Three-dimensional noncommutative Yukawa theory:  
Induced effective action and propagating modes

R. Bufalo\textsuperscript{1} and M. Ghasemkhani\textsuperscript{2}\textsuperscript{†}

\textsuperscript{1} Departamento de Física, Universidade Federal de Lavras,  
Caixa Postal 3037, 37200-000 Lavras, MG, Brazil  
\textsuperscript{2} Department of Physics, Shahid Beheshti University,  
G.C., Evin, Tehran 19839, Iran

February 23, 2017

Abstract

In this paper we establish the analysis of noncommutative Yukawa theory, encom-  
passing neutral and charged scalar fields. We approach the analysis by considering  
carefully the derivation of the respective effective actions. Hence, based on the obtained  
results, we compute the one-loop contributions to the neutral and charged scalar field  
self-energy, as well as to the Chern-Simons polarization tensor. In order to properly  
define the behaviour of the quantum fields, the known UV/IR mixing due to radiative  
corrections is analysed in the one-loop physical dispersion relation of the scalar and  
gauge fields.

PACS: 11.15.-q, 11.10.Kk, 11.10.Nx

\textsuperscript{*}E-mail: rodrigo.bufalo@dfi.ufla.br  
\textsuperscript{†}E-mail: ghasemkhani@ipm.ir
# Contents

1 Introduction 3

2 General discussion 5

3 Perturbative effective action 7
   3.1 Neutral scalar fields ........................................ 7
      3.1.1 \( \phi\phi \) contribution .......................... 8
      3.1.2 \( \phi\phi\phi \) contribution ......................... 9
   3.2 Charged scalar fields ...................................... 10
      3.2.1 \( \phi\phi A \) contribution .......................... 11
      3.2.2 \( \phi\phi AA \) contribution .......................... 12

4 Propagating modes scalar field 13

5 Propagating modes charged scalar fields 14
   5.1 Dispersion relation charged scalar fields ............... 15
   5.2 Dispersion relation gauge field .......................... 17

6 Concluding remarks 20

A Non-planar integrals 21

B Effective action 22
   B.1 \( \phi\phi A \) contribution ............................ 22
   B.2 \( \phi\phi AA \) contribution ............................ 23

C Dispersion relation 24
1 Introduction

The field theoretical model for description of the interaction between nucleons in particle physics was first proposed by H. Yukawa in 1935 [1], which led to the prediction of pion before its discovery from cosmic rays in 1947 [2]. The Yukawa term originates from the exchange of a massive scalar field that in the non-relativistic limit yields a Yukawa potential and hence the corresponding force has a finite range, which is inversely proportional to the mediator particle mass.

Since its proposal, the notion of Yukawa potential has been used in different areas in the description of several phenomena such as chemical process, astrophysics, fluid plasma system and especially in modern particle physics. More importantly, in the latter case, i.e. standard model, the Yukawa interaction of the Higgs field and massless quarks and leptons is the responsible coupling to give mass to these fermionic fields.

Due to its importance in the different physical phenomena, Yukawa theory has been used as a laboratory in the search of physics beyond standard model, or even to scrutinize the cornerstones of gauge theories. Furthermore, if we expand our scope and add to our interest the description of nature behavior at shortest distances [3, 4], i.e. a quantum theory of gravity, or even the so-called minimal length scale physics, one inexorably finds that noncommutative geometry is one of the most highly motivated and richer framework [5], including phenomenological inspirations [3, 6–9]. Space-time noncommutativity naturally emerges at Plank scale in attempts to accommodate quantum mechanics and general relativity in a common framework, one finds uncertainty principles that are compatible with non-commuting coordinates [10, 11].

The simplest realization of noncommutativity is given by the following canonical algebra

\[ [\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}. \] (1.1)
where \( \theta_{\mu\nu} \) is a constant skewsymmetric matrix of dimension of length squared. These commutation relations give an uncertainty relation among the coordinates: \( \Delta \hat{x}_\mu \Delta \hat{x}_\nu \gtrsim \frac{1}{2} |\theta_{\mu\nu}|. \) Notice, however, that the nonzero components of \( \theta_{\mu\nu} \) are arbitrary parameters that must be constrained by experiments, we can think as they being the resolution scale that can be taken to be, for instance, of the order of square of Planck length \( \ell_H \).

A suitable framework to compute quantities in a NC–QFT is by the use of Weyl-Moyal (symbol) correspondence [12]. This allows to define a classical (commutative) analogue of the noncommutative space, so that the following relation holds: \( \varphi(\hat{x}) \psi(\hat{x}) \rightarrow \varphi_W \ast \psi_W \), where \( \varphi_W \) is the so-called Weyl symbol of the operator \( \varphi(\hat{x}) \) [12]. Moreover, in this context, we have that the Moyal star product is defined as

\[ f(x) \ast g(x) = f(x) \exp \left( \frac{i}{2} \theta_{\mu\nu} \frac{\delta}{\delta x_\mu} \frac{\delta}{\delta x_\nu} \right) g(x). \] (1.2)

One common property of NC gauge theories that has been uncovered is that high-momentum modes (UV) affect the physics at large distances (IR) leading to the appearance of the so-called UV/IR mixing [13]. These “anomalies” involve non-analytic behavior in the noncommutativity parameter \( \theta \) making the limit \( \theta \rightarrow 0 \) singular. Despite of the many attempts to understand this issue in four and three-dimensional field theory models, see [14] and [15, 16], respectively, no complete description to handle it has yet been provided [17].
An important branch of interest regarding NC gauge theories is the study of how noncommutativity affects established properties of conventional theories, in particular a considerable effort has been expended in analyzing gauge theories defined in a three-dimensional noncommutative spacetime, this effort is highly supported by the fact that wandering into lower-dimensional models has been proved to be very fertile and stimulated significantly the development of our knowledge in the subject. Gauge theories defined in a three-dimensional spacetime are known to possess unique properties and are well motivated as providing a simple setting where important theoretical ideas are suitably tested. Noncommutative three-dimensional field theory, in particular gauge theory, can find application in the study of planar physics in condensed matter and statistical physics. After this observation, various perturbative aspects of the noncommutative Chern-Simons theory have been studied, NC Maxwell-Chern-Simons theory and NC QED, as well as its supersymmetric extension, where deviations of known phenomena and interesting new properties have been uncovered.

However, as we have extensively discussed, in addition to its importance in the Standard Model of particles, a noncommutative extension of the Yukawa field theory action should be fully considered, in particular how the noncommutative Higgs effective action can be generated. Some aspects for this theory have been discussed previously.

In particular, our present analysis will be twofold: first, we will consider the interaction between a neutral scalar field and dynamical fermionic fields, where the scalar field effective action is found by integrating out the fermionic modes, an additional derivative cubic coupling is found for the scalar field. Second, a more interesting case is considered, now we have the interaction among charged scalar fields with fermionic fields augmented by gauge fields, in which the effective action describing the interaction between the charged scalar and gauge fields is obtained. In the latter, we shall consider the dynamics of the gauge sector given by the higher-derivative (HD) Chern-Simons action, where new features are discussed. At last, in both cases, UV/IR mixing is analysed in the one-loop physical dispersion relation of the scalar and gauge fields due to radiative corrections, this is justifiable once this anomaly might modify significantly the behavior of the quantum field in the description of a given phenomenon and find room in interesting application.

Therefore, in this paper we will consider the effective action for two distinct Yukawa couplings: i) neutral scalar field with fermionic fields, and ii) charged scalar field with fermionic fields plus a gauge field. For this purpose, we will make use of the ideas outlined in Ref. and in which noncommutative fermionic effective actions were considered. The cornerstone of this approach is outlined in Sec. and consists in consider, in a formal way, the existence of an exact Seiberg–Witten map, valid to all orders in so that the noncommutative effects into the resulting outcome are in fact nonperturbative. We compute explicitly in section the respective effective action for the neutral and charged scalar field, where the presence of the new couplings is discussed. Based on the obtained results for the effective action, we proceed in Sec. to compute the one-loop correction to the self-energy of the neutral scalar field, in particular application to its dispersion relation. Next, in Sec. we consider the effective action obtained for the case of charged scalar fields minimally coupled with a HD Chern-Simons gauge field, a model that one can name as NC HD-Chern-Simons-Higgs model. In this case we carefully analyse the dispersion relation for both fields, by computing the set of diagrams for the respective one-loop self energy functions. Additionally to what we have already discussed, another physical application where the present study
can be employed, follows from Refs. [41–43] in which it is shown that a model where the Chern-Simons term coupled to the scalar field matter is a suitable framework for field theoretical description of the Aharonov-Bohm effect. At Sec. 6 we summarize our results and present our conclusion and prospects.

2 General discussion

Let us now define the noncommutative extension of fermionic fields interacting with a neutral scalar field, which can be named as a noncommutative Yukawa model. For this, we shall consider the following action

$$S = \int d^3x \left[ i \bar{\psi} \star \gamma^\mu \partial_\mu \psi - m \bar{\psi} \star \psi + g \bar{\psi} \star \phi \star \psi \right].$$

It should be remarked that we are working with a two-component representation for the spinors with the standard convention

$$\gamma^0 = \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^1 = i \sigma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad \gamma^2 = i \sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

where the $\gamma$-matrices satisfy $\gamma^\mu \gamma^\nu = \eta^{\mu\nu} - i \varepsilon^{\mu\nu\sigma} \gamma_\sigma$. We observe that the action (2.1) is invariant under a global $U(1)$ symmetry, $\psi \to e^{i\alpha} \psi$. Furthermore, on the behavior of this action under discrete transformations, parity (P), charge conjugation (C) and time reversal (T), we have prepared a detailed analysis in the following:

(i) Parity

The description of the parity transformation in 2 + 1 dimensions is given by $x_1 \to -x_1$ and $x_2 \to x_2$. Using the invariance of the kinetic part of the Dirac Lagrangian under parity, it is found that the fermionic field transforms as $\psi \to \gamma^1 \psi$. Hence, it is easily concluded that parity is broken by the fermion mass term, since $\bar{\psi} \psi \to -\bar{\psi} \psi$.

From (1.1), it is deduced that the noncommutative parameter changes sign under parity $\theta \to -\theta$, and hence we observe that the interaction term $\bar{\psi} \star \phi \star \psi$ transforms into the following

$$S_{int}^P = -g \int d^3x \ \bar{\psi} \star (\psi \star \phi_p),$$

in which we have used the anti-commuting property of the fermionic fields. If we assume a pseudo scalar field $\phi_p = -\phi$, similar to the three-dimensional Yukawa term in commutative space, then it is shown that the Yukawa coupling as considered here Eq. (2.1) is not parity invariant; however, it follows that we can construct a combination that is parity invariant

$$\tilde{S}_{int} = g \int d^3x \ \bar{\psi} \star \{ \phi, \psi \},$$

here $\{ , \} = [ , ]_+$. On the other hand, for the other choice $\phi_p = +\phi$, it is easily realized that

$$\tilde{S}_{int} = g \int d^3x \ \bar{\psi} \star [\phi, \psi],$$

is parity invariant.
(ii) **Charge conjugation**

Under a charge conjugation transformation in three dimensions, the spinor field changes as \( \psi \rightarrow C \gamma^0 \psi^* \), so that the operator \( C \) should satisfy the following relation

\[
C^{-1} \gamma^\mu C = -(\gamma^\mu)^T.
\]

Hence, by considering the above constraint and also the representation of the gamma matrices in (2.2), the appropriate choice for the charge conjugation operator in 2 + 1 dimensions is given by \( C = \gamma^2 \). Since \((\gamma^2)^* = \gamma^2\) and \((\gamma^0)^* = \gamma^0\) then \( \psi^* \rightarrow \gamma^2 \gamma^0 \psi \), so that consequently we can find the transformation of the fermion mass term as follows

\[
\bar{\psi}_c \psi_c = -\psi^T \gamma^0 \psi^* = \bar{\psi} \psi,
\]

where \((\gamma^2)^T = -\gamma^2\) and the anticommuting property of the spinors had been used. We thus see that the fermion mass term is \( C \)-invariant. Finally, if we consider that the scalar field satisfies \( \phi_c = \phi \), the noncommutative Yukawa interaction term transforms as

\[
S_{\text{int}}^c = g \int d^3x \bar{\psi} * (\psi^* \phi)
\]

We notice that there are two different choices to build a noncommutative Yukawa interaction term that these are related to each other by a charge conjugation transformation. This point was first mentioned in [14], including the study of the discrete symmetries in noncommutative \( \text{QED}_4 \). Furthermore, if we apply \( \theta \rightarrow -\theta \) to (2.8), we find that the interaction term is \( C \)-invariant.

(iii) **Time reversal**

Time reversal operator acts on the fermionic field as \( \psi \rightarrow \gamma^2 \psi \). Thus the fermion mass term, similar to the parity transformation, is not invariant under \( T \), since it behaves as \( \tilde{\psi} \psi \rightarrow -\tilde{\psi} \psi \). For the interaction term, we have

\[
S_{\text{int}}^T = g \int d^3x \bar{\psi} * (\psi^* \phi)
\]

in which it is supposed that \( \phi_T = -\phi \). Once again we observe that, similar to the charge conjugation transformation, adding the assumption \( \theta \rightarrow -\theta \) gives us a \( T \)-invariant interaction term.

Finally, we are able to establish the one-loop effective action for the scalar field \( \phi \) by integrating over the fermionic fields,

\[
i \Gamma [\phi] = \ln \frac{\det (i\slashed{D} - m + g\phi^*)}{\det (i\slashed{D} - m)} = -\sum_n \frac{1}{n^2} \text{tr} \left( (\slashed{D} + im)^{-1} i (g\phi^*) \right)^n,
\]

where we identify the differential operator as for the fermionic propagator,

\[
(\slashed{D} + im)^{-1} \delta (x - y) = \int \frac{d^3p}{(2\pi)^3} \frac{i (p^2 + m)}{p^2 - m^2 + i\varepsilon} e^{-ip(x - y)}.
\]
Nevertheless, due to our interest, we can rewrite (2.10) in a far more convenient form as

\[ i \Gamma [\phi] = \sum_{n} \int d^{3}x_{1} \ldots \int d^{3}x_{n} [\phi (x_{1}) \phi (x_{2}) \cdots \phi (x_{n})] \Gamma (x_{1}, x_{2}, \ldots, x_{n}), \]  

(2.12)

where perturbative calculation is readily obtained and we have defined

\[ \Gamma (x_{1}, x_{2}, \ldots, x_{n}) = -\frac{(-g)^{n}}{n} \int \prod_{i} \frac{d^{3}p_{i}}{(2\pi)^{3}} (2\pi)^{3} \delta \left( \sum_{i} p_{i} \right) \times \exp \left( -i \sum_{i} p_{i} x_{i} \right) \exp \left( -\frac{i}{2} \sum_{i<j} p_{i} \times p_{j} \right) \Xi (p_{1}, p_{2}, \ldots, p_{n-1}), \]  

(2.13)

in which we introduced the notation \( p \times q = \theta^{\mu\nu} p_{\mu} q_{\nu} \); by simplicity, the one-loop contributions are defined in the form

\[ \Xi (p_{1}, \ldots, p_{n-1}) = \int \frac{d^{3}q}{(2\pi)^{3}} \frac{\text{tr} \left[ (q + p_{1} + m)(q + m)(q - p_{2} + m) \cdots \left( q - \sum_{i=2}^{n-1} p_{i} + m \right) \right]}{((q + p_{1})^{2} - m^{2}) [q^{2} - m^{2}] \left( (q - p_{2})^{2} - m^{2} \right) \ldots \left( (q - \sum_{i=2}^{n-1} p_{i})^{2} - m^{2} \right)}. \]  

(2.14)

In order to rewrite (2.12) into the form (2.14) we have made use of the general result

\[ \int dx \ O_{1} (x) \star O_{2} (x) \ldots \star O_{n} (x) = \int \prod_{i} d^{3}x_{i} \prod_{i} \frac{d^{3}p_{i}}{(2\pi)^{3}} O_{1} (x_{1}) O_{2} (x_{2}) \ldots O_{n} (x_{n}) \times \exp \left( -i \sum_{i} p_{i} x_{i} \right) \exp \left( -\frac{i}{2} \sum_{i<j} p_{i} \times p_{j} \right) \delta \left( \sum_{i} p_{i} \right). \]  

(2.15)

With this result we finish our formal development where all the necessary information were carefully presented. In the next section we will proceed in computing explicitly the full effective action for two cases: first for a neutral scalar field, and second for a charged scalar field. In order to compute such contributions, we will concentrate in considering the leading contributions for the resulting expressions, this can be suitably achieved by means of the long wavelength limit (i.e., \( m^{2} > p^{2} \), where \( p \) is an external momentum).

3 Perturbative effective action

3.1 Neutral scalar fields

We shall now proceed in evaluating explicitly the contributions of two, three and four scalar fields to the effective action. Actually, the contribution of one scalar field is identically vanishing. In this case we will find that at the long wavelength limit we generate a full action for the neutral scalar field, in particular that no self-coupling is present at order higher than three, only derivative couplings are available.
3.1.1 $\phi\phi$ contribution

Let us consider the first nonvanishing contribution of the one-loop effective action, for this matter we take $n = 2$ in the Eq.(2.14), depicted in Fig. 1,

$$\Xi (p) = \int \frac{d^3q}{(2\pi)^3} \frac{tr [(q+p)+m] (q+m)}{[(q+p)^2 - m^2] [q^2 - m^2]}. \quad (3.1)$$

Moreover, the momentum integration can be readily evaluated by considering the Feynman parametrization of the denominator factors, and considering the change of variables $q \rightarrow q + xp$, so that

$$\Xi (p) = \int_0^1 dx \int \frac{d^3q}{(2\pi)^3} \frac{tr [(q+(1-x)p)+m] (q-xp+m)}{[q^2 + x(1-x)p^2 - m^2]^2}. \quad (3.2)$$

The trace of $\gamma$-matrices in the numerator of (3.2) can be computed with help of the results

$$tr (\gamma^\mu \gamma^\nu) = 2\eta^{\mu\nu}, \quad tr (\gamma^\mu \gamma^\nu \gamma^\beta) = 2i\varepsilon^{\mu\nu\beta}. \quad (3.3)$$

Finally, we write the above integral in a dimensional regularized form as

$$\Xi (p) = 2 \int_0^1 dx \int \frac{d^2q}{(2\pi)^2} \frac{q^2 - x(1-x)p^2 + m^2}{[q^2 + x(1-x)p^2 - m^2]^2}. \quad (3.4)$$

Hence, the integration in the momentum $q$ is rather straightforward and the resulting expression has no poles when $\omega \rightarrow 3^+$, so the result reads

$$\Xi (p) = \frac{i}{\pi} \int_0^1 dx \sqrt{m^2 - x(1-x)p^2}. \quad (3.5)$$

We note that no scalar Chern–Simons-like term, proportional to $\varepsilon^{\mu\nu\beta}$, is generated. This is understood since it is impossible to build a Lorentz invariant quadratic combination of $\phi$ and $\varepsilon^{\mu\nu\beta}$ at this order.

**Long wavelength limit**  Let us take a look at the remaining integration at the Eq.(3.5). Moreover, considering the case when $p^2 \ll m^2$, then we find that $\Xi (p) = -\frac{i}{12\pi |m|} (p^2 - 12m^2)$. As it is easily seen, this $O (m^{-1})$ term corresponds to the kinetic term of the Klein-Gordon action for the neutral scalar field.

---

1 This result for a scalar field is in contrast to the effective action for a vector field, where an induced Chern–Simons action is produced in three-dimensional QED, at the large fermion mass limit [31].
In the configuration space, if we replace the above result into the expression (2.13) we find after some manipulation that
\[
\Gamma (x_1, x_2) = -\frac{i g^2}{24\pi |m|} |m| \left( \Box + 12 m^2 \right) \delta (x_1 - x_2). \tag{3.6}
\]
As it is well-known, there is no noncommutativity effects for the case of two fields, since the phase factor in (2.13) vanishes. Finally, the analysis of the first non vanishing term in (2.12) is given by
\[
i \Gamma [\phi \phi] = \frac{i g^2}{24\pi |m|} \int d^3 x (\partial_\mu \phi \partial^\mu \phi - \mu^2 \phi^2) (x), \tag{3.7}
\]
and it leads to the radiatively induced Klein-Gordon action, where we have introduced a new square mass parameter \(\mu^2 = 12 m^2\).

3.1.2 \(\phi \phi \phi\) contribution

The calculation of the next contribution follows as in the previous analysis. We compute the \(n = 3\) contribution in the Eq.(2.14) and given in Fig. 2
\[
\Xi (p, k) = \int \frac{d^3 q}{(2\pi)^3} \frac{\text{tr} \left[ (q + p + m) (q + m) (q' - k + m) \right]}{[ (q + p)^2 - m^2] [q'^2 - m^2] \left[ (q' - k)^2 - m^2 \right]}.	ag{3.8}
\]
The momentum integral can be computed straightforwardly using the dimensional regularization, and realizing that \(\Gamma (2 - \frac{d}{2})\) and \(\Gamma (3 - \frac{d}{2})\) have no poles when \(\omega \to 3^+\), so we find
\[
\Xi (p, k) = \frac{i}{16\pi} \int d\xi \left[ \frac{N_1 (p, k; x, z)}{(m^2 - A^2 (p, k))^\frac{d}{2}} - \frac{N_2 (p, k; x, z)}{(m^2 - A^2 (p, k))^\frac{3}{2}} \right], \tag{3.9}
\]
where by simplicity we have defined \(s_\mu = x p_\mu - z k_\mu\), the measure \(\int d\xi = \int_0^1 dx \int_0^{1-x} dz\), and the quantity \(A^2 (p, k) = - (xp - zk)^2 + xp^2 + zk^2\), as well as the following quantities
\[
N_1 (p, k; x, z) = 3tr (-3s' - k' + y') + 18m, \tag{3.10}
\]
and
\[
N_2 (p, k; x, z) = -tr [(s' - y') (s - m) (s + k' - m)] + mtr [(s' - m) (s + k' - m)]. \tag{3.11}
\]
As usual, the full contribution of (3.9) gives rise to a full set of information, but here we are interested in the particular cases of the \(\mathcal{O} (m^0)\) and \(\mathcal{O} (m^{-2})\) contributions, in order to add these self-interacting terms to the kinetic contribution (3.7).
**Derivative coupling** Although the structure of the contribution \( \mathcal{O}(m^0) \) is rather complicated than those from the two fields, we can keep traced of the terms \( \mathcal{O}(m^0) \) and \( \mathcal{O}(m^{-2}) \) by paying careful attention to the contributions from the numerator and denominator at the long wavelength limit. For that matter, we shall focus on the \( \mathcal{O}(m^1) \) and \( \mathcal{O}(m^3) \) contributions from the quantities \( N_1(p,k;x,z) \) and \( N_2(p,k;x,z) \), and take the \( p^2 \ll m^2 \) limit. With such considerations the remaining integral can now be computed, so that the three fields contribution is simply given by

\[
\Xi(p,k) = \frac{i}{8\pi} \frac{m}{|m|} \left[ 4 + \frac{1}{6m^2} \left( 3k^2 + 2p^2 + 2(p,k) \right) \right].
\]

Finally, the interacting effective action \((2.12)\) of the noncommutative Klein-Gordon action for a neutral scalar field is found to be

\[
i\Gamma[^{\phi\phi\phi}] = \frac{ig^3}{6\pi} \frac{m}{|m|} \int d^3x \left( \phi \star \phi \star \phi + \frac{1}{3m^2} \partial^\alpha \phi \star \partial_\alpha \phi \star \phi \right)(x),
\]

where we have applied the identity \( 2 \int d^3x \partial^\alpha \phi \star \partial_\alpha \phi \star \phi = -\int d^3x \Box \phi \star \phi \star \phi \), found as a result of using the cyclic property of the Moyal product and performing an integration by part. We thus see that a noncommutative \( \lambda^3 \phi^6 \) interacting term is radiatively generated, in addition to a derivative coupling as well, where the coupling constant has dimension of \([\lambda] = [gm] \). It is worth to mention that due to theory's structure higher self-interacting contributions \( \lambda^6 \phi^n \) for \( n > 3 \) are absent in the effective action of a neutral scalar field at the long wavelength limit, even the well-known dimensionless coupling \( \lambda^6 \). In contrast, we find that only derivative couplings are present in this situation.

Once again, similarly to the case of \( \phi\phi \) contribution, we also see that a Chern–Simons-like term does not appear in the analysis of the \( \phi\phi \) contribution. This can be easily seen by considering the possible Chern–Simons-like expressions described by \( \int d^3x \varepsilon^{\mu\nu\beta}(\partial_\mu \phi) \star (\partial_\nu \phi) \star (\partial_\beta \phi) \) or \( \int d^3x \varepsilon^{\mu\nu\beta}(\partial_\mu \partial_\alpha \phi) \star (\partial_\nu \partial_\alpha \phi) \star (\partial_\beta \phi) \) that are apparently nonzero but are in fact both of them vanish, due to the integration by part and discarding the surface terms.

### 3.2 Charged scalar fields

For completeness, in addition to the discussion of a neutral scalar field, let us consider the case of charged scalar fields too. The fermionic action in this case is defined as

\[
S = \int d^3x \left[ i\bar{\psi} \star \gamma^\mu D^-_\mu \psi - m\bar{\psi} \star \psi + g\bar{\psi} \star \Phi \star \psi + h.c. \right],
\]

in which the covariant derivative is defined as \( D^-_\mu \psi = \partial \psi - ieA^-_\mu \psi \), and the h.c. term ensures the reality of the action. Moreover, this action is invariant under the local infinitesimal gauge transformations,

\[
\delta \psi = ig\lambda \star \psi, \quad \delta A^-_\mu = \partial_\mu \lambda - ie [A^-_\mu, \lambda].
\]

The one-loop effective action coming from \((3.14)\) are readily obtained,

\[
i\Gamma[A] = -\sum_n \frac{1}{n} tr \left( (\partial + im)^{-1} i(g\phi \star + eA^\star) \right)^n,
\]

10
where we have defined by simplicity the combination $\phi = \Phi + \Phi^\dagger$. The $n = 2$ contribution is exactly the same as the one obtained in (3.7), just with the previous replacement on the field $\phi$. We shall now proceed to analyse the interacting terms between the scalar and gauge fields coming from the $n = 3$ and $n = 4$ terms.

### 3.2.1 $\phi\phi A$ contribution

Let us start with the first contribution coming from $n = 3$ and depicted in Fig. 3. Among all these interacting terms coming from this expansion we shall concentrate in those giving a combination of $\phi\phi A$ fields. We thus find that three terms are present and have the following structure

$$

i\Gamma[\phi\phi A] \approx \int d^3x_1 d^3x_2 d^3x_3 \left[ [\phi(x_1) \phi(x_2) A_\mu(x_3)] \Gamma^\mu_{(a)}(x_1, x_2, x_3) + [\phi(x_1) A_\mu(x_2) \phi(x_3)] \Gamma^\mu_{(b)}(x_1, x_2, x_3) + [A_\mu(x_1) \phi(x_2) \phi(x_3)] \Gamma^\mu_{(c)}(x_1, x_2, x_3) \right],

$$

(3.17)

where we define and compute the general tensor quantities $\Gamma^\mu_{(i)}(x_1, x_2, x_3)$, for $i = a, b, c$, in the Appendix B.

Therefore, substituting the results (B.8) and (B.9) back into the expression (3.17), and after some straightforward integral manipulation we are finally able to write

$$
i\Gamma_{(a)}[\phi\phi A] = \frac{g^2 e}{36\pi |m|} \int d^3x \left[ \partial^\mu \phi \ast \phi \ast A_{\mu} - \phi \ast \partial^\mu \phi \ast A_{\mu} \right],

$$

(3.18)

$$
i\Gamma_{(b)}[\phi\phi A] = -\frac{g^2 e}{36\pi |m|} \int d^3x \left[ \partial^\mu \phi \ast A_{\mu} \ast \phi - \phi \ast A_{\mu} \ast \partial^\mu \phi \right],

$$

(3.19)

$$
i\Gamma_{(c)}[\phi\phi A] = \frac{g^2 e}{36\pi |m|} \int d^3x \left[ A_{\mu} \ast \partial^\mu \phi \ast \phi - A_{\mu} \ast \phi \ast \partial^\mu \phi \right].

$$

(3.20)

The complete contribution is found by summing the above three contributions, Eqs. (3.18)–(3.20). Thus, using the cyclic property of the Moyal product, we find

$$
i\Gamma[\phi\phi A] = i\Gamma_{(a)}[\phi\phi A] + i\Gamma_{(b)}[\phi\phi A] + i\Gamma_{(c)}[\phi\phi A],

$$

(3.21)

$$

= \frac{g^2 e}{12\pi |m|} \int d^3x \left[ A_{\mu} \ast \partial^\mu \phi \ast \phi - A_{\mu} \ast \phi \ast \partial^\mu \phi \right].

$$

(3.22)

At last, by using the definition $\phi \rightarrow \Phi + \Phi^\dagger$ and keeping the relevant terms, we can rewrite expression (3.21) into the following convenient form

$$
i\Gamma[\Phi\Phi^\dagger A] \approx \frac{g^2 e}{12\pi |m|} \int d^3x \left( [\Phi^\dagger, A_{\mu}] \ast \partial^\mu \Phi - \partial^\mu \Phi^\dagger \ast [A_{\mu}, \Phi] \ast \right).

$$

(3.22)
As we will see afterwards, this result is exactly the cubic interaction from the NC Higgs model, since the coupling in this case is given by $(\mathcal{D}_\mu \Phi)^\dagger \star \mathcal{D}^\mu \Phi$, where the covariant derivative is now written in its adjoint form $\mathcal{D}_\mu \Phi = \partial_\mu \Phi - ie [A_\mu, \Phi]_\star$. It is worth of mention that the generated couplings are in the adjoint representation and not in the fundamental one.\footnote{This fact might be closely related to our choice of interaction term in (2.1), perhaps because different couplings such as $\bar{\psi} \psi \phi$ or $\bar{\psi} \star [\phi, \psi]$ could give in principle different contributions to the effective action. However, this fact should be further elaborated and then analysed.}

### 3.2.2 $\phi \phi AA$ contribution

By means of complementarity, we now approach the one-loop $n = 4$ contribution to the charged scalar fields effective action, which graph is given in Fig. 4 where we shall find the last piece of the interacting sector.

The contribution proportional to the following structure of $\phi \phi AA$ fields is given by six terms

\begin{equation}
\begin{aligned}
    i \Gamma [\phi AA] &\simeq \int d^3 x_1 \cdots d^3 x_4 \left[ \phi (x_1) \phi (x_2) A_\mu (x_3) A_\nu (x_4) \right] \Gamma^{\mu \nu} (a) + \left[ \phi (x_1) A_\mu (x_2) \phi (x_3) A_\nu (x_4) \right] \Gamma^{\mu \nu} (b) \\
    &+ \left[ \phi (x_1) A_\mu (x_2) A_\nu (x_3) \phi (x_4) \right] \Gamma^{\mu \nu} (c) + \left[ A_\mu (x_1) \phi (x_2) \phi (x_3) A_\nu (x_4) \right] \Gamma^{\mu \nu} (d) \\
    &+ \left[ A_\mu (x_1) A_\nu (x_2) \phi (x_3) \phi (x_4) \right] \Gamma^{\mu \nu} (e) + \left[ A_\mu (x_1) A_\nu (x_2) \phi (x_3) \phi (x_4) \right] \Gamma^{\mu \nu} (f),
\end{aligned}
\end{equation}

where we define and compute the tensor quantities to each one of these contributions $\Gamma^{\mu \nu}_i$, in the Appendix B.

Hence, replacing the results (B.18) in conjunction with (B.11) and (B.12) back into the expression (3.23), we find out the following

\begin{equation}
    i \Gamma [\phi AA] = \frac{i g^2 e^2}{12 \pi |m|} \int d^3 x \left[ \phi \star A_\mu \star A^\mu \star \phi - \phi \star A_\mu \star \phi \star A^\mu \right].
\end{equation}

Now, making use again of $\phi \rightarrow \Phi + \Phi^\dagger$, we finally find the expression for the effective action

\begin{equation}
    i \Gamma [\Phi^\dagger AA] = \frac{i g^2 e^2}{12 \pi |m|} \int d^3 x \left( [\Phi^\dagger, A_\mu]_\star \star [A^\mu, \Phi]_\star \right).
\end{equation}

As we have anticipated, the expression (3.25) is precisely the last piece for the (minimal) interaction content of the NC Higgs model $(\mathcal{D}_\mu \Phi)^\dagger \star \mathcal{D}^\mu \Phi$, consisting in the quartic interaction among the scalar and gauge fields.

---

**Figure 4:** Relevant graph for the induced $\phi \phi AA$-term.
4 Propagating modes scalar field

From the obtained results, Eqs. (3.7) and (3.13), we can analyse the dynamics of the scalar fields by proposing the following effective Lagrangian for the noncommutative neutral scalar field

$$\mathcal{L} = \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 \right) + \frac{\lambda}{3!} \left( \phi \ast \phi \ast \phi + \frac{1}{3m^2} \partial_\mu \phi \ast \partial_\mu \phi \ast \phi \right). \quad (4.1)$$

Notice that the usual $\phi^3$ theory is recovered in the limit when the HD contribution decouples. The Feynman rules for this theory are readily obtained from the Lagrangian (4.1).

Moreover, we can establish the renormalization of the complete propagator, by writing the self-energy function as $\Sigma (p^2) = p^2 \Sigma_1 (p^2) + m^2 \Sigma_2 (p^2)$, that can be carried out as

$$S (p) = \frac{1}{p^2 - m^2 - \Sigma (p^2)} = \frac{1}{p^2 (1 - \Sigma_1 (p^2)) - m^2 (1 + \Sigma_2 (p^2))} = \frac{Z}{p^2 - m^2_{\text{ren}}}$$

where we have defined the renormalization constants as $Z^{-1} = 1 - \Sigma_1 (p^2)$ and $Z^{-1}_m = 1 + \Sigma_2 (p^2)$, so that the renormalized mass is defined as the following

$$m_{\text{ren}} = m \sqrt{\frac{Z}{Z_m}}. \quad (4.2)$$

Hence, the one-loop self-energy for the neutral scalar fields reads (see Fig. 5)

$$\Sigma (p) = -i \lambda^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2 - m^2 (q - p)^2 - m^2} \left[ 1 + \frac{1}{9m^2} \left( (p.q) + (q - p)^2 \right) \right]^2 \cos^2 \left[ \frac{p \times q}{2} \right], \quad (4.3)$$

We can work out the numerator of the above expression in order to simplify the dependence on the integrated momentum. Furthermore, we now compute separately the planar from the non-planar contribution by means of the identity $2 \cos^2 \left[ \frac{p \times Q}{2} \right] = 1 + \cos (p \times Q)$. First, the planar contribution results into the following

$$\Sigma_p (p) = \frac{1}{1296\pi} \frac{\lambda^2}{m} \left\{ \frac{1}{m} \int_0^1 dx \left[ 100 + 20x\beta + x^2 \beta^2 \right] \frac{1}{(1 - x) (1 - x) \beta^2} \right\}$$

$$+ \beta \int_0^1 dx \sqrt{1 - x (1 - x)} \frac{1}{\beta + 2 [20 + \beta]}, \quad (4.4)$$

where we have defined the notation $\beta = p^2/m^2$. Next, the non-planar contribution from
(4.3) can be computed with help of the results Eqs.(A.1) and (A.2), and yields

\[
\Sigma_{n-p} (p) = \frac{1}{1296 \pi m} \lambda^2 \left\{ \int_0^1 dx \left[ 100 + 20 x \beta + x^2 \beta^2 \right] e^{-m|\vec{p}| \sqrt{1-x(1-x)} } \right. \\
- \beta \frac{m}{|\vec{p}|} \int_0^1 dx e^{-m|\vec{p}| \sqrt{1-x(1-x)} } - 2 \left( 20 + \beta \right) \frac{1}{m |\vec{p}|} e^{-m|\vec{p}|} \right\}. 
\]

(4.5)

Finally, the complete contribution is given by the sum of (4.4) and (4.5), \( \Sigma (p) = \Sigma_p (p) + \Sigma_{n-p} (p) \).

Now with the one-loop self-energy we can analyse the renormalized mass expression structure. Thus, expanding (4.2) at leading order, we find that

\[
m_{\text{ren}} = m \sqrt{\frac{Z}{Z_m}} \simeq m + \Sigma^{(1)} + O (\alpha^2),
\]

(4.6)

where we have defined \( \Sigma^{(1)} = \frac{\lambda^2}{2} (\Sigma_1 (m^2) + \Sigma_2 (m^2)) \), with the previous coefficients given by

\[
\Sigma_1 (m^2) = \frac{1}{1296 \pi m^3} \lambda^2 \left\{ \int_0^1 dx \left[ \frac{20x + x^2}{\sqrt{1-x(1-x)}} \left[ 1 + e^{-m|\vec{p}| \sqrt{1-x(1-x)} } \right] + \right. \right. \\
+ \left. \left. \int_0^1 dx \left( \sqrt{1-x(1-x)} - \frac{1}{m |\vec{p}|} e^{-m|\vec{p}| \sqrt{1-x(1-x)} } \right) + 2 \left( 1 - \frac{1}{m |\vec{p}|} e^{-m|\vec{p}|} \right) \right\}. 
\]

(4.7)

and

\[
\Sigma_2 (m^2) = \frac{1}{1296 \pi m^3} \lambda^2 \left\{ \int_0^1 dx \left[ \frac{100}{\sqrt{1-x(1-x)}} \left[ 1 + e^{-m|\vec{p}| \sqrt{1-x(1-x)} } \right] + 40 \left( 1 - \frac{1}{m |\vec{p}|} e^{-m|\vec{p}|} \right) \right\}. 
\]

(4.8)

Hence, we see that the dispersion relation for the scalar field at this order reads

\[
\omega^2 = \vec{p}^2 + m_{\text{ren}}^2 \simeq \vec{p}^2 + m^2 + 2m \Sigma^{(1)} + O (\alpha^2).
\]

(4.9)

By means of illustration, we shall consider the infinitesimal noncommutative modification in the dispersion relation. Thus, if we additionally take the on-shell limit, i.e. \( |\vec{p}| = |\theta| \sqrt{\vec{p}^2} \rightarrow m |\theta| \), we can write the dispersion relation in the following simple form

\[
\omega^2 \simeq \vec{p}^2 + m^2 + \frac{1}{1296 \pi m} \lambda^2 \left( 86 + \frac{441}{2} \log 3 \right) - \frac{43}{1296 \pi m^3} \lambda^2 \theta + O (\lambda^2 \theta).
\]

(4.10)

Immediately we observe two features from the expression (4.10). First, we find a correction for the mass as \( m_{\text{eff}}^2 = m^2 \left[ 1 + \frac{\lambda^2}{\pi m^3} \left( \frac{43}{648} + \frac{49}{288} \log 3 \right) \right] \), meaning that the particle gets heavier. Second, the last term shows the presence of a UV/IR instability (with a 1/\( \theta \) behavior) caused by noncommutative perturbative effects.

5 Propagating modes charged scalar fields

In addition, we now consider the obtained results (3.22) and (3.23) (the gauge field effective action was obtained in [31]), so that we can propose the following effective Lagrangian for
the charged scalar fields coupled with a higher derivative Chern-Simons field

\begin{equation}
\mathcal{L} = (\mathcal{D}_\mu \Phi) \dagger \star \mathcal{D}^\mu \Phi - m^2 \Phi \dagger \star \Phi + \frac{m}{2} \varepsilon^{\mu\nu\sigma} A_\mu \left( 1 + \frac{\Box}{m^2} \right) \partial_\nu A_\sigma \\
- \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + \frac{me}{3} \varepsilon^{\mu\nu\sigma} A_\mu \star A_\nu \star A_\sigma + \partial^\mu \tau \star D_\mu \zeta, \tag{5.1}
\end{equation}

where the covariant derivatives are defined as such \( \mathcal{D}_\mu = \partial_\mu - ie [A_\mu, \_]. \) Based on the Lagrangian (5.1) we will study the dynamics of the scalar and gauge fields.

Notice that the model described by the Lagrangian (5.1) has a non-gauge invariant contribution, given by the higher derivative (HD) term. The generation of higher derivative terms and derivative couplings within the NC Chern-Simons theory were considered in Ref. [31]. Moreover, our interest in exploring the features of this HD term into the propagator is motivated by the possibility of finding non-trivial effects into the whole NC Chern-Simons-Higgs theory (5.1). For instance, as we will shortly show (see Sect 5.2) this solely HD contribution is responsible for obtaining nontrivial outcomes for the pure NC CS theory (which is a free theory without the HD term), so its effect on the complete theory is expected to be rather interesting. Moreover, it is easy to see that the commutative limit of this NC HD Chern-Simons theory is also a free theory.

We shall now approach two physical situations in the NC HD-Chern-Simons-Higgs theory: we consider first the dispersion relation of the scalar fields, where the two-point function renormalization takes place as before; second, we analyse the dispersion relation for the gauge field.

### 5.1 Dispersion relation charged scalar fields

For the one-loop self-energy contribution, we have the following two diagrams (Fig. 6), the whole contribution reads

\[
\Sigma (p) = \Sigma^{(a)} (p) + \Sigma^{(b)} (p),
\]

\[
= (\sqrt{2})^2 ie^2 \int \frac{d^3q}{(2\pi)^3} i\mathcal{D}_{\mu\nu} (q) \left[ \frac{(2p - q)^\mu (2p - q)^\nu}{(q - p)^2 - m^2} - 2\eta^{\mu\nu} \right] \sin^2 \left[ \frac{p \times q}{2} \right]. \tag{5.2}
\]

An important remark follows from (5.2), we observe that only the symmetric part of the gauge field propagator

\[
i\mathcal{D}_{\mu\nu} (k) = m \frac{ik^{\mu\nu}\xi}{k^2} \]}

\[
= \frac{i\varepsilon^{\mu\nu\lambda} k^\lambda}{k^2} + \xi \frac{k_\mu k_\nu}{k^4}. \tag{5.3}
\]
contributes at this perturbative order. Hence, only a non-physical gauge-dependent contribution is present in the case of a Chern-Simons gauge field. By completeness, we will discuss below the result for the NC Maxwell-Chern-Simons theory.

Nonetheless, making use of the propagator (5.3) and rewriting (5.2) with help of Feynman parametrization, we obtain

$$
\Sigma(p) = 2i\xi e^2 \mu^{3-\omega} \int \frac{d^d Q}{(2\pi)^d} \left[8 \int_0^1 dx (1-x) \left((p.Q)^2 + x^2 p^4 \right) \frac{1}{(Q^2 + x(1-x)p^2 - xm^2)} \right. \\
- 4 \int_0^1 dx \frac{xp^2}{(Q^2 + x(1-x)p^2 - xm^2)} + \frac{1}{Q^2 - m^2} \left[ \frac{2}{Q^2} \sin^2 \left( \frac{p \times Q}{2} \right) \right].
$$

(5.4)

Now, it is convenient to compute separately the planar from the non-planar contributions from (5.4), this is achieved by means of $$2 \sin^2 \left[ \frac{p \times Q}{2} \right] = 1 - \cos (p \times Q)$$. We have, for the planar contribution

$$
\Sigma_p(p) = -\frac{\xi e^2 m}{8\pi} \left[2 \beta \int_0^1 dx (1-x) \left[ \frac{1}{(x-x(1-x)\beta)\frac{1}{2}} - x^2 \beta \frac{1}{(x-x(1-x)\beta)\frac{1}{2}} \right] \right. \\
- 4\beta \int_0^1 dx \left[ \frac{1}{x-x(1-x)\beta}\frac{1}{2} + 2 \right].
$$

(5.5)

whereas, with help of (A.1) and (A.2), the non-planar contribution reads

$$
\Sigma_{n-p}(p) = \frac{\xi e^2 m}{4\pi} \left[-2 \beta \int_0^1 dx \left[ \frac{e^{-m|\vec{p}|}}{\sqrt{x-x(1-x)\beta}} - \frac{1}{m|\vec{p}|} e^{-m|\vec{p}|} + \frac{2}{m|\vec{p}|} \right] \\
+ \beta \int_0^1 dx (1-x) \left( 1 - x^2 \beta \frac{1 + m|\vec{p}|}{(x-x(1-x)\beta)} \right) \frac{e^{-m|\vec{p}|}}{\sqrt{x-x(1-x)\beta}} \right].
$$

(5.6)

The renormalization here follows closely the steps as of the neutral scalar field, so that the renormalized mass is also given by (4.2). Hence, we can proceed and decompose the one-loop self-energy in terms of its components, $$\Sigma(p^2) = p^2 \Sigma_1(p^2) + m^2 \Sigma_2(p^2)$$. Now, since the renormalization is performed on-shell, we have that these components are simply written as

$$
\Sigma_1(m^2) = -\frac{\xi e^2}{4\pi} \frac{1}{m} \left[ \frac{1}{m|\vec{p}|} - 1 - \frac{e^{-m|\vec{p}|}}{m|\vec{p}|} \right],
$$

(5.7)

$$
\Sigma_2(m^2) = -\frac{\xi e^2}{4\pi} \frac{1}{m} \left[ 1 + \frac{1}{m|\vec{p}|} e^{-m|\vec{p}|} - \frac{2}{m|\vec{p}|} \right].
$$

(5.8)

As we have already discussed, the renormalized mass is given by (4.2) $$m_{ren} = m \sqrt{1 + \Sigma_2(m^2)}$$, so the dispersion relation for the charged scalar fields are written as

$$
\omega^2 = \vec{p}^2 + m_{ren}^2 \simeq \vec{p}^2 + m^2 + m^2 (\Sigma_1(m^2) + \Sigma_2(m^2)) + O(\alpha^2).
$$

(5.9)

Finally, making use of (5.7) and (5.8), and recalling that at the on-shell limit we have $$|\vec{p}| \to m|\theta|$$, we thus find

$$
\omega^2 \simeq \vec{p}^2 + m^2 + \frac{\xi e^2}{4\pi} \frac{1}{m\theta} + O(\alpha^2).
$$

(5.10)
Some conclusions can be depicted from the expression (5.10). We first realize that in this framework no radiative correction for the mass is found. However, we still have a UV/IR instability caused by NC effects, in particular we see that this UV/IR instability is proportional to the gauge parameter $\xi$, and for the Landau gauge, this instability vanishes. We therefore conclude that this sector of the theory is empty of physical content.

In order to illustrate the physical content of the scalar sector, let us consider the usual Maxwell-Chern-Simons propagator (at Landau gauge, $\xi = 0$)

$$i D_{\mu \nu} (k) = \frac{1}{k^2 (k^2 - m^2)} \left( k^2 \eta_{\mu \nu} - k_\mu k_\nu + im \varepsilon_{\mu \nu \lambda} k^\lambda \right), \quad (5.11)$$

this gives the following expression for the scalar field dispersion relation

$$\omega^2 \simeq \tilde{p}^2 + m^2 - \frac{e^2 m}{\pi} \left( 1 + \frac{e^{-m^2 \theta}}{m^2 \theta} \right) + \mathcal{O} (e^2 \theta), \quad (5.12)$$

where we realize that in this framework of a Maxwell-Chern-Simons gauge field, a radiative correction for the mass of the scalar field is found, so that the effective mass reads

$$m_{\text{eff}}^2 = m^2 \left[ 1 - \frac{1}{\pi m^2} \right].$$

In the same way, we also have a UV/IR instability caused by NC effects. All these effects present in (5.12) are caused by the symmetric component of the propagator (5.11).

### 5.2 Dispersion relation gauge field

In general, we can consider that the self-energy $\Pi$-function in a three dimensional non-commutative spacetime has the following tensor form [29]

$$\Pi^{\mu \nu} = \left( \eta^{\mu \nu} - \frac{p^\mu p^\nu}{p^2} \right) \Pi_e + \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2} \tilde{\Pi}_e + i \Pi_0^A \varepsilon^{\mu \nu \lambda} p_\lambda + \Pi_0^S \left( \tilde{p}^\mu u^\nu + \tilde{p}^\nu u^\mu \right), \quad (5.13)$$

where we regard the basis composed by the vectors $p^\mu$, $\tilde{p}^\mu$ and $u_\mu = \epsilon_{\mu \alpha \beta} p^\alpha \tilde{p}^\beta$; moreover, the form factors $\Pi_e$, $\tilde{\Pi}_e$, $\Pi_0^A$ and $\Pi_0^S$ are determined by means of the following relations

$$\Pi_e = \eta_{\mu \nu} \Pi^{\mu \nu} - \frac{\tilde{p}_\mu \tilde{p}_\nu}{\tilde{p}^2} \Pi^{\mu \nu}, \quad (5.14)$$

$$\tilde{\Pi}_e = - \eta_{\mu \nu} \Pi^{\mu \nu} + 2 \frac{\tilde{p}_\mu \tilde{p}_\nu}{\tilde{p}^2} \Pi^{\mu \nu}, \quad (5.15)$$

$$\Pi_0^A = \frac{i}{2p^2} \epsilon_{\mu \nu \alpha} p^\alpha \Pi^{\mu \nu}, \quad (5.16)$$

$$\Pi_0^S = - \frac{1}{2 \tilde{p}^4 \tilde{p}^2} \left( u_\mu \tilde{p}_\nu + u_\nu \tilde{p}_\mu \right) \Pi^{\mu \nu}. \quad (5.17)$$

3This is justifiable as an example, since we have a symmetric operator multiplying the gauge field propagator in (5.2), so the skew-symmetric nature of the pure Chern-Simons propagator gives a vanishing result.
Figure 7: One-loop self-energy graphs for the gauge field: (a) scalar loop, (b) scalar tadpole loop, (c) gauge loop, (d) ghost loop.

Moreover, the general expression of the complete propagator for the CS gauge field (augmented with the HD term) can be put into the form [29]

\[
\begin{align*}
    iD_{\mu\nu} &= \frac{-\Pi_e + \tilde{\Pi}_e}{D} \eta_{\mu\nu} + \left(\frac{\Pi_e + \tilde{\Pi}_e}{D} + \frac{\xi}{p^2}\right) \frac{p_\mu p_\nu}{p^2} + \frac{\tilde{\Pi}_e p_\mu p_\nu}{D p^2} \\
    &+ \frac{\Pi_0^8}{D} (\tilde{p}_\mu u_\nu + u_\mu \tilde{p}_\nu) + \frac{m\left(1 - \frac{p^2}{m^2}\right) + \Pi_0^6}{D} i\epsilon_{\mu\nu\lambda} p^\lambda.
\end{align*}
\]

(5.18)

where by simplicity we have defined the quantity at the denominator

\[
D = \Pi_e \left(\Pi_e + \tilde{\Pi}_e\right) + p^2 \left[(p^2 \Pi_0^8)^2 - m\left(1 - \frac{p^2}{m^2}\right) + \Pi_0^6\right]^2.
\]

(5.19)

Before discussing the structure of the complete propagator we shall now compute the one-loop contribution to the polarization tensor.

The one-loop correction to the gauge field self-energy is given by the four contributions depicted in Fig. 7: the first and second contributions are from the scalar loops, graph (a) corresponds to the scalar loop, while graph (b) to the scalar tadpole loop, graph (c) corresponds to the gauge loop, and graph (d) corresponds to the ghost loop, respectively. Their explicit expressions are written as

\[
\begin{align*}
    \Pi_{\mu\nu}^{(a+b)}(p) &= -2ie^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2 - m^2} \left[\frac{(2q - p)^\mu (2q - p)^\nu}{(q - p)^2 - m^2} - 2\eta^{\mu\nu}\right] \sin^2 \left[\frac{p \times q}{2}\right], \\
    \Pi_{\mu\nu}^{(c)}(p) &= -im^4e^2 \int \frac{d^3q}{(2\pi)^3} \frac{q^\nu (q - p)^\mu + q^\mu (q - p)^\nu}{((q - p)^2 - m^2) q^2 (q^2 - m^2)} \sin^2 \left[\frac{p \times q}{2}\right], \\
    \Pi_{\mu\nu}^{(d)}(p) &= 2ie^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2 - (q - p)^2} \sin^2 \left[\frac{p \times q}{2}\right].
\end{align*}
\]

(5.20) - (5.22)

In the first line, by simplicity of notation, we have summed the two contributions coming from the scalar loops.

An interesting remark is now in place. In particular, in the limit \(m \to \infty\), that corresponds to the situation where the HD contribution is removed, we find that the scalar contributions \((5.20)\) are equal to zero, while the contributions \((5.21)\) and \((5.22)\) sum to

\[
\Pi_{\mu\nu}^{(c+d)}(p) = ie^2 \int_0^1 dy \int \frac{d^3Q}{(2\pi)^3} \frac{[Q^\nu p^\mu - p^\nu Q^\mu]}{[Q^2 + yp^2]^2} \sin^2 \left[\frac{p \times Q}{2}\right] = 0.
\]

(5.23)
This is a known result where we see that the NC Chern-Simons theory is a free theory \[24\]. However, we conclude that the HD contributions are sufficient to modify the character of the polarization tensor of the pure NC Chern-Simons theory, rendering a nonvanishing result.

Moreover, in order to evaluate the form factors (5.14)–(5.17) it is easier to compute the contraction of the expressions (5.20)–(5.22) with the operators $\eta_{\mu\nu}$, $\tilde{p}^\mu \tilde{p}^\nu / \tilde{p}^2$, $\varepsilon^{\mu\nu\lambda} p_\lambda$, and $(u^\mu \tilde{p}^\nu + u^\nu \tilde{p}^\mu)$. Surprisingly, we immediately find out that the following projections

$$
(u^\mu \tilde{p}^\nu + u^\nu \tilde{p}^\mu) \Pi_{\mu\nu}^{(a+b)} = (u^\mu \tilde{p}^\nu + u^\nu \tilde{p}^\mu) \Pi_{\mu\nu}^{(c)} = (u^\mu \tilde{p}^\nu + u^\nu \tilde{p}^\mu) \Pi_{\mu\nu}^{(d)} = 0,
$$

and

$$
\varepsilon^{\mu\nu\lambda} p_\lambda \Pi_{\mu\nu}^{(a+b)} = \varepsilon^{\mu\nu\lambda} p_\lambda \Pi_{\mu\nu}^{(c)} = \varepsilon^{\mu\nu\lambda} p_\lambda \Pi_{\mu\nu}^{(d)} = 0,
$$

vanish identically. In particular, we have made use of the identities $p.u = 0$ and $p.\tilde{p} = 0$ in order to obtain the previous results. So, the form factors $\Pi^A_0$ and $\Pi^S_0$, Eqs. (5.16) and (5.17), respectively, vanish at this order. Hence, we are left to compute only the projection of (5.20)–(5.22) onto the operators $\eta_{\mu\nu}$ and $\tilde{p}^\mu \tilde{p}^\nu / \tilde{p}^2$.

The planar and non-planar parts of the contractions $\eta_{\mu\nu} \Pi_{\mu\nu}$ ($p$) and $\tilde{p}^\mu \tilde{p}^\nu / \tilde{p}^2 \Pi_{\mu\nu}$ ($p$) are computed in the Appendix C.

Before computing the form factors $\Pi_e$ and $\tilde{\Pi}_e$, Eqs. (5.14) and (5.15), respectively, let us now establish the renormalizability of the theory. First, note that due to the fact that the form factors $\Pi^A_0$ and $\Pi^S_0$ are null, the expression of the complete propagator (5.18) reads

$$
i D_{\mu\nu} = -\frac{\Pi_e + \tilde{\Pi}_e}{D} \eta_{\mu\nu} + \frac{\xi}{p^2} \frac{p_\mu p_\nu}{p^2} + \frac{\tilde{\Pi}_e \tilde{p}_\mu \tilde{p}_\nu}{\tilde{p}^2} + m \left(1 - \frac{p^2}{m^2}\right) \frac{1}{D} i \varepsilon_{\mu\nu\lambda} p_\lambda,$$

with a simplified form $D = \Pi_e (\Pi_e + \tilde{\Pi}_e) - p^2 m^2 \left(1 - \frac{p^2}{m^2}\right)^2$. It is sufficient for our interest, in defining the physical pole of the NC HD Chern-Simons sector, to consider the CP odd term from (5.26)

$$
i D_{\mu\nu} \sim m \left(1 - \frac{p^2}{m^2}\right) \frac{1}{p^2} \frac{m \left(1 - \frac{p^2}{m^2}\right)}{p^2 \Pi_e (\Pi_e + \tilde{\Pi}_e) - m^2 \left(1 - \frac{p^2}{m^2}\right)^2} i \varepsilon_{\mu\nu\lambda} p_\lambda,$$

where we have performed the scaling $\{\Pi_e, \tilde{\Pi}_e\} \rightarrow p^2 \{\Pi_e, \tilde{\Pi}_e\}$. By a simple analysis and manipulation we can rewrite the expression (5.27) in its renormalized form

$$
i D_{\mu\nu} \sim \frac{m_{\text{ren}} \varepsilon_{\mu\nu\lambda}}{p^2 (p^2 - m_{\text{ren}}^2)} i \varepsilon_{\mu\nu\lambda} p_\lambda,$$

where the physical pole $p^2 = m_{\text{ren}}^2$ is localized so that the renormalized mass of the Chern-Simons gauge field and the respective renormalization constant were defined as

$$m_{\text{ren}} = z_{CP} m, \quad z_{CP} = \frac{1}{\sqrt{1 + \Pi_e (\Pi_e + \tilde{\Pi}_e)}}.$$
In order to finally compute the renormalized mass, it is worth to evaluate the form factors $\Pi_e$ and $\tilde{\Pi}_e$ in the highly noncommutative limit, i.e. $\beta = p^2/m^2 \to 0$ while $\tilde{k}^2$ is kept finite. This allows us to obtain the leading noncommutative effects onto the renormalized mass.

Hence, from the expression (5.14) we find that

$$\Pi_e(p) = \eta_{\mu\nu} \Pi^{\mu\nu}(p) - \frac{\tilde{p}_\mu \tilde{p}_\nu}{\tilde{p}^2} \Pi^{\mu\nu}(p)$$

$$= \frac{e^2}{12\pi} \frac{1}{m^2|\tilde{p}|^3} \left\{ 3 \left( 2 + m|\tilde{p}| \right)^2 e^{-m|\tilde{p}|} - \left( 12 + m^3|\tilde{p}|^3 \right) \right\}. \quad (5.30)$$

Moreover, due to the structure of the renormalization constant $Z_{CP}$ in (5.29), it shows convenient to compute the following combination

$$\Pi_e + \tilde{\Pi}_e = \frac{\tilde{p}_\mu \tilde{p}_\nu}{\tilde{p}^2} \Pi^{\mu\nu}(p)$$

$$= -\frac{e^2}{24\pi} \frac{1}{m^2|\tilde{p}|^3} \left\{ -24 + m^3|\tilde{p}|^3 + 3 \left( 8 + 8m|\tilde{p}| + 8m^2|\tilde{p}|^2 + 5m^3|\tilde{p}|^3 \right) e^{-m|\tilde{p}|} \right\}. \quad (5.31)$$

We can finally make use of the results (5.30) and (5.31) into the expression (5.29) in order to compute the renormalized mass

$$m_{\text{ren}} \simeq m - \frac{1}{16\pi^2} \frac{1}{\kappa^2 m|\tilde{p}|^2} + \frac{5}{32\pi^2} \frac{1}{\kappa^2 |\tilde{p}|} - \frac{1}{8\pi^2} \frac{m}{\kappa^2} + \mathcal{O} \left( \frac{m|\tilde{p}|}{\kappa} \right), \quad (5.32)$$

when writing this expression we have used the tree-level relation $e^2 \sim m/\kappa$. Moreover, we simplified the above terms by taking the leading noncommutative perturbations $m|\tilde{p}| \ll 1$. Furthermore, recalling that at the on-shell limit we have $|\tilde{p}| \to m|\theta|$, the dispersion relation for the Chern-Simons gauge field reads

$$\omega^2 \simeq |\tilde{p}|^2 + m^2 - \frac{1}{4\pi^2} \frac{m^2}{\kappa^2} - \frac{1}{8\pi^2} \frac{1}{\kappa^2 m^2} \frac{1}{\theta^2} + \frac{5}{16\pi^2} \frac{1}{\kappa^2} + \mathcal{O} \left( \frac{m^2\theta}{\kappa} \right). \quad (5.33)$$

In particular, we see that a correction is obtained for the massive mode, $m_{\text{eff}} = m^2 \left[ 1 - \frac{1}{4\pi^2} \frac{1}{\kappa^2} \right]$, and that a leading $1/\theta^2$ and subleading $1/\theta$ UV/IR mixing are present in this case.

### 6 Concluding remarks

In this paper we have considered the Yukawa field theory for neutral and charged scalar fields in the framework of a noncommutative three-dimensional spacetime. In order to study the dynamics of the scalar fields we have followed the effective action approach by integrating out the fermionic fields. The two-point function renormalization for both cases were carefully established, allowing us to define the physical dispersion relation and hence study the UV/IR anomaly.

Initially we established the main properties of the effective action approach for noncommutative field theories [31,39]. In particular, we defined the noncommutative Yukawa action for a neutral scalar field and computed its effective action, and showed that besides its kinetic terms, only a cubic interaction $\phi^3$ is present at the leading order $\mathcal{O}(m^0)$, and that $\phi^3$ (for
$n > 3$) interacting terms are absent in its effective action when defined in a three-dimensional spacetime. Additionally, we considered the case of charged scalar fields, where a gauge field is need in order to ensure a local gauge invariance. Hence, by computing the respective cubic and quartic terms, we found the minimal interacting parts of the NC Higgs model, i.e. charged scalar fields minimally coupled to a gauge field.

In order to illustrate the behavior of the neutral scalar field in a noncommutative three-dimensional spacetime, we considered the effective action with an additional derivative coupling supplementing the cubic interacting term. We then determined the respective Feynman rules and the one-loop correction to the self-energy, also the renormalized mass was established. After computing the planar and non-planar contributions to the one-loop self-energy we found the physical dispersion relation, where we have showed a correction to the bare mass and the presence of a $1/\theta$ singular term, as a result of UV/IR instability caused by noncommutative effects.

By completeness, we considered the effective action for charged scalar fields coupled with a dynamical Chern-Simons gauge field. We have considered a higher-derivative contribution to the kinetic term of the gauge field in order to discuss some novel features in the gauge field sector. By computing first the one-loop correction to the scalar field self-energy, we showed that this contribution is non-physical due to its dependence on the gauge parameter $\xi$ onto the dispersion relation; by means of illustration we presented how the dispersion relation is modified and became physical if a Maxwell-Chern-Simons propagator is considered instead of the HD-Chern-Simons propagator. Afterwards, we established the renormalization and analysed the one-loop dispersion relation for the gauge field. In the resulting expression we found the presence of a correction to the bare mass parameter, and that a leading $1/\theta^2$ and subleading $1/\theta$ UV/IR mixing are present in this case.

It is worth of mention that all the previous analysis were considered in the highly noncommutative limit of the Chern-Simons-Higgs theory, that also corresponds to the low-momenta limit. As we have discussed, this kind of effective models are nowadays of major interest for physical application in planar materials, in particular in the description of new materials in the framework of condensed matter physics, which allows the use of effective low-energy models. Moreover, this model can be an appropriate field-theoretical framework for study of Aharonov-Bohm effect. This framework also works in a way to provide a consistent scenario to scrutinize theoretical features in order to set stringent bounds on deviations of known field theory properties, for instance standard model symmetries.

Acknowledgements

We would like to thank M.M. Sheikh-Jabbari for discussion. R.B. thankfully acknowledges CAPES/PNPD for partial support, Project No. 23038007041201166.

A Non-planar integrals

Along the paper we have made use of some results involving momentum integration. We shall recall some of these results, in particular those involving a non-planar factor. The
simplest non-planar integration reads
\[ \int \frac{d^q q}{(2\pi)^q} \frac{1}{(q^2 - s^2)^a} e^{i k \wedge q} = \frac{2i}{(4\pi)^\frac{q}{2}} \frac{1}{\Gamma(a) (s^2)^{a-\frac{q}{2}}} \left( \frac{|k|}{|s|} \right)^{a-\frac{q}{2}} K_{a-\frac{q}{2}} \left( |k|/s \right) . \] (A.1)

Next, we have the tensor integration
\[ \int \frac{d^q q}{(2\pi)^q} \frac{q^\mu q^\nu}{(q^2 - s^2)^a} e^{i k \wedge q} = \eta^{\mu\nu} F_a + \frac{\tilde{k}^\mu \tilde{k}^\nu}{k^2} G_a, \] (A.2)

where we have introduced the following quantities
\[ \{ F_a, G_a \} = \frac{i}{(4\pi)^\frac{q}{2}} \frac{1}{\Gamma(a) (s^2)^{a-\frac{q}{2}}} \{ f_a, g_a \}, \] (A.3)

with
\[ f_a = \left( \frac{s|\tilde{k}|}{2} \right)^{a-1-\frac{q}{2}} K_{a-1-\frac{q}{2}} \left( |\tilde{k}|/s \right), \] (A.4)
\[ g_a = (2a - 2 - \omega) \left( \frac{s|\tilde{k}|}{2} \right)^{a-1-\frac{q}{2}} K_{a-1-\frac{q}{2}} \left( |\tilde{k}|/s \right) - 2 \left( \frac{s|\tilde{k}|}{2} \right)^{a-\frac{q}{2}} K_{a-\frac{q}{2}} \left( |\tilde{k}|/s \right) . \] (A.5)

B Effective action

We summarize in this appendix some useful and important long expressions related to the computation of the effective action in the Sec. 3.

B.1 \( \phi \phi A \) contribution

The three distinct contributions for the one-loop effective action for the \( \phi \phi A \) vertex (3.17) are given by
\[ \Gamma^\mu_{(i)} (x_1, x_2, x_3) = \frac{g^2 e}{3} \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 p_3}{(2\pi)^3} \left( \delta (p_1 + p_2 + p_3) \right) \exp \left[ -i (p_1 \cdot x_1 + p_2 \cdot x_2 + p_3 \cdot x_3) \right] \] \[ \times \exp \left[ -\frac{i}{2} (p_1 \times p_2 + p_1 \times p_3 + p_2 \times p_3) \right] \Xi^\mu_{(i)} (p_1, p_2), \] (B.1)

in which \( p \cdot q = \theta^\mu \nu p_\mu q_\nu \), where the explicit expressions \( \Xi^\mu_{(i)} \) are readily obtained from the graph Fig. 3.

Due to the structure of the momentum integral, and our interest in finding the leading contribution for the interaction terms, we see that those correspond to the overall \( \mathcal{O} (m^{-1}) \) terms, that are precisely the \( \mathcal{O} (m^2) \) and \( \mathcal{O} (m^0) \) terms in the numerator of these expressions, that will result in a (linear) derivative coupling. Hence, from the denominator of the above
expressions we find that

\[ tr[\ldots]^a \simeq 2m^2 (p - k - 3s) + \frac{2}{3}q^2 (3p - 3k - 5s), \quad (B.2) \]

\[ tr[\ldots]^b \simeq 2m^2 (p - k - 3s) - \frac{2}{3}q^2 (3k + 5s + p), \quad (B.3) \]

\[ tr[\ldots]^c \simeq 2m^2 (p - k - 3s) + \frac{2}{3}q^2 (k + 3p - 5s), \quad (B.4) \]

where we have used the identity \( q^\mu q^\nu \to \frac{1}{\omega^2} q^2 \eta^{\mu\nu} \). Therefore, with such considerations we can compute the momentum integration straightforwardly, and by realizing that \( \Gamma (2 - \frac{\omega}{2}) \) and \( \Gamma (3 - \frac{\omega}{2}) \) have no poles when \( \omega \to 3^+ \), we obtain

\[ \Xi_{(a)}^\mu (p, k) = \frac{i}{8\pi} \int d\xi \left[ \frac{(3p - 3k - 5s)}{(m^2 - A^2 (p, k))^\frac{3}{2}} - m^2 \frac{(p - k - 3s)}{(m^2 - A^2 (p, k))^\frac{3}{2}} \right], \quad (B.5) \]

\[ \Xi_{(b)}^\mu (p, k) = \frac{i}{8\pi} \int d\xi \left[ -\frac{(3k + 5s + p)}{(m^2 - A^2 (p, k))^\frac{3}{2}} - m^2 \frac{(p - k - 3s)}{(m^2 - A^2 (p, k))^\frac{3}{2}} \right], \quad (B.6) \]

\[ \Xi_{(c)}^\mu (p, k) = \frac{i}{8\pi} \int d\xi \left[ \frac{(k + 3p - 5s)}{(m^2 - A^2 (p, k))^\frac{3}{2}} - m^2 \frac{(p - k - 3s)}{(m^2 - A^2 (p, k))^\frac{3}{2}} \right]. \quad (B.7) \]

Finally, we can now determine the leading contributions by taking the long wavelength limit, i.e., the approximation \( m^2 \gg A^2 (p, k) \) in the above expressions. Hence, proceeding with this calculation and computing the remaining integration, we find

\[ \Xi_{(a)}^\mu (p, k) \simeq \frac{i}{12\pi |m|} (p - k)^\mu, \quad \Xi_{(b)}^\mu (p, k) \simeq -\frac{i}{12\pi |m|} (k + 2p)^\mu, \quad (B.8) \]

\[ \Xi_{(c)}^\mu (p, k) \simeq \frac{i}{12\pi |m|} (2k + p)^\mu. \quad (B.9) \]

### B.2 \( \phi\phi AA \) contribution

The contribution proportional to the structure of the \( \phi\phi AA \) vertex \([3.23]\) is given by six terms which have the general structure

\[ \Gamma_{(i)}^{\mu\nu} (x_1, x_2, x_3, x_4) = -\frac{g^2 e^2}{4} \int \frac{d^3 p_1}{(2\pi)^3} \cdots \frac{d^3 p_4}{(2\pi)^3} (2\pi)^3 \delta (p_1 + p_2 + \cdots + p_4) \]

\[ \times \exp [-i (p_1 x_1 + \cdots + p_4 x_4)] \exp \left( -\frac{i}{2} \sum_{i<j} p_i \times p_j \right) \Xi_{(i)}^{\mu\nu} (p_1, p_2, p_3), \quad (B.10) \]

where the integral expressions \( \Xi_{(i)}^{\mu\nu} \) are readily obtained from the graph Fig. 4.

It should be remarked that due to our interest in obtaining the last piece of the (minimal) coupling between scalar and gauge fields, we shall now concentrate in those contributions independent of the external momenta in order to complete the derivation of the effective action. In this way, it is easy to show the following equality among the contributions

\[ \Xi_{(a)}^{\mu\nu} (p, k, r) = \Xi_{(c)}^{\mu\nu} (p, k, r) = \Xi_{(d)}^{\mu\nu} (p, k, r) = \Xi_{(f)}^{\mu\nu} (p, k, r), \quad (B.11) \]

\[ \Xi_{(b)}^{\mu\nu} (p, k, r) = \Xi_{(e)}^{\mu\nu} (p, k, r), \quad (B.12) \]
where we find that
\[
\Xi^{\mu\nu}_{(a)} (p, k, r) = \Gamma (4) \int d\zeta \int \frac{d^3 q}{(2\pi)^3} \frac{\text{tr} [(q' + m) (q' + m) \gamma^\mu (q' + m) \gamma^\nu]}{[q^2 + M^2 - m^2]^4}, \tag{B.13}
\]
\[
\Xi^{\mu\nu}_{(b)} (p, k, r) = \Gamma (4) \int d\zeta \int \frac{d^3 q}{(2\pi)^3} \frac{\text{tr} [(q' + m) (q' + m) \gamma^\mu (q' + m) \gamma^\nu]}{[q^2 + M^2 - m^2]^4}, \tag{B.14}
\]
with the following definition
\[
\int d\zeta = \int_0^1 dx \int_0^{1-x} dz \int_0^{1-x-z} dw,
\]
\[
M^2 (p, k, r) = - (xp - (z + w) k - wr)^2 + xp^2 + zk^2 + w (r + k)^2. \tag{B.15}
\]
Now the numerator of both expressions can be computed with help of the results \(q^\mu q^{\nu} \to \frac{1}{\omega} q^2 \eta^{\mu\nu}\) and \(q^\mu q^{\nu} q^\sigma q^\rho \to \frac{1}{\omega (\omega + 2)} (q^2)^2 (\eta^{\mu\rho} \eta^{\sigma\nu} + \eta^{\mu\sigma} \eta^{\rho\nu} + \eta^{\mu\nu} \eta^{\rho\sigma})\). The resulting expressions are
\[
\text{tr} [...]^a = - \frac{10}{\omega (\omega + 2)} (q^2)^2 \eta^{\mu\nu} + 12m^2 \frac{1}{\omega} q^2 \eta^{\mu\nu} + 2m^4 \eta^{\mu\nu}, \tag{B.16}
\]
\[
\text{tr} [...]^b = \frac{30}{\omega (\omega + 2)} (q^2)^2 \eta^{\mu\nu} + 4m^2 \frac{1}{\omega} q^2 \eta^{\mu\nu} + 2m^4 \eta^{\mu\nu}. \tag{B.17}
\]
Once again we see that the momentum integration is finite, since \(\Gamma (2 - \omega/2), \Gamma (3 - \omega/2)\) and \(\Gamma (4 - \omega/2)\) have no poles when \(\omega \to 3^+\). Hence, the analysis follows as before, and considering the expansion \(m^2 \gg M^2 (p, k, r)\), we find that the leading contributions from Eqs. (B.13) and (B.14) at \(\mathcal{O} (m^{-1})\) terms are
\[
\Xi^{\mu\nu}_{(a)} (p, k, r) \approx - \frac{i}{12\pi |m|} \eta^{\mu\nu}, \quad \Xi^{\mu\nu}_{(b)} (p, k, r) \approx \frac{i}{6\pi |m|} \eta^{\mu\nu}. \tag{B.18}
\]

**C  Dispersion relation**

Therefore, we can compute with no problems the contraction of \((5.20) - (5.22)\) to the operators \(\eta^{\mu\nu}\), resulting the total contribution projection
\[
\eta^{\mu\nu} \Pi_{\mu\nu} (p) = 2ie^2 \mu^{3-\omega} \int \frac{d^3 q}{(2\pi)^3} \left\{ -m^4 \frac{1}{q^2 (q^2 - m^2)} \frac{1}{((q - p)^2 - m^2)} \right. \\
- m^4 \frac{p.(q - p)}{q^2 (q - p)^2 ((q - p)^2 - m^2)} + \frac{1}{q^2} + \frac{p.(q - p)}{q^2 (q - p)^2} \\
- \frac{2}{q^2 (q - p)^2 ((q - p)^2 - m^2)} + 2\omega \frac{1}{q^2 - m^2} \right\} \sin^2 \left[ \frac{p \times q}{2} \right]. \tag{C.1}
\]
while the projection \(\frac{\hat{p} \hat{p}^{\mu\nu}}{p^2} \Pi_{\mu\nu}\) yields the contribution
\[
\frac{\hat{p} \hat{p}^{\mu\nu}}{p^2} \Pi_{\mu\nu} (p) = 2ie^2 \mu^{3-\omega} \int \frac{d^3 q}{(2\pi)^3} \left\{ -m^4 \frac{q_{\mu} q_{\nu}}{q^2 (q - p)^2 ((q - p)^2 - m^2)} \right. \\
+ \frac{1}{q^2 (q - p)^2} - 4 \frac{q_{\mu} q_{\nu}}{(q^2 - m^2) ((q - p)^2 - m^2)} + \frac{2}{q^2 - m^2} \right\} \sin^2 \left[ \frac{p \times q}{2} \right]. \tag{C.2}
\]
Let us now compute separately the planar contribution from the non-planar one of the above projections, this is achieved by the identity \(2 \sin^2 \frac{p \cdot Q}{2} = 1 - \cos (p \times Q)\). First, by following the previous procedure to compute the momentum integration, we find for the planar part of (C.1) and (C.2), the following expressions

\[
(\gamma^{\mu \nu} \Pi_{\mu \nu})_p (p) = -\frac{e^2 m}{8\pi} \left\{ \frac{1}{2} \int_0^1 dz \int_0^{1-z} dx \frac{1}{(1 - x) - z (1 - z) \beta^2} + \frac{3}{4} \beta \int_0^1 dz \int_0^{1-z} dx \int_0^{1-z-x} dw \frac{x + z}{\Delta^5} \right. \\
- \left. \beta \int_0^1 dy \sqrt{-y (1 - y) \beta} \right\},
\]

and

\[
\left( \frac{\tilde{p}^{\mu} \tilde{p}^{\nu}}{\tilde{p}^2} \Pi_{\mu \nu} \right)_p = \frac{e^2 m}{8\pi} \left\{ -\frac{1}{4} \int_0^1 dz \int_0^{1-z} dx \int_0^{1-z-w} dw \frac{1}{\Delta^3} \\
- \int_0^1 dy \sqrt{-y (1 - y) \beta} + 4 \int_0^1 dy \sqrt{1 - y (1 - y) \beta} - 4 \right\},
\]

where we have introduced the notation \(\Delta^2 = (z + w) - (x + w) (1 - (x + w)) \beta\), and again \(\beta = p^2/m^2\). Next, we compute the non-planar contributions with help of the identities (A.1) and (A.2),

\[
(\eta^{\mu \nu} \Pi_{\mu \nu})_{n-p} (p) = \frac{e^2 m}{16\pi} \left\{ \int_0^1 dz \int_0^{1-z} dx \frac{e^{-m|\tilde{p}|\sqrt{(1-z)(1-z)(1-z)\beta}}}{((1-x)-z(1-z)\beta)^2} \right. \\
\left. \left( 1 + m|\tilde{p}|\sqrt{(1-x)-z(1-z)\beta} \right) + \frac{1}{2} \beta \int_0^1 dz \int_0^{1-z} dx \int_0^{1-z-x} dw \frac{x + z}{\Delta^5} (3 + 3m|\tilde{p}| + \Delta^2 m^2 \tilde{p}^2) e^{-m|\tilde{p}|} \\
- 2\beta \int_0^1 dy \sqrt{-y(1-y)\beta} - 8 \int_0^1 dy \left[ \sqrt{1 - y (1 - y) \beta} - \frac{1}{m|\tilde{p}|} \right] e^{-m|\tilde{p}|\sqrt{1-y(1-y)\beta}} \\
- 2\beta \int_0^1 dy (2y - 1)^2 \frac{e^{-m|\tilde{p}|\sqrt{1-y(1-y)\beta}}}{\sqrt{1 - y (1 - y) \beta}} - \frac{4}{m|\tilde{p}|} (6e^{-m|\tilde{p}|} + 1) \right\},
\]

and

\[
\left( \frac{\tilde{p}^{\mu} \tilde{p}^{\nu}}{\tilde{p}^2} \Pi_{\mu \nu} \right)_{n-p} = \frac{e^2 m}{8\pi} \left\{ \frac{1}{4} \int_0^1 dz \int_0^{1-z} dw \int_0^{1-z-w} dx \frac{1 + m|\tilde{p}| - \Delta^2 m^2 \tilde{p}^2}{\Delta^3} \right. \\
\left. e^{-m|\tilde{p}|\sqrt{1-y(1-y)\beta}} \\
+ \int_0^1 dy \sqrt{-y(1-y)\beta} e^{-m|\tilde{p}|\sqrt{-y(1-y)\beta}} \\
- 4 \int_0^1 dy \sqrt{1 - y (1 - y) \beta} e^{-m|\tilde{p}|\sqrt{1-y(1-y)\beta}} - \frac{4}{m|\tilde{p}|} e^{-m|\tilde{p}|} \right\}.
\]

References

[1] H. Yukawa, “On the Interaction of Elementary Particles I,” Proc. Phys. Math. Soc. Jap. 17, 48 (1935).
[2] C. M. G. Lattes, H. Muirhead, G. P. S. Occhialini and C. F. Powell, “Processes Involving Charged Mesons,” Nature 159, 694 (1947).

[3] G. Amelino-Camelia, “Quantum-Spacetime Phenomenology,” Living Rev. Rel. 16, 5 (2013) arXiv:0806.0339 [gr-qc]

[4] S. Hossenfelder, “Minimal Length Scale Scenarios for Quantum Gravity,” Living Rev. Rel. 16, 2 (2013) arXiv:1203.6191 [gr-qc]

[5] M. R. Douglas and N. Nekrasov, “Noncommutative field theory”, Rev. Mod. Phys. 73, 977 (2001) arXiv:hep-th/0106048
R. J. Szabo, “Quantum field theory on noncommutative spaces,” Phys. Rept. 378, 207 (2003) arXiv:hep-th/0109162

[6] I. Hinchliffe, N. Kersting and Y. L. Ma, “Review of the phenomenology of noncommutative geometry,” Int. J. Mod. Phys. A 19, 179 (2004), arXiv:hep-ph/0205040.

[7] P. Nicolini, “Noncommutative Black Holes, The Final Appeal To Quantum Gravity: A Review,” Int. J. Mod. Phys. A 24 (2009) 1229 arXiv:0807.1939 [hep-th].

[8] M. Chaichian, P. Presnajder, M. M. Sheikh-Jabbari and A. Tureanu, “Noncommutative standard model: Model building,” Eur. Phys. J. C 29 (2003) 413 arXiv:hep-th/0107055

[9] G. Amelino-Camelia, “Challenge to Macroscopic Probes of Quantum Spacetime Based on Noncommutative Geometry,” Phys. Rev. Lett. 111 (2013) 101301 arXiv:1304.7271 [gr-qc]

[10] S. Doplicher, K. Fredenhagen and J. E. Roberts, “Space-time quantization induced by classical gravity,” Phys. Lett. B 331 (1994) 39

[11] D. V. Ahluwalia, “Quantum measurements, gravitation, and locality,” Phys. Lett. B 339 (1994) 301 arXiv:gr-qc/9308007.

[12] N. L. Balazs and B. K. Jennings, “Wigner’s function and other distribution functions in mock phase spaces,” Phys. Rep. 104 (1984) 347

[13] S. Minwalla, M. Van Raamsdonk and N. Seiberg, “Noncommutative perturbative dynamics,” JHEP 0002 (2000) 020 arXiv:hep-th/9912072

[14] D. D’Ascanio, P. Pisani and D. V. Vassilevich, “Renormalization on noncommutative torus,” Eur. Phys. J. C 76 (2016) no.4, 180 arXiv:1602.01479 [hep-th]

[15] P. Vitale and J. C. Wallet, “Noncommutative field theories on $R^3_\lambda$: Toward UV/IR mixing freedom,” JHEP 1304 (2013) 115, Addendum: JHEP 1503 (2015) 115 arXiv:1212.5131 [hep-th]

[16] A. Gr, T. Juri and J. C. Wallet, “Noncommutative gauge theories on $R^3_\lambda$: perturbatively finite models,” JHEP 1512 (2015) 045 arXiv:1507.08086 [hep-th]
D. N. Blaschke, E. Kronberger, A. Rofner, M. Schweda, R. I. P. Sedmik and M. Wohlgenannt, “On the Problem of Renormalizability in Non-Commutative Gauge Field Models: A Critical Review,” Fortsch. Phys. 58 (2010) 364 arXiv:0908.0467 [hep-th].

S. Deser, R. Jackiw and S. Templeton, “Topologically Massive Gauge Theories,” Annals Phys. 140 (1982) 372, Annals Phys. 185 (1988) 406 Annals Phys. 281 (2000) 409.

L. Susskind, “The Quantum Hall fluid and noncommutative Chern-Simons theory,” arXiv:hep-th/0101029.

C. Bastos, O. Bertolami, N. C. Dias and J. N. Prata, “Noncommutative Graphene,” Int. J. Mod. Phys. A 28 (2013) 1350064 arXiv:1207.5820 [hep-th].

B. G. Sidharth, “Graphene and high energy physics,” Int. J. Mod. Phys. E 23 (2014) 5, 1450025.

G. H. Chen and Y. S. Wu, “One loop shift in noncommutative Chern-Simons coupling,” Nucl. Phys. B 593 (2001) 562, arXiv:hep-th/0006114.

A. A. Bichl, J. M. Grimstrup, V. Putz and M. Schweda, “Perturbative Chern-Simons theory on noncommutative R**3,” JHEP 0007 (2000) 046, arXiv:hep-th/0004071.

A. K. Das and M. M. Sheikh-Jabbari, “Absence of higher order corrections to noncommutative Chern-Simons coupling,” JHEP 0106 (2001) 028 arXiv:hep-th/0103139.

M. M. Sheikh-Jabbari, “A Note on noncommutative Chern-Simons theories,” Phys. Lett. B 510 (2001) 247 arXiv:hep-th/0102092.

R. Banerjee, T. Shreecharan and S. Ghosh, “Three dimensional noncommutative bosonization,” Phys. Lett. B 662 (2008) 231 arXiv:0712.3631 [hep-th].

E. A. Asano, L. C. T. Brito, M. Gomes, A. Y. Petrov and A. J. da Silva, “Consistent interactions of the 2+1 dimensional noncommutative Chern-Simons field,” Phys. Rev. D 71 (2005) 105005 arXiv:hep-th/0410257.

S. Ghosh, “Bosonization in the noncommutative plane,” Phys. Lett. B 563 (2003) 112 arXiv:hep-th/0303022.

M. Ghasemkhani and R. Bufalo, “Noncommutative Maxwell-Chern-Simons theory: One-loop dispersion relation analysis,” Phys. Rev. D 93 (2016) no.8, 085021 arXiv:1512.04094 [hep-th].

R. Bufalo, T. R. Cardoso and B. M. Pimentel, “Källén-Lehmann representation of noncommutative quantum electrodynamics,” Phys. Rev. D 89 (2014) 8, 085010 arXiv:1404.0941 [hep-th].

R. Bufalo and M. Ghasemkhani, “Higher derivative Chern-Simons extension in the noncommutative QED3,” Phys. Rev. D 91 (2015) 12, 125013 arXiv:1412.1635 [hep-th].

A. F. Ferrari, M. Gomes, J. R. Nascimento, A. Y. Petrov, A. J. da Silva and E. O. Silva, “On the finiteness of the noncommutative supersymmetric Maxwell-Chern-Simons theory,” Phys. Rev. D 77 (2008) 025002, arXiv:0708.1002 [hep-th].
[33] P. K. Panigrahi and T. Shreecharan, “Induced magnetic moment in noncommutative Chern-Simons scalar QED,” JHEP 0502 (2005) 045, arXiv:hep-th/0411132.

[34] D. N. Blaschke, S. Hohenegger and M. Schweda, “Divergences in non-commutative gauge theories with the Slavnov term,” JHEP 0511 (2005) 041, arXiv:hep-th/0510100.

[35] C. P. Martin, D. Sanchez-Ruiz and C. Tamarit, “The Noncommutative U(1) Higgs-Kibble model in the enveloping-algebra formalism and its renormalizability,” JHEP 0702 (2007) 065, arXiv:hep-th/0612188.

[36] R. Horvat, A. Ilakovac, P. Schupp, J. Trampetic and J. Y. You, “Yukawa couplings and seesaw neutrino masses in noncommutative gauge theory,” Phys. Lett. B 715 (2012) 340, arXiv:1109.3085 [hep-th].

[37] K. Bouchachia, S. Kouadik, M. Hachemane and M. Schweda, “One loop radiative corrections to the translation-invariant noncommutative Yukawa Theory,” J. Phys. A 48 (2015) 36, 365401, arXiv:1502.02992 [hep-th].

[38] A. Martín-Ruiz, M. Cambiaso and L. F. Urrutia, “Green’s function approach to Chern-Simons extended electrodynamics: An effective theory describing topological insulators,” Phys. Rev. D 92 (2015) 12, 125015, arXiv:1511.01170 [cond-mat.other].

[39] C. P. Martin and C. Tamarit, “Renormalisability of the matter determinants in noncommutative gauge theory in the enveloping-algebra formalism,” Phys. Lett. B 658 (2008) 170, arXiv:0706.4052 [hep-th].

[40] N. Seiberg and E. Witten, “String theory and noncommutative geometry,” JHEP 9909 (1999) 032, arXiv:hep-th/9908142.

[41] D. Bak and O. Bergman, “Perturbative analysis of non-Abelian Aharonov-Bohm scattering,” Phys. Rev. D 51, 1994 (1995), hep-th/9403104.

[42] M. Boz, V. Fainberg and N. K. Pak, “Chern-Simons theory of scalar particles and the Aharonov-Bohm effect,” Phys. Lett. A 207, 1 (1995).

[43] M. Boz, V. Fainberg and N. K. Pak, “Aharonov-Bohm scattering in Chern-Simons theory of scalar particles,” Annals Phys. 246, 347 (1996).

[44] M. M. Sheikh-Jabbari, “C, P, and T invariance of noncommutative gauge theories,” Phys. Rev. Lett. 84, (2000) 5265, hep-th/0001167.