Investigation into position deviation effect on micro newton force sensor

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Abstract. The position deviation effect on force sensor with a resolution of micro newton scale was investigated by theory model analysis and numerical simulation method. The finite element method and theoretical model were used to analyse the influence of the electrostatic micro force sensors when the electrode position of the cylindrical capacitor changed. Relationship between the sensor output micro force and the position deviation of the internal electrode in the cylindrical capacitance was obtained. It was found that different position deviation form had different degrees of influence on the sensor’s output result and the regions with little effect by position deviation were existed. For translation along Z-axis, the region was [-4 mm, 14 mm]. For translation along r-axis, the region was [-0.2 mm, 0.2 mm]. And for rotation around r-axis, the region was [-1°, 1°]. The corresponding maximum errors of the micro force output were 1.49%, 2.09% and 2.20%, respectively.

1. Introduction
With the fast advances of nanotechnology, the micro force measurement sensors are very important in various fields of science and technology [1~3]. In aerospace technology, the accurate and reliable micro force measurement can provide the microsatellites for its maneuvers of station keeping, orbital corrections, and attitude control [4,5]. In biotechnology, the biomechanical research and single-cell operation require accurate measurement and control of micro forces[6,7]. In material science, the micro force measurement sensors are used for determining the elastic modulus and nano-hardness of material [8,9]. In fundamental science, AFM (atomic force microscope) and Casimir force measurement are both related to the force measurement sensor with a resolution of micro newton scale [10,11]. Researchers found it was a good method to reproduce and measure micro forces by using the micro electrostatic force between capacitor plates. NIST (National Institute of Standards and Technology) developed an electrostatic balance, using cylindrical capacitors as micro force sensors [12,13]. NPL (National Physical Laboratory) had also proposed a micro force sensor device by using the electrostatic force balance between cylindrical capacitors [14]. The AFM calibration device researched by KRISS(Korea Research Institute of Standars and Science) [15,16] and the micro force measurement and value traceability device developed in NIM (National Institute of Metrology, China) [17] were all used the cylindrical capacitors as the micro force sensor.

The position deviation of cylindrical capacitors is one of limits for micro force sensor accuracy. However, very few research was focus on the position deviation effect on force measurement sensor. In
present work, theoretical and numerically simulated analysis on the position deviation effect on force measurement sensor had been carried out. Relationship between the micro force sensor output and the position deviation of internal and external electrodes of the cylindrical capacitance was investigated. The results is important for the manufacture and application of cylindrical capacitors in micro force sensors.

2. The basic theory of the cylindrical capacitance micro force measurement sensor
The structure of a cylindrical capacitor in the micro force measurement sensor is shown in Fig.1. When the sensor device is working, the external electrode is fixed, and the internal electrode can move freely along the axis direction. When the internal electrode moves, the internal and external electrode overlap length $Z$ changes, leading to the capacitance of the cylindrical capacitor changed. If the fringe effect is ignored, the capacitance of the cylindrical capacitor can be obtained:

$$ C = \frac{2\pi\epsilon Z}{\ln(D_i/d_o)} $$

where $D_i$ is the inner diameter of the external electrode; $d_o$ is the outer diameter of the internal electrode; $Z$ is the internal and external electrode overlap length; $\epsilon$ is the permittivity of the air between the internal and external electrode.

![Figure 1. Cylindrical capacitor in the micro force sensor](image)

When a constant voltage is maintained between the internal and external electrodes, the displacement $dZ$ of the internal electrode movement needs to do work:

$$ dW = F \cdot dZ = \frac{1}{2} U^2 dC $$

where $dW$ is the energy change; $F$ is the micro force acting on the internal electrode; $dZ$ is the relative position change of the internal and external electrode of the cylindrical capacitor; and $U$ is the potential difference between the internal and external electrode.

The force measurement sensor output value by theoretical model can be obtained:

$$ F = \frac{1}{2} U^2 \frac{dC}{dZ} = \frac{\pi \epsilon}{\ln(D_i/d_o)} U^2 $$

During the analysis of position deviation effect on micro force measurement sensor, the external electrode is fixed and the internal electrode is offset from the ideal location that is no position deviation. Figure 1 shows that when the external electrode is fixed, there are three ways about the positional deviation of the internal electrode, including translation along $Z$-axis (axial), along $r$-axis (radial) and rotation around $r$-axis.

3. Results and discussion

3.1. Internal electrode position deviation in $Z$-axis (axial)
Figure 2. Force sensor outputs and errors with internal electrode translation along the Z-axis

When the internal electrode moves along Z-axis in the external electrode, the output of micro force sensor is shown in Fig.2(a). The theoretical model predicts a constant micro force of 2.08 μN with the fringe effect ignored, which lies just above the data predicted by finite element method (FEM) in region I [-4 mm, 14 mm]. The range of micro force predicted by FEM is from 2.00 μN to 2.05 μN in region I, the max difference of 3.8% from the theoretical model. These results show that the data trend predicted by FEM matches the theoretical model well in region I that the fringe effect can be ignored. The internal electrode is controlled in region I during the sensor is working, in order to avoid the fringe effect on the sensor output. In this region, the micro force sensor output is stable. Fig.2(b) is shown the error of the sensor output when the internal electrode position deviation in Z-axis by FEM. The position that the internal electrode is in the middle of the external electrode in Z-axis (shown in Fig.1) was set as the ideal position without deviation, and the sensor force output at this position is the standard value. Fig.2(b) shows that the maximum error does not exceed 1.49% by FEM in region I. This result indicated that the internal electrode position deviation in Z direction has little effect on the micro force sensor output at this region.

3.2. Internal electrode position deviation in r-axis (radial)

The micro force sensor with cylindrical capacitor has a symmetrical structure. When the internal electrode is in an ideal location (that is the internal and external electrodes are coaxial), the internal electrode is only acted by the micro forces in axial direction. In the radial direction, due to the symmetry of the cylindrical capacitor, the micro force acting on the internal electrode is zero.

When the internal electrode is moving along the radial direction, the positions of the internal and external electrodes are no longer symmetrical about the Z axis. The internal electrodes will be acted by the micro electrostatic forces in the radial direction. However, the micro force measurement sensor with cylindrical capacitor only used the micro force in Z-axis (axial) direction, only force in this direction is considered in the paper.

Figure 3. Force sensor outputs and errors with internal electrode translation along the r-axis
Figure 3 shows the output characteristics of the sensor when the internal electrode is translating in the $r$-axis direction. Due to the symmetrical structure of the sensor’s cylindrical capacitor, when the internal electrode moves along the positive and negative directions of the $r$-axis, its output characteristics are symmetrically distributed with $r = 0$ mm. With the position deviation of internal electrode increased, the sensor’s output micro force increased rapidly. When the position deviation is limited to the region II (-0.2 mm, +0.2 mm), the output variation is small. It can be seen in Figure 3b that when the radial position deviation is controlled in region II, the micro force error of the sensor output is within 2.09%.

### 3.3. Internal electrode rotation around $r$-axis (radial)

![Figure 4](image)

Figure 4. Force sensor outputs and errors with internal electrode rotation around the $r$-axis

Besides the sensor’s internal electrode position deviation in $Z$-axis and in $r$-axis, the rotation around $r$-axis could occur during the sensor assembly. This position deviation would also affect the micro force output of the sensor. Fig. 4(a) shows the micro force output with the internal electrode rotating around $r$-axis. Fig. 4(b) is the output errors relative to the ideal position. The results show that the internal electrode rotation around $r$-axis has great impact on the sensor’s output. The error of micro force output could exceed 2.2% even if the internal electrode rotation angle is only 1°. The output micro force error would rapidly increase to 12.9%, when the internal electrode position deviation exceeds 2.5°. Therefore, position deviation with rotation around the $r$-axis should be strictly controlled as much as possible to reduce the output error of the sensor.

Only by limiting the position deviation within region III [-1°, +1°], the accuracy of the sensor output could be effectively guaranteed.

| position deviation form | region              | Max error |
|-------------------------|---------------------|-----------|
| translation along $Z$-axis | [-4 mm, 14 mm]    | 1.49%     |
| translation along $r$-axis | [-0.2 mm, 0.2 mm] | 2.09%     |
| rotation around $r$-axis | [-1°, 1°]          | 2.20%     |

Table 1 shows the regions with letter effect by the internal electrode position deviation. This table also indicates the error limitation required of the internal electrode during the micro force sensor manufacturing process.

### 4. Conclusions

Investigation into position deviation effect on force measurement sensor with a resolution of micro newton scale was carried out. The positional deviation of the internal electrode, including translation along $Z$-axis, along $r$-axis and rotation around $r$-axis were analyzed separately. The following conclusions were obtained:

1. The position deviation of the internal electrode in different directions would affect the output of the micro force sensor.
(2) Different position deviation form had different degrees of influence on the sensor’s output result. When the position deviation was translation along $Z$ axis, there was little effect on the sensor’s output. The internal electrode translate along or rotate around $r$ axis had great influence on the output of the sensor.

(3) The regions with little effect of position deviation had been found in every position deviation form, and there were $[-4 \text{ mm}, 14 \text{ mm}], [-0.2 \text{ mm}, 0.2 \text{ mm}]$ and $[-1^\circ, 1^\circ]$.

(4) When position deviation could be restrict into these regions, the maximum errors of the micro force output were $1.49\%$, $2.09\%$ and $2.20\%$, respectively.

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References
[1] Ma C Z, Du J S, Liu Y Y, et al. Overview of micro-force sensing methods[C]//Applied Mechanics and Materials. Trans Tech Publications, 2014, 462: 25-31.
[2] Wei Y, Xu Q. An overview of micro-force sensing techniques[J]. Sensors and Actuators A: Physical, 2015, 234: 359-374.
[3] Lay A, Wang D S, Wisser M D, et al. Upconverting nanoparticles as optical sensors of nano to micro Newton forces[J]. Nano letters, 2017, 17(7): 4172-4177.
[4] Kolbeck J, Porter T E, Keidar M. High Precision Thrust Balance Development at The George Washington[C]. Proceedings of the 35th International Electric Propulsion Conference, Georgia, USA, 2017.
[5] Karadag B, Cho S, Funaki I. Note: Precision balance for sub-milliNewton resolution direct thrust measurement[J]. Review of Scientific Instruments, 2018, 89(8): 086108.
[6] Metzger A, Baltisberger S, Burkhard H R, et al. Weighing cell based on the principle of electromagnetic force compensation with optoelectronic position sensor: U.S. Patent 9,086,315[P]. 2015-7-21.
[7] Panda B, Sidhu M S, Munjal P, et al. Time resolved nano Newton force spectroscopy in air and vacuum using a load cell of ultra micro balance[J]. Review of Scientific Instruments, 2019, 90(4): 043117.
[8] Molaei S. The measurement of Young’s modulus of thin films using secondary laser speckle patterns[J]. Measurement, 2016, 92: 28-33.
[9] Xiang D. Capacitive micro-force sensor as a transfer standard for verification and calibration of nanoindentation instruments[C]. Micro-and Nanotechnology Sensors, Systems, and Applications XI. International Society for Optics and Photonics, 2019, 10982: 1098239.
[10] Pratt J R, Kramar J A, Newell D B, et al. Review of SI traceable force metrology for instrumented indentation and atomic force microscopy. Measurement Science and Technology. 2005, 16(11): 2129-2018.
[11] Mohideen U, Roy A. Precision measurement of the Casimir force from 0.1 to 0.9 $\mu \text{ m}$[J]. Physical Review Letters, 1998, 81(21): 4549.
[12] Shaw G A, Stirling J, Kramar J A, et al. Milligram mass metrology using an electrostatic force balance[J]. Metrologia, 2016, 53(5): A86.
[13] Shaw G A. Current state of the art in small mass and force metrology within the international system of units[J]. Measurement Science and Technology, 2018, 29(7): 072001.
[14] Leach R, Chetwynd D, Blunt L, et al. Recent advances in traceable nanoscale dimension and force metrology in the UK[J]. Measurement Science and Technology, 2006, 17(3): 467.
[15] M. S. Kim, I. M. Choi, Y. K. Park, et al. Atomic force microscope probe calibration by use of a commercial precision balance. Measurement. 2007, 40(7): 741–745.
[16] Kim M S, Pratt J R, Brand U, et al. Report on the first international comparison of small force facilities: a pilot study at the micronewton level[J]. Metrologia, 2011, 49(1): 70.

[17] Hu G., Song L, Meng F, et al. Research and Development of Small Force Standards at NIM [C]. International Journal of Modern Physics: Conference Series. World Scientific Publishing Company, 2013, 24: 1360020.