On the simultaneous growth of multiple hydraulic fractures emanating from an inclined well

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Abstract. The primary focus of this paper is to investigate the interaction between simultaneously propagating multiple fractures, initiated from an inclined well. In particular, the aim is to better understand the influence of the well inclination angle on the stress shadow between the fractures and on the overall resulting geometry of individual cracks. To simplify the analysis, the paper assumes the limit of large perforation friction, which leads to a uniform flux distribution between the fractures. The mathematical model for multiple hydraulic fractures is constructed by coupling together the respective models for individual fractures, each representing a single planar fracture model. In this approach, the fracture induced stress or stress shadow from a previous time step is used as an input for a given single hydraulic fracture to propagate independently. Further, to reduce computational burden, the effects associated with tangential stresses and displacements are neglected, whereby the stress interaction between the fractures is solely described by the normal opening and the normal stress component. Numerical results are presented for the storage viscosity dominated regime, whereby the effects of toughness and leak-off are negligible. An interesting behaviour is observed, demonstrating that the well inclination angle plays a significant role on the overall fracture symmetry. For zero inclination, all the fractures are nearly symmetrical and identical. However, once well inclination is introduced, this breaks the symmetry, making a profound effect on the final result.

1. Introduction

Hydraulic fracturing is one of the key technologies for enhancing oil and gas production in low-permeability reservoirs [1]. To optimize the developments, multiple hydraulic fractures are often induced simultaneously from a single well. From the modeling perspective, in the case it is necessary to consider the induced stress field or stress shadow resulting from mutual mechanical interaction of the fractures. This topic has been studied thoroughly in the literature [2–6]. At the same time, such studies often assume that the fractures are initiated perpendicular to the well orientation, which is not always the case in practice. As a result, this study aims to further dig into this topic and to better understand the influence of well inclination.

The model for multiple hydraulic fractures is based on the coupling of individual fracture models, where each individual crack is described using the so-called Planar3D ILSA [7]. The Implicit Level Set Algorithm (ILSA) uses the universal asymptotic solution [8] in the vicinity of the fracture tip as a propagation condition, which allows us to smoothly track the fracture front and to accurately determine the fracture tip location on a relatively coarse mesh.
For the purpose of coupling the individual fractures, it is necessary to take into account the non-uniform flux distribution between individual cracks. However, in the case of large perforation friction, which is often the case in practice, the fluxes are assumedly distributed uniformly among the fractures. To simplify calculation of the mutual mechanical interaction between multiple cracks, the uncoupled formulation of the elasticity equation is considered. In this approach, the fracture induced stress from a previous time step is used to update each individual fracture independently. In addition, tangential displacement discontinuity is assumed to be zero at each fracture, whereby zero tangential stress boundary condition is not strictly satisfied at each fracture face. However, it is anticipated that the contribution of shear components is relatively small for planar fractures.

The paper is organized as follows. The mathematical formulation is presented first. Then, it is followed by the numerical model. Results of the simulations are presented next, which is wrapped up by conclusions.

2. Mathematical formulation

The left panel in Figure 1 shows the scheme for a single fluid-driven planar fracture. The elastic medium is characterized by uniform values for Young’s modulus $E$, Poisson ratio $\nu$, fracture toughness $K_{ic}$ and leak-off coefficient $C_L$. At the same time, it is assumed that there are $n$ layers with different values of the in-situ compressive stress $\sigma^0_h$. The solution of the hydraulic fracture problem involves the fracture width $w(t, x, y)$, the fluid pressure $p(t, x, y)$ and the location of the fracture front $C$.

The right panel in Figure 1 shows an inclined well with $N$ simultaneously growing hydraulic fractures. It is assumed that all fractures are parallel to the plane $z = 0$, and the perforations or initiation points of different fractures can be shifted along the well. The location of the perforations along the $z$-axis is governed by $z^k$ values that are determined by the inclination angle $\phi$.

To simplify mathematical expressions, it is convenient to introduce the following scaled parameters

$$E' = \frac{E}{1 - \nu^2}, \quad K' = 4 \left(\frac{2}{\pi}\right)^{1/2} K_{ic}, \quad C' = 2 C_L, \quad \mu' = 12 \mu,$$

as is commonly done in the literature on this topic [7].

Figure 1. Panel (a): Scheme of a planar hydraulic fracture in a layered media. Panel (b): Scheme for multiple parallel hydraulic fractures.

2.1. Model for a single fracture

The elasticity equation relates the fracture width $w$ (or normal displacement discontinuity) and stress field in the solid which in turn is related to the fluid pressure $p$. According to the displacement discontinuity method [9] it can be expressed by the following hyper singular integral equation

$$p(t, x, y) = \sigma_h(y) - \frac{E'}{8\pi} \int_{A(t)} \frac{w(t, x', y') \, dx' \, dy'}{[(x' - x)^2 + (y' - y)^2]^{3/2}}$$

(1)
Here, $A(t)$ is the fracture domain enclosed by the fracture front $C(t)$, and $\sigma_h$ is the prescribed compressive stress field that captures geological stress layering.

Combination of Poiseuille's law for laminar flow of Newtonian fluid and continuity equation yields in Reynolds's equation
\[
\frac{\partial w}{\partial t} - \text{div} \left( \frac{w^3}{\mu} \nabla p \right) = Q(t)\delta(x,y) - \frac{c'}{\sqrt{t-t_0(x,y)}},
\]
where $Q(t)$ is the injection flow rate and $t_0(x,y)$ is the time instant at which the fracture front was located at the point $(x,y)$. The last term in equation (2) represents fluid leakage according to Carter's model [10]. Note that the divergence and the gradient operators act in the $(x,y)$ plane.

As shown in [11], the fracture width in the vicinity of the fracture tip is governed by the model for a semi-infinite hydraulic fracture propagating with constant velocity $V$. This, combined with the zero-flux boundary condition, yields in:
\[
w(s) \approx w_a(s), \lim_{s \to 0} \frac{w^3 \partial p}{\partial s} = 0,
\]
where $s$ is the distance to the fracture front (see Figure 1(a)) and $w_a(s)$ is the universal tip asymptotic solution [7, 8], that captures the effects of fracture toughness, fluid viscosity, and leak-off in the tip region. This solution reduces to the toughness or dry fracture solution very close to the fracture tip, i.e., $w_a(s) \to (K'/E')s^{1/2}$ for $s \to 0$.

2.2. Model for multiple fractures

The multiple hydraulic fracture model is based on the single fracture model described above. However, modelling the growth of multiple parallel fractures additionally requires calculating the flux distribution between perforations and taking into account the mutual mechanical interaction between the fractures, typically referred to as stress shadowing [3].

In the case of incompressible fluid, the addition of the flow rates for each fracture is equal to the total flow rate in the well, i.e., $\sum_{i=1}^{N} q_i = Q_0$. Then, the pressure continuity in the well, together with the perforation friction [12] can be used to deduce the flux distribution, i.e., $p_i + p_{pf,i}(q_i) = p_w$. Here $p_i$ is the pressure inside the $i$th fracture at the location of the wellbore, $p_{pf,i}(q_i)$ is the corresponding perforation friction, and $p_w$ is the pressure inside the wellbore. To simplify the analysis, in this work we consider the case of large perforation friction, which leads to uniform flux distribution between the fractures $q_i = Q_0/N$.

When calculating stress interference between the fractures, we neglect the presence of tangential displacement discontinuities and consider only the dominant normal components. In this case, the conditions of mechanical equilibrium for fractures become:
\[
\sigma_{zz}(x,y,z^k) + \sigma_h(y) = p^k(x,y),
\]
where $p^k$ is fluid pressure of $k$th fracture and $\sigma_{zz}(x,y,z^k)$ is the induced normal stress component on the plane of the $k$th fracture. The latter may be computed as
\[
\sigma_{zz}(x,y,z^k) = \sum_{l=1}^{N} \int_{A(t)} \mathcal{C}_{zzz}(x-x',y-y',z^k-z^l)w^l(x',y')dx'dy',
\]
where $A^l$ denotes the footprint of $l$th fracture and $w^l(x',y')$ is the fracture width of the $l$th fracture. The integral kernel $\mathcal{C}_{zzz}(x-x',y-y',z^k-z^l)$ represents the normal stress component at the point $(x,y,z^k)$ due to a unit displacement discontinuity in the $z$ direction located at the point $(x',y',z^l)$ (see [9]).

3. Numerical scheme

We assume that the fractures grow in parallel planes that are perpendicular to the $z$ axis. Each plane is covered by rectangular elements with dimensions $\Delta x$ and $\Delta y$. A piece-wise constant approximation is used for the fracture width $w^k$ and fluid pressure $p^k$. In order to simplify the notation, bold symbols...
are used to gather mesh indices as follows $w_k = \{ w_{ij}^k \} (i,j)$ is all indices of $k$-th mesh for the $k$th fracture and $w = (w^1, \ldots, w^N)^T$ for all the fractures.

Using a piece-wise constant approximation, equation (3) and equation (4) for all fractures can be written as

$$p = \sigma_h + C_{zzz}w.$$  

The matrix $C_{zzz}$ has a block structure, where the block $C_{zzz}(k,l)$ is responsible for the elastic influence of the $k$th fracture on the $l$th fracture. The expressions for the components of matrix $C_{zzz}$ are given as an example in [2]. By taking the stress shadow from the previous time step for simplicity, the fluid pressure for the $k$th fracture can be expressed as

$$p^k = \sigma_h + \sum_{i=1}^{N} C_{kk} w^k, \quad \sigma^k_{ss} = C_{kk} w^k_{pr}.$$  

where $\sigma^k_{ss}$ is the additional compressive stress for the $k$th fracture from the $l$th fracture due to the stress shadowing, and $\sigma_{ss}^{k,l}$ is the fracture width of the $l$th fracture from the previous time step. Thus, the explicit scheme is used to model the mutual mechanical interaction between the fractures. It is worth noting that the matrix $C_{kk}$ can be obtained by discretizing the equation (1) for the $k$th fracture.

The Reynolds equations for each individual fracture are discretized using the finite volume method. For time integration, the backward Euler scheme is used to ensure stability. To track the fracture front, we apply the Implicit Level Set Algorithm (ILSA) similar to the one described in [7].

### 4. Numerical experiments

To investigate the effect of stress shadowing on the development of asymmetric fractures in an inclined well, the well axis is shifted by an angle $\theta$ relative to the $z$-axis as indicated in Figure 1(b). This shift corresponds to the shift of the perforation entry points along the $x$-axis, so that the actual fracture spacing is preserved. Three parallel fractures are considered in calculations and the distance between them is 20 meters, i.e., $z^1 = 0$ m, $z^2 = 20$ m, and $z^3 = 40$ m. We also assume that the reservoir is homogeneous with Young’s modulus $E = 30$ GPa, Poisson ratio $\nu = 0.25$, fracture toughness $K_{IC} = 1$ MPa $\cdot$ m$^{1/2}$, no fluid leakage into the formation $C_i = 0$, and no stress layers. Newtonian fluid with viscosity $\mu = 0.1$ Pa $\cdot$ s is pumped with the constant total injection rate of $Q_0 = 0.25$ m$^3$/s for 1000 seconds.

First consider the case of no well inclination, i.e., $\theta = 0$. Figure 2(a) shows the width $w^k(x,y)$ of all fractures. The black crosses show the injection points, and the white vertical dashed line separates the right and left wings of the fracture. In the absence of inclination, the fractures are symmetrical relative to the injection point and the shape of all fractures is radial. However, there is a visible width variation between the outer and the inner fractures. This is due to the presence of an additional compressive stress (5) caused by stress shadowing. The corresponding distribution of $\sigma_{ss}$ (stress shadow from other fractures) is shown in the third and fourth columns of Figure 2(a).

Then, the well inclination with the angle $\theta = \pi/4$ is considered, in which case the perforations are shifted by 20 meters along the $x$ axis. Figure 2(b) depicts the width $w^k(x,y)$ and the stress shadow $\sigma_{ss}^k(x,y)$ for all fractures. In the presence of a horizontal shift, the outer fractures become significantly asymmetric, while the inner fracture is elongated vertically and its width is suppressed near the injection point by the two outer fractures. This demonstrates that a relatively small lateral shift of the injection points can significantly affect fracture morphology even for the most benign viscosity dominated regime.

Figure 3 displays area $S$, length $l$ and height $h$ of the inner and outer fractures for the different well inclination angles $\theta$. As the well inclination angle $\theta$ increases, the area $S$ and the length $l$ of the fractures decreases. The height $h$ of the inner fracture increases, whereas the height of the outer fractures doesn’t change much. The presence of an additional compressive stress $\sigma_{ss}$ from the outer fractures result in the significant decrease in length $l$ and increase in height $h$ of the inner fracture. Large inclination angle $\theta$ corresponds to a shift along the $x$-axis of more than a hundred meters. In
this case, the fractures do not overlap, which leads to the growth of an identical radial cracks. It is worth noting that in the case of non-uniform flux distribution, figure 3 may be significantly changed.

Figure 2. Results of numerical simulations with three hydraulic fractures in an inclined well with angle $\varphi$. The first and third columns show the fracture width $w^k(x, y)$ and the stress shadow $\sigma_{ss}^k(x, y)$, respectively, for the outer fractures ($k = 1, 3$) accounting for symmetry relative to the line $x = 0$. The second and fourth columns show the width $w^2(x, y)$ and the stress shadow $\sigma_{ss}^2(x, y)$, respectively, for inner fracture. Panel (a) shows the results for $\varphi = 0$, while panel (b) corresponds to $\varphi = \pi/4$. Black crosses indicate location of the injection point, and vertical dashed lines separate the right and left wings of each fracture.

Figure 3. Fracture area $S$, length $l$ and height $h$ against the well inclination angle $\varphi$. Red solid line corresponds to the outer fractures, blue dashed line corresponds to the inner fracture.

Conclusions
In this paper, the problem of simultaneous propagation of multiple hydraulic fracturing from an inclined well has been investigated. The limit of quite large perforation friction assuming the uniform
flux distribution between the fractures has been considered. For the mutual mechanical interaction between the fractures, tangential displacement discontinuities are neglected for simplicity. This allows solving the elasticity problem for each fracture independently instead of using a fully coupled approach. Consequently, to further simplify the problem, the additional compressive stress field due the stress shadowing is calculated based on the fracture width from the previous time step.

Results of simulations demonstrate that in the case of no well inclination, the fractures are symmetrical and nearly radial. At the same time, well inclination and the corresponding lateral shift of the injection points can lead to very asymmetrical fractures and their shapes are significantly non-circular. The results are presented in the storage-viscosity-dominated regime.

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