On the Gaussian Multiple Access Wiretap Channel and the Gaussian Wiretap Channel with a Helper: Achievable Schemes and Upper Bounds

Rick Fritschek and Gerhard Wunder
Heisenberg Communications and Information Theory Group
Freie Universität Berlin,
Takustr. 9, D–14195 Berlin, Germany
Email: rick.fritschek@fu-berlin.de, g.wunder@fu-berlin.de

Abstract—We study deterministic approximations of the Gaussian two-user multiple access wiretap channel (G-MAC-WT) and the Gaussian wiretap channel with a helper (G-WT-H). These approximations enable results beyond the recently shown 2/3 and 1/2 secure degrees of freedom (s.d.o.f.) for the G-MAC-WT and the G-WT-H, respectively. While the s.d.o.f. were obtained by real interference alignment, our approach uses signal-scale alignment. We show achievable schemes which are independent of the rationality of the channel gains. Moreover, our results can differentiate between channel strengths, in particular between both users, and establishes secrecy rates dependent on this difference. We can show that the resulting achievable secrecy rates tend to the s.d.o.f. for vanishing channel gain differences. Moreover, we extend previous and develop new techniques to prove general s.d.o.f. bounds for varying channel strengths and show that our achievable schemes reach the bounds for certain channel gain parameters. We believe that our analysis is the next step towards a constant-gap analysis of the G-MAC-WT and the G-WT-H.

I. INTRODUCTION

The wiretap channel was first proposed by Wyner in [3], and solved in its degraded version. This result was later extended to the general wiretap channel by Csiszar and Körner in [4]. Moreover, the Gaussian equivalent was studied by Leung-Yan-Cheon and Hellman in [5]. The wiretap channel and its modified version served as an archetypical channel for physical-layer security investigations. However, in recent years, the network nature of communication, i.e. support of multiple users, became increasingly important. A straightforward extension of the wiretap channel to multiple users was done in [6], where the Gaussian multiple access wiretap channel (G-MAC-WT) was introduced. A general solution for the secure capacity of this multi-user wiretap set-up was out of reach and investigations focused on the secure degrees of freedom (s.d.of.) of these networks. Degrees of freedom are used to gain insights into the scaling behaviour of multi-user channels. They measure the capacity of the network, normalized by the single-link capacity, as power goes to infinity. This also means that the d.o.f. provide an asymptotic view on the problem at hand. This simplifies the analysis and enables asymptotic solutions of channel models where no finite power capacity results could be found. An example for a technique which yields d.o.f. results is real interference alignment. It uses integer lattice transmit constellations which are scaled such that alignment can be achieved. The intended messages are recovered by minimum-distance decoding and the error probability is bounded by usage of the Khintchine-Groshev theorem of Diophantine approximation theory. The disadvantage of the method is that these results only hold for almost all channel gains. This is unsatisfying for secrecy purposes since it leaves an infinite amount of cases where the schemes do not work, e.g. rational channel gains. Moreover, secrecy should not depend on the accuracy of channel measurements. Real interference alignment is part of a broader class of interference alignment strategies. Interference alignment (IA) was introduced in [7] and [8], among others, and its main idea is to design signals such that the caused interference overlaps (aligns) and therefore uses fewer signal dimensions. The resulting interference-free signal dimensions can be used for communication. IA methods can be divided into two categories, namely the vector-space alignment approach and the signal-scale alignment approach [9]. The former uses the classical signalling dimensions time, frequency and multiple-antennas for the alignment, while the latter uses the signal strength for alignment. Real interference alignment and signal-strength deterministic models are examples for signal-scale alignment. Signal-strength deterministic models are based on an approximation of the Gaussian channel. An example for such an approximation is the linear deterministic model (LDM), introduced by Avestimehr et al. in [10]. It is based on a binary expansion of the transmit signal, and an approximation of the channel gain to powers of two. The resulting binary expansion gets truncated at the noise level which yields a noise-free binary signal vector and makes the model deterministic. It has been shown that various Gaussian channels (i.e. [11], [12], [13], [14]) can be approximated by the LDM such that the deterministic capacity is within a constant bit-gap of the Gaussian channel. Moreover, layered lattice coding schemes can be used to transfer the achievable scheme to the Gaussian model.

Previous work and Contributions: Previous work on the wiretap channel in multi-user settings mainly utilized the real IA approach in addition to cooperative jamming, introduced in
The idea of using IA in a secrecy context is to cooperatively jam the eavesdropper, while aligning the jamming signal in a small subspace at the legitimate receiver. This resulted in a sum s.d.o.f characterization of \( \frac{h(K-1)}{K(K-1)+1} \) for the K-user case in [16]. The idea is that the users can allocate a small part of the signalling dimensions with uniformly distributed random bits. Those random bits are send such that they occupy a small space at the legitimate receiver, while overlapping with the signals at the eavesdropper. A specialized model is the wiretap channel with a helper. This model consists of the standard wiretap channel model, with a second independent user, whose only purpose is to jam the eavesdropper. In [17] and [18], the real IA approach was used on the wiretap channel with a helper (with and without CSIT, respectively) to investigate the s.d.o.f., therefore achieve results for the infinite SNR regime. Another branch of recent work [19] approached the problem, using a compute-and-forward decoding strategy, which leads to results for the finite regime that are optimal in an s.d.o.f sense. The next step is to transition from the s.d.o.f. results, to a secure constant-gap capacity result. We take a different approach and study the linear deterministic approximations of both models to gain insights leading to constant-gap capacity approximations. This approach has been used for example for wiretap channels in [20], [21], for relay networks [22] and for IC channels [23], [24]. It was also recently used in [25] for an s.d.o.f. analysis of the Gaussian diamond-wiretap channel, which is a multi-hop version of the G-MAC-WT. We show that the previously known \( \frac{1}{3} \) s.d.o.f. result of the wiretap channel with a helper [16] can be extended to a general (asymmetric) and finite SNR regime, independent of the channel gain being rational or not. Moreover, we develop a converse proof which shows a constant-gap for certain channel gain values. The converse proof converges to the s.d.o.f bound for vanishing channel gain differences. Furthermore, we use the same alignment methods to present an achievable scheme for the linear deterministic MAC-WT (LD-MAC-WT) and show that a rate can be achieved, which converges to \( \frac{1}{3} \) s.d.o.f., for vanishing receive signal power differences. We also extend the converse proof of [16] for the G-MAC-WT towards general receive signal powers, to match our achievable scheme for certain channel parameters. Moreover, we show that both achievable schemes can be translated to the Gaussian channel models, by using layered lattice codes to imitate bit-levels. We also combine previous techniques with new novel techniques to translate the results of both converse proofs to the Gaussian channel.

II. SYSTEM MODEL

The G-MAC-WT and the G-WT-H are defined as a system consisting of 2 transmitters and 2 receivers. Where \( X_1 \) and \( X_2 \) are the channel inputs of both user, communicating with the legitimate receiver, with channel output \( Y_1 \). Both channel inputs are also received by an eavesdropper with channel output \( Y_2 \). The channel itself is modeled with additive white Gaussian noise, \( Z_1, Z_2 \). Therefore the system equations can be written as

\[
Y_1 = h_{11}X_1 + h_{21}X_2 + Z_1 \tag{1a}
\]

\[
Y_2 = h_{12}X_2 + h_{22}X_1 + Z_2, \tag{1b}
\]

where the channel inputs satisfy an input power constraint \( E\{X^2_i\} \leq P \) for each \( i \). Moreover the Gaussian noise terms are assumed to be independent and zero mean with unit variance, \( Z_i \sim \mathcal{N}(0,1) \). The difference between both models is, that in case of the the G-WT-H, one of the users is just helping the other user. He is independently jamming both receivers to help achieving a secure communication to the legitimate receiver.

1) G-WT-H: A \( (2^{nR_i}, n) \) code will consist of an encoding and a decoding function. The encoder assigns a codeword \( x_i^n(w) \) to each message \( w \), where \( W \) is uniformly distributed over the set \( \{1 : 2^{nR_i}\} \), and the associated decoder assigns an estimate \( \hat{w} \in \{1 : 2^{nR_i}\} \) to each observation of \( Y_i^n \). A rate is said to be achievable if there exist a sequence of \( (2^{nR_i}, n) \) codes, for which the probability of error \( P_e^n(W) = (\hat{W} \neq W) \) goes to zero, as \( n \) goes to infinity \( \lim_{n \to \infty} P_e^n = 0 \). As opposed to the general Gaussian multiple access wiretap channel, the second channel input is used as a pure helper. This means, instead of sending codewords through \( X_2 \), it is used as a jamming signal. A message \( W \) is said to be information-theoretically secure if the eavesdropper cannot reconstruct the message \( W \) from the channel observation \( Y_2^n \). This means that the uncertainty of the message is almost equal to its entropy, given the channel observation:

\[
\frac{1}{n}H(W|Y_2^n) \geq \frac{1}{n}H(W) - \epsilon, \tag{2}
\]

which leads to \( I(W; Y_2^n) \leq \epsilon n \) for any \( \epsilon > 0 \). A secrecy rate \( r \) is said to be achievable if it is achievable while obeying the secrecy constraint (2).

2) G-MAC-WT: A \( (2^{nR_1}, 2^{nR_2}, n) \) code for the multiple access wiretap channel will consist of a message pair \( (W_1, W_2) \) uniformly distributed over the message set \( \{1 : 2^{nR_1}\} \times \{1 : 2^{nR_2}\} \) with a decoding and two randomized
encoding functions. Encoder 1 assigns a codeword $X^n_1(w_1)$ to each message $w_1 \in [1 : 2^{nR_1}]$, while the encoder 2 assigns a codeword $X^n_2(w_2)$ to each message $w_2 \in [1 : 2^{nR_2}]$. The decoder assigns an estimate $(\hat{w}_1, \hat{w}_2) \in [1 : 2^{nR_1}] \times [1 : 2^{nR_2}]$ to each observation of $Y^n_1$. A rate is said to be achievable if there exist a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes, for which the probability of error $P^e(n) = P[(W_1, W_2) \neq (\hat{W}_1, \hat{W}_2)]$ goes to zero, as $n$ goes to infinity. The channel observation. Considering both messages $W_1, W_2$, we have that

\[ \frac{1}{n} H(W_1, W_2 | Y^n_2) \geq \frac{1}{n} H(W_1, W_2) - \epsilon, \]

which leads to $I(W_1, W_2; Y^n_2) \leq \epsilon n$ for any $\epsilon > 0$. A secrecy rate $r$ is said to be achievable if it is achievable while obeying the secrecy constraint (3).

### III. THE LINEAR DETERMINISTIC MODEL SYSTEM

#### A. LD Wiretap with a Helper and LD-MAC-WT

As simplification we will investigate the corresponding linear deterministic model (LDM) of the system models as an intermediate step. The LDM models the signals of the channel as bit-vectors $X$, which is achieved by a binary expansion of the input signal $X$. The positions within the bit-vector are referred to as bit-levels. Furthermore, superposition of different signals is modeled by binary addition of the bit-levels itself. Carry over is not used to limit the superposition on the specific level where it occurs. Truncation of the bit-vector at noise level models the signal impairment of the Gaussian noise, which yields a deterministic approximation of the Gaussian model. Channel gains are included by shifting the bit-vector for an appropriate number of bit-levels. This shift is introduced by a shift-matrix $S$, which is defined as

\[
S = \begin{pmatrix}
0 & 0 & \cdots & 0 & 0 \\
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & 0 & 1
\end{pmatrix}. \tag{4}
\]

With $S$ an incoming bit vector can be shifted for $q-n$ positions with $Y = S^{q-n}X$, where $q := \max\{n\}$. The channel gain is represented by $n_{ij}$-bit levels which corresponds to $\frac{1}{2} \log \text{SNR}$ of the original channel. With this definitions, the model can be written as

\[
Y_1 = S^{q-n_1}X'_1 \oplus S^{q-n_2}X'_2, \tag{5a}
\]

\[
Y_2 = S^{q-n_2}X'_2 \oplus S^{q-n_1}X'_1, \tag{5b}
\]

where $q := \max\{n_{11}, n_{12}, n_{21}, n_{22}\}$. For ease of notation, we denote $X_1 = S^{q-n_1}X'_1$ and $X_2 = S^{q-n_2}X'_2$. Furthermore, we denote $S^{q-n_2}X'_2$ and $S^{q-n_1}X'_1$ by $X_2$ and $X_1$, respectively. We use the assumption that $n_{22} = n_{12} = n_E$. Thus, the Gaussian model. Channel gains are included by shifting the levels itself. Carry over is not used to limit the superposition of different signals is modeled by binary addition of the bit-levels. In the intermediate step. The LDM models the signals of the channel as bit-vectors $X$, which is achieved by a binary expansion of the input signal $X$. The positions within the bit-vector are referred to as bit-levels. Furthermore, superposition of different signals is modeled by binary addition of the bit-levels itself. Carry over is not used to limit the superposition on the specific level where it occurs. Truncation of the bit-vector at noise level models the signal impairment of the Gaussian noise, which yields a deterministic approximation of the Gaussian model. Channel gains are included by shifting the bit-vector for an appropriate number of bit-levels. This shift is introduced by a shift-matrix $S$, which is defined as

\[
S = \begin{pmatrix}
0 & 0 & \cdots & 0 & 0 \\
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & 0 & 1
\end{pmatrix}. \tag{4}
\]

With $S$ an incoming bit vector can be shifted for $q-n$ positions with $Y = S^{q-n}X$, where $q := \max\{n\}$. The channel gain is represented by $n_{ij}$-bit levels which corresponds to $\frac{1}{2} \log \text{SNR}$ of the original channel. With this definitions, the model can be written as

\[
Y_1 = S^{q-n_1}X'_1 \oplus S^{q-n_2}X'_2, \tag{5a}
\]

\[
Y_2 = S^{q-n_2}X'_2 \oplus S^{q-n_1}X'_1, \tag{5b}
\]

where $q := \max\{n_{11}, n_{12}, n_{21}, n_{22}\}$. For ease of notation, we denote $X_1 = S^{q-n_1}X'_1$ and $X_2 = S^{q-n_2}X'_2$. Furthermore, we denote $S^{q-n_2}X'_2$ and $S^{q-n_1}X'_1$ by $X_2$ and $X_1$, respectively. We use the assumption that $n_{22} = n_{12} = n_E$, and denote $n_1 - n_2 := n_\Delta$ with $n_{11} := n_1$ and $n_{21} := n_2$. For the LD-MAC-WT, due to symmetry, we may assume w.l.o.g. that $n_1 > n_2$, where we leave out the case that $n_1 = n_2$. The achievable secrecy rate of the linear deterministic Wiretap channel with a helper is

\[
R_{ach} = n_p + \left\lfloor \frac{n_c}{2n_\Delta} \right\rfloor n_\Delta + Q \tag{7}
\]

where

\[
Q = \begin{cases}
q_\Delta & \text{for } n_Q < n_\Delta, n_1 \geq n_2 \\
0 & \text{for } n_Q \geq n_\Delta, n_1 \geq n_2 \\
n_\Delta - n_Q & \text{for } n_Q < n_\Delta, n_1 < n_2 \\
n_Q & \text{for } n_Q \geq n_\Delta, n_1 < n_2
\end{cases} \tag{8}
\]

with $n_Q = n_c \mod 2n_\Delta$.

**Proof:**
1) Case \( n_1 \geq n_2 \): We denote the part of \( X_1 \) and \( X_2 \) in \( Y_{1,c} \) by \( X_{1,c} \) and \( X_{2,c} \), respectively. Moreover, we partition these common parts of the signals into \( 2\Delta \) bits partitions. We now utilize the first \( n_\Delta \) bits of every full partition in \( X_{1,c} \) for messages and leave the reminder free. And for \( X_{2,c} \) we utilize the first \( n_\Delta \) bits of every partition for jamming, while the rest is free. After partitioning, \( Y_{1,c} \) has a reminder part with
\[
q = n_c \mod 2n_\Delta \text{ bits.} \tag{9}
\]
bit-levels. The user signal in this reminder part follows the same rules as before, while the helper lets the first \( n_\Delta \) bits free and only utilizes the bits afterwards for jamming, until we have filled all \( q \) bits. The private part \( Y_{1,p} \) can be used completely by the user, and all of \( X_1 \) in this part can be used for messaging. The total achievable rate is the private rate \( r_p = n_p \) plus the common rate
\[
r_c = \frac{1}{2} \left( \left\lfloor \frac{n_c}{2n_\Delta} \right\rfloor 2n_\Delta \right) + Q, \tag{10}
\]
where \( Q \) is defined as in the theorem. The common rate follows from the fact, that we utilize half of the bits of all \( 2\Delta \) partitions, along with a reminder part \( Q \). In the reminder we utilize every bit, as long as \( q \) is smaller than \( n_\Delta \). If \( q \) is larger than \( n_\Delta \), we only utilize the first \( n_\Delta \) bits.

2) Case \( n_1 < n_2 \): We use the same strategy as before, except for the reminder part \( n_q \) (9) of \( Y_{1,c} \). In the reminder part, the first \( n_\Delta \) bits of the user are left free, and all bits afterwards are used for messaging, while the Helper only jams the first \( n_\Delta \) bits. The strategy is therefore the opposite as before. This yields a different \( Q \)-term, where for \( q < n_\Delta \) no rate is achieved, and for \( q \geq n_\Delta \) one can use \( q - n_\Delta \) bits for messaging. We note that the secrecy is provided by the Crypto-lemma and the fact that we use binary addition on each level as well as jamming signals chosen such that each bit is Bern(\( \frac{1}{2} \)) distributed. And we therefore have that
\[I(X_1^n; Z^n) = 0.\]

**Lemma 1** (Crypto-Lemma, [26]). Let \( G \) be a compact abelian group with group operation \( + \), and let \( Y = X + N \), where \( X \) and \( N \) are random variables over \( G \) and \( N \) is independent of \( X \) and uniform over \( G \). Then \( Y \) is independent of \( X \) and uniform over \( G \).

**C. Achievable Scheme for the LD-MAC-WT**

We use the same common and private part definitions as in the LD-WT with a helper case. However, there are some important differences. Note that the channel is symmetrical, i.e. both users can send messages and jam and we can therefore assume w.l.o.g that \( n_1 \geq n_2 \). We can show the following result:

**Theorem 2.** The achievable secrecy sum-rate \( R_{ach} \) of the linear deterministic multiple access wiretap channel with symmetric channel gains at the eavesdropper is
\[
R_{ach} = \left\lfloor \frac{n_p}{3n_\Delta} \right\rfloor 2n_\Delta + n_p + Q. \tag{11}
\]
where \( n_c = \min\{n_E + n_\Delta, n_1\} \), \( n_p = n_1 - n_c \) and
\[
Q = \begin{cases} q \quad \text{for } n_Q < n_\Delta \\ n_\Delta + q \quad \text{for } 2n_\Delta > n_Q \geq n_\Delta \end{cases} \tag{12}
\]
with \( n_Q = n_c \mod 3n_\Delta \) and \( q = n_Q \mod n_\Delta \).

**Proof:** First of all, we look at the case that \( n_2 \geq n_E \). Our strategy is the same as before, i.e. to deploy a cooperative jamming scheme such that minimal jamming is done to \( Y_2 \), while maximal jamming is received at \( Y_2 \). We partition the common signals, \( X_{1,c} \) and \( X_{2,c} \), into \( 3n_\Delta \)-bit parts and partition these parts again into \( n_\Delta \)-bit parts. For \( X_{1,c} \), in every \( 3n_\Delta \)-bit part we use the first \( n_\Delta \) bits for the message and the last \( n_\Delta \) bits for jamming, while the last \( n_\Delta \) bits will not be used. For \( X_{2,c} \), in every \( 3n_\Delta \)-bit part, the first \( n_\Delta \) bits will be used for jamming. The next \( n_\Delta \) bits will be used for the message and the last \( n_\Delta \) bits left free. There will be a reminder part with
\[
q = n_c \mod 3n_\Delta \text{ bits.} \tag{13}
\]
The reminder part follows the same design rules as the \( 3n_\Delta \) parts, except that \( X_{2,c} \) leaves the first \( n_\Delta \) bits free, then uses jamming on the next \( n_\Delta \) bits and utilizes the last \( n_\Delta \) bits for messaging, until \( q \) bits are allocated. The scheme is designed such that the jamming parts of \( X_{1,c} \) and \( X_{2,c} \) overlap at \( Y_{1,c} \), while the message parts of one signal overlap with the non-used part of the other signal. However, due to the signal strength difference \( n_\Delta \), the jamming parts overlap with the messages at \( Y_2 \), see Fig. 4. Secure communication is therefore provided by the Crypto-lemma, as long as we use a Bern(\( \frac{1}{2} \)) distribution for the jamming bits. The whole private part can be used for messaging and its sum-rate is therefore \( r_p = n_p \). The achievable secure rate for the common part consists of the rate for the \( 3n_\Delta \) partitions and the reminder part. It can be seen that every \( 3n_\Delta \)-part of \( Y_{1,c} \) allocates \( 2n_\Delta \) bits for the messages. This results in the common secrecy rate
\[
r_c = \left( \left\lfloor \frac{n_p}{3n_\Delta} \right\rfloor 3n_\Delta \right) + Q, \tag{14}
\]
where \( Q \) specifies the rate of the reminder term. In the remainder part we allocate all remaining bits as message bits, as long as \( n_Q < n_\Delta \). For \( 2n_\Delta > n_Q \geq n_\Delta \), we allocate the first \( n_\Delta \) bits of \( n_Q \) for the message. And for \( n_Q \geq 2n_\Delta \), we allocate the first \( n_\Delta \) bits as well as the last \( q \) bits, where \( q \) is defined as
\[
q = n_Q \mod n_\Delta. \tag{15}
\]
This results in
\[
Q = \begin{cases} q \quad \text{for } n_Q < n_\Delta \\ n_\Delta \quad \text{for } 2n_\Delta > n_Q \geq n_\Delta \\ n_\Delta + q \quad \text{for } n_Q \geq 2n_\Delta. \end{cases} \tag{16}
\]
Together with the private rate term, we achieve
\[
R = \frac{2}{3} \left( \left\lfloor \frac{n_p}{3n_\Delta} \right\rfloor 3n_\Delta \right) + n_p + Q.
\]
For \( n_2 \geq n_\Delta \) the achievable scheme is the same, except that we do not have a private part. We therefore have an achievable rate of
\[
R = \frac{2}{3} \left( \left\lfloor \frac{n_p}{3n_\Delta} \right\rfloor 3n_\Delta \right) + Q.
\]
which completes the proof.

Remark 2. The bit level shift between $Y_1$ and $Y_2$ of $n_{\Delta}$ bits makes it impossible to divide $Y_1$ in exclusively private and common parts. In our division, the bottom $n_{\Delta}$ bits of $x_{1,c}$ are only received at $Y_1$ and therefore private. Hence, the common rate $r_c$ is not purely made of common signal parts. Nevertheless, our choice of division reaches the upper bound and fits into the scheme.

Remark 3. Our scheme relies on the signal strength difference between both users. Our scheme would not work, if $n_1 = n_2$, while having equal channel gains at the eavesdropper. In that case we would not have any signal strength diversity to exploit which results in a singularity point where the secrecy rate is zero.

D. Converse for the LD-WT with a Helper

Theorem 3. The secrecy rate $R$ of the linear deterministic wiretap channel with one helper and symmetric channel gains at the wiretapper is bounded from above by

$$R \leq \min\{r_{ub1}, r_{ub2}, r_{ub3}\}$$

with

$$r_{ub1} = n_p + \frac{1}{2} n_c + \frac{1}{2} (n_1 - n_2)^+$$

$$r_{ub2} = n_1$$

$$r_{ub3} = n_2 + (n_1 - n_2 - n_E)^+ + [n_E - n_2 - (n_E - n_1 + n_2)^+]^+$$

Proof: The proof is in the same fashion as for the truncated deterministic model and therefore omitted. It can be found in [1].

E. Converse for the LD-MAC-WT

Theorem 4. The secrecy sum-rate $R_{ach}$ of the linear deterministic multiple access wiretap channel with symmetric channel gains at the eavesdropper is bounded from above by

$$r_{UB} = \begin{cases} \frac{2}{3} n_c + n_p + \frac{1}{3} n_{\Delta} & \text{for } n_2 \geq n_E, \\ \frac{2}{3} n_c + \frac{1}{3} n_{\Delta} & \text{for } n_E > n_2. \end{cases}$$

Proof: The proof is in the same fashion as for the truncated deterministic model and therefore omitted. It can be found in the preprint [2].

IV. THE GAUSSIAN WIRETAP CHANNEL WITH A HELPER

In this section we analyse the Gaussian wiretap channel with a helper. To get results we stick to the previously developed scheme in section III-B, and we will transfer the alignment and jamming structure to its Gaussian equivalent with layered lattice codes. This will lead to an achievable rate which is directly based on the deterministic rate. Moreover, we will make use of results in [27] to show that the mutual information of the Gaussian case can be upper bounded by an appropriate deterministic model. As a result, the deterministic bound in section III-E is a bound for the Gaussian model as well, with a constant bit-gap attached.

A. Achievable Scheme

Theorem 5. The achievable secrecy rate of the Gaussian wiretap channel with a helper is

$$r_{ach} = r^p + r^c + r^R$$

where $r^c := l_u \left( \frac{1}{2} \log \text{SNR}^{1-(1-\beta_1)} - \frac{1}{2} \right)$,

with

$$l_u := \left\lfloor \frac{\min\{1+\beta_2-\beta_1,1\}}{2(1-\beta_1)} \right\rfloor,$$

$$r^p := \frac{1}{2} \log (\max\{1,\text{SNR}^{\beta_1-\beta_2}\}),$$

$$r^R = \begin{cases} r_{R_1} & \text{for } r_{R_1} < r_{R_2}, \text{SNR}_1 \geq \text{SNR}_2 \\ r_{R_2} & \text{for } r_{R_1} \geq r_{R_2}, \text{SNR}_1 \geq \text{SNR}_2 \\ 0 & \text{for } r_{R_1} < r_{R_2}, \text{SNR}_2 \geq \text{SNR}_1 \\ r_{R_3} & \text{for } r_{R_1} \geq r_{R_2}, \text{SNR}_2 \geq \text{SNR}_1 \end{cases}$$

with

$$r_{R_1} := \frac{1}{2} \log \text{SNR}_1^{1-2\beta_1(1-\beta_1)} - \frac{1}{2} \log \text{SNR}_1^\min\{\beta_1^{-\beta_2},0\} - \frac{1}{2},$$

$$r_{R_2} := \frac{1}{2} \log \text{SNR}_1^{1-\beta_2} - \frac{1}{2},$$

$$r_{R_3} := r_{R_1} - r_{R_2}.$$

Proof: In the following, we look into the case that $\text{SNR}_1 \geq \text{SNR}_2$. For the achievable scheme, we need to partition the available power into intervals. Each of these intervals plays the role of an $n_{\Delta}$-interval of bit-levels in the linear deterministic scheme. Remember that we have $E\{X_1^2\} \leq P$ and $Z_1, Z_2 \sim \mathcal{N}(0,1)$, which means that $|h_{11}|^2P = \text{SNR}_1$ and $|h_{22}|^2P = \text{SNR}_2$ represent the power of both direct signals. As in the deterministic model, we assume that both signals at $Y_2$ are received with the same power and therefore $h_{12} = h_{22} = h_E$ which gives $|h_E|^2P = \text{SNR}_E$. We introduce the two parameters $\beta_1$ and $\beta_2$, which connects the SNR ratios with $\text{SNR}_2 = \text{SNR}_1^{\beta_1}$ and $\text{SNR}_E = \text{SNR}_1^{\beta_2}$. Now we can partition the received power at $Y_1$ into intervals of $\text{SNR}_1^{1-\beta_1}$. 
Each of the intervals has therefore signal power $\theta_l$ which is defined as

$$\theta_l = q_{l-1} - q_l = \text{SNR}_1^{1-(l-1)(1-\beta_1)} - \text{SNR}_1^{1-l(1-\beta_1)} \quad (18)$$

with $l$ indicating the specific level. The users decompose the signals $X_i$ into a sum of independent sub-signals $X_i = \sum_{l=1}^{l_{\text{max}}} X_{i,l}$. We will use n-dimensional nested lattice codes introduced in [28] which can achieve capacity in the AWGN single-user channel. A lattice $\Lambda$ is a discrete subgroup of $\mathbb{R}^n$ which is closed under real addition and reflection. Moreover, denote the nearest neighbour quantizer by $Q(\Lambda)(x) := \arg \min_{\mathbf{d} \in \Lambda} ||x - \mathbf{d}||$. The fundamental Voronoi region $V(\Lambda)$ of a lattice $\Lambda$ consists of all points which get mapped or quantized to the zero vector and the modulo operation for lattices is defined as $[x] \mod \Lambda := x - Q(\Lambda)(x)$.

1) Nested Lattice Codes: A nested lattice code is composed of a pair of lattices $(\Lambda_{\text{fine}}, \Lambda_{\text{coarse}})$, where $V(\Lambda_{\text{coarse}})$ is the fundamental Voronoi region of the coarse lattice and operates as a shaping region for the corresponding fine lattice $\Lambda_{\text{fine}}$. It is therefore required that $\Lambda_{\text{coarse}} \subset \Lambda_{\text{fine}}$. Such a code has a corresponding rate $R$ equal to the log of the nesting ratio. A part of the split message is now mapped to the corresponding codeword $u(l) \in \Lambda_{\text{fine}} \cap V(\Lambda_{\text{coarse}})$, which is a point of the fine lattice inside the fundamental Voronoi region of the coarse lattice. Note that $\Lambda_{l_{\text{max}}} \subset \cdots \subset \Lambda_1$. The code is chosen such that it has a power of $\theta_l$. The codeword $x_i(l)$ is now given as $x_i(l) = [u_i - d_i] \mod \Lambda_i$, where we either (shift) with $d_i \sim \text{Unif}(V(\Lambda_i))$ and reduce the result modulo-$\Lambda_i$. Transmitter $i$ now sends a scaled $x_i$ over the channel, such that the power per sub-signal $x_i(l)$ is $\frac{\theta_l}{h_{i1}^2}$ and receivers see a power of $\theta_l$. Due to the partitioning construction, the $x_i$ satisfy the power restriction of $P$ for user 1,

$$\sum_{l=1}^{l_{\text{max}}} \frac{\theta_l}{|h_{i1}|^2} \leq \frac{\text{SNR}_1}{|h_{i1}|^2} = P \quad (19)$$

and user 2

$$\sum_{l=2}^{l_{\text{max}}} \frac{\theta_l}{|h_{i2}|^2} \leq \frac{\text{SNR}_2}{|h_{i2}|^2} = P \quad (20)$$

Moreover, aligning sub-signals use the same code (with independent shifts). In [28] it was shown that nested lattice codes can achieve the capacity of the AWGN single-user channel with vanishing error probability. Viewing each of our power intervals as a channel, we therefore have that

$$R(l) = \frac{1}{2} \log \left( 1 + \frac{\theta_l}{N(l)} \right) \quad (21)$$

where $N(l)$ denotes the noise variance per dimension of the sub-sequent levels. Now, $X_1$ is used for signal transmission, while $X_2$ is solely used for jamming. As in the deterministic case, the objective is to align the signal parts of $X_2$ with the jamming of $X_2$ at $Y_2$, while allowing decoding of the signal parts at $Y_1$. Due to the signal scale based coding strategy and the equal receive power at $Y_2$, an alignment is achieved with the proposed scheme. We use a jamming strategy, where the jamming sub-codeword is uniformly distributed on $V$, therefore $x_{2,jam}(l) \sim \text{Unif}(V(\Lambda_i))$. Now, application of lemma 1 shows, that the received codeword $y = [x_1(l) + x_{2,jam}(l)] \mod \Lambda_i$ is independent of $x_1(l)$, therefore providing secrecy. We therefore just need to prove, that the signal can be decoded at $Y_1$. The decoding is done level-wise, treating subsequent levels as noise. Every level is treated as a Gaussian point-to-point channel with power $\theta_l$ and noise $1 + \text{SNR}_1^{1-l(1-\beta_1)}$, which consists of the base noise $N_1$ at $Y_1$ and the power of all subsequent levels of both signals. Successful decoding can be assured with a rate limitation (see (21)) of

$$r_l \leq \frac{1}{2} \log \left( 1 + \frac{\theta_l}{1 + \text{SNR}_1^{1-l(1-\beta_1)}} \right) \quad (22)$$

2) Achievable rate: As in the deterministic case, we have a private and common part. Common and private parts are defined as in the deterministic model. The common part depends on the strength of the received power at Eve (remember $n_c := \min\{n_\beta + n_\Delta, n_1\}$ for $n_1 \geq n_2$). The part $n_\beta + n_\Delta$ corresponds to $\text{SNR}_1^{1-\beta_1}$ in the Gaussian model. The opposing reminder is therefore $\text{SNR}_1^{1-\beta_2}$ and we get the common power as

$$P^c := \text{SNR}_1 - \max\{1, \text{SNR}_1^{1-\beta_1}\} \quad (23)$$

while the private part has a power of

$$P^p := \max\{1, \text{SNR}_1^{1-\beta_2}\} - 1 \quad (24)$$

The private part will not be partitioned further, since it can be used completely and without penalty. Moreover, it has only the base noise and a rate of $r^p = \frac{1}{2} \log(1 + P^p)$ can be achieved. For the common part, we also use the deterministic achievable scheme and need to partition the available power. All odd levels $l$ of $X_1$ will be used for signal transmission. Every level $l$ can handle a rate of $r_l$. We can simplify the rate of (22) with

$$\log \left( 1 + \frac{a - b}{1 + b} \right) = \log \left( \frac{a + 1}{1 + b} \right) \geq \log \left( \frac{a}{2b} \right)$$

where we used that $b > 1$ to get $r_l \geq \frac{1}{2} \log \text{SNR}_1^{1-\beta_1} - \frac{1}{2}$, where the $1-l(1-\beta_1)$-terms get cancelled in the last fraction. Since we use the same scheme as in the deterministic case, we have a total of

$$L_u := \left\lfloor \frac{\min\{1 + \beta_2 - \beta_1, 1\}}{2(1-\beta_1)} \right\rfloor \quad (25)$$

used levels in $X_1$, where the reminder term is not yet included. The alignment section of the common part has a total rate of

$$r^c = L_u \left( \frac{1}{2} \log \text{SNR}_1^{1-\beta_1} - \frac{1}{2} \right) \quad (26)$$

which corresponds to $\left\lfloor \frac{2L_u}{\Delta} \right\rfloor n_\Delta$ in the deterministic case. Moreover, we need to consider the reminder term, which is allocated between the alignment structure and the noise floor or the private part. Once again we use the deterministic scheme as a basis. We see in (8), that we have two cases for $n_1 \geq n_2$, which corresponds to $\text{SNR}_1 \geq \text{SNR}_2$. If the remainder has an available power of

$$P^R = \text{SNR}_1^{1-2L_u(1-\beta_1)} - \text{SNR}_1^{\min\{\beta_1-\beta_2, 0\}} \quad (27)$$

where

$$\text{SNR}_1^{1-2L_u(1-\beta_1)} - \text{SNR}_1^{1-2L_u(1-\beta_1)}$$
it can use the whole power and achieves a rate of
\[ r^R \geq \frac{1}{2} \log \text{SNR} \left(1 - 2^{\frac{1}{2} (1 - \beta_1)} - \frac{1}{2} \log \text{SNR}^{\min\{\beta_1 - \beta_2, 0\}}\right) - \frac{1}{2}. \]
Otherwise, it can only use a full partition, which leads to
\[ r^R \geq \frac{1}{2} \log \text{SNR}^{1 - \beta_1} - \frac{1}{2}. \]

We therefore get a total rate of \( r_{\text{ach}} = r^p + r^c + r^R \). The case for \( \text{SNR}_2 > \text{SNR}_1 \) can be shown similarly. Note that we are therefore within a gap of \( \frac{1}{4} + \frac{1}{2} \) bits at maximum, of the rates of the deterministic model (Theorem 1), by comparing via \( n = \frac{1}{2} \log \text{SNR} \).

**B. Developing a Converse from LD-Bounds**

For the converse, the goal is to bound the Gaussian mutual information terms by the ones of the deterministic model. Since we have a bound for the deterministic model, we immediately have a bound for the Gaussian model. Due to the G-WT-H consisting of MAC channels with security constraint, one could try to use the constant-gap bound of [11]. Unfortunately, the result of [11, Thm.1] for the complex Gaussian IC, which shows that the capacity is within a constant-gap of the deterministic IC capacity, depends on an assumption on the uniformity of the optimal input distribution to show that \( I(W; \mathcal{Y}^{n}_{G}) \leq I(W; \mathcal{Y}^{n}_{LD}) + cn \), where \( G \) stands for Gaussian model. We therefore need to introduce another approximation model first. It was shown in [27] that the integer-input integer-output model of the MAC-WT and WT-H, is within a constant-gap of the G-MAC-WT and G-WT-H. The system equations for the integer-input integer-output model can be written as
\[
\begin{align*}
\tilde{Y}_1 &= |h_{11}X_{1,1}| + |h_{21}X_{2,1}|, \\
\tilde{Y}_2 &= |h_{22}X_{2,2}| + |h_{12}X_{1,2}|,
\end{align*}
\]
where the \( \tilde{X}_D = \{0, 1, \ldots, \lfloor \sqrt{\text{SNR}} \rfloor \} \). One can construct these codewords easily from a set of given codewords for the Gaussian case by \( [X_G] \mod \lfloor \sqrt{P} \rfloor \). It was now shown in [27], that the mutual information terms for the integer-input integer-output channel (27) are within a constant-gap\(^1\) of the corresponding Gaussian model (1), which means that
\[
\begin{align*}
I(W; \mathcal{Y}^{n}_{1,G}) &\leq I(W; \mathcal{Y}^{n}_{1,D}) + nc, \\
I(W; \mathcal{Y}^{n}_{2,G}) &\leq I(W; \mathcal{Y}^{n}_{2,D}) + nc,
\end{align*}
\]
where \( c \) is a constant. The first equation follows from a proof in [11] and a more detailed version of the same ideas in [29]. The second equation builds on lemmata and ideas from [11], [29] and [30]. We therefore have that
\[
\begin{align*}
nR &= I(W; Y^{n}_{1,G}) - I(W; Y^{n}_{1,D}) + nc \leq I(W; \tilde{Y}^{n}_{1,D}) - I(W; Y^{n}_{1,D}) + n(c + \epsilon),
\end{align*}
\]
which shows that any bound for the integer-input integer-output model, can be used as an outer bound for the corresponding Gaussian model. Now, to bring the LDM ideas to

\(^1\)It was actually stated, that both terms are within \( o(\log P) \). However, the proof results also satisfy the stronger notion of a constant-gap.

the truncated model, we modify the form such that \( \tilde{X}_D \) is represented\(^2\) as
\[
X_{1,D} = 2^n \sum_{b=1}^{n} \tilde{X}_{1,b} 2^{-b} \in \{0, 1, \ldots, 2^n - 1\},
\]
where \( n = \lfloor \log (\sqrt{\text{SNR}}) \rfloor \). Note that the floor function around the logarithm, i.e. the quantization from integers to powers of two, reduces the cardinality of the input constellation by at most half plus one-half, which results in a maximum bit-gap of 2 bits in the capacity results for high-SNR. We can therefore work with the model
\[
\begin{align*}
Y_{1,D} &= |h_{11}X_{1,1}| + |h_{21}X_{2,1}|, \\
Y_{2,D} &= |h_{22}X_{2,2}| + |h_{12}X_{1,2}|,
\end{align*}
\]
where \( h_{ij}X_{i,D} = h_{ij}2^{n_{ij}} \sum_{b=1}^{n_{ij}} \tilde{X}_{1,b} 2^{-b}, h_{ij} \in [1,2] \) and the \( n_{ij} \) correspond to the bit-levels in the LD model. We therefore change the notation to include the assumption on equal received power at the wiretapper and write the model as
\[
\begin{align*}
Y_{1,D} &= |h_{11}X_{1,1}| + |h_{22}X_{2,2}|, \\
Y_{2,D} &= |h_{12}X_{1,2}| + |h_{21}X_{2,1}|.
\end{align*}
\]
We will call this model the truncated deterministic model (TDM). For the converse proofs we will also need the following lemmata. Note that the following lemmata results and ideas were already used for example in the converse proof in [31] but without rigorous justification. Moreover, the first lemma uses ideas from a proof in [11].

**Lemma 2.** For an arbitrary signal \( X_D \in \{0, 1, \ldots, 2^n - 1\} \), with \( n \in \mathbb{N} \) and channel gain \( h \in [1,2] \) we have that
\[
H(|hX_D|) = H(\tilde{X}_1, \tilde{X}_2),
\]
where \( \tilde{X}_i \in \mathbb{F}_2 \) are such that \( X_D = 2^n \sum_{i=1}^{n} \tilde{X}_i 2^{-i} \).

**Proof:** We denote the tuple \( (\tilde{X}_1, \ldots, \tilde{X}_n) \in \mathbb{F}_2^n \) by \( \tilde{X} \). There is a bijection \( f_1 : \mathbb{F}_2^n \to \{0, 1, \ldots, 2^n - 1\} \) which can be constructed as \( f_1(\tilde{X}) = 2^n \sum_{i=1}^{n} \tilde{X}_i 2^{-i} \). Now, the resulting integers are distance one apart. Therefore multiplying by \( h \in [1,2] \) does not lower the distance. Quantizing those scaled values to the integer part only introduces gaps in the support, but does not reduce the cardinality. We therefore have that \( f_2(X_D) = |hX_D| \) is again a bijection. Therefore, the composition of both functions \( f_3 = f_2 \circ f_1 \) is a bijection and we have that
\[
H(f_3(\tilde{X})) = H(\tilde{X})
\]
which shows the result.

**Lemma 3.** For an arbitrary signal \( X_D \in \{0, 1, \ldots, 2^n - 1\} \), with \( n, m \in \mathbb{N}, n < m, X_D = 2^n \sum_{i=1}^{n} \tilde{X}_i 2^{-i}, \tilde{X} \in \mathbb{F}_2^n \) and channel gain \( h \in [1,2] \) we have that
\[
H([h2^n \sum_{i=1}^{m} \tilde{X}_i 2^{-i}]) = H([h2^n \sum_{i=m+1}^{n} \tilde{X}_i 2^{-i}])
\]
\(^2\)For the time being, we use \( n \) as the index of the bit-level as well as the sequence index. This will be distinguishable later on, since the bit-level index will always have a subscript indicating the specific channel gain.
Proof: The first entropy term contains $2^n \sum_{i=1}^{n} \tilde{X}_i 2^{-i} \in \{0, 1, \ldots, 2^n - 1\}$. As argued previously, the support has distance one, and multiplying by the channel gain and taking the integer part only introduces gaps in the support and scales the values up, but the cardinality stays the same. Therefore, $|\text{supp}(X_D)| = |\text{supp}(\lfloor hX_D \rfloor)| = 2^n$. Now, the same is true for

$$X_D := 2^n \sum_{i=m+1}^{n} \tilde{X}_i 2^{-i} \in \{0, 1, \ldots, 2^n-m-1\}.$$ 

It also holds for

$$\tilde{X}_i 2^{-i} \in \{0, 2^{n-m}, 2^{n-m+1}, 2^{n-m}+2^{n-m+1}, \ldots, 2^{n-m}\},$$

where the distance is $2^n-m > 1$, since $n > m$. Moreover, we have that $X_D = \tilde{X}_D + \tilde{X}_D$. The cardinality of the support of $\tilde{X}_D$ is

$$|\text{supp}(\tilde{X}_D)| = 2^{n-m} - 2^{n-m-1} + 1 = 2^m.$$

Now, due to the structure\(^3\), the sum between $X_D$ and $\tilde{X}_D$ yields a Cartesian product between the support sets, and we therefore have that

$$|\text{supp}(\lfloor h(X_D + \tilde{X}_D) \rfloor)| = 2^n - 2^{n-m} - m$$

for the support of the sum-set. The same holds for the scaled integer parts, since they have the same scaling and therefore

$$|\text{supp}(\lfloor hX_D + \tilde{X}_D \rfloor)| = |\text{supp}(X_D + \tilde{X}_D)|$$

which proves the result.

Moreover, we introduce the function

$$f_{[a,b]}(\lfloor hX_D \rfloor) = \lfloor hX_D \rfloor_{[a,b]} = \lfloor h_{1,2^n} \sum_{k=a}^{b} \tilde{X}_k 2^{-k} \rfloor,$$

which restricts the exponents of the binary expansion inside the term to lie in the set of integers $\{a, a+1, \ldots, b\}$. The result of lemma 3 can then be written as

$$H(\lfloor hX_D \rfloor) = H(\lfloor hX_D \rfloor_{[1:m]} + \lfloor hX_D \rfloor_{[m+1:n]}).$$

If the term is a sum of two signals, then both get restricted relative to the stronger part. Therefore, a signal

$$Y_D = h_{1,2^n} \sum_{i=1}^{n} \tilde{X}_i 2^{-i} + h_{2,2^n} \sum_{i=1}^{m} \tilde{X}_i 2^{-i},$$

where $n > m$, can be restricted to

$$(Y_D)_{[1:a]} = \lfloor h_{1,2^n} \sum_{i=1}^{a} \tilde{X}_i 2^{-i} + h_{2,2^n} \sum_{i=1}^{a-(n-m)} \tilde{X}_i 2^{-i} \rfloor.$$ 

Moreover, we use the notation also on the bit-tuples to indicate that $(\tilde{X}_1, \ldots, \tilde{X}_n) \in \mathbb{F}_2^n$ by $\tilde{X}$ is restricted to the bits $a$ to $b$, such that $(\tilde{X}_a, \ldots, \tilde{X}_b)$ is denoted as $(\tilde{X})_{[a:b]}$. The notation is therefore the same as for the bit-vectors in the linear deterministic model.

We can now show Theorem 3 for the TD model which yields the following Theorem for the Gaussian equivalent.

**Theorem 6.** The secrecy rate $R$ of the Gaussian wiretap channel with one helper and symmetric channel gains at the wiretapper is bounded from above by

$$R \leq \min\{r_{ub1}, r_{ub2}, r_{ub3}\} + c$$

with

$$r_{ub1} = n_p + \frac{1}{2}n_c + \frac{1}{2}(n_1 - n_2)^+$$

$$r_{ub2} = n_1$$

$$r_{ub3} = n_2 + (n_1 - n_2 - n_1E)^+ + [n_1E - n_2 - (n_1 - n_1 + n_2)]^+,$$

where $c$ is a constant independent of the power $P$.

Proof: We start with equation (29) and convert the steps of the proof for the linear deterministic case to the truncated deterministic model.

$$n(R - \epsilon) = I(W; Y_{1,l}) - I(W; Y_{2,l}) \leq I(W; Y_{1,l}^n) - I(W; Y_{2,l}^n) + nc$$

$$= I(W; Y_{1,l}^n) - I(W; Y_{2,l}^n) + nc$$

$$= H(Y_{1,l}^n) - H(Y_{1,l}^n|W) - H((Y_{2,l}^n|_{[n_1:]})$$

$$+ H((Y_{2,l}^n|_{[n_2:]})|W) + nc$$

$$= H(Y_{1,l}^n) - H((Y_{2,l}^n|_{[n_2:]}) + H((|hE_{X_{2,l}}^n|_{[n_2:]})$$

$$- H(|hE_{X_{2,l}}^n|_{[n_2:]}) + nc,$$

where Fano's inequality and the secrecy constraint was used. Moreover, we used the fact that $I(W; Y_{2,l}^n) \geq I(W; f(Y_{2,l}^n))$ for arbitrary functions $f$, due to the data processing inequality. Note that for $n_2 \geq n_1$, we have that $(Y_{2,l}^n|_{[n_1:]}) = Y_{2,l}^n$. In the last line we used that $X_{1,l}$ is a function of $W$, and $X_{2,l}$ is independent of $W$, due to the Helper model assumptions. We remark that the first property does not hold in general, since jamming through the first user would result in a stochastic function. Now, for $n_2 \geq n_1$, both terms $|hE_{X_{2,l}}^n|_{[n_2:]})$ and $(|hE_{X_{2,l}}^n|_{[n_2:]})$ have the same bits, and we can use lemma 2 to show that

$$H((|hE_{X_{2,l}}^n|_{[n_2:]}) = 0$$

and for $n_2 < n_1$, the second term contains more bits, and we can therefore use the chain rule and lemma 2 and show that

$$H((|hE_{X_{2,l}}^n|_{[n_2:]}) = - H((\tilde{X}_{2})_{[n_2:]})_{[n_2:]})$$

$$(\tilde{X}_{2})_{[n_2:]})$$

We now split the received signal in common and private parts. Also, remember that $n_c$ is defined in equation (6). We start by adding two of the terms and split them apart

$$2(H(Y_{1,l}^n) - H(|hE_{X_{2,l}}^n|_{[n_2:]})$$

$$\leq 2(H(Y_{1,l}^n|_{[n_2:]}) + 2H(|hE_{X_{2,l}}^n|_{[n_1:]}) - 2H(|hE_{X_{2,l}}^n|_{[n_2:]})$$.
Note that the private part $H((Y_{1,D}^n)_{[n_1+1:n]})$ is zero for $n_1 \leq n_E$. Now, counting from top to bottom, for $n_1 \geq n_2$, $X_{1,D}^n$ has $n_c$ bit-levels in $(Y_{1,D}^n)_{[n_2]}$, while $X_{2,D}^n$ has $\eta := \min\{n_E, \min\{n_1, n_2\}\} = n_c - n_{\Delta}$ bit-levels. Therefore, $\eta$ represents the amount of bit-levels of the weaker signal in the common received signal part. Hence, for $n_2 > n_1$, $X_{1,D}^n$ and $X_{2,D}^n$ have $\eta$ and $n_c$ bit-levels in that term, respectively.

We need to account for this switch of indexing in the next part, where we analyse the entropy difference. We will use a method inspired by [32] to show the following (for $n_1 \geq n_2$)

\[
2H(Y_{1,D}^n) - H((Y_{2,D}^n)_{[n_2]}) \leq 2H(Y_{1,D}^n)_{[n_1+1:n]} + 2H((Y_{1,D}^n)_{[n_1]}) \\
- H((Y_{2,D}^n)_{[n_2]} - H((Y_{2,D}^n)_{[n_2]}) \\
= 2H(Y_{1,D}^n)_{[n_1+1:n]} + 2H((Y_{1,D}^n)_{[n_1]}) \\
- H((X_{1,D}^n)_{[n_2]} - H((X_{1,D}^n)_{[n_2]}) \\
= 2H(Y_{1,D}^n)_{[n_1+1:n]} + H((Y_{1,D}^n)_{[n_1]}) - H((X_{1,D}^n)_{[n_2]}) + H((f(h_1X_{1,D}^n, h_2X_{2,D}^n))_{[n_2]} - H((X_{1,D}^n)_{[n_2]}) \\
\leq 2H(Y_{1,D}^n)_{[n_1+1:n]} + H((Y_{1,D}^n)_{[n_1]}) - H((X_{1,D}^n)_{[n_2]}) + H((h_1X_{1,D}^n, h_2X_{2,D}^n))_{[n_2]} - H((X_{1,D}^n)_{[n_2]}) \\
= 2H(Y_{1,D}^n)_{[n_1+1:n]} + H((Y_{1,D}^n)_{[n_1]}) - H((X_{1,D}^n)_{[n_2]}) + H((X_{1,D}^n)_{[n_2]}) - H((X_{1,D}^n)_{[n_2]}) \\
= H((X_{1,D}^n)_{[n_2]}) - H((X_{1,D}^n)_{[n_2]}) \\

We now have for $n_1 \geq n_2$

\[
H((X_{1,D}^n)_{[n_1]}) - H((Y_{1,D}^n)_{[n_2]}) \leq n(n_c - \min\{n_2, n_E\})^+ \leq nn_{\Delta},
\]

and

\[
H((X_{1,D}^n)_{[n_2]}) - H((Y_{1,D}^n)_{[n_2]}) \leq n(\eta - \min\{n_2, n_E\})^+ = 0.
\]

And for $n_2 > n_1$ we get

\[
H((X_{1,D}^n)_{[n_2]}) - H((Y_{1,D}^n)_{[n_2]}) \leq n(\eta - \min\{n_2, n_E\})^+ = 0,
\]

and

\[
H((X_{1,D}^n)_{[n_2]}) - H((X_{2,D}^n)_{[n_2]}) \leq n(n_c - \min\{n_2, n_E\})^+.
\]

We remark that the last term gets $(n_2 - n_E)^+$ for $n_1 < n_E < n_2$, in which case we can use (35), which has a length of $(n_2 - n_E)$ bit-levels. Also for $n_E < n_1 < n_2$ we have that $n(n_c - \min\{n_2, n_E\})^+ = nn_{\Delta}$, by using (35) again, we see that for $n_2 > n_1$

\[
H((X_{2,D}^n)_{[n_2]}) - H((X_{2,D}^n)_{[n_2]}) \leq n(n_c - \min\{n_2, n_E\})^+ = 0.
\]

We therefore have an additional term of $nn_{\Delta}$ for $n_1 \geq n_2$.

Now one can divide all terms by two, resulting in

\[
H(Y_{1,D}^n) - H((Y_{2,D}^n)_{[n_2]}) \leq H((Y_{1,D}^n)_{[n_1+1:n]}) + \frac{1}{2}H((Y_{1,D}^n)_{[n_1]}) + \frac{n}{2}(n_1 - n_2)^+.
\]

Plugging all the results into the first equation yields

\[
n(R - \epsilon) \leq n(n_p + \frac{1}{2} n_c + \frac{1}{2}(n_1 - n_2)^+ + c).
\]

dividing by $n$ and letting $n \to \infty$ shows the result.

For the case that $n_2 > 2n_1$ we have that

\[
n(R - \epsilon) \leq H(Y_{1,D}^n) - H(Y_{2,D}^n) + H([h_EX_{2,D}^n]) \\
- H([h_2X_{2,D}^n]) + nc \\
\leq H([h_1X_{1,D}^n] + H([h_2X_{2,D}^n]) - H(Y_{2,D}^n)X_{1,D}^n) \\
- H([h_2X_{2,D}^n]) + H([h_EX_{2,D}^n]) + nc \\
= H([h_1X_{1,D}^n]) \leq nn_1
\]

and for the case that $3n_2 < 2n_1$ we have that

\[
n(R - \epsilon) \leq I(W; Y_{1,D}^n) - I(W; Y_{2,D}^n) + n(\epsilon + c) \\
\leq I(W; Y_{1,D}^n) - I(W; Y_{2,D}^n) + n(\epsilon + c) \\
\leq H(Y_{1,D}^n) - H(Y_{2,D}^n) + n(\epsilon + c) \\
H([h_EX_{2,D}^n]) - H([h_2X_{2,D}^n]) + n(\epsilon + c) \\
H([h_1X_{1,D}^n] - H([h_2X_{2,D}^n]) + n(\epsilon + c).
\]

One can show that

\[
H(Y_{1,D}^n)_{[n_1 - n_2 - n_E]^+} \leq n(n_1 - n_2 - n_E)^+
\]

and

\[
H([h_EX_{2,D}^n])_{[n_1 - n_2 - n_2]} - H([h_2X_{2,D}^n])_{[n_1 - n_2 - n_2]} \\
\leq n(\min\{n_1 - n_2, n_E\} - n_2) \\
= n(n_2 - n_2 - (n_2 - n_2)^+) \\
and H((Y_{1,D}^n)_{[(n_1 - n_2) + 1]; (Y_{1,D}^n)_{[n_1 - n_2]}]) \leq nn_2 \text{ which yields}
\]

\[
n(R - \epsilon) \leq n(n_1 - n_2 - n_2)^+ \\
+ n(n_2 - n_2 - (n_2 - n_2)^+) + n(\epsilon + c)
\]

dividing by $n$ and letting $n \to \infty$ shows the result.

V. THE GAUSSIAN MULTIPLE-ACCESS WIRETAP CHANNEL

In this section we analyse the Gaussian MAC-WT channel. As in the case for the WT channel with a Helper, we want to stick to the ideas of the corresponding linear deterministic model. This means we want to transfer the alignment and jamming structure to its Gaussian equivalent with layered lattice codes. This will lead to an achievable rate which is directly based on the deterministic rate. Moreover, we will make use of the previously developed ideas to convert the converse proof of the linear deterministic model, to the truncated model and therefore to the Gaussian model.

A. Achievable Scheme

**Theorem 7.** The achievable secrecy rate of the Gaussian multiple-access wiretap channel is

\[
r_{ach} = r^P + r^C + r^R
\]

where $r^C := l_a \frac{1}{2} \log SNR^{(1 - \beta_1)} - \frac{1}{2}$, with

\[
l_a := 2 \left\lfloor \min\{1 + \beta_2 - \beta_1, 1\} \right\rfloor / 3(1 - \beta_1).
\]
\[ r^p := \frac{1}{2} \log(\text{max}\{1, \text{SNR}_1^{\beta_1 - \beta_2}\}), \quad \text{and} \]
\[ r^R = \begin{cases} r^R_1 & \text{for } r^R_1 < r^R_2 \\ r^R_2 & \text{for } 2r^R_2 > r^R_1 \geq r^R_2 \\ r^R_1 + r^R_2 & \text{for } r^R_1 \geq 2r^R_2 \end{cases} \tag{38} \]

with
\[ r^R_1 := \frac{1}{2} \log \text{SNR}_1^{\frac{3}{2} l_u(1-\beta_1)} - \frac{1}{2} \log \text{SNR}_1^{\min(\beta_1 - \beta_2, 0)} - \frac{1}{2}, \]
\[ r^R_2 := \frac{1}{2} \log \text{SNR}_1^{(1-\beta_1)} - \frac{1}{2}. \]

**Proof:** We use the framework as for the wiretap channel with a helper in section IV-A. We therefore partition the available power into intervals with power \(\theta_i\), see eq. (18), where \(l\) indicates the level. Each of these intervals plays the role of an \(n_\Delta\)-interval of bit-levels in the linear deterministic scheme. We have that \(|h_{12}|^2 P = \text{SNR}_1, |h_{22}|^2 P = \text{SNR}_2\), as well as \(h_{12} = h_{22} = h_E\) which gives \(|h_E|^2 P = \text{SNR}_E\). We also use the two parameters \(\beta_1\) and \(\beta_2\), which connects the SNR ratios with \(\text{SNR}_2 = \text{SNR}_1^{\beta_1}\) and \(\text{SNR}_E = \text{SNR}_1^{\beta_2}\). We therefore partition the received power at \(Y_1\) into intervals \(\text{SNR}_1^{(1-\beta_1)}\). The users decompose the signals \(X_i\) into a sum of independent sub-signals \(X_i = \sum_{j=1}^{l_u} X_{ij}\). And each signal uses the layered lattice codes as defined in section IV-A.

### 1) Achievable rate
Note that, w.l.o.g. we look at the case \(\text{SNR}_1 > \text{SNR}_2\), which is \(\beta_1 < 1\). Due to the symmetry of the users the case \(\beta_1 \geq 1\) follows immediately by interchanging both signals. As in the deterministic case, we have a private and common part. The common part is defined as the bit-levels \(n_c := \min\{n_E + n_\Delta, n_1\}\). The part \(n_E + n_\Delta\) corresponds to \(\text{SNR}_1^{\beta_1 + (1-\beta_1)}\) in the Gaussian model. The opposing remainder is therefore \(\text{SNR}_1^{\beta_1 - \beta_2}\) and we get the common power as
\[ P^c := \text{SNR}_1 - \text{max}\{1, \text{SNR}_1^{\beta_1 - \beta_2}\}, \tag{39} \]
while the private part has a power of
\[ P^p := \text{max}\{1, \text{SNR}_1^{\beta_1 - \beta_2}\} - 1, \tag{40} \]
extactly as in the case of the wiretap channel with a helper. However, due to the modified scheme where both users jam and align their jamming signals at the legitimate receiver (see section III-C) we have a different number of used levels for messaging. We have
\[ l_u := 2 \left\lfloor \frac{\min\{1 + \beta_2 - \beta_1, 1\}}{3(1-\beta_1)} \right\rfloor \tag{41} \]
used levels for messaging, where each one supports a rate of \(r_i \geq \frac{1}{2} \log \text{SNR}_1^{(1-\beta_1)} - \frac{1}{2}\). And we therefore have a sum rate of
\[ r^c = l_u \left(\frac{1}{2} \log \text{SNR}_1^{(1-\beta_1)} - \frac{1}{2}\right), \tag{42} \]
for the whole common alignment part. Moreover, we need to consider the remainder term, which is allocated between the alignment structure and the noise floor or the private part. We see from the deterministic scheme that for \(1 - (\frac{3}{2} l_u + 1)(1 - \beta_1) < \min\{\beta_1 - \beta_2, 0\}\) we can achieve a rate of
\[ r^R \geq \frac{1}{2} \log \text{SNR}_1^{\frac{3}{2} l_u(1-\beta_1)} - \frac{1}{2} \log \text{SNR}_1^{\min(\beta_1 - \beta_2, 0)} - \frac{1}{2}. \]
Moreover, for \(1 - (\frac{3}{2} l_u + 1)(1 - \beta_1) < \min\{\beta_1 - \beta_2, 0\}\) we have
\[ r^R \geq \frac{1}{2} \log \text{SNR}_1^{(1-\beta_1)} - \frac{1}{2}, \]
and for \(\min\{\beta_1 - \beta_2, 0\} \leq 1 - (\frac{3}{2} l_u + 1)(1 - \beta_1)\) we have
\[ r^R \geq \frac{1}{2} \log \text{SNR}_1^{(1-\beta_1)} + \frac{1}{2} \log \text{SNR}_1^{\frac{3}{2} l_u(1-\beta_1)} - \frac{1}{2} \log \text{SNR}_1^{\min(\beta_1 - \beta_2, 0)} - 1. \]
We therefore get a total rate of \(r_{ach} = r^c + r^r + r^R\). The case for \(\text{SNR}_2 > \text{SNR}_1\) can be shown similarly. Note that we are therefore within a gap of \(1 + \frac{3}{2}\) bits of the rates of the deterministic model, by comparing via \(n = \left\lfloor \frac{1}{2} \log \text{SNR}_1 \right\rfloor\).

### B. Converse Bound for the G-MAC-WT
We use a similar approach as for the Gaussian WT with a helper, with the same framework developed in section IV-B. This means we also use the truncated deterministic model
\[ Y_{1,D} = |h_1 X_{1,D}| + |h_2 X_{2,D}| \tag{43a} \]
\[ Y_{2,D} = |h_E X_{2,D}| + |h_E X_{1,D}|, \tag{43b} \]
which can be shown to be within a constant gap to the Gaussian channel, see (28).

**Theorem 8.** The secrecy sum-rate \(R_{ach}\) of the Gaussian multiple access wiretap channel with symmetric channel gains at the eavesdropper is bounded from above by
\[ R = \begin{cases} \frac{2}{3} n_c + n_p + \frac{1}{3} n_\Delta + c & \text{for } n_2 \geq n_E \\ \frac{2}{3} n_c + \frac{1}{3} n_\Delta + c & \text{for } n_E > n_2, \end{cases} \tag{44} \]
where \(c\) is a constant independent of the signal power \(P\).

**Proof:** We begin with the following derivations
\[ n(R - \epsilon) \]
\[ = (I(W_1, W_2; Y^n_{1,G}) - I(W_1, W_2; Y^n_{2,G})) \]
\[ \leq I(W_1, W_2; Y^n_{1,D}) - I(W_1, W_2; Y^n_{2,D}) + n_c \]
\[ \leq I(W_1, W_2; Y^n_{1,D}, Y^n_{2,D}) - I(W_1, W_2; Y^n_{2,D}) + n_c \]
\[ \leq I(X^n_{1,D}; Y^n_{2,D}, Y^n_{2,D}) + n_c \]
\[ = H(Y^n_{1,D}; Y^n_{2,D}) - H(Y^n_{1,D}) | Y^n_{2,D}, Y^n_{1,D}, X^n_{1,D}, X^n_{2,D}) + n_c \]
\[ \leq H(Y^n_{1,D}) | Y^n_{2,D}) + n_c \]
\[ \leq H(Y^n_{1,D}, Y^n_{2,D}) + H(Y^n_{1,D}, Y^n_{2,D}, Y^n_{1,D}, c) + n_c \] (46)
where we used basic techniques such as Fano’s inequality and the chain rule. Step \((a)\) introduces the secrecy constraint (2), while we used the chain rule, non-negativity of mutual information and the data processing inequality in the following lines. Step \((b)\) follows from the fact that \(Y^n_{1,D}\) is a function of \((X^n_{1,D}, X^n_{2,D})\). Note that due to the definition of the common and the private part of \(Y^n_{1,D}\), it follows that \(H(Y^n_{1,D}, Y^n_{2,D}, Y^n_{1,D}, c) = 0\) for \(n_E \geq n_2\). For step \((c)\), we

\footnote{The common part is defined as \(Y^n_{1,D,c} = Y^n_{1,D}|n_c\), and the private part as \(Y^n_{1,D,p} = Y^n_{1,D}|n_c + 1\).}
used lemma 3, the data-processing inequality and the chain-rule. We now extend the strategy of [16], of bounding a single signal part, to asymmetrical channel gains

\[ n(R_1 - \epsilon_3) \leq I(X_{1,D}^n; Y_{1,D}^n) \tag{47} \]

\[ \leq I(X_{1,D}^n; Y_{1,D,c}^n) + I(X_{1,D}^n; Y_{1,D,p}^n|Y_{1,D,c}^n) \]

\[ = H(Y_{1,D,c}^n) - H(Y_{1,D,c}|X_{1,D}) + I(X_{1,D}^n; Y_{1,D,p}^n|Y_{1,D,c}^n) \]

\[ = H(Y_{1,D,c}^n) - H([h_E X_{2,D}]|\{n_e\}) + I(X_{1,D}^n; Y_{1,D,p}^n|Y_{1,D,c}^n) \]

and it therefore holds that

\[ H([h_E X_{2,D}]|\{n_e\}) \leq H(Y_{1,D,c}^n) \]

\[ + I(X_{1,D}^n; Y_{1,D,p}^n|Y_{1,D,c}^n) - n(R_1 - \epsilon_3). \tag{48} \]

The same can be shown for \( H([h_E X_{2,D}]|\{n_e\}) \), where it holds that

\[ H([h_E X_{2,D}]|\{n_e\}) \leq H(Y_{1,D,c}^n) \]

\[ + I(X_{1,D}^n; Y_{1,D,p}^n|Y_{1,D,c}^n) - n(R_1 - \epsilon_3). \tag{49} \]

Moreover, we have that

\[ I(X_{1,D}^n; Y_{1,D,p}^n|Y_{1,D,c}^n) \]

\[ = 2H(Y_{1,D,p}^n|Y_{1,D,c}^n) - H(Y_{1,D,p}^n|Y_{1,D,c}^n, X_{1,D}^n) \]

\[ - H(Y_{1,D,p}^n|Y_{1,D,c}^n, X_{1,D}^n) \]

\[ = 2H(Y_{1,D,p}^n|Y_{1,D,c}^n) - H([h_E X_{2,D}]|\{n_e\}) \]

\[ - H([h_E X_{2,D}]|\{n_e\}) \]

\[ = H(Y_{1,D,c}^n). \tag{50} \]

The key idea for the various cases is now to bound the term \( H(Y_{1,D,c}^n) \), or equivalently \( H(Y_{1,D}^n) \) for \( n_E > n_2 \), in an appropriate way, to be able to use (49) and (50) on (46).

We start with the first case:

1) Case \( n_2 \geq n_E \): Here we have a none vanishing private part, due to the definition of \( Y_{1,D,c}^n \) and therefore need to bound the term \( H(Y_{1,D,c}^n) \). Note that due to the definition of \( Y_{1,D,c}^n \) and the specific case, we have that

\[ H([h_E X_{2,D}]|\{n_e\}) = H([h_E X_{2,D}]) \]

We look into the first term of equation (46) and show that

\[ H(Y_{1,D,c}^n|Y_{2,D}) \]

\[ = H(Y_{1,D,c}^n, Y_{2,D}) - H(Y_{2,D}) \]

\[ \leq H(Y_{1,D,c}^n| [h_E X_{2,D}], [h_E X_{1,D}]) - H(Y_{2,D}) \]

\[ = H([h_E X_{2,D}], [h_E X_{1,D}]) - H(Y_{2,D}) \]

\[ + H(Y_{1,D,c}|[h_E X_{2,D}], [h_E X_{1,D}]) \]

\[ \leq H([h_E X_{1,D}]) + H([h_E X_{2,D}]) - H(Y_{2,D}|X_{2,D}) \]

\[ + H(Y_{1,D,c}|[h_E X_{2,D}], [h_E X_{1,D}]) \]

\[ = H([h_E X_{2,D}]) + H(Y_{1,D,c}|[h_E X_{2,D}], [h_E X_{1,D}]) \]. \tag{51} \]

Observe that the second term of equation (51) is dependent on the specific regime. We can bound this term by

\[ H(Y_{1,D,c}|[h_E X_{2,D}], [h_E X_{1,D}]) \leq n(n_e - n_E) = n \Delta. \tag{52} \]

Note that the choice of \([h_E X_{2,D}]\) in (51) as remaining signal part was arbitrary due to our assumption that both signals \([h_E X_{1,D}]\) and \([h_E X_{2,D}]\) have the same signal strength. Moreover, it follows on the same lines that

\[ H(Y_{1,D,c}^n|Y_{2,D}) \leq H([h_E X_{1,D}]) + n \Delta. \tag{53} \]

Looking at this result, it's intuitively that one can also show the stronger result

\[ H(Y_{1,D,c}^n|Y_{2,D}) \leq H([h_1 X_{1,D}]) \] \tag{54} \]

for the case that \( n_2 \geq n_E \). This can be shown by considering a similar strategy as in (51)

\[ H(Y_{1,D,c}^n|Y_{2,D}) \]

\[ = H(Y_{1,D,c}^n, Y_{2,D}) - H(Y_{2,D}) \]

\[ \leq H(Y_{2,D}(\{h_1 X_{1,D}\}|\{n_1\}) - H(Y_{1,D,c}^n, Y_{2,D}) \]

\[ \leq H(Y_{2,D}(\{h_1 X_{1,D}\}|\{n_1\})) - H(Y_{2,D}) \]

\[ + H(Y_{2,D}(\{h_2 X_{2,D}\}|\{n_2\}) \]

\[ \leq H([h_1 X_{1,D}]) \]

\[ \leq H([h_2 X_{2,D}]) \]

\[ = H([h_1 X_{1,D}]) \]

\[ \leq H([h_2 X_{2,D}]) \]

\[ = H([h_1 X_{1,D}]) \]

\[ \leq H([h_2 X_{2,D}]) \]

\[ \leq n(n_E - n_2)^+ = 0. \tag{56} \]

We combine one sum-rate inequality (46) with (51) and one with (55). Moreover, we plug (49) and (50) into the corresponding bound, which yields

\[ n(R_1 + R_2 - \epsilon_3) \leq H(Y_{1,D,c}^n) + I(X_{1,D}^n; Y_{1,D,p}^n|Y_{1,D,c}^n) \]

\[ + H(Y_{1,D,p}^n|Y_{2,D}, Y_{1,D,c}^n) \]

\[ - n(n_E - n_2)^+ \]

\[ = H(Y_{1,D,c}^n, Y_{2,D}) - H(Y_{2,D}) \]

\[ \leq H([h_E X_{2,D}], [h_E X_{1,D}]) - H(Y_{2,D}) \]

\[ + H(Y_{1,D,c}|[h_E X_{2,D}], [h_E X_{1,D}]) \]

\[ \leq H([h_E X_{1,D}]) + H([h_E X_{2,D}]) - H(Y_{2,D}|X_{2,D}) \]

\[ + H(Y_{1,D,c}|[h_E X_{2,D}], [h_E X_{1,D}]) \]

\[ = H([h_E X_{2,D}]) + H(Y_{1,D,c}|[h_E X_{2,D}], [h_E X_{1,D}]). \tag{51} \]

A summation of these results gives

\[ 3n(R_1 + R_2 - n \epsilon_8) \]

\[ \leq 2H(Y_{1,D,c}^n) + I(X_{1,D}^n; Y_{1,D,p}^n|Y_{1,D,c}^n) \]

\[ + I(X_{1,D}^n; Y_{1,D,p}^n|Y_{1,D,c}^n) + n \Delta \]

\[ + 2H(Y_{1,D,p}^n|Y_{2,D}, Y_{1,D,c}^n) \]

\[ = 2H(Y_{1,D,c}^n) + H(Y_{1,D,c}^n) + n \Delta \]

\[ + 2H(Y_{1,D,p}^n|Y_{2,D}, Y_{1,D,c}^n), \]

\[ n(n_E - n_2)^+ \]

\[ = n(n_E - n_2)^+ \]

\[ = 2mn_e + 3mn_p + n \Delta. \tag{58} \]

Dividing by 3n and letting \( n \to \infty \) shows the result.

\[ 3n(R_1 + R_2 - n \epsilon_8) \leq 2mn_e + 3mn_p + n \Delta. \tag{58} \]
2) Case \( n_E > n_2\): First, we assume that \( n_E \geq n_1 \), and include a short proof for \( n_1 > n_E \geq n_2 \) at the end of this subsection. For this case, the private part \( Y_{1,D,p}^{n} \) is zero, due to the definition of the private part and \( n_E > n_2 \). It follows that (46) is

\[
n(R_1 + R_2) \leq H(Y_{1,D}^{n} | Y_{2,D}^{n}). \tag{57}
\]

Moreover, we have that

\[
H([h_2 X_{2,D}],) = H(\{[h_2 X_{2,D}]\} | n_E) \leq H(\{h_E X_{2,D}\}),
\]

which is why we need to bound (57) by \( H([h_1 X_{1,D}],) \) and \( H(\{h_2 X_{2,D}\}) \). We therefore modify (55) to fit our case in the following way

\[
H(Y_{1,D}^{n} | Y_{2,D}^{n}) = H(Y_{1,D}^{n} | Y_{2,D}) - H(Y_{2,D}) \leq H([h_1 X_{1,D}], [h_2 X_{2,D}]) - H(Y_{2,D}) = H([h_1 X_{1,D}], [h_2 X_{2,D}]) - H(Y_{2,D}) + H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) = H([h_1 X_{1,D}], [h_2 X_{2,D}]) + H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) + H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) \leq H([h_1 X_{1,D}], [h_2 X_{2,D}]) + H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) = H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]),
\]

where \( Y_{2,D} = \{Y_{2,D}^{n} \} [n_1] \) and \( Y_{2,D}^{n} = \{Y_{2,D}^{n} \} [n_1 + 1] \). Now, we can show that

\[
H(Y_{1,D}^{n} | Y_{2,D}) \leq H([h_1 X_{1,D}], [h_2 X_{2,D}]) + H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) = H([h_1 X_{1,D}], [h_2 X_{2,D}]) - H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) \leq H([h_1 X_{1,D}], [h_2 X_{2,D}]) - H(Y_{2,D}),
\]

where the last inequality follows because we have that

\[
H([h_1 X_{1,D}], [h_2 X_{2,D}]) \leq H([h_1 X_{1,D}], [h_2 X_{2,D}]) - H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) \leq H([h_1 X_{1,D}], [h_2 X_{2,D}]) - H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) \leq H([h_1 X_{1,D}], [h_2 X_{2,D}]) - H(Y_{2,D}),
\]

which is why we need to bound (57) by \( H([h_1 X_{1,D}],) \) and \( H(\{h_2 X_{2,D}\}) \). We therefore modify (55) to fit our case in the following way

\[
H(Y_{1,D}^{n} | Y_{2,D}^{n}) = H(Y_{1,D}^{n} | Y_{2,D}) - H(Y_{2,D}) \leq H([h_1 X_{1,D}], [h_2 X_{2,D}]) - H(Y_{2,D}) = H([h_1 X_{1,D}], [h_2 X_{2,D}]) - H(Y_{2,D}) + H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) = H([h_1 X_{1,D}], [h_2 X_{2,D}]) + H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) + H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) \leq H([h_1 X_{1,D}], [h_2 X_{2,D}]) + H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]) = H(Y_{2,D} | [h_1 X_{1,D}], [h_2 X_{2,D}]),
\]

with the last step follows due to equation (59). Now we can bound one (57) with (58) and one with (60). Moreover, we use (49) and (50) on the result. Note that due to our regime, (49) becomes

\[
H([h_2 X_{2,D}]) \leq H(Y_{1,D}^{n}) - n (R_1 + \epsilon_3),
\]

while (50) becomes

\[
H([h_1 X_{1,D}]) \leq H(Y_{1,D}^{n}) - n (R_2 + \epsilon_4).
\]

Putting everything together results in

\[
3n(R_1 + R_2) - n \epsilon_8 \leq 2n n_{c} + n n_{\Delta}.
\]

Dividing by \(3n \) and letting \( n \to \infty \) shows the result. We need to modify a bound on \( H(Y_{1,D}^{n} | Y_{2,D}^{n}) \), if the signal strength \( n_E \) lies in between \( n_1 \) and \( n_2 \). In (58), we see that

\[
H([h_1 X_{1,D}]) - H(Y_{1,D}^{n} | X_{2,D}) \leq n (n_1 - n_E)^{+}.
\]

Moreover, we have that

\[
H([h_1 X_{1,D}]) - H(Y_{1,D}^{n} | X_{2,D}) \leq n (n_E - n_2)^{+}.
\]

Both changes cancel and we get the same result as (58). The result follows on the same lines as in the previous derivation.

VI. CONCLUSIONS

We have shown an achievable scheme for both, the Gaussian multiple-access wiretap channel and the Gaussian wiretap channel with a helper. We used the linear deterministic approximation of both models, to gain insights into the structure and devised novel achievable schemes based on orthogonal bit-aided alignment to achieve secrecy. These techniques can be summarized as signal-scale alignment methods, where we used jamming alignment at the eavesdropper in the signal-scale, while minimizing the negative effect at the legitimate receiver. Both results were then transferred to the Gaussian model, by utilizing layered lattice coding. Moreover, we developed converse proofs for both models, which achieve a constant-gap bound for certain signal power regimes. Those converse techniques were developed for the LD model and then transferred to a truncated deterministic model, which in turn is within a constant-gap of the integer-input integer-output model. The integer-input integer-output model yields converse proofs for the Gaussian models, by invoking a result of [27]. Since our results hold for asymmetrical channel gains and are dependent on those ratios, they can be seen as generalized s.d.o.f. and converge to the known s.d.o.f. results for the channel gain ratio approaching one. Looking into the figure 5, one can see the
achievable rate normalized by the single-link channel, with varying parameter $\beta_1$, i.e. channel gain configurations. One can see that the figure shows the s.d.o.f of $\frac{1}{2}$ for the G-WT-H, and $\frac{2}{3}$ for the G-MAC-WT for $\beta_1 \rightarrow 1$, which agrees with the results of [17]. We can also see, that the achievable rate of both models fluctuates between the upper bound and a lower bound, for the part where the bit-level alignment scheme is dominant. We believe that this is a result of the orthogonal bit-level alignment techniques which get transferred to the Gaussian model. A deterministic model with inter-dependent bit-levels, like the one used in [9], could help to completely reach the upper bound. This would give a constant-gap sum-capacity result for the whole range.

REFERENCES

[1] R. Fritsche and G. Wunder, “Towards a constant-gap sum-capacity result for the gaussian wiretap channel with a helper,” in IEEE International Symposium on Information Theory (ISIT), July 2016, pp. 2978–2982.

[2] ———, “On the deterministic sum-capacity of the multiple access wiretap channel,” arXiv preprint arXiv:1701.07380, 2017.

[3] A. D. Wyner, “The wire-tap channel,” Bell System Technical Journal, vol. 54, no. 8, pp. 1355–1387, 1975.

[4] I. Csiszar and J. Korner, “Broadcast channels with confidential messages,” IEEE Transactions on Information Theory, vol. 24, no. 3, pp. 339–348, May 1978.

[5] S. Leung-Yue-Cheong and M. Hellman, “The gaussian wire-tap channel,” IEEE Transactions on Information Theory, vol. 24, no. 4, pp. 451–456, Jul 1978.

[6] E. Tekin and A. Yener, “The gaussian multiple access wire-tap channel,” IEEE Transactions on Information Theory, vol. 54, no. 12, pp. 5747–5755, Dec 2008.

[7] V. Cadambe and S. Jafar, “Interference alignment and degrees of freedom of the k-user interference channel,” IEEE Transactions on Information Theory, vol. 54, no. 8, pp. 3425–3441, Aug 2008.

[8] M. Maddah-Ali, A. Motahari, and A. Khandani, “Communication over mimo x channels: Interference alignment, decomposition, and performance analysis,” IEEE Transactions on Information Theory, vol. 54, no. 8, pp. 3457–3470, Aug 2008.

[9] U. Niesen and M. Maddah-Ali, “Interference alignment: From degrees of freedom to constant-gap capacity approximations,” IEEE Transactions on Information Theory, vol. 59, no. 8, pp. 4855–4888, Aug 2013.

[10] S. Avestimehr, S. Diggavi, and D. Tse, “A deterministic approach to wireless relay networks,” in Proc. Allerton Conference on Communication, Control, and Computing, Monticello, IL, 2007.

[11] G. Bresler and D. Tse, “The two-user gaussian interference channel: a deterministic view,” European Transactions on Telecommunications, vol. 19, no. 4, pp. 333–354, 2008.

[12] G. Bresler, A. Parekh, and D. Tse, “The Approximate Capacity of the Many-to-One and One-to-Many Gaussian Interference Channels,” IEEE Transactions on Information Theory, vol. 56, no. 9, pp. 4566–4592, 2010.

[13] S. Saha and R. A. Berry, “Sum-capacity of a class of k-user gaussian interference channels within $o(k \log k)$ bits,” Allerton Conf. 2011, 2011.

[14] R. Fritsche and G. Wunder, “On multiuser gain and the constant-gap sum capacity of the gaussian interfering multiple access channel,” arXiv preprint arXiv:1705.04514, 2017.

[15] E. Tekin and A. Yener, “The general gaussian multiple-access and two-way wiretap channels: Achievable rates and cooperative jamming,” IEEE Transactions on Information Theory, vol. 54, no. 6, pp. 2735–2751, June 2008.

[16] J. Xie and S. Ulukus, “Secure degrees of freedom of one-hop wireless networks,” IEEE Transactions on Information Theory, vol. 60, no. 6, pp. 3359–3378, June 2014.

[17] ———, “Secure degrees of freedom of the gaussian wiretap channel with helpers,” in Proc. Allerton Conference on Communication, Control, and Computing, Oct 2012, pp. 193–200.

[18] ———, “Secure degrees of freedom of the gaussian wiretap channel with helpers and no eavesdropper csi: Blind cooperative jamming,” in 2013 47th Annual Conference on Information Sciences and Systems (CISS), March 2013, pp. 1–5.

[19] P. Babashaidarian, S. Salimi, and P. Papadimitratos, “Finite-snr regime analysis of the gaussian wiretap multiple-access channel,” in 2015 53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton), Sept 2015, pp. 307–314.

[20] M. El-Halabi, T. Liu, C. N. Georghiades, and S. Shamai, “Secret writing on dirty paper: A deterministic view,” IEEE Transactions on Information Theory, vol. 58, no. 6, pp. 3419–3429, June 2012.

[21] Y. Chen, H. Vogt, and A. Sezgin, “Gaussian wiretap channels with correlated sources: Approaching capacity region within a constant gap,” in 2014 IEEE International Conference on Communications Workshops (ICC), June 2014, pp. 794–799.

[22] E. Perron, S. Diggavi, and E. Telatar, “On cooperative wireless network secrecy,” in IEEE INFOCOM 2009, April 2009, pp. 1935–1943.

[23] M. Mohapatra and C. R. Murthy, “Secrecy in the 2-user symmetric deterministic interference channel with transmitter cooperation,” in 2013 IEEE 14th Workshop on Signal Processing Advances in Wireless Communications (SPAWC), June 2013, pp. 270–274.

[24] H. Vogt, Z. H. Awan, and A. Sezgin, “On deterministic ic with common and private message under security constraints,” in 2016 IEEE Global Conference on Signal and Information Processing (GlobalSIP), Dec 2016, pp. 947–952.

[25] S. H. Lee, W. Zhao, and A. Khisti, “Secure degrees of freedom of the gaussian diamond-wiretap channel,” IEEE Transactions on Information Theory, vol. 63, no. 1, pp. 496–508, Jan 2017.

[26] G. D. Forney Jr, “On the role of mmse estimation in approaching the information-theoretic limits of linear gaussian channels: Shannon meets wiener,” arXiv preprint cs0409053, 2004.

[27] P. Mukherjee, J. Xie, and S. Ulukus, “Secure degrees of freedom of one-hop wireless networks with no eavesdropper csi,” IEEE Transactions on Information Theory, vol. 63, no. 3, pp. 1898–1922, March 2017.

[28] U. Urez and R. Zamir, “Achieving $\frac{2}{3} \log(1+snr)$ on the awgn channel with lattice encoding and decoding,” IEEE Transactions on Information Theory, vol. 50, no. 10, 2004.

[29] A. G. Davoodi and S. A. Jafar, “Aligned image sets under channel uncertainty: Settling conjectures on the collapse of degrees of freedom under finite precision csi,” IEEE Transactions on Information Theory, vol. 62, no. 10, pp. 5603–5618, Oct 2016.

[30] S. Avestimehr, S. N. Diggavi, and D. N. C. Tse, “Wireless Network Flow: A Deterministic Approach,” IEEE Transactions on Information Theory, vol. 57, no. 4, pp. 1872–1905, 2011.

[31] C. Geng, H. Sun, and S. A. Jafar, “On the optimality of treating interference as noise: General message sets,” IEEE Transactions on Information Theory, vol. 61, no. 7, pp. 3722–3736, July 2015.

[32] R. Fritsche and G. Wunder, “Upper bounds and duality relations of the linear deterministic sum capacity for cellular systems,” in Proc. IEEE International Conference on Communications (ICC), Sydney, Australia, 2014.