\[ e^+e^- \rightarrow Z^0Z^0 \rightarrow b\bar{b}c\bar{c} \] events as model
independent probe of colour reconnection effects

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Abstract

According to the basic properties of QCD, colour reconnection effects
can occur in hadronic processes at high energies. The comparison of
\[ e^+e^- \rightarrow Z^0Z^0 \rightarrow q\bar{q}q'\bar{q}' \] events with the superposition of \[ Z^0 \rightarrow q\bar{q} \] and
\[ Z^0 \rightarrow q'\bar{q}' \] events from LEP1 would provide an unambiguous model-
independent probe. We show that at LEP2 energy, the background
processes are negligible if we select only \[ e^+e^- \rightarrow Z^0Z^0 \rightarrow b\bar{b}c\bar{c} \] events,
and limit the measurements in the phase space of on-shell \[ Z^0Z^0 \] events.

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The usual treatment of the hadronic processes in high energy collisions is divided into two phases, the perturbative parton cascade and the non-perturbative hadronization; and a phenomenological colour flow model (CFM) is used to assign the colour connections of the partons at the end of the first phase. These colour connections are the interface between the two phases, which is the starting point for the second phase. But it has been realized (see, e.g. [1] and references therein) that CFM is a good approximation only when \( N_C \to \infty \). With \( N_C = 3 \) as it is in QCD, colour connections of the partons can occur in many different ways. For example, for the \( q\bar{q} + n g (n > 1) \) system, as the final states of the perturbative phase in \( e^+e^- \) annihilation, strict PQCD calculations show [2] that many different parallel complete colour singlet sets can exist. Each of these sets can equivalently act as the bases of the colour space of the perturbative final state, but none of them is equivalent to the colour flow chain obtained from CFM [3]. PQCD cannot tell from which colour singlet set the hadronization starts. This implies that colour reconnection effects can be very significant in some cases.

A well known example which has been studied frequently in literature is the colour reconnection effects in \( e^+e^- \to W^+W^- \to \text{hadrons} \) at LEP2 energy. Here we have \( e^+e^- \to W^+W^- \to q_1\bar{q}_2q_3\bar{q}_4 \), and the two initial colour-singlet systems \( q_1\bar{q}_2 \) (from \( W^+ \)) and \( q_3\bar{q}_4 \) (from \( W^- \)) may be produced closely in space-time. The basic properties of QCD allow the colour reconnection to occur among partons from \( W^+ \) and \( W^- \) showers. Such effects destroy the naive picture of independent evolution and fragmentation of \( q_1\bar{q}_2 \) and \( q_3\bar{q}_4 \), respectively. This is one of the most important sources of the theoretical uncertainty in
determining the W mass and has attracted much attention in recent years (see [1, 4, 5, 6] and references therein). But up to now, the manifestations of colour reconnection effects in the final state hadrons can only be studied with the help of model-dependent Monte-Carlo events generators in which a number of approximations and/or assumptions are made. Some of these approximations and/or assumptions can be very sensitive to the colour reconnection effects. So the theoretical uncertainty is usually very large. Hence, a very crucial and urgent question in this study is how to probe such kind of effects in a model independent manner.

One candidate of such probe currently being discussed[3] is to compare pure hadronic decay $W^+W^- \rightarrow q_1\overline{q}_2q_3\overline{q}_4$, in which reconnection between $W^+$ and $W^-$ sources can occur, with semileptonic decay $W^+W^- \rightarrow q_1\overline{q}_2l\nu$ of the same kinematics. But as pointed out in [5], this comparison still suffers from strong dependence on the hadronization scenario and on the choice of model parameters. Moreover, results may be strongly sensitive to the adopted experimental strategy. On the other hand, above the $Z^0Z^0$ threshold at LEP2, the $e^+e^- \rightarrow Z^0Z^0 \rightarrow q\overline{q}'q'\overline{q}' \rightarrow hadrons$ events seem to be the most promising candidate in this aspect[7, 8]. Here the hadronic $Z^0Z^0$ events can contain similar colour reconnection effects as those in $W^+W^-$ events, while the single $Z^0 \rightarrow q\overline{q} \rightarrow hadrons$ data can be obtained from LEP1 without any theoretical uncertainty and with high precision. So the comparison of the experimental results in $e^+e^- \rightarrow Z^0Z^0 \rightarrow hadrons$ and the convolution of two single hadronic $Z^0$ event data from LEP1 will allow an unambiguous probe of the colour reconnection effects.
However, there is a great difficulty here, i.e. the background of \(Z^0 Z^0\) events is too large. For example, the cross sections of the corresponding \(W^+ W^-\) process and corresponding QCD process (\(e^+ e^-\) annihilating into any \(q\bar{q}\) pair) are both more than one order of magnitude larger than that of the signal process. Other electroweak processes like \(e^+ e^-\) annihilating through \(\gamma^* Z^0\) and \(\gamma^* \gamma^*\) into four quarks are also significant [9]. Can we reduce the background by selecting some specific type of events and/or limiting the measurement in some kinematic region? In this letter, we show that this question can be answered in the affirmative. By selecting the \(e^+ e^- \rightarrow Z^0 Z^0 \rightarrow b\bar{b} c\bar{c}\) events as the signal process and limiting the measurements in the phase space of on-shell \(Z^0 Z^0\) decays, which is the phase space for most of the \(Z^0 Z^0\) events above threshold, the contribution of background process can indeed be suppressed very much. Our numerical results show that they are even negligible compared to the signal process \(e^+ e^- \rightarrow Z^0 Z^0 \rightarrow b\bar{b} c\bar{c}\).

For the sake of explicity, we divide the background processes of hadronic \(Z^0 Z^0\) events into the following three types: 1.) the corresponding \(W^+ W^-\) process, 2.) the corresponding QCD process and 3.) other electroweak process. As pointed out above, the total cross section for them are more than one order of magnitude larger than that of the signal process. But these backgrounds can be greatly suppressed if the measurements are restricted in the following way.

First, \(Z^0\) is neutral, while \(W^+ W^-\) are charged. If we look only at

\[
e^+ e^- \rightarrow Z^0 Z^0 \rightarrow b\bar{b} c\bar{c}
\]  (1)
events as signal \( \text{(1)} \), the corresponding \( W^+W^- \rightarrow c\bar{b}b\bar{c} \) events are suppressed by the CKM matrix element \( |V_{cb}|^2 \sim 0.0016 \). This leads to a strong suppression of five order of magnitude. So we can neglect \( e^+e^- \rightarrow W^+W^- \rightarrow b\bar{b}c\bar{c} \) in comparison with the signal process(1) (hereafter, we will call the signal process(1) and 3.) other electroweak processes as all the EW process).

Second, comparing the signal process(1) to those in the remaining background processes, i.e.

2.) the QCD process

\[
e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow (b\bar{b})/(c\bar{c}) + g^* \downarrow (c\bar{c})/(b\bar{b})
\]

(2)

3.) other electroweak processes

\[
e^+e^- \rightarrow \gamma^*Z^0 \rightarrow b\bar{b}c\bar{c},
\]

(3)

\[
e^+e^- \rightarrow \gamma^*\gamma^* \rightarrow b\bar{b}c\bar{c},
\]

(4)

\[
e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow (b\bar{b})/(c\bar{c}) + \gamma^*/Z^0 \downarrow (c\bar{c})/(b\bar{b}).
\]

(5)

the \( b\bar{b} \) and \( c\bar{c} \) in \( \text{(1)} \) have the following important peculiarity: They originate predominantly from on-shell \( Z^0 \) bosons. This determines that the phase space of each of such kind of quarks and antiquarks is very limited: It is easy to see

\(^1\)Here we select \( b\bar{b}c\bar{c} \) events but not \( b\bar{b}b\bar{b} \) or \( c\bar{c}c\bar{c} \) to avoid identical particle effects. However, if some colour reconnection effects have no relation with identical particle effects, the \( b\bar{b}b\bar{b} \) and \( c\bar{c}c\bar{c} \) events, which will not be created via \( W^+W^- \) decay at all, can be included in those measurements. The following qualitative and quantitative discussions are quite the same, while the events which can be selected in experiments are increased to about 3 times.

\(^2\)We also note that heavy quarks are created only in the perturbative phase and can be easily identified in experiments.
that, at $\sqrt{S} = 2M_Z$ (where $M_Z$ is $Z^0$ mass), the velocities of the two on-shell $Z^0$'s, $\beta = \sqrt{1 - 4M_Z^2/S}$, are both zero. Thus this four quarks and antiquarks from on-shell $Z^0$ decay have the same energy $M_Z/2$. But for $\beta = 0$, the on-shell $Z^0Z^0$ cross section $\sigma_{ZZ} = 0$. Therefore we should study the process (1) above $Z^0Z^0$ threshold, i.e. $\sqrt{S} > 2M_Z$. In this case, if the quark mass is neglected, the quark energy $E_i (i = c, \bar{c}, b, \bar{b})$ in the on-shell $Z^0Z^0$ decay satisfies,

$$
\frac{M_Z (1 - \beta)}{2\sqrt{1 - \beta^2}} \leq E_i \leq \frac{M_Z (1 + \beta)}{2\sqrt{1 - \beta^2}}. \quad (6)
$$

This shows that $E_i$ is limited to a given small region. Obviously, this range becomes wider as $\sqrt{S}$ increases, but even at the highest energy of LEP2, i.e. $\sqrt{S} = 200 \text{ GeV}$, we have

$$29.5 \text{ GeV} \leq E_i \leq 70.5 \text{ GeV}, \quad (7)$$

which is still a very narrow region compared to the processes (2)-(5), in which the quark energy can range from 0 to $\sqrt{S}/2$, since there are non-resonant intermediate states (eg. $\gamma^*$, $Z^0^*$, $g^*$). If only the energy range given by Eq. (6) is considered in measurement, the phase space of the four quarks in background processes (2)-(5) is restricted into a small part of the total, so that their cross sections are all strongly suppressed.

To show these effects quantitatively, we calculate the cross sections and differential cross sections of processes (1)-(5) in groups. For the comparison of the signal process to its background, (1) should be studied separately. In the QCD process (2), one of the two quark pairs originates from a colour-octet gluon, so both $c\bar{c}$ and $b\bar{b}$ are in colour-octets. In all the other processes, they
are in colour-singlets. Therefore there is no interference between process(2) and the others. Hence the QCD process(2) can be studied separately. While all the other processes we consider here[(1) and (3)-(5)] can lead to the state with exactly the same quantum numbers, thus can interfere with each other, so we should include the contributions from all of the interference terms. We note that the scattering matrices for these processes can be calculated using the perturbation theory in the standard model for electroweak and strong interactions. The numerical results for the cross sections are obtained by integrated in the 8-dimensional phase space of four fermion system which is parameterized as usual(see, e.g.[11]).

Before we present the calculated results for the cross sections, we would like to mention the following. we note that in general, a quark fragments into a hadron jet which can be observed in experiments. But if a quark is soft or collinear with other quarks, this quark cannot form a resolvable hadron jet. There are several different schemes to define a resolvable hadron jet. We use the Durham scheme[12]. According to this scheme, a quark $i$ and another quark $j$ can form two resolvable jets if their energies $E_i, E_j$ and the angle $\theta_{ij}$ between their moving directions satisfy the condition $y_{ij} > y_{cut}$. Here

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{S}, \quad (i \neq j)$$

and $y_{cut}$ is taken as 0.0015, as an example, same as [7]. If $y_{ij} > y_{cut}$ for all possible permutations of $i$ and $j$, we say that these four quarks fragment into four different hadron jets. This criteria is applied in our calculation, thus the numerical results we present in the following should be understood as those
for $b\bar{c}\bar{c}$ four jets.

We now take $\sqrt{S} = 200\ GeV$ as an example and show the results for the total cross sections of the $Z^0Z^0$ process(1), the corresponding QCD process(2) and all EW processes[(1) and (3)-(5)] in Fig.1. The shaded areas represent the corresponding cross sections $\sigma_{ZZ}^c, \sigma_{EW}^c, \sigma_{QCD}^c$ with the constraint(7) for the phase space. From the figure, we see clearly that the $Z^0Z^0$ events dominate the $b\bar{c}\bar{c}$ four jet events, especially in the phase space as limited in (7). More precisely, we see that the $Z^0Z^0$ events in (7) take about 94% of the whole(including off-shell) $Z^0Z^0$ events, this implies our criteria has selected most of the $Z^0Z^0$ events; and that $\sigma_{ZZ}^c \sim \sigma_{EW}^c$, with $\delta_{EW} = \frac{\sigma_{EW} - \sigma_{ZZ}^c}{\sigma_{ZZ}^c} \sim 3\%$, this shows clearly that in the limited phase space contributions from the non-$Z^0Z^0$ EW processes and all interference between every two single processes are negligible. We also see $\delta_{QCD} = \frac{\sigma_{QCD}^c}{\sigma_{ZZ}^c} \sim 0.7\%$. This means that the corresponding QCD process is really negligible.

In Fig. 2 we show the corresponding energy distributions $d\sigma/dE$. In the case that the quark mass are neglected, the symmetry of the matrix element under the interchange of (anti)quark labels implies that the energy distributions are identical for the four quarks and antiquarks in the above mentioned processes. From this figure, we see again the dominance of $Z^0Z^0$ events in the energy range(7): The distribution curve of $Z^0Z^0$ process and that of all the EW processes show a high platform, and they almost coincide in this range. While outside range(7), the $Z^0Z^0$ curve drop very fast and the other process become dominant. For the QCD process, the energy distribution is peaked near the two edges: $E_j = 0$ and $E_j = \sqrt{S}/2$, since in these events we have quarks
both from the decay of $\gamma^*/Z^0*$ with virtuality $\sqrt{S}$ (and thus the quark energy $E_j$ is $\sqrt{S}/2$) and those from soft virtual gluons (i.e. $E_j \to 0$). but it is an order of magnitude lower than that of the $Z^0Z^0$ and the EW processes under the range (7). These results show clearly the efficiency of the restriction(7). It picks most of the signal events but drops a large part of the background events.

It may be also interesting to look at the angular distribution of these processes because of the following. In the $Z^0Z^0$ process (1), the colour reconnection may lead to two new colour singlets $b\bar{c}$ and $c\bar{b}$ if $b$ and $\bar{c}$ ($c$ and $\bar{b}$) are sufficiently close to each other in phase space. This means that the colour reconnection has large probability to occur if the angle $\theta$ between $b$ and $\bar{c}$ (or $c$ and $\bar{b}$) is small. We show in Fig. 3 the distribution $d\sigma/d\cos\theta$ versus angular separation between $c$ and $\bar{b}$ for the $Z^0Z^0$ process, the QCD process and the whole EW processes under condition (7). The angular distributions for the $Z^0Z^0$ process and the EW processes almost coincide, the same feature as in Fig 2. For the QCD process, the $c\bar{b}$ angular distribution is peaked in the back-to-back direction, which reflects the dominance of the back-to-back configuration for $c\bar{b}$ (and also $b\bar{c}$) with the virtual gluon preferentially emitted along the quark or antiquark directions. For $\cos\theta \to 1$, the distribution is strongly suppressed by $y_{cut}$, while for $\cos\theta \to -1$, it is slightly restricted by the condition (7). Obviously, when $\theta$ is less than about 53 degree, the differential cross section of the QCD process is at least three orders of magnitude lower than the $Z^0Z^0$ one, which makes the measurements of the possible colour reconnection effects in the real $Z^0Z^0$ process more feasible.
In summary, the study of the colour reconnection effects in hadronic reactions is of fundamental significance in understanding QCD. High statistical data above the $Z^0 Z^0$ threshold at LEP2 may allow a model-independent probe of these effects. The background processes are suppressed to a negligible level if we choose only $e^+ e^- \rightarrow Z^0 Z^0 \rightarrow b\bar{b}c\bar{c}$ as signal events and limit the measurement in the energy range given by (6). Qualitative analysis and quantitative results presented in this letter show the following: First, the greatest pollution to the signal process, the corresponding $W^+ W^-$ process, can be dropped by the CKM suppression for $c, \bar{b}$ ($b, \bar{c}$) pair. Second, in energy range(3), the pollution from the corresponding QCD process, that from other electroweak processes and that from all electroweak interference terms are negligibly small, while most of the $Z^0 Z^0$ events are picked. Furthermore, limiting the angle between $b$ and $\bar{c}$ (or between $c$ and $\bar{b}$) to small values, where colour reconnections occur with larger possibilities, the QCD background will be further suppressed. So comparing $e^+ e^- \rightarrow Z^0 Z^0 \rightarrow b\bar{b}c\bar{c}$ events with the superposition of the corresponding $Z^0 \rightarrow b\bar{c}$ and $Z^0 \rightarrow c\bar{c}$ events from LEPI would provide an unambiguous model-independent probe of the colour reconnection effects.

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FIGURE CAPTIONS

Fig. 1. The total cross section for $e^+e^- \to Z^0Z^0 \to b\bar{b}c\bar{c}$, that for the whole EW processes and that for the corresponding QCD process at $\sqrt{S} = 200GeV$. The shaded area represents the corresponding cross section under the restriction $29.5GeV < E_i < 70.5GeV$. In the calculations here and following $\alpha_s$ is set to 0.1.

Fig. 2. The energy distribution $\frac{d\sigma}{dE_b}$ of the $Z^0Z^0$ process (solid line), the whole EW processes (dashed line) and the QCD process (dotted line). The dash-dotted line denotes the edges of the energy range $29.5GeV < E < 70.5GeV$.

Fig. 3. The angular distribution $\frac{d\sigma}{d\cos\theta}$ of the $Z^0Z^0$ process (solid line), the whole EW processes (dotted line) and the QCD process (dashed line) under the restriction $29.5GeV < E < 70.5GeV$. 
Figure 1:
Figure 2:
Figure 3: