Time Dependent Modeling of the Markarian 501 X-ray and TeV Gamma-Ray Data Taken During March and April, 1997

H. Krawczynski$^{1,2}$, P.S. Coppi$^1$, F. Aharonian$^3$

$^1$ Yale University, P.O. Box 208101, New Haven, CT 06520-8101, USA
$^2$ Now at: Washington University in St. Louis, Phys. Dept., 1 Brookings Drive, Campus Box 1105, St. Louis, MO 63130, USA
$^3$ Max Planck Institut für Kernphysik, Postfach 103980, D-69029 Heidelberg, Germany

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ABSTRACT
If the high-energy emission from TeV blazars is produced by the Synchrotron Self-Compton (SSC) mechanism, then simultaneous X-ray and gamma-ray observations of these objects are a powerful probe of the electron (and positron) populations responsible for this emission. Understanding the emitting particle distributions and their temporal evolution in turn allows us to probe physical conditions in the inner blazar jet and test, for example, various acceleration scenarios. Furthermore, by constraining the SSC emission model parameters, such observations enable us to predict the intrinsic (unabsorbed) gamma-ray energy spectra of these sources, a major uncertainty in current attempts to use gamma-ray observations to constrain the intensity of the Diffuse Extragalactic Background Radiation (DEBRA) at optical/infrared wavelengths. As a next step in testing the SSC model and as a demonstration of the potential power of coordinated X-ray and gamma-ray observations, we model in detail the X-ray and gamma-ray light curves of the TeV blazar Mrk 501 during its April-May 1997 outburst with a time dependent SSC model. Extensive, quasi-simultaneous X-ray and gamma-ray coverage exists for this period. We discuss and explore quantitatively several of the flare scenarios presented in the literature. We show that simple two-component models (with a soft, steady X-ray component plus a variable SSC component) involving substantial pre-acceleration of electrons to Lorentz factors on the order of $\gamma_{\text{min}} = 10^5$ describe the data train surprisingly well. All considered models imply an emission region that is strongly out of equipartition and low radiative efficiencies (ratio between kinetic jet luminosity and comoving radiative luminosity) of 1 per-mill and less. Degeneracy in both, model variant and jet parameters, prevents us to use the time resolved SSC calculations to substantially tighten the constraints on the amount of extragalactic gamma-ray extinction by the DEBRA in the relevant 0.5-50 microns wavelength range, compared to earlier work.

Key words: galaxies: BL Lacertae objects: individual (Mrk 501) — galaxies: jets — X-rays: galaxies: — gamma rays: theory

1 INTRODUCTION

1.1 EGRET Blazar Observations

The EGRET detector on board the Compton Gamma-Ray Observatory showed that many blazars are copious gamma-ray emitters (Hartman et al. 1999), their power at gamma-ray energies being comparable to (for low luminosity sources, i.e. BL Lac objects) or dominating by a wide margin (for high luminosity sources, i.e., FSRQs, Flat Spectrum Radio Quasars, and OVVs, Optically Violently Variables) the power emitted at longer wavelengths. The nonthermal radiation component probably originates from a population of relativistic particles embedded in the collimated outflow (jet) from a super-massive ($10^8$ up to several times $10^9 M_\odot$) black hole. The nonthermal continuum emission is commonly explained with Synchrotron Compton (Ulrich et al. 1997; Sikora & Madejski 2001) models: embedded in a jet which approaches the observer with relativistic velocity, a population of high energy electrons emits Synchrotron radiation at longer wavelengths and at shorter wavelengths, Inverse Compton (IC) radiation of high energy electrons off lower energy seed photons. The origin of the seed photons
is still uncertain (e.g. Blazejowski et al. 2000). The seed photon source could be “external” to the jet, e.g., radiation scattered and reprocessed by ambient matter in the Broad Line Region near the black hole, or infrared radiation emitted by dust in the inner nucleus of the host galaxy (External Compton models). Alternatively, the dominant seed photons are synchrotron photons from the same electron population responsible for the IC scattering (SSC, Synchrotron Self Compton models). In a generic source, both external and internal seed photons could be important in producing the observed spectrum. In the following we use the term Synchrotron Compton models if we do not want to specify the source of the seed photons.

Alternative models, so-called “hadronic” models, invoke hadronic interactions of a highly relativistic outflow which sweeps up ambient matter (Pohl & Schlickeiser 2000), interactions of high energy protons with gas clouds moving across the jet (Dar & Laor 1997), or, interactions of extremely high energy protons with ambient photons (Mannheim 1998), with the jet magnetic field (Aharonian 2000), or with both (Mücke et al. 2002). If the reported fluxes of the diffuse infrared background between 60 and 100 micron (Lagache et al. 1999, Finkbeiner et al. 2000) correctly describe the DEBRA intensity in the far-infrared band, the “reconstructed” spectrum of Mrk 501, corrected for intergalactic absorption, may contain a sharp pile-up at and above 15 TeV. The latter cannot be explained by conventional Synchrotron Compton models. It has been argued that the presence of such a pile-up can be explained by bulk-motion comptonization (in the deep Klein-Nishina regime) of the ambient radiation by a ultra-relativistic conical cold outflow with a bulk Lorentz factor of $\gtrsim 10^7$, while the remaining part of the spectrum could be explained by a conventional SSC model (Aharonian et al. 2002).

All these models have some degree of success in explaining the overall Spectral Energy Distribution (SED) of gamma-ray blazars. However, one can break much of the apparent degeneracy between these models by taking advantage of the rapid, large-scale time variability these sources exhibit. Different models, for example, produce emission at a given frequency using particles of different energies, interaction cross-sections, and cooling times. The response of different models to changes in source conditions or the injection of fresh new particles is therefore different and in principle distinguishable – provided that one has sufficient time resolution to fully sample the flux variations and sufficient frequency coverage to constrain the different emission components that may be present.

In view of this potential payoff, considerable effort has been dedicated to carrying out multi-wavelength observations on powerful EGRET blazars like 3C 279 (Wehrle et al. 1998). While the campaigns have lent considerable support to Synchrotron Compton models, the results of the campaigns were not as conclusive as one might have hoped. The reasons for this are three-fold:

(i) These blazars turned out to be highly variable on timescales down to at least hours (Mattox et al. 1997; Wagner et al. 1997). Even for the brightest objects, the instrument available for the gamma-ray observations, EGRET, simply did not have enough collection area to track all the gamma-ray flux variations, let alone provide high quality energy spectra.

(ii) In typical models the electrons responsible for the GeV EGRET IC flux emit their synchrotron radiation at $\sim$-UV energies. However, UV observations are difficult if not impossible because of atmospheric and galactic absorption. Thus the simultaneous observations that were made, e.g., at gamma-ray and X-ray energies, tracked radiation from electrons with very different energies and different cooling times and thus potentially different time histories and perhaps even emission regions.

(iii) The observations showed that the gamma-ray emission in several EGRET blazars is not consistent with the SSC model, the simplest version of Synchrotron Compton models (see e.g. the comprehensive modeling of 3C 279 broadband data described by Hartman et al. 2001). The necessity to consider in External Compton models alternative seed photon fields substantially complicates the unambiguous interpretation of the data, especially since along our line of sight the beamed emission from the jet often dominates, making direct observation of these other photon fields difficult.

1.2 Potential of TeV Blazar Observations

The second class of gamma-ray emitting blazars that EGRET discovered, the low power BL Lac objects like Mrk 421, were initially passed over as targets for extensive multi-wavelength campaigns since they were too weak in the EGRET band. The arrival of ground-based gamma-ray detectors like Whipple, HEGRA, and CAT with detection areas on the order of $10^5$ m$^2$, however, now allows us to follow their gamma-ray fluxes on minute timescales (Gaidos et al. 1996) and to routinely obtain detailed spectral information on timescales down to one hour (Aharonian et al. 1999a). Besides their better accessibility at gamma-ray energies, these low power objects have several other important advantages. BL Lacs and their likely FR-I radio galaxy parent population appear to have underluminous accretion disks, i.e., “external” photon fields may not be important as seeds for IC scattering (Chiaberge et al. 1999). This together with the fact that their time-averaged SEDs have successfully been described with one-component SSC models, strongly suggest that SSC models which have much fewer free parameters than External Compton models indeed apply. Also, perhaps because of the lower internal and external radiation fields and thus lower radiative losses (Ghisellini et al. 1998), the characteristic electron energies appear to be higher for the lower power objects, moving their synchrotron emission peak out of the UV, squarely into the X-ray range, where individual flares strongly dominate the overall luminosity and can readily be observed with broad-band X-ray satellites like RXTE and BeppoSAX.”

In the SSC model, the IC peak then moves from GeV to $\sim$-TeV energies. Thus, simultaneous X-ray and TeV gamma-ray observations follow the evolution of the electron population responsible for the bulk of the source luminosity, and the observations are well-matched in the sense that they track the emission from the same electrons, providing tight constraints on the electron distribution and its time evolution. SSC models that apply to these objects are therefore testable, especially with the
next generation of X-ray and gamma-ray detectors coming on line in the next few years.

Proving whether an SSC model works or not has a potentially large payoff. If the model does not work, then we must significantly revise our understanding of the physical conditions and processes in these objects. If it does work, then we can use it for example to probe the acceleration processes at work in the innermost region of the jet. We can also use it to constrain the amount of extragalactic gamma-ray extinction due to pair production processes on the diffuse optical/infrared background (e.g., Schréd 1966; Stecker, De Jager & Salamon 1992), by comparing the predicted intrinsic TeV gamma-ray energy spectrum with the observed one (Coppi & Aharonian 1999, Krawczynski et al. 2000, called “Paper I” in the following).

Based on these considerations, the brightest TeV blazars, Mrk 421 (z = 0.031) and Mrk 501 (z = 0.034), have been the subject of increasingly intensive observing campaigns. This has led to the discovery of pronounced TeV gamma-ray / X-ray flux correlations for Mrk 421 (Buckley et al. 1996, Takahashi et al. 1996, Maraschi et al. 1999, Takahashi et al. 2000) and Mrk 501 (Pian et al. 1998, Djamati-Atai et al. 1999, Paper I, Sambruna et al. 2000, see also Fig. 1 of this paper).

1.3 Goal of this Paper and Relation to Previous Work

The goal of this paper is to extend the analysis of Paper I, which was a first joint analysis of an unprecedented set of X-ray/TeV monitoring data taken during the 1997 flare of Mrk 501. Using RXTE (X-ray) and HEGRA (TeV) observations that were simultaneous to within a few hours (i.e. less than the ~12 hour characteristic variability time scale of the source), we showed that the X-ray flux of the source, particularly above 10 keV, was strongly correlated with the TeV flux, in accord with Synchrotron Compton models. Moreover, we found that for that two month period the data were consistent with the quadratic relation expected between the X-ray and TeV fluxes by up to 30 times. It is highly probable that these individual flares were produced by single jet regions, rather than being the superposition of several, causally not connected events. In addition, the physical conditions in all the emission regions seemed to be very similar: the X-ray and TeV gamma-ray fluxes were well correlated during more than two months (see Paper I, and this paper Fig. 1), and the TeV energy spectrum stayed remarkably stable during more than 6 months (Aharonian 1999a-c). As a final justification of our approach, the analysis presented in Paper I showed that the X-ray and TeV gamma-ray fluxes typically varied on time scales of ~1 day with shortest flux rise and decay times on the order of half a day. Kataoka et al. (2001) and Tanihata et al. (2001) analyzed RXTE and ASCA data taken during the years 1997-2000 and find, in accord with our results, that Mrk 501 has a low duty cycle for flares on time scales of a few hours and shorter. Thus, the sampling of the data with 2 X-ray observations and several TeV gamma-ray observations per day was probably sufficient for giving a rough picture of how X-ray and TeV gamma-ray fluxes evolved in time.

Observational signatures of Synchrotron Compton models have been described by various authors (see e.g. the references given in Table 1). In the following we show for the first time an attempt to fit a prolonged sequence of X-ray and TeV gamma-ray data with a time dependent SSC code. This approach makes it possible to use the full information encoded in the correlated flux variability at different wavelengths. In contrast to parametric SSC fits (see e.g. Paper I, Tavecchio et al. 2001) the method uses a self-consistently evolved electron population which assures that the assumed electron energy spectrum is physically realizable from an initial acceleration spectrum (see also the discussion by Mastichiadis & Kirk 1997). We think that the approach of using a time resolved analysis to break model degeneracies will become increasingly more powerful and important as the sensitivity and energy coverage of X-ray and gamma-ray instruments continue to improve. Note that a thorough understanding of the SSC model is also a necessary prerequisite for the evaluation of External Compton models which always include a SSC component.

The rest of this paper is structured as follows. In Sect. 2 we introduce the data set and show an updated version of the X-Ray/TeV gamma-ray flux correlation. In Sect. 3 we describe the model calculations and in Sect. 4 the time dependent model fits. Finally, we discuss the results in Sect. 5.
2 THE DATA SET

During 1997 the BL Lac object Mrk 501 went into a remarkable state of continuous strong flaring activity and the source was intensively monitored at X-rays and TeV gamma-rays. During April and May, 1997 the source was regularly observed with the RXTE X-ray satellite, with typically two pointed observations per day of ~20 min duration (Paper I). Each pointing resulted in a high accuracy measurement of the 3-25 keV X-ray flux and photon index with an accuracy which was only limited by systematic effects. The curvature of the X-ray spectrum could be assessed for a couple of pointings with relatively high X-ray fluxes and long integration times. On three days (April 7, 13th, and 16) the source was also scrutinized with the BeppoSAX X-ray telescopes, revealing the X-ray energy spectrum of the source over the broad energy range from 0.1 keV to ~200 keV (Pian et al. 1998).

In Paper I we studied the correlation of the X-ray fluxes with the TeV gamma-ray fluxes as measured with the HEGRA Cherenkov telescope system (Aharonian et al. 1999a). For the present study we complemented the data set with the TeV fluxes from the HEGRA CT1 (Aharonian et al. 1999c), Whipple (Quinn et al. 1999), and CAT (Djannati-Atai et al. 1999) telescopes. In Paper I we found a very tight correlation between the 25 keV and 2 TeV fluxes. The flux variability amplitude was approximately 3 times larger at TeV than at X-ray energies, being consistent with a quadratic relationship. An updated version of the X-ray/TeV gamma-ray flux correlation is shown in Fig. 1. The additional X-ray/TeV gamma-ray flux pairs confirm the previous finding of a clear flux correlation. However, the quality of the correlation still does not allow us to differentiate between a quadratic X-ray/TeV gamma-ray relationship and a linear one with a non-zero X-ray flux offset.

The X-ray as well as TeV gamma-ray data are plagued by systematic errors. In the case of the BeppoSAX data the spectral index below 1 keV is not well determined due to uncertainties in the neutral hydrogen column density. Above 50 keV the scatter of the data points increases more than the uncertainties in the neutral hydrogen column density. Above 1 keV the scatter of the data points increases more than the statistical errors, indicating systematic uncertainties in the neutral hydrogen column density. Above 50 keV the scatter of the data points increases more than the statistical errors, indicating systematic uncertainties in the neutral hydrogen column density. Above 1 keV the scatter of the data points increases more than the statistical errors, indicating systematic uncertainties in the neutral hydrogen column density. Above 50 keV the scatter of the data points increases more than the statistical errors, indicating systematic uncertainties in the neutral hydrogen column density. Above 1 keV the scatter of the data points increases more than the statistical errors, indicating systematic uncertainties in the neutral hydrogen column density. Above 50 keV the scatter of the data points increases more than the statistical errors, indicating systematic uncertainties in the neutral hydrogen column density.

The spectral variability at TeV energies has been a matter of debate: the HEGRA group did not detect spectral changes with an accuracy (1-5 TeV photon index) of ~0.2 and 0.05 for diurnal and flux selected mean energy spectra, respectively. The CAT group reported the statistically significant detection of a hardness intensity correlation based on the $F(>900\text{ GeV})/F(>450\text{ GeV})$ hardness ratio, corresponding to a ~0.25 change in photon index. The two data sets overlapped only partially in time: the HEGRA group did i.e. not take data on April 16, 1997, which is the most important day in the CAT analysis. Konopelko et al. (1999) noted that the stability of the TeV energy spectra, evident in the HEGRA data, might be used to infer constraints on the intensity of the DEBRA. In the plots shown below, we cross-calibrated the BeppoSAX data relative to RXTE measurements taken at approximately the same time.

Compared to the results shown by Pian et al. (1998) we reduce the normalization of the BeppoSAX PDS data by up to 35% which eliminates the discontinuity of the joint BeppoSAX MECS, LECS, and PDS energy spectra at ~15 keV (between the energy coverage of the LECS and PDS instruments) and is then consistent with the spectral shapes simultaneously measured from 3 keV to 25 keV with RXTE. We also cross-calibrated the CAT, HEGRA CT1, and Whipple gamma-ray fluxes relative to the ones measured by the HEGRA CT System. Although we obtained a list of CAT fluxes as function of the integer MJDs of the observations, the fractional MJDs of the CAT observations are not known to us. In the following we centered the CAT observations at 12 am UTC.

3 MODELING

3.1 Time Dependent SSC Code

The SSC code (Coppi & Blandford 1990; Coppi 1992) assumes a spherical emission region of radius $R$ which is filled with an isotropic electron population and a randomly oriented magnetic field $B$ and which approaches the observer relativistically. The motion of the jet toward the observer can be characterized with the jet Doppler factor, defined by

$$\delta_j^{-1} = \Gamma(1 - \beta \cos(\theta)), \quad (1)$$

where $\Gamma$ is the Lorentz factor, $\beta$ is the bulk velocity of the jet, and $\theta$ is the angle between the jet axis and the line of sight.
with \( \Gamma \) the bulk Lorentz factor of the emission plasma, and \( \beta \) its bulk velocity in units of the speed of light, and \( \theta \) is the angle between jet axis and the line of sight in the observer frame. The TeV gamma-ray flux variability on time scale \( \Delta T_{\text{obs}} \approx 12 \text{ hr} \) (Aharonian et al. 1999a) together with causality arguments set an upper limit on the radius of the emission volume:

\[
R \lesssim \delta_1 c \Delta T_{\text{obs}}
\]  

(2)

If the jet moves along a curved path more rapid flares could result from a change of the Doppler factor as the jet’s radiation beam sweeps across the observer.

The kinetic equations, discretized in energy, take fully into account the non-continuous character of IC processes in the Klein-Nishina regime, and are evolved in time with a two step implicit scheme treating first the photon distribution and subsequently the electron distribution. The length of time steps is chosen such that the number of photons and particles per energy bin changes per step by less than 20%.

The kinetic equation for the photon density (per unit volume and energy) \( n_\gamma \) reads:

\[
\frac{\partial n_\gamma}{\partial t} = q_\gamma - p_\gamma - \frac{c}{R(1+\kappa)} n_\gamma
\]  

(3)

where \( q_\gamma \) and \( p_\gamma \) are the rate of photons being produced into and out of the energy interval \([\epsilon, \epsilon + d\epsilon]\) due to electromagnetic field, electron-photon and 2-photon interactions. The last term of the right hand side represents photons which escape from the emission region. The factor \( cR^{-1}\) in the last term assures that the photon density approaches steady state values only with a rise/decay constant longer than the light crossing time. The factor \( (1+\kappa(\gamma)) \) parameterizes the modification of the photon escape time by Compton processes (Coppi 1992); however, for all the models discussed in the following, we have always \( \kappa \ll 1 \). The Klein-Nishina effect decisively influences the resulting gamma-ray energy spectrum and proper modeling is imperative.

The kinetic equation of the electron (and possibly positron) density \( n_e \) reads:

\[
\frac{\partial n_e}{\partial t} = Q_e - \frac{\partial}{\partial \gamma} \left[ \gamma_{\text{cont}} n_e \right] + q_e - p_e - \frac{n_e}{\tau_{e,\text{esc}}} \]  

(4)

with \( Q_e(\gamma, t) \) from Eq. (5), \( \gamma_{\text{cont}} \) gives the decrease of an electron’s Lorentz factor per unit time due to continuous energy losses, and \( q_e \), \( p_e \), \( d\gamma \), and \( \tau_{e,\text{esc}} \) are the rate of particles being produced or scattered into and out of the Lorentz factor interval \([\gamma, \gamma + d\gamma]\) due to non-continuous energy loss processes, respectively. The last term of the right hand side represents an energy independent escape probability of electrons from the emission region.

To first order approximation our code takes the non-vanishing source extension into account through the last term in Eq. 3. As a consequence, the code is able to describe flux variations even on time scales on the order of \( R/c \) in a qualitatively correct way. We limit ourselves in this paper to describe the time variable emission component with a one-zone SSC model. A one-zone model is able to approximate multi-zone models as long as the spatial gradients of the magnetic field and the non-linear components in the properly modified kinetic equations (3) and (4) are small. Our code can i.e. mimic “linearized inhomogeneous models” as discussed by Kirk, Rieger & Mastichiadis (1998) and Chi-}

aberge & Ghisellini (1999). While External Compton models can be dominantly linear, the SSC model is inherently non-linear: the synchrotron component directly follows the evolution of the electron population, but the IC component results from the interaction of the electron population with the self-produced synchrotron photons. Since electrons and synchrotron photons traverse the emission region on a time scale of \( R/c \), one expects that the IC component lags the synchrotron component by approximately one light crossing time (Coppi & Aharonian 1999). This is the most drastic time lag effect expected in the SSC model. For Mrk 501 however no such time lag has been observed so far, the upper limit being about 12 hrs (Aharonian et al. 1999a; Aharonian et al. 1999c; Paper I; Sambruna et al. 2000). As long as instrumental resolutions do not permit to resolve this time lag, we think it is safe to use only one component to describe the time variable emission.

We fit the full two months data train using a single emission volume. As we will point out in the discussion, it might be that individual flares (of durations on the order of \( \sim 1 \) day) are produced by independent emission regions. Upon flaring, a region would expand adiabatically, and thus fade away quickly. Even in this case, our model should give reasonable results for two reasons: (i) as it turns out the best fitting models have particle escape times on the order of the flux variability time scale; (ii) the tight X-ray/TeV gamma-ray flux correlation argues for a very similar size of the emission regions. As a consequence, each flare is produced by freshly accelerated electron populations and modeling the flares with one emission region gives similar results as using several disjoint emission regions.

### 3.2 Treatment of Particle Acceleration

Given the sparse observational sampling of our data set in time and wavelength, we did not embark on modeling the acceleration process in detail but used instead an “external” acceleration function. We parameterize the production rate of freshly accelerated particles as function of electron Lorentz factor \( \gamma \), spectral index of particle acceleration \( p \), and normalization \( Q(\tau) \), minimum Lorentz factor \( \gamma_{\text{min}} \), and high energy cut-off \( \gamma_{\text{max}}(\tau) \) as follows:

\[
Q(\gamma, t) = Q_0(\tau)^{-p} \exp \left( -\frac{\gamma}{\gamma_{\text{max}}(\tau)} \right) \Theta(\gamma - \gamma_{\text{min}})
\]  

(5)

with \( \Theta(x) = 0 \) for \( x < 0 \) and \( \Theta(x) = 1 \) for \( x \geq 0 \). We use the canonical value of \( p = 2 \) expected for diffusive particle acceleration at strong shocks (Bell 1978; Blandford & Ostriker 1978) and do not consider the ramifications arising from the non-linear modification of the shock structure due to the backreaction of accelerated particles (Bell 1987) and mildly or ultra-relativistic shock velocities (see the recent review by Kirk & Duffy 1999).

The low-energy cutoff in the spectrum of accelerated electrons \( \gamma_{\text{min}} \) is a critical model parameter. If the radiative cooling time of electrons with Lorentz factor \( \gamma_{\text{min}} \) is shorter than all the other characteristic time scales of the system, the main break of the electron spectrum occurs at \( \gamma_{\text{min}} \). Thus, at high enough values (\( \sim 10^8 \)), \( \gamma_{\text{min}} \) determines the energies at which the synchrotron and IC SEDs peak. On theoretical grounds one expects much lower values of between 1 and the proton to electron mass ratio \( m_p/m_e = 1836 \) (Hoshino et al. 1992; Levinson 1996; McClements 1997). We
will use in the following a relatively low value of $\gamma_{\text{min}} = 1000$ as the fiducial value and will discuss higher values at several points.

We characterize the acceleration luminosity $l_c$ by the pair-compactness parameter (Coppi 1992):

$$l_c = \frac{L_\gamma \sigma_T}{R m_e c^3} = \frac{8\pi R^2 \sigma_T}{3c} \int \gamma_0 Q(\gamma_0) d\gamma_0$$

(6)

3.3 Treatment of Extragalactic Extinction

The TeV gamma-ray spectra are expected to be modified by extragalactic extinction due to pair production processes. The uncertain DEBRA intensity in the relevant 0.5-50 microns wavelength range introduces a major uncertainty in the modeling of the source. While earlier estimates of the DEBRA level predicted negligible extinction at gamma-ray energies below $\sim 1$ TeV, more recent observational and theoretical efforts suggest that this might not be true (Primack et al. 2001). We think that model estimates of the DEBRA still have not reached the reliability that we should limit our computations to a specific DEBRA model. Rather we will treat the modification of the TeV flux level and energy spectrum as not fully constrained. Clearly, the DEBRA extinction does not modify the relative TeV gamma-ray flux variations and we use the information encoded in the relative extinction does not modify the relative TeV gamma-ray flux spectrum as not fully constrained. Clearly, the DEBRA extinction volume.

3.4 Fitting Procedure

The free parameters of our model are the radius of the emission region $R$, the jet Doppler factor $\delta_j$, the mean magnetic field $B$, the escape time of relativistic electrons from the emission region $t_{\text{esc}}$, the normalization of the electron acceleration rate $Q_0$, and the minimum and maximum Lorentz factors of accelerated particles $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$. We fit the April and May, 1997 RXTE 10 keV fluxes and 3-25 keV photon indices and the 2 TeV fluxes derived from CAT, HEGRA, and Whipple measurements. Given a hypothesis of what causes the flaring activity (a variable $Q_0(t)$, $\gamma_{\text{max}}(t)$, and/or $\delta_j(t)$) we fit the data in 2 steps:

(i) For a set of parameters $(R, \delta_j, B, \gamma_{\text{min}}, t_{\text{esc}})$ we determine the simplest possible function $Q_0(t)$ and $\gamma_{\text{max}}(t)$, or for some models $\delta_j(t)$, which fit the X-ray flux amplitudes. Hereby, “the simplest possible functions” means that given the X-ray flux measurements at times $t_i$ (in the jet frame)
(ii) According to the mechanism that determines the energies at which the synchrotron and IC SEDs peak. The SED peak energies are either determined by the minimum Lorentz factor \( \gamma_{\text{min}} \) of accelerated particles, or, by the balance between radiative cooling times and the shorter of particle escape time and the characteristic duration of individual flares (sometimes referred to as injection time scale, or, dynamical time scale of the jet).

The modeling of the full data train is computationally very intensive and we therefore focused on exploring only the models which seemed most promising to us. While the time resolved analysis clearly rules out some models, it gives fits of very similar quality for others. Our difficulties to distinguish between models mainly derive from two facts:

(i) From the limitations of the data set, namely sparse observational sampling in time and energy, and systematic errors on X-ray energy spectra and TeV gamma-ray fluxes. Keeping these limitations in mind, we discuss the fit results with a focus on pointing out which models are capable of very similar quality for others. Our difficulties to distinguish between radiative cooling times and the shorter of particle escape time and the characteristic duration of individual flares (sometimes referred to as injection time scale, or, dynamical time scale of the jet).

(ii) From the unknown modification of the TeV gamma-ray energy spectra by extragalactic extinction.

4.1 One-Component Models

4.1.1 Time variability through \( Q_0(t) \)

We first consider time variability through a varying rate of accelerated particles. If \( Q_0(t) \) varies, the SSC mechanism automatically produces a more than linear increase of TeV flux as function of X-ray flux. Fig. 2 shows the observed and modeled X-ray and gamma-ray flux amplitudes and photon indices. In Paper I, we derived a lower limit on the Doppler factor of 6.3. In most of the following models we use a rather high Doppler factor of \( \delta = 45 \), for two reasons: (i) a high Doppler factor allowed us to fit the data with a wide range of magnetic field values; for lower Doppler factors, weak magnetic fields result in a strong overproduction of TeV gamma-rays (see the related discussion by Krawczynski et al. 2001); (ii) non-negligible extragalactic extinction of TeV gamma-rays seems highly probable; the high Doppler factor results in predicted TeV energy spectra that agree with the observed ones after correcting for extragalactic extinction (which steepens the TeV energy spectrum).

Varying \( \gamma_{\text{max}} \) alone we did not achieve a satisfactory fit to the data. A typical result is shown in Fig. 3. As before the X-ray fluxes can be described to arbitrary precision. While the model has no difficulty in producing the observed range of X-ray photon indices, it fails to describe the X-Ray / TeV gamma-ray flux correlation: the predicted TeV gamma-ray fluxes hardly vary at all. Combinations of Doppler factor and magnetic field where the TeV flux changes more strongly than the X-ray flux result in steeper than observed TeV energy spectra. The time variation of \( \gamma_{\text{max}} \) causes large flux and spectral variability only at energies \( \gg 10 \) TeV where the inherent TeV energy spectrum is extremely soft (photon index \( \lesssim 2.5 \)). Extragalactic extinction cannot remedy this shortcoming since it is believed to steepen and not to soften the TeV energy spectra.

Tavecchio et al. (2001) studied parametric SSC model fits to Mrk 501 snapshot data and concluded that the maximum Lorentz factor of accelerated particles is mainly re-
Figure 2. X-ray and TeV gamma-ray data (data points) from April, 1997 (left column) and May, 1997 (right column) with SSC model fit (lines). Here, model flares are caused by a time dependent $Q_0(t)$. The panels show from top to bottom: (i) the logarithm of the injection compactness $l_i$ (see Eq. 6), (ii) the logarithm of the 10 keV X-ray energy flux (CGS units), (iii) the X-ray 3-25 keV photon index, (iv) the logarithm of the 2 TeV energy flux (CGS units), and (v) the 1-5 TeV photon index. The gamma-ray fluxes are from CAT (squares), HEGRA CT System (solid lines), HEGRA CT 1 (asterisks), and Whipple (open circles). The value for April 16 has been inferred from the energy spectrum published by Djannati-Atai et al. (1999). The vertical dotted lines show the days with BeppoSAX observations (April 7, 11, and 16 which will be discussed in more detail further below). The model parameters are: $\delta_1 = 45$, $R = 1.1 \times 10^{16}$ cm, $B = 0.014$ G, $\tau_{esc} = 10 R c^{-1}$, $\gamma_{\min} = 10^3$, $\gamma_{\max} = 2.5 \times 10^7$, $\xi = 0.5$, $\eta = 0.2$.

Figure 3. Same as in Fig. 2, but with flaring activity through a time dependent maximum Lorentz factor of accelerated particles $\gamma_{\max}(t)$. The upper panel shows here $\gamma_{\max}(t)$. The model parameters are: $\delta_1 = 45$, $R = 1.5 \times 10^{16}$ cm, $B = 0.009$ G, $\tau_{esc} = 3 R c^{-1}$, $\gamma_{\min} = 10^3$, $\xi = 0.5$, $\eta = 0.2$.

4.1.3 Time variability through $Q_0(t)$ and $\gamma_{\max}(t)$

Models in which both, $Q_0$ and $\gamma_{\max}$, change with time have been invoked to account for the secular changes of the Mrk 501 X-ray (Pian et al. 1998; Sambruna et al. 2000) and TeV gamma-ray (Aharonian et al. 2001a) energy spectra. In models of diffusive electron acceleration at strong shocks the electron acceleration rate $Q_0$ is determined by the rate with

$$\gamma_{\max} = Q_0^\alpha$$

and treat the exponent $\alpha$ as an additional free parameter of the fit. The $\chi^2$-value of the X-ray photon indices show a pronounced minimum for a value of $\alpha = 2$, and Fig. 4 shows a SSC fit to the data. The model describes the RXTE and gamma-ray data rather satisfactorily.

During the first 3 days of the April campaign the model X-ray indices are by $\sim 0.1$ harder than the observed ones, a discrepancy which is shared also by all subsequent models. Compared to other days of similar X-ray flux levels, the X-ray spectrum of the first three days with RXTE coverage was very soft, indicating that the source properties did evolve during the 2 months campaign.

Although only 3 BeppoSAX observations were performed during 1997, the data is very constraining since it covers the broad energy range from 0.1 keV to $\sim 200$ keV. The long BeppoSAX pointings of $\geq 12$ hrs duration bracketed the RXTE and TeV gamma-ray observations. The
Figure 4. Same as in Fig. 3, but with flaring activity through time dependent $Q_0(t)$ and $\gamma_{\text{max}}(t)$ with $\gamma_{\text{max}} \propto Q_0^2$. The upper panel shows here $Q_0(t)$. The model parameters are: $\delta_1 = 45$, $R = 3.2 \times 10^{15}$ cm, $B = 0.035$ G, $t_{\text{esc}} = 3 R c^{-1}$, $\gamma_{\text{min}} = 10^3$, $\xi = 0.5$, $\eta = 0.1$.

Figure 5. For the model of Fig. 4 the upper panel shows the electron energy spectra ($E^2 dN/dE$, energy $E$ in the jet frame) responsible for the observations of April 7 (solid line), April 11 (dashed line) and April 16 (dotted line). The lower three panels compare the observed (points) with the modeled (solid line) SEDs (energy in observer’s frame). For illustrative purposes the dashed line shows the TeV gamma-ray energy spectra modified by extragalactic absorption as predicted by the DEBRA model “LCDM, Salpeter Stellar Initial Mass Function” of Primack et al. (2001). For April 7, the model under-predicts the TeV flux, and the dashed-dotted line shows the same absorbed gamma-ray spectrum, but normalized to the flux at 2 TeV to facilitate comparison of the spectral shapes. Asterisks show the BeppoSAX data normalized to the flux measured with RXTE. The solid points show the shape of the HEGRA 1997 time averaged Mrk 501 energy spectrum (Aharonian et al. 1999b; Aharonian et al. 2001b) normalized to the mean TeV gamma-ray flux measured on each day by CAT, HEGRA, and Whipple. The 2$\sigma$ upper limit at 100 MeV has been derived from EGRET observations between April 9th and April 15th, 1997 under the assumption of a constant emission level (Catanese et al. 1997).

4.2 Two-Component Models

The region near the presumed central black hole as well as various regions along the jet might emit X-rays in

Bremsstrahlung, IC, or Synchrotron processes without producing a comparable luminosity at gamma-rays (see also Blażejowski et al. (2000), Bicknell, Wagner & Groves (2001)). Thus it is conceivable, even probable, that the X-ray emission from Mrk 501 is “contaminated” by an emission component which varies on longer time scales than the TeV gamma-ray radiation. While all the one-component models described in the previous section failed to fully describe the data, we find that the addition of a quasi-stationary X-ray component substantially improves the situation. Varying contributions of the quasi-stationary soft and the time
variable hard component are able to account for the large spectral changes observed at X-rays.

Over the narrow spectral range from 3 keV to 25 keV we describe the quasi-steady X-ray component by a power law. We determined possible values of flux level and spectral slope of the quasi-stationary component from extrapolating the 10 keV vs. 2 TeV flux correlation toward zero gamma-ray flux, and the 10 keV flux vs. 3-25 keV photon index correlation (Paper I) toward zero X-ray flux, respectively. Due to the scatter of both correlations this criterion gave a range of allowed values. We chose the values which resulted in the best two-component SSC fits to the data, namely a 10 keV amplitude $\nu F_\nu = 10^{-10}$ erg cm$^{-2}$ s$^{-1}$ and a photon index of 2.2.

In the following we describe different incarnations of two-component models: two with variability through a time dependent rate of accelerated particles, and one with a time dependent Doppler factor of the SSC emission region.

4.2.1 Time variability through $Q_0(t), \gamma_{\text{min}} = 1000$

First we consider a two-component model with flares caused by varying $Q_0(t)$. Since the spectral variability at X-rays is produced by the varying dominance of the soft quasi-static and the hard time dependent components no additional spectral variability has to be produced by a changing

$\gamma_{\text{max}}$ and we use a fixed $\gamma_{\text{max}}$ corresponding to a high energy cut-off in the synchrotron spectrum at MeV energies. Fig. 6 shows the fit of the two-component model (see figure caption for model parameters). Compared to the one-component $Q_0(t)$ model shown in Fig. 2, the additional quasi-stationary soft component significantly improves the fit of the X-ray photon indices. The improvement is most pronounced for the April data. The model also describes well the TeV gamma-ray fluxes, with the notorious exception of MJD 50544.

Finally, we compare the model SEDs with the one measured by BeppoSAX. Based on the data of one of the three BeppoSAX observations (we used the observation of April 11), one can determine the spectrum of the quasi-stationary X-ray component outside the energy range covered by the RXTE observations. The other two observations can then be used to check the model predictions. The upper panel of Fig. 7 shows the electron energy spectra averaged over the integration time of the 3 BeppoSAX pointings. The lower three panels compare the modeled with the observed SEDs. By construction, the model describes the X-ray data of April 7; the fit to April 11 is also good, but the model spectrum of April 16 is too soft. More detailed inspection shows that the model fails to describe the temporal evolution of the $< 1$ keV fluxes, i.e. for April 16 it produces too much flux below 1 keV.
Remarkably, the predicted TeV energy spectrum, modified by extragalactic extinction according to the LSDM model of Primack et al. 2001 fits the HEGRA time averaged spectrum very well.

### 4.2.2 Time variability through $Q_0(t)$, $\gamma_{\min} = 10^5$

A similar model with a high value of $\gamma_{\min} \sim 10^5$ does not show these difficulties. In this case the break in the energy spectrum is more abrupt and the peak of the synchrotron SED of the time variable component is narrower than in the previous case. Viable models with high minimum Lorentz factors are located in a completely different region of parameter space: at Doppler factor 45, we infer a magnetic field of $B = 1.1$ G, $t_{\text{esc}} = 10^4 R c^{-1}$, $\gamma_{\min} = 10^5$, $\gamma_{\max} = 1.4 \times 10^7$, $\xi = 0.5$, $\eta = 0.00$.

Even without any extragalactic extinction the model of the TeV gamma-ray data is very soft and only barely consistent with the observed data below 10 TeV. Only above 10 TeV, the model implies a slight amount of extinction. Note the pronounced break of the IC spectrum at $\sim 2$ TeV. Obviously, fitting a power law to a small portion of such a spectrum and inferring the DEBRA intensity from the deviation of the observed spectrum from this power law will not produce correct results.

### 4.2.3 Variability through $\delta_j(t)$, $\gamma_{\min} = 5 \times 10^5$

In the framework of one-component models, an emitting blob with constant and isotropic emission in its rest frame but with a varying angle between its motion and the line of sight can not account for the 1997 X-ray and TeV gamma-ray flares. The reason is that the large variability of the peak energy of the synchrotron SED would imply a large change of the blob’s Doppler factor and as a consequence a much larger than observed flux variability (Paper I). In a two-component model however, a variable Doppler factor can explain the flux variability: the X-ray energy spectra mainly change due to the relative dominance of the quasi-stationary and the time-variable X-ray components. Figs. 10 and 11 show the two-component fit to the time resolved data and the broadband spectral data, respectively. The model gives an excellent fit to the data. This model is qualitatively very different from the other ones: time variability can be produced on small time scales by changing $\delta_j$, and the electron
spectrum is a steady state electron spectrum, and does not develop in time. We used here a rather low value of the Doppler-factor, $\delta_j = 10$. As a consequence, after correction for DEBRA extinction, the TeV energy spectra are steeper than the observed ones.

5 DISCUSSION

In this paper we describe the time resolved modeling of the X-Ray and TeV gamma-ray data of a 2 month observation campaign. The time resolved analysis is plagued by the sparse observational sampling and the unknown modification of the TeV gamma-ray energy spectrum by extragalactic extinction. However, modeling the X-ray fluxes and energy spectra and the relative changes of the TeV gamma-ray fluxes and photon indices allows us to exclude some hypothesis about the flare origin. Furthermore, we are able to verify that simple but self-consistently evolved SSC models based on canonical power-law energy spectra of accelerated electrons are able to account for the very detailed observational data. More specifically, our conclusions from the time dependent modeling are as follows:

(i) One-component models do not fully describe the data. While, by construction, the models succeed in accounting for the temporal evolution of the X-ray fluxes they do not adequately predict at the same time the range of observed X-ray spectral indices, the broadband 0.1 keV-200 keV energy spectra, and the variation of the TeV gamma-ray fluxes.

(ii) Two-component models give surprisingly good fits to the data. In these models, the X-rays originate from a superposition of a soft quasi-steady component and a hard rapidly variable component. We found two models which give an excellent fit: in the first model flares are produced by a time dependent rate of accelerated particles. In the second model, a changing Doppler factor causes the flares. In both models, changes of the observed X-ray energy spectrum mainly result from the relative dominance of the quasi-stationary and the time-variable X-ray component.

(iii) Accurate fits to the BeppoSAX broadband data require a large minimum Lorentz factor of accelerated particles on the order of $\gamma_{\text{min}} = 10^5$.

(iv) Degeneracy in both, model variant and jet parameters, prevents us to use the time resolved SSC calculations to substantially tighten the constraints on the amount of extragalactic TeV gamma-ray extinction, compared to earlier work (see e.g. Paper I, Guy et al. 2000, Vassiliev 2000, de Jager & Stecker 2002, and references therein). The gamma-ray SEDs of Figs. 7 and 11 are consistent with the LCDM DEBRA model of Primack et al. (2001). In contrast, the model of Fig. 9, implies negligible extinction below $\sim 10$ TeV. Especially the model with flux variability through the Doppler factor (Fig. 11) can produce very different intrinsic gamma-ray SEDs while perfectly fitting the X-ray...
Figure 12. Change of GeV/TeV gamma-ray optical depth as function of gamma-ray energy, inferred from comparing the model of Fig. 11 (lower solid lines, respectively) with the 1997 time averaged gamma-ray energy spectrum measured by HEGRA (Aharonian et al. 1999b, 2001a). The upper solid line is an upper limit for parameter combinations for which the 0.5-16 TeV energy spectrum approximates a power law of photon index 1.5. The upper limits include 2σ statistical errors and take the 15% uncertainty of the absolute HEGRA energy calibration into account. Allowed DEBRA models have to lie between the two solid lines. However, even if an absorption model lies between the two solid curves it does not imply that a valid SSC model exists such that the absorbed gamma-ray spectrum describes the data (the solid lines give a necessary but not sufficient condition that a DEBRA model is consistent with the SSC models and the data). The dashed and dotted lines show model predictions of Primack et al. 2001 (“LCDM” with Kennicutt and Salpeter Stellar Initial Mass Functions, respectively). All curves have been normalized to 1 TeV where the systematic errors on the HEGRA energy spectrum are small.

and gamma-ray flux variations and the X-ray photon indices. In this model, the data constrain only the absolute flux level and energy spectrum of the quasi-stationary component, and the relative changes of the Doppler factor. The absolute value of δ, as well as the parameters R_, B, and τ_esc remain degenerate.

An upper limit on the modification of the TeV gamma-ray energy spectrum by extragalactic extinction can be derived from the fact, that the emitted time averaged gamma-ray energy spectrum is unlikely to be harder than dN_g/ dE ∝ E−Γ with Γ ≈ 1.5. In Fig. 12 the range of allowed changes of gamma-ray optical depth with gamma-ray energy is shown and is compared to recent model calculations of Primack et al. (2001). A more accurate estimate of the amount of extragalactic extinction from SSC modeling of Mrk 501 requires to pin down the jet parameters. Some key-observations are discussed further below; a more detailed discussion will be given by Coppi et al. (2002).

(v) Table 2 lists for all studied models the electron to magnetic field energy density ratio r = (u_e / u_B) as well as the minimum kinetic luminosity L_k = π R^2 c Γ^2 (u_e + u_B) (Begelman et al. 1994) and we use Γ = δ. All models are strongly out of equipartition with r between 300 and 7500. Similar results, derived from an one-zone stationary SSC model, have recently been reported by Kino et al. (2002). The kinetic luminosities lie between 5 × 10^{42} erg s^{-1} and 2 × 10^{44} erg s^{-1}, about 1000 times and more than the co-moving radiative luminosities which are on the order of ~ 5 × 10^{39} erg s^{-1}. We computed the kinematic luminosity assuming a steady state jet with the same physical parameters as the SSC emission region. Although this assumption might overestimate the true kinematic luminosity by a factor of a few, our models clearly indicate that TeV blazars have rather powerful jets. Models with high \gamma_min-values are closest to equipartition and require the least power.

In SSC models the X-ray to TeV gamma-ray luminosity ratio strongly depends on the size and magnetic field of the emission region. As a consequence, most models that have been proposed to account for the flaring activity (as the internal shock model of Spada et al. 2001) do not naturally predict such a tight correlation of X-ray and TeV gamma-ray fluxes through a large number of distinct flares as evident in Fig. 1. The hypothesis that a single emission region of constant size produces a series of flares encounters several problems: (i) due to the strong dominance of particle pressure over magnetic field pressure, the emission region should quickly expand adiabatically and thus become undetectable; (ii) it is not clear how the energy required for sustaining a prolonged flaring phase could be fed into the emission region; (iii) during the flaring period that lasted more than Δt ∼ 2 months, the emission region would have advanced by ~ c Γ^2 Δt, that means by a distance of about ~100 pc. The stability of the radius of the emission region would thus imply a jet opening angle of ~ 10^{-2} rad, several orders smaller than radio observations indicate.

Our preferred interpretation is that flares originate from distinct emission regions with very similar characteristics, i.e. size and magnetic field. Such emission regions might form as the jet becomes radiative at a certain characteristic distance from the central engine. The fact that our models give particle escape times on the order of and shorter than the flux variability time scale indicates that the flare duration is limited by the adiabatic expansion of individual emission regions. The jet would naturally feed energy to the site where the flares originate. The conclusions presented here are not limited to SSC models. Also in External Compton models, the tight X-ray/TeV gamma-ray correlation indicates a preferred location for the production of individual flares: why else should the ratio of the jet frame magnetic field and external seed photon energy densities remain roughly constant during 2 months?

The preferred distance from the central engine could correspond to a characteristic length at which the jet becomes unstable. Alternatively, a change in ambient pressure could induce jet instabilities at a characteristic distance from the central engine. Note that a qualitatively different but similarly puzzling stability has been found in the hardness intensity correlation of Mrk 421 (Fossati et al. 2000) for measurements taken between days and years apart. One conclu-
sion from this discussion is that refined modeling should treat adiabatic expansion in more detail.

Since the modeling is computationally very intensive, we explored only a limited number of models. We did not consider External Compton models which historically have been applied to the more powerful EGRET blazars. For high jet Doppler factors even a very weak external photon field as e.g., IR radiation from dust, can be boosted and become significant in jet the frame. Depending on the seed photon energy spectrum, the radiative IC cooling of lower energy electrons might be stronger than for high energy electrons due to the Klein-Nishina effect. A possible consequence is that the energy spectra of External Compton models can be very rich behavior of External Compton models and the consequences of radiative cooling in the extremely “blue” TeV gamma-ray blazars can substantially differ from those in EGRET GeV blazars.

Crucial advances in fixing model parameters will only be possible by substantially extending the observational coverage in time and wavelength. The 1997 April and May observations had diurnal integration rates of typically 2 times 20 min. Pinning down the evolution of the source during several flares requires quasi-continuous monitoring over many days. Unfortunately, no sensitive X-ray all sky monitor with broadband spectroscopic capabilities will be available for the next several years or even longer. Such an instrument would be able to participate in intensive Multiwavelength campaigns on a large number of objects. The upcoming generation of Cherenkov telescopes CANGAROO III, H.E.S.S., MAGIC, and VERITAS will have energy thresholds of between 10 GeV and 50 GeV and a one order of magnitude higher sensitivity than present day instruments. The lower energy threshold is of crucial importance as it makes it possible to assess the IC component at low energies where extragalactic extinction is negligible ($z < 0.1$) or much less ($z = 0.5 - 1$) than at ~ 500 GeV. The new experiments should be able to reliably assess changes of the diurnal GeV/TeV energy spectra with a statistical and systematic accuracy in photon index of 0.05 or better, due to better gamma-ray statistics and improved detector calibration and atmospheric monitoring. Thus spectral changes as shown in Figs. 3, 4, and 10 will become measurable.

A key observation for fixing the jet parameters is to measure a time lag between the X-ray and the TeV gamma-ray flux variability. A general prediction of SSC models is a time delay of approximately a light crossing time $R c^{-1}$ between the leading X-ray and following gamma-ray fluxes. The measurement of this delay would allow one to determine the size of the emission region. The requirement that the DEBRA reduces the 2 TeV flux by a factor of 5 or less would break the degeneracy in $\delta$ and $B$. If the X-ray/TeV gamma-ray lag remains elusive it may be that the determination of the jet parameters of the TeV blazars detected so far has to wait until more reliable DEBRA estimates will be available derived from multiple blazar detections at redshifts between 0.05 and 1.

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### Table 1. Selected Blazar SSC Models Relevant to This Paper

| Authors | Objects Studied | Time Dependent? | SED Peak Determined By | Flare Mechanism | DEBRA Extinction? |
|---------|-----------------|-----------------|------------------------|-----------------|------------------|
| Inoue & Takahara (1996) | 3C 279, Mrk 421 | No | Cooling vs. Particle Escape | Not specified | No |
| Bednarek & Protheroe (1997; 1999) | Mrk 421, Mrk 501 | No | Not specified | Not specified | Yes |
| Böttcher et al. (1997) | Mrk 421 | No | $\gamma_{\text{min}}$ | $B$, $\gamma_{\text{min}}$ | No |
| Mastichiadis & Kirk (1997) | Mrk 421 | Yes | Cooling vs. Particle Escape | $Q_0$, $\gamma_{\text{max}}$, $B$ | No |
| Pian et al. (1997) | Mrk 501 | No | $\gamma_{\text{min}}$ | $\gamma_{\text{min}}$, $\gamma_{\text{max}}$ | No |
| Dermer et al. (1998) | generic | Yes | Cooling vs. Particle Escape and Plasmon Deceleration | $\delta_j$ | No |
| Chiaberge & Ghisellini (1999) | generic | Yes | Cooling vs. Particle Escape | $Q_0$ | No |
| Coppi & Aharonian (1999) | generic | Yes | Cooling vs. Particle Escape | $Q_0$, $B$, $\gamma_{\text{max}}$ | Yes |
| Kirk & Mastichiadis (1999) | generic | Yes | Cooling vs. Particle Escape | $Q_0$ | No |
| Kataoka et al. (2000) | PKS 2155-304 | Yes | Cooling vs. Particle Escape | $\gamma_{\text{max}}$ | No |
| Petry et al. (2000) | Mrk 501 | No | Cooling vs. Injection Time Scale | | No |
| Kusunose et al. (2000) | generic | Yes | Cooling vs. Particle Escape | $\gamma_{\text{max}}$ (through $t_{\text{esc}}$ and $t_{\text{acc}}$) | No |
| Tavecchio et al. (2001) | Mrk 501 | No | Not specified | Change of $\gamma_b$ | No |
| Krawczynski et al. (2001) | Mrk 421 | Yes | Cooling vs. Particle Escape | $\gamma_{\text{max}}$ | No |
| Sikora et al. (2001) | 3C 279, PKS 1406-076 | Yes | Cooling vs. Injection Time Scale or $\gamma_{\text{min}}$ | $Q_0$ | No |
| Kino et al. (2002) | Mrk 421, Mrk 501, PKS 2155-304 | No | Cooling vs. Particle Escape | Not specified | No |
| This work | Mrk 501 | Yes | Cooling vs. Particle Escape or $\gamma_{\text{min}}$ | $Q_0$, $\gamma_{\text{max}}$, $\delta_j$ | Yes |

### Table 2. Parameters of Models Shown in Figures

| Time Dependent Parameter | Comments | $\delta_j$ | $R$ [cm] | $B$ [G] | $t_{\text{esc}}$ [Rc$^{-1}$] | $\gamma_{\text{min}}$ | $\gamma_{\text{max}}$ | $\xi$ | $\eta$ | $u_e/u_B$ | $L_k$ [erg s$^{-1}$] |
|--------------------------|----------|-----------|----------|--------|-----------------|-----------------|-----------------|-----|-----|-------|-----------------|
| $Q_0(t)$ 1-component     |          | 45        | 1.1 x 10$^{16}$ | 0.014  | 10              | 1000            | 2.5 x 10$^7$   | 0.5 | 0.2 | 660   | 1.2 x 10$^{44}$ |
| $\gamma_{\text{max}}(t)$ | 1-component | 45        | 1.5 x 10$^{16}$ | 0.003  | 3               | 1000            | 1.6 x 10$^6$   | 0.5 | 0.2 | 1200  | 1.7 x 10$^{44}$ |
| $Q_0(t)$ 1-component     |          | 45        | 3.2 x 10$^{15}$ | 0.035  | 3               | 1000            | 1.6 x 10$^6$   | 0.5 | 0.1 | 860   | 8.3 x 10$^{43}$ |
| $Q_0(t)$ 2-component     |          | 45        | 3.4 x 10$^{15}$ | 0.014  | 3               | 1000            | 2.3 x 10$^7$   | 0.5 | 0.4 | 7470  | 1.3 x 10$^{44}$ |
| $Q_0(t)$ high $\gamma_{\text{min}}$ | 2-component | 45        | 4.5 x 10$^{13}$ | 1.12   | 10000          | 1.0 x 10$^5$   | 1.4 x 10$^7$   | 0.5 | 0.00 | 290   | 5.6 x 10$^{42}$ |
| $\delta_j(t)$ 2-component, high $\gamma_{\text{min}}$ |          | 10        | 10$^{15}$ | 0.16   | 10             | 4.5 x 10$^5$   | 2.9 x 10$^7$   | 0.5 | 0.04 | 2970  | 2.8 x 10$^{43}$ |