DISTRIBUTED DYNAMIC Platoons CONTROL AND JUNCTION CROSSING OPTIMIZATION FOR MIXED TRAFFIC FLOW IN SMART CITIES- PART II. STABILITY, OPTIMIZATION, AND PERFORMANCE ANALYSIS

Bohui Wang, Senior Member, IEEE, Rong Su, Senior Member, IEEE *

August 30, 2022

*This research is supported by A*STAR under its RIE2020 Advanced Manufacturing and Engineering (AME) Industry Alignment Fund C Pre Positioning (IAF-PP) (Award A19D6a0053).

†B. Wang and R. Su are with the Department of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 (e-mail: bhwang@ntu.edu.sg; rsu@ntu.edu.sg)
ABSTRACT

In part I, we have shown that despite the nonlinearity, non-smoothness, and uncertainty of mixed traffic flows in smart cities, the cooperative driving solution of safe, efficient, and fuel economic platoon management and junction crossing problem can be solved by our proposed automatic decision and smart crossing assistant system (ADSCAS), where the dynamic platoon size determination, the reference trajectory planning, and the fuel economic acceleration adjusting problems have been considered. In part II, we present a fully distributed nonlinear variable time headway space strategy to ensure the subsequent safe cruising and junction crossing, where the cooperative perception of multiple neighbors stimuli and the cooperative tracking of the follower connected automated vehicles (CAVs) to the leader CAV are developed, which will result in a heterogeneous traffic flow dynamic. Once the proper length of the platoon determined by the ADSCAS characterized in part I is formed, we propose a cooperative observer design to estimate the leader CAV’s acceleration adjustment which is affected by the unknown traffic lights. We shall show that the distributed and resilient nonlinear platoons control and junction crossing problem will be solved by a robust cooperative trajectory tracking optimization algorithm to ensure the fast formation and split of the platoons and safe junction cruising within the finite time horizons, taking into account the social driving behaviors (SDBs) of the surrounding vehicles (SVs), the dynamics of the follower CAVs, and an upcoming traffic signal schedule while minimizing the overall platoons fuel consumption. Performance analysis and case studies are presented to illustrate the effectiveness of the proposed approaches for multiple platoon dynamic management, which also show that the cooperation between CAVs and human-driven vehicles (HDVs) can further smooth out the driving trajectory, reduce the fuel consumption, and enhance the safety of the mixed traffic flow.

Index Terms—Connected automated vehicles (CAVs); mixed traffic flow; cooperative driving; social driving behaviors; optimization; distributed control; car-following model; formation control.

I. Introduction

In [1], we have concluded that vehicle platoon into land-based transportation has great benefits to carbonizing the European and global economy. Considering the expected increase in the oil and natural gas price and the need for maintaining the competitiveness of saving energy, developing new vehicle platoon technologies is a growing demand to get increasingly fuel-efficient, which will greatly promote the sustainable development of smart cities. For example, the fuel cost for a truck platoon could account roughly for one-third of the total cost of operating a heavy-duty vehicle, where a reduction of a few percent of the fuel consumption would lead to significant savings [2]. Therefore, a vehicle platoon is an effective method to reduce fuel consumption and greenhouse gas emissions.

In particular, vehicle platoon has been studied extensively over the past few years. The early works mainly are the theoretical study of the dynamics of a string of vehicles, i.e., string stability, focusing on the vehicle’s stability over space under the influence of a small perturbation originating from the first vehicle of the platoon. The typical work is the adaptive cruise control (ACC) [3] which only relies on standard onboard sensors to enhance driving safety. Then, cooperative adaptive cruise control (CACC) [4] tries to create a smaller headway distance by combining wireless communication and radar measurements. Considering the dynamic characteristics of vehicle information interaction, the bidirectional interaction topology policy [5] and the vehicle look-ahead topology policy [6] were proposed to establish a more flexible platoon model. With the advancement of consensus theory, the cooperative platoon control framework that is different from the string stability has been recently developed for connected automated vehicles (CAVs) [7], [8], which is a dynamic system and tends to design flexible communication topology and control protocol to improve the practicability by using Lyapunov stability analysis. Notably, although ACC and CACC can provide a simple control framework for CAVs control, the inter-CAV communication relies on a fixed communication pattern; cooperative platoon control can flexibly schedule communication resources, but all the CAVs controllers are based on linearized models with a combination of the trajectory, velocity, and acceleration of vehicles, which cannot fundamentally reflect the nonlinear relationship between actual driving behaviors and external stimuli. In other words, the above works only focus on stability characteristics of platoon systems under the influence of a small perturbation, while the large perturbation, such as sudden stop/slow-and-go decision of the leader CAV in the traffic congestion which is a very general character in a smart city, cannot be handled smoothly.

Notably, a car-following model as a nonlinear model, called “Optimal Velocity Model” (OVM) [9]–[11], can perfectly describe how one driver responds to a single stimulus from surrounding vehicles (SVs) by establishing a perception mechanism of CAVs. Although the above studies are very helpful in understanding the driving behaviors of CAVs, the car-following model only responds to a single stimulus without exploring cooperative behaviors, while multiple surrounding stimuli and the driver’s driving sensitivity could affect the platoon’s safety. We have shown an integrated approach in [12] that takes full advantage of cooperative control theory to handle multiple peripheral stimuli for purse CAVs in a perfect environment, but it cannot be applied to mixed traffic flow in a smart city where human-driven vehicles (HDVs) and traffic lights exist. Notably, CAVs and HDVs will coexist on urban roads, creating a competitive and mixed traffic flow environment, in which the platoon CAVs would inevitably encounter the free-cruising and smart crossing procedures (SCP’s) by the HDVs in neighboring roads or junctions,
which brings big challenges for dynamic platoon management and smart junction crossing of CAVs in a mixed traffic flow.

Whatever the CAVs models, the CAVs platoon aims to ensure that all CAVs remain desired speed and platoon spacing, where the platoon spacing strategy is one of the important components to ensure the safe following gap between successive CAVs. Generally, there are mainly three classes of space error strategies for platoon control. The first one is the constant spacing strategy, in which the desired platoon spacing is considered as a fixed constant [13], [14]. The second one is the constant time headway strategy, in which the platoon gap is designed as a linear function of the relative CAV’s speed [15], [16]. Notably, the two above space error strategies could cause a driving risk and low road utilization for insufficient platoon space and low cruising speed, respectively. Specifically, for complex traffic scenarios such as frequent acceleration or deceleration of the preceding vehicle, such two strategies may not ensure safety. The third one is the variable time headway strategy, in which a nonlinear function of the speed is considered to improve the flexibility of the platoon dynamics. The research [17] is shown that the variable time headway strategy based on the driver’s driving parameters has better traffic capacity and lower energy consumption than the constant time headway strategy. Based on this view, a new consensus-based control approach with the input saturation and variable time headway spacing strategy was proposed in [18]. Furthermore, a novel distributed cooperative approach for fixed platoon consisting of connected and automated vehicles and human-driven vehicles was addressed in [19]. However, the variable time headway strategy in the above works is based on the global information of the leader CAV’s speed and the acceleration of the leader CAV must be zero, which is a very ideal condition so cannot be applicable to the complex traffic scenarios, i.e., the CAVs should adjust the acceleration to respond to the social driving behaviors (SDBs) of the preceding HDVs and upcoming traffic lights in a smart city.

The aim of this article is to propose a distributed dynamic and resilient platoon control for the mixed traffic flows to respond to the SDBs of the HDVs and upcoming traffic lights while ensuring platoon safety and junction crossing effectiveness within a finite time horizon.

Our method has been carried out in general platoon management and automatic decision framework that fits with the decision studied in part I [1]. Compared with the existing methods [20]–[22] that address a simple scenario, we could formulate a full process automatic driving decision strategy by considering multiple platoons CAVs and the interaction of SDBs of the HDVs, making it more consistent with actual traffic conditions. Upon this, we first consider a fully distributed variable time headway space error strategy to respond to the SDBs of HDVs. Different from the existing works [18], [19], our space error strategy is fully distributed and allows a time-varying acceleration adjusting. Furthermore, we consider a distributed observer of the leader CAV’s acceleration which is derived from the safe and economic requirement of the SDBs of the preceding HDVs and the smart junction crossing for the upcoming traffic lights. Compared with the traditional ACC [3], CACC control [4] and cooperative platoon control [7], [8], we employ the cooperative perception, distributed observers, cooperative information interaction, and difference velocity models to dynamically adjust the inter-and exter-platoon gaps to improve the dynamic platoon performance and reduce the energy consumption and the influence of SDBs for the HDVs. Finally, we solve a robust cooperative trajectory tracking optimization problem within a finite time to ensure safe and efficient platoon control and junction crossing. Unlike existing works using ACC-based car-following models in an infinite horizon [23], [24], the driving safety and fuel efficiency of a fleet of CAVs with a reaction time delay in our framework are specifically addressed by designing a cooperative trajectory tracking optimization algorithm to ensure the dynamic platoon can be formed within a finite time.

Notably, our proposed control strategy may potentially become part of a broader fuel-economic solution for the effective management of CAV platoons in a large urban traffic network because we have shown in [1] that despite the nonlinear and nonsmooth character of complex traffic dynamics, the distributed dynamic platoons management and decisions are comprised of the reference trajectories planning and economic acceleration adjusting that can solve by a robust cooperative trajectory tracking optimization problem within a finite time. In this article, we shall show that the platoon dynamics under our proposed fully distributed variable time headway space error strategy will be more complex, where the heterogeneous dynamics will rely on the local velocity and acceleration of follower CAVs such that the stability analysis will be different from the works [7], [8], [12], [25], [26]. Specifically, for such a third-order system, the traffic flow stability analysis of multiple platoons for the existing methods [27], [28] is quite cumbersome, even not possible to derive the analytical stability results for the third-order system. To the best of our knowledge, the feasibility of this notion has not been explicitly discussed in the scenario of signalized intersections. To this end, we will also focus on the traffic flow stability of the entire mixed platoon system consisting of both the CAV and its preceding HDVs and seek to improve the overall performance via a distributed control and optimization framework for the CAVs only.

It should be noted that the methodology used for the derivations in the two-part article could be of independent technical interest. In part I [1], the automatic decision and smart crossing assistant system (ADSCAS) provide a cooperative driving decision that includes all the platoon cruising states and smart junction crossing procedures under the influence of preceding SDBs of the HDVs, which can be used to analyze all the smart junction problems for a single CAV or mixed traffic flows in a smart city. In part II, the distributed and resilient dynamic platoon control is carried out by developing a fully distributed variable time headway platoon space error policy, which is also applicable to the pure CAV control with a variable speed to characterize the general stability analysis of the platoon dynamics.

The part II article is organized as follows. Section II introduces the problem formulation and driver models that will be used within the article. Section III provides the fully distributed variable time headway space strategy to respond to the SDBs of the HDVs and upcoming traffic lights. Section IV proposes the stability analysis of dynamic platoon management strategy. Sec-
tion V presents a finite-time robust cooperative trajectory tracking optimization algorithm. Section VI illuminates the effectiveness of the proposed control framework with simulations. Finally, conclusions and discussions are given in Section VII.

II. Problem Formulation

We use the same cooperative driving mode and definitions as in part I of this article and we refer the reader to the preliminaries section therein. The cooperative driving mode including the OVM [9], [29], [30] and intelligent driver model (IDM) [31] with additional empirical driving parameters, will frequently be used throughout the article. Notably, the combination of the consensus-based platoon model (CBPM) and the OVM model is proposed to represent the dynamics of the platoon. For the sake of platoon control, the following third-order consensus-based platoon model (CBPM) and the OVM model is used to describe cooperative driving and perception of CAVs, and the IDM model is used to simulate the SDBs of the HDVs, respectively.

A. CBPM-OVM model

For all platoons, the dynamic of each CAV is inherently nonlinear, which is affected by the engine, brake system, aerodynamics drag, rolling resistance, gravitational force, etc. [32]. Based on some reasonable assumptions as made in [33], [34], we introduce the following first-order longitudinal dynamics of the acceleration for a group of follower CAVs to describe the reaction-time delay of the changes of dynamics of preceding CAVs

\[
\tau \hat{a}_{p,i}(t) + a_{p,i}(t) = u_{p,i}(t) + y_{p,i}(t) + g_{p,i}(t), \quad i \in F, p \in M,
\]

where \( a_{p,i}(t) \) is the acceleration of the \( i \)-th CAV for the \( p \)-th platoon, \( \tau \) is the reaction time delay, \( F = \{1, \ldots, N\} \) and \( M = \{1, \ldots, m\} \) are two index sets, \( u_{p,i}(t) \) is the cooperative control input, and \( y_{p,i}(t) \) is a car-following model which represents the optimal speed for CAVs with an adjustable sensitivity, and \( g_{p,i}(t) \) is a difference velocity model which reflects the difference error between the current CAV and its neighboring CAVs for the \( p \)-th platoon. For the sake of platoon control, following the third-order model is proposed to represent the dynamics of the \( i \)-th Follower CAV in the \( p \)-th platoon

\[
\dot{q}_{p,i}(t) = v_{p,i}(t),
\]

\[
v_{p,i}(t) = a_{p,i}(t),
\]

\[
\tau \dot{a}_{p,i}(t) + a_{p,i}(t) = u_{p,i}(t) + y_{p,i}(t) + g_{p,i}(t), \quad i \in F, p \in M,
\]

\[
y_{p,i}(t) = \tilde{\alpha} \sum_{j=0}^{N} \alpha_{ij}^p \left[ V_{p,i}(y_{p,i}(t)) - v_{p,i}(t) \right],
\]

\[
g_{p,i}(t) = \tilde{\alpha} \sum_{j=0}^{N} \alpha_{ij}^p \left( v_{p,j}(t) - v_{p,i}(t) \right), \quad i \in F, p \in M,
\]

where \( y_{p,i}(t) \) and \( v_{p,i}(t) \) are the position and velocity of the \( i \)-th CAV for the \( p \)-th platoon, \( \tilde{\alpha} \) and \( \tilde{\alpha} \) are the sensitivity constants, \( \alpha_{ij}^p \) is the adjacency weight of the inter-vehicle communication which will be defined later, and \( V_{p,i}(y_{p,i}(t)) \) is the nonlinear reaction function, also named as “Optimal Velocity Function (OVF)” [9], [29], [30], to capture the interactions between CAVs

We aim to design distributed control and optimization methods to ensure smart crossing and efficient platoon management with the reduction of the influence of different SDBs for the HDVs, which requires that the dynamics of the Leader CAV can be programmable based on the perception information and observed the trajectory of HDVs. Then, the following dynamics of the Leader CAV are proposed

\[
\dot{q}_{p,0}(t) = v_{p,0}(t),
\]

\[
v_{p,0}(t) = a_{p,0}(t),
\]

\[
\tau \dot{a}_{p,0}(t) + a_{p,0}(t) = f(AutoC(t), u_{p,0}(t)),
\]

where \( q_{p,0}(t) \), \( v_{p,0}(t) \) and \( a_{p,0}(t) \) are the position, velocity, and acceleration for the Leader CAV 0 in the \( p \)-th platoon, respectively, \( f(AutoC(t), u_{p,0}(t)) \) is a nonlinear regular function including a finite state machine function AutoC(t) and a planning demand \( u_{p,0}(t) \) which represents the braking or driving dynamics of the Leader CAV to describe the emergent acceleration, deceleration or constant speed to respond the traffic lights and the HDVs.

B. Extended IDM model

The IDM can describe car-following behaviors of manual driving accurately. Specifically, the acceleration assumed in the IDM is a continuous function of the velocity \( v_{h,i}(t) \), the gap \( s_{h,i}(t) \) between the \( i \)-th Target vehicle and the preceding vehicle, and the velocity difference (approach rate) \( \Delta v_{h,i}(t) \) to the preceding vehicle [31], [35], which is expressed as

\[
\dot{v}_{h,i}(t) = a_{h,\text{max}} \left[ 1 - \left( \frac{v_{h,i}(t)}{v_f} \right)^\delta - \left( \frac{s^*(v_{h,i}(t), \Delta v_{h,i}(t))}{s_{h,i}(t)} \right)^\gamma \right],
\]

\[
\Delta v_{h,i}(t) = v_{h,i}(t) - v_{h,i}(t-1),
\]

\[
\gamma = \frac{\Delta v_{h,i}(t)}{v_{h,i}(t)},
\]

where \( a_{h,\text{max}} \) is the maximum acceleration of the human driver, \( v_f \) is the free speed, \( s^*(\cdot) \) is the optimal braking function, \( s_{h,i}(t) \) is the bumper-to-bumper distance, \( \delta \) is the velocity decrease factor, and \( \gamma \) is the velocity difference factor. The parameters are shown in Table I.

| PARAMETERS CHOSEN FOR IDM | \( \text{parameters} \) | \( \text{value} \) |
|---------------------------|--------------------------|
| Vehicle length \( l_c \)  | 5m                       |
| Gap error sensitivity \( \tilde{\alpha} \) | 0.5s\(^{-1}\) |
| Speed difference sensitivity \( \tilde{\alpha} \) | 0.05s\(^{-2}\) |
| \( D_1 \)                 | 6.75m/s                  |
| \( D_2 \)                 | 7.91m/s                  |
| \( D_3 \)                 | 0.13m/s                  |
| \( D_4 \)                 | 1.59                     |

TABLE I
This expression is an interpolation of the tendency to accelerate with
\[ a_f(h_{h,i}(t)) = a_{h,max}[1 - \frac{(v_{h,i}(t))^\delta}{v_f}], \quad (8)\]
on a free road and the tendency to brake with deceleration
\[ b_{int}(s_{h,i}(t), v_{h,i}(t), \Delta v_{h,i}(t)) = -a_{h,max} \left( \frac{s^*(v_{h,i}(t), \Delta v_{h,i}(t))}{s_{h,i}(t)} \right), \quad (9)\]
when the HDV \( i \) comes too close to the vehicle in front. The deceleration term depends on the ratio between the “desired minimum gap” \( s^* \) and the actual gap \( s_{h,i}(t) \), where the desired gap varies dynamically with the velocity and the approach rate, which is defined as follows:
\[ s^*(v_{h,i}(t), \Delta v_{h,i}(t)) = s_0 + s_1 \sqrt{\frac{v_{h,i}(t)}{v_f}} + T_v v_{h,i}(t) + \frac{v_{h,i}(t)\Delta v_{h,i}(t)}{2a_{h,max}b_f}, \quad (10)\]
where \( \delta > 0 \) is the acceleration component, \( s_0 \) is the minimum safety distance and \( s_1 \) is the jam distance, \( T_v \) is the safety time headway, \( v_f \) and \( b_f \) are the desired velocity and deceleration. The actual gap \( s_{h,i}(t) \) is defined as follows:
\[ s_{h,i}(t) = h_{i}(t) - l_c, \quad (11)\]
where \( h_{i}(t) \) is the space headway between the preceding and the Target vehicle and \( l_c \) is the longitudinal length of the vehicle.

We here consider an extended IDM model (EIDM) as follows:
\[ \dot{q}_{h,i}(t) = v_{h,i}, \]
\[ \dot{v}_{h,i}(t) = a_{h,i}(t), \]
\[ \tau_h a_{h,i}(t) + a_{h,i}(t) = a_{h,max} \left[ 1 - \left( \frac{v_{h,i}(t)}{v_f} \right)^\delta \right. \]
\[ \left. - \frac{s^*(v_{h,i}(t), \Delta v_{h,i}(t))}{s_{h,i}(t)} \right]^2. \quad (12)\]
The parameters are shown in Table II [31], [35].

| Parameters chosen for IDM | Value |
|---------------------------|-------|
| Desired velocity \( v_f \) | 120 km/h |
| Maximum acceleration \( a_{h,max} \) | 1 m/s² |
| Desired deceleration \( b_f \) | 2 m/s² |
| Acceleration exponent \( \delta \) | 4 |
| Jam distance \( s_0 \) | 2 m |
| Jam distance \( s_1 \) | 0 m |
| Vehicle length \( l_c \) | 5 m |
| Reaction delay \( \tau_h \) | 0.3 s |

The velocity and the acceleration of all the vehicles are bounded by the following constraints
\[ v_{p,min} \leq v_{p,i}(t) \leq v_{p,max}, \quad (13)\]
\[ a_{p,min} \leq a_{p,i}(t) \leq a_{p,max}, \quad (14)\]
\[ v_{h,min} \leq v_{h,i}(t) \leq v_{h,max}, \quad (15)\]
\[ a_{h,min} \leq a_{h,i}(t) \leq a_{h,max}, \quad (16)\]
where \( v_{p,min}, v_{p,max}, a_{p,min} \) and \( a_{p,max} \) are the minimum and maximum speed and acceleration limits for all the CAVs, and \( v_{h,min} v_{h,max}, a_{h,min} \) and \( a_{h,max} \) are the minimum and maximum speed and acceleration limits for the HDVs, respectively.

C. Economic acceleration adjusting scheme

According to the part I [1], considering cooperative driving between the CAVs and the SDBs of the HDVs by using the CBPM-OVM model and Extended IDM model, the finite state machine function AutoC(t) in the ADSCAS can be designed by
\[ S0 : C-I, Case-III \land Case-IV, \]
\[ S1 : C-I \implies C-II, Case-V, \]
\[ S2 : C-II \implies C-III, Case-VI \land Case-VII \land Case-VIII, \]
\[ S3 : C-III \implies C-IV, Case-IX \land Case-X, \]
\[ S4 : C-IV \implies C-V, Case-XI \land Case-XII \]
\[ S5 : C-V \implies C-VI, Case-XIII \land Case-XIV \]
where “\( \implies \)” represent the switching scheme. In our ADSCAS, if the platoon is traveling with a constant velocity \( v_{p,c} \) under C-I, the acceleration of the Leader CAV will be zero. Our aim is to adjust the platoon velocity for a given time domain \([t^-, t^+]\) to ensure that the platoon reaches the planning demands AutoC(t) to accomplish the SCPs of multiple platoons and ensure safety and green. In general, we can use the maximum acceleration to adjust the traveling velocity to reach the reference velocity under C-II - C-VI. However, it may cause a risk to driving safety due to the existing possible SDBs of the HDVs. Therefore, we will try to design an optimal regulation time \( T \), where \( T \leq t_{++} \) with \( t_{++} = t^+ - t^- \) being a time interval between any two former and later switching moments, which is associated with the reference velocity adjustments \( v_{p,ref}^r(t) \) and \( v_{p,ref}^{int}(t) \) characterized in part I [1] and the stability of the platoon with the reference trajectory under C-II - C-IV in a finite time. Notably, \( T \) is free for the Cruising state because the Leader CAV will maintain the constant cruising velocity under C-I. For other cases, since the Leader CAV adjusts the status first under C-II - C-VI and then all the Follow vehicles track the preceding CAVs, the option of the finite time \( T \) will be the key factor to ensure the safety, economy, and comfortability of the platoon tracking and the SCPs with respect to the underlying SDBs of the HDVs. Therefore, the following problem and theory given in part I [1] will be recalled.
There exists two positive constants \( \rho^{**} \) and \( T^{*} \) with respect to the acceleration and velocity limits within the period \([t^*, t^* + \rho] \). If \( \rho^{**} \leq T^{*} \), the minimum fuel consumption \( J^u \) and the function of \( u_{p,0}(t) \) for the Problem 1 will be optimally determined.

For the case \( \rho^{**} > T^{*} \), the Leader CAV needs to use a constant acceleration \( \frac{v_{p,ref}(t) - v_{p,nil}(t)}{\rho^{**}} \) to reach the reference velocity \( v_{p,ref}(t) \) under C-II - C-IV as an economical acceleration adjusting under the sufficient regulation time \( \rho^{**} \) that satisfies

\[
\rho^{**} < \epsilon \rho^{**} \leq T^{*}, \quad 1 < \epsilon = \frac{T^{*}}{\rho^{**}},
\]

where \( \epsilon > 0 \) is a regulation parameter to value the compromise between driving safety and energy consumption, and it generally takes the value \((1, 10)\) for the comfort level of driving into account.

Notably, when the Leader CAV achieves the planning demands within the period \([t^*, t^* + \rho] \), all the Follower CAVs will need to form the platoon and ensure the safe crossing distance in a finite time \( T^{**} \). Therefore, the control horizon will need to be satisfied in the following constraints

\[
\begin{align*}
\rho^{**} < T^{**} &= \text{Free}, \quad \text{SET-0}, \\
\rho^{**} < T^{**} - t^{**} &= \text{SET-1}, \\
\rho^{**} < T^{**} - t^{**} < TT &= \text{SET-2}, \\
TT &= \begin{cases} 
    t_{g1}^{*}, & \text{if } \sigma = 1 \land \text{Case-I} \land (Q_{p,0}(t_{g1}^{*}) - t_{g1}^{*}) \leq v_{p,max}; \\
    t_{g2}^{*}, & \text{otherwise},
\end{cases}
\end{align*}
\]

\[
T_{g1}^{*} = K_{w} t_{cycle} - t_{r} - t - \rho^{**},
\]
\[
T_{g2}^{*} = (K_{w} + 1) t_{cycle} - t_{r} - t - \rho^{**},
\]

where \( t^{**} \) and \( t^{***} \) are two small adjustment parameters to ensure the safe crossing, and the first case is for the STATE 1-Free-cruising and the second case covers the STATE 2-Tracking-cruising, STATE 3-Platoon dynamic forming, STATE 5-Safe-check control, and STATE 6-Traffic flow control, and the third case is for the STATE 4-Smart crossing control within the upcoming traffic timing, respectively.

Then, the economic acceleration adjusting scheme is selected by a constant value during the adjustment period \([0, \rho^{**}] \) and zero in the non-adjustment period \([\rho^{**}, T^{*}] \). With the dynamic \( s(t) \), the acceleration profile of the Leader CAV under C-II - C-IV can be designed by

\[
\begin{align*}
J^u &= \int_{t^-}^{t^+} (a_{p,0}(t))^2 dt, \\
\text{Subject to} & : v_{p,ref}(t) + \int_{t^-}^{t^+} a_{p,0}(\tau)d\tau = v_{p,ref}(t),
\end{align*}
\]

where \( v_{p,ref}(t) \) and \( v_{p,ref}(t) \) are the vehicle’s reference velocities, and the function \( J^u \) captures the energy consumption.

**Theorem 2:** There exists two positive constants \( \rho^{**} \) and \( T^{*} \) with respect to the acceleration and velocity limits within the period \([t^*, t^* + \rho] \). If \( \rho^{**} \leq T^{*} \), the minimum fuel consumption \( J^u \) and the function of \( u_{p,0}(t) \) for the Problem 1 will be optimally determined.

**Problem 1 (Fuel economic optimization problem):**

\[
\begin{align*}
\text{min } J^u &= \int_{t^-}^{t^+} (a_{p,0}(t))^2 dt, \\
\text{Subject to} & : v_{p,ref}(t) + \int_{t^-}^{t^+} a_{p,0}(\tau)d\tau = v_{p,ref}(t),
\end{align*}
\]

where \( v_{p,ref}(t) \) and \( v_{p,ref}(t) \) are the vehicle’s reference velocities, and the function \( J^u \) captures the energy consumption.

A fully distributed variable time headway spacing strategy

Notably, the control execution varies depending on the role of the current vehicle. For the STATE 1-Free-cruising, the control action aims to form the platoon with a constant cruising speed due to the large external-space distance with respect to the HDVs. For the STATE 2-Tracking-cruising, the control action will need to maintain the following safe external-space distance with respect to the specific HDV. The main difference of the control actions among the STATE 3-Platoon dynamic forming, and STATE 4-Smart crossing control, STATE 5-Safe-check control, and STATE 6-Traffic flow control is different reference trajectory planning for the Leader CAV due to different roles of the Target vehicles.
and upcoming traffic lights. Then, letting $N = \tilde{N} - 1$ as the number of the **Following CAVs** and 0 as the **Leader CAV**, the following two criteria are proposed to design the cooperative controller for the Follower CAVs in different driving scenarios, by introducing a fully distributed variable time headway (FDVTH) spacing strategy

**Criterion 3 (C-V):** For the **Leader CAV** having the constant cruising velocity under C-I, the following controllers are designed for the **STATE 1-Free-cruising**

$$
\begin{align*}
    u_{p,i}(t) &= c^p K^p_1 \sum_{j=0}^{N} \alpha_{ij}^p (q_{p,j}(t) - q_{p,i}(t) + S^p_{j1}(t)) \\
    &+ c^p K^p_2 \sum_{j=0}^{N} \alpha_{ij}^p (v_{p,j}(t) - v_{p,i}(t)) \\
    &+ c^p K^p_3 \sum_{j=0}^{N} \alpha_{ij}^p (a_{p,j}(t) - a_{p,i}(t)),
\end{align*}
$$

where $h^p_{ij}$ and $c^p_{ij}$ are the parameters of the FDVTH spacing strategy for the $p$-th platoon, $K^p_1, K^p_2, K^p_3$ are the feedback gains, $c^p > 0$ is the coupling gain, and $A^p = [\alpha_{ij}^p]|_{N+1} \times (N+1)$ is the adjacency matrix of the inter-vehicle communication.

**Criterion 4 (C-VI):** For the **Leader CAV** having unknown planning demands under C-II-C-IV that are associated with the SDBs of the HDVs in the other five states, i.e., **STATE 2-Tracking-cruising**, **STATE 3-Platoon dynamic forming**, **STATE 4-Smart crossing control**, **STATE 5-Safe-check control** and **STATE 6-Traffic flow control**, the following controllers are further designed

$$
\begin{align*}
    \dot{u}_{p,i}(t) &= \xi_{p,i}(t) + \zeta_{p,i}(t) + u_{p,i}(t) \\
    &+ c^p \text{sgn}(K^p_1 \sum_{j=0}^{N} \alpha_{ij}^p (q_{p,j}(t) - q_{p,i}(t) + S^p_{j1}(t))) \\
    &+ K^p_2 \sum_{j=0}^{N} \alpha_{ij}^p (v_{p,j}(t) - v_{p,i}(t)) \\
    &+ K^p_3 \sum_{j=0}^{N} \alpha_{ij}^p (a_{p,j}(t) - a_{p,i}(t))),
\end{align*}
$$

where $\xi_{p,i}(t)$ is the distributed estimations of the $i$-th **Follower CAV** for the **Leader CAV**’s acceleration in the $p$-th platoon, and we denote $\xi_{p,0}(t) = a_{p,0}(t), c^p_0$ is the coupling gain, $\tilde{c} > 0$ is a scalar, and $Q^p$ is the observer gain which will be designed later.

**Remark 5:** In this paper, a new cooperative consensus-based control approach with a full distributed variable time headway (FDVTH) spacing strategy is proposed. The general VTH strategy is a nonlinear function of the global speed of the leader vehicle [36], which is more flexible compared with the constant spacing (CS) strategy and the constant time headway (CTH) strategy. In reality, for complex traffic scenarios such as frequent acceleration or deceleration of the preceding vehicle, the constant spacing strategy and constant time headway strategy do not perform well. For example, the platoon gap for constant spacing strategy may not be safe when the **Leader CAV** breaks suddenly, especially in the case of all the platoons reaching the intersection. To ensure safety, we require a large platoon gap, resulting in a problem of low road utilization. The performance of the constant time headway strategy may be better, however, which is mainly considered in the pure CAVs environments as made in [15], [16], [37]. Although the works [18], [19] develop variable time headway based on the global information of the leader CAV’s constant speed, in particular, we here consider that the car in front is an HDV with SDBs, where all the above strategies may cause the risk of an accident. Therefore, we develop a new fully distributed VTH space error strategy, which introduces a quadratic function of local speed that can improve the robustness and safety for multiple platoons management in a mixed traffic flow environment.

### IV. Stability analysis of dynamic platoon management

The goal of this paper is to design a dynamic platoon control algorithm to address the safe cruising and SCPs problems for multiple platoons, where each platoon forming will be affected by the finite state machine function and upcoming traffic lights. Based on the design in the above section, we have known that the **STATE 1-Free-cruising** is a special case of the finite state machine function AutoC$(t)$ in the ADSCAS, resulting in the stability analysis of the platoon under the controller (21) - (23) with C-VI to cover all the switching states. Therefore, for each vehicle $i \in \mathcal{F}$ of the $p$-th platoon, the platoon control goal can be expressed as solving the following consensus problem:

$$
\begin{align*}
    q_{p,i}(t) &\rightarrow q_{p,0}(t) - i \times l_c - D^p_{i0} - h^p_{i0}v_{p,i}(t) - r^p_{i0}v_{p,i}^2(t), \\
    v_{p,i}(t) &\rightarrow v_{p,0}(t), \\
    a_{p,i}(t) &\rightarrow a_{p,0}(t), \\
    \xi_{p,i}(t) &\rightarrow \xi_{p,0}(t),
\end{align*}
$$

Accordingly, the position, velocity, acceleration, and estimation errors with respect to the desired platooning equilibrium (24) can
be defined as follows:
\[
\begin{aligned}
\dot{q}_{p,i}(t) &= q_{p,i}(t) - q_{p,0}(t) + i \times l_c + D_{p0}^p + h_{i0}^p v_{p,i}(t) + c_{i0}^p v_{p,i}^2(t), \\
\ddot{v}_{p,i}(t) &= v_{p,i}(t) - v_{p,0}(t), \\
\ddot{a}_{p,i}(t) &= a_{p,i}(t) - a_{p,0}(t), \\
\dddot{\xi}_{p,i}(t) &= \xi_{p,i}(t) - \xi_{p,0}(t).
\end{aligned}
\] (25)

By defining \( y_{p,ij}^* = \frac{S_j}{j=1} \) and \( V_{p,i}(y_{p,ij}(t)) = v_{p,0}(t) \), it follows from (3) that
\[
\begin{aligned}
y_{p,i}(t) &= \hat{\alpha}^p \sum_{j=0}^{N} \alpha_{ij}^p [V_{p,i}(y_{p,ij}(t)) - V_{p,i}(y_{p,ij}^* (t)) \\
&+ V_{p,i}(y_{p,ij}^* (t)) - v_{p,i}(t)] \\
&= \hat{\alpha}^p \sum_{j=0}^{N} \alpha_{ij}^p [V_{p,i}(y_{p,ij}(t)) - V_{p,i}(y_{p,ij}^* (t)) - \ddot{v}_{p,i}(t)].
\end{aligned}
\] (26)

Specifically, it follows (4) that
\[
V_{p,i}(y_{p,ij}(t)) = V_{p,i}(y_{p,ij}^*(t)) + V_{p,i}^*(y_{p,ij}^*(t)) (y_{p,ij}(t) - y_{p,ij}^*(t)).
\] (27)

Based on (5), we have
\[
\begin{aligned}
V_{p,i}(y_{p,ij}(t)) - V_{p,i}(y_{p,ij}^*(t)) &= V_{p,i}(y_{p,ij}^*(t)) ((q_{p,j}(t) - q_{p,i}(t))/(j-i) - y_{p,ij}^*(t)) \\
&= V_{p,i}(y_{p,ij}^*(t)) (q_{p,j}(t) - q_{p,i}(t) + y_{p,ij}^*(t)) \\
&= V_{p,i}(y_{p,ij}^*(t)) (\ddot{q}_{p,i}(t) - \ddot{q}_{p,j}(t)) \\
&= \phi_{p,ij}(t) (\ddot{q}_{p,j}(t) - \ddot{q}_{p,i}(t)),
\end{aligned}
\] (28)

where \( \phi_{p,ij}(t) = V_{p,i}(y_{p,ij}^*(t))/(i-j) \).

It follows from (2), (9), (22) - (28) that
\[
\begin{aligned}
\dot{q}_{p,i}(t) &= \hat{\alpha}^p q_{p,i}(t) + h_{i0}^p a_{p,i}(t) + 2\xi_{p,i} v_{p,i}(t) a_{p,i}(t), \\
\ddot{v}_{p,i}(t) &= \ddot{a}_i(t), \\
\ddot{a}_{p,i}(t) &= -\frac{1}{\tau} a_{p,i}(t) + \frac{1}{\tau} \xi_{p,i}(t) + \frac{1}{\tau} a_{p,0}(t) - \frac{1}{\tau} u_{p,0}(t) \\
&= \frac{1}{\tau} (c_{i0}^p V_{p}^p) \sum_{j=0}^{N} \alpha_{ij}^p \ddot{q}_{p,i}(t) - \ddot{q}_{p,j}(t) \\
&+ c_{i0}^p V_{p}^p \sum_{j=0}^{N} \alpha_{ij}^p \ddot{v}_{p,i}(t) - \ddot{v}_{p,j}(t) \\
&+ c_{i0}^p V_{p}^p \sum_{j=0}^{N} \alpha_{ij}^p \ddot{a}_{p,i}(t) - \ddot{a}_{p,j}(t) \\
&- \frac{c_{i0}^p}{\tau} \text{sgn} \left( K_{2}^p \sum_{j=0}^{N} \alpha_{ij}^p q_{p,i}(t) - q_{p,j}(t) \right) \\
&+ c_{i0}^p K_{2}^p \sum_{j=0}^{N} \alpha_{ij}^p \ddot{v}_{p,i}(t) - \ddot{v}_{p,j}(t) \\
&+ c_{i0}^p K_{3}^p \sum_{j=0}^{N} \alpha_{ij}^p \ddot{a}_{p,i}(t) - \ddot{a}_{p,j}(t) \\
&- \frac{c_{i0}^p}{\tau} \sum_{j=0}^{N} \alpha_{ij}^p (\phi_{p,j}(t) (\ddot{q}_{p,i}(t) - \ddot{q}_{p,j}(t)) + \ddot{v}_{p,i}(t)) \\
&- \frac{c_{i0}^p}{\tau} \sum_{j=0}^{N} \alpha_{ij}^p ((\ddot{v}_{p,i}(t) - \ddot{v}_{p,j}(t)))).
\end{aligned}
\] (29)

**Remark 6:** It must be noted that the platoon dynamics will be affected by the parameters \( h_{i0}^p, c_{i0}^p \) of the developed FDVHT space strategy because we consider that the platoon stability of every Following CAV will depend on its local neighbors’ velocity \( v_{p,i} \) and acceleration \( a_{p,i} \), resulting in a fully distributed cooperative platoon control strategy. The existing works [18], [19] only employ the global information of the Leader CAV such as the global fixed cruising speed to design the platoon space strategy which ignores the influence of the varying cruising dynamics of the Leader CAV to form a homogeneous platoon system. Differently, the item \( h_{i0}^p a_{p,i}(t) + 2\xi_{p,i} v_{p,i}(t) a_{p,i}(t) \) will affect the platoon stability under our proposed distributed strategy, which results in additional heterogeneous system dynamics. Therefore, the proposed approach can handle the complex traffic scenarios in a smart city with a varying and distributed space strategy, making the platoon stability analysis more applicable and challenging.
Using a mathematical transformation, we have
\[
\begin{align*}
& h_{0i}^p a_{p,i}(t) + 2 v_{0i}^p v_{p,i}(t) a_{p,i}(t) \\
& = h_{0i}^p a_{p,i}(t) - h_{0i}^p a_{0i}(t) + h_{0i}^p a_{0i}(t) + 2 v_{0i}^p v_{p,i}(t) a_{p,i}(t) \\
& = h_{0i}^p a_{p,i}(t) + h_{0i}^p a_{0i}(t) + 2 v_{0i}^p v_{p,i}(t) a_{p,i}(t) \\
& - 2 v_{0i}^p v_{p,i}(t) a_{p,i}(t) + 2 v_{0i}^p v_{p,i}(t) a_{p,i}(t) \\
& = h_{0i}^p a_{p,i}(t) + h_{0i}^p a_{0i}(t) + 2 v_{0i}^p v_{p,i}(t) a_{p,i}(t) \\
& + 2 v_{0i}^p v_{p,i}(t) a_{p,i}(t).
\end{align*}
\]
where \( h_{0i}^p \) and \( v_{0i}^p \) are two headway parameters.

With the conditions (25), (29) and (30), we further have
\[
\begin{align*}
\dot{\hat{v}}_{p,i}(t) &= \dot{\tilde{v}}_{p,i}(t) + h_{0i}^p a_{p,i}(t) + h_{0i}^p a_{0i}(t) \ \\
&+ 2 v_{0i}^p v_{p,i}(t) \tilde{a}_{p,i}(t) + 2 v_{0i}^p v_{p,i}(t) a_{p,i}(t), \\
\dot{\hat{a}}_{p,i}(t) &= \dot{\tilde{a}}_{p,i}(t) \ \\
&- \frac{1}{\tau} \left[ \frac{1}{\tau} \tilde{a}_{p,i}(t) + \frac{1}{\tau} \dot{\tilde{a}}_{p,i}(t) + \frac{1}{\tau} a_{p,i}(t) - \frac{1}{\tau} a_{p,i}(t) \right] \\
&+ \frac{1}{\tau} \left( c_{P} K_{1}^p \sum_{j=0}^{N} \alpha_{ij}^p (\tilde{q}_{p,i}(t) - \tilde{q}_{p,j}(t)) \\
&+ c_{P} K_{2}^p \sum_{j=0}^{N} \alpha_{ij}^p (\tilde{a}_{p,i}(t) - \tilde{a}_{p,j}(t)) \right) \\
&- \frac{c_{F}}{\tau} \text{sgn} \left( K_{1}^p \sum_{j=0}^{N} \alpha_{ij}^p (\tilde{q}_{p,i}(t) - \tilde{q}_{p,j}(t)) \\
&+ K_{2}^p \sum_{j=0}^{N} \alpha_{ij}^p (\tilde{a}_{p,i}(t) - \tilde{a}_{p,j}(t)) \right) \\
&- \frac{1}{\tau} \sum_{j=0}^{N} \alpha_{ij}^p [\phi_{p,j,i}(t)(\tilde{q}_{p,i}(t) - \tilde{q}_{p,j}(t)) + \dot{\tilde{a}}_{p,i}(t)] \\
&- \frac{1}{\tau} \sum_{j=0}^{N} \alpha_{ij}^p (\tilde{v}_{p,i}(t) - \tilde{v}_{p,j}(t)),
\end{align*}
\]
the following platoon dynamics
\[
\begin{align*}
\chi_{p}(t) &= A_{p}^p(t) \chi_{p}(t) + C_{F} f(\chi_{p}(t), t) \\
&+ T_{2}^p \tilde{q}_{p,i}(t) + T_{3}^p(a_{p,i}(t) - u_{p,i}(t)) \\
&+ U_{p,i}(t) + c_{P} B_{p} sgn(\tilde{U}_{p,i}(t)) + A_{0i}^p(t) a_{p,i}(t),
\end{align*}
\]
where
\[
\begin{align*}
A_{p}^p(t) &= \begin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}, \\
B_{p} &= \begin{bmatrix} 0 \ 0 \end{bmatrix}, \\
C_{F} &= I_{3}, \\
T_{2}^p &= [0, 0, \frac{1}{T_{2}}]^T, \\
A_{0i}^p(t) &= [h_{0i}^p + 2 v_{0i}^p v_{p,i}(t), 0, 0]^T, \text{and}
\end{align*}
\]
and
\[
U_{p,i}(t) = - (c_{P} K_{1}^p \sum_{j=0}^{N} \alpha_{ij}^p (\tilde{q}_{p,i}(t) - \tilde{q}_{p,j}(t)) \\
+ c_{P} K_{2}^p \sum_{j=0}^{N} \alpha_{ij}^p (\tilde{a}_{p,i}(t) - \tilde{a}_{p,j}(t))) \\
+ K_{3}^p \sum_{j=0}^{N} \alpha_{ij}^p (\tilde{a}_{p,i}(t) - \tilde{a}_{p,j}(t)),
\]
\[
f(t) = [0, 0, \gamma_{3}(t)]^T,
\]
\[
\gamma_{3}(t) = - \frac{\hat{u}_{p}^p}{\tau} \sum_{j=0}^{N} \alpha_{ij}^p [\phi_{p,j,i}(t)(\tilde{q}_{p,i}(t) - \tilde{q}_{p,j}(t)) + \tilde{v}_{p,i}(t)] \\
- \frac{\hat{u}_{p}^p}{\tau} \sum_{j=0}^{N} \alpha_{ij}^p ((\tilde{v}_{p,i}(t) - \tilde{v}_{p,j}(t))).
\]
Now, we introduce the inter-vehicle communication model of the platoon CAVs. Since the Leader CAV will not be affected by the Follower CAVs within every platoon, according to (38), we can model a directed graph for the \( p \)-th platoon with \( (N + 1) \) CAVs including the Leader CAV indexed by \( p, 0 \) and \( N \) Follower CAVs as \( \{p1, \ldots, pN\} \). With this consideration, the problem of multiple platoon stability analysis will be transformed as a problem of internal stability of a single platoon plus an external disturbance including the large external space-safe planning with respect to the preceding platoon. Then, the inter-vehicle communication model for the \( p \)-th platoon can be described by \( \tilde{G}_{p} = \{V_{p}, \tilde{E}_{p}, \tilde{A}_{p}\} \) with a nonempty finite set of \( N + 1 \) nodes \( V_{p} = \{v_{0}, \ldots, v_{N}\} \), a finite set of edges \( \tilde{E}_{p} \subset V_{p} \times V_{p} \), and an adjacency matrix \( \tilde{A}_{p} = [a_{ij}^p] \in \mathbb{R}^{N+1 \times N+1} \) with nonnegative elements \( a_{ij}^p \) for all \( v_{i}^p, v_{j}^p \in \tilde{V}_{p} \), if and only if the system \( i \) can receive the information from the neighbor \( j \).
To obtain the results, the following assumption and lemma are introduced.

**Assumption 7:** For every platoon, the *Leader CAV* has a directed path to every *Follower CAV*, that is, the communication topology contains a directed spanning tree with the *Leader CAV* as a root node.

With Assumption 7, the Laplacian matrix $\bar{L}^p$ associated with the topology $G^p$ can be written as the following form

$$\bar{L}^p = \begin{pmatrix} 0 & \Theta^p \bar{L}_p^a & 0^T_N \\ L_0^p & 0 & L_1^p \end{pmatrix}, \quad (37)$$

where $L_0^p \in \mathbb{R}^N$, $L_1^p \in \mathbb{R}^{N \times N}$.

**Lemma 8:** With Assumption 7, there exists a positive vector $\theta^p = (\theta_1^p, \ldots, \theta_N^p)^T \in \mathbb{R}^N$, such that $L_0^p \theta^p = \mathbf{1}_N$ and

$$\Theta^p \bar{L}_p^a + (L_1^p)^T \Theta^p > 0, \quad (38)$$

where $\Theta^p = \text{diag}\{1/\theta_1^p, \ldots, 1/\theta_N^p\}$.

Now, we start to analyze the platoon’s stability. Define $\hat{\xi}^p(t) = [(\xi_{p,1}(t))^T, \ldots, (\xi_{p,N}(t))^T]^T$. Based on (31), we have

$$\dot{\hat{\xi}}^p(t) = (I_N \otimes I_N) \hat{\xi}^p(t) - c^p (L_1^p \otimes Q^p) \hat{\xi}^p(t) - c_{\max} (I_N \otimes I_N) \text{sgn}((L_1^p \otimes Q^p) \hat{\xi}^p(t)) + (I_N \otimes I_N) \hat{a}^p(t), \quad (39)$$

where $\hat{a}^p(t) = \frac{1}{2} a_{p,0}(t) + a_{p,0}(t) - \frac{1}{3} u_{p,0}(t) \leq (1 + \frac{2}{3}) a_{p,\max}$.

Then, by defining $\chi^p(t) = [(\chi_{p,1}^T)^T, \ldots, (\chi_{p,N}^T)^T]^T$, the error dynamics of platoon can be rewritten by the following compact form

$$\chi^p(t) = A^p(t) \chi^p(t) + (I_N \otimes \Theta^p) \hat{a}^p(t) + (I_N \otimes \Theta^p) d^p(t) - (\bar{L}_0^p \otimes M_1^p) \chi^p(t) + (\bar{L}_1^p \otimes M_1^p) \chi^p(t) - (D^p \otimes M_1^p) \chi^p(t) - \bar{c}^p (I_N \otimes I_N) \text{sgn}((L_1^p \otimes B^p K^p) \chi^p(t)) \quad (40)$$

where $A^p(t) = \text{diag}(A^p(t))$, $\hat{a}^p(t) = a_{p,0}(t) - u_{p,0}(t)$, $b = 2 + \lambda_{\max(h_{0,0} + 2 \xi_{p,i}^0 v_{p,0}(t))}$, $M_1^p = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, $M_2^p = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, $K^p = [K_1^p, K_2^p, K_3^p]$, $\phi_{p,0}(t) = \min_{1, \ldots, N} \phi_{p,i}(t)$, and $D^p = \text{diag}\{d^p_i\} \in \mathbb{R}^{N \times N}$ is the in-degree matrix.

**Remark 9:** It is clear that the error dynamics of the platoons are time-varying systems, where the stability will be affected by the $A^p(t)$ and $\hat{a}^p(t)$ which are based on the time-varying and unknown parameter $a_{p,0}(t)$, $u_{p,0}(t)$, and $v_{p,0}(t)$, making the stability analysis difficult. In this sense, the stability of the close-loop platoon system cannot be simply regarded as the static local feedback control by only using the criterions C-V-C-VI. Specifically, different from the existing works [18, 19, 40] that only apply to the constant speed constraint of the *Leader CAV* where the global information $v_0(t)$ is required to calculate the space error among the CAVs, for the first time, we propose a dynamic solution and fully distributed space error policy for platoon CAVs traveling on the smart city where the *Leader CAV* is time-varying and uncertain with the respect to the upcoming traffic lights to ensure the SCPs. Notably, the close-loop platoon dynamics and are fully different from the original system and for the developed time-varying space policy FDVHT will affect the cooperative behaviors of platoon CAVs directly, which cannot be ignored in the realistic crossing process, making a fully different stability analysis framework compared with the existing works [33, 34, 41]. Although solving the stability problem of the current close-loop platoon dynamics challenges, thanks to the boundedness of the vehicle dynamics, that is, all the CAVs will be limited in the realistic urban traffic environment to lead the bounded speed and acceleration, we are able to obtain the feasible solution for the distributed dynamic platoon control by introducing the contraction analysis and robust optimization theory as shown below.

Then, we present the first result for the distributed dynamic platoon management under an infinite time horizon and the finite-time convergence solution is obtained by proposing a robust cooperative trajectory tracking optimization which will be discussed later.

**Theorem 10:** With Assumption 7, the distributed dynamic platoon management problem is solved for an infinite time horizon by the protocols (22) and (23) under C-V-C-VI and the finite state function $Auto\text{C}(t)$ with the feedback gains $K^p = \kappa^p (B^p)^T (P^p)^{-1}$ and the observer gain $Q^p = (T^p)^{-1} P_0^p$, if the following equations have feasible solutions

$$P_0^p + (P_0^p)^T - \omega^p \theta_0^p P_0^p Q^p = -I_n, \quad (41)$$

$$\tilde{A}_i^p P^p + P^p \tilde{A}_i^p = -\lambda_i^p \theta_0^p M_1^p P^p - \lambda_i^p \theta_0^p M_2^p P^p - 2 \theta_0^p M_3^p P^p + \omega^p \theta_0^p B^p (P^p)^T + \beta P^p < 0, \quad (42)$$

where

$$\tilde{A}_i^p = \begin{bmatrix} 0 & 1 & 2 \theta_0^p v_{p,\max} \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, \quad (43)$$

$k > 0$ and $\beta > 0$ are two scalars, $\theta_0^p = \min \theta_i^p$ with $\theta_i^p$ being defined in Lemma 8, $d_0^p = \min d_0^p$, $\omega^p > 0$ is a coupling design parameter, $P_0^p$ is the positive definite matrix, $T^p = (T^p)^T \in \mathbb{R}^{n \times n}$ is to be selected positive definite matrix. Moreover, choosing the coupling strength $c_0^p = (1 + \frac{2}{3}) a_{p,\max}$, $\Theta^p \bar{L}_p^a = b a_{p,\max} c_0^p \bar{L}_p^a$ with $b = 2 + \lambda_{\max(h_{0,0} + 2 \xi_{p,i}^0 v_{p,0}(t))}$, $c_1^p = \lambda_{\min} ((B^p (B^p)^T)^T) / \kappa$, $c_2^p = \lambda_{\min} ((B^p (B^p)^T)^T) / \kappa P_2^p$, $\omega^p > \omega_0^p / \lambda_0^p$, where $\lambda_0^p = \lambda_{\min} (\Theta^p \bar{L}_p^a + (L_1^p)^T \Theta^p)$, $\Theta^p = \text{diag}\{1/\theta_1^p, \ldots, 1/\theta_N^p\}$, the protocols (22) and (23) will be constructed.

**Proof:** Based on Lemma 8 the matrix $L_0^p$ is invertible. Thereby, the distributed dynamic platoon management problem will be solved if and only if both error dynamics $\xi^p(t)$ and $\chi^p(t)$ converge synchronously to zero. Then, we would like to introduce
the following Lyapunov function candidates
\[ V((\hat{\xi}^p, \chi^p, t) = V_1(\hat{\xi}^p, t) + V_2(\hat{\xi}^p, \chi^p, t), \]
\[ V_1(\hat{\xi}^p, t) = 8\beta_{\lambda_{\text{max}}}((T^p_2)^\top (P^p)^{-1}T^p_2)(\hat{\xi}^p)^\top(t)(\Theta^p \otimes P_0^p)(\hat{\xi}^p(t), \]
\[ V_2(\hat{\xi}^p, \chi^p, t) = (\chi^p)^\top(t)(\Theta^p \otimes (P^p)^{-1})\chi^p(t). \]

(44)

The proof of the main results includes three parts.

Part i: taking the time derivative of \( V_1(\hat{\xi}^p, t) \) along the trajectories of system (39) gives
\[ \dot{V}_1(\hat{\xi}^p, t) \leq 8\beta_{\lambda_{\text{max}}}((T^p_2)^\top (P^p)^{-1}T^p_2) \]
\[ \times \left( (\hat{\xi}^p(t))^\top(\Theta^p \otimes (P^p_0 + (P^p_0))^\top) \right. \]
\[ \left. - 2\varphi(\Theta^p L^p_1 \otimes P_0^p Q^p) \right) \hat{\xi}^p(t) \]
\[ - 2\varphi(\hat{\xi}^p(t)) \left( \Theta^p \otimes P_0^p \right) \sgn((L^p_1 \otimes Q^p) \hat{\xi}^p(t)) \]
\[ + 2(\hat{\xi}^p(t))^\top(\Theta^p \otimes P_0^p) \bar{a}^p(t). \]

(45)

Part ii: the time derivative of \( V_2(\hat{\xi}^p, \chi^p, t) \) along the trajectories of the system (40) takes
\[ \dot{V}_2(\hat{\xi}^p, \chi^p, t) \leq \left( (\chi^p(t))^\top(\Theta^p \otimes (P^p)^{-1}A^p + (A^p)^\top(P^p)^{-1}) \right) \chi^p(t) \]
\[ - 2(\chi^p(t))^\top(\Theta^p L^p_1 \otimes (P^p)^{-1}M^p_1) \chi^p(t) \]
\[ - 2(\chi^p(t))^\top(\Theta^p D^p \otimes (P^p)^{-1}M^p_2) \chi^p(t) \]
\[ - 2\varphi(\hat{\xi}^p(t))^\top(\Theta^p L^p_2 \otimes (P^p)^{-1}T^p_2) \hat{\xi}^p(t) \]
\[ + 2(\chi^p(t))^\top(\Theta^p \otimes (P^p)^{-1}T^p_2) \varphi^2(t) \]
\[ - 2\varphi(\hat{\xi}^p(t))^\top(\Theta^p \otimes (P^p)^{-1}) \sgn((L^p_1 \otimes B^p K^p) \chi^p(t)) \]

(49)

With the condition (43), we have
\[ \dot{V}_2(\hat{\xi}^p, \chi^p, t) \leq (\chi^p(t))^\top(\Theta^p \otimes (P^p)^{-1}) \chi^p(t) \]
\[ - 2(\chi^p(t))^\top(\Theta^p L^p_1 \otimes (P^p)^{-1}M^p_1) \chi^p(t) \]
\[ - 2(\chi^p(t))^\top(\Theta^p D^p \otimes (P^p)^{-1}M^p_2) \chi^p(t) \]
\[ - 2\varphi(\hat{\xi}^p(t))^\top(\Theta^p L^p_2 \otimes (P^p)^{-1}T^p_2) \hat{\xi}^p(t) \]
\[ + 2(\chi^p(t))^\top(\Theta^p \otimes (P^p)^{-1}T^p_2) \varphi^2(t) \]
\[ - 2\varphi(\hat{\xi}^p(t))^\top(\Theta^p \otimes (P^p)^{-1}) \sgn((L^p_1 \otimes B^p K^p) \chi^p(t)) \]

(50)

Similarly, with \( K^p = \kappa (B^p)^\top(P^p)^{-1} \), we have
\[ - 2\varphi(\hat{\xi}^p(t))^\top(\Theta^p \otimes (P^p)^{-1}) \sgn((\kappa (L^p_1 \otimes B^p)^\top(P^p)^{-1} \chi^p(t) \]
\[ + 2(\chi^p(t))^\top(\Theta^p \otimes (P^p)^{-1}T^p_2) \varphi^2(t) \]
\[ \leq - 2\varphi(\hat{\xi}^p(t))^\top((\kappa (L^p_1 \otimes (P^p)^{-1}B^p)^\top(P^p)^{-1}) \chi^p(t) \]
\[ \times \sgn((\kappa (L^p_1 \otimes B^p)^\top(P^p)^{-1}) \chi^p(t) \]
\[ + 2(\chi^p(t))^\top((\kappa (L^p_1 \otimes (P^p)^{-1}B^p)^\top(P^p)^{-1}) \chi^p(t) \]
\[ \leq - 2(\chi^p(t))^\top(\Theta^p \otimes (P^p)^{-1}B^p)^\top(P^p)^{-1} \chi^p(t) \]
\[ \leq - 2(\chi^p(t))^\top(\Theta^p \otimes (P^p)^{-1}B^p)^\top(P^p)^{-1} \chi^p(t) \]

(51)

where \( c^1 = \lambda_{\text{min}}((B^p)^\top(P^p)^{-1}/\kappa) \), \( c^2 = \lambda_{\text{min}}((B^p)^\top(P^p)^{-1}/\kappa) \), and \( b = 2 + \frac{\lambda_{\text{max}}((\theta^p)^\top(\theta^p)^\top)}{\delta} \). Let \( \delta = b a_{\text{p, max}}(c_{\text{p, max}}^2) \). Then, we have
\[ \dot{V}_2(\hat{\xi}^p, \chi^p, t) \leq (\chi^p(t)) \left( (\Theta^p \otimes (P^p)^{-1}) \chi^p(t) \right. \]
\[ - 2(\chi^p(t))^\top((\kappa (L^p_1 \otimes (P^p)^{-1}B^p)^\top(P^p)^{-1}) \chi^p(t) \]
\[ - 2(\chi^p(t))^\top(\Theta^p L^p_1 \otimes (P^p)^{-1}M^p_1) \chi^p(t) \]
\[ - 2(\chi^p(t))^\top(\Theta^p D^p \otimes (P^p)^{-1}M^p_2) \chi^p(t) \]
\[ - \omega^2 \theta_{\text{p, max}}^2(\chi^p(t))^\top(\Theta^p \otimes (P^p)^{-1}T^p_2) \hat{\xi}^p(t) \]
\[ + 2(\chi^p(t))^\top(\Theta^p \otimes (P^p)^{-1}) \sgn((L^p_1 \otimes B^p K^p) \chi^p(t)) \]

(52)

Let \( \chi^p(t) = (\chi^p(t))^\top, \ldots, (\chi^p(t))^\top, i = 1, \ldots, N \). It follows from (44) and (45) -
Part iii: With Part i and Part ii, it follows from (44), (48) and (53) that
\[
\dot{V}(\hat{\chi}^p(t), \chi^p(t), t) \leq -\beta(\chi^p(t))^\top (\Theta^p \otimes P^{-1}) \chi^p(t) + \frac{3}{2}(\hat{\chi}^p(t))^\top (\Theta^p \otimes (P^p)^{-1} T_2^p) \hat{\chi}^p(t).
\] (54)

where \( \Theta^p = -8\beta^{-1}(\Theta^p \otimes (T_2^p)^\top (P^p)^{-1} T_2^p) \), \( \Theta^p = 2(\Theta^p \otimes (T_2^p)^\top (P^p)^{-1}) \), and \( \Theta^p = -\beta(\Theta^p \otimes (P^p)^{-1}) \). It is obtained that \( \Theta^p_{11} < 0, \Theta^p_{22} < 0 \) and \( \Theta^p_{11} - \Theta^p_{12}(\Theta^p_{22})^{-1}(\Theta^p_{12})^\top = \frac{\Theta^p}{2} < 0 \), and which is Schur equivalent to
\[
\Theta^p = \begin{bmatrix}
\Theta^p_{11} & \Theta^p_{12} \\
\Theta^p_{21} & \Theta^p_{22}
\end{bmatrix} < 0.
\] (55)

Then, it is resulting that both error dynamics \( \hat{\chi}^p(t) \) and \( \chi^p(t) \) converge synchronously to the origin as \( t \to \infty \). Therefore, the dynamic platoon management problem is solved by the proposed protocols (22) and (23). The proof is complete.

\[\Box\]

V. Finite-time robust cooperative trajectory tracking optimization

Although the stability analysis of dynamic platoon management in the above section provides a strategy to verify the feasible design of platoon controllers under C-V-C-VI, the performance of the mixed traffic flow may not be guaranteed in realistic scenarios because the strict stability of Theorem 10 relies on an infinite time horizon. Besides, according to Theorem 1 characterized in part I [1], the maximum platoon length is a function of the time \( t \), i.e., \( N(t) \), which however is derived without considering the actual platoon forming time needed during the SCPs. Let \( \hat{N} \) be the actual platoon size that needs to be determined. The system parameter \( \Theta^p \), which is associated with the scale of the CAVs in a platoon, appearing in Theorem 10 will be a function of \( \hat{N} \), thus, we simply write \( \Theta^p(\hat{N}) \) to denote this correlation relationship. In addition, the error signals \( \chi^p(t) \) and \( \hat{\chi}^p(t) \) are also affected by the choice of the cruising speed \( V_{p,0} \) of the Leader CAV, which is reflected in \( \chi^p(0) \) and \( \hat{\chi}^p(0) \). Thus, the Lyapunov function \( V(\chi^p, \hat{\chi}^p, t) \) is also affected by \( V_{p,0} \). We will simply write \( V(0, V_{p,0}) \) to reflect this correlation relationship. The planning phase starts at the moment the Leader CAV enters the entry point of the current zone and ends no later than the Leader CAV reaches the exit point of the current zone. We propose the following optimization problem:

To this end, we introduce the following optimization problem to guarantee that the dynamic platoon can be formulated in a finite time under a robust solution by introducing a tolerable level \( \tilde{\delta}^p > 0. \)

\[
\max \{\hat{T}, \hat{N}\} \tag{56}
\]

Subject to:

\[
\hat{T} < T^* \tag{57}
\]
\[
\hat{N} \leq \hat{N}(\hat{T}) \tag{58}
\]
\[
v_{p,0}^{\text{plan}} \leq v_{p,0}^{\text{max}} \tag{59}
\]
\[
\hat{T} = \frac{\ln(\delta^p)^2 \min(\hat{N}(\hat{T}))}{\lambda_{\min}(\hat{N}(\hat{T}))} - \ln V(0) \tag{60}
\]
\[
\hat{N}(\hat{T}) = \Theta^p(\hat{N}) \otimes (P^p)^{-1} (\hat{T}) \tag{61}
\]
\[
\hat{\chi}^p(0) = \Theta^p(\hat{N}) \otimes (P^p)^{-1} (\hat{T}) \tag{62}
\]
\[
\hat{\chi}^p(\hat{T}) = \Theta^p(\hat{N}) \otimes (P^p)^{-1} (\hat{T}) \tag{63}
\]
\[
\hat{\chi}^p(\hat{T}) = \Theta^p(\hat{N}) \otimes (P^p)^{-1} (\hat{T}) \tag{64}
\]
\[
\hat{\chi}^p(\hat{T}) = \Theta^p(\hat{N}) \otimes (P^p)^{-1} (\hat{T}) \tag{65}
\]
\[
\hat{\chi}^p(\hat{T}) = \Theta^p(\hat{N}) \otimes (P^p)^{-1} (\hat{T}) \tag{66}
\]
\[
\hat{\chi}^p(\hat{T}) = \Theta^p(\hat{N}) \otimes (P^p)^{-1} (\hat{T}) \tag{67}
\]
\[
\hat{\chi}^p(\hat{T}) = \Theta^p(\hat{N}) \otimes (P^p)^{-1} (\hat{T}) \tag{68}
\]
\[
\hat{T} \geq \frac{\phi_{p,0}}{v_{p,0}^{\text{plan}}} \tag{69}
\]

where \( P^p \) is derived in (42), \( \beta \) is a convergence rate specified in (42). Let \( \hat{T} \) be the optimal value of \( \hat{T} \), \( \phi_{p,0} \) be the resulting Leader CAV’s reference speed under the different switching states and C-I-C-VI. Then, the solution \( \hat{T} \) will be the final optimal time to implement the control stability.

Notably, the complexity of the solution \( \hat{T} \) of Equation (56) depends on the solution \( \hat{T}^* \) and \( P^p \) of Equations (41) and (42) which involves the nonlinear reaction functions \( V_{p,i}(y_{p,i,j}(t)) \), \( i, j \in \mathbb{F} \) (as shown in (3) and the FDVTH scheme (as shown in (22) and (9)), \( V(0, v_{p,0}^{\text{plan}}) \), the parameter \( \beta \), and the time-varying platoon size \( \hat{N}(t) \), to make the above optimization problem highly nonlinear and hard to solve. To simplify the computation, we present the following algorithm by iterating the platoon size \( \hat{N} \):

\textbf{Algorithm 1:} Estimated Realistic Platoon Size and Cruising Speed

\textbf{Step1:} Estimate the value of \( \hat{N}(\hat{T}) \) by setting \( t = \frac{\phi_{p,0}}{v_{p,0}^{\text{plan}}} \) according to the Theorem 1 in part I [1], and the constraints (13)-(16) and (60). We can check that \( \hat{N}(\hat{T}) \leq \hat{N}(t) \).

\textbf{Step 2:} Iterate on the values of \( \hat{N} = \hat{N}(t), \hat{N}(t) - 1, \ldots, 2 \). For each \( \hat{N} \), solving the following optimization problem:

\[
\max \{v_{p,0}^{\text{plan}}\}
\]

Subject to: constraints (57)-(69), and \( v_{p,0}^{\text{plan}} \geq v_{p,i,0}(0) \) for all \( i \in \mathbb{F} \).
If there is a solution of \( v_{\text{plan}} \), then output \( \hat{N} \) and \( v_{0}^{\text{plan}} \), and terminate; otherwise, set \( \hat{N} = N - 1 \). If \( \hat{N} = 2 \), then output \( \hat{N} \) and \( v_{0}^{\text{plan}} \) and terminate; otherwise, continue Step 2). □

Remark 11: Once the platoon size \( \hat{N} \) is determined, the parameters of the communication topologies, such as \( \lambda_{0}^{p} \), \( \Theta_{P} \), \( \Theta_{P} \), and the feasible solution \( P^{p} \) of the equation (42) can be checked offline. Specifically, substituting (58) - (60) into (69) yields

\[
v_{p,0}^{\text{plan}} \geq \frac{-\beta \delta_{p}}{\ln((\beta)^2 \lambda_{\min}(\Theta_{P}(N) \otimes (P^{p})^{-1}))}
\]

(70)

We can easily check that, \( v_{p,0}^{\text{plan}} \geq v_{p,i}(0) \) for all \( i \in \mathbb{F} \) is sufficient to ensure that inequality (70) is convex with respect to \( v_{p,0}^{\text{plan}} \), making the optimization problem in Step 2) a convex optimization problem that can be solved efficiently. Practically, to ensure \( v_{p,0}^{\text{plan}} \geq v_{p,i}(0) \) for all \( i \in \mathbb{F} \), all selected Follower CAVs need to be slower than the Leader CAV, which is implementable.

With the Leader CAV\’s cruising speed \( v_{p,0}^{\text{plan}} \) in the planning zone, we can calculate the time instant \( t^{*} = \delta_{p}/v_{p,0}^{\text{plan}} \) when the Leader CAV reaches the entry point of the optimization zone. Then we can determine the optimal zone crossing speed \( v_{p,0} \) determined by \( \delta_{\text{last}} \) with respect to the platoon size \( \hat{N}^{*} \) and the adjustment parameter \( \rho_{\text{last}} \), depending on whether the specified platoon CAVs will cross the junction in the current cycle or the next one. Once the maximum allowable lengths of all the platoons are determined at the first junction, the maximum allowable lengths of the platoons will be estimated in real-time at the following junctions. If a few follower CAVs cannot pass at the current traffic light, the platoons will use the developed algorithm to re-split the maximum allowable lengths of the platoon as several new platoons to ensure that the part of the platoon can cross the junction. After all the platoons have passed the specific junctions, they will be merged to form a multi-platoon control framework to guarantee that all the platoons can complete the SCPs at different junctions within the different traffic lights.

Then, we present the following result.

Theorem 12: The robust optimization problem of the cooperative trajectory tracking can be solved if the parameters \( \beta \) can be properly selected such that the constraints (57) - (69) can be satisfied.

Proof: To prove our result, we first formulate the tolerant level of the platoon error \( \delta_{\hat{P}} \) as follows

\[
\lim_{t \to T} \| \chi^{P}(t), \hat{\xi}(t) \| \leq \delta_{\hat{P}}.
\]

(71)

Then, according to (55) and (71), we have

\[
\dot{V}(\hat{\xi}^{P}, \chi^{P}, t) \equiv (\Xi^{P}(t))^{T} \Omega^{P}(\hat{N}^{*}) \Xi^{P}(t),
\]

(72)

where

\[
\Xi^{P}(t) = \left( \begin{array}{c} \hat{\xi}(t) \\ \chi^{P}(t) \end{array} \right), \quad \Omega^{P}(\hat{N}^{*}) = \begin{bmatrix} \Omega_{11}^{P}(\hat{N}^{*}) & \Omega_{12}^{P}(\hat{N}^{*}) \\ \Omega_{21}^{P}(\hat{N}^{*}) & \Omega_{22}^{P}(\hat{N}^{*}) \end{bmatrix}
\]

(73)

\[
\begin{bmatrix} \Omega_{11}^{P}(\hat{N}^{*}) & \Omega_{12}^{P}(\hat{N}^{*}) \\ \Omega_{21}^{P}(\hat{N}^{*}) & \Omega_{22}^{P}(\hat{N}^{*}) \end{bmatrix} < 0, \quad \Omega_{11}^{P}(\hat{N}^{*}) = -8\beta^{-1}(\Theta^{P}(N^{*}) \otimes (T_{2}^{p})^{T}(P^{p})^{-1}), \quad \Omega_{12}^{P}(\hat{N}^{*}) = 2(\Theta^{P}(N^{*}) \otimes (T_{2}^{p})^{T}(P^{p})^{-1}), \quad \Omega_{22}^{P}(\hat{N}^{*}) = -\beta(\Theta^{P}(N^{*}) \otimes (P^{p})^{-1}).
\]

(74)

It follows from the Lyapunov function (44) that

\[
V(\hat{\xi}^{P}, \chi^{P}, t) = (\Xi^{P}(t))^{T} \hat{\Omega}^{P}(\hat{N}^{*}) \Xi^{P}(t),
\]

(75)

where \( \hat{\Omega}^{P}(\hat{N}^{*}) = \begin{bmatrix} \Omega_{11}^{P}(\hat{N}^{*}) & 0 \\ 0 & \Omega_{22}^{P}(\hat{N}^{*}) \end{bmatrix} \). \( \hat{\Omega}^{P}(\hat{N}^{*}) = 8\beta^{-1} \lambda_{\max} ((T_{2}^{p})^{T}(P^{p})^{-1}T_{2}^{p}) (\Theta^{P}(\hat{N}^{*}) \otimes P^{p}), \quad \hat{\Omega}^{P}(\hat{N}^{*}) = \left( \Theta^{P}(\hat{N}^{*}) \otimes (P^{p})^{-1} \right) \)

(76)

With the condition (55), we have

\[
\dot{V}(\hat{\xi}^{P}, \chi^{P}, t) \leq -||\Xi^{P}(t)||^{2} \lambda_{\min}(\hat{\Omega}^{P}(\hat{N}^{*})) V(0) e^{\frac{-\lambda_{\min}(\hat{\Omega}^{P}(\hat{N}^{*})) t}{\lambda_{\min}(\hat{\Omega}^{P}(\hat{N}^{*}))}},
\]

(77)

(78)

We choose \( \hat{T} \in \mathbb{R}^{+} \) such that \( ||\Xi^{P}(\hat{T})|| \leq \delta_{\hat{P}}. \) Thus, we can sufficiently choose

\[
\hat{T} \geq \frac{\ln((\delta_{\hat{P}})^{2} \lambda_{\min}(\hat{\Omega}^{P}(\hat{N}^{*}))) - \ln V(0)}{\lambda_{\min}(\hat{\Omega}^{P}(\hat{N}^{*})) - \lambda_{\min}(\hat{\Omega}^{P}(\hat{N}^{*}))}
\]

(79)

To ensure a quick stability, we simply choose

\[
\hat{T} = \frac{\ln((\delta_{\hat{P}})^{2} \lambda_{\min}(\hat{\Omega}^{P}(\hat{N}^{*}))) - \ln V(0)}{\lambda_{\min}(\hat{\Omega}^{P}(\hat{N}^{*})) - \lambda_{\min}(\hat{\Omega}^{P}(\hat{N}^{*}))}
\]

(80)

Which is the condition (55) specified in the robust optimization problem. Then, we can select a proper convergence rate \( \beta \) to check the condition (55). The proof is completed.

VI. Performance analysis and simulation cases studies

This section presents a series of simulation cases to demonstrate the effectiveness of the proposed ADSCAS framework for the mixed traffic flows consisting of CAVs with the reaction-time delay and car-following dynamics, SDBs of the HDVs, and realistic inter-vehicle constraints on a straight signalized road. Different from the existing works \([7], [8]\) that only consider a pure CAV control in unsignalized roads, the dynamic platoon management in this paper will be affected by considering the SDBs of the SVs in the traffic network and calculating the unknown acceleration inputs from the connected vehicles environments in the ADSCAS. Specifically, we here formulate a finite state machine imposing the observation and planning information to determine the driving state of the platoons under the interaction...
with a specific HDV, where all the driving decisions will be described by a function $\text{AutoC}(t) = \{S0, S1, S2, S3, S4, S5\}$ representing the transformed state of the STATE 1-Free-cruising, STATE 2-Tracking-cruising, STATE 3-Platoon dynamic forming, STATE 4-Smart crossing control, STATE 5-Safe-check control, and STATE 6-Traffic flow control, respectively. Please see part I [1] for details. In simulations, the influence of the small differences of lateral dynamics for the platoon CAVs will be ignored in the straight lane. Besides, although the specific HDV is considered in case studies, the multiple HDVs are available by using our developed ADSCAS framework.

A. Platoon performance analysis

We first consider the platoon performance of our proposed FDVTH space error strategy with the CBPM-OVM model by using realistic inter-vehicle constraints under different reaction-time delays in the Cruising(CS1), as shown in [1] compared with:

- **CS2**: the existing works [7], [8] that addresses pure CAV control but does not involve the OVM model to describe the driver response to the stimulus from SVs; and
- **CS3**: our previous work [14] that considers pure CAV control the OVM model with a fixed platoon gap.

To start, the CAV communication is as follows: $L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, which can be easy to implement by C-V2X infrastructure. According to the proposed ADSCAS, when the cruising distance satisfies $D > D_c$, the traveling behaviors of the platoon CAVs under C-I will not be affected by the HDV and the platoon CAVs will be in the STATE 1-Cruising so $u_{p,0}(t) = 0$. According to the directed communication topology condition, we can obtain $\lambda_0 = 0.038$. With Lemma 3 we get $\theta_0 = 1$. By setting $\omega = 1$ and $\kappa = 1$, we can obtain $\alpha > \omega/\lambda_0 = 26.3158$, which is resulting in the setting $\alpha = 26.4 > 26.3158$. With such conditions, it is easy to check the feasible solution of the equation (43) and the protocol (22) will be constructed under C-V. Then, by setting $\tau = \{0,3\}$, the platoon gap error and velocity of the CAVs systems (2) and (6) under the optimal velocity functions $y_{p,i}(t)$ and difference velocity model $g_{p,i}(t)$ will be obtained, as shown in Figs. 2 (a) and (b), respectively. It is clear that the developed dynamic platoon control can be achieved in our proposed ADSCAS. Then, we shall show the dynamic platoon management of CAVs under the different reaction-time delays profile $\tau = \{0,1,0.2,0.3,0.4,0.5,0.6\}(s)$. By setting the initial parameters of displacement, velocity and acceleration, the platoon dynamics of the CAVs systems (2) and (6) under the optimal velocity functions $y_{p,i}(t)$ and difference velocity model $g_{p,i}(t)$ will be obtained, as shown in Figs. 2 (c) and (f), where Figs. 2 (c) and (d) represents the maximum and minimum platoon gap errors of CAVs, respectively. Specifically, the results of Figs. 2 (c) - (d) illustrate that the maximum velocities and platoon gap errors for all CAVs will be reduced, while the minimum velocities and platoon gap errors for all CAVs increases somewhat as the reaction time delays grow, which implies that the dynamic platoon management of the CAVs has been improved by the proposed algorithm for different reaction time delays. In particular, it is shown that our algorithm (CS1) has better performance than the cases CS2 and CS3.

To analyze the platoon performance, we would like to define the average tracking distance and platoon gap error as $\text{Error}_{\text{avg}}(t) = \frac{1}{6} \sqrt{\sum_{i=1}^{5} \|x_{p,i}(t) - x_{p,0}(t)\|^{2}}$ and $\text{Error}_{\text{ij}}(t) = x_{p,i}(t) - x_{p,j}(t)$, $i = 1, i = 1, \ldots, 6$, respectively. Besides, we also define the average control input and average control energy as $\text{Energy}_{\text{avg}}(t) = \frac{1}{6} \sqrt{\sum_{i=1}^{5} \|u_{p,i}(t)\|^{2}}$ and $\text{Energy}_{i}(t) = \sqrt{\sum_{i=1}^{5} \|u_{p,i}(t)\|^{2}}$, $i = 1, \ldots, 6$, respectively. Then, we need to introduce three evaluation indicators, i.e., mean, variance, and standard deviation, to value the platoon performance. It is shown that the values of mean, variance, and standard deviation of the average space error of all CAVs decrease but the average control energy increase on a smaller scale, as shown in Figs. 2 (g) and (h). The Figs. 2 (i) and (g) show that the maximum and minimum fuel economy of CAVs, where the maximum fuel economy of CAVs in our CS1 will be reduced compared with the CS2 and CS3 as the reaction time delays grow. In addition, the Figs. 2 (k) and (l) show the performance of average control input and total fuel economy for all CAVs under different reaction time delays, where the mean and standard deviation for all CAVs are similar but the variance of average control input will affect the variance of total fuel economy for different reaction time delays. The simulations show the great platoon performance of the proposed new car-following model compared with the existing cooperative platoon control that only considers the cooperative information, not cooperative perception.

B. Driving scenario I

We now consider the platoon performance in the mixed traffic flows environments, as shown in [1] where the preceding vehicle is an HDV and all the CAVs will travel in STATE 2-Tracking-cruising so that the larger external-space distance $D_{p,e}$ should be guaranteed. To analyze the impact of different communication modes on platoon performance, three classes of information flows are introduced: Topology A(TA): links intersections and Topology B(TB): limited connectivity using V2V infrastructure, and Topology C(TC): leader-and predecessor-following topology using C-V2X infrastructure, with the following Laplacian matrices $TA = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$, $TB = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$. 

Fig. 1. The traveling distance for the Leader CAV and SDBs of the HDVs.
Platoon performance under reaction-time delays {0.1, 0.2, 0.3, 0.4, 0.5, 0.6}

Performance of average tracking distances
The distance of seven connected automated vehicles

Maximum velocities
Performance of average control input
The velocity of seven connected automated vehicles
Leader CAV0
Follower CAV1
Follower CAV2
Follower CAV3
Follower CAV4
Follower CAV5
Follower CAV6

STATE 1-Free-cruising
D > D satisfies
The other parameters are the same as in the above analysis.

STATE 2-Tracking-cruising
will not be affected
p,

Standard deviation of total fuel economy with CS3
Variance of total fuel economy with CS2
Mean value of total fuel economy with CS1
Variance of total fuel economy
Mean value of average tracking distances with CS2
Variance of average tracking distances with CS2
Variance of average tracking distances with CS1
Mean value of average tracking distances
Variance of average control energy with CS3
Variance of average control energy with CS2
Variance of average control energy with CS1
Mean value of average control energy
Variance of average control input with CS3
Variance of average control input with CS2
Variance of average control input with CS1
Mean value of average control input

Rection-time delays
2
4
6
8
10
12
14

The performance of average tracking distance of CAVs;
The performance of average control energy of CAVs;
The performance of average control input of CAVs;
The performance of total fuel economy of CAVs;

Fig. 2. Platoon dynamics evolution with 7 CAVs under control protocol [2].

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & -12 \\
\end{bmatrix}
\cdot \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & 2 & 0 & 0 & 0 & 0 \\
-1 & -1 & -1 & 1 & 4 & 0 & 0 \\
-1 & -1 & -1 & -1 & 1 & 5 & 0 \\
-1 & -1 & -1 & -1 & -1 & 1 & 6 \\
\end{bmatrix}
\]

which are easy to implement by C-V2X infrastructure.

In this simulation, we consider that the preceding HDV is driving at a constant speed and far away from the platoon CAVs with a fixed reaction-time delay and realistic inter-vehicle constraints within the start stage [0, 0.4] (s) in a straight unsignalized road, where the initial velocities of the Leader CAV and the preceding HDV are assumed as $v_{p, o} = 20(m/s)$, $v_{b} = 18(m/s)$, and the initial headway of longitudinal is large than 70(m), respectively. The other parameters are the same as in the above analysis. According to the proposed ADSCAS, the initial cruising distance satisfies $D > D_{p,c}$ such that the traveling behaviors of the platoon CAVs under STATE 1-Free-cruising will not be affected by the preceding HDV. However, due to the larger CAV speed, the cruising distance will be reduced as $D < D_{p,c}$ after a few minutes so that the traveling behaviors of the platoon CAVs will be limited by the preceding HDV and the platoon CAVs will be in the STATE 2-Tracking-cruising. In this case, the ADSCAS will provide a reference speed for the Leader CAV so that the platoon can maintain a safe distance with respect to the preceding HDV. Then, for TA mode, we first plan the reference velocity for the Leader CAV under C-II. According to the conditions in part I [1], we get that $D_f = 14.3(m)$ under the parameters $\mu_1 = 0.1$ and $D_f0 = 12.5(m)$. Then, letting $\delta = 0.1$, the planning longitudinal reference velocity of the Leader CAV is $16.0699(m/s)$. With the parameter $\epsilon = 0.9$, $\kappa_p = 1$ and $\beta = 1$, and $t^{**} = 0(s)$, we have $a_{p,0} = -0.7840(m/s^2)$ and $T^{**} = 5.0125(s)$. Therefore, the Leader CAV will have the deceleration as $-0.7840(m/s^2)$ at $[40, 45.0125(s)]$ to adjust the cruising speed from 20m/s to 16.0699(m/s) such that the external-space distance $D_f = 14.3(m)$ can be ensured with the preceding HDV that travels with the constant speed 18(m/s). With the similar procedures for TB and TC modes, it is easy to check the feasible solutions of the equations (41) and (42) and the protocol (23) will be constructed under C-VI. Then, we can show the dynamic platoon management of CAVs under the communication profile. By setting the initial parameters of CAVs
and the tolerable level $\delta^p = 0.3$, the platoon gap error and the velocity of the CAVs systems under the optimal velocity functions $y_{p,i}(t)$ and difference velocity model $g_{p,i}(t)$ can be obtained, as shown in Figs 3 (a) - (f), respectively. The results show that, compared with the performance for TA and TC, the platoon gap errors in TB are smaller but the changes of the velocities for all CAVs are dramatic, and the smooth and safe platoon dynamics suggest the communication TC. In addition, the values of the maximum and minimum velocity and platoon gap error are given by Figs 3 (g) - (j), respectively, where the minimum platoon errors will be achieved in TC mode. Similarly, the performance of average tracking distance and average control energy are shown in Figs 3 (k) - (l), where TB mode employs great control energy to reduce the variance of average tracking distance of CAVs. The results of Figs. 3 (o) - (p) illustrate the relationship between average control input and total flue economy for different communication modes, where the smaller flue economy will cause the larger variance of control input.

C. Driving scenario II

We now consider the SCPs under the SDBs of the HDVs, where the different cruising speeds for SDBs will be considered as illustrated in Figure 4. In the simulations, we assume that the HDV is rude and has a longitudinal acceleration when the HDV crosses the junction as shown in Fig. 4 where the courteous HDVs always reduce the speed approaching the junction to ensure safety. Notably, whatever SDBs of HDVs, it will arise a challenge that the minimum safe distance between the HDV and the Leader CAV should be guaranteed as developed in Driving scenario II. In the simulations, due to limited space, we only test the rude HDVs and the platoon performance of SCPs for courteous HDVs can be achieved by using the same procedures according to our proposed ADSCAS. Since the HDVs are not controllable, the HDVs may accelerate with different driving speeds to accomplish the SCPs, and the platoon will be in the STATE 3-Platoon dynamic forming and STATE 4-Smart crossing control according to the proposed ADSCAS. Therefore, the Leader CAV needs to observe the trajectory of the HDVs to generate the reference trajectory planning and form the platoon in a finite time.

To do this, we first consider that the longitudinal velocity of the Leader CAV is a constant as $16.0699(m/s)$ and the HDV is a constant $18.0351(m/s)$ traveling on the roads, respectively. We assume that the HDV will accelerate for every junction with $x_f = 33(m/s)$ in the planning zone and the platoon forming will not be affected by the HDVs in the optimization zone. Firstly, we use Algorithm 1 to estimate the maximum allowable platoon size. Let the traffic parameters as follows: $l_1 = l_4 = 150(m), l_2 = l_3 = 50(m), Z_1 = 10(m), l_e = 5(m), t_1 = t_{g1} = 20(s), tr_2 = tg_2 = 10(s), tr_3 = tg_3 = 15(s), \rho^*_1 = 5(s), \rho^*_2 = 3(s), \rho^*_3 = 8(s)$ with being different acceleration regulation time corresponding to traffic lights $t_1, t_2, t_3$. Then, with Algorithm 1, it is obtained $N = 7, t = 6(s)$, and $T = 6.0254(s)$ under $\max(\delta) = 0.3$ for TA. Then, we can consider a CAV system consisting of seven vehicles to check the effectiveness of the proposed method under different communication topologies with the optimal cruising velocity $150/6.0254 = 24.8945(m/s)$ in the optimization zone. In the MATLAB simulations, the initial traffic light be a GREEN light. For the junctions 1#, 2#, and 3#, we can calculate the time interval of the current GREEN window to accomplish the SCPs for all CAVs and HDV according to the proposed ADSCAS respectively, as shown in Fig 5 (a), where the rude HDV always accelerates arriving the junctions as shown in Fig 5 (c) and (d). Clearly, the HDV and seven CAVs can finish the SCPs at the GREEN window, however, the safe external-space between the HDV and CAVs cannot be guaranteed, as shown in Figure 5 (b).

To solve this problem, we should consider the safe external-space in the STATE 4-Smart crossing control. With the same procedures, we have improved platoon performance as shown in Figs 5 (a) - (d), respectively, where the safe external-space between the HDV and CAVs can be guaranteed, as shown in Figs 6 (a) and (b). However, the maximum allowable number of the CAVs to accomplish the SCPs is changed, which causes two challenging problems: i) how to determine how many CAVs can finish the SCPs in the current GREEN window; and ii) how to manage multiple platoons to ensure the SCPs smoothly, as developed in our STATE 6-Traffic flow control.

To this end, for the first junction 1#, we can calculate the time interval of the current GREEN window at the planning zone. Then, the estimated allowable number of CAVs to accomplish the SCPs is five by using the developed algorithm and the platoon will be split as two platoons $p$ and $p + 1$, and the optimum target speeds of the leader CAVs will be designed for different GREEN windows. For the regulation, two leader CAVs will reach the optimum target speed by calculating an acceleration input signal $AutoC(t) = a_{p,0}(t)$, and all the follower CAVs need to estimate the unknown acceleration input signal to ensure the traffic performance and safety. With the numerical calculation, the speed and acceleration are satisfied with the inter-vehicle constraints. Then, for the current GREEN light $[0, 20](s)$, the platoon $p$ will cross the junction under the crossing time $20 - 6.025 = 13.975(s)$, i.e., the crossing displacements of the last CAV are larger than $390(m)$ under Topology A, and the platoon $p + 1$ will cross the junction under the next GREEN light $[40, 60](s)$ by using the time $45(s)$ by setting the $t^{**} = 15(s)$.

It is now assumed that the platoon will approach the second junction 2# after $30(s)$, where the rest time interval of the current GREEN window is $0(s)$, i.e., $[50, 60](s)$. Then, the platoon can only pass through the junction 2# in the next GREEN window. By the same procedures, we can estimate the allowable number of CAVs to accomplish the SCPs is six by using our developed algorithm and the platoon will be split into two platoons $p$ and $p + 1$. Then, after $d_1/v_{p,0} = 7.4111(s)$, the platoon $p$ will cross the junction under the next GREEN light $[60, 70](s)$ with the crossing time $20 - 7.4111 = 12.5889(s)$, and the platoon $p + 1$ will cross the junction under the next GREEN light $[80, 90](s)$ by using the time $35(s)$ by setting the $t^{**} = 5(s)$.

Furthermore, it is assumed that the platoon enters the optimization zone after $5(s)$, the junction 3# will change the traffic light from RED to GREEN, i.e., $[85, 90](s)$. By using the same calculation, we can estimate the allowable number of CAVs to accomplish the SCPs is seven by using our developed algorithm, and the platoon will be a whole $p$. That is, the platoon just needs to adapt the velocity of the leader CAV to ensure that all the CAVs can pass
Fig. 3. Platoon dynamics evolution with 7 CAVs under control protocol.
through the junction 3# in the nearest GREEN window. Then, after \(d_1/v_{p,0} = 6.2931(s)\), the platoon \(p\) will cross the junction under the next GREEN light \([90, 105](s)\) with the crossing time \(20–6.2931 = 13.7069(s)\).

Then, the platoon performance for SCPs under different communication topologies (TA, TB, TC), including the crossing displacements and platoon gap errors of CAVs systems under the car-following model \(u_{p,i}(t)\) and \(q_{p,i}(t)\) and acceleration input \(u_{p,0}(t)\) are shown in Figs. 7 (a) - (f), respectively. In addition, the maximum and minimum velocity and platoon gap errors, average control energy, control, tracking errors, and total fuel economy of platoon performance are also shown in Figs. 7 (f) - (p), respectively. Similar to the analysis in the Driving scenario I, the simulations show that our proposed framework can solve the platoon safe forming and fuel economic junction crossing problem under the different communication topologies by using C-V2X infrastructure, where smooth performance and lower fuel economy will be achieved for TC mode.

VII. Conclusion

In this article, the problems of distributed dynamic platoons control and junction crossing optimization are addressed for a mixed traffic flow of CAVs and social HDVs in a smart city. We have shown that despite the nonlinearity, non-smoothness, and uncertainty of mixed traffic flows in smart cities, the solution of safe, efficient, and fuel economic platoon control for the above problems are solved by our proposed ADSCAS framework that can provide the reference trajectory planning and the solution of the fuel economic optimization problems to ensure the SCPs for all HDVs and platoon CAVs. The cooperative controller design and stability analysis have been further characterized in part II, where a fully distributed nonlinear variable time headway space error strategy is developed to ensure the subsequent safe cruising, by exploring the cooperative perception of multiple neighbors stimuli and cooperative tracking of the leader CAV, also resulting in a heterogeneous traffic flow dynamic.

It is must be noted that although the proposed framework can respectively provide the reference trajectory to ensure the safe crossing for HDV and platoon, all the platoon CAVs may not accomplish the SCPs under the uncertain SDBs of the preceding HDVs due to the safe external-space requirements, which arises challenges for platoon splitting and merging and multiple platoon managements. With such designs, the problems of the heterogeneous traffic flow stability and the finite-time robust cooperative trajectory tracking optimization are studied. Various examples and applications are presented to illustrate the effectiveness of the developed approaches.

In addition, one of the limitations of the current works is that all the platoon CAVs travel only on a single lane, which may lead to a reduction in vehicle mobility. The maneuverability for platoons traveling on multiple lanes with SCPs, where the lanes management and leader CAV generation could be two challenging problems, will be developed in our future works.

References

[1] B. Wang and R. Su, “Distributed dynamic platoons control and junction crossing optimization for mixed traffic flow in smart cities-part I: fundamentals, theoretical and automatic decision framework,” IEEE Transactions on Automatic Control, p. Submitted, 2022.
[2] V. Turri, B. Besselink, and K. H. Johansson, “Cooperative look-ahead control for fuel-efficient and safe heavy-duty vehicle platooning,” IEEE Transactions on Control Systems Technology, vol. 25, no. 1, pp. 12–28, 2017.
[3] A. Vahidi and A. Eskandarian, “Research advances in intelligent collision avoidance and adaptive cruise control,” IEEE Transactions on Intelligent Transportation Systems, vol. 4, no. 3, pp. 143–153, 2003.
[4] J. Ploeg, N. Van De Wouw, and H. Nijmeijer, “Lp string stability of cascaded systems: Application to vehicle platooning,” IEEE Transactions on Control Systems Technology, vol. 22, no. 2, pp. 786–793, 2013.
[5] J. C. Zegers, E. Semsar-Kazerooni, J. Ploeg, N. van de Wouw, and H. Nijmeijer, “Consensus control for vehicular platooning with velocity constraints,” IEEE Transactions on Control Systems Technology, vol. 26, no. 5, pp. 1592–1605, 2017.
[6] K. Lidström, K. Sjöberg, U. Holmberg, J. Andersson, F. Bergh, M. Bjäde, and S. Mak, “A modular cacc system integration and design,” IEEE Transactions on Intelligent Transportation Systems, vol. 13, no. 3, pp. 1050–1061, 2012.
[7] S. Santini, A. Valente, A. Pescapè, M. Segata, and R. L. Cigno, “Platooning maneuvers in vehicular networks: A distributed and consensus-based approach,” IEEE Transactions on Intelligent Vehicles, vol. 4, no. 1, pp. 59–72, 2019.
[8] D. Zhang, Y.-P. Shen, S.-Q. Zhou, X.-W. Dong, and L. Yu, “Distributed secure platoon control of connected vehicles subject to doS attack: Theory and application,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, p. in press, 2020.
[9] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, “Dynamical model of traffic congestion and numerical simulation,” Physical review E, vol. 51, no. 2, p. 1035, 1995.
[10] D. Helbing and B. Tilch, “Generalized force model of traffic dynamics,” Physical review E, vol. 58, no. 1, p. 133, 1998.
[11] J. Sun, Z. Zheng, and J. Sun, “Stability analysis methods and their applicability to car-following models in conventional and connected environments,” Transportation research part B: methodological, vol. 109, pp. 212–237, 2018.
[12] B. Wang and R. Su, “Distributed observers-based cooperative platooning tracking control and optimization for connected automated vehicles with unknown jerk dynamics,” IEEE Transactions on Automatic Control, p. under review, 2021.
[13] Y. Li, L. Zhang, H. Zheng, X. He, S. Peeta, T. Zheng, and Y. Li, “Nonlane-discipline-based car-following model for electric vehicles in transportation-cyber-physical systems,” IEEE Transactions on Intelligent Transportation Systems, vol. 19, no. 1, pp. 38–47, 2017.
(a) The displacement of CAVs and HDV; (b) The platoon gap error of CAVs and safe external-space distance between HDV and CAVs;

(c) The velocity of CAVs; (d) The acceleration of CAVs.

Fig. 5. Platoon performance for SCPs with 7 CAVs under control protocol (23).

(a) The displacement of CAVs and HDV; (b) The platoon gap error of CAVs and safe external-space distance between HDV and CAVs;

(c) The velocity of CAVs; (d) The acceleration of CAVs.

Fig. 6. Improved platoon performance for SCPs with 7 CAVs under control protocol (23).

---

[14] Y. Li, C. Tang, S. Peeta, and Y. Wang, “Nonlinear consensus-based connected vehicle platoon control incorporating car-following interactions and heterogeneous time delays,” IEEE Transactions on Intelligent Transportation Systems, vol. 20, no. 6, pp. 2209–2219, 2019.

[15] A. Salvi, S. Santini, and A. S. Valente, “Design, analysis and performance evaluation of a third order distributed protocol for platooning in the presence of time-varying delays and switching topologies,” Transportation Research Part C: Emerging Technologies, vol. 80, pp. 360–383, 2017.

[16] A. Petrillo, A. Salvi, S. Santini, and A. S. Valente, “Adaptive multi-agents synchronization for collaborative driving of autonomous vehicles with multiple communication delays,” Transportation research part C: emerging technologies, vol. 86, pp. 372–392, 2018.

[17] B. Bayar, S. A. Sajadi-Alamdari, F. Viti, and H. Voos, “Impact of different spacing policies for adaptive cruise control on traffic and energy consumption of electric vehicles,” in 2016 24th Mediterranean Conference on Control and Automation (MED). IEEE, 2016, pp. 1349–1354.

[18] J. Chen, H. Liang, J. Li, and Z. Lv, “Connected automated vehicle platoon control with input saturation and variable time headway strategy,” IEEE Transactions on Intelligent Transportation Systems, vol. 22, no. 8, pp. 4929–4940, 2020.

[19] J. Chen, H. Liang, J. Li, and Z. Xu, “A novel distributed cooperative approach for mixed platoon consisting of connected and automated vehicles and human-driven vehicles,” Physica A: Statistical Mechanics and its Applications, vol. 573, p. 125939, 2021.

[20] R. Niroumand, M. Tajalli, L. Hajibabai, and A. Hajbabaie, “Joint optimization of vehicle-group trajectory and signal timing: Introducing the white phase for mixed-autonomy traffic stream,” Transportation research part C: emerging technologies, vol. 116, p. 102659, 2020.

[21] C. Chen, J. Wang, Q. Xu, J. Wang, and K. Li, “Mixed platoon control of automated and human-driven vehicles at a signalized intersection: dynamical analysis and optimal control,” Transportation Research Part C: Emerging Technologies, vol. 127, p. 103138, 2021.

[22] T. V. Baby, V. Bhattacharyya, P. K. Shahri, A. H. Ghasemi, and B. HomChaudhuri, “A suggestion-based fuel efficient control framework for connected and automated vehicles in heterogeneous urban traffic,” Transportation Research Part C: Emerging Technologies, vol. 134, p. 103476, 2022.

[23] L. Xiao, M. Wang, and B. Van Arem, “Realistic car-following models for microscopic simulation of adaptive and cooperative adaptive cruise control vehicles,” Transportation Research Record, vol. 2623, no. 1, pp. 1–9, 2017.

[24] C. Flores, P. Merdrignac, R. de Charette, F. Navas, V. Milanés, and F. Nashashibi, “A cooperative car-following/emergency braking system with prediction-based pedestrian avoidance capabilities,” IEEE Transactions on Intelligent Transportation Systems, vol. 20, no. 5, pp. 1837–1846, 2019.

[25] M. Di Bernardo, A. Salvi, and S. Santini, “Distributed consensus strategy for platooning of vehicles in the presence of time-varying heterogeneous communication delays,” IEEE Transactions on Intelligent Transportation Systems, vol. 16, no. 1, pp. 102–112, 2014.

[26] Ş. Sabău, C. Oară, S. Warnick, and A. Jadbabaie, “Optimal distributed control for platooning via sparse coprime factorizations,”
Fig. 7. Final performance of the platoons junction crossing for SCPs with 7 CAVs under control protocol (23) under TA, TB and TC modes.
IEEE Transactions on Automatic Control, vol. 62, no. 1, pp. 305–320, 2016.

[27] D. Jia and D. Ngoduy, “Platoon based cooperative driving model with consideration of realistic inter-vehicle communication,” Transportation Research Part C: Emerging Technologies, vol. 68, pp. 245–264, 2016.

[28] ——, “Enhanced cooperative car-following traffic model with the combination of v2v and v2i communication,” Transportation Research Part B: Methodological, vol. 90, pp. 172–191, 2016.

[29] M. Bando, K. Hasebe, K. Nakanishi, A. Nakayama, A. Shibata, and Y. Sugiyama, “Phenomenological study of dynamical model of traffic flow,” Journal de Physique I, vol. 5, no. 11, pp. 1389–1399, 1995.

[30] M. Bando, K. Hasebe, K. Nakanishi, and A. Nakayama, “Analysis of optimal velocity model with explicit delay,” Physical Review E, vol. 58, no. 5, p. 5429, 1998.

[31] M. Treiber, A. Hennecke, and D. Helbing, “Congested traffic states in empirical observations and microscopic simulations,” Physical review E, vol. 62, no. 2, p. 1805, 2000.

[32] B. HomChaudhuri, A. Vahidi, and P. Pisu, “Fast model predictive control-based fuel efficient control strategy for a group of connected vehicles in urban road conditions,” IEEE Transactions on Control Systems Technology, vol. 25, no. 2, pp. 760–767, 2016.

[33] Y. Zheng, S. E. Li, J. Wang, D. Cao, and K. Li, “Stability and scalability of homogeneous vehicular platoon: Study on the influence of information flow topologies,” IEEE Transactions on Intelligent Transportation Systems, vol. 17, no. 1, pp. 14–26, 2016.

[34] Y. Zheng, S. E. Li, K. Li, and L.-Y. Wang, “Stability margin improvement of vehicular platoon considering undirected topology and asymmetric control,” IEEE Transactions on Control Systems Technology, vol. 24, no. 4, pp. 1253–1265, 2016.

[35] T. Ruan, L. Zhou, and H. Wang, “Stability of heterogeneous traffic considering impacts of platoon management with multiple time delays,” Physica A: Statistical Mechanics and its Applications, vol. 583, p. 126294, 2021.

[36] J. Zhou and H. Peng, “Range policy of adaptive cruise control vehicles for improved flow stability and string stability,” IEEE Transactions on Intelligent Transportation Systems, vol. 6, no. 2, pp. 229–237, 2005.

[37] M. Di Bernardo, A. Salvi, and S. Santini, “Distributed consensus strategy for platooning of vehicles in the presence of time-varying heterogeneous communication delays,” IEEE Transactions on Intelligent Transportation Systems, vol. 16, no. 1, pp. 102–112, 2015.

[38] H. Zhang, F. L. Lewis, and A. Das, “Optimal design for synchronization of cooperative systems: state feedback, observer and output feedback,” IEEE Transactions on Automatic Control, vol. 56, no. 8, pp. 1948–1952, 2011.

[39] G. Wen, Z. Duan, G. Chen, and W. Yu, “Consensus tracking of multi-agent systems with lipschitz-type node dynamics and switching topologies,” IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 61, no. 2, pp. 499–511, 2014.

[40] S. Xiao, X. Ge, Q.-L. Han, and Y. Zhang, “Dynamic event-triggered platooning control of automated vehicles under random communication topologies and various spacing policies,” IEEE Transactions on Cybernetics, p. inpress, 2021.

[41] J. Hu, P. Bhowmick, F. Arvin, A. Lanzon, and B. Lennox, “Cooperative control of heterogeneous connected vehicle platoons: An adaptive leader-following approach,” IEEE Robotics and Automation Letters, vol. 5, no. 2, pp. 977–984, 2020.