Event-triggered $H_{\infty}$ filtering for discrete-time Markov jump delayed neural networks with quantizations

Tingting Zhang, Jinfeng Gao & Jiahao Li

To cite this article: Tingting Zhang, Jinfeng Gao & Jiahao Li (2018) Event-triggered $H_{\infty}$ filtering for discrete-time Markov jump delayed neural networks with quantizations, Systems Science & Control Engineering, 6:3, 74-84, DOI: 10.1080/21642583.2018.1531360

To link to this article: https://doi.org/10.1080/21642583.2018.1531360

© 2018 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group

Published online: 09 Oct 2018.

Article views: 290

View related articles

View Crossmark data

Citing articles: 1
Event-triggered $H_\infty$ filtering for discrete-time Markov jump delayed neural networks with quantizations

Tingting Zhang, Jinfeng Gao and Jiahao Li

Faculty of Mechanical Engineering and Automation, Zhejiang Sci-Tech University, Hangzhou, People’s Republic of China

ABSTRACT
The problem of event-triggered $H_\infty$ filtering for discrete-time Markov jump delayed neural networks with quantizations is investigated in this paper. Firstly, an event-triggered communication scheme is proposed to determine whether or not the current sampled data can be transmitted to the quantizer. Secondly, a quantizer is used to quantify the sampled data, which can reduce the data transmission rate in the network. Next, through the analysis of network-induced delay’s intervals, the discrete-time neural network, the event-triggered scheme and network-induced delay are unified into a discrete-time Markov jump delayed neural network. As a result, the sufficient conditions are obtained to guarantee the stability and $H_\infty$ performance of the augmented system and to present the $H_\infty$ filter design. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

I. Introduction

In the past decades, neural networks have attracted more and more research attention due to their extensive applications in various areas such as signal processing, image processing and artificial intelligence. Recently, lots of significant subjects including stability analysis, feedback control and passivity analysis for delay recurrent neural networks have stirred a great deal of research interests (Jian & Zhao, 2015; Lin, He, Zhang, & Wu, 2018; Ma, Sun, Liu, & Xing, 2016; Yang, Li, & Huang, 2016).

In general, the neural networks display a characteristics of network modes jumps and such jumps are commonly considered to be determined by a time homogeneous Markov chain. With the aid of analysis and synthesis methodologies in the area of Markov jump linear systems (MJLSs) (Oliveira, Vargas, DoVal, & Peres, 2014; Xia, Sun, Teng, & Zhang, 2014), the resulting Markov jump neural networks (MJNNs) attract widely research interests, and a great number of literature are carried out for MJNNs, for more details, see Stoica and Yaesh (2008), Zhang, Zhu, Shi, and Zhao (2015), Zhuang, Ma, Xia, and Zhang (2016), Ren, Liu, Zhu, Zhong, and Shi (2017). Moreover, the filtering problems for neural networks are extensively studied by many researchers via various methodologies (Bao & Cao, 2011; Huang, Huang, & Chen, 2015). Nowadays, several methods are proposed to solve the problem of the $H_\infty$ filter design (Liu, Liu, Cao, & Zhang, 2015; Wang, Xue, Wang, & Lu, 2017). The authors in Wang et al. (2017) investigate the problem of event-based $H_\infty$ filtering for the discrete-time Markov jump system with network-induced delay. In Liu et al. (2016), the problem of the adaptive event-triggered $H_\infty$ filter design for a class of T-S fuzzy systems with time delay is studied. Overall, the $H_\infty$ filter problem has received researchers’ attention for a long time. Nevertheless, the study on MJNNs only has a short history and a few of vital results on this topic appear in the literature. Therefore, it is essential to pay attention to filter design in the various aspects of the MJNNs.

As we all know, the control signals are transmitted in a shared communication network, and the bandwidth of this network is limited in networked control systems (NCSs). It is crucial to construct appropriate communication strategies to reduce the bandwidth occupation of the communication network. Compared with the traditional time-triggered scheme, the event-triggered scheme is utilized as an efficient way to reduce the burden of the communication network and improve the transmission efficiency. In the event-triggered communication scheme, the data is only transmitted if it meets certain conditions. Up to now, different event-triggered schemes have been proposed (Hu & Yue, 2012a; Liu, Tang, & Fei, 2016; Wang et al., 2017; Wang, Zhang, & Lu, 2018; Yuan, Wang, & Guo, 2017; Zha, Fang, Li, & Liu, 2017). The authors in Zha et al. (2017) are concerned with $H_\infty$
output feedback control of event-triggered Markov jump systems (MJSs) with measured output quantizations. In Wang et al. (2018), the authors discuss the problem of the event-triggered $H_\infty$ filter design for MJSs with output quantization. Hu Yue (2012a) investigate the problem of event-based $H_\infty$ filtering for networked systems with communication delay. In Liu et al. (2016), the problem of event-based $H_\infty$ filtering for discrete-time MJSs with network-induced delay is investigated. In Liu et al. (2016), the problem of the $H_\infty$ filter design for a class of neural network systems is studied with the event-triggered communication scheme and quantization. Motivated by the above results, it is necessary to design an event-triggered scheme to save the limited communication resources in the Markov jump delayed neural networks.

Signal quantization as another way to overcome this problem has been extensively researched. It is indispensable to quantize signal before it being transmitted through a communication channel with a limited bandwidth. In Hu and Yue (2012b), the authors are concerned with the control design problem of event-triggered networked systems with both state and control input quantizations. The $H_\infty$ state estimation problem for a class of discrete-time neural networks with time-varying delays, randomly occurring quantizations and missing measurements is addressed in Zhang, Wang, Ding, and Liu (2015). The problem of robust $H_\infty$ estimation for a class of MUNNs with transmission delay, measurement quantization and data packet dropout is studied in Zhuang et al. (2016). In Sasiirekha, Rakkiyappan, Cao, and Alsaedi (2017), $H_\infty$ state estimation of discrete-time Markov jump neural networks with general transition probabilities and output quantization is described. The effect of the quantization on the networked control systems is greater than the traditional control systems. To the best of the authors’ knowledge, little work has been done in event-triggered $H_\infty$ filtering for discrete-time Markov jump delayed neural networks with quantizations. This situation motivates our current investigation.

Inspired by the results mentioned above, we focus on the event-triggered $H_\infty$ filtering for a class of discrete-time Markov jump delayed neural networks with quantizations. The main work is as follows:

(a) Event-triggered scheme for discrete-time Markov jump delayed neural networks, to reduce network resource wastage;
(b) Quantization signals to save limited bandwidth and energy consumption;
(c) Reduce system conservatism and obtain sufficient conditions by using the Jenson inequality, to guarantee the stability with the $H_\infty$ performance index of the augmented system. Then, the filter parameters are designed.

The rest of the paper is organized as follows. In Section II, an $H_\infty$ filter design is addressed for the discrete-time Markov jump delayed neural networks with the event-triggered communication scheme and quantization. $H_\infty$ filter performance analysis and the method of filter design are presented in Section III. A numerical example to illustrate the effectiveness of the obtained results is proposed in Section IV. Section V is the conclusion.

**Notation:** $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote the $n$ dimensional Euclidean space, and the set of $n \times m$ real matrices; superscript $T$ and $−1$ represent the transposition of vectors or matrices and matrix inverse, respectively. The notation $P > 0 (\leq 0)$ means $P$ is real symmetric positive definite. $I$ and $0$ represent identity matrix and zero matrix, respectively. In addition, $*$ is used to denote the symmetry entries of symmetry matrices.

## II. Problem formulation

### A. System description

Consider a discrete-time $n$-neuron network with time-varying delays as follows:

$$
\begin{align*}
    x(k + 1) &= A(r_k)x(k) + B(r_k)f(x(k)) \\
    &\quad + E(r_k)g(x(k - d(k))) \\
    &\quad + D(r_k)\omega(k) \\
    y(k) &= C(r_k)x(k) \\
    z(k) &= L(r_k)x(k)
\end{align*}
$$

where $x(k) \in \mathbb{R}^n$ is the state vector of the neural network, $y(k) \in \mathbb{R}^m$ is the measured output, $z(k) \in \mathbb{R}^p$ denotes the neural signal to be estimated, $\omega(k) \in \mathbb{R}^q$ is the external disturbance with $\omega(k) \in L_2[0,\infty)$. The positive integer $d(k) \in [d_m,d_M]$ describes the known time-varying delay, and $d_m$ and $d_M$ are the lower and upper bound of $d(k)$, respectively. $f(x(k)) \in \mathbb{R}^n$ and $g(x(k)) \in \mathbb{R}^p$ are the neuron activation function. Respectively, $A(r_k), B(r_k), C(r_k), D(r_k), E(r_k), L(r_k)$ are known real constant matrices with appropriate dimensions. $r_k$ is the discrete-time Markov jump process taking values in a finite space $S = \{1,2,\ldots,N\}$. The transition probability matrix $\Pi = \pi_{ij}(i,j \in S)$ is given by

$$
\sum_{j=1}^{N} \pi_{ij} = 1
$$

where $\pi_{ij} \geq 0, \forall ij \in S$. The positive integer $\pi_{ij}$ is the probability of jumping from $i$ to $j$ in one step. The state transition time $T$ is assumed to be large enough to guarantee $T \geq \sum_{c=1}^{N} \pi_{cc}T$. The adjustable constant $\gamma$ and $\delta$ are the upper bound of $d(k)$ and $d(k) - d_{m}$, respectively.
The probability transfer matrix is as follows:
\[
\Pi = \begin{bmatrix}
\pi_{11} & \cdots & \pi_{1N} \\
\vdots & \ddots & \vdots \\
\pi_{N1} & \cdots & \pi_{NN}
\end{bmatrix}
\]

The following \( H_\infty \) filter will be adopted:
\[
x_f(k+1) = A_f(r_k)x_f(k) + B_f(r_k)y(k)
\]
\[
z_f(k) = C_f(r_k)x_f(k)
\]

where \( x_f(k) \in \mathbb{R}^n \) is the state vector of the filter, \( z_f(k) \in \mathbb{R}^p \) is the output of the filter, \( A_f(r_k), B_f(r_k), C_f(r_k) \) are appropriately dimensioned filter matrices to be designed.

For simplicity, when \( r_k = i, i \in S \), a matrix \( M_i : A_i(r_k) \) will be denoted by \( M_i : A(r_k) \) by \( A_i \) and \( B(r_k) \) by \( B_i \) and so on.

Time-delay is inevitable during signals transmit through limited network bandwidth. It is supposed that the time-varying delay in network communication is \( \tau_k \in [0, \bar{\tau}] \), \( \bar{\tau} \) is the maximal network-induced delay. The event generator sends out the signal at the time \( s_i \) \( (l = 0, 1, 2, \ldots, \infty) \) and reaches the controller at time instant \( s_j + \tau_{s_j} \). Consider the effect of zero-order (ZOH), the actual measurements can be described as
\[
\hat{y}(k) = y(s_i) = C_i x(s_i), k \in [s_j + \tau_{s_j}, s_{j+1} + \tau_{s_{j+1}} - 1]
\]

**B. Event-triggered scheme**

To reduce the communication burden of network and guarantee the performance of system, an event generator is introduced between the sensor and the quantizer. Similar to Hu and Yue (2012a), where the current sample data \( y(k) \) is directly transmitted to the event-triggered mechanism, whether the latest information can be sent out and transmitted via the communication channels depends on the following condition:
\[
[y(k+j) - y(k)]^T \Phi_j [y(k+j) - y(k)] \\
> \sigma_j (k+j) \Phi_j y(k+j)
\]
\[
(4)
\]

where \( \Phi_j \in \mathbb{R}^m \) are symmetric positive matrices to be designed and \( \sigma_j \in [0, 1) \) is a given scalar parameter. Only when the current sampled sensor measurement \( y(k+j) \), \( (j = 1, 2, \ldots) \) and the latest transmitted sensor measurement \( y(k) \) satisfying the inequality \( (4) \), the current sampled sensor measurement \( y(k+j) \) will be sent out and transmitted to the quantizer \( q(\cdot) \).

**Remark II.1:** The sensor measurement is sampled at time \( k \in N \) and the next sensor measurement is at time \( k+1 \). For simplicity, the instant \( (k+j) \) is replaced by \( k_j \) in follows.

According to Yue, Tian, and Han (2013), using the similar methods, \( \tau_{s_j} \) is the delay at the instant \( s_j, \tau_{s_j} \in [0, \bar{\tau}] \)

Case A: If \( s_j + 1 + \bar{\tau} \geq s_{j+1} + \tau_{s_{j+1}} - 1 \), define a function
\[
\tau(k) = k - s_j, k \in [s_j + 1 + \bar{\tau}, s_{j+1} + \tau_{s_{j+1}} - 1]
\]
\[
(5)
\]

Obviously
\[
\tau_{s_j} \leq \tau(k) \leq (s_{j+1} - s_j) + \tau_{s_{j+1}} - 1 \leq 1 + \bar{\tau}
\]
\[
(6)
\]

Case B: If \( s_j + 1 + \bar{\tau} < s_{j+1} + \tau_{s_{j+1}} - 1 \), consider the following intervals:
\[
[s_j + \tau_{s_j}, s_j + \bar{\tau}], [s_j + \bar{\tau} + h, s_j + \bar{\tau} + h + 1]
\]
where \( h \in \mathbb{Z}_+ \) and satisfies \( h \geq 1 \). It can be easily shown that there exists a positive integer \( m \), such that
\[
s_j + m + \bar{\tau} < s_{j+1} + \tau_{s_{j+1}} - 1 \leq s_j + m + 1 + \bar{\tau}
\]
\[
(7)
\]

and \( y(s_i), y(s_{i+1}) \) with \( h = 1, 2, \ldots, m \) satisfy
\[
[y(s_{i+1}) - y(s_i)]^T \Phi_{i+m} [y(s_{i+1}) - y(s_i)] \\
\leq \sigma_j (y(s_{i+1}) - y(s_i))^T \Phi_j y(s_{i+1}) - y(s_i))
\]
\[
(8)
\]

from (5) to (7), we can obtain
\[
\Omega_1 = [s_j + \tau_{s_j}, s_j + \bar{\tau} + 1]
\]
\[
\Omega_2^h = [s_j + \bar{\tau} + h, s_j + \bar{\tau} + h + 1], h = 1, 2, \ldots, m - 1
\]
\[
\Omega_3 = [s_j + m + \bar{\tau}, s_{j+1} + \tau_{s_{j+1}} - 1]
\]
\[
(9)
\]

Define
\[
\tau(k) = \begin{cases}
-k - s_j, & k \in \Omega_1 \\
-k - s_j - h, & k \in \Omega_2^h \\
-k - s_j - m, & k \in \Omega_3
\end{cases}
\]
\[
(10)
\]

Then it can be easily shown that
\[
\tau_{s_j} \leq \tau(k) \leq 1 + \bar{\tau} \leq \tau_M \leq \tau_{s_j}
\]
\[
(11)
\]

For Case A, \( k \in [s_j + \tau_{s_j}, s_{j+1} + \tau_{s_{j+1}} - 1] \), define the error vector \( e(k) = 0 \). For Case B, define
\[
e(k) = \begin{cases}
0, & k \in \Omega_1 \\
y(s_{i+1}) - y(s_i), & k \in \Omega_2^h \\
y(s_{i+1} + m) - y(s_i), & k \in \Omega_3
\end{cases}
\]
\[
(12)
\]

From the definition of \( e(k) \) and the triggering algorithm (4), it can be seen that when \( k \in [s_j + \tau_{s_j}, s_{j+1} + \tau_{s_{j+1}} - 1] \),
\[
e^T(k) \Phi_j e(k) \leq \sigma_j (y(s_{i+1}) - y(s_i))^T \Phi_j y(s_{i+1}) - y(s_i))
\]
\[
(13)
\]
C. Event-triggered quantized $H_\infty$ filtering problem

In order to reduce the communication burden, the quantizer is employed. According to Hu and Yue (2012b), the quantizer $q(\cdot)$ is defined as $q(y) = [q_1(y_1), q_2(y_2), \ldots, q_n(y_n)]^T$, where $q_i(y_i)$ $(s = 1, 2, \ldots, n)$ are chosen as logarithmic quantizers given by

$$ q_i(y_i) =\begin{cases} u_i^{(s)}, & \text{if } y_i < \frac{1}{1 + \delta_{q_i}} u_i^{(s)} \quad \text{or } y_i > \frac{1}{1 - \delta_{q_i}} u_i^{(s)}, \quad s = 1, 2, \ldots, n \end{cases}$$

(14)

where $\delta_{q_i} = (1 - \rho_{q_i})/(1 + \rho_{q_i})$, $0 < \rho_{q_i} < 1$, $\rho_{q_i}$ is the quantization density with $i$ being a given constant. Moreover, similar to Peng and Tian (2007), the quantification level is defined as

$$ U_s = \{ \pm u_0^{(s)}, u_1^{(s)}, u_2^{(s)}, \ldots, u_l^{(s)} \} \bigcup \{ 0 \} $$

(15)

where $u_0^{(s)} > 0$. Then define $\Delta_q = \text{diag}\{ \Delta_{q_1}, \Delta_{q_2}, \ldots, \Delta_{q_n} \}$, where $\Delta_{q_i} \in [-\delta_{q_i}, \delta_{q_i}], s = 1, 2, \ldots, n$, the actual input of filter $y(k)$ can be expressed by the sector bound method as (Fu & Xie, 2005)

$$ \tilde{y}(k) = (1 + \Delta_q) y(s) \quad \text{(16)} $$

Define the new state vector $\tilde{x}(k) = [\tilde{x}^T(k), \tilde{x}^T_s(k)]^T$ and the filtering error vector $\tilde{z}(k) = z(k) - z_t(k)$, then the augmented system can be obtained as

$$ \tilde{x}(k + 1) = \tilde{A}_i \tilde{x}(k) + \tilde{B}_i f(H \tilde{x}(k)) + \tilde{E}_i g(H \tilde{x}(k) - d(k)) + \tilde{D}_i w(k) $$

$$ \tilde{z}(k) = \tilde{L}_i \tilde{x}(k) $$

(17)

where

$$ \tilde{A}_i = \begin{bmatrix} A & 0 \\ 0 & A_{0i} \end{bmatrix}, \quad \tilde{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad \tilde{E}_i = \begin{bmatrix} E_i \\ 0 \end{bmatrix}, \quad \tilde{D}_i = \begin{bmatrix} D_i \\ 0 \end{bmatrix} $$

$$ \tilde{B}_{1i} = \begin{bmatrix} B_{1i} \sqrt{1 + \Delta_{q_i}} C_i \end{bmatrix}, \quad \tilde{B}_{2i} = \begin{bmatrix} 0 \\ -B_{2i} \sqrt{1 + \Delta_{q_i}} \end{bmatrix} \quad \text{(18)} $$

$$ H = \begin{bmatrix} I & 0 \\ 0 & \tilde{L}_i \end{bmatrix}, \quad \tilde{L}_i = \begin{bmatrix} L_i & -C_n \end{bmatrix} $$

(19)

Before ending this section, we recall the following definition and lemma, which will help us in deriving the main results.

Assumption II.1 (Li, Hu, Hu, & Li, 2012): The nonlinear functions $f(\cdot)$, $g(\cdot)$ in (1) satisfy $f(0) = g(0) = 0$ and the following sector-bounded condition:

$$ \begin{aligned} [f(x) - f(y) - U_1(x - y)]^T [f(x) - f(y) - U_2(x - y)] &\leq 0 \\ [g(x) - g(y) - V_1(x - y)]^T [g(x) - g(y) - V_2(x - y)] &\leq 0 \end{aligned} $$

(17)

for all $x, y \in \mathbb{R}^n$, where $U_1$, $U_2$, $V_1$ and $V_2$ are constant real matrices of appropriate dimensions.

Definition II.1: The augmented system (16) with $\omega(k) = 0$ is asymptotically stable in mean square, if for any initial conditions, such that

$$ \lim_{k \to \infty} E[\| \tilde{x}(k) \|^2] = 0 $$

(18)

Definition II.2: Given a scalar $\gamma > 0$, for all nonzero $\omega(k) \in L_2(0, \infty)$, the filtering system (16) is asymptotically stable with an $H_\infty$ performance index $\gamma$ if it is asymptotically stable and the filtering error $\tilde{z}(k)$ satisfies

$$ \sum_{k=0}^{+\infty} E[\| \tilde{z}(k) \|^2] \leq \gamma^2 \sum_{k=0}^{+\infty} E[\| \omega(k) \|^2] $$

(19)

Lemma II.1 (Xie, Fu, & De Sousa, 1992): For any given symmetric matrix $E_1$, and $E_2, E_3$ are the proper dimensions of the matrix, then

$$ E_1 + E_2 \Delta(k) E_3 + \bar{E}_3^T \Delta^T(k) \bar{E}_2^T < 0 $$

for all $\Delta(k)$ satisfying $\Delta^T(k) \Delta(k) \preceq I$, if and only if there exists a positive scalar $\epsilon > 0$, such that

$$ E_1 + \epsilon^{-1} E_2^T \bar{E}_2 + \epsilon \bar{E}_3^T E_3 < 0 $$

Lemma II.2 (Wu, Su, Chu, & Zhou, 2010): For any symmetric positive-definite matrix $M \in \mathbb{R}^{n \times n}$, integers $d_1$ and $d_2$ $(d_1 \leq d_2)$, vector function $w(k) \in \mathbb{R}^n$, $k \in [d_1, d_2]$, such that the sums in the following are well defined, then

$$ \left( \sum_{i=d_1}^{d_2} w(i)^T \right) M \left( \sum_{i=d_1}^{d_2} w(i) \right) \leq (d_2 - d_1 + 1) \left( \sum_{i=d_1}^{d_2} (w(k)^T M w(k)) \right) $$

III. Main results

A. $H_\infty$ filter performance analysis

In this section, the asymptotical stability analysis result for the augmented system (16) is obtained.
Theorem III.1: For given scalars $d_m, d_M$, $\tau_M$ and $0 \leq \sigma_i < 1$, the system (16) is stochastically stable with an $H_\infty$ performance index $\gamma$ under the event-triggered scheme (4), if there exist matrices $P_i > 0$, $\Phi_i > 0$ ($i \in S$), $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$ with appropriate dimensions and positive scalars $\alpha_1 > 0\alpha_2 > 0$, satisfying the following LMIs:

$$
\begin{bmatrix}
\Sigma_1 & \Sigma_2 \\
\ast & \Sigma_3
\end{bmatrix} < 0
$$

(20)

$$
\sum_{j=1}^{N} \pi_j P_j \leq P_i
$$

(21)

where

$$
\Sigma_1 = \begin{bmatrix}
\Sigma & 0 \\
\ast & -\gamma^2 I
\end{bmatrix}
$$

$$
\Sigma_2 = \begin{bmatrix}
\Gamma_1^T P_i & d_m \Gamma_1^T H^T R_1 & d_m \Gamma_1^T H^T R_2 & \tau_M \Gamma_1^T H^T R_3 & \Gamma_1^T
\end{bmatrix}
$$

$$
\Sigma_3 = \text{diag}(-P_i, -R_1, -R_2, -R_3, -I)
$$

$$
\Sigma = \begin{bmatrix}
\Sigma_{11} & \Sigma_{12} \\
\ast & \Sigma_{22}
\end{bmatrix}
$$

$$
\Sigma_{11} = \begin{bmatrix}
0 & H^T R_2 & 0 & H^T R_3 \\
\ast & \Pi_{22} & R_1 + R_2 & 0 \\
\ast & \ast & \Pi_{33} & R_1 \\
\ast & \ast & \ast & -Q_2 - R_1 \\
\ast & \ast & \ast & \ast & \Pi_{55}
\end{bmatrix}
$$

$$
\Sigma_{12} = \begin{bmatrix}
0 & -\alpha_1 \check{U}_2 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \alpha_2 \check{V}_2 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
\Sigma_{22} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
\ast & -\alpha_1 I & 0 & 0 \\
\ast & \ast & -\alpha_2 I & 0 \\
\ast & \ast & \ast & -\Phi
\end{bmatrix}
$$

$$
\Pi_{11} = -P_i + H^T Q_1 H + H^T Q_2 H + H^T Q_3 H \\
-H^T R_2 H - H^T R_3 H - \alpha_1 \check{U}_1
$$

$$
\Pi_{22} = -Q_1 - R_1 - R_2, \Pi_{33} = -2R_1 - 2R_2 - \alpha_2 \check{V}_1
$$

$$
\Pi_{55} = -2R_3 + \alpha_1 \Theta\Theta = \begin{bmatrix}
C_1^T \Phi_i C_i & 0 \\
0 & 0
\end{bmatrix}
$$

$$
\Gamma_1 = \begin{bmatrix}
\bar{A}_i & 0 & 0 & 0 & \bar{B}_i & \bar{E}_i & \bar{B}_{2i} & \bar{D}_i
\end{bmatrix}
$$

$$
\Gamma_2 = \begin{bmatrix}
\bar{A}_i - I & 0 & 0 & 0 & \bar{B}_i & \bar{E}_i & \bar{B}_{2i} & \bar{D}_i
\end{bmatrix}
$$

$$
\Gamma_3 = \begin{bmatrix}
\bar{L}_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
d_1 = d_M - d_m.
$$

Proof: For the augmented system (16), construct the following Lyapunov functional:

$$
V(k) = V_1(k) + V_2(k) + V_3(k)
$$

(22)

where

$$
V_1(k) = \check{x}^T(k) P_i \check{x}(k)
$$

$$
V_2(k) = \sum_{s = k - d_m}^{k-1} \check{x}^T(s) H^T Q_1 H \check{x}(s)
$$

$$
+ \sum_{s = k - d_M}^{k-1} \check{x}^T(s) H^T Q_2 H \check{x}(s)
$$

$$
+ \sum_{s = k - \tau_M}^{k-1} \check{x}^T(s) H^T Q_3 H \check{x}(s)
$$

$$
V_3(k) = (d_M - d_m) \sum_{s = k - d_m}^{k-1} \sum_{l = s}^{k-1} \eta^T(l) H^T R_1 H \eta(l)
$$

$$
+ \sum_{s = k - d_M}^{k-1} \sum_{l = s}^{k-1} \eta^T(l) H^T R_2 H \eta(l)
$$

$$
+ \tau_M \sum_{s = k - \tau_M}^{k-1} \sum_{l = s}^{k-1} \eta^T(l) H^T R_3 H \eta(l)
$$

$$
\eta(l) = \check{x}(l + 1) - \check{x}(l)
$$

with $P_i > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, and $R_1 > 0$, $R_2 > 0$, $R_3 > 0$.

$$
E[\Delta V_1(k)] = E[V_1(k + 1) - V_1(k)]
$$

$$
= E[\check{x}^T(k + 1) \sum_{j=1}^{N} \pi_j P_j \check{x}(k + 1) - \check{x}^T(k) P_i \check{x}(k)]
$$

(23)

$$
E[\Delta V_2(k)] = E[V_2(k + 1) - V_2(k)]
$$

$$
= E[\check{x}^T(k) H^T Q_1 H \check{x}(k) - \check{x}^T(k - d_m) H^T Q_1 H \check{x}(k - d_m)]
$$

$$
+ \check{x}^T(k) H^T Q_2 H \check{x}(k) - \check{x}^T(k - d_M) H^T Q_2 H \check{x}(k - d_m)
$$

$$
+ \check{x}^T(k) H^T Q_3 H \check{x}(k) - \check{x}^T(k - \tau_M) H^T Q_3 H \check{x}(k - \tau_M)
$$

(24)

$$
E[\Delta V_3(k)] = E[V_3(k + 1) - V_3(k)]
$$

$$
= E[(d_m - d_m)^2 \eta^T(l) H^T R_1 H \eta(l)]
$$

$$
- (d_m - d_m) \sum_{l = k - d_m}^{k-1} \eta^T(l) H^T R_1 H \eta(l)
$$

$$
+ d_m^2 \eta^T(k) H^T R_2 H \eta(k) - d_m \sum_{l = k - d_m}^{k-1} \eta^T(l) H^T R_2 H \eta(l)
$$
\[ + \sum_{i=k-\tau_M}^{k-1} \eta^T(l) H^T R_3 H \eta(l) \right) \]

(25)

Letting
\[
\xi^T(k) = [\tilde{x}^T(k), \tilde{x}^T(k - d_m) H^T \tilde{x}^T(k - d(k)) H^T, f^T(H \tilde{x}(k)) g^T(H \tilde{x}(k - d(k))), e^T(k)]
\]

(26)

When, \( \omega(k) = 0 \), it easily can be seen that
\[
\tilde{x}(k + 1) = \hat{\Gamma}_1 \xi(k)
\]
\[
\eta(k) = \tilde{x}(k + 1) - \bar{x}(k)
\]

(27)

where
\[
\hat{\Gamma}_1 = \begin{bmatrix} \bar{A}_i & 0 & 0 & 0 & \bar{B}_i & \bar{E}_i & \bar{B}_2 \end{bmatrix}
\]
\[
\hat{\Gamma}_2 = \begin{bmatrix} \bar{A}_i - I & 0 & 0 & 0 & \bar{B}_i & \bar{E}_i & \bar{B}_2 \end{bmatrix}
\]

By Assumption II.1, we can obtain
\[
\begin{bmatrix} \tilde{x}(k) \\ f(H \tilde{x}(k)) \end{bmatrix}^T \bar{U}_1 \tilde{U}_2 \begin{bmatrix} \tilde{x}(k) \\ f(H \tilde{x}(k)) \end{bmatrix} \leq 0
\]

(28)

\[
\begin{bmatrix} \tilde{x}(k - d(k)) \\ g(H \tilde{x}(k - d(k))) \end{bmatrix}^T \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} \begin{bmatrix} \tilde{x}(k - d(k)) \\ g(H \tilde{x}(k - d(k))) \end{bmatrix} \leq 0
\]

(29)

where
\[
\bar{U}_1 = H^T \bar{U}_1, \quad \bar{U}_2 = \frac{(U_1^T U_2 + U_2^T U_1)}{2}, \quad \bar{U}_2 = -H^T \bar{U}_2
\]
\[
\bar{V}_1 = \frac{(V_1^T V_2 + V_2^T V_1)}{2}, \quad \bar{V}_2 = \frac{(V_1^T + V_2^T)}{2}
\]

So for two positive scalars \( \alpha_1, \alpha_2 \), it is easy to obtain
\[
- \alpha_1 \begin{bmatrix} \tilde{x}(k) \\ f(H \tilde{x}(k)) \end{bmatrix}^T \bar{U}_1 \tilde{U}_2 \begin{bmatrix} \tilde{x}(k) \\ f(H \tilde{x}(k)) \end{bmatrix} \geq 0
\]

(30)

\[
- \alpha_2 \begin{bmatrix} \tilde{x}(k - d(k)) \\ g(H \tilde{x}(k - d(k))) \end{bmatrix}^T \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} \begin{bmatrix} \tilde{x}(k - d(k)) \\ g(H \tilde{x}(k - d(k))) \end{bmatrix} \geq 0
\]

(31)

According to Lemma II.2 and combining (13) and (22)–(31), the following inequality holds:
\[
E(\Delta V(k)) = \sum_{i=3}^{3} E(\Delta V_i(k)) \leq E(\hat{\xi}^T(k) \hat{\Pi} \hat{\xi}(k))
\]

(32)

where
\[
\hat{\Pi} = \Sigma + \hat{\Gamma}_1^T P_i \hat{\Gamma}_1 + (d_m - d_m)^2 \hat{\Gamma}_2^T R_2 \hat{\Gamma}_2
\]
\[
+ d_m^2 \hat{\Gamma}_2^T R_2 \hat{\Gamma}_2 + \tau_m^2 \hat{\Gamma}_2^T R_3 \hat{\Gamma}_2
\]

(33)

According to Definition 2.1 and the Schur complement lemma, it can be concluded that (20) guarantees \( \hat{\Pi} < 0 \), so the augmented system (16) with \( \omega(k) = 0 \) is stochastically stable.

Then, continue to demonstrate the \( H_\infty \) performance of the system (16). When external disturbance \( \omega(k) \neq 0 \)
\[
E(\Delta V(k)) = E(\Delta V(k) + \hat{\xi}^T(k) \hat{\xi}(k)) - \gamma^2 \omega^T(k) \omega(k)
\]
\[
\geq E(\hat{\xi}^T(k) \hat{\Pi} \hat{\xi}(k))
\]

(34)

where
\[
\hat{\xi}^T(k) = \begin{bmatrix} \xi^T(k) \\ \omega^T(k) \end{bmatrix}
\]

(35)

\[
\hat{\Pi} = \Sigma + \hat{\Gamma}_1^T P_i \hat{\Gamma}_1 + (d_m - d_m)^2 \hat{\Gamma}_2^T R_2 \hat{\Gamma}_2
\]
\[
+ d_m^2 \hat{\Gamma}_2^T R_2 \hat{\Gamma}_2 + \tau_m^2 \hat{\Gamma}_2^T R_3 \hat{\Gamma}_2 + \Gamma_3^T \Gamma_3
\]

(36)

\[
\Sigma = \begin{bmatrix} \Sigma & 0 \\ 0 & -\gamma^2 I \end{bmatrix}
\]

By using the Schur complement lemma, (20) implies that \( \hat{\Pi} < 0 \). Hence, the augmented system (16) is stochastically stable with an \( H_\infty \) performance index \( \gamma \) if (20) and (21) are satisfied. This completes the proof.

\section{B. \( H_\infty \) filter design}

In this sequel, we give the event-triggered \( H_\infty \) filter design for the augmented system (16) based on Theorem III.1.

\textbf{Theorem III.2:} For given positive parameters \( d_m, d_m, \tau_M \) and \( 0 \leq \alpha_i < 1 \), the filtering error system (16) is stochastically stable with an \( H_\infty \) performance index \( \gamma \) under the event-triggered scheme (4), if there exist the real matrices \( P_{i1} > 0, X_i > 0, \Phi > 0(i \in S), Q_1 > 0, Q_2 > 0, Q_3 > 0, R_1 > 0, R_2 > 0, R_3 > 0, \tilde{A}_i, \tilde{B}_i \) and \( \tilde{C}_i \) with appropriate dimension and positive scalars \( \alpha_1 > 0, \alpha_2 > 0 \) satisfy \( P_{i1} - X_i > 0 \) and the following LMIs:

\[
\begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \bar{Y}_{13} & \bar{Y}_{14} & \bar{Y}_{15} & \bar{Y}_{16} \\ * & \bar{Y}_{22} & 0 & 0 & 0 & 0 \\ * & * & -R_1 & 0 & 0 & 0 \\ * & * & * & -R_2 & 0 & 0 \\ * & * & * & * & -R_3 & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0
\]

(37)
\[
\sum_{j=1}^{N} \pi_j (p_{1j} - x_j) \leq p_{1i} - x_i
\]

where

\[
\tilde{\gamma}_{11} = \begin{bmatrix}
1, 1 & 1, 2 & * & (2, 2)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\tilde{\Pi}_{11} & 0 & \tilde{R}_2 & 0 & \tilde{R}_3 \\
* & \Pi_{22} & R_1 + R_2 & 0 & 0 \\
* & * & \Pi_{33} & R_1 & 0 \\
* & * & * & -Q_2 - R_1 & 0 \\
* & * & * & * & \Pi_{55}
\end{bmatrix}
\]

\[
\tilde{\gamma}_{12} = \begin{bmatrix}
\Psi_{11} & \Psi_{122}^T \\
\psi_{121} & \psi_{121}^T
\end{bmatrix}
\]

\[
\begin{bmatrix}
\tilde{\gamma}_{13} & \tilde{\gamma}_{132}^T \\
\psi_{131} & \psi_{131}^T
\end{bmatrix}
\]

\[
\begin{bmatrix}
\gamma_{12} & \gamma_{122} \\
\gamma_{121} & \gamma_{121}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Lambda_{21} & \Lambda_{21} \\
\Lambda_{21} & \Lambda_{21}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Lambda_{25} & \Lambda_{25} \\
\Lambda_{25} & \Lambda_{25}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Lambda_{27} & \Lambda_{27} \\
\Lambda_{27} & \Lambda_{27}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Lambda_{29} & \Lambda_{29} \\
\Lambda_{29} & \Lambda_{29}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Lambda_{31} & \Lambda_{31} \\
\Lambda_{31} & \Lambda_{31}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Lambda_{41} & \Lambda_{41} \\
\Lambda_{41} & \Lambda_{41}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Lambda_{51} & \Lambda_{51} \\
\Lambda_{51} & \Lambda_{51}
\end{bmatrix}
\]

Moreover, if the above conditions are feasible, the parameter matrices of the filter are given by

\[
A_f = X_i^{-1} A_f \\
B_f = X_i^{-1} B_f \\
C_f = C_f
\]

**Proof:** Now, we define \( J_1 = \text{diag}(P_i, P_{1i}^{-1}, P_{3i}^{-1}) \), \( J_2 = \text{diag}(J_1, J_1, J_1, J_1) \). Then, pre- and post-multiplying (20) by \( \text{diag}(J_{2i}, J_{3i}) \) and \( \text{diag}(J_{2i}, J_{3i})^T \), respectively. Next, define new variables

\[
\bar{X}_i = P_{2i}^{-1} P_{2i}, \bar{A}_f = P_{2i} A_f P_{2i}^{-1} P_{2i}^T, \bar{B}_f = P_{2i} B_f, \bar{C}_f = C_f P_{3i}^{-1} P_{2i}^T
\]

Therefore, LMIs (20) is equivalent to LMIs (37).

Applying the Schur complement, \( P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i} & P_{3i} \end{bmatrix} > 0 \) is equal to \( P_{1i} - X_i > 0 \). Since \( X_i = P_{2i} P_{3i}^{-1} P_{2i}^T > 0, P_{2i} \) and \( P_{3i} \) are nonsingular matrices. Note that \( P_{2i} \) and \( P_{3i} \) cannot be directly derived from the condition (37), the transfer function \( T \) from \( y(k) \) to \( z_L(k) \) can be described as

\[
T_{z_Ly(s)} = C_f (s I - A_f)^{-1} B_f
\]

\[
= \bar{C}_f (s I - A_f)^{-1} B_f
\]

So the parameter matrices (39) of the filter are readily obtained. This completes the proof. \( \blacksquare \)

In the following, we will deal with the nonlinear terms in Theorem III.2.

**Theorem III.3:** For given scalars \( 0 \leq \sigma_i < 1, \) \( d_{ii}, \) \( d_{ii}, \) \( r_{ii}, \) and appropriate scalar \( \delta, \) the filtering error system (16) is stochastically stable with an \( H_\infty \) performance index \( \gamma \) under the event-triggered scheme (4), if there exist the real matrices \( P_{1i} > 0, X_i > 0, \Phi_i > 0 (i \in S), Q_1 > 0, Q_2 > 0, \) \( Q_3 > 0, R_1 > 0, R_2 > 0, R_3 > 0, A_f, B_f, \) and \( C_f, \) with appropriate dimension and positive scalars \( \alpha_1 > 0, \alpha_1 > 0 \) satisfy \( P_{1i} - X_i > 0 \) and the following LMIs:

\[
\begin{bmatrix}
\gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} & \gamma_{15} & \gamma_{16} & \gamma_{17} & \gamma_{18} \\
\gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} & \gamma_{25} & \gamma_{26} & \gamma_{27} & \gamma_{28} \\
\gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} & \gamma_{35} & \gamma_{36} & \gamma_{37} & \gamma_{38} \\
\gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} & \gamma_{45} & \gamma_{46} & \gamma_{47} & \gamma_{48} \\
\gamma_{51} & \gamma_{52} & \gamma_{53} & \gamma_{54} & \gamma_{55} & \gamma_{56} & \gamma_{57} & \gamma_{58} \\
\gamma_{61} & \gamma_{62} & \gamma_{63} & \gamma_{64} & \gamma_{65} & \gamma_{66} & \gamma_{67} & \gamma_{68} \\
\gamma_{71} & \gamma_{72} & \gamma_{73} & \gamma_{74} & \gamma_{75} & \gamma_{76} & \gamma_{77} & \gamma_{78} \\
\gamma_{81} & \gamma_{82} & \gamma_{83} & \gamma_{84} & \gamma_{85} & \gamma_{86} & \gamma_{87} & \gamma_{88}
\end{bmatrix} < 0
\]

(41)
By the Schur complement, linear matrix inequality (41) is rewritten as the following:

\[
\hat{\Sigma} + L_{B} \Delta q L_{C} + L_{C}^{T} \Delta q L_{B}^{T} < 0
\]  

(43)

where

\[
\begin{bmatrix}
0, ..., 0, \hat{\Theta}_{12}^{T} \\
0, ..., 0, \hat{\Theta}_{12}^{T} \\
0, ..., 0, \hat{\Theta}_{12}^{T} \\
0, ..., 0, \hat{\Theta}_{21}^{T}
\end{bmatrix}
\]

\[
\hat{\Theta}_{12} = \begin{bmatrix}
\hat{\Psi}_{121} \\
\hat{\Psi}_{122}
\end{bmatrix}^{T}, \hat{\Psi}_{121} = \begin{bmatrix}
\Lambda_{21}^{T} \\
0 & 0 & 0 & \Lambda_{25}^{T}
\end{bmatrix}, \hat{\Psi}_{122} = \begin{bmatrix}
0 & \Lambda_{27}^{T} \\
\Lambda_{18}^{T} & \Lambda_{29}^{T} & \Lambda_{20}^{T}
\end{bmatrix}
\]

\[
\hat{\Lambda}_{25} = [C_{1}^{T} B_{h}^{T}, C_{1}^{T} B_{h}^{T}], \hat{\Lambda}_{29} = [-B_{h}^{T}, -B_{h}^{T}]
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & \delta \hat{B}_{h}^{T} \bar{\Sigma} \\
0 & 0 & 0 & 0 & \epsilon \hat{\Sigma}
\end{bmatrix}^{T}
\]

\[
\hat{B}_{h}^{T} = \begin{bmatrix}
\hat{B}_{h}^{T} \\
\hat{B}_{h}^{T}
\end{bmatrix}
\]

**Proof:** The matrix (37) can be rewritten as the following:

\[
\dot{\bar{\Sigma}} + L_{B} \Delta q L_{C} + L_{C}^{T} \Delta q L_{B}^{T} < 0
\]  

(43)

where

\[
L_{B}^{T} = \begin{bmatrix}
0, ..., 0, \hat{\Theta}_{12}^{T} \\
0, ..., 0, \hat{\Theta}_{12}^{T} \\
0, ..., 0, \hat{\Theta}_{12}^{T} \\
0, ..., 0, \hat{\Theta}_{21}^{T}
\end{bmatrix}
\]

\[
L_{C} = \begin{bmatrix}
0, ..., 0, C_{i} \\
0, ..., 0, -I \\
0, ..., 0, -I
\end{bmatrix}
\]

\[
\hat{\Sigma} = \begin{bmatrix}
T_{11} & T_{12} & T_{13} & T_{14} & T_{15} & T_{16} \\
T_{21} & T_{22} & T_{23} & T_{24} & T_{25} & T_{26} \\
T_{31} & T_{32} & T_{33} & T_{34} & T_{35} & T_{36} \\
T_{41} & T_{42} & T_{43} & T_{44} & T_{45} & T_{46}
\end{bmatrix}
\]

According to Lemma II.1, there exists \( \epsilon > 0 \) such that

\[
\dot{\bar{\Sigma}} + \epsilon^{-1} L_{B} \Delta q L_{C}^{T} + \epsilon L_{C}^{T} L_{B} < 0
\]

and

\[
\Delta_{q}^{2} \leq \delta^{2}/I
\]

By the Schur complement, linear matrix inequality (41) can be easily obtained. This completes the proof.

**IV. Numerical example**

In this section, a numerical example of system (16) is provided as follows:

**Mode 1:**

\[
A_{1} = \begin{bmatrix}
0.75 & 0 \\
0 & 0.75
\end{bmatrix}, B_{1} = \begin{bmatrix}
0.1 & -0.1 \\
-0.1 & 0.1
\end{bmatrix}
\]

\[
E_{1} = \begin{bmatrix}
0.1 & 0.1 \\
0.1 & 0.1
\end{bmatrix}, C_{1} = \begin{bmatrix}
1 & 1
\end{bmatrix}
\]

\[
D_{1} = 0.1, L_{1} = \begin{bmatrix}
0.5 & 0.6
\end{bmatrix}
\]

**Mode 2:**

\[
A_{2} = \begin{bmatrix}
-0.2 & -0.2 \\
-0.3 & -0.4
\end{bmatrix}, B_{2} = \begin{bmatrix}
0.2 & 0 \\
0 & 0.21
\end{bmatrix}
\]

\[
E_{2} = \begin{bmatrix}
0.3 & 0.3 \\
0.3 & 0.3
\end{bmatrix}, C_{2} = \begin{bmatrix}
0.9 & 0.3
\end{bmatrix}
\]

\[
D_{2} = 0.5, L_{2} = \begin{bmatrix}
0.4 & 1
\end{bmatrix}
\]

The corresponding transition probability matrices of the Markov process are supposed to be

\[
\Pi = \begin{bmatrix}
0.3 & 0.7 \\
0.85 & 0.15
\end{bmatrix}
\]

Given the initial condition \( x(0) = [0.7, -0.1]^{T} \), \( x_{1}(0) = [-0.5, 0.3]^{T} \) and the external disturbance is

\[
\omega(k) = \begin{bmatrix}
0.2e^{-2k} \cos(0.5k) \\
0.2e^{-0.1k} \cos(0.4k)
\end{bmatrix}
\]

The neuron activation functions are taken as

\[
f(x) = \begin{bmatrix}
0.4x_{1} + \tanh(0.1x_{1}) + 0.2x_{2} \\
0.8x_{2} - \tanh(0.6x_{2})
\end{bmatrix}
\]

\[
g(x) = \begin{bmatrix}
0.5x_{1} + \tanh(0.2x_{1}) + 0.1x_{2} \\
0.7x_{2} - \tanh(0.1x_{1}) + 0.2x_{1}
\end{bmatrix}
\]

It is easy to obtain \( U_{1}, U_{2} \) and \( V_{1}, V_{2} \) satisfying Assumption 1

\[
U_{1} = \begin{bmatrix}
0.3 & 0.2 \\
0 & 0.2
\end{bmatrix}, U_{2} = \begin{bmatrix}
0.4 & 0.2 \\
0 & 0.8
\end{bmatrix}
\]

\[
V_{1} = \begin{bmatrix}
0.3 & 0.1 \\
0.1 & 0.7
\end{bmatrix}, V_{2} = \begin{bmatrix}
0.5 & 0.1 \\
0.2 & 0.7
\end{bmatrix}
\]

Then, we consider two cases for this discrete-time Markov jump delayed neural networks. Case 1: \( \sigma_{1} = \sigma_{2} \) and Case 2: \( \sigma_{1} \neq \sigma_{2} \).

For Case 1, the event-triggered parameter \( \sigma_{1} = \sigma_{2} = 0.2 \), when \( d_{m} = 1, d_{M} = 4 \), the maximum delay in communication network \( \tau_{M} = 5 \) and \( \epsilon = 1, \gamma = 3 \), by using the LMI toolbox of Matlab, it is easy to obtain the following matrices:

\[
A_{1} = \begin{bmatrix}
0.7887 & 0.0073 \\
-0.0395 & 0.8184
\end{bmatrix}, B_{1} = \begin{bmatrix}
-0.3033 \\
-0.2636
\end{bmatrix}
\]

\[
C_{1} = \begin{bmatrix}
-0.4478 & -0.5476 \\
2.8538 & 0
\end{bmatrix}, \Phi_{1} = \begin{bmatrix}
0.2390 \\
-0.2206
\end{bmatrix}
\]

\[
A_{2} = \begin{bmatrix}
0.7864 & 0.0062 \\
-0.0378 & 0.5758
\end{bmatrix}, B_{2} = \begin{bmatrix}
-0.4478 & -0.5476 \\
2.8538 & 0
\end{bmatrix}
\]

\[
C_{2} = \begin{bmatrix}
0.7864 & 0.0062 \\
-0.0378 & 0.5758
\end{bmatrix}, \Phi_{2} = \begin{bmatrix}
0.2390 \\
-0.2206
\end{bmatrix}
\]

For Case 2, the event-triggered parameter \( \sigma_{1} = 0.2, \sigma_{2} = 0.1 \), when \( d_{m} = 1, d_{M} = 4 \), the maximum delay in communication network \( \tau_{M} = 5 \) and \( \epsilon = 1, \gamma = 3 \), by using the
LMI toolbox of Matlab, it is easy to obtain the following matrices:

\[
A_{f1} = \begin{bmatrix}
0.5457 & 0.0203 \\
-0.0256 & 0.7986
\end{bmatrix},
B_{f1} = \begin{bmatrix}
-0.3019 \\
-0.0301
\end{bmatrix},
C_{f1} = \begin{bmatrix}
-0.4940 & -0.5971
\end{bmatrix}, \Phi_1 = 3.0044
\]

\[
A_{f2} = \begin{bmatrix}
0.5358 & 0.0269 \\
-0.0127 & 0.7186
\end{bmatrix},
B_{f2} = \begin{bmatrix}
-0.4046 \\
-0.0076
\end{bmatrix},
C_{f2} = \begin{bmatrix}
0.2546 & 0.2081
\end{bmatrix}, \Phi_2 = 4.1218
\]

Then, Figures 1 and 5 show the probabilities of switching between modes. Figures 2 and 6 show the response of the measured output \(z(k)\) and its estimate \(z_f(k)\), it can be seen that although there exist slight differences of the trajectories, the final tracking effect is good. Figures 3 and 7 show the response of filter error \(e(k)\), from which we can see that the designed filter is effective. The event-triggered release instants and intervals are shown in Figures 4 and 8. The time of this simulation is 100 times, and Figures 4 and 7 are triggered 25 and 22 times, respectively. Thence, this method greatly reduces the waste of network resources. The results show that the event-triggered scheme and quantization can reduce the communication load in the neural network and gear up its efficiency.

**Remark IV.1:** Although some feasible results have been developed in the existing literature to deal with \(H_{\infty}\) filter (estimation) for neural networks with quantizations (Sasirekha et al., 2017; Zhang et al., 2015; Zhuang et al., 2016), they are difficult to be applied directly to deal with the case that exist in the event-triggered scheme. Comparing the existing literature and simulation results, we can easily conclude that the event-triggered mechanism can reduce the use of network bandwidth effectively.
Figure 5. Case 2: the probabilities of switching between modes.

Figure 6. Case 2: the measured output $z(k)$ and its estimation $z_f(k)$.

Figure 7. Case 2: the mode of estimation error $e(k)$.

Figure 8. Case 2: the event-triggering release instants and intervals.

Remark IV.2: In Liu et al. (2016), the authors designed a $H_\infty$ filter for a class of neural network systems with quantization and event-triggered schemes. The simulation results of this article can further illustrate that the event-triggered mechanism scheme and quantization can reduce the use of network resources, but there are no relevant research on discrete-time Markov jump delayed neural networks. This situation motivates our current investigation.

V. Conclusion

The event-triggered $H_\infty$ filter design problem for discrete-time Markov jump delayed neural networks with quantizations is studied in this paper. To reduce the communication bandwidth utilization, an event-triggered communication scheme and a quantizer are introduced to the framework. Based on the analysis of network-induced delay, a unified discrete-time Markov jump filter error system with time-delay is constructed to describe the event-triggered scheme, network-induced delays, quantizations and the neural network system together. The sufficient conditions of stochastically $H_\infty$ norm bound are obtained for the estimation error system with event-triggered scheme and quantization. Finally, a simulation example is presented to illustrate the effectiveness of the designed method.

Acknowledgement

The authors are grateful for the support of the National Natural Science Foundation of China (61374083).

Disclosure statement

No potential conflict of interest was reported by the authors.
References

Bao, H., & Cao, J. (2011). Delay-distribution-dependent state estimation for discrete-time stochastic neural networks with random delay. *Neural Networks the Official Journal of the International Neural Network Society*, 24(1), 19–28.

Fu, M., & Xie, L. (2005). The sector bound approach to quantized feedback control. *IEEE Transactions on Automatic Control*, 50(11), 1698–1711.

Hu, S., & Yue, D. (2012a). Event-based $H_{\infty}$ filtering for networked system with communication delay. *Signal Processing*, 92(9), 2029–2039.

Hu, S., & Yue, D. (2012b). Event-triggered control design of linear networked systems with quantizations. *ISA Transactions*, 51(1), 153–162.

Huang, H., Huang, T., & Chen, X. (2015). Further result on guaranteed $H_{\infty}$ performance state estimation of delayed static neural networks. *IEEE Transactions on Neural Networks & Learning Systems*, 26(6), 1335–1341.

Jian, J., & Zhan, Z. (2015). Global stability in lagrange sense for bam-type cohen grossberg neural networks with time-varying delays. *Systems Science & Control Engineering*, 3(1), 1–7.

Li, N., Hu, J., Hu, J., & Li, L. (2012). Exponential state estimation for delayed recurrent neural networks with sampled-data. *Nonlinear Dynamics*, 69(1–2), 555–564.

Lin, W. J., He, Y., Zhang, C. K., & Wu, M. (2018). Stability analysis of neural networks with time-varying delay: Enhanced stability criteria and conservatism comparisons. *Communications in Nonlinear Science and Numerical Simulation*, 54, 118–135.

Liu, J., Liu, Q., Cao, J., & Zhang, Y. (2016). Adaptive event-triggered $H_{\infty}$ filtering for t-s fuzzy system with time delay. *Neurocomputing*, 189, 86–94.

Liu, J., Tang, J., & Fei, S. (2016). Event-triggered $H_{\infty}$ filter design for delayed neural network with quantization. *Neural Networks*, 82, 39–48.

Ma, Z., Sun, G., Liu, D., & Xing, X. (2016). Dissipativity analysis for discrete-time fuzzy neural networks with leakage and time-varying delays. *Neurocomputing*, 175, 579–584.

Oliveira, R. C. L. F., Vargas, A. N., Do Val, J. B. R., & Peres, P. L. D. (2014). Mode-independent $H_2$-control of a dc motor modeled as a markov jump linear system. *IEEE Transactions on Control Systems Technology*, 22(5), 1915–1919.

Peng, C., & Tian, Y. C. (2007). Networked $H_{\infty}$ control of linear systems with state quantization. *Information Sciences*, 177(24), 5763–5774.

Ren, J., Liu, X., Zhu, H., Zhong, S., & Shi, K. (2017). State estimation of neural networks with two markovian jumping parameters and multiple time delays. *Journal of the Franklin Institute*, 354(2), 812–833.

Sasirekha, R., Rakkiyappan, R., Cao, J., Wan, Y., & Alsaeedi, A. (2017). $H_{\infty}$ state estimation of discrete-time markov jump neural networks with general transition probabilities and output quantization. *Journal of Difference Equations & Applications*, 23(3), 1–29.

Stoica, A. M., & Yaesh, I. (2008). Markovian jump delayed hopfield networks with multiplicative noise. *Automatica*, 44(8), 2157–2162.

Wang, H., Shi, P., & Zhang, J. (2015). Event-triggered fuzzy filtering for a class of nonlinear networked control systems. *Signal Processing*, 113, 159–168.

Wang, H., Xue, A., Wang, J., & Lu, R. (2017). Event-based $H_{\infty}$ filtering for discrete-time markov jump systems with network-induced delay. *Journal of the Franklin Institute*, 354(14), 6170–6189.

Wang, H., Zhang, D., & Lu, R. (2018). Event-triggered $H_{\infty}$ filter design for Markovian jump systems with quantization. *Nonlinear Analysis Hybrid Systems*, 28, 23–41.

Wu, Z., Su, H., Chu, J., & Zhou, W. (2010). Improved delay-dependent stability condition of discrete recurrent neural networks with time-varying delays. *IEEE Transactions on Neural Networks*, 21(4), 692–697.

Xia, J., Sun, C., Teng, X., & Zhang, H. (2014). Delay-segment-dependent robust stability for uncertain discrete stochastic markovian jumping systems with interval time delay. *International Journal of Systems Science*, 45(3), 271–282.

Xie, L., Fu, M., & De Souza, C. E. (1992). $H_{\infty}$ control and quadratic stabilization of systems with parameter uncertainty via output feedback. *IEEE Transactions on Automatic Control*, 37(8), 1253–1256.

Yang, S., Li, C., & Huang, T. (2016). Exponential stabilization and synchronization for fuzzy model of memristive neural networks by periodically intermittent control. *Neural Networks*, 75(3), 162–172.

Yuan, Y., Wang, Z., & Guo, L. (2017). Event-triggered strategy design for discrete-time nonlinear quadratic games with disturbance compensations: The noncooperative case. *IEEE Transactions on Systems Man & Cybernetics Systems*, 99, 1–12.

Yue, D., Tian, E., & Han, Q. L. (2013). A delay system method for designing event-triggered controllers of networked control systems. *IEEE Transactions on Automatic Control*, 58(2), 475–481.

Zha, L., Fang, J. A., Li, X., & Liu, J. (2017). Event-triggered $H_{\infty}$ output feedback control for networked markovian jump systems with quantizations. *Nonlinear Analysis Hybrid Systems*, 24, 146–158.

Zhang, J., Wang, Z., Ding, D., & Liu, X. (2015). $H_{\infty}$ state estimation for discrete-time delayed neural networks with randomly occurring quantizations and missing measurements. *Neurocomputing*, 148, 388–396.

Zhang, L., Zhu, Y., Shi, P., & Zhao, Y. (2015). Resilient asynchronous $H_{\infty}$ filtering for markov jump neural networks with unideal measurements and multiplicative noises. *IEEE Transactions on Cybernetics*, 45(12), 2840–2852.

Zhuang, G., Ma, Q., Xia, J., & Zhang, H. (2016). $H_{\infty}$ estimation for markovian jump neural networks with quantization, transmission delay and packet dropout. *Neural Processing Letters*, 44(2), 317–341.