On Stability of the Three 3-brane Model

Debajyoti Choudhury, Dileep P. Jatkar, Uma Mahanta and Subrata Sur

Mehta Research Institute of Mathematics and Mathematical Physics
Chhatnag Road, Jhusi, Allahabad 211 019, INDIA

We show that the Goldberger-Wise mechanism for the three 3-brane scenario proposed by Kogan et al. stabilizes the radion. We find that the system of 3-branes stabilizes in such a way that the loss in the scale factor is insignificant. That is, the negative tension brane chooses to stay close to the visible brane.

* e-mail: debchou,dileep,mahanta,subrata@mri.ernet.in
1. Introduction

In the quest to explore physics beyond the Standard Model (SM), certain new proposals\cite{1-3} have held considerable interest of the community. Amongst these is the proposal by Randall and Sundrum\cite{2} wherein the SM fields live on one of the two 3-branes which are end of the world branes of a five dimensional spacetime. This scenario can be embedded in higher dimensional spacetime as well. One of the attractive features of this proposal is the resolution of the hierarchy problem between the Planck scale and the electroweak scale. This is achieved by choosing the geometry of the embedding spacetime with an exponential warp factor for the four dimensional spacetime which is the worldvolume of the 3-brane. This exponential warp factor produces a difference in the mass scales between the two end of the world 3-branes.

There are several variants of the Randall-Sundrum (RS) model involving multiple branes\cite{4,5}, intersecting brane configurations\cite{6} and supersymmetry\cite{7-9}. Many of these models including the RS model have atleast one negative tension brane. This is demanded by the requirement of charge neutrality in the compact space. For example, in the original RS model, the visible world lives on the negative tension brane (i.e., the one with the induced cosmological constant $\Lambda_{\text{vis}} < 0$).

Any proposal for physics beyond the SM has to pass very stringent tests laid down by the SM itself as well as by the cosmological observations that have been made till date. Brane world cosmology has been studied in refs.\cite{10-17}. These papers conclude that the rate of expansion of the universe in the RS model is different from that of the familiar Friedman-Robertson-Walker cosmology in four dimensions. In particular, in RS models, the Hubble parameter is proportional to the energy density $\rho$, i.e., $H \sim \rho$. In contrast, in our universe, the Hubble parameter seems to behave as $H \sim \sqrt{\rho}$. It is, however, possible to circumvent this problem if one realizes that the total energy density $\rho$ is the sum of the vacuum energy density, i.e., the brane tension and the matter energy density $\rho_m$ which lives on the brane. In this case, dependence of the Hubble parameter on the energy density is $H \sim \sqrt{\Lambda_{\text{vis}} \rho_m}$. This resolution brings up a new problem though. If the RS model is to solve the hierarchy problem, the visible brane needs to have a negative tension. It turns out that the Hubble parameter on such a brane could be real only if the matter energy density is negative. However, this would lead to antigravity in the brane world, a consequence clearly at variance with experimental observations.

Another issue which was not dealt with in the original RS proposal\cite{2} was that of the stability of end of the world brane model. This was resolved by Goldberger and
They introduced a bulk scalar field into the model and showed that its (minimal) coupling to the bulk gravity stabilizes the two end of the world 3-branes at a critical distance $r_c$. For all parameters of $o(1)$ this can generate a warp factor of the order $10^{15}$ GeV thereby relating the Planck Scale to the SM scale.

Recently, Kogan et al.\cite{23} proposed a modified version of the RS model which contains three 3-branes with both the end of the world branes (one at $y = 0$ and the other at $y = L_2$) being of positive tension and the third (moving) brane with negative tension (at $y = L_1$). We will call this the three 3-brane model. The visible world lives on one of the end of the world branes (at $y = L_2$). This construction automatically ensures that the cosmological constant on our brane is positive and thereby avoids the problem of the Hubble parameter. This model has an exponential warp factor between either of the positive tension branes and the negative tension brane. The phenomenological consequences are rather striking and have been studied in refs.\cite{5,24}.

Stability of the Kogan et al.\cite{23} proposal is quite crucial. This is because the moving brane has negative tension and we have exponential warp factors growing from the negative tension brane to end of the world positive tension branes. Gain in the scale due to the warp factor between the positive tension brane at $y = 0$ and the negative tension brane at $y = L_1$ is lost partially by the time it reaches our universe at $y = L_2$. It is easy to see that if this construction of three 3-brane system stabilizes for $L_1 \sim L_2/2$ then the loss in the scale hierarchy is near total. It is, therefore, crucial that $L_1/L_2 \lesssim 1$.

In the light of this, we take up the issue of stability of the Kogan et al. model\cite{23} in this paper. We use the method of Goldberger and Wise\cite{18} of coupling a bulk scalar field to the brane system. We find that the system does stabilize, and remarkably enough, for $L_1/L_2 \approx 1$. We also find that this result is not very sensitive to small changes in the mass $m$ of the bulk scalar field as well as small fluctuations of the parameter $k$ appearing in the warp factor as long as $m \ll k$.

2. Stabilization of the three 3-brane Model

In this section, we will first briefly review the Kogan et al. model\cite{23} and will then proceed with the question of modulus stabilization. As mentioned in the introduction, the model of Kogan et al. contains three parallel 3-branes located at $L_0 = 0$, $L_1$ and $L_2$. The

\footnote{For other as well as related proposals of stabilization of the radion modulus see \cite{3,19,22}.}
fifth dimension $y$ has orbifold geometry $S^1/Z_2$ and $L_0$ and $L_2$ are orbifold fixed points. The action for this configuration is

$$S = \int d^4x \int_{-L_2}^{L_2} dy \sqrt{G} \{2M^3R - \Lambda_{\text{bulk}} \} - \sum_i \int_{y=L_i} d^4x V_i \sqrt{-\hat{G}^{(i)}}, \quad (2.1)$$

where $\hat{G}_{\mu\nu}^{(i)}$ is the induced metric on the branes and $V_i$ are their tensions. The five-dimensional metric ansatz is given by

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 dy^2. \quad (2.2)$$

This metric preserves four dimensional Poincare invariance. The warp factor $\sigma(y)$ is just a constant conformal scale factor for the induced four dimensional metric. Although it is constant on a given 3-brane, its numerical value is different on different 3-branes.

For the three 3-brane model, the warp factor grows on either side of the negative tension brane up to the positive tension branes. The warp factor $\sigma(y)$ which solves the equations of motion obtained from eq.(2.1) is

$$\sigma(y) = kr_c(L_1 - ||y| - L_1|) \quad (2.3)$$

when the brane tensions are $V_0 = -\Lambda/k$, $V_1 = \Lambda/k$ and $V_2 = -\Lambda/k$. Due to the warp factor, the physical mass $M$ on the visible brane situated at $y = L_2$ is related to the naive mass parameter $M_0$ of the four-dimensional Minkowski theory by $M = WM_0$, where,

$$W = \exp \left( -kr_c(2L_1 - L_2) \right) \quad (2.4)$$

is the warp factor between the visible brane and the other end of the world brane. As mentioned earlier, to solve the hierarchy problem between the Planck scale and the SM scale, $W$ should be of $o(10^{-15})$. Notice that in the three 3-brane model, if the moving brane located at $y = L_1$ stabilizes close to $L_1 = L_2/2$ then $W \sim 1$. In such a situation, we will not generate exponential scale factor in this model despite having the visible brane stabilized reasonably far from the other end of the world brane. Thus the ratio $L_1/L_2$ is as important, if not more, as the absolute stability of the three 3-brane system.

In order to study the stability of the three 3-brane model, consider coupling a bulk scalar field to the system of three 3-branes. Our technique is a straightforward generalization of the GW mechanism\[18\]. Consider the action for a bulk scalar field of mass $m$.

$$S_{\text{Bulk}} = \frac{1}{2} \int d^4x \int_{-L_2}^{L_2} dy \sqrt{G} (G^{AB} \partial_A \chi \partial_B \chi - m^2 \chi^2), \quad (2.5)$$
where $G_{AB}$ is the five dimensional metric given in eq.(2.2) with $\sigma(y)$ given in eq.(2.3).

Following ref.[18], we impose the condition that the bulk scalar field satisfies certain boundary conditions at the location of each of the branes. The boundary potentials are

$$
S_0 = \int d^4x \lambda_0 (\chi^2 - v_0^2)^2 \quad \text{at } y = 0,
$$

$$
S_{L_1} = \int d^4x \lambda_1 (\chi^2 - v_1^2)^2 \quad \text{at } y = L_1,
$$

$$
S_{L_2} = \int d^4x \lambda_2 (\chi^2 - v_2^2)^2 \quad \text{at } y = L_2.
$$

We consider only those configuration of the bulk scalar which solve the equations of motion subject to the condition that the boundary potentials (at $y = 0$, $L_1$ and $L_2$) are minimised. This essentially amounts to neglecting dynamics of $\chi$ along the directions tangential to any of the 3-branes. This assumption is reasonable because we are looking only at the stability of the three 3-brane system at the moment and are not studying the phenomenological consequence of possible coupling of the bulk scalar field $\chi$ to matter fields living on the branes. It, therefore, suffices to concentrate on the equation of motion of $\chi$ only in $y$ direction. Equation of motion for $\chi$ along $y$ is

$$
\partial^2_y \chi - 4\sigma'(y)\partial_y \chi - m^2 r_c^2 \chi = 0 \quad (2.7)
$$

where $\sigma'(y) = d\sigma(y)/dy$. It is straightforward to find the solution to this equation and it is given by

$$
\chi(y) = \exp (2\sigma'(y)y)[\bar{A}\exp (\sigma'(y)\nu y) + \bar{B}\exp (-\sigma'(y)\nu y)]
$$

(2.8)

where, $\nu = \sqrt{4 + (mr_c)^2/\sigma''(y)}$, $(\bar{A}, \bar{B}) = (A, B)$ for $0 \leq y \leq L_1$ and $(\bar{A}, \bar{B}) = (C, D)$ for $L_1 \leq y \leq L_2$. It is worth mentioning here that the formal solution (2.8) is a function of $\sigma'(y)$ but within any given range of values of $y$, e.g. $0 \leq y \leq L_1$, $\sigma'(y)$ is independent of $y$. The coefficients $A$, $B$, $C$ and $D$ are determined by demanding that $\chi$ minimizes the boundary potential. This gives,

$$
A = \frac{v_0 - v_1 X^{2-\nu}}{1 - X^{-2\nu}}, \quad B = \frac{v_1 X^{2-\nu} - v_0 X^{-2\nu}}{1 - X^{-2\nu}}
$$

$$
C = \frac{v_1 X^{\nu-2} - v_2 Y^{\nu-2}}{X^{2\nu} - Y^{2\nu}}, \quad D = \frac{-v_1 X^{\nu-2} Y^{2\nu} + v_2 X^{2\nu} Y^{\nu-2}}{X^{2\nu} - Y^{2\nu}}
$$

(2.9)

where, for notational convenience, we have made a change of variables from $L_1$ and $L_2$ to $X = \exp (-kr_c L_1)$ and $Y = \exp (-kr_c L_2)$. Substituting this solution into the action and
integrating out $y$ gives a four dimensional effective potential for $X$ and $Y$. Writing this effective potential in terms of $X$ and a new variable $R \equiv Y/X$, we have,

$$k^{-1}V(X, R) = \frac{1}{1 - X^{2\nu}} \left[ (\nu + 2)(X^{\nu}v_0 - X^{2\nu}v_1)^2 + (\nu - 2)(v_0 - X^{\nu+2}v_1)^2 \right]$$

$$+ \frac{X^4}{R^4(1 - R^{2\nu})} \left[ (\nu + 2)(R^2v_1 - R^{\nu}v_2)^2 + (\nu - 2)(v_2 - R^{\nu+2}v_1)^2 \right]. \quad (2.10)$$

An important thing to notice at this point is that, for arbitrary positive values of $\nu$, the potential (2.10) grows as $X \to 1$ or as $R \to 1$ as long as $v_1 \neq v_0$ and $v_1 \neq v_2$. These two limits correspond to the negative tension brane approaching the positive tension brane at $y = 0$ and at $y = L_2$ respectively. In fact, the potential has the following singularity in these limits: $V(X, R) \sim (1 - X^{2\nu})^{-1} > 0$ as $X \to 1$ and $V(X, R) \sim (1 - R^{2\nu})^{-1} > 0$ as $R \to 1$. This implies that the negative tension brane experiences repulsive forces exerted on it by both the positive tension end of the world branes and thus the three 3-brane model cannot reduce to the two 3-brane model. In other words, for a fixed value of $L_2$, $L_1$ lies strictly inside the interval $(0, L_2)$. However, there is an interesting caveat in this and that has to do with the choice of vev’s of the bulk scalar field on the 3-branes. If $v_1 = v_0$ and/or $v_1 = v_2$, then the leading singularity in the potential is removed and the subleading terms in the potential are attractive. In other words, for $v_1 = v_2$, the potential (2.10) is such that for $R$ close to 1, $V$ is attractive and assumes a finite value for $R = 1$. In this case, therefore, the two 3-brane limit is a stable one. The situation is similar, though not identical, for $v_1 = v_0$ and $X \to 1$. From here onwards we will work with the case when $v_0$, $v_1$ and $v_2$ take different numerical values. We still need to find out whether it is possible to stabilize $L_2$. An equally important question relates to the magnitude of $L_2 - L_1$. This number is important to get the correct metric scale factor on the visible brane.

We will first show how the three 3-brane system is stabilized by the bulk scalar field. In the process we will also be able to determine $L_2 - L_1$. Change of variables from $X$, $Y$ to $X$ and $R$ has simplified the form of the potential (2.10) to a considerable degree, yet, minimization remains a complicated task. The first point to notice here is that the potential is bilinear in $v_i$ and hence the results depend only on the ratios

$$r_0 \equiv \frac{v_0}{v_1}, \quad r_2 \equiv \frac{v_2}{v_1}. \quad (2.11)$$

Furthermore, this choice of variables is such that the Hessian matrix is automatically diagonal on the extremal locus. Extremizing $V$ with respect to $R$, we get

$$r_2^\mp (R) = \frac{\nu R^{2+\nu} \left[ (2 \mp \sqrt{\nu^2 - 4}) \left( R^{2\nu} - 1 \right) + \nu \left( 1 + R^{2\nu} \right) \right]}{2 \left( \nu^2 R^{2\nu} + 2 \left( R^{2\nu} - 1 \right)^2 + \nu \left( R^{4\nu} - 1 \right) \right)}. \quad (2.12)$$
Fig. 1: As the ratio $r_2$ of the vev of $\chi$ on the visible brane to that on the negative tension brane approaches 1, location of these two branes coincides. However, for $R < 1$ we have two possible values for $r_2$ as can be seen in this graph.

An analytic inversion of this equation is not possible on account of its transcendental nature. Numerical solutions are straightforward though and we present the results in Fig.1. For this purpose, instead of working with $\nu$, it is instructive to consider

$$\epsilon \equiv \nu - 2 \approx \frac{m^2}{4k^2}, \quad (2.13)$$

with the approximate equality holding for a light bulk scalar field.

Although the solutions of eq.(2.12) exist for a wide range of $\epsilon$, we are interested in small $\epsilon$’s and shall limit ourselves to this regime. In Fig.1, the left and right branches correspond to $r_2^-$ and $r_2^+$ respectively. At the cusp ($r_2 = 1$), the solutions move into unphysical domains. Looking at the figure, we surmise easily that $R \approx 1$ demands $r_2 \approx 1$. This is particularly pronounced for small $\epsilon$.

While the solutions eq.(2.12) represent minima in the $R$-direction, it remains to be seen whether simultaneous minima in the $X$-direction exist. We answer this question next. Extremizing with respect to $X$, we obtain

$$r_0^\pm(X; R, r_2) = \frac{X^{2-\nu}}{2\nu} \left[ 2 \left( 1 - X^{2\nu} \right) + \nu \left( 1 + X^{2\nu} \right) \pm \frac{X^{2+\nu}(1 - X^{2\nu})\sqrt{B}}{\sqrt{1 - R^{2\nu}}} \right], \quad (2.14)$$
where

$$\mathcal{B} = (1 - R^{2\nu}) \left( \nu^2 + 4\nu - 12 \right) - 8\nu + 16\nu R^{-2} r_2 + 16 r_2^2 - 4 r_2^4 \left( \nu + 2 \right) \left( R^{2\nu - 4} + 1 \right). \quad (2.15)$$

In each of eq. (2.14), $r_2$ assumes both of the values $r_2^\pm$. We thus have four possible branches in the solution space. Numerically though, for given $R$ and $X$, the four solutions for $r_0$ roughly split into two pairs (the two $r_0^-$'s on the one hand and the two $r_0^+$'s on the other), with the intra-pair splitting considerably smaller than the inter-pair one. More interestingly, for a given $\epsilon$,

- not the entire curve of Fig. 1 is admissible when confronted with eq. (2.14). Rather, only $r_2 \approx 1$ (and hence $R \approx 1$) can lead to a stable solution. For $r_2$ significantly different from unity, the quantity $\mathcal{B}$ in eq. (2.15) becomes negative resulting in unphysical values for $X$;

- for $r_0 = r_0^+$, $\partial^2 V / \partial X^2 < 0$ irrespective of the choice for $r_2$. Thus, each of these branches correspond to a sequence of saddle points.

**Fig. 2:** (a) For small values of $\epsilon$, the warp factor is not very sensitive to the vev ratio $r_2$. The dependence on $r_0$ is rather pronounced though. Warp factor of $10^{-15}$ can be obtained with both the ratios $r_0$ and $r_2$ of $o(1)$. (b) For small $\epsilon$, a warp factor of $10^{-15}$ can be obtained with both the ratios $r_0$ and $r_2$ of $o(1)$. However, for larger values of $\epsilon$, the ratio $r_0$ is required to be quite large. The numbers in the parentheses refer to $(\epsilon, N)$ where $N$ is the factor by which the $y$-axis has been rescaled for each curve.
In Fig.2(a), we exhibit the relation between $r_0$ and $r_2$ that must be satisfied to obtain a particular warp-factor for a given $\epsilon$. As is evident, $W$ is a very sensitive function of $r_0$. This is not very unexpected as, for small $\epsilon$, eq. (2.14) essentially gives $r_0$ to be polynomial function of $\ln X$ (and hence $\ln W$). This exponential dependence, a feature we share with the GW solution to the original RS model, could of course be termed a weakness of such stabilization schemes.

The dependence on $\epsilon$—see Fig.2(b)—is even more pronounced. This, again, is not unexpected as $\epsilon \sim o(1)$ implies $m \sim k$. For such large masses of a field propagating in the anti-de Sitter bulk, its wavefunction decays very fast. Consequently, its classical values at the two end of the world branes would be widely different.

3. Discussion

Brane world universe is one of the promising proposals for exploring the physics beyond the Standard model. There are several variants of the original proposal of Randall and Sundrum [2]. Many of them are invoked to avoid possible contradictions with our existing knowledge of the Standard model physics and the standard big bang cosmology. The proposal of Kogan et al. [23] is along the same lines. We have shown in this paper that the three 3-brane model can be stabilized by coupling the system to a bulk scalar field and using the Goldberger-Wise formalism [18].

We find that stability of the three 3-brane model is a more delicate problem than that for the Randall-Sundrum model. This is because gain in the scale due to the warp factor in the Randall-Sundrum model is partly offset by the moving brane. It is therefore not only necessary to have overall stabilization of the three 3-brane system but it is also necessary to have the ratio $L_1/L_2$ close to 1. We show that the generalization of the Goldberger-Wise mechanism to this model stabilizes the radion modulus in such a way that, for relatively small mass of the bulk scalar field or, equivalently, small $\epsilon$, $L_1/L_2 \sim 1$. Consequently, it is possible to generate a scale hierarchy between the Planck scale and the TeV scale even without finetuning the parameters.

As we have discussed earlier, the modulus ceases to be stabilized in the event that the vevs on two (adjacent) branes are exactly equal. Such a degeneracy might seem natural, at least in the classical limit. However, it should be realized that quantum corrections are likely to disturb such an equality. This is particularly so since the vev on a brane is affected, at one loop level and higher, by the particle spectrum on the brane and their
coupling to the bulk scalar field. Unless the spectrum is exactly alike on two adjacent branes, and unless their coupling to $\chi$ are exactly the same, it seems unlikely that the two vevs could be equal. And since it is only in this exactly equal vevs limit, that the potential becomes attractive, the point is, perhaps, a moot one.

**Acknowledgments:** We would like to thank A. Sen for discussions.
References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The hierarchy problem and new dimensions at a millimeter*, hep-ph/9803313, Phys. Lett B429 (1998) 263; *Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity*, hep-ph/9807344, Phys. Rev. D59 (1999) 086004;
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *New dimensions at a millimeter to a fermi and superstrings at a TeV*, hep-ph/9804398, Phys. Lett B436 (1998) 257.

[2] L. Randall and R. Sundrum, *A Large Mass Hierarchy from a Small Extra Dimension*, hep-th/9905221.

[3] L. Randall and R. Sundrum, *An Alternative to compactification*, hep-th/9906064.

[4] H. Hatanaka, M. Sakamoto, M. Tachibana and K. Takenaga, *Many brane extension of the Randall-Sundrum solution*, hep-th/9909070;
I. Oda, *Mass hierarchy and trapping of gravity*, hep-th/9909048; *Mass hierarchy from many domain walls*, hep-th/9908104.

[5] I. I. Kogan, G. G. Ross, *Brane universe and multigravity: modification of gravity at large and small distances*, hep-th/0003074.

[6] N. Kaloper, *Crystal manyfold universes in AdS space*, hep-th/9912125;
T.-J. Li, *Noncompact AdS$_5$ universe with parallel positive tension 3-branes*, hep-th/9911234.

[7] H. Verlinde, *Supersymmetry at large distance scales*, hep-th/0004003.

[8] N. Alonso-Alberca, P. Meessen and T. Ortin, *Supersymmetric brane worlds*, hep-th/0003248.

[9] R. Altendorfer, J. Bagger and D. Nemeschansky, *Supersymmetric Randall-Sundrum scenario*, hep-th/0003117.

[10] T. Nihei, *Inflation in the five-dimensional universe with an orbifold extra dimension*, hep-ph/9905487, Phys. Lett. B465 (1999) 81;
N. Kaloper, *Bent domain walls as brane-worlds*, hep-th/9905210, Phys. Rev. D60 (1999) 123506.

[11] C. Csáki, M. Graesser, C. Kolda and J. Terning, *Cosmology of one extra dimension with localized gravity*, hep-ph/9906513, Phys. Lett. B462 (1999) 34, hep-ph/9906513.

[12] J.M. Cline, C. Grojean and G. Servant, *Inflating intersecting branes and remarks on the hierarchy problem*, hep-ph/9909490, Phys. Rev. Lett. 83 (1999) 4245.

[13] H.B. Kim and H.D. Kim, *Inflation and gauge hierarchy in Randall-Sundrum compactification*, hep-th/9909053.

[14] R.N. Mohapatra, A. Perez-Lorenzana, and C.A. de Sousa Pires, *Cosmology of brane - bulk models in five-dimensions*, hep-ph/0003328.
[15] H. B. Kim, *Cosmology of Randall-Sundrum models with an extra dimension stabilized by balancing bulk matter*, hep-th/0001209.

[16] M. Dine, *Some reflections on moduli, their stabilization and cosmology*, hep-th/0001157.

[17] C. Grojean, J. Cline and G. Servant, *Supergravity inspired warped compactifications and effective cosmological constants*, hep-th/9910081; P. Kraus, *Dynamics of anti-de Sitter domain walls*, hep-th/9910149; A. Kehagias and E. Kiritsis, *Mirage cosmology*, hep-th/9910174; JHEP 9911 (1999) 022; T. Shiromizu, K. Maeda and M. Sasaki, *The Einstein equations on the 3-brane world*, gr-qc/9910076; E. Flanagan, S.H.H. Tye and I. Wasserman, *Cosmological expansion in the Randall-Sundrum brane world scenario*, hep-ph/9910498; D. Ida, *Brane World Cosmology*, gr-qc/9912003.

[18] W. D. Goldberger and Mark B. Wise, *Bulk Fields in the Randall-Sundrum Compactification Scenario*, hep-ph/9907218; *Modulus stabilization with bulk fields*, hep-ph/9907447.

[19] P. Kanti, K. A. Olive and M. Pospelov, *Static solutions for brane models with a bulk scalar field*, hep-ph/0002229.

[20] L. Pilo, R. Rattazzi and A. Zaffaroni, *The fate of the radion in models with metastable graviton*, hep-th/0004028.

[21] S. Ichinose, *An exact solution of the Randall-Sundrum model and the mass hierarchy problem*, hep-th/0003275.

[22] G. Amelino-Camelia, *On a possible quantum limit for the stabilization of moduli in brane world scenarios*, hep-th/0001207.

[23] I.I. Kogan, S. Mouslopoulos, A. Papazoglou, G.G. Ross and J. Santiago, *A Three three-brane Universe: New Phenomenology for the New Millennium?*, hep-ph/9912552.

[24] S. Mouslopoulos and A. Papazoglou, ‘+ − +’ brane model phenomenology, hep-ph/0003207.