Implications of New CDF-II $W$ Boson Mass on Two Higgs Doublet Model

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Abstract

We present the implications of the recent measurement of $W$ boson at CDF II on two Higgs doublet model. In the analysis, we impose theoretical bounds such as vacuum stability and perturbative unitarity, and several experimental constraints. In addition, we take into account the measurement of $\sin^2(m_Z)_{\text{MS}}$ on top of the CDF $W$ boson mass to investigate how the $S$ and $T$ parameters are determined. Using the results, we show how the parameter space is constrained.

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I. INTRODUCTION

Very recently, CDF announced a measurement of the $W$ boson mass \cite{1}

$$m_{W}^{CDF} = 80.43335 \pm 0.0094 \text{ GeV}. \quad (1)$$

This result represents two intriguing points. One is that it is the unprecedented high-precise measurement of $m_W$ ever made and the other is that it is in about $7\sigma$ tension with the prediction of the standard model (SM), which is $m_{W}^{SM} = 80.379 \pm 0.006 \text{ GeV}$ \cite{2}. Although the CDF result of $m_{W}^{CDF}$ also shows a significant shift compared to the PDG average of the LEP \cite{3}, ATLAS \cite{4} and the previous Tevatron \cite{5} results yielding $m_{W}^{PDG} = 80.379 \pm 0.012 \text{ GeV}$ \cite{6} as well as the result from LHCb leading to $m_{W}^{LHCb} = 80.354 \pm 0.031 \text{ GeV}$ \cite{7}, it may serve as a hint of new physics beyond the SM. Under the assumption that the CDF measurement will be confirmed in foreseeable future, it would deserve to explore its phenomenological implications.

The purpose of this work is to examine the implication of the recent measurement of the $W$ boson mass at CDF II on two-Higgs doublet model (2HDM). Recently, this possibility has been explored in other works \cite{8–13} with different approaches. The deviation of $m_{W}^{CDF}$ from its SM prediction can be parameterized in terms of the so-called Peskin-Takeuchi parameters, $S$ and $T$, which represent the contributions of new physics. The quantum corrections mediated by new scalar fields contribute to $S$ and $T$. It is common belief that the exquisite precision achieved in the measurements of $m_Z, m_W$ and $\sin^2 \theta_W$ makes it possible to explore the existence of new physics beyond the SM. In this regards, another important parameter to test the SM is the so-called weak mixing angle parameter, $\sin^2 \theta_W$. Instead of on-shell definition of $\sin^2 \theta_W$, it is more general to take it into account by employing the more theoretically motivated $\overline{MS}$ (modified minimal subtraction) prescription. All $Z^0$ pole measurements of $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ are usually averaged out to give

$$\sin^2 \theta_W(m_Z)_{\overline{MS}}^{\text{ave}} = 0.23124 \pm 0.00006. \quad (2)$$

Eq. (2) represents that the average of all $Z^0$ pole measurements is in consistent with the SM. But, the contributions of new physics to $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ can also be parameterized in terms of $S$ and $T$ parameters. Thus, the deviation of $m_W$ from its SM prediction may affect the prediction of $\sin^2 \theta_W(m_Z)_{\overline{MS}}$. In this work, we will examine whether there exist nontrivial $S$ and $T$ parameters accommodating both $m_{W}^{CDF}$ and $\sin^2 \theta_W(m_Z)_{\overline{MS}}^{\text{ave}}$. 
For our purpose, we first estimate how the deviation of $m_W^{\text{CPF}}$ can be accommodated in the framework 2HDM by scanning masses of new scalars, and then try to obtain allowed regions of the parameter space. Next, we will extract allowed regions of the parameter $S$ and $T$ from two observables $m_W$ and $\sin^2 \theta_W(m_Z)^{\overline{\text{MS}}}$, and then confront them with the parameter space obtained from the first stage. In the analysis, we impose the theoretical conditions and experimental results on the Higgs sectors can constrain the masses of Higgs fields and mixing parameters in 2HDM. The theoretical conditions taken into account are the vacuum stability, perturbativity and unitarity which are required to be satisfied up to a cut-off scale. Then one can obtain constraints on the couplings of the Higgs potential in 2HDM, which in turn lead to bounds on the masses of scalar bosons as well as mixing parameters. Although there are a few works on the estimation of bounds on the masses of scalar fields in 2HDM by applying the vacuum stability, perturbativity \cite{14, 16} and unitarity \cite{17, 18}, our new points are to show how the parameter spaces in 2HDM are constrained by those theoretical conditions applied up to a cut-off scale by identifying the 126 GeV Higgs boson as either lighter or heavier of CP even neutral scalar bosons, and to see how bounds on the masses of scalar bosons depend on the cut-off scale. In addition, we will examine how experimental constraints on the parameters of scalar bosons from the LEP can constrain the parameter spaces further. As expected, LEP results can severely constrain the parameter space for scalar bosons in the scenario that the new scalar boson observed at the LHC is the heavier CP even neutral scalar boson in the 2HDM. Finally, we will investigate whether the allowed regions of parameter space can accommodate the enhanced di-photon signals, $ZZ^*$ and $WW^*$ decay modes of the Higgs boson observed at the LHC, and examine the prediction of the signal strength of $Z\gamma$ decay mode for the allowed parameter regions.

II. TWO HIGGS DOUBLET MODEL

A. The Set-Up

Taking $\Phi_1$ and $\Phi_2$ are two complex $SU(2)_L$ Higgs doublet fields with $Y = 1$, the renormalizable gauge invariant scalar potential of 2HDM with softly broken $Z_2$ symmetry is written
\[
V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) \\
+ \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\
+ \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right\}.
\] (3)

We note that all the parameters in Eq.(3) to be real and the squared mass of pseudo-scalar \(m_A^2\) to be greater than \(|\lambda_5|v^2\) so as to keep CP symmetry in the scalar potential. As one can easily check, the dangerous FCNC is absent in the form given by Eq.(3) even if non-zero \(m_{12}^2\) softly breaking the \(Z_2\) symmetry is allowed. Depending on how to couple the Higgs doublets to the fermions, 2HDMs are classified into four types [19]. Among them, the Yukawa couplings of type II 2HDM arises in the minimal supersymmetric standard model which is one of the most promising candidates for the new physics model beyond the SM.

The spontaneous breaking of electroweak symmetry triggers the generation of the vacuum expectation values of the Higgs fields as follows,

\[
\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix},
\] (4)

where \(v^2 \equiv v_1^2 + v_2^2 = (246 \text{ GeV})^2\) with \(v_2/v_1 = \tan \beta\), and \(v_1\) and \(v_2\) are taken to be positive, so that \(0 \leq \beta \leq \pi/2\) is allowed. Among 8 degree of freedom, three are eaten by the gauge bosons and the remaining five become physical Higgs particles in 2HDMs: two CP-even Higgses \(h\) and \(H\) \((M_h \leq M_H)\), a CP-odd Higgs \(A\) and a charged Higgs pair \((H^\pm)\). Following [20], the squared masses for the CP-odd and charged Higgs states are calculated to be [20]

\[
M_A^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \lambda_5 v^2, \quad M_{H^\pm}^2 = M_A^2 + \frac{1}{2}v^2(\lambda_5 - \lambda_4),
\] (5)

and the squared masses for neutral Higgs \((M_H \geq M_h)\) are given by [20]

\[
M_{H,h}^2 = \frac{1}{2} \left[ P + Q \pm \sqrt{(P - Q)^2 + 4R^2} \right],
\] (6)

where \(P = \lambda_1 v_1^2 + m_{12}^2 t_\beta, Q = \lambda_2 v_2^2 + m_{12}^2 / t_\beta\) and \(R = (\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2 - m_{12}^2\) with \(s_\beta = \sin \beta, c_\beta = \cos \beta,\) and \(t_\beta = \tan \beta\).
B. W Boson Mass and $\sin^2\theta_W(m_Z)_{\overline{\text{MS}}}$

The contribution of new scalar fields to $T$ and $S$ parameters are given by [21–24]

$$T = \sqrt{2}G_F \frac{1}{16\pi^2\alpha_{\text{EM}}} \left\{ -F'(M_A, M_H^\pm) + \sin^2(\beta - \alpha) [F'(M_H, M_A) - F'(M_H, M_{H^\pm})] ight.$$ \nonumber
$$+ \cos^2(\beta - \alpha) [F'(M_h, M_A) - F'(M_h, M_{H^\pm})] \right\} \tag{7}$$

$$S = -\frac{1}{4\pi} [F(M_{H^\pm}, M_{H^\pm}) - \sin^2(\beta - \alpha) F(M_H, M_A)$$ \nonumber
$$- \cos^2(\beta - \alpha) F(M_h, M_A)], \tag{8}$$

where $\Delta\rho^\text{new} = \Delta\rho^\text{2HDM} - \Delta\rho^\text{SM}$ and the formulae for $\Delta\rho^\text{2HDM}$ as well as $\Delta\rho^\text{SM}$ are given in [25–27], and the functions $F$ and $F'$ are given by [21–23]

$$F(x, y) = -\frac{1}{3} \left[ 4 - \frac{x^2 \ln x^2 - y^2 \ln y^2}{x^2 - y^2} - \frac{x^2 + y^2}{(x^2 - y^2)^2} \left( 1 + \frac{x^2 + y^2}{2} - \frac{x^2 y^2}{x^2 - y^2} \ln \frac{x^2}{y^2} \right) \right]. \tag{9}$$

$$F'(x, y) = \frac{x^2 + y^2}{2} - \frac{x^2 y^2}{x^2 - y^2} \ln \frac{x^2}{y^2}. \tag{10}$$

Employing the precise measurements of QED coupling $\alpha$, $G_F$ and $m_Z$ accompanied by $m_t$ and $M_{\text{SM higgs}} = 126$ GeV, and allowing for loop effects mediated by heavy new particles via $S$ and $T$ parameters, We can recast the expressions for the predictions of $m_W$ and $\sin^2\theta_W(m_Z)_{\overline{\text{MS}}}$ as follows; [28]

$$m_W = 80.357 \text{ GeV}(1 - 0.0036S + 0.0056T), \tag{11}$$

$$\sin^2\theta_W(m_Z)_{\overline{\text{MS}}} = 0.23124(1 + 0.0157S - 0.0112T). \tag{12}$$

Plugging in experimental values for $m_W$ [1] and $\sin^2\theta_W(m_Z)_{\overline{\text{MS}}}$ [2] into (11) and (12) we obtain the allowed regions of $S$ and $T$ parameters as follows;

$$T = 0.3 \pm 0.062$$
$$S = 0.2 \pm 0.08 \tag{13}$$

Note that these values are in agreement with the values obtained in Ref. [13], to be precise, the value of $S$ is shifted $\sim 25\%$ in our work with the same error of roughly $50\%$. In the case of $T$ the shift is only $\sim 10\%$ and the errors (roughly $20\%$) are almost identical. In our numerical analysis, we extract allowed regions of parameters space by imposing the results [13].
III. THE BOUNDS

The vacuum stability of the scalar potential \(^{(3)}\) is guaranteed only if the following conditions are satisfied \(^{[16, 20]}\)

\[
\lambda_{1,2} > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}, \quad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1 \lambda_2}. \tag{14}
\]

Since radiative corrections lead to the modification of the couplings in the scalar potential, it is required that the stability conditions \(^{(14)}\) are valid for high energy up to cut-off scale \(\Lambda\). The stability conditions \(^{(14)}\) give rise to lower bounds on the couplings \(\lambda_i\) \(^{[20]}\), which in turn lead to bounds on the masses of the physical Higgs fields. In addition, we require that the quartic couplings \(\lambda_i\) in the scalar potential is perturbative and unitarity conditions \(^{[17]}\) are satisfied even at high scale up to the cut-off scale. Those theoretical conditions can constrain not only the masses of the Higgs fields but also mixing parameters \(\tan \beta\) and \(\alpha\) via the renormalization group (RG) evolutions.

On the other hand, we can consider the experimental constraints. in the case that the masses of light neutral Higgs bosons lie between 10 GeV and 150 GeV, the masses of Higgs bosons and the mixing parameters are constrained by the LEP data \(^{[29, 30]}\). The experimental lower bound on charged Higgs masses is 79.3 GeV \(^{[31]}\). The non-observation of \(Z \rightarrow hA\) in the LEP experiment gives rise to the condition that \(M_h + M_A > M_Z\) are kinematically allowed \(^{[32]}\). We also consider the Higgs pair production process, \(e^+e^- \rightarrow hA \rightarrow b\bar{b}b\bar{b}\) when the mass parameters are kinematically allowed. Non-observation of those Higgs pair productions can lead to the constraints on light neutral Higgs masses and mixing parameters as shown in \(^{[30]}\).

In addition to the above constraints explained, we take into account the measurement of \(R_b \equiv \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})\) \(^{[6]}\). The updated SM prediction of \(B_{\text{SM}}(\bar{B} \rightarrow X_s\gamma)\) \(^{[33]}\) and the Belle experiment \(^{[34]}\) give sever bound on \(M_{H^\pm}\) in the type II ; \(M_{H^\pm} > 570\) GeV for \(t_\beta \gtrsim 2\) at 95% C.L. The measurements of \(B - \bar{B}\) mixing also lead to the constraints on the \(M_{H^\pm} - \tan \beta\) plane but less severe ones in comparison with that from \(R_b\) \(^{[35]}\). Combining the theoretical constraints with the experimental ones as done in \(^{[36]}\), we can investigate how the masses of Higgs bosons and mixing parameters can be constrained.
FIG. 1. Contours for the S and T oblique parameters obtained with Eqs. (11) and (12). The black solid contours correspond to the value obtained using the value recently announced by the CDF Collaboration, the blue dashed contours is obtained using the previous average $80.379 \pm 0.012$ GeV. The interior (exterior) contour corresponds to $1\sigma$ ($3\sigma$).

IV. RESULTS AND DISCUSSION

In this section we present the analysis performed and discuss relevant details of the results. The contribution from the oblique parameter $U$ is expected to be negligible compared to the contributions from $S$ and $T$. Using Eqs. (12) we obtain the limits cited in Eq. (13). The two dimensional contours are shown in Fig. 1 including the value obtained using the previous average that is consistent with the SM. In what follows we describe the rest of the components of our numerical analysis.

V. METHODOLOGY

We begin by enforcing the conditions of unitarity, perturbativity and stability of the potential, by requiring that Eqs. (14) are respected. These theoretical conditions are applied as hard cuts to ensure that every sampled point is theoretically meaningful. We calculate the 2HDM contribution to the oblique parameters, $S$, $T$ and $U$, that will be responsible of the shift in the mass of the $W^\pm$ boson. Considering that the oblique parameters shift the
value for \( \sin^2 \theta_W (m_Z)_{\overline{\text{MS}}} \) as well, we require values for \( S, T \) and \( U \) that are consistent with both the recent measurement of the mass of the \( W^\pm \) by the CDF collaboration and with the current experimental average for the weak mixing angle. To calculate the theoretical constraints and oblique parameters, as well as other observables, in the 2HDM we employ 2HDMC-1.8.0 [18]. This tool is then interfaced with HiggsBounds [37] to incorporate several constraints from LEP, Tevatron and LHC on the Higgs sector and obtain a set of allowed at the 95\% confidence level. We use the Markov Chain Monte Carlo sampler emcee [38] to explore the parameter space. The free parameters of our model are given by the mass of the heavy Higgs, \( m_H \), the charged Higgs mass, \( m_{H^\pm} \), the mass of the pseudoscalar, \( m_A \), the mixing \( \cos(\beta - \alpha) \), the parameter \( \tan \beta \) and the squared mass parameter \( M_{12}^2 \). The limits of our scan are given by:

\[
\begin{align*}
    m_H / 1 \text{ TeV} : & \ [0.15, 1], \\
    m_{H^\pm} / 1 \text{ TeV} : & \ [0.15, 1], \\
    m_A / 1 \text{ TeV} : & \ [0.02, 1], \\
    \cos(\beta - \alpha) : & \ [-0.5, 0.5], \\
    \tan \beta : & \ [0.1, 20], \\
    M_{12}^2 / 1 \text{ GeV}^2 : & \ [200, 500^2].
\end{align*}
\]

The recent measurement of the mass of the \( W^\pm \), that we use to constrain our parameter space is given by [1]. We fix the mass of the lighter CP even Higgs to the current average central value of the SM Higgs \( m_h = 125.10 \text{ GeV} \) [6].

VI. PARAMETER SPACE AND THE MASS OF THE \( W^\pm \)

As is well known, the parameters \( S \) and \( T \), which give the largest contribution to the shift in \( m_W \), are largely affected by the mass splitting between scalars. Expectedly, we find that to predict the mass of the \( W^\pm \) boson in the range measured by the CDF Collaboration we require mass splittings with particular sizes. Different mass splittings vs the parameters \( S \) and \( T \) are shown in Fig. [2]. Notably, the splittings between the masses of the pseudoscalar and the heavy Higgs, \( M_A - M_H \), can be zero while all the other splittings are required \( \mathcal{O}(100 \text{ GeV}) \). The requirement that the splittings have the correct size to predict \( m_W \) results in the rough limits \( M_{H^\pm} - M_A \lesssim 300 \text{ GeV} \) and \( M_{H^\pm} - M_H \lesssim 350 \text{ GeV} \). One striking
FIG. 2. Dependence of the $S$ and $T$ parameters on several mass splittings. All the points are consistent with theoretical constraints, and are allowed at 95% CL, and predict $m_W$ in the range reported by the CDF Collaboration. The color represents the shift from the SM prediction for $m_W$.

Feature from Fig. 2 is the distribution in the shift $m_W^{\text{CDF}} - m_W^{\text{SM}}$, represented in color. While the shifts appear randomly for the $S$ plots, they follow a pattern for $T$. This demonstrates the dominance of $T$ in the contributions to the shift in the mass of the $W^\pm$. Since the size of the parameters $S$ and $T$ depends heavily on the splittings between scalars and pseudoscalar we can expect their mass ranges to depend notably on the masses of each other. This is illustrated in Fig. 3 where we can see that there is a strong dependence on the values the scalars and pseudoscalar masses can take. Other notable features of this figure, we can see that in our scan the charged Higgs mass took values $\gtrsim 200$ GeV while the heavy Higgs remained $\gtrsim 140$ GeV.
FIG. 3. Dependence of the scalars and pseudoscalar masses on each other. The points correspond to the same points of Fig. 2 and follow the same conditions. The orange dashed line indicates the lower limit $M_{H^\pm} > 570$ GeV in the type II 2HDM.

VII. CONCLUSION

In this work we attempted to constrain the 2HDM parameter space based on the new measurement of a $7\sigma$ deviation from the SM for the mass of the $W^\pm$ boson, $m_{W}$. Interestingly, we found that by combining the relationship of $m_{W}$ and $\sin^2(m_{Z})_{\text{MS}}$ with the $S$ and $T$ parameters we can constrain those parameters at a level consistent with more complete global fits [15]. We demonstrated that using this constraint, together with the usual theoretical conditions and several observations from LEP, Tevatron and LHC, there is a set of parameters that is compatible with the new measurement and, therefore, the 2HDM could successfully survive if the deviation on $m_{W}$ is confirmed in the future. In particular, we show how important the mass splittings are to shift the $W^\pm$ from its value predicted in the SM, mostly via the contribution from the oblique parameter $T$. Future experimental observations may bring more exciting clues about where to focus theoretical efforts.

[1] T. Aaltonen et al. [CDF], Science 376, no.6589, 170-176 (2022) doi:10.1126/science.abk1781
[2] M. Awramik, M. Czakon, A. Freitas and G. Weiglein, Phys. Rev. D 69, 053006 (2004) doi:10.1103/PhysRevD.69.053006 [arXiv:hep-ph/0311148 [hep-ph]].

[3] S. Schael et al. [ALEPH, DELPHI, L3, OPAL and LEP Electroweak], Phys. Rept. 532, 119-244 (2013) doi:10.1016/j.physrep.2013.07.004 [arXiv:1302.3415 [hep-ex]].

[4] M. Aaboud et al. [ATLAS], Eur. Phys. J. C 78, no.2, 110 (2018) [erratum: Eur. Phys. J. C 78, no.11, 898 (2018)] doi:10.1140/epjc/s10052-017-5475-4 [arXiv:1701.07240 [hep-ex]].

[5] T. A. Aaltonen et al. [CDF and D0], Phys. Rev. D 88, no.5, 052018 (2013) doi:10.1103/PhysRevD.88.052018 [arXiv:1307.7627 [hep-ex]].

[6] P. A. Zyla et al. [Particle Data Group], PTEP 2020, no.8, 083C01 (2020) doi:10.1093/ptep/ptaa104

[7] R. Aaij et al. [LHCb], JHEP 01, 036 (2022) doi:10.1007/JHEP01(2022)036 [arXiv:2109.01113 [hep-ex]].

[8] Y. Z. Fan, T. P. Tang, Y. L. S. Tsai and L. Wu, [arXiv:2204.03693 [hep-ph]].

[9] B. Y. Zhu, S. Li, J. G. Cheng, R. L. Li and Y. F. Liang, [arXiv:2204.04688 [astro-ph.HE]].

[10] H. Song, W. Su and M. Zhang, [arXiv:2204.05085 [hep-ph]].

[11] H. Bahl, J. Braathen and G. Weiglein, [arXiv:2204.05269 [hep-ph]].

[12] K. S. Babu, S. Jana and V. P. K., [arXiv:2204.05303 [hep-ph]].

[13] Y. Heo, D. W. Jung and J. S. Lee, [arXiv:2204.05728 [hep-ph]].

[14] G. Kreyerhoff and R. Rodenberg, Phys. Lett. B 226, 323 (1989); J. Freund, G. Kreyerhoff and R. Rodenberg, Phys. Lett. B 280, 267 (1992); B. M. Kastening, hep-ph/9307224; S. Nie and M. Sher, Phys. Lett. B 449, 89 (1999) [hep-ph/9811234]; S. Kanemura, T. Kasai and Y. Okada, Phys. Lett. B 471, 182 (1999) [hep-ph/9903289].

[15] C. T. Lu, L. Wu, Y. Wu and B. Zhu, [arXiv:2204.03796 [hep-ph]].

[16] P. M. Ferreira and D. R. T. Jones, JHEP 0908, 069 (2009) [arXiv:0903.2856 [hep-ph]].

[17] H. A. Weldon, Phys. Rev. D 30, 1547 (1984); S. Kanemura, T. Kubota and E. Takasugi, Phys. Lett. B 313, 155 (1993) [hep-ph/9303263]; A. G. Akeroyd, A. Arhrib and E. -M. Naimi, Phys. Lett. B 490, 119 (2000) [hep-ph/0006035]; I. F. Ginzburg and I. P. Ivanov, Phys. Rev. D 72, 115010 (2005) [hep-ph/0508020];

[18] D. Eriksson, J. Rathsman and O. Stal, Comput. Phys. Commun. 181, 189-205 (2010) doi:10.1016/j.cpc.2009.09.011 [arXiv:0902.0851 [hep-ph]].
G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, Phys. Rept. 516, 1 (2012) [arXiv:1106.0034 [hep-ph]].

J. F. Gunion and H. E. Haber, Phys. Rev. D 67, 075019 (2003) [hep-ph/0207010].

D. Toussaint, Phys. Rev. D 18, 1626 (1978).

S. Kanemura, Y. Okada, H. Taniguchi and K. Tsumura, Phys. Lett. B 704, 303 (2011) [arXiv:1108.3297 [hep-ph]].

M. Baak, M. Goebel, J. Haller, A. Hoecker, D. Ludwig, K. Moenig, M. Schott and J. Stelzer, Eur. Phys. J. C 72, 2003 (2012) [arXiv:1107.0975 [hep-ph]].

H. E. Haber and D. O’Neil, Phys. Rev. D 83, 055017 (2011) [arXiv:1011.6188 [hep-ph]].

John F. Gunion, Howard E. Haber, Gordon Kane, Sally Dawson, The Higgs Hunter’s Guide, Addison-Wesley Publishing Company, 1990.

K. Cheung and O. C. W. Kong, Phys. Rev. D 68, 053003 (2003) [hep-ph/0302111].

P. H. Chankowski, M. Krawczyk and J. Zochowski, Eur. Phys. J. C 11, 661 (1999) [hep-ph/9905436].

K. S. Kumar, S. Mantry, W. J. Marciano and P. A. Souder, Ann. Rev. Nucl. Part. Sci. 63, 237-267 (2013) doi:10.1146/annurev-nucl-102212-170556 [arXiv:1302.6263 [hep-ex]].

A. Sopczak, hep-ph/0502002.

S. Schael et al. [ALEPH and DELPHI and L3 and OPAL and LEP Working Group for Higgs Boson Searches Collaborations], Eur. Phys. J. C 47, 547 (2006) [hep-ex/0602042].

A. Heister et al. [ALEPH Collaboration], Phys. Lett. B 543, 1 (2002) [hep-ex/0207054].

T. V. Duong, E. Keith, E. Ma and H. Kikuchi, Phys. Rev. D 52, 5045 (1995) [hep-ph/9507276].

M. Misiak and M. Steinhauser, Eur. Phys. J. C 77, no.3, 201 (2017) doi:10.1140/epjc/s10052-017-4776-y [arXiv:1702.04571 [hep-ph]].

A. Abdesselam et al. [Belle], [arXiv:1608.02344 [hep-ex]].

P. M. Ferreira, H. E. Haber, R. Santos and J. P. Silva, arXiv:1211.3131 [hep-ph].

S. K. Kang, Z. Qian, J. Song and Y. W. Yoon, Phys. Rev. D 98, no.9, 095025 (2018) doi:10.1103/PhysRevD.98.095025 [arXiv:1810.0229 [hep-ph]]; T. Han, S. K. Kang and J. Sayre, JHEP 02, 097 (2016) doi:10.1007/JHEP02(2016)097 [arXiv:1511.05162 [hep-ph]]; S. Chang, S. K. Kang, J. P. Lee, K. Y. Lee, S. C. Park and J. Song, JHEP 09, 101 (2014) doi:10.1007/JHEP09(2014)101 [arXiv:1310.3374 [hep-ph]]; S. Chang, S. K. Kang, J. P. Lee, K. Y. Lee, S. C. Park and J. Song, JHEP 05, 075 (2013)
doi:10.1007/JHEP05(2013)075 [arXiv:1210.3439 [hep-ph]]; H. S. Cheon and S. K. Kang, JHEP 09, 085 (2013) doi:10.1007/JHEP09(2013)085 [arXiv:1207.1083 [hep-ph]].

[37] P. Bechtle, D. Dercks, S. Heinemeyer, T. Klingl, T. Stefaniak, G. Weiglein and J. Wittbrodt, Eur. Phys. J. C 80, no.12, 1211 (2020) doi:10.1140/epjc/s10052-020-08557-9 [arXiv:2006.06007 [hep-ph]].

[38] D. Foreman-Mackey, D. W. Hogg, D. Lang and J. Goodman, Publ. Astron. Soc. Pac. 125, 306-312 (2013). doi:10.1086/670067 [arXiv:1202.3665 [astro-ph.IM]].