Numerical simulation of overtopping in the earth-filled dam based on material point method

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Abstract. Dams breach by overtopping is a serious natural disaster. Numerical simulation, as a efficient research method, is widely used to study the failure mechanism and predict the destroyed process of earth dams. Traditional numerical methods such as the finite element method (FEM) only provide the capacity of small deformation analyses due to mesh distortion. The Material Point Method (MPM), a mesh-free method, has been worked well for large deformation analyses. In this paper, a double-point MPM is introduced to analysis the failure process of earth-filled dams due to overtopping. Finally, it shows that the MPM predicts the flow propagation, the development of breach cross section and breach longitudinal profile well. Moreover, MPM provides an alternative method for the risk assessment of dam breach.

1. Introduction
It is of a low probability and high risk to the dam failure, once the dams burst accidents, it will cause great damage to the downstream area. As the main dam type, the earth-filled dams accounted for about 30% of dams failed due to overtopping [1]. With the frequent occurrence of extreme weather in recent years, the water level rises and exceeds the original design level due to heavy rainfall, which is prone to overtopping damage. It is necessary to study the process of overtopping failure, reveal the failure mechanism, and carry out a more effective disaster prevention. The overtopping failure of earth-filled dams is a process of interaction between water and soil, which is comparatively complex. In order to investigate the mechanism of overtopping failure, scholars have done a lot of research on the interaction between water and soil. Compared to the experiment, numerical simulation is a flexible and convenient method to simulate the failure process of earth-filled dams. However, the current researches mainly focus on the mathematical model of overtopping failure, the simulations on the failure process are few [2-4]. Material point method (MPM) is a novel particle-based method, which can well model large deformation problems. It was first extended to solid mechanics by Sulsky et al in 1995 [5], but it has been widely used in latest ten years, especially in geological engineering [6]. In this study, Anura3D software (www.anura3d.com), which implements the double-point MPM formulation, is introduced to simulate the failure process of earth-filled dams due to overtopping.

2. The basic of MPM
With MPM, the continuum is discretized into a series of material points, which carry all the material information, such as mass, velocity, stress, strain. It combines the double advantage of the Euler mesh and Lagrange particle, and there is no relative motion between the material points and the grid, which
avoids the simulation difficult due to the mesh distortion [7]. The Hence, MPM is superior to mesh-based methods. Besides, it can process the complex boundary conditions because of the use of a background mesh, and the tensile instability don’t occur in the simulation [8]. The double-point MPM have two sets of material points, liquid phase material points and solid phase material points, which introduces a drag force to reflect the interaction between liquid and solid phase. With the double-point MPM, not only the large deformation of soil and the water flow can be simulate well, but the conversion between free water and pore water can be modeled. The more and computation scheme can be found in the literature [9, 10], and only the basic formulation of the double-point MPM is presented in this study. The motion of an object must satisfy the fundamental laws of conservation, such as mass conservation, momentum conservation and energy conservation. The governing equation is the mathematical description of these conservation laws. Since heat exchange and heat source are not considered in this study, and energy exchange is not involved, only the mass conservation equation and momentum conservation equation are quoted. The mass of each phase must be conserved in the calculation. The mass conservation law requires that the derivative of the mass of any region of the material with respect to time be 0. According to this law, the mass conservation equation for the solid and liquid phase respectively can be obtained as follow:

\[
\frac{D\bar{\rho}_s}{Dt} + \bar{\rho}_s \nabla \cdot \mathbf{v}_s = 0
\]

(1)

\[
\frac{D\bar{\rho}_l}{Dt} + \bar{\rho}_l \nabla \cdot \mathbf{v}_l = 0
\]

(2)

Where \( \bar{\rho}_s \) represents the relative density of solid, \( \bar{\rho}_l \) represents the relative density of liquid, \( \mathbf{v}_s \) represents the velocity vector of soil skeleton, and \( \mathbf{v}_l \) represents the velocity vector of liquid.

The relative acceleration of solid and liquid, darcy's law of permeability and the inertia of water are taken into account in order to capture the dynamic response of water and ensure the continuity of pressure. Solid velocity and liquid velocity are taken as basic unknowns. The momentum conservation equation of liquid is as follows:

\[
n \rho_l \mathbf{a}_l = -n \nabla p_l + n \rho_l \mathbf{b} - \frac{n^2 \gamma_l}{k} (\mathbf{v}_l - \mathbf{v}_s)
\]

(3)

Where the third term on the right represents the interaction between solid and liquid, \( \mathbf{v}_l, \mathbf{v}_s \) represents the velocity vector of liquid and solid, \( \rho_w, \gamma_w \) represents the density and bulk density of liquid respectively, \( n \) represents the porosity, \( k \) represents the permeability coefficient of soil, \( p_l \) represents the pore water pressure, and \( \mathbf{b} \) represents the physical force.

According to the mixed theory, momentum conservation equation of soil skeleton is as follows:

\[
(1 - n) \rho_s \mathbf{a}_s = \nabla \cdot \mathbf{\sigma} + (1 - n) \nabla p_l + \frac{n^2 \gamma_l}{k} (\mathbf{v}_l - \mathbf{v}_s) + (1 - n) \rho_s \mathbf{b}
\]

(4)

The momentum conservation equation of the mixture was obtained by linear superposition of (2) and (3):

\[
(1 - n) \rho_s \mathbf{a}_s + n \rho_l \mathbf{a}_l = \nabla \cdot \mathbf{\sigma} + \rho \mathbf{b}
\]

(5)

\[
\rho = \bar{\rho}_s + \bar{\rho}_l = (1 - n) \rho_s + n \rho_l
\]

(6)

\[
\mathbf{\sigma} = \mathbf{\sigma}' - p_l \mathbf{I}
\]

(7)

Where \( \rho \) represents the density of solid particles, \( \rho \) represents the density of mixture, \( \mathbf{a}_s, \mathbf{a}_l \) respectively represents the acceleration vector of solid and liquid phases, \( \mathbf{\sigma} \) represents the total stress vector, and \( \mathbf{\sigma}' \)represents the effective stress of solid.
3. Numerical simulation of overtopping
The earth-filled dam suffered an overtopping flow was modeled to study the failure process and mechanism, and a numerical case of water flowing through a porous dam has been conducted to validate the capacity of MPM by Zhao et al [11]. The details of a small-scale dam referred to this study is shown in figure 1. The height and base width is 0.4 m and 1.95 m, and the slope ratio is 1:2. The upstream water level is assumed to be 1.2 m high. The downstream slope is the focus of the study, so only the area A-B-C was modeled, the MPM model is illustrated in figure 2. The dams are filled of homogeneous sand, whose grain are 0.25 mm fine particles and 0.6 mm coarse particles respectively, and they are located on a impermeable and stiff foundation. The Mohr-Coulomb model is adopted for dams and the reservoir water is a Newtonian fluid, and their parameters are shown in the table 1.

![Figure 1. Details of dam](image1)

![Figure 2. MPM model of dam](image2)

| Table 1. Material parameters of dams and reservoir water |
|---------------------------------------------------------|
| Parameter                                              |
| Density [kg/m³]                                        | 2650                                      |
| Initial porosity [-]                                    | 0.3                                       |
| Maximum porosity [-]                                    | 0.5                                       |
| Friction angle [°]                                      | 30                                        |
| Cohesion [kPa]                                          | 0                                         |
| Young modulus [kPa]                                     | 1000                                      |
| Poisson ratio [-]                                       | 0.3                                       |
| Water density [kg/m³]                                  | 1000                                      |
| Water bulk modulus [kPa]                               | $2 \times 10^6$                           |
| Water viscosity [kPa·s]                                 | $8.905 \times 10^{-7}$                    |
4. The failure process of earth-filled dams due to overtopping

Through the analysis of numerical results, it showed that the failure processes of the earth-filled dam on the different conditions are similar in general. As a result, the failure process may be divided into three stages. The figure 3(a), 3(b) is the first stage, the figure 3(c), 3(d) is the second stage, and the figure 3(e) is the final stage. The specific characteristics of those stage are described as follows.

![Figure 3](image-url)

**Figure 3.** The gradual failure process of the earth dam due to overtopping

4.1. The first stage

After maintaining a high water level conditions, the soil became close to be saturated in the downstream slope of dam, and then a designated reservoir water level which was 0.3 m above the crest of dam was supplied to overtopping the earth-filled dams. As the case starts to simulate, the overtopping flow rushed at the soil of the dam crest instantaneously, the solid material points of that area are rolled up and carried to the downstream. The earth dam made of fine particles (0.25mm) is scoured more seriously than that made of coarse particles (0.6mm) in a certain time. The figure 4 shows two slope forms when the overtopping flow floods at 2.5 s, the water flows faster on the slope of fine particles than that of coarse particles, fine particles are washed farther than coarse particles as well. The porosity of soil made of coarse particles is bigger, so there will be more infiltration capacity. Therefore the velocity of overtopping flow will slow down because liquid permeates the earth dam. Besides, the anti-scouring property of the slope surface formed by coarse particles is stronger and the resistance of the overtopping flow tends to increase, it lead to slow down the velocity of the flow rush to the downstream. Because of the poor anti-scouring property of the fine particles, a portion of sediment particles will be washed away along the slopes surface with the water current.

![Figure 4](image-url)

**Figure 4.** Downstream slope at 2.5 s
4.2. The second stage
With the sediment of the dam crest are washed away continually, the crest is gradually turned to a scarp (as figure 3(b) shows). As the height of the scarp is increasing, the fall of hydraulic head become higher, and the scarp get steeper, the overtopping flow is stronger and stronger so that sediment of the scarp is rushed to the downstream. As the scarp deepens and moves down continually. Furthermore, the scarp gets instability and tends to collapse finally(as figure 3(e) shows). After the scarp collapsed, the hydraulic head of dam-breach control section increased suddenly, and then the dam-breach flow become larger. The increasing scouring lead to a second collapse. After that, the slope surface will form a uniform scouring phenomenon(as figure 3(d) shows). The high hydraulic head and flow velocity make the shear force bigger, as a result, the high velocity flow rush away more sediment accompanied by the waves breaking, and the flood peak value of dam flow appear in the stage.

4.3. The final stage
After the last stage, the hydraulic head of crest gets lower, and the flow velocity begin to slow down so that the ability of flow erosion reduces. As the dramatic scour in the second stage, the sediments are rushed to the toe of the slope, the slope gets extended and gentle and it turn to be a wave shape just like a retrograde longitudinal moving upstream(as figure 3(e) shows). It is similar to the experiment research of Chinnarasi et al [12]. As the hydraulic head of upstream reduces gradually, the velocity of the overtopping flow will be slower and slower until the flow stops moving. At last, the slope will reach a stable state if maintain a constant high overtopping level, and form a wedge deposit body that is flat and gently sloping on the basis of the original dam. At this time, the water surface profile parallel to the slope surface of the deposit body. This stage continue for a relatively long time, but it is harmless for flood control, and the flow pattern changes smoothly. According to the above, the anti-erosion performance of coarse particles and fine particles is different, so their final shape can not be the same. The height of the wedge deposit body made of fine particles is 0.51m, and that of coarse particles is 0.69m.

5. Conclusions
The onset and the progression of the earth-filled dam failure due to overtopping flow are successfully simulated in this study using the double-point MPM formulation. MPM predicts the flow propagation, the development of breach cross section and breach longitudinal profile well. It is shown that MPM can satisfactorily simulate the problem refer to the interaction between soil and water and large deformation. However, the small-strain FEM analysis can’t as it suffers from element distortion. Moreover, MPM provides an alternative method for the risk assessment of dam breach.

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