An Adaptive Method for Nonlinear Sea Level Trend Estimation by Combining EMD and SSA

Taoyong Jin1,2, Mingyu Xiao1, Weiping Jiang2,3, C. K. Shum4,5, Hao Ding1,2, Chung-Yen Kuo1, and Junkun Wan4

1School of Geodesy and Geomatics, Wuhan University, Wuhan, China, 2MOE Key Laboratory of Geospace Environment and Geodesy, Wuhan University, Wuhan, China, 3GNSS Research Center, Wuhan University, Wuhan, China, 4Division of Geodetic Science, School of Earth Sciences, The Ohio State University, Columbus, OH, USA, 5Institute of Geodesy and Geophysics, Chinese Academy of Sciences, Wuhan, China, 6Department of Geomatics, National Cheng Kung University, Tainan, Taiwan

Abstract  Adaptive and accurate trend estimation of the sea level record is critically important for characterizing its nonlinear variations and its study as a consequence of anthropogenic climate change. Sea level change is a nonstationary or nonlinear process. The present modeling methods, such as least squares fitting, are unable to accommodate nonlinear changes, including the choice of a priori information to help constrain the modeling. All these problems affect the accuracy and adaptability of nonlinear trend estimation. Here, we propose a method called EMD-SSA, that effectively combines adaptive empirical mode decomposition (EMD) and singular spectrum analysis (SSA). First, the sea level change time series is decomposed by EMD to estimate the intrinsic mode functions. Second, the periodic or quasiperiodic signals in the intrinsic mode functions can be determined using Lomb-Scargle spectral analysis. Third, the numbers of the identified periodicities/quasiperiodicities are used as embedding dimensions of SSA to identify possible nonlinear trends. Then, the optimal nonlinear trend with the largest absolute Mann-Kendall rank is selected as the final trend for the sea level change. Based on a comprehensive experiment using simulated sea level change time series, we concluded that the EMD-SSA method can adaptively provide better estimate of the nonlinear trend in a realistic sea level change time series with consistency or high accuracy. We suggest that EMD-SSA can be used not only to robustly extract nonlinear trends in sea level data, but also for trends in other geodetic or climatic records, including gravity, GNSS observed displacements, and altimetry observations.

1. Introduction

Sea level records exhibit complex nonlinear variability as a result of competing physical processes, some of which are largely unknown, and the available length of the record will also affect the trend estimation, therefore robust methods for determining consistent long-period, nonlinear trends are needed to effectively analyze different sea level records holistically, which is important for timely research and applications to address current and future coastal vulnerability and adaptations due to sea level rise (Ghil et al., 2002; Plattner & GanKasper, 2014; Zhang & Church, 2012). At least 30 methods classified into five categories, have been used to estimate or search for sea level trends (see Visser et al., 2015 for a detailed review), and some methods have been compared in terms of different aspects, including accuracy, standard deviation, computational cost, consistency, capacity to provide temporal information, resolution, complexity, and predictive performance (Watson, 2016a). Although there are differences between the results in terms of which methods are better, one could draw some common conclusions.

A simple parabola (Houston & Dean, 2011; Woodworth, 1990) or straight line is commonly used to fit the trend in sea level change time series (Douglas, 1991). However, the trend estimate provides limited, if any, information about transient variations of the trend, and the trend estimate is likely to be influenced by the particular data span (Parker et al., 2013). Some other predefined functions or models, such as piecewise functions (Fenoglio-Marc & Tel, 2010), exponential functions (Parker et al., 2013), and different autoregressive moving average models (Beenstock et al., 2015; Thompson, 1980), used to search for sea level trends are also not sufficiently universal to apply to all sea level change time series. Although the moving average is a simple method, data are lost from the two ends of the averaged record, leading to the absence of re-
Recent studies have proposed variants based on EMD, including ensemble EMD (Wu & Huang, 2009), complete ensemble EMD with adaptive noise (Torres et al., 2011), variational mode decomposition (Dragomiretskiy & Zosso, 2014), and empirical wavelet transform (Gilles, 2013). These decomposition results of these variants have improved in some cases, but additional parameters are needed; for example, the number of decompositions is needed for variational mode decomposition, while a priori noise information is needed for ensemble EMD and complete ensemble EMD with adaptive noise, which causes a loss of adaptability. EMD can be used without parameters, and without parameters, ensemble EMD is equivalent to EMD. As we want to take advantage of the adaptability of EMD relevant methods, we choose EMD instead of other variants. In addition, Ezer and Corlett (2012) proposed an EMD-HHT method which has improved the EMD method with bootstrapping simulation. The difference between the EMD and EMD-HHT method will be evaluated in this study.

SSA is another commonly used method for sea level data analysis (Haddad et al., 2013; Jevrejeva et al., 2006; Nitsche et al., 2005; Unal & Ghil, 1995; Watson, 2011, 2016a). It is a superior analytical tool capable of decomposing sea level records into the sum of interpretable components including slowly varying trends, oscillatory components with variable amplitudes and noise, and without a priori information on the time series structure. While SSA appears to be similar to EMD, it has physical meaning and a robust mathematical proof. The method is based on dynamic reconstruction of time series and is not constrained by the sine wave hypothesis. It can not only identify signals with different frequencies, but also strictly order those signals according to their energy, which is more advantageous than traditional spectral analysis (Vautard et al., 1992). SSA has also been proven to be a generalized power spectral analysis method with the ability
to stably identify and enhance periodic signals (Zabalza et al., 2014). With SSA applied to the analysis of tide gauge records and satellite altimetry time series, finer scales of periodic and quasiperiodic modes are identified (Haddad et al., 2013; Jevrejeva et al., 2006; Unal & Ghil, 1995). Trends of different global mean sea level reconstructions are estimated by SSA and compared to show more robust, convincing evidence of recent acceleration (Watson, 2016b). Based on simulated data tests, along with the maximal overlap discrete wavelet transform (MODWT) method, SSA is proven to be one of the two most recommended tools for sea level research (Watson, 2016a). The key parameter of SSA is the embedding dimension. The larger the embedding dimension is, the higher the resolution and the maximum detectable frequency, but an excessively large embedding dimension may result in spectral splitting, and a larger portion of the time series is affected by data boundaries (Vautard et al., 1992; Vautard & Ghil, 1989). In this case, it is not suitable to conduct time series analysis. Until now, there has been no unified principle for selecting the optimal embedding dimension, and the suggested ranges are based on experience and specific types of signals. Many methods have been developed based on SSA, including Auto-SSA (Haddad et al., 2013), multichannel SSA (Ghil et al., 2002), and Monte Carlo SSA (Allen & Smith, 1996). However, none of these methods achieve adaptive selection of the optimal embedding dimension. The essence of Monte Carlo SSA is still SSA, but noise is added as auxiliary information, like ensemble EMD. Auto-SSA improves the separation of decomposition results but is unable to find optimal embedding dimensions, and multichannel SSA is mostly used for two-dimensional signals. Here, we choose SSA instead of the other variants.

Although SSA is recommended, it is inappropriate to use the same embedding dimension for all sea level records with different data spans (Watson, 2016a). Different studies recommend different embedding dimensions: 1/5–1/2 of the data length (Nitsche et al., 2005), 1/10–11/5 (Ghil et al., 2002), 1/5–1/3 (Saraceno et al., 2014), 1/4–1/2 (Watson, 2016a), and 0–1/3 (Unal & Ghil, 1995). Vautard et al. (1992) suggested that a more accurate embedding dimension should be given according to the largest oscillatory cycle of the time series. As sea level change contains a variety of signals, including semiannual, annual, interannual, and interdecadal cycles, long-term trends and other unknown cycles (Douglas, 1991), the largest oscillatory cycle may not fall within these recommended intervals. Limited by the available length of the selected time series, ordinary spectral analysis (Fourier spectral analysis) cannot well identify the largest oscillatory cycle, primarily because of poor frequency resolution. In this study, we propose to find an adaptive method to determine the embedding dimensions for SSA. We believe a method has less adaptability when more prior parameters are needed and the parameters are complex. Thus, we believe EMD is more adaptive because it needs no parameters to be inputted for the algorithm.

In light of the above observations, we combine EMD and SSA, namely, the results of EMD are used as the input parameters for SSA. Here, we focus more on the robust methodology for estimating nonlinear trends in sea level time series. Hence, this study is not intended to explain the exact physical causes of sea level rise.

2. Methods

2.1. EMD-SSA Algorithm

Given that the trend estimated by SSA is more accurate than that estimated by EMD (Zabalza et al., 2014) but has weaker adaptability, we adapt SSA as the main algorithm and EMD as the reference to present a combined methodology named EMD-SSA to robustly estimate nonlinear sea level trends. The basic idea of EMD-SSA is to use the results of spectral analysis of IMFs as the embedding dimensions of SSA. The algorithmic steps are as follows and are also shown in Figure 1:

1. The original time series is decomposed by EMD (Huang et al., 1998) to obtain IMFs with different frequencies at different scales
2. Lomb-Scargle spectral analysis is applied to the IMFs to identify the main periodic signals contained in the corresponding IMFs
3. These periods are possibly suitable embedding dimensions, but some may not be integers, so the nearest integers for those periods are selected as the embedding dimensions
4. Each embedding dimension is used as the input of SSA to automatically identify the corresponding trend (Watson, 2018)
All possible trends are estimated, then the trend with the largest absolute MK (Hamed & Rao, 1998) is selected as the optimal estimated trend.

The steps of EMD, SSA methods and the computation of MK rank are shown in the supporting information (see supporting information text). Lomb-Scargle spectral analysis is selected because it is suitable for data with gaps. With the results from EMD as the input parameters, EMD-SSA greatly enhances the adaptability and accuracy of SSA. The appropriate embedding dimensions for SSA are related to the maximum period of the original signal, usually the period itself or its integer multiple (Vautard et al., 1992). It requires substantial expertise and is often almost impossible, because of high variance, to automatically perform ordinary spectral analysis on the initial time series to obtain the exact periodic signals without knowing the periodicities in advance. At present, EMD is one of the optimal adaptive methods for decomposing nonlinear or nonstationary signals into several narrow-band components, which is beneficial for determining the periods. EMD-SSA achieves nonparameterization and self-adaptation, and the method can automatically process time series of arbitrary length and greatly reduces the selection range of the embedding dimensions from an interval of ordinary SSA to several more meaningful values. For the evaluation of the accuracy of an estimated trend, it is impossible to achieve without knowing the true trend, so other methods are required for validation in the real world. Trend is defined in at least three aspects: (1) the trend is a smooth signal where the smoothness is chosen to filter out shorter-term natural fluctuations; (2) the trend is part of a series that provides the clearest indication of the future long-term movements in the series; (3) the trend is used for calibration of recent attempts to model historic geophysical processes (Visser et al., 2015). Here, we consider the first aspect that trend is a relatively smooth signal. Although smoothness itself is a vague concept, the relative relationship between smoothness and MK is clear: the larger the absolute MK, the smoother the trend is, and the better we think of the trend. As MODWT and SSA are the most recommended tools for sea level research (Watson, 2016a), EMD-SSA is compared with EMD, SSA, and MODWT in three aspects.

The computation of accuracy can be expressed as follows (Watson, 2016a):

\[
\text{Accuracy} = \frac{1}{n} \sum_{i=1}^{n} \left( X_i - \bar{X} \right)^2
\]

where \( n \) represents the number of data points, \( X_i \) represents the trend derived from a particular method, and \( \bar{X} \) represents the reference trend.

The consistency is a qualitative evaluation of changes in accuracy for different data lengths, while efficiency is quantified as the running time of the program. For the two quantitative assessments, the smaller the values, the more accurate and more efficient are the estimates.

3. Data

To verify the accuracy, consistency, and efficiency of EMD-SSA, simulated monthly data based on six tide gauge records from the Permanent Service for Mean Sea Level (PSMSL), whose station names and PSMSL IDs are LOWESTOFT (PSMSL ID: 78), STOCKHOLM (PSMSL ID: 97), FUNCHAL (PSMSL ID: 266), SEWARD (PSMSL ID: 754), BACKEVIK (PSMSL ID: 1.031), and CUXHAVEN 2 (PSMSL ID: 7), are used (Figure 2).
The method for the simulation is as follows. First, a harmonic function with possible periods including annual, semiannual, 1.19 years (Chandler Wobble, Trupin & Wahr, 1990), 7 years (roughly representing ENSO), 11 years (solar cycle), 18.6 years (lunar nodal tide), and 60 years (Chambers et al., 2012), and parabolic trend items is fitted to the tide gauge data. Since many possible periods are already considered, similar to previous studies (Houston & Dean, 2011; Woodworth, 1990), a parabolic trend is only used here to express the sea level trend, which will be used as the priori trend to evaluate the trends estimated by other methods. Then, an autoregressive moving average model is used to fit the residuals to simulate noise (Chambers et al., 2012) and added back to the above function to generate the simulated time series. The periodic terms longer than the data length are removed for that specific tide gauge to avoid aliasing with the trend. Considering the noise in the in situ data, we may say that the simulated data agree well with the observations in Figure 2. Specifically, compared to the standard deviations of the six tide gauges in situ data, the maximum biases between the simulated data and in situ data are smaller, and the root mean square error is reduced to a great extent, as shown in Figure 3.

All six simulated time series are used to assess the accuracy. Simulated data from tide gauge CUXHAVEN 2 is used to assess consistency, because CUXHAVEN 2 has the longest continuous record (to avoid influence of gap-filling procedure) of 176 years from January 1843 to December 2018, which is convenient for dividing initial data into several parts of the same length. To assess efficiency, the simulated CUXHAVEN 2 data is used too.
4. Results

4.1. Accuracy of Nonlinear Trend Estimation

Taking the parabolic trend in the simulation as the priori trend, using the formula 1, the accuracy of trends extracted by EMD, SSA, EMD-SSA, and MODWT for the simulated data from the five tide gauges are computed. For MODWT and EMD, the last component is selected as the trend. For SSA, we manually select the trend with the highest accuracy after a large amount of comparison with the priori trend. The accuracy of EMD, SSA, and EMD-SSA, and those trends’ absolute MKs are shown in Figure 4. The accuracy, absolute MK, embedding dimensions and acceptable intervals of embedding dimensions are listed in Table 1. Several conclusions can be drawn.

(a) MODWT is more accurate than EMD, and the accuracy of SSA depends on the embedding dimensions; an embedding dimension that is too small or too large may result in even worse performance than EMD. There is always an embedding dimension interval where SSA has nearly the highest accuracy compared to the fitted trend for each tide gauge, and within the interval, their difference is very small. In other words, the embedding dimensions in the interval are all acceptable.

(b) The acceptable interval of SSA is not fixed, and 0–1/2 of the data length are all possible. Thus, if the same embedding dimension is used for different sea level records, there could be a large difference in the results. For example, the commonly used interval 1/10–1/5 of the data length is suitable for tide gauge STOCKHOLM but not for the other four tide gauges, especially for SEWARD and BACKEVIK, and the difference in accuracy can be as large as two orders of magnitude. However, although the acceptable interval is not fixed, at least one of the embedding dimensions determined by EMD-SSA can be in the interval, which means that EMD-SSA could be more accurate than MODWT and EMD, and show little difference from the best result obtained by SSA.

(c) For all the embedding dimensions determined by EMD-SSA, the absolute MK of the embedding dimensions in the acceptable interval are larger than the others, so it is reasonable that we choose the optimal trend according to the largest absolute MK. Although the trend with the largest absolute MK may not be the most accurate, its corresponding embedding dimension is contained in the acceptable interval.

(d) As the embedding dimension selected by MK is contained in the acceptable interval but may not be the most accurate, the accuracy of the four methods can be ranked from high to low as: SSA, EMD-SSA, MODWT, EMD, and there is a small difference between SSA and EMD-SSA.

![Figure 3](image_url). Standard deviations of in situ data, the maximum bias and root mean square error between the simulated data and in situ data, for selected six tide gauges.
In addition to quantitative comparisons, the trends determined by EMD, MODWT, EMD-SSA, and SSA are plotted in Figure 5, as well as that determined by EMD-HHT (Ezer & Corlett, 2012). It can be seen most clearly from FUNCHAL and BACKEVIK stations that the deviations in the trends determined by EMD, MODWT, EMD-SSA, and SSA rank in the same order as the above conclusions (d). Moreover, the trend determined by EMD-HHT is similar to that of EMD but different from that of MODWT, EMD-SSA, and SSA, which accords with common sense because the difference between methods belonging to different categories is often larger than that of the same category, such as EMD-HHT is essentially EMD with a bootstrapping simulation but EMD-SSA is essentially SSA. To further verify this conclusion, in situ data from the Sewells Point tide gauge station which have already been used in (Ezer & Corlett, 2012), and trends estimated from EMD, MODWT, EMD-SSA, and EMD-HHT are plotted in Figure 6. SSA is excluded because with in situ data, it is difficult to determine which embedding dimension is better, since there is no priori trend. The trend of EMD-HHT is digitalized from (Ezer & Corlett, 2012). It can be seen that the trends of EMD-HHT and EMD are closer to each other, but have moderate difference between MODWT and EMD-SSA. And the trend by MODWT suffered from more serious end effects than that of EMD-SSA.

### 4.2. Consistency of Nonlinear Trend Estimation

It is important to understand how the relative accuracy changes in the estimation of trends from data sets with different data lengths (Watson, 2016a). Generally speaking, the more observations there are, the higher the accuracy of the estimation results of a specific method; however, it is not necessarily true if a linear
Table 1
The Accuracy (in $\text{mm}^2$), Absolute MK and Normalized Embedding Dimensions of the Four methods

| Tide gauges | Index | EMD   | MODWT | EMD-SSA | SSA |
|------------|-------|-------|-------|---------|-----|
| LOWESTOFT  | Accuracy | 36.9  | 34.6  | 30.4, 28.4, 28.2, 29.2, 30.0, 33.2 | 27.9 |
|            | Absolute MK | 0.45, 1.29, 2.13, 0.63, 0.58, 0.22 |       |         |     |
|            | Dimensions  | 0.009, 0.024, 0.035, 0.074, 0.089, 0.167 |       | 0–0.1  |     |
| STOCKHOLM  | Accuracy | 144.4 | 125.3 | 1,086, 532.2, 170.5, 116.7, 97.4 | 92.7 |
|            | Absolute MK | 49.6, 52.8, 56.7, 59.0, 59.1 |       |         |     |
|            | Dimensions  | 0.007, 0.017, 0.047, 0.088, 0.131 |       | 0.1–0.2 |     |
| FUNCHAL    | Accuracy | 126.3 | 95.7  | 599.1, 537.4, 325.6, 98.1, 71.1 | 68.0 |
|            | Absolute MK | 24.9, 25.6, 27.9, 34.2, 35.0 |       |         |     |
|            | Dimensions  | 0.018, 0.024, 0.057, 0.169, 0.266 |       | 0.2–0.3 |     |
| SEWARD     | Accuracy | 124.0 | 94.2  | 2,307, 2,081, 1,675, 696.8, 63.9, 66.0, 118.6 | 56.8 |
|            | Absolute MK | 8.17, 8.23, 9.12, 15.9, 37.3, 36.5, 33.9 |       |         |     |
|            | Dimensions  | 0.011, 0.018, 0.046, 0.129, 0.310, 0.387, 0.454 |       | 0.3–0.4 |     |
| BACKEVIK   | Accuracy | 539.7 | 380.6 | 8,640, 4,710, 3,671, 769.6, 124.7, 161.8 | 117.3 |
|            | Absolute MK | 3.58, 5.73, 6.37, 14.4, 30.1, 26.7 |       |         |     |
|            | Dimensions  | 0.015, 0.029, 0.049, 0.208, 0.420, 0.490 |       | 0.4–0.5 |     |

For SSA, the highest accuracy and acceptable intervals of the embedding dimensions are given. For EMD-SSA, all possible values from EMD are given.

Figure 5. Simulated priori true trend and estimated trends from EMD, MODWT, EMD-SSA, SSA, and EMD-HHT for six tide gauges.
estimation method is used for nonlinear time series. For example, we believe least squares fitting is less consistent than SSA in sea level trend estimation because it is unable to reflect the nonlinearity of sea level change and is likely to obtain different or even completely opposite results dependent on the selected data span. To show how the accuracy of the four methods changes within different data spans, simulated data of tide gauge CUXHAVEN 2 are divided evenly into two and four parts, and the accuracy of all time slices is computed in Table 2. Additionally, the monthly data are annualized to test the influence of annual average operation. Three conclusions can be drawn:

**Table 2**

*The Accuracy of EMD, MODWT, EMD-SSA, and SSA in Different Time Slices of Monthly and Annual Data (mm²)*

| Time slice | Time period       | EMD    | MODWT | EMD-SSA | SSA   |
|------------|-------------------|--------|--------|---------|-------|
| monthly    | as a whole        | 1843.1–2018.12 | 34.5   | 30.6    | 23.6  | 16.4  |
|            | 1st in 2 parts    | 1843.1–1930.12 | 97.2   | 57.8    | 32.6  | 18.8  |
|            | 2nd in 2 parts    | 1931.1–2018.12 | 78.4   | 50.2    | 33.1  | 17.2  |
|            | 1st in 4 parts    | 1843.1–1886.12 | 118.2  | 103.0   | 53.2  | 19.9  |
|            | 2nd in 4 parts    | 1887.1–1930.12 | 100.5  | 77.8    | 83.9  | 36.9  |
|            | 3rd in 4 parts    | 1931.1–1974.12 | 93.5   | 62.1    | 58.8  | 37.2  |
|            | 4th in 4 parts    | 1975.1–2018.12 | 175.1  | 144.8   | 121.3 | 80.8  |
| annual     | as a whole        | 1843.1–2018.12 | 28.9   | 27.4    | 15.6  | 10.8  |
|            | 1st in 2 parts    | 1843.1–1930.12 | 43.0   | 40.3    | 19.4  | 13.4  |
|            | 2nd in 2 parts    | 1931.1–2018.12 | 64.9   | 31.4    | 27.4  | 22.7  |
|            | 1st in 4 parts    | 1843.1–1886.12 | 113.9  | 88.7    | 59.8  | 19.2  |
|            | 2nd in 4 parts    | 1887.1–1930.12 | 87.6   | 73.9    | 60.5  | 28.9  |
|            | 3rd in 4 parts    | 1931.1–1974.12 | 79.7   | 68.4    | 42.3  | 32.4  |
|            | 4th in 4 parts    | 1975.1–2018.12 | 121.1  | 118.7   | 68.9  | 58.5  |
(a) From each row of the table, in most time slices, the accuracy of the four methods can be ranked from high to low as: SSA, EMD-SSA, MODWT, and EMD, and there is a small difference between SSA and EMD-SSA, which is the same as the conclusion (d) in Section 4.1.

(b) For each method (column of the table), the shorter the data length is, the less accurate the estimated trend, which is in line with common sense that higher accuracy is achieved with more observations. Thus, all four methods are consistent and able to extract nonlinear trends of sea level change. This conclusion shows the importance of longer records in improving the accuracy of sea level trend estimation.

(c) Comparing the corresponding values of monthly and annual parts, the accuracy of annual data is higher than that of monthly data because the annual data set provides a natural low-frequency smoothing and the seasonal influence is largely removed, so annual data is more recommended for nonlinear sea level trend estimation (Watson, 2016a).

4.3. Efficiency of Nonlinear Trend Estimation

Considering the empirical process of SSA, the shortest interval 1/10–1/5 of the data length is selected as the embedding dimension interval, and the test is repeated 100 times on the simulated data of tide gauge CUXHAVEN 2. For each time, the trend of SSA is determined after comparison with the priori trend using all possible embedding dimensions in the interval, while the trend of EMD-SSA is determined automatically. The operating system is Windows 8 × 32, the processor is an Intel® Core™ i5-4278u, the main frequency of the CPU is 2.6 G Hz, the internal memory is 2 GB and the software is MATLAB 2014a. The statistical calculation time required to obtain the trend of SSA and EMD-SSA is shown in Figure 7(a). It can be seen that EMD-SSA greatly reduces the runtime compared to SSA for choosing the optimal trend. Ensuring the characteristics of the simulated data remain unchanged, by changing the data length, the runtime can be obtained. As seen in Figure 7(b), as the data length increases, EMD-SSA improves the computational efficiency more substantially, which is very advantageous for large amounts of data of different lengths. Theoretically, the number of embedding dimensions that SSA needs to try increases linearly with an increase in data length n, and the operation time of a single embedding dimension is nearly cubic of the dimension.
thus the time complexity is approximately $\mathcal{O}(n^2)$. In contrast, regardless of how long the sea level record is, the number of IMFs obtained by EMD decomposition is not large. As each IMF is analyzed to obtain a main frequency, the total number of embedding dimensions provided by EMD is not large either. With invalid embedding dimensions eliminated, there are few embedding dimensions to try. Since the embedding dimensions are related to only the spectral properties of the time series, and not the length, they are not very large. Therefore, the time complexity of EMD-SSA can be estimated to be $\mathcal{O}(n)$. That is to say, the computation time of EMD-SSA is three orders of magnitude less than that of SSA to find the optimal trend.

4.4. Simple Application to the in Situ Data

Beyond the simulated data, Figure 8 displays the results of EMD-SSA with in situ data of the above selected tide gauges. Although no quantitative conclusions can be made because of no priori trends, it can be seen that the trends are nonlinear and reflect the overall characteristics; at the same time, the trends are smooth without relatively low-frequency signals. As the conclusions are not dependent on the selection of tide gauge, there is reason to believe that EMD-SSA can be applied to other measured data, including tide gauges and other geoscientific data.

5. Conclusions

Estimation of the nonlinear trend of sea level change plays an important role in sea level research. For trend estimation, most methods use predetermined models to estimate the trend, while others, such as wavelet analysis, EMD and SSA, do not. Among them, EMD does not need input parameters, and the parameters of wavelet analysis are complicated and diverse, while SSA needs only one input parameter. Combining
the adaptability of EMD and the accurate characteristics of SSA, this study proposes an adaptive nonlinear trend estimation method. Under the guarantee of accuracy of nonlinear trend extraction, the method is fully adaptive. The simulated sea level data of six tide gauges were used to verify the advantages of EMD-SSA compared to EMD, SSA, and MODWT in three aspects.

Although EMD still faces some serious problems, most of the periods determined are in the acceptable interval and deviate nonsignificantly from the best embedding dimension. In general, EMD-SSA is accurate, consistent and efficient. EMD-SSA is more accurate and consistent than EMD and MODWT, but slightly less accurate than SSA when the best embedding dimension is determined. However, the best embedding dimension is impossible to determine since the true trend is unknown; therefore, EMD-SSA is considerably more efficient than SSA actually. The process of nonlinear trend estimation by EMD-SSA is the same as that by SSA, but the advantages are that the optimal embedding dimension can be selected adaptively and the computational efficiency is greatly improved. In addition, it is an interesting finding that it is reasonable to select the optimal trend according to the MK. Nevertheless, there remains room for improvement for this algorithm. After determining the embedding dimension, Monte-Carlo SSA may be used to obtain uncertainty and confidence intervals. If the periods from EMD can be more accurate and the spectral analysis more refined, the embedding dimensions provided will be more accurate.

In conclusion, the EMD-SSA method has great potential in a wide range of applications for adaptive nonlinear trend estimation, such as changes in temperature, precipitation, and CO$_2$, and the 3-D displacements observed by GNSS stations. Based on the accurate nonlinear trend, the separation of periodic signals, denoising, prediction, interpolation, and identification of abnormal signals can be performed. Notably, this method is recommended for processing data in batches. For a specific timeseries, this method can be used as a reference, and alternative methods could be tested to fine-tune the estimations of optimal parameters for the time series analysis.

Data Availability Statement

The tide gauge data used in this study are provided by PSMSL, National Ocean Center, Liverpool, England (http://www.psmsl.org/data/obtaining/).

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