Calculations of antiproton-nucleus quasi-bound states using the Paris $\bar{N}N$ potential

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Abstract

An optical potential constructed using the $\bar{p}N$ scattering amplitudes derived from the 2009 version of the Paris $\bar{N}N$ potential is applied in calculations of $\bar{p}$ quasi-bound states in selected nuclei across the periodic table. A proper self-consistent procedure for treating energy dependence of the amplitudes in a nucleus appears crucial for evaluating $\bar{p}$ binding energies and widths. Particular attention is paid to the role of $P$-wave amplitudes. While the $P$-wave potential nearly does not affect calculated $\bar{p}$ binding energies, it reduces considerably the corresponding widths. The Paris $S$-wave potential supplemented by a phenomenological $P$-wave term yields in dynamical calculations $\bar{p}$ binding energies $B_{\bar{p}} \approx 200$ MeV and widths $\Gamma_{\bar{p}} \sim 200–230$ MeV, which is very close to the values obtained within the RMF model consistent with $\bar{p}$-atom data.

Key words: antiproton-nucleus interaction, Paris $\bar{N}N$ potential, antiproton-nuclear bound states

1 Introduction

The elastic part of an $\bar{N}N$ potential constructed from a boson exchange $NN$ potential using the G-parity transformation is strongly attractive (see, e.g. [1]). This fact stimulated speculations about existence of $\bar{p}$-nuclear bound states [2,3,4]. The $\bar{p}N$ and $\bar{p}$-nucleus interactions, as well as their capability of forming corresponding bound states have been explored extensively in LEAR experiments at CERN [5,6]. Complementary information about the $\bar{p}$ optical potential near threshold was acquired within the study of strong interaction energy shifts and widths in antiprotonic atoms [7,8,9]. Analyses of $\bar{p}$-atom data and $\bar{p}$ scattering off nuclei at low energies disclosed that the antiproton interaction with a nucleus is dominated by $\bar{p}$ annihilation which governs propagation...
of the antiproton in nuclear matter. It was found that the experimental data could be well fitted by a $\bar{p}$-nuclear optical potential, imaginary part of which greatly outweighs the strongly attractive real part. However, if the antiproton is deeply bound in the nuclear medium the situation might change. In fact, the phase space for $\bar{p}$ annihilation products gets considerably reduced, which could lead to relatively long $\bar{p}$ lifetime in nuclear matter [10]. Nonetheless, no definite evidence of forming a $\bar{p}N$ or $\bar{p}$-nucleus quasi-bound state has been reported so far.

The study of $\bar{p}$ interactions with a nucleon and the nuclear medium is still topical. One example worth mentioning is the very recent analysis of $J/\Psi$ events collected by the BESIII experiment which supports the existence of either a $\bar{p}p$ molecule-like state or a bound state [11]. It is to be noted that one of the observed resonant states, $X(1835)$, was described by the 2009 version of the Paris $\bar{N}N$ potential, assuming that it originates from a $\bar{p}p$ bound state [20,11]. Furthermore, the knowledge of $\bar{p}$-nucleus interaction at various densities and under different kinematical conditions will be utilized and further expanded in forthcoming experiments with $\bar{p}$-beams at FAIR [12]. Simulations of the considered processes in a wide range of $\bar{p}$-beam momenta, providing experimentalists with valuable hints, are being performed within the Giessen Boltzmann-Ueling-Uhlenbeck (GiBUU) transport model [13] in which the $\bar{p}$ potential in nuclear matter serves as input.

Properties of $\bar{p}$-nuclear quasi-bound states have been calculated within the Relativistic Mean Field (RMF) model [10,14,15,16,17] using the G-parity transformation of coupling constants involved. A scaling factor representing departure from G-parity together with a phenomenological imaginary part were introduced to construct an optical potential consistent with experimental data. In Ref. [17], the $\bar{p}$ annihilation was treated dynamically and fully self-consistently, taking into account the reduced phase-space for annihilation products as well as compression of the nuclear core caused by the antiproton. Though the calculated $\bar{p}$ widths in the nuclear medium were found to be suppressed significantly, they remained considerable. Recently, Gaitanos et al. have developed a non-linear derivative (NLD) model [18,19]. It incorporates momentum dependence of mean fields acting on $\bar{p}$ which yields the depth of the $\bar{p}$-nucleus potential in accord with experimental data, without introducing any additional scaling factor.

Several microscopic $\bar{N}N$ potential models, such as those based on one- and two-pion exchange [20,21] or chiral EFT [22,23] have been developed recently. Friedman et al. [24] confronted the 2009 version of the Paris $\bar{N}N$ potential [20] with the $\bar{p}$-atom data across the periodic table and antinucleon interactions with nuclei up to 400 MeV/c, including elastic scattering and annihilation cross sections. Their analysis revealed necessity to include the $P$-wave part of the $\bar{p}N$ interaction to make the real $\bar{p}$ potential attractive in the relevant
low density region of the nucleus, as required by experiment. However, it was found that the Paris $S$-wave potential supplemented by the contribution of the Paris $P$-wave amplitudes fails to achieve reasonable fit to $\bar{p}$ atom data. On the other hand, the Paris $S$-wave potential with a purely phenomenological $P$-wave term accounts well for the data on the low-density, near-threshold $\bar{p}$-nucleus interaction. From this point of view, it is tempting to apply the $S$-wave amplitudes derived from the Paris $\bar{N}N$ potential and either the Paris or phenomenological $P$-wave amplitudes to the description of $\bar{p}$ interactions in the nuclear interior, i.e., farther down below threshold and at higher nuclear densities.

In the present work we employ the 2009 version of the Paris $\bar{N}N$ model in the construction of an optical potential which is then used in calculations of $\bar{p}$-nuclear quasi-bound states for the first time. We demonstrate the role of a proper self-consistent treatment of the energy dependence of scattering amplitudes involved. The adopted procedure for evaluating the sub-threshold energy shift has been applied before in calculations of kaonic and $\bar{p}$ atoms, as well as $K^-$-, $\eta$- and $\bar{p}$-nuclear states [17-24,26,27,28,29,30,31,32]. We take into account the $P$-wave part of the $\bar{p}N$ potential aiming at exploring its impact on calculated $\bar{p}$ binding energies and widths. Finally, we compare present results with those obtained within the RMF approach constrained by $\bar{p}$-atom data [17].

The paper is organized as follows. In Section 2, we briefly describe applied methodology. We present construction of the in-medium $\bar{p}N$ $S$-wave amplitudes from the free-space amplitudes derived within the Paris $\bar{N}N$ potential. We discuss a self-consistent procedure for treating the energy dependence of the amplitudes and construct a relevant $\bar{p}$ optical potential. Moreover, we introduce the $P$-wave interaction term which supplements the $S$-wave part of the potential. In Section 3, we present selected results of our calculations of $\bar{p}$ quasi-bound states in various nuclei across the periodic table, illustrating dynamical effects in the nuclear core caused by the antiproton and the role of the $P$-wave part of the $\bar{p}N$ potential. Summary of the present study is given in Section 4.

2 Model

The binding energy $B_\bar{p}$ and width $\Gamma_\bar{p}$ of a $\bar{p}$ quasi-bound state in a nucleus are obtained by solving the Dirac equation

$$\left[-i\vec{\alpha} \cdot \vec{\nabla} + \beta m_\bar{p} + V_{\text{opt}}(r)\right]\psi_\bar{p} = \epsilon_\bar{p}\psi_\bar{p},$$

(1)
where $m_{\bar{p}}$ is the mass of the antiproton, $\epsilon_{\bar{p}} = -B_{\bar{p}} - i\Gamma_{\bar{p}}/2$, ($B_{\bar{p}} > 0$), and $V_{\text{opt}}(r)$ is a complex optical potential which enters the Dirac equation as a time component of a 4-vector.

### 2.1 S-wave interaction

First, we consider only the $S$-wave optical potential which is constructed in a ‘$t\rho$’ form as follows:

$$2E_{\bar{p}}V_{\text{opt}}^S(r) = -4\pi \left( F_0 \frac{1}{2} \rho_p(r) + F_1 \left( \frac{1}{2} \rho_p(r) + \rho_n(r) \right) \right),$$

where $E_{\bar{p}} = m_{\bar{p}} - B_{\bar{p}}$, $F_0$ and $F_1$ are isospin 0 and 1 in-medium amplitudes in the $\bar{p}$-nucleus frame, and $\rho_p(r)$ [$\rho_n(r)$] is the proton (neutron) density distribution calculated within the RMF model NL-SH \cite{33}. The amplitudes $F_0$ and $F_1$ entering Eq. (2) are constructed from the free-space $\bar{p}N$ amplitudes in the two-body frame using the multiple scattering approach of Wass et al. \cite{34} (WRW) which accounts for Pauli correlations in the nuclear medium.

$$F_1 = \frac{\sqrt{s} \overline{f}_{\bar{p}n}^S(\delta \sqrt{s})}{1 + \frac{1}{4} \xi_k \sqrt{s} \overline{f}_{\bar{p}n}^S(\delta \sqrt{s}) \rho(r)}, \quad F_0 = \frac{\sqrt{s} \overline{f}_{\bar{p}p}^S(\delta \sqrt{s}) - f_{\bar{p}p}^S(\delta \sqrt{s})}{1 + \frac{1}{4} \xi_k \sqrt{s} \overline{f}_{\bar{p}p}^S(\delta \sqrt{s}) - f_{\bar{p}p}^S(\delta \sqrt{s}) \rho(r)}.$$ (3)

Here, $f_{\bar{p}n}^S$ ($f_{\bar{p}p}^S$) denotes the free-space $\bar{p}n$ ($\bar{p}p$) $S$-wave two-body cm scattering amplitude as a function of $\delta \sqrt{s} = \sqrt{s} - E_{\text{th}}$, where $s$ is the Mandelstam variable and $E_{\text{th}} = m_N + m_{\bar{p}}$. The factor $\sqrt{s}/m_N$ transforms the amplitudes from the two-body frame to the $\bar{p}$-nucleus frame. The nuclear density distribution $\rho(r) = \rho_p(r) + \rho_n(r)$ and the Pauli correlation factor $\xi_k$ is defined as follows

$$\xi_k = \frac{9\pi}{k_F^2} \left( 4 \int_0^{\infty} \frac{dr}{r} \exp(ikr) j_1^2(k_Fr) \right),$$ (4)

where $j_1(k_Fr)$ is the spherical Bessel function, $k_F$ is the Fermi momentum, and $k = \sqrt{(\epsilon_{\bar{p}} + m_{\bar{p}})^2 - m_{\bar{p}}^2}$. The integral in Eq. (4) can be solved analytically. The

\begin{itemize}
  \item As a test, we solved the Schrödinger equation for the same potential $V_{\text{opt}}(r)$ and got $\bar{p}$ energies and widths which differ by less than 1 MeV from those obtained by solving Eq. (1).
  \item The NL-SH model contains non-linear scalar self-interactions comprising of the cubic and quadratic terms in the $\sigma$ field. The model has proven successful in reproducing the binding energies and charge radii of nuclei, as well as neutron-skin thickness. In addition, it describes well saturation properties of nuclear matter, such as the binding energy per nucleon $a_v = -16.33$ MeV, nuclear matter density $\rho_0 = 0.146$ fm$^{-3}$, and compressibility $K = 355$ MeV.
\end{itemize}
Fig. 1. Energy dependence of the Paris 09 $\bar{p}p$ (top) and $\bar{p}n$ (bottom) $S$-wave two-body cm amplitudes: in-medium (Pauli blocked) amplitudes for $\rho_0 = 0.17 \text{ fm}^{-3}$ (solid line) are compared with the free-space amplitude (dotted line).

resulting expression is of the form

$$\xi_k = \frac{9\pi}{k_F^2} \left[ 1 - \frac{q^2}{6} + \frac{q^2}{4} \left( 2 + \frac{q^2}{6} \right) \ln \left( 1 + \frac{4}{q^2} \right) - \frac{4}{3}q \left( \frac{\pi}{2} - \arctan \left( \frac{q}{2} \right) \right) \right], \quad (5)$$

where $q = -ik/k_F$.

The free-space $S$-wave scattering amplitudes are derived from the 2009 version of the Paris $\bar{N}N$ potential [20]. The $\bar{p}n$ and $\bar{p}p$ amplitudes are expressed as appropriate mixtures of isospin $T = 0$ and $T = 1 \bar{N}N$ amplitudes, evaluated as angular momentum averages of fixed-$T$ amplitudes [24].

In Fig. 1 the free-space $\bar{p}p$ (top panel) and $\bar{p}n$ (bottom panel) amplitudes plotted as a function of energy are compared with the in-medium amplitudes at saturation density $\rho_0 = 0.17 \text{ fm}^{-3}$. Both free and WRW modified amplitudes
manifest strong energy dependence for $\delta \sqrt{s} = E - E_{\text{th}} \leq 0$. While the in-medium $\bar{p}p$ amplitude is attractive in the entire energy range below threshold, the real part of the in-medium $\bar{p}n$ amplitude is attractive for $\delta \sqrt{s} \leq -70$ MeV and with slightly repulsive dip near threshold. The peaks of both in-medium amplitudes are lower in comparison with the free-space amplitudes and shifted by $\approx 30$ MeV towards the $\bar{p}N$ threshold.

We explored the effect of the WRW procedure on $\bar{p}$ binding energies and widths and performed, out of curiosity, also calculation with the free-space $S$-wave amplitudes. In Table 1 we present $1s$ $\bar{p}$ binding energies and widths in $^{208}$Pb calculated with the free-space amplitudes and WRW modified (in-medium) amplitudes, using static RMF densities. The in-medium modifications significantly reduce the $\bar{p}$ widths whereas the $\bar{p}$ binding energies are affected only slightly. This could be anticipated upon closer inspection of Fig. 1. The differences between the free-space and WRW-modified real $\bar{p}N$ amplitudes at $\delta \sqrt{s} = E - E_{\text{th}} \sim -200$ MeV (which is the energy shift relevant to static calculations) is almost negligible (see left panels). On the other hand, the imaginary amplitudes (right panels) are evidently reduced at $\delta \sqrt{s} \sim -200$ MeV when in-medium modifications are taken into account. Consequently, the $\bar{p}$ widths are reduced as well.

The energy argument of the $\bar{p}N$ scattering amplitudes is expressed by Mandelstam variable

$$s = (E_N + E_{\bar{p}})^2 - (p_N^2 + p_{\bar{p}}^2),$$

where $E_N = m_N - B_{N\text{av}}$ with $B_{N\text{av}} = 8.5$ MeV being the average binding energy per nucleon. In the two-body c.m. frame $p_N + p_{\bar{p}} = 0$ and Eq. (6) reduces to

$$\sqrt{s}_M = m_{\bar{p}} + m_N - B_{\bar{p}} - B_{N\text{av}}.$$

However, when the interaction of the antiproton with a nucleon takes place in a nucleus, the momentum dependent term in Eq. (6) does not vanish and gives rise to an additional downward energy shift [25]. Taking into account averaging over the angles $(p_N^2 + p_{\bar{p}}^2) \approx \bar{p}_N^2 + \bar{p}_{\bar{p}}^2$, Eq. (6) can be rewritten as

$$\sqrt{s}_J = E_{\text{th}} \left(1 - \frac{2(B_{\bar{p}} + B_{N\text{av}})}{E_{\text{th}}} + \frac{(B_{\bar{p}} + B_{N\text{av}})^2}{E_{\text{th}}^2} - \frac{T_{\bar{p}}}{E_{\text{th}}} - \frac{T_{N\text{av}}}{E_{\text{th}}} \right)^{1/2},$$

Table 1

|            | free | WRW |
|------------|------|-----|
| $B_{\bar{p}}$ (MeV) | 184.8 | 188.6 |
| $\Gamma_{\bar{p}}$ (MeV) | 318.5 | 233.8 |
where $T_{N\text{av}}$ is the average kinetic energy per nucleon and $T_{\bar{p}}$ represents the kinetic energy of the antiproton. The kinetic energies were calculated as the corresponding expectation values of the kinetic energy operator $\hat{T} = -\frac{\hbar^2}{2m_N}\Delta$.

We note that the $\bar{p}$ binding energy $B_{\bar{p}}$ appears as an argument in the expression for $\sqrt{s}$, which in turn serves as an argument for $V_{\text{opt}}$ in Eq (1). Therefore, $\sqrt{s}$ has to be determined self-consistently, namely its value obtained by solving the Dirac equation (1) should agree with the value of $\sqrt{s}$ which serves as input in Eqs. (2) and (3). An additional self-consistency scheme has to be considered in dynamical calculations: The RMF densities entering the expression (2) for $V_{\text{opt}}$ are modified by the $\bar{p}$ bound in a nucleus and thus by the solution of the Dirac equation (1).

The $\bar{p}N$ amplitudes are strongly energy and density dependent, as was shown in Fig. 1. Consequently, the depth and shape of the $\bar{p}$-nuclear potential depend greatly on the energies and densities pertinent to the processes under consideration. This is demonstrated in Fig. 2 where we present the $\bar{p}$ potential in $^{40}\text{Ca}$ calculated using the free-space amplitudes at threshold and in-medium Paris $S$-wave amplitudes in three different energy regions: At threshold, for $\delta\sqrt{s} = 0$, the $\bar{p}$ potential constructed using the free space amplitudes (denoted by ‘th free’) has a repulsive real part and fairly absorptive imaginary part. When medium modifications of the amplitudes are taken into account (‘th medium’), the $\bar{p}$ potential becomes attractive and more absorptive. At energies relevant to $\bar{p}$ atoms the $\bar{p}$ potential, constructed following Ref. [24], is more attractive and weakly absorptive. At energies relevant to $\bar{p}$ nuclei ($\sqrt{s}_J$), the $\bar{p}$ potential is strongly attractive, however, also strongly absorptive. Clearly, proper self-consistent evaluation of the energy shift $\delta\sqrt{s}$ is crucial.
2.2 *P*-wave interaction

Recent calculations of $\bar{p}$ atoms and scattering of 48 MeV antiprotons \cite{24} showed that a sizable contribution from the *P*-wave part of the $\bar{p}N$ interaction is needed to get reasonable description of the experimental data. In order to examine the effect of the *P*-wave interaction on the binding energies and widths of $\bar{p}$-nuclear quasi-bound states, we supplement the *S*-wave optical potential in Eq. (2) \[
q(r) = 2\bar{E}_{\bar{p}}V_{opt}^{S}(r)\]
by a gradient term which stands for the *P*-wave interaction \cite{24,35,36}
\[
2\bar{E}_{\bar{p}}V_{opt}^{P}(r) = q(r) + 3\vec{\nabla} \cdot \alpha(r)\vec{\nabla}.
\] (9)

The factor $2l + 1 = 3$ in the *P*-wave part is introduced to match the normalization of the Paris $\bar{N}N$ scattering amplitudes and
\[
\alpha(r) = 4\pi \frac{m_{N}^{N}}{\sqrt{s}} \left( f_{\bar{p}p}^{P}(\delta\sqrt{s})\rho_{p}(r) + f_{\bar{p}n}^{P}(\delta\sqrt{s})\rho_{n}(r) \right). (10)
\]

Here, $f_{\bar{p}p}^{P}(\delta\sqrt{s})$ and $f_{\bar{p}n}^{P}(\delta\sqrt{s})$ denote the *P*-wave $\bar{p}p$ and $\bar{p}n$ free-space scattering amplitudes, respectively. We assume that the *P*-wave interaction contributes mainly near the nuclear surface where the nuclear densities are relatively low, and further in the interior its effect should decrease due to gradient form of the *P*-wave potential. Therefore, we do not consider medium modifications of the Paris *P*-wave amplitudes. The free-space $\bar{p}p$ and $\bar{p}n$ *P*-wave scattering amplitudes derived from the latest version of the Paris $\bar{N}N$ potential are shown as a function of energy in Fig. 3. Again, we witness a strong energy dependence of the amplitudes.

The analysis of Friedman et al. \cite{24} revealed that the potential constructed from the Paris *S*- and *P*-wave amplitudes fails to fit the antiproton atom...
data and that it is through the fault of the $P$-wave part. Their analysis also showed that the potential based on the Paris $S$-wave and phenomenological $P$-wave amplitude $f_{PN}^P = 2.9 + i1.8 \text{ fm}^3$ [24] does fit the data well. Therefore, we performed calculations exploring the effect of the $P$-wave interaction using both the Paris and phenomenological $P$-wave interactions.

3 Results

In this section, we present selected results of our self-consistent calculations of $\bar{p}$ quasi-bound states in nuclei across the periodic table using an optical potential constructed from the $\bar{p}N$ scattering amplitudes derived from the 2009 version of the Paris $\bar{N}N$ potential [20]. First, we performed calculations using only the $S$-wave optical potential and explored its energy and density dependence. Then, we took into account the $P$-wave $\bar{p}N$ interaction and studied its effect on the $\bar{p}$ binding energies and widths.

We performed static, as well as dynamical calculations. In the static calculations, the nuclear core is unaffected by the presence of the antiproton and its structure thus remains the same. In the dynamical calculations, the $\bar{p}$ polarizes the nuclear core, causing changes in the nuclear density distribution and nucleon single-particle energies. In our previous calculations of $\bar{p}$ quasi-bound states within the RMF model [17] it was demonstrated that the nuclear core is significantly affected by the extra antiproton — the nuclear density in the central region reaches $2-3$ times the saturation density. Since the $\bar{p}$ optical potential is density dependent, such increase in the density would result in a considerable increase of the $\bar{p}$ binding energies and widths. In fact, there is a competing effect, energy dependence of the imaginary part of the phenomenological $\bar{p}N$ scattering amplitude, coming from the phase space suppression for the $\bar{p}$ annihilation products, which partly compensates the effect of the increased density. The corresponding lifetime of the $\bar{p}$ inside a nucleus is then $\sim 1 \text{ fm/c}$ [17]. However, the response of the nuclear core to the extra $\bar{p}$ is not instant — it could possibly last longer than the lifetime of $\bar{p}$ inside a nucleus [14,15]. As a result, the antiproton annihilates before the nuclear core is fully polarized. Our static and dynamical calculations of $\bar{p}$ binding energies and widths may thus be considered as two limiting scenarios.

As was shown in Figs.1 and 3, the $\bar{p}N$ scattering amplitudes strongly depend on energy. It is thus very important to evaluate the $\bar{p}$-nucleus potential self-consistently in the appropriate reference frame. This is demonstrated in Fig.4, where we present $1s$ $\bar{p}$ binding energies (left panel) and corresponding widths (right panel) in various nuclei calculated dynamically using the Paris $S$-wave potential and two forms of the energy factor: $\sqrt{s_M}$ [Eq. (24)] and $\sqrt{s_J}$ [Eq. (8)]. The two forms of $\sqrt{s}$ yield very different binding energies and widths. As for the $\sqrt{s_M}$, the $\bar{p}$ binding energies are sizable and show weak $A$-dependence. The
corresponding $\bar{p}$ widths are huge ($\leq 400$ MeV), much larger than the binding energies. Including momentum dependent terms in $\sqrt{s_{J}}$ causes additional considerable downward energy shift, which leads to lower values of $\bar{p}N$ scattering amplitudes (see Fig. 1; the relevant energy shift $\delta \sqrt{s} \leq -200$ MeV) and, consequently, shallower $\bar{p}$-nucleus optical potential. The $\bar{p}$ widths are strongly reduced, yet remain sizable. The $\bar{p}$ binding energies decrease as well (up to $\sim 20\%$) and are again almost $A$-independent. Finally, it is to be noted that in static calculations, the effect of the momentum dependent terms in $\sqrt{s_{J}}$ on $\bar{p}$ binding energies and widths is about half of that effect in the dynamical case.

Next, we consider the $P$-wave part of the $\bar{p}N$ interaction. We adopt the Paris $\bar{p}p$ and $\bar{p}n$ $P$-wave scattering amplitudes as well as the phenomenological $P$-wave potential fitted by Friedman and Gal to $\bar{p}$ atom data [24] and construct the $S+P$-wave $\bar{p}$-nucleus optical potential [Eq. (8)] which is further applied in self-consistent calculations of $\bar{p}$-nuclear quasi-bound states.

In Fig. 5 we present $1s\bar{p}$ binding energies (left) and widths (right) as a function of mass number $A$, calculated statically with the Paris $S$-wave (squares), Paris $S+P$-wave (triangles up), Paris $S$-wave + phen. $P$-wave (triangles down) potentials for $\sqrt{s_{J}}$ [Eq. (3)]. The $\bar{p}$ binding energies and widths calculated statically with a phenomenological optical potential (‘phen $V_{opt}$’, circles) [17] are shown for comparison. The real part of this $\bar{p}$-nucleus potential was constructed within the RMF approach using G-parity motivated $\bar{p}$-meson coupling constants which were multiplied by a scaling factor to account for available experimental data. The $\bar{p}$ absorption was described by the imaginary part of a purely phenomenological optical potential fitted to strong interaction energy shifts and widths in $\bar{p}$-atoms. The reduced phase space available for annihilation of $\bar{p}$ deeply bound in the nuclear medium was taken into account by introducing corresponding suppression factors (see Ref. [17] for more de-
Fig. 5. 1\textit{s} $\bar{p}$ binding energies (left panel) and widths (right panel) in various nuclei, calculated statically for $\sqrt{s}_J$ using S-wave Paris potential (red squares), including phenomenological $P$-wave potential (green triangles down), Paris $P$-wave potential (blue triangles up) and phenomenological RMF potential (black circles).

Table 2

| $^{208}\text{Pb}+\bar{p}$ | Paris $S$ | Paris $S + P$ | Paris $S + \text{phen. } P$ |
|--------------------------|-----------|---------------|----------------------------|
| $\delta\sqrt{s}_J$ (MeV) | -210.6    | -238.9        | -223.6                     |

As can be seen from Fig. 5, both $P$-wave interaction terms, Paris as well as phenomenological, do not affect much the $\bar{p}$ binding energies — they are comparable with binding energies evaluated using only the $S$-wave potential. On the other hand, the $\bar{p}$ widths decrease noticeably when the phenomenological $P$-wave term is included in the $\bar{p}$ optical potential. The effect is even more pronounced for the Paris $P$-wave interaction. We observe strong $A$-dependence of $\bar{p}$ widths for the Paris $S + P$-wave potential. On the contrary, the widths calculated with the phenomenological $P$-wave term, as well as only with the $S$-wave potential vary much less with $A$ (starting oxygen).

To better understand this behavior, we show in Table 2 the energy shifts $\delta\sqrt{s}_J$ in $^{208}\text{Pb}+\bar{p}$ evaluated self-consistently in static calculations with Paris $S$-wave, Paris $S + P$-wave and Paris $S$-wave + phen. $P$-wave potentials. The $S$-wave potential yields the smallest energy shift with respect to threshold, which implies stronger $\bar{p}N$ amplitudes (see Fig. 1) and thus larger $\bar{p}$ binding energies and widths. When the $P$-wave interaction is taken into account, the downward energy shift increases. As a result, the $S$-wave part of the $\bar{p}$ potential becomes weaker. However, this decrease of the $S$-wave attraction is more than compensated by the real part of the $P$-wave potential as illustrated in Fig. 6. Here we present the Paris $S$-wave, Paris $S + P$-wave and Paris $S$-wave
Fig. 6. The real (solid curves) and imaginary (dashed curves) parts of the $S$-wave Paris potential (red) and the local (Krell-Ericson [37]) forms of the Paris $S + P$-wave (green) and Paris $S$-wave + phen. $P$-wave (blue) potentials felt by $\bar{p}$ in $^{208}$Pb, calculated statically for $\sqrt{s}_J$ (see text for details).

As a result, the $\bar{p}$ binding energies shown in Fig. 5 are very close to each other. On the other hand, the weaker imaginary part of the $S$-wave potential is not fully compensated by the $P$-wave part, particularly the Paris $P$-wave which is very weakly absorptive for the corresponding $\delta\sqrt{s}_J$ (see Fig. 3 and Table 2). The imaginary part of the $S + P$ potential is thus shallower than that of pure $S$-wave potential. On top of that, the range of the Paris $S + P$-wave potential is smaller than the range of the Paris $S$-wave + phen. $P$-wave potential (see Fig. 6). Therefore, the $\bar{p}$ widths in heavier nuclei calculated using the Paris $S + P$-wave potential decrease considerably.

It is to be noted that the depth of the $S + P$-wave potential is a result of delicate interplay between the $S$- and $P$-wave parts which are linked together. Very important is also the balance between the real and imaginary parts of the $P$-wave amplitudes since their strength controls the range of the potential.

Dynamical effects are illustrated in Fig. 7 where we compare $1s$ $\bar{p}$ binding energies (left panel) and corresponding widths (right panel) in various nuclei, calculated statically and dynamically for $\sqrt{s}_J$ using the Paris $S$-wave and Paris $S$-wave + phen. $P$-wave optical potentials. In both cases, the binding energies $B_{\bar{p}}$ calculated dynamically are somewhat larger than those obtained in static calculations and the polarization effects decrease with the mass number $A$. In dynamical and static calculations alike, the $\bar{p}$ binding energies calculated for the Paris $S +$ phen. $P$-wave potential are comparable with those obtained with the Paris $S$-wave potential. The $P$-wave interaction slightly increases

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3 Fig. 6 shows local forms of the $S + P$-wave potentials obtained from nonlocal Kisslinger potential of Eq. (9) using the Krell-Ericson transformation [37].
Fig. 7. $1s$ $\bar{p}$ binding energies (left panel) and widths (right panel) in various nuclei, calculated statically (triangles) and dynamically (circles) for $\sqrt{s}$, using $S$-wave Paris potential (red) and including phenomenological $P$-wave potential (black).

the $\bar{p}$ binding energies in heavier nuclei ($^{40}$Ca, $^{90}$Zr, and $^{208}$Pb) and decreases them in light nuclei ($^{16}$O and $^{12}$C). The $\bar{p}$ widths calculated dynamically are noticeably larger than the widths calculated statically. It is caused mainly by the increase of the central nuclear density, which outweighs the decrease of the $\bar{p}N$ amplitudes due to the larger energy shift with respect to threshold ($\delta\sqrt{s}_{\text{dyn}} \sim -255$ MeV vs. $\delta\sqrt{s}_{\text{stat}} \sim -200$ MeV). The Paris $S$-wave + phen. $P$-wave potential yields again smaller $\bar{p}$ widths than the $S$-wave potential. Still, the $\bar{p}$ widths calculated dynamically are larger or at least comparable with the corresponding $\bar{p}$ binding energies. The lifetime of the antiproton inside the nucleus is consistent with $\simeq 1$ fm/c.

It is to be noted that in our previous RMF calculations [17] we found strong model dependence of the dynamical effects caused by the extra $\bar{p}$ inside the nucleus. It could be attributed to different values of nuclear compressibility given by applied RMF models (models with larger compressibility predict larger dynamical changes in $\bar{p}$ binding energies). In order to explore model dependence in the present study, we performed calculations also using the RMF model TM(1)2 [38]. We found that unlike the phenomenological RMF approach the present static as well as dynamical calculations based on Paris $\bar{N}N$ amplitudes yield quite similar results within the TM and NL-SH models, the differences in $\bar{p}$ binding energies and widths are up to 10 MeV. It is due to energy dependence of the $\bar{p}N$ amplitudes which compensates the increase of the nuclear density. Namely, larger dynamical changes imply larger subthreshold energy shift and thus weaker $\bar{p}N$ amplitudes (see Fig. [1]). We preferred the NL-SH model in the present work since the TM model consists of two different parameter sets – TM2 for light nuclei and TM1 for heavy nuclei.

Next, we compare the predictions for $\bar{p}$ binding energies and widths calculated dynamically using the 2009 version of the Paris $\bar{N}N$ potential with our former calculations based on the RMF model [17]. The $1s$ $\bar{p}$ binding energies (left)
and corresponding widths (right) in selected nuclei calculated using the phenomenological RMF approach (circle), Paris S-wave potential (square), Paris $S + P$-wave potential (triangle up) and Paris $S$-wave + phen. $P$-wave potential (triangle down) are shown in Fig. 8. The binding energies are very close to each other in all cases, $B_{\bar{p}} \sim 200$ MeV, and rather weakly $A$-dependent. The $\bar{p}$ widths exhibit considerably larger dispersion for the different potentials. The Paris $S$-wave potential yields sizable widths in all nuclei, $\Gamma_{\bar{p}} \sim 300$ MeV. The Paris $P$-wave interaction again reduces the $\bar{p}$ widths significantly, to less than one half. The Paris $S$-wave + phen. $P$-wave potential yields very similar $\bar{p}$ widths as the phenomenological approach. They are in the range of $\sim 200 - 230$ MeV and comparable with the corresponding binding energies. The agreement between the phenomenological RMF and Paris $S$-wave + phen. $P$-wave potentials is quite impressive.

One has to mention that in the dynamical calculations, the depths of the Paris $S + P$-wave and Paris $S$-wave + phen. $P$-wave potentials in the central region of all nuclei are very similar to each other. However, the range of the Paris $S + P$-wave potential (in the local form) is again much smaller than the range of the Paris $S +$ phen. $P$-wave potential. Consequently, the $\bar{p}$ widths calculated using the Paris $S + P$ potential are considerably smaller.

We may thus infer that the real and imaginary parts of the Paris $P$-wave amplitudes are not well balanced in the energy region relevant to $\bar{p}$-nuclear states calculations. Anyway, it was demonstrated by Friedman and Gal (see Table 1 in Ref. [24]) that the real and imaginary parts of the Paris $P$-wave had to be scaled by different factors in order to obtain satisfactory fit to $\bar{p}$ atom data.

Besides the $\bar{p}$ ground states we calculated also $\bar{p}$ excited states in selected nuclei. In Fig. 9 we compare the binding energies and widths of the $1s$ and $1p$ $\bar{p}$ states in $^{16}$O, calculated dynamically using the Paris $S$-wave + phen.

![Fig. 8. Binding energies (left panel) and widths (right panel) of $1s$ $\bar{p}$-nuclear states in selected nuclei, calculated dynamically for $\sqrt{s_J}$ using the Paris $\bar{NN}$ $S$-wave potential (red), Paris $S$-wave + phen. $P$-wave (green) and phenomenological approach within the RMF model NL-SH (black).](image-url)
Fig. 9. 1s and 1p binding energies (lines) and widths (boxes) of $\bar{p}$ in $^{16}\text{O}$ calculated dynamically within the NL-SH model for $\sqrt{s_J}$ with phenomenological $\bar{p}$ optical potential (left panel) and Paris S-wave + phen. P-wave potential (right panel).

Fig. 10. 1s, 1p and 1d binding energies (lines) and widths (boxes) of $\bar{p}$ in $^{40}\text{Ca}$ calculated dynamically within the NL-SH model for $\sqrt{s_J}$ with phenomenological $\bar{p}$ optical potential (left panel) and Paris S-wave + phen. P-wave potential (right panel).

$P$-wave potential (right) and within the phenomenological RMF approach (left). The Paris S-wave + phen. P-wave potential yields similar spectrum of the $\bar{p}$ bound states as the RMF potential, however the 1p binding energy is about 20% larger and the width is slightly smaller than in the RMF model. Nevertheless, the agreement of the two spectra, which were obtained within two different approaches, is surprisingly good.

Fig. 10 shows similar $\bar{p}$ spectra in $^{40}\text{Ca}$. The 1p and 1d binding energies calculated with the Paris S-wave + phen. $P$-wave potential are again slightly larger (and $s-p$ and $s-d$ level spacing smaller) than in the phenomenological RMF.
approach. It is due to a broader \( \bar{p} \) potential well of the Paris \( S \)-wave + phen. \( P \)-wave potential. Both approaches yield comparable \( \bar{p} \) widths.

It is to be noted that in the present calculations, \( V_{\text{opt}} \) is a central potential constructed from angular momentum-averaged scattering amplitudes and thus there is no spin-orbit splitting of \( p \) and \( d \) levels presented in Figs. 9 and 10. In the RMF approach, the \( \bar{p} \) binding energies in \( 1p \) and \( 1d \) spin doublets are nearly degenerate, the difference in \( \bar{p} \) energies (as well as \( \bar{p} \) widths) is up to \( \sim 1 \) MeV. These findings are in agreement with spin symmetry predicted for antinucleon spectra \[39\,40\,41\,42\]. In the left panels of Figs. 9 and 10 we show spin-averaged \( 1p \) and \( 1d \) \( \bar{p} \) binding energies and widths.

4 Summary

We performed fully self-consistent calculations of \( \bar{p} \)-nuclear quasi-bound states using an optical potential constructed from the \( S \)- and \( P \)-wave \( \bar{p}N \) scattering amplitudes obtained within the 2009 version of the Paris \( NN \) potential \[24\]. The free-space \( S \)-wave scattering amplitudes were modified by WRW procedure \[34\] in order to account for Pauli correlations in the medium. A self-consistent scheme for proper dealing with the energy and density dependence of the in-medium amplitudes was adopted in evaluation of the \( \bar{p} \)-nuclear optical potential. To our knowledge, such calculations based on a microscopic model were carried out for the first time. Previous studies of \( \bar{p} \)-nuclear states were performed within phenomenological (RMF) approaches \[4\,10\,14\,15\,16\,17\].

First, we explored the \( S \)-wave part of the \( \bar{p} \) optical potential and showed that its form depends strongly on energy and density at which it is evaluated. The potential derived from free-space \( \bar{p}N \) amplitudes is repulsive and moderately absorptive at threshold. After applying in-medium modifications of the amplitudes, the potential becomes strongly attractive and absorptive at energies and densities relevant to \( \bar{p} \)-nuclear states calculations. As a result, \( \bar{p} \) binding energies in the \( 1s \) state amount to almost 200 MeV, and the corresponding widths \( \Gamma_{\bar{p}} \sim 300 \) MeV in the dynamical calculations.

Then we took into account the \( P \)-wave part of the \( \bar{p} \) optical potential. Recent analysis by Friedman et al. \[24\] revealed that the optical potential based on the Paris \( S \)- and \( P \)-wave scattering amplitudes fails to fit the \( \bar{p} \) atom data. On the other hand, the Paris \( S \)-wave potential supplemented by a phenomenological \( P \)-wave term reproduces the data well. We adopted both the Paris and phenomenological \( P \)-wave terms in our calculations. We performed static calculations (neglecting modifications of the nuclear core) as well as dynamical calculations (nuclear core is polarized by \( \bar{p} \)) which yield lower and upper estimates of \( \bar{p} \) binding energies and widths. We found that the \( P \)-wave interaction almost does not affect the binding energies of \( \bar{p} \)-nuclear quasi-bound states.
This is in sharp contrast to the case of $\bar{p}$ atoms where it was found necessary to include the $P$-wave term of the Paris $\bar{p}N$ interaction in order to increase attraction of the $\bar{p}$ optical potential [24]. This again illustrates how the form of the potential depends on energy and density. The widths of $\bar{p}$-nuclear states are reduced substantially when the $P$-wave interaction part is considered. The Paris $P$-wave potential reduces the widths much more than the phenomenological one. It is a result of a delicate balance between the $S$- and $P$-wave parts of the total $\bar{p}$ optical potential. The strength of the $P$-wave part which acts mainly near the nuclear surface and thus controls the range of the optical potential seems to be decisive.

Finally, we compared results of our present calculations using the Paris $\bar{N}N$ potential with our previous calculations of $\bar{p}$-nuclear quasi-bound states performed within the RMF model tuned to the $\bar{p}$-atom data [17]. The $\bar{p}$ binding energies and widths calculated dynamically with the Paris $S$-wave potential supplemented by the phenomenological $P$-wave term were found to be in good agreement with the RMF model calculations. Both approaches yield the $1s$ $\bar{p}$ binding energies $B_{\bar{p}} \approx 200$ MeV and the widths $\Gamma_{\bar{p}} \sim 200 - 230$ MeV in considered nuclei. We find this agreement rewarding as it shows that the $\bar{p}$ atoms fits not only define the form of the $\bar{p}$ optical potential near threshold and at low density region but, moreover, quite sufficiently constrain extrapolations to higher densities and farther down below threshold — to the region relevant to $\bar{p}$-nuclear states.

In conclusion, it is to be noted that the present work based on the 2009 version of the Paris $\bar{N}N$ potential was inspired by the recent study of Friedman and Gal [24]. They examined this very potential in the analysis of experimental results for antiprotonic atoms across the periodic table as well as antinucleon interactions with nuclei up to 400 MeV/c. Other realistic $\bar{N}N$ models, such as the Bonn-Jülich chiral NNLO [22] and N$^3$LO [23] EFT potential models or Zhou-Timmermans model [21], could be applied in the study of $\bar{p}$ interactions with the nuclear medium. It would be desirable to perform such calculations and compare between different $\bar{N}N$ interaction models.

Acknowledgements

We thank E. Friedman, A. Gal and S. Wycech for valuable discussions, and B. Loiseau for providing us with the free $\bar{N}N$ amplitudes. This work was supported by the GACR Grant No.P203/15/04301S.
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