A distinguished Riemannian geometrization for quadratic Hamiltonians of polymomenta

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Abstract

In this paper we construct a distinguished Riemannian geometrization on the dual 1-jet space $J^1(T, M)$ for the multi-time quadratic Hamiltonian function

$$H = h_{ab}(t)g^{ij}(t, x)p^a_i p^b_j + U_{(i)}(t, x)p^a_i + F(t, x).$$

Our geometrization includes a nonlinear connection $N$, a generalized Cartan canonical $N$-linear connection $\mathcal{C}^\Gamma(N)$ (together with its local d-torsions and d-curvatures), naturally provided by the given quadratic Hamiltonian function depending on polymomenta.

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1 Short introduction

In the last decades, numerous scientists were preoccupied by the geometrization of Hamiltonians depending on polymomenta. In such a perspective, we point out that the Hamiltonian geometrizations are achieved in three distinct ways:

♦ the multisymplectic Hamiltonian geometry — developed by Gotay, Isenberg, Marsden, Montgomery and their co-workers (see [10], [9]);

♦ the polysymplectic Hamiltonian geometry — elaborated by Giachetta, Mangiarotti and Sardanashvily (see [7], [8]);

♦ the De Donder-Weyl Hamiltonian geometry — studied by Kanatchikov (see [11], [12], [13]).

In such a geometrical context, the recent studies of Atanasiu and Neagu (see the papers [4], [5] and [6]) are initiating the new way of distinguished Riemannian geometrization for Hamiltonians depending on polymomenta, which is in fact a natural "multi-time" extension of the already classical Hamiltonian geometry on cotangent bundles synthesized in the Miron et al.’s book [16]. Note
that our distinguished Riemannian geometrization for Hamiltonians depending on polymomenta is different one by all three Hamiltonian geometrizations from above (multisymplectic, polysymplectic and De Donder-Weyl).

2 Metrical multi-time Hamilton spaces

Let us consider that $h = (h_{ab}(t))$ is a semi-Riemannian metric on the "multi-time" (temporal) manifold $T^m$, where $m = \dim T$. Let $g = (g^{ij}(t^c, x^k, p^c_k))$ be a symmetric d-tensor on the dual 1-jet space $E^* = J^1(T, M^n)$, which has the rank $n = \dim M$ and a constant signature. At the same time, let us consider a smooth multi-time Hamiltonian function $E^* \ni (t^a, x^i, p^a_i) \to H(t^a, x^i, p^a_i) \in \mathbb{R}$, which yields the fundamental vertical metrical d-tensor

$$G^{(i)(j)}_{(a)(b)} = \frac{1}{2} \frac{\partial^2 H}{\partial p^a_i \partial p^b_j},$$

where $a, b = 1, \ldots, m$ and $i, j = 1, \ldots, n$.

**Definition 1** A multi-time Hamiltonian function $H : E^* \to \mathbb{R}$, having the fundamental vertical metrical d-tensor of the form

$$G^{(i)(j)}_{(a)(b)}(t^c, x^k, p^c_k) = \frac{1}{2} \frac{\partial^2 H}{\partial p^a_i \partial p^b_j} = h_{ab}(t^c)g^{ij}(t^c, x^k, p^c_k),$$

is called a **Kronecker h-regular multi-time Hamiltonian function**.

In such a context, we can introduce the following important geometrical concept:

**Definition 2** A pair $\text{MH}^n_m = (E^* = J^1(T, M), H)$, where $m = \dim T$ and $n = \dim M$, consisting of the dual 1-jet space and a Kronecker h-regular multi-time Hamiltonian function $H : E^* \to \mathbb{R}$, is called a **multi-time Hamilton space**.

**Remark 3** In the particular case $(T, h) = (\mathbb{R}, \delta)$, a "single-time" Hamilton space will be also called a **relativistic rheonomic Hamilton space** and it will be denoted by $\text{RRH}^n = (J^1(\mathbb{R}, M), H)$.

**Example 4** Let us consider the Kronecker h-regular multi-time Hamiltonian function $H_1 : E^* \to \mathbb{R}$ given by

$$H_1 = \frac{1}{mc} h_{ab}(t)\varphi^{ij}(x)p^a_i p^b_j,$$

where $h_{ab}(t) \ (\varphi_{ij}(x),$ respectively) is a semi-Riemannian metric on the temporal (spatial, respectively) manifold $T \ (M, \text{respectively})$ having the physical meaning of **gravitational potentials**, and $m$ and $c$ are the known constants from...
Theoretical Physics representing the mass of the test body and the speed of light. Then, the multi-time Hamilton space $G_{M^H_n} = (E^*, H_1)$ is called the multi-time Hamilton space of the gravitational field.

Example 5 If we consider on $E^*$ a symmetric d-tensor field $g^{ij}(t, x)$, having the rank $n$ and a constant signature, we can define the Kronecker h-regular multi-time Hamiltonian function $H_2 : E^* \to \mathbb{R}$, by setting

$$H_2 = h_{ab}(t) g^{ij}(t, x) p^a_i p^b_j + U^{(i)}_{(a)}(t, x) p^a_i + \mathcal{F}(t, x), \quad (2)$$

where $U^{(i)}_{(a)}(t, x)$ is a d-tensor field on $E^*$, and $\mathcal{F}(t, x)$ is a function on $E^*$. Then, the multi-time Hamilton space $G_{ED_{M^H_n}} = (E^*, H_2)$ is called the non-autonomous multi-time Hamilton space of electrodynamics. The dynamical character of the gravitational potentials $g_{ij}(t, x)$ (i.e., the dependence on the temporal coordinates $t^c$) motivated us to use the word "non-autonomous".

An important role for the subsequent development of our distinguished Riemannian geometrical theory for multi-time Hamilton spaces is represented by the following result (proved in the paper [4]):

Theorem 6 If we have $m = \dim T \geq 2$, then the following statements are equivalent:

(i) $H$ is a Kronecker h-regular multi-time Hamiltonian function on $E^*$.

(ii) The multi-time Hamiltonian function $H$ reduces to a multi-time Hamiltonian function of non-autonomous electrodynamic type. In other words we have

$$H = h_{ab}(t) g^{ij}(t, x) p^a_i p^b_j + U^{(i)}_{(a)}(t, x) p^a_i + \mathcal{F}(t, x). \quad (3)$$

Corollary 7 The fundamental vertical metrical d-tensor of a Kronecker h-regular multi-time Hamiltonian function $H$ has the form

$$G_{(a)(b)}^{(i)(j)} = \frac{1}{2} \frac{\partial^2 H}{\partial p^a_i \partial p^b_j} = \begin{cases} h_{11}(t) g^{ij}(t, x^b, p^1_i), & m = \dim T = 1 \\ h_{ab}(t^c) g^{ij}(t^c, x^b), & m = \dim T \geq 2. \end{cases} \quad (4)$$

We recall that the transformations of coordinates on the dual 1-jet vector bundle $J^1(T, M)$ are given by

$$\tilde{t}^a = \tilde{t}^a(t^b), \quad \tilde{x}^i = \tilde{x}^i(x^j), \quad \tilde{p}_i^a = \frac{\partial x^j}{\partial x^i} \frac{\partial \tilde{t}^a}{\partial t^b} p^b_j,$$

where $\det \left( \frac{\partial \tilde{t}^a}{\partial t^b} \right) \neq 0$ and $\det \left( \frac{\partial \tilde{x}^i}{\partial x^j} \right) \neq 0$. In this context, let us introduce the following important geometrical concept:

Definition 8 A pair of local functions on $E^* = J^1(T, M)$, denoted by

$$N = \left( N_{(a)}^{(i)b}, N_{(a)}^{(i)j} \right),$$
whose local components obey the transformation rules

\[ \tilde{N}^{(b)}_{1(j)c} \frac{\partial \tilde{c}}{\partial a} = N^{(c)}_{1(k)a} \frac{\partial \tilde{b}}{\partial \tilde{c}} \frac{\partial x^k}{\partial \tilde{x}^j} - \frac{\partial \tilde{p}^b}{\partial \tilde{t}}, \]

\[ \tilde{N}^{(b)}_{2(j)k} \frac{\partial x^k}{\partial x^l} = N^{(c)}_{2(k)c} \frac{\partial \tilde{b}}{\partial \tilde{c}} \frac{\partial x^k}{\partial \tilde{x}^j} - \frac{\partial \tilde{p}^b}{\partial \tilde{x}^l}, \]

is called a nonlinear connection on \( E^* \). The components \( N^{(a)}_{1(i)b} \) (resp. \( N^{(a)}_{2(i)j} \)) are called the temporal (resp. spatial) components of \( N \).

Following now the geometrical ideas of Miron from [14], the paper [4] proves that any Kronecker \( h \)-regular multi-time Hamiltonian function \( H \) produces a natural nonlinear connection on the dual 1-jet space \( E^* \), which depends only by the given Hamiltonian function \( H \):

**Theorem 9** The pair of local functions \( N = \left( N^{(a)}_{1(i)b}, N^{(a)}_{2(i)j} \right) \) on \( E^* \), where \( (\chi^a_{bc}) \) are the Christoffel symbols of the semi-Riemannian temporal metric \( h_{ab} \)

\[ N^{(a)}_{1(i)b} = \chi^a_{bc} \theta_c^i, \]

\[ N^{(a)}_{2(i)j} = \frac{h_{ab}}{4} \left[ \frac{\partial g_{ij}}{\partial x^l} \frac{\partial H}{\partial \tilde{p}^k_l} - \frac{\partial g_{ij}}{\partial x^l} \frac{\partial H}{\partial \tilde{p}^k_l} + g_{ik} \frac{\partial^2 H}{\partial x^j \partial \tilde{p}^k_l} + g_{jk} \frac{\partial^2 H}{\partial x^i \partial \tilde{p}^k_l} \right], \]

represents a nonlinear connection on \( E^* \), which is called the canonical nonlinear connection of the multi-time Hamilton space \( MH^m_n = (E^*, H) \).

Taking into account the Theorem 6 and using the generalized spatial Christoffel symbols of the d-tensor \( g_{ij} \), which are given by

\[ \Gamma^k_{ij} = \frac{g^{kl}}{2} \left( \frac{\partial g_{ij}}{\partial x^l} + \frac{\partial g_{ij}}{\partial x^l} - \frac{\partial g_{ij}}{\partial x^l} \right), \]

we immediately obtain the following geometrical result:

**Corollary 10** For \( m = \dim T \geq 2 \), the canonical nonlinear connection \( N \) of a multi-time Hamilton space \( MH^m_n = (E^*, H) \), whose Hamiltonian function is given by \( (3) \), has the components

\[ N^{(a)}_{1(i)b} = \chi^a_{bc} \theta_c^i, \]

\[ N^{(a)}_{2(i)j} = -\Gamma^k_{ij} \theta^a_k + T^{(a)}_{(i)j}, \]

where

\[ T^{(a)}_{(i)j} = \frac{h_{ab}}{4} \left( U_{ib} \theta_j + U_{jb} \theta_i \right), \]

and

\[ U_{ia} = g_{ik} U^{(k)}_{(i)b}, \quad U_{kb \theta r} = \frac{\partial U_{kb \theta r}}{\partial x^l} - U_{sb} \Gamma^s_{kr}. \]
3 The Cartan canonical connection $CT(N)$ of a metrical multi-time Hamilton space

Let us consider that $MH^m_n = (J^1(T, M), H)$ is a multi-time Hamilton space, whose fundamental vertical metrical d-tensor is given by (4). Let $N = \left( \begin{array}{cc} N_{(a)}^{(i)b} & N_{(a)}^{(i)(j)} \\ \end{array} \right)$ be the canonical nonlinear connection of the multi-time Hamilton space $MH^m_n$.

**Theorem 11 (the generalized Cartan canonical $N$-linear connection)**

On the multi-time Hamilton space $MH^m_n = (J^1(T, M), H)$, endowed with the canonical nonlinear connection $N$, there exists an unique $h$-normal $N$-linear connection $C\Gamma(N) = \left( \chi^a_{bc}, A^i_{jc}, H^i_{jk}, C^{ij(k)}_{j(c)} \right)$, having the metrical properties:

(i) $g_{ij|k} = 0$, $g^{ij|k}_{(c)} = 0$,

(ii) $A^i_{jc} = g^{il}_{(i)} \frac{\delta g_{lj}}{\delta t^c}$, $H^i_{jk} = H^j_{ik}$, $C^{ij(k)}_{j(c)} = C^{ji(k)}_{j(c)}$,

where $\nabla^a_{|c}$, $\nabla^i_{|c}$ and $\nabla^{ij(k)}_{|c}$ represent the local covariant derivatives of the $h$-normal $N$-linear connection $CT(N)$.

**Proof.** Let $CT(N) = \left( \chi^a_{bc}, A^i_{jc}, H^i_{jk}, C^{ij(k)}_{j(c)} \right)$ be an $h$-normal $N$-linear connection, whose local coefficients are defined by the relations

\[
A^a_{bc} = \chi^a_{bc}, \quad A^i_{jc} = g^{il}_{(i)} \frac{\delta g_{lj}}{\delta t^c},
\]

\[
H^i_{jk} = g^{ir}_{(i)} \left( \frac{\delta g_{jr}}{\delta x^k} + \frac{\delta g_{kr}}{\delta x^j} - \frac{\delta g_{jk}}{\delta x^r} \right),
\]

\[
C^{ij(k)}_{j(c)} = - g^{ir}_{(i)} \left( \frac{\partial g^{jr}}{\partial p^k} + \frac{\partial g^{kr}}{\partial p^j} - \frac{\partial g^{jk}}{\partial p^r} \right).
\]

Taking into account the local expressions of the local covariant derivatives induced by the $h$-normal $N$-linear connection $CT(N)$, by local calculations, we deduce that $CT(N)$ satisfies conditions (i) and (ii).

Conversely, let us consider an $h$-normal $N$-linear connection

\[
\hat{C}T(N) = \left( \hat{A}^a_{bc}, \hat{A}^i_{jc}, \hat{H}^i_{jk}, \hat{C}^{ij(k)}_{j(c)} \right)
\]

which satisfies conditions (i) and (ii). It follows that we have

\[
\hat{A}^a_{bc} = \chi^a_{bc}, \quad \hat{A}^i_{jc} = g^{il}_{(i)} \frac{\delta g_{lj}}{\delta t^c}.
\]
Moreover, the metrical condition $g_{ij|k} = 0$ is equivalent with

$$\frac{\delta g_{ij}}{\delta x^k} = g_{rj} \tilde{H}_{rk}^i + g_{ir} \tilde{H}_{jk}^r.$$  

Applying now a Christoffel process to indices $\{i, j, k\}$, we find

$$\tilde{H}_{jk}^i = \frac{g_{ir}^j}{2} \left( \frac{\delta g_{jr}}{\delta x^k} + \frac{\delta g_{kr}}{\delta x^j} - \frac{\delta g_{jk}}{\delta x^r} \right).$$  

By analogy, using the relations $C_{j(c)}^{i(k)} = C_j^{k(c)}$ and $g_{ij|c} = 0$, together with a Christoffel process applied to indices $\{i, j, k\}$, we obtain

$$\overline{C}_{j(c)}^{i(k)} = -\frac{g_{ir}}{2} \left( \frac{\delta g_{jr}}{\delta p^c_k} + \frac{\delta g^{jr}}{\delta p^c_j} - \frac{\delta g^{jk}}{\delta p^c_r} \right).$$  

In conclusion, the uniqueness of the generalized Cartan canonical connection $CT(N)$ on the dual 1-jet space $E^* = J^1(T, M)$ is clear.

**Remark 12** (i) Replacing the canonical nonlinear connection $N$ of the multi-time Hamilton space $MH^n_m$ with an arbitrary nonlinear connection $\hat{N}$, the preceding Theorem holds good.

(ii) The generalized Cartan canonical connection $CT(N)$ of the multi-time Hamilton space $MH^n_m$ verifies also the metrical properties

$$h_{ab/c} = h_{ab|k} = h_{ab|c} = 0, \quad g_{ij/c} = 0.$$  

(iii) In the case $m = \text{dim } T \geq 2$, the coefficients of the generalized Cartan canonical connection $CT(N)$ of the multi-time Hamilton space $MH^n_m$ reduce to

$$A_{bc}^a = \chi_{bc}^a, \quad A_{jc}^i = \frac{g_{il}^j}{2} \frac{\partial g_{lj}}{\partial t^c}, \quad H_{jk}^i = \Gamma_{jk}^i, \quad C_{j(c)}^{i(k)} = 0.$$  

### 4 Local d-torsions and d-curvatures of the generalized Cartan canonical connection $CT(N)$

Applying the formulas that determine the local d-torsions and d-curvatures of an $h$-normal $N$-linear connection $D\Gamma(N)$ (see these formulas in [22]) to the generalized Cartan canonical connection $CT(N)$, we obtain the following important geometrical results:

**Theorem 13** The torsion tensor $T$ of the generalized Cartan canonical connection $CT(N)$ of the multi-time Hamilton space $MH^n_m$ is determined by the local
where

(i) for $m = \dim \mathcal{T} = 1$, we have

$$T^r_{ij} = -A^r_{ji}, \quad P^r_{(j)i(1)} = C^r_{i(1)}, \quad P^{(1)}_{(r)i(1)} = \frac{\partial N^{(1)}_{r(1)}}{\partial p_j^r} + A^j_{r1} - \delta^j_{ri},$$

$$P^{(1)}_{(r)ij} = \frac{\delta N^{(1)}_{r(1)}}{\delta x^i} - \frac{\delta N^{(1)}_{r(1)}}{\delta x^j}.$$  

(ii) for $m = \dim \mathcal{T} \geq 2$, using the equality (5) and the notations

$$\chi^c_{fab} = \frac{\partial \chi^c_{fa}}{\partial \theta^b} - \frac{\partial \chi^c_{fb}}{\partial \theta^a} + \chi^d_{fa} \chi^c_{db} - \chi^d_{fb} \chi^c_{da},$$

$$\Omega^{rc}_{ki} = \frac{\partial \Gamma^r_{ki}}{\partial x^j} - \frac{\partial \Gamma^r_{kj}}{\partial x^i} + \Gamma^p_{ki} \Gamma^r_{pj} - \Gamma^p_{kj} \Gamma^r_{pi},$$

we have

$$T^r_{aj} = -A^r_{ja}, \quad P^{(f)}_{(r)a(b)} = \delta^f_{b} A^r_{ia}, \quad P^{(f)}_{(r)ab} = \chi^f_{gab} \eta^g_r,$$

$$R^{(f)}_{(r)aj} = -\delta^r_{aj} T^{(c)}_{(r)cj}, \quad R^{(f)}_{(r)ij} = -\Omega^{k}_{r} T_{(r)ij}.$$

**Theorem 14** The curvature tensor $\mathcal{R}$ of the generalized Cartan canonical connection $CT(N)$ of the multi-time Hamilton space $MB_m^N$ is determined by the
following adapted local curvature $d$-tensors:

| $h_T$ | $h_M$ | $v$ |
|-------|-------|-----|
| $m \geq 1$ | $m = 1$ | $m \geq 2$ |
| $m = 1$ | $m = 1$ | $m = 1$ |
| $m \geq 2$ | $m \geq 2$ | $m \geq 2$ |

$h_T h_T$ $\chi^d_{abc}$ $0$ $R^l_{ibc}$ $0$ $-R^{(d)(i)}_{(l)(a)bc}$

$h_M h_T$ $R^l_{i1k}$ $R^l_{ibk}$ $-P^{(1)(1)}_{(i)(1)1k} = -R^{(d)(i)}_{(l)(a)bk}$

$vh_T$ $P^l_{i(i11)(k)}$ $0$ $-P^{(1)(1)(1)}_{(i)(1)(1)(1)} = -P^{(k)}_{i(j)(1)(1)}$ $0$

$h_M h_M$ $R^l_{ijk}$ $g^l_{ijk}$ $-P^{(1)(1)(1)}_{(i)(1)(1)jk} = -R^{(d)(i)}_{(l)(a)jk}$

$vh_M$ $P^l_{i(k)(j)}$ $g^l_{ijk}$ $-P^{(1)(1)(1)}_{(i)(1)(1)jk} = -R^{(d)(i)}_{(l)(a)jk}$

$vv$ $S^l_{i(j)(k)}$ $g^l_{ijk}$ $-S^{(1)(1)(1)}_{i(j)(1)(1)} = -S^{(d)(i)(k)}_{(l)(i)j}$

where, for $m \geq 2$, we have the relations

$-R^{(d)(i)}_{(l)(a)bc} = \delta^d_{abc} - \delta^d_u R^u_{ibc},$ $-R^{(d)(i)}_{(l)(a)bk} = -\delta^d_u R^u_{ibk},$ $-R^{(d)(i)}_{(l)(a)jk} = -\delta^d_u g^l_{ijk},$

and, generally, the following formulas are true:

(i) for $m = \dim T = 1$, we have $\chi_{111} = 0$ and

\[ R^l_{i1k} = \frac{\delta A^l_{i1k}}{\delta x^k} - \frac{\delta H^l_{ik}}{\delta t} + A^l_{i1k} H^l_{rk} - H^l_{ik} A^l_{r1} + C^{(r)}_{(1)(1)} R^{(1)}_{(r)1k}, \]

\[ R^l_{ijk} = \frac{\delta H^l_{ij}}{\delta x^k} - \frac{\delta H^l_{ik}}{\delta x^j} + H^l_{ij} H^l_{rk} - H^l_{ik} H^l_{rj} + C^{(r)}_{(1)(1)} R^{(1)}_{(r)jk}, \]

\[ p^l_{i(i11)(k)} = \frac{\partial A^l_{i(i11)(k)}}{\partial p^l_k} - C^{(k)}_{i(i11)} + C^{(r)}_{i(i11)(r)} p^{(1)}_{i(i11)(1)}, \]

\[ p^l_{i(j)(k)} = \frac{\partial H^l_{ij}}{\partial p^l_k} - C^{(k)}_{i(j)(1)} + C^{(r)}_{i(j)(r)} p^{(1)}_{i(j)(1)}, \]

\[ g^l_{ij(k)(k)} = \frac{\partial C^{(k)}_{i(j)(1)}(1)}{\partial p^l_k} + \frac{\partial C^{(k)}_{i(j)(1)}(1)}{\partial p^l_j} + C^{(r)}_{i(j)(1)} C^{(k)}_{i(j)(r)} - C^{(k)}_{i(j)(1)} C^{(r)}_{i(j)(r)}(1), \]

(ii) for $m = \dim T = 2$, we have

\[ \chi^d_{abc} = \frac{\partial A^d_{abc}}{\partial t^c} - \frac{\partial A^d_{abc}}{\partial t^b} + A^d_{abc} f^d_{fc} - r_{abc} f^d_{db}, \]

\[ R^l_{ibc} = \frac{\partial A^l_{ibc}}{\partial t^c} - \frac{\partial A^l_{ibc}}{\partial t^b} + A^l_{ibc} A^l_{rc} - A^l_{ir} A^l_{cb}, \]

\[ R^l_{ibk} = \frac{\partial A^l_{ibk}}{\partial x^k} - \frac{\partial A^l_{ibk}}{\partial x^b} + A^l_{ibk} r_{rk} - r_{ik} A^l_{rb}, \]

\[ g^l_{ij(k)} = \frac{\partial r^l_{ij}}{\partial x^k} - \frac{\partial r^l_{ij}}{\partial x^j} + r^l_{ij} r_{rk} - r_{ik} r^l_{rj}, \]

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