Magnetic impurities in the one-dimensional spin-orbital model

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Using one-dimensional spin-orbital model as a typical example of quantum spin systems with richer symmetries, we study the effect of an isolated impurity on its low energy dynamics in the gapless phase through bosonization and renormalization group methods. In the case of internal impurities, depending on the symmetry, the boundary fixed points can be either an open chain with a residual spin or (and) orbital triplet left behind, or a periodic chain. However, these two fixed points are indistinguishable in the sense that in both cases, the lead-correction-to-scaling boundary operators (LCBO) only show Fermi-liquid like corrections to thermodynamical quantities. (Except the possible Curie-like contributions from the residual moments in the latter cases.) In the case of external (Kondo) impurities, the boundary fixed points, depending on the sign of orbital couplings, can be either an open chain with an isolated orbital doublet due to Kondo screening or it will flow to an intermediate fixed point with the same LCBO as that of the two-channel Kondo problem. Comparison with the Kondo effect in one-dimensional (1D) Heisenberg spin chain and multi-band Hubbard models is also made.

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I. INTRODUCTION

In the past few years, there have been intensive studies on the one-dimensional spin-orbital model both analytically and numerically. Part of the reasons stems from the belief that the unusual magnetic properties observed in some recently discovered quasi-one-dimensional spin gapped materials such as Na$_2$Ti$_2$Sb$_2$O$_7$ and Na$_2$V$_2$O$_7$ can be explained by a simple two-band Hubbard model at quarter filling. Owing to the strong Coulomb repulsion, the corresponding low energy effective Hamiltonian can then be mapped onto a quantum spin model:

$$H = \sum_i J_1 \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 T_i \cdot T_{i+1} + K (\mathbf{S}_i \cdot \mathbf{S}_{i+1})(\mathbf{T}_i \cdot \mathbf{T}_{i+1}), \quad (1.1)$$

where $\mathbf{S}_i$ and $\mathbf{T}_i$ are spin one-half operators representing spin and orbital degrees of freedom at each site, respectively. For generic couplings $J_1(2)$ and $K$, the model (1.1) has a SU(2)$_s$ $\otimes$ SU(2)$_t$ symmetry. However, at the special couplings $J_1 = J_2 = K/4$, the symmetry group is enlarged to SU(4), which is Bethe ansatz integrable. The low energy effective theory at this point is known to be described by the SU(4)$_2$ Wess-Zumino-Novikov-Witten (WZNW) model, with central charge $c = 3$, equivalent to three decoupled free bosons.

Besides being as a quantum spin model for quasi-one-dimensional materials, the spin-orbital model can also appear as a low energy effective theory in other context. An example of this is the spin-tube model studied recently by E. Origanc et al. In that case, the spin and orbital operators do not represent the real spin or orbital degrees of freedom, but are just mathematical objects used to describe the degenerate ground states obtained after projecting out the high energy states in the Hilbert space via renormalization group transformation.

Earlier studies of the model (1.1) around the SU(4) point were concentrated on the $\mathbb{Z}_2$ symmetric case $J_1 = J_2 = J$. The results show that when $J < K/4$ a small deviation from the SU(4) point is irrelevant, hence the low energy properties of the model is still controlled by the SU(4)$_2$ fixed point mentioned above. In contrast, for $J > K/4$, the deviation results in marginally relevant interactions which open a gap in the spectrum, and the ground states are dimerized with alternating spin and orbital singlets. The low-lying excitations are just the fermions of the SO(6) Gross-Neveu model.

Recently, the region where $J_1 \neq J_2$ has been explored in Ref. 2, and it was shown that the gapless phase can be extended to a large region around the original gapless line. (region V in Ref. 2 or phase B in Ref. 3.) Moreover, the low energy physics is described by SU(2)$_{2s} \otimes$SU(2)$_{2t}$ WZNW model, and the two level-two SU(2) WZNW models in general characterized by different velocities $u^*_s, u^*_t$. These conclusions are also consistent with those obtained through numerical studies.

On the other hand, the Kondo effect or more generally speaking, the quantum impurity problem in one-dimensional strongly correlated electron systems is one of the central topics in condensed matter physics over the past few years. It is known that the interacting environment developed around each local scattering center changes its character drastically. For example, a weak potential scatterer renormalizes into an infinitely strong blockade to transport, while a one-channel Kondo impurity develops properties reminiscent of the two-channel Kondo effect. From theoretical point of view, these problems usually provide interesting realizations of non-Fermi liquid physics, and the advances of nanofabrication techniques in the last few years make many of the above theoretical ideas can possibly be realized in laboratories.

In this paper, we would like to study the effect of local imperfection and Kondo problem of the one-dimensional spin-orbital model in its gapless phase. This can be viewed as a “stripped-down” version of quantum impurity problems where the charge degrees of freedom have been projected out, and may correspond to the Hubbard model.
II. BOSONIZATION FOR BULK AND BOUNDARY OPERATORS

The model (1.1) around the SU(4) point \((J_1 \simeq J_2 \simeq K/4)\) can be bosonized from the SU(4) Hubbard model at quarter filling \(\frac{\pi}{4a_0}\):

\[ H = \sum_i (-t\delta_{i+1}\sigma c_{i\sigma} + H.c.) + \frac{U}{2} \sum_{i\alpha\sigma} n_{i\alpha\sigma} n_{i\alpha\sigma'} (1 - \delta_{\alpha\sigma} \delta_{\alpha'\sigma'}) , \tag{2.1} \]

by introducing the left and right movers for low energy degrees of freedom around the Fermi points \(k_F = \pi/4a_0\):

\[ c_{i\sigma} \simeq R_{i\sigma}(x) \exp(ik_Fx) + L_{i\sigma}(x) \exp(-ik_Fx) . \]

At this point, we can bosonize the above slowly varying fields as usual through introducing four chiral bosonic fields \(\Phi_{\alpha\sigma R/L}\) using the Abelian bosonization formulæ:

\[ R_{\alpha\sigma} = \frac{\kappa_{\alpha\sigma}}{\sqrt{2\pi a_0}} \exp(i\sqrt{4\pi} \Phi_{\alpha\sigma R}) , \]
\[ L_{\alpha\sigma} = \frac{\kappa_{\alpha\sigma}}{\sqrt{2\pi a_0}} \exp(-i\sqrt{4\pi} \Phi_{\alpha\sigma L}) , \tag{2.2} \]

where the bosonic fields satisfy the commutation relation \([\Phi_{\alpha\sigma R}, \Phi_{\beta\sigma' L}] = \frac{1}{2} \delta_{\alpha\beta} \delta_{\sigma\sigma'}\), and the Klein factors \(\kappa_{\alpha\sigma}\) introduced here are used to insure the anticommutation relations between different flavors of fermions, which satisfies the following anticommutation rule \([\kappa_{\alpha\sigma}, \kappa_{\beta\sigma'}] = 2\delta_{\alpha\beta} \delta_{\sigma\sigma'}\). The physical properties of the system can be made more transparent by changing to a new basis:

\[ \Phi_c = \frac{1}{2} (\Phi_{1\uparrow} + \Phi_{1\downarrow} + \Phi_{2\uparrow} + \Phi_{2\downarrow}) , \]
\[ \Phi_s = \frac{1}{2} (\Phi_{1\uparrow} - \Phi_{1\downarrow} + \Phi_{2\uparrow} - \Phi_{2\downarrow}) , \]
\[ \Phi_f = \frac{1}{2} (\Phi_{1\uparrow} + \Phi_{1\downarrow} - \Phi_{2\uparrow} - \Phi_{2\downarrow}) , \]
\[ \Phi_{sf} = \frac{1}{2} (\Phi_{1\uparrow} - \Phi_{1\downarrow} - \Phi_{2\uparrow} + \Phi_{2\downarrow}) . \tag{2.3} \]

Umklapp scatterings arising at higher order perturbation theory will result in a Mott transition at finite value of \(U = U_c\), therefore for \(U >> U_c\), the charge field \(\Phi_c\) has a large gap, and only the spin-orbital part are left in the low energy sector. The remaining bosonized Hamiltonian can be further simplified by recombination through the introduction of six Majorana fermions \(\xi^a, a = 1 \ldots 6\):

\[ (\xi^1 + i\xi^2)_{R/L} = \frac{\eta_1}{\sqrt{\pi a_0}} \exp(\pm i\sqrt{4\pi} \Phi_{s R/L}) , \]
\[ (\xi^3 + i\xi^4)_{R/L} = \frac{\eta_2}{\sqrt{\pi a_0}} \exp(\pm i\sqrt{4\pi} \Phi_{f R/L}) , \]
\[ (\xi^5 + i\xi^6)_{R/L} = \frac{\eta_3}{\sqrt{\pi a_0}} \exp(\pm i\sqrt{4\pi} \Phi_{sf R/L}) , \tag{2.4} \]

where \(\eta_1, \eta_2, \eta_3\) are Klein factors. The resulting Hamiltonian can then be written as:

\[ \mathcal{H} = -\frac{iU_s}{2} (\xi_{s R} \partial_\xi_{s R} - \xi_{s L} \partial_\xi_{s L}) + (G_1 + G_3)(\kappa_1 + \kappa_2 + \kappa_6)^2 \]
\[ -\frac{iU_f}{2} (\xi_{i L} \partial_\xi_{i L} - \xi_{i R} \partial_\xi_{i R}) + (G_2 + G_3)(\kappa_3 + \kappa_4 + \kappa_5)^2 \]
\[ + 2G_3 (\kappa_1 + \kappa_2 + \kappa_6)(\kappa_3 + \kappa_4 + \kappa_5) , \tag{2.5} \]

where the spin and orbital triplets are defined as \(\xi_s = (\xi^2, \xi^4, \xi^6)\) and \(\xi^t = (\xi^1, \xi^3, \xi^5)\), \(\kappa_\alpha\) is defined as \(\xi^{\alpha,\beta}_{R/L}\).

It was shown in Ref. 2 and 3 that the Hamiltonian (2.3) contains several phases. Especially, there exist an equal to each other.

The spin and orbital density operators have the following general forms:

\[ S_i \sim J_{s R} + J_{s L} + (e^{i\pi x/2a_0} \mathcal{N}_s + H.c.) \]
\[ + (-1)^{x/a_0} a_s , \]
\[ T_i \sim J_{f R} + J_{f L} + (e^{i\pi x/2a_0} \mathcal{N}_t + H.c.) \]
\[ + (-1)^{x/a_0} a_t , \]
here $J_{s,t}$ are the smooth ($k \sim 0$) parts of the spin (or orbital) density, while $\mathcal{N}_{s,t}$ and $n_{s,t}$ are the $2k_F = \pi/2a_0$ and $4k_F = \pi/a_0$ parts.

The current operators can be expressed in terms of Majorana fermions:

$$J_{sR(L)} = \frac{i}{2} \xi_{sR(L)} \wedge \xi_{sR(L)} ,$$
$$J_{tR(L)} = -\frac{i}{2} \xi_{tR(L)} \wedge \xi_{tR(L)} .$$

(2.8)

The boson representations for $2k_F$ components $\mathcal{N}_{s,t}$ are:

$$\mathcal{N}^\pm \propto \exp \{ i\sqrt{\tau}(\Phi_s + \Phi_f + \Phi_{sf}) \} - \exp \{ i\sqrt{\tau}(\Phi_s - \Phi_f - \Phi_{sf}) \} + \exp \{ i\sqrt{\tau}(\Phi_s - \Phi_f + \Phi_{sf}) \} - \exp \{ i\sqrt{\tau}(\Phi_s + \Phi_f - \Phi_{sf}) \} .$$

(2.9)

where $\Theta_a$ are the dual fields of $\Phi_a$, and satisfy $[\Phi_a(x), \Theta_b(y)] = i\delta_{ab}\Theta(y - x)$.

The $2k_F$ components can be written in a more compact way by noting that the six Majorana fermions could be associated with six critical Ising models. Then using the order and disorder operators, $\sigma_a$, and $\mu_a$, of the Ising models, they can be expressed as follows:

$$\mathcal{N}^\pm \propto i\mu_1\mu_2\sigma_3\sigma_4\sigma_5\sigma_6 + \sigma_1\sigma_2\mu_1\mu_4\mu_5\mu_6 ,$$
$$\mathcal{N}^\pm \propto i\sigma_1\sigma_2\mu_3\mu_4\sigma_5\sigma_6 + \mu_1\mu_2\sigma_3\sigma_4\sigma_5\sigma_6 ,$$
$$\mathcal{N}^\pm \propto (\sigma_1\sigma_2 + i\mu_1\sigma_4)(\sigma_2\sigma_3\mu_4\mu_5\mu_6 - \sigma_1\sigma_2\mu_5\sigma_6) .$$

(2.10)

The $4k_F$ part of spin and orbital operators, generated from higher harmonics of bosonization due to interactions, can be written down by noting that these operators should transform as vectors under SO(3) and carry no chirality:

$$n_s \propto i\xi_{sR} \wedge \xi_{sL} ,$$
$$n_t \propto i\xi_{tR} \wedge \xi_{tL} .$$

(2.11)

Since the fixed point is governed by a SU(2)$_{2s}$SU(2)$_{2t}$ WZNW theory, it is better to rewrite the above operators in a way which makes the symmetry properties more transparent. This can be done by noting that each component of $S_i$ and $T_i$, should transform as a vector under spin and orbital SU(2) rotations, respectively. This means that each component of $S_i$ and $T_i$ should be primary fields of the SU(2)$_2$ WZNW model. It can then be immediately seen that $J_{s,t}$ are just the current operators of SU(2)$_{2s,t}$ WZNW models, and the $2k_F$ components correspond to the spin $1/2$ primary fields of it. The latter can be made evident from Eq. (2.10) by using the equivalence between a SU(2)$_2$ WZNW theory and three critical Ising models.

Especially, the spin $1/2$ primary field can be expressed as the product of three order or disorder operators of the corresponding three Ising models. We then have:

$$\mathcal{N}_s \sim \phi^{(2)}_a + ig^{(1)}_a t_0 ,$$
$$\mathcal{N}_t \sim \phi^{(2)}_a - ig^{(1)}_a t_0 ,$$

(2.12)

where $a = 1, 2, 3$ and $g^{(1,2)}_a$ are defined as:

$$g = \tau^a(\phi^{(1)}_a + ig^{(2)}_a) .$$

Here $\tau^a$ are Pauli matrices for $\alpha = 1, 2, 3$, $\tau^0$ is the identity matrix, and $g_{s,t}$ are spin one-half primary fields of SU(2)$_{2s,t}$ WZNW theory. The remaining primary fields with spin one $\Phi_{s,t}^1$ just correspond to the $4k_F$ components $n_{s,t}$.

After completing discussions about the fixed point theory and its operator contents, we turn to the bosonized forms of the above operators in open boundary condition. The open chain boundary condition introduces the following boundary conditions on the left- and right-moving fermion fields:

$$R_{aa}(0) + L_{aa}(0) = 0 ,$$

when transformed into boson language, it becomes

$$\Phi_{aR}(0) + \Phi_{aL}(0) = -\sqrt{\tau} .$$

We can then analytically continue the right-moving fields to left-moving fields by $\Phi_{aR}(x,t) = -\sqrt{\tau} - \Phi_{aL}(-x,t)$. In this way, we arrive at a description of the system in terms of chiral fields only.

With the above relations, we find for the boundary fields, we have:

$$\Phi_{aa}(x,t) = -\sqrt{\tau} + \Phi_{aL}(x,t) - \Phi_{aR}(-x,t)$$

$$\Rightarrow \Phi_{aa}(0,t) = -\sqrt{\tau} ,$$

$$\Theta_{aa}(x,t) = \sqrt{\tau} + \Phi_{aL}(x,t) + \Phi_{aR}(-x,t)$$

$$\Rightarrow \Theta_{aa}(0,t) = \sqrt{\tau} + 2\Phi_{aL}(0,t) ,$$

(2.13)

or in terms of Majorana fermions:

$$\xi_{R}^{a}(x,t) = \xi_{L}^{a}(-x,t) .$$

(2.14)

Substituting Eq. (2.13), Eq. (2.14) into Eq. (2.8), Eq. (2.9) and Eq. (2.11), it is easy to see that all components of spin (or orbital) operators are proportional to the current operators, i.e.:

$$S_{boundary} \propto J_{sL}(0) ,$$
$$T_{boundary} \propto J_{tL}(0) .$$

(2.15)

This completes our discussions about the bosonization formulas.
III. BOUNDARY CRITICAL BEHAVIOR

In this section, we shall apply the bosonization formulas obtained in previous sections to discuss the possible boundary fixed points. Two cases are considered here: local defects which result in a change of local coupling strength compared with the bulk value and an external local moment coupled to the bulk system (Kondo impurity).

A. Internal impurities

As discussed by Eggert and Affleck in the case of local defect, there are two important symmetries which distinguish the possible boundary fixed points of a Heisenberg chain: The site parity $P_S$ which is reflection of the chain about one site, and the link parity $P_L$ which reflects the chain about one link. Another important symmetry of the lattice system is the translation by one site $T$, and we have the relation $P_L = P_S \otimes T$. As we shall see later, it is exactly the same symmetries which distinguish the possible boundary critical behaviors in the case of spin-orbital model.

We first discuss the case where the local defect is invariant under link parity $P_L$, i.e. altering the coupling strength of one link slightly. The corresponding operators are $S_i \cdot S_{i+1}, T_i \cdot T_{i+1}$, and $(S_i \cdot S_{i+1}) \times (T_i \cdot T_{i+1})$. Using Eq. (3.1) and the following fusion rules for SU(2)$_2$ WZNW model:

\[
g \Phi \sim g,
\]

\[
J_L^z(z) g(\omega, \omega) \sim \frac{-\delta^a g}{z-\omega},
\]

\[
J_R^z(\bar{z}) g(\omega, \bar{\omega}) \sim \frac{g\delta^a}{\bar{z}-\bar{\omega}},
\]

where $g$ and $\Phi$ are spin-1/2 and spin-1 primary fields, respectively. It is then easy to see that the leading contribution from $S_i \cdot S_{i+1}$ and $T_i \cdot T_{i+1}$ is:

\[
\hat{O}_I = (i)^2 (\text{const.} \ tr g_s \cdot tr g_t + \text{const.} \ tr g_s^t \cdot tr g_t + \text{H.c.}) .
\] (3.2)

Since both $g_s$ and $g_t$ have conformal dimensions $(3/16, 3/16)$, the above operator $\hat{O}_I$ has scaling dimension $\Delta = 4/3$ and is a relevant boundary operator. Hence a small deviation of the coupling will be driven to strong coupling for either sign of $\delta J_{1,2}$. The remaining operator $(S_i \cdot S_{i+1}) \times (T_i \cdot T_{i+1})$ can be extracted from fusion between two $\hat{O}_I S$ and only contributes a marginal operator $\sim tr \Phi + tr \Phi$, which does not affect the RG flow. At this point, one important difference between the one-dimensional spin-orbital model and the usual antiferromagnetic Heisenberg chain should be noticed: For the spin-orbital model, the coupling constants between neighboring sites can be either antiferromagnetic (AF) or ferromagnetic (FM). With this in mind, we expect the following possible strong coupling behaviors:

- Case I. $J_{1(2)} > 0$ and $\delta J_{1(2)} > 0$. The couplings will flow toward strong AF coupling, and the local spin (orbital) degrees of freedom will form a singlet. The system becomes an open chain with two fewer spin (orbital) degrees of freedom. The stability of this strong coupling fixed point is guaranteed by the fact that the LCBO at this fixed point is just the product of two boundary spin (orbital) chiral current operators which have dimension two, hence are irrelevant.

- Case II. $J_{1(2)} < 0$ and $\delta J_{1(2)} < 0$. The couplings will flow toward strong FM couplings, and the local spin (orbital) degrees of freedom will form a triplet. The system becomes an open chain with an additional triplet degree of freedom left. The stability of this strong coupling fixed point is guaranteed by the fact that the residual coupling between the triplet moment and the open chain is FM, hence is marginally irrelevant.

- Case III. $J_{1(2)} > 0$ and $\delta J_{1(2)} < 0$ or $J_{1(2)} < 0$ and $\delta J_{1(2)} > 0$. In this case, the coupling will flow to zero and leave a residual spin-orbital coupling $K(S_0 \cdot S_1)(T_0 \cdot T_1)$. The fate of these degrees of freedom at impurity sites 0 and 1 depends on the details of combinations of various possibilities. For example, if spin $\in$ case I, orbital $\in$ case III, the two sites will first form a spin singlet with a residual orbital degrees of freedom described by $\frac{1}{4} K (T_0 \cdot T_1)$ which has a lower orbital triplet separated from a higher orbital singlet by a gap $\sim O(K)$. In this case, $J_2 < 0$, then the strong coupling fixed point is just an open chain with an orbital triplet. If $J_2 > 0$, the triplet will be further screened by neighboring sites due to Kondo screening and the strong coupling fixed point is just an open chain. On the other hand, if spin $\in$ case II, orbital $\in$ case III, the two sites will form a spin triplet with a residual orbital degrees of freedom described by $\frac{1}{4} K (T_0 \cdot T_1)$ which has a ground state with an orbital singlet separated from the higher triplet state. Then with the same reasoning as previous discussion, one expect the strong coupling fixed point is either an open chain or an open chain with a residual ferromagnetic coupling to a spin or orbital triplet.

The resulting possible boundary critical behavior is summarized in Table 1. Besides, we should mention that the appearance of a local triplet will not change the LCBOs, and only leads to an additional ground state degeneracy, hence an additional impurity entropy $S_{\text{imp}} = \ln 3$. Of course these asymptotically decoupled local moments will also add a Curie-like contribution to $C_\text{imp}$ and $\chi_\text{imp}$, in additional to logarithmic corrections characteristic of asymptotic freedom.

We now turn to the case where the local defect respects site parity $P_s$, i.e. varying the coupling strength of two adjacent links by the same amounts. In this case,
the leading boundary operator arises from the sum of \( S_i \cdot S_{i+1} \) and \( T_i \cdot T_{i+1} \) between two adjacent links. Due to the staggering factor in front of the \( 2k_F \) and \( 4k_F \) components, in the continuum limit, it is just the differential of Eq. (3.2):

\[
\frac{d}{dx} (\text{const.} \, \text{trg}_s \cdot \text{trg}_t + \text{const.} \, \text{trg}_s^\dagger \cdot \text{trg}_t + \text{H.c.}) ,
\]

which has scaling dimension \( 1 + \frac{3}{4} = \frac{7}{4} \). Therefore, we conclude that a small deviation of coupling strength of two adjacent sites is irrelevant and the low energy fixed point is just a periodic chain. Since the open chain with a decoupled spin or orbital doublet is stable only for ferromagnetic couplings, we arrive at the conclusion that when \( J_{1,2} > 0 \), the open chain will be unstable and flow to the stable periodic chain with the impurity site included. However, for \( J_{1,2} < 0 \), the open chain and periodic chain fixed points are not connected by a monotonous RG flow.

From the above discussions, we find that for a local defect, there can be two possible boundary critical behaviors similar to that of a Heisenberg chain:

- For impurities which violate the site parity \( P_s \), the infrared fixed point corresponds to an open boundary condition.
- For impurities which respect \( P_s \) (hence violate \( P_L \)), the local defect is irrelevant if the deviation of coupling strength is not too large and at low energy, the chain “heals”.

**TABLE I.** Possible boundary critical behaviors for a local defect respect link parity \( P_L \) (OC represents the open chain, \( T_s \) and \( T_t \) represent residual spin and orbital triplets, respectively):

| \( J_1 \) > 0 | \( J_1 \) > 0 | \( J_1 \) < 0 | \( J_1 \) < 0 | \( J_2 \) > 0 | \( J_2 \) > 0 | \( J_2 \) < 0 | \( J_2 \) < 0 |
|---|---|---|---|---|---|---|---|
| OC | √ | √ | √ | √ | OC+\( T_s \) | √ | √ | √ |
| OC+\( T_t \) | √ | √ | √ | | OC+\( T_s + T_t \) | √ | |

**B. External impurities**

In this section, we discuss the boundary critical behavior for an external spin \( 1/2 \) local moment coupled to the bulk system via AF exchange (Kondo) coupling. The Hamiltonian is decomposed as \( \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_K \) where \( \mathcal{H}_0 \) is Eq. (2.6) and \( \mathcal{H}_K \) is:

\[
\mathcal{H}_K = J_K S_\text{imp} \cdot S_0 ,
\]

\[
= S_\text{imp} \cdot \left\{ \lambda_F J_s(0) + \lambda_B (N_s(0) + N_s^\dagger(0)) + \lambda_{4k_F} n_s(0) \right\} .
\]

Here \( J_K > 0 \) is the Kondo coupling, \( \lambda_F, \lambda_B, \lambda_{4k_F} \) are the forward, backward, \( 4k_F \) scattering strength, respectively, and all are proportional to \( J_K \). Since the Kondo coupling is antiferromagnetic, all the above coupling constants are relevant. In fact, they satisfy the renormalization group equations:

\[
\frac{d\lambda_F}{d\ln L} = \frac{1}{2\pi u_s^2} (\lambda_F^2 + \lambda_{4k_F}^2) ,
\]

\[
\frac{d\lambda_B}{d\ln L} = \frac{1}{4} \lambda_B + O(\lambda_B \lambda_F) ,
\]

\[
\frac{d\lambda_{4k_F}}{d\ln L} = \frac{1}{\pi u_s^2} \lambda_F \cdot \lambda_{4k_F} .
\]

Note that the most crucial difference between forward scattering and other couplings is that the latter breaks chiral symmetry and they couple orbital degrees of freedom to the impurity spin. Also note that backward scattering is the most relevant one which will flow to strong coupling for both ferro- and antiferromagnetic couplings.

**For these purposes, we use the following coset construction**:

\[
SU(2)_a \otimes SU(2)_b \sim SU(2)_a \otimes \mathcal{G} ,
\]

where \( \mathcal{G} \) is the \( N = 1 \) SUSY unitary minimal model with \( c = 1 \). By matching the scaling dimensions and spin properties, we can establish the relations between the product of conformal towers in these two representations. For our present purpose, we only need the following:

\[
\left( \frac{1}{2} \right) \times \left( \frac{1}{2} \right) = (0) \times [\phi_{(2,1)}] + (1) \times [\phi_{(2,2)}] ,
\]

where \( [\phi_{(p,q)}] \) are conformal towers of \( \mathcal{G} \). The corresponding primary fields have conformal dimensions \( h_{p,q} = \frac{2}{4(p-1)} \) \( 1 - (-1)^{p-q} \). \( \langle j \rangle_k \) is the conformal tower of SU(2)_K WZNW theory with spin \( j \). With this new representation, the spin (orbital) current operator \( J_\text{s}(0) = J_\text{sL}(0) + J_\text{sR}(0) = J^1(0) + j^2(0) \equiv \mathcal{J}(0) \) is now the current operator of chiral SU(2)_a WZNW model, and the Kondo interaction can be rewritten as:

\[
\mathcal{H}_K = \left[ \lambda_F \mathcal{J}(0) + \lambda_B \Phi(0) \phi_{(2,2)} \phi_{(2,1)} \right] \cdot S_\text{imp} ,
\]

where \( \lambda_B \) is proportional to \( \lambda_B \), and \( \Phi \) is the spin one primary field of SU(2)_a WZNW theory. It can then be immediately seen that if both \( \lambda_B \) and \( \lambda_{4k_F} \) vanish, this problem can be solved as the usual Kondo problem.
where the remaining part of the first boundary operator has dimension $\phi$, Umklapp scattering opens a charge gap for the coupling $\lambda$. This case is known to be that of the four-channel Kondo problem whose LCBO is the Kac-Moody descendant of the spin one primary field $\mathbf{J}_{1-1} \cdot \Phi^{(1)}$, which leads to non-Fermi-liquid corrections to thermodynamical quantities. However, as we shall argue in the following, backward scattering will destabilize the four-channel Kondo intermediate fixed point. The system will be driven to the strong coupling open chain fixed point or a new intermediate fixed point depending on whether the coupling strength of orbital sector $J_2$ is ferro- or antiferromagnetic.

In fact, from eq. (3.5), we expect that $\lambda_I$ will scale to infinity first and dominate the low energy physics. To get a physical picture, we can do strong coupling analysis of the original lattice model. At strong coupling, the impurity spin forms a singlet with the spin at site zero and leaves an orbital doublet behind. For the case where $J_2 < 0$, the stability of this fixed point is guaranteed. We therefore conclude that in this case, the boundary fixed point is just an open chain. In other word, no LCBOs can come from the s=1 conformal tower of the spin sector. The leading operators are then the usual dimension two operators $\mathbf{J}^2$ which originate from exchange interaction between edge spins and orbitals. However, in the case where $J_2 > 0$, the strong coupling fixed point is unstable and the system will flow to some unknown intermediate fixed point. To determine the fate of the RG flow, we consider a special point in our parameter space with $J_2 > 0$, i.e. the SU(4) point where the underlying microscopic Hamiltonian can be considered as the SU(4) Hubbard model. Away from quarter filling, the Hilbert space can be decomposed into three parts: charge-, spin- and flavor conformal towers, and since the Kondo interaction breaks chiral invariance, operators with nonzero charge, spin and flavor quantum numbers can occur as long as they transform as singlets under the corresponding diagonal subgroups. The possible impurity critical behavior can be determined by identifying LCBOs which satisfy the following two conditions: (i) produce a noninteracting limit consistent with known results, and (ii) respect the symmetries of the Hamiltonian. By transforming to a basis with definite parity, it is easy to see that only channels with positive parity coupled to the impurity spin, hence the noninteracting limit of SU(4) Hubbard model corresponds to two-channel Kondo fixed point. Using this as a reference point, we can borrow the results of Ref. 11: The two leading boundary operators are $e^{i \sqrt{\pi/4} K_{\rho,sc}} \times \phi_s \times \phi_f$ and $\mathbf{J}_{1-1} \cdot \Phi^1 + \mathbf{J}_{2-1} \cdot \Phi^2$, where $K_{\rho}$ is the Luttinger liquid parameter and $\phi_s$ is the charge field. $\phi_{s,f}$ are singlet fields under the diagonal SU(2) subgroup and they are equal to $\phi_{(2,1)}$ in terms of our previous coset language. Upon approaching quarter-filling, Umklapp scattering opens a charge gap for $\phi_c$ and the remaining part of the first boundary operator has dimension smaller than one, therefore should be suppressed by selection rules. Only the second interaction independent boundary operator survives in our case. Since we do not expect any qualitative change when the parameters deviate from the SU(4) point slightly as long as we are still in the same phase, together with the above renormalization group analysis we conclude that the system flows to a two channel Kondo fixed point for antiferromagnetic orbital couplings.

### C. Thermodynamical behavior

In this section, we briefly discuss the corrections of specific heat and magnetic susceptibility induced by LCBOs, i.e. the corrections of LCBO $\lambda_I O(0)$ to the fixed point theory $\mathcal{H}$. The first type of LCBOs corresponding to open chain fixed point are the Virasoro descendants of identity operators : $\mathbf{J}^2_I$, $\mathbf{J}^3_I$, $\mathbf{J}_1 \cdot \mathbf{J}_2$. These operators will contribute to impurity free energy energy defined by $\delta f_{imp}(T, \lambda_I) = f_{imp}(T, \lambda_I) - f_{imp}(T, 0)$ in the first order of $\lambda_I$. Since these operators have dimension two, we expect that $\lambda_I$ is proportional to $1/T_K$, where $T_K$ is the temperature scale at which the coupling strength of the relevant perturbations becomes of order one. Because the relevant operators are of dimension 3/4, we have $T_K \propto J_{imp}/v$ for a small amount of initial change of coupling strength $\delta J$. Then from dimensional consideration, we expect the correction to the specific heat $C_{imp}$ should be proportional to $T/T_K$, and for the same reason, the correction to the magnetic susceptibility $\chi_{imp}$ should be proportional to $1/T_K$, i.e. at $T \rightarrow 0$, it produces a $T$ independent behavior. Note that this result is identical to that of the Heisenberg chain except the "Kondo temperature" $T_K$ has a different scaling relation with the coupling strength.

The second type of LCBO corresponding to periodic chain boundary condition is Eq. (3.3) which as we shall see, is a Virasoro primary operator. Consequently, its finite-temperature expectation value vanishes, and its contribution to $f_{imp}$ only starts from the second order of $\lambda_I$. In order to proceed the caculations, it is better to transform into a chiral representation as previous subsection. To do this, we first note that Eq. (3.3) can be written as $\mathbf{J}_{-1,s} \cdot \text{tr}_{g_I \sigma} \mathbf{g}_{t} + \text{tr}_{g_{-1,t}} \cdot \text{tr}_{g_I \sigma}$, where $\mathbf{J}_{-1} = \mathbf{J}_{R,-1} + \mathbf{J}_{L,-1}$ and is equal to $\mathbf{J}_{1-1} + \mathbf{J}_{2-1}$ in terms of chiral fields. At this point, we can use the coset relation (3.3) to cast the above operator into a simpler form. In fact, noting that $\text{tr}_{\mathbf{g}} = g_{1}^{\alpha} g_{2}^{\beta} g_{2}^{\alpha} \rightarrow \Phi \phi(2,3)$ and $\text{tr}_{\mathbf{g}} = g_{1}^{\alpha} g_{2}^{\alpha} \rightarrow \Phi \phi_{(2,1)}$, the leading irrelevant operator becomes $\mathbf{J}_{s,-1} \cdot \Phi_{s} \phi_{(2,3)} \phi_{(2,1)} + (s \leftrightarrow t)$. Following methods in Ref. 22, it can be shown that second-order perturbation theory results in an impurity specific heat : 

$$C_{imp} = \lambda_I^2 A \left( \frac{1}{u_s u_t} + \frac{1}{u_s u_t} \right) \left( \frac{2\Delta}{3(2\Delta - 3)} \right)^{2\Delta - 3} 2\Delta_{0}^{2-2\Delta} T_{0}^{2},$$

Where $\Delta = \frac{9}{4}$ is the dimension of $\mathbf{\hat{O}}_I$, $\tau_0$ is an infrared cutoff, and $A = 3(2 + \frac{1}{2}) = 12$. Similarly, the impurity magnetic susceptibility can be obtained.
\[ \chi_{\text{imp}} = \frac{\lambda^2 A'}{u_s \epsilon_{3/4}^{2\Delta} \epsilon_{1/4}^{3-2\Delta}} + O(\sqrt{T}), \]

where \( A' = 2(2 + \frac{2}{3})^2 = 32 \). Note that although this LCBO looks nontrivial, it only produces Fermi-liquid like behaviors because its dimension is “too high”.

The third type of LCBO is the Kac-Moody descendent of spin one primary field from the spin and orbital sector: \( J_{1,1}^1, \Phi^1 + J_{L,1}^2, \Phi^2 \) which appears in the case of Kondo impurity with antiferromagnetic orbital couplings and is equal to \( J_{1,1} \cdot \Phi \Phi^3 \) in terms of coset representation, where \( J \) and \( \Phi \) are now elements of \( k = 4 \) WZNW theory. It is exactly the same leading irrelevant operator as the two-channel Kondo problem except now it involves both spin and orbital sectors. It produces the following well-known form of impurity specific heat and magnetic susceptibility:

\[ C_{\text{imp}} \sim \lambda^2 T \ln (T_k/T), \quad \chi_{\text{imp}} \sim \lambda^2 T \ln (T_k/T). \]

IV. CONCLUSIONS AND DISCUSSIONS

To summarize, we have studied possible boundary critical behaviors for the one-dimensional spin-orbital model in its gapless phase with a magnetic impurity. For the case of internal impurities, there can be either an open chain or periodic chain fixed point. The underlying reason for the occurrence of these two different critical behaviors is similar to that of the Heisenberg spin chain: the leading instability for periodic chain is determined by the spin and orbital dimerization operators \( \epsilon_s(x) = i^j (S_j, S_{j+1}) \) and \( \epsilon_t(x) = i^j (T_j, T_{j+1}) \). Although they are allowed for impurities which violate the site parity \( P_s \), they are prohibited for impurities respecting \( P_s \), with \( \partial_x \epsilon_{s,t}(0) \) being the leading irrelevant operators. The new feature in this case is that due to the existence of additional orbital degrees of freedom and that the couplings between sites can be ferromagnetic, there can be spin (and) orbital triplet at the impurity site together with the open chain at the low energy fixed point.

For the case of Kondo impurity, we see that it can either flow to an open chain fixed point or an intermediate fixed point depending on the sign of orbital couplings. This should be compared with the case of Kondo effect in Luttinger liquid or Hubbard model at incommensurate filling. In that case, the backward scattering which breaks chiral symmetry will result in nontrivial leading irrelevant operators from the charge sector with \( Q_R - Q_L \neq 0 \). The operators arising from coset construction obtained via diagonal embedding (SU(2)_R \otimes SU(2)_L/SU(2)_{\text{diag}}) can also appear together with the ones from the charge sector through nontrivial selection rules. In fact, these leading irrelevant operators correspond to electron hopping with spin (orbital) flip between two ends in the open chain fixed point. In the present case, since the charge field is gapped, these processes cannot occur and only exchange interaction between edge spins (orbitals) can appear as LCBOs. A new feature in this case is that when orbital coupling is AF, the strong coupling fixed point will be destabilized and thermodynamical quantities at the intermediate fixed point show interesting temperature dependence similar to that of the two-channel Kondo effect.

Finally, since our theory is characterized by two different velocities, and in the case of periodic chain boundary fixed point, the contribution to \( \chi_{\text{imp}} \) and \( \chi_{\text{imp}} \) from the LCBO is the same as that of the dimension two Virasoro descendents \( J^2 \), a universal Wilson ratio can not be defined generally.

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