SEMI-EMPIRICAL FORMULA FOR BINDING ENERGY ON THE BASIS OF DEUTERON CLUSTERIZATION OF THE ATOMIC NUCLEUS

Isayev R.Sh. Email: Isayev653@scientifictext.ru

Isayev Rafael Shahbaz oglu – Master Student,
INSTITUTE OF NUCLEAR PHYSICS AND ENGINEERING,
NATIONAL RESEARCH NUCLEAR UNIVERSITY «MEPHI», MOSCOW

Abstract: this paper presents a mathematical justification for the deuteron clusterization of the atomic nucleus. As a result of the research, a semi-empirical formula for calculating the binding energy is obtained. Comparisons are made between experimental and theoretical results, with the error not exceeding 0.4 MeV. In the case of approximation of the chemical mass defect discrepancies in the results for the specific binding energy between experimental and theoretical values do not exceed 1 MeV. The obtained semi-empirical formula makes it possible to confirm the hypothesis that the neutron has a binding function in the atomic nucleus. Based on this, we can say that the mass defect is the amount of energy that a neutron from the ground state spends on the formation of nuclear forces. Keywords: deuteron clusterization, binding energy, mass defect, chemical mass defect.

1. Introduction.
With the development of the theory of nucleus structure and evolution of various models of the atomic nucleus, attempts were undertaken to create formulas for calculation of the mass and binding energies of the nuclei. These formulas are based on theoretical ideas about the structure of the nucleus, but the coefficients used in calculations are derived from experimental data on the mass of the nuclei. Such formulas, based both on theory and experimental data, are called semi-empirical formulas.

The semi-empirical formula obtained in this paper is based on the deuteron clustering of the nucleus. The idea behind it is that a nucleus consists of deuterium nuclei (clusters) and a certain number of neutrons that bind deuterons into a single structure. With this approach, a clear pattern is observed, which is related to the number of neutrons and deuteron clusters in the nucleus.

2. Binding energies based on nuclear mass defect.
The binding energy of a nucleus in relation to all nucleons is a matter of its strength, measured by the minimum amount of work to be performed in order to completely split the nucleus into its constituent protons and neutrons. By virtue of the relationship between mass and energy, the binding energy of the nucleus can be calculated by the formula (1) [3]

\[ E_b = \Delta mc^2 \]  

where \( \Delta m \) - mass defect, \( c \) – speed of light
The mass defect of the atomic nucleus is the difference between the sum of the remaining masses of nucleons, constituting the nucleus of a given nuclide, and the rest mass of the atomic nucleus of this nuclide.

\[ \Delta m = (ZM_p + NM_n) - M_{nuc} \]  

(2) [3]

where \( Z \) - number of protons, \( N \) - number of neutrons, \( M_p \) - proton mass, \( M_n \) - neutron mass, \( M_{nuc} \) - nuclide mass in amu

According to the formula (1) and (2), using experimental mass values [4] of 300 most stable isobars of chemical elements, the binding energy per nucleon \( (E_b/A) \) was calculated with the plot shown in Figure 1.

3. Binding energies based on liquid drop model.

The liquid drop model is one of the earliest models of the structure of the atomic nucleus. Based on this model Carl Weizsäcker obtained a semi-empirical formula for the binding energy of an atomic nucleus for the first time. According to this theory, the atomic nucleus can be represented as a spherical, uniformly charged drop of liquid from nuclear matter, which has incompressibility, saturation of nuclear forces, "evaporation" of nucleons, etc. Given the properties common to liquid and nuclear matter Weizsäcker obtained a semi-empirical formula (3) allowing calculation of the approximate values of the binding energy of the nucleus [3].

\[ E_b = \alpha A - \beta A^{2/3} - \gamma \frac{Z^2}{A^{1/3}} - \epsilon \frac{(A/2 - Z)^2}{A} + \delta \]  

(3) [3]

where \( A \) – mass number, \( Z \) – atomic number. The coefficients in formula (2) \( \alpha, \beta, \gamma, \) and \( \epsilon \) are obtained by statistical processing of experimental data. The coefficient \( \delta \) is determined from the concept of an even-odd nucleus.

![Fig. 1. Binding energy per nucleon versus mass number](image1)

![Fig. 2. Experimental values of binding energy per nucleon versus calculated by the formula Weizsäcker [4], [5]](image2)
The Weizsäcker formula allows to calculate the binding energy of a nucleus with an error of up to ~ 11 MeV as shown in Figure 2 [5]. The largest discrepancies between experimental and theoretically obtained are observed in the area of magic numbers. This is explained by the fact that in liquid drop model the non-uniformities of the distribution of nuclear matter due to the shell structure of atomic nuclei are not taken into account.

4. Binding energies based on deuteron clusterization of the atomic nucleus.

This paper presents a cluster model of nuclei, considering that nucleus consists of deuterium nuclei (clusters) and a certain number of neutrons. These neutrons bind clusters into a single nuclear structure [1] [2]. Based on this idea, a semi-empirical formula for the binding energy of the nucleus was obtained. If the masses are expressed in energy units, the formula will look as follows:

\[ E_b = M_n - \frac{M_{\text{nuc}} - \Delta M_D}{A} \times A - 2Z \]  

where \( M_n \) - neutron mass, \( M_{\text{nuc}} \) - nuclide mass, \( A \) - mass number, \( Z \) - atomic number, \( m_D \) - deuteron mass defect.

The values of the binding energy per nucleon for the 300 most stable isobars of chemical elements obtained by formula (4) are shown in the plot in Figure 3. The discrepancy between the experimental and theoretical values does not exceed 0.4 MeV.

\[ \Delta m = M_{\text{nuc}} - A \]  

Fig. 3. Experimental values of binding energy per nucleon versus calculated with use of the formula (4)

The values of \( M_{\text{nuc}} \) are determined experimentally by the method of mass spectrography and, therefore, the values expressed in terms of the masses of neutral atoms in amu are given in the tables instead of nuclear masses. For neutral atoms, it is necessary to take into account the masses of electrons of a given atom. Considering the above mentioned, the chemical defect mass will be:

\[ M_{\text{nuc}} = \Delta m + A \]  

Fig. 4. Chemical mass defect versus mass number based on the \(^{12}\)C

As can be seen from Fig. 4, the approximating line for the chemical mass defect [6] is:

\[ \Delta m = 5 \times 10^{-6} A^2 - 0.001A + 0.016 \]  

Fig. 4. Chemical mass defect versus mass number based on the \(^{12}\)C
Using formula (6), formula (5.b) will result the following:

$$M_{nuc} = 5 \times 10^{-6} A^2 + 0.999 A + 0.016 \quad (7)$$

Formula (7) makes it possible to convert formula (4), in order to avoid the use of the experimental values of the mass of the nucleus when calculating the binding energy:

$$E_b = M_n \frac{5 \times 10^{-6} A^2 + 0.999 A + 0.016}{A} - \frac{\Delta m_D A - 2Z}{A} \quad (9)$$

Recalculating the values of the binding energy using the formula (9), we obtain discrepancies between the experimental and theoretical results of not more than 1 MeV. This is due to the approximation in the calculation of the chemical mass defect, as shown in Figure 5.

$$\text{Fig. 5. Experimental values of binding energy per nucleon versus calculated by the formula (9)}$$

5. Conclusion.

Unlike the liquid drop model, the idea of deuteron clusterization allowed to find a semi-empirical formula with a small error of values for the binding energy independent from the symmetry (even-oddness) of the nucleus.

$$E_b = M_n \frac{5 \times 10^{-6} A^2 + 0.999 A + 0.016}{A} - \frac{\Delta m_D A - 2Z}{A}$$

In the formula for the binding energy of the deuteron clusterization, the $M_n$ term presents the neutron mass, characterizes the amount of energy that neutron loses on the formation of an intercluster bond, and the term $\frac{\Delta m_D A - 2Z}{A}$ characterizes the neutron energy loss due to the formation of a deuteron cluster. The shell structure of the clustering of the nucleus in which the deuteron wave functions overlap is also not excluded. One of the directions for further research will be related to this issue.

References / Список литературы

1. Isayev R. Statistical analysis of stable and long-lived isotopes using deuteron cluster //Problems of modern science and education, 2016. Vol. 10 (52). P. 10-16. [Electronic resource]. URL: https://doi.org/10.20861/2304-2338-2016-52-002/ (date of acces: 12.12.2018).
2. Isayev R. The logarithmic spiral of beta-stable odd isotopes //Problems of modern science and education. 2016. Vol. 13 (55). P. 23-19. [Electronic resource]. URL: https://doi.org/10.20861/2304-2338-2016-55-002/ (date of acces: 12.12.2018).
3. Sivukhin D.V. Obshchiy kurs fiziki. 3-e izdaniye, stereotipnoye. M.: Fizmatlit, 2006. V. Atomnaya i yadernaya fizika. S. 401-405.
4. Atomic mass evaluation by G. Audi and A.H. Wapstra// Nuclear Physics A595 Vol. 4 P.409-480, December 25, 1995. [Electronic resource]. URL: https://physics.nist.gov/PhysRefData/Compositions/mass_rmd.mas95round.txt/ (date of acces: 12.12.2018).
5. Calculator and Graph Engine for Atomic Nuclei Parameters / The Centre for Photonuclear Experiments Data of the Moscow State University. [Electronic resource]. URL:http://cdfe.sinp.msu.ru/services/calc_thr/calc_thr.html/ (date of acces: 12.12.2018).
6. Pourshahian, S.J. Am. Soc. Mass Spectrom. (2017) 28: 1836. [Electronic resource]. URL: https://doi.org/10.1007/s13361-017-1741-9/ (date of acces: 12.12.2018).