Enhancement of the $\bar{\nu}_e$ flux from astrophysical sources by two photon annihilation interactions

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(Dated: March 20, 2022)

The ratio of anti-electron to total neutrino flux, $\Phi_{\bar{\nu}_e} : \Phi_{\nu}$, expected from $p\gamma$ interactions in astrophysical sources is $\leq 1 : 15$. We point out that this ratio is enhanced by the decay of $\mu^+\mu^-$ pairs, created by the annihilation of secondary high energy photons from the decay of the neutral pions produced in $p\gamma$ interactions. We show that, under certain conditions, the $\Phi_{\bar{\nu}_e} : \Phi_{\nu}$ ratio may be significantly enhanced in gamma-ray burst (GRB) fireballs, and that detection at the Glashow resonance of $\bar{\nu}_e$ in kilometer scale neutrino detectors may constrain GRB fireball model parameters, such as the magnetic field and energy dissipation radius.

PACS numbers: 96.40.Tv, 14.60.Pq, 98.70.Rz, 98.70.Sa

I. INTRODUCTION

High energy neutrinos are expected to be produced in astrophysical sources mainly by $p\gamma$ interactions, leading to the production and subsequent decay of charged pions: $\pi^+ \to e^+ \nu_e\bar{\nu}_e\nu_\gamma$ (see, e.g., [1] for recent reviews). Neutrino oscillations lead in this case to an observed ratio of $\bar{\nu}_e$ flux to the total $\nu$ flux of $\simeq 1 : 15$ (or lower, in case muons suffer significant electromagnetic energy loss prior to decay [3]). For neutrinos produced in inelastic $pp$ ($pn$) nuclear collisions, where both $\pi^+$’s and $\pi^-$’s are produced, the ratio is $\simeq 1 : 6$, and it was suggested that measurements of the $\nu_e$ to $\bar{\nu}_e$ flux ratio at the $W$-resonance may allow one to probe the physics of the sources by discriminating between the two primary modes of pion production, $p\gamma$ and $pp$ collisions [2]. This test for discriminating between the two mechanisms is complicated by the fact that the ratio of $\Phi_{\bar{\nu}_e}$ to $\Phi_\nu$ produced in $p\gamma$ interactions can be enhanced to a value similar to that due to inelastic nuclear collisions in sources where the optical depth to $p\gamma$ interactions is large (e.g. [3]). In this case, neutrons produced in $p\gamma \to n\pi^+$ interactions are likely to interact with photons and produce $\pi^-$ before escaping the source, leading to production of roughly equal numbers of $\pi^+$’s and $\pi^-$’s.

In this paper, we point out that the $\Phi_{\bar{\nu}_e} : \Phi_\nu$ ratio from $p\gamma$ interactions may be enhanced above $1 : 15$ also in sources with small $p\gamma$ optical depth. Neutral pions, which are created at roughly the same rate as charged pions in $p\gamma$ interactions, decay to produce high energy $\gamma$-rays. These $\gamma$-rays typically carry $\sim 10\%$ of the initial proton energy, and may therefore interact with the low energy photons (with which the protons interact to produce pions) to produce $\mu^+\mu^-$ pairs. The decay of muons yields (after vacuum oscillations) $\Phi_{\bar{\nu}_e} : \Phi_\nu \simeq 1 : 5$, thus enhancing the $\bar{\nu}_e$ fraction.

We discuss below a specific example, the widely considered fireball model of GRBs. In this model, the observed $\gamma$-rays are produced by synchrotron radiation of shock accelerated electrons in the magnetic field which is assumed to be a fraction of the total energy (see [5] e.g. for reviews). The protons are expected to co-accelerate with electrons to ultra-high energy [6], and produce high energy neutrinos by $p\gamma$ interactions [7]. We calculate below the additional neutrino flux, due to the decay of muons produced by secondary photon annihilation, for a typical long duration GRB, and show that the enhanced $\bar{\nu}_e$ flux may be detectable at the Glashow resonance ($\bar{\nu}_e e \to W^- \to$ anything [6]) in kilometer scale neutrino detectors such as IceCube [6].

The enhancement of $\Phi_{\bar{\nu}_e} : \Phi_\nu$ due to $\gamma\gamma$ interactions in $p\gamma$ sources makes the discrimination between $p\gamma$ and $pp$ neutrino sources more difficult. On the other hand, it may provide a new handle on the physics of the source. We show below that for GRBs the enhancement of $\bar{\nu}_e$ flux depends on model parameters which are poorly constrained by observations, namely the magnetic field strength and the energy dissipation radius. Detection of $\bar{\nu}_e$’s at the Glashow resonance, in conjunction with $\gamma$-ray detection, may therefore constrain these parameters.

II. FIREBALL MODEL AND PHOTON SPECTRUM

The minimum observed GRB fireball radius $r$ may be estimated by requiring that it is optically thin to Thomson scatterings: $\tau_{Th} = \sigma_{Th}n'r' \lesssim 1$ (denoting the comoving and local lab. frame variables with and without a prime respectively). Here $n'$ is the density of scatterers in the fireball, $r' = r/\Gamma$ is the size of the interaction region and $\Gamma$ is the bulk Lorentz factor. The radius at which $\tau_{Th} \approx 1$ is the photospheric radius $r_{ph}$. For a kinetic luminosity $L_k$ of the fireball, mostly carried by the baryons, the number density of the baryons, and of the leptons which are coupled to the baryons, is $n_b \approx L_k/(4\pi r^2\Gamma^2 m_p c^3)$. The observed isotropic equivalent $\gamma$-ray luminosity of a long duration GRB is $L_{52} = L_\gamma/10^{52}$erg/s $\simeq 1$. Assuming $L_\gamma = \varepsilon_\gamma L_k$ with $\varepsilon_\gamma \sim 0.05\varepsilon_{-1.3}$ (a parametrization which is moti-
vated below), the photospheric radius is
\[ r_{ph} = \frac{\sigma_{Th} L_\gamma / \varepsilon_{e_\gamma}}{4 \pi \Gamma^3 m_e c^3} \approx 7.4 \times 10^{12} \frac{L_{52}}{\varepsilon_{e_\gamma - 1.3} \Gamma_{2.5}^3} \, \text{cm}, \] (1)
for \( \Gamma_{2.5} = \Gamma / 316 \approx 1 \). The radius at which the bulk kinetic energy dissipation occurs, e.g., by internal shocks, is in general \( r \gtrsim r_{ph} \).

The \( \gamma \)-ray spectrum of a GRB fireball at a dissipation radius \( r = 10^{14} \, \text{cm} \) peaks at a typical energy
\[ \epsilon_{\gamma, pk} = \frac{\hbar c \Gamma^2 (3 \gamma_{e_{\text{min}}}, B' / (2m_e c^2))}{\sim 500 (\varepsilon_{e_\gamma - 1.3} \varepsilon_{B_{-1}} \varepsilon_{L_{2.5}^2 / r_{14}^2})^{1/2} \, \text{keV}, \] (2)
due to synchrotron radiation by electrons with a Lorentz factor \( \gamma_{e_{\text{min}}} \approx \varepsilon_{e}(m_p / m_e) \). Here, \( \gamma_{e_{\text{min}}} \) is at the lower end of a \( \propto 1/\gamma^6 \) distribution of electron Lorentz factor, with \( p \geq 2 \), created by Fermi acceleration in the shock. The magnetic field is assumed to be \( B^2 / 8 \pi \approx \varepsilon_{B} L_k / (4 \pi r^2 c) \), where \( \varepsilon_{B} \approx 0.1 \varepsilon_{B_{-1}} \) is the equipartition value, currently unconstrained in the GRB prompt phase. Note that \( \epsilon_{\gamma, pk} \propto r^{-1} \) will be larger than the above value for \( r \gtrsim r_{ph} \), with other parameters fixed.

For a GRB at a luminosity distance \( d_L \) the observed \( \gamma \)-ray spectrum is generally approximated with a broken power-law Band fit \[ 11],
\[ \frac{dN_{\gamma}}{d\epsilon_{\gamma}} \approx \frac{L_\gamma}{4 \pi d_L^2 \epsilon_{\gamma, pk}^2} \left( \frac{\epsilon_{\gamma} - \epsilon_{\gamma, pk}}{\epsilon_{\gamma, sa} - \epsilon_{\gamma, pk}} \right); \epsilon_{\gamma} < \epsilon_{\gamma, pk} \]
\[ \frac{dN_{\gamma}}{d\epsilon_{\gamma}} \approx \frac{L_\gamma}{4 \pi d_L^2 \epsilon_{\gamma, sa}^2} \left( \frac{\epsilon_{\gamma} - \epsilon_{\gamma, sa}}{\epsilon_{\gamma, pk} - \epsilon_{\gamma, sa}} \right); \epsilon_{\gamma} > \epsilon_{\gamma, sa}. \] (3)
The spectrum deviates from this at low energy, becoming \( dN_{\gamma} / d\epsilon_{\gamma} \propto \epsilon_{\gamma}^{3/2} \) for \( \epsilon_{\gamma} \ll \epsilon_{\gamma, sa} \), the energy below which synchrotron self-absorption becomes dominant. Theoretical modeling indicates a value \[ 11
\[ \epsilon_{\gamma, sa} = 2.4 \left( \Gamma^2 \gamma_{e_{\text{min}}} n_{0} \epsilon_{\gamma} \hbar c B^2 / [m_e c^6] \right)^{1/3} \]
\[ \sim 8 \left( \varepsilon_{B_{-1}} \varepsilon_{L_{2.5}^2 / \Gamma_{2.5}^3} \right)^{1/3} \, \text{keV}, \] (4)
for \( p = 2 \). The differential number density of photons is
\[ \frac{dN_{\gamma}}{d\epsilon_{\gamma}} \approx \frac{L_\gamma}{4 \pi d_L^2 \epsilon_{\gamma, pk}^2} \left( \epsilon_{\gamma, sa} / \epsilon_{\gamma, pk} \right)^{-1} \left( \epsilon_{\gamma, pk} / \epsilon_{\gamma, sa} \right)^{3/2}; \epsilon_{\gamma} < \epsilon_{\gamma, sa} \]
\[ \times \left( \epsilon_{\gamma, pk} / \epsilon_{\gamma, sa} \right)^{2}; \epsilon_{\gamma, pk} > \epsilon_{\gamma, sa} \] (5)

Electron synchrotron radiation produces a power law \( \gamma \)-ray spectrum at energies above \( \epsilon_{\gamma, pk} \) [see Eq. 4] which depends on the maximum Lorentz factor. Other mechanisms can contribute to an extension of the \( \gamma \)-ray spectrum in Eq. 4 to high energies. High energy electrons can inverse Compton scatter synchrotron photons up to an energy similar to the maximum shock accelerated electron energy (which we derive shortly) in the Klein-Nishina limit in one mechanism. Here we consider ultra-high energy \( \gamma \)-rays from \( \pi^0 \) decays which are produced by \( p\gamma \) interactions of shock accelerated protons with synchrotron photons as \( p\gamma \rightarrow \Delta^+ \rightarrow p\pi^0 \rightarrow p\gamma\gamma \).

The maximum proton energy is calculated by equating its acceleration time \( t_{acc}' \approx \epsilon_{p}' / (qB') \) to the shorter of the dynamic time \( t_{dyn}' \approx r / (2 \Gamma c) \) and the synchrotron cooling time \( t_{syn}' \approx 6 \pi m_e c^3 / (\sigma_{Th} m_e^2 \epsilon_{p}' B'^2) \) as
\[ \epsilon_{p, max}' = \frac{(6 \pi m_e^4 q^2 \Gamma^2 / [\sigma_{Th} m_e^2 B']^{1/2}}{2} \approx 3.3 \times 10^{11} \left( \epsilon_{e_{\gamma} - 1.3} \Gamma_{2.5}^2 L_{2.5}^2 / [\epsilon_{B_{-1}} L_{52}]^{1/4} \right) \text{GeV}, \] (6)
respectively for \( t_{acc}' = t_{syn}' \) and \( t_{acc}' = t_{dyn}' \). For electrons, \( t_{dyn}' \gg t_{syn}' \) typically and the maximum electron energy, using Eq. 4, for electrons is \( \epsilon_{e, max}' \approx 10^{4} \epsilon_{e_{\gamma} - 1.3} \Gamma_{2.5}^2 L_{52}^{-1/4} \varepsilon_{B_{-1}}^{-1/4} \varepsilon_{L_{2.5}^2}^{-1/2} \varepsilon_{14}^{-1/2} \) GeV.

At a given incident proton energy \( \epsilon_{p} \), the threshold photon energy leading to a \( \Delta^+ \) resonance interaction is
\[ \epsilon_{\gamma, \Delta^+} \approx \frac{0.3 \Gamma^2}{(\epsilon_{p} / \text{GeV})} \, \text{GeV}. \] (7)
The optical depth for this interaction may be calculated using a delta-function approximation with a cross-section \( \sigma_{\pi^0} \approx 10^{-28} \, \text{cm}^2 \) as
\[ \tau_{\gamma p}^{'(\epsilon_{p}')} = \sigma_{\gamma p} r \left( \frac{dN_{\gamma, \Delta^+}}{d\epsilon_{\gamma, \Delta^+}} \right) d\epsilon_{\gamma, \Delta^+} \] (8)
using Eq. 6. Note that the target photon spectrum, within parenthesis, is now evaluated at the \( \Delta^+ \) resonance energy [see Eq. 7] for an incident proton energy \( \epsilon_{p}' \). In particular, Eq. 6 may be used to replace \( \epsilon_{\gamma, \Delta^+} \) by \( \epsilon_{p}' \) in Eq. 6. As a result, the optical depth in Eq. 8 is expressed as a function of \( \epsilon_{p}' \). The spectral shape of the optical depth is then \( \propto (\epsilon_{p}')^{q-1} \), where \( q \) is the spectral index, \( dN_{\gamma}/d\epsilon_{\gamma} \propto (\epsilon_{\gamma})^{-q} \), in Eq. 6. Also the order is reversed, i.e., \( q = 2, 1, -3/2 \) in \( \tau_{\gamma p}^{'(\epsilon_{p}')} \propto (\epsilon_{p}')^{q-1} \) instead of \( q = -3/2, 1, 2 \) in \( dN_{\gamma}/d\epsilon_{\gamma} \propto (\epsilon_{\gamma})^{-q} \), according to the condition in Eq. 6.

Protons lose \( \approx 20\% \) of their energy by \( p\gamma \) interactions to \( \pi^0 \) and \( \epsilon_{\gamma, \Delta^+} \approx 0.1 \epsilon_{p} \) for each secondary photon. With an equal probability to produce \( \pi^0 \) and \( \pi^+ \) in each \( p\gamma \) interaction, the resulting \( \pi^0 \) decay photon flux is
\[ \frac{dN_{\gamma}}{d\epsilon_{\gamma}} = \min(1, \epsilon_{\gamma}/\epsilon_{p}) \left( \frac{0.2 (\epsilon_{\gamma}/\epsilon_{p}) L_{\gamma}}{4 \pi d_L^2 \epsilon_{\gamma}^2} \right). \] (9)
Here \( \epsilon_{p} \) is the proton fraction undergoing shock acceleration. For \( \epsilon_{p} = 1 \) and \( \epsilon_{\gamma} = 0.05 \) we have \( \epsilon_{\gamma}/\epsilon_{p} \approx 1 \) which leads to the observed flux level in Eq. 9. Secondary pions from \( \Delta^+ \) decay, and subsequent decay photons and neutrinos follow the \( dN_{\gamma}/d\epsilon_{\gamma} \propto \epsilon_{\gamma}^{-q} \) spectral shape of the protons for a constant optical depth. For an optical depth of spectral shape \( \propto \epsilon_{\gamma}^{-1} \), the resulting pion, photon and...
neutrino spectra would be \( dN / dc \propto \epsilon^{-1-p} \). The \( \pi^0 \) decay photon spectrum in Eq. (10) is then \( dN_{\gamma} / d\epsilon \propto \epsilon^{-2} \) between \( \epsilon = 0.03 \Gamma / \epsilon_{\gamma, \text{pk}} \text{GeV}^2 \) and \( 0.03 \Gamma^2 / \epsilon_{\gamma, \text{sa}} \text{GeV}^2 \), \( \propto \epsilon^{-1} \) below \( \epsilon = 0.03 \Gamma^2 / \epsilon_{\gamma, \text{pk}} \text{GeV}^2 \) and \( \propto \epsilon^{-9/2} \) above \( \epsilon = 0.03 \Gamma^2 / \epsilon_{\gamma, \text{sa}} \text{GeV}^2 \) due to self-absorption following Eqs. (3) and (5).

Note that, the luminosity of shock-accelerated protons is \( 1 / \epsilon_e = 20 \) times the shock-accelerated electron luminosity. In the fast cooling scenario, valid in the GRB internal shocks, the electrons synchrotron radiate all their energy into observed \( \gamma \)-rays. Thus \( L_{\gamma} \approx L_{\gamma} / \epsilon_e \). In a single \( p \gamma \) interaction the secondary pion (charged or neutral) luminosity is \( L_{\pi} \approx 0.2 L_{\gamma} \approx 4 L_{\gamma} / \epsilon_{\pi, -1.3} \). The neutrino luminosity of all flavors from charged pion decay, assuming equal energy for all 4 final leptons, is \( L_{\nu} \approx (1/2)(3/4)L_{\pi} \approx 1.5 L_{\gamma} / \epsilon_{\pi, -1.3} \). The 1/2 factor arises from the equal probability of \( \pi^0 \) and \( \pi^+ \) production. The neutrinos carry away energy from the fireball, and the rest of the pion decay (\( e^+ \) from \( \pi^+ \) and \( \gamma \gamma \) from \( \pi^0 \)) energy is electromagnetic (e.m.), with a luminosity \( L_{e, \text{m}} \approx L_{\pi} - L_{\nu} \approx 2.5 L_{\gamma} / \epsilon_{\pi, -1.3} \). A significant fraction of the \( \pi^0 \)-decay \( L_{\gamma} \) [Eq. (9)] would be converted to muon pairs and subsequently to neutrinos, as we discuss next. A substantial (small) fraction of the rest of \( L_{e, \text{m}} \) would be emitted at \( r > r_{\text{ph}} \) (\( r \approx r_{\text{ph}} \)) as low energy photons with a luminosity not significantly above the observed \( \gamma \)-ray luminosity. These, however, do not affect substantially the neutrino flux calculated from very high energy \( \gamma \)-rays interacting with soft photons.

### III. TWO PHOTON PAIR PRODUCTION

High energy \( \gamma \)-rays can produce lepton pairs, \( l^+l^- \) (\( l = e, \mu \)), with other photons which are above a threshold energy \( \omega_{\text{th}} = m \mu c^2 \) in the center of mass (c.m.) frame of interaction. For an incident (target) photon of energy \( \epsilon_{\gamma, l} (\epsilon_{\nu, l}) \) in the comoving GRB fireball frame, \( \omega = (2 \epsilon_{\gamma, l} \epsilon_{\nu, l} / \epsilon_{\gamma, l}^2)^{1/2} \), and the cross-section for \( l^+l^- \) pair production may be written, ignoring the logarithmic rise factors at high energy, as \( \sigma_{\gamma l l l} \propto \pi \epsilon_{\gamma, l}^2 (m \mu c^2 / \omega)^2 \), where \( r_e \) is the classical electron radius. The corresponding optical depth is

\[
\tau_{\gamma \gamma}(\epsilon_{\gamma, l}) = \frac{r}{\Gamma} \int \sigma_{\gamma l l l}(\epsilon_{\gamma, l}; \epsilon_{\nu, l}) \frac{dN_{\gamma, l}}{d\epsilon_{\gamma, l}} d\epsilon_{\nu, l}.
\]

Given the power-law dependence of the photon distribution in Eq. (5), we may calculate the \( l^+l^- \) pair production opacities by integrating Eq. (10) piecewise as

\[
\tau_{\gamma\gamma \rightarrow l^+l^-}(\epsilon_{\gamma}) = r_{\gamma}^2 m \mu c^2 \frac{L_{\gamma}}{(8 \pi r_{\text{dec}}^2)} \epsilon_{\gamma} \left\{ \frac{1}{r_{\gamma}^2} - \frac{1}{r_{\text{th}}^2} \right\} \left[ \frac{\epsilon_{\text{pk}}^2}{\epsilon_{\gamma}} - \frac{1}{r_{\gamma}^2} \right] - \frac{1}{r_{\gamma}^2} \frac{\epsilon_{\gamma}}{r_{\gamma}^2} + \frac{1}{r_{\text{th}}^2} \frac{\epsilon_{\gamma}}{r_{\text{th}}^2} - \frac{1}{r_{\gamma}^2} \frac{\epsilon_{\gamma}}{r_{\gamma}^2}.
\]

(11)

Here we defined the threshold energy for lepton pair production as \( \epsilon_{\text{th}} = \epsilon_{\gamma}^2 c^2 / 2 \gamma \). Note that the high energy photons produce \( e^+e^- \) pairs dominantly at lower energy. The ratio of the two opacities \( \kappa = \tau_{\gamma\gamma \rightarrow l^+l^-} / \tau_{\gamma\gamma \rightarrow \nu\nu} \) becomes unity for higher energy photons, since the cross-section is the same for \( \mu^+\mu^- \) and \( e^+e^- \) pair productions above the muon pair production threshold energy.

#### A. Muon decay neutrino flux

Muon pairs decay to neutrinos as \( \mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \) and \( \mu^+ \rightarrow e^+ \bar{\nu}_e \bar{\nu}_\mu \), shortly after they are created in the c.m. frame of the \( \gamma\gamma \) collision. In the observer’s frame \( \epsilon_\mu \approx \epsilon_\gamma / 2 \), and the particle pairs move radially along the incident photon’s direction. For simplicity we assume that the \( \nu_e \) and \( \nu_\mu \) created from \( \mu^- \) decay carry \( 1/3 \) of the muon energy each. The observed neutrino energies are then \( \epsilon_{\nu} / 6 \) for each flavor. The neutrino source flux, which is the same for \( \nu_e, \nu_\mu, \bar{\nu}_e \) and \( \bar{\nu}_\mu \) previous to any flavor oscillation in vacuum, is

\[
\epsilon_{\nu} \Phi_{\nu, \gamma\gamma} \equiv \epsilon_{\nu} \frac{dN_{\nu}}{d\epsilon_{\gamma}} = \min\{1, \tau_{\gamma\gamma \rightarrow \mu^+\mu^-} / \kappa \epsilon_{\gamma} \frac{dN_{\gamma}}{d\epsilon_{\gamma}} \}.
\]

(12)

The high energy muons produced from \( \gamma\gamma \) interactions may lose a significant fraction of their energy by synchrotron radiation before they decay into neutrinos (with a decay time \( t_{\text{dec}} \)), if their energy is above a break energy

\[
\epsilon_{\mu, \text{sh}} = \left( \frac{6 \pi m_\mu c^3 \Gamma^2 / [t_{\text{dec}} \sigma_{\text{Th}} m_e^2 B']}{\epsilon_{\gamma}^3 \Gamma^2 / [t_{\text{dec}} \sigma_{\text{Th}} m_e^2 B']} \right)^{1/2} \approx 5 \times 10^7 \left( \epsilon_{e, -3} \Gamma_{1} \epsilon_{\gamma, 2} \epsilon_{\mu, 4} \right)^{1/2} \text{GeV}.
\]

(13)

The corresponding neutrino break energy from muon decay is \( \epsilon_{\nu, \text{sh}} = \epsilon_{\mu, \text{sh}} / 6 \). For \( \epsilon_\nu \gg \epsilon_{\nu, \text{sh}} \), the neutrino flux index would steepen by a factor 2 [12].

We have plotted in Fig. 1 the \( \nu_e \) flux at the source, \( \nu_\mu \Phi_{\nu, \gamma\gamma} \) (same for \( \nu_e, \nu_\mu, \bar{\nu}_e \) and \( \bar{\nu}_\mu \)), previous to any vacuum oscillation, arising from \( \gamma \gamma \rightarrow \mu^+\mu^- \) interactions and the associated muon decays, for a GRB of isotropic equivalent luminosity \( L_{\gamma} = 10^{52} \text{erg/s} \), which is average for a long GRB [4]. We assume a redshift of \( z \approx 0.1 \), which is near the low end of observed redshifts; there have been a few spectroscopic redshifts observed.
in the 0.1 – 0.2 in the past 8 years, and indirect redshift measures, such as time lags, indicate many more bursts in this range among the BATSE sample (see, e.g., Ref. 3 and references therein). Also plotted are the $\nu_e$ source flux: $\frac{e^2 \Phi_{\nu_e}}{c} = \min\{1, \tau_{\nu_e}'\}(0.2/8)L_{\gamma}/(4\pi d_{L}^2\epsilon_{e})$, from $p\gamma \rightarrow \Delta^+ \rightarrow n\pi^+$ interactions and subsequent $\pi^+$ and $\mu^+$ decays. Different panels are for different bulk Lorentz factor $\Gamma$ and dissipation radii $r$.

For antineutrinos $P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta}$ is the same as above. Different production mechanisms produce $\nu$ and $\bar{\nu}$ fluxes at the source with different flavor proportions. Their production ratios may be expressed as normalized vectors, shown in the left hand side of Eqs. (15 & 16). The corresponding flux ratios at Earth, using Eq. (14), are shown in the right hand side of Eqs. (15 & 16) below.

\[
p\gamma \rightarrow n\pi^+ \left[ \begin{array}{c} \Phi_{\nu_e}^s \\ \Phi_{\nu_\mu}^s \\ \Phi_{\nu_\tau}^s \end{array} \right] \Rightarrow \left[ \begin{array}{c} 1 \\ 1 \\ 0 \end{array} \right] \left[ \begin{array}{c} 0.8 \\ 0.6 \\ 0.4 \end{array} \right], \quad (15)
\]

\[
\gamma\gamma \rightarrow \mu^+\mu^- \left[ \begin{array}{c} \Phi_{\nu_e}^s \\ \Phi_{\nu_\mu}^s \\ \Phi_{\nu_\tau}^s \end{array} \right] \Rightarrow \left[ \begin{array}{c} 1 \\ 1 \\ 0 \end{array} \right] \left[ \begin{array}{c} 0.8 \\ 0.6 \\ 0.6 \end{array} \right], \quad (16)
\]

The source $\nu$-fluxes plotted in Fig. 4 will be modified accordingly. Note that $\Phi_{\bar{\nu}_e}(\gamma\gamma)/\Phi_{\nu_e}(p\gamma) \approx 4$ for the same initial $\gamma\gamma$ and $p\gamma$ flux levels. The $\bar{\nu}_e$-flux component is 1/5 (1/15) of the total $\nu$-flux from $\gamma\gamma$ ($p\gamma$). The observed ratio of $\bar{\nu}_e$ to total $\nu$ fluxes from both the single $p\gamma$ interactions and $\gamma\gamma$ interactions can be calculated using Eqs. (15 & 16) from the source fluxes as

\[
\Phi_{\bar{\nu}_e} = \Phi_\nu \frac{0.2\Phi_{\nu_e, p\gamma} + 0.8\Phi_{\nu_e, \gamma\gamma}}{3\Phi_{\nu_e, p\gamma} + 4\Phi_{\nu_e, \gamma\gamma}} \quad (17)
\]

We have plotted this ratio in Fig. 4 for different GRB model parameters. The ratio is enhanced from the canonical ($p\gamma$) 1/15 value in the energy range where $\gamma\gamma$ interactions contribute significantly (see Fig. 4). Interestingly, the enhancement takes place over a small energy range which may be explored to learn about the GRB model parameters as we discuss next.

### IV. NEUTRINO DETECTION

We consider here the anti-electron neutrino detection channel at the Glashow resonance energy $\epsilon_{\nu, \text{res}} = m_{\nu}^2 c^2 / 2m_e \approx 6.4$ PeV \( \approx 6 \times 10^{38} \) and the corresponding number of downgoing $\bar{\nu}_e$ events from a point source of flux $\Phi_{\bar{\nu}_e}$ is

\[
N_{\bar{\nu}_e} \approx \Delta t N_{\text{ice}, \text{eff}} \frac{\pi g^2 (hc)^2}{4m_e c^2} \Phi_{\bar{\nu}_e} (\epsilon_{\nu, \text{res}}).
\]

Here $g^2 \approx 0.43$ from the standard model of electro-weak theory, and $\Delta t$ is the duration of the emission.

We have plotted in Fig. 5 the expected number of $\bar{\nu}_e$ events at $\epsilon_{\nu, \text{res}}$ from a GRB fireball for various value of $r$, $\epsilon_B$ and $\Gamma$. We have assumed here that shock accelerated protons interact once with synchrotron photons (two top panels), losing $\approx 20\%$ of their energy. For very high $p\gamma$ opacity the protons can lose most of their energy through
the $p\gamma$ and $n\gamma$ interaction chains. This could lead to the $\nu_e$-fluxes from both $p\gamma$ and $\gamma\gamma$ plotted in Fig. 4. This is for a long solid duration of $L_c = 10^{32}$ erg/s at redshift $z \approx 0.1$, the solid, dashed, dot-dashed and dotted lines indicate magnetic field parameters $\varepsilon_B = 10^{-1}, 10^{-2}, 10^{-3}$ and $10^{-4}$ respectively, for different $\Gamma$ and $r$ combinations. Note that the 1/15 ratio from single $p\gamma \rightarrow n\pi^+$ interactions is enhanced by $\gamma\gamma \rightarrow \mu^+\mu^-$ interactions in certain energy ranges which depend upon the GRB model parameters.

The background for astrophysical $\bar{\nu}_e$ detection is mostly due to atmospheric prompt neutrinos from cosmic-ray generated charm meson decays. A parametrization for the $\nu_e$ and $\bar{\nu}_e$ atmospheric flux is given by

$$\Phi_{\nu_e+\bar{\nu}_e}^{\text{atm}} = \begin{cases} \frac{1.5 \times 10^{-5} \epsilon_{\nu}^{-2.77}}{2.3 \times 10^{-5} \epsilon_{\nu}^{0.02}} & \epsilon_{\nu} < 1.2 \times 10^6 \text{ GeV} \\ \frac{4.9 \times 10^{-4} \epsilon_{\nu}^{-3.2} \epsilon^{-0.2}}{1+1.5 \times 10^{-5} \epsilon_{\nu}} & \epsilon_{\nu} > 1.2 \times 10^6 \text{ GeV} \end{cases} \times 10^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}. \quad (19)$$

The corresponding background $\bar{\nu}_e$-events at the Glashow resonance energy is $\lesssim 10^{-7}$ for a GRB within a $\sim 100$ s time window, allowing the full directional uncertainty (2$\sigma$ sr), given the poor current knowledge of the $\nu_e$ or $\bar{\nu}_e$ signal reconstruction in neutrino Cherenkov detectors, from Eq. 12.

V. DISCUSSION

The results of Fig. 3 and Eqs. 14 & 15 show that $\gamma\gamma$ interactions in astrophysical sources can enhance the observed $\Phi_{\bar{\nu}_e} : \Phi_{\nu}$ flux ratio. A different source of enhancement of the $\bar{\nu}_e$ flux may be $pp$ interactions $[3, 4]$. In GRBs, however, the optical depth to $pp$ is low $[7]$, except in buried jets leading to $\nu$ precursors $[14]$, where $pp$ interactions are expected to lead to $\bar{\nu}$'s at energies $\sim \text{TeV}$. The number of resonant $\bar{\nu}_e$ events arising from $p\gamma$ interactions is essentially independent of $\varepsilon_B$ (for $\varepsilon_B \lesssim 10^{-2}$) for any $\Gamma$. On the other hand, the number of resonant $\bar{\nu}_e$ events arising from $\gamma\gamma$ interactions varies significantly with $\Gamma$ and $r$. It may become as large as the $p\gamma$ contribution for $10^{-2} \lesssim \varepsilon_B \lesssim 10^{-3}$, $100 \lesssim \Gamma \lesssim 300$ and $\tau_{\text{ph}} \lesssim r \lesssim 3r_{\text{ph}}$. For long bursts of average isotropic-equivalent luminosity at a redshift $\sim 0.1$, which from past experience are electromagnetically detected every few years, IceCube could probe the $\bar{\nu}_e$ enhancement, and thus the value of the magnetization parameter and dissipation radius, by measuring the $\Phi_{\bar{\nu}_e} : \Phi_{\nu}$ flux ratio. Finally, we note that a moderate excess of $\nu_e$ events compared to $\nu_\mu$ and $\nu_\tau$ events may also be an indication for the presence of a $\gamma\gamma$ component.

Acknowledgements

Work supported by NSF grant AST0307376. EW is partly supported by Minerva and ISF grants.
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