Research on the Influences of Inertial Properties on Body Freedom Flutter for an Airfoil

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Abstract. For some flying-wing aircraft with large aspect ratio, the pitching inertia is small, which makes the longitudinal short-period modal frequency higher. Structural flexibility makes the 1st bending mode frequency lower. This makes the pitching mode of rigid body easily coupled with the elastic mode, resulting in aeroelastic instability, which is called body freedom flutter. There are many factors affecting the body freedom flutter characteristics. The rigid body inertia and structural characteristics of the fuselage and wing all have an impact on it. Taking the flutter of an airfoil as an example, the structural dynamics model of the system is established. The flutter characteristics of the airfoil were solved by analytical method. The effects of pitching inertia and fuselage mass on the flutter speed, flutter frequency and flutter mode shape were studied.

1. Introduction
Flying wing configuration aircraft has inherent advantages in aerodynamic lift-drag ratio and structural efficiency, and becomes the preferred layout form of the next generation aircraft platform. In order to give full play to the advantages of structural weight, the new generation of flying-wing aircraft needs to use a large number of light composite materials, which makes the flying-wing aircraft have obvious structural flexibility. Especially for some flying-wing aircraft with large aspect ratio, the first natural frequency of structural elastic vibration is about 1 Hz or less. On the other hand, the tailless configuration of flying wing has small pitching inertia and high longitudinal short-period mode motion frequency. Elastic vibration and short-period mode coupling of rigid body motion are very easy to occur, and a dynamic instability phenomenon occurs at a flight speed far below the design limit speed, resulting in structural damage and crash. This dynamic instability phenomenon of rigid-elastic coupling is called Body Freedom Flutter (BFF). During the flight test of Lockheed Martin's next generation UAV, a typical body freedom flutter problem occurred. In 2003, the Helios prototype of ERASAT, a NASA high aspect ratio flying wing configuration verification aircraft, also suffered from aeroelastic instability due to the coupling of rigid body pitching motion and structural elastic motion during flight test, which led to the disintegration of the aircraft. At present, the body freedom flutter of flying-wing aircraft is seldom discussed in detail in public literature. Especially, there is no clear understanding and method on how to explain the mechanism of occurrence and put forward the key control measures. In this paper, the body freedom flutter on an airfoil was calculated theoretically, and the influence law of pitching inertia and fuselage mass parameters was analyzed.
2. Structural Dynamics Modelling

The two-dimensional airfoil’s body freedom flutter analysis model was established, considering the four degrees of freedom of pitch, plunge, bending and torsion. For the stability problem, the small disturbance linearization assumption was made, so that the X-direction displacement of the fuselage and wing caused by the pitch and torsion was not considered. The elastic deformation direction of the wing was always vertically downward. As shown in Fig. 1, the generalized coordinates of the four degrees of freedom were all established under the inertial system XY. \( H \) is the plunge displacement of the fuselage’s center of elastic, the downward direction is positive; \( \theta \) is the pitch displacement of the fuselage about the center of elastic, and nose up in a positive direction; \( h \) is the bending displacement of the wing's center of elastic, the downward direction is positive; \( \alpha \) is the torsion displacement of the wing about the center of elastic, and nose up in a positive direction. \( X_\theta \) is the distance from the center of mass of the fuselage to the center of elastic; \( X_\alpha \) is the distance from the center of mass of the wing to the center of elastic. \( R_\theta \) and \( R_\alpha \) are the radius of gyration of the fuselage and the wing pitching moment of inertia. \( M \) and \( m \) are the mass of the fuselage and the wing respectively. \( K_h \) and \( K_\alpha \) are the bending stiffness and torsional stiffness of the fuselage wing connection.

![Diagram of an airfoil with the fuselage model in inertial system](image)

Figure 1. Diagram of an airfoil with the fuselage model in inertial system

It was assumed that the aerodynamic force acting on the system came only from the wing. The fuselage was simulated as a particle, which only acted as inertia and had no effect on aerodynamic force. The dynamic equations considering the coupling of rigid body motion and elastic deformation were established by the Lagrange equation under the inertial system.

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} - \frac{\partial U}{\partial q_i} = Q (i = 1, 2, 3, \ldots, n) \tag{1}
\]

The kinetic energy of the system is:

\[
T = \frac{1}{2} m_f (\dot{H}^2 + 2X_\theta \dot{H} \dot{\theta} + X_\alpha^2 \dot{\theta}^2 + R_\theta^2 \dot{\theta}^2) + \frac{1}{2} m_a (\dot{h}^2 + 2X_\alpha \dot{h} \dot{\alpha} + X_\alpha^2 \dot{\alpha}^2 + R_\alpha^2 \dot{\alpha}^2) \tag{2}
\]

Considering the trim state, the lift and gravity are balanced, and the influence of the gravitational potential energy is ignored. The potential energy of the system is only the elastic potential energy:

\[
U = \frac{1}{2} K_h (h - H)^2 + \frac{1}{2} K_\alpha (\alpha - \theta)^2 \tag{3}
\]

Bring (2) and (3) into the Lagrange equation, we can get:
where $I_\theta = m_f (R_\theta^2 + X_\theta^2), I_\alpha = m_w (R_\alpha^2 + X_\alpha^2)$

The Theodorsen unsteady aerodynamic model was used on the right side of equation (4):

$$Q_h = L = \pi \rho b^2 \omega^2 \left[ L_h + \left( \frac{b}{2} + X_v \right) L_a \right] \alpha$$

$$Q_a = M_v = \pi \rho b^2 \omega^2 \left[ \left( M_h \frac{b}{2} + X_v \right) L_h + \left( M_a \frac{b}{2} + X_v \right) (L_a + M_a) + \left( \frac{b}{2} + X_v \right)^2 L_a \right] \alpha$$

Where:

$$L_h = 1 - i \frac{7}{2} \left( F(k) + i G(k) \right)$$

$$L_a = \frac{1}{2} - i \frac{1}{k} \left[ 2 \left( F(k) + i G(k) \right) \right] - \frac{7}{2} \left( F(k) + i G(k) \right)$$

$$M_h = \frac{1}{2}$$

$$M_a = \frac{3}{4} - i \frac{1}{k}$$

$$k = \frac{\omega \rho}{V}$$

The Theodorsen function can be approximately expressed as [15]:

$$F(k) + i G(k) = 1 - \frac{0.165}{1 - 0.041 \omega / k} - \frac{0.335}{1 - 0.32 \omega / k}$$

And we can get:

For this determinant, the condition that there is a non-zero solution is that the determinant is equal to zero. Expanding the determinant will result in a complex equation. It contains two equations of real and imaginary part and two unknowns of flutter velocity $V$ and flutter frequency $\omega$. It is difficult to solve the determinant directly. A set of values of $V$ can be given and the real part imaginary part equation can be brought to solve $\omega$. Then, an isopotential line whose imaginary part and the real part is equal to zero is drawn. The minimum velocity intersection point of the imaginary part and real part is the flutter point. According to this method, the variation of flutter speed and frequency with pitching inertia and fuselage mass ($I_{pitch} = I_\theta + I_\alpha$, $m_f$) is calculated. Keep the mass and inertia of the wing and fuselage equal. When the changing the pitching inertia, the fuselage mass remained unchanged. When changing the fuselage mass, the pitching inertia was kept constant by adjusting the radius of gyration.
3. Research on influences of pitching inertia and fuselage mass

The parameters of the selected benchmark model can be calculated in Table 1.

| Parameters                          | Values        | Parameters                          | Values        |
|-------------------------------------|---------------|-------------------------------------|---------------|
| Fuselage mass \( m_f \)            | 2–40/Kg       | Centroid position of wing          | 20\%c         |
| Wing mass \( m_w \)                | 2–40/Kg       | radius of gyration of fuselage \( r_\theta \) | 0.08–0.4/m   |
| Fuselage pitching moment of inertia \( I_{\theta} \) | 0.0272–0.6416/Kg\( \cdot \)m\(^2\) | radius of gyration of wing \( r_\alpha \) | 0.08–0.4/m   |
| Wing pitching moment of inertia \( I_{\alpha} \) | 0.0272–0.6416/Kg\( \cdot \)m\(^2\) | Bending stiffness \( K_h \) | 2/(N/mm)     |
| Airfoil chord length \( c \)       | 0.4/m         | Torsional stiffness \( K_\alpha \) | 600/(Nm/rad) |
| Wing segment length                 | 1.5/m         | Distance between Elastic center and Centroid position of fuselage \( X_\theta \) | 5\%c          |
| Elastic center position            | 15\%c         | Distance between Elastic center and Centroid position of wing \( X_\alpha \) | 5\%c          |
| Centroid position of fuselage      | 20\%c         |                                     |               |

3.1. The influence of pitching inertia and fuselage mass on bending and torsional mode frequencies

With the increase of pitching inertia, the frequency of torsional mode decreases and tends to the frequency of bending mode, and the ratio of bending to torsion mode frequency increases. With the increase of dimensionless mass \( \mu = \frac{m_f}{\pi \rho b^2} \), the frequency of bending mode decreases gradually, and tends to the frequency of bending mode in the fixed support, and the ratio of bending to torsion mode frequency decreases.

![Figure 2. Pitching inertia versus bending and torsional modes.](image)

![Figure 3. Fuselage mass versus bending and torsional modes.](image)

3.2. The influence of pitching inertia and fuselage mass on flutter velocity and flutter frequency

As can be seen from the previous section, increasing the pitching inertia will reduce the torsional mode frequency and increase the ratio of bending to torsion mode frequency. Fig. 4 shows the variation of...
flutter speed and frequency with the frequency ratio of bending and torsional modes when changing the pitching inertia. Three support conditions which are maintaining the freedom of plunge and pitch freedoms (sc1), maintaining the freedom of pitch freedoms (sc2) and fixed support (sc3) were considered in the calculation.

In the region where body freedom flutter occurs, the flutter velocity decreases first and then increases with the increase of pitching inertia when maintaining the freedom of plunge and pitch freedoms, and the flutter velocity has a minimum value. The same rule has appeared in reference [12]. When the plunge freedom is fixed, the flutter velocity decreases dramatically, and decreases monotonously with the increase of pitching inertia. When the bending-torsion frequency ratio is higher than 0.65, the flutter mode changes to bending-torsion coupling flutter. With the increase of pitching inertia, the flutter speed and frequency decrease. The flutter velocity of the free state is basically the same as that of the fixed state. Because of the lower bending frequency in the fixed state, the flutter frequency in the free state is higher than that in the fixed state.

![Figure 4](image.png)

**Figure 4.** Flutter characteristics with bending-torsion frequency ratio under the different support conditions

From the point of view of frequency coupling, when maintaining the freedom of plunge and pitch freedoms, the influence of pitching inertia on flutter characteristics has three aspects:

1) Increasing the pitching inertia will reduce the pitching frequency of rigid body. One approximation for the short-period frequency is proportional to $\frac{I_{\text{pitch}}}{0.5}$ [16]. Increasing the pitching inertia make pitching frequency away from the bending mode frequency, make it difficult to couple the pitching and bending modes, increase the flutter speed, and reduce the flutter frequency (zone 1). Therefore, by increasing the pitching inertia (such as increasing the length of the aircraft fuselage), body freedom flutter of can be avoided.

2) The increase of pitching inertia will also reduce the torsional mode frequency of the wing, make it close to the bending mode frequency gradually, and the flutter form will gradually change into the bending-torsion coupling flutter (zone 2). With the increase of pitching inertia, the flutter speed and frequency will decrease.

3) When the pitching inertia is too small, the pitch mode frequency increases with the decrease of the pitching inertia (zone 3). The pitch mode frequency is higher in this region. Continuous increase of the pitch mode frequency will make it difficult to couple with the bending mode, so the flutter speed increases and the flutter frequency increases.

When the plunge freedom was fixed, the flutter velocity decreases greatly and the minimum point of flutter velocity disappears. This rule can be studied by the influence of fuselage mass on flutter velocity. When the fuselage mass increases gradually, the displacement of plunge freedom decreases gradually. And when plunge freedom was fixed, the fuselage mass equivalently tends to be infinite. As can be seen from Fig. 5, the flutter speed decreases with the increase of dimensionless mass ratio, and the flutter speed gradually changes smoothly with the increase of bending-torsion frequency ratio, and the minimum flutter velocity moves to the left gradually.
Fig. 6 shows the variation of flutter velocity with dimensionless radius of gyration. It can be seen that the right half of the curve gradually merges into one line. It is shown that when the ratio of pitching inertia to fuselage mass reaches a certain degree, the influence of fuselage mass is weakened, and the flutter speed is only determined by dimensionless radius of gyration.

3.3. Divergent mode conversion

The root locus diagram of plunge, pitching, bending and torsional modes was observed when the body freedom flutter occurred (Fig. 7). It was found that besides bending-torsion coupling flutter occurring at very large inertia which diverged the torsion modes, there was also a transition from bending mode to pitching mode in the region where the body freedom flutter occurred, which corresponded to the minimum point of the flutter velocity curve in Fig. 4. The bending mode diverges when the dimensionless radius of gyration is on the left side of the minimum point, and the pitching mode diverges when the dimensionless radius of gyration is on the right side of the minimum point.

From the V-f curves of different dimensionless radius of gyration (Fig. 8), it can be seen that the pitching mode frequency curve intersects the bending mode frequency curve under small radius of rotation and small inertia. When the flutter occurs, the pitching mode frequency is higher than the bending mode frequency. So, when the pitching inertia continues to decrease at this situation, the pitching mode frequency increases, and moves away from the bending mode frequency, and becomes more difficult to couple, so the flutter speed increases. But at large inertia, the branch frequency curve of the two mode deviates from each other. When the flutter occurs, the pitching mode frequency is lower than the bending mode frequency. At this time, when increasing the pitching inertia, the pitching mode frequency decreases, and also moves away from the bending mode frequency, so the flutter speed also increases. The divergent mode branches of flutter tend to be modes with lower frequencies, so there is a divergent mode conversion at the minimum point.
4. Conclusion

With the increase of pitching inertia, the torsion mode frequency, bending-torsion frequency ratio and rigid body pitch mode frequency decrease when maintaining the freedom of plunge and pitch freedoms. The flutter velocity decreases at first and increases, but decreases at the time of transition to bending-torsion coupling flutter. There exists an extreme point in the flutter velocity of small pitching inertia, which is caused by the relationship between pitching mode frequency and bending mode frequency. When the pitching mode frequency is higher than the bending mode frequency, increasing the pitching inertia makes the pitching mode frequency lower, the two modes frequency closer, and the body freedom flutter speed decreases. When the pitching mode frequency is lower than the bending mode frequency, the pitching inertia is increased, which makes the two mode frequencies far away and the flutter speed increases. Therefore, there are extreme points. The divergent mode branches are usually the modes with smaller frequencies, so there is a divergent mode branch conversion at extreme points. When plunge freedom is fixed, the flutter velocity monotonously decreases with the increase of pitch inertia, and finally transits to the bend-torsion coupling flutter.

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