Dynamic modeling of process support by the influence of various factors on the activities of firms in a competitive environment

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Abstract. The paper builds and explores a dynamic model of a company’s development under the influence of both negative and positive factors in a competitive environment. It is assumed that the head of the company is interested in obtaining the maximum profit from his company in a finite period of time and, acting on the basis of his interests, wants to find the optimal strategy. The task of finding the optimal policy for maximum profit is considered. Labor resources are considered as positive factors in the competitive environment.

1. Introduction
During the life of the company, it is constantly exposed to the external and internal environment. Impact could be either positive or negative. Environmental factors can be divided into direct and indirect factors. The first group of elements of external factors usually include consumers, competitors, suppliers, laws and state authorities, trade unions. The second group consists of the state of the economy, scientific and technological progress, political and socio-cultural factors, international events. The elements of internal factors of a firm typically include the structure, goals, objectives, technology and human resources [1-5].

2. Deterministic model of the influence of various factors on the activities of the company

2.1. Informal statement of the problem of constructing and analyzing a deterministic model
Let \( \Gamma = (N, X = \{X_i\}_{i=1}^{m}, t = \{0, 1, ..., n\}, U = \{u_i\}_{i=1}^{m}, R = \{r_i\}_{i=1}^{m} \) - dynamic decision-making game by the head of a firm with complete information, where \( N \) is the head of the firm, the decision maker, \( X \) is the set of all the states of the firm, a discrete value denoting time, \( U \) - control function that determines the transition from state \( X_i \) to state \( X_{i+1} \), \( U_{i+1} : X_i \rightarrow X_{i+1} \), \( R \) rational function defined
on the set of all controls. Depending on what decision is made (management at the current stage) the development of the company occurs according to one or another scheme, which implies the action of the complex, as positive factors $\alpha_i$, and negative factors $\beta_i$ on the state of the company. Among the controls, leaving the point $X$ is required to choose a control with the maximum total capitalization. Capitalization is a process in which, under the influence of certain factors, an increase in the money supply occurs due to their investment in any active income-generating volumes [6-7].

2.2. Formalization of the task of constructing and analyzing a deterministic model
At the initial stage, the head of the company, being able, in a state $X_0$ to make a management decision $u_1 \in U_1$ etc., chooses one of the possible directions at the current stage. After that, the company moves to the next state $X_1$. Then, the firm begins to influence both external and internal factors that have either a positive or negative impact. Therefore, it is possible either to increase or decrease the capitalization of the company.

For example, hiring highly skilled workers leads to an increase in production. In turn, this event can lead to several outcomes. The most favorable of these outcomes is an increase in profits.

In the second step, the head again takes a management decision $u_2 \in U_2$ that chooses a control from a variety of possibilities. The company goes into the next state - $X_2$. After that, the firm begins to influence both external and internal factors that have either a positive or negative impact, etc.

In the step $n$ leader chooses $u_n \in U_n$ one of the possible controls at the current stage. After which the company goes into the final state. In the transition from the state $X_i$ to the state $X_{i+1}$, there is a growth or decline in the capitalization of the company. In the final state $X_n$ firm capitalization is defined as the sum of capitalizations at each previous step.

Thus, the problem will be solved when the capitalization of the company is determined for each end point and the maximum of them is found [8-10].

3. Stochastic model of the influence of negative and positive factors on the activities of the company

3.1. Informal statement of the problem of constructing and analyzing a stochastic model
Let $\Gamma = \Gamma \{N, X = \{X_i\}_{i=1}^m | t = \{0,1, ..., n\}, U = \{u_i\}_{i=1}^m, P = \{p_i\}_{i=1}^m, R = \{r_i\}_{i=1}^m\}$. The decision-making game by the owner of the company, where $N$ - head of the company, decision maker, $X$ - the set of all states of the company, $t$ - discrete value, time mark, determines the step number, $U$ - control function, defining transition from state $X_i$ to the state $X_{i+1}$, $U_{i+1} : X_i \rightarrow X_{i+1}$, $R$ - rational function, given on the set of all controls. The sum of the values of this function in the management is called the total capitalization of the company, $p_i$ transition probabilities from state $X_i$ to state $X_{i+1}$.

Among the controls, leaving the point $X$ is required to choose a control with the maximum total capitalization [11-12].

3.2. Formalization of the task of constructing and analyzing a stochastic model
The model is described as follows. At the initial stage, head of the company, decision maker, being able, in a state $X_0$ to make a decision Then, the firm is beginning to be affected by both external and internal factors, which have either a positive or a negative effect. The manager has information that the events of the second step from the set $X_2$ may occur with a certain probability $P$. The sum of the probabilities of events at each stage is 1. The manager knows the possible outcomes, their likelihood,
whether there will be a rise or fall in the capitalization of the company, depending on its decision, and by what amount. The uncertainty lies in the fact that the manager cannot determine what influence external factors will have on the choice of a particular decision [13-15].

4. Probabilistic - deterministic model of the influence of negative and positive factors on the activities of the company

4.1. Informal statement of the problem of constructing and analyzing a probabilistic - deterministic model

Let $\Gamma = \{N, X = \{X_i\}_{i=1}^n, t = \{0,1,...,n\}, U = \{u_i\}_{i=1}^n, R = \{r_k\}_{k=1}^c\}$ - dynamic decision-making game by the owner of the company, where $N$ - head of the company, decision maker, $X$ - the set of all states of the company, $t$ - discrete value, time mark, determines the step number, $U$ - control function, defining transition from state $X_i$ to the state $X_{i+1}$, $U_{i+1} : X_i \rightarrow X_{i+1}$, $R$ - rational function, given on the set of all controls.

4.2. Formalization of the problem of constructing and analyzing a probabilistic - deterministic model

Now suppose that the choice of control at the point $X$ determines not a state, but only a probability of distribution for this state.

Let $X_i$ and $U_i$ - arbitrary finite sets. Each $u$ of the $U_i$ mapped a probability of distribution $p(\cdot | u)$ on $X_i$. The function $p$ defining the law of transition from $U_i$ to $X_i$ will be called the transition function. It is natural to assume that the point of the set $X_0$ from which the game begins is also random, and its probability distribution $\mu$ (initial distribution) is given.

The transition from $x \in X_{i-1}$ to $U_i$, but from its subset $U(x)$, depending on the state $x$. The elements of a set $U(x)$ will be called controls at a point $x$. Sets $U(x)$ are defined and not empty for all non-final states $x$. It is assumed that the pairs $U(x)$ do not overlap and their sum over all $x \in X_{i-1}$ is equal to $U_i$. In other words, each control $u$ can be used in one and only one state. We denote this state $j(u)$, so the record $x = j(u)$ is equivalent to a record $u \in U(x)$. The set of all controls is set to current increase or decrease in firm capitalization $r(u)$, and on the set of final states - final growth or decline in firm capitalization $r(x)$.

The model is described as follows. At the initial moment of time the manager - the manager of the company - makes a decision, chooses the management - the set of transition probabilities at the current stage. Then the second participant - the employee chooses the next state. For the transition from state 0 to state 1, the manager receives a certain capitalization of the company. The purpose of the model is to obtain the maximum capitalization of the company [16-18].

Consider an example of the study of the influence of negative and positive factors on the activities of the company using a probabilistic-deterministic model. In Figure 1, the column $U_1$ shows three probability distributions on the set $X_1$, corresponding to the three controls leading from $X_0$. The column $U_2$ indicates five probability distributions on the set $X_2$ that correspond to the controls from the state $X_1$. The problem will be solved if we find the management with the maximum capitalization of the company gain [19-22].

$$X_0 \quad U_1 \quad X_1 \quad U_2 \quad X_2$$
In state III, the mathematical expectation is equal to: \( 0 + \frac{1}{5} \times 1 + \frac{4}{5} \times 2 = 0 + \frac{9}{5} = \frac{9}{5} \), when choosing the first control equals: \( 1 + \frac{2}{3} \times 1 + \frac{1}{3} \times 2 = 1 + \frac{4}{3} = \frac{7}{3} \), when choosing a second control, the estimate of state III is equal to the maximum of these two numbers. i.e. \( \frac{7}{3} \) and it is clear that in state III one should prefer the second control.

Similarly: \( v(IV) = \max(2 + \frac{2}{3} \times 1 + \frac{1}{3} \times 2; 1 + \frac{2}{5} \times 1 + \frac{3}{5} \times 2) = \max(2 + \frac{4}{3}; 1 + \frac{8}{5}) = \frac{10}{3} \), and in state IV, the first control is preferable.

\( v(V) = \max(3 + 1 \times 1 + 0 \times 2; 2 + \frac{1}{2} \times 1 + \frac{1}{2} \times 2) = \max(3 + 1; 2 + \frac{2}{3}) = 4 \).

**Figure 1.** Decision Tree Diagram.
In state V, the first control is more profitable. Next, choosing the first control in state I, and then, acting in an optimal way, we obtain an estimate: 

\[ 1 + \frac{1}{3} \times \frac{7}{3} + \frac{1}{2} \times \frac{10}{3} + \frac{1}{6} \times 4 = 1 + \frac{28}{9} = 4 \frac{1}{9}, \]

and choosing the second control:

\[ 0 + 0 \times \frac{7}{3} + \frac{1}{2} \times \frac{10}{3} + \frac{1}{2} \times 4 = 0 + \frac{11}{3} = 3 \frac{2}{3}. \]

![Figure 2. Final control states.](image)

The maximum of these two numbers is - \( \nu(I) \). In state I, you need to choose the arrow that led to the value \( \nu(I) \), i.e. first management. The controls selected in each non-final state in figure 2 describe the optimal mode of behavior.

5. Conclusion

Thus, the presented models allow us to find the optimal policy of the company for obtaining maximum profit under the influence of both negative and positive factors in a competitive environment. We look for the maximum of these two numbers. Initial moment of time the entrepreneur-owner

Acknowledgment

The work is partially supported by the RFBR grant # 18-01-00796.

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