Vector Leptoquark Pair Production in  
$e^+e^-$ Annihilation  

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Abstract  
The cross section for vector leptoquark pair production in $e^+e^-$ annihilation is calculated for the case of finite anomalous gauge boson couplings $\kappa_{\gamma,Z}$ and $\lambda_{\gamma,Z}$. The minimal cross section is found to behave as $\propto \beta^7$, leading to weaker mass bounds in the threshold range than in models studied previously.
Various extensions of the Standard Model predict bosonic states carrying both lepton and quark quantum numbers, the leptoquarks. If their couplings are baryon and lepton number conserving, cf. see [1], these states may even exist in the mass range accessible at high energy colliders. Stringent bounds on the leptoquark–fermion couplings $\lambda_{lq}$ are derived from low energy data [2]. In the mass range $M_V \leq 1$ TeV they are required to be much smaller than the electromagnetic coupling. On the other hand the bosonic leptoquark couplings cannot be arbitrarily small but are determined by the respective gauge couplings and eventual anomalous couplings.

In this note we extend the analysis performed in a previous investigation [3], where minimal vector couplings $\kappa = 1$ and $\lambda = 0$ were considered, to the general case of four independent anomalous couplings $\kappa_{\gamma}, \kappa_Z, \lambda_\gamma$ and $\lambda_Z$ for the process of vector leptoquark pair production in $e^+e^-$ annihilation, $e^+e^- \rightarrow \nabla V$ [4]. Due to the smallness of the fermionic couplings we will consider the bosonic contributions only in the following. The annihilation process dominates at not too large cms energies. Additional contributions are due to the photon–photon fusion processes, $e^+e^- \rightarrow e^+e^-\nabla VX$, which were studied in refs. [5, 4].

The tri-linear coupling of the photon and $Z$-boson to a pair of vector leptoquarks is given by

$$V_{\mu_1\mu_2\mu_3}^{\nabla}(Z)(k_1, k_2, k_3) = eQ_\gamma(Z)(V)\left[\hat{V}_{\mu_1\mu_2\mu_3} + \kappa_\gamma(Z)\hat{V}_\mu^{\kappa} + \frac{\lambda_\gamma(Z)}{M_V^2}\hat{V}_\mu^{\lambda}\right],$$

where

$$\hat{V}_{\mu_1\mu_2\mu_3}(k_1, k_2, k_3) = (k_1 - k_2)_{\mu_3}g_{\mu_1\mu_2} + (k_2 - k_3)_{\mu_1}g_{\mu_2\mu_3} + (k_3 - k_1)_{\mu_2}g_{\mu_3\mu_1},$$

$$\hat{V}_\mu^{\kappa}(k_1, k_2, k_3) = k_{\mu_3}g_{\mu_1\mu_2} - k_{\mu_2}g_{\mu_3\mu_1},$$

$$\hat{V}_\mu^{\lambda}(k_1, k_2, k_3) = (k_1, k_2)(k_{\mu_3}g_{\mu_1\mu_2} - k_{\mu_2}g_{\mu_3\mu_1}) + (k_2, k_3)(k_{\mu_1}g_{\mu_2\mu_3} - k_{\mu_3}g_{\mu_1\mu_2}) + (k_3, k_1)(k_{\mu_3}g_{\mu_1\mu_2} - k_{\mu_2}g_{\mu_3\mu_1}) + (k_{\mu_3}k_{\mu_2}g_{\mu_1\mu_2} - k_{\mu_2}k_{\mu_3}g_{\mu_1\mu_2}).$$

cf. [3, 7]. Here $Q_\gamma(V)$ denotes the electric charge of the vector leptoquark and the $Z$-coupling is $Q_Z(V) = |T_3(V) - Q_{em}(V)\sin^2\theta_W|/\cos\theta_W\sin\theta_W$, with $T_3$ being the third component of the weak isospin and $\theta_W$ the electroweak mixing angle. In the following we refer to the vector leptoquarks discussed in ref. [1]. Their quantum numbers can be found in ref. [3], table 1.

The differential scattering cross section in the center of momentum frame reads

$$\frac{d\sigma_{\nabla V}}{d\cos\theta} = \frac{3\pi\alpha^2}{2M_V^2} \beta^3 \sum_{a=L,R} \left\{ A_a^\gamma T_\gamma(\beta, \cos\theta, \kappa_\gamma, \lambda_\gamma) + A_a^{Z}(s)T_\gamma(\beta, \cos\theta, \kappa_Z, \lambda_Z) \right\},$$

where $M_V$ denotes the leptoquark mass, $\beta = \sqrt{1 - 4M_V^2/s}$, $s$ is the cms energy squared,

$$A_a^\gamma = Q_\gamma(V)Q_a^\gamma,$$

$$A_a^{Z}(s) = 2Q_\gamma(V)Q_a^{\gamma} Q_Z^{\gamma} Q_Z^{\gamma} \frac{(s - M_Z^2)s}{(s - M_Z^2)^2 + \Gamma_Z^2M_Z^2},$$

$$A_a^{Z}(s) = Q_Z^{\gamma}Q_Z^{\gamma} \frac{s^2}{(s - M_Z^2)^2 + \Gamma_Z^2M_Z^2}.$$
and
\[
\hat{T}_\mu(\beta, \cos \theta, \kappa, \lambda) = 1 - \kappa_\mu + \lambda_\mu + \frac{1}{4}(\kappa_\mu - \lambda_\mu)^2 + \frac{s}{16M_V^2} \sin^2 \theta [(1 - \kappa_\mu)^2 + 2\lambda_\mu^2] + \frac{1}{16} \sin^2 \theta [-1 - 3 \beta^2 - 2(\kappa_\mu - \lambda_\mu)^2 + 4\kappa_\mu],
\]

\[
\hat{T}_{\mu,\nu}(\beta, \cos \theta, \kappa, \lambda) = 1 - \frac{1}{2} (\kappa_\mu + \kappa_\nu - \lambda_\mu - \lambda_\nu) + \frac{s}{16M_V^2} \sin^2 \theta [(1 - \kappa_\mu)(1 - \kappa_\nu) + 2\lambda_\mu\lambda_\nu] + \frac{1}{16} \sin^2 \theta [-1 - 3 \beta^2 + 2(\kappa_\mu - \lambda_\mu)(\kappa_\nu - \lambda_\nu) + 4(\kappa_\mu + \kappa_\nu)].
\]

The lepton–gauge boson couplings are \(Q^L_R = -1\), \(Q^L_Z = -\frac{1}{2} + \sin^2 \theta_W\) for \(\cos \theta_W\), and \(Q^L_Z = \tan \theta_W\).

The angular distribution depends strongly on the value of the anomalous couplings. For symmetric couplings \(\kappa_\gamma = \kappa_Z\) and \(\lambda_\gamma = \lambda_Z\) the constant part vanishes for the case \(\kappa = \lambda = 2\). The differential cross section \(d\sigma/d \cos \theta\) is then proportional to \(\sin^2 \theta\) as in the case of scalar leptoquark pair production [3]. On the other hand, the differential distribution turns out to be rather flat for minimal vector couplings \(\kappa = 1, \lambda = 0\), cf. ref. [3].

For the integrated cross section one obtains
\[
\sigma_V(s) = \sigma_\gamma + \sigma_\gamma, \sigma_Z + \sigma_Z = 3\pi\alpha^2 M_V^2 \beta^3 \sum_{a=L,R} \left\{ A_\alpha^a T_\gamma(\beta, \kappa, \lambda) + A_\alpha^a T_\gamma Z(\beta, \kappa, \lambda, \gamma) + A_\alpha^a T_\gamma Z(\beta, \kappa, \lambda, \gamma) \right\}.
\]

Here the functions \(T_\mu(\beta, \kappa, \lambda)\) and \(T_{\mu,\nu}(\beta, \kappa, \lambda)\) are given by
\[
T_\mu(\beta, \kappa, \lambda) = \frac{(1 - \kappa_\mu)^2 + 2\lambda_\mu^2}{24} M_V^2 + \frac{(23 - 20\kappa_\mu + 4\kappa_\mu^2) - 3\beta^2}{24} + \lambda_\mu \left(1 - \frac{\kappa_\mu}{3} + \frac{\lambda_\mu}{6}\right),
\]
\[
T_{\mu,\nu}(\beta, \kappa, \lambda) = \frac{(1 - \kappa_\mu)(1 - \kappa_\nu) + 2\lambda_\mu\lambda_\nu}{24} M_V^2 + \frac{[23 - 10(\kappa_\mu + \kappa_\mu) + 4\kappa_\mu\kappa_\nu] - 3\beta^2}{24} + \frac{\lambda_\mu}{2} \left(1 - \frac{\kappa_\nu}{3} + \frac{\lambda_\nu}{6}\right) + \lambda_\nu \left(1 - \frac{\kappa_\mu}{3} + \frac{\lambda_\mu}{6}\right).
\]

The minimal cross section among all possible choices of the four anomalous couplings \(\kappa_\gamma, \kappa_Z, \lambda_\gamma\) and \(\lambda_Z\), is
\[
\sigma_{\min}^V(s, M_V^2) = \frac{\pi\alpha^2}{2M_V^2} \beta^7 \sum_{a=L,R} |\kappa_\alpha(s)|^2 \frac{1}{3(5 - 3\beta^2)},
\]

where
\[
\kappa_\alpha(s) = \sum_{B=\gamma, Z} Q_B^s \frac{s}{s - M_B^2 + iM_B\Gamma_B} Q_B(V).
\]

Unlike in the case of general values of the anomalous couplings \(\kappa\) and \(\lambda\) [13], the minimal cross section does not contain unitarity violating contributions \(\propto s/M_V^2\). The anomalous couplings determining the minimal cross section [13] are depicted in figure 1. Due to the symmetry of the
quadratic form (11) they are given by

\begin{align}
\kappa^\text{min}_\gamma &= \kappa^\text{min}_Z = \frac{1}{2} \left( \frac{15 - 10\beta^2 - \beta^4}{5 - 3\beta^2} \right), \\
\lambda^\text{min}_\gamma &= \lambda^\text{min}_Z = -\frac{1}{2} (1 - \beta^2) \left( \frac{5 - \beta^2}{5 - 3\beta^2} \right).
\end{align}

Only for large cms energies, \( \beta \to 1 \), \((S \gg 4M_V^2)\), the minimizing anomalous couplings approach the minimal vector couplings \( \kappa^\text{min} = 1 \) and \( \lambda^\text{min} = 0 \). On the other hand, for \( \beta \to 0 \) the couplings \( \kappa^\text{min} = 3/2 \) and \( \lambda^\text{min} = -1/2 \) are obtained.

If one would assume \( \lambda_\gamma \equiv \lambda_Z \equiv 0 \) and minimize the cross section for \( \kappa_\gamma \) and \( \kappa_Z \) only

\[ \sigma^\text{min}_{VV}(s, M_V^2) = \frac{\pi \alpha^2}{2M_V^2} \beta^3 \sum_{a=L,R} |\kappa_a(s)|^2 \frac{10 - 13\beta^2 + 10\beta^4 - 3\beta^6}{24(2 - \beta^2)^2} \]

is obtained, where

\[ \kappa^\text{min}_{\gamma,Z}(\lambda_\gamma,\lambda_Z \equiv 0) = \frac{7 - 5\beta^2}{2(2 - \beta^2)}. \]

i.e. \( \kappa^\text{min} = 7/4 \) for \( \beta \to 0 \). Here the cross section grows \( \propto \beta^3 \) near the threshold as also in the case of minimal vector couplings and Yang–Mills type couplings. The weaker rise \( \propto \beta^7 \) in the previous case is thus a consequence of considering non-zero values of \( \lambda_\gamma,\lambda_Z \) in eq. (1).

In figure 2 the mass dependence of the pair production cross section of the leptoquark \( U_1 \) is shown at \( \sqrt{s} = 1 \text{ TeV} \) for different choices of the anomalous couplings, upon which the accessible mass bounds strongly depend. Assuming a signal of 100 events at an integrated luminosity of \( \mathcal{L} = 10 \text{ fb}^{-1} \) for the minimizing anomalous couplings, a mass bound of 370 GeV can be reached, whereas the corresponding bound for minimal vector couplings is 470 GeV. A still larger cross section is obtained for Yang–Mills type couplings \( \kappa_{\gamma,Z} = \lambda_{\gamma,Z} \equiv 0 \).

In figures 3a–c the minimal values of the integrated cross sections are shown for all leptoquark states of ref. 4 at \( \sqrt{s} = M_Z, \ 190 \text{ GeV}, \) and 1 TeV. Accessible mass ranges are indicated assuming 100 signal events\(^4\) at integrated luminosities of \( \mathcal{L}(M_Z) = 150 \text{ pb}^{-1}, \mathcal{L}(190 \text{ GeV}) = 500 \text{ pb}^{-1}, \) and \( \mathcal{L}(1 \text{ TeV}) = 10 \text{ fb}^{-1} \), respectively. For all cms energies considered, the lowest mass bounds are estimated for the states \( U_1 \) and \( U_3(0) \), respectively, which have the same bosonic quantum numbers. The largest cross sections are obtained for \( U_3(-1) \) for \( \sqrt{s} = M_Z \) and \( U_3(1) \) at \( \sqrt{s} = 190 \text{ GeV} \) and 1 TeV, respectively.

In conclusion, we have shown that non-vanishing anomalous couplings \( \lambda_\gamma \) and \( \lambda_Z \) may yield much weaker search limits in the threshold range as obtained varying the couplings \( \kappa_\gamma \) and \( \kappa_Z \) only. This should be taken into account in forthcoming analyses of the LEP data both for \( \sqrt{s} = M_Z \), the energy range at LEPII, and at future linear colliders.

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\(^2\) See refs. 3, 4 for the notation.

\(^3\) The corresponding numbers are meant to be indicative and can not replace a detailed analysis accounting for all experimentally relevant aspects.

\(^4\) Previous analyses 5 considered only scalar leptoquarks.
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Figure 1: The minimizing couplings $\kappa^{\text{min}}$ and $\lambda^{\text{min}}$ as a function of $\beta = \sqrt{1 - 4M_V^2/s}$. The dashed line corresponds to the value of $\kappa$ minimizing the production cross section for $\lambda = 0$. 
Figure 2: Integrated pair production cross section for the vector leptoquark $U_1$, ref. [1], for different values of the anomalous couplings $\kappa_{\gamma,Z}$ and $\lambda_{\gamma,Z}$. Solid line: minimal cross section; dash–dotted line: minimal cross section for $\lambda_{\gamma} = \lambda_{Z} = 0$; dotted line: cross section for minimal vector couplings (MC) $\kappa_{\gamma,Z} = 1, \lambda_{\gamma,Z} = 0$; dashed line: cross section for Yang–Mills type couplings (YM) $\kappa_{\gamma,Z} = \lambda_{\gamma,Z} = 0$. The horizontal dotted line indicates the accessible search range at a luminosity of $\mathcal{L} = 10$ fb$^{-1}$ and 100 signal events.
Figure 3a: Mass dependence of the minimal integrated pair production cross section for the vector leptoquarks ref. [1] at $\sqrt{s} = M_Z$. The lines correspond to the production cross sections for the states $U_1 (U_3(0)), V_2(1/2), \tilde{V}_2(-1/2), \bar{U}_1, \tilde{V}_2(1/2), V_2(-1/2), U_3(1)$ and $U_3(-1)$ from left to right. The horizontal line indicates the accessible search range at a luminosity of $L = 150$ pb$^{-1}$ and 100 signal events.
Figure 3b: Mass dependence of the minimal integrated pair production cross section for the vector leptoquarks ref. [1] at $\sqrt{s} = 190$ GeV. The lines correspond to the production cross sections for the states $U_1 (U_3(0))$, $\tilde{V}_2(1/2)$, $\tilde{V}_2(-1/2)$, $V_2(-1/2)$, $V_2(1/2)$, $U_3(-1)(\tilde{U}_1)$ and $U_3(1)$ from left to right. The horizontal line indicates the accessible search range at a luminosity of $\mathcal{L} = 500$ pb$^{-1}$ and 100 signal events.
Figure 3c: Mass dependence of the minimal integrated pair production cross section for the vector leptoquarks ref. [1] at $\sqrt{s} = 1$ TeV. The lines correspond to the production cross sections for the states $U_1$ ($U_3(0)$), $\tilde{V}_2(1/2)$, $\tilde{V}_2(-1/2)$($V_2(-1/2)$), $U_3(-1)$($V_3(1/2)$), $\tilde{U}_1$ and $U_3(1)$ from left to right. The horizontal line indicates the accessible search range at a luminosity of $\mathcal{L} = 10$ fb$^{-1}$ and 100 signal events.