Energy dependence of the saturation scale and the charged multiplicity in pp and AA collisions

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A natural framework to understand the energy dependence of bulk observables from lower energy experiments to the LHC is provided by the Color Glass Condensate, which leads to a “geometrical scaling” in terms of an energy dependent saturation scale $Q_s$. The measured charged multiplicity, however, seems to grow faster ($\sim \sqrt{s}^{0.3}$) in nucleus-nucleus collisions than it does for protons ($\sim \sqrt{s}^{0.2}$), violating the expectation from geometric scaling. We argue that this difference between pp and AA collisions can be understood from the effect of DGLAP evolution on the value of the saturation scale, and is consistent with gluon saturation observations at HERA.

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I. INTRODUCTION

The unprecedented high energies of the LHC proton and nuclear beams provide us with new experimentals tests of QCD dynamics at high energy. On a fundamental level we know that also the bulk properties of the collision system such as the momentum spectra and correlations of all produced hadrons must follow from QCD. How this happens and to what extent the process can be understood in a weak-coupling approximation is still an open issue. However, the phenomenological success of the Color Glass Condensate (CGC, for reviews see e.g. [1]) makes one optimistic that this could in fact be possible. In the CGC framework the small $x$ degrees of freedom that dominate bulk particle production in hadronic collisions are described as nonperturbatively strong classical color fields. This picture leads naturally to the concept of gluon saturation and the saturation scale $Q_s$ as the dominant transverse momentum scale determining both the magnitude and the space and time dependence (i.e. the momenta of the gluons) of the small $x$ gluon fields.

The first LHC observable, in both proton-proton and nucleus-nucleus collisions, to give us information about QCD dynamics at high energy is the charged particle multiplicity [2,4]. As we shall discuss in more detail in Sec. VI the charged multiplicity is, to a very good approximation, proportional to $Q_s^2$. Thus the energy dependence of the multiplicity is an experimental probe of the $x$ dependence of the saturation scale $Q_s^2$, separately for nucleons and nuclei. The simplest model-independent way to see this is to realize that gluon saturation turns particle production into a one scale problem, with $Q_s$ as the only dimensionful scale apart from the size of the system. All bulk quantities, such as the multiplicity or the transverse energy, and correlations in the system can be understood in this way up to normalization constants parametrically of order 1.

In the CGC framework the energy dependence of the saturation scale follows from the JIMWLK [5] renormalization group equation or its mean field, large $N_c$ approximation, the BK equation [6]. At an intermediate scale, typically taken as $x \approx 0.01$, where one would start the BK or JIMWLK evolution, the typical nuclear saturation scale could be estimated as $Q_{sA}^2 \approx c A^{1/3} Q_{sp}^2$ with $c \approx A^{2/3} R_p^2/R_A^2$ a constant somewhat smaller than 1 (for a more detailed discussion see e.g. [7]). With a fixed coupling constant leading order JIMWLK or BK evolution would preserve the value of $Q_{sA}/Q_{sp}$ at all energies. Including the running of $\alpha_s$, however, changes this picture. At asymptotically high energies running coupling BK or JIMWLK evolution leads to a saturation scale that is independent of $A$ [8] and therefore grows more slowly for nuclei than for nucleons.

The arguments outlined above would lead to a primary gluon multiplicity that would have the same energy dependence in pp and AA collisions (fixed coupling), or that grows more slowly in AA than pp collisions (running coupling at asymptotical energies). The trend seen in the data on the final charged multiplicity is, however, the opposite one, posing a puzzle for attempts to understand...
it on the basis of gluon saturation. We argue in this paper that at least a part of the explanation lies in transient effects that do not follow directly from BK/JIMWLK evolution. We will show that, due to effects of the nuclear geometry and of DGLAP evolution, the growth of $Q_s$ with energy can actually be faster in nuclei than in protons. Thus a single parametrization of the dipole cross section can be in good agreement with the basic features of the experimental data in both protons and nuclei, as shown in Fig. 1. We will first discuss the energy dependence in different fits to HERA data in Sec. II before looking more closely at what happens in two particular dipole cross section parametrizations, “IPsat” in Sec. III and “bCGC” in Sec. IV. We will then, in Sec. V discuss in more detail the relation between the initial gluonic and final charged hadron multiplicity in nuclear and proton collisions.

II. ENERGY DEPENDENCE OF THE SATURATION SCALE

Let us now discuss what is known about the value of the saturation scale based on fits to HERA data. There exists by now a large amount of different saturated parametrizations of the dipole cross section, mostly fit to HERA or in some cases to RHIC dAu data. Usually the impact parameter dependence is factorized into a Gaussian profile multiplying the dimensionless scattering amplitude: this is the case in the well known GBW and IIM parametrizations and the more recent solutions of the running coupling BK equation. In these cases the fit seems to favor an $x$-dependence of the saturation scale that is faster than the observed energy dependence of the charged multiplicity in pp collisions. This is the case also for the BK evolution studied in [15], where the evolution speed is not a fit parameter, but follows directly from the evolution equation itself. Thus the value $\lambda \approx 0.3$ obtained from a simple power law fit $dN/d_b \sim \sqrt{s}^\lambda$, to the energy dependence of the multiplicity in AA collisions is in good agreement with the evolution speed obtained e.g. in the GBW fit to HERA data. Although some saturation calculations slightly underpredicted the AA multiplicity at the LHC, they still correctly predict a stronger growth than in pp collisions. The more recent application of running coupling BK evolution, with a more detailed inclusion of the nuclear geometry, reproduces the ALICE multiplicity data perfectly.

On the other hand, it appears that in dipole cross section parametrizations where the impact parameter dependence is not factorized out but included in the saturation scale itself, fits to HERA data prefer a slower increase of $Q_s$ with $x$. This is the case in both of the parametrizations, IPsat [17, 18] and bCGC [18, 19] that we shall analyze in more detail in Secs. III and IV. This is due to the functional form that intertwines the $r$ and $b$ dependence of the dipole cross section in such a way that the proton is allowed to grow with energy; leading to a growing total DIS cross sections even with a slower increase of the saturation scale. This growth is consistent with the increase of the total pp cross section with energy and the $t$-dependence of diffractive vector meson production at HERA. As discussed e.g. in [20], this is the kind of parametrization that one would generally prefer on theoretical grounds, since it causes the dipole cross section to saturate towards the correct black disk limit also for $b \neq 0$. However, it is not obvious if the particular functional form chosen in these parametrizations is the correct one. The slower growth of $Q_s$ in the IPsat and bCGC parametrizations has, in $k_T$-factorized calculations, yielded a good agreement with the charged multiplicity in LHC proton-proton collisions [21]. Also the multiplicity distributions and $p_T$-spectra in pp collisions have recently been analyzed in the $k_T$-factorization approach [22].

In conclusion, among the different CGC fits to HERA data there are ones that explain the multiplicity in pp collisions, and others that give a good description of the multiplicity in AA collisions. Neither the pp or AA multiplicity data separately is thus an indication against gluon saturation, but the apparent failure to describe both with the same parametrization is problematic. As we shall now see, this is in fact not the case. The IPsat parametrization, and also the bCGC (with an additional assumption that we will discuss), can in fact describe the $\sqrt{s}$ dependence of both the pp and AA multiplicities.

III. NUCLEAR EFFECTS IN EIKONALIZED DGLAP

The IPsat model [17, 18] is a modification of the idea (see e.g. [23]) of including multiple scatterings and enforce-
The parameter dependence is included in the saturation scale (or DGLAP gluon distribution), leading to the dipole cross section

\[
\frac{d\sigma_{\text{dip}}}{d^2b_T} = 2 \left[ 1 - \exp \left( -r^2 F(x, r) T_p(b) \right) \right].
\]  

Here \( T_p(b) = \exp \left( -b^2/2B_p \right) / (2\pi B) \) is the impact parameter profile function in the proton with \( B_p = 4.0 \text{ GeV}^2 \) and \( F \) is proportional to the gluon distribution

\[
F(x, r) = \frac{\pi^2}{2N_c} \alpha_s \left( \mu_0^2 + \frac{C}{r^2} \right) xg \left( x, \mu_0^2 + \frac{C}{r^2} \right).
\]

On a conceptual level, this formulation resums the multiple scatterings off the small-\( x \) gluons by assuming that they are independent and thus exponentiate. Strictly speaking it is not a “CGC” parametrization in the sense that it does include the correlations between the small-\( x \) gluons in the target that are included in the JIMWLK/BK equations. On the other hand it has the correct DGLAP behavior at large \( Q^2 \) (small \( r \)) and should be seen as a a good way to approach the saturation regime from the high \( Q^2 \) or large \( x \) direction.

A generalization of Eq. (1) to nuclei (including fluctuations in the positions of the nucleons) is very straightforward: one replaces the thickness by a sum over \( A \) nucleons as

\[
\frac{d\sigma_{\text{dip}}^A}{d^2b_T} = 2 \left[ 1 - \exp \left( -r^2 F(x, r) \sum_{i=1}^{A} T_p(b_T - b_{T_i}) \right) \right].
\]

This corresponds to treating the interactions with the separate nucleons in the nucleus as independent; consistently with the eikonalization idea. In terms of the S-matrix of the dipole scattering off the target Eq. (3) is equivalent to

\[
S_A(r, b_T, x) = \prod_{i=1}^{A} S_p(r, b_T - b_{T_i}, x).
\]  

The positions of the nucleons can then be averaged over to yield a Glauber-like averaged dipole cross section, written in the large \( A \) approximation as

\[
\frac{d\sigma_{\text{dip}}^A}{d^2b_T} \approx 2 \left[ 1 - \exp \left\{ -\frac{AT_A(b_T)}{2} \sigma_{\text{dip}}(r, x) \right\} \right].
\]

where \( \sigma_{\text{dip}}^p(r, x) \) is the nucleon dipole cross section of Eq. (1) integrated over the impact parameter \( b_T \).

The saturation scale \( Q_s \) characterizes the qualitative change between the dilute color transparency region \( r \to 0 \) and the black disk limit \( \frac{d\sigma_{\text{dip}}}{d^2b_T} \to 2 \) at large \( r \). We shall take here a model-independent definition of \( Q_s \) as the solution of

\[
\frac{d\sigma_{\text{dip}}}{d^2b_T} (x, r = 1/Q_s(x, b_T)) = 2(1 - e^{-1/4}).
\]  

The saturation scales defined as Eq. (6) for a proton and a lead nucleus in the IPsat parametrization are plotted in Fig. 2 multiplied by the color factor \( C_A/C_F \) appropriate for gluon production in pp or AA collisions. We are taking an effective value representing the average over the transverse plane by taking the saturation scale at \( b_{\text{med}} \), the median impact parameter of the total DIS cross section (the value such that half the cross section comes from \( b < b_{\text{med}} \)). The difference between protons and nuclei is striking: the energy dependence is \( Q_s^2 \sim x^{-0.31} \) for nuclei and \( Q_s^2 \sim x^{-0.20} \) for protons, in a manner which immediately evokes the \( \sqrt{x} \)-dependence of the charged multiplicity.

The reason for this difference lies in the behavior of the DGLAP-evolved gluon distribution, whose \( x \)-dependence gets steeper at higher \( Q \) and, because \( Q_s \) grows with \( A \) at higher \( A \). This feature was mentioned already in Ref. [7], although the discussion there is formulated in terms of the \( A \)-dependence at fixed \( x \) in stead of the \( x \)-dependence at fixed \( A \). The function \( F(x, r) \) of Eq. (2) is shown in Fig. 3. The gluon distribution at the initial scale \( \mu_0^2 \) in the IPsat model has a very mild \( x \)-dependence. The DGLAP evolution then drives the distribution to become much steeper at higher scales which, because of the \( A^{1/3} \) enhancement of \( Q_s^2 \) in nuclei, define the saturation region in a nucleus.

Let us try to estimate the saturation scales explicitly to illustrate how this happens. From Eqs. (1), (2) and (6) we can estimate the proton saturation scale at the center of the proton as

\[
Q_s^2(b = 0) = \frac{1}{4} \frac{F(x, r = 1/Q_{sp}(b = 0))}{2\pi B_p}.
\]  

Replacing, for purposes of illustration, the Woods-Saxon profile by a theta function \( T_A(b) \approx \theta(R_A - b)/(\pi R_A^2) \) we
get for the saturation scale in a nucleus
\[
\frac{\sigma_{\text{dip}}(r = 1/Q_{sA}, x)}{\pi R_A^2} = \frac{1}{4}. \tag{8}
\]

Now we know that \(Q_{sA} > Q_{sp}\) and therefore \(r = 1/Q_{sA}\) is in the dilute region for the proton dipole cross section. We can therefore take in Eq. (8) the small \(r\) approximation
\[
\sigma_{\text{dip}}(r = 1/Q_{sA}, x)/2 \approx \frac{1}{Q_{sA}^2} F(x, r = 1/Q_{sA}) \tag{9}
\]
and thus
\[
Q_{sA}^2(b = 0) = \frac{1}{4} \frac{AF(x, r = 1/Q_{sA}(b = 0))}{\pi R_A^2}. \tag{10}
\]
Equations (7) and (10) are still implicit equations that must be solved to obtain the saturation scales. It is, however, easy to see that because \(Q_{sA} > Q_{sp}\), the gluon distribution on the r.h.s. of Eqs. (7) and (10), and consequently the saturation scale on the l.h.s., evolves more rapidly with energy. We emphasize that this discussion is just an illustration of the origin of the different \(x\)-dependences in protons and nuclear, and the values in Fig. 2 are obtained from the full expressions.

Finally, relating these saturation scales to the charged multiplicities as discussed in Sec. V results in Fig. 4. There are two curves for protons and nuclei, differing by whether one keeps the coupling \(\alpha_s\) fixed at 0.33 or whether one allows it to run as \(\alpha_s(Q_{sA}^2)\). We see that the agreement with the experimental data is extremely good, considering the simplicity of this approach.

\section{Saturation Scale for Independent BK-Evolved Nucleons}

Whether the impact parameter profile is smooth (as in e.g. \cite{13}) or fluctuating \cite{16}, running coupling BK evolution leads to a slower increase of \(Q_s\) for a higher initial value, i.e. for nuclei. Let us here consider an approximation that the individual nucleons evolve according to BK, and are then combined into a nucleus using the assumption of independent scattering Eq. (14). Parametrically this ansatz could perhaps be justified at most in a moderate \(x\) regime where evolution does not yet happen coherently over the whole nucleus, and would certainly not be valid for asymptotically high energies. It is, however, interesting to see how a faster energy dependence in nuclei can arise also in this scenario.

To be more precise, the dipole cross section for a proton in the bCGC parametrization \cite{18, 19} is:
\[
\frac{d\sigma_{\text{dip}}^p}{d^2B_T} = 2N_0 \left( \frac{r Q_s^p}{2} \right)^2 \frac{2(\gamma_s + \frac{1}{x} \ln \left( \frac{2r Q_s^p}{Q_{sA}^2} \right))}{\pi R_A^2} \text{ for } rQ_s^p \leq 2
\]
\[
= 2 - 2 \exp \left( -A \ln^2 (BrQ_s^p) \right) \text{ for } rQ_s^p > 2. \tag{11}
\]

The saturation scales \(Q_s\) and \(Q_s'\) are conceptually the same quantity and their numerical values are of the same order, but we differentiate between them in order to maintain our model independent definition of \(Q_s\) in Eq. (6). The coefficients \(A\) and \(B\) in the can be determined uniquely from the condition that \(\frac{d\sigma_{\text{dip}}^p}{d^2B_T} = 0\) and its first derivative with respect to \(rQ_s'\) are continuous across \(rQ_s' = 2\). Here \(Y = \ln(1/x)\) is the rapidity, while \(\gamma_s = 0.628\) and \(\kappa = 9.9\) (which quantifies the geometric scaling violations in Eq. (11)) are obtained from leading logarithmic BKFL dynamics \cite{24}. The impact parameter dependence of the proton saturation scale is introduced into the bCGC model in the form
\[
Q_s'(x, b) = (\frac{x_0}{x})^\frac{\kappa}{2} \left[ \exp \left( -b^2/(2B_{\text{CGC}}) \right) \right]^\frac{\kappa}{2} \text{ GeV}. \tag{12}
\]

The parameters \(\lambda, x_0, N_0\) and \(B_{\text{CGC}}\) are fit to the data, with the fit resulting in \(\lambda = 0.159, x_0 = 5.95 \cdot 10^{-4}, N_0 = 0.417\) and \(B_{\text{CGC}} = 5.5\text{ GeV}^{-2}\).

We now use this parametrization for nuclei by assuming that the scatterings off the nucleons are independent, i.e. assuming Eq. (14) which leads to the Glauber form for the average gluon distribution \cite{3}. The resulting saturation scales, at the median impact parameter \(b_{\text{med}}\) are plotted in Fig. 2 for protons and nuclei. Due to the non-trivial functional form of the parametrization, the evolution speed for protons turn out to be 0.18, slightly larger than the parameter \(\lambda\) in the parametrization, but still slower than in \(b\)-independent fits or in BK evolution. The nuclear saturation scale grows faster, as \(Q_{sA}^2 \sim x^{-0.22}\).

Let us try to understand this difference analytically in the similar way as in Sec. IIII. It is easier here to use the definition of the saturation scale \(Q_s'\) that appears in the parametrization itself. Thus we have for the proton
\[
\frac{d\sigma_{\text{dip}}^p}{d^2B_T} (r = 2/Q_{sA}', x) = 2N_0 \tag{13}
\]
and in the nucleus
\[
\frac{d\sigma_{\text{dip}}^n}{d^2B_T} (r = 2/Q_{sA}', x) = 2N_0
\]
\[
= 2 \left[ 1 - \exp \left\{ -AT_A(b) \sigma_{\text{dip}}^n \left( r = \frac{2}{Q_{sA}^n}, x \right) \right\} \right]. \tag{14}
\]

We again replace the Woods-Saxon distribution by a theta function \(T_A(b) \approx \theta(R_A - b)/(\pi R_A^2)\). Because at the nucleus saturation scale one is in the dilute regime for the proton we can now approximate Eq. (14) by
\[
N_0 \approx \frac{AT_A(b)}{2} \sigma_{\text{dip}}^n (r = 1/Q_{sA}', x). \tag{15}
\]
We now assume that the integral over the impact parameter in the proton approximately factorizes into a constant \(\sigma_0 \approx 2\pi B_{\text{CGC}}\) to get
\[
\sigma_{\text{dip}}^n (r = 1/Q_{sA}', x) \approx 2\sigma_0 N_0 \left( \frac{Q_{sA}}{Q_{sA}^n} \right)^2 \left( \gamma_s + \frac{1}{x} \ln \left( \frac{2r Q_s^p}{Q_{sA}^n} \right) \right), \tag{16}
\]
leaving us with
\[
\frac{A\sigma_0}{\pi R_A^2} = \exp \left( \frac{\gamma_s + \frac{1}{2\kappa\lambda Y} \Delta}{\gamma_s + \Delta} \right),
\]
where we have denoted
\[
\frac{Q_\Delta}{Q_{sp}} = e^{\Delta/2}.
\]
Since the l.h.s of Eq. \((17)\) is independent of energy, it is obvious that \(\Delta\) must grow with the energy (i.e. with \(Y\)). Differentiating with respect to \(Y\) gives
\[
\Delta'(Y) = \frac{\Delta^2}{2\kappa\lambda Y^2(\gamma_s + \Delta/\gamma_s)}.
\]
Assuming \(\Delta \approx \ln A^{1/3} \approx 1.8\) and \(Y = \ln(1000) \approx 7\) we get \(\Delta' \approx 0.03\). In terms of the saturation scales this means that \(Q_{\Delta}^2 \sim Q_{sp}^2 x^{-0.03}\), which explains most of the effect seen in Fig. \(2\). The interpretation of this result is in fact the same as in the IPsat case. In the bCGC parametrization there is a logarithmic term in the exponent that violates geometric scaling. At smaller \(Y\), i.e. larger \(x\), the effective anomalous dimension \(\gamma_{\text{eff}} = \gamma_s + \ln(2/(rQ_{\Delta}^2))/(\kappa\lambda Y)\) is larger, i.e. closer to 1; thus the \(Q^2\)-dependence of the integrated gluon distribution is close to a logarithm. At large \(Y\) or small \(x\) one recovers the anomalous dimension \(\gamma_s\), which leads to a much faster increase of the integrated gluon distribution with \(Q^2\). This is precisely the scenario that lead to a faster growth of \(Q_{sA}\) in the case of the IPsat model. Another way to see this is to rewrite Eq. \((17)\) as
\[
\left( \frac{Q_{sA}}{Q_{sp}} \right)^2 \approx \left( \frac{A\sigma_0}{\pi R_A^2} \right)^{1/\gamma_{\text{eff}}} \sim (A^{1/3})^{1/\gamma_{\text{eff}}},
\]
At smaller \(x\) \(1/\gamma_{\text{eff}}\) is larger, thus the nuclear enhancement of \(Q_s\) is larger.

Again, using the procedure described in Sec. \(V\) results in the estimates for the charged multiplicity shown in Fig. \(4\). While the agreement with experiential data is not as good as with the IPsat parametrization, the general trend of a faster increase in AA than in pp is still seen.

V. RELATION BETWEEN \(Q_s\) AND \(N_{ch}\)

In any CGC calculation of gluon production in a collision of two hadronic objects the initial gluon multiplicity depends on the saturation scale parametrically in the same way. The theory of the CGC is based on weak coupling calculations, and for the consistency of the framework one assumes that \(\alpha_s(Q_s^2) \ll 1\). This means that \(Q_s\) is a semihard scale and we can assume that parametrically \(Q_s \gg \Lambda_{QCD}\). In the CGC, the saturation scale also defines the correlation length of the system in the transverse plane, \(\sim 1/Q_s\). In the limit when a weak coupling CGC calculation is justified in the first place, the correlation length is smaller than the size of the interaction region, \(1/Q_s^2 \ll \sigma\). Thus particle production happens locally in independent domains of size \(\sim 1/Q_s^2\) in the transverse plane. This picture leads to a gluon multiplicity that can be written as a local observable, where for dimensional reasons the number of gluons per unit area is proportional to the local \(Q_s^2\):
\[
\frac{dN_{\text{init},g}}{d^2x_T dy} = \frac{C_F Q_s^2}{2\pi^2 \alpha_s}.
\]
Here, following \[25, 26\], we have introduced the “gluon liberation coefficient” \(c\); a nonperturbative dimensionless constant that is parametrically of order 1, but depends on the detailed spectrum of the produced gluons. Its value in the MV model \[27\] has been determined using Classical Yang-Mills simulations \[28, 29\] to be \(c \approx 1.1\) (see \[30\] for a discussion of the CYM results parametrized in terms of \(c\) given here). As discussed e.g. in Ref. \[31\] the value of \(c\) remains very close to this value across a range of different models for the color charge distribution (or, equivalently, for the dipole cross section). There is also an analytical calculation \[20\] of the liberation coefficient with the result \(c = 2\ln 2 \approx 1.4\).

Phenomenological studies are often done using various \(k_T\)-factorized approximations to compute the gluon spectrum. Although one can derive a \(k_T\)-factorized formula for the dilute-dilute “pp” and dilute-dense “pA” cases, \(k_T\)-factorization yields the wrong gluon spectrum for the case of dense-dense or “AA” scattering \[32\]. Nevertheless, since the only dimensionful scale in the problem is still \(Q_s\), also the result from \(k_T\)-factorization can still be parametrized as \[21\], although the value of the coefficient \(c\) is incorrect.

In proton proton collisions, the proportionality between the initial gluon and final charged hadron multiplicities is based on the phenomenological success of
local parton-hadron duality (LPHD) \[33\]. The working assumption is that the final charged multiplicity is proportional to the initial partonic one. The constant of proportionality is a property of the independent fragmentation of partons into hadrons and should thus be basically independent of collision energy or centrality. In order to explain the slower growth of the charged multiplicity in pp-collisions by a modification of the LPHD hypothesis one would have to argue that as the energy increases, the number of charged hadrons produced from a primary parton decreases, which does not seem likely. Also the transverse area of the interaction region should, if anything, increase with energy. Therefore it seems that the proton saturation scale should grow with energy at most as the observed charged multiplicity, or even more slowly.

On the side of nucleus-nucleus collisions there is a wider range of plausible modifications to the relation between initial gluonic and observed hadronic multiplicities. One possible starting point is the LPHD assumption as in pp collisions. The fragmentation could then be argued to lead to a larger ratio of the final charged hadron to the initial gluon multiplicity, e.g. due to the larger initial gluon momentum \( \langle p_T \rangle \sim Q_s \); this is the argument in e.g. \[33\]. One potential problem for this approach comes from considering the first moment of the particle spectrum, i.e. the total transverse energy. For the initial gluons the typical initial transverse momentum is \( \langle p_T \rangle \sim Q_s \sim \sqrt{\langle dN/d\eta \rangle / S_\perp} \). A fragmentation process that leads to a larger final charged multiplicity for AA than pp collisions should thus lead to \( \langle p_T \rangle / \sqrt{\langle dN/d\eta \rangle / S_\perp} \) that decreases towards central collisions and towards higher energies.

To be more explicit, let us assume that in the LPHD picture one gluon produces \( n \) charged particles after fragmentation. To account for the faster growth of the multiplicity with \( \sqrt{s} \) in AA collisions, one would need \( n_A > n_p \). We now have for the initial gluons

\[
\frac{\langle p_T \rangle_g}{1} \frac{dN_g}{S_\perp d\eta} \sim Q_s^2 \tag{23}
\]

and for the final charged particles

\[
\frac{\langle p_T \rangle_{ch}}{1} \frac{dN_{ch}}{S_\perp d\eta} \sim nQ_s^2 \tag{25}
\]

since transverse momentum must be conserved during fragmentation. Now, if \( n_A > n_p \), the scaled mean \( p_T \)

\[
\frac{\langle p_T \rangle_{ch}}{\sqrt{1} \frac{dN_{ch}}{d\eta}} \sim \frac{1}{n\sqrt{n}} \tag{26}
\]

should be smaller in central AA collisions than for protons. This is indeed seen in the RHIC data \[10\]. However, if the increase in \( n \) is due the larger \( \langle p_T \rangle \sim Q_s \), the ratio \[26\] should also decrease with increasing collision energy. Between \( \sqrt{s} = 62.4 \text{ GeV} \) and \( \sqrt{s} = 200 \text{ GeV} \) no such decrease is seen at RHIC \[10\], while no firm conclusions for higher energies can yet be made from the LHC data.

The LPHD scenario is more or less based on neglecting all collective effects even in AA collisions, and thus assuming that no quark gluon plasma is formed. Ample experimental evidence points to the contrary. In the extreme case of strong interactions among the produced gluons, they thermalize into an isotropic plasma, which then expands in the transverse, and, more importantly, the longitudinal direction according to (nearly) ideal hydrodynamical equations of motion. In the ideal hydrodynamical case the entropy, and thus multiplicity, of the particles stays constant during the evolution. This leads to the picture where the final (total) multiplicity is, not only proportional, but nearly equal to the initial gluonic one. During the hydrodynamical expansion of a locally isotropic system the mean transverse momentum (or energy per particle) decreases by a large amount due to \( p d\Omega \) work done pushing the expanding plasma down the beampipe. Radial flow developing during the evolution also boosts the transverse momenta of the particles compared to pp collisions, consistently with the observed increase with centrality of \( \langle p_T \rangle \). The drawback of this scenario is that, in spite of much work, we do not have a quantitative theoretical understanding of the thermalization process. This fast thermalization argument is, however, strengthened by the results of explicit calculations of the initial gluon multiplicity in the CGC, which yield an initial gluon multiplicity that is close to the final total multiplicity. The initial transverse energy, on the other hand, is larger than the observed one by a large factor, which would be consistent with the hydrodynamical picture.

The true physical situation is most likely to lie somewhere between the two extreme scenarios of LPHD and ideal hydrodynamics, with some entropy production and thus increase in the multiplicity during the spacetime evolution of the plasma. However, based on this discussion it seems unlikely that final state effects would solve the problem of a faster growth of the multiplicity with \( \sqrt{s} \) in AA collisions than in pp.

Let us now use the assumption of fast thermalization and ideal hydrodynamical expansion to obtain quantitative estimates for the charged multiplicity. For central heavy ion collisions we take the following simple estimate for the final charged multiplicity: \[26\]

\[
\frac{2}{N_{\text{part}}} \frac{dN_{ch}}{d\eta} \approx \frac{2}{3} \frac{2}{N_{\text{part}}} \frac{dN_g}{d\eta} \sim \frac{2}{3} \frac{2C_F Q_s^2(x)}{2\pi^2\alpha_s} \frac{2S_\perp}{N_{\text{part}}} \tag{27}
\]

Here the factor \( 2/3 \) accounts for the fraction of charged particles of the total multiplicity (to a first approximation \( \pi^+, \pi^0 \)). We take here \( c \approx 1.1 \) as discussed above. The typical transverse area \( S_\perp \) per participant pair is taken as the value estimated in central gold-gold collisions by
The saturation scale as extracted from fits to DIS data depends on the momentum fraction $x$. In hadronic or heavy ion collisions the corresponding variable is a ratio of the transverse momentum to the collision energy. The $x$ and $Q_s(x)$ corresponding to each collision energy $Q_s(\sqrt{s})$ are solved from the relation

$$x = \frac{Q_s(x)}{\sqrt{s}}.$$  \hspace{1cm} (29)

We emphasize that, unlike in typical $k_T$-factorized calculations, there is no arbitrary normalization factor to adjust here. Once the saturation scale is known, it determines both the normalization and the shape of the $p_T$ spectrum of the produced gluons. The mean transverse momentum of the initial gluons is $\langle p_T \rangle$ depends slightly more on the precise $k_T$ dependence of the dipole cross section than the multiplicity. For the case of the MV model the spectrum is relatively hard, with $\langle p_T \rangle \approx 1.3 Q_s$ (following Refs. 29, 30). For RHIC this corresponds to $\langle p_T \rangle \approx 1.5$ GeV for the initial gluons. With rapid thermalization and (nearly) ideal (nearly) boost invariant hydrodynamical evolution this is then reduced (see e.g. [35]) by a factor 3 to match the final observed transverse energy of around 0.5 GeV per particle. At the 2.75 $A$ TeV collision energy at the LHC a similar estimate yields $\langle p_T \rangle \approx 2.0$ GeV for the gluons in the initial state which would be similarly reduced by hydrodynamical evolution.

Now this can be contrasted with the estimate based on $k_T$-factorization [16], where, after adjusting the normalization constant to reproduce the RHIC multiplicity, one obtains at the LHC a transverse energy $14$ TeV and a total multiplicity $\approx (3/2)1600 = 2400$, i.e. a transverse energy of 6 GeV per particle. Although a value for the transverse energy has not yet been released by the LHC experiments, based on the spectra published in Ref. 36 it seems safe to say that this would require a reduction of at least a factor of 6 between the initial gluons and the final state particles. This failure to calculate both the initial multiplicity and transverse energy without adjusting both by an arbitrary normalization constant follows from the incorrect gluon spectrum in $k_T$-factorization.

For protons we simply replace Eq. 27 with an expression that leaves out the scaling by the number of participant pairs

$$\frac{dN_{ch}}{d\eta} = \frac{2}{3} \frac{C_F Q_s^2(x)}{2\pi^2 \alpha_s} S_{\perp}. \hspace{1cm} (30)$$

Here, adjusting the normalization to data as always with LPHD, we take as the transverse area by a constant $S_{\perp} = 20$ mb for fixed $\alpha_s$ and $S_{\perp} = 24$ mb for running $\alpha_s$. This includes the conversion from gluons to final hadrons. This summarizes the simple procedure used to arrive at the multiplicity estimates in Figs. 1 and 4.

VI. CONCLUSIONS

The experimental data seems to be hinting that for bulk particle production at LHC energies one is not yet far enough from the asymptotic high energy, $Q_s \gg \Lambda_{QCD}$, regime for gluon saturation to work perfectly without additional finite $\sqrt{s}$ corrections. What these corrections are remains still somewhat an open issue. One possibility is that soft confinement scale physics remains to be dominant in pp collisions; in the weak coupling framework pursued in this paper we have not been able to analyze this option. The other possibility, more encouraging in terms of prospects for first principles understanding, is that the nuclear saturation scale is large enough at the LHC to be sensitive to the large $Q^2$ effects such as DGLAP evolution. Note that also in the BK equation the growth of the saturation scale with energy is slower in the preasymptotic regime close to the initial condition. In phenomenological applications [15, 37] this preasymptotic slower growth is essential for agreement with experimental data. Thus the energy dependence at LHC energies is to a large degree a consequence of the transverse momentum dependence in the initial condition, not only of the evolution itself.

We have in this note shown that the different energy dependences of the charged particle multiplicities in pp and AA collisions can be understood in the framework of gluon saturation. There is a well-tested and motivated impact parameter-dependent parametrization, based on an eikonalized, DGLAP-evolved gluon distribution, that very accurately describes both pp and AA multiplicities. The difference between protons and nuclei comes, in this parametrization, from effects of DGLAP evolution on the slope of the gluon distribution. These effects are not present in the pure JIMWLK/BK evolution formalism, whether at fixed or running coupling. Fully understanding the physics at play here requires incorporating higher transverse momentum physics not only into the initial condition, but also into nonlinear evolution itself. Some steps this direction have already been taken [38], but further investigation is needed to fully understand the consequences for bulk particle production at the LHC.

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\footnote{Note that this applies to the first version of Ref. [16], and in a subsequent version there is a different normalization coefficient for the energy than for the multiplicity.}
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