Multijets in $e^+e^-$ annihilation

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A review of multijet rates and event-shape variables sensitive to multijet configuration with applications to measurements of $\alpha_s$, QCD color factors and tests of power corrections is presented.

1. Introduction

A wealth of results on QCD in $e^+e^-$ annihilation emerged from the LEP experiments. The main emphasis of these studies was on high-statistics 3-jet configurations. However, in the second part of LEP running and with the advent of NLO calculations for $e^+e^- \rightarrow 4$jets, a new class of studied appeared yielding high precision results using multijet events.

The most basic observable characterising multijet configurations are jet rates. As an example the 5-jet rate measured at LEP1 by ALEPH is shown in Fig. 1. The presented data are corrected for acceptance and detector resolution effects to the hadron level. Hadronisation corrections, relevant for comparisons with NLO calculations,
are not applied at this stage but shown in Fig. 1 to indicate their size. The experimental systematic uncertainties are between one and two % in the central part and increasing toward the phase space edges. The measurements are compared to various leading-log generators, which generate higher jet multiplicity only in the cascade of parton showers. Again in the peak region a reasonable agreement is observed.

Jet rates exhibit a clear dependence on the value of $\alpha_s$, since at leading order $R_{n+2} \propto \alpha_n^s$. The evolution of jet rates at a fixed value of $y_{cut}$ with centre-of-mass energy hence reveals the running of $\alpha_s$, as demonstrated by a measurement of L3 shown in Fig. 2.

[Figure 2: The evolution of the 3-jet rate with $E_{cm}$]

2. 4-jet observables

Naturally the class of observables receiving most of attention beyond 3-jets are the 4-jet observables. 4-jet production receives contributions from double-bremsstrahlung and gluon splitting diagrams, the latter into gluonic and quarkonic final states, each of them contributing with a different color factor (ratio). This color structure can be measured by combining different angular observables constructed in the 4-jet system in order to extract the color factor ratios $C_A/C_F$ and $T_R/C_F$ with nominal QCD SU(3) values $C_A=3$, $C_F = 4/3$ and $T_R = 1/2$. On top of this residual dependence on the color factors the perturbative predictions for differential distributions receive an overall normalisation factor in powers of $\alpha_s$, yielding at NLO a prediction of the following form:

$$\frac{1}{\sigma} \frac{d\sigma}{dO_4} = \eta(\mu)^2 B(O_4) + \eta(\mu)^3 \left[ B(O_4) \beta_0 \ln x^2 + C(O_4) \right] + \mathcal{O}(\eta^4); \hspace{1cm} \eta(\mu) = \frac{\alpha_s(\mu) \cdot C_F}{2\pi},$$

for a generic 4-jet observable $O_4$. Therefore, the analyses determine usually the color factors and $\alpha_s$ simultaneously, using fits to both angular variables with a particular sensitivity to the color factor ratios and event-shape- / jetrate-type observables with better sensitivity to $\alpha_s$. The distributions of representative observables of each class are shown for the angular correlations in Fig. 3 and for the 4-jet rate in Fig. 4.

Simultaneous analyses have been carried out recently by ALEPH and OPAL yielding results summarised in Table 1. The results for the color factors are in excellent agreement with the expectation of SU(3) for QCD, as shown in Fig. 5. The precision in $\alpha_s$ obtained from simultaneous fits is rather limited though consistent with other determinations. Given the agreement of the color factor measurements with the QCD expectation, one can also assume the latter and extract $\alpha_s$ alone from 4-jet observables. It turns out that the systematic uncertainty of such a measurement is better than from 3-jet observables, given the quadratic dependence on $\alpha_s$ at lowest order

$$\frac{\Delta \alpha_s}{\alpha_s} \approx \frac{1}{2} \frac{\Delta O_4}{O_4}. \hspace{1cm} (2)$$
OPAL recently presented an analysis of various 4-jet observables, including the 4-jet event-shape variables thrust minor (Fig. 6) and D-parameter (Fig. 7) along these lines, using data at LEP1 and LEP2. On top of the NLO calculation, for one observable, the 4-jet rate in the Durham scheme, also an all-orders resummation is available. As observed already in the 3-jet case, this observable yields the best precision in terms of systematic uncertainty.

A similar work has also been published by DELPHI using pure NLO with optimised scales. DELPHI turned the measurement of $\alpha_s$ from the 4-jet rate into a test of the logarithmic slope $d\alpha_s^{-1}/d\log E$ of the energy evolution governing the running of $\alpha_s$, shown in Fig. 8.

All of these analyses agree in their competitive precision compared to other determinations of $\alpha_s$ mainly resulting from a reduced perturbative uncertainty. A summary of the results on $\alpha_s$ from the 4-jet rate obtained by the LEP collaborations is given in Table II.

| $\alpha_s(M_Z)$ | $C_A$ | $C_F$ |
|-----------------|-------|-------|
| ALEPH           | 0.119 ± 0.006 ± 0.026 | 2.93 ± 0.14 ± 0.58 | 1.35 ± 0.07 ± 0.26 |
| OPAL            | 0.120 ± 0.011 ± 0.020 | 3.02 ± 0.25 ± 0.49 | 1.34 ± 0.13 ± 0.22 |
| QCD             | 3     | 4/3   |

Table I: Results of NLO analyses of 4-jet observables, the first error is statistical and the second systematic.

| $\alpha_s(M_Z)$ + $\Delta_{stat}$ $\Delta_{sys}$ | data set | theory          |
|------------------------------------------------|---------|-----------------|
| OPAL 0.1182 ± 0.0003 0.0026                      | 91-209  | NLO+NNLA        |
| ALEPH 0.1170 ± 0.0001 0.0022                      | 91      | NLO+NNLA        |
| DELPHI 0.1175 ± 0.0005 0.0030                     | LEP1    | NLO+$x_m^{opt}$ |
| DELPHI Logarithmic slope = 1.14 ± 0.36 89-209    | QCD: 1.27 ± 0.10 |

Table II: Summary of results on $\alpha_s$ from the 4-jet rate.
3. Power corrections for multijets

Most of the effort in understanding non-perturbative effects in $e^+e^-$ annihilations went into 3-jet observables, as highlighted in various contributions to this conference. The most common case also applied by the experiments is the power law correction, where the non-perturbative correction is absorbed into a term scaling with powers of $1/Q$ and parameterised by the moment of an effective coupling $\alpha_0$ evaluated at a scale $\mu_I$ usually set to 2 GeV. With
the availability of NLO calculations for 4-jet observables and the calculation of the corresponding power correction terms, the scope of studies has been extended by an analysis of the D-parameter carried out by L3 \[2\]. In this case the full prediction including perturbative and power correction terms is given by

\[
\left< D \right> = \left< D_{\text{pert}} \right> + \left< D_{\text{pow}} \right>,
\]

\[
\left< D_{\text{pert}} \right> = B_D \left( \frac{\alpha_s}{2\pi} \right)^2 + D_D \left( \frac{\alpha_s}{2\pi} \right)^3,
\]

\[
\left< D_{\text{pow}} \right> = 195\frac{\alpha_s}{2\pi} P\left(1/Q\right).
\]

L3 performed a study of the mean values of event-shape variables measured over a large range of centre-of-mass energies and determined in simultaneous fits \(\alpha_s\) and \(\alpha_0\). The results of this analysis are summarised in Table III. The results from the D-parameter turn out to be only marginally consistent with overall combination (dominated by 3-jet event-shapes). The value of \(\alpha_s\) is lower and the one of \(\alpha_0\) much higher. This can also be seen in the confidence level contour plot in Fig. 9. The systematic uncertainties for the measurement using the D-parameter are also significantly larger. This seems to indicate that, although a reasonable qualitative description is obtained, additional terms are needed to achieve a good quantitative description.

| L3     | \(\alpha_s(M_Z)\)          | \(\alpha_0(2\text{ GeV})\) |
|--------|---------------------------|-----------------------------|
| D-parameter | 0.1046 ± 0.0078 ± 0.0096   | 0.682 ± 0.094 ± 0.018       |
| all combined | 0.1126 ± 0.0045 ± 0.0039   | 0.478 ± 0.054 ± 0.024       |

Table III: L3 analysis of power corrections from event-shape means, the first error is statistical and the second systematic.
4. Conclusions

Multijet observables, mostly based on 4-jet configurations, have given access to precision measurements of $\alpha_s$ and the QCD color factors. A wealth of experimental data on 4-jet event-shape variables is available from LEP and awaiting higher order QCD calculations to be analysed. Currently, only the D-parameter has been analysed for power corrections yielding mildly discrepant results. This is to be confirmed by further analyses of other 4-jet event shapes. Beyond 4-jets, jet rates have been measured for up to 6 jets and for certain multijet-configurations dedicated observables like the jet-resolution parameter $y_{ij}$ are available for 5- and 6-jets. The potential of these high jet multiplicity events is still to be explored.

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