Control of Energy Storage in Home Energy Management Systems: Non-Simultaneous Charging and Discharging Guarantees

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Abstract—In this paper we provide non-simultaneous charging and discharging guarantees for a linear energy storage system (ESS) model for a model predictive control (MPC) based home energy management system (HEMS) algorithm. The HEMS optimally controls the residential load and residually-owned power sources such as photovoltaic (PV) power generation and energy storage given residential customer preferences such as energy cost sensitivity and ESS lifetime. Under certain problem formulations with a linear ESS model, simultaneous charging and discharging can be observed as the optimal solution when there is high penetration of PV power. We present analysis that ensures non-simultaneous ESS charging and discharging operation in the given HEMS framework for a linear ESS model that captures both charging and discharging efficiency of the ESS. The energy storage system model behavior guarantees are shown for various electricity pricing schemes such as time of use (TOU) pricing and net metering. Simulation results demonstrating desirable ESS behavior are provided for each electricity pricing scheme.

Index Terms—model predictive control, optimization, home energy management systems, energy storage systems

NOMENCLATURE

α Penalty coefficient for ESS charging
β Penalty coefficient for ESS discharging
c_c Cost of $/kWh E ESS state of charge (kWh)
E_0 Initial ESS state of charge (kWh)
E_max Maximum energy storage in ESS (kWh)
E_min Minimum energy storage in ESS (kWh)
η_c ESS charging efficiency
η_d ESS discharging efficiency
P_c Curtailed solar power (kW)
P_dh Power drawn from ESS (kW)
P_dis Power drawn from ESS (kW)
P_grid Power consumed from the grid (kW)
P_{grid}^t Total residential load (kW)
P_max Maximum charging/discharging power (kW)
P_sol Available solar power (kW)

I. INTRODUCTION

INTEGRATING renewable energy into the power grid has led to both generation-side and demand-side solutions to address the intermittent nature of these renewable energy resources. Electric energy storage systems (ESS) are commonly used to cope with the variability in renewable energy resources. In particular, demand-side residential energy management solutions can be used to address stable renewable energy integration since residential buildings account for over 37.6% of total electricity consumption in the U.S. [1]. Home energy management systems (HEMS) provide residential demand-side energy management by coordinating residually-owned power sources, appliances, user preferences, and renewable energy forecasts [2]–[6]. Many HEMS systems are equipped with an ESS due to decreasing battery costs and gained flexibility in responding to demand response (DR), and energy cost savings for the customer in various electricity pricing schemes such as time-of-use (TOU) pricing [5]–[10]. However, when incorporating ESS models into HEMS, ensuring proper ESS dynamics can be limiting due to the use of lossless or non-convex ESS operation models, or the use of restrictive computation methods. In this work, we provide non-simultaneous battery charging and discharging operation guarantees for an ESS model that captures both charging and discharging efficiency for use in a model predictive control (MPC) based HEMS algorithm that coordinates power drawn from the grid, available solar power, an ESS, and total residential load.

To address this, we use several tools and techniques including the Karush-Kuhn-Tucker (KKT) optimality conditions to show that solutions to the convex MPC-based HEMS algorithm where the ESS is simultaneously charging and discharging are suboptimal under certain conditions. Similar analysis on ESS charging and discharging dynamics derived from the KKT conditions has been applied to a multi-period optimal power flow (OPF) problem with ESS assets [11]. For the OPF problem, the authors in [11] use the natural relation between locational marginal prices (LMPs) and the KKT conditions, to determine conditions for non-simultaneous ESS charging and discharging dynamics. We apply similar analysis to give guarantees on proper ESS dynamics in an MPC-based HEMS optimization framework for certain electricity pricing markets. Additionally, the authors in [12] provide analysis on simultaneous ESS charging and discharging for a distributed power system with multiple grid-connected storage systems.

The outline of this paper is as follows: in Section II, we survey various ESS models used in HEMS literature. In Section III, the mathematical HEMS models and the overall MPC-based HEMS optimization problem are introduced. In
Section IV, we present the main theoretical results of this paper showing that simultaneous ESS charging and discharging is suboptimal for various scenarios. In Section V, we provide simulation results highlighting proper ESS behavior for various electricity pricing schemes and net metering considered in this paper. We also provide simulation results showing when simultaneous ESS charging and discharging is an optimal solution. In Section VI, conclusions regarding the use of the proposed ESS model are discussed, as well as potential areas of future work.

II. COMMONLY USED ESS MODELS

Energy storage systems are often included in renewable energy research due to their energy management flexibility [13]. Additionally, an ESS can be used to account for uncertainties in renewable energy such as solar and wind energy [13], [14]. In HEMS, many homes are equipped with an ESS due to its flexibility to participate in grid services such as demand response [2], [5], [6] or help with consumer energy cost savings in real-time or demand pricing energy markets [3], [4], [7]. However, ensuring proper ESS dynamics (i.e. satisfying non-simultaneous ESS charging and discharging) when incorporating ESS models into the HEMS framework leads to the use of lossless, binary, or non-convex models.

To ensure non-simultaneous ESS charging and discharging, a lossless ESS model is often used. A common lossless model used in HEMS literature is:

\[
E(t+1) = E(t) + \Delta t P_{ess}(t), \quad \forall t, \quad (1a)
\]

\[
E_{\text{min}} \leq E(t+1) \leq E_{\text{max}}, \quad \forall t, \quad (1b)
\]

\[
-P_{\text{max}} \leq P_{\text{ess}}(t) \leq P_{\text{max}}, \quad \forall t, \quad (1c)
\]

where \(P_{\text{ess}}(t)\) is the power injected or drawn into the ESS and \(t \in \{1, \ldots, N\}\) is the time index [15], [16]. The model in (1a)-(1c) ensures non-simultaneous ESS charging and discharging since the power injected into the ESS and drawn from the ESS are captured in one variable \(P_{\text{ess}}(t)\) which represents discharging when negative and charging when positive. While this model is linear, it assumes perfect ESS charging and discharging efficiency, implying the roundtrip efficiency of the ESS is 100% [15], [17], which does not accurately model the non-negligible losses of the system.

Additionally, non-convex ESS models are used in HEMS research to capture ESS charging and discharging efficiency. A common non-convex ESS model that includes both charging and discharging terms and ensures the ESS does not simultaneously charge and discharge uses binary variables and is given by [3], [8], [9], [13], [18], [19]:

\[
E(t+1) = E(t) + \Delta t (\eta_c P_{ch}^{(t)} - \eta_d P_{dis}^{(t)}(1 - b^{(t)})), \quad \forall t, \quad (2a)
\]

\[
E_{\text{min}} \leq E(t+1) \leq E_{\text{max}}, \quad \forall t, \quad (2b)
\]

\[
0 \leq P_{ch}^{(t)}, P_{dis}^{(t)} \leq P_{\text{max}}, \quad \forall t, \quad (2c)
\]

\[
b^{(t)} \in \{0,1\}, \quad \forall t, \quad (2d)
\]

where \(t \in \{1, \ldots, N\}\). The use of separate terms for ESS charging and discharging, \(P_{ch}^{(t)}\) and \(P_{dis}^{(t)}\), respectively, allow for a roundtrip efficiency of less than 100% which accounts for losses in ESS-to-grid interactions [17], [19]. However, the use of binary variables in ESS models results in a non-convex model requiring the use of computationally restrictive methods such as mixed integer (non-)linear programming (MILP/MINLP) [3], [13], [18], [19]. Alternatively, the following nonlinear, non-convex ESS model without binary variables can be used:

\[
E(t+1) = E(t) + \Delta t \left(\eta_c P_{ch}^{(t)} - \eta_d P_{dis}^{(t)}\right), \quad \forall t, \quad (3a)
\]

\[
E_{\text{min}} \leq E(t+1) \leq E_{\text{max}}, \quad \forall t, \quad (3b)
\]

\[
0 \leq P_{ch}^{(t)}, P_{dis}^{(t)} \leq P_{\text{max}}, \quad \forall t, \quad (3c)
\]

\[
P_{ch}^{(t)}, P_{dis}^{(t)} = 0, \quad \forall t, \quad (3d)
\]

where \(t \in \{1, \ldots, N\}\) and the non-convex constraint in (3d) is included in the model to ensure non-simultaneous ESS charging and discharging. Similar to the ESS model with binary variables, the model in (3a)-(3d) also requires the use of computationally restrictive non-convex optimization solvers.

Thus, the common models used to ensure non-simultaneous ESS charging and discharging either assume perfect 100% roundtrip efficiency or are non-convex requiring the use of computationally limiting numerical methods. In this work, we provide non-simultaneous ESS charging and discharging guarantees for a linear ESS model that captures both charging and discharging efficiency. The main contributions of this work are the following:

- We present a formalized and thorough analysis which shows that the use of non-convex or mixed integer ESS models to ensure non-simultaneous charging and discharging are unnecessary under certain conditions in the HEMS optimization, including under various pricing schemes and net metering. We believe this to be the first manuscript that formally proves these claims.
- We present the situations under which simultaneous charging and discharging is optimal in the HEMS optimization, and provide a means to ensure an equivalent solution where simultaneous charging and discharging does not occur without adopting a non-convex ESS model.

III. PROBLEM FORMULATION

In this section, we provide the mathematical models for the HEMS and the overall MPC optimization problem. In this work, we assume that the HEMS must coordinate the
residential PV generation, the ESS, the total residential load, and power drawn from the grid needed to satisfy the load, as shown in Fig. 1.

The residential energy storage system state of charge (SOC) and power charged/discharged are modeled by:

\[ E(t+1) = E(t) + \eta_c \Delta t P_{ch}^t - \eta_d \Delta t P_{dis}^t, \quad (4a) \]
\[ E_{min} \leq E(t+1) \leq E_{max}, \quad (4b) \]
\[ 0 \leq P_{dis}^t, P_{ch}^t \leq P_{max}, \quad (4c) \]
\[ 0 < \eta_c < 1 < \eta_d, \quad (4d) \]

for all \( t \in \{1, \ldots, N\} \) where \( E_{min}, E_{max} \) define the SOC limits [2]. The charging efficiency is denoted \( \eta_c \) and the discharging efficiency is denoted \( \eta_d \). The condition in (4d) can be relaxed and the results we present in this paper hold for any \( \eta_c \) and \( \eta_d \) satisfying \( 0 < \eta_c, \eta_d < 1 \); however, we constrain \( \eta_c \) and \( \eta_d \) as in (4d) for a more practical roundtrip ESS-to-grid efficiency.

The available solar power generated from PV, denoted \( P_{sol}^t \), is a function of the solar irradiance, area of the PV array, and array and inverter efficiency. The curtailed solar power is denoted \( P_{cur}^t \). The overall power consumption from building loads is denoted \( P_{L,house}^t \). The power balance is given by:

\[ 0 = P_{grid}(t) + P_{L,house}(t) - (P_{sol}(t) - P_{e}(t) - P_{dis}^t + P_{ch}^t). \quad (5) \]

Next, we provide the overall MPC-based optimization problem for the HEMS. The objective function \( f_{cost}(\mathbf{x}_N) \), which captures both customer electricity price sensitivity and ESS lifetime considerations, is minimized at each step of the optimization and is given by:

\[ f_{cost}(\mathbf{x}_N) = \sum_{t=1}^{N} \left( c_e(t) P_{grid}^t + \alpha P_{ch}^t + \beta P_{dis}^t \right), \quad (6) \]

where \( N \) denotes the prediction horizon, \( c_e(t) > 0 \) for all \( t \in \{1, 2, \ldots, N\} \) reflects the unit cost of the energy from the grid, and \( \alpha, \beta \geq 0 \) reflect the lifetime cost of charging and discharging the battery, respectively. Other ESS lifetime prediction considerations are presented in [20]. The control variables at each time \( t \) are collected in the vector \( \mathbf{x}_N = [\mathbf{x}^{(1)} \ \mathbf{x}^{(2)} \ \ldots \ \mathbf{x}^{(N)}]^T \) where \( \mathbf{x}^{(i)} = \begin{bmatrix} P_{grid}^{(i)} & P_{ch}^{(i)} & P_{dis}^{(i)} & P_{e}^{(i)} \end{bmatrix}^T \). Based on these discussions, the overall linear optimization program is:

\[ \begin{align*}
\min_{\mathbf{x}_N} & \quad f_{cost}(\mathbf{x}_N) \\
\text{subject to} & \quad 0 \leq P_{grid}^t, \\
& \quad 0 \leq P_{ch}^t \leq P_{max}, \\
& \quad 0 \leq P_{dis}^t \leq P_{max}, \\
& \quad E(t+1) = E(t) + \eta_c \Delta t P_{ch}^t - \eta_d \Delta t P_{dis}^t, \\
& \quad E_{min} \leq E(t+1) \leq E_{max}, \\
& \quad 0 \leq P_{e}^t \leq P_{sol}^t, \\
& \quad 0 = -P_{grid}^t + P_{L,house}^t - (P_{sol}^t - P_{e}^t) - P_{dis}^t + P_{ch}^t. 
\end{align*} \quad (7a)-(7h) \]

for all \( t \in \{1, \ldots, N\} \). The MPC-based HEMS optimization problem given in (7a)-(7h), denoted (P₁), will be the basis of the analysis presented in Section IV for the proposed linear ESS model. We will omit the constraint in (7b) when addressing customer price sensitivity under net metering. Note that the above optimization problem is always feasible as the (traditional operating) point \( \mathbf{x}_N \) given by \( P_{grid}^t = P_{ch}^t = 0, \ P_{dis}^t = P_{sol}^t, \) and \( P_{ch}^t = P_{dis}^t \) for all \( t \) is a feasible point for this optimization problem.

IV. ESS MODEL BEHAVIOR

Next, we derive the KKT conditions for the optimization problem given in (P₁) and we prove that simultaneous charging and discharging in the ESS model (4a)-(4d) is suboptimal.

A. KKT Conditions

The Karush-Kuhn-Tucker (KKT) conditions provide necessary conditions which must be satisfied for the solution of a broad range of optimization problems to be optimal. These conditions will be leveraged to show, under various system conditions and pricing schemes, that simultaneous charging and discharging of the ESS model is suboptimal (i.e., solutions of this form violate the KKT conditions).

To aid in the presentation of the KKT conditions, we introduce additional notation for describing the inequality and equality constraints in the optimization problem given in (P₁). Notice that each of the inequality constraints in (7b)-(7g) describes \( N \) inequality constraints on each of the upper and lower bounds on each of the control variables and the ESS state of charge. Writing (P₁) in standard form [21], the functions on the left-hand side of the inequality constraints are denoted:

\[ f_{grid}^t(\mathbf{x}_N) = -P_{grid}^t, \]
\[ f_{ch}^t(\mathbf{x}_N) = -P_{ch}^t, \]
\[ f_{dis}^t(\mathbf{x}_N) = -P_{dis}^t, \]
\[ f_{sol}^t(\mathbf{x}_N) = -P_{sol}^t, \]

and the expressions on the right-hand-side of the equality constraint are denoted:

\[ \bar{f}_{soc}^t(\mathbf{x}_N) = E_{min} - E^0 + \Delta t \sum_{n=1}^{t} (\eta_d P_{dis}^{(n)} - \eta_c P_{ch}^{(n)}), \]
\[ \bar{f}_{soc}^t(\mathbf{x}_N) = E^0 + \Delta t \sum_{n=1}^{t} (\eta_c P_{ch}^{(n)} - \eta_d P_{dis}^{(n)}) - E_{max}, \]
\[ \bar{f}_{sol}^t(\mathbf{x}_N) = P_{e}^t - P_{sol}^t, \]

for all \( t \in \{1, \ldots, N\} \). Similarly, the equality constraint in (7b) describes \( N \) equality constraints. We denote the \( N \) expressions on the right-hand-side of the equality constraint (7b) by the function \( h^t(\mathbf{x}_N) \) for \( t \in \{1, 2, \ldots, N\} \).
Next, we derive the KKT conditions for the optimization problem given in (P_1). Let $\mathbf{x}_N$ denote an optimal solution for the problem. Then the KKT conditions are:

**Primal Feasibility:**

\begin{align}
    f_{\text{grid}}(\mathbf{x}_N) &\leq 0, \\
    f_{\text{ch}}(\mathbf{x}_N), f_{\text{dis}}(\mathbf{x}_N), f_{\text{soc}}(\mathbf{x}_N) &\leq 0, \\
    h^t(\mathbf{x}_N) &\leq 0, \\
    h^t(\mathbf{x}_N) &\leq 0, \\
    h^t(\mathbf{x}_N) &\leq 0,
\end{align}

(8a) (8b) (8c) (8d) (8e)

**Dual Feasibility:**

\begin{align}
    \lambda^t_{\text{grid}} &\geq 0, \\
    \lambda^t_{\text{ch}}, \lambda^t_{\text{dis}}, \lambda^t_{\text{soc}}, \lambda^t_{\text{sol}} &\geq 0, \\
    \lambda^t_{\text{ch}}, \lambda^t_{\text{dis}}, \lambda^t_{\text{soc}}, \lambda^t_{\text{sol}} &\geq 0, \\
    \lambda^t_{\text{ch}}, \lambda^t_{\text{dis}}, \lambda^t_{\text{soc}}, \lambda^t_{\text{sol}} &\geq 0.
\end{align}

(8f) (8g) (8h)

**Complementary Slackness:**

\begin{align}
    \lambda^t_{\text{grid}}f_{\text{grid}}(\mathbf{x}_N) &\leq 0, \\
    \lambda^t_{\text{ch}}f_{\text{ch}}(\mathbf{x}_N), \lambda^t_{\text{dis}}f_{\text{dis}}(\mathbf{x}_N) &\leq 0, \\
    \lambda^t_{\text{soc}}f_{\text{soc}}(\mathbf{x}_N) &\leq 0, \\
    \lambda^t_{\text{sol}}f_{\text{sol}}(\mathbf{x}_N) &\leq 0.
\end{align}

(8i) (8j) (8k) (8l)

**Stationarity:**

\begin{align}
\nabla f_{\text{cost}}(\mathbf{x}_N) + \sum_{t=1}^{N} \left( \lambda^t_{\text{grid}} \nabla f_{\text{grid}}(\mathbf{x}_N) + \lambda^t_{\text{ch}} \nabla f_{\text{ch}}(\mathbf{x}_N) + \lambda^t_{\text{dis}} \nabla f_{\text{dis}}(\mathbf{x}_N) + \lambda^t_{\text{soc}} \nabla f_{\text{soc}}(\mathbf{x}_N) + \lambda^t_{\text{sol}} \nabla f_{\text{sol}}(\mathbf{x}_N) + \mu_t \nabla h^t(\mathbf{x}_N) \right) = 0,
\end{align}

(8m)

where $0 \in \mathbb{R}^{4N}$, and $\lambda^t_{\text{const}}/\lambda^t_{\text{const}}$ is the Lagrange multiplier associated with the inequality constraint $f_{\text{const}}(\mathbf{x}_N)/f_{\text{const}}(\mathbf{x}_H)$, respectively, and $\mu_t$ is the Lagrange multiplier associated with the power balance equality constraint $h^t(\mathbf{x}_H)$ at time $t$ [21], [22]. Then, the condition in (8m) is decomposed into:

\begin{align}
    c^t_e - \lambda^t_{\text{grid}} - \mu_t &= 0, \\
    \alpha - \lambda^t_{\text{ch}} + \lambda^t_{\text{dis}} + \eta_c \Delta t \sum_{n=t}^{N} (\lambda^t_{\text{soc}} - \lambda^t_{\text{soc}}) + \mu_t &= 0, \\
    \beta - \lambda^t_{\text{dis}} + \lambda^t_{\text{dis}} + \eta_d \Delta t \sum_{n=t}^{N} (\lambda^t_{\text{soc}} - \lambda^t_{\text{soc}}) - \mu_t &= 0, \\
    - \lambda^t_{\text{soc}} + \lambda^t_{\text{soc}} + \mu_t &= 0,
\end{align}

(9a) (9b) (9c) (9d)

for all $t \in \{1, \ldots, N\}$. For convex optimization problems with differentiable objective functions, any solution that satisfies the KKT conditions given in (8a)-(8m) is optimal [21]. Thus, we leverage the KKT conditions to show that feasible points to the MPC-based HEMS optimization problem given in (P_1) where the ESS is simultaneously charging and discharging are suboptimal in certain situations.

**B. Convex ESS Model Behavior Guarantees**

Next, we show that feasible points where the ESS is charging and discharging simultaneously ($0 < P^t_{\text{ch}}, P^t_{\text{dis}} \leq P_{\text{max}}$) give rise to suboptimal solutions. Further, we show that a solution with simultaneous charging and discharging is suboptimal for various objective functions that capture customer energy cost sensitivity and ESS lifetime considerations. The customer energy cost sensitivity is studied for two cases: 1) the case where power is not allowed to be exported back to the grid, and 2) the net metering case. Under net metering, a consumer can provide a grid service by exporting excess available solar power to offset the price of power drawn from the grid, thus the constraint in (7b) is omitted in (P_1). This implies $P^t_{\text{grid}}$ can be negative, which represents the customer’s ability to export excess solar power to the grid. In each case, we assume that the optimal solution for some time $\tau \in \{1, \ldots, N\}$ in the prediction horizon is $\mathbf{x}^\tau = [P^\tau_{\text{grid}}, P^\tau_{\text{ch}}, P^\tau_{\text{dis}}, P^\tau_e]^T$, where $0 < P^\tau_{\text{ch}}, P^\tau_{\text{dis}} \leq P_{\text{max}}$, from which we obtain a contradiction to show that a solution with simultaneous ESS charging and discharging is suboptimal.

In Proposition 1, we show that a solution to (P_1) with simultaneous ESS charging and discharging is suboptimal in the following two cases: 1) when customers are not able to export excess power onto the grid and the optimal power drawn from the grid $P^t_{\text{grid}}$ is positive for some $t \in \{1, \ldots, N\}$, and 2) in a net metering situation.

**Proposition 1.** Assume $c^t_e > 0 \forall t$ and $\alpha, \beta \geq 0$ and $0 < c^t_e < 1 < \eta_e$. A solution to (P_1) where $0 < P^t_{\text{ch}}, P^t_{\text{dis}} \leq P_{\text{max}}$ for any $t \in \{1, \ldots, N\}$ is suboptimal in the following situations:

1) If customers are not able to export excess power onto the grid and $P^t_{\text{grid}} > 0 \forall t \in \{1, \ldots, N\}$,

2) If customers are in a net metering situation.

**Proof.** We prove this claim by contradiction. So, assume at some time $\tau \in \{1, \ldots, N\}$ the optimal solution to (P_1) is $\mathbf{x}^\tau = [P^\tau_{\text{grid}}, P^\tau_{\text{ch}}, P^\tau_{\text{dis}}, P^\tau_e]^T$, where $0 \leq P^\tau_{\text{grid}} \leq P^\tau_{\text{ch}}$, and $P^\tau_{\text{grid}} \leq P_{\text{max}}$.

First, we consider the case where excess power cannot be exported onto the grid with $P^\tau_{\text{grid}} > 0$ and electricity drawn from the grid is subject to a time-varying pricing schedule $c^\tau_e$ for power drawn from the grid such as TOU pricing. The objective function in this case is as in (6) where $c^\tau_e > 0$ and $\alpha, \beta \geq 0 \forall t$. We leverage the complementary slackness conditions at time $\tau$ in (8i)-(8l). Since $P^\tau_{\text{grid}} > 0$, therefore, $\lambda^\tau_{\text{grid}}, \lambda^\tau_{\text{ch}}, \lambda^\tau_{\text{dis}} = 0$. We also determine that $\lambda^\tau_{\text{ch}}, \lambda^\tau_{\text{dis}}, \lambda^\tau_{\text{soc}}, \lambda^\tau_{\text{sol}} \geq 0$ due to the assumptions $P^\tau_{\text{ch}}, P^\tau_{\text{dis}} \leq P_{\text{max}}$, the solar power curtailment satisfies (7g), and $\lambda^\tau_{\text{const}} \geq 0 \forall t$. Using the above conditions on the Lagrange multipliers at time $\tau$ together with (9a)-(9d), we have:

\begin{align}
    c^\tau_e &= \mu_t, \\
    \alpha + \lambda^\tau_{\text{ch}} + \eta_c \Delta t \sum_{n=\tau}^{N} (\lambda^\tau_{\text{soc}} - \lambda^\tau_{\text{soc}}) + c^\tau_e &= 0, \\
    \beta + \lambda^\tau_{\text{dis}} + \eta_d \Delta t \sum_{n=\tau}^{N} (\lambda^\tau_{\text{soc}} - \lambda^\tau_{\text{soc}}) - c^\tau_e &= 0.
\end{align}

(10) (11) (12)
Solving for $\mathcal{I} = \sum_{n=1}^{N} (\lambda_{n}^{a} - \lambda_{n}^{soc})$ in (11), and then replacing $-\mathcal{I}$ in (12), we get:

$$\beta + \bar{\lambda}_{\text{dis}} + \frac{\eta_d}{\eta_c} (\alpha + \bar{\lambda}_{\text{ch}}) + \left( \frac{\eta_d}{\eta_c} - 1 \right) c_{e}^{(\tau)} = 0. \quad (13)$$

Note that since $\alpha, \beta \geq 0$ and by dual feasibility in (8g), the first three terms in (13) are greater than or equal to zero, but the fourth term in (13) is positive since $c_{e}^{(\tau)} > 0$ and $0 < \eta_c < 1 < \eta_d$. Thus, we obtain a contradiction since the left hand side of (13) cannot equal 0. Therefore, in a situation where excess power cannot be exported to the grid, a solution to $(\mathcal{P}_{1})$ where $\tilde{P}_{\text{grid}} > 0$ and $0 < P_{\text{ch}}^{(\tau)}, \tilde{P}_{\text{dis}}^{(\tau)} \leq P_{\text{max}}$ for some $t \in \{1, \ldots, N\}$ is suboptimal because it does not satisfy the KKT conditions given in (8a)-(8m).

Next, we consider the situation where there is net metering. We again use contradiction to prove our claim. Assume that at time $\tau \in \{1, \ldots, N\}$ the optimal solution is $\tilde{x}_{\tau} = [\tilde{P}_{\text{grid}}^{(\tau)} \tilde{P}_{\text{ch}}^{(\tau)} \tilde{P}_{\text{dis}}^{(\tau)} \tilde{P}_{c}^{(\tau)}]^{T}$ where $0 < P_{\text{ch}}^{(\tau)}, \tilde{P}_{\text{dis}}^{(\tau)} \leq P_{\text{max}}$ and $0 \leq \tilde{P}_{c}^{(\tau)} \leq \tilde{P}_{\text{sol}}^{(\tau)}$. We use the complementary slackness conditions at time $\tau$ in (8j)-(8l). For this case, (8i) is omitted since constraint (7b) is excluded for net metering. Using similar analysis as in the proof of the first case above, we again obtain (13), which leads to a contradiction. Thus, under net metering, a solution to $(\mathcal{P}_{1})$ where the ESS simultaneously charges and discharges for any time $t \in \{1, \ldots, N\}$ is suboptimal.

Note that the same result in Proposition 1 is obtained for both cases with constant electricity prices by assuming $c_{e}^{(t)} > 0$ is constant for all $t \in \{1, \ldots, N\}$.

Next, we address the situations under which simultaneous charging and discharging is optimal in the HEMS optimization, and provide a means to ensure that a solution with non-simultaneous charging and discharges does occur. In Proposition 2, we show results for non-simultaneous ESS charging and discharging dynamics that are general, but in particular, address the case when $\tilde{P}_{\text{grid}}^{(t)} = 0$ for any $t \in \{1, \ldots, N\}$ where excess power cannot be exported to the grid. While simultaneous charging and discharging is a possible optimal solution in this case if $\alpha = \beta = 0$, we show that there exists an optimal solution where the ESS does not simultaneously charge and discharge. We also show that including nonzero battery cycling costs (i.e. $\alpha, \beta \geq 0$) leads to an optimal solution where the ESS does not simultaneously charge and discharge.

**Proposition 2.** Assume $c_{e}^{(t)} > 0 \ \forall t$ and $0 < \eta_c < 1 < \eta_d$. For any $\alpha, \beta \geq 0$, if excess power is not able to be exported to the grid, then there exists an optimal solution to $(\mathcal{P}_{1})$ where $\tilde{P}_{\text{ch}}^{(t)}$ and $\tilde{P}_{\text{dis}}^{(t)}$ are not simultaneously positive for any $t \in \{1, \ldots, N\}$.

**Proof.** To prove this statement, we will introduce some notation. Let $X^{*}$ be the set of all solutions of $(\mathcal{P}_{1})$. We say that a solution $\tilde{x}_{N}$ satisfies property $S$ if it satisfies the non-simultaneous charging and discharging at all times $t \in \{1, \ldots, N\}$. For a solution $\tilde{x}_{N} \in X^{*}$, if $\tilde{x}_{N}$ does not satisfy $S$, we let $\tau(\tilde{x}_{N})$ be the last time that we have simultaneous charging and discharging, otherwise, we let $\tau(\tilde{x}_{N}) = 0$.

Finally, for a solution $x_{N} \in X^{*}$, define its total charging $P_{ch}(x_{N})$ to be:

$$P_{ch}(x_{N}) = \sum_{t=1}^{N} P_{ch}^{(t)}. \quad (14)$$

Note that $X^{*}$ is a closed and bounded set and hence, a solution with minimum total charging always exists, i.e. the set argmin$_{x_{N} \in X^{*}} P_{ch}(x_{N})$ is non-empty. Let $\tilde{x}_{N}$ be the set argmin$_{x_{N} \in X^{*}} P_{ch}(x_{N})$.

We claim that $\tilde{x}_{N} = [\tilde{P}_{\text{grid}}^{(\tau)} \tilde{P}_{\text{ch}}^{(\tau)} \tilde{P}_{\text{dis}}^{(\tau)} \tilde{P}_{c}^{(\tau)}]^{T}$ satisfies property $S$. To show this, assume that the contrary holds. Then, for the last simultaneous charging and discharging time, we have $t^{*} = \tau(\tilde{x}_{N}) \in \{1, \ldots, N\}$. By the definition of $\tau(\tilde{x}_{N})$, either $\tilde{P}_{\text{ch}}^{(t^{*})} = 0$ or $\tilde{P}_{\text{dis}}^{(t^{*})} = 0$, for $t \in \{t^{*} + 1, \ldots, N\}$ while $\tilde{P}_{c}^{(t^{*})} > 0$. Therefore, for all $t > t^{*}$, $\tilde{P}_{c}^{(t)} = 0$.

(1) Case 1, $\tilde{P}_{\text{dis}}^{(t^{*})} > 0$: Let $\delta = \tilde{P}_{\text{dis}}^{(t^{*})} - \tilde{P}_{\text{ch}}^{(t^{*})}$. First, let us assume that for all time $t > t^{*}, \tilde{P}_{c}^{(t)} = 0$. In this case, define $\tilde{x}_{N} \in \mathbb{R}^{4N}$ by:

$$\tilde{x}_{N} = \begin{cases} \tilde{x}_{N}^{(t)} & \text{for } t \leq t^{*} \\ [\tilde{P}_{\text{grid}}^{(t)} \tilde{P}_{\text{ch}}^{(t)} 0 \delta \tilde{P}_{c}^{(t)}]^{T} & \text{for } t > t^{*} \end{cases} \quad (14)$$

i.e., $\tilde{x}_{N}$ and $\tilde{x}_{N}$ are identical except that $\tilde{P}_{c}^{(t)} = 0$ and $\tilde{P}_{\text{dis}}^{(t)} = \delta$. Note that since $\tilde{x}_{N} = \tilde{x}_{N}$ for all $t \neq t^{*}$, $\tilde{x}_{N}$ satisfies (7b), (7c), (7d), and (7f) for all $t \neq t^{*}$, Similarly, for $t = t^{*}$, since $\tilde{x}_{N}$ is a feasible point and $\tilde{P}_{\text{dis}}^{(t^{*})} - \tilde{P}_{\text{ch}}^{(t^{*})} = \tilde{P}_{c}^{(t^{*})}$, (7b) and (7f) hold for $t = t^{*}$. Using (7e), if we let $\tilde{E}^{(t)}$ be the state of charge for the solution $\tilde{x}_{N}$, then (7f) is satisfied for $t > t^{*}$ since $\tilde{x}_{t} = \tilde{x}_{t}$ for all $t < t^{*}$. For time $t = t^{*}$, we have:

$$\tilde{E}^{(t^{*}+1)} = \tilde{E}^{(t^{*})} + \eta_{d} \Delta t \tilde{P}_{c}^{(t^{*})} - \eta_{d} \Delta t \tilde{P}_{\text{dis}}^{(t^{*})}$$

$$= \tilde{E}^{(t^{*})} - \eta_{d} \Delta t \delta$$

$$= \tilde{E}^{(t^{*})} + \eta_{d} \Delta t \tilde{P}_{\text{ch}}^{(t^{*})} - \eta_{d} \Delta t \tilde{P}_{\text{dis}}^{(t^{*})} + (\eta_{d} - \eta_{c}) \Delta t \tilde{P}_{c}^{(t^{*})}$$

$$= \tilde{E}^{(t^{*}+1)} + (\eta_{d} - \eta_{c}) \Delta t \tilde{P}_{c}^{(t^{*})}. \quad (15)$$

The feasibility of $\tilde{x}_{N}$ and the equalities above imply that:

$$0 \leq \tilde{E}^{(t^{*}+1)} < \tilde{E}^{(t^{*})} \leq \tilde{E}^{(t^{*})} \leq E_{\text{max}}.$$
Similar to (14), define:  
\[
\hat{x}^{(t)} = \begin{cases} 
\hat{\tilde{x}}^{(t)}
& t \neq t^*, t^+ \\
[\hat{P}_{\text{grid}}^{(t*)^{-1}} \hat{P}_{\text{ch}}^{(t*)^{-1}} - p, \hat{P}_{\text{dis}}^{(t*)^{-1}} - p, \hat{P}_{c}^{(t*)^{-1}}] & t = t^* \\
[\hat{P}_{\text{grid}}^{(t*)^{-1}} - a, \hat{P}_{\text{ch}}^{(t*)^{-1}} - (\eta_p/\eta_c - 1), 0, 0, \hat{P}_{c}^{(t*)^{-1}} + b] & t = t^+
\end{cases}
\]

where \(a \in [0, \hat{P}_{\text{grid}}^{(t*)^{-1}}], b \in [0, P_{\text{sol}}^{(t*)^{-1}} - \hat{P}_{c}^{(t*)^{-1}}]\), such that \(a + b = (\eta_p/\eta_c - 1)p\). To show that such an \(a, b\) exist, using the power balance equation in (7h), we have:

\[
\hat{P}_{\text{grid}}^{(t*)^{-1}} + (P_{\text{sol}}^{(t*)^{-1}} - \hat{P}_{c}^{(t*)^{-1}}) \geq \hat{P}_{\text{ch}}^{(t*)^{-1}} \geq (\eta_p/\eta_c - 1)p,
\]

where the first inequality follows as \(P_{\text{L,house}}^{(t*)^{-1}} \geq 0\), and the second inequality follows from the choice of \(p \leq \eta_p/\eta_c - \eta_p/\eta_c\). Therefore, such an \(a\) and \(b\) as defined above exists.

Next, we show \(\hat{x}_{N}\) in (17) satisfies the constraints in (P1):

a. Constraint (7b): Note that \(\hat{P}_{\text{grid}}^{(t*)^{-1}} = \hat{P}_{\text{grid}}^{(t*)^{-1}} \geq 0\) for all \(t \neq t^*, t^+\). For \(t = t^*\), since \(a \in [0, \hat{P}_{\text{grid}}^{(t*)^{-1}}]\), it follows that \(0 \leq \hat{P}_{\text{grid}}^{(t*)^{-1}} - a = \hat{P}_{\text{grid}}^{(t*)^{-1}}\). Thus, \(\hat{P}_{\text{grid}}^{(t*)^{-1}}\) satisfies (7b) \(\forall t\).

b. Constraint (7c): Note that \(\hat{P}_{\text{ch}}^{(t*)^{-1}} = \hat{P}_{\text{ch}}^{(t*)^{-1}}\) for all \(t \neq t^*, t^+\) and \(0 \leq p \leq \hat{P}_{\text{ch}}^{(t*)^{-1}}\). Then, \(0 \leq \hat{P}_{\text{ch}}^{(t*)^{-1}} - p \leq \hat{P}_{\text{ch}}^{(t*)^{-1}} \leq P_{\text{max}}\) for \(t = t^*, t^+\), and we have \(0 \leq (\eta_p/\eta_c - 1)p \leq \hat{P}_{\text{ch}}^{(t*)^{-1}} \leq P_{\text{max}}\) for \(t = t^+, t^+\). Therefore, \(\hat{P}_{\text{ch}}^{(t*)^{-1}}\) satisfies (7c) \(\forall t\).

c. Constraint (7d): Recall \(\hat{P}_{\text{dis}}^{(t*)^{-1}} = \hat{P}_{\text{dis}}^{(t*)^{-1}} \forall t \neq t^*, t^+\). It is clear \(\hat{P}_{\text{dis}}^{(t*)^{-1}}\) satisfies (7d). For \(t = t^*\), \(0 \leq \hat{P}_{\text{dis}}^{(t*)^{-1}} - p \leq \hat{P}_{\text{dis}}^{(t*)^{-1}}\) (by assumption), and \(0 \leq \hat{P}_{\text{dis}}^{(t*)^{-1}} - p \leq \hat{P}_{\text{dis}}^{(t*)^{-1}} \leq P_{\text{max}}\). Thus, \(\hat{P}_{\text{dis}}^{(t*)^{-1}}\) satisfies (7d) \(\forall t\).

d. Constraint (7f): Since \(\hat{x}^{(t)} = \hat{x}^{(t)}\) for \(t < t^\star\), (7f) holds for \(t < t^\star\). For \(t = t^\star\), similar to (15) and using the definition of \(\hat{x}_{N}\) in (17), we have

\[
\hat{E}^{(t^\star+1)} = \hat{E}^{(t^\star)} + \eta_d \Delta t \hat{P}_{\text{ch}}^{(t^\star)} - \eta_d \Delta t \hat{P}_{\text{dis}}^{(t^\star)}
\]

\[
= \hat{E}^{(t^\star)} + \eta_d \Delta t (\hat{P}_{\text{ch}}^{(t^\star)} - p) - \eta_d \Delta t (\hat{P}_{\text{dis}}^{(t^\star)} - p)
\]

\[
= \hat{E}^{(t^\star)} + (\eta_d - \eta_c) \Delta t p.
\]

By the main assumption of Case 1, i.e., \(\hat{P}_{\text{ch}}^{(t^*)^{-1}} \geq \hat{P}_{\text{dis}}^{(t^*)^{-1}}\), using this fact and the first equality in (18), we have \(\hat{E}^{(t^\star+1)} \leq \hat{E}^{(t^\star)} = \hat{E}^{(t^\star)} \leq E_{\text{max}}\). From the last equality in (18), we get \(0 \leq \hat{E}^{(t^\star+1)} \leq E^{(t^\star+1)}\). Then, overall, we have:

\[
0 \leq \hat{E}^{(t^\star+1)} \leq \hat{E}^{(t^\star+1)} + (\eta_d - \eta_c) \Delta t p
\]

\[
= \hat{E}^{(t^\star+1)} \leq \hat{E}^{(t^\star)} \leq E_{\text{max}}.
\]

Therefore, (7f) holds for \(t = t^\star\). For \(t = t^\star+1, \ldots, t^\star+1\), we have \(\hat{P}_{\text{ch}}^{(t^\star)} = 0\) and hence,

\[
\hat{E}^{(t^\star+1)} = \hat{E}^{(t^\star)} - \eta_d \Delta t \hat{P}_{\text{dis}}^{(t^\star)}
\]

and inductively, we can show that

\[
E_{\text{max}} \geq \hat{E}^{(t^\star+1)} \geq \hat{E}^{(t^\star+2)} \geq \cdots \geq \hat{E}^{(t^\star)}
\]
the constraints (7a)-(7h), except possibly the SOC limits in (7f) for time \( t \geq t^* \). Focusing on the SOC limits, note that:

\[
E(t^{*}+1) = E(t^{*}) + \Delta t \eta_e \hat{P}_{\text{ch}}(t^{*}) - \Delta t \eta_d \hat{P}_{\text{dis}}(t^{*})
\]

\[
\approx \tilde{E}(t^{*}) + \Delta t \eta_e \left( \frac{\hat{P}_{\text{ch}}(t^{*})}{\eta_c} - \frac{\hat{P}_{\text{dis}}(t^{*})}{\eta_d} \right)
\]

\[
= \tilde{E}(t^{*}+1).
\]

Therefore, with this set of parameters, the state of charge will not be affected and hence, \( \tilde{X}_N \) is another feasible solution to (P1). Again, notice that

\[
f_{\text{cost}}(\tilde{X}_N) = f_{\text{cost}}(\tilde{X}_N) - c_e(t^*) a - \epsilon \left( \alpha \frac{\hat{P}_{\text{ch}}}{\eta_c} + \beta \right) \leq f_{\text{cost}}(\tilde{X}_N),
\]

and hence, \( \tilde{X}_N \in X^* \). But \( P_{\text{ch}}(\tilde{X}_N) = P_{\text{ch}}(\tilde{X}_N) - \frac{\eta_d}{\eta_c} \epsilon \). This contradicts the assumption that \( \tilde{X}_N \) is the minimizer of \( P_{\text{ch}}(\cdot) \) over \( X^* \). Therefore, \( \tilde{X}_N \) satisfies Property S. Thus, we have shown that there exists an optimal solution to (P1) such that the ESS does not simultaneously charge and discharge.

For Prop. 2, notice that when \( \alpha, \beta = 0 \), a solution with simultaneous ESS charging and discharging may be equivalent to a solution without simultaneous ESS charging and discharging behavior in certain situations. To ensure that there is an optimal solution without simultaneous ESS charging and discharging, choose \( \alpha, \beta \geq 0 \) such that \( (\alpha + \beta) > 0 \).

For the case that an energy utility charges \( c_e(t) = 0 \)/kWh for some time \( t \in \{1, \ldots, N\} \) to flatten grid power demand over time [23], we provide a remark on non-simultaneous ESS charging and discharging that follow directly from the propositions above.

Remark 1. In a situation where there is either net metering or excess power cannot be exported to the grid, if \( c_e(t) = 0 \)/kWh for any \( t \in \{1, \ldots, N\} \), we can ensure that the results of Props. 1-2 hold by further restricting \( \alpha, \beta \geq 0 \) such that \( (\alpha + \beta) > 0 \).

V. SIMULATIONS: ESS OPERATION

To demonstrate the charging and discharging behavior of the ESS model, we provide simulation results for the proposed MPC-based HEMS algorithm in (7a)-(7h) for each case described in Section IV. The MATLAB-based modeling system for solving disciplined convex programs, CVX, is used in this work. The simulation has a 24 hour prediction horizon with 1 hour time intervals.

The residential PV array size is 20m\(^2\) with a tilt of 30\(^\circ\) and an efficiency of 16\%. The solar irradiance forecasts were obtained from NOAA USCRN data [24]. The 5-kWh residential ESS system is restricted to 15\% to 85\% of the maximum SOC to preserve the ESS lifetime [2]. The initial ESS SOC is 2 kWh. The ESS inverter power limit is 3 kW with an inverter efficiency of 95\%. The ESS charging efficiency \( \eta_e \) is 95\%, and the discharging efficiency is \( \eta_d = \frac{1}{\eta_c} \).

A. Simulation Results

The proposed HEMS algorithm in (7a)-(7h) with the objective function in (6) is simulated for the case where the price of electricity is constant \( c_e(t) = 0.11$/kWh \forall t \), \( \alpha = \beta = 0 \), and excess power is not able to be exported to the grid. The simulation results for this case are provided in Fig. 2. From Fig. 2 (top), we can see that \( P_{\text{ch}}(t) = 0 \forall t \), and when \( P_{\text{grid}}(t) = 0 \), the excess solar power from satisfying the total load is able to be stored in the ESS, thus allowing \( \alpha, \beta = 0 \) for this case. Additionally, when \( P_{\text{grid}}(t) > 0 \), the ESS discharges to contribute power to satisfying the total load. The ESS operating behavior in Fig. 2 (middle) and (bottom) highlights that non-simultaneous ESS charging and discharging were optimal solutions at each time \( t \in \{1, \ldots, N\} \) with the convex ESS model in (4a)-(4d).

Next, we repeat the simulation above with the available solar increased by 150\% and with the same objective function in (6) where the price of electricity is constant \( c_e(t) = 0.11$/kWh \forall t \), there is no ESS usage penalty \( \alpha = \beta = 0 \), and excess power is not able to be exported to the grid. The simulation results for this case are shown in Fig. 3, demonstrating how simultaneous charging and discharging can occur when there is a high penetration of solar power and no ESS usage penalty included in the cost function. The ESS operating behavior in Fig. 3 (middle) shows simultaneous ESS charging and discharging in the linear model under these operating conditions. Building upon this, we provide an additional simulation in Fig. 4 with the same high penetration of solar but the cost function in (6) is modified such that the ESS charging and discharging penalty is included \( \alpha = 0.001 \), \( \beta = 0 \) (with the price of electricity still constant at \( c_e(t) = 0.11$/kWh \forall t \). The simulation results in Fig. 4 (middle) with non-simultaneous ESS charging and discharging demonstrate how including a penalty on ESS usage in the cost function ensures that a solution with simultaneous ESS charging and discharging is suboptimal.

Next, we show simulation results for the HEMS algorithm with the objective function in (6), where \( \alpha = 0.001 \) and \( \beta = 0 \), under a TOU pricing structure where excess power cannot be exported to the grid. The TOU pricing schedule is given in Table I, and is incorporated into the cost function. The simulation results for this case are provided in Fig. 5.
Fig. 3. HEMS simulation results for a constant electricity price where excess power is not exported to the grid with a high penetration of solar and no ESS usage penalty in the optimization cost function. Power profiles with HEMS optimization algorithm (top). Simultaneous ESS charging and discharging behavior (middle). ESS state of charge (bottom).

Fig. 4. HEMS simulation results for a constant electricity price where excess power is exported to the grid with a high penetration of solar and ESS usage penalty included in the optimization cost function. Power profiles with HEMS optimization algorithm (top). Non-simultaneous ESS charging and discharging behavior (middle). ESS state of charge (bottom).

Fig. 5. HEMS simulation results with TOU pricing and excess power when power export to the grid is not allowed. Power profiles with HEMS optimization algorithm (top). ESS charging and discharging behavior (middle). ESS state of charge (bottom).

Fig. 6. HEMS simulation results under net metering with TOU pricing. Power profiles with HEMS optimization algorithm (top). ESS charging and discharging behavior (middle). ESS state of charge (bottom).

From Fig. 5 (top), we see that when $P_{\text{grid}}^{(t)} = 0$, the ESS charges or discharges based on the TOU pricing schedule. The ESS charges when there is excess solar after satisfying the total residential load. During the On-Peak pricing period, once the house load surpasses the available solar power, the ESS discharges instead of using grid power. When $P_{\text{grid}}^{(t)} > 0$, the ESS charges or discharges depending on the TOU pricing period at that time. The ESS non-simultaneous charging and discharging behavior is shown in Fig. 5 (middle) and (bottom).

Lastly, we show simulation results for a HEMS algorithm under net metering with TOU pricing. The TOU pricing schedule is shown in Table II, and notice that $c_{\text{e}}^{(t)} = 0$ during the Off-Peak period. The TOU pricing schedule is incorporated into the cost function in (6), and $\alpha = \beta = 0.001$ to ensure non-simultaneous ESS charging and discharging during the Off-Peak period. The simulation for this case is shown in Fig. 6. From Fig. 6 (top), we can see the excess solar is exported to the grid once the ESS is full. During the Off-Peak period, the ESS charges when power can be drawn from the grid for free. During the On-Peak period, the ESS discharges to contribute to the residential load and, due to the net metering structure, available solar is exported onto the grid. The ESS charging and discharging behavior is shown in Fig. 6 (middle) and (bottom) to demonstrate non-simultaneous ESS charging and discharging in more detail.

TABLE I

| Time of Use | Pricing Period | Price of Electricity $c_{\text{e}}^{(t)}$ |
|------------|----------------|------------------------------------------|
| 9PM - 9AM  | Off-Peak       | $0.08$/kWh                               |
| 9AM - 2PM  | Shoulder       | $0.13$/kWh                               |
| 6PM - 9PM  | On-Peak        | $0.18$/kWh                               |

TABLE II

| Time of Use | Pricing Period | Price of Electricity $c_{\text{e}}^{(t)}$ |
|------------|----------------|------------------------------------------|
| 9PM - 9AM  | Off-Peak       | $0.00$/kWh                               |
| 9AM - 2PM  | Shoulder       | $0.13$/kWh                               |
| 6PM - 9PM  | On-Peak        | $0.18$/kWh                               |
VI. CONCLUSION

This work provides non-simultaneous charging and discharging guarantees for a linear ESS model in an MPC-based HEMS optimization algorithm for various cases. We used various tools and techniques, including the KKT conditions, to show that solutions to the optimization with simultaneous ESS charging and discharging were suboptimal in each situation studied in this work, meaning that nonconvex ESS models preventing simultaneous charging and discharging are unnecessary under these problem formulations. Additionally, we showed that including terms in the objective function \( f_{\text{cost}}(\mathbf{x}_N) \) to capture ESS lifetime considerations can improve the non-simultaneous charging and discharging guarantees for the ESS model in the situations studied in this work. If there are times when the cost of electricity \( c_e(t) = $0/\text{kWh} \) for some time \( t \), including terms in the objective function \( f_{\text{cost}}(\mathbf{x}_N) \) to capture ESS lifetime considerations are necessary to ensure proper ESS dynamics with the proposed ESS model.

Simulation results were provided to show the ESS charging and discharging behavior for each of the cases studied in this work.

Future work includes ensuring non-simultaneous charging and discharging in stochastic ESS models or in electric vehicle (EV) models where the EV battery is used for flexible storage. Additionally, this work can be extended to other electricity pricing schemes such as feed-in tariffs. The KKT analysis we applied to a MPC-based HEMS optimization can be expanded to other convex ESS models and renewable energy research settings equipped with an ESS.

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