Acoustic characteristics of interacting air-guns in marine seismic exploration

V I Gulenko¹, E I Zakharchenko¹, A V Rudakov², Yu I Zakharchenko¹

¹ c, Krasnodar, Russia
² JSC Yuzhmogeologiya, Gelendzhik, Russia

E-mail: v_gul@mail.ru

Abstract. There is still no exact theoretical description of the conditions for the coalescence of pulsating air bubbles in a liquid, based on complete (non-linearized) equations of hydrodynamics. A satisfactory mathematical model describing the mutual interaction of air-guns forming an array should be tailored to the dynamics of pulse chambers. The paper considers the interaction of air bubbles pulsating in a liquid resulting from an exhaust of compressed air during the operation of air-gun array used in a marine environment, as well as its effect on the acoustic characteristics of radiated pulses. The results of model calculations compared with experimental signals show that the resultant model describes quite adequately a sophisticated mutual effect of air-guns and can be used for practical purposes to calculate the configurations of air-gun arrays and the total pulses they cause.

1. Introduction
So far the conditions for the coalescence of pulsating gas bubbles in a liquid have not been described based on complete (non-linearized) equations of hydrodynamics. However, practical needs have called for a number of publications concerned with this phenomenon.

In some early publications on the issue of air-guns forming an array, mutual acoustic interaction was either completely ignored, or some fragmentary attempts were made to avoid mutual interaction by grouping air-guns on a large base. The researchers were mainly aimed at determining a minimum inter-gun spacing, at which the mutual interaction can be neglected [1, 2, 3].

Safar [2], considering the interaction of identical, weakly pulsating bubbles near the equilibrium position, suggested that the mutual interaction of air-guns can be neglected when the spacing is 10 times greater than the equilibrium radius of a bubble formed by released air.

Nootbooom [1] derived an empirical formula for calculating the critical spacing depending on the air pressure, and the volume of the firing chamber of a larger air-gun. As shown in [10], this spacing is 8.2 times the bubble radius in the equilibrium state.

The papers [3, 4] analyze the formulas proposed by Safar, Nooteboom, Giles and Johnston [5], which have a critical spacing of about 3 bubble radii in the equilibrium state and present the experimental results illustrating the mutual coupling of two air-guns depending on the inter-gun spacing.

Let us consider the main experimental facts. Figure 1 (a) shows the dependence of the 1\textsuperscript{st} peak amplitude of the total pressure output $P_{1\Sigma}$ on the combined volume of array $V_{\Sigma}$ formed by non-interacting 600V PAR air-guns with a volume of 0.65 dm\textsuperscript{3}.

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Figure 1 (b) shows a change in the firing cycle $\Delta T$ produced by a 600V PAR air-gun with a volume of 0.65 dm$^3$ mutually interacting with the same air-gun, depending on the inter-gun spacing $d$ [5]. As it follows from the experimental data presented in the paper, at small distances ($d < 0.7$ m), the mutual interaction is maximal. The bubbles coalesce, while the sum of amplitudes and firing cycles $P_{1\Sigma}$ and $T_{\Sigma}$ are close to the amplitude and firing cycle generated by a single air-gun of the total volume $V_{\Sigma}$:

$$P_{1\Sigma} \sim C \cdot (V_{\Sigma})^{1/3}, \quad T_{\Sigma} \sim T_1 \cdot (V_{\Sigma}/V_1)^{1/3}.$$

At distances between identical air-guns greater than 3.05 m ($d > 10$ feet), there is almost no mutual interaction, while the amplitude of the combined pulse produced by all air-guns is directly proportional to its total volume $V_{\Sigma}$, and the firing cycle $T_{\Sigma}$ practically coincides with that created by a single air-gun:

$$P_{1\Sigma} \sim C \cdot (V_{\Sigma}), \quad T_{\Sigma} \approx T_1.$$

The mutual interaction also has a significant effect on the amplitude of the second pressure peak (as well as subsequent peaks).

Figure 2 (a) shows the signature of combined pulses in a far-field range generated by two identical 1500C PAR air-guns with a volume of 4.75 dm$^3$ at different inter-gun distances $d$ [3]. Column (1)
shows the “far” pulses, column (2) – calculated without including mutual interaction, column (3) – the difference between pulses (1) and (2).

Figure 2 (b) shows, respectively, the relative changes in the first peak amplitude of the total pulse $P_{1\Sigma}$, the firing cycle $T_\Sigma$ and the amplitude ratio $Q = P_{1\Sigma}/P_{2\Sigma}$, depending on the distance $d$ therebetween [3].

Figure 2. Combined “far” pulse signature generated by two identical 1500C PAR air-guns with a volume of 4.75 dm$^3$ at different inter-gun distances $d$ – (a), relative changes in the first peak amplitude of the total pressure output $P_{1\Sigma}$, firing cycle $T_\Sigma$ and amplitude ratio $Q = P_{1\Sigma}/P_{2\Sigma}$ depending on distance $d$ between clustered guns[3]

The data indicate that $Q$ is the most vulnerable value to a change in the distance $d$: placing guns closer to one another maximizes suppression of successive oscillations and the ratio of amplitudes $Q$ increases (this effect, in particular, promotes the wide use of so-called “coupled air-guns”, or clusters of air-guns with the same total volume, as part of modern linear and areal heterogeneous groups of air-guns instead of large-volume guns.

Based on the results presented in [1–9], a satisfactory mathematical model to describe the mutual interaction of air-guns deployed in a configuration should respect the dynamics of pulse chambers. Consequently, from the experimental pressure output recorded from a single air-gun, it is impossible to accurately calculate its change from interaction with a neighboring air-gun with a known pressure output. The mutual interaction can be calculated theoretically from the equations of air chambers moving in a liquid. Therefore, it is clear that the more accurately these equations approximate the actual motion of the bubbles, and the more accurately we can simulate the pressure output from a single air-gun, the more accurately the mutual interaction will be calculated.

2. Simulation of single acoustic pulse signature

Figure 3 (a) shows an experimentally recorded acoustic “near” pulse generated by a Signal-5 air-gun with a volume of 1.0 dm$^3$ at an operating pressure of 15 MPa.

Figure 3 (b) shows the calculated signal obtained using the mathematical air-gun model described in [10]. It was assumed that after the air-gun was closed, the bubble oscillations in the form of a spherical gas layer occur with a polytropic change in the constant gas mass in the bubble.

The assumption of adiabatic gas changes in the bubble gives a pulse with a cycle shorter than the experimental one. With an isothermal gas change in the bubble, the calculated cycle turns out to be longer than the experimental one. Choosing a polytropic index equal to $n=1.15$ gives a calculated pulse, with a cycle being in good agreement with that of the experimental pulse. This value of the polytropic index agrees well with the value selected for other air-guns [9, 11] and equal to $n=1.13$. [9] was concerned with an air-gun model as a suddenly expanding sphere, where all compressed air was assumed to instantly flow from the air-gun into a bubble of the same volume as the air-gun chamber. The said case included the timing of compressed air overflow into the bubble from the air-gun chamber.
Figure 3. Acoustic “near” pulses generated by Signal-5 air-gun at $V_{01} = 1$ dm$^3$, $P_{01} = 15$ MPa and $h = 10$ m ($P_0 = 0.2$ MPa): (a) – experimental signal; (b) – theoretical at $n = 1.15$; $\alpha = 0$; (c) – theoretical at $n = 1.15$; $\alpha = 3 \cdot c^{-1}$

Close values of the polytropic indices indicate that the firing cycle in the air chamber depends mainly on the amount of incoming air. The timing of air flow into the bubble has little effect on the value of the polytropic index due to the fact that this time is a small part of the chamber firing cycle.

The acoustic pulses compared in Fig. 3 (a) and (b) show that the theoretical pulse differs from the experimental one by much less attenuation. The reason is energy losses for turbulization of spreading fluid flows and, possibly, non-spherical and unstable bubble shapes. The effect of water viscosity and surface tension on bubble oscillations, as it was established in [10], is negligible. Therefore, viscous suppression introduced in [8] should not be addressed. It is obvious, though, that the higher the velocity of the bubble wall, the greater the suppression caused by the turbulization of the spreading water flows, and over time these losses enhance. Therefore, a decrease in pressure in the liquid compared to the pressure inside the bubble, due to the turbulence of the liquid, basically can be described by the same equation:

$$P_w = P_b - (a t + b)R/R.$$
where \( P_b \) and \( P_w \) are pressures inside and outside the bubbles; 
\( a, b \) are attenuation coefficients; 
\( t \) is time.

Energy losses can also be included by multiplying the bubble radius function \( R(t) \) obtained from the bubble equation by a decreasing function, as is done, for example, in [9]:

\[
\bar{R}(t) = (R(t) - R_r) \cdot e^{-at} + R_r, \tag{1}
\]

where \( R(t) \) is the bubble radius derived from Keller and Kolodner’s equation [8]; 
\( \bar{R}(t) \) is the decaying bubble radius; 
\( R_r \) is the equilibrium bubble radius; 
\( \alpha \) is the attenuation coefficient; 
\( t \) is time.

Equation (1) satisfies the boundary conditions:

\[
\bar{R}(0) = R(0), \quad \bar{R}(\infty) = R_r.
\]

For the decaying velocity of the bubble wall and the changed gas pressure therein, we obtain:

\[
\bar{R}' = e^{-at} \left[ R' + \alpha(R - R_r) \right], \tag{2}
\]

\[
\bar{P} = P \left( \frac{R^3 - B^3}{R^3 - B^3} \right)^n,
\]

where \( B \) is the radius of the central rigid sphere (in [9], a gas sphere without a central body was considered).

The pressure pulse is calculated by the equation, which in new, decaying functions look like:

\[
P_t(r, t) = \rho_0 \frac{R}{r} \left( \frac{P - P_0}{\rho_0} + \frac{R^2}{2} \right) \left[ 1 - \frac{R' r^2}{c} \left( \frac{1 - R}{r} \right) \right] = -\rho_0 \frac{R^2}{2} \frac{R^4}{r^4}, \tag{3}
\]

where \( P_0 \) is the hydrostatic pressure.

The pressure pulse calculated for the Signal-5 source gun using a mathematical model and equations (1)-(3) is shown in Fig. 3 (c). The attenuation coefficient \( \alpha \) selected by enumerating its values, was taken equal to \( 3 \cdot c^{-1} \). As can be seen from Fig. 3 (c), the pulse correspondence is quite satisfactory. The shape of pressure pulses of the IGP-1 air-guns and other guns of the Signal type with different volumes of firing chambers is also well approximated by the value \( \alpha = 3 \cdot c^{-1} \) and the value of the polytropic index \( n = 1.15 \). The value of the attenuation coefficient \( \alpha = 1.1 \cdot c^{-1} \), obtained in [9], is attributed to the difference in the mathematical air-gun models.

3. System of differential equations describing the mutual interaction of air-guns arranged in groups

The mathematical model to describe interacting air-guns is based on the following assumptions:

1) the size of bubbles is small compared to the prevailing length of excited acoustic waves, so the bubbles can be considered as points, and the pressure is the same on all sides of the bubble at each moment of time;
2) during oscillations, the bubbles do not coalesce – this assumption introduces the minimum inter-gun spacing in the arrangement.

Mutual interaction is considered as the effect on the dynamics of each bubble in an alternating pressure field arising due to acoustic radiation produced by other air-guns in a cluster. As applied to the underwater compressed air exhaust, the dynamics of a single isolated chamber is described by a system of differential equations [10], which is a mathematical air-gun model. When air guns are deployed in close proximity to each other (at a distance of 1-3 m), acoustic pulses from surrounding
air-guns arrives at a time when each of them still remains open. The pulses travelling does not affect
the character of air outflow from the firing chamber and the dynamics of the moving element of the
air-gun, since during the entire process the chamber pressure significantly exceeds the bubble pressure
and, therefore, the mode of air run from the firing chamber is supercritical. As a consequence, bubble
expansion can be described by two differential equations, one of which being Keller and Kolodner’s
equation for an ideal bubble expansion with exponential decay (1), and the other describing the
pressure change therein with due allowance for the air flow from the firing chamber, and being
expressed through the flow function derived from the system of differential equations solved in [10].
The validity of the approach is confirmed by the close relationship of the calculated and experimentally recorded dependences of the air pressure drop in the firing chamber in the process of underwater exhaust.

Let us denote by $M_0$, $m_0$ and $m$, respectively, the initial gas masses in the firing chamber and in the
bubble and the gas mass flowing into the bubble. The current gas mass in the
bubble is equal to $M_0 - m$. With the Clapeyron-Mendeleev equation, we rewrite the equation of the bubble gas [10] like:

$$\frac{dP}{dt} = \gamma \frac{n-1}{\gamma-1} \frac{P_1V_1}{V(M_0-m)} G_1 + \frac{n}{\gamma-1} \frac{P}{m+m_0} G_1 - n \frac{P}{V} \frac{dV}{dt}. \quad (4)$$

The chamber gas pressure, in accordance with [10], is:

$$P_i = P_{1i} \left(1 - \frac{m}{M_0}\right)^\gamma,$$

where $P_{1i}$ is the chamber air pressure at the moment of opening the exhaust ports.

Since the gas flow rate into the bubble does not depend on the ambient liquid pressure, it can be
introduced in the form of a function obtained from the system of differential equations solved in [10].
Let us denote this function by $S_1(t)$. Then the mass of the gas flowing into the bubble will be equal to:

$$S_2(t) = m = \int_0^t S_1(t) \, dt.$$

The function $S_2(t)$, instead of the pneumatic air-gun model described by the system of differential
equations [10], considers the air-gun model in the form of a suddenly extending sphere with a given
variability in the gas mass inside.

When N air-guns in a group are actuated, to calculate the sum of signals based on the mutual
interaction, it is necessary to jointly solve a system of 2N differential equations:

$$\left(\frac{dR_i}{dt} - c_0\right) \left[R_i \frac{d^2R_i}{dt^2} + \frac{3}{2} \left(\frac{dR_i}{dt}\right)^2 - H_{0i}\right] - \left(\frac{dR_i}{dt}\right)^3 + 2 \frac{dR_i}{dt} H_{0i} + R_i \frac{dH_{0i}}{dt} = 0, \quad (5)$$

$$\frac{dP_i}{dt} = \gamma \frac{n-1}{\gamma-1} \frac{P_iV_i}{V_i(M_0-S_{2i})} \left(1 - \frac{S_{2i}}{M_0}\right)^\gamma S_{1i} + \frac{n}{\gamma-1} \frac{P_i}{V_i} S_{1i} - n \frac{P_i}{V_i} \frac{dV_i}{dt}, \quad i = 1, \ldots, N,$$

where $H_{0i} = \frac{P_i - P_{0i}}{p_0}$, $P_{0i}$ is ambient pressure of neighbouring $i$-th bubble.

The decaying radius of each bubble is calculated from equation (1), while the pressure in the
compression wave – from equation (3). The ambient pressure of each neighbouring bubble is
calculated as a superposition of pressure waves from each bubble and their reflections from the water-air surface, taking into account transit time delays and attenuation due to spherical divergence. Taking
the reflection coefficient from the “water-air” surface to be equal to −1, to calculate the pressure in the vicinity of the $i$-th bubble included in the expression for $H_{0i}$, we will have:

$$P_{0i}(t) = P_{ei} + \sum_{j=1}^{N} \sum_{k=1}^{N} P_{t_{ij}} \left(r_{1ij}, t - \frac{r_{ij}}{c_0}\right) - \sum_{j=i}^{N} P_{t_{ij}} \left(r_{2ij}, t - \frac{r_{2ij}}{c_0}\right), \quad (6)$$
where $P_{\infty i}$ is the hydrostatic pressure at the immersion depth of the $i$-th air-run; $r_{1ij}$, $r_{2ij}$, respectively, are the spacing between the $i$-th and $j$-th pulse sources and the spacing between the $i$-th and “imaginary” $j$-th sources.

Given that the air-guns are not actuated synchronously, but with some inter-gun time delay, the corresponding delay is considered when calculating the ambient pressure $P_{0i}(t)$.

The system of differential equations (5), together with auxiliary equations (1)-(3) and (6), is a mathematical model of a group of interacting air-guns.

The system of equations (5) was solved numerically on a computer by forth-order Runge-Kutta-Gill method. To find the differential of $H_{0i}$, the functions $P_{ij}(r,t)$ were numerically differentiated on time.

The system of equations (5) jointly solved gives the motion of bubbles, changed by the mutual interaction between the air-guns. Therefore, the total signal from a group of $N$ air-guns is calculated as a superposition of pulses from each air-gun.

4. Verifying the model and conducting a computational experiment

Figure 4 shows the calculated pulses for the cases $N = 1$ and $N = 2$, as well as the experimental pulses obtained from an IGP-1 air-gun with a volume of 3 dm$^3$ at an operating pressure of 12 MPa and immersion depths $h$ from 5 to 20 m. The air-guns were arranged on a square frame 2x2m. A generally accepted technique based on acoustic recording hardware (described in [6]) was used to record “near” ($r = 1,5 \div 2$ m) and “far” ($r = 70 \div 100$ m) pulses.

Figure 4. Calculated acoustic pulses vs experimental pulses for $N = 1$ (a) and $N = 2$ (b). IGP-1 air-gun, $V_{01} = 3$ dm$^3$; $P_{01} = 12$ MPa
As can be seen from Fig. 4, the correlation of the calculated and experimental pulses for two air-guns is of the same order as the correlation of the calculated and experimental pulses for a single air-gun and is considered as quite good. This confirms the legitimacy of the chosen method for calculating the mutual interaction of air-guns in a group and the possibility of its use for calculating the acoustic characteristics of groups of interacting air-guns for any arrangement that excludes the coalescence of bubbles.

To illustrate the effect of mutual interaction, Fig. 5 (a) shows a comparison of acoustic pulses generated by a pulse air-gun and by a pair of synchronously actuated identical air-guns placed at a distance of 1 m from each other.

Figure 5. Mutual acoustic interaction of two Signal-5 air-guns ($V_0 = 0.5$ dm$^3$; $P_0 = 12$ MPa, $h = 10$ m): (a) – calculated “near” pulses, (b) – relative changes in the first peak amplitude of the combined signal $P_{1Σ}$, the firing cycle $T_Σ$ and the ratio of the amplitudes $Q = P_{1Σ}/P_{2Σ}$ depending on the distance $r/R_r$ therebetween.
The calculated pulses were obtained for a model corresponding to Signal-5 air-gun with a volume of 0.5 dm³ at an operating pressure \( P_{01} = 12 \) MPa and an immersion depth \( h = 10 \) m. As can be seen from Fig. 5 (a), the mutual interaction increases the cycle and decreases the amplitude of signal oscillations. Figure 5 (b) illustrates the mutual acoustic interaction of two Signal air-guns depending on the inter-gun distance. It shows changing acoustic characteristics of the interacting air-gun, referred to the characteristics of a single air-gun (infinitely large distance between the air-guns) depending on a relative distance between the guns. The same graph has the plotted critical distances between the air-guns when mutual interaction can be neglected calculated by the formulas of Johnston [5], Nooteboom [1] and Safar [2]. As follows from Fig. 5 (b), the mutual interaction remains significant even if the inter-gun distances exceed the critical values. The same fact was experimentally proved in [1].

5. Conclusion
The paper describes single air-gun acoustic pulse signature simulated based on a spherical chamber model, in which the oscillations are described by Keller and Kolodner’s differential equation, taking into account the air flow from the firing chamber into the pulse chamber and the polytropic change therein. The system of differential equations describing the mutual interaction of pulse sources in a group assumes that the size of the bubbles is small compared to the prevailing length of the excited acoustic waves, and the pulsating bubbles do not coalesce. In this case, the mutual interaction of air-guns in the group is considered as the effect on the dynamics of each bubble in the field of variable ambient pressure arising from the acoustic radiation of other sources in the group. The results of model calculations and experimental signals show that the developed model quite adequately describes the sophisticated mechanism of mutual interaction of air-guns and can be used for practical purposes to calculate the configurations of air-gun arrays and the sum of pulses they emanate.

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