Determining the CKM Parameter $V_{cd}$ from $\nu N$ Charm Production

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The formalism for extracting the CKM parameter $V_{cd}$ from $\nu N$ production of charm is discussed in some detail. The various model assumptions needed are clearly pointed out. A direct determination from neutrino induced dimuon production requires $\nu N$ charm production data, $\nu N$ charm hadronization data, and the semi-muonic branching ratios for charmed hadrons. Hadronization data from FNAL E531 is re-analyzed to take advantage of better-determined properties of the charmed hadrons. A small bias in the original published result is removed. Neutrino induced charm fragmentation is compared to $e^+ e^-$ charm fragmentation functions; the data are consistent with a common hadronization scheme. An updated value of the mean semi-muonic branching ratio for charmed hadrons produced in $\nu N$ scattering for $E_n > 30$ GeV is obtained. This value is used to determine $V_{cd}$ and its associated uncertainties. Prospects for improving the $V_{cd}$ measurement to test the unitarity limit of the CKM matrix are described.

I. INTRODUCTION

The CKM parameters $|V_{cd}|$ and $|V_{us}|$ can presently only be measured via the neutrino production of charm at high energies. This paper summarizes the current state of knowledge of these CKM parameters and estimates the possible precision that will be achieved in future experiments. Much of the material has appeared previously in unpublished form [1], and a condensed summary of this article will appear in a forthcoming review [2]. This document is intended to update the previous result and to provide more details for specialists. The timing is propitious as NuTeV, Nomad, and Chorus should shortly produce new experimental results.

In the standard three generation CKM matrix, unitarity and the precise determinations of $|V_{ud}|$ and $|V_{us}|$ tightly constrain $|V_{cd}|$ and $|V_{cs}|$. This can be easily appreciated in the Wolfenstein parameterization [3] of the CKM matrix, in which the mixing between different generations is parameterized as $\sin \theta_{12} = \lambda$, $\sin \theta_{23} = \lambda^2$, and $\sin \theta_{13} e^{i \delta} = \lambda^3 (\rho + i \eta)$. In this scheme

$$\frac{|V_{cd}|}{|V_{us}|} = 1 + A^2 \lambda^4 (\rho - \frac{1}{2}) \simeq 1 + (2.4 \times 10^{-3}) A^2 (\rho - \frac{1}{2}),$$

and

$$\frac{|V_{cs}|}{|V_{ud}|} = 1 - \frac{1}{2} A^2 \lambda^4 \simeq 1 - (1.2 \times 10^{-3}) A^2.$$  

Since $A$ and $\rho$ are known to be of order one from measurements of $V_{cb}$ and $V_{ub}$ at CLEO and Argus, $|V_{cd}|$ and $|V_{cs}|$ must be within a few parts per thousand of $|V_{ud}|$ and $|V_{us}|$.

On the other hand, if three generation unitarity is not assumed the coupling of $|V_{cd}|$ to $|V_{us}|$ is not as tight. For example, in a four generation unitary CKM matrix, mixing between the second and fourth generation ($\sin \theta_{24}$) could allow $|V_{us}| - |V_{cd}| \leq 0.03$ while maintaining $|V_{ud}|^2 + |V_{us}|^2 = 1$. In summary, the standard model predicts $|V_{cd}|$ and $|V_{cs}|$ to a level of $\pm 0.1\%$. Any larger deviation would indicate new physics, and deviations as large as 10% are interesting.

Section 2 introduces the necessary formalism and points out assumptions that need to be made in the analysis of neutrino experiments and external input that is necessary. Section 3 summarizes the present state of knowledge of $|V_{cd}|$ and $|V_{cs}|$ and the auxiliary quantities that are needed to extract these parameters. Section 4 considers techniques of improving the measurements and possible systematic limitations; and estimates the sensitivity new experiments at Fermilab and CERN.
II. NEUTRINO PRODUCTION OF CHARM

A. Leading Order Quark-Parton Formalism

At a fixed reference 4-momentum transfer $Q_0^2$, the isoscalar cross section for the neutrino production of charm can be written to lowest order in QCD as

$$\frac{d\sigma(\nu N \rightarrow \mu^- e X)}{dxdy} = \frac{G_F^2 ME}{\pi} \times \left(1 - \frac{m_v^2}{2ME}\right) \cdot \Theta(y - \frac{m_v^2}{2ME}) \cdot \Theta(\xi - \frac{m_v^2}{2ME}) \cdot \Theta \left[2MEy(1-x) + M^2 - M_C^2\right] \times \left\{ |V_{cd}|^2 \left[u_v(\xi, Q_0^2) + d_v(\xi, Q_0^2) + u_S(\xi, Q_0^2) + d_S(\xi, Q_0^2)\right] + |V_{cs}|^2 2s(\xi, Q_0^2) \right\}$$

and for anti-neutrinos as

$$\frac{d\sigma(\bar{\nu} N \rightarrow \mu^+ e X)}{dxdy} = \frac{G_F^2 ME}{\pi} \times \left(1 - \frac{m_v^2}{2ME}\right) \cdot \Theta(y - \frac{m_v^2}{2ME}) \cdot \Theta(\xi - \frac{m_v^2}{2ME}) \cdot \Theta \left[2MEy(1-x) + M^2 - M_C^2\right] \times \left\{ |V_{cd}|^2 \left[\bar{u}(\xi, Q_0^2) + \bar{d}(\xi, Q_0^2)\right] + |V_{cs}|^2 2\bar{s}(\xi, Q_0^2) \right\}$$

where in the above two expressions:

- $G_F$ is the Fermi constant, $M$ is the nucleon mass and $E$ is the incident neutrino energy.
- $m_v$ is an effective charm mass parameter, and $M_C$ is the lowest mass charmed hadronic system allowed by conservation laws.
- $y$ is the fraction of neutrino energy transferred to the hadronic system, and $\xi$ is the fraction of the proton’s momentum carried by (massless) struck quark. In terms of the Bjorken scaling variable $x = \frac{Q_0^2}{2MEy}$, $\xi = \xi(x, Q_0^2) = x(1 + \frac{m^2}{Q_0^2})$.
- $\frac{1}{2} \left[u_v(\xi, Q_0^2) + d_v(\xi, Q_0^2)\right] = v(\xi, Q_0^2)$ is the valence quark momentum distribution of an isoscalar nucleon, and $\frac{1}{2} \left[u_S(\xi, Q_0^2) + d_S(\xi, Q_0^2)\right] = \bar{q}(\xi, Q_0^2)$ is the light sea quark distribution. Valence and sea quark distributions are practically defined via $d_v(\xi, Q_0^2) = d(\xi, Q_0^2) - \bar{d}(\xi, Q_0^2)$ and $d_S(\xi, Q_0^2) = \bar{d}(\xi, Q_0^2)$, and similarly for $u$-quarks.
- $s(\xi, Q_0^2) = \bar{s}(\xi, Q_0^2)$ is the strange quark momentum distribution inside the proton.

The second line in the two cross section formulas above incorporates threshold effects. The first factor $(1 - \frac{m_v^2}{2ME})$ is a kinematic factor reflecting the spin $\frac{1}{2}$ character of the quarks and leptons. The second factor $\Theta(y - \frac{m_v^2}{2ME})$ imposes the minimum inelasticity requirement for charm production. The third factor $\Theta \left[\xi - \frac{m_v^2}{2ME}\right]$ imposes the requirement that the neutrino-struck quark system have sufficient invariant mass to form a charm quark. The fourth factor $\Theta \left[2MEy(1-x) + M^2 - M_C^2\right]$ forces the invariant mass of the final state hadronic system to exceed the minimum that is compatible with the presence of a charmed hadron and a baryon. Real thresholds will not likely be so sharp; and these factors are probably approximations.

Under a number of assumptions detailed below, the major difference between neutrino and anti-neutrino production of charm is the possibility of production off the valence quarks in the neutrino case that is absent in the anti-neutrino mode. The sensitivity to $|V_{cd}|$ follows if one can isolate the valence quark contribution to charm production. This may be accomplished in principle by either measuring the charm
cross section at high \( x \), where sea quark distributions are small; or by subtracting the anti-neutrino cross section from the neutrino cross section. The CKM element \( V_{cs} \) always appears in combination with \( s(\xi, Q^2) \), which is not yet an independently measured quantity.

The model assumptions that must be addressed are divided into “partonic” and “hadronic” quantities below.

**B. Partonic Level Issues in \( \nu N \) Charm Production**

1. Higher Order QCD Effects

The most obvious question at the partonic level is that of higher order QCD effects. These have been addressed by a number of authors [4]. While the form of the cross sections becomes much more complex in detail, the essential structure remains the same as far as the CKM matrix elements are concerned, i.e.,

\[
\frac{d\sigma(\nu N \rightarrow \mu^- cX)}{dxdy} = \frac{G_F^2 ME}{\pi} \times \left\{ \begin{array}{l}
|V_{cd}|^2 F_V \left[ m_c, \alpha_S(Q^2), v(\xi, Q_0^2) \right] + \\
|V_{cd}|^2 F_S \left[ m_c, \alpha_S(Q^2), \bar{q}(\xi, Q_0^2), s(\xi, Q_0^2), G(\xi, Q_0^2) \right] + \\
|V_{cs}|^2 F_{SS} \left[ m_c, \alpha_S(Q^2), \bar{q}(\xi, Q_0^2), \bar{q}(\xi, Q_0^2), s(\xi, Q_0^2), G(\xi, Q_0^2) \right]
\end{array} \right\}
\]

and

\[
\frac{d\sigma(\bar{\nu} N \rightarrow \mu^+ \bar{c}X)}{dxdy} = \frac{G_F^2 ME}{\pi} \times \left\{ \begin{array}{l}
|V_{cd}|^2 \bar{F}_S \left[ m_c, \alpha_S(Q^2), \bar{v}(\xi, Q_0^2), \bar{s}(\xi, Q_0^2), G(\xi, Q_0^2) \right] + \\
|V_{cs}|^2 \bar{F}_{SS} \left[ m_c, \alpha_S(Q^2), \bar{v}(\xi, Q_0^2), \bar{q}(\xi, Q_0^2), s(\xi, Q_0^2), G(\xi, Q_0^2) \right]
\end{array} \right\}
\]

In these expressions, \( F_V, F_S, \bar{F}_S, \bar{F}_{SS} \) are calculable functionals of the parton distributions that depend on the running coupling constant \( \alpha_S(Q^2) \), the charm mass \( m_c \), and the parton distribution functions at the reference \( Q_0^2 \). The sea and strange sea functionals \( F_S \) and \( F_{SS} \) include contributions from the nucleon’s gluon momentum distribution \( G(\xi, Q_0^2) \) via W-gluon fusion. The gluon distribution also affects \( F_S \) and \( F_{SS} \), but not \( F_V \), via the normal QCD evolution of the structure functions. Higher order QCD processes do not affect the dominance of the valence quark distributions at high \( x \); thus \( |V_{cd}| \) is still measurable from the neutrino cross section alone. As long as \( F_S = \bar{F}_S \) and \( F_{SS} = \bar{F}_{SS} \), the valence contribution to charm production can still be isolated by subtracting the anti-neutrino cross section from the neutrino cross section.

These last assumptions seems quite reasonable, however it is not necessary for \( s(\xi, Q_0^2) = \bar{s}(\xi, Q_0^2) \), for example, at each value of \( \xi \), nor even for \( \int_0^1 dq^2 s(q^2, Q_0^2) = \int_0^1 dq^2 \bar{s}(q^2, Q_0^2) \). The only rigorous requirement is that \( \int_0^1 dq^2 s(q^2, Q_0^2) = \int_0^1 dq^2 \bar{s}(q^2, Q_0^2) \) since the parton distribution functions used here are defined as momentum distributions. A mechanism that could generate \( s(\xi, Q_0^2) \neq \bar{s}(\xi, Q_0^2) \) would be the presence of a \( \Lambda K^+ \) state in the proton’s wave function [3]. If the neutrino interacts with the proton while it is in this state, there will be a manifest asymmetry between the s-quark distribution of the \( \Lambda \) and the \( \bar{s} \) quark distribution of the \( K^+ \). It would be difficult to combine the neutrino and anti-neutrino data together if \( s(\xi, Q_0^2) \neq \bar{s}(\xi, Q_0^2) \) or \( d_s(\xi, Q_0^2) \neq d_s(\xi, Q_0^2) \). All that can be stated is that all measurements up to now are consistent with these assumptions.

The most serious theoretical issues in the treatment of higher order terms in neutrino charm production are probably the questions of scale dependence and the treatment of mass thresholds in the evolution of the running coupling constant. Whether these issues affect CKM matrix parameters depends on experimental design.
2. Partonic $p_T$ and Mass Effects

The parton model of deep inelastic scattering assumes that the struck quarks are massless objects moving colinear with the proton. Neither of these assumptions need be true. Quarks and gluons inside the nucleon will carry transverse momentum, both due to their confinement in the finite volume of the nucleon (“intrinsic” $p_T$) and due to hard gluon radiation and $q\bar{q}$ pair production. These effects appear in the longitudinal structure function of the nucleus

$$R_L(\xi, Q^2) = \frac{F_2(\xi, Q^2)}{2xF_1(\xi, Q^2)} \cdot (1 + \frac{4M^2\xi^2}{Q^2}). \quad (7)$$

The longitudinal structure function can be calculated perturbatively for high $Q^2$; at lower values of the momentum transfer, higher twist contributions may become important. From the point of view of charm production, the most serious question is the effect of the charm quark mass threshold on $R_L$.

The charged current charm production cross section contains a longitudinal piece even at lowest order just due to the charm mass. To see this simply, consider production off valence quarks, and note that

$$1 - \frac{m_c^2}{2ME\xi} \simeq 1 - \frac{m_c^2/Q^2}{1 + m_c^2/Q^2} \cdot y.$$ 

Then,

$$\frac{d\sigma(\nu d \rightarrow \mu^- eX)}{dxdy} \propto v(\xi, Q^2) \left[ (1 - y) + \frac{1}{1 + m_c^2/Q^2} \cdot \frac{y^2}{2} + \frac{1}{1 + m_c^2/Q^2} \cdot (y - \frac{y^2}{2}) \right].$$

The decomposition in $y$ isolates the structure functions, from which it follows, neglecting the $\frac{4M^2\xi^2}{Q^2}$ term, that

$$R_{LO}^{O-charm}(\xi, Q^2) \simeq 1 + \frac{m_c^2}{Q^2}.$$

While it seems reasonable to consider the $d$-quark to be massless, the strange quark might be expected to have a current quark mass of $\approx 300$ MeV/$c^2$. The modification of the kinematics due to this effect has been treated by Tung et al. [4], who have also considered the effects of the target proton mass. There is little effect on the differential cross section due to $m_s$ or $M$. Other mass effects could enter in a dependence of the $M_C^2$ threshold on the struck quark type. In the case of neutrino-nucleon scattering, the thresholds are $M_C = M_{\Lambda C} + M_\pi$, and $M_{\Xi C} + M_K$ depending on whether the struck quark is $d$-valence, $d$-sea, or $s$, respectively. Antineutrino charm production lacks the valence channel; and the thresholds are slightly higher for $\bar{d}$ and $\bar{s}$ struck quarks than for the corresponding neutrino case: $M_C = M_N + M_D$ for $\bar{d}$ and $M_C = M_N + M_{D_{\Xi}}$ for $\bar{s}$.

Both parton mass and $p_T$ effects are small at large $\xi$ where the valence quark distributions dominate; and corrections to the sea quark distributions are the same for neutrinos and antineutrinos. On the other hand, higher twist effects are concentrated at high $\xi$, and are thus of concern in a $|V_{cd}|$ extraction.

3. Nuclear Effects at the Parton Level

Most measurements of neutrino charm production are made using nuclear targets. One must correct for the neutron excess present in the heavier targets. In addition, there may be more subtle nuclear effects.

A standard assumption made in all deep inelastic scattering analyses is that strong isospin symmetry is obeyed, i.e., $u^p(x, Q^2) = d^n(x, Q^2)$, and $d^p(x, Q^2) = u^n(x, Q^2)$; where the superscripts $p$ and $n$ refer to proton and neutron, respectively. In addition, it is always assumed that $s^p(x, Q^2) = s^n(x, Q^2)$.
Recent measurements of the Gottfried sum rule by the NMC collaboration \cite{6} indicate that the stronger assumption $\bar{u}(x, Q^2) = \bar{d}(x, Q^2)$ is not true at small $x$. There are then two corrections due to the neutron excess in heavier targets, a valence correction for neutrinos only and a sea correction for neutrinos and anti-neutrinos:

$$\Delta_I^\nu = \frac{1}{2} \cdot \frac{(N - Z)}{(N + Z)} |V_{cd}|^2 \left\{ F_I^V \left[ u_V(\xi, Q^2_0) - d_V(\xi, Q^2_0) \right] + F_I^S \left[ \bar{u}(\xi, Q^2_0) - \bar{d}(\xi, Q^2_0) \right] \right\},$$  \hspace{1cm} (8)

and

$$\Delta_I^\bar{\nu} = \frac{1}{2} \cdot \frac{(N - Z)}{(N + Z)} |V_{cd}|^2 F_I^S \left[ \bar{u}(\xi, Q^2_0) - \bar{d}(\xi, Q^2_0) \right].$$  \hspace{1cm} (9)

Other nuclear effects are not as obvious; however, so long as all parton distribution functions used in charm production analysis are extracted from data using the same or similar nuclear targets, nuclear effects should not affect the results at the parton level. Nuclear partonic effects can be important if one must take parton distributions extracted from low atomic number targets and use them to make corrections in a heavy target. This happens in the case of the neutron excess correction described above. The differences between $u_V(\xi, Q^2_0)$ and $d_V(\xi, Q^2_0)$ are deduced by comparing scattering from hydrogen to that of deuterium \cite{7}. The results must then be extrapolated to heavier targets by correcting for the EMC effect. The EMC effect is determined from electroproduction measurements of the structure $F_2(x, Q^2)$, and not for the valence quark distributions separately.

C. Hadronization Effects

1. Charmed Quark Fragmentation

Experiments can only measure charmed hadron production, not charmed quark production. Some experiments, moreover, do not even detect the charmed hadrons; instead only the lepton (usually muon) from semi-leptonic charm decay is measured. Assuming factorization, the charmed hadron $(C)$ cross section can be connected to the charmed quark $(c)$ cross section via fragmentation functions:

$$\frac{d\sigma(\nu N \rightarrow \mu^- CX)}{dx dy dz dp_T^2} = \frac{d\sigma(\nu N \rightarrow \mu^- cX)}{dx dy} \cdot \sum_h f_h \cdot D_h^c(z, p_T^2),$$

$$\frac{d\sigma(\bar{\nu} N \rightarrow \mu^+ CX)}{dx dy dz dp_T^2} = \frac{d\sigma(\bar{\nu} N \rightarrow \mu^+ \bar{c}X)}{dx dy} \cdot \sum_h \bar{f}_h \cdot \bar{D}_h^\bar{c}(z, p_T^2).$$  \hspace{1cm} (10)

Here, $D_h^c(z, p_T^2)$ is the probability distribution for the charmed quark fragmenting into a charmed hadron of type $h$ carrying a fraction of the quark’s longitudinal momentum $z$ and transverse momentum $p_T$ with respect to the quark direction. The number $f_h$ is the mean multiplicity of the hadron $h$ in neutrino production of charm. The analogous objects for anti-neutrinos are indicated by the barred quantities. Since only one $c$-quark is produced in a charged current interaction, one can set the normalization conditions as

$$\int_0^1 dz \int_0^\infty dp_T^2 D_h^c(z, p_T^2) = \int_0^1 dz \int_0^\infty dp_T^2 \bar{D}_h^\bar{c}(z, p_T^2) = 1,$$  \hspace{1cm} (11)

and

$$\sum_h f_h = \sum_h \bar{f}_h = 1.$$  \hspace{1cm} (12)
2. Neutrino vs. Anti-neutrino Hadronization

It need not be the case that \( f_h = \bar{f}_h \) or \( D^0_c(z, p_T^2) = \bar{D}^0_c(z, p_T^2) \) since the remnant nucleon will not be the same in \( \nu N \) and \( \bar{\nu} N \) scattering at low energy. The threshold behavior, for example, differs:

\[
\nu N \rightarrow p^+ \Lambda_C, \mu^- \Sigma_C;
\]

but

\[
\bar{\nu} N \rightarrow p^+ \bar{\Lambda}_C, \mu^- \bar{\Sigma}_C.
\]

At sufficiently high energies, one expects the charm quark hadronization to become independent of the remnant nucleon, and consequently \( D^0_c(z, p_T^2) \rightarrow \bar{D}^0_c(z, p_T^2) \) and \( f_h \rightarrow \bar{f}_h \).

3. Quasi-elastic Charm Production

The cross section for \( \nu n \rightarrow p^+ \Lambda_C, \mu^- \Sigma_C \) were first calculated by Shrock and Lee [8]. Using SU(4) flavor symmetry to relate \( \sigma(\nu n \rightarrow p^+ \Lambda_C) \) to \( \sigma(\nu n \rightarrow \mu^- p) \), these authors calculated a cross section of \( \sigma(\nu n \rightarrow \mu^- \Lambda_C) = 2.4 \text{ fb} \) for \( E_\nu > 20 \text{ GeV} \). The Fermilab E531 experiment [10] observed three events consistent with quasi-elastic charm production, from which they obtained a cross section of \( (0.37^{+0.37}_{-0.23}) \text{ fb} \). The overestimate of the theoretical calculation is likely due to the substitution of charmed vector meson masses for the \( \rho \) and \( A_1 \) masses appearing in the \( \nu n \rightarrow \mu^- p \) cross section. Converting the E531 data into the upper limit \( \sigma(\nu n \rightarrow \mu^- \Lambda_C) < 1.0 \text{ fb} \) at 90% C.L., and assuming that the cross section is independent of energy, one finds that the fraction of neutrino charm production that appears in the quasi-elastic channel is less than 88%, 24%, 13%, 6%, and 2.5% for \( E_\nu = 10, 20, 30, 50, \) and \( 100 \text{ GeV} \), respectively. These numbers allow one to judge the extent to which the cross section factorizes into independent production and hadronization terms. Factorization is clearly not valid for \( E_\nu < 20 \text{ GeV} \).

Other non-quark-parton model calculations of neutrino charm production exist. Einhorn and Lee [9] considered a generalization of vector and axial vector meson dominance (VDM) to the charm sector in the same spirit as Shrock and Lee’s \( \Lambda_C \) calculation. The VDM contribution to the neutrino charm cross section calculated in this model is very large, due again due the replacement of light vector meson masses with charmed vector masses in the relevant form factors. The agreement of the data with the quark parton model of charm production at high energies and the failure of the VDM to describe the non-charm cross section indicates that VDM is not a dominant contribution at high energies. For \( E_\nu < 20 \text{ GeV} \), this mechanism could well be important.

4. Neutrino Induced Di-lepton Production

Emulsion experiments can, in principle, identify all charmed hadrons via their finite decay length. In practice, there will be efficiency variations according to the charged multiplicity of the modes; and acceptance for low \( z \) will be limited, particularly at low values of \( E_\nu \) or \( y \).

Other experiments that detect only leptons must de-convolve another layer from the decay process:

\[
\frac{d\sigma(\nu N \rightarrow \mu^- \mu^+ X)}{dxdydzdp_T^2dk^*d\cos\theta^*} = \frac{d\sigma(\nu N \rightarrow \mu^- cX)}{dxdy} \cdot \sum_h f_h \cdot D^0_c(z, p_T^2) \otimes B^h_c \cdot \Gamma^h_c(k^*, \cos\theta^*),
\]

(13)

with \( B^h_c \) the semi-leptonic branching fraction for the charmed hadron \( h \), and \( \Gamma^h_c(k^*, \cos\theta^*) \) the joint distribution function for the muon momentum in the hadron rest frame \( k^* \) and the decay angle of the muon with respect to the hadron direction in the hadron rest frame. The ‘\( \otimes \)’ symbol indicates that a boost must be performed along the charmed hadron direction to get to the hadron rest frame. To go
from a dimuon measurement to a charm production measurement thus requires knowledge of the relative production of the various charmed hadrons, their fragmentation functions, their semi-leptonic branching fraction, and their decay distributions.

It is possible that the nuclear environment could modify the charmed quark fragmentation functions for production off heavy targets relative to those measured in $e^+e^-$ experiments. Recent measurements \[11\] of $D$ meson production using hadron beams on nuclear targets are consistent with an $A^{1.0}$ behavior of the total production rate and of the $x_F$ and $p_T$ distributions. This is evidence against the presence of nuclear effects in either the production or fragmentation of charmed quarks.

Because of the fragmentation and decay parameters present, neutrino di-lepton experiments can only measure the quantity $\bar{B}_\mu \cdot |V_{cd}|^2$, where the mean semi-muonic branching ratio, defined via

$$\bar{B}_\mu = \sum_h f_h \cdot B_{\mu h}^h,$$  \hfill (14)

is assumed to be the same for neutrinos and antineutrinos. A calculation of $\bar{B}_\mu$ requires knowledge of both the production fractions $f_h$ and the semi-muonic branching fractions $B_{\mu h}^h$. In addition, the fragmentation functions $D_{\mu c}(z, p_T)$ and the decay distributions must be available in order to correct for acceptance and smearing. In practice, a mean fragmentation function to all charmed hadrons is used for this task.

D. Methods for Extracting $|V_{cd}|$ and $|V_{cs}|$

The most complete analyses for the extraction of the CKM parameters in neutrino scattering are those of the CCFR collaboration. Distributions of “visible” versions of the kinematic variables $E, x,$ and $z$ described above are fitted to a model with four parameters:

- $m_c$, the charm mass;
- $\alpha$, a parameter that describes the difference in shape between $s(\xi, Q^2)$ and $\bar{q}(\xi, Q^2)$;
- $\kappa$, the ratio of the nucleon’s momentum carried by $s$ quarks relative to that carried by $u$ and $d$ sea quarks, defined as

$$\kappa \equiv \frac{\int_0^1 s(\xi) d\xi}{\int_0^1 \bar{q}(\xi) d\xi}.$$  \hfill (15)

- $\bar{B}_\mu$, the mean semi-muonic branching ratio for charmed hadrons produced in neutrino anti-neutrino scattering.

The two analyses fit the same data to the cross section described in equation 13 using either leading order \[12\] or next-to-leading order \[13\] QCD. To extract values for $\kappa$ and $\bar{B}_\mu$, $|V_{cd}|$ and $|V_{cs}|$ are set to values implied by three generation CKM unitarity. If the unitarity constraints are dropped, then the CCFR fit parameters change from $\bar{B}_\mu \to \bar{B}_\mu \cdot |V_{cd}|^2$ and $\kappa \to \bar{B}_\mu \cdot \kappa^2 |V_{cs}|^2$.

The CCFR analyses use the maximum information in each dimuon event. However, the procedure couples the desired CKM matrix elements to the properties of the strange sea. Furthermore, the fits to absolute rates are sensitive to QCD scale uncertainties.

A cleaner way of obtaining $|V_{cd}|$ from data is to measure the ratio of differences \[14\]

$$R_{\mu\mu} = \frac{\sigma(\nu N \to \mu^- \mu^+ X) - \sigma(\nu N \to \mu^+ \mu^- X)}{\sigma(\nu N \to \mu^- X) - \sigma(\nu N \to \mu^+ X)}.$$  \hfill (16)

This can be recast in a more useful form for experiments:
Again, this can be expressed as a ratio of differences performed to the data with \( \bar{r} \) ratio from other data to obtain the CKM matrix element. The actual experimental procedure is to measure where \( r \) with \( \bar{r} \) is the well-measured \( \bar{\nu}/\nu \) charged current cross section ratio. The first two errors are the experimental statistical and systematic errors, respectively, the systematic error being dominated by the uncertainty in modeling the charmed quark fragmentation into charmed hadrons. The final error is attributed to the QCD factorization and renormalization scale uncertainties. This analysis also indicates that there is a negligible difference in the value of \( R_{\mu\mu}^- \) if the analysis

\[
R_{\mu\mu}^- = \frac{r_{\mu\mu} - r \cdot \bar{r}_{\mu\mu}}{1 - r^2},
\]

where \( r_{\mu\mu} = \frac{\sigma(\bar{\nu}N \rightarrow \mu^- X)}{\sigma(\nu N \rightarrow \mu^- X)} \) and \( \bar{r}_{\mu\mu} = \frac{\sigma(\bar{\nu}N \rightarrow \mu^+ X)}{\sigma(\nu N \rightarrow \mu^+ X)} \) are the normalized dimuon rates, and \( r = \frac{\sigma(\bar{\nu}N \rightarrow cX)}{\sigma(\nu N \rightarrow cX)} \) is the well-measured \( \bar{\nu}/\nu \) charged current cross section ratio.

To leading order in QCD,

\[
R_{\mu\mu}^- = \frac{3}{2} \bar{B}_\mu |V_{cd}|^2 \frac{K[m_c, E, v(\xi, Q^2)]}{1 + \frac{3}{2} |V_{cd}|^2 K[m_c, E, v(\xi, Q^2)]},
\]

where

\[
d\xi dv(1 - \frac{m_v^2}{2M^2})\Theta\left(y - \frac{m_v^2}{2M^2}\right)\Theta\left(\xi - \frac{m_v^2}{2M^2}\right)\Theta(2MEy(1 - x) + M^2 - M_C^2)v(\xi, Q^2).
\]

The actual experimental procedure is to measure \( R_{\mu\mu}^- \) as a function of neutrino energy. A fit can then be performed to the data with \( \bar{B}_\mu |V_{cd}|^2 \) and \( m_c \) the free parameters. One still must get the mean branching ratio from other data to obtain the CKM matrix element.

In an emulsion experiment, one could improve the situation by measuring directly the ratio

\[
R_c^- = \frac{\sigma(\nu N \rightarrow \mu^- cX) - \sigma(\bar{\nu} N \rightarrow \mu^+ X)}{\sigma(\nu N \rightarrow \mu^- X) - \sigma(\bar{\nu} N \rightarrow \mu^+ X)}.
\]

Again, this can be expressed as a ratio of differences

\[
R_c^- = \frac{r_c - r \cdot \bar{r}_c}{1 - r^2},
\]

with \( r_c = \frac{\sigma(\nu N \rightarrow \mu^- cX)}{\sigma(\nu N \rightarrow \mu^- X)} \) and \( \bar{r}_c = \frac{\sigma(\bar{\nu} N \rightarrow \mu^+ cX)}{\sigma(\bar{\nu} N \rightarrow \mu^+ X)} \). The leading order expression for \( R_c^- \) is the same as for \( R_{\mu\mu}^- \) with the substitution \( \bar{B}_\mu \rightarrow 1 \).

The nice property of \( R_{\mu\mu}^- \) and \( R_c^- \) is their functional dependence only on the valence quark momentum distribution. This fact has three virtues: the valence quark distribution is better measured; the valence quark contribution to charm production suffers less suppression from the charm mass; and the valence quark distribution has a theoretical simpler QCD evolution. Also, since \( R_{\mu\mu}^- \) depends only on ratios of cross sections, it should be less susceptible to QCD scale errors. The cleaner systematics is offset by the need for high statistics since \( R_{\mu\mu}^- \) is a ratio of differences.

**III. CURRENT STATE OF \( |V_{CD}| \) AND \( |V_{CS}| \)**

**A. Measurements of \( \bar{B}_\mu \cdot |V_{cd}|^2 \)**

The most up-to-date extraction of \( B_\mu \cdot |V_{cd}|^2 \) is from the CCFR collaboration [13]. Employing a next-to-leading order QCD formalism:

\[
\bar{B}_\mu \cdot |V_{cd}|^2 = (5.34 \pm 0.39 \pm 0.24^{+0.25}_{-0.31}) \times 10^{-3} \text{ (CCFR - NLO)}.
\]

The first two errors are the experimental statistical and systematic errors, respectively, the systematic error being dominated by the uncertainty in modeling the charmed quark fragmentation into charmed hadrons. The final error is attributed to the QCD factorization and renormalization scale uncertainties. This analysis also indicates that there is a negligible difference in the value of \( B_\mu \cdot |V_{cd}|^2 \) if the analysis
is performed to leading order or next-to-leading order in QCD. Accordingly, one may also use the older result from the CDHS collaboration [14].

\[
\bar{B}_\mu \cdot |V_{cd}|^2 = \left(4.1 \pm 0.7^{+0.19}_{-0.35}\right) \times 10^{-3} \quad \text{(CDHS – LO)}.
\]

The first error is the total experimental error. The second error is the QCD scale error, which is not given by the original analysis, but is instead assumed to be the same as in the CCFR measurement.

Combining the two results, assuming that all of the experimental errors are completely uncorrelated, but that the QCD scale error is totally correlated and equal to the CCFR value, yields

\[
\bar{B}_\mu \cdot |V_{cd}|^2 = \left(5.02^{+0.50}_{-0.69}\right) \times 10^{-3} \quad \text{(CCFR/CDHS)}.
\]

The CKM element \( V_{cs} \) is contained in the CCFR measurement

\[
\bar{B}_\mu \cdot \frac{\kappa}{\kappa + 2} |V_{cs}|^2 = \left(2.00 \pm 0.10 \pm 0.06^{+0.06}_{-0.14}\right) \times 10^{-3},
\]

where the uncertainties represent statistics, experimental systematics, and the QCD scale, respectively.

**B. Charm Production Fractions**

The charm fractions \( f_h \) have only been measured directly in one experiment, FNAL E531 [10]. In checking over the E531 result, a bias was detected in the way that they extracted their charmed hadron production fractions. Their data is re-fitted with the bias removed. If one believes that fragmentation functions are universal, one can check the re-fitted E531 measurements against similar fractions measured in \( e^+e^- \) experiments with similar kinematics. The results of this check are also presented.

1. **Reanalysis Of E531 Data**

In E531, charm could be tagged by the presence of a detached secondary vertex in the emulsion target. Some 122 events were tagged in this way, 119 which are neutrino induced and three induced by anti-neutrinos. The charge of each charmed hadron was determined by counting the number of prongs coming from its decay vertex. Under the assumption that the production of neutral charmed baryons is negligible at E531 energies, all neutral candidates are unambiguously identified as \( D^0 \) mesons. (There is one neutral particle in the sample that had an identified proton; this event is a candidate for a neutral charmed baryon, most likely the \( \Xi^0_c \), a \( csd \) state. However, the reconstructed lifetime of the track is very long, indicating that this event could be background, or that it could have been kinematically mis-fitted.) Charged charmed particles, on the other hand, could be one of three possibilities (again neglecting heavier baryons): a \( D^+ \), a \( D^+_s \), or a \( \Lambda^+_c \). A good fraction of the \( \Lambda^+_c \) could be identified by the presence of a proton in the emulsion. The majority of the remaining charged events could be a \( D^+ \) or \( D^+_s \) with essentially equal probability. The bias in the published E531 result is that they resolved this ambiguity by counting all of the “toss-up” events as \( D^+ \) mesons. They accounted for the possibility of a mistake in this procedure by increasing the error in the charmed fractions. Given the state of knowledge of charmed mesons at the time, this procedure was not unreasonable.

2. **Re-fitting E531 Data**

Since the complete data set from the experiment is readily available in the thesis of S. Frederikson [15], one can re-do the analysis to remove the bias. The procedure is to try to use all the available information about each event to construct a likelihood function. The relevant items are the relative
kinematic fit probabilities to the $D^+$ vs. $D^+_S$ and the proper lifetimes (assuming a particular mass hypothesis). Unfortunately, the kinematics offers essentially no separation because of the small $D^+_S$, $D^+$ mass difference. The lifetimes, on the other hand, are quite different for the two mesons. Accordingly, a likelihood function is constructed for each event using only the decay length information.

The form of the probability function for each event with a charged charmed hadron is taken to be

$$P(n) = \frac{\epsilon \left[ \ell(n) \right] \sum_i N_i w_i(n) \frac{M_i}{\tau_i q_i(n)} \sum_i N_i}{\sum_i N_i}.$$  \hspace{1cm} (22)

In this expression:

- The index $i$ takes on values $D^+$, $D^+_S$, $\Lambda^+_C$.

- The $N_i$ are the number of produced hadrons of each species type. These are the free parameters in the fit.

- The measured quantities $\ell(n)$ and $q_i(n)$ are the decay length and the momentum for the kinematic fit to the hadron $i$, respectively.

- The $M_i$ are the masses of the charmed hadrons. From the 1992 PDG:\[ M_{D^0} = 1.8645 \pm 0.0005 \text{ GeV}/c^2, \]
  \[ M_{D^+} = 1.8693 \pm 0.0005 \text{ GeV}/c^2, \]
  \[ M_{D^+_S} = 1.9688 \pm 0.0007 \text{ GeV}/c^2, \] and \[ M_{\Lambda^+_C} = 2.2849 \pm 0.0006 \text{ GeV}/c^2. \]

- The $\tau_i$ are the mean lifetimes of the particles. From the 1992 PDG: \[ \tau_{D^+} = (10.66 \pm 0.23) \times 10^{-13} \text{s}, \]
  \[ \tau_{D^0} = (4.20 \pm 0.08) \times 10^{-13} \text{s}, \]
  \[ \tau_{D^+_S} = (4.50 \pm 0.28) \times 10^{-13} \text{s}, \] and \[ \tau_{\Lambda^+_C} = (1.91 \pm 0.14) \times 10^{-13} \text{s}. \] Note that these are known much more accurately than when E531 ran.

- The $w_i$ are the acceptance weights for each hypothesis and each event,

$$w_i = Q_i(n) \cdot \left[ \int \frac{df}{c \tau_i} \cdot \frac{M_i}{q_i} e^{-\ell \cdot \frac{M_i}{q_i} \cdot \epsilon(\ell)} \right]^{-1}.$$ \hspace{1cm} (23)

These weights include the effects of the finite size and resolution of the emulsion, of a special $p_T$ cut for one prong charm events, and for general detector acceptance. Frederikson’s thesis gives the weight only for one hypothesis for each event. Weights for the other hypotheses are estimated by assuming that the dominant difference in the decay weights is due to the interplay of the lifetime and the minimum resolvable decay distance. If the minimum resolvable decay distance is $\ell_{\text{min}}$, then, approximately,

$$w_i^{-1} \sim e^{-\frac{M_i d_{\text{min}}}{q_i \tau_i}},$$ \hspace{1cm} (24)

so that

$$\frac{\log(w_i)}{\log(w_j)} = \frac{M_i q_i \tau_j}{M_j q_j \tau_i},$$ \hspace{1cm} (25)

Most of these measurements have now been improved in the 1996 PDG summary; however the relative change between the 1992 and 1996 PDG is very small compared to that between the E531 publication and the 1992 values used here.
independent of $\ell_{\text{min}}$. Thus, if the assumption is correct, one can scale the acceptance weights using the one given weight and the lifetimes, masses, and momenta. This scaling should take into account more complicated effects like the $p_T$ cut placed on one prong events. An explicit calculation using a minimum cut-off of $\ell_{\text{min}} = 15 \, \mu m$ and an analogous maximum cut-off of $\ell_{\text{max}} = 1.5 \, cm$ was also tried, with no change in the results. $Q_i(n)$ is set to 0 if some information about the event rules out a particular hypothesis. This occurs typically if no good kinematic fit existed for one of the hypotheses, or if the presence of an identified proton in the final state required the charmed hadron to be a baryon. Otherwise $Q_i(n) = 1$.

From the probability functions for each event, an extended log-likelihood function is constructed:

$$L = -\sum_n \log P_n = N_{\text{obs}} \log (N_D \bar{w}_D + N_{D_8} \bar{w}_{D_8} + N_{\Lambda_C} \bar{w}_{\Lambda_C}) + (N_D \bar{w}_D + N_{D_8} \bar{w}_{D_8} + N_{\Lambda_C} \bar{w}_{\Lambda_C})$$

The second and third terms above are the log of the Poisson probability function for observing $N_{\text{obs}}$ events given the produced events. The bars over the acceptance weights indicate that these quantities are averaged. The Poisson term incorporates the finite statistics of the experiment. The (negative) log-likelihood function is then minimized. The results of fits are given in the next section.

### 3. Results of Fits

The fit results are given in Table I in the form of charmed hadron fractions: $f_{D^0}, f_{D^+}, f_{D^+_S}$, and $f_{\Lambda_C}$. The fractions, their errors, and their correlations are obtained from MINUIT. (As a technical aside, the $N_i$ defined in the previous section are expressed in terms of the $f_i$ and the total number of produced charmed hadron events $N_C$. The $f_i$ and $N_C$ are then allowed to vary. The constraint $\sum f_i = 1$ keeps the number of free parameters the same.). The correlations between the fractions are give in Table II.

### Table I. E531 Re-fitted Production Fraction Results

| Energy (GeV) | $f_{D^0}$  | $f_{D^+}$  | $f_{D^+_S}$ | $f_{\Lambda_C}$ |
|-------------|------------|------------|-------------|-----------------|
| 5-20        | 0.32 ± 0.11| 0.05 ± 0.06| 0.18 ± 0.10 | 0.44 ± 0.12     |
| 20-40       | 0.50 ± 0.08| 0.10 ± 0.08| 0.22 ± 0.08 | 0.18 ± 0.07     |
| 40-80       | 0.64 ± 0.08| 0.22 ± 0.09| 0.09 ± 0.08 | 0.05 ± 0.04     |
| > 80        | 0.60 ± 0.11| 0.30 ± 0.11| 0.00 ± 0.06 | 0.09 ± 0.08     |
| > 40        | 0.61 ± 0.06| 0.27 ± 0.03| 0.04 ± 0.01 | 0.07 ± 0.02     |
| > 30        | 0.58 ± 0.06| 0.26 ± 0.06| 0.07 ± 0.05 | 0.07 ± 0.04     |
| > 20        | 0.56 ± 0.05| 0.20 ± 0.05| 0.11 ± 0.04 | 0.11 ± 0.04     |
| > 5         | 0.53 ± 0.05| 0.16 ± 0.04| 0.13 ± 0.04 | 0.17 ± 0.04     |

### Table II. Correlations Among Production Fractions

| Energy (GeV) | $C(D^0, D^+)$  | $C(D^0 D^+_S)$  | $C(D^0 \Lambda_C)$  | $C(D^+ D^+_S)$  | $C(D^+ \Lambda_C)$  | $C(D^+_S \Lambda_C)$  |
|-------------|----------------|----------------|---------------------|----------------|---------------------|---------------------|
| 5-20        | -0.165         | -0.332         | -0.572              | -0.116         | -0.227              | -0.451              |
| 20-40       | -0.166         | -0.280         | -0.229              | -0.810         | +0.034              | -0.332              |
| 40-80       | -0.497         | -0.302         | -0.239              | -0.557         | -0.082              | -0.205              |
| > 80        | -0.633         | -0.001         | -0.304              | -0.002         | -0.410              | -0.001              |
| > 40        | -0.619         | -0.166         | -0.288              | -0.382         | -0.250              | -0.150              |
| > 30        | -0.503         | -0.215         | -0.268              | -0.551         | -0.159              | -0.192              |
| > 20        | -0.404         | -0.284         | -0.375              | -0.544         | -0.181              | -0.138              |
| > 5         | -0.344         | -0.329         | -0.433              | -0.466         | -0.184              | -0.208              |

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4. Theoretical Expectations

One can make a simple model for what one expects the charm branching fractions to be:

\[ f_{D^0} = (1 - \lambda) \cdot (1 - \phi) \cdot \left( \frac{5 B^{00} + 2 B^{0+} + B^{+}}{V + 1} \right) \]
\[ f_{D^+} = (1 - \lambda) \cdot (1 - \phi) \cdot \left( \frac{5 B^{00} + 2 B^{0+} + B^{+}}{V + 1} \right) \]
\[ f_{D^+_s} = (1 - \lambda) \cdot \phi; \]
\[ f_{\Lambda^+} = \lambda. \]

The parameters above have the following meanings:

- \( \lambda \) is the fraction of time the c-quark fragments into a \( \Lambda^+ \).
- \( \phi \) is the relativity probability the c-quark fragments into a \( D^+_s \) vs. any charmed meson.
- \( V \) represents the relative probability of a c-quark fragmenting into a vector \( D^* \) meson compared to the probability of fragmenting directly into a pseudoscalar \( D \) meson.
- The \( B^{zz} \) are \( D^* \) branching ratios. From the 1992 PDG: \( B^{00} \equiv B(D^{*0} \rightarrow D^0X) = 1.00, B^{0+} \equiv B(D^{*0} \rightarrow D^+X) = 0.00, B^{++} \equiv B(D^{*+} \rightarrow D^+X) = 0.45 \pm 0.04, B^{+0} \equiv B(D^{*+} \rightarrow D^0X) = 0.55 \pm 0.04 \).

On the basis of spin counting rules, one expects \( V = 3 \). Studies of non-charm fragmentation in \( e^+e^- \) experiments indicate that \( \phi \approx 0.15 \) and \( \lambda \approx 0.10 \). The model thus predicts \( f_{D^0} = 0.54, f_{D^+} = 0.22, f_{D^+_s} = 0.13 \), and \( f_{\Lambda^+} = 0.10 \).

5. \( e^+e^- \) DATA

The fractions \( f \) can be obtained from CLEO data \cite{13} if one accepts the hypothesis that fragmentation functions obtained from \( e^+e^- \) experiments with \( \sqrt{s} = 10.55 \) GeV can be used to describe quark fragmentation in neutrino induced interactions with mean final state hadronic mass \( < W > \approx 10 \) GeV/\( c^2 \). CLEO measured the product of branching ratio times cross section, \( \sigma \cdot B \), for charged and neutral \( D \) and \( D^* \) mesons, \( D^+_s \) mesons, and \( \Lambda^+_c \) baryons for \( z = \frac{E_{T,\text{max}}}{E_{T}} > 0.5 \). They then extrapolated using a tuned Lund fragmentation model to all \( z \). It is possible to convert their \( \sigma \cdot B \) values into cross sections using up-to-date charmed hadron decay rates from the 1992 PDG. Table II summarizes the CLEO measurements of \( \sigma \cdot B \) and the branching ratio corrected cross sections integrated over all \( z \) and for \( z > 0.5 \).

| Mode            | \( \sigma \cdot B \) (pb) | \( \sigma \cdot B \) (pb) | B.R.(%)  | \( \sigma \) (nb) | \( \sigma \) (nb) |
|-----------------|--------------------------|--------------------------|----------|-------------------|-------------------|
|                 | \( z > 0.5 \)            | all \( z \)             | \( z > 0.5 \) | all \( z \)       | all \( z \)       |
| \( D^0 \rightarrow K^- \pi^+ \) | 27.0 ± 1.4               | 52 ± 6                  | 3.65 ± 0.21 | 0.74 ± 0.06       | 1.42 ± 0.19       |
| \( D^+ \rightarrow K^- \pi^+ \pi^+ \) | 29.3 ± 3.0               | 47 ± 7                  | 8.0 ± 0.8   | 0.37 ± 0.05       | 0.59 ± 0.11       |
| \( D^{*+} \rightarrow \pi^+ D^0(K^- \pi^+) \) | 10.9 ± 1.0               | 17 ± 2                  | 2.00 ± 0.18 | 0.54 ± 0.06       | 0.85 ± 0.13       |
| \( D^{*+} \rightarrow \pi^+ D^0(K^- \pi^+ \pi^-) \) | 23.1 ± 1.0               | 33 ± 3                  | 4.1 ± 0.3   | 0.56 ± 0.06       | 0.80 ± 0.12       |
| \( D^{*0} \rightarrow \gamma/\pi^0 D^0(K^- \pi^+) \) | 19.8 ± 4.0               | 30 ± 7                  | 3.65 ± 0.21 | 0.54 ± 0.11       | 0.82 ± 0.19       |
| \( D^{*0} \rightarrow \phi \pi^+ \) | 5.8 ± 1.0                | 7.2 ± 2.0               | 2.8 ± 0.5   | 0.21 ± 0.05       | 0.26 ± 0.09       |
| \( \Lambda^+_c \rightarrow pK^- \pi^+ \) | 8.6 ± 1.0                | 13.5 ± 4.0              | 3.2 ± 0.7   | 0.27 ± 0.07       | 0.42 ± 0.16       |

TABLE III. Charmed Hadron Production Measurements By CLEO
From the data in Table III one can extract the fractions $f_i$ as measured in $e^+e^-$ scattering at $\sqrt{s} = 10.55$ GeV. These fractions are summarized in Table IV.

| fraction | $z > 0.5$   | all $z$       |
|----------|-------------|---------------|
| $f_{D^0}$| $0.47 \pm 0.03$ | $0.53 \pm 0.05$ |
| $f_{D^+}$| $0.23 \pm 0.03$ | $0.22 \pm 0.03$ |
| $f_{D^+_s}$| $0.13 \pm 0.02$ | $0.10 \pm 0.06$ |
| $f_{\Lambda^+_c}$| $0.17 \pm 0.04$ | $0.16 \pm 0.05$ |

TABLE IV. Charm Hadron Fractions Measured By CLEO
Using the $D^*$ and $D$ data together, CLEO also determined

$$\frac{V}{V+1} = 0.85 \pm 0.11 \pm 0.17.$$  

This value is consistent with the model expectation of 0.75. Electron-positron data thus supports the simple model presented above and yields charmed hadron fractions in agreement with the re-fit E531 results, but not with the published E531 values.

### C. Updated Charm Branching Ratios

#### 1. Direct Extraction of $\bar{B}_\mu$

The charm branching ratios were measured in the early to mid eighties at SLAC. The measurements are not precise; one, the $D^+_S$ semi-muonic rate, is essentially not known at all. One can attempt to use a weak form of the spectator model of charm decays to improve the precision, but the correlations introduced result in no practical gain.

The semi-muonic branching ratios for most charmed particles have changed little recently, with one important exception. (Actually, only one measurement exists for the semi-muonic rate; the remainder are semi-electronic rates that are converted assuming $e-\mu$ universality.) All values below come from the 1996 PDG except for the $D^0$, which also includes a substantially improved number from CLEO [15], yielding $B_\mu(D^0) = (6.75 \pm 0.30)\%$. The $D^*$ rate is essentially that of the Mark III collaboration: $B_\mu(D^*) = (17.2 \pm 1.9)\%$. The $D^+_S$ rate is from a single poor measurement by Mark III [21]: $B_\mu(D^+_S) = (5.0 \pm 5.4)\%$. The $\Lambda^+_c$ rate is an ancient measurement from Mark II (at SPEAR) [22]: $B_\mu(\Lambda^+_c) = (4.5 \pm 1.7)\%$.

Using the direct measurements alone and the re-fitted E531 data yields

$$\bar{B}_\mu = 0.0919 \pm 0.0085_{\text{CF}} \pm 0.0041_{\text{BR}},$$

where the first error is the contribution due to the charmed hadron species fractions and the second error is due to the charmed hadron semi-muonic branching ratios. Charm hadron production fractions now dominate this error.

#### 2. Spectator Model Fits to Branching Fractions

The $D^+_S$ and $\Lambda^+_c$ branching ratios are poorly measured. One can try to improve the situation by imposing a weak form of the spectator model which requires the semi-electronic partial width $\Gamma_e$ be the same for all charmed particles. The semi-muonic branching fraction can then be related to the lifetime of a charmed hadron $X_n$ via

$$\bar{B}_\mu^h = \Gamma_e \cdot \tau_h.$$  

One can use the spectator model to fit for improved values of the branching ratios by minimizing

$$\chi^2 = \sum_k \left( \frac{\bar{B}_\mu^h - \bar{B}_\mu^h}{\sigma_{\bar{B}_\mu^h}} \right)^2 + \left( \frac{\bar{B}_\mu^h}{\Gamma_e \cdot \tau_h} - \tau_h \right)^2 \sigma_{\tau_h}^2$$

with respect to improved values of the branching ratios $\bar{B}_\mu^h$ and the common semi-electronic width $\Gamma_e$. The results of the fit are: $\bar{B}_\mu(D^0) = (6.76 \pm 0.30)\%$, $\bar{B}_\mu(D^*) = (17.2 \pm 0.8)\%$, $\bar{B}_\mu(D^+_S) = (7.6 \pm 0.4)\%$, and
\( \hat{B}_\mu (\Lambda^+_{c}) = (3.4 \pm 0.3)\% \). The spectator model fit introduces large correlations among the fit parameters: 
\[ C_{D^0D^+} = 0.930, C_{D^0D^+} = 0.613, C_{D^0A^+_c} = 0.698, C_{D^+D^+_s} = 0.598, C_{D^+A^+_c} = 0.681, \text{and } C_{D^+_sA^+_c} = 0.449. \]
The correlations must be included in calculating the error on the fitted mean branching ratio \( \hat{B}_\mu \). The result is \( \hat{B}_\mu = 0.0930 \pm 0.0088 \).

D. The CKM Parameters

1. \( |V_{cd}| \) from Neutrino Scattering

Using the updated value for \( \hat{B}_\mu \) obtained above:
\[ |V_{cd}| = 0.232^{+0.017}_{-0.019} \text{ (DIRECT)}. \]
The CKM parameter \( |V_{cd}| \) is thus known to \( \pm 9\% \) from direct measurement. Of this, the contribution from the uncertainty on \( \hat{B}_\mu \) is \( \pm 6.6\% \), from the experimental error on \( B_\mu \cdot |V_{cd}| \) \( 4.2\% \), and from the QCD scale uncertainty \( \pm 2.6\% \). The measurement is consistent with the value inferred from the assumed unitarity of the CKM matrix of
\[ |V_{cd}| = 0.221 \pm 0.003. \text{ (UNITARITY)}, \]
but the unitarity prediction is clearly not being tested.

2. Other Measurements of \( |V_{cd}| \)

There are no other direct measurements of \( |V_{cd}| \). The closest are the measurements of
\[ \left| \frac{V_{cd}}{V_{cs}} \right|^2 \cdot \left| \frac{f_\pi^+(0)}{f_\pi^K(0)} \right|^2 = \epsilon \cdot \frac{B(D \to \pi \ell \nu)}{B(D \to K \ell \nu)} \]
by Mark III in the neutral \( D \) mode \[23\] and CLEO in the charged \( D \) mode \[24\]. In this expression, \( B(D \to \pi \ell \nu) \) and \( B(D \to K \ell \nu) \) are the measured \( D_{s3} \) semi-leptonic branching fractions; \( \epsilon \) is a precisely known kinematic factor; and \( f_\pi^+(0) \) and \( f_\pi^K(0) \) are the \( D_{s3} \) transition form factors evaluated at zero momentum transfer to the \( \ell \nu \) system. The Mark III measurement is
\[ \left| \frac{V_{cd}}{V_{cs}} \right|^2 \cdot \left| \frac{f_\pi^+(0)}{f_\pi^K(0)} \right|^2 = 0.057^{+0.038}_{-0.015} \pm 0.005(\text{MarkIII}). \]
The CLEO measurement is
\[ \left| \frac{V_{cd}}{V_{cs}} \right|^2 \cdot \left| \frac{f_\pi^+(0)}{f_\pi^K(0)} \right|^2 = 0.085 \pm 0.027 \pm 0.014(\text{CLEO}). \]
Combining the two results yields
\[ \left| \frac{V_{cd}}{V_{cs}} \right|^2 \cdot \left| \frac{f_\pi^+(0)}{f_\pi^K(0)} \right|^2 = 0.069 \pm 0.020(\text{COMBINED}). \]
The ratio \( \left| \frac{f_\pi^+(0)}{f_\pi^K(0)} \right| \) is model dependent and ranges from 0.7 – 1.4 \[25\]. This translates into 30\% uncertainty in \( |V_{cd}| \). If the theoretical uncertainty in the form factor ratio could be reduced to 10\%, then the error on \( |V_{cd}| \) from \( D \) meson decay would be \( \pm 0.034 \) or 15\%. This is less precise than the neutrino measurement.
3. Limit on the Wolfenstein Parameters $A$ and $\rho$

The quantity $\left| \frac{V_{cd}}{V_{us}} \right| - 1$ can be used, via equation 1, to set the constraint

$$A^2 \left| \rho - \frac{1}{2} \right| < 173 \text{ at } 90\% \text{ C.L.}$$

This is obviously not a very exciting limit. There is no evidence for the third generation from quark mixing measurements involving only the first two generations.

4. Limit on $|V_{cs}|$

If one assumes that the strange sea carries no more momentum than the light quark sea, i.e., $\kappa \leq 1$, then it follows from the CCFR next-to-leading-order analysis that

$$|V_{cs}| > 0.74 \text{ at } 90\% \text{ C.L.}$$

If one arbitrarily assumes that $\kappa = 0.5 \pm 0.5$, then

$$|V_{cs}| = 1.04 \pm 0.42,$$

and

$$\left| \frac{V_{cd}}{V_{cs}} \right| = 0.23 \pm 0.09.$$  

The errors in these latter two quantities is dominated by the uncertainty in $\kappa$. Note that the ratio $|V_{cd}/V_{cs}|$ derived from neutrino and antineutrino scattering is independent of $\bar{B}_{\mu}$.

If $\kappa$ can be independently measured to an accuracy of $\pm 25\%$ from the structure function difference $xF_3^\nu - xF_3^{\bar{\nu}}$, then the errors on $|V_{cs}|$ and $\left| \frac{V_{cd}}{V_{cs}} \right|$ would be reduced to $10\%$.

IV. FUTURE MEASUREMENTS OF $|V_{CD}|$ IN $\nu N$ SCATTERING

Four high energy neutrino experiments are either now running or are approved to run in the next six years. While none of the experiments are optimized for the study of neutrino charm production, all have the potential to improve the CKM matrix element measurements. The experiments are summarized in Table V. The Nomad [26] and Chorus [27] experiments at CERN are designed to search for $\nu_\mu \rightarrow \nu_\tau$ oscillations. Nomad features a low mass target with very good tracking and electron identification. This experiment should be able to detect charm in both di-lepton modes ($\mu\mu$ and $\mu e$). Their excellent tracking may also allow for the identification of charm via the “$D^* \rightarrow D\pi$ trick”. Chorus is a hybrid emulsion spectrometer. It’s major virtue is its ability to reconstruct charm inclusively via the identification of the charm decay vertex. This feature serves to boost statistics, and, more importantly, largely eliminates the need to know the production, fragmentation, and decay properties of the charmed hadrons. Fermilab E815 [28] uses the E744/770 Lab E neutrino detector. The experiment is optimized for precision studies of neutral current interactions. The feature most relevant for charm studies is the new sign-selected neutrino beam. This will eliminate the $\nu/\bar{\nu}$ confusion in the dimuon channel and permit a cleaner measurement of $|V_{cs}|$, assuming that the strange sea is independently known by then.

The ultimate neutrino charm production experiments are COSMOS (FNAL E803) [29] and TOSCA [30] at CERN. Like Chorus, these experiments are designed for a high sensitivity search for $\nu_\mu \rightarrow \nu_\tau$ oscillations using hybrid emulsion spectrometers. E803 will have a factor of twenty higher statistics than
Chorus; and its spectrometer will have three times better resolution. The higher resolution is crucial to reduce backgrounds, particularly in one-prong decays of charm. E803 might be able to achieve a resolution of $\sim 2\%$ on $|V_{cd}|$. This is estimated by assuming: a sample of 50,000 reconstructed charm events, which reduces the statistical error to ±0.003; a $\times 5$ reduction in the experimental systematic errors due to the elimination of fragmentation uncertainties and background; a $\times 5$ reduction in the QCD scale error via the normalization of charm to single muon production that is possible with higher statistics; and a $\times 10$ reduction in production fraction and branching ratios achieved by the ability to inclusively reconstruct charm. The total ±0.004 error on $|V_{cd}|$ will be comparable to that on $|V_{us}|$; and one will thus be able to test the unitarity property of the CKM matrix at a level that is sensitive to new physics.

| Experiment     | Target   | Start | CC Sample   | Charm Sample                  |
|----------------|----------|-------|-------------|------------------------------|
| Nomad (CERN)   | low mass | 1994  | $1 \times 10^6$ | $2 \times 10^4$ (inclusive)  |
| Chorus (CERN)  | emulsion | 1994  | $3 \times 10^5$ | $2 \times 10^4$ (inclusive)  |
| NuTeV (FNAL)   | iron     | 1996  | $3 \times 10^6$ | $2 \times 10^4$ (inclusive)  |
| COSMOS (FNAL)  | emulsion | 2001  | $8 \times 10^6$ | $4 \times 10^5$ (inclusive)  |

TABLE V. Future Neutrino Experiments. Event samples are rough estimates.
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