Gravitational recoil from binary black hole mergers in scalar field clouds

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In vacuum, the gravitational recoil of the final black hole from the merger of two black holes depends exclusively on the mass ratio and spins of the coalescing black holes, and on the eccentricity of the binary. If matter is present, accretion by the merging black holes may modify significantly their masses and spins, altering both the dynamics of the binary and the gravitational recoil of the remnant black hole. This paper considers such scenario. We investigate the effects on the kick of the final black hole from immersing the binary in a scalar field cloud. We consider two types of configurations: one with non-spinning and unequal-mass black holes, and a second with equal mass and spinning holes. For both types, we investigate how the gravitational recoil of the final black hole changes as we vary the energy density of the scalar field. We find that the accretion of scalar field by the merging black holes could have a profound effect. For the non-spinning, unequal-mass binary black holes, the kicks are in general larger than in the vacuum case, with speeds of $\sim 1,200$ km/s for binaries with mass ratio 2:1, one order of magnitude larger than in vacuum. For equal mass, binaries with black holes with spins aligned with the orbital angular momentum, kicks larger than in vacuum are also found. For systems with spins in the super-kick configuration, the scalar field triggers a similar dependence of the kicks with the entrance angle at merger as in the vacuum case but in this case depending on the strength of the scalar field.

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I. INTRODUCTION

The gravitational waves (GWs) emitted during the inspiral and coalescence of a binary black hole (BBH) carry energy, angular momentum, and linear momentum \cite{1}. A net loss of linear momentum by the binary in certain direction implies a recoil of the final black hole (BH) in the opposite direction \cite{2,3}. In vacuum, this recoil or kick depends exclusively on the mass ratio and spins of the coalescing BHs, and if the binary is not in a quasicircular orbit, the recoil depends also on the eccentricity of the binary system \cite{5}. When matter is present, the situation is more complex. For instance, in mixed binary mergers, i.e. coalescences of BHs with neutron stars, the kick will depend also on any accretion of matter by the BH during the merger \cite{6}.

For this work, we focus on BH environments permeated by a scalar field. Scalar fields have been considered as sources of dark matter \cite{7}, in inflationary theories \cite{8,13}, and in the context of modified theories of gravity, such as scalar-tensor and $f(R)$ theories \cite{14,15}. In the presence of BHs, scalar fields have also been used to probe the transition from inspiraling BHs to a single perturbed BH \cite{17}. BBH systems in scalar-tensor \cite{18,19}, $f(R)$ \cite{20}, and Einstein-Maxwell-dilation \cite{21} theories have been also studied, as well as BBHs in dynamical Chern-Simons gravity \cite{22}, axion-like scalar fields \cite{23}, and scalar Gauss-Bonnet gravity \cite{24}.

Here, we are interested in investigating the effect that a scalar field may have on the kick of the final BH, an aspect not considered by the studies mentioned above. We focus on a simple scenario, a BBH immersed in a spherical shell of a massive scalar field and study two types of BBH configurations. One consists of un-equal mass binaries with non-spinning BHs, and in the other, binaries with equal-mass holes but spinning BHs. For the latter, we consider BH spins aligned with the orbital angular momentum (i.e., non-precessing binaries) and BH spins in the orbital plane in the super-kick configuration \cite{25,20}. In addition to the kick on the final BH, we also studied the characteristics of the GWs and the angular momentum radiated in GWs and by the scalar field.

The paper is organized as follows. In Sec. II, we present the method to construct initial data. Sec. III summarizes the equations of motion for the BBH with scalar field sources. Sec. IV presents the methodology to extract kicks, energy, and angular momentum radiated. The BBH configurations are given in Sec. V. Results for un-equal mass, non-spinning BHs binaries are given in Sec. VI and for equal mass, spinning BHs binaries in Sec. VII. Conclusions are found in Sec. VIII. Greek indices denote space-time indices, and Latin indices are used for spatial indices. We use geometrical units in which $G = c = 1$. A subscript 0 denotes initial values. Unless explicitly stated, we report results in units of $M_0$, the total initial mass of the BBH system.

II. INITIAL DATA

Under a 3+1 decomposition of the Einstein field equations \cite{27}, the initial data consist of $(\gamma_{ij}, K_{ij}, \rho, S_i)$, with $\gamma_{ij}$ the spatial metric and $K_{ij}$ the extrinsic curvature of
the constant time, space-like hypersurfaces. \( \rho \) and \( S_i \) are the energy and momentum densities, respectively. The initial data must satisfy the following equations:

\[
R + K^2 - K_{ij}K^{ij} = 16\pi\rho
\]

\[
\nabla_j K^j_i - \nabla_i K = 8\pi S_i,
\]

namely the Hamiltonian and momentum constraints, respectively. Here \( R \) is the Ricci scalar, and \( \nabla \) denotes covariant differentiation associated with \( \gamma_{ij} \). For our case of a massive scalar field:

\[
\rho = \frac{1}{2}\Pi^2 + \frac{1}{2}\nabla^j\nabla_j \phi + \frac{1}{2}m_\phi^2\phi^2,
\]

\[
S_i = -\Pi \partial_t \phi,
\]

with \( m_\phi \) the mass of the scalar field \( \phi \) and \( \Pi \) its conjugate momentum.

We solve the constraints (1) and (2) following the York-Lichnerowicz conformal approach \[28–31\] in which we impose \( \phi \) \( A \) solutions to the conformal factor, we require compact support. For simplicity, we set \( \sigma = 1 \).

\[
\Delta \psi + \frac{1}{8} \tilde{A}^{ij}\tilde{A}_{ij}\psi^{-7} = -\pi\Pi^2\psi^{-7} - \pi\psi \partial^i \phi \partial_i \phi
\]

\[
- \pi m_\phi^2 \partial^i \phi \psi^5
\]

\[
\partial_j \tilde{A}^j_i = -8\pi \tilde{\Pi} \partial_i \phi,
\]

where \( \Delta = \eta^{ij} \partial_i \partial_j \).

Since we are modeling BHs as punctures, the conformal factor \( \psi \) diverges at the punctures. Therefore, we will exploit the freedom for choosing initial data for \( \phi \) and \( \Pi \) and zero out the divergent terms proportional to \( \psi \) and \( \psi^5 \) in Eq. (7). We accomplish this by setting initially \( \phi = 0 \). With this assumption, (7) and (8) become

\[
\Delta \psi + \left( \frac{1}{8} \tilde{A}^{ij}\tilde{A}_{ij} + \pi\Pi^2 \right) \psi^{-7} = 0
\]

\[
\partial_j \tilde{A}^j_i = 0,
\]

respectively.

In Eq. (8), we use the Bowen-York solutions for \( \tilde{A}_{ij} \) \[34\]. Since we are interested in asymptotically flat solutions to the conformal factor, we require \( \tilde{\Pi} \) to have compact support. For simplicity, we set

\[
\tilde{\Pi}(r) = \Pi_0 \exp \left[ \frac{1}{2} \left( \frac{r - r_0}{\sigma} \right)^2 \right].
\]

That is, the scalar field source is a shell with radius \( r_0 \), thickness \( \sigma \), and amplitude \( \Pi_0 \). We solve Eq. (9) equation with the 2PUNCTURES solver \[35\], which was modified to include the \( \Pi^2 \) term.

## III. EVOLUTION EQUATIONS

The evolution equation for the scalar field is

\[
\Box \phi = m_\phi^2 \phi,
\]

with \( \Box = \nabla^\mu \nabla_\mu \) and \( \nabla_\mu \) covariant differentiation with respect to the space-time metric \( g_{\mu\nu} \). Under a 3+1 decomposition, the space-time metric is decomposed as

\[
g_{\mu\nu} = \gamma_{\mu\nu} - n_\mu n_\nu,
\]

with \( n^\mu = (\alpha^{-1}, -\beta^i \alpha^{-1}) \) the time-like unit normal vector to the \( t = \) constant space-like hypersurfaces. Here \( \alpha \) and \( \beta^i \) are the lapse function and shift vector, respectively. Given \[13\], we rewrite Eq. (12) as

\[
\frac{1}{\alpha} \partial_t \phi = -\Pi,
\]

\[
\frac{1}{\alpha} \partial_t \Pi = -\nabla^i \nabla_i \phi - \nabla_i \ln \alpha \nabla^i \phi + K \Pi + m_\phi^2 \phi,
\]

where \( \partial_o = \partial_t - \beta^i \partial_i \).

The evolution of the geometry of the space-like hypersurfaces, namely \( \gamma_{ij} \) and \( K_{ij} \), is handled with the BSSN formulation of the Einstein equations \[30\ 37\]. For a scalar field, the stress-energy tensor source in these equations is given by

\[
S_{ij} = \nabla_i \phi \nabla_j \phi + \frac{1}{2} \gamma_{ij} ( \Pi^2 - \nabla^k \nabla_k \phi - m_\phi^2 \phi^2 ).
\]

We used the moving puncture gauge \[38\ 39\] to evolve \( \alpha \) and \( \beta^i \). The resulting set of evolution equations is solved numerically using the MAYA code \[40\ 45\], our local version of the EINSTEIN Toolkit code \[46\].

## IV. PHYSICS EXTRACTION

The physical quantities of interest are the spin and masses of the BHs, as well as the properties of the radiated emission. The BH masses and spins are computed using the dynamical apparent horizons framework \[47\] as implemented in the EINSTEIN Toolkit \[46\]. On the other hand, the energy, linear and angular momentum radiated are computed from the Weyl scalar \( \Psi_4 \) as follows \[1\]:

\[
\frac{dE^{\text{rw}}}{dt} = \lim_{r \to \infty} \frac{r^2}{16\pi} \int \left| \int_{-\infty}^{t} \Psi_4 dt' \right|^2 d\Omega,
\]

\[
\frac{dP^{\text{rw}}}{dt} = \lim_{r \to \infty} \frac{r^2}{16\pi} \int \hat{l}_i \left| \int_{-\infty}^{t} \Psi_4 dt' \right|^2 d\Omega,
\]

\[
\frac{dJ^{\text{rw}}}{dt} = -\lim_{r \to \infty} \frac{r^2}{16\pi} \text{Re} \left[ \hat{j}_i \left( \int_{-\infty}^{t} \Psi_4 dt' \right) \right] \times \hat{j}_i \left( \int_{-\infty}^{t} \int_{-\infty}^{t'} \Psi_4 dt'' dt' \right) d\Omega,
\]

respectively.
where \( d\Omega = \sin \theta d\theta d\phi \), \( \hat{x}_i = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \), and \( \hat{J}_i \) is the angular momentum operator. Integration of \( [12] \) yields the recoil or kick of the final BH from the emission of GWs.

In addition to GW emission, we also have emission of energy, linear, and angular momentum associated with the scalar field. We compute this emission following the method in Ref. [24] as follows:

\[
\frac{dE_{\text{sf}}}{dt} = \lim_{r \to \infty} r^2 \int T_{tr} d\Omega, \tag{20}
\]

\[
\frac{dP_{\text{sf}}}{dt} = \lim_{r \to \infty} r^2 \int T_{\phi r} d\Omega, \tag{21}
\]

\[
\frac{dJ_{\text{sf}}}{dt} = \lim_{r \to \infty} r^2 \int T_{\phi \varphi} d\Omega, \tag{22}
\]

where the components of the stress-energy tensor are given by

\[
T_{\mu \nu} = \nabla_{\mu} \Phi \nabla_{\nu} \Phi - g_{\mu \nu} \left( \frac{1}{2} \nabla_{\alpha} \Phi \nabla^{\alpha} \Phi + \frac{1}{2} m_\phi^2 \Phi^2 \right). \tag{23}
\]

In all these fluxes, we evaluate the integrals at a finite radius and then extrapolate the values to infinity.

V. BINARY CONFIGURATIONS

The initial configuration for all BBH systems have the holes separated by a coordinate distance \( d = 8 M_0 \). The scalar field momentum shell has radius \( r_0 = 12 M_0 \) and thickness \( \sigma = 1 M_0 \). We also set the mass of the scalar field to \( m_\phi = 0.4/M_0 \). Each simulation was carried out with 8 levels of mesh refinements, outer boundary at 317.44 \( M_0 \), and resolution in the finest grid of \( M_0/64.5 \).

We considered two types of binaries. One is binaries with non-spinning BHs and initial mass ratios \( q_0 = m_1/m_2 = (2, 3, 4) \). The other type is binaries with equal mass BHs and their spins anti-aligned spins with magnitudes \( a = 0.6 \). For the spinning cases, we investigated two setups: one with the BH spins aligned with the orbital momentum (non-precessing binaries) and spins in the orbital plane (super-kick configuration). With the exception of the super-kick configuration binaries, we considered initial amplitude values of the scalar momentum \( \Pi_0 = \Pi \times M_0 \times 10^3 \) (5.0, 7.5, 10.0). On the other hand, for super-kick binaries, we have added more cases and set \( \Pi_0 = (1.25, 2.5, 3.75, 5.0, 6.25, 7.5, 8.50, 10.0) \). In order to do comparisons with the vacuum case, we did simulations with \( \Pi_0 = 0 \) for all types. The labeling of the simulations is as follows: A non-spinning, \( q_0 = x \) with \( \Pi_0 = y,z \) simulation is labeled \( xs-yz \). Similarly, an equal mass simulation with spins perpendicular and parallel to the orbital angular momentum with the same \( \Pi_0 \) are labeled \( a_{\pm}0yz \) and \( a_{\pm}0xy \), respectively.

Tables I and II show the scalar field energies \( E_\phi \) and total ADM energy \( E_{\text{ADM}} \) in the initial data for each of the cases. Notice that \( E_{\text{ADM}} \simeq E_{\text{ADM}}^{\text{rev}} + E_\phi \) where

\[
E_\phi = \int \rho \sqrt{\gamma} d^3x = \frac{1}{2} \int \Pi^2 \phi^6 \sqrt{\gamma} d^3x. \tag{24}
\]

VI. UN-EQUAL MASS, NON-SPINNING BH BINARY

Figure 1 shows the mode \( l = 2, m = 2 \) of the Weyl scalar \( \Psi_4 \) for the un-equal mass and non-spinning BH binaries. The top panels from left to right are for \( \Pi_0 = (5.0, 7.5, 10.0) \), respectively, with lines blue, red, and green corresponding to \( q_0 = (2, 3, 4) \), respectively. The bottom panels from left to right are for \( q_0 = (2, 3, 4) \), respectively, with lines blue, red, and green corresponding to \( \Pi_0 = (5.0, 7.5, 10.0) \), respectively. From the top
panels we see that, for a given $\hat{\Pi}_0$, the binary merges earlier for smaller $q_0$, as expected from the vacuum case, since the luminosity in GW during the inspiral scales as $q^2/(1+q)^4$ [27]. At the same time, for a given $q_0$, the larger the given value of $\hat{\Pi}_0$ is, the smaller the difference among the merger times.

From the bottom panels in Fig. 4 one sees that for a given $q_0$, the larger $\hat{\Pi}_0$, the earlier the binary merges. This is because the luminosity in GW also depends on the total mass of the binary $M$ as $M^2$ [27]. And as we shall see next, $M$ grows monotonically with $\hat{\Pi}_0$. Also, when one slices the data this way, we observe that the differences with $\hat{\Pi}_0$ in merger times remain roughly the same independently of $q_0$.

The accretion of the scalar field by the BHs modifies the total binary mass $M$ and its mass ratio $q$ as it evolves. Figure 2 shows the evolution of $m_1$, $m_2$, and $\hat{m}$ for each initial $q_0$. As expected, accretion starts when the scalar field shell reaches the BHs, approximately at a time $\sim r_0$. The bottom right panel also shows the evolution of $q$ due to the changes of the BH masses. In all panels, lines terminate at the time when the binary mergers, as signaled by the appearance of a common apparent horizon. The colors black, blue, red and green denote $\hat{\Pi}_0 = (0, 5.0, 7.5, 10.0)$, respectively. Figure 3 shows the corresponding BH accretion rates.

From Figs. 2 and 3 we observe that the BH masses and accretion rates grow monotonically with $\hat{\Pi}_0$ for a given initial $q_0$. Furthermore, the growth is such that the increase in $q$ is also monotonic with $\hat{\Pi}_0$. From Fig. 3, given a value of $\hat{\Pi}_0$, $\hat{m}_1 > \hat{m}_2$, similar to Bondi accretion behaviour in which the accretion rate is proportional to the mass of the accreting object. By taking into consideration the growth in $q$ observed in Fig. 2, namely $\dot{q} > 0$, one obtains that $\hat{m}_1 > q \hat{m}_2$.

Figure 3 shows the energy, angular momentum, and linear momentum radiated in GWs (dashed lines) and in the scalar field (solid lines) for the case $\hat{\Pi}_0 = 10.0$. We observe in the left panel that the energy radiated by the scalar field is higher than in GWs. This can be explained as follows: the ADM energy at the end of the simulations is given by $E_{\text{ADM}} = E_{\text{GW}}^\phi + E_{\text{rad}}^\phi + m f$, with $m f$ the mass of the final BH. For the case $q_0 = 2$ and $\hat{\Pi}_0 = 10.0$, we have from Table II that $E_{\text{ADM}} = 1.102 M_0$ and from Table III that $m f = 1.0147 M_0$; thus, $E_{\text{GW}}^\phi + E_{\text{rad}}^\phi = E_{\text{ADM}} - m f \simeq 0.087 M_0$. Since energy radiated in GWs is typically a few percent, in this case $E_{\text{GW}}^\phi \simeq 0.025 M_0$, we have that $E_{\text{rad}}^\phi \simeq 0.06 M_0$, consistent with the value in Fig. 3. Another characteristic in this figure is that, as with GWs, the energy radiated in the scalar field decreases monotonically with $q_0$.

The angular momentum radiated is depicted in the middle panel of Fig. 4. As expected, GWs carry away angular momentum and shrink the binary. The scalar field also extracts angular momentum but in smaller amounts. The reason why the scalar field angular momentum radiation is much smaller than the one in GWs is because initially the scalar field shell does not have any angular momentum. All the momentum generated is from the “stirring” of the scalar field by the binary.

The right panel in Fig. 4 shows the magnitude of linear momentum emitted, which for these non-precessing binaries lies in the $xy$-plane. As with the energy radiated, the emission of scalar field linear momentum is significantly larger than in the GWs. Also interesting is the oscillations in the scalar field linear momentum radiated, which are also observed in the energy and angular momentum but at a much smaller scale. The reason for this is because in systems of BBH with massive scalar fields, as it is in our case, the scalar fields develops long-lived modes due to the presence of an effective potential.

Table III shows the mass $m f$, spin $a f$, and kick velocity $v_{\text{kick}}$ of the remnant BH, where we have combined the emission of linear momentum by GWs and the scalar field to estimate the gravitational recoil. Independently of $q_0$, $m f$ grows monotonically with $\hat{\Pi}_0$. This is expected from the way the BHs accrete the scalar field, namely, the more massive the hole, the more it accretes.

Regarding the final spin, we found that for a given $\hat{\Pi}_0$, $a f$ decreases as $q_0$ increases. Which is the same trend observed in the vacuum case; that is, the scalar field modifies the spin magnitude but not its dependence with $q$. On the other hand, if one fixes the attention to the final spin for a given $q_0$, one sees monotonicity in the $q_0 = 3$ and 4, decreasing its value with $\hat{\Pi}_0$ increasing. At first look, this seems counter intuitive because one would think that, since the larger the value of $\hat{\Pi}_0$, the earlier the binary merger, there would be a larger residual of angular momentum that goes into the final spin. Yes, there is more angular momentum in the final BH, but one has to also remember that $a f = S f/m f^2$ is the dimensionless spin parameter, not the angular momentum $S f$. It is the growth in the final mass of the BH responsible for the decrease in $a f$. Since the growth in the masses for $q_0 = 2$ is not as large (see Fig. 2), the monotonicity of $a f$ with $\hat{\Pi}_0$ only shows for large values.

For the kick velocity, given a value of $q_0$, the recoil is larger than in the vacuum case and increases monotonically with $\hat{\Pi}_0$. In vacuum, the maximum kick velocity of the final BH in non-spinning, unequal-mass BBH occurs near $q_0 = 3$ [3]. In the presence of scalar field, we observe that the maximum kick for a given $\hat{\Pi}_0$ occurs for $q_0 \leq 2$, with $\hat{\Pi}_0 = 10.0$ reaching super-kick levels. For a given $\hat{\Pi}_0$, all the kicks are larger than in the vacuum case, the reason for this is because in these configurations the emission of linear momentum is larger through the scalar field channel. The initial momentum in the scalar field is not directly responsible for this since it does not have net linear momentum; it is spherically symmetric. It is through the interactions with the binary that linear momentum in the scalar field is redistributed and emitted in a particular direction. It turns out that this direction is aligned with that of the linear momentum emitted in GWs.
The binaries with equal mass and anti-aligned spinning BHs.

As mentioned before, we considered two setups for binaries with equal mass and anti-aligned spinning BHs. The $a_{\parallel}$ cases have BH spins along the direction of the orbital angular momentum (i.e., non-precessing binaries), and the $a_{\perp}$ cases have BH spins in the orbital plane in the super-kick configuration \cite{25,26}.

VII. EQUAL MASS, SPINNING BH BINARIES

Figure 5 shows the mode $l = 2$, $m = 2$ of the Weyl scalar $\Psi_4$. Panels from left to right are for $\Pi_0 = (5.0, 7.5, 10.0)$, respectively, with red lines for $a_{\parallel}$ and blue for $a_{\perp}$. It is interesting to notice that for $\Pi_0 = 10.0$ there is very little difference in the (2,2) mode between the $a_{\parallel}$ and $a_{\perp}$ case, this in spite of the large difference they have, as we shall see, in kicks produced. After all, the $a_{\perp}$ cases are in the super-kick class. This means that the differences are in the higher modes. We also observe from the waveforms in Fig. 5 that, as for the unequal mass and non-spinning BH binaries, the larger the value of $\Pi_0$, the earlier the binary merges, and the reasons are similar. The accretion of scalar field by the BH increases their masses and thus the luminosity of the binary.

Figure 6, shows from top to bottom the evolution of $m_1$, $m_2$, and $M$, respectively. Left panels are for the $a_{\parallel}$ cases and the right ones for $a_{\perp}$. The line colors black, blue, red, and green correspond $\Pi_0 = (0, 5.0, 7.5, 10.0)$, respectively. The behaviour in the growth of the masses is similar to that of unequal mass, non-spinning BH binaries. Namely, the growth is monotonic with $\Pi_0$. Interesting to point out that the growth in $m_1$ and $m_2$ is identical in the $a_{\perp}$; thus, $q$ remains unity. This is because for both holes, the orientation of their spins relative to the orbital angular momentum, are identical. On the other hand, since for the $a_{\parallel}$ cases, the BH with mass $m_1$ has its spin aligned with the orbital angular momentum and for the other anti-aligned, it is clear from panel top-

### TABLE III: Mass $m_f$, spin $a_f$ and kick of the final BH for the unequal mass, non-spinning BBH.

| Case | $m_f/M_0$ | $a_f$ | $v_{kick}$ (km/s) |
|------|-----------|------|-------------------|
| q2-000 | 0.9612 | 0.6232 | 146 |
| q2-050 | 0.9743 | 0.6218 | 550 |
| q2-075 | 0.9893 | 0.6230 | 946 |
| q2-100 | 1.0147 | 0.6267 | 1303 |
| q3-000 | 0.9712 | 0.5405 | 166 |
| q3-050 | 0.9869 | 0.5378 | 289 |
| q3-075 | 1.0055 | 0.5370 | 409 |
| q3-100 | 1.0337 | 0.5355 | 543 |
| q4-000 | 0.9777 | 0.4713 | 149 |
| q4-050 | 0.9942 | 0.4686 | 202 |
| q4-075 | 1.0137 | 0.4646 | 256 |
| q4-100 | 1.0422 | 0.4624 | 304 |
left and middle-left that there is a slight difference in the growth between hole $m_1$ and $m_2$. The BH with mass $m_2$ grows slightly more than $m_1$. This translates into mass ratios at merger of $q = 1.0049, 1.0073, 1.0102$ for $\hat{\Pi}_0 = 5.0, 7.5, 10$, respectively. This is consistent with accretion of spinning black holes immersed in a gaseous environment or circumbinary disks [49].

Figure 7 shows the energy, angular momentum, and linear momentum radiated as a function of time in GWs (blue lines), in the scalar field (red lines), and the total (black lines) for the $a_\parallel$ cases, with the top panels for $\hat{\Pi}_0 = 5.0$ and the bottom panels for $\hat{\Pi}_0 = 10.0$. We observe that the angular and linear momentum radiated in GWs is larger than in the scalar field for both $\hat{\Pi}_0$ values. This is not the case for the energy radiated. Not surprisingly, the larger the value of $\hat{\Pi}_0$, i.e. the larger the initial energy in the scalar field, the larger the energy emission. This does not imply that the remnant BH will have a smaller mass. As we can see from Table IV and saw from Fig. 6, the larger $\hat{\Pi}_0$, the larger the final BH because of the accretion of scalar field.

| Case | $m_f / M_0$ | $a_f$ | $|v|$ (km/s) |
|------|-------------|-------|-------------|
| $a_\parallel 0.000$ | 0.9512 | 0.6851 | 302 |
| $a_\parallel 0.050$ | 0.9603 | 0.6856 | 285 |
| $a_\parallel 0.075$ | 0.9691 | 0.6844 | 297 |
| $a_\parallel 0.100$ | 0.9850 | 0.6970 | 362 |

TABLE IV: Mass $m_f$, spin $a_f$ and magnitude of the kick of the final BH for equal mass, spinning BBHs in the $a_\parallel$ cases.

Regarding the radiated angular momentum from the middle panels of Fig. 7, the scalar field emission is sig-
FIG. 3: Unequal mass, non-spinning BBH: Mass accretion rates for each BH, top to bottom panels \( \hat{\Pi}_0 = (0, 5.0, 7.5, 10.0) \), blue, red, and green correspond to \( q_0 = (2, 3, 4) \), respectively.

The situation seems to reverse with the linear momentum radiated. Similar to the angular momentum radiated, it is still the case that, as the binary mergers earlier because of the presence of the scalar field, it does not “accumulate” as much kick as in the vacuum case (see kick values for \( a_{\parallel 050} \) and \( a_{\parallel 075} \) in Table IV). However, as we can see from the right panels in Figure 7, the kick contribution from the scalar field increases with \( \hat{\Pi}_0 \) and eventually turns things around. At \( \hat{\Pi}_0 = 10.0 \) this contributions is such that the kick becomes larger than in the vacuum case.

significantly smaller than from GWs. However, when comparing the emission in GWs from \( \hat{\Pi}_0 = 5.0 \) (top-middle panel) with that of \( \hat{\Pi}_0 = 10.0 \) (bottom-middle panel), the former is slightly larger. Since the initial configuration has mostly orbital angular momentum because the spins are anti-aligned, this implies that the spin of the final BH for \( \hat{\Pi}_0 = 5.0 \) will be smaller than for \( \hat{\Pi}_0 = 10.0 \), as we can see Table IV. This is consistent because the \( \hat{\Pi}_0 = 10.0 \) binary merges earlier (see Fig. 5), and thus it does not radiate as much angular momentum as with the \( \hat{\Pi}_0 = 5.0 \) case.
Figure 4 shows the energy, angular momentum, and linear momentum radiated as a function of time in GWs (blue lines), in the scalar field (red lines), and the total (black lines) for the $a_\parallel$ cases, with the top panels for $\tilde{\Pi}_0 = 5.0$ and the bottom panels for $\tilde{\Pi}_0 = 10.0$. It is interesting to observe that the spin configuration does not have a big effect on the energy and angular momentum radiated. Left and middle panels in Fig. 4 are very sim-
ilar to those in Fig. 4 for the $a_\parallel$ cases. The differences come in the linear momentum radiated (right panels in Fig. 5). Although the trend of which radiation dominates is similar to those in the $a_\parallel$ cases, the magnitude of the emission in the $a_\perp$ cases is much larger, after all these are super-kick setups.

Table V shows the mass, spin, and the $z$-component kick velocity (the most dominant in this cases) for the $a_\perp$ cases. Regarding the mass of the final BH, for the same reasons as all the previous binary types, $m_f$ increases monotonically with $\tilde{\Pi}_0$. There seems to be also monotonicity with $\tilde{\Pi}_0$ in $a_f$. The reason is because the larger the value of $\tilde{\Pi}_0$, the faster the binary merges thus the lower the angular momentum radiated and the larger residual angular momentum that goes into the final spin.

There is no monotonicity in the kicks. To help understand the situation, we plot the kicks as a function of $\tilde{\Pi}_0$ in Fig. 9. In this figure, we observe hints of an oscillatory trend in the $z$-component of the kick as a function of $\tilde{\Pi}_0$. The reason for this oscillatory behaviour is similar to the one found in the first studies of super-kicks, namely that the magnitude and direction of the kick is proportional to the cosine of the angle that the in-plane components of the spins make with the infall direction at merger [50]. In the vacuum case, this dependence is obtained by changing the be initial direction of the spins. In our case, it is the effect that the scalar field has on the mass growth of the holes, and thus its orbital dynamics, that produces the changes of the spin alignment relative to the infall direction.

| Case   | $m_f/M_0$ | $a_f$ | $v_z$ (km/s) |
|--------|-----------|-------|--------------|
| $a_\perp000$ | 0.9500 | 0.6797 | -2113        |
| $a_\perp0125$ | 0.9500 | 0.6786 | -2138        |
| $a_\perp0250$ | 0.9515 | 0.6801 | -1422        |
| $a_\perp0375$ | 0.9560 | 0.6860 | 1020         |
| $a_\perp0500$ | 0.9582 | 0.6802 | 2113         |
| $a_\perp0625$ | 0.9650 | 0.6834 | -1281        |
| $a_\perp0750$ | 0.9691 | 0.6829 | 335          |
| $a_\perp0875$ | 0.9734 | 0.6848 | 1669         |
| $a_\perp1000$ | 0.9841 | 0.6966 | -1576        |

TABLE V: Mass $m_f$, spin $a_f$ and $z$-component of the kick of the final BH for equal mass, spinning BBHs in the $a_\perp$ cases.

VIII. CONCLUSIONS

We have presented results from a numerical study of BBH mergers immersed in a scalar field cloud, focusing on the effects that the cloud has on the gravitational recoil, as well as on the spin of the final BH. We considered two initial configuration scenarios: binaries with non-spinning, un-equal mass BHs and binaries with equal mass BHs and their spins anti-aligned spins. For the later case we had to subcategories, one in which the BH
spins were parallel to the orbital angular momentum (i.e. non-precessing), and the other with the BH spins in the orbital plane in the so-called super-kick setup. The initial geometry of the scalar field cloud was a thin shell encapsulating the binary.

In all cases, because of scalar field accretion, the BHs gained mass, thus increased the emission of GWs, and as a consequence accelerated the merger. This also induced changes in the mass ratio of the binary, with the exception of the binaries in the super-kick configuration because the spin relative to the orbital momentum were the same.

We computed the radiated energy, angular momentum, and linear momentum emitted in both the GW and scalar field channels. For the un-equal mass BH binaries, we found that the scalar field emission was dominant in energy and linear momentum. Because of the later, the kicks were larger than in the vacuum case. Since the emission of angular momentum by the scalar field was smaller than from GWs, the spins varied very little from their vacuum counterparts. A similar situation took place with the equal mass, spinning BH binaries; the presence of the scalar field did not translate into significant changes in the spin of the final BH relative to the vacuum case. The main reason for this general situation is because the initial scalar field cloud did not have angular momentum that could be transferred via accretion to the BHs.

Regarding the gravitational recoil of the final BH, for the case of unequal mass, non-spinning BH binaries, we obtained that the kicks were larger that their vacuum counterparts because in these configurations the emission of linear momentum is larger via the scalar field channel. Some of the kicks reached super-kick levels of $\sim 1,300$ km/s.

For the binaries with equal mass BHs and spins aligned with the orbital angular momentum, we observed two effects competing against each other as we increased $\hat{\Pi}_0$. The scalar field accretion increased the BH masses and
accelerated the merger. This ameliorated the “accumulation” of the kick. Acting in the opposite direction was the linear momentum radiated in the scalar field, increasing with the value of $\hat{\Pi}_0$ and eventually yielding kicks larger than in the vacuum case.

Finally, for equal mass and spinning BH binaries in the super-kick configuration, we observed hints of the oscillatory behavior observed in the vacuum case. The reasons are similar; that is, the magnitude and direction of the kick is proportional to the cosine of the angle that the in-plane components of the spins make with the infall direction at merger $[50]$. However, instead of this dependence be from changing the initial direction of the spins, in our case, it is the change in the dynamics of the binary from the mass growth of the holes that produces the changes of the spin alignment relative to the infall direction.

One, of course, must take our results with a grain of salt regarding astrophysical implications. The purpose of our study was solely to investigate the sensitivity of BBH merger dynamics and the resulting final BH to the presence of a scalar field. Our results should be taken as a guide of the scale of energy in a scalar field necessary to imprint noticeably effects on the merger time of the binary and the gravitational recoil of the final black hole. In a subsequent study, we will focus on the impact in parameter estimation under the eyepiece of GW analysis.

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