Phase transitions in isolated strongly interacting systems

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Abstract

Thermodynamical properties of nuclear matter at sub-saturation densities were investigated using a simple van der Waals-like equation of state with an additional term representing the symmetry energy. First-order isospin-asymmetric liquid-gas phase transition appears restricted to isolated isospin-asymmetric systems while the symmetric systems will undergo fragmentation decay resembling the second-order phase transition. The density dependence of the symmetry energy scaling with the Fermi energy satisfactorily describes the symmetry energy at sub-saturation nuclear densities. The deconfinement-confinement phase transition from the quark-gluon plasma to the confined quark matter appears in the isolated systems continuous in energy density while discontinuous in quark density. A transitional state of the confined quark matter has a negative pressure and after hadronization an explosion scenario can take place which can offer explanation for the HBT puzzle as a signature of the phase transition.

Introduction

The knowledge of the phase diagram of nuclear matter is one of the principal open questions in modern nuclear physics with far reaching cosmological consequences. Detailed investigations have been carried out in the recent
years in particle-nucleus and nucleus-nucleus collisions in a wide range of projectile energies.

The process of multifragmentation was investigated at intermediate and high energies in order to study the properties of the expected liquid-gas phase transition at sub-saturation nuclear densities. For instance, using the calorimetry of the hot quasi-projectile nuclei formed in the damped nucleus-nucleus collisions (see Ref. [1] and Ref. [2] for a review of related methods), thermodynamical observables of the fragmenting system such as temperature and chemical potential were extracted and a set of correlated signals of the isospin-asymmetric liquid-gas phase transition was observed. On the other hand, various properties of the observed fragments can be interpreted as signals of criticality [3, 4, 5], thus indicating an underlying second order phase transition. The phase diagram at sub-saturation densities appears to be rather complex and detailed investigations are needed for clarification.

The recent experimental studies of the nucleus-nucleus collisions at ultra-relativistic energies provide vast amount of experimental observations which can be interpreted as signals of production and decomposition of the new state of matter with deconfined quarks, called quark-gluon plasma (QGP) [6]. Such matter exhibits many unique properties such as nearly perfect fluid behavior [7] and strong medium modification of high-energy particles with a typical Mach cone behavior [8]. One of the phenomena not yet understood there is the so-called ”HBT puzzle” [9], an unexpected behavior of the source radii extracted from particle-particle correlations, which appear to be almost isotropic, in contrast with intense elliptic flow observed in such collisions. It was suggested that spatial inhomogeneity (typically in the form of QGP droplets) can play a crucial role in its explanation [10]. The mechanism of formation of such inhomogeneity is one of the interesting open questions.

In the present work, the mechanism of a phase transition is investigated as a phenomenon reflecting the properties of the underlying interaction. A modified van der Waals equation of state is used for the nuclear medium at sub-saturation densities and the role of isospin asymmetry will be investigated within the limitation following from the evolution of an isolated system. An analogous treatment is used for schematic description of the system undergoing deconfinement-confinement phase transition.
Figure 1: Density dependence of a) pressure and b) internal energy, given by the nuclear equation of state from Eq. 1. The lines represent isotherms with temperatures increasing with the step 2.5 MeV, starting at zero.

Isospin asymmetric liquid-gas phase transition in the nuclear matter

The liquid-gas phase transition in the nuclear matter is supposed to occur after nuclear matter expands and medium becomes unstable toward density fluctuations. The process as initially suggested in Ref. [11] was supposed to be analogous to liquid-gas phase transitions in real gases, which is typically described by the van der Waals equation of state (EoS). However, unlike the van der Waals gas, nuclear matter is a two-component system and it thus possesses an additional degree of freedom related to proton and neutron concentrations. Additional energy term representing the symmetry energy has to be taken into account. Under assumption that the symmetry energy decreases with density, it can be favorable for the expanding system to store the excess of neutrons (protons) into a dilute phase and thus isospin-asymmetric liquid-gas phase transition can occur [12]. However, the expanding nuclear system is isolated and no heat can be transferred into or out of the system and a typical picture of phase transition under conditions of phase equilibrium, as represented by the well-known Maxwell construction, is not relevant due to necessity to transfer latent heat into or out of the system.

In order to study the mechanism of isospin asymmetric liquid-gas phase
transition in the isolated nuclear system, we adopt a simple EoS for the symmetric nuclear matter, closely reminding the van der Waals EoS, where the repulsive short-range cubic interaction term is introduced as a substitute for the minimum volume:

\[ p = \rho T + a \rho^2 + b \rho^3 \]  

The parameter values \( a = -263.7 \text{ MeV fm}^3 \) and \( b = 1674.94 \text{ MeV fm}^6 \) were chosen to fix zero pressure at saturation density taken as \( \rho_0 = 0.16 \text{ fm}^{-3} \) and preserve plausible values for both critical temperature (\( T_c \simeq 14 \text{ MeV} \)) and ground state energy (\( U_{gs} \simeq -21 \text{ MeV} \)). The equation of state thus can be considered as a reasonable approximation to the results obtained using the microscopic theory of nuclear matter. The resulting pressure and internal energy isotherms are shown in Fig. 1. The internal energy (obtained assuming the heat capacity \( c_V = 3/2 \)) exhibits a quadratic dependence on density with a minimum at saturation density, what is the expected result for the nuclear ground state.

Since the system is thermally isolated, the expansion of the system is an adiabatic process. The entropy does not change during system evolution and assuming that the heat capacity \( c_V \) is constant, the following expression for temperature as a function of density

\[ T = T_0 \left( \frac{V_0}{V} \right)^{1/c_V} = T_0 \left( \frac{\rho}{\rho_0} \right)^{1/c_V} \]  

(2)

can be derived, where \( T_0 \) is the temperature projected to saturation density \( \rho_0 \). The resulting isentropic density dependences of the pressure and internal energy are shown in Fig. 2a, b. One can identify spinodal region where pressure grows while density falls, however, the limiting temperature appears much smaller than the one extracted from Fig. 1. The resulting phase diagram constructed using the pressure and temperature at the spinodal contour is shown in Fig. 2c. The critical temperature appears to only slightly exceed 8 MeV. Such observation is in agreement with experimental results [13]. Two caloric curves are shown in Fig. 2d. The dashed line represents the standard caloric curve obtained using the Fermi-gas formula

\[ E^* = \tilde{a} T^2, \]  

(3)

where the level density parameter is chosen as \( \tilde{a} = A/(10 \text{ MeV}) \). Such dependence is widely used in various evaporation models and represents es-
sentially an evolution at ground state density. It is worthwhile to note that the temperature dependence in Eq. 3 is equivalent to the heat capacity $c_V$ linearly increasing with temperature. Such dependence is in agreement with theory of the Fermi gas at low excitation energies, however, its validity should be restricted only to the domain where $c_V$ does not exceed the ideal gas value $3/2$ (below temperature 7.5 MeV), since $c_V$ saturates at this value and thus assumption of its further growth is not correct. Solid line in Fig. 2d represents the caloric curve obtained by combining the temperature at which the system enters spinodal region with the estimate of initial excitation energy taking into account also expansion of the system during initial heating. The heat capacity $c_p$ was obtained from

$$c_p = c_V + \frac{T}{T + 2a\rho + 3b\rho^2},$$  \hspace{1cm} (4)

where $c_V$ was determined using the formula 3. The system expanded according to

$$\frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p = \frac{1}{T + 2a\rho + 3b\rho^2},$$  \hspace{1cm} (5)

which is closely related to $c_p$. At each step of the procedure the temperature was incremented by a small value, corresponding increase of the internal (excitation) energy was proportional to $c_p$ and the density was modified according to the calculated volume expansion coefficient. Finally, new value of the pressure was calculated from the equation of state using new values of density and temperature. The resulting evolution of the system is shown in Fig. 2b as a nearly horizontal line starting from the ground state. The pressure of the system appears to grow very slowly while the system expands. The resulting caloric curve is similar but much flatter than the one obtained using the Fermi gas formula. For real nuclei one can assume that the system will follow the dashed line until it has enough energy to pass the fragmentation barrier and expand toward spinodal region, thus approaching the solid line via a short plateau. Such a short plateau was observed in the experiment [1], where it correlated with the onset of thermal equilibrium with the nucleonic gas, as represented by agreement of the extracted double isotope-ratio temperature with the kinematic temperature of protons.

Furthermore, it is interesting to note that while the fermionic system expands its Fermi energy exhibits the $\rho^{2/3}$ density dependence, which is identical to density dependence of temperature during adiabatic expansion when
one assumes the saturation value $c_V = 3/2$. Thus the ratio of the Fermi energy to temperature, which is a parameter determining the occupation numbers, will remain the same during adiabatic expansion. The ideal gas approximation $c_V = 3/2$ appears justified for the isentropic expansion where such value was reached during heating, in the Fig. 2a such situation applies to pressure curves with initial temperatures above $T_0 = 9$ MeV.

![Figure 2](image)

Figure 2: Density dependence of a) pressure and b) internal energy for constant entropy. The lines represent adiabatically expanding systems with the temperature at saturation density $T_0$ increasing with the step 2.5 MeV, starting at zero. Curve in the panel a) represents the spinodal contour and the horizontal line heating process of the nucleus. In the panel c) is the p-T diagram for the spinodal contour and in the panel d) are the caloric curves for the spinodal contour (solid) and for the Fermi gas formula (dashed).

Nuclear matter is a two-component system and thus an energy term representing the symmetry energy has to be introduced in order to reflect the difference of proton and neutron concentrations.
\[ \frac{E_{asym}}{A} = a_a \left( \frac{\rho}{\rho_0} \right)^{2/3} \left( \frac{N - Z}{A} \right)^2, \]  
\[ (6) \]

where \( a_a \) is the symmetry energy coefficient from the Weizsäcker formula and a \( \rho^{2/3} \) density dependence is assumed from the behavior of the Fermi energy. Due to density dependence of the symmetry energy it can be favorable for the system to store the excess of neutrons (protons) into a dilute phase and thus the isospin-asymmetric liquid-gas phase transition can take place. The equation of state will be modified to the form

\[ p = \rho T + a \rho^2 + b \rho^3 + \frac{2 a_a \rho_0}{3} \left( \frac{\rho}{\rho_0} \right)^{5/3} \left( \frac{N - Z}{A} \right)^2, \]
\[ (7) \]

which allows us to investigate the behavior of the system as a function of density and isospin asymmetry \( I = (N - Z)/A \). The additional term in the equation of state will not modify the temperature dependence in Eq. 2.

Figure 3: a) Pressure contour plot, corresponding to the nuclear equation of state given in Eq. (7). The adiabatic expansion with the temperature at saturation density \( T_0 = 10 \text{MeV} \) is assumed. Thick curve represents the spinodal contour. In panel b) is shown a detailed view onto low density region.

The pressure contours determined by Eq. (7) using the value \( a_a = 25 \text{MeV} \) are shown in Fig. 3 as a contour plot. The resulting spinodal region is highlighted and its shape is similar to microscopic calculations [14]. The
asymmetric systems will enter the spinodal region at lower densities, on the other hand the isospin-asymmetric gas remains stable at much higher densities than the symmetric one.

Figure 4: The resulting values of a) proton concentrations $x_v$ (solid line), $x_l$ (dashed line) and b) vapor fraction $a_{vap}$ for an adiabatically expanding system with $T_0 = 10$ MeV and isospin asymmetry of the system $(N - Z)/A = 0.3$.

Metastable state inside the spinodal contour ($\frac{\partial p}{\partial V}_S > 0$) is unstable toward spontaneous compression to stable liquid state at the spinodal contour, but such process would create heat. Instead of heat, the released internal energy can be deposited into expansion of vapor into free volume, thus spending released energy into overcoming attractive nuclear force. When the whole system is isolated ($\delta Q_{tot} = 0$) the process of spinodal decomposition will be equivalent to fast transfer of particles from the metastable state at its constant pressure to the stable liquid and vapor states at their corresponding pressures and total enthalpy of the system will not change during phase transition and the relation

$$U_{ms} + p_{ms}V_{tot} = U_{liq} + p_{liq}V_{liq} + U_{vap} + p_{vap}V_{vap}$$  \hspace{1cm} (8)$$

will be preserved.

During fast transition the total volume remains constant ($dV_{tot} = 0$) and for a given combination of the stable liquid and gas states located at the spinodal contour the partial volumes of liquid and vapor are
Figure 5: The resulting values of a) proton concentrations $x_v$ (solid line), $x_l$ (dashed line) and b) vapor fraction $a_{\text{vap}}$ for an adiabatically expanding system with $T_0 = 10$ MeV and isospin asymmetry of the system $(N-Z)/A = 0.03$.

\[ V_{\text{tot}} = \frac{A_{\text{tot}}}{\rho_{\text{ms}}} = V_{\text{liq}} + V_{\text{vap}} = \frac{A_{\text{liq}}}{\rho_{\text{liq}}} + \frac{A_{\text{vap}}}{\rho_{\text{vap}}}, \tag{9} \]

where

\[ A_{\text{liq}} = A_{\text{tot}} - A_{\text{vap}} \tag{10} \]

and nucleonic fraction of the vapor is

\[ a_{\text{vap}} = \frac{A_{\text{vap}}}{A_{\text{tot}}} = \frac{\rho_{\text{vap}}}{\rho_{\text{ms}}} \frac{\rho_{\text{liq}} - \rho_{\text{ms}}}{\rho_{\text{liq}} - \rho_{\text{vap}}} \tag{11} \]

for which the charge conserving proton concentration ($x = Z/A$) is determined using proton concentration of a given liquid state $x_l$

\[ x_{\text{vc}} = \frac{x_{\text{ms}} - (1 - a_{\text{vap}})x_l}{a_{\text{vap}}}, \tag{12} \]

which should agree with the proton concentration $x_v$ of the vapor state at the spinodal contour used to determine $a_{\text{vap}}$. Such a self-consistent stable liquid and vapor states located at the spinodal contour can be determined numerically. Total enthalpy conservation relation can be written as
\[ H_{ms} - H_l - H_v = h_{ms} - a_{vap} h_v - (1 - a_{vap}) h_l = 0 , \]  

where \( h_{ms} \), \( h_l \) and \( h_v \) are values of the enthalpy per particle.

Figure 6: The resulting values of a) proton concentrations \( x_v \) (solid line), \( x_l \) (dashed line) and b) vapor fraction \( a_{vap} \) for an adiabatically expanding system with \( T_0 = 10 \text{ MeV} \) and isospin asymmetry of the system \( (N - Z)/A = 0.003 \).

The resulting values of \( x_v \), \( x_l \) and \( a_{vap} \) for an adiabatically expanding system with \( T_0 = 10 \text{ MeV} \) are shown in Figs. 4, 5 and 6 for the isospin asymmetries of the system \( (N - Z)/A = 0.3 \), 0.03 and 0.003, respectively. It can be seen that for the asymmetric system with \( (N - Z)/A = 0.3 \) there occurs a well pronounced isospin asymmetric liquid-gas phase transition with a sizable part of the system being converted into very isospin asymmetric vapor while the rest of the system becomes still more and more symmetric liquid. These trends become more prominent as the density of initial metastable state decreases. When the system becomes more symmetric (see Figs. 5 and 6) the vapor fraction drops very quickly while the vapor preserves its isospin asymmetry. Also the region of densities where isospin asymmetric liquid-gas phase transition can occur shrinks quickly. From the Figs. 4, 5 and 6 one can conclude that observable isospin asymmetric liquid-gas phase transition will occur for very asymmetric systems such as these observed in the experiment [1], while for symmetric systems (and for the remnants of liquid phase in asymmetric systems) another mechanism will play role. In order to reach spinodal region the system must pass the multifragmentation...
barrier and thus these systems will be unstable toward sudden decomposition into multiple fragments. Such a scenario can provide explanation for the somewhat confusing situation where signatures of both first and second order phase transitions can be extracted from observed data (even from the same set of data). The first-order behavior can be attributed to the distillation of isospin-asymmetric gas while the second-order signatures can be attributed to subsequent fragmentation (percolation) of the remaining symmetric liquid.

It is also interesting that the conclusions observed here are in good agreement with the assumptions used by the Statistical Model of Multifragmentation (SMM) \[15\] where the fragment partition is considered at equilibrium with nucleonic gas. Such a scenario is supported also by experimental data \[1\]. Other interesting point is that the \(\rho^{2/3}\) density dependence of the symmetry energy used in the present work appears as satisfactorily describing the properties of asymmetric nuclear matter at sub-saturation densities.

**Deconfinement-confinement phase transition in the quark matter**

The ultrarelativistic nucleus-nucleus collisions result in the very high energy densities where the quark matter can exist in thermally equilibrated deconfined state known as quark-gluon plasma. During the subsequent expansion energy density decreases until the critical value around 1 \(GeV/fm^{-3}\) where, according to the predictions of the quantum chromodynamical calculations performed on the lattice \[16\] the matter will revert to the confined state, where the interaction is dominated by a divergent confinement potential, linearly increasing with the distance. The potential energy, stored in the one-dimensional field tubes, called strings, will be subsequently released by string fragmentation in the form of hadronic gas. In order to investigate the phase diagram, one can use known equations of state of initial and final states of the quark-gluonic plasma and the hadronic gas and try to interpolate the equation of state in the spinodal region, as is done in Ref. \[17\]. However, since the process of hadronization typically proceeds via fragmentation of elongated strings it is also interesting to explore thermodynamical properties of the possible intermediate state of matter, where the particles of the quark gas interact via confinement potential. Such state of matter is referred here as the confined quark matter. The properties of intermediate states can be
important for the mechanism of phase transition, as was demonstrated in the
case of liquid-gas phase transition at sub-saturation densities. Here we will
explore thermodynamical properties of the confined quark matter in analo-
gous way as thermodynamical properties of the asymmetric nuclear matter
in previous section.

In order to investigate properties of the confined quark matter a confine-
ment potential is defined as

\[ V_{\text{conf}}(r) = \kappa r, \]  

(14)

where \( \kappa \) is the string tension with a typical value of 1 GeV/fm, as used
in the string fragmentation models [18]. In order to explore properties of
the confined quark matter, a 3-dimensional cubic lattice with a quark-quark
distance \( L \) can be assumed, where each quark interacts with six nearest sites.
Then the component of the internal energy, corresponding to confinement
potential, can be determined as

\[ U_{\text{conf}} = \frac{1}{2} N_q (6\kappa L) = 3N_q \kappa \rho^{-1/3}, \]  

(15)

where the number of quarks \( N_q \) is large enough to neglect surface effects.
However, an assumption that quark on the lattice interacts with all neighbors
must not necessarily be satisfied, since strings are essentially one-dimensional
objects. One can modify above formula by assuming that each quark on the
lattice interacts with \( w \) neighboring quarks and one obtains the formula

\[ U_{\text{conf}} = \frac{1}{2} N_q (w\kappa L) = N_q \kappa \frac{w}{2} \rho^{-1/3}. \]  

(16)

The pressure component corresponding to confinement potential can be
obtained as

\[ \Delta p_{\text{conf}} = -\frac{\partial U_{\text{conf}}}{\partial V} = -\kappa \frac{w}{6} \rho^{2/3} = -\frac{1}{3} U_{\text{conf}}, \]  

(17)

thus leading to negative pressure. In order to obtain the equation of state
of the confined quark matter, one has to introduce the pressure of the quark
gas which in the ultrarelativistic limit can be written as

\[ p = \frac{\epsilon}{3} = \frac{8\pi g_q T^4 \eta(4)}{h^3} = \frac{8\pi g_q T^4}{h^3} \frac{7\pi^4}{720}, \]  

(18)
where \( g_q \) is a degeneration factor and \( \eta \) is the Dirichlet Eta function. In order to write equation of state with the confinement potential one needs to find relation between energy density and quark density. That is possible for zero chemical potential (with \( g_q \) accounting for equal numbers of quarks and anti-quarks)

\[
\rho = \frac{N}{V} = \frac{8\pi g_q}{h^3} T^3 \eta(3) \tag{19}
\]

and after comparing the equations (18) and (19) one arrives to

\[
p = \rho T \frac{\eta(4)}{\eta(3)} - \frac{w}{6} \rho^{2/3} \tag{20}
\]

for the equation of state.

Figure 7: Dependence of the pressure on a) energy density and b) quark density for \( w = 2 \). Thick line represents quark-gluon plasma (Eqs. (18) and (19) with \( g_q = 24 \) and \( g_g = 16 \)), solid section corresponds to the region above phase transition. Thin solid line shows pressure of the confined quark matter, dashed line shows the quark gas contribution (Eqs. (18) and (19) with \( g_q = 24 \)), dash-dotted line shows the confinement potential contribution (Eq. (17)).

The resulting situation is shown in Fig. 7 for \( w = 2 \). At energy densities below critical value 1 \( GeV/fm^3 \) and corresponding quark densities the confined quark matter exhibits negative pressure and also violates the mechanical stability criterion. Such a medium is indeed unstable against collapse.
what on the other hand is a direct consequence of the interaction rather than an interplay of short- and long-range interactions, as in the liquid-gas phase transition. Since confined quark matter is primarily unstable toward hadronization it can be assumed that the survival time against hadronization is much shorter than collapse time and the medium can be considered as a quasi-stable transitional state.

The detailed mechanism of the deconfinement-confinement phase transition is beyond the scope of this work, the lattice QCD calculations \[16\] suggest a rapid crossover for the systems with small chemical potential produced at RHIC. As in the liquid-gas phase transition at sub-saturation densities, one can explore the global properties of the interaction and limitations on the phase transition resulting from the isolated system and, as a consequence, an isoenthalpic transition into confined quark matter can be considered. In the present case the hot quark-gluon plasma with high thermal pressure transforms into confined quark matter with much lower (negative) pressure. The hadronization phase transition then follows as a consequence of the fragmentation of the elongated strings.

Entalphy of the quark-gluon plasma with $u$ and $d$ quarks and antiquarks can be expressed as

$$U + pV = 4N_q\left(\frac{\eta(4)}{\eta(3)} + \frac{g_g \zeta(3) \zeta(4)}{g_q \eta(3) \zeta(3)}\right)T,$$  

(21)

where $g_q = 24$, $g_g = 16$ and $\zeta$ is the Riemann Zeta function. The enthalpy balance of the transition from quark-gluon plasma at temperature $T_0$ to confined quark matter at density $\rho$ and corresponding temperature $T$ can be written assuming ultrarelativistic fermionic gas also in the confined phase, with the additional component corresponding to confinement potential, obtained after combining the equations [16] and [17]

$$4N_q\left(\frac{\eta(4)}{\eta(3)} + \frac{g_g \zeta(4)}{g_q \eta(3)}\right)T_0 = 4N_q\frac{\eta(4)}{\eta(3)}T + 2N_q\frac{w}{6}\kappa\rho^{-1/3}$$

(22)

and using the relation [19] one arrives to equation

$$4\left(\frac{\eta(4)}{\eta(3)} + \frac{g_g \zeta(4)}{g_q \eta(3)}\right)T_0 = 4\frac{\eta(4)}{\eta(3)}\frac{h}{2(\pi g_q \eta(3))^{1/3}}\rho^{1/3} + \frac{w}{3}\kappa\rho^{-1/3}$$

(23)

which after assuming numerical values $T_0 = 170$ MeV (as suggested in the Refs. [6], [17]), $\kappa = 1000$ MeV/fm [18], $g_q = 24$ and $g_g = 16$ leads to
Figure 8: Solutions of the Eq. 23. Dependence of the a) parameter \( w \), b) energy density, c) pressure and d) temperature of the confined quark matter on quark density. Solid lines represent the low-density branch, dashed lines show the high-density branch.

a numerical equation which has solutions for \( w \leq 1.862 \). The solutions of Eq. (23) are shown in the panel Fig. 8a as a dependence of the parameter \( w \) on quark density. A low-density branch (solid line) and high-density branch (dashed line) can be identified, separated by the point at \( \rho = 0.921 \text{ fm}^{-3} \) corresponding to maximum value of the parameter \( w = 1.862 \). The resulting dependence of energy density on quark density is shown in the panel Fig. 8b and one can see that the solution for \( w = 1.862 \) corresponds approximately to initial energy density of quark-gluon plasma at \( T_0 = 170 \text{ MeV} \). The low-density branch corresponds to lower energy densities and thus represents possible states to which the system can transform since it corresponds to both quark and energy densities below the phase transition. The high-density branch represents the states with higher energy densities than the critical
density and thus can be considered unphysical since it is not obvious that the confinement potential is applicable to such states. From the Eq. (23) thus implies that the phase transition can be in principle continuous in the energy density, but discontinuous in the quark density. After reaching the critical energy density, further expansion will be achieved by transforming an increasing amount of the quark-gluon plasma into confined quark matter with the maximum of about two thirds of quark density, temperature below 150 MeV and negative pressure (see Fig. 8c). However, once the strings will fragment and a hadronic gas will be formed, potential energy will be released, pressure will increase dramatically and one can assume an explosion scenario. One can imagine a sudden expansion of the bubble of hadronic gas to the radius reaching the dimensions of the whole system. Such a scenario can offer a natural explanation to the "HBT puzzle" where the extracted radii are essentially equal for longitudinal, sideward and outward direction and suggest an isotropic source with a short emission time. HBT puzzle can thus be interpreted as a signature of the phase transition. It is also interesting to note formal similarity with the cosmologic inflation, a scenario leading to isotropic causality-connected universe, which is supposed to start from the patch of matter in the state of false vacuum at negative pressure. In the present case the confined quark matter could represent the false vacuum while the hadronic gas could be considered as the true vacuum. Thus, the ultrarelativistic nucleus-nucleus collisions may exhibit, apart from the "Little Bang" scenario [19] and possibility of the existence of mini black holes [20], another cosmological analogy in the form of "mini-inflation", as a possible explanation of the HBT puzzle.

It is also interesting to compare the results for the quark matter to the liquid-gas phase transition at sub-saturation density. Both systems will evolve spatial inhomogeneity with one phase at positive pressure and the other one at negative pressure, but different asymptotic behavior of the potential, one diverging at small distances while the other one at large distances, leads to reversion of some properties. While in the liquid-gas phase transition the dense liquid phase transforms into dilute gaseous phase, in the deconfinement-confinement phase transition the dense gaseous phase transforms into dilute liquid phase, which subsequently transforms into hadronic gas. The liquid consistence of the confined quark matter can be judged e.g. from its possible one-dimensional structure, as suggested by existing isoenthalpic solutions. In both cases the liquid phase is unstable toward subsequent decay by fragmentation mechanism. It is also interesting to in-
vestigate whether introduction of strangeness into the model description will lead to asymmetric phases. One can assume that for the strange quarks the string tension will increase and thus it will be preferable for strange particles to remain in the deconfined phase. The strangeness of the deconfined phase will increase and the strange hadrons will be formed in the latest stage of hadronization.

**Summary and conclusions**

Thermodynamical properties of the nuclear matter at sub-saturation densities were investigated using a simple van der Waals-like equation of state with an additional term representing symmetry energy. For the isolated system an enthalpy conservation rule was introduced resulting in significant limitations for the isospin-asymmetric liquid-gas phase transition, which is a sizable effect for very isospin-asymmetric systems but becomes negligible for symmetric systems. First-order phase transition thus appears restricted to asymmetric systems while for symmetric systems and symmetric remnants of the first-order phase transition the multifragmentation decay can exhibit similarity with second-order phase transition. The density dependence of the symmetry energy scaling with density dependence of the Fermi energy satisfactorily describes the symmetry energy at sub-saturation nuclear densities.

The limitations resulting from the isolated system were investigated also for the deconfinement-confinement phase transition from the quark-gluon plasma to quark matter with the confinement potential. It appears that the transition can be continuous in energy density while discontinuous in quark density. A transitional state of confined quark matter has a negative pressure and explosive scenario can take place during hadronization. Resulting scenario can offer an explanation for the HBT puzzle as a signature of the phase transition.

This work was supported through grant of Slovak Scientific Grant Agency VEGA-2/5098/25.

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