Workshop on Chiral Forces in Low Energy Nuclear Physics - the LENPIC Meeting

Jagiellonian University, Kraków, Poland
February 10-11, 2017

Jacek Golak, Roman Skibiński
M. Smoluchowski Institute of Physics, Jagiellonian University, 30-384 Kraków, Poland

ABSTRACT

These are the proceedings of the international workshop on ”Chiral Forces in Low Energy Nuclear Physics - the LENPIC Meeting” held at the Jagiellonian University, Kraków, Poland from February 10 to 11, 2017. The workshop focused on the new generation of chiral forces with the semi-local regularization and their applications to few- and many-nucleon systems. Each talk is represented by a short contribution.

* This workshop is a part of the LENPIC project activity and was supported by the Polish National Science Center under Grants No. DEC-2013/10/M/ST2/00420 and by the Faculty of Physics, Astronomy and Applied Computer Science of Jagiellonian University within the KNOW project.
# Contents

**Introduction**

E. Epelbaum: LENPIC : Status, Challenges and Perspectives  

K. Hebeler: Calculation of semilocal 3N interactions up to N3LO : status update and recent developments  

R. Roth: Ab Initio Calculations of Light Nuclei for Constraining Chiral NN+3N Interactions  

H. Witała et al.: Application of new N²LO 3NF’s in calculations of 3N reactions  

A. Calci: Probing and Constraining chiral Interactions in Nuclear Structure and Reaction Calculations  

P. Reinert et al.: Partial–Wave Analysis of NN scattering data at fifth order in chiral EFT  

J. P. Vary: No–Core Shell Model : Quantifying the Observables’ Uncertainties  

H. Kamada et al.: Relativistic Faddeev Calculations  

I. Ciepał et al.: Recent measurement of $^{12}$C(d,d) tensor and vector analyzing powers at the energy range 170–380 MeV in a view of the charged particles EDM studies  

H. Krebs: Nuclear currents in chiral effective field theory  

H. Le et al.: Jacobi No–Core Shell Model for Hypernuclei  

K. Topolnicki et al.: Operator form of nucleon–deuteron scattering  

K. Vobig: Advances in the In–Medium Similarity Renormalization Group  

Yu. Volkotrub et al.: The OPE–Gaussian force in elastic Nd scattering
Introduction

Chiral effective field theory (EFT) provides a powerful framework for analyzing low-energy nuclear structure and reactions in full agreement with the symmetries of quantum chromodynamics. It allows one to derive nuclear forces and currents in a systematically improvable way, using the so-called chiral expansion.

The chiral approach provides a natural explanation of the observed hierarchy of nuclear forces. In particular chiral EFT is expected to provide an ultimate theoretical solution of the long-standing three-nucleon force problem. This means that the chiral interactions are very attractive input for ab-initio studies of both nuclear structure and reactions. The recently developed new generation of the chiral potentials with semi-local regularization gives us hope for overcoming technical obstacles present for the older models of chiral interactions.

LENPIC is a worldwide collaboration that aims to develop chiral effective field theory nucleon-nucleon and many-nucleon interactions complete through fifth order in the chiral expansion. Using these new interactions, LENPIC intends to solve the structure and reactions of light nuclei including electroweak observables with consistent treatment of the corresponding single and many-nucleon currents.

LENPIC brings together scientists from the following institutions: Ruhr-University Bochum, Germany, University of Bonn, Germany, Technical University of Darmstadt, Germany, Jagiellonian University, Krakow, Poland, Iowa-State University, USA, Jülich Research Centre, Germany, Kyushu Institute of Technology, Japan, Ohio State University, USA, Orsay Institute of Nuclear Physics, France, and TRIUMF, Canada. More information can be found at the LENPIC website: www.lenpic.org.

This workshop took place at the Faculty of Physics, Astronomy and Applied Computer Science of the Jagiellonian University, Kraków, Poland, in February 2017. We would like to thank all the sponsors and the participants for making that meeting an exciting and lively event.

Jacek Golak and Roman Skibiński
LENPIC: Status, Challenges and Perspectives

Evgeny Epelbaum

Institut für Theoretische Physik II, Ruhr-Universität Bochum,
44780 Bochum, Germany

Chiral effective field theory (EFT) has become a standard tool to analyze low-energy reactions involving pions, nucleons and external electroweak sources. Most of the ab-initio calculations of light and medium-mass nuclei and nuclear reactions are nowadays performed based on the Hamiltonian and currents derived in chiral EFT. The Low Energy Nuclear Physics International Collaboration (LENPIC) aims to develop chiral nuclear forces and currents complete through (at least) fourth order (i.e. $N^3\text{LO}$) in the chiral expansion and to perform precision ab-initio studies of the structure and reactions of light and medium-mass nuclei [1].

Recently, we have presented a new generation of nucleon-nucleon (NN) potentials up to fifth order (i.e. $N^4\text{LO}$) in the chiral expansion [2]. These interactions employ a local coordinate-space regularization of the long-range interaction, which preserves the analytic structure of the amplitude and efficiently suppresses unphysical short-range components in the two-pion exchange making the spectral-function regularization obsolete. With all relevant $\pi N$ low-energy constants (LECs) taken from $\pi N$ scattering without any fine tuning, we were able to demonstrate a clear evidence of the chiral two-pion exchange. These new potentials, coupled with the novel approach to estimate truncation error formulated in [2], provide a solid basis for applications within LENPIC. We found promising results by applying these novel NN forces beyond the two-nucleon system to calculate nucleon-deuteron scattering and selected properties of light nuclei [3], electroweak NN and 3N reactions [4] and equations of state of nuclear matter [5]. In all cases considered so far, the observed discrepancies between the theoretical predictions and experimental data are in a good agreement with the size of the three-nucleon force (3NF) effects expected in Weinberg’s power counting scheme. Still, more work is needed to further test and/or adjust the algorithm for estimating uncertainty from the truncation of the chiral expansion. This applies, in particular, to the determination of the relevant expansion parameter in calculations of bound- and excited-state properties.

Clearly, the next step is the inclusion of the 3NF. The expressions for the three- and four-nucleon forces are available through $N^3\text{LO}$, see [6] for a review, and work is in progress towards completing the derivation of the remaining $N^4\text{LO}$ contributions. The numerical implementation of the 3NF is challenging and represents one of the central tasks of the LENPIC Collaboration. Specifically, the 3NF needs to be regularized in the way consistent with the NN potential and expressed in the partial-wave basis. Partial wave decomposition of a general 3NF can be performed numerically [7], and its computational cost can be further reduced in the case of local 3NFs [8]. Work is in progress towards the numerical
implementation of the regularization of the long-range parts of the 3NF at N³LO which represents the major challenge. To ensure that the results for 3NF matrix elements are numerically stable we follow different computational strategies by performing calculations independently in coordinate and momentum spaces. First benchmarks at the level of the N²LO 3NF have already been successively performed, and the implementation of the N³LO terms is in progress.

In parallel to these developments, we have also worked out the corresponding electromagnetic, axial and pseudoscalar nuclear currents up to N³LO [9], [10], see also [11], [12] and references therein for a related work by the JLab-Pisa group (whose results differ from ours). Similarly to the 3NFs, the resulting exchange currents need to be regularized and partial-wave decomposed. Special care is required to ensure that the symmetries, which manifest themselves in the form of the continuity equations, are not destroyed by regularization. This issue is particularly important for electroweak processes such as e.g. ³H β-decay and μ-capture on ²H, ³H and ³He, for which parameter-free predictions can be made at the level of N³LO (once the strength of the 3NF LEC cD is fixed in the strong sector).

In summary, the outline developments within the LENPIC Collaboration towards the implementation of the 3NFs and current operators derived in the framework of chiral EFT open the way for precision calculations of light and medium-mass nuclei with quantified theoretical uncertainties.

It is a pleasure to thank the whole LENPIC for enjoyable collaboration and the Kraków group for the excellent organization of the meeting.

References

[1] LENPIC Collaboration, www.lenpic.org
[2] E. Epelbaum, H. Krebs, U.-G. Meißen, Eur. Phys. J. A 51, no. 5, 53 (2015); Phys. Rev. Lett. 115, no. 12, 122301 (2015).
[3] S. Binder et al. [LENPIC Collab.], Phys. Rev. C 93, no. 4, 044002 (2016).
[4] R. Skibiński et al., Phys. Rev. C 93, no. 6, 064002 (2016).
[5] J. Hu et al., arXiv:1612.05433 [nucl-th].
[6] E. Epelbaum, PoS CD 15, 014 (2016).
[7] J. Golak et al., Eur. Phys. J. A 43, 241 (2010).
[8] K. Hebeler et al., Phys. Rev. C 91, no. 4, 044001 (2015).
[9] S. Kölling, E. Epelbaum, H. Krebs and U.-G. Meißen, Phys. Rev. C 80, 045502 (2009); Phys. Rev. C 84, 054008 (2011).
[10] H. Krebs, E. Epelbaum and U.-G. Meißen, Annals Phys. 378, 317 (2017).
[11] M. Piarulli et al., Phys. Rev. C 87, no. 1, 014006 (2013).
[12] A. Baroni et al., Phys. Rev. C 94, no. 2, 024003 (2016).
Calculation of semilocal 3N interactions up to N3LO: status update and recent developments

Kai Hebeler\textsuperscript{1,2}

\textsuperscript{1}Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany
\textsuperscript{2}ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany

During the recent years there has been considerable effort to derive novel nuclear interactions within chiral EFT. In particular, in Refs. \cite{1,2} a novel way of regularizing nuclear forces was proposed. In contrast to previous nuclear forces, which employed a non-local momentum-space regulator, the new way of regularizing the long-range parts of the interaction in a local way leads to significantly reduced cutoff artifacts and also preserve the analytic structure of the scattering matrix close to pion threshold. First calculations for light nuclei based on these new NN forces are very promising \cite{3}.

For consistent structure and reaction calculations up to N\textsuperscript{2}LO and N\textsuperscript{3}LO it will be crucial to also include contributions from three-nucleon (3N) interactions. Recently we made significant progress in generalizing the new semi-local regularization scheme to 3N forces. Using the new efficient framework for calculating unregularized 3N interactions presented in Ref. \cite{4} we implement the local coordinate-space regularization $V_{\text{reg}}(\mathbf{r}_{12}, \mathbf{r}_{23}, \mathbf{r}_{13}) = R(\mathbf{r}_{12}, \mathbf{r}_{23}, \mathbf{r}_{13})V(\mathbf{r}_{12}, \mathbf{r}_{23}, \mathbf{r}_{13})$ in form of convolution integrals in momentum space:

$$
\langle p'q'|V_{\text{reg}}|pq\rangle = \int \frac{d\mathbf{p}'}{(2\pi)^3} \frac{d\mathbf{q}'}{(2\pi)^3} \langle p'q'|R|\mathbf{p}'\mathbf{q}'\rangle \langle \mathbf{p}'\mathbf{q}'|V|pq\rangle .
$$

Here $r_{ij}$ represent the relative distances of the three particles, $R$ is the regulator function and $p$ and $q$ ($p'$ and $q'$) are the Jacobi momenta of the initial (final) state, respectively. Recently we resolved the numerical challenges related to the practical numerical calculation of the integrals and have now finished the calculation of all partial-wave matrix elements at N\textsuperscript{2}LO. Next we will explore their effects in few- and many-body calculations and then compute all matrix elements up to N\textsuperscript{3}LO.

References

\cite{1} E. Epelbaum, H. Krebs and U.-G. Meißner, Eur. Phys. J. A 51, (2015) 53.
\cite{2} E. Epelbaum, H. Krebs and U.-G. Meißner, Phys. Rev. Lett. 115, (2015) 122301.
\cite{3} S. Binder \textit{et al.}, Phys. Rev. C 93, (2016) 044002.
\cite{4} K. Hebeler, H. Krebs, E. Epelbaum, J. Golak and R. Skibinski, Phys. Rev. C 91 (2015) 044001.
Ab Initio Calculations of Light Nuclei
for Constraining Chiral NN+3N Interactions

Robert Roth

1Institut für Kernphysik, Technische Universität Darmstadt,
Schlossgartenstr. 2, 64289 Darmstadt, Germany

The ab initio description of light and medium-mass nuclei starting from two- and three-nucleon interactions derived within chiral effective field theory is one of the most dynamic fields in nuclear theory today. For light nuclei, i.e., the p-shell and selected sd-shell nuclei, the no-core shell model (NCSM) with importance truncation provides access to all relevant nuclei and observables [1],[2],[3],[4], including ground and excited-state energies, radii, density and momentum distributions, and all electromagnetic and weak observables. The sole limitation and source of uncertainty in the NCSM is the convergence of the observables with increasing size of the many-body model space. In order to enhance this convergence, we routinely use similarity renormalization group (SRG) transformations of the Hamiltonian and all relevant observables up to the three-nucleon level [5].

In combination, SRG-evolved operators and the NCSM provide a very efficient, robust, and universal tool to study ground-state and spectroscopic observables in all p-shell nuclei.

This is an ideal framework for characterizing and constraining next-generation chiral interactions beyond the few-body domain. Traditionally, few-body calculations for bound-state and scattering observables are being used to fit and test new NN+3N interactions. Ab initio NCSM calculations of p-shell nuclei access a new domain of observables that are highly sensitive to different aspects of the input interactions and, thus, provide important additional information on the performance of the interaction that is not accessible in the few-body sector. We can, e.g., probe the spin-orbit and tensor structure through excitation spectra and spectroscopy for states with larger angular momenta, the isospin dependence of the interaction through isospin chains, and the saturation properties through ground-state energies and radii in the upper p-shell.

In order to use p-shell nuclei for testing and constraining the consistent NN+3N interactions up to N3LO constructed within the LENPIC collaboration [6],[7],[8], we first need to identify a set of observables that depend sensitively on details of the interaction. In addition to sensitivity to the interaction, these observables should be easily accessible in a consistent way starting from the chiral EFT inputs. This makes ground-state and excitation energies particularly well suited, since they show a good convergence and need no additional input from chiral EFT, unlike electroweak observables that require two-body current contributions consistent with the interaction. Therefore, we explored ground-state and excitation energies of low-lying states with a range of different nuclei using a variety of previous-generation chiral NN+3N interactions. Using $^6$Li and $^{12}$C
as examples, we discussed results on the sensitivities of excitation energies \cite{9}. With preliminary versions of the consistent chiral NN+3N interactions at N2LO with semi-local regulators, we started to explore the dependence of the same excitation energies on the three-body low-energy constant (LEC) $c_D$, fixing the corresponding $c_E$ from a fit to the triton ground-state energy. These exploratory calculations confirm the picture of the previous survey with other interactions \cite{9} regarding the sensitivity of individual excited states on the interaction.

Working towards the final versions of the LENPIC NN+3N interactions up to N3LO, we propose to use ground-state and excitation energies of $p$-shell nuclei as additional diagnostic during the fit of the three-body LECs. Though the eventual fit will preferentially focus on few-body observables, monitoring these many-body observables will help to understand their dependence on individual LECs and the impact of the regulator scheme and scale. Furthermore, many-body observables can provide additional guidance for the choice of LECs, particularly if the few-body observables within their uncertainties do not constrain the LECs sufficiently. We advocate a comprehensive consideration of few- and many-body observables including their theoretical uncertainties when determining optimal parameter ranges for the LECs.

References

[1] P. Navratil, S. Quaglioni, I. Stetcu, B. Barrett, J. Phys. G 36, 083101 (2009).
[2] B. R. Barrett, P. Navratil, J. P. Vary, Prog. Part. Nucl. Phys. 69, 131 (2013).
[3] R. Roth, Phys. Rev. C 79, 064324 (2009).
[4] R. Roth, J. Langhammer, A. Calci, S. Binder, P. Navratil, Phys. Rev. Lett. 107, 072501 (2011).
[5] R. Roth, A. Calci, J. Langhammer, S. Binder, Phys. Rev. C 90, 024325 (2014).
[6] E. Epelbaum, H. Krebs, U.-G. Meißner, Phys. Rev. Lett. 115, 122301 (2015).
[7] K. Hebeler, H. Krebs, E. Epelbaum, J. Golak, R. Skibinski, Phys. Rev. C 91, 044001 (2015).
[8] S. Binder et al., Phys. Rev. C 93, 044002 (2016).
[9] A. Calci, R. Roth, Phys. Rev. C 94, 014322 (2016).
Application of new N\textsuperscript{2}LO 3NF’s in calculations of 3N reactions

Henryk Witała\textsuperscript{1}, Jacek Golak\textsuperscript{1}, Roman Skibiński\textsuperscript{1}, and Kacper Topolnicki\textsuperscript{1}

\textsuperscript{1}M. Smoluchowski Institute of Physics, Jagiellonian University, PL-30348 Kraków, Poland

Solving 3N scattering exactly in a numerical sense up to energies below the pion production threshold allows one to test the 3N Hamiltonian based on modern NN potentials and 3NF’s. At higher energies (above \(\approx 60\) MeV) for some observables large 3NF effects are predicted when standard NN interactions (AV18 \[1\], CDBonn \[2\], NijmI and II \[3\]) are combined with (semi)phenomenological models of 3NF’s such as TM \[4\] or Urbana IX \[5\]). The large discrepancy between the theory and experimental data for the total cross section and in the minimum of the elastic scattering cross section obtained with NN forces only, seen for energies above \(\approx 60\) MeV, is removed for energies below \(\approx 140\) MeV when 3NF’s, which reproduce the experimental triton binding energy, are included \[6\], \[7\]. A similar behavior shows up for the high energy deuteron vector analyzing power \(A_y(d)\) \[7\]. But there are many spin observables for which large 3NF effects are predicted and where the TM and the Urbana IX 3NF do not reproduce the data \[7\]. This is the case e.g. for the nucleon analyzing power \(A_y\) \[7\] and for the deuteron tensor analyzing powers \[7\]. In none of these cases the data can be reproduced by pure 2N force predictions.

The large discrepancies at higher energies between data and theory in elastic Nd scattering which cannot be removed by taking into account standard 3NF’s require to study the magnitude of relativistic effects. They were found to be small for the elastic scattering cross section and negligible for spin-observables at higher energies \[8\].

The small size of relativistic effects indicates that very probably the short range contributions to the 3NF are responsible for the higher energy elastic scattering discrepancies. The recently constructed new generation of chiral NN potentials up to N\textsuperscript{4}LO with an appropriate regularization in the coordinate space \[9\], \[10\] made it possible to reduce significantly finite-cutoff artifacts present when using the nonlocal momentum-space regulator employed in the chiral NN potentials of Refs. \[11\], \[12\]. Application of these new NN potentials does not lead to distortions in the cross section minimum of the higher energy elastic Nd scattering that were found in Ref. \[13\].

Application of improved chiral NN interactions up to N\textsuperscript{4}LO order of chiral expansion \[14\] combined with N\textsuperscript{2}LO 3NF’s supports conclusions obtained with this work was supported by the Polish National Science Center under Grant No. DEC-2013/10/M/ST2/00420. The numerical calculations have been performed on the supercomputer cluster of the JSC, Jülich, Germany.
standard forces. The observed pattern of higher energy discrepancies between data and theory resembles that obtained with standard forces. It can be expected that an application of consistent chiral NN and 3NF’s up to N^3LO will play an important role in understanding of elastic scattering and breakup reactions at higher energies.

At low energies effects of a 3NF are rather small and some serious discrepancies to data remain even when a 3NF is included. The prominent examples are the vector analyzing power in elastic Nd scattering and cross sections for symmetric-space-star (SST) configuration of the Nd breakup [15]. They present serious problem for explanation in terms of present day forces. For SST at 13 MeV the nd [16] and pd [17] breakup data clearly differ and are far away from theory. The calculations of the pd breakup with inclusion of the pp Coulomb force [18] revealed only very small Coulomb force effects for this configuration. Since at that energy the SST configuration is practically dominated by S-wave NN force components, the big difference between pd and nd data could suggest large charge-symmetry breaking in the 1S_0 partial wave. On the other hand the discrepancy to theory would imply that our knowledge of the 1S_0 pp and nn low energy forces is probably insufficient.

References

[1] R. B. Wiringa, V. G. J. Stoks, R. Schiavilla, Phys. Rev. C51, 38 (1995).
[2] R. Machleidt, Phys. Rev. C63, 024001 (2001).
[3] V. G. J. Stoks et al., Phys. Rev. C49, 2950 (1994).
[4] S. A. Coon et al., Nucl. Phys. A317, 242 (1979).
[5] B. S. Pudliner et al., Phys. Rev. C56, 1720 (1997).
[6] H. Witała et al., Phys. Rev. Lett. 81, 1183 (1998).
[7] H. Witała et al., Phys. Rev. C63, 024007 (2001).
[8] H. Witała et al.: Phys. Rev. C 77 (2008) 034004.
[9] E. Epelbaum, H. Krebs and U.-G. Meißner, Eur. Phys. J. A 51, no. 5, 53 (2015).
[10] E. Epelbaum, H. Krebs and U.-G. Meißner, Phys. Rev. Lett. 115, 122301 (2015).
[11] E. Epelbaum, Prog. Part. Nucl. Phys. 57, 654 (2006)
[12] R. Machleidt and D.R. Entem, Phys. Rep.503, 1 (2011).
[13] H. Witała et al., J. Phys. G 41, 094011 (2014).
[14] S. Binder et al.: Phys. Rev. C 93 (2016) 044002.
[15] W. Glöckle et al., Phys. Rep.274, 107 (1996).
[16] H. R. Setze et al., Phys. Lett. B388, 229 (1996).
[17] G. Rauprich et al. Nucl. Phys. A535, 313 (1991).
[18] A. Deltuva, A.C. Fonseca, and P.U. Sauer, Phys. Rev. C72, 054004 (2005).
Probing and Constraining chiral Interactions in Nuclear Structure and Reaction Calculations

Angelo Calci

1TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, V6T 2A3, Canada

In view of the current developments to derive nuclear interactions from chiral effective field theory [1],[2],[3],[4],[5],[6],[7],[8] the improvements of the No-Core Shell Model (NCSM) based ab initio approaches are of particular importance. Concepts such as using Importance-Truncated (IT) model spaces applied in the IT-NCSM [9],[10] allow us to study spectroscopy of bound-state systems throughout the p- and lower sd-shell within controlled approximations. In particular, the consideration of cluster formations in certain nuclei that is exploited in the NCSM with Continuum (NCSMC) [11],[12],[13] enables us to study weakly-bound systems, resonances and nuclear reactions on the same footing. The nuclear systems that can be computed with these ab initio methods often show a strong sensitivity to the details of the nuclear interactions and thus can be used to probe and constrain them. By introducing modifications of the chiral 3N interactions and studying chiral interactions that differ in the chiral order truncation, the regularization scheme and the fit procedure of the chiral constants [1],[2],[3],[4],[5],[6],[7],[8] we can perform a first step towards an uncertainty analysis in nuclear spectroscopy and moreover identify a strong correlated sensitivity of observables such as the excitation energy of the first $1^+$ state in $^{10}B$ and $^{12}C$ [14].

An accurate description of the low-lying spectrum in the $^{11}Be$ system that is dominated by an $n + ^{10}Be$ halo structure has been proven to be extremely challenging [15]. With state of the art ab initio NCSMC calculations we demonstrate that the reproduction of the parity inversion for the two weakly-bound states depend on the details of the chiral NN and 3N interactions. The best reproduction of the energy spectrum and E1 transitions in $^{11}Be$ including the parity-inversion properties is achieved with the $N^2LO_{SAT}$ interaction [8] that generally shows a good reproduction of long-range properties such as radii for this mass regime.

Due to the inclusion of explicit 3N interactions, the computational costs of the NCSMC calculations limits the applications range and number of interactions that can be investigated. However, by combining the concept of the NCSMC and the multi-reference normal-ordering (MR-NO) approach [16] allows us to include the 3N interactions in an approximative manner that is extremely accurate and reduces the computational costs by about two orders of magnitude. This new development allows us to increase the mass number and study as a first example the $^{12}N$ system that is dominated by the $p+^{11}C$ structure, but also revisit lighter reactions such as $n + ^4He$ for the large number of novel chiral potentials. The splitting of the $P_{3/2}$ and $P_{1/2}$ $n+^4He$ phase shifts shows a strong sensitivity to the 3N force and constitutes an ideal candidate to constrain future chiral interactions.
References

[1] D. R. Entem and R. Machleidt, *Phys. Rev. C*, vol. 68, p. 041001(R), 2003.
[2] P. Navrátil, *Few-Body Syst.*, vol. 41, pp. 117–140, 2007.
[3] E. Epelbaum, W. Glöckle, and U.-G. Meißner, *Eur. Phys. J. A*, vol. 19, p. 401, 2004.
[4] E. Epelbaum, W. Glöckle, and U.-G. Meißner, *Nucl. Phys. A*, vol. 747, p. 362, 2005.
[5] E. Epelbaum, H. Krebs, and U.-G. Meißner, *Phys. Rev. Lett.*, vol. 115, p. 122301, 2015.
[6] E. Epelbaum, H. Krebs, and U.-G. Meißner, *Eur. Phys. J. A*, vol. 51, no. 5, 2015.
[7] A. Ekström, G. Baardsen, C. Forssén, G. Hagen, M. Hjorth-Jensen, G. R. Jansen, R. Machleidt, W. Nazarewicz, T. Papenbrock, J. Sarich, and S. M. Wild, *Phys. Rev. Lett.*, vol. 110, p. 192502, 2013.
[8] A. Ekström, G. R. Jansen, K. A. Wendt, G. Hagen, T. Papenbrock, B. D. Carlsson, C. Forssén, M. Hjorth-Jensen, P. Navrátil, and W. Nazarewicz, *Phys. Rev. C*, vol. 91, p. 051301(R), 2015.
[9] R. Roth, *Phys. Rev. C*, vol. 79, p. 064324, 2009.
[10] R. Roth and P. Navrátil, *Phys. Rev. Lett.*, vol. 99, p. 092501, 2007.
[11] S. Baroni, P. Navrátil, and S. Quaglioni, *Phys. Rev. Lett.*, vol. 110, p. 022505, 2013.
[12] S. Baroni, P. Navrátil, and S. Quaglioni, *Phys. Rev. C*, vol. 87, p. 034326, 2013.
[13] P. Navrátil, S. Quaglioni, G. Hupin, C. Romero-Redondo, and A. Calci, *Phys. Scr.*, vol. 91, no. 5, p. 053002, 2016.
[14] A. Calci and R. Roth, *Phys. Rev. C*, vol. 94, p. 014322, 2016.
[15] A. Calci, P. Navrátil, R. Roth, J. Dohet-Eraly, S. Quaglioni, and G. Hupin, *Phys. Rev. Lett.*, vol. 117, p. 242501, 2016.
[16] E. Gebrerufael, A. Calci, and R. Roth, *Phys. Rev. C*, vol. 93, p. 031301, 2016.
Partial-Wave Analysis of NN scattering data at fifth order in chiral EFT

Patrick Reinert\textsuperscript{1}, Evgeny Epelbaum\textsuperscript{1}, and Hermann Krebs\textsuperscript{1}

\textsuperscript{1}Institut für Theoretische Physik II, Ruhr-Universität Bochum, 44780 Bochum, Germany

In effective field theories, free parameters known as low-energy constants (LECs) emerge, which are not constrained by symmetry and instead have to be fixed by experimental data. In the case of our improved chiral NN potential \cite{1,2}, the nucleon-nucleon (NN) contact LECs have (up to now) been determined by a phase shift fit to the Nijmegen partial-wave analysis \cite{3}. To overcome any possible model dependence of the Nijmegen analysis and to account for scattering data published after it, we determine the LECs here directly from experimental neutron-proton and proton-proton scattering data. We employ the same treatment of electromagnetic effects as in \cite{3}.

The experimental data are taken from the 2013 Granada database \cite{4} of mutually compatible scattering data. In order to extend the energy range of the fit to $T_{lab} = 0 - 300$ MeV and to account for the decreasing accuracy of the chiral expansion at higher energies, we estimate the theoretical uncertainties of the scattering observables, as detailed e.g. in \cite{1}, and combine them with the experimental errors.

A small set of high-precision proton-proton data at $T_{lab} > 140$ MeV can not be described to its experimental precision by the N$^4$LO predictions (although the deviation lies within the theoretical uncertainty). In order to probe the sensitivity of these observables to partial waves which are not parametrized at N$^4$LO, we introduce a N$^4$LO$^+$ potential, which differs from the one of \cite{1} by the inclusion of N$^5$LO contact terms in F-waves. We find that the description of the aforementioned outliers is significantly improved and we are now able to describe, after fitting, all proton-proton scattering data in the range of 0-300 MeV with a $\chi^2$/datum of 1.04, while for neutron-proton data we have a $\chi^2$/datum of 1.11.

The obtained phase shifts are in good agreement with the Nijmegen and Granada analyses and the description of experimental data at N$^4$LO$^+$ is comparable to the one of high-quality phenomenological NN potentials, in terms of precision.

References

\begin{itemize}
\item \cite{1} E. Epelbaum, H. Krebs and U.G. Mei\ss ner, Phys. Rev. Lett. 115, 122301 (2015)
\item \cite{2} E. Epelbaum, H. Krebs and U.G. Mei\ss ner, Eur. Phys. J. A 51, 53 (2015)
\end{itemize}
[3] V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester and J. J. de Swart, Phys. Rev. C 48, 792 (1993).

[4] R. Navarro Prez, J. E. Amaro and E. Ruiz Arriola, Phys. Rev. C 88, 064002 (2013)
Ab Initio No-Core Shell Model: Quantifying the Observables’ Uncertainties

James P. Vary

Department of Physics and Astronomy, Iowa State University
Ames, Iowa 50011, USA

Since our goal is to preserve predictive power with quantified uncertainties, it is important to have a solid grasp of all sources of theoretical and numerical uncertainty. I survey uncertainties that appear in current applications of the *ab initio* No-Core Shell Model (NCSM) [1],[2],[3] with Hamiltonians developed in Chiral Effective Field Theory (χEFT) [4],[5],[6],[7].

Under the banner of uncertainties associated with χEFT, I include several topics and comments:

- Fitting of LECs, NN data error propagation [8] (other LENPIC teams)
- Choice of regulator (results presented here are for R = 1.0 fm so far)
- Truncation at a fixed Chiral order (presenting here a revised method to estimate the uncertainty for the ground state energies of nuclei)
- Numerical uncertainty at fixed \([N_{\text{max}}, \hbar \Omega]\) (1 keV in total gs energy)
- Extrapolation uncertainty (new results here for gs energies) [9],[10]

Under the banner of uncertainties associated with the NCSM, I also include several topics and comments:

- Truncation vs OLS renormalization applied to the effective operators (large effects shown here)
- Rank of OLS-derived operator truncation (2-body, 3-body, . . . )
- Other approximations, if adopted, such as normal ordering approximation, importance truncation, . . .

The main conclusions are twofold:

- Uncertainty versus chiral order appears consistent when adopting an average relative momentum scale from the ground state total relative kinetic energy to define a nucleus-dependent chiral expansion parameter Q.
- OLS succeeds in renormalizing the IR and UV scales in initial applications to electroweak operators.
The outlook for these lines of investigation are promising and include:

- Novel approach to scattering is now established and used to predict the tetraneutron. This opens a path for scattering applications with chiral interactions in light nuclei [11], [12].

- Major additional efforts are needed to develop and apply these methods: effective Hamiltonians, effective electroweak operators, many-body methods, computational algorithms, . . .

I am especially indebted to Pieter Maris for many stimulating discussions of uncertainty quantification. I thank my students for diligent efforts to compute some of the effective operator results shown here. I thank the Cracow group for their warm hospitality and for their support of this meeting. This work is sponsored by the US NSF grant number PHY1516181 and by the US DOE DE-FG02-87ER40371.

References

[1] P. Navratil, J. P. Vary and B. R. Barrett, Phys. Rev. Lett. 84, 5728 (2000) [nucl-th/0004058].
[2] P. Navratil, J. P. Vary and B. R. Barrett, Phys. Rev. C 62, 054311 (2000).
[3] B. R. Barrett, P. Navratil and J. P. Vary, Prog. Part. Nucl. Phys. 69, 131 (2013).
[4] E. Epelbaum, W. Glöckle, and Ulf-G. Meißner, Nucl. Phys. A 637, 107 (1998); 671, 295 (2000).
[5] D. R. Entem and R. Machleidt, Phys. Rev. C 68 (2003) 041001 [nucl-th/0304018].
[6] R. Machleidt and D. R. Entem, Phys. Rept. 503, 1 (2011) [arXiv:1105.2919 [nucl-th]].
[7] E. Epelbaum, H. Krebs and U. G. Meißner, Nucl. Phys. A 808, 671 (2009) [nucl-th/0808.1468].
[8] R. Navarro Plesu, J. E. Amaro, E. Ruiz Arriola, P. Maris and J. P. Vary, Phys. Rev. C 92, no. 6, 064003 (2015) [arXiv:1510.02544 [nucl-th]].
[9] S. A. Coon, M. I. Avetian, M. K. G. Kruse, U. van Kolck, P. Maris and J. P. Vary, Phys. Rev. C 86, 054002 (2012) [arXiv:1205.3230 [nucl-th]].
[10] R. J. Furnstahl, G. Hagen and T. Papenbrock, Phys. Rev. C 86, no. 6, 064003 (2012) [arXiv:1207.6100 [nucl-th]].
[11] A. M. Shirokov, G. Papadimitriou, A. I. Mazur, I. A. Mazur, R. Roth and J. P. Vary, Phys. Rev. Lett. 117 (2016) 182502 [arXiv:1607.05631 [nucl-th]].
[12] A. M. Shirokov, A. I. Mazur, I. A. Mazur and J. P. Vary, Phys. Rev. C 94, no. 6, 064320 (2016) [arXiv:1608.05885 [nucl-th]].
Relativistic Faddeev Calculations

Hiroyuki Kamada\(^1\), Henryk Wita\(\acute{s}\)\(^2\), Jacek Golak\(^2\), Roman Skibiński\(^2\), Oleksandr Shebeko\(^3\), and Adam Arslanaliev\(^4\)

\(^1\)Department of Physics, Faculty of Engineering, Kyushu Institute of Technology, 1-1 Sensuicho Tobata, Kitakyushu 804-8550, Japan

\(^2\)M. Smoluchowski Institute of Physics, Jagiellonian University, 30048 Kraków, Poland

\(^3\)NSC Kharkov Institute of Physics and Technology, NAS of Ukraine, Kharkiv, Ukraine

\(^4\)The Karazin University, Kharkiv, Ukraine

Glöckle et al. started to study \(^1\) relativity in the three-nucleon (3N) system under the Bakamjian-Thomas formalism \(^2\), which belongs to the relativistic quantum mechanics and is dictated by the Poincaré algebra. Since a relativistic two-nucleon (2N) potential is not easily provided, we need some schemes which would allow us to transform a nonrelativistic potential into the corresponding relativistic one. Such schemes are required to fulfill the condition that the generated relativistic potential yields the same observables in the 2N system as the original nonrelativistic potential. There are two schemes which satisfy that condition. One was proposed by Coester et al. \(^3\) and we call it the CPS scheme. It requires a solution of a nonlinear integral equation, which can be achieved numerically by an iteration method \(^4\). The other scheme is a momentum scaling method (MSM) \(^5\), realized by an elaborate change of momentum variables. The above-mentioned schemes are examples of simple transformations, while we are actually interested in a comparison between the original nonrelativistic and the modified relativistic potential predictions in the 3N system. In the case of the triton binding energy we have already shown this comparison \(^6\), demonstrating that the difference is smaller when the CPS scheme is employed.

On the other hand, the Kharkov model \(^7\) provides directly the relativistic 2N potential so no transformation scheme is needed in this case.

At this workshop we present the relativistic results of the triton binding energies not only for the Kharkov potential \(^7\) but also for the new N4LO chiral potential \(^8\) and, additionally, for the older realistic CDBonn potential \(^9\). In Table \(^1\) the triton binding energies for these potentials, are demonstrated.

Using the CDBonn potential and the Tucson-Melbourne 3N force we have investigated the Nd elastic scattering \(^10\). Relativistic calculations \(^10,11,12\) show only small effects for elastic scattering cross sections and practically no effects for spin observables. Relativistic effects in the nucleon-induced deuteron

\(^1\)Only the 5-channel result of the Kharkov potential was already shown in \(^13\).
Table 1: The theoretical predictions for the triton binding energy (in MeV), resulting from the solutions of the relativistic and nonrelativistic Faddeev equations with 42 3N-partial-wave states \((j_{max} = 5)\). The numbers in brackets are obtained by the CPS scheme \([3]\), used to transform the nonrelativistic potentials into the relativistic ones, and for the opposite transition in the case of the Kharkov potential.

| Potential type     | Nonrelativistic calc. | Relativistic calc. | Difference |
|--------------------|-----------------------|--------------------|------------|
| CDBonn \([9]\)     | -8.249                | ( -8.150 )         | 0.099      |
| N4LO (R=0.9 fm)    | -7.832                | ( -7.706 )         | 0.126      |
| N4LO (R=1.0 fm)    | -7.867                | ( -7.748 )         | 0.119      |
| N4LO (R=1.1 fm)    | -7.847                | ( -7.733 )         | 0.115      |
| Kharkov \([7]\)    | ( -7.528 )            | -7.641             | 0.067      |

breakup have been investigated in \([14]\). The study of Nd scattering based on the Kharkov potential is also in progress.

This work was partially supported by the Polish National Science Center under Grants No. DEC-2013/10/M/ST2/00420, and by Grant-in-Aid for Scientific Research (B) No: 16H04377, Japan Society for the Promotion of Science (JSPS). The numerical calculations were partially performed on the interactive server at RCNP, Osaka University, Japan, and in the JSC, Jülich, Germany.

References

[1] W. Glöckle, T-S. H. Lee, and F. Coester, Phys. Rev. C 33, (1986) 709.
[2] B. Bakamjian, L.H. Thomas, Phys. Rev. 92, (1953) 1300.
[3] F. Coester, Steven C. Pieper, F.J.D. Serduke, Phys. Rev. C 11, (1975) 1.
[4] H. Kamada, W. Glöckle, Phys. Lett. B655, (2007) 119.
[5] H. Kamada, W. Glöckle, Phys. Rev. Lett. 80, (1998) 2457.
[6] H. Kamada, \textit{et al}., EPJ Web of Conf. 3, (2010) 05025.
[7] I. Dubovyk, O. Shebeko, Few-Body Syst. 48, (2010) 109.
[8] E. Epelbaum \textit{et al}., Eur. Phys. J. A51 (2015) 53.
[9] R. Machleidt, F. Sammarruca, Y. Song, Phys. Rev. C 53, (1996) R1483.
[10] H. Witała \textit{et al}., Phys. Rev. C 71, (2005) 054001.
[11] H. Witała, \textit{et al}., Phys. Rev. C 83, (2011) 044001 ; Erratum, Rev. C 88,(2013) 069904(E).
[12] K. Sekiguchi \textit{et al}., Phys. Rev. Lett. 95, (2005) 162301; Y. Maeda \textit{et al}., Phys. Rev. C 76, (2007) 014004.
[13] H. Kamada, O. Shebeko, and A. Arslanaliev, Few-Body Syst. 58, (2017) 70.
[14] H. Witała, J. Golak, R. Skibiński, Phys. Lett. B 634, 374 (2006)
Recent measurement of $^{12}$C(d, d) tensor and vector analyzing powers at the energy range 170-380 MeV in a view of the charged particles EDM studies

Izabela Ciepał and Ed Stephenson for the JEDI Collaboration

1Institute of Nuclear Physics Polish Academy of Sciences, PL-31342 Kraków, Poland,

2Indiana University Center for Spacetime Symmetries, Department of Physics 107 S. Indiana Avenue, Bloomington, Indiana 47405 USA

Permanent electric dipole moments (EDM) of fundamental particles violate both time invariance T and parity P. Assuming the CPT-theorem, this leads to CP violation, which is needed to explain the matter over antimatter dominance in the universe. Standard Model predictions for this EDM lead to extremely small value - typically many orders of magnitude below current experimental limits. However, extensions of the Standard Model, e.g. Supersymmetric Models, predict much larger EDM. The observation of non zero EDM would represent a clear sign of new physics beyond the Standard Model. Researchers have been searching for EDM in neutral particles, especially neutrons, for more than 50 years, by trapping and cooling particles in small volumes and using strong electric fields. Despite an enormous improvement in sensitivity, these experiments have only produced upper bound which currently is $10^{-26}$ e·cm for the neutron. Theoretical models predict many CP-violating mechanisms and therefore, not only the neutron EDM measurement is important, but so are other hadrons and (light) nuclei, e.g. the proton, deuteron, $^3$He, etc. This will help to isolate the specific CP-violating source(s). Recently, the two-nucleon contributions to the deuteron EDM were calculated in the framework of effective field theory up-to-and-including N2LO [1].

The JEDI (Jülich Electric Dipole moment Investigations) collaboration [2] has started investigations of a direct EDM measurement of charged hadrons at a storage ring [3]. The basic idea is to align the beam polarization along the momentum, and keep the beam circulating while interacting with the radial electric field always present in the particle frame. The EDM signal would then be detected as tiny changes (at the level of a microradian) in the vertical polarization during its storage time. To measure such small effects one needs to build a dedicated high precision polarimeter. To reach a required sensitivity of $10^{-29}$ e·cm, the polarimeter system must fulfill the following requirements: very stable long-term (long beam storage time), efficient (up to 1%), sensitive to the vertical and horizontal polarization components (high analyzing powers of at least 0.5) and strong radiation hardness. To meet to above expectations in a first step the $^{12}$C(d, d) tensor and vector analyzing powers in the energy range 170-380 MeV were measured with the use of the WASA Forward detector at COSY [5]. These data will be used to produce realistic Monte Carlo simulations of detector
responses for the polarimeter design. In parallel, the prototype of the polarimeter is under the development at COSY.

Two pure vector states \((-\frac{2}{3}, 0), (0, 0)\) and two mixed states \((-1, +1), (+\frac{1}{2}, -\frac{1}{2})\) were used in the experiment. The vector and tensor polarizations were first calibrated using the data at 270 MeV \(^6\). For the vector state the polarization was about 85% of maximum and for the tensor 50% and 83%, respectively. Using the so-called “cross ratio” \(^{[7]}\), which nominally cancels the systematic errors up to second order \(^{[8]}\), \(A_y\) and \(A_{yy}\) were calculated. In Fig. 1 the preliminary data obtained are presented, together with the data used for the calibration \(^6\). The vector and tensor analyzing powers were obtained at different energies of 270 MeV and above. In a peak \(A_y\) achieves value of about 0.8 and remains the same for all energies. In the case of \(A_{yy}\) the sensitivity is less.

Figure 1: Measurement of the deuteron elastic scattering \(A_y\) and \(A_{yy}\) analyzing powers at energies listed in the panel. The data at 270 MeV (orange) were used to calibrate the polarization values. The data are very preliminary.

References

[1] J. Bsaisou et al., Annals of Physics 359 (2015) 317.
[2] http://collaborations.fz-juelich.de/ikp/jedi/
[3] F.J.M. Farley et al., Phys. Rev. Lett. 93 (2004) 052001.
[4] G. Guidoboni, E.J. Stephenson, et al., Phys. Rev. Lett. 117 (2016) 054801.
[5] Chr. Bargholtz et al., Nucl. Instrum. Methods A594 (2008) 339.
[6] Y. Satou et al., Phys. Lett. B549 (2002) 307.
[7] G.G. Ohlsen, P.W. Keaton Jr., Nucl. Instrum. Methods A109 (1973) 41.
[8] N.P.M. Brantjes, P.W. Keaton Jr., Nucl. Instrum. Methods A664 (2012) 49.
Nuclear currents in chiral effective field theory

Hermann Krebs

Institut für Theoretische Physik II, Ruhr-Universität Bochum,
44780 Bochum, Germany

Chiral effective field theory (EFT) is a powerful tool for description of low energy nuclear phenomena. Based on the symmetries of Quantum Chromodynamics (QCD) it provides a systematic expansion of the nuclear forces in small momenta and masses scale.

Within chiral EFT nuclear vector and axial vector currents have been analyzed by different groups up to the order $Q^3$ in the chiral expansion ($N^3$LO). The currents have been calculated by using time-ordered perturbation theory (TOPT, see [1] and references therein) and a method of unitary transformations (UT, see [2] and references therein). In our recent work within UT [2] we analyzed the axial vector current and compared our results with TOPT results. We found discrepancies for two-pion exchange contributions. It remains to be seen if there exists a unitary transformation which would bridge TOPT and UT results.

Within UT method we found a large unitary ambiguity for the axial vector current which is mainly caused by pion-pole contributions. These contributions are coming from pion-production substructures. If we require pion-production substructures of the axial vector current and the three-nucleon forces to be the same (we call this requirement matching condition) and additionally require the currents to be perturbatively renormalizable we arrive at a unique result. All unitary phases become fixed by these two conditions.

In practical implementation the currents need to be regularized. Their regularization, however, should be chosen consistently with the nuclear forces. Matching condition suggests the same form of the regulator as in the nuclear forces. The chiral symmetry, on the other hand, provides an additional constraint which is the continuity equation. We propose for future investigations to force the continuity equation to be exactly (not only up to higher order terms) fulfilled for regularized currents (Siegert approach). Only in this case the cutoff dependence of the low energy constant $C_D$ in the axial vector current and the thee-nucleon forces will be the same. Numerical implementation within the proposed line are underway.

References

[1] D. O. Riska and R. Schiavilla, Int. J. Mod. Phys. E 26, no. 01n02, 1740022 (2017) doi:10.1142/S0218301317400225 [arXiv:1603.01253 [nucl-th]].
[2] H. Krebs, E. Epelbaum and U.-G. Meißner, Annals Phys. 378, 317 (2017) doi:10.1016/j.aop.2017.01.021 [arXiv:1610.03569 [nucl-th]].
Jacobi No-core Shell Model for Hypernuclei

Hoai Le1, Ulf-G Meißner1,2, and Andreas Nogga1,3

1Institut für Kernphysik, Institute for Advanced Simulation and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany

2Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany.

3Department of Physics and Astronomy, Ohio University, Athens, OH 45701, USA

Hypernuclei have recently attracted a lot of attention due to new experimental data. Ongoing experiments at international facilities like JPARC, JLAB, Mainz and FAIR are expected to provide more accurate information on hypernuclear structure. To establish a direct link between these data and properties of the hyperon-nucleon (YN) interaction, theoretical predictions for these hypernuclei based on realistic YN interaction models are vital.

In this talk, we presented our recent work on no-core shell model for light hypernuclei using a harmonic oscillator (HO) basis in Jacobi coordinates. The special properties of HO states allow an exact antisymmetrization of the basis [1]. Our calculations are based on realistic chiral nucleon-nucleon (NN) interactions up to fifth order [2] and YN ones up to second order [3], [4] in the chiral expansion. The similarity Renormalization Group (SRG) is applied to NN and YN interactions to speed up the convergence. We showed our first results for the Λ separation energies $E_b$ of $^4\Lambda$He, $^5\Lambda$He and $^7\Lambda$Li. We found that in both orders, zeroth (LO) and second (NLO), YN interactions with with SRG strongly overbind the light hypernuclei. NLO order in general leads to smaller separation energies than LO. Furthermore, the SRG for both YN interactions induces sizable higher-body (YNN, or even YNNN) forces. Similar results for LO order are also obtained in [5].

The impacts of NN interactions on the separation energy $E_b$ are also discussed in detail. The effect of NN forces and NN-SRG on $E_b$ is negligible for $^4\Lambda$He and $^5\Lambda$He, however it can be noticeable for $^7\Lambda$Li. We also found that $E_b$ is not strongly dependent on details of the core structure in $^4\Lambda$He and $^5\Lambda$He but somewhat more in $^7\Lambda$Li.

Our first results show the feasibility of Jacobi-NCSM calculations for hypernuclei. It will be very interesting to extend the J-NCSM to more complex single-strangeness systems as well as double-strangeness hypernuclei.

Acknowledgments This work is supported by DFG and NSFC through funds provided to the Sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD” (grant No. DBC01241). The numerical calculations have been
performed on JUQUEEN and JURECA of the JSC, Jülich, Germany.

References

[1] S. Liebig, U.-G. Meißner, A. Nogga, Eur. Phys. J. A 52 (2016) 103.
[2] E. Epelbaum, H. Krebs and U.-G. Meißner, Eur. Phys. J. A 51 (2015) no.5 53.
[3] H. Polinder, J. Heidenbauer, U.-G. Meißner, Nucl. Phys. A 779 (2006) 244.
[4] J. Heidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, Nucl. Phys. A 915 (2013) 24.
[5] R. Wirth, R. Roth. et al. Phys. Rev. Lett. 117 (2016) 182501.
Operator form of nucleon-deuteron scattering

Kacper Topolnicki, Jacek Golak, Roman Skibiński, Yuriy Volkotrub, and Henryk Witala

M. Smoluchowski Institute of Physics, Jagiellonian University, PL-30348, Kraków, Poland

The so-called “three-dimensional” approach is an alternative to partial wave calculations with applications in few-nucleon physics. It was successfully applied to various problems e.g. two-nucleon scattering [1], [2] or deuteron [1] and triton [3] bound state calculations. It was also used to calculate observables in processes that involve electro-weak probes like electron [4] and muon [5] induced deuteron disintegrations. Recently we performed “three-dimensional” calculations of neutron-deuteron scattering observables, in both the elastic and breakup channels, using an approximate solution of the Faddeev equation [6]. We compared partial wave and “three-dimensional” results and concluded that the partial wave observables converge slowly for certain kinematical configurations of the deuteron breakup at higher energies. This motivates us to construct complete “three-dimensional”, three-nucleon scattering calculations for the full solution of the Faddeev equation.

In order to perform practical calculations related to nucleon-deuteron scattering without resorting to partial wave decomposition we developed a general operator form of the scattering amplitude \( \langle pq | \hat{T} | \phi \rangle \) where \( \hat{T} \) is the three-nucleon transition operator, \( | \phi \rangle \) is an initial product state composed from a deuteron and a free nucleon, and \( p, q \) are Jacobi momenta in the final state. This amplitude is an element of the Faddeev equation:

\[
\hat{T} \mid \phi \rangle = i \hat{P} \mid \phi \rangle + \hat{G}_0 \hat{P} \hat{T} \mid \phi \rangle,
\]

where \( \hat{P} \) is a permutation operator and \( i \) is the two-nucleon transition operator satisfying the Lippmann-Schwinger equation. It has the following general operator form [7]:

\[
\langle pq | \hat{T} | \phi \rangle = \sum_{\gamma^{3N}} \sum_{r=1}^{64} \tau_r^{3N}(p, q, q_0) \mid \gamma^{3N} \rangle \otimes (\hat{O}_r(p, q, q_0) \mid \alpha \rangle \) (1)

with \( \tau_r^{3N}(p, q, q_0) \) being scalar functions of the two Jacobi momenta in the final state and the free nucleon momentum \( q_0 \), \( \hat{O}_r(p, q, q_0) \) being one of 64 spin operators [7] and \( \mid \gamma^{3N} \rangle \) being one of the possible three-nucleon isospin states.

Using the general form (1), the scattering amplitude is defined by the set of scalar functions \( \tau_r^{3N}(p, q, q_0) \). Rewriting the Faddeev equation using (1) leads to a significant reduction of numerical complexity and we hope that it will allow us to construct full “three-dimensional”, three-nucleon scattering calculations.
This will extend the applicability of the “three-dimensional” formalism to test three- and many-body nuclear forces derived from chiral effective field theory \cite{8,9,10,11}.

Acknowledgements: This work was supported by the Polish National Science Center under grants No. DEC-2016/21/D/ST2/01120 and DEC-2013/10/M/ST2/00420. Some numerical calculations were performed on the supercomputing clusters of the JSC, Jülich, Germany.

References

[1] J. Golak, W. Glöckle, R. Skibiński, H. Witała, D. Rozpędzik, K. Topolnicki, I. Fachruddin, Ch. Elster, A. Nogga, Phys. Rev. C 81, (2010) 034006
[2] J. Golak, R. Skibiński, H. Witała, K. Topolnicki, W. Glöckle, A. Nogga, H. Kamada, Few-Body Syst. 53, (2012) 237
[3] J. Golak, K. Topolnicki, R. Skibiński, W. Glöckle, H. Kamada, A. Nogga, Few-Body Syst. 54, (2013) 2427
[4] K. Topolnicki, J. Golak, R. Skibiński, A.E. Elmeshneb, W. Glöckle, A. Nogga, H. Kamada, Few-Body Syst. 54, (2013) 2233
[5] J. Golak, R. Skibiński, H. Witała, K. Topolnicki, A. E. Elmeshneb, H. Kamada, A. Nogga, L. E. Marcucci, Phys. Rev. C 90, (2014) 024001
[6] K. Topolnicki, J. Golak, R. Skibiński, H. Witała, C.A. Bertulani, Eur. Phys J. A 51, (2015) 132
[7] K. Topolnicki et. al., in preparation.
[8] E. Epelbaum, H. Krebs, Ulf-G. Meißer, Eur. Phys. J. A 51, (2015) 53
[9] E. Epelbaum, H. Krebs, Ulf-G. Meißer, Phys. Rev. Lett. 115, (2015) 122301
[10] D. R. Entem, N. Kaiser, R. Machleidt, Y. Nosyk, Phys. Rev. C 92, (2015) 064001
[11] H. Krebs, A. Gasparyan, E. Epelbaum, Phys. Rev. C 85, (2012) 054006
Advances in the In-Medium Similarity Renormalization Group

Klaus Vobig

1Institut für Kernphysik, Technische Universität Darmstadt, Schloßgartenstr. 2, 64289 Darmstadt, Germany

With the advent of nuclear interactions derived from chiral EFT and their early successes in the description of p-shell nuclei in the No-Core Shell Model (NCSM) [1], the \textit{ab initio} description of medium-mass nuclei with chiral NN+3N interactions moved into the focus of several research groups worldwide.

A very successful approach is the In-Medium Similarity Renormalization Group (IM-SRG) which aims at decoupling an $A$-body reference state $|\Psi_{\text{ref}}\rangle$ from all particle-hole excitations. This can be achieved via a continuous unitary transformation of the Hamiltonian. An important aspect of the IM-SRG is that the Hamiltonian is normal ordered with respect to the reference state and typically truncated at the normal-ordered two-body level. In this way, 3N interactions can be naturally included in a normal-ordered two-body approximation.

A great asset of the IM-SRG is the flexibility and simplicity of its basic concept. Through different choices of generators and decoupling patterns, the numerical characteristics and efficiency of the methods can be controlled and tailored for specific applications. The IM-SRG evolved Hamiltonian is easily accessible, hermitian, in contrast to e.g. coupled-cluster approaches, and can be used for subsequent calculations.

We have first applied this \textit{ab initio} method for the description of ground states of closed-shell nuclei with chiral NN [4] and NN+3N interactions [5]. Going beyond the closed-shell or single-reference version we have generalized the IM-SRG to an open-shell or multi-reference formulation [9]. This multi-reference IM-SRG is based on a multi-reference version of normal ordering and Wick’s theorem proposed by Kutzelnigg and Mukherjee in quantum chemistry [2]. Furthermore, we have merged the IM-SRG with the NCSM leading to the IM-NCSM. This novel \textit{ab initio} many-body approach uses multi-reference IM-SRG evolved operators as input for a subsequent NCSM calculation and, as we have shown in [3] comes along with a great improvement of the model-space convergence of the subsequent NCSM calculation. In Fig.2 we have used the IM-NCSM for the calculation of ground state energies of the even isotopes of the carbon and oxygen chain. Thus, we have advanced the IM-SRG to a versatile tool for the calculation of spectra and spectroscopy of medium-mass nuclei, both closed- and open-shell. We are investigating extensions of this IM-NCSM approach that will allow us to overcome some of its limitations which are related to the choice of the reference state. Overcoming these limitations would enable us to explore the whole medium-mass regime of the nuclear chart.
In the context of the consistent NN+3N interactions with semi-local regulators up to N^3LO developed within the LENPIC collaboration [6], we will apply the IM-SRG for exploring the order-by-order convergence of the chiral EFT expansion in the medium-mass regime for the first time. We will extract systematic EFT uncertainties for the ground-state and excitation energies as well as radii of medium-mass systems. Thus, we will be able to give a complete and systematic quantification of theory uncertainties from the chiral interaction up to the many-body method.

References

[1] R. Roth, J. Langhammer, et al., Phys. Rev. Lett. 107, 072501 (2011).
[2] W. Kutzelnigg and D. Mukherjee, J. Chem. Phys. 107, 432 (1997).
[3] E. Gebrerufael, K. Vobig, et al., Phys. Rev. Lett. 118, 152503 (2017)
[4] K. Tsukiyama, S. K. Bogner, et al., Phys. Rev. Lett. 106, 222502 (2011).
[5] H. Hergert, S. K. Bogner, et al., Phys. Rev. C 87, 034307 (2013).
[6] S. Binder, A. Calci, et al., Phys. Rev. C 93, 44002 (2016).
[7] D. R. Entem, R. Machleidt, et al., arXiv:1703.05454 (2017).
[8] A. Ekström, G. R. Jansen, et al., Phys. Rev. C 91, 051301 (2015).
[9] H. Hergert, S. K. Bogner, et al., Phys. Rev. C 90, 41302 (2014).
The OPE-Gaussian force in elastic Nd scattering

Yuriy Volkotrub, Roman Skibiński, Jacek Golak, Kacper Topolnicki, and Henryk Witała

M. Smoluchowski Institute of Physics, Jagiellonian University, 30-384 Kraków, Poland

A reliable estimation of theoretical uncertainties becomes an increasingly important issue in nuclear physics. The most relevant sources of such uncertainties are the inaccuracies of the potential parameters extracted from experimental data, the uncertainties related to theoretical approaches and finally the imprecision inherent to numerical methods. Truncation errors in studies performed with nuclear forces derived within the chiral effective field theory are an example of the errors arising from the applied theoretical framework. A prescription to estimate such uncertainties has been proposed recently in [1] for the two-nucleon (NN) system and in [2] for three-nucleon (3N) observables. An estimation of errors stemming from a given numerical scheme is also possible and many computer science methods can be applied here [3]. However, it is expected that numerical computations are performed with a high precision and these types of uncertainties can be relatively easily minimized.

The propagation of theoretical uncertainties from the NN force to many-body observables is an open question. The recently developed the One-Pion-Exchange-Gaussian (OPE-Gaussian) potential [4] delivers a unique opportunity to study this issue. This is because of extensive attention paid by the authors of Ref. [4] to determine statistically well defined uncertainties of the potential parameters.

In this contribution we announce our project for using the OPE-Gaussian force to study nucleon-deuteron (Nd) scattering and for investigating the uncertainties of 3N observables stemming from uncertainties of the potential parameters. At the current stage we have already obtained predictions for the Nd scattering observables using the central values of the interaction parameters.

The OPE-Gaussian force is a phenomenological potential, which employs the set of operators used in the AV18 potential [5] with few additional extensions. It can be decomposed as

\[ V(\vec{r}) = V_{\text{short}}(r)\theta(r_c - r) + V_{\text{long}}(r)\theta(r - r_c), \]

where \( r_c = 3 \text{ fm} \) and the \( V_{\text{long}} \) part is just the one-pion exchange force supplemented by the electromagnetic corrections in the case of the proton-proton force. The

This work is a part of the LENPIC project. It was supported by the Polish National Science Center under Grants No. DEC-2013/10/M/ST2/00420. The numerical calculations have been performed on the supercomputer cluster of the JSC, Jülich, Germany.
short-range force component can be written as

\[ V_{\text{short}}(\vec{r}) = \sum_{n=1}^{21} \hat{O}_n \left[ \sum_{i=1}^{N} V_{i,n} F_{i,n}(r) \right], \quad (2) \]

where \( \hat{O}_n \) are the operators from the extended AV18 basis [5]. The radial Gaussian functions \( F_{i,n}(r) \) depend on free parameters \( a_{i,n} \) which together with the \( V_{i,n} \) strength parameters are fixed from the NN data. It is worth mentioning that authors of Ref. [4] carefully analyzed the existing NN data and constructed the "3\( \sigma \) self-consistent" database which was necessary to obtain statistical uncertainties of the free potential parameters.

Working within the formalism of the Faddeev equation [6] we studied Nd elastic scattering up to incoming nucleon laboratory energy \( E=200 \) MeV. We confirm good behaviour of the new potential and exemplify this in Fig. 3 for \( E=13 \) MeV. We compare there predictions for various observables, obtained with the OPE-Gaussian force or with the AV18 interaction, and observe very good agreement between predictions based on both models. This is a starting point for our plans to estimate theoretical uncertainties for these observables.

Figure 3: The differential cross section \( \frac{d\sigma}{d\Omega} \), the nucleon analyzing power \( A_Y(N) \) and the deuteron analyzing powers \( iT_{11} \) and \( T_{21} \) for the Nd scattering at \( E=13 \) MeV. The black dashed (the red solid) curve represents the AV18 (the OPE-Gaussian) predictions. Data for the \( A_Y(N) \) are from Ref. [7].

References

[1] E.Epelbaum, H.Krebs, and Ulf-G.Meißner, Eur. Phys. J. A51, 53 (2015).
[2] S.Binder, et al., Phys. Rev. C93, 044002 (2016).
[3] M.L.Overton, Numerical Computing with IEEE Floating Point Arithmetic, SIAM, Philadelphia, 2001.
[4] R.Navarro Pérez, J.E.Amaro, and E.Ruiz Arriola, Phys. Rev. C89, 064006 (2014).
[5] R.B.Wiringa, V.G.J.Stoks, and R.Schiavilla, Phys. Rev. C51, 38 (1995).
[6] W.Glöckle et al., Phys. Rept. 274, 107 (1996).
[7] J.Cub, et al., Few-Body Syst. 6, 151 (1989).