Study on the Fuzzy Interval Evaluation for EPS Schemes

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Abstract. In order to evaluate vehicle steering feel of the electric power steering (EPS) system effectively, portability, returnability, the ISO lane change capability, and the on-center steering capability were adopted as test factors. On the basis of group multi-criteria decision-making and fuzzy analytical hierarchy process (AHP) method, the EPS hierarchic fuzzy evaluation model was established, and a simplified scale method was adopted to generate the consistent relation judgment matrices. The drivers' opinions were aggregated to form Triangular Fuzzy Number method with fuzzy Delphi method. Finally, EPS schemes were ranked with total utility values of fuzzy numbers and the optimal alternative could be selected. An example was given, under this approach, it shows that the decision-making process was a systematic and practical method for EPS schemes selection.

1. Introduction

At present, returnability and portability are the main subjective evaluation indicators. The evaluation of the steering feel, such as the steering flexibility, the sensitivity of the steering response, the vibration of the steering wheel, the on-center steering capability, and the degree of steering fatigue, are more dependent on many subjective tests [1]. The existence of personal factors in the subjective evaluation with many experiments. However, based on different drivers of a certain vehicle, the subjective evaluation trend is almost the same [2-4]. The experiments prove that the subjective evaluation of steering feel can be quantitatively evaluated and analyzed. At the same time, the results of the subjective evaluation may be scattered. However, the problem can be solved by standardizing the evaluation value or introducing fuzzy inference model [5-6]. Therefore, based on a combination of different test contents and results, the multi-index fuzzy evaluation system should be established to achieve the comprehensive evaluation of EPS schemes.

In this paper, the group multi-criteria decision-making [7] and fuzzy Analytical Hierarchy Process [8] will integrate into the evaluation of EPS schemes. The working steps are presented as follows:

Step 1: Based on the simplified scale method, the precedence relationship comparative matrix of evaluation criteria should be established, and then the comparative matrix was transformed into a fuzzy consistent judgment matrix.

Step 2: The fuzzy linear weighted method is used to integrate experts’ opinions, and then the criterion weights and sub-evaluation value intervals are obtained. The criterion weights represented by the extreme fuzzy numbers.

Step 3: The fuzzy total evaluation value interval of each scheme is obtained by integrated through multiple layer from inside to outside.

Step 4: Using the weighted average calculation to obtain the total priority order of each EPS scheme.
This approach can simplify the analysis process, improve the decision-making efficiency, and comprehensively reflect the experts’ opinions. Therefore, this approach can ensure the fairness and credibility of decision making.

2. Establishment of Fuzzy Hierarchical Structure Model

Based on the requirements of subjective evaluation, this paper adopts the hierarchical structure model [14-15] shown in Figure 1. The subjective evaluation indicators mainly include four aspects: portability, returnability, the on-center steering capability, and the ISO lane change capability. The subjective evaluation indicators have their own grading indicators. Among them, the portability and returnability are the subjective evaluation indicators when the vehicle is traveling at low speed. The on-center steering capability, the ISO lane change capability and its sub-indicators are the comprehensive evaluation indicators when the car is traveling at high or low speed. The steering feel can be evaluated comprehensively brought by the EPS scheme under different traveling speeds. The bottom layer is each EPS scheme.

![Figure 1. The hierarchical structure model of EPS steering feel](image)

3. Implementation of Fuzzy Comprehensive Evaluation Based on Group Multi-Criteria Decision-Making

Through the calculation of fuzzy weights and sub criticism values of each EPS scheme, and integration of the multiple layer from inside to outside. The total fuzzy evaluation value of each EPS scheme is obtained.

3.1. Establishment of Criterion Judgment Matrix and Integration of Fuzzy Weights

The establishment of the criterion judgment matrix is mainly based on three-scale method. The relative importance of each parent criterion layer \( B_1, B_2, \ldots, B_n \) is evaluated separately, and then the precedence relationship comparative matrix is constructed. The \( t-th \) test driver compares the criteria in pairs:

\[
B' = \left( b'_{ij} \right)_{n \times n} \quad (t = 1, 2, \ldots, T)
\]

\[
b'_{ij} = \begin{cases} 
1.0 & b_i = b_j \\
0.5 & b_i < b_j \\
0.0 & b_i > b_j 
\end{cases}
\]

(1)

Where: \( T \) is denoted as the total number of evaluators; \( n \) is denoted as the number of parent criteria layer; \( b_i \) and \( b_j \) are denoted as the relative importance of each parent criterion layer.
Next, the comparative matrix \( B' \) is summed by rows to obtain the consistent relation judgment matrices \( B'' \).

\[
r_i' = \sum_{j=1}^{n} b_{ij}' \quad (i = 1, 2, \ldots, n)
\]

\[
\overline{b_{ij}'} = \frac{|r_i' - r_j'|}{2n} + 0.5
\]

Where: \( r_i' \) is denoted as the sum of the \( i \)-th row; \( r_j' \) is denoted as the sum of the \( j \)-th row; \( b_{ij}' \) is transformed into \( \overline{b_{ij}'} \) by normalization processing.

After that, the elements in the consistency judgment matrix \( B'' \) are summed by rows and normalized by column, and then \( W_i'' \) is the evaluation of the importance of each criterion \( B_i' \) by each expert.

In order to reflect the uncertainty of decision-making, after integrating the test drivers' opinions, the result is represented by the fuzzy number extremum. The final weight of each criterion layer can be obtained, which is different from the traditional AHP.

\[
W_i = (O, Q) \quad (O = \min_j \{ W_i^j \}; \quad Q = \max_j \{ W_i^j \})
\]

Where: \( W_i \) is donated as the \( i \)-th criterion weight of the criterion layer.

Therefore, the fuzzy weight \( W_j \) of each sub-criterion under each criterion layer can be obtained.

\[
W_j = (o_j, q_j) \quad (j = 1, 2, \ldots, n_i)
\]

Where: \( W_j \) is donated as the fuzzy weight of the \( j \)-th sub-criteria under the \( i \)-th criterion layer; \( n_i \) is donated as the number of sub-criteria under the \( i \)-th criterion layer; \( o_j \) is donated as the weights of the \( j \)-th sub-criteria corresponding to \( O \); \( q_j \) is donated as the weights of \( j \)-th the sub-criteria corresponding to \( Q \).

### 3.2. Calculation and Integration of the Sub Criticism Values of EPS Schemes

The traditional fuzzy AHP method leads to a complicated process, when comparing the relative importance of each EPS scheme. Therefore, interval-valued fuzzy corresponding to the value of the linguistic variables in Table 1 are used, which represented the evaluation interval of each alternatives. The interval-valued fuzzy reflects the ambiguity of the evaluation object. The evaluation value is selected in the interval-valued fuzzy.

| Linguistic variables | Interval-valued fuzzy |
|----------------------|-----------------------|
| Very poor            | (0.00, 0.25)          |
| Difference           | (0.25, 0.50)          |
| General              | (0.50, 0.75)          |
| Good                 | (0.75, 1.00)          |
| Well                 | (1.00, 1.00)          |

In order to reduce the calculation process, facilitate the integration of experts' opinions of each test result, and reduce the generation of extreme opinions, the Triangular Fuzzy Number method is adopted to collect the expert weights. Then, calculate the YAI as the weight of expert opinions. Based on the test results of one EPS scheme compared with other EPS schemes, the algorithm is listed as follows.

\[
W = (w_{ij}^e, w_{ij}^m, w_{ij}^u)
\]
Where, based on the $t$-th expert in the $i$-th test, $w^{ij}_t$ is donated as the comparative evaluation weight of the schlussgruppe; $w^{im}_t$ is donated as the comparative evaluation weight of the mittelgruppe; $w^{in}_t$ is donated as the comparative evaluation weight of the spitzengruppe.

The YAI is used to represent the preferred EPS scheme, and as the final weight of the expert. The bias of each test result was eliminated, and the weight of each expert was calculated and listed as follows.

$$w_i = \left[ w^{ij}_t / (w^{ij}_t + w^{im}_t) \right] / \left[ \sum_t^{m} \left[ w^{ij}_t / (w^{ij}_t + w^{im}_t) \right] \right]$$  \hspace{1cm} (6)

Finally, the sub-evaluation value of each alternative is integrated.

$$V_{ijk} = \sum_{j=1}^{n} (W' \otimes V'_{ijk}) = (\sum_{j=1}^{n} W'_{ijk} e_{ijk}^j) \sum_{j=1}^{n} W'_{ijk} g_{ijk}^j) = (e_{ijk}^j, g_{ijk}^j) \quad (V'_{ijk} = (e_{ijk}^j, g_{ijk}^j))$$ \hspace{1cm} (7)

Where: $\otimes$ is donated as the fuzzy multiplication operator; based on the $j$-th sub-criteria under criterion $B_i$, $V_{ijk}$ is donated as the fuzzy evaluation value of the scheme $D_k (k = 1, 2, \cdots, m)$; $W'$ is donated as is the weight that reflects the decision-making ability of expert $P_i$; $V'_{ijk}$ is donated as the evaluation interval of expert $P_i$ under the sub-criteria; $e_{ijk}^j$ and $g_{ijk}^j$ are donated as the sub-evaluation value.

3.3. Calculation of the Fuzzy Total Evaluation Value Interval of EPS Scheme

According to the fuzzy hierarchical structure model in figure 1, the total evaluation value interval of each scheme can be obtained, which based on calculating of the multiple layer from inside to outside. Under the criterion, the weights of the sub-criteria are normalized. The weights are listed as follows.

$$W_q = w_q \otimes \left[ \sum_{j=1}^{n} w_q \right]^{-1} = (o_q, q_y)$$ \hspace{1cm} (8)

The evaluation interval $V_{ik}$ of the scheme $D_k$ is obtained for the criterion $B_i$.

$$V_{ik} = \sum_{j=1}^{n} (V_{ijk} \otimes W_q) = \left( \sum_{j=1}^{n} (e_{ijk}^j o_y^j), \sum_{j=1}^{n} (g_{ijk}^j q_y^j) \right) = (R_{ik}, T_{ik})$$ \hspace{1cm} (9)

The fuzzy total evaluation value of the alternative is listed as follows.

$$V_k = \sum_{i=1}^{m} (W_i \otimes V_{ik}) = \left( \sum_{i=1}^{m} (O_i \otimes R_{ik}), \sum_{i=1}^{m} (Q_i \otimes T_{ik}) \right) = (X_k, Z_k)$$ \hspace{1cm} (10)

The total evaluation value in the alternative is listed as follows.

$$\overline{V}_k = \frac{X_k + Z_k}{2}$$ \hspace{1cm} (11)

4. Implementation of the Weighted Ranking

After the fuzzy total evaluation value of each alternative is obtained, the fuzzy interval can be weighted ranking.

If the total evaluation value of EPS scheme is $\overline{V}_k$, then the weighted evaluation value is listed as follows:

$$\overline{Y}_k = \overline{W} \cdot \overline{V}_k$$ \hspace{1cm} (12)
\[
\overline{W} = \left( \sum_{j} w_{1,j} / N + \sum_{j} w_{2,j} / N + \cdots + \sum_{j} w_{n,j} / N \right) / M \tag{13}
\]

Where, \(\overline{W}\) is donated as the total evaluation weight value given by the expert; \(M\) is donated as the number of expert group; \(j\) is donated as the road experiment ordinal; \(N\) is donated as the total number of road experiments.

The larger the weighted value, the better the evaluation. Compared with other evaluation methods, this approach has a clear comparison of quantitative indicators. This approach can fully reflect the change of road experiment and experts’ opinions tendency of the alternative. Therefore, the ranking result is relatively more accurate.

5. Example Application Analysis
The subjective evaluation test of EPS steering feel was conducted at the Ministry of Communications. The test vehicle is a domestic sedan equipped with four different power-assisted EPS schemes. Seven professional male drivers are selected. In double lane change and slalom test, the speed of the test vehicle is \((90\pm5)\) km/h as stable as possible, and the test vehicle travels in a straight line. In the slalom test, the input angle is about 20° and the frequency is about 0.2 Hz. Before entering the test section, the zero line of each test variable is recorded. Then passing the test section, the time history curve of each test variable (sampling frequency is 100Hz) is recorded, and also the passing time of the effective marker area.

Based on each sub-criteria under each criterion layer, each driver used linguistic variables to evaluate the each alternative. Then integrate them by using formula (7). The subjective evaluation results of scheme D1 and the fuzzy numbers are listed as follows (see Table 2). The weights of each driver are listed as follows: 0.10, 0.10, 0.15, 0.15, 0.20, 0.20.

| Driver | Portability | Retumability | Sensitivity | Correction | Road feel | Steering veracity | Rolling feel | Fatigue degree |
|--------|-------------|--------------|-------------|------------|-----------|-------------------|-------------|---------------|
| 1      | General     | General      | Difference  | Good       | General   | Good              | General     | Well          |
| 2      | General     | Difference   | General     | Good       | General   | Good              | General     | Good          |
| 3      | General     | General      | Good        | Well       | General   | Good              | General     | General       |
| 4      | General     | General      | Good        | Good       | Well      | Difference        | Well        |
| 5      | General     | General      | Good        | General    | Well      | General           | General     |
| 6      | Good        | Difference   | General     | Good       | General   | Good              | Difference  |
| 7      | General     | Difference   | Good        | General    | General   | Good              | General     |
| Fuzzy number | 0.36, 0.83 | 0.17, 0.68 | 0.43, 0.78 | 0.50, 0.99 | 0.35, 0.84 | 0.49, 1.00 | 0.17, 0.65 | 0.65, 1.00 |

The relationship comparative matrix of the driver \(S_1\) weight is shown in Table 3. Based on the formula (2) and formula (3), the relationship comparative matrix can be converted into the consistent judgment matrix. The consistent judgment matrix of driver \(S_1\) weight is shown in Table 4. Therefore, the comparison results of other drivers can also possible to give.

| A       | B1    | B2    | B3    | B4    |
|---------|-------|-------|-------|-------|
| B1      | 0.50  | 1.00  | 0     | 1.00  |
| B2      | 0     | 0.50  | 1.00  | 0.50  |
| B3      | 1.00  | 0     | 0.50  | 0.50  |
| B4      | 0     | 0.50  | 0.50  | 0.50  |
Table 4. The consistent judgment matrix of the driver $S_1$ weight

|   | A   | B1  | B2  | B3  | B4  |
|---|-----|-----|-----|-----|-----|
| B1| 0.50| 0.56| 0.56| 0.62|
| B2| 0.44| 0.50| 0.50| 0.56|
| B3| 0.44| 0.50| 0.50| 0.56|
| B4| 0.38| 0.44| 0.44| 0.50|

Then, the consistent judgment matrix is summed by rows, and then normalized by column. The importance evaluation of each criterion is obtained. The test drivers’ opinions are integrated by using formula (4). The fuzzy number weights of each criteria $B_i (i = 1, 2, \ldots, 4)$ is obtained (see Table 5).

Table 5. The consistent judgment matrix of the driver $S_1$ weight.

|   | B    | S1  | S2  | S3  | S4  | S5  | S6  | S7  | Fuzzy weights       |
|---|------|-----|-----|-----|-----|-----|-----|-----|---------------------|
| B1| 0.28 | 0.19| 0.15| 0.25| 0.21| 0.34| 0.16| (0.16,0.28) |
| B2| 0.25 | 0.23| 0.21| 0.26| 0.26| 0.22| 0.16| (0.16,0.25) |
| B3| 0.25 | 0.28| 0.31| 0.29| 0.25| 0.31| 0.25| (0.25,0.34) |
| B4| 0.22 | 0.28| 0.31| 0.25| 0.23| 0.31| 0.21| (0.21,0.31) |

Each expert used fuzzy linguistic variables value to evaluate each EPS scheme. The evaluation value of each sub-criterion under each criterion layer is obtained through formula (7). The fuzzy interval of sub-evaluation value of scheme $D_1$ is shown in Table 5. Based on formula (8), formula (9) and formula (10), the total evaluation value of each scheme is obtained.

Taking scheme $D_1$ as an example, the relative weights of each sub-criterion are obtained by using formula (8). Then, the evaluation value of scheme $D_1$ under each criterion is obtained by using formula (9) (see Table 6).

Table 6. The consistent judgment matrix of the driver $S_1$ weight

|   | Criterion | Fuzzy weights corresponding to each sub-criterion | Sub-evaluation value | Evaluation value |
|---|-----------|-----------------------------------------------|---------------------|------------------|
| B1| (0.15,0.281) | 1                                             | 1                   | (0.34,0.89)     |
| B2| (0.15,0.25) | C31                                           | (0.28,0.39)         | (0.41,0.88)     |
|    |            | C32                                           | (0.22,0.39)         | (0.52,1.00)     |
|    |            | C33                                           | (0.22,0.45)         | (0.35,0.85)     |
|    |            | C41                                           | (0.28,0.44)         | (0.58,1.00)     |
| B3| (0.25,0.34) | C42                                           | (0.22,0.44)         | (0.16,0.66)     |
|    |            | C43                                           | (0.22,0.33)         | (0.65,1.00)     |
| B4| (0.21,0.31) |                                               |                     | (0.33,1.07)     |

The corresponding fuzzy interval of the total evaluation value of EPS scheme $D_1$ is obtained by using formula (10). Therefore, the corresponding fuzzy interval of the total evaluation value of other schemes is obtained (see Table 7).

Table 7. The consistent judgment matrix of the driver $S_1$ weight

|   | Total evaluation value of each scheme | Expert average weight value | Final weighted average |
|---|--------------------------------------|----------------------------|------------------------|
| D1| 0.68                                 | 0.95                       | 0.65                   |
| D2| 0.66                                 | 0.87                       | 0.57                   |
| D3| 0.67                                 | 0.94                       | 0.63                   |
| D4| 0.64                                 | 0.82                       | 0.52                   |
Based on ranked by weighted average, the order of the four schemes is $D_3$, $D_4$, $D_1$, $D_2$. The EPS scheme $D_3$ is optimal. The evaluation result is consistent with the subjective evaluation of the driver. Therefore, based on the fuzzy AHP method of group multi-criteria decision-making, the fuzzy interval and Triangular Fuzzy Number method, this approach can effectively evaluate and rank the steering feel of EPS schemes.

6. Conclusion
Firstly, based on the fuzzy linguistic variables, the three-scale method is used to construct the relationship comparative matrix of evaluation criteria. Secondly, the relationship comparative matrix is transformed into fuzzy consistency judgment matrix. Thirdly, based on the integration of multiple expert evaluations, fusion, decision and judgment on the decision results are given. Fourthly, the weighted average is used to rank the fuzzy evaluation intervals. Finally, the priority order of each EPS scheme is obtained. It effectively improves the credibility.

The evaluation results can better reflect the steering feel, and can be directly quantified into specific values. Therefore, this method has good practical value. Based on the system analysis modelling, the independence of the decision evaluation and the preciseness of the mathematical reasoning, this method is suitable for comparison of EPS steering feel between various vehicle models. Accurate and reliable ranking results can be obtained. Therefore, this method promotes the development of EPS and optimization of parameters.

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8. References
[1] Lin Yi, Zhang Xin and Shi Guobiao 2007 “Subjective evaluation method study on vehicle steering feel,” *Journal of Highway and Transportation Research and Development*, vol. 24, no. 2, pp. 130–134.
[2] Burton and A. W. 2003 “Innovation drives for electric power-assisted steering,” *IEEE Control Systems Magazine*, vol. 23, no. 6, pp. 30–39.
[3] Zhao Xianglei, Chen Hui, Gao Bolin, et al. 2012 Correlations Between Subjective and Objective Evaluations of On-Center Steering Feel. *Presented at FISITA 2012 World Automotive Congress--Volume 10: Chassis Systems and Integration Technology.*
[4] Misaji, Kazuhito, Tokunaga, Hiroyuki, Shimizu, Yasuo, et al. 2004 “Vehicle Dynamics Analysis by \"Analytical Method of Equivalent Linear System using the Restoring Force Model of Power Function Type\"” *Vehicle System Dynamics*, vol. 42, no. 1-2, pp. 119–131.
[5] Yeh C. H. and Chang Y. H. 2009 “Modeling subjective evaluation for fuzzy group multi-criteria decision-making,” *European Journal of Operational Research*, vol. 194, no. 2, pp. 464–473.
[6] Suzuki H., Harara M. and Kumakura H. 1994 “Study on Suitable Steering Feeling for Various Driving Conditions-By Controlling Power Steering Assist Characteristic,” *Jsae Review*, vol. 16, no. 2, pp. 212–212(1).
[7] Wang Huaji, Zong Changfu, Guan Xin, et al. 2011 “Method of Determining Weights of Subjective Evaluation Indexes for Car Handling and Stability Based on Fuzzy Analytic Hierarchy Process,” *China Mechanical Engineering*, vol. 47, no. 24, pp.87–94.
[8] Rouder J. N., Lu J., Speckman P., et al. 2005 “A Hierarchical Model for Estimating Response Time Distributions,” *Psychonomic Bulletin & Review*, vol. 12, no. 2, pp. 195–223.