What is the spectrum of cold dark matter particles on Earth?†

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Abstract

It is argued that the spectrum of cold dark matter particles on Earth has peaks in velocity space associated with particles falling onto the Galaxy for the first time and with particles which have fallen in and out of the Galaxy only a small number of times in the past. Estimates are given for the sizes and velocity magnitudes of the first few peaks. The estimates are based upon the secondary infall model of halo formation which has been generalized to include the effect of angular momentum.

†Invited talk at the DM96 Workshop on Sources and Detection of Dark Matter in the Universe, Santa Monica, CA, Feb. 14–16, 1996
1. INTRODUCTION AND OVERVIEW

This talk is based entirely upon work done in collaboration with J. Ipser and I. Tkachev and Y. Wang.

Experiments are under way which attempt to identify the nature of dark matter by direct detection on Earth. The candidates which are being searched for in this way are axions with mass in the $\mu$eV range and WIMPs (weakly interacting massive particles) with mass in the GeV range. Axions and WIMPs are the leading cold dark matter (CDM) candidates. Other forms of dark matter are baryons and neutrinos. From the point of view of galaxy formation, the defining properties of CDM are:

1. that CDM particles, unlike baryons, are guaranteed to interact with their surroundings only through gravity, and
2. that CDM particles, unlike neutrinos, have negligibly small primordial velocity dispersion.

Studies of large scale structure support the view that the dominant fraction of dark matter is CDM. Moreover, if some fraction of dark matter is CDM, it necessarily contributes to galactic halos by falling into the gravitational wells of galaxies and hence is susceptible to direct detection on Earth.

Motivated by the prospect that a direct search experiment may some day measure the spectrum of CDM particles on Earth, one may ask what can be learned from that spectrum about our galaxy and the universe. In particular, if a signal is found in the cavity detector of galactic halo axions, it will be possible to measure the CDM spectrum with great precision and resolution. In addition, there is the possibility that beforehand knowledge of some spectral properties helps in the discovery of a signal.

In many discussions of dark matter detection on Earth, it has been assumed that the dark matter particles in our galaxy have an isothermal velocity distribution. A strong argument in support of this assumption is the fact that a self-gravitating isothermal sphere always has a density profile $\rho(r)$ which falls off at large $r$ as $1/r^2$ and hence a flat rotation curve. However, it is easy to convince oneself that the velocity distribution of dark matter particles necessarily has a component which is not isothermal.

Indeed, consider the fact that our closest neighbor on the galactic scale, the galaxy M31 in Andromeda, at a distance of order 730 kpc from us, is falling towards our galaxy with a line-of-sight velocity of order 120 km/sec. This motion can be understood to be due to the mutual gravitational attraction between the two galaxies. We may use it as an indicator of the motion of any matter in our neighborhood. Now, if CDM exists, it is present everywhere because, by Liouville’s theorem, the 3-dim. sheet in 6-dim. phase-space on which the CDM particles lie can not be ruptured whatever its evolution may be. The thickness of that sheet is the tiny primordial velocity dispersion of the CDM particles. The implication is that, if CDM exists, there are CDM particles falling onto our galaxy continuously and from all directions. The motion of these particles gets randomized inside the galaxy by gravitational scattering off giant molecular clouds, globular clusters and other inhomogeneities but complete thermalization of their velocity distribution occurs only after they have fallen in and out of the galaxy many times.
As a result, there are peaks in the velocity distribution of CDM particles at any physical point in the galaxy\(^1\). One peak is due to particles falling onto the galaxy for the first time, one peak is due to particles falling out of the galaxy for the first time, one peak is due to particles falling in for the second time, and so on. Estimates have been obtained of the sizes and the velocity magnitudes (in a reference frame which is not rotating along with the disk) of these peaks using the secondary infall model of galactic halo formation\(^2\). The existing version of this model was generalized to allow the dark matter particles to have angular momentum. Indeed, as will be seen below, angular momentum has a large effect upon the peak sizes.

2. SELF-SIMILAR SECONDARY INFALL

In the secondary infall model of galactic halo formation\(^3\), a halo forms and grows around an initial overdensity because dark matter keeps falling onto it. The dark matter is initially receding from the overdensity, as part of the general Hubble expansion, but is gravitationally attracted to it. The dark matter is assumed to be non-dissipative and have zero initial velocity dispersion. The gravitational potential of the galaxy is taken to be spherically symmetric. Moreover, in the original formulation of the model, the dark matter particles are assumed to have zero angular momentum with respect to the center of the overdensity and they thus move on radial orbits through it. As is explained below, it is possible and, for the purpose of estimating velocity peaks, necessary to rid the model of this last assumption. But let’s keep the assumption for the moment for pedagogical purposes.

![Figure 1: The line is the location of dark matter particles in phase space at a fixed moment of time in the secondary infall model with \(\epsilon = 0.2\) and \(j = 0\). The dotted line corresponds to the Sun’s position if \(h = 0.7\).](image)

An initial overdensity profile \(\delta M_i(r)\) is assumed. The equations of motion for the radial coordinate \(r(\alpha, t)\) of each spherical shell (\(\alpha\) is a shell label, \(t\) is time) in the gravitational potential due to all the other shells must then be solved for initial conditions given by the Hubble expansion at some arbitrarily chosen but early time \(t_i: \dot{r}(\alpha, t_i) = H(t_i)\tau(\alpha, t_i)\). Much
progress in the analysis of the model came about as a result of the realization that the evolution of the galactic halo is self-similar\[7\] provided the initial overdensity has the following scale-free form:

\[
\frac{\delta M_i}{M_i} = \left( \frac{M_0}{M_i} \right)^\epsilon
\]

and provided \( \Omega = 1 \). \( M_i \) and \( \delta M_i \) are respectively the mass and excess mass interior to \( r_i \) at the initial time \( t_i \). \( \epsilon \) is a parameter in the range \( 0 < \epsilon \leq 1 \). Self-similarity means that the phase-distribution of the dark matter particles is time-independent after all distances have been rescaled by the overall size \( R(t) \) of the galactic halo and all masses by the mass \( M(t) \) interior to the radius \( R(t) \). \( R(t) \) is taken to be the “turn-around” radius at time \( t \), i.e., the radius at which particles have zero radial velocity for the first time in their history (see Fig. 1). In particular, the mass-profile of the halo \( M(r, t) = M(t) \mathcal{M} \left( \frac{r}{R(t)} \right) \). It was shown analytically\[4\] that \( \mathcal{M}(\xi) \sim \xi \) as \( \xi \to 0 \) if \( 0 < \epsilon \leq \frac{2}{3} \). Thus, in the range \( 0 < \epsilon \leq \frac{2}{3} \), the model produces flat rotation curves, i.e., it is in accord with the main feature of the galactic mass distribution.

Figure 2: The spectrum of velocity peaks at the Sun’s position for the case \( \epsilon = 0.2 \), \( j = 0 \) and \( h = 0.7 \).

To fit the model to our present galactic halo, one must choose appropriate values of \( R \) and \( M \). This was done\[2\] by matching the rotation velocity in the model to the observed one (220 km/sec) in our galaxy and choosing a value for the age of the universe: \( t_0 = \frac{1}{h} (6.52 \times 10^9 \text{ years}) \) where \( h \) parametrizes the present Hubble rate: \( H_0 = h \times 100 \text{ km sec}^{-1} \text{ Mpc}^{-1} \). For typical values of \( \epsilon \) and \( h \), \( R \) turns out to be in the 1 to 3 Mpc range. Figs. 1 and 2 show the phase-space diagram and the velocity peaks on Earth for \( \epsilon = 0.2 \) and \( h = 0.7 \). The rows labeled \( j = 0.0 \) in Table 1 give the density fractions and kinetic energies of the first five incoming peaks in the model without angular momentum. For each incoming peak there is an outgoing peak with approximately the same energy and density fraction (see Fig. 2).

However, one must question in this context the approximation of neglecting the angular momentum that the dark matter particles are expected to have. The model without angular momentum tends to overestimate the size of the peaks due to particles falling in and out of the
Figure 3: Rotational curves for the case $\epsilon = 0.2$, with and without angular momentum.

galaxy for the first time. Indeed, angular momentum has the effect of keeping infalling particles away from the galactic center and this effect is largest for the particles falling onto the galaxy last. Fortunately, there is a generalization of the model which takes angular momentum into account while still keeping the model tractable.

In this generalization[2], spherical symmetry is maintained by, in effect, averaging over all possible orientations of an actual physical halo. Each shell $\alpha$ is divided into subshells labeled by an index $k$. The particles in a given subshell $(\alpha, k)$ all have the same magnitude $\ell_k(\alpha)$ of angular momentum. At any point on a subshell, the distribution of angular momentum vectors is isotropic about the axis from that point to the galactic center. Thus the spherical symmetry of each subshell is maintained in time. Moreover, it was found that the evolution is self-similar provided:

$$\ell_k(\alpha) = j_k r_*(\alpha)^2 / t_*(\alpha)$$

(2)

where $r_*(\alpha)$ and $t_*(\alpha)$ are the turn-around radius and turn-around time of shell $\alpha$ and the $j_k$ are a set of dimensionless numbers characterizing the galaxy’s angular momentum distribution. In all cases presented here, the $j_k$ were taken to be distributed according to the density:

$$\frac{dn}{dj} = \frac{2j}{j_0^2} \exp \left( -\frac{j^2}{j_0^2} \right),$$

(3)

for which the average is $\bar{j} = \frac{\sqrt{\pi}}{2} j_0$. Fig. 3 shows the rotation curves for the cases $\epsilon = 0.2, j = 0$ and $\epsilon = 0.2, \bar{j} = 0.2$. It shows that the effect of angular momentum is to give a core radius to the halo, i.e., it makes the halo contribution to the rotation velocity go to zero for $r \to 0$. The ‘effective core radius’ $b$ is defined as the radius at which half of the rotation velocity squared is due to the halo.

Fig. 4 shows the velocity peaks for the case $\epsilon = 0.2, \bar{j} = 0.2$, and $h = 0.7$. Table 1 gives the values of the current turn-around radius $R$, the effective core radius $b$, the halo density at our location $\rho$ and the density fractions and energies of the five most energetic incoming
Table 1: Density fractions $f_n$ and kinetic energies $E_n$ of the first five incoming peaks for various values of $\epsilon$, $j$ and $h$. Also shown are the current turnaround radius $R$ in units of Mpc, the effective core radius $b$ in kpc, and the local density $\rho$ in $10^{-25}$ g cm$^{-3}$. The $f_n$ are in percent and the $E_n$ are in units of $0.5 \times (300 \text{ km s}^{-2})^2$.

| $\epsilon$ | $j$ | $h$ | $R$ | $b$ | $\rho$ | $f_1 (E_1)$ | $f_2 (E_2)$ | $f_3 (E_3)$ | $f_4 (E_4)$ | $f_5 (E_5)$ |
|------------|----|----|-----|-----|-------|--------------|--------------|--------------|--------------|--------------|
| 0.2        | 0.0| 0.7| 2.0 | 0.0 | 8.1   | 13 (4.0)     | 5.3 (3.2)    | 3.3 (2.7)    | 2.4 (2.4)    | 1.9 (2.2)    |
| 1.0        | 0.0| 0.7| 0.9 | 0.0 | 8.4   | 1.6 (3.4)    | 1.1 (3.2)    | 0.9 (3.0)    | 0.8 (2.9)    | 0.7 (2.8)    |
| 0.15       | 0.2| 0.7| 2.4 | 13  | 5.0   | 4.0 (3.1)    | 5.4 (2.3)    | 5.3 (1.8)    | 4.9 (1.5)    | 4.0 (1.3)    |
| 0.2        | 0.1| 0.7| 2.0 | 4.5 | 7.6   | 7.4 (3.8)    | 7.2 (3.0)    | 4.9 (2.5)    | 3.2 (2.2)    | 2.4 (2.0)    |
| "          | 0.2| 0.7| 12  | 5.4 | 3.1   | 4.1 (3.4)    | 4.3 (2.1)    | 4.1 (1.8)    | 3.6 (1.6)    |
| "          | 0.5| 2.8| 17  | 4.9 | 1.9   | 2.5 (2.7)    | 2.8 (2.3)    | 2.9 (2.0)    | 3.0 (1.7)    |
| "          | 0.9| 1.6| 9.3 | 6.0 | 4.4   | 5.3 (2.5)    | 5.1 (2.0)    | 4.5 (1.7)    | 3.6 (1.5)    |
| "          | 0.4| 0.7| 2.0 | 40  | 2.6   | 0.8 (2.5)    | 1.6 (1.8)    | 2.1 (1.4)    | 2.4 (1.1)    | 2.6 (0.9)    |
| 0.25       | 0.2| 0.7| 1.8 | 8.5 | 5.5   | 2.0 (3.5)    | 2.9 (2.8)    | 3.3 (2.4)    | 3.4 (2.1)    | 3.1 (1.8)    |
| 0.4        | 0.2| 0.7| 1.5 | 2.2 | 7.7   | 1.1 (4.0)    | 1.5 (3.4)    | 1.8 (3.0)    | 1.9 (2.8)    | 2.1 (2.5)    |

peaks for representative values of $\epsilon$, $j$ and $h$. The range of $\epsilon$ values chosen is motivated by models of large-scale structure formation\\cite{8} as well as by the fact that the rotation curve is flat in the range $0 < \epsilon \leq \frac{2}{3}$. The table shows that the contribution to the local halo density due to particles which are falling in and out of the galaxy for the first time or which have passed through the galaxy only a small number of times in the past, and which are therefore not thermalized, is not small since it comprises several percent per velocity peak.

Finally, let me emphasize that the actual peak sizes on Earth and anywhere else in the halo have a probabilty distribution which reflects the unknown distribution of angular momenta of the infalling dark matter particles. The peak sizes given in Table 1 and Fig. 4 are the average peak sizes for the angular momentum distribution of Eq.(3). More details will be given in a forthcoming publication\\cite{9}.

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Figure 4: The spectrum of velocity peaks for the case $\epsilon = 0.2, \bar{j} = 0.2$ and $h = 0.7$.

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