Incidence of a diffraction-free wave beam on a dielectric microcylinder: Subwavelength effects

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Abstract: It is known, that a linearly polarized wave beam or a plane wave when incident on a dielectric microcylinder convert into a photonic nanojet in the transmission area. We theoretically show, that a wave beam with antisymmetric polarization impinging the same microparticle experience a very different transform. First, it converts into a set of creeping waves which form a standing wave pattern with the locally enhanced concentration of the electric field in the maxima. On the rear side of the microparticle, we observe the subwavelength field concentration accompanied by a propagating-into-evanescent waves conversion. Depending on the particle permittivity and radius this effect may correspond to the enhancement of the local electric field which is maximal either in the hot spots located aside the axis of the incident/transmitted wave beam or in the hot spot centered by this axis. Outside the cylinder the hot spots have the subwavelength width. In all cases an increase of the longitudinal polarization has been obtained on the beam axis. This increase is a resonant effect: for a given cylinder radius a sharp maximum of the axial polarization versus permittivity is achieved.

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1. Introduction and problem formulation

Nine years ago a capacity to offer a label-free nanoimaging has been experimentally revealed for a simple dielectric microsphere [1]. If the object is complex, e.g. represents a pair of closely located dipole scatterers, a strongly subwavelength resolution was obtained (from \( \lambda/6 \) in [1] to \( \delta \sim \lambda/10 \) in recent works). Since this subwavelength image is strongly magnified it can be recorded without post-processing or developed by a usual microscope [2–9].

Before 2011 the functionality of the magnified subwavelength imaging without fluorescent labels was known only for the so-called hyperlens – a tapered/curved nanostructure with alternating plasmonic and dielectric constituents forming the so-called hyperbolic metamaterial [10–13]. Since a simple microsphere is incomparably cheaper, grants much finer resolution and much higher magnification that any available hyperlens, a spherical microlens has been commonly recognized a promising device for in-vivo label-free nanoimaging. It attracted a lot of attention of researchers, however, the mechanisms of its operation have not been fully understood up to now.

Attempts to refer the subwavelength imaging functionality of the microsphere to the resonances of the whispering gallery, multipole Mie resonances and other known resonances of a spherical cavity were done in works [2, 3, 6–8]. However, some of these works referred to special cases and cannot explain the majority of the experimental results, whereas the other works pretending to the general explanation mistakenly employed the reciprocity theorem [14].

In the recent work [15] the hyperlens functionality of the microsphere was presumably related with the normal polarization of the scattering object. The hypothesis of [15] assumed at least two mechanisms of the imaging related with the action of the normal polarization to the microsphere.
One of them, assumed for sufficiently large spheres \((kR \gg 10\), where \(k\) is the wave number in free space and \(R\) is the sphere radius\), is formation of a non-divergent radially polarized beam into which the near field of the radial dipole penetrating into the sphere is converted. Such a beam in the case of the azimuthal symmetry of its intensity does not experience the Abbe diffraction \([16]\). Therefore, this beam can be focused into a subwavelength spot by a usual microscope (with the \(f\)-number of the order of unity). Thus we obtain a subwavelength image of a radially polarized subwavelength object. Another mechanism, suggested in \([15]\) for a sphere with \(kR \sim 10\) was conversion of the near field produced by the radially polarized subwavelength source into a set of creeping waves propagating around the sphere on the internal side of the dielectric interface. For few creeping waves having the suitable wave numbers the perimeter of the sphere offers a resonance and the pattern of \(TM_{n,0}\) type resonance of the sphere is well pronounced. This pattern corresponds to the so-called pseudo-mode (resonant forced solution) that experiences the radiative leakage. In other words, the creeping waves forming this pattern eject to free space from two narrow areas located symmetrically with respect to the object. Corresponding wave beams compose a divergent imaging beam which is also almost free of the Abbe diffraction due to the symmetry of the electric and magnetic field vectors with respect to the axis of the source.

The validation of the first hypothesis by exact numerical calculations has not been done, yet. In \([15]\) the second hypothesis was validated for a microcylinder because the authors did not find a reliable tool to simulate the sphere with \(kR \geq 10\). In the present work, we aim to study a nearly reciprocal case to that numerically investigated in \([15]\). Namely, the same dielectric microcylinder as in \([15]\) is now impinged by a diffraction-free wave beam. This situation cannot be exactly reciprocal to that of \([15]\): since in both situations the evanescent waves are involved, the reciprocity does not yield to the inversion of the wave vectors. In the present work, the source is the so-called cosine wave beam. In practice, such a beam results from the transmission of two plane waves with the same frequency and opposite phases whose wave vectors form a sharp angle \(2\beta\) though a large diaphragm \(D \gg \lambda\). This transmission is shown in Fig. 1. After the aperture, the cosine beam experiences the Abbe diffraction only in the domain of its tails (it is so up to the distances exceeding the diaphragm width \(D\)). The diffraction spread does not occur in the paraxial domain of the beam where only the interference of two plane waves is observed. If the angle between two plane waves is small enough \((2\beta < \pi/kR)\) and the beam is TM-polarized the \(x\)-component of the electric field and the magnetic field grow versus \(|x|\) from zero almost linearly and the maximal intensity of the beam incident on the microparticle holds at its lateral edges \(x = \pm R, y = 0\).

Such the beam is the most adequate 2D analogue of an azimuthally symmetric non-divergent beam with radial polarization \([17]\). For our purposes, the presence of the diaphragm is not relevant. In our simulations, we saw that the power flux lines distant from the cylinder lateral edges by \(\Delta x > (1 - 2)\lambda\) do not feel the presence of the cylinder and the last one responds to that incident field which is in the domain \(-R - 2\lambda < x < R + 2\lambda\). This observation drastically saved our computation time, and in our further simulations the cosine beam has been mimicked by two plane waves without a diaphragm.

Our study represents both theoretical and practical interest. Theoretically, it is interesting to study the distribution of the electric and magnetic field in the hot spots which will arise due to the resonances of the pseudo-modes and to compare them with those corresponding to the so-called photonic nanojet \([18]\). Photonic nanojet is a wave beam attracting a lot of attention of researchers in the past decade. It results from a usual plane wave or a Gaussian wave beam incident on a dielectric microparticle (either as a cylinder or as a sphere). This wave beam has a waist of slightly subwavelength effective width formed at the rear side of the microparticle \([18–21]\).

In the case of the sphere, the waist effective width is as small as \(\lambda/3\) \([18–21]\), and in the case of the cylinder it is of the order of \(\lambda/2\) \([21]\). A so narrow wave beam experiences the Abbe diffraction due to which the waist length turns as small as nearly \(2\lambda\). However, in spite of
this far-field effect the waist could not subwavelength thin without excitation of the evanescent waves. The evanescent wave package responsible for the waist can be treated as the constructive interference of high-order spherical harmonics in [21], however, it is definitely a near-field effect. We wonder is the near-field effects drastically stronger in our case compared to that in the conventional case? What are the major differences between the case of the cosine incident beam and the single plane wave incidence?

Besides of this theoretical interest, our study has practical purposes. The first one is to obtain a high local enhancement of the incident electric field outside the microparticle. It is reasonable to expect this enhancement because in the imaging problem [15] the near field of the radially polarized dipole is efficiently converted into creeping waves which are propagating around the microparticle. If the evanescent waves convert into propagating ones efficiently, the inverse process should be presumably efficient, too. Thus we may expect the formation of the hot spots of electric field on the microparticle surface where the local intensity enhancement (LIE) may be high due to the propagating-into-evanescent waves conversion. High LIE achieved in free space can be used in different nanophotonic applications. For example, it can be used for the enhancement of the Raman scattering, in the case of biological analytes when the usual plasmonic SERS is not applicable [22]. Here we may define LIE normalizing the total field electric intensity

\[ I = E_x^2 + E_y^2, \]

\[ I_m = \frac{E_0^2}{2} \sin^2 \left( \frac{kR \sin \beta}{2} \right) / \sin^2 \beta, \]

\[ I_0 \approx E_0^2 \sin^2 (kR \sin \beta)/2 \sin^2 \beta. \]

Notice, that a study of a quite substantial glass microsphere impinged by a Bessel beam was reported in [23]. However, this microparticle did not show stronger propagating-into-evanescent waves conversion compared to the conventional nanojet. The authors theoretically and experimentally obtained several radially elongated hot spots with \( I/I_0 > 1 \). These hot spots were the waists of the multiple nanojets with the effective width nearly equal to \( \lambda/3 \). So, the
only implication of the replacement of a Gaussian beam by a Bessel beam in this work was an easily predictable birefringence of the photonic nanojet. Blow it is shown that the near-field effect we are looking for is sensitive to the radius of the microparticle and for given \( n \) there are specific values of \( kR \) corresponding to the maximal propagating-to-evanescent waves conversion. Parameters of the sphere from [23] (\( kR = 9.9 \), optical glass) did not offer this maximum.

## 2. Simulations results and discussion

In our calculations we use both commercial solver (COMSOL Multiphysics) and an exact analytical solution of the 2D diffraction problem for a cylinder impinged by a plane wave (see e.g. in [24]). An efficient algorithm of calculating the electromagnetic field for substantial dielectric spheres (\( kR > 1 \)) illuminated by a Bessel beam was suggested in the recent work [25]. A similar algorithm is helpful also for the 2D case when the Bessel beam is replaced by a cosine beam:

\[
H_i = H_i^0 = A \sin(k \rho \sin \beta \sin \phi) e^{-jk \cos(\beta \rho \cos \phi)}, \quad E_i(\phi = 0) = E_\rho^i = E_0 e^{-jk \cos(\beta \rho)},
\]

where \( E_0 \equiv -j A \sin \beta \). For this superposition of plane waves the exact solution in cylindrical functions converges faster than for a single plane wave, and in the case \( kR \gg 1 \) the high-frequency asymptotic methods reviewed in [24] and similar works are not necessary.

![Instantaneous pictures of the wave movies simulated by COMSOL for the electric field (a) and magnetic field (b). Here \( kR = 10, n = 1.7 \).](image)

Fig. 2. Instantaneous pictures of the wave movies simulated by COMSOL for the electric field (a) and magnetic field (b). Here \( kR = 10, n = 1.7 \).

![Local intensity enhancement on the back side of the cylinder surface (a) and polarization of the electric field inside the cylinder and behind it (b).](image)

Fig. 3. Local intensity enhancement on the back side of the cylinder surface (a) and polarization of the electric field inside the cylinder and behind it (b).

In Fig. 2 we present instantaneous pictures of the wave movies simulated by COMSOL for the electric field (a) and magnetic field (b) scattered by the cylinder with refractive index \( n = 1.7 \) and optical length of the radius \( kR = 10 \) illuminated by a beam with \( \beta = 0.01 \). In accordance to these movies, the creeping waves are efficiently excited by the incident beam near the points \( x = \pm R \) and travelling around the cylinder form a standing wave pattern. In Fig. 2(a) we see that at the back side of the cylinder several hot spots arise but it is difficult to treat them as the waists of the nanojets. Moreover, a noticeable electric field (with evidently longitudinal polarization exceeding \( E_0 \)) arises on the optical axis \( y \). Whereas these external electric hot spots represent
the extension of the electric field into free space, the hot spots of magnetic field are basically confined inside the particle and the magnetic field keeps zero on the optical axis.

The distribution of the normalized intensity $I/I_0$ over the back side of the cylinder surface is shown in Fig. 3(a). In this plot, we can see that the electric hot spots (resulting from the creeping waves propagating inside the cylinder) when extend to free space turn more pronounced and their maxima along the azimuthal angle $\phi$ become sharper. Namely, the sharpest hot spots observed in free space at the angles $\phi = \pm 56.5^\circ$ have the effective angular width $\Delta \phi$ (determined, in accordance to the Rayleigh criterion, on the level 70% of LIE value at the local maximum), close to $6^\circ = 0.105$ rad. It corresponds to strongly subwavelength linear width of the external hot spots $R\Delta \phi \approx \lambda/6$. This subwavelength field concentration is a feature of the strong near field effect i.e. domination of the evanescent waves in the hot spots. The domination of evanescent waves is confirmed by the simulation of the electric field polarization whose instantaneous picture is presented in Fig. 3(b). The standing-wave pattern of creeping waves produces the vortices of the electric field inside the particle and these internal vortices generate also the vortices of $\mathbf{E}$ in the transmission area at the wave distances from the particle. This picture has nothing to do with the electric polarization in a conventional photonic nanojet [19]. Comparing to the case of a sphere $kR = 9.9$, $\varepsilon = 2$ impinged by the Bessel beam in work [23] the mean width of the hot spots on the particle surface and their period around it in our case are half as much, whereas the LIE is twofold.

Fig. 4(a) illustrates the enhancement of the electric field axial component in the coordinates $(\rho, \phi)$. Therefore, the magnitude of the electric field vector is here normalized by $E_0$. In this plot the values $E^2/E_0^2 = I/E_0^2$ vary with a step i.e. the color map is discrete. Due to the problem symmetry we present only the domain $\phi > 0$. We shared with a dashed line a region centered on the optical axis and stretched along it by $\lambda/2$ and across it by $\lambda/3$. In this region the axial component of $\mathbf{E}$ has a local maximum. This local maximum is distant by $0.44\lambda$ from the rear

Fig. 4. Electric field enhancement behind the cylinder normalized to the axial component of the incident beam, discrete color map (a) and local intensity enhancement, continuous color map (b). Here $kR = 10$, $n = 1.7$.

Fig. 5. Distribution of the phase shift between $E_x$ and $E_y$ in the area covering two lateral maxima of the electric field marked by red in the inset (a). Phase shift between the electric and magnetic field vectors over the same area (b). Here $kR = 10$, $n = 1.7$. 
edge of the cylinder. In spite of the substantial distance and the length of the domain where the longitudinal electric field is concentrated, it is a near-field effect in which the field of the transmitted propagating waves interfere with the evanescent fields of the rear hot spot. In Fig. 4() we present the color map of $LIE/I_0$ in the Cartesian coordinates. Being calculated in COMSOL this color map very well fits the plot in Fig. 4(a) obtained from the exact solution. The locations of two paraxial hot spots in Fig. 4(b) corresponds to the angles $\phi = \pm 0.21$ rad. Two paraxial maxima detectable in Fig. 4(a) occur at $\phi = \pm 0.19$. Two next hot spots in Fig. 4(b) hold at $\phi = \pm 0.40$ whereas in Fig. 4(a) they occur at $\phi = \pm 0.37$. Further, the region where the axial field is locally enhanced outlined in Fig. 4(a) corresponds to the region of nearly same sizes outlined in Fig. 4(b). Finally, values of $E^2/E_0^2$ in the hot spots match the corresponding values for LIE mapped in Fig. 4(b).

![Fig. 6. Electric intensity distribution on the axis $z$ behind the cylinder with $kR = 10$ for different permittivities of its material.](image)

Properties of the electromagnetic field in the hot spots are illustrated by Fig. 5. Fig. 5(a) depicts the distribution of the phase shift $\Delta\Psi_{xy}$ between two Cartesian components of the electric field over the part of the microparticle surface marked by red in the inset. This area covers two lateral hot spots in which $E_x$ and $E_y$ have nearly the same magnitude and the phase shift $\Delta\Psi_{xy} \equiv \text{phase}(E_x/E_y) = 90^\circ$ means the circular polarization. Angular coordinates of two hot spots are marked on the plot as $\Delta\phi_1$ and $\Delta\phi_2$. In the center of the first electric field maximum, the polarization is close to the circular one, and in general, the electric polarization is essentially elliptic. The electric polarization in the second hot spot is very different. The mean value of $\Delta\Psi_{xy}$ equals zero in this area, however, this phase shift varies across this hot spot from $-\pi$ to $\pi$, i.e. the polarization is locally elliptic and even circular. Meanwhile the magnetic polarization keeps transverse $H = H_z$ everywhere, i.e. this polarization transformation is a near-field effect.

![Fig. 7. LIE for a cylinder with $kR = 10$, $n = 1.4$ as a color map (a) and as a function of the azimuthal angle on the back surface of the cylinder (b).](image)

The last hypothesis is confirmed by Fig. 5(b). In Fig. 5(b) we observe the noticeable phase shift $\Delta\Psi$ between the oscillations of the electric and magnetic field vectors. Our phase shift takes
into account the ellipticity of the field polarization. It is known that for all propagating waves \( \Delta \Psi = 0 \) whereas for evanescent waves it is equal \( \pm 90^\circ \). We see that \( \Delta \Psi = 0 \) holds only in the dark area where it this phase shift is not interesting, whereas in both hot spots the values of \( \Delta \Psi \) show the approximate parity of the evanescent and propagating field components within the hot spots.

The near-field effect in the rear area of the cylinder is very sensitive to the radius and the refractive index of the cylinder. In Fig. 6 we see how the variation of the sphere permittivity \( \varepsilon = n^2 \) eliminates the local maximum of the axial polarization behind the sphere. This maximum is very pronounced for \( \varepsilon = 3 \) and disappears when \( \varepsilon = 2.98 \). Similarly, it disappears versus a small variation of the frequency: this near-field effect is strongly resonant.

We also found for the same \( kR \) the regime of the resonant nanofocusing when the LIE is maximal in the rear hot spot. In In Fig. 7 we see both the color map of LIE in the Cartesian coordinates and the plot of LIE as a function of the azimuthal angle on the back surface of the cylinder with \( kR = 10, n = 1.4 \). In this case LIE does not attain so high values (\( I/I_0 = 14 - 15 \)) as in the case \( n = 1.7 \), however demonstrates the absolute maximum at the rear edge of our microparticle. It corresponds to the huge enhancement of the longitudinal component at this point: \( I/I_0 \approx 200 \). Imagine that our cosine beam is produced by a subwavelength source in presence of another microcylinder i.e. results from the first mechanism of the imaging assumed in [15]. This the cylinder \( kR = 10, n = 1.4 \) can be considered as a lens which may form the true subwavelength image of the object – the angular width of the rear hot spot is \( \Delta \phi = 8^\circ \) i.e. the linear width of this nanojet waist is about 0.25\( \lambda \). Of course, this nanofocusing is not a lensing effect but a strong near-field effect. In the maximum of this spot \( \mathbf{E} \) is polarized longitudinally, and it is evidently a near-field effect. Aside this point, i.e. at \( 0 < |\phi| < \Delta \phi/2 \) the polarization is elliptic, however, in this part of the focal spot evanescent waves also dominate. In Fig. 8 we depict for this case the similar phase shifts \( \Delta \Psi_{xy} \) and \( \Delta \Psi \) as were presented for \( n = 1.7 \) in Fig. 5. In the present case, \( \Delta \Psi_{xy} \approx 50^\circ \) in the region \( 2^\circ < |\phi| < 4^\circ \), where \( |E_x| \sim |E_y| \) and \( E \sim \eta H \). The
phase shift $\Delta \Psi$ between the electric and magnetic field vectors is in this region equal $47 - 52^\circ$. A so large $\Delta \Psi$ means the domination of the evanescent waves.

Varying both $kR$ and $n$ we have found the regime where the subwavelength focusing of our beam in the rear spot is even sharper. In Fig. 9 we show the color map and the angular distribution of LIE for the case $kR = 20$, $n = 1.7$. In this case $\Delta \Phi = 4.5^\circ$ and the linear width of the rear hot spot is as small as $0.22\lambda$. Probably, this case is not yet a limit one for the reduction of the focal spot size. The phase shifts $\Delta \Psi_{xy}$ and $\Delta \Psi$ in this case are distributed similarly to the similar plots in Fig. 8, whereas $\Delta \Psi$ attains in the rear hot spot almost $70^\circ$, i.e. evanescent waves in this spot dominate strongly.

To conclude this section: we have also studied the variations of all reported effects versus $\beta$. This parameter has no noticeable impact until $\beta \approx \pi/2kR$ that is the threshold value for our near-field effects. If the cross section of the particle covers several spatial oscillations of the incident beam intensity all these effects disappear, and the only difference of the cosine beam incidence from the single plane wave incidence is the birefringence of the usual photonic nanojet, already reported for a sphere in work [23].

3. Conclusions

In this work we have studied the incidence of a diffraction-free beam (cosine beam) on a dielectric microparticle in the 2D case, when the particle is a cylinder. If the beam intensity has the sharp minimum on its optical axis and varies slowly enough over the particle cross section and if the particle is substantial enough, the strong and unusual near field effects arise in free space behind the beam. Besides of the birefringence of the usual nanojet in the case of the cosine beam incidence (that was expected in the case of the cosine beam incidence), hot spots (the waists of the partial nanojets) in free space

- have strongly subwavelength width (are 2-3 times narrower than the usual photonic nanojet waist is);
- show a strong presence of the evanescent waves (in the case of a usual nanojet they are confined inside the particle);
- manifest the local intensity enhancement which attains 15 (triply higher that a usual nanojet may offer);
- manifest the strong presence of the evanescent waves (in the case of a usual nanojet they are confined inside the particle);

We have also found two new near-field effects which arise in a narrow band of parameters are represent sharp resonances:

- on the optical axis a huge resonant enhancement of the longitudinal electric field arises in a spot which is substantially shifted from the sphere and has substantial length (usually, resonant near-field effects hold in much smaller spots);
- it is possible to collect the most part of the incident beam energy into a subwavelength spot centered on the optical axis (usually, lateral hot spots in the case of the photonic nanojet birefringence are as important as the rear hot spot).

We believe that these new resonances of dielectric microparticles deserve further studies and an experimental check. The two last effects can find applications in the label-free subwavelength imaging and in affordable trapping of the charged particles.
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