Permeability Estimation of Porous Rock by Means of Fluid Flow Simulation and Digital Image Analysis

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Abstract. Permeability plays an important role to determine the characteristics of how fluids flow through a porous medium which can be estimated using various methods. Darcy’s law and the Kozeny-Carman equation are two of the most utilized methods in estimating permeability. In Darcy’s law, permeability can be calculated by applying a pressure gradient between opposing sides of inlet-outlet of a certain direction. The permeability then depends on the fluid viscosity and the flowrate. The Kozeny-Carman equation is an empirical equation which depends on several parameters such as shape factor of the pore, tortuosity, specific surface area, and porosity to determine the permeability. For both methods, digital image obtained by means of Micro CT-Scan is used. In this research, the permeability estimation using the Darcy’s law was conducted by simulating fluid flow through the digital image using Lattice Boltzmann Method (LBM). As for the Kozeny-Carman equation, digital image analysis was used to obtain the required parameters. Two Kozeny-Carman equations were used to calculate the permeability of the samples. The first equation (KC1) depends on pore shape factor, porosity, tortuosity, and specific surface area while the second equation (KC2) only depends on pore radius, porosity, and tortuosity. We investigate the methods by first testing on three simple pipe models which vary in the radii. By using the result from Darcy’s law as a reference, we compare the results from the Kozeny-Carman equations. From the calculation, KC2 yield smaller difference to the reference. The three methods were then applied to the Fontainebleau sandstone to verify the previous result.

1. Introduction
Permeability is one of petrophysical properties of a sedimentary rock that is significant in evaluating the quality of a reservoir. Permeability quantitatively describes the flow characteristic of fluids which flow through a porous medium. Permeability can be measured using laboratory-based measurement apparatus on core plug samples from well core. Filomena et al. [1] describe the advantages of using gas-driven permeameters, i.e., the measurement can be quickly performed and the samples are not contaminated. In some cases such as in a clay-bearing samples, the gas permeameter does not affect the sample, in contrary to the liquid driven ones which might swell and destroy the sample. Alternatively, permeability can also be calculated indirectly by utilizing a digital image of a porous rock sample. Permeability can then be calculated on the sample by means of simulating fluid flow through the sample using methods such as Lattice Boltzmann Method (LBM), or it can also be

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estimated using various empirical equations by first evaluating the physical parameters of the digitized data of the sample using Digital Image Analysis (DIA). LBM uses Darcy’s principle in calculating permeability, while DIA can be applied to numerous empirical equations such as the well-known Kozeny-Carman (KC) equations. In this study, we compare the result of permeability calculation using LBM and the permeability estimation using two empirical equations derived from the KC equations. Performance and accuracy of the methods were evaluated to analyze the applicability of the methods on various cases.

2. Samples and methods

2.1. Digital samples
For both LBM and DIA approach, a digitized data of a porous rock is essential. The digital image can be obtained using 3D imaging device such as X-Ray Micro-CT [2-5]. X-Ray Micro-CT scanner has the same principle as the medical CT scanner. However, in medical CT scanner, the detector and the X-Ray source rotates while in Micro-CT, the object rotates. A digital image can also be produced from computer modelling. The advantage of computer modelling, one can easily adjust the required parameters accordingly, and various models of porous media can be generated, ranging from a simple capillary cylindrical tube to a complex structured sedimentary rock model.

In this study we first generate two sets of models to compare the result obtained from DIA and LBM. The first set are three cubes with sidelength of 150 pixel which have a cylindrical shaped pipe as the pore space at the center of the cube. The diameters of the pipes are 100, 70, and 40 pixel (see figure 1).

![Figure 1](image_url). Top: the cube model of porous medium with simple cylindrical hollow as the pore space (white is the solid part, the gray colored area is the cylindrical pore wall). Bottom: 2D slice view of the sample with different diameter (white: solid, black: pore).
The second set are also three cubes with different sidelength which also have cylindrical shaped pipe as the pore space at the center of the cube. The sidelength of the cubes are varied as well as the diameter of the cylindrical pore as follows: 100:150, 90:60, and 60:40 (see figure 2). We also used the digital image of Fontainebleau sandstone to perform the permeability analysis. The digital sample of Fontainebleau sandstone has the size of 300×300×299 pixels (see figure 3), with spatial resolution of 7.5 μm/pixel. Fontainebleau sandstone is originated from France which has unique characteristics. It composed of 99% quartz, virtually devoid of clay, well recognized as a well-sorted, medium-grained, which sometimes denoted as a good example of clean sandstone. Fontainebleau sandstone is often used as a calibration experiments, usually in the calculation of porosity and permeability.

\[
B_1, \quad \frac{d}{l} = 100:150 \\
B_2, \quad \frac{d}{l} = 60:90 \\
B_3, \quad \frac{d}{l} = 40:60
\]

Figure 2. The second set of the model: three cubes which have the same sidelength:pore diameter of 2:3

\[
F_1
\]

Figure 3. The third model – Fontainebleau Sandstone

2.2. Permeability
Permeability was first described quantitatively from an experiment carried out by Darcy in 1856 [6] which relates the pressure gradient of a fluid-saturated porous medium with the corresponding fluid flowrate as follows:

\[
Q_x = -A \frac{\kappa}{\eta} \frac{\partial P}{\partial x}, \quad (1)
\]

where \(Q_x\) is the volumetric fluid flowrate in \(x\) direction, \(\kappa\) is the permeability of the medium, \(\eta\) is the dynamic viscosity of the fluid, \(P\) is the fluid pressure, and \(A\) is the cross-sectional area perpendicular to the pressure gradient. LBM simulates Darcy’s experiment by applying a pressure gradient to the inlet-outlet of a saturated porous medium. We use Palabos, a parallel Lattice Boltzmann solver (www.palabos.org) to perform the simulation [7]. Palabos utilized the D3Q19 scheme which describes motion in three dimensions and 19 associated velocity vectors. Permeability is then calculated using Darcy’s law:

\[
\kappa = \frac{Q_x \eta}{A} \frac{1}{\frac{\partial P}{\partial x}}. \quad (2)
\]

Volumetric fluid flowrate in a certain direction (in this case is \(x\) direction) can be expressed as the rate volume of fluid passing through the cross-sectional area \(A\) perpendicular to the flow direction as:

\[
Q_x = \frac{V}{l} = \frac{Al}{l}, \quad (3)
\]

where \(l\) is the length of the medium. Equation (2) can then be rewritten as:
Equation (4) is practically modified to conform with Palabos as follows:

$$\kappa = \frac{Al}{t} \eta \frac{1}{A \frac{\partial P}{\partial x}} = -l \eta \frac{1}{\partial P / \partial x}. \quad (4)$$

Equation (4) is practically modified to conform with Palabos as follows:

$$\kappa = \frac{\langle v \rangle \eta}{\partial P / \partial x}. \quad (5)$$

where $\langle v \rangle$ denotes the average magnitude of the intrinsic velocity over the entire system volume.

Permeability has units of area (m$^2$ in SI units), but sometimes in terms of geophysical aspect, it is more common to use the Darcy where 1 Darcy = $0.987 \times 10^{-12}$ m$^2$ or also millidarcy (mD) where 1 mD = 0.001 Darcy. There are some assumptions that are used in Darcy’s law: only 1 phase of fluid exist in the pore, the fluid system is isothermal, the fluid is incompressible, the flow is laminar, the fluid is Newtonian, the fluid does not interact with the pore walls, and the fluid flow is steady state.

In this study, we also estimate the permeability using two empirical equations derived from the work of Kozeny-Carman. The equations contain general parameters which can be calculated from the digital image using image analysis. In this study we use CT Analyser software (Bruker-MicroCT) to calculate the general parameters such as specific surface area and porosity. The Kozeny-Carman equations are expressed as [8, 9]

$$\kappa = \frac{1}{2} \phi \frac{S}{\tau^2}, \quad (6)$$

$$\kappa = \frac{1}{8} \phi \frac{R^2}{\tau^2}. \quad (7)$$

where $\kappa$ is permeability, $\phi$ is porosity, $\tau$ is tortuosity, $S$ is specific surface area, and $R$ is the average pore radius. Equation (6) and (7) are derived semi-analytically by comparing the flux in a pipe having circular cross-section with the Darcy’s law [9]. We differentiate the symbol of permeability from each method as $\kappa_{LBM}$ for permeability calculated using LBM, $\kappa_{KC1}$ for permeability estimated using equation (6), and $\kappa_{KC2}$ for permeability estimated using equation (7). Calculated permeability from equation (6) and (7) can also be expressed in lattice unit squared (lu$^2$) which can later be converted to m$^2$ if the spatial resolution of the digital data is available by multiplying the permeability in lu$^2$ with spatial resolution squared.

2.3. Error analysis

To evaluate the accuracy of the methods, we use analytical calculation of equation (6) and (7) and compare the result with the ones produced using DIA from the generated models. Analytical calculation can be easily done by using the predetermined parameters of the pore space i.e., the cylindrical pipe diameter and length, to which we can further analytically calculate porosity and specific surface area. The error, which is defined as the deviation of the result with regard to the analytical calculation is calculated using equation (8) for both $\kappa_{KC1}$ and $\kappa_{KC2}$

$$\varepsilon = \frac{\left| \kappa_{KC, DIA} - \kappa_{KC, ANA} \right|}{\kappa_{KC, ANA}}. \quad (8)$$
where $k_{KC1,DIA}$ is permeability calculation using digital image analysis, $k_{KC1,ANA}$ is permeability from analytical calculation for both equation (6) and (7) separately. Equation (8) is also used to calculate the error of the permeability calculated using LBM for dataset A and B by replacing $k_{KC1,DIA}$ with $k_{LBM}$.

This definition of error is appropriate since $k_{KC1,ANA}$ was actually derived semi-analytically for the cylindrical pipe model. As for the Fontainebleau sample, the errors are calculated by comparing to a known permeability of 1230 mD as calculated by Latief et al. [10] as follows:

$$E = \frac{|k_{CALC} - k_{REF}|}{k_{REF}},$$

where $k_{CALC}$ represents permeability obtained for the Fontainebleau sandstone by means of DIA using equation (6), equation (7), and from LBM ($k_{KC1}, k_{KC2},$ and $k_{LBM}$).

### 3. Result and discussion

Table 1 shows the results from calculation on sample A1, A2, and A3 and the corresponding calculated errors are listed in table 2. The permeability from analytical calculation ($k_{KC1,ANA}$) using equation (6) and (7) yield the same results for all datasets A and datasets B since both equations are derived for the same geometrical sample.

| Methods | $A_1$ ($\mu^3$) | $A_2$ ($\mu^3$) | $A_3$ ($\mu^3$) | $B_1$ ($\mu^3$) | $B_2$ ($\mu^3$) | $B_3$ ($\mu^3$) | $F_t$ (mD) |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|--------------|
| KC1     | $k_{KC1,ANA}$  | 109.083        | 26.191         | 2.793          | 109.083        | 39.270         | 17.453       | 2318.470 |
|         | $k_{KC1,DIA}$  | 13.325         | 1.096          | 0.044          | 13.325         | 1.101          | 2.164        |           |
| KC2     | $k_{KC2,ANA}$  | 109.083        | 26.191         | 2.793          | 109.083        | 39.270         | 17.453       | 4165.613  |
|         | $k_{KC2,DIA}$  | 109.257        | 26.239         | 2.824          | 109.340        | 34.192         | 17.761       |           |
| LBM     | $k_{LBM}$      | 108.683        | 26.071         | 2.791          | 108.683        | 38.984         | 17.446       | 1261.000  |

Table 2. First model error between LBM and DIA compared to Analytic

| Methods | $A_1$ (%) | $A_2$ (%) | $A_3$ (%) | $B_1$ (%) | $B_2$ (%) | $B_3$ (%) | $F_t$ (%) |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| KC1     | 87.78     | 95.82     | 98.42     | 87.78     | 92.12     | 87.60     | 88.49     |
| KC2     | 0.16      | 0.18      | 1.11      | 0.24      | 12.94     | 1.76      | 238.67    |
| LBM     | 0.37      | 0.46      | 0.07      | 0.37      | 0.73      | 0.04      | 2.52      |

The DIA approach for KC2 generally produce good results in contrary to the ones obtained using KC1 where the errors are ranged from 87% to 98%. The huge errors are caused by the characteristic of equation (6) which depends on three parameters compared to equation (7) which only depends on two parameters. The digital image analysis approach to estimate the parameters ($\phi$, $r$, $S$ and $R$) might also cause the great difference of the calculated results. Porosity calculated from analytical and DIA generally do not differ much because the calculation using DIA uses simple summation. However, calculation of specific surface area and pore radius are more complex. Edge detection method for calculating the surface area produces overestimated value which later yield underestimated value of permeability if the image is pixelated. On the other hand, the estimated pore radius from DIA mostly do not differ much from the predefined analytical one.

From table 2 we can also observe that the LBM commonly produced consistent results which are similar to the ones calculated using analytical KC for datasets A and B. For the Fontainebleau ($F_t$) sample, the LBM also produced much better result compared to both KC equations. This simple analysis could give a straightforward conclusion, that LBM could provide us a more accurate result.
compared to the other two methods for both simple structure and complex structure. The drawback of implementing LBM to a complex structured porous media, is that the method could take hours to run, while using DIA, despite its lack of accuracy, could visualize and characterize the sample in a matter of a few minutes. However, KC equations are derived semi-analytically which means the equations can still be modified accordingly depending on the applied medium structure. For example, a more generalized form of equation (6) can be used by replacing term $\frac{1}{2}$ with a more general term $\frac{1}{B}$ where $B$ is a geometric factor that accounts for the irregularities of pore shapes. In the case of sandstone, one can use value of $B$ ranged from 2 – 5.

4. Conclusion and future works
From the performed analysis, we can draw several conclusions. The difference of the result from analytical calculation and estimation using DIA of the empirical equation (6) is larger than of the empirical equation (7). It is concluded that the equation (6) is not suitable for the model, however, equation (7) fits well with the generated models. Result from LBM and equation (7) are quite similar for the datasets A and B. As for the Fontainebleau sandstone, aside from LBM none of the KC equations are suitable for the sample due to the complexity of the pore structure of the porous medium. For the future works, a more reliable results from DIA could be provided by modifying the Kozeny-Carman equations with regard to the characteristic of the real sample.

5. Acknowledgment
This research was funded by the research program, “Riset Desentralisasi DIKTI-ITB 2015”.

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