Wiedemann-Franz violation in the vortex state of a \( d \)-wave superconductor

Wonkee Kim\(^a\), F. Marsiglio\(^a\) and J. P. Carbotte\(^b\)

\(^a\)Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2J1
\(^b\)Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, Canada, L8S 4M1

We show that the Wiedemann-Franz law is violated in the vortex state of a \( d \)-wave superconductor at zero temperature. We use a semiclassical approach, which includes the Doppler shift on the quasiparticles as well as the Andreev scattering from a random distribution of vortices. We also show that the vertex corrections to the electrical conductivity due to the anisotropy of impurity scattering become unimportant in the presence of a sufficiently large magnetic field.

The Wiedemann-Franz (WF) law is one of the fundamental laws which is well-obeyed in many solid state systems. The law is so universal that its violations are considered to be very interesting phenomena. The WF law can be violated depending on the nature of impurity scattering and the presence of a magnetic field in a \( d \)-wave superconductor.

Applying a magnetic field \((H)\) along the \( c \) direction to a \( d \)-wave superconductor, we study the in-plane quasiparticle transport properties at zero temperature \((T = 0)\). Since nodal quasiparticles give the dominant contribution to the conductivity at \( T = 0 \), we consider only quasiparticles in the nodal areas in momentum space. The magnitude of \( H \) is assumed to be \( H_{c1} < H < H_{c2} \), where \( H_{c1(2)} \) is the lower (upper) critical field.

At zero field \((H = 0)\) and \( T = 0 \), the D.C. conductivity in a \( d \)-wave superconductor is universal \(^1\): namely, it is independent of the impurity scattering rate: \( \sigma_{00} = 2(e^2/\pi^2)(v_f/v_g) \), where \( v_f(g) \) is the magnitude of the Fermi (gap) velocity.

Similarly, one can show that the thermal conductivity is also universal: \( \kappa_{00}/T = (2/3)(v_f/v_g + v_g/v_f) \) as \( T \to 0 \). Taking the ratio of \( \kappa_{00}/T \) and \( \sigma_{00} \), we immediately find that the Lorenz number \( L = \kappa/(T\sigma) \approx L_{00} \), where \( L_{00} = \pi^2/(3e^2) \), because \( v_f \gg v_g \) for a high-\( T_c \) superconductor. This means that the WF law is obeyed. However, it has been shown \(^3\) that the WF law is violated at finite \( T \) and the violation can depend on the nature of the impurity scattering in the absence of a magnetic field. In this work we show that the WF law is violated in the vortex state at zero temperature based on the semiclassical approach, which includes the Doppler shift as well as Andreev reflection \(^4\); therefore, the total scattering is given by the impurity scattering and the quasiparticle-vortex scattering \(^5\) which we take to be proportional to the magnetic energy \( E_H \) with a proportionality constant \( b \).

In Fig. 1, we plot the normalized Lorenz number \( L/L_{00} \) as a function of \( E_H/\gamma_{00} \), where

\[
E_H \simeq \sqrt{H/H_{c2}} \Delta_0
\]

with a superconducting gap \( \Delta_0 \), and \( \gamma_{00} \) is the impurity scattering rate in the absence of the magnetic field. If \( L/L_{00} = 1 \), the WF law is obeyed. Thus the difference between \( L/L_{00} \) and unity indicates the degree of the WF law violation. As one can see, the WF law is violated for a sufficiently high field such that \( E_H \gg \gamma_{00} \) regardless of the nature of the impurity scattering. Note that in this plot (as well as in everything we have said so far), we have considered only the bare bubble diagram for transport coefficients without including the vertex corrections due to the anisotropy of the impurity scattering. The effect of the vertex corrections is the issue we investigate next. To do this we take into account the
ladder diagrams as was done in Ref. [2].

We consider both the vertex corrections to the D.C. conductivity and to the thermal conductivity. As in Ref. [2], we introduce the scattering anisotropy parameters $R_2$ and $R_3$ for the adjacent-node and opposite-node scattering, respectively. $R_2 = R_3 = 1$ corresponds to the isotropic case for which the vertex corrections vanish. In Fig. 2, we show effects of the vertex corrections on the D.C. conductivity by plotting $\sigma_{VC}/\sigma$, where $\sigma_{VC}$ ($\sigma$) is the D.C. conductivity as a function of $E_H/\gamma_{00}$ with (without) vertex corrections. We choose $R_2 = 0.9$ and $R_3 = 0.8$. The top panel is for the Born limit while the bottom panel is for the unitary limit. As one can see, the vertex corrections are significant particularly for the unitary limit. However, in the presence of a sufficiently large field ($E_H \gg \gamma_{00}$), the corrections become relatively unimportant. We found that the vertex corrections to the thermal conductivity are negligible in the presence of a magnetic field as in the zero field case of Ref. [2].

In conclusion, the WF law is violated in the vortex state at zero temperature. The vertex corrections to the conductivity become unimportant in the presence of a sufficiently high field and may enhance the WF violation.

REFERENCES

1. P. A. Lee, Phys. Rev. Lett. 71, 1887 (1993).
2. A. C. Durst and P. A. Lee, Phys. Rev. B 62, 1270 (2000).
3. M. J. Graf, S.-K. Yip, J. A. Sauls, and D. Rainer, Phys. Rev. B 53, 15147 (1996).
4. F. Yu, M.B. Salamon, A.J. Leggett, W.C. Lee, and D.M. Ginsberg, Phys. Rev. Lett. 74, 5136 (1995).
5. W. Kim, F. Marsiglio, and J. P. Carbotte, unpublished.