Constrained optimization via simulation models for new product innovation

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Abstract. We consider the problem of constrained optimization where the decision makers aim to optimize the primary performance measure while constraining the secondary performance measures. This paper provides a brief overview of stochastically constrained optimization via discrete event simulation. Most review papers tend to be methodology-based. This review attempts to be problem-based as decision makers may have already decided on the problem formulation. We consider constrained optimization models as there are usually constraints on secondary performance measures as trade-off in new product development. It starts by laying out different possible methods and the reasons using constrained optimization via simulation models. It is then followed by the review of different simulation optimization approach to address constrained optimization depending on the number of decision variables, the type of constraints, and the risk preferences of the decision makers in handling uncertainties.

1. Introduction

All models are wrong but some are useful [1]. Decision makers use models to assist them in thinking more clearly and weighing various trade-offs that are ubiquitous in this world of limited resources. For example, in developing new printer, we need to consider various key performance indicators or performance measures such as how fast it can print per minute, how many pages it can print given an ink cartridge, how much the cost per page is, how good the output image quality is, how long it can work before it fails, and how noisy it is. These are examples of stochastic performance measures as they depend on the printing usage behavior and the printing content of a printer user. At the same time, there are non-random constraints such as the dimensions or size and the weight of a printer that depends on the targeted user profiles.

There are various models that decision makers can use. Two powerful models are optimization and simulation [2]. Optimization helps the decision makers to list down clearly the decision variables involved, the primary performance measures to be optimized, and the constraints in terms of both decision variables and secondary performance measures. Simulation is useful to account for the external environment such as uncertainties that affect the performance measures.

This paper considers the problem of selecting the best feasible alternatives in the presence of stochastic constraints. In this case, the decision makers would like to optimize the primary performance measure while constraining the secondary performance measures. In some situations, both the primary and secondary performance measures need to be estimated via Discrete Event Simulation as there is no closed form expression to evaluate them. There are two challenges to address the problem. First there is a need to efficiently generate alternatives when the decision space is huge.
Second, multiple simulation replications are needed to estimate the performance measures. As stochastic simulation is computationally intensive, the decision makers have the choice to focus on the worst-case scenario or on average scenario as these correspond to different amount of simulation time.

This paper aims to complement existing surveys on simulation optimization methods by providing a specific overview on the methods for stochastically constrained optimization via simulation. There are many excellent reviews on simulation optimization methods [3-7]. However, most review papers focus on the methods first before breaking down into possible approaches of formulating the problem. This paper takes the other side of the approach, which is to look at the constrained optimization problem first and look at the literatures in addressing it. The outline of this paper is as follows. Section 2 provides possible reasons on why decision makers model problems as constrained optimization together with the different types of possible constraints. Section 3 takes it further by listing the benefits for decision makers to use stochastic model instead of only optimization and when they need to use Discrete Event Simulation instead of other types of simulation model. Section 4 reviews the literatures addressing stochastically constrained optimization while Section 5 concludes this paper.

2. Constrained optimization
Optimization model is one of the tools to make decision. The idea is to find the best possible alternative instead of only aiming for improvement or finding feasible solutions. Each decision maker is given various options on how to model a problem depending on the characteristic of the problem. In the case where all performance measures can be lumped into one performance measure such as cost or profit, we can use a single objective optimization model. In most cases, it is not easy to convert to monetary measures and so finding a good weight for each of the performance measure is difficult. This motivates the use of multi-objective optimization models which can be further divided into either unconstrained multi-objective optimization of which objective is to find the non-dominated solutions or Pareto optimal set or constrained multi-objective optimization. In this paper, we focus on the case where there is only one primary performance measure while the secondary performance measures are constrained. It should be noted that all models including the unconstrained optimization model usually have constraints on the decision variables as resources are limited.

There are at least two reasons for decision makers to model their problem as constrained optimization problems. First it provides clear boundaries and focus on what to optimize and what act as constraints. For example, in the printing context, if the target is office users, one may want to optimize the print speed while constraining other performance measures such as print quality and noise. Secondly, by formulating as constrained optimization, decision makers are clear which of the three to optimize while constraining the other two. For example, typically cost will be optimized while quality and time are modeled as constraints. This is because there is usually no external regulations on the cost. Quality is demanded by both customers and authorities, while time is crucial as exceeding the time means that the project is not completed.

There are two possible types of penalty or consequences when the constraints are violated: fixed and variable. Fixed penalty refers to pass or fail criteria such as the constraints imposed by authorities. This can be called as hard constraint. Another type of penalty is variable penalty where there are different impacts to the decision makers depending on how much the constraint is exceeded. In the second case, sometimes decision makers may want to convert into the primary performance measure if the weight age of that penalty can be determined.

3. Stochastically constrained optimization
Many models are better than a single model [10]. It is possible to only do optimization without considering uncertainties. For example, if the cost of incorporating simulation models is high, decision
makers may opt to use optimization only to find the best alternative. On the other hand, some may only want to use simulation model to evaluate the performance measures of a pre-determined alternative in the presence of uncertainties instead of attempting to find the best alternative. Those are valid options. However, if decision makers want to increase the quality of their decision, using two models such as marrying simulation and optimization to consider the uncertainties are recommended.

There can be many ways to incorporate uncertainties into the model. For example, robust optimization is excellent to consider the uncertainties in the parameters of the formula to evaluate the performance measures. At the same time, there are common cases where the performance measures need to be evaluated using a black-box approach of simulation model. For problems with no time aspect, one can use Monte Carlo simulation which can evaluate the performance of a betting decision given there are various input parameters from different distributions. However, when there is time aspect such as queuing with uncertain arrival and service time; and interaction between the systems we try to model, Discrete Event Simulation is needed. In most cases such as queuing network and printing system, the components are behaving more consistently compared to agents such as human and can be treated as a black box. Otherwise, one may want to use Agent-Based Modeling or combining the Discrete Event Simulation with the concept of agents which can act differently inside the model.

Figure 1 and Table 1 summarize the general decision making model. The simplest case of decision making is when the alternatives are pre-determined, the evaluation effort is minimum such as subjective assessment by a single individual, and there is no uncertainty. When there is no uncertainty, the evaluating and comparing alternatives can be done at the same time. When there is uncertainty in the evaluation and there are no closed-form expressions, the challenge would be on determining the sample size for simulation over different possible scenarios. At the comparing alternatives stage, the risk profile of the decision maker needs to be characterized. A risk-neutral person may prefer central tendency measures such as mean, median, or mode while a risk-averse person may want to include both the central tendency and the spread to model certain percentile level. In addition, the confidence level of the simulation results needs to be specified in determining the sample size of the simulation.

![Figure 1. Generalized decision making model.](image)

### Table 1. Possible types for each building block.

| Determining Alternatives | Evaluating Alternatives | Comparing Alternatives |
|--------------------------|-------------------------|------------------------|
| Pre-determined           | Closed form expressions | Weighted Aggregation   |
| Search on Discrete Space | Computer Simulation     | Ranking                |
| Continuous Space         | Physical Simulation     |                        |
|                          | Qualitative Assessment  |                        |


4. Literatures on stochastically constrained optimization

4.1. Methods
As both simulation and optimization require significant amount of computing budget, we need to balance the effort between evaluating via simulation and searching for which next sets of alternatives to evaluate [11]. In the case of continuous optimization, Bhatnagar et al. [12] proposed stochastic approximation algorithms when there are stochastic constraints while Kleijnen et al. [13] tackled the problem using response surface method. For the discrete case, there are some Ordinal-Optimization related works [14-16]. In addition, Park and Kim [17] handled the presence of multiple stochastic constraints using a penalty function where the penalty parameter converges to infinity. Luo and Lim [18] used the Lagrangian method where stochastic approximation is applied to the Lagrangian.

When the number of alternatives is finite, it may be possible to evaluate all of the alternatives for at least once that there is no need to spend the computing budget. This is called a constrained ranking and selection problem which decides on the amount of simulation samples to collect for each alternative [19]. For constrained ranking and selection, we have the choice to formulate the problem. This depends on the risk profile of the decision makers. If they want to guarantee the probability of correct selection for the worst case scenario, they can specify the minimum number of simulation samples needed for each alternative [19, 20]. However if they prefer to focus on the general cases, it is more efficient to formulate the simulation budget allocation by either maximizing the probability of correct selection subject to the given budget or to minimize the total simulation budget given the desired probability of correct selection. These two turns out to be dual problems to each other [21]. Another approach would be to consider other criteria such as the expected opportunity cost [22]. In the case of the probability correct selection, asymptotic approach is usually used to get the closed-form solution [23]. Otherwise, we need to use some kind of solver [24]. Another approach for improving the simulation efficiency is to minimize the switching cost in running the simulations between alternatives [25]. It is possible to let certain alternatives to be dormant temporarily to save simulation cost [26]. Another approach is to formulate as a chance-constrained selection problem [27].

4.2. Applications
There are several applications of stochastically constrained optimization via discrete event simulation. First, it can be applied for planning remanufacturing systems [15]. Another application is on designing the optimal water quality monitoring network [28]. Discrete Event Simulation is commonly applied in health care [29]. One example of constrained optimization is for determining the best bed allocation policy [30].

5. Conclusions
This paper aims to provide a brief overview on the stochastically constrained optimization via discrete event simulation. This is done by looking at the context where decision makers want to use constrained optimization model to make decision and the respective literatures which provide various methods and examples of applications.

One possible future research direction is to evaluate the impact of using different models in solving the same problem. This is especially when various models are conflicting in terms of the recommendation given. Another research gap is to build a model where different models can be incorporated. For example, the key performance measures may consist of both objective values and the qualitative assessments from experts. In addition, it is possible to incorporate some kinds of weights using multi-criteria decision making and combine them with the ranking approach in comparing the simulation results.

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