Wavelet sparse transform optimization in image reconstruction based on compressed sensing

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Abstract. The high image sparsity is very important to improve the accuracy of compressed sensing reconstruction image, and the wavelet transform can make the image sparse obviously. This paper is the optimization method based on wavelet sparse transform in image reconstruction based on compressed sensing, and we have designed a restraining matrix to optimize the wavelet sparse transform. Firstly, the wavelet coefficients are obtained by wavelet transform of the original signal data, and the wavelet coefficients have a tendency of decreasing gradually. The restraining matrix is used to restrain the small coefficients and is a part of image sparse transform, so as to make the wavelet coefficients more sparse. When the sampling rate is between 0.15 and 0.45, the simulation results show that the quality promotion of the reconstructed image is the best, and the peak signal to noise ratio (PSNR) is increased by about 0.5dB to 1dB. At the same time, it is more obvious to improve the reconstruction accuracy of the fingerprint texture image, which to some extent makes up for the shortcomings that reconstruction of texture image by compressed sensing based on the wavelet transform has the low accuracy.

1 Introduction
Image sparsity means that some large coefficients of image transform carry most of the energy and information of the image, while the other coefficients are zero or close to zero, which means that the use of a small amount of bits can represent the image. Fourier transform (FT), discrete cosine transform (DCT) and wavelet transform (WT) are the important methods of image sparsity. Compared with the previous two methods, wavelet transform has better sparsity. The rapid development of information technology makes people increase the demand for information, signal conversion from analog to digital has always been strict compliance with the requirements of the Nyquist sampling theorem, the sampling rate must reach more than 2 times the bandwidth of the signal, which make the signal can be accurately reconstructed. With the ability of sensing system to acquire data improving, the amount of data that needs to be processed is also increasing, which puts forward higher requirements to the ability of signal processing, and it also brings great challenges to the corresponding hardware devices. In recent years, there has been a new theory—compressed sensing (or CS, sampling compressive) [1,2]. The signal can be recovered with the sampling rate which is far lower than the Nyquist sampling rate. It greatly reduces the sampling frequency of the signal, the time of signal processing and the cost of the computation and the storage and transmission of data. The history of compressed sensing theory is not long, and there are still a lot of problems to be solved. At present, lots of research institutions and universities have started the research work about CS. The sampling measurement data of the signal can be obtained by
compressed sensing. If the signal is compressed and transmitted, it only needs to be processed on the basis of the measurement data, which can achieve a high compression ratio and transmission efficiency of the original signal. Based on wavelet transform, the compressed sensing method for image reconstruction can get the transform coefficients of low sampling rate and high compression ratio [10]. So image reconstruction based on the wavelet transform and CS has become a hot research topic with good research and development prospects.

2 Basic Theoretical Analysis
The core of compressed sensing is the linear measurement process, set \( x(n) \) as the original signal with N length, \( x(n) \) multiplied by the measurement matrix \( \Phi \) gets the measurement value, \( y(m) \) with M length (M < N). If \( x(n) \) signal is not sparse, we can get the sparse form \( s(k) \) by the orthogonal sparse transform, that is \( x = \Psi s \). The measurement process is rewritten as \( y = \Theta s \), and \( \Theta = \Phi \Psi (M \times N) \), which is called the sensing matrix, the process is shown in Figure 1.

![Figure 1: Linear measurement process in compressed sensing](image)

The compressed sensing theory mainly includes three aspects: design of measurement matrix, reconstruction algorithm and sparse representation of signal [3].

2.1 Construction of Measurement Matrix
Candès and Tao [6] proved the measurement matrix must satisfy the Restricted Isometry Property (RIP) condition: for k-sparsity signal \( x \) and constants \( \delta_k \), if measurement matrix \( \Phi \) satisfies the RIP condition, Equation (1) [4], the original signal will be restored.

\[
(1 - \delta_k)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1 + \delta_k)\|x\|_2^2, (0 < \delta_k < 1)
\]  

(1)

The RIP condition is a sufficient condition to ensure that the signal can be reconstructed. A lot of mathematical theory and practice prove that the random Gauss matrix, the Bernoulli matrix (two valued random measurement matrix) and the Fourier random matrix satisfy the RIP condition with high probability, so these matrices are used as the measurement matrix, and the two valued random measurement matrix (0 and 1 have the same probability distribution) is used in this paper.

2.2 Signal Reconstruction Algorithm
Signal reconstruction algorithm is defined as the process of reconstructing the sparse signal \( x \) with N length by the measure vector \( y \) with M length (M < N). The problem of signal reconstruction can be solved by solving the minimum \( l_0 \) norm problem, Equation (2) [9].

\[
\hat{x} = \arg \min \|x\|_0 \quad \text{s.t} \quad \Phi x = y
\]  

(2)

But the minimum \( l_0 \) norm problem is a NP-hard problem and needs to list \( c_N^k \) kinds nonzero permutation of \( x \), which can not be solved [11]. So we use the suboptimal algorithm to solve the NP-hard problem, including the minimum L1 norm method [7], the matching pursuit algorithm, the iterative threshold method [13,14], the total variation minimization method [12], and so on. This paper uses the orthogonal matching pursuit algorithm (OMP) [5].
2.3 Sparse Representation of Signal and the Improved Wavelet Transform

We got the simulation results of the one-dimensional sparse data with 256 length by MATLAB, and explored the relationship between the data reconstruction precision and the sparsity. The difference between the reconstructed data and the original data is lower than a threshold value, which is regarded as the successful reconstruction of the data, and 1000 reconstruction experiments were performed in each case. The relationship between the success rate of reconstruction and the number of measurements (N) and the sparsity (m) is shown in Figure 2, and d is the length of signal [8].

Figure 2: The percentage of 1000 input signals correctly recovered as a function of the sparsity level m for different numbers N of measurements in dimension d=256

From Figure 2, we can see that the enough sparsity is very important to improve the accuracy of data reconstruction. Therefore, the improvement of data reconstruction in this paper is to create a sparse transform with the higher sparsity. The wavelet transform is used to make the original signal data with 4096 length get sparse, as shown in Figure 3 and Figure 4.

Figure 3: Original signal data

Figure 4: Sparse distribution of coefficients in wavelet domain
It can be seen from Figure 3 and Figure 4 that the wavelet transform can make the data get very sparse, but the sparsity is still not ideal for the data reconstruction of compressed sensing. The coefficients data in wavelet domain shows the trend of gradually decreasing, so we can improve the sparsity of wavelet coefficients in a way of restraining the small coefficients. Thus the wavelet coefficient restraining matrix is designed, as shown in Figure 5.

\[
\begin{pmatrix}
1 & 0 & \cdots & \cdots & 0 \\
0 & \ddots & 0 & 0 & 0 \\
\vdots & 0 & \frac{m+1}{n} & 0 & \vdots \\
\vdots & 0 & 0 & \frac{m}{n} & \vdots \\
0 & \cdots & \cdots & 0 & \frac{1}{n}
\end{pmatrix}
\]

Figure 5: The wavelet coefficient restraining matrix

The wavelet coefficients restraining matrix in Figure 5 is the diagonal matrix with n dimensions, n is the length of signal data, and diagonal elements is the arithmetic sequence with the first term of 1 and common difference of -1/n. A new coefficients vector is obtained by multiplying the restraining matrix with the coefficients in the wavelet transform domain, as shown in Figure 6.

Figure 6: Distribution of wavelet coefficients multiplied by restraining matrix

It can be seen from Figure 6 that the sparsity of wavelet coefficients has been greatly improved compared with the original wavelet coefficients in Figure 4, which gets the effect of restraining small coefficients. From Figure 1 we can know that the old wavelet transform matrix is \(\Psi_0\)' and the restraining matrix is named W, so the equation is written as \(W\Psi_0'x=s\). \(W\Psi_0'\) is the wavelet transform matrix improved.

3 Simulation Experiment
The images of Lena(512*512) and fingerprint(512*512) are used to perform simulation experiments of image reconstruction. In order to eliminate the randomness of the experiment, the mean of the five experimental results under the same sampling rate was obtained. After improvement, the relationship between the PSNR of image reconstruction and the sampling rate is shown in Figure 7 and Figure 8.
Figure 7: The relationship between peak signal-to-noise ratio (PSNR) of Lena image reconstruction and the sampling rate

Figure 8: The relationship between peak signal-to-noise ratio (PSNR) of Fingerprint image reconstruction and the sampling rate

According to Figure 7 and Figure 8, in the case of the same sampling rate, the PSNR of reconstructed image is improved to a certain extent. For Lena image, when the sampling rate is between 0.15 and 0.45, this method has a better reconstruction effect, the PSNR is increased by about 1dB. For the texture image of higher reconstruction difficulty, such as Fingerprint image, when the sampling rate is between 0.15 and 0.45, the PSNR is increased by about 0.5dB. For these two kinds of images, the sampling rate interval with good reconstruction effect is both distributed between 0.15 and 0.45, and the sampling rate interval is the most commonly used sampling interval of engineering application for compressed sensing in image reconstruction. So this method can be put into engineering practice more quickly. When the sampling rate is 0.25, the global and local images of Lena and Fingerprint are used to do the simulation experiments respectively, and the reconstruction results are shown in Figure 9, Figure 10, Figure 11 and Figure 12.
According to Figure 9 and Figure 10, image reconstruction accuracy has been significantly improved after optimization. Especially for texture image like fingerprint, texture details before improvement are difficult to obtain, and the reconstruction image can’t be used for fingerprint recognition. After the
improvement, the fingerprint texture details become clearer and basically meet the requirements of fingerprint identification. The local image area (64*64) is extracted from the global image and reconstructed. The reconstruction results are shown in Figure 11 and Figure 12. From Figure 11 and Figure 12, we know that local image reconstruction effect has been greatly improved, images are smoother and less noise. Relative to the global image, the PSNR of local reconstruction image is higher, and the details information can also be basically presented.

4 Conclusions
Based on the above theoretical analysis, in this paper we have designed a kind of restraining matrix which is easy to implement and add it to the wavelet sparse transform, the image becomes more sparse. When the sampling rate is between 0.15 and 0.45, the image reconstruction accuracy is improved better, and the PSNR is increased by about 0.5dB to 1dB. This method also has a good effect on fingerprint texture image reconstruction, and to some extent makes up for the shortcomings that reconstruction of texture image by compressed sensing based on the wavelet transform has the low accuracy. However, this method can only be used in the wavelet transform domain, and can not be applied to other transform, and lack the universality. The PSNR of the reconstructed image is still low, which can not satisfy the engineering application well. These problems still need to be explored and studied, so the image reconstruction based on compressed sensing still has a broad prospect.

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