Integration of electromagnetic methods of intubation of stratified mediums on the basis of direct and alternating currents.

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Abstract. In this work the integration of two methods of electroinvestigation is considered. One method represents intubation by a direct current, the second intubation by alternating current. The integration is carried out for the purpose of increase in accuracy of results of the solution of the inverse task, the problem is solved in a twodimensional approximation. The direct task for the first and second method is solved numerically. The received values were compared with the experimental datas. The inverse task is formulated as a problem of minimization with the functional considering the experimental values received by both the first and second method of electroinvestigation. The problem of optimization is solved on a compact (for each parameter are set top and bottom border).

1. Introduction
Electromagnetic fields and their application in geoinvestigation it is studied in operations [1-18] Main objective of a research – prediction of an inner pattern and material composition of a geological section by results of processing of physical measurements. Application of geophysical methods, in particular electroinvestigations, allows not only to simplify and accelerate search of minerals, but also to save in search time Earth and its subsoil in a primitive status. It is the major aspect of the considered discipline. Now there is a large number of installations, for search of minerals at different depths.

In this operation the symmetric four-pointed installation (Schlumberger electrode spread), and dipole arrangement is considered, for each of them the mathematical model is described and the task of distribution of an electromagnetic field in the earth with different frequencies in a case is solved to dipole arrangement, and in case of different dispersions in case of Schlumberger electrode spread. Both dipole arrangement and Schlumberger electrode spread work with some error therefore measurements turn out not always exact that leads to the wrong decision of the reverse task. For the solution of this problem in this operation, integration of these two methods in case of the decision of the reverse task is used.
As the characteristic of the environment, the value of apparent resistance is used. In case of Schlumberger, electrode spread apparent resistance will depend on a dispersion, and in case of dipole arrangement from frequency. The experimental data used are in this operation by means of the mathematical model described for each of installations.

2. Direct task

2.1. Schlumberger electrode spread

The horizontally stratified medium consisting of a number of layers of terminating power with a \( h_i \) specific resistance \( \rho_i \). It is necessary to find potential in any layer and mainly on an environment surface. In the “Fig. 1”, the environment model is represented.

![Figure 1. Model of a horizontally stratified medium](image)

Figure 1. Model of a horizontally stratified medium.

Calculation of the field in such environment and establishment of the quantitative communication between components of the field and its parameters \( (h_i, \rho_i) \) it is a main problem of electric intubation.

Let us choose a cylindrical frame \( r, \varphi, z \) with a datum in a point A and the axis z directed down. At such choice of a frame environment and the field have a rotational symmetry with the center on an axis z owing to what required potential functions would depend only on variables \( r \) and \( z \). Let us designate a depth of a sole each layer \( z_i \) where \( i = 1, 2, 3, ... \) – its number one after another from top to down. We will not limit quantity of layers. We will write down a potential function \( U_1 \) for the first coat in the form of two items

\[
U_1(r, z) = \frac{c_1}{r} \sqrt{r^2 - z^2} + U_1'(r, z).
\]

(1)

The first of them represents potential in the neighborhood of a point A (a condition near a source); \( c_1 = I \rho_1 / 2\pi \) – the coefficient of an emission of current, an augend symbolizes the abnormal part of the field caused by influence of borders of layers.
Potential functions $U_i(r, z)$ in each layer are terminating, continuous and address in zero in points at infinity. All of them including $U'_i(r, z)$ satisfy to the equation of Laplace which common decision for the rotationally symmetric environment for looks:

$$U_i(r, z) = \int_0^\infty \left( a_i e^{-mz} + b_i e^{mz} \right) J_0(mr) \, dm$$

(2)

On a half-space surface current flows along border therefore:

$$\frac{\partial U_i}{\partial z} \bigg|_{z=0} = 0.$$  (3)

On a sole of any layer at $z = z_i$ boundary conditions, have to be satisfied

$$U_i = U_{i+1}, \quad \frac{1}{\rho_i} \frac{\partial U_i}{\partial z} = \frac{1}{\rho_{i+1}} \frac{\partial U_{i+1}}{\partial z}.$$  (4)

As the characteristic of the environment, we will use the apparent resistance which for this to installation looks as follows:

$$\rho_k(r) = \rho_i \left(1 + r^2 \int_0^\infty \left[ R_i(m) - 1 \right] m J_1(mr) \, dm \right)$$

(5)

Where $r$ – a rating; $J_1(mr)$ – a cylindrical function from real argument $m$ – space frequency; $R_i(m)$ – the space characteristic of a stratified medium including all data on specific electrical resistances $\rho_p$ and capacities $h_p$ of layers of a geoelectric section; $p$ – number of a layer. Feature of an integrand is that it consists of performing two functions: $R_i(m)$ and $J_1(mr)$. The first of them depends on parameter of a geoelectric section, and the second – only on a rating. Therefore, two stages are provided in the scheme of calculation $\rho_k(r)$. At first calculate function $R_i(m)$ in the wide range of space frequencies of $m$, and then carry out a numerical integration and find $\rho_k(r)$ for the fixed rating of $r$ [10].

Function $R_i(m)$ is calculated by means of the recurrence relation on a formula Vanyana:

$$R_i(m) = \frac{1 - \Phi_{i+1}(m)}{1 + \Phi_{i+1}(m)}$$

$$\Phi_{i+1}(m) = \frac{1 - \rho_{i+1} R_{i+1}(\omega)}{1 + \rho_{i+1} R_{p+1}(\omega)} e^{-2mh_i}.$$  (6)

As the experimental, we will choose environment model with the following electro physical properties.

$$\rho_1 = 100 \quad \text{Ohm} \cdot \text{m}; \quad h_1 = 5 \quad \text{m}$$

$$\rho_2 = 1000 \quad \text{Ohm} \cdot \text{m}; \quad h_2 = 200 \quad \text{m}$$

$$\rho_3 = 1 \quad \text{Ohm} \cdot \text{m}; \quad h_3 = \infty$$

The curve of apparent resistance for this environment is represented in the “Fig. 2”. 


2.2. Dipole arrangement.

In the “Fig. 3” the surface of the stratified half-space consisting of horizontal layers with physical properties $h_p, \rho_p, \mu_p$ where p-number of a layer, there is a vertical magnetic dipole – a loop or a frame with a total area of rounds of Q on which alternating current $I = I_0 e^{-i\omega t}$ circulates is represented. Dipole moment $M^* = IQ$.

Let us enter padding designations: $k_p$ – a wave number in p layer; $d_p$ – depth of its sole. Let us choose a cylindrical frame $r, \varphi, z$ with a datum in the center of a dipole and the axis $r$ directed down i.e. on a normal to borders of layers. In this case the field can be described by means of a vertical component of a vector potential $A$. Owing to an apparent rotational symmetry $\partial A / \partial \varphi = 0$. Let's write down the main set of equations in a look:
Considering that \( \text{div} A = \partial A_{\hat{z}} / \partial z \), we will make two groups of boundary conditions: for an interface of surface-to-air and a sole of each layer

\[
\begin{align*}
A_{z0}^* &= A_{z1}^* \\
\frac{1}{\mu_0} \frac{\partial A_{z0}^*}{\partial z} &= \frac{1}{\mu_1} \frac{\partial A_{z1}^*}{\partial z} \quad (z = 0),
\end{align*}
\]

\[
\begin{align*}
A_{zp}^* &= A_{z p+1}^* \\
\frac{1}{\mu_p} \frac{\partial A_{zp}^*}{\partial z} &= \frac{1}{\mu_{p+1}} \frac{\partial A_{z p+1}^*}{\partial z} \quad (z = d_p).
\end{align*}
\]

We will calculate apparent resistance for this installation on the following formulas:

\[
\rho_T / \rho_1 = R_1(\omega) = \left| \frac{\rho_T}{\rho_1} e^{i\varphi} \right|, \tag{8}
\]

where function \( R \) for a layer of \( p \) is calculated by a formula:

\[
R_p(\omega) = \text{th}(k_p h_p + \text{arth}(\rho_{p+1} \rho_p R_{p+1}^{}(\omega)))).
\]

This expression is the basis for an algorithm. With its function \( R(\omega) \) can be counted from a sole for a roof of the allocated layer. As a matter of convenience calculation we will transform it to a linear-fractional ratio [8, 15]:

\[
R_p(\omega) = \frac{1 - \Phi_{p+1}^{}(\omega)}{1 + \Phi_{p+1}^{}(\omega)},
\]

\[
\Phi_{p+1}^{}(\omega) = \frac{1 - \sqrt{\rho_{p+1} / \rho_p R_{p+1}^{}(\omega)}}{1 + \sqrt{\rho_{p+1} / \rho_p R_{p+1}^{}(\omega)}} e^{-k_p h_p}
\]

Let's accept that

\[
\begin{align*}
\frac{h_p}{\rho_p} &= h_p \mu_p \\
\frac{\rho_p}{\rho_p} &= \rho_p \mu_p.
\end{align*}
\]

Then

\[
\begin{align*}
R_1(\omega, h_p, \rho_p) &= \text{Re}_1 + i \text{Im}_1 \\
R_1(\omega, h_p^*, \rho_p^*) &= \overline{\text{Re}_1 + i \text{Im}_1}
\end{align*}
\]

Apparent resistance then is calculated on a formula

\[
\rho_T = \rho_1(\text{Re}_1^2 + i \text{Im}_1^2).
\]

As experimental, we will use the same environment, as for Schlumberger electrode spread. By means of mathematical model for dipole arrangement, we calculated value of apparent resistance depending on the period “Fig. 4”.
3. Inverse task

3.1. Task definition

The problem of selection of required parameters of a section can be reduced to minimization of the function $F$ consisting of two items. The first item is equal to the sum of squares of a difference of deviations of theoretical values from the experimental when using Schlumberger electrode spread, and the second when using dipole arrangement.

$$F(n, h_1, ..., h_n, \rho_1, ..., \rho_n) = \sum_{j=1}^{k} \left( \sum_{i=1}^{n} \left(\omega_i - h_i, h_n, \rho_1, ..., \rho_n \right) - \rho_j \right)^2 + \sum_{i=1}^{k} \left( \sum_{j=1}^{n} \left(\omega_i - h_i, h_n, \rho_1, ..., \rho_n \right) - \rho_j \right)^2 = \min$$

This task is a problem of optimization and it is solved if for the varied sizes borders of their measurements are set. For the solution of a task the algorithm of the complete search with the modification allowing to accelerate time of searching [11, 12] was used.

3.2. The solution of the inverse task for each method separately

For a start, we will solve the inverse problem for each of installations separately for this purpose we will exclude from a target functional (8) in turn the second and first item. The solution of the inverse task are values of parameters of the environment. We will call the received environment theoretical.

In “Fig. 5” the schedule received at the solution of the inverse task with use of an algorithm for the solution of a direct task by method of horizontal electric intubation is submitted. The curve of red color corresponds to the solution of a direct task. The curve of green color is received as a result of the solution of the inverse task. Apparently, on graphics, the algorithm of the solution of the inverse task was insufficiently precise at small values of dispersion.
In “Fig. 6” the schedule received at the solution of the inverse task with use of an algorithm for the solution of a direct task by method of dipole sources is submitted. The curve of blue color corresponds to the solution of a direct task. The curve of green color is received as a result of the solution of the inverse task.

![Figure 6. Theoretical and experimental curves for dipole arrangement](image)

Figure 6. Theoretical and experimental curves for dipole arrangement

Apparently, on graphics, the algorithm of the solution of the inverse task was insufficiently precise at small value of the period.

3.3. Integration of methods.
As shown in point B the solution of the inverse task for each of methods of electroinvestigation does not give the chance precise true values of parameters of a horizontally stratified medium. The value of the apparent resistance received at the solution of the inverse task significantly disperses from the experimental values.

For the solution of this problem, it is offered to integrate methods of horizontally electric intubation and electrical dipole moments. Also at an integration of methods, the number of layers from the category of the fixed passes into the category of varied. In order that the number of layers received at the solution of the inverse task corresponded to physical sense, for this parameter logical restrictions (to consider layers with the specific electrical resistance differing on 10-20 Ohms one layer) are introduced, also to put restriction for quantity of layers – 5.

As a result of an integration of methods of electroinvestigation at the solution of the inverse task we received more precise solutions of the inverse task. In the “Fig. 7” the curve of apparent resistance for the theoretical environment, depending on a rating (Schlumberger electrode spread) is displayed. In the “Fig. 8” the curve of apparent resistance of the same theoretical environment is represented, but is more narrow depending on the period (dipole arrangement).
3.4. Conclusions. Analysis of efficiency of an integration of methods

The integration of methods has led to increase in accuracy of results. The minimum difference between theoretical and experimental values has decreased by 8 times. The translation of number of layers from among fixed values in the varied sizes hasn't led to deterioration in the received results. In all the made experiments the number of layers has been defined truly. The developed algorithm has also been tested on environment models with two and four layers. Increase number of layers didn't influence the accuracy of the received results, but influenced time of program execution.

The integration of methods hasn't led to essential increase in time of search of true values of electrophysical parameters of horizontally layered environment.

In general, the integration of methods is optimum for the solution of the inverse tasks as allows to find values of parameters of the environment with a high precision, but for larger amount of time. The carried-out tests showed that increase in a run time of the program is not critical and it is expedient for accuracy of results.

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