User online consumption behaviour based on fractional differential equation

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Abstract

User consumption behaviour is a subject worthy of study. Because consumers’ consumption behaviours are dynamic and with individual differences, various factors need to be considered when establishing a fractional differential equation model of users’ online consumption behaviour. The two elements, namely advertising and price are more evident in influencing consumer behaviour. Therefore, the paper establishes a product diffusion fractional differential equation model of price and advertising presence or absence to study the impact of these two factors on consumer behaviour. It turns out that ignoring the advertisement and the cost of the product is related to the characteristics of the consumer network. When there are advertising and price factors, product awareness is related to price constraints.

Keywords: complex network, fractional differential equation, user network consumption, product diffusion, marketing strategy

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1 Introduction

Partnerships in social networks influence users’ decision-making behaviours. Nowadays, consumers increasingly rely on social networks to buy some products, especially when purchasing some high-tech or experiential products. There are two possible reasons: First, the advancement of modern network communication technology has reduced the difficulty and cost for consumers to obtain product/service-related information. The second is that some products have high prices, complex technical parameters, or limited information that consumers can

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get from advertisements and other mass media. Still, consumers’ purchase decisions require a large amount of high-quality product/service information.

Most classic product diffusion studies assume that the network between consumers is a regular network of interconnections [1]. However, the actual consumer social network is dynamically changing, and if the negative impact is significant, the product may not be able to spread entirely in the consumer social network. On the one hand, the continuous influx of new consumers with purchase needs in the consumer social system has supplemented the number of potential consumers. On the other hand, in some social networks such as forums or e-commerce sites, positive or negative word-of-mouth products can affect consumers’ purchasing decisions, especially when negative word-of-mouth has a more significant impact than positive word-of-mouth. As a result, consumers may lose their desire to buy and withdraw from consumer social interaction – the internet.

This article also considers the dynamics of consumer social networks and the heterogeneity of consumer network structures and studies the proliferation patterns of monopolistic firms’ products. The dynamic nature of consumer social networks mainly refers to the continuous entry or exit of potential consumers and the withdrawal of purchasers from the web [2]. The heterogeneity of consumer social network structure refers to the difference in the number of consumers communicating, the difference in node degree. This article first constructs a product diffusion model without considering advertising or price strategies and analyses the threshold conditions for continuous product diffusion. Then we extend the research question to the situation where the company adopts only the advertising strategy, and to the situation where the company adopts both advertising and pricing strategies. Finally, we use numerical simulation to verify the correctness of the theoretical analysis.

2 Product diffusion model

2.1 Diffusion models that do not consider advertising or price strategies

2.1.1 Dynamic consumer social network

Assuming that the size of the nodes in the network is N, undirected connections between nodes represent the interaction and exchange of information between individuals. Some nodes in the network are occupied by potential consumers and consumers who have purchased products [3]. Each consumer occupies only one node, and the remaining nodes are empty. We assign a state value to each node, represented by 0, 1 and 2. They correspond to three states, respectively, where 0 means an empty node, 1 indicates a potential consumer, and 2 represents a purchaser.

At time t, each new potential consumer entering the network will occupy an empty node. The pointless node changes to a node occupied by potential consumers with probability c, and the node state changes from state 0 to state 1. Some potential consumers lose interest in buying due to negative word-of-mouth or reduced expected utility of the product and exit the consumer social network with probability \( \mu \). Their node state changes from state 1 to state 0. Another part of potential consumers is affected by positive word of mouth to purchase products with probability a, and the node state changes from state 1 to state 2. The purchasers exit the consumer social network with probability \( \gamma \) due to not publishing product information or publishing invalid information [4]. This part of the purchasers who withdrew from the consumer social network will not join the network again. The node becomes an empty node and may be occupied by new potential consumers. That is, the state of the node changes from state 2 to state 0. The change of node state in the consumer social network is shown in Figure 1.

This paper uses the node degree distribution of consumer social networks as the key feature. The number of consumers who choose to communicate information indicates the node degree of consumers [5]. The node degree distribution function is \( p(k) \), which satisfies \( \sum_{k=1}^{N} p(k) = 1 \). Any node in the network has at least one node connected to it, and the difference in the degree of consumer nodes indicates the heterogeneity of consumer social network structure.
2.1.2 Product diffusion rules

Potential consumers are affected by any of the existing purchasers to purchase the product with probability a. $x_k(t)$, $y_k(t)$ and $z_k(t)$ represent the density of potential consumers, the density of buyers, and the density of empty nodes with node degree k at time t, $x_k(t) + y_k(t) + z_k(t) = 1$. The conditional probability of a potential consumer with node degree k at the same node degree $k'$ is $p(k'|k)$. The likelihood of a single consumer adopting a product per unit time is $ka \sum_{k'=1}^{M} p(k'|k)y_{k'}(t)$. The proportion of potential consumers with a node degree of k is $x_k(t)$, and the balance of products purchased per unit time affected by purchasers is $ka x_k(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t)$. Therefore, the total number of potential consumers in the consumer social network at time t is $x(t) = \sum_{k=1}^{N} x_k(t)p(k)$. The proportion of the total number of purchasers is $y(t) = \sum_{k=1}^{N} y_k(t)p(k)$.

2.1.3 The basic model of product diffusion

When the company does not implement advertising or price strategies, the change in the proportion of potential consumers with a node degree of k per unit time comes from three parts: the first is the new proportion $c(1 - x_k(t) - y_k(t))$ of potential consumers. The second is the reduction ratio $ka x_k(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t)$ of potential consumers who purchase products affected by the existing purchasers [6]. Third, the percentage of potential consumers who lose their desire to buy and withdraw from the internet due to negative word-of-mouth or lower expected utility of the product has decreased $\mu x_k(t)$. The change in the proportion of purchasers with a node degree of k in a unit time comes from two parts: one is the new proportion $ka x_k(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t)$ of the number of purchasers who have changed from the purchase of products by potential consumers. The second is the reduction ratio $\gamma y_k(t)$ of the number of people who have already purchased not to publish information or publish invalid information to exit the network. Therefore, when companies do not implement advertising or price strategies, the basic model of monopolistic product diffusion in dynamic consumer social networks is

$$\begin{align*}
\frac{dx_k(t)}{dt} &= c(1 - x_k(t) - y_k(t)) - ka x_k(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t) - \mu x_k(t) \\
\frac{dy_k(t)}{dt} &= ka x_k(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t) - \gamma y_k(t), \quad k = 1, 2, \ldots, N
\end{align*}$$

Available when the degree of the network is irrelevant

$$\theta = \sum_{k'=1}^{M} p(k'|k)y_{k'}(t) = \frac{\sum_{k'=1}^{M} k'p(k'|k)y_{k'}(t)}{\langle k \rangle}, \quad k = 1, 2, \ldots, N$$

Fig. 1 Diagram of node status changes in consumer social networks.
Among them, $\langle k \rangle = \sum_{k=1}^{M} kp(k)$ represents the average degree of consumer social networks.

### 2.2 Diffusion model considering advertising strategy

The classic diffusion theory defines consumers only affected by advertising and other mass media to purchase products as innovators [7]. Assuming that potential consumers buy products with probability $b$ being influenced by advertising, the change is that the number of potential consumers with node degree $k$ in a unit time affected by advertising and internal word-of-mouth changes is $\eta x_{k}(t)$.

The price when the company implements skimming pricing and penetration pricing, respectively, is $p_{k}(k')$.

The number of purchasers is reduced by $bx_{k}(t)$. The proportion of new purchasers corresponding to the number of people who have already purchased that do not publish information or publish invalid details and exit the network. When companies adopt advertising and skimming pricing strategies, the product diffusion model is as follows

$$
\begin{aligned}
\frac{dx_{k}(t)}{dt} &= c(1 - x_{k}(t) - y_{k}(t)) - kax_{k}(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t) - bx_{k}(t) + \mu x_{k}(t) \\
\frac{dy_{k}(t)}{dt} &= kax_{k}(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t) - bx_{k}(t) - \gamma y_{k}(t), \quad k = 1, 2, \ldots, N
\end{aligned}
$$

### 2.3 Diffusion model considering advertising and price strategies

We assume that the price response function of the product is $g_{i} = (\omega_{i}/\omega_{h} - 1)$. Where $i \in \{H, L\}$, $\omega_{h}, \omega_{L}$ represents the price when the company implements skimming pricing and penetration pricing, respectively. $\omega_{h}$ represents the benchmark price of the product $\eta > 1$. The proportion of the number of potential consumers with a node degree of $k$ in a unit time affected by advertising and internal word-of-mouth changes is $g_{H} \left[ kax_{k}(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t) + bx_{k}(t) \right]$, and the rest of the change mechanism is the same as the diffusion model that does not consider advertising or price strategies. Therefore, when companies adopt advertising and penetration pricing strategies, the product diffusion model is as follows

$$
\begin{aligned}
\frac{dx_{k}(t)}{dt} &= c(1 - x_{k}(t) - y_{k}(t)) - g_{L}[kax_{k}(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t) + bx_{k}(t) - \mu x_{k}(t)] \\
\frac{dy_{k}(t)}{dt} &= g_{L}[kax_{k}(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t) + bx_{k}(t)] - \gamma y_{k}(t), \quad k = 1, 2, \ldots, N
\end{aligned}
$$

When companies implement skimming pricing, high prices affect the market potential of consumer social networks. The market potential, the number of new potential consumers, and the number of existing potential consumers are all affected by the skimming pricing strategy. Therefore, the market potential of node degree $k$ becomes $g_{H} = (x_{k}(t) + y_{k}(t))$. The change in the proportion of potential consumers with a node degree of $k$ in a unit time comes from three parts: One is the new proportion $c_{g_{H}} = [1 - g_{H}(x_{k}(t) + y_{k}(t))]$ of potential consumers. The second is the reduction ratio $g_{H} \left[ kax_{k}(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t) + bx_{k}(t) \right]$ of the number of potential consumers who purchase products affected by advertising and internal word-of-mouth [8]. Third, the number of potential consumers who have lost the desire to buy and quit the network decreased by the negative word-of-mouth or the reduced expected utility of the product $\mu g_{H} x_{k}(t)$. The change in the proportion of purchasers with a node degree of $k$ in a unit time comes from two parts: one is the change from potential consumers to the new ratio of purchasers $g_{H} \left[ kax_{k}(t) \sum_{k'=1}^{M} p(k'|k)y_{k'}(t) + bx_{k}(t) \right]$. The second is reducing the balance $\gamma y_{k}(t)$ of people who have already purchased that do not publish information or publish invalid details and exit the network. When companies adopt advertising and skimming pricing strategies, the product diffusion model is as...
follows
\[
\begin{aligned}
\frac{dx(t)}{dt} &= c\theta [1 - gH(x_k(t) + y_k(t))] - kagHx_k(t) + p(k')y_k(t) - bgHx_k(t) - \mu gHx_k(t) \\
\frac{dy(t)}{dt} &= kagHx_k(t) \sum_{k'} p(k')y_k(t) + bgHx_k(t) - \gamma y_k(t), \quad k = 1, 2, \cdots N
\end{aligned}
\]  
\tag{5}

3 Model analysis

3.1 Diffusion model analysis that does not consider advertising or price strategies

**Proposition 1.** When potential consumers in the dynamic consumer social network are internally influenced by the adoption probability \(a > \frac{\gamma(c+\mu)[k]}{(c+k)^2}\) of buying products, there is always a positive balance point in the system of formula (1). Prove that the right side of Eq. (1) is equal to 0, then we have
\[
\begin{aligned}
c(1 - x_k(t) - y_k(t)) - kax_k(t) \sum_{k'} p(k')y_k(t) - \mu x_k(t) &= 0 \\
kax_k(t) \sum_{k'} p(k')y_k(t) - \gamma y_k(t) &= 0
\end{aligned}
\]  
\tag{6}

Two equilibrium solutions \(E_0 = \left( \frac{c}{c+\mu}, \frac{P}{c+\mu} \right), E^*_k = \left( \frac{x_k(t)}{y_k(t)} \right) = \frac{c}{c+\mu} + \frac{k\theta}{\gamma} \right) \) of the equation system (6) can be obtained. By substituting \(y_k(t)\) in \(E^*_k\) into Eq. (2), we can bring the self-consistent equation \(\theta = \frac{c}{\gamma} \sum_{k'} p(k') = \theta \) obviously \(\theta = 0, F(0) = 0, \) and 
\[
\frac{d^2F(\theta)}{d\theta^2} = -2 \sum_{k'=1}^M \frac{c^2\theta p(k')}{\gamma(c+\mu)+(c+\gamma)\theta} \leq 0.
\]
Among them, \(\theta\) changes in \([0, 1]\), and the value ranges of \(a, c, \mu\) and \(\gamma\) are both \((0, 1)\), so \(\frac{d^2F(\theta)}{d\theta^2} < 0, F\theta\) is a concave function on \(0 \leq \theta \leq 1\). At the same time
\[
F(1) = 1 - \frac{1}{\gamma(c+\mu) + (c+\gamma)\theta} - 1 < 0
\]  
\tag{7}

Therefore, the necessary and sufficient condition for the existence of a unique positive solution of formula (1) is \(\frac{d^2F(\theta)}{d\theta^2} |_{\theta=0} > 0\), that is, \(\frac{d^2F(\theta)}{d\theta^2} |_{\theta=0} = \frac{ca(k^2)}{\gamma(c+\mu)[k]} - 1 > 0\). Where \(\langle k^2 \rangle = \sum_{k=1}^N \langle k^2 \rangle p(k)\) represents the second moment of the node degree of the consumer’s social network. Therefore, when \(a > \frac{\gamma(c+\mu)[k]}{(c+k)^2}\), there is always a positive equilibrium point in the system (1).

**Proposition 2.** When \(R_0 = \frac{ca(k^2)}{\gamma(c+\mu)[k]} > 1\), there is only one positive equilibrium point in the social system of formula (1), which makes the products in the system continue to spread for a long time. It is almost impossible for consumers to communicate with all purchasers in the network to obtain information [9]. Therefore, suppose that the maximum number of contacts of potential consumers is \(Z\). When \(k > Z, p(k) = 0\). For the subspace \(\Omega = \{ (x_1, y_1, x_2, y_2, \cdots, x_Z, y_Z)^T \in R^{2Z} | x_k \geq 0, y_k \geq 0, x_k + y_k \leq 1, 1 \leq k \leq Z \} \) of system (1). It is easy to verify that the space \(\Omega\) is a positively invariant set and a compact convex set.
\[
(\lambda + c + \mu)Z(\lambda + \gamma)Z^{-1}(\lambda + \gamma - \sum_{j=1}^Z b_j) = (\lambda + c + \mu)^2(\lambda + \gamma)^{Z-1}(\lambda + \gamma - \gamma \frac{ca\langle k^2 \rangle}{\gamma(c+\mu)\langle k \rangle - 1})
\]  
\tag{8}

Therefore, when \(R_0 = \frac{ca(k^2)}{\gamma(c+\mu)[k]} > 1\), the Jacobian matrix of the system (1) at the equilibrium point \((\frac{c}{c+\mu}, 0)^T\) has a unique positive eigenvalue. According to the Perron-Frobenius principle, the most significant fundamental
part of all the eigenvalues of the Jacobian matrix when $R_0 > 1$ is a positive number. Therefore, from Lemma 2, we know that Proposition 2 is established.

Proposition 1 and Proposition 2 show that when potential consumers’ decision to purchase products is only affected by the purchasers. Therefore, the probability of potential consumers’ purchases of products affected by the internal influence of the consumer’s social network must be at least higher than a critical value to ensure the monopoly firm’s Products continue to increase in dynamic consumer social networks [10]. On the other hand, when the adoption probability is lower than the critical value, the monopolistic firm’s products cannot continue to spread, and the spread fails. This critical value is related to dynamic consumer social networks (node degree distribution and network scale). Therefore, the dynamics of consumer social networks (the proportion of new potential consumers, the probability of potential consumers losing their desire to buy, and the non-purchasing probability of publishing information or publishing invalid input) are related.

3.2 Diffusion model analysis considering advertising and price strategies

**Proposition 3.** When companies implement advertising and penetration pricing strategies, there are no constraints on the continuous diffusion of products in dynamic consumer social networks. The proportion of new purchases per unit time is more significant than when only advertising strategies are implemented [11]. Prove that the right side of Eq. (4) is equal to 0 to obtain the positive equilibrium solution of the equations

\[
\begin{align*}
\left( x_k(t) \right) \\
y_k(t)
\end{align*}
\]

\[
\begin{align*}
\left( x_k(t) \right) = \frac{c}{(c + \mu) + (c + \gamma) g_L(ak\theta + b)} \left( \frac{1}{g_Lak\theta + b} \right)
\end{align*}
\]  

(9)

Assuming $a_L = a'$, $b_L = b'$, the positive equilibrium solutions of Eqs. (4) and (3) are similar. Therefore, when we consider both advertising and penetration pricing strategies, our conclusions are identical to the findings when we only consider advertising strategies. That is, there are no restrictions on the continuous spread of products in dynamic consumer social networks. In system (3), in which the enterprise adopts the advertising strategy, the proportion $y_1(t)$ of buyers with node degree $k$ gradually increases with time and finally reaches a stable state. Because of $g_L > 1$, $a_L = a' > a$, $a_L = b' > b$, the balance of newly added purchasers affected by advertising and internal influence in the system (4) per unit time is more significant than that in the case of only implementing the advertising strategy, and the proportion coefficient that affects the exit of purchasers from the system has not changed. Therefore, the proportion of newly added purchasers in the system (4) per unit time is more significant than that in the system (3).

Proposition 3 shows that when companies implement advertising and penetration pricing strategies simultaneously, products continue to spread in dynamic consumer social networks. However, penetration pricing amplifies the effect of communication between advertising and consumers and makes product diffusion faster.

**Proposition 4.** In a dynamic consumer social network where the degree distribution of the network obeys a power-law distribution, when companies implement advertising and skimming pricing strategies at the same time, $\frac{\lambda}{\mu} \geq \frac{1}{3}$ or $\frac{\lambda}{\mu} < \frac{1}{3}$ and $\omega_H < \omega_1$ make the product continue to spread [12]. Prove that the right side of Eq. (5) is equal to 0 to obtain the positive equilibrium solution of the system of equations

\[
\begin{align*}
\left( x_k(t) \right) = \frac{c_H}{(c_H + \mu)g_H + (c_H^2 + \gamma) g_H} \left( \frac{1}{g_H(ak\theta + b)} \right)
\end{align*}
\]  

(10)

We substitute $y_k(t)$ in Eq. (10) into Eq. (2):

\[
\theta = \frac{1}{\langle k \rangle} \sum_{k'=1}^M k' p(k') \frac{c_H(ak'\theta + b)}{\gamma(c_H + \mu) + (c_H^2 + \gamma)(b + ak'\theta)}
\]  

(11)
The degree distribution of the network obeys the power-law distribution of dynamic consumer social networks, which can be obtained from Eq. (12)

\[ F(\theta) = \frac{1}{\langle k \rangle} \sum_{k' = 1}^{M} k' p(k') \left( \frac{c\theta + b}{\gamma(c\theta + b + \mu)(c\theta + b + \mu + k)} \right) - \theta. \]

Since \( F(0) = \frac{cH}{\gamma(c\eta + \mu + (c\eta + \mu + k)} > 0, \) \( \frac{d^2 F(\theta)}{d\theta^2} < 0, \) where the value ranges of \( a, b, c, \mu \) and \( \gamma \) are both \( (0, 1), g_H > 0, \) the necessary and sufficient conditions for \( F(0) = 0 \) to have a unique positive solution in \( \theta \in [0, 1] \) are:

\[ F(1) = \frac{1}{\langle k \rangle} \sum_{k' = 1}^{M} \frac{k' p(k') c_H (a' + b)}{\gamma(c_H + \mu) + (c_H + \mu + k)(a' + b + \theta)} - 1 \leq 0 \]  \hspace{1cm} (12)

The degree distribution of the network obeys the power-law distribution of dynamic consumer social networks, which can be obtained from Eq. (12)

\[ g_H^2 - g_H + \frac{\gamma}{c} \geq 0 \]  \hspace{1cm} (13)

When \( \frac{\gamma}{c} \geq \frac{1}{4} \) formula (13) always holds. At this time, the price change range of the company’s implementation of the skimming pricing strategy only needs to ensure that the proportion of new purchasers is not too small.

Proposition 5 shows that when companies implement advertising and skimming pricing strategies simultaneously, they must ensure that the company’s products continue to spread in the dynamic consumer social network where the network’s degree distribution obeys the power-law distribution. There are constraints on product prices [13]. This price constraint has nothing to do with the network characteristics of the dynamic consumer social network. Still, it is only related to the proportion of new potential consumers in the dynamic consumer social network and the probability that the consumers who have purchased the product will not publish information or publish invalid information.

4 Simulation

This section first compares and analyses the diffusion patterns of products in dynamic consumer social networks under different corporate strategies. Then it verifies the existence of critical conditions for continuous diffusion or diffusion failure when the enterprise does not adopt any approach. Finally, it demonstrates the impact of changes in the characteristics of dynamic consumer social networks on product proliferation [14].

The number of nodes in the network is \( N = 200, \) and the node degree of consumer social networks obeys the power-law distribution \( p(k) = mk^{-\gamma}. \) Assume \( r_1 = 3, r_2 = 2 \) has a new potential consumer proportion coefficient \( c = 0.01, \) internal influence coefficient \( a = 0.01, \) advertising influence coefficient \( b = 0.005, \) and the proportion of potential consumers who lose their desire to buy and withdraw from the consumer social system \( \mu = 0.003. \) The balance of buyers removed from the network \( \gamma = 0.005 \) \( \omega_L/\omega_H = 0.75 \) is used for penetration pricing, \( \omega_H/\omega_L = 4/3 \) is used for skimming pricing, and the price influence coefficient is \( \eta = 1.2. \) To facilitate comparison and analysis, we select the same number of simulation periods.

First of all, the simulation diagram without considering advertising or price strategy is shown in Figure 2a. Next, a simulation diagram that only considers advertising strategies is shown in Figure 2b. Third, Figure 2c shows a simulation diagram that considers both advertising and penetration pricing strategies. Finally, the simulation diagram of the diffusion model considering both advertising and skimming pricing strategies is shown in Figure 2d.

The proportion of potential consumers in dynamic consumer social networks has continued to decline. Although the balance of the number of buyers has continued to rise, the trend of change gradually slowed down and finally reached a stable state. Unlike the classic Bass model, all potential consumers in the Bass model will eventually adopt the product ultimately. In the end, the proportion of potential consumers is equal to 0.

The comparison between Figure 2a and b shows that the time required for diffusion to reach a steady-state under the latter strategy is shortened. The proportion of the number of purchasers in the constant state is also relatively high. Comparing the slope changes of the curves in Figure 2c and 2d, it can be found that the company only implements the advertising strategy. Implements the advertising and penetration pricing strategy simultaneously, the product diffusion rate per unit time is faster under the latter approach.
5 Conclusion

This article also considers the dynamics of consumer social networks and the heterogeneity of the structure of consumer social networks. We have constructed a differential equation model for the product diffusion of monopolistic firms. The study found that the threshold conditions for continuous product diffusion without considering advertising or price strategies are related to the network characteristics of consumer social networks and the dynamics of consumer social networks. If innovators are in the network, the product will continue to spread faster. When companies implement advertising and skimming pricing simultaneously, the continued proliferation of products is related to price constraints.

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