Study on Calculation Method of Compressional Velocities Based on Field Well Logs

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Abstract

In the past, most of the studies for compressional velocities are based on experimental measurements, which lack the support of field data. The purpose of this study is to estimate the compressional velocities based on well log data of delta front subfacies of Lower Tertiary ages of Ji-Dong oil field, China. At initial stage, we have chosen the well log parameters (effect factors) which strongly influence on compressional velocities and established a new modified equation for compressional velocities, which is based on these effect factors. Then Gardner, De-hua Han and this newly established equation were utilized to calculate the compressional velocities in each well. Finally, Least-square regression was carried out to check the fitting of each equation. Regression results clearly indicate that our purposed equation shows better fitting as compared to Gardner and De-hua Han equations.

Keywords

Compressional Velocities, Well Logs, Calculation Method

1. Introduction

Accurate and precise calculation of acoustic velocities can effectively improve the accuracy of processing, inversion and interpretation of seismic data. Different methods for the calculation of acoustic velocities have been studied by a number of scientists.

Wyllie et al. [1] [2] proposed the time-average equation which established a relation between compressional velocities and porosities. At atmospheric pressure and at a temperature of about 25°C, compressional velocities
of water saturated clean sandstones having porosities range from 10% - 20% commonly show good matching with the calculated results of the time-average equation. Subsequently, De Martini et al. [3], Tosaya and Nur [4], and Kowallis et al. [5] found that in shaly sandstones and shales the time-average equation significantly overestimates velocities.

Based on the time-average equation, De-hua Han et al. [6] investigated the effect of clay content and porosity on the acoustic velocities in sandstones systematically. He measured velocities of 75 samples with its porosities range from 2% to 30% and clay contents range from 0% to 50%. The results indicate that the compressional velocities are linearly related to porosities and clay contents in water-saturated shaly sandstones, a small amount of clays (1% to 2%) can significantly soften the sandstone matrix and reduce velocities, other factors (pore geometry, grain size, grain contacts, cementation, type of clay, distribution of clays, and mineralogy), compare with clay contents, have much smaller influences on velocities. The purposed equation can be written as

$$V_p = 5.59 - 6.93\phi - 2.18V_{sh}$$  \hspace{1cm} (1)

where $V_p$ (km/s) is the P-wave velocity, $\phi$ (f) is the porosity, and $V_{sh}$ (f) is the clay content. Equation (1) commonly shows good fitting with measured data. Nevertheless Equation (1) can’t be applied reliably to shaly sandstones containing clay minerals [7] and as the clay content in sandstones increased, the P-wave velocity decreased systematically in both well and poorly consolidated sediments.

Gardner et al. [8] obtained an equation between compressional velocities and densities of rock samples through a wide range of basins, geologic ages, and depths. The equation is

$$\rho = 0.23V_p^{0.25}$$  \hspace{1cm} (2)

where $\rho$ (g/cm$^3$) is the density, $V_p$ (ft/s) is the P-wave velocity. Until now, Equation (2) was used in the interpretation and inversion of seismic data prevalently. But Equation (2) is based on statistics and obtained through analyzing the distribution rules of velocity-density cross plot without considering the effect of clay content, pressure, and so on. The accuracy of compressional velocities calculated by Equation (2) is doubtful.

Equations (1) and (2), which are based on experimental measurements and statistics, have its own limitations. First, there are differences between experimental and subsurface circumstances, such as confining pressure, pore pressure, temperature, and so on. Equations (1) and (2) are purely based on experimental data and do not show good matching with the field measured data. Second, core samples are obtained from different depths and only contain information about those particular interval, not about the whole interval.

Thus, because of the limitations of experimental measurements, this study is based on field well logs, which record continuous variations of the target formations and investigate systematically the calculation method of compressional velocities in delta front subfacies of Lower Tertiary ages of Ji-Dong Oil field.

First we will find the influence factors of compressional velocities and then combine them in equation format, then we use least-squares regression to fit compressional velocities of all wells in the target formations and compare correlation coefficient with Equations (1) and (2).

2. Background of Research Area

2.1. Lithology of the Target Formations

The target formations can be divided into four sub intervals (Es1, Ed3, Ed2, and Ed1) from bottom to top respectively. Es1 (thickness ranges from 172 m to 455 m) is mainly composed of grey shale, light grey packsand, and light grey siltstone. Ed3 (thickness ranges from 275 m to 487 m) is mainly composed of dark grey shale within grey packsand and siltstone. Ed2 (thickness ranges from 287 m to 540 m) is composed of grey shale within packsand and siltstone, whereas Ed1 (thickness ranges from 230 m to 520 m) is mainly composed of packsand, siltstone, and shale.

2.2. Well Logs and Locations

Well logs of 19 wells are used as samples. Well names, target formations, type of well logs, and true vertical depths are listed in Table 1. Wells G37x3, GS1, NP4-66, NP4-68, and NP43-4704 contain all target formations, whereas L166x1, NP2-52, NP2-53, NP2-58, NP4-21, NP4-32, NP4-65, and NP403x1 contain all target formations except Es1, and wells NP4-31, NP4-33, NP4-38, NP4-39, NP43-4988, and NP403x2 contain all target
Table 1. Well information of research area.

| No. | Well name | Target formation | Well logs | TVD (m) |
|-----|-----------|------------------|-----------|---------|
| 1   | G37x3     | Ed1, Ed2, Ed3, Es1 | V_p, DEN, GR | 2435.1 - 4254.4 |
| 2   | GS1       | Ed1, Ed2, Ed3, Es1 | V_p, DEN, GR | 2512.0 - 4570.0 |
| 3   | L166x1    | Ed1, Ed2, Ed3     | V_p, DEN, GR | 2493.8 - 3918.2 |
| 4   | NP2-52    | Ed1, Ed2, Ed3     | V_p, DEN, GR | 2499.1 - 3613.9 |
| 5   | NP2-53    | Ed1, Ed2, Ed3     | V_p, DEN, GR | 2548.0 - 3412.0 |
| 6   | NP2-58    | Ed1, Ed2, Ed3     | V_p, DEN, GR | 2532.0 - 3724.5 |
| 7   | NP4-21    | Ed1, Ed2, Ed3     | V_p, DEN, GR | 2407.0 - 3982.7 |
| 8   | NP4-31    | Ed1, Ed2          | V_p, DEN, GR | 2583.8 - 3461.8 |
| 9   | NP4-32    | Ed1, Ed2, Ed3     | V_p, DEN, GR | 2553.3 - 3748.2 |
| 10  | NP4-33    | Ed1, Ed2          | V_p, DEN, GR | 2605.3 - 3582.9 |
| 11  | NP4-38    | Ed1, Ed2          | V_p, DEN, GR | 2500.7 - 3619.8 |
| 12  | NP4-39    | Ed1, Ed2          | V_p, DEN, GR | 2664.6 - 3573.5 |
| 13  | NP4-65    | Ed1, Ed2, Ed3     | V_p, DEN, GR | 2930.0 - 3935.0 |
| 14  | NP4-66    | Ed1, Ed2, Ed3, Es1 | V_p, DEN, GR | 2528.6 - 4076.8 |
| 15  | NP4-68    | Ed1, Ed2, Ed3, Es1 | V_p, DEN, GR | 2619.3 - 4033.4 |
| 16  | NP43-4704 | Ed1, Ed2, Ed3, Es1 | V_p, DEN, GR | 2547.8 - 4125.5 |
| 17  | NP43-4988 | Ed1, Ed2          | V_p, DEN, GR | 2719.6 - 3480.3 |
| 18  | NP403x1   | Ed1, Ed2, Ed3     | V_p, DEN, GR | 2626.4 - 3487.5 |
| 19  | NP403x2   | Ed1, Ed2          | V_p, DEN, GR | 2801.3 - 3712.1 |

$V_p$: P-wave velocity, km/s; DEN: Density, g/cm³; GR: Natural Gamma, API; TVD: True Vertical Depth, m.

formations except Es1 and Ed3. Well logs of compressional velocity, density, and natural gamma are available in all wells. Locations of all these 19 wells are shown in Figure 1.

3. Clay Content, Porosity, and Formation Pressure

In this paper, we proposed a new equation to calculate compressional velocities, and compared its correlation coefficient with Equations (1) and (2). The parameters used in the new equation and Equations (1) and (2) have been calculated as follow.

3.1. Clay Content

Natural gamma ray log was used to calculate clay content [9] [10]. First, we found the maximum and minimum values of natural gamma in each well (Table 2). Then, we have chosen 17.8 API and 193.6 API as the lowest minimum and highest maximum values which respectively represent to pure sand and shale in the target formation. Equations used to calculate clay content are

$$I_{GR} = \frac{GR - GR_{\min}}{GR_{\max} - GR_{\min}}$$  \hspace{1cm} (3)

$$V_{sh} = \frac{2^{e_{\text{GR}}} - 1}{2^e - 1}$$  \hspace{1cm} (4)

where, \( GR \), \( GR_{\min} \) and \( GR_{\max} \) are natural gamma values of the target formation, pure sand, and shale re-
Figure 1. Locations of 19 wells in the research area. The background are subfacies distribution of Ed2 which contains delta plain, delta front, lake faces, and subaqueous channel. All wells locate in the delta front subface.

Table 2. Maximum and minimum GR values of each well.

| No. | Well name  | Maximum(API) | Minimum(API) |
|-----|------------|--------------|--------------|
| 1   | G37x3      | 160.4        | 25.7         |
| 2   | GS1        | 148.0        | 50.5         |
| 3   | L166x1     | 170.1        | 17.8         |
| 4   | NP2-52     | 159.8        | 36.0         |
| 5   | NP2-53     | 153.6        | 45.2         |
| 6   | NP2-58     | 154.2        | 48.3         |
| 7   | NP4-21     | 147.3        | 30.5         |
| 8   | NP4-31     | 179.6        | 49.7         |
| 9   | NP4-32     | 154.2        | 47.8         |
| 10  | NP4-33     | 157.4        | 47.4         |
| 11  | NP4-38     | 168.1        | 49.2         |
| 12  | NP4-39     | 159.7        | 41.0         |
| 13  | NP4-65     | 193.6        | 32.6         |
| 14  | NP4-66     | 145.2        | 41.8         |
| 15  | NP4-68     | 148.3        | 51.1         |
| 16  | NP43-4984  | 148.9        | 52.4         |
| 17  | NP43-4704  | 168.1        | 44.9         |
| 18  | NP403x1    | 151.8        | 41.7         |
| 19  | NP403x2    | 163.5        | 47.7         |
respectively, $c$ is a coefficient which is obtained from core data analysis, which is equal to 3.7 in the Tertiary period and 2 for old formations, we choose 2 in this research, and $V_{sh}$ represent to clay content.

### 3.2. Porosity

Density log was used to calculate the porosity. First, we built up a physical model for the target formation. We supposed that the target formation is comprised of sand, shale, and porosity which is filled with water (Figure 2).

Then the relation among sand volume ratio ($V_{sand}$), shale volume ratio ($V_{sh}$), and porosity ($\phi$) can be written as

$$V_{sand} + V_{sh} + \phi = 1.$$  \hfill (5)

Second, setting up a relationship between porosity and density. We supposed the measured density ($\rho$) equals to a sum of multiplication of the density of sand ($\rho_{sand}$), shale ($\rho_{sh}$), and water ($\rho_{f}$) with their own volume ratio respectively. The relation is

$$\rho = \rho_{sand} V_{sand} + \rho_{sh} V_{sh} + \rho_{f} \phi.$$ \hfill (6)

Finally, we combined Equations (5) and (6) in order to obtain the porosity expression as follow

$$\phi = \frac{\rho - \rho_{sand}}{\rho_{f} - \rho_{sand}} V_{sand} - \frac{\rho_{sh} - \rho_{sand}}{\rho_{f} - \rho_{sand}} V_{sh}.$$ \hfill (7)

### 3.3. Formation Pressure

The formation pressure is equal to hydrostatic gradient multiplied by the true vertical depth. This simple relationship can be written as

$$P_0 = G_0 \times TVD$$ \hfill (8)

where $P_0$ (Mpa) is the formation pressure, $G_0$ (Mpa/m) is the pressure gradient which is equal to 0.01 Mpa/m in this research, and $TVD$ (m) represent to true vertical depth.

### 4. Calculation Formula of Compressional Velocities

Taking well L166x1 as an example, four cross-plots between compressional velocities versus density, porosity, clay content, and formation pressure have been generated as shown in Figure 3.

Figure 3 shows that the compressional velocities increases with the increase in density and formation pressure, whereas it decreases with the increase in porosity and clay content up to 0.35 and then decreases with the increase in clay content. Due to significant scattering of data points in the cross-plots, linear function may does not fit precisely, so we also have used exponential function to fit the compressional velocities of well L166x1.

In this paper, we hold the opinion that the key of establishing an equation of the compressional velocities depends upon choosing the right influence factors and combining them in equation format, rather than obtaining an equation with constant coefficients from experimental measurements of some specific core samples.

At first, exponential functions were used to fit the compressional velocities, and we analyzed seven equation formats: (1) choosing density as the base; (2) Gardner Equation (Equation (2)); (3) choosing porosity as the base;
Figure 3. Cross-plots of the compressional velocities versus the density, porosity, clay content, and formation pressure of well L166x1.

(4) choosing density and clay content as the bases; (5) choosing porosity and clay content as the bases; (6) choosing density, clay content, and formation pressure as the bases; (7) choosing porosity, clay content, and formation pressure as the bases. The exponential functions, coefficients, and correlation coefficients (Appendix) are listed in Table 3. The results suggest that (a) the correlation coefficient of the equation which chooses the density as the base is slightly higher than Equation (2); b) the correlation coefficient of the equation which chooses the density, clay content, and formation pressure as the bases is the highest in the above discussed seven equation formats, the equation format is

\[ V_p = a \rho^b V_{sh}^c P_0^d \]  

Then, linear functions were used to fit the compressional velocities, and we also analyzed seven equation formats: 1) choosing porosity as variable; 2) choosing density as variable; 3) choosing porosity and clay content as variables; 4) De-hua Han Equation (Equation (1)); 5) choosing density and clay content as variables; 6) choosing porosity, clay content, and formation pressure as variables; 7) choosing density, clay content, and formation pressure as variables. The linear functions, coefficients, and correlation coefficients are listed in Table 4. The results suggest that (a) the correlation coefficient of the equation which chooses porosity and clay content as variables is slightly higher than Equation (1); (b) the correlation coefficient of the equation which chooses density, clay content, and formation pressure as variables is the highest in the seven equation formats, the equation format is

\[ V_p = a + b \rho + c V_{sh} + d P_0 \]
Table 3. Fitting results of exponential functions.

| Exponential functions                  | Coefficients | R   |
|----------------------------------------|--------------|-----|
| $V_r = a \rho^b$                       | 1.47         | 1.02| 0.4659|
| $\rho = 0.23 V_r^{0.25}$               |              |     | 0.4631|
| $V_r = a \phi^b$                       | 3.48         | -0.02| 0.2933|
| $V_r = a \rho^b V_{\phi}^{c}$          | 0.69         | 1.67| -0.13| 0.6142|
| $V_r = a \phi^b V_{\rho}^{d}$          | 3.02         | -0.04| -0.09| 0.3925|
| $V_r = a \rho^b V_{\phi}^{c} P_i^{d}$  | 0.32         | 1.09| -0.11| 0.39| 0.8250|
| $V_r = a \phi^b V_{\rho}^{d} P_i^{e}$  | 0.65         | -0.03| -0.09| 0.45| 0.7894|

R: Correlation coefficient.

Table 4. Fitting results of linear functions.

| Linear functions                  | Coefficients | R   |
|-----------------------------------|--------------|-----|
| $V_r = a + b \phi$                | 3.92         | -2.19| 0.4294|
| $V_r = a + b \rho$                | -0.005       | 1.51| 0.4621|
| $V_r = a + b \phi + c V_{\phi}^{d}$ | 4.69       | -4.13| -1.89| 0.6340|
| $V_r = 5.59 - 6.93 \phi - 2.18 V_{\rho}$ |           |     |     | 0.6059|
| $V_r = a + b \rho + c V_{\rho}^{c}$ | -1.94       | 2.50| -1.67| 0.6340|
| $V_r = a + b \phi + c V_{\phi} + d P_i^{a}$ | 3.01       | -2.68| -1.45| 0.04| 0.8391|
| $V_r = a + b \rho + c V_{\rho} + d P_i^{a}$ | -1.29       | 1.63| -1.30| 0.04| 0.8391|

R: Correlation coefficient.

We conclude from Table 3 and Table 4 that a) the correlation coefficients of the equations with constant coefficients are less than the equations with non-constant coefficients; b) the correlation coefficients of the equations which choose density, clay content, and formation pressure as the influence factors are higher than the equations with the same former and choose only one, two or three other influence factors; c) the correlation coefficients of the linear functions are higher than the exponential functions with the same influence factors. Finally, Equation (10) was chosen to fit the compressional velocities of the research area.

5. Application Results

Equations (1), (2) and (10) were applied to fit the compressional velocities of all wells. The results are listed in Table 5, and the correlation coefficient of Equation (10) is the highest in the three equations.

The three equations of Table 5 were used to calculate compressional velocities of each well respectively and the correlation coefficients of each well are listed in Table 6 and visually displayed in Figure 4. The first column is well name, the second, third, and forth columns are correlation coefficients calculated by using Equations (1), (2) and (10) respectively. From Table 6 we conclude that a) the correlation coefficients of the second column are higher than the third column, except wells NP2-58, NP4-33, NP4-65, and NP403x2; b) the correlation coefficients of the second column are higher than the forth column, except well NP4-39. So we can conclude that the regression results of Equation (10) to fit the compressional velocities of each well are better than Equations (1) and (2).
**Table 5.** Fitting results of the compressional velocities of all wells.

| Functions | Coefficients | R   |
|-----------|--------------|-----|
| $V_p = a + b \rho + c V_a + dP_o$ | a | b | c | d | 0.6301 |
| $V_p = a \rho^b$ | 1.83 | 0.77 | 0.3300 |
| $V_p = a + b \phi + c V_a$ | 4.42 | -2.15 | -1.57 | 0.4456 |

R: Correlation coefficient.

**Table 6.** Correlation coefficients of each well.

| No. | Well name | $V_p = 0.7426 \rho + 0.5170 V_a + 0.04 P_o$ | $V_p = 4.42 - 2.15 \phi - 1.57 V_a$ | $V_p = 1.83 \rho^{0.77}$ |
|-----|-----------|--------------------------------------|--------------------------------------|---------------------|
| 1   | G37x3     | 0.7426                               | 0.5170                               | 0.3287              |
| 2   | GS1       | 0.7992                               | 0.4304                               | 0.2516              |
| 3   | L166x1    | 0.8328                               | 0.5636                               | 0.4628              |
| 4   | NP2-52    | 0.5486                               | 0.4103                               | 0.3868              |
| 5   | NP2-53    | 0.7090                               | 0.6606                               | 0.5868              |
| 6   | NP2-58    | 0.5869                               | 0.6514                               | 0.4607              |
| 7   | NP4-21    | 0.7814                               | 0.3595                               | 0.4951              |
| 8   | NP4-31    | 0.4727                               | 0.4358                               | 0.4585              |
| 9   | NP4-32    | 0.5188                               | 0.3847                               | 0.1171              |
| 10  | NP4-33    | 0.4778                               | 0.5332                               | 0.4750              |
| 11  | NP4-38    | 0.5534                               | 0.4462                               | 0.4108              |
| 12  | NP4-39    | 0.2368                               | 0.2234                               | 0.2372              |
| 13  | NP4-65    | 0.5429                               | 0.5904                               | 0.3383              |
| 14  | NP4-66    | 0.7498                               | 0.6967                               | 0.5244              |
| 15  | NP4-68    | 0.5573                               | 0.5101                               | 0.4322              |
| 16  | NP4-704   | 0.3445                               | 0.2427                               | -0.1645             |
| 17  | NP4-4988  | 0.4944                               | 0.4882                               | 0.4657              |
| 18  | NP403x1   | 0.6491                               | 0.5905                               | 0.5233              |
| 19  | NP403x2   | 0.7838                               | 0.7956                               | 0.6179              |
6. Conclusions

In the research, we hold the opinion that the key of establishing an equation of the compressional velocities depends upon choosing the right influence factors and combining them in equation format.

The correlation coefficients of the equations for which we have chosen density, clay content, and formation pressure as influence factors are higher than those which depend upon only one, two or three influence factors.

We also found that the fitting results of the compressional velocities by using linear equations are better than the exponential equations for the same influence factors.

Moreover, the fitting results of the compressional velocities through Equation (10) are better than Equations (1) and (2) which are proposed by De-hua Han and Gardner respectively in delta front subfacies of Lower Tertiary ages of Ji-Dong oil field.

Shear-wave velocity is the same important as compressional velocity in processing, inversion and interpretation of seismic data. But it’s expensive to obtain shear-wave velocity in the field and shear-wave data of single well is less than compressional-wave data. In the next step, we are going to transfer our attention to calculate shear-wave velocity.

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References

[1] Wyllie, M.R.J., Gregory, A.R. and Gardner, L.W. (1956) Elastic Wave Velocities in Heterogeneous and Porous Media. Geophysics, 21, 41-70. http://dx.doi.org/10.1190/1.1438217

[2] Wyllie, M.R.J., Gregory, A.R. and Gardner, L.W. (1958) An Experimental Investigation of Factors Affecting Elastic Wave Velocities in Porous Media. Geophysics, 23, 459-493. http://dx.doi.org/10.1190/1.1438493

[3] De Martini, D.C., Beard, D.C., Danburg, J.S. and Robinson, J.H. (1976) Variation of Seismic Velocities in Sandstones and Limestones with Lithology and Pore Fluid at Simulated in Situ Conditions. Proc. EGPC Exploration Seminar.

[4] Tosaya, C. and Nur, A. (1982) Effects of Diagenesis and Clays on Compressional Velocities in Rocks. Geophysical Research Letters, 9, 5-8. http://dx.doi.org/10.1029/GL009i001p00005

[5] Kowallis, B., Jones, L.E.A. and Wang, H.F. (1984) Velocity-Porosity-Clay Content; Systematics of Poorly Consolidated Sandstones. Journal of Geophysical Research, 89, 10355-10364. http://dx.doi.org/10.1029/JB089iB12p10355

[6] Han, D.-H., Nur, A. and Morgan, D. (1986) Effects of Porosity and Clay Content on Wave Velocities in Sandstones. Geophysics, 51, 2093-2107. http://dx.doi.org/10.1190/1.1442062

[7] Klimontos, T. (1991) The Effects of Porosity-Permeability-Clay Content on the Velocity of Compressional Waves. Geophysics, 56, 1930-1939. http://dx.doi.org/10.1190/1.1443004

[8] Gardner, G.H.F., Gardner, L.W. and Gregory, A.R. (1974) Formation Velocity and Density—The Diagnostic Basics for Stratigraphic Traps. Geophysics, 39, 770-780. http://dx.doi.org/10.1190/1.1440465

[9] Huang, K., Yang, X.H. and Xu, Q.Z. (1998) The Relation among Porosity, Shaliness and P- and S-Wave Velocity of Seismic Wave. Xinjiang Petroleum Geology, 19, 462-464. (In Chinese with English Abstract)

[10] Hunt, E. and Pursell, D. (1997) Fundamentals of Log Analysis; Part 7, Determining Shaliness from Logs. World Oil, 218, 55-58.
Appendix

\[ R = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \overline{x})^2 \cdot \sum_{i=1}^{n} (y_i - \overline{y})^2}} \]  

(11)

where \( x_i \) is a value which was measured by the instruments; \( y_i \) is a value which was calculated by the equation; \( \overline{x} \) is a mean value of \( x_i \); \( \overline{y} \) is a mean value of \( y_i \). The higher of \( R \) the better correlation between measured and calculated values.