Scalar field models for an accelerating universe

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Abstract. I describe a new class of quintessence+CDM models in which late time scalar field oscillations can give rise to both quintessence and cold dark matter. Additionally, a versatile ansatz for the luminosity distance is used to reconstruct the quintessence equation of state in a model independent manner from observations of high redshift supernovae.

1. A new model of quintessence and cold dark matter

The supernova-based discovery that the universe may be accelerating can be explained within a general relativistic framework provided one speculates the presence of a matter component with negative pressure, the most famous example of which is the cosmological constant ‘Λ’ (Perlmutter et al. 1998, 1999; Riess et al. 1999). Λ runs into formidable fine tuning problems since its value must be set ~ 10^{123} times smaller than the energy density in the universe at the Planck time in order to ensure that Λ dominates the total energy density at precisely the present cosmological epoch. This involves a fine tuning of one part in 10^{123} at the Planck scale or one part in 10^{53} at the Electroweak scale.

One way around this difficulty is to make Λ time-dependent, perhaps by using scalar field models which successfully generate a time-dependent Λ-term during an early Inflationary epoch. In this context the exponential potential provides an interesting illustration, since the density in the φ-field tracks the background matter/radiation density when the latter is cosmologically dominant (Ratra & Peebles 1988, Wetterich 1988, Ferreira & Joyce 1997):

\[
\frac{\rho_\phi}{\rho_B + \rho_\phi} = \frac{3(1 + w_B)}{p^2\lambda^2} \ll 1
\]

\( (w_B = 0, \ 1/3 \ \text{respectively for dust, radiation}). \) This behaviour allows \( \rho_\phi \) to be fairly large initially. Based on this property we introduce a new class of cosmological models which can describe both a time-dependent Λ-term (quintessence) and cold dark matter (CDM) within the unified framework of the class of potentials (Sahni & Wang 2000)

\[ V(\phi) = V_0(\cosh \lambda \phi - 1)^p. \]

\( V(\phi) \) has asymptotic forms:

\[
V(\phi) \approx \tilde{V}_0 e^{-p\lambda \phi} \text{ for } |\lambda \phi| \gg 1 \ (\phi < 0),
\]

\[
V(\phi) \approx \tilde{V}_0 (\lambda \phi)^{2p} \text{ for } |\lambda \phi| \ll 1
\]
where \( \tilde{V}_0 = V_0/2^p \). The exponential form of \( V(\phi) \) guarantees that the scalar field equation of state mimics background matter at early times so that \( w_\phi \simeq w_B \). At late times oscillations of \( \phi \) lead to a mean equation of state

\[
\langle w_\phi \rangle = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)} = \frac{p - 1}{p + 1},
\]

resulting in cold dark matter with \( \langle w_\phi \rangle \simeq 0 \) if \( p = 1 \), or in quintessence with \( \langle w_\phi \rangle \leq -1/3 \) if \( p \leq 1/2 \). We therefore have before us the attractive possibility of describing CDM and quintessence in a common framework by the potential

\[
V(\phi, \psi) = V_\phi (\cosh \lambda_\phi \phi - 1)^{p_\phi} + V_\psi (\cosh \lambda_\psi \psi - 1)^{p_\psi}
\]

where \( p_\psi = 1 \) in the case of CDM and \( p_\phi \leq 0.5 \) in the case of quintessence. In figure 1 we show a working example of this model which agrees well with observations of high redshift supernovae and does not suffer from the fine tuning problem faced by \( \Lambda \), since \( \rho_\phi \) can be fairly large initially. We should add that most models of quintessence usually work under the assumption that the three matter fields: baryons, CDM & quintessence need not be related in any fundamental way and might even have different physical origins. If this is indeed the case then it remains somewhat of a mystery as to why \( \Omega_\phi, \Omega_m \), (and possibly \( \Omega_b \)) have comparable magnitudes at the present time. By combining quintessence and CDM within a single class of potentials we make a small step in answering this question by showing that unified models of quintessence and CDM are conceivable (Sahni & Wang 2000).

An intriguing property of cold dark matter based on (6) is that it can have a large Jeans length which leads to suppression (frustration) of clustering on kiloparsec scales. Frustrated Cold Dark Matter (FCDM) redresses certain shortcomings of the standard CDM scenario and might provide a natural explanation for the dearth of dwarf galaxies seen in our local neighborhood (Sahni & Wang 2000).

Other quintessence potentials include \( V(\phi) \propto \phi^{-\alpha} \) (Ratra & Peebles 1988), \( V(\phi) \propto e^{3\phi^2}/\phi^{-\alpha} \) (Brax & Martin 2000) and \( V(\phi) \propto \sinh^{2p}(\phi + \phi_0) \) (Sahni & Starobinsky 2000). The latter describes quintessence which maintains a constant equation of state \( w = -(1 + p)^{-1} \) throughout the matter dominated epoch and later, during acceleration.

### 2. Reconstructing quintessence from supernova observations

Although a large class of scalar potentials can describe a time dependent \( \Lambda \)-term, no unique potential has so far emerged from a consideration of high energy physics theories such as supergravity or M-theory. (The situation in many respects resembles that faced by the Inflationary scenario, for a review see Sahni & Starobinsky 2000.) It is therefore meaningful to try and reconstruct \( V(\phi) \) directly from observations in a model independent manner. This is easy to do if one notes that, in a flat FRW universe, the luminosity distance determines the Hubble parameter uniquely (Starobinsky 1998, Saini et al. 2000)

\[
H(z) \equiv \frac{\dot{a}}{a} = \left[ \frac{d}{dz} \left( \frac{D_L(z)}{1 + z} \right) \right]^{-1}.
\]
The Einstein equations can be written in the suggestive form

\[ \frac{8 \pi G}{3 H_0^2} V(x) = \frac{H^2}{H_0^2} - \frac{x}{6 H_0^2} \frac{dH^2}{dx} - \frac{1}{2} \Omega_M x^3, \]  
\( (8) \)

\[ \frac{8 \pi G}{3 H_0^2} \left( \frac{d\phi}{dx} \right)^2 = \frac{2}{3 H_0^2} \frac{d\ln H}{dx} - \frac{\Omega_M x}{H^2}, \]  
\( (9) \)

where \( x \equiv 1 + z \). Thus knowing \( D_L \), we can determine both \( H(z) \) and \( dH(z)/dz \), and hence \( V(\phi) \). The cosmic equation of state can also be reconstructed from \( D_L \) since

\[ w_\phi(x) \equiv \frac{p}{\rho} = \frac{(2x/3)d\ln H/dx - 1}{1 - (H_0^2/H^2) \Omega_M x^3}. \]  
\( (10) \)

In order to apply our method to observations we use the following rational ansatz for the luminosity distance

\[ \frac{D_L}{x} \equiv \frac{2}{H_0} \left[ \frac{x - \alpha \sqrt{x} - 1 + \alpha}{\beta x + \gamma \sqrt{x} + 2 - \alpha - \beta - \gamma} \right]. \]  
\( (11) \)

where \( \alpha, \beta \) and \( \gamma \) are fitting parameters. This function reproduces the exact analytical form of \( D_L \) when \( \Omega_\phi = 0, \Omega_M = 1 \) and when \( \Omega_\phi = 1, \Omega_M = 0 \). It also has the correct asymptotic behaviour \( H(z)/H_0 \to 1 \) for \( z \to 0 \), and \( H(z)/H_0 \to \)

Figure 1. The evolution of the dimensionless density parameter for the CDM field \( \Omega_\psi \) (dashed line) and quintessence field \( \Omega_\phi \) (thin solid line). Baryon (dash-dotted line) and radiation densities (thick solid line) are also shown. For more details see Sahni and Wang (2000).
Figure 2. The equation of state parameter $w(\phi) = p_\phi/\rho_\phi$ as a function of redshift. The solid line corresponds to the best-fit values of the parameters. The shaded area covers the range of 68% errors, and the dotted lines the range of 90% errors. The hatched area represents the region $w_\phi \leq -1$, which is disallowed for a minimally coupled scalar field (from Saini et al. 2000).

$(1 + z)^{3/2}$ for $z \gg 1$. Applying a maximum likelihood technique to $D_L$ given by (11) and $D^\text{obs}_L$ obtained from observations of high redshift supernovae, we can reconstruct $H(z)$, $V(\phi)$ and $w_\phi(z)$. Our results for $w_\phi$ shown in fig. 2 indicate some evidence of possible evolution in $w_\phi$ with $-1 \leq w_\phi \lesssim -0.80$ preferred at the present epoch, and $-1 \leq w_\phi \lesssim -0.46$ at $z = 0.83$, the farthest SN in the sample (both at 90% CL). However, a cosmological constant with $w = -1$ is also consistent with the data.

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