New Probe of Gravity: Strongly Lensed Gravitational-wave Multimessenger Approach

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Abstract

Strong gravitational lensing by galaxies provides us with a unique opportunity to understand the nature of gravity on galactic and extra-galactic scales. In this paper, we propose a new multimessenger approach using data from both the gravitational wave (GW) and the corresponding electromagnetic (EM) counterpart to infer the constraint of the modified gravity (MG) theory denoted by the scale dependent phenomenological parameter. To demonstrate the robustness of this approach, we calculate the time-delay predictions by choosing various values of the phenomenological parameters and then compare them with that from general relativity (GR). For the third generation ground-based GW observatory, with one typical strongly lensed GW+EM event, and assuming that the dominated error from the stellar velocity dispersions is 5%, the GW time-delay data can distinguish an 18% MG effect on a scale of tens of kiloparsecs with a 68% confidence level. Assuming GR and a Singular Isothermal Sphere mass model, there exists a simplified consistency relationship between time-delay and imaging data. This relationship does not require the velocity dispersion measurement, and hence can avoid major uncertainties. By using this relationship, the multimessenger approach is able to distinguish an 8% MG effect. Our results show that the GW multimessenger approach can play an important role in revealing the nature of gravity on galactic and extra-galactic scales.

Key words: cosmology: theory – gravitation – gravitational lensing: strong – gravitational waves

1. Introduction

Einstein’s general relativity (GR) has been precisely tested on solar system scales (Bertotti et al. 2003; Shapiro et al. 2004), such as the Eddington’s measurement of light deflection during the solar eclipse of 1919 (Dyson et al. 1920), the observation of the gravitational redshift (Pound & Rebka 1960), the successful operation of the Global Positioning Satellites (Ashby 2003), the measurements of the Shapiro time-delay (Shapiro 1964), and the verification of energy loss via gravitational waves (GWs) in the Hulse–Taylor pulsar (Taylor et al. 1979). We refer to the living review (Will 2014) for updated theoretical and observational progress in testing gravity. However, the long-range nature of gravity on the extra-galactic scale is still poorly understood. Testing gravity with higher accuracy has been continuously pursued over the past decades. The purpose of these activities is not only to examine a specific model, but also to reveal the nature of gravitational phenomena, such as dark matter and dark energy, on cosmological scales. The parameterized post-Newtonian (PPN) framework (Thorne & Will 1971) provides us with a systematic way to quantify the deviation from GR. The scale independent post-Newtonian parameter γPPN represents the ratio between dynamical mass and lensing mass. The former is the mass determining the motion of nonrelativistic objects, such as the stellar velocity dispersion, while the latter is the mass determining the path deflection of relativistic particles, such as the Einstein radius of strong lensing image and Shapiro time-delay. GR predicts the equivalence of these two masses (γPPN = 1), while the alternative models break it due to the extra scalar degree of freedom, which mediates the fifth force. Here we note that we do not consider the massive graviton or the anomaly of the speed of GW, which is tightly constrained by the recent GW detection of Abbott et al. (2017a).

Strong gravitational lensings around galaxies provide us with a unique opportunity to probe modifications to GR over a range of redshifts and on/above kiloparsec scales (Bolton et al. 2006; Smith 2009; Schwab et al. 2010; Cao et al. 2017). Recently, Collett et al. (2018) estimated γPPN to be 0.97 ± 0.09 at a 68% confidence level by using a nearby lens, ESO 325-G004. In this analysis, the dynamical mass is estimated by the spectroscopic measurement of the stellar velocity dispersion of the lens galaxy, and the lensing mass is reconstructed by the measurement of the Einstein radius. All of these data are obtained by using the electromagnetic signals.

The first GW event GW150914 from the merger of the binary black hole, detected by the LIGO Scientific Collaboration5 and the Virgo Collaboration,6 opened a new observational window to explore our universe (Abbott et al. 2016). Moreover, the following event, GW170817, from a binary neutron star combined with the electromagnetic (EM) counterpart, announced the beginning of the golden era of multimessenger astronomy (Abbott et al. 2017b). With the development of multimessenger observations, searching the strongly lensed GW event associated with the EM counterpart should be attractive. Recently, several researches have investigated this issue for the LIGO events (Smith et al. 2018; Hannuksela et al. 2019; Broadhurst et al. 2019). In the context of the Laser Interferometer Space Antenna (LISA) interferometric detector in space,7 strong lensing of LISA target sources (supermassive BHs) and its application on

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5 https://ligo.org
6 http://www.virgo-gw.eu
7 https://www.lisamission.org
cosmology have also been discussed (Sereno et al. 2010, 2011). The third generation of ground-based GW observatories, such as the Einstein Telescope (Punturo et al. 2010), is expected to detect $10^4$–$10^5$ events per year and 50–100 among them are strongly lensed (Piórkowska et al. 2013; Biesiada et al. 2014; Ding et al. 2015). The redshift of the unlensed event is expected to reach around $z \sim 2$. While, due to the magnification by intervening galaxies, the redshift range would extended to $z \sim 8$ (Biesiada et al. 2014). The observation of a strongly lensed GW with an EM counterpart should be very promising. The time delays of GW from different paths are expected to be a few months; while the duration of each signal is less than 0.1 s. The transient nature of this double compact object merger event would lead to a very accurate measurement of the time-delay compared to the traditional light-curve measurement in the optical domain (~3%); hence, we can ignore this uncertainty of time-delay measurement from GW. Furthermore, the lensing mass reconstruction from the EM counterpart images has a systematic uncertainty of around the 0.6% level (Liao et al. 2017). While, for the traditional lensed quasar system, the uncertainties of the time-delay and lensing mass reconstruction are both of order $O(3\%)$ (Liao et al. 2015; Suyu et al. 2017). This improvement could be illustrated as follows. For the optical quasar system, the Fermat potentials are recovered from the lensed host galaxy image. However, in the central part of the active galactic nucleus (AGN), due to the overly saturated exposure, even a tiny mismatch, when extracting the AGN images as the scaled point spread functions (PSFs), would lead to a nonnegligible discrepancy. Ding et al. (2017) carried out a data challenge and found that even if the perfect PSF is given, a significant residual in the central AGN area is still inevitable. Fortunately, one does not encounter these difficulties while studying the lensed GW+EM events since these systems do not possess the bright point images. Liao et al. (2017) performed the simulations and used state-of-the-art lens modeling techniques; they found that the relative uncertainty of the Fermat potential reconstruction could reach the 0.6% level. This improves the traditional optical 3% level by a factor of 5. Recently, Liao et al. (2017) forecasted the robustness of Hubble parameter estimation by using the strongly lensed GW+EM signals. They found that 10 such systems are able to provide a Hubble constant uncertainty of 0.68% for a flat $\Lambda$ cold dark matter (ΛCDM) universe by using the Einstein Telescope. The possibility of exploring the GW speed anomaly was investigated by Fan et al. (2017).

Using the GW+EM multimessenger approach to test GR has been investigated by many studies (Baker et al. 2017; Abbott et al. 2019; Nishizawa & Arai 2019). The strongly lensed GW+EM should also provide us with a very promising tool to test GR. The optical strong lensing test of GR under the PPN framework has been proposed in Bolton et al. (2006), Smith (2009), Schwab et al. (2010), and Cao et al. (2017). All of these studies are limited only in the EM domain, which is out of date in the richness of GW+EM multimessenger observation today. Having the significant benefits from strongly lensed GW addressed above, it inspired us to consider the strongly lensed GW+EM system as a new probe to the nature of gravity on kiloparsec scales.

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2. Model Setup

In the limit of a weak gravitational field, consider the perturbed Friedmann–Lemaître–Robertson–Walker metric in the conformal Newtonian gauge

$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right)c^2dt^2 + a^2\left(1 - \frac{2\Psi}{c^2}\right)dx^2,$$

where $a$ is the background scale factor, $\Psi$ and $\Phi$ are the Newtonian potential and spatial curvature perturbation. Non-relativistic particle motion only responds to a spatial gradient of the Newtonian potential, $\Phi_\pm = (\Phi + \Psi)/2$. GR predicts the equivalence of Weyl potential $\Psi$ and the spatial curvature perturbation $\Phi$, namely $\gamma_{PPN} \equiv \Phi/\Psi = 1$. In contrast, the alternative models of gravity typically contain an additional scalar degree of freedom that mediates the fifth force, and hence breaks this degeneracy.

On kiloparsec scales, strong gravitational lensing, combined with stellar kinematics of the lens, allows a test of the weak field metric of gravity. Measurements of the stellar velocity dispersion determine the dynamical mass, whereas measurements of the image positions of the lens and time-delay between the multiple images determine the lensing mass. They reflect the nature of Newtonian and Weyl potentials of the underlying gravity, respectively. In GR, the two potentials are the same, but many alternative gravity models predict the ratio of the two potentials to be scale dependent. For example, for the generic dark energy/modified gravity (MG) models, the unified parameterization of linear perturbations under the quasi-static approximation is given by Pogosian & Silvestri (2016).

On the nonlinear scale, a parameterization of MG is given by Lombriser (2016), which unifies different screening mechanisms. All of these indicate that the MG effect is a scale dependent phenomenon. There exist plenty of parameter spaces to allow for spatial scale dependent modifications to standard gravity. However, the analysis made in references Bolton et al. (2006), Smith (2009), Schwab et al. (2010), Cao et al. (2017), and Collett et al. (2018) assumed a constant PPN parameter over the length scales relevant for their studies. In this paper, we directly parameterize the Weyl potential, which is the most relevant parameter to the lensing. As we know, the viable models of MG have to shield the fifth force under a certain scale, namely the screening scale (Joyce et al. 2015). The MG effect only arises above these scales.

In this paper, we introduce a scale dependent phenomenological parameter to denote the deviation of the Weyl potential from the Newtonian one, that is, $\Sigma(r) = \Phi_\div \Phi_{\text{GM}}$ in real space. Generally speaking, the MG effect shall also depend on the morphology of environment of the galaxies. However, most of the studies of screening mechanism in the current literature (see Koyama 2016 for a review) are assuming spherical symmetry. This is due to the fact that, on the one hand, spherical symmetry is computationally feasible; and, on the other hand, the nonsphericity, such as ellipticity, is not the major issue in the MG modeling for now. Our current galaxy survey data quality cannot tell us more than the spherical model in the gravity test. As for our studies, we explicitly assumed that the size of the screening scale of the MG effect is larger than the disk size of
the lens galaxy. Hence, the symmetry of the lens is irrelevant to its environment (screening scale).

The reason we want to test the scale dependence of the $\Sigma$ function is the following. As shown in Figure 1 of Pogosian & Silvestri (2016), the scale dependence of the $\Sigma$ function is crucial for the gravity test. Take the generalized Brans–Dicke model as an example, which includes $f(R)$ gravity (Capozziello et al. 2003; Carroll et al. 2004; Appleby & Battye 2007; Hu & Sawicki 2007; Starobinsky 2007), chameleon (Khoury & Weltman 2004), symmetron (Hinterbichler & Khoury 2010), and dilaton (Damour & Polyakov 1994; Brax et al. 2011) models. The extra scalar field could give rise to sizeable modifications to the Newtonian potential ($\mu$) below its Compton wavelength. But the modifications to Weyl potential ($\Sigma$) are usually tiny. As shown in Equation (21) of Pogosian & Silvestri (2016), in some subset of Horndeski (1974) models, Weyl potential can significantly deviate from unity on the cosmological scale. Moreover, Lombriser (2016) proposed a unified parameterization of the screening mechanism. Combining them, we can model the scale dependent MG dynamics from the linear down to the nonlinear regimes. Hence, if a sizeable deviation of $\Sigma$ from unity was detected, we can not only rule out GR, but also all kinds of generalized Brans–Dicke models. In addition, a scale dependent MG effect would be very helpful to break the degeneracy between the dynamical mass and the MG effect, especially when we have information of both the time-delay and image positions of many lensed events in the future. Thus the motivation of our paper should be stated clearly here. We want to provide a new approach with the exciting and fashionable strongly lensed multimessenger to test the GR on kiloparsec (galaxy) scales.

The photons and gravitons emitted from the source galaxies cumulate the MG effect along the line of sight when they approach us. Analogous to the parametric forms of $\mu$ and $\gamma$ (Bertschinger & Zukin 2008; Hojjati et al. 2012; Pogosian & Silvestri 2016), which denote modifications to $\Psi$ and $\Phi$, we set the parametric form of $\Sigma(r)$ as

$$\Sigma(r) = \begin{cases} 
1, & 0 < r < r_{01} \\
1 + \alpha_1 \left( \frac{r-r_{01}}{w_{01}} \right)^2, & r \geq r_{01}, \end{cases}$$

where $\alpha_1 / \alpha_2$ describes the magnitude of the modification of gravitational coupling, $r_{01}$ denotes the transition scale, and $w_{01}$ represents the width of the transition step. We should note that the symbol $\Sigma$ here should not be confused with the surface density of the lens galaxy, which we usually adopted. In the following, we will use another symbol to denote the surface density parameter. The typical Einstein radius of the galaxy lens is around 10 kpc. We can imagine that the galaxy lensing data shall be sensitive to the MG effect, which arises on similar scales. In this paper, we choose two specific transition scales, 10 and 20 kpc, for comparison. This means that the MG effect vanishes below the galactic scale. A bonus is that we can adopt the standard lens modeling, such as SIS, without modifying the complicated galaxy dynamics. More specifically, as we know, the scale above kiloparsecs to megaparsecs is a blank window for gravity tests (precise tests; Koyama 2016). In this paper, we are not aiming to test a specific model, but using a phenomenological parameterization, which may cover a rather large model space. The two transition scales we give here are for comparison to show the robustness of this GW+EM time delay approach for the gravity test. Hence, the MG effect comes into this system via a very clean way, namely the line-of-sight integral. Take the deflection angle as an example. For a point mass lens, the MG effect contributes to two aspects. The first term of Equation (3), which is similar to $\gamma_{\text{PPN}}$, corresponds to the enhancement/suppression of gravitational coupling of the relativistic species. The second term is due to the spatial variation of the gravitational coupling, which is absent in the constant $\gamma_{\text{PPN}}$ case. Notice that differential measurement of $dz$ here is not redshift, but the Cartesian coordinate infinitesimal increment along the “$z$” direction (line of sight)

$$|\hat{\alpha}| = \frac{4GM}{c^2} b \int_0^\infty \left( \frac{\Sigma}{r^3} - \frac{\Sigma_r}{r^2} \right) dz. \quad (3)$$

Set $I(b) = \int_0^\infty \left( \frac{\Sigma}{r^3} - \frac{\Sigma_r}{r^2} \right) dz$, the deflection angle can be written as

$$|\hat{\alpha}| = \frac{4GM}{c^2} b I(b). \quad (4)$$

In GR, $I(b) = 1/b^2$, thus the deflection angle should be recovered to $\frac{4GM}{c^2} \frac{1}{b^2}$.

Given the lensing model and gravity theory, we can derive the multiple image positions and the time delay. The MG theory with $\Sigma$ deviating from unity shall lead to a modification of the deflection angle. For a general lens model, the deflection angle is, basically, the integration of the point mass over the corresponding lensing mass distribution. Hence, for any specific lens model with a given mass profile, we can calculate the deflection angle for a given impact parameter $b$. Unlike the GR case, the time-delay in the MG case does not only depend on the geometric term, but also the effective lensing potential. The deviation of $\Sigma$ from unity represents the size of the MG effect. It produces both modifications to deflection angle and effective lensing potential, hence the final anomaly of time delay. After propagating all of the systematic uncertainties to the errors of time delay, we can compare the anomaly of time delay caused by MG to the systematic uncertainties to see how precisely we can distinguish MG theory from GR.

The time delays from GW are the “multiple sounds” and the multiple images from EM counterpart are the “multiple images.” For a given system, the “multiple sounds” and “multiple images” can be derived simultaneously and respond to the same lensing mass. Thus, if we could measure the dynamical mass (via velocity dispersions) accurately, the constraint on MG is straightforward. Both the time delay and image positions can be used, respectively, to estimate the difference between the lensing and dynamical mass. In our paper, we choose the time delay from GW as the first part of our work in Section 3 to infer the constraints of the MG parameters. However, the image information from the EM counterpart should also be used as a supplement to improve the constraints, especially for a scale dependent MG effect. This is the second part of our work in Section 4.

3. Time Delay from GW

For a given lensed GW+EM event by a galaxy with a specific mass profile, the time delay between two images is
given by
\[
\Delta u_{ij} = \frac{1 + z_f D_l D_i}{c} \frac{D_i D_j}{D_i} \Delta \phi_{ij},
\]
here \( \Delta \phi_{ij} = [(\theta_i - \beta)^2/2 - \psi(\theta_i)] - [(\theta_j - \beta)^2/2 - \psi(\theta_j)] \) is the Fermat potential difference between different images at angular positions \( \theta_i \) and \( \theta_j \). The source is located at angular \( \beta, \psi \) is the effective two-dimensional lensing potential, which is the integral of the Weyl potential along the line of sight. The MG effect will enter the Fermat potential, and thus give the anomaly of the time-delay compared to the GR case.

Identifying the lensed GW+EM events is a great challenge. In principle, the crucial part is a cross-confirming procedure in both GW and EM domains. A single detection in one domain should trigger an associated search in the data from the other one. For example, if GW data analysis provides a pair of events suspected of being lensed, it should trigger a search for lensed (repeated) EM transients in the sky location strip of the GW source. Conversely, if a lensed kilonova event is observed in a large survey telescope, this should trigger confirmation searches in the GW signal database for coherent waveforms and time delay, which are synergistic with EM signal (Liao et al. 2017). After getting the measurements of the source and lens galaxy redshifts \((z_s, z_l)\) as well as the spectroscopic measurement of the stellar velocity dispersion \((\sigma_v)\) of the lens galaxy, we can calculate the time-delay of the two images for different source positions. The lens equation here should be
\[
\beta = \theta - \alpha(\theta),
\]
where \( \alpha \) is the reduced deflection angle. From Equation (4), the reduced deflection angle for a point mass lens model is
\[
\alpha_p(\xi) = \frac{D_l}{D_i} \frac{4GM}{c^2} \xi \mathcal{I}(\xi).
\]
Here, \( \mathcal{I} = D \theta \) is the impact factor. However, in a real galaxy lens model, the deflection angle should be obtained by summing the contribution of all the mass elements. Then the reduced angle is
\[
\alpha(\xi) = \frac{4G}{c^2} \frac{D_l}{D_i} \int \Theta(\xi') (\xi - \xi') \mathcal{I}(\xi - \xi') d^2 \xi'.
\]
Here we denote the surface mass density of the lens galaxy as \( \Theta(\xi) = \int \rho(\xi, z) dz \). For a specific lens model, given any source position, we can solve the lens equation to obtain the solution of image position from Equations (6) and (8). The angular diameter distance of the source and lens are restricted to the background LCDM model. Using the relation between the effective lensing potential and the reduced deflection angle \( \nabla \psi = \alpha \), we can calculate the Fermat potential and then the final time-delay from Equation (5). The main goal of this work is to compare the time-delay predictions between the GR and MG theory in order to detect the MG effect.

The uncertainty of this lensed GW+EM system is dominated by the measurement uncertainty of velocity dispersions. With the current lensing image reconstruction technique, for lensed quasars, the uncertainty is of order \( O(3\%) \) (Liao et al. 2015; Suyu et al. 2017). The velocity dispersion measurement uncertainty is of order \( O(10\%) \) (Jee et al. 2015; Birrer et al. 2019). As demonstrated by Liao et al. (2017) and Wei & Wu (2017), the uncertainty of image reconstruction of EM counterparts of GW events \((\sigma_v)\), such as short Gamma-ray burst and kilonovae, is of order \( O(0.6\%) \). Due to the better control of PSF in the EM counterpart of the GW event, the uncertainty of the velocity dispersion measurement \((\sigma_v)\) could be improved to \( O(5\%) \) (Liao et al. 2017; Wei & Wu 2017). In addition, other matter structures along the line of sight might contribute an extra systematic uncertainty \((\sigma_{\text{LOS}})\) around the 1% level (Suyu et al. 2010, 2017). As we have demonstrated above, the uncertainty of time-delay measurement from the lensed GW events can be neglected. Thus the total uncertainty translated to the final time delay is straightforward,
\[
\sigma_{tE} = \sqrt{\sigma_{2}^2 + \sigma_{0}^2 + \sigma_{\text{LOS}}^2},
\]
which is dominated by the measurement of stellar velocity dispersions. Then we can calculate how much the deviations of MG from GR can propagate to the anomalies of the time delay and compare them with the systematic errors to see whether we can distinguish them.

In this paper, we want to verify the viability of using lensing time-delay to distinguish the MG effect. We focus on the error propagation from the MG to the anomaly of time delay. Thus here we calculate only one typical strong lensing system and choose a simple lens galaxy model. As an example, we specify a typical strong gravitational lensing system according to the sample collected by Cao et al. (2015) with \( z_s = 0.3, z_l = 1, \) and \( \sigma_v = 300 \text{ km s}^{-1}. \) The lens galaxy mass profile is set to be the SIS model with a 10 kpc radius. The background model is fixed to the LCDM model with \( H_0 = 67.8 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( \Omega_m = 0.3. \) Since the SIS lens model produces double images only when the source position is inside the Einstein radius, we plot the results with \( \beta < \theta_E. \) As shown in Figure 1, when the MG starts to take effect above the \( r_{01} = 10 \) (20) kpc scale, a \( \alpha_1/\alpha_2 = 18\% \) (50\%) deviation can be discovered by one typical strong lens (red/blue curve). Comparing the purple curve \((r_{01} = 10 \text{ kpc}, \alpha_1/\alpha_2 = 50\%)\) with the former two, we can conclude that a smaller screening scale or a larger deviation from GR can result in easier detection of the MG signal.
4. Consistency Relationship between Time Delay and Multiple Images

Using the time delay to test gravity is limited by large uncertainty of the measurement of stellar velocity dispersions. Remember that we have two observations, that is, the time-delay and multiple images. These two observations both reflect the information of the lensing potential. In Section 3 we only use the time delay from GW to infer the MG effect embedded in the lensing potential. This is a straightforward way. Here, we can use the image information as a supplement to help improve the constraints of MG parameters. The basic idea is that, for a scale dependent modification of the lensing potential, the response to the MG effect from GW and EM domain should be significantly different compared to the GR case. This inspires us that we can check the consistency relationship between the time delay and the multiple images under the MG framework. Similarly, for simplicity, we also assume an SIS lens model to verify the viability of our methodology. Hence, we can use this consistency relation, Equation (9), to test GR and also to successfully avoid the dispersion measurement

\[ \Delta t_{ij} = \frac{1 + z_i}{2c} \frac{D_i D_s}{D_s} (\theta_i^2 - \theta_j^2). \]  

(9)

If we assume that the “true” universe is governed by the law of MG, applying a typical lensed GW+EM system as before, we can obtain the time-delay and positions of multiple images. Then we can compare the time delay between the “true” one with the one calculated from the \( \Delta t-\theta \) relation under the GR+SIS assumption. If the difference exceeds the systematic uncertainty, the MG signal is detectable. Here the systematic uncertainties are only in the image position measurements and the mass along the line-of-sight contributions, \( \sigma_{\Delta t} = \sqrt{\sigma_{\theta}^2 + \sigma_{\text{LOS}}^2} \).

In this paper, we are looking for some scale dependent MG effect, namely the deviation of \( \sigma \) from unity is a function of scales. For this type of MG effect, mathematically, we do not need the measurement of dynamical mass as long as we have two (or more) different tracers of the lensing mass, for example, GW time delay and associated EM lensing image. The reasons are the followings. Take the simplest case as an example, GR plus SIS lens modeling. Equation (9) predicts a specific prediction of the EM lensing image (position angle) and the GW time delay. The benefit of this relation is that it does not involve the Weyl potential. However, any modifications to gravity or lens mass modeling will break this unique consistency relation. In this section, we are aiming to use this consistency relationship as the smoking gun of MG gravity. For simplicity, we do not consider the degeneracy between the MG effect and lens mass modeling, i.e., insisting on the SIS model. Of course, the dynamical mass input can always help us and gives an additional contribution to constrain MG parameters. This is exactly what we did in Section 3. Although ignoring this degeneracy is aggressive and dangerous, as in the first trial, we want to quantify the most optimistic limit of this approach.

Figure 2 shows the result using both time-delay and image positions. We can see that the time-delay difference is more sensitive to the magnitude than the screening scale. For a typical strongly lensed GW+EM event, an 8% deviation of MG from GR can be detected. Furthermore, because the time-delay can also arise due to the background expansion, we need to quantify the degeneracy between MG effect and Hubble expansion. To do so, we choose two very different values of present Hubble constant, \( H_0 \), namely \( \sim 67 \text{ km s}^{-1} \text{ Mpc}^{-1} \) from Planck (Ade et al. 2016) and \( \sim 73 \text{ km s}^{-1} \text{ Mpc}^{-1} \) from SNe Ia (Riess et al. 2016). If the time-delay difference is larger than those from the \( H_0 \) variation, we can conclude that this effect is unlikely to be due to the uncertainty of the cosmic expansion measurement. We need to have significant modifications to gravity on the kiloparsec scale in order to fully account for the current tension in the Hubble parameter. As seen from Figure 2, the latter can contribute an 8% uncertainty at most. Thus, if we have a 60% deviation from GR, we would have strong confidence to claim the deviation from the \( \Delta t-\theta \) consistency relation is caused by MG rather than the inaccurate measurement of \( H_0 \).

5. Discussion

To measure the GW time-delay, we need to record the time sequence when the maximum peak arrives. Compared with the standard sirens method (Schutz 1986), our approach evades the complicated wave form calculation. Furthermore, due to the much smaller uncertainties of time-delay and image position measurements, the strongly lensed GW+EM system can give a better estimation of lensing mass compared with the lensed quasar.

In Section 3 we propagate all the errors (including velocity dispersion) to the time-delay and calculate the difference of time-delay between GR and MG and derive an error estimation (18%) of the MG detection. This method does not rely on the specific lens model. Both the time delay and lensing image response to the same Weyl potential, but the relation between them varies among different models of gravity. In Section 4, we propose to test the simplest relation, Equation (9), which is valid in the GR+SIS assumption. The goodness of this relation is that it avoids the measurement of velocity dispersion of the lens galaxy, which is one of the major uncertainties of this approach. However, this assumption is aggressive because it breaks when either the GR or SIS assumption is invalid. The reason we still test this relationship is that we want to quantify...
the upper limit of this approach in the optimistic lens modeling assumption.

The PPN parameter is constrained up to the order of 10% by Collett et al. using the ESO 325-G004. We should note that the uncertainty of the dynamical mass they derived is about 4% which is smaller than the one we adopted, conservatively (5%). Instead of assuming a constant PPN parameter, we use a scale dependent Σ function to parameterize the MG effect. Since the gravitational lensing, in principle, is the redistribution of the relativistic radiation, the spatial derivative is essential. We argue that the constant PPN parameter is oversimplified in the lensing data analysis. Furthermore, the traditional method of using the optical strong lensing system is highly limited by the spectroscopic measurement. Take the ESO 325-G004 lens as an example, its image data were obtained by the Hubble Space Telescope in 2006, but the IFU spectroscopic data from Multi Unit Spectroscopic Explorer were obtained nearly 10 yr later. More importantly, to obtain high quality spectroscopic data, we are limited to the the low redshift lens, around zl ∼ 0.03. Even with such high quality data, we can only test gravity up to the 3 kpc scale with 10% accuracy (Collett et al. 2018). By using the method proposed in Section 4, although its assumption is aggressive, it is completely independent from the spectroscopic measurement. Hence, it can be used at higher redshift. The sample numbers can be greatly increased. If some significant modifications to gravity, say a >10% effect on Weyl potential, are detected, they can trigger further studies on the gravity test.

In our calculation, we set the galaxy radius to 10 kpc. The SIS model ρ(r) = σ2/(2πgr2) with σr = 300 km s−1, gives the total mass 6.57 × 1011M⊙. Most of the galaxy radii are about 0.5–50 kpc, which correspond to the total mass 1010–1012M⊙. Thus the galaxy we set here is a typical one.

Though some MG theories would be excluded on some specific scales. For example the generalized Brans–Dicke (GBD) models predict a Σ, which is strongly constrained to be close to unity in models with universal coupling to different matter species. However, the deviation from GR on any scale should attract our attention, especially for the galactic scale on which the GR are poorly constrained. A measurement of Σ ∼ 1 would rule out all universally coupled GBD theories, such as the f(R), chameleon, symmetron, and dilaton models. In this paper, we propose the idea of using the strongly lensed multimessenger to distinguish any deviation of the phenomenological parameter from the GR prediction.

We would emphasize here that our approach is more general and robust compared to the traditional method. The upcoming strongly lensed multimessenger system, such as the GW+EM events, may not be well aligned with the observer. Thus the traditional way to utilize the Einstein radius is impractical. In this paper, we take the SIS lens model as an example to demonstrate how much the MG effect can be detected. For a more general lens model, such as singular isothermal ellipsoid, the calculation is straightforward. Our approach is not limited by the specific source type or the lens model.

Although GW lensing is an exciting and relatively novel topic, it faces many challenges. Up to now, we do not observe any lensed GW event. There are many challenges facing the realistic GW lensing data analysis. One of the major issue is finding GW lensed sets in the time stream from a GW detector. Considering the confusion of background (unresolved) sources and variable noise/duty cycles of a detector, this identification procedure is far from straightforward. If a set can be done, there is another challenge of finding a unique galactic host in the large localization area. If a GW signal has been strongly magnified, the “effective redshift” inferred (assuming no magnification) will be very different from the host redshift, making identification more sophisticated. And depending on source-lens-observer setup, it is possible for signals to sometimes get demagnified, lowering the GW amplitude and potentially rendering them undetectable. We cannot tell more about this issue because the corresponding pipelines have not been built. However, in this paper, we proposed a new methodology that can be adopted in the future test of gravity.

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References
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2016, PhRvL, 116, 061102
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017a, ApJL, 848, L13
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017b, PhRvL, 119, 161101
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2019, PhRvL, 123, 011102
Ade, P. A. R., Aghanim, N., Arnaud, M., et al. 2016, A&A, 594, A13
Appleby, S. A., & Battye, R. A. 2007, PhLB, 654, 7
Ashby, N. 2003, LRR, 6, 1
Baker, T., Bellini, E., Ferreira, P. G., et al. 2017, PhRvL, 119, 251301
Bertiotti, B., Jess, L., & Tortora, P. 2003, Natur, 425, 374
Bertschinger, E., & Zakin, P. 2008, PhRvD, 78, 020145
Biesiada, M., Ding, X., Piorkowska, A., & Zhu, Z.-H. 2014, JCAP, 1410, 080
Birrer, S., Treu, T., Rusu, C., et al. 2019, MNRAS, 484, 4726
Bolton, A. S., Rappaport, S., & Burles, S. 2006, PhRvD, 74, 061501
Brax, P., van de Bruck, C., Davis, A.-C., Li, B., & Shaw, D. J. 2011, PhRvD, 83, 104026
Broadhurst, T., Diego, J. M., & Smoot, G. F. 2019, arXiv:1901.03190
Cao, S., Biesiada, M., Gavazzi, R., Piorkowska, A., & Zhu, Z.-H. 2015, ApJ, 806, 185
Cao, S., Li, X., Biesiada, M., et al. 2017, ApJ, 835, 92
Capozziello, S., Carloni, S., & Troisi, A. 2003, Recent Res. Dev. Astron. Astrophys., 1, 625
Carroll, S. M., Duvvuri, V.,Trodden, M., & Turner, M. S. 2004, PhRvD, 70, 043526
Collett, T. E., Oldham, L. J., Smith, R. J., et al. 2018, Sci, 360, 1342
Damour, T., & Polyakov, A. M. 1994, NuPhB, 423, 532
Ding, X., Biesiada, M., & Zhu, Z.-H. 2015, JCAP, 1512, 006
Ding, X., Liao, K., Treu, T., & et al. 2017, MNRAS, 465, 4634
Dyson, F. W., Eddington, A. S., & Davidson, C. 1920, RSpTA, A220, 291
Fan, X.-L., Liao, K., Biesiada, M., Piorkowska-Kurpas, A., & Zhu, Z.-H. 2017, PhRvL, 118, 091102
