Applying filtering for determining the angular orientation of spinning objects during interference

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Abstract. This article discusses the application of user navigation equipment of satellite radionavigation systems (GLONASS/GPS) for measuring the inclination of the aerial axis of rotation. The authors have demonstrated that the required accuracy of the angular inclination is achieved only in a relative phase mode, which is not always feasible. The application of filtering for measured parameters of navigation equipment has namely been used for planimetric coordinates. Filtering is performed using the second order Kalman filter. It has significant effect; thus, even at a speed of 1 rpm, there is no disruption in the autonomous coded mode; for the relative phase mode, the required accuracy is achieved at 3 rpm.

Keywords: GLONASS/GPS satellite radionavigation system, inclination from the aerial axis of rotation, second order Kalman filter.

1. Introduction

Currently, user navigation equipment of the GLONASS radionavigation system has found intense application for improving the mobility of aerial and space defense systems. As the demand for highly-accurate radiolocation data produced by mobile radiolocation devices grows, it becomes universal not only for topographic positioning, but for the precision of leveling antenna systems in triaxial mobile radiolocation devices, which greatly affects the instrumental accuracy for determining the altitude of aerial objects. The leveling error, using existing methods, usually exceeds 3 arc minutes; the altitude measurement error at a distance of 100 km is over 87 m.

It is preferable to position the antennas of user navigation equipment on the antenna system...
of the mobile radiolocation unit. This is necessary to provide the radio coverage of the upper hemisphere and reduce the error of the multibeam signal detection. In this case, using single-antenna user navigation equipment, it is possible to determine angular velocity and deviations from vertical of the aerial axis of rotation.

A method for determining the angular position of a rotating object and errors for autonomous coded, deferential and phased modes for user navigation equipment functioning is described in [1, 2]. The accuracy analysis in [1] demonstrates that using single-antenna equipment in an autonomous coded mode does not allow us to determine, with a sufficient degree of accuracy, the inclination angle of the aerial axis of rotation, in comparison with conventional optical methods of leveling mobile radiolocation unit antennas. Differential and relative phase modes in user navigation equipment require at least two user antennas to be positioned on the mobile radiolocation unit antenna system – an operation that is not always possible. This adds to the relevance of filtering measured parameters in user navigation equipment for the purpose of increasing the accuracy for determining the inclination angle of the aerial axis of rotation.

2. Determining the angular orientation for spinning objects

Let’s look at the algorithm for determining the inclination of an antenna’s aerial axis of rotation. The position of the axis of rotation is determined by trajectory points; the current angular position is determined in the plane of rotation.

![Figure 1. Determining the module of the angular velocity vector](image)

The coordinate increment vectors are noncollinear and orthogonal to the axis of rotation, hence the coordinates of the axis of rotation are determined through a cross product of antenna coordinate increment vectors, as seen in Fig. 1.

\[ \mathbf{n} = \Delta \mathbf{B}_1 \times \Delta \mathbf{B}_2. \] (1)
Using expression (1), it is also possible to determine the direction of rotation. By definition, the angular velocity vector is directed so that from its end the rotation of an object is counterclockwise. In order to find the direction of the angular velocity vector, vector $\Delta \mathbf{B}_1$ must precede vector $\Delta \mathbf{B}_2$ in time [3, 4].

The $\mathbf{n}$ vector module equals:

$$|\mathbf{n}| = |\Delta \mathbf{B}_1| \cdot |\Delta \mathbf{B}_2| \cdot \sin \omega_0,$$

(2)

The angle of $\omega_0$, as follows from Fig. 1, is equal to the following angle of pitch:

$$\omega_0 = \frac{\omega_1 + \omega_2}{2} = \arcsin \frac{|\mathbf{n}|}{|\Delta \mathbf{B}_1| \cdot |\Delta \mathbf{B}_2|}.$$  

(3)

The module of the angular velocity vector:

$$|\omega| = \frac{\omega_0}{\Delta t} = \frac{\left(\arcsin \frac{|\Delta \mathbf{B}_1 \times \Delta \mathbf{B}_2|}{|\Delta \mathbf{B}_1|^2}\right)}{\Delta t}.$$  

(4)

To determine the module of the angular velocity using formula (4) it is not necessary to know the length of the base. Moreover, the given method is functional, even if the base does not lie in the plane of rotation.

The proposed algorithm can be built using coded measurements in an autonomous mode, using a differential mode or angular phase observations. These modes have different errors for antenna coordinate increment measurements. (Fig. 2).

![Figure 2. Measuring coordinates X, Z: a – for an autonomous coded mode; b – for a differential mode; c – for a relative phase mode](image)

During determining the vector of the axis of rotation, pictured in Fig. 3, the distance between the user’s equipment antennas was 4 m; the speed of the antenna rotation was 6 revolutions per
minute. An analysis of the demonstrated figures shows that the accuracy for measuring the inclination of the axis of rotation is much higher for differential and relative phase modes than for autonomous coded modes. Less irregular measurements have been observed in the third mode. For a more or less accurate determination of the inclination of the axis of rotation, it is necessary to conduct a large series of observations, i.e. to find the average of the measured parameters.

Figure 3. Determining the vector of the axis of rotation: a – for an autonomous coded mode; b – for a differential mode; c – for a relative phase mode

Fig. 4 demonstrates errors in measuring the inclination angle of the antenna from a vertical axis at different rotation velocities.

Figure 4. Results of modeling the measurement error for the angle of inclination for an aerial axis of rotation
The standard inclination is calculated, using the following formula:

$$\sigma_{Sy} = \frac{\sigma_{in}}{|\Delta B|^2 \sin(\omega \cdot \Delta t)} = \frac{\sigma_{in}}{4|B|^2 \sin^2(\omega \cdot \Delta t / 2) \sin(\omega \cdot \Delta t)}.$$  \hspace{1cm} (5)

where $\sigma_{in}$ is the standard deviation for measuring planimetric coordinates, which, in this case, are equal to: 0.7 m for an autonomous mode; 4 cm for a differential mode and 5 mm for a relative phase mode.

Analyzing the results of the measurements, we can see that in order for the algorithm to be accurate enough, it is necessary for the antenna to rotate at a high speed, especially in the automated mode. High precision is achieved only in a relative phased (or angular) mode; in an autonomous mode, even using a long arm, the accuracy of the measurements does not exceed one degree. On the other hand, to determine the deviation of the axis of rotation and its special orientation, it is possible to use only three trajectory points.

By using filtering for measurable parameters, it is possible to significantly increase the accuracy for determining the inclination of the aerial axis of rotation. For such problems it is practical to use the filtering of measured $X$, $Z$ coordinates using the second order Kalman filter [5, 6]. During the development of the Kalman filter, the closest function has been selected: circumference $R^2 = X^2 + Z^2$.

**Figure 5.** Measuring and filtering coordinates $X$, $Z$: a – for an autonomous coded mode; b – for a differential mode; c – for a relative phase mode

Fig. 5 demonstrates that in result of filtering the determined coordinates $X$ and $Z$, the trajectory of the antenna path became more similar to a circle, without irregular measurements. Results in Fig 6 confirm a significant reduction in the error of determining the vector of the axis of rotation.
Figure 6. Determining the vector of the axis of rotation and its filtering: a – for an autonomous coded mode; b – for a differential mode; c – for a relative phase mode.

Figure 7. Results of simulating errors of determining the angle of the aerial axis of rotation inclination using a second order Kalman filter.

Fig. 7 demonstrates the results of simulating errors of determining the inclination of the aerial axis of rotation in various functioning modes of the user navigation equipment for radionavigation systems, filtering the measured coordinates with a second order Kalman filter. Thus, based on the simulation results, we can conclude that filtering measurable planimetric coordinates yields a significant result. For instance, even at spinning speeds of 1 revolution per minute there are no failures in an autonomous mode; if the relative phase mode
is employed, the desired accuracy may be obtained at a revolution of 3 rpm. In reality, the filtering level is selected, depending on the objects behavior.

4. Conclusion

The discussed method for determining the inclination of the aerial axis of rotation for mobile radiolocation devices, using user navigation equipment and filtering of measured parameters, enables us to perform immediate control over the antenna’s position in space. The method significantly improves the maneuverability of mobile radiolocation devices and the accuracy of determining the altitude of aerial objects. It is possible to use not only differential and relative phase modes for operating the user navigation equipment, but the autonomous coded mode, enabling us to use serial and standard user navigation equipment for determining the inclination of the angle of the axis of rotation.

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