Orthotropic ductile fracture criterion based on linear transformation

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Abstract. Accurate modelling of orthotropic ductile fracture is key to carry out reliable numerical prediction of rupture in plastic deformation of lightweight metals, such as ultra high strength steel, aluminum alloys, titanium alloys and magnesium alloys. Experiments are conducted for an aluminum alloy in shear, uniaxial tension, plane strain tension along the rolling direction, the diagonal direction and the transverse direction. Loading processes are recorded and fracture strain is measured by analysis of deformation with digital image correlation. First, isotropic fracture behavior is modeled by both linear model (Maximum Shear Stress (MSS) plus mean stress) and nonlinear model (Hosford yield function plus mean stress) considering different triaxiality conditions. It is observed that the mean stress model shows significant difference in the compression area compared to Mohr Coulomb-based normal stress model and a new isotropic model with the mean stress term shows a good correlation for AA 6k21. This approach is extended to an anisotropic ductile fracture criterion based on linear transformation. The anisotropic ductile fracture criterion is applied to model orthotropic fracture strain in shear, uniaxial tension and plane strain tension. The predicted anisotropy in ductile fracture is compared with experimental results for the verification of its accuracy. The comparison indicates that the proposed anisotropic ductile fracture criterion accurately models orthotropic ductile fracture in various loading conditions in shear, uniaxial tension and plane strain tension.
1. Introduction

One approach is to characterize fracture limits in terms of the true stresses. This approach is reasonable because stress-based models of necking have been shown to explain the nonlinear path dependency of strain-based necking limits, greatly simplifying the prediction of necking in manufacturing, which frequently results in critical conditions involving nonlinear strain paths. So a stress-based approach may offer a similar benefit for prediction of fracture under nonlinear deformations. The stress-based approach also provides a self-consistent picture to describe post-fracture deformation, in which the additional stretchability of newly created sheared edges can be easily explained from the reduction in stress caused by the loss of load across the newly formed sheared edge. The initial reduction of stress allows the metal to be stretched more if the lower stress is below the fracture limit. In addition, since the applied true stress must be proportional to the local forces acting on the chemical bonds that hold the material together, and fracture is the consequence of overcoming these bonds, it is reasonable to limit the fracture model to functions of the applied true stress.

Stoughton and Yoon [1] noticed a strong correlation of the fracture with the calculated maximum shear stress for a wide range of test conditions reported by Wierzbicki et al. [2] suggesting a fracture criterion bases on the maximum shear stress. However, Stoughton and Yoon [3] later noticed a weaker but linear anti-correlation with the calculated stress normal to the plane of maximum shear, and proposed to characterize fracture using an adaptation of the Mohr-Coulomb Model. In addition to the other arguments for a stress-based model, the Mohr-Coulomb model has two additional attractions from a mechanistic view. First, the dominant term is the shear stress, which is the driver for plastic slip and the primary mechanism of deformation. For stable plastic slip, bonds are obviously broken and reestablished in another configuration. So it’s reasonable that at some level of the shear stress the released energy during slip will be too large to enable the bonds to sufficiently recover between the shear planse, resulting in microcracks and eventually macro-scale fracture. So the connection between shear stress and fracture is intuitive. Second, with the friction coefficient parameter μ of the Mohr-Coulomb Model much less than 1, as it is found to be for steel and aluminum alloys, the limit on the shear fracture stress is reduced or increased by the positive or negative value of the stress normal to the shear slip plane. Intuitively, this also seems correct from the perspective of the mechanisms since it causes the slip to be more likely to fracture if the normal stress is positive, pulling the slip planes apart, and less likely to fracture if the stress is negative, due to the external forces holding the slipping planes together.

However, the Mohr-Coulomb Model has two limitations that make it unattractive for application to sheet metal forming analysis. First, it is an isotropic model, which is not consistent with engineering practice in metal forming analyses, which use anisotropic constitutive laws to characterize the stress-strain relationship for sheet metal forming. Second, it does not work for hardening models that saturate in stress, such as the Voce Law. Depending on how the fracture model is calibrated using these hardening laws, one can predict fractures at forbidden states of stress (which means that the metal would not fracture for loading conditions in the direction of the forbidden stress), or one can predict fracture in some stress states at unrealistically low strains. While these problems have not been observed with hardening laws that do not saturate, and doesn’t always happen under the Voce Law, either, particularly when the yield behavior is characterized by non-quadratic functions that more closely resemble the shape of the Mohr-Coulomb Model, from an application perspective, it is unacceptable that the fracture model could result in nonphysical behaviors depending on the choice of the hardening law.

In this paper, we describe a modified Mohr-Coulomb Model based on the mean stress modification. This 3D model is useful for tube and sheet hydroforming applications, as well as contact areas in sheet forming where the contact pressure is high. Although the through-thickness shear stresses are not rigorously included, these components are implicitly included in a term that represents the pseudo
principal stress normal to the plane of the sheet, so that the model can be used also in solid element analysis. It is observed that the mean stress model shows significant difference in compression area compared to normal stress model and a new isotropic model with mean stress term shows a good correlation for aluminum alloy application. This approach is extended to a general anisotropic ductile fracture criterion which is based on Yoon et al [4]'s model based on linear transformation. The anisotropic ductile fracture criterion is applied to model orthotropic fracture strain in uniaxial tension, plane strain tension and shear. The comparison shows that the proposed anisotropic ductile fracture criterion accurately models orthotropic ductile fracture in various loading conditions.

2. Modified Mohr-Coulomb fracture model for isotropy

The original Mohr-Coulomb Model describes a linear relationship between the two extremes of the principal stresses given by,

$$\max(\sigma_1, \sigma_2, \sigma_3) \leq \frac{1 - \mu}{1 + \mu} \min(\sigma_1, \sigma_2, \sigma_3) + \frac{2}{1 + \mu} \sigma_1. \quad (1)$$

When applied to model fracture, the parameter $\sigma_s$ the experimentally determined fracture stress in pure shear and $\mu$ is the so-called Mohr-Coulomb friction coefficient for fracture, which can be experimentally defined as,

$$\mu = \left( \frac{2\sigma_s}{\sigma_u} \right) - 1 \quad (2)$$

where $\sigma_u$ is the experimentally determined fracture stress in uniaxial tension. The model is similar to the Tresca function ($\mu=0$) but instead of a 6 sided polygonal surface with rectangular facets extending to infinity, the facets of the Mohr-Coulomb Model are triangular and collapse to a single point under pure shear and hydrostatic stress equal to $\sigma_s/\mu$. Since $\mu$ is typically less than 0.10 for steel and aluminum, this hydrostatic stress is typically much higher than 1 GPa, so it not an unreasonable value for a fracture under pure positive hydrostatic stress. In the other direction, the Mohr-Coulomb Model predicts the metal will never fracture under pure hydrostatic pressure, which also seems to be a reasonable prediction.

The MC fracture model can be simply defined as

$$\sigma_{MC} = \sigma_{MSS} + \mu \sigma_n \quad (3)$$

where $\mu$ and $\sigma_{MC}$ are material parameters to be identified; $\sigma_{MSS}$ is the maximum shear stress which is in the following form:

$$\sigma_{MSS} = \frac{\sigma_1 - \sigma_3}{2} \quad (4)$$

$\sigma_n$ is the normal stress which is defined by:

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} \quad (5)$$

Stoughton and Yoon (2013) proposed the following form of fracture model based on the mean stress :

$$\sigma_{MC} = \sigma_{MSS} + \mu \sigma_m \quad (6)$$
where
\[ \sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \]

(7)

Figure 1 shows the RMS errors measured for AA 2024-T351 reported by Wierzbicki, et al. [2]. RMS errors predicted from both the normal and mean stress models have the similar level of 15 Mpa which is much better than the RMS error of 26 Mpa observed from MSS (Maximum Shear Stress) criterion.

\[ \frac{\sigma_{\text{mss}}}{350.8} + \frac{\sigma_{\text{normal}}}{5105.9} = 1, \quad \text{normal} = 15.7 \]

(a)

\[ \frac{\sigma_{\text{mss}}}{351.4} + \frac{\sigma_{\text{m}}}{3897.5} = 1, \quad m = 15.6 \]

(b)

**Figure 1.** Normal Stress vs. Mean Stress for AA 2024-T351 by Wierzbicki, et al. [2]

**Figure 2.** MMS-Normal and MSS-Mean models: (a) uniaxial tension & shear input (b) plane strain & shear input

A further analysis has been performed for AA 6k21 material. Fig. 2 shows the results from the linear models based on “MMS-Normal” and “MSS-Mean” to fit either uniaxial & shear points or plane strain & shear points. Significant difference between two models can be observed when uniaxial & shear data have been used.
A nonlinear model has been proposed by Mohr and Marcadet [5] by replacing MSS term with Hosford yield function based on polycrystal observation. It leads to

$$\sigma_{MC} = \bar{\sigma}_h + \mu \sigma_n$$

(8)

where

$$\bar{\sigma}_h = \left( \frac{|\sigma_1 - \sigma_2|^a + |\sigma_1 - \sigma_3|^a + |\sigma_2 - \sigma_3|^a}{2} \right)^{1/a}$$

(9)

In Eq.9, $a$ is crystallographic related parameter which is 6 for bcc materials and 8 for fcc materials.

For the consistency with Stoughton and Yoon [3] in Eq.6, the following model can be considered as

$$\sigma_{MC} = \bar{\sigma}_h + \mu \sigma_m$$

(10)

The above two nonlinear models are identified by uniaxial tension, plane strain and shear tests. Figure 3 shows the comparison between the two models. It is observed that a nonlinear model with the mean stress shows a better correlation than the normal stress model. A partial reason is that the mean stress is directly related to triaxiality, which has a major contributor in ductile fracture.

![Figure 3. Isotropic fracture models based on Hosford-Normal and Hosford-Mean.](image)

3. **Extension to anisotropy**

In order to extend the fracture criterion to anisotropy, the yield function proposed by Yoon et al. [4] has been utilized. The yield function describes the anisotropic plastic deformation for orthotropic metals in a form of

$$f(\bar{\sigma}_p) = \tilde{I}_3 + \left( J_2^{\nu/2} - \sigma_3^{\nu/2} \right)^{\nu/\nu}$$

(11)

with
where $\bar{J}_1$ represents the anisotropic-weighted first stress invariant for pressure sensitivity, $J'_2$ is the second stress invariant of an isotropic plastic equivalent (IPE) transformed stress tensor of $s'$, and $J''_3$ is the third stress invariant of another IPE transformed stress tensor of $s''$. By using invariant forms, it is not necessary to calculate any principal stress values in Eqs.13 and 14, which leads to a simple form of yield function derivatives. The axes of $x$, $y$ and $z$ represent the rolling direction (RD), transverse direction (TD), and normal direction (ND) of cold rolled metals, respectively. Two IPE stress tensors of $s'$ and $s''$ are transformed from the stress tensor $\sigma$ by two fourth-order linear transformation tensors of $L'$ and $L''$ as follows:

$$s' = L' \sigma, \quad s'' = L'' \sigma$$

with

$$L' = \begin{bmatrix}
(c'_1 + c'_2)/3 & -c'_1/3 & -c'_2/3 & 0 & 0 & 0 \\
-c'_1/3 & (c'_1 + c'_2)/3 & -c'_3/3 & 0 & 0 & 0 \\
-c'_2/3 & -c'_3/3 & (c'_1 + c'_2)/3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c'_4 \\
0 & 0 & 0 & 0 & 0 & c'_5 \\
0 & 0 & 0 & 0 & 0 & c'_6 \\
\end{bmatrix}, \quad L'' = \begin{bmatrix}
(c''_1 + c''_2)/3 & -c''_1/3 & -c''_2/3 & 0 & 0 & 0 \\
-c''_1/3 & (c''_1 + c''_2)/3 & -c''_3/3 & 0 & 0 & 0 \\
-c''_2/3 & -c''_3/3 & (c''_1 + c''_2)/3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c''_4 \\
0 & 0 & 0 & 0 & 0 & c''_5 \\
0 & 0 & 0 & 0 & 0 & c''_6 \\
\end{bmatrix}$$

$$\sigma' = \begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} & \sigma_{yx} & \sigma_{yz} & \sigma_{zx} \\
\end{bmatrix}$$

In this paper, anisotropic fracture model can be defined as

$$\sigma_{MC} = \bar{\sigma}_{f2/3} + \mu \sigma_m$$

where

$$\bar{\sigma}_{f2/3} = a(J''_3 - cJ''_3^2)^{1/6}$$

With $a = \frac{3}{(27-4c)^{1/6}}$

The input data for the anisotropic fracture models are the fracture stress from uniaxial tension and plane strain tension and shear for 0, 45, 90 degrees. Table 1 shows the coefficients obtained from the optimization. The C coefficients related to 4 and 5 are assumed to be isotropic which is 1.0. Fig.4 shows a proposed anisotropic fracture surface compared to Hosford-Mean isotropic fracture model. It can be seen that anisotropic model makes a good agreement with the experimental data. Directionality plots for uniaxial tension, plane strain and shear have been provided in Fig.5. It can be seen that the anisotropic fracture stresses for uniaxial tension, plane strain and shear are well represented.

| $\sigma_{MC}$ (MPa) | $\mu$ | $c'_1$ | $c'_2$ | $c'_3$ | $c'_4$ | $c'_5$ | $c'_6$ | $c''_1$ | $c''_2$ | $c''_3$ | $c''_4$ | $c''_5$ | $c''_6$ |
|---------------------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| MMC                 | 629.8 | 0.08   | 1.62   | 1.70   | 1.41   | 2.20   | 1.89   | 1.79   | 1.07   | 0.016  |        |        |        |

Table 1 Anisotropic fracture model parameters
Figure 4. Anisotropic fracture surface model compared with Hosford-Mean isotropic model.

Figure 5. Directionality plots: (a) Uniaxial tension, (b) Plane strain, (c) Shear
4. Conclusion
Isotropic and anisotropic fracture models in the stress space have been proposed. It is observed that the mean stress model shows significant difference in compression area compared to M-C based normal stress model and a new isotropic model with mean stress term shows a good correlation. This approach is extended to an anisotropic ductile fracture criterion based on linear transformation. The anisotropic ductile fracture criterion is applied to model orthotropic fracture strain in shear, uniaxial tension and plane strain tension. The predicted anisotropy in ductile fracture is compared with experimental results for the verification of its accuracy. The comparison indicates that the proposed anisotropic ductile fracture criterion accurately models orthotropic ductile fracture in various loading conditions in shear, uniaxial tension and plane strain tension.

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