General CP Violation in Minimal Left-Right Symmetric Model and Constraints on the Right-Handed Scale

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Abstract

In minimal left-right symmetric theories, the requirement of parity invariance allows only one complex phase in the Higgs potential and one in the Yukawa couplings, leading to a two-phase theory with both spontaneous and explicit CP violations. We present a systematic way to solve the right-handed quark mixing matrix analytically in this model and find that the leading order solution has the same hierarchical structure as the left-handed CKM matrix with one more CP-violating phase coming from the complex Higgs vev. Armed with this explicit right-handed mixing matrix, we explore its implications for flavor changing and conserving processes in detail, low-energy CP-violating observables in particular. We report an improved lower bound on the $W_R$ mass of 2.5 TeV from $\Delta M_K$ and $\Delta M_B$, and a somewhat higher bound (4 TeV) from kaon decay parameters $\epsilon$, $\epsilon'$, and neutron electric dipole moment. The new bound on the flavor-changing neutral Higgs mass is 25 TeV.
I. INTRODUCTION

The physics beyond the Standard Model (SM) has been the central focus of high-energy phenomenology for more than three decades. Many proposals, including supersymmetry, technicolor, little Higgs, and extra dimensions, have been made and studied thoroughly in the literature, tests are soon to be made at the Large Hadron Collider (LHC). One of the earliest proposals, the left-right symmetric model (LRSM), was motivated by the hypothesis that parity is a perfect symmetry at high-energy, and is broken spontaneously at low-energy due to the asymmetric vacuum \cite{1}. Asymptotic restoration of parity has a definite aesthetic appeal \cite{2}. The model has a number of additional attractive features, including a natural explanation of weak hyper-charge in terms of baryon and lepton numbers, existence of right-handed neutrinos and entailed seesaw mechanism for neutrino masses, possibility of spontaneous CP (charge-conjugation-parity) violation, and natural solution for the strong CP problem. The model can be constrained strongly by low-energy physics and predicts clear signatures at colliders. It so far remains a decent possibility for new physics.

The LRSM is best constrained at low-energy by flavor-violating mixing and decays, particularly CP violating observables. In making theoretical predictions, the major uncertainty comes from the unknown right-handed quark mixing matrix, similar in spirit to that of the left-handed quark Cabibbo-Kobayashi-Maskawa (CKM) mixing. The new mixing is a unitary matrix, depending on 9 real parameters: 6 CP violation phases and 3 rotational angles. All are physical after the left-handed CKM mixing is rotated into a standard 4-parameter form.

Historically, two special CP violation scenarios in LRSM have been considered. The first one, “the manifest left-right symmetry”, assumes that there is no spontaneous CP violation i.e. all Higgs vacuum expectation values (vev’s) are real. Then the quark mass matrices are hermitian, and the left- and right-handed quark mixings become identical, modulo the sign uncertainty of the elements from negative quark masses. The second scenario, “pseudo-manifest left-right symmetry”, assumes that the CP violation comes entirely from spontaneous symmetry breaking (SSB) of the vacuum and all Yukawa couplings are real \cite{3}. Here the quark mass matrices are complex and symmetric, implying that the right-handed quark mixing is related to the complex conjugate of the CKM matrix multiplied by additional CP phases. There are few studies of the model with general CP violations in the literature \cite{4, 5}. It has been noted that there are problems with both manifest and pseudo-manifest scenarios. The manifest LRSM with real potential and vev’s always provides more minimization conditions than the number of vev’s, thus has a fine-tuning problem. In any case, in the non-supersymmetric LRSM, the assumption of all parameters of the model being real is not technically natural since loop corrections lead to one of the parameters being complex. On the other hand, the pseudo-manifest LRSM, where exact CP is assumed before symmetry breaking and all couplings in the Higgs potential are real, leads in the decoupling limit to either a model with light triplet Higgs, which is already excluded by SM precision test, or a two Higgs doublet model, which is excluded by experiment due to large tree level flavor-changing neutral current \cite{6}. Therefore, neither of the two special scenarios can be realistic.

In a recent paper \cite{7}, we reported a systematic approach to analytically solving the right-handed quark mixing in the minimal LRSM where only the requirement of parity invariance is imposed prior to symmetry breaking, leaving automatically only one CP phase in the Higgs potential and one in the Yukawa couplings and leading to a theory with both explicit
and spontaneous CP violations. This model therefore falls in-between the above two extreme cases and is free of the problems described above. Our approach is based on the observation that in the absence of any fine tuning, $m_t \gg m_b$ implies that the ratio of the two vev’s of the Higgs bi-doublet, $\xi = \kappa'/\kappa$, is small and is of the order of $m_b/m_t$. In the leading-order in $\xi$, we find a linear equation for the right-handed quark mixing matrix which can be readily solved. We present an analytical solution of this equation valid to $O(\lambda^3)$, where $\lambda = \sin \theta_C$ is the Cabibbo mixing parameter. The leading right-handed quark mixing is nearly the same as the left-handed CKM matrix, except for additional phases which are fixed by $\xi$, spontaneous CP phase $\alpha$, and the quark masses.

Our work is similar in spirit to the detailed numerical study of the general CP LRSM made by Kiers et al., with the two vev’s ratio $\kappa'/\kappa$ fixed exactly to $m_b/m_t$. It is interesting that the gross feature of the right-handed CKM was obtained already in the appendix of that paper, in particular the hierarchical structure of flavor mixing and the magnitude of the Dirac CP phase. However, realistic studies were made numerically for lack of an explicit solution with known precision. In fact, much of the numerical work of Ref. goes into solving the right-handed CKM in the presence of 11 input parameters, which must be scanned through using Monte Carlo to obtain the physical quark masses and the left-handed CKM mixing. Because the extensive nature of numerical study, it is difficult to see some of the physics in a clear way, in particular, the interplay between the explicit and spontaneous CP violations in physical observables. Our explicit analytic solution for the right-handed CKM makes extensive analytical studies simple and straightforward.

In this paper, we first give the detailed method obtaining the analytical solution to righthanded quark mixing. With this explicit right-handed mixing, we study the neutral kaon and B-meson systems and the neutron electric dipole moment (EDM) to obtain the lower bound on right-handed W-boson mass scale. The neutral kaon mass mixing provides a rigorous lower bound $M_{W_R} > 2.5 \text{ TeV}$, with the use of the new lattice QCD calculations of the four-quark matrix elements and the strange quark mass. The indirect CP violation $\epsilon$ receives large contributions from both explicit and spontaneous CP phases, and from both the gauge-boson box diagram and flavor-changing neutral Higgs (FCNH). There are strong cancelations among all the contributions, which in turn constrain severely the relation among the spontaneous CP phase, $M_{W_R}$, and FCNH mass $M_H$. We use the cancelation condition to fix the spontaneous CP phase, which is then used to predict the neutron EDM in terms of the model parameters. Using the experimental bound on the EDM, we obtain a strong lower bound on $M_{W_R}$, which can be improved with better calculations of the hadronic matrix elements and more precise experimental data. Furthermore, we study implications of direct CP violation in $K^0$ and $B^0$ decays. In the former case, a strong lower bound on $M_{W_R}$ is obtained under the factorization assumption for the four-quark matrix elements. The CP violating observables in the kaon and B systems and the neutron EDM provide competitive or even stronger bounds than the well-known kaon mass mixing. We also present a detailed study of the Higgs sector in the presence of the spontaneous CP phase, including the mass spectrum, the neutral and charged couplings to the quarks, and the bound on the FCNH mass.

The paper is organized as follows. We first briefly introduce the minimal LRSM in Sec. II. In Sec. III, we report our method in solving for the righthanded CKM matrix in the scenario of generic CP violation, including both spontaneous and explicit phases as is the case with only parity invariance. In Sec. IV, we study the Higgs sector, including the mass spectrum and Higgs couplings to quarks. In Sec. V, we explore the well-known neutral
kaon mass difference to find out updated constraints on the right-handed $W$ mass and the FCNH mass. We also consider similar constraints from the neutral B-meson system. The CP violations in various processes are discussed in Sec. VI, including $\epsilon$, $\epsilon'$, neutron EDM and CP asymmetry in $B \to J/\psi K_S$, to constrain the mass of $W_R$ and the spontaneous CP phase $\alpha$. We find that they place consistently strong lower bounds on $M_{W_R}$ and $M_H$. We conclude the paper in Sec. VII, with a table summarizing various constraint on the right-handed scales.

II. THE MINIMAL LEFT-RIGHT SYMMETRIC MODEL

The minimal LRSM is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Parity is assumed to be a good symmetry at the Lagrangian density level, and is broken spontaneously by vev’s of Higgs fields. The electric charge formula can be written as a generalized Gellmann-Nishijima formula including the third component of the right-handed isospin, $T_{3L}$ and $T_{3R}$, and the difference between baryon and lepton numbers $[8]$, 

$$ Q = T_{3L} + T_{3R} + (B - L)/2. $$

This in turn gives an explicit explanation for the standard model $U(1)$-hypercharge in terms of physical quantum numbers rather than an arbitrarily adjustable quantum number $Y$.

In the matter sector, the left-handed fermions form fundamental representations of the $SU(2)_L$ gauge group, while right-handed ones form the representations of the $SU(2)_R$ gauge group. The right-handed neutrinos are introduced automatically so that the right-handed leptons also form doublets under $SU(2)_R$

$$ Q_L = \left( \begin{array}{c} u_L \\ d_L \end{array} \right) \in (2, 1, \frac{1}{3}), \quad Q_R = \left( \begin{array}{c} u_R \\ d_R \end{array} \right) \in (1, 2, \frac{1}{3}), $$

$$ L_L = \left( \begin{array}{c} \nu_L \\ l_L \end{array} \right) \in (2, 1, -1), \quad L_R = \left( \begin{array}{c} \nu_R \\ l_R \end{array} \right) \in (1, 2, -1), $$

where the quantum numbers are those of the above gauge groups. From these, we can easily write down the gauge-coupled Lagrangian density for fermions,

$$ \mathcal{L}_{\text{fermion}} = \bar{Q}_{Li} \gamma^\mu \left( i \partial_\mu - \frac{g_L}{2} \bar{W}_{L\mu} \cdot \bar{\tau} - \frac{g'}{6} B_\mu \right) Q_{Li} $$

$$ + \bar{Q}_{Ri} \gamma^\mu \left( i \partial_\mu - \frac{g_R}{2} \bar{W}_{R\mu} \cdot \bar{\tau} - \frac{g'}{6} B_\mu \right) Q_{Ri} $$

$$ + \bar{L}_{Li} \gamma^\mu \left( i \partial_\mu - \frac{g_L}{2} \bar{W}_{L\mu} \cdot \bar{\tau} + \frac{g'}{2} B_\mu \right) L_{Li} $$

$$ + \bar{L}_{Ri} \gamma^\mu \left( i \partial_\mu - \frac{g_R}{2} \bar{W}_{R\mu} \cdot \bar{\tau} + \frac{g'}{2} B_\mu \right) L_{Ri}, $$

where the index $i = 1, 2, 3$ labels fermion generation with all the fields being in flavor eigenstates. $\bar{W}_{R,L\mu}$ and $B_\mu$ are the gauge fields associated with the above gauge groups, with corresponding couplings $g_L = g_R$ and $g'$. $\bar{\tau}$ are Pauli matrices for isospins. The right-handed currents couple to the gauge bosons $W_R$ in a way symmetric to the left-handed counterparts.
The Higgs sector contains a bidoublet $\phi$, belonging to the $(2, 2, 0)$ representation of the gauge group, which is the left-right symmetric version of the SM Higgs doublet and two triplets $\Delta_{L,R}$ belonging to $(3, 1, 2)$ and $(1, 3, 2)$, respectively,

\[
\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}, \quad \Delta_L = \begin{pmatrix} \delta_L^+ / \sqrt{2} & \delta_L^{++} \\ \delta_L^- / \sqrt{2} & -\delta_L^- / \sqrt{2} \end{pmatrix}, \\
\Delta_R = \begin{pmatrix} \delta_R^+ / \sqrt{2} & \delta_R^{++} \\ \delta_R^- / \sqrt{2} & -\delta_R^- / \sqrt{2} \end{pmatrix}.
\]

The Higgs boson’s kinetic energy and coupling to the gauge fields are canonical,

\[
\mathcal{L}_{\text{Higgs kin}} = \text{Tr}
[D_{\mu}\Delta_L]^{\dagger}(D^\mu\Delta_L) + \text{Tr}[D_{\mu}\Delta_R]^{\dagger}(D^\mu\Delta_R) + \text{Tr}[D_{\mu}\phi]^{\dagger}(D^\mu\phi),
\]

where the covariant derivatives are

\[
D_{\mu}\phi = \partial_{\mu}\phi + ig_L \frac{2}{g_{L}} \bar{W}^3_L \cdot \vec{\tau} \phi - ig_R \frac{2}{g_{R}} \bar{W}^3_R \cdot \vec{\tau} \phi,
\]

\[
D_{\mu}\Delta_{(L,R)} = \partial_{\mu}\Delta_{(L,R)} + i \frac{g_{(L,R)}}{2} \left[ \bar{W}^3_{(L,R)\mu} \cdot \vec{\tau}, \Delta_{(L,R)} \right] + ig'B_{\mu}\Delta_{(L,R)}.
\]

Again, left-right symmetry is explicit.

Like the SM, we use the vev’s of neutral Higgs fields to break the gauge symmetry. The symmetry group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is first broken to $SU(2)_L \times U(1)_Y$ by the vev $\langle \Delta_R \rangle = v_R$ at TeV or multi-TeV scale. Then electroweak symmetry breaking is induced by the vev’s of $\phi$. With a generic Higgs potential, all the neutral components of the Higgs get vev’s,

\[
\langle \phi \rangle = \begin{pmatrix} \kappa e^{i\alpha_1} & 0 \\ 0 & \kappa' e^{i\alpha_2} \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_1} & 0 \end{pmatrix}, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R e^{i\theta_2} & 0 \end{pmatrix},
\]

where there are 4 neutral complex components and so generically 4 phases. At first it appears that 3 of them can be eliminated through gauge symmetry because we have 3 generators, $T_{3L}$, $T_{3R}$, and $B - L$, commuting with the electromagnetic charge operator $Q$. In reality, however, we can eliminate only 2. If the transformation parameters associated with the above 3 operators are $\theta_L$, $\theta_R$ and $\theta_{B-L}$, the Higgs fields transform as,

\[
\langle \phi \rangle \rightarrow e^{iT_{3L}\theta_L} \langle \phi \rangle e^{-iT_{3R}\theta_R}, \\
\langle \Delta_L \rangle \rightarrow e^{iT_{3L}\theta_L} \langle \Delta_L \rangle e^{-iT_{3L}\theta_L} e^{i\theta_{B-L}}, \\
\langle \Delta_R \rangle \rightarrow e^{iT_{3R}\theta_R} \langle \Delta_R \rangle e^{-iT_{3R}\theta_R} e^{i\theta_{B-L}}.
\]

This leads a transformation of the vev phases,

\[
\alpha_1 \rightarrow \alpha_1 + \frac{1}{2}\theta_L - \frac{1}{2}\theta_R, \quad \alpha_2 \rightarrow \alpha_2 - \frac{1}{2}\theta_L + \frac{1}{2}\theta_R,
\]

\[
\theta_{1,2} \rightarrow \theta_{1,2} - \theta_{L,R} + \theta_{B-L}.
\]
It is clear that there are two independent combinations of transformation parameters, allowing removing two phases only. Conventionally the phases of $\kappa$ and $v_R$ are set to zero, and thus the general form of Higgs vev’s is simplified to

$$\langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{pmatrix},$$

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 1 \\ v_L e^{i\phi_L} & 0 \end{pmatrix}, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \quad (7)$$

If one chooses $v_L = 0$, the only remaining phase $\alpha$ is physically relevant.

Since the bidoublet $\phi$ transforms non-trivially under both $SU(2)_L$ and $SU(2)_R$, the gauge bosons $W_L$ and $W_R$ are not the mass eigenstates after SSB. From (3) we get the $W$-boson mass matrix,

$$-\mathcal{L}^{W-\text{Mass}} = \begin{pmatrix} W_{L\mu}^- & W_{R\mu}^- \end{pmatrix} \begin{pmatrix} \frac{1}{2}g^2(\kappa^2 + \kappa'^2 + 2v_L^2) - g^2 \kappa \kappa' e^{-i\alpha} & g^2 v_L^2 \\ -g^2 \kappa \kappa' e^{i\alpha} & g^2 v_R^2 \end{pmatrix} \begin{pmatrix} W_{L\mu}^+ & W_{R\mu}^+ \end{pmatrix}, \quad (8)$$

where $W_{L,R}^\pm = (W_{L,R}^1 \mp i W_{L,R}^2)/\sqrt{2}$, and $g = g_L = g_R$ by parity. Since $v_L$ is at most on the order of the left-handed neutrino masses and $\sqrt{\kappa^2 + \kappa'^2}$ on the SM scale, $v_L \ll \sqrt{\kappa^2 + \kappa'^2}$, and we can neglect $v_L^2$ in Eq. (8). The mass matrix can be diagonalized by a unitary transformation

$$\begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix} = \begin{pmatrix} \cos \zeta & \sin \zeta e^{-i\lambda} \\ \sin \zeta e^{i\lambda} & \cos \zeta \end{pmatrix} \begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix}, \quad (9)$$

where $W_1^+$ and $W_2^+$ are mass eigenstates (1 and 2 subscripts shall not be confused with the Cartesian components of the isospins), with masses $M_{W_1} \simeq M_{W_L} = g\sqrt{\kappa^2 + \kappa'^2}/\sqrt{2}$ and $M_{W_2} \simeq M_{W_R} = g v_R$. The parameters $\zeta$ and $\lambda$ are related to the CP phase $\alpha$ and the masses of $W_1$, $W_2$, $\kappa$, and $\kappa'$

$$\lambda = -\alpha, \quad \tan \zeta = -\frac{\kappa \kappa'}{v_R^2} \simeq -\frac{2\xi}{\kappa^2} \left( \frac{M_{W_L}}{M_{W_R}} \right)^2, \quad (10)$$

where again $\xi = \kappa'/\kappa$. If there is no cancelation in generating quark masses, $\xi$ has a natural size $m_b/m_t$, and thus $\zeta$ is suppressed by both $(M_{W_L}/M_{W_R})^2$ and $m_b/m_t$ and is smaller than $4 \times 10^{-5}$, which is much smaller than the current experimental bound $10^{-2}$. Even so this tiny mixing will be the dominating contribution to the neutron EDM, as we will discuss below. In terms of the mass eigenstates, the charged current couplings in the quark sector are

$$\mathcal{L}^{W-\text{current}} = -\frac{g_L}{\sqrt{2}} \bar{u}_{L\mu} \gamma^\mu (\cos \zeta W_{1\mu}^+ - \sin \zeta e^{i\lambda} W_{2\mu}^+) d_{Li}$$

$$-\frac{g_R}{\sqrt{2}} \bar{u}_{Ri} \gamma^\mu \left( \sin \zeta e^{-i\lambda} W_{1\mu}^+ + \cos \zeta W_{2\mu}^+ \right) d_{Ri} + \text{h.c.}. \quad (11)$$

The above expression is in quark flavor basis. In the next section, we will consider the quark mass basis in which it will be modified by the CKM mixing matrices.
III. RIGHT-HANDED QUARK MIXING MATRIX

In this section, we will focus on the quark sector, solving for the right-handed quark mixing matrix compatible with the observed quark masses and the left-handed mixing. The solution is valid in the general CP violation scenario in which both explicit and spontaneous CP breakings are allowed. The only significant assumption we will make is that bidoublet higgs vev’s and related Yukawa couplings have a hierarchical structure and there is no large cancelation in generating quark masses, and the solution can be made in a systematic expansion of the relevant small parameter.

The most general Yukawa coupling of the quark fields with the Higgs bidoublet $\phi$ is given by

$$L_Y = Q_L^i (h_{ij} \phi + \bar{h}_{ij} \bar{\phi}) Q_R^j + \text{h.c.},$$

(12)

where $\bar{\phi} = -i \tau_2 \phi^* i \tau_2$ and flavor indices $i, j = 1, 2, 3$. Parity symmetry, under which $\phi \leftrightarrow \phi^*$ and $Q_L^i \leftrightarrow Q_R^i$, constrains $h$ and $\bar{h}$ to be hermitian matrices. After SSB, the above Lagrangian density yields the following up- and down-type quark mass matrices,

$$M_U = \kappa h + \kappa' e^{-i \alpha} \bar{h},$$
$$M_D = \kappa' e^{i \alpha} h + \kappa \bar{h}.$$  

(13)

There are two terms in each, and we assume that there is no fine-tuned cancelation to generate a quark mass scale. Since the top quark mass is much larger than that of the bottom quark, the assumption implies that $h$ and $\bar{h}$, $\kappa$ and $\kappa'$ should not be on the same order. Without loss of generality, we take $\kappa' \ll \kappa$ and $\bar{h} \ll h$. To leading order in $\kappa'/\kappa$, we have

$$M_U \simeq \kappa h,$$
$$M_D = \kappa' e^{i \alpha} h + \kappa \bar{h}.$$  

(14)

The two terms in the down-type quark masses can be on the same order, however. Because of the flavor independence of the gauge coupling and the hermiticity of $h$, we can work in the basis where $M_U$ is diagonal,

$$M_U = S_U \begin{pmatrix} m_u & m_c & m_t \end{pmatrix} \equiv S_U \hat{M}_U,$$

(15)

in which $S_U = \text{diag}\{s_u, s_c, s_t\}$ is the sign of the up-type quark masses. It is present because the eigenvalues of a hermitian matrix can either be positive or negative, and by convention we take all $m_i$ positive. In this basis, $M_D$ is not diagonal and is related to

$$\hat{M}_D = \text{diag}\{m_d, m_s, m_b\}$$

via left- and right-handed CKM rotations. Since the phase factor $e^{i \alpha}$ in $M_D$ is generically non-zero, $V_L^{\text{CKM}} \neq V_R^{\text{CKM}}$.

$$M_D = V_L^{\text{CKM}} \hat{M}_D V_R^{\text{CKM} \dagger} S_U.$$

(16)

For simplicity, we will omit the superscript CKM henceforth. From Eqs. (14), (15) and (16), we have

$$\kappa \bar{h} = V_L \hat{M}_D V_R^\dagger S_U - \frac{\kappa'}{\kappa} S_U \hat{M}_U e^{i \alpha}.$$

(17)
Two comments are in order. First, through the phase transformations that are chirally independent but isospin-dependent, $u^i_{L,R} \rightarrow e^{i\alpha}u^i_{L,R}$ and $d^i_{L,R} \rightarrow e^{i\beta}d^i_{L,R}$, one can bring $V_L$ to a standard form with only 4 parameters (3 rotations and 1 CP violation phase) and the above equation remains the same. The $\hat{h}$ matrix, however, will be subjected to a unitary transformation and remains hermitian. Second, after the transformation, all parameters in the unitary matrix $V_R$ must be physical, including 3 rotations and 6 CP-violating phases.

To make further progress, one uses the hermiticity condition for $\hat{h}$, which leads to the following equation,

$$\hat{M}_D \hat{V}_R^\dagger - \hat{V}_R \hat{M}_D = 2i \xi \sin \alpha \ V_L^\dagger \hat{M}_U S_U V_L ,$$  \hspace{1cm} (18)

where $\hat{V}_R$ is the quotient between the left and right mixing $V_R = S_U V_L \hat{V}_R$. There are a total of 9 equations in the above expression, which is just enough to solve 9 parameters in $\hat{V}_R$. It is interesting to note that if there is no spontaneous CP violation, $\alpha = 0$, we recover the solution $V_R = S_U V_L S_D$, in which $S_D = \text{diag} \{ s_d, s_s, s_b \}$ is the sign matrix for down-type quark masses. This is just the manifest LRS case.

The above linear equations can be readily solved utilizing the hierarchy between down-type quark masses. We find an analytical expression for $V_R$ up to order of $O(\lambda^3)$, where $\lambda$ is sine of Cabibbo angle. We begin directly from (18) and the left side is anti-hermitian, and we can write it explicitly

$$\begin{pmatrix}
-2im_d \ \text{Im} \hat{V}_{R11} & -m_s \hat{V}_{R12} & -m_b \hat{V}_{R13} \\

m_s \hat{V}_{R12} & -2im_s \ \text{Im} \hat{V}_{R22} & -m_b \hat{V}_{R23} \\

m_b \hat{V}_{R13} & m_b \hat{V}_{R23} & -2im_b \ \text{Im} \hat{V}_{R33}
\end{pmatrix} ,$$  \hspace{1cm} (19)

where we have used $m_d \ll m_s \ll m_b$. The right-hand side of Eq. (18) depends on the physical quark masses, the standard CKM matrix, and the spontaneous CP violation parameter $\xi \sin \alpha$. Thus we can solve $\hat{V}_R$ in terms of these up to $O(\lambda^3)$

$$\text{Im} \hat{V}_{R11} = -r \sin \alpha \frac{m_b m_c}{m_d m_t} \lambda^2 \left( s_c + s_t \frac{m_t}{m_c} A^2 \lambda^4 \right) ,$$  \hspace{1cm} (20)

$$\text{Im} \hat{V}_{R22} = -r \sin \alpha \frac{m_b m_c}{m_s m_t} \left( s_c + s_t \frac{m_t}{m_c} A^2 \lambda^4 \right) ,$$  \hspace{1cm} (21)

$$\text{Im} \hat{V}_{R33} = -r \sin \alpha s_t ,$$  \hspace{1cm} (22)

$$\hat{V}_{R12} = 2ir \sin \alpha \frac{m_b m_c}{m_s m_t} \lambda \left( s_c + s_t \frac{m_t}{m_c} \lambda^4 A^2 \right) ,$$  \hspace{1cm} (23)

$$\hat{V}_{R13} = -2ir \sin \alpha A \lambda^3 (1 - \rho + i\eta) s_t ,$$  \hspace{1cm} (24)

$$\hat{V}_{R23} = 2ir \sin \alpha A \lambda^2 s_t ,$$  \hspace{1cm} (25)

where $r \equiv (m_t/m_b) \xi$, and $\lambda$, $A$, $\rho$, and $\eta$ are Wolfenstein parameters for $V_L$. The above solution exists only when $|r \sin \alpha| \leq 1$, which is an interesting and unexpected constraint. Since the natural size of $\xi$ is $m_b/m_t$, $r \sim 1$, allowing angle $\alpha \sim 1$. Given the physical values of various parameters, we find the following power counting: $\text{Im} \hat{V}_{R11} \sim \lambda$, $\text{Im} \hat{V}_{R22} \sim \lambda$, $\hat{V}_{R12} \sim \lambda^2$, $\hat{V}_{R13} \sim \lambda^3$ and $\hat{V}_{R23} \sim \lambda^2$. Using unitarity condition for $\hat{V}_R$, we can solve all other elements.
Defining new phases \( \sin \theta_i = S_{Dii} \text{Im} V_{Rii} \), where \( i = 1, 2, 3 \), we have up to \( \mathcal{O}(\lambda^3) \),

\[
\begin{align*}
\hat{V}_{Rii} &= S_{Dii} e^{i\theta_i}, \\
\hat{V}_{R21} &= -s_d s_s \hat{V}_{R12}^* e^{i(\theta_1 + \theta_2)}, \\
\hat{V}_{R31} &= -s_d s_b \hat{V}_{R13}^* e^{i(\theta_1 + \theta_3)}, \\
\hat{V}_{R32} &= -s_s s_b \hat{V}_{R23}^* e^{i(\theta_2 + \theta_3)}.
\end{align*}
\]

Therefore, we can write the righthanded CKM matrix in a more compact form

\[
V_R = P_U \tilde{V}_L P_D,
\]

in which \( P_U = \text{diag}(s_u, s_e e^{2i\theta_2}, s_t e^{2i\theta_3}) \), \( P_D = \text{diag}(s_d e^{i\theta_1}, s_s e^{-i\theta_2}, s_b e^{-i\theta_3}) \), and

\[
\tilde{V}_L = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & \lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & \lambda^2 e^{-2i\theta_2} \\
\lambda^3(1 - \rho - i\eta) & -\lambda^2 e^{2i\theta_2} & 1
\end{pmatrix},
\]

which differs from \( V_L \) by a small phase in 23 and 32 elements.

The phases \( \theta_i \) are functions of parameter \( r \sin \alpha \) and the signs of the quark masses \( s_i \) and \( \bar{s}_i \). Numerically we have,

\[
\begin{align*}
\theta_1 &= -\sin^{-1}[0.31(s_d s_c + 0.18 s_d s_t) r \sin \alpha], \\
\theta_2 &= -\sin^{-1}[0.32(s_s s_c + 0.25 s_s s_t) r \sin \alpha], \\
\theta_3 &= -\sin^{-1}[s_s s_t r \sin \alpha],
\end{align*}
\]

where we have taken from the Particle Data Group the central values of the quark masses \( m_u = 2.7 \text{ MeV} \), \( m_d = 5 \text{ MeV} \), \( m_s = 98 \text{ MeV} \), \( m_c = 1.25 \text{ GeV} \), \( m_b = 4.2 \text{ GeV} \), and \( m_t = 174 \text{ GeV} \) at scale 2 GeV [9]. It shall be noted that since only the quark mass ratios enter the mixing matrix and the quark masses run multiplicatively, the result is independent of quark mass scale. The parameters for the left-hand quark mixing are taken as \( \lambda = 0.2272 \), \( A = 0.818 \), \( \rho = 0.221 \), and \( \eta = 0.34 \).

A few remarks about the above result are in order. First, the hierarchical structure of the right-handed mixing is similar to that of the left-handed CKM, namely 1-2 mixing is of order \( \lambda \), 1-3 order \( \lambda^3 \) and 2-3 order \( \lambda^2 \). Second, every element now has a substantial CP phase. When \( r \) is of order 1, the elements involving the first two families have CP phases of order \( \lambda \), and the phases involving the third family are of order 1. These phases are all related to the single spontaneous CP-violating phase \( \alpha \), and generate rich phenomenology for \( K \) and \( B \) meson systems as well as the neutron EDM. Finally, from (27) and (29), it is clear that the final solution is a function of sign bi-products \( s_i s_j \). We can always fix one of them, say \( s_u \), to be positive, then we are left with \( 2^5 = 32 \) distinct sectors. The actual physical choice must be determined by phenomenology, as we will illustrate in the following sections.

**IV. HIGGS POTENTIAL, MASS SPECTRUM AND COUPLINGS**

In this section, we discuss several issues related to the Higgs sector. In particular, we consider the possibility of spontaneous CP violation from the Higgs potential, the mass
spectrum of the Higgs bosons, and the Higgs couplings to the quark sector. The results are useful for phenomenological studies in the following sections. Some of the results presented here have appeared in the literature before, and we include them for completeness.

The most general renormalizable Higgs potential invariant under parity is given by

\[ V(\phi, \Delta_L, \Delta_R) = -\mu_1^2 \text{Tr}(\phi^\dagger \phi) - \mu_2^2 \left[ \text{Tr}(\tilde{\phi}^\dagger \phi) + \text{Tr}(\phi^\dagger \tilde{\phi}) \right] - \mu_3^2 \left[ \text{Tr}(\Delta_L \Delta_L^\dagger_L) + \text{Tr}(\Delta_R \Delta_R^\dagger_R) \right] 
+ \lambda_1 \left[ \text{Tr}(\phi^\dagger \phi) \right]^2 + \lambda_2 \left( \left[ \text{Tr}(\phi^\dagger \phi) \right]^2 + \left[ \text{Tr}(\tilde{\phi}^\dagger \phi) \right]^2 \right) 
+ \lambda_3 \text{Tr}(\tilde{\phi}^\dagger \phi) \text{Tr}(\tilde{\phi}^\dagger \phi) + \lambda_4 \text{Tr}(\phi^\dagger \phi) \text{Tr}(\phi^\dagger \phi) + \text{Tr}(\phi^\dagger \phi) 
+ \rho_1 \left( \left[ \text{Tr}(\Delta_L \Delta_L^\dagger_L) \right]^2 + \left[ \text{Tr}(\Delta_R \Delta_R^\dagger_R) \right]^2 \right) 
+ \rho_2 \left[ \text{Tr}(\Delta_L \Delta_L^\dagger_L) \text{Tr}(\Delta_L^\dagger \Delta_L) + \text{Tr}(\Delta_R \Delta_R^\dagger_R) \text{Tr}(\Delta_R^\dagger \Delta_R) \right] 
+ \rho_3 \text{Tr}(\Delta_L \Delta_L^\dagger_L) \text{Tr}(\Delta_R \Delta_R^\dagger_R) + \rho_4 \left[ \text{Tr}(\Delta_L \Delta_L^\dagger_L) \text{Tr}(\Delta_R \Delta_R^\dagger_R) + \text{Tr}(\Delta_L \Delta_L^\dagger_R) \text{Tr}(\Delta_R \Delta_R^\dagger_L) \right] 
+ \alpha_1 \text{Tr}(\phi^\dagger \phi) \left[ \text{Tr}(\Delta_L \Delta_L^\dagger_L) + \text{Tr}(\Delta_R \Delta_R^\dagger_R) \right] 
+ \left\{ \alpha_2 e^{i\delta_2} \left[ \text{Tr}(\phi^\dagger \phi) \text{Tr}(\Delta_L \Delta_L^\dagger_L) + \text{Tr}(\phi^\dagger \phi) \text{Tr}(\Delta_R \Delta_R^\dagger_R) \right] + \text{h.c.} \right\} 
+ \alpha_3 \left[ \text{Tr}(\phi^\dagger \Delta_L \Delta_L^\dagger_L) + \text{Tr}(\phi^\dagger \Delta_R \Delta_R^\dagger_R) \right] + \beta_1 \left[ \text{Tr}(\phi \Delta_R \phi^\dagger \Delta_L^\dagger_L) + \text{Tr}(\phi^\dagger \phi \Delta_R \Delta_R^\dagger_R) \right] 
+ \beta_2 \left[ \text{Tr}(\phi \Delta_R \phi^\dagger \Delta_L^\dagger_L) + \text{Tr}(\phi^\dagger \phi \Delta_R \Delta_R^\dagger_R) \right] + \beta_3 \left[ \text{Tr}(\phi \Delta_R \phi^\dagger \Delta_L^\dagger_R) + \text{Tr}(\phi^\dagger \phi \Delta_R \Delta_R^\dagger_L) \right], \tag{30} \]

where there are a total of 18 parameters, \( \mu_1^{2,3}, \lambda_1, \lambda_2, \lambda_3, \rho_1, \rho_2, \alpha_1, \alpha_2, \) and \( \beta_1, \beta_2. \) Due to the left-right symmetry (LRS), only one of them, \( \alpha_2 \) can become complex and all other couplings are real. We have included an explicit phase \( e^{i\delta_2} \) in \( \alpha_2, \) introducing an explicit CP violation in the Higgs potential.

After SSB, the Higgs fields acquire vev’s, and the potential is minimized with respect to them. The six minimization conditions are

\[ \frac{\partial V}{\partial \alpha} = \frac{\partial V}{\partial \alpha'} = \frac{\partial V}{\partial v_L} = \frac{\partial V}{\partial v_R} = \frac{\partial V}{\partial \theta_L} = \frac{\partial V}{\partial \theta_R} = 0, \tag{31} \]

which lead to six relations among the vev’s and coefficients in the Higgs potential

\[ \frac{\mu_1^2}{v_R} = \frac{\alpha_1}{2} \left( 1 + \frac{v_L^2}{v_R^2} \right) - \frac{\alpha_3 \xi^2}{2(1 - \xi^2)} \left( 1 + \frac{v_L^2}{v_R^2} \right) + \left[ \lambda_1 (1 + \xi^2) + 2 \lambda_4 \xi \cos \alpha \right] \frac{v_L/v_R}{1 - \xi^2}, \tag{32} \]

\[ \frac{\mu_2^2}{v_R^2} = \frac{\alpha_2 \cos \theta_L - \beta_3 \xi^2 \cos(\theta_L - 2\alpha)}{2 \cos \alpha} \left[ \cos(\alpha + \delta_2) + \cos(\alpha - \delta_2) \frac{v_L^2}{v_R^2} \right] + \frac{\alpha_3 \xi}{4(1 - \xi^2)} \cos \alpha \left( 1 + \frac{v_L^2}{v_R^2} \right) + \left[ 2 \lambda_2 \xi \cos 2\alpha + \lambda_3 \xi + \frac{1}{2} \lambda_4 (1 + \xi^2) \right] \frac{\xi^2}{\cos \alpha} \frac{v_L/v_R}{1 - \xi^2}, \tag{33} \]
\[ \frac{\mu_3^2}{v_R^2} = \rho_1 \left( 1 + \frac{v_L^2}{v_R^2} \right) + \frac{1}{2} \left[ \alpha_1 (1 + \xi^2) + \alpha_3 \xi^2 \right] \epsilon^2 \]

\[ + 2\alpha_2 \left[ \cos(\alpha + \delta_2) - \cos(\alpha - \delta_2) \frac{v_L^2/v_R^2}{1 - v_L^2/v_R^2} \right], \] (34)

\[ \left( 2\rho_1 - \rho_3 \right) - \frac{8\alpha_2 \xi^2 \sin \alpha \sin \delta_2}{1 - v_L^2/v_R^2} \frac{v_L}{v_R} = [\beta_1 \xi \cos(\theta_L - \alpha) + \beta_2 \cos \theta_L + \beta_3 \xi^2 \cos(\theta_L - 2\alpha)] \epsilon^2 \]

\[ 0 = \beta_1 \xi \sin(\theta_L - \alpha) + \beta_2 \sin \theta_L + \beta_3 \xi^2 \sin(\theta_L - 2\alpha) \] (35)

\[ 2\alpha_2 (1 - \xi^2)(1 - v_L^2/v_R^2) \sin \delta_2 = \{2\xi \sin(\theta_L - \alpha)(\beta_2 + \beta_3) \]

\[ + \left[ \sin \theta_L + \xi^2 \sin(\theta_L - 2\alpha) \right] \beta_1 \} \frac{v_L}{v_R} + \xi \sin \alpha \left[ \alpha_3 (1 + v_L^2/v_R^2) + (4\lambda_3 - 8\lambda_2)(1 - \xi^2) \epsilon^2 \right] \] (36)

\[ 2\alpha_2 (1 - \xi^2)(1 - v_L^2/v_R^2) \sin \delta_2 = \{2\xi \sin(\theta_L - \alpha)(\beta_2 + \beta_3) \]

\[ + \left[ \sin \theta_L + \xi^2 \sin(\theta_L - 2\alpha) \right] \beta_1 \} \frac{v_L}{v_R} + \xi \sin \alpha \left[ \alpha_3 (1 + v_L^2/v_R^2) + (4\lambda_3 - 8\lambda_2)(1 - \xi^2) \epsilon^2 \right] \] (37)

where \( \epsilon = \kappa/v_R \) represents a hierarchy in symmetry breaking. The above equations can be solved for the Higgs vev’s in terms of the parameters in the Higgs potential.

Historically, two special cases of the general potential have been studied in the literature, namely “manifest” and “pseudo-manifest” LRS limits. The manifest LRS assumes real Higgs potential i.e. \( \delta_2 = 0 \), and in addition, no spontaneous CP violation, \( \alpha = \theta_L = 0 \). The only source of CP asymmetry is from the Yukawa couplings. In this case, the quark mass matrices are hermitian due to parity invariance, and the left- and right-handed CKM matrices are identical up to quark mass signs. Most early studies were made based on this simplification. At the level of Higgs potential, this scenario necessitates fine-tuning: From the neutrino and quark mass hierarchy, we have \( v_L \ll \kappa' < \kappa \ll v_R \). Taking all the phases to zero in Eqs. (32)-(37), the following relations are found at leading order in \( \epsilon^2 \) [10]

\[ \frac{\mu_1^2}{v_R^2} = \frac{\alpha_1}{2} - \frac{\alpha_3 \xi^2}{2(1 - \xi^2)} , \quad \frac{\mu_2^2}{v_R^2} = \frac{\alpha_2}{2} + \frac{\alpha_3 \xi^2}{4(1 - \xi^2)} , \quad \frac{\mu_3^2}{v_R^2} = \rho_1. \] (38)

There are three equations for only two vev’s \( v_R \) and \( \xi \), implying a relation among parameters in the Higgs potential, which can only be achieved through fine-tuning.

On the other hand, pseudo-manifest LRS requires P and CP invariance of the Lagrangian (\( \delta_2 = 0 \)), with the complex vev phase \( \alpha \) alone to explain the source of CP violation in the quark sector. The Higgs potential is real when \( \delta_2 = 0 \), but the vev could be complex. The Yukawa couplings are real and symmetric. The right-handed CKM matrix is related to the complex conjugate of its left-handed counterpart with additional diagonal phase matrices multiplied on both sides. However, when Higgs potential is real, the spontaneous CP phase is proportional to \( \sim \left( \frac{m_{W_R}}{m_{W_L}} \right)^2 \) and therefore goes to zero in the \( v_R \rightarrow \infty \) limit [12]. If one allowed for fine tuning of parameter, one can generate a large enough phase [13] but at the price of large flavor changing neutral current. Phenomenology of these models have been extensively studied in literature [14,15], and it has been established that this scenario fails to produce large enough CP asymmetry in \( B \rightarrow \psi K \) decay in the decoupling limit even
when the maximum spontaneous CP phase is allowed. Away from the decoupling limit, the scenario has been ruled out by the sign correlation between $\epsilon$ and the above B-decay CP asymmetry [14].

In view of these results, for the minimal LRSMs to be realistic and natural, both explicit and spontaneous CP phases must be taken into account. Anyway, as noted this is precisely what happens in the minimal model. In Ref. [11], an approximate relation was derived between the spontaneous CP phase $\alpha$ and the explicit CP phase $\delta_2$ in the Higgs potential,

$$\alpha \sim \sin^{-1}\left(\frac{2|\alpha_2| \sin \delta_2}{\alpha_3 \xi}\right),$$

(39)

where small $\xi$ requires a hierarchy between $\alpha_2$ and $\alpha_3$, and/or small $\delta_2$. Clearly, when $\delta_2 = 0$, one has $\alpha \sim 0$. A pioneering numerical study of the general CP scenario has been made in Ref. [5]. We will consider the Higgs spectrum and coupling in this general case in the remainder of the section.

A. Higgs Mass Spectrum

With the Higgs vev’s in Eq. (7) and the minimization conditions in Eqs. (32)-(37), the Higgs mass spectrum can be found in the presence of the CP phase $\delta_2$ in the Higgs potential as well as the spontaneous phase $\alpha$. We further restrict to the case $\kappa' \ll \kappa$, $v_L = 0$. Thus $\theta_L$ becomes irrelevant and all $\beta_i$ decouple. We will keep only terms linear in $\epsilon$ and $\xi$ for simplicity.

In the minimal LRSM, there are 20 scalar degrees of freedom in the Higgs fields $\phi$, $\Delta_L$ and $\Delta_R$, including 2 double-charged, 4 single-charged and 4 complex neutral Higgs bosons. After SSB, the mass eigenstates are linear combinations of those. Two single-charged and two real neutral Higgs bosons get absorbed and become longitudinal components of $W_L$, $W_R$, $Z$ and $Z'$.

$$G^+_L = (-\xi e^{-i\alpha} \phi^+_2 + \phi^+_1) ,$$

$$G^+_R = \left(-\frac{1}{\sqrt{2}} \epsilon \phi^+_2 + \delta^+_R \right) ,$$

$$G^0_{Z'} = \sqrt{2} \text{Im} \delta^0_L ,$$

$$G^0_Z = \sqrt{2} \text{Im} (\phi^*_1 + \xi e^{-i\alpha} \phi^*_2) ,$$

(40)

where we have neglected terms of order $\epsilon^2$ and $\xi^2$. Among the remaining 14 fields, only one real and neutral component $h^0$ acquires mass at the electroweak scale $\kappa$, identified as the SM Higgs boson, while the other Higgs fields have masses of order $v_R$. The physical Higgs states and their masses are collected in Table I. In the limit $\alpha \rightarrow 0$, $\delta_2 \rightarrow 0$ and $\kappa, \kappa' \ll v_R$, our results agree with those in Ref. [16], except for the SM Higgs mass.

The calculation of SM Higgs mass is a bit involved and warrants a little further discussion. From the (tree-level) Higgs potential, one can write down the mass matrix for 8 neutral Higgs components in the basis of $\{\text{Re} \phi^0_1, \text{Im} \phi^0_1, \text{Re} \phi^0_2, \text{Im} \phi^0_2, \text{Re} \phi^0_3, \text{Im} \phi^0_3, \text{Re} \phi^0_4, \text{Im} \phi^0_4\}$. The rows and columns containing $\text{Re} \delta^0_L$, $\text{Im} \delta^0_L$ and $G^0_{Z'} = \text{Im} \delta^0_R$ are already diagonal and hence decouple.

The remaining $5 \times 5$ sub-matrix $M^2$ is somewhat complicated. Since the major components have $v_R$-scale masses, we can work in perturbative expansion with respect to 3
\[
\begin{pmatrix}
0 & 0 & 0 & 0 & \kappa^2 + \kappa_0^2 v_R^2 \\
0 & \kappa_0^2 v_R^2 & 0 & 2\kappa_1 \epsilon \\
0 & 0 & \kappa_0^2 \epsilon^2 & 0 \\
\kappa_0^2 \epsilon^2 & 0 & 0 & 0 \\
0 & 2\kappa_1 \epsilon & 0 & 0 \\
\end{pmatrix}
\]

\begin{table}
\begin{tabular}{|c|c|}
\hline
\textbf{Higgs state} & \textbf{Mass}^2 \\
\hline
$h^0 = \sqrt{2} \Re (\phi_1^0 + \xi e^{-i\alpha} \phi_2^0)$ & \(4\lambda_1 - \frac{\alpha_2^2}{\rho_1}\) \(\kappa^2 + \alpha_3 v_R^2 \xi^2\) \\
$H_1^0 = \sqrt{2} \Re (-\xi e^{i\alpha} \phi_1^0 + \phi_2^0)$ & \(4\alpha v_R^2\) \\
$H_2^0 = \sqrt{2} \Re \phi_R^0$ & \((\rho_3 - 2\rho_1)^2 v_R^2\) \\
$A_1^0 = \sqrt{2} \Im (-\xi e^{i\alpha} \phi_1^0 + \phi_2^0)$ & \(\alpha_3 v_R^2\) \\
$A_2^0 = \sqrt{2} \Im \phi_R^0$ & \((\rho_3 - 2\rho_1)^2 v_R^2\) \\
\hline
\end{tabular}
\end{table}

TABLE I: Physical Higgs states and mass spectrum at leading order in minimal LRSMs. We assume \(v_L = 0\) and keep only linear terms in \(\epsilon = \kappa/v_R\) and \(\xi = \kappa'/\kappa\). All fields but \(h^0\) have masses on the \(v_R\) scale. \(h^0\) is identified as the SM Higgs boson.

parameters: the spontaneous phase \(\alpha, \xi = \kappa'/\kappa\) and \(\epsilon = \kappa/v_R\). As we shall see later, in order to satisfy the CP constraints and to have the right-handed scale at TeV, they must be approximately of the same order: \(\alpha, \xi, \epsilon \sim O(10^{-2})\). Naively, the SM Higgs mass squared should be of electroweak scale \(\kappa^2 = \epsilon^2 v_R^2\), and we must work up to the order \(\epsilon^2\) in diagonalization: \(M^2(\alpha, \xi, \epsilon) = M^2_{(0)} + M^2_{(1)} + M^2_{(2)} + \cdots\). At zeroth order, in the basis of \(\{\Re \phi_1^0, \Im \phi_1^0, \Re \phi_2^0, \Im \phi_2^0, \Re \phi_R^0\}\),

\[
M^2_{(0)} = v_R^2 \begin{pmatrix}
0 & 0 \\
0 & \alpha_3 \\
\alpha_3 & 0 \\
0 & 4\rho_1 \\
\end{pmatrix}
\]

which is already diagonal. \(\Re \phi_1^0\) and \(\Im \phi_1^0\) have zero mass and the other three have masses at scale \(\sim v_R\). This means \(\Re \phi_1^0\) and \(\Im \phi_1^0\) are the dominant components of the SM Higgs \(h^0\) and the Goldstone boson \(G_Z\). At the first order, only the first two rows of \(M^2_{(1)}\) are relevant,

\[
M^2_{(1)} = v_R^2 \begin{pmatrix}
0 & 0 & 0 & 0 & 2\alpha_1 \epsilon \\
0 & \alpha_3 \xi & 0 \\
0 & 0 & \alpha_3 \xi & 0 \\
\end{pmatrix}
\]

and at the second order, only the upper \(2 \times 2\) block of \(M^2_{(2)}\) is needed

\[
M^2_{(2)} = v_R^2 \begin{pmatrix}
\alpha_3 \xi^2 & 4\epsilon^2 \lambda_1 & 0 \\
0 & \alpha_3 \xi^2 & 0 \\
\end{pmatrix}
\]

Using the standard formula in perturbation theory, the SM Higgs boson state and mass up to the second order in the expansion are

\[
h^0 \simeq \sqrt{2} \Re (\phi_1^0 + \xi e^{-i\alpha} \phi_2^0),
\]

\[
m^2_{h^0} = \left(4\lambda_1 - \frac{\alpha_2^2}{\rho_1}\right) \kappa^2 + \alpha_3 \xi^2 v_R^2.
\]
There are two contributions to \( m_{h_0} \). One is the usual electroweak breaking mass \( 4\lambda_1\kappa^2 \), and the other is the mass shift \( \alpha_3\xi^2v_R^2 - \frac{\alpha_1^2}{\rho_1}\kappa^2 \), due to additional couplings in the Higgs potential. Because \( \xi = m_b/m_t \sim \epsilon = \kappa/v_R \), the two parts are roughly comparable when \( \lambda_1 \sim \alpha_3 \sim \mathcal{O}(1) \).

### B. Charged and Neutral Currents

In this subsection, we present the charged and neutral couplings between quarks and Higgs mass eigenstates. From the Yukawa coupling term \( \text{[12]} \), we can express \( h \) and \( \tilde{h} \) in terms of the vev’s and quark mass matrices

\[
h = \frac{M_u\kappa - M_u\kappa'e^{-i\alpha}}{\kappa^2 - \kappa'^2}, \quad \tilde{h} = \frac{M_d\kappa - M_d\kappa'e^{i\alpha}}{\kappa^2 - \kappa'^2}.
\]

Using

\[
\tilde{\phi} = \tau_2\phi^*\tau_2 = \left( \begin{array}{c} \phi_2^\ast \ - \phi_1^+ \\ \phi_2^- \ \phi_1^\ast \end{array} \right),
\]

one can write

\[
\mathcal{L}_Y = \overline{Q}_L(h_{ij}\phi + \tilde{h}_{ij}\tilde{\phi})Q_H + \text{h.c.} \equiv \mathcal{L}_N + \mathcal{L}_C.
\]

The neutral Higgs-quark coupling part is

\[
\mathcal{L}_N = \overline{u}_L(h_{ij}\phi_1^0 + \tilde{h}_{ij}\phi_1^0)u_R + \overline{d}_L(h_{ij}\phi_2^0 + \tilde{h}_{ij}\phi_1^0)\bar{d}_R + \text{h.c.}
\]

\[
= \left(\sqrt{2}G_F\right)^{1/2}\overline{u}_L\overline{M}_U[i\phi_1^0 + \tilde{h}_{ij}\phi_1^0]u_R + \overline{d}_L[i\phi_2^0 + \tilde{h}_{ij}\phi_1^0]d_R + \text{h.c.}
\]

\[
= \left(\sqrt{2}G_F\right)^{1/2}\overline{u}_L\overline{M}_U[i\phi_1^0 + \tilde{h}_{ij}\phi_1^0]u_R + \overline{d}_L(i\phi_2^0 + \tilde{h}_{ij}\phi_1^0)d_R + \text{h.c.}
\]

\[
= \left(\sqrt{2}G_F\right)^{1/2}\overline{u}_L\left(V_L\overline{M}_D\overline{V}_R\right)_{ij}(H_1^0 - iA_1^0)u_R + \overline{d}_L(i\phi_2^0 + \tilde{h}_{ij}\phi_1^0)d_R + \text{h.c.}
\]

where \( V_{L,R} \) are left- and right-handed CKM matrices. \( h^0 \) has the known SM Higgs couplings to the quark fields. The second term in Eq. \( \text{[13]} \) changes the quark flavors through \( H_1^0 \) and \( A_1^0 \) bosons. They are called the flavor changing neutral Higgs (FCNH) bosons in LRSMs. The dominant contribution comes from the intermediate top, bottom and charm quark masses. Taking into account the CKM hierarchy, we found that the transitions from \( d \) to \( s \) with intermediate charm quark mass and from \( b \) to \( s \) with intermediate top quark mass are most significant.

The charged Higgs-quark coupling part of Lagrangian density is

\[
\mathcal{L}_C = \overline{u}_L(h_{ij}\phi_1^+ + \tilde{h}_{ij}\phi_1^+)d_R + \overline{d}_L(h_{ij}\phi_1^- + \tilde{h}_{ij}\phi_1^-)u_R + \text{h.c.}
\]

\[
= \left(\sqrt{2}G_F\right)^{1/2}\overline{u}_L\left(V_R\overline{M}_D - 2\xi e^{-i\alpha}V_L\overline{M}_D\right)_{ij}d_R \overline{H}_2^+ 
\]

\[- \overline{u}_R\left(V_R\overline{M}_D - 2\xi e^{-i\alpha}V_L\overline{M}_D\right)_{ij}d_L \overline{H}_2^+ 
\]

\[- \overline{u}_L\left(V_L\overline{M}_D\right)_{ij}d_R \overline{G}_L^+ + \overline{u}_R\left(V_R\overline{M}_L\right)_{ij}d_L \overline{G}_L^+ + \text{h.c.}
\]

(49)
Again, the couplings are proportional to quark masses and hence the heavy-quark contributions stand out. With the above couplings, we will study their contributions to various flavor changing and conserving processes in the minimal LRSM.

V. \(K^0 - \bar{K}^0\) AND NEUTRAL B-MESON MASS MIXING

In this section, we consider the neutral kaon and \(B\)-meson mass mixing in the minimal LRSM, using the righthanded quark mixing matrix obtained in the previous section. We first study the \(W_L - W_R\) mixing-box contribution to the \(K_L - K_S\) mass difference \(\Delta m_K\) and derive an improved bound (2.5 TeV) on the mass of right-handed gauge boson \(W_R\), using the updated hadronic matrix element and strange quark mass. Historically, the kaon mass mixing provided the most stringent constraint upper bound (1.6 TeV) on the mass scale of the right-handed \(W_R\) boson [17]. With our new right-handed CKM mixing, the conclusion does not change significantly, although in the literature, quite different mixings have been speculated upon and the result did change dramatically, and we rule these possibilities out.

In the past few years, significant progress has been made in hadronic physics through lattice QCD simulations, helping to tighten the bound. We also consider the contribution from the FCNHs and constraint on their masses. Because the box and FCNH contributions are additive, the bounds are valid independently. In the last subsection we explore \(B_d - \bar{B}_d\) and \(B_s - \bar{B}_s\) mass mixing. Because the hadronic contributions arise dominantly from short distance, the bounds on \(M_{W_R}\) and \(M_H\) turn out to be significant as well.

It is useful to provide our convention for neutral meson mixing at the beginning. For a pair of neutral mesons, \(|P_0\rangle\) and \(|\bar{P}_0\rangle\), we assume under CP transformation \(|P_0\rangle = |P_0\rangle\). If the effective hamiltonian in the basis of \(|P_0\rangle\) and \(|\bar{P}_0\rangle\) is \(H_{ij} = M_{ij} - i\Gamma_{ij}/2\), a pair of eigenstates are \(|P_{1,2}\rangle = p|P_0\rangle \pm q|\bar{P}_0\rangle\). The ratio \(q/p\) is chosen to be \(\sqrt{(M_{12}^* - i\Gamma_{12}^*/2)/(M_{12} - i\Gamma_{12}/2)}\).

The mass difference \(M_2 - M_1 = -2\text{Re}(q/p(M_{12} - i\Gamma_{12}/2))\), and the width difference \(\Gamma_2 - \Gamma_1 = 4\text{Im}(q/p(M_{12} - i\Gamma_{12}/2))\).

A. Kaon Mixing and the Boxing Diagram

In SM, the leading-order short-distance \(\Delta S = 2\) process comes from the box diagram with \(W_L\) boson and up-type quark exchanges. Flavor change happens at the vertices via the CKM mixing matrix. The short distance contribution comes mainly from internal loop momentum flow at the scales around \(c\) and \(t\) quark masses, whereas the momentum region around \(W_L\)-boson mass is suppressed due to the celebrated Glashow-Iliopoulos-Maiani (GIM) mechanism. The long-distance contribution comes from one or two up-quark exchanges and must be calculated using non-perturbative methods. It has been generally accepted that this latter contribution does not dominate over the short distance one. In fact, a chiral perturbation calculation [18] puts the long distance contribution at about half of the experimental mass difference.

In the LRSM, there are new box-diagram contributions which turn out to be quite large [17, 19]. The dominant one comes from \(W_L - W_R\) interference with one internal vector-boson being the lefthanded \(W_L\) and the other righthanded \(W_R\). As such, the chirality of the internal as well as external quarks must be flipped, and the contribution is proportional to the internal quark masses, as shown in Fig. 1. There is no GIM suppression even in the...
manifest LRS scenario: $V_L = V_R$. The effective Lagrangian from the $W_L - W_R$ box-diagram is

$$\mathcal{L}_{LR} = -\frac{G_F^2 M_W^2}{4\pi^2} \sum_{ij} \lambda_i^{LR} \lambda_j^{RL} \sqrt{x_i x_j} [(4 + \eta x_i x_j)$$

$$\times I_1(x_i, x_j, \eta) - (1 + \eta) I_2(x_i, x_j, \eta)] \hat{O}_{LR} + \text{h.c.} ,$$

(50)

in which $\lambda_i^{LR} = V_{L2}^* V_{R1}^i$, $\eta = \left( \frac{M_{W_R}}{M_{W_L}} \right)^2$, and $x_i = \left( \frac{m_i}{M_{W_L}} \right)^2$, $i = u, c, t$. The four quark operator is

$$\hat{O}_{LR}(M_{W_R}^2) = (\bar{s}_P L d)(\bar{s}_P R d) ,$$

(51)

where $\bar{P}_{L,R} = (1 \mp \gamma_5)/2$ are chiral projection operators. The loop-related integrals are

$$I_1(x_i, x_j, \eta) = \eta \ln(1/\eta)$$

$$\frac{x_i \ln x_i}{(1 - \eta)(1 - x_i \eta)(1 - x_j \eta)}$$

$$+ \left[ \frac{x_i \ln x_i}{(x_i - x_j)(1 - x_i)(1 - x_i \eta)} + (i \leftrightarrow j) \right] ,$$

$$I_2(x_i, x_j, \eta) = \ln(1/\eta)$$

$$\frac{x_i^2 \ln x_i}{(1 - \eta)(1 - x_i \eta)(1 - x_j \eta)}$$

$$+ \left[ \frac{x_i^2 \ln x_i}{(x_i - x_j)(1 - x_i)(1 - x_i \eta)} + (i \leftrightarrow j) \right] .$$

(52)

The above contribution is evaluated in 't Hooft-Feynman gauge. Generally, we have other contributions from the exchange of single-charged Higgs bosons as well as the Goldstone bosons if the calculation is not in unitary gauge. In fact, to maintain gauge invariance, we have also to include the triangle diagrams with the FCNH bosons and the related self-energy corrections [20]. It turns out that in the gauge we use, the box diagram with $W_L$ and $W_R$ interchanges alone gives the dominant contribution.

The Wilson coefficient in Eq. (50) is proportional to the internal quark masses and the corresponding CKM matrix elements. The up-quark contribution is negligible and the top-quark contribution is suppressed by the off-diagonal CKM elements. Therefore the charm-
quark exchange dominates the effective interaction which simplifies

\[ \mathcal{H}_{LR} = \frac{G_F^2 M_W^2}{4\pi^2} 2\eta \lambda_e^{LR} \lambda_e^{RL} x_c \left[ 1 + \ln x_c + \frac{1}{4} \ln \eta \right] \times \left[ (\bar{\sigma}_d d) - (\bar{\sigma}_5 \gamma_5 d)^2 \right]. \]  

(53)

The hadronic matrix element of the four-quark operator is expressed in the vacuum saturation form,

\[ < K_0 | \bar{d}(1 - \gamma_5)s | K_0 > = -2 M_K F_K^2 B_4(\mu) \left( \frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2, \]

(54)

where the kaon decay constant \( F_K = 113 \text{ MeV} \) and \( B_4 = 1 \) corresponds to vacuum saturation approximation. In this form, the matrix element diverges in the chiral limit, an important reason for the enhanced contribution of the box diagram. The correction factor, \( B_4 \), can be and has been calculated using lattice QCD. In a recent calculation \[21\], the domain-wall fermion was used, and \( B_4 = 0.81 \) was found at \( \mu = 2 \text{ GeV} \) in naive dimensional regularization scheme. In the same scheme and scale, the strange quark mass is \( m_s = 98(6) \text{ MeV} \), which is smaller than what one has naively expected in the past.

### B. An Improved Lower Bound on \( M_{W_R} \)

Considering only the box diagram, the new contribution to the mass difference of \( K_L - K_S \) can be expressed as

\[ \Delta m_K = 2 \text{ Re } \eta_4(\mu) \langle K^0 | \mathcal{H}^{\Delta S = -2}(\mu) | \bar{K}^0 \rangle, \]

where \( \eta_4 \) is a factor characterizing the QCD radiative correction in scale running from \( M_{W_R} \) to \( \mu \sim 2 \text{ GeV} \) \[22, 23, 24\]. There are several enhancement factors here comparing to the SM box diagram. First, due to absence of the GIM mechanism, the Wilson coefficient is about a factor of 30 larger. Second, the hadronic matrix element is chirally enhanced by a factor of 20. Finally, the short-distance QCD correction \( \eta_4 = 1.4 \) gives another enhancement. The only suppression comes from the difference between the left and right-handed symmetry breaking scales, \( \eta = \left( \frac{M_{W_L}}{M_{W_R}} \right)^2 \). Therefore the new contribution can be approximated by

\[ \Delta M_{K-LR} \sim 10^3 \times \left( \frac{M_{W_L}}{M_{W_R}} \right)^2 \times \Delta M_{K-SM}. \]

(56)

The sign can both be positive or negative depending on the product \( s_t s_c \). With the standard criteria that the new contribution should not exceed the experimental value \[17\], we find a lower bound for \( M_{W_R} \),

\[ M_{W_R} > 2.5 \text{ TeV}. \]

(57)

On the other hand, the SM contributions from both long and short distances have the same sign as the experimental number and account for more than one-half of its value. Therefore, a less conservative bound is obtained if requiring the new physics contribution is less than one-half of the experimental data. If this new standard is adopted, the above bound change to 4 TeV. Giving the long-distance uncertainty in \( \Delta M_K \), this bound shall be used at less confidence level. Nonetheless, as we shall see in the next section, the CP violating observables are providing equally competitive bounds albeit with a significant hadronic physics uncertainty.

17
C. Tree-Level FCNH Contribution and A Lower Bound on $M_H$

In the LRSM, there is also a new contribution to the $K^0 - \bar{K}^0$ mixing mediated by the FCNH. The FCNH boson is a complex field and can be expressed in terms of the two real fields $H^0_1$ and $A^0_1$. The effective lagrangian follows from Eq. (48)

$$L_{FCNH} = \frac{G_F}{\sqrt{2}} \left[ \left( \sum_i \frac{\lambda^R_i + \lambda^L_i}{2} m_i \right)^2 \left[ \frac{(\bar{s}d)^2}{m^2_{H_1^0}} - \frac{(\bar{s}\gamma_5 d)^2}{m^2_{A_1^0}} \right] - \left( \sum_i \frac{\lambda^R_i - \lambda^L_i}{2} m_i \right)^2 \left[ \frac{(\bar{s}d)^2}{m^2_{A_1^0}} - \frac{(\bar{s}\gamma_5 d)^2}{m^2_{H_1^0}} \right] \right].$$  \hspace{1cm} (58)

The corresponding Feynman diagram is shown in Fig. 2. According to our previous discussion, the two scalar fields $H^0_1$ and $A^0_1$ have the same masses, roughly corresponding to the righthand scale, $m^2_{H_1^0} \approx m^2_{A_1^0} \approx \alpha_3 v^2_R$. Therefore, it is convenient to rewrite Eq. (58) in a more compact form

$$L_{FCNH} \approx -\frac{G_F}{\sqrt{2} m^2_{H_1^0}} \sum_{i,j} m_i m_j \lambda^L_i \lambda^R_j \left[ (\bar{s}d)^2 - (\bar{s}\gamma_5 d)^2 \right].$$  \hspace{1cm} (59)

\[\text{FIG. 2: } \Delta S = 2 \text{ effective interaction induced by flavor-changing neutral Higgses.}\]

It is easy to check that the FCNH and the box diagram contributions have the same sign because $4(1 + \ln x_c) + \ln \eta < 0$, and thus they cannot cancel each other, even allowing possible freedom in choosing the quark mass sign. Therefore, the lower bound on the righthanded-$W$ boson mass remains. One can also derive a lower bound on the masses of $H^0_1$ and $A^0_1$ using $\Delta M_K$. A straightforward calculation shows that if demanding the FCNH contribution is less than the experimental data,

$$M_{H^0_1}, M_{A^0_1} > 15 \text{ TeV},$$  \hspace{1cm} (60)

which is about twice as large as in [5]. One can obtain this value presumably by a large $\alpha_3$ parameter in the Higgs potential. However, one cannot make $\alpha_3$ arbitrarily large. As we shall discuss later, large $\alpha_3$ not only causes naturalness problem, but also leads to a large SM Higgs mass which threatens the perturbative unitarity [25].
D. Constraints From Neutral $B_d$ and $B_s$ Mass Mixing

The physics of $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing in SM is similar to that of $K^0 - \bar{K}^0$ mixing, coming from the $W$-boson box diagram and FCNH. However, in the former case, the intermediate top quark contribution dominates almost entirely due to its mass and CKM couplings. Because of this, the SM calculation can be done quite accurately with the help of the lattice QCD matrix elements. In fact, in many global CKM fits, the mass differences $\Delta M_{B_d}$ and $\Delta M_{B_s}$ have been used to determine the top CKM couplings $V_{td}$ and $V_{ts}$. However, there are still appreciable uncertainties in the lattice calculations and global CKM fits, and the beyond SM physics could contribute as much as 20% of the mass differences without running into conflict with the present SM calculations and experimental data. We will use this possible discrepancy as a constraint on the LRSM.

The LRSM contribution to the mixing can be taken directly from (50)

$$H^q_{LR}(\mu_b) \simeq \frac{G_F M^2_{W_L}}{4\pi^2} \eta_S \eta (V^R_{tb} V^L_{tq})(V^R_{tb} V^L_{tq}) x_t \left[(4 + \eta x_t^2) \times \xi_1(x_t, x_t, \eta) - (1 + \eta) \xi_2(x_t, x_t, \eta) \right] \bar{b}Lq \bar{b}Rq + \text{h.c.},$$

where the leading-logarithmic running factor $\eta_S = 2.112$ [22] and the functions $I_i$ are defined in Eq.(52). The hadronic matrix element can be defined using a factorization,

$$\langle B_q |qL^P b \bar{q}R^P b| B_q \rangle >= -M_{B_q} f_{B_q}^2 B_q^3(\mu) \left[ \frac{1}{12} + \frac{1}{2} \left( \frac{m_{B_q}}{m_b + m_q} \right)^2 \right],$$

where the minus arises from our definition of the CP transformation for the meson states. The decay constants have been calculated in lattice QCD: $f_{B_d} = 216$ MeV, $f_{B_s} = 1.20 f_{B_s}$ [26] and the non-perturbative $B$-factors are $B^d_4 = 1.16$ and $B^s_4 = 1.17$ [27]. The ratio of the new contribution to the SM one is $\sim 10^2 M^2_{W_L}/M^2_{W_R}$, which is smaller than the kaon mixing case in the absence of chiral enhancement.

The experimental values for the mass differences of $B_d$ and $B_s$ are $(0.507 \pm 0.12)\text{ps}^{-1}$ and $(17.77 \pm 0.12)\text{ps}^{-1}$, respectively [9], with central values shown as horizontal lines in Fig. 3.
The shades around the central values are within 20\%, and are considered as the combined experimental and SM theory error. In the same figure, we also plot the mass differences as a function of $M_{W_R}$, calculated as a sum of the LRSM contribution and the experimental central values. The agreement between experiment and theory is reached only when $M_{W_R}$ is larger than 2.5 TeV. The sign of the LRSM contribution is related to that of $s_d s_b$ and $s_s s_b$, which we have chosen to be $+1$. If taking as $-1$, approaching to the central value comes from below.

Thus the constraint on the $M_{W_R}$ mass from the neutral $B$-meson mixing is roughly comparable to the kaon case, due to a better theoretical understanding of the SM physics. The future improvement can come from a better determination of $V_{td}$ from other sources and a better determination of $B$-parameter and decay constants.

We have also studied the constraint on the FCNH mass, $M_H$. We find that from $\Delta M_{B_d}$, the bound is 12 TeV, and for $\Delta M_{B_s}$, the bound is 25 TeV.

VI. CP-VIOLATING OBSERVABLES IN LRSM

As mentioned before, in a generic yet minimal LRSM we have both explicit and spontaneous CP violations. A prominent feature of right-handed quark mixing $V_R$ obtained in the previous section is its phases, which are entirely determined by the Dirac phase $\delta_{CP}$ and the spontaneous CP phase $\alpha$. These physical phases generate interesting effects in various CP-violating observables. The phenomenology of CP violation in LRSM is rich, which in turn constraints the model severely. In this section, we will explore a number of CP violating observables including $\epsilon$, $\epsilon'$, neutron EDM and CP asymmetry in $B \to J/\psi K_S$, to place constrains on the mass of $W_R$ as well as the spontaneous CP phase $\alpha$. Some results here have appeared before in a Rapid Communication paper \cite{7} and there are important updates in this more extensive study.

The indirect CP violation $\epsilon$ receives large contributions from both explicit and spontaneous CP phases. Unless there is a strong cancelation, the right-handed $W_R$ mass must be larger than 15 TeV. We use the cancelation condition to fix the spontaneous CP phase, which is then used to predict the neutron EDM. Using the experimental bound on the EDM, we obtain a strong lower bound on $M_{W_R}$, which can be improved with better calculations of the hadronic matrix elements and more precise experimental data. We obtain a strong lower bound on $M_{W_R}$ from the direct CP violation parameter $\epsilon'$, calculated under the factorization assumption for the four-quark matrix elements. Therefore, we find that the CP violating observables in the kaon system and neutron EDM provide competitive bounds as the well-known kaon mass mixing. These bounds can be improved further with better knowledge of the non-perturbative hadronic physics.

A. Indirect CP violation $\epsilon$ in Kaon Decay

We first study the CP violating parameter $\epsilon$ in kaon mixing. This indirect CP violation parameter is related to the flavor mixing interaction by,

$$
\epsilon = -\frac{e^{i\pi/4}}{\sqrt{2}} \frac{\text{Im}(K^0|\mathcal{H}|\Delta S=2|K^0)}{\Delta m_K},
$$

(63)
where we have neglected the direct CP contribution $\xi_0$ from kaon decay, which can be justified posteriori \cite{5}. In LRSM, according to the previous section, both the $W_L - W_R$ box diagram in Eq. (53) and the tree level FCNH exchange in Eq. (59) can make significant contributions. In the present case, their signs can be different due to both charm and top quark contributions, in contrast to the mass mixing \cite{5}. For simplicity, we ignore that latter contribution and consider the constraint from the box diagram alone.

There are two sources of CP phases in $V_R$ which enter the $W_L - W_R$ box diagram: the Dirac phase $\delta_{CP}$ inherited from $V_L$, and the spontaneous phase $\alpha$. In the manifest LRS case, only $\delta_{CP}$ is present and there is a very tight lower bound on mass of $W_R$ which we find no lighter than $15 - 20$ TeV (see below). If the spontaneous CP phase is also present, one can seek for certain cancelation between the two contributions to lower the bound on $M_{WR}$. In fact, one can roughly estimate the size of $r \sin \alpha$ for a cancelation. The Dirac phase $\delta_{CP}$ appears in the expression $\epsilon$ proportional to $V_{ts}^R V_{td}^R \sim \lambda^5 \sin \delta_{CP}$ via top quark exchange in the box diagram, while the spontaneous CP phase contributes through $V_{cs}^R V_{cd}^R \sim \lambda^2 r \sin \alpha$. Hence $r \sin \alpha$ should be of order $\lambda^3 \sin \delta_{CP} \sim 0.01$ when cancelation happens.

The present experiment value is $|\epsilon|_{\text{expt}} = (2.232 \pm 0.007) \times 10^{-3}$ \cite{9}. In SM, $\epsilon$ can be calculated quite accurately because the top quark dominates the box diagram and the main contribution is due to short-distance QCD physics. The only large uncertainty comes from the CKM matrix element $V_{td}$ and the hadronic matrix element related to $K^0 - \bar{K}^0$ mixing. Because of this, we assume that the new contribution accounts for less than 1/4 of the experimental value.

Using the box-diagram result in the previous section, we find an approximate expression for $\epsilon_{LR}$ valid for $M_{WR} > 200$ GeV ,

$$\epsilon_{LR} = 1.58 \left( \frac{1 \text{ TeV}}{M_{WR}} \right)^2 s_s s_d \Im \left[ g(M_{WR}, \theta_2, \theta_3) e^{-i(\theta_1 + \theta_2)} \right],$$

(64)
where
\[
g(M_{WR}, \theta_2, \theta_3) = -2.22 + [0.076 + (0.030 + 0.013i) s_c s_t] \times \cos 2(\theta_2 - \theta_3) \ln \left( \frac{M_{W_L}}{M_{WR}} \right)^2,
\]  
(65)

where \(\theta_i\) are given in Eq. (29) and the QCD running correction has been taken into account. When \(\theta_i = 0\), the dominant CP violating contribution comes from charm-top interference in the box diagram.

We search for the allowed region in \(M_{WR}\)-\(r\sin \alpha\) plane shown in Fig. 4. The spontaneous CP violation effect can always cancel the effect of the Dirac phase, thus \(\epsilon\) itself places no bound on \(M_{WR}\). The cancelation depends on choices of the quark mass signs. The 32 choices of the signs (for fixed \(s_u = 1\)) roughly correspond to two scenarios: small \(r\sin \alpha\) solutions for \(s_s = s_d\) and large \(r\sin \alpha\) solutions for \(s_s = -s_d\). The existence of two scenarios can be readily seen through the above approximate expression. According to (29), \(\theta_1\) and \(\theta_2\) are similar in size and their relative sign depends on the bi-product \(s_d s_s\). When \(s_s = s_d\), \(\theta_1\) and \(\theta_2\) have the same sign and add constructively in the exponential in (64). So \(\alpha\) itself must be small in order for \(\theta_1 + \theta_2\) to cancel the phase in \(g(M_{WR}, \theta_2, \theta_3)\). This generates the small \(r\sin \alpha\) solutions. On the other hand, if \(s_s = -s_d\), \(\theta_1 + \theta_2\) cancels and is proportional to \(r\sin \alpha\) multiplied with a small coefficient. So \(r\sin \alpha\) has to be large to cancel the phase in \(g(M_{WR}, \theta_2, \theta_3)\). This corresponds to the large \(r\sin \alpha\) solutions. For the small \(r\sin \alpha\) case, we find \(r\sin \alpha \simeq \pm 0.05\), but for large \(r\sin \alpha\), it can take several different positive and negative values. As we will see in the next subsection, if one includes the constraint from neutron EDM, only small \(r\sin \alpha\) is phenomenologically viable.

![FIG. 5: Constraint on \(M_{WR}\) and the spontaneous phase \(\alpha\) from kaon decay parameter \(\epsilon\), with \(s_d = s_s = -1\) and all other \(s_i = 1\). Red triangles are for \(M_H = \infty\), blue squares are for \(M_H = 75\) TeV, and green dots for \(M_H = 20\) TeV.](image)

Of course, one has to consider the FCNH contribution which has been known to be large \[28\]. In fact, with just the Dirac CP phase in the FCNH contribution, \(\epsilon\) places a limit on the Higgs mass on the order of 100 TeV. With the new spontaneous CP contribution, there is a possibility of cancelation. In fact, one can make similar plots as in Fig. 4 in which the \(\epsilon\) bound can be satisfied even for very low \(M_H \sim 1\) TeV. However, the required \(r\sin \alpha\) for the
cancelation, ±0.2, is very different from that needed for the box diagram. The conclusion is that there is no bound on $M_H$ coming from $\epsilon$ when FCNH contribution is considered alone.

Because of the conflict in the spontaneous CP phase required for individual cancelations in the box and FCNH contributions, one might expects their combined contribution places a joint bound on $M_{W_R}$ and $M_H$. This, however, is not the case, because the two contributions again cancel each other, as was first found in [5]. In fact, with the analytical solution for the right-handed mixing, we arrive at an even stronger conclusion: For any given pair of $M_{W_R}$ and $M_H$, we can always find a $r \sin \alpha$ such that the total contribution to $\epsilon$ vanishes. This situation is extremely interesting, because it implies that $\epsilon$ itself, unlike the kaon mass difference, is completely useless in constraining the individual parameters in the new contribution. It does, however, provide a correlation among different parameters, as shown in Fig. 5 where for several different values of $M_H = \infty, 75, 25$ TeV, we plotted the allowed regions in the $M_R$ and $r \sin \alpha$ plane. Because of the FCNH contribution, the pattern of the correlation changes considerably as $M_H$ changes. The general trend is that the spontaneous CP parameter $r \sin \alpha$ increases toward 0.2 as $M_H$ becomes smaller, consistent with the cancelation pattern in FCNH contribution to $\epsilon$ found above.

**B. Neutron EDM**

The neutron EDM imposes another constraint on the new CP phases in $V_R$ and $M_{W_R}$. Non-zero EDM implies both P and T (or equivalently CP in local quantum field theories) violations. At the quark level, sources of flavor-neutral CP violation are mainly from the penguin diagrams in the SM, and from the tree level $W_L - W_R$ exchange in the LRSM [29, 30]. Generally, there are several contributions to the neutron EDM, including valence quark EDM, quark chromomagnetic dipole moment (CDM) induced EDM, dimension-6 pure gluonic operator contribution, as well as the contributions at hadronic level. The present experimental upper bound on neutron EDM is $3.1 \times 10^{-26} e$ cm [9].

In the SM, contributions to the neutron EDM mainly come from the CP violating penguin diagrams. The flavor changing nature of CKM CP violation means that the leading contribution is at least second order in weak interaction ($\sim G_F^2$). The predicted neutron EDM is well within the experimental bound—about $10^{-33} e$ cm.

\[ L_{uq\rightarrow uq} = -2\sqrt{2}G_F \sin \zeta e^{-i\alpha} V_{uq}^L V_{uq}^{R*} (O^q - O^q_+) + \text{h.c.} , \] (66)
where \( q = d, s \), and

\[
O_{q}^{L} = \bar{d}_{\gamma 5}^{R} L q q_{\gamma 5}^{R} P R u - \frac{2}{3} \bar{u}_{\gamma 5}^{P R} u q L q ,
\]

\[
O_{q}^{R} = \frac{2}{3} \bar{u}_{\gamma 5}^{P R} u q L q .
\]  

(67)

At low energy, short-distance QCD effect enhances the operator \( O_{q}^{q} \) and suppresses \( O_{q}^{q} \). The effective Lagrangian reduces to \([31]\)

\[
\mathcal{L}_{ud(s)-ud(s)} = -i \frac{2\sqrt{2} G_{F}}{3} \eta_{-} \sin \zeta \left[ \text{Im} \left( e^{-i\alpha V_{u d}^{L} V_{u d}^{R}} \right) \left( \bar{d}_{\gamma 5}^{u d d} - \bar{u}_{u d}^{d} \right) + \text{Im} \left( e^{-i\alpha V_{u d}^{L} V_{u d}^{R}} \right) \left( \bar{u}_{\gamma 5}^{u u s s} - \bar{u}_{s s}^{u u s s} \right) \right] ,
\]

(68)

where the leading-log QCD factor \( \eta_{-} = \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M_{W_L}^2)} \right)^{\frac{5}{6}} \simeq 3.5 \). The CP violating \( \pi n n \) coupling is proportional to the hadronic matrix element \( \bar{g}_{\pi n n} = \langle \pi n | \mathcal{L}_{ud(s)-ud(s)} | n \rangle \) which, in the factorization approximation, is

\[
\langle \pi n | \bar{q}_{\gamma 5}^{q} q' q' | n \rangle \simeq \langle \pi | \bar{q}_{\gamma 5}^{q} | 0 \rangle \langle n | q' q' | n \rangle ,
\]  

(69)

for \( q = u, d \) and \( q' = u, d, s \). From the SSB of the chiral symmetry, \( 2m_{u} \langle \pi | \bar{u}_{\gamma 5}^{u} | 0 \rangle = -2m_{d} \langle \pi | \bar{d}_{\gamma 5}^{d} | 0 \rangle = -i F_{\pi} m_{\pi}^{2} \), with \( F_{\pi} = 93 \text{ MeV} \). \( \langle n | \bar{u} u | n \rangle \simeq 4 \) and \( \langle n | \bar{d} d | n \rangle \simeq 5 \) can be fixed from the neutron-proton mass difference and the \( \pi N \sigma \)-term: \( \sigma_{N} = \frac{1}{2} (m_{u} + m_{d}) \langle n | \bar{u} u + \bar{d} d | n \rangle \approx 45 \text{ MeV} \) \([32]\), where we use quark masses \( m_{u} = 2.7 \text{ MeV}, m_{d} = 5.0 \text{ MeV} \) at \( \mu = 2 \text{ GeV} \). We neglect \( \langle n | \bar{s} s | n \rangle \ll \langle n | \bar{u} u | n \rangle \).

The neutron EDM from the Feynman diagram in Fig. 8 is

\[
d_{n} = \frac{e \bar{g}_{\pi n n} \bar{g}_{\pi n n} \mu_{N} F_{\pi}(s)}{8\pi^2} \left( \frac{m_{N}^2}{m_{n}^2} \right) ,
\]

(70)

where \( \mu_{N} = -1.91 \) is the neutron anomalous magnetic dipole moment and the loop function is

\[
F(s) = \frac{3}{2} - s - \frac{3s - s^2}{2} \ln s + \frac{s(5s - s^2) - 4s}{2\sqrt{s - s^2/4}} \arctan \frac{s - s^2/4}{s/2} .
\]

(71)

The contribution is suppressed by the mixing angle \( \zeta \) between \( W_{L} \) and \( W_{R} \), but is enhanced by the chiral logarithmic factor \( \ln \left( \frac{\Lambda_{X}}{m_{n}} \right)^2 \). Putting in all the known physical parameters, we arrive at an approximate formula

\[
|d_{n}^{k}| \simeq 3 \times 10^{-19} \sin \zeta \text{ Im} \left( e^{-i\alpha V_{u d}^{L} V_{u d}^{R}} \right) \text{ cm} ,
\]

(72)

which is approximately a function of \( r \sin \alpha \) for small \( \alpha \).

For a fixed \( M_{H} \), the neutron EDM and \( \epsilon \) can be used to give a lower bound for \( M_{W_R} \) as well as a corresponding solution for \( r \sin \alpha \). On the left panel in Fig. 8 we have shown the neutron EDM constraint as a function of \( r \sin \alpha \) and \( M_{W_R} \), and it is obvious that the EDM limit prefers small \( r \sin \alpha \). The smallest spontaneous CP phase is obtained when \( M_{H} \) is large and decouples, and then

\[
M_{W_R} > 8 \text{ TeV} ,
\]

(73)
FIG. 7: A dominant contribution to the neutron EDM through chiral pion exchange. The shaded blob is CP violating coupling $\bar{g}_{\pi nn}$, and the black dot is the strong coupling $g_{\pi nn}$. The neutron couples to photon via its anomalous magnetic moment.

which is a very tight bound. At this point, we also fix the product

$$r \sin \alpha \simeq 0.05 .$$

(74)

For a lower $M_H$, the spontaneous CP phase must be large from the $\epsilon$ constraint, and the corresponding lower bound on $M_W$ increases considerably. For example, when $M_H = 75$ TeV, $r \sin \alpha$ is now greater than 0.1, and the lower bound on $M_W$ becomes $\sim 18$ TeV.

We note that there is considerable hadronic uncertainty in the evaluation of CP-violating coupling $\bar{g}_{\pi nn}$ and, to the less extent, chiral perturbation expansion. However, even one allows a factor of 5 over-estimate in the hadronic calculation, the combined $\epsilon$ and EDM will still provide a strong constraint on $M_W$ on the order of order 4 TeV, as shown on the right panel in Fig. 8. If $M_H = 25$ TeV, the lower bound on $M_W$ becomes 8 TeV. A future improvement on the neutron EDM data and theoretical calculation can strengthen this bound considerably.

We finally comment on the Higgs boson exchange contributions to the neutron EDM. According to Sec. IV B, the Higgs bosons $H_0^0$, $A_1^0$ and $H_2^-$ have CP violating couplings (Eq. (48) and Eq. (49)) to the quark fields. The valence quark EDM receives contributions from virtue $H_1^0$, $A_1^0$ and $H_2^-$ exchange as shown in Fig. 9. Potentially large contribution also comes from neutral Higgs boson exchange and virtual top-quark effect at two loops [33], as well as two loop pure gluonic operators due to charged/neutral Higgs exchange [34]. A complete analysis of these contributions has been carried out in a pseudomanifest LRS limit with two doublets Higgs fields instead of triplets considered here [35]. With the explicit form of right-handed CKM mixing, we re-evaluate these contributions in the general case of CP violation. As discussed in Sec. VC, $H_1^0$ and $A_1^0$ must be heavy enough to suppress their contribution to kaon mixing, and we take their masses to be 15 TeV. In this case, the charged-Higgs $H_2^+$ exchange dominates, whose contribution to d-quark EDM is approximated by

$$d_d \simeq \frac{2e m_t G_F}{3} \frac{2 \zeta}{4 \sqrt{2} \pi^2} \frac{m_t^2}{m_{H_2^+}^2} \ln \left( \frac{m_{H_2^+}^2}{m_t^2} \right) \eta_d \text{Im} \left( V_{td}^L V_{td}^R e^{-i\alpha} \right) ,$$

(75)

where the scaling factor $\eta_d \simeq 0.12$. The d-quark CDM $f_d$ come from a similar diagram with photon replaced by gluon leg, $e f_d \simeq \frac{3 \eta_f}{2 \eta_d} d_d$. The contribution to u-quark EDM and CDM
FIG. 8: Constraints on the mass of $W_R$ and the spontaneous CP violating parameter $\alpha$ from kaon decay parameter $\epsilon$ ($M_H = \infty$, red triangle; $M_H = 75$ TeV, blue square; $M_H = 20$ TeV, large green dots) and neutron EDM (yellow dots). In the right panel, the theoretical EDM result is reduced by a factor of 5.

is suppressed by a factor $\frac{m_b}{m_t}$ and is negligible compared to that of $d$-quark. Meanwhile, the two loop diagrams are found to be negligibly small. To a certain level of approximation, the neutron EDM can be related to the quark EDM and CDM through the $SU(6)$ relation:

$$d_n^e = \frac{1}{3} (4d_d - d_u) + \frac{1}{3} \left( \frac{4}{3} f_d + \frac{2}{3} f_u \right).$$

A more accurate relation would use the tensor charges of the nucleon.

FIG. 9: Higgs exchange contributions relevant to neutron EDM. The dashed lines include both charged and neutral Higgs bosons exchanges. The two-loop diagrams contain closed top-quark loops.

In Fig. 10 we plot the Higgs exchange contributions to the neutron EDM as a function of $r$ for different values of $\alpha$. We choose the FCNH mass to be 15 TeV, and the charged
FIG. 10: One scenario (all $s_q = 1$) for the higgs exchange contribution to neutron EDM as a function of $r$ for different values $\alpha = 0.05$ (solid line), $\alpha = 0.5$ (long dashed line) and $\alpha = 1$ (short dashed line). We choose the FCNH mass to be 15 TeV and the charged Higgs mass equal to 3 TeV.

Higgs mass be 3 TeV. We find the contributions are always smaller than $10^{-26} e\:cm$, well within the experimental bound. Therefore one can neglect the Higgs exchange contribution without altering the $W$-mass bound for the neutron EDM.

C. Direct CP Violation $\epsilon'\epsilon$

The direct CP violation in neutral kaon to $\pi\pi$ decay is calculated via

$$\epsilon' = \frac{i}{\sqrt{2}} \omega \left( \frac{q}{p} \right) \left( \frac{\text{Im} A_2}{\text{Re} A_2} - \frac{\text{Im} A_0}{\text{Re} A_0} \right) e^{i(\delta_2 - \delta_0)},$$

where the decay amplitudes $A_0$ and $A_2$ are defined as the matrix elements of the $\Delta S = 1$ effective Hamiltonian between the neutral-K meson and the isospin $I = 0$ and 2 $\pi\pi$ states,

$$\langle (2\pi)_I | (-i) H_{\Delta S=1} | K^0 \rangle = A_I e^{i\delta I}.$$  

(78)

$\delta_I$ is the strong phase for $\pi\pi$ scattering, $\omega \equiv A_2/A_0$ and $p$, $q$ are the mixing parameters for $K^0 - \bar{K}^0$. To an excellent approximation, $\omega$ can be taken as real and $q/p = 1$. Therefore, we focus on calculating the imaginary part of the decay amplitudes.

In the SM, the contributions to $\epsilon'$ come from both QCD and electromagnetic penguin diagrams [36]. The QCD penguin contributes exclusively to $\Delta I = 1/2$ decay, whereas the electromagnetic penguin is mainly responsible for $\Delta I = 3/2$ decay. Both contributions are important but have opposite signs. Therefore, the final result depends on precision calculation of hadronic matrix elements. The state-of-art chiral perturbation theory [24, 37, 38, 39] and lattice QCD calculations [40, 41] have not yet been sufficiently accurate to reproduce the experimental result [42]. A review of the standard model calculation can be found in Ref. [43, 44].

In LRSM, each element in the righthanded CKM matrix has a substantial CP phase. As a consequence, there are tree level contributions to the phases of $A_2$ and $A_0$. Following closely the work by Ecker and Grimus [23], the $\Delta S = 1$ effective Hamiltonian from Eq. (11) and the tree-level Feynman diagram in Fig. 11 is
\[ \mathcal{H}_{\Delta S=1}^{\text{tree}} = \sqrt{2} G_F \lambda_u^{LL} \left[ \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M_L^2)} \right)^{-\frac{5}{2}} O_{+}^{LL}(\mu) + \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M_L^2)} \right)^{\frac{5}{7}} O_{-}^{LL}(\mu) \right] 
\quad + \sqrt{2} G_F \frac{M_L^2}{M_R^2} \lambda_u^{RR} \left[ \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M_R^2)} \right)^{-\frac{5}{2}} O_{+}^{RR}(\mu) + \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M_R^2)} \right)^{\frac{5}{7}} O_{-}^{RR}(\mu) \right] 
\quad + 2 \sqrt{2} G_F \sin \zeta \lambda_u^{LR} e^{i\alpha} \left[ \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M_L^2)} \right)^{\frac{5}{7}} O_{-}^{LR}(\mu) - \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M_L^2)} \right)^{-\frac{5}{7}} O_{+}^{LR}(\mu) \right] 
\quad + 2 \sqrt{2} G_F \sin \zeta \lambda_u^{RL} e^{-i\alpha} \left[ \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M_L^2)} \right)^{-\frac{5}{7}} O_{+}^{RL}(\mu) - \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M_L^2)} \right)^{\frac{5}{7}} O_{-}^{RL}(\mu) \right], \tag{79} \]

where we have taken into account the leading-logarithm QCD corrections with renormalization scale \( \mu \) taken to be around 1 GeV, and \( b = 11 - 2 N_f/3 \) with \( N_f \) the number of active fermion flavors. The mixing coupling \( \lambda_u^{AB} = V_{Aud}^\dagger V_{Bus}^* \), \( A, B \) are \( L, R \). The four quark operators are

\[ O_{+}^{LL,RR} = d\gamma^\mu \mathbb{P}_{L,R} u \mathbb{P}_{L,R} \gamma_\mu \mathbb{P}_{L,R} s \pm d\gamma^\mu \mathbb{P}_{L,R} s \mathbb{P}_{L,R} \gamma_\mu \mathbb{P}_{L,R} u, \]
\[ O_{+}^{LR,RL} = 2 d\gamma^\mu \mathbb{P}_{R,L} s \mathbb{P}_{L,R} \gamma_\mu \mathbb{P}_{L,R} u, \]
\[ O_{-}^{LR,RL} = 2 d\gamma^\mu \mathbb{P}_{R,L} u \mathbb{P}_{L,R} \gamma_\mu \mathbb{P}_{L,R} u, \tag{80} \]

where \( \mathbb{P}_{L,R} \) are projection operators. There are also new penguin diagrams involving the right-handed gauge boson contributing to \( \epsilon' \). However, these contributions are suppressed by loop factors and are neglected here.

\[ \begin{array}{ccc}
    & u & s \\
\hline
    d & W_L & \ \ \\
    d & W_R & \ \ \\
\end{array} \quad \begin{array}{ccc}
    & u & s \\
\hline
    d & W_R & \ \ \\
\end{array} \]

**FIG. 11**: New tree-level contribution to the \( \Delta S = 1 \) interaction from LRSM.

The hadronic matrix elements of the four-quark operators are calculated using the fac-
torization assumption,

\[
\langle (2\pi)_{I=0}|O^{LL,RR}_+|K^0\rangle = \pm \frac{X}{3\sqrt{6}},
\]

\[
\langle (2\pi)_{I=2}|O^{LL,RR}_+|K^0\rangle = \pm \frac{2X}{3\sqrt{3}},
\]

\[
\langle (2\pi)_{I=0}|O^{LL,RR}_-|K^0\rangle = \pm \frac{X}{2\sqrt{6}},
\]

\[
\langle (2\pi)_{I=2}|O^{LL,RR}_-|K^0\rangle = 0,
\]

\[
\langle (2\pi)_{I=0}|O^{LR,RL}_+|K^0\rangle = \pm \frac{4X}{9\sqrt{6}},
\]

\[
\langle (2\pi)_{I=2}|O^{LR,RL}_+|K^0\rangle = \pm \frac{2X}{9\sqrt{3}},
\]

\[
\langle (2\pi)_{I=0}|O^{LR,RL}_-|K^0\rangle = \mp \frac{1}{\sqrt{6}} \left( \frac{X}{18} + \frac{Y}{2} + \frac{Z}{6} \right),
\]

\[
\langle (2\pi)_{I=2}|O^{LR,RL}_-|K^0\rangle = \mp \frac{1}{6\sqrt{3}} \left( \frac{X}{6} - Z \right),
\]

where the parameters \(X, Y\) and \(Z\) are

\[
X \equiv -\langle \pi^- | \bar{d} \gamma_\mu \gamma_5 u | 0 \rangle \langle \pi^+ | \bar{u} \gamma^\mu s | K^0 \rangle,
\]

\[
= i\sqrt{2} F_\pi (m_K^2 - m_\pi^2) \simeq 0.03i\ \text{GeV}^3
\]

\[
Y \equiv -\langle \pi^+ \pi^- | \bar{u} u | 0 \rangle \langle 0 | \bar{d} \gamma_5 s | K^0 \rangle,
\]

\[
= i\sqrt{2} F_K A^2 (1 - m_K^2/m_\pi^2)^{-2} \simeq 0.273i\ \text{GeV}^3
\]

\[
Z \equiv -\langle \pi^- | \bar{d} \gamma_5 u | 0 \rangle \langle \pi^+ | \bar{s} u | K^0 \rangle,
\]

\[
= i\sqrt{2} F_\pi A^2 (1 - m_\pi^2/m_K^2)^{-2} \simeq 0.18i\ \text{GeV}^3,
\]

where \(A = m_K^2/(m_s + m_d)\), \(F_\pi = 93\ \text{MeV}\) and \(F_K = 1.22\ F_\pi\).

The numerical estimates are taken from Ref. [13]. \(Y\) and \(Z\) are much bigger than \(X\) due to chiral enhancement. Clearly the factorization approximation must be improved as indicated by the empirical \(\Delta I = 1/2\) rule, which is beyond the scope of this paper. We note, however, that for our estimation of the bound on \(M_{W_R}\), a multiplicative uncertainty factor on the matrix elements is reduced by a square-root.

To calculate the weak phases of the decay amplitudes \(A_0\) and \(A_2\), we use the experimental value for the real parts of \(A_0\) and \(A_2\): \(\text{Re} A_0 \simeq 3.33 \times 10^{-7}\) and \(\omega \simeq 1/22\). The dominant new contribution is from the \(W_L - W_R\) mixing term due to enhanced hadronic matrix elements and larger CP violation phase \(\alpha\). In fact, because the phase \(\alpha\) in the apparent factor \(e^{i\alpha}\) is much larger than the phase in \(\lambda^{RL,LR}_{u}\) which is typically of order \(\theta_1\) and \(\theta_2\) given in Eq. (29), \(\epsilon'\) is approximately a function of \(r \sin \alpha\), rather than \(r\) and \(\sin \alpha\) independently. Since \(r \sin \alpha\) has been fixed by \(\epsilon\) and \(d_n^e\) in the previous subsections, \(\epsilon'\) is approximately a function of \(M_{W_R}\) only. In Fig. [12] we plot \(\epsilon'\) as a function of \(M_{W_R}\) for \(\alpha = \arcsin\frac{1}{10}\), \(r = 0.5\) and \(s_d s_s = 1\) which is required by the neutron EDM calculation. [All \(s_i = 1\].] Requiring that the new contribution should be no larger than \(|\epsilon'_{\text{expt}}| = 3.92 \times 10^{-6}\) [9], we get a lower bound

\[
M_{W_R} > 4.2\ \text{TeV},
\]
Here we obtain a slightly tighter bound on $M_{W_R}$ than that from kaon mixing. However, because of the $\Delta I = 1/2$ rule, the factorization assumption might have overestimated the phase of $A_2$. If we take $r \sin \alpha = 0.15$, as required by low $M_H$, the bound changes to 7.4 TeV.

Finally, there are also tree-level FCNH contributions to $H_{\Delta S=1}$, as one can see from the lagrangian in Eq. (48) [15, 23, 30]. Since the relevant coupling is suppressed by either the Cabibbo angle or the quark masses, their contribution is negligible.

**D. CP Violation in $B_d \to J/\psi K_S$ Decay: $S_{J/\psi K}$**

The CP violation in B-meson decay was first observed in $B_d \to J/\psi K_S$. In SM, the decay proceeds mainly through the tree-level $b \to c \bar{c}s$ and the penguin contributions are expected to be suppressed by CKM and/or loop factors. The tree-level diagram is shown in the left panel in Fig. 13 with an intermediate $W_L$ exchange.

![Diagram](image)

FIG. 13: Tree level Feynman diagrams for $B_d \to J/\psi K_S$ from SM and LRSM.

The relevant CP asymmetry is defined as

$$S_{J/\psi K_S} = \frac{2 \text{Im} \lambda_d}{1 + |\lambda_d|^2} ,$$  

(84)
where

\[ \lambda_d = \left( \frac{q}{p} \right)_B \frac{\mathcal{A}(B_d \to J/\psi K_S)}{\mathcal{A}(B_d \to J/\psi K_S)} \]

\( \mathcal{A} \) is a decay amplitude and \( (q/p)_B \) is from the B-meson mixing.

The magnitude of \( \lambda_d \) is close to 1 and thus \( S_{J/\psi K_S} \sim \text{Im} \lambda_d \). In the SM, \( S_{J/\psi K_S} \) predominantly comes from \( B - \bar{B} \) mixing, and is related to the \( \beta \) angle of the unitary triangle because the ratio of the decay amplitude is independent of the hadronic matrix element when the tree operator dominates. Experimentally, \( \sin 2\beta_{\text{expt}} = 0.673 \pm 0.028 \).

In the LRSM, the effective \( \beta \) angle will receive new contributions from initial and final neutral meson mixings and from the new tree-level decay operators through \( W_R \) exchange and \( W_R - W_L \) mixing, as shown in Fig. 13. We will not consider the kaon mixing contribution for the following reason: Since \( K_S \) is dominantly CP-even \( |K_S\rangle = |p|K^0\rangle + q|\bar{K}^0\rangle \) and \( \bar{B}^0 \) decay involves \( \bar{K}^0 \), \( \lambda_d \) is proportional to \( (q/p)_K^* \). The imaginary part of \( (q/p)_K^* \) is proportional to the imaginary part of \( \epsilon \) which is known to be on the order of \( 10^{-3} \), much smaller than the phase in \( (q/p)_B \). The \( \epsilon \) constraint on the LR symmetric model has already been studied independently and we decouple the kaon-mixing effect from \( S_{J/\psi K_S} \). Thus we write

\[ 2\beta^H \approx 2\beta + \arg \left( 1 + \frac{M_{B^d,LR}^{12}}{M_{B^d,SM}^{12}} \right) \]

\[ + \arg \left( 1 + \frac{\langle J/\psi K_0 | \mathcal{H}^{LR} | B_0 \rangle}{\langle J/\psi K_0 | \mathcal{H}^{SM} | B_0 \rangle} \right) \left( 1 + \frac{\langle J/\psi K_0 | \mathcal{H}^{LR} | B_0 \rangle}{\langle J/\psi K_0 | \mathcal{H}^{SM} | B_0 \rangle} \right)^{-1}. \] (85)

The \( \mathcal{H}^{LR} \) operator is similar to that in Eq. (79), with substitutions \( s \to b, u \to c \), and \( d \to s \).

The ratio \( M_{B^d,LR}^{12}/M_{B^d,SM}^{12} \) can be calculated from Eq. (58) and a corresponding expression from the SM. Its magnitude is around \( 10^2 M_{W_L}^2/M_{W_R}^2 \) and carries a phase factor \( V_{tb}^* V_{tb} V_{td}^* V_{td} = s_b s_b e^{-i(\theta_2 + \theta_3)} \). The hadronic matrix element for the decay is less known.

In the naive factorization approximation for \( \langle J/\psi K_0 | \mathcal{H}^{SM} | B_0 \rangle \), the decay rate is under predicted by an order of magnitude. Therefore, the non-factorization contribution must be significant. In the ratio of matrix elements, we expect the factorization approach work better. Using this approximation, we find

\[ \frac{\langle J/\psi K_0 | \mathcal{H}^{LR} | B_0 \rangle}{\langle J/\psi K_0 | \mathcal{H}^{SM} | B_0 \rangle} = \frac{M_{W_L}^2}{M_{W_R}^2} \frac{\lambda_c^{RR}}{\lambda_c^{LL}} \frac{2 [\alpha_S(m_b)/\alpha_S(M_{W_L})]^{2/b} - [\alpha_S(m_b)/\alpha_S(M_{W_R})]^{2/b}}{2 [\alpha_S(m_b)/\alpha_S(M_{W_L})]^{2/b} - [\alpha_S(m_b)/\alpha_S(M_{W_R})]^{2/b}} \] (86)

where \( \lambda_c^{AB} = V_{cA}^* V_{cB} \) for \( A, B = L, R \), so \( \lambda_c^{RR}/\lambda_c^{LL} = s_b s_b e^{-i(\theta_2 + \theta_3)} \). The non-perturbative \( B \to K \) form factors \( F_+(m_{J/\psi}) \) has been canceled out and the result is independent of hadronic parameters. The magnitude of the ratio is \( O(M_{W_L}^2/M_{W_R}^2) \) and hence is much smaller than the mixing contribution to \( 2\beta_{\text{eff}} \). This conclusion remains valid even if we underestimate this ratio by an order of magnitude.

The modified CP asymmetry in LRSM depends on the righthanded scale \( M_{W_R} \) and \( r \sin \alpha \). We take \( r \sin \alpha \simeq 0.05 \) as determined from the \( \epsilon \) constraint. There are two independent choices for quark mass signs which generate different predictions. We take either \( s_1 = +1 \) or \( s_1 = -1 \), and the results are shown in Fig. 13. Demanding \( \sin 2\beta_{\text{eff}} \) to lie within the experimental error bar, we get a moderate lower bound on \( M_{W_R} \),

\[ M_{W_R} > 0.7 \text{ TeV}. \] (87)
FIG. 14: Predicted CP asymmetry in $B^0_d \to J/\psi K_S$ as a function of $M_{W_R}$, for $r \sin \alpha = 0.05$. The solid line corresponds to the sign choice $s_i = 1$ and the dashed line corresponds to $s_i = 1$ except $s_t = -1$. The shaded part is allowed by the experimental data.

As we have commented above, this constraint comes predominantly from the $B_d - \bar{B}_d$ mixing contribution.

VII. CONCLUSION

In this paper, we have made a comprehensive study of CP violating observables in the low-energy sector of the minimal LRSM with the only assumption of parity invariance imposed on the theory. This is made possible by an explicit solution for the right-handed quark mixing matrix with explicit dependence on the spontaneous CP violation phase $\alpha$. Although the hadronic physics uncertainty is still large, the CP observables do provide significant and strong constraint on the right-handed $W$-boson mass scale $M_{W_R}$ and the FCNH mass scale $M_H$. In fact, a new experiment result and/or improved theoretical calculation on EDM might provide the strongest bound yet on the right-handed gauge boson mass.

We stress the point that CP must be violated both explicitly and spontaneously in the minimal model, with one bidoublet and two triplets. Up to $O(\lambda^3)$, we can write $V_{CKM}^R$ in a compact form as in Eqs. [27], [28] and [29]. We find that $V_R$ has the same hierarchical structure as $V_L$, i.e., elements of the two mixing matrices are suppressed by the same orders of Cabibbo angle $\lambda = 0.22$. And because of the spontaneous CP phase $\alpha$, each element in $V_{CKM}^R$ acquires a phase angle proportional to $r \sin \alpha$. Therefore, the phenomenology related to CP violation in the minimal LRSM turns out to be very rich. We explored mass differences of neutral kaon and B-meson with updated lattice results, and found an updated lower bound on righthanded $W$-boson mass: $M_{W_R} > 2.5 \text{ TeV}$. With the CP violating processes, we find a combined bound $M_{W_R} > 4 \sim 8 \text{ TeV}$ from $\epsilon$ and neutron EDM constraints when the FCNH contribution is ignored. And we can also fix $r \sin \alpha \simeq 0.05$ from the combined bound. When the FCNH contribution is added to $\epsilon$, the bound is stronger when the $M_H$ becomes small. We go on to study $\epsilon'$ and CP asymmetry in $B_d \to J/\psi K_S$ decay. By applying the experimental constraints and theoretical uncertainty, we conclude that the lower bound on righthanded $W$-boson mass surviving from all above experimental constraint is about $4 \text{ TeV}$. If the $W_R$ really have a mass close to this lower bound, it is possible to detect its signal in the upcoming LHC.
TABLE II: A summary of bounds on $M_{W_R}$ and $M_H$ from different physical observables. Stars on the items indicate input values. For the neutron EDM case, the numbers in the parenthesis are direct result from Eq. (72) and those without are obtained by reducing theoretical values by a factor of 5.

We also find that the lower bound on $M_H > 25$ TeV is tighter than the bound previous bounds [19, 28]. Perhaps, this suggests that one should have two bidoublets so that one can invoke cancelation between them, as in the spontaneous CP violation model discussed in Ref. [47]. In that case for our analytic solution to remain valid, the second bi-doublet should develop vev. All bounds are shown in Table II for easy reading of the results of the paper.

Finally, we would like to comment on the constraint on $r$ arising from the consideration of the mass shift of SM Higgs boson in LRSM. According to discussions in Sec. IV the SM Higgs mass is

$$m_h^2 = \frac{4\lambda_1}{\rho_1} \kappa^2 + \alpha_3 \xi^2 v_R^2 , \quad (88)$$

to second order in $\alpha, \xi$ and $\epsilon$. The shift in mass due to LRS is $\alpha_3 \xi^2 v_R^2 - \frac{\alpha^2_1}{\rho_1} \kappa^2$ and can be expressed in terms of the masses of FCNH

$$\Delta m_{h^0} \simeq \xi M_{H_1^0} . \quad (89)$$

From the discussions below Eq. (59), FCNH masses have to be large enough to suppress the tree level contribution to kaon mixing. On the other hand, the SM Higgs mass should not exceed TeV scale in order to preserve perturbative unitarity. A recent analysis [25] yields

$$m_{h^0}^2 < 870 \text{ GeV} . \quad (90)$$

The lower bound on the FCNH boson mass $M_{H_1^0} > 25$ TeV yields an upper bound on $r = \xi m_t/m_h < 1.44$. From discussions on CP violating observables, $0.05 < |r \sin \alpha| < 0.15$, this translates into a lower bound on the spontaneous phase $|\alpha| > 0.035$.

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