1. Noise Analysis

To test the circuits for robustness against noise, noise analysis was performed both on the individual ts-WTA cell and the Disparity Selective Cell. In the design of the ts-WTA cell, the common source node from which the tunnel and injection feedback (that modifies the floating gate voltages) originates is the most sensitive node. Therefore, the resistance to noise at this node is critical for circuit performance. Noise was applied at this node and both the frequency and amplitude of noise was varied from 0.01s to .1s and ± 0.1 mV to 100mV respectively. It was observed that while there wasn’t much effect of the frequency of noise, the amplitude of noise started affecting the circuit performance beyond ± 90mV. Beyond this it was unable to distinguish between the two inputs coming to the two synapses. For the Disparity Selective Cell, noise was applied to the common source node of each of the 9x9 ts-WTA cells. For the disparity cell also, the noise frequency did not have any effect however, the circuit’s performance or its ability to learn a particular disparity pattern degraded beyond ± 90mV which is fairly high. We can say that both the circuits are fairly robust to noise.

2. Temperature Analysis

To test the robustness of the ts-WTA cell and the Disparity Selective Cell under temperature variation, simulations were performed for different temperatures in the range -45 to 85 °C. It was found that the ts-WTA worked well between -45 °C and 65 °C, however with an increase in temperature there was an increase in the learning time. This delayed learning time in the ts-WTA at high temperatures seemed to be affecting the Disparity Cell's performance. It appears that the increased learning time increases the diffusion or the neighborhood influence on each cell, as a result of which the pattern that the disparity cell learns is a not a unique pattern but a reflection of many input patterns. This can be adjusted by changing the diffusion resistances, however this aspect has not been taken into account in the current circuit.

During the detection phase, the circuit's performance remains unaltered for low temperatures. However for higher temperatures, although the detection of disparity happens correctly, there is a reduction in the output voltage range.

3. Monte Carlo Analysis of Disparity Selective Cell

To test the robustness of our Disparity Selective Cell, Monte Carlo Analysis with random parameter value variations was performed. The Disparity Selective Cell's performance heavily depends on the injection and tunnel currents. Any variation in these currents can affect the equilibrium of the circuit and affect the circuit’s learning and response behavior. Our models for injection and tunnel currents are based on the equations described in [Rahimi et.al, 2001] also described below.

The tunnel current varies according to the below equation and depends on the floating gate voltage, the tunnel voltage and a factor $V_t$ that depends on the oxide thickness. Our model assumes an oxide thickness of 70Å. $I_{ox}$is a pre-exponential current. The typical values of these parameters are listed in table I.
The injection current varies according to the below equation. It depends on the gate (floating)to drain and source to drain voltages. Here η, β and δ are fit parameters and \( \lambda = 1 \) for units consistency. \( I_s \) is the source current which is \( \sim 10 \text{nA} \) and can be ignored for all practical purposes. The typical values of the parameters η, β and δ are listed in table I.

\[
I_{\text{injection}} = F_{\text{inj}}(V_d, I_s, V_{fg}) = I_s \times W \times L \times \exp \left( -\frac{V_f}{V_{Tun} - V_{fg}} \right)
\]

\[\text{(1)}\]

\[
I_{\text{injection}} = \eta \times I_s \times \exp \left( -\frac{\beta}{(V_{fg} - V_d + \delta)^2} + \left[ \lambda \times (V_s - V_d) \right] \right)
\]

\[\text{(2)}\]

| Parameter | \( I_{\text{to}} \,(\text{A/m}^2) \) | \( V_f \,(\text{V}) \) | η | β | δ |
|-----------|---------------------------------|----------------|---|---|---|
| Base Value | \( 9.35 \times 10^8 \) | 368.04 | \( 1.30 \times 10^{-5} \) | 155.75 | 0.702 |

A single Disparity Cell is made from 9x9 ts-WTA cells. During fabrication, the variation in parameters can happen in two ways.

i). There could be variation in the parameter base values over the whole IC

ii). There could be minor variations in parameter values across the same IC

To test the robustness of our design under these two situations we performed Monte-Carlo analysis at two levels. First, by randomly varying the base values of all the parameters and applying the same (randomly generated parameters) to all the 9x9 ts-WTA cells and second, by randomly varying the base values and applying different parameter values to all 9x9 ts-WTA cells. The first analysis determines the extent to which the disparity cell is resilient to changes in the base values of parameters and the second analysis checks for how resilient the circuit is to variations in parameters across the 9x9 ts-WTA cells over the same IC. The following sections describe the detailed analysis.

### 3.1 Performance under parameter base value variation

To check for the first case, a MATLAB code was written to generate random values of all the parameters. The parameters were varied by 10%, 5% and 3% from the base values listed in table I. Some sample values of the parameters are listed in tables 2a, 2c and 2e.
3.1.1 Performance under 10% parameter variation

Multiple simulations were performed on the Disparity cell keeping the inputs and initial conditions the same but varying the parameters and applying the same to all the 9x9 ts-WTA cells.

Table 2a. 10% variation in device parameters

| Case No | $I_0 (A/m^2)$ | $V_f (V)$ | $\eta$ | $\beta$ | $\delta$ |
|---------|---------------|-----------|--------|---------|---------|
| 1       | 987114888.228762 | 362.893919 | 0.000013 | 145.053742 | 0.677823 |
| 2       | 875529355.768128 | 364.84702  | 0.000014 | 140.724017 | 0.663066 |
| 3       | 925538930.197264 | 343.556415 | 0.000012 | 166.233249 | 0.736087 |
| 4       | 947322216.259480 | 396.207331 | 0.000014 | 144.205576 | 0.730049 |
| 5       | 1016157953.038652 | 385.248023 | 0.000013 | 161.530224 | 0.663095 |
| 6       | 844396973.116741 | 358.564317 | 0.000014 | 169.719006 | 0.699921 |
| 7       | 849825326.820521 | 350.038661 | 0.000012 | 159.342702 | 0.764429 |
| 8       | 88287929.914150  | 362.929978 | 0.000014 | 148.026625 | 0.680760 |
| 9       | 966722055.874403 | 374.177837 | 0.000014 | 145.640335 | 0.747585 |
| 10      | 856079594.191899 | 358.490477 | 0.000012 | 163.461846 | 0.692258 |

Analysis: The cell’s learning response remained fairly stable for cases 1, 2 and 8 however for rest of the cases the response was significantly altered. Prior analysis done on a single ts-WTA cell reported in [Markan, C.M., Gupta, P., Bansal, M., 2013] shows that ts-WTA is stable under 10% variations in all parameters except the value of parameter $V_f$. However, as can be seen from equation(1), even when $V_f$ changes, the overall effect of the exponential term in the tunnel current can be kept constant by changing the tunnel voltage($V_{tun}$) appropriately. By modifying $V_{tun}$ for all the cases except for case 4, the response of the cell could be made normal. Therefore, it seems that the circuit is not very stable to 10% variation in $V_f$. However, in 90% of the cases, we can recover from this unstable response by adjusting $V_{tun}$. The exact variation in $V_{tun}$ can be seen in table 2b.

Table 2b. Change in the value of $V_{tun}$from the original value of 13.6v

| Case No | Modified $V_{tun}$(volts) | $\delta V_{tun}$(volts) |
|---------|--------------------------|------------------------|
| 3       | 13                       | -0.6                   |
| 5       | 14                       | 0.4                    |
| 6       | 13.3                     | -0.3                   |
| 7       | 13.3                     | -0.3                   |
| 9       | 13.8                     | 0.2                    |
| 10      | 13.3                     | -0.3                   |

3.1.2 Performance under 5% parameter variation
To check if the performance of the disparity cell improves with a lower percentage parameter variation, we varied the parameters by 5%. Some of the sample parameter values used in the simulations are recorded in Table 2c.

**Table 2c.** 5% variation in device parameters

| Case No | $I_0$ (A/m$^2$) | $V_f$ (V) | $\eta$ | $\beta$ | $\delta$ |
|---------|-----------------|-----------|--------|---------|---------|
| 1       | 903448852.215753| 351.649108| 0.000013 | 155.776379 | 0.734687 |
| 2       | 94500047.304039 | 380.988332| 0.000013 | 160.131355 | 0.696717 |
| 3       | 888654705.346983| 377.145651| 0.000013 | 158.178012 | 0.706970 |
| 4       | 95365366.477950 | 360.011766| 0.000013 | 158.136659 | 0.733318 |
| 5       | 91118212.488790 | 368.653780| 0.000013 | 148.067916 | 0.719875 |
| 6       | 924005782.434137| 358.665951| 0.000013 | 154.696435 | 0.708818 |
| 7       | 912696144.559396| 360.404296| 0.000013 | 148.440872 | 0.681787 |
| 8       | 945380010.191412 | 383.577475| 0.000013 | 149.460724 | 0.718531 |
| 9       | 945049412.311427 | 362.462831| 0.000013 | 152.474247 | 0.711202 |
| 10      | 914740138.349817 | 375.717765| 0.000013 | 159.974569 | 0.683564 |

**Analysis:** The cells response remained fairly stable for 60% of the cases; however, in 40% cases (e.g. case 1, 2, 3 and 8) the learning was altered moderately. In these cases also the response could be corrected by modifying $V_{tun}$ appropriately. The exact change in $V_{tun}$ is listed in Table 2d.

**Table 2d.** Change in the value of $V_{tun}$ from the original value of 13.6v

| Case No | 1 | 2 | 3 | 8 |
|---------|---|---|---|---|
| Modified $V_{tun}$ (volts) | 13.3 | 13.8 | 13.8 | 14 |
| $\delta V_{tun}$ (volts) | -0.3 | 0.2 | 0.2 | 0.6 |

**3.1.3 Performance under 3% parameter variation**

To find the range of parameter variation within which the cell works perfectly (without having to change $V_{tun}$) we then varied the parameters by 3%. Some sample values are listed in Table 2e.

**Table 2e.** 3% variation in device parameters

| Case No | $I_0$ (A/m$^2$) | $V_f$ (V) | $\eta$ | $\beta$ | $\delta$ |
|---------|-----------------|-----------|--------|---------|---------|
| 1       | 942305113.731676| 357.863394| 0.000013 | 155.912888 | 0.700073 |
| 2       | 919794968.390765| 361.952084| 0.000013 | 154.635215 | 0.704436 |
| 3       | 956106758.514632| 376.302343| 0.000013 | 151.379141 | 0.702056 |
| 4       | 917102887.966741| 372.542313| 0.000013 | 151.709425 | 0.702733 |
| 5       | 912307666.991632| 359.245728| 0.000013 | 154.072777 | 0.681407 |
| 6       | 938824854.542024| 365.572080| 0.000013 | 155.638304 | 0.706797 |
| 7       | 947710289.185848| 365.296566| 0.000013 | 157.926790 | 0.700346 |
| 9       | 933122974.181908| 360.571770| 0.000013 | 153.492817 | 0.721009 |
| 10      | 943420647.202232| 362.339327| 0.000013 | 155.784875 | 0.690321 |

**Analysis:** It was found that in all the cases the cell’s behavior was as expected. For case 3, the learning took slightly longer, however the output or receptive field did converge to the expected pattern. For all other cases the learning was normal.
3.2 Performance under parameter variation across the same IC

To test how robust our circuit is to parameter variations between the different 9x9 ts-WTA cells a MATLAB code was written to generate random parameters for all the 81 ts-WTAs. The parameter variation range was varied from ±2% to ±10%. It was found that when the parameters varied within ±3% of the base values, the cell performed normally, however, for larger limits the cell’s performance deteriorated. Sample values with a ±3% random parameter variation across all 9x9 ts-WTA cells are listed in table 3.

| tsWTA | \( I_0 \, (\text{A/m}^2) \) | \( V_f \, (\text{V}) \) | \( \eta \) | \( \beta \) | \( \Delta \) |
|-------|---------------------------|---------------------------|---------|---------|---------|
| 1     | 9169.26258.638770         | 374.388765                 | 0.000013 | 154.220189 | 0.686514 |
| 2     | 938.1568.395454           | 373.868665                 | 0.000013 | 158.703084 | 0.703177 |
| 3     | 943198321.797023          | 371.029368                 | 0.000013 | 153.853756 | 0.685235 |
| 4     | 936013995.315444          | 363.245351                 | 0.000013 | 152.389354 | 0.710212 |
| 5     | 943216764.482654          | 360.545362                 | 0.000013 | 151.369412 | 0.681080 |
| 6     | 94960249.771276           | 370.054359                 | 0.000013 | 152.911303 | 0.685080 |
| 7     | 942163018.938486          | 367.354815                 | 0.000013 | 158.703084 | 0.703177 |
| 8     | 93497692.456907           | 359.570799                 | 0.000013 | 159.771959 | 0.680987 |
| 9     | 936013995.315444          | 363.245351                 | 0.000013 | 152.389354 | 0.710212 |

Table 3. Sample values of Monte-Carlo analysis with ±3% parameter variation over all 81 ts-WTA cells together
5. Conclusions

i) The cell is fairly robust to noise

ii) Works well between -45 °C and 65 °C

iii) The two stage Monte-Carlo analysis performed on the Disparity Selective Cell brings forth the following conclusions

a. The cell is fairly stable under a ±3% variation in parameter base values.
b. For a variation greater than 3% but less than 10%, the response of the cell gets altered but it can easily be recovered by changing the tunnel voltage $V_{\text{tun}}$ appropriately.

b. The cell is also resilient up to a $\pm 3\%$ parameter mismatch between the 9x9 ts-WTA cells forming the disparity cell.

4. References

[1]. Rahimi, K., Diorio, C., Hernandez, C., & Brockhausen, M. D. (2002). A simulation model for floating-gate MOS synapse transistors. In Circuits and Systems, 2002. ISCAS 2002. IEEE International Symposium on (Vol. 2, pp. II-532). IEEE.

[2]. Markan, C. M., Gupta, P., & Bansal, M. (2013). An adaptive neuromorphic model of Ocular Dominance map using floating gate ‘synapse’. Neural Networks.