Comments on the Covariant \( Sp(2) \)-Symmetric Lagrangian BRST Formalism

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Abstract

We give a simple geometrical picture of the basic structures of the covariant \( Sp(2) \) symmetric quantization formalism – triplectic quantization – recently suggested by Batalin, Marnelius and Semikhatov. In particular, we show that the appearance of an even Poisson bracket is not a particular property of triplectic quantization. Rather, any solution of the classical master equation generates on a Lagrangian surface of the antibracket an even Poisson bracket. Also other features of triplectic quantization can be identified with aspects of conventional Lagrangian BRST quantization without extended BRST symmetry.

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1 Introduction

An $Sp(2)$-symmetric Lagrangian BRST quantization prescription reminiscent of the conventional Batalin-Vilkovisky formalism has been known for a few years. Just as ordinary Batalin-Vilkovisky Lagrangian quantization can be derived from the underlying principle of imposing the Schwinger-Dyson BRST symmetry at all stages of the quantization procedure (and thus ensuring correct Schwinger-Dyson equations at the level of BRST Ward Identities), the $Sp(2)$-symmetric scheme of ref. [2] can be derived from imposing the $Sp(2)$-symmetric version of the Schwinger-Dyson BRST symmetry. See also the alternative derivation in [6].

Very recently, Batalin and Marnelius have proposed a modified $Sp(2)$-symmetric scheme, which reproduces all results of the earlier method of ref. [2]. The main advantage of the new proposal is that it can be readily generalized to a covariant formulation, a task carried out by Batalin, Marnelius and Semikhatov. Here, “covariant” refers to the supermanifold of fields (including all necessary ghosts, auxiliary fields, etc.) and antifields (and even further fields, see below). They have called this new formulation “triplectic quantization”. On top of the usual doubling of fields (by the introduction of antifields), triplectic quantization involves an additional tripling. For $N$ fields $\phi^A$, the dimension of the supermanifold in question is thus $6N$.

In contrast to covariant formulations of conventional Batalin-Vilkovisky quantization, the covariant $Sp(2)$-symmetric formalism of ref. [8] involves a number of new complications that makes it quite involved and which to some extent obscure its geometric meaning. It may also appear surprising that in its precise formulation, the triplectic formalism does not include the minimal solution found in ref. [2].

The purpose of the present short letter is to provide some observations which we believe will make this new covariant $Sp(2)$-symmetric formalism more transparent. In so doing, we shall demonstrate that several of the new features of the covariant $Sp(2)$-symmetric quantization scheme have direct analogues also in conventional Lagrangian BRST quantization (without this extended BRST symmetry). We shall also show how the conditions for $Sp(2)$-symmetric quantization can be generalized in a simple manner to include the formulation of Batalin, Lavrov and Tyutin.

The main ingredients in triplectic quantization are a pair of antibrackets $(\cdot,\cdot)^a$, a pair of operators $\Delta^a$, and a pair of odd vector fields $V^a$. A number of consistency conditions involving these objects need to be satisfied. These will be reviewed below. It turns out that these consistency conditions can be compactly formulated in terms of a conventional antibracket defined on the $Sp(2)$-enlarged set of fields and antifields.

2 A Poisson bracket from the antibracket

We start with the conventional antibracket formalism, without at first imposing the additional requirement of $Sp(2)$ symmetry. Also, we will discuss the most
general setting, not necessarily restricted to Darboux coordinates. Thus, let \( M_0 \) be a \( 2N \)-dimensional supermanifold with local coordinates \( x^A \). The bilinear differential operation

\[
(f, g) = \partial^r f \partial x^A \partial^l f \partial x^B
\]

from differentiable functions \( f(x) \) and \( g(x) \) on \( M_0 \) defines the antibracket if it in addition satisfies the following 3 conditions. First, it changes Grassmann parity:

\[
\epsilon((f, g)) = \epsilon(f) + \epsilon(g) + 1.
\]

(2.2)

Second, it satisfies the exchange relation

\[
(f, g) = -(-1)^{(\epsilon(f)+1)(\epsilon(g)+1)} (g, f),
\]

(2.3)

and, third, it fulfills the generalized Jacobi identity

\[
(-1)^{(\epsilon(f)+1)(\epsilon(h)+1)} (f, (g, h)) + \text{cycl. perm.} = 0.
\]

(2.4)

If in addition one has a volume element \([dv] = \rho(x)dx]\) defined on \( M \), then one can introduce a generalized \( \Delta \)-operator in covariant form [9]:

\[
\Delta f = \frac{1}{2} \text{div}_\rho(f, \cdot) \equiv \frac{1}{2} \mathcal{L}_{(f, \cdot)}dv,
\]

(2.5)

where \( \mathcal{L}_{(f, \cdot)} \) denotes the Lie derivative along the anti-Hamiltonian vector field \((f, \cdot)\). Locally, the \( \Delta \)-operator (2.5) takes the form

\[
\Delta f = \frac{1}{2} \frac{\partial^r}{\partial x^A} (x^A, f) + \frac{1}{2\rho}(\rho, f).
\]

(2.6)

Some basic relations between the antibracket and the generalized \( \Delta \)-operator follows straightforwardly from the Leibnitz rule and the Jacobi identity:

\[
(-1)^{\epsilon(g)}(f, g) = \Delta(fg) - f \Delta g - (-1)^{\epsilon(g)}(\Delta f)g
\]

(2.7)

\[
\Delta(f, g) = (f, \Delta g) + (-1)^{\epsilon(g)+1}(\Delta f, g).
\]

(2.8)

Consider next two volume elements \([d\tilde{v}]\) and \([dv]\) which are related by \([d\tilde{v}] = e^{2S(x)}[dv]\) (with \( \epsilon(S) = 0 \)). The \( \tilde{\Delta} \)-operator associated with \([d\tilde{v}]\) is related to the \( \Delta \)-operator as follows:

\[
\tilde{\Delta} f = \Delta f + (S, f) = e^{-S} \Delta(e^S f),
\]

(2.9)

\[
\tilde{\Delta}^2 f = \Delta^2 f + (e^{-S} \Delta e^S, f).
\]

(2.10)

From these relations follow, particularly, that the squares of the \( \Delta \)-operators coincide if the function \( S(x) \) satisfies the “quantum Master Equation” :

\[
\Delta e^S = 0 \iff \Delta S + \frac{1}{2}(S, S) = 0.
\]

(2.11)

So the generalized \( \Delta \)-operator is not nilpotent with an arbitrary volume element. We will assume that the nilpotency condition holds with respect to the
volume element \( dv \). Assuming in addition that \( \Delta \) is a 2nd order operator, one can define the antibracket according to eq. (2.7).

After these general remarks, we are now ready to show how one can define an ordinary even Poisson bracket from the antibracket. To this end, assume that in addition to the above objects we can supply an odd, nilpotent, vector field \( V \) which anticommutes with \( \Delta \):

\[
V \Delta = -\Delta V .
\]

(2.12)

It follows that this vector field differentiates the antibracket as:

\[
V(f, g) = (Vf, g) + (-1)^{\epsilon(f)+1}(f, Vg) .
\]

(2.13)

Furthermore, let \( u(x), v(x), w(x), ... \) be functions on \( M \), which commute with respect to the antibracket:

\[
(u, v) = 0 .
\]

(2.14)

Consider on this set of functions the operation

\[
\{u, w\} \equiv (u, Vw) .
\]

(2.15)

It is easy to check, using \( V^2 = 0 \), eq. (2.13) and the Jacobi identity, that this operation satisfies

\[
(-1)^{\epsilon(u)\epsilon(v)}\{u, v\} = \{v, u\}
\]

\[
(-1)^{\epsilon(u)\epsilon(w)}\{\{u, v\}, w\} + \text{cycl. perm.} = 0 .
\]

(2.16)

Furthermore, using the derivation property of the antibracket it is straightforward to see that the bracket (2.15) satisfies the Leibnitz rule

\[
\{u, vw\} = \{u, v\}w + (-1)^{\epsilon(u)\epsilon(v)}v\{u, w\} .
\]

(2.17)

The expression (2.15) thus defines an even Poisson bracket-like operation on the set functions that satisfy (2.14). Particularly important realizations of the \( V \)-fields are Hamiltonian vector fields (with respect to the antibracket), generated by solutions of the classical Master Equation:

\[
V = (S_0, \cdot) ; \quad (S_0, S_0) = 0 , \quad \epsilon(S_0) = 0 .
\]

(2.18)

For any non-degenerate antibracket an arbitrary \( V \)-field can at least locally be put in such a Hamiltonian form. The classical Master Equation thus generates an even Poisson bracket on an arbitrary isotropic surface of a non-degenerate antibracket.

### 3 The triplectic formalism

We now turn to the main ingredients of the triplectic quantization scheme proposed in refs. [7, 8]. In this scheme one requires the existance of a pair...
of $\Delta$-operators $\Delta^a$ and odd vector fields $V^a$ ($a = 1, 2$) satisfying the following consistency conditions:

\begin{align*}
\Delta^{\{a}\Delta^b} &= 0, \quad \epsilon(\Delta^a) = 1 \quad (3.1) \\
V^{\{a}V^b\} &= 0, \quad \epsilon(V^a) = 1 \quad (3.2) \\
V^a\Delta^b + \Delta^bV^a &= 0 \quad (3.3)
\end{align*}

Here and in the following the curly bracket denote symmetrization with respect to the indices $a$ and $b$. The operators $\Delta^a$ generate, due to (2.7), a pair of antibrackets $(\cdot, \cdot)^a$. The above consistency conditions then imply:

\begin{align*}
(-1)^{[\epsilon(f)+1]([\epsilon(h)+1]}(f, (g, h)^{\{a}\}^{b}) + \text{cycl. perm.} &= 0 \quad (3.4) \\
\Delta^{\{a}(f, g)^{b)} &= (f, \Delta^{\{a}g^{b)} + (-1)^{\epsilon(g)+1}\Delta^{\{a}f^{b)} \quad (3.5) \\
V^{a}(f, g)^{b} &= (V^a f, g)^{b} + (-1)^{\epsilon(f)+1}(f, V^a g)^{b} \quad (3.6)
\end{align*}

The partition function in the triplectic formalism is defined by the expression

$$Z = \int [dv][d\lambda]\, e^{i\bar{\hbar}[W(x) + X(x, \lambda)]}, \quad (3.7)$$

where $W(x)$ is viewed as the quantum action of the theory, and $X(x, \lambda)$ is considered the gauge fixing term. This division into two pieces of the gauge fixed action is obviously to a large extent arbitrary, but we shall follow the conventions of refs. [7, 8]. Some background for this split into $W$ and $X$ can be found in the recent work of Batalin and Tyutin [9]. The gauge-fixing function $X$ restricts the partition function to the “space of effective fields” needed to describe the quantum dynamics, and $\lambda$ are some additional parametric field variables. They are best thought of as auxiliary fields in the path integral. They become simple Lagrangian multipliers of gauge constraints when $X$ depends on them linearly.

To define the pair of antibrackets, Batalin, Marnelius and Semikhatov [7, 8] extend the dimension of the supermanifold from $(n|m)$ (the submanifold of fields $\phi^A$) to $(2n + 4m|2m + 4n)$.

The partition function (3.7) is gauge independent if the following “quantum Master Equations” hold (using here for convenience the formulation in which $\text{div}V^a = 0$):

\begin{align*}
(\Delta^a + V^a)e^W &= 0, \quad (\Delta^a - V^a + \ldots)e^X = 0. \quad (3.8)
\end{align*}

In the last equation the dots indicates the extra terms that are required due to the variation of the parametric fields variables $\lambda$. These terms are not of fundamental importance for the formalism[1], and we shall therefore not display them in detail here. The crucial difference between the Master Equation for $W$ and the analogous one for $X$ is the sign in front of the “transport term” induced by the vector fields $V^a$.

We shall now provide an interpretation of the consistency conditions imposed between the pair of antibrackets $(\cdot, \cdot)^a$, the pair of $\Delta^a$ operators, and the

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1In particular, they can be removed from the above Master Equations by imposing an additional Master Equation condition on the $\lambda$-fields.
pair of odd vector fields $V^a$, $a = 0, 1$. We shall also give a simple picture of the rôle played by the vector fields $V^a$ in ensuring the $Sp(2)$-invariant BRST symmetry. Once these vector fields $V^a$ are appropriately interpreted, we will see that the emergence of an additional Poisson bracket structure within this $Sp(2)$ invariant scheme follows naturally from the Poisson bracket defined through the antibracket, as in the previous section.

The central ingredients on which many of our subsequent considerations are based, are the following: Consider a $\Delta$-operator and a $V$-field which both depend on a pair of real parameters $k_a$:

$$\Delta_k = \sum_{a=0,1} k_a \Delta^a \quad (3.9)$$

$$V_k = \sum_{a=0,1} k_a V^a, \quad (3.10)$$

with $k_a = \text{const.}$ and $\epsilon(k_a) = 0$. The antibracket generated by eq. (2.7) then splits into the sum

$$(f, g)_k = \sum_{a=0,1} k_a (f, g)^a. \quad (3.11)$$

Remarkably, requiring the nilpotency condition $\Delta_k^2 = 0$ to hold for all $k_a$ is equivalent to the condition (3.1), and similarly the relation (2.7) between (3.11) and (3.9) coincides with the condition (1.5). In addition, from the Jacobi identity for the antibracket (3.11) for all $k_a$ follows the consistency condition (3.4) for the pair of antibrackets in the triplectic formalism.

Now, from the fact that $V_k$ anticommutes with $\Delta_k$ (eq. (2.12)), we find

$$\Delta^a \{ a \} V^b + V^a \{ a \} \Delta^b = 0. \quad (3.12)$$

It similarly follows from eq. (2.13) that

$$V^a \{ a \} (f, g)_b^b = (V^a(f, g)_b^b) + (-1)^{\epsilon(f)+1}(f, V^a(g)_b^b). \quad (3.13)$$

It is also clear that the $V_k$-field generates a Poisson bracket as described in the previous section. The generation of such an even bracket on isotropic surfaces of the antibracket is therefore not a special property of triplectic quantization.

One important observation should be mentioned at this point. The relations (3.13) are (3.12) are more general than the analogous (2.6) and (3.3) of the triplectic formalism proposed in ref. [8]. However, all other relations which follow from the above construction coincide with the ones of triplectic quantization. In fact, these plus the more general conditions (3.13) and (3.12) seem to be sufficient for the construction of an $Sp(2)$ symmetric quantization formalism. The original scheme of Batalin, Lavrov and Tyutin [2] is indeed based on these more general conditions.

The extended BRST symmetry of the $Sp(2)$-symmetric partition function (3.7) now acquires a very simple interpretation. First, the combined invariance
under both BRST and anti-BRST transformations \[8\] takes in the notation above the form
\[
\delta x = \sum_{a=0,1} k_a [(W - X, x)^a + 2V^a] = (W - X, x)_k + 2V_k . \tag{3.14}
\]

Explicitly, invariance leads to the following condition:
\[
[\Delta_k W + V_k W + (W, W)_k] + [\Delta_k X - V_k X + (X, X)_k] + \text{div}V_k = 0 . \tag{3.15}
\]

Thus, if the Master Equations (3.8) hold, and \(\text{div}V_k = 0\), then eq. (3.14) is a symmetry.

Second, consider the case where \(V_k\)-field is Hamiltonian with respect to the antibracket (3.11), \(i.e.,\) has the form (2.18) for some solution \(S_0\) of the classical Master Equation which is independent of \(k_a\). We can then define
\[
\tilde{W} \equiv W - S_0 , \quad \tilde{X} \equiv X + S_0 . \tag{3.16}
\]

The partition function (3.7) is trivially invariant under the replacement \(X \to \tilde{X}, W \to \tilde{W}\). This simply reflects the unavoidable ambiguity in defining in this way what is meant by the “quantum action” \((W)\) and the “gauge fixing” \((X)\) before boundary conditions have been imposed. But the BRST symmetry of the gauge-fixed partition function depends on the difference \(W - X\), and this would seem to imply that the BRST transformations should change under the above substitutions. We will resolve this apparent contradiction below.

The extended BRST symmetry (3.14) now acquires a very simple interpretation. Namely, it takes in the above notation the form
\[
\delta x = (\tilde{W} - \tilde{X}, x)_k . \tag{3.17}
\]

Next, taking into account the last equality in eq. (2.9), we can rewrite the Master Equations (3.8) in the form
\[
\Delta_k e^{\tilde{W}} = 0 , \quad \Delta_k e^{\tilde{X}} = 0 . \tag{3.18}
\]

So the triplectic formalism can be brought in a form analogous to the conventional Batalin-Vilkovisky quantization (when expressed with the help of \(W\) and \(X\)). Of course, the inverse of this statement does not hold, since the supermanifold in question in general needs to be larger than that of conventional Batalin-Vilkovisky quantization. However, there can exist instances where a hidden \(Sp(2)\) symmetry is present even in the conventional space fields and antifields within conventional Batalin-Vilkovisky quantization.

The expressions (3.18) and (3.14) formally coincide with the corresponding ones in the generalized BV-formalism \[8\]. In this sense the triplectic formalism can be viewed as the special case of the antibracket formalism in which the antibracket depends on a parameter of the circle \(S^1\), and where an additional solution \(S_0\) of the classical Master Equation, independent of these parameters, exists.
4 A Related Formulation without $Sp(2)$ Symmetry

At this point we would like to make more direct contact with the conventional Batalin-Vilkovisky Lagrangian quantization (without extended BRST symmetry). It is suggested in the concluding remarks of ref. [8] that there might exist a formulation without $Sp(2)$ symmetry that still involves a nilpotent odd vector field $V$ which differentiates the antibracket. At first sight there may not seem to much room for such an extension of the conventional Batalin-Vilkovisky formalism. As formulated in ref. [1], there is indeed no obvious candidate for such a new vector field $V$, simply by ghost number conservation. However, the most general formulation of Lagrangian BRST quantization which has been derived in ref. [4] only reduces to the conventional Batalin-Vilkovisky formalism in the special case where the integration over the ghost partners $c^A$ of the antighosts (now viewed as “antifields”) $\phi_A^*$ can be integrated out of the path integral to leave a $\delta$-function constraint on the antighosts $\phi_A^*$. If one leaves the fields $c^A$ in the extended action in full generality, the quantum Master Equation for the quantum action $S$ is in fact not the one of ref. [1], but rather (in Darboux coordinates) [4]:

$$\frac{1}{2}(S, S) = -\frac{\delta S}{\delta \phi_A^*} c^A + i\hbar \Delta S .$$

(4.19)

Only when assuming the simple ansatz $\tilde{S}[\phi, \phi^*, c] = S^{BV}[\phi, \phi^*] - \phi_A^* c^A$ does this complete Master Equation reduce to the conventional Batalin-Vilkovisky Master Equation for $S^{BV}$. Clearly,

$$V = (-1)^{\epsilon_A+1} c^A \frac{\delta}{\delta \phi_A^*}$$

(4.20)

satisfies $V^2 = 0$. It also differentiates the antibracket in the sense of eq. (2.13), and it anticommutes with the $\Delta$-operator as in eq (2.12).

In this notation, the full quantum Master Equation [4] takes the form

$$\frac{1}{2}(S, S) = VS + i\hbar \Delta S ,$$

(4.21)

in perfect analogy with the $Sp(2)$-symmetric Master Equation proposed in ref. [8]. In hindsight, it is not at all surprising that such a formulation exists, nor is it surprising that one needs to enlarge (slightly) the set of fields (by keeping the ghosts $c^A$ instead of integrating them out in the path integral) in order to find it. The ghosts $c^A$ of ref. [4] are precisely what become the $Sp(2)$-symmetric partners in an $Sp(2)$-invariant formulation. It is interesting that even the “third set of fields” ($\phi_A$ in the notation of [4]) have a completely natural place in the conventional Lagrangian BRST quantization scheme (without extended BRST symmetry). They are simple linear combinations of the collective fields that are needed to derive the Schwinger-Dyson BRST symmetry [2] through shifts $\phi^A \rightarrow \phi^A - \varphi^A$ (where the fields $\varphi^A$ are linear combinations of the fields $\phi_A$ that are required in the $Sp(2)$-symmetric formulation). In conventional Lagrangian BRST quantization one normally integrates these fields out of the path integral.
But one could easily keep them, in which case the formalism of Batalin, Mar-nelius and Semikhatov [7, 8] would look even less different from the quantization scheme that does not impose $Sp(2)$ symmetry [4].

The ghosts $c^A$ are “spectator fields” in the antibracket, and there is likewise no need in conventional Lagrangian BRST quantization for an antibracket associated with the collective fields. Such an additional antibracket appears only if one insists that even the collective fields themselves shall obey their correct Schwinger-Dyson equations at the level of BRST Ward Identities. Continuing iteratively in this way, one can clearly double the number of fields as many times one wishes, each time introducing a new antibracket for the new collective fields. The final outcome is clearly unaltered by such an unnecessary complication. In the $Sp(2)$-symmetric formulation it is however imperative that at least the “first-stage” collective fields are kept. Otherwise the Schwinger-Dyson BRST–anti-BRST operator is not nilpotent even when just restricted to the set of fields $\phi^A$.

The vector field of eq. (4.20) is generated by $S_0 = \phi^*_A c^A$ in the sense that $V = (S_0 , \cdot )$. Indeed, the disappearance of this vector field from the Master Equation when substituting the ansatz $S[\phi, \phi^*, c] = S^{BV}[\phi, \phi^*] - S_0[\phi^*, c]$ can be understood in the same simple manner as discussed in the previous section for the $Sp(2)$-symmetric case. Again, subtracting the generator of $V$ from the action in general changes the boundary conditions. In the formulation of ref. [4] this change in boundary conditions is automatically avoided precisely because it involves an additional set of ghost fields (the $c$'s). Of course, in the formulation based on $S^{BV}$ one should not integrate over the $c^A$-fields in the partition function.

Remaining within the framework of Darboux coordinates, we note that the natural condition of the vector field $V$ being divergence-free,

$$ \text{div } V = 0 , \quad (4.22) $$

is the simple statement that the functional measure of the $\phi^A$-fields is invariant under arbitrary local shifts (a trivial but nevertheless implicit assumption in conventional Batalin-Vilkovisky quantization when restricted to Darboux coordinates). When the functional measure is not invariant under such shifts, one can still follow the procedure of [4] through (see the last reference in [4]), and one then derives the covariant version of Batalin-Vilkovisky quantization. This covariant description also includes the additional vector field $V$ in its most general formulation.

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