Cosmic-ray electron injection from the ionization of nuclei

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We show that the secondary electrons ejected from the ionization of heavy ions can be injected into the acceleration process that occurs at supernova remnant shocks. This electron injection mechanism works since ions are ionized during the acceleration when they move already with relativistic speed, just like ejected electrons do. Using the abundances of heavy nuclei measured in cosmic rays at Earth, we estimate the electron/proton ratio at the source to be \( \sim 10^{-4} \), big enough to account for the nonthermal synchrotron emission observed in young SNRs. We also show that the ionization process can limit the maximum energy that heavy ions can reach.

Supernova Remnant (SNR) are believed to be the primary sources of Cosmic Rays (CR) in the Galaxy. The theory of Diffusive Shock Acceleration (DSA) applied to SNR blast waves propagating in the interstellar medium provides the most comprehensive framework for the explanation of the CR spectrum measured at Earth. Nevertheless some aspects of DSA still remain unsolved and one of the most fundamental issues concerns how electrons can be injected into the acceleration process. The presence of accelerated electrons is a well established fact, deduced from direct observations of young SNRs, where both Radio and X-ray emission are interpreted as synchrotron radiation of highly relativistic electrons. According to theoretical estimates and observational constraints, the electron/proton ratio at the source to be \( \sim 10^{-4} \), big enough to account for the nonthermal synchrotron emission observed in young SNRs.

In this paper we show that the ionization of heavy nuclei during acceleration can inject a number of mildly relativistic electrons large enough to account for the synchrotron radiation observed in young SNRs. In fact, nuclei heavier than hydrogen in the ISM where SNR shocks propagate, are never fully ionized simply because the typical ISM temperature is not large enough, being of the order of \( 10^4 - 10^5 \) K. This statement is also supported by the presence of Balmer-dominated filaments observed in several young remnants, showing that even the hydrogen in the ISM is not fully ionized.

Once the shock encounters a partially ionized atom, the atom can start DSA in the same way protons do. We note that a correct computation of the injection of heavy ions involves the knowledge of the initial charge and the aggregate state of atoms and their downstream temperature, which are very difficult to predict. We neglect such complications and we assume that the injection of heavy elements occurs, simply because they are observed in the CR spectrum. The key point we want to stress here is that, once the acceleration begins, atoms are not stripped immediately, because the ionization time turns out to be large enough to allow them to reach relativistic energies before complete ionization. When atoms move relativistically, the ionization can occur either via Coulomb collisions or via photoionization. In both cases, the mean kinetic energy of ejected electrons, measured in the ion rest frame, is negligible with respect to the electron mass energy. Hence ejected electrons move, in the plasma rest frame, approximately along the same direction and with the same speed of the parent atoms. In this case, the momentum of ejected electrons can easily exceed \( p_{\text{inj}} \). In order to prove this statement we start comparing the acceleration with the ionization time.

Let us consider a single partially ionized species \( N \), with mean charge \( Z_{\text{eff}} \), atomic charge \( Z \) and mass \( m_N = 2Zm_p \). For simplicity we compute the acceleration time in the framework of linear shock acceleration theory, i.e. neglecting the dynamical role of accelerated particles, which are very difficult to predict.

\[
\frac{\gamma_{\text{inj}}}{3-30} \text{for a typical } T_{p,2} \text{ in the range } 10^6-10^8 \text{ K. It is therefore hard to imagine how electrons can come from the thermal component: if we assume that electrons upstream of the shock thermalize downstream their bulk kinetic energy, i.e. } K_{B}T_{e,2} = \frac{2}{3}m_eu_{\text{shock}}^2, \text{ then the mean thermal electron momentum is } p_{e,\text{th}} = (m_e/m_p)p_{p,\text{th}}, \text{ Even assuming some mechanism able to equilibrate quickly electrons and protons at the same temperature, the mean electron momentum rises only up to } p_{e,\text{th}} = \sqrt{m_e/m_p}p_{p,\text{th}}. \]

Most proposed solutions for this injection problem involve some kind of pre-acceleration mechanism able to accelerate electrons from thermal energies up to mildly relativistic energies. Some studies predict that electrons can be effectively pre-accelerated by electrostatic waves generated in the shock layer. These mechanisms are difficult to study analytically, because their understanding requires the knowledge of the complex microphysics that regulates collisionless shocks. A better way to investigate them is through particle-in-cell and Monte Carlo simulations, which, unfortunately, are still not able to provide firm conclusions on the electron injection efficiency.
and for a plane shock geometry. If a particle with momentum $p$ diffuses with a diffusion coefficient $D(p)$, the well known expression for the acceleration time is \( t_{\text{acc}} = \frac{3}{u_1 - u_2} \left( \frac{D_1(p)}{u_1} + \frac{D_2(p)}{u_2} \right) \), \( t_{\text{acc}}(p) \), \( p \)

where $u$ is the plasma speed in the shock rest frame, and the subscript 1 (2) refers to the upstream (downstream) quantities (note that $u_{\text{shock}} = u_1$). The downstream and upstream plasma speeds are related through the compression factor, $u_2 = u_1/r$. We limit our considerations to strong shocks, which have compression factor $r = 4$, and we assume Bohm diffusion coefficient, i.e. $D_B = r_L \beta c/3$, where $\beta c$ is the particle speed and $r_L = pc/(ZeR B)$ is the Larmor radius. The turbulent magnetic field responsible for particle diffusion is assumed to be compressed downstream according to $B_2 = r B_1$. Even if this relation applies only for the magnetic component parallel to the shock plane, choosing a different compression rule does not affect our main results. Applying previous approximations, Eq. \( 1 \) becomes:

\[
t_{\text{acc}}(\gamma) = 1.7 \left( \gamma - \gamma^{-1} \right) B_{\mu G} u_8^{-2} (Z/Z_{\text{eff}}) \text{ yr} , \]

where $\gamma$ is the particle Lorentz factor. Here $B_{\mu G}$ is the upstream magnetic field expressed in $\mu G$ and the shock speed is $u_1 = u_8 \times 10^8 \text{cm/s}$. In order to compute the energy reached by particles when ionization occurs, we compare Eq. \( 2 \) with the ionization time. As already mentioned, ionization can occur either via Coulomb collisions with thermal particles or via photoionization by background photons. Whether the former process dominates on the latter depends on the ratio between the thermal particle and the ionizing photon densities.

We consider first the role of collisions. A full treatment of collisional ionization is hard because of the complex cross section involved and it is beyond the scope of this work; our purpose can be achieved using the following approximation. Let us consider the process in the rest frame of the target atom, which is bombarded with point-like charged particles (in our case protons or electrons) with kinetic energy $E_{\text{kin}}$. The classical ionization cross section has a maximum when $E_{\text{kin}}$ is twice the ionization energy, $I$, and the value is:

\[
\sigma_{\text{coll}} = \alpha_0^2 N_e I_{\text{Ryd}}^{-2} , \]

where $\alpha_0$ is the Bohr radius, $I_{\text{Ryd}}$ is the ionization potential expressed in Rydberg units and $N_e$ is the number of electrons in the considered atomic orbital. For $E_{\text{kin}} > 2I$ the cross section decreases like $E_{\text{kin}}^{-2}$ and reaches a minimum when relativistic effects become important, while, in the full relativistic regime $\sigma_{\text{coll}} \propto \log(E_{\text{kin}})$ (see e.g. \( 11 \)). Hence we can use Eq. \( 3 \) as a good upper limit for the collisional cross section in a wide range of incident particle energy.

Accelerated ions collide mainly with thermal protons and electrons, whose densities are assumed to be equal, $n_e = n_p$. The total ionization time is the average between the upstream and downstream contribution, weighted for the respective residence time, i.e. $\tau_{\text{coll}} = (t_1 + t_2)/(t_1/\tau_{\text{coll},1} + t_2/\tau_{\text{coll},2})^{-1}$, where $t_i = 4D_i/cu_i$. The final result is:

\[
\tau_{\text{coll}} = (c\sigma_{\text{coll}} n_1 (1 + r))^{-1} = 0.0024 I_{\text{Ryd}}^2 n_1^{-1} \text{ yr} , \]

where $n_1$ is the upstream proton density in $\text{cm}^{-3}$. Now, equating $\tau_{\text{coll}}$ with the acceleration time in Eq. \( 2 \), we get the value of the Lorentz factor, $\gamma_{\text{coll}}$ that ions have when the collisional ionization occurs. Using typical parameters for a young SNR, the result reads:

\[
\gamma_{\text{coll}} \simeq 9 \left( \frac{0.1 \text{ cm}^{-3}}{n_1} \right) \left( \frac{B_1}{25 \mu G} \right) \left( \frac{u_1}{5000 \text{ km/s}} \right)^2 I_{\text{Ryd}}^2 . \]

An upstream magnetic field around $25 \mu G$ is expected if magnetic amplification occurs, but the condition $\gamma_{\text{coll}} > \gamma_{\text{inj}}$ can be easily fulfilled even for magnetic field as low as the mean galactic value, i.e. $\sim 5 \mu G$.

In the atom rest frame ejected electrons can have kinetic energy ranging from 0 up to $E_{\text{kin}} - I$. But, due to the long-range nature of Coulomb interaction, events with a small momentum transfer are highly favored and the majority of ejected electrons have kinetic energy $E \ll n_e e^2$ \( 11 \). As a consequence when parent atoms move relativistically with respect to the plasma rest frame, ejected electrons move approximately in the same direction and with the same Lorentz factor computed in Eq. \( 3 \). The same is true when the electron is ejected by photoionization.

Photoionization can occur only when atoms collide with photons whose energy is larger than the ionization potential $I$. Atoms moving relativistically see a distribution of photons peaked in the forward direction of motion, with a mean photon energy $\epsilon' = \gamma \epsilon$, where $\epsilon$ is the photon energy in the plasma rest frame. In order to estimate the photoionization time we adopt the simplest approximation for the $K$-shell cross section \( 12 \), i.e.:

\[
\sigma_{\text{ph}}(\epsilon') = 64 \alpha^{-3} \sigma_T Z^{-2} (I/\epsilon')^{7/2} , \]

where $\sigma_T$ is the Thompson cross section and $\alpha$ is the fine structure constant. The factor $Z^{-2}$ is due to the nuclear charge dependence of K-type orbitals dimension. In order to get the full photoionization time we need to integrate over the total photon energy spectrum, i.e.:

\[
\tau_{\text{ph}}^{-1}(\gamma) = \int d\epsilon \frac{dn_{\text{ph}}(\epsilon)}{d\epsilon} c \sigma_{\text{ph}}(\gamma \epsilon) , \]

where $dn_{\text{ph}}/d\epsilon$ is the photon spectrum as seen in the plasma rest frame. The relevant ionizing photons are only those with energy $\epsilon \simeq I/\gamma$ (measured in the plasma frame) because the photoionization cross section rapidly decreases with increasing photon energy. The corresponding numerical value of photoionization time is:

\[
\tau_{\text{ph}}(\gamma) \simeq 0.01 Z^2 (u_{\text{ph}} I/\gamma)/\text{cm}^{-3} \text{ yr} . \]
Comparing $\tau_{\text{ph}}$ with Eq. (1) for the last inner orbital (which has $I_{\text{Ryd}} = Z^2$) we see that photoionization generally dominates over the collisional ionization for heavy ions, namely when $n_{\text{ph}} \gtrsim 4n_1Z^{-2}$.

In Fig. 1 we compare the acceleration time with both the collisional and the photoionization time. The two panels show the case of ionization for two hydrogen-like ions, He$^+$ and C$^{5+}$, so that $Z_{\text{eff}} = Z - 1$. Each characteristic time is shown for two different choices of the parameters: $t_{\text{acc}}$, plotted with solid lines, is shown for $u_1 = 3000$ km/s, $B_1 = 3\mu$G (upper line) and for $u_1 = 10^4$ km/s, $B_1 = 20\mu$G (lower line); $\tau_{\text{coll}}$ (dot-dashed) is shown for $n_1 = 0.01$ (upper line) and $1 \text{ cm}^{-3}$ (lower line); finally $\tau_{\text{ph}}$ (dashed line) is computed according to Eq. (7), using the Galactic interstellar radiation field (ISRF) plus the cosmic microwave background. We use the ISRF as calculated in [13], which includes the photons produced by the Galactic center [13]. We neglect the high energy radiation coming from the remnant itself because the typical number density of the X-ray photons is negligible and does not exceed $10^{-3} \text{ph/cm}^2 \cdot \text{s}$. Fig. 1 shows that the most relevant contribution to photoionization comes from the optical photons, which produces the first dip present of the dashed curves. The value of $\gamma$ where $\tau_{\text{coll}}$ and $\tau_{\text{ph}}$ intersect $t_{\text{acc}}$, identifies the Lorentz factor of ejected electrons. From the upper panel we see that even electrons from He$^+$ can be ejected with $\gamma > 10$ and can easily undergo DSA.

A remarkable consequence of the ionization process is that, under appropriate circumstances, heavy elements reach a maximum energy lower than $Z \times E_{\text{proton}}$, the value predicted by the shock acceleration theory. This because they cannot be completely ionized in a time less than the Sedov time of a typical SNS. In fact the photons, which produces the first dip present of the dashed curves. The value of $\gamma$ where $\tau_{\text{coll}}$ and $\tau_{\text{ph}}$ intersect $t_{\text{acc}}$, identifies the Lorentz factor of ejected electrons. From the upper panel we see that even electrons from He$^+$ can be ejected with $\gamma > 10$ and can easily undergo DSA.

![FIG. 1: Comparison between acceleration (solid lines), photoionization (dashed) and collisional ionization time (dot-dashed) as functions of the ion’s Lorentz factor. The top and lower panels show the results for the hydrogen-like ions He$^+$ and C$^{5+}$, respectively. For each time two curves are shown, representing two different set of parameters, as explained in the text.](image-url)
In order to get the electron spectrum, $f_e(p)$, we need to solve first the transport equations for all partially ionized species which take part in the acceleration process and release electrons. Then we can use the ions distribution functions as source terms for the electron transport equation. For the sake of simplicity let us consider the simplest case where the acceleration involves only one hydrogen-like species which inject one electron per atom, $N^+ \rightarrow N^{++} + e^-$, as could be the case for He$^+$. The acceleration of all the three components can be described using the well known transport equation [10] but adding a “decay term” to take into account the ionization process, i.e.:

$$\frac{\partial f_i}{\partial x} = D(p) \frac{\partial^2 f_i}{\partial x^2} + \frac{1}{3} \frac{du_i}{dp} \frac{\partial f_i}{\partial p} + Q_i - S_i, \quad (9)$$

where the index $i = e, N^+, N^{++}$ identifies the species. $Q_i$ is the source term while $S_i$ is the “decay term” due to the ionization. We assume that the injection of ions $N^+$ occurs only at the shock position and at a fixed momentum $p_{nj}$, hence $Q_{N^+}(x,p) = K \delta(p-p_{nj}) \delta(x)$, where the normalization constant $K$ is determined by the total number of ions injected per time unit. The decay term is $S_{N^+} = f_{N^+}(x,p)/\tau_{ion}(p)$, where the total ionization time is $\tau_{ion} = (\tau_{coll} + \tau_{pl})^{-1}$. For electrons and $N^{++}$ the decay terms vanish while the injection terms can be approximated as follows:

$$Q_i(x,p) = \int_p^\infty \delta(p') \frac{f_{N^+}(x,p')}{\tau_{ion}(p')} \delta^3(p - \xi_i p'), \quad i = e, N^{++}. \quad (10)$$

Because both $N^{++}$ and $e$ move approximately with the same Lorentz factor of $N^+$, we can set $\xi_e = 1$ for $N^{++}$ and $\xi_e = m_e/m_N$ for electrons. In the case of linear shock acceleration theory, Eq. (10) can be solved using standard techniques [10] and we will show the detailed procedure in a future paper. We define $p_0$ as the momentum value where the ionization time for $N^+$ is comparable to its diffusion time $\sqrt{4D/\xi_i^2}$. It is easy to show that $f_{N^+}(p_N)$ and $f_e(p_e)$ both become a power law $\propto p^{-s}$ for $p_N > p_0$ and $p_e > p_0 \propto \frac{m_e}{m_N}$, respectively. The index $s$ is only a function of the compression factor, $s = 3r/(r - 1)$, and $s \rightarrow 4$ when the shock is strong. In the limit $p \gg p_0$ the ratio between electrons and ions has the following expression:

$$K_{eN} \equiv \lim_{p \gg p_0} \frac{f_e(p)}{f_{N^{++}}(p)} = \frac{Z}{2Z - 1} \left( \frac{m_e}{m_N} \right)^{s - 3}. \quad (11)$$

Here the factor $Z/(2Z - 1)$ is due to the different diffusion coefficient for electrons and ions, while the ratio $m_e/m_N$ is due to the different momentum they have when the ionization occurs. In order to give an approximated estimate for $K_{eN}$, we need to multiply Eq. (11) by the total number of ejected electrons, i.e. $(Z - Z_{eff})$, and sum over all atomic species present in the accelerator, i.e. $K_{eN} \approx \sum N K_{Np} (Z_N - Z_{N,eff}) K_{eN}$, where $K_{Np}$ are the abundances of ions measured at the source in the range of energy where the ionization occurs. Even if the values of $K_{Np}$ are widely unknown, we can estimate it using the abundances measured at Earth and adding a correction factor to compensate for propagation effects, namely the fact that particles with different $Z$ diffuse in a different way. The diffusion time in the Galaxy is usually assumed to be $\tau_{diff} \propto (p/Z)^{-\delta}$, with $\delta \approx 0.3 - 0.6$ (see [10] for a review on recent CR experiments). If $K_{Np,0}$ is the ion/proton ratio measured at Earth, than the same quantity measured at the source is $K_{Np} = K_{Np,0} Z_N^{-\delta}$. Hence the final expression for the electron/proton ratio at the source is: $K_{eN} \approx \sum N K_{Np,0} Z_N^{-\delta} (Z_N - Z_{N,eff}) K_{eN}$.

Using the abundances of nuclei measured at 1 TeV [17] and assuming that $Z/2$ is the number of electrons effectively injected by each species, we have $K_{eN} \sim 10^{-4}$. Remarkably this number is exactly the order of magnitude required to explain the X-ray emission in the context of efficient CR acceleration. We note that the present estimate is based on linear shock acceleration theory, while a correct treatment requires the inclusion of non linear effects.

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\begin{thebibliography}{10}
\bibitem{1} S. P. Reynolds, Ann. Rev. Ast. Astrophy. \textbf{46}, 89 (2008)
\bibitem{2} H. Kang, T. W. Jones and U. D. J. Gieseler, ApJ \textbf{579}, 337 (2002)
\bibitem{3} P. Blasi, S. Gabici and G. Vannoni, MNRAS \textbf{361}, 907 (2005)
\bibitem{4} G. Morlino, E. Amato and P. Blasi, MNRAS \textbf{392}, 240 (2009)
\bibitem{5} A. A. Galeev, Sov. Phys., JETP \textbf{59}, 965 (1984)
\bibitem{6} A. Levinson, MNRAS \textbf{278}, 1018 (1996)
\bibitem{7} T. Amano and M. Hoshino, ApJ \textbf{661}, 190 (2007)
\bibitem{8} M. G. Baring and E. J. Summerlin, Ap&SS \textbf{307}, 165 (2007)
\bibitem{9} J. Solierman et al., A&A \textbf{407}, 249 (2003)
\bibitem{10} L. O. Drury, Rep. Prog. Phys. \textbf{46}, 973 (1983)
\bibitem{11} M. Inokuti, Rev. Mod. Phys. \textbf{43}, 297 (1971)
\bibitem{12} W. Heitler, \textit{The Quantum Theory of Radiation}, Oxford University Press, London (1954)
\bibitem{13} T. A. Porter and A. W. Strong, proc. 29th ICRC \textbf{4}, 77 (2005)
\bibitem{14} F. Aharonian et al. [HESS Coll.], A&A \textbf{449}, 223 (2006)
\bibitem{15} E. G. Berezhko, L. T. Ksenofontov and H. J. Völk, A&A \textbf{395}, 943 (2002)
\bibitem{16} P. Blasi, arXiv:0801.4534, rapporteur Paper - OG1 session of the 30th ICRC, Merida, Mexico (2007).
\bibitem{17} B. Wiebel-Sooth, P. L. Biermann and H. Meyer, A&A \textbf{330}, 389 (1998)
\end{thebibliography}