A moving platform photometric observation based attitude and angular velocity estimation method for space target

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Abstract—Based on the photometric information, a method is proposed for estimating space target attitude and angular velocity under the condition of moving observation platform. Firstly, according to the dynamically changing relative pose relationship among the target, the observation platform and the sun, photometric measurement model is established. Then, in the framework of Unscented Kalman Filter, the joint estimation of target attitude and angular velocity is realized by combining use of quaternion and Modified Rodriguez Parameters. Finally, the feasibility and estimation accuracy of the method are verified by numerical simulation experiments. The results show that the proposed method achieves high accuracy and has certain value in practical application.

1. Introduction

Attitude estimation plays an important role in satellite recognition, anomaly detection and specific direction determination[1]. The common active measurement method usually uses telemetry to receive the attitude data sent by the target, or observe the target attitude by radar. The common passive measurement method usually adopts optical detection which mainly include photometric observation based on time series, spectral observation based on low dispersion and optical imaging[2].

Optical images based method is directly processing the obtained images, and roughly estimating the target attitude by segmentation, geometric feature extraction and matching[3]. This kind of method has a large amount of calculation and demands high resolution target images. It is suitable for observing nearby targets.

For distant space targets, by processing photometric information, the target attitude can be obtained by using an inversion method. Some scholars have analyzed the periodic change of the target photometric curve and successfully distinguished the rolling or triaxial stable state of space targets[4]. Furthermore, the rotation axis and rotation cycle of the target can be extracted by time series analysis method[2, 4].

In this paper, it presents a method to estimate the target attitude and rotational angular velocity using photometric information of space target under the condition of moving observation platform. Numerical simulation experiment result shows good performance of the proposed method.
2. Joint estimation of space target attitude and angular velocity based on nonlinear filtering

2.1. Target photometric measurement model under the condition of moving observation platform

A space target can be assumed to be composed of a finite number of planes, and its radiation received by a station is the sum of the solar radiation reflected by each surface. Considering time variation and movement of the observation platform, we establish a dynamic target photometric measurement model. The dynamic target photometric measurement model is

\[
\mathbf{q}_{\text{sun}} = \begin{bmatrix} \cos(\lambda^s) \\ \cos(e^s) \sin(\lambda^s) \\ \sin(e^s) \sin(\lambda^s) \end{bmatrix}^T
\]

(1)

\(\lambda^s\) is longitude of ecliptic and \(e^s\) is obliquity of the ecliptic at the current time. For a moving target in space, its position \(\mathbf{p}_T\) can be calculated from the number of orbit elements.

\[
\mathbf{p}_T = f_{\text{sat}}(\mathbf{L}, \omega)
\]

(2)

Where, \(\mathbf{L}\) represents orbit elements of the target, Kepler orbit elements or Two Line Elements can be used, \(f_{\text{sat}}(\cdot)\) is a position predicting function for the target according to the orbital elements.

For the \(i\)-th surface, its current unit normal vector is

\[
\mathbf{u}_i^h = \mathbf{A}^T(\mathbf{w}_i, \Delta t) \mathbf{u}_i^0
\]

(3)

Where, \(\mathbf{u}_i^0\) is its initial normal vector, \(\mathbf{A}^T(\mathbf{w}_i, \Delta t)\) represents the target attitude matrix calculated from its attitude angle calculated by its angular velocity \(\mathbf{w}_i\) and the interval time \(\Delta t\).

Position of the observation platform is measured by its navigation equipment, expressed as \(\mathbf{P}_{\text{O}}\). Then, the unit direction vector pointing from the target to the platform is

\[
\mathbf{u}_{\text{sun}}^h = (\mathbf{P}_{\text{O}}^t - \mathbf{P}_i^t) \left(\left\| \mathbf{P}_{\text{O}}^t - \mathbf{P}_i^t \right\| \right)^{-1}
\]

(4)

The current distance is \(r_i = \left\| \mathbf{P}_{\text{O}}^t - \mathbf{P}_i^t \right\|\). According to [4], the dynamic magnitude is

\[
m^T = -26.7 - 2.5 \log r_i^2 \sum \rho_i A_i (\mathbf{u}_i^h \cdot \mathbf{u}_{\text{sun}}^h) (\mathbf{u}_i^h \cdot \mathbf{u}_{\text{sun}}^h)
\]

(5)

Where, \(A_i\) is the area of the \(i\)-th surface of the target, \(\rho_i\) is the reflection coefficient which is related to the surface material. Let

\[
n_i^T = \rho_i A_i (\mathbf{u}_i^h \cdot \mathbf{u}_{\text{sun}}^h) (\mathbf{u}_i^h \cdot \mathbf{u}_{\text{sun}}^h)
\]

(6)

Obviously, when the angle between the surface normal vector and the observation normal vector is greater than 90°, the sunlight reflected by the target surface cannot be received by the observation platform. Similarly, when the angle between the sun direction vector and the surface normal vector is greater than 90°, the sun cannot shine on the surface. Therefore, in practice, there should be

\[
n_i^T = \begin{cases} 0 & (\mathbf{u}_i^h \cdot \mathbf{u}_{\text{sun}}^h) \leq 0 \text{ or } (\mathbf{u}_i^h \cdot \mathbf{u}_{\text{sun}}^h) \leq 0 \\ (\mathbf{u}_i^h \cdot \mathbf{u}_{\text{sun}}^h) & \text{else} \end{cases}
\]

(7)

The dynamic target photometric measurement model is

\[
m^T = -26.7 - 2.5 \log r_i^2 \sum n_i^T
\]

(8)

2.2. Space target attitude estimation based on nonlinear filtering

As its advantages of large angle manoeuvrability and avoiding singular problems, quaternion is often adopted to describe the attitude change of space target. It is defined as \(\mathbf{q} = [p^T \ q_3]^T\), where \(p = [q_1 \ q_2 \ q_3]^T\) is imaginary coefficient vector, \(q_3\) is the real part and \(\mathbf{q}^T\). Its differential equation is

\[
\dot{\mathbf{q}} = \frac{1}{2} \begin{bmatrix} q_3 \mathbf{I}_{3 \times 3} + [\mathbf{p}^T \mathbf{p}] \end{bmatrix} \mathbf{w} + \mathbf{w}_1(t)
\]

(9)

\(\mathbf{w} = [\mathbf{o}_w \ 0 \ 0]^T\) is target angular velocity, \(\mathbf{w}_1(t)\) and \(\mathbf{w}_2(t)\) are Gaussian noise vectors. By time integrating equation (9), the discrete dynamic model of quaternion can be obtained, as

\[
\mathbf{p}_{k+1} = F(\mathbf{p}_k, \mathbf{w}_k) + \mathbf{G}_k
\]

(10)

Where, \(\mathbf{G}_k\) is a three-dimensional column vector representing the uncertainty of the dynamic model, which can be represented by white noise. And \(F(\mathbf{p}_k, \mathbf{w}_k) = \Omega(\mathbf{w}_k) \cdot \mathbf{p}_k\), where
\[ \Omega(\omega_k) = \begin{bmatrix} \cos \left( \frac{1}{2} \| \omega_k \| \Delta t \right) I_{3\times3} & \Psi_k \\ -\Psi_k^T \cos \left( \frac{1}{2} \| \omega_k \| \Delta t \right) \end{bmatrix} \] \tag{11}

\( \Delta t \) is the time interval, \( \omega_k \) is angular velocity at time \( k \). And

\[ \Psi_k = \sin \left( \frac{1}{2} \| \omega_k \| \Delta t \right) \omega_k (\| \omega_k \|)^{-1} \] \tag{12}

It can be seen from equation (10) that the joint estimation of attitude angle and angular velocity is not a linear process. It is proposed to adopt a nonlinear filtering method for the estimation. In this paper, Unscented Kalman Filter (UKF) is adopted[5]. UKF uses unscented transformation (UT) which avoids the linear approximation of nonlinear function in traditional methods and does not need to derive the nonlinear function to calculate Jacobian matrix, resulting in high calculation accuracy[6].

Due to the normalization limitation of quaternions in UT, Modified Rodriguez Parameters (MRPs) are used during UT transformation[7].

To quaternion \( q \), its corresponding MRPs

\[ p_{MRPs} = \frac{4p}{1+q_4} \] \tag{13}

And to MRPs \( p_{MRPs} \), their corresponding quaternion

\[ \begin{aligned} q_4 &= \frac{16-p_{MRPs}^T p_{MRPs}}{16+p_{MRPs}^T p_{MRPs}} \\ p &= \frac{1+q_4}{4} q_{MRPs} \\ q &= [p^T, q_4]^T \end{aligned} \] \tag{14}

The state is defined as a 6-dimension column vector \( X = [p_{MRPs}^T, \omega_1, \omega_2, \omega_3]^T \). Suppose that the state vector at time \( t \) is \( X' \) and its corresponding covariance matrix is \( P' \). The state transition equation is

\[ X'^{t+\Delta t} = F(X', \Delta t) + \Gamma_X \] \tag{15}

Where, \( \Gamma_X \) represents process noise. The output of function \( F(X', \Delta t) \) contains two parts, \( p_{MRPs}^{t+\Delta t} \) and \( \hat{p}_{MRPs}^{t+\Delta t} = \hat{\omega}_t \) and \( \hat{p}_{MRPs}^{t+\Delta t} \) needs to be converted by quaternion. Firstly, \( p_{MRPs}^{t+\Delta t} \) is converted into its corresponding quaternion \( q' \) according to equation (14), then \( q'^{t+\Delta t} \) is calculated according to equation (10), and finally \( \hat{p}_{MRPs}^{t+\Delta t} \) is inversely calculated according to equation (13).

During the state prediction stage, according to UT, take \( 2n+1 \) sigma sampling points near \( X' \) \( (n \) is the dimension of \( X \), here \( n=6 \)), expressed as \( \gamma_i \) \((i=1, \ldots, 13) \), and their corresponding mean weight and variance weight are \( W_i^m \) and \( W_i^v \) respectively.

The state transfer is carried out according to equation (15). It is obtained that the predicted state at each sigma sampling point is \( \gamma_i \), then the predicted state of UKF is

\[ \hat{X}'^{t+\Delta t} = \sum_{i=1}^{13} W_i^m \gamma_i \] \tag{16}

Its corresponding covariance matrix

\[ P'^{t+\Delta t} = \sum_{i=1}^{13} W_i^m \left( \gamma_i, X'^{t+\Delta t} \right) \left( \gamma_i, X'^{t+\Delta t} \right)^T + Q' \] \tag{17}

Where, \( Q' \) is covariance matrix of \( \Gamma_X \).

During the measurement update stage, the quaternion is used to calculate the target at each sigma sampling point, and then the target photometric prediction value \( \hat{z}_i \) is calculated according to photometric measurement model described in equation (8). The predicted mean measurement is

\[ \hat{m}'^{t+\Delta t} = \sum_{i=1}^{13} W_i^m z_i \] \tag{18}

Its corresponding covariance matrix

\[ \begin{aligned} P_{mm}'^{t+\Delta t} &= \sum_{i=1}^{13} W_i^m \left( \hat{z}_i, \hat{m}'^{t+\Delta t} \right) \left( \hat{z}_i, \hat{m}'^{t+\Delta t} \right)^T + R' \\
P_{nm}'^{t+\Delta t} &= \sum_{i=1}^{13} W_i^m \left( \gamma_i, \hat{X}'^{t+\Delta t} \right) \left( \gamma_i, \hat{X}'^{t+\Delta t} \right)^T \end{aligned} \] \tag{19}
Where, $R_t$ is measurement error variance. Then the gain matrix is

$$K_{t+\Delta t} = P_{Xm}^{-1}(P_{mm} t + \Delta t)^{-1}$$

(20)

The state at time $t+\Delta t$ is

$$\hat{X}_{t+\Delta t} = \hat{X}_t + K_{t+\Delta t} \left( m^{t+\Delta t} - \hat{m}^{t+\Delta t} \right)$$

(21)

Where, $m^{t+\Delta t}$ is real value of target photometric measurement. Its covariance matrix

$$P^{t+\Delta t} = P^{t+\Delta t} \cdot \left( K_{t+\Delta t} \right)^T$$

(22)

3. Simulation Experiment

Assume a space target whose orbit height is about 36000km, its main body is cube structure with side length of 2m, $\rho_i=0.5$ $(i=1, \ldots, 6)$, in addition, it has two solar panels with 8m length and 2m width, $\rho_i=0.7$ $(i=7, \ldots, 10)$. The initial attitude angle of the target is $[0 \ 0 \ 0]$ rad and rotates at the angular speed of $[0.001 \ 0.001 \ 0.003]$ rad/s. The observation platform moves at a speed of 15 m/s on the ground, and the photometric measurement error is expressed by Gaussian white noise with mean value of 0 and standard deviation of 0.07. According to the photometric measurement model, the theoretical value of target photometry and its observation value are obtained, as shown in Figure 1.

![Figure 1. Theoretical value and observation value of target photometry.](image)

Using the joint estimation method of attitude and angular velocity proposed in this paper, the filtering result of one observation process is shown in Fig. 2. It can be seen that the estimated values of target attitude and angular velocity can converge to the true value, the attitude error is stable within 0.05 rad, and the angular velocity error is less than $3.5 \times 10^{-5}$ rad/s. Further, 1000 tests are repeated, and the average error value at each moment is calculated. The result is shown in Fig. 3. After filtering convergence, the measurement accuracy is consistent with that of one single experiment.

![Figure 2. Result of one test (left: angle, right: angular velocity).](image)
4. Conclusion
A method is proposed for simultaneously estimating the target attitude and angular velocity by using the target photometric measurement information under the condition of moving observation platform. By combining use of quaternion and Modified Rodriguez Parameters, the nonlinear filtering process is simplified and standardization limitation of quaternion in the filtering process is avoided. Numerical simulation results show high measurement accuracy of the proposal method. Further research can be carried out in combination with real observation data.

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References
[1] Chen S, Liu H, Liu X and Yu Q, (2019) Non-cooperative maritime target position and velocity measuring method based on monocular trajectory intersection for video satellite. Proceedings of the Institution of Mechanical Engineers Part G Journal of Aerospace Engineering, 233: 44-56.
[2] Shan B, Liang Y and Li H, (2017) Attitude and Angular Speed Estimation of Spacial Objects Based on Photometric Observation. Acta Optica Sinica, 37: 512002.
[3] Sun X, Liu X, Su A, Chen S and Yu Q, (2016) Objectness to assist salient object detection. IET IMAGE PROCESS, 10: 391-397.
[4] Linares R, Jah M K, Crassidis J L and Nebelecky C K, (2014) Space Object Shape Characterization and Tracking Using Light Curve and Angles Data. J GUID CONTROL DYNAM, 37: 13-25.
[5] John, Crassidis, Landis and Markley, (2003) Unscented Filtering for Spacecraft Attitude Estimation. Journal of Guidance Control & Dynamics, 26: 536-542.
[6] Chang L, Hu B, Li A and Qin F, (2013) Transformed Unscented Kalman Filter. IEEE T AUTOMAT CONTR, 58: 252-257.
[7] Chang L, Hu B and Chang G, (2014) Modified Unscented Quaternion Estimator Based on Quaternion Averaging. Journal of Guidance Control Dynamics, 37: 305-309.