The radion potential and supersymmetry breaking in detuned RS

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We study radiative corrections to the radion potential in the supersymmetric “detuned RS model”, with supersymmetry broken by boundary conditions. Classically, the radion is stabilized in this model, and the 4d theory is AdS$_4$. With a few bulk hypermultiplets, the one-loop correction to the cosmological constant is positive. For small warping, this correction can (almost) cancel the classical result. The loop expansion is still reliable in this limit. The graviphoton zero-mode, which controls supersymmetry breaking, is a modulus of the classical theory, but is stabilized at one-loop. Both unbroken supersymmetry and maximal supersymmetry breaking are stable ground-states of the quantum theory.

1 Introduction

An essential ingredient of brane-world models is fixing the brane distance, or stabilizing the radion field. In the “detuned Randall-Sundrum” (RS) model, where the brane tensions are different from their RS values, this happens automatically. The supersymmetric version of this model also allows for supersymmetry breaking by boundary conditions, unlike in RS. Classically, however, the resulting 4d theory has a negative cosmological constant, so it is not suitable for phenomenology. Here I will describe quantum corrections to the radion potential, since they require supersymmetry breaking, which is non-local, these corrections are finite, and in the presence of a few bulk hypermultiplets, give positive contributions to the potential. Furthermore, in models with very small warping, the loop corrections can be very significant, and substantially reduce the classical cosmological constant.

At low energies, the detuned model gives rise to an effective radion theory with a superpotential and Kähler potential which are very similar to those obtained in models of flux compactification, but whose origin is purely perturbative. Of the two scalars in this simple example, only the radion is stabilized classically, while its partner is stabilized at one loop.
2 The classical theory

The supersymmetric detuned RS model was studied in \[2,6\]. We will now review its main elements. Starting with the RS model and allowing arbitrary brane tensions, the position of the second brane is determined by the jump conditions,

\[ R = \frac{1}{2\pi k} \ln \left( \frac{(T + T_0)(T + T_\pi)}{(T - T_0)(T - T_\pi)} \right), \]

where \( T_0 \) and \( T_\pi \) are the brane tensions, \( k \) is the AdS\(^5\) curvature, and \( T \) is defined in terms of \( k \) and the 5d fundamental scale as \( T \equiv 6M_5^3k \). The theory can be supersymmetrized provided that

\[ |T_{0,\pi}| \leq T. \]

The resulting 4d theory is either AdS\(_4\) or Mink\(_4\) with the metric

\[ ds^2 = a^2(x_5)\hat{g}_{\mu\nu}dx^\mu dx^\nu - dx^2_5, \]

where \( \hat{g}_{\mu\nu} \) denotes the standard AdS\(_4\) or Mink\(_4\) metric in Poincaré coordinates, and where the warp factor is given by

\[ a(x_5) = e^{-kx_5} + \frac{1}{4k^2L^2}e^{kx_5}. \]

Here \( L \) is the 4d curvature radius, given by

\[ \frac{1}{4k^2L^2} = \frac{T - T_0}{T + T_0}. \]

It's easy to see that when the inequality \[2\] is violated, \( L^2 \) is negative, so the background is given instead by dS\(_4\), which is not compatible with supersymmetry. In fact, the origin of \[2\] is the need to introduce brane “mass terms” for the gravitini in order for the brane plus bulk action to be supersymmetric. These brane masses are given by,

\[ |\alpha_0|^2 = \frac{T - T_0}{T + T_0} = \frac{1}{4k^2L^2}, \hspace{1em} \text{and} \hspace{1em} |\alpha_\pi| = |\alpha_0|e^{k\pi R}, \]

so \[2\] must hold. Furthermore, \( \mathcal{N} = 1 \) supersymmetry is broken for \( \arg(\alpha_\pi) \neq \arg(\alpha_0) \). Clearly, this is not the case in RS, for which \( \alpha_{0,\pi} \) vanish. Working in the “downstairs” picture, supersymmetry breaking translates to breaking by boundary conditions.

Since the 5d theory has a local U(1)\(_R\) symmetry under which the gravitino is charged, a phase difference of \( \alpha_0 \) and \( \alpha_\pi \) can be compensated by an \( x_5\)-dependent U(1)\(_R\) transformation. The fifth component of the graviphoton, which is the U(1)\(_R\) gauge field, is then nonzero. This can also be seen in the low energy effective theory. At low energies, the theory contains, apart from the 4d SUGRA multiplet, a chiral \( \mathcal{N} = 1 \) supermultiplet \( T \), whose scalar component is \( r + ib \) where \( b \) is the zero mode of the fifth component of the graviphoton. The superpotential and Kähler potential are

\[ W = \frac{1}{1 - e^{-2k\pi R}} \frac{M_4^2}{L} \left( 1 - e^{k\pi R}e^{-3k\pi T} \right), \]

\[ K = -3M_4^2 \ln \left( \frac{1 - e^{-k\pi(\mathcal{T} + \bar{\mathcal{T}})}}{1 - e^{-2k\pi R}} \right), \]

up to \( \mathcal{O}(1/(M_4L)^2) \) corrections (where \( M_4 \) is the 4d Planck scale). Supersymmetry is then broken if,

\[ D_T W \propto \left( 1 - e^{i(\phi - 3k\pi b)} \right) \neq 0, \]
namely, for non-zero $b$. The field $b$, however, is a modulus of the classical theory. The potential derived from (7) depends on $r$ only, with a minimum at $r = R$.

The superpotential of (7) is of the same form as in KKLT models. Similarly, the Kähler potential of (7) coincides with the KKLT Kähler potential for small $k$.

3 Radiative corrections

When supersymmetry is broken, loop corrections can modify the 4d negative cosmological constant. Because supersymmetry breaking is non-local, loops are cutoff by the compactification scale, and are therefore finite. Since the computation of loop corrections in curved space is quite complicated, we will consider small detuning, or $1/(kL) << 1$, and perturb around the RS model. To leading order in $1/L^2$, we will then be able to use flat space propagators. Supersymmetry will further simplify the calculation.

3.1 KK contributions

In the supersymmetric RS model, each KK level contains two degenerate gravitini states, as well as bosonic states from the 5d graviton and graviphoton. Upon detuning the brane tensions, the 4d theory becomes AdS$_4$, so that the masses of each KK supermultiplet, and in particular the two gravitini, are split, with the splitting proportional to $1/L$. If in addition supersymmetry is broken, by a nonzero phase difference $\phi$ of the gravitino brane terms $\alpha_0$ and $\alpha_\pi$, the gravitini masses shift further. For small $\phi$, these shifts are proportional to $\phi/L$. The factor $1/L$ appears because supersymmetry is restored as the detuning goes to zero.

For unbroken supersymmetry, or $\phi = 0$, the contribution of each KK supermultiplet to the potential vanishes. Therefore, we can calculate the correction to the potential by considering only the gravitini KK tower,

$$\Delta V(\phi) = \Delta V_{\text{bosons}}(\phi) + \Delta V_{\text{fermions}}(\phi) = \Delta V_{\text{fermions}}(\phi) - V_{\text{fermions}}(\phi = 0).$$

(10)

Writing the $n$-th level gravitini masses as,

$$m^{(n)\pm} = m_0^{(n)} \pm \frac{1}{L} \left[ c_1^{(n)} + c_{1,SB}^{(n)} \phi^2 \right] + \frac{1}{kL^2} \left[ c_2^{(n)} + c_{2,SB}^{(n)} \phi^2 \right] + \mathcal{O}(\phi^4),$$

(11)

where the dimensionless coefficients $c$, which depend on $k$ and $R$, are calculated in[14] and where $m_0^{(n)}$ is the RS mass, the one-loop potential is

$$\Delta V = 4 \frac{1}{L^2} \phi^2 \times$$

$$\sum_n \int \frac{d^4p}{(2\pi)^4} \left[ 2 \left( \frac{m_0^{(n)}}{p^2 + (m_0^{(n)})^2} \right)^2 - \frac{c_1^{(n)} c_{1,SB}^{(n)}}{p^2 + (m_0^{(n)})^2} - \frac{c_0^{(n)} c_{2,SB}^{(n)}}{p^2 + (m_0^{(n)})^2} - \frac{c_1^{(n)} c_{1,SB}^{(n)}}{p^2 + (m_0^{(n)})^2} \right],$$

(12)

up to $1/L^4$ terms. In general this can only be calculated numerically. But it is easy to see from (12) that the correction is linear in the supersymmetry breaking scale. Since supersymmetry breaking is non-local, the result scales as the warp factor $\exp(-k\pi R)$. It is therefore significant only for small warping, $kR << 1$. In this limit, the one-loop potential (12) can be calculated analytically and reduces to,

$$\Delta V = -\frac{3\zeta(3)}{2^5 \pi^2} \frac{1}{(\pi R)^4} \frac{1}{(kL)^2} \phi^2.$$

(13)
3.2 4d radion theory

We can also calculate the one-loop correction to the potential using the 4d radion effective theory. The superpotential of \( \mathcal{W} \) is not renormalized at one-loop. Since it’s proportional to the AdS\(_4\) curvature \( 1/L \), its contribution to the potential starts as \( 1/L^2 \). Therefore, all we need is the one-loop Kähler potential to zeroth order in \( 1/L \), that is, the one-loop correction to the RS Kähler potential \( \mathcal{K} \). This correction depends on the radion superfield only through the combination \( \mathcal{T} + \bar{\mathcal{T}} \) and when combined with the superpotential of \( \mathcal{W} \), generates a \( b \)-dependent potential. The \( b \) dependence is of the form \( \pm \sin^2(3k\pi b/2) \), where the sign depends on the matter content of the theory. Pure supergravity gives a negative contribution, while hypermultiplets give a positive contribution. The simplest hypermultiplet to consider has bulk mass parameter \( c = 1/2 \), which gives minus a half of the gravity contribution. For four or more such multiplets, the net contribution is positive.

The loop correction stabilizes the modulus \( b \). The point \( b = 0 \) is then the global minimum of the potential, with unbroken supersymmetry. At \( b = 2/(3k) \), supersymmetry is maximally broken. This is a maximum along the \( b \) direction. Nonetheless, it is stable because the \( b \) mass is above the BF bound, as long as the net cosmological constant is negative.

As we saw above, the loop correction is suppressed for large warping, because it involves the warp factor \( \exp(-k\pi R) \). For small warping however, it can be significant. It is easy to see this by thinking about what happens as \( k \) is reduced, for constant \( kL \). The tree level potential decreases with \( k \), because there is no classical potential in the flat limit. The one-loop contribution on the other hand does not go to zero in this limit, because there is a non-vanishing Casimir energy even in the flat limit. In fact, the result \( V_\text{loop} \) reproduces the Casimir energy of flat orbifold models with supersymmetry broken by brane superpotentials proportional to \( 1/(kL) \) (for small supersymmetry breaking). Indeed,

\[
\frac{V_\text{loop}}{V_\text{tree}} \propto \frac{1}{(kR)^2}, \tag{14}
\]

which can easily compete with the loop suppression. Perturbation theory is still reliable, because higher loop corrections are suppressed compared to the one-loop result by the usual loop factors, as long as the number of bulk hypermultiplets is small.

We are thus led to consider the regime

\[
\frac{1}{L} \ll k \ll \frac{1}{R}. \tag{15}
\]

In principle, we can make the 4d cosmological constant arbitrarily small by a suitable choice of \( kR \). As long as the 4d cosmological constant is negative, the saddle point with \( b = 2/(3k) \) is still stable. For the purposes of model building however, it is sufficient to reduce the net cosmological constant to below the typical MSSM contribution, which is roughly a TeV\(^4\). If we embed the MSSM into the model, the soft masses generated are at most \( 1/L \). So the relevant question to ask is whether we can tune the parameters such that the net cosmological constant is comparable to \( 1/L^4 \). Note that the scale \( L \) only appears as a common overall factor, so the only relevant free parameter, when only \( c = 1/2 \) hypermultiplets are present, is \( kR \). In this simple model, the net cosmological cosmological cannot be made smaller than \( 1/L^4 \). But more complicated models with different hypermultiplets may be more successful in this regard, since they would involve several free parameters, and the resulting potentials would contain several terms with different \( r \) dependence.

\[\text{In principle, } M_4 \text{ and } R \text{ are independent parameters, but taking } R \text{ large relative to the inverse Planck scale does not help typically}\]
4 Conclusions

The supersymmetric detuned RS models has several attractive features. Classically, the radion is stabilized and supersymmetry can be broken spontaneously. With broken supersymmetry, loop corrections can reduce the classical 4d curvature. For small warping, these loop corrections can be very large, so this setup may be used as a starting point for constructing (practically) flat extra dimension models.

From a theoretical point of view, the model we considered here is an interesting example of moduli stabilization in AdS compactifications. The classical modulus of the theory is stabilized by loop corrections. Without any additional ingredients, radiative corrections can lift the tree-level vacuum energy towards zero with all moduli stabilized. The new ground state is a saddle point, corresponding to a minimum along the $r$ direction and a maximum along the $b$ direction. The field $b$ remains stable for arbitrarily small vacuum energy, since its mass is above the BF bound.

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