Kinematics Modelling of Tendon-Driven Continuum Manipulator with Crossed Notches

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Abstract. Single port surgical robot (SPSR) is a giant leap in the development of minimally invasive surgical robot. An innovative manipulator with high control accuracy and good kinematic dexterity can reduce wound, expedite recovery, and improve the success rate. This paper presents a tendon-driven continuum manipulator with crossed notches. This manipulator has two degrees of freedom (DOF), which possesses good flexibility and high capacity. Then based on cantilever beam theory, a mechanics model is proposed, which connects external force and deformation of a single flexible ring (SFR). By calculating the deformation of each SFR, the manipulator is considered as a series robot whose joint numbers is equal to SFR numbers, and the kinematics model is established through Denavit-Hartenberg (D-H) procedure. In this paper, the total manipulator is described as a curve tube whose curvature is increased from tip to base. Experiments were conducted and the comparison between theoretical and actual results proved the rationality of the models.

1. Introduction
Minimally invasive surgery (MIS) is featured by small trauma, less bleeding, light pain, and quick recovery [1]. As the second generation surgical robot, SPSR needs only one wound on patient’s body instead of 3 to 5 wounds necessarily in traditional MIS. The limitation of wound reduces the injuries to people, but innovative manipulators need to be designed to perform essential motion [2]. Mainstream SPSRs can be divided into two types, one contains traditional discrete joint arm, and the other one adopts novel continuum manipulator. The first type has high positional accuracy and easy to control, but it needs relatively large wound. Wortman [3] devised an in vivo robot which contains two operation arms, whose motors are on the joints. Petroni [4] developed the SPRINT surgical platform including image system. Quaglia [5] presented SPRINT 2.0 with motors installed outside and tendons adopted to transmit force. The second type has miniature size and flexible movement, but control is complex and laden capacity is limited. Webster [6] and Wu [7] proposed the snake robots made of concentric pre-curved tubes. Xu [8] and Li [9] introduced a kind of manipulator in which the backbone is a continuous elastic structure in the centre and drivers are tendons distributed uniformly around. Du [10] and Gao [11] built continuum manipulators with different notches.
Continuum manipulators have excellent flexibility and biocompatibility, but the existing ones still have the problem of poor control accuracy and insufficient laden capability. This paper designs a continuum manipulator with high rigidity, and a relatively precise kinematics model is established for control.
2. Overall design of the continuum manipulator
Analysing of existing SPSRs, this section presents a novel structure, a notched flexible continuum manipulator driven by tendons.

2.1. Design of the structure
Flexible hinges have many advantages such as no back, friction, gap and wear, small size and so on, which is widely used in many fields [12]. These features are also expected for the manipulator of SPSR, so this paper designs SFR based on the idea of flexible hinge. The crossed notches are fabricated on hyperelastic nitinol tube through wire cut electrical discharge machining (WEDM), and the continuum manipulator is a combination of several crossed SFRs whose bending planes are perpendicular to each other. The continuum manipulator is driven by two sets of drive tendons, each consisting of two tendons, which uniformly distribute in the tube. Each set of tendons controls forward and reverse bending on a plane. Combination of two sets can realize universal bending. The tendons are fixed at the top of the continuum manipulator and the flexible skeleton is installed internally to fit tendons, as shown in Figure 1.

2.2. Features of the continuum manipulator
The notched continuum manipulator achieves overall processing, and no connection friction extends the service life span. The manipulator has good flexibility and biocompatibility, which can avoid unnecessary damage to organs and issues during surgery. The tendons minimize the size, weight, and volume of the manipulator. Compared with our previous design, the crossed SFRs make one manipulator achieve two DOF, which improves control and movement dexterity. Also, according to the first-order natural frequency, the proposed manipulator overcomes insufficient rigidity which causes low capability. Figure 2 shows the results of finite element analysis by ANSYS, and each structure has four deformation units and similar key parameters.

3. Mechanics modelling of continuum manipulator
The mechanical properties of continuum manipulator are deducted. A SFR is divided into rigid region and elastic region. Stiffness characteristics are built using the theory of cantilever beam, and deformation for SFR is calculated by the iterative algorithm.

3.1. Analysis of SFR stiffness characteristics
Because of the complex shape of SFR, the following assumptions are put forward: deformation mainly concentrates on the rounded straight flexible part, and deformation outside this part is ignored; interference between different SFRs can be neglected because deformation is very small; the SFR is assumed to be fixed at one end and subjected to torque and force on the other end. The flat projection and structural parameters of SFR are shown in Figure 3. R is the outer radius of tube, b is the wall.
thickness, \( t \) is the minimal width of flexible hinge, \( l \) is the length of straight hinge, \( r \) is the radius of rounded circle and \( h \) is the vertical distance between two SFRs.

According to the actual loading situation, the main external forces on SFR are exerted by tendons. To describe stiffness characters, two variables are introduced: bending stiffness \( K_\theta \) is defined as the ratio of bending moment to corresponding bending angle; tension/compression stiffness \( K_l \) is defined as the ratio of the tension/compression force to corresponding translation displacement [13]. \( E \) is the elastic modulus, which of nitinol is affected by the processing method, so actual elastic modulus is obtained by parameter identification. According to statics, \( K_\theta \) and \( K_l \) of FNR can be calculated as follows.

\[
K_\theta = \int_{\pi/2}^{\pi/2} \frac{Ebt^3}{6} dx + \int_{-\pi/2}^{\pi/2} \frac{Eb(t + 2r - 2r \cos \alpha)^3}{6r \cos \alpha} d\alpha
\]

\[
K_l = \frac{2Ebt}{l} + \int_{-r/2}^{r/2} \frac{2Ebt(2r + t - 2r \cos \alpha)}{r \cos \alpha} d\alpha
\]

Using calculus theory, the comparison between bending deformation and translation displacement under the same force is calculated, and the results show that the former one is much bigger than the latter one. To simplify the model, the following calculation assumes that the deformation only occurs on the bending plane.

### 3.2. Angle deformation of SFR

Angle deformation for the \( i \)th notch \( \theta_i \) is calculated using the iterative algorithm, and the computing process is as follows: for initial calculation, assume that there is no lateral pressure, then the bending angle \( \theta_i(0) \) for SFR is obtained; at this time lateral pressure \( F_{ni}(0) \) and friction force \( F_f(0) \) can be calculated; because of the aforementioned two forces, new deformation is derived, which is written as \( \theta_i(n) \) for the \( n \)th calculation; the difference between two adjacent calculations is viewed as objective functions; repeat the above process until the objective function tends to zero so the final angle deformation \( \theta_i \) is calculated. Here \( \mu \) is the friction coefficient which is given by experience. The force diagram of SFR is shown in Figure 4.

**Figure 3.** Flat projection of SFR.

**Figure 4.** Force diagram of SFR.

Equivalent bending moment equals to:

\[
M(n) = F_x L_x + F_y L_y
\]

Where \( F_x \) and \( F_y \) are the decomposition of all the forces in Cartesian coordinate system, \( L_x \) and \( L_y \) are the corresponding force arm, all of these related to deformation. The bending angle for the \( n \)th iterative algorithm can be calculated according to \( M(n) \) and \( K_\theta \). Due to frictional forth and lateral pressure,
tendon force varies in different segments. Angle deformation for all SFRs can be obtained according to the varied equivalent tendon forces.

4. Kinematics modelling of continuum manipulator
The kinematics model of continuum manipulator proposed in this paper regards the whole manipulator as a series robot, which divides the manipulator into several equivalent rigid links and joints. The initial subcoordinate system is established at the end of base SFR, the final subcoordinate system is established at the top of tip SFR, and other subcoordinate systems are established at the intersection of two tangents of adjacent SFRs. The coordinate system is shown in Figure 5.

The deformation angle of each SFR can be obtained according to the mechanics model in section 3, and the D-H method can be applied to build kinematics model. This paper uses the Craig rule and D-H parameters are shown in Table 1. \( \alpha_i \) is the twist angle of the link, \( l_i \) is the length of the link, \( d_i \) is the offset of the link, and \( \beta_i \) is the rotation angle of the link.

| Parameters | Number of notches |
|-----------|------------------|
| \( \alpha_i \) | 1/2  | -\pi/2 | \pi/2 | \ldots | \pi/2 | -\pi/2 |
| \( l_i \) | 1  | \( l_{2i-1} \) | \( l_{2i} \) | \ldots | \( l_{2n-1} \) | \( l_{2n} \) |
| \( d_i \) | 0 | 0 | 0 | \ldots | 0 | 0 |
| \( \beta_i \) | \( \theta_{2i-1} \) | \( \theta_{2i} \) | \( \theta_{2i} \) | \ldots | \( \theta_{2n-1} \) | \( \theta_{2n} \) |

Where \( \beta_i \) is equal to the angle deformation \( \theta_i \) of the \( i \)th SFR, and \( l_i \) can be calculated as follows:

\[
l_i = \begin{cases} 
    h + \beta_i^{-1}(l + 2r)\tan(\beta_i/2) & \text{if } i = 1 \\
    h + \beta_{i-1}^{-1}(l + 2r)\tan(\beta_{i-1}/2) + \beta_i^{-1}(l + 2r)\tan(\beta_i/2) & \text{if } 1 < i \leq 2n \\
    \beta_{i-1}^{-1}(l + 2r)\tan(\beta_{i-1}/2) & \text{if } i = 2n + 1
\end{cases}
\]  

(4)

The transformation matrix between adjacent equivalent rigid joints can be expressed as:
The overall transformation matrix of the continuum manipulator from base coordinate system \( \{0\} \) to tip coordinate system \( \{2n\} \) can be written as:

\[
{^0}_T^{2n} = {^0}_T^{T_1} {^1}_T^{T_2} \ldots {^{i-1}_T^{T_i}}
\]  

(6)

The end position of the continuum manipulator can be obtained by equation (6). Calculated working space is shown in Figure 6.

5. Experiments and results
The test bed was constructed as shown in Fig.7. A prototype was machined for experiment, which had 8 notches in one bending direction (16 notches in total), and the effective length is 87.6mm. The multiple strand tendon adopted in experiment has a diameter of 0.45mm. The loading was exerted by the counterweight. Two CCD cameras were used in the experiment to take photos, and then angle deformation and spatial position were obtained by image processing using MATLAB software.

5.1. Validation of mechanics model
One tendon was loaded each time, so the bending occurred in the plane. The load was applied from 0 to 2 kg by an increase of 0.2 kg, and then from 2 kg to 0 kg by a decrease of 0.2 kg. The actual bending angles under different loads were the average value of 5 cycles. The result between theoretical value and actual value is shown in Figure 8. The maximum deviation is 3.41%, which comes from model error, manufacturing error and image processing error.

5.2. Validation of kinematics model
Two tendons of different sets were loaded each time, so the bending occurs in the space. Different combinations of two tendons were applied. Each position two photos from different cameras were needed and the actual deformation displacement was obtained. The result between theoretical value and actual value is shown in Figure 9. The maximum deviation is 3.72%, which also comes from model error, manufacturing error and image processing error.
Figure 8. Bending angle under one-direction load.

Figure 9. Deformation displacement under two-direction load.

6. Conclusion
The tendon-driven continuum manipulator with crossed notches proposed in this paper is tractable, small, light, safe, and dexterous which can be used in SPSR. The mechanics model describes the manipulator as a variable curvature tube, while the kinematics model considers the manipulator as an equivalent series robot. Both of two models are relatively accurate when depict mechanical and kinematic properties characteristics respectively, and verified experimentally. Future study should focus on increasing accuracy and optimization. Inverse kinematics, force feedback and other issue also need to be considered in the future.

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8. Reference
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