Lattice QCD gluon propagators near transition temperature

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Landau gauge gluon propagators are studied numerically in the SU(3) gluodynamics as well as in the full QCD with the number of flavors \(n_F = 2\) using efficient gauge fixing technique. We compare these propagators at temperatures very close to the transition point in two phases: confinement and deconfinement. The electric mass \(m_E\) has been determined from the momentum space longitudinal gluon propagator. Gribov copy effects are found to be rather strong in the gluodynamics, while in the full QCD case they are weak ("Gribov noise"). Also we analyse finite volume dependence of the transverse and longitudinal propagators.

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I. INTRODUCTION

The transition from the confinement to the deconfinement phase is one of the most interesting features of QCD at finite temperature. This transition separates a low-temperature confinement phase from a high-temperature – quark-gluon plasma – phase, where color charges should be deconfined. The existence of this transition has been confirmed by recent observations of the collective effects in ultrarelativistic heavy-ion collisions (see, e.g., the review [1]).

The non-perturbative study of gauge variant propagators is of interest for various reasons. These propagators are expected to show different behavior in each phase and, therefore, to serve as a useful ‘order parameters’, detecting the phase transition point \(T_c\). One expects that their study can shed the light on the mechanism of the confinement-deconfinement transition [2,3]. Another reason is that for the reliable phenomenological analysis of high-energy heavy-ion collision data, it is important to obtain information on the momentum dependence of the gluon propagators, especially in the infrared region (see, e.g., [3–10]). Finally, the non-perturbatively calculated lattice propagators are to be used to check the correctness of various analytical methods in QCD, for example, Dyson-Schwinger equations method at finite temperature [11,13].

Last years a number of papers have been dedicated to the lattice study of the finite temperature SU(2) pure gauge gluon propagators in Landau gauge (see, e.g., papers [13–20]). However, the finite temperature SU(3) gluon propagators - especially in the presence of dynamical fermions - are much less studied (see papers [18,21,23] for the pure gauge theory and [24] for the full QCD).

In this paper we study the transverse (magnetic) and longitudinal (electric) gluon propagators both in the pure gauge SU(3) theory and in the full QCD with the number of flavors \(n_F = 2\). The main goal of our work is to compare the momentum behavior of these propagators very close to the transition point, slightly above \(T_c\) (deconfinement phase) and slightly below \(T_c\) (confinement phase). We compare our results for the gluon propagators computed in gluodynamics and in full QCD at coexisting values of the ratio \(T/T_c\), where \(T_c\) is a corresponding transition temperature. For both theories we have chosen temperatures \(T\) very close to the corresponding transition point: \(T/T_c = 0.97\) and \(T/T_c = 1.02\). Of special interest in this study are the infrared mass scale parameters (‘screening masses’): \(m_E\) (‘electric’) and \(m_M\) (‘magnetic’).

We apply effective gauge fixing algorithm - simulated annealing - and generate few gauge copies for every configuration to reduce Gribov copy effects. For gluodynamics we apply additionally flips between sectors of the Polyakov loops [25]. The flips were applied to this theory for the first time. Also we check finite volume effects comparing results for two lattice sizes.

Section II contains main definitions as well as some details of simulations and gauge fixing procedure we use. Volume and temperature dependence of the propagators for both pure gauge theory and for QCD are discussed in Section III. Section IV is dedicated to the discussion of the screening masses and Section V is reserved for conclusions and discussion.
II. GLUON PROPAGATORS: THE DEFINITIONS AND SIMULATION DETAILS

For the study of the gluon propagators in the pure gauge theory we employ the standard Wilson action $S_W$ with $\beta = 6/g_0^2$ where $g_0$ is a bare coupling constant. To define the spacing $a$ as a function of $\beta$ in the pure gauge case we have used Necco–Sommer parametrization $[20]$ with the popular choice of the Sommer scale $r_0 = 0.5\text{fm}$. In what follows we will refer to this theory as $n_F = 0$ theory.

To study the effect of the quarks on the gluon propagator we computed propagators on configurations generated with the gauge action $S_W$ and $n_F = 2$ dynamical flavors of nonperturbatively $O(a)$ improved Wilson fermions (clover fermions). The configurations were produced by the DIK collaboration $[27]$ with BQCD code $[28]$. The dimensionless quantities $r_0/a$ and $m_{\pi}/r_0$ are taken from results of QCDSF collaboration. To convert them into physical units we use the Sommer scale $F_{\pi}$.

In the pure gauge case we have used Necco–Sommer parameterization $[20]$ where link variables $U_\mu \in SU(3)$ transform under gauge transformations $g_x \in SU(3)$ as follows:

$$ U_{\mu} \rightarrow g_x U_{\mu} g_x^{-1} $$

The Landau gauge condition is given by $\langle \partial A_x \rangle = \sum_{\mu=1}^4 (A_{x \mu} - A_{x - \mu}) = 0$ which is equivalent to finding an extremum of the gauge functional

$$ F_U(g) = \frac{1}{4V} \sum_{x \mu} \frac{1}{3} \text{Re} \text{Tr} U_{\mu x}^g, $$

with respect to gauge transformations $g_x$.

The bare gluon propagator $D_{\mu \nu}^{ab}(p)$ is given by

$$ D_{\mu \nu}^{ab}(p) = \frac{g_0^2}{g_0} \langle \vec{A}_\mu(k) \vec{A}_\nu^b(-k) \rangle, $$

where $\vec{A}(k)$ represents the Fourier transform of the gauge potentials according to Eq. (1) after having fixed the gauge. The physical momenta $p$ are given by $p_i = (2/a) \sin (\pi k_i/L)$, $p_4 = (2/a) \sin (\pi k_4/L)$, $k_i \in (-L/2, L/2]$, $k_4 \in (-L_4/2, L_4/2]$.

In what follows we consider only soft modes $p_4 = 0$. The hard modes ($p_4 \neq 0$) have an effective thermal mass $2\pi Tn$ and behave like massive particles.

On the asymmetric lattice there are two tensor structures for the gluon propagator $[32]$:

$$ D_{\mu \nu}^{ab}(p) = \delta_{ab} (P_T^{\mu \nu}(p) D_T(p) + P_{\mu \nu}^L(p) D_L(p)), $$

where (symmetric) orthogonal projectors $P_T^{\mu \nu}(p)$ are defined as $p = (\vec{p} \neq 0; p_4 = 0)$ as follows

$$ P_T^{ij}(p) = (\delta_{ij} - \frac{p_i p_j}{p^2}), \quad P_T^{ij}(p) = 0; \quad P_{\mu \nu}^L(p) = 1; \quad P_{\mu \nu}^L(p) = 0. $$

Therefore, two scalar propagators - longitudinal $D_T(p)$ and transverse $D_L(p)$ - are given by

$$ D_T(p) = \frac{1}{16} \sum_{a=1}^8 \sum_{i=1}^3 D_{ii}^a(p); \quad D_L(p) = \frac{1}{8} \sum_{a=1}^8 D_{44}^a(p). $$

For $\vec{p} = 0$ propagators $D_T(0)$ and $D_L(0)$ are given by

$$ D_T(0) = \frac{1}{24} \sum_{a=1}^8 \sum_{i=1}^3 D_{ii}^a(0); \quad D_L(0) = \frac{1}{8} \sum_{a=1}^8 D_{44}^a(0). $$

The transverse propagator $D_T(p)$ is associated to magnetic sector, and the longitudinal one $D_L(p)$ - to electric sector.

In the case of the $n_F = 0$ theory we employ for gauge fixing the $Z(3)$ flip operation as has been proposed in $[22]$. It consists in flipping all link variables $U_{x \mu}$ attached and orthogonal to a 3d plane by multiplying them with exp $\{\pm 2\pi i/3\}$. Such global flips are equivalent to non-periodic gauge transformations and represent an exact symmetry of the pure gauge action $S_W$. At finite temperature we apply flips only to directions $\mu = 1, 2, 3$. As for the 4th direction, we stick to the sector with $|\arg P| < \pi/3$ which provides maximal values of the functional $F$. Therefore, the flip operations combine for each lattice field configuration the $3^3$ distinct gauge orbits of strictly periodic gauge transformations into one larger gauge orbit.

All details of our gauge fixing procedure - FSA algorithm - are described in our recent papers $[19, 33, 35]$. The features of the simulated annealing algorithm application specific for $SU(3)$ group are essentially same as in $[36]$ where this algorithm was applied to $SU(3)$ theory for the first time. For every configuration we produce two gauge copies per flip-sector; therefore we have in total $N_{copy} = 54$ copies. We take the copy with maximal value of the functional $[2]$ as our best estimator of the global maximum and denote it as best (“bc”) copy.
In the case of the \( n_F = 2 \) theory the global \( Z(3) \) transformations do not anymore make part of the symmetry group of the action (only one flip-sector remains). In this case we have made \( N_{\text{copy}} = 10 \) gauge copies.

To suppress 'geometrical' lattice artifacts, we have applied the "\( \alpha \)-cut" \[27\], i.e. \( \pi k_i/L_s < \alpha \), for every component, in order to keep close to a linear behavior of the lattice momenta \( p_i \approx (2\pi k_i)/(aL_s) \), \( k_i \in (-L_s/2, L_s/2) \). We have chosen \( \alpha = 0.5 \). Obviously, this cut influences large momenta only. We did not employ the cylinder cut in this work.

The values of \( T/T_c, \beta, \kappa \) and lattice sizes are given in Table I. Number of independent configurations in all cases equals \( \sim 200 \).

\[
\begin{array}{|c|c|c|c|c|}
\hline
T/T_c & L_4 \cdot L_s^3 & \beta & \beta & \kappa \\
\hline
0.97 & 8 \cdot 16^3 & 6.044 & 5.25 & 0.1341 \\
1.02 & 8 \cdot 16^3 & 6.075 & 5.25 & 0.1345 \\
0.97 & 8 \cdot 24^3 & 6.044 & 5.25 & 0.1341 \\
1.02 & 8 \cdot 24^3 & 6.075 & 5.25 & 0.1345 \\
\hline
\end{array}
\]

TABLE I: Run parameters.

For \( n_F = 0 \) theory the critical value \( \beta_c \) (corresponding to the 1st order phase transition) has been taken from \[38\].

In the case of \( n_F = 2 \) theory the simulations were made \[27\] at fixed \( \beta = 5.25 \) and varying \( \kappa \). The pseudocritical values \( \kappa_c \) (and thus the transition temperatures \( T_c \)) of the deconfining and chiral symmetry restoration transitions have been determined \[27\] by the maximum of the Polyakov loop and chiral condensate susceptibilities, respectively. It has been found \[27\] that these two maxima coincide within numerical precision at \( \kappa_c = 0.1343 \).

The pion is heavy at values of \( \kappa \) we have chosen \((m_\pi \sim 1 \text{ Gev})\). However, it is known that the effect of the quarks is rather strong even at these values of \( m_\pi \). The transition is a crossover, rather than a 1st order phase transition. Furthermore, the transition temperature is shifted to substantially lower values in comparison to the gluodynamics \[27, 39, 10\] and the string breaking phenomenon is observed at \( T < T_c \) \[41, 42\].

III. GLUON PROPAGATORS : NUMERICAL RESULTS

We define the renormalized propagators \( D_{T,ren}^F(p) \) and \( D_{L,ren}^F(p) \) in such a way that their dressing functions are equal to unity at the normalization point \( \mu = 2.5 \text{ Gev} \). In the rest of this paper we will omit the subscript 'ren'; therefore, \( D_T(p) \) and \( D_L(p) \) will denote transverse and longitudinal renormalized propagators, respectively.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
n_F & T/T_c & L_4 \cdot L_s^3 & Z_M & Z_E & Z_M & Z_E \\
\hline
0 & 0.97 & 8 \cdot 16^3 & 1.03(1) & 1.00(2) & 1.12(2) & 1.14(2) \\
 & 1.02 & 8 \cdot 16^3 & 1.01(1) & 1.00(1) & 1.13(2) & 1.16(2) \\
1 & 0.97 & 8 \cdot 24^3 & 1.04(1) & 1.00(1) & 1.14(1) & 1.14(1) \\
 & 1.02 & 8 \cdot 24^3 & 1.04(1) & 1.03(1) & 1.15(1) & 1.17(1) \\
\hline
\end{array}
\]

TABLE II: Renormalization constants \( Z_{M/E} \).

To compute the renormalization constants \( Z_M \) and \( Z_E \) we fitted the propagators at momenta \( p \) in the range between 2.3 and 2.7 GeV to a polynomial function. They are shown in Table II for both \( n_F = 0 \) and \( n_F = 2 \) theories. As expected, there is no volume dependence of these constants. In the case of the pure gauge theory all \( Z_M \) and \( Z_E \) are very close to unity, which is in agreement with the results for the bare gluon propagators at zero temperature in pure gauge SU(3) theory \[43\]. In the \( n_F = 2 \) case all renormalization constants are substantially larger. This deviation (~15%) might be explained, at least, partially, by the quark contribution. Another possible explanation might be the larger spacing (or smaller \( \beta \)) effect.

For both theories every constant, i.e., \( Z_M \) and \( Z_E \), is practically the same above and below \( T_c \). This means that ultraviolet parts of \( D_T(p) \) and \( D_L(p) \) are not 'phase-sensitive'.

A. On Gribov copy effects

In our study of the Gribov copy effects we follow the approach described briefly in Section I and - in more details - in our papers \[13, 27, 33, 34, 44\] dedicated to the Gribov copy effects in SU(2) gluodynamics at zero and nozero temperatures.

We find that in the case of the \( n_F = 0 \) theory, similar to SU(2) gluodynamics, the Gribov copy influence still remains a serious problem in the lattice calculations, at least, at small momenta. These effects are very strong for \( D_T(p) \) in the infrared and weak for \( D_L(p) \) for all momenta.

It is worthwhile to note that the effect of different flip-sectors plays a very important role in this case, even more important than additional gauge copies in each sector. The details of this study will be published elsewhere.

In contrast, in the case of dynamical fermions \( n_F = 2 \) the Gribov copy effects look completely negligible as compared to the pure gauge case. The propagators do not change within errorbars ("Gribov noise").
However, it is worthwhile to note that our lattice sizes are comparatively small. One cannot exclude that these effects might become stronger for larger lattice sizes and/or smaller lattice spacings.

### B. Volume dependence

We find that for the transverse propagators $D_T(p)$ the finite volume effects are rather strong at zero momentum but they become very weak with increasing $|p|$. As an illustration, in Figure 1 we show the momentum dependence of the $n_F = 0$ and $n_F = 2$ transverse propagators at comparatively small momenta ($|p| < 1.5$ GeV) at $T/T_c = 1.02$ on the lattices $8 \cdot 16^3$ and $8 \cdot 24^3$. No volume dependence is observed also for $|p| > 1.5$ GeV. The similar situation takes place for $T/T_c = 0.97$.

The volume dependence of the longitudinal propagators $D_L(p)$ is different. One can see from Figure 2 that the sign of the finite volume effects differs from that for $D_T(p)$ case: while the transverse propagator decreases with increasing volume, the longitudinal one is increasing. This is in agreement with $SU(2)$ theory \textsuperscript{19}. Another observation is that the finite volume effects are much more substantial for $n_F = 0$ theory than for $n_F = 2$ one. At $p = 0$ the quenched theory propagator increases by factor two when $L_s$ increases from 16 to 24 as can be seen from Figure 2 at $T/T_c = 0.97$. This increase is similar to that observed in $SU(2)$ theory \textsuperscript{17, 19}. The finite volume effects decrease fast with increasing $|p|$ both for $n_F = 0$ and for $n_F = 2$ theories. At $T/T_c = 1.02$ the corresponding figure looks in a similar way.

In what follows we show data for $L_s = 24$ lattices only.

### C. Temperature dependence

We find that the transverse propagators $D_T(p)$ are not very much sensitive to the crossing of the transition point $T_c$, at least, at $p \neq 0$. In Figure 3 where we present results for $D_T(p)$ at $T/T_c = 0.97$ and $T/T_c = 1.02$ for both $n_f = 0$ and $n_f = 2$ theories. We observe that in the case of $n_f = 0$ theory the temperature dependence is very weak at all momenta. (Similar conclusion was made in \textsuperscript{18}.) The same is true for $n_f = 2$ theory at nonzero momenta. But, contrary to $n_f = 0$ case, for $p = 0$ substantial change in the propagator is observed as can be seen from Figure 3. This is one of the clear effects of dynamical quarks.

Another interesting observation is that one can not see any substantial influence of the dynamical fermions on the momentum dependence of the transverse propagators for $|p| \neq 0$. Moreover, for the momenta $|p| \gtrsim 0.8$ GeV all four propagators coincide within errorbars. It would be interesting to see how this situation will change for smaller values of the pion mass $m_\pi$. The values of propagators obtained at $p = 0$ suggest that in $n_f = 2$ theory the transverse propagator is infrared enhanced in comparison with the one in $n_f = 0$ theory. This is in contrast with $T = 0$ results \textsuperscript{42, 46}. But this conclusion should be checked in future studies on larger volumes.

In contrast, the longitudinal propagators $D_L(p)$ demonstrate much more pronounced temperature dependence for both theories, especially in the infrared region $|p| \leq 1.5$ GeV. For both theories, i.e., $n_F = 0$ and $n_F = 2$, the propagators $D_L(p)$ differ significantly also for nonzero values of momenta when temperature...
FIG. 3: The momentum dependence of the transverse propagator \(D_T(p)\) at two temperatures for \(|p| \leq 1.5\text{ Gev.}\)

FIG. 4: The momentum dependence of the longitudinal propagator \(D_L(p)\) at two temperatures for \(|p| \leq 1.5\text{ Gev.}\)

FIG. 5: The momentum dependence of the longitudinal propagator \(D_L(p)\) at two temperatures for \(|p| > 1.5\text{ Gev.}\)

FIG. 6: The momentum dependence of \(D_T(p)\) and \(D_L(p)\) at two temperatures for \(|p| < 1.5\text{ Gev.}\)

IV. ON THE SCREENING MASSES

One of the interesting features of the finite temperature physics is the appearance of the infrared mass scale parameters: \(m_E\) (‘electric’) and \(m_M\) (‘magnetic’). These parameters (or ‘screening masses’) define screening of electric and magnetic fields at large distances and, therefore, control the infrared behavior of \(D_L(p)\) and \(D_T(p)\). The electric screening mass \(m_E\) has been computed in the leading order of perturbation theory long ago: 
\[ m_E^2/T^2 = (1 + n_F/6) \alpha_s^2(T) \]
for \(SU(3)\) theory with \(n_F\) flavors [17, 48]. But at the next order the problem of the infrared divergencies has zero (within errorbars) for \(|p| \geq 1.2\text{ Gev.}\).
been found. On the other hand, the magnetic mass $m_M$ is entirely nonperturbative in nature. Thus a first-principles nonperturbative calculations in lattice QCD should play an important role in the determination of these quantities.

In the deep infrared region the momentum dependence of the longitudinal propagator is expected to fit the pole-type behavior

$$D_L(p) \simeq \frac{C}{m_E^2 + p^2} ; \quad p \sim 0 \ .$$  \hspace{1cm} (9)

Indeed, in the case of the pure gauge $SU(2)$ theory we observed the linear behavior of the inverse longitudinal propagator $D_L^{-1}(p)$ as a function of $p^2$ for small enough values of momenta \[^{19}\]. Moreover, this linear dependence has been observed both above and below $T_c$ and even at $T \simeq T_c$.

In the present work our lattice sizes $L_s$ do not permit us to reach small enough values of $p^2$ where the linear $p^2$-behavior of $D_L^{-1}(p)$ could be found. Instead we have used for the fit a somewhat simplified Stingl-like parametrization of the longitudinal propagator \[^{19, 50}\]

$$D_L^{-1}(p) = C^{-1} \cdot (m_E^2 + p^2 + b |p|^4) \ .$$  \hspace{1cm} (10)

We employed this formula in the comparatively large momentum interval, up to $p^2 \lesssim 2 \text{ Gev}^2$, where 6 data points were used for the fit for every data set (see Figure 7). The parameter $b$ indicates the deviation from the simple pole-type behavior. We use the parameter $\hat{m}_E$ as an estimator of the electric screening mass $m_E$.

The values of the fit parameters are presented in the Table III. In the case of the pure gauge theory

we can compare our value of $\hat{m}_E/T$ calculated at $T/T_c = 1.02$ with $m_E/T$ obtained in \[^{23}\] for nearby temperature $T/T_c = 1.05$. Both values are well consistent within errorbars. Instead, in the unquenched QCD case our value of $\hat{m}_E/T$ at $T/T_c = 1.02$ differs by factor 1.5 approximately from $m_E/T$ calculated in $n_F = 2$ QCD with KS fermions at the same value of $T/T_c$ \[^{24}\]. The origin of this deviation is still to be clarified.

As is well-known, one can hardly rely on the perturbation theory at $T \sim T_c$. However, it is interesting to note that the ratio of two values $m_E^2/T^2$ (for $n_F = 2$ and $n_F = 0$) is consistent with the factor $(1 + n_F/6)$.

Our main observation is that the dimensionless parameter $\hat{m}_E/T$ is practically not sensitive (within errorbars) to the crossing of the transition point $T_c$.

This statement is true not only for $n_F = 2$ theory where the crossover (or higher order phase transition) is expected but also for the pure gauge theory where the existence of the 1st order phase transition is firmly established. Therefore, we conclude that the electric mass can hardly be considered as an 'order parameter' indicating the transition point $T_c$.

Note, that a similar situation have been observed in the case of the finite temperature pure gauge theory \[^{23}\] where the values of the parameter $m_E/T$ coincide for $T/T_c = 0.9$ and $T/T_c \simeq 1$. \[^{19}\]

In contrast, dimensionless factors $C$ are much more sensitive to the crossing of the transition point $T_c$. This is true for both $n_F = 0$ and $n_F = 2$ theories. The difference between $n_F = 0$ and $n_F = 2$ cases is more quantitative than qualitative: the temperature variation of the parameter $C$ in the pure gauge theory is much more strong as compared to the full QCD case.

In fact, the only quantity which demonstrates clear difference between $n_F = 0$ and $n_F = 2$ cases is the dimensionless parameter $bT^2$. Indeed, for the pure gauge theory the temperature jump of $bT^2$ is rather strong, while for the $n_F = 2$ case values of $bT^2$ above and below $T_c$ coincide within errorbars.

The calculation of the magnetic screening mass $m_M$ looks a somewhat more delicate problem. In the case of pure gauge $SU(2)$ theory it has been shown that at

| $T/T_c$ | $n_F$ | $\hat{m}_E/T$ | $C$ | $bT^2$ |
|---|---|---|---|---|
| 0.97 | 0 | 1.725(126) | 5.03(49) | 0.091(14) |
| 0.97 | 2 | 2.09(15) | 3.17(14) | 0.054(6) |
| 1.02 | 0 | 1.851(77) | 3.24(15) | 0.050(5) |
| 1.02 | 2 | 2.24(9) | 2.68(16) | 0.045(7) |

TABLE III: Values of parameters $\hat{m}_E$, $C$ and $b$ obtained from fits to eq. (10).
$T > T_c$ the transverse propagator $D_T(p)$ has a maximum at $p \neq 0$. This has been found also in $SU(3)$ gluodynamics. Moreover, in our recent paper it has been shown that $D_T(p)$ has a maximum not only at high temperatures but even at $T < T_c$. The position of this maximum at $T$ close to $T_c$ was found to be at about 400 Mev. Therefore, the transverse propagator $D_T(p)$ has a form which is not compatible with the simple pole-type behavior, so for $m_M$ another, different from pole mass, definition is necessary. This definition has been proposed in [19].

In paper [51] it has been suggested that the proximity of the Gribov horizon at finite temperature forces the transverse gluon propagator $D_T(p, p_4 = 0)$ to vanish at zero three-momentum. If this is the case, then the finite-temperature analog of Gribov formula $\langle |\vec{p}|^2/|\vec{p}|^4 + M_H^2 \rangle = 1/(|\vec{p}|^2 + m_{eff}^2(\vec{p}))$ suggests that the effective magnetic screening mass $m_{eff}(\vec{p})$ becomes infinite in the infrared (so called, magnetic gluons ‘confinement’).

In the case of $SU(3)$ gauge group, both for $n_f = 0$ and $n_f = 2$ theories considered in this paper, we do not see a maximum of $D_T(p)$ at $p \neq 0$. The transverse propagators $D_T(p)$ look somewhat similar to the longitudinal ones. The most probable reason for this is that our volumes are not large enough. Note that also in the case of $SU(2)$ theory we did not observe the maximum at $p \neq 0$ on smaller lattices; this maximum appeared only with increasing of lattice size $L_s$. To fit $D_T(p)$ we employed the same formula given in eq. (10) and same fitting range as for $D_L(p)$. We found that the fit is good when $p = 0$ is excluded. Otherwise the $\chi^2/ndf$ increases up to 6 and becomes significantly larger than in the case of $D_L(p)$. We consider this as an indication of the maximum to be seen when the volume is increased.

We conclude that for reliable definition of the magnetic screening mass $m_M$ one needs larger values of the lattice size $L_s$.

V. CONCLUSIONS

In this work we studied numerically the behavior of the Landau gauge transverse and longitudinal gluon propagators in the pure gauge $SU(3)$ theory and in the theory with dynamical fermions with the number of flavors $n_F = 2$. We compare these two theories in the close vicinity of the transition temperature $T_c$, slightly above and slightly below corresponding transition temperature. For both theories we have chosen temperatures $T$ in such a way that ratios $T/T_c$ are equal to 0.97 and 1.02. To our knowledge, this is the first study where two theories are compared in confinement and deconfinement phases.

Let us summarize our findings.

In the case of the pure gauge theory the Gribov copy effects are rather strong for $D_T(p)$ in the infrared and weak for $D_L(p)$ for all momenta. In contrast, in the case of dynamical fermions the Gribov copy effects look completely negligible (the so called “Gribov noise”).

The renormalization constants $Z_M$ and $Z_E$ are practically the same above and below $T_c$ for both theories. It confirms that ultraviolet parts of $D_T(p)$ and $D_L(p)$ are not ‘phase-sensitive’.

Both below and above $T_c$ the finite volume effects are rather strong for the transverse propagators $D_T(p)$ at zero momentum but they become weak fast with increasing $|p|$. This weakening is slower for $n_F = 2$ theory. The volume dependence of the longitudinal propagators $D_L(p)$ is different. First, the sign of the effect is different from that for $D_T(p)$ case. Second, it is much more substantial for $n_F = 0$ theory than for $n_F = 2$ one. At $p = 0$ the quenched theory propagator increases by factor two with increasing volume as can be seen from Figure 2. This increase is similar to that observed in $SU(2)$ theory [12, 19]. However, at $p \neq 0$ this dependence is not very much pronounced, both for $n_F = 0$ and for $n_F = 2$ theories.

We computed the electric mass $m_E$ defined in eq. (10). It is practically not sensitive to the crossing of the transition point $T_c$. This is true not only for $n_F = 2$ theory where the crossover (or higher order phase transition) is expected but also for the $n_F = 0$ case where the existence of the 1st order phase transition is well established. Therefore, $m_E$ can hardly be considered as an ‘order parameter’ indicating the transition point $T_c$.

In contrast, factors $C$ defined in eq. (10) are much more sensitive to the confinement-deconfinement transition for both $n_F = 0$ and $n_F = 2$ theories. However, the difference between $n_F = 0$ and $n_F = 2$ cases is more quantitative than qualitative: the temperature variation of $C$ in the pure gauge theory is much more strong as compared to QCD case. Note, that temperature dependence for $m_E$ and $C$ computed in $SU(2)$ gluodynamics [13] was similar to those found here in $n_F = 0$ theory.

The only (dimensionless) parameter which demonstrates clear difference between $n_F = 0$ and $n_F = 2$ theories near $T_c$ is $bT^2$. For the pure gauge theory the confinement-deconfinement variation of $bT^2$ is rather strong. Instead, for the full QCD case values of $bT^2$ above and below $T_c$ coincide within errorbars.

Calculation of the magnetic screening mass $m_M$ is somewhat more delicate problem. Our caution is based on the experience of $m_M$ study in the $SU(2)$ theory. Indeed, in the case of the pure gauge $SU(2)$ theory the transverse propagator $D_T(p)$ has a local maximum at $p \neq 0$ in the deconfinement and even in the confinement phases when lattice size $L_s$ is large enough and Gribov copies are properly handled. In our study of $SU(3)$ theories we do not see a maxi-
mum of $D_T(p)$ at $p \neq 0$ and the transverse propagators $D_T(p)$ look similar to the longitudinal ones. It is very probable that this is because our lattice sizes are not large enough. We conclude that for reliable definition of $m_M$ one needs larger values of $L_z$ and $\beta$’s.

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[1] M. Gyulassy and L. McLerran, Nucl. Phys. A750, 30 (2005), nucl-th/0405013.
[2] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. 66, 1 (1979).
[3] V. N. Gribov, Nucl. Phys. B139, 1 (1978).
[4] D. Zwanziger, Nucl. Phys. B364, 127 (1991).
[5] D. Zwanziger, Nucl. Phys. B412, 657 (1994).
[6] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne, and D. Schiff, Nucl. Phys. B483, 291 (1997), hep-ph/9607355.
[7] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne, and D. Schiff, Nucl. Phys. B484, 265 (1997), hep-ph/9608322.
[8] M. Gyulassy, I. Vitev, X.-N. Wang, and B.-W. Zhang (2003), nucl-th/0302077.
[9] M. A. Kovner and U. A. Wiedemann (2003), hep-ph/0304151.
[10] X.-N. Wang, Phys. Lett. B485, 157 (2000), nucl-th/0003033.
[11] A. Maas, J. Wambach, and R. Alkofer, Eur. Phys. J. C42, 93 (2005), hep-ph/0504019.
[12] A. Maas, Mod. Phys. Lett. A20, 1797 (2005), hep-ph/0506066.
[13] A. Cucchieri, A. Maas, and T. Mendes, Phys. Rev. D75, 076003 (2007), hep-lat/0702022.
[14] U. M. Heller, F. Karsch, and J. Rank, Phys. Lett. B355, 511 (1995), hep-lat/9505016.
[15] U. M. Heller, F. Karsch, and J. Rank, Phys. Rev. D57, 1438 (1998), hep-lat/9710033.
[16] A. Cucchieri, F. Karsch, and P. Petreczky, Phys. Rev. D64, 036001 (2001), hep-lat/0103009.
[17] A. Cucchieri and T. Mendes, PoS LAT2007, 297 (2007), 0710.0412.
[18] C. S. Fischer, A. Maas, and J. A. Mueller (2010), 1003.1968.
[19] V. G. Bornyakov and V. K. Mitrjushkin (2010), 1011.1479.
[20] A. Cucchieri and T. Mendes, PoS LATTICE2010, 280 (2010), 1101.4537.
[21] J. I. Kapusta, Cambridge University Press, New York, NY p. 70 (1979).
[44] T. D. Bakeev, E. M. Ilgenfritz, V. K. Mitrjushkin, and M. Müller-Preussker, Phys. Rev. D69, 074507 (2004), hep-lat/0311041.
[45] P. O. Bowman, U. M. Heller, D. B. Leinweber, M. B. Parappilly, and A. G. Williams, Phys. Rev. D70, 034509 (2004), hep-lat/0402032.
[46] E. M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, and A. Schiller (2006), hep-lat/0601027.
[47] S. Nadkarni, Phys. Rev. D27, 917 (1983).
[48] S. Nadkarni, Phys. Rev. D38, 3287 (1988).
[49] M. Stingl, Phys. Rev. D34, 3863 (1986).
[50] U. Habel, R. Konning, H. G. Reusch, M. Stingl, and S. Wigard, Z. Phys. A336, 435 (1990).
[51] I. Zahed and D. Zwanziger, Phys. Rev. D61, 037501 (2000), hep-th/9905109.