Composite vortex model of the electrodynamics of high-$T_c$ superconductor

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Abstract

We propose a phenomenological model of vortex dynamics in which the vortex is taken as a composite object made of two components: the vortex current which is massless and driven by the Lorentz force, and the vortex core which is massive and driven by the Magnus force. By combining the characteristics of the Gittleman-Rosenblum model (Phys. Rev. Lett. 16, 734 (1966)) and Hsu’s theory of vortex dynamics (Physica C 213,305 (1993)), the model provides a good description of recent far infrared measurements of the magneto-conductivity tensor of superconducting YBa$_2$Cu$_3$O$_{7-\delta}$ films from 5 cm$^{-1}$ to 200 cm$^{-1}$.

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With the discovery of high-$T_c$ superconductors and the recognition of their potential for high field applications much attention has been devoted to the study of vortex dynamics. Nevertheless, the dissipation mechanism associated with vortex motion remains poorly understood. From surface impedance measurements in the MHz and GHz ranges the vortex pinning and viscosity parameters have been reported based on the Gittleman-Rosenblum (G-R) model of vortex dynamics [1–3]. The observation of magneto-optical activity was reported in far infrared (FIR) experiments, [4,5] which suggests a connection between vortex dynamics and cyclotron resonance [3]. This observation greatly enriches the phenomenology of electrodynamics of vortices. Recently, quantized vortex core levels in YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) have been probed directly using scanning tunneling microscopy [7]. A level spacing within the FIR frequency range was reported. Also evidence for vortex core excitations has been reported in FIR experiments [8], but the role of vortex core structure in vortex dynamics is not yet well understood. The use of terahertz spectroscopy [9,10] and polarized FIR laser sources [11] have recently filled in the gap of measurements between microwave and FIR frequencies. Therefore, a more satisfactory description of vortex dynamics over wide frequency range is needed, in order to describe these experimental observations.

Microwave surface impedance studies [1–3,9] have demonstrated a loss band near zero frequency whose width is characterized by the depinning frequency $\omega_d (= \kappa/\eta)$. There is remarkable consistency in the reported pinning force constant $\kappa$ for YBCO within a factor of two. However, the viscosity $\eta$ was found to vary much more widely. This leads to a wide range for $\omega_d$. From measurements of the conductivity up to 800GHz $\omega_d \sim 300GHz$ (10 cm$^{-1}$) has been reported for YBCO films [9]. At FIR frequencies, optical activity has been observed in magneto-transmission measurements using broadband and laser sources throughout the 5 cm$^{-1} \leq \omega \leq 200$ cm$^{-1}$ frequency range [4,5,11]. The conventional theories based on a Lorentz force acting on the vortices, e.g. the G-R model [12] and Coffey-Clem Model [13], which are widely used to describe microwave surface impedance data, can not properly describe these FIR measurements since they do not produce optical activity [3]. A model of vortex dynamics proposed by Hsu [14] based on the microscopic BCS theory...
in the clean limit gives a Magnus force acting on vortices and successfully describes the high frequency \((\omega \geq 25 \text{ cm}^{-1})\) optical activity in terms of hybridization of the cyclotron resonance and the vortex core resonance with the pinning resonance \([7,15]\). The hybridized pinning-cyclotron resonance has been fully resolved in recent FIR laser measurements \((\omega \geq 5 \text{ cm}^{-1})\) \([11]\). However, Hsu’s theory fails to describe the low frequency absorption band \([11]\) observed in the microwave studies \([1,3,9]\).

In this paper we present a new phenomenological model to describe the electrodynamics of type II superconductors. The two different inertial masses that have been discussed in vortex dynamics \([16]\) suggests a model consisting two coupled equations of motion which is intended to capture the qualities of G-R model at low frequencies and Hsu’s theory at higher frequencies. We take the vortex to be a two component composite object (see Fig.1). One component is the vortex current outside the vortex core, which has a G-R type equation of motion, and the other component is the vortex core with an equation of motion based on Hsu’s model \([14,12]\). The resulting two coupled equations of motion produces a conductivity tensor with a London term and three Lorentzian oscillators: one low frequency oscillator similar to the G-R loss band and two finite frequency chiral oscillators as in Hsu’s model. These three resonances are illustrated schematically in Fig.2. This composite vortex model provides a good description of the measured magneto-transmission and the magneto-conductivity tensor from \(5 \text{ cm}^{-1} \leq \omega \leq 200 \text{ cm}^{-1}\) \([11]\).

The motion of a vortex line in type-II superconductor under the influence of an alternating current is limited by frictional and pinning forces. The simplest description of linear vortex dynamics is \([12]\)

\[
\vec{J} \times \hat{\phi}_0 = \eta v_v + \kappa r_v
\]

where \(\vec{J} = nev_s\) is the driving transport current density \((v_s\) is the superfluid velocity), \(v_v\) \((r_v)\) is the average vortex velocity \((\text{displacement})\), \(\kappa\) is the pinning force constant, \(\eta\) is the viscosity and \(\hat{\phi}_0\) is the flux quantum, \(hc/2e\), which is expressed as a vector in the direction of the applied magnetic field \((\hat{z})\). (A more precise interpretation of \(v_v\) will be discussed later).
Eq. (1) does not contain an inertial term which corresponds to an assumption of massless dynamics of the vortex current pattern. This equation of motion produces a pole in the conductivity at zero frequency of width $\omega_d = \kappa/\eta$ (Peak 2 in Fig. 2) in addition to a London delta function reduced in strength from its zero field value. This model which is based on classical hydrodynamic analogies is valid only in those regions outside the core where the superfluid can be described by the order parameter in the time dependent Ginzberg-Landau (TDGL) theory [16,17]. Suhl [18] has considered the inertial mass of the vortex within the TDGL theory and concluded that it is order $(\Delta/\epsilon_F)^2$ smaller than the naive estimate $M_L \simeq mn\pi\xi^2$.

A Hall force term $v_v \times \vec{z}$ can be added to Eq. (1) leading to

$$\Lambda (v_s - \beta v_v) \times \vec{z} - \eta v_v - \kappa r_v = 0$$

where $\Lambda = nh/2$ the strength of Lorentz force and $\beta$ characterizes the relative strength of the Hall force to the Lorentz force. When $\beta = 1$ the first term corresponds to the Magnus force. A Hall term is required to describe the anomalous Hall effect observed in both conventional and high-$T_c$ superconductors in the mixed state where the sign of dc Hall effect changes if $\beta$ changes its sign [17,19]. Otterlo et al. [16] have argued that, in a fermionic system, the strength of the Hall force depends on the electronic structure through the derivative of the density of states at the Fermi level. Their theory leads to a small $\beta$ whose magnitude and sign depends on the details of the electronic band structure at the Fermi level.

From an analysis of vortex motion within the BCS theory Hsu [14] proposed an equation of motion which has a kinetic term, $\dot{v}_C$ corresponding to a non-negligible vortex core mass, a Magnus force $(v_s - v_C) \times \vec{z}$ and a ‘gauge term’ $\dot{v}_s$ which couples the vortex motion to the external electric field. Ao and Thouless [20] have shown that a Magnus force is a general property of vortex motion from a Berry phase argument. Hsu’s equation of motion, normalized by the vortex core mass, is written as:

$$\dot{v}_C = \dot{v}_s + \Omega (v_s - v_C) \times \vec{z} - \frac{1}{\tau} v_C - \alpha^2 r_C$$

(3)
where \( h\Omega \approx \Delta^2/\epsilon_F \) is the energy spacing of the quasi particle levels in the vortex core \([21]\), \( v_C (r_C) \) is the core velocity (displacement), \( \frac{1}{\tau} \) is the damping rate associated with the core motion and \( \alpha \) is the bare pinning frequency. Due to the finite vortex core mass implicit to Eq.(3) two finite frequency poles of finite width are produced in the conductivity tensor in addition to a London term. One pole is a hybridized pinning-cyclotron resonance (Peak (1) in Fig.2) and the other is a hybridized vortex core-pinning resonance (Peak (3) in Fig.2). However, the vortex core resonance, which is quenched in the absence of pinning (leaving only the cyclotron resonance, in accord with Kohn’s theorem \([3]\)), remains weak for physically reasonable pinning.

The moving vortices produce a Josephson field \([22]\) which couples to the superfluid by a modification of the London equation:

\[
\dot{v}_s = \frac{e\vec{E}}{m} + \omega_c v_L \times \hat{z} \tag{4}
\]

where \( \omega_c = eH/\rho c \) is the cyclotron resonance frequency and \( v_L \) is the average vortex velocity. Elementary arguments show that the Josephson field due to a moving vortex is proportional to the gradient of the local magnetic field distribution, \( E \sim \nabla B(r) \). Since the curvature of the magnetic field is almost zero in the vortex core \([23]\), most of the contribution of the Josephson field should come from the current flow pattern outside the core, \( r \approx \lambda_L \) (London penetration length), where there is maximum curvature in the magnetic field distribution of the vortex. Consequently, we expect \( v_L \) to be the velocity of the vortex current pattern relative to the lattice. Therefore,

\[
v_L = v_v + v_C \tag{5}
\]

where \( v_v \) is interpreted as the velocity of the vortex current pattern relative to the core and \( v_C \) is the velocity of the core relative to the pinning center, as illustrated in Fig.1. In this picture the pinning force \( \kappa r_v \) acting on the vortex current pattern can be thought of as arising from the distortion of the vortex through its tendency to restore itself to the equilibrium configuration. Therefore, in this model \( \kappa \) is an intrinsic property of the vortex.
As pointed out by Hsu [14], there is an additional contribution to the current due to the moving vortex core which acts like a charged particle

\[ \vec{J} = ne (A v_s + (1 - A) v_C) \]  

(6)

We determine \( A \) by requiring the response function to obey the energy conservation law. First we neglect the damping terms \( \frac{1}{\tau} v_C \) and \( \eta v_v \) and make the product \( \vec{E} \cdot \vec{J} \) energy conserving. The only nonzero terms are the kinetic energy terms \( \frac{d}{dt} \left( \frac{1}{2} v_s^2 \right), \frac{d}{dt} \left( \frac{1}{2} v_C^2 \right), \frac{d}{dt} \left( \frac{1}{2} v_v^2 \right) \), and pinning potential terms \( \frac{d}{dt} \left( \frac{1}{2} r_s^2 \right), \frac{d}{dt} \left( \frac{1}{2} r_v^2 \right) \). Setting all other cross product terms to be zero, we obtain \( A = \Omega / (\Omega + \omega_c) \) which is similar to that obtained by Hsu [14]. We also find that we have to add a term, \(-\omega_c v_v \times \hat{z}\), to the right hand side of Eq.(3). Combined with the ‘gauge term’ \( \dot{v}_s, \dot{v}_v - \omega_c v_v \times \hat{z} = \frac{e \vec{E}}{m} + \omega_c v_C \times \hat{z} \). The interpretation is that the total electric field acting on the vortex core is equal to the external electric field plus the Josephson field from the current flow pattern, but the Josephson field from the core itself is excluded. However, this is only a small correction since \( \omega_c \ll \Omega \).

From these equations, we can calculate the conductivity function. First from Eq.(2) and Eq.(3) (in left circular polarization) we define

\[ g_v(\omega, H) = \frac{v_v}{v_s} = \frac{i \Lambda}{i \beta \Lambda + \eta + i \kappa / \omega} \]  

(7)

and

\[ g_C(\omega, H) = \frac{v_C}{v_s} = \frac{-i \omega + i \Omega - \omega_c g_v(\omega, H)}{-i \omega + i \Omega + \frac{1}{\tau} + i \alpha^2 / \omega} \]  

(8)

The conductivity function is then

\[ \sigma^+(\omega, H) = \frac{\vec{J}}{\vec{E}} = \frac{ne^2}{m} \frac{[\Omega + \omega_c g_C(\omega, H)] / (\Omega + \omega_c)}{-i \omega - i \omega_c [g_C(\omega, H) + g_v(\omega, H)]} \]  

(9)

from Eq.(4) and Eq.(6). \( \sigma^- \) is obtained from \( \sigma^+ \) through the time reversal symmetry relation \( \sigma^- (\omega, H) = \sigma^+ (-\omega, H)^* \). This relation suggests a canonical representation of \( \sigma \) in which \( \sigma^+ \) is plotted for positive frequencies and \( \sigma^- \) is plotted for negative frequencies.

The conductivity function Eq.(8) is causal (\( Re[\sigma] \geq 0 \)), satisfies the oscillator strength sum rule, reduces to cyclotron resonance in the clean limit (in accord with Kohn’s theorem
and further reduces to the London conductivity if the field is set to zero. With regard to the first two properties, we note that the conductivity tensor can be resolved into a finite sum of Lorentzian oscillators $\sigma^+(\omega, H) = ne^2/m \sum_{q=0}^3 f_q \left/ (-i(\omega - \omega_q) + \Gamma_q) \right.$ but with complex oscillator strengths, $f_q$, (except $f_0$ the superfluid oscillator strength is real, see Eq.(10) below.) and $\sum_{q=0}^3 f_q = 1$ as in G-R model and Hsu’s theory. However, these complex oscillator strengths are unusual and may be an indication that the model needs further refinement. To obtain cyclotron resonance, we use the Bardeen-Stephen theory in which $\eta = B_{c2} \phi_0 \sigma_n$ and $\sigma_n = \tau_{qp} c^2 / 4\pi \lambda_L^2$ where $\sigma_n$ is the normal state conductivity, and $\tau_{qp}$ is the quasiparticle scattering rate. In the clean limit where $\tau_{qp} \rightarrow \infty$ and $\alpha^2 = 1/\tau = 0$, the conductivity reduces to $-ne^2/m \cdot (\omega - \omega_c)$.

It is also interesting to consider the normal state limit of the model. In the flux flow regime where $\alpha^2 = 0$ but $1/\tau \neq 0$, and near $T_c$ the superfluid carrier is depleted so that $\Lambda$, $\Omega \rightarrow 0$ (or $\ll \eta, 1/\tau$), we obtain $\sigma^+(\omega, H) \equiv ne^2/m \left/ (-i(\omega - \omega_c) + 1/\tau) \right.$ which is the form of the metallic cyclotron resonance. However, a contribution from the quasiparticles should also be included, which would have the same form but proportional to $n_q \equiv (n_{total} - n)$. Therefore, the model can have a smooth transition into the normal state.

Recently, the FIR magneto-transmission coefficient, $T^{\pm}(\omega, H)$ for YBCO thin films has been measured over a wide range of frequencies by the combination of broadband Fourier Transform Spectroscopy (FTS) and CO$_2$ pumped FIR laser measurements. The magneto-conductivity tensor was obtained from the magneto-transmission spectra by a Kramers-Kronig analysis. Fig.3 shows the transmission amplitude ratio $|T^{\pm}(\omega, H)/T(\omega, 0)|^{1/2}$ at $H=9T$ and 4 K (Panel (a)) and the magneto-conductivity tensor $Re[\sigma^+(\omega, H)]$ (Panel (b)) in the canonical coordinate representation. A fit to the composite model (the dotted line) is also shown in Fig.3 and is seen to be in very good agreement with the data except for some small structures above 50 cm$^{-1}$. These features are induced by the presence of 45° misaligned grains in these samples as discussed in Ref [8]. The parameters of the best fit are $\alpha = 45$ cm$^{-1}$, $1/\tau = 58$ cm$^{-1}$, $\Omega = 49$ cm$^{-1}$, $\omega_c = 4.6$ cm$^{-1}$ [25], for the vortex core, and $\beta = -0.086$, $\kappa = 6 \times 10^5$ N/m$^2$, $\eta = 5.6 \times 10^{-7}$ kg/m sec, for the vortex current,
assuming that $\Lambda = nh/2$ and $ne^2/m = c^2/4\pi\lambda_0^2$ where $\lambda_0 = 1850\text{Å}$ \[11\]. By associating the Magnus force constant $M_V\Omega$ ($M_V$ is the vortex mass) with $nh/2$ \[20\], we can also estimate $M_V \approx 6 \times 10^8$ electron mass per cm. The corresponding estimate for the coherence length $\xi_V \approx 26\text{Å}$ by taking $M_V = mn\pi\xi_V^2$ \[20\]. The fitting also gives $\omega_d = \kappa/\eta \approx 10\text{ cm}^{-1}$, very close to the value reported by Parks et al. \[3\].

The resulting conductivity function is seen to be dominated by two major oscillators in addition to the London term ($\omega_0 = 0, \Gamma_0 = 0$) \[11\]: a low frequency oscillator at $\sim 3\text{ cm}^{-1}$ and width of $10\text{ cm}^{-1}$, associated with the vortex currents; and a high frequency chiral oscillator at $-24\text{ cm}^{-1}$ and width of $17\text{ cm}^{-1}$ in the hCP mode which is associated with the vortex core (the hybridized pinning-cyclotron resonance). Another oscillator has been identified at about $65\text{ cm}^{-1}$ in the eCP mode (the hybridized vortex core-pinning resonance), but it is very weak because of motional quenching \[8\].

The best fit also gives a negative $\beta$ which may be related to the sign reversal of the dc Hall effect \[16,17,19\]. The presence of the Hall force ($\beta \neq 0$) in Eq. (2) shifts the low frequency oscillator to finite frequency $\omega_1 \approx -\beta\Lambda\Gamma_1/\eta$ (neglecting the contributions related to the vortex core, which is a fairly good approximation in this case.), where $\Gamma_1 \approx (\kappa + \Lambda\omega_c)/\eta$ is the width of this oscillator. $\beta < 0$ results from $\omega_1 > 0$, which means that the low frequency oscillator is slightly electron-like (eCP). In the experimental paper \[11\] we also pointed out that the superconvergent sum rule, $\int_0^\infty \ln |T^+(\omega, H)/T^-(\omega, H)| \frac{d\omega}{\omega} = 0$, which is a consequence of the supercurrent, is not satisfied unless the low frequency oscillator is shifted toward positive frequencies (eCP) to avoid the excessive low frequency weight in the optical activity.

Finally we discuss $f_0$ the oscillator strength of the superfluid condensate within the composite vortex model. In the low frequency limit $\lim_{\omega \to 0} \sigma^+(\omega, H) = \frac{ne^2}{m} f_0(H)/(i\omega)$, where

$$f_0(H) = \frac{\Omega/(\Omega + \omega_c)}{1 + \omega_c(\Omega/\alpha^2 + \Lambda/\kappa)}.$$ (10)

Note that $f_0(H) = \lambda_2^2/\lambda^2(H)$ which can be measured in microwave experiments and that
$f_0$ is independent of the damping parameters $\eta$ and $1/\tau$. In the G-R model, $f_0(H) = (1 + n\hbar\omega_c/2\kappa_{GR})^{-1}$. Comparing Eq. (10) with the G-R result we can define an effective “$\kappa_{GR}$” in the low field limit where $f_0(H) \cong 1 - \omega_c(\Omega/\alpha^2 + \Lambda/\kappa + 1/\Omega)$. Taking $M_V\Omega = \Lambda = n\hbar/2$ we find $1/\kappa_{GR} = 1/\kappa + 1/\kappa_C + 1/\Lambda\Omega$ where $\kappa$ is the force constant associated with the force between the vortex currents and the core and $\kappa_C = M_V\alpha^2$ is the force constant associated with the force between the core and the pinning center. In our model $\kappa$ is an intrinsic property of the vortex. A review of the published experiment data [1–3,9] indicates that the observed $\kappa$ is universal within a factor of two, which is difficult to understand in terms of pinning to random defects in the lattice. The last term $1/\Lambda\Omega$ is also intrinsic and comes from the reduction in the condensate density due to the vortex core volume ($\kappa/\Lambda\Omega \equiv 4\pi\kappa m\xi_V^2/n\hbar^2 \approx 0.12$). In general, we see that the small “force constant” dominates $f_0$. Therefore for strong pinning, such as occurs in YBCO films, the intrinsic pinning may determine “$\kappa_{GR}$” $\cong \kappa$ (in our case, $\kappa/\kappa_C \approx 0.13$), which may explain the observed universal value reported in the literature. As $\kappa_C$ gets small, for example in high quality single crystals, the extrinsic pinning $\kappa_C$ may dominate and “$\kappa_{GR}$” would become smaller and sample dependent.

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This $\omega_c$ corresponds to the effective mass $m^*$ of $1.8 \ m_e$. In high frequency measurements, we have deduced a mass of $3.1 \ m_e$. This discrepancy, which is outside of experimental error, is related to $\beta \neq 0$ in the composite vortex model, which also gives some high frequency optical activity. The interpretation of this is not yet clear.

$\pi \xi_\ell^2$ assumes the cylinder approximation for the vortex core.
FIGURES

FIG. 1. The vortex core (grey circle in the center) is pinned to the lattice defect (cross). The circulating vortex current pattern (concentric annular rings) is bound to the vortex core by the intrinsic restoring force (zigzag springs).

FIG. 2. Three poles in the conductivity of the composite vortex model. Peak (1) and (3) come from the magneto-response of the vortex core. Peak (2) comes from the massless vortex current flow around the vortex core. The peak positions and oscillator strengths are the same as in Fig. 3(b) but made much sharper for illustration purposes. hCP (eCP) stands for hole (electron) cyclotron resonance polarization mode.

FIG. 3. (a) The transmission amplitude ratio $|T^\pm(\omega, H)/T(\omega, 0)|^{1/2}$ as a function of frequency $\omega$ for YBCO thin films at $H=9$ T and 4 K. (b) The magneto-conductivity $\text{Re}[\sigma^+(\omega, H)]$ as a function of frequency $\omega$. The solid line represents the broadband data from 30 cm$^{-1}$ to 200 cm$^{-1}$ and the triangular points represent data from the laser source from 24.5 cm$^{-1}$ down to 5.26 cm$^{-1}$. The dotted line is the fit to the composite vortex model. The dashed line in (b) is the residual metallic background $\text{Re}[\sigma(\omega, 0)]$. 
Fig. H.-T. S. Lihn et al.
