RELAXATION OF BLAZAR-INDUCED PAIR BEAMS IN COSMIC VOIDS

FRANCESCO MINIATI 1 AND ANDRII ELIV 2, 3

1 Physics Department, Wolfgang-Pauli-Strasse 27, ETH Zürich, CH-8093 Zürich, Switzerland; fm@phys.ethz.ch
2 Institut d’Astrophysique et de Géophysique, Université de Liège, B-4000 Liège, Belgium
3 Main Astronomical Observatory, Academy of Sciences of Ukraine, 27 Akademika Zabolotnogo St., 03680 Kyiv, Ukraine

Received 2012 August 4; accepted 2013 April 25; published 2013 May 23

ABSTRACT

The stability properties of a low-density ultrarelativistic pair beam produced in the intergalactic medium (IGM) by multi-TeV gamma-ray photons from blazars are analyzed. The problem is relevant for probes of magnetic field in cosmic voids through gamma-ray observations. In addition, dissipation of such beams could considerably affect the thermal history of the IGM and structure formation. We use a Monte Carlo method to quantify the properties of the blazar-induced electromagnetic shower, in particular the bulk Lorentz factor and the angular spread of the pair beam generated by the shower, as a function of distance from the blazar itself. We then use linear and nonlinear kinetic theory to study the stability of the pair beam against the growth of electrostatic plasma waves, employing the Monte Carlo results for our quantitative estimates. We find that the fastest growing mode, like any perturbation mode with even a very modest component perpendicular to the beam direction, cannot be described in the reactive regime. Due to the effect of nonlinear Landau damping, which suppresses the growth of plasma oscillations, the beam relaxation timescale is found to be significantly longer than the inverse Compton loss time. Finally, density inhomogeneities associated with cosmic structure induce loss of resonance between the beam particles and plasma oscillations, strongly inhibiting their growth. We conclude that relativistic pair beams produced by blazars in the IGM are stable on timescales that are long compared with the electromagnetic cascades. There appears to be little or no effect of pair beams on the IGM.

Key words: gamma rays: general – instabilities – intergalactic medium – plasmas – radiation mechanisms: non-thermal – relativistic processes

Online-only material: color figures

1. INTRODUCTION

Streaming relativistic particles are common in tenuous astrophysical plasma and their propagation and stability properties are a recurrent theme. The examples include type III solar radio burst (Benz 1993), quasars’ jets (Lesch & Schlickeiser 1987), cosmic rays streaming out of star-forming galaxies (Miniati & Bell 2011), and cosmic-ray transport in the intracluster medium (Ensslin et al. 2011). Propagation and stability properties, related in particular to the excitation of plasma waves, are a subject of attentive investigation as they can play a crucial role in the interpretation of observational data.

Ultrarelativistic beams of $e^+e^-$ pairs are also generated in the intergalactic medium (IGM) by very high energy gamma rays from distant blazars by way of photon–photon interactions with the extragalactic background light (EBL; Gould & Schrédé 1967; Schlickeiser et al. 2012). While blazars’ spectra, and in particular their multi-TeV cutoff features, have been studied in detail to constrain the EBL (e.g., Aharonian et al. 2006), recently multi-GeV and TeV blazar observations have also been used to constrain magnetic field in cosmic voids for the first time (Neronov & Vovk 2010; Tavecchio et al. 2011). In fact, for a flat enough blazar spectra, the electromagnetic cascade should produce an observable spectral bump at multi-GeV energies. The absence of such a bump in a number of observed blazars is ascribed to the presence of a sufficiently strong magnetic field, $B_r \gtrsim 10^{-18}$ G (Dermer et al. 2011; Taylor et al. 2011). The required filling factor of the magnetic field is about 60% (Dolag et al. 2011). Other potential effects of a magnetic field in voids on the electromagnetic cascades have also been investigated, including extended emission around gamma-ray point-like sources (Aharonian et al. 1994; Neronov & Semikoz 2007; Dolag et al. 2009; Elyiv et al. 2009; Neronov et al. 2010a) and the delayed echoes of multi-TeV gamma-ray flares or gamma-ray bursts (Plaga 1995; Takahashi et al. 2008; Murase et al. 2008; Murase 2009).

However, in principle the pair beam is subject to various instabilities, in particular microscopic plasma instabilities of the two-stream family. On this account, Broderick et al. (2012) conclude that transverse modes of the two-stream instability act on much shorter timescales than inverse Compton scattering, effectively inhibiting the cascade and invalidating the above magnetic field measurements. In addition, as a result of the beam’s relaxation, a substantial amount of energy would be deposited into the IGM, with dramatic consequences for its thermal history (Chang et al. 2012; Pfrommer et al. 2012).

In this paper, we reanalyze the stability of blazar-induced ultrarelativistic pair beams. In particular, we use a Monte Carlo model of the electromagnetic shower to quantify the beam properties at various distances from the blazar, and analyze the stability of the produced beam following the work of Breizman, Ryтов, and collaborators (reviewed in Breizman & Rytov 1974; Breizman 1990). We find that even for very modest perpendicular components of the wavevector, the analysis of the instability requires a kinetic treatment. We thus estimate the maximum growth rate of the instability and find that for bright blazars (with an equivalent isotropic gamma-ray luminosity of
10^{45} \text{ erg s}^{-1}) it is suppressed by Coulomb collisions at distances $D \gtrsim 50$ and 20 physical Mpc at redshifts 0 and 3, respectively. Importantly, the growth rate of plasma oscillations is found to be severely suppressed by nonlinear Landau damping so that even at closer distances to the blazar, the beam relaxation timescale remains considerably longer than the inverse Compton cooling time. Finally, the resonance condition cannot be maintained in the presence of density inhomogeneities associated with cosmological structure formation, which also act to dramatically suppress the instability. Thus, our findings support the magnetic field based interpretation of the gamma-ray observational results and rule out effects of the blazar beam on the thermal history of the IGM. Broderick et al. (2012) did not consider the role of density inhomogeneities and concluded that nonlinear Landau damping is unimportant, although they did not present a quantitative analysis of the process.

The rest of this paper is organized as follows. Section 2 summarizes the physical properties of pair beams produced by blazars and present the results of the Monte Carlo model. The two-stream instability in both the reactive and kinetic regimes is discussed in Section 3, where the maximum growth rate of the instability is also given and compared to the collisional rate. Nonlinear effects are considered in Section 4, where the timescales for the beam relaxation are derived. Finally, Section 5 briefly summarizes the results.

2. PAIR BEAMS IN VOIDS

In order to carry out the analysis of the stability of ultrarelativistic pair beams produced by blazars, we need to estimate characteristic quantities of the beam, including its density contrast to the IGM, the Lorentz factor, and the angular and velocity spread. These quantities are derived from the energy and number density of the pair producing photons, i.e., the blazar’s spectral flux, $F_{\gamma}$, and the EBL model. Pair production has been studied extensively in the literature (e.g., Gould & Schrédé 1967; Bonometto & Rees 1971; Schlickeiser et al. 2012) and in the following we briefly summarize its qualitative features, which we then use to describe the results of our Monte Carlo model of a blazar-induced cascade.

2.1. Basic Qualitative Features

Pairs are most efficiently created just above the energy threshold for production, i.e., where

$$s \equiv E_{\gamma} E_{\text{EBL}} (1 - \cos \phi)/2m_e^2c^4 \geq 1,$$  

with $\phi$ the angle between the interacting photons, $E_{\gamma}$ and $E_{\text{EBL}}$ the energy of the incident and target EBL photon, respectively, and the relativistic invariant, $s$, the center of mass energy square in units $m_e^2c^4$. The mean free path for the process depends on the details of the EBL model (Kneiske et al. 2004; Franceschini et al. 2008) but is approximately

$$\ell_{\gamma\gamma} \simeq 0.8 \left(\frac{E_{\gamma}}{\text{TeV}}\right)^{-1} (1 + z)^{-1} \text{Gpc},$$  

with $\zeta = 4.5$ for $z \leq 1$, $\zeta = 0$ otherwise (Neronov & Semikoz 2009; Broderick et al. 2012). The particle number density of the beam, $n_b$, is set by the balance of pair production rate, $2F_{\gamma}/\ell_{\gamma\gamma}$, evaluated close to production threshold, and energy loss rate. If inverse Compton losses dominate then, at a distance $D$ from the blazar such that $F_{\gamma} = L_{\gamma}/4\pi D^2$, 

$$n_b \simeq 2 \frac{F_{\gamma}}{c} \frac{\ell_{\gamma\gamma}}{\ell_{\gamma\gamma}} \simeq 3 \times 10^{-25} \text{cm}^{-3} \left(\frac{E_{\gamma}L_{\gamma}}{10^{45} \text{ erg s}^{-1}}\right) \times \left(\frac{D}{\text{Gpc}}\right)^{-2} \left(\frac{E_{\gamma}}{\text{TeV}}\right)(1 + z)^{\zeta-4},$$  

where $E_{\gamma}L_{\gamma}$ is an estimate of the blazar’s equivalent isotropic gamma-ray luminosity for a source at distance $D$ (see Broderick et al. 2012). Each pair particle carries about half the energy of the incident gamma ray, so the beam Lorentz factor is

$$\Gamma = \frac{E_{\gamma}}{2m_e c^2} \simeq 10^5 \left(\frac{E_{\gamma}}{\text{TeV}}\right).$$  

Another important characteristic quantity of the beam is its angular spread, $\Delta \theta$, determined by the distribution of angles $\theta$ between the pair produced particles and the parent photon’s direction. This can be found to be related to $\Gamma$ and the relativistic invariant as

$$\Delta \theta \leq \frac{s}{\Gamma} \left(1 - \frac{1}{s}\right)^{1/2}. $$  

2.2. Monte Carlo Model

In this section, we compute the characteristic quantities of a blazar-induced pair beam, using a Monte Carlo model of the electromagnetic cascade, fully described in Elyiv et al. (2009) and also applied in Neronov et al. (2010). For this purpose, the blazar’s spectral emission is a typical power-law distribution of primary gamma-ray photons, $dn_{\gamma}/dE_{\gamma} \propto E_{\gamma}^{-\delta}$, in the range $10^3 \leq E_{\gamma}/m_e c^2 \leq 10^6$ and with $\delta = 1.8$ (Abdo et al. 2010). In addition, we use the nominal model of Aharonian (2001) for the EBL in the range 0.1–1000 $\mu$m. For the cosmic background radiations beyond 0.1 $\mu$m we used data from Hauser & Dwek (2001). Contrary to Elyiv et al. (2009), we did not consider the deflection of $e^+ e^-$ pairs in the extragalactic magnetic field as well as the inverse Compton interactions with cosmic microwave background (CMB) photons. Here we took into account pair distributions resulting from just the first double photon collisions. The energy distribution and cross section of the relevant reactions were taken from Aharonian (2003).

Given the primary gamma-ray photon spectrum, we generated random gamma–photon interaction events. These are characterized by the random distance from the blazar at which the double photon collision occurs based on the EBL dependent mean free path of the high-energy photons. Next, we randomly generated the energies $E_{\pm}$ of the produced $e^+ e^-$ pairs and evaluated the proper angle $\theta$ between each pair and the direction of the incident gamma-ray photon. For these quantities we used the analytical expressions for $\mu_e = \cos(\theta)$ in Schlickeiser et al. (2012).

The results of the calculation are shown in Figure 1 for a blazar with equivalent isotropic gamma-ray luminosity of $10^{45}$ erg s$^{-1}$. The top panel shows the spectral energy distribution of the production rate of the pairs at several distances from the blazar, ranging from 3.5 Mpc (top, black line) to 1 Gpc (bottom, yellow line). The bottom panel shows the peak (dashed line) and mean (solid line) energy of the generated pairs, in units of $m_e c^2$, as a function of distance from the blazar. The Lorentz factor of the pairs is a monotonically decreasing function of distance in the range $\Gamma = 10^5$–$10^8$ up to a Gpc away from the
Figure 1. Top: energy distribution of the beam pairs generated at distances, from top to bottom, 3.5 (black), 9.1 (red), 23.3 (green), 60 (blue), 153 (cyan), 390 (magenta), and 1000 (yellow) Mpc from a blazar with equivalent isotropic gamma-ray luminosity of $10^{45} \text{ erg s}^{-1}$. Bottom: peak (dashed line) and mean (solid line) pair beam energy as a function of distance from the blazar for an equivalent isotropic gamma-ray luminosity of $10^{45} \text{ erg s}^{-1}$. The vertical bars correspond to 68% of the energy spread about the median.

(A color version of this figure is available in the online journal.)

Figure 2. Angular distribution of beam pairs generated at 244 Mpc (solid curve) from the blazar. The colored curves, from right to left, indicate the contribution to the angular distribution from pairs in the energy range $10^3$–$2.1 \times 10^4$ (dotted green), $2.1 \times 10^4$–$6 \times 10^3$ (dotted cyan), $4.6 \times 10^3$–$10^4$ (dotted red), and $10^4$–$2.1 \times 10^5$ (dotted blue), which dominate the large-angle end, and $10^5$–$2 \times 10^6$ (dash blue), $2.1 \times 10^6$–$4 \times 10^5$ (dash red), $4.6 \times 10^5$–$10^6$ (dash cyan), and $10^6$–$2.1 \times 10^7$ (dash green), which dominate the small-angle end.

(A color version of this figure is available in the online journal.)

Angular distribution of beam pairs generated at distances 244 Mpc (black dash) from the blazar. This shows that pair production typically peaks at near-infrared ($E_{\text{EBL}} \simeq 0.1 \text{ eV}$) EBL target photons interacting with gamma rays with energy $E_\gamma \sim 0.1$–$1 \text{ TeV}$. The vertical bars indicate the energy range encompassing 68% of the particles. In fact, there is considerable energy spread about the mean value, which decreases toward larger distances as the energy range of gamma-ray photons interacting with the EBL is also reduced. However, the beam particles always remain ultrarelativistic, a detail relevant in the analysis below.

Figure 2 shows the angular distribution of beam pairs at distances 244 Mpc (black dash) from the blazar. The beam angular spread is of order $\Delta \theta \simeq 10^{-5}$, consistent with Equation (5) and the value of $\Gamma$ estimated above. The colored (dashed and dotted) curves show the angular distribution of pairs in eight different energy bins, indicating that the beam angular spread is primarily determined by pairs with $\Gamma \sim 10^1$–$10^5$.

The distribution that we use for the analysis of the pair beam stability below is the steady state one obtained by balancing the production rate given in Figure 1 with inverse Compton losses, i.e.,

$$f(\Gamma) = \int_0^\Gamma \frac{\tau_{\text{IC}}(\varepsilon)}{\varepsilon} \frac{dn}{d\varepsilon}(\varepsilon) \ v_{\gamma\gamma} \ d\varepsilon,$$

where $\tau_{\text{IC}}(\Gamma) = \frac{\ell_{\text{IC}}}{c}$ is the energy dependent timescale for inverse Compton losses.

In Table 1, we report as a function of distance, $D$, from the blazar, the main properties of the beam, which are relevant to the analysis below. These include the number density of the beam pairs, $n_b$; the mean value of the inverse of the pairs’ Lorentz factor, $(\Gamma^{-1})^{-1}$, which enters the estimate of the maximum growth rate of the instability; the mean value of the pairs’ Lorentz factor, $\langle \Gamma \rangle$, which determines the mean energy of the beam; the mean square value of the pairs’ Lorentz factor divided by the mean value of the same, $(\langle \Gamma^2 \rangle / \langle \Gamma \rangle)$, which enters the estimate of the inverse Compton timescale; and the rms of the opening angle of the pairs, $\Delta \theta$, which also enters the growth rate of the instability. These Lorentz gamma factors differ considerably, and they all monotonically decrease with distance as in the bottom panel of Figure 1.

Although the results in this and the next sections assume a blazar equivalent isotropic gamma-ray luminosity of $10^{45} \text{ erg s}^{-1}$, they can be generalized to other luminosities by rescaling the pair beam number density according to

$n_b \rightarrow n_b (E_\gamma L_\gamma / 10^{45} \text{ erg s}^{-1})$.

Table 1

| $D$ (Mpc) | $n_b$ ($10^6$) | $(\Gamma^{-1})^{-1}$ | $\langle \Gamma \rangle$ | $(\langle \Gamma^2 \rangle / \langle \Gamma \rangle)$ | $\Delta \theta$ |
|----------|---------------|----------------------|-------------------------|---------------------------------|-------------|
| 0.87     | 2.81e-18      | 1.52                 | 1.56                    | 5.65                            | 6.43        |
| 1.39     | 1.17e-18      | 1.28                 | 1.53                    | 5.50                            | 8.30        |
| 2.22     | 4.73e-19      | 1.48                 | 1.52                    | 5.13                            | 6.06        |
| 3.55     | 1.79e-19      | 1.39                 | 1.48                    | 4.65                            | 6.32        |
| 5.68     | 7.48e-20      | 1.32                 | 1.39                    | 4.01                            | 7.07        |
| 9.09     | 2.93e-20      | 1.33                 | 1.35                    | 3.28                            | 7.60        |
| 14.55    | 1.14e-20      | 1.35                 | 1.28                    | 2.55                            | 7.75        |
| 23.28    | 4.48e-21      | 1.28                 | 1.20                    | 1.94                            | 7.50        |
| 37.25    | 1.65e-21      | 1.30                 | 1.13                    | 1.50                            | 7.29        |
| 59.60    | 5.25e-22      | 1.44                 | 1.11                    | 1.26                            | 8.76        |
| 95.37    | 1.86e-22      | 1.39                 | 1.00                    | 0.99                            | 9.14        |
| 152.59   | 6.31e-23      | 1.34                 | 0.86                    | 0.75                            | 8.88        |
| 244.14   | 2.03e-23      | 1.27                 | 0.72                    | 0.52                            | 9.67        |
| 390.63   | 6.13e-24      | 1.19                 | 0.58                    | 0.32                            | 11.0        |
| 625.00   | 1.75e-24      | 1.12                 | 0.47                    | 0.18                            | 11.0        |
| 1000.00  | 4.71e-25      | 1.04                 | 0.39                    | 0.12                            | 11.8        |

2.3. IGM in Voids

The analysis of the pair beam instability also depends on the thermodynamic properties of the plasma in cosmic voids, namely, the number density of free electrons and their temperature. The number density of free electrons can be expressed as $n_e \simeq 2 \times 10^{-7}(1 + \delta)(1 + z)^3$ cm$^{-3}$. The typical overdensity $\delta$ is taken to be the value at which the cumulative distribution of the IGM gas is 0.5. Using the simulation results presented in Section 4.2, and in particular the insets in Figure 9, we estimate as representative value for the voids $\delta_e = -0.9(1 + z)$, where the redshift dependence is approximate but sufficient for our
purposes. Note that this implies a redshift evolution of the bulk IGM density in voids, $n_v \propto (1+z)^4$. As for the gas temperature, we assume $T_v \simeq 10^4 \text{K}$, which reproduces the IGM temperature at mean density of a few $10^4$ at redshift 3. This redshift dependence, while again a rough approximation, is acceptable for our purposes.

3. BEAM INSTABILITY: REACTIVE VERSUS KINETIC

The blazar-induced pair beam is subject to microscopic instabilities, in particular two-stream-like instabilities, of both electrostatic and electromagnetic nature. The beam is neutrally charged so no return current is induced. In the following, we assume a sufficiently weak magnetic field, such that $\omega_H \ll \omega_p$, where $\omega_H$ is the cyclotron frequency, $\omega_p = (4\pi n_e e^2/m_e)^{1/2}$ is the plasma frequency of the IGM in voids, and $e$ is the electron’s charge. In this case, the instability is predominantly associated with Cherenkov emission of Langmuir waves, which operates under the resonant condition

$$\omega - k \cdot v = 0,$$

where $k$ is the wavevector of the perturbation mode and $v$ is the beam particle velocity. The pair particles contribute equally to the dielectric function as they have the same mass, number density, velocity distribution, and plasma frequency, $\omega_{p,b} = (4\pi n_b e^2/m_e)^{1/2}$. After separating the contributions from the background plasma and the beam particles, the dispersion relation for Langmuir waves, valid in the relativistic case, can be written as (Breizman 1990)

$$1 - \frac{\omega_p^2}{\omega^2} - \frac{4\pi e^2}{k^2} \int \frac{k \cdot f}{k \cdot v - \omega} dp = 0,$$

where $f(p)$ is the distribution function of the beam particles. There are two important regimes that characterize the unstable behavior of the beam, namely, reactive and kinetic. In the reactive case, the beam’s velocity spread, $\Delta v$, is negligible so all particles can participate in the unstable behavior and the growth rate of the instability is therefore fastest. In the kinetic regime, on the other hand, the velocity spread is considerable and only the resonant particles contribute to the growth of Langmuir waves, so the growth rate is slower than in the reactive case. Formally, the reactive regime is applicable when (Breizman & Mirnov 1970)

$$|k \cdot \Delta v| \ll \gamma_r,$$

where $\gamma_r$ is the reactive growth rate. In this case, the integral in Equation (8) can be solved in a simplified way, which involves neglect of the velocity spread around the mean value. This leads to the estimate of the reactive growth rate that, maximized along the longitudinal component of the wavevector, reads (Fainberg et al. 1970)

$$\gamma_r \simeq \omega_p \left( \frac{n_b}{\Gamma n_v} \right)^{1/2} \left( \frac{k^2}{k_T^2} + \frac{k^2}{k^2} \right)^{-1/2},$$

with $k_T = \omega_p/v$ and $k_1, k_2$ being the components of the wavevector parallel and perpendicular to the beam direction, respectively. It is well known that, for an ultrarelativistic beam ($\Gamma \gg 1$), the fastest growing modes in the reactive regime are those quasi-perpendicular to the beam. This is due to the large suppression caused by relativistic inertia along the longitudinal direction (Fainberg et al. 1970). However, as shown later, for quasi-perpendicular directions of the wavevector, the reactive regime is not applicable.

When the approximation (Equation (9)) is not valid, the growth rate is evaluated from a pole of the integrand in the dispersion relation (Equation (8)), namely,

$$\gamma_k = \frac{2\pi e^2}{k^2} \int \frac{\partial f}{\partial p} \delta(\omega_p - k \cdot v) dp.$$

If, as is the case here, despite the energy spread the particles remain ultrarelativistic and $|v| = c$ can be assumed, the above integral can be simplified to (Breizman & Mirnov 1970)

$$\gamma_k = -\omega_p \pi n_b \left( \frac{\omega_p}{ke} \right)^3 \int_{\mu_-}^{\mu_+} d\mu \frac{2g + \left( \mu - k_c/\omega_p \right) \frac{\partial g}{\partial \mu}}{[(\mu_+ - \mu) (\mu - \mu_-)]^{1/2}},$$

where the integration variable $\mu$ is the angle between the particles and the beam direction, and

$$\mu_{\pm} = (\omega_p/ke)(k_1/k \pm k_2/k) \sqrt{k^2 c^2/\omega_p^2 - 1},$$

$$g(\theta) = \frac{m_e c}{n_b} \int pf(p, \theta) dp \simeq (\Gamma^{-1}) \frac{1}{\Delta^2} e^{-\frac{\Delta^2}{\omega^2}}.$$

The second equality for $g(\theta)$ in Equation (14) is found to be a good approximation based on results of the Monte Carlo model of the cascade. The integral for the growth rate in Equation (12) can be evaluated numerically. The qualitative behavior of the growth rate, $\gamma_k$, on the plane $k_1-k_2$ is summarized in Figure 3 (Breizman & Mirnov 1970) for a beam at a Gpc from the blazar. Outside the narrow resonant region of $k$-space, corresponding in the plot to the uniform color, the growth rate is effectively null. In the narrow resonant region around $k_1 = \omega_p/c$, the growth can be positive (bright), negative (dark), and null, and for large enough values of $k_2$, it carries the sign of $\omega_p/k_2 - c$ (see below). Within the resonant region, the growth rate as a function of $k_2$ has typically two extrema, a maximum and a minimum. This is shown in Figure 4 for values of $k_2$ of interest, i.e., where

![Figure 3](image-url)
Figure 5. Normalized growth rate as a function of $k_c/\omega_p$ inside the resonant region for values of $k_c$ where $\gamma_k$ reaches its maximum values.

$\gamma_k \simeq \omega_p (\Gamma^{-1}) n_b \frac{1}{n_v} \frac{\omega_p^2}{\omega_p^2 + k_{\perp}^2 c^2}$, \hspace{1cm} (15)

which we will be using in the following.

Figure 6. Ratio of instability maximum growth rate, $\gamma_{\text{max}}$, to Coulomb collision rate, $v_c$, for redshifts $z = 0$ (solid curve), $z = 1$ (short dashed curve), and $z = 3$ (long dashed curve). The shaded area corresponds to the absolutely stable region where $\gamma_{\text{max}} \ll v_c$.

### 3.1. Fastest Growing Modes versus Coulomb Collisions

For particles of an ultrarelativistic beam with modest angular spread, $\Delta \theta \ll 1$, we can assume $v_\parallel \simeq c$ and $v_\perp \simeq c\Delta \theta$. If the energy spread $\Delta E/E \ll 1$, then the longitudinal velocity spread of the beam is

$$\Delta v_\parallel \simeq c\left(\frac{\Delta E}{(\Gamma - 1)^2 E}\right) + c\Delta \theta^2.$$ \hspace{1cm} (16)

Thus, for the angular spread and Lorentz factor characteristic of the blazar-induced beam, the longitudinal velocity spread is negligible with respect to the perpendicular velocity spread. It turns out that the first and second terms in Equation (16) are comparable to within a factor of a few, so for the sake of simplicity in the following we retain the second term only, neglecting any fudge factor. If we then use Equations (9) and (16), with the estimates for the beam angular spread and bulk Lorentz factor from the previous section, we find that virtually all modes require a kinetic description unless

$$\frac{k_{\perp}}{k} \lesssim 10^{-5} \left(\frac{n_b/n_v}{10^{-15}}\right) \left(\frac{\langle \Gamma \rangle}{10^2}\right)^{-1} \left(\frac{\Delta \theta}{10^{-5}}\right)^{-3}.$$ \hspace{1cm} (17)

The maximum growth rate occurs at $k_{\perp}$ provided by the above estimate. For smaller values, we enter the reactive regimes and relativistic inertia increases. For larger values, we are in the kinetic regime where the growth rate decreases due to the increasing velocity spread along $k$ although the decrease becomes significant only for $k_{\perp} \gtrsim \omega_p/c$. The fastest growth rate for modes with $k_{\perp} \lesssim \omega_p/c$ is therefore given by

$$\gamma_{\text{max}} \simeq \omega_p (\Gamma^{-1}) n_b \frac{1}{n_v} \frac{\omega_p^2}{\omega_p^2 + k_{\perp}^2 c^2} \times \left(\frac{\langle \Gamma \rangle}{10^2}\right)^{-1} \left(\frac{\Delta \theta}{10^{-4}}\right)^{-2} \left(\frac{D}{\text{Gpc}}\right)^{-2},$$ \hspace{1cm} (18)

where we have taken $n_b \simeq 10^{-24} \text{ cm}^{-3}$ at a Gpc from a blazar of luminosity $E_y L_y = 10^{45} \text{ erg s}^{-1}$. A basic condition for the growth of an instability is that its growth rate exceeds the collisional damping rate, i.e., $\gamma_{\text{max}} \gg v_c$, where (Huba 2009)

$$v_c \simeq 10^{-11} \text{ s}^{-1} \left(\frac{n_v}{2 \times 10^{-8} \text{ cm}^{-3}}\right) \left(\frac{T_v}{3 \times 10^3 \text{ K}}\right)^{-2},$$ \hspace{1cm} (19)

and for the Coulomb logarithm we have used $\Lambda_e = 27.4$ (Huba 2009). In Figure 6, we plot the ratio $\gamma_{\text{max}}/v_c$ as a function of distance from the blazar, using the values reported in Table 1,
which again applies for a blazar of equivalent isotropic gamma-ray luminosity of $10^{45}$ erg s$^{-1}$. The solid, dashed, and long-dashed curves correspond to redshifts $z = 0$, $z = 1$, and $z = 3$, respectively. The redshift dependence is obtained by using the void average density and temperature redshift dependences discussed in Section 2.3, together with the redshift dependence of $n_0$ given in Section 2.1. The shaded area corresponds to the region where the instability is inhibited by collisions. The plot shows that the instability can only develop at distances of less than a 50 Mpc at redshift $z = 0$ and about 20 physical Mpc $z = 3$.

4. BEAM STABILIZATION

As shown in the previous section, pair beams within a certain distance of the parent blazar may be unstable due to the excitation of Langmuir waves. In this section we further analyze these unstable conditions. In particular, we consider nonlinear effects on plasma waves due to scattering off thermal ions and density inhomogeneities. We begin, however, with a brief outline of the main features of the relaxation process (for ions and density inhomogeneities. We begin, however, with a brief outline of the main features of the relaxation process discussed in Section 2.3, together with the redshift dependence of $n_0$ given in Section 2.1. The shaded area corresponds to the region where the instability is inhibited by collisions. The plot shows that the instability can only develop at distances of less than a 50 Mpc at redshift $z = 0$ and about 20 physical Mpc $z = 3$.

4.1 Nonlinear Landau Damping

In this section, we consider in some detail the main process that we believe compensates the growth of Langmuir waves, i.e., induced scattering off plasma ions, also known as nonlinear Landau damping (Tsytovich & Shapiro 1965; Breizman et al. 1972; Lesch & Schlickeiser 1987). In this process, a thermal ion, with characteristic velocity $v_{ti}$, interacts with the beam wave produced by two Langmuir oscillations, $\omega(k)$, $\omega(k')$, under the condition for Cherenkov interaction, i.e.,

$$\omega(k) - \omega(k') = (k - k') \cdot v_{ti}. \quad (20)$$

The rate of induced scattering of Langmuir waves off thermal ions in a Maxwellian plasma with number density $n$ and ion/electron temperature $T_i/T_e$, respectively, is (e.g., Melrose 1989)

$$\gamma_{nl}(k) = \frac{3(2\pi)^{\frac{1}{2}}}{2} \frac{T_i T_e}{(T_i + T_e)^{2}} \int \frac{d^3 k'}{(2\pi)^3} \frac{\hat{W}(k')}{n m e v_{ti}} \times \left( \frac{k \cdot k'}{k k'} \right)^2 \frac{k'^2 - k^2}{|k' - k|^2} \exp \left[ -\frac{1}{2} \frac{3}{2} \frac{v_{te}^2}{\omega_{pe} v_{ti}} \frac{k'^2 - k^2}{|k' - k|^2} \right]. \quad (21)$$

where $\hat{W}(k)$ indicates the spectral energy density of Langmuir waves. The growth rate, $\gamma_{nl}(k)$, bears the sign of $(k' - k)$. This indicates that as a result of induced scattering, Langmuir waves cascade toward regions of phase space of lower wavevectors, i.e., lower energies, the energy difference being absorbed by the thermal ions. Eventually, the wave energy is transferred to modes with wavenumber, $k$, small enough that the wave phase-speed, $\omega/k > c$, exceeds the speed of light, and resonance with the beam particles is lost. The wavenumbers allowed in the scattering process are constrained by the integral expression in Equation (21). In particular, the following condition must be fulfilled:

$$\left| \frac{k'^2 - k^2}{|k' - k|^2} \right| \leq \frac{1}{\omega_p v_{ti}} \approx 35 \times \frac{\omega_p}{c} \left( \frac{T_v}{3 \times 10^4 K} \right)^{-\frac{1}{2}}. \quad (22)$$

The above constrain is satisfied for the case of differential scattering, i.e., $\Delta k/k \ll 1$, whereby $k \sim k'$ and $k' \sim -k$. In this case, Langmuir waves, generated with wavenumber, $k \sim \omega_p/c$, parallel to the beam, are isotropized. This reduces the level of resonant energy density by a factor $\sim \Delta \theta^2/4\pi$, somewhat increasing the lifetime of the beam. However, given the low temperature of the IGM in voids, the more efficient integral scattering, with $k' \ll k$, is also allowed. In this case, Langmuir waves are mostly kicked out of resonance in a single scattering event, dramatically reducing the level of resonant energy density and suppressing the instability.

The general solution for the evolution of the energy density in plasma waves is a non-trivial task as it requires solving for integro-differential equations that describe the detailed energy transfer of the wave energy across different modes. However, for a conservative estimate, we can neglect differential scattering and evaluate the rate of induced integral scattering from Equation (21) using the condition $k' \ll k$. We thus obtain

$$\gamma_{nl} \approx \frac{W_{nr}}{n k_B T_v v_{ti} c}. \quad (23)$$
where $W_{nr}$ is the total energy density in Langmuir waves at $k \ll \omega_p/c$, i.e., non-resonant with the beam. This energy density is excited by the nonlinear scattering process and for the most part is dissipated by Coulomb collisions at a rate $v_e$ (see further discussion below). Thus it evolves according to

$$\frac{\partial W_{nr}}{\partial t} = 2\gamma_{nl} W_{nr} - v_e W_{nr},$$  \hspace{0.5cm} (24)

where we have used $\gamma_{nl} \equiv \gamma_{nl} / W_{nr} = \omega_p (1 / n_b T_c) (\nu_c / v_e) c$ and $W_r$ is the total energy density in Langmuir waves at $k \sim \omega_p / c$, i.e., resonant with the beam. The latter obviously evolves according to

$$\frac{\partial W_r}{\partial t} = 2\gamma_{max} W_r - 2\gamma_{nl} W_{nr} W_r,$$  \hspace{0.5cm} (25)

where we have neglected the role of collisions (i.e., we assume $\gamma_{max} \gg v_e$, as required for the existence of the instability). Equations (24) and (25) form a well-known Lotka–Volterra system of coupled nonlinear differential equations, which has stable periodic solutions, with the following average values for the energy densities:

$$\overline{W_{nr}} = \frac{\gamma_{max}}{\gamma_{nl}}, \quad \overline{W_r} = \frac{v_e}{2\gamma_{nl}}.$$  \hspace{0.5cm} (26)

In this regime, the transfer rate of Langmuir waves out of resonance by nonlinear Landau damping equals on average their production rate, i.e., $\gamma_{nl} \approx \gamma_{max}$. Thus, the beam emission of Langmuir waves is only linear in time, with an average power $P(W_r) = 2\gamma_{max} W_r$, and the beam relaxation timescale at redshift $z = 0$ is

$$\tau_{beam} \simeq \frac{n_b (\Gamma) m_e c^2}{2\gamma_{max} \overline{W_r}} = 1.5 \times 10^9 \text{yr} \left(\frac{n_v}{2 \times 10^{-3} \text{cm}^{-3}}\right)^{-1} \times \left(\frac{(1-\Gamma)}{10^{-4}}\right) \left(\frac{\Gamma}{10^5}\right) \left(\frac{\Delta \theta}{10^{-2}}\right)^2 \left(\frac{T_v}{3 \times 10^3 \text{K}}\right).$$  \hspace{0.5cm} (27)

The above timescale should be compared with the pairs’ cooling time on the CMB, $\tau_{IC} = \ell_{IC} / c \approx 3 \times 10^9 (E_{\text{peak}} / \text{TeV})^{-1} (1+z)^4 \text{yr}$. The ratio of these timescales is plotted in Figure 7 using the values reported in Table 1 as a function of distance from our reference blazar with isotropic gamma-ray luminosity of $10^{45}$ erg s$^{-1}$. At redshift $z = 0$ (solid line), the beam appears to be stable on significantly longer timescales than the inverse Compton emission energy loss timescale, particularly within 100 Mpc from the blazar, where the average value of $\Gamma$ of the pairs tends to be higher. This conclusion is reinforced at higher redshifts (dashed line for $z = 3$), where the redshift dependence is inferred as in Section 3.1.

The above analysis works in the weak turbulence regimes, which require that the energy density of resonant Langmuir waves be a small fraction of the beam energy density. For the typical values of IGM gas and beam parameters used above, this requirement is readily fulfilled as $\overline{W_{nr}} / n_b T_c m_e c^2 \simeq 3 \times 10^{-6}$, warranting our approach limited to second-order processes.

Another consistency check to be performed concerns the assumption of collisional dissipation of the long-wavelength Langmuir waves. In fact, accumulation of energy in these non-resonant waves can generate modulation instability of the background plasma if (Breizman 1990)

$$\frac{\overline{W_{nr}}}{n_v k_B T_v} \geq k^2 \lambda_D^2 \sim k_B T_v / m c^2.$$  \hspace{0.5cm} (28)

In Figure 8, the ratio on the left-hand side of the above equation is plotted as a function of distance from our reference blazar using the values in Table 1. The solid line corresponds to redshift $z = 0$ and the horizontal line is the nominal threshold value for the onset of the modulation instability. For redshifts higher than $z = 0$, we plot for comparison with the same threshold line the same ratio on the left-hand side of Equation (28) but divided by a factor $(1+z)^3$ to account for the IGM temperature redshift dependence (see Section 2.3).

The plot shows that the assumption of collisional dissipation of the long-wavelength Langmuir waves is always valid except at short distances from low-redshift blazars. The modulation instability deserves more attention than the scope of the current paper can afford. Here we notice that, while the modulation instability could help stabilize the beam (Nishikawa & Ryutov 1976), it could also provide an effective dissipation rate that is more efficient than collisions. In this case, the level of energy density of resonant waves, $W_r$, will increase with consequent reduction of the beam lifetime (see Equations (26) and (27)). However, because the threshold condition for triggering the modulation instability depends quadratically on the temperature (see Equation (28)), should the background plasma suffer even modest heating caused by the beam relaxation, the modulation instability will quickly stabilize, restoring the conditions for collisional dissipation of the non-resonant waves.

Therefore, in conclusion, from the above analysis together with the findings in Section 3.1, it appears that the beam is stable at basically all relevant distances from the blazar, and the beam...
instability plays only a secondary role on the electromagnetic shower, the beam dynamics, and the thermal history of the IGM.

4.2. Plasma Inhomogeneities

In addition to the kinetic effects described above, the energy density of the plasma waves evolves in time due to spatial gradient effects according to (Kaplan & Tsytovich 1973)

$$\frac{d}{dt} \hat{W}(x, t, k) = \frac{\partial \hat{W}}{\partial t} + v_\parallel \hat{\nabla}_k \hat{W} - \hat{\nabla}_k \omega_\parallel \nabla_k \hat{W},$$

(29)

where for the rate of change of the wavevector we have used the equation of geometric optics

$$\frac{dk_\parallel}{dt} = -\hat{\nabla}_x \omega_\parallel.$$  

(30)

The first and second terms on the right-hand side of Equation (29) describe, as usual, explicit time dependence and the effects of spatial gradients discussed at the beginning of Section 4. The last term describes the change in $\hat{W}$ associated with modifications of the waves' wavevector as a result of inhomogeneities. This term is important because, just like induced scattering by thermal ions, it transfers the excited Langmuir waves to wavemodes that are out of resonance with the beam particles, therefore suppressing the instability (Breizman & Ryutov 1971; Nishikawa & Ryutov 1976).

For Langmuir waves, the most important contribution to $\nabla_x \omega_\parallel$ comes from density inhomogeneities. In addition, the beam stabilization mainly results from changes in the longitudinal component of the wavevector. Therefore, we restrict our analysis to this case only, and write

$$\frac{dk_\parallel}{dt} \equiv \frac{1}{\lambda_1} \frac{\omega_p}{\lambda_1},$$

(31)

with $\lambda_1 = n_0/(\hat{\nabla}_x n_0)_\parallel$, the length scale of the density gradient along the beam.

In order to estimate the scale lengths of IGM density gradients, $\lambda$, we have carried out a cosmological simulation of structure formation including hydrodynamics, dark matter, and self-gravity as described in Miniati & Colella (2007). For the cosmological model, we adopted a flat $\Lambda$CDM universe with the following parameters: total mass density, normalized to the critical value for closure, $\Omega_m = 0.2792$; baryonic mass density, $\Omega_b = 0.0462$; normalized vacuum energy density, $\Omega_\Lambda = 1 - \Omega_m = 0.7208$; Hubble constant $H_0 = 70.1$ km s$^{-1}$ Mpc$^{-1}$; spectral index of primordial perturbation, $n_s = 0.96$; and rms linear density fluctuation within a sphere of comoving radius of 8 h$^{-1}$ Mpc, $\sigma_8 = 0.817$, where $h \equiv H_0/100$ (Komatsu et al. 2009). The computational box has a comoving size $L = 50 h^{-1}$ Mpc, which is discretized with 512$^3$ comoving cells, corresponding to a nominal spatial resolution of 100 h$^{-1}$ comoving kpc. The collisionless dark matter component is represented with 512$^3$ particles with mass $6 \times 10^8 h^{-1} M_\odot$.

Figure 9 shows the range of scale lengths of IGM density gradients as a function of IGM gas overdensity for three different cosmological redshifts, $z = 0$ (top), $z = 1$ (middle), and $z = 3$ (bottom). Accordingly, the distance covered by the beam particles during the fastest growth time, $\sim c\gamma_{\text{max}}^{-1} \simeq 1$ kpc, is much shorter than the typical scale length of density gradients. This corresponds to the case of regular, as opposed to random, inhomogeneities.

It is clear that in order for the excited waves to have an effect on the beam, the beam–wave interaction under resonant conditions must continue for a sufficiently long time. Therefore, the condition for wave excitation is expressed as (Breizman & Ryutov 1971)

$$\frac{\Delta k_\parallel}{|d k_\parallel/dt|} = \frac{2\omega_p}{\omega_p} \frac{\Delta k_\parallel}{\omega_p} > \Lambda_\gamma,$$

(32)

where $\Lambda_\gamma$ is the Coulomb logarithm and $\Delta k_\parallel$ the change in longitudinal component of the wavevector allowed by the resonant condition (Equation (7)). Using Equation (7) (and neglecting the term $E/E^2$) to obtain $\Delta k_\parallel \lesssim (\omega_p/c)\Delta \theta^2 + k_\perp \Delta \kappa_\perp$, Equation (32) can be solved to express the condition for wave excitation in terms of $\lambda_1$, i.e.,

$$\lambda_1 \gtrsim \frac{c}{2\omega_p} \left( \Gamma^{-1} \right) \frac{n_a}{n_b} \Lambda_\gamma \left( 1 + \frac{k_\perp}{k_\parallel \Delta \theta} \right)^{-1} > 10^6 \text{kpc}$$

$$\times \left( \frac{D}{\text{Gpc}} \right)^2 \left( \Gamma^{-1} \right) \left( \frac{\Delta \theta}{10^{-4}} \right) \left( \frac{\Lambda_\gamma}{30} \right) (1 + z)^2,$$

(33)

where in the second inequality we have used $k_\perp \simeq \kappa_1$, which corresponds to the most favorable case for the instability growth in the presence of inhomogeneities, and again the redshift dependence is derived as described in Section 3.1. We can again plot the minimal values of $\lambda_1$ allowed for the growth of the beam instability using the parameter values for the beam from Table 1. This is shown by the oblique lines in Figure 10 for redshifts $z = 0$ (solid line), $z = 1$ (dashed line), and $z = 3$ (long dashed line). For the same redshifts (and with line-style matching that of the corresponding oblique lines), the three horizontal thin lines represent the mean scale length of density inhomogeneities at typical void overdensity (i.e., where the cumulative gas distribution function is 0.5), extracted from Figure 9. The figure shows that the growth of Langmuir waves...
is constrained by the presence of inhomogeneities, to distances $D < 30, 6, \text{and} 1 \text{Mpc}$ from the blazar, for $z = 0, 1, \text{and} 3$, respectively. Inhomogeneities provide another independent argument against the growth of Langmuir waves and the unstable behavior of the pair beam. While nonlinear Landau damping weakens with distance from the blazar (see Figure 6), the impact of inhomogeneities becomes stronger (see Figure 10), so that the stabilization effects of the two processes compensate each other at different distances.

5. CONCLUSION

We have considered the stability properties of a low-density ultrarelativistic pair beam produced in the IGM by multi-TeV gamma-ray photons from blazars. The physical properties of the pair beam are determined through a Monte Carlo model of the electromagnetic cascade. The stability analysis is based on linear and nonlinear kinetic plasma theory. In summary, we find that the combination of kinetic effects, nonlinear Landau damping, and density inhomogeneities appear to considerably stabilize blazar-induced ultrarelativistic beams over the inverse Compton loss timescale, so that the electromagnetic cascade remains mostly unaffected by the beam instability. This implies that the lack of a bumpy feature at multi-GeV energies in the gamma-ray spectrum of distant blazars cannot be attributed to such instabilities and can in principle be related to the presence of an intergalactic magnetic field. Finally, heating of the IGM by pair beams appears negligible.

We are thankful to an anonymous referee for valuable comments on the manuscript. F.M. acknowledges useful discussions with B. N. Breizman and A. Benz, and comments from D. D. Ryutov and R. Schlickeiser. We are grateful to D. Potter for making grafic++ package available for cosmological initial conditions. This work was supported by a grant from the Swiss National Supercomputing Center (CSCS) under Project ID S275.

REFERENCES

Abdo, A., Ackermann, M., Ajello, M., et al. 2010, A&A, 710, 1271
Aharonian, F. A., Coppi, P. S., & Volk, H. J. 1994, ApJL, 423, L5
Aharonian, F. A. 2001, arXiv:astro-ph/0112314
Aharonian, F. A. 2003, Very High Energy Cosmic Gamma Radiation (Singapore: World Scientific)
Aharonian, F. A., Akhperjanian, A. G., Bazer-Bachi, A. R., et al. 2006, Natur, 440, 1018
Benz, A. 1993, Plasma Astrophysics (Kluwer:Dordrecht)
Bonometto, S., & Rees, M. R. 1971, MNRAS, 152, 21
Breizman, B. N. 1990, RvPP, 15, 61
Breizman, B. N., & Mimov, V. V. 1970, Ge&Ar, 10, 34
Breizman, B. N., & Ryutov, D. D. 1971, JETP, 33, 220
Breizman, B. N., & Ryutov, D. D. 1974, NucFu, 14, 873
Breizman, B. N., Ryutov, D., & Chebotaev, P. Z. 1972, JETP, 35, 72
Broderick, A. E., Chang, P., & Pfommer, C. 2012, ApJ, 752, 22
Chang, P., Broderick, A. E., & Pfommer, C. 2012, ApJ, 752, 23
Dermer, C. D., Cavallini, M., Razzaque, S., et al. 2011, ApJL, 733, L21
Dolag, K., Kachelriess, M., Ostapchenko, S., & Tomas, R. 2009, ApJ, 703, 1078
Dolag, K., Kachelriess, M., Ostapchenko, S., & Tomás, R. 2011, ApJL, 727, L4
Elyiv, A., Neronov, A., & Semikoz, D. V. 2009, PhRvD, 80, 023010
Ensslin, T. A., Pfommer, C., Miniati, F., & Subramanian, K. 2011, A&A, 527, A99
Fainberg, Ya. B., Shapiro, V. D., & Shechonkov, V. I. 1970, JETP, 30, 528
Franceschini, A., Rodighiero, G., & Vaccari, M. 2008, A&A, 487, 837
Gould, R. J., & Schréder, G. P. 1976, NucFu, 5, 228
Goud, R., & Dwark, E. 2001, A&A, 39, 249
Grong, R., & J.-M. 1975, AuPh, 28, 731
Hauser, M., & Dwek, E. 2001, A&A, 39, 249
Huba, J. D. 2004, NRL Plasma Formulary, NRL/PU/679009-523 (Washington, DC: Naval Research Laboratory)
Kaplan, S. A., & Tsytovich, V. N. 1973, Plasma Astrophysics (Oxford: Pergamon)
Kneiske, T. M., Bretz, T., Mannheim, K., & Hartmann, D. H. 2004, A&A, 413, 807
Komatsu, E., Dunkley, J., Nolta, M. R., et al. 2009, ApJS, 180, 330
Lesch, H., & Schlickeiser, R. 1987, A&A, 179, 93
Melrose, D. B. 1989, Instabilities in Space and Laboratory Plasmas (Cambridge: Cambridge Univ Press)
Miniati, F., & Bell, A. R. 2011, ApJ, 729, 73
Miniati, F., & Colella, P. 2007, JCoPh, 227, 400
Murase, K. 2009, PhRvL, 103, 081102
Murase, K., Takahashi, K., Inoue, S., Ichiki, K., & Nagataki, S. 2008, ApJL, 686, L67
Neronov, A., & Semikoz, D. V. 2007, JETP Lett., 85, 473
Neronov, A., & Semikoz, D. 2009, PhRvd, 80, 123012
Neronov, A., Semikoz, D., Kachelriess, M., Ostapchenko, S., & Elyiv, A. 2010, ApJL, 719, L130
Neronov, A., & Vovk, I. 2010, Sci, 328, 7
Nishikawa, K., & Ryutov, D. D. 1976, IPSI, 41, 1757
Pfrommer, C., Chang, P., & Broderick, A. E. 2012, ApJ, 752, 24
Plaga, R. 1995, Natur, 374, 430
Schlickeiser, R., Elyiv, A., Ibscher, D., & Miniati, F. 2012, ApJ, 758, 101
Takahashi, K., Murase, K., Ichiki, K., Inoue, S., & Nagataki, S. 2008, ApJL, 687, L5
Tavecchio, F., Ghisellini, G., Bonnoli, G., & Foschini, L. 2011, MNRAS, 414, 3566
Taylor, A. M., Vovk, I., & Neronov, A. 2011, A&A, 529, A144
Tsytovich, V. N., & Shapiro, V. D. 1965, NucFu, 5, 228