Kaon and antikaon properties in cold nuclear medium

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Abstract. We present results of a self-consistent calculation for the kaon and antikaon spectral functions in cold nuclear matter, using as input the kaon-nucleon and antikaon-nucleon scattering amplitudes of the vacuum. We investigate the effect of in-medium pion dressing on the antikaon-nucleon scattering amplitudes and antikaon spectral function. We find the influence of pion dressing to be minor on the antikaon spectral function and limited on the hyperon resonances causing only a small additional broadening. An exception is the Σ(1690). At nuclear saturation density an attractive mass shift of about 20 MeV and width of about 130 MeV is obtained. The kaon shows a repulsive mass increase of 36 MeV and a small width of the quasiparticle peak at saturation density.

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1. Introduction

Interest in kaon properties in nuclear medium increased greatly after it was suggested [1] that antikaons may condense in the interior of neutron stars. Describing kaonic atoms [2] and strangeness production in heavy-ion reactions [3] also requires knowing the in-medium spectral function of kaons and antikaons.

The theoretical work on kaon properties in the nuclear medium in the last decade was extensive [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17]. For the antikaons (K$^-$ and K$^0$) the results confirmed a considerable softening expected on the basis of K-matrix analysis [18], although quantitative differences remain in the shape of the spectral function and its average shift towards smaller energy. The kaons (K$^+$ and K$^0$), on the other hand, show a positive mass shift with little
broadening of the spectral function.

2. Formalism

The computational scheme consists of a generalization of the self-consistent approach presented in ref. [16]. It uses the in-medium kaon-nucleon and antikaon-nucleon scattering amplitude, which take into account the medium’s influence on the particles’ propagation. This is achieved by solving the in-medium Bethe-Salpeter (BS) equation which contains the in-medium propagators, but is based on the vacuum BS kernel. Inclusion of relevant meson-hyperon channels leads to the matrix BS equation in vacuum, written in compact way as:

\[ T = K + K \cdot G \cdot T, \]

(1)

where \( T \) is the scattering amplitude, \( K \) the kernel and \( G \) denotes the two-particle propagator. In the medium the same equation takes the form:

\[ T = K + K \cdot \mathcal{G} \cdot T. \]

(2)

Under the assumption that the kernels are the same, \( K = K \), eq. (2) can be rewritten:

\[ T = T + T \cdot \Delta \mathcal{G} \cdot T, \quad \Delta \mathcal{G} \equiv \mathcal{G} - G. \]

(3)

More explicitly, the (pseudo)scalar-meson–spin-1/2 baryon on-shell scattering amplitude is written as

\[
\langle M(\bar{q}) B(\bar{p}) | T | M(q) B(p) \rangle = (2\pi)^4 \delta^4(q + p - \bar{q} - \bar{p}) \times \bar{u}_B(\bar{p}) T_{MB\to MB}(\bar{q}; \bar{p}; q, p) u_B(p),
\]

(4)

and satisfies the BS equation:

\[
T(k, k; w) = K(\bar{k}, k; w) + \int \frac{d^4 l}{(2\pi)^4} K(\bar{k}, l; w) G(l; w) T(l, k; w),
\]

\[
G(l; w) = -i S_N(\frac{1}{2} w + l) D_M(\frac{1}{2} w - l),
\]

\[
w = p + q = \bar{p} + \bar{q}, \quad k = \frac{1}{2} (p - q), \quad \bar{k} = \frac{1}{2} (\bar{p} - \bar{q}),
\]

(5)

containing the free space baryon propagator \( S_B(p) = 1/(\bar{p} - m_B + i \epsilon) \) and the meson propagator \( D_M(q) = 1/(q^2 - m_M^2 + i \epsilon) \). The in-medium eq. (2) has the same form, apart from the fact that the 4-point Green function \( T(\bar{k}, k; w, u) \) and the 2-particle propagator depend also on the 4-velocity \( u_\mu \) characterizing the nuclear matter frame. For nuclear matter moving with a velocity \( \bar{v} \):

\[
u_\mu = \left( \frac{1}{\sqrt{1 - \bar{v}^2/c^2}}, \frac{\bar{v}/c}{\sqrt{1 - \bar{v}^2/c^2}} \right), \quad u^2 = 1.
\]

(6)
For the calculation of kaon and antikaon self energies in the nuclear medium we need the kaon-nucleon and antikaon-nucleon in-medium scattering amplitudes (more precisely, the 4-point Green functions). In the case of antikaons a successful description of free-space scattering data requires the inclusion of various inelastic meson-baryon channels \( X = \pi \Lambda, \pi \Sigma, \eta \Lambda, \eta \Sigma, K \Xi \) with strangeness \(-1\) (see e.g. ref. [19]). As one sees from eq. (3) the need for the explicit inclusion of such channels for the in-medium computation depends on size of the modification of the meson-baryon propagator but also on the size of the relevant transition amplitude \( T_{\bar{K}N \rightarrow X} \). Note that the contribution of such channels involving the free propagators is already taken care of in \( T_{\bar{K}N \rightarrow \bar{K}N} \). Since we do not consider nuclear densities high enough for hyperon condensation, only meson propagator modification should be considered. It has been claimed in ref. [14] that the medium-modified pion influences significantly the antikaon properties. It is known that the pion spectral function changes considerably in the medium, however its influence on the antikaon-nucleon channel depends on the \( T_{\pi Y \rightarrow \bar{K}N} \) transition amplitudes. The higher mass of the \( \eta \)-meson leads to a smaller expected effect since already the influence of the \( \eta Y \) channels are of minor for the low-energy antikaons-nucleon scattering. Motivated by these considerations we do not include \( \eta Y \) channels in our calculation, dropping as well the kaon-cascade channels.

We work with definite isospin amplitudes and thus consider separately the \( I = 0 \) and \( I = 1 \) cases. We have three coupled channels for \( I = 1 \): \( \bar{K}N, \pi\Lambda(1116), \pi\Sigma(1195) \), and only two for \( I = 0 \): \( \bar{K}N \) and \( \pi\Sigma(1195) \). The meson-baryon propagator matrix is diagonal and for \( I = 1 \) has the form: \( \Delta G^{(0)} = \text{diag}(\Delta G_{\bar{K}N}, \Delta G_{\pi\Sigma}, \Delta G_{\pi\Lambda}) \), while for \( I = 0 \) only the first two entries are present. Since we consider only isospin-symmetric medium, the propagators in the two channels are identical. When solving the matrix BS equation we use the fact that the pion dressing is performed based on the pion-nucleon scattering amplitude, which means that it is independent of the antikaon dressing.

We illustrate the method on the case of the \( I = 0 \) channel. For simplicity we denote the channel index \( \bar{K}N \) by “1” and \( \pi\Sigma \) by “2”. Then writing out the [11] component of the matrix BS equation gives:

\[
\mathcal{T}_{11} = T_{11} + T_{11} \Delta G_1 \mathcal{T}_{11} + T_{12} \Delta G_2 \mathcal{T}_{21},
\]

(7)

where \( \Delta G_1 \) denotes \( \Delta G_{\bar{K}N} \) and \( \Delta G_2 \) means \( \Delta G_{\pi\Sigma} \). Similarly, the [21] component allows us to solve for the \( \mathcal{T}_{21} \) in terms of \( \mathcal{T}_{11} \):

\[
\mathcal{T}_{21} = (1 - T_{22} \Delta G_2)^{-1} \left[ T_{21} + T_{21} \Delta G_1 \mathcal{T}_{11} \right],
\]

(8)

Substituting eq. (8) into (7) we get:

\[
\mathcal{T}_{11} = \hat{T}_{11} + \hat{T}_{11} \Delta G_1 \mathcal{T}_{11},
\]

(9)

where we introduced

\[
\hat{T}_{11} \equiv T_{11} + T_{12} \Delta G_2 (1 - T_{22} \Delta G_2)^{-1} T_{21}.
\]

(10)
The form of eq. (9) is the same as for the case with no pion dressing, only the vacuum scattering amplitude $T_{11}$ has been replaced by the right-hand side of (10). An analogous, though somewhat more involved scheme can be used for the $I = 1$ channel. This means we have to calculate the effect of pion dressing only once and then perform the iterative, self-consistent calculation with antikaon dressing using the obtained input which, however, has the full in-medium structure (containing mixing of partial waves) and depends on the energy and magnitude of the 3-momentum.

3. Results

For the dressed pion propagator we use the results of a self-consistent calculation [20] using as an input the vacuum pion-nucleon scattering amplitudes, as well as the relativistic delta-hole model of ref. [21]. The main difference between the two results is that the second one provides somewhat more attraction, leading to a more pronounced softening of the pion spectrum in the medium. As explained in the previous section we first perform a calculation taking into account the pion dressing and then use that as input for the self-consistent computation of the antikaon spectral function. A hint for the importance of pion dressing is given already by these input amplitudes, i.e. their departure from the vacuum ones. In fig. 1 we show the amplitudes corresponding to the $\Lambda(1405)$ and the $\Sigma(1385)$ as obtained after including the pion dressing (gray lines) and both pion and antikaon dressing (black lines). The thin black lines show the vacuum amplitudes. For fig. 1 we used the pion spectral function as obtained in a relativistic delta-hole model [21], which produces slightly more pronounced effect than the pion spectral function from a self-consistent calculation of ref. [20]. As compared to our previous results [16] the $\Lambda(1405)$ resonance is basically unchanged only for the $\Sigma(1385)$ a somewhat larger attractive mass shift is obtained. Further results not shown in the figures concern the $\Sigma(1690)$ resonance for which no significant effect was found previously ref. [15]. At saturation density pion dressing shifts the resonance mass down by 20 MeV and leads to a sizeable increase of its decay width ($\approx 130$ MeV).

Based on the observation that the antikaon properties in the nuclear medium are determined mostly by the properties of the s-wave $\Lambda(1405)$ resonance and to a smaller effect by the p- and d-wave resonances, we expect the inclusion of pion dressing not to produce dramatic changes in the antikaon spectral function. These expectations are confirmed by the full self-consistent calculation, whose results for the antikaon spectral function are shown in fig. 2. We observe that the pion dressing has a small effect on the antikaon spectral function, limited to momenta below 400 MeV. We do not confirm the striking influence of the dressed pion on the antikaon in-medium spectrum obtained in ref. [14]. Possible reasons for this discrepancy can reside in the different pion spectral functions and transition amplitudes coupling the antikaon-nucleon channel to pion-hyperon channels. The pion spectral function [15] used in ref. [14] shows very strong softening (more than 100 MeV at 400 MeV momentum), which was not obtained in self-consistent schemes [22, 23, 20].
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Fig. 1. The reduced scattering amplitudes corresponding to the Λ(1405) and Σ(1385) at zero 3-momentum in nuclear medium of density \( \rho = 0.17 \text{fm}^{-3} \). Full lines show the imaginary part, dotted lines the real part. The black lines show the effect of both dressed pion and antikaon, the gray lines of dressed pion only. The thin black lines show the vacuum amplitudes for comparison.

and the relativistic delta-hole model [21].

We now turn to the kaon (K\(^+\), K\(0\)) properties in cold isospin-symmetric nuclear medium. The computational procedure is identical to the one used for the antikaons [16], the difference being only the input vacuum scattering amplitudes, which now refer to kaon-nucleon scattering. They are again taken from ref. [19], based on an extensive fit to experimental data. Figure 3 shows the kaon spectral function at saturation density (black lines) and twice the saturation density (gray lines), for momenta 0, 200 MeV, 400 MeV and 600 MeV. We do not get any structure in the spectral function, the medium effect shows up as a repulsive shift of the quasiparticle peak, which also acquires a finite width. The latter is quite small (less than 5 MeV) for momenta below 200 MeV at saturation density, but increases to 15 MeV at 400 MeV momentum. The positions of the peaks are a few MeV below the value given by the \( \sqrt{M^2 + \vec{p}^2} \) for momenta \(|\vec{p}| > 200\text{ MeV}\), where \( M \) is the peak’s position for \(|\vec{p}| = 0\).

In fig. 4 we show the kaon-mass shift as a function of nucleon density. The circles show the mass shift obtained with vacuum scattering amplitudes, while the squares correspond to the full self-consistent solution, based on the in-medium scattering amplitudes. The lines are drawn simply as smooth connections of points. The self-consistent mass shift exceeds the one based on the vacuum scattering amplitude, but shows signs of beginning saturation above 1.5 \( \rho_0 \), however the results should be regarded with caution for densities reaching 2 \( \rho_0 \), since neglected nucleon correlations may affect them.
Fig. 2. The antikaon spectral function for different momenta: 0 (solid line), 200 MeV (dotted line), 400 MeV (dashed line), and 600 MeV (dash-dot-dot line). Black lines show the result of both dressed pion and antikaon, gray lines previous results [16] without pion dressing, in the nuclear matter at saturation density.

4. Conclusions

We studied the properties of kaons and antikaons in cold nuclear medium by solving self-consistently the in-medium Bethe-Salpeter equation. For the antikaons we included the effect of the dressed pion in the coupled pion-hyperon channels. The pion dressing did not change the in-medium properties of hyperons and antikaons in a significant way, introducing in general slightly more broadening for the hyperons and smoothing somewhat the antikaon spectral function at small momenta. The only exception is the Σ(1690) which was affected very little by the dressing of the antikaon, but now suffers a shift of −20 MeV and significant broadening, its width increasing to about 130 MeV at zero momentum. The minor effect of the pion dressing is in contrast with the result of ref. [14] where a significant change of the antikaon spectrum was reported, when using a pion spectral function with a large shift of the main maximum towards smaller energy.

For the kaons (K⁺, K⁰) we obtain a repulsive mass shift of 36 MeV at saturation density and only a modest broadening of the quasiparticle peak. The position of the peak is a few MeV below the value of √√M^2 + |p|^2 for momenta |p| > 200 MeV, where M^* is the position for |p| = 0. The self-consistently obtained mass shift is larger than the one obtained from vacuum scattering amplitude, the difference increasing from 11 MeV at saturation density to 22 MeV at twice the saturation density.
Fig. 3. The kaon spectral function for different momenta: 0 (solid line), 200 MeV (dotted line), 400 MeV (dashed line), and 600 MeV (dash-dot-dot line). Black lines show the result at saturation density, $\rho = 0.17 \, \text{fm}^{-3}$, gray lines at twice saturation density, $\rho = 0.34 \, \text{fm}^{-3}$.

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Fig. 4. The kaon mass shift as function of the nucleon density, with $\rho_0$ denoting the saturation density. The circles show the results based on the vacuum kaon-nucleon scattering amplitude, while the squares correspond to the self-consistent result. The lines smoothly connect the points.

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