An explanation of the muon puzzle of ultrahigh-energy cosmic rays and the role of the Forward Physics Facility for model improvement

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We investigate the observed muon deficit in air shower simulations when compared to ultrahigh-energy cosmic ray (UHECR) data. Based upon the observed enhancement of strangeness production in high-energy hadronic collisions reported by the ALICE Collaboration, the concomitant $\pi \leftrightarrow K$ swap is considered as the keystone to resolve the muon anomaly through its corresponding impact on the shower development. We construct a toy model in terms of the $\pi \leftrightarrow K$ swapping probability $F_\pi$. We present a parametrization of $F_\pi$ in terms of the pseudorapidity that can accommodate the UHECR data. Looking to the future, we explore potential strategies for model improvement using the massive amounts of data to be collected by LHC neutrino detectors, such as FASERv and experiments at the Forward Physics Facility. We calculate the corresponding sensitivity to $F_\pi$ and show that these experiments will be able to probe the model phase space.

I. INTRODUCTION

Ultra-high-energy ($10^8 \leq E/\text{GeV} \leq 10^{11}$) cosmic ray (UHECR) collisions have center-of-mass energies ($50 \leq \sqrt{s}/\text{TeV} \leq 450$) well beyond those achieved at collider experiments, and thereby provide an invaluable probe of particle interactions below the fermi distance [1]. Of particular interest here, the highest energy cosmic rays currently observed by the Pierre Auger Observatory (Auger) [2–4] and the Telescope Array [5–7] show a significant discrepancy in the shower muon content when compared to predictions of LHC-tuned hadronic event generators [8]. More concretely, the analysis of Auger data suggests that the hadronic component of showers (with primary energy $10^{9.8} < E/\text{GeV} < 10^{10.2}$) contains about 30% to 60% more muons than expected. The significance of the discrepancy between Auger data and model prediction is somewhat above 2.1$\sigma$ [4]. Auger findings have been recently confirmed studying air shower measurements over a wide range of energies. The muon deficit between simulation and data, dubbed the muon puzzle, starts at $E \sim 10^8$ GeV increasing noticeably as primary energy grows, with a slope which was found to be significant at about $8\sigma$ [9–11].

Certainly, in solving the muon puzzle one has to simultaneously get a good agreement with the measurements of the distribution of the depth of shower maximum $X_{\text{max}}$ [12], and the fluctuations in the number of muons [13]. A thorough phenomenological study has shown that an unrivaled solution to the muon deficit, compatible with the observed $X_{\text{max}}$ distributions, is to reduce the transfer of energy from the hadronic shower into the electromagnetic shower, by reducing the production or decay of neutral pions [14]. Several models have been proposed to accommodate this effect, including those wherein strangeness production suppresses the pion-to-kaon ratio [15–17]. This modification could have a compounded effect on the hadronic cascade, so that only a comparably small reduction of $\pi^0$ production is required.

We note in passing that the proposed enhancement of strangeness production in high-energy hadronic collisions was observed by ALICE in the mid-rapidity region [18]. Specifically, ALICE observations show an enhancement of the yield ratio of strange and multi-strange hadrons to charged pions as a function of multiplicity at mid-rapidity not only in PbPb and XeXe collisions but also in $pp$ and PbPb scattering [19]. It goes without saying that none of the hadronic interaction models currently used in air shower simulations correctly reproduce ALICE data [20]. Assuming that the observed enhancement of strangeness production in high-energy hadronic collisions is at the core of the muon puzzle in this paper we study the concomitant $\pi \leftrightarrow K$ swap impact on the development of extensive air showers (EASs), using phenomenological toy models implemented in AIRES (version 19.04.08) [21]. After that, we discuss the prospects to constrain our model using forward neutrino flux measurements at FASERv [22, 23] and future experiments at the Forward Physics Facility (FPF) [24].

There are two points worth noting at this juncture: (i) The mid-rapidity region in which the ALICE Collaboration reported a universal strangeness enhancement in $pp$, $\text{PbPb}$ and $\text{PbPb}$ collisions is not directly relevant for air showers experiments. It has not been observed experimentally yet whether these effects could also be seen in hadrons produced at forward rapidities. This is the main assumption of our model, which will be directly tested at the FPF. (ii) Accommodating the muon deficit between simulations and data can be virtually reduced to a constant factor, which is independent of the primary
energy [25]. In our toy model this factor is taken to be related to the $\pi \leftrightarrow K$ swapping probability.

The layout of the paper is as follows. In Sec. II we first discuss general aspects of a toy model and describe the (input and output) AIRES module interface. Armed with the new AIRES module we confront the toy model with Auger data. We perform a parameter scan using results of EAS simulations and determine the phase space boundaries of the $\pi \leftrightarrow K$ swapping probability from experimental data. In Sec. III we improve our toy model to transform it into a predictive model. We present a parametrization of the $\pi \leftrightarrow K$ swapping probability in terms of the pseudorapidity that can accommodate the UHECR data. In Sec. IV we investigate the sensitivity to the $\pi \leftrightarrow K$ swapping probability at FASERν and the FPF and demonstrate that a direct test of the model predictions is indeed feasible. The paper wraps up with some conclusions presented in Sec. V.

II. A TOY MODEL

To describe the shower evolution we adopt the AIRES simulation engine [21] which provides full space-time particle propagation in a realistic environment. The features of the AIRES version used for this work (19.04.08) are explained in detail in Ref. [21].

For the present analysis, we prepared a new module to account for the possible enhancement of strangeness production in high-energy hadronic collisions. Every time a hadronic collision is processed, the list of secondary particles obtained from the external event generator invoked (for our analysis we adopt SIBYLL 2.3d [26]) is scanned by the new module before passing it to the main particle propagating engine. The main characteristics of the new AIRES module are as follows.

A. Model Parameters

Swapping fraction $f_s$ Controls the kind and number of secondary particles that are affected by change of identity: $-1 \leq f_s \leq 1$. In this zeroth-order approximation we take the swapping probability $F_s = f_s$.

Projectile energy range $E_{p_{min}} - E_{p_{max}}$ Particle swapping is performed only in hadronic collisions where the projectile kinetic energy verifies $E_{p_{min}} \leq E_{proj} < E_{p_{max}}$. $E_{p_{min}}$ must be larger than 900 MeV and less than $E_{p_{max}}$. We set $E_{p_{max}} \rightarrow \infty$ unless otherwise specified.

Secondary energy range $E_{s_{min}} - E_{s_{max}}$ Secondary particles with kinetic energies out of the range $[E_{s_{min}}, E_{s_{max}}]$ are always left unchanged. $E_{s_{min}}$ must be larger than 600 MeV and less than $E_{s_{max}}$. We set $E_{s_{min}} = 1$ TeV and $E_{s_{max}} \rightarrow \infty$ unless otherwise specified.

B. Logics of Hadronic Collision Post-Processing

During shower simulation, hadronic collisions are processed via calls to an event generator; we adopt SIBYLL 2.3d [26]. The input parameters for these calls are the projectile identity $p_{i_{proj}}$, its kinetic energy $E_{proj}$, and the target identity. On return, the generator provides a list of $N_{sec}$ particles, specifying their identity $i_{sec}$, energy $E_{sec}$, momentum, etcetera, with $i = 1, \cdots, N_{sec}$.

All the returned secondary particle lists undergo a post-processing process, just before they are stacked into the particle stacks for further propagation. The post-processing algorithm obeys the following rules:

1. If $f_s = 0$ or $E_{proj} < E_{p_{min}}$ or $E_{proj} > E_{p_{max}}$ then no action is taken; the secondary particle list remains unchanged.

2. If $f_s \neq 0$ and $E_{p_{min}} \leq E_{proj} \leq E_{p_{max}}$ then the list of secondaries is scanned, and processed as follows:

   (a) If $f_s > 0$, all the secondary pions whose kinetic energies lie within the interval $[E_{s_{min}}, E_{s_{max}}]$ are considered for identity swapping. Each of them is randomly selected with probability $|f_s|$. In case of positive selection, the identity is changed with the following criteria:

   i. Each $\pi^0$ is transformed onto $K^0_S$ of $K^0_L$, with 50% chance between them.

   ii. Each $\pi^+$ ($\pi^-$) is transformed onto $K^+$ ($K^-$).

   (b) If $f_s < 0$, all the secondary kaons whose kinetic energies lie within the interval $[E_{s_{min}}, E_{s_{max}}]$ are considered for identity swapping. Each of them is randomly selected...
FIG. 1: \( z(R_{\mu}) \), \( z(N_{\text{max}}) \), and \( z(X_{\text{max}}) \) as a function of \( f_s \), for \( E_{\text{proj}} = 10 \text{ EeV}, E_{\text{min}} = 1 \text{ TeV}, \) and \( E_{\text{pmin}} = 1 \text{ PeV} \). We have run 1600 (20000) showers per point for ground muons (longitudinal development), setting at each case the thinning algorithm parameters to get a more detailed simulation of the hadronic or the electromagnetic cascade, respectively.

FIG. 2: \( z(R_{\mu}) \) versus \( f_s \) (left), \( z(N_{\text{max}}) \) versus \( f_s \) (middle), and \( z(R_{\mu}) \) versus \( z(N_{\text{max}}) \) (right), for varying \( E_{\text{min}} \) (upper), and \( E_{\text{pmin}} \) (lower).
with probability \( |f_i| \). In case of positive selection, the identity is changed with the following criterion:

i. Each \( K^0 \) or \( K^0 \) is transformed onto \( \pi^0 \).

ii. Each \( K^+ (K^-) \) is transformed onto \( \pi^+ (\pi^-) \).

3. The kinetic energy of swapped particles is set so as to keep total energy conserved.

C. Air Shower Simulations

To characterize the possible cross-correlation among selected observables we have simulated more than a million showers with incident zenith angles of 45° and 67°. The shower observables relevant to our analysis are:

- the depth of maximum shower development \( X_{\text{max}} \) and its fluctuations \( \sigma X_{\text{max}} \);
- the dimensionless muon content \( R_\mu = N_\mu/N_{\mu,19} \) and its fluctuations \( \sigma R_\mu \), where \( N_\mu \) is the total number of muons (with \( E_\mu > 300 \text{ MeV} \)) at ground level and \( N_{\mu,19} = 1.455 \times 10^7 \) is the average number of muons in simulated proton showers at 10\(^3\) eV with incident angle of 45°;
- the number of charged particles at the shower maximum \( N_{\text{max}} \).

For each observable \( O \), we evaluate

\[
    z(O) = \frac{(O(f_i))}{(O(f_i = 0))},
\]

(1)
to work with normalized variables.

In Fig. 1, we show \( z(R_\mu) \), \( z(N_{\text{max}}) \), and \( z(X_{\text{max}}) \), as a function of \( f_i \), for \( E = 10 \text{ EeV} \), \( E_{\text{min}} = 1 \text{ TeV} \), and \( E_{\text{pmin}} = 1 \text{ PeV} \), with both \( E_{\text{max}} \) and \( E_{\text{pmax}} \) set to infinite. Note that this particular \( E_{\text{pmin}} \) corresponds to hadronic interactions at \( \sqrt{s_{\text{NN}}} = 1.41 \text{ TeV} \), just below the energy \( (\sqrt{s_{\text{NN}}} \approx 2.76 \text{ TeV}) \) where the ALICE Collaboration reported a smooth rise of the hyperon-to-pion ratio [27].

Note also that for \( f_i < 0 \), kaons are changed into pions, whereas for \( f_i > 0 \), pions are changed into kaons, with progressive probability equal to \( |f_i| \). The simulations to evaluate \( X_{\text{max}} \) are always carried out using inclined showers at 45°. The variations in \( X_{\text{max}} \) fluctuations (not shown in the figure) are very small: \( |z(\sigma X_{\text{max}}) - 1| < 0.03 \) for all \( f_i \in [-1,1] \). Taking \( f_i \approx 0.4 \) as fiducial we observe a change in \( R_\mu \) of roughly 20% for showers initiated by protons and 10% in those initiated by iron. These variations correspond to a reduction of \( N_{\text{max}} \) by about 3%. In the right panel of Fig. 1 we can see that the model predictions on \( X_{\text{max}} \) vary less than 1.5% when compared to the \( f_i = 0 \) result. Similarly, the fluctuations \( \sigma X_{\text{max}} \) vary by less than 3%. Our analysis thus corroborates the results presented in [14], which show that by suppressing the \( \pi^0 \) energy fraction we can obtain an increase in the number of muons at ground without coming into conflict with \( X_{\text{max}} \) observations.

To study the model dependence with \( E_{\text{min}} \) and \( E_{\text{pmin}} \), we use proton induced showers. In Fig. 2, we show the dependences of \( R_\mu \) and \( N_{\text{max}} \) with \( E_{\text{min}} \) (upper row) and \( E_{\text{pmin}} \) (lower row). We can see that the change of \( E_{\text{min}} \) leads to negligible effects, and that there is virtually no difference between \( E_{\text{pmin}} = 90 \text{ GeV} \) and \( E_{\text{pmin}} = 10 \text{ TeV} \), indicating a saturation effect; see Appendix A. These are, however, unrealistic energy thresholds. A linear dependence between the two observables is evident, especially for \( z(R_\mu) \approx 1 \). The physically unrealistic case of \( E_{\text{pmin}} = 90 \text{ GeV} \) is the one that presents the largest departure from linearity.

In the spirit of [25], we now incorporate the change of the nuclear composition of the cosmic ray primary [28] and study the variation of \( (R_\mu)/(E/10 \text{ EeV}) \). As displayed in Fig. 3, the effect of increasing \( R_\mu \) yields a flattening of the curve when compared to the \( f_i = 0 \) prediction. Even though \( f_i \approx 0.4 \) seems to roughly accommodate the data around \( E \approx 10^3 \text{ eV} \), it is clear from the shape of the best-fit curve that to describe the muon anomaly in a larger energy range we would need an energy-dependent \( f_i \); see Fig. 4.

We note, however, that this zeroth order approximation should be understood as an effective (macroscopic) description of the entire shower evolution, rather than a collection of individual interactions generated by a homogeneous beam of projectiles. In this approach \( E_{\text{pmin}} \) is no less important than \( E_{\text{min}} \) and for a \( 10^{10} \text{ GeV} \) proton shower with \( f_i = 0.7 \) the number of pions effectively swapped barely exceeds 0.5% of the total number of secondaries generated in shower. Global observables, such as the number of muons at ground level, were obtained after adding and averaging heaps of individual contributions, a process in which statistics erases many “microscopic” details.

III. MODEL REFINEMENT

In the previous section we have shown that the zeroth order approximation toy model gives a fair description of all shower observables. However, there are two important caveats with this toy model. Firstly, heavy flavor production should be enhanced in kinematic regimes where quark masses may be insignificant. This implies that a more realistic parametrization of \( F_s \), which can accurately describe single particle collisions, should depend on pseudorapidity. Secondly, the shape of the best-fit curve to Auger data is driven by both strangeness effects, and that there is virtually no dependence between the two observables is evident; especially for \( z(R_\mu) \approx 1 \). The physically unrealistic case of \( E_{\text{pmin}} = 90 \text{ GeV} \) is the one that presents the largest departure from linearity.
and so for simplicity, we will neglect A-induced effects in our study. Future LHC data (including pO and OO collisions [30]) will provide new insights to reduce these uncertainties and guide software development.

The Lorentz transformation between the center-of-mass (CM) and laboratory (LAB) systems is given by

$$E_{LAB} = \gamma (E_{CM} + \beta p_{long,CM}),$$

where $\gamma$ is the Lorentz factor and $\beta$ the velocity of the CM with respect to the LAB frame. For ultrarelativistic particles, $\beta \sim 1$ and $p_{long,CM} \sim E_{CM} \cos \theta_{CM}$, where $\theta_{CM}$ is the angle of the secondary particle’s momentum with respect to the axis where the projectile of the collision moves (i.e. direction of the beam). A straightforward substitution leads to

$$E_{LAB} \sim \gamma E_{CM} (1 + \cos \theta_{CM}).$$

At first sight one may conjecture that the imposed lower limit on $E_{min}$ in our toy model is inconsistent with the description of hadronic collisions as $0 < E_{LAB} < 2\gamma E_{CM}$. To inspect the forward-backward directions in the CM frame we conveniently work with the pseudorapidity

$$\eta_{CM} = -\ln \left( \tan \left( \frac{\theta_{CM}}{2} \right) \right).$$

The forward-backward symmetry of Eq. (3) is evident in the pion pseudorapidity distributions shown in the upper row of Fig. 5. We note that the toy model approximation $E_{min} = 1$ TeV breaks this symmetry when going into the LAB frame; see the lower row of Fig. 5. In particular, pions with $\eta_{CM} < -4$ are not considered for swapping in the AIRES module described in Sec. II. The relation between the CM and LAB pseudorapidity is displayed in the scatter plots of Fig. 6. It is important to stressed that the densities of dots in different places of these plots may not accurately represent the actual number of secondaries produced in the collision has been represented. The sampling was performed trying to obtain a uniform coverage of the entire range of CM pseudorapidities of the secondaries. To this end, the $-\infty < \eta_{CM} < \infty$ axis is partitioned in consecutive intervals, with extremes at the points $-\infty, -10, -7, -5, -4, -3, -2, 0, 2, 3, 4, 5, 7, 10, \infty$, and then the entire set of secondary pions emerging from
As the shower develops in the atmosphere, the hadrons propagate through a medium with an increasing density while the altitude decreases and the hadron-air cross section rises slowly with energy. Thereby, the probability for interacting with the air molecules before decay increases with rising energy. Furthermore, the relativistic time dilation increases the decay length by a factor $E_h/m_h$, where $E_h$ and $m_h$ are the energy and mass of the produced hadron. The $\pi^0$'s, with a lifetime of $\approx 8.4 \times 10^{-17}$ s, do decay promptly to two photons, feeding the electromagnetic component of the shower. To see how neutral kaons could suppressed this process, it is instructive to estimate the critical energy at which the chances for interaction and decay are equal for other longer-lived mesons. For a vertical transversal of the atmosphere, the critical energy is found to be: $\xi_{\pi^\pm} \sim 115$ GeV, $\xi_{K^0} \sim 850$ GeV, $\xi_{K^0_S} \sim 210$ GeV, $\xi_{K^0_L} \sim 30$ TeV [31]. The dominant $K^+$ branching ratios are to $\mu^+\nu_\mu$ (64%), to $\pi^+\pi^-\pi^0$ (21%), to $\pi^+\pi^-\pi^0$ (6%), and to $\pi^+\pi^0\pi^0$ (2%), whereas those of the $K^0_S$ are to $\pi^+\pi^-\pi^0$ (60%), to $\pi^0\pi^0$ (30%), and for $K^0_L$ we have $\pi^+\pi^-\nu_\ell$ (40%), $\pi^+\pi^-\nu_\mu$ (27%), $\pi^0\pi^0\pi^0$ (19%), $\pi^+\pi^-\pi^0$ (12%) [32]. Using these branching fractions, to a first approximation we can estimate that in each generation of particles about 25% of the energy is transferred to the electromagnetic shower, and all hadrons with energy $\gtrsim \xi_{\pi^\pm}$ interact rather than decay, continuing to produce the hadronic shower [33, 34]. Eventually, the electromagnetic cascade dissipates around 90% of
the primary particle’s energy and the remaining 10% is carried by muons and neutrinos. Even though these numbers depend on the incident zenith angle of the primary cosmic ray we note that very low energy kaons will decay before interacting to feed the electromagnetic showers in similar way neutral pions do. Therefore, the required symmetry with respect to the CM pseudorapidity seems to indicate that there must be swapping of some pions which do not produce an overall effect on the shower evolution. Taking these considerations into account, we are ready to amend the AIRES module.

Before proceeding, we pause to note that we have verified that there is no significant difference in the scattering predictions by changing the hadronic interaction model. For a direct comparison, in Appendix B we show the pion, kaon, and nucleon bivariate distributions for the same particle collisions, but simulated with EPOS-LHC [35].

In what follows we refer to the measurements/tunes performed in the “central” and “forward” regions, as defined with respect to the CM pseudorapidity of the particles. The central pseudorapidity region is defined as $|\eta_{CM}| \leq 2.5$, corresponding to the ATLAS [36], CMS [37] and ALICE [38] acceptances, and the forward pseudorapidity region as $|\eta_{CM}| \geq 2.5$. It is generally thought that the observed differences between data and simulation originate, in most part, due to the model extrapolation from the central pseudorapidity region, in which the hadronic event generators adopted in UHECR shower simulations are mainly tuned. We therefore assume herein that the enhancement of strangeness production is negligible for $|\eta_{CM}| < 4$ (more on this below). The free parameters of the refined model are defined as follows:
Swapping probability \( F_s(\eta_{CM}) \) \( \ldots \ldots \) Controls the number of secondary pions that are affected by change of identity. \( F_s \) depends on the centre of mass pseudorapidity of the secondary particles, \( \eta_{CM} \), and must verify \( 0 \leq F_s \leq 1 \). Unless otherwise specified, we use

\[
F_s(\eta_{CM}) = \begin{cases} f_s & \text{if } -\infty < \eta_{CM} < -4 \\ 0 & \text{if } -4 \leq \eta_{CM} \leq 4 \\ f_s & \text{if } 4 < \eta_{CM} < \infty \end{cases}
\]

with \( 0 \leq f_s \leq 1 \).

Minimum projectile energy \( \ldots \ldots \) \( E_{p\text{min}} \)

Particle swapping is performed in hadronic collisions whose projectile kinetic energy is larger than this energy. \( E_{p\text{min}} \) must be larger than 900 MeV. As in our toy model we take \( E_{p\text{min}} = 1 \text{ PeV} \).

Minimum secondary energy \( \ldots \ldots \) \( E_{s\text{min}} \)

Secondary particles with kinetic energies below this threshold are always left unchanged. \( E_{s\text{min}} \) must be larger than 600 MeV. To sample the entire CM pseudorapidity region we take \( E_{s\text{min}} = 1 \text{ GeV} \).

The logics of the hadronic collision post-processing remains the same to that discussed in Sec. II B.

In Fig. 7 we show \( z(R_p), z(\sigma R_p), z(N_{\text{max}}) \), and \( z(X_{\text{max}}) \) as a function of \( f_s \), for \( E = 10 \text{ EeV}, E_{s\text{min}} = 1 \text{ GeV} \), and \( E_{p\text{min}} = 1 \text{ PeV} \). We can see that there are no significant changes with respect to the results shown in Fig. 1 for the toy model. It is remarkable that \( \forall f_s \) we have \( \sigma R_p < R_p \), in agreement with Auger observations [13]. In addition, for the fluctuations of \( X_{\text{max}} \) (not shown in the figure) we reobtain that \( |z(\sigma X_{\text{max}})| < 0.03 \) for all \( f_s \in [0, 1] \). This is because the secondaries emitted in the central pseudorapidity region have minimal impact on the evolution of the shower. This is visible in Fig. 8 where we show \( z(R_p) \) as a function of \( f_s \), but with varying limits of the periferic (pl) and central (cl) regions; namely,

\[
F_s^{\text{pl}}(\eta_{CM}) = \begin{cases} f_s & \text{if } -\infty < \eta_{CM} < -\eta_{pl} \\ 0 & \text{if } -\eta_{pl} \leq \eta_{CM} \leq \eta_{pl} \\ f_s & \text{if } \eta_{pl} < \eta_{CM} < \infty \end{cases}
\]

and

\[
F_s^{\text{cl}}(\eta_{CM}) = \begin{cases} 0 & \text{if } -\infty < \eta_{CM} < -\eta_{cl} \\ f_s & \text{if } -\eta_{cl} \leq \eta_{CM} \leq \eta_{cl} \\ 0 & \text{if } \eta_{cl} < \eta_{CM} < \infty \end{cases}
\]

respectively. Moreover, the plots in Fig. 8 clearly show that setting \( \eta_{pl} = 3 \) or \( 4 \) return virtually the same results. For \( \eta_{pl} > 4 \), the impact of \( \pi \rightarrow K \) swapping diminish with increasing \( \eta_{pl} \), as expected, until presenting a virtually zero impact for \( \eta_{pl} = 12 \). Complementary, the curves displayed in the right panel show that the impact of \( \pi \rightarrow K \) swapping increases monotonically as long as the “central” region considered gets progressively wider. For \( \eta_{cl} < 4 \), the central region provides a negligible contribution to \( z(R_p) \).

In Fig. 9 we show \( \langle R_p \rangle/(E/10 \text{ EeV}) \) considering the variation of UHECR composition shown in Fig. 3 and \( F_s(\eta_{CM}) \) as defined in Eq. (5). As expected from the discussion above, there is no significant differences with the results displayed in Fig. 3 for the toy model of Sec. II.

A few crosschecks on these considerations are in order. In Tables I and II we provide a summary of the global counters of shower simulations using the toy model and the refined model, respectively, with \( f_s = 0.7 \). It is interesting to note that the percentage the pions produced above \( E_{p\text{min}} \) remains the same and is slightly smaller than 2%. In addition, the number of collisions and consequently the number of secondaries being produced, decreases when considering the refined model. This is because in the toy model we consider secondary neutral pions from the central region with LAB energy above 1 TeV, and if these pions mutate into kaons they would most likely interact before decaying, yielding more collision.
FIG. 7: \( z(R_{\mu}) \), \( z(\sigma R_{\mu}) \), \( z(N_{\text{max}}) \) and \( z(X_{\text{max}}) \) as a function of \( f_s \), for \( E_{\text{prim}} = 10 \text{ EeV}, E_{\text{min}} = 1 \text{ GeV}, \) and \( E_{\text{pmin}} = 1 \text{ PeV} \). We have run 8000 (20000) showers per point for ground muons (longitudinal development), setting at each case the thinning algorithm parameters to get a more detailed simulation of the hadronic or the electromagnetic cascade, respectively.

However, the percentage of the number of pions considered for swapping increases in the refined model with a ratio of 40% \( \div \) 96%. This is because by lowering the \( E_{\text{min}} \) there are many more pions that can be swapped (some of them with \( \eta_{\text{CM}} < 0 \)). Looking at the final figures of pions actually swapped, it shows up that the number of swapped pions with respect to the number of scanned pions is more or less the same, and it is actually lower in the refined model; the ratio is 28% \( \div \) 25%. The number of swapped pions when compared with the number considered for swapping is roughly 70% in the toy model and reduces to 27% in the refined model. Obviously, the ratio of swapped pions to the effective number of pions considered for swapping (i.e., those with \( |\eta_{\text{CM}}| > 4 \)) is \( f_s = 0.7 \). Finally, the number of scanned pions with respect to the total number of secondaries produced with \( E_{\text{proj}} > E_{\text{pmin}} \) is roughly 51%. Note that the fraction of
IV. SENSITIVITY TO \( F_s \) WITH LHC NEUTRINO EXPERIMENTS

During the next two decades, the LHC will lengthen the energy frontier into both higher energies and much higher luminosities. Most general-purpose LHC detectors, such as ATLAS, CMS, and ALICE, are committed to high-\( p_T \) physics, featuring events with small cross section: \( O(\text{fb}, \text{pb}, \text{nb}) \). However, the total cross section of LHC collisions is \( O(100 \text{ mb}) \). Curiously, most of this...
cross section as well as most of the highest energy particles produced in these collisions are in the far forward region, \textit{viz.} at low $p_T$. This implies that there is an entire physics program in the far forward region which remains to be explored and can indeed be exploited during the LHC high luminosity (HL) era.

One challenge that far-forward detectors in or close to the LHC beam pipe have to face are the large particle fluxes and radiation levels, essentially restricting their operation to short low-luminosity runs. Another possibility is to make use of the large flux of LHC neutrinos, which can be probed in low-background environments at a safe distance away from the interaction point and accelerator infrastructure. Indeed, the LHC produces an intense and strongly collimated beam of high energy neutrinos in the far-forward direction. These neutrinos are mainly produced in the decay of charged pions, kaons, hyperons and charmed hadrons, making the measurement of the neutrino flux a complimentary probe of forward particle production compared to the neutral
pion and neutron measurements performed at LHCf.

The feasibility of such LHC neutrino measurements has recently been demonstrated by the FASER collaboration, which reported the observation of the first neutrino interaction candidates at the LHC [39]. Building on this experience, the FASERV neutrino detector [22, 23], which is part of the FASER experiment [40, 41], will start its operation already with the LHC Run 3 in 2022. With a target mass of about 1.2 tons and an anticipated luminosity of $150 \text{ fb}^{-1}$ a total of $O(10^6)$ muon neutrino and $O(10^5)$ electron neutrino interactions are expected to be observed. During the HL-LHC, additional far-forward neutrino experiments have been proposed in the context of the FPF [24]. In particular, this includes an emulsion-based neutrino detector with target mass of about 20 tons called FASERv, a liquid argon based neutrino detector with target mass of about 10 tons called FLArE and an electronic neutrino detector called AdvSND. With their higher target masses and the HL-LHC luminosity of $3000 \text{ fb}^{-1}$ a large event rate of roughly $10^7$ electron neutrino and $10^6$ muon neutrino interactions are expected to be observed.

Both FASERν in the near future and the FPF neutrino experiments during the HL-LHC would provide a profitable arena to measure the pion-to-kaon ratio through the shape of differential neutrino flux distributions. In particular, the pion-to-kaon ratio can be inferred by measuring the ratio of electron-to-muon neutrino fluxes. This is because pions primarily decay into muon neutrinos, whereas kaon decays yield a flux of both muon and electron neutrinos. Moreover, neutrinos from different parent mesons populate a different energy range, and so this can be used to disentangle the fluxes. In addition, since $m_\pi < m_K$, neutrinos from pion decay are more concentrated around the line-of-sight than those of kaon origin, and consequently neutrinos from pions obtain less additional transverse momentum than those from kaon decays. Hence, the closeness of the neutrinos to the line-of-sight, or equivalently their rapidity distribution, becomes a compelling signal to trace back the neutrino origin to measure the pion-to-kaon ratio.

In Fig. 10, we show the expected number of neutrino interactions with the FASERν detector, assuming a $25 \text{ cm} \times 25 \text{ cm}$ cross sectional area and a 1.2 ton target mass, as a function of the neutrino energy. Here, we have used SIBYLL 2.3d [26] as primary generator and use the fast LHC neutrino flux simulation introduced Ref [42] to describe the propagation and decay the long-lived hadrons in the LHC beam pipe. The origin of the neutrinos is indicated by the different line colors: red for pion decay, blue for kaon decay, magenta for hyperon decay, and green for charm decay. As explained above, the neutrinos from pions and kaons populate different regions of phase space, which can be used to disentangle pion and kaon production. In Fig. 11, we also show the results for the FLArE detector at the FPF, which is assumed to have a $1 \text{ m} \times 1 \text{ m}$ cross sectional area and a $10 \text{ ton}$ target mass.

In Fig. 10 and Fig. 11, we also show how a $\pi \leftrightarrow K$ swapping as defined in Eq. (5) changes the expected neutrino fluxes and event rates for the considered experiments. As expected, positive values of $f_s$ lead to a suppression of the neutrino flux from pions as well as a larger relative enhancement of the neutrino flux from kaons. This is due to the initially roughly 10 times larger flux of pions, such that even a small rate of $\pi \leftrightarrow K$ swapping can substantially increase the neutrino flux from the kaon decays. This leads to the remarkable result that already for $f_s = 0.1$ ($f_s = 0.2$) the predicted electron neutrino flux at the peak of the spectrum is a factor of 1.6 (2.2) larger. These differences are significantly larger than the anticipated statistical uncertainties at the FPF [24, 42]. This let’s us conclude that LHC neutrino flux measurements with new forward detectors at the LHC will provide invaluable complementary information to test our model and its improvements, together with eventual alternative ones, addressing the muon puzzle via strangeness enhancement.

V. CONCLUSIONS

We have examined the influence of $\pi \leftrightarrow K$ swapping on the development of extensive air showers. We constructed an empirical testable model, based on ALICE observations of the enhancement of strangeness production in high-energy hadronic collisions, which can accommodate the muon deficit between simulations and Auger data.1 We derived a parametrization of the $\pi \leftrightarrow K$ swapping probability in terms of the pseudorapidity and the nucleus baryon number.

We have also explored potential strategies for model improvement using the massive amounts of data to be collected at the FASERV and future LHC neutrino experiments at the FPF. We have shown that these experiments will attain sensitivity to probe the model phase space.

Within this decade, ongoing detector upgrades of existing facilities, such as AugerPrime [43] and IceCube-Gen2 [44], will enhance the precision of air shower measurements and reduce uncertainties in the interpretation of muon data. In particular, as a part of the upcoming AugerPrime upgrade each surface station will have additional detectors that will provide complementary measurements of the incoming shower particles, consequently leading to improved reconstruction of muons and electromagnetic particles [43]. This will allow for the measurement of the properties of extensive air showers initiated by the highest energy cosmic rays with unprecedented precision. As we have shown in this paper, future Auger measurements will be highly complemented

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1 One possible realization of our phenomenological model may be obtained by considering collective statistical hadronization effects into the standard string fragmentation process [17].
FIG. 12: $z(R_{\mu})$ as a function of $E_{\text{plim}}$ (a) and $E_{\text{slim}}$ (b) for fixed $f_s = 0.5$. (a) Varying projectile energy range, with fixed secondary energy range. $[E_{\text{min}}, E_{\text{max}}]$ is always kept fixed and equal to $[1 \text{ TeV}, \infty]$ (solid symbols) or $[1 \text{ GeV}, \infty]$ (open symbols). Each blue circle (red square) in the figure corresponds to simulations run with $[E_{\text{pmin}}, E_{\text{pmax}}] = [E_{\text{plim}}, \infty]$ ($[E_{\text{pmin}}, E_{\text{pmax}}] = [90 \text{ GeV}, E_{\text{plim}}]$), $100 \text{ GeV} \leq E_{\text{plim}} \leq 10.05 \text{ EeV}$. (b) Fixed projectile energy range, with varying secondary energy range. $[E_{\text{pmin}}, E_{\text{pmax}}]$ is always kept fixed and equal to $[1 \text{ PeV}, \infty]$. Each blue circle (red square) corresponds to simulations run with $[E_{\text{smin}}, E_{\text{smax}}] = [E_{\text{slim}}, \infty]$ ($[E_{\text{smin}}, E_{\text{smax}}] = [1 \text{ GeV}, E_{\text{slim}}]$), $1 \text{ GeV} \leq E_{\text{slim}} \leq 10 \text{ EeV}$.

by observations at the LHC neutrino experiments which will provide a unique determination of the pion-to-kaon ratio at LHC energies. Altogether this will provide a powerful test of models addressing the muon puzzle via strangeness enhancement.

Appendix A: Limiting Projectile and Secondary Energies

In this Appendix we analyze the variation of $z(R_{\mu})$ with both projectile and secondary energies for fixed $f_s$. To this end we introduce the new variables $E_{\text{plim}}$ and $E_{\text{slim}}$ to limit the maximum and minimum energies of the projectile $E_{\text{proj}}$ and secondary $E_{\text{sec}}$, respectively. In Fig. 12 we show $z(R_{\mu})$ as a function of $E_{\text{plim}}$ and $E_{\text{slim}}$ for fixed $f_s = 0.5$. By analyzing the variation of $z(R_{\mu})$ with $E_{\text{plim}}$ and $E_{\text{slim}}$ we conclude that:

- The impact of the substitution of $\pi$’s by $K$’s reaches a maximum when $0 < E_{\text{pmin}} \leq 10 \text{ TeV}$.
- In (a), at $E_{\text{plim}} \approx 10^{19} \text{ eV}$, both the blue and red sets show pairs of points significantly apart: they correspond to values of $E_{\text{plim}}$ slightly smaller or larger than the primary energy ($10^{19} \text{ eV}$), that respectively prevents or not the application of the swapping algorithm to the first hadronic interaction at the beginning of the shower development. This reveals that the first interaction has, by itself, a finite impact of the final number of muons at

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There are no significant differences between the open and solid symbols plots included in (a). This means that swapping of low energy pions ($E_{\text{sec}}$ lower than 1 TeV) has no visible impact on $z(R_\mu)$. This also shows up clearly in (b) where the blue points remain around the maximum value for $E_{\text{slim}} \lesssim 1$ TeV.

Appendix B: EPOS-LHC

In this Appendix we report on the results of simulated particle collisions with EPOS-LHC [35]. In Fig. 13 we show bivariate distributions of secondary pions. From a comparison with Fig. 5 we see that there are no major differences in the distributions, but just a small deviation of the predicted multiplicity in the central region.

We have shown elsewhere [25] that the discrepancy between Auger data and air shower simulations with SIBYLL 2.3d is slightly smaller than the discrepancy obtained from simulations with EPOS-LHC 1909. For showers process with QGSJetII-04 hadronic event generator [45], the discrepancy between data and simulations is even larger [4]. This justifies the choice of SIBYLL 2.3d in our study.
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