On the Interaction between Autonomous Mobility on Demand Systems and Power Distribution Networks — An Optimal Power Flow Approach

Alvaro Estandia*, Maximilian Schiffer, Federico Rossi, Justin Luke, Emre Kara, Ram Rajagopal, Marco Pavone

Abstract—In future transportation systems, the charging behavior of electric Autonomous Mobility on Demand (AMoD) fleets, i.e., fleets of electric self-driving cars that service on-demand trip requests, will likely challenge power distribution networks (PDNs), causing overloads or voltage drops. In this paper, we show that these challenges can be significantly attenuated if the PDNs’ operational constraints and exogenous loads (e.g., from homes or businesses) are accounted for when operating an electric AMoD fleet. We focus on a system-level perspective, assuming full coordination between the AMoD and the PDN operators. From this single entity perspective, we assess potential coordination benefits. Specifically, we extend previous results on an optimization-based modeling approach for electric AMoD systems to jointly control an electric AMoD fleet and a series of PDNs, and analyze the benefit of coordination under load balancing constraints. For a case study of Orange County, CA, we show that the coordination between the electric AMoD fleet and the PDNs eliminates 99% of the overloads and 50% of the voltage drops that the electric AMoD fleet would cause in an uncoordinated setting. Our results show that coordinating electric AMoD and PDNs can help maintain the reliability of PDNs under added electric AMoD charging load, thus significantly mitigating or deferring the need for PDN capacity upgrades.

Index Terms—Electric Autonomous Mobility on Demand, Network Flow, Smart Grid, Unbalanced Optimal Power Flow.

I. INTRODUCTION

A. Estandia is with Marain Inc., Palo Alto, CA 94306, USA. He worked on this paper while he was a visiting student at Stanford University. Email: alvaro@marain.com.

M. Schiffer is with the TUM School of Management, Technical University of Munich, Munich 80333, Germany. Email: schiffer@tum.de.

F. Rossi is with the NASA Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, USA. He worked on this paper while he was a Ph.D. student at Stanford University. Email: federico.rossi@jpl.nasa.gov.

J. Luke is with the Department of Civil and Environmental Engineering, Stanford University, Stanford, CA 94035, USA. Email: jthluke@stanford.edu.

E. Kara is with eIQ Mobility, Oakland, CA 94612, USA. He worked on this paper while he was with the SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA. Email: eck@fastmail.com.

R. Rajagopal is with the Department of Civil and Environmental Engineering, Stanford University, Stanford, CA 94035, USA. Email: ramr@stanford.edu.

M. Pavone is with the Department of Aeronautics and Astronautics, Stanford University, Stanford, CA 94035, USA. Email: pavone@stanford.edu.

*Corresponding author.

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to control the AMoD system. Often, network flow models using fluidic relaxations, i.e., allowing for fractional vehicles and fractionally serviced trip requests are used [7]. Models of this type have been extended to consider road capacities and congestion [10].

To control an electric AMoD system, an operator must keep track of a vehicle’s state-of-charge (SoC) and recharge a vehicle’s battery accordingly. Again, some heuristic approaches exist [11], [12]. Optimization-based algorithms are so far not amenable to large-scale problems as they rely on mixed-integer linear programs (MILPs) with discretized SoCs [13], [14].

At its core, the operation of an electric AMoD system induces a coupling between the power network and the transportation system. Specifically, the electric AMoD fleet represents a controllable load in time and space. All previously mentioned studies neglect the impact of an electric AMoD system on the power grid, despite the fact that even a moderate amount of EVs may significantly increase electricity prices [15] and may negatively influence the power grid’s reliability [16], [17]. A few recent studies consider such a coupling implicitly via available capacities [18] or prices [19], but the proposed control algorithms for the electric AMoD fleet do not explicitly account for the fleet impact on the power network. Only Rossi, Iglesias, Alizadeh, et al. [20] consider the fleet impact on the power network explicitly, introducing the Power in the Loop Autonomous Mobility on Demand (P-AMoD) model, a linear model that combines a network flow model for the electric AMoD system and a balanced single-phase DC model of a transmission network. However, this model does not consider the power distribution network, which is the more appropriate grid stage to analyze mesoscopic EVs fleet operations [21]. Notably, a single-phase DC model is not sufficient to model a PDN as it assumes a constant voltage magnitude, and neglects reactive power and link resistances [22]; instead, a three-phase model is necessary [23, Ch. 1].

So far, PDNs were only considered when determining optimal charging schedules for privately owned EVs which have to reach a certain SoC by the end of a given planning horizon [5], [6], [24] as opposed to centrally coordinated fleets. Here, an instance of the Optimal Power Flow (OPF) problem can be solved to balance necessary charging loads with PDN-specific constraints.

In summary, individual aspects of the control problem addressed in this paper, such as the control of an electric AMoD system or considering PDN models to optimally charge private EVs, have been addressed in the literature. However, to the best of our knowledge, no studies that tightly couple an electric AMoD system and PDN models currently exist.

This work addresses this gap. Specifically, our contribution is threefold. First, we present a benchmark of convex three-phase PDN power flow approximations and identify a model compatible with the characteristics of the electric AMoD problem. We then extend the mesoscopic model in [20] to capture the operations of and interaction between an electric AMoD system and a series of unbalanced PDNs. Second, we embed this model within an optimization problem that assesses achievable benefits with respect to full cooperation between the two systems. The mesoscopic optimization’s solution enables comprehensive analyses to identify bottlenecks in PDNs and inform operator decisions in the day-ahead electricity market. Third, we provide a case study of Orange County, CA where we study the impact of an electric AMoD system on the PDNs and evaluate the benefits of coordination.

The remainder of this paper is structured as follows: Section II reports the mesoscopic model for an electric AMoD system used in previous work for self-consistency. Section III surveys existing PDN models and identifies a suitable model for the electric AMoD application. Section IV discusses the interaction between the electric AMoD system and a series of PDNs. Section V details our case study of Orange County, CA, and presents results that characterize the impact of electric AMoD systems on PDNs, highlighting the improvement potential stemming from coordination. Section VI concludes this paper with a summary of its main findings and an outlook on future research. Finally, the Appendix summarizes our notation and nomenclature.

II. MODELING ELECTRIC AMoD SYSTEMS

In an AMoD system, a fleet of autonomous vehicles service customer transportation requests, i.e., picks up customers at their origin and brings them to their destination [2]. A fleet operator controls the AMoD fleet by assigning vehicles to customer requests and by routing each vehicle. Besides origin-destination trips of customers, the routing may comprise rebalancing trips in-between two customer trips as spatial and temporal mismatches between origins and destinations of different customer requests arise. In an electric AMoD problem, the fleet operator additionally controls vehicle charging schedules and rebalances vehicles based on anticipated spatial-temporal variations of vehicle SoCs and electricity prices.

We model an electric AMoD system with a network flow model as originally presented in [20], reported in this section for self-consistency. Sections III and IV then detail our main contribution by integrating this model with PDNs. To avoid integer variables, the model uses i) a fluidic vehicle approximation and ii) a road graph expanded along two dimensions: discrete-time and vehicles’ SoC.

a) General road network representation: We model the road network as a graph $G_R = (V_R, A_R)$ with a set of vertices $v \in V_R$ and a set of road segment arcs $(v, w) \in A_R$. Each arc $(v, w) \in A_R$ is characterized by a distance $d_{v,w}$, a traversal time $t_{v,w}$, and an energy consumption $c_{v,w}$.

We consider a set $T = \{1, \ldots, T\}$ of discrete equidistant time steps (each of duration $\Delta t \in \mathbb{R}^+$), and a set $C = \{1, \ldots, C\}$ of equidistant discrete battery charge levels (each of energy $E_c \in \mathbb{R}^+$).

While some vertices in $G_R$ merely represent intersections or access points, others represent charging stations $S \subseteq V_R$ that allow recharging of vehicles. Each charging station $s \in S$ has a charging rate $\delta_{C,s} \in \{1, \ldots, C\}$ that denotes the amount of SoC that can be recharged in a single time step. Additionally, charging stations have a certain number of charging plugs $S_s \in \mathbb{N}^+$ which limits the number of concurrently charging vehicles.

We model congestion using a threshold model, i.e., we assume that vehicles drive at the road’s free-flow speed as long
as their number is less than the road’s capacity \( \hat{f}_{v,w} \in \mathbb{R}^+ \), as detailed in [20].

b) Expanded graph representation: We use an expanded graph to model a vehicle’s location and SoC over time. The expanded graph \( G = (V, A) \) is directed and has a vertex set \( V \subseteq V_R \times T \times C \). Each vertex \( v \in V \) is defined by a tuple \((v_t, t_c, s_c)\) that represents a vertex \( v \) of \( V_R \) at a specific time \( t_v \) with a specific SoC \( s_c \). Figure 1 (left) illustrates the concept of SoC expansion; for ease of representation, the time expansion is not shown. The resulting arc set \( A \) consists of two subsets \( A_T \cup A_S = A \). Arcs \((v, w) \in A_T \) represent travel in the road network and must meet the following condition

\[
A_T = \{ (v, w) \in A \mid (v, v_w) \in A_R, \quad t_w - t_v = t_{v,v_w}, c_v - c_w = c_{v,v_w} \},
\]
i.e.: i) \((v, v_w)\) is a road arc, ii) the time expansion \( t_w - t_v \) equals its traversal time \( t_{v,v_w} \), and iii) the SoC expansion \( c_w - c_v \) equals its consumption \( c_{v,v_w} \). Arcs \((v, w) \in A_S \) represent recharging at a charging station and must meet the following condition

\[
A_S = \{ (v, w) \in A \mid v_v = v_w = s, c_v - c_w = (t_w - t_v) \delta_{C,s} \},
\]
i.e.: i) \(v_v\) and \(v_w\) are equal and correspond to a charging station, and ii) the SoC difference \( c_w - c_v \) equals the amount of energy recharged, that is \((t_w - t_v) \delta_{C,s}\).

c) Customer trip requests: In addition to this graph representation, we define a set of customer trip requests \( \mathcal{M} = \{1, \ldots, M\} \). Each trip \( m \in \mathcal{M} \) is defined by a quadruple \((v_m, w_m, t_m, \lambda_m)\) \( \in V_R \times V_R \times T \times \mathbb{R}^+ \) that denotes its origin \( v_m \), its destination \( w_m \), its departure timestep \( t_m \), and the number of customer trip requests \( m \). We assume a deterministic setting in which these requests are known or predicted for all timeslots. To reduce the number of decision variables, we use precomputed vehicle routes for customer-carrying vehicles, corresponding to shortest-time paths \( v_m \rightarrow w_m \) that do not violate the congestion constraints. As we use a threshold congestion model, we can straightforwardly precompute such feasible shortest-time paths by solving a network flow problem as in [20]. Each shortest-time path has a traveling time \( t_{v_m \rightarrow w_m} \) and a charge requirement \( c_{v_m \rightarrow w_m} \). We denote \( \lambda_{m,\text{dep}} \) as the number of vehicles with charge \( c \) departing to serve customer trip request \( m \) and \( \lambda_{m,\text{arr}} \) as the number of vehicles with charge \( c \) arriving at time \( t \) after serving customer trip request \( m \). Thus,

\[
\lambda_{m,\text{arr}} = \begin{cases} 
\lambda_m & \text{if } t_m = t - t_{v_m \rightarrow w_m} \\
0 & \text{otherwise}
\end{cases} \quad \forall t \in T, \ c \in C, \forall m \in \mathcal{M}.
\]

d) Electric AMoD model: We introduce \( f_0(v, w) : A \rightarrow \mathbb{R}^+ \) to represent the flow of customer-empty vehicles on arc \((v, w)\), which includes both rebalancing and charging vehicles. Further, \( N_f(v) \) denotes the initial location of the vehicles, i.e., the number of vehicles available at vertex \( v \) with charge level \( c_v \) at \( t_v \) = \( t \) and is zero for all other timeslots. Analogously, \( N_F(v) \) denotes the desired final location of the vehicles, i.e., the number of vehicles that must be at node \( v \) with charge level \( c_v \) at \( t_v \) = \( T \). With this notation, a multi-commodity flow representation of the electric AMoD model is given by:

\[
\sum_{(v, w) \in A} f_0(v, w) + \sum_{m=1}^{M} \sum_{c=1}^{C} \mathbb{1}_{v_m = v_w} \mathbb{1}_{t_v = t_m} \lambda_{m,\text{dep}}^c + N_f(v) = 0 \quad (2)
\]

\[
= \sum_{(u, v) \in A} f_0(u, v) + \sum_{m=1}^{M} \sum_{c=1}^{C} \mathbb{1}_{v_m = v_w} \lambda_{m,\text{arr}}^c + N_f(v) \quad \forall v \in V,
\]

\[
C \sum_{c=1}^{C} \lambda_{m,\text{dep}}^c = \lambda_m \forall m \in \mathcal{M}, \quad \sum_{t=1}^{T} \sum_{c=1}^{C} \lambda_{m,\text{arr}}^c = \lambda_m \forall m \in \mathcal{M} \quad (3)
\]

Here, \( \mathbb{1}_x \) is the indicator function. Equation (2) secures flow conservation for rebalancing and charging vehicles, ensures a sufficient number of empty vehicles in each vertex to cover originating trip requests, and enforces initial and final conditions on the vehicle locations through \( N_f \) and \( N_F \). Equation (3) distributes the demand for a given trip request \( m \) to vehicles with different SoC, and accumulates vehicles arriving at different times with different SoC for request \( m \).

e) Electric AMoD problem: We now extend the basic constraints of the electric AMoD model to a full electric AMoD model. Specifically, we optimize the vehicles’ rebalancing routes and charging schedules in order to minimize the cost of operating the electric AMoD system, that is:

\[
\begin{align}
\text{minimize} & & V_D \sum_{(v, w) \in A_T} d_{v,v_w} f_0(v, w) \\
& & \left[ \sum_{(v, w) \in A_T} \left( \lambda_{m,\text{arr}}^c + \sum_{c=1}^{C} \mathbb{1}_{v_m = v_w} \delta_{C,s} f_0(v, w) \right) \right] + \sum_{(v, w) \in A_S : v=v_w} \left( \lambda_{m,\text{arr}}^c + \sum_{c=1}^{C} \mathbb{1}_{v_m = v_w} \delta_{C,s} f_0(v, w) \right) \\
& & \begin{cases} 
\text{subject to} \\
\text{Eqs. (1) to (3)}
\end{cases}
\end{align} \quad \text{Electric AMoD model} \quad (4a)
\]

\[
\sum_{(v, w) \in A_T : v=v_w, t_v = t} f_0(v, w) \leq \bar{f}(v, w, t) \quad \forall (v, w) \in A_T, \ t \in T \quad (4b)
\]

\[
\sum_{(v, w) \in A_S : v=v_w, t_v = t} f_0(v, w) \leq \bar{s}_s \quad \forall s \in S, t \in T \quad (4c)
\]

\[
g_I(N_f) = 0, \quad g_F(N_F) = 0 \quad \text{Electric AMoD model} \quad (4d)
\]

Here, we use the previously introduced concept of expanded graph vertices: each vertex \( v \in V \) is defined by a tuple \((v_t, t_v, s_v)\) \( \in V_R \times T \times C \). The objective function Eq. (4a) minimizes the operational cost of the electric AMoD system, considering time-invariant operational cost per unit distance (e.g., discounted cost for maintenance, tires, depreciation) \( V_D \in \mathbb{R}^+ \) for rebalancing vehicles and time-varying electricity costs \( V_{el,s} \in \mathbb{R} \) for recharging vehicles at a charging station \( s \in S \). Figure 1 depicts example arcs that model such rebalancing and charging flows \( f_0 \), as well as \( \lambda_{m,\text{dep}}^c \) and \( \lambda_{m,\text{arr}}^c \) for an example trip \( m \) marked with bold arrows. Equations (1) to (3) impose general flow conservation while Eq. (4b) applies the threshold congestion model to rebalancing flows. As customer-carrying flows are fixed, we do not consider these directly in Eq. (4b). Instead, we use the residual road capacity \( \bar{f}(v, w, t) \)
which results from subtracting the customer carrying flow on road arc \((v,w)\) at time step \(t\) from the corresponding road capacity \(f_{v,w}^t\). The pre-routed vehicles may congest a road link. In this case, we set the residual capacity \(\bar{f}_{v,w}^t\) for that link to zero. Thus, customer-carrying flows and residual capacity are fixed and constant with respect to the optimization of rebalancing flows. Equation (4c) limits the number of vehicles that can use a charging station concurrently according to the number of charging plugs at each station. We impose initial and final conditions on vehicles with the generic functions \(g_I\) and \(g_F\) in Eq. (4d). The brackets in the decision variables denote concatenation. We will use this convention in the rest of the paper.

The electric AMoD problem Eq. (4) has \(TC(|A_R| + |S|) + CM + TC|V_R| + C|V_R|\) decision variables. The dominant term is \(CM\): there could be at most one customer trip request from every origin to every destination at every time step such that \(M \leq |V_R|^2T\). It follows that \(CM \in O(|V_R|^2T)\).

A few comments are in order. First, we consider discrete time steps as well as discrete SoC levels. From a mesoscopic viewpoint, these discretizations bear sufficient accuracy while improving the model’s computational tractability significantly. Second, the network flow model treats vehicles and customers as fractional flows; accordingly, it is not readily suitable for real-time control of electric AMoD fleets. Again, this accuracy loss is acceptable at a mesoscopic level and is compatible with our goal of assessing the achievable performance stemming from the coordination between electric AMoD and PDN operators. Note that our solution can still be used as a reference plan for a lower-level microscopic controller [cf. 25]. Third, we limit the vehicle flow on a given road link to its capacity and assume vehicles travel at free-flow speed accordingly. Such a threshold congestion model is in line with the accuracy requirements of our mesoscopic viewpoint. If necessary, more sophisticated congestion models can easily be integrated into our modeling approach, at the cost of computational tractability. Fourth, our model does not explicitly account for congestion from non-AMoD traffic. However, this type of traffic can be considered by subtracting the corresponding flow from the residual road capacity \(\bar{f}_{v,w}^t\). Fifth, we assume that future trip requests are known or estimated with a high degree of accuracy. While the development of tools to estimate AMoD demand is beyond the scope of this paper, remarkably accurate algorithms are available in the literature [e.g., 26]. Sixth, we optimize only rebalancing trips and fix customer trips to their shortest-time-paths. In principle, including the optimization of customer-carrying trips could yield solutions with lower cost; however, our prior work has shown that the inclusion of customer-carrying trips in the optimization problem results in a small decrease in cost at the price of a significant increase in computational complexity [20]. Also, note that although the route of customer-carrying trips is fixed, the SoC of customer-carrying vehicles is part of the optimization problem. Finally, the electric AMoD problem Eq. (4) may become infeasible if the number or the distribution of customer trip requests exceeds the customer-carrying capacity of the electric AMoD system. Here, we assume that the problem is always feasible as the fleet operator can reject or postpone trip requests to ensure feasibility. This is in line with common practice in today’s taxi or ride-hailing platforms. Nonetheless, a mechanism to decide which trips should be rejected or postponed is beyond the scope of this paper.

### III. Modeling Unbalanced Power Distribution Networks

This section provides the basics for modeling unbalanced PDNs and presents the identification of a compatible convex power flow surrogate to model the integration of PDN into an electric AMoD model under a unified notation framework. First, we introduce an unbalanced PDN model in Section III-A. Then, we define the optimal power flow problem in Section III-B. Finally, we compare convex power flow surrogates in Section III-C and justify the selected surrogate.

#### A. Unbalanced Power Distribution Network Model

In the following, we consider only radial network structures which is the typical configuration for PDNs [23, Ch. 1.1] and base our notation on [22]. A radial PDN is modeled as a directed graph \(P = (N, E)\) with a tree topology, consisting of a set of buses \(N = \{0, \ldots, N\}\) and a set of links \(E \subset N^2\). Each PDN has a reference bus which typically denotes a substation that connects the PDN to the transmission network. The set \(N^+ = N \setminus 0\) contains all buses other than the reference bus 0. Buses are connected by links (e.g., power lines, transformers, regulators), such that \((n,o) \in E\) represents a link between \(n\) and \(o\) for which \(n\) lies in the single path between the reference bus 0 and bus \(o\). Note that there is only one such path because, by assumption, \(P\) is a tree.

We consider unbalanced PDNs with three phases \(\Phi = \{a, b, c\}\). In line with this, \(\Phi_{n,o} \subseteq \Phi\) is the set of phases in link \((n,o) \in E\). Further, the set of phases in bus \(n \in N\) comprises the phases of all links connected to the bus:

\[
\Phi_n = (\cup_{(m,n) \in E} \Phi_{m,n}) \cup (\cup_{(n,o) \in E} \Phi_{n,o}) \quad \forall n \in N.
\]

Each bus \(n\) has a time-invariant shunt admittance matrix \(Y_n \in \mathbb{C}^{|\Phi_n| \times |\Phi_n|}\), representing the admittance between the bus and ground. Further, each link \((n,o)\) has a time-invariant impedance matrix \(Z_{n,o} \in \mathbb{C}^{|\Phi_{n,o}| \times |\Phi_{n,o}|}\).

We consider a discrete-time model that tracks a series of steady states in the power network and neglects dynamic effects. This is appropriate if the discretization time is substantially longer than the time scale for the dynamic effects (i.e., in the order of minutes). We consider a timespan \(T = \{1, \ldots, T\}\) with time steps \(t \in T\), each having a length \(\Delta t \in \mathbb{R}^+\). Each bus \(n\) has a time-dependent complex voltage \(v_n^t\) and a complex power injection \(s_{i,inj,n}^t \in \mathbb{C}\) for each of its phases. Concurrently, each link shows a time-dependent current for each of its phases \(i_{n,o}^t \in \mathbb{C}\). For brevity, we use vectors for per-phase quantities: \(v_n = [v_n^a \in \Phi_n, v_n^b \in \Phi_n, v_n^c \in \Phi_n]\), \(s_{i,inj,n} = [s_{i,inj,n}^a \in \Phi_n, \ldots, s_{i,inj,n}^c \in \Phi_n]\), and \(i_{n,o} = [i_{n,o}^a \in \Phi_{n,o}, \ldots, i_{n,o}^c \in \Phi_{n,o}]\). Herein, superscripts represent the projection onto specific phases.

The current on each link obeys Ohm’s law, that is:

\[
i_{n,o}^t = Y_{n,o}((v_n^t)_{\Phi_{n,o}} - (v_o^t)_{\Phi_{n,o}}) \quad (n,o) \in E, t \in T,
\]
with $Y_{n,n} = Z_{n,n}^{-1}$ [22]. Each bus is either specified by its voltage or by its power injection such that the remaining quantity is a dependent variable [27, Ch. 6.4]. We refer to specified variables as direct variables and to those that are dependent as indirect variables. The reference bus specifies the reference voltage $v_0^\phi[t] \in \mathbb{R}$ for the network:

$$v_0^\phi[t] = v_0^\phi[t] \in \Phi_0, t \in \mathcal{T}.$$  

(5)

Accordingly, the complex voltage $v_0$ is the direct variable and the complex power injection $s_{sij,0}$ remains dependent.

For all other buses $n \in \mathcal{N}^+$, the complex power injection $s_{sij,n}$ is the direct variable, whereas the complex voltage $v_n$ remains dependent. These buses are called PQ buses since the active ($p$) and reactive power injection ($q$) are the direct variables. Herein, each PQ-bus has a time-varying uncontrollable load with complex power $s_{unc,n}[t] \in \mathbb{C}^{\mathcal{G},n}$. These loads represent electricity demand from residential and commercial customers. We consider uncontrollable loads to be exogenous but known in advance within timespan $\mathcal{T}$.

Controllable loads $\ell \in \mathcal{L} = \{1, \ldots, L\}$ are defined by a tuple $(s_{con,\ell}[t], n_{\ell}) \in \mathcal{C}^{\mathcal{G},n_{\ell}} \times \mathcal{N}$ denoting their complex power $s_{con,\ell}$ and its corresponding bus $n_{\ell}$. These loads represent dispatchable generators or loads that can be throttled. With this notation, the power injections at PQ-buses are

$$s_{sij,n}[t] = -s_{unc,n}[t] - \sum_{\ell=1}^{L} 1_{n=n_{\ell}} s_{con,\ell}[t] n \in \mathcal{N}^+, t \in \mathcal{T}. \tag{6}$$

Note that we model generators as negative loads without loss of generality. Further, we consider only wye-connected constant power loads which may require performing delta-to-wye conversions for some loads and approximating constant current and constant impedance loads as constant power ones. This simplification is common in optimization frameworks [28].

Dependent variables result from the network topology and its controllable and uncontrollable loads. Specifically, they are related by the power flow equation [29]

$$s_{sij,n}[t] = \text{diag}(v_n[t] v_n[t]^H Y_n^H) + \sum_{n:(n,o) \in E} \text{diag}(v_n^{\Phi_o}[t] (v_n^{\Phi_o}[t] - v_0^{\Phi_o}[t]) Y_{n,o}^{H}) \in \mathcal{T}. \tag{7}$$

Collectively, these equations allow us to model a radial time-invariant unbalanced PDN with time-varying controllable and uncontrollable loads.

A few comments are in order. First, we consider a discrete-time model that tracks a series of steady states in the power network. As we are not interested in dynamic effects, this model is appropriate, and the level of aggregation is aligned with our mesoscopic transportation model. Second, we consider a time-invariant PDN which cannot model control elements, e.g., step voltage regulators. Optimization frameworks commonly neglect these elements (see [24], [29]) as their inclusion substantially increases complexity while their omission results in a more conservative optimization. This simplification is appropriate for the purposes of a mesoscopic system-level analysis. Third, we assume that high-quality estimates of uncontrollable electrical loads are available. While deriving such estimates exceeds our scope, techniques to accurately estimate future power demand exist [e.g., 30].

### B. Optimal Power Flow problem

The Optimal Power Flow (OPF) problem Eq. (8) optimizes a power network’s state subject to its operational constraints and is often used to support grid-related decisions, e.g., operational or strategic planning, and pricing [31]. Here, we use an OPF problem for operational planning and decide on the controllable loads while optimizing a generic objective function $f(\cdot)$ subject to the power flow equation Eq. (7) and additional operational constraints:

$$\begin{align*}
\text{minimize} & \quad f(\cdot) \quad \text{(8a)} \\
\text{subject to} & \quad \text{Eq. (5)} \quad \text{Voltage at reference bus} \\
& \quad \text{Eq. (6)} \quad \text{Power injections} \\
& \quad \text{Eq. (7)} \quad \text{Power flow equation} \\
& \quad |v_n^\phi[t]| \geq v_{\min,n}^\phi, \quad \phi \in \Phi_n, n \in \mathcal{N}^+, t \in \mathcal{T} \quad \text{(8b)} \\
& \quad |v_n^\phi[t]| \leq v_{\max,n}^\phi, \quad \phi \in \Phi_n, n \in \mathcal{N}^+, t \in \mathcal{T} \quad \text{(8c)} \\
& \quad \sum_{\phi \in \Phi} s_0^\phi[t] \leq s_0 \quad t \in \mathcal{T} \quad \text{(8d)} \\
& \quad p_{\text{con,min},\ell}^\phi \leq p_{\text{con,max},\ell}^\phi \quad \phi \in \Phi_n, \ell \in \mathcal{L}, t \in \mathcal{T} \quad \text{(8e)} \\
& \quad q_{\text{con,min},\ell}^\phi \leq q_{\text{con,max},\ell}^\phi \quad \phi \in \Phi_n, \ell \in \mathcal{L}, t \in \mathcal{T} \quad \text{(8f)}
\end{align*}$$

Equations (5) to (7) denote the general power network model. Equations (8b) and (8c) constrain the voltage magnitude $|v_n^\phi[t]|$ to be within a minimal $v_{\min,n}^\phi \in \mathbb{R}$ and a maximal $v_{\max,n}^\phi \in \mathbb{R}$ value, according to regulations (e.g., ANSI C84.1). Equations (8d) limits the apparent power injected to the reference bus to be less than $s_0 \in \mathbb{R}^+$, typically, to respect the rating of the substation transformer. Equations (8e) and (8f) model the characteristics of controllable loads through lower and upper bounds on active power ($p_{\text{con,min},\ell}^\phi \leq p_{\text{con,max},\ell}^\phi \in \mathbb{R}$) and reactive power ($q_{\text{con,min},\ell}^\phi \leq q_{\text{con,max},\ell}^\phi \in \mathbb{R}$). The AMoD-OPF joint problem described in Section IV-C will leverage approximations of the operational constraints in Eq. (8) and include an electricity cost objective term.

This OPF problem is non-convex because of i) the power flow equation Eq. (7) and ii) lower bound constraints on voltage magnitudes Eq. (8b). Even the optimization of a balanced single-phase approximation of this problem remains an NP-hard problem [32].

### C. Convex power flow surrogates

We desire the joint AMoD-OPF problem to be convex and ideally linear to preserve computational tractability. Hence, we convexify the OPF problem Eq. (8) using a power flow surrogate that approximates the power flow equation Eq. (7) with a convex proxy, making the problem formulation computationally tractable. Using such a power flow surrogate, we lose exact knowledge of the indirect variables.

Given the high relevance of the OPF problem, a vast literature on power flow surrogates exists [31], [33]. However,
most of these surrogates, as well as comparative studies, consider only balanced single-phase models as typically used in transmission networks [34].

For unbalanced three-phase models, only a few power flow surrogates exist, and, to the best of our knowledge, no survey or benchmark classifies the suitability of these surrogates for specific problem structures, such as integration with the electric AMoD problem. To close this gap, we analyzed and compared three promising surrogates.

We compared a convex, semi-definite program (SDP) surrogate [22], the branch flow model SDP (BFM-SDP), against two linear surrogates: the branch flow model LP (BFM-LP) [22], [35] and the linearized power flow manifold LP (LPFM-LP) [36]. We used the charger maximization problem, which maximizes the power delivered to a series of charging stations across a distribution network as a benchmark, as it challenges the surrogates by pushing the network’s operational constraints to its limits. For each surrogate, we evaluated its accuracy in approximating the indirect variables used in Eqs. (8b) to (8d). Additionally, we analyzed the resulting constraint violations and computational times. We detail the methodology of our comparison in [37] but omit it in this paper due to space limitations. In summary, BFM-LP yielded exact solutions on small instances, but performed significantly worse than the other two surrogates. In summary, BFM-SDP yielded exact solutions on small instances but performed significantly worse than the other two surrogates. In summary, BFM-SDP yielded exact solutions on small instances but performed significantly worse than the other two surrogates. In summary, BFM-SDP yielded exact solutions on small instances but performed significantly worse than the other two surrogates.
A. Multi-OFP problem

The multi-OFP problem couples $D$ instances of the OPF problem and results straightforwardly by extending the constraints for each instance $d \in D = \{1, \ldots, D\}$.

We neglect couplings upstream of PDN substations through the transmission network as this paper focuses solely on the interaction between an electric AMoD system and a series of PDNs. Couplings between the electric AMoD system and the power network at the transmission and distribution level occur on very different spatial scales (tens of kilometers vs. hundreds of meters), and result in largely orthogonal effects: specifically, couplings at the transmission level mainly influence bulk electricity prices [20], whereas couplings at the distribution level influence bus voltages and power losses. Accordingly, due to the orthogonal nature of the two couplings, we envision that a nested optimization approach could be used to first address transmission-level couplings through existing algorithms [e.g., 20], and then optimize distribution-level couplings through the tools proposed in this paper.

B. Coupling of the electric AMoD system and power distribution networks

The charging stations, which appear as controllable loads in the PDNs, couple the electric AMoD system to the PDNs (see Fig. 1). Formally, this coupling is established by two functions, $M_{S,A_S}$ and $M_{S,L}$, defined below.

The function $M_{S,A_S} : S \times T \to A_S$ maps a charging station $s \in S$ for each time step $t \in T$ to all arcs in $A_S$ that represent charging vehicles at this station:

$$M_{S,A_S}(s,t) = \{(v, w) \in A_S | v = v = s, c_v < c_w, t_v \leq t \leq t_w\}.$$ 

Then, the load at charging station $s$ is given by

$$p_s(t) = E_c \delta_{C,s} \sum_{(v, w) \in M_{S,A_S}(s,t)} f_{0}(v, w) \quad s \in S, t \in T \quad (13)$$

The function $M_{S,L} : S \to (L \times D)$ maps a charging station $s \in S$ to the associated controllable load $l \in L$ and distribution network $d \in D$. It follows that charging station $s$ is attached to bus $n_{M_{S,L}}(s)$ in PDN $d_{M_{S,L}}(s)$. As we consider three-phase charging stations, we assume equally distributed loads, that is

$$s_{\text{con},M_{S,L}}(s)[t] = s_{\text{con},M_{S,L}}(s)[t] = \frac{1}{3} p_s(t) \quad s \in S, t \in T \quad (14)$$

Note that we can model inverters that control the load power factor since $q_{\text{con},M_{S,L}}(s)$ must not necessarily be zero. Although the charging station load is distributed equally among phases, loads in distribution networks are inherently unbalanced, which requires an unbalanced distribution model [23, Ch1.3] Also note that charging stations are commonly modeled to operate at unity power factor (no reactive power consumption) [39].

C. AMoD-OPF problem

The joint AMoD-OPF problem results from coupling the electric AMoD problem Eq. (4) with the multi-OFP problem through Eqs. (13) and (14), namely:

$$\min \sum_{s \in S} d_{0,v,w} f_{0}(v, w) \quad (v, w) \in A$$

subject to

Eqs. (1) to (3) and (4b) to (4d) Electric AMoD system

Eqs. (5), (6), (9) and (10) PDNs

Eqs. (8b) to (8f) Coupling from charging stations.

The objective Eq. (15a) captures operating costs for both the electric AMoD fleet and the PDNs since we consider full cooperation between both operators. Analogously to the isolated electric AMoD problem Eq. (4a), we consider only rebalancing costs for the AMoD fleet given fixed customer flows. In each distribution network $d \in D$, we account for the electricity cost that results from charging vehicles, uncontrollable loads, and power losses.

Note that our joint problem formulation treats both operators as a single entity, assuming complete information and cooperation. This assumption is in line with our mesoscopic view and scope to estimate the achievable benefits of coordination and cooperation between the two systems. We leave the study of game-theoretical aspects to future work where we intend to develop pricing and coordination mechanisms to align the goals of the electric AMoD operator and the PDN operators, and to leverage distributed optimization algorithms to compute a solution to the AMoD-OPF problem Eq. (15) in a distributed manner. Further, our joint model assumes that the electric AMoD system is the dominant means of electric transportation, which is in line with our system-level perspective [3]. However, the model can readily accommodate other EVs by including their traffic flow as residual capacity in Eq. (4b) and their charging as exogeneous loads in Eq. (8).

V. CASE STUDY IN ORANGE COUNTY, CA

We evaluate the impact of an electric AMoD system on the PDNs and the benefit of optimized joint coordination through a case study in Orange County, CA. Our case study considers commuting trips within the cities of Fountain Valley, Irvine, North Tustin, Orange, Santa Ana, Tustin, and Villa Park. In the following, we detail our data (V-A), outline the experimental design (V-B) and, finally, discuss our results (V-C).

A. Model parameters

We focus on an eight-hour commuting cycle from 5 am to 1 pm on July 3, 2015 discretized into six-minute time steps, such that $|T| = 80$. As we do not consider future grid storage devices, which would charge/discharge over the span of a day, an eight-hour horizon is sufficient to model the power system. We chose the time discretization to be close to the traversal
time of the shortest road link. As the power system considers hourly prices and excludes transient effects, six-minute time steps are more than sufficient to model PDNs for a mesoscopic analysis. For this period, we model the charging station and transportation networks at a mesoscopic aggregation level that allows a sufficient level of detail to analyze the interaction between an electric AMoD system and the PDNs, and ensures computational tractability.

- **Transportation network data**: we derive trip demand from Census Tract Flow data from the 2006-2010 American Community Survey. From these data, we take the estimated commuting flows between the 143 census tracts that are part of our case study. To align the granularity of aggregated charging station network representations and census tracts, we cluster the 143 census tracts into 20 larger areas using a k-means algorithm. We neglect commuting flows if they start or end outside the area of our case study or if they start and end within the same cluster since these types of flows cannot be accurately represented in our model. Our planning horizon comprises 122,219 trips (32.8% of the total daily trips).

The problem of fleet sizing for (electric) AMoD systems [40] is beyond the scope of this paper. For this case study, we heuristically selected a sufficient fleet size, large enough to keep the AMoD-OPF problem Eq. (15) feasible with only a small number of idle vehicles and corresponding to 140% of the peak concurrent number of passenger-carrying trips.

We create an aggregated road network based on OpenStreetMap data with the same granularity as the trip demand data. For this, we select the road network vertices closest to the centroids of the census tract clusters and add arcs between those vertices if a connection exists in the real road network. We obtain an aggregated road network with 20 vertices and 76 arcs (see Fig. 2), which captures vehicle travel and charging between the separate PDNs of the case study region. Note that computational complexity limits our model to coarse road networks; this is discussed in detail below. For each aggregated road network vertex, we consider three-phase 50-kilowatt DC fast charging stations with \( S_s = 40 \) plugs in total. Accordingly, each vertex has a charging station with a maximum load of two megawatts (0.66 megawatts per phase).

- **Electric vehicle data**: we consider a homogeneous vehicle fleet based on the characteristics of the 2018 Nissan Leaf which has a 40-kilowatt-hour battery and a range of 240 kilometers. Based on fast-charging guidelines, we reduce a vehicle’s battery capacity and its range to 80 percent of their original values [11], and discretize this effective battery capacity into \( C = 40 \) levels, resulting in energy discretizations of 0.8 kWh which remains close to the energy necessary to traverse the lowest energy road link. To account for the possibility that vehicles might not start the day with fully charged batteries, we set the SoC at \( t = 1 \) to 50%. Furthermore, we require vehicles to recharge the amount of energy used over a planning horizon such that the final SoC must be at a minimum 50% again. We set the vehicle operation cost per unit distance (excluding electricity) to \( V_D = 0.3 \text{USD/km} \) [41].

- **Power distribution networks data**: we use a GridLAB-D model of the PL-1 distribution network, a primary feeder operated by the Pacific Gas and Electric Company (PG&E) available for research purposes [42], as a proxy for (sub-)urban distribution networks. The network comprises 322 buses and operates at a nominal voltage of 12.6 kilovolts. We set the uncontrollable loads to the model’s time-varying loads.

We take the location of substations from the utility’s data [43] and attach a model of the PL-1 distribution network to each substation. We set the electricity price at each substation to the corresponding locational marginal price [44] and conservatively assume a base load utilization of 75 percent at the substation transformer. Typically, distribution networks are operated at 50 to 75 percent of their load capacity so that loads can be transferred from one distribution network to another if needed [45]. Accordingly, we set the substation transformer rating \( s_0 \) to 1/0.75 times the value of the peak base load (i.e., without charging stations), yielding \( s_0 = 10.42 \text{MVA} \). In addition, we set the lower voltage magnitude limit to 0.96 per-unit and the upper limit to 1.04 per-unit, which is 0.01 per-unit tighter than required by ANSI C84.1 to allow for the voltage drop in the secondaries of the network.

We connect each charging station to the distribution network whose substation is nearest. Since no data on the coordinates of the distribution network buses exist, we randomly attach the charging station to one of the PDN buses. Thus, the PDN is the same for each substation, except for the varying number and location of charging stations. In total, we consider 14 distribution networks, each with one or two charging stations.

We set the price of electricity at each charging station to be equal to the electricity price at the respective substation, such that \( V_{el,s}[t] = V_{el,d_{MS,C(s)}}[t] \) holds. Since we focus on the total benefit from a system perspective and treat both operators as a single entity, only the spatial variation of electricity prices that are closely linked to the substation prices affects our solution.

Some comments on the distribution network modeling are in order. First, we used the same network model and load values for each distribution network, considering loads from a single summer day. As PDNs are treated as critical infrastructure and load data is usually confidential to protect customers, more accurate data is not publicly available for research purposes [46]. However, our model can be rerun with more accurate
data at any time. Second, we set the electricity price at each substation to the corresponding locational marginal price. Locational marginal prices result from the power consumption at the transmission grid level. As our focus is on the interaction of the electric AMoD fleet with the distribution grids and the power used for recharging represents only a negligible fraction at the transmission grid level, neglecting the impact of this consumption on the marginal prices only minimally affects the accuracy of our results. Third, we assume the electricity price for charging at a certain station to be equal to the electricity price at the respective substation. Neglecting the possible difference in electricity prices among nodes in a single distribution network is consistent with our mesoscopic transportation model.

The resulting AMoD-OPF problem has 6,224,240 decision variables, 1,463,600 from the electric AMoD part and 4,760,640 from the multi-OPF part. Since the multi-OPF part comprises $D$ PDNs, the number of variables in it admits the upper bound $O(TD|N|)$. Thus, the number of decision variables in the whole AMoD-OPF problem admits the following upper bound: $O(T(C|V_R|^2 + D|N|))$. Recall that the complexity of solving the LP with an interior point method is polynomial in the number of variables with an exponent lower than 3.5 (depending on the implementation) [47]. Namely, the size of the electric AMoD part of the problem increases quadratically with the number of road vertices. However, if more vertices are added for the same area, the road segment arcs will become shorter, and $T$ and $C$ should be increased to capture the reduced travel duration and energy consumption in the shorter road segment arcs. Thus, in practice, the electric AMoD part of the problem grows more than quadratically with the number of road vertices. This limits our formulation to coarse road networks. In future work, we will explore methods that improve the scalability of the AMoD-OPF problem, extending its applicability to finer networks.

**B. Experimental design**

To quantify the impact of an electric AMoD system on the PDNs and the benefit of optimized joint coordination, our experiments consider two cases. First, we analyze the impact of an electric AMoD system on the PDNs without coordination, i.e., the uncoordinated case. This study shows how electric AMoD systems can negatively affect PDNs. Then, we focus on the coordinated case in which the electric AMoD system and the distribution networks are jointly optimized. Comparing the results of both cases allows us to quantify the potential of optimized coordination between these systems. In both cases we generate results as follows:

1) Computing controllable loads: we determine the load at each charging station that results from the operation of the electric AMoD system. Depending on the studied case, we solve either Eq. (4) (uncoordinated) or Eq. (15) (coordinated).

2) Solving the power flow equation: to assess the quality of a solution from step (a), we solve the exact power flow equation Eq. (7) to derive the true values of the indirect variables (i.e., complex power injection at the reference bus and complex voltage in all other buses).

c) Evaluating constraint violations: in step (a), we determine controllable loads without an exact model of the PDNs as it is either neglected (uncoordinated case) or approximated (coordinated case). Hence, it is often the case that solutions do violate some of the constraints. To quantify these violations, we evaluate integral constraint violations as we consider a time-variant model. Specifically, regulations require voltage magnitudes to be kept within a given percentage of a nominal value (e.g., ANSI C84.1). Hence, we analyze the integral absolute voltage magnitude constraint violation

$$u_{\text{viol}, \text{int}} = \Delta t \sum_{t \in T} \sum_{d \in D} \sum_{n \in N_d^+} \sum_{\phi \in \Phi_{n,d}} |u^{\phi}_{\text{viol}, n,d}[t]|$$

where

$$u^{\phi}_{\text{viol}, n,d}[t] = \min(u^{\phi}_{n,d}[t] - u^{\phi}_{\text{min}, n,d}, 0) + \max(u^{\phi}_{n,d}[t] - u^{\phi}_{\text{max}, n,d}, 0)$$

is the voltage magnitude constraint violation at phase $\phi \in \Phi_n$ in bus $n \in N$. Note that $u^{\phi}_{\text{viol}, n,d}$ is negative when the voltage magnitude is lower than $u^{\phi}_{\text{min}, n,d}$ positive when it is larger than $u^{\phi}_{\text{max}, n,d}$ and zero when it is in-between. Additionally, substations typically connect distribution networks to the higher-voltage transmission network, requiring a transformer to lower the voltage. To avoid overloading this transformer, the power draw must be less than the transformer rating. Hence, we analyze the integral substation transformer rating violation

$$\hat{s}_{0, \text{viol}, \text{int}} = \Delta t \sum_{t \in T} \sum_{d \in D} \hat{s}_{0,d,\text{viol}}[t]$$

where

$$\hat{s}_{0,d,\text{viol}}[t] = \max(|\sum_{\phi \in \Phi_{n,d}} s^{\phi}_{0,d}[t] - \hat{s}_{0,d}, 0)$$

is the substation transformer rating violation for $d \in D$.

d) Evaluating energy consumption and cost: we analyze the energy consumption of the electric AMoD system and its cost. The total energy consumption $E_{\text{total}, \text{base}}$, which includes the energy consumed by exogenous loads and the electric AMoD system, results from summing the energy draw of all substations. The total energy consumption in the base case $E_{\text{total,base}}$ results analogously without considering an electric AMoD system. Consequently, the difference of $E_{\text{total}}$ and $E_{\text{total,base}}$ represents the additional energy consumption caused by the electric AMoD system:

$$E_{\text{AMoD}} = E_{\text{total}} - E_{\text{total,base}} = \sum_{t \in T} \sum_{d \in D} \phi \in \Phi_{n,d} (p^{\phi}_{0,d}[t] - p^{\phi}_{\text{base},d}[t]).$$

Here, $p^{\phi}_{\text{base},d} \in \mathbb{R}$ is the power drawn in phase $\phi \in \Phi_0$ from substation $d \in D$ in the base case.

Due to losses in the distribution networks, not all of $E_{\text{AMoD}}$ relates to charging stations. The energy delivered to the charging stations is given by

$$E_{\text{charge,AMoD}} = \sum_{t \in T} \sum_{d \in D} \phi \in \Phi_{n,d} \sum_{\phi \in \Phi_{n,d}} p^{\phi}_{\text{con,}\ell,d}[t].$$
The difference between $E_{\text{AMoD}}$ and $E_{\text{charge,AMoD}}$ represents the link losses caused by the electric AMoD system:

$$E_{\text{loss,AMoD}} = E_{\text{AMoD}} - E_{\text{charge,AMoD}}.$$  

Analogously, the cost of these losses is given by

$$V_{\text{el,loss,AMoD}} = V_{\text{el,AMoD}} - V_{\text{el,charge,AMoD}}$$  

where $V_{\text{el,AMoD}}$ is given by

$$V_{\text{el,AMoD}} = \sum_{t \in T} \Delta t \sum_{d \in D} V_{\text{el},d}[t] \sum_{\phi \in \Phi_{o,d}} (p_{\phi,d}[t] - p_{\phi,\text{base},d}[t])$$

and $V_{\text{el,charge,AMoD}}$ is the cost of $E_{\text{charge,AMoD}}$:

$$V_{\text{el,charge,AMoD}} = \sum_{t \in T} \Delta t \sum_{d \in D} V_{\text{el},d}[t] \sum_{\ell \in \mathcal{L}_d} \sum_{\phi \in \Phi_{n,v,d}} p_{\phi,\text{con},\ell,d}[t].$$

Our implementation builds on top of the authors’ AMoD Toolkit\(^1\) which relies on YALMIP \(^2\) to formulate and solve electric AMoD problems. Additionally, we built a general codebase for unbalanced OPF problems, the Unbalanced OPF Toolkit\(^3\). To support future research in this field, we released both the AMoD Toolkit and the Unbalanced OPF Toolkit under an open-source license.

\(\text{C. Results and discussion}\)

Following our experimental design, we evaluate constraint violations (Fig. 3), as well as energy consumption and costs. Table I summarizes the key results.

Figure 3a shows a histogram with all voltage magnitude constraint violations $u_{\text{viol},n,d}^q$, each event represents the constraint being violated in one phase during one of the six-minute time steps. The base case shows no violations and, hence, is not plotted. In contrast, violations appear in both cases that include the electric AMoD system. ANSI C84.1, the power quality standard for voltage ranges used across the United States, advises that service voltage violations must be limited in extent, frequency, and duration. Optimized coordination between the electric AMoD system and the PDNs helps to decrease voltage constraint violations significantly. The number of voltage constraint violations is reduced by 3.85 percent in the coordinated case, from 46,910 to 45,106.

Notably, coordination reduces the number of serious violation events (i.e., those exceeding 0.005 p.u. which are the most concerning, see Fig. 3a) by 74.85 percent, from 21,734 to 5,467. All in all, there is a 50.28 percent reduction in integral absolute voltage magnitude constraint violation, from 24.04 per-unit hour to 11.95 per-unit hour. Consequently, coordination between the two systems helps to achieve better compliance with regulations that require the voltage magnitude to be kept close to its nominal value.

Figure 3b shows a histogram with all substation transformer rating violations $s_{0,d,\text{viol}}$; each event represents the constraint being violated in one substation transformer during one of the six-minute time steps. Optimized coordination nearly eliminates substation capacity constraint violations, reducing their count by 94.05 percent from 168 to 10. The number of substations that experience a transformer rating violation is reduced from six to two. All in all, there is a 99.71 percent reduction in integral substation transformer rating violation, from 7.89 mega volt-ampere hour to 0.02 mega volt-ampere hour.

Transformers represent a significant investment by utilities. For example, installing a transformer with a rating similar to the one used in this case study ($s_0 = 10.42 \text{ MVA}$) has a cost in the order of 1.7 million USD \(^4\). Given transformers’ substantial cost, increasing their useful life by reducing transformer capacity threshold violations (as done by coordination) can lead to significant monetary savings for utilities. We leave the precise quantification of these savings for future research.

Figure 4 shows the load at one representative substation along with the applicable transformer rating. The load is shown for the three cases: base, uncoordinated and coordinated. The base case represents the substation load arising from the uncontrollable loads. The other two cases show higher loads due to the recharging vehicles. In the uncoordinated case, there is a significant transformer rating violation between 8 am and 11 am. Coordination helps to resolve the violation, as charging loads that exceed the capacity constraint are shifted to later time steps.

Figure 5 shows the number of charging vehicles and the electricity price over time. The coordinated case shows steady charging activity after 11 am. In contrast, charging activities decrease significantly after 11 am in the uncoordinated case. The charging activity mirrors the substation load in Fig. 4 which is higher for the coordinated case in later time steps.

The increased charging activity later in the day and the ensuing load leads to increased electricity expenditure as the electricity price is higher later in the day.

Table I shows the impact of coordinating an electric AMoD fleet with PDNs. The total operational costs of the electric AMoD system during the studied 8-hour time span increase slightly by 3.13 percent (3,329.61 USD). Rebalancing costs show an increase of 3.28 percent (3,206.47 USD) as vehicles charge at more distant charging stations due to an increase in rebalancing detours. The shift of charging activity to later in the day due to coordination causes electricity costs to increase by 1.42 percent (123.15 USD). The small increase in operational costs reflects the price paid for reducing system constraint violations, which improves voltage profiles and prolongs transformer life.

| Case          | Number of Charging Vehicles | Electricity Price Increase |
|---------------|-----------------------------|---------------------------|
| Base          | 100                         | 3.13%                     |
| Uncoordinated | 150                         | 3.28%                     |
| Coordinated   | 100                         | 1.42%                     |

The energy delivered to the charging stations (see Table I) increases by 1.82 megawatt-hour (0.68 percent) in the coordinated case because of increased rebalancing detours. However, the energy attributable to the electric AMoD system consumed at the substations increases only by 1.24 megawatt-hour (0.44 percent). The difference of 0.58 megawatt-hour is due to energy losses being reduced by 5.24 percent. Reduced energy losses reflect more efficient power distribution: a greater share of the energy leaving the substations reaches the charging stations in the coordinated case (96.29 percent compared with 96.07 percent).

The optimization was performed on an AWS r4.xlarge instance (4 vCPU at 2.3 GHz, 30.5 GB RAM). The AMoD-OPF

\(^1\)https://github.com/StanfordASL/AMoD-toolkit
\(^2\)https://github.com/StanfordASL/unbalanced-opf-toolkit
\(^3\)https://github.com/StanfordASL/unbalanced-opf-toolkit
\(^4\)USD
Table I: Impact of coordinating an electric AMoD fleet with PDNs. Coordination significantly reduces constraint violations at the cost of slightly higher operational costs.

|                     | Unit   | Uncoord | Coord | Change       |
|---------------------|--------|---------|-------|--------------|
| Voltage violation   | p.u. h | 24.04   | 11.95 | −50.28%      |
| Capacity violation  | MVAh   | 7.89    | 0.02  | −99.71%      |
| Electricity cost, charging | USD    | 8.35k   | 8.49k | 1.67%        |
| Electricity cost, losses | USD    | 0.35k   | 0.33k | −4.59%       |
| Electricity cost, AMoD | USD    | 8.60k   | 8.82k | 1.42%        |
| Rebalancing cost    | USD    | 97.79k  | 101.00k | 3.28%      |
| Total cost, AMoD    | USD    | 106.49k | 109.82k | 3.13%      |
| Energy, charging    | MWh    | 268.82  | 270.63 | 0.68%       |
| Energy, losses      | MWh    | 11.01   | 10.43  | −5.24%      |
| Energy, AMoD        | MWh    | 279.82  | 281.06 | 0.44%       |

We investigated the impact of an electric AMoD system on the PDNs. Herein, we especially focused on the benefits of coordination between the two systems and discussed results for a case study in Orange County, CA. We showed that in an uncoordinated system, the electric AMoD fleet negatively affects the distribution networks: the charging behavior of the electric AMoD vehicles caused overloads at substation transformers and violated (lower) voltage magnitude limits. Furthermore, we showed that a coordinated system helps to balance the load in the PDNs in time and space. Specifically, link losses were slightly reduced, substation overloads were nearly eliminated, and voltage violations were halved. Nonetheless, these reductions in constraint violations increased the cost of operating the electric AMoD system by 3.13 percent caused by vehicles driving to charge in less congested but more distant stations and charging when electricity prices are higher. This indicates that distribution networks can support more electric vehicles before upgrades are needed if the vehicles are charged in coordination with exogenous loads in the PDNs. Due to our system-optimal objective, these findings remain an assessment of the overall benefit of coordination between an electric AMoD fleet and PDNs.

VI. CONCLUSION

We presented the AMoD-OPF problem, which integrates an electric AMoD problem with a multi-OPF problem. In this context, we discussed power flow surrogates to obtain a computationally tractable convex problem formulation. The resulting AMoD-OPF problem allows one to assess the achievable benefit of coordinating an electric AMoD system and a series of PDNs. With this methodological framework, we investigated the impact of an electric AMoD system on the PDNs. Herein, we especially focused on the benefits of coordination between the two systems and discussed results for a case study in Orange County, CA. We showed that in an uncoordinated system, the electric AMoD fleet negatively affects the distribution networks: the charging behavior of the electric AMoD vehicles caused overloads at substation transformers and violated (lower) voltage magnitude limits. Furthermore, we showed that a coordinated system helps to balance the load in the PDNs in time and space. Specifically, link losses were slightly reduced, substation overloads were nearly eliminated, and voltage violations were halved. Nonetheless, these reductions in constraint violations increased the cost of operating the electric AMoD system by 3.13 percent caused by vehicles driving to charge in less congested but more distant stations and charging when electricity prices are higher. This indicates that distribution networks can support more electric vehicles before upgrades are needed if the vehicles are charged in coordination with exogenous loads in the PDNs. Due to our system-optimal objective, these findings remain an assessment of the overall benefit of coordination between an electric AMoD fleet and PDNs.

Our findings open the field for multiple directions of future research. First, our AMoD-OPF problem is mesoscopic and assumes perfect knowledge of future loads and trip requests. To design a real-time algorithm, the integration of forecasts to capture the stochastic nature of the problem is an interesting avenue for further research. Second, we modeled the operators of the AMoD fleet and the PDNs as a single entity, implying full cooperation. In future work, one should address...
the interplay between these two stakeholders, with the goal of designing incentive mechanisms, and investigate market dynamics, e.g., the price of stability and the price of anarchy. Third, our case study provides preliminary results about the benefit of coordinating electric AMoD fleets with PDNs. To provide decision support to practitioners, additional case studies that capture different PDNs, different road network characteristics, varying instance sizes, and distributed renewable energy generation are required. Fourth, our case study did not consider the EVs potential to feed power back into the PDN. Hence, extending our modeling approach for vehicle-to-grid options, evaluating regulation and operating reserve potentials, remains a promising avenue for future research.

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APPENDIX A

NOMENCLATURE

AMoD system

\( \mathcal{A} \) set of expanded graph arcs

\( \mathcal{A}_R \) set of road arcs

\( \mathcal{A}_S \) set of expanded graph arcs representing a recharging process

\( \mathcal{A}_T \) set of expanded graph arcs representing a physical time-dependent movement in the road network

\( \mathcal{C} \) set of discrete battery charge levels

\( c_{v,w} \) SoC associated to expanded vertex \( v \in \mathcal{V} \)

\( d_{v,w} \) energy consumption for traversing road arc \( (v, w) \in \mathcal{A}_R \)

\( E_v \) distance of road arc \( (v, w) \in \mathcal{A}_R \)

\( f_n \) network flow for rebalancing vehicles

\( f_{v,w,t} \) maximum capacity of road arc \( (v, w) \in \mathcal{A}_R \)

\( f_{v,w,t}^{\text{residual}} \) residual road capacity

\( \mathcal{M}_s \) set of customer trip requests

\( \mathcal{M}_{S,s} \) maps a charging station \( s \) for each time step \( t \) to all arcs in \( \mathcal{A}_S \) that represent charging vehicles at this station

\( \mathcal{M}_{S,C} \) maps a charging station \( s \) to the associated controllable load \( \ell \) and distribution network \( d \)

\( \mathcal{S} \) set of chargers in the road network

\( S_s \) number of charging plugs in charging station \( s \in \mathcal{S} \)

\( T \) set of time steps

\( m \) departure timestep of trip request \( m \in \mathcal{M} \)

\( v \) time step associated to expanded vertex \( v \in \mathcal{V} \)

\( w \) time to traverse road arc \( (v, w) \in \mathcal{A}_R \)

\( \mathcal{V} \) set of expanded graph vertices

\( V_d \) vehicle operation cost per unit distance (excluding electricity)

\( V_{d,l} \) price of electricity at the substation of network \( d \in \mathcal{D} \)

\( V_{d,s} \) price of electricity in charging station \( s \in \mathcal{S} \)

\( v_m \) origin of trip request \( m \in \mathcal{M} \)

\( \mathcal{V}_R \) set of road vertices

\( w_c \) road vertex associated to expanded vertex \( v \in \mathcal{V} \)

\( w_m \) destination of trip request \( m \in \mathcal{M} \)

\( \delta_{C,s} \) charging rate of charger \( s \in \mathcal{S} \)

\( \Delta t \) length of a time step

\( \lambda_{c,\text{dep}} \) number of vehicles with charge \( c \) departing to serve customer trip request \( m \)

\( \lambda_{c,\text{arr}} \) number of vehicles with charge \( c \) arriving at time \( t \) after serving customer trip request \( m \)

OPF problem

\( \mathcal{D} \) set of distribution networks

\( \mathcal{E} \) set of links

\( \mathcal{E}_n \) complex current through link \( (n, o) \in \mathcal{E} \)

\( \mathcal{F}_n \) set of controllable loads

\( \mathcal{N} \) set of buses

\( n_l \) reference bus of controllable load \( l \in \mathcal{L} \)

\( s_{\text{cont},l} \) complex power of controllable load \( l \in \mathcal{L} \)

\( s_{\text{inj},l} \) complex power injection at phase \( \phi \in \Phi_n \) in bus \( n \in \mathcal{N} \)

\( s_{\text{unc},l} \) complex power of uncontrollable load in bus \( n \in \mathcal{N}^+ \)

\( v_{\phi,l} \) complex voltage at phase \( \phi \in \Phi_n \) in bus \( n \in \mathcal{N} \)

\( \mathcal{Y}_n \) shunt admittance matrix of bus \( n \in \mathcal{N} \)

\( Z_{n,o} \) impedance matrix of link \( (n, o) \in \mathcal{E} \)

\( \Phi \) set of phases

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Alvaro Estandia is a Software Engineer at Marain Inc. He earned an MSc in Robotics in 2018 and a BSc in Mechanical Engineering in 2015, both from ETH Zurich. He develops software to simulate and algorithms to control fleets of electrical autonomous vehicles providing mobility-on-demand in urban environments. More broadly, he is interested in the applications of optimization for improving the performance of transportation systems and the power network.
Maximilian Schiffer is an Assistant Professor of Operations and Supply Chain Management at Technical University of Munich. He received a Ph.D. degree in Operations Research from RWTH Aachen University in 2017. His main research interests are in operations research, machine learning, and intelligent systems, with an emphasis on transportation and logistics topics, especially electric vehicles and autonomous systems. He is a recipient of the INFORMS TSL Dissertation Prize and the GOR Doctoral Dissertation Prize.

Federico Rossi is a Robotics Technologist at the Jet Propulsion Laboratory, California Institute of Technology. He earned a Ph.D. in Aeronautics and Astronautics from Stanford University in 2018, a M.Sc. in Space Engineering from Politecnico di Milano, and the Diploma from the Alta Scuola Politecnica in 2013. His research focuses on optimal control and distributed decision-making in multi-agent robotic systems, with applications to planetary exploration and coordination of fleets of self-driving vehicles for autonomous mobility-on-demand.

Justin Luke is a Ph.D. Candidate at Stanford University in the Autonomous Systems Laboratory and Sustainable Systems Laboratory. He earned a B.S. in Energy Engineering from the University of California, Berkeley in 2018. His research focuses on optimization methods for integration of electric autonomous mobility-on-demand fleets into the electricity grid, particularly in scenarios with high-penetration of renewable generation.

Emre Kara received the Ph.D. degree from the Carnegie Mellon University focusing on infrastructure systems, machine learning, and data science. He is currently leading the engineering and data science efforts with eIQ Mobility. His research interests include data-driven methods to integrate HVAC, electric vehicles, and battery storage systems into the electricity grid as flexibility assets.

Marco Pavone is an Associate Professor of Aeronautics and Astronautics at Stanford University, where he is the Director of the Autonomous Systems Laboratory. He received a Ph.D. degree in Aeronautics and Astronautics from MIT in 2010. His main research interests are in the development of methodologies for the analysis, design, and control of autonomous systems, with an emphasis on self-driving cars, autonomous aerospace vehicles, and future mobility systems. He is currently an Associate Editor for the IEEE Control Systems Magazine.

Ram Rajagopal is an Associate Professor of Electrical Engineering as well as Civil and Environmental Engineering at Stanford University, where he directs the Sustainable Systems Lab, focused on large-scale monitoring, data analytics, and stochastic control for infrastructure networks, in particular, power networks. He received the Ph.D. degree in Electrical Engineering and Computer Sciences from the University of California, Berkeley. His current research interests are in the integration of renewables, smart distribution systems, and demand-side data analytics.