Probing the gravitational wave background from cosmic strings with the alternative LISA-TAIJI network

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Abstract As one of the detection targets of all gravitational wave detectors today, the stochastic gravitational wave background (SGWB) provides an important means of understanding the evolution of our universe. In this paper, we explore the feasibility of detecting the SGWB generated by the loops arising throughout the cosmological evolution of the cosmic string network, using both individual space detectors (LISA and TAIJI) and joint space detectors (LISA-TAIJI). For the joint detectors, we choose three different configurations of TAIJI, namely TAIJI_m, TAIJI_p, and TAIJI_c, to form the LISA-TAIJI networks, and we investigate their ability to detect the SGWB. By comparing the power-law sensitivity (PLS) curves of individual space detectors and joint detectors with the energy density spectrum of SGWB, we find that LISA-TAIJI_c has the best sensitivity for detecting the SGWB from cosmic string loops. It thus holds promise for further constraining the tension of cosmic strings $G_{\mu} = O(10^{-17})$.

1 Introduction

Gravitational waves (GWs), which are generated from violent movement and changes in matter and energy, carry very important information about their sources. At its beginning, the universe was full of dense matter, so that gravitational waves generated by collisions between particles were immediately absorbed by other particles. In the inflationary stage of the rapid expansion of the universe, a sudden decrease in density occurred, and the released gravitational waves were no longer absorbed. Since then, those primitive disturbances have spread throughout space to form a stochastic gravitational wave background (SGWB). The SGWB is a superposition of many incoherent GWs, including cosmological sources such as phase transitions [1–4], cosmic strings [5–7], and inflation models [8,9]. The SGWB has an extremely wide frequency band ($10^{-18}$–$10^{10}$Hz) [10,11], so that the wave sources of all detectors contain SGWB. NANOGrav reported a stochastic process from the 12.5-year data set [12], and some scientists believe that NANOGrav detected an inflationary SGWB, provided a sufficiently low reheating scale [13,14], thus raising our expectations for the detection of the SGWB.

Cosmic strings are one-dimensional defect solutions of field theories [15], and can also be regarded as cosmologically stretched fundamental strings of string theory [16,17]. Networks of cosmic string were generated in the early universe and are thought to exist throughout cosmological history. Cosmic string loops are a kind of SGWB [18–20]. Generally, these loops have been generated in abundance throughout cosmological history due to the frequent interactions between strings [21]. Once produced by the cosmic string network, these loops decay and emit energy in the form of GWs.

The Laser Interferometer Space Antenna (LISA) spaceborne detector is scheduled for launch in the 2030s and is aimed at detecting gravitational wave frequencies on the order of millihertz [22]. The detection of stochastic gravitational wave events generated by cosmic strings [23] in LISA has been investigated systematically, which has shown that, compared with current pulsar timing array (PTA) observations, LISA is expected to further constrain cosmic string tension by six orders of magnitude. TAIJI is another spaceborne detector which is proposed for a LISA-like mission to observe GWs in the same period as LISA [24]. The joint LISA-TAIJI networks have been studied for their potential benefit in SGWB detection [25–29].
Researchers [29–32] have investigated the detectability of four typical spectral shapes of SGWB generated from astrophysical sources: power-law (PL), flat, broken power-law (BPL), and single peaked (SP). The SGWB generated by cosmic string networks can be regarded as a PL spectral shape, but it is different from the PL shape of the SGWB generated from astrophysical sources. In this paper, we explore the capability of joint LISA-TAIJI networks to further constrain the tension $G\mu$ (where $G$ is the gravitational constant) in cosmic string networks. Analytical approximation is used to calculate the energy density of the SGWB [33] generated by cosmic string networks. For a power-law SGWB, we use a special curve, called the power-law sensitivity (PLS) curve [34], to express the corresponding performance of the space detectors. For joint LISA-TAIJI, we consider the three different configurations of TAIJI (TAIJIm, TAIJIp, TAIJIc), so the joint network is called LISA-TAIJIx ($x=m,p,c$) [29]. Comparing the PLS curves of the joint space-borne detectors and the SGWB, LISA-TAIJIc has the best sensitivity for SGWB generated by cosmic string networks, around 1 mHz, and constrains the upper limit of tension $G\mu = O(10^{-17})$.

The remainder of this paper is organized as follows. In Sect. 2, we introduce the calculation of the SGWB from cosmic strings and its spectral shape. In Sect. 3, we introduce the joint networks composed of the three configurations of TAIJI and evaluate the equivalent energy density of the single and joint detectors. In Sect. 4, we compare the PLS curves of the single and joint space detectors with the energy density spectrum of the SGWB produced by cosmic string loops, and discuss the detectability of the cosmic string SGWB. A summary is provided in Sect. 5.

2 The SGWB from cosmic strings

The SGWB generated by the evolving universe has been studied in many works ([5,18–20,35–51,55]). It is typically expressed as a fraction of the critical density of the GW in a logarithmic interval of frequency

$$\Omega_{gw}(t_0, f) = \frac{8\pi G}{3H_0^2} \frac{d\rho_{gw}}{df}(t_0, f), \quad (2.1)$$

where $H_0$ is the Hubble constant at the current time, and $\frac{d\rho_{gw}}{df}(t_0, f)$ is the energy density of gravitational waves per unit frequency at the current time.

This paper focuses on SGWBs generated by cosmic strings, which have been well established in [21,52–61]. Cosmic strings are described by the Nambu–Goto (NG) model. For Eq. (2.1) with a given frequency of GW at the current time, all the GWs emitted by the loops that are contributing at that frequency should be integrated throughout the history of the universal evolution to obtain the GW energy at this frequency.

2.1 The principle of the GW energy density calculation in the string network model

Taking into account the redshift of the GW frequency from emission to the present time, the energy density of gravitational waves observed today at a particular frequency $f$ is [21]

$$\frac{d\rho_{gw}}{df}(t_0, f) = G\mu^2 \int_0^{t_0} dt \left( \frac{a(t)}{a_0} \right)^3 \times \int_0^\infty d\ln(l) P \left( \frac{a_0}{a(l)} f l \right), \quad (2.2)$$

where $G\mu^2$ is a dimensionless unit of energy, $G\mu$ is the cosmic string tension ($G$ is the gravitational constant, $\mu \approx \eta^2$ is the energy per unit length of string, $\eta$ is the characteristic energy scale), and $a(t)$ is a scale factor. At this time, its value is $a_0$, and $n(l, t)$ is the loop number density, $P(f l)$ is the average power spectrum of the GW, and $l$ is the length of loop.

We can use an approximate estimation. Assuming that loops have periodic behavior in a flat space, the emission should be discrete

$$\omega_n = \frac{2\pi n}{T}, \quad (2.3)$$

where $T = l/2$ is the oscillation period, $n = 1, 2, 3, \ldots$ Thus we can replace $P(f l)$ with another function $P_n$ of the harmonic wave with $n$. A single loop generated by some special events (e.g., cusps and kinks in cosmic strings) can emit a gravitational wave. The power spectrum of this wave is [35,52–54]

$$P_n = \frac{\Gamma}{\eta(q)} n^{-q}, \quad (2.4)$$

where $\eta(q)$ is the Riemann zeta function, and $\Gamma = \sum_{n=1}^{\infty} P_n$ is the total emitted energy. In this paper we take $\Gamma \sim 50$ [55–60], and the index $q = 5/3$, $4/3$, $2$, corresponds to the kink, cusp, or kink-kink oscillations, respectively. The energy density can therefore be expressed as [55,58]

$$\frac{d\rho_{gw}}{df}(t_0, f) = G\mu^2 \sum_{n=1}^{\infty} C_n(f) P_n, \quad (2.5)$$

$$C_n = \frac{2n}{f} \int_0^\infty \frac{dz}{H(z)(1+z)^{q+1}} \left( \frac{2n}{1+z} f l(z) \right). \quad (2.6)$$

The number of harmonic waves is finite. However, its total number needs to be well converged for cosmic background results. For the standard universe, $\Omega_{gw}(t_0, f)$ will converge in the case of integrated $10^3$–$10^5$ models. This number depends on the power index $q$, and more models must be integrated at high frequency [61]. In our work, the evolution of the universe is assumed as a standard $\Lambda C D M$ model whose underlying parameters are $H_0 = 100$ h km/s.
In order to calculate the energy density of GW from cosmic strings, it will be necessary to fix the number density function \( n(l, t) \) of their loops, which can be written as [33,62–65]

\[
n(l, t) = \int_{l_i}^{l} \frac{\alpha f(t')}{\bar{a}(t')} d(t')^3, \tag{2.7}
\]

where \( f(l, t) \) is the production function of the non-self-interacting loops. The above equation can be further specified by numerical simulation of cosmic strings [55]. Thus the number densities in the three universe evolution phases are as follows:

\[
n_r(l, t) = \frac{0.18}{t^{3/2}(l + \Gamma G \mu t)^{3/2}} \Theta(0.1 - l/t), \tag{2.8a}
\]

\[
n_{r,m}(l, t) = \frac{0.18(2\sqrt{\Omega_A}^{3/2}(1 + z)^3)}{t [l + \Gamma G \mu t]^{3/2}} \Theta (0.09 t_{eq}/t - \Gamma G \mu - l/t), \tag{2.8b}
\]

\[
n_m(l, t) = \frac{0.27 - 0.45(l/t)^{0.31}}{2l(l + \Gamma G \mu t)^2} \Theta(0.18 - l/t), \tag{2.8c}
\]

where the subscript \( r \) stands for loops generated in the radiation era and decay in the radiation era; \( r, m \) means that the loops are generated in the radiation era but emit GWs in the matter era; and \( m \) represents loops generated and emitted GWs in the matter era. \( \Theta(x) \) is the Heaviside step function, through which we can simply find the limit of the loop size in each era, because when \( x < 0 \), the Heaviside function will be zero. In Eqs. (2.8a) and (2.8c), the cutoff values are obtained from the maximum scales that the loops can reach in the corresponding eras. But the cutoff value of Eq. (2.8b) is derived from the loops which are generated in the radiation era and decay in the matter era, which means that the loops must exist in a certain size until they are in the matter era.

### 2.2 Spectrum of the SGWB from cosmic string loops

Since the analytical method can calculate the density of loops throughout the history of the universe and can determine the power spectrum of the loops, we use the analytical method to calculate the cosmic strings in this work. In addition, several related studies have demonstrated that there is little difference in the conclusions between the analytical and numerical simulation methods [21].

From [33] we can obtain the density of the SGWB energy spectrum in three different phases. The energy spectrum of the SGWB with respect to loops generated and decaying in the radiation era is

\[
\mathcal{P}_{gw}^{\gamma}(f) = \frac{128}{9} \pi \beta G \mu \left[ \frac{G \mu}{\epsilon_r} \left( \frac{f (1 + \epsilon_r)}{B_r \Omega M / \Omega A + f} \right)^{3/2} - 1 \right],
\]

where

\[
\epsilon_r = \frac{\alpha \xi_r}{\Gamma G \mu}, \quad \beta = \frac{\sqrt{2}}{\xi^3_m / \epsilon_m}, \quad \alpha = \frac{\alpha \xi_m}{\Gamma G \mu},
\]

and we obtain \( \epsilon_m = 2/3. \xi_m = 0.625. \theta_m = 0.583. \beta_m = 0.39 F. \mathcal{P}_{gw}^{\gamma} \) is dominant when \( \alpha \gg \Gamma G \mu \), and its dominance decreases with the reduction of \( \alpha \) until \( \mathcal{P}_{gw}^{\gamma} \sim \alpha^{1/2} \). When \( \alpha \) is sufficiently small, \( \mathcal{P}_{gw}^{\gamma} \) is dominant in the low-frequency region. The spectra of the three eras are shown in Fig. 1.
Fig. 1 Simulation of the SGWB for cosmic string production. We choose $G\mu = 10^{-10}$, $\alpha = 0.1$. The red dashed line indicates the contribution from loops generated and decaying in the matter era, the green dashed line is the contribution from loops generated in the radiation era and decaying in the matter era, and the yellow dashed line is the contribution from loops generated and decaying in the radiation era, and the gray curve is the total SGWB.

Fig. 2 The spectrum of the SGWB varying with $G\mu$ and the free parameter constant $\alpha = 0.1$.

For $\alpha \geq \Gamma G\mu$ and $f < 3.5 \times 10^{10}/(1 + \epsilon_r)$ Hz, SGWB can be well approximated by the form [33]

$$\Omega_{gw}(f) = \Omega_{gw}^r(f) + \Omega_{gw}^{rm}(f) + \Omega_{gw}^{m}(f).$$

(2.15)

The spectra of the SGWB are displayed in Fig. 1. For small strings, i.e., $\alpha < \Gamma G\mu$, the loops decay rapidly so that no loops can be produced in the radiation era and exist in the matter era. Therefore, the SGWB formulation can be further simplified as [33,49]

$$\Omega_{gw}(f) = \frac{64}{3} \pi G\mu \Omega_A A_r + 54\pi \frac{H_0 \Omega^2 M}{\epsilon_m F^3} \frac{A_m}{f} \times \left[1 - \frac{B_m}{\epsilon_m f}\right].$$

(2.16)

Similarly, if we take different values of $\alpha$ and $G\mu$, we will get different curves. In this simulation, we choose 0.1 as the value of $\alpha$, and the spectrum of the SGWB varying with $G\mu$ is shown in Fig. 2.

3 Detection of the SGWB from cosmic strings in the space detectors

The LISA and TAIJI detectors are designed to detect gravitational wave signals in space, and as the missions of both detectors are of the same duration, we are looking forward to exploring the SGWB emanating from cosmic strings through their joint utilization. LISA, launched by the ESA (European Space Agency), comprises three spacecraft positioned behind the Earth as it orbits the Sun. These three spacecraft relay laser beams back and forth in the channels connecting the spacecraft, and their signals are combined to search for GWs around 1 mHz. TAIJI, as a LISA-like detector, shares the same geometry and path, but its arm length is $3 \times 10^6$ km, positioned ahead of the Earth.

For LISA-like detectors, time-delay interferometry (TDI) can suppress the laser frequency noise, which combines multiple time-shifted interferometric links to obtain an equivalent equal path for two interferometric laser beams [29].

Using the system symmetry of a single LISA-like triangular unit, it is possible to construct three data channels $A$, $E$, and $T$ without correlated noise interference [26,66], since the $T$ channel has a negligible effect in our frequency range of interest [25,26,66,67]. Those studies show that the response of the $T$-channel only reaches the response level of the $A$ and $E$ channels by an order of magnitude near 0.1 Hz, but around the mHz detection frequency, the response of the $T$-channel is typically two or three orders of magnitude lower than that of the $A$ and $E$ channels. Therefore, we only consider $A$ and $E$ channels in the following discussion.

3.1 The noise in each detector

The equivalent energy density can characterize the sensitivity of the detector to SGWB. For a LISA-like mission, this can be evaluated as [29]

$$\Omega_{\text{mission}}(f) = \frac{4\pi^2 f^3}{3H_0^2} \left(\sum_{i=A,E} R_i(f) N_i(f)\right)^{-1},$$

(3.1)

where $R_i(f)$ is the response function of the relevant channel, and $N_i(f)$ is the noise spectrum. According to Ref. [67],

$$R_{A,E} \approx \frac{9}{20} \left|W\right|^2 \left[1 + \left(\frac{f}{4f_s/3}\right)^2\right]^{-1},$$

(3.2a)

$$W = 1 - e^{-2if_s/f_s},$$

(3.2b)

where $f_s$ is the characteristic frequency of a single detector, and $f_s = c/(2\pi L)$.

The noise spectrum $N_i(f)$ can be found in the LISA Science Requirements Document [68]. The noise of the $A$ and
E channels is [69]

\[ N_{AE} \equiv |W|^2 \left( 6N_a(f) + 24N_o(f) \right). \tag{3.3} \]

The primary noise in this idealized model is acceleration noise \( N_a \) and optical path perturbation noise \( N_o \), which can be expressed as

\[ N_a = \frac{N_I}{4(2\pi f)^4}, \quad N_o = N_{II}, \tag{3.4} \]

where

\[ N_I = 4 \left( \sqrt{\delta a^2}/L \right)^2 \left( 1 + (f_1/f)^2 \right) \]
\[ = 5.76 \times 10^{-48} \times (1 + (f_1/f)^2) \text{ s}^{-4} \text{ Hz}^{-1}, \tag{3.5} \]

\[ N_{II} = \left( \sqrt{\delta x^2}/L \right)^2 = 3.6 \times 10^{-41} \text{ Hz}^{-1}, \tag{3.6} \]

in which \( f_1 = 0.4 \text{ mHz} \), and the rms magnitudes are

\[ \sqrt{\delta a^2} = 3 \times 10^{-15} \text{ m/s}^2, \quad \sqrt{\delta x^2} = 1.5 \times 10^{-11} \text{ m}. \tag{3.7} \]

\( L = 2.5 \times 10^6 \text{ km} \) is the arm length of LISA, and \( L = 3.0 \times 10^6 \text{ km} \) is used to calculate TAIJI (considering that the arm length of each detector is constant in the following text). TAIJI has the same acceleration and optical path perturbations as LISA [67].

Combining equations (3.1)–(3.7), the equivalent energy density equation of a single detector can be expressed as

\[ \Omega_{\text{mission}}(f) = \frac{20\pi^2 f^3}{3H_0^2} \times \left( \frac{5.76 \times 10^{-48} (f^2 + f_1^2)}{16\pi^4 f^6} + 3.6 \times 10^{-41} \right) \times R(f), \tag{3.8} \]

where

\[ R(f) = 1 + \left( \frac{f}{f_2} \right)^2, \tag{3.9} \]

\[ f_2 = \frac{4f_o}{3}. \tag{3.10} \]

3.2 Cross-correlation analysis

For cross-correlation, i.e., the LISA-TAIJI network, we adopt LISA and three alternative TAIJI orbital deployments to construct the network [29]:

(a) LISA, trailing the Earth by \( \sim 20^\circ \), and its formation plane has an inclination angle with respect to the ecliptic plane about \( \sim +60^\circ \).

(b) TAIJJp, leading the Earth by \( \sim 20^\circ \), and its formation plane has an inclination angle with respect to the ecliptic plane about \( \sim +60^\circ \).

(c) TAI JJc, leading the Earth by \( \sim 20^\circ \), and its formation plane has an inclination angle with respect to the ecliptic plane about \( \sim +60^\circ \).

(d) TAI JJp, leading the Earth by \( \sim 20^\circ \), and its formation plane has an inclination angle with respect to the ecliptic plane about \( \sim +60^\circ \).

(e) TAI JJc, trailing the Earth by \( \sim 20^\circ \), and its formation plane has an inclination angle with respect to the ecliptic plane about \( \sim +60^\circ \).

The equivalent energy density of the LISA-TAIJI network can be evaluated as [29],

\[ \Omega_{\text{cross}}(f) = \frac{4\pi^2 f^3}{3H_0^2} \left( \sum_{i,j=A,E,T} S_{n,i,T}\gamma_{n,j}(f) \right)^{(-1/2)}, \tag{3.11} \]

the overlap reduction function \( \gamma_{ij}(f) \) can be written as [26]

\[ \gamma_{ij}(f) = \Gamma_{abcd} D_{i,ab} D_{j,cd}, \tag{3.12} \]

and the tensor \( \Gamma_{abcd} \) can be written as a formula of Kronnecker’s delta and unit vector \( m_i \)

\[ \Gamma_{abcd} = b_0 \delta_{ac} \delta_{bc} + b_1 \delta_{ac} m_b m_d + b_2 m_a m_b m_d. \tag{3.13} \]

The subscript \( abcd \) is the four arms of the two effectively L-shaped interferometers \( A \) and \( E \), and the coefficients \( b_0, b_1, b_2 \) are given by the spherical Bessel function \( j_i = j_i(y), y = 2\pi f d/c \), where \( d \) is the separation distance between the two detectors

\[ b_0(y) = 2 \left( j_0 - \frac{10}{7} j_2 + \frac{1}{14} j_4 \right), \tag{3.14} \]

\[ b_1(y) = 4 \left( \frac{15}{7} j_2 - \frac{5}{14} j_4 \right), \tag{3.15} \]

\[ b_2(y) = \frac{5}{2} j_4. \]

\( D_{i,ab} \) and \( D_{j,cd} \) are the tensors of the two detectors. Considering the L-shaped interferometer \( A \) for a single triangular unit \( X \), and using the orthonormal unit vector \( \{m_a, m_b\} \), the detector tensor for the \( A \) channel is

\[ D_{A,ab} = \left( m_a \times m_a - m_b \times m_b \right)/2. \tag{3.16} \]

The \( A \) and \( E \) channels can be effectively regarded as two L-shaped interferometers with an offset angle of \( 45^\circ \) [26]. Therefore, the detector tensor for the \( E \) channel is

\[ D_{A,ab} = \left( m_a \times m_b + m_a \times m_b \right)/2. \tag{3.17} \]

\( m_a, m_b \) are the orientations of the two arms about an L-shaped interferometer (\( m_a \times m_b = 0 \)). We can ignore the effect of the \( T \) channel when calculating the equivalent energy density of the joint network, since its effect is not significant [26]. That is, the joint network we are studying consists of four parts, namely \( AA, AE, EA, EE \) (without considering the beam pattern function). The noise models used in the joint network in [70] are expressed as follows:

\[ S_{n,i}(f) = \frac{4}{3R_E^2} \left[ P_{o1} + 2 \left( 1 + \cos (f/f_s)^2 \right) \frac{P_{a1}}{(2\pi f)^3} \right]. \]
where
\[
P_{a1} = 9.0 \times 10^{-30} \left[ 1 + (4 \times 10^{-4}/f)^2 \right] \\
\times \left( 1 + \left[ f/(8 \times 10^{-3}) \right]^4 \right) m^2 s^{-4}/Hz,
\]
\[
P_{a1} = 2.25 \times 10^{-22} \left[ 1 + \left( 2 \times 10^{-3}/f \right)^4 \right] Hz^{-1},
\]
Note that for LISA and TAIJI, the characteristic frequency \( f_s \) is not the same, and \( f_s \) is related to the arm length of the detector and \( R_L = 2.5 \times 10^6 \) km, \( R_T = 3.0 \times 10^6 \) km. Since the noise of the A and E channels is the same, we can define the total response function as
\[
Y(f) = \frac{\gamma_A^2 + \gamma_{AE}^2 + \gamma_{EA}^2 + \gamma_E^2}{\gamma_A^2 + \gamma_{AE}^2 + \gamma_{EA}^2 + \gamma_E^2}.
\]
By combining Eqs. (3.12)–(3.16) and Eq. (3.21) we can obtain that
\[
Y(f) = \sum_{e=0}^{2} \sum_{f=0}^{2} b_e(y) b_f(y) X_{ef}.
\]
\( X_{ef} \) can be obtained by the tensor of the detector and the unit orientation vector \( m_i \). For instance,
\[
X_{02} = \sum_{i}^{AE} \sum_{j}^{AE} \left( \delta_{ai} \delta_{bd} D_{i,ab} D_{j,cd} \right) \\
\times (m_r m_s m_t m_u D_{i,rs} D_{j,tu}).
\]
The total response function and tensor factors are rotationally invariant [26], and the tensor factor \( X_{ef} \) for acquiring \( Y(f) \) is the key point. For calculating \( X_{ef} \), there are three cosines
\[
c_l = \hat{e}_l \cdot m, \quad c_{tx} = \hat{e}_{tx} \cdot m, \quad c_{lx} = \hat{e}_l \cdot \hat{e}_{tx},
\]
where \( \hat{e}_l, \hat{e}_{tx} \) are the normalized unit vectors and \( m \) is the unit direction vector of LISA and TAIJI(m,p,c). The subscript \( l \) represents LISA and \( tx \) indicates the different orbital deployments of TAIJI, \( x = m, p, c \). According to [26], \( Y(f) \) can be simplified as
\[
Y = \sum_{e=0}^{2} \sum_{f=0}^{2} b_e(y) b_f(y) X_{ef}(c_l, c_{tx}, c_{lx}).
\]
The detailed expansion of \( X_{ef} \) is given by Eqs.(40)–(45) in Ref. [26]. The values of the unit vectors for TAIJI(m,p,c) and normalized unit vectors are as follows:

(A) For LISA-TAIJIc, the orbital and structural design gives \( d_c = 0 \), and the directional vector \( m = (0, 1, 0) \). The normalized unit vector of the detector for this joint network is
\[
\hat{e}_{lm} = \left( -\frac{\sqrt{3}}{2} \cos 20^\circ, -\frac{\sqrt{3}}{2} \sin 20^\circ, \frac{1}{2} \right).
\]

(B) For LISA-TAIJJp, the orbital and structural design gives \( d_p = 1AU \times 2 \) sin 20°, and the directional vector \( m = (0, 1, 0) \). The normalized unit vector of the detector for this joint network is
\[
\hat{e}_{lp} = \left( -\frac{\sqrt{3}}{2} \cos 20^\circ, -\frac{\sqrt{3}}{2} \sin 20^\circ, \frac{1}{2} \right).
\]

(C) For LISA-TAIJIc, the orbital and structural design gives \( d_c = 0 \), and the directional vector \( m = (0, 0, 1) \). The normalized unit vector of the detector for this joint network is
\[
\hat{e}_{lc} = \left( -\frac{\sqrt{3}}{2} \cos 20^\circ, \frac{\sqrt{3}}{2} \sin 20^\circ, \frac{1}{2} \right).
\]

Combining Eqs. (3.11)–(3.25) and ignoring the effect of the \( T \) channel we can obtain the equivalent energy density of the joint LISA-TAIJI network. Since the noise in the A and E channels is the same, we can express them as such and simplify the subscripts \( ni, nj \) to \( n \)
\[
\Omega_{cross}(f) = \frac{4\pi^2 f^3}{3H_0^2} \\
\times \left( \sum_{e=0}^{2} \sum_{f=0}^{2} b_e(y) b_f(y) X_{ef}(c_l, c_{tx}, c_{lx}) \right)^{-1/2}.
\]
The subscript cross can be taken as cross $m$, cross $p$, and cross, denoting the joint observation of LISA and TAIJI(m,p,c), respectively. The full spectrum of equivalent energy density is shown in Fig. 3.

4 Detection of the SGWB from cosmic strings by the space detectors

4.1 PLS to detectors

Generally, the power-law sensitivity (PLS) is used to express the detectability of a power-law SGWB in a GW detector [29,34]. By comparison with the spectrum of the power-law SGWB energy density, the PLS curve can reveal whether the detector is able to detect the SGWB. There are two physical quantities important for calculating the PLS: (1) the signal-to-noise threshold $\rho_t$ and (2) the observation time $T_{ob}$.

Based on the $\rho_t$ and $T_{ob}$, the PLS of the detector can be calculated as

$$\Omega_\kappa = \frac{\rho_t}{\sqrt{27T_{ob}}} \left( \int_{f_{\min}}^{f_{\max}} df \frac{(f/f_{ref})^{2\kappa}}{\Omega_{\text{mission}}(f)^2} \right)^{-1/2},$$

(4.1)

where the subscript mission in Eq. (4.1) represents a single detector or a joint detector, which can be replaced by cross when LISA-TAIJI(m,p,c). The limits $f_{\min}$ and $f_{\max}$ are the bandwidths of the detector for a set of power-law indices $\kappa \in \{-8, -7, \ldots, 7, 8\}$. The reference frequency $f_{ref}$ can be chosen arbitrarily and will not affect the PLS curve [34]. $\Omega_\kappa$ can be derived for each value of $\kappa$ and different values of $f_{ref}$. Then the power-law sensitivity $\Omega_{\text{PLS}}$ can be given by the following equation [29,34]

$$\Omega_{\text{PLS}}(f) = \max_{\kappa} \left[ \Omega_\kappa \left( \frac{f}{f_{ref}} \right)^\kappa \right].$$

(4.2)

Figure 4 shows the PLS curves for different detection methods of the individual or joint detectors. An observation time of 4 years is chosen, and we assume that LISA and TAIJI will be in co-operational mode for 4 years during their mission. With $\rho_t = 10$, the PLS plots of different detection methods show that LISA-TAIJIc still has the best sensitivity without considering the effect of the T channel, and consistently shows the best detection capability under the joint network.

We will use the PLS diagram to express the possibility of observing the gravitational wave signals generated by cosmic strings, by comparing $\Omega_{gw}$ in Fig. 2 with the PLS in Fig. 4. Figure 5 can help us to judge whether the SGWB signals can be detected and whether the joint detection can further constrain the tension $G_\mu$.

From the results, it can be seen that a single space detector can detect the SGWB signal at $G_\mu = 10^{-15}$, but not at $G_\mu = 10^{-17}$. However, the combined detection of LISA- TAIJIc is able to achieve an SNR $> 10$ for the SGWB with tension $G_\mu = 10^{-17}$ around 1 mHz. Also, within the PLS diagram, LISA-TAIJIc still shows the most favorable sensitivity, while LISA-TAIJIp still has greater capability of detection than the individual detectors in the low-frequency region.

4.2 SNR of the SGWB from cosmic strings

In order to determine the detection capability of space GW detectors probing for cosmic strings, we calculate the PLS curves of the detectors to compare with the energy density of cosmic strings, and we also calculate the signal-to-noise ratio of the probes for this signal. As mentioned earlier, only the A and E channels are considered for single detectors and for LISA- TAIJI networks.

For LISA-TAIJIx, we calculate the corresponding four data sets for each of the three TAIJI orbits, with the following total SNR [26,29]:

$$\text{SNR} = \text{total SNR}.$$
for different TAIJI orbital constructions and single detectors. LISA-TAIJIc can reach the threshold for LISA-TAIJIc network maintains the highest variation with network is the best network when compared to two single SNR2

\[ \text{SNR}^2 = 2T_{ob} \left( \frac{3H_0^2}{4\pi^2} \right)^2 \left[ \int_0^\infty df \frac{\gamma(f) \Omega_{\text{GW}}(f)^2}{f^6 S_{\text{LISA}}^2(f) S_{\text{TAIJI}}^2(f)} \right] \]

\[ \text{(4.3)} \]

For a single detector, we calculated two sets of data for the A and E channels, with the single detector SNR set as [67]

\[ \text{SNR}^2 = T_{ob} \int_0^\infty df \frac{\Omega_{\text{GW}}(f)^2}{E_{\text{mission}}^2(f)} \]

\[ \text{(4.4)} \]

For further investigation on the limits of cosmic string tension \( G \mu \), we compute the corresponding SNR varying with \( G \mu \) in Fig. 6 to compare the detectability of the SGWB for different TAIJI orbital constructions and single detectors.

Figure 6 shows that for SGWB detection, the LISA-TAIJIc network is the best network when compared to two single detectors and the other two networks, because the SNR in the LISA-TAIJIc network maintains the highest variation with \( G \mu \). And considering the detection threshold SNR>10, only LISA-TAIJIc can reach the threshold for \( G \mu = 10^{-17} \), which also indicates that it is possible for LISA-TAIJIc to detect SGWBs from cosmic strings with \( \alpha = 0.1 \) and tensor \( G \mu = 10^{-17} \).

5 Conclusion

In this paper, we analyze the detection ability of the single and joint space detectors for the SGWB generated by cosmic string loops. By comparing the PLS curves with the energy density spectrum of SGWB and the SNR of SGWB produced by different detectors for cosmic strings of different \( G \mu \), the single and joint detectors are all capable of capturing the SGWB with \( G \mu \geq 10^{-15} \) and are able to reach SNR>10. Among them, LISA-TAIJIp are LISA-TAIJIm are less sensitive than the single detectors (LISA and TAIJI), which is consistent with the results in Ref. [29]. The LISA-TAIJIc detector network has the best sensitivity for detecting the SGWB from cosmic string loops. It may be able to detect SGWB with loop size \( \alpha = 0.1 \) and tension \( G \mu = 10^{-17} \) with SNR>10. The corresponding code is obtained from [71]. In our study, the sensitivity of LISA-TAIJIc surpasses that of TAIJI, which is slightly different from the result in Fig. 2 of Ref. [29]. This is because we use an analytical approximation to calculate the SGWB generated by cosmic string loops, yielding some divergence from the numerical results. In addition, in the calculation of the spectrum of equivalent energy density and PLS curves, we ignore the effect of the T channel and use an analytical method rather than a numerical simulation.

This work provides a future scientific goal for probing the SGWB during LISA and TAIJI operations. As another important GW space-borne detector, TIANQIN [26,72–74] can be added to the detection network. Such studies will be an important topic in our follow-up research, with potential to further restrict the parameters of the SGWB generated by cosmic string loops or other theoretical models.

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Data availability My manuscript has data included as electronic supplementary material.

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