Superconductivity in doped two-leg ladder cuprates

Jihong Qin
Department of Physics, Beijing University of Science and Technology, Beijing 100083, China

Feng Yuan
Department of Physics, Qingdao University, Qingdao 266071, China

Shiping Feng
Department of Physics, Beijing Normal University, Beijing 100875, China

Within the $t$-$J$ ladder model, superconductivity with a modified d-wave symmetry in doped two-leg ladder cuprates is investigated based on the kinetic energy driven superconducting mechanism. It is shown that the spin-liquid ground-state at the half-filling evolves into the superconducting ground-state upon doping. In analogy to the doping dependence of the superconducting transition temperature in the planar cuprate superconductors, the superconducting transition temperature in doped two-leg ladder cuprates increases with increasing doping in the underdoped regime, and reaches a maximum in the optimal doping, then decreases in the overdoped regime.

In recent years two-leg ladder cuprates have attracted great interest since their ground state may be a spin liquid state with a finite spin gap.\textsuperscript{1–3} This spin liquid state may play a crucial role in superconductivity of the planar cuprate superconductors as emphasized by Anderson.\textsuperscript{4} When carriers are doped into two-leg ladder cuprates, a metal-insulator transition occurs.\textsuperscript{5–7} Although the ambient pressure ladder superconductivity was not observed until now, superconductivity in one of the doped two-leg ladder cuprate Sr$_{1-x}$Ca$_x$Cu$_2$O$_{4.1}$ has been observed under high pressure,\textsuperscript{5–7} which is the only known superconducting (SC) copper oxide without a square lattice. It has been shown that the most important role of pressure for realizing superconductivity is the doped hole redistribution between chains and ladders,\textsuperscript{5–7} and then the number of charge carriers on the ladders is increased.\textsuperscript{8,9} Moreover, the structure under high pressure remains the same as the case in ambient pressure,\textsuperscript{7} and the spin background in this SC phase does not drastically alter its spin gap properties. The particular geometrical arrangement of the Cu ions in two-leg ladder cuprates provides a playground for the normal- and SC-state studies of low-dimensional strongly correlated materials.\textsuperscript{1–3} This follows from the fact that all planar cuprate superconductors found up now contain square CuO$_2$ planes,\textsuperscript{10} whereas doped two-leg ladder cuprates consist of two-leg ladders of other Cu ions and edge-sharing CuO$_2$ chains.\textsuperscript{1–3} By virtue of the nuclear magnetic resonance and nuclear quadrupole resonance, particularly inelastic neutron scattering measurements, it has been shown that there is a region of parameter space and doping where doped two-leg ladder cuprates in the normal state is an antiferromagnet with the commensurate short-range order.\textsuperscript{3,11} Moreover, transport measurements on doped two-leg ladder cuprates in the same region of parameter space and doping indicate that the resistivity is linear with temperatures,\textsuperscript{8} one of the hallmarks of the exotic normal state properties found in the planar cuprate superconductors.\textsuperscript{10} These experimental results have revealed some close analogies between the doped planar cuprates and doped two-leg ladder cuprates. The normal-state of doped two-leg ladder cuprates exhibits a number of anomalous properties which is due to the charge-spin separation (CSS), while the SC state may be characterized by the charge-spin recombination.

Theoretically, there is a general consensus that the charge carrier pair of doped two-leg ladder cuprates in the SC-state are created on the ladders,\textsuperscript{2} i.e., superconductivity develops mainly within the ladders, with a minor role played by the interladder hopping amplitude. Within the $t$-$J$ ladder model, many authors have shown that the charge carrier pair correlation is very robust,\textsuperscript{12} clearly indicative of a ground-state dominated by strong SC tendencies. Moreover, it has been shown in the renormalized mean-field (MF) theory that superconductivity should exist in the d-wave channel,\textsuperscript{13} which has been confirmed by variety of numerical simulations.\textsuperscript{14} Within the framework of the CSS fermion-spin theory,\textsuperscript{15} we have developed a kinetic energy driven SC mechanism,\textsuperscript{16} where the dressed holons interact occurring directly through the kinetic energy by exchanging the spin excitations, leading to a net attractive force between the dressed holons, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation reveals the SC ground-state. This SC-state is controlled by both SC gap function and quasiparticle coherence, then the maximal SC transition temperature occurs around the optimal doping, and decreases in both underdoped and overdoped regimes.\textsuperscript{17} In particular, this kinetic energy driven SC mechanism does not depend on the fine details of a lattice structure, and the main ingredient was identified into a charge carrier pairing mechanism involving the internal spin degree of freedom.\textsuperscript{16} Therefore this SC mechanism shows that the strong electron correlation favors superconductivity.\textsuperscript{16,17} Since there is a remarkable resemblance in the normal-
state properties between the doped planar cuprates and doped two-leg ladder cuprates as mentioned above, and the strong electron correlation is common for both these cuprate materials, then two systems may have similar underlying SC mechanism, i.e., it is possible that superconductivity in doped two-leg ladder cuprates is also driven by the kinetic energy. In this paper, we discuss superconductivity in doped two-leg ladder cuprates along with the kinetic energy driven SC mechanism. We show that in analogy to the doping dependence of the SC transition temperature in the planar cuprate superconductors, the SC transition temperature in doped two-leg ladder cuprates increases with increasing doping in the underdoped regime, and reaches a maximum in the optimal doping, then decreases in the overdoped regime.

The basic element of two-leg ladder materials is the two-leg ladder, which is defined as two parallel chains of ions, with bonds among them such that the interchain coupling is comparable in strength to the couplings along the chains, while the coupling between the two chains that participates in this structure is through rungs. In this case, the t-J ladder model on the two-leg ladder is expressed as,

\[ H = -t \sum_{ \vec{r}_{\lambda \sigma} } C_{\lambda \sigma}^\dagger C_{\lambda + \vec{r}_{\lambda \sigma} \sigma} - t_L \sum_{ \sigma } (C_{1 \downarrow \sigma}^\dagger C_{2 \sigma} + \text{H.c.}) - \mu \sum_{ \lambda \sigma } C_{\lambda \sigma}^\dagger C_{\lambda \sigma} + J_\parallel \sum_{ \vec{r}_{\lambda \sigma} } S_{\lambda \uparrow} \cdot S_{\lambda + \vec{r}_{\lambda \sigma} \downarrow} + J_L \sum_{ i } S_{i \uparrow} \cdot S_{i + \vec{r}_{i \sigma} \downarrow} \]

where \( \vec{r} = \pm c_0 \hat{x}, c_0 \) is the lattice constant of the two-leg ladder lattice, which is set as unity hereafter, \( i \) runs over all rungs, \( \sigma(=\uparrow, \downarrow) \) and \( \alpha(=1, 2) \) are spin and leg indices, respectively. \( C_{i \sigma} \) (\( C_{i \alpha \sigma} \)) are the electron creation (annihilation) operators, \( S_{\lambda \sigma} = C_{\lambda \sigma}^\dagger \Phi C_{\lambda \sigma} / 2 \) are the spin operators with \( \Phi = (\sigma_x, \sigma_y, \sigma_z) \) as the Pauli matrices, and \( \mu \) is the chemical potential. This t-J ladder Hamiltonian (1) is subject to an important on-site local constraint \( \sum_{ \sigma } C_{i \sigma}^\dagger C_{i \sigma} \leq 1 \) to avoid the double occupancy. In the materials of interest, the exchange coupling \( J_\parallel \) along the legs is close to the exchange coupling \( J_L \) across a rung, and the same is true of the hopping \( t_\parallel \) along the legs and the rung hopping strength \( t_L \). Therefore, in the following discussions, we will work with the isotropic system \( J_\parallel = J_L = J, t_\parallel = t_L = t \). On the other hand, the single occupancy local constraint in the t-J ladder Hamiltonian (1) can be treated properly in analytical calculations within the CSS fermion-spin theory. The spinful fermion operator \( h_{i \sigma} = e^{-\phi_{\sigma}^i} h_{i \sigma} \) describes the charge degree of freedom together with some effects of the spin configuration rearrangements due to the presence of the doped hole itself (dressed holon), while the spin operator \( S_{i \sigma} \) describes the spin degree of freedom (spin), then the electron local constraint for single occupancy, \( \sum_{ \sigma } C_{i \sigma}^\dagger C_{i \sigma} = S_{i \uparrow}^\dagger h_{i \uparrow} h_{i \uparrow}^\dagger S_{i \downarrow} - S_{i \downarrow} h_{i \downarrow} h_{i \downarrow}^\dagger S_{i \uparrow} = h_{i \uparrow} h_{i \downarrow} (S_{i \uparrow}^\dagger S_{i \downarrow} + S_{i \downarrow}^\dagger S_{i \uparrow}) = 1 - h_{i \uparrow}^\dagger h_{i \downarrow} \leq 1 \), is satisfied in analytical calculations, and the double spinful fermion occupancies \( h_{i \uparrow} h_{i \downarrow} h_{i \downarrow}^\sigma = e^{\phi_{\sigma}^i} h_{i \sigma}^\dagger h_{i \sigma} e^{\phi_{\sigma}^i - \sigma} = 0 \) and \( h_{i \uparrow} h_{i \downarrow} h_{i \downarrow} = e^{-\phi_{\sigma}^i} h_{i \sigma} h_{i \sigma} e^{-\phi_{\sigma}^i - \sigma} = 0 \), are ruled out automatically. Since these dressed holons and spins are gauge invariant, and then in this sense, they are real and can be interpreted as the physical excitations. Although in common sense \( h_{i \sigma} \) is not a real spinful fermion, it behaves like a spinful fermion. In this CSS fermion-spin representation, the low-energy behavior of the t-J ladder Hamiltonian (1) can be expressed as,

\[ H = t \sum_{ \vec{r}_{\lambda \sigma} } (h_{i \uparrow + \vec{r}_{\lambda \sigma} \uparrow} h_{i \downarrow + \vec{r}_{\lambda \sigma} \downarrow} S_{i \uparrow}^\dagger S_{i \downarrow}^\dagger + h_{i \downarrow + \vec{r}_{\lambda \sigma} \uparrow} h_{i \uparrow + \vec{r}_{\lambda \sigma} \downarrow} S_{i \downarrow}^\dagger S_{i \uparrow}^\dagger) + t \sum_{ \vec{r}_{\lambda \sigma} } (h_{i \uparrow + \vec{r}_{\lambda \sigma} \uparrow} h_{i \downarrow + \vec{r}_{\lambda \sigma} \downarrow} S_{i \uparrow}^\dagger S_{i \downarrow}^\dagger + h_{i \downarrow + \vec{r}_{\lambda \sigma} \uparrow} h_{i \uparrow + \vec{r}_{\lambda \sigma} \downarrow} S_{i \downarrow}^\dagger S_{i \uparrow}^\dagger) + h_{i \uparrow} \sum_{ \sigma } S_{i \sigma} \cdot \Phi_{i \sigma} + J_\parallel \sum_{ \vec{r}_{\lambda \sigma} } S_{i \uparrow} \cdot S_{i + \vec{r}_{\lambda \sigma} \downarrow} + J_L \sum_{ i } S_{i \uparrow} \cdot S_{i + \vec{r}_{i \sigma} \downarrow} \]

with \( J_\parallel = J \) and \( \delta = (h_{i \uparrow}^\dagger h_{i \downarrow} h_{i \downarrow}^\sigma) = \langle h_{i \sigma} \rangle \) is the hole doping concentration. In this CSS fermion-spin representation, the kinetic terms have been expressed as the dressed holon-spin interactions, which reflect even the kinetic energy terms in the t-J ladder Hamiltonian have the strong Coulombic contribution due to the restriction of no doubly occupancy of a given site. As in the planar cuprate superconductors, the SC state in doped two-leg ladder cuprates is also characterized by electron Cooper pairs, forming SC quasiparticles. Because there are two coupled t-J chains in the two-leg ladder cuprates, therefore the order parameters for the electron Cooper pair is a matrix \( \Delta = \Delta_L + \sigma_x \Delta_T \), with the longitudinal and transverse order parameters are defined as,

\[ \Delta_L = \langle C_{i \sigma}^\dagger C_{j \sigma}^\dagger \rangle - \langle C_{i \sigma}^\dagger C_{j \sigma} \rangle = \langle h_{i \sigma} h_{j \sigma} S_{i \sigma}^\dagger S_{j \sigma} - h_{i \sigma} h_{j \sigma} S_{i \sigma}^\dagger S_{j \sigma} \rangle = -\langle S_{i \sigma}^\dagger S_{j \sigma} \rangle \]

\[ \Delta_T = \langle C_{i \sigma}^\dagger C_{j \sigma}^\dagger \rangle - \langle C_{i \sigma}^\dagger C_{j \sigma} \rangle = \langle h_{i \sigma} h_{j \sigma} S_{i \sigma}^\dagger S_{j \sigma} - h_{i \sigma} h_{j \sigma} S_{i \sigma}^\dagger S_{j \sigma} \rangle = -\langle S_{i \sigma}^\dagger S_{j \sigma} \rangle \Delta_T \]

respectively, where the longitudinal and transverse dressed holon pairing order parameters are expressed as,

\[ \Delta_{hL} = \langle h_{j \sigma} h_{i \sigma} \rangle - \langle h_{j \sigma} h_{i \sigma} \rangle \]

\[ \Delta_{hT} = \langle h_{j \sigma} h_{i \sigma} \rangle - \langle h_{j \sigma} h_{i \sigma} \rangle \]

In this case, the physical properties of doped two-leg ladder cuprates in the SC state are essentially determined by
the dressed holon pairing state, i.e., the SC order parameters are determined by the dressed holon pairing amplitude, and are proportional to the number of doped holes, and not to the number of electrons.

For discussions of superconductivity in the doped two-leg cuprates, we now introduce the dressed holon normal and anomalous Green’s functions and spin Green’s functions as, $g(k, \omega) = g_L(k, \omega) + \sigma_z g_T(k, \omega)$, with $\xi_{\nu k} = 2t \chi_{\parallel} \cos k_x + \mu + \chi_{\perp} t (-1)^{\nu+1}$, (7a)

$$\omega_{\nu k}^2 = \frac{1}{2} \alpha \epsilon_{\|}^2 \lambda^2 A_1 \cos^2 k_x + [X_1 + X_2 (-1)^{\nu+1}] \cos k_x$$

$$+ X_3 + X_4 (-1)^{\nu+1}, \quad \omega_{\nu k}^2 = \alpha \lambda^2 \chi_{\parallel} \cos k_x + [Y_1 + Y_2 (-1)^{\nu+1}] \cos k_x$$

$$+ Y_3 + Y_4 (-1)^{\nu+1}, \quad (7b)$$

where $X_1 = -\epsilon_\perp \lambda^2 \chi \lambda + \epsilon_\perp (C_\perp + \epsilon_\perp) / 2$, $X_2 = \alpha \lambda \lambda (\epsilon_\perp C_\perp + \epsilon_\perp (C_\perp + \epsilon_\perp) / 2)$, $X_3 = \lambda^2 [A_3 - \alpha \epsilon_\perp A_1 / 4 + \epsilon_\perp / 4] + \alpha \lambda \lambda (\epsilon_\perp C_\perp + 2 \epsilon_\perp / 4) + \epsilon_\perp C_\perp / 2 - \epsilon_\perp J_{\perp}^2 / 2$, $X_4 = -\epsilon_\perp \lambda^2 \chi_\parallel (\epsilon_\perp C_\perp + \epsilon_\perp (C_\perp + \epsilon_\perp) / 2) - \epsilon_\perp \lambda \lambda (\epsilon_\perp C_\perp + \epsilon_\perp (C_\perp + \epsilon_\perp) / 2)$.

Within the Eliashberg’s strong coupling theory, it has been shown that the dressed holon-spin interaction in the doped planar cuprates can induce the dressed holon pairing state (then the electron Cooper pairing state) by exchanging the spin excitations in the higher power of the doping concentration. Following their discussions, the self-consistent equations that satisfy the full dressed holon normal and anomalous Green’s functions in doped two-leg ladder cuprates are expressed as,

$$g(k, \omega) = g^{(0)}(k, \omega) + g^{(0)}(k, \omega) \Sigma^{(h)}_{\nu L}(k, \omega) g(k, \omega)$$

$$- \Sigma_{\nu L}^{(h)}(-k, -\omega) \Sigma^{(h)}_{\nu L}(k, \omega),$$

$$3^{(h)}(k, \omega) = g^{(0)}(-k, -\omega) \Sigma^{(h)}_{\nu L}(-k, -\omega) \Sigma^{(h)}_{\nu L}(-k, -\omega) + \Sigma_{\nu L}^{(h)}(-k, -\omega) g(k, \omega),$$

where the self-energy functions $\Sigma^{(h)}_{\nu L}(k, \omega) = \Sigma^{(h)}_{\nu L}(k, \omega) + \sigma_z \Sigma_{\nu L}^{(h)}(k, \omega)$ and $\Sigma_{\nu L}^{(h)}(k, \omega) = \Sigma_{\nu L}^{(h)}(k, \omega) + \sigma_z \Sigma_{\nu L}^{(h)}(k, \omega)$, with the longitudinal and transverse parts are evaluated as,

$$\Sigma_{\nu L}^{(h)}(k, i\omega_n) = \frac{1}{N^2} \sum_{p, p'} \langle \gamma_{p, p'+k}^2 + t^2 \rangle$$

$$\times \frac{1}{\beta} \sum_{ip_m} g_L(p + k, ip_m + i\omega_n) \Pi_LL(p, p', ip_m)$$

$$+ 2t_{p, p'+k} \frac{1}{\beta} \sum_{ip_m} g_T(p + k, ip_m + i\omega_n)$$

$$\times \Pi_{TL}(p, p', ip_m),$$

$$\Sigma_{\nu T}^{(h)}(k, i\omega_n) = \frac{1}{N^2} \sum_{p, p'} \langle \gamma_{p, p'+k}^2 + t^2 \rangle$$

$$\times \frac{1}{\beta} \sum_{ip_m} g_T(p + k, ip_m + i\omega_n) \Pi_{TT}(p, p', ip_m).$$
\[ g_L(k,\omega) = \frac{1}{2} \sum_{\nu=1,2} Z^{(\nu)}_{FA}(k) \left( \frac{U^2_{\nu k}}{\omega - E_{\nu k}} + \frac{V^2_{\nu k}}{\omega + E_{\nu k}} \right) \] (10a)

\[ g_T(k,\omega) = \frac{1}{2} \sum_{\nu=1,2} (-1)^{\nu+1} Z^{(\nu)}_{FA}(k) \left( \frac{U^2_{\nu k}}{\omega - E_{\nu k}} + \frac{V^2_{\nu k}}{\omega + E_{\nu k}} \right) \] (10b)

\[ \Delta^{(\nu)}_{h_z}(k) = \frac{1}{2} \sum_{\nu=1,2} Z^{(\nu)}_{FA}(k) \Delta^{(\nu)}_{h_z}(k) \left( \frac{1}{\omega - E_{\nu k}} - \frac{1}{\omega + E_{\nu k}} \right) \] (10c)

\[ \Delta^{(\nu)}_{h_z}(k) = \frac{1}{2} \sum_{\nu=1,2} (-1)^{\nu+1} Z^{(\nu)}_{FA}(k) \Delta^{(\nu)}_{h_z}(k) \left( \frac{1}{\omega - E_{\nu k}} - \frac{1}{\omega + E_{\nu k}} \right) \] (10d)

where the dressed holon quasiparticle coherence factors
\[ U^2_{\nu k} = [1 + \xi_{\nu k}/E_{\nu k}] / 2 \] and
\[ V^2_{\nu k} = [1 - \xi_{\nu k}/E_{\nu k}] / 2, \]
the renormalized dressed holon excitation spectrum \( \xi_{\nu k} = Z^{(\nu)}_{FA}(k)\xi_k \), the renormalized dressed holon pair gap function \( \Delta^{(\nu)}_{h_z}(k) = Z^{(\nu)}_{FA}(k)\Delta_{h_z}(k) + (-1)^{\nu+1}\Delta_{h_T}(k) \), and the dressed holon quasiparticle dispersion
\[ E_{\nu k} = \sqrt{[\xi_{\nu k}]^2 + [\Delta^{(\nu)}_{h_z}(k)]^2}. \] Although \( Z^{(1)}_{FA}(k) \) and \( Z^{(2)}_{FA}(k) \) still are a function of \( k \), the wave vector dependence may be unimportant, since everything happens at the electron Fermi surface. In this case, we need to estimate a special wave vector \( k_0 \) that guarantees \( Z^{(\nu)}_{FA}(k_0) \) near the electron Fermi surface. Following the discussions in the case of the planar cuprate superconductors, this special wave vector can be obtained as \( k_0 = k_A - k_p \), with \( k_A = \pi \) and \( k_p \approx (1 - x)\pi/2 \), then we only need to calculate \( Z^{(\nu)}_{FA}(k) \) near \( k_0 \). On the other hand, many authors have shown that superconductivity in doped two-leg ladder cuprates possesses a modified d-wave symmetry, and the gap function in this modified d-wave symmetry can be expressed as
\[ \Delta^{(1)}_{h_z}(k) = \Delta_{h_L}(k) + \Delta_{h_T}(k) = 2\Delta_{h_L} \cos k_x + \Delta_{h_T} \]
and \( \Delta^{(2)}_{h_z}(k) = \Delta_{h_L}(k) - \Delta_{h_T}(k) = 2\Delta_{h_L} \cos k_x - \Delta_{h_T} \) for the antibonding and bonding cases, respectively. In this case, the dressed holon effective gap parameters and quasiparticle coherent weights in Eq. (9) satisfy four equations.

\[ \Delta_{h_L} = -\frac{1}{32N^3} \sum_{k,p,p',\nu',\nu} \sum \cos(k_x - p_x + q_x) \times \Delta^{(1)}_{\nu\nu',\nu'}(k, q, p) \] (11a)

\[ \Delta_{h_T} = -\frac{1}{32N^3} \sum_{k,p,p',\nu',\nu} (-1)^{\nu + \nu' + \nu'' + 1} \times \Delta^{(1)}_{\nu\nu',\nu''}(k, q, p) \] (11b)
\[
\frac{1}{Z_{F1}} = 1 + \frac{1}{32N^2} \sum_{q,p} \sum_{\nu',\nu''} \Lambda^{(2)}_{\nu',\nu''}(q,p), \quad (11c)
\]
\[
\frac{1}{Z_{F2}} = -\frac{1}{32N^2} \sum_{q,p} \sum_{\nu',\nu''} (1+\nu'') (1+\nu''+1) \times \Lambda^{(2)}_{\nu',\nu''}(q,p), \quad (11d)
\]
where the kernel functions \(\Lambda^{(1)}_{\nu',\nu''}(k,q,p)\) and \(\Lambda^{(2)}_{\nu',\nu''}(q,p)\) are evaluated as,
\[
\Lambda^{(1)}_{\nu',\nu''}(k,q,p) = \frac{Z_{F_A}^{(0)}}{\omega_{\nu'}^{(0)}} B_{\nu'}^{\nu''} B(q) \gamma_{kq+k \nu' + (1-\nu''+1) \times (1+H_{\nu'}^{(2)}(q,p) + H^{(3)}_{\nu',\nu''}(q,p)}} \times \] 
\[
\Lambda^{(2)}_{\nu',\nu''}(q,p) = \frac{Z_{F_A}^{(0)} B_{\nu'}^{\nu''} B(q) \gamma_{kq+k \nu' + (1+\nu''+1) \times (1+H_{\nu'}^{(2)}(q,p) + H^{(3)}_{\nu',\nu''}(q,p)}} \times \] 
respectively, where \(F^{(1)}_{\nu',\nu''}(q,p) = n_B(\omega_{\nu'}^{(0)}) + n_B(\omega_{\nu''}^{(0)}) + 2n_B(\omega_{\nu'}^{(0)})n_B(\omega_{\nu''}^{(0)})\), \(F^{(2)}_{\nu',\nu''}(q,p) = 2n_B(\omega_{\nu'}^{(0)}) - 1\)} 
\[
\delta = \frac{1}{4N} \sum_{\nu,k \nu} Z_{F_A}^{(0)} B_{\nu'k} B_{\nu''k} \rho_{\nu,k} \rho_{\nu,k}, \quad (13c)
\]
\[
\chi = \frac{1}{4N} \sum_{\nu,k \nu} \cos k \rho_{\nu,k} \cot \frac{1}{2} \beta \omega_{\nu,k}, \quad (13d)
\]
\[
C = \frac{1}{4N} \sum_{\nu,k \nu} \cos \tilde{k} \rho_{\nu,k} \cot \frac{1}{2} \beta \omega_{\nu,k}, \quad (13e)
\]
\[
\Delta_{hL} = \frac{1}{2N} \sum_{\nu,k \nu} Z_{F_A}^{(0)} \cos k \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k}, \quad (13f)
\]
\[
\Delta_{hT} = \frac{1}{2N} \sum_{\nu,k \nu} (-)^{\nu-1} Z_{F_A}^{(0)} \cos k \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k}, \quad (13g)
\]
then all the above dressed holon effective gap parameters, quasiparticle coherent weights, dressed holon particle-hole order parameters, decoupling parameter \(\alpha\), spin correlation functions, and chemical potential \(\mu\) are determined by the self-consistent calculation. With the help of the above discussions, we now obtain the longitudinal and transverse parts of the dressed holon pair order parameter in Eq. (4) in terms of the dressed holon anomalous Green’s functions (10c) and (10d) as,
\[
\Delta_{hL} = \frac{1}{2N} \sum_{\nu,k \nu} Z_{F_A}^{(0)} \cos k \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k} \rho_{\nu,k}, \quad (13f)
\]
As in the case of the planar cuprate superconductors, this dressed holon pairing state originating from the kinetic energy terms by exchanging the spin excitations in doped two-leg ladder cuprates also leads to form the electron Cooper pairing state. For discussions of superconductivity in doped two-leg ladder cuprates, we need to calculate the electron anomalous Green’s function \(\Gamma^{(1)}(k,\omega) = \Gamma^{(1)}(k,\omega) + \alpha \Gamma^{(1)}(k,\omega)\), with the longitudinal and transverse parts are defined as, \(\Gamma^{(1)}(i - j, t - t') = \langle \langle C_{i a l}^{(1)}(t); C_{j a l}^{(1)}(t')\rangle\rangle\) and \(\Gamma^{(1)}(i - j, t - t') = \langle \langle C_{i a l}^{(1)}(t); C_{j a l}^{(1)}(t')\rangle\rangle\) \((\alpha' \neq \alpha)\). These longitudinal and transverse parts of the electron anomalous Green’s func-
tion are the convolutions of the corresponding longitudinal and transverse parts of the dressed holon anomalous Green’s function and spin Green’s function in the CSS fermion-spin theory, and reflect the charge-spin recombination\textsuperscript{22}. In terms of the MF spin Green’s functions in Eqs. (6c) and (6d) and dressed holon anomalous Green’s functions (10c) and (10d), we obtain the longitudinal and transverse parts of the electron anomalous Green’s function, then the longitudinal and transverse parts of the SC gap parameter are evaluated and bonding cases, respectively, then the longitudinal and transverse parts of the SC gap parameter \( \Delta(k) = \Delta_L(k) + \sigma_x \Delta_T(k) \) are evaluated as,

\[
\Delta_L(k) = \frac{1}{16N} \sum_{p,v,v'} Z_F^{(v')}(p-k) \frac{E_{v'}^{(v')}}{E_{v'}^{(v')} - \omega_{v'}} \beta \rho \text{coth} \left( \frac{1}{2} \beta \Delta_{h}^{(v')} \right), \tag{15a}
\]

\[
\Delta_T(k) = \frac{1}{16N} \sum_{p,v,v'} (-1)^{v+v'} Z_F^{(v')} \frac{E_{v'}^{(v')}}{E_{v'}^{(v')}} \omega_{v'} \beta \rho \text{coth} \left( \frac{1}{2} \beta \Delta_{h}^{(v')} \right), \tag{15b}
\]

which shows that the symmetry of the electron Cooper pair in doped two-leg ladder cuprates is essentially determined by the symmetry of the dressed holon pairs. In this case, the SC gap function is written as \( \Delta^{(1)}(k) = \Delta_L(k) + \Delta_T(k) = 2\Delta_L \cos \delta_x + \Delta_T \) and \( \Delta^{(2)}(k) = \Delta_L(k) - \Delta_T(k) = 2\Delta_L \cos \delta_x - \Delta_T \) for the antibonding and bonding cases, respectively, then the longitudinal and transverse parts of the SC gap parameter are evaluated in terms of Eqs. (15) and (14) as \( \Delta_L = -\chi^L_{h} \Delta_{h,L} \) and \( \Delta_T = -\chi^T_{h} \Delta_{h,T} \). In Fig. 1, we plot the longitudinal (solid line) and transverse (dashed line) parts of the dressed holon pairing (a) and SC (b) gap parameters as a function of the doping concentration \( \delta \) for parameter \( t/J = 2.5 \) at temperature \( T = 0.0001J \). Our result shows that both longitudinal and transverse parts have almost the same amplitude, and the longitudinal (transverse) part of the dressed holon pairing parameter has a similar doping dependent behavior of the longitudinal (transverse) part of the SC gap parameter. In particular, the value of the longitudinal part of the SC gap parameter \( \Delta_L \) increases with increasing doping in the underdoped regime, and reaches a maximum in the optimal doping \( x_{opt} \approx 0.07 \), then decreases in the overdoped regime.

Our result in Eq. (15) also shows that the SC transition temperature \( T_c \) occurring in the case of the SC gap parameter \( \Delta = 0 \) is identical to the dressed holon pair transition temperature occurring in the case of the dressed holon pairing gap parameter \( \Delta_h = 0 \). In correspondence with the SC gap parameter, the SC transition temperature \( T_c \) as a function of the hole doping concentration \( \delta \) for \( t/J = 2.5 \) is plotted in Fig. 2. For comparison, the experimental result of the SC transition temperature \( T_c \) in the doped two-leg ladder cuprate \( Sr_{0.4}Ca_{13.6}Cu_{24}O_{11} \) as a function of pressure\textsuperscript{7} is also shown in Fig. 2 (inset). Experimentally, it has been shown that the main effect of pressure in doped two-leg ladder cuprates is to reduce the distance between the ladders and chains, which leads to the doped hole redistribution between chains and ladders\textsuperscript{5-7}. However, the structure at ambient pressure with clearly defined ladders and chains remains stable under the pressure needed to induce the SC regime\textsuperscript{7}, and the spin background in this pressure induced SC phase does not drastically alter its spin gap properties\textsuperscript{2}. On the other hand, when Ca is doped upon the original Sr-based Ca-undoped phase, the interatomic distance ladder-chain was found to be reduced by Ca substitution, leading to a redistribution of holes originally present only on the chains\textsuperscript{8,9}. These experimental results show that an increase of the pressure may corresponding to an increase in the number of charge carriers on the ladders\textsuperscript{7-9}. In other words, the doping dependence of the SC transition temperature should be similar to the pressure dependence of the SC transition temperature. In this sense, our present result of the doping dependence of the SC transition temperature is qualitatively consistent with the experimental result\textsuperscript{7}. The

\[ \text{FIG. 1. The longitudinal (solid line) and transverse (dashed line) dressed holon pairing (a) and SC (b) gap parameters as a function of the doping concentration for } t/J = 2.5 \text{ with } T = 0.0001J. \]
maximal SC transition temperature $T_c$ occurs around the optimal doping concentration $x_{opt} \approx 0.07$, and then decreases in both underdoped and overdoped regimes. In particular, this domed shape of the doping dependence of the SC transition temperature is same as that of the doping dependence of the longitudinal part of the SC gap parameter, which shows that superconductivity is mainly produced by the development of the pairing correlation along legs, and is consistent with the one-dimensional charge dynamics under high pressure.\(^7\)\(^,\)\(^2\) Furthermore, $T_c$ in the underdoped regime is proportional to the hole doping concentration $\delta$, and therefore $T_c$ in the underdoped regime is set by the hole doping concentration, which reflects that the density of the dressed holons directly determines the superfluid density in the underdoped regime.

The essential physics of superconductivity in the present doped two-leg ladder cuprate superconductors is the same as that in the planar cuprate superconductors\(^16\),\(^17\), i.e., the SC-order in doped two-leg ladder cuprate superconductors is controlled by both gap function and quasiparticle coherence, which is reflected explicitly in the self-consistent equations (11). The dressed holons (then electrons) interact by exchanging the spin excitations and that this interaction is attractive. This attractive interaction leads to form the dressed holon pairs (then electron Cooper pairs). The parent compound of doped two-leg ladder cuprates is a Mott insulator, when holes are doped into this insulator, there is a gain in the kinetic energy per hole proportional to $t$ due to hopping, but at the same time, the spin correlation is destroyed, costing an energy of approximately $J$ per site, therefore the doped holes into the Mott insulator can be considered as a competition between the kinetic energy ($\delta t$) and magnetic energy ($J$), and the magnetic energy decreases with increasing doping. In the underdoped and optimally doped regimes, the magnetic energy is rather large, then the dressed holon (then electron) attractive interaction by exchanging the spin excitations is also rather strong to form the dressed holon pairs (then electron Cooper pairs) for the most dressed holons (then electrons), therefore the number of the dressed holon pairs (then electron Cooper pairs) and SC transition temperature are proportional to the hole doping concentration. However, in the overdoped regime, the magnetic energy is relatively small, then the dressed holon (then electron) attractive interaction by exchanging the spin excitations is also relatively weak, in this case, not all dressed holons (then electrons) can be bounden as dressed holon pairs (then electron Cooper pairs) by this weak attractive interaction, and therefore the number of the dressed holon pairs (then electron Cooper pairs) and SC transition temperature decrease with increasing doping.

In summary, we have discussed superconductivity with the modified d-wave symmetry in doped two-leg ladder cuprates based on the kinetic energy driven SC mechanism. It is shown that the spin-liquid ground-state at the half-filling evolves into the SC ground-state upon doping. In analogy to the doping dependence of the SC transition temperature in the planar cuprate superconductors, the SC transition temperature in doped two-leg ladder cuprates increases with increasing doping in the underdoped regime, and reaches a maximum in the optimal doping, then decreases in the overdoped regime.

When this work was finished we became aware of the discovery of the analogous quasi-one dimensional cuprate superconductor\(^23\) $\text{Pr}_2\text{Ba}_4\text{Cu}_7\text{O}_{15-\delta}$ at ambient pressure. It has been shown\(^23\) that in addition to the CuO$_2$ planes, the cuprate superconductor $\text{Pr}_2\text{Ba}_4\text{Cu}_7\text{O}_{15-\delta}$ contains two CuO chains. In particular, these two single chains bound together like the two-leg ladder in two-leg ladder cuprates. This cuprate superconductor $\text{Pr}_2\text{Ba}_4\text{Cu}_7\text{O}_{15-\delta}$ is called a double chain SC system, since the quasi-one dimensional double chain turns into the SC-state at ambient pressure, while the planes remain insulating even below the SC transition temperature. Although the Cu ions in the double chain do not form the two-leg ladder structure but a zigzag chain, the quasi-one dimensional nature of the double chain in the cuprate superconductor $\text{Pr}_2\text{Ba}_4\text{Cu}_7\text{O}_{15-\delta}$ is the same as that of the two-leg ladder in two-leg ladder cuprates. This experimental measurement\(^23\) on $\text{Pr}_2\text{Ba}_4\text{Cu}_7\text{O}_{15-\delta}$ provides an evidence that the doped quasi-one dimensional cuprates can become SC at ambient pressure when the doped charge carriers distribute properly along the chains. These and related issue is under investigation now.

![FIG. 2. The SC transition temperature as a function of the doping concentration for $t/J = 2.5$. Inset: the experimental result taken from Ref. [7].](image)
ACKNOWLEDGMENTS

The authors would like to thank Dr. Ying Liang, Dr. Tianxing Ma, Dr. Bin Liu, and Dr. Huaiming Guo for the helpful discussions. This work was supported by the National Natural Science Foundation of China under Grant Nos. 10547104, 10125415, and 90403005, and the funds from the Ministry of Science and Technology of China under Grant No. 2006CB601002.

1 E. Dagotto and T.M. Rice, Science 271, 618 (1996), and references therein.
2 E. Dagotto, Rep. Prog. Phys. 62, 1525 (1999), and references therein.
3 S. Katano, T. Nagata, J. Akimitsu, M. Nishi, and K. Katsumata, H. Eisaki, N. Motoyama, S. Uchida, S.M. Shapiro, and G. Shirane, Phys. Rev. B 54, 12199 (1996).
4 P.W. Anderson, in Frontiers and Borderlines in Many Particle Physics, edited by R.A. Broglia and J.R. Schrieffer (North-Holland, Amsterdam, 1987), p. 1; Science 235, 1196 (1987).
5 M. Uehara, T. Nagata, J. Akimitsu, H. Takahashi, N. Mori, and K. Kinoshita, J. Phys. Soc. Jpn. 65, 2764 (1996).
6 T. Nagata, M. Uehara, J. Goto, J. Akimitsu, N. Motoyama, H. Eisaki, S. Uchida, H. Takahashi, T. Nakanishi, and N. Mori, Phys. Rev. Lett. 81, 1090 (1998).
7 M. Isobe, T. Ohta, M. Onoda, F. Izumi, S. Nakano, J.Q. Li, Y. Matsui, E. Takayama-Muromachi, T. Matsumoto, and H. Hayakawa, Phys. Rev. B 57, 613 (1998).
8 T. Ohta, M. Onoda, F. Izumi, M. Isobe, E. Takayama-Muromachi, and A.W. Hewat, J. Phys. Soc. Jpn. 66, 3107 (1997).
9 M. Kato, K. Shiota, and Y. Koike, Physica C 258, 284 (1996); N. Motoyama, T. Osafune, T. Kakeshita, H. Eisaki, and S. Uchida, Phys. Rev. B 55, R3386 (1997); T. Osafune, N. Motoyama, H. Eisaki, and S. Uchida, Phys. Rev. Lett. 78, 1980 (1997).
10 See, e.g., M.A. Kastner, R.J. Birgeneau, G. Shirane, and Y. Endoh, Rev. Mod. Phys. 70, 897 (1998).
11 K. Magishi, S. Matsumoto, Y. Kitaoka, K. Ishida, K. Asayama, M. Uehara, T. Nagata, and J. Akimitsu, Phys. Rev. B 57, 11533 (1998); S. Ohsugi, K. Magishi, S. Matsumoto, Y. Kitaoka, T. Nagata, and J. Akimitsu, Phys. Rev. Lett. 82, 4715 (1998).
12 E. Dagotto, J. Riera, and D.J. Scalapino, Phys. Rev. B 45, 5744 (1992).
13 M. Sigrist, T.M. Rice, and F.C. Zhang, Phys. Rev. B 49, 12058 (1994).
14 J.A. Riera, Phys. Rev. B 49, 3629 (1994); H. Tsumetsugu, M. Troyer, and T.M. Rice, Phys. Rev. B 51, 16456 (1995); C.A. Hayward, D. Poilblanc, R.M. Noack, D.J. Scalapino, and W. Hanke, Phys. Rev. Lett. 75, 926 (1995).
15 Shiping Feng, Jihong Qin, and Tianxing Ma, J. Phys. Condens. Matter 16, 343 (2004).
16 Shiping Feng, Phys. Rev. B 68, 184501 (2003).
17 Shiping Feng, Tianxing Ma, and Huaiming Guo, Physica C 436, 14 (2006); Shiping Feng and Tianxing Ma, Phys. Lett. A 350, 138 (2006); Shiping Feng and Tianxing Ma, in New Frontiers in Superconductivity Research, edited by B.P. Martins (Nova Science Publishers, New York, 2006), Chapter 12, in press, cond-mat/0603148.
18 R.B. Laughlin, Phys. Rev. Lett. 79, 1726 (1997); J. Low. Temp. Phys. 99, 443 (1995).
19 See, e.g., C.C. Tsuei and J.R. Kirtley, Rev. Mod. Phys. 72, 969 (2000).
20 Jihong Qin, Yun Song, Shiping Feng, and Wei Yue Chen, Phys. Rev. B 65, 155117 (2002); Jianhui He, Shiping Feng, and Wei Yue Chen, Phys. Rev. B 67, 94402 (2003); Jihong Qin, Shiping Feng, and Feng Yuan, Phys. Lett. A 335, 477 (2005).
21 G.M. Eliashberg, Sov. Phys. JETP 11, 696 (1960); D.J. Scalapino, J.R. Schrieffer, and J.W. Wilkins, Phys. Rev. 148, 263 (1966).
22 P.W. Anderson, Phys. Rev. Lett. 67, 2092 (1991); Science 288, 480 (2000); Physica C 341-348, 9 (2000); cond-mat/0108522.
23 S. Sasaki, S. Watanabe, Y. Yamada, F. Ishikawa, K. Fukuda, and S. Sekiya, cond-mat/0603067.