Flat-Directions in Grand Unification
With $U(1)_R$ Symmetry

(a) S. M. Barr\textsuperscript{1}, (b) Bumseok Kyae\textsuperscript{2}, and (a) Qaisar Shafi \textsuperscript{3}

\textsuperscript{(a)} Bartol Research Institute, Department of Physics,
University of Delaware, Newark, DE 19716, USA
\textsuperscript{(b)} School of Physics, Korea Institute for Advanced Study,
207-43, Cheongnyangni-Dong, Dongdaemun-Gu, Seoul 130-722, Korea

Abstract

It is shown that in $SO(10)$ and $SU(5)$ models having a $U(1)_R$ symmetry, the requirement of breaking the unified group to the Standard Model leads to flat directions in the scalar potential. These can lead to a “cosmological modulus problem”. This is relevant to grand unified models of inflation, where $U(1)_R$ symmetries are often used to insure the flatness of the inflaton potential. A way that the modulus problem might be avoided is discussed.

\textsuperscript{1}smbarr@bxclu.bartol.udel.edu
\textsuperscript{2}bkyae@kias.re.kr
\textsuperscript{3}shafi@bartol.udel.edu
1 Introduction

Grand unified theories (GUTs) are among the most promising extensions of the standard model. They can explain a number of low energy phenomena, such as electric charge quantization, neutrino oscillation, and certain fermion mass relations \[1\] as well as giving unification of gauge interactions and of matter multiplets.

This encourages the search for the answers to various questions in cosmology also within the GUT framework — questions such as the origin of cosmological inflation and baryon asymmetry.

In Refs. \[2\], one possible approach to these questions was proposed in the context of supersymmetric (SUSY) GUT theory. In particular, it was noted there that in a certain class of SUSY models inflation is intimately associated with the spontaneous breaking of a gauge symmetry at the GUT scale, in such a way that \(\frac{\delta T}{T}\) is proportional to \(\left(\frac{M}{M_{\text{Planck}}}\right)^2\), where \(M\) denotes the symmetry breaking scale and \(M_{\text{Planck}}\) (\(\equiv 1.2 \times 10^{19}\) GeV) denotes the Planck mass. Thus, from measurements of \(\delta T/T\), \(M\) is estimated to be of order \(10^{16}\) GeV \[2, 3\], which is very close to the SUSY GUT scale.

The scalar spectral index \(n_s\) in these models is very close to unity in excellent agreement with recent fits to the data \[4\]. The vacuum energy density during inflation is of order \(10^{14}\) GeV, so that the gravitational contribution to the quadrupole anisotropy is essentially negligible. Furthermore, the inflaton field in this scenario eventually decays into right-handed neutrinos, whose out of equilibrium decays lead to the observed baryon asymmetry via (non-thermal) leptogenesis \[5\]. It is, therefore, worthwhile to realize this inflationary scenario within a grand unified framework. Realistic inflationary models along this line were presented, based on the \(SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) \[6\], \(SU(4)_c \times SU(2)_L \times SU(2)_R\) \[7\], \(SU(5) \times U(1)\) \[8\], \(SU(5)\) \[9\], and \(SO(10)\) \[10\].

In the construction of these inflationary models, which are of the “F-term inflation” type, a global \(U(1)_R\) symmetry plays the essential role of guaranteeing the
flatness of the inflaton potential. Indeed, $U(1)_R$ symmetry is associated with $N = 1$ SUSY, and so it can reside in any $N = 1$ SUSY model. This $U(1)_R$ symmetry is important also in “D-term” inflationary scenario for the same reason. The $U(1)_R$ symmetry, however, makes it more difficult to build SUSY GUT models with $SU(5)$, $SO(10)$, and other unified groups. In $SU(5)$ and $SO(10)$ etc, at least an adjoint Higgs field needs to achieve vacuum expectation values (VEVs) of order $O(M_G)$ in order to break the symmetry to the standard model group. Unfortunately, mechanisms to give VEVs to adjoint Higgs in SUSY GUT models with $U(1)_R$ symmetry usually leave scalar fields with VEVs of order $O(M_G)$, whose potentials are flat in the SUSY limit.\footnote{This difficulty is readily avoided in models based on gauge groups such as flipped $SU(5)$ and $SU(4) \times SU(2) \times SU(2)$ which employ Higgs fields in the tensor and bi-fundamental representations respectively [8, 7].} Such fields, which we shall call “flat-directions,” potentially correspond to moduli. Moduli, here, means relatively light scalar fields developing superlarge VEVs, whose presence are quite problematic in cosmology. In this paper, we shall prove that emergence of a flat-direction is not avoidable in SUSY $SO(10)$ (and also $SU(5)$) model with $U(1)_R$ symmetry in section 2, and then propose a resolution in section 3. In section 4, we conclude.

2 Flat-Direction in Grand Unification

Before we prove rigorously a theorem about the connection between $U(1)_R$ symmetry, adjoint Higgs VEVs, and flat-direction in the context of $SO(10)$, it may be helpful first to illustrate the problem with a very simple example. Let us denote the adjoint (45) Higgs in $SO(10)$ by $A_H$. A simple set of terms which would generate a VEV for it would be $W_A = M \text{tr}(A_H^2) + \alpha \text{tr}(A_H^4)/M$, where $M = O(M_G)$. However, if there is $U(1)_R$ symmetry, then every term in $W$ must have the same $R$ charge (which we shall henceforth take to be 1). Obviously this is not possible for the terms in $W_A$, unless we allow one of the coefficients to be replaced by a field that has $R \neq 0$. For example, we may consider instead $W'_A = X \text{tr}(A_H^2) + \alpha \text{tr}(A_H^4)/M$, with $R(X) = \frac{1}{2}$.
and $R(A_H) = \frac{1}{4}$. That means that some terms must exist to fix the VEV of $X$ to be of order $M_G$. One possibility would be $W_X = S(X \overline{X} - M^2)$. The existence of $\overline{X}$ is necessitated by the requirement of $U(1)_R$ invariance. We immediately see the problem: this term fixes the VEV of the product $X \overline{X}$, but leaves unfixed the relative magnitudes of $X$ and $\overline{X}$. That is, there remains a flat direction.

This simple example illustrates another problem that is relevant to our later considerations. Introducing the singlet field $X$ into $W'_A$ means that there is an $F$-term equation for $X$ to be satisfied. In this case, the equation $F_X = 0$ implies that $\text{tr}(A^2_H) + S \overline{X} = 0$. Since, by hypothesis $A_H \neq 0$, and by the $F_X = 0$ condition $S = 0$, one has a contradiction. A simple way to avoid this difficulty would be to have two adjoints, one of which has vanishing VEV. For example, one might consider $W''_A = \text{tr}(A A_H) + \alpha \text{tr}(A A^3_H)/M$, where $\langle A \rangle = 0$. The $U(1)_R$ invariance is ensured by the choices $R(A) = 1$ and $R(A_H) = 0$. The equation $F_{A_H} = 0$ is satisfied by the vanishing of $A$, while $F_A$ fixes $A_H$. This seems to obviate the difficulty encountered before, in that we have not had to introduce the singlet $X$ to satisfy $U(1)_R$ invariance. However, the two terms in $W''_A$ are insufficient to give mass to all the fields in $A$ and $A_H$. In particular, there are several color-triplet fields that remain light, and that would be disastrous for the running of the gauge couplings. All fields in the adjoints can be made heavy if an additional term is introduced: 
$X \text{tr}(A^2) + M \text{tr}(A A_H) + \alpha \text{tr}(A A^3_H)/M$. Note the crucial point that the singlet field $X$ has had to be introduced again to satisfy $U(1)_R$ invariance.

The lessons of these simple examples can in fact be generalized to a theorem, which we now state, and shall then prove: In $SO(10)$, if the superpotential has $U(1)_R$ symmetry, and if the adjoint Higgs fields have $O(M_G)$ VEVs and contain no goldstone or pseudo-goldstone components, then there must exist fields with $O(M_G)$ VEVs that have a flat direction. In short, fixing the VEVs of the adjoints when there is $U(1)_R$ symmetry leads to “flat-directions”.

We shall prove the theorem first in the simplified context of $SO(10)$ models whose
Higgs sector consists of the adjoints $A_H$ and $A$ and some number of singlets. We shall distinguish between two kinds of fields that appear in the Higgs superpotential: those that have $O(M_G)$ VEVs we shall call “Higgs fields”, while those that have vanishing VEVs in the supersymmetric limit we shall call “null fields”. The Higgs fields will therefore consist of $A_H$ and some number $N$ of singlets that we shall denote $\phi_i$, $i = 1, \ldots, N$. The null fields will consist of $A$ (possibly) and some number $N_S$ of singlets that we shall denote $S_a$, $a = 1, \ldots, N_S$.

We shall try to fix the VEVs of all these fields (i.e. try to avoid the existence of flat directions), but shall find that this cannot be done if the superpotential has $U(1)_R$ symmetry. One can distinguish three kinds of terms in the Higgs superpotential: Type 0 terms contain no null fields; Type 1 terms are first order in null fields; and Type 2 are second order or higher in null fields. The first thing to be noted is that terms of Type 2 in the superpotential make no contribution to $F$ terms at the minimum and thus have no effect on the minimization of the Higgs potential or the VEVs of the fields. Therefore, in fixing the VEVs we need only consider terms of Type 0 and Type 1. First we shall deal with the case where $W$ contains no Type 0 terms.

### 2.1 Case with no Type 0 terms

If we assume that the superpotential contains no Type 0 terms, it can be taken to have the form:

$$W = W_S + W_A + W_2,$$

where $W_2$ represents Type 2 terms that can be neglected,

$$W_S = \sum_{a=1}^{N_S} S_a P_a(\phi_i, A_H),$$

and

$$W_A = X_1 \text{tr}(A^2) + X_2 \text{tr}(A A_H) + X_3 \text{tr}(A A_H^3) + h.o.$$
The $P_a$ in Eq. (2) are polynomials in the Higgs fields. The factors $X_1$, $X_2$ and $X_3$ in Eq. (3) represent either singlet Higgs fields ($\phi_i$) or products of Higgs fields ($\phi_i$ and $A_H$) divided by appropriate powers of $M_G$ to make the terms dimensionally correct. We include the term $X_1 \text{tr}(A^2)$ even though it is of Type 2 and does not affect the minimization, because it must be there to give mass to all the color-triplet fields in the adjoints (as noted earlier), and because it shall play a crucial role in the proof by constraining the $U(1)_R$ charges of fields. The "h.o." in Eq. (3) represents terms containing $\text{tr}(A^n A_H^m)$ with $n$ larger than 3. These can be included but would not affect the analysis.

Each of the three terms in Eq. (3) must have $R = 1$ and must be neutral under all local $U(1)$ symmetries. Consider, then, the product

$$
\Pi \equiv [X_1]^{-1}[X_2]^3[X_3]^{-1}.
$$

(4)

This product obviously has the same charges as the following product of terms in Eq. (3): $[X_1 \text{tr}(A^2)]^{-1}[X_2 \text{tr}(A A_H)]^3[X_3 \text{tr}(A A_H^2)]^{-1}$. Since each term in Eq. (3) must have gauge charge zero and $R = 1$, it must be that $\Pi$ has $R = -1 + 3 - 1 = 1$ and is neutral under all local $U(1)$ symmetries. Moreover, since it is made up of powers of the Higgs fields $\phi_i$ and $A_H$, it has a VEV that is $O(M_G^3)$. The crucial question will be whether such a product can exist if there are no flat directions.

There are only two kinds of terms in the Higgs potential available to fix the VEVs of the $\phi_i$ and $A_H$ fields: $D$-terms corresponding to whatever local $U(1)$ symmetries exist in the model, and $F$-terms corresponding to the null fields $S_a$ and $A$. (It is clear from the fact that there are no Type 0 terms in the Higgs superpotential that $F_{\phi_i}$ and $F_{A_H}$ automatically vanish, since they necessarily have at least one power of a null field. That means that only the $F$-terms $F_{S_a}$ and $F_A$ affect the minimization.)

Let the gauged $U(1)$ groups be denoted $U(1)_K$, $K = 1, \ldots, N_D$. The charges of the Higgs fields $\phi_i$, $i = 1, \ldots, N$, and $A_H$ under the group $U(1)_K$ can be thought of as a vector in an $(N + 1)$-dimensional space: $
abla Q^K = (Q^K(\phi_1), \ldots, Q^K(\phi_N), Q^K(A_H))$. 

5
There are $N_D$ such vectors, one for each $U(1)_K$. However, it may be that not all of these vectors are independent. Suppose that $\tilde{N}_D$ of them are independent. Then the $D$-terms provide $\tilde{N}_D$ independent conditions on the VEVs of the Higgs fields. We can also think of the $R$ charges of the fields as forming an $(N + 1)$-component vector in the same space: $\vec{R} = (R(\phi_1), ..., R(\phi_N), R(A_H))$.

There are $(N_S + 1)$ conditions (of the form $P_a = 0$) on the VEVs of $\phi_i$ and $A_H$ coming from $F_{S_a} = 0$, $a = 1, ..., N_S$, and $F_A = 0$. It is easily seen that these $F$-term conditions must be independent. (If some $P_a$ were expressible as linear combinations of others, that would obviously mean that there were exact relationships among some of the coefficients in the $P_a$, which would require fine-tuning.) Thus, there are a total of $(\tilde{N}_D + N_S + 1)$ conditions on the VEVs of $(N + 1)$ fields. Consequently, if all Higgs field VEVs are to be fixed in the supersymmetric limit, i.e. if there are to be no flat directions, it must be that $\tilde{N}_D + N_S \geq N$. We will now show that this would imply that the $U(1)$ charges of Higgs fields are so highly constrained that no product $\Pi$ of them can be constructed that has $R(\Pi) = 1$ and $Q^K(\Pi) = 0$, as needed to write down the terms in Eq. (3).

Consider one of the polynomials $P_a$ in Eq. (2). In order to satisfy the relation $-F_{S_a}^* = P_a = 0$, the polynomial $P_a$ must have $p$ terms, where $p \geq 2$, since each term in $P_a$ is a product of fields with non-zero VEVs. Then the requirement that these $p$ terms all have the same charge under a $U(1)$ gives $p - 1$ homogeneous linear relations on the charges of the Higgs fields under that $U(1)$. (Note that this is true also for $U(1)_R$.) Consequently, for any $U(1)$ group, the terms in $W_S$ give at least $N_S$ homogeneous linear relations on the charges of the Higgs fields under that group.

Moreover, for any $U(1)$, the requirement that the three terms displayed in Eq. (3) have the same charge under that group (in particular 0 for a gauge $U(1)$ and 1 for $U(1)_R$) gives two homogeneous linear relations that must be satisfied by the charges of the fields $A$, $A_H$, and $\phi_i$ under that $U(1)$. Eliminating the charge of the null field $A$ from these relations, one is left with exactly one homogeneous linear relation on
the charges of the the Higgs fields under the $U(1)$. The “other terms” in Eq. (3) may give additional linear relations.

We thus have that the components of any “charge vector” (whether it be $\vec{Q}^K$ or $\vec{R}$) must satisfy at least $N_S + 1$ homogeneous linear relations. Therefore, all the charge vectors lie in a subspace that has dimension $\leq (N + 1) - (N_S + 1) = (N - N_S)$. Since it has already been shown that $\tilde{N}_D + N_S \geq N$ (in order to fix all the VEVs), the subspace in which all the charge vectors lie must have dimension $\leq \tilde{N}_D$. However, the number of vectors $\vec{Q}^K$ is $\tilde{N}_D$ and they are all independent, so they must span this subspace. Consequently, $\vec{R}$, which also lies in this subspace, must be expressable as a linear combination of the $\vec{Q}^K$: i.e. $\vec{R} = \sum_K \alpha_K \vec{Q}^K$. That means that any product of powers of the Higgs fields $\phi_i$ and $A_H$ that has vanishing charge under all the gauge groups $U(1)_K$ must also have vanishing $R$ charge. However, this contradicts the requirement for writing down the terms in Eq. (3), namely that some product $\Pi$, given in Eq. (4), has $Q^K(\Pi) = 0$ and $R(\Pi) = 1$. We have thus proven the theorem in the case where $W$ contains no Type 0 terms. Now we turn to the case where there are Type 0 terms.

### 2.2 The case with Type 0 terms

Before doing the general case, consider a few simple examples. Let $W_0$ comprise the Type 0 terms in $W$, and let them all depend on only one field, a gauge-singlet Higgs field that we will denote $H$. If $W_0$ contains only a single monomial in $H$, say $H^p$, then the condition $F_H = 0$ will force $\langle H \rangle$ to vanish, which is a contradiction since $H$ would then be a null field appearing in $W_0$, which by definition contains no such fields. There must therefore be at least two different monomials in $W_0$, e.g. $W_0 = aH^p + bH^q$, with $p \neq q$, and $a$ and $b$ being some coefficients. However, it is then obviously impossible to make both terms in $W_0$ have $R = 1$. This illustrates the general difficulty that if there are enough terms in $W_0$ to fix the VEVs of all the Higgs that it contains, then there are too many constraints on the $R$ charges to be satisfied. To put it the other way, if the $R$ charges can be consistently assigned, there must be at least one VEV that does not get fixed, i.e. a flat direction. A simple illustration of this is the
following. Let \( W_0 = H_1 H_2 H_3 + M H_2 H_3 + M^2 H_3 + (1/M) H_1 H_2^2 H_3 \). Then \( R \) invariance is satisfied by assigning \( R(H_1) = R(H_2) = 0 \) and \( R(H_3) = 1 \). Moreover, \( F_{H_1} = 0 \) gives \( H_2 = -M \); and \( F_{H_2} = 0 \) gives \( H_1 = M \). However, there is a flat direction, since the remaining condition \( F_{H_3} = 0 \) gives \( H_1 H_2 + MH_2 + M^2 + (1/M) H_1 H_2^2 = 0 \), which is automatically satisfied independently of the value of \( H_3 \). Note that the flat direction corresponds to the \( R \) charge assignments. (That is, only \( H_3 \) has a non-zero \( R \) charge, and the flat direction is the \( H_3 \) direction in field space.) This result generalizes, as we now show.

Let \( W_0 \) depend on \( N_0 \) gauge-singlet Higgs fields, which we will denote \( H_\alpha, \alpha = 1, ..., N_0 \). Let \( W_0 \) have the form

\[
W_0 = \sum_{n=1}^{N_T} T_n(H_\alpha), \quad n = 1, ..., N_T. \tag{5}
\]

where each term \( T_n \) is a monomial in the Higgs fields: \( T_n = c_n(H_1)^{a_1 n}...(H_{N_0})^{a_{N_0} n} \). There are \( N_0 \) conditions \( F_{H_\alpha} = 0 \). It is convenient to write them in the form (no sum over \( \alpha \)):

\[
H_\alpha \frac{\partial}{\partial H_\alpha} W_0 = 0, \quad \alpha = 1, ..., N_0. \tag{6}
\]

Obviously the other terms in \( W \) besides \( W_0 \) do not contribute to these conditions since they all contain at least one null field. The operator \( H_\alpha \frac{\partial}{\partial H_\alpha} \) acting on any term \( T_n \) in Eq. (5) just multiplies that term by a number, \( a_{\alpha n} \). Consequently, the equations given in Eq. (6) are just homogeneous linear equations in the terms \( T_n \):

\[
H_\alpha \frac{\partial}{\partial H_\alpha} W_0 = \sum_n H_\alpha \frac{\partial}{\partial H_\alpha} T_n = \sum_n a_{\alpha n} T_n = 0. \tag{7}
\]

We may assume that these \( N_0 \) equations are all linearly independent. For, if they were not, it would mean that some of the fields \( H_\alpha \) only appeared in \( W_0 \) in certain product combinations. (For example, if \( H_1 \) and \( H_2 \) only appeared in the combination \( H_1^p H_2^q \), then the \( F_{H_1} \) equation and \( F_{H_2} \) equation would be proportional.) We could then take those product combinations to be new singlet fields \( H'_\alpha \). The equations that
resulted from differentiating $W_0$ with respect to these new fields would be linearly independent. We can assume that we have already performed such a reduction to get Eq. (5), so that the $N_0$ equations in Eq. (6) are linearly independent.

Now, suppose that that all the terms in $W_0$ have $R = 1$ and that $W_0$ contains enough terms that there is a solution to Eq. (6) with all the $T_n \neq 0$. (We have already given an example of this with $N_0 = 3$ above.) Let the fields have values $\overline{H}_\alpha$ at this solution. Consider scaling this solution by powers of some complex number $\lambda$ in the following way: $H_\alpha = \lambda^{R(H_\alpha)}\overline{H}_\alpha$. Since each term in $W_0$ has $R(T_n) = 1$, every term $T_n$ will scale by a factor of exactly $\lambda$. Therefore, the equations given in Eq. (7) are all still satisfied, which in turn implies that the direction parametrized by $\lambda$ is $F$-flat. (And this general result is verified in the example given above, where the scaling would affect only the field $H_3$, which is indeed the flat direction.)

We have assumed that the fields $H_\alpha$ are singlets. However, the same argument easily generalizes to non-singlet fields. For example, suppose we allow one of these fields to be an adjoint $A_H$. Only traces of an even power of $A_H$ can appear because the adjoint of $SO(10)$ is antisymmetric. We may write all the traces of powers of $A_H$ that appear in $W_0$ as combinations of $\text{tr}(A_H^2)$ and ratios of the form $R_p \equiv \text{tr}(A_H^{2p})/(\text{tr}(A_H^2))^{p}$. Now given a certain form of the VEV of $A_H$ these ratios $R_p$ just give numbers. Moreover, in the equation $A_H \partial W_0/\partial A_H = 0$ from Eq. (6) the factors of $R_p$ make no difference, since $A_H \partial R_p/\partial A_H = 0$. Thus, we may effectively take the $R_p$ to be pure numbers and $\text{tr}(A_H^2)$ to be a singlet field in the argument we made before.

Now, all we have shown up to this point is that there is a direction that is undetermined (i.e. flat) by the conditions $F_{H_\alpha} = 0$. That does not imply that when all the $F$ and $D$ terms are taken into account there must be a flat direction. To prove the theorem requires a few more steps. First, let us prove that there are at least $N_0$ independent terms in $W_0$. (A set of monomials in the fields $H_\alpha$ is independent if none of them can be expressed as a product of powers of the others.) Let us suppose that of the $N_T$ terms in $W_0$, $M$ are independent. Then the remaining ($N_T - M$) terms
can be written in terms of the independent ones as follows:

\[ W_0 = \sum_{n=1}^{M} T_n + \sum_{\ell=M+1}^{N_T} c_{\ell} (T_1^{\rho_1 \ldots \rho_M \ell}) . \]  

(8)

Then Eq. (6) becomes

\[ 0 = \sum_{n=1}^{M} a_{\alpha n} T_n + \sum_{\ell=M+1}^{N_T} a_{\alpha n} p_{n\ell} T_{\ell} = \sum_{n=1}^{M} a_{\alpha n} \left( T_n + \sum_{\ell=M+1}^{N_T} p_{n\ell} T_{\ell} \right) . \]  

(9)

This is a set of \( N_0 \) independent linear equations in the \( M \) quantities \( \tilde{T}_n \equiv T_n + \sum_{\ell=M+1}^{N_T} p_{n\ell} T_{\ell} \). There can be no solution unless \( M \geq N_0 \), which is what we wanted to show. We are now in a position to complete the proof of the theorem.

Consider the case where the Higgs superpotential consists of the terms given in Eqs. (1), (2), (3), and (5), where the \( N_0 \) fields \( H_\alpha \) appearing in Eq. (5) are a subset of the \( (N+1) \) fields \( \phi_i \) and \( A_H \). As was shown previously, there are \( (N_S + 1) \) conditions on the VEVs coming from \( F_{S_a} = 0 \) and \( F_A = 0 \), and \( \tilde{N}_D \) conditions coming from the \( D \) terms. And we have shown that there are no more than \( (N_0 - 1) \) conditions on the VEVs coming from \( F_{H_\alpha} = 0 \) (not \( N_0 \) because of the direction that is left flat by those conditions). Altogether, then, there are no more than \( (N_S + N_0 + \tilde{N}_D) \) conditions, and these must be sufficient to fix the VEVs of the \( (N+1) \) fields \( \phi_i \) and \( A_H \). That implies that \( (N_S + N_0 + \tilde{N}_D) \geq (N + 1) \).

On the other hand, we showed that coming from the terms in \( W_S \) and \( W_A \) there are \( (N_S + 1) \) homogeneous linear relations on the charges of the Higgs fields under any \( U(1) \) group. In addition, because there are at least \( N_0 \) independent terms in \( W_0 \), the fact that these terms must all have the same charges (i.e. 0 for gauge \( U(1) \) and 1 for \( U(1)_R \)), yields at least another \( (N_0 - 1) \) homogeneous linear relations on the charges of the Higgs fields under any \( U(1) \). Altogether, then, there are at least \( (N_S + N_0) \) homogeneous linear relations on the charges of the \( (N+1) \) Higgs fields. That means that each “charge vector” lies in a subspace of dimension \( \leq (N + 1 - N_S - N_0) \leq \tilde{N}_D \). That implies that the \( \tilde{N}_D \) independent vectors \( \vec{Q}^K \) must span this subspace, and that therefore \( \vec{R} \) is a linear combination of the \( \vec{Q}^K \). As before, that means that the product
Π in Eq. (4) cannot have both $Q^K(\Pi) = 0$ and $R(\Pi) = 1$ as required to write the terms in Eq. (3).

There is one final case to consider. Suppose that the Higgs superpotential consists of $W_S$ and $W_0$, but has no terms of the form $W_A$ given in Eq. (3). (So there is no $A$ field.) That is, suppose the VEV of $A_H$ is set by $W_0, W_S$ and $D$ terms, rather than by $W_A$. In that case, there are $N_S + (N_0 - 1) + \tilde{N}_D$ conditions on the $(N + 1)$ Higgs VEVs, so that $(N_S + N_0 + \tilde{N}_D) \geq (N + 2)$. On the other hand, the terms in $W_S$ and $W_0$ yield at least $N_S + (N_0 - 1)$ independent linear relations on the charges of the Higgs under any $U(1)$ group. Thus the charge vectors lie in a subspace of dimension $(N + 1) - (N_S + N_0 - 1) \leq \tilde{N}_D$. The argument then proceeds as before.

The foregoing argument applies to $SU(5)$ as well as $SO(10)$. In $SU(5)$, one is allowed to have a cubic coupling of an adjoint Higgs, so that the breaking of $SU(5)$ can be achieved with the terms $M \text{tr}(A A_H) + \text{tr}(A A_H^2)$, where $A$ and $A_H$ are in the $24$. However, it is easy to show that these terms are not enough to give mass to all the un-eaten modes of the adjoints. Thus, to avoid goldstone modes, one must add a term like $\text{tr}(A^2)$, as in the $SO(10)$ case. The reasoning then proceeds in complete analogy with the $SO(10)$ case, the only (and insignificant) exception being that there is a cubic rather than a quartic term in $W_A$.

### 3 Flat-Direction and Modulus

So far we have proven that in SUSY GUTs based on $SO(10)$ and $SU(5)$ (and other groups that require an adjoint to get a VEV to break to the Standard Model) $U(1)_R$ symmetry leads to flat directions. A direction that is flat in the supersymmetric limit can lead to a “cosmological modulus problem”. The point is that soft SUSY-breaking terms will give a tiny mass in the flat direction and pick out some point in that direction as the true minimum. However, in the early universe this “modulus” field may find itself “initially” far from the true minimum — indeed, one might typically expect that it finds itself a distance of order $M_G$ from the minimum. In that case,
after SUSY breaking, the modulus field would oscillate about its true minimum, and
the energy in these oscillations would have disastrous consequences for standard Big
Bang cosmology.

However, even if there is a flat direction, a cosmological catastrophe would be
avoided if the “modulus” field happened “initially” to find itself at the true minimum.
What we mean by the “initial” value of the modulus field is its value after inflation.
This value is determined by temperature-dependent terms in the effective potential
that lift the degeneracy in the flat direction. On the other hand, the minimum at low
temperature is determined by the soft SUSY-breaking terms. The question is whether
the high-temperature terms select out the same point along the flat direction as the
low-energy SUSY-breaking terms. If so, there is no problem.

Let us illustrate with a toy model how the modulus might find itself at the true
minimum initially. Consider a superpotential of three scalar superfields $S$, $X$ and
$Y$ of the form $W = S\left(\frac{1}{M_{(a+b-2)}} X^a Y^b - M_{X}^2\right)$. The flat direction is given by $Y = (M_{X}^2 M_{1}^{(a+b-2)})^{1/b} X^{-a/b}$. Suppose the soft SUSY-breaking terms are “universal”, i.e.
of the form $m_0^2 (|X|^2 + |Y|^2 + ...)$. (Ignore $A$ terms.) This would pick out the minimum
$X = (a/b)^{b/2(a+b)} (M_{X}^2 M_{1}^{(a+b-2)})^{1/(a+b)}$ and $Y = \sqrt{b/a} X$. Now, consider the situation
during inflation. If the Kähler potential is of the minimal form, then during inflation
all scalars can have an equal contribution to their mass-squared given by $M_{\text{infl}}^2 (|X|^2 +
|Y|^2 + ...)$, with $M_{\text{infl}} \sim \Lambda^2 / M_P$. $\Lambda$ is the vacuum energy density during inflation and
$M_P (= 2.4 \times 10^{18} \text{ GeV})$ is the reduced Planck mass. If $M_{\text{infl}}$ is large enough, the fields
$X$ and $Y$ will be trapped at the origin during inflation. However, the crucial point
is that both the $M_{\text{infl}}$ term and the soft SUSY-breaking terms that dominate at low
energy have the same “universal form”. That means that as $X$ and $Y$ evolve away
from zero after inflation they will maintain the ratio $Y = \sqrt{b/a} X$ and go directly
to the true minimum. In other words, because the high-$T$ and low-$T$ mass-squared
terms have the same form, the modulus finds itself “initially” at the true minimum,
thus avoiding the cosmological difficulties coming from energy trapped in oscillations
about that minimum.

Somewhat more generally, assume the flat-direction is one dimensional line in two dimensional field space \((\psi_1, \psi_2)\), as shown in Fig. 1.

![Diagram](image)

**FIG. 1:** By including SUSY breaking soft terms in the scalar potential, VEVs of a flat-direction \(F\) are assumed to be determined at \(\vec{v}_a\) (a) and \(\vec{v}_b\) (b), respectively. We neglect deformations of \(F\) by soft terms.

The line “F” in Fig. 1(a) and 1(b) denotes the flat-direction, along which the scalar potential vanishes in the SUSY limit. By including SUSY breaking soft terms, the vacuum state is assumed to be fixed on \(\vec{v}_a\) [in (a)] or \(\vec{v}_b\) [in (b)]. Soft terms would deform the flat-direction just by \(\mathcal{O}(m_{3/2})\), which we may neglect. Once the vacuum is determined, the physical mass eigenstates on that vacuum state can be found. \(\psi'_1, \psi'_2\) and \(\psi''_1, \psi''_2\) correspond to the mass eigenstates in Fig. 1(a) and 1(b), respectively. Since \(\psi'_2\) and \(\psi''_2\) are parallel to the tangent lines \(T_a\) and \(T_b\) on \(\vec{v}_a\) and \(\vec{v}_b\), their masses are \(\mathcal{O}(m_{3/2})\) [zero in the SUSY limit]. On the other hand, the masses of \(\psi'_1\) and \(\psi''_1\), which are orthogonal to \(\psi'_2\) and \(\psi''_2\) respectively, should be \(\mathcal{O}(M_G)\). The important difference between (a) and (b) arises from the VEVs of \((\psi'_1, \psi'_2)\) and \((\psi''_1, \psi''_2)\). In case (a), the VEVs of both \(\psi'_1\) and \(\psi'_2\) are \(\mathcal{O}(M_G)\), so that the oscillations of the light scalar \(\psi_2\)
would generally give rise to the cosmological modulus problem. However, in type (b), the VEV of the light field $\psi''$ vanishes, as can be seen from Fig. 1(b), and no modulus problem results, as long as the “initial” value of $\psi''$ also vanishes. Note that the tangent line “$T_b$” of the flat-direction “$F$” at $\vec{v}_b$ is also a tangent line to the circle “$C$” whose center is located at the origin. If the terms that lift the degeneracy of the flat direction $F$ are of “universal” form, then their effect, of course, is to minimize the distance to the origin, If both the high-temperature and the low-temperature terms that lift the degeneracy of $F$ are minimal in form, then the fields will evolve along the straight line from the origin to the point on $F$ that is closest to the origin. As can be seen from Fig. 1(b), this means that the “modulus” mode is never excited, and no cosmological modulus problem arises.

The toy model involving the superfields $S, X$, and $Y$ that we gave at the beginning of this section is a realization of case (b). (As the temperature falls in that example, the fields evolve from the origin directly in a straight line $Y = \sqrt{b/a}X$ toward the true minimum, so that the field we call $\psi''$ in Fig. 1(b) always vanishes.)

It remains to ask whether the high-temperature and low energy effective mass terms can be of minimal form. If the Kähler potential itself has a minimal form, then this can be the case. Moreover, in gauge-mediated SUSY breaking scenario, the “A-terms” (which we neglected in the above analysis) are generally small. And even in some special cases of gravity mediation the “A-term” contributions to the scalar potential $V(\psi_1, \psi_2)$ may be suppressed [10]. Suppose that the VEV of the superpotential in which $\psi_1$ and $\psi_2$ are involved ($\equiv \langle W(\psi_1, \psi_2) \rangle$ is cancelled by another term ($\equiv \langle W_c \rangle$). If the dimensions of $\langle W(\phi) \rangle$ and $\langle W_c \rangle$ are the same, then the “A-term” contribution to $V(\psi_1, \psi_2)$ with the minimal Kähler potential would effectively also cancel at the leading order.

The above discussion can obviously be extended to the more general case with any number of fields. If the tangent space at a vacuum point of an N-dimensional flat-direction coincides with the tangent space at that point of the sphere whose center is
at the origin, one has a situation of type (b).

4 Conclusion

We have proved a theorem that in models based on $SO(10)$ and $SU(5)$ the existence of a $U(1)_{R}$ symmetry together with the requirement of breaking the unified group to the standard model leads to the existence of flat directions in the scalar potential in the SUSY limit. Such light modes (which generally obtain masses of order $m_{3/2}$ when supersymmetry breaks) can lead to the well known “cosmological modulus problem”. These observations are relevant to the problem of building grand unified models of inflation, where a $U(1)_{R}$ symmetry is typically required to ensure the flatness of the inflaton potential. We have noted in the previous section that there is a way that the cosmological modulus problem can in principle be avoided, even if there are flat directions, if the modulus modes find themselves at the true minimum of their potential after inflation. We have discussed conditions under which this may be the case.

Acknowledgments

S.M.B and Q.S. are partially supported by the DOE under contract No. DE-FG02-91ER40626.

References

[1] For instance, see C. H. Albright and S. M. Barr, Phys. Rev. D 58 (1998) 013002 [arXiv:hep-ph/9712488]; C. H. Albright, K. S. Babu and S. M. Barr, Phys. Rev. Lett. 81 (1998) 1167 [arXiv:hep-ph/9802314]; C. H. Albright and S. M. Barr, Phys. Lett. B 452 (1999) 287 [arXiv:hep-ph/9901318]; S. M. Barr, Phys. Rev. Lett. 92 (2004) 101601 [arXiv:hep-ph/0309152]; S. M. Barr and B. Kyae, Phys. Rev. D 70 (2004) 075005 [arXiv:hep-ph/0407154].
[2] G. R. Dvali, Q. Shafi and R. K. Schaefer, Phys. Rev. Lett. 73 (1994) 1886 [arXiv:hep-ph/9406319]. For a review and additional references, see G. Lazarides, Lect. Notes Phys. 592 (2002) 351 [arXiv:hep-ph/0111328]. See also D. H. Lyth and A. Riotto, Phys. Rept. 314 (1999) 1 [arXiv:hep-ph/9807278].

[3] V. N. Senoguz and Q. Shafi, Phys. Lett. B 567 (2003) 79 [arXiv:hep-ph/0305089]; V. N. Senoguz and Q. Shafi, Phys. Rev. D 71 (2005) 043514 [arXiv:hep-ph/0412102].

[4] D. N. Spergel et al., Astrophys. J. Suppl. 148 (2003) 175 [arXiv:astro-ph/0302209]; C. L. Bennett et al., Astrophys. J. Suppl. 148 (2003) 1 [arXiv:astro-ph/0302207]; H. V. Peiris et al., Astrophys. J. Suppl. 148 (2003) 213 [arXiv:astro-ph/0302225]. See also G. F. Smoot et al., Astrophys. J. 396 (1992) L1; C. L. Bennett et al., Astrophys. J. 464 (1996) L1 [arXiv:astro-ph/9601067].

[5] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45. For non-thermal leptogenesis, G. Lazarides and Q. Shafi, Phys. Lett. B 258 (1991) 305.

[6] G. R. Dvali, G. Lazarides and Q. Shafi, Phys. Lett. B 424 (1998) 259 [arXiv:hep-ph/9710314].

[7] R. Jeannerot, S. Khalil, G. Lazarides and Q. Shafi, JHEP 0010 (2000) 012 [arXiv:hep-ph/0002151]; S. F. King and Q. Shafi, Phys. Lett. B 422 (1998) 135 [arXiv:hep-ph/9711288].

[8] B. Kyae and Q. Shafi, arXiv:hep-ph/0510105

[9] B. Kyae and Q. Shafi, Phys. Lett. B 597 (2004) 321 [arXiv:hep-ph/0404168].

[10] B. Kyae and Q. Shafi, Phys. Rev. D 72 (2005) 063515 [arXiv:hep-ph/0504044]; arXiv:hep-ph/0510300.
[11] P. Binetruy and G. R. Dvali, Phys. Lett. B 388 (1996) 241

arXiv:hep-ph/9606342.