Magnetic Field Spectral Evolution in the Inner Heliosphere

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Abstract

Parker Solar Probe and Solar Orbiter data are used to investigate the radial evolution of magnetic turbulence between 0.06 < R < 1 au. The spectrum is studied as a function of scale, normalized to the ion inertial scale. In the vicinity of the Sun, the inertial range is limited to a narrow range of scales and exhibits a power-law exponent of \( \alpha_B = -3/2 \), independent of plasma parameters. The inertial range grows with distance, progressively extending to larger spatial scales, while steepening toward a \( \alpha_B = -5/3 \) scaling. It is observed that spectra for intervals with large magnetic energy excesses and low Alfvénic content steepen significantly with distance, in contrast to highly Alfvénic intervals that retain their near-Sun scaling. The occurrence of steeper spectra in slower wind streams may be attributed to the observed positive correlation between solar wind speed and Alfvénicity.

Unified Astronomy Thesaurus concepts: Solar wind (1534); Magnetohydrodynamics (1964); Interplanetary turbulence (830); Space plasmas (1544); Plasma astrophysics (1261)

1. Introduction

The solar wind flow transports a wide range of magnetic field and plasma fluctuations (Coleman 1968; Velli et al. 1989). Because fluctuations are predominantly Alfvénic (i.e., magnetic field and velocity fluctuations exhibit the correlations typical of outwardly propagating Alfvén waves; Bruno & Carbone 2013), and relative density fluctuations are very small, solar wind turbulence is usually discussed within the phenomenologies of incompressible magnetohydrodynamic (MHD).

During the expansion, nonlinear interactions result in a cascade of the energy toward smaller scales (Matthaeus & Velli 2011). Therefore, the energy injected into the solar wind at large scales, likely of solar origin, cascades downwards until it reaches ion scales, at which point the dynamics involve kinetic processes and structures such as ion cyclotron damping, kinetic Alfvén waves, kinetic scale current sheets, etc. (Leamon et al. 1999; Cranmer 2001; Dmitruk et al. 2004; Karimabadi et al. 2013; TenBarge & Howes 2013). Turbulence is thought to be one of the main processes contributing to the nonadiabatic expansion, as well as the generation of the solar wind (SW; Matthaeus & Velli 2011). MHD turbulence phenomenologies predict different power-law exponents depending on prevailing characteristics of turbulence, such as spatial wavenumber anisotropy (Goldreich & Sridhar 1995, 1997), intermittency (Mininni & Pouquet 2009; Chandran et al. 2015), and the scale-dependent correlation between velocity and magnetic field (Boldyrev 2006; Beresnyak & Lazarian 2010). The variability of solar wind turbulence properties in the inner heliosphere reflects the diversity of solar coronal sources, which modulate the density, velocity, temperature, and ion composition of the plasma. As a result, several factors, including the role played by large-scale gradients (Velli et al. 1989; Chandran & Perez 2019); the proximity to the heliospheric current sheet (HCS; Chen et al. 2021; Shi et al. 2022b); the presence of magnetic field switchbacks (Bourouaine et al. 2020; Martinovic et al. 2021; Shi et al. 2022a); large-scale velocity shear in the SW (Coleman 1968), strongly influence the properties of turbulence, resulting in a wide range of spectral scalings. By means of fitting the power spectrum within a constant...
range in the frequency domain, recent statistical studies of Parker Solar Probe (PSP) data, have recovered a nonevolving velocity spectral index close to $-3/2$, independent of the radial distance from the Sun (Shi et al. 2021), while the magnetic field spectrum steepens from a $-3/2$ slope at $\sim0.2$ au to a $-5/3$ slope at $\sim0.6$ au (Alberti et al. 2020; Chen et al. 2020; Shi et al. 2021).

Two scales are crucial to understanding the radial evolution of turbulence in the solar wind (1) the ion inertial scale $d_i = V_{di}/\Omega_i$, and (2) the thermal ion gyroradius, $\rho_i = V_{thi}/\Omega_i$, where, $\Omega_i = e|B|/m_p$, is the proton gyrofrequency, $e$ is the elementary charge, $|B|$ is the magnitude of the magnetic field, and $m_p$ is the mass of the proton. With increasing heliocentric distance, both physical scales ($d_i$, $\rho_i$) increase (Duan et al. 2020; Cuesta et al. 2022).

It is thus natural to expect that the relative physical scale of fluctuations of a given frequency decreases as the solar wind expands. Here we aim to understand the radial evolution of magnetic turbulence and to study the basic features of scaling laws for solar wind fluctuations in terms of properly normalized physical scales. High resolution data from PSP (Fox et al. 2016), and Solar Orbiter (SO; Müller et al. 2020) covering heliocentric distances $13 R_\odot \lesssim R \lesssim 220 R_\odot$ are utilized and the radial evolution of the magnetic spectral index as a function of normalized wavenumber is investigated.

It is shown that closer to the Sun the magnetic field power spectrum exhibits a poorly developed inertial range that is characterized by a $-3/2$ spectral index. The inertial range extends to larger and larger scales as the solar wind expands into the interplanetary medium, with the inertial range spectral index steepening toward a $-5/3$ value. We demonstrate that the rate at which the steepening occurs is strongly dependent on magnetic energy excess and Alfvénicity of the fluctuations.

2. Radial Evolution of Magnetic Field Spectral Index

We considered overlapping intervals of duration $d = 24$ hr such that the beginnings of adjacent intervals are 8 hr apart. For each interval, the Fourier trace power spectral density $F(f_{sc})$ was calculated, smoothed by averaging over a sliding window of a factor of 2, and transformed into a wavenumber spectrum expressed in physical units $E(\kappa^*)$ by virtue of the modified TH:

$$E(\kappa^*) = \frac{V_{sc}}{2\pi} F(f_{sc}) \cdot \xi,$$

where $\kappa^* = \kappa \cdot \xi = \frac{2\pi f_{sc}}{V_{sc}} \cdot \xi$, and, $\xi = d_i/\rho_{ci}$.

The radial evolution of the power spectrum as a function of spacecraft frequency and normalized by $d_i$ is presented in Figures 1(a), (b), respectively. Due to the expansion of the solar wind but also in part because of the turbulent cascade, a decrease of $\sim4$ orders of magnitude in magnetic power is observed with increasing heliocentric distance.

The spectral index, $\alpha_B$ is obtained by taking a sliding window of one decade in the spacecraft-frame frequency (wavenumber) domain, over the smoothed spectra and calculating the best-fit linear gradient in log-log space over this window. For clarity, 10 radial bins have been used, and the median value of the spectral index as a function of frequency has been estimated for intervals that fall within the same bin. The color of the curve is keyed to the mean value of the distance $R$ corresponding to the intervals within each bin. The results of this analysis are presented in Figure 2(a). In the inertial range, an energy cascade rate that is independent of scale is expected, reflecting on the power spectrum in the form of a constant spectral index over this range of scales. In light of this, it can be seen that close to the Sun (dark blue line in Figure 2(a)), the inertial range is limited into a narrow range of frequencies ($2 \times 10^{-2} - 2 \times 10^{-1}$ Hz). As the solar wind expands in the interplanetary medium (1) a universal steepening (i.e., across all frequencies) is observed for the spectral index, $\alpha_B$ at a constant $f_{sc}$; (2) the curves shift horizontally to lower and lower frequencies. As illustrated in Figure 2(a), the frequency range over which the spectral index is constant is migrating to the left while steepening with increasing distance, from $\alpha_B \approx -3/2$ to $\alpha_B \approx -5/3$. Similar behavior is observed at the largest scales. Closer to the Sun for $f_{sc} \lesssim 2 \times 10^{-2}$ Hz, the spectrum gets progressively shallower at lower frequencies and obtains a value of $\alpha_B \approx -1$ at $f_{SC} = 3 \times 10^{-4}$ Hz. As heliocentric distance increases, this low-frequency part of the spectrum gradually steepens, with all the frequencies approaching a $-5/3$ scaling. Therefore, as the solar wind propagates outward, the inertial range of the spectrum develops gradually, extending from higher frequencies to progressively lower and lower frequencies. Additionally, in accordance with Duan et al. (2020) the ion scale break, separating the inertial from the kinetic range is observed to migrate to lower frequencies with distance.

To cast the results in terms of relevant physical scales, we considered the evolution of $\alpha_B$ into the wavenumber domain...
normalizing by either the ion inertial length ($d_i$) or the ion gyroradius ($\rho_i$).

The evolution of the spectral index as a function of distance ($R$) in the wavenumber domain normalized by $d_i$ is illustrated in Figure 2(b). It is readily seen that the vertical shifting of the curves to lower frequencies, observed in Figure 2(a), has vanished: all the curves roll over at $\kappa d_i \approx 0.1$ and overlap at smaller scales. The normalization does not appear to have a substantial impact on the radial development of the spectral index at large scales, $\kappa \gtrsim 8 \times 10^{-2}$, since a steepening that closely resembles Figure 1(a) is obtained. On the other hand, as shown in Figure 1(b), the small scale break, demarcating the beginning of the transition region, $\kappa^* \approx 9 \times 10^{-2} (\rho_i^{-1})$, does not show any remarkable evolution with distance and stays constant in physical space. We do not show plots using $\rho_i$ as normalization because the spectra do not collapse as clearly into one curve at small scales, demonstrating that $d_i$ is the more appropriate scale for such a normalization.

2.1. Dependence of $\alpha_B$ on Plasma Parameters

To disentangle the spectral variation with distance from changes due to the differing plasma parameters of different solar wind streams the dependence of $\alpha_B$ on the normalized cross-helicity $\sigma_c$.

$$\sigma_c = \frac{E_r - E_i}{E_r + E_i},$$

a measure of the relative amplitudes of inwardly and outwardly propagating Alfvén waves, and the normalized residual energy $\sigma_r$,

$$\sigma_r = \frac{E_r - E_b}{E_r + E_b},$$

indicating the balance between kinetic and magnetic energy is examined; $E_q = \frac{1}{2} (\mathbf{q} \cdot \mathbf{q})$ denotes the energy associated with the fluctuations of the field $\mathbf{q}$. In particular, $E_{\alpha,i}$ can be estimated using Eötvös variables, defining outward and inward propagating Alfvénic fluctuations (Velli et al. 1991; Velli 1993)

$$\delta Z_{\alpha,i} = \delta V \mp \text{sign}(B_0^\parallel) \delta b,$$

with $B_0$ the background magnetic field, $\delta b = \delta B / \sqrt{\mu_0 \sigma_r}$ the magnetic fluctuations in Alfvén units, and $B_0^\parallel$ the ensemble average of $B_R$, are utilized to determine the polarity of the radial magnetic field (Shi et al. 2021). The variation of $\alpha_B$ with $V_{SW}$, the ratio of magnetic to thermal pressure, $\beta \equiv n_e k_B T / (B^2 / 2 \mu_0) \ll 1$, and the field/flow angle $\theta_{BF}$ was also examined. Though we do not focus on $\beta$, and $\theta_{BF}$ here, we will comment on these in Section 3. The evolution of $\alpha_B$ is investigated by fitting the magnetic spectrum over a constant range $(10^{-3} - 3 \times 10^{-2} d_i^{-1})$. To ensure that the plasma parameters under study do not vary significantly within the interval the duration of intervals has been reduced to $d = 1$ hr.

2.2. Solar Wind Speed, $V_{SW}$

As shown in Figure 4(a), within 30 $R_s$, no significant differences in spectral index with solar wind speed are found with an inertial range scaling, $\alpha_B \approx -3/2$. As the solar wind expands, the dependence on solar wind speed becomes more evident: steepening occurs regardless of solar wind speed, but it is more efficient for slower solar wind streams. As a result, at $R \approx 1$ au, the dependence of the spectral index on speed is clear, with the spectral index being consistent with a K41 scaling in the fast wind and a steeper scaling of $\sim -1.8$ for the slowest winds. Categorizing the spectral index as a function of $\tau_{adv}$, Figure 4(b), instead of radial distance one finds that for $\tau_{adv} < 40$ hr no clear trend is observed for the spectral index as a function of wind speed. Beyond, this point, though steepening is monotonic with $\tau_{adv}$ at all wind speeds. Overall, lower speed intervals display a significant radial steepening as compared to faster winds that only display a slight steepening. Closer to the Sun, however, there seems to be no dependence on wind speed on the spectral index, suggesting that the spectra are initially similar regardless of the wind speed.

2.3. Normalized Cross Helicity, $\sigma_c$, and Normalized Residual Energy $\sigma_r$

The joint $\sigma_c - \sigma_r$ distribution, estimated using 1 minute long moving averages of the respective time series is presented in Figure 3. The median and mean value of $\sigma_c$ and $\sigma_r$ for each bin are also shown as red and black crosses, respectively. The gray
circle defines fluctuations with perfect alignment between velocity and magnetic field, given by $\sigma_v^2 + \sigma_B^2 = 1$. Closer to the Sun (0.06–0.1 au) turbulence is highly Alfvénic, dominated by outwardly propagating waves ($\sigma_v \approx 0.85$), and in slight excess of magnetic energy ($\sigma_B \approx -0.15$). A small population of strongly magnetically dominated intervals characterized by very low Alfvénic content (i.e., $\sigma_v \approx -1$, and $\sigma_B \approx 0$), mostly associated with HCS crossings is also observed (see Shi et al. 2022b). At larger heliocentric distances the mean/median value of $\sigma_v$ progressively decreases (Chen et al. 2020; Shi et al. 2021). Several mechanisms have been proposed to explain the diminishing dominance of outwardly propagating waves with increasing heliocentric distance due to wave reflection, including velocity shears (Buvassano et al. 1982) and the parametric decay instability (Tenerani & Velli 2013; Shoda et al. 2019). At 1 au, $\sigma_v$ is clearly more negative than in the near-Sun environment, but it does not show a clear trend with radial distance. In the distance range of 0.6–1 au, most of the data points are concentrated in the lower half, with a few intervals having slightly positive $\sigma_v$ values. In addition, data points located in the bottom left quadrant are increasing with radial distance, indicating a radially decreasing dominance of waves propagating outward.

The power-spectra of the fluctuating fields $\delta b$, $\delta V$, $\delta Z_{\alpha\beta}$ have been obtained and both $\sigma_v$, and $\sigma_B$ have been estimated by integrating the resulting spectra over a constant range ($10^{-3} - 5 \times 10^{-2} d_i^{-1}$) in the wavenumber domain normalized by the ion inertial length. The dependence of the spectral index on $|\sigma_v|$ and $\sigma_B$ as well as the radial distance ($R$) is presented in Figures 4(c), (d) for $\sigma_v$ and $\sigma_B$, respectively. These show how highly Alfvénic ($|\sigma_v| \approx 1$) and energetically equipartitioned intervals display little spectral evolution, while evolution to significantly steeper spectra is associated with low $|\sigma_v|$ and/or large magnetic energy excess, with the data at large distances consistent with 1 au results (Podesta & Borovsky 2010; Chen et al. 2013; Bowen et al. 2018).

3. Conclusions

Using PSP and SO data from the inner heliosphere we have analyzed (1) how the statistical signatures of turbulence evolve with heliocentric distance and (2) the plasma parameters driving the evolution.

Identifying a plasma scale that grows radially at the same rate as the high frequency break point is crucial for this study in order to anchor the spectrum in normalized wavenumber space and enable meaningful comparison between intervals sampled at different heliocentric distances. Since the high frequency break point exhibits a power-law radial dependence with a scaling exponent $1.08 \pm 0.03$ (Duan et al. 2020; Lotz et al. 2022), then $d_i$, which grows radially as $\propto R^{0.04}+0.01$, provides a better normalization than $\rho_i$, which is characterized by a $\propto R^{0.68}+0.02$ radial scaling. It is important to emphasize that the goal here is not to find the quantity that has 1:1 correspondence with the high frequency break point. In fact, it is well known that intervals characterized by low ion $\beta$ values exhibit a magnetic power spectral density that breaks at the ion inertial length ($d_i$), while high $\beta$ intervals are characterized by a small scale break at the thermal ion gyroradius ($\rho_i$) (Chen et al. 2014). Thus, the pinning of the power spectrum at a constant $\kappa d_i$ scale, with increasing distance may be explained by the fact that plasma $\beta$ values remain rather low in the inner heliosphere; see Figure 5. Another plasma scale that is correlated with the high frequency break point and should be considered by future investigations is the proton cyclotron resonance (Woodham et al. 2018).

Additionally, our analysis indicates that closer to the Sun, the inertial range of the magnetic field power spectrum is poorly developed i.e., the range of scales over which $\alpha_B$ remains constants is limited; its value is closer to $\alpha_B = -3/2$. As the solar wind expands into the interplanetary medium, the inertial range extends to progressively larger scales, while at
the same time the inertial range spectral index steepens toward $\alpha_B = -5/3$.

We demonstrate that the rate at which $\alpha_B$ steepens is strongly dependent on the normalized residual energy and normalized cross helicity of the intervals under study. In particular, intervals with high Alfvénic content ($\sigma_c \approx 1$), and equipartitioned in $E_V - E_B$ ($\sigma_r \approx 0$) seem to retain their near-Sun scaling, and show a minor steepening with radial distance. In contrast, magnetically dominated and balanced intervals are observed to strongly steepen, resulting in anomalously steep inertial range slopes at 1 au, consistent with previous studies (Pouquet 2009; Borovsky 2010; Chen et al. 2013; Bowen et al. 2018).

While $|\sigma_c| \approx 1$ and $\sigma_r \approx 0$ values may be found in slow wind streams, especially closer to the Sun, they are statistically less relevant than in fast winds (Shi et al. 2021). As a result, the occurrence of steeper spectral indices in slower wind streams may be attributed to the observed positive correlation between solar wind speed and $\sigma_c$, $\sigma_r$.

Intervals with large magnetic energy excess closer to the Sun do not display the steep spectra observed at 1 au, attributed by Bowen et al. (2018) to the correlation between magnetic coherent structures and highly negative $\sigma_r$ values (Mininni & Pouquet 2009). Recent studies (Cuesta et al. 2022; Sioulas et al. 2022), suggest that magnetic field intermittency is strengthened with increasing heliocentric distance in the inner heliosphere, but no similar analysis has been conducted for the velocity field. However, velocity spectra do not display radial evolution (Shi et al. 2021) and exhibit a scaling of $a_v = -3/2$ at 1 au (Chen et al. 2013). Based on our results, we expect that both the magnetic and velocity field spectra display a $-3/2$ scaling closer to the Sun, with the evolution of the magnetic spectrum related to the in situ generation of magnetic coherent structures during expansion. A study of the evolution of $\alpha_B$ and $a_v$ as a function of radial distance as well as intermittency is ongoing. Turbulence in the solar wind is anisotropic with respect to the mean magnetic field (see, e.g., reviews by Schekochihin et al. 2009; Horbury et al. 2012; Oughton et al. 2015, and references therein). Horbury et al. (2008), Wicks et al. (2010), and Kiyani et al. (2012) have shown that when the field/flow angle $\Theta_{BV}$ is $\Theta_{BV} = 90^\circ$, then the inertial range range scales like either $\alpha_B \approx -5/3$, or sometimes $\approx -3/2$, consistent with a critical balance cascade and dynamical alignment models respectively. In the parallel direction, $\Theta_{BV} = 0^\circ$, it is nearer $\alpha_B \approx -2$. In contrast, when a global magnetic field is utilized to estimate $\theta_{BV}$ no anisotropy in the spectral index as a function of $\Theta_{BV}$ is observed (Tessein et al. 2009; Chen et al. 2011). Though it is not shown here, we find no correlation between $\theta_{BV}$ and $\alpha_B$, using a global magnetic field. A similar result was obtained when considering the dependence of $\alpha_B$ on plasma $\beta$, suggesting that these two parameters are not related to the steepening of the spectrum. Further work to clarify the debate between a local, scale-dependent and global background magnetic field and its role on the spectral evolution is presented in a companion paper (Sioulas et al. 2023).

Our findings will help us gain a better understanding of how solar wind turbulence is generated and transported and will guide future models of solar wind turbulence.

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Appendix
Data Selection and Processing

We analyze magnetic field data from the Flux Gate Magnetometer (Bale et al. 2016), as well as plasma moment data from the Solar Probe Cup (SPC) and Solar Probe Analyzer (SPAN) part of the Solar Wind Electron, Alpha, and Proton suite between 2018 January 1, and 2022 June 15, encompassing the first 12 perihelia (E1–E12) of the PSP mission. SPC data have been utilized for E1–E8, while SPAN data for E9–E12. Quasi-thermal noise derived data (Moncuquet et al. 2020), have been preferred over SPC or SPC data when available. Additionally, magnetic field and particle moment measurements from the Magnetometer instrument (Horbury et al. 2020), prioritizing burst data when available, and the Proton and Alpha Particle Sensor (SWA-PAS; Owen et al. 2020) on board the SO mission between 2018 June 1, to 2022 March 1, were considered. Following consideration of quality flags, time intervals that were found to be missing >1% and/or >10%, in the magnetic field and particle time series have been omitted from further analysis. The remaining intervals have been resampled linearly to the highest cadence possible, based on their initial resolution. Finally, in order to eliminate spurious spikes, a Hampel filter (Davies & Gather 1993) was applied to the plasma time series.

Converting the spacecraft-frame frequency derived PSD, \( F(f_{sc}) \) to a wavenumber PSD, \( E(\kappa) \), far from the Sun is possible by means of Taylor’s hypothesis (TH; Taylor 1938), \( \kappa = 2\pi f_{sc}/V_{SW} \), which becomes questionable when both the Alfvén and spacecraft velocity are comparable to the velocity of the solar wind. Therefore, a modified version of TH that accounts for both wave propagation and spacecraft velocity is adopted (Klein et al. 2015): in the above expression for \( \kappa V_{sw} \) is replaced by \( V_{tot} = |V_{sw} + V_a - V_{sc}| \) where \( V_{sc} \) is the spacecraft velocity, where turbulence is assumed to be dominated by outwardly propagating Alfvén waves. Note that the TH remained either moderately or highly valid for the majority of time intervals examined, with only \( \sim 1.5\% \) of the intervals under study exhibiting \( M_A < 1.5 \), including a number sub-Alfvénic intervals during PSP E8–E12 (\( \sim 0.45\% \) of the entire data set). Figure 5 illustrates the radial evolution of ion inertial length \( d_i \), ion gyroradius \( \rho_i \), and ion plasma \( \beta \), quantities relevant to this study.

Figure 5. The radial evolution of (a) the ion inertial length \( d_i \), (b) the ion gyroradius \( \rho_i \), (c) ion plasma \( \beta \).
