I. INTRODUCTION

The study of non classical correlations has not only turned out to be an important tool in probing the basic features which make a quantum system different from a classical system, but has also provided potential resources for future quantum technologies. The non classical correlations can be quantified in many ways. The celebrated Bell inequalities [1] serve as a test for the local realism. Quantum steering [2] allows allows one party to change the state of the other by local measurements. It emerges from our work that the intervening noise would be the observable $\hat{Q}$ at different times $t_i$ and $t_j$; similar conclusions follow.

Temporal quantum correlations, which exist between different measurements made on a single system at different times, have attracted lot of attention in recent years. Popular among these are the Leggett-Garg inequalities (LGIs). Leggett-Garg inequality, in its different forms, has been studied in various systems and has received a lot of attention both from the theoretical [17–28] and experimental [29–35] fronts. Leggett-Garg inequalities codify two important macroscopic notions: macrorealism and non-invasive measurability [21, 23]. Macrorealism means that a system, which has available to it two or more macroscopically distinct states, must be in one of these states at any given time. Non-invasive measurability means that the act of measurement reveals the state of the system without disturbing its future dynamics. Both these assumptions are not respected by quantum systems; the superposition principle violates the first and the collapse of the wavefunction under measurement defies the second. With these two assumptions, the simplest form of LG inequality in terms of the LG parameter

$$K_3 = C_{01} + C_{12} - C_{02},$$

is given by $-3 \leq K_3 \leq 1$. Here $C_{ij} = \langle Q(t_i)Q(t_j) \rangle$ is the two time correlation for the dichotomic observable $Q(t) = \pm 1$.

Suppose Alice and Bob measure observables $\hat{A}$ and $\hat{B}$, obtaining outcomes $a, b \in \{\pm 1\}$, then, for any input state $\rho$, one finds after straightforward calculation that:

$$P_{ab|\hat{A}(t_i)\hat{B}(t_j)} = \text{Tr} \left( \frac{1 + b\hat{B}(t_j) + a\hat{A}(t_i) + a\hat{A}(t_i)}{2} \right)$$

$$= \frac{1}{4} + \frac{a}{4} \text{Tr}(\hat{A}(t_i)\rho) + \frac{b}{8} \text{Tr}(\hat{B}(t_j)\rho) + \frac{ab}{8} \text{Tr}(\{\hat{A}(t_i), \hat{B}(t_j)\}\rho) + \frac{b}{8} \text{Tr}(\hat{A}(t_i)\hat{B}(t_j)\hat{A}\rho),$$

from which it follows that the correlator $C_{ij} = \langle \hat{A}(t_i)\hat{B}(t_j) \rangle = \sum_{a,b} ab \langle \hat{A}(t_i)\hat{B}(t_j) \rangle = \frac{1}{2} \langle [\hat{A}(t_i), \hat{B}(t_j)] \rangle = \hat{A}(t_i) \cdot \hat{B}(t_j)$, where $\hat{A}(t) = \hat{A}(t) \cdot \hat{\sigma}$ and $\hat{B}(t) = \hat{B}(t) \cdot \hat{\sigma}$ [33, 37], where $\sigma$ are the Pauli matrices. Thus, the correlators $C_{ij}$ are independent of the input state, if the two measurements are projective. In the context of Leggett-Garg inequalities, $\hat{A}$ and $\hat{B}$ would be the observable $\hat{Q}$ at different times $t_i$ and $t_j$; similar conclusions follow.

It emerges from our work that the intervening noise between two measurements is relevant for the evolution of the LG parameter. This can be understood equally well by absorbing the noise into the measurements, which can then be regarded as a noise-induced POVM [38], and no longer projective measurements.

In this work, we study the violations of the LG inequality in noise channels like Random Telegraph Noise (RTN) and Ornstein-Uhlenbeck Noise (OUN) by using appropriate Kraus operators in the two cases. The resulting dynamics of the system can be Markovian or non-Markovian depending on the absence or presence of memory effects. The transition between these two regimes is sensitive to the channel parameters. Efforts
have been made to look at the characteristic of non-Markovian behavior from the quantum correlation perspective \[39,40\]. A sufficient but not necessary measure for non-Markovianity in terms of a temporal steerable weight is given in \[40\]. Thus temporal correlations seem to be intimately related to the non-Markovian nature of the system. We find a similar role of non-Markovianity with a given family of noise, say RTN or OUN.

The plan of this work is as follows: In Sec. (II A) we give a general formalism of constructing the two-time correlations using Kraus operators, while Sec. (II B) is devoted to a brief description of the RTN and OUN models and in Sec. (II C), the generalized measurement settings used in the work is discussed. We construct the LG system. We find a similar role of non-Markovianity with intimately related to the non-Markovian nature of the probability of obtaining outcome \(a\) at time \(t_i\), is given by

\[
q^{(b_t_j|a_t_i)} = Tr\left(\Pi^b \sum_{\nu,\mu} K_{\nu}(t_j - t_i) \Pi^a K_{\mu}^\dagger(t_i) \rho(0) K_{\mu}^\dagger(t_i) \Pi^a \right),
\]

(5)

Therefore, a generic term in the right hand side of Eq.(3) becomes

\[
p^{(a_t_i)}q^{(b_t_j|a_t_i)} = Tr\left(\Pi^b \sum_{\nu} K_{\nu}(t_j - t_i) \Pi^a \times K_{\mu}(t_i) \rho(0) K_{\mu}^\dagger(t_i) \Pi^a \times K_{\nu}^\dagger(t_j - t_i) \right).
\]

(6)

Using \(\Pi^+ = \Pi - \Pi^-\), after some algebra, the two time correlations turn out to be

\[
C_{ij} = 1 - 2p^{(t_1)} - 2p^{(t_2)} + 4Re[g(t_1, t_2)],
\]

(7)

where

\[
g(t_j, t_i) = Tr\left(\Pi^+ \sum_{\nu} K_{\nu}(t_j - t_i) \Pi^+ \rho(t_i) K_{\nu}^\dagger(t_j - t_i) \right).\]

(8)

II. BRIEF INTRODUCTION OF CONCEPTS

Here we provide a brief introduction to Leggett-Garg inequality, the noise models used in the work and the generalized measurement scheme employed.

A. Leggett-Garg inequality

The two time correlations \(C_{ij}\) appearing in Eq. (1) can be written in terms of the conditional probabilities as

\[
C_{ij} = p^{(a_t_i})q^{(b_t_j|a_t_i)} - p^{(a_t_i)}q^{(b_t_j|a_t_i)}
\]

where \(p^{(a_t_i)}\) is the probability of obtaining the result \(a = \pm 1\) at \(t_i\), and \(q^{(b_t_j|a_t_i)}\) is the conditional probability of getting result \(b = \pm 1\) at time \(t_j\), given that result \(a = \pm 1\) was obtained at \(t_i\).

We calculate the two time correlations \(C_{ij}\) using the Kraus operator formalism \[41\]. It is clear that the probability of obtaining outcome 'a' at time \(t_i\) is

\[
p^{(a_t_i)} = \frac{Tr\{\Pi^a \rho(t_i)\}}{Tr\{\Pi^a \rho(t_i)\}} = \frac{\Pi^a \sum_{\mu} K_{\mu}(t_i) \rho(0) K_{\mu}^\dagger(t_i) \Pi^a}{p^{(a_t_i)}}.
\]

(4)

this state evolves until \(t_j\), when the state of the system looks like \(\sum_{\nu} K_{\nu}(t_j - t_j) p^{(i_t)} K_{\nu}^\dagger(t_j - t_i)\), so that the probability of obtaining outcome \(b\) at time \(t_j\), given that \(a\) was obtained at time \(t_i\), is given by

\[
q^{(b_t_j|a_t_i)} = \frac{Tr\left(\Pi^b \sum_{\nu,\mu} K_{\nu}(t_j - t_i) \Pi^a K_{\mu}^\dagger(t_i) \rho(0) K_{\mu}^\dagger(t_i) \Pi^a \right),}{p^{(a_t_i)}}
\]

(5)

Therefore, a generic term in the right hand side of Eq.(3) becomes

\[
p^{(a_t_i)}q^{(b_t_j|a_t_i)} = Tr\left(\Pi^b \sum_{\nu} K_{\nu}(t_j - t_i) \Pi^a \times K_{\mu}(t_i) \rho(0) K_{\mu}^\dagger(t_i) \Pi^a \times K_{\nu}^\dagger(t_j - t_i) \right).
\]

(6)

Using \(\Pi^+ = \Pi - \Pi^-\), after some algebra, the two time correlations turn out to be

\[
C_{ij} = 1 - 2p^{(t_1)} - 2p^{(t_2)} + 4Re[g(t_1, t_2)],
\]

(7)

where

\[
g(t_j, t_i) = Tr\left(\Pi^+ \sum_{\nu} K_{\nu}(t_j - t_i) \Pi^+ \rho(t_i) K_{\nu}^\dagger(t_j - t_i) \right).\]

(8)

B. RTN and OUN noise models

Consider a system in state \(\rho\) and completely uncorrelated with its environment at time \(t = 0\). Both system and environment evolve unitarily and subsequently get entangled. Such noisy processes are described by the linear maps \[41\]

\[
\Phi_t(\rho) = \sum_n K_n^\dagger(\rho) K_n(0),
\]

where \(\sum_n K_n^\dagger(\rho) K_n = 1\) ensures the conservation of probabilities.

In \[43\] a two level quantum system interacting with the environment having the properties of random telegraph signal noise (RTN) was studied. The dynamical map is described by the following master equation

\[
\frac{d\rho_s(t)}{dt} = K\mathcal{L}\rho_s(t).
\]

(10)

The action of \(K\) operator is defined by \(K\psi = \int_0^t k(t-t')\psi(t')dt'\), where \(k(t-t')\psi(t')\), the kernel function, determines the type of memory in the environment. The dynamics of Eq. (10) is often studied in the context of the following time dependent Hamiltonian

\[
H(t) = \sum_k \Gamma_k(t)\sigma_k.
\]

(11)

The von-Neumann equation \(\dot{\rho}_s(t) = (1/i\hbar)[H(t), \rho_s(t)]\), immediately leads to the solution

\[
\rho_s(t) = \rho_s(0) - i \int_0^t \sum_k \Gamma_k(t')\sigma_k\rho_s(t')dt'.
\]

(12)
Putting this back in the von-Neumann equation, we have the following master equation:

$$\dot{\rho}_s(t) = -\int_0^t e^{-(t-t')/\tau_k} a_k^\dagger \sigma_k [\sigma_k, [\sigma_k, \rho_s(t)]] dt', \quad (13)$$

such that the correlation function of the random variable $\Gamma(t)$ is given by $\langle \Gamma_1(t)\Gamma_k(t') \rangle = a_k^2 e^{-(t-t')/\tau_k}$. It was shown in [43] that Eq. (13) preserves complete positivity (CP) if two of the $a_k$’s are zero. This dynamical process is completely positive dephasing with a colored noise, such that

$$\rho_s(t) = \sum_n K_n(t) \rho_s(0) K_n^\dagger(t), \quad (14)$$

where the Kraus operators $K_n(t)$ are given by

$$K_1(\nu) = \sqrt{\frac{1 + \Lambda(\nu)}{2}} I, \quad K_2(\nu) = \sqrt{\frac{1 - \Lambda(\nu)}{2}} \sigma_z. \quad (15)$$

Here $\Lambda(\nu) = e^{-\nu} \left[ \cos(\mu \nu) + \frac{\sin(\mu \nu)}{\mu} \right], \mu = \sqrt{(4\alpha \tau)^2 - 1}$ and $\nu = \frac{\tau}{2\tau} = \gamma t$ is a dimensionless parameter. One can show that the dephasing dynamics governed by the operators in Eq. (15) is non-Markovian if $\frac{d\Lambda(\nu)}{d\nu} > 0$ [44].

Ornstein-Uhlenbeck (OUN) is stationary, Gaussian and, in general, non-Markovian process with a well defined Markov limit. It is a prototypical example of a noisy relaxation process. The Kraus operators for the OUN noise are

$$\tilde{K}_1 = |0\rangle \langle 0| + q(t) |1\rangle \langle 1| \quad \tilde{K}_2 = \sqrt{1 - q^2(t)} |1\rangle \langle 1|, \quad (16)$$

where $q(t) = \exp \left[ -\frac{\nu}{2} (t + \frac{e^{-\gamma t} - 1}{\gamma}) \right]$. Here, $\gamma$ specifies the noise bandwidth and $\Gamma$ is the effective relaxation time.

### C. Measurement settings

In order to develop the two time correlations in the context of general measurement settings, we choose the measurement such that the system is first rotated by the following unitary operated at time $t$

$$R = \left( \begin{array}{cc} \cos(\theta/2) & e^{i\phi} \sin(\theta/2) \\ -e^{-i\phi} \sin(\theta/2) & \cos(\theta/2) \end{array} \right),$$

such that $-\pi \leq \theta < \pi$; $\pi/2 \leq \phi \leq \pi/2$ [45]. Next, we take the projection on $\sigma_z$ and finally perform a back rotation $R^\dagger$. Thus the complete operator $O = R^\dagger \sigma_z R$ becomes

$$O = \left( \begin{array}{cc} \cos(\theta) & e^{i\phi} \sin(\theta) \\ -e^{-i\phi} \sin(\theta) & -\cos(\theta) \end{array} \right). \quad (17)$$

One can write the spectral decomposition $O = |\psi_+\rangle \langle \psi_+| - |\psi_-\rangle \langle \psi_-|$, with

$$|\psi_{\pm}\rangle = \frac{\cos \theta \pm 1}{\sqrt{\sin^2 \theta + (\cos \theta \pm 1)^2}} e^{i\phi}, \quad (18)$$

For $\theta = \pi/2$ and $\phi = 0$, the operator $O$ reduces to Pauli operator $\sigma_x$ and the basis $|\psi_{\pm}\rangle = |\pm\rangle$, with the usual notation $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$.

### III. LGI UNDER THE INFLUENCE OF RTN AND OUN

Leggett-Garg inequality for a qubit under unitary evolution can be constructed easily. For observable $\hat{Q} = \sigma_x$, and the time evolution governed by the Hamiltonian $\hat{H} = \Omega \sigma_z/2$, one can show that the two time correlations turn out to be $C(t_i, t_j) = \cos(\Omega (t_j - t_i))$. Therefore, under the equal time assumption $t_j - t_i = \Delta t$, the LGI becomes

$$K_3^{\text{unitary}} = 2 \cos(\Omega \Delta t) - \cos(2\Omega \Delta t). \quad (19)$$

A violation of LGI would mean $K_3 > 1$.

We will now formulate the Leggett-Garg inequality for non-unitary time evolution of the qubit system. Specifically, we consider the evolution under the RTN and OUN channels discussed in Sec. [45]. The Kraus operators for

![Graph](image-url)

**FIG. 1.** (color online) RTN: Depicting LG parameter $K_3$, as defined in Eq. 23. (a) Non-Markovian: Parameters used are $a = 0.05$, $\gamma = \frac{5}{2\tau} = 0.001$, so that $\tau = 500$, therefore $\sigma t = 25 > 0.25$. Also, $\mu \approx 100$ (b) Markovian: Parameters used are $a = 0.05$, $\gamma = 0.5$ so that $\sigma t = 0.025 < 0.25$ and $\mu \approx \sqrt{-1}$.}
Here $K_\tau = 0$ with respect to $\tau$, the constant time between two measurements. (a) Non-Markovian: Parameters used are $\gamma = 0.01$, $\Gamma = 0.1$. (b) Markovian regime with $\gamma = 1.0$, $\gamma = 10^2$ and $\Gamma = 0.1$.

**RTN noise** are given by

\[
K_1 = \sqrt{\frac{1 + \Lambda(\nu)}{2} [0\langle 0] + 1\langle 1] = k_+(1 0 1),
\]

\[
K_2 = \sqrt{\frac{1 - \Lambda(\nu)}{2} [0\langle 0] - 1\langle 1] = k_-(1 0 0).
\]

Here $k_+ = \sqrt{\frac{1 + \Lambda(\nu)}{2}}$, $k_- = \sqrt{\frac{1 - \Lambda(\nu)}{2}}$ and $\nu = \gamma t$ is a dimensionless parameter. Consider a general state of a two level system at time $t_0 = 0$ given by

\[
\rho(0) = \begin{pmatrix} |\alpha|^2 & \alpha \beta^* \\ \alpha^* \beta & |\beta|^2 \end{pmatrix},
\]

with $|\alpha|^2 + |\beta|^2 = 1$. The state at some later time $t_1 > 0$ will be

\[
\rho(t_1) = K_1(t_1)\rho(0)K_1^+ + K_2(t_1)\rho(0)K_2^+(t_1),
\]

\[
= \begin{pmatrix} |\alpha|^2 & \alpha \beta^* \Lambda(t_1) \\ \alpha^* \beta \Lambda(t_1) & |\beta|^2 \end{pmatrix}.
\]

At a later time $t_2 > t_1$, $\rho(t_1)$ becomes

\[
\rho(t_2) = K_1(t_2 - t_1)\rho(t_1)K_1^+(t_2 - t_1)
\]

\[
+ K_2(t_2 - t_1)\rho(t_1)K_2^+(t_2 - t_1),
\]

\[
= \begin{pmatrix} |\alpha|^2 & \alpha \beta \Lambda(t_1) \Lambda(t_2 - t_1) \\ \alpha^* \beta \Lambda(t_1) \Lambda(t_2 - t_1) & |\beta|^2 \end{pmatrix}.
\]

The dichotomic operator $O$ defined in Eq. (17) can be written as $O = \Pi^+ - \Pi^-$ with $\Pi^\pm = |\nu_\pm\rangle\langle \nu_\pm|$ being the projectors corresponding to the outcome $\pm 1$. Using the formalism sketched in Sec. (II A), the two time correlations, under the evolution governed by RTN, become:

\[
C_{01} = \cos^2 \theta + \sin^2 \theta \Lambda(t_1 - t_0)
\]

\[
C_{12} = \cos^2 \theta + \sin^2 \theta \Lambda(t_2 - t_1)
\]

\[
C_{02} = \cos^2 \theta + \sin^2 \theta \Lambda(t_2 - t_0)
\]

Similar structure of two time correlations is found for OU/N noise with $\Lambda(t)$ replaced by $q(t)$. It is worth noting that the two time correlations remain state independent under the noisy evolution. That the correlation $C_{12}$ does not depend on the parameter $\Lambda(t_1 - t_0)$ is a consequence of this fact. With above two time correlations, the LG parameter becomes:

\[
K_3(\Delta t, \theta) = C_{01} + C_{12} - C_{02},
\]

\[
= \cos^2 \theta + \sin^2 \theta \left[ \Lambda(t_1 - t_0) + \Lambda(t_2 - t_1) - \Lambda(t_2 - t_0) \right].
\]

In this work, we are going to assume equal time measurements, $t_2 - t_1 = t_1 - t_0 = \Delta t$. For the state independent condition, $\Lambda = 1 (q = 1)$, all the two time correlations are equal to 1, yielding $K_3 = 1$. Therefore, in this model, the particular structure of the Kraus operators is not only responsible for the state evolution, but also causes the violation of LG inequality. For $\theta = \pi/2$, we have the following simple form of the LG parameter:

\[
K_3(\Delta t, \theta = \pi/2) = 2\Lambda(\Delta t) - \Lambda(2\Delta t).
\]

As discussed in Sec. (IV), it turns out that this form of LG parameter obtained for $\theta = \pi/2$ is very special. It corresponds to the case of maximum violation of LGI.

**COMPLEMENTARY FORMS OF LGI**

One can exploit the symmetry properties of LGIs and derive further inequalities by redefining the observable independently at various times. For example, a redefinition of the observable $Q(t_2) \rightarrow -Q(t_2)$, leads to the inequality $-3 \leq K'_3 \leq 1$, with $K'_3 = -C_{01} - C_{12} - C_{02}$. Using Eq. (21) one can write

\[
K'_3(\Delta t, \theta = \pi/2) = -2\Lambda(\Delta t) - \Lambda(2\Delta t).
\]

The two forms of LG parameter, that is, $K_3$ and $K'_3$ show complementary behavior in the sense that one shows violations in regions in which other does not, so that together both cover the entire parameter space [21].
IV. RESULTS AND DISCUSSION

Figure 1 depicts the evolution of the LG parameter, Eq. (23), under RTN noise both for Markovian and non-Markovian regimes. Violation are observed in non-Markovian regime suggesting that the non-Markovian behavior enhances the quantumness of the system. This is consistent with earlier findings [46], where LGI in a different set up was studied. Figure 2 shows the same quantity for the case of OUN noise model. Again, we find that the violation of LGI occurs in the non-Markovian regime. This is consistent with our understanding that in the case of Markovian phenomena, information goes out of the system and is lost in the environment; whereas in the non-Markovian scenario, there is a possibility of the system dynamics getting updated by the information coming back from the environment, resulting in the signature recurrent behavior.

In the large $\gamma$ limit, Fig. 2, the LGI is not violated. This is in accord with the fact that in this limit, the OUN reduces to the standard classical white noise. We also get an understanding in what sense the modified OUN could be considered as a non-Markovian noise; it allows for the violation of the LG parameter. This is significant in that the standard well known signatures of non-Markovianity, such as the Bruer measure [47] and RHP [48], when applied on the modified OUN are unable to discern its non-Markovian behavior [49, 50].

Figures 3 and 4 exhibit the complementary behavior of the two forms of the LG parameter, $K_3$ and $K'_3$, defined by Eqs. (23) and (24), respectively. A similar behavior is seen for a qubit undergoing unitary evolution [21].

The parameter $\mu$ in the expression of $\Lambda(\nu)$ is important in differentiating the Markovian and non-Markovian regimes. Depending on whether $\mu$ is (real) imaginary (say $\mu = i\mu_0$, $\mu_0$ is real), the evolution is (non-)Markovian. This implies the following forms of $\Lambda(\nu)$ in the two domains:

\[
\Lambda(\nu) = \begin{cases} 
  e^{-\nu} \left[ \cosh(\mu_0 \nu) + \frac{\sinh(\mu_0 \nu)}{\mu_0} \right], & \text{Markovian} \\
  e^{-\nu} \left[ \cos(\mu \nu) + \frac{\sin(\mu \nu)}{\mu} \right], & \text{non-Markovian}
\end{cases}
\]

Therefore the LG parameter, Eq. (23), in the two regimes becomes (with $\nu = \gamma \Delta t$)


The existence of the extrema of function $K_3$ demands the following conditions in the two regimes

\[
2e^{-\nu} \cosh(\mu_0 \nu) = 1 \quad \text{Markovian,} \tag{26}
\]

\[
2e^{-\nu} \cos(\mu \nu) = 1 \quad \text{non-Markovian.} \tag{27}
\]

A graphical representation of these equations is given in Figs. 5 (a) and (b), respectively. For the Markovian case, $\mu$ is imaginary. It is clear that Eq. (26) has no solution for $|\mu| = \mu_0 > 1$. 

Figure (6) shows the variation of the maximum of $K_3$ with respect to $\Delta t$ for different values of the parameter $\mu$ in the non-Markovian regime. One can see that the number of violations increases as $\mu$ is increased. Since

\[
\mu = \sqrt{\left(\frac{2a}{\gamma}\right)^2 - 1}, \quad \text{where} \quad a \text{ denotes the strength of the }
\]

system environment coupling; increasing $\mu$ implies the increase in the system-reservoir coupling. One can therefore say, that the violations of LGI increase as a function of the system-environment coupling, indicating stronger entanglement between the system and its environment.
The expression of LG parameter for the OUN noise turns out to be
\[
K_3 = 2 \exp\left[\frac{1}{2} \left(\Delta t \pm \frac{e^{-\gamma\Delta t} - 1}{\gamma}\right) - 1\right] - \exp\left[\frac{1}{2} \left(2\Delta t \pm \frac{e^{-2\gamma\Delta t} - 1}{\gamma}\right)\right].
\] (28)

Here, \(\Delta t\) as defined above, is the equal time separation between the successive measurements made on the system. The LG function given by Eq. (28) is plotted in Fig. (3). The Markovian limit of OUN is obtained by taking sufficiently large values of the bandwidth \(\gamma\). One can see that the violation is sustained for longer time in non-Markovian case. This is consistent with what was seen in the context of spatial quantum correlations for evolution under these noises [49].

Figure (7) depicts \(F(\Delta t)\) as a function of \(\Delta t\). The condition for the existence of maximum is satisfied when \(F(\Delta t) = 1\). There is only one solution in this case for all values of \(\gamma\), unlike the RTN scenario. This thus sheds light into the recurrent versus non-recurrent behavior of the dynamics for RTN and OUN, respectively.

We now investigate the behavior of the LG parameter in Eq. (22) with the general two time correlations given by Eq. (21). The LG parameter depends on the measurement variables \(\theta\) apart from the function \(\Lambda(t)\) for RTN and \(q(t)\) for the OUN case. To generate all possible measurements, we constrain the measurement variables to \(-\pi \leq \theta \leq \pi\). Figure (8) gives the variation of the LG parameter with respect to the time separation \(\Delta t\) and the measurement angle \(\theta\) for RTN noise model, in the non-Markovian regime. It is clear that the LG parameter attains its maximum at the detector setting \(\theta = \pm \pi/2\). A similar feature is found in the case of OUN noise model in Fig. (9), with the difference that the LG parameter shows no recurrence behavior, in time. This feature of the maximum of \(K_3\) occurring at these particular measurement settings is clearly state independent.

\[\text{FIG. (8). (color online) RTN (non-Markovian): Showing the variation of the LG parameter, } K_3(\Delta t, \theta, \phi), \text{ as defined in Eq. (22), with } \theta \text{ and } \phi \text{ characterizing the measurement given by Eq. (17). Bottom plot is an exaggerated portion of the top plot. One can see that the maximum of } K_3 \text{ occurs for } \theta = \pm \pi/2, \text{ for all } -\pi/2 \leq \phi \leq \pi/2.\]

V. CONCLUSION

In this work, we have studied the violations of the LG inequality under the effect of noise models like RTN and OUN. The LG inequality is found to be violated both in RTN and OUN models. Within a family of noisy channels, the non-Markovian case demonstrates an enhanced violation of the LGI, and hence a greater degree of nonclassicality, as compared with the corresponding Markovian case. The strong nonclassical behavior observed in the RTN case may be attributed to entanglement between the system and the environment degrees of freedom. We are also able to discern the recurrent versus non-recurrent behavior of the dynamics under RTN and OUN, respectively. For the case of RTN, the degree of violations as well as the number of times the violations happen, increase with \(\mu\). In the limit \(\mu \to \infty\), \(\text{Max}(K_3) = 1.5\), the quantum bound. Since, \(\mu\) denotes the coupling strength between the system and its environment, therefore, the violation of LGI increases as a function of the system-environment coupling, indicating stronger entanglement between the system and its environment. We optimize over the general measurement setting characterized by the parameters \(\theta \in [0, \pi]\) and \(\phi \in [0, 2\pi]\). The maximum of \(K_3\) is attained for \(\theta = \pm \pi/2\), irrespective of the value \(\phi\). In the limit \(\Lambda \to 1\), the two time correlations \(C_{ij} \to 1\) leading to \(K_3 = 1\). Thus the noise parameter \(\Lambda\) is not only responsible for the state updating of the qubit, but also makes the violation of LG inequality possible. This, therefore brings forth the surprising point that here noise helps to highlight the quantum nature of the system [51].
FIG. 9. (color online) OUN (non-Markovian): Showing the variation of $K_3 = K_3(\theta, \phi, \Delta t)$, as defined in Eq. (22), with $\Delta t$ and the angle $\theta$. In analogy to RTN case, we find that $K_3$ attains its maximum value for $\theta = \pm \pi/2$, for all $-\pi/2 \leq \phi \leq \pi/2$. Bottom panel depicts an exaggerated portion of the top panel.

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