Non-abelian flavour symmetry and $R$-parity

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Abstract

If $R$-parity violation turns out to be a true aspect of Nature, a speculation about its possible origin could add a new dimension to the supersymmetric flavour problem. It has been shown in the past by Barbieri, Hall and their collaborators that the small breaking parameters of an approximate non-abelian flavour symmetry could govern the light quark and lepton masses and at the same time could account for the near degeneracies of squarks and sleptons. A possible connection of the above feature to the natural suppressions of $R$-parity-violating couplings has been investigated here. With some modifications of the approximate flavour symmetry, a supersymmetric theory without $R$-parity has been motivated that has testable experimental signatures.

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Is it possible to reconcile the conventional notion of flavour physics in supersymmetry concerning masses and mixings and the scenario of $R$-parity violation? In this paper, we seek for a phenomenologically viable solution to this question within the framework of a non-abelian flavour symmetry. $R$-parity is a discrete symmetry, defined as $(-1)^{3B+L+2S}$, where $B$ and $L$ are the baryon- and lepton-numbers and $S$ is the intrinsic spin of a particle \( [1] \). It is +1 for all Standard Model particles and −1 for their superpartners. Recall that neither $L$- nor $B$-conservation is ensured by gauge invariance. But their uncontrolled violation leads to rapid proton decay and speeds up many other physical processes at unwanted rates: these prompted to impose $R$-parity in canonical supersymmetric theories. However, violating $R$-parity \( [2, 3] \) in a controlled way has rich phenomenological consequences that in recent times have received considerable attention. An attempt to link $R$-parity violation to the origin of masses and mixings was made in the past by invoking a horizontal U(1)-symmetry where charges dictated by fermion masses and mixings are shown to produce sufficient suppression in $R$-parity-violating \( \tilde{R} \) couplings \( [4] \). Here we are concerned with a non-abelian flavour symmetry, conjectured first \( [5] \) to realise the conventional supersymmetric theory of flavour, generalized now to admit \( \tilde{R} \) interactions as well. In addition to maintaining the existing consistencies and predictions \( [4, 6, 7] \), our generalization predicts $R$ couplings that are within the level of phenomenological tolerance and lead to detectable signatures. In the present analysis we consider only the $L$-violating interactions and leave aside the $B$-violating ones.

In a nutshell, flavour-problem in a supersymmetric theory addresses the question as how to relate the flavour structure of the fermions and scalars to each other by the same symmetry principle. An approximate U(2)-symmetry, which after all descends from a strong breaking of U(3), through the following step-wise breaking

\[
U(2) \rightarrow U(1) \rightarrow 0,
\]  

has been shown, in the context of $R$-parity-conserving supersymmetry, to reproduce the observed patterns of masses and mixings, where $\epsilon$ and $\epsilon'$ are small dimensionless breaking-parameters. The three generations of matter fields transform as $2 \oplus 1$, i.e. $\psi = \psi_a + \psi_3$ ($a = 1, 2$) and the ‘flavon’ fields, whose vacuum expectation values (VEVs), after spontaneous breaking of flavour symmetry, order the mass hierarchies, have the representations $\phi^a$, $S^{ab}$ (symmetric tensor) and $A^{ab}$ (antisymmetric tensor). The upper indices in flavons indicate U(1)-charge opposite to that of $\psi_a$.

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\( \epsilon \) and \( \epsilon' \), are responsible for the near degeneracies of the squarks and slepton masses, leading to a “super-GIM” mechanism. With \( \epsilon \simeq 0.02 \) and \( \epsilon' \simeq 0.004 \), all observed masses and mixing patterns are qualitatively well understood.

If we now assume that the same flavour-symmetry is responsible also for an exact \( R \)-parity, the strengths of the \( R \) interactions are governed by \( \epsilon \) and \( \epsilon' \). Do the magnitudes of \( \epsilon \) and \( \epsilon' \), dictated by the fermion masses and mixings, inflect the desirable suppressions to the \( R \) interactions so as to make the scenario phenomenologically viable? Before attempting to answer this question, we set up our notations that we follow hereafter. Recalling that \( H_d \) (the Higgs doublet superfield responsible for the masses of isospin \(-1/2\) fermions) and \( L \) (the lepton doublet superfield) have identical gauge quantum numbers, the \( \mu H_d H_u \)-term in the superpotential can now be generalized to include 3 more similar terms; in compact notation,

\[
\mu_\alpha L_\alpha H_u \quad (\alpha = 0, i),
\]

where \( L_0 \equiv H_d, \mu_0 = \mu \) and \( L_i \) \((i = 1, 2, 3)\) correspond to the three lepton flavours. One also has the following trilinear \( L \)-violating interactions in the superpotential:

\[
\frac{1}{2} \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k,
\]

where \( L_i \) and \( Q_j \) are lepton and quark doublet superfields and \( E^c_k \) and \( D^c_k \) are charged lepton and down quark singlet superfields; \( i, j, k \) run from 1 to 3. \textit{A priori}, without any suppression (\textit{e.g.} from a horizontal symmetry), the natural expectation is \( \mu_i \sim O(m_Z) \); \( \lambda, \lambda' \sim O(1) \) and during electroweak breaking \( (\bar{\nu}_i) \sim O(m_Z) \). But these overwhelmingly violate the laboratory upper limits of the neutrino (Majorana) masses \( \tilde{m} \) (all at 95\% C.L.)

\[
m_\nu \leq 15 \text{ eV}, \quad m_{\bar{\nu}_\tau} \leq 170 \text{ KeV} \quad \text{and} \quad m_{\nu_e} \leq 24 \text{ MeV},
\]

and overshoot the stringent upper limits (indirect) on various combinations of \( \lambda \)- and \( \lambda' \)-couplings by many orders of magnitude. The most relevant and stringent constraints are shown in Table 1 (For an extended list of product couplings, see ref. [9] for example). A way out to have naturally suppressed neutrino masses was suggested in ref. [3] through a mechanism that approximately aligns \( \mu_\alpha \) with \( v_\alpha \) (the VEVs of the neutral scalars in \( L_\alpha \)). A perfect alignment can be achieved if (i) the supersymmetry-breaking \( B_\alpha \propto \mu_\alpha \) and (ii) \( \mu_\alpha \) is an eigenvector of \( \tilde{m}^2_{\alpha \beta} \), the soft scalar mass matrix that arises after supersymmetry breaking; even though misalignment creeps in through radiative corrections [10]. Breaking an abelian horizontal \( U(1) \) symmetry, with charges appropriately chosen, was shown [10] to yield \( m_{\nu_e} \leq 10 \text{ eV} \) (a hot dark matter candidate) and generate the \( \lambda \)- and \( \lambda' \)-couplings with required suppressions so as not to violate any experimental constraint.

How does an approximate \( U(2) \) symmetry fare to achieve the desired goal? Since with a non-abelian horizontal symmetry the theory is much more constrained than with \( U(1) \), the task is much more challenging and, as we will see below, it faces unavoidable experimental obstructions, yet gives hints as how to generalize and search for a plausible solution. The \( R \) bilinear and trilinear terms in the superpotential can be obtained by appropriately contracting the superfields appearing in eqs. (1) and (3) with the flavons. Given the flavon representations and the hierarchy of their VEVs during the step-wise breaking of \( U(2) \) down to nothing as mentioned earlier, the order of magnitude of the \( R \) couplings are given by (to their leading order)[4],

\[
\mu_i \sim 0, \quad \mu_2 \sim \epsilon \mu, \quad \mu_3 \sim \mu;
\]

\[1\text{All } R \text{ couplings involve flavour indices in the weak basis. For our order of magnitude estimates, a distinction between the weak basis and the mass basis is not important.}\]
\(\lambda_{ijk}\)-couplings:

\[
(121), (131), (133) \sim 0; \quad (123), (132), (231) \sim \epsilon'; \quad (232), (233) \sim \epsilon; \quad (122) \sim \epsilon' \epsilon;
\]  

(6)

\(\lambda'_{ijk}\)-couplings:

\[
(111)', (121)', (131)', (112)', (113)', (133)', (211)', (311)', (331)', (313)' \sim 0; \\
(123)', (132)', (231)', (213)', (321)', (312)' \sim \epsilon'; \quad (122)', (221)', (212)' \sim \epsilon' \epsilon; \\
(223)', (232)', (323)', (322)', (332)' \sim \epsilon; \quad (222)' \sim \epsilon^2; \quad (333)' \sim 1.
\]  

(7)

There are two major phenomenological obstacles in the above formulation. First, \(\langle \tilde{\nu}_\tau \rangle\) and \(\mu_3 \sim m_Z\), while neutrino-neutralino mixings constrain them to be \(\lesssim m_{\nu} m_Z \lesssim 1\) GeV (assuming \(\mu \sim m_Z\)) and second, \(\lambda'_{321} \lambda'_{312} \sim \epsilon^2 \sim 10^{-5}\) and \(\lambda'_{231} \lambda'_{213} \sim \epsilon^2 \sim 10^{-5}\) exceeding the constraints from \(\Delta m_K\) and \(\Delta m_B\) (see Table 1) by a few orders of magnitude.\(^2\)

The above difficulties are unreparable and strongly suggest towards the consideration of \(U(3)\), the ultimate flavour symmetry. However, \(U(3)\) has to be ‘strongly’ broken to account for the heavy top quark. On the other hand, the failure with \(U(2)\) guides us to the necessity of having an additional suppression factor for the third generation lepton superfield during \(U(3) \rightarrow U(2)\) solving the ‘\(\mu_3\)-problem’, that is as well expected to inflict suppressions in \(U(2)\)- and \(U(1)\)-breaking parameters curing the product-couplings’ overshooting. So in the lepton sector \(U(3)\) needs to be ‘weakly’ broken. Then how about treating leptons and quarks differently in flavour-space?\(^3\)

Following the above line of arguments, we consider the flavour symmetry \(U(3)_l \otimes U(3)_q\), where lepton and quark superfields transform under different unitary groups. \(U(3)_q\) is anyhow strongly broken to \(U(2)_q\). The complete breaking configuration is

\[
U(3)_l \otimes U(3)_q \rightarrow U(3)_l \otimes U(2)_q \xrightarrow{\epsilon_3} U(2)_l \otimes U(2)_q \xrightarrow{\epsilon_1} U(1)_l \otimes U(1)_q \xrightarrow{\epsilon', \epsilon''} 0,
\]  

(8)

where ‘\(\ast\)’ indicates a strong breaking of \(U(3)_q\). A triplet flavon \(\tilde{\phi}_1\), with VEV assignments \(\langle \tilde{\phi}_1 \rangle = \epsilon_3\), \(\langle \tilde{\phi}_2 \rangle = \langle \tilde{\phi}_1 \rangle = 0\), breaks \(U(3)_l\) to \(U(2)_l\). The subsequent breaking of \(U(2)_q\) and \(U(2)_l\) are assisted by the VEVS of two different sets of flavon fields (one for quarks and the other for leptons) which are straightforward three dimensional extensions of the \(\phi\), \(S\) and \(A\)-fields introduced in the context of a general \(U(2)\) having analogous VEV patterns. For those VEVS related to the lepton sector we assign a suffix \(l\).

Before proceeding further, we must first ensure that the observed fermion masses and mixings are successfully reproduced. A crucial assumption at this point is called for that, instead of one pair, there are two pairs of Higgs doublet superfields. Considering the two \(H_d\)-type Higgs superfields, we assume that one \((H_d^1)\) couples only to leptons and the other \((H_d^2)\) only to quarks and there is a non-trivial mixing between them. The physical state that acquires a VEV during electroweak breaking is assumed to be the one that dominantly couples to the leptons and is given by

\[
H_d \simeq H_d^1 + \xi H_d^2,
\]  

(9)

while the orthogonal state (assumed too heavy) does not acquire any VEV. The mass matrices of the charged leptons and the down quarks assume the following form:

\[
\mathcal{M}_l = \begin{pmatrix}
0 & \epsilon_1' & 0 \\
-\epsilon_1' & \epsilon_1 & \epsilon_1 \\
0 & \epsilon_1 & \epsilon_3\epsilon_1
\end{pmatrix} v_d, \\
\mathcal{M}_d = \begin{pmatrix}
0 & \epsilon' & 0 \\
-\epsilon' & \epsilon & \epsilon \\
0 & \epsilon & 1
\end{pmatrix} \xi v_d.
\]  

(10)

The mixing angle \(\xi\) is adjusted as \(\xi \approx \epsilon_3 m_b / m_\tau\). Choosing \(v_d = v / \sqrt{2} \approx 174\) GeV (where \(v\) is the standard model VEV), we obtain \(\epsilon_3 m_b / v_\tau \approx 0.01\), \(\epsilon_1 \approx \epsilon_3 m_\mu / m_\tau \approx 6.10^{-4}\), \(\epsilon_1' \approx \epsilon_1 \sqrt{m_e / m_\mu} \approx 4.10^{-5}\), \(\epsilon \approx m_\tau / m_\mu \approx 0.03\) and \(\epsilon' \approx \epsilon \sqrt{m_d / m_\tau} \approx 9.10^{-3}\). Note that a ‘strong’ breaking of \(U(3)_q\) keeps the values of \(\epsilon\) and \(\epsilon'\) the same as in a general \(U(2)\)-hypothesis; thus all the consistencies and observable predictions of

\(^2\)The contribution to \(\epsilon_K\) vanishes as \(\lambda_{12} = -\lambda_{21} \) following from the antisymmetric nature of \(A\)-flavons.

\(^3\)This is indeed against the idea of unification, but nevertheless a viable option.
the latter related to $B$- and $K$-physics automatically apply to our scenario. On the contrary, a ‘weak’ breaking of $U(3)_f$ inflicts a suppression of 2 orders of magnitude in the $(33)$-element of the charged lepton Yukawa matrix that is led to $\mu_3$ and the $U(2)$- and $U(1)$-breaking parameters in the lepton sector; we will see later that quantitatively these fit to our requirement. The rôle of Higgs-mixing is obvious now: despite the ‘strong’ breaking of $U(3)_f$ vis-à-vis the ‘weak’ breaking of $U(3)_f$, it pulls $m_b$ relative to $m_t$ sufficiently low as to place it close to $m_\tau$.

Now we are all set to check the consistencies as regards the $R$ couplings. First, we present the order of magnitude estimates of $\mu_i$, $\lambda_{ijk}$ and $\lambda_{ijk}'$ (to their leading order) in the present scenario:

\[
\begin{align*}
\mu_i &\sim 0, \quad \mu_2 \sim \epsilon_1 \mu, \quad \mu_3 \sim \epsilon_{3\mu}; \\
\lambda_{ijk} &\sim 0; \quad (123), (132), (231) \sim \epsilon_l \epsilon_{3\mu}; \quad (321), (232), (233) \sim \epsilon_l \epsilon_{3\mu}; \quad (122) \sim \epsilon_l' \epsilon_l; \\
\lambda_{ijk}' &\sim 0; \quad (1j\bar{k})', (211)', (231)', (311)', (313)', (221)', (212)', (321)', (232)', (332)', (333)' \sim \epsilon_l \epsilon_l'; \quad (322)', (233)', (332)' \sim \epsilon_{3\mu}; \quad (333)' \sim \epsilon_{3\mu}.
\end{align*}
\]

By putting values of the breaking parameters and comparing the predictions for the various product-couplings with their experimental upper limits, we observe that the compatibility has improved considerably compared to the $U(2)$-scenario. The prediction $\lambda_{321}' \lambda_{312}' \sim 7.10^{-9}$ is in a marginally tight position with respect to the limit from $\Delta m_K$. But the entries in the Yukawa matrices are always subject to $O(1)$ uncertainties that one can exploit to stretch the breaking parameters for accommodating the above constraint. The $\epsilon_K$-constraint is trivially satisfied as in the case of a general $U(2)$. The other constraints (including those which are not listed in Table 1) are comfortably satisfied.

Now we turn our attention to the issue of neutrino mass and its decay. Neutrino mass arises due to neutrino-neutrino mixings (photino is irrelevant in the context of neutrino mass) and in the basis $\left(\tilde{H}^0, H^0, \tilde{Z}\right)$ it has the following form $(gW = g/2 \cos \theta_W$ and a tilde on a superfield denotes its fermionic component):

\[
\mathcal{M}_n = \begin{pmatrix}
0_{4 \times 4} & \mu_\alpha & gWv_\alpha \\
\mu_\alpha & 0 & -gWv_u \\
gWv_\alpha & -gWv_u & m_2^2
\end{pmatrix},
\]

where $v_u = (H^0_u)$. The zeros in the first $(4 \times 4)$-block can be lifted by non-renormalizable terms in the superpotential of the form $LLH_u H_u / M$, which of course can be arranged to have a negligible correction assuming $M \gg m_Z$. The above $(6 \times 6)$-matrix has two zero eigenvalues that can be identified with the physical $\nu_e$ and $\nu_\mu$ masses, while the physical $\nu_\tau$ is massive and its mass is determined by the extent to which $\nu_3$ is misaligned with $\mu_3$ (neglecting, for the sake of simplicity, the misalignment between $v_2$ and $\mu_2$ which turns out to be much smaller: in comparison to the $U(2)$-case mentioned earlier, that this does not change the conclusions drawn above). Assuming for an illustration (good enough for an order of magnitude estimate) that $B$ is universal and the origin of a possible misalignment is only an off-diagonal entry $\Delta m^2 = \tilde{m}^2 m_{L_3}$ in the scalar lepton mass matrix, an explicit scalar potential minimization yields

\[
v_3 = \kappa_\mu \mu_3 + \kappa' v_d,
\]
where $\kappa = B\nu_\alpha/\tilde{m}^2$ and $\kappa' = \Delta m^2/\tilde{m}^2$ ($\tilde{m}$ is a common diagonal soft scalar mass). It also follows from the scalar potential minimization that to a very good approximation $v_d \simeq \kappa \mu$. Therefore, a non-zero $\kappa'$ is responsible for the deviation from $\nu_\alpha \propto \mu_\alpha$ alignment giving rise to a neutrino mass. Now, $\nu_\mu$-mass is obtained by taking the ratio of the determinant of the $(4 \times 4)$ mass matrix in the $(\nu_\tau, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{Z})$ basis to the determinant of the $(3 \times 3)$ mass matrix in the $(\tilde{H}_d^0, \tilde{H}_u^0, \tilde{Z})$ basis. The leading behaviour turns out to be

$$m_{\nu_\mu} \sim \frac{g^2}{4 \cos^2 \theta_W} \frac{\varepsilon_d^2 v_d^2}{m_\tilde{Z}},$$

where we have used $\Delta m^2 \approx \epsilon_d \tilde{m}^2$ following from U(3)$^c$-breaking. Thus for $m_\tilde{Z} \sim v_d$, $m_{\nu_\mu} \sim O(1 \text{ MeV})$ lying in the range of detectability, for example, at a tau-charm factory [1].

However, this massive $\nu_\tau$ is not stable and before we discuss its decay properties, a few remarks on the cosmological constraints that apply on it are in order [12]. The age and the present energy density of the universe restricts the lifetime of a 1 MeV $\nu_\tau$ to be less than $\sim 10^8$ s. A stronger constraint (lifetime less than $\sim 10^3$ s) follows from the requirement that $\nu_\tau$ should decay before the recombination time ($t_{\text{rec}} \lesssim 10^7 s t_U$, where $t_U$ is the age of the universe being $10^{10} y$), i.e. when matter could start forming. The nucleosynthesis upper bound on the lifetime of a 1 MeV neutrino is $\sim 10^2 s$, unless it has additional annihilation channels besides those in the Standard Model. When the dominant decays are in visible channels (e.g. radiative decays), practically all otherwise allowed neutrino masses are excluded [4].

Within our framework, $\nu_\tau$ has three types of decay modes:

(i) Invisible decay $\nu_\tau \to \nu_\mu f$, where $f$ is a familon [13, 14] (a massless Nambu-Goldstone boson arising from the breaking of the family symmetry U(3)). The effective operator $LLH_u H_u / M$ induces this decay (recall that a familon does not carry any overall lepton number) and the loop-driven decay graph involves two $R$ Yukawa couplings (e.g. $\lambda'_{333}$ and $\lambda_{333}$) generating $\Delta L = 2$;

(ii) Invisible decay to three light neutrinos, $\nu_\tau \to 3\nu$ (Z-mediated), following from the frustration of GIM-mechanism due to neutrino – zino mixing [4];

(iii) Visible radiative decay $\nu_\tau \to \nu_\mu + \gamma$, induced by $\lambda'_{333}$ and $\lambda_{333}$ (for example).

For superparticle masses around 100 GeV, the lifetime in channel (i) is $\sim 10^{16}$ s with $V \sim 6.10^9$ GeV (global U(3), breaking scale) while the lifetimes in channels (ii) and (iii) are $\sim 10^{12} - 10^{13}$ s. It should be noted though that the lack of finding a fast enough decay channel of a massive neutrino is a somewhat generic problem that has been noticed in the past in different contexts [12, 16]. We observe that we cannot advance any solution to this general problem in a scenario where approximate non-abelian horizontal symmetries have been assumed to control both the $R$ Yukawa couplings and the structure of the supersymmetry breaking soft terms.

If we instead assume that family symmetries govern only the Yukawa couplings through their hierarchical breaking and do not control the structure of the soft masses at the supersymmetry breaking scale ($\Lambda_U$), this indeed results in a loss of generality. But this is aimed to avoid the difficulties related to the rather long lifetime of the massive neutrino by bringing its mass below 100 eV making it cosmologically stable [12]. Let us assume the following: (i) soft terms are universal at $\Lambda_U$, i.e. $\tilde{m}_{\alpha\beta}^2 = \tilde{m}^2 \delta_{\alpha\beta}$, (ii) $B_\alpha = B\mu_\alpha$ and finally (iii) the supersymmetric $\mu$ parameter is non-zero in only one direction, namely, $\mu_L L H_u \equiv \mu H_d H_u$: this is not unjustified as there is an in-built distinction between $H_d$ and $L_i$, since the former is a singlet under family group while the latter transforms under U(3)$_i$. Assumption (iii) therefore relies on a property of the theory that its superpotential could sense that distinction and chooses the ‘singlet direction’ for the $\mu$-term. Still a question remains: even if one starts with a universal boundary condition on the scalar masses at $\Lambda_U$, how much sneutrino-Higgs mixing is generated by renormalization group (RG) running of the soft parameters down to low energy? Singling out the dominant effects, an approximate (nevertheless quite reasonable for an order of magnitude estimate) expression of the mass of $\nu_\tau$ induced by such misalignment is obtained as [14, 17]

$$m_{\nu_\tau}^{\text{RG}} \sim \frac{g^2}{4 \cos^2 \theta_W} \frac{v_d^2}{m_\tilde{Z}} \left[\frac{3t_U m_b}{8\pi^2 v}\right]^2 \left(3 + \frac{A^2}{\tilde{m}^2} + \frac{A}{B}\right)^2 \lambda_{333}^2,$$

(17)

---

7 See e.g. Fig. 2 of Gelmini and Roulet in ref. [12].
8 Charged lepton – chargino mixing will trigger flavour-changing Z decays into light leptons, $Z \to l_i l_j$, the rates of which, we have checked, are much below their experimental upper limits [1].
9 This lower limit follows from the non-observation of the $\mu \to e\gamma$ decay [12].
where \( t_U = \ln(\Lambda_U/m_Z) \) and \( A \) is the universal trilinear soft parameter at \( \Lambda_U \). By comparing eqs. (14) and (17) one obtains an idea of the relative sizes of the RG-induced effect on the neutrino mass and the \( U(3) \)-breaking contribution discussed earlier. Let us consider, for the sake of simplicity and illustration, \( A \ll m, B \). Then, (i) for \( \Lambda_U = 10^{16} \) GeV, \( m_{\nu_{\tau}}^{\text{RG}} \) is at the level of a few KeV and (ii) for \( \Lambda_U = 10^{5} \) GeV, \( m_{\nu_{\tau}}^{\text{RG}} \) is \( \mathcal{O}(100 \) eV). In case (i), even by exploiting the \( \mathcal{O}(1) \) uncertainty in \( \lambda_{333} \), it is difficult to bring the neutrino mass below 100 eV for natural choices of soft parameters, while in case (ii), which corresponds to low energy gauge-mediated supersymmetry breaking [18], there is more breathing space to accomplish it mainly because of less RG-running\(^{10}\). At this level it becomes important to evaluate the one-loop contribution to the neutrino mass induced by (dominantly) the \( \lambda_{333} \) coupling. The leading term reads [19]

\[
m_{\nu_{\tau}}^{\text{loop}} \approx \frac{3m_b m_{LR}^2}{8\pi^2 m^2} \lambda_{333}^2,
\]

where assuming the left-right squark mixing \( m_{LR}^2 = m_b \tilde{m} \), we obtain, for \( \tilde{m} = 100 \) GeV, \( m_{\nu_{\tau}}^{\text{loop}} \sim 1 \) KeV. Again, it is possible to arrange the squark masses and mixings and/or \( \epsilon_{\text{soft}} \)-scaling such that \( m_{\nu_{\tau}}^{\text{loop}} \) becomes \( \mathcal{O}(100 \) eV). It is noteworthy that for low energy supersymmetry breaking \( m_{\nu_{\tau}}^{\text{RG}} \) becomes comparable or even less than \( m_{\nu_{\tau}}^{\text{loop}} \), while for \( \Lambda_U \sim 10^{16} \) GeV the dominant contribution comes from misalignment. In any case, we have exhibited that it is possible to design a scenario (particularly with gauge-mediated supersymmetry breaking) reconciling \( R \)-parity violation with conventional flavour physics that, in addition to having passed the laboratory tests, is also cosmologically viable.

In the scenario discussed above, the cosmologically stable neutrinos are hot dark matter candidates. Axions, that have resulted from breaking non-abelian, continuous and global family symmetries, could constitute cosmologically interesting cold dark matter [3]. The other candidates for cold dark matter in \( R \)-parity-conserving supersymmetry are neutralinos, which are not stable here in cosmological scales. Given the predictions of the \( R \)-couplings in eqs. (12) and (13), the most striking collider signatures of this scenario are: (i) [if the lightest neutralino is the lightest supersymmetric particle (LSP)] like-sign di-muon final states [2] from LSP-decays after a rather long flight (\( \sim 1 \) m) close to the detector edge and (ii) [in the sneutrino-LSP scenario] \( \tilde{\nu} \) decaying to 2 jets inside the detector through \( \lambda_{333} \) couplings [21]. We note in passing that the particular couplings (\( \lambda_{333} \)) relevant to explain the recent HERA anomaly [22] are vanishing in our case and so if those anomalous events turn out to be real in future, they cannot be explained within our framework. In any case, if \( R \)-parity-violation turns out to be a true feature of Nature, we believe that its possible ancestral link with masses and mixings could constitute a complete theory of flavour. Our effort is an attempt in that direction.

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