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Bayesian Inference and Prediction of Wave-induced Ship Motion based on Discrete-frequency Model Approximations

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Abstract: In this paper, we investigate the use of a discrete-frequency approximation for stochastic processes, modelling wave-induced ship motion and assess its prediction performance. The proposed estimator is obtained by adapting Bayesian spectral inference methods. We study the relationship between the lag window and the prediction performance and suggest a minimum requirement on window length for minimising the prediction error. We show an application to prediction of the pitch, heave, and roll motion from experiments of a scale-model container ship.

Keywords: Statistical Inference; Detection Algorithms; Bayesian Inference

1. INTRODUCTION

Wave-induced motion can significantly affect both the performance of personnel on board ships as well as the operation of equipment (Lloyd, 1989). As a consequence, motion control systems, called ride-control systems, are used in ships to reduce the effect of wave-induced motion (Perez, 2005). This motion is considered to be a disturbance and is often characterised in terms of stochastic processes (St Denis and Pierson, 1953). The spectral characteristics of these processes change significantly due to sea state, vessel speed, and vessel heading relative to the waves. Therefore, to extend the effectiveness of motion control systems over a desired envelope of operational conditions of ships, it is desirable to be able to characterise the frequency spectrum of this disturbance online. Such information can be used to switch or to adapt the motion control laws, and in some cases as part of control designs based on internal-model-control techniques (Perez, 2005).

Two alternatives are commonly used to represent the wave-induced motion. One is based on a representation in terms of discrete-frequency spectrum, which is very commonly used in the area of naval architecture and offshore engineering (Faltinsen, 1990). The other is based on the spectral factorisation representation, which considers the process to be a filtered version of an uncorrelated process - see, for example, Astrom (1970). In this paper, we take the former approach and use Bayesian techniques to infer frequencies, amplitudes, and phases of a multi-

sine representation of the process. The proposed on-line estimator is motivated by the work of Bretthorst (1988), which considers the marginal posterior distribution of the frequencies as a representation of the spectrum of the signal. This approach has been shown to have superior power in frequency discrimination when compared to direct discrete-Fourier-Transform methods (Ó Ruanaidh and Fitzgerald, 1996).

The contributions of this paper are the adaptation of the offline Bayesian inference technique to an online process for discrete-spectrum estimation, and the investigation of its properties in relation to prediction of ship motion in waves. To the best of the authors’ knowledge, the use of Bretthorst’s techniques have not been investigated for the application of ship motion inference despite the common use of discrete spectra in ship motion simulation. The results of the paper thus make a link between the techniques used for simulation in naval architecture and the inference needs for adaptive ship-ride control design. The use of Bayesian spectrum estimation for wave-induced ship motion can also find application in decision making about ship heading in order to avoid conditions conducive to parametric roll resonance. Indeed, the characterisation of uncertainty naturally encoded in the posterior distribution of the frequency naturally suits the problem of decision under uncertainty.

2. SHIP MOTION IN WAVES

Ocean waves are the consequence of complex interactions between the wind and the ocean water surface (Ochi, 1998). Waves propagate on the surface due to energy ex-
change from the wind, which produces oscillating pressure and velocity fields distributed in time and space. When a ship, or other floating object, is subjected to waves, the passing of the waves and the wave diffusion due to the presence of the body, together with the interference due to body-motion-generated waves, create pressure variations on the surface of the body, which induce forces and moments that result in motion.

In deep water conditions, the sea surface elevation, and thus the pressure field, can be considered a realisation of a zero-mean correlated Gaussian stochastic process. The stationarity of this process can be assumed for periods between one to four hours depending on the location (Haverre and Moan, 1985). Under these assumptions, the sea-surface elevation is fully characterised by the power-spectral density $S_{ww}$(ω). Standard formulae for the wave spectrum have derived from data commonly used in oceanography (Ochi, 1998).

Under certain particular assumptions of wave length and height relative to the size of the ship, it is common to consider a linear mapping from the sea surface to the motion in the different degrees of freedom. Due to the linearity, and power spectral density of motion $S_{\eta\eta,\ell}(\omega)$, for a stationary ship the motion in the degrees of freedom $\ell = 1, 2, \ldots, 6$ can be expressed as:

$$S_{\eta\eta,\ell}(\omega) = |H_{\ell}(j\omega)|^2 S_{ww}(\omega).$$

The frequency response $H_{\ell}(j\omega)$ is called the response amplitude operator in the naval architecture literature. This response depends on the shape of the hull, ship mass distribution, and the direction in which the waves approach the ship.

When the ship moves with forward speed, there is a Doppler effect that shifts the wave frequency experienced by the ship. This shifted frequency is called the encounter frequency and there is a nonlinear mapping between the frequency experienced by a stationary body and that frequency and there is a nonlinear mapping between the motion by the real scalar function:

$$\text{frequency experienced by a stationary body and that frequency and there is a nonlinear mapping between the motion by the real scalar function:}$$

The problem then amounts to computing the marginal posterior distributions:

$$p(\omega_i|D_W), p(a_i|D_W), p(b_i|D_W),$$

which are conditional on the data $D_W$ resulting from the sampling of $f(t)$ over a window of $W$ samples. We seek a computationally feasible algorithm that infers $N$ frequencies and predicts the motion $L$ steps ahead, where $L \geq 0$.

4. BAYESIAN FREQUENCY ESTIMATOR

We propose a recursive method for estimating the frequencies and associated amplitudes from data. We first revisit the offline Bayesian method developed in Bretthorst (1988) and Ó Ruanaidh and Fitzgerald (1996).

4.1 Frequency Estimation from a Bayesian Perspective

We consider a sampled measurement model with data

$$d_i = f_t + e_t,$$

where $t = k, k-1, \ldots, k-W+1$, $k > W$, $f_t = f(Tt)$ with sampling period $T$ and $e_t$ are i.i.d. with each $e_i \sim \mathcal{N}(0, \sigma_e^2)$ and mutually independent from $f_t$. The signal model (4) can be expressed as

$$D = G(\Omega) B + E,$$

where $D = [d_t, \ldots, d_{t-W}]^T$, $B = [a_1, \ldots, a_N, b_1, \ldots, b_N]^T$, $E = [e_t, \ldots, e_{t-W}]^T$, $\Omega = [\omega_1, \ldots, \omega_N]^T$, and

$$G(\Omega) = \begin{bmatrix} c_{1,1} & \cdots & c_{1,N} & s_{1,1} & \cdots & s_{1,N} \\ c_{2,1} & \cdots & c_{2,N} & s_{2,1} & \cdots & s_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{W,1} & \cdots & c_{W,N} & s_{W,1} & \cdots & s_{W,N} \end{bmatrix},$$

with $c_{ij} = \cos(\omega_j(t-i+1))$ and $s_{ij} = \sin(\omega_j(t-i+1))$. Then the likelihood function becomes

$$p(D|\Omega, B, \sigma) = (2\pi\sigma_e^2)^{-W} \exp \left\{ -\frac{E^T E}{2\sigma_e^2} \right\}.$$ (7)

The desired posterior distributions are obtained by the use of Bayes theorem and marginalisation (Ó Ruanaidh and Fitzgerald, 1996):

$$p(\Omega|D) = \int_0^\infty \int_{\mathbb{R}^{2N}} p(\Omega, B, \sigma|D) dB d\sigma,$$

where the integration domains stem from the choice of prior distributions. Since $\sigma$ is a scale parameter, we encode our background information into a Jeffrey’s prior and the background information about $B$ into a uniform (improper) prior (Bretthorst, 1988). These priors adhere to the Maximum Entropy Principle (Kapur, 1989). There is a corresponding marginal distribution $p(B|D)$, see for example, (Ó Ruanaidh and Fitzgerald, 1996).

As a point estimate of the frequency vector, we chose the maximum of the marginal posteriors:

$$\hat{\Omega} = \sup_{\Omega} p(\Omega|D), \quad \hat{B} = \sup_{B} p(B|D).$$ (10)

Due to the improper prior distribution, the latter reduces to the ML estimate, which due to the linearity of the
measurement results in LS estimate (Ó Ruanaidh and Fitzgerald, 1996):
\[ \hat{B} = (G^T G)^{-1} G^T D, \quad (11) \]
using the point estimate of the frequency vector \( G = G(\hat{\Omega}) \).

4.2 Proposed Narrow Band Predictor

In our approach, we choose a maximum model order \( N \) and a lag window of \( W \) samples, and compute the estimates described in the previous section at each time sample. Our algorithm computes the different frequency component estimates in \( \Omega \) iteratively. That is,
\[ \hat{\omega}_i = \sup_{\omega \in (0, \pi)} p(\hat{\omega}_1, \ldots, \hat{\omega}_{i-1}, \omega|D), \quad i = 1, \ldots, N. \quad (12) \]
The estimates \( \hat{\Omega} \) will match the most dominant frequencies in the data window \( t = [k, k-1, \ldots, k-W+1] \).

In the next section, we conduct a numerical study using different lag windows and data representative of wave-induced ship motion. In a subsequent section, we use the proposed algorithm with experimental data from a ship. As a measure of performance, we assess the \( L \)-steps ahead prediction error.

5. SIMULATION STUDIES

We consider two simulation studies. In the first, the data is generated from the model (4), whereas in the second the data is generated from filtered white noise, namely a continuous spectrum. We refer to our algorithm as a Discrete-Frequency Predictor and use the short-hand notation \( \text{DFP}(N,W,L) \), where \( N \) is the number of discrete frequencies, \( W \) is the number of samples in the lag window, and \( L \) is the number of prediction steps. The performance of the predictor is assessed in terms of the \( L \)-step-ahead prediction error.

5.1 Discrete-spectrum Data

We consider five frequencies \( \Omega = [0.2, 0.5, 1.1, 1.7, 2.1] \) [rad/s] with associated amplitudes \([1.3, 0.5, 2, 1.5, 0.7]\) and phases \([2, -2.9, 1, -1.4, 0.3]\) [rad]. The sampling period is \( T = 0.1s \), and the measurement noise has a variance \( \sigma^2 = 1 \).

Figure 1 shows a realisation of the model-generated data, and the filtered versions \((L = 0)\) based on \( \text{DFP}(5,t,0) \) (dashed line) and filtered estimate \( \text{DFP}(3,t,0) \) (dot-dashed line), namely approximations with 5 and 3 frequencies. The size of the window is increasing in order to show the asymptotic behaviour of the filter. After 100 samples the filtered estimate is very close to the original data.

Figure 2 shows the filtering error between the original signal and the filtered estimate, which reduces close to 0 after 200 samples. Note: when the model structure matches that of the model used to generate the data, and \( \sigma^2 = 0 \), the error approaches zero with increasing lag-window size.

5.2 Continuous-spectrum Data

The data is generated by filtering Gaussian white noise through a second order low pass filter with natural frequency 1 rad/sec and damping ratio 0.1. The sampling period was set at \( T = 0.25 \) seconds for \( 0 \leq t \leq 5000 \) samples. This data has a continuous frequency spectrum.

Figure 3 shows the filtering and prediction results based on a lag widow of 80 samples and an approximating model with five frequencies. The filtering results indicate good accuracy while, as expected performance deteriorates as we increase the prediction horizon. Figure 4 shows the mean square error for the filtering and two predictor cases for different lag windows. For short windows, the error of forward prediction increases as the filter fits the noise. The prediction error reduces as we increase the window. These results are based on 50 realisations. Figure 5 shows the evolution of the five frequency estimates. The dominant frequency \( \omega_1 \) is close to the modal frequency of the continuous spectrum. Figure 6 shows the corresponding frequency estimates as we increase the size of the lag window. It is interesting to notice that as the window increases all the frequency estimates tend to converge towards the dominant frequency.

If we consider lower order approximations, namely \( N < 5 \), the behaviour of the error and that of the frequency estimates is similar to that shown in Figures 4 and 5 although the error values increase.

![Fig. 1. Data (solid line), filtered with increasing window: DFP(5,t,0) (dashed line) and DFP(3,t,0) (dot-dashed line). After 200 samples (20s), the estimate of DFP(5,t,0) matches the data.](image)

![Fig. 2. Filtering Error. The error of DFP(5,t,0) (solid line) becomes small, compared to the error of DFP(3,t,0) (dashed line) which becomes periodic reflecting the two undetermined frequencies in the data.](image)
6. EXPERIMENTAL TOWING-TANK SHIP DATA

In this section, we consider the performance of our predictor on experimental data of ship motion in waves. The data is from experiments conducted on a 1:45 scale model of a 281m container ship in head seas—see Figure 7. The experiments were conducted at the former Centre for Ships and Ocean Structures at NTNU in Norway (Holden et al., 2007).

The set of experimental data contain experiments in regular (R) or sinusoidal waves (experiments R-1173, R-1189) and irregular (I) waves (experiment I-1194, I-1195) with a prescribed narrow-banded spectrum. The experiments chosen match those used by Belleter et al. (2015), in which the authors seek to infer only the dominant frequency using an observer based on a deterministic nonlinear dynamical model.

Figure 8 shows the time series of the pitch angle of the ship, and Figure 9 shows the time series of heave. This data has been scaled to correspond to the full-scale ship, and two regular wave experiments have been concatenated at time 15min. At this time, the encounter frequency of the waves reduces due to the change in speed of the vessel, which starts from stationary forward motion.

In our analysis, we first low-pass-filter and downsampled (to avoid aliasing) to achieve a similar sampling rate to that used in Section 5.2. We utilised the filter, DFP(1,200,0), to produce a frequency estimate at every time point. Figures 10 and 11 show the dominant frequency estimates. The estimate of the pitch frequency quickly converges to the true encounter frequency, and switches promptly between the two experiments. Similar to the pitch frequency estimate, the frequency estimate using the heave data quickly reaches the dominant encounter frequency, however, as there is more noise in the heave data the estimate oscillates around the true value. The reduction in frequency estimate at time 15min, is in
Fig. 8. Pitch data obtained from two towing tank experiment of a 1:45 scale model container ship, experiments R-1173 and R-1189 (Holden et al., 2007)

Fig. 9. Heave data obtained from two towing tank experiment of a 1:45 scale model container ship, experiments R-1173 and R-1189 (Holden et al., 2007)

Fig. 10. Frequency estimation using DFP(1,200,0) corresponding to the pitch shown in Figure 8.

Fig. 11. Frequency estimation using DFP(1,200,L) corresponding to the heave data shown in Figure 9.

Fig. 12. Heave data in irregular waves; experiment I-1195 (Holden et al., 2007)

Fig. 13. Frequency estimation using DFP(1,200,0) corresponding to the heave data shown in Figure 12.

Figures 14 and 15 show the pitch and roll time series for experiment I-1194 in irregular waves. In this experiment, the ship develops parametric roll resonance in two instances as a result of the wave-frequency synchronisation between pitch and roll and the particular wave length, which is similar to the length of the vessel (Holden et al., 2007). The frequency estimates are shown in Figure 16, with the roll frequency estimate doubled to show the synchronisation effect. The pitch dominant frequency estimate is very close to twice the roll frequency estimate. This suggests it could...
of wave-induced motion in ships. We propose that in the context of this application, the estimates can be used for aiding the performance of motion controllers.

The algorithm is based on an extension of a Bayesian approach to spectrum inference, and the estimates correspond to the maximum of the marginal posterior of the discrete frequencies. We conducted numerical studies to highlight properties of the estimator and then we applied the estimator to experimental data from a model ship. By analysing the result of applying the algorithm to the experimental data set, it is shown that an estimator could be used as means to detect parametric roll resonance.

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REFERENCES

Astrom, K. (1970). *Introduction to Stochastic Control*. Academic Press, New York and London.

Belleter, D.J., Galeazzi, R., and Fossen, T.I. (2015). Experimental verification of a global exponential stable nonlinear wave encounter frequency estimator. *Ocean Engineering*, 97, 48–56.

Brethorst, G.L. (1988). *Bayesian Spectrum Analysis and Parameter Estimation*, volume 48 of *Lecture Notes in Statistics*. Springer-Verlag Berlin and Heidelberg GmbH & Co. K.

Faltinsen, O. (1990). *Sea Loads on Ships and Offshore Structures*. Cambridge University Press.

Haverre, S. and Moan, T. (1985). Probabilistic Offshore Mechanics, chapter On some uncertainties related to short term stochastic modelling of ocean waves. Progress in Engineering Science. CML.

Holden, C., Galeazzi, R., Rodriguez, C., Perez, T., Fossen, T.I., Blanke, M., De Almeida, M., and Neves, S. (2007). Nonlinear Container Ship Model for the Study of Parametric Roll Resonance. *Modeling, Identification and Control*, 28(4), 87–103.

Kapur, J.N. (1989). *Maximum-entropy Models in Science and Engineering*. John Wiley & Sons.

Lloyd, A.R.J.M. (1989). Seakeeping; Ship Behavior in Rough Water. Ellis Horwood Ltd.

Ó Ruanaidh, J.J.K. and Fitzgerald, W.J. (1996). *Numerical Bayesian Methods Applied to Signal Processing*. Statistics and Computing. Springer New York.

Ochi, M. (1998). *Ocean Waves: The Stochastic Approach*. Ocean Technology Series. Cambridge University Press.

Perez, T. (2005). *Ship Motion Control*. Springer Verlag.

St Denis, M. and Pierson, W. (1953). On the motion of ships in confused seas. *SNAME Transactions*, 61, 280–332.