Supersymmetry of Black Strings in $D = 5$
Gauged Supergravities

D. Klemm$^1$ and W. A. Sabra$^2$

$^1$ Dipartimento di Fisica dell’Università di Milano and INFN, Sezione di Milano, Via Celoria 16, 20133 Milano, Italy.

$^2$ Center for Advanced Mathematical Sciences (CAMS) and Physics Department, American University of Beirut, Lebanon.

Abstract

Supersymmetry of five dimensional string solutions is examined in the context of gauged $D = 5$, $N = 2$ supergravity coupled to abelian vector multiplets. We find magnetic black strings preserving one quarter of supersymmetry and approaching the half-supersymmetric product space $AdS_3 \times H^2$ near the event horizon. The solutions thus exhibit the phenomenon of supersymmetry enhancement near the horizon, like in the cases of ungauged supergravity theories, where the near horizon limit is fully supersymmetric. Finally, product space compactifications are studied in detail, and it is shown that only for negative curvature (hyperbolic) internal spaces, some amount of supersymmetry can be preserved. Among other solutions, we find that the extremal rotating BTZ black hole tensored by $H^2$ preserves one quarter of supersymmetry.

$^*$email: dietmar.klemm@mi.infn.it

$^†$email: ws00@aub.edu.lb
1 Introduction

The conjectured equivalence between string theory on anti-de Sitter (AdS) spaces (times some compact manifold) and certain superconformal gauge theories living on the boundary of AdS \[1\] has led to an increasing interest in black objects in asymptotically anti-de Sitter spaces. On one hand, this interest is based on the fact that the classical supergravity solution can furnish important information on the dual gauge theory in the large \(N\) limit, \(N\) being the rank of the gauge group. An example of this is the Hawking-Page phase transition \[2\] from thermal AdS space to the Schwarzschild-AdS black hole, which was later reconsidered by Witten in the spirit of the AdS/CFT correspondence \[3\]. There it was observed that it can be interpreted as a transition from a low-temperature confining to a high temperature deconfining phase in the dual field theory.

On the other hand, the proposed AdS/CFT equivalence opens the possibility to a microscopic understanding of the Bekenstein-Hawking entropy of asymptotically anti-de Sitter black holes. This route was pioneered by Strominger \[4\], who used the central charge of the AdS\(3\) asymptotic symmetry algebra \[5\] to count the microstates giving rise to the BTZ black hole entropy. Of particular interest in this context are black objects in AdS space which preserve some fraction of supersymmetry. On the CFT side, these supergravity vacua could correspond to an expansion around non-zero vacuum expectation values of certain operators. Supersymmetry of Reissner-Nordström-AdS black holes in four dimensions was first studied by Romans in the context of \(N = 2\) gauged supergravity \[6\]. These considerations have been extended later in various directions \[7, 8, 9, 10, 11, 12, 13, 14, 15, 16\]. One common feature of all results is the appearance of naked singularities in the BPS limit\(^1\). This means that the theory on the bulk side is ill-defined in the limit of small distances, and stringy corrections have to be taken into account.

Although many string- and brane solutions in ungauged supergravity theories are known, very little is known on the corresponding objects in the gauged case. To remedy this will be the main purpose of the present paper. In particular, we will derive supersymmetric black string solutions with various topologies in five dimensional \(N = 2\) supergravity theories coupled to vector

\(^1\)It is worth pointing out, however, that by including rotation \[8, 10\], or by allowing for different event horizon geometries \[10, 11\], one can get genuine BPS black holes in anti-de Sitter space.
multiplets [14]. The theory of ungauged five-dimensional $N = 2$ supergravity coupled to abelian vector supermultiplets can be obtained by compactifying eleven-dimensional supergravity, the low-energy theory of M-theory, on a Calabi-Yau three-fold [18]. Gauged supergravity theories are obtained by gauging a subgroup of the R-symmetry group, the automorphism group of the supersymmetry algebra. The gauged $D = 5, N = 2$ supergravity theories are obtained by gauging the $U(1)$ subgroup of the $SU(2)$ automorphism group of the superalgebra [17]. The Lagrangian of the theory is obtained by introducing a linear combination of the abelian vector fields already present in the ungauged theory, i.e. $A_\mu = V_I A^I_\mu$, with a coupling constant $g$. The coupling of the Fermi-fields to the $U(1)$ vector field breaks supersymmetry, and therefore gauge-invariant $g$-dependent terms must be introduced in order to preserve $N = 2$ supersymmetry. In a bosonic background, this amounts to the addition of a $g^2$-dependent scalar potential $V$ [1, 17].

Our work in this paper is organized as follows. Section 2 contains a brief review of $D = 5, N = 2$ gauged supergravity. In 3, the supersymmetric string solutions are derived, and their near horizon limit is studied. In section 4, general supersymmetric product space compactifications of $D = 5, N = 2$ gauged supergravity are considered. Finally, our results are summarized and discussed in 5.

2 $D = 5, N = 2$ Gauged Supergravity

The bosonic part of the gauged supersymmetric $N = 2$ Lagrangian which describes the coupling of vector multiplets to supergravity is given by

\[ e^{-1} \mathcal{L} = \frac{1}{2} R + g^2 V - \frac{1}{4} G_{IJK} F^{I\mu\nu} F^{J\rho\sigma} - \frac{1}{2} G_{ij} \partial_i \phi^i \partial_j \phi^j + \frac{e^{-1}}{48} \epsilon^{\mu\rho\sigma\lambda} C_{IJK} F_{I\mu\nu} F_{J\rho\sigma} A^K_\lambda, \]

where $\mu, \nu$ are spacetime indices, $R$ is the scalar curvature, $F^{I\mu\nu}$ denote the abelian field-strength tensors, and $e = \sqrt{-g}$ is the determinant of the F"{u}nfbein $e^a_\mu$. The scalar potential $V$ is given by

\[ V(X) = V_I V_J \left( 6 X^I X^J - \frac{9}{2} G^{ij} \partial_i X^I \partial_j X^J \right). \]
where $X^I$ represent the real scalar fields satisfying the condition $V = \frac{1}{6} C_{IJK} X^I X^J X^K = 1$. The physical quantities in (1) can all be expressed in terms of the homogeneous cubic polynomial $V$ which defines a “very special geometry” \cite{19}. We also have the relations

$$
G_{IJ} = -\frac{1}{2} \partial_I \partial_J \log V \bigg|_{V=1},
$$

$$
G_{ij} = \partial_i X^I \partial_j X^J G_{IJ} \bigg|_{V=1},
$$

where $\partial_i$ and $\partial_I$ refer, respectively, to a partial derivative with respect to the scalar field $\phi_i$ and $X^I = X^I(\phi^j)$.

Note that for Calabi-Yau compactification of M-theory, $V$ is the intersection form, $X^I$ and $X^I$ correspond to the size of the two- and four-cycles and $C_{IJK}$ are the intersection numbers of the Calabi-Yau threefold.

The supersymmetry transformations of the gravitino $\psi_\mu$ and the gauginos $\lambda_i$ in a bosonic background read \cite{9}

$$
\delta \psi_\mu = \left( D_\mu + \frac{i}{8} X_I (\Gamma_{\mu \rho} - 4 \delta_{\mu \rho} \Gamma^\gamma) F^{\gamma I} - \frac{i}{2} g \Gamma_\mu X^I V_I - \frac{3i}{2} gV_I A^I_\mu \right) \epsilon, \tag{3}
$$

$$
\delta \lambda_i = \left( \frac{3}{8} \Gamma_{\mu \nu} F^{I}_{\mu \nu} \partial_i X_I - \frac{i}{2} G_{ij} \Gamma^\mu \partial_\mu \phi^j + \frac{3i}{2} g V_I \partial_i X^I \right) \epsilon, \tag{4}
$$

where $\epsilon$ is the supersymmetry parameter and $D_\mu$ is the covariant derivative.

### 3 Supersymmetric String Solutions

As a general ansatz for the supersymmetric solutions we consider metrics of the form

$$
ds^2 = -e^{2V} dt^2 + e^{2T} dz^2 + e^{2U} dr^2 + F(r)^2 d\sigma^2, \tag{5}
$$

where $V, T, U$ are functions of $r$ only, and we consider either $F(r) = R = \text{constant}$. $d\sigma^2$ denotes the metric of a two-manifold $S$ of constant Gaussian curvature $k$. Without loss of generality we restrict ourselves to the

\footnote{We use the metric $\eta^{ab} = (-, +, +, +, +)$, $\{\Gamma^a, \Gamma^b\} = 2 \eta^{ab}$, $D_\mu = \partial_\mu + \frac{1}{2} \omega_{\mu ab} \Gamma^a b$, $\omega_{\mu ab}$ is the spin connection, and $\Gamma^{a_1 a_2 \ldots a_n}$ is the spin connection, and $\Gamma^{a_1 a_2 \ldots a_n} = \frac{1}{n!} \Gamma^{[a_1} \Gamma^{a_2} \ldots \Gamma^{a_n]}$.}
cases $k = 0, \pm 1$. Clearly $S$ is a quotient space of the universal coverings $S^2 (k = 1)$, $H^2 (k = -1)$ or $E^2 (k = 0)$. Explicitly, we choose

$$d\sigma^2 = d\theta^2 + f(\theta)^2 d\phi^2,$$

where

$$f(\theta) = \begin{cases} 
\sin \theta, & k = 1, \\
1, & k = 0, \\
\sinh \theta, & k = -1.
\end{cases}$$

The case $k = 1$, $F = r$ has been considered in [16]. There, a spherically symmetric magnetic string solution was obtained, which contains a naked singularity.

For the metric (3), the fünfbein and its inverse can be chosen as

$$e_0^t = e^V, \quad e_1^z = e^T, \quad e_2^r = e^U, \quad e_3^\theta = F, \quad e_4^\phi = Ff,$$

$$e_0^t = e^{-V}, \quad e_1^z = e^{-T}, \quad e_2^r = e^{-U}, \quad e_3^\theta = \frac{1}{F}, \quad e_4^\phi = \frac{1}{Ff}.$$  (8)

The nonvanishing components of the spin connection are given by

$$\omega_{t}^{02} = V'e^{-U}, \quad \omega_{z}^{12} = T'e^{-U}, \quad \omega_{\theta}^{23} = -F'e^{-U}, \quad \omega_{\phi}^{24} = -F'fe^{-U}, \quad \omega_{\phi}^{34} = -f'.$$  (9)

In five dimensions, strings can carry magnetic charges under the one-form potentials $A^I$, so we assume that the gauge fields have only a magnetic part, i.e.

$$F_{b\phi}^I = kq^lf(\theta), \quad A_{\phi}^I = kq^l \int f(\theta)d\theta.$$  (10)

Furthermore, we are looking for solutions with constant moduli $X^I$, which are chosen to minimize the magnetic central charge $Z = q^lX_l$, as in the case of the double extreme solutions in the ungauged theory [20, 21]. This means that one has

$$\partial_i(Z) = \partial_i(q^lX_l) = \frac{1}{3}C_{IJK}X^I\partial_i(X^J)q^K = 0.$$  (11)
Moreover, we make the choice

\[ X^I V_I = 1. \]  

(12)

Using (11) and (12), the gaugino transformations (4) can be easily seen to vanish identically.

Plugging the spin connection (9) and the magnetic fields (10) into (3), we obtain for the supersymmetry variation of the gravitino

\[ \delta \psi_t = \left( \partial_t + \frac{1}{2} V' e^{V-U} \Gamma_{02} + \frac{ik}{4} Z \frac{e^V}{F^2} \Gamma_{034} + \frac{g}{2} e^V \Gamma_0 \right) \epsilon, \]

\[ \delta \psi_z = \left( \partial_z + \frac{1}{2} T' e^{T-U} \Gamma_{12} + \frac{ik}{4} Z \frac{e^T}{F^2} \Gamma_{134} + \frac{g}{2} e^T \Gamma_1 \right) \epsilon, \]

\[ \delta \psi_r = \left( \partial_r + \frac{ik e^U}{4} \frac{Z}{F^2} \Gamma_{234} + \frac{g}{2} e^U \Gamma_2 \right) \epsilon, \]

\[ \delta \psi_\theta = \left( \partial_\theta - \frac{1}{2} F' e^{-U} \Gamma_{23} - \frac{ik Z}{2F} \Gamma_4 + \frac{g}{2} F \Gamma_3 \right) \epsilon, \]

\[ \delta \psi_\phi = \left( \partial_\phi - \frac{1}{2} F' f e^{-U} \Gamma_{24} - \frac{1}{2} f \Gamma_{34} + \frac{ik Z f}{2F} \Gamma_3 + \frac{g}{2} F f \Gamma_4 - \frac{3i}{2} g V_I q^I k \int f d\theta \right) \epsilon. \]

In what follows, we will consider three different cases.

3.1 Solutions With Flat Transverse Space

Let us first consider the case \( k = 0, \ F = r, \) i.e. flat transverse space. Our choice (10) implies that we have vanishing gauge fields for \( k = 0. \) The vanishing of the gravitino supersymmetry transformations (13) then yields the Killing spinor equations

\[ \left( \partial_t + \frac{1}{2} e^V \Gamma_0 (V' e^{-U} \Gamma_2 + g) \right) \epsilon = 0, \]

\[ \left( \partial_z + \frac{1}{2} e^T \Gamma_1 (T' e^{-U} \Gamma_2 + g) \right) \epsilon = 0, \]

\[ \left( \partial_r + \frac{g}{2} e^U \Gamma_2 \right) \epsilon = 0, \]

\[ \left( \partial_\theta + \frac{1}{2} \Gamma_3 (e^{-U} \Gamma_2 + g r) \right) \epsilon = 0, \]
\[
\left( \partial_\phi + \frac{1}{2} \Gamma_4 (e^{-U} \Gamma_2 + gr) \right) \epsilon = 0. \tag{14}
\]

From the integrability conditions of these equations one gets
\[
e^V = e^T = e^{-U} = gr. \tag{15}\]

Plugging these results into (14) and introducing the projectors \( P_\pm = \frac{1}{2} (1 \pm \Gamma_2) \), we obtain for the Killing spinors
\[
\epsilon = r^{-\frac{1}{2}} \epsilon_+^0 - gr^2 (g \Gamma_0 t + g \Gamma_1 z + \Gamma_3 \theta + \Gamma_4 \phi) \epsilon_+^0 + r^2 \epsilon_-^0,
\]

where \( \epsilon_\pm^0 \) are constant spinors satisfying \( P_\pm \epsilon_\pm = 0 \). From (13) it is clear that the solution we found is locally \( AdS_5 \) (written in horospherical coordinates). (16) tells us that this spacetime, as it should be, is fully supersymmetric. However, one may wish to compactify the \((\theta, \phi)\) sector to a cylinder or a torus, considering thus a quotient space of \( AdS_5 \). In this case, the surviving Killing spinors are those which respect the identifications performed in the \((\theta, \phi)\) sector. These are
\[
\epsilon = r^{\frac{1}{2}} \epsilon_-^0, \tag{17}\]

so that the considered \( AdS_5 \) quotient space preserves half of the supersymmetries.

Note that the above supersymmetric string solution is a limiting case of a family of nonextremal black strings, whose metric is given by
\[
ds^2 = -e^{2V} dt^2 + r^2 dz^2 + e^{-2V} dr^2 + r^2 (d\theta^2 + d\phi^2), \tag{18}\]

where
\[
e^{2V} = -\frac{m}{r^2} + g^2 r^2, \tag{19}\]

\( m \) denoting an integration constant related to the mass of the black string. Considering \( z \) as a coordinate of transverse space, (18) clearly can also be interpreted as a black hole, it is the solution found in [22].

### 3.2 Hyperbolic Transverse Space

We turn now to the more interesting case of hyperbolic transverse space, i.e. \( k = -1, F = r \). As supersymmetry breaking conditions we take
\[
\Gamma_3 \Gamma_4 \epsilon = i \epsilon,
\]
\[
\Gamma_2 \epsilon = -\epsilon. \tag{20}\]
Then, the transformations (3) reduce to

\[
\begin{align*}
\delta \psi_t &= \left( \partial_t - \frac{1}{2} (V'e^V - U - Z \frac{V}{2r^2} - ge^V) \Gamma_0 \right) \epsilon, \\
\delta \psi_z &= \left( \partial_z - \frac{1}{2} (T'e^T - U - Z e^T) \Gamma_1 \right) \epsilon, \\
\delta \psi_r &= \left( \partial_r - \frac{e^U}{2} \left( \frac{Z}{2r^2} + g \right) \right) \epsilon, \tag{21} \\
\delta \psi_\theta &= \left( \partial_\theta - \frac{1}{2} (e^{-U} + \frac{Z}{r} - gr) \Gamma_3 \right) \epsilon, \\
\delta \psi_\phi &= \left( \partial_\phi - \frac{i}{2} (1 - 3gq'I) \cosh \theta - \frac{1}{2} (e^{-U} + \frac{Z}{r} - gr) \sinh \theta \Gamma_4 \right) \epsilon.
\end{align*}
\]

The vanishing of the above equations implies the following conditions on the supersymmetry spinor \( \epsilon \),

\[
\begin{align*}
\partial_t \epsilon &= 0, \\
\partial_z \epsilon &= 0, \\
\partial_\theta \epsilon &= 0, \\
\partial_\phi \epsilon &= 0, \\
3gq'IV_I &= 1, \\
e^{-U} - \frac{Z}{r} + gr &= 0, \\
e^{-U}T' + \frac{Z}{2r^2} + g &= 0, \\
e^{-U}V' + \frac{Z}{2r^2} + g &= 0. \tag{22}
\end{align*}
\]

The last two equations in (22) imply that one should set \( T = V \). From the sixth equation of (22), one immediately obtains

\[
e^{-U} = -\frac{Z}{r} + gr. \tag{23}
\]

Using the last equation of (22), we obtain a differential equation for \( V \),

\[
V'e^{-U} = g + \frac{Z}{2r^2}. \tag{24}
\]
The above differential equation can be easily solved by noticing that it can be rewritten in the form

\[
\frac{dV}{dr} = \frac{d}{dr} \log(e^{-U}) - \frac{1}{4} \frac{d}{dr} \log\left(\frac{e^{-U}}{gr}\right).
\]  

(25)

(Recall that the central charge \(Z\) takes a constant value to be determined). (25) yields the following solution for \(V\),

\[e^V = e^{-\frac{3U}{4}} (gr)^\frac{1}{4}.
\]  

(26)

Let us now return to the minimization condition (11) of the magnetic central charge. It implies that the critical values of \(X^I\) and its dual are given by

\[X^I = \frac{q^I}{Z}, \quad X_I = \frac{1}{6} \frac{C_{IJK}q^J q^K}{Z^2},
\]  

(27)

and thus the critical value of the magnetic central charge is

\[Z = \left(\frac{1}{6} C_{IJK} q^I q^J q^K \right)^\frac{1}{3}.
\]  

(28)

Using the conditions \(X^I V_I = 1\) and the fifth relation of (22), one obtains a generalized Dirac quantization condition

\[
\left(\frac{1}{6} C_{IJK} q^I q^J q^K \right)^\frac{1}{3} = \frac{1}{3g}.
\]  

(29)

For the case of pure supergravity where only the graviphoton charge \(q^0\) is present, one obtains \(q^0 = 1/(3g)\). A similar condition was obtained in [6, 10].

To summarize, the BPS magnetic black string solution to \(D = 5, N = 2\) gauged supergravity coupled to vector multiplets is given by

\[ds^2 = (gr)^\frac{1}{2} e^{-\frac{3U}{4}} (-dt^2 + dz^2) + e^{2U} dr^2 + r^2 \left(d\theta^2 + \sinh^2 \theta d\phi^2\right),
\]

\[e^{-U} = -\frac{1}{3gr} + gr,
\]  

(30)

while the gauge fields and the scalars are

\[A^I_\phi = -q^I \cosh \theta, \quad X^I = 3g q^I.
\]  

(31)
The Killing spinor is independent of the angular variables, and its radial dependence is obtained by solving for its radial differential equation, which reads
\[
\left( \partial_r - \frac{e^U}{2} \left( g + \frac{1}{6g r^2} \right) \right) \epsilon = 0.
\] (32)
Using the relation (24), the above differential equation can be written in the simple form
\[
\left( \partial_r - \frac{1}{2} V' \right) \epsilon = 0,
\] (33)
and thus we get
\[
\epsilon(r) = e^{\frac{1}{2} V} \epsilon_0,
\] (34)
where \( \epsilon_0 \) is a constant spinor satisfying the constraints
\[
\Gamma_3 \Gamma_4 \epsilon_0 = i \epsilon_0, \quad \Gamma_2 \epsilon_0 = -\epsilon_0.
\] (35)
As the Killing spinors do not depend on the coordinates \( \theta, \phi \) of the transverse hyperbolic space, one could also compactify the \( H^2 \) to a Riemann surface \( S_n \) of genus \( n \), and the resulting solution would still preserve one quarter of supersymmetry.
Whereas the spherical BPS magnetic string found in [16] contains a naked singularity, the hyperbolic black string (30) has an event horizon at \( r = r_+ = 1/(g \sqrt{3}) \). This is analogous to the \( AdS_4 \) case, where BPS magnetic black holes with hyperbolic event horizons have been found [11], whereas for spherical topology one gets supersymmetric naked singularities [3].
Note that the black strings (30) are solitonic objects in the sense that the limit \( g \to 0 \) (we recall that \( g \) is the coupling constant of the gauged theory, coupling the \( U(1) \) vector fields to the fermions) does not exist.
In the near horizon region, (30) reduces to the product manifold \( AdS_3 \times H^2 \). This is easily seen by introducing the new radial coordinate \( \rho = (r-r_+)^{1/4} \). In the next subsection, we will see that in the near horizon limit, supersymmetry is enhanced.
\[3.3\] Hyperbolic Transverse Space and Constant Warping Function

Let us now consider the case \(k = -1\) and \(F = R\), where \(R\) is a constant. We also choose \(T = V\). As supersymmetry breaking condition we take

\[\Gamma_3\Gamma_4\epsilon = i\epsilon.\] (36)

The Killing spinor equations following from (3) are then

\[
\begin{align*}
\delta\psi_t &= \left( \partial_t + \frac{1}{2} V'e^{-V-U}\Gamma_{02} + \frac{1}{4} Z\frac{e^V}{R^2}\Gamma_0 + \frac{g}{2} e^V \Gamma_0 \right) \epsilon = 0, \\
\delta\psi_z &= \left( \partial_z + \frac{1}{2} V'e^{-V-U}\Gamma_{12} + \frac{1}{4} Z\frac{e^V}{R^2}\Gamma_1 + \frac{g}{2} e^V \Gamma_1 \right) \epsilon = 0, \\
\delta\psi_r &= \left( \partial_r + \frac{1}{4} Z\frac{e^U}{R^2}\Gamma_2 + \frac{g}{2} e^U \Gamma_2 \right) \epsilon = 0, \\
\delta\psi_\theta &= \left( \partial_\theta - \frac{Z}{2R} \Gamma_3 + \frac{g}{2} R \Gamma_3 \right) \epsilon = 0, \\
\delta\psi_\phi &= \left( \partial_\phi - \frac{i}{2} \cosh \theta - \frac{Z}{2R} \sinh \theta \Gamma_4 + \frac{gR}{2} \sinh \theta \Gamma_4 + \frac{3i}{2} g V_l q^l \cosh \theta \right) \epsilon = 0.
\end{align*}\] (37)

The integrability conditions for these equations imply that

\[3gV_l q^l = 1,\] (38)

and that the central charge is related to the compactification radius by \(Z = gR^2\). Furthermore, one obtains

\[\partial_r(V'e^{-V-U}) = \frac{9}{4} g^2 e^{V+U}, \quad V'e^{-V} = \frac{3}{2} g.\] (39)

Defining a new radial coordinate \(\rho\) by \(g\rho = e^V\), one immediately sees that the three-dimensional part of spacetime is \(AdS_3\) in horospherical coordinates. We have thus obtained a supersymmetric product space \(AdS_3 \times H^2\), i.e. the near-horizon geometry of (30). Plugging the relations (38) and \(Z = gR^2\) into (37), one obtains that the Killing spinors are independent of \(\theta, \phi\). The remaining system is solved by

\[
\epsilon = e^{-\frac{1}{2}V}\epsilon^0_+ - \frac{3}{2} g e^{\frac{1}{2}V}(\Gamma_0 t + \Gamma_1 z)\epsilon^0_+ + e^{\frac{1}{2}V}\epsilon^0_-,\] (40)
where \( \epsilon^0_\pm \) are constant spinors satisfying
\[
(1 \mp \Gamma_2)\epsilon^0_\pm = 0, \quad \Gamma_3\Gamma_4\epsilon^0_\pm = i\epsilon^0_\pm.
\] (41)

The product space \( \text{AdS}_3 \times H^2 \) is thus one half supersymmetric. This supersymmetry enhancement near the horizon of the BPS black string is analogous to the case of ungauged supergravity theories, where usually in the near-horizon limit supersymmetry is fully restored.

## 4 General Product Space Compactifications

In this section we consider general product space compactifications of gauged \( D = 5, \ N = 2 \) supergravity coupled to vector multiplets. Spacetime is assumed to be a product \( M_3 \times M_2 \), where, as above, \( M_2 \) denotes a two-manifold of constant curvature. We are interested in the conditions imposed by supersymmetry on \( M_{2,3} \). To this end, we perform a 3 + 2 decomposition of the gamma matrices\(^3\) in the following way
\[
\Gamma^a = (\Gamma^{\hat{\alpha}}, \Gamma^i) = (\gamma^{\hat{\alpha}} \otimes \sigma^3, 1 \otimes \Sigma^i),
\] (42)

where early Greek letters \( \alpha, \beta, \ldots \) are \( M_3 \) spacetime indices, and \( i, j, \ldots \) are \( M_2 \) spacetime indices. The hatted indices refer to the corresponding tangent spaces. The \( \gamma^{\hat{\alpha}} \) and \( \Sigma^i \) denote Dirac matrices in three and two dimensions respectively. To be concrete, we make the choice \( \Sigma^1 = \sigma^2, \Sigma^2 = \sigma^1 \), where the Pauli matrices are chosen to be
\[
\begin{align*}
\sigma^1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\
\sigma^2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \\
\sigma^3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\end{align*}
\] (43)

The supersymmetry parameter \( \epsilon \) in five dimensions is decomposed as \( \epsilon = \eta \otimes \chi \). Note that \( \sigma^3 \) plays the role of a chirality operator for the spinors \( \chi \) in two dimensions. Some useful relations needed below are
\[
\Gamma^{\hat{\alpha}\hat{\beta}} = \gamma^{\hat{\alpha}\hat{\beta}} \otimes 1, \quad \Gamma^{ij} = 1 \otimes \Sigma^{ij}, \quad \Gamma^{\hat{\alpha}\hat{\beta}}\Gamma^i\Gamma^j = \gamma^{\hat{\alpha}\hat{\beta}} \otimes \sigma^3\Sigma^i\Sigma^j.
\] (44)

For the field strength of the abelian vectors we make the ansatz
\[
F^I = q^I \epsilon,
\] (45)

\(^3\)See the appendix of [23] for a nice summary of gamma matrix decomposition in Kaluza-Klein compactifications.
where \( \epsilon \) denotes the volume form on \( \mathcal{M}_2 \). The gaugino variation \( \mathcal{P} \) vanishes as before, provided that \( (11) \) and \( V_I X^I = 1 \) are satisfied. (Note that we still assume the moduli \( X^I \) to be constant). Inserting the decomposition of the Dirac matrices and the supersymmetry parameter, as well as the ansatz for the field strength into the gravitino variation \( \mathcal{Q} \) yields the relations

\[
\left( \partial_\alpha + \frac{1}{4} \omega^\hat{\alpha}_\beta \gamma_{\hat{\alpha} \hat{\beta}} + \frac{1}{4} X_I q^I \gamma_\alpha + \frac{g}{2} \gamma_\alpha \right) \eta \otimes \chi_+ = 0
\]

and

\[
\left( \partial_i + \frac{1}{4} \omega^i_j \Sigma_{ij} - \frac{i}{2} X_I q^I \epsilon_{ij} \Sigma^j + \frac{g}{2} \Sigma_i - \frac{3i}{2} g V_I A_i^I \right) \chi = 0,
\]

where \( \chi_\pm = \frac{1}{2}(1 \pm \sigma^3)\chi \) denote the chirality projections of the two-dimensional spinor \( \chi \). From \( (10) \) we get that either \( \chi_- \) and the bracketed expression in the first line have to vanish, or \( \chi_+ \) and the term in brackets in the second line are zero. Without loss of generality, we assume the first possibility. Plugging \( \chi_- = 0 \) into \( (17) \), one obtains

\[
(g \Sigma_i - i X_I q^I \epsilon_{ij} \Sigma^j) \chi_+ = 0.
\]

In order to have nontrivial solutions to this equation, the determinant of \( g \Sigma_i - iZ \epsilon_{ij} \Sigma^j \) has to vanish. This implies

\[
Z = \pm g
\]

for the magnetic central charge \( Z \). One can easily show that the lower sign is incompatible with the condition \( \sigma^3 \chi_+ = \chi_+ \), so the upper positive sign is chosen and \( (18) \) is then automatically satisfied. The remaining Killing spinor equation for \( \chi_+ \) reads

\[
\mathcal{D}_i \chi_+ = 0,
\]

with the gauge- and Lorentz-covariant derivative \( \mathcal{D}_i \) given by

\[
\mathcal{D}_i = \partial_i + \frac{1}{4} \omega^i_j \Sigma_{ij} - \frac{3i}{2} g V_I A_i^I.
\]

\(^{4}\text{We apologize for using the same symbol for the volume form and the supersymmetry parameter, but the meaning should be clear from the context.}\)
The integrability condition for (50) is
\[ [\mathcal{D}_i, \mathcal{D}_j] \chi_+ = 0, \quad (52) \]
or, equivalently,
\[ \left( \frac{1}{4} R_{ijkl} \Sigma^{kl} - \frac{3i}{2} g V_I q^I \epsilon_{ij} \right) \chi_+ = 0. \quad (53) \]
Taking into account that \( M_2 \) is of constant curvature, we have
\[ R_{ijkl} = R \left( g_{ik} g_{jl} - g_{il} g_{jk} \right) \quad (54) \]
for the Riemann tensor of \( M_2 \). Using this in (53), one immediately obtains
\[ R + 6 g V_I q^I = 0 \quad (55) \]
for the scalar curvature \( R \) of \( M_2 \). From \( q^I X_I = g \) we have
\[ X^I = \frac{q^I}{g} \quad (56) \]
for the moduli, and thus
\[ R = -6 g V_I q^I = -6 g^2 V_I X^I = -6 g^2 < 0 \quad (57) \]
for the scalar curvature \( R \). This means that, to preserve some supersymmetry, \( M_2 \) must be diffeomorphic to hyperbolic space \( H^2 \) or to a quotient thereof.

Using the second Cartan equation, one obtains that the spin connection \( \omega^{12} \) on \( M_2 \) is related to the vector potential \( A^I \) by
\[ q^I \omega^{12} = A^I \frac{R}{2}. \quad (58) \]
Using this, (51) reduces to \( \partial_i \chi_+ = 0 \), so that \( \chi_+ \) is independent of the coordinates on \( M_2 \). The remaining equation to solve is the Killing spinor equation on \( M_3 \) for \( \eta \), i. e.
\[ \left( \partial_\alpha + \frac{1}{4} \omega^{\dot{\alpha} \dot{\beta}} \gamma_{\dot{\alpha} \dot{\beta}} + \frac{3}{4} g \gamma_\alpha \right) \eta = 0. \quad (59) \]
The integrability conditions for Eq. (59) yield that $M_3$ must be an Einstein space with cosmological constant $\Lambda = -(3g/2)^2$. As we are in three dimensions, this means that $M_3$ is also of constant curvature, i.e. a quotient space of $AdS_3$. Note that the chirality condition $\chi = \chi_+ \pm$ breaks half of supersymmetry. The amount of supersymmetry preserved by $M_3 \times M_2$ is then determined by the solutions of Eq. (59). If $M_3 = AdS_3$, then the whole solution $AdS_3 \times H^2$ is half supersymmetric, in agreement with what we found above. However, we can also choose $M_3$ to be e.g. the BTZ black hole. If we take the extremal rotating BTZ black hole, which preserves one half of the $AdS_3$ supersymmetries [24], then the solution $BTZ_{\text{extr}} \times H^2$ preserves one quarter of the supersymmetries. We would like to point out that $BTZ \times H^2$ compactifications of $D = 5$ anti-de Sitter gravity without gauge fields were previously considered in [25]. However, as we showed above, due to the relation $Z = g$ these configurations cannot be supersymmetric unless some gauge fields are turned on.

5 Summary and Discussion

To sum up, we presented supersymmetric string solutions of gauged $D = 5$, $N = 2$ supergravity coupled to abelian vector multiplets. The main result is the construction of a BPS black string with hyperbolic transverse space, preserving one quarter of supersymmetry. The curvature of the $H^2$ is supported by a nonvanishing field strength of the vector fields. The black strings are thus magnetically charged. In the near-horizon limit, their geometry approaches the half-supersymmetric product space $AdS_3 \times H^2$, so we encounter supersymmetry enhancement near the horizon. This behaviour is similar to the case of ungauged supergravity theories. Note however, that in the ungauged case, usually supersymmetry is fully restored near the event horizon. As the near horizon geometry contains an $AdS_3$ factor, it should be possible to use the $AdS_3$ asymptotic symmetry algebra [3] in order to count the microstates yielding the Bekenstein-Hawking entropy of the extremal black string. This was done in [14] for the BTZ black hole, and subsequently generalized in [26, 27, 28, 29] to higher-dimensional black holes containing a BTZ factor near the horizon. In our case, a similar procedure is hindered by the fact that we get the $M = J = 0$ BTZ black hole in the near-horizon limit, so using Cardy’s formula we would obtain an incorrect result for the entropy. Similar difficulties have been encountered in [30], where a state counting for
extremal black strings in three dimensions was performed. This suggests that a similar approach to that in [30] must be used in our case, in order to overcome the above mentioned difficulties.

It is also of interest to investigate the role of the BPS magnetic black strings in the AdS/CFT correspondence. Note that the $U(1)^3$ truncation of gauged $D = 5$, $N = 2$ supergravity can be embedded into type IIB supergravity [31]. This means that our solutions can be lifted to ten dimensions, with an internal five-sphere that is distorted by the one-form gauge fields $A_I$. That breaks the isometry group $SO(6)$ of the $S^5$ down to a smaller subgroup. In the dual CFT, which is $\mathcal{N} = 4$ SYM on $\mathbb{R}^2 \times H^2$ (or $\mathbb{R} \times S^1 \times H^2$, if the coordinate $z$ parametrizes a compact space), the $S^5$ isometry group becomes the $R$-symmetry. On the CFT side, we are now dealing with the presence of nonvanishing background $U(1)^3$ currents, which break the global $SO(6)$ $R$-symmetry. In principle, it should also be possible to count the microstates giving rise to the black string entropy using the dual SYM theory on $\mathbb{R}^2 \times H^2$ in the presence of these global background $U(1)^3$ currents. As the near-horizon geometry is $AdS_3 \times H^2$, the presented magnetic black string solutions may also have a holographic interpretation in the sense that the four-dimensional field theory discussed above flows to a two-dimensional CFT in the IR. These issues are currently under investigation [32].

Finally, in the present paper, we also investigated $3 + 2$ product compactifications, and showed that only if the internal two-manifold is diffeomorphic to $H^2$, some amount of supersymmetry can be preserved. As an example we found the one quarter supersymmetric $BTZ_{\text{extr}} \times H^2$ configuration, where $BTZ_{\text{extr}}$ denotes the extremal rotating BTZ black hole. Considering the equations of motion following from (11), one easily sees that one also can have the nonextremal BTZ black hole tensored by $H^2$ as a solution in presence of magnetic gauge fields. Presumably, these configurations arise as the near-horizon limit of the nonextremal generalization of (30).

**Acknowledgements**

The authors would like to thank K. Khuri-Makdisi and A. Zaffaroni for useful discussions.
References

[1] J. M. Maldacena, The large $N$ limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1998) 231; Int. J. Theor. Phys. 38 (1999) 1113; E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2 (1998) 253; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Gauge theory correlators from non-critical string theory, Phys. Lett. B428 (1998) 105; O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Large $N$ field theories, string theory and gravity, hep-th/9905111.

[2] S. W. Hawking and D. N. Page, Thermodynamics of black holes in anti-de Sitter space, Commun. Math. Phys. 87 (1983) 577.

[3] E. Witten, Anti-de Sitter space, thermal phase transition, and confinement in gauge theories, Adv. Theor. Math. Phys. 2 (1998) 505.

[4] A. Strominger, Black hole entropy from near-horizon microstates, JHEP 9802 (1998) 009.

[5] J. D. Brown and M. Henneaux, Central charges in the canonical realisation of asymptotic symmetries: An example from three-dimensional gravity, Commun. Math. Phys. 104 (1986) 207.

[6] L. J. Romans, Supersymmetric, cold and lukewarm black holes in cosmological Einstein-Maxwell theory, Nucl. Phys. B383 (1992) 395.

[7] L. A. J. London, Arbitrary dimensional cosmological multi-black holes, Nucl. Phys. B434 (1995) 709.

[8] A. Kostelecky and M. Perry, Solitonic black holes in gauged $N = 2$ supergravity, Phys. Lett. B371 (1996) 191.

[9] K. Behrndt, A. H. Chamseddine and W. A. Sabra, BPS black holes in $N = 2$ five dimensional AdS supergravity, Phys. Lett. B442 (1998) 97.

[10] M. M. Caldarelli and D. Klemm, Supersymmetry of anti-de Sitter black holes, Nucl. Phys. B545 (1999) 434.
[11] D. Klemm, *BPS black holes in gauged N = 4, D = 4 supergravity*, Nucl. Phys. B545 (1999) 461.

[12] K. Behrndt, M. Cvetic and W. A. Sabra, *Non-extreme black holes of five dimensional N = 2 AdS supergravity*, Nucl. Phys. B553 (1999) 317.

[13] M. J. Duff and J. T. Liu, *Anti-de Sitter black holes in gauged N = 8 supergravity*, Nucl. Phys. B554 (1999) 237.

[14] J. T. Liu and R. Minasian, *Black holes and membranes in AdS7*, Phys. Lett. B457 (1999) 39.

[15] W. A. Sabra, *Anti-de Sitter BPS black holes in N = 2 gauged supergravity*, Phys. Lett. B458 (1999) 36.

[16] A. H. Chamseddine and W. A. Sabra, *Magnetic strings in five dimensional gauged supergravity theories*, [hep-th/9911195](https://arxiv.org/abs/hep-th/9911195).

[17] M. Günyaydin, G. Sierra and P. K. Townsend, *Gauging the D = 5 Maxwell-Einstein supergravity theories: More on Jordan algebras*, Nucl. Phys. B253 (1985) 573.

[18] P. S. Howe and P. K. Townsend, *Supermembranes and the modulus space of superstrings*, Talk given at Trieste Conference on Supermembranes and Physics in 2 + 1 Dimensions, Trieste, Italy, Jul 17 - 21, 1989, Published in Trieste Supermembr. (1989) 165-172.

[19] B. de Wit and A. Van Proeyen, *Broken sigma-model isometries in very special geometry*, Phys. Lett. B293 (1992) 94.

[20] S. Ferrara and R. Kallosh, *Universality of supersymmetric attractors*, Phys. Rev. D54 (1996) 1525.

[21] A. H. Chamseddine and W. A. Sabra, *Calabi-Yau black holes and enhancement of supersymmetry in five dimensions*, Phys. Lett. B460 (1999) 63.

[22] D. Birmingham, *Topological black holes in anti-de Sitter space*, Class. Quant. Grav. 16 (1999) 1197.
[23] H. Lü, C. N. Pope, and J. Rahmfeld, *A construction of Killing spinors on $S^n$*, hep-th/9805151.

[24] O. Coussaert and M. Henneaux, *Supersymmetry of the 2+1 black holes*, Phys. Rev. Lett. **72** (1994) 183.

[25] Y. Kiem and D. Park, *BTZ black holes from the five-dimensional general relativity with a negative cosmological constant*, Phys. Lett. **B450** (1999) 41.

[26] V. Balasubramanian and F. Larsen, *Near horizon geometry and black holes in four dimensions*, Nucl. Phys. **B528** (1998) 229.

[27] M. Cvetič and F. Larsen, *Near horizon geometry of rotating black holes in five dimensions*, Nucl. Phys. **B531** (1998) 239.

[28] M. Cvetič and F. Larsen, *Microstates of four-dimensional rotating black holes from near-horizon geometry*, Phys. Rev. Lett. **82** (1999) 484.

[29] M. Cvetič, *Microscopics of rotating black holes: Entropy and greybody factors*, Based on talks given at PASCOS ’98, Strings ’98, and Buckow ’98, hep-th/9810142.

[30] N. Kaloper, *Entropy count for extremal three-dimensional black strings*, Phys. Lett. **B434** (1998) 285.

[31] M. Cvetič, M. J. Duff, P. Hoxha, J. T. Liu, H. Lu, J. X. Lu, R. Martinez-Acosta, C. N. Pope, H. Sati and T. A. Tran, *Embedding AdS black holes in ten and eleven dimensions*, hep-th/9903214.

[32] D. Klemm and W. A. Sabra, in preparation.