Joule-Thomson Expansion of Born-Infeld AdS Black Holes

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Abstract

In this paper, the Joule-Thomson expansion of Born-Infeld AdS black holes is studied in the extended phase space, where the cosmological constant is identified with the pressure. The Joule-Thomson coefficient, the inversion curves and the isenthalpic curves are discussed in detail by 4-dimensional black hole. The critical point of Born-Infeld black hole is depicted with varying parament $\beta$ and the charge $Q$. In $T - P$ plane, the inversion temperature curves and isenthalpic curves are obtained with different parament $\beta$ and the charge $Q$. It shows that feature of the missing negative slope is still conserved in Born-Infeld black hole. The critical temperature and the minimum of inversion temperature are compared, the ratio is asymptotically $1/2$ as $Q$ increases or $\beta \to \infty$.

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I. Introduction

The pioneer work [1, 2] of Hawking and Bekenstein makes it possible to study black holes from the perspective of thermodynamics. Since the four laws of black hole mechanics were established [3], much attention has been paid to reveal the deep and fundamental relationships between the laws of general relativity, quantum field theory and thermodynamics [4–7]. In view of the key role played in the research of quantum gravity [8], surrounding the topic of black holes many ideas collide and new insights have extensively enriched our understanding of the universe, such as the holographic superfluids and superconductors [8–11], the quantum version in condensed matter theory [12–15], etc.

Hawking and Page showed the phase transition between the Schwarzschild AdS black hole and the thermal AdS space[16]. It has been shown that black holes in AdS space share common properties with general thermodynamic systems, and this relation was further enhanced in the extended phase space[17], where cosmological constant and its conjugate quantity are treated as thermodynamic pressure and volume respectively

\[ P = -\frac{\Lambda}{8\pi} = \frac{(d-1)(d-2)}{16\pi l^2}, \quad V = \left( \frac{\partial M}{\partial P} \right)_{S,Q}, \]

(1)

where \( l \) is the AdS space radius and the black hole mass \( M \) is understood as the enthalpy [18]. Then a number of papers explored the various thermodynamic aspects of black holes such as phase transition [19, 20], heat engine’s efficiency [21, 22], compressibility [23, 24], critical phenomenon [25–27], weak cosmic censorship conjecture [28, 29], and a brief review is given in ref. [18].

The Born-Infeld electrodynamics encode the low-energy dynamics of D-branes, which incorporates maximal electric fields and smoothes divergences of the electrostatic self-energy of point charges. The Born-Infeld AdS black hole solution was first obtained in [30]. And the thermodynamic behavior and phase transitions of these black holes were studied both in the canonical [31] and the grand canonical ensemble [32]. Besides, we can consider the string theoretical correction on the thermodynamic properties of charged AdS black holes by introducing the Born-Infeld action [33–36].

The Joule-Thomson expansion of black holes was investigated in [37] for the first time, then this subject was comprehensively studied in the literature [38–54]. The inversion curves that separate heating-cooling regions in \( T - P \) plane for isenthalpic curves with different parameters are present. The results of these papers show that the inversion curves \( T(P) \) for different black hole systems
are similar. In this paper, we would like to generalize the current research of Joule-Thomson expansion to the case of the Born-Infeld AdS black holes.

This paper is organized as follows. In Sec. II we briefly review the thermodynamics of the Born-Infeld AdS black holes in $D$-dimensional spacetime. Then in Sec. III we discuss the Joule-Thomson expansion of the Born-Infeld AdS black hole in 4-dimensional spacetime, which include the Joule-Thomson coefficient, the inversion curves and the isenthalpic curves. Furthermore, we compare the critical temperature and the minimum of inversion temperature, and the influence of nonlinear electrodynamics parameter and charge $Q$ on inversion curves is discussed. Finally we make a conclusion in Sec. IV. We will use the units $G = \hbar = k_B = c = 1$.

II. Black Hole Thermodynamics

The Einstein-Hilbert-Born-Infeld action for $D$-dimensional ($D \geq 4$) spacetime could be described as follows

$$S = \frac{1}{16\pi} \int d^D x \sqrt{-g} (R - 2\Lambda + \mathcal{L}(F)), \quad (2)$$

where the cosmological constant is related to the AdS space radius $l$ by

$$\Lambda = -\frac{(D - 1)(D - 2)}{2l^2}. \quad (3)$$

This action is a nonlinear generalization of the Maxwell action in string theory [33, 34]

$$\mathcal{L}(F) = 4\beta^2 \left(1 - \sqrt{1 + \frac{F^2}{2\beta^2}}\right), \quad (4)$$

where the parameter $\beta \sim 1/2\pi\alpha'$ relates to the Regge slope. In the zero-slope limit ($\beta \to \infty$), the action degenerates into Einstein-Maxwell theory,

$$\mathcal{L}(F) = -F^2 + O\left(\frac{1}{\beta}\right). \quad (5)$$

The metric of $D$-dimensional Born-Infeld AdS black hole is[30, 31, 55]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{D-2}^2, \quad (6)$$

with

$$f(r) = 1 - \frac{m}{r^{D-3}} + \frac{r^2}{l^2} + \frac{4\beta^2 r^2}{(D - 1)(D - 2)} \times \left(1 - \sqrt{1 + \frac{(D - 2)(D - 3)q^2}{2\beta^2 r^{2D-4}}}\right)$$
\[2F_1(a, b, c, z)\] is the hypergeometry function, the parameters \(m\) and \(q\) are related to the black holes mass and charge respectively as \[M = \frac{(D-2)\omega_{D-2}}{16\pi} m, \quad Q = \sqrt{2(D-2)(D-3)} \frac{\omega_{D-2}}{8\pi} q.\]

The event horizon \(r_+\) is the solution of \(f(r_+) = 0\), and the mass can be rewritten as

\[M = \frac{(D-2)\omega_{D-2}}{16\pi} r_+ D-3 \left\{ 1 + \frac{r_+^2}{L^2} + \frac{4\beta^2 r_+^2}{(D-1)(D-2)} (1 - \sqrt{1-z}) \right\} + \frac{2(D-2)q^2}{(D-1)r_+^{2D-6}} 2F_1 \left[ \frac{D-3}{2D-4}, \frac{1}{2}, \frac{3D-7}{2D-4}, z \right].\]

here

\[z = -\frac{(D-2)(D-3)q^2}{2\beta^2 r_+^{2D-4}},\]

The first law of black hole thermodynamics reads

\[dM = T dS + \Phi dQ + V dP,\]

and the connected Smarr relation is

\[M = 2(TS - VP) + \Phi Q,\]

where the electric potential

\[\Phi = \sqrt{\frac{D-2}{2(D-3)} \frac{q}{r_+^{D-3}} 2F_1 \left[ \frac{D-3}{2D-4}, \frac{1}{2}, \frac{3D-7}{2D-4}, z \right]},\]

and the Hawking temperature \(T\), entropy \(S\), thermodynamic volume \(V\) are given by \([30, 55, 56]\)

\[T = \frac{1}{4\pi} \left[ \frac{(D-1)r_+}{L^2} + \frac{D-3}{r_+} + \frac{4\beta^2 r_+}{(D-2)} (1 - \sqrt{1-z}) \right],\]

\[S = \frac{\omega_{D-2}}{4} r_+^{D-2}, \quad V = \frac{\omega_{D-2}}{D-1} r_+^{D-1}.\]

Bringing Eq. (1) and (3) into Eq. (15) one obtain the equation of state

\[P(V, T) = \frac{D-2}{4r_+} \left\{ T - \frac{D-3}{4\pi r_+} - \frac{\beta^2 r_+}{\pi(D-2)} (1 - \sqrt{1-z}) \right\}.\]
The critical point \( r_c \) obeys
\[
\frac{\partial P}{\partial r_c} |_{r=r_c} = \frac{\partial^2 P}{\partial r_c^2} |_{r=r_c} = 0,
\] (18)
and it can be exactly solved in \( D = 4 \) case, then
\[
T_c = \frac{1}{2\pi r_c} - \frac{Q^2}{\pi r_c^3} \frac{1}{\sqrt{1 + Q^2 / \beta^2 r_c^2}},
\] (19)
\[
p_c = \frac{1}{8\pi r_c^2} - \frac{Q^2}{2\pi r_c^4} \frac{1}{\sqrt{1 + Q^2 / \beta^2 r_c^2}} - \frac{\beta^2 r_c}{4\pi r_c} \left( 1 - \sqrt{1 + \frac{Q^2}{\beta^2 r_c^2}} \right).
\] (20)

We can plot the critical temperature with varying \( \beta \) and \( Q \) in Fig. 1, and we find that it is cut when \( \beta Q \) is \( \sqrt{2}/4 \), which is in agreement with the result of [57]. Furthermore, one can easily go back to the RN-AdS case as in [37] by simply taking the limit \( \beta \to \infty \).

![Diagram](image)

Fig. 1: The critical point \( r_c \) and Critical temperature \( T_c \) versus charge \( Q \) in \( D = 4 \), from bottom to top, the curves correspond to \( \beta = 1, 2, \infty \).

### III. Joule Thomson Expansion of Born-Infeld AdS Black Hole

In Joule-Thomson expansion, gas at a high pressure passes through a porous plug or small valve while keeping thermally insulated so that no heat is exchanged with the environment. One can describe temperature change with the Joule-Thomson coefficient
\[
\mu = \left( \frac{\partial T}{\partial P} \right)_H = \frac{1}{C_p} \left[ T \left( \frac{\partial V}{\partial T} \right)_P - V \right].
\] (21)
The zero point of Joule-Thomson coefficient has important physical significance in the study of Joule-Thomson Expansion. It is the inversion point that distinguishes the cooling process from the
heating process. So \( \mu = 0 \) gives the inversion temperature

\[
T_i = V \left( \frac{\partial T}{\partial V} \right)_P.
\]  

(22)

A. Van de Waals fluid

The van der Waals equation

\[
p = \frac{T}{v-b} - \frac{a}{v^2},
\]  

(23)

generalize the ideal gas equation, and is regarded as the appropriate description of real fluids. \( v = V/N \) is the specific volume and \( a,b \) measure the attraction between particles and molecule volume, respectively. The enthalpy is given by

\[
H(T,v) = \frac{3}{2}T + \frac{T v}{v-b} - \frac{2a}{v}.
\]  

(24)

The Joule-Thomson expansion is a process during which the enthalpy is keep constant, so it would also be interesting to study the isenthalpic curves of the Born-Infeld AdS black holes. One can spare lots of efforts to get the inversion temperature as a function of inversion pressure as follows

\[
T_i = \frac{2 \left( 5a - 3b^2 p_i \pm 4\sqrt{a^2 - 3ab^2 p_i} \right)}{9b}.
\]  

(25)

The isenthalpic and inversion curves are presented in Fig. 2. In the \( T - p \) plane, when the isoenthalpy curve crosses the inversion curve, its slope changes sign. The slope of the isenthalpy curve in the inversion curve is positive, otherwise it is negative. One can know that the Joule-Thomson coefficient is positive inside the inversion curve and the cooling occurs in this region.
Fig. 2: The colored isenthalpic curve lines from top to bottom are corresponding to $H$ starting from 1 to 6.5 with a interval of 0.5. Black empty diamonds pin on the maximum value points, and purple solid curve together with dashed orange one seperates the cooling region and heating region. We have set the parameters $a = b = 1$.

**B. Born Infeld AdS Black Hole**

Now we consider Joule-Thomson expansion of 4-D Born-Infeld AdS Black Hole. Thermodynamic quantities can be acquired with the relations Eq. (12), (13), (15) and (16), the heat capacity at constant pressure is

\[ C_p = T \left( \frac{\partial S}{\partial T} \right)_{P,Q,\beta} = \frac{2\pi r_+^4 (8\pi Pr_+^2 + 1 + 2\beta^2 r_+^2 (1 - \sqrt{1 - z_4}))}{8\pi Pr_+^4 - r_+^2 + 2\beta^2 r_+^4 (1 - \sqrt{1 - z_4}) + 4Q^2/\sqrt{1 - z_4}}, \]  

(26)

where

\[ z_4 = z(d = 4) = -\frac{Q^2}{\beta^2 r_+^4}, \]  

(27)

and one can derive

\[ \mu = \frac{1}{C_p} \left[ T \left( \frac{\partial V}{\partial T} \right)_{P,\beta} - V \right] = \frac{8r^2 + 32\pi Pr^4 - 8Q^2/\sqrt{1 - z_4} + 8\beta^2 r^4 (1 - \sqrt{1 - z_4})}{3r (1 + 8\pi Pr^2 + 2\beta^2 r^2 (1 - \sqrt{1 - z_4}))}, \]  

(28)
Fig. 3: Joule-Thomson coefficient $\mu$ and Hawking temperature $T$ versus the event horizon $r_+$, here $\beta = 20$, $P=1$, from left to right, the curves correspond to $Q = 1, 2, 10$.

The Joule-Thomson coefficient $\mu$ versus the horizon $r_+$ shown in Fig. 3. We fixed black hole parameter $\beta = 20$, pressure $P = 1$ and charge $Q$ as 1, 2, 10 in order. There exist both a divergent point and a zero point in different $Q$. By comparing the above two figures, we can easily see that the divergence point of the Joule-Thomson coefficient is consistent with the zero point of Hawking temperature. The divergent point here reveals the information of Hawking temperature and corresponds to the extremal black hole.

Taking $D = 4$ in Eq. (17), one obtains the equation of state

$$T = 2 r_+ P + \frac{1}{4 \pi r_+} + \frac{\beta^2 r_+}{2 \pi} \left( 1 - \sqrt{1 - z_4} \right), \quad \text{(29)}$$

and the inversion temperature is given by

$$T_i = V \left( \frac{\partial T}{\partial V} \right)_p = r_+ \left\{ \frac{\beta^2}{2 \pi} \left( 1 - \sqrt{1 - z_4} \right) + \frac{Q^2}{\pi r_+^4 \sqrt{1 - z_4}} + 2P - \frac{1}{4 \pi r_+^2} \right\}. \quad \text{(30)}$$

Bringing Eq. (30) into Eq. (29) at $P = P_i$ gives

$$P_i = -\frac{1}{4 \pi r^2} - \frac{\beta^2}{4 \pi} + \frac{Q^2}{2 \pi r^2 \sqrt{1 - z_4}} + \frac{\beta^2}{4 \pi} \sqrt{1 - z_4}, \quad \text{(31)}$$

$$T_i = -\frac{1}{4 \pi r} + \frac{Q^2}{2 \pi r^3 \sqrt{1 - z_4}}, \quad \text{(32)}$$

Eq. (31) and (32) make up the parameter equation of the inversion curve. Since Eq. (31) and (32) may have not a analytically solution, we use numerical solutions to plot inversion curves in $T - P$ plane.
$\beta = 10$
$\beta = 15$
$\beta = 20$
$\beta = \infty$

(a) $Q=1$
(b) $\beta=20$

Fig. 4: Inversion curves for Born-Infeld AdS Black Holes in T-P plane. From bottom to top, the left curves correspond to $Q = 1$ and $\beta = 10, 15, 20, \infty$, the right curves correspond to $\beta = 20$ and $Q = 1, 2, 10, 20$.

The inverse curve for different values of $\beta$ and charge $Q$ are shown in Fig. 4. The inversion temperature increases monotonously with the inversion pressure, but the slope of inversion curve decreases with the inversion pressure. Besides, the inversion temperature increases with the charge $Q$ and parameter $\beta$, and we can go back to the case of RN-AdS black hole as $\beta$ approaches to infinity. Compared with van der Waals fluids, it can be know from Fig. 4 that the inversion curve is not closed, and only exists one inversion curve that corresponds to the lower dashed orange curve of Fig. 2. As a result the Joule-Thomson coefficient is positive above the inversion curves and cooling occurs inside this region. It means that the Born-Infeld AdS black holes always cool above the inversion curve during the Joule-Thomson expansion, which is similar with the case of RN-AdS black hole [37], Kerr-AdS black hole [38] and other black holes in other previous works.

Now we want to get the minimum of inversion temperature, it can be obtained by Eq. (32),

$$T_i^{\min} = -\frac{1}{4\pi r_+^{\min}} + \frac{Q^2}{2\pi r_+^{\min^3}} \frac{1}{\sqrt{1 + \frac{Q^2}{\beta^2 r_+^{\min^4}}}},$$

where $r_+^{\min}$ is obtained by setting $P_i = 0$ in Eq. (31). The minimum inversion temperature can be obtained numerically, then we calculate the $Q$ dependence of minimum inversion temperature with different $\beta$ in Fig. 5(a). We also can return to the case of RN-AdS black hole as $\beta \to \infty$. It is significant to calculate the ratio between the minimum inversion temperature $T_i^{\min}$ and the critical temperature $T_c$. The previous work showed that this ratio turned out to be $\frac{1}{2}$[37]. The minimum inversion temperature and critical temperature can be obtained numerically and displayed by Fig. 1.
and Fig. 5(a), then we show the $Q$ dependence of the ratio $T_{i,\text{min}}/T_c$ with different $\beta$ in Fig. 5(b). We can see that the ratio is not always $\frac{1}{2}$, but the curves show that the ratio tends to $\frac{1}{2}$ as $Q$ increases. And when $\beta \to \infty$, it degenerates into RN AdS black hole again and the ratio always $\frac{1}{2}$.

![](image1.png)

(a) The minimum inversion temperature versus the charge $Q$

(b) The ratio $T_{i,\text{min}}/T_c$ versus the charge $Q$

Fig. 5: The minimum inversion temperature and the ratio $T_{i,\text{min}}/T_c$ versus the charge $Q$. From bottom to top, the curves correspond to $\hat{\beta} = 1, 2, 10, \infty$.

Other thermodynamic quantities can be acquired with the relation Eq. (12) and (13). Using Eq. (10), we can get the mass of this black hole in terms of $r_+$ in $D = 4$,

$$M = \frac{r_+}{2} \left\{ \frac{1}{3} + \frac{8\pi Pr_+^2}{3} + \frac{2\beta^2 r_+^2}{3} \left( 1 - \sqrt{1 - z^4} \right) + \frac{4Q^2}{3r_+^2} \text{F}_{1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, z \right] \right\}. \quad (34)$$

Since the Joule-Thomson expansion is a constant enthalpy process, it is significance to study the isenthalpic curves of Born-Infeld AdS Black Holes. In the extended phase space, the mass could be interpreted as enthalpy. So we can plot isenthalpic curves in $T - P$ plane by fixing the mass of the black hole. The isenthalpic curves are given by Eq. (29) with $r_+$ being the larger root of Eq. (34) at given $M$. We show the isenthalpic curves and the inversion curves of Born-Infeld AdS Black Holes in Fig. 6, this result is consistent with that in Fig. 4. The inversion curve is the dividing line between heating and cooling. The isenthalpic curve has a positive slope above the inversion curve, so there is cooling above this inversion curve. On the contrary, the sign of slope changes and heating occurs below the inversion curve.
IV. Conclusion

In this paper, we have studied the Joule-Thomson expansion for Born-Infeld AdS black hole in the extended phase space, where the cosmological constant is identified with the pressure. Since the black hole mass is interpreted as enthalpy, it is the mass that does not change during the Joule-Thomson expansion. The Joule-Thomson coefficient $\mu$ versus the horizon $r_+$ shown in Fig. 3. There exists both a divergent point and a zero point in different $\beta$ and we can easily see that the divergent point of the Joule-Thomson coefficient is consistent with the zero point of Hawking temperature which corresponds to the extremal black hole. In addition, one can easily go back to the RN-AdS case as in [39] by taking the limit $\beta \to \infty$. 

Fig. 6: The different solid lines are isenthalpic curves with parameters given below the figures. Black empty diamonds pin on the maximum value points, and purple dotdashed lines are the invesion curves. The lower panel denotes the RN-AdS case.
The inversion curves depend on the charge $Q$ and parameter $\beta$ are investigated in Born-Infeld AdS black holes, the results are depicted in Fig. 4. We also show the isenthalpic curves and the inversion curves in Fig. 6. It shows that the inversion curve always has positive slope, which is same with previous work. This means that black hole always cools above the inversion curve during the expansion. And we can use inversion curve distinguish the cooling and heating regions for different values of $\beta$ and $Q$. Furthermore, we checked the ratio of critical temperature and the minimum of inversion temperature in Fig. 5, the ratio is asymptotically $1/2$ as $Q$ increases or $\beta \to \infty$.

**Acknowledgement**

We are grateful to thank Bo Ning, Peng Wang and Wei Hong for useful discussions. This work is supported by NSFC (Grant No.11947408).

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