MEMBRANE PREGEOMETRY and the VANISHING of the COSMOLOGICAL CONSTANT

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Submitted to Class. Quantum Gravity
ABSTRACT

We suggest a model of induced gravity in which the fundamental object is a relativistic \textit{membrane} minimally coupled to a background metric and to an external three index gauge potential. We compute the low energy limit of the two-loop effective action as a power expansion in the surface tension. A generalized bootstrap hypothesis is made in order to identify the physical metric and gauge field with the lowest order terms in the expansion of the vacuum average of the composite operators conjugate to the background fields. We find that the large distance behaviour of these classical fields is described by the Einstein action with a cosmological term plus a Maxwell type action for the gauge potential. The Maxwell term enables us to apply the Hawking-Baum argument to show that the physical cosmological constant is “probably” zero.
The *induced gravity* programme, pioneered by Zel’dovich and Sakharov in the late sixties, provides an ingenious way to “sweep under the carpet” the long standing problem of quantizing General Relativity \[1\]. The basic idea is to consider General Relativity as an *effective theory* describing the large distance spacetime structure, rather than a fundamental theory of gravity at every length scale. The analogy which comes immediately to mind is with the non-renormalizable Fermi theory of weak interactions which represents only the low-energy approximation of the $SU(2)_L \otimes U(1)_Y$ electro-weak, renormalizable, gauge theory. In the same spirit one would like to derive the Einstein theory as a macroscopic limit of some suitable gauge theory which unifies gravity with other gauge interactions and provides a reliable description of short distance gravitational phenomena \[2\].

As a by-product, this approach would provide finite, unambiguous values of the macroscopic gravitational constants, i.e. the Newton and the cosmological constants, in terms of the gauge charges and vacuum condensates of the underlying fundamental theory.

However, there are, at least, two basic problems with this idea:

i) the fundamental field theory one starts with is of the gauge type, with no dimensional coupling constants, in order to implement renormalizability at the quantum level. But then, the induced Newton constant should arise through some dynamical mechanism breaking the original gauge, or Weyl, symmetry at large distances. This kind of process is essentially *non-perturbative* and is presumably similar to the hadronization process of the fundamental QCD degrees of freedom. Any attempt to describe this type of phenomena, beside technical problems, introduces regularization ambiguities which spoil the predictability of the induced
Newton constant $^{[3]}$. 

ii) The value of the induced cosmological constant should be very small, and possibly vanishing, in order to be consistent with the experimental bound $\lambda_{\exp} \leq 10^{-120}(\text{Planck Mass})^4$. On the contrary, the typical value one usually obtains is of the order $\langle \text{Planck Mass} \rangle^4$!

The current attitude towards ultra-short distance physics is to replace local fields with extended objects, mainly strings, as fundamental constituents of matter and to treat particle physics below some (string)energy-scale as a local limit of the fundamental theory. Extended objects, or p-branes, carry a proper mass, or length scale, related to the (hyper-) tension $\rho$ by the relation $l_{p-\text{brane}} \sim (\rho)^{-1/(p+1)}$. Moreover, the spatial extension of the object should improve its ultraviolet behaviour, leading ultimately to a finite or at least renormalizable quantum theory. Thus, one hopes that this is a good framework to derive unambiguously the dimensional coupling constants of the low energy effective theory.

In part because of the above considerations and in part because of the increasing relevance of relativistic membranes both in particle physics and cosmology $^{[4]}$, it seems pertinent to ask if and how General Relativity may arise as the low energy limit of a quantum theory of relativistic membranes. Our objective is to show that this is indeed the case: the gravitational and gauge forces acting on the membrane are generated by the membrane itself and the macroscopic dynamics of the classical fields is self-consistently induced by the quantum dynamics of the membrane in the long-wavelength approximation. The large distance behaviour of these classical fields is described by the Einstein action with a cosmological term plus a Maxwell type action for the three index gauge potential, and the presence of this latter term
enables us to show that the physical cosmological constant most likely is zero. Our starting point is the (euclidean) Nambu-Goto action for a relativistic closed membrane interacting with two background fields: a symmetric, non-degenerate, tensor $J_{\mu\nu}(X)$, and a totally antisymmetric tensor $\star K_{\mu\nu\rho}(X)$

$$S_{NG} = \frac{1}{l_3} \left[ \int_{\mathcal{H}} d^3 \sigma \sqrt{\frac{1}{3!} J_{\mu\nu} J_{\nu\rho} J_{\rho\sigma} X_{\mu\nu} X_{\nu\rho} X_{\rho\sigma} \dot{X}_{\mu} \dot{X}_{\nu} \dot{X}_{\rho}} + \frac{1}{3!} \int_{\mathcal{H}} d^3 \sigma \dot{X}_{\mu\nu\rho} K_{\mu\nu\rho}(X) \right],$$

where $\mathcal{H}$ stands for a domain in the space of the parameters $\sigma^a = (\sigma^1, \sigma^2, \sigma^3)$ which represents the euclidean membrane manifold and $\dot{X}_{\mu\nu\rho}$ stands for the tangent three-vector at each point of the embedded submanifold $x^\mu = X^\mu(\sigma)$ which represents the world-history of the membrane in the (euclidean) spacetime. Finally, for later convenience, we have expressed the gauge coupling constant in terms of the surface tension $1/l_3$, and rescaled the gauge field $K$ so that it becomes adimensional, $[K] = 1$.

We remark the absence of any kinetic term for $J_{\mu\nu}$ and $K_{\mu\nu\rho}$. Our final goal is just to recover these terms from the quantum dynamics of the membrane.

Usually, this action is interpreted as describing the classical dynamics of the extended object under the combined effects of the external gauge potential $K_{\mu\nu\rho}$ and the pre-assigned gravitational field $J_{\mu\nu}$. Thus, it corresponds to the limit of Classical Bubble Dynamics \cite{5} where both gauge and gravitational degrees of freedom are included.

\* In string theories the graviton and the Kalb-Ramond gauge potential are present in the string spectrum. Whether the membrane spectrum contains massless states at all is an open (model-dependent) question we shall not address here. For our purposes, we can look at equation (1) as a generalization of the the generally covariant action of a point-particle coupled to an external electromagnetic field.
freedom are frozen. However, at this early stage, the tensors $J_{\mu\nu}$ and $K_{\mu\nu\rho}$ play simply the role of auxiliary field variables introduced to endow the model with:

i) general covariance in “target space”, and,

ii) extended gauge invariance $K_{\mu\nu\rho} \rightarrow K_{\mu\nu\rho} + l_m \partial_{[\mu} \Lambda_{\nu\rho]}$.

Indeed these two requirements alone are sufficient to determine the form of the effective lagrangian which, in turn, will provide a physical interpretation of $J_{\mu\nu}$ and $K_{\mu\nu\rho}$. Variation of the action with respect to the external fields gives us the corresponding “current densities”:

\[
\frac{\delta S_{\text{NG}}}{\delta J_{\mu\nu}} = \frac{1}{2} T_{\mu\nu}(X) = \frac{1}{l_m^3} \sqrt{\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu \tag{2a}
\]

\[
\frac{\delta S_{\text{NG}}}{\delta K_{\mu\nu\rho}} = \frac{1}{l_m^3} \dot{X}^{\mu\nu\rho} = \frac{1}{l_m^3} \delta^{[abc]} \partial_a X^\mu \partial_b X^\nu \partial_c X^\rho, \tag{2b}
\]

where $T_{\mu\nu}(X)$ is the membrane energy-momentum tensor-density, $\gamma_{ab}$ is the induced metric on the membrane world-tube, i.e. $\gamma_{ab} = \partial_a X^\mu \partial_b X^\nu J_{\mu\nu}$, and the “gauge-current” is represented by the tangent three-vector $\dot{X}^{\mu\nu}$. Therefore, $J_{\mu\nu}$ and $K_{\mu\nu\rho}$ can also be considered as external sources for the composite operators $T_{\mu\nu}(X)$ and $\dot{X}^{\mu\nu}$.

From now on, we shall make no distinction between background fields and external sources.

The generating functional corresponding to the classical action (1) is

\[
Z[J_{\mu\nu}, K_{\mu\nu\rho}] = \sum_{\text{topologies}} \int \prod_\xi \sqrt{\det J_{\mu\nu}(X)} \mathcal{D}[X] \exp \left[ -S_{\text{NG}} \right], \tag{2}
\]

where the Jacobian $\sqrt{\det J_{\mu\nu}(X)}$ has been inserted into the functional measure to preserve general covariance in the target space; gauge fixing and ghost terms.
required for the reparametrization invariance of the world-track are understood but were purposely omitted in order to avoid an unnecessarily complicated formalism.

From our vantage point, the most relevant property of the partition function of the quantum membrane is that an integration over spacetime points is implicit in the functional integral. In fact, the free membrane term is insensitive to the position of the membrane “centre”, that is, the action (1) is invariant under the transformation $X^\mu(\sigma) \rightarrow X^\mu(\sigma) + x^\mu$ where $x^\mu$ is constant, i.e., independent of the world-tube coordinates $\sigma$. Hence, the free membrane partition function contains the zero-mode contribution (the spacetime volume) as a factor. But, in the presence of background fields, translational invariance is broken and the four dimensional zero-mode integral becomes non-trivial \cite{6}. Then, we find it useful to extract the, integration over the membrane “centre” from the very beginning by separating the zero mode contribution as follows, $X^\mu(\sigma) = x^\mu + \eta^\mu(\sigma)$; then,

$$
\int \mathcal{D}[X] F[X] = \int d^4x \int \mathcal{D}[\eta] F[x + \eta] ,
$$

$$
\mathcal{D}[X] = \mathcal{D}[\eta] \delta^4(P^\mu[x, \eta]) \Delta_{FP}[x, \eta] , \tag{3}
$$

Here $P^\mu[x, \eta] = 0$ is the gauge fixing condition breaking the invariance under $\eta \rightarrow \eta + \text{const.}$, and $\Delta_{FP}[x, \eta]$ is the corresponding ghost determinant. With the aid of equation (3) we can write $Z$ as

$$
Z = \int d^4x \sqrt{J(x)} L_{\text{eff}}[J, K] \nonumber
$$

$$
L_{\text{eff}}[J, K] = \sum_{\text{topologies}} \int \mathcal{D}[\eta] \exp \left[ -S(x + \eta, K) \right] . \tag{4}
$$

In principle, $L_{\text{eff}}$ depends on all powers of the derivatives of the fields multiplied
by suitable powers of $l_m$. However, the gauge symmetries of the model force the derivatives of $J_{\mu\nu}(x)$ and $K_{\mu\nu\rho}(x)$ to appear only through the curvature tensor of the $J_{\mu\nu}$-metric and the field strength $K_{\mu\nu\rho\sigma} = \partial_{[\mu}K_{\nu\rho\sigma]}$. The $\sqrt{J(x)}$ factor comes from the zero-mode contribution to the functional covariant measure in equation (2).

On general grounds, $L_{\text{eff}}$ will be a non-local quantity difficult to compute. However, our purpose is to determine the “low energy” approximation of $L_{\text{eff}}$, i.e., the leading terms in the $l_m$ power expansion $^\star$, which, at some energy scale below the Planck energy, dominate the local part of the effective action. Furthermore we do not consider branching and rejoining processes and take into account only the contribution to the functional integral coming from “free” membranes emerging from a point and finally recollapsing to a point. In this approximation we interpret equation (2) as the generating functional for the vacuum-to-vacuum amplitude in the presence of the background fields $J$ and $K$ and the general form of $L_{\text{eff}}$ is determined by the requirements of general covariance and gauge symmetry alone:

$$Z[J, K] = \frac{1}{l_m^4} \int d^4x \sqrt{J} \left[ 2\Lambda + l_m^2 \left( a J^{\mu\nu} R_{\mu\nu}(J) - b K_{\mu\nu\rho\sigma} K_{\mu'\nu'\rho'\sigma'} J^{\mu\mu'} J^{\nu\nu'} J^{\rho\rho'} J^{\sigma\sigma'} \right) + O(l_m^4) \right],$$

in terms of three positive numerical constants $a$, $b$, and $\Lambda$. It is worth noticing that similar approach has been adopted for string as well $^6$, but in that case the abovementioned constants have an unpleasant cut-off dependence. In this case however, it turns out that the constants are finite which makes membranes more appealing pregeometric objects.

$^\star$ This approximation is similar to the inverse mass power expansion of the effective action in chiral gauge theories $^7$. The mass of the field is here replaced by the membrane tension.
Variation of $Z[J,K]$ with respect to the external sources provides now the corresponding vacuum average of the composite field operators coupled to them, i.e. the so called classical fields:

\[ J_{\text{cl}}^{\mu\nu}(J_{\rho\sigma}; K_{\rho\sigma\tau}) \equiv \frac{\delta Z}{\delta J_{\mu\nu}(x)} \]

\[ K_{\text{cl}}^{\mu\nu\rho}(J_{\rho\sigma}; K_{\rho\sigma\tau}) \equiv \frac{\delta Z}{\delta K_{\mu\nu\rho}}. \]

Thus, an effective dynamics for the background fields emerges at the quantum level. In fact, it is customary to derive the “effective action” for the classical fields by exchanging the external sources in favour of the classical fields by means of the functional Legendre transform

\[ \Gamma_{\text{eff}}(J_{\text{cl}}^{\mu\nu}, K_{\text{cl}}^{\mu\nu\rho}) = \int d^4x \left[ J_{\text{cl}}^{\mu\nu} J_{\mu\nu} + K_{\text{cl}}^{\mu\nu\rho} K_{\mu\nu\rho} \right] - Z(J_{\mu\nu}, K_{\mu\nu\rho}), \]

and then deriving effective field equations for the classical fields by varying $\Gamma_{\text{eff}}$.

Before we attempt to implement this algorithm, we should perhaps emphasize that while the background fields are put in “by hand” in the action in order to implement some symmetry principle, the classical fields are introduced dynamically in the theory as vacuum averaged values of suitable composite operators. In principle these c-number quantities can be expressed perturbatively in terms of the external sources once $Z(J,K)$ is determined in powers of $l_m$.

Our basic assumption, then, is the following generalized bootstrap hypothesis: at the lowest order in the expansion parameter, both the classical and the background fields coincide with the macroscopic fields in which the membrane moves. In other
words, over distances much larger than $l_m$, we can no longer distinguish between external sources, vacuum expectation values and the “classical forces” acting on the membrane.†

From eq. (7) we obtain the explicit form of the classical fields in powers of $l_m^{-2}$

$$J_{cl}^{\mu\nu} = \frac{\Lambda}{l_m^4} \sqrt{J} J^{\mu\nu} - \frac{1}{l_m^2} \sqrt{J} \left[ a \left( R^{\mu\nu} - \frac{1}{2} J^{\mu\nu} R \right) + 4 b \left( K^{\mu\lambda}_\rho \lambda^{\nu\rho\sigma} - \frac{1}{8} J^{\mu\nu} K_{\lambda\rho\sigma\tau} K^{\lambda\rho\sigma\tau} \right) \right] + \ldots$$

$$K_{cl}^{\mu\nu\rho} = \frac{48 b}{l_m^2} \sqrt{J} \nabla_\lambda K^{\lambda\mu\nu\rho} + \ldots$$

(10)

where $\nabla_\lambda$ is the torsion free covariant derivative compatible with the $J_{\mu\nu}$ metric.

Correspondingly, we expand the sources as

$$J_{\mu\nu} = g_{\mu\nu} + l_m^2 j_{\mu\nu} + \ldots$$

$$J^{\mu\nu} = g^{\mu\nu} - l_m^2 g^{\mu\mu'} g^{\nu\nu'} j_{\mu\nu'} + \ldots$$

$$\sqrt{J} = \sqrt{g} \left( 1 + \frac{l_m^2}{2} g^{\mu\nu} j_{\mu\nu} \right)$$

$$K_{\mu\nu\rho} = K_{\mu\nu\rho}^{(0)} + l_m^2 K_{\mu\nu\rho}^{(1)} + \ldots$$

(11)

and, following our bootstrap hypothesis, set

$$J_{cl}^{\mu\nu} \equiv \frac{\Lambda}{l_m^4} \sqrt{g} g^{\mu\nu}$$

$$K_{cl}^{\mu\nu\rho} \equiv \frac{48 b}{l_m^2} \sqrt{g} \nabla_\lambda H^{\lambda\mu\nu\rho}$$

$$H_{\lambda\mu\nu\rho} \equiv \partial_{[\lambda} B_{\mu\nu\rho]}$$

(12)

where the covariant derivative is now computed by means of the macroscopic metric.

† A similar self-consistency criterion has been recently applied in the framework of a “mean field” quantization of a GL(4) gauge theory of gravity. Also in this case one has both the soldering form as dynamical variable and an auxiliary metric introduced into the model only to allow perturbative calculations. At the end the vacuum expectation value of the composite operator describing the physical metric is self-consistently identified with the background metric itself.
$g_{\mu \nu}$. Then, we find at the $l_m^2$ order,

$$K^{(0)}_{\mu \nu \rho} = B_{\mu \nu \rho}$$  \hspace{1cm} (13a)

$$\Lambda j^{\mu \nu} = -a R^{\mu \nu}(g) + 4b \left[ K^{\mu \rho \sigma \tau} K^{\nu \rho \sigma \tau} - \frac{3}{8} g^{\mu \nu} K^{\lambda \rho \sigma \tau} K_{\lambda \rho \sigma \tau} \right].$$  \hspace{1cm} (13b)

Now, we can evaluate the Legendre transform (8) and compute the effective action for the classical fields $g$ and $B$ up to the order $l_m^2$:

$$\Gamma_{\text{eff}}(g_{\mu \nu}, B_{\mu \nu \rho}) = \int d^4x \sqrt{g} \left[ \left( \frac{4 \Lambda}{l_m^4} - \frac{a}{l_m^2} R(g) - \frac{4b}{l_m^2} H_{\mu \nu \rho \sigma} H^{\mu \nu \rho \sigma} \right) - \frac{2b}{l_m^2} H_{\mu \nu \rho \sigma} H^{\mu \nu \rho \sigma} \right]$$

$$\quad - \frac{1}{l_m^4} \int d^4x \sqrt{g} \left[ 2 \Lambda + l_m^2 \left( a g^{\mu \nu} R_{\mu \nu}(g) - b H_{\mu \nu \rho \sigma} H^{\mu \nu \rho \sigma} \right) \right.$$  

$$\quad \left. - l_m^2 \left( a g^{\mu \nu} R_{\mu \nu}(g) + 2b H_{\mu \nu \rho \sigma} H^{\mu \nu \rho \sigma} \right) + 0(1) \right].$$  \hspace{1cm} (14)

If we define the induced constants as

$$\frac{a}{l_m^2} \equiv \frac{1}{16\pi G_N},$$  \hspace{1cm} (15a)

$$\frac{2\Lambda}{l_m^4} \equiv \frac{2\lambda_{\text{ind}}}{16\pi G_N},$$  \hspace{1cm} (15b)

and rescale the generalized Maxwell field strength according to

$$\frac{b}{l_m^2} H_{\mu \nu \rho \sigma} H^{\mu \nu \rho \sigma} \equiv \frac{1}{2 \cdot 4!} F_{\mu \nu \rho \sigma} F^{\mu \nu \rho \sigma},$$  \hspace{1cm} (16)

we obtain that the low energy limit of the membrane effective action is the Einstein action with a cosmological term, coupled to the $F$-field:

$$\Gamma_{\text{eff}}(J^{\mu \nu}_{\text{cl}}, K^{\mu \nu \rho}_{\text{cl}}) = - \int d^4x \sqrt{g} \left[ \frac{1}{16\pi G_N} \left( R(g) - 2\lambda_{\text{ind}} \right) + \frac{1}{2 \cdot 4!} F_{\mu \nu \rho \sigma} F^{\mu \nu \rho \sigma} \right].$$  \hspace{1cm} (17)

In order to reproduce the correct value of $G_N$ the constant $l_m$ must be of the order
of the Planck length, i.e. \( l_m \sim L_P \), and our pregeometric membranes have surface tension \( \sim (\text{Planck Mass})^3 \); correspondingly, the induced Cosmological Constant turns out to be, as usual, enormously large: \( \lambda_{\text{ind}} \sim L_P^2 \). However, this result is not as bad as it would seem at first sight because \( \lambda_{\text{ind}} \) is not the physical cosmological constant! In fact, the generalized Maxwell field strength, in four dimensions, has no propagating modes associated to it, rather it represents a constant energy density background which shifts the value of the cosmological constant. The Maxwell equation derived from eq.(17) admits a solution of the form

\[
\bar{F}_{\mu\nu\rho\sigma} = m^2 \delta_{[\mu\nu\rho\sigma]}, \quad m = \text{const}, \quad [m] = \text{mass}.
\]  

(18)

When inserted back in \( \Gamma_{\text{eff}} \), the solution (18) gives the Einstein action with a new effective cosmological constant, \( \lambda_{\text{phys}} \), given by

\[
\lambda_{\text{phys}}(m) = \lambda_{\text{ind}} - 8\pi G_N m^4.
\]  

(19)

Thus, the physical spacetime emerging from the underlying quantum dynamics is a solution of the vacuum Einstein equations with a cosmological term

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left( R - 2\lambda_{\text{phys}}(m) \right) = 0
\]  

(20)

The actual value of \( \lambda_{\text{phys}}(m) \) can now be fixed by the Hawking-Baum argument \([9,10]\). Supplemented with the Hartle-Hawking boundary condition \([11]\) the effective Einstein equation (20) admits as solution the 4-sphere of radius \( r = \left( \frac{3}{\lambda_{\text{phys}}(m)} \right)^{1/2} \) to which correspond an action

\[
\Gamma_{\text{eff}}(S^4) = -\frac{3\pi}{G_N \lambda_{\text{phys}}(m)}.
\]  

(21)

If one interprets \( \exp(-\Gamma_{\text{eff}}) \) as a probability distribution for the value of the
physical cosmological constant, then the peak for $\lambda_{\text{phys}}(m) \to 0^+$ suggests that the most probable value of the physical cosmological constant is zero. The key property which allows us to apply the Hawking-Baum argument is the general property that $(p+1)$ classical gauge forms in a $(p+2)$ dimensional spacetime describe a constant background energy distribution rather than propagating degrees of freedom $^{[4]}$. In particular, the possibility that four-forms may be used to circumvent the problem of the cosmological constant, either in conjunction with the Baum-Hawking mechanism or with Coleman’s mechanism $^{[12]}$, has been suggested by several authors $^{[13,14,15,16]}$. To our knowledge, however, the model discussed in this paper is the first one in which the Maxwell term for a four-form arises naturally in the low energy effective lagrangian for induced gravity; our discussion makes it clear that the presence of this term can be traced back to the use of a relativistic membrane, rather than a local field, or even a string, as the basic pregeometric object in the action (1).

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