MPC Without the Computational Pain: 
The Benefits of SLS and Layering in Distributed Control

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Abstract—The System Level Synthesis (SLS) approach facilitates distributed control of large cyberphysical networks in an easy-to-understand, computationally scalable way. We present a case study motivated by the power grid, with communication constraints, actuator saturation, disturbances, and changing setpoints. This simple but challenging case study necessitates the use of model predictive control (MPC); however, MPC incurs significant online computational cost and often scales poorly to large systems. We overcome these challenges by combining various SLS-based techniques, including SLS-based MPC, in a layered controller. This controller achieves performance that is within 3% of the centralized MPC performance, requires only 5% of the online computational resources of distributed MPC, and scales to systems of arbitrary size. For the unfamiliar reader, we also present a review of the SLS approach and its associated extensions in nonlinear control, MPC, adaptive control, and learning for control.

I. INTRODUCTION

The control of large cyberphysical systems is important to today’s society. Relevant examples include power grids, intelligent transport systems, and IoT. In each of these fields, emerging technology (e.g., renewables, autonomous vehicles) and increasingly complex architectures present new challenges, and theoretical frameworks and algorithms are needed to address them. Generally speaking, these widespread large-scale systems require a distributed control framework that offers scalable, structured solutions. The resulting large-scale distributed control problems benefit from layered architectures, in which the central problem is decomposed into sub-problems which are assigned to different layers [1].

The recently introduced System Level Synthesis (SLS) framework [2]–[4] provides theory and scalable algorithms to deal with complex control systems that are structured, distributed and subject to perturbations. SLS is suitable for large-scale networks, tackling challenges of distributed control in a transparent manner via complex optimization; these challenges include disturbance containment, inter-subsystem coupling, and communication constraints both in terms of delay and locality (i.e. how many neighboring subsystems each subsystem can communicate with). Moreover, it enables distributed synthesis and implementation; the runtime of the synthesis algorithm is independent of the network size [5], and each subsystem can synthesize its own controller, bypassing the need for centrally coordinated controller synthesis and making this framework highly scalable.

Many large cyberphysical systems benefit from model predictive control (MPC); to address this, a recent SLS-based work presents a distributed and localized MPC algorithm [6]. However, running MPC on a system requires constant online computation and is often unrealistic. To bypass this problem, we propose a two-layer SLS-based controller that achieves near-optimal performance while reducing online computation. We demonstrate this on an example system motivated by a power grid, which is subject to communication constraints, actuator saturation, disturbances, and setpoint changes that result from intermittently shifting optimal power flows (OPFs). In this problem, the presence of actuator saturation necessitates the use of MPC. Using the two-layer controller, which combines intermittent MPC-based setpoint tracking with offline disturbance rejection, we obtain MPC-level performance at a fraction of the computation cost of both centralized MPC and single-layer SLS-based MPC.

The proposed layered controller relies on the SLS framework and its derivative works. Since its inception, many extensions of the SLS framework have been developed, including works on nonlinear plants, online control, adaptive control, and learning; the core SLS ideas have proved themselves useful and applicable to a variety of settings. These SLS-based methods are effective as standalone controllers, and are also candidates for use in distributed layered control as demonstrated in our example in Section IV. We anticipate that most readers will be unfamiliar with SLS; to this end, we review the core mathematics of SLS and survey SLS-based techniques in Sections II and III, respectively.

The main contributions of this paper are twofold. Firstly, we present a scalable layered controller that successfully approximates MPC performance while using much less online computation; this provides a new way to combine existing tools to both maximize performance and minimize computational cost. Secondly, this paper is the first to present a comprehensive overview of all SLS-based methods; it forms a useful introductory reference for the system-level approach to distributed control.

II. THE SLS PARAMETRIZATION

We introduce the basic mathematics of SLS. The following is adapted from §2 of [4]. For simplicity, we focus on the finite-horizon state feedback case; analogous results for infinite-horizon and output feedback can be found in §4 and §5 of [4], respectively.
A. Setup

We will work with the discrete-time linear time varying (LTV) system

\[ x(t + 1) = A_t x(t) + B_t u(t) + w(t), \]

where \( x(t) \in \mathbb{R}^n \) is the state, \( w(t) \in \mathbb{R}^n \) is an exogenous disturbance, and \( u(t) \in \mathbb{R}^p \) is the control input. The control input is generated by a causal LTV state-feedback controller

\[ u(t) = K_t(x(0), x(1), ..., x(t)) \]

where \( K_t \) is some linear map. Let \( Z \) be the block-downshift operator. By defining the block matrices

\[ \hat{A} := \text{blkdiag}(A_1, A_2, ..., A_T, 0) \quad \text{and} \quad \hat{B} := \text{blkdiag}(B_1, B_2, ..., B_T, 0), \]

the dynamics of system (1) over the time horizon \( t = 0, 1, ..., T \) can be written as

\[ x = Z \hat{A} x + Z \hat{B} u + w \]

where \( x, u, \) and \( w \) are the finite horizon signals corresponding to state, disturbance, and control input respectively. The controller (2) can be written as

\[ u = K x \]

where \( K \) is the block-lower-triangular matrix corresponding to the causal linear map \( K_t \).

B. System Responses

Consider the closed-loop system responses \( \{ \Phi_x, \Phi_u \} \), which map the disturbance to the state and control input, respectively, i.e.

\[ x := \Phi_x w \quad \text{(5a)} \]
\[ u := \Phi_u w \quad \text{(5b)} \]

By combining (3) and (4), we easily see that

\[ \Phi_x = (I - Z(\hat{A} + \hat{B} K))^{-1} \quad \text{(6a)} \]
\[ \Phi_u = K(I - Z(\hat{A} + \hat{B} K))^{-1} = K \Phi_x \quad \text{(6b)} \]

**Definition 1.** \( \{ \Phi_x, \Phi_u \} \) are **achievable** system responses if there exists a block-lower-triangular matrix \( K \) (i.e. causal linear controller) such that \( \Phi_x, \Phi_u, \) and \( K \) satisfy (6). If such a \( K \) exists, we say that it achieves system responses \( \{ \Phi_x, \Phi_u \} \).

In the SLS framework, we work with the system responses \( \{ \Phi_x, \Phi_u \} \) directly. During controller synthesis, we optimize over the set of achievable system responses to find optimal controllers for a structured system in a convex manner. We are able to do this because the set of achievable closed-loop system responses is fully parametrized by an affine subspace, as per the core SLS theorem:

![Diagram](image)

**Theorem 1.** For the dynamics (1) evolving under the state-feedback policy \( u = K x \), where \( K \) is a block-lower-triangular matrix, the following are true

1. The affine subspace of block-lower-triangular \( \{ \Phi_x, \Phi_u \} \) parametrizes all achievable system responses (9).
2. For any block-lower-triangular matrices \( \{ \Phi_x, \Phi_u \} \) satisfying (7), the controller \( K = \Phi_u \Phi_x^{-1} \) achieves the desired response (5a) from \( w \mapsto (x, u) \).

**Proof.** See Theorem 2.1 of [4].

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\[ \text{Fig. 1. Controller implementation} \]

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Theorem 1 allows us to formulate an optimal control problem over state and input pairs \( (x, u) \) as an equivalent problem over system responses \( \{ \Phi_x, \Phi_u \} \). As long as the system responses satisfy (7) (which can be interpreted as a generalization of controllability), part 2 of Theorem 1 guarantees that we will also have a controller \( K \) to achieve these system responses. Thus, a general optimal control problem can be formulated in the SLS framework as

\[ \min_{\Phi_x, \Phi_u} f(\Phi_x, \Phi_u) \]
\[ \text{s.t.} \quad \Phi_x \in X, \Phi_u \in U, \]

where \( f \) is any convex objective function and \( X \) and \( U \) are convex sets. Details of how to choose \( f \) for several standard control problems is provided in §2 of [4]. Common specifications for distributed control, such as disturbance containment, communication delay, localized communication, and actuation delay, can be enforced by sparsity patterns on \( X \) and \( U \). Suitable specifications are discussed at length in [4] and in the SLS extensions presented in the next subsection. Additionally, for many classes of \( f \) (e.g. \( H_2 \) objective), (8) decomposes into smaller subproblems that can be solved in parallel, making the SLS formulation scalable.

C. Controller Implementation

Once we have solved (8) and obtained the optimal system responses \( \{ \Phi_x, \Phi_u \} \), we can implement a controller \( K \) as per part 2 of Theorem 1. Instead of directly inverting \( \Phi_x \), we use the feedback structure shown in Fig. 1 which is described by

\[ u = \Phi_x \tilde{w}, \quad \tilde{x} = (\Phi_x - I)\tilde{w}, \quad \tilde{w} = x - \tilde{x} \]

where \( \tilde{x} \) can be interpreted as a nominal state trajectory, and \( \tilde{w} = Zw \) is a delayed reconstruction of the disturbance. This implementation is particularly useful because structural constraints (e.g. sparsity) imposed on the system responses
\{\Phi_\delta, \Phi_u\} translate directly to structure in the controller implementation. Thus, constraints on information sharing between controllers can be enforced as constraints on \(X\) and \(U\) in \(\mathcal{G}\).

III. SLS-BASED TECHNIQUES & EXTENSIONS

Here, we provide an overview of SLS-based techniques, ongoing work on the core SLS methods, and extensions that make use of the SLS parametrization. All works described in this section exploit the fact that the SLS parametrization provides a transparent approach to analyzing and synthesizing controllers for closed-loop systems. Some works use the parametrization to carry out theoretical analyses and obtain bounds, while others take advantage of the distributed information-sharing constraints so that the resulting controllers are distributed and enjoy the core scalability benefits of SLS.

A. Standard SLS

The SLS parametrization was first introduced for state feedback, as described in the previous section, and subsequently extended to output feedback [5], [7]–[10]. A more detailed analysis of the SLS parametrization can be found in [3], with a companion paper [2] describing its scalability benefits. We refer the interested reader to [11] for a brief tutorial paper, or [4] for a comprehensive review of standard SLS.

B. Robust SLS

A robust variant of state feedback SLS is derived in [12], and is also included in [4]. These results extend to the output feedback case [13]. Robust SLS is also used to analyze sparsity preservation in discretization (sparsity approximation); it is shown that robust SLS works effectively with such approximations [14]. More recent results on the robustness of the generalized SLS parametrization can be found in [15]. Informally, the main idea of these works deals with the case in which the synthesized matrices \(\{\Phi_\delta, \Phi_u\}\) do not describe the actual closed-loop system behavior, either by design or due to uncertainty. In these cases, robust SLS methods and analyses provide guarantees on the behavior of the closed-loop system. These guarantees are used as a baseline in the development of many of the SLS methods we will describe in the remainder of this section.

In a setting with minimal uncertainty, robust SLS can be used when overly strict controller-motivated constraints on \(\{\Phi_\delta, \Phi_u\}\) result in infeasibility during synthesis; this was the original goal of the robust derivation. An alternative ‘two-step’ approach is presented in [16], whose results allow for separation of controller and closed-loop system constraints in the state-feedback setting.

C. SLS for Nonlinearities & Saturations

A recent extension of SLS addresses general nonlinear systems with time-varying dynamics [17]. This work generalizes the notion of system responses to the nonlinear setting, in which they become nonlinear operators. No constraints are considered; instead, the author focuses on the relationship between achievable closed-loop maps and realizing state feedback controllers.

Nonlinear SLS can be applied to saturating linear systems [18]. The resulting work provides an anti-windup implementation that is able to deal with saturation constraints on both the state and the input. An alternative approach to dealing with saturations uses no nonlinear analysis, instead relying on the application of robust optimization techniques to SLS [19]. In this work, a primal-dual optimization approach is provided for distributed synthesis. In the \(L_1\) case, with no coupling in the saturation constraints, the nonlinear method achieves superior performance compared to the linear method; however, the linear method deals with saturations in a more general setting.

D. Distributed Model Predictive Control

A popular method well-suited to deal with nonlinearities, saturations, and general input and state constraints is the model predictive control (MPC) approach. Based on the SLS parametrization, a distributed and localized MPC algorithm is developed, and is applicable to networks of arbitrary size [6]. In this work, the authors provide a closed-loop MPC algorithm distributed and localized in both in the synthesis and in the implementation of the controller. This is, to the best of our knowledge, the first closed-loop MPC scheme that is distributed in both the synthesis and in the implementation. Since the nominal approach is already closed-loop, this MPC scheme can be readily extended to a robust setting using robust optimization techniques from [19]; this will allow the scheme to preserve its distributed and localized synthesis and implementation.

Computation of this distributed MPC method can be significantly sped up via explicit solutions; so far, explicit solutions are available for the case of quadratic cost and saturation constraints [20]. In addition to forming the basis for this MPC algorithm, the SLS parametrization is also used to perform robustness analysis and guarantees on the general MPC problem [21].

E. Adaptive Control & Machine Learning

SLS – especially robust SLS – is also applicable to the fields of adaptive control and machine learning. In particular, [22] provides a method to adapt online dynamic robust controllers in a stable manner with the goal of improving performance; this is achieved by collecting measurements at every timestep to reduce uncertainty about system parameters. An application of this work is used to synthesize and implement SLS controllers that are robust to package dropouts [23]. A different adaptive SLS algorithm deals with networks that switch between topological configurations according to a finite-state Markov chain [24].

In the area of machine learning, SLS has been used to analyze safety and provide theoretical bounds. SLS is especially beneficial for this application due to its ability to relate model uncertainty with stability and performance suboptimality [25]. The SLS parametrization is used to
analyze the linear quadratic regulator (LQR) problem in the case where dynamics are unknown [26], [27]. These works provide safe (robust) learning algorithms with guarantees of sub-linear regret, and study the interplay between regret minimization and parameter estimation. Additionally, the SLS parametrization forms the basis of a framework for constrained LQR with unknown dynamics, where system identification is performed through persistent excitation while guaranteeing the satisfaction of state and input constraints [28]. SLS is also used to provide complexity analysis sharp end-to-end guarantees for the stability and performance of unknown sparse linear time invariant systems [29].

SLS-based analyses have been applied to the output feedback setting as well [30]. Motivated by vision-based control of autonomous vehicles, this work solves the problem of controlling a known linear system for which partial state information is extracted from complex and nonlinear data; a safe set and robust controller is designed under mild assumption. SLS also underpins the sample complexity analysis for Kalman filtering of an unknown system [31].

F. Additional works

SLS is extended to the spatially invariant setting in [32], [33]. In particular, it is shown that infinite-dimensional spatially-invariant systems can be formulated as a model-matching problem with finitely many transfer function parameters. The limitations of SLS in the case of relative feedback are analyzed in [34], [35], where the authors explore realizable structured controllers via SLS, and propose a relaxation of the structured SLS problem to analyze the feasibility in the presence of relative feedback.

Lastly, several SLS publications focus on optimizing computational solutions for SLS. Explicit solutions to the general SLS problem are described in [36], and [37] uses dynamic programming to solve for SLS synthesis problems 10 times faster than using a conventional solver. [38] describes deployment architecture for SLS, and is used in the construction of the SLSpy software framework [39]. In addition to this Python implementation, a MATLAB-based toolbox is also available [40]; these open-source SLS implementations are available at https://github.com/sls-caltech/sls-code. Techniques implemented include standard and robust SLS as well as the two-step SLS algorithm and SLS-based distributed MPC method. The examples in the next section make use of these publicly available SLS tools.

IV. CASE STUDY: POWER GRID

We demonstrate SLS-based approaches on a system motivated by the power grid, with three key features

1) Periodic setpoint changes, induced by changing optimal power flows (OPFs) in response to changing load profiles
2) Frequent, small disturbances
3) Actuator saturation

We start with a randomly generated connected graph over a 5x5 mesh network, shown in Fig. 2. Edges in the graph represent connections between buses. Interactions between neighboring buses are governed through the undamped linearized swing equations (10); this is similar to the example used in [4].

A. System Setup

begin{equation}
x_i(t + 1) = A_{ii} x_i(t) + \sum_{j \in N(i)} A_{ij} x_j(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} w_i(t) + u_i(t) \end{bmatrix}
\end{equation}

\begin{equation}
A_{ii} = \begin{bmatrix} -\frac{1}{b_{ii}} \Delta t & \Delta t \\ \frac{1}{m_i} & -1 \end{bmatrix}
\end{equation}

\begin{equation}
A_{ij} = \begin{bmatrix} 0 & \frac{1}{m_i} \Delta t \\ \frac{1}{m_j} & 0 \end{bmatrix}
\end{equation}

where the state of bus $i$ includes phase angle, $x_i^{(1)}$, relative to some setpoint, and $x_i^{(2)}$, the associated frequency, i.e.

\begin{equation}
x_i(t) = \begin{bmatrix} x_i^{(1)}(t) \\ x_i^{(2)}(t) \end{bmatrix}
\end{equation}

$m_i$, $w_i$, and $u_i$ are the inertia, external disturbance, and control action of the controllable load of bus $i$. $b_{ij}$ represent the line susceptance between buses $i$ and $j$; $b_i = \sum_{j \in N(i)} b_{ij}$. $b_{ij}$ and $m_i^{-1}$ are randomly generated and uniformly distributed between [0.5, 1], and [0, 10], respectively. The large values of $m_i^{-1}$ render the system unstable; for this example, the spectral radius of the system matrix is 1.5. Inertia-related instability is a well-known challenge associated with adding renewable sources to the power grid, as renewables are typically low inertia [41].

Periodically, we generate a new load profile at random and solve a centralized DC-OPF problem, using the load profile as input, to generate the optimal setpoint $x^*$. We then send each sub-controller their individual optimal setpoint, and allow each subsystem to reach this setpoint in a distributed manner; sub-controllers are only allowed to communicate with their immediate neighbors and the neighbors of those

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{example_system.png}
\caption{Topology of example system. We will plot the time trajectories of states, disturbances, and input for the red square node.}
\end{figure}
neighbors (i.e. localized communication). In addition to tracking setpoint changes, subsystems must also reject randomly generated disturbances. The overall system setup is shown in Fig. 3.

Fig. 3. Architecture of example system. For ease of visualization, we depict a simple 4-node topology instead of the 25-node mesh we’ll be using. Grey dotted lines indicate periodic communications; the OPF solver sends $x^*$ only on the timesteps when it receives a new load profile.

Additionally, we enforce actuator saturation constraints of the form

$$|u_i(t)| \leq u_{max}$$  \hspace{1cm} (12)

Actuator saturation is a ubiquitous constraint, not only in power systems but in other distributed systems as well (e.g. transport networks).

B. Toward a Layered Controller

In the absence of actuator saturation, distributed setpoint movement can be easily and optimally done with a variety of SLS methods, including standard SLS. With saturation, the linear controller can incur significant costs due to integral windup and may lose stability. In a standard disturbance-rejection problem, this can be ameliorated by the SLS-based methods that deal with saturation [18], [19]. However, offline methods deal with worst-case scenarios and are too conservative to deal with the large setpoint changes that are characteristic of our system; specifically, the combination of expected large setpoint changes and relatively tight saturation bounds render synthesis of such a offline controller infeasible. Thus, we turn to the distributed and localized SLS-based MPC [6], which mitigates windup effects at the cost of significant online computation, since we must solve an optimal control problem at every timestep.

Here, the combination of large setpoint changes, small disturbances, and actuator saturation present us with a performance-computation trade-off. With offline methods, computation costs are minimal but performance costs are large and possibly infinite, in the case of stability loss; with MPC, computation cost is large but performance cost is minimized. To bypass this trade-off, we propose a distributed layered controller. Roughly speaking, we decompose the main problem into two subproblems – reacting to large setpoint changes and rejecting disturbances – and assign each to a layer.

The top layer of the two-layer controller is an MPC-based trajectory generator. The primary purpose of this controller is to optimally track setpoint changes while satisfying saturation constraints. Every $T_{MPC}$ timesteps, the top layer will solve an optimal control problem with initial condition equal to the current state, i.e. $x_0 = x(t)$, subject to saturation constraints. This generates a safe trajectory of states $x$ and inputs $u$ for the next $T_{MPC}$ steps. The top layer is online, but is computationally cheaper than a standard MPC controller; while standard MPC solves an optimal control problem every time step, the trajectory generator solves an optimal control problem every $T_{MPC}$ steps – this gives a $T_{MPC}$-fold decrease in computational cost. To maximize performance, we time the top layer trajectory generation to coincide with the periodic setpoint changes. We can also run the top layer periodically in between setpoint changes, which may be useful for dealing with unexpectedly large disturbances.

The trajectory generator alone is not sufficient. Since it runs only once every $T_{MPC}$ timesteps, disturbances can persist or even amplify between adjacent runs, severely compromising performance. This could potentially be improved by a robust MPC formulation, although it is unclear whether it is advisable to rely solely on any online method that only runs once every $T_{MPC}$ timesteps, especially given that we would like to choose a large $T_{MPC}$ in order to minimize the computational cost. Thus, we add an offline standard SLS controller to the bottom layer to reject disturbances and preserve performance. The bottom layer controller receives trajectory information from the top layer and outputs a control signal $u$ that tracks the desired trajectory while rejecting disturbances. The two-layer controller is shown in Fig. 4.

Fig. 4. Architecture of layered sub-controller for a single subsystem. The grey dotted line indicates periodic setpoint changes sent by the OPF solver. The grey dashed line indicates periodic sensing; every $T_{MPC}$ timesteps, the top layer accesses the sensor info and uses it to generate a sequence of trajectories for the next $T_{MPC}$ timesteps. The top layer then feeds this trajectory to the bottom layer controller and waits for $T_{MPC}$ timesteps before it runs again.
Like its constituent controllers, the layered controller is distributed; each subsystem synthesizes and implements both layers of its own controller. The synthesis of the two layers is independent of one another, although some synthesis parameters may be shared. Each top layer sub-controller must communicate to the corresponding bottom layer sub-controller, but no other cross-layer communication is required, i.e. top layer controllers never communicate with bottom layer controllers from a different subsystem and vice versa. Since both the top and bottom layer controllers can be synthesized in a distributed manner, the overall layered controller, like its constituent controllers, is well-suited for systems of arbitrary size.

C. Results & Discussion

We compare the performance of the layered controller (‘SatLocLayered’) with the performance of the SLS-based distributed and localized MPC controller (‘SatLocMPC’). For additional comparison, we include three centralized controllers: the optimal linear controller (‘UnsatCentLin’) which is subject to no saturation; the same controller, subject to saturation (‘SatCentLin’), and MPC (‘SatCentMPC’). Note that the centralized controllers are not subject to any communication constraints, while the distributed controllers may only communicate locally. In this example, we set $T_{MPC} = 20$, which means that the layered controller uses only 5% as much online computation as the distributed MPC controller. The resulting LQR costs, normalized by the non-saturating optimal centralized cost, are shown in Table I. We plot the trajectories of the red node from Fig. 2 in Fig. 5, focusing on a small window of time during which only one setpoint change occurs. The non-saturating controller is omitted from the plot.

| Controller      | Actuator Saturation | Centralized | LQR cost |
|-----------------|---------------------|-------------|----------|
| UnsatCentLin    | No                  | Yes         | 1.00     |
| SatCentLin      | Yes                 | Yes         | 1.93     |
| SatCentMPC      | Yes                 | Yes         | 1.93     |
| SatLocMPC       | Yes                 | No          | 1.93     |
| SatLocLayered   | Yes                 | No          | 1.93     |

For this setpoint change, windup effects induced by actuator saturation result in oscillations of increasing size from the saturated linear controller, causing it to lose stability and incur an astronomical cost. Windup effects are mitigated by all three online controllers, which perform similarly despite drastically different computational requirements. For a fixed horizon size, per time-step, centralized MPC scales with $O(N^2)$, where $N$ is the number of states in this system (equal to 50 in this example). Conversely, the distributed and localized SLS-based MPC algorithm scales with $O(d^2)$ where $d$ is the communication locality size (equal to 2 in this example) [6]. Clearly, centralized MPC does not scale well to large networks while localized MPC scales to networks of arbitrary size. The layered controller uses the localized MPC algorithm but additionally reduces the cost by a factor of $T_{MPC}$ by utilizing a lower layer offline controller; in this example, the layered controller uses 5% as much computation as the localized MPC controller and still performs within 3% of the centralized MPC cost. As desired, the proposed layered structure achieves near-optimal performance while drastically reducing computational time.

To check general behavior and confirm that our results are not peculiar to this particular example, we re-run the simulation 30 times with different randomly generated grids, plant parameters, disturbances, and load profiles for the OPF. In the interest of time, we do not include the centralized MPC algorithm. The resulting LQR costs, normalized by the non-saturating centralized optimal cost in each run, are shown in Table II. As before, we use $T_{MPC} = 20$.

| Controller      | LQR cost |
|-----------------|----------|
| UnsatCentLin    | 1.00     |
| SatCentLin      | 1.3257   |
| SatLocMPC       | 1.16     |
| SatLocLayered   | 1.17     |

Compared to the 30-trial averages, the example presented...
in Table I is an extreme example. However, the general observations still stand. Over 30 examples, the layered controller achieves performance that is within 1% of the standard localized MPC performance, using only 5% of the online computation time. Online methods again demonstrate enormous improvement over the saturated linear controller, which often loses stability after large setpoint changes and incurs huge costs; stability is lost in 4 of 30 runs. We observe that performance differences predominately arise from reactions to setpoint changes; when the saturated linear controller manages to maintain stability after a setpoint change, responses to disturbance are similar across all methods. In these cases, the layered controller still performs a few percent better than the saturated linear controller. Lastly, we note that both the distributed, localized online-based methods’ costs are within 17% of the centralized optimal cost without actuator saturation.

The concept of layering multiple distributed controllers extends beyond this case study. Our choice of features – namely, large setpoint changes, small disturbances, and actuator saturation – make MPC and standard SLS obvious candidates for the layered structure. For a system with different features, e.g. plant uncertainty, we can use different tools, such as distributed adaptive control in the top layer and robust SLS in the bottom layer. The general idea is to put computationally expensive, high-performance methods in the top layer, where they are run sporadically, while using a basic offline algorithm in the bottom layer to remain stable. Furthermore, instead of periodically running the top layer, we could conditionally trigger runs; in this example, trajectory generation could be triggered by receiving a new OPF setpoint. Like standard SLS methods, this layered framework is also compatible with distributed hardware codesign, whereby we locally update sensing, actuation, and communication between subsystems and locally update the corresponding controllers as well [4], [42]. More broadly, layered controllers are found not only in cyberphysical systems but in biological systems as well [43]; SLS-based distributed layered control has potential applications toward producing models in this domain.

V. CONCLUSIONS

In this paper, we reviewed the core concept of SLS, the body of SLS-based works, and demonstrated an effective combination of SLS-based methods in a novel layered controller. This layered controller performed similarly to centralized MPC while drastically reducing computational cost. We anticipate that the notion of the layered distributed controller can make use of a variety of controllers beyond those presented in our case study, and plan to extend to more general settings in future work.

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