A study of $S$-wave $DK$ interactions in the chiral SU(3) quark model

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The $DK$ interaction is relevant to the interpretation of the $D_{sJ}(2317)$. We dynamically investigate $S$-wave $DK$ interactions in the chiral SU(3) quark model by solving the resonating group method equation. The numerical results show an attraction between $D$ and $K$, which is from boson exchanges between light quarks. However, such an attraction is not strong enough to form a $DK$ molecule. Meanwhile, $S$ partial wave phase shifts of $DK$ elastic scattering are obtained. The case of $S$-wave $D^*K$ is rather similar to that of $DK$. To draw a definite conclusion whether a molecular state exists in $DK$ or $D^*K$ system, more details of dynamics should be considered in further study.

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In 2003 BaBar collaboration reported a narrow positive-parity scalar meson $D_{sJ}(2317)$ [1], which was confirmed by CLEO later [2]. In the same experiment CLEO observed the $1^+$ partner state at 2460 MeV [2]. These two states lie below $DK$ and $D^*K$ thresholds, respectively. So far there have been lots of experimental investigations of the $D_{sJ}(2317)$ and $D_{sJ}(2460)$.

The discovery of these two states has triggered heated discussion on their nature [4], and the key point is how to interpret their low masses. It's tempting to explain these two states as $(0^+, 1^+)$ $P$-wave $cs$ doublet [5], tetraquark states [6,7], or the $cs(0^+, 1^+)$ spin parity partners of the $(0^-, 1^-)$ doublet in the framework of chiral symmetry [8]. However, the predicted masses in quark model or in lattice QCD calculation [9] are higher than the low experimental data. In addition, it is pointed out [10] that $D_{sJ}(2317)$ might receive a large component of $DK$. From the experience with $a_0/ f_0(980)$, the low mass of $D_{sJ}(2317)$ could arise from the mixing between the $0^+ cs$ state and the $DK$ continuum [11]. $D_{sJ}(2317)$ is also proposed to be a dominantly $I = 0$ $DK$ state with some $I = 1$ admixture [12].

As mentioned above, it is worthwhile to study $DK$ interactions dynamically with various methods to further understand the nature of the $D_{sJ}(2317)$. In this paper, we will investigate $S$-wave $DK$ interactions in the chiral SU(3) quark model by solving the resonating group method (RGM) equation [13].

The chiral SU(3) quark model [14] is a useful tool in connecting the QCD theory and the experimental observables, especially for the light quark systems. In this model, the quark-quark interaction contains confinement, one gluon exchange (OGE) and pseudoscalar and scalar meson exchanges. It has been proved successful in reproducing the energies of the baryon ground states, the binding energy of the deuteron, the nucleon-nucleon($NN$) scattering phases and the hyperon-nucleon($YN$) cross sections. Recently, the chiral SU(3) quark model has been extended to study the baryon-meson interactions [15], baryon-antibaryon system [16], and states including heavy quarks [7,17]. In the present letter, we will follow the methods in above works to study $DK$ systems.

The paper is organized as follows. Firstly, we briefly describe the theoretical frame including the model hamiltonian and parameters. Then numerical results are shown and discussed, and the summary is presented finally.

The chiral SU(3) quark model has been widely described in the literature [7,14-17] and we just give its salient feature here. The Hamiltonian of the $DK$ system can be written as

$$H = \sum_i T_i - T_G + \sum_{i<j=1}^4 V_{ij},$$

where $T_G$ is the kinetic energy operator for the c.m. motion, and $V_{ij}$ represents the interactions between $qq$, $q\bar{q}$ or $q\bar{q}$.

As for $qq$ pair,

$$V_{qq}(ij) = V_{qq}^{conf} + V_{qq}^{OGE} + V_{qq}^{ch},$$

where the confinement potential $V_{qq}^{conf}$ is taken as linear form in this work. $V_{qq}^{ch}$ represents the interaction from chiral field coupling, which includes scalar and pseudoscalar boson exchanges in the chiral SU(3) quark model,

$$V_{qq}^{ch}(ij) = \sum_{a=0}^8 V_{\sigma_a}(r_{ij}) + \sum_{a=0}^8 V_{\pi_a}(r_{ij}),$$

where $\sigma_0, ..., \sigma_8$ are the scalar nonet fields, and $\pi_0, ..., \pi_8$ the pseudoscalar nonet fields.
Replacing the color part \((\lambda_i^c \cdot \lambda_j^c)\) in \(V_{qq}^{conf}\) and \(V_{qq}^{OGE}\) by \((\lambda_i^{c*} \cdot \lambda_j^{c*})\), we can obtain \(V_{qq}^{OGE}\) and \(V_{qq}^{conf}\). \(V_{qq}^{ch}\) has the same form as \(V_{qq}^{ch}\).

The interaction of \(q\bar{q}\) pair includes two parts: direct interaction and annihilation part

\[
V_{q\bar{q}} = V_{q\bar{q}}^{dir} + V_{q\bar{q}}^{ann},
\]

\[
V_{q\bar{q}}^{dir} = V_{q\bar{q}}^{conf} + V_{q\bar{q}}^{OGE} + V_{q\bar{q}}^{ch}.
\]

For a preliminary investigation, we neglect the contribution of annihilation part in the present work. \(V_{q\bar{q}}^{dir}\) can be obtained from \(V_{q\bar{q}}\). As for \(V_{q\bar{q}}^{conf}\) and \(V_{q\bar{q}}^{OGE}\), the transformation from \(V_{q\bar{q}}\) to \(V_{q\bar{q}}\) is given by \(\lambda_i^q \lambda_j^q \rightarrow -\lambda_i^{c*} \lambda_j^{c*}\), while

\[
V_{q\bar{q}}^{ch} = \sum_j (-1)^{G_j} V_{q\bar{q}}^{ch,j}.
\]

Here \((-1)^{G_j}\) represents the G parity of the \(j\)th meson. The detailed expressions can be found in Refs. [7,14-17].

Note that for the heavy-light quark pairs, the Goldstone boson exchanges will not be considered as a primary study. We use the same cutoff \(\Lambda\) for various mesons. Its value is around the scale of chiral symmetry breaking (~1 GeV).

**TABLE I: Model parameters for the light quark pairs.** The meson masses are: \(m_u = m_c = 980\) MeV, \(m_s = 138\) MeV, \(m_K = 495\) MeV, \(m_{\eta} = 549\) MeV, \(m_{\eta'} = 957\) MeV.

| Model Parameters | \(b_u\) (fm) | \(m_u\) (MeV) | \(m_s\) (MeV) | \(g_{ch}\) | \(m_{\sigma}\) (MeV) | \(\theta^{ps}\) | \(\theta^s\) |
|------------------|------------|-------------|-------------|----------|----------------|-------------|---------|
| 0.5              | 313        | 470         | 2.621       | 959      | -23°          | 0°          |

The parameters for the light quark pairs are taken from the previous work [14], which can give a satisfactory description of the energies of the baryon ground states, the binding energy of deuteron, the NN scattering phase shifts, and NY cross sections. For simplicity, we only show them as Table I where the harmonic-oscillator width parameter \(b_u\) = 0.50 fm. The up (down) quark mass \(m_{u(d)}\) and the strange quark mass \(m_{s}\) are taken to be the usual values: \(m_{u(d)} = 313\) MeV and \(m_{s} = 470\) MeV. The coupling constant for scalar and pseudoscalar chiral field coupling, \(g_{ch}\), is determined according to the relation

\[
g_{ch}^2 = \left(\frac{3}{5}\right)^2 \frac{g_{N\pi}^2}{4\pi} \frac{m_{\pi}^2}{M_N^2},
\]

with empirical value \(g_{N\pi}^2/4\pi = 13.67\). The mass of the phenomenological \(\sigma\) meson is treated as an adjustable parameter, and we take \(m_\sigma = 595\) MeV in the chiral SU(3) quark model. For other meson masses, we use the experimental values. \(\eta, \eta'\) mesons are mixed by \(\eta_1, \eta_8\)

\[
\eta = \eta_8 \cos \theta^{ps} - \eta_1 \sin \theta^{ps},
\]

\[
\eta' = \eta_8 \sin \theta^{ps} + \eta_1 \cos \theta^{ps},
\]

and the mixing angle \(\theta^{ps}\) is taken to be the usual value with \(\theta^{ps} = -23°\). Usually \(\sigma, \epsilon\) mesons are mixed by \(\sigma_1, \sigma_8\)

\[
\sigma = \sigma_8 \sin \theta^s + \sigma_1 \cos \theta^s,
\]

\[
\epsilon = \sigma_8 \cos \theta^s - \sigma_1 \sin \theta^s.
\]

The mixing angle \(\theta^s\) is an open problem because the structure of the \(\sigma\) meson is unclear and controversial. Firstly the scalar meson mixing is not considered, i.e. \(\theta^s = 0°\).

To investigate the heavy quark mass dependence, we take several typical values \(m_c = 1430\) MeV [7], \(m_c = 1550\) MeV [18], \(m_c = 1870\) MeV [19].

The OGE coupling constants and the confinement strengths can be derived from the masses of ground state baryons and heavy mesons [7,14]. Between the two color-singlet clusters \(D(c\bar{u})\) and \(K(u\bar{s})\), there is no OGE interaction and the confinement potential scarcely contributes any interaction. Therefore these values will not affect the final results and we do not present them here.

To explore the effect of the cutoff, we use two values \(\Lambda = 1100\) MeV and \(\Lambda = 1500\) MeV.

With the parameters determined, the S-wave \(DK\) system can be dynamically studied in the framework of the RGM, a well established method for detecting the interaction between two clusters. The details of solving the RGM equation can be found in Refs. [13,15,16]. By solving the RGM equation, one gets the energy of the system, the relative motion wave function, and the elastic scattering phase shifts.

As mentioned above, in our present study two parts are not considered: (1) the chiral field induced interactions between heavy and light quarks; (2) the s-channel annihilation interactions between \(q\) and \(\bar{q}\). In addition, in the chiral SU(3) quark model, only scalar and pseudoscalar meson exchanges are involved.

Firstly, we apply the RGM calculation to the S-wave \(DK\) isospin \(I = 0\) system. Before the numerical evaluation, let’s take a look at the effective potential

\(V(s) = V^{L=0}(s, s)\),

where the generator coordinate \(s\) can qualitatively describe the distance between the two clusters. The potentials corresponding to various considerations with \(\Lambda = 1100\) MeV are illustrated in Fig. [1]. From this figure, we can see that the total potential \(V(s)\) (Line ‘V’ in Fig. [1]) is attractive, which is relies on meson exchanges. \(\sigma\) and \(\sigma'\) mesons provide considerable attractions, while the interactions due to \(K, \epsilon, \kappa\) are weakly repulsive with comparable amplitudes. And \(\pi, \eta, \eta'\) have no contribution.

The further numerical calculation shows that all potentials are independent of the mass of \(c\) quark \(m_c\). It
is independent of the pseudoscalar mesons mixing angle \( \theta \). On the other hand, we find the attractions become weaker when \( \Lambda = 1100 \) MeV. From top to bottom, the curves correspond to the contributions from \( K, \epsilon, \kappa, \sigma' \) mesons, total contribution of all mesons \( V \), and \( \sigma \) meson.

It is obvious since we do not consider meson exchanges between \( c \) and light quarks. The results with \( \Lambda = 1500 \) MeV are similar to those demonstrated in Fig. 1 but amplitudes of all curves are a little bigger, which lead to the attractive potential \( V \) at most \( 4 - 6 \) MeV stronger.

To present the effect of the mixing angle, we take two more values \( \theta^s = 35.264^\circ \) and \( \theta^s = -18^\circ \) [20]. As shown in Fig. 2, the attractions become weaker when \( \theta^s \neq 0^\circ \). On the other hand, we find the \( DK \) effective potential is independent of the pseudoscalar mesons mixing angle \( \theta^p \), since \( \eta, \eta' \) mesons have no contribution (as shown in Fig. 1).

Then it is natural for us to wonder whether such an attraction can form a \( DK \) bound state. The binding energy of the \( DK \) system [21] is calculated by solving RGM equation. After exploring all possible combinations of the parameters in the former section, we fail to get a bound state of \( DK \).

In order to get more information, we study the \( DK \) elastic scattering processes, and the phase shifts of \( S \) partial waves are shown in Fig. 3 where \( \Lambda = 1100 \) MeV. This figure indicates that the interactions are weakly attractive in the middle energy range. When we take \( \Lambda = 1500 \) MeV, the curves are a little higher shifted, implying the attractions are a little stronger. We also find that the mass of \( c \) quark gives little contribution. The analysis of \( S \) partial waves phase shifts is qualitatively consistent with that of \( V(s) \).

All results we have presented above are based on \( DK \) \( I = 0 \). Our further calculations suggest that the results are isospin independent, i.e., the case of \( I = 1 \) are the same as that of \( I = 0 \). A part reason is \( \sigma \) exchange plays dominant role in the \( DK \) interactions, which is isospin independent. Another possible reason is meson exchanges between \( c \) and light quarks are not included.

In addition, we perform the same calculation to the \( S \)-wave \( D^*K \) system, and the rather similar results are obtained.

In this work we have dynamically studied the interactions of \( S \)-wave \( DK \) system by solving RGM equation in the chiral SU(3) quark model, including bound state problem and elastic scattering phase shifts. We have obtained some useful information. In our present calculation the potentials between \( D \) and \( K \) two clusters come from meson exchanges. By taking parameters shown in Table I we find the attractions provided by \( \sigma \) and \( \sigma' \) are stronger than the repulsions from \( K, \epsilon, \kappa \), which results in \( DK \) interaction is attractive. However, such an
attraction is not strong enough to form a $DK$ bound state. Moreover, the values of $\Lambda$ and $m_c$ offer little help to the $DK$ interaction, the scalar mesons mixing angle $\theta_s \neq 0$ can weaken the attractions, and the results are independent of isospin. The information extracted from the $S$ partial phase shift of $DK$ is qualitatively consistent with that of bound state problem. Additionally, the case of $S$-wave $D^*K$ is rather similar to that of $S$-wave $DK$ system.

In order to determine the nature of $DK$ interactions and whether $DK$ or $D^*K$ molecule exists, we will take detailed study in future, including: (1) to involve the vector meson exchanges, which are expected to contribute more attractions; (2) to consider the chiral field induced interactions between heavy and light quarks; (3) to study the $s$-channel annihilation interactions between $q$ and $\bar{q}$.

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[1] BaBar Collaboration, Aubert B et al. 2003 Phys. Rev. Lett. 90 242001.
[2] CLEO Collaboration, Besson D et al. 2003 Phys. Rev. D 68 032002.
[3] e.g. Belle Collaboration, Krokovny P et al. 2003 Phys. Rev. Lett. 91 262002; Belle Collaboration, Mikani Y et al. 2004 Phys. Rev. Lett. 92 012002; BaBar Collaboration, Aubert B et al. 2004 Phys. Rev. Lett. 93 181801; 2004 Phys. Rev. D 69 031101.
[4] To read review papers, e.g. Zhu S L 2008 Int. J. Mod. Phys. E 17 283.
[5] Godfrey S and Isgur N 1985 Phys. Rev. D 32 189; Godfrey S and Kokoski R 1991 Phys. Rev. D 43 1679; Matsuki T and Torii T 1997 Phys. Rev. D 56 5646; Dipierro M and Eichten 2001 E Phys. Rev. D 64 114004.
[6] Cheng H Y and Hou W S 2003 Phys. Lett. B 566 193; Terasaki K 2003 Phys. Rev. D 68 011501; Chen Y Q and Li X Q 2004 Phys. Rev. Lett. 93 232001; Vijande J, Fernandez F and Valcarce A 2006 Phys. Rev. D 73 034002.
[7] Zhang H X, Zhang M and Zhang Z Y 2007 Chin. Phys. Lett. 24 2533; Zhang H X, Wang W L, Dai Y B and Zhang Z Y 2008 Commun. Theor. Phys. 49 414.
[8] Bande W A, Eichten E J and Hill C T 2003 Phys. Rev. D68 054024.
[9] Bali G S 2003 Phys. Rev. D 68 071501; Dougall A, Kennedy R D and McNeile C M 2003 Phys. Lett. B 569 41; Lin H W, Ohya S, Soni A and Yamada N 2006 Phys. Rev. D 74 114506.
[10] Bali G S 2005 Phys. Rev. D 71 114513.
[11] Van Beveren E and Rupp G 2003 Phys. Rev. Lett.91 012003; 2004 Euro. Phys. J. C 32 493.
[12] Barnes T, Close F E and Lipkin H J 2003 Phys. Rev. D 68 054006.
[13] Wildermuth K and Tang Y C 1977 A Unified Theory of the Nucleus (Vieweg, Braunschweig); Kamimura M 1977 Suppl. Prog. Theor. Phys. 62 236; Oka M and Yazaki K 1981 Prog. Theor. Phys. 66 556; Straub U, Zhang Z Y, Brauer K, Faessler A, Kardikiar S B and Lubeck G 1988 Nucl. Phys. A 483 686.
[14] Zhang Z Y, Yu Y W, Shen P N, Dai L R, Faessler A and Straub U 1997 Nucl. Phys. A625 59;
[15] e.g. Huang F and Zhang Z Y 2004 Phys. Rev. C 70 064004; Huang F, Zhang Z Y and Yu Y W 2004 Phys. Rev. C 70 044004; Huang F, Zhang Z Y and Yu Y W 2005 ibid. 71 064001; Huang F, Wang W L, Zhang Z Y and Yu Y W 2007 ibid. 76 018201; Wang W , Huang F, Zhang Z Y, Yu Y W and Liu F 2007 Eur. Phys. J. A 32 293.
[16] Zhang D, Huang F, Dai L R , Yu Y W and Zhang Z Y 2007 Phys. Rev. C 75 024001.
[17] Zhang H X, Zhang M and Zhang Z Y 2007 Chin. Phys. Lett. 24 2533; Zhang H X, Wang W L, Dai Y B and Zhang Z Y 2008 Commun. Theor. Phys. 49 414.
[18] Bande W A, Eichten E J and Hill C T 2003 Phys. Rev. D68 054024.
[19] Bali G S 2003 Phys. Rev. D 68 071501; Dougall A, Kennedy R D and McNeile C M 2003 Phys. Lett. B 569 41; Lin H W, Ohya S, Soni A and Yamada N 2006 Phys. Rev. D 74 114506.
[20] Bali G S 2005 Phys. Rev. D 71 114513.
[21] Van Beveren E and Rupp G 2003 Phys. Rev. Lett.91 012003; 2004 Euro. Phys. J. C 32 493.