Effect of two loop correction in the formation of QGP droplet

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The effect of two loop correction in the formation of quark-gluon plasma (QGP) droplet is studied with the introduction of the two loop correction factor in the mean field potential. Due to the correction factor it shows stability in the droplet formation of QGP indicating at different parametrization factors of the QGP fluid. The correction factor in the potential also shows gluon parameter factor shifts to a larger value from its earlier value of gluon factor of one loop correction in obtaining the stable droplets. The results show decreasing in the observable QGP droplets and droplet sizes are found to be $1.5 - 2.0$ fm radii with the two loop correction. It indicates that there is parameter like Reynolds's number which can control the dynamics of QGP droplet formation and the stability of droplet in the case of droplet formation with the two loop correction factor.

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I. INTRODUCTION

Lattice theory indicates about the phase transition [1] from a deconfined phase of free quarks and gluons to a confined phase of hadrons. The deconfined state of matter is broadly known as quark-gluon plasma (QGP), that is probably obtained in the early universe formation. The study has made the theorist and experimentalist highly busy in the last two decades in search of identifying the formation of QGP. During these decades, numbers of highly energetic laboratories are set up around the globe and focusing to find out how the early universe started to create. They believe that matter present in the earlier time was as deconfined of free quarks and gluons known as quark-gluon plasma (QGP), and if it is formed then it would expand hydrodynamically with subsequent cooling lead to formation of confined colorless matter of quarks of hadrons. So the process of early universe creation is considered to be a complicated phenomena and this complicated nature indicates the study of QGP fireball in Ultra Relativistic Heavy-Ion Collisions an exciting field in the present day of heavy ion collider physics [2]. There are a number of phenomenological methods which try to solve these complicated phenomena. We also try to solve the problem by considering a phenomenological potential model. In the model we try to create free energy evolution through the different quark and gluon flow parameters forming various sizes of droplet. The formation of droplet differs with the change of temperature and somehow with the parametrization values can make a few stable droplets. This indicates that droplet formation with the paramatization value of quark and gluon is dependent on the temperature. The droplets determine the critical size when transforming the phase from a quark-gluon to a confined phase of hadron droplets. On the basis of the critical radius of the droplet, we also calculate the surface tension considered to be a parameter making the sharp boundary between these two phases. Moreover the calculation of surface tension gives another important property of liquid drop model in determining the stability of droplet formation in the system.

So, in this paper, we focus on the stability of QGP droplet formation through the different quark and gluon parameters incorporating the two loop correction in mean field potential. To evaluate the droplet formation, we use the thermodynamic partition function with the correction of two loop potential in the Hamiltonian of the system. The partition function is correlated through Gibbs free energy which is developed through density of state [3,4]. The density of state can be established through the earlier process of Thomas and Bethe model incorporating the one and two loop correction in the potential. The correction in the potential with loops affect in the droplet size and impacts in the stability of droplet formation with the variation of the dynamical quark and gluon flow parameters.

The paper is organised as: In section II, it briefly tries to construct the Hamiltonian of the system incorporating one loop correction extending to the two loop correction factor in the potential and set up the Gibbs free energy of the two loop correction. In section III the free energy evolution of system is discussed. In section IV it presents the surface tension of the system symbolising the stable droplet formation of QGP. In section V, the analytical solutions as results are discussed. At last, the conclusion with the details of stable droplet formation of QGP with different flow parametrization values is presented.

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II. HAMILTONIAN AND MEAN FIELD POTENTIAL WITH TWO LOOP CORRECTION

The dynamical behaviour of quarks-antiquarks and gluons in QGP enforce us in identifying the interacting potential among quark-quark, quark-antiquark, quark and gluon, which in turn, give the bulk thermodynamical and hydrodynamical properties of the particles of the system. So the effective mean field potential for QGP is calculated through the thermal mass formalism and the corresponding thermal Hamiltonian leading to the confining/de-confining potential among them. The thermal Hamiltonian is obtained as \([5, 6]\):

\[
H(k, T) = \left[ k^2 + m^2(T) \right]^{1/2} = k + m^2(T)/2k \text{ for large } k
\]

(1)

\[
H(k, T) = k + m_0^2/2k - \{m_0^2 - m^2(T)\}/2k
\]

(2)

where,

\[
m^2(T) = \frac{16\pi}{k} \gamma_{q,g} \alpha_s(k)T^2 \left[ 1 + \frac{\alpha_s(k)}{4\pi} a_1 + \frac{\alpha_s^2(k)}{16\pi^2} a_2 \right].
\]

(3)

It is thermal mass obtained after one and two loop corrections being introduced in the potential. The co-efficients used in the thermal mass \(a_1\) and \(a_2\) are the one and two loop correction factors which are obtained through the interactions among the constituent particles. They are defined numerically depending on the number of quark flavours and they are given as:

\[
a_1 = 2.5833 - 0.2778 n_l, \quad (4)
\]

\[
a_2 = 28.5468 - 4.1471 n_l + 0.0772 n_l^2 \quad (5)
\]

where \(n_l\) is considered to be the number of light quark elements \([7, 8]\). \(k\) is the quark (gluon) momentum and \(m_0\), the dynamic rest mass of the quark. \(T\) is temperature. \(\alpha_s(k)\) is QCD running coupling constant defined as:

\[
\alpha_s(k) = \frac{4\pi}{(33 - 2n_f) \ln(1 + k^2/\Lambda^2)},
\]

(6)

in which \(\Lambda\) is QCD parameter. The value is taken to be 0.15 GeV. \(n_f\) is degree of freedom of quark and gluon.

So the interacting mean-field potential \(V_{\text{conf}}(k)\) is now obtained with inclusion of two loop correction factor from simple confining potential obtained through the Hamiltonian and it is modified from the earlier potential. The modified potential is now expressed through the expansion of strong coupling constants of two loop factor within the perturbation theory as \([11–13]\):

\[
V_{\text{conf}}(k) = \frac{8\pi}{k} \gamma_{q,g} \alpha_s(k)T^2 \left[ 1 + \frac{\alpha_s(k)a_1}{4\pi} \right.
\]

\[
+ \frac{\alpha_s^2(k)a_2}{16\pi^2}] - \frac{m_0^2}{2k},
\]

(7)

where the loop co-efficients \(a_1\) and \(a_2\) play the roles for involving in the creation of interacting potential. In the expression again, quark and gluon parametrization factors are defined as \(\gamma_q = 1/14\) and \(\gamma_g = (48 - 60) \gamma_q\). These factors play many functional roles with the creation of droplets. First it determines the critical droplet formation. It then increases the dynamics of QGP flow and it also enhances in the process to transform QGP droplet to hadron droplets. Now the density of states in phase space with loop corrections in the interacting potential is modified and obtained through a generalized Thomas-Fermi model as \([3, 15]\):

\[
\rho_{q,g}(k) = v/\pi^2 \left[ -V_{\text{conf}}(k) \right]^2 \frac{dV_{\text{conf}}}{dk},
\]

(8)

or,

\[
\rho_{q,g}(k) = \frac{v}{\pi^2} \left[ \gamma^3_{q,g} T^2 \frac{G_0(k)}{2} \right]^3 A,
\]

(9)
where

\[ A = \left\{ 1 + \frac{\alpha_s(k)a_1}{\pi} + \frac{\alpha_s^2(k)a_2}{\pi^2} \right\}^2 \times \left( \frac{1 + \alpha_s(k)a_1/\pi + \alpha_s(k)^2a_2/\pi^2}{k^4} \right) + \frac{2(1 + 2\alpha_s(k)a_1/\pi + 3\alpha_s(k)^2a_2/\pi^2)}{k^2(k^2 + \Lambda^2)\ln(1 + \frac{k^2}{\Lambda^2})} \]  

(10)

and \( v \) is the volume occupied by the QGP and \( G^2(k) = 4\pi\alpha_s(k)(1 + \frac{\alpha_s(k)a_1}{4\pi}) \).

FIG. 1: The free energy vs. R at \( \gamma_q = 1/14 \), \( \gamma_g = 48\gamma_q \) for various values of temperature.

FIG. 2: The free energy vs. R at \( \gamma_q = 1/14 \), \( \gamma_g = 50\gamma_q \) for the various values of temperature.

III. THE FREE ENERGY EVOLUTION

The free energy of quarks and gluons is defined in the following with the modified density of states as [14]:

\[ F_i = -\eta T g_i \int dk \rho_{q,g}(k) \ln(1 + \eta e^{-(\sqrt{m_i^2 + k^2})/T}) \],

(11)

where \( \eta = +ve \) gives the bosonic particle and \( \eta = -ve \) gives the contribution from the fermionic particles. The minimum potential cut off in terms of momentum is obtained as:

\[ V(k_{\text{min}}) = (8a_1\gamma_{g,q}N^4T^2\Lambda^4/27\pi^2)^{1/6}, \]

(12)
where \( N = (4/3) [12\pi/(33 - 2n_f)] \). The minimum cut off in the model leads the integral to a finite value by avoiding the infra-red divergence while taking the magnitude of \( \Lambda \) and \( T \) as of the same order of lattice QCD. \( g_i \) is degeneracy factor (color and particle-antiparticle degeneracy) which is 6 for quarks and 8 for gluons. The inter-facial energy obtained through a scalar Weyl-surface in Ramanathan et al. [10, 16] with a suitable modification to take care of the hydrodynamic effects is given as:

\[
F_{\text{interface}} = \frac{1}{4} \gamma R^2 T^3. \tag{13}
\]

This interfacial energy is used to replace the bag energy of MIT model and it minimizes the drawback produced by MIT model. \( \gamma \) is root mean square value in terms of quark \( \gamma_q \) and gluon parameter \( \gamma_g \). The hadron free energy is [17]

\[
F_h = (d_i T / 2\pi^2) v \int_0^\infty k^2 dk \ln(1 - e^{-\sqrt{m_h^2 + k^2}/T}). \tag{14}
\]

where \( d_i \) is the degeneracy factor for the different light hadron particles and \( m_h \) is the light hadron corresponding masses. Because we considered only light hadrons as they are produced as maximum amount in the reaction plane. To calculate the total free energies, the particle masses are taken as: quark masses \( m_u = m_d = 0 \) MeV and \( m_s = 0.15 \) GeV [11]. Now we can compute the total modified free energy \( F_{\text{total}} \) as,

\[
F_{\text{total}} = \sum_i F_i + F_{\text{interface}} + F_h, \tag{15}
\]

where \( i \) stands for \( u, d \) and \( s \) quark and gluon.
IV. SURFACE TENSION WITH EFFECT OF TWO LOOP CORRECTION

The surface tension with two loop correction in the potential is calculated using the difference relation between free energy of QGP phase and the light element hadrons phase. It will indicate a sharp boundary between hadron and quark phase, balancing the pressure and keeping chemical equilibrium in the mixed phase which is normally treated by the Gibbs condition of a simple bulk calculation of free energy. However the calculation of surface tension can be highly affected by the mixed phase in which large finite size effects are included [18, 19]. To exclude the effects of finite size, we exclude the mixed phase system in the present calculation. So the difference in energy of the two phases define critical phase transition of liquid drop model after neglecting the finite size effects and shape contribution. It is therefore given as:

\[ \Delta F = - \frac{4\pi}{3} R^3 [ P_{\text{had}}(T) - P_{q,g}(T)] + 4\pi R^2 \sigma \]  

(16)

where the first term represents pressure difference and second term represents the contribution from the surface tension. The surface tension is calculated by minimizing the above expression with respect to droplet size. So, the surface tension formula is obtained as:

\[ R_c = \frac{2\sigma}{\Delta p} \quad \text{or} \quad \sigma = \frac{3\Delta F}{4\pi R_c^2} \]  

(17)

where, \( \Delta F \) is the change in the free energy and \( R_c \) is the corresponding critical radius obtained at the transition point from quark droplet to hadron droplet.
V. RESULTS:

The effects of QGP droplet formation with the inclusion of two loop correction factor in the interacting mean-field potential is numerically calculated. Due to the inclusion of the two loop correction, the QGP droplet changes a lot from one loop correction and without loop correction. The modifications in the droplet sizes are replicated in the figures showing in the free energies. The results in the free energies are modified by the quark and gluon flow parameters involved in the two loop correction as interacting parameter. In Fig.1, we can see stability evolution of droplet at the particular quark and gluon flow parametrization factor $\gamma_q = 1/14$, $\gamma_g = 48\gamma_q$. There is a stable droplet formation at all the temperatures forming the droplet size of 2.0 fm radius with the effects of two loop correction. Now we keep on increasing the gluon flow parameter fixing the quark flow parameter. We get again the stable droplet for all the temperatures at another gluon flow parameter. The size of the droplet is found to be around $R \leq 2.0$ fm and gluon parametrization what we found is to be $\gamma_g = 50\gamma_q$. It means that stability of droplet is really observed for all the different temperatures at the certain range of gluon parameters. In these two droplets we obtain the free energy amplitude less than 2.0 GeV and at lesser gluon parameter the energy amplitude is larger with large stable droplet size. In Fig.3 we further increase the gluon flow parameter. We obtain a slightly stable droplet with the increase of gluon flow parameter $\gamma_g < 52\gamma_q$ and the amplitude of the free energy is found to be lesser. This indicates that adopting the flow parameter in the range $\gamma_q = 1/14$ and $\gamma_g <= 52\gamma_q$, we obtained stability of the QGP droplet and the amplitude of the free energies drop down with increasing gluon flow parameter. In Fig.4, we further increase gluon parameter up to $\gamma_g = 56\gamma_q$ then we observe unstable droplet formation as we increase the gluon parameter from $\gamma_g = 52\gamma_q$ to higher value $\gamma_g = 56\gamma_q$. It indicates that instability starts forming from the gluon parameter $\gamma_g = 52\gamma_q$ with QGP droplet formation with the effect of two loop correction in the potential. Such type of effects are obtained earlier in one loop correction and the droplet sizes are bigger. With the addition of two loop correction in the potential, the size of the droplet decrease and stable droplet formation are tightly bound in comparison to the earlier droplet formation of one loop correction. It implies that no further stability is observed with the increasing gluon flow parameter. So stable droplet formation is specially found in the range of parametrization factor $48\gamma_q \leq \gamma_g \leq 52\gamma_q$ and the amplitude of the free energy with the stability is modified by these quark and gluon flow parameter.

We again calculate the surface tension at these particular quark and gluon flow parameter where more stable droplets are obtained. Calculating the surface tension of droplet is the characteristic feature of fluid to determine the stability of the droplet. On the basis of this character, the stable droplet features are shown in Figs.5 – 7. The Fig.5 indicates the increasing strength of surface tension with increasing temperature at the parametrization values of $48\gamma_q \leq \gamma_g \leq 52\gamma_q$. In Fig.6, we again observe the decreasing order of surface tension with the increasing critical radius of droplets. As the size of droplet is smaller we get larger surface tension so that QGP droplets are tightly bounded and more stable. Increasing the size of the droplet the surface tension is bound to be lesser. In Fig.7, we again plot the ratio of surface tension to the cube of critical temperature showing constancy of $\sigma/T_c^3$ with the temperature [20]. It is to show the comparative result with the lattice data. The result is found to be $\sigma = 0.173 T_c^3$ which is almost near the lattice result $\sigma = 0.2 T_c^3$ [21,22]. So the inclusion of two loop correction in the mean field potential with these parametrizations improve and enhance the stability of QGP droplet.
VI. CONCLUSION:

The results show the effects on the stability of droplet in the presence of two loop correction in the mean field potential. The effects of the stability is increased when the droplet size decreases as indicated by Fig(6). The size of droplet is more affected by the gluon flow parameter. If the parameter is increased beyond $\gamma_g \geq 52\gamma_q$ then unstable droplet starts forming and size of droplet is difficult to predict. In the range of the gluon flow parameters say $48\gamma_q \leq \gamma_g \leq 52\gamma_q$ the stable droplets are formed and the stability is more in the case of two loop correction in comparison to the one loop correction [9]. It indicates that two loop correction with the dynamical flow parameter can enhance the stable droplet formation. This is another possible indication that evolution of QGP fireball is steady dynamics depending on some kind of dynamical parameter which plays in forming the stable droplets.

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