NON–LOCAL THERMODYNAMIC EQUILIBRIUM MODEL OF NGC 6543’s CENTRAL STAR AND ITS RELATION TO THE SURROUNDING PLANETARY NEBULA

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ABSTRACT

We analyze the chemical composition of the central star of the planetary nebula NGC 6543 based on a detailed NLTE model of its stellar wind. The logarithmic abundances by number are $H = 12.00$, $He = 11.00$, $C = 9.03$, $N = 8.36$, $O = 9.02$, $Si = 8.19$, $P = 5.53$, $S = 7.57$, and $Fe = 7.24$. Compared with the solar abundances, most of the elements have solar composition with respect to hydrogen, except $C$, which is overabundant by 0.28 dex, and $Fe$, which is depleted by $\sim 0.2$ dex. Contrary to most previous work, we find that the star is not H-poor and has a normal He composition. These abundances are compared with those found in the diffuse X-ray plasma and the nebular gas. Compared to the plasma emitting in diffuse X-rays, the stellar wind is much less depleted in iron. Since the iron depletions in the nebular gas and X-ray plasma are similar, we conclude that the plasma emitting diffuse X-rays is derived from the nebular gas rather than the stellar wind. Excellent agreement is obtained between the abundances in the stellar wind and the nebular recombination line abundances for $He$, $C$, and $O$ relative to $H$. On the other hand, the derived stellar $N$ abundance is smaller than the nebular $N$ abundance derived from recombination lines and agrees with the abundance found from collisionally excited lines. The mean temperature variation determined by five different methods indicates that the difference in the nebular abundances between the recombination lines and collisionally excited lines can be explained as due to the temperature variations in a chemically homogeneous medium.

Subject headings: planetary nebulae: individual (NGC 6543)

1. INTRODUCTION

The generally accepted scenario of planetary nebula formation involves the interaction between the stellar material expelled at low velocity during the asymptotic giant branch (AGB) phase (slow wind) and at high velocity at some later moment (fast wind; Kwok et al. 1978). A direct consequence of this scenario is the presence of shock fronts propagating inward into the fast wind and outward into the slow wind. The shocked gas of the fast wind should be heated to temperatures that correspond to its kinetic energy. The typical fast wind velocity is $\sim 1000$ km s\(^{-1}\) and the mass loss rate is $1 \times 10^{-8} M_\odot$ yr\(^{-1}\), which implies temperatures $\sim 10^7$ K. At these temperatures the gas should emit in X-rays; that emission has been detected recently (Chu et al. 2001; Guerrero et al. 2005). However, the observed temperatures of the X-ray gas are $(1-3) \times 10^6$ K, an order of magnitude lower than expected. Several scenarios have been proposed to explain this discrepancy (Soker & Kastner 2003), but none of them are easily accepted as the best one.

In our previous paper (Georgiev et al. 2006b), we tried to learn more about the X-ray emitting region using optical coronal lines of iron. Unfortunately, these lines were not detected, which set an upper limit on the iron composition of the X-ray emitting gas, a limit far below the solar metallicity. This result was confirmed by Kastner et al. (2006), who did not detect iron lines in their high-resolution X-ray spectrum of BD +30 36939. The peculiar chemical composition of the X-ray-emitting gas leads to a possible trace of its origin. If the compositions of the stellar wind, the nebula, and the X-ray emitting gas are compared, one can decide which of them are related, and that might indicate the origin of the X-ray gas and perhaps provide clues to the physical processes that leading to its low temperature.

Some planetary nebulae (PNe), NGC 6543 among them, have iron abundances that are very depleted with respect to the solar value (Perinotto et al. 1999; Marcolino et al. 2007), most probably due to depletion into dust grains. The depleted iron content in both the nebular and the X-ray-emitting gas points to a possible relation between them. Unfortunately, during the AGB phase of evolution, iron atoms can capture slow neutrons (the $\nu$-processes; Busso et al. 1999). The $\nu$-processes deplete iron and one needs a good estimate of the composition of the stellar wind before arriving at any conclusion about the relation between the hot gas and its surroundings.
In this paper, we model the stellar wind of the central star of NGC 6543, one of the brightest planetary nebulae emitting X-rays, to derive its chemical composition. Section 2 presents the observed spectrum and § 3 describes the selection of the model parameters. In §§ 4 and 5, we compare the chemical composition of the stellar wind with that of the nebular gas and the X-ray plasma, respectively. Section 6 considers the evolution of the progenitor of NGC 6543 while § 7 presents our conclusions.

2. OBSERVATIONS

To construct a good stellar atmosphere model that includes the stellar wind, we need information as wide a range of wavelengths as possible. We constructed the spectrum of NGC 6543’s central star from four sources, three covering the ultraviolet and one covering the optical.

First, we extracted the Far Ultraviolet Spectroscopic Explorer (FUSE) spectrum \( (R \sim 15,000-20,000) \) using program Q108 (PI Vidal-Madjar, A) obtained on 2000 October 1 at 21 hr 51 minutes. The spectrum was constructed from SiC 2A, SiC 2B, LiF 1A, and LiF 1B fragments. All parts of the spectrum were shifted in wavelength and scaled to match the LiF 1B segment. The resulting spectrum was wavelength shifted and scaled to match the overlapping part of the Space Telescope Imaging Spectrograph (STIS) spectrum. Second, two Hubble Space Telescope STIS spectra \( (R \sim 100,000) \) from program 9736 (PI: R. Williams), taken on 2004 April 7, were used to cover the wavelength region from 1190 to 1500 \( \AA \). Unfortunately, the third spectrum from the same program, which covers the C iv 12548/1250 region, has a problem with the wavelength calibration and is not useful. Third, we used International Ultraviolet Explorer (IUE) spectrum SWP33504 \( (R \sim 10,000) \) to cover the region 1500–2000 \( \AA \). The two LWP high-resolution spectra of NGC 6543 are too noisy and we did not use them in the diagnostic procedure. These three fractions of the spectrum were finally combined, and the resulting spectrum was used as a temperature diagnostic and for determining the reddening. After the temperature and reddening were determined, we normalized the observed spectrum to the predicted model continuum scaled to match the line-free region between 1750 and 1900 \( \AA \).

Finally, the fourth set of spectra were in the optical and consisted of spectra obtained by us at the Observatorio Astronómico Nacional in the Sierra of San Pedro Martir (SPM) using the REOSC echelle spectrograph \( (R \sim 20,000) \). A blue spectrum was obtained on 2004 May 26. The data and their reduction are described in Georgiev et al. (2006b). This spectrum was obtained to maximize signal-to-noise ratio (S/N). All of the strong nebular lines are saturated and the spectrograph was set so that H\( \alpha \) was outside the observed wavelength interval. Red spectra were obtained on 2004 June 6 with short exposures so that the hydrogen and oxygen lines were not saturated. The spectrum was reduced using MIDAS (Munich Image Data Analysis System). After the standard CCD reductions, the spectrum was extracted only in the part of the slit covered by the star. Two additional spectra were extracted in adjoining windows. The signal from these windows was averaged, scaled, and subtracted from the stellar spectrum. As a result, most of the nebular spectrum was removed, except in the cores of the strong lines arising from both the nebula and the central star. For these stellar lines, the profiles are severely perturbed, but, keeping in mind that the stellar lines are much broader than the nebular lines, the residuals of the strong nebular lines do not affect the wings of the stellar lines, and so they may still be compared with model line profiles. This procedure worked well for the blue spectrum. The red spectrum is strongly contaminated by scattered light from H\( \alpha \). We corrected the spectrum for this contamination, but we expect H\( \alpha \) to be overcorrected. Therefore, the observed intensity of H\( \alpha \) is expected to be less than the model prediction. After the spectrum was corrected for the nebular contamination, it was normalized to the continuum by fitting polynomials to the line-free regions.

3. MODEL

The central star of NGC 6543 is known to be variable. A detailed analysis of the spectral variability was published by Prinja et al. (2007). They found that “Mostly the flux changes are at \( \sim 10\% - 20\% \) of the continuum level and occur over localized blueward velocity regions, as opposed to the flux increasing or decreasing simultaneously over the entire absorption trough.” For the rest of the spectrum the authors concluded that “Any changes in the other spectral lines are rather subtle if present at all.” Therefore, our analysis, which is based on the overall line profile and, in most cases, on weak absorption lines, is not affected by the variability.

3.1. Helium Abundance

The helium content of the wind is one of the fundamental parameters that has to be determined at least roughly beforehand. The central star of NGC 6543 has been found to be hydrogen-poor (Bianchi et al. 1986; de Koter et al. 1996), so we tested this assumption as the first step of our analysis. Due to the similarity of He ii and H i ions, the He ii lines arising from the transition \( 4 \rightarrow n \) where \( n \) is even have wavelengths close to the wavelengths of the H i lines of the Balmer series. The lines where \( n \) is odd fall between the Balmer lines and are not affected by the hydrogen component. When the wind is hydrogen-poor, the intensities of the lines where \( n \) is odd and even decrease smoothly with increasing \( n \). On the other hand, if the abundance of H i is not negligible, the intensities of the Balmer lines are higher than the He ii lines where \( n \) is odd (Smith 1973). From Figure 1 it is obvious that the central star of NGC 6543 cannot be H-poor. A series of models suggests that the He/H ratio should be approximately solar. We adopt \( N(\text{H})/N(\text{Ne}) = 0.10 \) in the following analysis.

3.2. Stellar Parameters

The model of a stellar wind depends on a large number of parameters. In addition to the effective temperature and the gravity, which describe a plane parallel, static atmosphere, a wind requires parameters describing the velocity and the mass loss rate, and a parameter describing the clumpiness of the wind. The task of fitting all of the parameters is difficult. We proceeded by fixing some of the parameters while fitting the others and then iterated, changing the parameters to be fitted.

The parameters describing the star are not independent. If we describe the star as an opaque nucleus with radius \( R_\ast \) and temperature \( T_\ast \), then its luminosity is \( L = 4\pi R_\ast^2 \alpha T_\ast^4 \). It is known (Schmutz et al. 1989) that models with the same temperature \( T_\ast \), velocity law, and transformed radius,

\[
R_\ast = R_* \left( \frac{V_{\infty}}{2500} \right)^{2/3},
\]

have very similar emitted spectra. Substituting the radius \( R_\ast \), one obtains a scaling rule for the mass loss rate if a different luminosity is necessary,

\[
\frac{\dot{M}_1}{\dot{M}_2} = \left( \frac{L_1}{L_2} \right)^{3/4}.
\]
We describe the velocity of the wind using the standard velocity law given by

\[ V(r) = V_\infty (1 - R/r)^{3/2}, \]

(3)
fitted to a hydrostatic atmosphere with a gravity \( g \), as described in Hillier et al. (2003). The terminal velocity \( V_\infty \) was set to 1340 km s\(^{-1}\) as determined from the blue wing of the P\( \nu \lambda 1118/\lambda 1128 \) doublet. The wind of NGC 6543 is not simple. Strong lines of different ions with P Cyg profiles require different terminal velocities. Ions with higher ionization potential tend to have P Cyg lines with a more extended blue wing. In addition, Prinja et al. (2007) showed that the wind variability is present in the absorption component of the P\( \nu \lambda 1118/\lambda 1128 \) doublet but that O\( \nu \lambda 1032/\lambda 1038 \) is more stable. The formation of the strong P Cyg lines is obviously complicated, and there is no easy explanation for their behavior. We avoided this problem by excluding lines with strong P Cyg profiles from our analysis. We account for the presence of different blue wings of the P Cyg lines by increasing the error in \( V_\infty \) to 200 km s\(^{-1}\). With the velocity law fixed, we continued with the determination of the temperature.

There are lines of several elements with two or more consecutive ionization stages which, in principle, could be used as temperature indicators. Unfortunately, it is not easy to use any of them in practice. The O\( \nu \lambda 1339/\lambda 1341 \) doublet and O\( \nu \lambda 1371 \) show different terminal velocities. In addition, the O\( \nu \lambda 1371 \) line is heavily blended with iron lines and its intensity strongly depends on the iron composition. The same is true for the N\( \nu \lambda 4930 \) and N\( \nu \lambda 4944 \) lines. Instead of using ion ratios, we selected several temperature-sensitive features in the spectrum and searched for a temperature that reproduces most of them. These features usually depend on the mass loss rate also, so one needs to determine the correct combination of \( T_{\text{eff}} \) and \( \dot{M} \) that reproduce the measured fluxes simultaneously. To do that, we ran a grid of models covering...
temperatures from 60 to 75 kK and mass loss rates from $2.8 \times 10^{-8}$ to $7.8 \times 10^{-8} \, M_\odot \, \text{yr}^{-1}$, setting the clumping factor to $f = 0.1$ (see below) and the luminosity to $L = 5200 \, L_\odot$ (as in previous work; de Koter et al. 1996; Georgiev et al. 2006a). The temperature limits were set by the C iii and P v lines. Below 60 kK the C iii $\lambda 1179$ line is too strong. Above 75 kK the P v $\lambda 1118/\lambda 1128$ doublet is too weak. Once the grid is calculated, one can construct a surface for the value of any parameter measured in the synthetic spectra as a function of both temperature and $\dot{M}$. The observed value of the parameter defines an isoline on that surface. If the isolines defined by several parameters are drawn on the same coordinates $(T_{\text{eff}}, \dot{M})$, their crossing point defines the combination of $T_{\text{eff}}$ and $\dot{M}$, which reproduces all of the parameters simultaneously. In practice, the isolines do not cross in a point, but rather in a region, whose size defines the error in the parameters. We ran grids for the velocity law, setting the parameter $\beta$ equal to 1.2, and 3. The isolines cross in a smaller region for $\beta = 2.0$, so we used that value in the subsequent analysis.

The left panel of Figure 2 shows the isolines for the intensity of He ii $\lambda 4686$, H$\beta$, and their ratio. The right panel of the same figure shows several other diagnostics. All of the isolines cross in the vicinity of $T = 66750 \pm 500 \, \text{K}$ and $M = 3.2 \pm 0.05 \times 10^{-8} \, M_\odot \, \text{yr}^{-1}$. We adopted these parameters for further modeling. Once the temperature and the mass loss rate are determined, the absolute flux calculated by the model can be compared with the observed UV spectrum. The difference provides an estimate of the extinction. We used the R-dependent Galactic extinction curve from Fitzpatrick (1999) for $R = 3.1$. The best fit of the line-free regions of the spectrum is given by $E(B-V) = 0.025 \pm 0.005 \, \text{mag}$ (Fig. 3), which is close enough to the value $E(B-V) = 0.07 \, \text{mag}$ used by Bernard-Salas et al. (2003). The error in the reddening is the formal error of the fit. The actual error is larger, due to its dependence on the model parameters. The scaling factor between the observed UV spectrum and the calculated continuum implies a distance of $1.81 \pm 0.05 \, \text{kpc}$, where the error reflects the error in the reddening. Reed et al. (1999) determined the distance to NGC 6543 of $\sim 1.0 \, \text{kpc}$. To get results in agreement with this distance, we scaled down the luminosity to $L = 1585 \, L_\odot$ and the mass loss rate $\dot{M} = 1.86 \pm 0.3 \times 10^{-8} \, M_\odot \, \text{yr}^{-1}$, according to equation (2), where the error also reflects the uncertainty in $V_\infty$. We stress that the model itself does not provide a distance due to the degeneracy between the luminosity and the mass loss rate mentioned above.

Finally, we have to address two additional parameters. First is the stellar mass. The assumed luminosity $L = 1585 \, L_\odot$ and the adopted temperature implies a value of $\log g = 5.3$ if the stellar mass is $0.6 \, M_\odot$. The mass of the star can vary by a factor of 1.3, which leads to changes in $\log g$ of $\pm 0.12$. We did test runs with higher and lower gravity. There were no significant changes in the emitted spectrum, so we think that a different value of $\log g$ (necessary for a different luminosity) would not change the conclusions made in the next section. On the other hand, the wings of the hydrogen lines are well reproduced (Fig. 5), which indicates that the adopted value of $g$ is adequate.

Another parameter that can change the emitted spectrum is the wind clumpiness. The winds of massive stars are clumpy, as deduced from the electron scattering wings of strong lines (Hillier 2005). The same lines in the spectrum of NGC 6543 are not strong enough to permit us to determine the clumping factor $f$. In general, models with the same $M/\sqrt{f}$ produce similar spectra. We ran test models with $f = 0.05, 0.1, 0.5$, and 1.0, with corresponding changes in the mass loss rates. Only a few lines changed, between them the strong C iv $\lambda 5801/\lambda 5812$ doublet. Since we did not use these lines in the composition analysis, we set the clumping factor to $f = 0.1$, which is similar to the value $f = 0.08$ obtained by Prinja et al. (2007). A comparison between the stellar parameters obtained in this paper and by other authors is given in Table 1.

### 3.3. Chemical Composition

Once the parameters of the star are determined, the composition of the elements is determined by comparing the predicted and observed line fluxes. As mentioned above, the strong resonance lines are poorly reproduced and are not useful for abundance determinations. The following analysis is based on weaker lines that are formed deep in the wind and in a smaller volume. These lines are better reproduced by the model and we expect that they are less sensitive to the perturbations of the wind structure and geometry. The model reproduces most of the lines correctly, but we could not reproduce everything. Some of the observed features are missing in the model spectrum. One has to keep in mind that the model presented here includes most of the important physics and chemistry, but not all of them. Given this consideration, a comparison between observations and models with different abundances yields the errors presented in Table 2.

### TABLE 1

| Reference                  | $V_\infty$ (km s$^{-1}$) | $\beta$ | $T_{\text{eff}}$ (K) | $L/L_\odot$ | Distance (pc) | $M$ ($M_\odot$ yr$^{-1}$) |
|----------------------------|--------------------------|--------|----------------------|-------------|---------------|--------------------------|
| Castor et al. (1981)........| 2150                     | 1.0    | 43000               | 2000        | 1170          | $9 \times 10^{-8}$       |
| Bianchi et al. (1986).......| 1900                     | 2.0    | 80000               | 15100       | 1390          | $32 \times 10^{-8}$      |
| Perinotto et al. (1989).....| 1900                     | 1.5    | 60000               | 5600        | 1440          | $4 \times 10^{-8}$       |
| de Koter et al. (1996)......| 1600                     | 1.5    | 48000               | 5200        | ...           | $16 \times 10^{-8}$      |
| This work...................| 1340                     | 2.0    | 66750               | 1585        | 1000          | $1.86 \times 10^{-8}$    |
Oxygen is found mainly as O\textsuperscript{iv} and O\textsuperscript{v}. We determined its abundance using the O\textsuperscript{iv} doublet at 3560 Å/3563 Å. This line is blend-free and has a good S/N (Fig. 4). The line intensity is reproduced with an oxygen abundance of 12 + log N(O)/N(H) = 9.02 dex. As an additional criteria to check the O abundance, we used the O\textsuperscript{v} λ1419, O\textsuperscript{v} λ1423, and O\textsuperscript{v} λ5114 lines (Figs. 5 and 6), which were well reproduced. In addition, there are a few O\textsuperscript{v} lines around 965 Å (Fig. 7) that are overestimated, which points to an overabundance of O\textsuperscript{v} in the model with respect to the observations. This is in agreement with the additional ionization of O\textsuperscript{v} (see below) and we adopt the abundance derived from O\textsuperscript{iv} lines.

In addition to the O\textsuperscript{iv} and O\textsuperscript{v} lines, the FUSE spectrum shows a very strong O\textsuperscript{iii} λ1032/λ1038 resonance doublet. The model with the temperature and mass loss rate adopted above underestimates these lines by an order of magnitude. One needs a temperature above 90 kK and a higher mass loss rate to reproduce these lines, such as the λ3831/λ3834 doublet, not present in our spectra. Test models with higher temperature, which reproduce O\textsuperscript{iii} λ1032/λ1038, also show an observable O\textsuperscript{iii} λ3811/λ3834 doublet. These arguments imply an additional ionization source in the external part of the wind, which does not affect the internal, denser, part of the wind. The density in the highly ionized part of the wind is very low, meaning that only the resonance lines have enough opacity to form an observable feature, i.e., producing O\textsuperscript{vi} λ1032/λ1038, but not O\textsuperscript{v} λ3811/λ3834. One could speculate that the additional ionization source is the diffuse X-ray plasma in the nebula shell. Fitting the O\textsuperscript{vi} λ1032/λ1038 lines with shock-generated or other X-ray emission is beyond the scope of this paper. The abundance analysis based on weak lines of low ionization stages should not be affected by the X-ray ionization and should be correct.

Carbon is represented by two ionization stages, with C\textsuperscript{iv} being the dominant one. Lines of C\textsuperscript{iii} and C\textsuperscript{iv} are observed in the FUSE, IUE, and optical regions of the spectrum. The C\textsuperscript{iv} resonance doublet λ1548/λ1550 behaves similarly to O\textsuperscript{v} λ1032/λ1038. It has a higher $V_{\infty}$ and a larger observed intensity than the model predicts. We determined the carbon composition mainly from the C\textsuperscript{iii} λ1176 and λ977 lines (Figs. 9 and 7). These two lines, together with C\textsuperscript{iv} λ1169, were used as a temperature diagnostic. Our final estimate is 12 + log N(C)/N(H) = 9.03 dex.

Nitrogen is mainly in the form of N\textsuperscript{v} and, like O\textsuperscript{v} and C\textsuperscript{iv}, it is also apparently strongly affected by the X-ray ionization. We estimate its composition using the N\textsuperscript{v} λ955, N\textsuperscript{v} λ1718, N\textsuperscript{iv} λ4058, and N\textsuperscript{v} λ4944 lines (Figs. 7, 5, and 6). The lines are reproduced with 12 + log N(N)/N(H) = 8.36 dex.
The Si iv resonance doublet 1398 Å/1402 Å is usually present in many objects. Due to the high temperature of the star, the lines are weak and heavily blended with Fe vi lines. Instead of using the resonance doublet, we estimated the silicon abundance using the Si iv λ4089 and λ4116 lines (Fig. 5). The lines are reproduced with 12 + log N(Si)/N(H) = 8.19 dex, a factor of 5 higher than the solar value. The model with this abundance reproduces several other Si iv lines as well. We have to stress that the Si iv ion is not the dominant stage of silicon at a temperature of ~70 kK. Only 0.2% of the silicon is in Si iv. Therefore, the estimated overabundance depends strongly on the atomic data and it is highly uncertain. On the other hand, Hultzsch et al. (2007) found a similar silicon overabundance in the central star of M1-37. Silicon overabundance is an unexpected result which has no explanation for the moment.

In addition to the above-mentioned elements, the model contains sulfur and phosphorus. The sulfur line S v λ1503 was fitted with 12 + log N(S)/N(H) = 7.57 dex. We checked that this abundance also reproduces the S vi λ4162 (Fig. 5) line well. The phosphorus lines P v λ1118/λ1128 are well fitted with an abundance of 12 + log N(P)/N(H) = 5.53 dex.

Finally, we analyzed the iron content. A first look at the spectrum reveals strong iron lines between 1250 Å and 1450 Å. The presence of these lines rules out a significant depletion of iron (Georgiev et al. 2006a), but the precise modeling showed that these features are not very sensitive to variations in the iron abundance. A similar result was reported by Marcelino et al. (2007) for BD+30°36939. Therefore, we decided to apply the classical curve of growth method. Once we had the wind parameters and composition fixed, we ran several models with different iron abundances, starting from the solar value and finishing with 1/10 of the solar value. We selected about 10 absorption lines, mainly Fe v and Fe vii (Figs. 8 and 9), that showed the strongest variation with composition. The equivalent widths of the lines both in the observed spectrum and in the models were measured using a Gaussian fit. Using the curve of growth method, we derived the Fe abundance for each line. The median value yields 12 + log N(Fe)/N(H) = 7.24 dex which is ~0.2 dex lower than the solar value.

The model also includes lines of argon, neon and nickel. The abundances of these elements were set to their solar values and were not determined by line fitting.

Our elemental abundances for the central star’s wind are presented in Table 2, along with those for the Sun and the Orion nebula. Note that the Ne and Ar abundance determinations for the Orion nebula are more accurate than those for the Sun. The uncertainties in the abundances for NGC 6543, except in the case of iron, are derived from models with increased and decreased abundances. The uncertainty reflects the changes in the abundances that produce changes greater than the noise in the data. The uncertainty in the iron abundance is the standard deviation of the abundance derived for each of the lines analyzed.

Finally, we stress that our model of the central star’s wind is undoubtedly only one of several possible solutions. Several combinations of parameters produce similar spectra and therefore increase the uncertainty in the stellar parameters, which is reflected by the errors in the derived abundances. In addition, the observed spectrum shows features that are not present in our model. Nonetheless, the precision of the chemical composition presented here is sufficiently robust for the purposes of this paper. A more refined model, dealing with the formation of O vi lines, the identification of the observed features, and the difference in the wind velocity shown by different resonance lines, will be treated in a forthcoming paper. A final conclusion on the evolutionary status of the object and the interrelation between its components (star, nebula, X-ray-emitting gas) require a self-consistent model including the central star and the nebula (C. Morisset & L. N. Georgiev, in preparation).

4. RELATIONSHIP BETWEEN THE STELLAR AND NEBULAR ABUNDANCES

In most planetary nebulae, the abundances for heavy elements derived from recombination and collisionally excited lines do not agree. Explanations for this result depend fundamentally on
large temperature variations that cannot be explained by simple photoionization models (e.g., Liu 2006; Peimbert & Peimbert 2006 and references therein). The temperature variations explain the large differences between the chemical abundances derived from recombination lines and those derived from collisionally excited lines when a constant temperature is used. The difference between these types of abundances is called the abundance discrepancy factor, ADF. There are two different ideas that explain the ADFs: (1) temperature variations are present in a chemically inhomogeneous medium (e.g., Torres-Peimbert et al. 1990; Liu 2006 and references therein); and (2) temperature variations are present due to other causes in a chemically homogeneous medium (e.g., Peimbert & Peimbert 2006 and references therein).

According to the two-abundance nebular model by Liu and collaborators, PNe present two components: (1) a low-density component that has most of the mass and is relatively hot, emits practically all the intensity of the H lines and of the forbidden lines in the visual and the UV, as well as part of the intensity of the He i lines; and (2) a high-density component that has only a small fraction of the total mass, is relatively cool, H-poor, rich in heavy elements, and emits part of the He i and all of the recombination lines of the heavy elements, but emits practically no H nor any collisionally excited lines from heavy elements. Chemically inhomogeneous nebulae can be produced by H-poor stars that eject H-poor material into H-rich nebulae. That is the case in A30 and A78 (Jacoby 1979; Hazard et al. 1980; Jacoby & Ford 1983; Manchado et al. 1988; Wesson et al. 2003). This type of situation might occur in those cases where the central star has an H-poor atmosphere. According to Görny & Tylenda (2000), about 10% of the central stars of PNe are H-poor. Studies based on the Sloan project find similar results: based on 2065 DA and DB white dwarfs, Kleinman et al. (2004) find that 1888 are nonmagnetic DAs and 177 are nonmagnetic DBs. Therefore, we conclude that about 10% of Galactic PNe have a H-poor central star, and might show He-, C-, and O-rich inclusions in their expanding shells. We consider it unlikely that PNe with H-rich central stars would contain significant amounts of H-poor material in their associated nebulae.

4.1. Nebular Collisional and Recombination Abundances

In Table 3, we present abundances derived from collisionally excited lines from different studies. In the last column, we present our adopted values based on the values presented in the previous columns. For O\(^{++}\), the value taken from Wesson & Liu (2004) is based only on their [O \text{III}] \lambda 4959 determination, since the 52 and 88\(\mu\)m lines have a different dependence on temperature than the [O \text{III}] \lambda 4959 and 5007 lines. In addition, the presence of density variations affects the O abundance determination based on the 52 and 88\(\mu\)m lines.

In Table 4, we present abundances derived from recombination lines from different studies. In the last column we present our adopted values. The \((\text{N(He)}/(\text{N(H)}) ratio was derived by us from helium recombination lines and uses more recent atomic data than those used in other studies. The detailed abundance determination is based on maximum likelihood method, MLM, and is discussed in §4.2. For C, the adopted value is just the average of the three determinations. For O\(^{++}\), the adopted abundance value is based on the determination of Wesson & Liu (2004) but with two modifications: (1) we weighted the contribution of each multiplet according to its effective recombination coefficient; and (2) we eliminated multiplet V12, which might be contaminated by other emission lines. Our determination yields

**TABLE 3**

| Element | (1) | (2) | (3) | (4) | (5) |
|---------|-----|-----|-----|-----|-----|
| C       | 8.50| 8.40| ... | 8.30| 8.40 ± 0.10 |
| N       | 8.50| 8.36| ... | ... | 8.79 ± 0.06 |
| Ne      | 8.27| 8.28| 8.20| ... | 8.25 ± 0.06 |
| S       | 7.09| 7.11| 7.05| ... | 7.08 ± 0.06 |
| Ar      | 6.53| 6.62| ... | ... | 6.57 ± 0.06 |
| Cl      | 5.40| ... | ... | ... | 5.40 ± 0.10 |

**TABLE 4**

| Element | (1) | (2) | (3) | (4) | (5) | (6) |
|---------|-----|-----|-----|-----|-----|-----|
| He      | 11.0 ± 0.01 | 11.07| 11.03| 11.09| 11.05| 11.05 |
| C       | 8.90 ± 0.00 | 8.90| 8.92| 8.87| 8.90| 8.90 |
| N       | 8.83 ± 0.20 | 8.83| ... | ... | 8.83| 8.83 |
| O       | 9.30 ± 0.12 | 9.30| 9.40| 9.19| 9.19| 9.19 |
| Ne      | 8.67 ± 0.10 | 8.67| ... | ... | ... | ... |

**Notes**—In units of \(10^{\text{12} + \log (N(\text{X})/N(\text{H}))}\). Col. (1): Zhang et al. (2004), Col. (2): Bernard-Salas et al. (2003), Col. (3): Kingsburgh et al. (1996), Col. (4): Rola & Stasinska (1994) and Peimbert et al. (1995), Col. (5): Adopted values.
N\(\text{O}^{++}\)/N(\(\text{H}^+\)) = 1.42 \times 10^{-3}.\) Note that V1, the brightest multiplet, yields 1.18 \times 10^{-3} for this ratio. Adopting an ionization correction factor of 1.09 derived from the [O ii] \(\lambda\lambda 3726,3729\) lines and the N\(\text{O}^{++}\)/N(\(\text{H}^+\)) recombination ratio, we obtain an oxygen abundance 12 + log N(O)/N(H) = 9.19 dex. At first sight the Ne/H ratio derived from recombination lines seems to be high (see Table 4), but the values for log Ne/O values for the CELs and RLs amount to –0.52 and –0.54 dex, respectively, in good agreement with the values derived for a large number of PNe by Torres-Peimbert & Peimbert (1977) and Kingsburgh & Barlow (1994), who find an average log Ne/O ratio of –0.59 dex.

### 4.2. \(\text{r}^2\) Value Determinations

In Table 5, we present various temperature determinations for NGC 6543. \(T(\text{Bac})\) is the temperature determined from the intensity ratio of the Balmer continuum to a Balmer line. \(T(\text{He ii})\) is the temperature derived from the \(\text{He i}\) recombination line ratios (see next subsection). \(T[\text{O iii}]\) and \(T[\text{O ii}]\) are the temperatures derived from the ratio of the auroral and nebular line intensities for the corresponding ions. \(T[\text{O iii}], [\text{O ii}]\) is the representative temperature for the forbidden lines, where we are assuming that 92% of the emission originates in the \(\text{O}^{++}\) zone and 8% in the \(\text{O}^+\) zone. The differences among the various temperatures imply the presence of temperature variations within the observed volume. In addition, the differences between the collisional and recombination abundances, the ADF values, for C, N, O, and Ne also imply the presence of temperature variations.

To reconcile the differences among the various temperatures and between the collisional and recombination abundances it is possible to characterize the temperature structure by an average temperature, \(T_0\) and a mean square temperature variation, \(r^2\). These quantities are given by

\[
T_0(N_e,N_i) = \frac{\int T_e(r)N_e(r)N_i(r)dr}{\int N_e(r)N_i(r)dr}
\]

and

\[
r^2 = \frac{\int (T_e - T_0)^2N_eN_idV}{T_0^2\int N_eN_idV},
\]

where \(N_e\) and \(N_i\) are the electron and the ion densities, respectively, of the observed emission line and \(V\) is the observed volume (Peimbert 1967).

Under the assumption of chemical homogeneity to derive a \(r^2\) value, we need two independent temperature determinations, and the temperature dependence of the line or continuum intensities used to derive the temperature. It is also possible to derive a \(r^2\) value for a particular ion by reconciling the abundances derived for this ion derived from the intensities of collisional and recombination lines (Peimbert et al. 2004 and references therein). The result is correct even in the presence of chemical inhomogeneities.

In Table 6, we present five independent \(r^2\) determinations. The first two were obtained from the comparison of two temperatures representative of the whole observed volume, and the other three were derived under the assumption that the collisional and the recombination abundances had to be the same. The temperature dependencies of the recombination lines of \(\text{C}^{++}\), \(\text{O}^{++}\), and \(\text{Ne}^{++}\), needed to determine the \(r^2\) values were obtained from Davey et al. (2000), Storey (1994), and Kisielius et al. (1998), respectively.

### 4.3. Physical Conditions Derived from the Helium Recombination Lines

To obtain N(\(\text{He}^+\))/N(\(\text{H}^+\)) values, we need a set of effective recombination coefficients for the helium and hydrogen lines, an estimate of the optical depth effects for the \(\text{He i}\) lines, and the contribution to the \(\text{He i}\) line intensities due to collisional excitation. We used the hydrogen recombination coefficients from Storey & Hummer (1995), the helium recombination coefficients from Porter et al. (2005) with the interpolation formulae provided by Porter et al. (2007), and the collisional contribution to the \(\text{He i}\) lines by Sawey & Berrington (1993) and Kingdon & Ferland (1995). The optical depth effects in the triplet lines were estimated from the computations by Benjamin et al. (2002).

To derive the physical conditions associated with the helium ionized region, we have used a maximum likelihood method, MLM (Peimbert et al. 2000, 2002). To determine \(N_e(\text{He ii}), T_e(\text{He ii}), N(\text{He}^+)/N(\text{H}^+),\) and the optical depth in the \(\text{He i}\) \(\lambda\lambda 3889\) line, (\(\tau_{3889}\)), self-consistently, we used as inputs a characteristic density from the forbidden line ratios of \(N_e = 4000 \pm 2000\) cm\(^{-3}\) and 13 \(I(\text{He i})/I(\text{He ii})\) line ratios observed by Wesson & Liu (2004; the 13 \(\text{He i}\) lines are \(\lambda\lambda 3880, 3889, 3965, 4026, 4387, 4438, 4471, 4713, 4922, 5876, 6676, 7065,\) and 7281). Each of the 14 observational constraints depends on \(T_e(\text{He ii}), N_e(\text{He ii}), N(\text{He}^+)/N(\text{H}^+),\) and \(\tau_{3889}\), each dependence being unique. Therefore, we have a system of 14 equations and four unknowns. We obtain the best value for the four unknowns and \(r^2\) by minimizing \(\chi^2\). In this way, we obtained \(r^2 = 0.035 \pm 0.014,\)

### Table 6

| Method | \(r^2\) |
|--------|--------|
| \(T(\text{Bac})\) and \(T_e([\text{O iii}]],[\text{O ii}])\) | 0.028 ± 0.009 |
| \(T(\text{He ii})\) and \(T_e([\text{O iii}],[\text{O ii}])\) | 0.035 ± 0.014 |
| \(N(\text{C}^{++})_{\text{RL}}\) and \(N(\text{C}^{++})_{\text{CEL}}\) | 0.036 ± 0.010 |
| \(N(\text{O}^{++})_{\text{RL}}\) and \(N(\text{O}^{++})_{\text{CEL}}\) | 0.024 ± 0.008 |
| \(N(\text{Ne}^{++})_{\text{RL}}\) and \(N(\text{Ne}^{++})_{\text{CEL}}\) | 0.022 ± 0.010 |
| Average | 0.028 ± 0.005 |
emitting gas cannot be related to the stellar wind. On the other hand, the nebular gas is iron-depleted. The iron abundance in the nebular gas is estimated to be depleted by a factor of 11 compared to the solar abundance (Perinotto et al. 1999). Therefore, the hot gas appears to be of nebular origin.

At least for NGC 6543, therefore, one can interpret the formation of the hot, X-ray-emitting plasma as the result of heating nebular material. One possible mechanism to accomplish this is thermal conduction, as suggested by Soker (1994) and Zhekov & Perinotto (1996, 1998). This mechanism also explains the low temperature of the X-ray-emitting gas. In this case, the observed temperature is not directly related to the deposition of the mechanical energy by the wind, but rather to the efficiency of thermal conduction. The cold nebular gas is heated by thermal conduction to the observed temperature of the hot plasma and so emits in diffuse X-rays.

The same scenario can be extended to at least one other object with diffuse X-ray emission. Marcolino et al. (2007) showed that the central star of BD +30 3639 has an iron abundance that is only moderately depleted. They concluded that the iron forest in the UV is reproduced with an iron content equal to 1/4 of the solar value, but they cannot rule out the solar composition either. Previously, we have found the X-ray-emitting plasma to be depleted in iron by a factor of 8 (Georgiev et al. 2006b). This result again implies that the hot gas arises from nebular material rather than from the stellar wind.

6. THE EVOLUTION OF THE PROGENITOR OF NGC 6543

In addition to the iron abundance in the stellar wind, we derive the abundances of CNO and some other elements that constrain the evolution of the star before it became a planetary nebula. The most important abundance anomaly is the carbon abundance. Compared to the solar composition, carbon is enriched significantly, while oxygen and nitrogen are almost normal. In addition, iron is depleted. These abundances imply that the progenitor star had a mass below 4.0 $M_\odot$, but higher than 1.8 $M_\odot$. The upper limit follows from the normal nitrogen composition. Stars with masses above 4.0 $M_\odot$ experience hot bottom burning and show large nitrogen and helium enhancements (Herwig 2005). On the other hand, carbon is overabundant, which requires a carbon-rich intershell, formed only in stars with masses above 1.8 $M_\odot$. The depletion of iron points to the presence of effective s-processes. We cannot observe any of the elements heavier than iron and cannot determine their abundances, but the iron depletion is an indirect indicator of these processes.

7. CONCLUSIONS

We have obtained a detailed atmospheric non-LTE model of the central star of NGC 6543 that includes a stellar wind and that matches a large number of the observed emission and absorption lines. The main physical parameters of the model are $T_{\text{eff}} = 66750$ K, $R_\text{star} = 1.97 \times 10^{10}$ cm, $L_\text{eff} = 1585$ $L_\odot$, log $g = 5.3$ cm s$^{-2}$, $M_\text{lon} = 1.86 \times 10^{-8}$ $M_\odot$ yr$^{-1}$ (adopting a clumping factor $f = 0.1$), and $V_\infty = 1340$ km s$^{-1}$. For this model we assumed a distance of 1 kpc. Changes in the distance and the mass of the central star affect the value of log $g$ but do not appreciably modify the spectrum or the chemical abundances that we derive. The chemical composition of the stellar wind is presented in Table 2. The He/H ratio has been reliably determined and implies that the central star is not He-rich, contrary to the results obtained by most other authors. The chemical composition of the stellar wind may be compared with the compositions of other components in the planetary

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**Table 7**

| Element | Stellar | Nebular (RL) | Nebular (CEL) |
|---------|---------|--------------|---------------|
| He      | 11.00 ± 0.04 | 11.05 ± 0.01 | ...           |
| C       | 9.03 ± 0.10  | 8.90 ± 0.10  | 8.40 ± 0.10   |
| N       | 8.36 ± 0.10  | 8.83 ± 0.20  | 8.43 ± 0.20   |
| O       | 9.02 ± 0.10  | 9.19 ± 0.12  | 8.79 ± 0.06   |
| S       | 7.57 ± 0.10  | ...          | 7.08 ± 0.06   |
| Ne      | ...        | 8.67 ± 0.10  | 8.25 ± 0.06   |

Note.—In units of $12 + \log N(X)/N(H)$. 

$T_e$(He ii) = 6674 ± 559 K, $N_e$(He ii) = 3383 ± 2100 cm$^{-3}$, $N_\text{HI}$ = 9.4 ± 0.9, and $N$(He$^+$/H$^+$) = 0.1130 ± 0.0023.
nebula system. In particular, the iron abundance in the stellar wind is much higher than that found for the nebular gas or the hot plasma that emits in X-rays. Therefore, it would appear that the plasma emitting diffuse X-rays in NGC 6543 must arise from heated nebular material. The same conclusion is reached regarding the X-ray-emitting plasma in BD+30°3639.

We have also derived the chemical composition of the nebula surrounding the star based on recombination lines (RLs) and collisionally excited lines (CELs). The abundances of C, O, and Ne relative to hydrogen derived from recombination lines are from 0.4 to 0.5 dex higher than the abundances derived from collisionally excited lines. The difference has been called the ADF. From five different methods involving emission lines of H, He, C, O, and Ne, we have found a mean square temperature variation $i^2 = 0.028 \pm 0.005$. Supposing spatial temperature variations of this amplitude in a chemically homogeneous nebula, it is possible to reconcile the CEL and RL abundances. In this situation, we find excellent agreement between the stellar and the nebular RL abundances for He/H, C/H, and O/H. On the other hand, the stellar N/H value is about 0.4 dex smaller than the nebular RL abundance and agrees with the nebular CEL abundance.

This is the first paper in which we make a detailed comparison between the chemical composition of the central star and of the surrounding nebula of a planetary nebula. We consider it imperative to compare the chemical composition of the central stars of planetary nebulae with those of their surrounding nebulae to advance the study of the evolution of intermediate-mass stars. This comparison is also paramount to test different hypotheses regarding the origin of a variety of observed properties, among them the large temperature variations present in many planetary nebulae and the origin of the X-ray emission.

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