THE SHORT-RANGE BARYON-BARYON INTERACTION IN
A CHIRAL CONSTITUENT QUARK MODEL

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Abstract

The previous analysis of the short-range NN repulsion originating from the Goldstone boson exchange hyperfine interaction between constituent quarks is revisited. We study in which respects the repulsion depends on the radial form of the spin-spin quark-quark force. We show that while the radial form affects the structure of the 6Q wave function, the short-range repulsion in the NN system persists in any case. We extend our analysis to other YN and YY (flavor octet-octet) systems and demonstrate that the flavor-spin hyperfine interaction implies a short-range repulsion in these B=2 systems as well.

I. INTRODUCTION

An intricate question of the intermediate energy nuclear physics is about the origin of the short-range repulsion in the nucleon-nucleon (NN) system or, more generally, in baryon-baryon systems. By now it is clear that the mechanism describing the NN interaction
should be related with the QCD dynamics responsible for the low-energy properties of the nucleon. It is also now evident that the most important QCD phenomenon in this case is the spontaneous breaking of chiral symmetry, which implies that at momenta below the chiral symmetry breaking scale the relevant quasiparticle degrees of freedom are constituent quarks and pseudoscalar mesons, which are Goldstone bosons of the broken chiral symmetry. Assuming that in this regime the dominant interaction between confined constituent quarks is due to Goldstone boson exchange (GBE) one can understand the structure of the whole low-lying baryon spectrum [1,2]. A GBE interaction between constituent quarks is a natural interpretation of the t-channel iterations of point-like gluonic interactions between quarks which are responsible for dynamical breaking of chiral symmetry in QCD vacuum [3].

In a previous work [4] it was shown that the short-range part of the flavor-dependent spin-spin force between constituent quarks, which is due to GBE, and which is reinforced by the short-range part of vector-meson exchange [5] (correlated two-GBE [7]), induces a strong short-range repulsion in the $NN$ system. The same interaction also implies a strong short-range repulsion in the $6Q$ system with “$H$-particle” quantum numbers [5], suggesting that a very existence of a deeply bound $H$-particle is impossible within the given picture. In the present work we extend our analysis to other flavor octet-octet $B=2$ systems and show that the short-range repulsion persists as a general case.

In section II we revisit the $NN$ interaction and discuss in which respects the predictions of [4] depend on the radial form of the short-range flavor-spin interaction. Section III is devoted to the classification of $6Q$ states relevant for the hyperon-nucleon ($YN$) and hyperon-hyperon ($YY$) systems and to a qualitative estimate of the short-range repulsion appearing in these systems.
II. THE NN INTERACTION AT SHORT RANGE - REVISITED

The results of Ref. [4] are based on the flavor-spin hyperfine interaction between two constituent quarks $i$ and $j$ which has a short-range part of the form

$$-\lambda^F_i \cdot \lambda^F_j \vec{\sigma}_i \cdot \vec{\sigma}_j,$$

(1)

where $\lambda^F$ are the flavor Gell-Mann matrices and an implicit summation over $F = 1, 2, \ldots, 8$ is understood. The operator (1) represents the short-range part of the Goldstone boson exchange interaction [1]. Two correlated Goldstone bosons (vector meson exchange) enhance the effect of the short-range part of the one-boson exchange interaction, as shown in [3,7], so it can be incorporated in (1) as well. In Ref. [4] it was shown that such a flavor-spin interaction leads to a short-range repulsion in the $NN$ interaction when the latter is treated as a 6Q system. This repulsion results from the fact that the energy of the most favourable compact 6Q configuration, $s^4p^2|42\rangle_O|51\rangle_{FS}$, is much above the two-nucleon threshold. Here and below $[f]_O$, $[f]_{FS}$, and $[f]_F$ denote Young diagrams specifying the permutational orbital (O), $SU(6)_{FS}$ (FS), and $SU(3)_F$ (F) symmetries and it is always assumed that the center-of-mass motion is removed from the shell model configurations. In [4] the nonrelativistic Hamiltonian with the parametrization of the hyperfine interaction of Ref. [8] has been used. That parametrization, being successful for baryon spectra, may lead, however, to some undesirable effects in the $NN$ system because it contains the shift parameter $r_0 = 0.43$ fm of the short-range hyperfine interaction from the origin:

$$4\pi \delta(\vec{r}_{ij}) \Rightarrow \frac{4}{\sqrt{\pi}} \alpha^3 \exp(-\alpha^2(r - r_0)^2).$$

(2)

This shift enhances the quark-quark matrix elements with $1p$ relative motion and affects the coupling between the symmetry states chosen for the diagonalization of the Hamiltonian. Here we try to find out to which extent the results of Ref. [4] are modified when one takes $r_0 = 0$. Actually the baryon spectra can be described as well without such a shift with a properly chosen parametrization [2].
To estimate the strength of the short-range $NN$ interaction we use the adiabatic (Born-Oppenheimer) approximation

$$V_{NN}(R) = \langle H \rangle_R - \langle H \rangle_{\infty},$$  \hspace{2cm} (3)

where $R$ is a generator coordinate, defined as the distance between two harmonic oscillator wells, each associated asymptotically to a $3Q$ cluster. In Eq. (3) $\langle H \rangle_R$ is the lowest eigenvalue resulting from the diagonalization of the Hamiltonian at fixed $R$. Presently we are interested in the effective potential at $R = 0$ only. In this case in the S-wave relative motion only the $s^6[6]_O$ and $s^4p^2[42]_O$ shell-model configurations are allowed [4]. So to estimate the strength of the repulsion we diagonalize the Hamiltonian in the basis of the following four most important configurations [4]:

$$|1> = |s^6[6]_O[33]_{FS} >$$

$$|2> = |s^4p^2[42]_O[33]_{FS} >$$

$$|3> = |s^4p^2[42]_O[51]_{FS} >$$

$$|4> = |s^4p^2[42]_O[411]_{FS} >$$ \hspace{2cm} (4)

The Hamiltonian and the confinement potential are taken from the ref. [8]. Here we modify the radial dependence (see eqs. (17)-(18) of ref. [4]) of the meson-exchange potential as follows:

(i) we drop the long-range Yukawa part $\mu^2 \gamma \exp(-\mu \gamma r_{ij}) / r_{ij}$ of the potential, as it contributes very little at short-range in the 6Q system;

(ii) in the short-range part of the hyperfine interaction we put $r_0 = 0$;

(iii) we keep the ratio of the singlet ($g_0$) to octet ($g_8$) coupling constants as in Refs. [4,8] but readjust the absolute value of each coupling constant in order to reproduce the $\Delta - N$ mass splitting keeping in mind the modifications (i) and (ii).

The root-mean-square matter radius $\beta$ of the $s^3$ nucleon and $\Delta$, which coincides with the harmonic oscillator parameter in the 6Q basis, is obtained from the nucleon stability condition.
\[
\frac{\partial}{\partial \beta} \langle N | H | N \rangle = 0.
\]  

(5)

This procedure gives \( g_\beta^2 / 4\pi = 2.11 \), which should be regarded as an effective coupling constant simulating the combined effect of both the pseudoscalar- and the vector-meson-like exchange short-range interaction \( \Pi \) with a nonrelativistic \( s^3 \) ansatz for the baryon. The minimum of \( m_N = \langle N | H | N \rangle = 1.324 \) GeV is achieved at \( \beta = 0.373 \) fm. The absolute value of the “nucleon mass” \( m_N \) is unimportant in the present context as it identically cancels out in (3). The calculated \( m_N \) can also be shifted to a physical value by simply adding a constant contribution to the effective confining interaction.

The results of the diagonalization for the \( ^3S_1 \) and \( ^1S_0 \) NN partial waves are shown in Tables I and II, which should be compared with Tables II and III of Ref. [4] or more precisely to Tables V and VI of Ref. [10] because the latter tables show the diagonalization results of a \( 4 \times 4 \) matrix, as here. In Ref. [10] the 5th basis vector used in [4] has also been removed because it has no correspondence in the molecular orbital basis employed there.

We see that with the present parametrization of the short-range QQ interaction the \( s^4p^2[51]_F \) configuration is again the lowest one with an energy of roughly 1 GeV below the energy of the orbitally unexcited configuration \( s^6[33]_F \). In this sense the conclusion of Ref. [4] is reconfirmed. The lowest eigenvalue is about 1.4 GeV above the \( 2m_N \) threshold, which shows that the strong short-range repulsion in the \( NN \) system persists with this new parametrization. However, the numerical values of the off-diagonal matrix elements are now different compared to those obtained with the parametrization of Refs. [4] or [10]. As a consequence the amplitude of the configuration \( s^4p^2[51]_F \) is somewhat smaller and one finds more mixing among the configurations \( |1\rangle - |4\rangle \) in the lowest state eigenvector, in contrast to Refs. [4] or [10]. But the configuration \( s^4p^2[51]_F \) remains dominant in the lowest state and it can induce additional effective repulsion as discussed in [4] (see also [11,12]). While the mixing does not affect the conclusion about the repulsive core, it is important for the behaviour of the 6Q wave function at short range. When one projects this wave function on the \( NN \) channel according to the procedure described in [4] the node, predicted there
(see Fig. 1), nearly disappears. Now the behaviour of the projection is similar to the $NN$ wave function obtained with usual repulsive core potentials. This behaviour comes from the destructive interference of the excited $s^4p^2$ and nonexcited $s^6$ configurations. Such a destructive interference has been observed earlier in other microscopical models [13,14] and can be considered as a substantiation of the repulsive core in $NN$ potentials. However, contrary to any simple $NN$ potential model, the 6Q wave function is much richer and contains not only the $NN$ component, but also a variety of other components such as $NN^*, N^*N^*, \Delta\Delta, \Delta\Delta^*, \ldots$ [15].

Next we adress the question to which extent the height of the repulsion in the $NN$ system is sensitive to the contribution of the flavor-singlet $\eta'$-like exchange. The $\eta'$-like exchange tends to decrease the $\Delta - N$ splitting, while the $\pi$-like exchange works just in opposite direction. It means that in reproducing the $\Delta - N$ splitting the octet coupling constant will become smaller once the $\eta'$-exchange interaction is dropped. We take the extreme limit $g^2_0/4\pi = 0$ and repeat the steps (i)-(iii) from above. One obtains $\beta = 0.522$ fm, $g^2_8/4\pi = 1.29$ and $m_N = 1.4657$ GeV. The results for the $NN$ system are given in Tables III and IV. Comparing them with those of Tables I and II we conclude that the height of the repulsive core is essentially smaller than before and the structure of the ground state eigenvector is changed. In the present case the configurations $s^6[33]_{FS}$ and $s^4p^2[51]_{FS}$ become approximately degenerate and still about 900 MeV above the $2m_N$ threshold. The lowest eigenvalue is about 600 MeV above the threshold, showing that the strength of the repulsion is reduced compared to the previous case, but still important.

The calculation of the $NN$ interaction at short range within the adiabatic approximation above should be taken with some caution, however. In fact it represents only the diagonal kernel of a dynamical treatment such as the resonating group method (RGM). So an ultimate conclusion could only be drawn from the behaviour of the $NN$ phase shifts calculated beyond the adiabatic approximation. Such phase shifts, obtained within an extended resonating group method, do indicate the presence of a very strong repulsion even in the case without any $\eta'$-exchange interaction [16].
III. SHORT-RANGE 6Q CONFIGURATIONS IN HYPERON-NUCLEON AND HYPERON-HYPERON SYSTEMS

In this section we discuss the issue whether or not the short-range repulsion in the \( NN \) system, implied by the flavor-spin hyperfine interaction \( (1) \), persists in other baryon-baryon (flavor octet-octet) systems. We first construct the lowest possible symmetry states \( [f]_O[f]_F[f]_{FS} \) compatible with the asymptotic two-baryon channels. For simplicity we consider the \( SU(3)_F \) limit (see Eq. (6) below).

Assuming that the orbital wave function of any of the octet baryons is described by the \([3]_O\) permutational symmetry, only two states \([6]_O\) and \([42]_O\) are allowed in the S-wave relative motion of a two-baryon system \( [9,11] \). Applying the inner product rules of the symmetric group both for \([f]_O \times [f]_C\), where \([f]_C = [222]_C\) is fixed by the color-singlet nature of the two-baryon system, and \([f]_F \times [f]_S\), where \([f]_S\) is fixed by the total spin \( S \) of the 6Q system, one arrives at the symmetry states listed in Tables V and VI. We recall that for a given \([f]_S\) of \( SU(2) \) where \( f_1 \) and \( f_2 \) represent the number of boxes in the first and second rows of the Young diagram \([f]_S\) respectively, with \( f_1 + f_2 = 6 \) in the present case, the spin is given by \( S = 1/2(f_1 - f_2) \).

When one considers a schematic model \( [3] \), where the interaction Hamiltonian is approximated as

\[
H_\chi = -C_\chi \sum_{i<j} \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j,
\]

and the constant \( C_\chi = 29.3 \text{ MeV} \) is extracted from the phenomenological \( \Delta - N \) splitting, then the expectation value of the interaction \( (6) \) for all symmetry states in Tables V and VI can be evaluated through the Casimir operator eigenvalues, see Appendix A of ref. \( [4] \). The results are given in Tables V and VI in units of \( C_\chi \). One can immediately conclude from these Tables that far the lowest state is \( |s^4p^2[42]_O[321]_F[51]_{FS} > \). This state is not allowed in the \( NN \) system. Here it appears due to the presence of three distinct flavors \( u, d \) and \( s \). Its contribution is lower than that of the vector \( |3 > \) of the list \( (3) \) by about 10 \( C_\chi \) units (recall
that in the $NN$ case the flavor symmetry is $[42]_F$ for $S = 0$ S-wave and $[33]_F$ for $S = 1$ S-wave). By itself the $|s^4p^2[42]_O[321]_F[51]_{FS}>$ state guarantees that a strong effective repulsion will persist in the $YN$ and $YY$ systems, again related to a specific symmetry structure of the type $s^4p^2[51]_{FS}$. The ground state configuration $s^6[6]_O$ becomes even more “forbidden” by dynamics than in the $NN$ system (i.e. the weight of $s^6[6]_O$ should be expected to become smaller). This effect, however, cannot be obtained within the simple approximation considered below where only the expectation value of the $|s^4p^2[42]_O[321]_F[51]_{FS}>$ state is calculated.

To have a rough qualitative idea about the strength of the interaction at short range in $YY$ and $YN$ systems we thus calculate the diagonal matrix element

$$<s^4p^2[42]_O[321]_F[51]_{FS}|H_0 + H_{conf} + H_\chi |s^4p^2[42]_O[321]_F[51]_{FS}>$$

and compare it with the two-baryon threshold. With the coupling constant $g_\Lambda^2/4\pi = 2.11$, and the ratio $g_\chi^2/g_\Lambda^2$, fixed in the previous section, we now use a harmonic oscillator parameter $\beta = 0.403$ fm, which provides an equilibrium value for $\Lambda$, as it follows from Table VII.

In all cases we describe the kinetic energy of a $6Q$ system in a simple way

$$<s^4p^2|H_0|s^4p^2> = \frac{19}{4}\hbar\omega,$$  

$$\hbar\omega = \frac{\hbar^2}{m_{ave}\beta^2}. \tag{9}$$

where $m_{ave}$ is an average quark mass defined for each system and the center-of-mass motion is removed. For example the $\Lambda\Lambda$ system has an average mass $m_{ave} = (4m + 2m_s)/6$, where $m = 0.340$ GeV and $m_s = 0.440$ GeV.

All the contributions from $H_\chi$ and $H_{conf}$ are calculated with the help of the fractional parentage technique, similar to [4,5] and described in detail in Ref. [17]. The $SU(3)$ Clebsch-Gordan coefficients for the flavor part of the wave function are taken from Ref. [18]. By this technique one can reduce the six-quark matrix elements to linear combinations of two-quark
matrix elements which allow immediate integration in the spin-flavor space by use of Eq. (3.3) of Ref. [1]. In particular, for the confinement part $H_{\text{conf}}$ the orbital matrix elements can be easily calculated analytically. This gives

$$\langle H_{\text{conf}} \rangle = \frac{71}{6} \sqrt{\frac{2}{\pi}} C \beta.$$  \hspace{1cm} (10)

where $C$ is the string tension taken from [8] and $\beta$ has been specified above. In an analogue way the confinement energy of a ground state baryon is

$$\langle H_{\text{conf}} \rangle = 6 \sqrt{\frac{2}{\pi}} C \beta.$$  \hspace{1cm} (11)

Thus the difference between the $6Q$ and two times the $3Q$ confinement energy is $-\sqrt{\frac{2}{\pi}} \frac{C \beta}{6}$. This gives about $-5$ MeV, which proves that the confinement contribution nearly cancels out in the baryon-baryon potential in the present approximation.

Results for $\langle H_0 \rangle$, $\langle H_\chi \rangle$ and $\langle H \rangle$ are exhibited in Tables VIII and IX for $S = 0$ and $S = 1$ repectively. We define the separation energy (last column) of either table as the difference between $\langle H \rangle$ and the lowest threshold two-baryon mass, calculated with the same hamiltonian, associated to a given $YI$. The lowest thresholds are $\Lambda \Lambda$, $\Sigma \Sigma$, $N \Lambda$ and $\Lambda \Xi$ for $YI = 00$, $01$, $1 \ 1/2$ and $-1 \ 1/2$ respectively. The separation energy is roughly interpreted as the value taken by the baryon-baryon interaction potential at zero separation distance in the Born-Oppenheimer approximation. The last column of Tables VIII and IX indicates repulsion of the order of 1 GeV in all cases. When one uses a Hamiltonian without $\eta'$-exchange, like at the end of the above section, then the repulsion is weakened by roughly 200 - 300 MeV.

**IV. CONCLUSIONS**

This paper closes a series of papers [2][3] devoted to a qualitative study of the short-range baryon-baryon interactions within a chiral constituent quark model relying on meson-exchange dynamics. Our results indicate that the short-range flavor-spin interaction (1)
between constituent quarks implies a strong short-range repulsion in NN and other flavor octet-octet systems. While we observe a strong repulsive core we cannot insist on numerical values for this repulsion because of the use of simplified approximations. The next stage of the study should invoke more detailed and refined dynamical approximations as well as the incorporation of multiple correlated GBE interactions (scalar and vector meson exchanges) in order to provide a realistic description of the middle- and long-range physics in baryon-baryon systems with all the necessary components, like tensor and spin-orbit forces.

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TABLES

TABLE I. Results of the diagonalization for IS = (01). Column 1 - basis states, column 2 - diagonal matrix elements (GeV), columns 3 - eigenvalues (GeV) in increasing order for a 4 x 4 matrix, column 4 - components of the lowest state. The results correspond to $\beta = 0.373$ fm. The diagonal matrix elements and the eigenvalues are relative to $2 m_N = 2.648$ GeV.

| State                  | Diag. elem - $2 m_N$ | Eigenvalues - $2 m_N$ | Lowest state amplitudes |
|------------------------|----------------------|-----------------------|--------------------------|
| $|s^6[6]O[33]_{FS}>$     | 2.461                | 1.403                 | 0.32885                  |
| $|s^4p^2[42]O[33]_{FS}>$| 3.119                | 1.914                 | -0.25317                 |
| $|s^4p^2[42]O[51]_{FS}>$| 1.513                | 3.192                 | 0.90351                  |
| $|s^4p^2[42]O[411]_{FS}>$| 3.171                | 3.755                 | -0.10691                 |

TABLE II. Results of the diagonalization for IS = (10). See legend of Table I.

| State                  | Diag. elem - $2 m_N$ | Eigenvalues - $2 m_N$ | Lowest state amplitudes |
|------------------------|----------------------|-----------------------|--------------------------|
| $|s^6[6]O[33]_{FS}>$     | 3.108                | 1.890                 | 0.32335                  |
| $|s^4p^2[42]O[33]_{FS}>$| 3.572                | 2.368                 | -0.27724                 |
| $|s^4p^2[42]O[51]_{FS}>$| 2.009                | 3.778                 | 0.89853                  |
| $|s^4p^2[42]O[411]_{FS}>$| 3.743                | 4.397                 | 0.10896                  |
TABLE III. Results of the diagonalization for IS = (01) without $\eta'$-like exchange. Column 1 - basis states, column 2 - diagonal matrix elements (GeV), columns 3 - eigenvalues (GeV) in increasing order for a $4 \times 4$ matrix, column 4 - components of the lowest state. The results correspond to $\beta = 0.522$ fm. The diagonal matrix elements and the eigenvalues are relative to $2m_N=2.9314$ GeV.

| State                  | Diag. elem - $2m_N$ | Eigenvalues - $2m_N$ | Lowest state amplitudes |
|------------------------|----------------------|-----------------------|--------------------------|
| $|s^6[6]O[33]_{FS}>$    | 0.873                | 0.565                 | 0.72185                  |
| $|s^4p^2[42]O[33]_{FS}>$| 1.326                | 1.032                 | -0.43487                 |
| $|s^4p^2[42]O[51]_{FS}>$| 0.912                | 1.416                 | 0.53626                  |
| $|s^4p^2[42]O[411]_{FS}>$| 1.410                | 1.509                 | -0.04737                 |

TABLE IV. Results of the diagonalization for IS = (10) without $\eta'$-like exchange. See legend to the Table III.

| State                  | Diag. elem - $2m_N$ | Eigenvalues - $2m_N$ | Lowest state amplitudes |
|------------------------|----------------------|-----------------------|--------------------------|
| $|s^6[6]O[33]_{FS}>$    | 0.913                | 0.594                 | -0.71686                 |
| $|s^4p^2[42]O[33]_{FS}>$| 1.352                | 1.056                 | 0.44305                  |
| $|s^4p^2[42]O[51]_{FS}>$| 0.942                | 1.4566                | -0.53765                 |
| $|s^4p^2[42]O[411]_{FS}>$| 1.445                | 1.545                 | 0.02746                  |
TABLE V. Expectation value \(\langle H_\chi \rangle\) of the operator (1) for the lowest symmetry states \(|f\rangle |f\rangle_F |f\rangle_{FS}\) with \(S = 0\) compatible with asymptotic two baryon channels.

| State | \(\langle V_\chi \rangle/C_\chi\) |
|-------|-----------------|
| \(|6\rangle_O [222]_F [33]_{FS}\) | -24 |
| \(|42\rangle_O [321]_F [51]_{FS}\) | -42 |
| \(|42\rangle_O [42]_F [51]_{FS}\) | -32 |
| \(|42\rangle_O [222]_F [33]_{FS}\) | -24 |
| \(|42\rangle_O [321]_F [33]_{FS}\) | -18 |
| \(|42\rangle_O [42]_F [33]_{FS}\) | -8 |

TABLE VI. Same as Table V but for spin \(S = 1\).

| State | \(\langle V_\chi \rangle/C_\chi\) |
|-------|-----------------|
| \(|6\rangle_O [321]_F [33]_{FS}\) | -46/3 |
| \(|6\rangle_O [33]_F [33]_{FS}\) | -28/3 |
| \(|6\rangle_O [411]_F [33]_{FS}\) | -28/3 |
| \(|42\rangle_O [321]_F [51]_{FS}\) | -118/3 |
| \(|42\rangle_O [33]_F [51]_{FS}\) | -100/3 |
| \(|42\rangle_O [411]_F [51]_{FS}\) | -100/3 |
| \(|42\rangle_O [42]_F [51]_{FS}\) | -88/3 |
TABLE VII. Variational solution for low lying $S = 1/2$ baryons as compared to the experimental masses.

| Baryon | Variational parameter $\beta$ (fm) | Variational solution (GeV) | Experimental mass (GeV) |
|--------|----------------------------------|---------------------------|------------------------|
| $N$    | 0.373                            | 1.324                     | 0.940                  |
| $\Lambda$ | 0.403                           | 1.4702                    | 1.1156                 |
| $\Sigma$ | 0.433                            | 1.5050                    | 1.1930                 |
| $\Xi$  | 0.457                            | 1.6120                    | 1.3181                 |

TABLE VIII. Expectation value (7) of the $6Q$ hamiltonian for the lowest state $|42\rangle_{\lambda}[321]_{F}[51]_{FS}$ with $S = 0$. The harmonic oscillator parameter is chosen equal to that of $\Lambda$ from Table 1. Column 1 gives the quantum numbers $YI$ compatible with $[321]_{F}$ of $SU(3)$ and the last column the separation energy above the lowest the threshold as implied by Table I.

| $YI$ | System | $\langle H_0 \rangle$ (GeV) | $\langle H_\chi \rangle$ (GeV) | $\langle H \rangle$ (GeV) | Separation energy (GeV) |
|------|--------|-----------------------------|-------------------------------|-----------------------------|-------------------------|
| 00   | $\Lambda\Lambda, \Sigma\Sigma, N\Xi$ | 3.0536                      | -1.6056                      | 4.0424                     | 1.102                   |
| 01   | $\Sigma\Sigma, N\Xi$           | 3.0536                      | -1.4163                      | 4.2317                     | 1.222                   |
| $1\frac{1}{2}$ | $N\Lambda, N\Sigma$       | 3.1963                      | -2.0200                      | 3.6706                     | 0.876                   |
| $-1\frac{1}{2}$ | $\Lambda\Xi$         | 2.9231                      | -1.1647                      | 4.4528                     | 1.371                   |
| $YI$ | System         | $\langle H_0 \rangle$ (GeV) | $\langle H_\chi \rangle$ (GeV) | $\langle H \rangle$ (GeV) | Separation energy (GeV) |
|------|----------------|-----------------------------|-------------------------------|---------------------------|-------------------------|
| 00   | $\Lambda\Lambda, \Sigma\Sigma, N\Xi$ | 3.0536                      | -1.7287                       | 3.9193                    | 0.979                   |
| 01   | $\Sigma\Sigma, N\Xi$               | 3.0536                      | -1.4263                       | 4.2217                    | 1.212                   |
| 1$\frac{1}{2}$ | $N\Lambda, N\Sigma$      | 3.1963                      | -2.0258                       | 3.6649                    | 0.871                   |
| $-1\frac{1}{2}$ | $\Lambda\Xi$             | 2.9231                      | -1.2964                       | 4.3212                    | 1.239                   |