The parameterization of nice and $Q$-nice polynomials with four roots

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Abstract. A univariable polynomial $p(x)$ is said to be nice if all of its coefficients as well as all of the roots of both $p(x)$ and its derivative $p'(x)$ are integers. $p(x)$ is called $Q$-nice polynomial if the coefficients, roots, and critical points are rational numbers. This paper concentrates on the problem of finding, constructing, and classifying parameterized family of nice and $Q$-nice polynomials with four roots.

1. Introduction
A polynomial $p(x)$ of degree $d$ is totally nice if the roots of $p(x), p'(x), ..., p^{d-1}(x)$ are integers. The known examples of nice polynomials with distinct roots are limited to quadratic, cubic, quartic, quintic, and sextic polynomials. We often work with nice polynomials $p(x)$. However, in some cases we may require $p(x)$ to have rational coefficients, roots, and critical points. Such polynomials are called $Q$-nice polynomials.

Caldwell [2] found that if $p(x)$ is a nice polynomial of degree $d$, and $a, b, c$ are arbitrary integers such that $a \neq 0, c \neq 0$, then $g(x) = a^d[p((x+b)/a)]$ is also a nice polynomial of degree $d$. Thus, given a single nice polynomial $p(x)$ of degree $d$, we readily obtain infinitely many nice polynomials of degree $d$. Any two nice polynomials that are obtained in this manner from the same nice polynomial will be considered 'equivalent'.

The problem of finding properties, characterizations, and methods of construction of polynomials with coefficients, roots, and critical points in the ring of rational integers is on the list of unsolved problems published by Nowakowski [8]. Bucholz and MacDougall [1] considered the problem of classifying all univariate polynomials, defined over a domain $k$, with the property that they and their derivatives have all their roots in $k$. From a number of theoretic perspective, the most interesting cases for this problem are $k = \mathbb{Z}$ or $k = \mathbb{Q}$. In this paper, we want the polynomials to have roots and critical points in integer domain or rational domain.

Evard [4] considered the relations between roots and critical points of polynomials to present equivalences of nice polynomials which reduce the search of just one representant in each equivalence class. A key relations to deal with nice polynomials, namely, the system of equations has been established. Also, some of the results have been generalized to $\mathbb{Z}$ domain. For simplicity,
p(x) ∈ ℤ. Choudry [3] found nice nonsymmetric quartic, quintic, and sextic polynomials whose coefficients are given in parametric terms and whose roots are all distinct.

Groves [5], [6] and [7] considered the problem of finding, constructing, and classifying nice polynomials. In these papers, complete solutions to the symmetric three and four roots cases and also the general three roots case have been presented. However, the general four roots case has not been solved. The author gave the relations between the roots and critical points for all polynomials with four roots as follow:

In this section, we will give the parameterization of a nice polynomial with four roots. Consider all nice polynomials \( p(x) \) with four distinct roots. When one root is shifted at zero, \( p(x) \) has the form

\[
p(x) = x^{m_0}(x-a)^{m_1}(x-b)^{m_2}(x-c)^{m_3}
\]

and the derivative has the form

\[
p'(x) = dx^{m_0-1}(x-a)^{m_1-1}(x-b)^{m_2-1}(x-c)^{m_3-1}(x-q)(x-r)(x-s).
\]

**Lemma 1.1.** A polynomial \( p(x) = x^{m_0}(x-a)^{m_1}(x-b)^{m_2}(x-c)^{m_3} \) of degree \( d \) with integer coefficients and with four integer roots is nice if and only if there exist integers \( q, r, \) and \( s \) such that

\[
\begin{align*}
(d - m_1)a + (d - m_2)b + (d - m_3)c &= d(q + r + s) \quad (3) \\
(m_0 + m_3)ab + (m_0 + m_2)ac + (m_0 + m_1)bc &= d(qr + qs + rs) \quad (4) \\
m_0abc &= dqr. \quad (5)
\end{align*}
\]

The lemma above is needed to solve the general four roots case. In this paper, we consider the problem of finding, constructing, and classifying parameterized family of nice and \( Q \)-nice polynomials with four roots.

2. Results and discussion

In this section, we will give the parameterization of nice polynomial with four roots.

**Theorem 2.1.** Let \( a, b, c, q, r \) and \( s \) be integers, a polynomial \( p(x) = x^{m_0}(x-a)^{m_1}(x-b)^{m_2}(x-c)^{m_3} \) of degree \( d = m_0 + m_1 + m_2 + m_3 \) with four roots is nice if and only if

\[
a = -d(-d^2a^2m_0 + da^2m_0^2 + da^2m_0m_1 + da^2m_0m_2 + da^2m_0m_3 - a^2m_0m_1m_2 + d^2um_1
\]
\[
- d^2um_3 - dum_0m_1 + dum_0m_3 - dum_1^2 - dum_2m_3 + dum_3^2 + um_0m_1m_2
\]
\[
- um_0m_2m_3 + um_1^2m_2 - um_2m_3^2 + d^3 - d^2m_0 - d^2m_1 - 2d^2m_2 - d^2m_3 + dm_0m_2
\]
\[
+ dm_1m_2 + dm_2^2 + 2dm_2m_3 - m_2^2m_3)(-dum_0 + d^2 - dm_2 - dm_3 + m_2m_3) \quad (6)
\]

\[
b = (-dum_0 + d^2 - dm_2 - dm_3 + m_2m_3)(du - um_1 - d)(d - m_0 - m_2)m_2d \quad (7)
\]

\[
c = \frac{1}{u}d(du - um_1 - d)(d^2m_0 - d^2m_0^2 + d^2m_0m_1 + d^2m_0m_2 + d^2m_0m_3
\]
\[
- u^2m_0m_1m_2 + d^2um_1 - d^2um_3 - dum_0m_1 + dum_0m_3 - dum_1^2 - dum_1m_2 + dum_2m_3
\]
\[
+ dum_3^2 + um_0m_1m_2 - um_0m_2m_3 + um_1^2m_2 - um_2m_3^2 + d^3 - d^2m_0 - d^2m_1 - 2d^2m_2
\]
\[
- d^2m_3 + dm_0m_2 + dm_1m_2 + dm_2^2 + 2dm_2m_3 - m_2^2m_3)(d - m_2)u \quad (8)
\]
q = \frac{1}{u} d(-d^2 u^2 m_0 + d u^2 m_0^2 + d u^2 m_0 m_1 + d u^2 m_0 m_2 + d u^2 m_0 m_3 - u^2 m_0 m_1 m_2 + d^2 u m_1 \\
- d^2 u m_3 - d u m_0 m_1 + d u m_0 m_2 - d u m_1 m_2 + d u m_2 m_3 + d u m_3^2 + u m_0 m_1 m_2 \\
- u m_0 m_2 m_3 + m_1^2 m_2 - u m_2 m_3^2 + d^3 - d^2 m_0 - d^2 m_1 - 2 d^2 m_2 - d^2 m_3 + d m_0 m_2 \\
+ d m_1 m_2 + d m_2^2 + 2 d m_2 m_3 - m_2^2 m_3) (-d u m_0 + d^2 - d m_2 - d m_3 + m_2 m_3) \quad (9)

r = (-d u m_0 + d^2 - d m_2 - d m_3 + m_2 m_3) (d u - u m_1 - d) (d - m_0 - m_2) m_2 (d - m_2) \quad (10)

s = m_0 (d u - u m_1 - d) d (-d^2 u^2 m_0 + d u^2 m_0^2 + d u^2 m_0 m_1 + d u^2 m_0 m_2 + d u^2 m_0 m_3 \\
- u^2 m_0 m_1 m_2 + d^2 u m_1 - d^2 u m_3 - d u m_0 m_1 + d u m_0 m_3 - d u m_1^2 - d u m_1 m_2 \\
+ d u m_2 m_3 + d u m_3^2 + u m_0 m_1 m_2 - u m_0 m_2 m_3 + u m_1 m_2 - u m_2 m_3^2 + d^3 - d^2 m_0 \\
- d^2 m_1 - 2 d^2 m_2 - d^2 m_3 + d m_0 m_2 + d m_1 m_2 + d m_2^2 + 2 d m_2 m_3 - m_2^2 m_3) \quad (11)

where u is a nonzero arbitrary parameter.

Proof. \(\Rightarrow\) The relations between the roots and critical points of nice polynomial with four roots are given in (3), (4), and (5). From (5) and by parameterization technique, we let

\[ a = qu, b = rv, c = \frac{ds}{(m_0 u v)} \quad (12) \]

where u and v are arbitrary nonzero rational parameters. By substituting these values in (11), we get

\[ (d - m_1) q u + (d - m_2) r v + \frac{(d - m_3)s}{m_0 u v} = dq + dr + ds. \]

Simplify the above equation and solving for s, we have

\[ s = \frac{(dm_0 - (d - m_1) m_0 u)p + (dm_0 - (d - m_2) m_0 v)q}{d(-uvw m_0 + d - m_3)}. \quad (13) \]

On substituting (12) and (13) in (4), we obtain

\[ (m_0 + m_3) q u r v + \frac{(m_0 + m_2)q((dm_0 - (d - m_1) m_0 u)q + (dm_0 - (d - m_2) m_0 v)r)}{(-uvw m_0 + d - m_3)m_0} \]
\[ + \frac{(m_0 + m_1)r((dm_0 - (d - m_1) m_0 u)q + (dm_0 - (d - m_2) m_0 v)r)}{(-uvw m_0 + d - m_3)m_0} \]
\[ = q\frac{((dm_0 - (d - m_1) m_0 u)q + (dm_0 - (d - m_2) m_0 v)r)uv}{-uvw m_0 + d - m_3} \]
\[ + r\frac{((dm_0 - (d - m_1) m_0 u)q + (dm_0 - (d - m_2) m_0 v)r)uv}{-uvw m_0 + d - m_3} + dq r. \quad (14) \]

By simplifying (14), we have the following quadratic equation in terms of q and r:

\[ -u(v m_0 - m_0 - m_2)(d u - u m_1 - d)q^2 + (u^2 v^2 m_0^2 + u^2 v^2 m_0 m_3 - d u^2 v m_0 - d u v^2 m_0) \]
\[ + u^2 v m_0 m_1 + u^2 v m_0 m_2 + 2 d u v m_0 + d u m_1 + d u m_2 - d u m_3 - u m_0 m_1 - u m_0 m_2 \]
\[ + u m_0 m_3 - u m_1^2 - u m_2^2 + u m_3^2 - d m_0 - d m_2 - d m_0 - d m_1 + d^2 - d m_3) q r \]
\[ - v(u m_0 - m_0 - m_1)(d v - v m_2 - d)q^2 = 0. \quad (15) \]
One solution of (15) is obtained quite simply by taking $v = \frac{d}{d-m_2}$ that is when the coefficient of $r^2$ in this equation vanishes. Therefore,

\[
(d-m_2) um_2 (d-m_0-m_2) (du-um_1-d) q^2 + d(-d^2 u^2 m_0 + du^2 m_0^2 + du^2 m_0 m_1 \\
+ du^2 m_0 m_2 + du^2 m_0 m_3 - u^2 m_0 m_1 m_2 + d^2 u m_1 - d^2 u m_3 - dum_0 m_1 + dum_0 m_3 - dum_1^2 \\
- dum_1 m_2 + dum_2 m_3 + dum m_3^2 + um_0 m_1 m_2 - um_0 m_2 m_3 + um_1^2 m_2 - um_2 m_3^2 + d^3 - d^2 m_0 \\
- d^2 m_1 - 2 d^2 m_2 - d^2 m_3 + dm_0 m_2 + dm_1 m_2 + dm_2^2 + 2 dm_2 m_3 - m_2^2 m_3)q = 0. \\
(16)
\]

By simplifying (16), we obtain

\[
(d-m_2) um_2 (d-m_0-m_2) (du-um_1-d) q = -d(-d^2 u^2 m_0 + du^2 m_0^2 + du^2 m_0 m_1 \\
+ du^2 m_0 m_2 + du^2 m_0 m_3 - u^2 m_0 m_1 m_2 + d^2 u m_1 - d^2 u m_3 - dum_0 m_1 + dum_0 m_3 - dum_1^2 \\
- dum_1 m_2 + dum_2 m_3 + dum m_3^2 + um_0 m_1 m_2 - um_0 m_2 m_3 + um_1^2 m_2 - um_2 m_3^2 + d^3 \\
- d^2 m_0 - d^2 m_1 - 2 d^2 m_2 - d^2 m_3 + dm_0 m_2 + dm_1 m_2 + dm_2^2 + 2 dm_2 m_3 - m_2^2 m_3)r. \\
(17)
\]

Substitute $v = \frac{d}{d-m_2}$ into (13), we get

\[
q = \frac{(d-m_2) s}{(dm_0 - (d-m_1) m_0 u)} u \left( -\frac{dum_0}{d-m_2} + d - m_3 \right). \\
(18)
\]

Then by substituting (18) into (17), we have

\[
r(-dum_0 + d^2 - dm_2 - dm_3 + m_2 m_3)(du-um_1-d)(d-m_0-m_2)m_2(d-m_2) \\
= m_0 (du-um_1-d)\left(-d^2 u^2 m_0 + du^2 m_0^2 + du^2 m_0 m_1 + du^2 m_0 m_2 + du^2 m_0 m_3 \\
- u^2 m_0 m_1 m_2 + d^2 u m_1 - d^2 u m_3 - dum_0 m_1 + dum_0 m_3 - dum_1^2 - dum_1 m_2 \\
+ dum_2 m_3 + dum m_3^2 + um_0 m_1 m_2 - um_0 m_2 m_3 + um_1^2 m_2 - um_2 m_3^2 + d^3 \\
- d^2 m_0 - d^2 m_1 - 2 d^2 m_2 - d^2 m_3 + dm_0 m_2 + dm_1 m_2 + dm_2^2 + 2 dm_2 m_3 - m_2^2 m_3) s. \\
(19)
\]

Based on (19), we let

\[
r = (-dum_0 + d^2 - dm_2 - dm_3 + m_2 m_3)(du-um_1-d)(d-m_0-m_2)(d-m_2) m_2 \\
and
s = dm_0 (du-um_1-d)\left(-d^2 u^2 m_0 + du^2 m_0^2 + du^2 m_0 m_1 + du^2 m_0 m_2 + du^2 m_0 m_3 \\
- u^2 m_0 m_1 m_2 + d^2 u m_1 - d^2 u m_3 - dum_0 m_1 + dum_0 m_3 - dum_1^2 - dum_1 m_2 \\
+ dum_2 m_3 + dum m_3^2 + um_0 m_1 m_2 - um_0 m_2 m_3 + um_1^2 m_2 - um_2 m_3^2 + d^3 - d^2 m_0 \\
- d^2 m_1 - 2 d^2 m_2 - d^2 m_3 + dm_0 m_2 + dm_1 m_2 + dm_2^2 + 2 dm_2 m_3 - m_2^2 m_3). \\
(20)
\]

By working backwards, substitute (20) into (16), we get

\[
(d-m_2)(d-m_0-m_2)(du-um_1-d)um_2 = -d(-d^2 u^2 m_0 + du^2 m_0^2 + du^2 m_0 m_1 + du^2 m_0 m_2 \\
+ du^2 m_0 m_3 - u^2 m_0 m_1 m_2 + d^2 u m_1 - d^2 u m_3 - dum_0 m_1 + dum_0 m_3 - dum_1^2 - dum_1 m_2 \\
+ dum_2 m_3 + dum m_3^2 + um_0 m_1 m_2 - um_0 m_2 m_3 + um_1^2 m_2 - um_2 m_3^2 + d^3 - d^2 m_0 - d^2 m_1 \\
- 2 d^2 m_2 - d^2 m_3 + dm_0 m_2 + dm_1 m_2 + dm_2^2 + 2 dm_2 m_3 - m_2^2 m_3)(-dum_0 + d^2 - dm_2 \\
- dm_3 + m_2 m_3)(du-um_1-d)(d-m_0-m_2)(d-m_2) m_2. \\
(22)
\]
Next, suppose \( m \) whose derivative polynomials with distinct roots can be obtained. By taking \( u \)
where \( q, r, \) \((a, b, c, q, r, \) 
\( p \) \)
\( p \)
\( 1 \), we have
\[
q = -\frac{1}{u}(d(-d^2 u^2 m_0 + du^2 m_0^2 + du^2 m_0 m_1 + du^2 m_0 m_2 + du^2 m_0 m_3 - u^2 m_0 m_1 m_2 + d^2 u m_1 \\
- d^2 u m_2 - d u m_0 m_1 + d u m_0 m_3 - du^2 m_1 m_2 + d u m_2 m_3 + d u m_3^2 + u m_0 m_1 m_2 \\
- u m_0 m_2 m_3 + u m_1^2 m_2 - u m_2 m_3^2 + d^3 - d^2 m_0 - d^2 m_1 - 2d^2 m_2 - d^2 m_3 + d m_0 m_2 \\
+ d m_1 m_2 + d m_2^2 + 2d m_2 m_3 - m_2^2 m_3)(-du m_0 + d^2 - d m_2 - d m_3 + m_2 m_3).
\]

Then, substitute these values of \( q, r, \) and \( s \) into (12) and we will get (6), (7), and (8).

With these values of \( a, b, c, q, r, \) and \( s \) where \( u \) is an arbitrary parameter, a nice polynomial \( p(x) = x^{m_0}(x-a)^{m_1}(x-b)^{m_2}(x-c)^{m_3} \) with degree \( d = m_0 + m_1 + m_2 + m_3 \) is obtained. \( \square \)

We now give numerical examples based on Theorem(2.1). Suppose \( m_0 = 2, m_1 = 7, m_2 = 1, m_3 = 3, d \) is equal to 13. Then, by substituting these values into (6), (7), (8), (9), (10), and (11), we have
\[
a = -13 (-14 u^2 + 48 u + 36) (-26 u + 120)
\]
\[
b = 130 (-26 u + 120) (6 u - 13)
\]
\[
c = \frac{1}{u} (156(6 u - 13)(-14 u^2 + 48 u + 36))
\]
\[
q = \frac{1}{u} (-13(-14 u^2 + 48 u + 36)(-26 u + 120))
\]
\[
r = 120 (-26 u + 120) (6 u - 13)
\]
\[
s = 26 (6 u - 13) (-14 u^2 + 48 u + 36)
\]
where \( u \) is an arbitrary parameter. With the above values of \( a, b, c, q, r, \) and \( s \), a family of nice polynomials with distinct roots can be obtained. By taking \( u = -2, \)
\[
p(x) = x^{13} - 578032 x^{12} - 301395346304 x^{11} + 267677073858621440 x^{10}
- 14989796321150600867840 x^9 - 28196997010479904654377680896 x^8
+ 5695056813936063574491874991276032 x^7
+ 89769736067642997636825483391063713824 x^6
- 33783556041861327677581002806666753804697600 x^5
+ 5979742472606639729494908929533306043923169280000 x^4
+ 609914969052403265480009893566570740376788795392000000 x^3
- 51097320740613880690799939972139937091601097241395200000000000 x^2
= x^2 (x - 259376)^7 (x + 559000) (x + 226200)^3
\]
whose derivative
\[
p'(x) = 13 x (x - 259376)^6 (x + 226200)^2 (x + 129688) (x + 516000) (x - 75400).
\]
Next, suppose \( m_0 = 3, m_1 = 5, m_2 = 1, m_3 = 2, d \) is equal to 11. Then, by substituting these
values into (6), (7), (8), (9), (10), and (11), we have

\[
\begin{align*}
a &= -11 \left( -15 u^2 + 30 u + 20 \right) \left( -33 u + 90 \right) \\
b &= 77 \left( -33 u + 90 \right) \left( 6 u - 11 \right) \\
c &= \frac{1}{u} \left( 110 \left( 6 u - 11 \right) \left( -15 u^2 + 30 u + 20 \right) \right) \\
q &= \frac{1}{u} \left( -11 \left( -15 u^2 + 30 u + 20 \right) \left( -33 u + 90 \right) \right) \\
r &= 70 \left( -33 u + 90 \right) \left( 6 u - 11 \right) \\
s &= 33 \left( 6 u - 11 \right) \left( -15 u^2 + 30 u + 20 \right)
\end{align*}
\]

where \( u \) is an arbitrary parameter. With the above values of \( a, b, c, q, r, \) and \( s \), a family of nice polynomials with distinct roots is given by (1). Taking \( u = 5 \) leads to nice polynomial,

\[
p(x) = x^{11} + 1126730 x^{10} + 549889518475 x^9 + 151697103462135000 x^8 \\
+ 2584959705531136054875 x^7 + 2783637270245179776269531250 x^6 \\
+ 184833774548161256914727783203125 x^5 \\
+ 6913845657207983745816915893554687500 x^4 \\
+ 111482044101157658340144103775024414062500 x^3 \\
= x^3 \left( x + 169125 \right)^5 \left( x + 109725 \right) \left( x + 85690 \right)^2
\]

whose derivative

\[
p'(x) = 11 x^2 \left( x + 169125 \right)^4 \left( x + 85690 \right) \left( x + 33825 \right) \left( x + 99750 \right) \left( x + 128535 \right).
\]

Now, consider the case where \( p(x) \in \mathbb{Q} \) is a \( \mathbb{Q} \)-nice polynomial with four roots. Lemma 1.1 can be used to derive the relations for the rational roots and critical points for this kind of polynomial. However, instead of integer numbers, we solve system (3), (4), and (5) for all rational numbers \( a, b, c, q, r, \) and \( s \). Theorem 2.1 gives formulas for all \( \mathbb{Q} \)-nice polynomials with four roots. Since we require \( a, b, c, q, r, \) and \( s \) to be rational numbers, we choose \( u \) that satisfies the relations so that a \( \mathbb{Q} \)-nice polynomial can be constructed.

The following example is the illustration of this theorem in \( \mathbb{Q} \)-nice polynomial. Suppose \( m_0 = 1, m_1 = 1, m_2 = 2, m_3 = 2, d \) is equal to 6. Then, by substituting these values into (6), (7), (8), (9), (10), and (11), we have

\[
\begin{align*}
a &= -6 \left( -2 u^2 - 8 u + 16 \right) \left( -6 u + 16 \right) \\
b &= 36 \left( -6 u + 16 \right) \left( 5 u - 6 \right) \\
c &= 24 \left( 5 u - 6 \right) \left( -2 u^2 - 8 u + 16 \right) \frac{u}{u} \\
q &= -6 \left( -2 u^2 - 8 u + 16 \right) \left( -6 u + 16 \right) \frac{u}{u} \\
r &= 24 \left( -6 u + 16 \right) \left( 5 u - 6 \right) \\
s &= 6 \left( 5 u - 6 \right) \left( -2 u^2 - 8 u + 16 \right)
\end{align*}
\]

where \( u \) is an arbitrary rational parameter. Taking \( u = \frac{1}{7} \) leads to \( \mathbb{Q} \)-nice polynomial,
\[ p(x) = x^6 + \frac{11464056 x^5}{343} + \frac{632336829504 x^4}{16807} + \frac{1370714248302930432 x^3}{823543} + \frac{124058747960773611651072 x^2}{40353607} + \frac{3825719190959175975597337214976 x}{1977326743} \]
\[ = x \left( x + \frac{461736}{343} \right) \left( x + \frac{141192}{49} \right)^2 \left( x + \frac{644688}{49} \right)^2 \]

whose derivative
\[ p'(x) = 6 x^5 + \frac{57320280 x^4}{343} + \frac{25293347318016 x^3}{16807} + \frac{4112142744908791296 x^2}{823543} + \frac{24811749592154723302144 x}{40353607} + \frac{3825719190959175975597337214976}{1977326743} \]
\[ = 6 \left( x + \frac{141192}{49} \right) \left( x + \frac{644688}{49} \right) \left( x + \frac{461736}{49} \right) \left( x + \frac{94128}{49} \right) \left( x + \frac{161172}{343} \right) \]

3. Conclusion
In this paper, parameterized family of nice and \( Q \)-nice polynomials with four roots have been obtained. By using the results of the Lemma 1.1 and Theorem 2.1, we can construct nice and \( Q \)-nice polynomials with four roots.

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