On probing the anomalous magnetic moments of vector leptoquarks in $ep$ collisions.

V. Ilyin, A. Pukhov, V. Savrin, A. Semenov,  
Institute of Nuclear Physics of Moscow State University, 119899 Moscow, Russia  
and  
W. von Schlippe  
Queen Mary & Westfield College, London, England

Abstract

We study a possibility to measure the anomalous magnetic moment of vector leptoquarks in reactions with single leptoquark production associated with hard photon emission in $ep$ collisions. For this purpose we propose to use the radiative amplitude zero effect. We find that an exact radiative zero in the angular distribution of the emitted photon is present only for pure Yang-Mills coupling of photons to leptoquarks. As a result the cross section is sensitive to the anomalous magnetic moment $\kappa$ of the leptoquark. This effect offers a possibility to measure $\kappa$ with rather high accuracy. For LEP+LHC we establish upper bounds in the plane leptoquark mass–anomalous magnetic moment.

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1 Introduction.

In this paper we continue the investigation of single leptoquark (LQ) production with emission of a hard photon in electron-proton collisions, 

\[ e^\pm + p \rightarrow \gamma + LQ + X. \]  

(1)

In [1] we found that a radiative amplitude zero (RAZ) effect is present for some types of leptoquarks. Of particular interest is the case of vector leptoquarks which have an exact radiative amplitude zero only in the case of a Yang-Mills structure of coupling to the photon. In this paper we develop the application of this effect to the measurement of the anomalous magnetic moment of the vector leptoquark. We present the numerical analysis for the case of a possible LEP+LHC experiment.[2][3]

2 Anomalous magnetic moment

The electromagnetic interaction of vector leptoquarks is described by the following Lagrangian:

\[ \mathcal{L}_\gamma^V = i e Q \left[ A^\mu \Phi^{+\mu\nu} \Phi_\nu - A^\mu \Phi^{+\mu} \Phi^\nu + (1 - \kappa) A^\mu \Phi^{+\nu} \Phi^\mu \right] - (e Q)^2 \left[ A^2 (\Phi^{+\mu} \Phi^\mu) - (A^\mu \Phi^{+\mu})(A^\nu \Phi^\nu) \right]. \]  

(2)

Here \( \Phi_{\mu\nu} = \partial_\nu \Phi_\mu - \partial_\mu \Phi_\nu, A_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu \); \( \Phi_\mu \) denotes the vector leptoquark; \( e \) is the elementary electric charge; \( Q \) is the charge of the leptoquark in units of \( e \).

This Lagrangian contains a free dimensionless parameter \( \kappa \), the anomalous magnetic moment of the leptoquark. The value \( \kappa = 0 \) corresponds to the Yang-Mills structure of the interaction, while \( \kappa = 1 \) corresponds to minimal coupling. The measurement of this parameter could help us in the understanding of the nature of this new massive vector boson.

A value of \( \kappa = 0 \) would indicate that the vector leptoquark belongs to the family of gauge bosons. Such particles were introduced in grand unified theories (GUT) (see the corresponding references in [4]). In these models vector leptoquarks are represented by some components of a unified gauge field that provides the interaction of all matter fields, while the other components represent the known gauge bosons, i.e. photons, gluons, \( W \) and \( Z \) bosons. Therefore in these theories the Yang-Mills structure of the LQ-photon vertex is defined by the selfinteraction of the unified gauge field.

On the other hand if it were found that \( \kappa = 1 \), then we should consider this leptoquark to be a new charged matter field rather than a gauge field. In this case the minimal coupling to the electromagnetic field is also consistent with the principle of local gauge invariance similarly to the electromagnetic interaction of leptons and quarks. However, in this case there arises a serious problem of how to ensure the unitarity and renormalizability of the theory. Thus from a theoretical point of view the deviation from the Yang-Mills structure is a most interesting effect and we organize our numerical analysis in such a way as to study its possible manifestations.

We can conclude that the measurement of the leptoquark magnetic moment will arise as a fundamental problem when these massive vector bosons are discovered. In the literature the possible existence of vector leptoquarks with relatively small masses (hundreds of GeV) is under discussion. Searches for leptoquarks have been carried out at HERA and lower limits on their masses have been established at the level \( \sim 200 \) GeV, depending on the leptoquark type[3]. Let us note that some higher limits have been derived by indirect methods from low energy experiments[4], being stronger for the vector leptoquarks. In [3] the minimal coupling was investigated in leptoquark pair production in \( e^+e^- \) collisions. In [5] the Lagrangian with anomalous magnetic moment (and with anomalous quadrupole electric moment) was considered in connection with probing these anomalous couplings in \( e^+e^- \), \( \gamma e \) and \( \gamma\gamma \) collisions. In the present paper

1At HERA the cross sections for these processes are too small to be useful for this purpose even for electroweak values of the LQ-fermion coupling constant \( \lambda = 0.3 \).

2Here we are citing also some reviews where leptoquarks were considered in the framework of different extensions of the Standard Model.
we analyse the potential of conceivable LEP+LHC experiments for the measurement of the anomalous magnetic moment from reactions with single leptoquark production associated with the emission of a hard photon.

3 Squared matrix element and RAZ

The hard subprocess underlying reaction (1) is

$e^\pm + q \rightarrow \gamma + \text{LQ}$,  

(3)

where $q$ is a constituent quark of the proton. We consider the case when the leptoquark interacts only with quarks of the first generation, i.e. $q = (u, \bar{u}, d, \bar{d})$.

From a phenomenological point of view the most general Lagrangian for fermion-LQ vertices was proposed in [9] where leptoquarks of all possible quantum numbers were discussed. Below we use the notation introduced for leptoquarks in [9] (see also the detailed table of leptoquark quantum numbers in [1]).

In [1] we have obtained the analytical formulas for the squared matrix element of hard subprocess (3):

$$|A|^2 = \frac{e^2 \lambda^2}{(\xi - 1)^2} \left[ (Q_q \tau - Q_e \nu)^2 K_0 + Q(Q_q \tau - Q_e \nu)K_1 \kappa + Q^2 K_2 \kappa^2 \right],$$  

(4)

$$K_0 \equiv \frac{\xi^2 + 1 - 2\tau \nu}{\tau \nu}, \quad K_1 \equiv \tau - \nu, \quad K_2 \equiv \frac{\tau \nu}{2} + \frac{\xi}{8}(\tau^2 + \nu^2).$$

Here $Q_q$ and $Q_e = \mp 1$ are the quark, electron and positron charges, respectively, in units of $e$; $Q = Q_q + Q_e$.

We also use normalized dimensionless Mandelstam variables ($M$ is the leptoquark mass):

$$\xi \equiv \frac{s}{M^2} = \frac{(p_e + p_q)^2}{M^2}, \quad \tau \equiv \frac{t}{M^2} = \frac{(p_\gamma - p_e)^2}{M^2}, \quad \nu \equiv \frac{u}{M^2} = \frac{(p_{\text{LQ}} - p_e)^2}{M^2}.$$

We see that the first two terms in (4) vanish at some value of $\tau$ and $\nu$ due to the factor $(Q_q \tau - Q_e \nu)$. This means in particular that for $\kappa = 0$ (Yang-Mills case) there is no photon radiation in some direction depending on the electric charge of the leptoquark (radiative amplitude zero – RAZ effect). In [3] we have investigated the RAZ effect in detail and arrived at some conclusions about the possibility of observing the RAZ effect at HERA and LEP+LHC and to use this effect for the determination of quantum numbers of the discovered leptoquarks.

The fact that the $\kappa^2$ term has no RAZ factor $(Q_q \tau - Q_e \nu)$ makes the cross section sensitive to the value of the leptoquark anomalous magnetic moment. In Fig. 1 we show a typical angular distribution with a clear dependence on $\kappa$ in the RAZ region. In the next section we analyse numerically the possibility to get an upper bound on the value of the anomalous magnetic moment from reaction (1). We show that the RAZ effect, if it is present, enables one to establish much stronger upper bounds on the value of $\kappa$ then in the cases without RAZ.

4 Cross sections

We have calculated the contributions of separate (quark) constituents to the integrated cross sections by convoluting the differential cross sections of the hard subprocesses with the corresponding parton distribution functions:

$$\sigma(s) = \int_{x_{\text{min}}}^{1} dx \ q(x, Q^2) \int_{-1}^{1} d\cos \vartheta_\gamma \frac{d\sigma (s, \cos \vartheta_\gamma, \kappa)}{d\cos \vartheta_\gamma} \Theta_{\text{cuts}}(E_\gamma, \vartheta_\gamma).$$  

(5)

$^3$The result in the case of $U_3^{\pm 1}$ leptoquarks are greater by a factor of two.
Here $\hat{\sigma}$ is the cross section of the corresponding subprocess \[^1\]; $s$ is the squared CMS energy of the electron-proton system; the squared CMS energy of the hard subprocess is $\hat{s} = x s$; the quark distribution function is denoted by $q(x,Q^2)$ and the 4-momentum transfer scale is taken to be $Q^2 = \hat{s}$. We denote the photon emission angle by $\vartheta$; the direction $\vartheta = 0$ is along the proton beam. The function $\Theta_{c_{\text{cuts}}}(E_\gamma, \vartheta)$ introduces the necessary kinematical cuts. The process under discussion is infrared divergent, therefore we have to introduce a cut $E_\gamma > E_\gamma^0 > 0$; for our numerical analysis we use $E_\gamma^0 = 1$ GeV. Also we introduce a cut on the photon emission angle, $\vartheta_{\text{min}} < \vartheta < \vartheta_{\text{max}}$, to exclude the unobservable forward and backward cones and to ensure the optimal conditions for probing $\kappa$ (see next section). As the lower bound $x_{\text{min}}$ we use values which will also ensure the optimal conditions for probing $\kappa$.

For our calculations, both analytical for the squared matrix elements and numerical for the integrands including the convolution with parton distributions and $\Theta_{c_{\text{cuts}}}$, we have used the CompHEP package \[^{10}\]. As Monte Carlo integrator and event generator we have used the BASES/SPRING package \[^1\]. For the parton densities we used the parametrizations CTEQ2p \[^{12}\] and MRS-A \[^{13}\] which take account of recent HERA data. Both parametrizations gave the same results within calculation errors.

The numerical analysis of the corresponding cross sections is given for the LEP+LHC collider with $\sqrt{s} = 1740$ GeV, electron beam energy of 100 GeV and an integrated luminosity of $1 \text{fb}^{-1}$. Of course in reality the leptoquarks decay into two fermions, a lepton and a quark. So we have to take account of complete sets of diagrams, including the Standard Model background consisting of deep inelastic scattering associated with hard photon emission. In \[^3\] we have calculated all contributions and found that the Standard Model background was less than 1% of the signal. This result was obtained for some set of kinematical cuts to reduce the Standard Model background; the most effective cut was the cut on the invariant mass of the lepton-quark system around the leptoquark mass.

5 Probing the anomalous magnetic moment

In this section we analyse numerically the possibility of getting an upper bound on $\kappa$ from experiment \[^1\].

One of the general restrictions on the LQ-fermion interaction is the chirality of the lepton (see \[^9\] and references therein). Therefore we consider only either coupling with left-handed leptons or with right-handed ones. In both cases we have used an electroweak value for this constant, $\lambda = 0.3$. For other values of $\lambda$ numerical estimates can be obtained by rescaling our results from the figures (the cross sections have $\lambda^2$ as a factor). As to leptoquark masses we investigated the range $200 \text{GeV} < M < 1 \text{TeV}$.

Let us consider as a criterion for the value $\kappa \neq 0$ to be detectable the condition that the number of events in this case has to be different from those of the Yang-Mills case with 95$\%$ CL. Therefore the following relation must be satisfied: $N(\kappa) - N(0) > 1.96\sqrt{N(0)}$. Here $N(\kappa)$ is the number of events corresponding to cross section \[^3\] with luminosity 1 fb$^{-1}$. It is clear that the cross section is a quadratic function of $\kappa$. We found from our numerical analysis that the minimum of this function lies near the Yang-Mills value $\kappa = 0$ and in the region under consideration its exact position does not strongly depend on the leptoquark mass. Therefore we consider only the positive branch of $\kappa$. The analysis for negative values gives practically the same results as for positive values; the difference is no more than 20$\%$. Thus, we should keep in mind that if a deviation from the Yang-Mills cross section is observed in the experiment it will show evidence only for an absolute value of $\kappa$ but will not allow us to determine its sign. We denote by $\kappa_2$ the lowest positive value of $\kappa$ for which the above criterion is satisfied. It is clear that larger values of $\kappa_2$ mean a lower sensitivity of the experiment to $\kappa$; we have therefore chosen $\kappa_2$ as a representative characteristic of our analysis.

As we already noted the RAZ effect should increase the sensitivity to $\kappa$. Indeed, we are going to observe the contribution of the terms proportional to $\kappa$ and to $\kappa^2$ in \[^3\] with a value of $\kappa$ as small as possible. So regions, where the contribution of the Yang-Mills term (the first term in \[^3\]) is smaller, are more promising for our analysis. It is clear that the region near the RAZ, where $Q_{\eta \tau} = Q_{e \nu}$, has to be very sensitive to the value of $\kappa$. We conclude that the strongest bound on the value of $\kappa$ can be obtained for leptoquarks with RAZ, i.e. for $V_2$ and $U_3$ in the left-hand sector and for $V_2$ and $U_1$ in the right-hand
sector. In Fig. 2 we show a typical dependence of parameter $\kappa_2$ on the cut over the photon emission angle in the case of RAZ. We see from these figures that an optimal cut could be given by

$$10^\circ < \vartheta_\gamma < 90^\circ.$$  \hspace{1cm} (6)

The term with the anomalous magnetic moment in Lagrangian (2) contains derivatives of the electromagnetic field. Therefore the contribution of the $\kappa$ terms to the cross section is smaller for lower energies of the emitted photon. This point is important for our analysis because the region near the threshold where the quasi-resonant peak lies turns out to be insensitive to $\kappa$. So to ensure a maximal sensitivity to $\kappa$ we have to cut off the threshold region. On the other hand, the introduction of too large a cut on $x$ could remove all events. The implication is that we have to find the optimal value of $x_{\text{min}}$. In Fig. 3 we show $\kappa_2$ as a function of $x_{\text{min}}$ for three values of the leptoquark mass and with the angular cuts (6) applied. The dependence of $\kappa_2$ for large values of $x_{\text{min}}$ is rather smooth. We found that the optimal values for this cut could be parametrized by the following empirical formula

$$x_{\text{opt}}^{\text{min}} = (M + 150 \text{ GeV})^2/s.$$  \hspace{1cm} (7)

Our final results are represented on the Fig. 4 where the dependence of the $\kappa_2$ parameter is given as a function of the leptoquark mass, on the left-hand side in the RAZ case and on the right-hand side for other leptoquarks. Points above the curves can be measured with 95\% $\text{CL}$. Here we have used the cuts (4) and (5). In Table 1 we give the upper bounds on the leptoquark masses for two values of the anomalous magnetic moment, $\kappa = 1$ and $\kappa = 3$ in the RAZ cases. For lower masses the corresponding values of $\kappa$ can be measured with 95\% $\text{CL}$. Let us stress again that only an absolute value of $\kappa$ is discussed here.

In this paper we have carried out the RAZ analysis only for reactions with electron decay modes of the leptoquarks. So only some isospin components of the leptoquarks were considered. The analysis for the complementary isospin components has to be based on reactions with neutrino decay modes. These channels could be added also in the total statistics to establish stronger bounds on $\kappa$. The corresponding analysis will be presented elsewhere.

6 Conclusions

The proposed analysis of cross sections of vector leptoquark production in $ep$ collisions shows a possibility to measure the leptoquark anomalous magnetic moment $\kappa$. The corresponding experiments will be more sensitive for those leptoquarks which reveal the RAZ effect.

The value $\kappa = 1$ (minimal coupling) can be measured at LEP+LHC up to $M = 400 \text{ GeV}$ for $V_2^-\tilde{1}$, and up to $M = 600 \text{ GeV}$ for $U^{-1}_3$ and $\tilde{U}_1$ types of leptoquarks. In the case of $U_3$ and $\tilde{U}_1$ leptoquarks, the positron-proton channel is more sensitive than the electron-proton channel.

The anomalous magnetic moment of leptoquarks for which there is no RAZ effect can be measured only on the level of several units or even $\kappa \sim 10$.

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4We refer to [3] where a typical distribution in $x$ is shown, with a sharp quasi-resonant peak near the threshold $x_0 = M^2/s$ even with the cut $E_\gamma > 1 \text{ GeV}$.
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Table

| $\ell q$ channel | $e^+d$ | $e^-\bar{u}$ | $e^-d$ | $e^+u$ |
|------------------|--------|-------------|--------|--------|
| left-handed lepton | $V_2^+ U_3^{-1}$ | $V_2^{-1} U_3^+$ | $V_2^{-1}$ | $\tilde{U}_1$ |
| right-handed lepton | $V_2^{-1}$ | $\tilde{U}_1$ | $V_2^{+}$ | $\tilde{U}_1$ |

$\kappa = 1$ | 310 | 370 | 390 | 590 |
$\kappa = 3$ | 530 | 590 | 660 | 890 |

Table 1: Upper bounds on the LQ mass, for lower masses the corresponding values $\kappa = 1$ and $\kappa = 3$ can be measured with 95% CL.

Figure captions

Figure 1: RAZ effect for $V_2^{-\frac{1}{2}}$ with $M = 300$ GeV. Here $x_{\text{min}} = 0.04$. For $\kappa = 0$ (1) the cross section equals $\sigma = 2.41$ (2.49) pb.

Figure 2: Left-hand side: $\kappa_2$ vs $\vartheta_{\gamma\text{min}}$ with $\vartheta_{\gamma\text{max}} = 90^\circ$; Right-hand side: $\kappa_2$ vs $\vartheta_{\gamma\text{max}}$ with $\vartheta_{\gamma\text{min}} = 10^\circ$; different curves correspond to: 1) $M = 300$ GeV, $x_{\text{min}} = 0.067$; 2) $M = 400$ GeV, $x_{\text{min}} = 0.1$; 3) $M = 500$ GeV, $x_{\text{min}} = 0.14$.

Figure 3: $\kappa_2$ vs $x_{\text{min}}/x_0$. Here $\vartheta_{\gamma\text{min}} = 10^\circ$, $\vartheta_{\gamma\text{max}} = 90^\circ$; different curves correspond to: 1) $M = 300$ GeV; 2) $M = 400$ GeV and 3) $M = 500$ GeV.

Figure 4: $\kappa_2$ vs LQ mass. Here $\vartheta_{\gamma\text{min}} = 10^\circ$, $\vartheta_{\gamma\text{max}} = 90^\circ$; $x_{\text{min}} = x_{\text{opt/min}}$. On the left-hand side processes with RAZ are presented, on the right-hand side RAZ effect is absent.
Figures

Figure 1:

Figure 2:
Figure 3:

Figure 4: