Low temperature superfluid stiffness of \(d\)-wave superconductor in a magnetic field

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The temperature and field dependence of the superfluid density \(\rho_s\) in the vortex state of a \(d\)-wave superconductor are calculated using a microscopic model in the quasiclassical approximation. We show that at temperatures below \(T < \sqrt{H}\), the linear \(T\) dependence of \(\rho_s\) crosses over to a \(T^2\) dependence different from the behavior of the effective penetration depth, \(\lambda_{\text{eff}}^2(T)\). We point out that the expected dependences could be probed by a mutual-inductance technique experiment.

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\textit{Introduction.} While the mechanism of superconductivity and unusual nature of the normal state in high-temperature superconductors (HTSC) are not yet understood, there is a consensus that the zero field superconducting state has a \(d\)-wave superconducting energy gap, with nodes along the diagonals of the Brillouin zone \(\mathcal{B}\). The presence of the nodes results in a density of low energy quasi-particle excitations large compared with conventional \(s\)-wave superconductors even at the temperatures much smaller than the transition temperature, \(T < T_c\). Although these excitations are reasonably well described by Landau quasiparticles their presence brings in a qualitatively new quasi-particle phenomenology not encountered in conventional superconductors. Among many aspects of this new physics a major role is played by these low energy excitations in the mixed (or vortex) state \(\mathcal{B}\). Since all HTSCs are extreme type-II superconductors a huge mixed phase extends from the lower critical field, \(H_{c1} \sim 10 - 100\) Gauss to the upper critical field, \(H_{c2} \sim 100\) T. An important property of the vortex state was pointed out by Volovik \(\mathcal{B}\), who predicted, that in contrast to conventional superconductors, in \(d\)-wave systems the density of states (DOS) is dominated by contribution from excited quasi-particle states rather than the bound states associated with vortex cores. It was shown that in an applied magnetic field \(H\) the extended quasi-particles DOS, \(N(\omega = 0, H) \sim \sqrt{H}\) rather than \(\sim H\) as in the conventional case. This result was confirmed by specific heat measurements on high quality single crystals \(\mathcal{B}\). Subsequently the semiclassical treatment of \(\mathcal{B}\) was incorporated into the Green’s functions formalism extended to include the effects of impurity scattering \(\mathcal{B}\). Accounting for the impurity scattering which violates a simple \(\sqrt{H}\) dependence has improved the agreement between the theory and measurements of the electronic specific heat (see Refs. in \(\mathcal{B}\)).

A similar to the Volovik effect, the weak \((H < H_{c1})\) field response was studied by Yip and Sauls \(\mathcal{B}\). They predicted a direction dependent \textit{nonlinear} Meissner effect also associated with quasiclassical shift of the excitation spectrum due to the superflow created by the screening currents. Although initially an experimental evidence for such an effect was reported, the subsequent experiments \(\mathcal{B}\) did not confirm this effect.

Another interesting result was obtained by the \(\muSR\) measurements of temperature and field dependences of the effective in-plane penetration depth, \(\lambda_{\text{eff}}\) in single crystals \(YBa_2Cu_3O_6.95\) by Sonier et al. \(\mathcal{B}\). At high magnetic fields, they observed a flattening of \(\lambda_{\text{eff}}^2\) (defined in these experiments as the width of magnetic field distribution) at low temperatures in contrast to the linear \(T\) behavior expected in a clean \(d\)-wave superconductor. If one assumes \(\lambda_{\text{eff}}^2\) is proportional to the superfluid density, then such a flattening could, in principle, indicate an opening of a secondary gap at the nodes of a \(d\)-wave superconducting gap as was already suggested after the measurements of thermal conductivity \(\mathcal{B}\).

It was argued by Amin et al. \(\mathcal{B}\) that the simple relation \(\lambda_{\text{eff}}^2 \propto \rho_s\) is not valid for the penetration depth extracted in \(\muSR\) experiments at finite fields, so that using the proper definition \(\mathcal{B}\) of \(\lambda_{\text{eff}}\), which corresponds to the \(\muSR\) measured penetration depth, the observed behavior of \(\lambda_{\text{eff}}^2\) can be explained by a nonlocal London model for a \(d\)-wave superconductor. The behavior of the superfluid stiffness, defined as a second derivative of the free energy of the system with respect to a vector potential, in this case cannot be addressed by the \(\muSR\).

Nevertheless the superfluid density in itself could also be extracted, for example, from the measurements of the low frequency complex sheet conductance as done in two-coil mutual-inductance technique measurements on thin films \(\mathcal{B}, \mathcal{B}\). The superfluid density (stiffness) in this case is found from the inductive part \(\sigma_2(\omega)\) of the conductivity \(\mathcal{B}\)

\[
\frac{\rho_s(T)}{m} = \frac{c^2}{4\pi\varepsilon^2\lambda_{\text{eff}}^2(T)} = \frac{1}{c^2} \lim_{\omega \to \infty} \omega \sigma_2(\omega, T),
\]

where \(c\) is the electron charge, \(m\) is its mass (or, more generally, a mass of the charge carrier) and \(c\) is the velocity of light. It is very important to make a distinction between the London penetration depth \(\lambda_L\) appearing in \(\mathcal{B}\) and \(\lambda_{\text{eff}}\) deduced from the \(\muSR\) measurements \(\mathcal{B}, \mathcal{B}\).

Thus in this work we investigate the influence of the magnetic field directly on the superfluid stiffness in HTSC using the quasiclassical approximation at \(H_{c1} < H < H_{c2}\). Our result is that at low temperatures there is also a flattening of \(\rho_s\) which is not related to an opening
of the gap and is just the result of a Doppler shift of the quasiparticle energies in the vortex state.

Although a quantum mechanical treatment is certainly needed for a consistent treatment of the vortex state [1], the semiclassical approximation always provides a good starting point when a new physical property of the system is involved. Furthermore, as argued in [3] for the parameter range relevant to the study of HTSC, this approximation reproduces the energy spectrum of the near-nodal quasiparticles in a vortex state to a high degree of accuracy allowing also to include the effect of impurity scattering into the analysis. In particular, the thermal conductivity was studied using the semiclassical approximation [1, 22].

Superfluid stiffness in the presence of impurities. Superfluid stiffness is given by

$$\rho_s^\alpha (T, H) = \rho_0^\alpha - \rho_0^\alpha (T, H).$$

The normal fluid density, \(\rho_n\) in [1], calculated within the “bubble approximation” with dressed fermion propagators (i.e., with self-energy \(\Sigma\) due to the scattering on impurities included) but neglecting the vertex and Fermi liquid corrections, is

$$\rho_n^\alpha = \int \frac{d^2 k}{(2\pi)^2} \int_{-\infty}^{\infty} d\omega \frac{\omega v_F^\alpha v_F^\beta}{2T} \frac{\partial \omega}{4\pi i}$$

$$\times \text{tr} [G_A(\omega, k)G_A(\omega, k) - G_R(\omega, k)G_R(\omega, k)].$$

As shown in [22] the vertex corrections can be neglected if the impurity scattering potential is isotropic in \(k\)-space. Likewise the Fermi liquid corrections can be taken into account along the lines of Ref. [22]. In [3] \(G_{R,A}(\omega, k)\) are the retarded and advanced Green’s functions

$$G_{R,A}(\omega, k) = \frac{\omega \pm i\Gamma}{(\omega \pm i\Gamma)^2 - \xi^2(k)} - \Delta^2(k)$$

with the dispersion law \(\xi(k)\), the \(d\)-wave superconducting gap \(\Delta(k)\), and the Fermi velocity is \(v_F = \partial \xi(\omega)/\partial k\). The width \(\Gamma = -\text{Im} \Sigma(\omega)\) in [3] is the scattering rate due to impurities and other sources, e.g., disordered vortex lattice (see below). Following [22] we assume that around the nodes \(\Gamma\) is frequency and momentum independent, so that the presence of impurities is modeled by a widening of \(\delta\)-like quasiparticle peaks by a Lorentzian

$$A(\omega, k) = \frac{1}{2\pi i} [G_A(\omega - i0, k) - G_R(\omega + i0, k)].$$

$$\rho_s^\alpha (T, H) = \frac{\rho_0^\alpha}{\pi v^\alpha}$$

where \(\rho_0 = \pi v^\alpha\) is the scattering rate due to impurities and other sources, e.g., disordered vortex lattice (see below). Following [22] we assume that around the nodes \(\Gamma\) is frequency and momentum independent, so that the presence of impurities is modeled by a widening of \(\delta\)-like quasiparticle peaks by a Lorentzian

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Employing the low temperature regime, \(T \ll T_c\) the nodal approximation [22] we obtain that

$$\rho_s^\alpha (T, H) = \frac{v_F \delta_{\alpha \beta}}{\pi v^\alpha} J,$$

where \(v_\Delta = \partial \Delta(k)/\partial k\) is the gap velocity and

$$J = \frac{1}{\pi} \int_0^\infty d\omega \frac{\omega v_F}{2 T}$$

$$\times \left[ \arctan \frac{\Gamma^2 - \omega^2}{2 \omega \Gamma} - \arctan \frac{\Gamma^2 - \omega^2 + \Gamma_0^2}{2 \omega \Gamma} \right]$$

$$- \frac{\Gamma_0}{(\Gamma_0 - \omega)^2 + \Gamma^2} \left[ \frac{\Gamma_0^2}{(\Gamma_0 - \omega)^2 + \Gamma^2} \right]$$

with the cutoff energy \(\Gamma_0\) (as estimated in [1], \(\Gamma_0 \sim 1500K\)). Taking the no impurities limit, \(\Gamma \to 0\) one can recover from (6), (7) the known linear \(T\) dependence [22]

$$\rho_n(T) = \frac{2 \ln 2 v_F T}{\pi v^\alpha},$$

where we set \(\hbar = k_B = 1\). It is essential, however, to take into account the presence of impurities which modifies the low temperature \(T\) dependence of \(\rho_s\) from linear to \(T^2\) [21, 22]. Indeed in the limit \(\Gamma \gg T\) we get from (6), (7) that

$$\rho_n(T) = \frac{2 \ln 2 v_F T}{\pi v^\alpha}.\]$$

For clean YBCO monocrystals the estimated [21] value of the scattering due to impurities \(\Gamma_0 \sim 1 - 2K\), but in thin films the quadratic temperature dependence of \(\rho_s(T)\) is observed over a wide temperature range [10] implying that the value of \(\Gamma_0\) could be bigger, so that for our estimates we use \(\Gamma_0 = 10K\). However, as we will see below the obtained dependences on \(H\) are in fact even stronger for smaller values of \(\Gamma_0\).

Doppler shift. In the semiclassical approach to the vortex state the presence of a superflow is accounted for by introducing the Doppler shift into the energy \(\omega \to \omega - \epsilon(k, r)\) [1, 3], where \(\epsilon(k, r) = v_s(r) \cdot k\), and \(v_s(r)\) is the supervelocit field at a position \(r\) created by all vortices. There are several ways to treat the problem depending on the assumptions we made about the vortex lattice.

1. Completely disordered vortex state. Following [21] we assume that the vortex lattice is disordered, which is not unreasonable for thin films [10]. Then on the length scales large compared with the intervortex distance \(a_0 = \sqrt{\Phi_0/\pi B}\) (here \(\Phi_0 = h c/2 e\) is the flux quantum and \(B\) is the internal field), propagation of quasiparticles is described by the Green’s function averaged over the vortex positions \(\{R_i\}\). This averaging can be done using the probability density

$$\mathcal{P}(\eta) = \langle \delta(\eta - v_s(r) \cdot k) \rangle_{\{R_i\}},$$

(10)
so that the averaged Green’s function

$$\langle G(\omega, k) \rangle = \int_{-\infty}^{\infty} d\eta \mathcal{P}(\eta) G^0(\omega - \eta, k), \quad (11)$$

where $G^0(\omega, k) = G(\omega, k)$ from Eq. (1) with $\Gamma = 0$. For typical accessible fields $H_\alpha < H \ll H_\alpha$ the magnetization due to the vortex lattice is small, and the internal field $B$ can be replaced by the applied field, $H$ directed along the $c$-axis.

Assuming that the vortex positions are random and uncorrelated, one would get a Gaussian distribution

$$\mathcal{P}(\eta) = (2\pi E_H^2)^{-1/2} e^{-\eta^2/2E_H^2}$$

with

$$E_H^2 = k_\alpha k_\beta (v^\alpha_s(r)v^\beta_s(r)) \approx \left( \frac{h v_F}{2a_v} \right)^2 = \frac{\pi^2 v^2}{4} \frac{H}{\Phi_0}, \quad (12)$$

where we used the expression $|v_s(r)| = h/2mr$ (the Planck’s constant $h$ is restored in these expressions) for a supervelosity field created by a single vortex and took $r = a_v$. In what follows we will use an estimate $E_H[K] \sim 30\sqrt{H/T}$ obtained for the typical values of $v_F \approx (1.5-2.5) \times 10^7$cm/s in YBCO. To make an analytical calculations possible we follow Ref. [21] and replace the Gaussian probability distribution by a Lorentzian $\mathcal{P}(\eta) = \pi^{-1} E_H/(\eta^2 + E_H^2)$, so that

$$\langle G_R(\omega, k) \rangle = G^0_R(\omega + iE_H, k). \quad (13)$$

Thus within employed approximation the effect of quasiparticle scattering from the supervelosity field $v_s(r)$ created by a completely disordered vortex lattice also results in a widening of the quasiparticle peaks with a constant width $E_H$. Although this property may not be valid for other forms of the distribution functions, we find it useful for a physical insight to the problem.

We note that the used above Lorentzian distribution has another property that the average of the product of the Green’s functions is equal to the product of averaged Green’s functions, so that the individual Green’s function can be averaged over the vortex positions. For any other type of the distribution function this property is not valid and averaging should be done for the product of the Green’s functions itself (see below).

Eq. (13) may be used to calculate the field induced DOS, $N(\omega, H) = 1/(2\pi^2) \int d^2k tr A(\omega, k)$, so that $N(\omega = 0, H) \propto E_H n_0/\sqrt{H} \sim \sqrt{H}$ in the region $H \gg T$ with $H_0 \sim 2500$T. This is just DOS from [22] with $\Gamma$ replaced by $E_H$. Since $p_0 \gg E_H$ this result agrees with the dependence $N(\omega = 0, H) \sim \sqrt{H}$ obtained by Volovik [6]. Furthermore, as we will see below, the extra ln factor is caused by the assumption we made about the completely disordered vortex state. Note also, that the first temperature independent term of Eq. (4) may be identified as a DOS contribution to $\rho_s$.

Assuming also a Matthiessen type rule for the impurity and vortex contributions to the lifetime [27], we finally obtain that the field dependence of the normal fluid density is described by Eqs. (6), (7) with $\Gamma = \Gamma_0 + E_H$.

and its asymptotics for $T \gg \Gamma$ and $T \ll \Gamma$ are given by Eqs. (8) and (9), respectively. These results are presented in Fig. (2) (we used for computation some typical values of the parameters [23], e.g. $v_F/v_\Delta = 15$), where one can clearly see a crossover from $T^2$ dependence of $\rho_s(T)$ to the linear $T$ dependence at $T \sim \Gamma_0 + E_H$. For a clean system ($\Gamma_0 = 0$) and a single vortex averaging this crossover of $\rho_s(T)$ was also obtained in [22]. Thus this behavior of $\rho_s(T)$ indeed differs from the effective penetration depth $\lambda_{eff}^2(T)$ [13] which also crosses over from the linear $T$ dependence, but to a $T^3$ dependence at (for a clean system) $T^* \propto \sqrt{H}$. Note also that since the nodal approximation was used to arrive at Eqs. (14), (15) one cannot approach the region $\rho_s(T, H)/\rho_s(0, 0) \ll 1$, where the dependence of the superconducting gap on temperature should be taken into account.

In Fig. (3) we show the dependence of $\rho_s$ on the applied field $H$. It appears to be not very different from $\sim \sqrt{H}$ dependence obtained in [16] for the periodic vortex lattice at $T = 0$. The reason for this coincidence becomes more clear after we consider a vortex liquid case.

2. Vortex liquid. To consider a more ordered vortex liquid state we adopt a method used in [4]. In this case one introduces a Doppler shift $\epsilon_n(r) = v_s(r) \cdot k_n$ at the node $k_n = -k_3$ and $k_2 = -k_4$ for a $d$-wave superconductor, so that $\epsilon_1 = -\epsilon_3$ and $\epsilon_2 = -\epsilon_4$ to approximate the Doppler shift for the entire nodal region. Then if we know how to express a physical quantity $F$ in terms of the Green function $G(\omega, k)$ we can compute its local value $F(r)$ with the local “Doppler shifted” Green’s function, $G(\omega, k; \epsilon(k, r)) = G(\omega - \epsilon_n(r), k)$. We then approximate the field-dependent measured value $F(H)$ by the spatial average which depends on the magnetic field $H$ through $\epsilon_n$ as

$$A^{-1} \int d^2r F(\epsilon_1(r), \epsilon_2(r)) = \int d\epsilon_1 d\epsilon_2 F(\epsilon_1, \epsilon_2) \mathcal{L}(\epsilon_1, \epsilon_2),$$

where the first integral is taken over the part of a unit cell of the vortex lattice (with the area $A$) in real space where the Doppler shift is
we choose $P\ Lionel + E_{L}$. The dotted line is obtained using the first term of Eq. (14), i.e. for $\rho_{n} = \frac{\nu_{F}}{\pi v_{F}}E_{H}$.

much smaller than the gap maximum and in the second integral we used the distribution function

$$\mathcal{L}(\epsilon_{1}, \epsilon_{2}) = A^{-1} \int d^{2}r \delta(\epsilon_{1} - v_{s}(r)k_{1})\delta(\epsilon_{2} - v_{s}(r)k_{2}),$$

(14)

where $k_{1}$ and $k_{2}$ label two nearest nodes. If $\mathcal{L}(\epsilon_{1}, \epsilon_{2})$ depends on a single variable $\epsilon_{1}^{2} + \epsilon_{2}^{2}$ then a more simple distribution function $P(\epsilon) = \frac{1}{2(E_{H}^{2} + \epsilon^{2})^{3/2}}$ which is the most convenient for analytical calculations. Using this function one can obtain the following field induced DOS: $N(\omega = 0, H) \propto \sqrt{H}$ in the strong field, $\Gamma_{0} \ll |\epsilon_{n}|$ limit and $N(\omega = 0, H) \propto H \ln H_{0}/H$ in the impurity dominated, $\Gamma_{0} \gg |\epsilon_{n}|$ regime. This shows that this distribution function which allows to reproduce the original $\sim \sqrt{H}$ Volovik's result is indeed better for the description of a more ordered vortex glass or even lattice state.

Hence the local quantity $J$ from Eq. (5) has to be replaced by its local value $J(\epsilon_{n})$ and

$$\frac{\rho_{n}}{m} = \frac{\nu_{F}}{\pi v_{F}} \int_{-\infty}^{\infty} d\epsilon J(\epsilon)P(\epsilon).$$

(15)

One can obtain that the impurity dominated limit ($T \ll |\epsilon_{n}| \ll \Gamma_{0}$) is described by Eq. (5) with $\Gamma = \Gamma_{0}$, while in the field dominated regime, $T \ll \Gamma_{0} \ll |\epsilon_{n}|$ regime

$$\frac{\rho_{n}}{m} = \frac{\nu_{F}}{\pi v_{F}} \left[ E_{H} + \frac{2\Gamma_{0}}{\pi} \ln \frac{\rho_{0}}{E_{H}} + \frac{\pi T^{2}}{3} \frac{\pi}{2E_{H}} \right].$$

(16)

Thus irrespectively of the vortex structure we assumed the last $T^{2}$ term of (16) differs only by a factor $\pi/2$ from Eq. (5) taken with $\Gamma = \Gamma_{0} + E_{H} \sim E_{H}$ for the field dominated regime, $E_{H} \gg \Gamma_{0}$. It also agrees with [29], where, however, there is no second term related to impurities. The first term of (16) is already $\sim \sqrt{H}$, not $\sim \sqrt{H} \ln H_{0}/H$, exactly as one would expect from the DOS calculation and obtains for a single vortex averaging [29] and the vortex lattice as $T = 0$.[30]

To conclude we have shown using the quasiclassical approximation and the simplest form for the impurity scattering that the temperature $T^{*} \sim \Gamma_{0}$ of the crossover from the linear to the $T^{2}$ dependence of the superfluid stiffness $\rho_{s}(T, H)$ increases in the presence of magnetic field to the temperature $T^{*} \sim \Gamma_{0} + E_{H}$. The dependence of $\rho_{s}(T)$ is different from the effective penetration depth, $\lambda_{\text{eff}}^{2}(T)$. The experimental measurements of $\rho_{s}(T, H)$ are necessary to check our predictions against them.

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