The Counterintuitive Concept of Ergodicity in the Context of a Business Simulation Game

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Abstract. The concept of ergodicity as described in the London Mathematical Laboratory seems very counterintuitive. Something is ergodic when its time average equals to its expectation value. Transferring this sentence into a more suitable real-life situation, one can say pooling and sharing earnings will increase one’s wealth steadily over time. One may wonder why should they cooperate with each other when they all have bad exterior conditions? Will cooperation not make everything worse? We developed a Business Simulation Game, New Rising of the Ylsung concept, which tests different parameters to visualize the concept of ergodicity. We distinguished that in most of the cases it brings a greater advantage to collaborate than to work by oneself. Moreover, the simulation helps high school students to understand the research of the London Mathematical Laboratory in a playful manner. Hence, they do not have to read research papers, but simply play through our Business Simulation Game while adjusting the parameter sets and analyzing the graphs. Our main aim is to find out to what extent the application of different parameters between cooperation and non-cooperation differ from each other and how the application of those looks in real-life scenarios.

Keywords: Ergodicity · Business simulation · Game · Playful learning · Economic concept · Ergodicity economics

1 Introduction and Motivation

Marc Elsberg’s book “Gier” published in 2019 focuses on the works of the London Mathematical Laboratory (LML) scientists. The thriller builds on inequality, injustice, polarization, division of society and the great dissatisfaction with how politics and businesses deal with those aspects. Furthermore, the thriller presents a scientific approach to the discomfort of bringing the LML’s approach closer to people [4]. The LML has several research areas from economics to inference, from models to limits and to learning. The concept of the Farmer’s Fable, hence the concept of ergodicity, can be found within the economics research [16]. The Farmer’s Fable within “Gier” is a fictional story describing the economic concept of ergodicity. In this
story, two farmers cooperate and share the earnings of their farming, whereas the other two farmers do not want to cooperate and consequently keep their whole earnings [4]. The advantage of cooperation lies in the concept of not risking everything you own. You always have somebody to rely on in difficult times. Even though you might have to give away most of your earnings, over time the profit will be larger. Whereas if you never share, you might make high profits for a couple of years, considering if any unexpectedly happens, like a natural disaster, you are at risk of losing everything you worked for.

We created the Business Simulation Game (BSG) New Rising of the Ylsung based on the Farmer’s Fable. This project is a part of the Business Simulation Design module at the Technical University of Munich, with the target group of teenagers and young adults in middle and high schools. For this purpose, we constructed a single player game in Unity with C# and integrated Python. The BSG presents valuable information about the economic phenomenon of ergodicity.

Moreover, we conducted a user study and developed a quiz, which are not part of this paper, to get a better understanding of how the simulation is perceived by outsiders and which parts we can improve during the development process. The group of play testers consisted of pupils at the Staatliche Fachober- und Berufsoberschule Technik München, a technical college in Bavaria, and of students at the Technical University of Munich. This quiz is designed to function as a pre- and post-test, to examine how well the participants understand the concept of ergodicity before and after playing the BSG and whether they are able to apply it to real-life situations.

It is important to consider that the Farmer’s Fable takes up a span of several years [4]. When taking only a couple of years into account, they might not discover any significant advantages towards neither cooperating nor working alone. For this reason, we implemented an adjustable timeline. The user can regulate it between a span of 10 up to 250 years and observe the different outcomes.

2 Background

In the following sub-sections we define a simulation and the concept of ergodicity.

2.1 Simulation

The process of reproducing systems in various application fields is called “Simulation” [11]. These simulations may analyze possible issues within the system, without putting anyone or anything at risk [11]. A business simulation facilitates a faster growing of the learning curve, by creating an immersive feeling and with no risk of damaging a working system [3, 8, 9]. It allows to gain an overview over the project, while highlighting bottlenecks and raising questions for improvement of the overall product [3, 8, 9]. Furthermore, taking a closer look onto our target group, teenagers and young adults between the ages 14 to 25, it is of high importance for us to create a simulation that is easily understandable as well as enjoyable. Compared to only reading about a system or a process, the user has a first-hand experience and gains some hands-on experience. Since games, in particular serious games and business simulation games are able to
create a link between entertainment and learning content, the BSG *New Rising of Ylsung* aims to be an interactive and playful way of studying economic principles and learning to adapt them quickly.

### 2.2 The Concept of Ergodicity

Ergodicity is a part of probability theory and related to statistics [16]. Both are difficult topics that students in German schools usually have to study during their mathematics classes at high school. We created the BSG *New Rising of the Ylsung* to make the concept of ergodicity more intuitive for this target group.

#### 2.2.1 A Definition of Ergodicity

Ergodicity can be found in equilibrium statistical mechanics. For a system to be ergodic, it is assumed that the time average of an observable equals its expectation value. Even though, this concept is applied a lot in economics, it does not hold in most cases, since wealth is not ergodic [16].

We have to take a closer look on dynamical systems to find ergodicity. Now, a dynamical system is used as an object in physics to model certain phenomena. These objects can have states, which will develop over time. Three main themes can be distinguished within dynamical systems, namely the predictive ones, diagnostic ones and ones that aim to explain physical phenomena by theories [1]. Moreover, attention on the systems that are not only dynamical, but also indecomposable dynamical, has to be paid. Indecomposability means that something cannot be partitioned into several parts [13]. Our aim is to reach the points in a set of another orbit. Allowing the system to be broken down into smaller sets would make this impossible [1].

#### 2.2.2 New Rising of the Ylsung

In this subsection, we give an overview of our BSG. At the start of the Game (see Fig. 1), the user has the option to choose between four and up to 20 farmers that can either cooperate or farm by themselves. Moreover the user can apply conditions to the farmer’s fields. These conditions vary between good, bad and random (see Sect. 4.1). The adjustable timeline of the BSG ranges from 10 to 250 years.

While the BSG runs through, one can watch the farmers running around their fields, as shown in Fig. 1. Meanwhile a graph is depicted on the top left corner, which shows how much profit each farmer respectively each collaboration of farmers makes. This is useful to grasp the idea of the business concept in real time. To get a more elaborate explanation, the user receives more graphs when the simulation is completed. Those graphs are ordered in the shape of spider webs to explain different parameter sets that act upon the profits of the farmers, simultaneously. The impact of various parameter sets will be discussed in Sect. 4.1.
2.2.3 Ergodicity in New Rising of the Ylsung

According to the philosopher Thomas Hobbes’ conception of a human being, a human being can be understood as selfish by nature [7]. Moreover, interpreting John Locke’s philosophies, man is altruistic [12]. At the beginning of the BSG, the user is asked which and how many farmers should collaborate for a certain amount of time. A first intuition makes us wonder why the farmers should cooperate in the first place. It is a competitive world that we live in, hence, would not it make more sense for the farmers to work on their own and keep the whole amount of profit they make? Nowak argues in his paper on *Five rules for the evolution of cooperation*: “[…] cooperation is the decisive organizing principle of human society […]” [1] and that “Cooperation is needed for evolution to construct new levels of organization” [1]. So in contrary to the first intuitive belief, it should be beneficial, not only for “genes [to] cooperate in genomes” [14] and for “chromosomes [to] cooperate in eukaryotic cells” [14], but for farmers to cooperate in New Rising of the Ylsung, so that they can increase their profits.

This paper explores how in every cooperation one entity has more of a certain aspect, such as profits in our case, than the other cooperating entity can provide. This means that one entity will decrease their previous value when they share [14, 18]. However, considering cooperation in a long-run, it is quite possible that giving and receiving will balance out between the entities. In research of the LML, Adamou and Peters [18] describe this idea as reciprocity and relatedness. Reciprocity describes the balancing of giving and receiving [18]. In some cases, entity_1, in our game called a farmer, will share their profits and in other cases entity_1 will gain from their cooperation partner, entity_2, as well a farmer. The latter, relatedness, aims to spread genetic material that entity_2 is carrying [14, 18]. This case is of lower importance for the BSG. We will not explicitly look at the biological aspects, as the farmers are not related to each other, neither are their crops.

3 Research Questions and Methodology

The LML provides a great knowledge base, which we try to support with our BSG. For our research, we posed the following two research questions:

**RQ1:** How does a parameter set have to look like to support the *London Mathematical Laboratory* concept of ergodicity by using a business simulation?
RQ2: For which real human scenarios is the concept of ergodicity a particularly suitable solution?

For the first research question, we define some test cases within our BSG and apply different parameter sets. These parameter sets cover a variation of extreme weather conditions among six different farmers, whereby two farmers respectively have the same conditions for their fields in order to be able to compare harvest yields retrospectively. Moreover, we conducted a minor informal user study and a presentation at a Games Symposium at the Technical University of Munich to analyze our simulation and the parameter outcomes.

However, in real-life one cannot influence the weather as they like, neither do farmers have unlimited field sizes. These reasons lead us to the next question, RQ2. More situations than farming exist where one encounters the concept of ergodicity. To answer RQ2, a literature research according to [19] was done by using the following keywords on the website of the LML and in Google Scholar: “ergodicity, applied ergodicity, ergodicity in nature, ergodicity in real-life”. According to Peters [17] ergodicity economics predict functional forms, given the wealth dynamics. However, wealth dynamics is not a well-known or well predictable factor. Therefore, ergodicity economics is not a trivial concept in mathematics. Apart from the economics factor, the question leads to other real-life scenarios where ergodicity has been applied, can be applied or will be applied in future – even if only partially.

4 Results

In the following sub-sections, we will answer RQ1 with the findings of our BSG and RQ2 with literature research.

4.1 Applying Parameter Sets

To analyze and answer RQ1, various sets of parameters were applied and the results were compared, accordingly. Not fixating the BSG to the original story of Marc Elsberg [4], where he explains the concept with the use of four farmers but giving the user the opportunity to change the parameters, the BSG becomes more interactive. This interaction helps users to learn more thoroughly, since they can manipulate the game and observe the different outcomes, instead of following a strict presentation of the outcomes as they would do in frontal education. As Zimmermann and Miliband explain in their paper on self-regulated personalized learning (SRPL), the student, in our case the user, holds the responsibility for learning themselves [20].

While playing through the BSG, an intuition might lead to the Prisoner’s Dilemma [10], where the most common strategy for winning is tit-for-tat. Both parties rely on each other’s answer, where one will cooperate only if the other does the same. However, they cannot know each other’s reaction, and might fall back into egoistical thinking due to mistrust in the other party. While changing the strategy in the midst of the game is not possible in the BSG, the next simulation cycle offers the opportunity to
change the arrangement of the cooperating farmers, such that adequate results of the various strategies can be drawn.

Finally, to answer RQ1 the BSG was tested multiple times with different parameter sets. Two interesting sets are explained in further detail in the following. For our examples we use six farmers. Thus, Farmer 1 and 2 have the same weather conditions, as Farmer 3 and 4 and Farmer 5 and 6 do. While Farmer 1, Farmer 3 and Farmer 5 do not cooperate, Farmer 2, Farmer 4 and Farmer 6 share their earnings after each year.

An important factor of the BSG lies in the adjustable timeline, since the most significant results are achieved after a huge time span. It is important to consider several generations of farmers, rather than just one generation. To conclude our results, we sat the timeline to 50 years, 100 years, and 250 years and compared the results under the same weather conditions. For our exemplary cases, we are setting the timeline to 100 years.

Please note that farmers might be able to predict on average how the climate is going to change in the future. However, they are not able to make accurate predictions about the weather [6]. It can stay the same over a period of time or change rapidly. The decision of whether the farmers want to collaborate for the next certain amount of time gets tougher when they are not able to distinguish how the exterior factors will act upon their fields.

In the following table (see Table 1), we distinguish the influence of the weather conditions on the fields by their multiplying factors displayed as float values.

| Weather Conditions | Multiplying Factor |
|--------------------|--------------------|
| Sunny              | 1.33               |
| Cloudy             | 1.15               |
| Rainy              | 0.97               |
| Stormy             | 0.79               |
| Deer attacks       | 0.6                |

These values only apply if the user influences the weather. Otherwise, a range between 0.6 and 1.5 is given for the random weather condition. Nevertheless, these results, which might be adaptable to real-life situations, could be derived within the prefixed range.

Table 2 is an exemplary observation for six different simulations with different parameters that run with an annual time span of 100 years. The same parameters are also used for simulations lasting 50 and 250 years, but they are not shown here.
4.1.1 Cooperation Under Random Exterior Conditions

Since random weather conditions are closest to reality, we will outline an exemplary case in this section. For all simulations, we used 1.00 kg as initial value. In a real-life situation, farmers would use tons of crops and not only 1.00 kg. We can extract the values of profit with a total of ten decimal digits, but decided to use only two decimal digits in this paper.

For our first observation (see Fig. 2) with random weather conditions, we consider a lifetime of 100 years for the simulation cycle. In the first 40 years, the growth curves of all farmers look quite similar. Only Farmer 1, who is not collaborating, cannot expand their profits and is staying stable at a very low level. After 80 years, they finally start to grow, only to reach their peak of 15.81 kg after a time span of 100 years.

Farmer 3 has a highly fluctuating growth curve. They have several peaks, for instance, at 43 years and again at 60 years. However, every few years they must deal with great losses. Their curve is not stable at all and is continuously decreasing, down to 4.00 kg after 100 years. This value is very close to their initial starting value.

Farmer 5 can never reach their opponent Farmer 3. We observe that after the major loss from year 45 to year 55 Farmer 5 manages to develop a more stable growth. They are even able to outperform Farmer 3 and reach their top performance with 21.99 kg after 100 years.

The most remarkable are the collaborating Farmers 2, 4 and 6. Even though they are outperformed by Farmer 3 between year 40 and 65, they never have to deal with great losses. Their growth curve is continuously increasing. We observe the huge differences between the respective farmers. Those who do not cooperate have to deal with great losses and keep their profits quite low, whereas the cooperating farmers are steadily increasing their value. They even reach 118.49 kg after 100 years.
Hence, we conclude that under random influences it is of advantage to cooperate. This might seem counterintuitive in today’s society, since everybody wants to grow their own profits without sharing. However, as the LML observes with their research: “[…] sharing increases the long-time growth rate for cooperating entities, meaning that cooperators outgrow similar non-cooperators” [18].

4.1.2 Cooperation Under Bad Exterior Conditions
Since we heavily influenced the parameters in this case, the outcomes are observed in more detail. We set the parameters such that the farmers only have bad weather conditions through the whole iteration. The time frame of the simulation is set to 100 years. Rapidly we can distinguish a trend: it is of advantage to cooperate.

We fixed following exterior conditions for the fields of our farmers:

- **Farmer 1 and 2**: only deer attacks
- **Farmer 3 and 4**: only storm
- **Farmer 5 and 6**: only rain

That explains why the weather influences for each variant are depicted as straight lines in the spider web graphs (see Fig. 3).

In Fig. 4, **Farmer 1**, 3 and 5 are only depicted as straight lines, which means they have a constant value of zero. Even if, within the 100 years of farming, they might have raised their profits a little bit, they dropped rather quickly to zero again. Those minor
fluctuations are so insignificant that they are depicted as the straight lines we see at the bottom. Moreover, the farmers even lost their initial value and could not raise it again. On the contrary, the collaborating Farmers 2, 4 and 6 who had bad weather conditions, as well, managed to make profits as high as 120 kg.

![Fig. 4. Second cycle. Profits of harvest after 100 years.](image)

We distinguish that if the farmers work together, they can share even the little amount of harvest they make and hence, grow their profits. About 30 years into the simulation, it already shows the advantage of collaborating farmers. One might think it would be better to hold on to the little profit they make and to try to expand it on their own. Nevertheless, the simulation shows the exact opposite. Ergodicity might seem very counterintuitive, especially when the exterior conditions are bad for all participating teams. However, it certainly improves the situation for everybody when they share. Again, we conclude that it is of advantage to cooperate when we know that we are going to encounter consecutive years of bad exterior conditions. To answer RQ1, a parameter set should consist of randomized weather conditions, which means uncertainty for the future of the farmers, or of bad weather conditions for all farmers, where they depend on each other to cooperate and share their profits.

### 4.2 Ergodicity as a Solution for Real-Life Scenarios

In evolution, pooling and sharing of resources is a commonly spread behavior. It is omnipresent, in the cells, organisms and more, because the development respectively emergence of cells and cellular organisms are based on cooperation and for evolution, cooperation is indispensable [14, 18]. On the other hand, humanity is intuitively seen as profit-driven, as individual human beings are looking for the best way to ensure their wealth (see 2.2.2). No one would share for only altruism purposes, since they cannot gain anything in return. Apart from this counterintuitive wealth factor, as asked in RQ2, can we apply ergodicity also on further human made situations? As defined by [15] ergodicity means: “In an ergodic scenario, the average outcome of a group is the same as the average outcome of the individual over time.” For instance, coin toss is an ergodic situation as if 100 people flip a coin once or one-person flips a coin 100 times the outcome is the same. Even though consequences of these outcomes are typically
not ergodic. To find real-life scenarios we must compare two outcomes, the individual’s and the group of individuals’, and when they have the same outcome, it is an ergodic scenario. Since almost every human scenario is non-ergodic, we must have a closer look at how non-ergodic situations can be made more ergodic. Thus, diversification plays an important role, since it is defined as “the process of starting to include more different types or things”, according to the Cambridge Dictionary [2]. There are various human scenarios that we could take on, but in order not to go beyond the scope of this paper, we use an example of an investment strategy that we create with the help of diversification ergodic. A businessman called Bill wants to invest his money in a secure way but also with profit. So, he could combine two different investing approaches for example treasury bonds with stocks. What matters is not the average correlation, but correlation during a crisis, e.g. the actual Covid-19 crisis. It is suggested that Bill should put, during a timespan of his life, around 80 percent into something less risky, such as real estates, and 20 percent into something much riskier, such as starting another business [15]. This strategy of diversification is called “Barbell Approach” and it can be inferred that the outcome of Bill’s investment is more ergodic when he splits the risk management into smaller risk factors instead of taking one big risk [15].

As an answer to RQ2, one can say, there is not that one scenario where the concept of ergodicity is a suitable solution but with diversification, you can make almost every scenario more ergodic than it was before.

5 Conclusion

While answering RQ1 we found out that the concept of ergodicity from the London Mathematical Laboratory can be supported with the random and bad weather condition parameter sets inside our simulation. However, there are much more factors which are not considered for today’s farmers, such as limited fields, planting space, and investing the profit into better agricultural vehicles and crops. With these factors in mind, we can relate to RQ2, which explains that there is no real ergodic scenario, but it is possible to improve a scenario and make it more ergodic than it was before. With this research one can say, that a world ruled by the concept of the Farmer’s Fable is a utopia. However, with the concept of ergodicity partially applied to real-life scenarios one can raise their outcome over time. This might be especially helpful for teenagers and young adults. By applying different parameter sets themselves, they gain hands-on experiences. Furthermore, the classroom should provide the students with a safe space to discuss their findings and dive deep into the topic of ergodicity economics. Our aim is not to impose a fixed strategy on them, but we rather aim at provoking their thoughts to develop suitable strategies themselves and to be able to think further of how they can exceed the scope of the Business Simulation Game to real-life scenarios. Moreover, the students gain familiarity with the concept of not being always right. They have to try out several parameter sets to find the best suitable one, sometimes there might be more and sometimes they might not find even a single one, with their respective strategy. As mentioned previously we conducted a case study during the development of the Business Simulation Game. The outcome of further improvement of the game, such as
game balancing, and discussion of the case study combined with bringing statistics and stochastic closer to students will be a topic of another paper. For instance, the students should learn the fundamental concept of ergodicity economics and be able to scrutinize its value for real-life scenarios. As a famous (disputed) quote from Richard Feynman says: “I would rather have questions that can’t be answered than answers that can’t be questioned” [5].

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