MULTIJET FINAL STATES IN \( \text{e}^+\text{e}^- \) ANNIHILATION

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We review the current status of analyses of multijet final states in \( \text{e}^+\text{e}^- \) annihilation. Results for jet observables from LEP 1, LEP 2 and from the reanalysis of the PETRA experiment JADE will be presented. A determination of the \( b \)-quark mass using jet observables will be discussed and tests of power correction models will be shown. Finally, determinations of the QCD colour factors from an analysis of event shape distributions at several energy points using power corrections will be discussed.

1 Introduction

Hadronic final states in \( \text{e}^+\text{e}^- \) annihilation events are the subject of many experimental and theoretical studies. The structure of the hadronic final states in \( \text{e}^+\text{e}^- \) annihilation is characterised by the presence of a small number of so-called jets, i.e. clearly separated and collimated sprays of particles. As a consequence, hadronic events may be classified by e.g. the number of jets after a jet finding algorithm has been defined. Alternative classification schemes are event shape observables, where the reconstructed momenta of the final state particles are combined in a way which characterises the structure of the event in a single number.

Section 2 presents results on jet production for centre-of-mass (cms) energies from 35 to 189 GeV, while section 3 describes a measurement of the mass of the \( b \)-quark at a scale of \( \sqrt{s} \). Section 4 contains a brief summary of experimental tests of power corrections. Section 5 shows a test of the gauge structure of QCD and, finally, section 6 gives a summary of the report.

2 Jet Production from 35 to 189 GeV

Jet production was studied comprehensively using data from the JADE and OPAL experiments at the PETRA and LEP \( \text{e}^+\text{e}^- \) colliders at cms energies from 35 to 189 GeV. The large range of cms energies allows detailed studies of scale dependent effects predicted by QCD.

Figure 1 (left) shows the 3-jet fraction measured using the JADE algorithm at cms energies 35 (triangles), 91 (points) and 189 GeV (squares) as functions of the jet resolution parameter \( y_{\text{cut}} \). In the JADE algorithm the invariant masses \( m_{ij} \) of all pairs of final state particles are calculated and
the pair with the smallest value is combined into a pseudo-particle by adding their 4-vectors. This process is repeated until no invariant masses below a cut value remain: \( m_{ij}^2/s > y_{\text{cut}} \) for all \( i, j \). The decrease of the 3-jet rate at large \( y_{\text{cut}} \) is clearly visible. The distributions are well described by Monte Carlo simulation programs tuned to OPAL data recorded at \( \sqrt{s} = M_Z \) [3, 4] including QCD coherence effects in the parton shower, PYTHIA [5] (solid), HERWIG [7] (dashed) and ARIADNE [6] (dotted). A simulation program without coherence effects, COJETS [8] (dash-dotted), describes the data less well.

Figure 1 (right) shows the dependence of the 3-jet fraction \( R_3(y_{\text{cut}} = 0.08) \) using the JADE algorithm as a function of cms energy \( \sqrt{s} \), corrected for detector and for hadronisation effects. In leading order, we have \( R_3(\sqrt{s}) \sim \alpha_s(\sqrt{s}) \) for \( y_{\text{cut}} = 0.08 \) [9]; the data provide convincing evidence for the running of \( \alpha_s \) as required by QCD.

3 Running b-Quark Mass

QCD predictions are generally calculated for massless quarks. This is a good approximation for the light (\( O(10 - 100) \) MeV) u-, d- and s-quarks. However, for the heavy c- and b-quarks the masses are comparable to energy scales
where perturbative QCD calculations are expected to be valid and thus quark mass effects may be significant. Quark masses in the QCD Lagrangian are free parameters like the strong coupling $\alpha_s$ and have to be renormalised to obtain finite predictions, see e.g. [13]. The renormalised quark masses are expected to “run”, i.e. to depend on the energy scale of the process, because they must obey a renormalisation group equation (RGE). The main effect of a heavy quark mass in QCD is the suppression of gluon radiation from the heavy quark which leads to the expectation of a reduced 3-jet rate.

The ALEPH collaboration presented an analysis to test the prediction of a running b-quark mass based on jet observables determined for b- and light quark events [14]. After correcting for experimental effects, b-tagging biases and hadronisation effects, the ratio of the jet observable measurements in b- over d-quark events is calculated, e.g. for the 3-jet rate $R_3$ one has $R_{\text{bd}}^{\text{pert}}(R_3) = R_{3,b}/R_{3,d}$. Figure 2 (left) presents $R_{\text{bd}}^{\text{pert}}(R_3)$ at several values of $y_{\text{cut}}$ compared with NLO QCD predictions for $m_b = 3$ and 5 GeV and Monte Carlo simulations. The data clearly prefer the lower value of $m_b$ while the simulations are in slight disagreement at low $y_{\text{cut}}$.

The final measurement of $m_b(M_{Z^0})$ is performed using the observable with the smallest hadronisation corrections and systematic uncertainties; this
Figure 3. The figure on the left shows as solid lines \( \mathcal{O}(\alpha_s^2) \) QCD fits with power corrections to 1st moments of event shape observables \([15]\). The figure on the right shows a summary of all results for \( \alpha_0 \) and \( \alpha_s(M_{Z^0}) \) from various analyses \([15–19]\).

is the 1st moment of the differential 2-jet rate distribution. The result \( m_b(M_{Z^0}) = 3.27 \pm 0.52 \) GeV is presented in figure 2 (right) together with other measurements at the \( Z^0 \) scale and at low scales. The QCD prediction of a running b-quark mass starting from the value at low scale is in good agreement with the measurements at \( M_{Z^0} \). Assuming the QCD description of heavy quark effects to be correct the ratio \( \alpha_s^b/\alpha_s^{uds} = 0.997 \pm 0.009 \) is determined and provides a precise test of the flavour independence of the strong coupling \( \alpha_s \).

4 Power Corrections

Most measurements in QCD studies have to correct for the discrepancy between perturbative QCD calculations and the quantities calculated from the observed hadrons. These corrections are commonly carried out using Monte Carlo models of the hadronisation process like JETSET/ PYTHIA, HERWIG or ARIADNE. An alternative approach to the problem of hadronisation are analytical QCD based models of hadronisation, the power corrections.

In the Ansatz of Dokshitzer, Marchesini and Webber (DMW) \([20]\), the
effects of gluons with transverse momentum \( k_t \sim \Lambda_{\text{QCD}} \), so-called gluers, are calculated. The model must assume that the strong coupling \( \alpha_s \) is finite in the region of the Landau Pole leading to a new free parameter \( \alpha_0 \) in the model: 

\[
\alpha_0 = \int_0^{\mu_I} \alpha_s(k)dk.
\]

The variable \( \mu_I \) is the infrared matching scale where non-perturbative and the perturbative evolution of \( \alpha_s \) are merged.

For the differential distributions of the event shape observables Thrust, Heavy Jet Mass, \( C \)-parameter and Total and Wide Jet Broadening, the model predicts that hadronisation effects are described by a shift of the perturbative prediction: 

\[
F(y) = F_{\text{PT}}(y - c_y P) \quad \text{where} \quad y \text{ is the value of the observable}.
\]

For the 1st and 2nd moment one obtains 

\[
\langle y \rangle = \langle y \rangle_{\text{PT}} + c_y P \quad \text{and} \quad \langle y^2 \rangle = \langle y^2 \rangle_{\text{PT}} + 2\langle y \rangle_{\text{PT}} c_y P + O(1/Q^2).
\]

The quantity \( c_y \) depends on the observable while \( P \sim M_{\mu_I}/Q\alpha_0(\mu_I) \) is universal and the Milan factor \( M \) takes account of two-loop effects \[21\]. The shift is inversely proportional to the hard scale \( Q \) usually identified with the cms energy.

A study using 1st moments of event shape observables by DELPHI is shown in figure 3 (left) \[15\] using DELPHI data from LEP 1 and 2 and data from various experiments at lower energies. The fits of \( O(\alpha_s^2) \) QCD predictions with power corrections (solid lines) describe the data well. The dashed lines represent the perturbative part, such that it becomes apparent that hadronisation corrections are important even at large cms energies.

A direct test of the power corrections using differential distributions of the event shape observables is presented in \[18\]. Data measured at \( \sqrt{s} = 35 \) to 183 GeV are fitted simultaneously with only \( \alpha_s(M_Z) \) and \( \alpha_0 \) as free parameters. The fitted predictions describe the data well within the fitted regions.

Results from power correction analyses for \( \alpha_s(M_Z) \) and \( \alpha_0 \) from many recent analyses are summarised in figure 3 (right) \[15,16,17\]. The results for \( \alpha_s(M_Z) \) are generally consistent with the world average value \( \alpha_s(M_Z) = 0.119 \pm 0.003 \) \[12\] while the results for \( \alpha_0 \) are in agreement with each other at the 20% level, as expected theoretically \[21\]. The results for \( B_W \) from distributions are not as consistent with universality of \( \alpha_0 \) as the other results.

5 QCD Colour Factors

A study of the QCD colour factors using fits of \( O(\alpha_s^2) + \text{NLLA QCD} \) predictions with power corrections to distributions of \( 1-T, C, B_T \) and \( B_W \) was presented in \[24\]. The QCD colour factors \( n_iT_F, C_A \) and \( C_F \) represent the relative

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\(^a\) Results for \( \alpha_0 \) based on the old erroneous value of the Milan factor \( M = 1.795 \) have been scaled to correspond to the correct value of \( M = 1.49 \) \[23\].
contributions of the QCD vertices of quark-pair production from a gluon, gluon-radiation of a gluon (triple gluon vertex) and gluon radiation of a quark, respectively. In the product $n_\ell T_F$, $n_\ell$ is the number of active quark flavours and $T_F$ is the actual colour factor. The QCD colour factors are determined by the choice of the gauge symmetry group, SU(3) in the case of QCD, and are expected as $n_\ell = 5$ for $T_F = 1/2$, $C_A = 3$ and $C_F = 4/3$.

The analysis follows [18] but uses more recent data. The dependence of the complete QCD predictions on the colour factors is made explicit such that the colour factors $n_\ell$, $C_A$ or $C_F$ can be varied in the fits, in addition to $\alpha_s(M_Z)$ and $\alpha_0$. The main sensitivity comes from the running of $\alpha_s$, which in $O(\alpha_s)$ reads $\alpha_s(Q) = \alpha_s(\mu)/(1 - 2\beta_0 \alpha_s(\mu) \ln(\mu/Q))$, where $\beta_0 \sim 11C_A - 2n_\ell$. Using the power correction calculations as the hadronisation model reduces potential biases from hadronisation corrections, because the power corrections depend explicitly on the QCD colour factors.

Figure 4 (left) shows the results of fits with $\alpha_s(M_Z)$, $\alpha_0$ and one of the colour factors $n_\ell$, $C_A$ or $C_F$ as free parameters. The results for $\alpha_s(M_Z)$ are consistent with the world average, except the result from $B_W$ when fitting $C_F$. The results for $\alpha_0$ are consistent with those presented in section 4. The results for the colour factors are consistent with the SU(3) QCD expectation.
with five active flavours, indicated by the vertical dotted lines.

Figure 4 (right) displays the unweighted averages of results of $1 - T$ and $C$ from simultaneous fits with $\alpha_s(M_{Z^0})$, $C_A$ and $C_F$ as free parameters; the non-perturbative parameter $\alpha_0$ was fixed at $\alpha_0 = 0.543 \pm 0.058$. Also shown are the expectations for various alternatives for the gauge symmetry group in QCD, in particular $U(1)^3$ is the representation for a theory with three different neutral gauge bosons in direct analogy to QED. The measurement agrees well with standard QCD with the SU(3) symmetry group.

The analysis is complementary to the traditional approach of using angular correlations in 4-jet final states [25–27] and has similar total uncertainties. Under the assumption that QCD based on the SU(3) gauge symmetry group is the correct theory of strong interactions, the analysis provides a successful consistency check of the power correction model.

6 Summary

We have shown experimental studies of jet production in $e^+e^-$ annihilation. Jet production as measured from PETRA to LEP 2 energies is well described by QCD models and by perturbative QCD calculations. A measurement of the b-quark mass at the $Z^0$ peak provided evidence for the running of the b-quark mass as predicted by QCD. Investigations of power corrections were discussed and it was found that the model successfully predicts the hadronisation effects for a number of event shape observables. The free non-perturbative parameter $\alpha_0$ is observed to be universal within the theoretically expected uncertainty of about 20%. A measurement of the QCD colour factors using power correction calculations was presented. This analysis is complementary to traditional analyses of angular correlations in 4-jet final states at the $Z^0$ peak and of similar accuracy.

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