Study of Light Scalar Meson Structure in D1 decay

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1. Introduction
1-1. Hadronic Phenomena

QCD Lagrangian causes many Hadronic Phenomena:

QCD Properties:
- Spontaneous Chiral Symmetry Breaking
- Quark Confinement

The ordinary hadrons

The exotic hadrons

The mass of "us" (the mass of hadrons)

Let's pay attention to the exotic hadrons.
1-2. Candidates of Exotic Hadrons

\( \Theta^{+}(1530) \) SPring-8 (LEPS, 2003)

\( \Xi(3872) \) Belle (2003)

\( Z(4430) \) Belle (2007)

Other Candidates:
\( Y(4260), X(3940), Y(3940) \)…
1–3. Other: Scalar Meson Puzzle

The light scalar mesons

- $a_0(1450)$
- $a_0(980)$
- $f_0(980)$
- $f_0(1370)$
- $f_0(1500)$
- $f_0(1710)$
- $\kappa(800)$
- $\sigma(600)$
- $K_0^-(1430)$
If Scalar Meson is $q\bar{q}$, then from number of strange quark

$$m(a_0) < m(f_0)$$

If Meson is $qq\bar{q}\bar{q}$, then

$$m(a_0) = m(f_0)$$

This picture seems to be good.

|       | $a_0^0$ | $f_0$ |
|-------|---------|-------|
| 2-quark states: | $(u\bar{u} - d\bar{d})/\sqrt{2}$ | $s\bar{s}$ |
| 4-quark states: | $(\bar{d}s\bar{s} - \bar{u}s\bar{u})/\sqrt{2}$ | $(\bar{d}s\bar{s} + \bar{u}s\bar{u})/\sqrt{2}$ |

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- $\bar{K}(800)$
1-4. Our Goal

Our Goal is to see $\sigma$ meson Structure in $D_1 \rightarrow D \pi \pi$ Decay (Energy region $\leq 560\text{MeV}$):

$\sigma$ meson = \begin{cases} 
\text{Two quark dominant ?} \\
\text{or} \\
\text{Four quark dominant ?}
\end{cases}$
Our Goal is to see $\sigma$ meson Structure in $D_1 \rightarrow D\pi\pi$ Decay (Energy region $\lesssim 560\text{MeV}$):

\[ D_1 \rightarrow D\pi + \pi \]

\[ D_1 \rightarrow D^* \pi \]

\[ D_1 \rightarrow D\pi \]

\[ \sigma \text{ meson} = \begin{cases} \text{Two quark dominant} \\ \text{or} \\ \text{Four quark dominant} \end{cases} \]
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\{ or \}
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1. Introduction
2. 2–4quark Mixing and ππ Scattering
3. \( D1 \rightarrow D\pi \pi \) Decay under \( U(1)A \) Symmetry
4. Results : Scalar Meson Mixing Effects in \( D1 \rightarrow D\pi \pi \) Decay
2. Two-Four quark Mixing and $\pi \pi$ Scattering
2-1. 2quark and 4quark States

3×3 Flavor Matrix Fields $M$ and $M'$ in Linear Sigma Model:

- Two quark fields $\sim q_L q_R$
  \[ M = S + i\phi \]
  Scalar Pseudo scalar

- Four quark fields $\sim q_R q_R \bar{q}_L \bar{q}_L$
  \[ M' = S' + i\phi' \]
  Scalar Pseudo scalar

They have the same transformation properties under $SU(3)_L \times SU(3)_R$:

- $M \rightarrow U_L M U_R^\dagger$
- $M' \rightarrow U_L M' U_R^\dagger$

But different transformation properties under $U(1)_A$:

- $M \rightarrow M e^{+2i\nu}$
- $M' \rightarrow M' e^{-4i\nu}$
2-2. Physical States

If there is U(1)\textsubscript{A} Symmetry, Two and Four quark state are not mixed, but U(1)\textsubscript{A} Symmetry is

- Explicitly Broken by Anomaly.
- Spontaneously Broken by chiral sym. breaking.

Then they are mixed:

\[
\begin{align*}
I=0 & & \quad P=+ \quad \text{(Mass)} \sim 140 \text{MeV} \\
I=0 & & \quad P=- \quad \text{(Mass)} \sim 1300 \text{MeV}
\end{align*}
\]
2-2. Physical States

If there is $U(1)_A$ Symmetry, Two and Four quark state are not mixed, but $U(1)_A$ Symmetry is

- Explicitly Broken by Anomaly.
- Spontaneously Broken by chiral sym. breaking.

Then they are **mixed**:

\begin{align*}
\text{Physical States} & \quad \text{π} \quad \text{Lighter} \\
\text{π} \quad \text{Heavier} \\
\text{σ} \quad \text{Lightest} \\
\text{f}_0 \quad 2^{\text{nd}} \\
\text{f}_0 \quad 3^{\text{rd}} \\
\text{f}_0 \quad \text{Heaviest}
\end{align*}

\text{Mass} \approx \begin{cases} 140 \text{MeV} \\ 1300 \text{MeV} \\ 600 \text{MeV} \\ 980 \text{MeV} \\ 1370 \text{MeV} \\ 1570 \text{MeV} \end{cases}
If there is $U(1)_A$ Symmetry, Two and Four quark state are not mixed, but $U(1)_A$ Symmetry is

\[
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\text{Spontaneously Broken by chiral sym. breaking.}
\end{align*}
\]

Then they are mixed:

\[
\begin{pmatrix}
\phi_\pi \\
\phi'_\pi
\end{pmatrix}
= 
\begin{pmatrix}
\cos \theta_\pi & -\sin \theta_\pi \\
\sin \theta_\pi & \cos \theta_\pi
\end{pmatrix}
\begin{pmatrix}
\pi_{\text{Lighter}} \\
\pi_{\text{Heavier}}
\end{pmatrix}
\]

\[
\begin{pmatrix}
u \bar{u} + dd \\
ss \\
nuss + dsd\bar{s} \\
udud
\end{pmatrix}
= 
\begin{pmatrix}
U_{1\sigma} & \cdots & 4 \times 4 \text{ matrix}
\end{pmatrix}
\begin{pmatrix}
\sigma \\
(f_0)_2 \\
(f_0)_3 \\
(f_0)_4
\end{pmatrix}
\]

Physical States

\[
\begin{align*}
\text{Lightest} \\
2^{\text{nd}} \\
3^{\text{rd}} \\
\text{Heaviest}
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\sin \theta_{\pi} & \cos \theta_{\pi}
\end{pmatrix}
\begin{pmatrix}
\pi_{\text{Lighter}} \\
\pi_{\text{Heavier}}
\end{pmatrix}
\]

\[
\begin{pmatrix}
u\bar{u} + d\bar{d} \\
ss \\
u\bar{s}u\bar{s} + dsd\bar{s} \\
u\bar{d}u\bar{d}
\end{pmatrix} =
\begin{pmatrix}
U_{1\sigma} & \ldots & \text{4 \times 4 matrix}
\end{pmatrix}
\begin{pmatrix}
\sigma \\
(f_0)_2 \\
(f_0)_3 \\
(f_0)_4
\end{pmatrix}
\]

Lightest \hspace{1cm} 2^{\text{nd}} \hspace{1cm} 3^{\text{rd}} \hspace{1cm} \text{Heaviest
2-3. Effective Lagrangian

Effective Lagrangian for Light Mesons part by using Linear Sigma Model:

\[
\mathcal{L} = -\frac{1}{2} \text{Tr}(\partial_\mu M \partial_\mu M^\dagger) - \frac{1}{2} \text{Tr}(\partial_\mu M' \partial_\mu M'^\dagger)
\]

\[-V_0(M, M') - V_{\text{anom}}(M, M') - V_{SB}\]

\[V_0: \text{SU}(3)_L \times \text{SU}(3)_R \text{ invariant, U}(1)_A \text{ invariant.}\]

\[V_{\text{anom}}: \text{SU}(3)_L \times \text{SU}(3)_R \text{ invariant, U}(1)_A \text{ breaking (anomaly).}\]

\[V_{SB}: \text{Explicit SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)_A \text{ breaking terms.}\]

(Effect of Current quark masses)

This part is matched by QCD.
2-4. Relations between Couplings

There are relations between couplings as follows:

\[
\sum_{j=1}^{4} \frac{1}{m_j^2} \left( \frac{\partial^3 V}{\partial \pi^0 \partial \pi^0 \partial f_j} \right)^2 = \frac{1}{3} \left( \frac{\partial^4 V}{\partial \pi^0 \partial \pi^0 \partial \pi^0 \partial \pi^0} \right) \]

\[
\sum_{j=1}^{4} \frac{1}{m_j^4} \left( \frac{\partial^3 V}{\partial \pi^0 \partial \pi^0 \partial f_j} \right)^2 = \frac{2}{F_\pi^2}
\]

Π Π Scattering Amp.

\[
\sim \frac{2}{F_\pi^2} s
\]

\( F_\pi \) is the pion decay constant.

Π Π Scattering amplitude can be written as a function of Σ Π Π coupling and Σ mass:

\[
A(s, t, u) = - \frac{g_{001}^2}{m_1^2} + \sum_{j=1}^{4} \frac{f_j}{s} \sim - \frac{g_{001}^2}{m_1^2} + \left( \frac{2}{F_\pi^2} - \frac{g_{001}^2}{(m_1^2)^2} \right) s
\]

\[
expansion \ in \ \left( \frac{s}{m_j^2} \right) \ (j \geq 2, \sqrt{s} \lesssim 560 \text{MeV})
\]

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π π Scattering data can be reproduced by $m(\sigma) = 580\text{MeV}$, π π σ coupling $= 1.9$ up to 560MeV ($\sim$ Upper limit of $D_1 \rightarrow D\pi\pi$):
3. $D_1 \rightarrow D \pi \pi$ Decay under $U(1)_A$ Symmetry
3-1. Heavy Meson Chiral Partner

We assume a chiral partner structure for 2-quark Heavy Mesons as follows:

- $D$ and $D_0^*$ are chiral partner.
- $D_1$ and $D_1^*$ are chiral partner.
- $D_0^*$ and $D_1$ are chiral partner.
3-2. U(1)A Sym. Effect

By using Linear Sigma Model and assuming D’s are 2-quark meson, D1 and D couple to

$$\pi \quad \pi \quad \pi$$

$$\pi \quad \pi \quad \pi$$

By using Linear Sigma Model and assuming D’s are 2-quark meson, D1 and D couple to
By using Linear Sigma Model and assuming D’s are 2–quark meson, D1 and D couple to ONLY 2–quark light state under U(1)A Symmetry:

Only 2-quark Scalar is Coupled
3-2. U(1)A Sym. Effect

By using Linear Sigma Model and assuming D’s are 2-quark meson, D1 and D couple to ONLY 2-quark light state under U(1)A Symmetry:

And we can expand Light Scalar Propagator as the same way as \(\pi\ \pi\) Scattering, then...
3-3. $D_1 \rightarrow D \pi \pi$ Decay Amplitude

we can get $D_1 \rightarrow D \pi \pi$ Decay Amplitude as a function of $\sigma$. And There is $U(1)A$ Sym. Effect as Sigma–2quark mixing $U_{1\sigma}$:

![Diagram showing the decay process $D_1 \rightarrow D \pi \pi$ and the effect of $U_{1\sigma}$]
Since $D_1$ and $D^*$ are chiral partners, $D_1$D scalar coupling and $D^*$Dπ coupling are related:

\[ g_{D_1D\sigma} = g_A U_{1\sigma} \]

\[ g_{D^*D\pi} = g_A \cos \theta_{\pi} \]

\[ g_{D_1D\sigma} = \frac{g_{D^*D\pi}}{\cos \theta_{\pi}} = g_A \frac{U_{1\sigma}}{\cos \theta_{\pi}} \]
Then we can determine Heavy–Light couplings by experiments:
4. Results: Scalar Meson Mixing Effects in $D_1 \rightarrow D \pi \pi \pi$ Decay
There are different lines which corresponds to ratio $R = \frac{U_1\sigma}{\cos \theta_\pi}$:

- $R = 0$ (\(\sigma\) is Pure 4quark)
- $R = 0.5$
- $R = 1.0$ (\(U = \cos \theta\))
- $R = 1.5$ (\(U > \cos \theta\))

$m(\sigma) = 580\text{MeV}$, \(\pi \pi \sigma\) coupling $= 1.9$
(determined by \(\pi \pi\) scattering)

ex.) $\cos \theta = 1$: \(\pi\) is Pure 2quark state.
4. Results: Scalar Meson Mixing Effects in $D^1 \to D\pi\pi\pi$ Decay

There are different lines which corresponds to ratio $R = \frac{U_1\sigma}{\cos \theta_\pi}$:

- $R = 0$ (it is Pure 4quark)
- $R = 1.0$ ($U = \cos \theta$)
- $R = 1.5$ ($U > \cos \theta$)

$m(\sigma) = 580$ MeV, $\pi\pi\sigma$ coupling $= 1.9$
(determined by $\pi\pi\pi$ scattering)

Future Data $\rightarrow$ Determination of Scalar Mixing Angle.

$m_{\pi\pi}$ (GeV)
6. Summary
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We assumed that:
Linear Sigma Model,
U(1)A Symmetry is broken only by anomaly,
Chiral Partner Structure of Heavy Mesons.

And Input is $\pi \pi$ Scattering.

We predicted $D1 \rightarrow D\pi \pi$ Decay width under U(1)A Symmetry for several values of the ratio $R = \frac{U_{1\sigma}}{\cos \theta_\pi}$.
THANK YOU!
App: Heavy-Light Meson Interaction under U(1)\textsubscript{A} Symmetry

Our model under SU(3) × SU(3) × U(1)\textsubscript{A} and Heavy Quark Symmetry by using Linear Sigma Model:

\[ \mathcal{L}_{\text{heavy}} = -\frac{1}{2} \text{Tr}[\bar{\mathcal{H}}_L \partial \mathcal{H}_L] - \frac{1}{2} \text{Tr}[\bar{\mathcal{H}}_R \partial \mathcal{H}_R] - \Delta \frac{1}{2} \text{Tr}[\bar{\mathcal{H}}_L \mathcal{H}_L + \bar{\mathcal{H}}_R \mathcal{H}_R] \]

\[ - \frac{g_\pi}{4} \text{Tr}[M^\dagger \bar{\mathcal{H}}_L \mathcal{H}_R + M \bar{\mathcal{H}}_R \mathcal{H}_L] + \frac{c_1}{2f_\pi} \text{Tr}[\phi M^\dagger \bar{\mathcal{H}}_L \mathcal{H}_R + \phi M \bar{\mathcal{H}}_R \mathcal{H}_L] \]

\[ + i \frac{g_A}{2f_\pi} \text{Tr}[\gamma^5 \phi M^\dagger \bar{\mathcal{H}}_L \mathcal{H}_R - \gamma^5 \phi M \bar{\mathcal{H}}_R \mathcal{H}_L] \]

\[ + \frac{c_3}{2f_\pi} \text{Tr}[MM^\dagger \bar{\mathcal{H}}_L \mathcal{H}_L + M^\dagger MM \bar{\mathcal{H}}_R \mathcal{H}_R] + \frac{c_4}{2f_\pi} \text{Tr}[M'M'^\dagger \bar{\mathcal{H}}_L \mathcal{H}_L + M'^\dagger M' \bar{\mathcal{H}}_R \mathcal{H}_R] \]
App: Heavy-Light Meson Interaction under U(1)A Symmetry

Our model under SU(3) × SU(3) × U(1)A and Heavy Quark Symmetry by using Linear Sigma Model:

\[
\mathcal{L}_{\text{heavy}} = -\frac{1}{2} \text{Tr}[\mathcal{H}_L \partial \mathcal{H}_L] - \frac{1}{2} \text{Tr}[\mathcal{H}_R \partial \mathcal{H}_R] - \frac{\Delta}{2} \text{Tr}[\mathcal{H}_L \mathcal{H}_L + \mathcal{H}_R \mathcal{H}_R]
\]
\[
- \frac{g_\pi}{4} \text{Tr}[M^\dagger \mathcal{H}_L \mathcal{H}_R + M \mathcal{H}_R \mathcal{H}_L] + \frac{c_1}{2f_\pi} \text{Tr}[\phi M^\dagger \mathcal{H}_L \mathcal{H}_R + \phi M \mathcal{H}_R \mathcal{H}_L]
\]
\[
+ i \frac{g_A}{2f_\pi} \text{Tr}[\gamma^5 \phi M^\dagger \mathcal{H}_L \mathcal{H}_R - \gamma^5 \phi M \mathcal{H}_R \mathcal{H}_L]
\]
\[
+ \frac{c_3}{2f_\pi} \text{Tr}[MM^\dagger \mathcal{H}_L \mathcal{H}_L + M^\dagger M \mathcal{H}_R \mathcal{H}_R] + \frac{c_4}{2f_\pi} \text{Tr}[MM'^\dagger \mathcal{H}_L \mathcal{H}_L + M'^\dagger M' \mathcal{H}_R \mathcal{H}_R]
\]

Up to Five mass dimension operator, Only this term has Light Four Quark states – Heavy Meson Interaction.
App: Heavy-Light Meson Interaction under U(1)\textsubscript{A} Symmetry

Our model under SU(3) × SU(3) × U(1)\textsubscript{A} and Heavy Quark Symmetry by using Linear Sigma Model:

\[
\mathcal{L}_{\text{heavy}} = -\frac{1}{2} \text{Tr}[\overline{\mathcal{H}}_L i\gamma_5 \partial \mathcal{H}_L] - \frac{1}{2} \text{Tr}[\overline{\mathcal{H}}_R i\gamma_5 \partial \mathcal{H}_R] - \frac{\Delta}{2} \text{Tr}[\overline{\mathcal{H}}_L \mathcal{H}_L + \overline{\mathcal{H}}_R \mathcal{H}_R]
- \frac{g_\pi}{4} \text{Tr}[M^\dagger \overline{\mathcal{H}}_L \mathcal{H}_R + M \overline{\mathcal{H}}_R \mathcal{H}_L] + \frac{c_1}{2f_\pi} \text{Tr}[\phi M^\dagger \overline{\mathcal{H}}_L \mathcal{H}_R + \phi M \overline{\mathcal{H}}_R \mathcal{H}_L]
+ i\frac{g_A}{2f_\pi} \text{Tr}[\gamma^5 \phi M^\dagger \overline{\mathcal{H}}_L \mathcal{H}_R - \gamma^5 \phi M \overline{\mathcal{H}}_R \mathcal{H}_L]
+ \frac{c_3}{2f_\pi} \text{Tr}[M M^\dagger \overline{\mathcal{H}}_L \mathcal{H}_L + M^\dagger M \overline{\mathcal{H}}_R \mathcal{H}_R] + \frac{c_4}{2f_\pi} \text{Tr}[M' M'^\dagger \overline{\mathcal{H}}_L \mathcal{H}_L + M'^\dagger M' \overline{\mathcal{H}}_R \mathcal{H}_R]
\]

When chiral symmetry is broken, \( M \to \langle M \rangle \), this terms make mass splittings of Heavy Mesons.