A Search for the Fourth SM Family Fermions and $E_6$ Quarks at $\mu^+\mu^-$ Colliders

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Abstract

The potential of $\mu^+\mu^-$ colliders to investigate the fourth SM family fermions predicted by flavour democracy has been analyzed. It is shown that muon colliders are advantageous for both pair production of fourth family fermions and resonance production of fourth family quarkonia. Also isosinglet quarks production at $\mu^+\mu^-$ colliders has been investigated.

I. INTRODUCTION

The mass spectrum and the mixing of fundamental fermions are the most important unsolved problems of the particle physics. According to the Standard Model (SM), these masses and mixings arise from the interaction with the Higgs doublet via spontaneous symmetry breaking.

In the framework of SM, fermions with the same quantum numbers (electric charge, weak isospin, etc.) are indistinguishable before the symmetry breaking. Therefore, in the fermion-Higgs interaction, the Lagrangian terms corresponding to fermions with the same
quantum numbers should come with equal strength. As a result, one deals with singular mass matrices after the spontaneous symmetry breaking.

According to DMM (Democratic Mass Matrix) approach \[1–5\] in the case of \(n\) SM families \((n-1)\) families are massless and \(n^{th}\) family fermions have masses \(na\eta\) (here \(a\) is the common strength of Higgs-fermion interactions). Taking the real mass spectrum of the third family fermions into account necessarily leads to the assumption that at least a fourth SM family must exist \([6,8]\) (for recent situation see Ref. \[9\]).

The existence of the fourth SM family and masses of the fourth SM family quarks will be determined as a result of experiments done at LHC \([10,11]\). In our opinion, muon colliders will be advantageous for investigation of the fourth SM family leptons and quarkonia.

II. THE PRODUCTION OF THE FOURTH SM FAMILY FERMIONS AT \(\mu^+\mu^-\) COLLIDERS

It is clear that direct pair production of the fourth family fermions will be possible at future high energy colliders only, since their predicted masses lie between 300 \(GeV\) and 700 \(GeV\) \([7]\). Therefore, lepton colliders with \(\sqrt{s} \geq 1.5\) \(TeV\) and sufficiently high luminosity will give the opportunity to search for all fermions from the fourth SM family.

Linear \(e^+e^-\) colliders with high energy are ones of the necessary devices to search the fundamental ingredients of matter and interactions of them. But the advantage of \(\mu^+\mu^-\) colliders with respect to \(e^+e^-\) colliders is that, they have more monochromatic particle beams. For example, while the energy spread of \(e^+e^-\) colliders is more than 1%, that of \(\mu^+\mu^-\) colliders is between 0.1% and 0.014%. In addition, since the mass of muon is 207 times more than the mass of electron, the energy uncertainty from effect of the opposite beam can be ignored. Design values of \(\mu^+\mu^-\) colliders are \(\sqrt{s} = 4\) \(TeV\) and \(L = 5 \times 10^{33} cm^{-2}s^{-1}\) or \(\sqrt{s} = 30\) \(TeV\) and \(L = 3 \times 10^{35} cm^{-2}s^{-1}\) \([12]\).

The cross section for the process \(\mu^+\mu^- \rightarrow f \bar{f}\) has the form

\[
\sigma = \frac{2\pi\alpha^2}{3s} \xi\beta \left\{ Q_f (Q_f - 2\chi_1 v v_f) (3 - \beta^2) + \chi_2 (1 + v^2) \left[ v_f^2 (3 - \beta^2) + 2\beta^2 a_f^2 \right] \right\}
\]  (1)
where
\[ \chi_1 = \frac{1}{16 \sin^2 \theta_W \cos \theta_W} \frac{s(s-M_Z^2)}{(s-M_Z^2)^2+\Gamma_{\mu\mu}^2 M_Z^2} \]
\[ \chi_2 = \frac{1}{256 \sin^4 \theta_W \cos \theta_W} \frac{s^2(s-M_Z^2)}{(s-M_Z^2)^2+\Gamma_Z^2 M_Z^2} \]
\[ v = -1 + 4 \sin^2 \theta_W \]
\[ a_f = 2T_3f \]
\[ v_f = 2T_3f - 4Q_f \sin^2 \theta_W \]
\[ \beta = \sqrt{1 - 4m_{Q_f}^2/s} \]
\[ T_3 = \frac{1}{2} \text{ for } \nu_4 \text{ and } u_4, \quad T_3 = -\frac{1}{2} \text{ for } l_4 \text{ and } d_4 \]
\[ \xi = 1 \text{ for leptons}, \quad \xi = 3 \text{ for quarks.} \]

The production cross section values and corresponding event numbers (with \( \sqrt{s} = 4 \text{ TeV} \) and \( L^{int} = 50 \text{ fb}^{-1} \)) are given in Table I.

### III. THE PRODUCTION OF THE FOURTH SM FAMILY \( \psi_4 (3S_1) \) QUARKONIA AT \( \mu^+\mu^- \) COLLIDERS

Differing from t-quarks, fourth family quarks can form the quarkonia since \( u_4 \) and \( d_4 \) are almost degenerate and their decays are suppressed by small CKM mixings [8].

The cross section for the formation of the fourth family quarkonium and its decay into any \( X \) state is given with the relativistic Breit-Wigner equation

\[
\sigma \left( \mu^+\mu^- \rightarrow (Q \bar{Q}) \rightarrow X \right) = \frac{12\pi(s/M^2) \Gamma_{\mu\mu} \Gamma_X}{(s-M^2)^2 + M^2\Gamma^2}, \tag{2}
\]

where \( X \) corresponds to final state particles, \( M \) is the mass of the fourth family quarkonium, \( \Gamma_{\mu\mu}, \Gamma_X \) and \( \Gamma \) correspond to partial decay width to \( \mu^+\mu^- \), \( X \) state particles and the total decay width of the fourth family quarkonium, respectively.

Since the \( \mu^+\mu^- \) colliders has the certain energy spread, the average cross section can be estimated from

\[
\sigma^{\text{ave}} = \frac{\Gamma_{\text{tot}}}{\Delta E_{\text{coll}}} \sigma^{\text{res}} \left( \mu^+\mu^- \rightarrow (Q \bar{Q}) \right), \tag{3}
\]
where $\sigma^{\text{res}}$ is the resonance value of the cross section [13].

The energy spread is $\Delta E_{\text{coll}} \approx 10^{-3} \sqrt{s}$ for the $\mu^+\mu^-$ collider with $\sqrt{s} = O(\text{TeV})$. The estimated cross section values for $\psi_4 (u_4 \bar{u}_4)$ are presented in the Table II. Corresponding values for $\psi_4 (d_4 \bar{d}_4)$ are approximately the same.

The value of the luminosity at the resonance, used in calculations, has been estimated as

$$L (\sqrt{s_{\text{res}}}) = \frac{\sqrt{s_{\text{res}}}}{4\text{TeV}} L (4\text{TeV}).$$

As a result, we obtain number of events per year which are given in the last column of the Table II.

In this study we consider only $\psi_4 (3S_1)$ quarkonia state. Using corresponding formulae from [13], we obtain decay widths for main decay modes of $\psi_4 (u_4 \bar{u}_4)$ which are given in Table III. One can see that dominant decay modes for $\psi_4$ quarkonia are $\psi_4 \rightarrow W^+W^-$, $\psi_4 \rightarrow Z^0\gamma$ and $\psi_4 \rightarrow \gamma H$.

**IV. THE PRODUCTION OF THE E$_6$ QUARKS AT $\mu^+\mu^-$ COLLIDERS**

Another way to explain the relation $m_{b,\tau} << m_t$ is the introduction of exotic fermions. Let us consider as an example the extension of the SM fermion sector which is inspired by $E_6$ GUT model initially suggested by F. Gursey and collaborators [14,15]. It is known that this model is strongly favored in the framework of SUGRA (see Ref. [19] and references therein). For illustration let us restrict ourselves by quark sector:

\[
\begin{align*}
(u^0_L, u^0_R, d^0_R) & , & (c^0_{L}, c^0_R, s^0_R) & , & (t^0_L, t^0_R, b^0_R) \\
D^0_{1L}, D^0_{1R} & , & D^0_{2L}, D^0_{2R} & , & D^0_{3L}, D^0_{3R}.
\end{align*}
\]

According to Flavor Democracy the down quarks’ mass matrix has the form:
\[ M^0 = \begin{pmatrix} a\eta & a\eta & a\eta & a\eta & a\eta \\ a\eta & a\eta & a\eta & a\eta & a\eta \\ a\eta & a\eta & a\eta & a\eta & a\eta \\ M & M & M & M & M \\ M & M & M & M & M \\ M & M & M & M & M \end{pmatrix}, \]

where \( M \) is the scale of "new" physics which determines the masses of the isosinglet quarks. As the result we obtain 5 massless quarks and the sixth quark has the mass \( 3M + m_t \).

\( E_6 \) quarks can be produced at \( \mu^+ \mu^- \) colliders. To estimate the production cross sections, one can use equation (1) with following minor changes. For the isosinglet \( D_1 \) quark, \( a_f = 0 \) and \( v_f = -4Q \sin^2 \theta_W \). The result of estimations shows that the production cross section changes between 1.91 fb and 1.86 fb (which corresponds to events rates between 95 and 93 per year) for the isosinglet quark mass between 0.1 TeV and 1 TeV.

Additionally, we estimated production cross sections and events numbers per year for isosinglet \( \psi \) quarkonia. They are given in Table IV. Decay widths for main decay modes of \( \psi_{D_1} \) are given in Table V, where we use \(|V_{D_1 t}| \approx 0.01\).

V. CONCLUSION

We have shown that \( \mu^+ \mu^- \) colliders with \( \sqrt{s} = \mathcal{O}(TeV) \) is a good place to investigate both fourth family fermions, quarkonia and \( E_6 \) isosinglet quarkonia. In this study we have concentrated on \( \psi_4 (^3S_1) \) state, other quarkonium states will be considered on future study.
REFERENCES

[1] H. Harari, H. Haut and J. Weyers, Phys. Lett. B 78 (1978) 459.

[2] H. Fritzsch, Nucl. Phys. B 155 (1970) 189.

[3] H. Fritzsch, Phys. Lett. B 184 (1987) 391.

[4] H. Fritzsch and J. Plankl, Phys. Lett. B 237 (1990) 451.

[5] H. Fritzsch and D. Holtmannspötter, Phys. Lett. B 338 (1994) 290.

[6] A. Datta and S. Raychaudhuri, Phys. Rev. D 49 (1994) 4762.

[7] A. Çelikel, A.K. Çiftçi and S. Sultansoy, Phys. Lett. B 342 (1995) 257.

[8] S. Atağ et al., Phys. Rev. D 54 (1996) 5745.

[9] S. Sultansoy, arXiv:hep-ph/0004271 (2000).

[10] ATLAS Detector and Physics Performance Technical Design Report, CERN/LHCC/99-15 (1999), p. 663.

[11] E. Arik et al., ATLAS Internal Note ATL-PHYS-98-125 (1999).

[12] B.J. King and A. Caldwell, 6-Month Feasibility Study on HEMC, http://pubweb.bnl.gov/people/bking/mucoll.

[13] V. Barger et al., Phys. Rev D 35 (1987) 11.

[14] F. Gursey, P. Ramond and P. Sikivie, Phys. Lett. B 60-(1976) 177.

[15] F. Gursey and M. Serdaroglu, Lett. Nuovo Cim. 21 (1978) 28.

[16] J. L. Hewett and T. G. Rizzo, Phys. Reports 193 (1989) 193.
TABLES

TABLE I. The production cross section values for the fourth SM family fermions.

| $M_4$ (GeV) | $\mu^+\mu^- \rightarrow u_4\bar{u}_4$ | $\mu^+\mu^- \rightarrow d_4\bar{d}_4$ | $\mu^+\mu^- \rightarrow l_4^+\bar{l}_4^-$ | $\mu^+\mu^- \rightarrow \nu_4\bar{\nu}_4$ |
|-------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
|             | $\sigma$ (fb) | $\Gamma_{\text{tot}}$ (Ev./year) | $\Delta E_{\text{coll}}$ (MeV) | $\sigma$ (fb) | $\Gamma_{\text{tot}}$ (Ev./year) | $\Delta E_{\text{coll}}$ (MeV) | $\sigma$ (fb) | $\Gamma_{\text{tot}}$ (Ev./year) | $\Delta E_{\text{coll}}$ (MeV) |
| 300         | 9.8            | 5.0                                | 6.1                                | 9.8            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                |
| 375         | 9.7            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                |
| 450         | 9.7            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                |
| 525         | 9.6            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                |
| 675         | 9.5            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                |
| 750         | 9.4            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                | 1.3            | 5.0                                | 6.1                                |

TABLE II. The production cross section values and event numbers per year for the fourth SM family ($u_4\bar{u}_4$) quarkonia.

| $M_{\psi_4}$ (GeV) | $\sigma^{\text{res}}$ (pb) | $\Gamma_{\text{tot}}$ ($\psi_4$) (MeV) | $\Delta E_{\text{coll}}$ (GeV) | $\sigma^{\text{ave}}$ (fb) | $\Gamma_{\text{tot}}$ (Ev./year) |
|---------------------|----------------------------|----------------------------------------|-------------------------------|----------------------------|----------------------------------|
| 600                 | 68.2                       | 8.3                                    | 0.60                          | 0.94                       | 7100                             |
| 750                 | 19.5                       | 21.1                                   | 0.75                          | 0.55                       | 5200                             |
| 900                 | 6.8                        | 46.9                                   | 0.90                          | 0.35                       | 4000                             |
| 1050                | 2.8                        | 93.3                                   | 1.05                          | 0.25                       | 3300                             |
| 1200                | 1.3                        | 170.5                                  | 1.20                          | 0.18                       | 2800                             |
| 1350                | 0.6                        | 291.6                                  | 1.35                          | 0.13                       | 2200                             |
| 1500                | 0.3                        | 472.6                                  | 1.50                          | 0.09                       | 1800                             |
TABLE III. Decay widths for main decay modes of $\psi_4 (u \bar{u}_4)$, for $m_H = 150$ GeV.

| $M_{\psi_4}$, GeV | $\Gamma(\psi_4 \rightarrow \ell^+ \ell^-)$, $10^{-2}$ MeV | $\Gamma(\psi_4 \rightarrow u \bar{u})$, $10^{-2}$ MeV | $\Gamma(\psi_4 \rightarrow d \bar{d})$, $10^{-2}$ MeV | $\Gamma(\psi_4 \rightarrow Z\gamma)$, MeV | $\Gamma(\psi_4 \rightarrow ZZ)$, $10^{-2}$ MeV | $\Gamma(\psi_4 \rightarrow ZH)$, $10^{-2}$ MeV | $\Gamma(\psi_4 \rightarrow \gamma H)$, $10^{-2}$ MeV | $\Gamma(\psi_4 \rightarrow W^+W^-)$, MeV |
|------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|
| 600              | 1.4                           | 2.3                           | 1.0                           | 0.4                           | 4.0                           | 4.3                           | 36.1                           | 6.8                           |
| 750              | 1.6                           | 2.6                           | 1.2                           | 0.7                           | 7.6                           | 7.9                           | 66.3                           | 18.9                           |
| 900              | 1.8                           | 3.0                           | 1.3                           | 1.1                           | 12.6                          | 13.0                          | 108.5                          | 43.6                           |
| 1050             | 2.0                           | 3.3                           | 1.5                           | 1.7                           | 19.2                          | 19.6                          | 164.2                          | 88.7                           |
| 1200             | 2.1                           | 3.5                           | 1.6                           | 2.4                           | 27.6                          | 28.1                          | 234.8                          | 164.4                          |
| 1350             | 2.3                           | 3.8                           | 1.7                           | 3.3                           | 38.0                          | 38.5                          | 321.9                          | 283.6                          |
| 1500             | 2.5                           | 4.1                           | 1.8                           | 4.4                           | 50.1                          | 51.1                          | 426.8                          | 462.4                          |

TABLE IV. The production cross section values and event numbers per year for the $\psi(D_1 \bar{D}_1)$ quarkonia of $E_6$ isosinglet quarks.

| $M_{\psi_{D_1}}$ (GeV) | $\sigma^{res}$ (pb) | $\Gamma_{\text{tot}} (\psi_4)$ (MeV) | $\Delta E_{\text{coll}}$ (GeV) | $\sigma^{ave}$ (pb) | Ev./year |
|------------------------|---------------------|--------------------------------------|--------------------------------|---------------------|----------|
| 300                    | 26.6                | 13.8                                 | 0.3                            | 1.22                | 4590     |
| 600                    | 19.2                | 7.1                                  | 0.6                            | 0.23                | 1700     |
| 900                    | 1.76                | 44.6                                 | 0.9                            | 0.087               | 980      |
| 1200                   | 0.31                | 169.8                                | 1.2                            | 0.044               | 660      |
| 1500                   | 0.082               | 478.0                                | 1.5                            | 0.026               | 490      |
| 1800                   | 0.028               | 1111.7                               | 1.8                            | 0.017               | 390      |
| 2100                   | 0.011               | 2270.1                               | 2.1                            | 0.012               | 310      |
TABLE V. Decay widths for main decay modes of $\psi(D_1 D_1)$, for $m_H = 150$ GeV.

| $M_{\psi_{D_1}}$, GeV | 300  | 600  | 900  | 1200 | 1500 | 1800 | 2100 |
|-----------------------|------|------|------|------|------|------|------|
| $\Gamma(\psi_4 \rightarrow \ell^+\ell^-)$, keV | 2.3  | 3.4  | 4.3  | 5.2  | 6.0  | 6.8  | 7.5  |
| $\Gamma(\psi_4 \rightarrow u \bar{u})$, keV     | 2.6  | 3.8  | 4.9  | 5.9  | 6.8  | 7.7  | 8.5  |
| $\Gamma(\psi_4 \rightarrow d \bar{d})$, keV     | 0.9  | 1.2  | 1.5  | 1.8  | 2.0  | 2.3  | 2.5  |
| $\Gamma(\psi_4 \rightarrow ZH)$, keV             | 4    | 27   | 82   | 177  | 321  | 524  | 792  |
| $\Gamma(\psi_4 \rightarrow \gamma H)$, keV       | 12   | 90   | 271  | 587  | 1067 | 1739 | 2627 |
| $\Gamma(\psi_4 \rightarrow W^+W^-)$, MeV         | 0.3  | 7    | 44   | 166  | 465  | 1082 | 2217 |