Tests of fundamental quantum mechanics and dark interactions with low-energy neutrons

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Abstract | Among the known particles, the neutron is special because it provides experimental access to all of the four fundamental forces and a wide range of hypothetical interactions. Despite being unstable, free neutrons live long enough to be used as test particles in interferometric, spectroscopic and scattering experiments probing low-energy scales. Recognized already in the 1970s, fundamental concepts of quantum mechanics can be tested in neutron interferometry using silicon perfect single crystals. Besides enabling tests of uncertainty relations or Bell inequalities, neutrons offer the opportunity to observe the effects of gravity and hypothetical dark forces acting on extended matter wavefunctions. Such tests gained importance in the light of recent discoveries of inconsistencies in the understanding of cosmology and the incompatibility between quantum mechanics and general relativity. Experiments with low-energy neutrons are, thus, indispensable tools for probing fundamental physics and represent a complementary approach to particle colliders. In this Review, we discuss the history and experimental methods used at this low-energy frontier of physics and overview the current bounds and limits on quantum mechanical relations and dark energy interactions.
Key points

- The neutron is an excellent probe of various interactions, because it is sensitive to all four fundamental forces and hypothetical beyond-standard-model interactions.
- Matter-wave interferometry with neutrons offers several advantages, such as macroscopic beam separation, individual control of the sub-beams and long interaction and coherence times at room temperature.
- The neutron as a single-particle quantum system is almost ideal to study fundamental concepts of quantum mechanics, such as entanglement, weak values or uncertainty relations, because it can be prepared, manipulated and detected with high efficiency and accuracy.
- Tensions between observation and the standard model of Big Bang cosmology and the ongoing quest to understand the nature of the dark sector are a strong motivation for new physics.
- The neutron as a single-particle quantum system is almost ideal to study fundamental interaction and coherence times at room temperature. The neutron is an excellent probe of various interactions, because it is sensitive to all four fundamental forces and hypothetical beyond-standard-model interactions. (Sagnac effect)\(^1\), motional effects on the wavefunction (Fizeau effect)\(^2\) and spin–rotation coupling (Sagnac–Mashhoon effect)\(^3\) were also demonstrated. In more recent experiments, Newton’s inverse square law of gravity was tested at micron distances on an energy scale of \(10^{-14}\) eV searching for hypothetical fifth forces. Gravity resonance spectroscopy (GRS) with ultracold neutrons\(^4\)–\(^6\) (UCNs) and neutron interferometry with thermal neutrons\(^7\) provide constraints on many possible new interactions, such as screened DE chameleon and symmetron fields.

**Matter-wave interferometry.** The superposition of de Broglie matter waves, commonly referred to as matter-wave interferometry, was first observed for electrons\(^8\) in the mid-1950s, followed by thermal neutrons\(^9\)–\(^11\) in the 1970s and atoms two decades later\(^12\)–\(^15\). Then, even larger objects, such as molecules (with masses exceeding 10,000 amu) were used in interference experiments\(^16\). Neutron interferometry\(^17\), which makes use of the Bragg diffraction by successive slices cut from a single (perfect) crystal of silicon, provides a powerful tool for matter-wave interference. Neutrons are particularly well suited for applications in matter-wave interferometry for several reasons. First, neutron interferometers offer a macroscopic beam separation of several centimetres. Although a macroscopic beam separation is also found in atom interferometry, neutron interferometry provides the unique possibility to manipulate the sub-beams individually by inserting various neutron optical devices (see Box 1). Furthermore, neutrons can be detected with an efficiency close to one, and, owing to the relatively large de Broglie wavelength of thermal neutrons (see wavelength/energy spectrum in Box 1), neutrons can be manipulated coherently and efficiently over a long distance, for instance, local realistic\(^18\) or non-contextual hidden variables\(^19\), have been ruled out, and no indications have been found for any contradiction of QM\(^20\). A century after its inception, QM has found many practical applications and formed the basis for new technologies, such as quantum computation.

**Relativity and inertia in quantum mechanics.** As every massive object, the neutron is affected by Newton’s gravitational force. The resulting interaction appears not only in the trajectories of motion but also in the phase of the wavefunction, when the neutron is in a spatial superposition of different gravitational potentials. The first experimental demonstration of the associated gravitational phase shift was measured\(^21\) in 1975 using a neutron interferometer. Soon afterwards, the Earth’s rotation affecting the orbital angular momentum (Fizeau effect)\(^22\) and spin–rotation coupling (Sagnac–Mashhoon effect)\(^23\) were also demonstrated. In more recent experiments, Newton’s inverse square law of gravity was tested at micron distances on an energy scale of \(10^{-14}\) eV searching for hypothetical fifth forces. Gravity resonance spectroscopy (GRS) with ultracold neutrons\(^4\)–\(^6\) (UCNs) and neutron interferometry with thermal neutrons\(^7\) provide constraints on many possible new interactions, such as screened DE chameleon and symmetron fields.

**Box 1 Neutron’s properties and interaction with matter and fields**

Neutrons are usually classified into regions, depending on their energy as depicted below. At all energy scales relevant for this Review, a neutron can be described by the non-relativistic Schrödinger equation:

\[
-\frac{\hbar^2}{2m}\nabla^2 + V(r, t) \psi(r, t) = i\hbar \frac{\partial}{\partial t}\psi(r, t).
\] (9)

Interactions with nuclei and matter are defined by the potential \(V\). For nuclei located at \(r\), we have \(V_{\text{nuc}}(r) = 2\pi\hbar^2b_i/m_n\delta(r - r_i)\) depending on the coherent scattering length \(b_i\). At lower energies (cold and ultracold neutrons), the wavelength of the neutron exceeds the interatomic distances and only the optical (average pseudo-Fermi) potential \(V_{\text{opt}} = V_i = 2\pi\hbar^2b_i\rho/m_n\) is important. If magnetic fields are involved, we have, in addition, \(V_{\text{mag}} = -\mu \cdot B(r, t) = -\mu_0 \sigma \cdot B(r, t)\), with the neutron’s magnetic dipole moment \(\mu\). Gravity and hypothetical potentials also have to be included.

The interaction potentials and typical numbers are tabulated below. \(B\) is the magnetic field strength, \(g\) is the gravitational acceleration, \(m_n\) is the neutron mass and \(V_{\text{opt}}\) denotes the optical (average pseudo-Fermi) potential of silicon.

### Table

| Interaction        | Potential         | Typical numbers |
|--------------------|-------------------|-----------------|
| Nuclear (optical)  | \(2\pi\hbar^2b_i\delta(r)/m_n\) | \(V_i \approx 50\,\text{neV}\) |
| Gravitational      | \(m_n g \cdot r\) | \(\approx 100\,\text{neV per m}\) |
| Magnetic           | \(-\mu \cdot B(r, t)\) | \(\approx 60\,\text{neV per T}\) |

| Range     | Ultracold | Cold | Thermal | Epithermal | Fast | Relativistic |
|-----------|-----------|------|---------|------------|------|-------------|
| Velocity (m s\(^{-1}\)) | 300 \(\text{neV}\) | 10 \(\text{meV}\) | 300 \(\text{meV}\) | 1\(\text{keV}\) | 10\(\text{keV}\) | 20 MeV |
| Wavelength (m) | \(10^{-6}\) | \(10^{-9}\) | \(10^{-10}\) | \(10^{-11}\) | \(10^{-12}\) | \(10^{-15}\) |
| Energy (eV) | \(10^{-4}\) | \(10^{-9}\) | \(10^{-10}\) | \(10^{-12}\) | \(10^{-15}\) | \(10^{-18}\) |
Box 2 | Single perfect crystal neutron interferometry

A silicon perfect crystal neutron interferometer (figure, top right) is geometrically analogous to the optical Mach–Zehnder interferometer (figure, top left). On the left side, BS stands for beam splitter, M stands for mirror, PS is a phase shifter, D for detector, and a and b denote the two interfering beams. The right side depicts a neutron interferometer, consisting of a single silicon perfect crystal, cut in such a way that the incoming monochromatic neutron beam is split by Bragg diffraction at angle $\theta_B$ at the first plate. Behind the first plate, the single neutron’s wavefunction is found in a coherent superposition of the partial waves $\psi_I$ and $\psi_r$, belonging to paths I and II, respectively. By rotating the phase shifter plate by an angle $\eta$, the phase shift $\chi$ can be tuned systematically, owing to the change of the relative optical path length through the phase shifter in paths I and II. After a second beam splitter (acting as a mirror), the interfering sub-beams are recombined at the third plate, yielding $\pi$-shifted interference fringes in intensities $I_o$ and $I_m$, in the forward (O) and refracted (H) directions.

Neutrons are affected by all four fundamental interactions, which are used in realizing neutron optical components, as shown in the toolboxes for nuclear (nuc), gravitational (grav) and magnetic (mag) phase shifts (figure, lower panel), where $V$ denotes the respective potentials and $\phi$ the respective phases. Phase shifters: by inserting a phase shifter plate (or phase flag) into the interferometer, the phase relation between the two sub-beams, belonging to paths I and II, can be varied. The phase shift is given by $\chi = N_d \phi_0$, with atom density $N_d$ in the phase shifter plate of thickness $D$, the coherent scattering length $b_c$, and the neutron wavelength $\lambda$.

Rotation stages: after rotation of the entire interferometer by an angle $\alpha$ around the incident beam direction by employing a tilting stage, the neutron’s split wave packets travel at different heights, and, therefore, at different gravitational potentials. This yields a dispersive phase shift, which depends on the angle $\alpha$, the area $A$ enclosing the sub-beams and the wavelength $\lambda$. Spin rotators: the neutron couples to magnetic fields via its permanent magnetic dipole moment $\mu$, where static as well as time-dependent magnetic fields $B(r,t)$, applied over a field region of length $l$, are used to realize arbitrary spinor rotations, due to the Larmor precession of the neutron’s polarization vector. With time-dependent magnetic fields, not only the neutron’s spin but also its total energy can be manipulated owing to photon absorption or emission.

Neutron optical elements

- Phase shifter
  - $V_{nuc} = \frac{2\hbar b_c \phi_0}{m_n}$
  - $\phi_{nuc} = N_b \lambda A D$

- Rotation stage
  - $V_{grav} = m_n g r$
  - $\phi_{grav} = m_n g \lambda A \sin \alpha/(2 \hbar)$

- (RF) spin rotator
  - $V_{mag} = -\mu \cdot B(r,t)$
  - $\phi_{mag} = \pm \mu B(r,t)/(2 \hbar)$
polarizability is many orders of magnitude smaller than that of atoms. Neutron interferometry offers fairly stable (stationary) experimental conditions, owing to the monolithic structure of the interferometer crystal, resulting in comparatively high contrast of over 90%\(^6\),\(^4\). Atom interferometry is also used to study fundamental physics, such as atomic masses\(^5\),\(^6\),\(^7\) and the fine-structure constant \(\alpha\) (\textit{REF.} \(^8\)), Newton’s gravitational constant \(G\) (\textit{REF.} \(^9\)), tests of the equivalence principle\(^1\),\(^2\), and DM and DE searches (such as chameleon models)\(^3\),\(^4\). Many of these topics are also investigated by autonomously operating atom interferometers in drop towers (under microgravity)\(^5\),\(^6\),\(^7\), on sounding rockets\(^8\), or in free space (for example, on the International Space Station)\(^9\). See \textit{REF.} \(^10\) for a recent review on atom interferometry.

**Dark matter.** The particle content of the standard model describes only a small fraction of the matter and energy distribution in our universe. In the 1930s, Fritz Zwicky postulated the existence of DM, which is now well established\(^11\). Although the origin of DM is still unknown, it certainly does not lie within the standard model. Possible candidates for DM have been found in hypothetical particles. A prime example is the axion\(^12\),\(^13\),\(^14\),\(^15\),\(^16\). This scalar particle was introduced as the consequence of an elegant mechanism proposed to solve the so-called strong charge parity (CP) problem\(^17\),\(^18\),\(^19\). From robust astrophysical constraints, it is known that the axion mass has to be very small\(^1\) (\(\lesssim 10\) meV). This implies long-range forces, which are, in principle, observable in laboratory experiments\(^20\). Heavier axions can be ruled out, since they would have produced observable effects in astrophysical objects or terrestrial experiments. Lighter axions, however, are not ruled out experimentally and even have a rather strong theoretical motivation\(^21\),\(^22\). The search for axions gained momentum when it was realized that they provide a possible DM candidate\(^23\),\(^24\),\(^25\),\(^26\),\(^27\),\(^28\). Other proposals for similar light bosons are collectively called axion-like particles (ALPs)\(^29\),\(^30\),\(^31\),\(^32\),\(^33\),\(^34\),\(^35\) (in \textit{REF.} \(^36\), experimental results of nuclear electron dipole moment searches including those for new spin-dependent forces have been compared in order to distinguish the effects of the original axion from generic ALPs).

**Dark energy.** Contrary to expectations, the expansion of the universe was found to be accelerating\(^37\),\(^38\),\(^39\),\(^40\). The standard theory describing the evolution of the universe, general relativity, can account for the accelerated expansion only when a so-called cosmological constant is included\(^41\). But this inclusion leads to a severe fine-tuning problem\(^42\), which indicates the incompleteness of the theory. However, the obvious solution to modify general relativity at cosmological scales gives rise to theoretical inconsistencies. Another way out is to add hypothetical new fields, which then describe the substance called DE. Quintessence denotes a proposal to describe the accelerated expansion of the universe by mimicking the effect of a time-dependent cosmological ‘constant’ via the potential energy of a hypothetical new scalar field\(^43\). Such an additional field (sometimes referred to as ‘cosmon’) generically leads to new interactions, so-called fifth forces, that are now strongly constrained by manifold observational tests at distance scales of the Solar System and below\(^44\). A way to avoid observational constraints while still having DE on cosmological scales can be achieved through the so-called ‘screening mechanisms’. These mechanisms suppress the scalar fields or their interaction with standard model matter in regions of high matter density, whereas they allow the scalar field to prevail in interstellar space, where it can effectively drive the cosmic expansion. Among the several different possible classes of mechanisms, screening via non-linear self-interaction terms, that let the effective potential (and, thereby, the field profile and interaction strength) depend on the local ambient energy density, has gained popularity. To this class belong the so-called chameleon models\(^45\),\(^46\),\(^47\),\(^48\),\(^49\). By now, several laboratory experiments have searched for chameleons. Another similar model is known as the symmetron model, which uses an effective potential similar to the Higgs potential\(^50\),\(^51\),\(^52\). Unfortunately, these scalar fields still require a certain amount of fine-tuning and the addition of a non-zero cosmological constant\(^53\).

**Tests of quantum-tuning and the addition of a non-zero cosmological constant\(^53\).**

**Tests of quantum-mechanics.** Neutron interferometry and its tools to study quantum mechanics. In the 1960s, progress in semiconductor technology made it possible to produce monolithic perfect crystal silicon ingots, with a diameter of several inches, from which a single-crystal interferometer can be cut out. In 1974, such an interferometer was illuminated with thermal neutrons, which resulted in the first observed interference fringes\(^54\) at the 250-kW TRIGA research reactor at the Atomistitut, Technische Universität Wien. In this experiment, a beam of neutrons was split by amplitude division and superposed coherently after passing through different regions of space. During these macroscopic spatial separations, the neutron’s wavefunction can be modified in phase and amplitude in various ways. In the interferometer, neutrons exhibit self-interference, since, at any given instant, at most a single neutron propagates through the interferometer. The interferometer is geometrically analogous to the well-known Mach–Zehnder interferometer in photonics, as illustrated in Box 2. A matter-wave interferometer (with completely separated sub-beams) was first realized by using neutrons; this is due to the following facts: first, thermal neutrons have moderate de Broglie wavelength of \(~2\) Å, which is comparable to widely used X-ray wavelength, and, second, charge neutrality of neutrons provides robustness against the disturbance from electronic interactions, which allows a higher degree of coherence to be maintained for a prolonged time. This type of interferometer enables the realization of a quantum optical experiment using matter waves on a macroscopic scale; this has opened up a new era of investigations concerning fundamental quantum mechanical phenomena using matter waves. In the following years, numerous remarkable experiments on the foundations of QM have been carried out using neutron interferometry, for example, the verification of the 4\(\pi\) spinor symmetry\(^55\), which is, nowadays, applied in quantum gates, where a \(2\pi\) rotation of the applied system is performed to induce a \(\pi\) phase shift. Furthermore, the
superposition of different spin orientations of the two interferometer arms and the phase shift arising from a lateral confinement of a neutron beam passing through a narrow slit system have been investigated.

**Entanglement and contextuality.** The counter-intuitive nature of QM, especially its probabilistic character, has been called into question from its very beginning. Albert Einstein, Boris Podolsky and Nathan Rosen argued that QM is incomplete, with a hidden, but more complete and deterministic, physics underlying it. In 1964, John S. Bell proved in his celebrated theorem that all hidden-variable theories, or realistic theories based on the assumptions of locality and realism, lead to conflicts with the predictions of QM. Bell introduced inequalities that hold for the predictions of any local hidden-variable theory but are violated by QM. In parallel with Bell’s work, Simon Kochen and Ernst Specker (KS) devised another powerful argument, which shows that logical contradictions occur if non-contextual (realistic) assumptions are considered. This theorem shows that the predictions of QM are incompatible with the following assumptions: a definite value of the measurements, that is, observables \( A \) and \( B \) have predefined values \( v(A) \) and \( v(B) \) prior to a measurement, and non-contextuality, that is, the outcome of a measurement is assumed to be independent of the experimental context. This means that the measured value is the same, irrespective of whether any commuting observables are measured jointly. Quantum contextuality tests with neutrons are achieved by entangling the neutron’s different degrees of freedom (DOF), which is referred to as intra-particle entanglement and is depicted in Fig. 1. The first experimental realization of a spin-path entangled neutron state was demonstrated by violating a Bell-like inequality reported in ref. 23. The Bell-like single-neutron state consists of a spin-path entanglement, which is realized by placing a spin flipper (depicted BOX 2) in one arm of the interferometer. The neutron’s state is described by a tensor product Hilbert space \( \mathcal{H} = H_\alpha \otimes H_\chi \), where the former corresponds to the spin wavefunction and the latter to the spatial wavefunction. Since observables of the spatial part commute with those of the spin part, one can prepare a Bell-like state \( \left| \Psi^\text{Bell}_n \right\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle \otimes |\uparrow\rangle + |\downarrow\rangle \otimes |\downarrow\rangle \right) \), where \(|\uparrow\rangle \) and \(|\downarrow\rangle \) correspond to the spin up and down eigenstates and \(|\uparrow\rangle \) and \(|\downarrow\rangle \) represent the two path eigenstates in the interferometer. Measurements of the joint expectation value of spin and path \( E(\alpha, \chi) \), where \( \alpha \) and \( \chi \) denote the measurement directions of spin and path, respectively, yield a violation of the realistic Bell-like inequality \(-2 \leq S^\text{KS}_{\text{real}}(\alpha, \chi) \leq 2\). This is in contrast to the predictions of QM, which yield \( S^\text{QM}(\alpha, \chi) = 2\sqrt{2} \). In the actual experiment, the figure of merit \( S^{\text{KS}}_{\text{real}}(\alpha, \chi) = E(\alpha, \chi) + E(\alpha, \chi) - E(\alpha, \chi) + E(\alpha, \chi) \) is determined by applying the respective measurement direction for spin and path \( \alpha \) and \( \chi \) (with \( i = 1, 2 \)). The results of the Bell experiments reported in refs. 23-27, where newly developed spin rotators made it possible to obtain data with higher accuracy, are given in the respective light blue box in Fig. 1.
Furthermore, in Ref.5, a test of the KS theorem was carried out with a neutron interferometer, where a combination of six observables has been evaluated30,5. A contradiction with the predictions of non-contextual hidden-variable theories was obtained owing to the contextual nature of QM. For the proof of the KS theorem, one should once again consider single neutrons prepared in a maximally entangled Bell-like state denoted as \( |\Psi_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle \otimes |\uparrow\rangle + |\downarrow\rangle \otimes |\downarrow\rangle) \). The proof is based on the six observables \( \hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z, \hat{d}_p^x, \hat{d}_p^y, \hat{d}_p^z \), where \( s \) and \( p \) denote the spin and path degrees of freedom of the respective Pauli matrices \( \sigma \) and \( d_p \). The inconsistency between a non-contextual hidden-variable theory and QM arises in any attempt to ascribe the predefined values \(-1\) or \(+1\) to each of the six observables. Since experiments never show perfect (anti-)correlations owing to their finite precision, one has to derive an inequality that can be experimentally tested. It can be shown that, in any non-contextual hidden-variable theory, the relation \( S_{\text{KS}}^{\text{real}} = (\hat{\sigma}_x \cdot \hat{d}_p^x) - (\hat{\sigma}_y \cdot \hat{d}_p^y) - (\hat{\sigma}_z \cdot \hat{d}_p^z) \leq 1 \) holds, in contrast to the quantum mechanical prediction \( S_{\text{QM}}^{\text{real}} = 3 \). A violation of this inequality directly reveals quantum contextuality. The final experimental results in favour of the quantum mechanical predictions, thereby ruling out non-contextuality, from Ref.30 are given in the light blue box in Fig. 1.

However, in addition to the spin and path DOF, the (total) energy of a neutron can also be entangled, generating tripartite entangled states, such as the so-called Greenberger–Horne–Zeilinger (GHZ) state30,5, which has already been generated in a neutron interferometer. Using a radio frequency spin-flipper in one path of the interferometer, one can manipulate the neutrons’ total energy, thereby realizing a triple entanglement between the path, spin and energy DOF, \( |\Psi_{\text{GHZ}}\rangle \equiv (|\uparrow\rangle \otimes |\uparrow\rangle \otimes |\uparrow\rangle + |\downarrow\rangle \otimes |\downarrow\rangle \otimes |\downarrow\rangle) \otimes (|\uparrow\rangle \otimes |\uparrow\rangle \otimes |\uparrow\rangle - |\downarrow\rangle \otimes |\downarrow\rangle \otimes |\downarrow\rangle) \), where \( E_0 \) denotes the neutron's initial (total) energy. The setup is schematically illustrated in Fig. 1. In the 1990s, Mermin analysed the GHZ argument in detail and derived an inequality suitable for experimental tests to distinguish between predictions of QM and realistic theories30. According to non-contextual hidden-variable theories, one can calculate a limit for the sum of certain observables’ expectation values, which can be tested in an experiment. For the expectation value of \( M_{\text{GHZ}} = \hat{\sigma}_x \hat{d}_p^x + \hat{\sigma}_y \hat{d}_p^y - \hat{\sigma}_z \hat{d}_p^z \), non-contextual hidden-variable theories set a maximum limit of \( M_{\text{GHZ}}^{\text{real}} = 2 \). In contrast, quantum theory predicts an upper bound of \( M_{\text{GHZ}}^{\text{QD}} = 4 \). Consequently, any measured value of \( M_{\text{GHZ}} > 2 \) decides in favour of quantum contextuality. The results obtained in Ref.30 are given in the light blue box in Fig. 1 and again illustrate the inconsistency between the quantum mechanical predictions and a non-contextual (realistic) model.

In a more recent experiment, the Larmor spin-echo instrument located at the second target station of the ISIS Neutron and Muon Source, which is part of the Rutherford Appleton Laboratory in the UK, has been used to perform Bell tests and to study GHZ states5. The main difference to the interferometric approach is that, in a neutron spin-echo interferometer, the trajectories of the individual spin states are manipulated by using refraction through magnetic fields. The spatial separation of trajectories thereby induced ranges from nanometres to microns, whereas the beam separation in a silicon perfect crystal interferometer is on the order of several centimetres (see Fig. 1 for results).

The quantization of orbital angular momentum (OAM) — another DOF — of bound massive particles and free photons was discovered a long time ago. However, in recent years, stable OAM has also been observed for electrons in terms of so-called electron vortex beams100. Furthermore, it was demonstrated that mixed OAM states can be prepared in free thermal neutrons using a spiral phase plate101 and magnetic gradients102. However, with these methods, it is not possible to create pure OAM states, owing to the small coherence length of thermal neutrons. OAMs of mixed type are usually referred to as extrinsic OAM, since each neutron in the beam has the same OAM with respect to the optical beam axis but a different intrinsic OAM. So, a different experimental approach to generate pure OAM states is required. Among the theoretically developed new methods for generating OAM in neutrons are magnetic quadrupoles that can create neutron OAM entangled to the neutron spin (spin–orbit states)103, a series of perpendicular linear magnetic gradients forming a spin–orbit lattice, parity-violating neutron–nucleus interactions, where the neutron spin is rotated slightly around the momentum vector owing to the weak interaction104, and static homogeneous electric fields, since, in an electric field, the particle spin couples to the cross product between the electric field strength and the particle momentum. This procedure involves the preparation of a spin–orbit textured lattice of vortices wavefront. A variety of textures can be generated using this method105, including skyrmion-like geometries. In Ref.105, an experimental procedure is presented, whereby a static homogeneous electric field, polarized along the direction of particle propagation, induces longitudinal spin–orbit states, while a transversely polarized electric field generates transverse spin–orbit states. The latter type of OAM has not yet been observed in massive free particles. OAM is foreseen to be used in multipartite entanglement, where four neutron DOF, namely, spin, path, total energy and OAM, contribute to a multipartite entangled single-neutron state.

Weak values and weak measurements. Another example of counter-intuitive prediction of QM concerns the introduction of a new concept of quantum measurements, such as weak measurements, with the resulting weak values, as first proposed106 by Yakir Aharonov in 1988. The first corresponding experiment was realized using an optical setup107. The peculiarity of the weak value is that its value may lie far outside the range of an observable’s eigenvalues and that it allows information to be extracted from a quantum system with only minimal disturbance. The weak value of a Hermitian operator \( \hat{A} \) is defined as \( \langle \hat{A} \rangle_{\omega} = \langle \psi \rangle \hat{A} \langle \psi \rangle \) (the indices \( i \) and \( f \) stand for initial and final states, respectively)
and the corresponding weak measurement involves three steps: first, the preparation of an initial quantum state $|\psi\rangle$ (pre-selection) of the system; second, a weak coupling of this system with a probe, that is, another quantum system and described by an interaction Hamiltonian; the interaction is supposed to be sufficiently weak, so that the system is only minimally disturbed; third, the post-selection of the final quantum state $|\psi_f\rangle$, by performing a standard projective measurement of another observable $\hat{B}$ of the system. Finally, a measurement on the probe is performed, usually referred to as pointer readout, yielding the weak value $\langle \hat{A} \rangle_w$.

The weak value has been found to be useful as a technique that aims to amplify small signals\textsuperscript{108-111}, also demonstrated with neutrons (see setup in FIG. 2a), where all aspects of the weak value of the neutron's Pauli spin operator $\hat{\sigma}_z$ that is, its real component, as well as the imaginary component, are experimentally determined, as reported in ref. \textsuperscript{35}. The real part of the weak value exhibits values lying outside the usual range of spin eigenvalues, that is ±1, ranging from −3.2 to 3.4. Furthermore, non-zero values for the imaginary part of the weak value $\langle \hat{A} \rangle_w$ have been observed, which can be seen in FIG. 2b. These results are an unambiguous quantum mechanical effect, since no classical theory can describe the observed weak measurement results. This new experimental scheme allows the full determination of the weak value, which can be used to characterize the evolution of the neutron’s wavefunction inside an interferometer, just as in a report of a photonic double-slit experiment\textsuperscript{114}.

Another application of the weak value concerns the estimation of quantum states\textsuperscript{36,114-119} (see ref. \textsuperscript{120} for a recent review). Quantum state tomography is a well-known approach to reconstruct a quantum state. However, quantum state tomography involves a lot of computational data post-processing. In 2011, a new, more direct, tomographical method was established that allows a quantum state to be determined via a weak measurement without the post-processing\textsuperscript{115}. However, that new method had a drawback: because of this weakness of the measurement, the information gain is very low for each measurement and these have to be repeated several times. A new measurement scheme, where the system’s quantum state is characterized via direct
measurements (without the need for computational post-processing), using strong interactions, was proposed and demonstrated in a neutron interferometric experiment\(^\text{133}\). The use of strong measurements significantly reduces the measurement time and enables the determination of the quantum state with higher precision and accuracy compared with weak measurements. The results obtained are not limited to that particular quantum system, for example, neutrons, but are generally valid and, as such, may be applied to many other quantum systems as well.

In addition, weak values and weak measurements have been successfully applied to quantum paradoxes such as the three-box problem\(^\text{122}\), Hardy’s paradox\(^\text{123,125}\), the quantum pigeonhole principle\(^\text{126}\) and the quantum Cheshire cat\(^\text{14,127}\). The last two have been investigated with neutron interferometric experiments. Before discussing the experimental results, we recall the quantum Cheshire cat in more detail. In the book ‘Alice’s Adventures in Wonderland’, the smile of the Cheshire cat can exist in the absence of the cat’s body. Such a phenomenon, which, at first, seems absurd, is actually possible in a quantum mechanical sense for the quantum Cheshire cat in a Mach–Zehnder interferometer. Here, the cat itself is located in one beam path, while its grin is located in the other\(^\text{127}\). An artistic depiction of this behaviour can be seen in FIG. 2c.

In the neutron interferometric realization of the quantum Cheshire cat\(^4\), the neutron’s path plays the role of the cat and the cat’s grin is represented by the neutron’s spin component along the z-direction. The system is initially prepared in an entangled state, given by \(\psi = \frac{1}{\sqrt{2}}(\langle \uparrow \rangle \langle \uparrow \rangle + \langle \downarrow \rangle \langle \downarrow \rangle)\). For an observation of the quantum Cheshire cat, after pre-selection of an ensemble, a weak measurement of the neutron’s population in a given path on the one hand and of the value of the spin in a given path on the other is performed. Subsequently, the ensemble is post-selected in the final state, which is the product state \(\psi_f = \frac{1}{\sqrt{2}}[(\langle \uparrow \rangle + \langle \downarrow \rangle)\langle \uparrow \rangle + (\langle \uparrow \rangle - \langle \downarrow \rangle)\langle \downarrow \rangle]\). The weak values of the projection operators on the neutron path eigenstates \(\Pi_j = \langle j \mid j \rangle\), with \(j = 1\) and \(1\), yield \(\langle \Pi_1 \rangle_w = 0\) and \(\langle \Pi_1 \rangle_w = 1\). The first expression indicates that a weak interaction coupling of the spatial wavefunction to a probe localized on path I has no effect on the probe on average — the system behaves as if there were no neutrons travelling on path I. The weak value of the spin component along each path \(j\) suggests the location of the neutron’s spin component. The appropriate observable of neutrons’ spin component in path \(j\) is given by \(\langle \hat{\sigma}_z \Pi_j \rangle_w\), which yields \(\langle \hat{\sigma}_z \Pi_1 \rangle_w = 1\) and \(\langle \hat{\sigma}_z \Pi_1 \rangle_w = 0\), for paths I and II, respectively. The experimentally obtained values of the weak measurements are determined by the observed intensities, plotted in FIG. 2d, and are explicitly given by \(\langle \Pi_1 \rangle_w = 0.14(4),\ (\Pi_1 \rangle_w = 0.96(6),\ (\langle \hat{\sigma}_z \Pi_1 \rangle_w = 1.07(25)\) and \(\langle \hat{\sigma}_z \Pi_1 \rangle_w = 0.02(24)\) for path and spin, respectively. The results have been published in REF. 14. Later photonic realizations of the quantum Cheshire cat were reported in REF. 15.

Uncertainty relations. In his original paper\(^\text{130}\) from 1927, Werner Heisenberg proposed a reciprocal relation for the mean error of a position measurement and disturbance on the momentum measurement, thereby predicting a lower bound on the uncertainty of a joint measurement of incompatible observables. However, this relation also sets an upper bound on the accuracy with which the values of non-commuting observables can be simultaneously prepared. Although, in the past, these two statements have often been mixed, they are now clearly distinguished as measurement uncertainty and preparation uncertainty relations, which is schematically illustrated in FIG. 3a, b, respectively. Heisenberg was inspired by Einstein’s realistic view to base a new physical theory only on observable quantities (elements of reality), arguing that terms such as velocity or position make no sense without defining an appropriate apparatus for a measurement. By considering solely the Compton effect, Heisenberg proposed the famous gamma ray microscope thought experiment. It gives a heuristic estimate for the product of the inaccuracy (error) of a position measurement \(p_i\) and the disturbance \(q_i\) induced on the particle’s momentum, denoted as \(p_i q_i \sim \hbar\).

Heisenberg’s paper presented his idea only heuristically. The first rigorously proven uncertainty relation for position \(Q\) and momentum \(P\) was introduced by Earle Kennard\(^\text{131}\) as \(\Delta(Q)\Delta(P) \geq \frac{\hbar}{2}\), in terms of standard deviations defined as \(\Delta^2(A) = \langle \psi | A^2 | \psi \rangle - \langle \psi | A | \psi \rangle^2\) for an observable \(A\). However, the gamma ray microscope sets a lower bound for the product of the measurement error and the disturbance in a joint measurement of position \(Q\) and momentum \(P\). Hence, the position–momentum uncertainty relation in terms of standard deviations quantifies how precise one can be prepared with respect to the observables of interest. In 1929, Howard Robertson extended\(^\text{132}\) Kennard’s relation to arbitrary pairs of observables \(A\) and \(B\) as \(\Delta(A)\Delta(B) \geq \frac{1}{\epsilon} \langle \psi | [A, B] | \psi \rangle\), with commutator \([A, B] = AB - BA\). The corresponding generalized form of Heisenberg’s original error-disturbance uncertainty relation (measurement uncertainty relation) would read \(\epsilon(A)\eta(B) \geq \frac{1}{2\epsilon} \langle \psi | [A, B] | \psi \rangle\) (see definition of \(\epsilon\) and \(\eta\) below). However, certain measurements do not obey this relation, as was pointed out\(^\text{133}\) in 1988. Consequently, in 2003, the correct form of a generalized error-disturbance uncertainty based on rigorous theoretical treatments of quantum measurements was introduced\(^\text{134,138}\):

\[
\epsilon(A)\eta(B) + \epsilon(A)\Delta(B) + \Delta(A)\eta(B) \geq \frac{1}{2}\langle \psi | [A, B] | \psi \rangle.
\]  

(1)

In this relation (known as the Ozawa relation), \(\epsilon(A)\) denotes the root mean square (r.m.s.) error of an arbitrary measurement for an observable \(A\), \(\eta(B)\) is the r.m.s. disturbance on another observable \(B\) induced by the measurement, and \(\Delta(A)\) and \(\Delta(B)\) are the standard deviations of \(A\) and \(B\), respectively, in the state \(|\psi\rangle\) before the measurement. The first measurement of observable \(A\) (with error \(\epsilon(A)\)) disturbs the subsequent second measurement of observable \(B\), modifying it to an effective \(B\) measurement (with disturbance \(\eta(B)\)), due to the change of the initial state from \(|\psi\rangle\) to \(|\psi\rangle\), which is caused by the first measurement. In principle, the first measurement can be made with arbitrary accuracy,
which means that $\epsilon(A) = 0$; however, in that case, the disturbance of the second measurement will be maximal, such that no information can be gained from the second measurement. Hence, instead of exactly measuring $A$, an approximate measurement, denoted as $O_x$, is performed, aiming to reduce the disturbance on the second measurement. Although universally valid, Ozawa’s relation is not yet optimal and it was tightened by Cyril Branciard via:

$$\epsilon^2(A)\Delta^2(B) + \Delta^2(A)\eta^2(B) + 2\epsilon(A)\eta(B)\sqrt{\epsilon^2(A)\Delta^2(B) - C_{AB}^2} \geq C_{AB}^2,$$

with $C_{AB} = \frac{1}{2}\left|\langle[\psi|A, B]\rangle\right|$. A tight error-disturbance uncertainty relation, describing the optimal trade-off relation between error $\epsilon(A)$ and disturbance $\eta(B)$, is obtained in the special case of $\Delta(A) = \Delta(B) = 1$ and replacing $\epsilon(A)$ and $\eta(B)$ by $\epsilon(A) = \epsilon(A)\sqrt{\frac{C_{AB}^2}{4}}$ and
The interest in measurement uncertainty relations revealed that the Robertson–Schrödinger uncertainty relation lacks an irreducible or state-independent lower bound. Consequently, Ref. 110 proposed state-independent uncertainty relations for arbitrary Pauli observables $A = \mathbf{a} \cdot \sigma$ and $B = \mathbf{b} \cdot \sigma$ of a two-level system (qubit), expressed as $\Delta(A) + \Delta(B) \geq |\mathbf{a} \mathbf{b}|$. However, this relation is not tight in general. Hence, Ref. 112 proposed a state-independent tight preparation uncertainty expressed as:

$$\Delta^2(A) + \Delta^2(B) + 2 |\mathbf{a} \cdot \mathbf{b}| \sqrt{1 - \Delta^2(A) \sqrt{1 - \Delta^2(B)}} \geq 1 + |\mathbf{a} \cdot \mathbf{b}|^2,$$

for standard deviations $\Delta(A)$ and $\Delta(B)$. Applying the neutron polarimetric setup depicted in Fig. 5c, Eq. (5) was tested and reported in Ref. 112. In Fig. 5f, experimental results in Ref. 113 tight-state-independent preparation uncertainty, together with the lower bound from Ref. 114 and the (trivial) bound from Ref. 115, can be seen.

**Tests of gravity and dark interactions**

Another topic in which neutrons have contributed significantly concerns the investigation of (modified) gravity, DM and DE. Testing the plethora of DM and DE models requires a variety of different experiments. Because of their small dielectric polarizability and vanishing electric charge and dipole moment, neutrons are not susceptible to most electrostatic background effects affecting other test particles. For this reason, neutron experiments give the tightest constraints on parameters for some models at interaction ranges between femtometers and micrometres. In the following, we briefly review the most important experimental methods used in neutron physics to perform precision tests of dark interactions.

**Neutron scattering.** Neutron scattering (see reviews in Refs 103,114) on solid or gaseous targets is a well-established method to not only measure scattering parameters but also to derive limits on non-Newtonian forces. In the experiment, thermal neutrons with wavelengths of a few Ångström units and momentum $k_n$ are directed onto a solid or fluid target, where they interact with the nuclei of the material. Since these nuclei have femtometre diameters, the interaction is well described in the Born approximation as point-like. Accordingly, the isotropically or Bragg (s-wave) scattered intensity $I(q)$ is recorded by large-scale detectors and, thus, depends exclusively on the difference $\mathbf{q} = \mathbf{k}_\text{out} - \mathbf{k}_\text{in}$ by which the neutron momentum is changed in the interaction. The independence of the geometric angle renders neutron scattering robust, as $q$ can be determined conveniently from time-of-flight information.

Apart from a factor $C_{\text{exp}}$ determined by the experimental setup, the intensity can be calculated exactly from $I(q) = C_{\text{exp}} |\mathbf{b}(q)|^2$, with a factor $S$ that equals unity in the case of gases and the Born approximation, the total scattering length $b$ can be written as a Fourier integral:

$$b(q) = \frac{m_n}{2\pi\hbar^2} \int d^3r V(r) e^{i\mathbf{q} \cdot \mathbf{r}},$$

depending on the interaction potential $V(r)$ between a neutron and an atom at distance $r$. Now, $b$, or, respectively, $V(r)$, can be separated into different contributions described in detail in the literature116. Firstly, the nuclear interaction $b_n(q)$ can be approximated by a Fermi delta potential in the slow neutron regime $k_n r \ll 1$. Secondly, the neutron's magnetic dipole moment and spin give rise to electromagnetic interactions $b_m(q)$ with the field and spin of the nucleus. Thirdly, the neutron contains the distributed charges of its quarks, leading to a non-vanishing electromagnetic potential in the slow neutron regime $k_n r \ll 1$. Additionally, the associated electrostatic $b_e(q)$ and polarization $b_p(q)$ terms are at least two orders of magnitude smaller than nuclear and magnetic contributions, for which they are mostly neglected in experimental evaluations. For measurements with unpolarized neutrons or nuclei, the magnetic term is already strongly reduced. In practice, the different contributions are rewritten in the form:

$$b(q) = b_n(q) + b_m(q) [\hbar^2 I(I + 1)]^{1/2} \sigma \cdot 1 + \text{magnetic terms},$$

with the coherent and incoherent scattering lengths $b_n(q)$ and $b_m(q)$, respectively, the neutron spin vector $\sigma$ and the nucleus spin $F$ with eigenvalue $\hbar^2 I(I + 1)$. Additional minor corrections are discussed in the literature116. Experimentally, the total scattering length is determined from the intensity via Eq. (7). For Bragg diffraction, $q$ is fixed by the scattering angle $2\theta$ to $q = 4\pi\lambda \sin \theta$, whereas for the transmission method, $q$ can be varied over a wide range. Theoretical and experimental accuracy in the determination of $b(q)$ are both...
**Box 3 | Potentials of hypothetical dark interactions**

Hypothetical dark matter interactions typically involve spin-0 or spin-1 bosons. The exchange of a spin-0 particle between two fermions results in the potentials:

\[ V_{SS}(r) = \frac{g_{s1} g_{s2} \hbar c}{4\pi} \frac{e^{-r/\lambda}}{r} \quad \text{scalar-scalar} \quad (10) \]

\[ V_{SP}(r) = \frac{g_{s1} g_{p2} \hbar^2 s \cdot \hat{r}}{4\pi m_2} \left[ \frac{1 + \frac{1}{\lambda}}{r} + (1 \leftrightarrow 2) \right] \quad \text{scalar-pseudoscalar} \quad (11) \]

depending on the separation \( r \) between fermions 1 and 2 with unit vector \( \hat{r} = \mathbf{r}/r \). (neutron) spin \( s = (\hbar/2)\mathbf{p} \) and interaction range \( \lambda \). Permutation symmetry requires the addition of the same potential with exchanged indices \( (1 \leftrightarrow 2) \) for the potentials arising from two different vertices. Exchange of a massive spin-1 boson induces the position-space interaction potentials (where \( \mathbf{p} \) are the configuration space momentum operators and \( \{ , \} \) denotes the anti-commutator).

\[ V_{AV}(r) = \frac{g_{a1} g_{a2} \hbar^2 s \cdot \hat{r}}{4\pi m_2} \left[ \frac{1 + \frac{1}{\lambda}}{r} + (1 \leftrightarrow 2) \right] \text{ vector-axial vector,} \]

\[ V_{AA}(r) = \frac{g_{a1} g_{a2} \hbar^2 s \cdot (\mathbf{v} \times \hat{r})}{8\pi m_2} \left[ \frac{1 + \frac{1}{\lambda}}{r} + \frac{1}{\lambda} \right] \text{ dual axial vector} \quad (13) \]

We note that, in neutron experiments, only limits for the potentials \( V_{AV} \) and \( V_{AA} \) have been derived. Expressions for the remaining potentials \( V_{AN}, V_{AN} \) and massless spin-1 boson exchange can be found in Ref. 129 but have not been considered in neutron experiments. Historically, generic Yukawa interactions have directly been related to modifications of the Newtonian potential with the identification \( a = - q^2 \hbar c/(4\pi G_N m_1 m_2) \), where either \( m_1 = m_2 \) for neutron scattering or \( m_1 = m_2 \) for most other experiments. We also give limits for the screened quintessence models chameleon and symmetron. For low densities, they are given by the lagrangians 75:

\[ \mathcal{L}_{\text{Cha}} = \frac{1}{2} \frac{\partial \phi}{\partial \xi} \frac{\partial \phi}{\partial \xi} - \frac{\alpha^2 q}{\rho}\frac{\partial \phi}{\partial \xi} - \frac{\alpha^2}{\rho^2} \frac{\partial \phi}{\partial \xi} - \frac{\alpha^2}{\rho^2} \frac{\partial \phi}{\partial \xi} \]

\[ \mathcal{L}_{\text{Sym}} = \frac{1}{2} \frac{\partial \phi}{\partial \xi} \frac{\partial \phi}{\partial \xi} - \frac{1}{2} \frac{\rho}{\rho^2 - \mu^2} \frac{\partial \phi}{\partial \xi} - \frac{1}{4} \frac{\partial \phi}{\partial \xi} \]

where \( \phi \) denotes the scalar field, \( \alpha, \beta, M \) and \( \mu \) are model parameters, \( \rho \) is the mass density of the environment and \( M_0 = 1/\sqrt{8\pi G_N} \) is the reduced Planck mass. Depending on the details of the experimental setup, the scalar field takes on a field profile and induces a potential \( V(r) \).

The first limits on non-Newtonian forces on the basis of \( ^{208}\text{Pb} \) neutron scattering data were reported in Ref. 139. A more detailed analysis of neutron scattering data from \( ^{208}\text{Pb} \) in the 1.26–24 keV range with respect to limits on dark interactions was reported in Ref. 137. Reference 160 combined data from 13 independent measurements of forward and total cross sections to obtain limits \( a q^2 \lambda^2 [1 + (q\lambda)^2] \leq 0.0013 \text{ fm}^2 \) in the range 1 pm–5 nm. Reference 161 performed Bragg scattering on polycrystalline silicon and extracted limits from their data. The best current limits on general Yukawa forces from neutron scattering have been obtained using a gaseous Xe target 155,161.

Neutron scattering offers insight into a range of parameters, among which the coherent scattering length is only one. The downside is that not only do all four fundamental forces contribute to the measured intensity but also that the dependence on \( q \) needs to be modelled accurately. The low-energy tails of resonances at higher energies need to be known and taken into account via the Breit–Wigner formalism, which further complicates data evaluation. For this reason, more direct optical methods have gained popularity and are discussed below. Neutron interferometry is described separately in another section.

**Neutron optics.** There are several different methods in this category that commonly use neutrons at energies <1 eV. Such neutrons have wavelengths larger than the interatomic spacings and mainly interact with the optical (average pseudo–Fermi) potential \( V_f = (2\hbar^2/m)\mathbf{p}^2 \), where \( \rho \) denotes the atomic density and \( b_f = b_0 + b_R + b_Z[1 - f(q)] \) receives contributions from neutron–nucleon interactions \( b_R \), electric polarizability \( b_0 \) and interaction with the distributed charge of the nucleus \( b_Z \). The latter is scaled by the charge number \( Z \) of the nucleus and the form factor \( f(q) \). An extensive review of the relevant theory can be found in Ref. 164. Descriptions of the experimental methods total reflection, Christiansen filter and prism refraction can be found in Refs 139,142. Neutron gravity refractometry (NGR) 139,142 is one of the most precise methods for coherent scattering length measurements. Cold neutrons are admitted to freely fall down a well-defined height \( h \) onto a target surface, where they are reflected from \( V_r \). For heights larger than the critical height \( h_c \), the reflected intensity rapidly declines and the scattering length is extracted from \( V_r = m_g h_c \). An additional hypothetical long-range interaction would be indistinguishable from \( V_r \) and, therefore, influence respective measurements.

One major difference between the results of neutron scattering and neutron optics is that the former depend on the scattering length at \( q = 0 \), while for the latter, \( q = 0 \) is most important. Neutron optical effects are described in the same way as their counterparts in photon optics via a refractive index:

\[ n^2 = \left( \frac{k_m}{k_{out}} \right)^2 = 1 - \frac{4\pi \rho N}{m_g} b_0(0) \xi, \]

where \( \xi \) is a correction factor for multiple scattering and local field effects 165,166 at the \( 10^{-4} \) level. The reason why \( n \)
only depends on $b_1$ at $q = 0$ is that, in diffraction (transmission) theory\textsuperscript{166,167}, the transmitted wave is created by coherence between the incident $k_{n}$ and refracted wave $nk_{m}$ in the material, which only gives a contribution of around\textsuperscript{167} $|n - 1| \ll 1$. At $q = 0$ also, most form factors reduce to unity, which simplifies the analysis and reduces uncertainties.

A long-range Yukawa interaction with a $q$-dependence described by Eq. (7) would influence $b_{n,\gamma}$ measured in neutron optics at $q = 0$ differently than $b_{n,\gamma}$ determined in scattering experiments at $q \neq 0$. The various experiments on $b_{1}$ are, thus, differently sensitive to hypothetical interactions. A limit on $\alpha$ can be extracted by demanding\textsuperscript{118} $V_{c} < (2n\hbar^{2}/m_{n})(b_{\gamma,\gamma} - b_{\gamma,\gamma})$ from neutron-nucleon and neutron electron scattering data\textsuperscript{168}. Statistically, it is advantageous to combine results from different measurements\textsuperscript{40}.

**Relativity and inertia in neutron interferometry.** Early in the history of neutron interferometry, it was recognized\textsuperscript{169,170} that the vertical difference $\Delta h$ between the two paths of a neutron interferometer leads to a measurable phase shift $\Delta \phi_{v}$. By rotating the interferometer around the neutron axis, a precise differential measurement of this phase shift was implemented. However, the results of this first experiment only agreed with the theoretical prediction at 88(3)%. Most of the deviation was attributed to bending of the interferometer crystal under its own weight during rotation. Soon after, the measurement was improved\textsuperscript{171}. The authors considered the simultaneous effects of gravity, inertia and the quantum mechanical propagation of neutrons, leading to a remaining deviation of 4.0(2)%, which was later reduced further\textsuperscript{172} to 1.0(1)%. An alternative to single-crystal neutron interferometers are grating interferometers operating with very cold neutrons instead of thermal neutrons (see BOX 1). The longer wavelength results in a higher sensitivity and less influence of deformation during the rotation. However, despite these obvious merits, a measurement of the gravitational phase in modern spectroscopic methods. The strength of GRS is that it does not rely on the collective Fermi pseudopotential $V_{F}$ of a material. As $V_{F}$ is typically larger than 100 neV for most materials, UCNs can reflect specularly from surfaces and even be stored in ‘bottles’\textsuperscript{177}. On horizontal surfaces, UCNs form bound states in the potential well created by $V_{F}$ below and the (in good approximation locally linear) gravitational potential — a system known as the quantum bouncer (QB)\textsuperscript{178} or quantum bouncing ball. The neutron QB has been investigated experimentally and theoretically\textsuperscript{180,181}. The first experimental evidence for the existence of the quantized solutions of Eq. (12) was given in REF.\textsuperscript{182} and analysed in detail\textsuperscript{183–185}. From these data, limits on non-Newtonian interactions were derived\textsuperscript{186,187}. The analysis\textsuperscript{187} was re-evaluated later\textsuperscript{188}. Subsequently, the development of a new technique called GRS\textsuperscript{14} (a nomenclature coined in analogy to magnetic resonance spectroscopy, as adopted by electric dipole moment experiments\textsuperscript{189}) proved to be the key to more sensitive measurements utilizing the non-equidistant eigenstates of the QB in modern spectroscopic methods. The strength of GRS is that it does not rely on electromagnetic interactions. Using neutrons as test particles offers the advantage of bypassing the electromagnetic background induced by van der Waals and Casimir forces and other polarizability effects. The peV

| Material | $Z$ | $b_{\gamma}$ | $b_{\gamma}$ | $b_{\alpha}$ | $b_{\alpha}$ | $b_{\alpha}/\alpha$ |
|----------|----|--------|--------|-------|-------|----------------|
| $^{208}$Pb | 82 | 9.49 | 0.113 | -0.110 | 0.047 | 2.32 x 10^{-24} |
| Xe (natural) | 54 | 4.69 | 0.075 | 0.073 | 0.024 | 1.12 x 10^{-26} |

For details of the terms and calculations, see REF.\textsuperscript{184}. We use charge radii from REF.\textsuperscript{185}. The Yukawa scattering length is given for $\lambda = 1$ nm.
Box 4 | Gravity resonance spectroscopy

The quantum bouncer can be described by a one-dimensional Schrödinger equation in the vertical $z$-direction, with three different potentials:

$$V(z) = m_g z^2 t + V_{\Theta}(z - z_{m}) + V_{\nu}(z)$$

(14)

for a neutron of mass $m_n$ at height $z$ above a horizontal mirror at height $z_m$, under the influence of the local gravitational acceleration $g$. For $V_{\nu}(z) = 0$, these potentials permit analytical solutions for the eigenstates $\psi(z,t) = \psi(z) \exp(-iE_n t/\hbar)$ with non-equidistant energies $E_n$ (see the figure). Characteristically for the quantum bouncer, the spatial solutions take the form of Airy functions $\psi(z) = N_A \text{Ai}(z/z_0 - E_n / \hbar)$ with normalization constant $N$. $z(t)$ represents the height above the resting or vibrating lower mirror. The natural scalings of the problem $E_0 = [\hbar^2 m_g g^2 / 2]^{1/3} = 0.602 \text{peV}$ and $z_0 = [\hbar^2 / (2 m_n g^2)]^{1/3} = 5.87 \mu\text{m}$ already indicate that ultracold neutrons are macroscopic objects. Since the probability distributions of states shown in the figure below have different vertical extensions, a hypothetical $V_{\nu}(z)$ would shift the energy of each state $|n\rangle$ — and, thereby, the transition frequencies — differently by an amount $\Delta E_n = \langle n | \nu(z) | n \rangle$. Details on the computation of the transmission probability as a function of excitation frequency can be found in refs.\textsuperscript{16,19,15.}

Panel a shows the lowest energy states and wavefunctions of an ultracold neutron above a horizontal mirror. Vibrations of the mirror trigger transitions between these states at frequencies indicated in the lower left table. Panels b,c illustrate the exemplary transmission spectrum measured in the qBOUNCE setup in Rabi configuration\textsuperscript{16}. The dips in the transmission at 463 Hz and 647 Hz correspond to the transitions $|1\rangle \rightarrow |3\rangle$ and $|1\rangle \rightarrow |4\rangle$, respectively. In the experiment, the transition frequencies are measured by fitting a theoretical model to the recorded spectroscopic data.

a Probability distribution

![Graph showing the probability distribution of neutron transmission as a function of height above the mirror, with energy levels $E_1$ to $E_5$ and frequencies 254.672 Hz to 561.229 Hz.]

b Measured transmission

![Graph showing the measured transmission relative to the theoretical model fit, with transition frequencies indicated at 350 to 700 Hz.]

differences between the eigenstate's energies correspond to acoustic frequencies. State transitions were demonstrated using controlled mechanical oscillations of the lower boundary (mirror)\textsuperscript{14.}

The qBOUNCE collaboration has demonstrated Rabi spectroscopy using purely mechanical excitations\textsuperscript{11}. In their setup, UCNs are velocity-selected using an aperture system before entering a sandwich structure consisting of two glass mirrors separated by a precisely calibrated gap of ~25 μm. The upper mirror is roughened, leading to chaotic scattering of higher states. Only the lowest few states (see the figure in BOX 4) are allowed to pass. In a second ‘region’ consisting of a flat mirror, controlled vibrations at variable frequency are used to induce transitions from the initial state $|i\rangle$ to the target state $|f\rangle$. A third region similar to the first one filters neutrons in state $|f\rangle$ such that the detector placed after this second filter only detects neutrons not excited by the vibration. The measured neutron transmission rate $T(\omega)$ as a function of the excitation frequency thus shows clear dips at the transition frequencies (see BOX 4). If an additional $z$-dependent potential $V_{\nu}(z)$ (in Eq. (12)) existed, it would shift the energy of each state differently and, hence, also shift the transition frequencies. By not detecting such a shift, limits can be set on the maximum strength of $V_{\nu}(z)$.

The GRANIT experiment\textsuperscript{190} intends to excite resonant transitions magnetically between quantum states. Two modes of operation are foreseen. First, using a space-periodic and static magnetic field gradient, the frequency of the excitation seen by a neutron will, thus, vary according to its horizontal velocity. Second, using a homogeneous gradient oscillating in time, which should allow a more direct probe of the resonances\textsuperscript{191}. 
The spin, being a pure quantum DOF, extends the parameter space for measurements with neutrons. As measurements are possible with and without spin polarization, GRS is a versatile method that makes it possible to test a wide range of potentials and models, including scalar-pseudoscalar axion interactions\textsuperscript{11}, Cartan gravity\textsuperscript{192,193}, screened DE chameleon and symmetron fields\textsuperscript{15,16}. GRS may also be used to search for a non-vanishing electric charge of the neutron\textsuperscript{149} or violations of the weak equivalence principle. The sensitivity of GRS scales with the interaction time $\tau$ of neutrons with the setup. An implementation of Ramsey-type GRS increases\textsuperscript{195} $\tau$ by a factor of 4. Four orders of magnitude improvement seems possible if neutrons are stored instead of passing through the setup\textsuperscript{196}.

The QB has also been realized in the horizontal direction, where the gravitational potential is replaced by the centrifugal potential\textsuperscript{197} $V(r) = rm, v^2/2$. Here, $v$ is the tangential velocity of the UCNs and $r$ is the circular mirror’s radius. In the same way as for the gravitational QB, the wavefunctions in the radial direction are also quantized. However, the states are velocity-dependent and there is always a non-vanishing probability for tunnelling through the radial barrier\textsuperscript{188}.

Other searches for new physics. Neutron storage experiments were originally designed to determine the lifetime of free neutrons, but can also be used to search for new physics if polarized neutrons are used. CP-violating interactions $\propto \sigma \cdot \tau$ cause a rotation of the neutron spin (represented by the Pauli spin matrix $\sigma$) on every wall bounce. By comparing the expected depolarization rate with the one measured in neutron bottles\textsuperscript{196,204}, limits on the respective coupling parameters $g_s$ can be derived. A different way to obtain such limits in a storage chamber has been followed by the nEDM collaboration. They used Ramsey spectroscopy to measure the ratio of the precession frequencies of UCNs and $^{199}$Hg atoms\textsuperscript{202} for different holding fields. Any spin-dependent interaction between the unpolarized walls and the two particle species would influence this ratio. The following experiments could probe vector-axial vector or double axial-vector interactions, but an error in the analysis has invalidated derived limits (see BOX 3). Reference\textsuperscript{202} let a slow polarized neutron beam pass parallel to an unpolarized flat source mass. A hypothetical interaction $\propto \sigma \cdot (v \times r)$ would lead to an effective magnetic field normal to the neutron velocity and source plate normal vector, in which the neutron spin would precess, leading to a phase shift of the observed Ramsey pattern. The same hypothetical interaction was tested in polarimeters\textsuperscript{185}. Here, a neutron beam is blocked by two orthogonally oriented polarizers. Exchange of a spin-1 boson between the neutron beam and a test mass placed between polarizers would induce a minute rotation of the particle spin, thereby allowing some neutrons to pass the second polarizer. From not observing such transmission, limits on the coupling parameter $g^\lambda_\lambda$ were derived. A further method to probe double axial-vector and vector axial-vector interactions is spin-echo small-angle neutron scattering\textsuperscript{203} (SESANS). Here, two neutron paths are coherently separated and recombined using magnetic fields, resulting in a higher sensitivity as compared with perfect crystal neutron interferometers relying on Bragg scattering. Since neutrons in different interferometer paths have different polarization, it was suggested that the discrepancy between interferometer data and theory was due to a hypothetical spin-dependent interaction. Reference\textsuperscript{194} excluded this conjecture but the analysis suffers from the assumption of an invalid potential.

Apart from hypothetical forces, tests of the Newton equivalence principle (NEP) have also been performed with neutrons. Reference\textsuperscript{204} compared the gravitational energy $m_{\text{grav}}gh$ of neutrons falling a height $h$ onto a rotating diffraction gratating of reciprocal lattice vector $q$ with the kinetic energy $\hbar \omega / (2m_{\text{grav}}) / (2k, q - q_f)$ acquired in the reflection from the grating. From knowledge of the neutron horizontal wave vector $k_2$, the change $q$ in momentum and the rotational frequency $\omega$, limits on the ratio between the inertial $m_\text{inert}$ and the gravitational $m_{\text{grav}}$ mass of the neutron can be derived. Tests of the equivalence of $m_{\text{grav}}$ and $m_{\text{inert}}$ can be performed using interferometry\textsuperscript{170,205} or GRS. Various limits from older neutron experiments and measurements at higher energies were reviewed in REF\textsuperscript{5,206}.

Experimental limits on non-Newtonian interactions. In FIG. 4, we attempt to collect all limits obtained with neutron experiments on the various classes of Yukawa interactions\textsuperscript{207} with coupling parameters $a = g\sim \sqrt{\hbar c / (4\pi G_N m^2_{\text{n}})}$, $g_\text{sp}g_\text{sp}^\lambda g_\text{ep}^\lambda$ and $g_\text{sp}g_\text{ep}^\lambda$, as well as the parameter $\beta$ of chameleon interactions with $\Lambda$ fixed to the DE scale, and cuts $\lambda(M)$ through the three-parameter space of symmetron DE at different values of $\mu$. Details of the potentials can be found in BOX 3. Limits from neutron experiments are given in colour, while other limits included for comparison are given in grey. Red and pink indicate scattering experiments or SESANS, orange stands for spectroscopy, green for neutron-optical methods, turquoise for spin rotation and blue for QB and GRS. The most widely tested class is scalar interactions, where neutron experiments provide the tightest high-confidence limits in a wide interaction range $10^{-14} \text{ m} < \lambda < 10^{-6} \text{ m}$, as shown in FIG. 4a. In the same range, atomic spectroscopy permits the derivation of limits from the comparison of measured transition frequencies with quantum electrodynamics calculations in light elements\textsuperscript{208,209}. Presently, only few such limits are competitive\textsuperscript{198}, but the field shows rapid progress. Constraints for spin-dependent interactions are given in FIG. 4b. We recalculate the results from different publications to the potentials $V_N(r)$ (scalar) and $V_V(r)$ (scalar pseudoscalar) given in Eqs (10) and (11). Limits on double axial-vector and vector-axial vector interactions are shown in FIG. 4c,d. For the latter interaction, we focus on the subset $g_\text{sp}g_\text{sp}^\lambda$, where the axial vector coupling is to the neutron spin\textsuperscript{211} or a nucleus dominated by the neutron spin\textsuperscript{122,213} and $g_\text{sp}^\lambda = 2(g_\text{sp}^\lambda + g_\text{sp}^\lambda + g_\text{sp}^\lambda)$ (REF\textsuperscript{211}). We note that limits on $g_\text{sp}^\lambda$ and $g_\text{sp}^\lambda$ from references\textsuperscript{174,211-215} are based on the potentials from REF\textsuperscript{207}, for which some issues have been pointed out\textsuperscript{194} (see BOX 3). Strictly speaking, in order to properly relate experimental constraints to a specific theoretical model, the physical details of the parts of the individual experimental setup, which act as the source
of the hypothetical interaction, must be fully considered. Depending on the theoretical model, the coupling to protons, neutrons and electron differs, which is also true for different macroscopic materials. Experimental constraints on the various non-Newtonian potentials should, therefore, be disentangled concerning the vertex of the interaction, which may contain derivatives or not as emphasized in Refs.\textsuperscript{9,217} and reviewed in Ref.\textsuperscript{218}.

Owing to the lack of information for many experiments, we refrain from such disentanglement in this Review and, rather, compare limits for the same potential, as given in the original publications. Hypothetical chameleon interactions have been searched for using neutron interferometry\textsuperscript{17,219} and GRS\textsuperscript{15}, but neutron limits are not competitive at the moment. Figure 4e shows neutron and other limits on the coupling parameter to matter.

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**Fig. 4** *Compiled limits on non-Newtonian interactions. a | Limits on Yukawa-type modifications of Newtonian gravity with range $\lambda$ and strength $\alpha$. 1a and 1b: scattering and optical limits 90% C.L. (Ref.\textsuperscript{18}); 2: neutron scattering 90% C.L. (Ref.\textsuperscript{22}); 3a and 3b: comparison of forward and backward neutron scattering, and scattering asymmetry in noble gases 95% C.L. (Ref.\textsuperscript{18}); 5 and 6: Xe-neutron scattering 95% C.L. (Refs.\textsuperscript{25-27}); 7: silicon powder neutron scattering 67% C.L. (Ref.\textsuperscript{18}); 13: gravitational quantum states 90% C.L. (Ref.\textsuperscript{28}); 14: bouncing neutrons 95% C.L. (repulsive). Non-neutron limits: 8: torsion balance 95% C.L. (Ref.\textsuperscript{21}); 9: torsion balance 95% C.L. (Ref.\textsuperscript{21}); 10: torsion balance 95% C.L. (Ref.\textsuperscript{21}); 11: micro torsion balance 95% C.L. (Ref.\textsuperscript{21}); 12: Casimir force between sinusoidally corrugated surfaces 95% C.L. (Ref.\textsuperscript{21}); 37: Lamb shift of the 1S–2S transition in muonic H/D (no C.L.). b | Scalar-pseudoscalar interactions with coupling parameters $g_s$ and $g_p$. 18: Rabi-GRS with UCNs 95% C.L. (Ref.\textsuperscript{21}); 19: $n$/$^{199}$Hg comagnetometer 95% C.L. (Ref.\textsuperscript{21}); 20: UCNs transmission experiment 95% C.L. (Ref.\textsuperscript{21}); 21: depolarization in UCN–wall interactions 67% C.L. (Ref.\textsuperscript{21}) (we only show their line 7); 22: depolarization in UCN–wall interactions 95% C.L. (Ref.\textsuperscript{21}); 23: $\lambda$ depolarization 95% C.L. (Ref.\textsuperscript{21}); 24: $^{129}$Xe $^{131}$Xe NMR frequency shift 67% C.L. (Ref.\textsuperscript{21}); 35: $^{4}$He $^{131}$Xe spin precession 95% C.L. (Ref.\textsuperscript{21}); 40: $^{4}$He spin relaxation 67% C.L. (Ref.\textsuperscript{21}). c | Vector-axial vector interactions with coupling parameter $g_A$ and range $\lambda$. 16: neutron spin precession 95% C.L. (Ref.\textsuperscript{21}); 30: neutron polarimeter 95% C.L. (Ref.\textsuperscript{21}); 31a: neutron spin echo 67% C.L. (Ref.\textsuperscript{21}). Non-neutron limit: 32: K/He comagnetometer\textsuperscript{18} (curve extracted from Fig. 18 of Ref.\textsuperscript{21}). d | Vector-axial vector interactions with coupling parameters $g_A$ and $g_p$, and range $\lambda$. 15: neutron spin rotation in $^3$He 67% C.L. (Ref.\textsuperscript{21}); 31b: neutron spin echo 67% C.L. (Ref.\textsuperscript{21}). Non-neutron limit: 33: combined limits from $^3$He polarization and torsion balance with no assumptions 67% C.L. (Ref.\textsuperscript{21}). Note the comment in Box 1 regarding the validity of $\bar{V}_{AA}$ and $V_{AA}$. e | Chameleon interactions with parameters $\beta$ and $n$. 18b: Rabi-GRS 95% C.L. (Ref.\textsuperscript{21}); 25: neutron interferometry 95% C.L. (Ref.\textsuperscript{21}); 26: neutron interferometry 95% C.L. (Ref.\textsuperscript{21}). Non-neutron limits: 27: atom interferometry 95% C.L. (Ref.\textsuperscript{21}); 28: torsion balance 95% C.L. (Ref.\textsuperscript{21}) (curve taken from Fig. 3b of Ref.\textsuperscript{21}); 36: neutron spin rotation in $^3$He 67% C.L. (Ref.\textsuperscript{21}). f | Symmetron interactions with parameters $\lambda$, $\mu$ and $\beta$. 3a–d: Rabi-GRS $\mu = (10^4, 10^3, 10^2, 10^1)$ eV, solid: Fermi-screened, dashed: micron-screened 95% C.L. (Ref.\textsuperscript{21}); 37a–e: Casimir experiments 95% C.L., $\mu = (10^5, 10^4, 10^3, 10^2, 10^1)$ eV (Ref.\textsuperscript{21}); 38a–c: torsion balance $\mu = (10^3, 10^2, 10^1)$ eV 67% C.L. (Ref.\textsuperscript{21}); 39a, b: atom interferometry $\mu = (10^2, 10^1)$ eV 90% C.L. (Ref.\textsuperscript{21}). GRS, gravity resonance spectroscopy; UCN ultracold neutron.*
β with the potential $V(\phi) = \Lambda^4 + \Lambda^4/\phi^6$ fixed by the DE scale $\Lambda_0 = 2.4$ meV [REF 23]. Very competitive limits on symmetron interactions have been derived from older GRS data [REF 21]. We present cuts $\lambda(M)$ of the symmetron's three-parameter space of limits with different fixed $\mu$ as indicated.

The best current limits on the NEP from neutron experiments have been reported in REF 221, where $1 - m_{\gamma}/m_n = \beta < 1.1 \pm 1.7 \times 10^{-4}$ was obtained by comparing NGR results for the coherent scattering length with those from scattering measurements on carbon. Reference 230 achieved $\beta < 1.8 \pm 2.3 \times 10^{-5}$. These limits for the NEP are not directly comparable to the best weak equivalence principle limits $\eta = 2[(m_\gamma/m_p)^{\text{Pl}} - (m_\gamma/m_p)^{\text{Pl}}/((m_\gamma/m_p)^{\text{Pl}} + (m_\gamma/m_p)^{\text{Pl}})] \leq -1 \pm 27 \times 10^{-14}$ obtained from the MICROSCOPE space mission222.

**Outlook**

In many fields of fundamental physics, the neutron as a massive quantum particle gives access to numerous parameters that would be hard to probe in other ways. At low energies, neutron experiments allow for accurate tests of fundamental quantum statistical theorems and to elucidate the behaviour of matter waves. The experiments presented here have been performed at neutron sources in different countries; we just mention the ILL (Grenoble, France), Atominstitut (Vienna, Austria), ISIS (Oxfordshire, UK), NIST (Gaithersburg, MD, USA) and KURRI (Kyoto, Japan). These worldwide efforts are reflecting an unquenchable thirst for insights into the fundamental issues of QM, gravity and DE. We emphasize that most of the presented experiments testing fundamental concepts of QM have been performed using thermal neutrons206–208. However, using higher (epithermal) energies may offer further experimental opportunities not used to date. Another option to improve precision is to use objects of larger mass. Matter wave optics are also used in atom interferometry, which can be used to test the weak equivalence principle with high precision25. Decoherence and interference near the classical limit have been studied with macro–molecules. One particularly challenging task in the future is to test possible extensions of QM overcoming the probabilistic character of the theory. Intrinsically, QM only allows for predictions on the entire ensemble and does not give information about individual ‘events’. Besides the classical EPR approach21, possible extensions include the time–symmetric interpretation of QM224,225 or Bohmian mechanics226. However, to date, no experiment exists that can distinguish between QM and such alternative theories.

The neutron is also the smallest practically available massive electrically neutral particle that allows for accurate tests of Newtonian gravity and possible modifications thereof. Neutron scattering still gives the tightest limits on scalar Yukawa interactions in the range 10 fm–10 nm, but both theory and experiment are accurate at the 10^{-4} level in measured scattering lengths. Any improvement requires advancements on the experimental and theoretical sides. However, neutron–nucleon interactions remain hard to model and, on the experimental side, isotopic composition of scattering target, perfect control of various environmental parameters216 and background10,115 are limitations. The combination of data sets may statistically improve limits40. Reference 227 speculates that accurate definition of the falling height in neutron gravity refractometry could improve scattering length measurements by 1–2 orders, which would improve limits on Yukawa interactions proportionally, as the theory is more under control in this case. Anyway, the mentioned interaction range can, at present, only be covered by neutron experiments. For scalar-pseudoscalar interactions, neutron comagnetometry may yield some improvements201 below 100 μm, whereas at larger interaction ranges, atom-based methods228,229 are clearly more sensitive. Prospects for vector-axial vector and double axial-vector interactions have to be evaluated after careful re-analysis of the involved potentials216. However, as neutrons already give the tightest limits for $\tilde{g}_{\omega n}$ at $\lambda < 1$ cm, it may be interesting to look at measurements where both the neutron and the involved target (for example, an extended mass distribution) are spin-polarized, as this would enable the full $V_{\omega n}$ and $V_{\omega n}$ (REF 216) that are not suppressed by $\kappa / c$ to be probed. With respect to symmetron interactions, gravity resonance spectroscopy already covers a large parameter space25,226 and bears substantial potential for improvement. However, both neutron interferometry and GRS are not competitive for chameleon interactions at the moment, but the situation would change if neutrons were able to be stored219. GRS has also been proposed as a test of Einstein–Cartan gravity192, as well as the NEP and weak equivalence principle. The latter would represent the first test of the weak equivalence principle with hadrons and would, independently of the achievable precision, be in a class of its own. Reference 231 proposed to test the NEP using Bragg diffraction in large crystals at large angle, but anomalous absorption has hampered the extraction of limits so far25. As scattering experiments are sensitive to a wide range of energies, the possibility of detecting extra dimensions with neutrons was discussed231.

In conclusion, new unexpected aspects of particle-wave duality may emerge from neutron interferometry. Furthermore, prospects for the exclusion or possible detection of hypothetical interactions are bright in next-generation neutron gravity experiments.
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