Abstract

We extend the Lebesgue theorem (on covers of cubes) and the Knaster-Kuratowski-Mazurkiewicz theorem (on covers of simplices) to different classes of convex polytopes (colored in the sense of M. Joswig). Those classical theorems play an important role in other mathematical disciplines because they are deeply connected with Sperner's lemma and Brouwer's fixed point theorem. We also show that the $n$-dimensional Hex theorem admits a generalization where the $n$-dimensional cube is replaced by a $n$-colorable simple polytope. The use of quasitoric manifolds offers great flexibility and versatility in applying the general method.