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QUASI-LINEAR CALCULATION OF ION TAILS AND NEUTRON RATES IN A D-T PLASMA DUE TO LOWER HYBRID WAVES

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Abstract. Quasi-linear theory is used to calculate the distribution function and neutron rate of a D-T plasma being heated by a lower hybrid wave. It is shown that Q-values approaching 1 are possible for sufficiently high bulk temperatures and for small deuterium concentrations.

Lower hybrid wave heating experiments have been carried out on several tokamaks; in several cases the production of fast-ion tails due to RF injection was reported [1-5]. Specifically, the Alcator-A lower hybrid experiment observed the formation of an energetic ion tail in the plasma centre having a tail temperature $T_T > 15$ keV and extending out to energies $E > 50$ keV [4, 5]. Such ion tails are predicted by the quasi-linear theory of Karney [6]. It has been shown that this theory is not inconsistent with the ion tail observations of the Wega experiment [7]. Furthermore, a Monte Carlo quasi-linear ion heating code has shown consistency between its results and the Alcator-A RF-produced ion tails [8, 9]. These results demonstrate a reasonable agreement between experimental results and the quasi-linear theory of lower hybrid ion heating.

Stix [10] has employed quasi-linear theory to calculate the neutron rates and Q-values of a D-T plasma being heated by ion cyclotron waves. In this paper, we calculate the Q-values of a D-T plasma being heated by lower hybrid waves, employing the quasi-linear formalism of Ref. [6]. Here we shall show that Q-values approaching 1 are achievable with lower hybrid heating by reducing the deuterium fraction and increasing the bulk plasma temperature.
Karney [6] has shown that in steady state the ion distribution function can be approximated as

\[ f(\xi) = F(u_\perp) \frac{\omega^2}{2\pi^2} \exp \left( -m_i \frac{\omega^2}{2T_i} \right) \]

where

\[ F(u_\perp) = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \frac{u_\perp}{\sqrt{1 + D_i(u_\perp)/C_i(u_\perp)}} \exp \left( -\frac{u_\perp^2}{2(1 + D_i(u_\perp)/C_i(u_\perp))} \right) \]

The following definitions are from Refs [8, 9]:

\[ D_i(u_\perp) = \frac{e^2 E_0^2}{2m_i^2} \frac{\omega^2}{u_\perp^2} \int_{k_{\perp,\min}}^{k_{\perp,\max}} \frac{dk_{\perp}}{\Delta k_{\perp}} \frac{G(k_{\perp})}{k_{\perp}^2 v_{\perp,\max}^2 - \omega^2} \]

\[ C_i(u_\perp) = \sum_i \left( \frac{1}{2} \frac{v_i^0}{v_{\perp,\max}^2} + \frac{1}{4} \frac{v_i^\alpha}{T_i/m_i} \right) \]

\[ v_i^\alpha \text{ and } v_i^0 \text{ are defined in Ref. [11], } v_i^0 = T_i/m_i, \text{ and all species temperatures are equal. The summation is over all ion species and electrons, and} \]

\[ \int G(k_{\perp})dk_{\perp} = \Delta = k_{\perp,\max} - k_{\perp,\min} \]

\[ G(k_{\perp}) \text{ is the spectral shape of } E_0^2(k_{\perp}). \text{ For a } D-T \text{ plasma,} \]

\[ C_D(u_\perp) = \frac{6\pi e^4 \ln A}{m_i^2 T_i^4} \times \left[ \frac{v_i^\alpha}{v_{\perp,\max}^2} \left( \delta + \frac{2}{3} \left( \frac{1}{2} + \frac{m_D}{m_T} \right) (1 - \delta) \right) + \frac{v_D^2}{v_D^0} \right] \]

\[ C_T(u_\perp) = \frac{6\pi e^4 \ln A}{m_i^2 T_i^4} \times \left[ \frac{v_T^\alpha}{v_T^0} \left( (1 - \delta) + \frac{2}{3} \left( \frac{1}{2} + \frac{m_T}{m_D} \right) \delta \right) + \frac{v_T^0}{v_T^D} \right] \]

\[ \delta = n_D/n_T \]

where

\[ u_D = 6.99 u_D \quad u_T = 7.48 u_T \]

Here we have let \( T_e = T_i = T \). The last term in brackets in Eq.(2) is due to drag on electrons and decreases as \( T_e \) increases (this term dominates at high \( v_{\perp,\max} \)). The power dissipated by the wave is [6]:

\[ P_d = \frac{2\pi n_e}{T} \left[ \int_0^{u_D} \frac{du_D F_D(u_D) D_D(u_D) v_D^3 m_D^2}{1 + D_D(u_D)/C_D(u_D)} \right] \]

\[ + (1 - \delta) \int_0^{\infty} \frac{du_T F_T(u_T) D_T(u_T) v_T^3 m_T^2}{1 + D_T(u_T)/C_T(u_T)} \]

The neutron rate per unit volume due to tail-bulk collisions is then

\[ R_N = n_D^2 (1 - \delta) \times \int_0^{\infty} \frac{2\pi u_D du_D \sigma_{DT}(u_D) (F_D(u_D) + F_T(u_D))}{u_D^2 a_D + \gamma_D u_D^2 + v_D^2} \]

where \( \sigma_{DT}(u_D) \) is the neutron production cross-section and is a function of the relative ion velocity. We then see that the ratio between neutron and alpha-particle production power and the RF power dissipation is

\[ Q = R_N E_N / P_d, \text{ where } E_N = 17.6 \text{ MeV. For } (v_T) \gg \omega / k_{\perp,\max}, \text{ Eq.(1) becomes:} \]

\[ F_D(u_\perp) \approx F_D \left( \frac{\omega}{k_{\perp,\max}} \right) \times \exp \left[ -\int_{u_D/a_D}^{\infty} \frac{du_D (u_D/a_D)^2 (v_D^2 a_D + v_D^2)^2}{u_D^2 a_D + \gamma_D u_D^2 + v_D^2} \right] \]

where

\[ \gamma_D = \delta + \frac{2}{3} \left( \frac{1}{2} + \frac{m_D}{m_T} \right) \]

\[ \gamma_D = \frac{1}{2} \frac{e^2 E_0^2}{m_D^4 k_{\perp,\max}^3 v_D^2 C_D} \]

\[ C_D = \frac{6\pi e^4 \ln A}{m_i^2 T_i^4} \]

Thus, the shape of the ion tail is strongly characterized by \( \gamma_D \), and its amplitude is determined by \( \omega / k_{\perp,\max} \).

Figure 1 shows graphs of typical deuterium and tritium ion tails versus \( E_\perp \), the perpendicular ion energy for \( \gamma_D = 7.05 \) and \( \gamma_D = 113 \). Here, parameters similar to those anticipated in the Alcator-C lower hybrid heating experiment [12] are used. \( k_{\perp,\max} \) and \( k_{\perp,\min} \) are determined from the dispersion relation

\[ k_i^4 E_{\pi z,0} + k_i^5 E_{\pi z,0} + k_i^2 E_{\pi z,0} = 0 \]

where

\[ E_{\pi z,0} = 1 + \frac{\omega_{po}^2}{\omega_{ee}^2} - \sum_i \frac{4\pi n_i e^2 \delta_i}{m_i \omega_i^2} \]

\[ E_{\pi z,0} = 1 - \frac{\omega_{po}^2}{\omega_{ee}^2} \]

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\[ \epsilon_{ee2} = -\frac{3}{4} \frac{T}{n_e^2} - 3 \sum_i \frac{4 \pi n_i^2 \delta_i \omega^2}{m_i \omega^2} \]

\[ \delta_i = n_i/n_e \]

and where the electric field spectrum is chosen to extend from \( k_{z\text{min}} = 2.75 \omega/c \) to \( k_{z\text{max}} = 3.25 \omega/c \). We see that, as \( \gamma_D \) is increased, the extent in energy of the ion tail increases and, in this case, \( Q \) increases from 0.049 to 0.246. We also note that the deuterium tail is generally of greater amplitude than the tritium tail. This ratio of the tritium tail amplitude to the deuterium tail amplitude at \( \nu_i \) just greater than \( \omega/k_{l\text{max}} \) is

\[ \text{RATIO} = \exp \left( -\frac{\omega^2}{2k_{l\text{max}}^2} (m_T - m_D) \right) \] (7)

Nevertheless, the tritium tail will be flatter since \( \gamma_T \) is larger (\( \gamma_T \approx m_i \)).

\[ Q = \frac{E_D(\epsilon_{ee2} + 2m_e k_{l\text{max}}^2)}{8 \pi k_{l\text{max}}} \] (9)

For \( \delta_A \sim 1 \) the RF power would be absorbed in the order of one pass. For \( \delta_A \ll 1 \), many passes would be required to absorb the RF power (which might result in edge plasma heating), whereas, for \( \delta_A > 1 \), the RF power would be absorbed at the edge of \( \Delta V \). For \( \delta_A \sim 1 \), the following calculation of \( Q \) is meaningful. (A ray tracing calculation would be necessary to calculate heating details and the fraction of incident RF power that can penetrate to the plasma centre.) For a tokamak, \( \delta_A = (P_D/S) t_s \) (where \( t_s \) is the minor radius of the heating volume).

Figure 2 shows graphs of \( Q \), \( \delta_A \) and \( R \), the ratio between the RF-tail-produced neutron rate and the thermal plasma neutron rate for \( \delta = 0.5 \) and \( 0.1 \). We see that higher values of \( Q \) are achievable with lower \( \delta \). This is easily explainable by noting that both the RF power dissipation and the neutron rate are dominated by the deuterium tail. From Eqs (1—4), we can then deduce \( Q \) for \( \gamma_D > 1 \):

\[ Q \approx \frac{(1 - \delta) m_D E_D}{6 \pi \epsilon^2 k_{l\text{max}}} \left[ \frac{\int_{\nu_i}^{\infty} d\nu_i \nu_i F_D(\nu_i) V_D(\nu_i) \sigma_{DT}(\nu_i)}{\int_{\nu_i/k_{l\text{max}}}^{\infty} d\nu_i F_D(\nu_i)(1 + \frac{\nu_i}{u_0})} \right] \] (10)

From Eq. (10) we see that \( Q \) is not directly dependent on \( n_e \), except that, as \( n_e \) increases, \( k_{l\text{max}} \) increases (from Eq. (6)), which then increases \( P_D \) and lowers \( Q \). \( Q \) is proportional to \( (1 - \delta) \) and will increase as \( T \) increases, which increases \( V_D \) and lowers the electron drag on the deuterium ions. Lowering \( n_e \) sufficiently will raise \( Q \) by lowering \( k_{l\text{max}} \); however, as this is done, \( \delta_A \) also rapidly decreases and the absorption becomes too weak for the resulting high \( Q \)-values to have any real meaning. Also, as \( \gamma_D \) continues to increase, \( Q \) will decrease, as \( \sigma_{DT}(\nu_i) \) decreases at sufficiently large energy while the electron drag term grows. Finally, these results are only useful when \( R \gg 1 \), as only then is the deuterium tail sufficiently large.
FIG. 2. $Q, \delta_A$, and $\log_{10}(R)$ versus $\gamma_D$ for $f = 4.6$ GHz, $T = 2$ keV, $\Delta r = 5$ cm, $k_{z\text{max}} = 3.25 \omega/c$, $k_{z\text{min}} = 2.75 \omega/c$ and $B_T = 12$ T. (a) $\delta = 0.5$ and $n_e = 9 \times 10^{14}$ cm$^{-3}$; (b) $\delta = 0.1$ and $n_e = 1.2 \times 10^{15}$ cm$^{-3}$. In both cases, $n_e$ has been optimized for the best $Q$ consistent with $\delta_A \sim 1$.

FIG. 3. $Q, \delta_A$, and $\log_{10}(R)$ versus $\gamma_D$ for $f = 4.6$ GHz, $T = 5$ keV, $\Delta r = 5$ cm, $k_{z\text{max}} = 3.25 \omega/c$, $k_{z\text{min}} = 2.75 \omega/c$, $B_T = 12$ T, $\delta = 0.1$ and $n_e = 6 \times 10^{14}$ cm$^{-3}$. Compared with the thermal tail to make a consideration of an RF $Q$-value meaningful.

Figure 3 is a graph of $Q$ versus $\delta_D$ for $T = 5$ keV. We see that, for larger $T$, $Q$ can approach 1. While in this case $\delta_A < 1$, it is proportional to $\Delta r$; picking a larger heating minor radius would increase it linearly. Figure 3 employs $2.75 < k_z c/c < 3.25$; these values of $k_z$ are not strictly appropriate, as they would subject the lower hybrid wave to strong electron Landau damping when $T_e = 5$ keV. Figure 4 shows a similar graph, with $2.0 < k_z c/c < 2.5$ and with $n_e$ now increased to $1.0 \times 10^{15}$ cm$^{-3}$. The previous $Q$-values are approximately recovered; the higher density is only necessary to restore $k_{z\text{max}} \sim 160$/cm.

In conclusion, we have calculated the lower hybrid wave damping, neutron rates and $Q$-values of a D-T plasma. For properly selected $n_e$, $T > 5$ keV, and small deuterium fraction, values of $Q \sim 1$ can be achieved.

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ALPHA-PARTICLE DIAMAGNETIC DRIFT EFFECTS ON TOKAMAK BALLOONING STABILITY

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ABSTRACT. The diamagnetic drift frequency impact on the ballooning stability of INTOR-like tokamaks that contain a Maxwellian energetic ion species is investigated. The energetic ion diamagnetic drift frequency contribution enhances the critical beta ($\beta_c$) imposed by ballooning modes with toroidal mode numbers $n \geq 7$ by a factor of 1.2 to 1.3 above the corresponding limits obtained when the plasma consists of only thermal species.

Ideal magnetohydrodynamics (MHD) has predicted that ballooning instabilities with an infinite toroidal mode number ($n \rightarrow \infty$) can set the limiting beta values to be achieved in tokamaks [1–5]. However, this class of instability is characterized by wavelengths that can be comparable to the Larmor radii of the ionic species of the plasma. Therefore, kinetic modifications to the MHD predictions can be very important [6–9]. Significant stabilizing effects have been reported from finite diamagnetic drift frequency [10] and other kinetic effects [11] in model tokamak equilibria with a large-aspect-ratio circular cross-section. A similar stabilizing trend is observed with diamagnetic drift frequency effects in realistic tokamak equilibria [12–13]; yet, these estimates have been limited to thermal electron single-ion-species plasmas. The estimates of the contributions of the injected fast ions in present-day, high-power neutral-beam-heated tokamaks such as the Impurity Study Experiment (ISX-B) and in tokamak reactor designs of alpha particles such as the International Tokamak Reactor (INTOR) typically exceed 20% of the total beta of the device. The contributions of the energetic ions can exceed 30% of the total beta at the magnetic axis. In addition, the energy content per particle of the fast ion species exceeds the energy content per particle of the thermal ion species by roughly a factor of 20 in ISX-B and 90 in INTOR. Therefore, the Larmor radii of the fast ions can be significantly larger than the Larmor radii of the background thermal ions. It is thus important to