The effect of wave crest speed in breaking waves

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Abstract. The present work focuses on the evolution of the crest speed around large focal type water wave events. The phenomenon of crest slowdown around the point of maximum elevation is studied numerically using a boundary element model. The model is used to generate large design waves based on realistic ocean spectra. The evolution in terms of coupling between crest speed reduction and increase in particle velocities is studied up until initiation of wave breaking. It is shown that this coupling is a viable mechanism that leads to the formation of large deep water unidirectional breaking waves regardless of spectral shape and bandwidth.

1. Introduction
The global oceans and seas have always been an indispensable resource for the world's population. The global economy is dependent on the ever-increasing human activity in the oceans. Structures that range from offshore wind farms, high-voltage direct current converter platforms, wave energy converters and oil and gas platforms ensure and satisfy our energy demands. Furthermore, bulk carriers, tankers, vessels and container ships all roam the oceans to maintain global trade and transport of goods.

The design of reliable, fit for purpose and cost-effective offshore infrastructure requires a grasp of the harsh environmental conditions and safety factors. An overconservative design represents a considerable increase in the cost of marine structures in the order of millions of dollars. This in turn can render a project commercially infeasible. To balance the costs of the project with acceptable levels of failure probabilities, modern codes of design and practice were formalised and are used today. However, the importance of wave breaking is unfortunately not emphasised enough in any of the current standards. As breaking occurs at the crest of steep waves, it causes maximum loads from both the highest fluid particle kinematics and largest moment-arm about the sea bed. Slamming loads due to wave breaking pose a serious threat to sub-structures with sizeable vertical members as well as super-structures. For existing structures, especially those affected by subsidence, the main concerns related to wave breaking originate from loss of air gap and the resulting wave-in-deck loads as well as large dynamical excitations.

All the above emphasise the engineering importance associated with the understanding of breaking waves. However, the present understanding around the process of wave breaking is limited due to its complexity. The turbulent two-phase flow characterised by different length scales of interacting eddies is beyond the modelling capabilities of many numerical approaches and difficult to completely measure and describe from laboratory experiments. The prediction and detection of wave breaking remains one of the greatest challenges for metocean engineers.

The scientific community has adopted several approaches in an attempt to study and predict wave breaking. The study of [1] summarised that advancements have been made in describing wave breaking geometry, energy dissipation and wave breaking onset criteria. While a universal geometric criterion seems improbable, dynamic and kinematic criteria more in keeping with the physics of the process are showing promising results. Moreover, recent studies into the reduction of crest speed in large wave events [2] have prompted the reassessment of the classical kinematic wave breaking criterion which states that breaking occurs when the horizontal velocity of the free surface particle (u) exceeds the propagation velocity of the wave crest (C).
After this introductory section, the present paper continues with a brief outline of the background work in Section 2 around the effects of crest slowdown on the formation of wave breaking. Following this, the wave cases generated using the fully-nonlinear boundary element model (BEM) of [4] will be presented in Section 3 together with the methodologies applied in computing the crest speed. Finally, Section 4 presents the current findings while Section 5 draws the relevant conclusions.

2. Background
A kinematic breaking criterion is based on accurately determining the fluid particle velocity and comparing it to some threshold value above which wave breaking would occur. According to [5] the kinematic criteria are based on wave motions such as fluid particle velocity or wave phase speed. The basic kinematic criteria frequently used is that breaking occurs when the maximum horizontal water particle velocity, $u_{\text{max}}$, exceeds the phase velocity, $c$; essentially meaning that the water travels faster than the wave, leading to a jet. As the process of wave breaking is highly nonlinear, [1] states that in the context of random waves, the application of these criteria is difficult due to the need to accurately calculate particle velocities and define the crest speed. While for monochromatic waves the phase speed $c$ is clearly defined, this is not the case for random waves. For random waves, the definition of a wave crest speed $C$ is preferred, but accurate measurements of this quantity (especially in field conditions) is difficult.

There have been several studies that suggested that breaking initiation occurs well below the unity threshold. Numerical work of [6] found that the maximum horizontal velocity at the wave crest $u_{\text{max}}$ computed from a third-order numerical formulation was 38% smaller than the crest velocity for waves which in the laboratory were seen to break. The study concluded that the horizontal components of the water particle velocity remained smaller than the crest propagation velocity for the studied cases. This would seem to suggest that the basic kinematic criteria defined at the beginning of this section does not always hold. On the other hand, this result could also be a limitation of the third-order numerical scheme implemented in [6], that uses a semi-Lagrangian method which cannot model breaking waves. The experimental studies of [7] backup the findings of [6]. Using both PIV measurements of laboratory generated breaking waves and field data from the GOTEX project, [7] conclude that the maximum particle velocity remains well below the local wave speed ($u_{\text{max}} < c$).

A recent set of studies show that the wave crest slows down near a maximum point such as focusing. The study of [8] includes the measurements after breaking initiation of the surface profile. Having the measured time series at two closely spaced measuring stations allowed them to estimate the speed of the breaking crests ($C_{br}$) and compare that to a prediction by the linear phase speed ($c$). Based on a sample size of 30 waves and measuring passing time of the crest between two stations, the relationship of $C_{br}/c = 0.9$ was established. Effectively, a linear prediction of the phase speed seems to overestimate the actual speed of the crest. This is because as the wave group approaches focusing, there is a well-known transfer of energy towards the high frequency range [9]. This range is characterised by short waves that travel at low speeds. The increased energy levels in this region coupled with the low propagation speed of these wave components results in the overall slowdown of the crest relative to linear theory.

Within the context of these new findings, it has been discussed that the classical kinematic criterion of unity might indeed be sufficient for breaking, but breaking initiation may occur at lower threshold values. This phenomenon of crest slowdown has been the focus of several numerical and experimental based publications [2,10–12]. The most promising result that stems from this recent finding related to the evolution of large wave crests is the dynamic breaking threshold proposed by [13]. This newly derived breaking criterion simplifies to a kinematic criterion at the free surface of water waves and is further detailed in Section 4.

On the topic of kinematic wave breaking criteria there is much research ongoing. A case in point is that there is still no clear understanding relating the effects of wave nonlinearity on the evolution of crest speed. Furthermore, the topic of the reduction in crest speed near large wave events has not been extensively studied in the context of realistic ocean wave spectra. This phenomenon has a clear implication in the formation of breaking waves, but the implications in the context of engineering design
are less understood. This can be put in the context of engineering challenges such as wave-in-deck and slamming loads, where focused or NewWave [14] wave events are generally used in design. As such, this study will present a numerical investigation based on boundary element modelling into the evolution of steep waves to breaking from the perspective of a kinematic criterion.

3. Methodology

For the present study unidirectional deep water focused wind wave groups based on the NewWave theory developed by [14–16] were simulated. Describing the free wave components that are directly proportional to the autocorrelation function of the surface elevation, NewWave generates the most probable shape of the largest waves in the field. They typically represent design wave cases. The BEM was employed to generate these wave groups within a numerical wave tank at a given location. Full details regarding the set-up of the numerical model can be found in [17]. The equivalent linear surface elevation was generated using linear random wave theory (LRWT), which employed the identical space-time domain as the BEM simulations.

3.1. Wave cases

Wind waves that are locally influenced by weather conditions (fetch limited) are typically described using the JONSWAP spectra of [18] while swell dominated seas are described using Gaussian spectra. Within the present study, focused waves were generated such that they replicate laboratory experiments corresponding to a length scale of 1:100 and a time scale of 1:10. The peak spectral component for both spectra had a period of $T_p = 1.2 \text{ s}$. For the JONSWAP spectra, three typical values for the peak enhancement factor $\gamma$ of 1, 2.5 and 5 were used. These effectively control the peakedness of the spectra and slightly modify bandwidth. For the Gaussian spectra the bandwidth was controlled using three values for standard deviation of $\sigma$ of 0.15 Hz, 0.25 Hz and 0.45 Hz.

The depth of the numerical domain was fixed at 1.5 m, setting the depth regime of the cases at $kpd=4.2$ ($\tanh(kpd) = 0.999$ - deep water), where $kp$ is the wavenumber of the spectral peak. All the wave cases studied are summarised in Table 1. The linear amplitude sum ($A$ - which is the largest elevation wave event within a linear wave group) was systematically increased until wave breaking was initiated. A typical example of the spatial surface elevation of focused events normalised by $A$ obtained for the Gaussian spectra with $\sigma = 0.15 \text{ Hz}$ for increasing steepness is shown in Figure 1. The figure illustrates the typical nonlinear amplifications above the linear input sum $A$ as waves evolve towards breaking.

| Spectrum  | Bandwidth | $k_c$ [rad/m] | $A_kc$ range |
|-----------|-----------|---------------|--------------|
| JONSWAP   | $\gamma=1$| 4.8           |              |
|           | $\gamma=2.5$| 4.3           | $A_kc \leq 0.31$ |
|           | $\gamma=5$| 3.9           |              |
| Gaussian  | $\sigma=0.15$ Hz | 2.9           | $A_kc \leq 0.26$ |
|           | $\sigma=0.25$ Hz | 3.1           | $A_kc \leq 0.29$ |
|           | $\sigma=0.45$ Hz | 3.8           | $A_kc \leq 0.31$ |

An estimate of the nonlinearity of a given wave group that incorporates the bandwidth of the spectrum can be computed using a weighted average quantity termed the central wavenumber $k_c = \sum a_n k_n / \sum a_n$ (where $a_n$ and $k_n$ are the amplitude and linear wave number of individual components respectively). Hence, the global steepness of all wave cases is described using $A_kc$.

To test convergence of the generated BEM focused wave events, the input location of the numerical wave paddle $x_l$ in the computational domain was iteratively changed and the focused wave events compared. An example of the focused events obtained from four different domain sizes is shown in Figure 2a for the JONSWAP case with $\gamma = 1$ and $A_kc = 0.24$ and in Figure 2b for the JONSWAP $\gamma = 5$.
2.5 case with Akc = 0.215. The two figures show that for xl ≤ -9 m there are no discernible differences in the focused event. The location of xl = -9 m was chosen such that the computational effort was minimised, and more spatial information of the wave group was available, especially for the narrow-banded cases. Additionally, the input was far enough from the focus location so that interactions above second-order (which confer the asymmetry to the focused crest) were allowed enough time to develop. The horizontal and vertical asymmetry of the largest crest and downshift of the location of ηmax is evident in Figure 2.

Figure 1. Spatial surface elevation of focused wave event normalised by the linear amplitude sum A for Gaussian spectra with σ = 0.15 Hz and increasing steepness Akc (shown as increasingly darker shades of grey). For comparison all focused events are shifted to x = 0 m.

Figure 2. Comparison between focused wave events for four different domain sizes.

As the wave group becomes steeper and approaches breaking, both [19] and [20] showed the importance of using a correct reference frame within the BEM. As such, the BEM employs a switching scheme
between a semi-Lagrangian and a fully-Lagrangian reference frame [17]. This switch is done at a user specified time, and in the case of breaking waves, [20] notes that the switching time becomes essential in obtaining an appropriate nodal resolution. The fully-Lagrangian frame allows the BEM model to accurately describe steep waves and clear plunging breaking waves that form a jet up until the point of surface reconnection.

For a linear wave group [19] showed that the numerical predictions between a semi-Lagrangian and a fully-Lagrangian reference frame are identical. Furthermore, the fully-Lagrangian results are independent of the switching times. For the cases studied here, Figure 3a shows that for the JONSWAP case with $\gamma = 1$ and $A_{kc} = 0.24$ the differences between the two reference frames are small. However, as the nonlinearity of the wave group is increased in Figure 3b, the changes in predictions become important and the fully-Lagrangian frame is needed to capture the appropriate shape of the wave event.

![Figure 3](image_url)

**Figure 3.** Comparison of the BEM predicted surface elevation in the vicinity of the focused event in a fully-Lagrangian frame (dashed blue line) and a semi-Lagrangian
frame (continuous red line). The surface profiles from both frames of reference are shown at the same simulation time.

It is clear that to obtain both accurate estimates of the crest propagation speed and particle kinematics, the fully-Lagrangian frame should be used for very steep waves. Figure 3c shows the evolution of the focused crest for the steep JONSWAP $\gamma = 1$ case with $A_{k_c} = 0.312$. The increased steepness and amplitude of the fully-Lagrangian simulation is mostly affecting the crest region, where significantly higher particle velocities will develop.

3.2. Calculation of crest speed
As the BEM simulations are characterised by a high temporal ($\Delta t \approx 0.002s$) and spatial discretisation ($\Delta x = 0.025m$), this allowed accurate calculations of the crest speed. A first step in the crest speed estimation is to identify the space-time trajectory of the wave crest. In this study, the location of the crest was determined from time-signals of the free surface $\eta(t)$ at each individual computational node in the domain. The instantaneous characteristic crest speed is thus given by the slope $\frac{\Delta x_c}{\Delta t_c}$ of the crest space-time trajectory. Using this approach, $\Delta t_c$ measures the time taken for the crest to travel the distance between two measuring points $\Delta x_c$. It must be noted that with the use of a Lagrangian frame, the computational nodes are free to move both horizontally and vertically. To keep a constant $\Delta x_c$, interpolation of the spatial free surface profiles onto a fixed grid is done in the post-processing stage. An example of typical crest trajectories obtained is shown in Figure 4. The crest speed calculated using the above approach will be termed $C_{BEM}$ when it is obtained from the BEM simulations and $C_{LRWT}$ when it is obtained from the equivalent LRWT simulation.

![Figure 4. Example of identified crest trajectories with highlights for maximum elevation at a given location $x_i$ (thick continuous blue line) and elevation of focal event (red circle).](image)

In addition to the crest speed, the location of the centroid of the wave above still water level (SWL) was also computed from the simulations. The x-coordinate of the centroid ($X_G$) shown in Figure 5 was calculated by geometric decomposition of the wave above SWL into small trapeziums based on the
spatial discretisation of the BEM simulation. The coordinate was then used to compute an estimate of the propagation speed of the centroid of the wave to compare with the results obtained from the tracking of the actual crest. This was only done for the non-breaking waves.

4. Results and discussion

Within the present study, the BEM simulations were used to observe the trajectory of the crest within the space-time domain that would lead to the focused event. The reasoning behind this was discussed in Section 2 in the context of the wave crest speed evolution and the application of the NewWave theory for the design of offshore structures.

Within a wave group, the variation in crest speed will be present within all wave crests as they evolve and decay [2,11]. Panel (a) of Figure 6 concentrates on the crest speed of the largest crest within the wave group as well as the two adjacent crests around the focal event for the JONSWAP case with $\gamma = 1$ and $A_{kc} = 0.096$. The results of the wave crests obtained from both LRWT and BEM are illustrated. Immediately visible is a considerable reduction in the crest speed of the waves as they reach their maximum elevation.

Together with the crest speed, the speed calculated by tracking the centroid of the wave is presented, (as shown in Figure 5). It is noticeable that while the entire body of the wave slows down as a crest reaches its maximum elevation, there is a considerable additional slowdown in the crest region. This figure confirms the finding of [11] that the crest slowdown is a highly localised process, and this occurs in all crests within the wave group. As breaking is also a highly localised process that occurs high-up in the wave crest, the significance of this local crest slowdown becomes apparent. In this panel, as the steepness is low, the LRWT results are in good agreement with the BEM simulations.

To complement Figure 6a, Figures 6b and 6c illustrate the same results but for steeper cases of $A_{kc} = 0.192$ and 0.29. In both figures it can be observed that in the focal region, the centroid of the wave crest has a significantly larger speed compared to the crest. Focusing only on the focal event at $t/T_p = 0$ and comparing all three panels of Figure 6 it can be seen that the minimum crest speed attained reduces from $C = 1.2$ m/s to 1 m/s. Therefore, there is evidence to demonstrate that steeper focused waves are characterised by smaller minimum crest speeds. Additionally, there is an asymmetry in the evolution of the crest speed profile with increasing nonlinearity. The reduction of the crest speed with increasing $A_{kc}$ is also noticeable by comparison with the LRWT results. This reduction of crest speed with increasing nonlinearity has also been observed experimentally by [21].

In addition to all previous observations, Figure 6c shows the BEM predicted maximum horizontal particle velocity at the free surface $U_{\eta_{max}}$ along the trajectory of the largest crest. A systematic slowdown of the crest as the elevation increases is associated with an increase in the particle velocity. The simultaneous occurrence of reduced crest speed and increased particle velocity has obvious implications in the formation of deep water breaking waves.
The well-known kinematic breaking criterion of \( u = C \) at breaking inception can be employed here to study the effects observed in Figure 6 within the NewWave formulation of generating steep design waves. However, very recent studies such as [13] have promoted the use of dynamic breaking criteria based on the energy fluxes of the unsteady evolving wave.
Knowing that breaking occurs in the crests of very steep waves and is a highly localised process, [13] have studied the excess wave energy flux within the crest of steep waves in order to generate a breaking criterion. The starting point in deriving the breaking threshold proposed by [13] is the mechanical energy balance equation that relates the local rate of change of the energy density $E$ shown in Equation (1) with the divergence of the local energy flux $F$ in Equation (2).

$$E = \rho g(z - z_0) + \frac{1}{2} \rho \|u\|^2. \quad (1)$$

$$F = u((p - p_0) + \rho g(z - z_0) + \frac{1}{2} \rho \|u\|^2. \quad (2)$$

In the above equations, $\rho$ is the fluid density, $p$ and $p_0$ are the pressure and atmospheric pressure respectively, $g$ is the gravitational acceleration, $z$ and $z_0$ are the vertical coordinate and the origin of the $z$ coordinate respectively and $\|u\|$ is the modulus of the fluid velocity vector. Taking the atmospheric pressure as reference, the ratio of $F/E$ can be computed as:

$$\frac{F}{E} = \frac{u(E + p)}{E}, \quad (3)$$

and [13] chose the instantaneous crest velocity as the normalising velocity $c$ for this flux ratio. The reasoning behind this choice stems from the relevance of this quantity with respect to wave breaking as well as the fact that within an unsteady moving frame of reference with velocity $c$, at the free surface, $(u-c)$ is the flux velocity that transports the energy within the evolving crest. Using this result, the parameter proposed is:

$$B = \frac{F}{E\|c\|}. \quad (4)$$

Additionally, on the free surface, the pressure is equal to the atmospheric pressure and hence from Equation (3), $F/E = u$. In applying this parameter, one can calculate the norm of the two vector quantities or project along the dominant propagation direction. In unidirectional waves at the free surface $F/E = u_x$, and hence the threshold simplifies to a kinematic criterion $B_x = u_x/c_x$. The threshold value given by [12] is $0.85 < B_x < 0.86$.

This parameter has been validated experimentally in the work of [22,23]. Additionally, [24] applied the parameter as a wave breaking onset criteria within their high-order spectral model. Within the present study, this threshold was applied to the BEM simulations with two purposes: to identify the limiting steepness waves and separate them from the incipient breaking cases and to validate the threshold of [13] for deep water wave groups based on realistic ocean spectra. The definition of limiting steepness is: a wave is said to be at its limiting steepness if any additional increase in the linear amplitude sum (A) used for its generation would cause it to break, by spilling.

The results are shown in Figure 7 for the $\gamma = 1$ JONSWAP spectra, for which the domain of the BEM simulations was kept the same as throughout the present study and the corresponding linear amplitude sum was incrementally increased. The horizontal velocity of the water particle at the free surface $u_x = U_{\eta_{\text{max}}}$ at the largest crest along the tracked trajectory was used within Figure 7. Note that, for very steep plunging waves, as shown in [18] and [19], the largest horizontal velocity $U_{\text{max}}$ does not correspond to the horizontal velocity at the largest elevation $U_{\eta_{\text{max}}}$ due to the development of large Lagrangian drift.
velocities. The choice of $u_x = \frac{U_{\eta_{\text{max}}}}{\eta}$ and not $u_x = \frac{U_{\text{max}}}{\eta}$ in this study is due to the definition of the $B_x$ parameter. Furthermore, $c_x$ was taken as $C_{BEM}$.

It can be seen in panels (a) and (b) of Figure 7 that the value of the $B_x$ parameter is just under the breaking initiation threshold. In panel (c), for the case with $A_k c = 0.312$, the parameter exceeds the threshold, signifying wave breaking. Indeed, the BEM simulation crashes shortly afterwards due to node clustering, as the BEM model of [17] does not perform filtering, re-gridding or redistribution of the nodes. The present results for limiting wave steepness are in good agreement with the numerical work of [25] based on OpenFOAM as well as [19] using BEM and the laboratory investigations of [20]. The evolution of the $B_x$ parameter with nonlinearity for the JONSWAP cases studied in the present work is plotted in Figure 8a.

![Figure 7. Evolution of $B_x = \frac{U_{\eta_{\text{max}}}}{C_{BEM}}$ parameter of [12] for increasingly steeper JONSWAP waves with $\gamma = 1$. In all panels thick grey line indicates threshold value $0.85 < B_x < 0.86$.](image-url)
Figure 8. Evolution of $B_x$ parameter with increasing nonlinearity for JONSWAP and Gaussian cases. In all panels thick grey line indicates threshold value $0.85 < B_x < 0.86$. In panel (a) dashed line indicates limiting $A_k c$ for JONSWAP spectra. It can be seen that the evolution is consistent regardless of the parameter used to describe the spectral bandwidth of the JONSWAP spectra. At this point it would seem that the characteristic $A_k c$ value for a given wave group would also be a good indicator for breaking initiation for JONSWAP waves. As noted by studies such as [26], the global steepness $A_k c$ at breaking initiation is dependent on the spectral bandwidth. This is illustrated when studying the results for the Gaussian spectra. These results are plotted in Figure 8b and show a clear dependence on spectral bandwidth for the limiting global steepness values. The difference in results between the two spectra might be explained by the fact that the $\gamma$ parameter does not alter the bandwidth of the JONSWAP spectra as much as the standard deviation does in a Gaussian spectrum. Hence, the Gaussian spectra is preferred when studying the effects of bandwidth.

Coupling the free surface wave profile obtained from the BEM with the estimation of $B_x$ for a given wave event allows accurate separation between breaking and non-breaking events. Examples of wave events that have just surpassed the breaking threshold for Gaussian spectra are illustrated in Figure 9.

Figure 9. Spatial surface profiles $\eta(x)$ for Gaussian spectra waves past their breaking initiation point.

Having demonstrated in Figure 8 that the breaking threshold value of the $B_x$ parameter is not sensitive to the bandwidth of the spectrum, it is interesting to highlight the evolution of the crest speed and horizontal particle velocity for waves characterised by different spectral bandwidths but the same value of the $B_x$ parameter. Essentially, this would highlight the difference in spectral bandwidth for two wave groups that are at the same level of nonlinearity compared to their breaking limit. The comparison is made in Figure 10 for a narrow-banded Gaussian spectrum $\sigma = 0.15$ Hz and a broad-banded spectrum with $\sigma = 0.45$ Hz characterised by a maximum $B_x$ value of 0.717.

![Figure 8](image.jpg)

![Figure 9](image.jpg)

![Figure 10](image.jpg)
While the minimum crest speed for the narrow-banded spectra is considerably larger than the broad-banded spectra, the horizontal velocity of the water particles is also significantly higher. Hence, a broad-banded spectrum generates slower crests but also generates smaller particle velocities. It is thus clear that taken individually, neither the increased particle velocities nor the reduction in crest speed are sufficient for the initiation of wave breaking. It is the combination of the two nonlinear behaviours that eventually results in wave breaking.

If the classical kinematic breaking criteria is used (essentially $B_x = 1$), a further increase in the linear amplitude sum compared to Figure 7 would be needed. For example, using this approach for the JONSWAP spectrum with $\gamma = 1$, an increase of the global steepness up to $A_{kc}=0.34$ was needed before the criteria $u = C$ was met. As observed in both the studies of [25] and [19], further increases in the linear amplitude shift the location of the maximum elevation and breaking crest to the wave just before the focal event.

The combination of reduced crest speed and increased particle kinematics, present in all crests within the wave group, as shown in Figure 6c, can be used to explain this effect. The combination of these two mechanisms that leads to wave breaking was first suggested in [27] and is re-emphasized in the present study.

To generate a vigorous spilling or even plunging breakers, the linear amplitude sum needs to be increased. Two new cases for the JONSWAP spectrum with $\gamma = 1$ were generated. The cases presented are a spilling ($A_{kc}=0.35$) and a plunging ($A_{kc}=0.44$) breaker that occur in the same crest, which precedes the focused event. The only change needed for the new simulations to be accurate is an appropriate switching time between reference frames. The iterations needed in the selection of this switch time is only done to achieve satisfactory nodal resolution so that the breaking event is well described. For the present cases the selected switching time was $t_{sw} = 2.3$ s.

The comparison between the two wave cases in terms of both crest speed and particle kinematics are given in Figure 11. In both panels of the figure, the maximum horizontal particle velocity at any elevation at the location of the crest is plotted. Comparing the panels of Figure 11 with the results presented and 8c highlight the fact that as the linear amplitude sum is increased, the slowdown of the crest and the corresponding increase in the horizontal particle velocity results in the breaking events occurring in the crest preceding the focused event. For these cases, the theoretical focal location is $t = 0$ s. From Figure 11 it can also be observed that as the nonlinearity is increased, the breaking event occurs earlier in the space time domain and the drop in the crest speed immediately before the breaking event occurs much more abruptly. It was shown in studies such as [28] and [20], that for very steep waves the maximum horizontal velocity does not occur on the particle at the free surface, but rather at an elevation of around $0.9 \eta_{max}$. This is a characteristic feature in the formation of the ejecting jet of fluid that gives the characteristic shape of an overturning wave.
The present results highlight an important limitation of the NewWave theory, especially if used to generate very steep waves. As such, if the linear amplitude sum used to generate the waves is large enough, the expected focused wave event will never occur, and energy will be dispersed via breaking in the wave crest preceding the main event. Additionally, it was observed that the focused event based on JONSWAP spectra had a reversed asymmetry compared to the preceding event. This is due to the different relationship between the phasing of the components in the focused event and the preceding spilling breaker.

Figure 12 compares the spilling event obtained in the case of $\gamma = 1$ and $A_{k_c}=0.35$ at the previous crest (shown in Figure 11a) and the spilling event obtained for a case with $A_{k_c}=0.34$, in the focused crest. While the comparison between the two cannot be made directly due to the different phasing of the components, it indicates the type of asymmetry needed to obtain clear spilling and even plunging breaking waves.

Compared to the focused wave event, the preceding crest has inverted asymmetry and the characteristic wave breaking profile shown in studies such as [26] and [1]. For comparison, the events have been shifted to $x = 0$ m. The difference in shape between the two waves is obvious, while difference in the maximum elevation is less than 2%.
Figure 11. Comparison between C\textsubscript{BEM} (thin continuous red line) and \(U_{\text{max}}\) (thin dashed blue line) until breakdown of the BEM simulations for JONSWAP cases with \(\gamma=1\). Shape of breaking crest is illustrated in the inset figures.

![Figure 11](image)

Figure 12. Spatial surface elevation of spilling events for cases with \(A_{Kc}=0.34\) (dashed line) that correspond to the focused wave and \(A_{Kc}=0.35\) (continuous line) that corresponds to the wave preceding focus for JONSWAP spectrum with \(\gamma=1\). As the two surface profiles correspond to different waves they have been shifted to \(x=0\) m for comparison.

5. Conclusions
The work presented in the present study has addressed the effects of crest speed reduction in the formation of deep water breaking waves. Deep water unidirectional focused wave groups were generated using the BEM formulation of [4]. The numerical wave tank was used to generate waves based on realistic JONSWAP ocean spectra and Gaussian spectra.

When comparing the propagation speed of the wave centroid with the propagation speed of the point of maximum elevation in the crest, it was found that there is an additional slowdown of the top part of the crest compared to the main body of the wave. This is expected, as the highly unstable nature of breaking waves is localised high up in the water column. This small note confirms the findings of [11] who concluded that the noticeable slowdown observed is a localised crest effect.

Looking over the entire space-time domain of the wave group propagation, it was found that the generic slowdown of the crest combined with the increase in particle kinematics as a large crest develops is a potential mechanism that leads to the process of wave breaking. In this context, the newly proposed dynamic breaking parameter \(B_x\) of [13] was validated for deep water waves based on realistic ocean spectra. When comparing two waves characterised by a similar \(B_x\) value but with different underlying spectral bandwidth, the broad-banded spectrum gives a smaller crest speed but also lower horizontal particle velocities.

In terms of engineering relevance, the study offers further insight into the use of focused wave groups to generate very steep and even breaking waves that can be used as design waves in the context of deep-water offshore structures. Further work is needed to assess the implication of the coupling between crest speed reduction and particle kinematics increase in steep waves in the context of directionally spread sea-states.

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