Enhancement of localization length for two interacting kicked rotators

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Abstract

We study the effect of coherent propagation of two interacting particles in a disordered potential. The dependence of the enhancement factor for coherent localization length due to interaction is investigated numerically in the model of quantum chaos. The effect of interaction for two particles in many dimensions is also discussed.

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I. INTRODUCTION

The quantum localization of dynamical chaos has received a great deal of attention during the last years [1], [2]. It has been understood that quantum interference effects lead to a suppression of diffusive spreading in the action space in spite of the chaotic dynamics of the correspondent classical model. An important consequence of this phenomenon is the exponential localization of quantum eigenfunctions over the unperturbed levels. A close correspondence has been established between this dynamical localization and the Anderson localization in a random potential for solid state systems [3]. One of the most studied models in this field is the kicked rotator model (KRM), which in the classical limit corresponds to the Chirikov standard map [4] (CSM), a common paradigm of classical chaos. Although KRM seems to be at a first glance a pure mathematical model, it has however found important applications for real physical systems, as for example the process of microwave ionization of Rydberg atoms [5]. Another useful property of KRM is that it can be studied very efficiently in numerical simulations allowing to investigate its properties in great details.

Anyway KRM describes the one particle quantum dynamics and in many respects it is quite similar to one particle localization in a quasi one-dimensional random potential [6]. The last problem has been intensively studied in solid state physics and it is well understood from the theoretical point of view. On the contrary the case of interacting particles is much more complicated and a clear theoretical picture is still lacking (see for instance the recent Review [7]). Usually this problem is studied near the ground state and it is a common lore that a repulsive interaction would result in a even stronger localization [8,9]. However, the recent investigation of two interacting particles in a quasi-one-dimensional random potential [10] has shown that, even in the case of repulsive particles, interaction leads to an enhancement of localization length and a coherent propagation of two particles on a distance \( l_c \) much larger than the one-particle localization length \( l_1 \). According to [10] the enhancement factor is given by:
$l_c/l_1 \sim l_1 M \frac{U^2}{32V^2}$ (1.1)

where $U$ is the strength of on site interaction between two particles, $V$ is the one-particle hopping element between nearest sites which determines the size of one-particle energy band and $M$ is the number of transverse channels so that by itself $l_1 \sim M$ ($M < l_1$). In (1.1) the intersite distance $a$ is taken to be equal one and energy is taken near the center of the band so that $k_F \sim 1/a = 1$. Interaction $U$ is considered to be less or comparable with $V$.

The equation (1.1) is valid in the regime when the enhancement factor is larger than one $l_c/l_1 > 1$.

The physical reasons according to which two particles propagates together, forming an effective pair, can be understood in the following way. Since the interaction couples only nearby sites (or on site) then the particles initially located on a distance $r \gg l_1$ have only exponentially small effective coupling with each other due to the exponential decay of localized one-particle eigenstates. For such type of states two interacting particles (TIP) remain localized near their initial positions. These states are localized and form the majority of all states. However, there are other states in which TIP are initially close so that $r < l_1$. As it follows from the previous case $r \gg l_1$, the initially close particles ($r < l_1$) cannot become separated on a distance much larger than $l_1$ (otherwise we would contradict the previous case $r \gg l_1$). Therefore, they always remain on a distance $r \sim l_1$. On this distance interaction between particles is important. Qualitatively, the motion of one particle with respect to the other produces some kind of noise on it. This noise gives destruction of interference effects which have produced one-particle localization. The destruction of interference allows particles to propagate on a distance $l_c$ which is much larger than the distance between particles $l_1$. This propagation can take place only for two particles since as soon as they become separated ($r \gg l_1$) they become localized. In some sense the way of TIP propagation in a random potential is similar to the method used by Münchhausen to save himself from a swamp.

The functional dependence in (1.1) can be understood in the following way. The enhance-
ment factor $l_c/l_1$ is proportional to the probability to mix one-particle states by interaction. In fact $l_c/l_1 \sim \Gamma \rho$ where $\Gamma \sim U_s^2 \rho$ is the rate of transition and $\rho$ is the density of coupled states \[^{[10]}\]. The coupling matrix element is $U_s \sim U/N^{3/2}$ \[^{[10]}\] where $N = Ml_1$ is the number of unperturbed components in one-particle eigenstate. The density of states is $\rho = 1/\Delta E \sim N^2/V$ so that the enhancement factor is $l_c/l_1 \sim (U_s/\Delta E)^2 \sim N(U/V)^2$ in agreement with \[^{[11]}\]. It is interesting to mention that such type of estimate is quite similar to the derivation of statistical enhancement for weak interaction and parity violation in neutron-nucleus reactions discussed by Sushkov and Flambaum \[^{[11]}\]. However in \[^{[11]}\], even enhanced, the interaction remains relatively weak giving only small corrections while in our case the enhanced interaction $U_{eff} \sim (Ml_1)^{1/2}U$ is not small and it leads to significant changes in the properties of the system.

Another way of derivation of \[^{[11]}\] is based on the reduction of the TIP problem to a problem of superimposed band random matrices (SBRM) \[^{[10]}\]. There, the interaction, even if repulsive, creates an effective thick wire along the diagonal $n_1 = n_2$ in the two-dimensional plane $(n_1, n_2)$ of indices corresponding to two particles. The effective width of the wire (effective number of transverse channels) is determined by the number of levels $Ml_1$ coupled by interaction in one-particle basis. Outside of this width interaction is exponentially small and can be neglected at least in the first approximation. The large number of effective transverse channels $M_{eff} = Ml_1$ leads to enhancement of localization length with the enhancement factor proportional to $M_{eff}$.

All the above approaches were based on the assumption of statistical independence of transition matrix elements and eigenenergies in one-particle basis. This approximation seems to be reasonable due to randomness of potential and finite radius of interaction. However, it is very important to have a direct check and to verify the prediction \[^{[11]}\]. Some numerical checks were presented in \[^{[11]}\]. Here we present the results of a more detailed numerical investigation which we carried out for the model of interacting kicked rotators which had been discussed in \[^{[11]}\]. We also present numerical results for a model with finite radius of interparticle interaction.
The paper is constructed as follows. In section II we introduce the model and present the main results for on site interacting kicked rotators. The case of finite radius of interaction is discussed in section III. Conclusions and discussions of results are presented in section IV.

II. THE “ON SITE” INTERACTION MODEL

To investigate the effect of enhancement of localization length by interaction we used the model of two interacting kicked rotators introduced in [10]. The model represents two particles on a ring perturbed by kicks periodic in time. The evolution of the wave function $\psi$ on one period of perturbation is described by the unitary operator (Floquet operator):

$$\hat{S} = e^{-iT(\hat{n}_1^2 + \hat{n}_2^2)/2 + iT\delta_{n_1,n_2}} \times e^{-i(k(\cos \theta_1 + \cos \theta_2))}$$

(2.1)

with $\hat{n}_{1,2} = -i\partial/\partial \theta_{1,2}$. For $U = 0$ we have two noninteracting kicked rotators which had been intensively studied during the last years [1], [2], [3]. The classical dynamics is chaotic and diffusive for the chaos parameter $K = kT > 1$ [4]. The diffusion rate is approximately $D = n^2/t = k^2/2$ for $K \gg 1$. Quantum interference effects lead to suppression of this diffusion for typical irrational values of $T/4\pi$ and to exponential localization of eigenstates so that the averaged probability distribution over unperturbed levels decays as $|\psi_n|^2 \approx \exp(-2|n-n_0|/l_1)/l_1$. The localization length in the region of strong chaos is approximately given by $l_1 \approx D \approx k^2/2$. Quasiclassical regime corresponds to $k \gg 1, T \ll 1, kT = \text{const}$ and $l_1 > 1$.

For $U \neq 0$ interaction between particles is switched on. Using the Bessel expansion, eq. (2.1) can be written as:

$$\hat{S} = e^{-iT(\hat{n}_1^2 + \hat{n}_2^2)/2 + iT\delta_{n_1,n_2}} \times \sum_{m_{1,2}} J_{n_{1}-m_1}(k)J_{n_{2}-m_2}(k)(-i)^{n_1+n_2-m_1-m_2}e^{im_1\theta_1+im_2\theta_2}$$

(2.2)

Since $J_{n-m}(k)$ is exponentially small when $|n-m| > k$, at each iteration of $\hat{S}$ many states ($\sim 2k$) are coupled. Inter-particles interaction acts only when the two particles have the same momentum, namely when they occupy the same site on the momentum grid, on site
interaction, if we adopt the solid state terminology. Due to the presence in the exponent, the interaction $U$ can only take values in the interval $(0, 2\pi)$. Due to interaction the two particles are able to propagate coherently on a distance $l_c$ much larger than the original one particle localization length $l_1$, as was anticipated in the introduction. Of course this can happen if they are initially started within a distance $r < l_1$. Even if very close with the TIP problem in 1d Anderson model [10], our model has three different features. Indeed no randomness is here acting and the interaction is neither attractive, nor repulsive. In addition the perturbation couples many levels at each iteration (kick).

The quantum dynamics was investigated in numerical simulations for symmetric configurations with an effective number of unperturbed levels from 1000 to 2000. Antisymmetric configurations of two particles do not feel the on site interaction $U$ and are not interesting. We iterated the quantum operator $\hat{S}$ starting from two particles initially at the same site for different parameters values. The spreading of the wave function in the 2-d space $(n_1, n_2)$ was studied through the second moments along the diagonal line $n_1 = n_2$:

$$\sigma_+(t) = \frac{1}{4} \langle (|n_1| + |n_2|)^2 \rangle_t$$

and across it

$$\sigma_-(t) = \langle (|n_1| - |n_2|)^2 \rangle_t$$

as a function of the iteration time $t$. In any investigated case $\sigma_+$ was observed to saturate at an higher value than in absence of interaction, see Fig.1. On the other side $\sigma_-$ keeps the same order of magnitude as $l_1^2$ (as it should be in absence of interaction). This means that the localization length is strongly enhanced along the diagonal $n_1 = n_2$ while it remains localized, with roughly the same localization length, across the diagonal. This is even more evident if one looks on the probability distribution $P(n_1, n_2) = |\psi(n_1, n_2)|^2$ at a fixed time $t \gg t^*$, where $t^* \approx l_1$ is the localization time, see for instance Fig.2. In this picture a local averaged distribution function is represented in the quarter of space $n_1, n_2 > 0$, in a semilog plot. The channel of propagation along the diagonal $n_1 = n_2$ is manifested in the contour lines drawn at the surface basis.
From the distribution function important information can be extracted by computing the following distributions:

\[ P_{\pm}(n_{\pm}) = \sum_{|n_1\pm n_2|=n} |\psi(n_1, n_2)|^2 \]

represented in Fig.3 as a function of \( n_{\pm} = |n_1 \pm n_2|/\sqrt{2} = n/\sqrt{2} \). These distributions give a measure of the “perturbed” localization lengths along (+) and across (−) the principal diagonal. It is relatively easy to derive from them the respective localization lengths \( l_{\pm} \) by the usual best fitting procedure. Indeed the distributions are quite close to exponential curves \( P_{\pm} \sim 2 \exp(-2n_{\pm}/l_{\pm}^2) \) as can be inferred from Fig.3. In the same picture we show the probability distribution in absence of interaction \( P_0^{\pm} = 8n_{\pm} \exp(-2^{3/2}n_{\pm}/l_1)/l_1^2 \) with \( l_1 = k^2/2 \). This noninteracting distribution is quite similar to \( P_{-} \). The localization lengths \( l_{\pm} \) are then plotted vs \( l_1 = k^2/2 \) in order to check the validity of Eq. (1.1). For sake of comparison the lines with power 1 and 2 are drawn. The dependence of coherent localization length \( l_c = l^+ \) on one-particle length \( l_1 \) can be satisfactory described by \( l^+ \approx 0.5l_1^2 \) at \( U = 2 \) while \( l^- \approx 1.5l_1 \). However, the least square fit for the data of Fig.4 gives \( l_{\pm} \sim l_1^{\alpha_{\pm}} \) with \( \alpha_+ = 1.44 \pm 0.29 \) and \( \alpha_- = 1.14 \pm 0.07 \). We attribute the difference from the theoretical values 2 and 1 to the insufficiently large interval of variation of \( l_1 \) (only 4 times). Further more detailed numerical investigations should be done to extract more accurate values of \( \alpha_{\pm} \). Another interesting point following from the Figs.1-4 is that for the same length \( l_1 \) the coherent length \( l_c \) is significantly larger than in TIP in 1d Anderson model considered in [10]. This can be seen by direct comparison of \( \sigma_+ \) values. One of the reasons for this difference could be the different type of hopping in KRM where one kick couples many levels.

To determine the numerical factor in the dependence of \( l_c \) on both \( U \) and \( l_1 \) one should also study the problem at small values of \( U \ll 2 \). However, here for observation of the enhancement \( l_c/l_1 \) one should work at much larger values of \( l_1 \) than we used in Figs.1-4. This requires a sharp increase of the basis and makes the numerical calculations too difficult. Therefore, to investigate the dependence on \( U \) we did the following. According to (1.1) we expect that it should exist a critical \( U_{cr} \) given by \( U_{cr}\sqrt{l_1} > C \) with \( C \sim 1 \). To
check this we consider the same model but with random rotating phases, which means that
\[ T(n_1^2 + n_2^2)/2 \] in the first exponent is replaced by \( f(n_1) + f(n_2) \) with \( f(n) = f(-n) \) being a
random function in the interval \((0, 2\pi)\).

In this way we can change configuration varying the random realization and obtaining
results for the average behaviour. The results averaged over 10 realizations of disorder are
presented in Fig.5. The asymptotic value reached by the second moments \( \sigma_{\pm}^\infty = \lim_{t \to \infty} \sigma_{\pm}(t) \)
are plotted in units of the same value in absence of interaction \((U = 0)\). Error bars are due
to fluctuations in varying the random configuration. For small \( U \), \( \sigma_+^\infty \) and \( \sigma_-^\infty \) are both
increasing up to double their value without interaction. When \( U > U_{cr} \) full (\( \sigma_+ \)) and open
(\( \sigma_- \)) circles start to deviate one from each other thus indicating the presence of a sharp
transition. In our case the transition starts at one particle localization length \( l_1 = 8 \) which
approximately agrees with the observed critical value \( U_{cr} \approx 0.3 \) and \( C \approx 1 \). We were not
able to extract a more precise information on the dependence of \( l_c \) on \( U \) due to the heaviness
of numerical simulations.

III. THE MODEL WITH FINITE RADIUS OF INTERACTION

In this section we analyzed the effect of a finite range interaction on the dynamics. To
be more precise we chose in (2.1), instead of the former on site interaction \( U \delta_{n_1,n_2} \) a more
general, finite radius interaction :

\[ U \eta(n_1, n_2) \theta(b - |n_1 - n_2|) \]

where \( \theta(x) \) is the usual step function which is zero for \( x < 0 \) and one for \( x \geq 0 \). The phase
\( \eta \) is a random number in the interval \((-1, 1)\) which depends only on \( n_1 \) if \( n_1 < n_2 \) and only
on \( n_2 \) if \( n_2 < n_1 \). It is quite clear that for \( b = 0 \) it becomes the previous one with diagonal
disorder. The diagonal disorder creates some difference from the model of section II since
now the interaction depends not only from the difference \( n_1 - n_2 \). However, physically it is
clear that diagonal disorder in interaction will not change too much the results. Indeed, the
main point is to have some coupling between two particles and the sign of interaction is not very important for the destruction of interference, since the one-particle random potential is already acting. Our numerical results confirm that disordered one site interaction gives qualitatively the same effects as for on site interaction $U\delta_{n_1,n_2}$. We usually investigated the cases with different interaction radius $R = 2b + 1$ and $U = \pi$.

Our main interest is to investigate the effect of interaction with finite radius $R$. From the theoretical point of view we can expect that for interaction radius $R < l_1$ the equation (1.1) is still valid since the particles are effectively coupled on a distance $l_1$. However, for $R > l_1$ the size of the effective thick wire on the lattice $n_1, n_2$ is defined by $R$ so we can expect that the enhancement factor will become larger $l_c/l_1 \sim (R + l_1)$. This expression should remains valid up to values of $R \ll l_{12}$ where $l_{12}$ is the 2d localization length for infinite radius $R$: $\ln l_{12} \sim l_1$. Indeed, for $R \gg l_{12}$ with the chosen type of interaction one should have the same localization length as in 2d.

The results of our numerical simulations for finite interaction radius $R$ are presented in Figs.6-9. In Fig.6 and Fig.7 we show $\sigma_+$ and $\sigma_-$ for three different $b$ values ($b = 0, 4, 16$) which roughly agree with the above estimates. In agreement with the above picture the enhancement factor remains practically unchanged for $R < l_1$. Only the case $b = 16$ has $R > l_1$ and this produces a significant growth in $\sigma_+$ (and even in $\sigma_-$). The distributions $P_{\pm}(n)$, as defined in the previous section, are shown in Fig.8 and demonstrate a sharp increase of $l^+\pm$ comparing to $U = 0$. In the same way we took the asymptotic values $\sigma_{\pm}^\infty$ reached by $\sigma_{\pm}(t)$ at large time $t$ and we plot in Fig.9 their square root as a function of the radius of interaction $R$ (the values of $l^{\pm}$ have a similar behaviour). This figure confirms the above arguments that the enhancement starts to grow only for $R > l_1$. However, it should be mentioned that the increase of $R$ leads to a growth not only of $\sigma_+$ but also of $\sigma_-$. Indeed, $\sigma_+$ and $\sigma_-$ are growing in the same way: from the same Fig.9 one can see that the ratio $\sigma_+^\infty/\sigma_-^\infty$ is approximately constant as a function of the interaction radius $R$. The physical explanation of this similar growth is quite simple: the increase of $R$ leads not only to the increase of coherent propagation length but also to an increase of the effective size of
the pair which becomes of the order of $R \gg l_1$. Unfortunately, we were not able to study numerically the regime $R \gg l_1$ (in our case the maximal ratio $R/l_1 \approx 2$) and it was not possible to check the dependence $l_c \sim R l_1$.

**IV. CONCLUSIONS AND DISCUSSIONS**

Above we presented the results of our numerical investigation about two interacting kicked rotators in the domain of quantum chaos. They clearly demonstrate that on site interaction between two rotators in momentum space leads to large enhancement for localization length comparing to noninteracting case (Figs. 1-3). The localization length for coherent propagation of two particles $l_c = l^+$ is significantly larger than the distance between them $l^- \approx l_1$. The maximal ratio $l^+/l^-$ in our numerical simulations was near 10 (Fig.4) which justifies the fact of effective enhancement of localization length for coherent propagation of two particles. The direct check of the relation (1.1) shows that the coherent localization length $l_c = l^+$ grows approximately as $l^+ \sim l_1^2$ but more detailed numerical calculations are necessary to have a more accurate check of the power (see also discussion below).

Another part of our investigations was devoted to the effects of a final radius of interaction $R$ between particles. They definitely show that for $R < l_1$ the enhancement is not sensitive to the value of $R$ (Fig.9). The physical reason is quite clear. Indeed on site interaction couples one-particle states in a radius of $l_1$ and therefore interaction with $R < l_1$ does not give significant changes. For $R \gg l_1$ the enhancement factor starts to grow with $R$. One can expect that in the regime $R \gg l_1$ the radius $R$ will play the role of number of coupled states $Ml_1 = R$ in an effective thick wire so that $l_c/l_1 \sim R$. Of course, this growth can continue only up to $R < l_{12}$ where $l_{12}$ is one-particle localization length in two dimensions and $\ln l_{12} \sim l_1 \gg 1$. While our results definitely show the increase of enhancement with $R$ the power of growth is around 0.25 and is significantly less than 1. We attribute this difference to the fact that the ratio $R/l_1$ was not big enough (in Fig.9 $R/l_1 < 2.1$) and the
asymptotic regime was not yet reached. The further increase of $R$ is quite difficult since $l_c$ becomes comparable with the size of the basis.

In general our results confirm the relation (1.1) but a more detailed verification of this equation is still desirable.

Let us now discuss in more details the different consequences of the result (1.1). First we start from different dimensions $d$. For $d = 2$ the length $l_1$ in (1.1) should be understood as one-particle localization length in 2 dimensions. The number of transverse channels $M$ is approximately equal to $l_1$ so that finally $l_c \sim l_1^3$. For dimension $d = 3$ an interesting situation appears below Anderson transition for one particle [12]. Indeed, it is possible to realize a random potential in which all one-particle eigenstates are localized for the hopping strength $V < V_c$ (a shift of mobility edge by interaction is not a very interesting case). As a typical example let us consider the Lloyd model with diagonal disorder $E_{n_1,n_2,n_3} = \tan \theta_{n_1,n_2,n_3}$ and hopping $V$ on a cubic lattice, where $\theta_{n_1,n_2,n_3}$ are random phases homogeneously distributed in the interval $[0, \pi]$. In this case $V_c \approx 0.2$ and below this value all states are localized. For two interacting particles in such random potential the effective strength of hopping for a pair will be strongly enhanced $V_{eff} \sim \sqrt{NU}$. Here $U$ is on site (or nearby site) interaction and $N \sim l_1^3$ is the effective number of states coupled by interaction. Since $l_1$ can be quite large near (but below) one-particle transition point $V_c$ then two particles, even if characterized by repulsive interaction, can be delocalized when all one-particle states are exponentially localized [12]. Another way to see this effect is to say that the pair feels the disorder averaged over the size of the pair $l_1$ which gives a strong effective decrease of disorder. Since in 3d delocalization takes place for $V_{eff} > V_c$ generally there is no requirement to have $l_1 \gg 1$ and it is not necessary to take $V$ very close to $V_c$. The condition $V_{eff} > V_c$ gives the boundary of pair delocalization $U l_1^3/2/V > 1$.

The appearance of delocalization for a pair in 3d leads to quite interesting properties of energy spectrum. Indeed, for particles located on a distance $r \gg l_1$ from each other the effective interaction is exponentially small ($\sim \exp(-2r/l_1)$) due to the small overlapping of one-particle states. Therefore, such states remain localized while the delocalization will take
place only for the states with interparticle distance \( r < l_1 \). Since the localized states with \( r \gg l_1 \) form an everywhere dense spectrum this would mean that the continuous spectrum, corresponding to a delocalized pair, is embedded into the pure point spectrum of almost noninteracting one-particle states.

Generally speaking such kind of spectrum is unstable with respect to small coupling between quasi-degenerate levels. In the present case the coupling is exponentially small but nevertheless it can change in principle the structure of the spectrum. The physical reason of such possible change can be understood in the following way. The delocalized pair propagates in a random potential which acts as some effective noise. This can increase the size of the pair even if the matrix elements for transitions with \( r = n_- \gg l_1 \) are exponentially small. Due to this noise the size of the pair will grow in time. The rate of growth can be estimated as \( D_- = n_-^2/t \sim l_1^2 \exp(-2n_-/l_1) \). This gives a logarithmically slow growth of the pair size \( n_- \sim (l_1/2) \ln t \). At the moment it is not quite clear what will be the effect of the pair size growth on pair propagation in \( n_+ \). At minimum, the displacement of the pair should become slower than diffusive \( n_+^2 \approx (n_1 - n_2)^2 \sim t/\ln t \). However, it is quite possible that sticking at \( n_- \gg l_1 \) will produce a more significant effect on the growth of \( n_+ \) since in the region \( n_- \gg l_1 \) the matrix elements for transitions in \( n_+ \) are also exponentially small. It is interesting to note that even in the case of strong attraction between particles the coupled state should be destroyed during the propagation in a random potential. Indeed, during the displacement of the pair disorder leads to transitions from the coupled state to continuum leading to the destruction of pair. Usually, the destruction rate is proportional to the squared amplitude of disorder and this can make the life time of coupled state relatively short. Contrary to this case the effective life time of a pair of repulsive particles discussed above can be much larger since \( n_- \) grows only logarithmically with time. In some sense the interference creates exponentially high barriers which effectively push particles to stay together. In quasi-one-dimensional case with \( l_1 \gg 1 \) the effects of slow pair size growth can also lead to the appearance of logarithmic corrections in the expression of the enhancement factor in (1.1). For example, we expect \( l_c/l_1 \sim l_1/(\ln l_1)^\nu \) with \( \nu \sim 1 \) for \( M \sim 1 \). The
effects of TIP in 3d systems below Anderson transition when all one-particle eigenstates are localized are quite interesting and at present, we try to study them in numerical simulations with effective 3d models \cite{14}. Recently, an interesting approach to the TIP problem in $d$-dimensions was introduced in \cite{14}.

Up to now we discussed the effects of interaction only for two particles. However, for solid state systems the natural question is what will happens for a finite particles density $\rho_e$. As it was discussed in \cite{10} the above picture of TIP can be quite useful in the regime of small density $l_1 \ll 1/\rho_e \ll l_c$. In this case the interaction is mainly reduced to interaction between two isolated particles. If all the particles are separated from each other by a distance $L \sim 1/\rho_e \gg l_1$ then the interaction is exponentially small, all particles are localized and the current through such sample is exponentially small. However, it is possible to have another type of configuration when the particles are distributed by pairs of size $l_1$. In this case pairs can easily propagate on a distance $l_c \gg L \sim 1/\rho_e \gg l_1$. Collisions of pairs will go in a random way and will destroy interference effects for a pair. These collisions will lead to delocalization and appearance of finite conductivity in an infinite system. It is interesting to note that it is enough to have only one pair when all other particles are well separated by the distance $L \gg l_1$. Then the collisions will allow to transfer the charge through the whole sample. However the above consideration, based on \cite{13} and being correct for particle energy at the center of the band ($E \sim V$), should be applied more accurately for low energies near the ground state. Indeed, as it was discussed in \cite{10} at low energies one should consider a transition from a lattice to a continuous system in which the enhancement factor should be proportional to $l_c/l_1 \sim (k_F l_1) M$ since $k_F l_1$ determines the number of independent components in a localized state (for $M \gg 1$ the factor $M$ should be replaced by $k_F a_t$ where $a_t$ is the transverse width of the sample). Near the ground state $\rho \sim k_F$ and it seems that condition $l_1 \ll 1/\rho_e \sim 1/k_F$ implies that the enhancement does not work at low energies. Due to that at small densities there is no formal contradiction with the results \cite{8} according to which repulsive interaction reduces the localization length near the ground state. To have a better understanding of the situation at small $\rho_e$ a more exact analysis should be
carried out to obtain a more precise expression for $l_c$ in the continuous limit. In principle, the average difference of energies for two repulsing particles ($k_F \sim 1/a = 1$) on a distance $r \gg l_1$ ($E_\infty$) and $r < l_1$ ($E_{l_1}$) is of the order of $|E_\infty - E_{l_1}| \sim U/l_1$ and is not very large for large $l_1$. In fact this difference is even less than the amplitude of disorder $W$ (we take the case of 1d Anderson model discussed in [10] with diagonal disorder in the interval $\pm W$ where near the center of the band $l_1 \approx 25(V/W)^2$). It is possible that for investigation of continuous limit $k_F a \ll 1$ at low energy the approach used in [15] for two particles with strong attraction can be useful after some extension.

The most interesting case with density $\rho_e \sim 1$ formally cannot be analysed on the basis of the result (1.1) for TIP. However, it is possible to think that interaction between quasi-particles can be studied in the same way as for TIP and that at small density of quasi-particles $\rho_q$ with $k_F \sim 1/a = 1$ ($l_1 \ll 1/\rho_q \ll l_c \sim l_1^2$) conductivity will be not exponentially small for one-dimensional samples with a size $l_{\text{sam}} \gg l_c \gg l_1$. In 3d for a ”gas” of quasi-particles the possible slow growth of the pair size should be less important since collisions between pairs give rise to destruction of interference and finite conductivity in the regime where all quasi-particles are localized. Due to existence of exact connection between localization in 1d and 1d disordered spin systems [9] it would be interesting to understand possible manifestations of the analog of two particles interaction for spin systems.

Finally, let us briefly discuss the possibilities of application of the observed enhancement for explication of large persistent currents observed in the experiments with small metallic rings [16]. Formally the coherent localization length (1.1) is strongly enhanced in presence of interaction. Nevertheless, the direct estimates for the model of interacting kicked rotators and numerical results (see Fig.1 and [10]) clearly show that the diffusion rate on the time scale $l_1 \ll t \ll l_c$ is not larger than the classical rate at $t \ll l_1$. It follows that the time to cross a sample will be not decreased by interaction. However, the magnitude of persistent current depends not only on the diffusion rate but also on the density of levels which in principle can become very large for multi-particles systems. Therefore, the possibility of enhancement of persistent current due to interaction is still open and should be studied in
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REFERENCES

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[1] Chirikov B.V. 1991 in: Chaos and Quantum Physics, Eds. Giannoni M.-J., Voros A. and Zinn-Justin J., North-Holland, Amsterdam, p.443.

[2] Izrailev F.M. 1990 Phys. Reports 196, 299.

[3] Fishman S. 1993 in: Quantum Chaos, Eds. Casati G., Guarneri I. and Smilansky U., North-Holland, Amsterdam, p.187.

[4] Chirikov B.V. 1979 Phys. Reports, 52, 263; 1969 Preprint INP N 267, Novosibirsk.

[5] Shepelyansky D. L. 1993 in: Quantum Chaos, Eds. Casati G., Guarneri I. and Smilansky U., North-Holland, Amsterdam, p.221.

[6] Lee P.A. and Ramakhrishnan T.V. 1985 Rev. Mod. Phys. 57, 287.

[7] Belitz D. and Kirkpatrick T.R. 1994 Rev. Mod. Phys. 66, 261.

[8] Giamarchi T. and Schulz H.J. 1988 37, 325.

[9] Doty C.A. and Fisher D.S. 1992 Phys. Rev. B 45, 2167.

[10] Shepelyansky D.L. 1994 Phys. Rev. Lett. 73, 2607.

[11] Sushkov O.P. and Flambaum V.V. 1982 Usp. Fiz. Nauk 136, 3 [1982 Sov. Phys. Usp. 25, 1].

[12] Shepelyansky D.L. 1994 unpublished.

[13] Borgonovi F. and Shepelyansky D.L. 1995 in preparation.

[14] Imry Y. 1995 to appear in Europhys. Lett. .

[15] Dorokhov O.N. 1990 Sov. Phys. JETP 71(2), 360.

[16] Levy L.P., Dolan G., Dunsmuir J. and Bouchiat H. 1990 Phys. Rev. Lett. 64, 2074.
FIGURES

FIG. 1. Dependence of second moments on time in model (1.1) with \( k = 7, \quad K = kT = 5, \quad U = 2; \) upper curve is \( \sigma_+, \) lower is \( \sigma_- \). At \( t = 0 \) both particles are at \( n_1 = n_2 = 0, \) basis is \(-800 \leq n \leq 800. \) For \( U = 0 \) \( \sigma_+(t) \approx 600 \) for large \( t. \)

FIG. 2. Probability distribution for two particles in the case of Fig.1 at \( t = 8 \times 10^4. \) Different contours show different probability levels.

FIG. 3. Probability distribution as a function of \( n_\pm = 2^{-1/2}(n_1 \pm n_2) \) for the case of Fig.2: \( P_+(n_+) \) (full line); \( P_-(n_-) \) (dashed); dotted line is the theoretical distribution \( P^0_+(n_+) \) for \( U = 0. \)

FIG. 4. Dependence of localization length \( l^+ \) (full circles) and \( l^- \) (open circles) on one-particle localization length \( l_1 = k^2/2 \) for \( K = 5, \) \( U = 2 \) and \( 4 \leq k \leq 8. \) Full line shows dependence \( l_c = l^+ \propto l^2_1, \) dashed line marks \( l^- \propto l_1. \)

FIG. 5. Dependence of enhancement for \( \sigma_\infty^\pm \) on \( U \) for the model (2.1) with random rotation phases (see section II), \( k = 4. \) Error bars are obtained from \( \sigma_\infty^\pm \) for 10 different realizations of disorder.

FIG. 6. Dependence of \( \sigma_+ \) on time for the model with finite interaction radius; \( k = 5.7, \) \( K = 5, \) \( U = \pi; \) \( R = 1 \) (full curve), \( 9 \) (dotted), \( 33 \) (dashed). Initial conditions are as in Fig.1, basis is \(-500 \leq n \leq 500. \) The noninteracting case \( U = 0 \) has \( \sigma_+ \approx 250 \approx l_1^2 \) (see Fig.3 in [10]).

FIG. 7. Same as in Fig.6 but for \( \sigma_- \).

FIG. 8. Probability distribution as a function of \( n_\pm = 2^{-1/2}(n_1 \pm n_2) \) for the case of Fig.6 and \( R = 9: \) \( P_+(n_+) \) (full line), \( P_-(n_-) \) (dashed line). \( P^0_+(n_+) \) (dotted line) is the theoretical distribution for \( U = 0. \)

FIG. 9. Dependence of \( \sigma_\infty^\pm \) (full circles) and \( \sigma^- \infty \) (open circles) on interaction radius \( R; k = 5.7, \) \( K = 5, \) \( U = \pi. \)