An application of almost marginal conditional stochastic dominance (AMCSD) on forming efficient portfolios

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Abstract. Investors always seek an efficient portfolio which is a portfolio that has a maximum return on specific risk or minimal risk on specific return. Almost marginal conditional stochastic dominance (AMCSD) criteria can be used to form the efficient portfolio. The aim of this research is to apply the AMCSD criteria to form an efficient portfolio of bank shares listed in the LQ-45. This criteria is used when there are areas that do not meet the criteria of marginal conditional stochastic dominance (MCSD). On the other words, this criteria can be derived from quotient of areas that violate the MCSD criteria with the area that violate and not violate the MCSD criteria. Based on the data bank stocks listed on LQ-45, it can be stated that there are 38 efficient portfolios of 420 portfolios where each portfolio comprises of 4 stocks and 315 efficient portfolios of 1710 portfolios with each of portfolio has 3 stocks.

1. Introduction
Investments are made on investment objects or assets, one of which is stock. A collection of the assets is called portfolio. In fact, investors prefer an efficient portfolio. The portfolio is said to be efficient if it has maximum return on a specific risk or minimal risk in specific return. Therefore, many methods have been developed to obtain the efficient portfolio.

In 1994, Shalit and Yitzhaki [6] introduced the marginal criteria conditional stochastic dominance (MCSD) to solve the problem of second order stochastic criteria in identifying efficiency portfolio. MCSD criteria states the probabilistic conditions under which all risk averse individuals. Given a portfolio of asset, investors prefer to increase the share of one risky asset over the other. Furthermore, Denuit et al. [3] introduced criteria almost marginal conditional stochastic dominance (AMCSD) by replacing the utility function of second order stochastic dominance used in MCSD with utility functions of almost second order stochastic dominance. AMCSD criteria is used when there is violation area which violates the rule of MCSD. The value of AMCSD criteria is ratio of area which violates the rule of MCSD with area which violates and/or not violates the rule of MCSD. AMCSD criteria is therefore used to determine the efficiency portfolio and this provides solutions if portfolio is not efficient (Denuit et al. [3]).

The banking sector is the foundation of economy. Some stocks of banking sector are listed in the LQ−45. These stocks are the stocks of Bank BRI, Bank BTN, Bank Mandiri, Bank BCA, and Bank BNI. Stocks listed in the LQ−45 is the best stocks because they have good liquidity (BEI [1]). In fact, investors are interested in investing the stocks with good liquidity (Levy [4]).
2. Research Method

In this research, we apply the AMCSD criteria to form efficient portfolios which consist of banking stocks in the LQ-45. The steps are as follows. Firstly, we apply the AMCSD criteria to determine the area which violates the rule of MCSD. Next, we determine the value of $\varepsilon$ which is ratio of area which violates the rule of MCSD with area which violates and/or not violates the rule of MCSD. For the application of AMCSD, the first step is to calculate the stock returns and the portfolio return. After that, we calculate the value of Absolute Concentration Curve (ACC) for every stocks. The next step is to calculate the value of $\varepsilon$ of any comparison of two stocks in a portfolio. Finally, we conclude whether the portfolio is efficient or not.

3. Almost Marginal Conditional Stochastic Dominance (AMCSD)

Almost marginal conditional stochastic dominance (AMCSD) is a decision-making criteria when there is violation of MCSD criteria. Utility functions used in AMCSD is utility function $U_2^*(\varepsilon_2)$, defined by

$$U_2^*(\varepsilon_2) = \{u : u'(x) \geq 0, u''(x) \leq 0, u'(x) \leq \inf \{-u''(x)\} \frac{1}{\varepsilon_2} - 1, \forall x \in S\}.$$ 

Conditional expected return of $R_i$ when the portfolio’s return equals $r_+$ is defined by $E[R_i|R_+ = r_+]$. Furthermore, this is denoted by $\mu_i(r_+)$. Suppose we take two assets in the portfolio i.e asset $k$ and asset $j$. Asset $k$ dominates asset $j$ by MCSD if $ACC_k^\alpha(\xi) \geq ACC_j^\alpha(\xi)$. The value of $ACC_k^\alpha(\xi)$ is denoted by $ACC_k^\alpha(\xi) = \int_{-\infty}^{F_{R_+}^{-1}(\xi)} \mu_k(r_+)f(r_+)dr$. Then,

$$ACC_k^\alpha(\xi) = E[R_i|R_+ \leq F_{R_+}^{-1}(\xi)]f(r_+), \quad (1)$$

where $F_{R_+}^{-1}(\xi) = \inf \{r_+\epsilon R_+|F_{R_+}(r_+) \geq (\xi)\}$. If return is bounded in interval $[a, b]$, the difference of ACC’s value of asset $k$ and asset $j$ is defined by

$$B(t) = \int_a^t (\mu_k(r_+) - \mu_j(r_+))f(r_+)dr_+. \quad (2)$$

If asset $k$ dominate asset $j$ by MCSD then we get $B(t) \geq 0$.

Given the condition that asset $k$ does not dominate asset $j$, because for the area $L_2$, it can be found that $ACC_k^\alpha(\xi) \leq ACC_j^\alpha(\xi)$ so that the criteria of MCSD can not be used. Suppose $\Omega$ is defined as a set of areas that violate the rules of MCSD as $\Omega = \{te[a, b]|B(t) < 0\}$. The area $L_2$ is an area which violate the MCSD’s criteria because $B(t) < 0$. The area of $L_2$ can be obtained as

$$L_2 = \int_\Omega -B(t)dt. \quad (3)$$

Furthermore, the total area that lies between the curve of ACC’s asset $k$ and the curve of ACC’s asset $j$ is written as

$$(L_1 + L_3) + L_2 = \int_a^b |B(t)|dt. \quad (4)$$

The ratio of area which violates the rule of MCSD with area which violates or not violates the rule of MCSD is denoted by $\varepsilon$. Therefore, $\varepsilon$ can be expressed as

$$\varepsilon = \frac{\int_\Omega -B(t)dt}{\int_a^b |B(t)|dt}. \quad (5)$$

In the economy, the critical areas of AMCSD’s criteria is 0,032 (Levy et al. [5]).

Based on Shalit and Yitzhaki [6], if there is no assets dominates in portfolio, then the portfolio is an efficient portfolio. However, if there is asset $k$ which dominates asset $j$ by AMCSD in the portfolio with $k \neq j$, then the portfolio is not an efficient potfolio.
4. APPLICATION OF AMCSD ON FORMING EFFICIENT PORTFOLIOS

The data used in this research are the monthly stocks price data of the stocks of Bank BMRI, BBCA, BBRI, BBNI, and BBTN respectively. The data taken are data from January 2011 to October 2015. In this research, the portfolios formed consists of 5 stocks, 4 stocks and 3 stocks. Then, AMCSD will be used to analyze portfolio with any portion of stocks. For the illustration of calculation, we calculated the BBRI’s stock returns in February 2011 using Brigham and Houston [2]. The return is

\[ R(t) = \frac{5.800-5.850.66}{5.850.66} \times 100\% = 0.8659\% . \]

In this example, the BMRI’s stock price in January 2011 is 5.850, 66 and BMRI’s stock price in February 2011 is 5.800. Random test was performed according to Wackerly et al. [7]. With a significance level of 0.05, it was obtained that stock returns of BMRI, BBCA, BBRI, BBNI, and BBTN are random. Jarque-Bera normality test was performed with a significance level of 0.05 based on the stock returns data and it obtained that the stock returns data of BMRI, BBCA, BBRI, BBNI, and BBTN are not normally distributed. Therefore, AMCSD criteria can be used.

In this research, the lowest portion of stocks in a portfolio consists of 5 stocks, 4 stocks, and 3 stocks were at 10%, 10%, and 5%, respectively. The portfolios formed are 126 portfolios that consist of 5 stocks with different portion of each stock. There are 5 combinations stocks that can be formed which consist of 4 stocks. Each combination consist of 84 portfolios with different portions, so that the number of portfolios which has 4 stocks is 420 portfolios. There are 10 combinations of portfolios that consists of 3 stocks. Each combination consists of 171 portfolios with different portions, so that the number of portfolios which has 3 stocks is 1710 portfolios. For example, portfolio that has 3 stocks is a portfolio consisting of 70% BMRI’s stock, 5% of BBRI’s stock, and 25% of BBNI’s stock. Furthermore, these portfolios will be analyzed whether the portfolios are efficient or not. Returns of the portfolio for February 2011 can be calculated, and the result is

\[ R_+ = 70\% \times (-0.8659\%) + 5\% \times (-3.0928\%) + 25\% \times (10.0775\%) = 1.7586\% . \]

To calculate the ACC of BBRI when the cumulative probability of portfolio’s return is 0.0702, we use the equation of (1), and we obtained 0.8951.

It is known that \( E[R_{BBRI}] \geq E[R_{BBNI}] \geq E[R_{BMRI}] \). Therefore, we just need to compare between stocks of BBRI with BBNI, BBRI with BMRI, and BBNI with BMRI. Then, \( B(t) \) was calculated using equation (2). In comparison of stock BBRI with BBNI, the value of \( B(t) \) is negative which means that there is violation of MCSD’s criteria. Furthermore, the value of \( \varepsilon_{BBRI.BBNI} \) was calculated using equation (5) and the result is 0.1580. The same way, the value of \( \varepsilon_{BBRI.BMRI} \) and \( \varepsilon_{BBNI.BMRI} \) are obtained at 0.0181 and 0.1266, respectively. The value of \( \varepsilon_{BBRI.BMRI} \) is less than 0.032 so that it can be said that BBRI’s stock dominates BMRI’s stock by AMCSD. The portfolio can be said to be an inefficient portfolio.

Table 1 shows five examples of portfolios with each portfolio consists of stock BMRI, BBRI, and BBNI. In similar way, there are no efficient portfolios out of 126 portfolios which each portfolio has 5 stocks. Furthermore, there are 38 efficient portfolios out of 420 portfolios which each portfolio of 38 portfolios consists of BMRI, BBRI, BBNI, and BBTN. Then, there are 315 efficient portfolios out of 1710 portfolios. Out of 315 efficient portfolios, there are 119 efficient portfolios with each portfolio consist of BMRI’s stock, BBRI’s stock, and BBNI’s stock. There are 614 efficient portfolios with each portfolio consists of BMRI’s stock, BBRI’s stock, and


Table 1. The Analysis of Portfolio.

| No | BMRI | BBRI | BBNI | ε_{BBRI, BBNI} | ε_{BBRI, BMRI} | ε_{BBNI, BMRI} | Result |
|----|------|------|------|-----------------|-----------------|-----------------|--------|
| 1  | 90   | 5    | 5    | 0.3892          | 0.0066          | 0.0524          | not    |
| 2  | 85   | 10   | 5    | 0.4704          | 0.0177          | 0.0526          | not    |
| 3  | 80   | 15   | 5    | 0.6120          | 0.0210          | 0.0526          | not    |
| 4  | 75   | 20   | 5    | 0.6467          | 0.0392          | 0.0576          | Efficient |
| 5  | 70   | 25   | 5    | 0.7018          | 0.0823          | 0.0786          | Efficient |

BBTN’s stock, and there are 76 efficient portfolios with each portfolio consists of BMRI’s stock, BBNI’s stock, BBTN’s stock. There are 59 efficient portfolios with each portfolio consists of BBRI’s stock, BBNI’s stock, and BBTN’s stock.

5. Conclusion

Based on the discussion, it can be concluded that:

(i) The criteria of almost marginal conditional stochastic dominance is given by

\[ \int_{\Omega} B(t) dt \leq \varepsilon \int_{a}^{b} |B(t)| dt, \]

where \(0 < \varepsilon < 0.032\).

(ii) Based on the banking stocks data listed on the LQ–45, there are 38 efficient portfolios out of 420 portfolios with each portfolio has 4 stocks and 315 efficient portfolios out of 1710 portfolios with each portfolio has 3 stocks.

References

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