Generalized proportional integral disturbance observer-based fuzzy sliding mode control for active magnetic bearing system

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Abstract. This paper presents a generalized proportional integral disturbance observer-based combination of a fuzzy logic control and sliding mode control for the suspension active magnetic bearing system. Firstly, a generalized disturbance observer based on proportional and integral is built to estimate the parameters variation and the outside disturbances. In order to guarantee the system-state will be stabilized on pre-defined states, a disturbance feedback is constructed with a low pass filter. Subsequently, the proportional integral derivative surface sliding mode control is designed. Finally, the Fuzzy logic controller is constructed to determine the hitting controller gain. The stability analysis is given following the Lyapunov law. The proposed control structure can guarantee the system transient response quite good, no overshoot value, and settling time is quite narrow. Two testing cases of the proposed controller with and without the generalized disturbance observe is compared.

1. Introduction
The active magnetic bearing system offer a noncontact working between the stator and rotor [1-4]. However, the active magnetic bearing system is a highly unstable system. In order to control this system previous published paper as our research [3] proposed the fuzzy sliding mode control for the suspension active magnetic bearing system. The fuzzy is used to regulate the controller current value, this research given good chattering rejection method, with small overshoot, narrow settling time, free-chattering value, but the research ignored the outside disturbance and uncertainty values. Chen et al. [5] proposed the neural network to approximate the uncertainty value of the active magnetic bearing system, their proposed method is very good at tracking with non-periodic trapezoidal and periodic trapezoidal signals. Zad et al. [6] presented a design and adaptive sliding-mode control of hybrid magnetic bearings, the results of their paper are good but the settling time still high. Due to the sensitive with the output disturbance of the suspension active magnetic bearing system, this paper uses a general proportional integral to estimate the output disturbance and inside uncertainty values. The convergence of the system state is guaranteed by a low-pass-filter. The concept of disturbance observer was presented and has been developed by many researchers. Kim et al. [7] said that the disturbance is estimated by a general disturbance observer, the original work of their paper is based on the concept of the friction observer. Chen et al. [8] applied the general proportional integral observer-based composition control method for robotic thermal tactile sensor with disturbances, their paper is well achieved with the regulation temperature of the robotic thermal tactile under unknown time varying disturbances. In [9] Wang et al. present the same method to estimate the output disturbance for
the induction motor. The results of their control method are very good at reject the output disturbance and parameter uncertainty. This paper proposes the sliding mode control with a proportional-integral-derivative surface is used to construct the controller for the active magnetic bearing system.

The concept of the sliding mode was initially developed in mid-1950s. In [10] Utkin presented the concept and the stability of a sliding mode controller. Sliding mode control is a type of the variable structure controller [11]. The stability of system should be satisfied the Lyapunov condition. The control signals of sliding mode controller are consisted the equivalent value and the switching value. The switching control value is used to force the system-state converge on the pre-defined surface, and the equivalent control signal is used to stabilize the system-state on that surface. In order to reduce the chattering value of the sliding mode control, this paper use the saturation function to replace the sign function of the switching control part. The fuzzy output is used to approximate the hitting control gain of the switching control part. The archived results will be shown in the subsequent part.

2. Mathematical modeling of the SAMB system

The control structure of the suspension active magnetic bearing system is included two opposite poles, one rotor, and one embedded thrust disk, one computer includes a multi channels analog to digital card with 16 bits revolution. Multi channels digital to analog card with 16 bits revolution, an eddy-current position sensor, and an amplifiers 0.5 A/V. The details of the system are presented as Fig. 1 below.

![Figure 1. Active magnetic bearing control system](image)

The mathematical model of the SAMB is built from the literature [4] as

\[
F_x = F_+ - F_- = K \left( \frac{(i_0 + i_x)^2}{(x_0 - x)^2} - \frac{(i_0 - i_x)^2}{(x_0 + x)^2} \right) \tag{1}
\]

where \(F_+\) and \(F_-\) are the upper and lower forces of upper and lower the magnet coils, respectively. \(K\) is the coefficient of the force values. \(i_0\) and \(i_x\) are the initial current and controlled current values. Using Taylor expansion of the Eq. (1) leads to

\[
F_x = k_p \cdot x + k_i \cdot i_x \tag{2}
\]

With \(k_p\), \(k_i\) are the amplification factors of the rotor position and the magnet coil currents, respectively. Where

\[
k_p = \left. \frac{\partial F_x(x, i_x)}{\partial x} \right|_{x=0, i_x=0} = \frac{2k(i_0 + i_x)^2}{(x_0 - x)^3} - \frac{-2k(i_0 + i_x)^2}{(x_0 + x)^3} = 4k \frac{i_x^2}{x^3}
\]
\[ k_i = \left. \frac{\partial F_x(x, i_x)}{\partial i_x} \right|_{x=0, i_x=0} = \left( \frac{2k(i_0 + i_x)}{(x_0 - x)^2} - \frac{2k(i_0 + i_x)}{(x_0 - x)^2} \right) = 4k \frac{i}{x^2} \]

Follows the Newton II law:

\[ m \ddot{x} = F_x(t) - f_{dt x(t)} \]  

where \( m \) is the mass of the inside rotor and an embedded thrust disk, \( F_x \) is the Lorentz force, and \( f_{dt x(t)} \) is the unexpected output disturbance. The Eq. (3) can be written as

\[ \ddot{x}(t) = \frac{1}{m} (-c \dot{x} + k_x \dot{x}(t) - i_x(t) - f_{dt x(t)}) \]  

\[ \dot{x}(t) = (A_n + \Delta A) \cdot \dot{x}(t) + (B_n + \Delta B) \cdot x(t) + (C_n + \Delta B) \cdot \dot{x}(t) - \gamma \cdot f_{dt x(t)} \]

where \( A_n \), \( B_n \), and \( C_n \) are known system state matrices. The values of \( \Delta A \in [\Delta A_1, \Delta A_h] \), \( \Delta B \in [\Delta B_1, \Delta B_h] \), and \( \Delta C \in [\Delta C_1, \Delta C_h] \) are unknown parameters, it represents the uncertainties values of the system and parameter variation. The system model is represented as

\[ \ddot{x}(t) = (A_n) \cdot \dot{x}(t) + (B_n) \cdot x(t) + (C_n) \cdot \dot{x}(t) + d \]

The Eq. (6) can be written as

\[ \dot{X}(t) = \begin{bmatrix} 0 & I \\ B_n & A_n \end{bmatrix} \cdot X(t) + \begin{bmatrix} 0 \\ C_n \end{bmatrix} \cdot \dot{x}(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} \cdot d \]

where \( d = (\Delta A \cdot \dot{x}(t) + \Delta B \cdot z(t) + \Delta B \cdot i_x(t) - \gamma \cdot f_{dt x(t)}) < \beta \), and \( X(t) = [x(t), \dot{x}(t)]^T \) \( \beta \) is a positive constant upper boundary of the unknown disturbance of the system. The system parameters are shown in Table 1 below.

| Parameters | Description | Value | Unit |
|------------|-------------|-------|------|
| \( m \) | Mass of rotor | 2.565 | kg |
| \( k_{ai} \) | Current stiffness of electromagnetic force of thrust disk AMB | 40 | N/A |
| \( k_{ap} \) | Position stiffness of electromagnetic force of thrust disk AMB | 25200 | N/M |
| \( z_0 \) | Nominal air gap where thrust disk is centered | 1 | mm |
| \( T \) | Thrust disk mass | 0.38 | kg |
| \( c \) | Damping constant | 0.001 | N.s/m |
| \( v_0 \) | Reference voltage | 1.4 | V |
| \( i_0 \) | Amplifier range | 0.5 | A/V |

3. Proposed approach

A highly unstable like the suspension active magnetic bearing system could be provided a robust controller. The disturbance observer can be used to reject the outside disturbance and system
parameters variation. A generalized proportional integral disturbance is applied to reject the unexpected values and guarantee the asymptotic convergence of the rotor position to the pre-defined position.

3.1. Generalized proportional integral disturbance observer
In facts, the disturbance rejection is one of the most importance thing to estimate the controller performance. This paper presents a GPIDO as follows

\[ d = \sum_{m=0}^{n} a_m t \]  

(8)

the \( a_m \) are unknown constant values. The observer system mathematical model is described as

\[
\dot{x} = \begin{bmatrix} 0 & I \\ B_n & A_n \end{bmatrix} \cdot x(t) + \begin{bmatrix} 0 \\ C_n \end{bmatrix} \cdot i_x(t) + \begin{bmatrix} 0 \\ \dot{\delta}_i \end{bmatrix} \cdot g_i
\]

\[ \dot{d} = \begin{bmatrix} 0 \end{bmatrix} \cdot g_i \]  

(9)

where \( i = 1 \pm n \), and

\[ g_i = \begin{cases} \frac{1}{T} & t > t_k \\ 0 & \text{otherwise} \end{cases} \]

(10)

The estimate disturbance value is asymptotically tracking the lump of disturbance and uncertainty when the disturbance gain is resulted the \( p(s) \) are Hurwitz stable, where

\[ p(s) = s^{n+1} + \delta_0 s^{n} + \ldots + \delta_{n-1} s + \delta_n a \]

(11)

For reducing the noise of high frequency term, this study used the low-pass-filter gain to estimate disturbance value. The low-pass-filter is

\[ G(s) = \frac{1}{Ts + 1} \]

(12)

A suitable low-pass-filter time constant need to be chosen.

3.1.1. The stability of proposed observer
In this section the detail analysis of the stability of the disturbance estimation will be revealed. By denoting \( \tilde{d} = \hat{d} - d \), then the derivative of the estimation disturbance error is

\[ \dot{\tilde{d}} = \dot{\hat{d}} - \dot{d} = (\delta_0 \dot{g}_0 + \delta_1 \dot{g}_1 + \ldots + \delta_n \dot{g}_n) - \dot{d} = (-\dot{\hat{d}} + \delta_1 \dot{g}_1 + \ldots + \delta_n \dot{g}_n) - \dot{d} \]

(13)

After \( n + 1 \) steps of derivation the estimated disturbance error value will leads to

\[ \tilde{d}^{n+1} = -(\delta_0 \tilde{d} + \ldots + \delta_n \tilde{d}) - d^{n+1} \]

(14)

or

\[ \tilde{d}^{n+1} + (\delta_0 \tilde{d} + \ldots + \delta_n \tilde{d}) = d^{n+1} \]

(15)

From Eq. (8) after \( n + 1 \) steps of derivation, the term \( d^{n+1} \) goes to zero. The Eq. (15) could be written as
\[ d^{n+1} + (\delta_0 d^n + \ldots + \delta_n d) = 0 \]  \hspace{1cm} (16)

The derivative error of Eq. (8) is asymptotically stable, and the estimated disturbance can well track the disturbance and uncertainty only when \( p(s) = s^{n+1} + \delta_0 s^n + \ldots + \delta_{n-1}s + \delta_n \) are Hurwitz. A filter with approximate bandwidth as a low-pass-filter is used to guarantee the disturbance error value goes to zero. The stability is proved as

\[
d = d - L^{-1}(G(s)) \times \dot{d}
\]  \hspace{1cm} (17)

The disturbance and uncertainty are accurately approximated by

\[ d \times L^{-1}(G(s)) = \dot{d} \]  \hspace{1cm} (18)

The Eq. (17) then could be

\[ \dd = L^{-1}(1-G(s)) \times d \]  \hspace{1cm} (19)

The transformation of the term

\[ L^{-1}(1-G(s)) = 1 - \frac{t}{T} e^{-t/T} \rightarrow 0 \]  \hspace{1cm} (20)

3.2. Sliding mode control

The sliding mode control are included an equivalent control value, and a switching control value. Every system-state will slide and converge on a pre-defined state. This paper proposes the sliding surface by

\[ s(t) = k_d \cdot \dot{e}(t) + k_p \cdot e(t) + k_i \cdot \int_0^t e(\tau) \cdot d\tau \]  \hspace{1cm} (21)

refer \( e(t) = x_r - x_m \) where \( x_r \) is the reference distance value, and \( x_m \) is the measured distance. \( k_p, k_i, k_d \) are positive constant values and it should be chosen such that the real parts of the roots then sliding surface is satisfied Lyapunov law as

\[ V(t) = s(t) \cdot s(t) < 0 \]  \hspace{1cm} (22)

Taking the derivative of the Eq. (21)

\[ \dot{s}(t) = k_d \cdot \ddot{e}(t) + k_p \cdot \dot{e}(t) + k_i \cdot e(t) \]  \hspace{1cm} (23)

Combining Eq. (10) and Eq. (20), the current is calculated as

\[ I_{cref}(t) = \frac{1}{k_d} \left[ k_d \cdot x_r(t) - k_d \cdot (A_n \cdot x_m(t) + B_n \cdot x_m(t) + d) + k_p \cdot \dot{e}(t) + k_i \cdot e(t) + k \cdot sign(s(t)) \right] \]  \hspace{1cm} (24)

where the derivative surface value is chosen by term of switching control value as \( \dot{s}(t) = -k \cdot sign(s(t)) \). Then the control value is divided by two terms of values as following

\[ I_{cref} = I_{eq} + I_{sw} \]  \hspace{1cm} (25)
In term of reduction chattering value from switching control part, the saturation is used to replace the sign function. Where \( sat(s) = \text{sign}(s) \cdot \min \{1,|s|\} \) Or

\[
\begin{aligned}
    sat(s) &= \begin{cases} 
        1 & \text{if } s > \varepsilon \\
        -s & \text{if } s \in [-\varepsilon,\varepsilon] \\
        -1 & \text{if } s < -\varepsilon 
    \end{cases} 
\end{aligned}
\]  

(26)

Then Eq. (21) could be written as

\[
I_{\text{ref}}(t) = \frac{1}{k_d} \cdot \frac{1}{C} \cdot \left( (k_d \cdot \dot{x}_r(t) - k_d \cdot (A \cdot \dot{x}_m(t) + B \cdot x_m(t) + L) + k_p \cdot \dot{e}(t) + k_i \cdot e(t) + k \cdot sat(s(t))) \right)
\]  

(27)

The Eq. (22) can be written to obtain the disturbance and uncertainty value as

\[
s(t) \cdot \dot{s}(t) = s \cdot \left( k_d \cdot \dot{x}_r(t) + k_p \cdot \dot{e}(t) + k_i \cdot e(t) \right)
\]  

(28)

or

\[
s(t) \cdot \dot{s}(t) = s \cdot (k_d \cdot \dot{x}_r(t) - (A_n + \Delta A) \cdot \dot{x}_m(t) + (B_n + \Delta B) \cdot x_m(t) + L + (C_n + \Delta C)(I_{eq} + I_{sw})) + k_p \cdot \dot{e}(t) + k_i \cdot e(t)
\]

(29)

The inequality (34) is satisfied when the hitting control gain should be defined as

\[
k = \left( C_n + \Delta C \right)^{-1} \cdot \left( [-\Delta A + C_n^{-1} \cdot A_n \cdot \Delta C] \cdot \dot{x}_m(t) + ([-\Delta B + C_n^{-1} \cdot B_n \cdot \Delta C] \cdot \dot{x}_m(t)) \right)
\]

(31)

The hitting control gain should by approximate for the chattering as small as possible. The paper provides the fuzzy logic controller to implement the work.

3.3. Fuzzy logic control
Fuzzy logic is another practical mathematical addition to classic Boolean logic [12]. The Fuzzy rules were chosen somehow the output of the fuzzy is satisfied the Lyapunov condition. The hitting control gain is output of the fuzzy system. The sliding mode stability provides \( s(t) \cdot \dot{s}(t) \leq -k_f |s(t)| \), with \( k_f \) is the fuzzy output. In case \( s(t) > 0 \) the \( k_f \) increase will lead to \( \dot{s}(t) \) decrease and vice versa. Otherwise, in case \( s(t) < 0 \) the \( k_f \) decrease will leads to \( \dot{s}(t) \) increase and vice versa. The fuzzy membership function includes five distinguished partitions as NB (Negative Big), NM (Negative Medium), ZO (Zero), PM (Positive Medium), and PB (Positive Big). The fuzzy rules are constructed as Table 2 and Fig. 2 below.

| \( s(t) \) | NB | NM | ZO | PM | PB |
|----------|----|----|----|----|----|
| NB       | VH | H  | M  | H  | VH |
| NM       | H  | H  | M  | H  | H  |
| ZO       | M  | M  | L & H | M | M |
| PM       | L  | L  | M  | L  | L  |
| PB       | ZO | L  | M  | ZO | L  |

Fuzzy logic membership function are determined as

(a) sliding mode control surface

(b) derivative of the sliding mode surface

(c) fuzzy output signal Sliding mode surface

**Figure 2.** Fuzzy rules

The control system is constructed as Fig. 3 below.

**Figure 3.** GPI disturbance observer based fuzzy sliding mode control for AMB system

The proposed method results are given by the next part. The control parameters are in Table 3.
### Table 3. The controller parameters are

| Parameter | Description | Value |
|-----------|-------------|-------|
| $p_k$     | Proportional coefficient | 3000  |
| $i_k$     | Integral coefficient    | 3000  |
| $d_k$     | Derivative coefficient  | 1     |
| $k_f$     | Positive gain           | $k_f$ |
| $\varepsilon$ | Saturation coefficient | 0.8   |
| $T$       | Low-pass-filter time    | 0.001 |
| $\delta_0$ | Disturbance coefficient | 50    |
| $\delta_1$ | Disturbance coefficient | 100   |

#### 4. Proposed approach

The approximation fuzzy system used centroi-weight defuzzification to operate the fuzzy system. The stability of the system now is combination of Eqs. (24) and (36) with a hitting control gain is determined by $k_f$. With a case of testing the outside disturbance. This section brief describes the effectiveness of the proposed control method by these performances. An illustrative example of distance tracking response is most important impact, their properties is including the settling time, overshoot value, and steady state, the disturbance response is important also.

**4.1. The Fuzzy sliding mode control performance**

This section gives some result of the case without generalized proportional integral disturbance observer, a disturbance signal is given from outside to test the performance of the proposed method was implemented. The given output signal determined that the proposed control method is good at tracking the flexible signal. The uncertainty and disturbance are rejected, but not completely rejected the output disturbance signal. The given output are shown in Fig. 4 below.

![Graphs showing the proposed method's performance](image-url)
The control current value

![Graph showing control current value](image)

**Figure 4.** Fuzzy sliding mode control for AMB system given output signals

### 4.2. The GPI disturbance observer based fuzzy sliding mode control performance

The given output signal are shown in Fig. 5 below.

![Graph showing GPI disturbance observer based fuzzy sliding mode control performance](image)

**Figure 5.** GPI Disturbance observer based fuzzy sliding mode control for AMB system output signals
This section will present the performance of the GPI disturbance observer-based fuzzy sliding mode control for the active magnetic bearing system. The outside disturbance is completely rejected by a feedback gain. The results are represented as

The given output signals figure out that the performance of the proposed method is quite good at tracking the sinusoidal reference signal, the settling time just approximately 3ms in case of no disturbance observer, and 2ms in case of disturbance observer is equipped, respectively. There are no overshoot values in both case, and the average distance tracking error values are equal to 1.344um. The maximum of the distance tracking error value is approximately equalled 17.366mm and 11.662mm in case 1 and case 2, respectively. The testing disturbance is responded by a lump of disturbance observer, which means the exogenous disturbance is automatically rejected.

5. Proposed approach
This study used a proportional integral derivative to construct the sliding mode control surface, a fuzzy logic control is used to construct the hitting control gain. There are somehow the Lyapunov is guaranteed for system stability. A general proportional integral disturbance observer is equipped to estimate the unknown outside disturbance and unknown parameters variation. The design methodology is utilized to control the suspension active magnetic bearing system. The archived result are figured out that proposed methodology is good at tracking the dynamic input signal, the proposed controller can reject the output disturbance and the variation of the parameters. The main advantages are, the settling time of proposed controller is very small, no overshoot value, and the average of the distance tracking error value is quite small. The energy of control value is strongly small. The proposed fuzzy controller is suitable for building the hitting control gain. In comparison of two cases is aimed to figure out that the proposed controller is good at rejecting the outside disturbance and system parameters variation.

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