Spin switching: From quantum to quasiclassical approach

A. Chudnovskiy*, Ch. Hübner, B. Baxevanis, and D. Pfannkuche

I. Institut für Theoretische Physik, Universität Hamburg, Jungiusstr. 9, 20355 Hamburg, Germany

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*Corresponding author: e-mail achudnov@physik.uni-hamburg.de, Phone: +49 40 428382433, Fax: +49 40 428386798

In this paper, we review the theory of spin switching for systems of different sizes, from a quantum mechanical master equation approach for small atomic clusters to the quasiclassical description of spin dynamics by a stochastic Landau–Lifshitz–Gilbert equation in metallic nanoscale magnetic systems. We present characteristic results emphasizing the role of the quantum character of spin at different scales. Comparing the quantum mechanical and the quasiclassical approach to spin switching, we draw analogies between the factors affecting quantum and classical spin dynamics. We analyze assumptions used in each approach and emphasize the limits of applicability of the quantum mechanical and of the quasiclassical descriptions.

1 Introduction

The process of spin switching induced by a spin polarized electric current provides the foundation of reading and writing information in magnetic memory devices [1]. The early observation of the spin-switching by Awshalom et al. [2], spawned a great deal of theoretical as well as experimental activity during the last decades [3–5]. Spin switching occurs in systems with magnetic anisotropy which creates a potential landscape for magnetization with two energy minima separated by a barrier, as shown in Fig. 1. The presence of the energy barrier stabilizes the spin direction if no external field or current is applied. Controlled by a spin-polarized current, switching between the two energy minima is the basic operation in an all-electric memory device [6].

Miniaturization of spintronic devices up to the size of few atoms brings along new phenomena entering the switching process. In particular, for small micro- and nano-size devices and even more for clusters of magnetic atoms, the quantum nature of spin becomes apparent. In micromagnetic systems, the discrete nature of spin projections on the quantization axis leads to the appearance of nonequilibrium spin-shot-noise under the influence of an external spin-polarized current. Of similar origin as charge shot noise in microelectronic devices, spin-shot-noise shows up as stochastic fluctuations of the magnetization. It provides a mechanism for activated spin switching [7, 8]. The corresponding stochastic character of the spin switching process has been observed in a number of experiments [9–16]. Appropriate for a theoretical description of spin-shot-noise in micromagnetic metallic devices is a quasiclassical approach that is based on an extension of classical equations of motion for the magnetization in the form of stochastic Langevin equations [8, 17–22].

In devices of even smaller size, such as clusters of few magnetic atoms, the discreteness of energies of different spin states and the possibility of quantum spin tunneling starts to play an important role in the spin switching dynamics. Quasiclassical approach to spin-tunneling based on path-integral formulation has been developed in Refs. [23–25]. The incorporation of external electron reservoirs in that approach requires further work though. Coupling the magnetic cluster to external spin reservoirs can result in the suppression of spin tunneling analogous to the Caldeira–Leggett effect [26], and one expects to observe a change from quantum tunneling to noise activation as the dominant mechanism governing spin switching. At the same time, the Kondo correlations between the electron spins in the external leads and the spin of the magnetic cluster can enhance the spin tunneling at low temperatures [27, 28]. Moreover, Kondo correlations in combination with magnetic anisotropy lead to a wide variety of new physical regimes, such as non-Fermi-liquid behavior of the equilibrium ground state, as well as splitting of the Kondo peak at finite bias voltage [29–31]. Along with
spin coordinates [44]. The magnetic anisotropy field created by interactions of the magnetic cluster with the crystal-field of the non-magnetic substrate and the exchange interaction with electrons in the tip and on the surface play a crucial role for the spin dynamics. The giant spin Hamiltonian

\[ \hat{H}_S = D \hat{S}_z^2 + \mathcal{E} (\hat{S}_z^2 - \hat{S}_z^4) \]  

originates from the crystal-field of the underlying surface reflecting its symmetry properties. It is derived under the giant spin approximation, in which the lowest-lying magnetic level is assumed not to interact with higher-lying spin multiplets. This allows to restrict the Hamiltonian to the second order in spin operators [51]. For \( D < 0 \) and \( \mathcal{E} = 0 \), \( \hat{S}_z \) commutes with \( \hat{H}_S \). The eigenstates in this case are the ones of the operator \( \hat{S}_z \). Two degenerate ground states with \( |S_z| = S \) are separated by a barrier that is displayed in Fig. 1. For half-integer spin the barrier height is of the order of \( |D|(S^2 - 1/4) \) in the case of \( |\mathcal{E}/D| \ll 1/3 \). For a non-vanishing value of \( \mathcal{E} \), \( \hat{H}_S \) does not commute with \( \hat{S}_z \) anymore, resulting in mixing of states across the barrier. However, the eigenstates of \( \hat{H}_S \) mix more at the top of the barrier compared to its valleys. This is even more dominant for integer spin systems, which manifests itself in a zero magnetic field splitting [32].

In the following, we investigate a small ferromagnetic cluster composed of five Fe atoms. If the ferromagnetic exchange coupling is sufficiently strong, the low-lying spectrum of the cluster can be modeled by an effective collective spin \( \hat{S} \). The effective anisotropy constants for the cluster spin can be derived from the constants of a single atom using symmetry considerations [52]. In case of a five-Fe cluster on a Co(111) surface the anisotropy constants have been determined to be of the order of \( \mathcal{D} \approx -0.1 \) and \( \mathcal{E} \approx 0.02 \) meV [49]. Furthermore, it has been found that the effective total spin is given by \( S \approx 15/2 \) which will be used in the following considerations.

2.2 Transport

Magnetism in the Fe cluster arises from strongly interacting electrons in the d orbitals. In a STM experiment, the cluster is probed by electrons tunneling between the tip and the surface. The main conduction channel in the magnetic atoms is established by s-electrons, which gives rise to a model where localized d-levels are coupled to a non-interacting bath of s-conduction electrons.

The interaction between the conduction electron spins and the spin of the atomic cluster can be described by an Appelbaum Hamiltonian [53]

\[ \hat{H}_t = \frac{1}{2} \sum_{n'k'\mathbf{m}'\nu} v_{\nu} v_{\nu'} a_{n\sigma\mathbf{m}}^\dagger a_{n'\sigma'\mathbf{m}'} \mathbf{S}_{\nu} \mathbf{S}_{\nu'} \]  

with the annihilation (creation) operators \( a_{\nu\mathbf{m}}^\dagger \) in tip and substrate and the vector of Pauli matrices \( \sigma_{\nu\sigma'} \) associated with the spin of the tunneling electrons. The electrons in the tip and in the substrate are described by Fermi liquid Hamiltonians \( \hat{H}_{\text{tip}} \) and \( \hat{H}_{\text{surface}} \), respectively.

The strength of the tunnel coupling between the tip and the cluster \((r = T)\) or the substrate and the cluster \((r = S)\)
is parametrized by $v_r$. In the following, we are interested in the regime in which the tip weakly perturbs the state of the cluster, $v_r \ll v_S$.

The dynamics of the total system consisting of the tip, the giant spin, and the surface is determined by von Neumann’s equation

$$i\hbar \frac{d}{dt} \rho = [\hat{H}, \rho] ,$$

which describes the time evolution of the statistical operator $\rho$, called the density matrix. The Hamiltonian $\hat{H} = \hat{H}_S + \hat{H}_t + \hat{H}_{sp} + \hat{H}_{tune}$ includes all subsystems and their interactions. Since we are not interested in the properties of the total system we trace over all bath degrees of freedom and get the reduced density matrix of the spin. The interaction of the cluster with the substrate leads to dephasing of the spin with a substrate-mediated dephasing rate $1/T_2 \sim v_S^2$, which is faster than the tunnel rate of electrons through the cluster, $\gamma \sim v_T^2 v_S^2$ [54]. In the regime with $v_T \ll v_S$ the large dephasing rate leads to a fast decay of the off-diagonal elements in the reduced density matrix in comparison to the electron tunneling time. This motivates a description in which tunneling electrons induce transitions between eigenstates $|\alpha \rangle$ of the Hamiltonian $\hat{H}_S$. Written in those eigenstates, the von Neumann equation will involve the diagonal elements of the reduced density matrix $P_{\alpha}$ only. The weak coupling regime $v_T, v_S \ll k_b T$, in which the level broadening is smaller than the thermal energy also suggests a perturbative expansion in $\hat{H}_S$, which we perform to the first non-vanishing order [48, 53, 55, 56]. The resulting equation of motion under the influence of electron reservoirs has the form of a master equation,

$$\frac{d P_{\alpha}}{dt} = \sum_{\beta} (\mathcal{W}_{\alpha \beta} P_{\beta} - \mathcal{W}_{\beta \alpha} P_{\alpha}) .$$

In this equation, the diagonal elements of the density matrix are propagated in time by multiplication with the kernel $\mathcal{W}$. In the lowest non-vanishing order the kernel

$$\mathcal{W}_{\alpha \beta} = \sum_{kk',\alpha',\alpha''} \Gamma_{kk',\alpha',\alpha''}(\epsilon_{kk'}) \left[ 1 - f_{\epsilon_{kk'}}(\epsilon_{kk''}) \right]$$

describes all transitions between spin states $|\alpha \rangle$ and $|\beta \rangle$ induced by the inelastic tunneling of electrons from reservoir $r$ to $r'$ [57]. $f_{\epsilon_{kk'}}(\epsilon_{kk''})$ denotes the equilibrium distribution function (Fermi-Dirac distribution) of the reservoir $r$. The transition rate is calculated according to the Fermi’s golden rule

$$\Gamma_{kk',\alpha',\alpha''} = \frac{2\pi}{\hbar} \left| \frac{1}{2} \langle \epsilon_{kk'} | v_r v_T | \sigma_{\alpha',\alpha} \cdot \hat{S} | \beta \rangle \right|^2 \times \delta(\epsilon_{kk'} + E_\alpha - \epsilon_{kk''} - E_\beta) .$$

Here, $E_\alpha$ is the eigenvalue of $\hat{H}_S$ for the eigenvector $|\alpha \rangle$. In particular, we label the two ground states of $\hat{H}_S$, as $|g_+ \rangle$ and $|g_- \rangle$ with ± addressing the orientation of their magnetic moment along the $z$-axis. Integration over reservoir energies simplifies the kernel to the form

$$\mathcal{W}_{\alpha \beta} = \pi \sum_{\nu' \in \{\uparrow, \downarrow\}} |v_r v_T|^2 \sum_{\alpha'} \xi \left( \mu_\nu - \mu'_\nu - \Delta_{\alpha \alpha'} \right)$$

with the energy difference $\Delta_{\alpha \alpha'}$ between state $|\alpha \rangle$ and state $|\beta \rangle$. In course of the integration we assumed that the coupling $v_r$ to the leads is energy-independent. The convolution of the Fermi functions gives rise to an energy dependent contribution

$$\xi(x) = \frac{x}{1 - \exp(-\frac{x}{k_b T})} .$$

The temperature $k_b T$ is assumed to be equal in tip and substrate. The transition probabilities are proportional to the spectral weights

$$\sum_{\alpha'} = |\langle \alpha | \hat{S}_z | \beta \rangle|^2 \rho_{\uparrow \uparrow} \rho_{\downarrow \downarrow} + \frac{1}{2} \rho_{\uparrow \downarrow} \rho_{\downarrow \uparrow} + \left( \rho_{\uparrow \downarrow} \rho_{\downarrow \uparrow} + \rho_{\uparrow \uparrow} \rho_{\downarrow \downarrow} \right) .$$

Each spectral weight comprises a $\hat{S}_z$-dependent term giving rise to tunnel magnetoresistance and spin-flip terms that are proportional to the tip polarization $P = \rho_{\uparrow \uparrow} - \rho_{\downarrow \downarrow}$. Depending on the bias voltage between the tip and the surface, the spin of the sample is excited or de-excited. For a bias voltage smaller than the energy of the first spin excitation in the sample, inelastic spin excitations are suppressed ($\propto \xi(\Delta_{M,M'} - eV)$) while linearly dependent on the bias for $eV \gg \Delta_{M,M'}$.

The master equation is solved to obtain the time-dependent population probabilities for the spin states that further will be used to extract the switching rates. The probability to remain in the state $|\alpha \rangle$ after the time $t$ is given by $P_{\alpha}(t)$ if one chooses $P_{\alpha}(0) = 1$ to be the initial condition. The spin dynamics obtained in the master equation approach follows a multi-exponential law which is defined by the interplay of inelastic spin excitations by tunneling electrons and spin relaxation by interactions with the surface. In the present model, the strong coupling of the cluster to the surface $v_S/v_T = 10$ leads to a fast spin relaxation into one of the two ground states. In this regime, the dwell time of the spin in one of the ground states is long compared to the tunneling time of electrons and we find a slow spin switching between the two ground states. The dependence of the switching rate on the parameters of the system will be examined in the following. The rates will be given in units of the total spin flip rate $\Gamma_0 = \pi \hbar^2 v_T^2 \rho_{\uparrow \downarrow} (\rho_{\uparrow \downarrow} + \rho_{\downarrow \uparrow}) / |D|$, induced by inelastically tunneling from the tip to the surface.

### 2.3 Voltage dependence

By choosing a sufficiently small coupling to the tip $v_T/v_S = 0.1$, the regime in which the spin of the cluster slowly switches between the two ground states is approached and we find that occupation probabilities of the two ground states, $|g_+ \rangle$ and $|g_- \rangle$, sum up to one, $P_{g_+} + P_{g_-} \approx 1$. Thus the occupation of the excited spin states
states.

For bias voltages

vanishingly small but finite switching rate

of the cluster is negligibly small. In that regime, we find a vanishingly small but finite switching rate \( \Gamma \) between the two ground states (Fig. 2).

The switching rate between the two ground states shows a strong dependence on the applied bias voltage between the tip and the surface featuring a sharp onset of the switching rate at a distinct bias voltage \( eV = \Delta_{01} \), corresponding to the excitation energy \( \Delta_{01} = E_s - E_g \) from the ground state (the energy \( E_s \)) to the first excited state (the energy \( E_g \)). The onset suggests that the spin of the cluster is driven by inelastic spin excitation due to the electron flow through the cluster. In the case of low temperature \( k_B T \ll \Delta_{01} \), the transition from the ground state to the first excited state cannot be thermally induced. Therefore a quantum mechanical spin remains in the ground state until the tunneling electrons have enough energy to induce the first excitation. This is in contrast to switching of a classical magnetic moment which will be dealt with in the following section and which does not exhibit such voltage dependent threshold. The energy difference between the ground state and the first exited state in the case of \( \mathcal{E} = 0 \) is given by \( \Delta_{01} = (2S - 1)D \). The subsequent transitions between excited levels require less energy. A reversal of the spin direction is thus possible if a cascade of independently tunneling electrons rotates the spin across the barrier. Only the electrons tunneling from the tip to the surface (from higher to lower chemical potential) can provide the energy and magnetic moment needed for the transition out of the ground state. Increasing the voltage enables more conduction electrons to perform inelastic spin excitations and results in a larger switching rate. Another quantum mechanical effect stems from the mixing of \( S_z \) eigenstates due to the hard axis anisotropy \( \mathcal{E} \). As previously mentioned, the mixing of states is stronger for states near the top of the barrier compared to the low-lying states. This leads to an effectively lowered barrier, which increases the probability for transitions from one ground state to the other. This behavior can be observed in Fig. 2 where the rate at a fixed voltage increases with increasing value of \( \mathcal{E} \).

2.4 Temperature dependence

The temperature dependence of the switching rate exhibits a crossover between two regimes. In Fig. 3 the switching rate between the ground states versus the temperature is shown. For temperatures \( k_B T > 2D \), the spin of the cluster is thermally excited leading to a classical Néel-Brown type behavior, which has also been observed in larger clusters consisting of a few-hundred atoms [58, 59]. At small temperatures, however, thermal switching is increasingly suppressed and the switching rate begins to deviate from an exponential law. In fact, in the low-temperature regime we find a linear dependence of the switching rate on the temperature due to the interplay of spin-flips by substrate electrons and in-plane anisotropy. Inelastic excitation is not responsible for the linear dependence because the associated rate is proportional to \( \zeta(eV - \Delta) \approx eV - \Delta \) for \( eV \gg \Delta \), which is independent of temperature. However, the interaction of the cluster with surface electrons leads to a transfer of spin between degenerate states which is proportional to temperature [54] and can explain the low-temperature behavior. This effect is conceptually different to the coherent effect of quantum tunneling of magnetization (QTM) observed in molecular magnets which is temperature independent [32, 60, 61]. QTM originates in the coherent quantum tunneling between degenerate molecular spin states [26]. While in molecules sufficiently long coherence times at low temperature can be established, in STM experiments the surface strongly interacts with the sample atoms leading to fast decoherence. For this reason, coherences have been explicitly neglected in the present master equation approach.
The investigated regime, \( eV \simeq \Delta_{01} \) and \( k_{B}T \ll \Delta_{01} \), is characterized by switching between the two ground states \(|g_{+}\rangle\) and \(|g_{-}\rangle\) with a switching time much smaller than the mean electron tunneling time. In this regime, the magnetic cluster effectively resembles a two-level system and the transition probability for \(|g_{+}\rangle \rightarrow |g_{-}\rangle \) \((|g_{-}\rangle \rightarrow |g_{+}\rangle)\) during a time interval \( \Delta t \) is given by \( \Gamma_{1}\Delta t / \Gamma_{2}\Delta t \) [62]. For such a two-level system, the zero-frequency current noise can be directly related to the switching rate \( \Gamma = \Gamma_{1} + \Gamma_{2} \) by [63, 64]

\[
\lim_{\omega \rightarrow 0} \int d\tau \langle I(t)I(t + \tau)\rangle e^{i\omega t} \sim 2(I_{2} - I_{1})^{2} \Gamma_{1} \Gamma_{2} / \Gamma_{3}^{3}
\]

with the current \( I_{1} (I_{2}) \) flowing through the cluster in state \(|g_{+}\rangle (|g_{-}\rangle)\). The small switching rates obtained for the magnetic cluster result in a large current noise and can be experimentally observed as random telegraph noise in the STM signal [49].

The effects described in this section rely on the quantized nature of the cluster spin and are relevant for current driven spin switching in small magnetic clusters [49].

3 Quasiclassical approach to spin switching process

The spin quantum state \(|S, m\rangle\), where \( m \) denotes the spin projection on the quantization axis, can be associated with a spin-vector precessing under the angle \( \theta_{m} = \arccos(m/S) \) to the quantization axis. Using that analogy one can see, that the difference between the precession angles corresponding to two adjacent spin states diminishes with total spin as \( \Delta \theta \sim \frac{\pi}{2S} \). For a large spin, the chain of quantum mechanical transitions between the spin states, that leads to the switching of the spin as it was explained in the previous section, looks like an almost continuous change of the precession angle. Similarly, and particularly important for spin switching, the energy difference between the adjacent spin states diminishes relative to the total height of the energy barrier separating the two stable spin configurations. For the anisotropic system described by the Hamiltonian (1), the relation of the spin excitation energy \( \Delta_{01} \) to the height of the energy barrier decreases as \( 2/S \) for a large spin. Therefore, the energy quantization becomes also unimportant for systems with large spin. Then, a quasi-classical description of spin switching in terms of continuous variables is justified. Moreover, increase of the total number of spin states of a magnetic system with its total spin brings about a corresponding increase of the number of master equations (4), which eventually makes the fully quantum mechanical approach unpractical for magnetic systems of micrometer size.

The quasiclassical approach to spin switching treats the giant spin of a magnetic domain as a classical vector. In contrast to the quantum mechanical approach, used in the previous section, the switching dynamics is described as a continuous time evolution of the magnetization vector between the two opposite directions, rather than in terms of transition rates between different quantum mechanical spin states. However, for conduction electrons, establishing a spin polarized current within the magnet, this approxi-

mation cannot be made. Their discrete spin projection onto the quantization axis gives rise to nonequilibrium spin-shot noise accompanying the dynamics of magnetization under the influence of a spin-polarized electric current. The phenomenon of spin-shot noise is similar to the shot noise produced by the electric current due to the discretization of charge [20]. While the noise is automatically incorporated in the master equation of the quantum approach, Eq. (4), the treatment of noise in the classical approach requires an extension of the classical equations of motion by stochastic noise terms. Those terms have to be derived from consequent quantum mechanical considerations that are reviewed in this section.

Landau–Lifshits–Gilbert (LLG) equation forms the basis of the quasi-classical approach to magnetization dynamics. This equation has the form

\[
\dot{M} = M_{LL} + M_{dd} + M_{ST}.
\]

It includes the three main sources for a change of the magnetization direction: the precession in an effective magnetic field \( M_{LL} \), the damping term \( M_{dd} \) usually called Gilbert damping that accounts for the attenuation of the precession due to dissipative processes, and the spin torque term \( M_{ST} \) that describes the reaction of magnetization on the spin-polarized current.

The precessional term has a standard form

\[
\dot{M}_{LL} = -\gamma [M \times H_{eff}],
\]

where \( \gamma \) is the gyromagnetic ratio. The effective magnetic field is given by the gradient of the magnetic energy \( E(M) \) with respect to the magnetization, i.e., \( H_{eff} = -\nabla_{M} E(M) \). In this section the magnetic system under investigation is a magnetic junction device consisting of different magnetic layers stacked on top of each other. As for the atomic cluster on a surface, the energy of a single magnetic layer typically includes a combination of an easy plane and the in-plane easy axis anisotropy. Here, additionally the effect of an externally applied magnetic field will be considered

\[
E(M) = \mu_{0} \left[ -\frac{H_{c}^{2}}{2M_{s}} (M \cdot \hat{z})^{2} + \frac{H_{c}^{2}}{2M_{s}} (M \cdot \hat{x})^{2} - H_{ext} \cdot M \right].
\]

Here, \( H_{c}^{e} \) represents the strength of the easy axis anisotropy, chosen in the \( z \)-direction, while \( H_{c}^{o} \) represents the strength of the easy \((yz)\)-plane anisotropy field, and \( M_{s} \) denotes the absolute value of the magnetization. The \( x \)-axis represents a hard direction that is avoided at all costs. The anisotropy fields create an energy landscape with two low-energy states with magnetization orientation parallel to the easy axis \( z \). Those two states are separated by an energy barrier (see Fig. 1). One of the energy minima can be lowered with respect to the other by application of the external magnetic field \( H_{ext} \). Note that the Hamiltonian (13) can be brought to the form Eq. (1)
by a coordinate transformation $y \rightarrow z, z \rightarrow y, x \rightarrow -x,$ and identification $E = \frac{1}{\kappa} (H^f + H^d), D = \frac{1}{\kappa} (H^f - H^d)$.

The Gilbert damping term takes into account the dissipative processes phenomenologically. It has a form

$$M_{GD} = \frac{\alpha}{M_s} [M \times \dot{M}]$$

(14)

and results in the attenuation of precession and an alignment of the magnetization along the effective magnetic field. The strength of the Gilbert dissipation is proportional to the dimensionless damping constant $\alpha$. In modern nanomagnetic devices its value may be as small as $\alpha = 0.01$ [65–67], allowing for dozens precession cycles prior to equilibration. The Gilbert damping constant acquires contributions both from the intrinsic spin-relaxation processes in the micromagnetic device as well as from the dissipation due to the coupling to external spin-reservoirs. In what follows, the intrinsic Gilbert damping is treated phenomenologically.

The contribution to the Gilbert damping constant from external reservoirs is calculated explicitly based on the model of the magnetic tunnel junction (MTJ). It is expressed through the spin-dependent conductances of the junction (see below).

The influence of an external spin polarized current on the magnetization known as spin torque (ST) effect was first described by Slonczewski [68] and Berger [69], who extended the LLG equation by the spin torque term

$$M_{st} = \gamma \frac{M_s}{M_s^2} [M \times \hat{J}_s \times M],$$

(15)

which tends to align the magnetization along the spin polarization of the electric current. The spin-current vector $\hat{J}_s$ is directed along the spin polarization. Its absolute value is proportional to the difference of the electric currents of spin-up and spin-down electrons, $I_s = (I_{\uparrow} - I_{\downarrow}) \hbar / 2e$. Eqs. (13), (14), and (15) provide a classical description of current driven magnetization dynamics. Those equations do not include magnetization noise though, the latter necessarily accompanies the Gilbert damping according to the fluctuation-dissipation theorem (FDT). In thermodynamical equilibrium the noise is proportional to the temperature of the system. In frame of the LLG equation, noise can be introduced as stochastic fluctuations of a magnetic field, as it first has been done by Brown [58]. The fluctuating magnetic field plays the role of a Langevin force for the magnetization. It has zero average, the strength of fluctuations is proportional to the temperature and to the Gilbert damping

$$\langle h(t) h^{\dagger}(t') \rangle = 2 \delta_{t, t'} \delta(t - t') \frac{\alpha k_B T}{M_s \gamma}.$$ 

(16)

The effect of noise becomes progressively important with diminishing the size of the magnetic structure. In addition to the thermal noise, application of an external current produces a nonequilibrium contribution to the magnetization noise. The nonequilibrium noise enhances the probability of activated spin switching, it also modifies the magnetization dynamics during the switching process. The noise strength can be calculated explicitly for magnetic tunnel junctions, revealing the general features of the nonequilibrium noise, such as its proportionality to the current and its dependence on the angle between the spin polarization of itinerant electrons and the instantaneous direction of magnetization [8, 20]. This calculation also allows to express the noise characteristics in terms of experimentally measurable conductances of the device. Below we review the main results for the nonequilibrium noise in MTJ.

The MTJ is modeled by two ferromagnets, one with fixed magnetization direction (the fixed layer) and the other one exhibiting magnetization dynamics as a result of the spin torque effect (the free layer). The fixed layer is used to produce the spin polarized current (see Fig. 4). When an electric current is passed through the fixed layer, it becomes polarized along the direction of the fixed layer. Once it enters the free layer the polarized spin-current induces a spin torque and thus may change the magnetization direction of the layer, allowing for a number of dynamic regimes [70–80].

The Hamiltonian of MTJ consists of three parts describing an isolated fixed and free layers along with the tunneling of electrons between them:

$$H = H_{\text{fixed}} + H_{\text{free}} + H_{\text{tun}}.$$ 

(17)

The majority ($\sigma = +$) and minority ($\sigma = -$) bands of itinerant electrons are described by the operators $c_{i\sigma}^{\dagger}, c_{i\sigma}$ for the fixed ferromagnet and $d_{i\sigma}^{\dagger}, d_{i\sigma}$ for the free layer.

The Hamiltonian of the free layer accounts for the interactions of the itinerant electrons in the free layer with its total...
\[ \mathbf{s} = \frac{\mathbf{M}}{\gamma} \text{ given by} \]
\[ H_{\text{free}} = \sum_{\mathbf{d}_\alpha} c_{\mathbf{d}_\alpha}^\dagger c_{\mathbf{d}_\alpha} - J \mathbf{s} \cdot \mathbf{S} - \gamma \mathbf{S} \cdot \mathbf{H}_{\text{eff}}, \tag{18} \]

where \( \mathbf{s} = \frac{1}{2} \sum_{\mathbf{d}_\alpha} d_{\mathbf{d}_\alpha}^\dagger \mathbf{\sigma}_{\mathbf{d}_\alpha} d_{\mathbf{d}_\alpha} \) is the spin of itinerant electrons and \( J \) is the Heisenberg exchange interaction constant. It is equivalent to the Hamiltonian used for the description of the atomic magnetic cluster in the previous section. The electrons in the free layer play the role of an external electron bath (the tip and the substrate in the model of a magnetic cluster) for the giant total spin \( \mathbf{S} \) while the exchange interaction is equivalent to the Appelbaum Hamiltonian (2). The effects of anisotropy are captured by the last term on the right hand side of Eq. (18), which is equivalent to the giant spin Hamiltonian Eq. (1).

The fixed layer is described by a Fermi-liquid Hamiltonian
\[ H_{\text{fixed}} = \sum_{k, \sigma} \varepsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma}. \tag{19} \]

Finally, the tunneling term in the Hamiltonian is given by
\[ H_{\text{tun}} = \sum_{\mathbf{k}, \mathbf{k}'} W_{\mathbf{k} \mathbf{k}'} c_{\mathbf{k}'}^\dagger c_{\mathbf{k}} + h.c. \tag{20} \]

Here, the spin indices of the operators \( c_{k\sigma}^\dagger \) and \( d_{\mathbf{d}_\alpha} \) denote the spin projections along the magnetization directions in the fixed and in the free layer, respectively. The direction of magnetization determines the quantization axis for electrons in each layer. Because of a finite angle between the two magnetization directions the tunneling matrix elements become spin-dependent. They are given by \( W_{\mathbf{k} \mathbf{k}'} = \langle \sigma | \mathbf{\sigma} | \sigma' \rangle W \), where the spin-transformation matrix is \( \langle \sigma | \mathbf{\sigma} | \sigma' \rangle = e^{-i \theta/2} \cos \theta/2 \) and \( \langle \sigma | \mathbf{\sigma} | \sigma' \rangle = e^{i \theta/2} \sin \theta/2 \). The angles \( (\theta, \phi) \) are the polar and azimuthal angles that denote the direction of the magnetization in the free layer in the reference frame with \( z \)-axis pointing in the direction of the magnetization of the fixed layer. Note, that in contrast to the micromagnetic MTJ considered here, there is no relaxation of electrons inside the small magnetic cluster considered in the previous section. For that reason, the spin quantization axis for the electrons interacting with a small magnetic cluster is determined by the (constant in time) quantization directions in the two external reservoirs (the tip and the substrate), see Eq. (6), rather than by the instantaneous position of the cluster spin.

The main source of non-equilibrium noise is the discrete nature of the angular momentum transfer between conduction electrons and magnetization. Indeed, each electron tunneling at a random time may undergo a spin-flip with a probability depending on the mutual orientation of the quantization axes in the two layers. Since each event transfers exactly the unit of angular momentum \( h \), the direction of an ensuing magnetization rotation is random due to the uncertainty principle (i.e., non-commutativity of the three components of the angular momentum operator). Therefore, the electron transport is accompanied by a stochastic force acting on the magnetization. Its nature is similar to the charge shot noise. While the latter is due to the discreteness of charge, the former is due to the quantization of the angular momentum. We call it thus the spin shot noise [8, 18, 20].

To introduce this noise into the semiclassical equation of motion we write the model specified by Eq. (17) as a path integral on the Keldysh contour [8, 20, 81] and integrate out all fermionic degrees of freedom, while keeping the dynamics of the macroscopic spin \( \mathbf{S} \). The integration is facilitated by the tunneling approximation, i.e., keeping only the lowest non-vanishing terms in the coupling \( W \). (A similar procedure leads to the equation of motion for the reduced density matrix of the small magnetic cluster in the previous section.) As a result, the spin shot noise can be cast into the form of random fluctuations of the spin current vector \( \delta \mathbf{I}_s \) entering the spin torque term (15). The noise correlator can be expressed in terms of the MTJ conductance for parallel \( G_p \) and antiparallel \( G_{AP} \) magnetization orientation of the two layers \( (G_p \geq G_{AP}) \)
\[ \langle \delta I_s(t) \delta I_s(t') \rangle = 2M_s^2 D(\theta) \delta_{ij} \delta(t - t'), \tag{21} \]

where
\[ D(\theta) = \frac{\alpha_0}{M_s \gamma} k_B T + \frac{\hbar}{2M_s^2} L_\nu(\theta) \coth \left( \frac{eV}{2k_B T} \right). \tag{22} \]

Here, \( \alpha_0 \) is the bare Gilbert damping of the isolated free layer and \( V \) is a voltage bias between the two ferromagnets. The first term in Eq. (22) describes the equilibrium noise due to intrinsic relaxation processes in the free layer, it is therefore proportional to the intrinsic Gilbert damping constant \( \alpha_0 \). The second term describes the additional noise due to the coupling of the free layer to the fixed layer which in this case plays the role of the external reservoir of spins. This term depends on the angle \( \theta \) between the magnetization of the two layers, which makes the noise strength dependent on the instantaneous orientation of the free layer magnetization \( D = D(\mathbf{M}) \). The external contribution to the noise is proportional to the spin-flip current \( I_\nu(\theta) \). The latter counts the total number of spin flips irrespective of the direction of the ensuing magnetization change (as opposed to the spin current \( I_s \)). In the magnetic tunnel junction setup we found for the spin-flip conductance
\[ \frac{dI_\nu(\theta)}{dV} = \frac{\hbar}{4e} \left[ G_p \sin^2 \left( \frac{\theta}{2} \right) + G_{AP} \cos^2 \left( \frac{\theta}{2} \right) \right]; \tag{23} \]
\[ G_p = G_{++} + G_{--} \quad \text{and} \quad G_{AP} = G_{+-} + G_{-+}, \]

where we adopted notations of [5] for the partial conductances \( G_{\alpha\beta} \) between the spin-polarized bands of the two ferromagnets. The external spin noise is accompanied by the renormalization of the Gilbert damping constant, which now depends on the instantaneous magnetization of the free layer through the angle \( \theta \) it forms with the magnetization of the
fixed layer

$$\alpha(\theta) = \alpha_0 + \frac{\hbar}{eM_s} \left( \frac{dI_s(\theta)}{dV} \right) = \alpha(M), \quad (24)$$

where the spin-flip differential conductance is given by Eq. (23).

Note that in the case of a small magnetic cluster considered in Section 2, the spin of the cluster can also relax its energy due to inelastic spin-flips with itinerant electrons which provides an analog to the Gilbert damping [82]. There is no intrinsic Gilbert damping though. The Gilbert damping has only extrinsic contributions related to the tunneling conductances between the cluster and the electronic reservoirs (the tip and the substrate).

For low voltages, $eV < k_B T$, the tunneling contribution to the noise is proportional to the temperature and the total noise satisfies the fluctuation-dissipation theorem with the renormalized Gilbert damping constant $\alpha(M)$, [18, 20]. For voltages exceeding the temperature $eV > k_B T$, the system is essentially in nonequilibrium which can be read of the noise being proportional to the applied voltage and independent of temperature. This corresponds to the low-temperature spin switching regime in small magnetic clusters, where the switching rate exhibits only a weak temperature dependence (see Fig. 3).

The conductances $G_{xx}$, $G_y$, and $G_{xy}$ allow the complete characterization of the electric and magnetic properties of MTJ. So, the electric conductance of the MTJ in an arbitrary orientation is given by

$$dI_s(\theta)/dV = G_y \cos^2(\theta/2) + G_{xy} \sin^2(\theta/2). \quad (25)$$

Notice that the spin-shot noise is minimal for a parallel orientation and maximal for an antiparallel one – exactly opposite to the charge current and the charge shot noise. In contrast to the spin-flip current introduced above, the spin current $I_s$ is governed by the spin conductance [5]

$$\frac{dI_s}{dV} = \frac{\hbar}{4e} \left( G_{++} - G_{--} + G_{+-} - G_{-+} \right). \quad (26)$$

Taking noise into account, one arrives at the stochastic equation of motion for the magnetization

$$\dot{\mathbf{M}} = -\gamma \left[ \mathbf{M} \times \mathbf{H}_{\text{ext}} \right] + \frac{\alpha(\theta)}{M_s} \left[ \mathbf{M} \times \dot{\mathbf{M}} \right]$$

$$+ \frac{\gamma}{M_s^2} \left[ \mathbf{M} \times \left( \dot{\mathbf{I}}_s + \delta I_0(t) \right) \times \mathbf{M} \right]. \quad (27)$$

The equivalence of the two forms of description of the magnetization noise, either as a stochastic spin current or as a stochastic magnetic field, is expressed by the relation of the corresponding cumulants

$$\langle h'(t) h'(t') \rangle = \frac{1}{M_s^2} \langle \delta I(t) \delta I(t') \rangle$$

$$= 2 D(M) \delta_{i,j} \delta(t - t'). \quad (28)$$

Notice that the noise correlator itself (22) as well as the renormalized Gilbert damping (24) depend on the instantaneous angle between the free and fixed layer magnetization direction. The angular dependence of the nonequilibrium noise leads to the appearance of “cold” and “hot” regions of magnetization precession, according to high and low noise strength, as illustrated in Fig. 5.

3.1 Energy-angle variables The dynamics of the spin switching process at moderate driving currents exhibits a separation of time scales between the rapid spin precession in the effective magnetic field and a slow change of the energy and form of the precessional orbit. To make the separation of the time scales explicit, it is convenient to formulate the equation of motion in the energy-angle variables [21]. Thereby the angular variable characterizes the instantaneous position of the magnetization vector on the precessional orbit of constant energy, which is called the Stoner–Wohlfarth (SW) orbit. The precession along a SW orbit is governed by Eq. (12). Introducing an angular variable $\varphi$ along the orbit, and the slowly varying energy $E$ of the orbit, one derives from equation (11) the corresponding equations of motion in the energy-angle variables (for the detailed derivation see Ref. [21]). The equation for the slowly varying energy acquires the form

$$\dot{E} = -F(E, \varphi) + \dot{\varphi} \cdot \mathbf{V}(E, \varphi). \quad (29)$$

The Gilbert damping and spin-torque terms of the LLG equation give rise to the two *generalized forces* on the right.
correspondingly. Hereafter, $\mathbf{M}_{\perp}$ denotes the conservative LL part, Eq. (12). The precessional energy conserving term $\mathbf{M}_{\perp}$ does not contribute to the equation for the energy. On the other hand, Gilbert damping drops from the equation for the angular variable $\varphi$ and its dynamics is mostly governed by the uniform precession

$$\dot{\varphi} = \Omega_E + \Omega_E \frac{\mathbf{J} \cdot [\mathbf{M} \times \mathbf{H}_{\text{eff}}]}{[\mathbf{H}_{\text{eff}} \times \mathbf{M}]^2}.$$ \hfill (32)

Here, $\Omega_E$ denotes the precession frequency along the SW orbit with the energy $E$. Averaging of Eq. (29) for the slow energy variable over the precession period [4, 7], we arrive at

$$\dot{E} = -F_E + \hat{T}_s \cdot \mathbf{V}_E,$$ \hfill (33)

where the averages of the generalized forces are given by

$$F_E = \frac{1}{M_s P_E} \oint \alpha [\mathbf{M} \times \mathbf{H}_{\text{eff}}] \cdot \mathbf{M},$$ \hfill (34)

$$\mathbf{V}_E = \frac{1}{M_s^2 P_E} \oint [\mathbf{M} \times \mathbf{M}],$$ \hfill (35)

correspondingly. The integrals here run along the SW orbit with energy $E$.

A stochastic magnetic field adds a Langevin force term $\mathbf{M}_{\text{stoch}} = \gamma_1 [\mathbf{h}(t) \times \mathbf{M}]$ to the equation of motion (11) for the magnetization. It leads to Langevin force terms in the equations of motion (29) and (32) for the energy-angle variables, which results in stochastic equations of motions with the multiplicative noise

$$\dot{E} = -F(E, \varphi) + \hat{T}_s \cdot \mathbf{V}(E, \varphi) + \mathbf{g}_E(E, \varphi) \cdot \mathbf{h}(t),$$ \hfill (36)

$$\dot{\varphi} = \Omega_E + \gamma_1 \mathbf{g}_n(E, \varphi) \cdot \mathbf{h}(t).$$ \hfill (37)

Here, the two mutually orthogonal noise-multiplying vectors are given by

$$\mathbf{g}_E = \mathbf{M} = \Omega_E \partial_E \mathbf{M};$$ \hfill (38)

$$\mathbf{g}_n = \gamma_1 \Omega_E [\mathbf{M} \times \mathbf{M}] / ||\mathbf{M}||^2 = -\Omega_E \partial_E \mathbf{M}.$$ \hfill (39)

The deterministic equation of motion (33) allows one to identify the energy-dependent critical spin-current

$$I_c(E) = \frac{F_E}{\mathbf{E} \cdot \mathbf{V}_E},$$ \hfill (40)

at which the right hand side of Eq. (33) vanishes. Each energy-dependent critical spin-current corresponds to the compensation between the angular momentum pumped into the system by the spin-torque effect and the loss of angular momentum due to the Gilbert damping. At such current, the magnetization exhibits a precession in the effective magnetic field without change of its energy. This regime is called a steady state magnetization precession (SSMP). After the spin-current is switched on, magnetization starts to precess, slowly increasing the energy of the precession orbit. This process terminates when the energy $E$ of the orbit corresponds to the SSMP for the given spin-current strength, $I_c = I_c(E)$. Let us denote the height of the energy barrier between the two energy minima corresponding to the opposite magnetization directions along the easy axis, see Fig. 1, as $E_0$. If the energy of the SSMP orbit is lower than $E_0$, the magnetization comes back to the initial low energy state after the spin-current is switched off. No spin switching occurs in that situation. On the other hand, if the energy of the SSMP orbit exceeds the energy barrier, then it is possible that the magnetization relaxes to the position of the other energy minimum after the spin current is switched off, and spin switching takes place. Therefore, we can define the largest of all energy-dependent critical spin-currents as the critical current for the spin switching or simply the critical spin-current $I_c = \max[I_c(E) | 0 < E < E_0]$, so if the applied spin-current $I = I_c(E)$, it forces the magnetization direction to switch to a new stable energy minimum. The corresponding deterministic switching time is given by

$$t_{sw}(E_{\text{ini}}) = \int_{E_{\text{ini}}}^{E_0} \frac{dE}{z \cdot \mathbf{V}_E (I_c - I_c(E))},$$ \hfill (41)

where $E_{\text{ini}}$ is an initial energy. This expression diverges as $(I_c - I_c)^{-1}$ and such a tendency was indeed observed in experiment [83]. This divergence is however cured by the stochasticity, since even for currents somewhat less than critical the switching does occur due to the fluctuations of the spin-current, albeit taking exponentially long waiting time. The proper description of the switching time must therefore rely on the probability distribution to undergo the irreversible switch during time $t_{sw}$. Figure 6 presents probability distributions calculated by Monte Carlo simulations of Eq. (27) for several values of the spin-current $I_c$. Having such distributions, one may evaluate e.g., the average switching time, plotted in Fig. 6b as a function of the applied spin-current. One observes that the switching time indeed grows exponentially at $I_c < I_c$.

Langevin Eqs. (36) and (37) imply the Fokker–Planck (FP) equation for the probability density of the magnetization $P(E, \varphi, t)$:

$$\mathcal{P} = \partial_E \left[ (F - \hat{T}_s \cdot \mathbf{V}(E, \varphi) + D_E \partial_E \ln \Omega_E) \mathcal{P} + D_E \partial_E \mathcal{P} \right] + \partial_{\varphi} \left[ -\Omega_E \mathcal{P} + D_\varphi \partial_{\varphi} \mathcal{P} \right],$$ \hfill (42)
where the two diffusion coefficients are \( D_0(E, \phi) = D |g_\phi|^2 \).

The periodicity of \( \mathcal{P}(E, \phi, t) \) in the angular directions suggests its representation as a Fourier series

\[
\mathcal{P}(E, \phi, t) = \sum_{m=-\infty}^{\infty} \mathcal{P}_m(E, t) e^{im\phi}.
\]

Assuming a fast equilibration of the distribution in the angular direction, one concludes that only the lowest \( m = 0 \) term of the Fourier-series dominates at long times, \( \mathcal{P}_0 \gg \mathcal{P}_m \) where \( m \neq 0 \). As a result one can write a closed FP equation for \( \mathcal{P}_0(E, t) \) in the following form

\[
\partial_t \mathcal{P}_0 = D_\theta \left[(F_E - \hat{J}_s \cdot \mathbf{V}(E) + D_\theta \partial_E \ln \Omega)\mathcal{P}_0 + D_\theta \partial_E \mathcal{P}_0 \right],
\]

where the average generalized force \( F_E \) is given by Eq. (34) and the averaged over the period energy diffusion coefficient is

\[
D_\theta = \frac{1}{2\pi} \int_0^{2\pi} \mathbf{D} \mathbf{\Omega}_E \left| \partial_E \mathbf{M} \right|^2 d\phi = \frac{1}{P_E} \int \mathbf{D} \mathbf{M} \times \mathbf{H}_{\text{eff}} \cdot \mathbf{M}.
\]

Equation. (43) is the quasiclassical analog of the quantum mechanical master equation for diagonal elements of the reduced density matrix Eq. (4). In equilibrium it follows from the fluctuation-dissipation theorem (FDT) that \( D = \frac{\alpha k_B T}{M_s} \gamma \) and thus \( D_E = k_B T F_E \), cf. Eq. (34). As a result, the equilibrium stationary solution of the 1D FP Eq. (43) takes the universal form

\[
\mathcal{P}_0(E) = \frac{1}{Z} \exp \left[ -\frac{E - k_B T \ln \Omega^{-1}}{k_B T} \right] = \frac{P_E}{2\pi Z} \exp \left[ -\frac{E}{k_B T} \right],
\]

where \( Z \) is a normalization factor. The very last term here is the Boltzmann exponent. It is crucial that the entropy \( S(E) = k_B \ln \Omega^{-1} \) has emerged, which may be traced back to the density of states along the SW orbit given by \( P_E \).

Furthermore, Eq. (43) can be represented as a continuity relation in the energy direction

\[
\mathcal{J}_0(E, t) = -\left[ F_E - \hat{J}_s \cdot \mathbf{V}_E + D_\theta \partial_E \ln \Omega \right] \mathcal{P}_0 - D_\theta \partial_E \mathcal{P}_0 = 0,
\]

The stationary solution of the FP Eq. (46) plays a special role in several regimes, such as activated magnetization switching [7] or steady state magnetization precession [84]. This solution is obtained by putting \( \mathcal{J}_0 = 0 \) in Eq. (46) and reads as

\[
\mathcal{P}_0(E) \propto P_E \exp \left[ -\int E \mathbf{F}_E - \hat{J}_s \cdot \mathbf{V}_E + D_\theta \partial_E \ln \Omega \right] dE.
\]

Notice that in equilibrium, i.e., \( \hat{J}_s = 0 \) and \( D(M) = \alpha(M) k_B T / M_s \gamma \), one has \( D_E = k_B T F_E \), even for the magnetization-dependent damping constant and the noise correlator! As a result the Boltzmann form (45) with the entropic factor given by \( P_E \) still holds. However, away from equilibrium the stationary energy distribution (47) may be rather different from the Boltzmann shape.

4 Conclusions

Miniaturization of spintronics devices puts the noise effects in the magnetization dynamics on the cutting edge of investigations. Thereby the influence of nonequilibrium noise on the magnetization grows with the diminishing size of the magnetic system. An adequate theoretical description thus requires a change from the quasiclassical approach for micromagnetic systems to the fully quantum mechanical consideration for clusters of few magnetic atoms. In this paper, we reviewed quantum mechanical and quasiclassical theoretical approaches and demonstrated their application to spin switching in magnetic atomic clusters and micromagnetic spin-valve devices.

Considering the switching behavior of a small ferromagnetic cluster coupled to a spin-polarized tip and a...

Figure 6  Numerical simulations of the switching process are shown for \( H_c^F = 0.028 M_s \), \( H_o^F = M_s \), and \( T = 300 \text{K} \). (Upper panel) Switching probability as a function of time for spin-current values (from left to right): Green = 3 \( \times \), Yellow = 1.55 \( \times \), Red = 1.08 \( \times \), Blue = 0.71 \( \times \). (Lower panel) Average switching time as a function of spin-current relative to the critical current, \( I_c \). Time is measured in units of \( (\gamma M_s)^{-1} \) (from Refs. [8, 21]).
non-magnetic surface through tunnel contacts, we obtained the time-dependent solution of the master equation and extracted the switching rate of the spin between its two ground states. Due to well-resolved quantization of spin energy levels in the atomic cluster, there is a finite voltage threshold for the onset of a switching rate that corresponds to the excitation energy between the ground state and the first excited spin-state of the cluster. Additionally, we found that a finite transversal anisotropy leads to an effectively lowered energy barrier by inducing transitions between degenerate excited spin-states. Increasing the temperature one observes a crossover from the linear temperature dependence of the switching rate to an exponential one. The former characterizes a nonequilibrium spin switching regime where the external spin-polarized current acts in close interplay with thermal excitations, whereas the latter is characteristic for a purely thermally activated spin switching.

The energy quantization of the spin-states becomes less important with increase of the total spin and size of the magnetic system. It becomes largely negligible in the micromagnetic systems. At the same time, the fully quantum mechanical description of micromagnetic systems turns out to be overcomplicated because of the large number of spin states. For such systems, the quasi-classical description becomes more appropriate. It ignores the energy quantization, considering the magnetization as a large classical vector. Nevertheless, the discreteness of a possible spin transfer by conduction electrons is kept, which opens the possibility of including the magnetization noise. In contrast to the quantum mechanical master equations that include noise effects by construction, extension of the classical equation of motion by noise terms requires a detour from purely classical to quantum mechanical considerations. In frame of the quasiclassical approach, the magnetization noise enters as stochastic Langevin terms in the equations of motion for the magnetization. We applied those stochastic equations for the magnetization to the description of spin switching processes in spin-valve devices and determined the spin switching time from it. The common feature of magnetization dynamics in micromagnetic systems is the time scale separation between the fast and almost energy conserving magnetization precession and the slow energy change. It allows a simplification of the equations of motion by averaging over the fast magnetization precession. This approach culminates in the derivation of a one-dimensional Fokker–Planck equation for the probability density of the energy of a precessional orbit. The one-dimensional character of the obtained equation greatly simplifies its treatment in comparison to the initial FP equation for the magnetization vector. At the same time, this equation is a quasiclassical pendant to the quantum mechanical master equation for diagonal elements of the density matrix.

Comparison of the results for spin switching in small magnetic clusters and in micromagnetic systems reveals some common features. The switching time decreases (the rate increases) with the applied spin current due to an increase of spin transfer processes. The switching time is also strongly influenced by the energy barrier due to magnetic anisotropy, increasing with the height of the barrier. In both systems, the thermally activated switching is observed at high temperatures. At the same time, at low temperatures the energy quantization of the spin states in small magnetic cluster leads to a linear temperature dependence of the switching rate that is not observed in micromagnetic spin-valve systems. Furthermore, an interesting feature of both systems is the onset of the spin switching with applied current. In the quasiclassical micromagnetic device, the onset of substantial spin switching rate takes place at the critical value of the spin current. For spin currents lower than the critical one, the spin switching has a noise activated character with exponentially small switching rate. In contrast, in the quantum mechanical small magnetic cluster there is a threshold voltage for the spin switching that is determined by the energy of the first excited spin state. No spin switching occurs below the threshold at zero temperature. Above the threshold, the switching rate grows rapidly with applied voltage. Therefore, in a quantum mechanical system, the threshold voltage plays a similar role to that of the critical spin current in the quasiclassical system. (Keep in mind, however, that the two are not related by some kind of Ohm’s law!)

Together, quantum mechanical and quasiclassical approaches provide tools for a comprehensive theoretical treatment of magnetization dynamics for systems of different sizes. It seems especially interesting to apply both approaches to atomic clusters with very large spin that take an intermediate position between small quantum mechanical and large quasiclassical systems. Comparison of results obtained by different descriptions will reveal their strengths and weaknesses. It can also show the way for further improvement of both approaches.

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