PDF uncertainties in the extraction of $M_W$ at the LHC: a Snowmass Whitepaper

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Abstract

The precision measurement of the $W$ boson mass is an important milestone for the LHC physics program in the coming years. An accurate measurement of $M_W$ allows to perform stringent consistency tests of the Standard Model by means of global electroweak fits, which in turn are sensitive to New Physics at scales potentially higher than the ones explored in direct searches. From the theoretical point of view, our limited knowledge of PDFs will be one of the dominant sources of uncertainty in ongoing and future LHC determinations of $M_W$. In this whitepaper, we have quantified the impact of PDF uncertainties in the $W$ mass extractions from the transverse mass distribution at the LHC. The calculation has been performed using the NNPDF2.3 set, which includes direct constrains on the $W$ boson production kinematics with data for electroweak gauge boson production from the LHC. Our results confirm previous estimates that PDF uncertainties in the determination of $M_W$ from the $m_T^W$ distribution are moderate, around 10 MeV at most. We briefly discuss also the case of the lepton $p_T$ distribution.
The very precise measurement of the $W$ mass at the Tevatron \cite{1} allows to perform stringent consistency tests of the Standard Model by means of global electroweak fits, which in turn are sensitive to New Physics at scales potentially higher than the ones explored in direct searches. The continuous improvement in experimental accuracy for the extractions of $M_W$ at hadron colliders implies that parton distribution functions (see \cite{2} for a recent benchmark comparison) are now one of the dominant theoretical uncertainties. With this motivation, it becomes important to quantify in detail the role of PDF uncertainties and to explore new avenues to reduce their impact in the final $M_W$ determination.

In order to determine the contribution to $\delta M_W$ arising from PDF uncertainties at hadron colliders, Ref. \cite{3} performed an analysis using a template-fit method, a strategy similar to the one used by the Tevatron collaborations. In a first step, we generated pseudo-data for the $W$ transverse mass distribution $m_{WT}$, for various NLO PDF sets and their corresponding error sets: CTEQ6.6 \cite{4}, MSTW08 \cite{5} and NNPDF2.1 \cite{6}. Then, with the central CTEQ6.6 as input, we generated a large number of accurate templates varying $M_W$ from the reference value in a suitable range. The shift in $M_W$ corresponding to each PDF error set is determined by the template which leads to a better $\chi^2$ agreement with the corresponding pseudo-data. Templates were generated with HORACE \cite{7, 8} at LO and with DYNNLO \cite{9} at NLO.

An essential ingredient of our approach was to normalize the templates to the total integral of the distribution in the fit region, since in this way the PDF uncertainty is substantially reduced, without losing sensitivity to the value of $M_W$. In fact, the PDF sensitivity enters mostly through the normalization of the $m_{WT}$ distribution, while the sensitivity to $M_W$ arises from the shape, and is quite independent of the overall normalization. Our results from Ref. \cite{3} are summarized in Fig. 1. The conservative estimate was that PDF uncertainties in $M_W$ fits at the LHC would not be larger than 20 MeV. Our analysis did not support previous studies \cite{10}, which claimed that achieving a 10 MeV accuracy on $M_W$ at the LHC was out of reach precisely due to PDF uncertainties.

None of the PDF sets studied in Ref. \cite{3} included the recent constraints from the LHC.

Figure 1: PDF uncertainties in the $W$ boson determination from the $m_{WT}$ distribution at the Tevatron and the LHC, taken from Ref. \cite{3}. The templates for different $M_W$ values have been generated with the CTEQ6.6 set.
measurements of electroweak boson production, which determine the PDFs for the same flavor combinations and the same kinematical regions relevant for $M_W$ determinations. In order to update our study taking this information into account, we have revisited the determination of $\delta M_W$ at NLO-QCD of Ref. [3], with DYNNLO for the theory modeling, but now the NNPDF2.3 set [11]. This set is particularly suited for the determination of the $W$ mass at the LHC since it already includes constraints from $W$ and $Z/\gamma^*$ production data from ATLAS, CMS and LHCb.

In order to reduce the statistical fluctuations, a dedicated set of $N_{\text{rep}} = 1000$ replicas of NNPDF2.3 NLO has been produced and used to compute the theory predictions for the $W$ transverse mass distribution at NLO-QCD. In addition, the templates have been computed with very high-statistics runs, in order to tame Monte Carlo fluctuations as much as possible, below the size of PDF uncertainties. All the results shown below correspond to $W^+$ production, but as discussed in [3], they should apply in a similar way to $W^-$ production.

![NNPDF2.3, Distribution of best-fit $M_W$ over replicas](image)

Figure 2: The distribution of best-fit $M_W$ values obtained from the comparison of the 1000 replicas of NNPDF2.3 with the reference templates. We show the results both in the case of unnormalized and in the case of normalized templates.

|                  | Absolute Templates | Normalized Templates |
|------------------|--------------------|----------------------|
| $M_W \pm \delta_{\text{pdf}} M_W$ | $80.359 \pm 0.010$ | $80.359 \pm 0.004$ |

Table 1: PDF uncertainties in the determination of $M_W$ at the LHC 7 TeV using the template fits to the $m_{T W}$ distribution with NNPDF2.3 $N_{\text{rep}} = 1000$.

For each of the $N_{\text{rep}} = 1000$ PDF replicas, the fit determines which is the value of $M_W$ which maximizes the agreement with the template distributions. The distribution of best-fit $M_W$ over the NNPDF2.3 replicas, for both cases of normalized and unnormalized distributions, is shown in Fig. 2. It is clear that the distribution obtained from the normalized templates is narrower, indicating reduced sensitivity to PDF uncertainties. The mean and the width of these histograms are reported in Table 1. PDF uncertainties are halved when normalized templates are used, and an uncertainty of $\delta_{\text{pdf}} M_W \sim 4$ MeV.
with NNPDF2.3 is found in this case. Note also that our estimate applies to the \( m_W \) distribution fits only, and to derive the final result we would need as well the results obtained with the MSTW and CT10 sets, which can lead to an increase of the total PDF uncertainty by up to a factor two.

In order to determine if a particular PDF combination is responsible for the bulk of the PDF uncertainties in \( M_W \), it is useful to compute the correlations \([12]\) between the 1000 PDF replicas of NNPDF2.3 and the 1000 determinations of \( M_W \) obtained from the template fits for each replica. The results are shown in Fig. 3 for the unnormalized templates (left plot) and for the normalized templates (right plot). In the case of the unnormalized templates, the correlation between PDFs and \( M_W \) is similar to the case of the inclusive \( W^+ \) cross section \([6]\). On the other hand, for the normalized templates the correlations are much smaller, showing that the normalization effectively decorrelates the \( M_W \) fits with respect to the PDFs. It is clear that there is not a particular range of Bjorken-\( x \) or a particular quark flavor that dominates the \( M_W \) measurement. This implies that, in order to further reduce the PDF uncertainty in the \( M_W \) measurement from \( m_W \), one needs new data constraining all quark flavors and gluons in the broadest possible \( x \) range.

![Figure 3: Correlations between different PDF flavours and the \( M_W \) determination at LHC 7 TeV, as a function of Bjorken-\( x \), for unnormalized (left plot) and normalized (right plot) templates. The predictions from the 1000 replicas of NNPDF2.3 have been used in the computation.](image)

The results that we have just discussed were based on the determination of \( M_W \) from the \( W \) transverse mass distribution. This distribution receives small higher-order QCD corrections, but its accurate measurement at the LHC will be challenging, in view of a competitive \( M_W \) measurement. Now we report on preliminary work towards the extension of the results of \([3]\) to a template-fit analysis of the lepton transverse momentum distribution, which has also been successfully used at the Tevatron to measure \( M_W \).

At variance with the transverse mass distribution, the lepton transverse momentum, \( p_{lT} \), is substantially modified by higher-order QCD corrections, given its strong correlation with the \( W \) boson transverse momentum, \( p_{W}\). For this distribution the use of resummed calculations for the \( W \) boson \( p_{W} \) is required, either using analytical \( p_{W} \) QCD resummation \([13]\) or NLO-(QCD+EW) calculations matched to (QCD+QED) parton showers \([14,15]\), with a significant increase in the amount of CPU time needed to generate the theory templates.

The relevance of NLO-QCD corrections implies that the gluon PDF leads also to a
more important contribution to the PDF uncertainty on $M_W$ than in the transverse mass case. In order to confirm this, in Fig. 4 we show the contribution of quark-antiquark terms to the total PDF uncertainty in the transverse mass and lepton $p_T$ distributions, computed at NLO-QCD with DYNNLO. Therefore, for the lepton $p_T$ distribution the contribution of the quark-gluon subprocess is substantial, in particular near the Jacobian peak.

It should be stressed that the results shown in Fig. 4 have been obtained at fixed order, whereas, as mentioned above, a fully resummed calculation is necessary in the lepton $p_T$ case; furthermore, the quark-antiquark contribution alone provides only a correct estimate of its PDF uncertainty; only the results that include all the partonic subprocesses are sensible in terms of physical distributions. With these two caveats in mind, it is clear that a dedicated analysis should be pursued in order to limit as much as possible the contribution to $\Delta M_W$ due to the gluon PDF. For example, ratios of $W$ over $Z$ distributions provide a significant cancellation of contributions which are common in the two cases, such e.g. the quark-gluon initiated subprocesses, strongly reducing the corresponding contribution to the PDF uncertainty.

![Figure 4: The total relative PDF uncertainty and the separate contribution of quark-antiquark diagrams for the transverse mass (left plot) and the lepton $p_T$ (right plot) distributions, computed at NLO with DYNNLO.](image)

As a final remark, let us mention that PDFs with QED contributions included should be taken into account to consistently assess the corrections to the $M_W$ fits induced by QED effects. The recently released NNPDF2.3 QED set is especially suitable for this purpose, since it includes NNLO QCD combined with LO QED corrections, and also the most recent constraints from electroweak gauge boson production data at the LHC. The implications of NNPDF2.3 QED for $M_W$ determinations should be the topic of detailed studies in the near future.

To conclude, LHC is collecting a huge amount of high-quality data that should be fully exploited in order to improve our knowledge of the proton parton densities. Taking into account present and future information on PDFs, as well as recent improvements in theory modeling, the measurement of the $W$ mass at the LHC with a PDF uncertainty as small as $\delta_{pfr}M_W \sim 5 - 10$ MeV is certainly within reach. The experimental uncertainties are expected to be of a similar size. To further improve on this result, future lepton colliders will be required, such as the recent TLEP proposal.
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