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An Effective Lagrangian for Low-Scale Technicolor

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Abstract

We present an effective Lagrangian for low-scale technicolor. It describes the interactions at energies $\lesssim M_{\rho_T}$ of the lowest-lying bound states of the lightest technifermion doublet — the spin-one $\rho_T, \omega_T, a_T, f_T$ and the corresponding technipions $\pi_T$. This Lagrangian is intended to put on firmer ground the technicolor straw-man phenomenology used for collider searches of low-scale technicolor. The technivectors are described using the hidden local symmetry (HLS) formalism of Bando, \textit{et al.} The Lagrangian is based on $SU(2) \otimes U(1) \otimes U(2)_L \otimes U(2)_R$, where $SU(2) \otimes U(1)$ is the electroweak gauge group and $U(2)_L \otimes U(2)_R$ is the HLS gauge group. Special attention is paid to the higher-derivative standard HLS and Wess-Zumino-Witten interactions needed to describe radiative and other decays of $a_T$ and $\rho_T/\omega_T$, respectively.

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I. Introduction and Motivation

This paper is devoted to constructing an effective Lagrangian for low-scale technicolor and discussing its main predictions for the Large Hadron Collider. To begin, we believe that technicolor, if it describes electroweak symmetry breaking, must have technihadron states at a low scale — at just a few hundred GeV. This section explains why we think this is so and our plan for the paper.

Technicolor [1, 2, 3, 4] was invented to provide a natural and consistent quantum-field-theoretic description of electroweak (EW) symmetry breaking — without elementary scalar fields. Extended technicolor [5, 6] was invented to complete that natural description by including quark and lepton flavors and their symmetry breaking as interactions of fermions and gauge bosons alone. At the outset, ETC was recognized to have a problem with flavor-changing neutral current interactions, especially those that induce $K^0 – \bar{K}^0$ mixing. The problem is that very high ETC scales, of $O(1000 \text{ TeV})$, are required to suppress these interactions to an acceptable level while — making plausible QCD-based assumptions for the magnitudes of technifermion condensates $\langle \bar{T}T \rangle$ — ETC masses this large imply quark and lepton masses of at most a few MeV. Walking technicolor [7, 8, 9, 10] was invented to cure this FCNC problem. The cure is that the QCD-based assumptions do not apply to technicolor after all. In walking TC the gauge coupling decreases very slowly, staying large for 100s, perhaps 1000s, of TeV and remaining near its critical value for spontaneous chiral symmetry breaking. Then, the $\bar{T}T$ anomalous dimension $\gamma_m \simeq 1$ over this large energy range [11], so that $\langle \bar{T}T \rangle_{\text{ETC}} \gg \langle \bar{T}T \rangle_{\text{TC}}$ and reasonable fermion masses result.\footnote{Except for the top quark, which needs an interaction such as topcolor to explain its large mass [12].}

Because it implies strong dynamics very different from QCD, a walking TC gauge coupling may solve another problem, one of TC, not ETC. This is the apparent conflict with precision electroweak measurements, especially with the value of the $S$-parameter [13, 14, 15, 16] extracted from these measurements. The technicolor contribution to $S$ is defined in terms of polarization functions of the technifermion electroweak currents and their spectral representation by

$$S = 16\pi \frac{d}{dq^2} \left[ \Pi_{33}(q^2) - \Pi_{3q}(q^2) \right]_{q^2=0} = 4\pi \int \frac{dm^2}{m^4} \left[ \sigma^3_V(m^2) - \sigma^3_A(m^2) \right]. \quad (1)$$

Here, for $N_D$ electroweak doublets of technifermions, the currents are $j_{3\mu}^L = \sum_{i=1}^{N_D} \frac{1}{2} \bar{T}_i \gamma_\mu \tau_3 T_i$, etc., and the $\sigma^3_{V,A}$ are vector and axial vector spectral functions for the isovector currents $j_{\mu,5\mu}^3$. Experimentally, the $S$-parameter is consistent with zero or slightly negative [17]. If the spectral functions in Eq. (1) can be represented as a sum over a tower of narrow isovector $\rho_T$ and $a_T$ resonances, with masses $M_{\rho_T}$ and $M_{a_T}$ and couplings $1/g_{\rho_T}$, $1/g_{a_T}$ to $j_{\mu,5\mu}^3$ so that, e.g., $\sigma^3_V(m^2) \simeq \sum_{i=1}^{N_D} \sum_{\text{tower}} M_{\rho_T i}^4 / g_{\rho_T i}^2 \delta(m^2 - M_{\rho_T i}^2)$, the $S$-parameter is given by

$$S \simeq 4\pi \sum_{i=1}^{N_D} \sum_{\text{tower}} \left[ \frac{1}{g_{\rho_T i}^2} - \frac{1}{g_{a_T i}^2} \right]. \quad (2)$$
The usual assumptions made to estimate the TC contribution to $S$ are based on analogy with the way QCD actually works. These assumptions are invalid in walking technicolor [18, 19]. In particular, in QCD the lowest lying $\rho$ and $a_1$ saturate the integrals appearing in Weinberg’s spectral function sum rules [20, 21]. Then, $\Pi_{33} - \Pi_{3q}$ falls off like $1/q^4$ for $q^2 \gtrsim 1\ GeV^2$, and the spectral integrals for the sum rules and $S$ converge very rapidly. This “vector meson dominance” of the spectral integrals is related to the precocious onset of asymptotic freedom in QCD. The $1/q^4$ behavior is consistent (up to logs) with the operator product expansion. The leading OPE term for $\Pi_{33} - \Pi_{3q}$ is essentially $\langle T T \bar{T} T \rangle q^4$, and it dominates above $\sim 1\ GeV$. Here, the $q$-subscript indicates the scale at which the operator is renormalized. In walking TC, however, $\langle T T \bar{T} T \rangle q \sim (q^2/\Lambda_{TC}^2)\langle T T \bar{T} T \rangle_{\Lambda_{TC}}$ for $q$ below the scale at which asymptotic freedom finally sets in. To account for this in terms of spin-one technihadrons, the tower of $\rho_T$ and $a_T$ must extend to very high energy and contribute substantially to the spectral function sum rules and to $S$. Lacking experimental knowledge of these states, and even whether a tower of states is the proper description of the spectral functions, it is at least as difficult to estimate $S$ reliably for TC as it would have been for QCD before the $\rho$ and $a_1$ were discovered. Undaunted, some theorists in the past decade suggested how walking (or near-conformal) dynamics might solve the $S$-parameter problem; see, e.g., Refs. [22, 23, 24, 25, 26]. These proposals, in their simplest realization, amount to there being near equality of the partner $\rho_T$–$a_T$ masses and couplings to the weak currents. For related work, see [27, 28, 29]. Thus, $S$ may be small, and even negative if

$$M_{a_T} \simeq M_{\rho_T} \quad \text{and} \quad g_{a_T} \simeq g_{\rho_T}. \quad (3)$$

This is an interesting and reasonable assumption, and it may be more plausible than requiring large cancellations among the many TC contributions to $S$. But, just how walking technicolor produces this result is a knotty theoretical problem\(^2\). We shall see that, depending on the relative size of couplings in our effective Lagrangian, there can be tension between Eq. (3) and the phenomenology of low-scale technicolor.

A walking TC gauge coupling with $\gamma_m \simeq 1$ for a large energy range occurs if the critical coupling for chiral symmetry breaking lies just below a value at which there is an infrared fixed point [31, 32]. This requires a large number of technifermions. That may be achieved by having $N_D \gg 1$ doublets in the fundamental representation $N_{TC}$ of the TC gauge group, $SU(N_{TC})$, or by having a few doublets in higher-dimensional representations [33, 34]. In the latter case, the constraints on ETC representations [7] almost always imply other technifermions in the fundamental representation as well. In either case, then, there generally are technifermions whose technipion ($\pi_T$) bound states have a decay constant $F_1^2 \ll F_\pi^2 = (246\ GeV)^2$. This low scale implies there also are technihadrons $\rho_T$, $\omega_T$, $a_T$, etc. with masses well below a TeV. We refer to this situation as low-scale technicolor (LSTC) [33, 35, 36]. While, in the past, we preferred to assume the alternative of many TC\(^2\)

\(^2\)It is possible that some sort of duality connecting walking TC to a weakly coupled theory, perhaps in higher dimensions, can explain this. See e.g., Refs. [24, 25]. Contrarily, see e.g. Ref. [30].
fundamentals, the effective Lagrangian we present below is applicable to either situation.\(^3\)

We stress two important consequences of this picture of walking TC. First, to restate what we just said, \(N_D > 1\) technifermion doublets implies the existence of physical technipions, some of which couple to the lightest technivector mesons. Second, since \(M_{\pi_T}^2 \propto \langle \bar{T}TTT \rangle_{\text{ETC}}\), walking TC enhances the masses of technipions much more than it does other technihadron masses. Thus, it is very likely that the lightest \(M_{\rho_T} < 2M_{\pi_T}\) and that the two and three-\(\pi_T\) decay channels of the light technivectors are closed [33]. This further implies that these technivectors are very narrow, a few GeV or less, because their decay rates are suppressed by phase space and/or small couplings (see below). Technipions are a distinctive feature of LSTC and finding them in the decays of technivectors is an important way of distinguishing it from other scenarios of dynamical electroweak symmetry breaking, such as Higgsless models in five dimensions [39][40][30][41], deconstructed models [42][43], a walking TC model with \(N_{TC} = 2\) and just one doublet of technifermions [44][34][26], and the BESS and DBESS models [45][28].

A simple phenomenology of LSTC is provided by the Technicolor Straw-Man Model (TCSM) [46][47][29]. The TCSM’s ground rules and major parameters are these:

1. The lightest doublet of technifermions \((T_U, T_D)\) are color-\(SU(3)_C\) singlets.\(^4\)

2. The decay constant of the lightest doublet’s technipions is \(F_1 = (F_\pi = 246\ \text{GeV}) \cdot \sin \chi\).
   In the case of \(N_D\) fundamentals, \(\sin^2 \chi \approx 1/N_D \ll 1\). In the case of two-scale TC, \(F_\pi = \sqrt{F_1^2 + F_2^2} = 246\ \text{GeV}\) with \(F_1^2/F_2^2 \approx \tan^2 \chi \ll 1\).

3. The isospin breaking of \((T_U, T_D)\) is small. Their electric charges are \(Q_U\) and \(Q_D = Q_U - 1\). In Refs. [47][29] the rates for several decay modes of the technivectors to transversely-polarized electroweak gauge bosons \((\gamma, W^\pm_1, Z^0_1)\) plus a technipion or longitudinal weak boson \((W^0_L \equiv W^\pm_L, Z^0_L)\) and for decays to a fermion-antifermion pair depend sensitively on \(Q_U + Q_D\).

4. The lightest technihadrons are the pseudoscalars \(\pi^{\pm,0}_{T1}(I = 1)\), \(\pi^{0}_{T1}(I = 0)\) and the vectors \(p^{\pm,0}_T(I = 1)\), \(\omega_T(I = 0)\) and axial vectors \(a^{\pm,0}_T(I = 1)\), \(f_T(I = 0)\). Isospin symmetry and quark-model experience strongly suggest \(M_{p_T} \approx M_{\omega_T}\) and \(M_{a_T} \approx M_{f_T}\).

5. Since \(W^\pm_L\) are superpositions of all the isovector technipions, the \(\pi_{T1}\) are not mass eigenstates. This is parameterized in the TCSM as a simple two-state admixture of \(W_L\) and the lightest mass-eigenstate \(\pi_T\):

\[
|\pi_{T1}\rangle = \sin \chi |W_L\rangle + \cos \chi |\pi_T\rangle.
\]

\(^3\)Two other walking-TC scenarios have been proposed and these need not have low-mass technifermions. Ref. [37] employed electroweak singlet technifermions to make the TC coupling walk. Ref. [38] considered an \(SU(2)_{TC}\) with technifermions in the vector representation and a TC-singlet lepton doublet.

\(^4\)Colored technifermions get a substantial contribution to their mass from \(SU(3)_C\) gluon exchange. We also assume implicitly that, in the case of \(N_D\) fundamentals, ETC interactions split the doublets substantially.
Thus, technivector decays involving $W_L$, while nominally, strong interactions, are suppressed by powers of $\sin \chi$. In a similar way, $|\pi_{T1}^0\rangle = \cos \chi' |\pi_{T}^0\rangle + \ldots$, where $\pi_{T}^0$ is the lightest isoscalar technipion, $\chi'$ is another mixing angle, and the ellipsis refer to other isoscalar bound states of technifermions needed to eliminate the two-technigluon anomaly from the $\pi_{T1}^0$ chiral current. It is unclear whether $\pi_T^0$ and $\pi_{T}^0$ will be approximately degenerate as $\rho_T$ and $\omega_T$ are. While they both contain the lightest $\bar{T}T$ as constituents, $\pi_{T}^0$ must contain other heavier technifermions because of the anomaly cancellation.

6. The lightest technihadrons, $\pi_T$, $\rho_T$, $\omega_T$ and $a_T$, may be studied in isolation, without significant mixing or other interference from higher-mass states. This is the most important of the TCSM’s assumptions. It is made to avoid a forest of parameters, and it is in accord with the “simplicity principle” for our effective Lagrangian, discussed below. In the absence of actual data on technihadrons, there is no way to know its validity.

7. In addition to these technihadrons and $W_L^\pm$, $Z_L^0$, the TCSM involves the transversely-polarized $\gamma$, $W_L^\pm$ and $Z_L^0$. The principal production process of the technivector mesons at hadron and lepton colliders is Drell-Yan, e.g., $\bar{q}q \rightarrow \gamma, Z^0 \rightarrow \rho^0_T, \omega_T, a^0_T \rightarrow X$. This gives strikingly narrow $s$-channel resonances at $M_X = M_{\rho^0_T, \omega_T, a^0_T}$ if $M_X$ can be reconstructed.

8. Technipion decays are mediated by ETC interactions and are therefore expected to be Higgs-like, i.e., $\pi_T$ preferentially decay to the heaviest fermion pairs they can. There are two exceptions. Something like topcolor-assisted technicolor [12] is required to give the top quark its large mass. Then, the coupling of $\pi_T$ to top quarks is not proportional to $m_t$, but more likely to $O(m_b)$ [12]. We shall take this into account in Sec. V (see Eq. (37)). Second, the two-gluon decay mode of $\pi_{T}^0$ can be appreciable which would make it difficult to discover it at a hadron collider. In this paper we shall assume that the $\pi_{T}^0$ is heavier than the other LSTC hadrons. Then it is not interesting phenomenologically and we shall not study the details of its interactions.

This TCSM phenomenology was tested at LEP (see, e.g., Refs. [48, 49]) and the Tevatron [50, 51, 52] for certain generic values of the parameters. So far there is no compelling evidence for TC, but there are also no significant restrictions on the masses and couplings commonly used in the TCSM search analyses carried out so far ($M_{\rho_T} \gtrsim 225\text{–}250 \text{ GeV}$, $M_{\pi_T} \gtrsim 125\text{–}145 \text{ GeV}$, $\sin \chi = 1/3$ and $Q_U \simeq 1$). On the other hand, the more general idea of LSTC makes little sense if the limit on $M_{\rho_T}$ is pushed past 600–700 GeV. Therefore, we believe that the LHC can discover it or certainly rule it out [53]. If LSTC were found at the LHC, it would be a field day for a linear collider such as the ILC or CLIC with $\sqrt{s} \simeq M_{\rho_T, \omega_T, a_T}$. Such a collider may be able to separate the closely spaced $\rho_T^0$ and $\omega_T$ and, perhaps, $a_T^0$ resonances. Furthermore, precision measurements, essentially free of
background, of the rates and angular distributions of these states’ decays into gauge boson and $\ell^+\ell^-$ pairs could yield valuable information on LSTC masses and couplings.

The TCSM described above was incorporated into Pythia \cite{54} and used in the recent CDF study \cite{52}. Nowadays, however, many physicists prefer the versatility of programs such as CalcHEP \cite{55} MadGraph \cite{56} and SHERPA \cite{57} to generate new physics signal and background events at the parton level. CalcHEP et al. require inputting a set of Feynman rules, consistent with all relevant gauge and global symmetries. From these, they generate scattering amplitudes that can be interfaced with such programs as Pythia and HERWIG \cite{58} for decays and hadronization.

The Feynman rules, of course, require a Lagrangian. So far, however, a Lagrangian has not been written down for the TCSM or any other variant of LSTC. This is because of the way it was formulated and implemented in Pythia. To guarantee a massless photon pole, kinetic mixing was used in inverse propagator matrices describing the coupling between gauge and technivector bosons. These large matrices must then be inverted at each value of $\hat{s}$ for use in the amplitudes for processes enhanced by $\rho_T$, $\omega_T$ and $a_T$ poles such as $\bar{q}q' \rightarrow W^\pm \pi_T$ and $W^\pm Z^\circ$. Another feature difficult to include in a Lagrangian is the way the TCSM described production of longitudinal weak bosons. Amplitudes involving $W_L$ treated them as spinless particles which, although not a bad approximation at LHC energies, is exact only when $\sqrt{\hat{s}} \gg M_W$. This treatment makes it especially difficult in the TCSM to discuss properly, e.g., the $\rho_T$-enhancement of $W_L^+W_L^-$ in $e^+e^- \rightarrow W^+W^-$. 

The purpose of this paper is to provide an effective Lagrangian, $\mathcal{L}_{\text{eff}}$, for low-scale technicolor. It includes all the LSTC states listed above plus the quarks and leptons. This $\mathcal{L}_{\text{eff}}$ is “effective” not only in being valid just in the energy region in which one can consider the lowest-lying technihadrons in isolation. In LSTC, typical momenta are of the order of the scale — which we shall call $F_1$ — which “suppresses” higher derivative terms, so there seems no systematic way to limit the terms included. Much the same is true of an effective Lagrangian for QCD if the $\rho$ and $a_1$ mesons are included. We shall adhere to a “principle of simplicity”: we keep only the lowest-dimension operators sufficient to describe the phenomenologically important processes of LSTC. Thus, as in the Pythia implementation of the TCSM, we strive to minimize the number of adjustable parameters in $\mathcal{L}_{\text{eff}}$.

We adopt the hidden local symmetry (HLS) formalism of Bando, et al. \cite{55, 56} to describe the technivector mesons, electroweak bosons and technipions. This method guarantees that the photon is massless and the electromagnetic current conserved. The “naive” form of the HLS Lagrangian, $\mathcal{L}_\Sigma$, in which terms with no more than two covariant derivatives are kept, also guarantees that production of longitudinal electroweak bosons via annihilation of massless fermions is well-behaved at all energies in tree approximation. This is important,\footnote{http://theory.sinp.msu.ru/~pukhov/calchep.html} \footnote{madgraph.hep.uiuc.edu/} \footnote{http://projects.hepforge.org/sherpa/dokuwiki/doku.php} \footnote{http://hepwww.rl.ac.uk/theory/seymour/herwig/herwig65.html}\footnote{The reason for this is that, in the absence of $\mathcal{L}_\Sigma$, the gauge structure of the Lagrangian, including...}
because many of the most experimentally accessible LSTC processes at colliders involve production of one or more \( W_L \) in the final state. Elastic \( W_L W_L \) scattering still behaves at high energy as it does in the standard model without a Higgs boson, i.e., the amplitude \( \sim s/F_\pi^2 \) at large cm energy \( s \). Of course, this violation of perturbative unitarity signals the strong interactions of the underlying technicolor theory.

Unfortunately, the naive HLS formalism is too restrictive. Its two-covariant-derivative structure and its symmetries imply relations for interaction operators which are untrue for bound states such as \( \rho_T \) (see, e.g., Refs. [57, 56, 58, 59]). Important LSTC processes, such as \( a_T^\pm \to \gamma \pi_T^\pm, \gamma W^\pm \) and \( \omega_T, \rho_T \to \gamma \pi_T^0, \gamma Z^0 \) do not occur in the Lagrangian. For the radiative and other \( a_T \) decays, we shall apply our “simplicity principle” to choose one particular four-derivative operator of many possible ones. One would expect this operator to spoil the high energy behavior of amplitudes to which it contributes. As we will show in Sec. II, while amplitudes involving only standard model (SM) particles may be modified by this new term, their large-\( s \) behavior is unaltered.

The absence of \( \rho_T \) and \( \omega_T \) radiative decays from the naive Lagrangian is more serious. As in QCD, it happens because the Lagrangian has a parity symmetry not present in the underlying theory. And, as in QCD, the remedy is found in Wess-Zumino-Witten (WZW) terms [60, 61]. They implement the effects of anomalously nonconserved symmetries of the high-energy theory — in QCD, the Adler-Bardeen-Jackiw anomaly. In our case, the question of the anomalies of the high-energy theory is even more subtle. Partly, this is because the HLS Lagrangian seems to require a more extensive set of fermions in the underlying theory than just the lightest doublet \((T_U, T_D)\), so the anomalies in question are less obvious. In addition, there is nothing to cancel the anomalies of the HLS gauge interaction so, unlike the LSTC theory it is supposed to represent, it is truly nonrenormalizable. A somewhat similar problem was considered by Harvey, Hill and Hill [62] (extending and improving earlier work of Kaymakcalan, Rajeev and Schechter [63]). They constructed a gauge-invariant WZW interaction for the standard model in the presence of \( \rho \) and \( a_1 \), which they treated as background fields. An obstacle for us was determining how to apply Ref. [62] to our non-renormalizable theory, in which the HLS fields are dynamical and mix with the electroweak ones. To our knowledge, this has not been done previously for a theory with anomaly-free, renormalizable gauge symmetries and anomalous hidden local symmetries involving vector and axial vector mesons.

The HLS formalism has also been used in BESS models [45, 28], a minimal model of walking technicolor [29], and in deconstructed versions [42, 43, 64, 65, 66] of five-dimensional Higgsless models [39, 40, 30, 41]. However, these papers did not include higher-derivative interactions needed for \( a_T \) decays nor the WZW interactions for \( \rho_T \) and \( \omega_T \) decays.

The rest of this paper is organized as follows: Sec. II specifies the symmetries and gauge and Goldstone fields used to construct \( \mathcal{L}_{\text{eff}} \). Then, we use LSTC dynamics (and phenomenol-
ogy) to motivate the two-derivative terms we allow in the naive $\mathcal{L}_{\text{eff}}$. The resulting Lagrangian is similar, but not identical, to those used in Refs. [57] for QCD and [28] for strong electroweak symmetry breaking. We differ from them in that we included the $U(1)_Y$ gauge boson and its couplings to the technihadrons consistent with arbitrary $(T_U, T_D)$ charges $Q_U$ and $Q_D = Q_U - 1$. Also, as we emphasized, other treatments of strong electroweak symmetry breaking do not include technipions; we expect them to occur in any realistic low-scale technicolor model. Finally, we added an interaction to describe $a_T \rightarrow \gamma \pi_T$ and $\gamma W/Z$. A similar interaction occurs in Ref. [57]. In Sec. III we transform to the unitary gauge and present the vector boson mass matrices, eigenvalues and eigenstates. The connections with the masses and mixings of the electroweak and technivector bosons in the TCSM [47, 29] are discussed. We describe the shifting of gauge fields necessary to eliminate mixed gauge-technipion kinetic terms. The WZW interaction needed at low energy to describe certain important technivector decays is treated in Sec. IV. As a test of the prescription we use to determine it, we show that it produces the expected form for $\pi_T^0 \rightarrow \gamma \gamma$. Technipion masses and their couplings to quarks and leptons are given in Sec. V.

In Sec. VI we compare the predictions of our $\mathcal{L}_{\text{eff}}$ with the TCSM phenomenology outlined above for the technihadron decay amplitudes that are important at the Tevatron and LHC. There is, in fact, no a priori guarantee that our $\mathcal{L}_{\text{eff}}$ reproduces the TCSM because, as we noted above, it is not clear that the two have the same underlying technicolor theory. Nevertheless, we find that they agree. In particular, terms in $\mathcal{L}_{\text{eff}}$ related by the replacement of $\pi_T^{\pm,0}$ by $W_L^{\pm,0}$ stand in the ratio $\cos \chi : \sin \chi$, and amplitudes for processes such as $\rho_T^{\pm,0} \rightarrow \gamma \pi_T^{\pm,0}$ and $\omega_T \rightarrow \gamma \pi_T^0$ differ only by simple valence-quark-model-like “Clebsch” factors. Indeed, requiring that $\mathcal{L}_{\text{eff}}$ reproduce the TCSM in this way has been a valuable check on our calculations.

In Sec. VII we use $\mathcal{L}_{\text{eff}}$ to calculate the low-scale technihadrons’ contributions to the precision electroweak parameters $S$, $T$, $W$ and $Y$ [13, 67, 68, 69] in tree approximation. We see that, thanks to the higher-derivative term added to account for $a_T$ decays, it is possible to make this contribution to $S$ small. The matter of this term’s contribution to TCSM phenomenology is under investigation and will be the subject of a future paper. Finally, some projects for future study are described in Sec. VIII.

Three appendices are attached. Appendix A summarizes the TCSM predictions amplitudes for technivector decay to a pair of technipions and/or electroweak bosons and is useful for comparison with the implications of $\mathcal{L}_{\text{eff}}$. Appendix B contains the eigenvectors for the mass-eigenstate gauge bosons and the coefficients $\zeta$ in the shifts of the gauge fields, $g_G G_\mu^\alpha \rightarrow g_G G_\mu^\alpha + \partial_\mu \zeta G^\alpha / F_1$, needed to rectify their kinetic energy terms. Appendix C is a list of $\mathcal{L}_{\text{eff}}$’s adjustable parameters and suggested defaults.
II. LSTC Symmetries and the Effective Lagrangian

Our Lagrangian is based on the hidden local symmetry formalism \[55, 56\] with gauge group \( G = SU(2)_W \otimes U(1)_Y \otimes U(2)_L \otimes U(2)_R \). The first two groups are the standard electroweak gauge symmetries, with primordial couplings \( g \) and \( g' \) and gauge bosons \( W = (W^1, W^2, W^3) \) and \( B \), respectively. The latter two are the “hidden local symmetry” groups. We use \( U(2)_{L,R} \) instead of \( SU(2)_{L,R} \) for the HLS groups because we expect the isoscalar \( \omega_T \) to be important phenomenologically. Furthermore, radiative decays of \( \rho_0^T \) and \( \omega_0^T \) to the same final state differ only in a factor of \( Q_U + Q_D \) versus \( Q_U - Q_D = 1 \). Thus, they can in principle tell us about technifermion charges. We assume that the underlying TC interactions are parity-invariant, so that their zeroth-order couplings are equal, \( g_L = g_R = g_T \). The assumed equality of the \( SU(2)_{L,R} \) and \( U(1)_{L,R} \) couplings reflect the isospin symmetry of TC interactions and the expectation that \( M_{\rho_T} \approx M_{\omega_T} \) and \( M_{\alpha_T} \approx M_{\rho_T} \). The gauge bosons \( (L, L^0) \) and \( (R, R^0) \) contain the primordial technivector mesons, \( V, V_0, A, A_0 \approx \rho_T, \omega_T, \alpha_T, \rho_T \). We shall see in Sec. III that we can identify \( g_T \approx \sqrt{2} g_{\rho_T} \), where \( g_{\rho_T} \) is the \( \rho_T \) coupling to the isospin current (see Eq. (2)).

To describe the lightest \( \pi_T \) and \( \pi_0^T \), and to mock up the heavier TC states that contribute most to electroweak symmetry breaking (i.e., the isovector technipions of the other \( N_D - 1 \) technifermion doublets or the higher-scale states of a two-scale TC model), and to break all the gauge symmetries down to electromagnetic \( U(1) \), we use nonlinear \( \Sigma \)-model fields \( \Sigma_2 \), \( \xi_L \), \( \xi_R \) and \( \xi_M \). Under \( G \)-transformations (see the moose diagram in Fig. [1]):

\[
\begin{align*}
\Sigma_2 &\rightarrow U_W \Sigma_2 U_Y^\dagger \\
\xi_L &\rightarrow U_W U_Y \xi_L U_L^\dagger \\
\xi_M &\rightarrow U_L \xi_M U_R^\dagger \\
\xi_R &\rightarrow U_R \xi_R U_Y^\dagger .
\end{align*}
\]
Their covariant derivatives are
\[
\begin{align*}
D_\mu \Sigma_2 &= \partial_\mu \Sigma_2 - ig t \cdot W_\mu \Sigma_2 + ig' \Sigma_2 t_3 B_\mu, \\
D_\mu \xi_L &= \partial_\mu \xi_L - ig (g t \cdot W_\mu + g' y_1 t_0 B_\mu) \xi_L + ig T \xi_L t \cdot L_\mu, \\
D_\mu \xi_M &= \partial_\mu \xi_M - ig T (t \cdot L_\mu \xi_M - \xi_M t \cdot R_\mu), \\
D_\mu \xi_R &= \partial_\mu \xi_R - ig t \cdot R_\mu \xi_R + ig \xi_R (t_3 + y_1 t_0) B_\mu, \\
\end{align*}
\]
where \( t \cdot L_\mu = \sum_{\alpha=0}^{3} t_\alpha L_\mu^\alpha \) and \( t = \frac{1}{2} \tau, \ t_0 = \frac{1}{2} \mathbf{1} \). The hypercharge \( y_1 = Q_U + Q_D \) of the TCSM. The field \( \Sigma_2 \) contains the technipions that get absorbed by the \( W \) and \( Z \) bosons. We represent them as an isotriplet of \( F_2 \)-scale Goldstone bosons, where \( F_2 = F_\pi \cos \chi \gg F_1 \) and \( \chi \) was introduced in Sec. 1\(^{10}\). It may be parameterized as \( \Sigma_2(x) = \exp (2i t \cdot \pi_2(x)/F_2) \).

We define \( \Sigma_1 = \xi_L \xi_M \xi_R \). Then
\[
\begin{align*}
\Sigma_1 &\rightarrow U_W \Sigma_1 U_Y^\dagger, \\
D_\mu \Sigma_1 &= \partial_\mu \Sigma_1 - ig t \cdot W_\mu \Sigma_1 + ig' \Sigma_1 t_3 B_\mu. \\
\end{align*}
\]

To construct an effective Lagrangian of manageable size, we first include only two-derivative terms of the nonlinear fields. There is not much justification for this in LSTC because the momenta of technivector decay products are typically of order \( F_1 \). There are still \( \mathcal{O}(10) \) possible \( \text{Tr}(|D_\mu \Sigma|^2) \) terms. We limit them by requiring that \( g_t \)-strength interactions (only!) are consistent with the assumptions of their underlying LSTC dynamics: In particular, they are isospin and parity-invariant. To reduce their number, we employ the TCSM assumption that the lowest-lying technihadrons in isolation. This implies that the interactions we allow arise only from two-technifermion irreducible (i.e., Zweig-allowed) graphs\(^{11}\). Then the naive version of the nonlinear Lagrangian is
\[
\mathcal{L}_\Sigma = \frac{1}{4} F_2^2 \text{Tr}|D_\mu \Sigma_2|^2 + \frac{1}{4} F_1^2 \left \{ a \text{Tr}|D_\mu \Sigma_1|^2 + b \left[ \text{Tr}|D_\mu \xi_L|^2 + \text{Tr}|D_\mu \xi_R|^2 \right] \right \} + c \text{Tr}|D_\mu \xi_M|^2 + d \text{Tr}(\xi_L^\dagger D_\mu \xi_L D_\mu \xi_M \xi_R^\dagger D_\mu \xi_R \xi_M^\dagger) \right \}
\]

The couplings \( a, b, c, d \) are nominally of order one in magnitude.

The interaction in Eq. (8) does not contain terms for the phenomenologically important decays \( a_T^\pm \rightarrow \gamma \pi_T^\pm \) (and, of especial importance at the LHC, \( a_T^\pm \rightarrow \gamma W^\pm \)). Gauge invariance and parity conservation require these be mediated by terms of the form \( F_{\mu \nu} a_T^\pm F_{\mu \nu}^\dagger (\gamma) \pi_T \) and, so, we must include higher derivative terms in \( \mathcal{L}_\Sigma \) to do the job. The same problem was faced for QCD in Ref. \(^{56}\). Unlike those authors, we have no experimental input to guide us, so we assume our “simplicity principle” and add just one four-derivative term to \( \mathcal{L}_\Sigma \):
\[
\begin{align*}
\mathcal{L}_\Sigma &= \frac{1}{4} F_2^2 \text{Tr}|D_\mu \Sigma_2|^2 + \frac{1}{4} F_1^2 \left \{ a \text{Tr}|D_\mu \Sigma_1|^2 + b \left[ \text{Tr}|D_\mu \xi_L|^2 + \text{Tr}|D_\mu \xi_R|^2 \right] \right \} + c \text{Tr}|D_\mu \xi_M|^2 + d \text{Tr}(\xi_L^\dagger D_\mu \xi_L D_\mu \xi_M \xi_R^\dagger D_\mu \xi_R \xi_M^\dagger) \\
&- \frac{if}{2g_T} \text{Tr}(D_\mu \xi_M \xi_M^\dagger D_\nu \xi_M \xi_M^\dagger t \cdot L_{\mu \nu} + \xi_M^\dagger D_\mu \xi_M \xi_M^\dagger D_\nu \xi_M t \cdot R_{\mu \nu}).
\end{align*}
\]

\(^{10}\)Chivukula, et al. \(^{70}\) recently used an HLS construction with multiple scales and a parameter analogous to \( \chi \) to discuss a Higgsless model with topcolor for top-quark mass generation.

\(^{11}\)This eliminates many interactions, e.g., \( \text{Tr}|D_\mu \Sigma_1 \Sigma_2^\dagger|^2 \) and \( \text{Tr}(D_\mu \Sigma_1 \Sigma_1^\dagger)|\text{Tr}(D_\mu \Sigma_2 \Sigma_2^\dagger)| \).
As with the other constants, we expect $f = \mathcal{O}(1)$. The normalization of the $f$-term is chosen to make its contribution to $\rho_T \to \pi_T \pi_T$ easy to compare with that from other terms. As we shall see in Sec. VI, the decays $a_T \to W/Z + \pi_T$ and $a_T^\pm \to W^\pm Z^0$ also proceed through the $f$-term. Several of these modes will be sought at the Tevatron and the LHC \[53\].

One expects higher-derivative operators such as this $f$-term to spoil the high energy behavior of standard-model processes. Fortunately, while the $f$-term may modify their form and field structure, as we explain now, it does not alter the dependence of $SM \to SM$ amplitudes on the cm energy $s$ at high energy: All mixing among the gauge bosons is induced by the two-derivative terms in $\mathcal{L}_\Sigma$. If there were no mixing, the $f$-term would not contribute to any $SM \to SM$ process, and the high energy behavior of such amplitudes would be as in the standard model without a Higgs boson. In particular, the amplitudes for massless fermion-antifermion annihilation to a pair of longitudinally polarized gauge bosons would be constant at high energy and the $W^+_L W^-_L \to W^+_L W^-_L$ amplitude would grow linearly with $s$. Suppose we turn on the mixing between the primordial EW gauge bosons, $W$ and $B$, and the $R, L = (V \pm A)/\sqrt{2}$ bosons in such a way that all acquire the same mass. Then, the unitary transformation matrices from the gauge basis to the mass basis are undefined, and nothing, including the effect of the $f$-term on $SM \to SM$ processes, can depend on them. In this case then, the $f$-term still does not contribute to these processes. Now, allowing different masses for the gauge bosons, it is clear that the $f$-term contribution to $SM \to SM$ amplitudes must involve differences of gauge boson propagators, differences which vanish when the bosons are degenerate. This reduces the high-$s$ behavior of these amplitudes by one power of $s$ from naive power-counting, and so they have the large-$s$ dependence expected in the standard model (without a Higgs). In particular, $f \bar{f} \to WW, WZ \sim \text{constant}$ and $W^+_L W^-_L \to W^+_L W^-_L \sim s/F^2_\pi$ at large $s$.

The effect of the $f$-term (as well as other terms in $\mathcal{L}_\Sigma$) on these standard-model processes and on triple gauge boson vertices remains to be worked out. These are under investigation and will be the subject of future papers (see Sec. VIII).

Still, $\mathcal{L}_\Sigma$ does not allow $\pi^0_T \to \gamma \gamma$ and $\rho_T, \omega_T \to \gamma \pi_T$. The reason for this is that the $g_T$-strength interactions in $\mathcal{L}_\Sigma$ are invariant under more than isospin and space inversion. Under ordinary parity, $P$: $r \to -r, t \to t$, and $\xi_{L,R} \leftrightarrow \xi_{R,L}^\dagger, \xi_M \leftrightarrow \xi_M^\dagger, \Sigma_i \leftrightarrow \Sigma_i^\dagger$ and $R^\alpha \leftrightarrow (-1)^{(1+g_\omega)\alpha} L^\alpha$. Generalizing the discussion in Ref. \[61\], the strong interactions in Eq. (8) are also invariant under $P_0$: $r \to -r, t \to t, (R, L)_\mu \to (-1)^{1+g_\omega}(R, L)_\mu$ and separately under the non-spatial interchanges $\mathcal{P}$: $\xi_{L,R} \leftrightarrow \xi_{R,L}^\dagger, \xi_M \leftrightarrow \xi_M^\dagger, \Sigma_i \leftrightarrow \Sigma_i^\dagger,$ and $R^\alpha \leftrightarrow L^\alpha$. This $\mathcal{P}$ can be enlarged to include electromagnetic interactions: keeping $A_\mu = \sin \theta_W W^3 \mu + \cos \theta_W B_\mu$ while setting other electroweak gauge fields to zero, $\mathcal{L}_\Sigma$ remains invariant under $\mathcal{P}$ with $eA_\mu \to eA_\mu$. Thus, this symmetry forbids, e.g., $\pi^0_T \to \gamma \gamma$ and $\rho^0_T, \omega_T \to \gamma \pi^0_T$ and $\gamma Z^0$. As in QCD, there is no reason to expect that TC respects this symmetry and these decays should occur. The WZW interaction discussed in Sec. IV violates $\mathcal{P}$ and induces these processes.

The complete effective Lagrangian is

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\Sigma + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{ff} + \mathcal{L}_{\text{WZW}} + \mathcal{L}_{M^2} + \mathcal{L}_{\pi_T ff}. \quad (10)$$
The gauge-field Lagrangian has the standard form,
\[
\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \left[ W^a_{\mu\nu} W^{a,\mu\nu} + B_{\mu\nu} B^{\mu\nu} + R^a_{\mu\nu} R^{a,\mu\nu} + L^a_{\mu\nu} L^{a,\mu\nu} \right],
\]
where \( a = 1, 2, 3 \). Quark and lepton couplings to gauge bosons involve only the primordial \( W^a \) and \( B \). This is important in controlling the energy dependence of \( SM \rightarrow SM \) amplitudes and in calculating the oblique parameters \( S, T, W, Y \) in Sec. VII. With an obvious condensed notation,
\[
\mathcal{L}_{ff} = \sum_{j=1}^{3} \left[ \bar{u}_j \gamma^\mu D_\mu u_j + \bar{d}_j \gamma^\mu D_\mu d_j + \bar{\psi}_j \gamma^\mu D_\mu \psi_j \right].
\]

As shown for Higgsless models in Refs. [43, 64, 65], it is possible to reduce the value of the \( S \)-parameter by introducing special couplings of the standard model fermions to the \( L \) gauge bosons. However, these couplings must be finely tuned, and this is antithetical to our technicolor philosophy. Finally, the Lagrangian \( \mathcal{L}_{M2} \) includes ETC-induced \( \pi_T \) masses and \( \mathcal{L}_{\pi_Tff} \) the \( \pi_T \) couplings to fermion-antifermion. They are discussed in Sec. V.\(^{12}\)

### III. Vector Boson States and Masses

To transform to the unitary gauge, first make an \( SU(2)_W \) transformation with \( U_W = \Sigma_2^f(x) \), bringing \( \Sigma_2 \) to the identity, \( \xi_L \) to \( \xi_L' = \Sigma_2^f \xi_L \), and \( \xi_M \) and \( \xi_R \) unchanged, so that \( \Sigma_1' = \Sigma_2^f \Sigma_1 \). Then make \( U(2)_L \) and \( U(2)_R \) transformations with \( U_L = \xi_L' \) and \( U_R = \xi_R' \equiv \xi_R \). This takes those two fields to the identity and \( \xi_M' \) and \( \Sigma_1' \) to
\[
\begin{align*}
\xi_M'' &= \xi_L' \xi_R' = \Sigma_1' ; \\
\Sigma_1'' &= \Sigma_1' \equiv \exp(2it \cdot \tilde{\pi}/F_1).
\end{align*}
\]

In the second equation, \( \tilde{\pi}_a \) are the not-yet-canonically-normalized LSTC technipions. We relate them to \( \pi_T \) and \( \pi_T^0 \) in Eq. (32) below. Dropping the primes, \( \mathcal{L}_\Sigma \) now has the form
\[
\mathcal{L}_\Sigma = \frac{1}{4} F_2^2 \text{Tr} \left[ g \cdot W_\mu - g' t_3 B_\mu \right]^2 + \frac{1}{4} F_1^2 \left\{ a \text{Tr} \left[ i \partial_\mu \Sigma_1 + g t \cdot W_\mu \Sigma_1 - g' t_3 B_\mu \right]^2 + b \text{Tr} \left[ g t \cdot W_\mu + g' y t_0 B_\mu - g_t \cdot L_\mu \right]^2 + c \text{Tr} \left[ g' (t_3 + y t_0) B_\mu - g_t \cdot R_\mu \right]^2 \right. \\
+ d \text{Tr} \left[ g t \cdot W_\mu \Sigma_1 - g' t_3 B_\mu + g_t (\Sigma_1 t \cdot R_\mu - t \cdot L_\mu \Sigma_1) \right] \\
- \frac{1}{2g_T} \text{Tr} \left[ (\Sigma_1 + g_t (\Sigma_1 t \cdot R_\mu - t \cdot L_\mu \Sigma_1)) (\Sigma_1 + g_t (\Sigma_1 t \cdot R_\mu - t \cdot L_\mu \Sigma_1)) + (\Sigma_1 + g_t (\Sigma_1 t \cdot R_\mu - t \cdot L_\mu \Sigma_1)) (\Sigma_1 + g_t (\Sigma_1 t \cdot R_\mu - t \cdot L_\mu \Sigma_1)) \right].
\]

\(^{12}\)The shifts in \( W^a \) and \( B \) discussed in Eq. (31) also induce \( \pi_T \) couplings to quarks and leptons. These are also discussed in Sec. V.
The charged and neutral gauge boson mass matrices can be read off from $\mathcal{L}_\Sigma$ by putting $\Sigma_1 \rightarrow 1$. Defining
\[ x^2 = \frac{g^2}{2g_T^2}, \] (16)
the charged mass matrix is (with rows and columns labeled, in order, by the primordial $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$, $V^\pm = (R^1 + L^1 \mp i(R^2 + L^2))/2$, $A^\pm = (R^1 - L^1 \mp i(R^2 - L^2))/2$):
\[ M_{\pm}^2 = \frac{1}{4}g_T^2F_1^2 \left( \begin{array}{ccc}
2x^2 \left( \frac{F_2^2}{F_1^2} + a + b \right) & -x(b + d) & x(b + d) \\
-2x^2 \left( \frac{F_2^2}{F_1^2} + a \right) t_W & -bx & 0 \\
-bx & -btx_W & 0 \\
0 & -2bxy_W & 0 \\
(b + d)x & 0 & 0
\end{array} \right). \] (17)
The $5 \times 5$ neutral mass matrix has rows and columns labeled by $W^3$, $B$, $V^3 = (R^3 + L^3)/\sqrt{2}$, $V^0 = (R^0 + L^0)/\sqrt{2}$ and $A^3 = (R^3 - L^3)/\sqrt{2}$. The isoscalar axial vector $A^0 \equiv f_T$ does not mix with these and, consequently, will not be produced as an $s$-channel resonance in colliders. For this reason, we will not study its phenomenology in this paper. The neutral mass matrix is
\[ M_0^2 = \frac{1}{4}g_T^2F_1^2 \left( \begin{array}{ccc}
2x^2 \left( \frac{F_2^2}{F_1^2} + a + b \right) & -2x^2 \left( \frac{F_2^2}{F_1^2} + a \right) t_W & -bx & 0 & (b + d)x \\
-2x^2 \left( \frac{F_2^2}{F_1^2} + a \right) t_W & 2x^2 \left( \frac{F_2^2}{F_1^2} + a + b(1 + 2y^2) \right) t_W^2 & -btx_W & -2bxy_W t_W & (b + d)x t_W \\
-bx & -btx_W & 0 & b & 0 \\
0 & -2bxy_W & 0 & b & 0 \\
(b + d)x & 0 & 0 & b + 2(c + d)
\end{array} \right), \] (18)
where $t_W \equiv \tan \theta_W = g'/g$. This matrix has a zero-mass eigenstate, the photon. The $f_T$ mass is
\[ M_{f_T}^2 = \frac{1}{4}g_T^2(b + 2(c + d))F_1^2. \] (19)
So long as $|a|, \ldots, |d|$ are at most $\mathcal{O}(1)$ and $F_2^2 \gg F_1^2$, we see that $M_{V,A}^2 \sim \frac{1}{4}g_T^2F_1^2$ and $M_W^2 \sim \frac{1}{4}g_T^2F_2^2$. Then, in order that $M_{V,A}^2 \gg M_{W,Z}^2$
\[ \frac{g_T^2}{g^2} \gg \frac{F_2^2}{F_1^2} \gg 1 \gg x^2. \] (20)
From $M_{\pm,0}^2$ we can read off the approximate mixings of the technivectors with the primordial electroweak bosons — $W^\pm$, photon $A = W^3 \sin \theta_W + B \cos \theta_W$, and $Z = W^3 \cos \theta_W - B \sin \theta_W$.\(^\text{13}\)
\[ f_{AV^3} \simeq \frac{M_{AV^3}}{M_{V^3V^3}} = -2x \sin \theta_W = -\sqrt{2}e \frac{ey_W}{g_T}, \quad f_{AV^0} \simeq -\sqrt{2}e y_W \frac{1}{g_T}, \quad f_{AA^3} = 0; \] (21)
\[ f_{ZV^3} \simeq -\sqrt{2}e \cot \theta_W \frac{1}{g_T}, \quad f_{ZV^0} \simeq \sqrt{2}e y_W \tan \theta_W \frac{1}{g_T}, \quad f_{ZA^3} \simeq \sqrt{2}e D \frac{1}{B g_T \sin 2\theta_W}; \] (22)
\[ f_{W^\pm V^3} \simeq -\frac{e}{\sqrt{2}g_T \sin \theta_W}, \quad f_{W^\pm A^3} \simeq \frac{e D}{\sqrt{2}B g_T \sin \theta_W}; \] (23)
\(^\text{13}\) All the gauge bosons are canonically normalized.
where \( e = g \sin \theta_W = g' \cos \theta_W \) and \( y_1 = Q_U + Q_D \). For convenience, we are introducing the following combinations of \( a, b, c, d \):

\[
A = a(b + 2(c + d)) + bc - \frac{1}{2}d^2 \equiv aB + bc - \frac{1}{2}d^2 , \\
B = b + 2(c + d) , \quad C = 2c + d , \quad D = b + d \equiv B - C .
\]

(24)

The mixing parameters in the TCSM corresponding to Eqs. (21) are (from Refs. [47, 29]).

\[ f_{\gamma \rho} = -e/g_{\rho T} \] and \( f_{\rho T} = -e y_1 / g_{\rho T} \). The coupling of \( \rho_T \) to the weak isospin current, \( j^3_\rho \), is \( M_{\rho T}^2 / g_{\rho T} \), and the coupling of \( a_T^0 \) to \( j^3_\mu \) is \( M_{a_T}^2 / g_{a_T} \). Then, to leading order in \( x \), \( \mathcal{L}_{\text{eff}} \) produces the TCSM mixings if we identify the HLS gauge coupling \( g_T \) to be

\[
g_T \simeq \sqrt{2} g_{\rho T} \simeq \frac{\sqrt{2} D}{B} g_{a_T} .
\]

(25)

One numerical estimate of \( g_{\rho T} \) may be obtained, rather cavalierly, by scaling from QCD using large-\( N_{TC} \). Using the QCD value \( \alpha_\rho = 2.16 \), extracted from the rate for \( \tau \rightarrow \rho \nu_\tau \) [17], this gives \( g_{\rho T} = \sqrt{4 \pi (2.16)(3/N_{TC})} \). With this identification,

\[
x^2 \simeq \alpha_{EM} / (4 \alpha_{\rho T} \sin^2 \theta_W) = 0.52 \times 10^{-2}
\]

(26)

for \( N_{TC} = 4 \).

The condition \( g_{a_T} \simeq g_{\rho T} \) that the \( F_1 \)-scale contribution \( S_1 \) to the \( S \)-parameter in Eq. (2) is \( B \simeq D \), i.e.,

\[
C = 2c + d \simeq 0
\]

(27)

We shall confirm this in Sec. VII. The condition \( M_{a_T} \simeq M_{\rho T} \) (which, strictly speaking, we don’t need for small \( S_1 \)) implies that \( c + d \simeq 0 \). Together, these are the condition \( c = d = 0 \) used in the DBESS model to make the \( S \)-parameter small [28]. The enhanced symmetry implied by this condition is discussed in Ref. [23]. Neither of these papers employed the \( f \)-interaction, so their conclusions about the consequences of \( c = d = 0 \) for \( \rho_T \rightarrow WW, WZ \) do not apply to us.

Diagonalizing the charged mass matrix through \( \mathcal{O}(x^2) \) and for \( c, d \neq 0 \), we obtain

\[
M_{W^\pm}^2 = \frac{1}{4} g^2 \left[ F_2^2 + \frac{A}{B} F_1^2 \right] \equiv \frac{1}{4} g^2 F_\pi^2 ,
\]

\[
M_{\rho T^\pm}^2 = \frac{1}{4} g_{\rho T}^2 F_1^2 b(1 + x^2) ,
\]

\[
M_{a_T^\pm}^2 = \frac{1}{4} g_{a_T}^2 F_1^2 B \left( 1 + \frac{x^2 D^2}{B^2} \right) ,
\]

(28)

where we introduced the fundamental electroweak scale of the LSTC described by \( \mathcal{L}_{\text{eff}} \), \( F_\pi = \sqrt{F_2^2 + A F_1^2 / B} = 246 \text{ GeV} \). The “mixing angle” \( \chi \) characterizing the contribution of the low \( F_1 \)-scale to electroweak symmetry breaking is

\[
\sin \chi = \sqrt{\frac{A}{B} F_1 / F_\pi}.
\]

(29)

\footnote{The signs of these \( f \)'s are opposite those in these references; their overall sign is purely a matter of convention.}
Note the additional factor of $\sqrt{A/B}$ (expected to be $O(1)$) relative to the TCSM definition, $F_1/F_π$. This is due to our having defined $F_1$ as the decay constant of the non-canonically normalized $\tilde{\pi}$-fields in Eq. (13) (see Eq. (32) below). The nonzero neutral eigenmasses, through $O(x^2)$ and $O(g_1^2 \sin^4 \theta_W)$, are given by (again, for $c, d \neq 0$)

$$M_{Z^0}^2 = \frac{1}{4} (g^2 + g'^2) F_π^2 = \frac{M_W^2}{\cos^2 \theta_W},$$

$$M_{ρ^0}^2 = \frac{1}{4} g_π^2 F_1^2 \left[ 1 + \frac{x^2 (1 + 4 g_1^2 \sin^4 \theta_W)}{\cos^2 \theta_W} \right],$$

$$M_{ω^±}^2 = \frac{1}{4} g_π^2 F_1^2 \left( 1 + 4 x^2 g_1^2 \sin^2 \theta_W \right),$$

$$M_{α^±}^2 = \frac{1}{4} g_π^2 F_1^2 B \left[ 1 + \left( \frac{x D}{B \cos \theta_W} \right)^2 \right],$$

$$M_{f^±}^2 = \frac{1}{4} g_π^2 F_1^2 B.$$

Note that the zeroth-order $V_3 \cong ρ^0$ and $V_0 \cong ω_T$ masses are equal, $\frac{1}{2} g_π \sqrt{b} F_1$ and are split only by terms of $O(x^2)$. Thus, the phenomenology of our $\mathcal{L}_{\text{eff}}$ has very nearly degenerate $ω_T$ and $ρ^0$. If we wish to split them by more than $O(x^2)$, it is necessary to use $U(1)_{L,R}$ couplings $g_′_T \neq g_T$. That is an easy modification to adopt, but we shall not do so in this paper. The eigenvectors in the charged and neutral sectors are in Appendix B.[15]

The last step in preparing the Lagrangian with properly normalized fields requires eliminating gauge-technipion kinetic terms. In going to unitary gauge, we removed mixing between gauge and unphysical Goldstone bosons, but not those involving the $\pi_T$. To eliminate $G µ \partial^ν π_T$ terms, we shift the gauge fields by linear functions $ζ$ of the $\pi_T$:

$$gCG_µ^α \rightarrow gCG_µ^α + \partial_µ ζ_α /F_1, \quad (G^α = W^α, B, V^α = (R + L)^α /\sqrt{2}, A^α = (R - L)^α /\sqrt{2}),$$

Unlike the transformation to unitary gauge, the Lagrangian is not invariant under these shifts. Therefore, we must include them in all the terms in Eq. (10). Once the shifts are done, we can read off the coefficients of $\frac{1}{2} (\partial_µ \pi_T)^2$ and scale the $π_T$ appropriately. The shift fields $ζ_α$ are in Appendix B. The $\tilde{\pi}_α$ are related to the canonically-normalized $π_α \equiv (π_T, π_0^T)$ by

$$\tilde{π}_α = \sqrt{B/A} \cos \chi π_α \equiv η π_α, \quad \tilde{π}_0 = \sqrt{B/A} π_0 \equiv η' π_0,.$$  \hspace{1cm} (32)

Finally, we record the electroweak parameters $c_R^2$ and $(\tan^2 θ_W)_R$ through $O(x^2)$, are

$$\frac{1}{c_R^2} = \frac{1}{c^2} \left[ 1 + 4 x^2 (1 + g_1^2) \sin^2 θ_W \right]$$

$$(\tan^2 θ_W)_R = \tan^2 θ_W \left[ 1 + x^2 \left( 1 + 2 \cos 2θ_W - 4 g_1^2 \sin^2 θ_W - \frac{D^2}{B^2} \right) \right].$$

[15] The point $c = d = 0$ is a singular one for the mass matrices. In that case, the charged eigenstates are slightly ($O(x)$) mixed $W^\pm$ and $L^\pm$ with masses $M_W^2 = \frac{1}{4} g_π^2 F_1^2$ and $M_{L^±}^2 \cong \frac{1}{4} g_π^2 F_1^2 b (1 + 2x^2)$, and $R^±$ with mass $M_{R^±}^2 = \frac{1}{4} g_π^2 b$. The neutral eigenstates are the massless photon, slightly mixed $Z$ and $L^3$ with masses $\frac{1}{4} g_π^2 F_1^2 / \cos^2 θ_W$, $R^3$ with mass $\frac{1}{4} g_π^2 F_1^2 b (1 + 2x^2)$ and a degenerate $ω_T$ and $f_T$ with mass $M_{ω_T,f_T}^2 = \frac{1}{4} g_π^2 F_1^2 b$. 

15
IV. The Wess-Zumino-Witten Interaction

As we discussed in Sec. II, the HLS interaction $\mathcal{L}_\Sigma$ has a symmetry, $\mathcal{P}$, that forbids $\pi^0_T \rightarrow \gamma\gamma$ and $\omega_T, \rho_T \rightarrow \gamma\pi_T$. The interaction’s $SU(2)$ gauge structures also forbid $\omega_T \rightarrow \gamma Z^0$ and $\rho_T \rightarrow \gamma Z^0, \gamma W$. There is no reason not to expect such decays in LSTC and, moreover, they may be of considerable phenomenological importance. For example, $\omega_T \rightarrow \gamma Z^0$ is likely to be the discovery channel for $\omega_T$ at the LHC [53]. Such processes might be found in $\mathcal{P}$-violating Wess-Zumino-Witten interactions induced by anomalously-nonconserved symmetries of the underlying TC theory.

It is, in fact, not clear how to construct $\mathcal{L}_{WZW}$ for the theory whose chiral Lagrangian is $\mathcal{L}_\Sigma$. A general approach for discussing the WZW terms for an effective theory of pions and vector and axial-vector mesons was developed by Kaymackcalan, Rajeev and Schechter [63] and by Harvey, Hill and Hill (HHH) [62]. Ref. [63] was concerned with the electromagnetic interactions of these mesons; HHH generalized this to include full ($SU(2) \otimes U(1)$)$_{EW}$ gauge invariance. The situation studied by HHH is similar to ours. They considered the standard model with one doublet each of quarks and leptons, and addressed the question of constructing $\mathcal{L}_{WZW}$ when the quarks had been integrated out. Their effective Lagrangian describes the $U(2)_L \otimes U(2)_R$-invariant interactions of pions and $\rho, \omega, a_1, f$. They treated the spin-one mesons as nondynamical background fields. The essence of Ref. [62] is the determination of counterterms needed to maintain the local $SU(2) \otimes U(1)$ invariance of $\mathcal{L}_{WZW}$ in the presence of the global $U(2)_L \otimes U(2)_R$ symmetry. If we follow their method exactly, our WZW action would be given by their Eqs. (69) with $A_{L,R} = A_{L,R} + B_{L,R}$ where $A_{L,R}$ are the appropriate $SU(2) \otimes U(1)$ fields, $W$ and $B$, and $B_{L,R}$ are the $U(2)_{L,R}$ background fields $L$ and $R$. One important difference is that their $\Gamma_{AAA}$ and $\Gamma_{AAAA}$ would be absent from our WZW action because we integrated out all the technifermions and so there are no anomalies associated with the electroweak symmetries.

However, this approach is inappropriate for us. Our $L$ and $R$ are dynamical fields, not backgrounds. More importantly, if we think of $L$ and $R$ as composites of underlying fermions, the ones composing $\xi_{L,M,R}$, those fermions are not just our just light technifermion doublet, $(T_U, T_D)_{L,R}$, whose isospin indices are gauged in the electroweak group and not in $U(2)_{L,R}$. Furthermore, the $U(2)_L \otimes U(2)_R$ gauge currents composed of the additional fermions are anomalously nonconserved. There is nothing to cancel these anomalies, and the HLS gauge interaction is nonrenormalizable. To our knowledge, the problem of determining $\mathcal{L}_{WZW}$ for such a theory has not been discussed before.

A second approach we considered, therefore, follows HHH, but $\mathcal{L}_{WZW}$ was constructed just for the $U(2)_L \otimes U(2)_R$ center of the moose in Fig. 1 i.e., treating $L$ and $R$ as dynamical ($A$-type), not background ($B$-type) gauge fields. The motivation for this is that, having integrated out $(T_U, T_D)$, the only anomalies of the theory should be those associated with $U(2)_L \otimes U(2)_R$ and that of the baryon number current, $\bar{T}_U \gamma_\mu T_U + \bar{T}_D \gamma_\mu T_D$. This procedure fails because it breaks electroweak gauge invariance. Since $L, R$ are dynamical fields mixing with the $SU(2) \otimes U(1)$ fields, the mass-eigenstate electroweak bosons, including the photon,
are admixtures involving $R \pm L$. The breakdown of EM gauge invariance is manifest in this $\mathcal{L}_{WZW}$; it contains terms inducing $a^0_T$, $f_T \to \gamma\gamma$, a violation of Yang’s theorem.

To circumvent that problem, we employed HHH’s procedure on each of the three sub-mooses in Fig. 1. That is, we used their Eq. (69), successively taking $A_{L,R} = A_{L,R}$ with $A_{L\mu} = W_\mu$ and $A_{R\mu} = L_\mu$; $A_{L\mu} = L_\mu$ and $A_{R\mu} = R_\mu$; $A_{L\mu} = R_\mu$ and $A_{R\mu} = B_\mu$ including, implicitly, the shifts as in Eq. (31)\textsuperscript{16}. We then added the resulting WZW interactions. This calculation was done in the unitary gauge in which $\xi_M = \Sigma_1 = \exp \left(2it \cdot \pi/F_1 \right)$.

Several phenomenologically important WZW interactions resulting from our procedure are presented in Sec. VI. Here we discuss the interaction for the isovector $\pi^0_T \to \gamma\gamma$. Its strength determines that of all the other WZW interactions. It is given by

$$\mathcal{L}(\pi^0_T \to \gamma\gamma) = -\frac{e^2y_1N_{TC}(1 - \frac{2}{3}\sin^2\chi)}{16\pi^2}\frac{\pi^0_T F_{\mu\nu}\tilde{F}_{\mu\nu}}{A/BF_1\cos\chi},$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ and we assumed that the fermions $(T_U, T_D)$ transform according the fundamental $\mathcal{N}_{TC}$ representation of $SU(N_{TC})$. This is just what we expect to leading order in $\sin^2\chi$. In QCD, the coefficient of $\pi^0_T F_{\mu\nu}\tilde{F}_{\mu\nu}$ is $\frac{1}{2}\cdot N_{TC}\cdot(Q_u^2 - Q_d^2)e^2/(8\pi f_\pi)$. The corresponding factor here is $\frac{1}{2}\cdot N_{TC}\cdot y_1e^2/(8\pi\sqrt{A/BF_1}\cos\chi)$ where we used Eq. (32), $\pi_a = \sqrt{A/B}\cos\chi \tilde{\pi}_a$, and $\langle \Omega | \frac{1}{2} \bar{T}\gamma_\mu\gamma_5\tau_3 T | \pi_3(q) \rangle = iF_1 q_\mu$.

V. Technipion Masses and Couplings to Fermions

Technipion masses are generated mainly by ETC interactions [5]. As in the TCSM, we assume for simplicity that technipion masses are isospin symmetric but, as explained earlier, there is no need for $M_{\pi^0_T}$ to equal $M_{\pi_T}$. We describe their masses by the simple Lagrangian

$$\mathcal{L}_{M_T} = -\frac{1}{4\eta^2}M^2_T F^2_1\text{Tr}(\Sigma_1 + \Sigma_1^\dagger) + \frac{1}{32\eta'^2}M^2_{\pi_T} F^2_1|\text{Tr}(\Sigma_1 - \Sigma_1^\dagger)|^2,$$

where $\eta$ and $\eta'$ are the normalization constants of Eq. (32). Then,

$$M^2_{\pi_T} = M^2_T \quad M^2_{\pi^0_T} = M^2_T + M^2_{\pi_T}.$$

We shall assume that $M^2_{\pi_T} \gg M^2_T$ and not discuss $\pi^0_T$ phenomenology further.

Technipion decays to fermion pairs are also induced by ETC interactions. In the absence of an explicit ETC model, we can only guess at the form of the $\pi_T$-decay Lagrangian. Because the same ETC bosons induce $\pi_T f_i f_j R$ and the $f_i f_j R$ mass term, we expect that the couplings are Higgslike, i.e., approximately proportional to the fermions’ masses. To maintain consistency with the way the decays are modeled in PYTHIA, we take the effective

\textsuperscript{16}We did not close the moose in a circle by also taking $A_{L\mu} = W_\mu$ and $A_{R\mu} = B_\mu$ because, having integrated out all the technifermions coupling to these fields, the corresponding $\Gamma_{AAA} = \Gamma_{AAAA} = 0$. 

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Lagrangian for the coupling of a technipion to a pair of fermions \( f_i \bar{f}_j \) with masses \( m_i, m_j \) (renormalized at the mass of the technipion) to be

\[
\mathcal{L}_{\pi_T f f} = \sum_{\pi_T, i, j} \frac{C_{\pi_T, ij}(m_i + m_j)}{\sqrt{A/B} F_1 \cos \chi} \pi_T \bar{f}_i f_j + \text{h.c.}
\]  

(37)

Here, we assume that \( C_{\pi_T, ij} \) is a constant of \( \mathcal{O}(1) \), without CKM-like mixing angle suppression, unless one or both fermions are top quarks. For light fermions, \( C_{\pi_T, ij} = 1 \) if \( \pi_T = \pi_T^0 \) and \( C_{\pi_T, ij} = \sqrt{2} \) if \( \pi_T = \pi_T^\pm \). If either fermion (or both) is a top quark, \( m_i \) is to be replaced by \( m_b \), reflecting the fact that ETC interactions probably contribute at most \( \sim 5 \text{ GeV} \) to the top’s mass \[12\].

The shifts of the primordial \( W \) and \( B \)-fields shifts discussed in Sec. III induce another coupling of \( \pi_T \) to quarks and leptons. As can be seen from Eq. (84), they are of order \((m_i + m_j) \sin^2 \chi (\sqrt{A/2} F_1 \cos \chi)\) for the \( W^\pm \) shift. Since \( \sin^2 \chi \ll 1 \) in LSTC, they can be important only for the \( \pi_T \) couplings to \( t \bar{b} \) and \( t \bar{t} \). The CDF limit on \( t \to H^+ b \) (with \( H^+ \) assumed to decay to \( c \bar{s} \)) is \( B(t \to H^+ b) \lesssim 0.20 \pm 0.10 \) \[71\]. This puts no meaningful restriction on \( \sin \chi \) for the current CDF limit of \( M_{\pi_T^\pm} \gtrsim 125 \text{ GeV} \) \[52\].

VI. \( \mathcal{L}_{\text{eff}} \) at the Tevatron and LHC and Comparison with the TCSM

Walking technicolor dynamics probably close off the two and three-\( \pi_T \) decay channels of \( \rho_T \), \( \omega_T \) and \( a_T \) \[33\]. This makes them very narrow, with striking decay signatures and favorable signal to background ratios. To repeat, we assume in this paper that the isosinglet \( \pi_T^0 \) is too heavy to appear in technivector decays. Then, at the Tevatron the most promising decay channels likely are \( \rho_T \to W \pi_T \), \( \omega_T \to \gamma \pi_T^0 \), \( a_T^0 \to W \pi_T \) and and \( a_T^\pm \to \gamma \pi_T \), \( W \pi_T \), \( Z \pi_T \) \[47\] \[29\]. The weak bosons are sought in their decay to electrons or muons and missing energy. Technipions accessible at the Tevatron must be lighter than top quarks, so the expected signatures there are \( \pi_T^\pm \to q \bar{b} \) and \( \pi_T^0 \to b \bar{b} \). As discussed in Sec. V, the Tevatron limits on \( t \to H^+ b \) are consistent with our assumption that the \( \pi_T \) coupling to the \( t \)-quark is small, probably of order \( m_b / F_1 \).

At the LHC, the backgrounds to technivector decays to \( \pi_T \) plus an electroweak gauge boson depend on how important the decay modes \( \pi_T \to t \bar{q} \) and \( t \bar{t} \) are. If they are unimportant, hadronic backgrounds, especially \( t \bar{t} \) production, make \( \rho_T \to W \pi_T \to \text{ leptons} + (b + q)-\text{jets unobservable} \[17\]. No studies have been carried out yet on backgrounds when \( \pi_T \to t \bar{q} \) is substantial. Thus, at the LHC, the most promising discovery channels appear to be the low-rate, but relatively background-free, modes \( \rho_T^\pm \to W^\pm Z^0 \), \( \omega_T \to \gamma Z^0 \) and \( a_T^\pm \to \gamma W^\pm \), with \( W, Z \to e, \mu \)-leptons. The weak bosons in these decays are expected to be mainly longitudinally polarized, providing technivector decay angular distributions that are indicative of their underlying dynamical origin \[29\] \[53\].

\[17\] At high luminosity, \( \mathcal{O}(100 \text{ fb}^{-1}) \), \( \rho_T^+ \to \pi_T^0 Z \to q \bar{b} e^+ \ell^- \) appears to be observable above background \[53\].
For use in calculating the important technivector decay rates, we record in this section the relevant on-mass-shell operators from $\mathcal{L}_{\text{eff}}$.\(^{18}\) They were calculated to leading order in $x^2 = g^2/2g_T^2$, $\sin^2 \chi = AF_1^2/BF_\pi^2$ and $y_1 \sin^2 \theta_W$. All fields are mass-eigenstates. In general, we simplified the interactions by using leading-order equations of motion such as
\[
\partial_\mu \rho_{\mu \nu} = -M^2_{\rho_T} \rho_{\mu \nu} - \frac{1}{4}bg^2_{T}T^2(1 + \mathcal{O}(x^2)) = -\frac{1}{4}bg^2_{T}(B/A)F^2_\pi \sin^2 \chi \rho_{\mu \nu}(1 + \mathcal{O}(x^2)),
\]
where $\rho_{\mu \nu} = \partial_\mu \rho_{\nu \mu} - \partial_\nu \rho_{\mu \mu}$, and $\partial_\mu \rho_{\nu \nu} = 0$, and by dropping total derivatives.

At the end of this section we will compare these decay operators with what is expected from the TCSM. As we have said, it is not clear — especially from our consideration of the underlying theory’s anomalies and the WZW interactions they imply — that our effective Lagrangian is based just on a theory with a single technifermion doublet, $(T_U, T_D)$. Whether it is or not (and the WZW discussion suggests it isn’t), we find complete agreement between these operators and those that occur in the TCSM summarized in App. A \cite{47, 29}.

We start with the operators for $\rho^0_T$ two-body decays to technipions and weak bosons.

\[
\mathcal{L}(\rho^0_T \rightarrow \pi^+_T \pi^-_T) = -\frac{ig_T bCY}{2\sqrt{2}A} \rho^0_{T\mu} \pi^+_T \partial_\mu \pi^-_T + \frac{2i[C(C \cos^2 \chi + 2B \sin^2 \chi) - f D^2 \cos^2 \chi]Y \sqrt{2}g_T(BF_\pi \sin \chi)^2}{\rho^0_{T\mu} \partial_\mu \pi^+_T \partial_\nu \pi^-_T} \approx -ig_{\rho_T \pi_T \pi_T} \cos^2 \chi \rho^0_{T\mu} \pi^+_T \partial_\mu \pi^-_T.
\]

\(^{38}\)Here, $Y = (1 - 2y_1^2 \sin^4 \theta_W)$, the difference from unity being a measure of weak isospin violation. Under the reasonable presumption that $Y \approx 1$, we defined the $\rho_T \pi_T \pi_T$-coupling

\[
g_{\rho_T \pi_T \pi_T} = \frac{b[C(B + D) + f D^2]}{4\sqrt{2}AB}g_T \approx \frac{M^2_{\rho_T}}{\sqrt{2}g_T F^2_\pi \sin^2 \chi} \left[1 + (f - 1) \frac{M^2_{A_2}}{M^2_{A_1}}\right],
\]

where the mass parameters $M_{A_1}$ and $M_{A_2}$ are defined in Eqs. (67, 68) below. This corresponds to $g_{\rho_T}$ in Eq. (7) of the TCSM paper \cite{47} (and not necessarily to the coupling of $\pi_T$ to the axial isospin current in Eq. (2)). In the above approximations, the $\rho^0_T \rightarrow W^\pm \pi_T^\mp$ and $W^+W^-$ transitions are considered.
interactions are (with $Y \cong 1$):

\[
\mathcal{L}(\rho_T^0 \rightarrow W^\pm \pi_T^\mp) \cong \frac{i g g_T b C F_\pi}{4 \sqrt{2} A \cos \chi} \rho_T^0(\pi_T^\mp W^-_\mu - \pi_T^\pm W^+_\mu) \\
+ \frac{igD}{\sqrt{2} g_T B^2 F_\pi} [\rho_T^0(\partial_\mu \pi_T^\mp W^-_\nu - \partial_\nu \pi_T^\pm W^+_\mu) + \rho_T^0(W^+_{\mu\nu} \partial_\mu \pi^-_T - W^-_{\mu\nu} \partial_\mu \pi^+_T)] \\
+ \frac{igf}{\sqrt{2} g_T B^2 F_\pi} \rho_T^0(\partial_\mu \pi_T^\mp W^-_\nu - \partial_\nu \pi_T^\pm W^+_\mu) \\
- \frac{e g T(B + D) N_{TC} y_1 \sin \theta_W \cos \chi}{32 \sqrt{2} A B \pi^2 F_1} \rho_T^0(\tilde{W}^-_{\mu\nu} \pi^-_T + \tilde{W}^+_{\mu\nu} \pi^+_T) \\
\rightarrow + ig_{\rho_T \pi_T \pi_T} F_{\pi} \sin \chi \cos \chi \rho_T^0 \left(\pi_T^\mp \partial_\mu \Pi_T + \Pi_T^\mp \partial_\mu \pi_T \right); \quad (40)
\]

where $\tilde{W}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} W^{\rho\sigma}$ and, in accord with Eq. [20], we dropped terms of relative $\mathcal{O}(M_W^2/M_{\rho_T}^2)$. Next,

\[
\mathcal{L}(\rho_T^0 \rightarrow W^+ W^-) \cong - \frac{ig^2 C(B + D)}{2 \sqrt{2} g_T B^2} \left[\rho_T^0 \left(W^+_\mu W^-_\nu + \rho_T^0 \left(W^+_{\nu\mu} W^-_\mu - W^-_{\nu\mu} W^+_\mu\right) \right)\]
\]

\[
\rightarrow - \frac{ig^2 f D^2}{2 \sqrt{2} g_T B^2} \rho_T^0 W^+_\mu W^-_\nu \\
+ \frac{e g T(B + D) N_{TC} y_1 \sin \theta_W}{32 \sqrt{2} B \pi^2} \rho_T^0 \left(W^-_\nu \tilde{W}^+_\mu + W^+_\nu \tilde{W}^-_\mu\right) \\
\rightarrow - ig_{\rho_T \pi_T \pi_T} \sin^2 \chi \rho_T^0 \left(\Pi_T^\mp \partial_\mu \Pi_T\right) \\
- \frac{ie f D^2 \cos \chi}{2 \sqrt{2} A B B g_T F_1} \rho_T^0 \left(W^+_{\mu\nu} \Pi_T^\mp - W^-_{\mu\nu} \Pi_T\right) \\
+ \frac{e g T(B + D) N_{TC} y_1 \sin \theta_W \sin \chi}{32 \sqrt{2} A B \pi^2 F_1} \rho_T^0 \left(\tilde{W}^+_\mu \Pi_T^\mp + \tilde{W}^-_{\mu\nu} \Pi_T\right); \quad (41)
\]

\[
\mathcal{L}(\rho_T^0 \rightarrow Z^0 \pi_T^0) \cong \frac{e g T(B + D) N_{TC} y_1 \sin^2 \theta_W \cos \chi}{16 \sqrt{2} A B \pi^2 F_1} \rho_T^0 \tilde{Z}^0_{\mu\nu} \pi_T^0; \quad (42)
\]

\[
\mathcal{L}(\rho_T^0 \rightarrow Z^0 Z^0) \cong - \frac{g^2 g_T(B + D) N_{TC} y_1 \sin^2 \theta_W \tan^2 \theta_W}{16 \sqrt{2} B \pi^2} \rho_T^0 Z^0_{\nu\mu} \tilde{Z}^0_{\nu\mu} \\
\rightarrow - \frac{e g T(B + D) N_{TC} y_1 \sin^2 \theta_W \tan \theta_W \sin \chi}{16 \sqrt{2} A B \pi^2 F_1} \rho_T^0 \tilde{Z}^0_{\mu\nu} \pi_T^0. \quad (43)
\]

Note the $f$-term $\rho_T^0 W_{\mu\nu} \pi_T$ and WZW $\rho_T^0 \tilde{W}_{\mu\nu} \pi_T$ forms in Eqs. (40) [43]. These correspond to the $FF\pi_T$ and $FF\pi_T$ terms in Eq. (78) of App. A and will be discussed below.
In Eqs. (40,41,43) we indicated their “TCSM limit”. In that limit, amplitudes involving weak gauge bosons are dominated at large $M_{\rho_T}/M_W$ by the emission of their longitudinally-polarized components with $W_{L\mu}^{\pm,0} \approx \partial_\mu \Pi_T^{\pm,0}/(gF_\pi)$, where $\Pi_T$ is the unphysical Goldstone boson. Note that $W_{\mu\nu}$ has no large $W_L$-piece.

The corresponding charged $\rho_T$ decay operators are:

$$\mathcal{L}(\rho_T^\pm \to \pi_T^\pm \pi_T^0) \cong -\frac{i\Gamma b C}{2\sqrt{2} A} \left[ \rho_T^+ \rho_T^- \partial_\mu \pi_T^0 + \rho_T^- \rho_T^+ \partial_\mu \pi_T^0 \right]$$

$$+ 2i[C(C \cos^2 \chi + 2B \sin^2 \chi) - f D^2 \cos^2 \chi] \left[ \rho_T^{\pm} \partial_\mu \pi_T^0 \partial_\nu \pi_T^0 + \rho_T^{\pm} \partial_\mu \pi_T^0 \partial_\nu \pi_T^+ \right]$$

$$\cong -ig_{\rho_T \pi_T \pi_T} \cos^2 \chi [\rho_T^+ \pi_T^0 \partial_\mu \pi_T^0 + \rho_T^- \pi_T^0 \partial_\mu \pi_T^+] . \quad (44)$$

and

$$\mathcal{L}(\rho_T^\pm \to W^\pm \pi_T^0) \cong -ig_{\rho_T \pi_T \pi_T} F_\pi \sin \chi \cos \chi (\rho_T^+ W^- - \rho_T^- W^+ ) \pi_T^0$$

$$+ \frac{ie f D^2 \cos \chi}{2\sqrt{2} AB g_T F_1 \sin \theta_W} (\rho_T^+ W^- - \rho_T^- W^+ ) \pi_T^0$$

$$\rightarrow ig_{\rho_T \pi_T \pi_T} \sin \chi \cos \chi (\rho_T^+ \Pi_T^0 \partial_\mu \pi_T^0 + \rho_T^- \Pi_T^0 \partial_\mu \pi_T^+) . \quad (45)$$

and

$$\mathcal{L}(\rho_T^\pm \to Z^0 \pi_T^0) \cong ig_{\rho_T \pi_T \pi_T} F_\pi \sin \chi \cos \chi (\rho_T^+ \pi_T^0 - \rho_T^- \pi_T^+ ) Z^0$$

$$- \frac{ie f D^2 \cos \chi}{\sqrt{2} AB B g_T F_1 \sin 2\theta_W} (\rho_T^+ \pi_T^0 - \rho_T^- \pi_T^+ ) Z^0$$

$$+ \frac{e g_T (B + D) \theta_T \cos \chi (\rho_T^+ \pi_T^0 + \rho_T^- \pi_T^+ ) Z^0}{32\sqrt{2} AB \pi^2 F_1}$$

$$\rightarrow ig_{\rho_T \pi_T \pi_T} \sin \chi \cos \chi (\rho_T^+ \Pi_T^0 \partial_\mu \pi_T^0 + \rho_T^- \Pi_T^0 \partial_\mu \pi_T^+) . \quad (46)$$

and

$$\mathcal{L}(\rho_T^\pm \to W^\pm Z^0) \cong -\frac{i g^2 C}{2\sqrt{2} g_T B^2 \cos \theta_W} [(W^+ \rho_T^- - W^- \rho_T^+ ) Z^0$$

$$+(\rho_T^+ W^- - \rho_T^- W^+ ) + W^+ \rho_T^- - W^- \rho_T^+ ) Z^0]$$

$$- \frac{ie f D^2}{2\sqrt{2} g_T B^2 \cos \theta_W} (\rho_T^+ W^- - \rho_T^- W^+ ) Z^0$$

$$- \frac{e g_T (B + D) \theta_T \cos \chi (\rho_T^+ W^- + \rho_T^- W^+ ) Z^0}{32\sqrt{2} B \pi^2}$$

$$\rightarrow -ig_{\rho_T \pi_T \pi_T} \sin^2 \chi [\rho_T^+ \Pi_T^0 \partial_\mu \Pi_T^0 + \rho_T^- \Pi_T^0 \partial_\mu \Pi_T^+]$$

$$\frac{ie f D^2 \sin \chi}{2\sqrt{2} AB g_T F_1 \sin \theta_W} (\rho_T^+ W^- - \rho_T^- W^+ ) \Pi_T^0$$

$$\frac{ie f D^2 \sin \chi}{\sqrt{2} AB g_T F_1 \sin 2\theta_W} (\rho_T^+ \Pi_T^0 - \rho_T^- \Pi_T^0 ) Z^0$$

$$- \frac{e g_T (B + D) \theta_T \cos \chi (\rho_T^+ \Pi_T^0 + \rho_T^- \Pi_T^0 ) Z^0}{32\sqrt{2} AB \pi^2 F_1} . \quad (47)$$
Note that there is no WZW term in Eq. (45).

We include with these the $\omega_T \rightarrow W^\pm \pi_T^\mp$, $W^+W^-$, $Z^0\pi_T^0$ and $Z^0Z^0$ operators, all of which arise from $L_{WZW}$:

$$L(\omega_T \rightarrow W^\pm \pi_T^\mp) \approx \frac{eg_T(B + D)N_{TC}}{64\sqrt{2AB} \pi^2 F_1 \sin \theta_W} \omega_{\mu\nu}(\bar{W}_{\mu\nu}^T\pi_T^\mp + \bar{W}_{\mu\nu}^T\pi_T^\mp); \quad (48)$$

$$L(\omega_T \rightarrow W^+W^-) \approx \frac{eg_T(B + D)N_{TC}}{64\sqrt{2B} \pi^2 \sin \theta_W} \omega_{\mu}(W_\nu^\dagger\bar{W}_{\mu\nu}^T + W_\nu^\dagger\bar{W}_{\mu\nu}^T)$$

$$\approx \frac{eg_T(B + D)N_{TC}}{64\sqrt{2AB} \pi^2 F_1 \sin \theta_W} \omega_{\mu\nu}(\bar{W}_{\mu\nu}\Pi_T^+ + \bar{W}_{\mu\nu}\Pi_T^+); \quad (49)$$

$$L(\omega_T \rightarrow Z^0\pi_T^0) \approx \frac{-eg_T(B + D)N_{TC}}{32\sqrt{2}AB \pi^2 F_1} \cot 2\theta_W \cos \chi \omega_{\mu\nu}(\bar{W}_{\mu\nu}\pi_T^0); \quad (50)$$

$$L(\omega_T \rightarrow Z^0Z^0) \approx \frac{g^2eg_T(B + D)N_{TC}}{64\sqrt{2B} \pi^2 \cos^2 \theta_W} \omega_{\mu\nu}Z_\nu^0\pi_T^0$$

$$\approx \frac{-eg_T(B + D)N_{TC}}{32\sqrt{2}AB \pi^2 F_1} \cot 2\theta_W \sin \chi \omega_{\mu\nu}(\bar{W}_{\mu\nu}\Pi_T^0). \quad (51)$$

Next, we list phenomenologically interesting couplings of $a_T$ to $\pi_T$ and a weak boson. To the order we calculated these, the parity-violating couplings $a_T \rightarrow \pi_TW_L$ do not occur.

$$L(a_T^T \rightarrow Z^0\pi_T^T) \approx \frac{igg_T\sqrt{BC}F_1 \cos 2\theta_W \cos \chi}{4\sqrt{2}A \cos \theta_W} (a_{\mu\nu}^T\pi_T^T - a_{\mu\nu}^T\pi_T^T)Z_\mu$$

$$- \frac{igC \cos 2\theta_W \cos \chi}{\sqrt{2AB} g_T F_1 \cos \theta_W} [(a_{\mu\nu}^T \partial_{\mu}^T - a_{\mu\nu}^T \partial_{\mu}^T)Z_\nu^0$$

$$+(a_{\mu\nu}^T \partial_{\nu}^T - a_{\mu\nu}^T \partial_{\nu}^T)Z_\mu]$$

$$- \frac{igfD \cos 2\theta_W \cos \chi}{\sqrt{2AB} g_T F_1 \cos \theta_W} (a_{\mu\nu}^T \partial_{\nu}^T - a_{\mu\nu}^T \partial_{\nu}^T)Z_\mu^0$$

$$\approx - \frac{iefD \cot 2\theta_W \cos \chi}{\sqrt{2AB} g_T F_1} (a_{\mu\nu}^T \partial_{\mu}^T - a_{\mu\nu}^T \partial_{\mu}^T)Z_\nu^0. \quad (52)$$

This decay is of particular interest at the LHC because it is so far the only one involving a technipion that has been shown to be visible above backgrounds. Note that, in our $L_{\text{eff}}$, only the $f$-term has the derivative structure to contribute to this and other $a_T$ decays of interest. Similarly,

$$L(a_T^T \rightarrow W^\pm \pi_T^0) \approx \frac{iefD \cos \chi}{2\sqrt{2AB} g_T F_1 \sin \theta_W} (a_{\mu\nu}^T W_{\mu\nu}^T - a_{\mu\nu}^T W_{\mu\nu}^T)\pi_T^0; \quad (53)$$

and

$$L(a_T^0 \rightarrow W^\pm \pi_T^T) \approx \frac{iefD \cos \chi}{2\sqrt{2AB} g_T F_1 \sin \theta_W} a_{\mu\nu}^0 (W_{\mu\nu}^T\pi_T^T - W_{\mu\nu}^T\pi_T^T), \quad (54)$$

22
and
\[
\mathcal{L}(a_T^0 \to W^+ W^-) \cong -\frac{ig^2 f D}{2\sqrt{2} g_T B} a_T^0 (W_{\mu\nu}^+ W_{\nu\mu}^- - W_{\mu\nu}^- W_{\nu\mu}^+) \\
\quad \to -\frac{ie f D \sin \chi}{2\sqrt{2} A g_T F_1 \sin \theta_W} a_T^0 (W_{\mu\nu}^+ \Pi_T^- - W_{\mu\nu}^- \Pi_T^+) .
\]

Finally, the decay $a_T^\pm \to W^\pm Z^0$ may be sought in the $\rho_T^\pm \to W^\pm Z^0$ analysis:
\[
\mathcal{L}(a_T^\pm \to W^\pm Z^0) \cong \frac{ig^2 f D}{2\sqrt{2} g_T B \cos \theta_W} [(a_T^+ W_{\nu\mu}^- - a_T^- W_{\nu\mu}^+) Z_{\mu\nu}^0 \cos 2\theta_W \\
\quad + (W_{\mu\nu}^+ a_{T\mu}^- - W_{\mu\nu}^- a_{T\mu}^+) Z_{\mu\nu}^0] \\
\quad \to \frac{ie f D \cot 2\theta_W \sin \chi}{2\sqrt{2} A B g_T F_1} (a_T^+ \Pi_T^- - a_T^- \Pi_T^+) Z_{\mu\nu}^0 \\
\quad - \frac{ie f D \sin \chi}{2\sqrt{2} A B g_T F_1 \sin \theta_W} (a_T^+ W_{\mu\nu}^- - a_T^- W_{\mu\nu}^+) \Pi_T^0 .
\]

The $a_T^\pm$ radiative decays, which were an important motivation for including the $f$-interaction in Eq. (9) are listed next.
\[
\mathcal{L}(a_T^\pm \to \gamma \pi_T^\pm) \cong -\frac{ie f D \cos \chi}{\sqrt{2} A B g_T F_1} (a_T^+ \pi_T^- - a_T^- \pi_T^+) F_{\mu\nu} ;
\]
and
\[
\mathcal{L}(a_T^\pm \to \gamma W^\pm) \cong -\frac{ie g f D}{\sqrt{2} g_T B} (a_T^+ W_{\mu\nu}^- - a_T^- W_{\mu\nu}^+) F_{\mu\nu} \\
\quad \to \frac{ie f D \sin \chi}{\sqrt{2} A B g_T F_1} (a_T^+ \Pi_T^- - a_T^- \Pi_T^+) F_{\mu\nu} .
\]

Here, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where $A_\mu$ is the photon field.

Finally, there are the interactions for the main radiative decays of $\omega$ and $\rho_T$. They arise from the WZW terms in the effective Lagrangian.
\[
\mathcal{L}(\omega_T \to \gamma \pi_T^0) \cong -\frac{eg_T N_{TC} (B + D) \cos \chi}{32\sqrt{2} A B \pi^2 F_1} \omega_{T\mu\nu} \bar{F}_{\mu\nu} \pi_T^0 ;
\]
and
\[
\mathcal{L}(\omega_T \to \gamma Z^0) \cong \frac{eg_T N_{TC} (B + D)}{32\sqrt{2} \pi^2 B \cos \theta_W} \omega_{T\mu\nu} Z_{\nu\mu}^0 F_{\mu\nu} \\
\quad \to \frac{eg_T N_{TC} (B + D) \sin \chi}{32\sqrt{2} A B \pi^2 F_1} \omega_{T\mu\nu} F_{\mu\nu} \Pi_T^0 ;
\]
and
\[
\mathcal{L}(\rho_T^0 \to \gamma \pi_T^0) \cong -\frac{eg_T N_{TC} (B + D) y_1 (1 + 2 \sin^2 \theta_W) \cos \chi}{32\sqrt{2} A B \pi^2 F_1} \rho_{T\mu\nu} \bar{F}_{\mu\nu} \pi_T^0 ;
\]
and
\[
\mathcal{L}(\rho^0_T \rightarrow \gamma Z^0) \approx \frac{eg_T N_{TC}(B + D)y_1 (1 + 2 \sin^2 \theta_W)}{32 \sqrt{2} \pi^2 B \cos \theta_W} \rho^0_{T,\mu} Z^0_{\mu} \tilde{F}_{\mu \nu}
\rightarrow \frac{eg_T N_{TC}(B + D)y_1 (1 + 2 \sin^2 \theta_W) \sin \chi}{32 \sqrt{2}AB \pi^2 F_1} \rho_{T,\mu} \tilde{F}_{\mu \nu} \Pi^T_{\mu \nu} .
\]  
(62)

For \(\rho^\pm_T\),
\[
\mathcal{L}(\rho^\pm_T \rightarrow \gamma \pi^\pm_T) \approx -\frac{eg_T N_{TC}(B + D)y_1 \cos \chi}{32 \sqrt{2}AB \pi^2 F_1} [\rho^+_T, \pi^-_T + \rho^-_T, \pi^+_T] \tilde{F}_{\mu \nu} ;
\]  
(63)

and
\[
\mathcal{L}(\rho^\pm_T \rightarrow \gamma W^\pm) \approx \frac{eg_T N_{TC}(B + D)y_1}{32 \sqrt{2} \pi^2 B} [\rho^+_T, W^-_\nu + \rho^-_T, W^+_\nu] \tilde{F}_{\mu \nu}
\rightarrow \frac{eg_T N_{TC}(B + D)y_1 \sin \chi}{32 \sqrt{2}AB \pi^2 F_1} [\rho^+_T, \Pi^-_T + \rho^-_T, \Pi^+_T] \tilde{F}_{\mu \nu} .
\]  
(64)

Let us now compare these decay operators with the TCSM amplitudes in App. A (see Eqs. 78,79 and the table of \(V_{V_T,G_\perp,\pi_T}\) and \(A_{V_T,G_\perp,\pi_T}\) factors). Although not listed in that table, it is clear that the operators for \(\rho_T \rightarrow \pi_T \pi_T\), \(W_L \pi_T\) and \(W_L W_L\) are consistent with both isospin symmetry and the replacement of \(\cos \chi\) by \(\sin \chi\) for each replacement of \(\pi_T\) by \(W_L\). (Actually, there is a peculiar change in the sign of \(\sin \chi\) relative to the TCSM, but this has no observable consequence.)

Consider the WZW interactions above. The mass scale suppressing these interactions is
(65)
\[
M_{V_1} = \frac{16 \sqrt{2}AB \pi^2 F_1}{g_T N_{TC}(B + D)} \approx \frac{4 \pi \sqrt{2}AB/bM_{\rho_T}}{\alpha_{\rho_T} N_{TC}(B + D)} .
\]

Using the estimate in Eq. (26) and assuming all \(\mathcal{L}_\Sigma\) strengths are \(\mathcal{O}(1)\), we have \(M_{V_1} \simeq 2M_{\rho_T}\). This estimate is in reasonable accord with QCD where the corresponding mass for \(\rho, \omega \rightarrow \gamma \pi^0\) is \(M_V \simeq 700\text{ MeV} \simeq M_{\rho}\). Moreover, it would not be surprising to find that \(\alpha_{\rho_T}\) is twice as large as the naive scaling from QCD in Eq. (26) suggests, so that \(M_{V_1} \simeq M_{\rho_T}\) then.

The strengths of these WZW interactions agree with the amplitude factors \(V_{V_1,G_\perp,\pi_T}\) in App. A except for two pairs of operators. In the TCSM table, \(V_{\rho_T^0 W_\perp, \pi_T^\pm} = V_{\rho_T^0 W_\perp, W_L^\pm} = 0\,\text{ but they are proportional to } y_1 \sin \theta_W\) in Eqs. (40,41). And \(V_{\rho_T^0 Z_\perp, \pi_T^\pm}, V_{\rho_T^0 Z_\perp, W_L^\pm} \propto -y_1 \tan \theta_W\) in the TCSM, but they are proportional to \(-2y_1 \sin^2 \theta_W \tan \theta_W\) in Eqs. (42,43). The reason for these discrepancies is this: In the TCSM, no mixing was allowed between \(\rho_T^0\) and \(\omega_T\) because, it was argued, this mixing is negligibly small in QCD. This argument is plausible \textit{unless} the zeroth-order \(\rho_T^0\) and \(\omega_T\) — our \(V_3\) and \(V_0\) — are degenerate. And that is exactly what happens in our HLS model with equal \(SU(2)_{LR}\) and \(U(1)_{LR}\) couplings; see \(M_3^2\) in Eq. (18). Thus, any amount of nonzero mixing can have a significant, \(\mathcal{O}(x^0)\), effect. From the eigenvectors in Eq. (83) of App. B, we see that
\[
|\rho_T^0\rangle \approx \cos \epsilon |V_3\rangle + \sin \epsilon |V_0\rangle , \quad |\omega_T\rangle \approx -\sin \epsilon |V_3\rangle + \cos \epsilon |V_0\rangle ;
\]  
(66)
where \( \epsilon \simeq 2y_1 \sin^2 \theta_W \) is presumed small compared to one.\(^{19}\) Thus, \( V_{\rho T}^2 w_{\perp \pi T}/w_L \simeq V_{3 T} w_{\perp \pi T}/w_L + 2y_1 \sin^2 \theta_W V_0 w_{\perp \pi T}/w_L \), where the amplitude factors for \( V_3 \) and \( V_0 \) are in App. A. Since the \( V_0 \) amplitude factors in the table are of zeroth order in \( y_1 \sin \theta_W \), they are equal, to the order in \( \epsilon \) we calculated, to the corresponding \( \omega_T \) decay strengths we found in Eqs. (48–51).

All the \( \rho_{T \mu \nu} G_{\perp \mu \nu \pi T} \) and \( a_{T \mu \nu} G_{\perp \mu \nu \pi T} \) operators are also in accord with the corresponding TCSM \( A_{\rho \pi T} \) factors (up to that pesky sign of \( \sin \chi \)). We deduce that the mass scale \( M_{A_1} \) suppressing \( \rho_{T \mu \nu} G_{\perp \mu \nu \pi T} \) operators is

\[
M_{A_1} = \frac{\sqrt{2} A B g_T F_1}{2 f D^2} = \frac{g_T B^2 F_\pi \sin \chi}{\sqrt{2} f D^2} \simeq \frac{\sqrt{2} A B M_{a_T}}{f D^2}.
\]

(67)

For \( \mathcal{L}_\Sigma \) couplings of \( \mathcal{O}(1) \), \( M_{A_1} \simeq M_{a_T} \simeq M_{\rho_T} \), which is what we would naively expect. The characteristic scale suppressing the \( a_T \) decay interactions is

\[
M_{A_2} = \frac{\sqrt{2} A B g_T F_1}{2 f D} = \frac{g_T B F_\pi \sin \chi}{\sqrt{2} f D} \simeq \frac{\sqrt{2} A M_{a_T}}{f D}.
\]

(68)

For \( \mathcal{L}_\Sigma \) couplings of \( \mathcal{O}(1) \), \( M_{A_2} \simeq M_{a_T} \). Note, from Eq. (67), that \( M_{A_1} = M_{A_2} \) when \( B = D \), i.e., \( 2c + d = 0 \). This is almost the same as the condition \( c = d = 0 \) for \( M_{\rho_T} = M_{a_T} \). It is quite remarkable, despite our uncertainty about the theory underlying our \( \mathcal{L}_{\text{eff}} \), how closely it tracks the valence-quark-model-inspired TCSM. Naive dimensional analysis fixes the TCSM’s arbitrary mass parameters \( M_V \) and \( M_{A_i} \) in Eqs. (63–67) to be what one would expect merely by scaling from QCD.

**VII. \( F_1 \)-Scale Contribution to \( S, T, W, Y \)**

In this section we present the results of a calculation of the \( F_1 \)-scale contribution to the precision electroweak parameters \( S, T, W \) and \( Y \).\(^{13} \)\(^{67} \)\(^{68} \)\(^{69} \). The complete parameters are defined in terms of the technicolor contribution to the polarization functions of electroweak currents as follows:

\[
S = 16\pi (\Pi_{33}'(0) - \Pi_{3Q}'(0)), \quad T = \frac{16\pi}{M_Z^2 \sin^2 \theta_W} (\Pi_{11}(0) - \Pi_{33}(0))
\]

\[
W = \frac{1}{2} g^2 M_W^2 \Pi_{33}'(0), \quad Y = \frac{1}{2} g^2 M_W^2 \Pi_{YY}'(0),
\]

(69)

where \( \Pi'(0) = (d\Pi(q^2)/dq^2)_{q^2=0} \). Barbieri, et al., argued that these four quantities describe the most important effects on standard-model processes at energies well below the technivector masses. They are well-constrained by \( e^+e^- \) data at the \( Z^0 \) and above\(^{17} \)\(^{68} \):

\[
S = -0.04 \pm 0.09 (-0.07), \quad T = 0.02 \pm 0.09 (+0.09)
\]

\[
W = (-2.7 \pm 2.0) \times 10^{-3}, \quad Y = (4.2 \pm 4.9) \times 10^{-3},
\]

(70)

---

\(^{19}\)This is an instructive problem in degenerate perturbation theory. See K. Gottfried, *Quantum Mechanics: Fundamentals*, First Edition, 1966, p. 397, Problem 1, and J. J. Sakurai, *Modern Quantum Mechanics*, Revised Edition, 1993, p. 348, Problem 5.12. The exact result for the mixing angle is \( \epsilon = \frac{1}{2} \tan^{-1}(4y_1 \sin^2 \theta_W/(1 - 4y_1^2 \sin^2 \theta_W)) \).
where, for $S$ and $T$, $U = 0$ was assumed and the central value corresponds to subtracting out the contribution of a standard model Higgs boson of mass 117 GeV; the correction to the central value when the Higgs mass is increased to 300 GeV is given in parentheses.

To calculate $S_1, \ldots, Y_1$, we follow the method described in Refs. [68 69]. It applies here because quarks and leptons couple only to primordial $W$ and $B$ and only in the standard way. We use the technivectors’ lowest-order equations of motion to integrate them out of $\Pi(q^2)$ and then canonically re-normalize the $W^{\pm,0}$ and $B$ fields’ kinetic terms by dividing the fields by the square roots of

$$
\mathcal{N}_W = 1 + \frac{(B^2 + D^2)x^2}{B^2}, \quad \mathcal{N}_B = 1 + \frac{(B^2(1 + 4y_1^2) + D^2)x^2\tan^2 \theta_W}{B^2},
$$

where $A, \ldots, D$ were defined in Eq. (24). We obtain, to leading order in $M_W^2/M_{\rho_T}^2$ and $x^2$:

$$
S_1 = \frac{8\pi(B^2 - D^2)}{g_T^2 B^2}, \quad T_1 = 0 \quad (72)
$$

$$
W_1 = \frac{x^4(B^3 + bD^2)}{2bB^3} \left[2 \left( \frac{F_2^2}{F_1^2} + a + b \right) - b - \frac{D^2}{B} \right] \approx \frac{x^4F_2^2}{bF_1^2} \left[ 1 + \frac{bD^2}{B^3} \right] \quad (73)
$$

$$
Y_1 = \frac{x^4(B^3(1 + 4y_1^2) + bD^2)\tan^2 \theta_W}{2bB^3} \left[2 \left( \frac{F_2^2}{F_1^2} + a + b \right) - b - \frac{D^2}{B} \right] \quad (74)
$$

$$
\approx \frac{x^4F_2^2\tan^2 \theta_W}{bF_1^2} \left[ 1 + 4y_1^2 + \frac{bD^2}{B^3} \right]. \quad (75)
$$

Leave $S_1$ aside for a moment. That $T_1 = 0$ (and $U_1 = 0$) at tree level is guaranteed by the model’s built-in custodial isospin symmetry. For $|A|, \ldots, |D| = \mathcal{O}(1)$ and the estimate $x^2 \approx 0.5 \times 10^{-2}$ we made in Sec. III, $W_1$ and $Y_1 = \mathcal{O}(x^4)$ are well within experimental bounds.

Regarding $S_1$, first note that it is what we would get from Eq. (2) using Eq. (25). Then under the same assumptions on $B$ and $D$, $S_1$ is likely to be an order of magnitude too large. However, we can make $S_1$ small by choosing $B^2 \approx D^2$. Positivity of $M_{\rho_T}^2$ and $M_{\rho_T}^2$ require $b, B > 0$. Then, it seems likely that $D = b + d > 0$ also, so that the condition for small $S_1$ is

$$
C = B - D = 2c + d \cong 0. \quad (76)
$$

This is implied by the condition $c = d = 0$ assumed in Refs. [28 23] to make $S$ small in their models. In those references, $C = 0$ implies the vanishing of $g_{\rho_T \pi^+ \pi^-}$ so that $W_L W_L$ scattering in the $J = 1$ channel has no $\rho_T$ pole to unitarize it. For us, the $f$-term in $\mathcal{L}_\Sigma$ gives $g_{\rho_T \pi^+ \pi^-} \approx bB f g_T/\sqrt{2}A$ and a $\rho_T$ coupling to $W_L W_L$ of $g_{\rho_T \pi^+ \pi^-} \sin^2 \chi$.\(^\text{20}\) Finally, if

\(^\text{20}\)Another way to make $S_1$ small is to note that, if $g_{\rho_T \pi^+ \pi^-}$ is held fixed, a decrease of $B^2 - D^2$ by a multiplicative factor $\epsilon < 1$ is compensated by $g_T \rightarrow g_T/\epsilon$ and $f \rightarrow \epsilon f$. This decreases $S_1$ by $\epsilon^3$. It is also possible, of course, that $S_1$ is not small, but is canceled by the contributions to $S$ from other technifermion doublets. This course seems less natural to us.
$|C| \ll |B + D|$, Eqs. (28,30) imply

$$\frac{M^2_{\alpha_T}}{M^2_{\rho_T}} \simeq \frac{b + d}{b},$$

so that the condition $M_{\alpha_T} \simeq M_{\rho_T}$ further implies that $d$ and, hence, $c$ are both small.

**VIII. Future Projects**

Several projects flow immediately from the effective Lagrangian developed in this paper. We summarize them here.

1. The ALEPH Collaboration at LEP searched for a $\rho_T$ enhancement in $e^+e^- \rightarrow W^+_L W^-_L$ and claimed a limit of $M_{\rho_T} > 600$ GeV [49]. Eichten and Lane pointed out that the ALEPH analysis does not apply to the TCSM because the $\rho^0_T \rightarrow W^+_L W^-_L$ coupling is proportional to $\sin^2 \chi \ll 1$ and, using a simplified version of the HLS model discussed here, showed that ALEPH set no meaningful limit on LSTC [29]. That analysis will be redone with the $\mathcal{L}_{\text{eff}}$ developed here.

2. The HLS effective Lagrangian provides a way to test an assumption on which the TCSM relies heavily — the validity of the approximation $W^\pm_{L\mu} \approx \partial_\mu \Pi^\pm_T/M_W = 2 \partial_\mu \Pi^\pm_T/(g_F \pi)$ and the dominance of longitudinally polarized weak bosons in such processes as $\rho_T \rightarrow W \pi_T$ and $a_T \rightarrow \gamma W$ — and an important consequence of this approximation, the angular distributions in resonant production of $WZ$, $\gamma W$ and $\gamma Z$ [29]. In a future paper, we shall examine these processes and study the $f$-term’s effect on them at the resonance mass.

3. Precision measurements of triple gauge boson vertices at LEP and the Tevatron [17] and, hopefully, soon at the LHC may be sensitive to the presence of technivector poles and to the non-standard triple gauge boson vertices in the $f$-term of $\mathcal{L}_\Sigma$. These studies at the LHC can provide complementary information to the direct technivector searches. An analysis of these effects seems worthwhile, therefore. This will be a generalization of the study in item 1 above.

4. If low-scale technicolor is discovered at the LHC, a high energy linear $e^+e^-$ collider such as the ILC or CLIC offers an excellent possibility to study the resonant contributions to $e^+e^- \rightarrow \ell^+\ell^-$, $W^+W^-$ and $\gamma \pi^0_T/Z^0$. The energy resolution of the collider and its detectors could make it possible to resolve the $\rho^0_T$, $\omega_T$ and $a^0_T$ in their $\ell^+\ell^-$ decay channels. The linear collider is also likely to be the best place to analyze the angular distributions in these channels and, perhaps, determine the sum of charges, $y_1 = Q_U + Q_D$, of the constituent technifermions. It would be interesting to study the $s$-dependence of $W^+W^-$ as it passes through the resonance region. And, if backgrounds at the Tevatron and LHC prove daunting, the linear collider will be the only place to observe $\omega_T \rightarrow \gamma \pi^0_T$, an important process because it involves a technipion.
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Appendix A: Table of TCSM Couplings

| Process                        | $V_{V_T}/a_T G_{\perp \pi T}$ | $A_{V_T}/a_T G_{\perp \pi T}$ |
|--------------------------------|--------------------------------|--------------------------------|
| $\omega_T \to \gamma\pi_T^0$  | $\cos \chi$                    | 0                             |
| $\to \gamma Z_T^0$            | $\sin \chi$                    | 0                             |
| $\to W_{\perp \pi_T^0}$       | $\cos \chi/(2 \sin \theta_W)$ | 0                             |
| $\to W_{\perp W_T^0}$         | $\sin \chi/(2 \sin \theta_W)$ | 0                             |
| $\to Z_{\perp \pi_T^0}$       | $\cos \chi \cot 2 \theta_W$   | 0                             |
| $\to Z_{\perp Z_T^0}$         | $\sin \chi \cot 2 \theta_W$   | 0                             |
| $\rho_T^0 \to \gamma\pi_T^0$  | $y_1 \cos \chi$                | 0                             |
| $\to \gamma Z_L^0$            | $y_1 \sin \chi$                | 0                             |
| $\to W_{\perp \pi_T^0}$       | 0                               | $\pm \cos \chi/(2 \sin \theta_W)$ |
| $\to W_{\perp W_L^0}$         | 0                               | $\pm \sin \chi/(2 \sin \theta_W)$ |
| $\to Z_{\perp \pi_T^0}$       | $-y_1 \cos \chi \tan \theta_W$ | 0                             |
| $\to Z_{\perp Z_L^0}$         | $-y_1 \sin \chi \tan \theta_W$ | 0                             |
| $\rho_T^{-} \to \gamma\pi_T^{-}$ | $y_1 \cos \chi$                | 0                             |
| $\to \gamma W_L^{\pm}$        | $y_1 \sin \chi$                | 0                             |
| $\to Z_{\perp \pi_T^{-}}$     | $-y_1 \cos \chi \tan \theta_W$ | $\pm \cos \chi/(\sin 2 \theta_W)$ |
| $\to Z_{\perp W_L^{\pm}}$     | $-y_1 \sin \chi \tan \theta_W$ | $\pm \sin \chi/(\sin 2 \theta_W)$ |
| $\to W_{\perp \pi_T^{0}}$     | 0                               | $\mp \cos \chi/(2 \sin \theta_W)$ |
| $\to W_{\perp Z_L^{0}}$       | 0                               | $\mp \sin \chi/(2 \sin \theta_W)$ |
| $a_T^{0} \to W_{\perp \pi_T^{0}}$ | 0                               | $\mp \cos \chi/(2 \sin \theta_W)$ |
| $\to W_{\perp W_L^{\pm}}$     | 0                               | $\mp \sin \chi/(2 \sin \theta_W)$ |
| $a_T^{\pm} \to \gamma\pi_T^{\pm}$ | 0                               | $\mp \cos \chi$               |
| $\to \gamma W_L^{\pm}$        | 0                               | $\mp \sin \chi$               |
| $\to W_{\perp \pi_T^{0}}$     | 0                               | $\pm \cos \chi/(2 \sin \theta_W)$ |
| $\to Z_{\perp \pi_T^{0}}$     | 0                               | $\mp \cos \chi \cot 2 \theta_W$ |
| $\to W_{\perp \pi_T^{0}}$     | 0                               | $\pm \sin \chi/(2 \sin \theta_W)$ |

The table above presents the amplitude factors in the TCSM for $V_T(=\rho_T, \omega_T)$ and $a_T$ decay into a technipion plus a transversely-polarized electroweak boson or one transverse and one longitudinal electroweak boson \[17\] \[29\]. The amplitudes are defined in terms of the following matrix elements:

$$
\mathcal{M}(V_T/a_T(p_1) \to (p_2)\pi_T(p_3)) = \frac{e V_{V_T,a_T} G_{\perp \pi T}}{2 M_{V_1,2}} \tilde{F}_{1}^{\lambda \mu} F_{2}^{\star \lambda \mu} + \frac{e A_{V_T,a_T} G_{\perp \pi T}}{2 M_{A_1,2}} F_{1}^{\lambda \mu} F_{2}^{\star \lambda \mu} \tag{78}
$$

Here, $F_{n\lambda \mu} = \epsilon_{n \lambda} p_{\mu} - \epsilon_{n \mu} p_{\lambda}$ and $\tilde{F}_{n\lambda \mu} = \frac{1}{2} \epsilon_{n \lambda \mu \rho} F_{\rho}^{\star}$. The TCSM mass parameters $M_{V_1}$ and $M_{A_1}$ are expected to be of order $M_{\rho_T} \approx M_{\omega_T}$ while and $M_{V_2}, M_{A_2} = \mathcal{O}(M_{a_T})$. The factors
$V_{VT,AT}G_{\pi T}$ and $A_{VT,AT}G_{VT}$ are given by:

\begin{align}
V_{VTG_{\perp\pi T}} & = 2 \text{Tr}(Q_{VT}\{Q_{G\pi T}, Q_{\pi T}\}), \quad A_{VTG_{\perp\pi T}} = 2 \text{Tr}(Q_{VT}[Q_{G\pi T}, Q_{\pi T}]) \quad (79) \\
V_{aTG_{\perp\pi T}} & = 2 \text{Tr}(Q_{aT}\{Q_{G\pi T}, Q_{\pi T}\}), \quad A_{aTG_{\perp\pi T}} = 2 \text{Tr}(Q_{aT}[Q_{G\pi T}, Q_{\pi T}]) \quad (80)
\end{align}

In the TCSM, with electric charges $Q_{U}, Q_{D}$ for $T_{U}, T_{D}$, and $y_1 = Q_{U} + Q_{D}$, the generators $Q$ in Eq. (79) are given by

\begin{align}
Q_{\omega T} & = \left( \begin{array}{ccc}
\frac{1}{2} & 0 & 0 \\
0 & 0 & \frac{1}{2}
\end{array} \right), \\
Q_{\rho_{T}^+, a_{T}^0} & = \left( \begin{array}{ccc}
\frac{1}{2} & 0 & 0 \\
0 & 0 & -\frac{1}{2}
\end{array} \right); \quad Q_{\rho_{T}^+, a_{T}^0} = Q_{\rho_{T}^+, a_{T}^0} = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 0
\end{array} \right), \\
Q_{\pi_{T}^0} & = \cos \chi \left( \begin{array}{ccc}
\frac{1}{2} & 0 & 0 \\
0 & 0 & -\frac{1}{2}
\end{array} \right); \quad Q_{\pi_{T}^+} = Q_{\pi_{T}^+} = \frac{\cos \chi}{\sqrt{2}} \left( \begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 0
\end{array} \right), \\
Q_{\pi_{T}^0'} & = \cos \chi' \left( \begin{array}{ccc}
\frac{1}{2} & 0 & 0 \\
0 & 0 & \frac{1}{2}
\end{array} \right), \\
Q_{\gamma V} & = \left( \begin{array}{ccc}
Q_{U} & 0 & 0 \\
0 & 0 & Q_{D}
\end{array} \right); \quad Q_{\gamma A} = 0, \\
Q_{Z V} & = \frac{1}{\sin \theta_W \cos \theta_W} \left( \begin{array}{ccc}
\frac{1}{4} - Q_{U} \sin^2 \theta_W & 0 \\
0 & -\frac{1}{4} - Q_{D} \sin^2 \theta_W
\end{array} \right), \\
Q_{Z A} & = \frac{1}{\sin \theta_W \cos \theta_W} \left( \begin{array}{ccc}
-\frac{1}{4} & 0 & 0 \\
0 & 0 & \frac{1}{4}
\end{array} \right), \\
Q_{W_{V}^+} & = Q_{W_{V}^+} = -Q_{W_{A}^+} = -Q_{W_{A}^+} = \frac{1}{2\sqrt{2} \sin \theta_W} \left( \begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 0
\end{array} \right). \quad (81)
\end{align}

**Appendix B: Gauge Boson Mass Eigenstates and Shift Parameters**

The components of the mass eigenstate vectors in the charged sector are (assuming $c + d$ is not small):

\begin{align}
\hat{v}_{W^\pm} & = \left\{ 1 - \frac{1}{2} \left[ 1 + \frac{D_x^2}{B^2} \right] x^2, x, -\frac{D_x}{B} \right\}, \\
\hat{v}_{\rho_T^\pm} & = \left\{ -x, 1 - \frac{1}{2} x^2, \frac{Dx^2}{2(c + d)} \right\}, \\
\hat{v}_{a_T^\pm} & = \left\{ \frac{Dx}{B}, -\frac{bDx^2}{2(c + d)B}, 1 - \frac{D^2 x^2}{2B^2} \right\}, \quad (82)
\end{align}

where the elements are labeled by the primordial gauge bosons $\hat{W}^\pm \equiv (W^1 \mp iW^2)/\sqrt{2}, V^\pm, A^\pm$. In the same approximation, the neutral sector eigenvectors, labeled by $W^3, B, V^3, V^0,$
A3, are\textsuperscript{21}

\begin{align*}
\dot{\nu}_\gamma &= \left\{ \sin \theta_W \left[ 1 - 2x^2(1 + y_1^2) \sin^2 \theta_W \right], \cos \theta_W \left[ 1 - 2x^2(1 + y_1^2) \sin^2 \theta_W \right], 2x \sin \theta_W, 2xy_1 \sin \theta_W, 0 \right\}; \\
\dot{\nu}_Z &= \left\{ \cos \theta_W - \frac{x^2}{2 \cos \theta_W} \left[ 1 + \frac{2D^2}{B^2} - 4(1 - y_1^2) \sin^4 \theta_W \right], \\
& \quad - \sin \theta_W \left[ 1 + \frac{x^2}{2 \cos^2 \theta_W} \left( \cos 2\theta_W(1 + 2 \cos^2 \theta_W) - \frac{D^2}{B^2} - 4y_1^2 \sin^2 \theta_W(1 + \cos^2 \theta_W) \right) \right], \\
& \quad x \cos \theta_W, -2xy_1 \sin \theta_W \tan \theta_W, -\frac{Dx}{B \cos \theta_W} \right\}; \\
\dot{\nu}_{\rho_t} &= \left\{ -x(1 - 2y_1^2 \sin^4 \theta_W), -x \tan \theta_W \left[ 1 + 2y_1^2 \sin^2 \theta_W(1 + \cos^2 \theta_W) \right], \\
& \quad 1 - 2y_1^2 \sin^4 \theta_W - \frac{x^2}{2 \cos^2 \theta_W} \left( 1 - 2y_1^2 \sin^4 \theta_W(1 + 4 \cos 2\theta_W) \right), \\
& \quad 2(1 - 2x^2)y_1 \sin^2 \theta_W, -\frac{Dx^2}{2(c + d) \cos^2 \theta_W} \left( \cos 2\theta_W - 2(1 + 2 \cos^2 \theta_W)y_1^2 \sin^4 \theta_W \right) \right\}; \\
\dot{\nu}_{\omega_T} &= \left\{ 2xy_1 \sin^2 \theta_W, -xy_1 \sin 2\theta_W, -2y_1 \sin^2 \theta_W \left( 1 - \frac{x^2(1 + 2 \cos 2\theta_W)}{2 \cos^2 \theta_W} \right), \\
& \quad 1 - 2y_1^2 (\sin^4 \theta_W + x^2 \tan^2 \theta_W \cos 2\theta_W), -\frac{2Dx^2y_1 \sin^2 \theta_W}{c + d} \right\}; \\
\dot{\nu}_{\alpha_t} &= \left\{ \frac{Dx}{B}, -\frac{Dx \tan \theta_W}{B}, -\frac{bDx^2 \cos 2\theta_W}{2(c + d)B \cos^2 \theta_W}, \frac{bDx^2y_1 \tan^2 \theta_W}{(c + d)B}, 1 - \frac{D^2x^2}{2B^2 \cos^2 \theta_W} \right\}. \quad (83)
\end{align*}

Note that all “mixing angles” are of $\mathcal{O}(x^2)$, as would be expected from the mass matrices in Eqs. (17, 18), except for $\rho_T^a - \omega_T$. The reason is that their zeroth-order masses are equal so that the diagonalization of these two states is a problem in degenerate perturbation theory. The mixing between these two states vanishes entirely when $y_1 = 0$.

The shift fields $\zeta$ defined in Eq. (31) are given in terms of the non-canonically normalized $\tilde{\pi}_T$ by

\begin{align*}
\zeta_{W^\pm} &= \frac{2AF_1^2}{(BF_2^2 + AF_1^2)} \tilde{\pi}^\pm = 2 \sin^2 \chi \tilde{\pi}^\pm, \\
\zeta_{V^\pm} &= \sqrt{2} \sin^2 \chi \tilde{\pi}^\pm, \quad \zeta_{A^\pm} = -\sqrt{2} \left( \sin^2 \chi + \frac{C}{B} \cos^2 \chi \right) \tilde{\pi}^\pm; \\
\zeta_{W^3} &= 0, \quad \zeta_B = -2 \sin^2 \chi \tilde{\pi}_3, \quad \zeta_{V^3} = -\sqrt{2} \sin^2 \chi \tilde{\pi}_3, \quad \zeta_{V^0} = -2 \sqrt{2} y_1 \sin^2 \chi \tilde{\pi}_3, \\
\zeta_A &= -\sqrt{2} \left( \sin^2 \chi + \frac{C}{B} \cos^2 \chi \right) \tilde{\pi}_3, \quad \zeta_0 = -\frac{C}{\sqrt{2}B} \tilde{\pi}_0. \quad (84)
\end{align*}

\textsuperscript{21}The exact form of the massless photon eigenvector of $M_0^2$ is $\dot{\nu}_\gamma = (g' \kappa, g \kappa, 2xg' \kappa, 0, 2xy_1 g' \kappa)$ where $\kappa = \{(g^2 + g'^2)(1 + 4x^2(1 + y_1^2) \sin^2 \theta_W)\}^{-1/2}$. This form guarantees that the EM current of the standard-model fermions is proportional to $j_L^\mu + j_Y^\mu$.  

31
Appendix C: Adjustable Parameters in $\mathcal{L}_{\text{eff}}$ with Suggested Defaults

We present two schemes for choices of the adjustable parameters in $\mathcal{L}_{\text{eff}}$. The first is the more general and makes essentially no approximations. The second drops terms of $\mathcal{O}(x^2)$ in the technivector masses. In all cases, $F_\pi = \sqrt{F_2^2 + A/B F_1^2} = 2^{-1/4}G_F^{-1/2} \approx 246$ GeV is fixed.

**C.1 General Scheme for Parameters with $c+d \neq 0$**

While it would be convenient to use the technivector masses as inputs, this is is not practical if one wishes to keep the $\mathcal{O}(x^2)$ terms in their masses and assume that the $\mathcal{L}_\Sigma$ couplings $c,d$ are not very small; see Eqs. (28,30). In this case, we recommend the following choice of independent input parameters:

\[
\begin{align*}
a, b, c, d, f; \\
g_T, y_1, N_{TC}, \sin \chi = \sqrt{A/B} F_1 / F_\pi; \\
\text{quark and lepton masses (at } M_{\pi_T}) : m_{u_i}, m_{d_i}, m_{\ell_i}, \ i = 1, 2, 3; \\
\text{technipion masses } M_{\pi_T} \equiv M_{\pi_T^+} = M_{\pi_T^0} = M_1, \ M_{\pi_T^{\prime 0}} = \sqrt{M_1^2 + M_2^2}; \\
\pi_T \text{ couplings to quarks and leptons; see Eq. (37).}
\end{align*}
\]  

In terms of these, 
\[
A = aB + bc - \frac{1}{2} d^2, \quad B = b + 2(c + d), \quad C = 2c + d, \quad D = b + d = B - C, \quad x^2 = g^2 / 2 g_T^2, \quad F_1 = \sqrt{B / \bar{A} F_\pi} \sin \chi, \quad \text{and } g_{\rho_T \pi_T \pi_T} = b(B^2 + (f - 1) D^2) g_T / (4 \sqrt{2} A B).
\]

In general, one would experiment with these parameters to determine a set that gives the desired $\rho_T$, $\omega_T$ and $a_T$ masses. Suggested defaults, corresponding roughly to recently studied masses using Pythia (and assuming $a,b,d > 0$), are

\[
\begin{align*}
a &= b = f = 1, \quad d \approx \pm 2c = +0.10; \\
g_T &= \sqrt{8 \pi (2.16)(3 / N_{TC})} \text{ with } N_{TC} = 4; \\
y_1 &= 1 \text{ or } 0; \\
\sin \chi &= 1/3 \text{ (reasonably } 1/4 - 1/2) \\
\text{quark and lepton masses at } M_{\pi_T} \text{ as in Pythia} \\
M_{\pi_T} \equiv M_{\pi_T^+} = M_{\pi_T^0} = (1/2 - 2/3) M_{\rho_T}, \ M_{\pi_T^{\prime 0}} \gg M_{\pi_T^0}; \\
\pi_T \text{ couplings to quarks and leptons; see Eq. (37).}
\end{align*}
\]

These correspond to $M_{\rho_T} \approx M_{\omega_T} \approx 258$ GeV, $M_{a_T} \approx 294$ GeV, $F_1 = 84$ GeV, $g_{\rho_T \pi_T \pi_T} = 1.09$, $M_{V_1} = 390$ GeV, $M_{A_1} = 517$ GeV, $M_{A_2} = 437$ GeV and $S_1 = 0.18$ for $d = 2c = 0.10$, and to $M_{\rho_T} \approx M_{\omega_T} \approx 269$ GeV, $M_{a_T} \approx 282$ GeV, $F_1 = 84$ GeV, $g_{\rho_T \pi_T \pi_T} = 1.19$, $M_{V_1} = 360$ GeV, $M_{A_1} = M_{A_2} = 370$ GeV and $S_1 = 0$ for $d = -2c = 0.10$; in both cases, $\sin \chi = \frac{1}{3}$ and $y_1 = 1$. 

32
C.2 Scheme for Parameters with \( x^2 = 0 \) Masses and \( c + d \neq 0 \)

Since the technivector masses hardly depend on \( x^2 \) for “reasonable” values of \( g_T \), we can use them as inputs and solve for some \( \mathcal{L}_\Sigma \) parameters in terms of them. We recommend the following choice of independent input parameters:

\[
M_{\rho T} = M_{\omega T} = \frac{1}{2} g_T F_1 \sqrt{b}, \quad M_{a_T} = \frac{1}{2} g_T F_1 \sqrt{B};
\]

\( a, b, f; \)

\( g_T, y_1, N_{TC}, \sin \chi = \sqrt{A/B} F_1 / F_\pi; \)

quark and lepton masses (at \( M_{\pi_T} \): \( m_{u_i}, m_{d_i}, m_{\ell_i}, i = 1, 2, 3; \)

technipion masses \( M_{\pi_T} = M_1, \quad M_{\pi_T}^0 = \sqrt{M_1^2 + M_2^2}; \)

\( \pi_T \) couplings to quarks and leptons; see Eq. \( \text{(37)} \).

Solving for the other parameters, we obtain:

\[
F_1 = \frac{2M_{\rho T}}{g_T \sqrt{b}} = \sqrt{\frac{B}{A}} F_\pi \sin \chi;
\]

\[
B = \frac{M_{\omega T}^2 b}{M_{a T}^2} \implies A \equiv aB + bc - \frac{1}{2} d^2 = \left( \frac{b g_T M_{\omega T} F_1 \sin \chi}{2 M_{\rho T}} \right)^2;
\]

\[
c + d = \frac{1}{2} (B - b) = \frac{1}{2} b \left[ \left( \frac{M_{\omega T}}{M_{\pi_T}} \right)^2 - 1 \right];
\]

\[
bc - \frac{1}{2} d^2 = A - aB \implies d = -b \pm \sqrt{bB - 2(A - aB)}.
\]

(87)

To resolve the quadratic ambiguity, we always take \( b > 0 \) so that \( M_{\rho T}^2 > 0 \); then, so long as \( b \geq |d| \), so that \( D > 0 \), the positive square root is the correct solution. Once \( d \) is determined in this way, \( c = \frac{1}{2} (B + b) - \sqrt{bB - 2(A - aB)}. \)

A set of parameters, based on Case A in Ref. \[53\], is the following:

\[
M_{\rho T} = M_{\omega T} = 300 \text{ GeV}, \quad M_{a_T} = 330 \text{ GeV};
\]

\( a = b = f = 1; \)

\( g_T = \sqrt{8\pi(2.16)(3/N_{TC})} \) with \( N_{TC} = 4; \)

\( y_1 = 1 \) or \( 0; \)

\( \sin \chi = 1/3 \) (reasonably \( 1/4 - 1/2 \))

quark and lepton masses at \( M_{\pi_T} \) as in Pythia

\[ M_{\pi_T} \equiv M_{\pi_T^0} = M_{\pi_T^0} = (1/2 - 2/3) M_{\rho T}; \quad M_{\pi_T^0} \gtrsim M_{\rho T}; \]

\( \pi_T \) couplings to quarks and leptons; see Eq. \( \text{(37)} \).

(89)

Taking \( \sin \chi = \frac{1}{3}; \) these lead to \( F_1 = 94 \text{ GeV}, B = 1.21, A = 0.92, d = 0.34, c = -0.23, D = 1.34, C = -0.13, g_{\rho T \pi_T \pi_T} = 1.48, M_{V_1} = 340 \text{ GeV}, M_{A_1} = 303 \text{ GeV}, M_{A_2} = 335 \text{ GeV}\) and \( S_1 = -0.14. \)
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