Hadron Formation and the Phase Diagram of QCD Matter

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We investigate hadronic species freeze-out systematics in A+A collisions at low SPS energies, corresponding to a baryochemical potential above 300 MeV, analyzing NA49 hadron production data in the framework of the statistical hadronization model, and in the UrQMD hadronic transport model. Observing no deviation from universal grand canonical hadro-chemical equilibrium freeze-out, we argue that the observed hadronic freeze-out points should universally signal the boundary line of the hadronic phase in the QCD matter phase diagram.

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The phase diagram of strongly interacting matter represents the most challenging open problem of quantum chromodynamics (QCD). Its key-feature, the confinement-deconfinement line between hadrons and partons interpolates between the regions in the \([T, \mu_B]\) plane where the temperature \(T\) is high and the baryochemical potential \(\mu_B\) (the net baryon density) low, to the domain of low \(T\) and high \(\mu_B\). Lattice QCD studies at finite temperature and \(\mu_B = 0\) have described the phase transformation as a rapid crossover in the vicinity of \(T_c = 170 \text{ MeV}\) [1] which coincides with Hagedorn’s former estimate of the limiting temperature \(T_H\), expected for matter consisting of hadrons and their resonances [2]. Toward higher \(\mu_B\), the position of the parton-hadron coexistence line has been estimated recently by extrapolations of lattice QCD to finite net baryon density [3, 4, 5], and by former considerations of the QCD process of chiral symmetry restoration, occurring at the border of hadronic matter [6, 7, 8] which is both confined, and exhibits strongly broken chiral symmetry. These two properties disappear at the QCD phase boundary.

This phase boundary is traversed in relativistic nucleus-nucleus \((A+A)\) collisions at sufficiently high \(\sqrt{s}\) (above about 5-10 GeV). It is the goal of such studies to elucidate the properties of QCD matter as sampled along the dynamical trajectory of the collisional “fireball”. The dynamics in such collisions can be sketched [9, 10, 11] by an initial stage of matter heating and compression by processes occurring at the microscopic level (such as breakup of the parton structure functions), to be followed by a slowdown phase broadly analogous to a classical turning point, during which partonic equilibrium should be approached and the system first appears on the \([T, \mu_B]\) plane of QCD matter, in various domains depending on \(A\) and \(\sqrt{s}\). The dynamical trajectory enters the phase diagram well above \(T_c\). A subsequent expansion phase transports the contained matter across the confinement and chiral symmetry breaking phase border(s), giving rise to hadronization, and hadronic “freeze-out” to a high \(T\) hadron-resonance system. A last stage, of essentially elastic and resonant rescattering, then leads to a final de-

FIG. 1: The phase diagram of QCD matter in the grand canonical variables of temperature \(T\) vs. baryochemical potential \(\mu_B\). Also shown is a conjectured critical point [3, 4, 5] and the hadronic freeze-out points [12, 14, 15, 16] (see text).
scribing hadron production multiplicities in elementary $e^+e^-$ collisions \[19\], suggesting that statistical equilibrium production is a general property of the hadronization process itself \[12, 16, 20, 21, 22, 23, 24\]. The resulting idea - that the hadro-chemical freeze-out data in $A+A$ collisions at various $\sqrt{s}$ thus should result in a succession of points that universally locate the QCD hadronization transition in the $[T, \mu_B]$ phase diagram - will be pursued below.

Fig. 1 illustrates our present view \[3, 25, 26\] of the QCD phase diagram. A critical point on the deconfinement line marks the end of the cross-over domain \[1\] at lower $\mu_B$, the transition becoming first order toward higher $\mu_B$ \[6, 7, 8\]. There, we also sketch a further phase transition line which defines the border between hadron-resonance matter and the quarkyonium phase recently postulated by McLerran and Pisarski \[27\]. It is suggested to represent the large number of color ($N_c$) limit of QCD below $T = T_c$, consisting of massive quarks plus thermal excitations of glueballs and mesons. Whereas the deconfinement line at low $\mu_B$, and an estimate of the critical point position are adopted from recent lattice calculations \[3\], the border line of the quarkyonic phase is a mere guess. Following a suggestion by McLerran \[28\] we have sketched it such that it intersects the points corresponding to hadron-resonance freeze-out at various $\sqrt{s}$, as determined from a statistical model analysis to be described below. We thus imply that hadron-resonance freeze-out occurs, at high $\mu_B$, at the phase boundary of a new state of QCD matter, interpolating between the lattice confinement line and hadronization - such as the conjectured quarkyonic phase.

This choice is in line with our above ascertation that hadronic freeze-out universally coincides with hadronization. This was first argued \[12, 14, 15, 16, 23, 24\] only for low $\mu_B$ where we see, in Fig. 1, that the freeze-out points merge closely with the lattice phase boundary of the quark-gluon plasma (QGP). However, the freeze-out points fall well below this boundary at $\mu_B \geq 300$ MeV, leading to the question of an intervening state of matter that could cool down the system toward hadronic freeze-out.

It is the aim of this letter to show, first, that the grand canonical statistical model gives an excellent description of the hadron multiplicities obtained by NA49 \[23, 30\], in central $Pb + Pb$ collisions at 40 and 30 $AGeV$ ($\sqrt{s} = 8.7$ and 7.6 $GeV$) where $\mu_B > 300$ MeV. There is no deviation from a synchronous freeze-out in grand canonical species equilibrium. This indicates a freeze-out driven by a phase transition to the hadron-resonance state - as conjectured above. Second, we shall show that, in particular, the system state above the freeze-out points and below the confinement line can not merely be a more dense, hotter hadron-resonance matter as described, e.g., in transport models such as UrQMD \[31\] because such a state would decouple to frozen-out hadron yields in a sequential pattern, in order of species specific inelastic mean free path. We thus seem to require a further non-hadronic phase above the hadron freeze-out points, which decays into hadrons in a phase transition. The quarkyonic phase would provide for this.

Turning to analysis of total $4\pi$ hadron production multiplicities reported by the SPS experiment NA49 \[23, 30\], for central $Pb + Pb$ collisions at 40 and 30 $AGeV$, we show in Fig. 2 the results of a statistical grand canonical ensemble model calculation. The formal and practical details of this analysis are described in ref. \[20\]. We note, in particular, that the so-called "strangeness undersaturation factor" $\gamma_s \leq 1$ is employed here, as a necessary ingredient of the fit procedure of total $4\pi$ multiplicities. It has been recently demonstrated for $A+A$ collisions at SPS and RHIC \[32, 33\] that this fudge factor may stem from surface effects: a significant fraction of the participating nucleons from target and projectile nuclei interact.
only once due to the dilute surface of nuclear density profiles, thus experiencing canonical strangeness suppression characteristic of elementary $p+p$ collisions (core-corona model) \cite{33}. They do not participate in the bulk, grand canonical fireball. However, we can not make use of this quantitative model here as the elementary nucleon-nucleon corona cross sections are only poorly known at such low energies.

The statistical model fits for the two energies illustrated in Fig. \ref{fig:1} are quite satisfactory, with resulting $[T, \mu_B] = [148, 387]$ and $[145, 434]$ and $\gamma_s = [0.86, 0.79]$ respectively. Their quality is equal to that encountered at higher energies, toward top SPS and RHIC energies \cite{17, 18, 19, 20}. This holds, in particular, for the prediction of hyperon yields which, thus, appear to emerge \cite{17, 18, 19, 20}. This holds, in particular, for the prediction of hyperon yields which, thus, appear to emerge from freeze-out at the respective common bulk hadron temperatures. These temperatures fall well below the lattice QCD estimates of $T_c$ (see Fig. \ref{fig:1}), at these high values of $\mu_B$, but the system arrives at freeze-out in a global equilibrium of species, with no indications of a sequential freeze-out.

The two resulting freeze-out points are included in Fig. \ref{fig:1}. The remaining points are taken from a former, similar analysis \cite{34}. Our results for the chemical freeze-out temperature and baryon-chemical potential are in fairly good agreement with those presented in \cite{18, 35, 36}.\cite{18, 35, 36}

We conclude that the equilibrium distribution of hadron-resonance species at high $\mu_B$ freeze-out reflects a system expansion trajectory from $T_c$ to $T_{freeze-out}$, through a state of fireball matter that can cool down the species distribution while maintaining equilibrium. We shall show, next, that this state can not be, merely, a hadron-resonance population undergoing binary inelastic rescattering at the microscopic level, at density and temperature approaching $T_c$. Such a system can not cool down the species distribution to a uniform equilibrium population, freezing-out at $T$ well below $T_c$.

This property of a dense hadron-resonance gas can be studied with the microscopic hadron-resonance transport model UrQMD \cite{31}. The absence of cooling the species distributions established at an initial phase transition at $T_c$ (where the hadron-resonance population is generated via the Cooper-Frye formalism \cite{32}), is illustrated in Fig. \ref{fig:2}. In the top panel we employ results of a former UrQMD calculation by Dumitru and Bass \cite{38}, for central $Au+Au$ collisions at RHIC energy $(\sqrt{s} = 200 \text{ GeV})$. The hadronic yield distribution derived from grand canonical Cooper-Frye hadronization at an assumed $T_C = 170 \text{ MeV}$ is, on the one hand, shown as if it were emitted into vacuum, at this stage, without further interaction. Alternatively, a UrQMD binary hadron collision expansion phase is attached, as an "afterburner". Within exceptions concerning some inevitable final state rearrangement of baryon and antibaryon yields, the overall primordial yield distribution is preserved, frozen-out throughout the hadronic expansion phase.

This analysis refers to $\sqrt{s} = 200 \text{ GeV}$ where $\mu_B$ is small (of order 20 $\text{MeV}$ \cite{18}). The bottom part of Fig. \ref{fig:3} illustrates results of a similar procedure \cite{38}, carried out at $\sqrt{s} = 8.7 \text{ GeV}$, $\mu_B = 380 \text{ MeV}$: the region of interest to the present study. The Cooper-Frye hadronization mechanism is employed, according to an average critical energy density criterion, with resulting hadronization temperatures of 155-160 $\text{MeV}$. Again, the bulk hadronic species yields (e.g. $\pi, K, \Lambda$ and even $\Xi$ and $\Omega$) survive the hadronic expansion phase, as modeled by UrQMD, essentially unchanged. However, the anti-hyperon yields exhibit significant annihilation, in this case. This results in a distortion of the global equilibrium distribution, which seems to be absent in the statistical model analysis illustrated in Fig. \ref{fig:2}. We conclude, for now, that the mode of expansion modeled by UrQMD can apparently not cool down the system to a new equilibrium state, at a lower temperature, but seems to merely lead away from
an initial equilibrium. We leave these observations to the forthcoming, systematic investigation of the UrQMD model predictions.

At low $\mu_B$ the statistical model freeze-out points recover the parton-hadron phase boundary at $T_c (\mu_B)$, as predicted by lattice QCD (see Fig. 1). The primordial hadron-resonance populations emerging at the QGP phase boundary thus are, apparently, not cooled down to below $T_c$. This observation supports the hypothesis of a phase transformation from a QGP to a hadron-resonance phase \( T_c \), occurring at $\mu_B \lesssim 200 \, \text{MeV}$. Conversely, however, the statistical model analysis at high $\mu_B$ determines a hadronization equilibrium temperature well below the lattice QCD estimate of $T_c$, as we have seen in Figs. 1 and 2. Our above considerations show that an intervening hadron-resonance state of system expansion can not explain these observations. It can not cool down the bulk species distributions to below $T_c$. A different state of QCD matter thus seems to be required, to shift the process of hadronization toward $T_c (\text{freeze-out}) < T_c$ at high $\mu_B$, via a further phase transition.

In summary, we have revisited the hadronic species freeze-out data at low SPS energies where $\mu_B \approx 400 \, \text{MeV}$, comparing them to the statistical model and to the UrQMD hadron-resonance transport model. The hadronic yield distributions do not exhibit aspects of a sequential freeze-out. On the contrary, freeze-out occurs in a global equilibrium grand canonical ensemble, similar to the situation encountered at $\mu_B \rightarrow 0$. We have argued that this can not be the consequence of a mere expansive dilution as it occurs in UrQMD.

It has been pointed out [12, 14, 15, 16, 24, 41] that hadronic freeze-out should coincide with hadronization of a quark-gluon phase, at low $\mu_B$. The universality of statistical equilibrium among the hadronic species suggests that the hadrons produced in $A + A$ collisions universally freeze out as the consequence of a hadronization phase transition, freeze-out thus always coinciding with hadronization. This implies that the observed freeze-out temperatures at high $\mu_B$ also coincide with the appropriate Hagedorn limiting hadronic temperature \( T_c \). At high $\mu_B$ the phase boundary $T_c$ predicted by lattice QCD falls well above the freeze-out points as shown in Fig. 1. One thus expects a further, intervening phase of QCD which would postpone the hadronization transition, such as the quarkonia phase predicted by Mellarran and Pisarski [27, 28].

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