Josephson effect in superfluid atomic Fermi-gases

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We consider an analog of the internal Josephson effect in superfluid atomic Fermi-gases. Four different hyperfine states of the atoms are assumed to be trapped and to form two superfluids via the BCS-type of pairing. We show that Josephson oscillations can be realized by coupling the superfluids with two laser fields. Choosing the laser detunings in a suitable way leads to an asymmetric below-gap tunneling effect for which there exists no analogue in the context of solid-state superconductivity.

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Cooling of trapped gases of Fermionic atoms well below the Fermi temperature makes it reasonable to anticipate the achievement of the predicted BCS-transition. The existence of a gap in the excitation spectrum of the superfluid Fermi-gas will be the first issue to address, and several methods for detecting it have already been proposed. Trapped atomic Fermi-gases will allow to study and test fermion-fermion pairing theories in a tunable, controlled manner. For example, the classic problem of the BCS-BEC crossover when the interparticle attraction varies could be studied using the possibility of tuning the interatomic scattering length using Feshbach resonances. Besides the standard superfluid phenomenology, gases of Cooper-paired atoms are expected to have properties which are specific to atomic gases only and not present, or not easily realizable, in metallic superconductors or Helium. For instance, the trapping potential has a major effect on the characteristic lengths of the superfluid Fermi-gas.

In this paper we propose a way to investigate the Josephson effect in trapped superfluids of Fermionic atoms. We find a phenomenon that is unique to atomic Fermi-gases, namely an asymmetry in the Josephson current spectrum. Cooling the superfluid Fermi-gas will be the first issue to address, and several methods for detecting it have already been proposed. Trapped atomic Fermi-gases will allow to study and test fermion-fermion pairing theories in a tunable, controlled manner. For example, the classic problem of the BCS-BEC crossover when the interparticle attraction varies could be studied using the possibility of tuning the interatomic scattering length using Feshbach resonances. Besides the standard superfluid phenomenology, gases of Cooper-paired atoms are expected to have properties which are specific to atomic gases only and not present, or not easily realizable, in metallic superconductors or Helium. For instance, the trapping potential has a major effect on the characteristic lengths of the superfluid Fermi-gas.

The two superfluids are coupled by driving laser-induced transitions between the states and with the laser Rabi frequency and detuning δ, and between the states and with the Rabi frequency and detuning δ'. For a Raman-process these are effective quantities. If several lasers are used then in order to be able to see the Josephson oscillations they should maintain their phase coherence for a time much longer than the inverse of the detunings. In the case of metals, the two superconductors are spatially separated and connected by a tunneling junction. In our scheme, the superfluids share the same spatial region and are connected by the laser-coupling of the atoms’ internal states; this resembles the internal Josephson effect in atomic Bose-Einstein condensates or in superfluid 3He-A.

For metallic superconductors the a.c. Josephson current is driven by applying a voltage over the junction – here the role of the voltage is played by the laser detunings. The difference is that the detunings can be different for the two states forming the pair; in the metallic superconductor analogy this would mean having a different voltage for the spin-up and spin-down electrons, a situation which has not been investigated in the context of metallic superconductors. There is an interesting connection to recent experiments on superconductor-ferromagnet proximity effects, where the chemical potentials of the spin up and down electrons are slightly different in the ferromagnet due to the exchange interaction.

We consider a system described by the standard BCS-theory. The laser interaction is assumed to be a small perturbation and its effect is calculated using linear response theory. The observable of interest is the change
in the number of particles in one of the states, say $|e\rangle$ or $|e'\rangle$.

In the rotating wave approximation the interaction of the laser light with the matter fields can be described by a time-independent Hamiltonian in which the detunings $\delta$ and $\delta'$ play the role of an externally imposed difference in the chemical potential of the two states. The total Hamiltonian becomes then $\hat{H} = \hat{H}_0 + \hat{H}_T$, where

$$\hat{H}_0 = \hat{H}_{BCS} + \left(\mu_e + \frac{\delta}{2}\right) \int d\vec{r} \hat{\psi}_{e'}^\dagger(\vec{r}) \hat{\psi}_e(\vec{r}) + \left(\mu_g - \frac{\delta}{2}\right) \int d\vec{r} \hat{\psi}_g^\dagger(\vec{r}) \hat{\psi}_g(\vec{r})$$

$$+ \left(\mu_e + \frac{\delta'}{2}\right) \int d\vec{r} \hat{\psi}_{e'}^\dagger(\vec{r}) \hat{\psi}_{e'}(\vec{r}) + \left(\mu_g - \frac{\delta'}{2}\right) \int d\vec{r} \hat{\psi}_g^\dagger(\vec{r}) \hat{\psi}_g'(\vec{r})$$

(the definition for $I_e$ is similar) and can be further evaluated with the help of the Schrödinger equation $i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$ as

$$I_e = i \int d\vec{r} \langle \Psi(t)| \Omega^*(\vec{r}) \hat{\psi}_e^\dagger(\vec{r}) \hat{\psi}_{e'}(\vec{r}) - \Omega(\vec{r}) \hat{\psi}_{e'}^\dagger(\vec{r}) \hat{\psi}_e(\vec{r}) |\Psi(t)\rangle.$$  

(3)

In the following we call $I_e$ the current in analogy to metallic superconductors where the flux of electrons out of the superconductor constitutes the electrical current.

We introduce an interaction representation with respect to $\hat{H}_0$ and use linear response theory with respect to $\hat{H}_T$. Validity of the linear response theory requires that the laser intensity is small and the transfer of atoms can be treated as a perturbation.

We split the result for the current $I_e$ into a part which corresponds to the Josephson current $I_{eJ}$ and to the part which describes normal single-particle current $I_{eS}$, $I_e = I_{eS} + I_{eJ}$. The single-particle current can be evaluated at finite temperature using the standard techniques of superconducting Green’s functions; we present however only the result for $T = 0$ and positive detunings:

$$I_{eS} = -2\pi \sum_{n,m} \left| \int d\vec{r} \Omega(\vec{r}) v_e^\dagger(\vec{r}) u_m^\dagger(\vec{r}) \right|^2 \delta(e_e^e + e_m^g - \delta).$$

Here the triplet $(u_n, v_n); \epsilon_n$ is a solution of the (nonuniform) Bogoliubov-de Gennes equations for superconductors $|\Psi\rangle$ and $\delta = \mu_e - \mu_g + \delta$. This is the standard Fermi Golden rule result and very similar to the ones obtained in $[15]$. The current $I_{eS}$ is zero when $\delta < 2\Delta$ since pair breaking is required for single particle excitations. Next we concentrate on the Josephson current – this is non-zero also for detunings $\delta$ that are smaller than twice the gap energy.

The Josephson current becomes...
\[ I_{e,J} = -2\text{Im} \left[ e^{-i(\tilde{\delta} + \tilde{\delta}')} \sum_{n,m} \int d\vec{r} d\vec{r}' \Omega^* (\vec{r}) \Omega^* (\vec{r}') u^n_m (\vec{r}) u^n_m (\vec{r}') v^n_m (\vec{r}) v^n_m (\vec{r}') \left( \frac{1}{\delta' + \epsilon^g_n + \epsilon^m + i\eta} - \frac{1}{\delta' - \epsilon^g_n - \epsilon^m + i\eta} \right) \right]. \]

(4)

The current \( I_{e,J} \) is the same only that \( \tilde{\delta} \) and \( \tilde{\delta}' \) are interchanged. Note that the oscillating term is proportional to both of the detunings whereas the rest of the expression is proportional only to \( \tilde{\delta}' \). For the choice of a homogeneous system (large trap, local density approximation) and a constant laser profile the expression simplifies into

\[ I_{e,J} = I_0 (\tilde{\delta}') \sin((\tilde{\delta} + \tilde{\delta}') t) \] (5)

\[ I_{e,J} = I_0 (\tilde{\delta}) \sin((\tilde{\delta} + \tilde{\delta}') t). \] (6)

Both partners of the pair thus oscillate in phase, with the same frequency \( \tilde{\delta} + \tilde{\delta}' \). But the amplitudes are different whenever the detunings \( \tilde{\delta} \) and \( \tilde{\delta}' \) differ. This means that more atoms are transferred, say, in the \(|g\rangle - |e\rangle \) oscillation than in the \(|g'\rangle - |e'\rangle \) one.

A simple expression for \( I_0(\delta) \) can be derived when we assume identical superfluids, that is \( \Delta = \Delta' \) and \( \mu_g = \mu_e \equiv \mu \):

\[ I_0(\delta) = \frac{\sqrt{2mV}}{\pi^2} \Delta^2 \Omega^2 \int_{-\mu}^{\infty} \frac{d\xi \sqrt{\mu + \xi}}{\sqrt{\xi^2 + \Delta^2 (4\xi^2 + 4\Delta^2 - \delta^2)}}. \]

where \( V \) is the volume of the sample and the variable \( \xi \) is the continuous version of \( \xi_k = \frac{k^2}{2m} - \mu \). Since \( \Delta \ll \mu \), the result can also be written as

\[ I_0(\delta) = \frac{\sqrt{2mV}}{\pi^2} \Delta^2 \Omega^2 \int_{-\mu}^{\infty} \frac{d\xi \sqrt{\mu + \xi}}{\sqrt{\xi^2 + \Delta^2 (4\xi^2 + 4\Delta^2 - \delta^2)}}. \]

A plot of the intensity \( I_0 \) as a function of the detuning \( \delta \) is shown in Fig. 2. This result shows a divergence at \( \delta = 2\Delta \), which reflects the divergence of the density of states for the two superconductors at the gap. For quite a large range of detunings, the amplitude \( I_0 \) of the Josephson current is approximately constant — thus no asymmetry effect will be visible. The asymmetry is most pronounced when the timescale of the oscillation, that is, \( 1/(\delta + \delta') \) is close to \( 1/(2\Delta) \). Note that \( 1/(2\Delta) \) can also be understood as the Cooper pair correlation time based on the uncertainty principle.

According to the conventional intuitive picture of the Josephson effect, the particles forming a Cooper-pair tunnel “together” through the junction. Therefore our result seems counterintuitive at first glance. The physics becomes, however, more transparent by a closer look at the equation (4). For simplicity, we consider here the transfer process to one direction only, from the superfluid \(|gg'\rangle\) to \(|ee'\rangle\), which corresponds to the first denominator in (4). In the initial state \(|g\rangle\) is paired with \(|g'\rangle\), in the final state \(|e\rangle\) with \(|e'\rangle\). The process has, however, an intermediate state as indicated by the second-order form of the observable, and the intermediate states corresponding to the observables \( I_{e,J} \) and \( I_{e,J}' \) are different: For \( I_{e,J} \), \(|g\rangle - |e\rangle \) has been transferred into \(|e'\rangle \). Therefore its pairing partner \(|g\rangle\) is left as an excitation in the superfluid \(|gg'\rangle\) with the energy \( \epsilon^g_n \) and \(|e'\rangle\) becomes an excitation in the superfluid \(|ee'\rangle\) with the energy \( \epsilon^e_m \). In contrast, for \( I_{e,J}' \), the atom in \(|g'\rangle\) remains as a quasiparticle of the energy \( \epsilon^g_n \) in the superfluid \(|gg'\rangle\) and \(|e\rangle\) becomes an excitation in the superfluid \(|ee'\rangle\). For \( I_{e,J}' \), the initial energy of the Cooper pair was \( (\mu_g - \frac{\Delta}{2}) + (\mu_g - \frac{\Delta}{2}) \) and the energy of the intermediate state is \( [(\mu_g - \frac{\Delta}{2}) + \epsilon^g_n] + [(\mu_e + \frac{\Delta}{2}) + \epsilon^e_m] \) (for explanation, see Fig. 3). The relative energy of the intermediate state with respect to the initial state is \( \epsilon^g_n + \epsilon^e_m + \delta' \), which is precisely the first denominator in (4). For \( I_{e,J}' \), the initial energy of the pair is again \( (\mu_g - \frac{\Delta}{2}) + (\mu_g - \frac{\Delta}{2}) \), but the intermediate state has an energy \( [(\mu_g - \frac{\Delta}{2}) + \epsilon^g_n] + [(\mu_e + \frac{\Delta}{2}) + \epsilon^e_m] \), or a relative energy \( \epsilon^g_n + \epsilon^e_m + \delta \). In summary, the intermediate states of the transfer processes for “spin up” and “spin down” atoms have different energies and this results in different amplitudes for \( I_{e,J} \) and \( I_{e,J}' \).

The asymmetry in the currents implies the existence of excitations in the superfluids. We have analyzed the many-body wavefunction of the system in the Schrödinger picture and indeed it contains excitations corresponding to the asymmetry. Specifically, the analysis confirms that the so-called Fermi surface polarization
where $\langle \langle N_e \rangle \rangle = \langle \langle N_e^2 \rangle \rangle / (\langle N_e \rangle + \langle N_e^2 \rangle) \simeq (\langle N_e \rangle - \langle N_e' \rangle) / (\langle N_e \rangle + \langle N_e' \rangle)$ is non-zero and oscillates as $f(\delta, \delta') \cos(\delta + \delta' t)$ where $f(\delta, \delta') = 0$. We also found that time-independent perturbation theory is not sufficient to reveal the asymmetry in the amplitudes: the simple treatment of [19] applied to our system results in symmetric currents because the ansatz used does not allow any excitations. This and the fact that the oscillation is most pronounced for timescales of the order of the Cooper-pair correlation time indicates that the effect is related to the dynamics of the superfluid state.

To observe the Josephson effect one should be able to measure the number of particles in two of the states, e.g. $|e\rangle$ and $|e'\rangle$, at different stages of the oscillations, either destructively or non-destructively. The scale of the gap energy is for typical systems $1$-$100$kHz, which means that the highest time resolution needed should be at least somewhat above $10\mu$s. Measuring the number of particles accurately is the more challenging part of the observation. In [14] we considered laser probing of the superfluid Fermi-gas, where the laser was creating excitations in the BCS state. The number of particles transferred was directly reflected in the absorption of the light. Here one can use similar techniques to detect the Josephson oscillations in a simple way.

In summary, we propose a method to realize Josephson oscillations in superfluid atomic Fermi-gases. The coupling between two superfluids is provided by laser light, and the laser detuning plays the same role as voltage over metallic superconductor junctions. Detunings that affect the two atomic internal states involved in pairing can be chosen to be different – this would correspond to different voltage for spin-up and spin-down electrons. This leads to asymmetry in the oscillation amplitudes of the two states. The asymmetry is pronounced when the timescale of the oscillation is the same order of magnitude as the Cooper-pair correlation time. This is an effect unique to atomic Fermi-gases in the superfluid state.

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