Deep Policy Iteration with Integer Programming for Inventory Management

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Problem Definition: In this paper, we present a Reinforcement Learning (RL) based framework for optimizing long-term discounted reward problems with large combinatorial action space and state dependent constraints. These characteristics are common to many operations management problems, e.g., network inventory replenishment, where managers have to deal with uncertain demand, lost sales, and capacity constraints that result in more complex feasible action spaces. Our proposed Programmable Actor Reinforcement Learning (PARL) uses a deep-policy iteration method that leverages neural networks (NNs) to approximate the value function and combines it with mathematical programming (MP) and sample average approximation (SAA) to solve the per-step-action optimally while accounting for combinatorial action spaces and state-dependent constraint sets. Results: We then show how the proposed methodology can be applied to complex inventory replenishment problems where analytical solutions are intractable. We also benchmark the proposed algorithm against state-of-the-art RL algorithms and commonly used replenishment heuristics and find that the proposed algorithm considerably outperforms existing methods by as much as 14.7% on average in various supply chain settings. Managerial Insights: We find that this improvement in performance of PARL over benchmark algorithms can be directly attributed to better inventory cost management, especially in inventory constrained settings. Furthermore, in a simpler back order setting where optimal replenishment policy is tractable, we find that the RL based policy also converges to the optimal policy. Finally, to make RL algorithms more accessible for inventory management researchers, we also discuss the development of a modular Python library that can be used to test the performance of RL algorithms with various supply chain structures. This library can spur future research in developing practical and near-optimal algorithms for inventory management problems.

Key words: Multi Echelon Inventory Management, Inventory Replenishment, Deep Reinforcement Learning

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1. Introduction

Inventory and supply chain management have seen a tremendous shift over the past decade. With the advent of online e-commerce, supply chains have become more and more complex and global with increasingly connected physical flows (Young 2022). Naturally, the cost of managing these supply chains have increased over the years. Furthermore, the pandemic has lead to an increased
spending in managing such complex supply chains. In fact, a recent Wall Street Journal report states that US business logistics costs increased as much as 22% year-on-year \cite{Young2022}. One potential promising direction for managing such complex supply chains is to use Artificial Intelligence (AI) based inventory management solutions and explore the benefits it can provide.

AI and Reinforcement learning (RL) has led to considerable breakthroughs in diverse areas such as games \cite{Mnih2013}, robotics \cite{Kober2013} and others. RL provides a systematic framework to solve sequential decision making problems with very limited domain knowledge. In fact, one can leverage several state-of-the-art, open-source RL methods to learn a very good policy that maximizes long-run rewards of many sequential decision problems at hand. Therefore, it is not surprising that RL has recently also been applied to similar problems in various domains such as healthcare \cite{Yu2019}, supply chains \cite{Gijsbrechts2018, Oroojlooyjadid2021, Sultana2020} and more. Yet enterprise level operations applications of RL remains limited and quite challenging.

Consider typical operations management (OM) problems such as inventory management and network revenue management. These problems are generally characterized by large action spaces, often well-defined state-dependent action constraints and rewards, and underlying stochastic transition dynamics. For example, a firm managing the inventory across a network of nodes in the supply chain has to decide how much inventory to place across the different nodes of the network. To accomplish this efficiently, the firm has to overcome various challenges. This includes accounting for (i) the uncertain demand across the nodes in the network; (ii) a large set of locally feasible (and often combinatorial) actions since the firm decides on the number of units to allocate to different nodes; (iii) a large number of state-dependent constraints to ensure that the complete allocation vector is feasible; (iv) the trade-off between the immediate and the long term reward of actions.

RL methods use an environment, a live or simulated one, to sample the underlying uncertainty to generate reward trajectories, and in-turn estimates of different action policies by understanding the trade-off between long term and short term rewards. Nevertheless, large combinatorial action spaces with state-dependent constraints, as in the case of the OM problems described above, render enumeration based RL techniques over the action space, computationally intractable. Hence, in this paper, we present a specialized RL algorithm that resolves these challenges. In particular, we present a deep-policy iteration method that leverages neural networks (NNs) to approximate the value function and combines it with mathematical programming (MP) and sample average approximation (SAA) to solve the per-step-action optimally while accounting for combinatorial action spaces and state-dependent constraint sets. From an RL perspective, one can view this as replacing the actor in an actor-critic method with a math-programming based actor. We use this modified RL approach to provide benchmark solutions to inventory management problems.
with complexities that make analytical solutions even in simpler networks intractable (e.g. lost sales, fixed costs, dual sourcing, lead times in multi-echelon networks) and compare them with state-of-the-art RL methods and popular supply chain heuristics.

1.1. Contributions

We make the following contributions through this work:

1. We present a policy iteration algorithm for dynamic programming problems with large action spaces and underlying stochastic dynamics that we call Programmable Actor Reinforcement Learning (PARL). In our framework, the value-to-go is represented as a sum of immediate reward and sum of future discounted rewards. The future discounted rewards are approximated with a NN that is fitted by generating value-to-go rewards from Monte-Carlo simulations. Then, we represent the NN and the immediate reward as an integer program which is used to optimize the per-step action. As the approximation improves, so does the per-step-action, which ensures that the learned policy eventually converges to the unknown optimal policy. The approach overcomes the enumeration challenges and allows for easy implementation of known contextual state dependent constraints.

2. We apply the proposed methodology to the problem of optimal replenishment decisions of a retailer with a network of warehouses and retail stores. Focusing on settings where analytical tractability is not guaranteed, we analyze and compare the performance of PARL under different supply chain network structures (multi-echelon distribution networks with and without dual-sourcing and with lost sales) and network sizes (single supplier and three retailers, to up to 20 heterogeneous retailers with multiple intermediary warehouses). We find that PARL is competitive, and in some cases outperforms, state of the art methods (14.6% improvement on average across different settings studied in this paper). Our numerical experiments provide a comprehensive benchmark results in these settings using different RL algorithms (SAC, TD3, PPO and A2C), as well as commonly used heuristics in various supply chain settings (specifically, base stock policies on an edge and on a serial path). We also perform additional numerical experiments to analyze the structure of the learned RL based replenishment policies and find that (i) in a simpler back-order setting, the RL policy is near optimal as it also learns an order up-to-policy and (ii) in the other more complex settings, the higher profitability is on account of improved cost management across the supply chain network.

3. Finally, we open source our supply chain environment as a Python library to allow researchers to easily implement and benchmark PARL and other state-of-the-art RL methods in various supply chain settings. Our proposed library is modular and allows for researchers to design supply chains networks with varying complexity (multi-echelon, dual sourcing), size (number of retailers and warehouses) and reward structure (back order and lost sales settings). While our objective is similar
to Hubbs et al. (2020), we focus on inventory management problems specifically and provide more flexibility in the network design. We believe this library will make RL algorithms more accessible to the OR and inventory management community and spur future research in developing practical and near-optimal algorithms for inventory management problems.

1.2. Selected Literature Review

Our work is related to the following different streams of literature: (1) parametric policies, (2) approximate dynamic programming (ADP), (3) reinforcement learning (RL) and (4) mathematical programming based RL. As the literature in these topics, in general and even within the context of supply chain and inventory management specifically, is quite vast, we describe the state-of-the-art with select related literature that is most related to the current work.

**Parametric policies for inventory management:** The topic of inventory management has been studied for many years and it has a long history. The seminal work of Scarf (1960) shows that for a single node sourcing from a single supplier with infinite inventory, a constant lead time and an ordering cost with fixed and variable components, the optimal policy for back-ordered demand has a \((s,S)\) structure where \(S\) is referred to as the order-up-level based on the inventory position (sum of on-hand inventory plus that in the pipeline, i.e., a collapsed inventory state) and \(s\) an inventory position threshold, below which orders are placed. This \((s,S)\) policy is commonly referred to as the base stock policy.

For lost-sales demand settings, i.e., wherein demand excess of inventory is lost, when the lead times are non-zero, the structure of the optimal policy is unknown (Zipkin 2008a,b) even in the single node setting sourcing from a single retailer. Moreover, base stock policies generally perform poorly (Zipkin 2008a), except when penalty costs are large, when inventory levels are high and stock-outs are rare. In general, the optimal policy depends on the full-inventory pipeline, unlike the state-space collapse that is possible in back order settings, and the complexity grows exponentially in lead time. In these settings, Huh et al. (2009), Goldberg et al. (2016) respectively prove the following asymptotic optimality results (1) regarding the basestock policy when the penalties are high and (2) regarding the constant ordering policies when the lead times are large. Xin (2021) combines these results to propose a capped base stock policy that is asymptotically optimal as the lead time increases and shows good empirical performance for smaller lead times.

Sheopuri et al. (2010) prove that the lost sales problem is a special case of dual sourcing problem (a retail node has access to 2 external suppliers), and hence base stock policies are not optimal, in general. In the dual sourcing setting, various heuristic policies similar to the lost sales setting extend the base stock policy with a constant order and/or cap that splits the order between the two suppliers depending on the inventory position across each have been independently proposed.
These include the Dual-Index (Veeraraghavan and Scheller-Wolf 2008), Tailored Base-Surge (Allon and Van Mieghem 2010) and Capped Dual Index policies (Sun and Van Mieghem 2019).

Multi-echelon networks are those wherein there are multiple nodes, stages or echelons that hold inventory. Base stock policies are optimal only in special cases with back ordered demands without fixed costs with additional restrictive assumptions such as the a serial chain with back order penalty at the demand node (Clark and Scarf 1960) or the inability to hold demand in the warehouse in a 2-echelon distribution network (Federgruen and Zipkin 1984). We refer the reader to de Kok et al. (2018) for an extensive review multi-echelon models studied based on variety of modeling assumptions and supply chain network structures.

Despite the non-optimality of base stock policies (the use of \((s,S)\) policies with a collapsed state space, i.e., via inventory positions), they are popular both in practice and in the literature. For example, Özer and Xiong (2008) and Rong et al. (2017) propose competitive heuristics that compute the order-up to base stock levels for the multi-echelon distribution (tree) networks without fixed costs but with service level constraints and demand back ordering costs respectively, and show asymptotic optimality in certain dimensions in the 2-echelon case. Agrawal and Jia (2019) propose a learning-based method to find the best base stock policy in a single node lost sales setting with regret guarantees. Pirhooshyaran and Snyder (2020) develop a DNN-based learning approach to find the best order up-to levels in each link of a general supply chain network.

In this work, we present a deep RL approach and leverage it to solve a certain class of cost-based stochastic inventory management problems in settings where parametric optimal policies do not exist or are unknown, such as the multi-echelon supply chains with fixed costs, capacities, lost sales and dual sourcing. In these settings, we provide new empirical benchmarks wherein the algorithm we propose is able to outperform commonplace heuristics that are based on the base-stock policy.

**Approximate Dynamic Programming (ADP) and Reinforcement Learning (RL):** Our work is also related to the broad field of ADP (Bertsekas 2017, Powell 2007). This generally focuses on solving the Bellman’s equation (1) and has been an area of active research for many years. ADP methods typically use an approximation of the value function to optimize over computationally intractable dynamic programming problems. Popular algorithms can be classified into two types: model-aware and model-agnostic. Model-aware approaches use information on underlying system dynamics (known transition function and immediate reward) to estimate value-to-go and popular approaches include value iteration and policy iteration. These algorithms essentially start with arbitrary estimates of the value-to-go and iteratively improve these estimates to eventually converge to the unknown optimal policy. We refer the interested readers to Gijsbrechts et al. (2018) for an excellent review of ADP based approaches for inventory management.
Model-agnostic approaches circumvent the issue of partial or no knowledge of the underlying system dynamics by trial-and-error using an environment that generates immediate-reward and state-transitions, given the current state and a selected action. This latter framework is popularly referred to as Reinforcement Learning (RL) \cite{SuttonBarto2018}. Note that RL methods themselves are also categorized into model-based RL and model-free RL wherein in process of learning the optimal policy via the environment, the transition functions are also learned in the former method and not learned in the latter method.

In this paper, we focus on model-free RL methods. One of the popular model-free RL algorithms is Q-learning, wherein the value of each state-action pair is estimated using the collected trajectory of state and rewards based on different actions and then the optimal action is obtained by an exhaustive search. In classical RL methods, a set of features are chosen and polynomial functions of those features are used to approximate the value function \cite{VanRoy1997}. This had limited success initially, but after the deep-learning revolution and the availability of significant compute, the deep-RL (DRL) methods regained significant popularity and success as neural-nets were used to approximate the value function, thereby automating the step of feature and function selection. This led to many algorithmic breakthroughs including the development of a family of policy gradient methods called the actor-critic method \cite{Mnih2016}, in which neural networks are used to approximate both the value function and the policy itself - the policy with an actor network which encodes a distribution over actions. Despite their successes, DRL and actor-critic approaches suffer from several challenges, such as lack of robust convergence properties, high sensitivity to hyper parameters, high sample complexity, and function approximation errors in their networks leading to sub-optimal policies and incorrect value estimation (see, for discussions, \cite{Haarnoja2018, Maei2009, Fujimoto2018, Duan2016, Schulman2017, Henderson2018, Lillicrap2016}). To address these different issues, new variations of the actor-critic approach continue to be proposed, such as Proximal Policy Optimization (PPO), which tries to avoid convergence to a sub-optimal solution while still enabling substantial policy improvement per update by constraining the divergence of the updated policy from the old one \cite{Schulman2017}, and Soft Actor-Critic (SAC), which tries to improve exploration via entropy regularization to improve hyper parameter robustness and prevent convergence to bad local optima while accelerating policy learning overall \cite{Haarnoja2018}.

The current work is complimentary to this literature since we provide a principled way of factoring in known constraints and immediate reward explicitly in training, as opposed to having to implicitly infer/learn them. Additionally, our framework, as mentioned earlier, can be viewed as replacing the actor in the actor-critic method with a mixed-integer program. These aspects of our proposed approach potentially help address the known issues with actor critic methods, such
as reducing the sample complexity, improving robustness and reducing the risk of convergence to poor solutions, and removing the dependence on function approximation of the policy network which may be inaccurate due to over or under fitting or sampling from the data (e.g., due to under-exploration or sub-optimal convergence).

RL for inventory management: Early work that shows the benefits of RL for multi-echelon inventory management problems include [Van Roy et al. (1997), Giannoccaro and Pontrandolfo (2002), Stockheim et al. (2003)]. There has been a recent surge in using DNN-based reinforcement learning techniques to solve supply chain problems ([Gijsbrechts et al. 2018, Oroojlooyjadid et al. 2021, Sultana et al. 2020, Hubbs et al. 2020]). A DNN-based actor-critic method to solve the inventory management problem was studied in [Gijsbrechts et al. (2018)] for the case of single node lost sales and dual sourcing settings, as well as multi-echelon settings, and showed improved performance in the latter setting. [Oroojlooyjadid et al. 2021] show how RL can be used to solve the classical bear game problem where agents in a serial supply chain compete for limited supply. More recently [Sultana et al. 2020] use a multi-agent actor-critic framework to solve an inventory management problem for a large number of products in a multi-echelon setting. Similarly, [Qi et al. 2020] develop a practical end-to-end method for inventory management with deep learning. [Hubbs et al. 2020] show the benefit of RL methods over static policies like base stock in a serial supply chain for a finite horizon problem. We also refer the interested readers to an excellent overview and roadmap for using RL for inventory management in [Boute et al. 2021]. Unlike these papers, we adopt a mathematical programming-based RL actor and show the benefit over vanilla DRL approaches in the inventory management setting.

Mathematical programming (MP) based RL actor: MP techniques have recently been used for optimizing actions in RL settings with DNN-based function approximators and large action spaces. They leverage MP to optimize a mixed-integer (linear) problem (MIP) over a polyhedral action space using commercially available solvers such as CPLEX and Gurobi. A number of papers show how trained ReLU-based DNNs can be expressed as an MP with [Tjandraatmadja et al. 2020, Anderson et al. 2020] also providing ideal reformulations that improve computational efficiencies with a solver. [Ryu et al. 2019] propose a Q-learning framework to optimize over continuous action spaces using a combination of MP and a DNN actor. [Delarue et al. 2020, van Heeswijk and La Poutré 2019, Xu et al. 2020] show how to use ReLU-based DNN value functions to optimize combinatorial problems (e.g., vehicle routing) where the immediate rewards are deterministic and the action space is vast. In this paper, we apply it to inventory management problems where the rewards are uncertain.
2. Model and Performance Metrics

Model and notation: We consider an infinite horizon discrete-time discounted Markov decision process (MDP) with the following representation: states $s \in \mathcal{S}$, actions $a \in \mathcal{A}(s)$, uncertain random variable $D \in \mathbb{R}^{\text{dim}}$ with probability distribution $P(D = d|s)$ that depends on the context state $s$, reward function $R(s, a, D)$, distribution over initial states $\beta$, discount factor $\gamma$ and transition dynamics $s' = T(s, a, d)$ where $s'$ represents the next state. A stationary policy $\pi \in \Pi$ is specified as a distribution $\pi(\cdot|s)$ over actions $\mathcal{A}(s)$ taken at state $s$. Then, the expected return of a policy $\pi \in \Pi$ is given by $J_\pi = E_{s \sim \beta} V_\pi(s)$ where the value function is defined as

$$V_\pi(s) = \sum_{t=0}^{\infty} \mathbb{E}[\gamma^t R(s_t, a_t, D_t)|s_0 = s, \pi, P, T],$$

where the expectation is taken over both the immediate reward as well as the transition state. The optimal policy that maximizes the long term expected discounted reward is given by $\pi^* := \arg \max_{\pi \in \Pi} J_\pi$. Finally, by Bellman’s principle, the optimal policy is a unique solution to the following recursive equation:

$$V_{\pi^*}(s) = \max_{a \in \mathcal{A}(s)} \mathbb{E}_D \left[ R(s, a, D) + V_{\pi^*}(T(s, a, D)) \right]. \quad (1)$$

As discussed in §1.2, solving (1) directly is computationally intractable due to the curse of dimensionality. We take a hybrid approach where a model determines the immediate reward but value-to-go is model-free and is determined using trial-and-error in a simulation environment. We discuss the algorithm in detail next.

2.1. Algorithm

We propose a monte-carlo simulation based policy-iteration framework where the learned policy is the outcome of a mathematical program which we refer to as PARL: Programming Actor Reinforcement Learning (see Algorithm 1 and an illustrative block diagram (Figure 1).

As is common in RL based methods, our framework assumes access to a simulation environment that generates state transitions and rewards, given an action and a current state. PARL is initialized with a random policy. The initial policy is iteratively improved over epochs with a learned critic (or the value function). In epoch $j$, policy $\pi_{j-1}$ is used to generate $N$ sample paths, each of length $T$. At every time step, a tuple of $\{\text{state } (s_n^t), \text{reward } (R_n^t), \text{next-state } (s_{n+1}^t)\}$ is also generated from the environment that is then used to estimate the value-to-go function $\hat{V}_{\pi_j}^{\pi_j-1}$. The value-to-go function is represented using a neural network parametrized by $\theta$ which is generated by solving the following error minimization problem:

$$\min_{\theta} \sum_{n=1}^{N} \sum_{t=1}^{T} \left( R_{\text{cum},n}^t - \hat{V}_{\theta}^{\pi_j}(s_n^t) \right)^2,$$
where the target variable $R_{t}^{cum,n} := \sum_{i=1}^{T} \gamma^{i-t} R^*_i$ is the cumulative discounted reward from the state at time $t$ in sample path $n$, generated by simulating policy $\pi_{j-1}$. Once a $\hat{V}$ is estimated, the new policy using the trained value-to-go function is simply

$$\pi_j(s) = \arg \max_{a \in A(s)} \mathbb{E}_{D} \left[ R(s, a, D) + \gamma \hat{V}_{\theta}^{\pi_{j-1}}(T(s, a, D)) \right]. \quad (2)$$

Problem (2) resembles Problem (1) except that the true value-to-go is replaced with an approximate value-to-go. Since, each iteration leads to an updated subsequent improved policy, we call it a policy iteration approach. Next, we discuss how to solve Problem (2) to get an updated policy in each iteration.

Problem (2) is hard to solve because of two main reasons. First, notice that $\hat{V}^{\pi_{j-1}}$ is a neural network which makes enumeration based techniques intractable, especially for settings where the actions space is large and combinatorial. And second, the objective function involves evaluating expectation over the distribution of uncertainty $D$ that is analytically intractable to compute. We next discuss how PARL addresses each of these complexities.

2.2. Optimizing over a neural network

We first focus on the problem of maximization of the objective in (2). To simplify the problem we start by considering the case where demand $D$ is deterministically known to be $d$. Then, Problem (2) can be written as

$$\max_{a \in A(s)} R(s, a, d) + \gamma \hat{V}_{\theta}^{\pi_{j-1}}(T(s, a, d)),$$
where we have removed the expectation over the uncertain demand $D$. Notice that the decision variable, $a$ is an input to the value-to-go estimate represented by $\hat{V}$. Hence, optimizing over $a$ involves optimizing over a neural network which is non-trivial. We take a math programming based approach to solve this problem. First, we assume that the value-to-go function is a trained $K$-layer feed forward ReLU-network with input state $s$ and satisfies the following equations $\forall k = 2, \ldots, K$,

$$z_1 = s, \quad \hat{z}_k = W_{k-1} z_{k-1} + b_{k-1}, \quad z_k = \max\{0, \hat{z}_k\}$$

$$\hat{V}_\theta(s) := c^T \hat{z}_K.$$  

Here, $\theta = (c, \{(W_k, b_k)\}_{k=1}^{K-1})$ are the parameters of the value-to-go estimator. Particularly, $(W_k, b_k)$ are the multiplicative and bias weights of layer $k$ and $c$ is the weight of the output layer (see Figure 2). Finally, $\hat{z}_k, z_k$ denotes the pre- and post-activation values at layer $k$. ReLU activation at each neuron allows for a concise math programming representation that uses binary variables and big-M constraints (Ryu et al. 2019, Anderson et al. 2020). For completeness, we briefly describe the steps.

![Figure 2](image)  

**Figure 2** A representative NN that takes as an input, a 6-dimensional state space, and considers a 10 layer NN (including the output layer) with each internal layer containing 9 neurons. Each neuron is defined by weights (W) and bias (b), except for the output layer that is defined with parameter vector $c$. The output of each neuron uses ReLU activation and passes it as an input to the neurons in the subsequent layer.

Consider a neuron in the neural-network with parameters $(w, b)$. For example, in layer $k$ neuron $i$’s parameters are $(W_i^k, b_i^k)$. Assuming a bounded input $x \in [l, u]$, the output $z$ of that neuron can be obtained by solving following MP representation:
\[ P(w, b, l, u) := \arg \max_{x, z, y} \begin{align*}
&0 \\
z &\geq w^T x + b \\
z &\geq 0 \\
z &\leq w^T x + b - M^- (1 - y) \\
z &\leq M^+ y \\
x &\in [l, u] \\
y &\in \{0, 1\}. 
\end{align*}\]

Here,
\[ M^+ = \max_{x \in [l, u]} w^T x + b \quad \text{and} \quad M^- = \min_{x \in [l, u]} w^T x + b, \]
are the maximum and minimum outputs of the neuron for any feasible input \( s \). Note that \( M^+ \) and \( M^- \) can be easily calculated by analyzing the component-wise signs on \( w \). For example, let
\[ \tilde{u}_i = \begin{cases} u_i & \text{if } w_i \geq 0 \\
l_i & \text{otherwise.} \end{cases} \]

Similarly, let
\[ \tilde{l}_i = \begin{cases} l_i & \text{if } w_i \geq 0 \\
u_i & \text{otherwise.} \end{cases} \]

Then, simple algebra yields that \( M^+ = w^T \tilde{u} + b \) and \( M^- = w^T \tilde{l} + b \). Starting with the bounded input state \( s \), the upper and lower bounds for subsequent layers can be obtained by assembling the \( \max\{0, M^+\} \) and \( \max\{0, M^-\} \) for each neuron from its prior layer. We will refer to them as \([l_k, u_k]\) for every layer \( k \). This MP reformulation of the neural network that estimates the value-to-go is crucial in our approach. Particularly, since we can also represent the immediate reward directly in terms of the decision \( a \), and the feasible action set for any state is a polyhedron, we can now use the machinery of integer programming to solve Problem (2). We will make this connection more precise in §3 in the context of inventory management.

Next, we discuss how to tackle the problem of estimating expectation in Problem (2).

### 2.3. Maximizing expected reward with a large action space:

The objective in Problem (2) has an expectation that is taken over the uncertainty \( D \). Note that the uncertainty in \( D \) impacts both the immediate reward as well as the value-to-go via the transition function. Evaluating this expectation could be potentially hard since \( D \) generally has a continuous distribution and we use a NN based value-to-go approximator. Hence, we take a SAA approach [Kim et al. 2015] to solve it. Let \( d_1, d_2, \ldots, d_\eta \) denote \( \eta \) independent realizations of the uncertainty \( D \). Then, SAA posits approximating the expectation as follows
\[
\mathbb{E}_D[R(s, a, D) + \gamma \hat{V}_\pi^{\pi_j - 1}(\mathcal{T}(s, a, D))] \approx \frac{1}{\eta} \sum_{i=1}^\eta R(s, a, d_i) + \gamma \hat{V}_\pi^{\pi_j - 1}(\mathcal{T}(s, a, d_i)).
\]
Hence, Problem (2) becomes

$$\hat{\pi}^{\eta}_{j}(s) = \arg \max_{a \in \mathcal{A}(s)} \frac{1}{\eta} \sum_{i=1}^{\eta} R(s, a, d_i) + \gamma \hat{V}^{\pi_{j-1}}_{\theta}(T(s, a, d_i)).$$

(4)

Problem (4) involves evaluating the objective only at sampled points instead of all possible realizations. Assuming that for any $\eta$, the set of optimal actions is non empty, we show that as the number of samples, $\eta$ grows, the estimated optimal action converges to the optimal action. We make this statement precise in Proposition 1. The proof follows through standard results in the analysis of SAA and is provided in Appendix §A.

PROPOSITION 1. Consider epoch $j$ of the PARL algorithm with a ReLU-network value function estimate $\hat{V}^{\pi_{j-1}}_{\theta}(s)$ for some fixed policy $\pi_{j-1}$. Suppose $\pi_j, \hat{\pi}^{\eta}_{j}$ are the optimal policies as described in Problem (2) and its corresponding SAA approximation respectively. Then, $\forall$ $s$,

$$\lim_{\eta \to \infty} \hat{\pi}^{\eta}_{j}(s) = \pi_j(s).$$

Proposition 1 shows that the quality of the estimated policy improves as we increase the number of demand samples. Nevertheless, the computationally complexity of the problem also increases linearly with the number of samples: for each demand sample, we represent the DNN based value function estimation using binary variables and the corresponding set of constraints.

Algorithm 1 PARL

1: Initialize with random actor policy $\pi_0$.
2: for $j \in [\bar{T}]$ do
3:   for (epoch) $n \in [N]$ do
4:     Play policy $\pi_{j-1}$ for $T(1 - \epsilon)$ and random action for $\epsilon T$ steps starting with state $s_n^0 \sim \beta$.
5:     Let $R_{i}^{\text{cum},n} = \sum_{t=i}^{T} \gamma^{i-t} R_{t}^{n}$ and store tuple $\{s_{t}^{n}, R_{i}^{\text{cum},n}\}$ $\forall t = 1,..,T$.
6:   end for
7:   Approximate a DNN value function approximator by solving

$$\hat{V}_j = \arg \min_{\theta} \sum_{n=1}^{N} \sum_{i=1}^{T} (R_{i}^{\text{cum},n} - f(s_{t}^{n}, \theta))^2$$

8: Sample $\eta$ realizations of the underlying uncertainty $D$ and obtain a new policy (as a lazy evaluation, as needed) by solving Problem (4).
9: end for
Remark 1 (Incorporating Extra Information on the Underlying Uncertainty).

In many settings, the decision maker might have extra information on the underlying uncertainty $D$. In these cases, one can use specialized weighting schemes to estimate the expected value to go for different actions, given a state. For example, consider the case when the uncertainty distribution $P(D = d)$ is known and independent across different dimensions. Let $q_1, q_2, \ldots, q_\eta$ denote $\eta$ quantiles (for example, evenly split between 0 to 1). Also let $F_j$ and $f_j$, $\forall j = 1, 2, \ldots, \text{dim}$, denote the cumulative distribution function and the probability density function of the uncertainty $D$ in each dimension respectively. Let $d_{ij} = F_j^{-1}(q_i)$, $\forall i = 1, 2, \ldots, \eta, j = 1, 2, \ldots, \text{dim}$ denote the uncertainty samples and their corresponding probability weights. Then, a single realization of the uncertainty is a $\text{dim}$ dimensional vector $d_i = [d_{i1}, \ldots, d_{i,\text{dim}}]$ with associated probability weight $w_i^\text{pool} = w_{i1} \cdot w_{i2} \cdots \cdot w_{i,\text{dim}}$. With $\eta$ realizations of uncertainty in each dimension, in total there are $\eta^{\text{dim}}$ such samples. Let $Q = \{d_i, w_i^\text{pool}\}$ be the set of demand realizations sub sampled from this set along with the weights (based on maximum weight or other rules) such that $|Q| = \eta$. Also let $w_Q = \sum_{i \in Q} w_i^\text{pool}$. Then Problem (4) becomes

$$\hat{\pi}^\eta_j(s) = \arg \max_{a \in A(s)} \sum_{d_i \in Q} w_i \left( R(s, a, d_i) + \gamma \hat{V}_{\theta}^\eta_{j-1}(T(s, a, d_i)) \right),$$

(5)

where $w_i = w_i^\text{pool} / w_Q$. The computational complexity of solving the above problem remains the same as before but since we use weighted samples, the approximation to the underlying expectation improves.

3. Inventory Management Application

We now describe the application of PARL to an inventory management problem. We consider a firm managing inventory replenishment and distribution decisions for a single product across a network of stores (also referred to as nodes) with goal to maximize profits while meeting customer demands. As we will discuss next, our objective is to account for practical considerations in the inventory replenishment problem by accounting for (i) lead times in shipments; (ii) fixed and variable costs of shipment; and (iii) lost sales of unfulfilled demand. These problems are notoriously hard to solve and there are very few results on the structure of the optimal policy (see §1.2), but as we describe next, RL and particularly, PARL can be used to generate well performing policies on these problems.

3.1. Model and System Dynamics

The firm’s supply chain network consists of a set of nodes ($\Lambda$), indexed by $l$. For example, these nodes can denote the warehouses, distribution centers or retail stores of the firm. Each node in the supply chain network can produce inventory units (denoted by random variable $D_l^p$) and/or
generate demand (denoted by random variable $D^d_l$). Inventory produced at each node can be stored at the node, or it can be shipped to other nodes in the supply chain. For node $l$, $O_l$ denotes the upstream nodes that can ship to node $l$. At any given time, each node can fulfill demand based on the available on-hand inventory at the node. We assume that any unfulfilled demand is lost (lost-sales setting).

The firm’s objective is to maximize revenue (or minimize costs) by optimizing different fulfillment decisions across the nodes of the supply chain. Fulfillment at any node can happen both through transshipment between nodes in the supply chain, or from an external supplier. Each trans-shipment from node $l$ to $l'$ has a deterministic lead time $L_{ll'} \geq 0$ and is associated with a fixed cost $K_{ll'}$ and a variable cost $C_{ll'}$. The fixed cost could be related to hiring trucks for the shipment, while the variable cost could be related to the physical distance between the nodes. Each node $l$ has a holding cost of $h_l$. Each inventory unit sold at different nodes generates a profit (equivalently revenue) of $p_l$.

We discuss the system dynamics in detail next. Note that the dependence on the time period $t$ is suppressed for ease of exposition.

1. In each period $t$, the firm observes $I$, the inventory pipeline vector of all nodes in the supply chain. The inventory pipeline vector for node $l$ stores the on-hand inventory as well as the inventory that will arrive from upstream nodes. By convention, we denote $I^0_l$ to be the on-hand inventory at node $l$.

2. The firm makes trans-shipment decision $x_{l'l}$ which denotes the inventory to be shipped from node $l'$ to $l$ and incurs a trans-shipment cost ($\text{tsc}$) of

$$\text{tsc}_l = \sum_{l' \in O_l} [K_{ll'} \mathbb{1}_{x_{l'l} > 0} + C_{ll'} x_{l'l}] ,$$

for node $l$.

3. The available on hand-inventory at each node is updated so as to account for units that are shipped out, as well as units that arrive from other nodes, and units that are produced at this node. We let $\bar{I}^0_l$ denote this intermediate on-hand inventory, which is given by

$$\bar{I}^0_l = I^0_l + I^1_l + D^p_l + \sum_{l' \in O_l} x_{l'l} \mathbb{1}_{L_{ll'} = 0} - \sum_{\{l' \in A \mid l' \in O_{l'}\}} x_{ll'} .$$

4. The stochastic demand at each node, $D^d_l$ is realized and the firm fulfills demand from the intermediate on-hand inventory, generating a revenue from sales ($\text{rs}$) of

$$\text{rs}_l = p_l \min\{D^d_l, \bar{I}^0_l\} .$$
5. Excess inventory (over capacity) gets salvaged, and the firm incurs holding costs on the leftover inventory. The firm incurs holding-and-salvage costs (\(hsc\)) given by

\[
hsc_l = h_l \min \left\{ \bar{U}_l, \left[ \bar{I}^0_l - D^d_l \right]^+ \right\} + \delta_l [\bar{I}^0_l - D^d_l - \bar{U}_l]^+,
\]

where \(\bar{U}_l\) is the storage capacity of node \(l\). We let \(I^0_l := \min \left\{ \bar{U}_l, \left[ \bar{I}^0_l - D^d_l \right]^+ \right\}\) for ease of notation.

6. Finally, inventory gets rotated at the end of the time period:

\[
I^{i+1}_l = I^i_l + \sum_{l' \in O_l} x_{l'l} \mathbb{1}_{L(l') = j}, \quad \forall \, 1 \leq j \leq \max_{l' \in O_l} L(l').
\]

The rotated inventory becomes the inventory pipeline for the next time period. That is, \(I_{t+1} = I_t\).

The problem of maximizing profits can be now written as a MDP. In particular, the state space \(s\) is the pipeline inventory vector \(I\); the action space is defined by the set of feasible actions; the transition function \(T\) is defined by the set of next-state equations (which depend on the distribution of demand \(D^d_l\)); and the reward from each state is the profit minus the inventory holding and transshipment costs in each period. We can write the optimization problem using Bellman recursion. Let

\[
R_l(I, x, D) = rs_l - tsc_l - hc_l,
\]

denote the revenue per node as a function of the pipeline inventory, the trans-shipment decisions, and the stochastic demand and production. Then, the total revenue generated from the supply chain in each time period is

\[
\bar{R}(I, x, D) = \sum_{l \in \Lambda} R_l(I, x, D).
\]

Similarly, the optimization problem can be written as

\[
V(I) = \max_{x \in A(I)} \mathbb{E}_D [\bar{R}(I, x, D) + \gamma V(I')],
\]

where \(I'\) is implicitly a function of \(I, x\) and \(D\). As discussed before, this inventory replenishment problem takes exactly the same form as the general problem of §2. It can be solved using Bellman recursion which is unfortunately computationally intractable. Hence, in what follows we will discuss how the framework developed in §2.1 can be effectively used to solve such problems.
3.2. PARL for Inventory Management

Recall that the PARL algorithm solves Problem (2) to estimate an approximate optimal policy. In the inventory management context, this problem becomes

$$
\pi(\mathbf{I}) = \arg \max_{x \in \mathcal{A}(\mathbf{I})} \frac{1}{\eta} \sum_{i=1}^{\eta} \hat{R}(\mathbf{I}, x, d_i) + \gamma \hat{V}(\mathbf{I}') ,
$$

where as discussed before the next state $\mathbf{I}'$ is a function of the current state, action as well as the demand realization. We describe how this problem can be written as an integer program. First, since we consider the lost-sales inventory setting, we define auxiliary variables $s_{\mathbf{a}li}$ that denotes the number of units sold for node $l$ and demand sample $i$. Then, constraints

$$
\begin{align*}
    s_{\mathbf{a}li} &\leq d_{\mathbf{p}li} \\
    s_{\mathbf{a}li} &\leq \tilde{I}^0_{\mathbf{li}},
\end{align*}
$$

ensure that the number of units sold are less than the inventory on-hand and demand. Note that with discounted reward and time-invariant prices/costs, opportunities for stock hedging in future time periods due to the presence of a sales variable are not present, and that sales will exactly be the the minimum of demand and inventory. We also define auxiliary variables $B_{\mathbf{li}}$ that denotes the number of units salvaged at node $l$ for demand sample $i$. Then, constraints

$$
\begin{align*}
    \tilde{I}^0_{\mathbf{li}} &= I^0_l + I^1_l + d_{\mathbf{p}li} + \sum_{l' \in O_l} x_{\mathbf{l}l'1} L_{l'1=0} - \sum_{l' \in \Lambda \cap O_l} x_{\mathbf{l}l'1} \\
    I^0_{\mathbf{li}} &= \tilde{I}^0_{\mathbf{li}} - s_{\mathbf{li}} - B_{\mathbf{li}} \\
    I^{j+1}_{\mathbf{li}} &= I^j_l + \sum_{l' \in O_l} x_{\mathbf{l}l'1} L_{l'1=j}, \quad 1 \leq j \leq \max_{l' \in O_l} L_{l'1}, \\
    B_{\mathbf{li}} &\geq 0,
\end{align*}
$$

(9)

capture the next state transition, for each demand realization. Finally, the objective function has two components: the immediate reward, and the value-to-go. The immediate reward, in terms of the auxiliary variables can be written as

$$
R_l(\mathbf{I}, x, d_i) = p_l s_{\mathbf{a}li} - \sum_{l' \in O_l} \left[ K_{l'l} g_{l'1} + C_{l'l} x_{l'1} \right] - \left( h_l \tilde{I}^0_{l} + \delta B_{\mathbf{li}} \right).$$

Note that $g_{l'1}$ is a binary variable that models the fixed cost of ordering. Next, let $\mathbf{I}'_l$ denote the next state under the $i^{th}$ demand realization. Assume that the NN estimator for value-to-go has $\Psi$ fully connected layers with $N_k$ neurons in each layer with ReLU activation. Let $\text{neu}_{j,k}$ denote the $j^{th}$ neuron in layer $k$. Then, the outcome from the neurons in the first layer can be represented as

$$
(z_j, y_{j1}) := P(W_{1k}, b_{1k}, \mathbf{I}'^1_l, \mathbf{I}'^1_l), \forall j \in [1, \ldots, N_1].
$$
The output from this layer becomes the input of the next layer. Hence, let \( Z_1 := [z_{11}, z_{12}, \ldots, z_{1N_1}] \) denote the outcome of layer 1. Then, the output of each neuron of layer 2 can be now written as

\[
(z_{j2}, y_{j2}) := P(W_{2k}, b_{2k}, Z_1, Z_1), \quad \forall j \in [1, \ldots, N_2].
\]

Note that we have suppressed \( Z_1 \) from the outcome to de-clutter notation since it is fixed as an input to the problem (see Problem (3) for details). Continuing this iterative calculation, we have that \( \forall k = 2, \ldots, \Psi - 1 \)

\[
(z_{jk}, y_{jk}) := P(W_{2k}, b_{2k}, Z_{k-1}, Z_{k-1}), \quad \forall j \in [1, \ldots, N_k].
\]

Finally, letting \( c \) denote the weight vector of the output layer, we have that

\[
V(I') = c^T Z_{N_{\Psi}},
\]

where note that the value-to-go is implicitly a function of the decisions \( x \) since they impact the next-state \( I' \). Using this value-function approximation, the inventory fulfillment problem for each time period can be now written as

\[
\max_{x_{lt} \in Z^+, u_{kt} \leq x \leq u^H} \frac{1}{n} \sum_{i=1}^{n} \left[ R(I, x, d_i) + \gamma c^T z_{\Psi} \right]
\]

where

\[
R(I, x, d_i) = \sum_{l \in \Lambda} R_l(I_l, x_l, d_{li})
\]

\[
\text{safety}_li \leq d_{li} \quad \forall l \in \Lambda, i,
\]

\[
safety_i \leq \tilde{P}_{li} \quad \forall l \in \Lambda, i,
\]

\[
g_{li} \leq \alpha_{li} \quad \forall l \in \Lambda, i,
\]

\[
x_{li} \leq U_{li} g_{li} \quad \forall l \in \Lambda, i,
\]

\[
\tilde{P}_{li} = I_l^0 + I_l^1 + \delta_l^0 + \sum_{l' \in O_l} x_{l'l} - \sum_{l' \in O_l} x_{l'}, \quad \forall l \in \Lambda, i,
\]

\[
\tilde{I}_{li}^0 = \tilde{I}_{li}^0 - s_{li} - B_{li}^0, \quad \forall l \in \Lambda, i,
\]

\[
\tilde{I}_{li}^j = \tilde{I}_{li}^{j+1} + \sum_{l' \in O_l} x_{l'l} I_{l'l} = j - B_{li}^j, \quad \forall 1 \leq j \leq \max_{l' \in O_l} L_{l'l}, l \in \Lambda, i,
\]

\[
B_{li}^j \geq 0, \quad \forall j = 0, \ldots, \max_{l' \in O_l} L_{l'l}, l \in \Lambda, i,
\]

\[
(z_{2q}, y_{2q}) \in P(W_{1q}, b_{1q}, I', I') \quad \forall q \in N_1,
\]

\[
(z_{k+1,q}, y_{k+1,q}) \in P(W_{kq}, b_{kq}, z_{k,i}, z_{k,i}) \quad \forall q \in N_k, k = 2, \ldots, \Psi - 1.
\]
Remark 2 (Alternate Q-Learning Based Approach: ). We note that the proposed PARL methodology is based on using the value function estimate $\hat{V}(s)$ to optimize actions. Alternatively, one could consider solving the following problem

$$\pi_j(s) = \arg \max_{a \in A(s)} \hat{Q}^{\pi_j-1}(s, a),$$

where $\hat{Q}^{\pi_j-1}(s, a)$ is the estimated expected discounted reward from selecting action $a$ when in state $s$. We decided not to pursue the Q-learning approach due to multiple reasons. First and foremost is because of the larger size of the neural network needed to estimate the Q values. In particular, assume that the supply chain network is fully connected and that in each time period, the centralized planner decides on the number of units to be shipped from each supply chain node to another. Then, the action decision $a \in \mathbb{R}^{|\Lambda|^2}$. Hence, the input layer of the neural network approximating the Q-function will be of size $|I| + |\Lambda|^2$. In comparison, the the input layer in the value function based approach we propose is of size $|I|$, which is much smaller. Note that larger the input space, larger is the DNN network size used for approximation. Since the number of integer variables increase linearly with the network size, smaller networks are preferred to ensure computational feasibility. Second, in a Q-learning based approach, directly modeling the immediate reward information is infeasible unless one solves two optimization problems per action step. Next, we present results from various numerical experiments using the proposed algorithm.

4. Numerical Results

In this section we present numerical results on the performance of the proposed PARL algorithm on various supply chain settings. The objective is two-fold: (i) to benchmark the proposed algorithm and compare its performance with state-of-the-art RL and inventory management policies, and (ii) to discuss the usage of a open-source Python library to create inventory management simulation environments with implementation of various RL algorithms, for easy benchmarking in supply chain applications.

We model a representative supply chain network in this environment with at most three different node types:

- **Supplier**: Supplier node (denoted by $S$) in the network produce (or alternatively order) inventory to be distributed across the network. For example, these can be big or manufacturing sites that produce inventory units or a port where inventory from another country lands.

- **Retailer**: Retailer nodes (denoted by $R$) in the network consume inventory by generating demand for the product. For example, these can be retail stores that directly serve customers who are interested in purchasing the inventory units.
Figure 3 Example of different multi-echelon supply chain networks. In 1S-3R, a single supplier node serves a set of 3 retail nodes directly. In 1S-2W-3R, the supplier node serves the retail nodes through two warehouses. In 1S-2W-3R (dual sourcing), each retail node can is served by two distributors.

- **Warehouse**: Warehouse nodes (denoted by W) are intermediate nodes that connect supplier node to retailer nodes. They hold inventory and ship it to downstream retail nodes. Each of the nodes are associated with holding costs, holding capacities and spillage costs, while retailers are additionally associated with price, demand uncertainties and a lost-sales/backorder demand type, and suppliers with production uncertainties. The directed link between the nodes forms the supply chain network. Each link is associated with order costs, lead time and maximum order quantity. The environment executes on the ordering and distribution actions specified by the policy by first ensuring its feasibility using a proportional fulfillment scheme (as it cannot send more than the inventory in a node), samples the uncertainties, accumulates the reward (the revenue from fulfillment less the cost of ordering and holding), and returns the next state.

With this overview, we start by first describing the various supply chain networks that we will consider for the numerical study.

4.1. **Network structure and settings used for numerical experiments**

We consider 3 different multi-echelon supply chain network structures, inspired from real-world retail distribution networks.

1. **Two-echelon networks**: This network consists of one supplier that is connected with a set of heterogeneous retailers that differ from one another in terms of the holding costs, as well as the lead time to ship from the supplier to the retailer. We consider networks with varying number of retailers (3, 10 and 20 retailers), as well as a setting with high-versus-low production. This is done so as to test the performance of the algorithms in resource constrained settings where inventory units available to be shipped could be lesser than the demand generated across the network. Hence, the policy has to decide where to ship available inventory units. This setting is particularly inspired from fast-fashion retail where the number of units per stock keeping unit (SKU) is very low. These settings are henceforth referred to as the 1S-nR networks. We specifically study the 1S-3R, 1S-10R, 1S-20R in the low production setting and 1S-3R-High for the relatively higher production setting.
2. Tree-distribution networks: We study distribution networks with a tree-structure, specifically networks consisting of one supplier, two intermediate warehouses, and three retailers that are each served from one of the two warehouses. This models networks where in retailers own big warehouses that are geographically dispersed with each serving a set of retailer stores nearby. We analyze two different settings: \(1S-2W-3R\) where the supplier inventory is constrained, and \(1S^{inf}-2W-3R\) which is the more traditional infinite inventory setting, where the supplier is not constrained by the inventory. Note that only when the supplier inventory is infinite, denoted by \(S^{inf}\), we do not include its inventory level as part of the state space.

3. Distribution networks with dual-sourcing: This setting is similar to the prior setting, except that the retail store is connected to not a single, but to multiple warehouses, and in this case two. It models multi-echelon supply chains where retailers have big warehouses, sometimes farther away from demand centers with longer lead times and smaller regional warehouses that also serve the demand with shorter lead times but relatively higher costs. Hence, the retailer has to decide not only how much inventory to store, but also where to ship this inventory from amongst the two warehouses. We refer to this settings as \(1S-2W-3R\) (DS).

In Table 1 we present different parameters of the supply chain in each setting. We discuss some salient features of these settings below and we follow it by additional detail related to notations and other specifics to better understand the table.

1. Highly uncertain demand: In many real world settings (e.g., fast-fashion retail), demand for products is highly uncertain. Hence, we let demand distribution to have a low signal-to-noise ratio.

2. Non-symmetric retailers: The retailer node parameters are selected so that they differ in terms of (i) the holding costs (ii) the lead time from the upstream warehouse. This asymmetry implies that commonly used heuristics such as learning an approximate policy for one retailer, and identically applying it to other retailers would be highly sub-optimal.

3. Fixed ordering costs, holding capacities, non-zero leadtimes and lost sales: All these settings are very common in practice. For example, in retail B2C distribution networks, retailers lose sales opportunities if the item is not in the shelf. There is also limited shelf space compared to upstream warehouses. Furthermore, these are settings where optimal policies structures have not been characterized and analytical tractability is not guaranteed.

In all the settings, we assume deterministic, constant per-period production and that the variability is only in the demand. The parameters are provided by node type - Retailer (R), Supplier (S or \(S^{inf}\)) and Warehouse (W) - and then by links between them. Whenever they are provided in a list format, they correspond to the retailers and warehouses in a chronological order (i.e., \(R_1, R_2, R_3\) or \(W_1, W_2\)). Also, when there are more nodes or links than the parameters (few elements in the list specified in the table), it implies that the parameters list in repeated in a cyclic fashion.
Table 1 Environment parameters for different supply chains studied.

| Parameters                                      | 1S-3R-High | 1S-3R | 1S-10R | 1S-20R | 1S-2W-3R | 1S-2W-3R (DS) | 1S"w"-2W-3R |
|-------------------------------------------------|------------|-------|--------|--------|----------|---------------|-------------|
| Retailer demand distribution                    | [N(2,10)] | [N(2,10)] | [N(2,10)] | [N(2,10)] | [N(2,10)] | [N(2,10)] | [N(2,10)] |
| Retailer revenue per item                       | [50]       | [50]  | [50]   | [50]   | [50]     | [50]          | [50]        |
| Retailer holding cost                           | [1,2,4]    | [1,2,4] | [1,2,4,8] | [1,2,4,8] | [1,2,4]   | [1,2,4]      | [1,2,4]    |
| Retailer holding capacity                       | [50]       | [50]  | [50]   | [50]   | [50]     | [50]          | [50]        |
| Supplier production qty per step                | 15         | 10    | 25     | 40     | 10       | 10            | 100         |
| Supplier holding capacity                       | 100        | 100   | 150    | 300    | 100      | 100           | 500         |
| Warehouse holding cost                          | -          | -     | -      | -      | [0.5]    | [0.5, 0.1]    | [0.5]       |
| Warehouse holding capacity                      | -          | -     | -      | -      | [150]    | [150]         | [150]       |
| Spillage cost at S,W,R                           | [10]       | [10]  | [10]   | [10]   | [10]     | [10]          | [10]        |
| Lead time (S or W to R)                         | [1,2,3]    | [1,2,3] | [1,2,3] | [1,2,3] | [1,2,3]  | (1,5),(2,6),(3,7) | [1,2,3]    |
| Fixed order cost (S or W to R)                  | [50]       | [50]  | [50]   | [50]   | [50]     | [50]          | [50]        |
| Fixed order cost (S to W)                       | -          | -     | -      | -      | [0]      | [0]           | [0]         |
| Variable order cost (any link)                  | [0]        | [0]   | [0]    | [0]    | [0]      | [20]          | [20]        |
| Maximum order (any link)                        | [50]       | [50]  | [50]   | [50]   | [50]     | [50]          | [50]        |
| Initial inventory distribution (node or link)   | [U(0,4)]   | [U(0,4)] | [U(0,4)] | [U(0,4)] | [U(0,4)] | [U(0,4)]     | [U(0,4)] |

For example the lead time (S or W to R) for the environment 1S-10R is given by [1,2,3] and this implies that the lead time for links [S-R1, S-R2,..., S-R10] is (1,2,3,1,2,3,1,2,3,1). The notation for the distributions used are $N(\mu, \sigma)$ for a normal distribution with mean $\mu$ and standard deviation $\sigma$ and $U(a,b)$ discrete uniform between $a$ and $b$. Note that because demand is discrete and positive, when we use a normal distribution, we round and take the positive parts of the realizations. The lead-time list has a tuple representation in the dual-sourcing setting to represent the lead time of a retailer from the two different warehouses. For example (1,5) in the list represents the lead time for W1-R1 and W2-R2.

Next, we discuss different benchmark algorithms that we tested in this paper.

4.2. Benchmarks

We compare PARL with four state-of-the-art, widely used RL algorithms: PPO [Schulman et al. 2017], TD3 [Fujimoto et al. 2018], SAC [Haarnoja et al. 2018], and A2C [Mnih et al. 2016].

For the RL algorithms we used the tested and reliable implementations provided by Stable-Baselines3 [Raffin et al. 2019] under the MIT License. We made all our environment compatible with OpenAI Gym [Brockman et al. 2016] and to implement PARL we built on reference implementations of PPO provided in SpinningUp [Achiam 2018] (both MIT License). We ran RL baselines on a 152 node X 26 (average) CPU cluster (individual jobs used 1 CPU and max <1GB RAM), and PARL on a 13 nodes X 48 (average) CPU cluster (individual PARL job uses 16 CPUs for trajectory parallelization and CPLEX computations and average <4GB RAM). We use version 12.10 of CPLEX with a time constraint of 60s per decision step with 2 threads.
We additionally compare PARL with commonly used order-upto heuristics in supply chains: a popularly used \((s, S)\) base stock (BS) policy \(\text{(Scarf 1960)}\), here implemented for each link (see Appendix B.1 for implementation details); and a decomposition-aggregation (DA) heuristic \(\text{(Rong et al. 2017)}\) to evaluate echelon order-up to policies \((S,S)\) for a serial path in near closed form (see Appendix B.2 for implementation details). The latter heuristic is designed for tree-networks with backordered demand and no-fixed costs in the presence of infinite supply and it has near-closed form expressions with asymptotic guarantees under those settings. In the former heuristic, a simulation based grid search using the environment is performed to identify the best \((s, S)\) pair for every link assuming infinite supply but with all other complexities (lost sales and fixed costs) in-tact. In fact, due to these differences, we believe that the BS heuristic tends to outperform the DA heuristic in the settings we study, as we will see in the next section.

4.3. Testing Framework

Evaluating RL algorithms can be tricky as most methods have a large number of hyper parameters or configurations that could be tuned, and care must also be taken to capture the actual utility of an RL method via the evaluation and avoid misleading conclusions occurring due to improper evaluation \(\text{[Henderson et al. 2018, Agarwal et al. 2021]}\). In particular, since the same simulation environments are used to train and evaluate (test) a model, if a model is chosen based on its performance on that environment during a particular training run and the exact same model (model weights) used to then evaluate on the same environment for test scoring, this can lead to misleading conclusions. The particular model class and set of hyper parameters may only have performed so well due to random chance (i.e., the random initialization, action sampling, and environment transitions) during the particular training run, and due to selecting the best result throughout training. Nevertheless, this may not at all characterize how well different models or hyper parameters may enable learning good policies in general or work in practice. It is therefore important to use separate model hyper parameter selection, training and evaluation runs for each model. Additionally it is important to use multiple runs in each case to characterize the distribution of results one can expect with a particular algorithm. We follow such rigorous procedures with our testing framework and in the results reported here.

Specifically, for a given RL algorithm, we first use a common technique of random grid search over the full set of hyper parameters to narrow down a smaller set of key tuning hyper parameters and associated grid of values for a given domain (using multiple random runs with different random seeds per hyper parameter combination). Then, for each environment and RL method, we use multiple random training runs (10 in this paper) for each hyper parameter combination to evaluate each hyper parameter combination. We then select the set of hyper parameters giving the best
average result across the multiple random runs as the best hyper parameters to use for that method and environment. Finally, we perform the evaluation runs. For the selected best hyper parameters for a method and environment, we run a new set of 10 random training runs with new randomly instantiated models to get our final set of trained models. Then for each of the 10 models we evaluate the model on multiple randomly initialized episodes of multiple steps (20 episodes and 256 used in our experiments) to get a mean reward per model run. Then given the set of mean rewards from multiple model runs, we report the overall mean, median, and standard deviation of mean rewards for the particular model (model class and set of best hyper parameters) and environment - which characterizes what kind of performance we would expect to see for that given RL model approach on that environment, and how it varies through random chance due to random initialization and actions/ trajectories). This gives a more thorough and accurate evaluation and comparison of the different RL algorithms. Additionally we performed extensive hyper parameter tuning and focused on bringing out the best performance of the baseline DRL models - searching over 4700 hyper parameter combinations with the initial random grid search to narrow down smaller grids, and using final grids of 32-36 hyper parameter combinations (varying 3 or more hyper parameters) per method to perform final hyper parameter optimization per environment and method. Note that more details on the hyper parameter tuning is provided in §4.4.3.

4.4. Performance, policy analysis and parameter tuning

We start by first presenting the results of different algorithms in different settings and then give more details on the learned policy by different RL algorithms, as well as the hyper parameter tuning performed for each algorithm.

4.4.1. Performance: In Table 2 we present the average per step reward (over test runs) of the different algorithms and compare them to PARL in 7 different settings described earlier. We additional provide the percentage improvement over two widely used methods: PPO and BS. We observe that PARL is a top performing method, in fact it outperforms all benchmark algorithms in all but one setting, 1Sinf-2W-3R. On average across the different supply chain settings we study, PARL outperforms the best performing RL algorithm by 14.7% and the BS policy by 45%.

Notably, the improvements are higher in supply chain settings that are more complex (1S-20R, 1S-10R, 1S-2W-3R and 1S-2W-3R (DS)) amongst the settings tested in the paper. While in the 10R and 20R settings, the retailer has to optimize decisions over a larger network with larger action space, 1S-2W-3R and 1S-2W-3R (DS) are multi-echelon settings with more complex supply chain structure. Similarly, in the 1S-3R setting, the supplier is more constrained than the 1S-3R-High setting, which makes the inventory allocation decision more complex. PARL’s ability to explicitly factor in known
state-dependent constraints enables it to out-perform other methods in these settings. Settings 1S-3R-High and 1S\textsuperscript{inf}-2W-3R have relatively high supplier production, where BS and related heuristics like DA are known to work well. Here PARL is on par with the BS heuristic (within one standard deviation of the BS heuristic’s performance) and out-performs other RL methods. In these settings, the combination of surplus inventory availability, coupled with low holding cost and high demand uncertainty, encourage holding high inventory levels over long time-horizons. This makes reward attribution for any specific ordering action harder, and thus these settings are harder to learn for RL algorithms.

| Setting         | SAC  | TD3  | PPO  | A2C  | BS   | DA   | PARL   | PARL over PPO / BS |
|-----------------|------|------|------|------|------|------|--------|-------------------|
| 1S-3R-High      | 398.0 ± 3.2 | 329.6 ± 45.2 | 307.0 ± 1.6 | 392.4 ± 4.4 | 313.7 ± 4.1 | 303.2 ± 2.2 | 400.3 ± 3.3 | 513.0 / 0.3% |
| 1S-10R          | 870.5 ± 68.9 | 744.4 ± 71.4 | 786.1 ± 40.5 | 660.5 ± 2.1 | 651.9 ± 1.6 | 513.0 / 0.3% |
| 1S-20R          | 1216.1 ± 25.2 | 1098.0 ± 43.3 | 1079.8 ± 63.4 | 1117.3 ± 37.5 | 882.7 ± 3.0 | 851.9 ± 1.5 | 1379.2 ± 29.5 | 1015.7 / 9.6% |
| 1S-2W-3R        | 374.2 ± 4.7 | 361.1 ± 15.4 | 377.5 ± 3.7 | 360.2 ± 21.2 | 300.8 ± 5.4 | 287.2 ± 1.9 | 398.3 ± 2.5 | 285.3 / 9.9% |
| 1S-2W-3R (DS)   | 344.1 ± 20.6 | 259.3 ± 32.3 | 287.8 ± 5.3 | 327.5 ± 32.7 | 166.2 ± 3.8 | 158.3 ± 1.7 | 405.4 ± 2.0 | 55.5 / 32.4% |
| 1S\textsuperscript{inf}-2W-3R | 40.6 ± 59.8 | 21.0 ± 57.0 | 136.8 ± 18.4 | 62.9 ± 33.7 | 208.8 ± 5.2 | 163.2 ± 4.8 | 206.1 ± 9.0 | 4.5 / 143.9% |

Table 2: Average per-step-reward with standard deviation and median (next line) of different benchmark algorithms, averaged over different testing runs. We bold all top performing methods: those with performance not statistically significantly worse than the best method, using one standard deviation.

We also analyze the rate of learning of different algorithms during training. In Figure 4 we plot the average per-step reward over training steps from 3 different environments. We find that in each case, the PARL actor performs much worse in the initial, very early training steps on account of optimizing over a poorly trained critic. Once the critic improves in accuracy, PARL is able to recover a very good policy during training. Furthermore, we generally observe improved sample complexity of PARL compared to the DRL baselines from these learning curves. That is, we observe that PARL often converges to its final high-reward solution in fewer total steps with the environment (in some cases much fewer steps) compared to the DRL baselines.

Finally, we report algorithm run-times. The average per-step run time of the PARL algorithm is 0.178, 0.051, 0.050, 0.089, 0.051, 0.044 and 0.042 seconds in the 1S-3R-High, 1S-3R, 1S-10R, 1S-20R, 1S-2W-3R, 1S-2W-3R (DS) and 1S\textsuperscript{inf}-2W-3R settings, respectively. The average per step run time is highest in the 1S-3R-High setting. This is due to the larger feasible action set in this setting. In all other settings, the run time remains below 0.10 seconds. We note that during training, we use 8 parallel environments to gather training trajectories, and use 2 CPLEX threads per environment. The run time can improve further by increasing parallelization. In contrast, the average per-step
run time of PPO (the DRL algorithm that performs the best in most settings) is 0.007, 0.005, 0.010, 0.008, 0.008, 0.008, and 0.007 seconds respectively. Clearly, PPO outperforms PARL in terms of run-time. This is because while per-step action in PARL is an outcome of an integer-program, DRL algorithms take gradient steps that are computationally much faster.

4.4.2. Analyzing learned replenishment policy from RL algorithms: The numerical results from the experiments above show that RL algorithms can provide substantial gains in complex supply chain settings. Naturally, the next important question is how are RL algorithms achieving such improved performance and how good are these policies? We answer both these questions in what follows.

Why are RL algorithms performing better in some settings?: In Figure 7, we plot the costs, revenue and reward breakdown of different algorithms in different supply chain network settings. In the 1S-3R-High setting (see Figure 5a), we find that while PARL incurs lower total costs (ordering plus holding), but nevertheless all algorithms perform equally well in terms of the overall reward since BS and PPO are able to compensate for higher inventory costs with higher revenue. Similar insights continue to hold in the other high inventory 1Sinf-2W-3R setting (see Figure 5c), with the exception of the PPO based policy, whose performance deteriorates in comparison to the other policies. This is on account of lost sales which results in lower revenue and hence rewards. As we decrease available inventory (see Figure 5b), the performance of the BS policy deteriorates since it incurs high ordering costs due to proportional fulfillment. Note that in this setting, both PARL and PPO are better able to trade-off between costs and revenue. As we increase the complexity of the underlying network, PARL’s performance improves mainly due to lower costs. In particular, in the largest network with 20 retailers (see Figure 5d), PARL incurs much lower ordering cost in comparison to the other algorithms. Nevertheless, its holding costs are higher than both the other algorithms. This in turn implies that PARL makes fewer orders (but with larger order sizes) which ensure low overall inventory costs. Cost and reward comparison for other settings are provided in §B of the Appendix.
Bench-marking with known optimal policy: All the settings considered so far in the paper have no analytical optimal solution. Hence, while PARL performs better than known heuristics, its performance might still be far from the optimal policy. Therefore, to compare PARL’s performance with an optimal policy and to easily visualize the learnt policy, in what follows, we consider a simplified setting with one retailer and one supplier with infinite inventory (1S\textsuperscript{inf}-1R) with back-ordered demands, non-zero lead time, and no-fixed costs (see Appendix B.3 for full details on the different parameters). In this setting, it is well known that the optimal policy is a order up-to base-stock policy (i.e., order in each period to maintain an inventory position, which is on-hand plus pipeline, up to this level). The optimal policy is easy to compute as it has a closed form solution. Therefore, we compare PARL’s learnt policy with the optimal underlying base stock policy. For the chosen parameters, in this setting, the order up-to-level is 27 (see details of this calculation in §B.3 of the Appendix). In Figure 6, we present the learned policy from two different DRL algorithms. On the x-axis we plot the inventory position (sum of on-hand and pipeline inventory), and on the y-axis we plot the optimal action taken by the policy. Notice that interestingly, the optimal policy learned by the DRL algorithms is also an order-up-to policy. While this result is not surprising for existing DRL algorithms, especially given the recent success of using off-the-shelf RL methods for inventory management, the analysis highlights the near optimal performance of PARL and also demonstrates that the learned optimal policy mimics the optimal parametric policy in this setting.
Figure 6  Comparing the optimal ordering policy of PARL and SAC in the $1S^{inf}$-1R backorder setting. Here, the optimal static policy is an order-up to policy with 27 units being the order up to level. Observe that both PARL and SAC are able to learn the optimal order-up-to policy across various on-hand inventory states observed in test (99.9% of the time). Higher variance in comparison to SAC for PARL can be attributed to its deterministic policy structure that optimizes over the learned critic with a four dimensional input state.

Having discussed the overall performance of the various benchmark algorithms, we discuss hyper-parameter tuning next, that plays a very important role in the performance of different algorithms.

4.4.3. Hyper-parameter tuning of RL algorithms: Given the number of hyperparameters that need to be selected for running RL algorithms, we divide them in two sets: (i) parameters that were fixed to be the same across different benchmark algorithms and (ii) parameters that were tuned as we trained different benchmark algorithms.

**Fixed hyper parameters:** Here we report the fixed set of hyper parameters used by all methods. We fix batch size, the NN architecture, the neuron activation function, the state space representation\(^1\) the action representation, as well as the epoch length for generating trajectories. These were determined based on two factors: (1) the commonly used settings across the RL literature (for example 64x64\(^2\) NN architecture and batch size of 64 is most commonly used across many

---

\(^1\) States and actions can be represented in different ways, which affect modeling outcomes and so can be treated as a hyper parameter. Discrete means each possible state (i.e., combination of inventory values) or action (i.e., combination of order actions per entity) is given a unique index (so a policy network would output a probability for each possible index - i.e., state or action variable value combination) and a policy network gives a probability for each discrete action. Multi-discrete means each different state or action variable is encoded with its own set of discrete values and the policy network gives a probability for each state of each variable (e.g., output a probability over a discrete set for each order action to be taken). Finally, continuous (and normalized continuous) means the state and action variables are treated as continuous values - so the policy network outputs a single value for each action variable (i.e., each order action) and the probability is modeled via a continuous distribution, most often Gaussian.

\(^2\) 64x64 represents a NN with two layers, each of fully connected 64 neurons.
different problems and methods), and by sampling random combinations from a large grid of hyper parameters and comparing results trends to narrow down the set of hyper parameters to consider to consistently well-performing values and reasonable ranges.

As such, this was an iterative process where we tried a range of hyper parameters, then refined. For example, for the network architecture size, we also tried larger sizes including 128x128, 512x512, 1024x1024, 128x128x128, 512x256, 1024x512, 1024x512x256, 128x32, 512x128, 512x256x64. Nevertheless, larger sizes did not see considerable improvement, but led to significant increase in the run-time. Similarly, we tried batch sizes of 32, 64, and 128. But, we found no significant differences in the performance and hence selected the most commonly used 64 as the batch size. To select the activation function, we tested both ReLU and tanh activation. We found that ReLU activation performs as good or better than tanh (overall it gave close but slightly better results). Hence, we fixed the activation to ReLU across methods for fair comparison, and for ease of implementation with more efficient optimization methods [Glorot et al. 2011, Hara et al. 2015]. Similarly, for the state and action space representation, we tested discrete, multi-discrete and continuous representations (normalized between -1 and 1). We found that both continuous state and action representations work best in these settings.

Finally, we also comment on some algorithm specific hyper parameters. For the number of internal training iterations per collected buffer (PPO-specific setting) we tried 10, 20, and 40, and 70 and found the best results with 10-20, so fixed this parameter to be 20. For the number of steps per epoch / update (PPO and A2C-specific setting) we tried 512, 1024 and 2048 and found the larger number to give better results generally so fixed this to 2048. For PPO, we also found early stopping the policy update per epoch, based on KL-divergence threshold of default 0.15, to consistently provide better results than not using this. The final set of fixed hyper-parameters used across all experiments, models, runs, and environments are given in Table 3.

| Batch size | Net arch. (hidden layers per net) | Activation | State representation | Action representation | Epoch length |
|------------|-----------------------------------|------------|----------------------|----------------------|--------------|
| 64         | 64x64                             | ReLU       | continuous (normalized) | continuous (normalized) | 2048         |

Hyper-parameter search: Next, we discuss the set of hyper-parameters that were tuned for each supply chain setting during training for different algorithms. Table 6 of Appendix §C describes the set of parameters for PPO and A2C. Similarly, Table 7 of Appendix §C describes the set of parameters for SAC and TD3. For PARL, a common set of hyper-parameters were used across all
settings, with the exception of the discount factor, \( \gamma \), for the infinite supplier inventory setting, since we observed universally higher gamma being necessary for the baseline DRL methods (see Table 5). The discount factor was set to 0.75 (0.99 for the infinite supplier inventory setting only), learning-rate was set to 0.001, and the sample-averaging approach used was quantile sampling with 3 demand-samples per step. In each case, this set of hyper parameters was then used for the evaluation of the RL model - by retraining 10 different times with different random seeds using those best hyper parameters for each method, and reporting statistics on the 20-episode evaluations of the best epoch model across the 10 runs. Besides the parameters mentioned here, all other parameters were set at their default values in the Stable Baselines 3 implementation (see API3 for more details). Note that our experiments revealed that standard gamma value of 0.99 consistently gave much poorer results than smaller gamma values for most environments (we experimented with 0.99, 0.9, 0.85, 0.8 and 0.75) - so we included smaller gamma values in our hyper-parameter grids but still kept the option of the traditionally used 0.99 gamma for the benchmark RL methods in case they were able to factor in longer-term impact better. Finally, note, default optimizers are used: ADAM (Kingma and Ba 2015) for all except A2C which uses RMSprop (Hinton et al. 2012) by default.

**Evaluation of hyper parameters:** In Table 5 of Appendix C, we show the selected set of best hyper parameters used for each benchmark RL method and environment. These were selected based on the parameter setting that gave the best average reward (maximum over the training epochs), averaged across 10 different model runs for that hyper-parameter combination.

### 4.5. Open source tool for bench-marking

In this section, we briefly discuss the development and usage of an open-source Python library that can be used to facilitate the development and benchmarking of reinforcement learning on diverse supply chain problems4.

The library is designed to make it easy to define arbitrary, customizable supply chain environments (such as used in this paper) and to easily plug in different RL algorithms to test them on the environments. Different supply chain configurations can easily be defined via code or configuration files, such as varying supply chain network structures, capacities, holding costs, lead times, demand distributions, etc. For example the configuration file for the library defining the 1S-2W-3R supply chain environment used in this paper, with one supplier, 2 warehouses and three retailers, under a lost sales setting is shown in Listing 1.

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3 https://stable-baselines3.readthedocs.io/en/master/

4 We plan to release the library as open source upon publication.
[conf_type]
conf_type = graph  # Specify network via graph or list

[env_params]
env_type = pdr  # pdr (define producers, distributors, and retailers) or 1sMr
state_rep = N  # Normalized continuous state representation
action_rep = MD  # Multi-discrete action representation
quant = 1  # Order action quantization amount
reset_max_entity_inv = 4  # Max initial inventory randomly generated on reset
reset_max_connection_inv = 4
back_order = False  # Set to True for back order at retailers setting instead of lost sales

[supply_chain_general_params]
max_order_action = 50  # Maximum order amount

# Next, for each of producers, distributors and retailers, define list of IDs and associated lists of settings for each
# Note: default distribution for all entities is (truncated, rounded) stationary Gaussian - other distributions can be easily specified and plugged in (Poisson, non-stationary versions, and inventory-dependent are already available options)

[supply_chain_producer_params]
id_list = P1
prod_daily_prod_avg_list = 10  # Mean production per producer
prod_daily_prod_std_list = 0.  # Std. dev. of production per producer
holding_cost_list = 0
holding_capacity_list = 100
overorder_penalty_list = 0
max_start_inv = -1  # if below 0, set equal to prod_holding_cap

[supply_chain_distributor_params]
id_list = D1, D2
holding_cost_list = 0.5, 0.1
holding_capacity_list = 150, 150
overorder_penalty_list = 10, 10
max_start_inv = 60, 60

[supply_chain_retailer_params]
id_list = R1, R2, R3
demand_avg_list = 2, 2, 2  # Mean demand per retailer
demand_std_list = 10, 10, 10  # Std. dev. of demand per retailer
revenue_list = 50, 50, 50
holding_cost_list = 1, 2, 4
overorder_penalty_list = 10, 10, 10
holding_capacity_list = 50, 50, 50
max_start_inv = 12, 12, 12
backorder_penalty_list = 0, 0, 0

[supply_chain_connection_params]
# Define connections and their parameters (costs and lead times) between defined entities
upstream_id_list = P1, P1, D1, D1, D1, D2, D2, D2
downstream_id_list = D1, D2, R1, R2, R3, R1, R2, R3
L_list = 2, 2, 1, 2, 3, 5, 6, 7  # Specify lead times for each connection
order_cost_per_item_list = 0, 0, 0, 0, 0, 0, 0, 0
This library is based around OpenAI Gym environment API \cite{Brockman2016}, so that standard RL implementations, like Stable Baselines 3 \cite{Raffin2019} used here, can work directly with defined environments. Listing 2 shows an example of instantiating the 1S-2W-3R environment and exercising it - taking actions and getting rewards and next states from the environment, and loading and training a DRL baseline with the environment. This also demonstrates how the library supports easily specifying different representation encodings for the states and actions (common representation as mentioned in an earlier footnote: discrete, multi-discrete, continuous, and normalized continuous). Additionally, this example illustrates enabling logging so all states and inventory levels, actions and orders, rewards, demands, and costs per network entity are logged and can be exported after taking actions in the environment.

```
from supply_chain_rl.run.env_setup_from_cfg import get_env_fun
from stable_baselines3 import PPO
from supply_chain_rl.model.parl import PARL

# Instantiate the environment with normalized continuous action representations
env_function = get_env_fun("env.cfg", action_rep="n")
env = env_function()

# Instantiate a baseline DRL model or PARL model - randomly initialized
model = PARL(env=env)

# Fit the model to the env
model.learn(total_timesteps=1e6)

# Evaluate model for some number of steps and log costs and rewards
# Reset environment, send action from model to env to get reward and next state
state = env.reset()
env.logging_enable()

for step in range(256):
    action, _ = model.predict(state)
    state, reward, done, info = env.step(action)
    print(step, reward)

# Export log of evaluation to a file:
env.logging_write_json("trajectory_log.json")
```

Listing 2: Example Python code using the library to load a supply chain environment and train and evaluate PARL or a baseline RL algorithm on the environment.
To the best of our knowledge such a reinforcement learning library and set of environments focused on broadly enabling supply chain environments for reinforcement learning does not currently exist. OpenAI provides the Gym library which provides an interface for RL environments and a set of video game environments. Various different libraries exist with RL algorithms that leverage video game environments. However, there are no libraries focused on RL for supply chain management that facilitate easy creation of different supply chain management environments. There is an open source library that provides a set of general environments for a variety of operations research problems, ORGym, but it is more broadly focused on OR as a whole and only includes a couple environments for supply chains under specific settings. We believe our framework is complementary to this and essentially - providing a full set of environments for different practical common supply chain scenarios as well as the capability to flexibly define custom environments and specify environment variations of interest, from different network, cost, and fulfillment structures, to different demand distributions including non-stationary and supply chain settings including lost sales settings.

5. Conclusions and future research directions

Reinforcement learning has lead to considerable break-throughs in diverse areas such as robotics, games and many others in the past decade. In this work, we present a RL based approach to solve some analytically intractable problems in supply chain and inventory management. Many real world problems have large combinatorial actions and state-dependent constraints. Hence, we propose the PARL algorithm that uses integer programming and SAA to account for underlying stochasticity and provide a principled way of optimizing over large action spaces. We then discuss the application of PARL to inventory replenishment and distribution decision across a supply chain network. We demonstrate PARL’s superior performance on different settings which incorporate some real-world complexities including, heterogeneous demand across the demand nodes in the network, supply lead times, and lost sales. Finally, to make the work more accessible, we also detail the development of a Python library that allows easy implementation of various benchmark RL algorithms along with PARL for different inventory management problems. This work also opens up various avenues for future research. First and foremost, by making RL algorithms more accessible via the Python library, we believe that researchers will be able to easily benchmark and also propose new RL inspired algorithms for other complex supply chain problems. For example, one potential interesting research direction is to use this methodology in settings where complete demand distribution is unknown. In these cases, one can use historical sales data to estimate demand distributions and improve these forecasts over time using online learning techniques. The improved forecasts can then be directly used for generating replenishment policies using the proposed framework. Another
interesting direction could be to extend this framework for order fulfillment, wherein the demand is realized and the decision is where to fulfill it from. Finally, since IP based methods still suffer from longer run times, coming up with alternate near-optimal formulations that are computationally efficient, remains an open area of research.

References
Joshua Achiam. Spinning Up in Deep Reinforcement Learning. [https://github.com/openai/spinningup](https://github.com/openai/spinningup), 2018.

Rishabh Agarwal, Max Schwarzer, Pablo Samuel Castro, Aaron C Courville, and Marc Bellemare. Deep reinforcement learning at the edge of the statistical precipice. *Advances in neural information processing systems*, 34:29304–29320, 2021.

Shipra Agrawal and Randy Jia. Learning in structured mdps with convex cost functions: Improved regret bounds for inventory management. In *Proceedings of the 2019 ACM Conference on Economics and Computation*, pages 743–744, 2019.

Gad Allon and Jan A Van Mieghem. Global dual sourcing: Tailored base-surge allocation to near-and offshore production. *Management Science*, 56(1):110–124, 2010.

Ross Anderson, Joey Huchette, Will Ma, Christian Tjandraatmadja, and Juan Pablo Vielma. Strong mixed-integer programming formulations for trained neural networks. *Mathematical Programming*, pages 1–37, 2020.

Dimitri Bertsekas. *Dynamic programming and optimal control: Volume I and II*. Athena scientific, 2017.

Robert N Boute, Joren Gijsbrechts, Willem van Jaarsveld, and Nathalie Vanvuuchelen. Deep reinforcement learning for inventory control: A roadmap. *European Journal of Operational Research*, 2021.

Greg Brockman, Vicki Cheung, Ludwig Pettersson, Jonas Schneider, John Schulman, Jie Tang, and Wojciech Zaremba. OpenAI Gym. *arXiv:1606.01540*, 2016.

Andrew J Clark and Herbert Scarf. Optimal policies for a multi-echelon inventory problem. *Management science*, 6(4):475–490, 1960.

Ton de Kok, Christopher Grob, Marco Laumanns, Stefan Minner, Jörg Rambau, and Konrad Schade. A typology and literature review on stochastic multi-echelon inventory models. *European Journal of Operational Research*, 269(3):955–983, 2018.

Arthur Delarue, Ross Anderson, and Christian Tjandraatmadja. Reinforcement learning with combinatorial actions: An application to vehicle routing. *Advances in Neural Information Processing Systems*, 33, 2020.

Yan Duan, Xi Chen, Rein Houthooft, John Schulman, and Pieter Abbeel. Benchmarking deep reinforcement learning for continuous control. In *International conference on machine learning*, pages 1329–1338. PMLR, 2016.

Awi Federgruen and Paul Zipkin. Approximations of dynamic, multilocation production and inventory problems. *Management Science*, 30(1):69–84, 1984.

Scott Fujimoto, Herke Hoof, and David Meger. Addressing function approximation error in actor-critic methods. In *International Conference on Machine Learning*, pages 1582–1591, 2018.

Ilaria Giannoccaro and Pierpaolo Pontrandolfo. Inventory management in supply chains: a reinforcement learning approach. *International Journal of Production Economics*, 78(2):153–161, 2002.

Joren Gijsbrechts, Robert N Boute, Jan A Van Mieghem, and Dennis Zhang. Can deep reinforcement learning improve inventory management? performance and implementation of dual sourcing-mode problems. *Performance on Dual Sourcing, Lost Sales and Multi-Echelon Problems*, 2018.

Xavier Glorot, Antoine Bordes, and Yoshua Bengio. Deep sparse rectifier neural networks. In *Proceedings of the fourteenth international conference on artificial intelligence and statistics*, pages 315–323. JMLR Workshop and Conference Proceedings, 2011.
David A Goldberg, Dmitriy A Katz-Rogozhnikov, Yingdong Lu, Mayank Sharma, and Mark S Squillante. Asymptotic optimality of constant-order policies for lost sales inventory models with large lead times. *Mathematics of Operations Research*, 41(3):898–913, 2016.

Tuomas Haarnoja, Aurick Zhou, Pieter Abbeel, and Sergey Levine. Soft actor-critic: Off-policy maximum entropy deep reinforcement learning with a stochastic actor. In Jennifer Dy and Andreas Krause, editors, *Proceedings of the 35th International Conference on Machine Learning*, volume 80 of *Proceedings of Machine Learning Research*, pages 1861–1870. PMLR, 10–15 Jul 2018.

Kazuyuki Hara, Daisuke Saito, and Hayaru Shounou. Analysis of function of rectified linear unit used in deep learning. In *2015 International Joint Conference on Neural Networks (IJCNN)*, pages 1–8, 2015.

Peter Henderson, Riashat Islam, Philip Bachman, Joelle Pineau, Doina Precup, and David Meger. Deep reinforcement learning that matters. In *Proceedings of the AAAI conference on artificial intelligence*, volume 32, 2018.

Geoffrey Hinton, Nitish Srivastava, and Kevin Swersky. Lecture 6e-rmsprop: Divide the gradient by a running average of its recent magnitude. *COURSERA: Neural networks for machine learning*, 4(2):26–31, 2012.

Christian D Hubbs, Hector D Perez, Owais Sarwar, Nikolaos V Sahinidis, Ignacio E Grossmann, and John M Wassick. Or-gym: A reinforcement learning library for operations research problems. *arXiv preprint arXiv:2008.06319*, 2020.

Woonghee Tim Huh, Ganesh Janakiraman, John A Muckstadt, and Paat Rusmevichientong. Asymptotic optimality of order-up-to policies in lost sales inventory systems. *Management Science*, 55(3):404–420, 2009.

Sujin Kim, Raghu Pasupathy, and Shane G Henderson. A guide to sample average approximation. *Handbook of simulation optimization*, pages 207–243, 2015.

Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In *ICLR*, 2015.

Jens Kober, J Andrew Bagnell, and Jan Peters. Reinforcement learning in robotics: A survey. *The International Journal of Robotics Research*, 32(11):1238–1274, 2013.

Timothy P Lillicrap, Jonathan J Hunt, Alexander Pritzel, Nicolas Heess, Tom Erez, Yuval Tassa, David Silver, and Daan Wierstra. Continuous control with deep reinforcement learning. In *ICLR*, 2016.

Hamid Maei, Csaba Szepesvari, Shalabh Bhatnagar, Doina Precup, David Silver, and Richard S Sutton. Convergent temporal-difference learning with arbitrary smooth function approximation. *Advances in neural information processing systems*, 22, 2009.

Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Alex Graves, Ioannis Antonoglou, Daan Wierstra, and Martin Riedmiller. Playing atari with deep reinforcement learning. *arXiv preprint arXiv:1312.5602*, 2013.

Volodymyr Mnih, Adria Puigdomenech Badia, Mehdi Mirza, Alex Graves, Timothy Lillicrap, Tim Harley, David Silver, and Koray Kavukcuoglu. Asynchronous methods for deep reinforcement learning. In *International conference on machine learning*, pages 1928–1937. PMLR, 2016.

Afsin Oroojlooyjadid, MohammadReza Nazari, Lawrence V Snyder, and Martin Takáč. A deep q-network for the beer game: Deep reinforcement learning for inventory optimization. *Manufacturing & Service Operations Management*, 2021.

Özalp Özer and Hongxia Xiong. Stock positioning and performance estimation for distribution systems with service constraints. *IIE Transactions*, 40(12):1141–1157, 2008.

Mohammad Pirhooshyaran and Lawrence V Snyder. Simultaneous decision making for stochastic multi-echelon inventory optimization with deep neural networks as decision makers. *arXiv preprint arXiv:2006.05608*, 2020.

Warren B Powell. *Approximate Dynamic Programming: Solving the curses of dimensionality*, volume 703. John Wiley & Sons, 2007.

Meng Qi, Yuanyuan Shi, Yongzhi Qi, Chenxin Ma, Rong Yuan, Di Wu, and Zuo-Jun Max Shen. A practical end-to-end inventory management model with deep learning. *Available at SSRN 3737780*, 2020.
Antonin Raffin, Ashley Hill, Maximilian Ernestus, Adam Gleave, Anssi Kanervisto, and Noah Dormann. Stable Baselines3. https://github.com/DLR-RM/stable-baselines3, 2019.

Ying Rong, Zümübil Atan, and Lawrence V Snyder. Heuristics for base-stock levels in multi-echelon distribution networks. Production and Operations Management, 26(9):1760–1777, 2017.

Moonkyung Ryu, Yinlam Chow, Ross Anderson, Christian Tjandraatmadja, and Craig Boutilier. Caql: Continuous action q-learning. arXiv preprint arXiv:1909.12997, 2019.

Herbert Scarf. The optimality of (s, s) policies in the dynamic inventory problem. 1960.

John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms. arXiv preprint arXiv:1707.06347, 2017.

Alexander Shapiro, Darinka Dentcheva, and Andrzej Ruszczyński. Lectures on stochastic programming: modeling and theory. SIAM, 2014.

Alexander Shapiro. Monte carlo sampling methods. Handbooks in operations research and management science, 10:353–425, 2003.

Anshul Sheopuri, Ganesh Janakiraman, and Sridhar Seshadri. New policies for the stochastic inventory control problem with two supply sources. Operations research, 58(3):734–745, 2010.

Tim Stockheim, Michael Schwind, and Wolfgang Koenig. A reinforcement learning approach for supply chain management. In 1st European Workshop on Multi-Agent Systems, Oxford, UK, 2003.

Nazneen N Sultana, Hardik Meisher, Vinita Baniwal, Somjit Nath, Balaraman Ravindran, and Harshad Khadikar. Reinforcement learning for multi-product multi-node inventory management in supply chains. arXiv preprint arXiv:2006.04037, 2020.

Jiankun Sun and Jan A Van Mieghem. Robust dual sourcing inventory management: Optimality of capped dual index policies and smoothing. Manufacturing & Service Operations Management, 21(4):912–931, 2019.

Richard S Sutton and Andrew G Barto. Reinforcement learning: An introduction. MIT press, 2018.

Christian Tjandraatmadja, Ross Anderson, Joey Huchette, Will Ma, Krunal Patel, and Juan Pablo Vielma. The convex relaxation barrier, revisited: Tightened single-neuron relaxations for neural network verification. arXiv preprint arXiv:2006.14076, 2020.

Wouter van Heeswijk and Han La Poutré. Approximate dynamic programming with neural networks in linear discrete action spaces. arXiv preprint arXiv:1902.09855, 2019.

Benjamin Van Roy, Dimitri P Bertsekas, Yuchun Lee, and John N Tsitsiklis. A neuro-dynamic programming approach to retailer inventory management. In Proceedings of the 36th IEEE Conference on Decision and Control, volume 4, pages 4052–4057. IEEE, 1997.

Senthil Veeraraghavan and Alan Scheller-Wolf. Now or later: A simple policy for effective dual sourcing in capacitated systems. Operations Research, 56(4):850–864, 2008.

Linwei Xin. Understanding the performance of capped base-stock policies in lost-sales inventory models. Operations Research, 69(1):61–70, 2021.

Shenghe Xu, Shivendra S Panwar, Murali Kodialam, and TV Lakshman. Deep neural network approximated dynamic programming for combinatorial optimization. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 34, pages 1684–1691, 2020.

Liz Young. Companies face rising supply-chain costs amid inventory challenges, Jun 2022.

Chao Yu, Jiming Liu, and Shamim Nemati. Reinforcement learning in healthcare: A survey. arXiv preprint arXiv:1908.08796, 2019.

Paul Zipkin. Old and new methods for lost-sales inventory systems. Operations Research, 56(5):1256–1263, 2008.

Paul Zipkin. On the structure of lost-sales inventory models. Operations research, 56(4):937–944, 2008.
Appendix A: Proof of Proposition 1

Proof of Proposition 1 Consider any state \( s \) and let \( g(s, a, d) = R(s, a, d) + \gamma \hat{V}^{\pi}_{\theta} - \theta T(s, a, d) \). We start by showing that \( g(\eta, s, a, d) \) uniformly converges to \( \mathbb{E}[g(s, a, D)] \) with probability 1. We prove this result by proving two main properties of \( g(s, a, d) \): (i) \( g(s, a, d) \) is continuous in \( a \) for almost every \( d \in D \), and (ii) \( g(s, a, d) \) is dominated by an integrable function. To prove (ii), we show that \( g(s, a, d) \leq C < \infty \) w.p. 1 \( \forall a \in A(s) \).

First, notice that \( g(s, a, d) \) is an affine function of the immediate reward \( R(s, a, d) \) and NN approximation of the value-to-go function. By assumption, the immediate reward follows these properties. Hence, to show these properties for \( g(s, a, d) \), we only need to illustrate that the value-to-go estimation also follows these properties.

Consider the value-to-go approximation, simply denoted as \( \hat{V}_{\theta}(T(s, a, d)) \) with \( \theta = (c, \{(W_k, b_k)\}_{k=1}^{K-1}) \) denoting the parameters of the \( K \)-layer ReLU-network. As \( T(s, a, d) \) is continuous and \( \hat{V}_{\theta}(s) \) is continuous, \( \hat{V}_{\theta}(T(s, a, d)) \) is continuous. Note that \( T(s, a, d) \) lies in a bounded space for any realization of the uncertainty \( d \). Furthermore, since the parameters of the NN \( \theta \) are bounded, the outcome of each hidden layer, and subsequently the outcome of the NN are also bounded. This proves that the NN is uniformly dominated by an integrable function. Then, following Proposition 8 of Shapiro (2003), we have uniform convergence of \( g(\eta, s, a, d) \) to \( \mathbb{E}[g(s, a, D)] \) w.p. 1. Finally, convergence of the optimal solution follows from a direct application of Theorem 5.3 of Shapiro et al. (2014), where we have used the fact that for all \( s \) the set of feasible actions is a bounded polyhedron \( A(s) \) and that for any \( \eta \), the set of optimal actions \( \hat{\pi}^\eta(s) \) is non-empty. This proves the final result.

While the above proof assumes that the action space is continuous, one can extend the results in the case of discrete action spaces as well. See Kim et al. (2015) for a discussion on the techniques used for extending the analysis to this setting.

Appendix B: More details on benchmark algorithms and parameter settings

B.1. Base Stock heuristic policy

The seminal work of Scarf (1960) shows the optimality of the parametric \((s, S)\) base stock policies for retail nodes with infinite capacity upstream supplier in backordered demand settings. This optimality holds even in the case with fixed costs and constant lead times. In this policy, if \( I \) is the inventory pipeline vector for a firm, the inventory position is defined as \( IP = \sum_{i=0}^{L} I^i \), where \( L \) is the lead time from the supplier, and the order quantity is \( \max\{0, S - IP\} \) as long as \( IP < s \) and 0 otherwise.

Due to the popularity of these policies, we implement a heuristic base stock policy for the multi-echelon networks we study as follows. At the high level, we construct a base stock policy for every link assuming the upstream entity’s supply is unconstrained. For the 2 echelon \( 1S - nR \) environments, we identify the best base stock policy via grid search for each link using a \( 1S - 1R \) environment with unconstrained supply. The reason we do a grid search is because we are in the lost sales setting. Note that we observe that the constrained supply setting also yields very similar results and hence restrict ourselves to the unconstrained supply setting. For the 3-echelon environments, we use the same strategy for the \( W - R \) links but computing inventory positions \( IP \) based on the lead time for that link (note that inventory pipelines can be longer than
leadtime in the dual sourcing setting. For the $S - W$ links we use environments that treat the warehouse as a retailer with demand equal to the sum of the downstream (lead time) retail demands to find the optimal parameters for that link.

B.2. Decomposition-Aggregation (DA) heuristic (Rong et al. 2017)

In this section, we describe our implementation of the DA heuristic. Note that we adaptation of the DA heuristic to the case of a Normal demand distribution as the authors discuss the method in the case of a Poisson demand distribution.

For 1S-nR environments, for every retailer $r$ compute its respective order up to level $S_r = F_{D_r}^{-1}\left[\frac{b_r}{b_r+h_r}\right]$ where $F_{D_r}^{-1}$ is the inverse cumulative demand distribution (cdf) of the random variable $D_r \sim N(\mu_r(L_r+1), \sigma_r\sqrt{L_r+1})$. Here $b_r$ is the retailer revenue per item less the variable ordering cost from the supplier and $h_r$ is the holding cost at the retailer. In every period, the retailer orders $S_r - IP_r$ where $IP_r$ is the retailer’s inventory position which is the sum of the retailer’s on-hand inventory and that in the pipeline vector.

For the 1S-2W-nR environments, we first decompose by sample paths $1S - 1W - 1R$.

For each such sample path, we compute $S_r = F_{D_r}^{-1}\left[\frac{b_r}{b_r+h_r}\right]$ where $F_{D_r}^{-1}$ is the inverse cdf of the random variable $D_r \sim N(\mu_r(L_r+1), \sigma_r\sqrt{L_r+1})$. Here $b_r$ is the retailer revenue per item less the variable ordering cost from the supplier, $h_r$ is the holding cost at the retailer and $h_w$ is the holding cost of the warehouse in the sample path of interest.

We then compute $q_w = F \left[0.5F^{-1}\left[\frac{b_r}{b_r+h_r}\right] + 0.5F^{-1}\left[\frac{b_r}{b_r+h_w}\right]\right]$ where $F$ and $F^{-1}$ refers to the cdf and inverse cdf of the standard normal distribution $N(0,1)$. We use this to compute echelon order up to level of the warehouse $S_w = F_{D_w}^{-1}\left[q_w\right]$ where $D_w$ is distributed as $N(\mu_r(L_r+L_w+1), \sigma_r\sqrt{L_r+L_w+1})$. An expected shortfall is computed which is $Q_{D_w}(s_w) = E_{D_w,r}(D_w - s_w)$ where $s_w = S_w - S_r$.

Next we aggregate across sample paths to recompute the order up to level at common warehouse $w$ using a back-order matching method described as follows: $S_w = Q_{D_w}^{-1}\left(\sum_{r|w_r=w} Q_{D_{w,r}}(s_{w_r})\right)$ where $D_w \sim N\left(\sum_{r|w_r=w} \mu_r L_w + \sqrt{\sum_{r|w_r=w} \sigma_r^2 L_w}\right)$ and $Q_{D_w}^{-1}(y) = \min\{S|E_{D_w}[D_w - S] \leq y\}$.

In every period, the retailer orders $S_r - IP_r$ from the warehouse and the warehouse orders $S_w - IP_w$ from the supplier where $IP_r, IP_w$ are the retailer’s and warehouse’s respective inventory positions which is the sum of the on-hand inventory and that in the pipeline vector.

B.3. Parameters and policy for the 1Sinf-1R environment

The 1Sinf-1R environment is a setting with back ordered demand and no-fixed costs. The lead time ($L$) is 4, the holding cost ($h$) is 0.8 and the back order penalty ($b$) is 7. The demand ($D$) is assumed to be normal $N[\mu, \sigma]$ where $\mu = 5$ and $\sigma = 0.8$. In this setting the optimal policy is an order-up to policy with closed form solution $F_D^{-1}\left(\frac{b}{b+h}\right)$ which is 26.48. As the environment only allows ordering integer quantities, we tested both 26 and 27 order up-to levels and 27 outperformed on average across 10 (identical) model runs each with 20 test runs with trajectory length of 10K steps.
B.4. Comparison of quantile and random sampling in PARL

Here we compare the use of quantile sampling and random sampling to generate realizations of the uncertainty in (4). For the five settings under consideration, we compare the per-step reward and per-step training time across the two sampling approaches.

As can be seen in Table 4, random sampling yields per-step rewards which are close to those obtained via quantile sampling. In terms of training time, random sampling is slower in certain settings (e.g. 1S-3R-High and 1S-10R), with higher per-step train-time average and variance.

| Setting               | PARL-quantile | PARL-random | PARL-quantile | PARL-random |
|-----------------------|---------------|-------------|---------------|-------------|
|                       | per-step reward | per-step reward | per-step train-time (s) | per-step train-time (s) |
| 1S-3R-High            | 514.8 ± 5.3   | 505.3 ± 11.0 | 0.178 ± 0.06  | 0.457 ± 0.26 |
| 1S-3R                 | 400.3 ± 3.3   | 399.5 ± 2.8  | 0.051 ± 0.01  | 0.053 ± 0.01 |
| 1S-10R                | 1006.3 ± 29.5 | 1005.4 ± 21.1| 0.089 ± 0.03  | 0.12 ± 0.07  |
| 1S-2W-3R (DS)         | 398.3 ± 2.5   | 395.3 ± 3.1  | 0.051 ± 0.01  | 0.050 ± 0.01 |
| 1S-2W-3R              | 405.4 ± 2.0   | 398.9 ± 9.7  | 0.044 ± 0.01  | 0.043 ± 0.01 |

Table 4 Comparison of quantile and random sampling in PARL.

B.5. Cost and reward comparison for other settings

Figure 7 Breakdown of rewards in test across BS, PPO and PARL algorithms for settings not discussed in the main text. Note that ordering costs include fixed and variable costs of ordering, revenue refers to the revenue earned from sales and reward refers to the revenue net costs incurred (see §3 and Table 1 for more details).
Appendix C: Parameter Tuning Details

Table 5  Best hyper parameters selected for each environment and method, for the benchmark RL methods. See Tables 6 and 7 for hyper parameter abbreviations.

| method setting | SAC | TD3 | PPO | A2C |
|----------------|-----|-----|-----|-----|
| 1S-3R-High     | G=0.9, LR=0.01, EO=True | G=0.9, LR=0.0003, EO=False | G=0.9, LR=0.003, VFC=1.0 | G=0.8, LR=0.003, VFC=0.5 |
| 1S-3R          | G=0.75, LR=0.003, EO=True | G=0.8, LR=0.0003, EO=False | G=0.8, LR=0.003, VFC=1.0 | G=0.8, LR=0.003, VFC=0.5 |
| 1S-10R         | G=0.8, LR=0.003, EO=True | G=0.9, LR=0.0003, EO=False | G=0.8, LR=0.010, VFC=1.0 | G=0.9, LR=0.003, VFC=1.0 |
| 1S-20R         | G=0.9, LR=0.003, EO=False | G=0.75, LR=0.0003, EO=False | G=0.9, LR=0.003, VFC=1.0 | G=0.9, LR=0.003, VFC=1.0 |
| 1S-2W-3R (DS)  | G=0.8, LR=0.003, EO=False | G=0.9, LR=0.0003, EO=False | G=0.8, LR=0.003, VFC=1.0 | G=0.8, LR=0.003, VFC=1.0 |
| 1Sinf-2W-3R    | G=0.99, LR=0.003, EO=False | G=0.99, LR=0.003, EO=False | G=0.99, LR=0.010, VFC=0.5 | G=0.99, LR=0.003, VFC=3.0 |

Table 6  Tuning hyper parameters and additional fixed hyper parameters for PPO and A2C - we vary γ, learning rate, and value function coefficient - resulting in 36 hyper parameter combinations

| Hyper Parameters for PPO and A2C | Value(s) |
|----------------------------------|----------|
| Discount Factor - γ (G)          | 0.99, 0.9, 0.80, 0.75 |
| Learning rate (LR)               | 0.01, 0.003, 0.0003 |
| Value function coefficient (in loss) (VFC) | 0.5, 1.0, 3.0 |
| Number of steps to run per update (epoch length) | 2048 |
| Max gradient norm (for clipping) | 0.5 |
| GAE lambda (trade-off bias vs. variance for Generalized Advantage Estimator) | 0.95 (PPO) and 1.0 (A2C) (defaults) |
| Number of epochs to optimize surrogate loss (internal train iterations per update - PPO only) | 20 |
| KL divergence threshold for policy update early stopping per epoch (PPO only) | 0.15 (“target_kl”=0.1) |
| Clip range (PPO only)            | 0.2 |
| RMSprop epsilon (A2C only)       | 1e-05 |
Table 7 Tuning hyper parameters and additional fixed hyper parameters for SAC and TD3 - we vary gamma, learning rate, and exploration options - resulting in 32 hyper parameter combinations

| Hyper Parameters for SAC and TD3 | Value(s)       |
|---------------------------------|----------------|
| Discount Factor - $\gamma$ (G)  | 0.99, 0.9, 0.80, 0.75 |
| Learning rate (LR)              | 0.01, 0.003, 0.0003, 0.00003 |
| Use generalized State Dependent Exploration vs. Action Noise Exploration (SAC) or Action Noise vs. not (TD3) (EO) | True, False |
| Tau (soft update coefficient)   | 0.005          |
| Replay buffer size              | 10^5           |
| Entropy regularization coefficient (SAC only) | auto           |