Entropy/Area spectra of the charged black hole from quasinormal modes

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Abstract

With the new physical interpretation of quasinormal modes proposed by Maggiore, the quantum area spectra of black holes have been investigated recently. Adopting the modified Hod’s treatment, results show that the area spectra for black holes are equally spaced and the spacings are in a unified form, $\Delta A = 8\pi \hbar$, in Einstein gravity. On the other hand, following Kunstatter’s method, the studies show that the area spectrum for a nonrotating black hole with no charge is equidistant. And for a rotating (or charged) black hole, it is also equidistant and independent of the angular momentum $J$ (or charge $q$) when the black hole is far from the extremal case. In this paper, we mainly deal with the area spectrum of the stringy charged Garfinkle-Horowitz-Strominger black hole, originating from effective action that emerges in the low-energy string theory. We find that both methods give the same results—that the area spectrum is equally spaced and does not depend on the charge $q$. Our study may provide new insights into understanding the area spectrum and entropy spectrum for stringy black holes.

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I. INTRODUCTION

The quantization of the black hole horizon area has become a fascinating subject since the famous Bekenstein conjecture [1] was presented. Regarding the black hole horizon area as an adiabatic invariant, Bekenstein obtained an equidistant area spectrum $A_n = \epsilon \hbar \cdot n (n = 0, 1, 2, ...).$ After the obtainment of the equally spaced area spectrum, many attempts have been made to derive the area spectrum and entropy spectrum directly from the dynamical modes of the classical theory [2–6]. However, the area spectrum has been somewhat controversial. In [2], the area spectrum was given as $A_n = 4 \ln(k) \hbar \cdot n$ $(k = 2, 3, ...)$, but other methods [7–9] produced $A_n = 8\pi \hbar \cdot n$. In particular, in loop quantum gravity, the area spectrum is in the form of $A = 8\pi \gamma \Sigma_j \sqrt{j(j + 1)}$, with $\gamma$ the Immirzi parameter and $j = 0, 1/2, 3/2, ...$ [10–12].

An important step in this direction was made by Hod ten years ago. He suggested that the spacing $\epsilon \hbar$ of the area spectrum can be determined by utilizing the quasinormal mode frequencies of an oscillating black hole [13]. On the other hand, Kunstatter pointed out that, for a system with energy $E$ and vibrational frequency $\Delta \omega(E)$, the ratio $\frac{E}{\Delta \omega(E)}$ is a natural adiabatic invariant [14]. Interpreting the vibrational frequency $\Delta \omega(E)$ as the real part of the quasinormal mode frequencies and replacing the energy $E$ with the mass $M$, the area spectrum of the Schwarzschild black hole was calculated as $A_n = 4\hbar \ln 3 \cdot n$, which is consistent with Hod’s result. This rejuvenates a great interest in the investigation of the black hole area spectrum via the interpretation of the quasinormal mode frequencies [13, 15–38].

Recently, Maggiore suggested that, in the high damping limit, the proper frequency of the equivalent harmonic oscillator, which is interpreted as the quasinormal mode frequency $\omega(E)$, should be of the form [8]

$$\omega(E) = \sqrt{|\omega_R|^2 + |\omega_I|^2}. \quad (1)$$

This form of proper frequency for the quasinormal mode frequency was first pointed out in [39]. Obviously, when $\omega_I \ll \omega_R$, one could get $\omega(E) = \omega_R$ approximately (adopted in the unmodified Hod’s treatment), which has been extended to other black holes [13, 15, 17]. However, at highly excited quasinormal modes (i.e., $\omega_R \ll \omega_I$), it is natural to get $\omega(E) = |\omega_I|$. With the choice that the vibrational frequency $\Delta \omega(E) = |\omega_I|_n - |\omega_I|_{n-1}$, the area spectrum of the Kerr black hole was obtained by Vagenas [20] with the modified Hod’s
treatment and Kunstatter’s method, respectively. The area spectrum calculated with the modified Hod’s treatment is equally spaced, while it is nonequidistant and depends on the angular momentum parameter $J$ when employing Kunstatter’s method. The methods disagree with each other in describing the Kerr black hole. At the same time, Medved also pointed out the disagreement between these two methods [21]. He argued that the quantum number $n$ appearing in the Bohr-Sommerfeld quantization condition is actually a measure of the areal deviation from extremality. So, the calculation of Kunstatter’s method is restricted to small value of $J$, which means that the black hole is far from the extremal black hole, i.e., $M^2 >> J$. In the spirit of this idea, the two methods are found to coincide with each other and both give an equally spaced and angular momentum parameter independent area spectrum when the black hole is far from the extremal case. For other gravity theories, for example, five-dimensional Gauss-Bonnet (GB) gravity, both methods will give the result that the area spectrum is nonequidistant [22, 40, 41], but the entropy spectrum is equidistant as first pointed out by Kothawala et.al. [22]. However, if one sets the GB coupling parameter $\alpha_{GB} \rightarrow 0$, the area spectrum would be equidistant. A similar conclusion can also be found in [30, 34] for Hořava-Lifshitz gravity. So, it is natural to make the following conclusions (from the new physical interpretation of quasinormal modes proposed by Maggiore): (1) For a nonrotating black hole with no charge, both the modified Hod’s treatment and Kunstatter’s method can reproduce an equally spaced area spectrum. (2) For rotating black holes (i.e. the Kerr black hole and three-dimensional spinning black hole), the two methods agree with each other when the black holes are far from the extremal case and an equally spaced area spectrum can be reproduced. (3) For other non-Einstein gravity theories, the two methods meet each other and give the same result that the area spectrum is not equally spaced (details can be seen in [22, 30, 34, 40, 41]).

Although much work has been done and many issues have been clarified in this field, another type of black hole solutions, obtained from string theory, has not been discussed. These black holes have some properties which are very different from the vacuum solutions obtained from the frame of Einstein gravity. Take the Garfinkle-Horowitz-Strominger (GHS) black hole, for example: its temperature is the same as the Schwarzschild black hole. However, it has charge-dependent area and entropy. Compared to the Reissner-Nordstrom (RN) black hole, the stringy charged black hole exhibits several different properties. First, it has only one horizon, while the RN black hole has two. Second, the extremal cases for this black
holes are different. They occur at $q^2 = 2M^2 e^{2\phi_0}$ for the charged GHS black hole and $q^2 = M^2$ for the RN black hole. Based on these differences, it is therefore worthwhile to investigate the area spectrum and entropy spectrum for the stringy black hole. In this paper, we will study these spectra for the charged GHS black hole. Employing the new interpretation of the quasinormal modes, we obtain the area spectrum and entropy spectrum for the charged GHS black hole using the modified Hod’s treatment and the Kunstatter’s method, respectively. We find that the two methods reproduce the same spacing of the area spectrum for the GHS black hole. The area spectrum and entropy spectrum are also found to be independent of the charge $q$. Our results are also in agreement with the universal spacing in [9], where the author proposed an alternative method for calculating the area spacing and demonstrated that this method works not only for the Schwarzschild black hole, but also for other charged and/or rotating black holes.

The paper is organized as follows. In Sec. II we briefly review the thermodynamics of the charged GHS black hole and show that the Bekenstein-Hawking entropy/area law holds. In Sec. III with the new interpretation of the quasinormal modes, we calculate the area spectrum and entropy spectrum of the charged GHS black hole with the modified Hod’s treatment and Kunstatter’s method, respectively. Finally, the paper ends with a brief summary.

II. THERMODYNAMIC PROPERTIES OF THE CHARGED GHS BLACK HOLE

The GHS dilaton black hole is depicted by the following four-dimensional low-energy action obtained from string theory:

$$I = \int d^4x \sqrt{-g} \left( -R - 2(\nabla \phi)^2 + e^{-2\phi} F^2 \right),$$

where $\phi$ is a dilaton field and the Maxwell field $F_{\mu\nu}$ is associated with a $U(1)$ subgroup of $E_8 \times E_8$ or $Spin(32)/Z_2$. The charged black hole solution is given by [42]

$$d\sigma^2 = -\left( 1 - \frac{2M}{r} \right) dt^2 + \left( 1 - \frac{2M}{r} \right)^{-1} dr^2 + r \left( r - \frac{q^2 e^{-2\phi_0}}{M} \right) d\Omega^2,$$

$$e^{-2\phi} = e^{-2\phi_0} \left( 1 - \frac{q^2 e^{-2\phi_0}}{M r} \right),$$

$$F = q \sin \theta d\theta \wedge d\phi,$$
where \( M \) and \( q \) are related to the mass and charge of the black hole, respectively. \( \phi_0 \) is the asymptotic constant value of \( \phi \) at \( r \to \infty \). The metric (3) will become the Schwarzschild metric as the charge \( q \to 0 \). The radius of the event horizon is determined by \( g_{tt} = \frac{1}{g_{rr}} = 0 \), which gives

\[
    r_+ = 2M. \tag{6}
\]

This result is consistent with that of the Schwarzschild black hole. The area of the event horizon is calculated as

\[
    A = \int \sqrt{g_{\theta\theta} g_{\phi\phi}} d\theta d\phi \\
    = 4\pi r_+ \left( r_+ - \frac{q^2 e^{-2\phi_0}}{M} \right) \\
    = 4\pi r_+^2 - 8\pi q^2 e^{-2\phi_0}. \tag{7}
\]

Note that the area goes to zero at \( r_+ = \frac{q^2 e^{-2\phi_0}}{M} \) or \( q^2 = 2M^2 e^{2\phi_0} \), which is related to the extremal black hole. The Hawking temperature of the GHS black hole is similar to the Schwarzschild case with \( T = \frac{1}{8\pi M} \). The electric potential computed on the horizon of the black hole is

\[
    V_+ = \frac{q}{r_+} e^{-2\phi_0}. \tag{8}
\]

Employing the first law of black hole thermodynamics, the entropy is calculated as

\[
    S = \int \frac{dM - V_+ dq}{T} \\
    = 4\pi M^2 - 2\pi q^2 e^{-2\phi_0} + S_0, \tag{9}
\]

where \( S_0 \) is an integral constant. Substituting Eqs. (6) and (8) into Eq. (9), we obtain the entropy/area law

\[
    S = \frac{A}{4} + S_0. \tag{10}
\]

Setting the constant \( S_0 \to 0 \), the result exactly confirms the standard Bekenstein-Hawking entropy/area law. In other word, if the relation (10) holds, the first law of black hole thermodynamics will be satisfied naturally. For the fixed charge, the heat capacity is expressed as

\[
    C_q = \left( \frac{\partial M}{\partial T} \right)_q = -8\pi M^2. \tag{11}
\]

This negative heat capacity implies that this black hole could not stably exist in a heat bath.
III. QUASINORMAL FREQUENCIES AND AREA SPECTRUM

Introducing a coordinate transformation $\rho = \sqrt{r(r - 2b)}$, the metric (3) can be expressed as

$$ds^2 = -\left(1 - \frac{2M}{b + \sqrt{b^2 + \rho^2}}\right) dt^2 + \left(1 - \frac{2M}{b + \sqrt{b^2 + \rho^2}}\right)^{-1} \frac{\rho^2}{b^2 + \rho^2} d\rho^2 + \rho^2 d\Omega^2,$$

(12)

where $b = q^2 e^{-2\phi_0} / (2M)$. The event horizon of the black hole now locates at

$$\rho_+ = 2\sqrt{M^2 - \frac{1}{2} q^2 e^{-2\phi_0}},$$

(13)

while the Hawking temperature is still given by $T = \frac{1}{8\pi M}$.

In the dilaton spacetime, the general perturbation equation for a coupled massless scalar field is given by

$$\nabla^2 \Phi - \zeta R \Phi = 0,$$

(14)

where $\Phi$ is the scalar field and $R$ is the Ricci scalar curvature. The parameter $\zeta$ denotes the coupling between the scalar field and the gravitational field. The Laplace-Beltrami operator is

$$\nabla^2 = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu).$$

(15)

Adopting the WKB approximation $\Phi = e^{-i\omega t} r^{-1} f(r) Y(\theta, \varphi)$ and imposing the proper boundary conditions, the asymptotic quasinormal frequencies were obtained in [43]:

$$\frac{i\omega}{T} = \ln(1 + 2 \cos(\sqrt{2}\zeta \pi)) + i(2k + 1)\pi, \quad k \to \infty.$$

(16)

It is worth noting that the quasinormal frequencies not only depend on the parameters of the black hole, but also on the coupling constant $\zeta$. Ignoring the second term in Eq. (14) or taking the limit $\zeta \to 0$, the real part of the quasinormal frequencies will become $T \ln 3$, which is consistent with the Schwarzschild case. For other black holes, the quasinormal modes were studied in [44–48].

Now, we will study the area spectrum of the charged GHS black hole via the quasinormal frequencies (16) and try to find out whether the area spectrum is equidistant. First, we will consider the area spectrum of the GHS black hole with the modified Hod’s treatment,
where the variations of the black hole parameters are regarded as finite quantities. Thus, the variations of the parameters satisfy

$$\Delta M - V_+\Delta q = \hbar \Delta \omega(E). \quad (17)$$

With the interpretation of identifying the vibrational frequency $\Delta \omega(E)$ as the real part of the quasinormal modes, much work have been done (e.g., [13, 15, 17]). However, as proposed by Maggiore, when $\omega_R \ll \omega_I$, we should take $\omega(E) = |\omega_I|$, approximately. Following this choice, the $\Delta \omega$ that appeared in (17) is calculated as

$$\Delta \omega = |\omega_I|_k - |\omega_I|_{k-1} = 2\pi T. \quad (k \gg 1) \quad (18)$$

From the formulas (6), (8) and (18), we get

$$4M\Delta M - 2q e^{-2\phi_0} \Delta q = \hbar. \quad (19)$$

Recalling the area (7), it is easy to obtain the spacing of the area spectrum

$$\Delta A = 8\pi \hbar. \quad (20)$$

So far, we have obtain a charge-independent spacing of the area spectrum for the charged GHS black hole which is in full agreement with that of the Schwarzschild black hole given by Maggiore. It might be worth mentioning that the black hole here is not restricted to the case of a far-from-extremal black hole. However, we will see in the following that the black hole must be a far-from-extremal one in the calculation using Kunstatte's method.

Next, following Kunstatte's method, we would like to reconsider the area spectrum of the charged GHS black hole. Here, we regard the variations of the black hole parameters as infinite quantities, which can ensure that the formula could be written in an integral form. Given a system with energy $E$ and vibrational frequency $\Delta \omega(E)$, a natural adiabatic invariant quantity proposed by Kunstatte [14] is

$$I = \int \frac{dE}{\Delta \omega(E)}. \quad (21)$$

At the large $n$ limit, the relation between the adiabatic invariant quantity $I$ and Bohr-Sommerfeld quantization is of the form

$$I \approx n\hbar, \quad n \to \infty \quad (22)$$
Here we need to note that the application of the Bohr-Sommerfeld quantization condition requires \( n \) to be a very large number, which also implies that the black hole must be far away from extremality, as the number \( n \) is a measure of the areal deviation from extremality. In general, the adiabatic invariant quantity \( I \) of a black hole can be expressed as

\[
I = \int \frac{dE}{\Delta \omega} = \int \frac{dM - \Omega dJ - V_+ dq}{\Delta \omega},
\]

(23)

where \( \Omega \) and \( V_+ \) are the angular velocity and the electric potential on the horizon, respectively. In the second step, the first law of black hole thermodynamics is used. For the charged GHS black hole, the adiabatic invariant quantity (23) reduces to

\[
I = \int \frac{dE}{\Delta \omega} = \int \frac{dM - V_+ dq}{\Delta \omega}.
\]

(24)

The vibrational frequency \( \Delta \omega \) is given in (18). Substituting \( \Delta \omega \) into (24), we obtain the adiabatic invariant quantity

\[
I = \int 4MdM - \int \frac{4Me^{-2\phi_0}}{r_+} dq = \frac{1}{2} r_+^2 - q^2 e^{-2\phi_0}.
\]

(25)

Using the Bohr-Sommerfeld quantization condition (22), at the large \( n \) limit, we obtain

\[
2M^2 - q^2 e^{-2\phi_0} = n\hbar.
\]

(26)

Recalling the area from (7), the area spectrum of this black hole is given by

\[
A_n = 8\pi \hbar \cdot n,
\]

(27)

with the spacing \( \Delta A = A_n - A_{n-1} = 8\pi \hbar \). This spacing is consistent with that of (20), which is equally spaced. It is also obvious that the spacing of the area spectrum is independent of the charge \( q \) of the black hole. Recalling the relationship (10), one could get the entropy spectrum

\[
S_n = 2\pi \hbar \cdot n + S_0,
\]

(28)

with the spacing

\[
\Delta S = S_{n+1} - S_n = 2\pi \hbar.
\]

(29)

The entropy is also equidistant and the spacing is independent of the charge \( q \) of the black hole. On the other hand, if one interprets the vibrational frequency \( \Delta \omega(E) \) as the real part
of the quasinormal modes, i.e. $\Delta \omega(E) = T \ln(1 + 2 \cos(\sqrt{2\zeta} \pi))$, one could obtain the area spectrum $A_n = 4 \ln(1 + 2 \cos(\sqrt{2\zeta} \pi))\hbar \cdot n$. Taking the coupling $\zeta = 0$, the spectrum is $A_n = 4 \ln(3)\hbar \cdot n$, which is the same as in Schwarzschild black hole.

We are almost finished with the calculations of this paper. For completeness, we would like to give a brief review on the study of the area spectra for the charged black holes. A reasonable discussion about our results and other charged black holes is also given.

In [18], Setare studied the area spectrum of the four-dimensional extreme RN black hole by regarding that the real part of the quasinormal frequency for the extreme RN black hole is the same as that of the Schwarzschild black hole. The area spectrum and entropy spectrum are given as $A_n = \ln 3 \hbar \cdot n$ and $S_n = \frac{\ln 3}{4} \hbar \cdot n$, respectively. The area and entropy spectra are both equally spaced. However the values of the spacings are not consistent with our results. For the nonextreme RN black hole, Hod showed the equally spacings $\Delta A = 4 \hbar \ln 2$ and $\Delta A = 4 \hbar \ln 3$ with two distinct families of quasinormal frequencies [36, 37].

Using the reduced phase-space quantization, Barvinsky et.al. [5] also calculated the area spectrum for the charged black hole. For the four-dimensional RN black hole, they found the area spectrum $A_{n,p} = 4\pi \hbar(2n + p + 1)$ with two quantum numbers $n, p = 0, 1, 2, ..., $ where the charge quantum number $p$ corresponds to $Q = \pm \sqrt{\hbar p}$. Although this area spectrum is quantized, it is charge-dependent as the charge quantum number $p$ comes into the spectrum.

After this paper was completed, some results about the area spectrum for the $d$-dimensional RN black hole were obtained in [49]. For the black hole far from the extremal case (small charge limit), it is found that the area spectrum is equally spaced with $A_n = 8\pi \hbar \cdot n$, following Kunstatter’s method.

Our results show that the modified Hod’s treatment and Kunstatter’s method give a unified description of the area spectrum of the charged GHS black hole. The area spectrum and entropy spectrum are found to be equally spaced, in agreement with the universal one. The spacing is also found to be charge-independent, so there are some differences from that of [5]. However, this quasinormal frequencies method is not easy to generalize to the other charged and/or rotating black holes owing to the fact that their quasinormal frequencies are generally not in a closed form unless some approximations are taken. However, we can claim that, the two methods work for the charged GHS black hole, and the equally spaced area spectrum and entropy spectrum are obtained. We believe that our study into the area spectrum of the charge GHS black hole should provide new insights on understanding the
IV. SUMMARY

In this paper, we mainly deal with the area spectrum and entropy spectrum of a stringy charged GHS black hole, which originate from the effective action that emerges in low-energy string theory. The black hole have some properties that are very different from the vacuum solutions obtained from the frame of Einstein gravity. With the new physical interpretation of quasinormal modes, the area spectrum and entropy spectrum for the black hole are obtained following the modified Hod’s treatment and the Kunstatter’s method, respectively. Despite the different properties between the black holes in the low-energy string theory and in Einstein gravity, the area spectrum and the entropy spectrum of the black hole are found to be equally spaced, which is consistent with the results for the charged RN black hole \[49\]. Furthermore, these spectra are also found to be independent of the charge \( q \), which is different from the results of \[5, 18, 36, 37\], obtained with a different physical interpretation of quasinormal modes or other methods. This quasinormal modes method is hard to extend to other charged and/or rotating black holes, as the quasinormal modes generally can not be obtained in a closed form. However, we believe that our study on the charged GHS black hole should provide new insights into understanding the area spectrum and entropy spectrum for other stringy black holes.

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