Research of Available Transfer Capacity of Power Grid Considering Optimal Location-allocation of STATCOM

Bin Zhang¹, Hanbin Wu²,*

¹School of Electrical Engineering, Naval University of Engineering, Wuhan 430033, China
²State Grid Hubei Wuhan Jiangxia Power Supply Company, Wuhan 430029, China

*Corresponding author: 2014202070048@whu.edu.cn

Abstract. The rational planning of STATCOM can effectively improve the power transmission capacity of the system. In the research of grid transmission capacity considering N-1 safety check, the nonlinear problem is difficult to solve. This paper proposes a calculation method of transmission capacity based on Benders decomposition method. Firstly, the sub-question is added to the core accident screening process to fit the actual grid operation scenario. Secondly, the parallel problem calculation method is used in the sub-problem solving to improve the calculation efficiency. The example shows that the proposed method solves the transmission capacity of the asynchronous grid, and the convergence speed is faster, which verifies the improvement effect of STATCOM on the transmission capacity of the asynchronous grid. The method in this paper can provide calculation aids for grid optimization planning and operation.

1. Introduction

With the deep reform of power system and the rapid development of renewable energy, a great change in pattern of economics, technology and structure is taking place around the world. The uncertainty of load and renewable energy increases the transmission line's transfer power and the circulation current, which improves the risk of power congestion in the system, and severely constrained the available transmission capacity [1].

The available transfer capacity depicts the active power of lines. The ATC should be converged under the base case and N-k contingency case [2]. As FACTS technology's rapid development, there is a novel technique to control and manage the power flow based on FACTS devices. The FACTS changes the power flow by altering the system's structure parameters with FACTS equipment, which can reduce the probability of transmission congestion and effectively improve the power transmission capacity of the grid. Meanwhile, the scale of new energy integrate to the power system is mounting to astronomical figures and shows no signs at present of ceasing to rise, which deteriorates the problem of transmission congestion. For this reason, the ATC research considering the FACTS integrated system becomes more and more important. During such research, three major difficulties are hard to handle: non-convex optimization of AC power flow, N-1 safety check constraints, and FACTS locations and settings.

Three main types of the traditional method have been widely used in terms of ATC calculation: continuous power flow method [3-4], repeated power flow method [5-8] and optimal power flow method...
For instance, literature [11] constructed an AC power flow model of ATC and solved the ATC based on a continuous power flow method and optimal power flow method. Literature [12] introduces the multi-state of the power system based on Monte Carlo simulation and uses DC power flow to obtain ATC considering a multi-index model of ATC; Literature [13] studies the method of solving ATC under the condition of wind power integrated system through interval planning and strong duality theory, meanwhile, a DC power flow model of ATC is obtained considering the uncertainty of wind power. In [14], ATC is obtained while an optimal dispatch plan is trying to be sought by constructing an economic dispatch model and an optimal power flow method, but research on N-k conditions is ignored. Literature [15] analyzes the impact of ATC detailly from the perspective of power marketization and separation of management and control. Obviously, most of the ATC studies use the DC power flow model to calculate ATC. Although some scholars use AC power flow to calculate ATC, two factors weigh heavily against the effectiveness of the research. Firstly, the N-k safety check is generally neglected. Simultaneously, the difficulty and the low efficiency of solving AC power flow are usually ignored.

To improve the reliability of the power supply and utilization efficiency of existing power resources, many scholars regard the FACTS device as an effective method to improve the system's ATC. Literature [16] express a detailed review of the methods that FATCS improve ATC. Literature [17] adopted the optimal planning of UPFC parameters and the equivalent injection model of UPFC, and the ATC is solved by the optimal power flow method. Literature [18] proposed a continuously repeated optimal power flow method. This method only focuses on the placement of one UPFC and adjusts the power flow through repeated iterations. Literature [19] researched the DG and D-FACTS together. In this study, the optimal output of DG and the optimal configuration of D-FACTS are obtained by using the N-1 constraint and the decomposition method.

In this paper, STATCOM is introduced to study the transmission capacity. Firstly, an ATC model is constructed considering the ground state power flow and the N-1 condition. To find the solution of the non-convexity of the power flow equation, a second-order cone programming model is introduced to convert the non-convex power flow equation to a convex optimization model. Meanwhile, to tackle the problem of the dimensional explosion caused by the ground state power flow and the N-1 condition, this paper introduces the Benders decomposition method to convert the original ATC problem into the main problem which accounts for the ground state flow and the Sub-problems considering the power flow check which involves the N-1 condition respectively. In the end, the optimal solution of ATC is obtained through repeated iterations between the main and sub-problems.

2. Mathematical model of ATC with STATCOM integration

Transmission capacity is described as the sum of active power on the transmission section under satisfying static safety constraints. The general procedure is to increase the generator output and load in the same proportion at both sides of the section until the safety constraints are violated.

With the rapid growth of load and the expansion of distributed generator (DG), transmission congestion is a frequent occurrence. STATCOM can reduce the frequency of transmission congestion by adjusting the distribution of reactive power in the system, which plays an important role in improving the reliability of the power supply.

This paper redistributes the reactive power in the system and adjusts the distribution of power flow by introducing STATCOM, which can improve the power transmission capacity of the grid.

2.1. Objective function

The objective function of the transmission capacity considering the STATCOM integration is to maximize the sum of the active power of the transmission lines on the section, and its mathematical model is:

$$\max J = \sum_{i \in S} P_i$$

(1)
Where $J$ is the transmission capacity of the system, $P_l$ is the active power of the transmission line $l$, and $S$ is the set of section lines.

2.2. Constraints

The ATC calculation involves the ground state power flow and the power flow under the N-1 condition. Therefore, the constraint conditions should consider the boundary operation constraints of the ground state and N-1 condition simultaneously. Meanwhile, the installation location, quantity, and capacity of STATCOM should also be accounted for in the model. The ATC constraints considering the STATCOM integration include ground state power flow constraints, unit output constraints, upper and lower load limits, STATCOM installation capacity constraints, STATCOM installation quantity constraints, and N-1 operating conditions constraints.

1) Ground state power flow constraints

Power balance constraints:

$$P_{Di} - (1 + \lambda_i)P_{Di} = \sum_{j=l} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij})$$

(2)

$$Q_{Di} + Q_{STAT,i} - (1 + \lambda_i)Q_{Di} = \sum_{j=l} V_j (G_{ij} \sin \theta_{ij} + B_{ij} \cos \theta_{ij})$$

(3)

Where $\lambda_i$ is the growth coefficient of load, $P_{Di}/Q_{Di}$ is the active/reactive load of generator, $P_{Di}/Q_{Di}$ is the active/reactive output of generator, $Q_{STAT,i}$ is the capacity of STATCOM, $V_i$ is the voltage amplitude of node, $\theta_{ij}$ is the phase difference of angle, $G_{ij}/B_{ij}$ is the line parameter, and $I$ is the collection of nodes which connected to node $i$.

Voltage constraints:

$$V_{i_{min}} \leq V_i \leq V_{i_{max}}$$

(4)

Where $V_{i_{min}}/V_{i_{max}}$ is the upper and lower bounds of voltage.

Phase difference of angle constraint:

$$\theta_{i_{min}} \leq \theta_i \leq \theta_{i_{max}}$$

(5)

Where $\theta_{i_{min}}/\theta_{i_{max}}$ is the upper and lower bounds of the phase difference of angle.

Phase angle of the balanced node constraint:

$$\theta_{i_{ref}} = 0$$

(6)

Where $\theta_{i_{ref}}$ is the phase angle of the balanced node.

Transmission power constraint of the Line:

$$\sqrt{P_l^2 + Q_l^2} \leq S_{stab,l}$$

(7)

Where $l$ is the number of the transmission line, $P_l/Q_l$ respectively represents the active power/reactive power of line $l$, and $S_{stab,l}$ is the limit of transmission power.
2) Unit output and load constraints

Generator output constraints:

\[ P_{Gi}^{\text{min}} \leq P_{Gi} \leq P_{Gi}^{\text{max}} \]  
\[ Q_{Gi}^{\text{min}} \leq Q_{Gi} \leq Q_{Gi}^{\text{max}} \]  

Where \( P_{Gi}^{\text{min}} \) is the upper and lower limits of generator \( i \)'s active power output, and \( Q_{Gi}^{\text{min}} \) is the upper and lower limits of generator \( i \)'s reactive power output.

3) STATCOM constraints

STATCOM capacity constraints:

\[ \mu_{\text{STAT},i} Q_{\text{STAT},i}^{\text{min}} \leq Q_{\text{STAT},i} \leq \mu_{\text{STAT},i} Q_{\text{STAT},i}^{\text{max}} \]  

Where \( \mu_{\text{STAT},i} \) is a binary variable, and when its value is 1, STATCOM is installed. \( Q_{\text{STAT},i}^{\text{max}} \) is the upper and lower limits of STATCOM installation capacity respectively.

STATCOM installation quantity constraints:

\[ \sum_{i} \mu_{\text{STAT},i} \leq \eta \]  

Where \( \eta \) is the number of STATCOM installations.

4) power flow constraint under N-1 condition

Power balance constraints under N-1 condition:

\[ P_{Di} - (1 + \lambda_{i}) P_{Di} = V_{i}^{p} \sum_{j \in i} V_{j}^{p} (G_{ij}^{p} \cos \theta_{ij}^{p} + B_{ij}^{p} \sin \theta_{ij}^{p}) \]  
\[ Q_{Di} + Q_{\text{STAT},i} - (1 + \lambda_{i}) Q_{Di} = V_{i}^{p} \sum_{j \in i} V_{j}^{p} (G_{ij}^{p} \sin \theta_{ij}^{p} + B_{ij}^{p} \cos \theta_{ij}^{p}) \]  

The state variable constraints under N-1 condition include voltage constraints, phase difference of angle constraints, phase angle constraints of balanced node, and transmission power constraints of line under N-1 conditions:

\[ V_{i}^{\text{min}} \leq V_{i}^{p} \leq V_{i}^{\text{max}} \]  
\[ \theta_{ij}^{\text{min}} \leq \theta_{ij}^{p} \leq \theta_{ij}^{\text{max}} \]  
\[ \theta_{ij}^{p} = 0 \]
3. Two-stage Benders solution method of ATC

The original ATC model considering STATCOM integration is a mixed-integer nonlinear programming model. The model includes ground state power flow and the power flow under the N-1 condition. The power flow constraint is a non-convex optimization problem, which is difficult to solve. With the expansion of the system, the difficulty of solving the ATC model will increase sharply. The second-order cone programming (SOCP) model can convert the original non-convex model to a convex optimization model without breaking the coupling relationship of the physical quantities, representing the character of power flow, and achieves a great advance in the process of calculation. Therefore, this paper transforms the original model into a two-stage convex optimization model based on the Benders decomposition method and SOCP, which can quickly improve the efficiency and accuracy of the solution.

3.1. Second-order cone programming model

AC power flow constraint is a non-convex programming problem, and the convergence is difficult to guarantee. The second-order cone programming model can transform the power flow model into a convex optimization model by satisfying the coupling relationship between the internal state quantities of the power flow and approximating the original power flow. The second-order cone constraints of power flow are as follows:

\[
\sum_{e=1}^{n_e} p_e + (1 + \lambda_b) p_{b_e} + g_i v_{ie}^2 = p_{g_i} + \sum_{j=1}^{n_j} (p_{ij} - r_{ij} h_{ij}) \\
\sum_{e=1}^{n_e} q_e + (1 + \lambda_b) q_{b_e} + b_i v_{ie}^2 = q_{g_i} + q_{STAT_j} + \sum_{j=1}^{n_j} (q_{ij} - x_{ij} h_{ij})
\]

\[
v_{ie}^2 = v_{ij}^2 - 2(r_{ij} p_{ij} + x_{ij} q_{ij}) + (r_{ij}^2 + x_{ij}^2) h_{ij}
\]

\[
(P_{ij}^2 + Q_{ij}^2) \leq h_{ij} v_{ij}^2
\]

\[
(\theta_i - \theta_j) = \frac{x_{ij} p_{ij} - r_{ij} q_{ij}}{v_{ij} v_{ij}}
\]

Where \( p_e \) and \( q_e \) are the active and reactive power which flowing out of the node \( i \) on line \( ie \); \( p_{ij} \) and \( q_{ij} \) are the active and reactive power which flowing into node \( i \) on line \( ij \); \( r_{ij} \) and \( x_{ij} \) are the resistance and reactance of line \( ij \); \( g_i \) and \( h_{ij} \) are the conductance of node \( i \) which connected to ground and the susceptance; \( h_{ij} \) is the square of the branch current modulus.

3.2. Benders two-stage method

Obviously, the ATC model considering the N-1 condition involves the ground state power flow and power flow under N-1 conditions. The dimensional explosion problem will occur if the calculation is
performed synchronously, which is difficult to solve. This paper transforms the ATC model into a two-stage model based on the Benders decomposition method. The main problem is the ground state power flow model, and the sub-problem is the safety check model of power flow under the N-1 condition. Meanwhile, the power flow models all adopt the second-order cone programming model considering FACTS integration. Thereby the original non-convex optimized ATC model can be converted into a two-stage convex optimization model, which greatly reduces the calculation dimension of the ATC model, and improved the efficiency greatly.

The main problem of ground state power flow:

\[
\max f_{\text{down}} = \sum_{s} P_{s} + \phi
\]

where

\[
\phi_{\text{down}} = \text{objective function of the main problem, }
\]

\[
\phi_{\text{down}} = \text{lower bound of the original ATC model as well, and Phi is a variable parameter.}
\]

sub-problem of N-1 security check:

\[
\min z = M \sum_{i} (\alpha_{i}^{\alpha} + \alpha_{i}^{\beta} + \beta_{i}^{\alpha} + \beta_{i}^{\beta})
\]

\[
\sum_{e} P_{ie} + (1 + \lambda_{i})P_{de} + g_{i}V_{i}^{2} = P_{gi} + \alpha_{i}^{\alpha} - \alpha_{i}^{\beta} + \sum_{j \neq i} (P_{ij} - r_{ij}h_{ij})
\]

\[
\sum_{e} Q_{ie} + (1 + \lambda_{i})Q_{de} + h_{i}V_{i}^{2} = Q_{gi} + \beta_{i}^{\alpha} - \beta_{i}^{\beta} + \sum_{j \neq i} (Q_{ij} - x_{ij}h_{ij})
\]

Where \( M \) is the penalty function, \( \alpha_{i}^{\alpha} / \alpha_{i}^{\beta} \) is the active slack variable, and \( \beta_{i}^{\alpha} / \beta_{i}^{\beta} \) is the reactive slack variable.

Benders cut is:

\[
\phi_{\text{up}} = \phi_{\text{down}} + \sum_{s} P_{s}
\]

The convergence criterion of the Benders two-stage method considering STATCOM integration is

\[
\max f_{\text{down}} = \sum_{s} P_{s} + \phi
\]

\[
\min z = M \sum_{i} (\alpha_{i}^{\alpha} + \alpha_{i}^{\beta} + \beta_{i}^{\alpha} + \beta_{i}^{\beta})
\]

\[
\sum_{e} P_{ie} + (1 + \lambda_{i})P_{de} + g_{i}V_{i}^{2} = P_{gi} + \alpha_{i}^{\alpha} - \alpha_{i}^{\beta} + \sum_{j \neq i} (P_{ij} - r_{ij}h_{ij})
\]

\[
\sum_{e} Q_{ie} + (1 + \lambda_{i})Q_{de} + h_{i}V_{i}^{2} = Q_{gi} + \beta_{i}^{\alpha} - \beta_{i}^{\beta} + \sum_{j \neq i} (Q_{ij} - x_{ij}h_{ij})
\]

Where \( \alpha_{i}^{\alpha}, \alpha_{i}^{\beta}, \beta_{i}^{\alpha}, \beta_{i}^{\beta} \geq 0 \)

Where \( x \) is the decision variable of the main problem. \( x^{\text{iter}} \) is the iterative value of the decision variable, \( x = \{P_{ie}, Q_{ie}, P_{de}, Q_{de}, Q_{\text{STAT}_i}\} \) and \( \lambda^{\text{iter}} \) is the corresponding Lagrangian multiplier. All variables in the sub-problems are follow the N-1 conditions, and all superscripts \( p \) are omitted.

In the Benders two-stage problem, the upper bound of the objective function is expressed as:

\[
\phi_{\text{up}} = \phi_{\text{down}} + \sum_{s} P_{s}
\]
Where $\varepsilon$ is an arbitrarily small positive number.

The ATC algorithm flow chart based on Benders decomposition is shown in Figure 1:

\[
\frac{|\phi^{up} - \phi^{down}|}{\phi^{up}} \leq \varepsilon
\]  

(27)

4. Simulation analysis

Effectiveness of this method proposed in this article is carried out using the IEEE-118 node system. The IEEE-118 system is composed of 118 nodes, 186 lines and 54 generators. The total load of the system under steady state is 4242MW, and the generator capacity is 9966MW. The ATC model considering STATCOM integration is established on the YALIMIP platform. The main problem and sub-problems are solved using CPLEX12.0, and the threshold is 0.0001.

Suppose the IEEE-118 system is divided into three areas and two transmission sections. As shown in Figure 2,
4.1. The impact of the number of STATCOM installations on ATC

Taking the power transmission section between area 2 and area 3 as an example. The section consists of 4 lines. At this time, the installed capacity of STATCOM is set to a fixed value, which is 10Mvar.
As indicated in Fig. 3, the transmission capacity of the transmission section increases when the number of STATCOM installations increases. The reason lies in that the system will adjust the voltage and reactive power in each area after the STATCOM is installed flexibly, which will reduce the
probability of transmission congestion in the local grid. Meanwhile, the growing number of STATCOMs in the system can improve the efficiency of the algorithm. On the one hand, the current number of STATCOM installation schemes is sufficient to eliminate the transmission congestion system. On the other hand, the installation cost of STATCOM is not considered, which makes the model only considering the optimal configuration of STATCOM from reactive power, thereby reducing the computational difficulty.

4.2. Impact of STATCOM installation capacity on ATC
Taking the power transmission section between area 2 and area 3 as an example. The section is composed of 5 tie lines. The installation position is the same as that in Figure 2, and the capacity of STATCOM is uncertain.

| \( \eta \) | STATCOM configuration scheme (node) ATC | ATC       |
|---------|----------------------------------------|-----------|
| 1       | 97(16 MVar)                           | 1129.72 MW|
| 2       | 97(14 MVar), 106(8 MVar)              | 1243.58 MW|
| 3       | 97(12 MVar), 104(6 MVar), 106(13 MVar)| 1309.63 MW|

As shown in Table 2 and Figure 2, it is apparently that the transmission capacity of the system increases as the configured capacity of STATCOM increases. This is because the reactive power compensation of STATCOM can adjust the voltage distribution and improve the stability of the system.

4.3. The impact of upper and lower limits of voltage on STATCOM-ATC
The voltage stabilization of STATCOM makes it possible to play a significant role in the adjustment of power flow, but the reactive power compensation of STATCOM is limited by the upper and lower limits of the voltage. Therefore, based on the concept of Q/V sensitivity, adjusting the upper and lower limits of the voltage to make \( V_i \in [0.85, 1.15] \).

| \( \eta \) | STATCOM configuration scheme (node) ATC | ATC       |
|---------|----------------------------------------|-----------|
| 1       | 97(8.3 MVar)                           | 1860.46 MW|
| 2       | 97(4.2 MVar), 106(6.8 MVar)            | 1963.89 MW|
| 3       | 97(7.6 MVar), 104(3.5 MVar), 106(4.9 MVar)| 2013.36 MW|

As shown in Table 3, with the increase of the voltage adjustment range, STATCOM greatly improves the power transmission capacity of the system and can play a vital role with a smaller configuration capacity. The reason lies in that when the range of voltage variation increases, the critical state of the system is further amplified, and the reactive power compensation of STATCOM further expands the feasible range of power flow, thereby effectively decreasing the congestion phenomenon and ATC is released.

5. Conclusion
To find a solution to the transmission capacity under optimal STATCOM integration, this paper first proposes the ATC model of the STATCOM integration. It uses the second-order cone programming to convert the non-convex model of the ground state power flow and the power flow under the N-1 condition to a convex model. Meanwhile, the Benders decomposition is introduced to convert the original second-order cone ATC model into a two-stage convex programming model, which greatly improves the efficiency of the algorithm. Later, the conclusion that STATCOM can effectively improve the power transmission capacity of the system is verified by using the IEEE-118 node system. At the
same time, it is found that the method proposed in this paper can effectively solve ATC with high efficiency and accuracy. However, the installation cost of STATCOM is ignored in the research of this paper, and only considering the installation quantity and capacity constraints of STATCOM. In the future, we will investigate ways to solve ATC, considering the cost of STATCOM.

References.
[1] TANG Xisheng, DENG Wei, LI Ningning, et al, Control technologies of micro-grid operation based on energy storage, Electric Power Automation Equipment. 32(2012) 99-103.
[2] Ejebco G C, Tong J, Waight J G, et al, Available transfer capability calculations, IEEE Transactions on Power systems. 13(1998) 1521-1527.
[3] Ou Y, Singh C, Assessment of available transfer capability and margins, IEEE Transactions on Power Systems. 17(2002) 463-468.
[4] Guo Qi, Zhao Jinquan, Zhang Boming, etal, A Method for On-Line Computation of Total Transfer Capability, Proceeding of the CSEE. 5(2006) 1-5.
[5] Luo Gang, Shi Dongjuan, Chen Jinfu, Improved available transmission capability algorithm based on parameterized power flow model, Electric Power Automation Equipment. 34(2014) 19-25.
[6] Ding Ping, Zhou Xiaoxin, Yan Jianfeng, etal, calculation of online total transfer capability in bulk interconnected grid integrating rationality and security principles, Proceeding of the CSEE. 30(2010) 1-6.
[7] Li G, Sun Y, Wang L, Interregional available transfer capability research based on interior point method with consideration of wind power penetration level, Electric Power Automation Equipment. 34(2014) 1-7, 15.
[8] Fei Y, Shixin L, Haixia W, et al, A Decomposition Method to Calculate Available Transfer Capability of Large Interconnected Power Grid, Power System Technology. 2014.
[9] Zhou Ming, Zhan Zhongjie, Li Genying, etal, Available Transfer Capability Determination for AC/DC Transmission Systems Based on Power Increase, Proceeding of the CSEE. 31(2011) 48-55.
[10] Zhang Jian, Ji Ruifang, Li Guoqing, Study of enhancement of available transfer capability using TCSC optimal allocation, Power System Protection and Control. 40(2012) 23-28.
[11] Pan Xiong, Xu Guoyu, OPF Based ATC Calculation With Static Voltage Stability Constraints, Proceeding of the CSEE. 12(2004) 90-95.
[12] Li Genying, Gao Yajing, Zhou Ming, Sequential Monte Carlo Simulation Approach for Assessment of Available Transfer Capability, Proceeding of the CSEE. 25(2008) 74-79.
[13] Kou X, Li F F, Interval Optimization for Available Transfer Capability (ATC) Evaluation Considering Wind Power Uncertainty, IEEE Transactions on Sustainable Energy. 2018.
[14] Wang B, Fang X, Zhao X, et al, Bi-level optimization for available transfer capability evaluation in deregulated electricity market, Energies. 8(2015) 13344-13360.
[15] Adewolu B O, Saha A K, Determination and Analyses of Available Transfer Capability: Deregulated and Restructured Power Systems Perspective, 2019 Southern African Universities Power Engineering Conference/Robotics and Mechatronics/Pattern Recognition Association of South Africa (SAUPEC/RobMech/PRASA), IEEE. 2019: 504-509.
[16] Albatsh F M, Mekhilef S, Ahmad S, et al, Enhancing power transfer capability through flexible AC transmission system devices: a review, Frontiers of Information Technology & Electronic Engineering. 16(2015) 658-678.
[17] Rajabi-Ghahnavieh A, Fotuhi-Firuzabad M, Othman M, Optimal unified power flow controller application to enhance total transfer capability, IET Generation. 2015, 9(2015) 358-368.
[18] Azadani E N, Hosseinian S H, Janati M, et al, Optimal placement of multiple STATCOM, 2008 12th International Middle-East Power System Conference, IEEE. 2008, pp. 523-528.
[19] H. Mori and H. Tani, "Two-staged tabu search for determining optimal allocation of D-FACTS in radial distribution systems with distributed generation,” Transmission and Distribution
Conference and Exhibition 2002: Asia Pacific. IEEE/PES, 2002, pp. 56-61 vol.1.