The baffle influence on sound radiation characteristics of a plate

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Abstract. The acoustic radiation characteristics of the baffle plates and unbaffle plates are calculated and compared by single-layer potential and double-layer potential. Based on the boundary integral equation, the sound pressure integral equation of the baffle and the baffle are deduced respectively. According to the boundary compatibility condition, the sound pressure and the vibration velocity of the plates are obtained. Further, the dynamic equation of the structure is substituted into the vibration equation in the form of the baffle plate and the baffle plate. The sound pressure difference and the displacement of a plate surface are in the form of the vibration mode superposition and the acoustic radiation impedance of the double integral form is obtained, which determines vibration mode coefficient and sound radiation parameters. The effect of the baffle on the acoustic radiation characteristics of the thin plate is analyzed by comparing the acoustic radiation parameters with the simple and simple rectangular plate in water.

1 Introduction

Many of the structures used in engineering can be treated as a finite plate approximately, and the acoustic radiation characteristics such as sound efficiency and sound power are widely concerned by engineers. When analyzing the acoustic radiation characteristics of a thin plate, a simple rectangular plate is usually used as the solution model. Not only because of the simple vibration pattern of simple-supported rectangular thin plate, it is convenient to use analytic solution to express its surface vibration velocity and sound radiation characteristic parameters, and its analytical solution can be compared with numerical calculation method such as finite element method to reveal the structural sound radiation general rule.

In the early study, the acoustic radiation characteristics of the simply supported rectangular plate are usually placed on an infinite baffle. Only the single-layer potential is needed to be calculated, and the integral term exhibits weak singularity, which can simplify the computational complexity. In the 1960s, Maidanik¹ first proposed the approximate equation of the acoustic radiation resistance of a simple rectangular plate, which provided a theoretical basis for later researchers. Wallace² then used the integral of the far field sound intensity to deduce a series of approximate integral equations of acoustic radiation resistance, and successfully solved the modal sound radiation power of the structure with an critical frequency. Heck³ uses Fourier transform to study the acoustic radiation problem of the plate in the wavenumber segment. Leppington⁴ obtained several approximate equations to solve the vibration mode efficiency of the large wavenumber domain, but the above scholars neglected the influence of crossmodal on the sound power. To this end, using the modal radiation efficiency and taking into account the coupling between the modals, Snyder and Tanaka⁵ calculated the low-frequency structural sound power. Li and Gibeling⁶-⁷ used the integral transformation method to transform the acoustic radiation resistance from the quadratic integral form into the form of double integral and successfully transformed into a form of single integral, and obtained the self-radiation resistance, mutual-radiation resistance. For the acoustic radiation problem of underwater plate, because the need to consider the role of fluid on the structure, it can be attributed to the sound and vibration coupling problem. Crighton⁸-⁹ analyzes the acoustic radiation of infinite plate in water under point excitation. Leibowitz¹⁰ proposed a method for calculating the acoustic properties of finite plates in water. M.L.Rumerman¹¹ studied the effect of water load presence on the sound radiation efficiency. B.E. Sandman¹² further validated the correctness of the theory of plate vibration and acoustic radiation under fluid loading by experiments.

However, in the actual project, the infinite baffle is not present, so there is the need to study the sound properties of unbaflled plates. Williams¹³ used the fast Fourier transform (FFT) to give the sound pressure of the simply supported thin plate in the air by the iterative operation. However, the convergence of the acoustic radiation impedance in high-order modes is poor. Atalla¹⁴ calculated the sound characteristics of an unbaflled plate in air under any boundary conditions using Kirchhoff-Helmholtz integral equation, but ignores the impact of pressure fluctuations on both sides of the plate resulting in large error in high frequency.

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Oppenheimer and Dubrowsky\cite{15} used experiments to modify the acoustic radiation parameters of baffled plates to match unbaffled plates and gave empirical formulas for the acoustic radiation of an unbaffled plate.

In this paper, the acoustic radiation characteristics of the baffled and unbaffled plates are analyzed. The sound pressure integral equations of the baffled and unbaffled plates are deduced by the boundary integral equation, and sound pressure and velocity in double integral form are obtained according to boundary compatibility condition. Further, the structural dynamic equation is substituted into the sound pressure equation of the baffled and unbaffled plates. Through expanding sound pressure difference and the displacement of plates in the form of the vibration mode superposition, the acoustic radiation impedance in the double integral form is obtained to solve the vibration modal coefficient, and determine the sound radiation characteristics. The effect of the baffle on the acoustic radiation characterstics to a plate is analyzed by comparing the acoustic radiation parameters with the simply rectangular plate in water.

2 Plate dynamics equations and fundamentals of acoustics

The acoustic radiation characteristics of simply supported rectangular thin plates are studied in this paper. The density of the plate is \( \rho_s \), its dimensions are \( a \times b \).

There is an excitation force \( F(x, y) \) of angular frequency \( \omega \) which is perpendicular to the plate shown in figure 1. From the classical dynamics, the transverse vibration equation of the thin plate is shown as

\[
\mathbf{K} \mathbf{w} = -\mathbf{M} \mathbf{\ddot{w}} + \mathbf{F}(x, y)
\]

where \( \mathbf{K} \) is the stiffness matrix, \( \mathbf{M} \) is the mass matrix, \( \mathbf{F}(x, y) \) is the modal force amplitude which can be expressed as

\[
F_{mn}(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \phi_{mn}(x, y)
\]

where \( A_{mn} \) is the modal force amplitude and \( \phi_{mn}(x, y) \) is the modal shape function which can be expressed as

\[
\phi_{mn}(x, y) = \sin(m\pi x/a) \sin(n\pi y/b)
\]

for simply supported plate. Set \( L = m\pi / a \) and \( L = n\pi / b \), then \( \phi_{mn}(x, y) = \sin(Lx) \sin(Ly) \). External forces can also be expressed as follows similary:

\[
F(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} F_{mn} \phi_{mn}(x, y)
\]

where \( F_{mn} \) is the modal force amplitude and \( \Delta \rho(x, y) \) is the difference between the sound pressure on the upper and lower surfaces of the plate shown as

\[
\Delta \rho(x, y) = \rho^- (x,y) - \rho^+ (x,y)
\]

where \( \rho^- (x,y) \) is the sound pressure acting on the lower surface of the plate, \( \rho^+ (x,y) \) is the sound pressure acting on the higher surface of the plate.

![Figure 1. Schematic diagram of a simply supported plate and associated coordinate system](image)

The sound pressure of the plate in the medium satisfies the classical Helmholtz integral equation, expressed as

\[
\nabla^2 \rho(x, y) + k^2 \rho(x, y) = 0
\]

where \( \rho(x, y) \) is the sound pressure at any point in the medium, \( k \) is the wave number, \( c \) is the velocity of sound. Based on Euler equation, the compatibility condition is:

\[
\frac{\partial \rho(x, y)}{\partial z} \bigg|_{z=\pm h} = \rho_s \omega^2 w(x, y)
\]

where \( \rho_s \) is the density of the medium.

Using the Kirchhoff-Helmholtz integral formula to an infinite space of a bounding plate structure, the sound pressure at any point in the space can be expressed as

\[
C(M_0)\rho(M_0) = -\int_S \rho \phi y(M) G(M, M_0) dS + \int_S \frac{\partial G\rho}{\partial n} (M, M_0) dS
\]

where \( C(M_0) \) is the sound pressure coefficient. When \( M_0 \) located outside the structure, \( C(M_0) \) is taken as 1, when \( M_0 \) located on the smooth structure, \( C(M_0) \) is taken as 1/2. The first and second terms on the right side of the equation are called single-layer potential and double-layer potential respectively. \( n \) denotes the outer normal direction of the structural surface, and \( G(M, M_0) \) is Green function in free space, satisfying the following equation:

\[
\nabla^2 G(M, M_0) + k^2 G(M, M_0) = \delta(x - x_0) \delta(y - y_0) \delta(z - z_0)
\]

where \( \delta \) is Delta function, \( G(M, M_0) \) is Green function in free space shown as

\[
G(M, M_0) = \frac{e^{ikr}}{4\pi r}
\]

where \( r = \sqrt{(x' - x)^2 + (y' - y)^2 + z'^2} \) means the distance between \((x', y', z')\) in space and \((x, y, z)\) on the plate. Using two-dimensional Fourier integral transformation to
the above formula, the Green’s function can be expressed as:

\[
G(M, M') = \frac{i}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i k_x (x - x')} e^{i k_y (y - y')}}{k_x k_y} \, dx \, dy
\]  

where \( k_x \), \( k_y \) is the Fourier variations in the two-dimensional wavenumber space, \( k_x = \sqrt{k^2 - k_y^2} \), \( k_y \) adopts the following form:

\[
\begin{align*}
k_x &= \sqrt{k^2 - k_y^2}, \quad k^2 \geq k_y^2, \\
k_y &= j\sqrt{k_x^2 + k_y^2}, \quad k^2 \leq k_x^2 + k_y^2
\end{align*}
\]

The normal direction of the upper and lower surfaces of the thin plate is opposite, and for this purpose the uniform direction of Green function is defined as positive direction of \( z \) axial. The coordinates are shown in figure 2.

![Figure 2. The cross section of a simply supported plate](image)

3 The calculation of a baffled plate

For a baffled plate, since in Eq. (7) \( \partial G / \partial n_p = 0 \), the sound pressure generated at a point \( r \) above the plate is equal to the integral of the points on the plate.

\[
p(M) = \frac{i \omega \rho}{2\pi} \int_{S} v(M') e^{-\frac{i k |r - r'|}{\rho}} \, dS
\]

Substituting the above formula into Eq. (1), one obtains

\[
B^\top \omega w(x, y) - \omega^2 \omega w(x, y) = \\
F(x, y) - \frac{\omega^2}{2\pi} \int_{S} w(M') e^{-\frac{i k |r - r'|}{\rho}} \, dS
\]

Seen from Eq. (2), the deflection of the rectangular plate can be expressed as a linear superposition of the vibration mode, so Eq. (13) can be transformed into

\[
\sum_{n=1}^{N} \sum_{m=1}^{M} A_{mn} \left[ B_l l_n + 2B_{\phi l_n} e^{ikx} + l_n \right] - \omega^2 \omega \phi_{mn}(x, y) = \\
F - \frac{\omega^2}{2\pi} \sum_{n=1}^{N} \sum_{m=1}^{M} A_{mn} \int_{S} \phi_{mn}(x', y') e^{-\frac{i k |r - r'|}{\rho}} \, dS
\]

Take Fourier inverse transformation to the above equation

\[
\begin{align*}
a b A_{mn} \left( B_{\phi l_n} - \omega^2 \omega \phi_{mn} \right) + 4i \omega \rho \phi_{mn} \sum_{n=1}^{N} \sum_{m=1}^{M} Z_{mnmn} = \\
4 \int_{S} \int_{S} F \phi_{mn}(x', y') \, dS
\end{align*}
\]

where \( Z_{mnmn} = \frac{1}{2\pi} \int_{S} \int_{S} \phi_{mn}(x', y') \, dS \)

\[
Z_{mnmn} = \frac{k^2}{2\pi} \int_{S} \int_{S} \phi_{mn}(x', y') \, dS
\]

is defined as the radiation impedance of \( (pq, mn) \).

Through coordinate transformation, Suppose \( u = \frac{x - x'}{a} \), \( v = \frac{y - y'}{b} \), \( v' = \frac{y'}{b} \), \( Z_{mnmn} \) can be transformed into a double integral form

\[
Z_{mnmn} = \\
\int_{a}^{b} \int_{b}^{b} J_{pq}(v) J_{q}(v') \, dv \, dv'
\]

where \( J_{pq}(v) = \int_{-\infty}^{1} \sin \rho \pi(u + u') \sin m \rho v' \, du' \),

\[
J_{q}(v') = \int_{-\infty}^{1} \sin \rho \pi(v + v') \sin n \rho v' \, dv',
\]

\[
\Omega(v, v') = \frac{\omega^2}{(u^2 + b^2 v'^2 / a^2)^{1/2}}.
\]

The transformed acoustic radiation impedance is substituted into Eq.(15), a matrix form can be expressed as

\[
(K + i \omega \rho - \omega^2 \Omega) A = F
\]

where \( K \) is stiffness matrix, \( \rho_{mn} = 4a i \omega \rho e Z_{mnmn} \), \( \Omega \) is impedance matrix, \( R_{mn} = 4a i \omega \rho e Z_{mnmn} \), \( M \) is mass matrix, \( F = ab \Omega a \), \( F \) is external force array.

4 The calculation of an unbaffled plate

In order to calculate the sound pressure at \( M_0 \), the position of point \( M_0 \) needs to be determined. The Green function in Eq. (10) can be expressed as:

\[
\begin{align*}
G(M_0, M') &= \frac{k^2}{2\pi} \int_{S} \int_{S} \phi_{mn}(x', y') \, dS
\end{align*}
\]

For any point \( M_0 \) in space, Eq. (7) can be discretized as:

\[
\begin{align*}
C(M_0, M) p(M) &= \int_{S} - \frac{\partial G}{\partial x} M' + \rho \omega \sum_{n=1}^{N} \sum_{m=1}^{M} Z_{mnmn} \, dS + \\
& \int_{S} \frac{\partial G}{\partial z} M' + \rho \omega \sum_{n=1}^{N} \sum_{m=1}^{M} Z_{mnmn} \, dS
\end{align*}
\]

In the formula, \( p(M) \) represents the sound
pressure and the velocity of point \( M^+ \) in the upper surface, and \( p(M^+) \), \( v(M^+) \) represents the sound pressure and the velocity of point \( M^- \) in the lower surface. When the plate is quite thin, the single-layer potential disappears

\[
\int_S v(M^-)G(M^+, M_n)\,dS^+ - \int_S v(M^+)G(M^-, M_n)\,dS^- = 0
\]

(20)

where \( \frac{\partial G}{\partial Z_{x'}}(M^+, M_n) \) and \( \frac{\partial G}{\partial Z_{y'}}(M^-, M_n) \) can be expressed as

the same approximately in thin plate, then Eq.(19) can be reduced to

\[
\frac{1}{2} \rho(M_n) = \int_S \frac{\partial G}{\partial Z_{y'}}(M, M_n)\,dS
\]

(21)

Substituting Eq.(1) into the above formula, the sound pressure expression of \( M_n \) is

\[
C(M_n)\rho(M_n) = \int_S \frac{\partial G}{\partial Z_{y'}}(M, M_n)\,dS
\]

(22)

By the Euler formula, deriving on both sides of the above equation, one obtains

\[
\rho_c \phi^2 C(M_n)w(x, y) = \int_S \frac{\partial G}{\partial Z_{x'}}(M, M_n)\,dS
\]

(23)

The equation for the plate surface displacement is established. Expand vibration modes on the items of above equation, one obtains

\[
\rho_c \phi^2 \sum_{a=1}^{\infty} \sum_{b=1}^{\infty} A_{ab} \phi_{mn}(x, y) = \int_S \sum_{a=1}^{\infty} \sum_{b=1}^{\infty} [B_{1a}^2 + 2B_{1a}B_{2b} + B_{2b}^2] + \frac{1}{2} \int_S \frac{\partial G}{\partial Z_{x'}}(M, M_n)\,dS
\]

(24)

For the point force acting on \((x_0, y_0)\), \( F_{an} \) can be expressed as

\[
F_{an} = \frac{AF_c}{S\phi_{mn}(x_0, y_0)}
\]

(25)

where \( F_c \) is the magnitude of the force. Using the orthogonality of the vibration modes, Eq.(24) can be reduced to:

\[
\rho_c \phi^2 \frac{S}{4} A_{pq} = \sum_{a=1}^{\infty} \sum_{b=1}^{\infty} \left[ \omega_{mn}^2 - \omega_{pq}^2 \right] A_{pq} - F_{an} \beta_{pq}
\]

(26)

where \( \omega_{mn}^2 = (\beta/\pi)^2 \left[ (\alpha b\pi/\beta)^2 + (n\pi/b)^2 \right]^2 \) represents the square of the first natural frequency of the simply supported rectangular plate

\[
\beta_{pq} = \int_S \int_S \phi^2_{pq}(x_0, y_0)\phi_{pq}(x, y) \frac{\partial G}{\partial Z_{x'}}(M, M_n)\,dS\,dS
\]

(27)

Exchanging the order of the structural surface integral, \( \beta_{pq} \) can be expressed as:

\[
\beta_{pq} = \frac{j}{8\pi^2} \int_{-\infty}^{+\infty} k \phi_{pq}^*(k_x, k_y) \phi_{pq}(k_x, k_y) \,dk_x\,dk_y
\]

(28)

where \( \phi_{pq}^* \) is the conjugate complex form of \( \phi_{pq} \), \( \phi_{pq} \) is the Fourier transform of \( \phi_{pq} \) defined by:

\[
\phi_{pq} = \int_{-\infty}^{+\infty} e^{j\omega t} \phi_{pq}(t) \,dt
\]

(29)

When the elements \( \beta_{pq} \) are determined, \( A_{pq} \) can be solved by the formula (26), the matrix form is expressed as:

\[
\mathbf{m} \mathbf{B} \Omega - \frac{1}{4} \rho_c \phi^2 S \mathbf{A} = \mathbf{B} \mathbf{F}
\]

(30)

where \( \mathbf{B} \) is the matrix for the formed matrix. \( \Omega \) is expressed as:

\[
\Omega = \begin{bmatrix}
\phi_{11} - \phi_{22} & 0 & \ldots & 0 \\
0 & \phi_{22} - \phi_{23} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \phi_{nn} - \phi_{nn}^2
\end{bmatrix}
\]

Referring to the definition of acoustic radiation impedance in above section, the acoustic radiation impedance of the unbaffled plate is defined as

\[
Z_{pq} = j\omega \phi_{pq} \frac{S}{4} \beta_{pq}^{-1}
\]

(31)

5 Numerical Analysis

Set length \( a = 1 \text{m} \), width \( b = 1.2 \text{m} \), thickness \( h = 0.01 \text{m} \), the sound speed \( c = 1400 \text{m/s} \) in the water, regardless of board damping, and reference sound power \( W_r = 10^{-12} \text{W} \). In the center of the plate excited by a normal excitation force, the amplitude is \( 1\text{N} \). The pictures below show the comparison of the sound radiation power, mean square velocity and sound radiation efficiency of the plate.

Figure 3. The comparison of the sound power
The figure above shows that the presence of the baffle causes the acoustic radiation impedance to be different, which causes the change of resonance frequency of the structure. The existence of the baffle causes the resonance frequency of the structure to be further reduced, and the sound radiation power of the structure is increased. This is due to the existence of the baffle, that the flat surface compresses one side medium which can not flow to the other side medium, resulting in structural vibration energy being more effectively converted into sound energy, thereby increasing the energy of acoustic radiation. So at the same frequency, the radiation power and efficiency of a baffled plate are higher than the unbaffled plate. The radiation from the baffled plate is similar to that of the monopole, and the acoustic radiation of the unbaffle plate is similar to that produced by the dipole. The acoustic radiation resistance of the baffled plate is higher than that of the unbaffled plate, and the acoustic resistance is equivalent to the additional mass on the surface of the structure. Therefore, it can be seen from the figure that the mean square velocity of the baffled plate is lower than that of the unbaffled one.

6 Conclusion

By using the boundary integral equation, the formulas of the acoustic radiation of the baffled plate and the unbaffled one are established respectively. Their respective acoustic radiation impedances are analyzed, and the radiated sound power, mean square velocity and the radiation efficiency of them are compared. It can be concluded that:

1. The presence of the baffle causes the sound radiation power and efficiency of the structure to be improved, similar to the conversion of the structure of the dipole radiation to the monopole radiation;
2. The presence of the baffle also changes the resonance frequency of structures in heavy medium, resulting in a reduced resonant frequency of the structure;
3. In general, the mean square velocity of the baffled plate is lower than the unbaffled one.

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