The Phenomenology of Single Top Quark Production at the Fermilab Tevatron

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Abstract

Single top quark production at the Fermilab Tevatron Run II (a $p\bar{p}$ collider with $\sqrt{s} = 2$ TeV) proceeds dominantly via two sub-processes, a $t$-channel $W$-gluon fusion process, and an $s$-channel $W^*$ process. We show that these two sub-processes have different sensitivities to new physics effects in the top quark’s electro-weak interactions. The $W^*$ process is sensitive to new heavy charged resonances, such as a $W'$ boson, while the $W$-gluon fusion process is more sensitive to modifications to the top’s interaction, including flavor-changing neutral currents involving the top quark. We examine the implications of these results on our ability to measure $V_{tb}$ with confidence, and propose a quantity $R = \sigma_{W^*}/\sigma_W$, which may be studied in order to characterize the confidence one may place upon a given measurement of $V_{tb}$ from single top production.

PACS numbers: 14.65.Ha, 12.39.Fe, 12.60.-i

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1 Introduction

With the discovery of the top quark by the CDF and D∅ collaborations [1], it has become natural to consider its properties, such as its couplings to the other particles of the Standard Model (SM). The top quark is singled out by its large mass ($m_t \approx 175$ GeV), the same order as the electro-weak symmetry breaking (EWSB) scale, $v = 246$ GeV. Thus it seems reasonable that the top may prove valuable in probing the mechanism of mass generation in the Standard Model of particle physics. Indeed, if there is some type of new physics associated with the generation of mass, it may be more apparent in the top quark sector than for any of the other, lighter, known fermions, in accordance with the mass hierarchy.

While production of $t\bar{t}$ pairs [2] provides an excellent opportunity to probe the top’s QCD properties, in order to carefully measure the top’s electro-weak interactions it is also useful to consider single top production, in addition to studying the decay of the top quark in $t\bar{t}$ events. Single top production at the Tevatron occurs within the SM in three different channels, the $s$-channel $W^*$ production, $qq' \rightarrow W^* \rightarrow t\bar{b}$ [3, 4, 5, 6, 7], the $t$-channel $W$-exchange mode, $bq \rightarrow tq' [7, 8, 9, 10, 11, 12, 13, 14]$ (sometimes referred to as $W$-gluon fusion), and through $tW^-$ production [15]. These three sub-processes have very different kinematics and experimental signatures [4, 9, 11, 12], and as we shall discuss later, are sensitive to different types of new physics in the top quark sector. Thus they provide complimentary information about the properties of the top quark.

In this paper, we quantify the observation that the modes of single top production can be used to provide complimentary information about the top quark. In Section 2, we discuss the properties of these separate modes of single top production in the context of the SM, summarizing the positive and negative aspects of each for probing the top quark sector at the Tevatron Run II. In Section 3 we show the effect of an additional heavy resonance on each sub-process, and in Section 4 we examine the effects from possible modifications to the top’s couplings to the other particles of the SM. Finally, we examine what the possibility of new physics in the top quark sector implies about our ability to measure $V_{tb}$ with confidence.

2 Single Top Production at a Hadron Collider

Single top production at a hadron collider occurs dominantly through three sub-processes. The $W^*$ mode of production shown in Figure 1 occurs when a quark and an anti-quark fuse into a virtual $W$ boson, which then splits into a $t$ and $\bar{b}$ quark. The $W$-gluon fusion mode shown in Figure 1 occurs when a $b$ quark fuses with a...
Figure 1: Feynman diagram for $q\bar{q} \rightarrow W^* \rightarrow t\bar{b}$ at the leading order.

Figure 2: Representative Feynman diagrams for the $W$-gluon fusion mode of single-top production. The gluon splitting diagram is not really an $\alpha_s$ QCD correction, but rather a $1/\ln(m_t^2/m_b^2)$ correction coming from the definition of the $b$ PDF. When combining the contributions from these two diagrams, it is necessary to subtract the part in the gluon splitting diagram where the gluon becomes collinear with the $b$ parton, to avoid double counting this region of kinematics.

Figure 3: Feynman diagrams for $gb \rightarrow tW^-$. 

$W^+$ boson, producing a top quark. The $tW^-$ mode occurs when a $b$ quark radiates a $W^-$, and is shown in Figure 3. This mode may be important at the Large Hadron Collider (LHC), but is highly suppressed at the Tevatron because of the massive $W$ and $t$ particles in the final state. Because of its low cross section of about 0.1 pb (for $m_t = 175$ GeV) at the Tevatron Run II [11, 15], and relative insensitivity to the new physics effects we will be considering in this work, we do not present detailed studies of its cross section.

The three single top production processes contain the $W$-$t$-$b$ vertex of the SM, and thus are sensitive to any possible modification of this vertex from physics beyond

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3\footnote{The LHC is a $pp$ collider with $\sqrt{s} = 14$ TeV.}
the SM, and to the Cabibbo-Kobayashi-Maskawa (CKM) parameter $V_{tb}$. It has been shown [4, 9, 11, 12] that because of the difference in kinematics, it is possible to statistically disentangle the two sub-processes from each other, and from the SM backgrounds expected at the Tevatron Run II.

The total cross section for the $W^{*}$ production sub-process ($\sigma_{W^{*}}$) has been studied at next-to-leading-order (NLO) in QCD corrections (order $\alpha_{S}^{3}$), by including NLO corrections to both the $q\bar{q}'$ initial state (c.f. Figure 4) and the $t\bar{b}$ final state (c.f. Figure 5). Detailed studies for the kinematics of this process at leading order (LO) [4, 7, 12] exist in the literature, as well as more realistic studies in which the effects of soft gluons on the kinematics have been re-summed [6]. This sub-process is an attractive mode for studying top quark properties, because the initial state partons are quarks with relatively large momentum fraction, $x$, and thus the parton densities are well understood. On the other hand, this mode suffers from a smaller cross section than the $W$-gluon fusion mode, and a larger percentage dependence on the mass of the top quark [3]. In order to estimate the uncertainty in the NLO theoretical prediction for the $W^{*}$ production rate, we examine the dependence of the result on the factorization and renormalization scales (which in principle are separate quantities, however we will work with $\mu_{R} = \mu_{F} = \mu$), and the dependence on the choice of the parton distribution functions (PDF) used. A physical observable cannot depend on the scale $\mu$, and a theoretical calculation to all orders in perturbation theory must similarly be independent of the value of $\mu$ used in the calculation. However, at fixed order in the perturbation series the theoretical prediction will still show some residual dependence on $\mu$. Thus, by varying $\mu$, one obtains an estimate of the dependence of the calculation on the uncalculated, higher order terms in the perturbation series. In [5, 6] it was found that in order to cancel potentially large logarithms in the NLO
perturbation coefficients, a scale choice of $\mu = \sqrt{\hat{s}}$ is appropriate, where $\hat{s}$ is the center of mass energy of the incoming partons. Thus we vary the scale between $\mu = 2\sqrt{\hat{s}}$ and $\mu = \sqrt{\hat{s}}/2$ in order to estimate the scale dependence of the calculation. To examine the PDF dependence of the result, we choose the CTEQ4M [16] and MRSS(R1) [17] PDF. For each of various values of $m_t$, we combine all of these predictions (due to varied PDF and scale) to determine the upper and lower bounds on $\sigma_{W^*}$ for each top mass under consideration, by taking the combination of PDF and scale choice which gives the highest and lowest result respectively. In this way we can study the correlated effect obtained by varying the PDF and scale (and $m_t$). We also present the central value of $\sigma_{W^*}$, obtained as the point midway between the upper and lower bounds, and the mean, which is the average of the two PDF sets with the scale choice $\mu = \sqrt{\hat{s}}$. The mean value represents the best theoretical estimate of $\sigma_{W^*}$. The fact that the mean and central values are the same for most of the top masses considered shows that the result does not depend more strongly on either raising or lowering the scale, and thus indicates that the choice of scale is appropriate. Our results are presented in Table 1 and are shown graphically in Figure 6. These results include the full CKM elements for the light quark fusion into $W^*$, but do not include any CKM parameters in the $W$-$t$-$b$ vertex$^3$. The numbers include the rate of both $t\bar{b}$ and $\bar{t}b$ production through the $W^*$ mode. If we define the variable,

$$\Delta m_t \equiv m_t - 175 \text{ GeV}, \quad (1)$$

we can parameterize the curves in Table 1 (in units of pb) as follows,

$$\sigma_{W^*}^{\text{mean}}(\Delta m_t) = 0.84 - (2.0 \times 10^{-2})\Delta m_t + (8.9 \times 10^{-6})\Delta m_t^2 -$$

$$- (3.6 \times 10^{-5})\Delta m_t^3 + (8.9 \times 10^{-6})\Delta m_t^4$$

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$^3$This is equivalent to studying $t\bar{b}$ production and not $ts$ or $t\bar{d}$ production, under the assumption that $V_{tb} = 1$. 

Figure 5: Representative Feynman diagrams for NLO corrections to $q\bar{q} \rightarrow W^* \rightarrow t\bar{b}$ arising from final state corrections.
\[ \sigma_{W^*}^{\text{central}}(\Delta m_t) = 0.84 - (2.0 \times 10^{-2}) \Delta m_t + (6.6 \times 10^{-5}) \Delta m_t^2 - (8.3 \times 10^{-5}) \Delta m_t^3 + (1.4 \times 10^{-5}) \Delta m_t^4 \]

\[ \sigma_{W^*}^{\text{upper}}(\Delta m_t) = 0.93 - (2.1 \times 10^{-2}) \Delta m_t + (6.6 \times 10^{-5}) \Delta m_t^2 - (8.3 \times 10^{-5}) \Delta m_t^3 + (1.4 \times 10^{-5}) \Delta m_t^4 \]

\[ \sigma_{W^*}^{\text{lower}}(\Delta m_t) = 0.75 - (1.9 \times 10^{-2}) \Delta m_t + (8.7 \times 10^{-4}) \Delta m_t^2 + (4.8 \times 10^{-5}) \Delta m_t^3 - (1.9 \times 10^{-5}) \Delta m_t^4 . \]

| \( m_t \) (GeV) | \( \sigma_{W^*} \) (mean) (pb) | \( \sigma_{W^*} \) (central) (pb) | \( \sigma_{W^*} \) (upper) (pb) | \( \sigma_{W^*} \) (lower) (pb) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 170             | 0.95            | 0.96            | 1.06            | 0.85            |
| 171             | 0.92            | 0.93            | 1.02            | 0.83            |
| 172             | 0.90            | 0.90            | 0.99            | 0.81            |
| 173             | 0.88            | 0.88            | 0.97            | 0.79            |
| 174             | 0.86            | 0.86            | 0.95            | 0.77            |
| 175             | 0.84            | 0.84            | 0.93            | 0.75            |
| 176             | 0.82            | 0.82            | 0.91            | 0.73            |
| 177             | 0.80            | 0.80            | 0.89            | 0.72            |
| 178             | 0.78            | 0.78            | 0.86            | 0.70            |
| 179             | 0.76            | 0.76            | 0.83            | 0.69            |
| 180             | 0.74            | 0.74            | 0.81            | 0.67            |
| 181             | 0.72            | 0.72            | 0.79            | 0.65            |
| 182             | 0.71            | 0.71            | 0.78            | 0.63            |

Table 1: The central and mean values of \( \sigma_{W^*} \) (in pb), along with the upper and lower bounds derived by varying the scale and by considering the PDF sets CTEQ4M and MRRS(R1). The central value is the value midway between the upper and lower bounds, while the mean is the averaged result of \( \sigma_{W^*} \) calculated using the CTEQ4M and MRRS(R1) PDF with the canonical scale choice discussed in the text.

The \( W \)-gluon fusion mode has also been studied at NLO in QCD (order \( \alpha_{EW}^2 \alpha_S^1 \)) \[14\]. This production mode has the advantage of a larger cross section (\( \sigma_{Wg} \)) and a smaller percentage dependence on \( m_t \) than the \( W^* \) process, though the absolute dependence on \( m_t \) is actually comparable for the two rates (c.f. Fig [3]). This mode is also of interest because within the SM, it provides a way to directly probe the partial width of the top quark, \( \Gamma(t \rightarrow W^+ b) \), through the effective-\( W \) approximation [18], valid at energies much larger than the \( W \) mass (\( m_W \)), in which the \( W \) boson is treated as a parton within the proton. Using this approximation, \( \sigma_{Wg} \) can be related
Figure 6: The dependence of the mean values of the NLO total cross sections, $\sigma_{Wg}$ (upper curve) and $\sigma_{W^*}$ (lower curve), on the mass of the top quark. The dashed lines represent the upper and lower bounds on $\sigma_{Wg}$, and the dotted lines represent the upper and lower bounds on $\sigma_{W^*}$.

to the width $\Gamma(t \to W^+b)$ by the equation \[1\]

$$\sigma_{Wg} \simeq \sum_{\lambda=0,+,--} \int dx_1 \, dx_2 \, f_\lambda(x_1) \, b(x_2) \left[ \frac{16\pi^2 m_t^2}{s(m_t^2 - M_W^2)} \right] \Gamma(t \to W^+b), \quad (3)$$

where $x_1 x_2 = \hat{s}/S$, $f_\lambda(x_1)$ is the distribution function for $W$ bosons within the proton carrying momentum fraction $x_1$, $b(x_2)$ is the $b$ quark PDF, and $\lambda$ is the polarization of the $W$ boson. Once this partial width has been extracted from a measurement of $\sigma_{Wg}$, it can be combined with a measurement of the branching ratio $(BR)$ of $t \to W^+b$ (obtained from examining top decays within $tt$ production) to get the top quark’s full width ($\Gamma(t \to X)$, where $X$ is anything) via the relation \[1\],

$$\Gamma(t \to X) = \frac{\Gamma(t \to W^+b)}{BR(t \to W^+b)}. \quad (4)$$

This method relies on the fact that within the SM there are no flavor-changing neutral current (FCNC) interactions, and the CKM elements $V_{ts}$ and $V_{td}$ are very small \[14\]; thus the $t$-channel single top production involves fusion of only the $b$ parton with a $W^+$ boson\[4\].

The major drawback of the $W$-gluon fusion mode is that it suffers from a larger theoretical uncertainty due to the uncertainty in the $b$ quark parton density (through its dependence on the gluon density). It has been known for some time \[11, 12\].

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\[4\]Assuming a top mass of $m_t = 175$ GeV, including the non-zero contributions from $V_{td} = 0.009$ and $V_{ts} = 0.04$ \[19\] increases the $W$-gluon fusion cross section by less than 0.5%.
Figure 7: Representative Feynman diagrams for $\alpha_S$ NLO corrections to the $W$-gluon fusion mode of single top production coming from the light quark line.

that the treatment of the $b$ quark as a parton must be done carefully, and that a realistic calculation of the $W$-gluon fusion rate must include the initial $b$-quark diagram as well as the gluon splitting diagram (c.f. Figure 2) with the overlap between the two diagrams when the $b$ quark becomes collinear with the initial gluon properly subtracted out, to avoid double counting this region (which has already been resummed into the LO rate through the definition of the $b$ PDF). We refer to the heavy line gluon splitting contribution as the gluon splitting diagram, minus the kinematic region where the $b$ parton is collinear to the incoming gluon. It was demonstrated in [11, 14] that the corrections due to the gluon splitting diagram are actually of order $1/\ln(m_t^2/m_b^2)$ rather than order $\alpha_S$. In that work the genuine order $\alpha_S$ corrections to the light quark line (c.f. Figure 7) and heavy quark line (c.f. Figure 8) were also computed.

In order to estimate the theoretical uncertainty in $\sigma_{Wg}$, we examine its dependence on the choice of scale and PDF. It is expected that the uncertainties in the gluon PDF will reflect themselves in a strong dependence of individual contributions to the NLO cross section from the LO and gluon splitting pieces (shown in Figure 2), the $\alpha_S$ corrections to the light quark line (shown in Figure 7), and the $\alpha_S$ corrections to the heavy quark line (shown in Figure 8). However, because of the QCD sum rules governing the PDF, we expect that the sum of all of these contributions to the NLO rate will actually show much less sensitivity than the individual pieces do [11, 12]. In order to illustrate this point, we examine the individual contributions to the NLO rate.
Figure 8: Representative Feynman diagrams for $\alpha_S$ NLO corrections to the $W$-gluon fusion mode of single top production coming from the heavy quark line.

mentioned above for the CTEQ4M, CTEQ4A4, CTEQ4A5 and CTEQ4HJ PDF\textsuperscript{5} as examples of PDF sets with different shapes for the gluon density, and in the case of CTEQ4HJ, different parameterization for the gluon density. The results are presented in Table 2. From this comparison, we see that while individual pieces of the NLO cross section can vary by about 4%, the total NLO rate varies by less than 1%. Also, further data from Deeply Inelastic Scattering (DIS) experiments is expected to provide stronger constraints on the gluon PDF, and thus with time it is expected that we will better understand and reduce this uncertainty in the $W$-gluon fusion rate.

| Quantity                  | CTEQ4M (pb) | CTEQHJ (pb) | CTEQ4A4 (pb) | CTEQ4A5 (pb) |
|---------------------------|-------------|-------------|--------------|--------------|
| LO rate                   | 2.65        | 2.66        | 2.63         | 2.60         |
| $1/\ln(m_t^2/m_b^2)$      | -0.55       | -0.56       | -0.56        | -0.54        |
| Heavy line $\alpha_S$     | 0.30        | 0.30        | 0.31         | 0.31         |
| Light line $\alpha_S$     | 0.04        | 0.04        | 0.04         | 0.04         |
| Total NLO rate            | 2.44        | 2.44        | 2.41         | 2.42         |

Table 2: The contributions to $\sigma_{Wg}$ from the LO rate, the gluon splitting into $b$ diagram, and genuine $\alpha_S$ corrections to the light and heavy quark lines, for the CTEQ4M, CTEQ4A4, CTEQ4A5, and CTEQ4HJ PDF. Despite the fact that separate contributions can vary at the 4% level, the over-all cross section remains stable to less than 1%.

\textsuperscript{5} We do not use the MRRS(R1) PDF for this analysis, as it differs from the CTEQ4 PDF in its treatment of the bottom quark evolution. Since we are trying to examine the effects of similar PDF fits for the gluon distribution using different gluon density parametrizations, we feel it is best to consider the same $b$ quark evolution scheme in order to avoid confusion by the different modelling of the $b$ quark mass threshold effects.
process contain logarithms which indicate that the appropriate choice of scale is \( \mu = \sqrt{Q^2} \) for the light line corrections, and \( \mu = \sqrt{m_t^2 + Q^2} \) for the heavy line corrections. In that work it was argued that due to the strong analogy between \( W \)-gluon fusion and the DIS process by which most of the experimental understanding of the PDF is derived, it is not necessary to vary the scale of the light line PDF to estimate the uncertainty in the calculation. However, in order to be conservative, we examine the result of simultaneously varying \( \mu \) between \( 2\sqrt{Q^2} \) and \( \sqrt{Q^2/2} \) in the light line quantities, and between \( 2\sqrt{m_t^2 + Q^2} \) and \( \sqrt{m_t^2 + Q^2}/2 \) in the heavy line quantities, for the CTEQ4M and MRRS(R1) PDF\(^6\). These PDF differ in that the MRRS(R1) set has a different treatment of the charm and bottom PDF evolution near their mass thresholds. This is not expected to be important in the heavy line contribution to \( W \)-gluon fusion, where \( \mu = \sqrt{m_t^2 + Q^2} \gg m_b, m_c \). However, it is also necessary to modify the usual \( \overline{\text{MS}} \) subtraction term for the gluon splitting diagrams involving the \( c \) and \( b \) quarks in order to correctly deal with the way the \( b \) PDF is derived in the MRRS formalism \([7]\). From this we arrive at the theoretical estimate for the total \( W \)-gluon fusion rate, as well as the upper and lower bands for its variation, and the central value between these bounds, in the same way that those for the \( W^* \) cross section were obtained. This allows us to examine the correlated effect to the \( W \)-gluon fusion rate due to PDF choice, scale, and \( m_t \). Once again, we see that the central and mean values are very close, indicating that the scale choices are appropriate. These results, for various values of the top mass are presented in Table 3 (and shown graphically in Figure 3). In obtaining these results, we have included the full effects of the CKM matrix in the light quark vertex, but have ignored them in the heavy quark vertex. This prescription effectively amounts to assuming \( V_{td} = 1 \), since the contributions from \( V_{td} \) and \( V_{ts} \) are negligible anyway (see footnote 4). Using the variable \( \Delta m_t \) defined above, one may parameterize these curves (in units of pb) as,

\[
\sigma^{\text{mean}}_{Wg}(\Delta m_t) = 2.35 - (3.2 \times 10^{-2})\Delta m_t - (2.4 \times 10^{-3})\Delta m_t^2 - (3.2 \times 10^{-4})\Delta m_t^3 + (8.0 \times 10^{-5})\Delta m_t^4
\]

(5)

\[
\sigma^{\text{central}}_{Wg}(\Delta m_t) = 2.37 - (3.1 \times 10^{-2})\Delta m_t - (1.5 \times 10^{-3})\Delta m_t^2 - (3.0 \times 10^{-4})\Delta m_t^3 + (5.4 \times 10^{-5})\Delta m_t^4
\]

\[
\sigma^{\text{upper}}_{Wg}(\Delta m_t) = 2.55 - (2.3 \times 10^{-2})\Delta m_t - (4.5 \times 10^{-3})\Delta m_t^2 - (8.3 \times 10^{-4})\Delta m_t^3 + (1.6 \times 10^{-4})\Delta m_t^4
\]

\[
\sigma^{\text{lower}}_{Wg}(\Delta m_t) = 2.19 - (3.1 \times 10^{-2})\Delta m_t - (1.6 \times 10^{-3})\Delta m_t^2 -
\]

\(^6\)We have also examined the predictions of the CTEQ4A4, CTEQ4A5 and CTEQ4HJ PDF for \( \sigma_{Wg} \) in the range of top masses we are considering. We find that all of these PDF give results for \( \sigma_{Wg} \) that differ from CTEQ4M by less than 1% (Table 2 illustrates this for \( m_t = 175 \text{ GeV} \)). Thus, the limits on the range of \( \sigma_{Wg} \) we derive are consistent with the full set of PDF MRRS(R1), CTEQ4M, CTEQ4HJ, CTEQ4A4, and CTEQ4A5.
(1.9 \times 10^{-4}) \Delta m_t^3 + (4.8 \times 10^{-4}) \Delta m_t^4.

| $m_t$ (GeV) | $\sigma_{Wg}$ (mean) (pb) | $\sigma_{Wg}$ (central) (pb) | $\sigma_{Wg}$ (upper) (pb) | $\sigma_{Wg}$ (lower) (pb) |
|-------------|-----------------|-----------------|-----------------|-----------------|
| 170         | 2.54            | 2.56            | 2.76            | 2.36            |
| 171         | 2.48            | 2.50            | 2.67            | 2.31            |
| 172         | 2.44            | 2.46            | 2.62            | 2.28            |
| 173         | 2.41            | 2.43            | 2.59            | 2.25            |
| 174         | 2.38            | 2.40            | 2.57            | 2.22            |
| 175         | 2.35            | 2.37            | 2.55            | 2.19            |
| 176         | 2.32            | 2.34            | 2.52            | 2.16            |
| 177         | 2.28            | 2.30            | 2.48            | 2.12            |
| 178         | 2.23            | 2.26            | 2.43            | 2.08            |
| 179         | 2.18            | 2.22            | 2.37            | 2.04            |
| 180         | 2.14            | 2.18            | 2.32            | 2.00            |
| 181         | 2.11            | 2.14            | 2.28            | 1.96            |
| 182         | 2.09            | 2.11            | 2.27            | 1.94            |

Table 3: The central value of $\sigma_{Wg}$, along with the mean value (for the canonical scale choice described in the text) and the upper and lower bounds derived by varying the scale and by considering the PDF sets CTEQ4M and MRRS(R1).

Studying single top production at the Tevatron Run II (and beyond) is expected to yield the first direct measurements of $V_{tb}$, the top-bottom CKM matrix parameter \cite{11}. While the assumption of three generations, combined with unitarity considerations, allows one to constrain $V_{tb}$ to within a few percent of $V_{tb} = 0.9991$ \cite{19}, this bound disappears if one allows for more than three generations. Thus, it is important to directly measure $V_{tb}$, as it may provide a clue concerning the existence of a fourth generation of fermions which mixes with the third family through the CKM matrix. A method to accomplish this at the Tevatron Run II has been proposed, using the single top $W$-gluon fusion \cite{11} and $W^*$ \cite{20} rates. The method relies upon $t\bar{t}$ production to measure the branching ratio for $t \to bW$, and then uses this information in conjunction with a measurement of single top production (through either mode), which is proportional to $|V_{tb}|^2$, to extract $|V_{tb}|$ independently from the top quark decay mode. However, as we shall show in Section 3, the presence of new physics in the top quark sector can result in a large modification of either $\sigma_{W^*}$ or $\sigma_{Wg}$, which could distort the value of $V_{tb}$ obtained in this way. We will return to this question in Section 4, after examining how an additional heavy resonance or modified top quark interactions can affect $\sigma_{Wg}$ and $\sigma_{W^*}$.

In attempting to estimate the total uncertainty in the cross sections of $\sigma_{W^*}$ and $\sigma_{Wg}$ due to both theoretical and statistical uncertainties, we assume a top mass of $m_t = 175$ GeV, and that the actual physical values of the cross sections are the same as the mean values at $m_t = 175$ GeV. We have seen from the results in Tables 1 and
that the effect on the uncertainty of the cross sections from the uncertainties due to the scale dependence, PDF choice, and uncertainty in the top mass are actually correlated with one another. For the theoretical uncertainty, we see from Tables 1 and 2 that for a top mass of $m_t = 175 \pm 2$ GeV, the expected values and uncertainties of $\sigma_{W^*}$ and $\sigma_{Wg}$ are,

$$\sigma_{W^*} = 0.84 \, \text{pb} \pm 15\%$$

$$\sigma_{Wg} = 2.35 \, \text{pb} \pm 10\%.$$

However, this theoretical uncertainty is not correlated with the statistical uncertainty in the measurement, and thus these two fractional uncertainties may be added in quadrature to arrive at the total theoretical and statistical uncertainty in $\sigma_{W^*}$ and $\sigma_{Wg}$. In deriving the projected statistical uncertainty, we assume a top quark decay of $t \rightarrow b \ W \rightarrow b \ \ell \ \bar{\nu}_\ell$ with $\ell = e$ or $\mu$, and thus include a branching ratio of $2/9$ for this decay mode. In order to include an estimation of the detection efficiencies at the Tevatron Run II, we include the LO efficiency factors of 9% for the $W^*$ production mode and 33% for the $W$-gluon fusion mode. This is somewhat crude, as the NLO results include kinematic configurations where an additional jet is present in the final state, and thus the actual efficiencies are probably slightly different from these estimates. Nonetheless, when considering new physics effects in single top production, we will not rely on detailed studies of the final state kinematics in our analysis, and thus this should not have a large effect on our conclusions. From these results we derive the projected statistical uncertainty in the measured cross sections for an integrated luminosity of 2, 10, and 30 fb$^{-1}$. The results, including the theoretical uncertainties derived above, and the total over-all uncertainty obtained by adding the fractional uncertainties in quadrature, are summarized in Table 4. It is also interesting to examine the theoretical uncertainties derived by varying the top mass, scale, and PDF, without taking into account the correlations in the dependence of the cross sections on these quantities, though as we have argued above, this is not the correct way to estimate the full theoretical uncertainty. Assuming the canonical scale choices for each mode described above, $m_t = 175$ GeV, and the CTEQ4M PDF, we vary each of these quantities individually by the same range used in our correlated analysis above, and examine the effect on the cross section. The resulting fractional uncertainty in the cross sections is presented in Table 5. From this we see clearly that if one considers the separate contributions to the theoretical uncertainty as uncorrelated (and thus adds them in quadrature), one under-estimates the true uncertainty.

\footnote{We assume that with 2 fb$^{-1}$ of integrated luminosity or more, the uncertainty in the top mass will be $\delta m_t = \pm 2$ GeV.}

\footnote{This efficiency for detecting the $W^*$ mode includes a factor of 36% for double $b$-tagging when both $b$ quarks have $p_{Tb} \geq 20$ GeV and $|\eta_b| \leq 2$, based on an estimated 60% efficiency for single $b$-tagging of a $b$ quark in this kinematic region.}

\footnote{This estimate for the $W$-gluon fusion detection efficiency include a 60% single $b$-tagging efficiency when the $b$ quark has $p_{Tb} \geq 35$ GeV and $|\eta_b| \leq 2$.}

\footnote{The theoretical uncertainty in the production rate may also depend on the kinematic cuts imposed to enhance the signal-to-background ratio. This requires a study on the detailed distributions of the final state particles in the single top events, and is thus beyond the scope of this work.}
shown in Table 4. We also see that the correlations in these quantities are some-what stronger for $\sigma_{W^*}$ than for $\sigma_{Wg}$. We note that our estimate of the theoretical uncertainty in the $W^*$ rate is some-what more pessimistic than the estimate of $\pm10\%$ in [5], where the complete correlation in the theoretical uncertainties was not taken into account.

| Quantity          | $\delta\sigma_{W^*}$ | $\delta\sigma_{Wg}$ |
|-------------------|-----------------------|---------------------|
| Theoretical       | $\pm15\%$            | $\pm10\%$          |
| Statistical (2 fb$^{-1}$) | $\pm17\%$ (34 Events) | $\pm5\%$ (345 Events) |
| Statistical (10 fb$^{-1}$) | $\pm8\%$ (168 Events) | $\pm2\%$ (1723 Events) |
| Statistical (30 fb$^{-1}$) | $\pm4\%$ (504 Events) | $\pm1\%$ (5170 Events) |
| Total (2 fb$^{-1}$)   | $\pm23\%$            | $\pm11\%$          |
| Total (10 fb$^{-1}$)  | $\pm17\%$            | $\pm10\%$          |
| Total (30 fb$^{-1}$)  | $\pm16\%$            | $\pm10\%$          |

Table 4: The correlated theoretical uncertainty from scale, PDF, and uncertainty in $m_t$ for $\sigma_{W^*}$ and $\sigma_{Wg}$, as well as the projected statistical uncertainty for various integrated luminosities assuming a semi-leptonic top decay into an electron or muon, and the detection efficiencies of 9% for the $W^*$ process and 33% for the $W$-gluon fusion process obtained from LO studies. A top mass of 175 GeV with an uncertainty of $\pm2$ GeV is assumed.

| Quantity Varied | $\delta\sigma_{W^*}$ | $\delta\sigma_{Wg}$ |
|-----------------|-----------------------|---------------------|
| $\delta m_t$    | $\pm6\%$             | $\pm3\%$            |
| Scale           | $\pm5\%$             | $\pm4\%$            |
| PDF             | $\pm6\%$             | $\pm7\%$            |
| Total added in Quadrature | $\pm10\%$ | $\pm9\%$ |
| Total added linearly | $\pm18\%$ | $\pm14\%$ |
| Correctly Correlated Total | $\pm15\%$ | $\pm10\%$ |

Table 5: The uncorrelated theoretical uncertainty from scale, PDF, and uncertainty in $m_t$ for $\sigma_{W^*}$ and $\sigma_{Wg}$, and the totals obtained from adding these uncertainties in quadrature (assuming they are correlated), linearly (assuming 100% correlation), and the result including the true correlation from Table 4.

From these results, we learn that for 2 fb$^{-1}$ of integrated luminosity, the $W$-gluon fusion sub-process has an uncertainty which is about a factor of 2 better than that of the $W^*$ mode. This is because the $W^*$ mode’s larger sensitivity to the top mass, and the strong correlations between this uncertainty and those arising from the PDF and scales, and smaller cross section (and detection efficiency) act to compensate for
the larger uncertainty in the $W$-gluon fusion mode due to the parton distribution functions. For the larger data samples considered, the improvements in available statistics allow the uncertainties of the two rates to become comparable, although the $\sigma_{W^*}$ uncertainty remains dominated by that mode's larger percentage dependence on the top mass, and thus is larger. These results must be considered with some care, as no systematic experimental uncertainties are included, and these are likely to be different for the two production modes.

3 New Physics in the Form of an Additional Heavy Resonance

One possible form of new physics in the top quark sector is an additional resonance, beyond those required by the SM, which couples to the top quark. In particular, a resonance with electric charge $Q = +1$ could couple to the top and bottom quarks, and thus could contribute to single top production. Generically, a heavy vector boson with charge $Q = +1$ (which we shall refer to as a $W'$) can affect the rate of single top production (in either the $s$ or $t$ channel processes) by contributing additional diagrams in which the $W'$ is exchanged, such as those shown in Figures 9 and 10. Because the initial and final states are the same for both the $W$ exchange and $W'$ exchange diagrams, they can interfere at the amplitude level, and thus the effect of the $W'$ could either raise or lower the single top cross sections, depending on the relative sign of the couplings between the $W$ and the fermions, and the $W'$ and the fermions.

We expect that the two modes of single top production will show a very different sensitivity to the presence of a $W'$. The $s$-channel $W^*$ process can show a large sensitivity, because the time-like momentum of the exchanged $W'$ can be close to on-shell, thus providing an enhancement from the $W'$ propagator in the $s$-channel matrix element (c.f. Figure 9). This also suggests that one way to look for the presence of a $W'$ is to examine the distribution of $\sigma_{W^*}$ with respect to the invariant mass of the $t\bar{b}$ system ($M_{t\bar{b}}$) for bumps from the $W'$ resonance, or for signs of a $W'$ tail. In fact the authors of [1] found that in order to distinguish the $W^*$ signal from the large $Wb\bar{b}$ background at the Tevatron, it is necessary to require a large invariant mass between the $b$ and $\bar{b}$ quarks in the final state, $M_{t\bar{b}} \geq 110$ GeV. Since the final state $b$ quark results from the decay of the top, this is equivalent to requiring that the invariant mass $M_{t\bar{b}}$ is large, and thus the observed $W^*$ signal comes mostly from the region of $M_{t\bar{b}}$ which is most sensitive to the effect of the $W'$. However, if the mass of the $W'$ is too large, or its width too broad, the effect in $M_{t\bar{b}}$ can be washed out at the Tevatron,

\footnote{There is an ambiguity in reconstructing $M_{t\bar{b}}$ when the top decays semi-leptonically, as we are assuming in this work, because the final state neutrino is not observed, and thus the component of its momentum along the beam axis is not measured. However, it is possibly to reconstruct the top's momentum in a statistical way [4, 6, 12], thus allowing a similar reconstruction of $M_{t\bar{b}}$.}
Figure 9: Feynman diagram showing how an additional heavy charged vector particle $(W')$ can contribute to the $s$-channel process of single top production.

Figure 10: Representative Feynman diagram showing how an additional heavy charged vector particle $(W')$ can contribute to the $W$-gluon fusion mode of single top production.

preventing one from identifying the presence of the $W'$ in this way.

On the other hand, the $t$ channel process requires a space-like momentum for the $W'$ boson (c.f. Figure 10), and thus can never experience this type of resonant propagator enhancement. The additional $t$-channel diagram containing the $W'$ (c.f. Figure 10) will be suppressed by $1/M_{W'}^2$. Furthermore, the kinematic distribution of the invariant mass $M_{q'q}$, where $q'$ is the light quark in the final state (c.f. Figure 4), shows the same characteristics as the SM distribution, and thus does not provide much additional information beyond that contained in the total rate, $\sigma_{Wg}$. The $tW^-$ process is also not expected to show an effect from the presence of a $W'$, because the large mass of the $W'$ forbids direct production of a top and a $W'$. Similarly, the large mass of the $W'$ will prevent it from affecting top quark decays since $M_{W'}$ must be greater than $m_t$ in order to respect current limits on a $W'$ mass.

For the purposes of illustrating these points, we will consider the (light) top-flavor model [22, 23, 24], in which the third family of fermions undergoes a separate SU(2) weak interaction from the first two families. In the top-flavor model the overall gauge symmetry is SU(2)$_h \times$ SU(2)$_l \times$ U(1)$_Y$, and thus there are three additional weak bosons ($W'^\pm$ and $Z'$). The first and second generation fermions transform under SU(2)$_l$, while the third generation fermions transform under SU(2)$_h$. A set of scalar fields transforming under both SU(2)$_l$ and SU(2)$_h$ acquire a vacuum expectation value (v.e.v.), $u$, and break the symmetry to SU(2)$_{l+h} \times$ U(1)$_Y$. From here the usual electroweak symmetry breaking can be accomplished by introducing a scalar doublet which

\footnote{A comprehensive introduction to this model, may be found in [23]. We are using the same conventions as were used in that work.}
acquires a v.e.v. $v$, further breaking the gauge symmetry to $U(1)_{EM}$. The model is parameterized by two quantities, $x = u/v$, and $\sin^2 \phi$, which characterizes the mixing between the heavy and light SU(2) gauge couplings. At leading order, the heavy bosons are degenerate in mass,

$$M^2_{Z',W'} = M_0^2 \left( \frac{x}{\sin^2 \phi \cos^2 \phi} + \frac{\sin^2 \phi}{\cos^2 \phi} \right),$$

(7)

where $M_0^2 = \frac{\epsilon^2 v^2}{4 \sin^2 \theta \cos \theta}$, and $\epsilon$ and $\theta$ are the $U(1)_{EM}$ coupling and the weak mixing angle, respectively. We can thus parameterize the model by the heavy boson mass, $M_{Z'}$, and the mixing parameter $\sin^2 \phi$. Low energy data requires that the mass of these heavy bosons, $M_{Z'}$, be greater than about 1.1 TeV [23]. The impact of this type of model on the $W^*$ sub-process \footnote{For $\sin^2 \phi \leq 0.04$, the third family fermion coupling to the heavy gauge bosons can become non-perturbative. Thus we restrict ourselves to considering $0.95 \geq \sin^2 \phi \geq 0.05$. (c.f. Ref. [23]).} of single top production through the observable \footnote{In this section, we use "$W^*$ process" to denote $qq' \rightarrow W, W' \rightarrow t\bar{b}$. The results we present will also include single anti-top production through $qq' \rightarrow W, W' \rightarrow \bar{t}b$.} $R_\sigma = \frac{\sigma(qq' \rightarrow W^*, W' \rightarrow t\bar{b})}{\sigma(qq' \rightarrow W, W' \rightarrow \ell\nu)}$ was considered in [23]. The conclusions drawn there about the possibility of detecting top-flavor at the Tevatron Run II by observing single top production are similar to ours.

To illustrate these ideas concerning the effect of a $W'$ on the single top production rates, we examine the prediction of the top-flavor model for single-top production at NLO in both the $s$ and $t$ channels, in the region of top-flavor parameter space not ruled out by low energy data. We assume a top mass of $m_t = 175$ GeV. We find that in this region of parameter space, the $W^*$ production mode is generally increased by about 15%, for heavy boson masses in the region around $M_{Z'} = 1.1$ TeV (c.f. Figure 11). We also examine the effects of this model to the $W$-gluon fusion rate. As expected, we find that the $W$-gluon fusion cross section is quite insensitive to the presence of the $W'$, deviating from the SM value by less than 2% in the entire range of allowed parameters.

As mentioned previously, one could look for the effect of the heavy resonance in the distribution of $M_{tb}$ for $W^*$ single-top events. With enough statistics, it is likely that examining this distribution could allow one to further constrain the top-flavor model for regions of parameter space where the $W'$ is not too heavy, and its width is not too broad. However, in the region shown in Figure 11 the effect is washed out by the heavy $W'$ mass, and its broad width. Further, with limited statistics it may not be practical to consider distribution of $M_{tb}$. To illustrate this point, in Figure 12 we show $\frac{d\sigma_{W^*}}{dM_{tb}}$ for the SM as well as for a typical choice of allowed top-flavor parameters which modify the total rate $\sigma_{W^*}$ by at least 15%. From this result one can see that the $M_{tb}$ effect can be washed out by the large mass, and broad width of the $W'$.

These results reveal a limitation in relying solely on the $W^*$ process to measure $V_{tb}$. An additional charged resonance can have a large effect on the $W^*$ cross section, which could lead one to mis-measure $V_{tb}$. While for the specific example of the top-flavor model, the effect of the $W'$ was to increase the $W^*$ cross section, and thus
Figure 11: The region of top-flavor parameter space allowed by low energy data (above the solid curve), and modifying the $\sigma_{W^*}$ rate by at least 15% (below the dashed line).
Figure 12: The distribution of $d\sigma_W/dM_{tb}$ for the SM (solid curve), and the top-flavor model (dotted curve) with $\sin^2 \phi = 0.05$ and $M_{Z'} = 1.2$ TeV. (The vertical scale of the histogram is in arbitrary units.) Both calculations assume a top mass of $m_t = 175$ GeV.
would be likely to lead to a measurement of $V_{tb} \geq 1$ (which would in itself signal that there is a problem in using the $W^*$ rate to measure $V_{tb}$), in a general model where the $W'$ couplings to the fermions have an arbitrary sign relative to the $W$ couplings, the amplitudes for $W$ and $W'$ can interfere, leading to a decrease in $\sigma_{W^*}$ (and thus decreasing the value of $V_{tb}$ one would extract from such a measurement). This could lead one to conclude, erroneously, that there was evidence for a fourth generation of fermions mixed with the third generation via the CKM matrix. Thus, to be sure one is measuring $V_{tb}$ accurately, it is not enough to rely completely on $\sigma_{W^*}$.

Other types of heavy resonance can be added to the SM, and analyzed in a similar fashion. If the additional particle has charge +1 (such as the $W'$ considered here), and couples to both heavy and light quarks, it can contribute to the single top rate through the $W^*$ mode, but should not have a large effect on the $W$-gluon fusion process because of suppression by the heavy mass and space-like momentum. Other types of new particles, such as e.g., those found in $R$-parity-conserving super-symmetry (SUSY), are unlikely to affect $\sigma_{W^*}$ or $\sigma_{W^g}$ in a large way, though they could modify the top’s decay width and branching ratios. For example, a charged Higgs, $H^\pm$ could modify the top width by allowing decays such as $t \to H^+ b$, but would not affect the $W^*$ or $W$-gluon fusion rates because the Higgs couples very weakly to the light fermions. This example provides an illustration of the sense in which the $W$-gluon fusion process is a measure of the top quark’s partial width, $\Gamma(t \to W^+ b)$. In the case of an additional $W'$, the top’s width was not modified, and we also saw that the $W$-gluon fusion rate was not modified. In the case in which there is a charged Higgs $H^+$, the top’s total width is modified because the decay $t \to H^+ b$ becomes allowed (assuming $m_t > M_{H^+}$), but the partial width $\Gamma(t \to W^+ b)$ is unchanged, and so we find that the $W$-gluon fusion process is not modified. Thus it still can be used to extract the partial width $\Gamma(t \to W^+ b)$ via Equation (3). One could then use Equation (4) and $BR(t \to W^+ b)$, extracted from the $t\bar{t}$ event sample, to calculate the full width of the top quark.

4  New Physics in the Form of Modified Top Quark Interactions

It is also possible that the top quark may couple differently to light particles from what is predicted by the SM [26, 27, 28, 29, 30]. As an example to illustrate the general features of this type of new physics, we will consider a flavor-changing neutral current (FCNC) coupling the top to the charm quark and the $Z$ boson. To introduce such an interaction, we consider the SM Lagrangian as an effective theory, allow the $SU(2)_L \times U(1)_Y$ gauge symmetry to be realized nonlinearly [31], and include the dimension four FCNC term [27],

$$\Delta L^{eff} = \frac{\kappa Z_{tc}}{2 \sin \theta_W \cos \theta_W} Z^\mu \left[ \bar{t} \gamma_\mu c + \bar{c} \gamma_\mu t \right],$$

Equation (8)
where $\kappa_{tc}^Z$ parameterizes the strength of the anomalous term for $Z$-$t$-$c$ coupling, and is assumed to be real. As shown in [27], low energy data requires $|\kappa_{tc}^Z| \leq 0.29$ (at 95% C.L.).

This form of new physics will allow new decay modes for the top quark, and thus will modify the top quark’s full decay width, $\Gamma(t \to X)$, and cause its branching ratio $BR(t \to bW^+)$ to deviate from the SM prediction of $\sim 100\%$, by allowing new decay modes such as, e.g. $t \to Zc$, through diagrams such as that pictured in Figure 13. Also, a FCNC cannot modify the rate of $tW^-$ production, though it does allow a new mode of single top production by allowing a $tZ$ final state (c.f. Figure 14). However, the measured rate of $tW^-$ is not sensitive to a $Z$-$t$-$c$ vertex, and thus does not provide information about the strength of such an anomalous coupling.

Should the top’s couplings involving the partons of the proton (i.e. the $u$, $d$, $s$, $c$, and $b$ quarks, and the gluons) be modified, this may also have an impact on the production of single tops in the $t$-channel mode through diagrams such as that shown in Figure 15, where an incoming charm parton experiences a flavor-changing neutral current involving a $Z$ boson, producing a top quark and a forward jet in the final state. Since the distribution of charm quarks in the proton is larger than distribution of bottom quarks relevant for the SM $t$-channel production mechanism (c.f. Figure 4), this contribution may be visible even if the FCNC $Z$-$t$-$c$ vertex coupling strength is not large. In this case we also see that in a sense the $W$-gluon fusion process is sensitive to the top quark width; a modification of the top’s couplings to the light particles found as partons inside the proton will result in a change of the top decay branching ratios, and the rate of $W$-gluon fusion production. Both properties of the top are thus sensitive to the same type of new physics. However, in the case in which a FCNC is present, it is no longer possible to use the $W$-gluon fusion process to
directly measure the partial width $\Gamma(t \to W^+b)$, as could be done for a purely SM top, or in a scenario in which an additional heavy charged resonance is present.

At the Tevatron Run II, we estimate the effect on the $W$-gluon fusion cross section to be $\delta \sigma_{Wg} = 1.36$ pb for $\kappa_{tc}^Z = 0.29$, and $m_t = 175$ GeV. This would constitute a $3\sigma$ deviation from the expected SM cross section, and thus it is possible to use single top production to constrain the size of such a FCNC term at the 99% C.L. Requiring that no $3\sigma$ deviation in the $W$-gluon fusion rate is observed allows one to derive the constraint $|\kappa_{tc}^Z| \leq 0.22$ for $L = 2 \text{ fb}^{-1}$, or $|\kappa_{tc}^Z| \leq 0.21$ with 10 fb$^{-1}$ of integrated luminosity. In [27], it was found that by studying the $BR$ of the decay $t \to Zc$, it is possible to constrain (at the 99% C.L.) $|\kappa_{tc}^Z| \leq 0.3$ with $L = 2 \text{ fb}^{-1}$, or $|\kappa_{tc}^Z| \leq 0.16$ with $L = 10 \text{ fb}^{-1}$. Thus, the $W$-gluon fusion rate provides a comparable means to explore $\kappa_{tc}^Z$ to that provided by studying $t \to Zc$. However, in order to use the branching ratio $BR(t \to Zc)$ to constrain $\kappa_{tc}^Z$ as was done in [27], it is necessary to make the strong assumption that there is no other new physics modification to the top decays besides $t \to Zc$, so that one can directly convert the branching ratio $BR(t \to Zc)$ into the partial width $\Gamma(t \to Zc)$. Since the anomalous contribution to the $W$-gluon fusion cross section is proportional to $|\kappa_{tc}^Z|^2$, single top production provides a way to constrain $\kappa_{tc}^Z$ without relying on strong assumptions about the possibility of other forms of new physics in the top quark sector.

It is also possible for this FCNC vertex to modify the rate of $W^*$ single top production through diagrams such as that shown in Figure 14. However, this contribution is unlikely to significantly affect the $W^*$ rate measured at the Tevatron, because in [4] it was found that in order to isolate the $W^*$ mode from the large $Wjj$ background and from the $W$-gluon fusion mode, it is necessary to tag both the $\bar{b}$ produced when the $W^*$ decays and the $b$ produced from the top decay, $t \to Wb$. Thus if another type of quark is produced with the top (such as the charm quark pictured in Figure 16), the additional events will not be included in the $W^*$ measurement[4]. Thus, the $s$-channel single top production process is insensitive to a modification of the top’s couplings in the form of a FCNC.

One could also imagine new physics which directly modifies the $W-t-b$ vertex. In this case, all three modes of single top production will show effects from the

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$^{15}$ It could be possible to constrain the size of such an anomalous operator by studying the $W^*$ mode of single top production, but employing other search strategies than those used in [4].
Figure 16: Representative Feynman diagram showing how an anomalous $Z$-$t$-$c$ vertex can contribute to the $s$-channel $W^*$ production mode of single top production.

modification, as will the top’s total decay width $\Gamma(t \to X)$, though the branching ratio $BR(t \to bW^+)$ will remain close to the SM value of $\sim 100\%$. Thus, it would again be possible to use the $W$-gluon fusion mode to directly probe the top’s partial width $\Gamma(t \to W^+b)$, and from Eq. (8), compute the full width of the top quark. In this case, one could examine the $W^*$ production, and $W$-gluon fusion rates separately in order to probe the dependence of the modified vertex on $Q^2$, the momentum of the $W$ boson, in the time-like and space-like regions, respectively. With enough statistics, the $tW^-$ production mode could also be helpful in this regard, since this process is not sensitive to additional resonances, nor to FCNC interactions.

5 $V_{tb}$ and the Possibility of New Physics

As we have seen, it is possible that new physics effects in the top quark sector can have a large effect on the production of single top quarks in the $W^*$ mode (through an additional charged heavy resonance) and in the $W$-gluon fusion mode (through modifications of the top quark’s couplings to the partons within the proton). This naturally leads us to consider how one can measure the CKM parameter $V_{tb}$ with confidence. That is, how one can be sure that a given measurement is accurately extracting $|V_{tb}|$, and not being mislead by a new physics effect manifest in single top production.

Since both modes of single top production are sensitive to new physics effects, we conclude that neither one alone is enough to provide a confident extraction of $V_{tb}$. Thus, we propose the following ratio,

$$ R = \frac{\sigma_{Wg}}{\sigma_{W^*}}, $$

of the rates for the two sub-processes. Since both cross sections are proportional to $|V_{tb}|^2$, this quantity does not depend on $V_{tb}$. Further, it is sensitive to a new physics effect in either cross section, and as we have shown above, each cross section is independently sensitive to different sources of new physics effects. Thus this quantity provides one with a cross-check on the confidence with which one may regard an extraction of $V_{tb}$ from single top production. Assuming a top mass of $m_t = 175 \text{ GeV}$, a deviation of $R$ from its SM prediction of 2.79 by more than $\pm25\%$ (for $2 \text{ fb}^{-1}$), $\pm20\%$ (for $10 \text{ fb}^{-1}$), or $\pm19\%$ (for $30 \text{ fb}^{-1}$) would indicate the possible presence of
new physics\footnote{The fractional uncertainty in $R$ is straightforward to calculate from the uncertainties in $\sigma_{W^*}$ and $\sigma_{Wg}$, added in quadrature.} in single top production, and thus call into question the validity of using it to extract $V_{tb}$. In that case, one can look at the two modes for deviation from SM values in order to identify the likely form of the new physics effect (i.e. either a heavy charged resonance, or a modification of the top’s couplings).

If new physics modifies the $W-t-b$ vertex itself, it will affect both of the cross sections, and could cause $R$ to remain near its SM prediction. However, this will only be the case if the modified $W-t-b$ vertex continues to be a trivial function of $Q^2$, the momentum of the $W$ boson. Since the two modes of single top production are sensitive to different regions of $Q^2$, any modification of the $W-t-b$ vertex with a non-trivial $Q^2$ dependence will likely still cause a deviation in $R$ from its SM prediction. In this case, detailed study of the kinematics of the $W^*$ and $W$-gluon fusion modes could serve to identify the $Q^2$ dependence of the modified $W-t-b$ coupling in the time-like and space-like regions, respectively.

It may also be useful to consider the correlation of the two cross sections, $\sigma_{W^*}$ and $\sigma_{Wg}$, in a two dimensional plane. From Tables 1 and 3 one can locate the point for the SM in the $\sigma_{W^*} - \sigma_{Wg}$ plane. For example, assuming a top mass of $m_t = 175$ GeV, the location of the SM point is at $(\sigma_{W^*}, \sigma_{Wg}) = (0.84, 2.35)$. The curves corresponding to a constant probability for deviation from the SM values will be ellipses in this plane, as shown in Figure 17. From looking at this plane, and using what we have learned about the sensitivity of the two cross sections to new physics effects, one can also decide on a likely cause for a given deviation. A deviation more along the $x$-axis is due to a larger shift in $\sigma_{W^*}$, and thus as we have seen is more likely to be due to an additional heavy charged resonance, while a deviation more along the $y$-axis comes from a shift in $\sigma_{Wg}$ and thus is more likely to result from a modification of the top quark’s couplings. The ratio $R$ considered above is equivalent to considering a line with unit slope running through the SM point in this plane. By comparing an experimental measurement of the point in this plane favored by the data with the predictions of specific models, it could also be possible to constrain those models.

6 Conclusions

In this article we have investigated the inclusive production of single top quarks at the Tevatron Run II, in both the $W^*$ and $W$-gluon fusion processes, including the first analysis of the correlated effects of uncertainties due to top mass uncertainty, scale dependence, and PDF choice on the uncertainties of the cross sections. We have also examined the sensitivity of these processes to various types of new physics effects. Because the two modes of production are sensitive to different regions of $Q^2$, we find that they are sensitive to different types of new physics effects. The $s$-channel $W^*$ mode is more sensitive to an additional heavy charged resonance, while the $W$-gluon fusion mode, which provides a measure of the top quark’s decay width, is more
Figure 17: The location of the $m_t = 175$ GeV SM point (the diamond) in the $\sigma_{W^*} - \sigma_{Wg}$ plane, as well as the curve representing a $3\sigma$ deviation from this point. Also plotted are the results for the top-flavor model (described in Section 3) with $M_{Z'} = 1.2$ TeV and $\sin^2 \phi = 0.05$ (the square) and the effective Lagrangian containing the FCNC $Z$-$t$-$c$ vertex (described in Section 4) with $\kappa_{tc}^{Z} = 0.29$ (the circle).
sensitive to modifications of the top quark’s couplings to the other SM particles. As examples of these kinds of new physics, we have considered the effect of top-flavor models with additional electro-weak gauge bosons, and an anomalous $Z$-$t$-$c$ coupling to the rate of single top production through both modes.

The possibility of new physics in the top quark sector makes it difficult to use single top production to directly measure the CKM parameter $V_{tb}$. However, because the two modes are sensitive to different forms of new physics, one can obtain a measure of the confidence with which one can regard an extraction of $V_{tb}$ from the single top rate by considering their ratio, $R = \sigma_{Wg}/\sigma_{W^*}$. A large deviation of this ratio from its SM prediction of about 2.79 can signal the presence of new physics in the top sector, and implies that extracting $V_{tb}$ from single top production requires careful study to be sure that what is being measured is actually $V_{tb}$, and not a new physics effect. In analyzing single top data, because of the different sensitivities to different types of new physics effects of the two production modes, it is useful to consider the experimental data in the $\sigma_{W^*}$-$\sigma_{Wg}$ plane. Comparison of the predictions of explicit models with the experimental point on this plane could be used to rule out or constrain these models. We have also seen that the $tW^-$ mode of single top production is insensitive to the types of new physics effects we have considered, and thus could provide a safe way to measure $V_{tb}$ provided enough statistics or a carefully tuned search strategy compensate for its low cross section.

Thus we conclude that it is important to study single top production at the Tevatron Run II, in both the $W^*$ and $W$-gluon fusion modes separately, as these two modes provide complimentary information about the top quark. Single top production provides an excellent opportunity to directly measure $V_{tb}$, and to search for possible signs of the new physics associated with the top quark.

For completeness, we present the results for single top production in the $W$-gluon fusion and $W^*$ modes at the LHC, using an analysis identical to that performed for the two modes at the Tevatron in Section 2. Our results are presented in Tables 6 and 7. We also note that the $tW^-$ mode will also present another way of examining the single top rate at the LHC, and due to the fact that it shows different sensitivities to the types of new physics we have considered, is also complimentary to the other two modes in probing the properties of the top quark.

Acknowledgments

The authors would like to thank C. Balazs for helpful discussion. T. Tait is grateful for conversations with E. L. Berger, S. Mrenna, and S. Murgia. C.–P. Yuan wishes to thank the CTEQ collaboration for useful discussions. This work was supported in part by the NSF grant No. PHY-9507683. Part of T. Tait’s work was completed at Argonne National Laboratory, in the High Energy Physics division and was supported in part by the U.S. Department of Energy, High Energy Physics Division, under Contract W-31-109-Eng-38.
| $m_t$ (GeV) | $\sigma_{W^*}$ (mean) (pb) | $\sigma_{W^*}$ (central) (pb) | $\sigma_{W^*}$ (upper) (pb) | $\sigma_{W^*}$ (lower) (pb) |
|------------|------------------|------------------|------------------|------------------|
| 170        | 12.2             | 12.2             | 12.6             | 11.7             |
| 171        | 11.9             | 11.9             | 12.3             | 11.4             |
| 172        | 11.7             | 11.6             | 12.0             | 11.2             |
| 173        | 11.4             | 11.3             | 11.7             | 10.9             |
| 174        | 11.2             | 11.1             | 11.5             | 10.7             |
| 175        | 11.0             | 10.9             | 11.3             | 10.5             |
| 176        | 10.7             | 10.7             | 11.0             | 10.3             |
| 177        | 10.5             | 10.5             | 10.8             | 10.1             |
| 178        | 10.3             | 10.3             | 10.6             | 9.9              |
| 179        | 10.1             | 10.1             | 10.4             | 9.7              |
| 180        | 9.9              | 9.9              | 10.2             | 9.5              |
| 181        | 9.7              | 9.7              | 9.9              | 9.3              |
| 182        | 9.5              | 9.4              | 9.7              | 9.1              |

Table 6: The central and mean values of $\sigma_{W^*}$ (in pb) at the LHC, along with the upper and lower bounds derived by varying the scale and by considering the PDF sets CTEQ4M and MRRS(R1). The central value is the value midway between the upper and lower bounds, while the mean is the averaged result of $\sigma_{W^*}$ calculated using the CTEQ4M and MRRS(R1) PDF with the canonical scale choice discussed in the text.

| $m_t$ (GeV) | $\sigma_{W^g}$ (mean) (pb) | $\sigma_{W^g}$ (central) (pb) | $\sigma_{W^g}$ (upper) (pb) | $\sigma_{W^g}$ (lower) (pb) |
|------------|------------------|------------------|------------------|------------------|
| 170        | 253              | 249              | 261              | 236              |
| 171        | 250              | 248              | 259              | 234              |
| 172        | 246              | 247              | 257              | 233              |
| 173        | 244              | 246              | 255              | 231              |
| 174        | 241              | 244              | 253              | 230              |
| 175        | 239              | 243              | 251              | 228              |
| 176        | 237              | 241              | 249              | 226              |
| 177        | 236              | 239              | 247              | 225              |
| 178        | 234              | 237              | 246              | 224              |
| 179        | 234              | 235              | 244              | 222              |
| 180        | 233              | 232              | 242              | 221              |
| 181        | 233              | 229              | 241              | 219              |
| 182        | 232              | 226              | 239              | 218              |

Table 7: The central and mean values of $\sigma_{W^g}$ (in pb) at the LHC, along with the upper and lower bounds derived by varying the scale and by considering the PDF sets CTEQ4M and MRRS(R1). The central value is the value midway between the upper and lower bounds, while the mean is the averaged result of $\sigma_{W^g}$ calculated using the CTEQ4M and MRRS(R1) PDF with the canonical scale choice discussed in the text.
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