Parameter Appendix

Bo Tan, John Thompson

May 5, 2014

This is a support document which describes the properties of the cable and parameters of the formulas in [1]. The cable parameters help the reader build powerline channel according to the transmission line theory. The document also presents the parameters which describe the distribution of the number of path, path magnitude, path interval and the cable loss feature of the powerline channel. By using the parameters in this document, readers can model the powerline channel according to the methodology proposed in [1].

1 Cable Property

Figure 1 shows the structure of a power cable which is used in [2]. Table 1 and Table 2 show the geometric parameters and the electromagnetic parameters of NAYY150 and NAYY35 cables. The insulator between conductors is PVC. When feeding signals into two adjacent conductors, most of the electric field is concentrated between these two conductors. The lumped parameters of the cable can be calculated by the geometric dimensions and material electrical properties.

![Figure 1: The Cross-Section of a Four Conductor Power Cable, with three live connections (L1, L2 and L3) and one neutral connection (N). The scalar $a$ represents the space between two adjacent conductors which is filled by PVC. Scalar $r$ is the radius from geometry center to the outer edge of the conductor.](image)

Table 1: Cable Geometry Properties

|        | NAYY150 (mm) | NAYY35 (mm) |
|--------|--------------|-------------|
| $a$    | 1.8          | 1.2         |
| $r$    | 6.9099       | 5.9161      |

The lumped parameters such as capacitance ($C$), inductance ($L$), resistance ($R$) and conductance ($G$) per unit length can be calculated by applying the parameters in Table 1 and 2 to equation (1) to
Table 2: Cable Electrical Properties

| Property                          | Value                        |
|----------------------------------|------------------------------|
| Conductivity of Cooper $\kappa$  | $58 \times 10^6$ S/m        |
| Dissipation of PVC $\tan\delta$   | 0.025                        |
| Relative permittivity of PVC $\varepsilon_r$ | 4                           |
| Free space permittivity $\varepsilon_0$ | $8.5419 \times 10^{-12}$ F/m |
| Relative permeability of Cooper $\mu_r$ | 1                           |
| Free space permeability $\mu_0$   | $1.2566 \times 10^{-6}$ [H/m] |

Table 3: Parameters for the Path Number distributions

| $i=2$ | $p_{i1}$ | $p_{i2}$ | $p_{i3}$ | $q_{i1}$ | $q_{i2}$ | $q_{i3}$ |
|-------|----------|----------|----------|----------|----------|----------|
|       | 1.623    | 0.08596  | 0        | -0.3818  | -0.8461  | 1.592    |
| $i=3$ | 3.913    | 0.0968   | 0        | -0.005983| 1.033    | 1.783    |
| $i=4$ | 6.169    | 0.09686  | 0        | -0.4401  | -0.6989  | 2.007    |
| $i=5$ | 8.684    | 0.1688   | 0        | -8.725   | -0.06764| 10.98    |

\[ C = \varepsilon_0 \varepsilon_r \frac{r}{a} \]  \hspace{1cm} (1)
\[ L = \mu_0 \mu_r \frac{r}{a} \]  \hspace{1cm} (2)
\[ R = \sqrt{\frac{\pi \mu_0}{\mu_r^2 f}} \]  \hspace{1cm} (3)
\[ G = 2\pi f C \tan\delta \]  \hspace{1cm} (4)

where, $f$ in equation (3) and (4) denotes the frequency with Hz as unit.

2 Parameters for Number of Paths Distribution

The number of the paths for the channel of the $i$th Class and the $k$th Cluster can be described by equation (7) in [1] which is a Gaussian distribution:

\[ N_{i,k} = \lceil N(\mu_{i,k}, \sigma_{i,k}^2) \rceil \]  \hspace{1cm} (5)

where \([\cdot]\) means to round towards the nearest integer, parameters $\mu_{i,k}$ and $\sigma_{i,k}^2$ are the expectation and standard deviation of the Gaussian distribution. The value $\mu$ and $\sigma$ in each Class increase as power function of Cluster Index. The Power function can be written as:

\[ \mu_{i,k} = p_{i1} k^{p_{i2}} + p_{i3} \]  \hspace{1cm} (6)
\[ \sigma_{i,k}^2 = q_{i1} k^{q_{i2}} + q_{i3} \]  \hspace{1cm} (7)

The parameters $p_{i1}$, $p_{i2}$, $p_{i3}$, $q_{i1}$, $q_{i2}$ and $q_{i3}$ are shown in Table 3:
Table 4: Parameters for the First Arrival Path Magnitude distribution

| i=1  | a_i^M = 0.4815 | b_i^M = -0.0821 | c_i^M = 0.4103 | d_i^M = -0.02408 |
| i=2  | a_i^M = 0.2601 | b_i^M = -0.1214 | c_i^M = 0.4948 | d_i^M = -0.03241 |
| i=3  | a_i^M = 0.1841 | b_i^M = -0.1246 | c_i^M = 0.3628 | d_i^M = -0.03334 |
| i=4  | a_i^M = 0.1221 | b_i^M = -0.1515 | c_i^M = 0.2736 | d_i^M = -0.03445 |
| i=5  | a_i^M = 0.1721 | b_i^M = -0.1517 | c_i^M = 0.0905 | d_i^M = -0.01979 |

3 Parameters for Magnitude Distribution

3.1 First Arrival Path

For the channels in a particular Class, the magnitude of the first arrival path of the ith Class follows a double exponential decay distribution with the increase of the Cluster Index k. In [1], equation (8) is used to describe the double exponential decay which can be written as:

$$I_{i,k} = a_i^M e^{b_i^M k} + c_i^M e^{d_i^M k}$$

where $a_i^M$, $b_i^M$, $c_i^M$ and $d_i^M$ are the double exponential parameters, and are shown in Table 4:

3.2 Other paths

The magnitude of the other paths are dependent on the arrival time of the path. The decay profile with the time sampling index can be also demonstrated by the double exponential function which is also can be seen in [1] as equation (9):

$$I_{k,j} = a_k^O e^{b_k^O j} + c_k^O e^{d_k^O j}$$

where $k$ is the cluster index. $a_k^O$, $b_k^O$, $c_k^O$ and $d_k^O$ are the double exponential parameters, and are shown in Table 5:

4 Parameters for Path Interval Distribution

For the channels in a particular Class, the path interval can be described by a Generalized Extreme Value (GEV) distribution. The PDF of GEV distribution for ith Class and kth Cluster is given in [1] equation (10) which can be written as:

$$f_{\text{gev}}(x; \xi_{i,k}, \eta_{i,k}, \xi_{i,k}) = \frac{1}{\eta_{i,k}} \left(1 + \frac{x - \xi_{i,k}}{\eta_{i,k}}\right)^{-\frac{1}{\xi_{i,k}} - 1} e^{-\left(1 + \frac{x - \xi_{i,k}}{\eta_{i,k}}\right)^{-\frac{1}{\xi_{i,k}}}}$$

Parameters $\xi$ and $\eta$ in Class V should be described by power functions of cluster Index. Except for the 2 special cases in Class V, the other parameters can be written as linear functions of the cluster Index.

5 Parameters for Cable Losses

In equation (11) of [1], the cable loss of the powerline cable is described as a function of frequency and signal propagation distance.

$$A(f) = e^{-\left(a_0 + a_1 \cdot f^k\right)} e^{-j b_0 f}$$
Table 5: Parameters for the Other Path Magnitude distribution

| k   | $a_k^c$    | $b_k^c$    | $c_k^c$    | $d_k^c$    |
|-----|------------|------------|------------|------------|
| 1   | 0.4194     | -1.270     | 0.0328     | -0.0083    |
| 2   | 0.4388     | -1.355     | 0.0487     | -0.0207    |
| 3   | 0.4647     | -1.353     | 0.0502     | -0.0206    |
| 4   | 0.4542     | -1.329     | 0.0562     | -0.0235    |
| 5   | 0.4381     | -1.244     | 0.0521     | -0.0229    |
| 6   | 0.4632     | -1.253     | 0.0571     | -0.0249    |
| 7   | 0.4677     | -1.163     | 0.0422     | -0.0196    |
| 8   | 0.5124     | -1.120     | 0.0457     | -0.0213    |
| 9   | 0.4262     | -1.1032    | 0.0327     | -0.0171    |
| 10  | 0.4419     | -1.1004    | 0.0287     | -0.0151    |
| 11  | 0.5116     | -1.1046    | 0.0292     | -0.0149    |
| 12  | 0.4604     | -0.964     | 0.0257     | -0.0140    |
| 13  | 0.4501     | -0.925     | 0.0223     | -0.0126    |
| 14  | 0.4968     | -0.946     | 0.0238     | -0.0134    |
| 15  | 0.5187     | -0.950     | 0.0243     | -0.0136    |
| 16  | 0.5242     | -0.915     | 0.0207     | -0.0116    |
| 17  | 0.5355     | -0.896     | 0.0188     | -0.0109    |
| 18  | 0.6164     | -0.934     | 0.0224     | -0.0125    |
| 19  | 0.5288     | -0.852     | 0.0180     | -0.0108    |
| 20  | 0.5829     | -0.864     | 0.0175     | -0.0099    |

Table 6: GEV parameters for Class V:

| Expression | Fitted Result |
|------------|---------------|
| $\xi_{4,k} = a k^2 + c$ | $a = 0.4063, b = 0.2886, c = 1.061$ |
| $\mu_{5,k} = a k + b$ | $a = 0.0002687, b = 0.2033$ |

Table 7: GEV parameters for Class IV:

| Expression | Fitted Result |
|------------|---------------|
| $\xi_{4,k} = a k^2 + b$ | $a = 0.0000972, b = 2.734$ |
| $\eta_{4,k} = a k + b$ | $a = 0.00009786, b = 0.9539$ |
| $\epsilon_{4,k} = a k + b$ | $a = 0.0001653, b = 0.3061$ |

Table 8: GEV parameters for Class III:

| Expression | Fitted Result |
|------------|---------------|
| $\xi_{3,k} = a k^2 + b$ | $a = 0.0006167, b = 2.3937$ |
| $\eta_{3,k} = a k + b$ | $a = 0.0005993, b = 0.8095$ |
| $\epsilon_{3,k} = a k + b$ | $a = -0.00009132, b = 0.571$ |
Table 9: GEV parameters for Class II:

| Expression             | Fitted Result               |
|------------------------|----------------------------|
| $\xi_{2,k} = ak + b$  | $a = 0.001143$, $b = 2.211$ |
| $\eta_{2,k} = ak + b$ | $a = 0.0008684$, $b = 0.6979$ |
| $\epsilon_{2,k} = ak + b$ | $a = -0.00003362$, $b = 0.5586$ |

The parameters for the current cables can be written as the function of the path propagation distance:

\[
\begin{align*}
    a_0 &= 0.0002086 \cdot d + 0.0008739 \quad (12) \\
    a_1 &= 0.00002644 \cdot d - 0.00004644 \quad (13) \\
    k  &= -0.00009098 \cdot d + 0.8876 \quad (14) \\
    b_0 &= -0.0006432 \cdot d - 0.00001126 \quad (15)
\end{align*}
\]

where, $d$ is the path propagation distance. The unit of $f$ is MHz.
References

[1] B. Tan, J. Thompson, "Powerline Communications Channel Modelling Methodology Based on Statistical Features," *IEEE Transactions on Power Delivery*, Submitted

[2] M. Zimmermann, K. Dostert, "A multipath model for the powerline channel," *IEEE Transactions on aCommunications*, vol.50, no.4, pp.553-559, Apr. 2002