Ergodic Layered Erasure One-Sided Interference Channels

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Abstract—The sum capacity of a class of layered erasure one-sided interference channels is developed under the assumption of no channel state information at the transmitters. Outer bounds are presented for this model and are shown to be tight for the following sub-classes: i) weak, ii) strong (mix of strong but not very strong (SnVS) and very strong (VS)), iii) ergodic very strong (mix of strong and weak), and (iv) a sub-class of mixed interference (mix of SnVS and weak). Each sub-class is uniquely defined by the fading statistics.

I. INTRODUCTION

The capacity region of interference channels (IFCs), comprised of two or more interfering links (transmitter-receiver pairs), remains an open problem. The sum capacity of a non-fading two-user IFC is known only when the interference is either stronger or much weaker at the unintended than at the intended receiver (see, for e.g., [1–5], and the references therein). Recently, the sum capacity and optimal power policies for two-user ergodic fading IFCs are studied in [6] and [7] under the assumption that the instantaneous fading channel state information (CSI) is known at all nodes. A sum capacity analysis for K-user ergodic fading channels using ergodic interference alignment is developed in [8] and [9]. In general, however, the instantaneous CSI is not available at the transmitters and often involves feedback from the receivers. Thus, it is useful to study the case in which only receivers have perfect CSI and the transmitters are strictly restricted to knowledge only of the channel statistics.

The sum capacity of multi-terminal networks without transmit CSI remains a largely open problem with the capacity known only for ergodic fading Gaussian multiaccess channels (MACs) without transmit CSI. For this class of channels, it is optimal for each user to transmit at its maximum average power in each use of the channel (see for e.g., [10] or [11]). The receiver, with perfect knowledge of the instantaneous CSI, decodes the messages from all transmitters jointly over all fading realizations.

Recently, the sum capacity of ergodic fading two-receiver broadcast channels (BCs) without transmit CSI has been studied in [12]. The authors first develop the sum capacity achieving scheme for an ergodic layered erasure BC where the channel from the source to each receiver is modeled as a time-varying version of the binary expansion deterministic channel introduced in [13]. In this model, the transmitted signal is viewed as a vector (layers) of bits from the most to the least significant bits. Fading is modeled as an erasure of a random number of least significant bits and the instantaneous erasure levels, or equivalently the number of received layers (or levels), are assumed to be known at the receivers. For a layered erasure fading BC, the authors in [12] show that a strategy of signaling independently on each layer to one receiver or the other based only on the fading statistics achieves the sum capacity. Furthermore, the authors also demonstrate the optimality of their achievable scheme to within 1.44 bits/s/Hz of the capacity region for a class of high-SNR channel fading distributions.

In this paper, we introduce an ergodic fading layered erasure one-sided (two-user) IFC in which, in each channel use, one of the receivers receives a random number of layers from its intended transmitter while the other receiver receives a random number of layers from both transmitters. One can view this channel as a time-varying one-sided version of a two-user binary expansion deterministic IFC introduced and studied in [14]. The model in [14] is a subset of the class of deterministic IFCs whose capacity region is developed in [15]. More recently, in [16], the sum capacity of a class of one-sided two-user and three-user IFCs in which each transmitter has limited information about its connectivity to the receivers is developed. For the ergodic layered erasure one-sided IFC considered here, we develop outer bounds and identify fading regimes for which the strategies of either decoding or ignoring interference at the interfered receiver is tight. We classify the capacity achieving regimes based on the fading statistics of the direct and interfering links as follows: i) weak, ii) strong (mix of strong but not very strong (SnVS) and very strong (VS)), iii) ergodic very strong (mix of SnVS, VS, and weak), and (iv) a sub-class of mixed interference (mix of SnVS and weak).

The paper is organized as follows. In Section II we introduce the channel model. In Section III, we develop the capacity region of a layered erasure multiple-access channel. In Section IV, we develop outer bounds for the layered erasure IFC and identify the regimes where these bounds are tight using in part the results developed in Section III. We conclude in Section V.
II. CHANNEL MODEL AND PRELIMINARIES

A two-user IFC consists of two point-to-point transmitter-receiver links where the receiver of each link also receives an interfering signal from the unintended transmitter. In a deterministic IFC, the input at each transmitter is a vector of \( q \) bits. We write \( X^q_k = [X_{k,1}^q \ldots X_{k,n}^q]^T \), \( k = 1, 2 \), to denote the input at the \( k^{th} \) transmitter such that \( X_{k,1} \) and \( X_{k,2} \) are the most and the least significant bits, respectively. Throughout the sequel, we refer to the bits as levels or layers, and write the input at level \( n \) for transmitter \( k \) as \( X_{k,n} \), for all \( n = 1, 2, \ldots, q \). The received signal of user \( k \), is denoted by the \( q \)-length vector \( Y^q_k = [Y_{k,1}^q \ldots Y_{k,2}^q \ldots Y_{k,q}^q]^T \).

Associated with each transmitter \( k \) and receiver \( j \) is a non-negative integer \( n_{jk} \) that defines the number of bit levels of \( X_k \) observed at receiver \( j \). The maximum level supported by any link is \( q \). Specifically, an \( n_{jk} \) link erases \( q - n_{jk} \) least significant bits of \( X^q_k \) such that only \( n_{jk} \) most significant bits of \( X^q_k \) are received as the \( n_{jk} \) least significant bits of \( Y^q_k \). The missing entries \( X_{k,n_{jk}+1}, \ldots, X_{k,q} \) have been erased by the fading channel. Thus, we have [13]

\[
Y^q_k = \begin{bmatrix} 0 & \ldots & 0 & X_{k,1} & X_{k,2} & \ldots & X_{k,n_{jk}} \end{bmatrix}^T
\]

(1)

where \( S^{q-n_{jk}} \) is a \( q \times q \) shift matrix with entries \( S_{m,n} \) that are non-zero only for \((m, n) = (q-n_{jk}+n, n), n = 1, 2, \ldots, n_{jk}\).

In a layered erasure IFC, we model each of the four transmit-receive links as a \( q \)-bit layered erasure channel. A \( q \)-bit layered erasure channel is defined in [12] and summarized below.

**Definition 1 ([12]):** A \( q \)-bit layered erasure channel has input \( X^q \in \mathbb{F}_2^q \) and output \( Y^q = [0 \ldots 0 X^N] \) where \( N \) is an integer channel state that is independent of \( X^q \) and satisfies \( \Pr[N \geq 0] = 1 \) and \( \Pr[N \geq q + 1] = 0 \).

From Definition 1, in every use of the channel, the received signal \( Y^q_j \), \( j = 1, 2 \), of a layered erasure IFC is given by

\[
Y^q_j = S^{q-N_{1j}} X^q_1 \oplus S^{q-N_{2j}} X^q_2, \quad j = 1, 2,
\]

(3)

where \( \oplus \) denotes the XOR operation, \( N_{11} \) and \( N_{22} \) are the random variables representing the fading channel states over the direct links, and \( N_{21} \) and \( N_{12} \) are the random variables representing the cross-link fading states. The one-sided IFC considered in this paper is obtained by setting \( N_{12} = 0 \), i.e., receiver 1 sees no interference from transmitter 2. One can visualize the resulting one-sided channel as an ‘S-IFC’.

As a first step towards developing the sum capacity of a layered erasure one-sided IFC, we will develop the ergodic sum capacity of a two-user layered erasure multiple-access channel (MAC), consisting of one receiver and two transmitters. For this MAC, the received signal \( Y^q_j \), \( j = 1 \), is given by (3). For simplicity, we eliminate the subscript 1 and write \( N_1 \) and \( N_2 \) to denote the fading states of transmitters 1 and 2 to the receiver, respectively.

For a random variable \( N \), we write \( \Pr[N = n] \) to denote the probability mass function and \( F_N(n) \) to denote the complementary cumulative distribution function (CDF). It is straightforward to verify that

\[
\mathbb{E}[N] = \sum_{n=1}^{q} F_N(n) = \sum_{n=1}^{q} \Pr[N \geq n].
\]

(4)

We also write \( x^+ = \max(x, 0) \). All logarithms are taken to the base 2 and the rates are in units of bits per channel use. Throughout the sequel we use the words transmitters and users interchangeably.

III. LAYERED FADING MAC: SUM CAPACITY

Consider a multiple access channel with the two transmitters transmitting \( X_1 \) and \( X_2 \) respectively, and a received signal \( Y \) given by

\[
Y = X_1^{N_1} \oplus X_2^{N_2},
\]

(5)

where \( N_1 \) and \( N_2 \) are the channel states for the links from the two transmitters to the receiver respectively. Both random variables \( N_1 \) and \( N_2 \) satisfy \( \Pr[N_i \geq 0] = 1 \) and \( \Pr[N_i \geq q + 1] = 0 \).

**Theorem 1:** The capacity region for the layered erasure multiple access channel is given by

\[
R_1 \leq \mathbb{E}[N_1],
\]

(6)

\[
R_2 \leq \mathbb{E}[N_2],
\]

(7)

\[
R_1 + R_2 \leq \mathbb{E}[\max(N_1, N_2)].
\]

(8)

**Proof:** We will describe the achievability here since the converse is straightforward. We prove the achievability of a corner point given by the rate pair \( (\mathbb{E}[N_1 - N_2])^+, \mathbb{E}[N_2] \). The capacity region can then be achieved by interchanging the coding schemes over the two links and by time sharing.

The first user uses a codebook of rate \( \Pr[N_1 - N_2 \geq n] \) to transmit a message at level \( n \). For a given level \( n \), the second user uses a codebook of rate \( \Pr[N_2 \geq n] \) to transmit its message. Codebooks are independent across layers at both users. Across all channel states, i.e., on average, the receiver receives \( \mathbb{E}[N_1 - N_2] = \Pr[N_1 - N_2 \geq n] \) bits from level \( n \) of user 1, where we have used the fact that the expected value of an indicator function of an event is the probability of that event. The codebook rate of user 1 at this level therefore allows the receiver to reliably decode the message of user 1. After decoding the messages of user 1 its contribution from the received signal can be canceled and the remaining contribution of the second user can be decoded reliably. Thus, across all levels, the average transmission rates of \( \mathbb{E}[(N_1 - N_2)^+] \) and \( \mathbb{E}[N_2] \) at users 1 and 2, respectively, enable reliable communications.

**Example 1:** Consider a layered MAC with \( q = 4 \) and two fading states: the first state with \( N_1 = 4 \) and \( N_2 = 3 \) occurs with probability \( p \), and the second state with \( N_1 = 2 \) and \( N_2 = 4 \) with probability \( 1-p \). The above achievability scheme for rate pair \((p, 4-p)\) reduces to the following. At transmitter 1, a rate \( p \) code is used on the first level while nothing is transmitted on the remaining levels. At transmitter 2, a rate 1 code is used at the top three levels while a rate \( 1-p \) code is used on the fourth level. Note that in this case, whenever
the channel is in the first state \((N_1 = 4, N_2 = 3)\) the top bit of user 1 reaches the receiver noiselessly. Hence, the rate \(p\) codeword of user 1 can be decoded from the occurrences of state 1. Thus, the contribution of the first transmitter can be cancelled by the receiver across both states. Following this, the receiver uses the top 3 levels of the second transmitter that are interference-free in both the states and hence a rate of 1 bit/channel use can be achieved for each of the three levels. The fourth level reaches the receiver whenever the system is in the second state \((N_1 = 2, N_2 = 4)\) which happens with probability \(1 - p\), and thus the codebook of rate \(1 - p\) can be decoded by the receiver from the occurrences of the second state.

IV. LAYERED ERASURE ONE-SIDED IFC

A. Outer Bounds

Outer bounds on the capacity region of a class of deterministic IFCs, of which the binary expansion deterministic IFC is a sub-class, are developed in [15]. For a time-varying (ergodic) layered erasure IFC with perfect CSI at the receivers, we follow the same steps as in [15, Theorem 1] while including the CSI as a part of the received signal at each receiver. The following theorem summarizes the outer bounds on the capacity region of layered erasure one-sided IFCs.

**Theorem 2:** An outer bound of the capacity region of an ergodic layered erasure one-sided IFC is given by the set of all rate tuples \((R_1, R_2)\) that satisfy

\[
R_1 \leq \mathbb{E}[N_{11}] \\
R_2 \leq \mathbb{E}[N_{22}] \\
R_1 + R_2 \leq \mathbb{E}[\max(N_{11}, N_{22}, N_{21}, N_{11} + N_{22} - N_{21})].
\]

B. Optimality of Outer Bounds

We now prove the tightness of the sum capacity outer bounds for specific sub-classes of ergodic layered erasure IFCs. For the very strong sub-class, the achievable scheme also achieves the capacity region. For the remaining sub-classes, we achieve a corner point of the capacity region.

1) Very Strong IFC:

**Theorem 3:** For a class of very strong layered erasure IFCs for which \(N_{21} \geq N_{11} + N_{22}\) holds with probability 1, the sum capacity is \(\mathbb{E}[N_{11} + N_{22}]\) and the capacity region is given by \(R_1 \leq \mathbb{E}[N_{11}]\) and \(R_2 \leq \mathbb{E}[N_{22}]\).

**Proof:** Consider the following achievable scheme: at level \(n\), the first user uses a codebook of rate \(\Pr(\min(N_{11}, (N_{21} - N_{22})^+) \geq n) = \Pr(N_{11} \geq n)\), i.e., at each level, the first user transmits at the erasure rate supported by that level at the receiver. On the other hand, at level \(n\), the second user uses a codebook of rate \(\Pr(N_{22} \geq n)\) to transmit its message. At both users, encoding is independent across layers. The message of user 1 can be reliably decoded at receiver 1 and the average rate achieved is

\[
R_1 = \sum_{n=1}^{q} \Pr(N_{11} \geq n) = \sum_{n=1}^{q} F_{N_{11}}(n) = \mathbb{E}[N_{11}].
\]

The second receiver acts like a multi-access receiver and at each level, it first decodes the message of user 1. Thus, across all channel states, it can, on average, reliably decode the message from level \(n\) at a rate \(\Pr(\min(N_{11}, (N_{21} - N_{22})^+) \geq n)\). After decoding all levels of user 1, receiver 2 eliminates the contribution of user 1 from its received signal thereby decoding the messages from user 2 interference-free at an average rate of \(\mathbb{E}[N_{22}]\).

2) Strong but not Very Strong IFC:

**Theorem 4:** The sum capacity of a class of very strong layered erasure IFCs for which \(N_{11} \leq N_{21} \leq N_{11} + N_{22}\) with probability 1 is \(\mathbb{E}[\max(N_{21}, N_{22})]\).

**Proof:** The proof is very similar to that of Theorem 3, and is hence omitted.

3) Strong IFC: For the two sub-classes considered thus far, it sufficed to use independent coding across the layers. However, for the sub-class with a mix of SnVS and VS states, joint coding across the layers is required as shown in the following theorem.

**Theorem 5:** If \(N_{21} \geq N_{11}\) with probability 1, the sum capacity is given by \(\min(\mathbb{E}[N_{11} + N_{22}], \mathbb{E}[\max(N_{21}, N_{22})])\).

**Proof:** Let \(\mathbb{E}[(N_{21} - N_{22})^+] \leq \mathbb{E}[N_{11}]\). In this case, the first user forms a codebook of rate \(\mathbb{E}[(N_{21} - N_{22})^+]/q\). The transmitter at level \(n\) sends data from this codebook while the second user at level \(n\) uses a codebook of rate \(\Pr(N_{22} \geq n)\) to transmit the data.

The decoding scheme proceeds as follows. The first receiver receives across all channel states, i.e., on average, \(\sum_{n=1}^{q} \Pr(N_{11} \geq n) = \mathbb{E}[N_{11}]\) bits from all the levels of user 1 and is thus able to decode data at the lower rate of \(\mathbb{E}[(N_{21} - N_{22})^+]\). Similarly, the second receiver receives across all channel states, i.e., on average, \(\sum_{n=1}^{q} \Pr(N_{21} - N_{22} \geq n) = \mathbb{E}[(N_{21} - N_{22})^+]\) bits reliably from all the levels of user 1 and is thus able to decode. After decoding user 1, receiver 2 eliminates the contribution of user 1 from its received signal thereby decoding the messages from user 2 interference-free at an average rate of \(\mathbb{E}[N_{22}]\).

One can proceed similarly for \(\mathbb{E}[(N_{21} - N_{22})^+] \geq \mathbb{E}[N_{11}]\). In this case, the first user forms a codebook of rate \(\mathbb{E}[N_{11}]\) and the same strategy achieves the sum capacity.

4) Ergodic Very Strong IFC: More generally, one can also consider the sub-class of IFCs with a mix of all types of sub-channels, i.e., a mix of weak, SnVS, and VS. In the following
theorem we develop the sum capacity for subset of such a sub-class in which on average the conditions for very strong are satisfied.

**Theorem 6:** If $E[\max(N_{21}, N_{22})] \geq E[N_{11} + N_{22}]$, then the sum capacity is $E[N_{11} + N_{22}]$. 

**Proof:** The first user forms a codebook of rate $E[N_{11}]/q$. The transmitter at level $n$ sends data from this codebook while the second user at level $n$ uses a codebook of rate $Pr(N_{22} \geq n)$ to transmit the data. The first receiver receives across all channel states, i.e., on average, it receives $E[N_{11}]$ bits from all the levels of user 1 and is thus able to decode. Similarly, the second receiver receives across all channel states, i.e., on average, it receives $E[(N_{21} - N_{22})^+] \geq E[N_{11}]$ bits from all the levels of user 1 and is thus able to decode. After decoding user 1, receiver 2 eliminates the contribution of user 1 from its received signal thereby decoding the messages from user 2 interference-free at an average rate of $E[N_{22}]$. 

5) Weak IFC:

**Theorem 7:** If $N_{21} \leq N_{11}$ with probability 1, then the sum capacity is $E(\max(N_{11}, N_{11} + N_{22} - N_{21}))$. 

**Proof:** Consider the following achievable scheme: at level $n$, the first user uses a codebook of rate $Pr(N_{11} \geq n)$, i.e., at each level, the first user transmits at the erasure rate supported by that level at its receiver. On the other hand, at level $n$, the second user uses a codebook of rate $Pr(N_{22} - N_{21} \geq n)$ to transmit its message. The second receiver receives across all channel states, i.e., on average, it receives $E[1_{N_{22} - N_{21} \geq n}] = Pr(N_{22} - N_{21} \geq n)$ bits reliably from all level $n$ of user 2 and is thus able to decode. 

6) Mixed IFC:

**Theorem 8:** For a layered erasure one-sided IFC, the following sum rate can be achieved:

$$E[N_{22}] + \sum_{n=1}^{q} (Pr(N_{11} \geq n) - Pr(N_{21} \geq n, N_{21} - N_{22} < n))^+.$$  

(11) 

**Proof:** The transmission scheme is as follows: user 1 transmits on a subset $I_1$ of levels on which it is more likely to be received at its intended receiver than it is to interfere with user 2, i.e.,

$$I_1 = \{ n \in [1, q] : Pr(N_{11} \geq n) \geq Pr(N_{21} \geq n, N_{21} - N_{22} < n) \}.$$ 

Furthermore, user 1 transmits at level $n$ (independent coding across levels) using a codebook of rate $Pr(N_{11} \geq n)$ for $n \in I_1$ and does not transmit on the remaining levels such that

$$R_1 = \sum_{n \in I_1} Pr(N_{11} \geq n).$$  

(12) 

Since user 1 is transmitting at the erasure rate for any level $n \in I_1$, receiver 1 can decode the data of the first transmitter with asymptotically negligible error probability. The second user transmits a message encoded across all layers. This in turn allows receiver 2 to decode the message of the second user jointly across those layers that do not experience interference from the first user on average. Consider a level $n \in I_1$ at the first transmitter. This level interferes with the data of the second user at the second receiver when $N_{21} \geq n$ and $N_{21} - N_{22} < n$. Thus, all the levels of the first user interfere on an average with $E[\sum_{n \in I_1} 1_{N_{21} \geq n, N_{21} - N_{22} < n}] = \sum_{n \in I_1} Pr(N_{21} \geq n, N_{21} - N_{22} < n)$ bits. Hence, for reliable reception, transmitter 2 needs to transmit at an average rate

$$R_2 = E[N_{22}] - \sum_{n \in I_1} Pr(N_{21} \geq n, N_{21} - N_{22} < n)$$  

(13) 

bits/channel use across all levels. The sum-rate is then given by (11). 

**Lemma 1:** For every $n \in [1, q]$ let

$$A_1(n) = 1_{N_{21} \leq n} \cup (\min(N_{11}, N_{21} - N_{22}) \geq n)$$  

(14a) 

$$A_2(n) = 1_{N_{21} < n, N_{21} \geq n, N_{21} - N_{22} < n}$$  

(14b) 

s.t. $E[A_1(n)] - E[A_2(n)]^+ = E[A_1(n)].$  

(14c) 

Given (14), the sum-rate in (11) simplifies to

$$E[\min(N_{11} + N_{22} + (N_{11} - N_{21})^+, \max(N_{11}, N_{21}, N_{22}, N_{11} + N_{22} - N_{21})].$$  

(15) 

**Remark 1:** Choosing $N_{11}$ as deterministic is a sufficient condition for (14). 

**Proof:** Consider the $n^{th}$ term in the summation over $I_1$ in (11) in Theorem 8. Using the fact that for any two sets $A$ and $B$, $Pr(A \cup B) = Pr(A \setminus B) - Pr(B \setminus A)$, we have

$$Pr(N_{21} \geq n) - Pr(N_{21} \geq n, N_{21} - N_{22} < n) = \min(N_{11}, N_{21} - N_{22}) \geq n)$$  

(16) 

Substituting (14) into (16), every term within the summation in (16) simplifies as

$$Pr(N_{21} < n \leq N_{11} \cup \min(N_{11}, N_{21} - N_{22}) \geq n)$$  

$$= Pr(N_{21} < n \leq N_{11}) + Pr(\min(N_{11}, N_{21} - N_{22}) \geq n)$$  

(17) 

Summing over all $n \in [1, q]$ and adding $E[N_{22}]$, the sum-rate in (11) then simplifies as

$$E[N_{22}] + E[(N_{11} - N_{21})^+]$$  

(18) 

$$+ E[\min(N_{11}, (N_{21} - N_{22})^+)]$$  

(19) 

$$= E[\max(N_{11} - N_{21}, 0)]$$  

$$+ E[\min(N_{11} + N_{22}, \max(N_{21}, N_{22})]$$  

(20) 

$$= E[\min(N_{11} + N_{22} + (N_{11} - N_{21})^+, \max(N_{11}, N_{21}, N_{22}, N_{11} + N_{22} - N_{21})].$$  

(21) 

**Theorem 9:** The sum capacity of a class of mixed layered erasure IFCs for which the condition (14) of Lemma 1 is satisfied and $N_{21} \leq N_{11} + N_{22}$ with probability 1 is given by

$$E[\max(N_{11}, N_{21}, N_{22}, N_{11} + N_{22} - N_{21})].$$  

(22) 

**Example 2:** (Ergodic Very Strong) Consider a layered IFC with $q = 4$ and two fading states: the first state with $N_{11} = 2$, $N_{21} = 1$ and $N_{22} = 4$ occurs with probability $1/2$, and the
second state with $N_{11} = 1$, $N_{21} = 4$ and $N_{22} = 1$ with probability $1/2$. The first state is weak while the second is very strong, but overall the net mixture is ergodic very strong. Thus, the sum capacity of 4 bits/channel use can be attained. We now present two examples for the mixed IFC. For the first, the sum capacity is given by Theorem 9; for the second, we present a new sum capacity achieving strategy.

**Example 3:** (Mixed) Consider a layered IFC with $q = 4$ and two fading states: the first state with $N_{11} = 2$, $N_{21} = 1$ and $N_{22} = 2$ occurs with probability $1/2$, and the second state with $N_{11} = 3$, $N_{21} = 4$ and $N_{22} = 1$ with probability $1/2$. The first state is very strong while the second is strong, but overall the net mixture satisfies all the conditions in Theorem 9. (Note that although $N_{11}$ is not deterministic, the condition in Lemma 1 is satisfied.) Thus, the ergodic sum capacity of $7/2$ bits/channel use can be attained.

**Fig. 2.** The layered IFC in Example 4 in each of the two states

**Example 4:** (Mixed) Consider a layered IFC with $q = 4$ and two fading states: the first state with $N_{11} = 2$, $N_{21} = 4$ and $N_{22} = 1$ occurs with probability $1/2$, and the second state with $N_{11} = 2$, $N_{21} = 1$ and $N_{22} = 2$ with probability $1/2$. The first state is very strong while the second is weak though the IFC is not ergodic very strong. The states satisfy the condition in Lemma 1, and thus, the sum rate of $5/2$ bits/channel use can be achieved. However, applying Theorem 2 the outer bound on sum capacity is 3 bits/channel use. We here present an alternate achievable strategy that achieves this outer bound. At its second level, the first transmitter sends a message at a rate of 1 bit/channel use which its intended receiver can always decode but the second receiver cannot. Suppose receiver 2 does not decode this second level in either channel state. Thus, with respect to receiver 2, the equivalent channel has two fading states: the first state $N_{11} = 1$, $N_{21} = 3$ and $N_{22} = 1$ with probability $1/2$, and the second state $N_{11} = 1$, $N_{21} = 1$ and $N_{22} = 1$ with probability $1/2$. This is an ergodic strong IFC and hence a sum capacity of 2 bits/channel use can be achieved. Combining that with the rate sent to receiver 1 from the second level of transmitter 1, we achieve a sum capacity of 3. Note that our strategy uses a public and a private message from the first transmitter at the first and second levels, respectively. Thus, while the second level from the first transmitter is received at the second receiver half of the time, the message on this level is considered private from the second user. This is in contrast with the deterministic interference channel where the message reaching the other receiver is always public.

**V. CONCLUDING REMARKS**

We have developed inner and outer bounds on the sum capacity of a class of layered erasure ergodic fading IFCs. We have shown that the outer bounds are tight for the following sub-classes: i) weak, ii) strong, iii) ergodic very strong (mix of strong and weak), and (iv) a sub-class of mixed interference (mix of SnVS and weak), where each sub-class is uniquely defined by the fading statistics. Our work demonstrates that for layered erasure IFCs with sub-channels that are not uniquely of one kind, i.e., that are not all strong but not very strong or very strong or weak, joint encoding is required across layers. Of immediate interest is extending these results to the ergodic fading Gaussian IFCs without transmitter CSI. Furthermore, we are also exploring extending the results of Theorem 9 to both general layered IFCs as well as ergodic fading Gaussian IFCs.

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