the calculated resistance of the masonry is $R=0.45\text{MPa}$ with a coefficient of $k=2$ and the elastic characteristics of the masonry $\alpha =750$. Then the masonry deformation modulus is $337.5\text{MPa}$. The Poisson ratio for stone structures during operation is a variable [14], so we take $\nu=0.25$. We transfer the load from the floor to half of all the upper nodes, that is, through 7 middle nodes out of 13 possible, thereby we will assume that they created the most unfavorable combination of loads in a simplified design scheme. The nodal load value will be $p l^2/(4\cdot7)=12.86\text{kN}$. The displacements of the upper nodes of the conditional wall (figure 2) after rounding are equal to 1 mm. Therefore, the maximum displacements $Z_0$ of the contour joints for the floor slab will be 1 mm from the abutment on the wall.

Let us modify the initial calculation scheme so that the walls and columns bear the load, and the floor slab is still in the free position. We find a solution to the problem using the finite element FE 55 [15]. It allows you to simulate a linear supple vertical bond, without regard to its length. Let us place it vertically from below under the joints of the contour. To determine the linear stiffness of the bond $R_z$, we carry out a preliminary calculation, where instead of FE 55 we use the usual universal cane element of general purpose FE 10. Based on the calculation results, we determine the number of FE 55 groups with different stiffness values. The values in the groups can be determined, for example, by the order of the number. In the case considered, two groups can be distinguished: compressed elements with negative values (44 main elements) and stretched elements with positive values (4 corner auxiliary elements).

Since the plate is simply supported, the corners of the plate lift, then 4 auxiliary elements have a minimum rigidity, we assume $R_{z0}=1\text{kN}/\text{m}$. 44 basic elements will bear the main load. Since the distance between the joints of the support contour is the same, preliminary force $F_z$ in them will be $pl^2/44=8.18\text{kN}$. The maximum displacements of the joints of the contour of the floor slab from supporting on the wall are $Z_0=1\text{mm}$. We also set a coefficient that takes into account the total deflection, consisting of the deflection of the contour from supporting on the wall and the deflection of the plate corners upwards, for the considered design scheme it is equal to $k_z=1.7$. Then the linear stiffness $R_z$ is equal to:

$$R_z=k_z F_z / Z_0=13900\text{kN}/\text{m}.$$  

(1)
Figure 3. A free-lying slab in a PC, supported flexibly along the contour onto an elastic bond along the Z axis: a) Mx stress isofield; b) Mxy stress isofield; c) displacements along the Z axis of the nodes of one side of the contour; d) the mosaic of the reinforcing the slab along the X axis at the upper face; e) the mosaic of reinforcing the slab along the X axis at the bottom face; f) Z axis displacements.
After calculation, the bending moments $M_x$ in the span of the slab increased (figure 3.a) compared to the original calculation scheme, and the torques $M_{xy}$ (figure 3.b) almost halved. The deflections of the contour joints, as expected, are equal to 1 mm from the abutment on the wall (figure 3.c).

A similar solution is obtained by the analytical method for a square slab, which satisfies the elastic surface equation, the Sophie Germain equation [16, 17]. The supple contour in the classical problem is a system of contour beams. Identical beams are rigidly supported at the tops of the slab. The origin is located in the center of the slab. The boundary conditions are obtained under the condition that the bending moments on the supports are zero and the deflections of the beams correspond to the deflections of the slab along the contour. It was also taken into account that the support contour is not defined by beams, but by its suppleness, i.e., by displacements, and one more boundary condition is obtained that determines the beam stiffness through the displacement value. The deflection of the beam in the middle is 0.00162 m. This value is obtained on the basis that the deflections of the conditional beams modeling the supple contour are determined relative to the vertices. Accordingly, as was established above, this deflection consists of the deflection of the contour from leaning on the wall and the deflection of the corners of the slab upwards and is refined when determining the coefficients of the series of equations of deflection of the plate.

After obtaining the equation of the plate, the maximum deflection and bending moment in the center of the plate are equal to:

$$w=0.00601pI^2/D=6.736\text{mm}, \quad M_x=0.0500584pI^2=18.021\text{kNm/m}. \quad (2)$$

Comparing the calculation by the numerical method with the analytical method, we obtain the calculation error for deflections of 3% and bending moments of 1%.

Comparing the calculation according to the proposed method with the calculation according to the original scheme, we see that the bending moments $M_x$ increased by 12%, and the torques $M_{xy}$ decreased by 1.9 times. At the same time, it is not necessary to reinforce the upper zone of the slab, but the reinforcement of the lower zone of the slab increased. Considering the anchoring of the reinforcement according to building standards [18], we find that the mass of the reinforcement for the slab, calculated by the proposed method, differs from the mass calculated by the original method by only 1%.

3. Conclusions

We performed the calculation of a simply supported slab with a supple support contour in the calculation software package. The support contour of a simply supported slab is formed by two-joint finite elements, which allow modeling a linear vertical supple bond. When modeling a supple contour, we proposed to form groups of elements of a linear supple bond with values corresponding to the features of the group. The magnitude of the suppleness of the contour from prefabricated elements was 1 mm without taking into account the bends of the corners of the simply supported slab.

We obtained the forces and reinforcement of a simply supported square slab according to the proposed method. As a result, the bending moments $M_x$ increased by 12%, and the torques $M_{xy}$ decreased by 1.9 times. In this case, reinforcement of the upper zone of the slab is not required, but the reinforcement of the lower zone of the slab is increased. The change in the demand for reinforcement for the slab according to the proposed method is insignificant, in the considered example, the excess demand of the reinforcement was 1% compared to the original scheme and was caused by the redistribution of reinforcement from the upper zone of the slab to the lower. We took into account the possibility of changing the design scheme due to the subsidence of the walls, thereby ensuring the safety and reliability of the construction of floor slabs, simply supported along the contour.

An analytical solution, where the dependencies are associated with a rectangular coordinate system, becomes time-consuming with a free-form contour. Calculation in the software package does not change the complexity of the solution. For the free form of the contour, one should do a preliminary analysis described above, determine groups of finite elements, assign stiffnesses and find a solution, make an assessment of the obtained deflections.
In the calculation of spatial schemes, when floor slabs and prefabricated walls are calculated together, tensile forces due to the rise in the corners of the slab weakly affect the stresses in the walls.

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