Hadron and Nuclear Physics on the Light Front

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(Received July 31, 2019)

The QCD light-front (LF) Hamiltonian equation

\[ H_{\text{LF}} | \Psi \rangle = M^2 | \Psi \rangle \]

derived from quantization at fixed LF time \( \tau = t + z/c \) provides a causal, frame-independent, method for computing hadron spectroscopy as well as dynamical observables such as structure functions, transverse momentum distributions, and distribution amplitudes. The LF formalism also leads to novel nuclear phenomena, such as “hidden color”, “color transparency”, “nuclear-bound quarkonium” and the shadowing and antishadowing of nuclear structure functions. For example, there are five distinct color-singlet Fock state representations of the six color-triplet quarks of the deuteron. The hidden color Fock states become manifest when the deuteron is probed when it has small transverse size, as in measurements of the deuteron form factor at large momentum transfer. The QCD Lagrangian with zero quark mass has no explicit mass scale. However, as shown by de Alfaro, Fubini, and Furlan (dAFF), a mass scale can appear in the equations of motion without affecting the conformal invariance of the Action. When one applies the dAFF procedure to the QCD LF Hamiltonian, it leads to a color confining potential \( \kappa^4 \zeta^2 \) for mesons, where \( \zeta \) is the LF radial variable conjugate to the \( q\bar{q} \) invariant mass squared. The same result, including spin terms, is obtained using LF holography (AdS/QCD) – the duality between LF dynamics and AdS5 – if one modifies the AdS5 Action by the dilaton \( \exp + \kappa^2 \zeta^2 \) in the fifth dimension \( z \). If this procedure is generalized using superconformal algebra, the resulting LF eigensolutions provide unified Regge spectroscopy of mesons, baryons, and tetraquarks, including remarkable supersymmetric relations between the masses of mesons, baryons and tetraquarks with a universal Regge slope. One also obtains empirically viable predictions for spacelike and timelike hadronic form factors, structure functions, distribution amplitudes, and transverse momentum distributions. AdS/QCD also predicts the analytic form of the nonperturbative running coupling \( \alpha_s(Q^2) \propto \exp (-Q^2/4\kappa^2) \), in agreement with the effective charge measured from measurements of the Bjorken sum rule. The mass scale \( \kappa \) underlying hadron masses can be connected to the parameter \( \Lambda_{\text{QCD}} \) in the QCD running coupling by matching the nonperturbative dynamics to the perturbative QCD regime. The result is an effective coupling \( \alpha_s(Q^2) \) defined at all momenta.

KEYWORDS: Light Front, Holography, Superconformal Algebra, Hadronic Spectroscopy, Nuclear Physics, running coupling, structure functions, form factors...

1. Introduction

One of the most powerful theoretical tools for the application of quantum chromodynamics (QCD) to hadron physics is “light-front” (LF) quantization, based on the use of Dirac’s “front-form” time \( \tau = t + z/c \), where \( (t, \vec{x}) \) are the usual space-time coordinates. The LF wavefunctions (LFWFs) of hadrons and nuclei are the eigensolutions of the QCD Hamiltonian \( H_{\text{LF}} | \Psi_H \rangle = M^2 | \Psi_H \rangle \), where \( M \) is the mass of the state \( | \Psi_H \rangle \). Unlike ordinary Hamiltonian field theory quantized at fixed “instant time” \( t \), the LFWFs of the eigenstates \( \Psi_n | x_i, k_L, i, k_i \rangle = \langle \Psi_H | n \rangle \) are causal and Poincaré invariant; i.e., they are independent of the hadron’s relative motion. Here \( x_i = \frac{k^i}{M^2} = \frac{k^0 + k^3}{M^2} \) is the boost-invariant momentum fraction of a constituent in an \( n \)-particle Fock state \( | n \rangle \). There is no physical Lorentz contraction
of a bound-state in motion since the LFWFs do not depend on the choice of Lorentz frame.

As first noted by Terrell [2] and Penrose [3] (see also Weisskopf [4]), the Lorentz contraction of a moving object is unobservable. For instance, the structure of a proton measured in deep inelastic lepton-proton scattering $e p \rightarrow e' X$, does not depend on whether it is measured with the proton frame at rest as in fixed-target experiments, or in motion as in an electron-proton collider. Factorization theorems such as the Drell-Yan cross-section for lepton pair production are independent of the observer’s Lorentz frame. Lorentz contraction could only apply if one could observe an object of a finite size at a single fixed time $t$, but this boundary condition violates causality.

LF quantization is the natural formalism for relativistic quantum field theory since one obtains a fundamental, boost-invariant dynamical description of bound-states. Measurements of hadron structure, such as deep inelastic lepton-proton scattering, are made at fixed LF time $\tau$, analogous to a flash photograph, not at a single “instant time” $t$. Virtually every observable used in particle and nuclear physics is based on LF quantization. Structure functions are the squares of LFWFs, and form factors are computed from the overlaps of LFWFs. They also underly “hadronization”, the conversion of quark and gluon quanta to a hadron at the amplitude level. The boost invariance of the LFWFs ensures that measurements in the center of mass (CM) frame, e.g. for an electron-ion collider, give the same results as measurements in the target rest frame.

As Dirac emphasized [1], a Lorentz boost in the “instant-form” formalism necessarily introduces a boost operator with mixed kinematical and dynamical dependence. In fact, in the ordinary instant-form formalism, dynamical corrections to the boosted wavefunction (WF) of a moving bound-state are necessary to compute form factors, Compton amplitudes, and other observables. The boost of the instant-form WF cannot be resolved within the framework of the instant-form alone. For example, the boost of the deuteron instant-form WF is not the product of separate Wigner (or Melosh) boosts [5] of the proton and neutron. Consequently, an apparently ad-hoc spin-orbit correction is needed in order to restore the low energy theorem (LET) [6] since even an infinitesimal boost in the instant-form involves contributions which depend on the internal dynamics. Similarly, the derivation of the Gerasimov-Drell-Hearn (GDH) sum rule [7–9] fails unless one computes the matrix elements of the electromagnetic current using the correct, dynamically boosted instant-form WF of the target [10].

The required boost for atomic systems in quantum electrodynamics (QED) can be derived by adopting a covariant approach such as the Bethe-Salpeter bound-state formalism, but this approach is outside the instant-form framework; thus the effects of the dynamical contributions to the boost in the instant-form have to be parametrized as a “fictitious force” – in the sense that it is an inertial force rather than of fundamental origin. The problem of constructing the boosted instant-form WF is more complex in QCD, since the required boost of a hadronic or nuclear bound-state combines non-perturbative dynamics with kinematics. The boost of a bound-state WF in QCD is generally not calculable in the instant-form, which in turn makes an analytic analysis intractable.

There is a further complication when one computes the form factors and other observables for bound-states using the instant-form. The evaluation of form factors and other matrix elements of currents cannot be computed just from the integrated overlap of the instant-form WFs since there are additional contributions from the current of pairs of charged constituents which arise from the vacuum. For example, in QED one must couple the external current $J^\mu$ to the leptons which are created or annihilated from the vacuum by the $e^+ e^- \gamma$ interaction. These “vacuum-induced” contributions to the current appear in the instant-form when one relates Feynman diagrams to the sum of “old-fashioned” time-ordered perturbation theory diagrams, as in Wick’s theorem. The correct Poincaré invariant results for form factors cannot be obtained in the instant-form without these contributions. In contrast, they do not appear in the front-form since all particles have positive $k^+$. 
2. Novel QCD Nuclear Phenomena

The LF formalism leads to many novel nuclear phenomena, such as “hidden color” [11] “color transparency” [12], “nuclear-bound quarkonium” [13], “nuclear shadowing and antishadowing” of nuclear structure functions, etc. For example, there are five distinct color-singlet Fock state representations of the six color-triplet quarks of the deuteron. These hidden-color Fock states become manifest when the deuteron fluctuates to a small transverse size, as in measurements of the deuteron form factor at large momentum transfer. One can also probe the hidden-color Fock states of the deuteron by studying the final state of the diffractive dissociation of the deuteron on a nucleus DA → XA, where X can be Δ++ + Δ−, six quark jets, or other novel final states.

More generally, one can measure the LFWFs of a hadron by using the “Ashery method” [14]: in the diffractive dissociation of a high energy hadron into quark and gluon jets by two-gluon exchange, the cross-section measures the square of the second transverse derivative of the projectile LFWF. Similarly, the dissociation of a high energy atom such as positronium or “true muonium” ((µ+µ−)) can be used to measure the transverse derivative of its LFWFs.

LFWFs provide the input for scattering experiments at the amplitude level, encoding the structure of a projectile at a single LF time τ. For example, consider photon-ion collisions. The incoming photon probes the finite size structure of the incoming nucleus at fixed LF time, like a photograph – not at a fixed instant time, which is acausal. Since the nuclear state is an eigenstate of the LF Hamiltonian, its structure is independent of its momentum, as required by Poincaré invariance. One gets the same answer in the ion rest frame, the CM frame, or even if the incident particles move in the same direction, but collide transversely. There are no colliding “pancakes” using the LF formalism.

The resulting photon-ion cross-section is not point-like; it is shadowed: σ(γA → X) = Aασ(γN → X), where A is the mass number of the ion, N stands for a nucleon, and the power α ≈ 0.8 reflects Glauber shadowing. The shadowing stems from the destructive interference of two-step and one-step amplitudes, where the two-step processes involve diffractive reactions on a front-surface nucleon which shadow the interior nucleons. Thus the photon interacts primarily on the front surface. Similarly a high energy ion-ion collision A1 + A2 → X involves the overlap of the incident frame-independent LFWFs. The initial interaction on the front surface of the colliding ions can resemble a shock wave.

In the case of a deep inelastic lepton-nucleus collision γ∗A → X, the two-step amplitude involves a leading-twist diffractive deep inelastic scattering (DDIS) γ∗N1 → V∗N1 on a front surface nucleon N1 and then the on-shell propagation of the vector system V∗ to a downstream nucleon N2 where it interacts inelastically: V∗N2 → X. If the DDIS involves Pomeron exchange, the two-step amplitude interferes destructively with the one-step amplitude γ∗N1 → X thus producing shadowing of the nuclear parton distribution function at low xbj < 0.1 where xbj is the Bjorken scaling variable. On the other hand, if the DDIS process involves I = 1 Reggeon exchange, the interference is constructive, producing flavor-dependent leading-twist antishadowing [15] in the domain 0.1 < xbj < 0.2.

3. Light-front Holography

Five-dimensional AdS5 space provides a geometrical representation of the conformal group. Remarkably, AdS5 is holographically dual to 3 + 1 spacetime at fixed LF time τ [16]. A color-confining LF equation for mesons of arbitrary spin J can be derived [17] from the holographic mapping of the “soft-wall model” modification of AdS5 space for the specific dilaton profile e±b2z2, where z is the fifth dimension variable of the five-dimensional AdS5 space. A holographic dictionary maps the fifth dimension z to the LF radial variable ζ, with ζ2 = b2(1 − x). The same physics transformation maps the AdS5 and (3 + 1) LF expressions for electromagnetic and gravitational form factors to each other [17].
A key tool is the remarkable de Alfaro, Fubini, and Furlan (dAFF) [18] principle which shows how a mass scale can appear in a Hamiltonian and its equations of motion while retaining the conformal symmetry of the action. When one applies the dAFF procedure to LF holography, a mass scale \( \kappa \) appears which determines universal Regge slopes, and the hadron masses. The resulting “LF Schrödinger Equation” incorporates color confinement and other essential spectroscopic and dynamical features of hadron physics, including Regge theory [19], the Veneziano formula [20], a massless pion for zero quark mass and linear Regge trajectories with the universal slope in the radial quantum number \( n \) and the internal orbital angular momentum \( L \). The combination of LF dynamics, its holographic mapping to AdS\(_5\) space, and the dAFF procedure provides new insight into the physics underlying color confinement, the nonperturbative QCD coupling, and the QCD mass scale. The \( q\bar{q} \) mesons and their valence LFWFs are the eigensolutions of the frame-independent a relativistic bound-state LF Schrödinger equation. The formalism is summarized in Fig. 1. The mesonic \( q\bar{q} \) bound-state eigenvalues for massless quarks are

$$M^2(n, L, S) = 4\kappa^2(n + L + S/2).$$

This equation predicts that the pion eigenstate \( n = L = S = 0 \) is massless for zero quark mass. When quark masses are included in the LF kinetic energy \( \sum_i k_i^2 + m^2 \), the spectroscopy of mesons are predicted correctly, with equal slope in the principal quantum number \( n \) and the internal orbital angular momentum \( L \). A comprehensive review is given in Ref. [16].

4. Superconformal Algebra and Hadron Physics

If one generalizes LF holography using superconformal algebra the resulting LF eigensolutions yield a unified Regge spectroscopy of mesons, baryons and tetraquarks, including remarkable supersymmetric relations between the masses of mesons and baryons of the same parity [21, 22]. LF Holography, together with superconformal algebra, not only predicts meson and baryon spectroscopy consistent with measurements, but also hadron dynamics; this includes vector meson electroproduction, hadronic LFWFs, distribution amplitudes, form factors, and valence structure functions. Applications to the deuteron elastic form factors and structure functions is given in Refs. [23, 24].

QCD is not supersymmetrical in the usual sense –the QCD Lagrangian is based on quark and gluonic fields– not squarks or gluinos. However, its hadronic eigensolutions conform to a representation of superconformal algebra, reflecting the underlying conformal symmetry of chiral QCD and its Pauli matrix representation. A comparison with experiment is shown in Fig. 2.

The eigensolutions of superconformal algebra predict the Regge spectroscopy of mesons, baryons, and tetraquarks of the same parity and twist as equal-mass members of the same 4-plet representation with a universal Regge slope [27–29]. The \( q\bar{q} \) mesons with orbital angular momentum \( L_M = L_B + 1 \) have the same mass as their baryonic partners with orbital angular momentum \( L_B \) [25, 29].

The predictions from LF holography and superconformal algebra can also be extended to mesons, baryons, and tetraquarks with strange, charm and bottom quarks. Although conformal symmetry is strongly broken by the heavy quark masses, the basic underlying supersymmetric mechanism, which transforms mesons to baryons (and baryons to tetraquarks), still holds and gives remarkable mass degeneracy across the entire spectrum of light, heavy-light and double-heavy hadrons. One also obtains viable predictions for spacelike and timelike hadronic form factors, structure functions, distribution amplitudes, and transverse momentum distributions [30, 31].

5. The QCD Running Coupling at all Scales

The QCD running coupling \( \alpha_s(Q^2) \) sets the strength of the interactions of quarks and gluons as a function of the momentum transfer \( Q \). The dependence of the coupling \( Q^2 \) is needed to describe hadronic interactions at both long and short distances [32]. The QCD running coupling can be defined [33] at all momentum scales from a perturbatively calculable observable, such as the coupling...
Confinement scale: \( \kappa \simeq 0.5 \ \text{GeV} \)

\( \alpha_s^\text{LF}(Q^2) \), which is defined using the Bjorken sum rule [34], and determined from the sum rule prediction at high \( Q^2 \) and, below, from its measurements [35–37]. At high \( Q^2 \), such “effective charges” satisfy asymptotic freedom, obey the usual pQCD renormalization group equations, and can be related to each other without scale ambiguity by commensurate scale relations [38].

The dilaton \( e^{+\kappa^2 z^2} \) soft-wall modification of the AdS\(_3\) metric, together with LF holography, predicts the functional behavior of the running coupling in the small \( Q^2 \) domain [39]: \( \alpha_s^\text{LF}(Q^2) = \pi e^{-Q^2/4\kappa^2} \).

Measurements of \( \alpha_s^\text{LF}(Q^2) \) [40, 41] are remarkably consistent with this predicted Gaussian form; the best fit gives \( \kappa = 0.513 \pm 0.007 \ \text{GeV} \), see Fig. 3. We have also shown [39, 42, 43] with de Téramond how the parameter \( \kappa \), which determines the mass scale of hadrons and Regge slopes in the zero quark mass limit, can be connected to the mass scale \( \Lambda_3 \) controlling the evolution of the perturbative QCD coupling. The high \( Q^2 \) dependence of \( \alpha_s^\text{LF}(Q^2) \) is predicted by pQCD. The matching of the high and low \( Q^2 \) regimes of \( \alpha_s^\text{LF}(Q^2) \) – both its value and its slope – then determines a scale \( Q_0 \) which sets the interface between perturbative and nonperturbative hadron dynamics. In the \( \overline{\text{MS}} \) scheme, \( Q_0 = 0.87 \pm 0.08 \ \text{GeV} \). This connection can be done for any choice of renormalization scheme. The result of this perturbative/nonperturbative matching is an effective QCD coupling defined at all momenta. The predicted
value of $\Lambda_{\overline{MS}} = e^{-a} \sqrt{2/a} = 0.339 \pm 0.019 \text{ GeV}$ (with $a = 4[\sqrt{\ln(2)/2} + 1 + \beta_0/4 - \ln(2)]/\beta_0 + O(\beta_1)$) from this analysis agrees well the measured value [44] $\Lambda_{\overline{MS}} = 0.332 \pm 0.017 \text{ GeV}$. These results, combined with the AdS/QCD superconformal predictions for hadron spectroscopy, allow one to compute hadron masses in terms of $\Lambda_{\overline{MS}}$: $m_p = \sqrt{2} \kappa = 3.21 \Lambda_{\overline{MS}}$, $m_{\rho} = \kappa = 2.2 \Lambda_{\overline{MS}}$ and $m_{\omega} = \sqrt{2} m_p$, meeting a challenge proposed by Zee [45]. The value of $Q_0$ can be used to set the factorization scale for DGLAP evolution of hadronic structure functions [46–48] and the ERBL evolution of distribution amplitudes [49, 50]. We have also computed the dependence of $Q_0$ on the choice of the effective charge used to define the running coupling and on the renormalization scheme used to compute its behavior in the perturbative regime. The use of the scale $Q_0$ to resolve the factorization scale uncertainty in structure functions and fragmentation functions, in combination with the scheme-independent principle of maximum conformality (PMC) [51] for setting renormalization scales, can greatly improve the precision of pQCD predictions for collider phenomenology.

6. Summary

The new approach to Poincaré invariant hadron dynamics based on light-front holography discussed here has many attractive features, including color confinement, the analytic form of the LF
confinement potential, a massless quark-antiquark pion bound state in the chiral limit. The incorporation of supersymmetric algebra leads to 4-plets of mass degenerate $q\bar{q}$ mesons, quark-diquark baryons and diquark/antidiquark tetraquarks. All of the Regge Trajectories have universal slopes in $n$ and $L$. The resulting causal boost-invariant LF wavefunctions predict hadronic form factors, structure functions, transverse momentum distributions, and other dynamical observables such as counting rules and hadronization at the amplitude level. We also obtain the form of the QCD coupling at all scales, consistent with experiment, thus connecting perturbative and nonperturbative hadron dynamics.

Acknowledgments

The physics results reported here are based on work developed with our collaborators, including Guy de Téramond, H. Guenter Dosch, Marina Nielson, Cedric Lorcè, Josh Erlich, and R.S. Sufian. This work is based in part upon work supported by the U.S. Department of Energy, the Office of Science, and the Office of Nuclear Physics under contract DE-AC05-06OR23177. This work is also supported by the Department of Energy contract DE–AC02–76SF00515. SLAC-PUB 17464.

References

[1] P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949).
[2] J. Terrell, Phys. Rev. 116, 1041 (1959).
[3] R. Penrose, Proc. Cambridge Phil. Soc. 55, 137 (1959).
[4] V. F. Weisskopf, Physics Today 13, 9, 24 (1960).
[5] E. P. Wigner, Annals Math. 40, 149 (1939) [Nucl. Phys. Proc. Suppl. 6, 9 (1989)].
