Gauge-invariant models of interacting fields with spins 3, 1 and 0

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Abstract

New local gauge-invariant models of interacting fields with spins 3, 1 and 0 are found. The construction of the models is completely based on the new approach to the deformation problem proposed in our papers (Buchbinder and Lavrov in JHEP 06: 097, 2021; Buchbinder and Lavrov in Eur. Phys. J. C 81:856, 2021; Lavrov in Eur. Phys. J. C 82:429, 2022). The approach allows to describe the deformation procedure for theories with gauge freedom in an explicit and closed form in terms of special anticanonical transformations of the Batalin-Vilkovisky formalism. The action of the models appears as a local part of the deformed action which is represented by a functional of the fourth order in fields. This functional is invariant under original gauge transformations.

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1 Introduction

It is well-known that the Batalin-Vilkovisky (BV) formalism \[1, 2\] is the powerful method not only for quantization of general gauge theories but an important tool for solving different problems on classical and quantum levels. So, almost thirty years ago it was proposed to include the deformation procedure which is considered as a way to construct suitable interactions between fields, in solutions to the classical master-equation of the BV formalism \[3, 4\]. It was proposed to search solutions in the form of Taylor expansion with respect to a deformation parameter. In general, it causes infinite series of equations which are analysed with the help of cohomological methods (for recent application and references see, for example, \[5\]). Recently, a new approach to find solutions of the deformation procedure within the BV-formalism has been proposed \[6, 7, 8\]. The approach uses the fact that the classical master-equation is formulated in terms of the antibracket. In turn, the antibracket obeys the property of invariance under anticanonical transformations. In particular, it means that any two solutions of the classical master-equation can be connected to each other by an appropriate anticanonical transformation. In \[6, 7, 8\], it was proved that the deformation procedure can be solved with the help of special anticanonical transformations acting non-trivially in the sector of initial fields of a given gauge theory. In turn, it allows to describe the deformation procedure in an explicit and closed form. The deformation looks like the result of an explicit summation of a series of the standard approach based on cohomological methods. In fact, it opens new possibilities in construction of gauge-invariant models of interacting fields. Using new approach, in \[9\], the first new gauge-invariant model with interaction has been constructed. This model presents the massless spin 3 field interacting with a real scalar field. The action of the model is a local functional of the fourth order in fields which is invariant under original gauge transformations. This model is of particular interest due to the fact that it cannot be reproduced using the standard Noether procedure, which is accepted as the main method for describing interactions in the theory of higher spin fields \[10, 11, 12, 13\]. In connection with the locality problem\(^2\) for interactions of higher spin field, the model may have some interest because of the local nature of quartic vertices.

In this paper, we continue studying the interaction of a massless spin 3 field with a real scalar field for the case when a massive vector field is additionally involved. As in \[9\], the interaction appears due to deformations of initial gauge action under special anticanonical transformations \[6, 7, 8\]. The initial action is sum of the Fronsdal action for massless spin 3 fields \[16\] and the actions for a massive vector field and a real scalar field. The generating function of anticanonical transformations is chosen in a form that preserves gauge symmetry after deformations. In turn, it allows to extract a local functional from the deformed action which is invariant under original gauge transformations. This functional defines explicitly the gauge-invariant model and depends on fields up to the fourth order. By passing, it is proved that cubic vertices invariant under original gauge transformations are forbidden.

The plan of the paper is as follows. Section 2 contains a formulation of an initial free gauge model of massless spin 3 field and the massive vector and scalar fields. It is shown that local cubic vertices invariant under original gauge transformations are forbidden. In section 3, local gauge-invariant quartic vertices are constructed. In section 4, a new local gauge-invariant model with mixed quartic vertices is proposed. Section 5 is summarized the results.

The DeWitt’s condensed notations are systematically used \[17\]. The right functional derivatives are marked by special symbols ” ← “. Arguments of any functional are enclosed in square brackets \[\]\, and arguments of any function are enclosed in parentheses, ( ).

\(^2\)For quite recent discussions of the locality problem in the theory of higher spin fields see \[14, 15\] and references therein.
2 Cubic vertices

Our starting point is the free model of massless spin 3, vector and scalar fields in flat Minkowski space of dimension \( d \) with the action

\[
S_0[\varphi, A, \phi] = S_0[\varphi] + S_0[A] + S_0[\phi],
\]

where \( S_0[\varphi] \) is the Fronsdal action for completely symmetric third rank tensor \( \varphi^{\mu\nu\lambda} = \varphi^{\mu\nu\lambda}(x) \),

\[
S_0[\varphi] = \int dx \left( \varphi^{\mu\nu\rho} \Box \varphi_{\mu\nu\rho} - 3 \eta_{\mu\nu} \eta^{\rho\sigma} \varphi^{\mu\nu\delta} \Box \varphi_{\rho\sigma\delta} - \frac{3}{2} \varphi_{\mu\nu\lambda} \eta^{\mu\nu} \partial^{\lambda} \partial^{\alpha} \varphi_{\alpha\beta\gamma} \eta_{\beta\gamma} - 3 \varphi^{\mu\rho\sigma} \partial_{\mu} \varphi_{\nu\rho\sigma} + 6 \eta_{\mu\nu} \varphi^{\mu\nu\delta} \partial^{\rho} \partial^{\sigma} \varphi_{\rho\sigma\delta} \right),
\]

\( S_0[A] \) and \( S_0[\phi] \) are the actions of a massive vector field \( A^\mu = A^\mu(x) \) and real scalar field \( \phi = \phi(x) \), respectively,

\[
S_0[A] = \int dx \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_0^2 A_\mu A^\mu \right), \quad S_0[\phi] = \frac{1}{2} \int dx (\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2).
\]

The action (1) is invariant under the following gauge transformations

\[
\delta \varphi^{\mu\nu\lambda} = \partial^{(\mu} \xi^{\nu\lambda)}, \quad \delta A^\mu = 0, \quad \delta \phi = 0, \quad \eta_{\mu\nu} \xi^{\mu\nu} = 0,
\]

where gauge parameters \( \xi^{\mu\nu\lambda} \) are arbitrary symmetric functions of space-time coordinates. As it was explained in [9], algebra of such gauge transformations is Abelian and belongs to the class of first-stage reducible theories in the terminology of BV-formalism.

Now, we consider the special class of deformations of initial action using the approach proposed in our papers [6, 7, 8] when anticanonical transformations of the BV-formalism are ruled out the gauge-invariant deformations of an initial gauge theory. Here, we restrict ourself to the case of anticanonical transformations acting effectively in the sector of fields \( \varphi^{\mu\nu\lambda} \) only. It means that the generating function of anticanonical transformations should be completely symmetric third rank tensor \( h^{\mu\nu\lambda} = h^{\mu\nu\lambda}(\varphi, A, \phi) \). In construction of suitable generating functions \( h^{\mu\nu\lambda} \), we have to take into account the dimensions of quantities involved in the initial action \( S_0[\varphi, A, \phi] \),

\[
\text{dim}(\varphi^{\mu\nu\lambda}) = \text{dim}(A^\mu) = \text{dim}(\phi) = \frac{d - 2}{2}, \quad \text{dim}(\xi^{\mu\nu}) = \frac{d - 4}{2}, \quad \text{dim}(\partial_\mu) = 1, \quad \text{dim}(\Box) = 2.
\]

The generating function \( h^{\mu\nu\lambda} \) should be non-local with the dimension equals to \((d - 2)/2\). The non-locality is achieved by using the operator \( \Box \). Here, we shall be interested in cubic vertices of the form \( \sim \varphi A \phi \) to continue our knowledge about cubic vertices in theories of spin 3 fields [9]. It means that the \( h^{\mu\nu\lambda} \) can be constructed with the help of terms proportional to \( \sim A \phi \). The tensor structure of \( h^{\mu\nu\lambda} \) is obeyed by using partial derivatives \( \partial_\mu \), the metric tensor \( \eta_{\mu\nu} \) and vector fields \( A^\mu \). The minimal number of derivatives equals to 2. Therefore, the more general form of \( h^{\mu\nu\lambda} = h^{\mu\nu\lambda}(A, \phi) \) satisfying requirements listed above reads

\[
h^{\mu\nu\lambda} = a_0 \Box \left( c_1 \partial^{(\mu} \phi \partial^{\nu} A^{\lambda)} \phi + c_2 \partial^{(\mu} \partial^{\nu} \phi A^{\lambda)} + c_3 \partial^{(\mu} \phi \partial^{\nu} A^{\lambda)} + c_4 \eta^{(\mu\nu} A^{\lambda)} \phi + c_5 \Box \phi \eta^{(\mu\nu} A^{\lambda)} + c_6 \eta^{(\mu\nu} \partial_\sigma A^{\lambda)} \partial^{\sigma} \phi + c_7 \eta^{(\mu\nu} \partial^{\lambda)} \phi \partial^{\sigma} A_\sigma \right),
\]

where \( a_0 \) is the coupling constant with \( \text{dim}(a_0) = -(d - 2)/2 \) and \( c_i, \quad i = 1, 2, ..., 7 \) are arbitrary dimensionless constants. Local part of the deformed action has the form

\[
S_{loc}[\varphi, A, \phi] = S_0[\varphi, A, \phi] + S_{1\ loc}[\varphi, A, \phi]
\]
where \( S_{1\,\text{loc}} = S_{1\,\text{loc}}[\varphi, A, \phi] \),

\[
S_{1\,\text{loc}} = 2a_0 \int dx \varphi_{\mu\nu\lambda} \left[ c_1 \partial^{(\mu} \partial^{\nu} A^{\lambda)} \phi + c_2 \partial^{(\mu} \partial^{\nu} \phi A^{\lambda)} + c_3 \partial^{(\mu} \phi \partial^{\nu} A^{\lambda)} - 
- \left( c_1 + c_4(d + 1) \right) \eta^{(\mu\nu} \Box A^{\lambda)} \phi - \left( c_2 + c_6(d + 1) \right) \Box \phi \eta^{(\mu\nu} A^{\lambda)} - 
- \left( c_3 + c_8(d + 1) \right) \eta^{(\mu\nu} \partial_{\sigma} A^{\lambda)} \partial^{\sigma} \phi - \left( c_4 + c_7(d + 1) \right) \eta^{(\mu\nu} \partial_{\sigma} \phi \partial_{\sigma} A^{\lambda)} - 
- 2c_1 \eta^{(\mu\nu} \partial^{\sigma} A_{\sigma} \phi - 2c_2 \eta^{(\mu\nu} \partial^{\sigma} \phi A_{\sigma} \right].
\] (8)

corresponds to possible cubic vertices. Due to the special structure of generating functions of anticanonical transformations, the matrix \((M^{-1})^i_j\) (for details see \([6, 8]\)) can be found in the explicit form

\[
(M^{-1})^i_j = \begin{pmatrix}
E_{\rho\sigma\gamma}^{\mu\nu\lambda} & -h_{\mu\nu\lambda}(A, \phi) \delta A_{\sigma} \delta \phi & -h_{\mu\nu\lambda}(A, \phi) \delta A_{\sigma} \\
0 & \delta A_{\sigma} & 0 \\
0 & 0 & 1
\end{pmatrix},
\] (9)

where \(E_{\rho\sigma\gamma}^{\mu\nu\lambda}\) are elements of the unit matrix in the space of third rank symmetric tensors. From the structure of matrix \((9)\), it follows that the deformed gauge generators coincide with initial ones.

Consider the variation of \(S_{1\,\text{loc}}\) under the gauge transformations \([4]\). We obtain the result

\[
\delta S_{1\,\text{loc}} = -6a_0 \int dx \xi_{\nu\lambda} \left[ -2c_1(d + 1) \Box \partial^\nu A^\lambda \phi + \left( 2c_6 - 2c_6(d + 1) \right) \Box \partial^\nu A^\lambda \partial_{\sigma} \phi - 
- 3c_1 \partial_{\nu} \partial^\nu A^\lambda \phi + c_1 \partial_{\nu} \partial^\nu A^\lambda \partial_{\sigma} \phi - 2c_6(d + 1) \Box \partial^\nu \phi A^\lambda + 
+ (2c_2 - 2c_6(d + 1)) \partial^\nu \partial^\nu \partial^\sigma \phi \partial_{\nu} A^\sigma - 3c_2 \partial^\nu \partial^\nu \partial^\sigma \phi A_{\sigma} + 
+ (c_2 - 2c_3 - 2c_7(d + 1)) \partial^\nu \partial^\nu \phi \partial_{\nu} A^\sigma + (c_3 - 2c_2 - 2c_6(d + 1)) \Box \phi \partial^\nu A^\lambda + 
+ (c_3 - 2c_1 - 2c_4(d + 1)) \Box A^\nu \partial^\nu \phi + (c_3 - 4c_2) \partial_{\sigma} \partial^\nu \phi \partial^\nu A^\sigma - 
- (c_3 + 4c_1 + 2c_7(d + 1)) \Box \phi \partial^\nu \phi \partial^\nu A^\sigma \right].
\] (10)

The requirement \(\delta S_{1\,\text{loc}} = 0\) leads to the conditions

\[
c_i = 0, \quad i = 1, 2, \ldots, 7.
\] (11)

It means that

\[
S_{1\,\text{loc}}[\varphi, A, \phi] = 0.
\] (12)

Therefore, we have proved impossibility in the theory of massless spin 3 field interacting with massive vector and scalar fields to construct cubic vertices \(\sim \varphi A \phi\) being invariant under initial gauge transformations \([4]\). In this point, the situation is similar with cubic vertices studied in \([9]\).

### 3 Quartic vertices

In the paper \([9]\), it was proved that although gauge-invariant cubic vertices \(\sim \varphi \phi \phi\) are forbidden in the theory of massless spin 3 and scalar fields, nevertheless, local quartic gauge-invariant vertices \(\varphi \phi \phi \phi\) can be constructed. Here, we meet the similar situation, namely, local gauge-invariant quartic vertices \(\sim \varphi A \phi \phi\) can be constructed. Repeating the main arguments that led
to the construction of the generating function (6) of the anticanonical transformation, the most
general form of the generating function \( h^{\mu\nu\lambda} = h^{\mu\nu\lambda}(A, \phi) \) with two derivatives responsible for
the generation of quartic vertices reads

\[
h^{\mu\nu\lambda} = a_1 \left[ c_1 \partial^{(\mu} A^{\nu A^{(\lambda)}} \phi^2 + c_2 \partial^{(\mu} A^{\nu A^{(\lambda)}} \phi + c_3 A^{(\mu} \partial^{(\nu} \partial^{(\lambda)}} \phi + c_4 A^{(\mu} \partial^{(\nu} \partial^{(\lambda)}} \phi + + c_5 \eta^{(\mu\nu A^{(\lambda)}} \phi^2 + c_6 \eta^{(\mu\nu A^{(\lambda)}} \phi + c_7 \Box \phi \eta^{(\mu\nu A^{(\lambda)}} \phi + c_8 \eta^{(\mu\nu A^{(\lambda)}} \phi + + c_9 \eta^{(\mu\nu A^{(\lambda)}} \partial^{(\nu} \partial^{(\lambda)}} \phi + c_{10} \eta^{(\mu\nu A^{(\lambda)}} \partial^{(\nu} \partial^{(\lambda)}} \phi + + c_{11} \eta^{(\mu\nu A^{(\lambda)}} \partial^{(\nu} \partial^{(\lambda)}} \phi + + c_{12} \eta^{(\mu\nu A^{(\lambda)}} \partial^{(\nu} \partial^{(\lambda)}} \phi + + c_{13} \eta^{(\mu\nu A^{(\lambda)}} \phi A^{(\sigma} \partial^{(\lambda)}} \phi \right],
\]

(13)

where \( a_1 \) is a coupling constant with \( \text{dim}(a_1) = -(d-2) \) and \( c_i, \ i = 1, 2, ..., 13 \) are arbitrary dimensionless constants. For the local addition, \( S_{2, \text{loc}} = S_{2, \text{loc}}[\varphi, A, \phi] \), to the initial action (1), we obtain

\[
S_{2, \text{loc}} = 2a_1 \int dx \varphi_{\mu\nu\lambda} \left[ c_1 \partial^{(\mu} A^{\nu A^{(\lambda)}} \phi^2 + c_2 \partial^{(\mu} A^{\nu A^{(\lambda)}} \phi + c_3 A^{(\mu} \partial^{(\nu} \partial^{(\lambda)}} \phi + c_4 A^{(\mu} \partial^{(\nu} \partial^{(\lambda)}} \phi - - (c_1 + c_5(d + 1)) \eta^{(\mu\nu A^{(\lambda)}} \phi^2 - (c_2 + c_6(d + 1)) \eta^{(\mu\nu A^{(\lambda)}} \phi - - (c_3 + c_7(d + 1)) \Box \phi \eta^{(\mu\nu A^{(\lambda)}} \phi - (c_4 + c_8(d + 1)) \eta^{(\mu\nu A^{(\lambda)}} \phi - - (2c_1 + c_9(d + 1)) \eta^{(\mu\nu A^{(\lambda)}} \partial^{(\nu} \partial^{(\lambda)}} \phi - (c_2 + c_10(d + 1)) \eta^{(\mu\nu A^{(\lambda)}} \partial^{(\nu} \partial^{(\lambda)}} \phi - - (c_3 + c_11(d + 1)) \eta^{(\mu\nu A^{(\lambda)}} \partial^{(\nu} \partial^{(\lambda)}} \phi - (2c_3 + c_12(d + 1)) \eta^{(\mu\nu A^{(\lambda)}} \partial^{(\nu} \partial^{(\lambda)}} \phi - - (2c_4 + c_13(d + 1)) \eta^{(\mu\nu A^{(\lambda)}} \phi A^{(\sigma} \partial^{(\lambda)}} \phi \right].
\]

(14)

Notice that as in previous case, the gauge generators do not transform under anticanonical transformations generated by functions (13).

Using integration by parts and performing simple algebraic calculations, we obtain the variation \( S_{2, \text{loc}} \) under gauge transformations (1) in the form

\[
\delta S_{2, \text{loc}} = -6a_1 \int dx \xi_{\nu A^\lambda} \left[ -2c_5(d + 1) \Box \partial^{\nu} A^\lambda \phi^2 + (4c_1 - c_2 - 2c_6(d + 1)) \partial^{\nu} \partial^{\rho} A^\lambda \partial^{\sigma} \phi \phi - - (3c_1 + c_9(d + 1)) \partial^{\nu} \partial^{\lambda} A^\nu \phi^2 + 2(c_1 - c_2 - c_11(d + 1)) \partial^{\nu} \partial^{\lambda} A^\nu \partial^{\rho} \phi \phi + + (c_2 - c_4 + 4c_5(d + 1)) \partial^{\nu} \partial^{\rho} \phi \phi - (c_2 - c_3 + 2c_6(d + 1)) \partial^{\nu} A^\nu \partial^{\rho} \phi \phi - - (c_2 - c_4 + 2c_6(d + 1)) \partial^{\nu} \partial^{\rho} \phi \phi - - (8c_1 + c_2 + 4c_9(d + 1) + 2c_{10}(d + 1)) \partial^{\nu} \partial^{\rho} \phi \phi - - (c_2 + 4c_3 + 2c_{11}(d + 1) + 2c_{12}(d + 1)) \partial^{\nu} \partial^{\rho} \phi \phi - - (c_2 + 4c_4 + 2c_{11}(d + 1) + 2c_{13}(d + 1)) \partial^{\nu} \partial^{\rho} \phi \phi + + (c_2 - c_3 - 2c_7(d + 1)) \partial^{\nu} A^\lambda \partial^{\rho} \phi \phi + (c_2 - c_4 - 2c_8(d + 1)) \partial^{\nu} A^\lambda \partial^{\rho} \phi \phi - - 2c_7(d + 1) A^\lambda \partial^{\nu} \phi \phi + (c_3 - c_4 - c_8(d + 1)) A^\lambda \partial^{\nu} \partial^{\rho} \phi \phi - - (2c_2 - c_3 + 2c_{10}(d + 1)) \partial^{\nu} A^\lambda \partial^{\rho} \phi \phi - (3c_3 + c_{12}(d + 1)) A^\lambda \partial^{\nu} \partial^{\rho} \phi \phi + + (c_3 - 4c_4 - c_{13}(d + 1)) A^\lambda \partial^{\nu} \partial^{\rho} \phi \phi + - 2c_3 + c_4 + c_{12}(d + 1) + c_{13}(d + 1) A^\lambda \partial^{\nu} \partial^{\rho} \phi \phi - - (2c_2 - c_4 + 2c_{10}(d + 1)) \partial^{\nu} A^\lambda \partial^{\rho} \phi \phi \right],
\]

(15)

where the relation \( \eta_{\mu\nu} \xi^{\mu\nu} = 0 \) was systematically used.
Note that the system of algebraic equations
\begin{align*}
4c_1 - c_2 - 2c_9(d + 1) &= 0, \quad 3c_1 + 2c_9(d + 1) = 0, \quad c_5 = 0, \quad c_1 - c_2 - c_{11}(d + 1) = 0, \\
c_2 - 4c_1 - 4c_5(d + 1) &= 0, \quad c_2 - 2c_3 + 2c_6(d + 1) = 0, \quad c_2 - 2c_4 + 2c_6(d + 1) = 0, \\
8c_1 + c_2 + 4c_9(d + 1) + 2c_{10}(d + 1) &= 0, \quad c_2 + 4c_3 + 2c_{11}(d + 1) + 2c_{12}(d + 1) = 0, \\
c_2 + 4c_4 + 2c_{11}(d + 1) + 2c_{13}(d + 1) &= 0, \quad c_2 - 2c_3 - 2c_7(d + 1) = 0, \quad c_7 = 0, \\
c_2 - 2c_4 - 2c_5(d + 1) &= 0, \quad 2c_2 - c_3 + 2c_{10}(d + 1) = 0, \quad 3c_3 + 2c_{12}(d + 1) = 0, \\
c_3 - 4c_4 - 2c_{13}(d + 1) &= 0, \quad c_3 - c_4 + c_7(d + 1) = 0, \quad 2c_2 - c_4 + 2c_{10}(d + 1) = 0, \\
2c_3 + c_4 + c_{12}(d + 1) + c_{13}(d + 1) &= 0, \quad c_3 - c_4 - 2c_8(d + 1) = 0,
\end{align*}
has non-trivial solution
\begin{align*}
c_2 &= 4c_1, \quad c_3 = c_4 = 2c_1, \quad c_5 = c_6 = c_7 = c_8 = 0, \\
c_9 &= -\frac{3}{2(d + 1)} c_1, \quad c_{10} = c_{11} = c_{12} = c_{13} = -\frac{3}{d + 1} c_1.
\end{align*}
Therefore, the functional
\begin{align*}
S_{2\text{ loc}} &= 2a_1 \int dx \varphi_{\mu\lambda} \left[ \partial^{(\mu} \partial^{\nu} \partial^{\lambda\nu} \phi \right]^2 + 4\partial^{(\mu} \partial^\nu A^\nu \partial^{\lambda)} \phi \phi + 2A^{(\mu} \partial^{\nu} \partial^{\lambda)} \phi \phi + 2A^{(\mu} \partial^{\nu} \partial^{\lambda)} \partial^{\nu} \phi \partial^{\lambda} \phi \\
&- \eta^{(\mu\nu\partial^\lambda \phi^2 - 4\eta^{(\mu\nu} \partial^\sigma A^\lambda) \partial^{\sigma} \phi \phi - 2\Box \phi \eta^{(\mu\nu A^\lambda) \partial^{\sigma} \phi \partial^{\sigma} \phi} \\
&- \frac{1}{2} \eta^{(\mu\nu \partial^\lambda \partial^\sigma A^\sigma \phi^2 - \eta^{(\mu\nu \partial^\lambda \phi \partial^\sigma A^\sigma \phi - \eta^{(\mu\nu \partial^\lambda \partial^\sigma \phi \partial^\sigma A^\sigma \phi \phi - \eta^{(\mu\nu \partial^\lambda \phi \partial^\sigma A^\sigma \phi - \eta^{(\mu\nu \partial^\lambda \phi \phi - \eta^{(\mu\nu \partial^\lambda \phi \partial^\sigma A^\sigma \partial^\sigma \phi} + \eta^{(\mu\nu \partial^\lambda \phi \partial^\sigma A^\sigma \partial^\sigma \phi} \right].
\end{align*}
is gauge invariant,
\begin{equation}
\delta S_{2\text{ loc}} = 0,
\end{equation}
and presents the quartic vertices.

The local action
\begin{equation}
S_{\text{loc}}[\varphi, A, \phi] = S_0[\varphi, A, \phi] + S_{2\text{ loc}}[\varphi, A, \phi]
\end{equation}
describes the model of interacting $\varphi^{\mu\lambda}$, $A^\mu$ and $\phi$ fields which is invariant under gauge transformations \cite{4} and belongs to the class of first-stage reducible theories. The action \cite{20} is described in the explicit form by the functional of finite order in fields (the fourth in the case under consideration). In turn, this action can be considered as a simple but not trivial model in the theory of interacting higher spin fields. The gauge invariance of this action is checked with the help of the explicit, closed and finite (in fields) relation. It seems the obtained result cannot be reproduced within the Noether’s procedure being the main method in studies of interacting higher spin fields (see, for example, \cite{12} \cite{13}).

## 4 Gauge-invariant model with mixed vertices

Results of \cite{9} and obtained above allow us to formulate the local action
\begin{equation}
S[\varphi, A, \phi] = S_0[\varphi, A, \phi] + S_{\text{mix}}[\varphi, A, \phi]
\end{equation}
where \( S_0[\varphi, A, \phi] \) is defined in (11) and
\[
S_{\text{int}}[\varphi, A, \phi] = a_1 \int dx \varphi_{\mu\nu\lambda} \left[ \partial^{(\mu} A^{\nu)} A^\lambda \phi^2 + 4 \partial^{(\mu} A^{\nu} \partial^\lambda \phi \phi + 2 A^{(\mu} \partial^{\nu)} \partial^\lambda \phi \phi + 2 A^{(\mu} \partial^{\nu)} \phi \partial^\lambda \phi - \eta^{(\mu\nu} A_\lambda) \phi^2 - 4 \eta^{(\mu\nu} \partial_\sigma A^\lambda) \partial^\tau \phi \phi - 2 \square \phi \eta^{(\mu\nu} A_\lambda) \partial^\tau \phi - 2 \eta^{(\mu\nu} A_\lambda) \partial_\sigma \phi \partial^\tau \phi - \frac{1}{2} \eta^{(\mu\nu} \partial_\lambda) \partial_\sigma \phi A^\sigma \phi \partial^\tau \phi - \eta^{(\mu\nu} \partial_\lambda) \partial_\sigma \phi A^\sigma \partial^\tau \phi \right] + b_1 \int dx \varphi_{\mu\nu\lambda} \left[ \partial^{(\mu} \partial^{\nu)} A^\lambda \phi^2 + 2 \partial^{(\mu} \partial^{\nu)} \phi \partial^\lambda \phi \phi + 2 \partial^{(\mu} \partial^{\nu)} \phi \partial^\lambda \phi \partial^\tau \phi - \frac{1}{2} \eta^{(\mu\nu} \partial^\lambda) \square \phi \phi^2 - 2 \eta^{(\mu\nu} \partial^\lambda) \partial_\sigma \phi \partial^\tau \phi \phi - \eta^{(\mu\nu} \partial^\lambda) \partial_\sigma \phi \partial^\tau \phi \phi \right]. \tag{22}
\]

Here, \( a_1, b_1 \) are coupling constants with the dimensions \( \text{dim}(a_1) = -(d-2) \), \( \text{dim}(b_1) = -(d-1) \). The action is invariant under the gauge transformations (4).
\[
\delta S[\varphi, A, \phi] = 0. \tag{23}
\]

The gauge invariance holds in the limit \( m = 0 \) for scalar field, but it does not maintain for massless vector field when additional gauge invariance of the initial action appears, \( \delta A_\mu = \partial_\mu \xi \). In this case, the deformed gauge transformations in the sector of spin 3 fields become non-local,
\[
\tilde{\delta} \varphi_{\mu\nu\lambda} = \partial_{(\mu} \xi_{\nu\lambda)} - h_{\mu\nu\lambda}(A, \phi) \left[ \partial_{A_\sigma} \partial_{\sigma} \xi \right]. \tag{24}
\]

In turn, it causes the appearance of additional local part in variation of deformed action which compensates exactly the variation of \( S_{\text{int}}[\varphi, A, \phi] \) under gauge transformations of vector field \( A^\mu \). Note that the deformed action \( \tilde{S}[\varphi, A, \phi] = S_0[\varphi + h, A, \phi] \) is described by a functional that has an explicit form and contains vertices up to the eighth order. This functional is invariant under non-local gauge transformations (24) accompanied by relations \( \tilde{\delta} A_\mu = \partial_\mu \xi, \tilde{\delta} \phi = 0, \eta_{\mu\nu} \xi^\mu = 0 \). These transformations have an explicit and finite form.

## 5 Conclusion

In the present paper, we have continued our study \[9\] in construction of new gauge-invariant models using recently proposed approach to the deformation procedure considering as a tool to describe interactions among different fields \[\[6, 7, 8\]. Our starting point was a free gauge-invariant model containing massless spin 3 field, massive vector and scalar fields. Interaction among the fields was introduced with the help of a special anticanonical transformation in the BV formalism acting non-trivially in the sector of spin 3 fields. Moreover, the generating function of the anticanonical transformation was chosen to be independent of the spin 3 field. In turn, this led to the important consequence that the deformation procedure did not affect the original gauge algebra. Further specification of the generating function was dictated by our desire to study local cubic and quartic vertices containing spin 3 field, vector field and one or two scalar fields, respectively. Then, it was proved that the cubic vertices which must be invariant under original gauge transformations due to locality reasons are forbidden while local gauge-invariant quartic vertices have been constructed. The situation is similar with the case of local cubic and quartic vertices describing the interactions of the spin 3 field with the scalar fields studied in \[9\].

Taking into account the results of paper \[9\] and those obtained above in Sec. 3, a new gauge-invariant model with mixed quartic vertices of fields with spins 3, 1 and 0 has been proposed. The action of this model has been described in a closed and explicit form by a local
functional of fourth order in fields. The construction of the model was completely based on a new approach to solving the deformation problem using anticanonical transformations in the BV formalism [6, 7, 8]. It was noted that this model cannot be reproduced within the framework of the standard Noether procedure, which is the main method for studying interactions in the theory of higher spin fields.

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