Harnessing ferroelectric domains for negative capacitance

I. Luk’yanchuk,1,2 Y. Tikhonov,3 A. Sené,1 A. Razumnyay,1,3 and V. M. Vinokur4
1University of Picardie, Laboratory of Condensed Matter Physics, Amiens, 80039, France
2Landau Institute for Theoretical Physics, Moscow, Russia
3Faculty of Physics, Southern Federal University, 3 Zorge str., 344090 Rostov-on-Don, Russia
4Materials Science Division, Argonne National Laboratory, 9700 S. Cass Ave, Argonne, IL 60439, USA

The pressing quest for overcoming Boltzmann tyranny in low-power nanoscale electronics revived the thoughts of engineers of early 1930-s on the possibility of negative circuit constants. The concept of the ferroelectric-based negative capacitance (NC) devices triggered explosive activity in the field. However, most of the research addressed transient NC, leaving the basic question of the existence of the steady-state NC unresolved. Here we demonstrate that the ferroelectric nanodot capacitor hosts a stable two-domain state realizing the static reversible NC device thus opening routes for the extensive use of the NC in domain wall-based nanoelectronics.

Over 40 years ago Rolf Landauer raised the question whether the capacitance can be negative [1], that is if the increase in the charge of the capacitor can decrease its voltage. Having stated that ferroelectrics (FE) can harbor the negative capacitance (NC) during the transient processes of switching, Landauer indicated that the very existence of the static NC as a part of steady state is challenged by instability against spontaneous domain formation. The idea of ferroelectric-based negative capacitance (NC) devices[2, 3] brought back the suggestion of engineers of early 1930-s on the exploration of circuits with negative elements[4–6] and ignited an explosive research activity, see [7–9] for a review. The NC would naturally arise in the Landau description of the uniformly polarized FE, where the free energy, \( F \) as a function of polarization, \( P \), or capacitor charge, \( Q \), assumes a double well form, see Fig. 1a. A standard approach based on the minimization of \( F \) at the given voltage \( V \) yields the dependence \( Q(V) \), where the induced charge represents the system’s response to the applied voltage. The resulting charge-voltage characteristic has the form of an S-shaped curve, GABCDEF, shown in Fig. 1b. Being driven by the applied voltage, the capacitor switches between the two stable branches GAB and DEF, corresponding to the up- and down-oriented FE polarizations, in a hysteretic fashion. The differential capacitance, \( C = dQ/dV \), is positive when the system falls into either of these two polarized states. Remarkably, \( C \) reverses its sign when the system traverses along the intermediate branch BCD, having the negative slope. However, since the BCD path of the \( Q(V) \) dependence corresponds to the concave segment in the energy profile (Fig. 1a), this intermediate branch of the S-curve is unstable, hence the static NC regime is unreachable for a voltage-driven capacitor. In an experiment, the switching instability may occur also even before reaching the turning points B and D as a result of the reverse domain nucleation and accelerated growth. The corresponding time-dependent evolution of the charge and voltage drop would then follow the dynamical \( Q(V) \) trajectory, depicted by B’F and D’G dashed segments in Fig. 1b, with the negative slope at the beginning of the switching[9, 10]. It is this negative slope that is commonly viewed as a manifestation of NC, which, however, in this case, has a transient and irreversible character.

What can stabilize the static NC, is setting the charge, \( Q \), as a driving parameter controlling the state of the capacitor, so it is \( V \) that becomes the system’s response. This would reverse the S-shaped \( Q(V) \) dependence of the monodomain FE state into an N-shaped \( V(Q) \) function that maintains the negative sign of \( dQ/dV \) at low \( Q \) but has no bistability anymore. Yet, however attractive, this approach may not to straightforwardly work since the charge-driven system would likely split into the multidomain state[1]. For instance, the paraelectric uniform state \( C \) at \( Q = 0 \) will break up into the lower-energy FE states A and E comprising the domains with alternating oppositely oriented polarization. This effect is often thought to be detrimental to steady state NC [11, 12].

However the situation appears not hopeless once the delicate balance between the domain wall (DW) and electric field energies is taken into account. To see that, let us consider the mechanism of domain formation in a FE sample more in details. The spontaneous polarization, \( P_0 \), inside the monodomain FE flat cell with uniaxial anisotropy generates the depolarization surface charges, \( +P_0 \), at the top and \( -P_0 \) at the bottom of the cell, which inducethe depolarization field \( E_{dep} = -P_0/\varepsilon_0\varepsilon_{ff} \), directed opposite to the polarization (Fig. 1c). Here \( \varepsilon_0 \) is the permittivity of the FE material along \( P \), and \( \varepsilon_{ff} \) is the absolute permittivity of the vacuum. An additional energy associated with this field disfavors the uniform polarization and splits the system into the alternating sequence of the up/down polarized domains (Fig. 1d). Such a domain texture was found in ferroelectric-dielectric (FE-DE) heterostructures [13–17]. It was demonstrated,[18–20] that the FE layers harboring domains possess a negative permittivity, i.e. the average intrinsic field there is directed opposite to the electrostatic displacement. However this negative permittivity effect as is remains hard to utilize for a NC device, since the metallic electrodes when placed on the surfaces of the FE layer would effectively screened the depolarization fields induced by domains and the domains themselves would disappear right off (Fig. 1e).

Here we step into the breach and demonstrate that the problem of the implementation of the steady reversible NC is achieved by about 10 nm nanodot FE capacitor, endowed with the two-domain ground state. Splitting the monodomain state...
FIG. 1. Ferroelectric capacitor with negative capacitance. (A) The plot of the Landau free energy for the FE capacitor, $F$, as function of its charge, $Q$. (B) The corresponding charge-voltage characteristic, $Q$-$V$. The blue lines stand for the monodomain state, while the orange lines stand two-domain state with static NC. The dashed green curves depict the polarization switching process manifesting the transient NC. (C) The FE monodomain sample. Termination of polarization, $P$, at the surface induces surface depolarization charges and the depolarization field $E$. (D) The periodic domain structure. (E) The FE sample with short-circuited electrodes, which recovers the monodomain structure. (F) Formation of two equal domains in a FE confined between disconnected electrodes redistributes the free charges and cancels the depolarization field. (G) Top view of the two-domain structure with disconnected electrodes. (H) Adding the charge, $Q$, pushes the DW towards the edge. (I) Top view of the FE capacitor with the displaced DW. (K) Negative differential permittivity $\varepsilon_f$ of the cylindrical FE PbTiO$_3$ nanodot capacitors of different radii as functions of the charge density, $Q/S$. The jumps in $\varepsilon_f$ from the negative to positive value mark the full polarization of the nanodot.

into the two-domain ground state and the subsequent motion of the DW, results in the modified free energy curve and corresponding modification of the $Q(V)$ dependence shown by orange curves in Fig. 1a,b. These characteristics retains the features of the $S$-curve of the monodomain system, hence the steady state NC. We consider, for concreteness, the cylindrical FE nanodot of radius $R$ and thickness $d_f$ sandwiched between the disconnected metallic electrodes. Let the charge at the electrodes of the FE capacitor be zero, $Q = 0$, Fig. 1f. Then the energy of the system is indeed minimized by splitting the system into the two equal-size domains with equal surfaces $S_1 = S_2 = S/2$, with $S = \pi R^2$, and down/up directed polarizations, $-P_0$, and $+P_0$ (Fig. 1g). These oppositely oriented polarizations generate surface depolarization charges of the corresponding densities $\mp P_0$ at the top surface of the FE cell and the charges of the opposite signs at the bottom surface. This, in turn, induces redistribution of the free charges in the electrodes to compensate electric fields of the surface depolarization charges. Since the domains are of the equal size, the electroneutrality of the electrodes, $Q = 0$, is preserved and the average polarization and the corresponding depolarization charges are zero. As the price to pay for eliminating the depolarization energy is the DW energy, the least energy cost is provided by splitting the bulk of the FE capacitor into the two equal size domains with the minimal area of the DW. Since the depolarization energy is proportional to the volume of the system, whereas the DW energy cost is proportional to its area, the splitting into domains provides the gain in energy and drives the system into the stable state.

Adding the charge on electrodes, shifts the DW, which finds the corresponding new equilibrium position (Fig. 1h). To immediately see that the FE capacitor harboring the two-domain state has a negative capacitance we note, that if one short-circuited the plates, allowing for the free electrons to flow between the plates and charge the capacitor to allow to compensate the uniform depolarization field, the DW exits from the system which thus transits into the monodomain state with the lower energy Fig. 1e. This self-charging effect manifests the emergence of the NC. For the evaluation of the NC, let us place the small test charges, $+Q$ on the top electrode and $-Q$ on the bottom electrode of the FE capacitor, Fig. 1h, (the electrodes are disconnected) and follow its response. Charging the capacitor causes the displacement of the DW in such a way that the depolarization field arising due to the misbalance of the domain surfaces, $S_1$ and $S_2$, (Fig. 1i) would screen the electrical field induced by the test charges. However, the DW overshoots the position of the exact field compensation and reverses the internal electrical field $E$ since the extra displacement of the DW decreases further its surface, hence the total surface energy, due to geometrical finiteness of the system. This is the physical origin of the NC. To carry out a quantitative analysis, we calculate the internal field, $E$, and find the integral capacitance as $\tilde{C}_f \equiv Q/V$ where $V = -d_f E$ is the potential difference between the top and bottom electrodes; we use the tilde to distinguish $\tilde{C}_f$ from the differential capacitance $C_f \equiv dQ/dV$.

We obtain the integral FE capacitance in a standard form with the negative, integral permittivity $\tilde{\varepsilon}_f$ (see Appendix):

$$\tilde{C}_f = \varepsilon_0 \tilde{\varepsilon}_f \frac{S}{d_f}, \quad \tilde{\varepsilon}_f = -\frac{\pi}{4\varepsilon_0} \psi(Q) R \xi_0.$$

The integral permittivity, $\tilde{\varepsilon}_f$, is a fundamental characteristic of the FE nanodot and appears also in the expression for the polarization curve, $P(E) = \varepsilon_0(\tilde{\varepsilon}_f - 1)E$. The dimensionless
The DW width is taken of order of the FE coherence length \( \xi \), which is computed by the phase-field calculations, described in Methods. The behavior of the differential permittivity, as a function of the charge density, is shown in Fig. 1k for cylindrical nanodots of PbTiO\(_3\) of radius \( R = 2, 5 \) and 10 nm, respectively. The interval of the NC is extended over all range of the domain existence. The jumps in the \( \varepsilon_f \) from the negative to the positive values correspond to the complete polarization of the FE capacitor to the monodomain state.

We carry out the phase field modelling of the FE capacitor, consisting of the cylindrical FE nanodot of PbTiO\(_3\) sandwiched between two thin metallic electrodes and uniaxially-strained by the SrTiO\(_3\) substrate. The approach rests on the relaxation minimization of the strain-renormalized Landau-Devonshire functional, see Methods. The response of the 5 nm thick cylindrical FE capacitor of radius \( R = 5 \) nm with the two-domain structure to the variations in charge \( Q \) placed on the electrodes, is shown in Fig. 2a. In compliance with the model considerations described above, the DW departs from its bisection position at \( Q = 0 \) (state (1)), traversing the sample with simultaneous bending (state (2)). Finally the DW exits the sample (state (3)), leaving behind the uniformly-polarized monodomain state. The corresponding \( P-E \) characteristics of the FE capacitors of different sizes and shapes are shown in the Fig. 2b. Importantly, the negative slope of the \( P(E) \) dependencies, that manifest the NC, maintains well beyond the linear regime at \( E \sim 0 \) and holds during the entire process of the motion of the DW. Therefore, the NC extends over the entire electric field interval from zero to the coercive threshold where the DW leaves the sample and the \( P-E \) characteristics turn around into the standard behavior of the monodomain sample. At variance, the multidomain-state nucleation switching process[9, 10], resulting in the transient differential NC as well as in the charge-controlled single nano-domain formation[21], resulting in the static differential NC, occur in the intermediate vicinity of the coercive field and far from the zero-field steady-state, see Fig. 1b. Inset in Fig. 2b demon-

The DW width is taken of order of the FE coherence length \( \xi \), which is computed by the phase-field calculations, described in Methods. The behavior of the differential permittivity, as a function of the charge density, is shown in Fig. 1k for cylindrical nanodots of PbTiO\(_3\) of radius \( R = 2, 5 \) and 10 nm, respectively. The interval of the NC is extended over all range of the domain existence. The jumps in the \( \varepsilon_f \) from the negative to the positive values correspond to the complete polarization of the FE capacitor to the monodomain state.

We carry out the phase field modelling of the FE capacitor, consisting of the cylindrical FE nanodot of PbTiO\(_3\) sandwiched between two thin metallic electrodes and uniaxially-strained by the SrTiO\(_3\) substrate. The approach rests on the relaxation minimization of the strain-renormalized Landau-Devonshire functional, see Methods. The response of the 5 nm thick cylindrical FE capacitor of radius \( R = 5 \) nm with the two-domain structure to the variations in charge \( Q \) placed on the electrodes, is shown in Fig. 2a. In compliance with the model considerations described above, the DW departs from its bisection position at \( Q = 0 \) (state (1)), traversing the sample with simultaneous bending (state (2)). Finally the DW exits the sample (state (3)), leaving behind the uniformly-polarized monodomain state. The corresponding \( P-E \) characteristics of the FE capacitors of different sizes and shapes are shown in the Fig. 2b. Importantly, the negative slope of the \( P(E) \) dependencies, that manifest the NC, maintains well beyond the linear regime at \( E \sim 0 \) and holds during the entire process of the motion of the DW. Therefore, the NC extends over the entire electric field interval from zero to the coercive threshold where the DW leaves the sample and the \( P-E \) characteristics turn around into the standard behavior of the monodomain sample. At variance, the multidomain-state nucleation switching process[9, 10], resulting in the transient differential NC as well as in the charge-controlled single nano-domain formation[21], resulting in the static differential NC, occur in the intermediate vicinity of the coercive field and far from the zero-field steady-state, see Fig. 1b. Inset in Fig. 2b demon-

The DW width is taken of order of the FE coherence length \( \xi \), which is computed by the phase-field calculations, described in Methods. The behavior of the differential permittivity, as a function of the charge density, is shown in Fig. 1k for cylindrical nanodots of PbTiO\(_3\) of radius \( R = 2, 5 \) and 10 nm, respectively. The interval of the NC is extended over all range of the domain existence. The jumps in the \( \varepsilon_f \) from the negative to the positive values correspond to the complete polarization of the FE capacitor to the monodomain state.

We carry out the phase field modelling of the FE capacitor, consisting of the cylindrical FE nanodot of PbTiO\(_3\) sandwiched between two thin metallic electrodes and uniaxially-strained by the SrTiO\(_3\) substrate. The approach rests on the relaxation minimization of the strain-renormalized Landau-Devonshire functional, see Methods. The response of the 5 nm thick cylindrical FE capacitor of radius \( R = 5 \) nm with the two-domain structure to the variations in charge \( Q \) placed on the electrodes, is shown in Fig. 2a. In compliance with the model considerations described above, the DW departs from its bisection position at \( Q = 0 \) (state (1)), traversing the sample with simultaneous bending (state (2)). Finally the DW exits the sample (state (3)), leaving behind the uniformly-polarized monodomain state. The corresponding \( P-E \) characteristics of the FE capacitors of different sizes and shapes are shown in the Fig. 2b. Importantly, the negative slope of the \( P(E) \) dependencies, that manifest the NC, maintains well beyond the linear regime at \( E \sim 0 \) and holds during the entire process of the motion of the DW. Therefore, the NC extends over the entire electric field interval from zero to the coercive threshold where the DW leaves the sample and the \( P-E \) characteristics turn around into the standard behavior of the monodomain sample. At variance, the multidomain-state nucleation switching process[9, 10], resulting in the transient differential NC as well as in the charge-controlled single nano-domain formation[21], resulting in the static differential NC, occur in the intermediate vicinity of the coercive field and far from the zero-field steady-state, see Fig. 1b. Inset in Fig. 2b demon-
strates the jerky character of the dynamics of the DW due to pinning effects.

For cylindrical nanodots, the negative slope of $P(E)$ decreases with the decreasing nanodot’s radius. The theoretical dependencies $P(E)$, calculated from Eq. (1), are shown by solid lines. They perfectly describe the results of simulations except for the smallest 2 nm sample where the DW width is of order of the sample size. Similarly, it is the finite size of the DW that results in the slight deviation of the experimental behaviors from the theoretical predictions at the moment of the exit when the width of the DW compares to the size of the already nearly disappearing residual domain.

For the rectangular nanodot, the nearly infinite slope of $P(E)$ reflects that the energy of the parallel-displaced straight DW almost does not depend on its position. The jump to the more shallow negative-slope dependence $P(E)$ in the pre-critical region corresponds to the abrupt re-orientation of the DW from the parallel to the edge- to the corner-enclosing configuration (see animation).

Having found the $Q$-$V$ characteristics of the FE capacitor we are now equipped to include it into an amplifying NC circuit. A sketch of the proposed device is shown in Fig. 3a. The capacitor consists of DE and FE cylindrical nanodots, consequently grown on the insulating substrate and partitioned by the platelet metallic electrodes. The equivalent circuit (Fig. 3b), which is a customary proposed model implementation of the NC low-dissipation field effect transistor [2, 11], comprises the two in-series capacitors, FE and DE with capacitances $C_f$ and $C_d$, respectively. The operating voltage, $V_{in}$, applied to the top gate electrode, controls the state of the system, and in particular, the output voltage drop on the DE layer, $V_{out}$, is collected from the middle electrode. The bottom electrode is grounded. Once $C_f$ is negative and $C_d$ is positive, then applying the input voltage, $V_{in}$, results in an amplification of $V_{out}$ given by $V_{out} = \tilde{m}^{-1}V_{in}$ where $\tilde{m} = 1 + C_d/C_f < 1$ is the so-called body factor [2] and $C_f(Q)$ is given by Eq. (1) with $Q = C_dV_{out}$.

Figure 3c shows, how the output voltage $V_{out}$ depends on the driving voltage $V_{in}$ for PbTiO$_3$/SrTiO$_3$ cylindrical nanodots of radii $R$ = 2, 5 and 10 nm, in which the DE layer is about half thin compared to the FE one, and $C_d = \varepsilon_0\varepsilon_dS/d_d$ with $d_d \approx 300$ for SrTiO$_3$. The working range of the device is confined between the negative and positive reverse points in the middle branch of $V_{out}(V_{in})$ curves where the two-domain state is locally stable. The reversal segments correspond to the unstable situation when the DW comes out from the sample. The upper and lower branches of the curves reflect the monodomain states with the positive differential capacity. As the working part of the $V_{out}(V_{in})$ dependence corresponding to the two-domain state lies above the $V_{out} = V_{in}$, the device exhibits the amplification effect, which, in the linear approximation, is given by the body factor

$$\tilde{m} = 1 - \frac{3\pi\nu \varepsilon_d}{8} \frac{d_f \xi_0}{d_d} \frac{1}{R} .$$  \hspace{1cm} (2)

For nanodots with $R = 5$ nm the estimate gives $\tilde{m} \approx 0.69$, which can be improved by further material and geometry optimization, for instance, via selecting the proper ferroelectric material with the low permittivity and high DW energy (expressed through the factor $\nu$), while the DE layer is to be done as thin as possible and should have the highest permittivity available. At the same time, the nanodot radius should be chosen as small as possible, bearing in mind the intertwined relationships between the parameters (that e.g. $2R > d_f, \xi_0$ should hold). Further improvement can be achieved by going into the nonlinear regimes near instability reversal points in the $V_{out}(V_{in})$ dependencies, i.e. to the parts corresponding to the flattering of $P(E)$ curves in Fig. 2b. There, the respective values of the differential body factor, $m = (dV_{out}/dV_{in})^{-1}$, become appreciably smaller, as shown in the inset to Fig. 3c.

![Image](image_url)
APPENDIX

Phase-field calculations

The description of the uniaxially-strained perovskite FE platelet, rests on the minimization of the functional,

$$ F = \int \left[ f_{GL}(T, u_m, P_i) + f_{grad}(\partial_i P_j) - \frac{1}{2} P_i E \right] d^3r, \quad (S1) $$

where $i, j = x, y, z$, the uniform electric field is produced by the electrode- and depolarization surface charges, $E = -\left(Q/S + \overline{P}_t\right)/\varepsilon_0\varepsilon_r$, and the bar denotes averaging of the polarization at the platelet surface. The uniform Ginzburg-Landau part of the functional, $f_{GL}$, is taken from the strain-renormalized form with the appropriate for the PbTiO$_3$ material coefficients and the compressive strain $u_m = -0.013$ is produced by the SrTiO$_3$ substrate. The gradient part of the functional, $f_{grad}$, is taken from with the PbTiO$_3$-specific coefficients.

Numerical phase-field calculations were performed using the FERRET package for the open-source multiphysics object-oriented finite-element framework MOOSE by solving time-dependent relaxation equation $-\gamma \partial P_i/\partial t = \delta F/\delta P_i$ with free boundary conditions for $P_i$ at the sample surface, coupled with electrostatic equations. Random small distribution of $P_i$ was chosen as initial condition.

Domain wall geometry

In this section, we find the dependence of the DW length $L$ on the misbalance factor $s = (S_1 - S_2)/S$, where $S_1$ and $S_2$ are the surfaces of domains with down- and up-oriented polarization, respectively, and $S = S_1 + S_2 = \pi R^2$ is the total cross-section area of the FE capacitor, $R$ is the nanodot radius. The sketch of the cross-section plane $xy$ (Fig. S4A) presents the geometry of the system. The DW has a cylindrical-arc shape minimizing the DW surface tension energy at fixed $s$. The arc makes the right contact angle with the nanodot surface to eliminate the tangential drag caused by the surface tension. The bended DW can be seen as the segment ABG obtained by the crossing of the FE nanodot circular boundary, $C_f$, with an external circle $C_d$ of radius $r$, both circles being mutually orthogonal. Selecting the origin at the center of $C_d$ and axis $x$ and $y$ along and perpendicular to the line, connecting the centers $O_f$ and $O_d$ of the circles $C_f$ and $C_d$ respectively, we write the geometrical equations for these circles as

$$ y_j^2 + x_j^2 = R^2, \quad (S2) $$

$$ y_d^2 + (x_d - l)^2 = r^2. $$

Here $l = |O_fO_d| = \sqrt{R^2 + r^2}$ is the distance between the centers of the circles. A simple geometrical consideration shows that the length of the DW can be expressed as $L = 2Rl(\rho)$ using the dimensionless function:

$$ l(\rho) = \rho^{-1} \arctan \rho, \quad (S3) $$

of the dimensionless parameter $\rho = R/r$. The surface $S_2$, is calculated as area confined by the segments ABG and ADG as $S_2 = 2 \int_{x_0}^{x_f} y_f(x_f) dx_f + 2 \int_{x_0}^{x_d} y_d(x_d) dx_d$, where the dependencies $y_f(x_f)$ and $y_d(x_d)$ are given by Eqs. (S2) and $x_0 = l - r$, $x_f = R^2/l$ and $x_D = R$. Taking $S_1 = S - S_2$ we obtain:

$$ s = \frac{2}{\pi} \left[ 0.5 + \left(1 - \rho^{-2}\right) \arctan \rho \right]. \quad (S4) $$

Equations (S3), (S4) provide the dependence $L(s)$ parameterically where the running parameter $\rho$ changes from from 0, which corresponds to the straight bisectoral DW at $s = 0$, to $\infty$, which corresponds to the small semi-circle DW, leaving the sample at $s = 1$. The graphic presentation of $l(s)$ is shown in Fig. S4B. The behaviour of $l(s)$ at small s is found by the proper expansions of expressions (S3) and (S4):

$$ l(s) \approx 1 - \frac{3\pi^2}{64} s^2. \quad (S5) $$

Distribution of charge and field in FE capacitor

The depolarization charges at the surface of FE cell have the densities $\sigma_{1,2}^{dep} = -P_0$ and $\sigma_{1,2}^{dep} = +P_0$ for the domains with the down- and up-oriented polarization, $-P_0$ and $+P_0$, respectively (we consider here the upper surface for definiteness). The test charge, placed at the electrodes is redistributed between areas $S_1$ and $S_2$ with the densities $\sigma_{1}^{el}$ and $\sigma_{2}^{el}$, keeping the value of the test charge, $Q$, constant,

$$ Q = \sigma_1^{el} S_1 + \sigma_2^{el} S_2. \quad (S6) $$

This distribution should ensure the equality of the electrostatic potential over the whole electrode area. The latter condition is equivalent to the uniformity of the distribution of the total charge, $q$, that includes both the depolarization and free charges and to the uniformity of the distribution of the field inside the FE capacitor. Accordingly, the density of the total charge is equal at the surfaces $S_1$ and $S_2$ and is given by

$$ \sigma^{tot} = q/S = \sigma_1^{el} - P_0 = \sigma_2^{el} + P_0. \quad (S7) $$

Equations (S6) and (S7) allow to calculate $q$,

$$ q = Q - sSP_0, \quad (S8) $$

where the depolarization charge entering with the weight $s$ that accounts for the alternation of the polarization orientation in domains. Finally we find the internal field in FE capacitor,

$$ E(s) = -\frac{\sigma^{tot}}{\varepsilon_0 |\overline{P}_t|} = -\frac{q}{S \varepsilon_0 |\overline{P}_t|} = -\frac{1}{\varepsilon_0 |\overline{P}_t|} \left(Q/S - sP_0\right) \quad (S9) $$

as function of the factor $s$. 
Calculation of the capacitance

The energy of the system reads:

$$ E = \frac{V_f}{2} \varepsilon_0 \varepsilon_l E^2 + d_f L w_{DW}, \quad \text{(S10)} $$

where the first term describes the electrostatic energy contribution, while the second term represents the energy of the DW. Here $V_f = \pi R^2 d_f$ is the FE capacitor volume, $w_{DW} = 2 \xi_0 P_0^2 / \varepsilon_0 \varepsilon_l$ is the surface tension of DW. $L$ is the DW length, and $d_f L$ is the DW area. Both, the field, $E$, and the domain length, $L$, are parametrized via the misalliance factor $s$. The dependence $E(s)$ is linear and is given by Eq. (S9). The DW length is expressed via the shown in Fig. S4B function $l(s)$ as $L(s) = 2Rl(s)$.

Considering $s$ as a variational parameter and minimizing $E$ we obtain the equation

$$ s + \frac{4 \nu \xi_0}{\pi R} f(s) = \frac{Q}{SP_0}, \quad \text{(S11)} $$

that can be solved by recurrence, since the second term in the right-hand side is smaller than the first one. (It tends to zero when the DW energy associated with $\nu$ vanishes).

After decomposition $s = s_0 + s_1$, with $s_0 = Q/SP_0$ and $|s_1| \ll |s_0|$ we obtain

$$ s = \frac{Q}{SP_0} - \frac{4 \nu \xi_0}{\pi R} f(s_0) = \frac{Q}{SP_0} \left(1 + \frac{4 \nu \xi_0}{\pi \psi(s_0) R}\right) \quad \text{(S12)} $$

where the function $\psi(s)$ is defined from $l(s)$ as $\psi(s) \equiv -s/l'(s)$. Its graphical presentation is given in Fig. S4B. At small $s$ $\psi(s) \approx \frac{s^2}{2 \nu R} \approx 1.08$ as it follows from Eq. (S5).

From (S9) we obtain the internal field

$$ E = \frac{1}{\varepsilon_0 \varepsilon_l} \frac{Q}{5 \pi \psi(s_0) R} \quad \text{(S13)} $$

and, finally

$$ C_f = \frac{Q}{V} = \frac{Q}{-d_f E} = -\varepsilon_0 \varepsilon_l \frac{S \pi}{d_f 4 \nu \psi(s_0) R} \quad \text{(S14)} $$

ACKNOWLEDGMENTS

This work was supported by H2020-RISE-ENGIMA action (I.L. A.S. and A.R.), by internal grant VnGr-07/2017-23 of Southern Federal University, Russia (A.R. and Y.T.) and by the U.S. Department of Energy, Office of Science, Basic Energy Sciences, Materials Sciences and Engineering Division (V.M.V. and partially I.L.).

[1] R. Landauer, Collective Phenomena 2, 167 (1976).
[2] S. Salahuddin, S. Datta, Nano Lett. 8, 405 (2008).
[3] A. I. Khan, et al., Science 345, 646 (2014).
[4] L. C. Verman, Proceedings of the Institute of Radio Engineers 19, 676 (1931).
[5] L. Behr, R. Tarpley, Proceedings of the Institute of Radio Engineers 20, 1101 (1932).
[6] F. E. Terman, Variable reactance circuit (1934). US Patent 1,950,759.
[7] G. Catalan, D. Jiménez, A. Gruverman, Nature Materials 14, 137 (2015).
[8] K. Ng, S. J. Hillenius, A. Gruverman, Solid State Comm. 265, 12 (2017).
[9] M. Hoffmann, et al., Journ. of Appl. Phys. 123, 184101 (2018).
[10] S. Smith, K. Chatterjee, S. Salahuddin, IEEE Transactions on Electron Devices 65, 295 (2017).
[11] A. Cano, D. Jiménez, Appl. Phys. Lett. 97, 10 (2010).
[12] M. Hoffmann, M. Pešić, S. Slesazeck, U. Schroeder, T. Mikołajick, Nanoscale 10, 10891 (2018).
[13] A. Bratkovsky, A. Levanyuk, Phys. Rev. Lett. 84, 3177 (2000).
[14] S. K. Streiffer, et al., Phys. Rev. Lett. 89, 067601 (2002).
[15] I. Kornev, H. Fu, L. Bellaiche, Phys. Rev. Lett. 93, 196104 (2004).
[16] V. Stephanovich, I. Luk’yanchuk, M. Karkut, Phys. Rev. Lett. 94, 047601 (2005).
[17] P. Zubko, N. Stucki, C. Lichtensteiger, J.-M. Triscone, Phys. Rev. Lett. 104, 187601 (2010).
[18] A. Bratkovsky, A. Levanyuk, Phys. Rev. B 63, 132103 (2001).
[19] P. Zubko, et al., Nature 534, 524 (2016).
[20] I. Luk’yanchuk, A. Sené, V. M. Vinokur, Phys. Rev. B 98, 024107 (2018).
[21] T. Sluka, P. Mokry, N. Setter, *Appl. Phys. Lett.* **111**, 152902 (2017).

[22] N. Pertsev, A. Zembilgotov, A. Tagantsev, *Phys. Rev. Lett.* **80**, 1988 (1998).

[23] Y. Li, S. Hu, Z. Liu, L. Chen, *Appl. Phys. Lett.* **78**, 3878 (2001).

[24] J. Mangeri, *et al.*, *Nanoscale* **9**, 1616 (2017).

[25] D. Gaston, C. Newman, G. Hansen, D. Lebrun-Grandie, *Nucl. Eng. and Design* **239**, 1768 (2009).