Abstract

Social networks contain data on both actor attributes and social connections among them. Such connections reflect the dependence among social actors, which is important for individual's mental health and social development. To investigate the potential mediation role of a social network, we propose a mediation model with a social network as a mediator. In the model, dependence among actors is accounted by a few mutually orthogonal latent dimensions. The scores on these dimensions are directly involved in the intervention process between an independent variable and a dependent variable. Because all the latent dimensions are equivalent in terms of their relationship to social networks, it is hardly to name them. The intervening effect through an individual dimension is thus of little practical interest. Therefore, we would rather focus on the mediation effect of a network. Although the scores are not unique, we rigorously articulate that the proposed network mediation effect is still well-defined. To estimate the model, we adopt a Bayesian estimation method. This modeling framework and the Bayesian estimation method is evaluated through a simulation study under representative conditions. Its usefulness is demonstrated through an empirical application to a college friendship network.

Keywords: Friendship network, Mediation analysis, Social network analysis, Latent space modeling, Bayesian estimation

1 Introduction

Network analysis is an interdisciplinary research topic of mathematics, statistics, and computer sciences (Wasserman and Faust, 1994). It has been adopted in diverse fields to address different research interests (Grunspan et al., 2014). Different techniques are used for tasks in a network analysis (Carrington et al., 2005). Graph theories is often used by mathematicians to examine the network structure (Newman et al., 2002). Fast and efficient algorithms are developed by computer scientists and statisticians to detect network communities (Zhao et al., 2012; Yang et al., 2013). Statistical models are also built to understand the formation of connections between actors in a network (Holland et al., 1983; Hoff et al., 2002; Choi et al., 2012; Paul and Chen, 2016).

Network analysis has also a long history in psychology and sociology, which is called social network analysis since it focuses on social relations among actors (Wasserman and Faust, 1994; Westaby et al., 2014). Social relations have been traditionally studied in psychology and sociology (House et al., 1988; McCamish-Svensson et al., 1999; Umberson et al., 2010; Cacioppo and Cacioppo, 2014). Social
relations influence people’s subjective well-being over the life course (House et al., 1988; Gurung et al., 1997; McCamish-Svensson et al., 1999; Seeman, 2001). Close social relations such as marriage and friendship predict late-life health and well-being (Waldinger et al., 2015). Social relations with peers also influence individual’s health behaviors (Broman, 1993; Umberson et al., 2010). Social cohesion can facilitate smoking cessation (Reitzel et al., 2012). Because of the importance of social connections, explaining its formation is of enormous interests to researchers. Actor attributes such as personalities are found to be closely related to close relations like friendship and marriage (Asendorpf and Wilpers, 1998; McCrae et al., 2008; Harris and Vazire, 2016). Similarity in academic achievement predicts the friendship ties among students (Flashman, 2012).

A social network contains data on both social relations and actor attributes (Wasserman and Faust, 1994). It provides a platform for researchers to study traditional research questions, such as the association between a network and actor attributes, from a network perspective. In our empirical study, we collected data on a college student’s friendship network. Both network data and behavior data were recorded. We are interested in how a student’s attributes (e.g., gender) influence his/her friendship network and in turn affects his/her health behaviors (e.g., smoking). Such research interests can be addressed by a mediation analysis. Mediation analysis is a common framework for statistical analysis of the causal mechanism of the observed relationship between an independent variable and a dependent variable (Baron and Kenny, 1986; Hayes, 2009). It is a popular research discipline in epidemiology, psychology, sociology, and related fields (e.g., Fritz and MacKinnon, 2007; Richiardi et al., 2013). The object of a mediation analysis is to determine whether the relation between two variables is due, wholly, or in part, to a mediating variable that transmits the effect of an independent variable on the dependent variable (MacKinnon et al., 2007; MacKinnon, 2012). In classical mediation analysis, regression analysis is the widely used technique to study the relationship between mediator, independent and dependent variables.

Over the years, mediation analysis has achieved great progress along two equally important methodological lines: more reliable methods for hypothesis testing of mediation effects and new models for conducting mediation analysis under different contexts. Many methods have been proposed to test mediation effects in the frequentist framework (MacKinnon et al., 2002). The joint significance test approach (MacKinnon et al., 2004) and bootstrap approach (Preacher and Hayes, 2008) are recommended over the Sobel test because they have higher power and more accurate Type I error rates (MacKinnon et al., 2002; Preacher and Selig, 2012). Bayesian methods are also used in mediation analysis and they are expanding their applications in this field (Yuan and MacKinnon, 2009; Wang and Zhang, 2011; Enders et al., 2013; Wang and Preacher, 2015; Miočević et al., 2018). Bayesian methods can result in more efficient parameter estimates with prior information. In addition, Bayesian inference of mediation effect is based on its posterior distribution, but not the distribution of the test statistic. Thus, it is especially useful with small sample sizes when asymptotic distributions of test statistics are not available (Yuan and MacKinnon, 2009).

Despite the great progress in mediation analysis and the great substantive interests in the network mediation effect, little progress has been achieved in statistical methodologies for social network mediation analysis. As far as we know, there is no rigorous modeling framework and estimation methods available yet in the methodological literature. This is largely due to the mismatch of dimensions of network data and actor attribute data. For instance, in our empirical study with \( N \) students, each student’s network variable \( m \) consists of \( N - 1 \) dyads indicating whether he/she is a friend of any others or not. In total, there are \( \binom{N}{2} \) dyads for an undirected network. Both gender (independent variable) and smoking behavior (outcome variable), however, are univariate. Thus, the dimensions of variables do not match. Therefore, the regression analysis technique used in ordinary mediation analysis cannot be directly used in network mediation analysis. In addition, dyads in a network are not independent with each other. Particularly, two dyads sharing a common node depends on each other. Therefore, the
traditional mediation techniques such as regression analysis are not directly applicable in studying the mediation effect of a social network.

The goal of this study is thus to address current gaps in the literature by proposing a mediation model with a social network $M$ as the mediator and developing an estimation method to obtain statistical inference on the network mediation effect. To fulfill the goal, dealing with the conflict in dimensions of a network and a dependent and an independent variables is a crucial task. One possible solution is to reduce the dimension of a social network by extracting key information out of a network represented by quantities in fewer dimensions.

In the current study, the social network mediation analysis is built on the latent space model proposed by Hoff et al. (2002). Latent space modeling maps actors in a network into an unknown latent space with a few dimensions. The distance of two actors in the latent space predicts the existence of connections between them. Therefore, the latent positions of actors largely explain the observed network structure. In our mediation model, the latent positions are directly involved in the intervention process between the independent and dependent variables as actual “mediators.” Hence, our model contains both a measurement model explaining an observed network using actor’s positions in the latent space and a mediation model with the latent positions as actual mediators.

Because all the latent dimensions are equivalent to each other in terms of their association with the social network, it is hardly to name them. The intervening effect through an individual dimension is thus of little practical interest. Therefore, we would rather focus on the mediation effect of a network. Although the latent positions are not unique for a given network, we rigorously articulate that the proposed network mediation effect is well-defined given the number of dimensions of latent space. To estimate the model, we adopt a Bayesian estimation method, which is also used in simple mediation analysis (Yuan and MacKinnon 2009).

The remainder of this article is structured as follows. First, we briefly introduce the mediation model in the context of simple mediation analysis. Next, we present our network mediation model using a latent space modeling approach. We then define the network mediation effect and show that it is well-defined. After that, we explain the settings in a Bayesian estimation method for obtaining model parameter estimates and a simulation study is conducted to evaluate the performance of the Bayesian estimation method in estimating the network mediation model. A detailed empirical example is used to demonstrate the application of this model. We conclude the study with a discussion section on the current study and future works.

2 A Brief Introduction to Mediation Analysis

In this section, we briefly introduce the mediation analysis. The basic mediation framework involves a three-variable system in which an independent variable $X$ explains some of the variance of a dependent variable $Y$ via regression models, which is demonstrated by the diagrams in Figure 1. The diagram on top the panel of Figure 1 portrays the total relation between the independent and dependent variables and the regression equation is as follows:

$$Y_i = i_1 + cX_i + \varepsilon_{i,1}$$  \hspace{1cm} (1)

where the coefficient $c$ is the total effect of the independent variable $X$ on the dependent variable $Y$ without considering a third variable, $i_1$ is the intercept of the model and $\varepsilon_{i,1}$ is the error term associated with case $i$. The diagram on the bottom panel of Figure 1 is a mediation model with the variable $M$ as the mediator or intervening variable. To study the indirect effect of $X$ on $Y$ through an mediator...
variable $M$, one needs to regress $M$ on $X$ and then $Y$ on both $X$ and $M$,

Model 2: \[ M_i = i_2 + aX_i + \varepsilon_{i,2} \]  
Model 3: \[ Y_i = i_3 + bM_i + c'X_i + \varepsilon_{i,3} \]

where $i_2$ and $i_3$ are the intercepts of the two regression models. The parameter $a$ is the coefficient of the relation between $X$ and $M$, $b$ is the coefficient relating the mediator $M$ to $Y$ while controlling $X$, and $c'$ is the coefficient quantifying the relation between $X$ and $Y$ while controlling $M$. The two terms $\varepsilon_{i,2}$ and $\varepsilon_{i,3}$ are errors associated to case $i$ in these two models.

The indirect effect is the estimate of reduction in the predictor effect on the outcome variable when the mediator is included in the model, that is $\hat{c} - \hat{c}'$ given a sample. In general, it holds that $\hat{c} - \hat{c}' = \hat{a} \times \hat{b}$ when the three variables are linearly related to each other. The rationale behind this method is that mediation effect depends on the extent to which the predictor changes the mediator, represented by the coefficient $a$ and the extent to which the mediator affects the outcome variable, represented by the coefficient $b$.

3 Proposed Network Mediation Model

In this section, a network mediation model will be proposed based on the latent space model developed by [Hoff et al. (2002)](Hoff2002). The network mediation model is analogous to the simple mediation model, but the mediator becomes a social network. Due to the mismatch of the dimensions of network data and the dependent and independent variables, it is impossible to use the network data directly in the analysis. Instead, we use the latent space model to get the information on the latent positions of the actors, which act as actual mediators involved in the causal path. The newly proposed model thus consists of two parts: a latent space model explaining a social network using a few latent dimensions and a mediation model with scores in those dimension as mediators.

In the current study, we consider undirected binary social networks. Let $M$ be a social network with $N$ actors and $m_{ij}$ be the ties between actor $i$ and actor $j$. Therefore, $m_{ij} = 1$ indicates that actor $i$
and \( j \) are connected and 0, otherwise. Because the relation between two actors are undirected, it holds that \( m_{ij} = m_{ji} \) for all \( i \) and \( j \) from \( \{1, 2, \ldots, N\} \).

### 3.1 Latent space model

A latent space model assumes that each actor holds a position in a \( D \)-dimensional Euclidean space \( \mathbb{R}^D \) with \( D \) being a given natural number. Each axis of the Euclidean space represents a latent factor influencing the formation of connections between actors. The actor positions in the latent space are measured by a social network and the relation between them are modeled by a latent space model,

\[
\text{Latent space model:} \begin{cases} 
  m_{ij} & \sim \text{Bernoulli}(p_{ij}) \\
  \logit(p_{ij}) & = \alpha - |z_i - z_j|
\end{cases}
\]

where \( z_i = (z_{i1}, \ldots, z_{iD})^t \) and \( z_j = (z_{j1}, \ldots, z_{jD})^t \) are the latent positions of actors \( i \) and actor \( j \), which represent their locations in the underlying social space; \(|z_i - z_j|\) is the Euclidean distance between actors \( i \) and \( j \), such that

\[
|z_i - z_j| = \sqrt{\sum_{d=1}^{D}(z_{id} - z_{jd})^2}.
\]

Because of the use of Euclidean distance, the latent dimensions are equivalent with each other in predicting a social network. Therefore, the dimensions of the latent space have no specific meaning and we are unable to label them.

As in most statistical models, local independence is assumed in a latent space model. Specifically, the dyads are conditionally independent with each other given latent positions,

\[
P(m_{ik} = 1, m_{jk} = 1 | z_i, z_j, z_k) = P(m_{ik} = 1 | z_i, z_k)P(m_{jk} = 1 | z_j, z_k),
\]

which indicates that the dependence among dyads are totally explained by actor’s latent positions.

### 3.2 Mediation model

Actor positions in the latent social space are the underlying variables explaining the observed network. A part/full effect of the independent variable \( X \) on the outcome variable \( Y \) is explained by the latent positions forming the social network.

The second part of our model is thus a mediation model with multiple mediators,

\[
\text{Mediation model:} \begin{cases} 
  z_i & = i_1 + ax_i + \varepsilon_{i1} \\
  y_i & = i_2 + b^t z_i + c'x_i + \varepsilon_{i2}
\end{cases}
\]

where \((\cdot)^t\) is the transpose of a vector or matrix, \( z_i = (z_{i1}, \ldots, z_{iD})^t \) is the latent position of actor \( i \) in the Euclidean space \( \mathbb{R}^D \), \( i_1 = (i_{11}, i_{21}, \ldots, i_{1D})^t \) is the column vector of intercepts of the regression model from the independent variable to latent positions, \( i_2 \) is the intercept of the regression model from latent positions to the outcome variable \( Y \). In Equation (5), \( a \) and \( b \) are both column vectors of slope parameters from the independent variable \( X \) to mediators \( z \) and from \( z \) to the outcome variable \( Y \). Because the latent positions are in the \( D \)-dimensional Euclidean space, both \( a \) and \( b \) are of length \( D \), i.e., \( a = (a_1, a_2, \ldots, a_D)^t \) and \( b = (b_1, b_2, \ldots, b_D)^t \). In the model, \( c' \) is the direct effect of \( X \) on \( Y \) after controlling latent positions. \( \varepsilon_{i1} \) is a \( D \times 1 \) vector of residuals of case \( i \) when mediators are regressed on the independent variable and \( \varepsilon_{i2} \) is the residual of case \( i \) when the outcome variable is regressed on both the independent variable and mediators. Following the literature in mediation analysis,
both \( \varepsilon_1 \) and \( \varepsilon_2 \) are assumed to follow (multivariate) normal distributions with mean 0, and variances \( \text{var}(\varepsilon_1) = \text{diag}(\sigma^2_{1,d}, d = 1, 2, \ldots, D) \) and \( \text{var}(\varepsilon_2) = \sigma^2_2 \). Figure 2 is the path diagram of the model defined by Equations (4) and (5).

Figure 2: Path diagram of a network mediation model with \( D \) dimensions in the latent space

4 Network Mediation Effect

Given the dimension of a latent space \( D \), the latent position describing the intervening process are \( z = (z_1, z_2, \ldots, z_D)^t \). The coefficients from the independent variable to latent positions and from the latent positions to the outcome variable are \( a = (a_1, a_2, \ldots, a_D)^t \) and \( b = (b_1, b_2, \ldots, b_D)^t \). The direct effect from the independent variable to the dependent variable is \( c' \).

Because there are multiple latent variables joining the intervention process as in Figure 2, a primary question to ask is what kind of effects are well-defined and testable. In the newly proposed mediation model, there are potentially three types of effects of interests. The first one is the product of the coefficients in each dimension, i.e., \( a_d b_d \), which can be used to describe the “indirect effect” of a specific dimension. The second one is the direct effect of an independent variable \( X \) on a dependent variable \( Y \), represented by \( c' \) (Figure 2). The third effect that can be obtained using the current model formulation is the “indirect effect of a social network” as a whole. Because multiple latent mediators are orthogonal to each other and they are measured by a given social network, a candidate for the network mediation effect is

\[
\text{med} = a^t b = \sum_{d=1}^{D} a_d b_d, \quad (6)
\]

which is the number of units change on \( Y \) with respective to a unit change on \( X \) that relates to the observed network.

As discussed in Hoff et al. (2002), the latent distance \( d_{ij} = |z_i - z_j| \) directly predicts dyads in a social network. However, the Euclidean distance is invariant to the operations of translation, rotation, and reflection. Specifically, if a set of actors positions \( \{z_i\}_{i=1}^N \) are the optimal positions predicting a social network, then another set of positions \( \{z_i^*\}_{i=1}^N \) translated, rotated, or reflected from \( \{z_i\}_{i=1}^N \) are also
optimal because their pairwise distances are the same. However, regression coefficients in the network mediation model (Equation (5)) are not necessarily the same, when a different set of latent positions are used. Consequently, it is still unclear whether the three types of effects are uniquely determined or not. In the following, the identification issue of the three types of effects are discussed one by one.

4.1 Indirect effect in each dimension
In the proposed mediation model, the indirect effect of each latent dimension, i.e., $a_d b_d$, is not well-defined. When the latent positions are rotated around origin clockwise 90 degrees, the actors coordinates in the latent space shifts to lefts. The positions of actors becomes $z^*_i = (z_{i,2}, z_{i,2}, \cdots, z_{i,D}, z_{i,1})^t$. Therefore, the regression coefficients using $\{z^*_i\}_{i=1}^N$ are $a^* = (a_2, a_2, \cdots, a_D, a_1)^t$ and $b^* = (b_2, b_3, \cdots, b_D, b_1)$. Thus the indirect effect along each individual dimension varies when the latent positions are rotated around the origin.

In addition, all latent dimensions are measured by the same network, they are thus equivalent to each other in terms of the relations to the observed network. As a result, it is impossible to name the latent dimensions. Hence it is not meaningful to study the indirect effect of an individual dimension.

4.2 Indirect effect of a social network and direct effect
Nonetheless, it is still reasonable to study the mediation effect of a social network as a whole. In the current study, the network mediation effect is based on the inner product of coefficients,

$$med = a^t b = \sum_{d=1}^D a_d b_d.$$  

When $D = 1$, there are only one latent variables underlying the social network. The mediation effect is $a_1 b_1$, which is similar to the conventional mediation analysis with a latent mediator. When $D > 1$, there are multiple latent variables. The mediation effect defined in Equation (7) is the change on $Y$ resulted from a unit change on $X$ that passes through the social network.

Because there are multiple sets of latent positions maximizing the likelihood in predicting the social network, it is yet unknown whether the network mediation effect in Equation (7) is uniquely defined or not for a given social network. In the following, we are going to show that the quantity defined in Equation (7) is invariant to translation, rotation, and reflection given the latent dimension $D$.

4.2.1 Translation
The operation of translation is to slide a position to a new position. For instance, moving a position left or right, up or down are cases of translation. Given a constant column vector $t = (t_1, t_2, \cdots, t_D)^t$, let $T_t$ be a translation operator such that

$$z^*_i = T_t(z_i) = z_i + t,$$

for any actor $i = 1, \cdots, N$,  

where $z^*_i$ is the “new” position of actor $i$ after translation, and $t = (t_1, t_2, \cdots, t_D)^t$ is the difference between the two positions before and after the transition. When fitting the mediation model (Equation (5)) to the new latent positions, we have

$$\begin{align*}
{i^*_1} & = i_1^* + a^* x_i + \varepsilon_{i1}^* \\
i_2^* & = i_2^* + (b^*)^t z^*_i + \varepsilon_{i2}^* \\
i_3^* & = i_3^* + \varepsilon_{i3}^* \\cdots & \cdots \\
Y_i & = i_D^* + \varepsilon_{iD}^*
\end{align*}$$
Because \( z_i^* = z_i + t \), then the above regression models become

\[
\begin{align*}
\begin{cases}
z_i + t &= i_1^* + a^* x_i + \epsilon_{i1}^* \\
y_i &= i_2^* + (b^*)^t (z_i + t) + c^* x_i + \epsilon_{i2}^*
\end{cases}
\end{align*}
\tag{10}
\]

By comparing the two sets of coefficients in Equations (5) and (10), it is clear that

\[
a^* = a \text{ and } b^* = b \text{ and } c^* = c
\tag{11}
\]

Therefore, \( med^* = a^* b^* = a' b = med \). Hence, both the mediation effect defined by Equation (7) and the indirect effect (i.e, \( c' \)) are invariant to the operator of translation.

### 4.2.2 Rotation

Let \( R \) be a \( D \) by \( D \) rotation matrix. In general, a rotation matrix is an orthogonal matrix whose inverse and transpose matrices are the same, and its determinant is 1. Let \( R^{-1} \) and \( R^t \) be the inverse and transpose matrices of \( R \), respectively. Let \( z_i^* \) be the new position of actor \( i \) after translation, i.e., \( z_i^* = R z_i \) for actor \( i \).

To study the association between the independent variables, new latent positions, and the outcome variable, we fit the mediation model using the new positions such that

\[
\begin{align*}
\begin{cases}
R z_i &= i_1^* + a^* x_i + \epsilon_{i1}^* \\
y_i &= i_2^* + (b^*)^t z_i + c^* x_i + \epsilon_{i2}^*
\end{cases}
\end{align*}
\tag{12}
\]

Thus

\[
\begin{align*}
\begin{cases}
z_i &= R^{-1} i_1^* + R^{-1} a^* x_i + R^{-1} \epsilon_{i1}^* \\
y_i &= i_2^* + (R^t b^*)^t z_i + c^* x_i + \epsilon_{i2}^*
\end{cases}
\end{align*}
\tag{13}
\]

By comparing the coefficients in Equation (13) with those in Equation (5), we find,

\[
R^{-1} a^* = a \text{ and } R^t b^* = b \text{ and } c'^* = c
\tag{14}
\]

or equivalently using the fact that \( R^{-1} = R^t \),

\[
a^* = Ra \text{ and } b^* = Rb.
\tag{15}
\]

The mediation effect of the new latent positions is

\[
med^* = a^* b^* = (Ra)^t (Rb) = a' R^t R b = a' b = med
\tag{16}
\]

using the factor that the rotation matrix \( R \) is a orthogonal matrix such at \( R^t = R^{-1} \). Therefore, the mediation effects before and after translation of the latent positions through rotation are the same. The indirect effect also does not change.

### 4.2.3 Reflection

A reflection operator maps each position to a symmetry image about some hyperplane in \( \mathbb{R}^D \). The most imaginable planes are those with one coordinate being 0. Without generality, consider a plane formed by the first \( D - 1 \) axises and all points on this plane have \( z_D = 0 \). For convenience, we name this hyperplane \( P_D \). For a position \( z_i = (z_{i1}, z_{i2}, \cdots, z_{iD})^t \), its symmetric position about plane \( P_D \)
\( z_i^* = (z_{i,1}, z_{i,2}, \cdots, z_{i,D-1}, -z_{i,D}) \). Specifically, the first \( D - 1 \) coordinates do not change, and the last one reflects its sign.

When fitting the mediation model Equation (5) to the new position, the new coefficients \( a^*, b^* \), and \( c'^* \) are the same as those obtained using the original latent positions,

\[
a^* = a \text{ and } b^* = b \text{ and } c'^* = c'
\]

Thus, the mediation effect does not change before and after reflection.

For a general hyperplane \( P \), it can be transformed from \( P_D \) through a series of translations and rotations, which do not influence the the quantity \( a'b \) and \( c' \). As a consequence, the mediation effect defined as the inner product of coefficients (Equation (7)) and the direct effect \( c' \) do not change when the latent positions are reflected in general.

Based on the above articulation, both the mediation effect (Equation 7) of a social network and the direct effect are invariant to translation, rotation, and reflection of the latent positions. Therefore, they are well-defined.

The total effect\(^1\) is defined as,

\[
c = c' + a'b = c' + \sum_{d=1}^{D} a_d b_d.
\]

Throughout the remainder of this article, the analysis will focus on the mediation effect of a network as a whole, the indirect effect, and the total effect.

5 Model Estimation

To estimate the model, a Bayesian estimation method is used, which is also used in standard mediation analysis (Yuan and MacKinnon 2009; Wang and Zhang 2011; Enders et al. 2013; Wang and Preacher 2015; Miocević et al. 2018). Bayesian inference on parameters \( \theta \) is based on the posterior distribution given data \( x \) and priors distributions of model parameters (Gelman et al. 2014; Kruschke 2014),

\[
p(\theta|x) \propto p(x|\theta)p(\theta),
\]

where \( p(\theta|x) \) is the posterior distribution, \( p(x|\theta) \) is the likelihood, and \( p(\theta) \) is the joint prior distribution of model parameters.

Given the Euclidean space \( \mathbb{R}^D \), into which the actors in a network mapped, the likelihood functions are listed below,

\[
z_i \sim \text{MVN}(ax_i, \text{diag}(\sigma_i^2, d = 1, \cdots, D))
\]

\[
\text{logit} (m_{ij} = 1) = \alpha - |z_i - z_j|
\]

\[
y_i \sim \text{N}(c'x_i + b'z_i, \sigma_z^2).
\]

For the variance parameters of the latent positions and residual variance of the outcome variable \( Y \), inverse Gamma prior is used. Specifically, both \( \sigma_i^2 \) and \( \sigma_{1,d}^2 (d = 1, 2, \cdots, D) \) follow inverse Gamma distribution with both shape and position parameters 0.001. The regression coefficients \( a_d, b_d \) \((d = 1, \cdots, D)\) and indirect effect parameter \( c' \) and slope parameter in the latent space mode \( \alpha \) all take normal prior with mean 0 and standard deviation 1000.

\(^1\)The total effect is purely the sum of the indirect effect and direct effect. It may not be the same as if regressing \( Y \) on \( X \) directly. This is because the model complexity changes and the information used in estimating the model is also different.
Because the posterior distribution has no closed form, we thus use Markov Chain Monte Carlo method to draw samples of parameters from the posterior distribution. For each parameter, a total of 20,000 samples are drawn from its marginal posterior distribution. Samples for the mediation effect and the total effect parameters are computed using the samples of $a_d, b_d, c'$ through Equations (7) and (18). For each Markov Chain, the burn-in phrase is 6,000 iterations and summary statistics are computed based on the remaining part of the chain.

In both simulation and empirical studies, posterior means are used as point estimates of model parameters. The 95% credible intervals are reported and they are used to evaluate whether the parameter estimates are significant or not as done in the frequentest framework.

6 Simulation Study

In this section, we will conduct a simulation study to evaluate the performance of the proposed model and Bayesian estimation methods. We will first explain the simulation design and evaluation criteria, and then present the simulation results.

6.1 Simulation design

All the data sets are generated from the network mediation model in Equations (4) and (5). In the simulation, the independent variable is generated from the standard normal distribution, that is $X \sim \mathcal{N}(0,1)$. The dimensions of the latent space considered are 1, 2, and 3.

When $D = 1$, the mediation model has one mediator. Four values for the regression coefficients are used: $a = b = 0, .14, .39,$ and $.59$. These values correspond to 0, small (S), medium (M), and large coefficients (L), respectively (Fritz and MacKinnon 2007). When the dimension of the latent space $D > 1$, the coefficient values for $a$ and $b$ are scaled by $\sqrt{D}$, thus $a = b$ takes values from 0, $.14/\sqrt{D}, .39/\sqrt{D},$ and $.59/\sqrt{D}$. This parameter specification guarantees that the network mediation effect, i.e., $\sum_{d=1}^{D} a_d b_d$, is the same even the number of latent dimensions varies. The corresponding network mediation effects are thus 0, .0196, .1521, and .3481 in the population model. For the direct effect parameter $c'$, two values $c' = 0.14$ and $c' = 0$, are considered, corresponding to the partial and complete mediation, respectively.

All intercept parameters $(i_{1,1}, \cdots, i_{1,D})^t, i_2,$ and $\alpha$ are set to be 0 in the data generating model. To make both latent factors ($z = (z_1, \cdots, z_D)^t$) and the outcome variable ($Y$) to have unit variances, the residual variance of latent factors (mediators) is computed by

$$\sigma^2_{1,d} = \text{var}(\varepsilon_{i1,d}) = 1 - a^2_d \quad \text{for } d = 1, \cdots, D. \quad (20)$$

And the residual variance of the outcome variable is calculated as

$$\sigma^2_2 = \text{var}(\varepsilon_2) = 1 - (\sum_{d=1}^{D} a_d b_d + c')^2 - \sum_{d=1}^{D} b^2_d (1 - a^2_d). \quad (21)$$

It is important to note that both the latent factors $z_d(d = 1, 2, \cdots, D)$ and $Y$ have unit variances using the current setup of variance parameters. Therefore, the network mediation effect $a' b = \sum_{d=1}^{D} a_d b_d$ is the summation of $D$ standardized indirect effects. Because the network mediation effect is the summation of $D$ standardized indirect effect, it is thus not standardized. The sample sizes considered are 50, 100, 150, 200, 250, and 300 in the simulation.

Combining all the factors, there are $4 \times 3 \times 2 \times 6 = 144$ different conditions. For each condition, 500 data sets are generated and model parameters for the model in Equations (4) and (5) are obtained using the Bayesian estimation method introduced in the previous section.
6.2 Evaluation Criteria

Summary statistics are based on the remaining 14,000 Markov samples after burn-in. The posterior mean based on the samples is computed as

$$\hat{\theta} = \frac{1}{14000} \sum_{i=6001}^{20000} \theta^{(i)}. \quad (22)$$

Given a significance level $\alpha$, a posterior credible interval of $r$th replication is defined as interval $[L_r, R_r]$ such that

$$\frac{\#\{\theta^{(i)} : \theta^{(i)} < L_r\}}{14000} = \frac{\#\{\theta^{(i)} : \theta^{(i)} > R_r\}}{14000} = \alpha/2. \quad (23)$$

Let $\theta$ be an arbitrary parameter in the model to be estimated and also its population value. Let $\hat{\theta}_r$ and $[L_r, R_r]$ be the posterior mean and 95% credible interval from the $r$th replication ($r = 1, 2, \cdots, 500$). Let

$$\bar{\theta} = \frac{1}{500} \sum_{r=1}^{500} \hat{\theta}_r. \quad (24)$$

which is the average of parameter estimates across 500 replications.

The accuracy of parameter estimates is evaluated using “relative bias”, which is a ratio of bias (different between point estimate and true value of a parameter) to the true value in percentage,

$$\text{relative bias}_{\theta} = \begin{cases} \frac{\hat{\theta} - \theta}{|\theta|} \times 100\% & \text{if } \theta \neq 0 \\ (\hat{\theta} - \theta) \times 100\% & \text{otherwise}. \end{cases} \quad (25)$$

The root mean squared error based on the 500 replications is

$$\text{RMSE}_{\theta} = \sqrt{\frac{1}{500} \sum_{r=1}^{500} (\hat{\theta}_r - \theta)^2}. \quad (26)$$

Moreover, the coverage probability of Bayesian credible intervals are also reported, which is proportion of replications whose posterior credible intervals cover the true parameter value $\theta$,

$$\text{CR}_{\theta} = \frac{1}{500} \sum_{r=1}^{500} I(\theta \in [L_r, R_r]) \times 100$$

where $[L_r, R_r]$ is the 95% Bayesian credible interval. The coverage rate is often used to assess the validity of Bayesian credible intervals. A coverage rate close to the nominal one (i.e., 0.95) indicates the statistical inference based on the credible intervals is trustworthy.

6.3 Results

The primary goal of a network mediation analysis is to assess the intervention effect, the direct effect, and the total effect of a social network. The performance of the Bayesian estimation method is examined from three aspects: (1) accuracy of parameter estimates, (2) root mean squared error (RMSE), and (3) the coverage rates of Bayesian credible intervals. In the following, results with 1, 2 and 3 latent dimensions are reported in Table 1, 2, and 3, respectively.

When the latent space has two dimensions, the estimates of the mediation effect are biased less than 5% with a sample size 150 or large. With a sample size 100, there are several cases with relative
biases of the mediation effect larger than 10%. With a sample size 50, all estimates of mediation effect are larger than 10% except when both the direct and mediation effect are 0.

As expected, RMSE of all parameter estimates decreases when the sample size increases. The change is prominent between the sample size 50 and 100. When the sample size is as small as 50, the RMSE increases when the latent space has more dimensions.

The reported quantities are coverage rates in percentage, thus a coverage rate close to 95% is acceptable. In general, the coverage rates are in the acceptable range [92.5%, 97.5%]. When the true mediation effect is 0, its coverage rates are close to 100%, which are similar to those found in traditional mediation analysis (Yuan and MacKinnon, 2009).

In our simulation study, we conclude that we have to consider the sample size (node sizes) when we determines the number of dimension of latent spaces. Note that as the number of sample size increases, the coverage probability for $D = 1$ with large mediation effect decreases. In addition, as the dimension size increases, the overall performances for small sample size cases become worse. This implies that we have to set the dimension size to be small when our sample size is not large and increase the number of dimension as the sample size increases.

7 **Empirical Example**

The data were collected by the Lab for Big Data Methodology at the University of Notre Dame in 2017. The participants were 162 students of a four-year college in China. There were 90 female and 72 male students. Their average age was 21.64 years (SD=0.86). During the data collection, each student was asked to indicate whether other students were his/her friend or not. The data on friendship were recorded in a $N$ by $N$ matrix $M$, which was illustrated in Figure (3). Each dot represents a student and a gray line indicates the existence of friendship between two students. In addition to the information on the friendship, each student also reported whether he/she smoked cigarette or not. Among the 162 students, 43 students reported they had smoked during the past 30 days.

In the study, we are interested in how student’s gender impacts their friendship network and how the friendship network affects student’s smoking behavior in turn. To answer this question, we would
Table 1: Relative bias (%), RMSE and coverage probability of parameter estimates under the conditions with $D = 1$. Note that (1) a bold number in relative bias (%) is a relative bias larger than 10% and (2) a bold number in a coverage rate out of the range [92.5%, 97.5%].
| Par | TR | N=50 | RMSE | Coverage | Bias | RMSE | Coverage | Bias | RMSE | Coverage | Bias | RMSE | Coverage | Bias | RMSE | Coverage | Bias | RMSE | Coverage | Bias | RMSE | Coverage | Bias | RMSE | Coverage |
|-----|----|------|------|---------|------|------|---------|------|------|---------|------|------|---------|------|------|---------|------|------|---------|------|------|---------|------|------|---------|------|------|---------|
| $c^0$ | 0 | -4.22 | 0.93 | **92.0** | -0.94 | 0.29 | 97.57 | 0.52 | 0.08 | 95.8 | $c^0$ | 0.14 | -117.44 | 0.92 | **91.6** | -12.13 | 0.27 | 94.6 | -2.66 | 0.08 | 94.0 |
| med | 0 | 4.87 | 0.92 | 93.0 | 0.48 | 0.28 | **98.8** | -0.02 | 0.02 | **99.6** | med | 0 | 16.48 | 0.91 | 92.6 | 2.03 | 0.24 | **98.6** | -0.09 | 0.01 | **100.0** |
| tot | 0 | 0.65 | 0.15 | 95.2 | -0.41 | 0.10 | 96.8 | 0.49 | 0.08 | 95.4 | tot | 0.14 | 0.28 | 0.14 | 95.6 | 2.38 | 0.10 | 96.4 | -3.33 | 0.08 | 93.6 |
| $c^0$ | 0 | -13.52 | 1.15 | **92.2** | -2.41 | 0.34 | 97.57 | 0.52 | 0.08 | 95.4 | $c^0$ | 0.14 | -49.68 | 0.89 | **91.8** | 0.26 | 0.25 | 95.4 | -2.87 | 0.02 | 99.0 |
| med | 0.02 | 672.62 | 1.13 | 94.4 | 88.03 | 0.32 | **99.0** | 0.52 | 0.02 | **99.0** | med | 0.02 | 309.02 | 0.87 | 93.6 | -20.94 | 0.23 | **98.4** | -2.87 | 0.02 | **99.0** |
| tot | 0.02 | -3.43 | 0.15 | 94.0 | -32.32 | 0.10 | 95.4 | 1.61 | 0.08 | 95.0 | tot | 0.16 | -8.84 | 0.14 | 95.6 | -2.39 | 0.10 | 96.0 | -0.73 | 0.08 | 95.6 |
| $c^0$ | 0 | -27.28 | 1.30 | **89.6** | -1.10 | 0.26 | 95.2 | -0.27 | 0.08 | 97.2 | $c^0$ | 0.14 | -160.57 | 1.04 | **90.5** | -16.94 | 0.25 | 95.7 | 1.38 | 0.08 | 93.6 |
| med | 0.1568 | 174.37 | 1.30 | 91.4 | 6.52 | 0.25 | 96.0 | 1.13 | 0.05 | 97.2 | med | 0.1568 | 144.90 | 1.03 | **91.8** | 6.52 | 0.25 | 97.2 | 1.91 | 0.05 | 96.0 |
| tot | 0.1568 | 0.42 | 0.14 | 95.0 | -0.47 | 0.10 | 96.0 | -0.58 | 0.08 | 96.6 | tot | 0.2968 | 0.81 | 0.14 | 96.6 | -2.87 | 0.10 | 96.0 | 1.66 | 0.08 | 93.4 |
| $c^0$ | 0 | -24.09 | 0.96 | **91.0** | -3.83 | 0.25 | 95.0 | -0.50 | 0.08 | 95.0 | $c^0$ | 0.14 | -256.31 | 1.13 | **85.8** | -6.80 | 0.19 | 96.4 | -6.88 | 0.07 | 95.0 |
| med | 0.3528 | 68.79 | 0.96 | **90.8** | 10.99 | 0.25 | 95.0 | 0.91 | 0.07 | 95.4 | med | 0.3528 | 101.95 | 1.03 | **91.8** | 9.69 | 0.25 | 97.2 | 1.91 | 0.05 | 96.0 |
| tot | 0.3528 | 0.49 | 0.14 | 95.6 | 0.14 | 0.09 | 97.6 | -0.51 | 0.08 | 95.4 | tot | 0.4928 | 0.17 | 0.13 | 95.8 | 0.18 | 0.09 | 94.4 | 0.25 | 0.07 | 97.0 |

Table 2: Relative bias (%), RMSE and coverage probability of parameter estimates under the conditions with $D = 2$. Note that (1) a bold number in relative bias (%) is a relative bias larger than 10% and (2) a bold number in a coverage rate out of the range [92.5%, 97.5%].
Table 3: Relative bias (%), RMSE and coverage probability of parameter estimates under the conditions with \( D = 3 \). Note that (1) a bold number in relative bias (%) is a relative bias larger than 10% and (2) a bold number in a coverage rate out of the range [92.5\%, 97\%].
study the mediation effect of the friendship network in explaining the effect of “gender” on “smoking.”

A network mediation analysis was thus conducted using the newly proposed model in Equations (4) and (5). Because the outcome variable $Y$ is binary (0=not smoking, 1=smoking), latent variable analysis was used and it assumed that there was an underlying continuous variable $Y^*$ such that the binary outcome variable was obtained by dichotomizing it given a threshold,

\[
\text{Mediation model: } \begin{cases} 
  z_i = i_1 + a x_i + \varepsilon_{i1} \\
  y^*_i = b' z_i + c' x_i + \varepsilon_{i2}
\end{cases}
\]  

(27)

where $\varepsilon_{i2}$ follows the standard normal distribution. The mediation effect is

\[
\text{med} = a'b = \sum_{d=1}^{D} a_d b_d
\]

with $D$ being the number of latent dimensions.

In practical situations, however, it is hardly to know how many latent dimensions would be sufficient to explain the existing social relations. As a result, latent space models (Equation (4)) were fit to the binary friendship network in an exploratory manner varying the number of latent dimensions. The model was estimated using the R package “latentnet" (Krivitsky and Handcock [2017]). The false positive rates, false negative rates, rates of correct prediction, as well as BICs (Schwarz et al., 1978) were shown in Figure (4).

[Figure 4: (a) false positive rate, (b) false negative rate, (c) correct prediction rate, (d) Bayesian Information Criterion (BIC) of Latent Space Models fit to a Binary Friendship Network]

Based on plots (a), (b), and (c), better prediction was achieved with more latent dimensions. However, the model parsimony suffers. Considering also the model complexity, the optimal model was the one with smallest Bayesian Information Criterion (BIC) index. According to plot (d), it was most plausible to have 5 dimensions in the latent Euclidean space. Thus the network mediation model (Equations (27)) with $D = 5$ was fit to the friendship network and the model parameters estimates were shown in Table 4.

The estimated total effect (tot) was -2.17 with the 95% credible interval $[-2.982, -1.476]$ excluding 0, thus the total effect was significant. The estimated mediation effect (med) was -1.191 with the 95% credible interval $[-2.254, -0.417]$, it was also significant. Similarly, the estimated direct effect was -0.980 with the 95% credible interval $[-1.781, -0.065]$, which was also significant. This analysis result
implies that female student smoke significantly less than male students on average and gender influences individual’s friendship network and, in turn, the friendship network affects individual’s smoking behavior.

Table 4: Parameter estimates fitting the mediation model to the college friendship network

| D  | Par | Est  | 2.5%  | 97.5% | DIC |
|----|-----|------|-------|-------|-----|
|    | c'  | -0.980 | -1.781 | -0.065 |     |
| 5  | med | -1.191 | -2.254 | -0.417 | -4244 |
| tot | -2.17 | -2.982 | -1.476 |       |     |

8 Discussion

The purpose of this study was to develop a model to conduct social network mediation analysis and a Bayesian approach to obtain model parameter estimates. To deal with the conflict in dimensions between variables, a latent space model was used to extract underlying variables explaining the existence of an observed network. In latent space modeling, each actor holds a position in a Euclidean social space. The relative distance of two actors determines the propensity for them to be connected. The position of an actor represents his/her attributes predicting social relations with others. Such attributes may intervene the association of other actor attributes. Therefore, they are the actual variables involved in the intervention process. Although the latent space is invariant to rotation, translation, and projection, we rigorously showed that the proposed network mediation effect is well-defined.

To evaluate the modeling framework and Bayesian estimation method, we conducted a simulation study. According to the results from the simulation study, the Bayesian estimation method could provide accurate parameter estimates. With $D = 1$, the relative bias was less than 10% with a few exceptions under the conditions with a sample size as small as 50. With more dimensions in the latent space, more instances with biases larger 10% occurred for a given small sample size such as 50 or 100. When the sample size is above 100, the parameter estimates were accurate. As expected, the mean squared error decreased as the sample size increased, and they increased when the number of latent dimensions increased. The latter was due to the more complex of models with higher number of dimensions in the latent space. The coverage rate of 95% credible intervals are mostly in the good range from $[92.5\%, 97.5\%]$ with a few exceptions with the true mediation effect being 0, which was also observed in ordinary mediation analysis as discussed in Yuan and MacKinnon (2009). From our simulation studies, we can conclude that the sample size should be considered to determine the number of dimensions of latent spaces.

We used the newly developed model to analyze a college friendship network. We found that female student smoked significantly less than male students on average. A part of the gender difference in smoking behavior passed through the friendship network. Hence, gender influences individual’s friendship network and in turn, the friendship network affects individual’s smoking behavior. In addition, gender directly influences individual’s smoking behavior when the friendship network was controlled.

The effect of latent spaces on the probability of having edges between nodes is heavily influenced by the definition of latent spaces. In our future research, we would develop other network mediation analysis framework by adopting other specification in latent effects - for example, latent spaces based on principles of eigen-analysis (Hoff 2008). Our modeling framework also can be extended to the longitudinal frameworks (Jose 2016; Roth and MacKinnon 2012) and causal inferences (VanderWeele 2015).
References

Asendorpf, J. B. and Wilpers, S. (1998) Personality effects on social relationships. *Journal of Personality and Social Psychology, 74*, 1531–1544.

Baron, R. M. and Kenny, D. A. (1986) The moderator–mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of personality and social psychology, 51*, 1173.

Broman, C. L. (1993) Social relationships and health-related behavior. *Journal of Behavioral Medicine, 16*, 335–350.

Cacioppo, J. T. and Cacioppo, S. (2014) Social relationships and health: The toxic effects of perceived social isolation. *Social and personality psychology compass, 8*, 58–72.

Carrington, P. J., Scott, J. and Wasserman, S. (2005) *Models and methods in social network analysis*, vol. 28. Cambridge university press.

Choi, D. S., Wolfe, P. J. and Airoldi, E. M. (2012) Stochastic blockmodels with a growing number of classes. *Biometrika, 99*, 273–284.

Enders, C. K., Fairchild, A. J. and MacKinnon, D. P. (2013) A bayesian approach for estimating mediation effects with missing data. *Multivariate behavioral research, 48*, 340–369.

Flashman, J. (2012) Academic achievement and its impact on friend dynamics. *Sociology of education, 85*, 61–80.

Fritz, M. S. and MacKinnon, D. P. (2007) Required sample size to detect the mediated effect. *Psychological science, 18*, 233–239.

Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A. and Rubin, D. B. (2014) *Bayesian data analysis*, vol. 2. CRC press Boca Raton, FL.

Grunspan, D. Z., Wiggins, B. L. and Goodreau, S. M. (2014) Understanding classrooms through social network analysis: A primer for social network analysis in education research. *CBE — Life Sciences Education, 13*, 167–178.

Gurung, R., Sarason, B. and Sarason, I. (1997) Close personal relationships and health outcomes: A key to the role of social support. *Handbook of personal relationships: Theory, research and interventions (2nd ed) Chichester, UK: Wiley*, 547–573.

Harris, K. and Vazire, S. (2016) On friendship development and the big five personality traits. *Social and Personality Psychology Compass, 10*, 647–667.

Hayes, A. F. (2009) Beyond baron and kenny: Statistical mediation analysis in the new millennium. *Communication monographs, 76*, 408–420.

Hoff, P. (2008) Modeling homophily and stochastic equivalence in symmetric relational data. In *Advances in Neural Information Processing Systems*, 657–664.

Hoff, P. D., Raftery, A. E. and Handcock, M. S. (2002) Latent space approaches to social network analysis. *Journal of the american Statistical association, 97*, 1090–1098.
Holland, P. W., Laskey, K. B. and Leinhardt, S. (1983) Stochastic blockmodels: First steps. *Social networks, 5*, 109–137.

House, J. S., Landis, K. R. and Umberson, D. (1988) Social relationships and health. *Science, 241*, 540–545.

Jose, P. E. (2016) The merits of using longitudinal mediation. *Educational Psychologist, 51*, 331–341.

Krivitsky, P. N. and Handcock, M. S. (2017) latentnet: Latent Position and Cluster Models for Statistical Networks. The Statnet Project ([http://www.statnet.org](http://www.statnet.org)). URL: [https://CRAN.R-project.org/package=latentnet](https://CRAN.R-project.org/package=latentnet).

Kruschke, J. (2014) *Doing Bayesian data analysis: A tutorial with R, JAGS, and Stan*. Academic Press.

MacKinnon, D. (2012) *Introduction to statistical mediation analysis*. Routledge.

MacKinnon, D. P., Fairchild, A. J. and Fritz, M. S. (2007) Mediation analysis. *Annu. Rev. Psychol., 58*, 593–614.

MacKinnon, D. P., Lockwood, C. M., Hoffman, J. M., West, S. G. and Sheets, V. (2002) A comparison of methods to test mediation and other intervening variable effects. *Psychological methods, 7*, 83.

MacKinnon, D. P., Lockwood, C. M. and Williams, J. (2004) Confidence limits for the indirect effect: Distribution of the product and resampling methods. *Multivariate behavioral research, 39*, 99–128.

McCamish-Svensson, C., Samuelsson, G., Hagberg, B., Svensson, T. and Dehlin, O. (1999) Social relationships and health as predictors of life satisfaction in advanced old age: results from a swedish longitudinal study. *The International Journal of Aging and Human Development, 48*, 301–324.

McCrae, R. R., Martin, T. A., Hrebickova, M., Urbánek, T., Boomsma, D. I., Willemsen, G. and Costa, P. T. (2008) Personality trait similarity between spouses in four cultures. *Journal of personality, 76*, 1137–1164.

Miočević, M., Gonzalez, O., Valente, M. J. and MacKinnon, D. P. (2018) A tutorial in bayesian potential outcomes mediation analysis. *Structural Equation Modeling: A Multidisciplinary Journal, 25*, 121–136.

Newman, M. E., Watts, D. J. and Strogatz, S. H. (2002) Random graph models of social networks. *Proceedings of the National Academy of Sciences, 99*, 2566–2572.

Paul, S. and Chen, Y. (2016) Consistent community detection in multi-relational data through restricted multi-layer stochastic blockmodel. *Electronic Journal of Statistics, 3807–3870*.

Preacher, K. J. and Hayes, A. F. (2008) Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. *Behavior research methods, 40*, 879–891.

Preacher, K. J. and Selig, J. P. (2012) Advantages of monte carlo confidence intervals for indirect effects. *Communication Methods and Measures, 6*, 77–98.

Reitzel, L. R., Kendzor, D. E., Castro, Y., Cao, Y., Businelle, M. S., Mazas, C. A., Cofta-Woerpel, L., Li, Y., Cinciripini, P. M., Ahluwalia, J. S. et al. (2012) The relation between social cohesion and smoking cessation among black smokers, and the potential role of psychosocial mediators. *Annals of Behavioral Medicine, 45*, 249–257.
Richiardi, L., Bellocco, R. and Zugna, D. (2013) Mediation analysis in epidemiology: methods, interpretation and bias. *International journal of epidemiology, 42*, 1511–1519.

Roth, D. L. and MacKinnon, D. P. (2012) Mediation analysis with longitudinal data. In *Longitudinal data analysis: A practical guide for researchers in aging, health, and social sciences* (eds. J. T. Newsom, R. N. Jones and S. M. Hofer), 181–216. Taylor & Francis Group.

Schwarz, G. et al. (1978) Estimating the dimension of a model. *The annals of statistics, 6*, 461–464.

Seeman, T. (2001) *How do others get under our skin? Social relationships and health*. Oxford University Press.

Umberson, D., Crosnoe, R. and Reczek, C. (2010) Social relationships and health behavior across the life course. *Annual review of sociology, 36*, 139–157.

VanderWeele, T. J. (2015) *Explanation in causal inference : methods for mediation and interaction*. Oxford University Press.

Waldinger, R. J., Cohen, S., Schulz, M. S. and Crowell, J. A. (2015) Security of attachment to spouses in late life: Concurrent and prospective links with cognitive and emotional well-being. *Clinical Psychological Science, 3*, 516–529.

Wang, L. and Preacher, K. J. (2015) Moderated mediation analysis using bayesian methods. *Structural Equation Modeling: A Multidisciplinary Journal, 22*, 249–263.

Wang, L. and Zhang, Z. (2011) Estimating and testing mediation effects with censored data. *Structural Equation Modeling, 18*, 18–34.

Wasserman, S. and Faust, K. (1994) *Social network analysis: Methods and applications*, vol. 8. Cambridge university press.

Westaby, J. D., Pfaff, D. L. and Redding, N. (2014) Psychology and social networks: A dynamic network theory perspective. *American Psychologist, 69*, 269.

Yang, J., McAuley, J. and Leskovec, J. (2013) Community detection in networks with node attributes. In *Data Mining (ICDM), 2013 IEEE 13th international conference on*, 1151–1156. IEEE.

Yuan, Y. and MacKinnon, D. P. (2009) Bayesian mediation analysis. *Psychological methods, 14*, 301.

Zhao, Y., Levina, E., Zhu, J. et al. (2012) Consistency of community detection in networks under degree-corrected stochastic block models. *The Annals of Statistics, 40*, 2266–2292.