WE REVIEW ONE-PARTICLE INCLUSIVE PRODUCTION OF HEAVY-FLAVORED HADRONS IN A FRAMEWORK WHICH RESUMS THE LARGE COLLILINEAR LOGARITHMS THROUGH THE EVOLUTION OF THE FFs AND PDFs AND, AT THE SAME TIME, RETAINS THE FULL DEPENDENCE ON THE HEAVY-QUARK MASS WITHOUT ADDITIONAL THEORETICAL ASSUMPTIONS. THE MAIN FOCUS IS ON THE PRODUCTION OF $D^*$ MESONS IN DEEP INELASTIC ELECTRON–PROTON SCATTERING AT HERA. WE SHOW RESULTS, NEGLECTING FOR THE TIME BEING THE HEAVY-QUARK MASS TERMS, FOR DEEP INELASTIC $D^*$ MESON PRODUCTION AT FINITE TRANSVERSE MOMENTA. WORK TO IMPLEMENT THIS PROCESS IN THE ABOVE MENTIONED MASSIVE QCD FRAMEWORK IS IN PROGRESS.

1. Introduction

One-particle inclusive production processes provide extensive tests of perturbative quantum chromodynamics (pQCD). In contrast to fully inclusive processes, it is possible to study distributions in the momentum of the final state particle and to apply kinematical cuts which come close to the experimental situation. On the other hand, contrary to even more exclusive cases, QCD factorization theorems $^1, ^2$ still hold stating that this class of observables can be computed as convolutions of universal parton distribution functions (PDFs) and fragmentation functions (FFs) with perturbatively calculable hard scattering cross sections. As is well-known, it is due to the factorization property that the parton model of QCD has predictive power. Hence, tests of the universality of the PDFs and FFs are of crucial importance for validating this QCD framework. At the same time, lowest order expressions for the hard scattering cross sections are often not sufficient for meaningful tests and the use of higher order computations is mandatory.

The perturbative analysis is becoming more involved and interesting
if the observed final state hadron contains a heavy (charm or bottom) quark. In this case, the heavy-quark mass $m_h$ enters as an additional scale. Clearly, the conventional massless formalism, also known as zero-mass variable-flavor-number scheme (ZM-VFNS), can also be applied to this case, provided the hard scale $Q$ of the process is much bigger than the heavy-quark mass such that terms $m_h/Q$ are negligible. However, at present collider energies, most of the experimental data lie in the kinematic region $Q \gtrsim m_h$ and it is necessary to take the power-like mass terms into account in a consistent framework.

The subject of this article is the theoretical description of one-particle inclusive production of heavy-flavored hadrons $X_h = D, B, \Lambda_c, \ldots$ in a massive variable-flavor-number scheme (GM-VFNS). In such a scheme the large collinear logarithms of the heavy-quark mass $\ln \mu/m_h$ are subtracted from the hard scattering cross sections and resummed through the evolution of the fragmentation functions (FFs) and parton distribution functions (PDFs). At the same time, finite non-logarithmic mass terms $m_h/Q$ are kept in the hard part and fully taken into account.

In order to test the pQCD formalism, in particular the universality of the FFs, it is important to provide a description of all relevant processes in a coherent framework. Therefore, it is important to work out the GM-VFNS at next-to-leading order (NLO) of QCD for all the relevant processes. Previously, the GM-VFNS has been applied to the following processes:

- $\gamma + \gamma \to D^* + X$ (direct part) \textsuperscript{3}, $\gamma + \gamma \to D^{**} + X$ (single resolved part) \textsuperscript{4}, $\gamma + p \to D^{**} + X$ (direct part) \textsuperscript{5}, $p + \bar{p} \to (D^0, D^{**}, D^+, D_s^+) + X$ \textsuperscript{6,7,8}, where the latter results for hadron–hadron collisions also constitute the resolved contribution to the photoproduction process $\gamma + p \to X_h + X$.

In this contribution, we will review the production of heavy-flavored hadrons $X_h$ in hadron–hadron, photon–proton and deep inelastic electron–proton collisions where the main focus will be on the electroproduction case. We will show results, neglecting for the time being the $m_h/Q$ mass terms, for deep inelastic $D^*$ meson production at finite transverse momenta.

### 2. Theoretical Framework

#### 2.1. GM-VFNS

The differential cross sections for inclusive heavy-flavored hadron production can be computed in the GM-VFNS according to the familiar factorization formulae, however, with heavy-quark mass terms included in the hard scattering cross sections \textsuperscript{9}. Generically, the physical cross sections are
expressed as convolutions of PDFs for the incoming hadron(s), hard scattering cross sections, and FFs for the fragmentation of the outgoing parton into the observed hadron. All possible partonic subprocesses are taken into account. The massive hard scattering cross sections are constructed in a way that in the limit \( m_h \to 0 \) the conventional ZM-VFNS is recovered. A more detailed discussion of the GM-VFNS and the construction of the massive hard scattering cross sections can be found in Refs. 6, 7 and the conference proceedings 10, 11.

2.2. Fragmentation Functions

A crucial ingredient entering these calculation are the non-perturbative FFs for the transition of the final state parton into the observed hadron \( X_h \). For charm-flavored mesons, \( X_c \), three sets of FFs have been employed in our analyses: (i) BKK98-D 12: For \( X_c = D^{*+} \), such FFs were extracted at LO and NLO in the \( \overline{\text{MS}} \) factorization scheme with \( n_f = 5 \) massless quark flavors several years ago 12 from the scaled-energy \( (x) \) distribution \( d\sigma/dx \) of the cross section of \( e^+e^- \to D^{*+} + X \) measured by the ALEPH 13 and OPAL 14 Collaborations at CERN LEP1. (ii) KK05-D 15: Recently, the BKK98-D analysis was extended 15 to include \( X_c = D^0, D^+, D_s^+, \Lambda_c^+ \) by exploiting appropriate OPAL data 16. (iii) KK05-D2: In Refs. 12, 15, the starting scales \( \mu_0 \) for the DGLAP evolution of the \( a \to X_c \) FFs in the factorization scale \( \mu'_F \) have been taken to be \( \mu_0 = 2 m_c \) for \( a = g, u, d, s, c, \bar{c} \) and \( \mu_0 = 2 m_b \), with \( m_b = 5 \text{ GeV} \), for \( a = b, \bar{b} \). The FFs for \( a = g, u, d, s, \bar{s}, \bar{c} \) were assumed to be zero at \( \mu'_F = \mu_0 \) and were generated through the DGLAP evolution to larger values of \( \mu'_F \). For consistency with the \( \overline{\text{MS}} \) prescription for PDFs, we repeated the fits of the \( X_c \) FFs for the choice \( \mu_0 = m_c, m_b \). This changes the c-quark FFs only marginally, but has an appreciable effect on the gluon FF, which is important at Tevatron energies, as was found for \( D^{*+} \) production in Ref. 6. These new FFs will be presented elsewhere.

2.3. Input Parameters

If not stated otherwise, the following parameters have been chosen for the numerical results presented below. For the proton PDFs we have employed the CTEQ6M/CTEQ6.1M PDFs from the CTEQ Collaboration 17 and the KK05-D2 FFs. We have set \( m_c = 1.5 \text{ GeV} \), \( m_b = 5 \text{ GeV} \) and have used the two-loop formula for \( \alpha_s^{(n_f)}(\mu_R) \) in the \( \overline{\text{MS}} \) scheme with \( \alpha_s^{(5)}(m_Z) = 0.118 \). Our theoretical predictions depend on three scales, the renormalization scale \( \mu_R \), and the initial- and final-state factorization scales \( \mu_F \) and \( \mu'_F \),
respective. Our default choice for hadro- and photoproduction has been $\mu_R = \mu_F = \mu'_F = m_T$, where $m_T = \sqrt{p_T^2 + m_h^2}$ is the transverse mass. The scale choice in the electroproduction case will be specified below.

### 3. Hadroproduction

Recently, the CDF collaboration has published first cross section data for inclusive production of $D^0$, $D^+$, $D^{*+}$, and $D_s^+$ mesons in $p\bar{p}$ collisions $^{18}$ obtained in Run II at the Tevatron at center-of-mass energies of $\sqrt{S} = 1.96$ TeV. The data come as distributions $d\sigma/dp_T$ with $y$ integrated over the range $|y| \leq 1$ and the particle and antiparticle contributions are averaged.

Our theoretical predictions in the GM-VFNS are compared with the CDF data for $D^*$ mesons on an absolute scale in Fig. 1 (left) and in the data-over-theory representation with respect to our default results in Fig. 1 (right). We find good agreement in the sense that the theoretical and experimental errors overlap where the experimental results are gathered on the upper side of the theoretical error band, corresponding to a small value of $\mu_R$ and large values of $\mu_F$ and $\mu'_F$, the $\mu_R$ dependence being dominant in the upper $p_T$ range. As is evident from Fig. 1 (right), the central data points tend to overshoot the central QCD prediction by a factor of about 1.5.
at the lower end of the considered $p_T$ range, where the errors are largest, however. This factor is rapidly approaching unity as the value of $p_T$ is increased. The tendency of measurements of inclusive hadroproduction in Tevatron run II to prefer smaller renormalization scales is familiar from single jets, which actually favor $\mu_R = p_T/2$ \cite{19}.

For more details and a comparison with the data for the $D^0$, $D^+$, and $D_s^+$ mesons we refer to Ref. 8. A comparison of NLO predictions for $p+\bar{p} \rightarrow B^+ + X$ in the GM-VFNS with recent CDF data \cite{20} is in preparation \cite{21}.

4. Photoproduction

Inclusive photoproduction of $D^*$ mesons, $\gamma + p \rightarrow D^* + X$, has been studied in Ref. 5 where the direct part has been computed in the GM-VFNS whereas the resolved part has been included in the ZM-VFNS. In this analysis the BKK98-D FFs have been utilized and, for the resolved contribution, the GRV92 photon PDFs \cite{22}. The other parameters have been chosen as specified in Sec. 2.3. In Fig. 6 of Ref. 5, the central numerical predictions for the transverse momentum ($p_T$) distributions of the $D^*$ meson have been compared with preliminary ZEUS data \cite{23}. There exist similar data by the H1 collaboration \cite{24} which have not been used in this analysis. As can be seen in this figure, the agreement of the $p_T$-distributions with the data is quite good down to $p_T \simeq 2m_c$ and the mass effects turn out to be small.

In order to extend the range of applicability of the GM-VFNS into the region $p_T < 3$ GeV more work on the matching to the 3-fixed flavor theory would be needed. The Figs. 7–9 of Ref. 5, showing results for the rapidity ($y$), invariant mass ($W$) and inelasticity ($z(D^*)$) distributions, have to be taken with a grain of salt since they receive large contributions from the transverse momentum region $1.9 < p_T < 3$ GeV which is outside the range of validity of the present theory.

With the work in Ref. 6, it is now possible to include also the resolved part in the GM-VFNS. It will be interesting to compare the complete GM-VFNS framework at NLO of QCD, combined with updated FFs, in more detail with ZEUS and H1 photoproduction data once they are finalized.

5. Electroproduction

Recently, the single inclusive electroproduction of light hadrons at finite transverse momenta has attracted quite a lot of interest, where the outgoing hadron is required to carry a non-vanishing transverse momentum ($p_T^*$) in the center-of-mass system (CMS) of the virtual photon and the incoming
proton. The following partonic subprocesses contribute at leading order (LO) and are of the order $O(\alpha_s)$: $\gamma^* + q \to q + g$ and $\gamma^* + g \to q + \bar{q}$. Very recently, the NLO ($O(\alpha_s^2)$) corrections to this process have been accomplished by three independent groups. Suffice to say here that the NLO corrections increase the LO results in certain kinematical regions by large factors and are essential for bringing theory in agreement with the experimental results. For more details see Ref. 28.

Endowed with appropriate fragmentation functions, the computation in Ref. 27 has been employed to obtain predictions in the ZM-VFNS for the production of heavy-flavored hadrons in electron–proton collisions at HERA, $e + p \to e + X_h + X$. The electron and proton energies have been set to $E_e = 27.5$ GeV and $E_p = 820$ GeV (Fig. 2) resp. 920 GeV (Figs. 3, 4) in the laboratory frame. Furthermore, the renormalization and factorization scales have been chosen as $\mu^2_R = \mu^2_F = \mu^{'2}_F = \xi Q^2 + (p^* T)^2$. The dimensionless parameter $\xi$ has been varied between $1/2$ and $2$ about the default value 1 in order to estimate the scale uncertainties of the theoretical predictions. The other input parameters have been chosen as described in Sec. 2.3. The following cuts have been imposed on the numerical results in Figs. 2–4: $2 < Q^2 < 100$ GeV$^2$, $0.05 < y < 0.7$, $1.5 \leq p_{T, Lab}(D^*) \leq 15$ GeV, and $|\eta_{Lab}(D^*)| < 1.5$ where the momentum transfer $Q^2$, the inelasticity $y$, and the pseudorapidity $\eta$ are defined as usual. Furthermore, in Figs. 3–4 we have asked for the additional constraint $p_{T, Lab}(D^*) > 2$ GeV where $p_{T, Lab}(D^*)$ is the transverse momentum of the $D^*$ meson in the $\gamma^* p$-CMS. This cut is essential to avoid the collinear singularities as $p_{T, Lab} \to 0$. The results have been calculated with $n_f = 5$ flavors and include the contribution where a bottom quark fragments into the $D^*$ meson via $b \to B \to D^*$. No distinction is made between $D^{*+}$ and $D^{*-}$. Figure 2 shows the $p_{T, Lab}$ spectrum in comparison with H1 data, collected in the years 1994 to 1996 with 27.5 GeV positrons colliding with 820 GeV protons at CMS energies of $\sqrt{S} = 300$ GeV. The data points are reasonably well described if one keeps in mind that the theory is expected to work for larger $p_{T, Lab} \gtrsim 2$ GeV. Furthermore, in the ZM-VFNS the heavy-quark mass terms are missing which are potentially important in the region of small $p_{T, Lab}$. These terms will be included as soon as ongoing work to implement this process in the GM-VFNS has been completed.

Finally, results for other distributions ($p_{T, Lab}$, $\eta_{Lab}$, $W$, $z$, $Q^2$, and $x_{Bj}$) are presented in Figs. 3–4.
6. Summary

We have discussed one-particle inclusive production of heavy-flavored hadrons in hadron–hadron, photon–proton, and electron–proton collisions in a massive variable-flavor-number scheme (GM-VFNS). The importance of a unified treatment of all these processes, based on QCD factorization theorems, has been emphasized, in order to provide meaningful tests of the
universality of the FFs and hence of QCD. At the same time, it is necessary to incorporate heavy-quark mass effects in the formalism since many of the present experimental data points lie in a kinematical region where the hard scale of the process is not much larger than the heavy-quark mass. This is achieved in the GM-VFNS, which includes heavy-quark mass effects in a rigorous way and still relies on QCD factorization. We have discussed numerical results for the three reactions. In general, the description of the transverse momentum spectra is quite good down to transverse momenta $p_T \approx 2m_b$. Extending the range of applicability of our scheme to smaller $p_T$ would require more work on the matching to the corresponding theories in the fixed flavor number schemes. Our ZM-VFNS results for the electroproduction of $D^*$ mesons indicate that future experimental results by the H1 collaboration can be nicely described. All this leads us to the expectation that a good overall description of the data can be reached in the future.
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