Astrophysical S-factor of the $^{7}\text{Be}(p, \gamma)^{8}\text{B}$
reaction from Coulomb dissociation of $^{8}\text{B}^*$

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Abstract

The Coulomb dissociation method to obtain the astrophysical S-factor, $S_{17}(0)$, for the $^{7}\text{Be}(p, \gamma)^{8}\text{B}$ reaction at solar energies is investigated by analysing the recently measured data on the breakup reaction $^{208}\text{Pb}(^{8}\text{B}, ^{7}\text{Be}p)^{208}\text{Pb}$ at 46.5 MeV/A beam energy. Breakup cross sections corresponding to E1, E2 and M1 transitions are calculated with a theory of Coulomb excitation that includes the effects of the Coulomb recoil as well as relativistic retardation. The interplay of nuclear and Coulomb contributions to the breakup process is studied by performing a full quantum mechanical calculation within the framework of the distorted-wave Born Approximation. In the kinematical regime of the present experiment, both nuclear as well as Coulomb-nuclear interference processes affect the pure Coulomb breakup cross sections very marginally. The $E2$ cross sections are strongly dependent on the model used to describe the structure of $^{8}\text{B}$. The value of $S_{17}(0)$ is deduced with and without $E2$ and $M1$ contributions added to the $E1$ cross sections and the results are discussed.

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The radiative capture reaction $^7\text{Be}(p, \gamma)^8\text{B}$ plays a crucial role in the so called “Solar neutrino puzzle”. The cross sections of this reaction at solar energies ($E_{\text{cm}} \approx 20$ keV where $E_{\text{cm}}$ is the center of mass relative energy of the $p + ^7\text{Be}$ system) are directly related to the flux of the high energy neutrinos (emitted in the subsequent $\beta$ decay of $^8\text{B}$) to which the $^{37}\text{Cl}$ and Kamiokande detectors are most sensitive [1]. The direct measurement of this reaction, at such low relative energies, is strongly hindered because of the Coulomb barrier. Therefore, the cross sections measured at relatively higher values of $E_{\text{cm}}$ are extrapolated to solar energies using a theoretically derived energy dependence of the low energy cross sections. However, absolute values of the S-factors obtained by various workers by following this method differ from one another considerably [2,3,4].

The method of Coulomb dissociation provides an alternate indirect way of determining the cross sections of the radiative capture reaction at low relative energies. In this procedure it is assumed that the break-up reaction $a + Z \rightarrow (b + x) + Z$ proceeds entirely via the electromagnetic interaction; the two nuclei $a$ and $Z$ do not interact strongly. By further assumption that the electromagnetic excitation process is of first order, one can relate [5] directly the measured cross-sections of this reaction to those of the radiative capture reaction $b + x \rightarrow a + \gamma$. Thus, the astrophysical S-factors of the radiative capture processes can be determined from the study of break-up reactions under these conditions. A few reactions e.g. $\alpha + d \rightarrow ^6\text{Li} + \gamma$, $\alpha + t \rightarrow ^7\text{Li} + \gamma$, and $^{13}\text{N} + p \rightarrow ^{14}\text{N} + \gamma$ have been studied both experimentally and theoretically using this method [see e.g. Ref. [6] for a recent review].

A first attempt has been made recently by Motobayshi et al. [8], to study the Coulomb dissociation of $^8\text{B}$ into the $^7\text{Be} - p$ low energy continuum in the field of $^{208}\text{Pb}$ with a radioactive $^8\text{B}$ beam of 46.5 MeV/A energy [7]. Assuming a pure E1 excitation, the Monte Carlo simulation of their data predicts a $S_{1\gamma}(0) = 16.7 \pm 3.2$ eV barn, which is considerably lower than the value of $22.4 \pm 2.0$ eV barn used by Bahcall and Pinsonneault [8] in their standard solar model (SSM) calculations.

In this letter we perform a more rigorous analysis of the data of Ref. [7] by using a theory of Coulomb excitation which simultaneously includes the effects of Coulomb recoil and relativistic retardation by solving the general classical problem of the motion of two relativistic charged particles [9]. Under the kinematical condition of the present experiment, $E2$ transitions may be disproportionately enhanced in the Coulomb dissociation process. There has been quite some debate in the literature recently [10,11,12] about the extent of E2 contribution to the data of Ref. [7]. We would, therefore, like to reexamine this issue carefully, as this has important consequences for the $S_{1\gamma}(0)$ extracted from the data. We also include the contributions of the $M1$ transition which may be important for $E_{\text{cm}}$ in the vicinity of 0.633 MeV which corresponds to a $1^+$ continuum resonant state in $^8\text{B}$. Furthermore, we investigate the role of the nuclear excitations. Although the cross sections of the pure nuclear breakup may be small as compared to those of the Coulomb break-up, their interference may still have some effect on the angular distributions. The usefulness of the Coulomb dissociation method in extracting the reliable astrophysical $S$ factor from the breakup data depends on this term having negligible influence on the calculated break-up cross sections. We report here the result of the first full quantum
mechanical calculation of the Coulomb and nuclear excitations and their interference for this reaction, performed within the framework of the distorted wave Born approximation (DWBA). This is expected to highlight whether or not the nuclear effects alter appreciably the prediction of a pure Coulomb breakup process.

The double differential cross-section for the Coulomb excitation of a projectile from its ground state to the continuum, with a definite multipolarity of order $\pi\lambda$ is given by [5,6]

\[ \frac{d^2 \sigma}{d\Omega dE_{\gamma}} = \sum_{\pi\lambda} \frac{1}{E_{\gamma}} \frac{dn_{\pi\lambda}}{d\Omega} \sigma_{\pi\lambda}^{\pi\lambda}(E_{\gamma}), \]  

where $\sigma_{\pi\lambda}^{\pi\lambda}(E_{\gamma})$ is the cross-section for the photodisintegration process $\gamma + a \rightarrow b + x$, with photon energy $E_{\gamma}$, and multipolarity $\pi = E$ (electric) or $M$ (magnetic), and $\lambda = 1, 2...$ (order), which is related to that of the radiative capture process $\sigma(b + x \rightarrow a + \gamma)$ through the theorem of detailed balance. In terms of the astrophysical S-factor, we can write

\[ \sigma(b + x \rightarrow a + \gamma) = \frac{S(E_{cm})}{E_{cm}} \exp(-2\pi\eta(E_{cm})), \]  

where $\eta = \frac{Z_b Z_x e^2}{hv}$, with $v$, $Z_b$ and $Z_x$ being the relative center of mass velocity, and charges of the fragments $b$ and $x$ respectively.

In most cases, only one or two multipolarities dominate the radiative capture as well as the Coulomb dissociation cross sections. In Eq. (1) $n_{\pi\lambda}(E_{\gamma})$ represents the number of equivalent (virtual) photons provided by the Coulomb field of the target to the projectile, which is calculated by the method discussed in Ref. [9]. $S(E_{cm})$ can be directly determined from the measured Coulomb dissociation cross-sections using Eqs. 1 and 2.

The quantal treatment of the Coulomb and nuclear excitations is well known [13]. The expression for the differential cross sections corresponding to a transition of multipolarity $\lambda$ can be schematically represented as

\[ \frac{d\sigma}{d\Omega} \propto \sum_{\lambda=+\mu}^{\lambda=-\mu} \left| \int_0^{\infty} dr R_{Li}(k_ir)(F_C^\lambda + F_N^\lambda)R_{Lf}(k_f r) \right|^2 \]  

In Eq. (3), indices $i$ and $f$ refer to the incoming and outgoing channels. This equation also involves summation over the partial waves $L_i$ and $L_f$. $F_C^\lambda$ and $F_N^\lambda$ denote the form factors for the Coulomb and nuclear excitations, respectively. The expression for $F_C^\lambda$, which is determined entirely by the corresponding B(E\lambda) value, is given in Refs. [13,14]. For $F_N^\lambda$, we take the usual collective model expression with the value of the “nuclear deformation parameter” being the same as that of the Coulomb one. $R_{Li}$ and $R_{Lf}$ define the wave functions of the relative motion in the incoming and outgoing channels, respectively, which are obtained by solving the Schrödinger equation with appropriate optical potentials. In our calculations we have used the potentials given in table 3 of Ref.[15] (set D). The same set of potentials were used for the incoming and outgoing channels. For a Glauber-model calculation of the Coulomb and nuclear breakup we refer to Ref. [16].
In Fig. 1, we show the results of our full quantal calculations for the E1 breakup of $^8$B on $^{208}$Pb target at the beam energy of 46.5 MeV/A for $E_{cm}$ of 1.0 MeV. We have chosen a higher value of $E_{cm}$ where nuclear effects may be larger; the calculation reported in Ref. [16] is at a very low value of $E_{cm}$ (100 keV). The solid line shows the result obtained with the coherent sum of nuclear and Coulomb excitations as well as their interference. The dashed (dotted) line depict the cross sections for pure nuclear (Coulomb) excitation. We have used 5000 partial waves ($L_i$ and $L_f$) in the calculation of the Coulomb excitation cross section. We note that the pure Coulomb excitation angular distribution is a smooth curve approaching zero as $\theta \to 0$ (the so called adiabaticity limit). This reflects the semiclassical nature of the process, thus the pure Coulomb excitation process should be amenable to the semiclassical methods [17]. The peak in this distribution corresponds to an angle $\theta_{min} \approx \frac{2aE_{cm}^2}{\hbar^2}$, where $a$ is half the distance of closest approach in a head on collision) below which the adiabaticity condition sets in. On the other hand, the pure nuclear cross sections show the typical diffraction pattern, and are at least two orders of magnitude smaller than the pure Coulomb one. The important point to note is that pure Coulomb cross sections are very similar to full calculations. Similar results are obtained
also for the values of $E_{cm}$ of 0.8 MeV and 0.6 MeV. Thus, nuclear effects modify the total amplitudes very marginally in the entire kinematical regime of the data of Ref. [7] (except for the region very close to $\theta = 0$). Thus the Coulomb dissociation method can be used to extract reliable the astrophysical S factor for the $^7\text{Be}(p, \gamma)^8\text{B}$ reaction from this data.

In Fig. 2, we show the comparison of our calculated Coulomb dissociation double differential cross sections with the corresponding data of Ref. [7] as a function of the scattering angle $\theta_{cm}$ of the excited $^8\text{B}$ (center of mass of the $^7\text{Be}+p$ system) for three values of the $E_{cm}$. The calculated $E_1$, $E_2$ and $M_1$ cross sections are folded with an efficiency matrix provided to us by Naohito Iwasa (one of the authors of Ref. [7]). This matrix accounts for the efficiency and geometry of the detectors (including energy and angular spread)\(^3\). The solid lines in Fig. 2 show the calculated $E_1$ cross sections obtained with S-factors ($S_{17}$) that provide best fit to the data (determined by $\chi^2$ minimization procedure). These are $(17.58 \pm 2.26)$ eV barn, $(14.07 \pm 2.67)$ eV barn and $(15.59 \pm 3.49)$ eV barn at $E_{cm} = 0.6$ MeV, 0.8 MeV and 1.0 MeV respectively.

The contributions of the $E_2$ and $M_1$ excitations are calculated by using the radiative capture cross sections $\sigma(p+^7\text{Be} \rightarrow ^8\text{B}+\gamma)$, given by the models of Typel and Baur (TB) [18] and Kim, Park and Kim (KPK) [19]. We have used as input the corresponding S factors averaged over energy bins of experimental uncertainty in the relative energy of the fragments. In Fig. 2, the dashed (dashed dotted) line shows the $E_1$ (with best fit $S_{17}$) + $E_2$ + $M_1$ cross sections, with $E_2$ and $M_1$ components calculated with TB (KPK) capture cross sections . It should be noted that the contribution of $M_1$ component is substantial for $E_{cm} = 0.6$, while at 0.8 MeV and 1.0 MeV it is relatively quite small. This multipolarity was not included in the analysis presented in Ref. [10] and also in the calculations of the angular distributions shown in Ref. [20]. The $E_2$ capture cross sections of KPK and TB are quite different from each other, while those of $M_1$ multipolarity are (within 10-15%) model independent. If the KPK model is correct then $E_2$ contributions to the data of Ref. [7] is not negligible. On the other hand, with the TB model the calculated ($E_1+E_2+M_1$) cross sections are within the experimental uncertainties of the data.

The best fit “$E_1$ only” $S_{17}$ factors, as described above, give a $S_{17}(0) = (15.5 \pm 2.80)$ eV barn, if we use the direct extrapolation procedure adopted in Ref. [7] together with the TB capture model. It should be noted that in this model the $E_1$ capture from both $s$ and $d$ waves to the $p$-wave ground state is included. In the model of Tombrello [21], which is the basis of extrapolation in Ref. [7], the $d$ wave contributions are ignored. This can lead to a 10-15% difference in the extrapolated S factor. To see the effect of the $E_2$ and $M_1$ components on the extracted astrophysical S factor we refitted the data of Ref. [7] with ($E_1$(best fit) + $E_2$ + $M_1$) multiplied by a parameter $\xi(E_{cm})$, which is determined by $\chi^2$ minimization procedure. The best fit values of the “correction factor” $\xi$ are $(0.70 \pm 0.17)$, $(0.73 \pm 0.10)$ and $(0.75 \pm 0.15)$ with the KPK model and $(0.85 \pm 0.12)$, $(0.88 \pm 0.16)$ and $(0.91 \pm 0.20)$ with the TB model, at the relative energies of 0.6 MeV, 0.8 MeV and 1.0 MeV respectively. The values of the parameter $\xi(E_{cm})$ translated into $E_1$ S factors give a

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\(^3\)In Ref. [7], the detector response was taken into account by putting the theoretical cross sections as input to a Monte Carlo simulation program.
Figure 2: Comparison of experimental and theoretical Coulomb dissociation yields (cross section × detector efficiency) as a function of $\theta_{cm}$ for the $E_{cm}$ values of 0.6 MeV, 0.8 MeV and 1.0 MeV. Solid lines show the calculated pure $E1$ Coulomb dissociation cross sections obtained with best fit values of S factors as discussed in the text. The dashed and dashed dotted curves represent the sum of $E1$, $E2$ and $M1$ contributions with latter two components calculated with capture cross sections given in the models of TB [18] and KPK [19] respectively. The experimental data is taken from Ref. [7].
corrected \( S_{17}(0) \) of 11.20 \( \pm \) 2.02 (14.0 \( \pm \) 2.45) for KPK (TB) model. Thus, with the KPK model, the \( E2 \) and \( M1 \) contributions reduce the “\( E1 \) only S-factor” by more than 25%, while with TB model this reduction is limited to only about 10 − 12%.

In fig. 3, the calculation for the angle integrated cross sections is compared with the data. The solid line shows the results for the \( E1 \) breakup calculated with a constant astrophysical S-factor of 15.5 eV barn. The dashed (dashed dotted) line shows the sum of the \( E1, E2 \) and \( M1 \) contributions with latter two components calculated with the capture cross sections of TB (KPK). It may be noted that unlike the case in Ref. [20], we have used efficiencies corresponding to \( E2 \) and \( M1 \) multipoarities to fold the calculated cross sections of these transitions. We observe that with the KPK model the cross sections up to \( E_{cm} \approx 1.2 \text{ MeV} \) are modified by the \( E2 \) and \( M1 \) components, while with the TB model only the point at 600 keV is appreciably modified.

Thus we see that \( E2 \) corrections to the data of Ref. 7 is strongly model dependent and it is difficult to draw any definite conclusion about their contributions. In the past the KPK model has been criticised by Barker [22] for their incorrect treatment of the resonant contribution due to \( M1 \) and \( E2 \) transitions. On the other hand Typel and Baur have neglected the capture from \( f \)-wave relative initial states, which could lead to a smaller \( E2 \) capture cross section. The calculations of several other authors also differ in their predictions of the \( E2 \) cross section [23]. It is therefore important to develop a reliable model to calculate the \( E2 \) capture cross sections for the \( ^{7}\text{Be}(p, \gamma)^{8}\text{B} \) reactions.

Nevertheless, measurements performed at higher beam energies are likely to provide a kinematical regime in which Coulomb dissociation cross section are less dependent on the nuclear structure model, as \( E2 \) components in this regime are appreciably weaker than their \( E1 \) counterpart. In Fig. 4 we show the ratio \( d\sigma/d\Omega((E1 + E2)/E1) \) at the beam energies of 46.5 MeV/A and 200 MeV/A as a function of \( \theta_{cm} \), with \( E2 \) cross sections calculated within the KPK (dotted lines) and TB (solid lines) models for the \( E_{cm} \) value of 0.6 MeV. The \( E1 \) angular distributions \( (d\sigma/d\Omega) \) decrease as \( \theta_{cm} \) increases beyond \( \theta_{min} \) while those of \( E2 \) multipolarity remains flat (in the angular region considered in this figure). Thus this ratio increases with angle; the rate of the increase being determined by the magnitude of the \( E2 \) component. We notice that the value of this ratio is around 3.0 (1.2) at the beam energy of 46.5 MeV/A (200 MeV/A) with \( E2 \) cross sections calculated in the KPK model. This implies that at the beam energies around 200 MeV and at very forward angles the \( E2 \) contributions calculated even in the KPK model will introduce relatively small modifications to pure \( E1 \) cross sections (they are small in the TB model anyway). Therefore, higher beam energies and very forward angles are expected to be better suited for extracting the astrophysical S-factors independent of the uncertainties in the nuclear structure model of \( ^{8}\text{B} \). Furthermore, the higher order effects (eg. the so called post-acceleration) which can distort the the relative energy spectrum of \( ^{7}\text{Be} + p \) system are expected to be insignificant at these energies [20].

In summary, we analysed the recently measured data [7] of the breakup of \( ^{8}\text{B} \) on \( ^{208}\text{Pb} \) target at the beam energy of 46.5 MeV/A by the Coulomb dissociation method in order to extract the astrophysical S factors for the radiative capture reaction \( ^{7}\text{Be}(p, \gamma)^{8}B \). We used the first order perturbation theory of Coulomb excitation which included both the effects
Figure 3: Comparison of the experimental and calculated angle integrated Coulomb dissociation yields as a function of fragment relative energy. The solid curve represents pure $E1$ cross sections calculated with a constant $S$ factor of 15.5 eV barn. Dashed and dashed dotted lines show the sum of the $E1$, $E2$ and $M1$ components with latter two being calculated with capture cross sections predicted by TB and KPK respectively. The data is from Ref. [7].
Figure 4: The ratio of differential cross sections for $E1 + E2$ transitions to pure $E1$ transition (calculated with a $S$ factor of 15.5 eV barn) as a function of $\theta_{cm}$ for the reactions of Fig. 1 at the beam energies of 46.5 MeV/A (upper part) and 200 MeV/A (lower part). The solid (dashed) line corresponds to the case where $E2$ component has been calculated with the capture cross sections of TB (KPK).
of the relativistic retardation as well as Coulomb recoil simultaneously. We considered the breakup due to $E1$, $E2$ and $M1$ transitions. Full quantum mechanical calculations within the framework of the DWBA, were performed for the pure Nuclear breakup, pure Coulomb breakup and their interference and it is shown that nuclear break-up contributions affect the total break-up amplitudes only very marginally in the kinematical regime of the data of Ref. [7]. Thus the assumptions of the Coulomb dissociation method are well fulfilled for this case.

The $S_{17}(0)$ value deduced in our analysis considering only the $E1$ cross sections is in agreement with the lower values reported from the extrapolation of the direct capture cross sections, and with that extracted from a recent indirect theoretical method [24]. This is significantly smaller than the value used by Bahcall and Pinsenault [8] and Turck-Chieze et al. [1] in their SSM calculations. The $E2$ break-up cross sections can reduce the “$E1$ only S factors”, bringing the $S_{17}(0)$ further down. However, the magnitude of the $E2$ contributions to the data of Ref. [7] depend significantly on the nuclear structure model used to calculate the corresponding capture cross sections and at this stage it is not possible to make a definite prediction. Experiments performed at higher beam energies are expected to be relatively less affected by the $E2$ component, and therefore they provide a better kinematical regime where the extracted S factors are likely to be less model dependent.

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