Possible Origin of Antimatter Regions in the Baryon Dominated Universe

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We discuss the evolution of $U(1)$ symmetric scalar field at the inflation epoch with a pseudo Nambu–Goldstone tilt revealing after the end of exponential expansion of the Universe. The $U(1)$ symmetry is supposed to be associated with baryon charge. It is shown that quantum fluctuations lead in natural way to baryon dominated Universe with antibaryon excess regions. The range of parameters is calculated at which the fraction of Universe occupied by antimatter and the size of antimatter regions satisfy the observational constraints, survive to the modern time and lead to effects, accessible to experimental search for antimatter.

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I. INTRODUCTION

The statement that our Universe is baryon asymmetric as a whole is quite firmly established observational fact of contemporary cosmology. Indeed, if large regions of matter and antimatter coexist, then annihilations would take place at the borders between them. If the typical size of such a domain was small enough, then the energy released by these annihilations would result in a diffuse $\gamma$–ray background, in distortions of the spectrum of the cosmic microwave radiation and light element abundance, neither of which is observed (see for review e.g. [1]). Recent analysis of this problem [2] for baryon symmetric Universe demonstrates that the size of regions should exceed 1000 Mpc., being comparable with the modern cosmological horizon. It therefore seems that the Universe is fundamentally matter–antimatter asymmetric. However the arguments used in [2] do not exclude the case when the Universe is composed almost entirely of matter with relatively small insertions of primordial antimatter. Thus we may expect the existence of macroscopically large antimatter regions in the Universe, that differs drastically from the case of baryon symmetric Universe. We call the region filled with antimatter in the baryon dominated Universe, as antizillah. Of course the existence of antizillahs is not rigorous requirement of baryosynthesis, but some modification of baryogenesis scenarios will result in formation of domains with different sign of baryon charge (see for example [3]). The only condition which is necessary to satisfy is the amount of antibaryons within antizillahs must be small comparing to the total baryon number of the Universe.

At the first glance it is not difficult to have some amount of antizillahs if we simply suppose that in the early Universe when the baryon excess is generated the C–and CP–violation have different sign in different space regions [4]. This may be achieved, for example, in models with two different sources of CP–violation, explicit and spontaneous [4] one. However, any spontaneous CP–violation processes are a result of early phase transition of first or second order what implies very small size of primordial antizillahs [5]. For example if the antizillahs are formed in the second order phase transition, their size at the moment of formation is determined by $l_i \simeq 1/(\lambda T_c)$, where $T_c$ is so called Ginsburg temperature (the critical temperature at which the phase transition take place) and $\lambda$ is the selfinteraction coupling constant of field which breaks CP symmetry [6]. In the result of expansion the modern sizes of domains would reach $l_0 \simeq l_i (T_c/T_0) = 1/(\lambda T_0) \simeq 10^{-21}pc/\lambda$, where $T_0$ is the present temperature of the background radiation.

On the other hand it has been revealed [7] that the average displacement of the antizillah’s boundary caused by annihilation with surrounding matter is about 0.5pc at the end of radiation dominated (RD) epoch. Therefore any primordial antizillah having initial size up to 0.5pc or more at the end of RD stage is survived to the contemporary epoch and in the case of successive homogeneous expansion has the size $\simeq 1kpc$ or more. Any primordial antizillah with scale less then critical survival size
l_c \simeq 1 \text{kpc} at contemporary epoch must be eaten up by annihilation process. Thus it is the serious problem which any model with thermal phase transition encounters to create primordial antizillah with the size exceeding the critical survival size $l_c$ to avoid complete annihilation.

There is an additional problem for baryosynthesis with surviving antizillah’s sizes. The point is that any phase transition is accompanied by formation of topological defects. If we blow-up the region with different signs of charge symmetry, we automatically blow-up the scale of respective topological defect structure. If the structure decays sufficiently late in the observable part of Universe, the contribution of energy density of such topological defects could be sufficiently high to contradict with observations. It can be easily estimated that the structure with the scale corresponding to the survival size enters the horizon and starts to decay at $T \lesssim 0.1\text{MeV}$, i.e. in the period of Big bang nucleosynthesis. To remove these unwanted relics sufficiently early it is necessary to have a mechanism for symmetry restoration. This mechanism implies that the baryogenesis is going on within rather narrow time interval $\Delta t$.

In the present paper we have elaborated the issue for inhomogeneous baryosynthesis without the difficulties pointed above. The proposed approach is based on the mechanism of spontaneous baryogenesis [9]. This mechanism implies the existence of complex scalar field carrying baryonic charge with explicitly broken $U(1)$ symmetry. The baryon/antibaryon number excess is produced, when the phase of this additional field moves along the valley of its potential [10].

It is supposed that the vacuum energy responsible for inflation is driven by any scalar inflaton field, and additional complex field coexists with the inflaton. Due to the fact that vacuum energy during inflational period is too large, the tilt of potential is vanished. This implies that the phase of the field behaves as ordinary massless Nambu–Goldstone (NG) boson and the radius of NG potential is firmly established by the scale of spontaneous $U(1)$ symmetry breaking. Owing to quantum fluctuations of massless field at the de Sitter background [21,22] the phase is varied in different regions of the Universe. When the vacuum energy decreases the tilt of potential becomes topical, and pseudo NG (PNG) field starts oscillate. As the field rolls down in one direction during the first oscillation, it preferentially creates baryons over antibaryons, while the opposite is true as it rolls down in the opposite direction. Thus to have globally baryon dominated Universe one must have the phase sited in the point, corresponding to the positive baryon excess generation, just at the beginning of inflation (when the size of the modern Universe crosses the horizon). Then subsequent quantum fluctuations can move the phase to the appropriate position causing the antibaryon excess production. If it takes place not too late after the inflation begins, the size of antizillah may exceed the critical surviving size $l_c$.

The main idea of proposed issue is based on the existence of quantum fluctuations along the effectively massless angular direction of baryonic charged scalar field. Thus, more general, the considered issue of generation of antizillas is applicable practically to all mechanisms of baryogenesis where the number density and sign of baryon asymmetry depend on the angular component of complex scalar field. The advantage of the mechanism of spontaneous baryogenesis [9] considered here is the quite simple unambiguous inflation dynamics of scalar field generated baryon charge. This fact allows to establish quantitatively definite relationship between the effects of inflation and generation of baryon (antibaryon) excess in inhomogeneous baryogenesis. However, this relationship may be too rigid for the realistic model of antimatter domain formation compatible with the whole set of astrophysical constraints. The consistent picture may need more sophisticated scenarios. The principal possibility for such scenario can be considered on the base of Affleck Dine (AD) [12] baryogenesis mechanism that still receives a lot of attention [12,17].

AD baryogenesis also involves the cosmological evolution of effective scalar field, which carries baryonic charge, being composed of supersymmetric partners of electrically neutral quark and lepton combinations. The important feature of supersymmetric extensions of standard model is the existence of ”flat directions” in field space, on which the scalar potential vanishes [11]. We will refer for the definiteness to the flat directions of minimal standard supersymmetric model (MSSM) [13,15]. Thus, if the some component of scalar field lies along a flat direction, this component can be considered as a free massless complex scalar so called AD field [12,13]. At the level of renormalizable terms, ”flat directions” are generic, but supersymmetry breaking and nonrenormalizable operators lift the ”flat directions” and sets the scale for their potential. During the inflational period the AD field develops non-zero vacuum expectation value and subsequently when the Hubble rate becomes of the order of the curvature of AD potential, the condensate starts to oscillate around its present minimum. Baryon asymmetry can be induced in such condensate only if there exists phase shift between real and imaginary parts of the AD field. Such shift and consequently B and CP violation is provided by A–term in the potential which parameterizes MSSM ”flat direction” [12,13]. The resulting sign and number density of baryon asymmetry depends on the magnitude of initial phase of AD field and on phase shift created by A-term at the relaxation period [12,14]. Therefore the de–Sitter fluctuations can generate antizillas in the baryon asymmetric Universe in the similar way to the spontaneous baryogenesis if the angular direction of AD field is characterized the mass that is much smaller that the Hubble constant $H$ during inflation. It takes place if there are no of order $H$ corrections to the A–term [17].

The early dynamics of AD field is quite complicated [10] owing to the non–trivial background energy density driving inflation in MSSM. Moreover AD potential can
get corrections from the vacuum energy that removes its minimum from the original one \([13,14,16,17]\). In general there are two types of inflation in MSSM, D–term or F–term inflation (see for review [19]), depending on the type of vacuum contributing the energy density during de–Sitter stage. In the case of D–term inflation AD fields and inflaton slow roll coherently \([13,14,16]\) (in the absence of order \(H^2\) corrections to the mass squared term of AD potential). It implies that the radius of effectively massless angular AD direction is determined by the immediate value of inflaton field. For the case of F–term inflation the AD scalar will get an order \(H^2\) negative mass squared term \([13,14,17]\) causing the minimum of AD potential. The AD field is closed to the minimum during the F–inflation stage \([16]\) and this minimum determines the radius of circle valley of effectively massless angular direction.

The conclusion from this explicit example based on the MSSM is following. For any complicated inflation dynamics of baryon charged field it is possible to simulate appropriate massless direction that behaves similar to the circle valley of NG potential. This fact makes the proposed issue for generation of antizillahs viable not only for spontaneous baryogenesis mechanism, but for the all mechanisms dealing with effectively massless angular directions during inflation \([20]\).

The paper is organized as to following. In section II we discuss the quantum behavior of nondominant \(U(1)\) symmetric scalar field at the inflation period. We estimate the amplitude and space scale of fluctuations of the phase for this field without PNG tilt. The size distribution of these fluctuations determines the size distribution of antizillahs. The section III contains calculations of baryon/antibaryon net excess production at the relaxation of phase when the tilt of Mexican hat potential becomes topical. We summarize our conclusions and discuss some problems of the considered scenarios in section IV.

II. PHASE DISTRIBUTION FOR NG FIELD AT THE INFLATION PERIOD

We start our consideration with the discussion of evolution of \(U(1)\) symmetric scalar field which coexists with inflaton at the inflation epoch. The quantum fluctuations of such field during the inflation stage cause the perturbations for the phase marking the Nambu–Goldstone vacuum. In our model this phase determines the sign and value of baryon excess, so the size distribution of domains containing the appropriate phase values, caused by that fluctuations, coincide with the size distribution of antizillahs.

Thus to estimate the number density of antimatter regions with sizes exceeding the critical survival size \(l_*\) in the baryogenesis model under consideration we have to deal with long – wave quantum fluctuations of the NG boson field at the inflation period. Various aspects of this question have been examined in the numerous papers \([23,24]\) in the connection with cosmology of invisible axion. Also the de–Sitter quantum fluctuations have been analyzed in the framework of AD baryogenesis \([13,14]\).

The effective potential of the complex field is taken in the usual form

\[
V(\chi) = -m^2\chi^*\chi + \lambda_\chi(\chi^*\chi)^2 + V_0,
\]

where the field \(\chi\) can be represented in the form

\[
\chi = \frac{f}{\sqrt{2}} \exp\left(\frac{i\alpha}{f}\right)
\]

The \(U(1)\) symmetry breaking implies that the radial component of the field \(\chi\) acquires a nonvanishing classical part, \(f = m_\chi/\sqrt{\lambda_\chi}\) and field \(\alpha\) in eq. (3) becomes a massless NG scalar field with a vanishing effective potential, \(V(\alpha) = 0\). In this case, \(\chi\) has the familiar Mexican–hat potential, and the degenerated vacua correspond to the circle of radius \(f\). Throughout present paper we deal with dimensionless angular field \(\theta = \alpha/f\).

We concern here the possibility to store appropriate phase value in the domain with the size exceeding the critical survival size. Such value of phase plays the role of starting point for clockwise movement, which is going to generate antibaryon excess when the tilt of potential breaking \(U(1)\) explicitly, will turn to be topical.

We assume that the Hubble constant varies slowly during inflation. Also we use well established behavior of quantum fluctuations on the de–Sitter background \([28]\). It implies that vacuum fluctuations of every scalar field grow exponentially in the inflating Universe. When the wavelength of a particular fluctuation becomes greater than \(H^{-1}\) the average amplitude of this fluctuation freezes out at some nonzero value because of the large friction term in the equation of motion of the scalar field, whereas its wavelength grows exponentially. In the other words such a frozen fluctuation is equivalent to the appearance of classical field that does not vanish after averaging over macroscopic space intervals. Because the vacuum must contain fluctuations of every wavelengths, inflation leads to the creation of more and more new regions containing the classical field of different amplitudes with scale greater than \(H^{-1}\). The averaged amplitude of such NG field fluctuations generated during each time interval \(H^{-1}\) is given by \([21]\)

\[
\delta\alpha = \frac{H}{2\pi}
\]

During such time interval the universe expands by a factor of \(e\). Since the NG field is massless during inflation period (the PNG tilt is vanish yet), one can see that the amplitude of each frozen fluctuation is not changed in time at all and the phases of each wave are random. Thus the quantum evolution of NG field looks like one-dimensional Brownian motion \([23,24]\) along the circle valley corresponding to the bottom of NG potential. This
that means that dispersion grows as $\sqrt{\langle (\delta\theta)^2 \rangle} = \frac{H}{2\pi} \sqrt{N}$, where $N$ is the number of e–folds. In the other words the phase $\theta$ makes quantum step with the scale $\frac{H}{2\pi}$ at each e–fold, and the total number of steps during some time interval $\Delta t$ is given by $N = H\Delta t$.

Let us consider the scale $k^{-1} = H_0^{-1} = 3000 h^{-1} Mpc$ which is the biggest cosmological scale of interest. We suppose that Universe is baryon asymmetric in this scale which leaves the horizon at definite e–fold $N = N_{\text{max}}$. On the other side this scale is the one entering the horizon now, namely $a_{\text{max}} H_{\text{max}} = a_0 H_0$ where the subscript 0 indicates the contemporary epoch. This implies that:

$$N_{\text{max}} = \ln \frac{a_{\text{end}} H_{\text{end}}}{a_0 H_0} - \ln \frac{H_{\text{end}}}{H_{\text{max}}}$$

the subscript $\text{end}$ denotes the epoch at the end of inflation. The slow-roll paradigm tells us that the last term of (5) is usually $\leq 1$. The first term depends on the evolution of scale factor $a$ between the end of slow-roll inflation and the present epoch. Assuming that inflation ends by short matter dominated period, which is followed by RD stage lasting until the present matter dominated era begins, one has [22]

$$N_{\text{max}} = 62 - \ln \frac{10^{16} GeV}{\sqrt{H_{\text{end}} M_p}} - \frac{1}{3} \ln \frac{\sqrt{H_{\text{end}} M_p}}{\rho_{\text{reh}}}$$

where $\rho_{\text{reh}}^{1/4}$ is the reheating temperature when the RD stage is established. With $H_{\text{end}} \approx 10^{13} GeV$ and instant reheating this gives $N_{\text{max}} \approx 62$, the largest possible value. However, if one has to invoke supersymmetry to prevent the flatness of the inflation potential, for example like as in the case of AD baryogenesis, the $\rho_{\text{reh}}^{1/4}$ should not exceed then $10^{16} GeV$ to avoid too many gravitino over-production [23], and one have $N_{\text{max}} = 58$, perhaps the biggest reasonable value. Through the paper we will use $N_{\text{max}} = 60$. The smallest cosmological scale of antizillah that is survived after annihilation is $k_c^{-1} = l_c \approx 8 h^2 kpc$ [5]. It is 9 order of magnitude smaller then $H_0^{-1}$, that corresponds to

$$N_c \approx N_{\text{max}} - 13 - 3 \ln h \approx 45$$

Thus the $l_c$ should left horizon at 45–folds before the end of inflation.

\[ \pi - \frac{15H}{2\pi f} \leq \theta_{60} \leq \frac{15H}{2\pi f} \]
Consider initially the case of exact equalities in expression (8) when the main part of antimatter is contained in the antizillahs of size \(l\). The number of domains containing the equal values of phase at the 45 e–folds before the end of inflation is given by the following expression

\[ n_{45} \approx (\frac{c^3}{2})^{15} \approx 10^{15}. \]  

(9)

Then the probability that every domain of size \(l\) would not be separated into \(e^3\) domains with size one order of magnitude less then \(l\) at the next e–fold is given by \(P \approx (1/2)e^3 \approx 10^{-6}\). Thus the number of domains serving as the prototypes for antizillahs of size \(l\) looks like

\[ \bar{n} = n_{45}P \approx 10^9 \]  

(10)

There are about \(10^{11}\) galaxies in the Universe. Thus, according to such simple consideration, we reveal that 1% of volume boxes corresponding to each galaxy contains the region of size \(l\), filled with antimatter of highest possible antibaryonic density if the \(\theta_60\) coincides with left side of inequality (8) or lowest one in the case if the opposite inequality is held.

We are able also to find the size distribution for antizillahs. For this purpose it is necessary to study the inhomogeneities of phase introduced by \(\delta\). It has been well established that for any given scale \(l = k^{-1}\) large scale component of the phase value \(\theta\) is distributed in accordance with Gauss’s law \[\begin{align*} P(\theta_0, l) \approx \frac{1}{\sqrt{2\pi\sigma_l^2}} e^{-\frac{(\theta_0 - \theta)^2}{2\sigma_l^2}}. \end{align*}\]. The quantity which will be especially interesting for us is the dispersion \(\sigma_l^2\) for quantum fluctuations of phase with moments from \(k = H^{-1}\) to \(k_{\min} = l_{\max}^{-1}\) (where the \(l_{\max}\) is the biggest cosmological scale that corresponds to 60 e–folds). This quantity can be expressed in the following manner

\[ \sigma_l^2 = \frac{H^2}{4\pi^2} \int_{k_{\min}}^k d\ln k = \frac{H^2}{4\pi^2} \ln \frac{l_{\max}}{l} = \frac{H^2}{4\pi^2f^2}(60 - N_l), \]  

(11)

where \(N_l\) is the number of e–folds which relates the biggest cosmological scale to the given scale \(l\). This means that the distribution of phase has the Gaussian form

\[ P(\theta, l) = \frac{1}{\sqrt{2\pi\sigma_l^2}} e^{-\frac{(\theta_0 - \theta)^2}{2\sigma_l^2}} \]  

(12)

Suppose that at e–fold \(N_l\) before the end of inflation the volume \(V(\bar{\theta}, N_l)\) has been filled with phase value \(\bar{\theta}\). Then at the e–fold \(N_l + \Delta t = N_l - \Delta N\) the volume filled with phase \(\bar{\theta}\) will follow iterative expression

\[ V(\bar{\theta}, N_l + \Delta t) = e^{3\bar{\theta}N_l} + (V_{\bar{U}}(N_l) - e^{3\bar{\theta}N_l})P(\bar{\theta}, N_l + \Delta t)h. \]  

(13)

Here the \(V_{\bar{U}}(N_l) \approx e^{3N_l}H^{-3}\) is the volume of the Universe at \(N_l\) e–fold. Expression (13) makes it possible to calculate the size distributions of domains filled with appropriate value of phase numerically. In order to illustrate quantitatively the number distribution of domains, we present here the numerical results for definite values of \(\theta_60\) and \(h = \frac{H}{\pi f}\). The table contains the results concerning to number of domains with average phase \(\bar{\theta}\) at e-fold number \(N\).

**TABLE I.** The sample of distribution of proto–antizillahs by sizes and numbers of e–folds at \(\theta_60 = \frac{\pi}{2}; \bar{\theta} = -0; h = 0.026\)

| \(N\)  | \(N_{\text{antizillahs}}\) | \(L_{\text{antizillah}}\) |
|-------|-----------------|-----------------|
| 59    | 0               | 1103Mpc         |
| 55    | 5.005 \times 10^{-14} | 37.7Mpc         |
| 54    | 7.91 \times 10^{-10}   | 13.9Mpc         |
| 52    | 1.291 \times 10^{-3}    | 1.9Mpc          |
| 51    | 0.499             | 630kpc          |
| 50    | 74.099            | 255kpc          |
| 49    | 8.966 \times 10^{3}  | 94kpc           |
| 48    | 8.012 \times 10^{7}  | 35kpc           |
| 47    | 5.672 \times 10^{7}  | 12kpc           |
| 46    | 3.345 \times 10^{9}  | 4.7kpc          |
| 45    | 1.705 \times 10^{11} | 1.7kpc          |
The fraction of the Universe filled with phase $\theta$ appears to be equal to $7.694 \times 10^{-9}$. Thus we see that the distribution of domains with size is very abrupt and should be peaked at smallest value of size. Adjusting the free parameters $\theta_{o0}$ and $h$ we are able to achieve the situation that volume box corresponding to each galaxy contains $(1 \div 10)$ regions with appropriate phase $\theta$. The sizes of such regions are larger or equal to critical surviving size. In spite of the sufficiently large total number of antizillas only the small part of our Universe will be occupied by antizillas (see the last line in the presented table).

The nontrivial question on the actual forms of astrophysical objects antizillas can have in the modern Universe needs spacial analysis, which, in general, strongly depends on the assumed form of the nonbaryonic dark matter, dominating in the period of galaxy formation. However, based on the early analysis $[6,37,38]$ the two extreme cases can be specified, when the evolution of antizillas is not strongly influenced by the dark matter content. In the first case, the antibaryon density within the antizillah is by an order of magnitude higher than the average baryon density, so that the over-density inside this region can exceed the dark matter density and rapid evolution of such an antizillah with the size exceeding the surviving scale can provide formation of compact antmatter stellar system (globular cluster (see for review [34])) which can survive in galaxy $[35,37,38]$. The other extreme case is antizillah with extremely low internal antibaryon density $\Omega_B < 10^{-5}$. Then the diffused antiverse is realized, when no compact antmatter objects are formed and antizillas evolve into low density antiproton-positron plasma regions in voids outside the galaxies $[4,35]$. 

III. SPONTANEOUS BARYOGENESIS MECHANISM

The following element of our scenario of inhomogeneous baryogenesis should contain a conversion of the phase $\theta$ into baryon/antibaryon excess. We consider the ansatz of spontaneous baryogenesis mechanism $[\overset{}{}1]$. The basic feature of this mechanism is that the sign of baryon charge created by relaxation of energy of PNG field critically depends on the direction that the phase is rotated on the bottom of Mexican heat potential. It provides us to convert the domains containing the appropriate phase value, caused by fluctuations, to the antizillas at the period when the NG potential gets the tilt.

The one of reasonable issue to the spontaneous baryogenesis $[\overset{}{}1]$ has been considered in the work $[\overset{}{}10]$. Let us briefly discuss it. It was assumed that in the early Universe a complex scalar field $\chi$ coexists with inflaton $\phi$ responsible for inflation. This field $\chi$ has non vanishing baryon number. The possible interaction of $\chi$ that violates lepton number can be described by following Lagrangian density (see e.g. $[\overset{}{}10]$)

$$L = -\partial_\mu \chi \partial^\mu \chi - V(\chi) + i \bar{Q} \gamma^\mu \partial_\mu Q + i \bar{L} \gamma^\mu \partial_\mu L - m_Q \bar{Q} Q - m_L \bar{L} L + (g \chi \bar{Q} L + h.c.) \quad (14)$$

The fields $Q$ and $L$ could represent heavy quark and lepton, coupled to the ordinary quark and lepton matter fields. Since fields $\chi$ and $Q$ possess baryon number while the field $L$ does not, the couplings in the $[\overset{}{}10]$ violate lepton number $[\overset{}{}10]$. The $U(1)$ symmetry that corresponds to baryon number is expressed by following transformations

$$\chi \rightarrow \exp(i\beta) \chi, \quad Q \rightarrow \exp(i\beta) Q, \quad L \rightarrow L \quad (15)$$

The effective Lagrangian density for $\theta$, $Q$ and $L$ eventually has the following form after symmetry breaking $[\overset{}{}10]$

$$L = -\frac{f^2}{2} \partial_\mu \theta \partial^\mu \theta + i \bar{Q} \gamma^\mu \partial_\mu Q + i \bar{L} \gamma^\mu \partial_\mu L - m_Q \bar{Q} Q - m_L \bar{L} L + \left( \frac{g}{\sqrt{2}} f \bar{Q} L + h.c. \right) + \partial_\mu \theta \bar{Q} \gamma^\mu Q \quad (16)$$

At the energy scale $\Lambda \ll f$, the symmetry $[\overset{}{}10]$ is explicitly broken and the Mexican–hat circle gets a little pseudo NG tilt described by the potential

$$V(\alpha) = \Lambda^4(1 - \cos \theta) \quad (17)$$

This potential, of high $2\Lambda^4$, has a unique minimum at $\theta = 0$. Of course, in the most cases, the potential $[\overset{}{}17]$ is the lowest–order approximation to a more complicated expressions emerged from particle physics models (see e.g. $[\overset{}{}10]$ and Refs. therein).

The important parameter for spontaneous baryogenesis is the curvature of $[\overset{}{}17]$ in the vicinity of its minimum, which is determined by the mass of PNG field

$$m_\theta^2 = \frac{\Lambda^4}{f^2} \quad (18)$$

As it was mentioned above the field $\chi$ is an additional field with nondominate energy density contribution to the Hubble constant deriving by de Sitter stage. Suppose that the tilt was formed during inflation. Then the order of magnitude estimation for fluctuations induced by large–scale inhomogeneity of oscillations of the field $\chi$ gives $\frac{\delta f}{f} = \frac{1}{3} \frac{\delta \rho}{\rho} \left( \frac{\Lambda}{T} \right)^4$. Thus, for $T = H/2\pi$ and reasonable value $\Lambda \simeq 10^{-5}H$ (see the end of this section) the thermal electromagnetic background fluctuations are within the observational limits.

Also we assume that the field $\theta$ behaves as massless NG field during inflation implying that the condition

$$m_\theta \ll H \quad (19)$$

is valid, where $H$ is the Hubble constant during the inflation. After the end of inflation condition $[\overset{}{}13]$ is violated and the oscillations of field $\theta$ around the minimum of potential $[\overset{}{}17]$ are started. The energy density
\[ \rho_0 \simeq \theta^2 m^2 f^2 \] of the PNG field which has been created by quantum fluctuations of \( \theta \) during the inflation converts to baryons and antibaryons \[ \text{[11]} \]. The sign of baryon charge depends on the initial value of phase from which the oscillations are started.

Let us estimate the number of baryons and antibaryons produced by classical oscillations of field \( \theta \) with an arbitrary initial phase \( \theta_i \). The appropriate expression for the density of produced baryons (antibaryons) \( n_B(n) \) is represented in \[ \text{[10]} \]

\[ n_B(n) = \frac{g^2}{\pi} m_Q + m_L \int_0^\infty \omega d\omega \left| \int_{-\infty}^{\infty} dt \chi(t) e^{\pm 2i\omega t} \right|^2, \tag{20} \]

that is valid if \( \chi(t \to -\infty) = \chi(t \to +\infty) = 0 \). General case can be obtained in the limits \( \chi(t \to -\infty) \neq 0; \chi(t \to +\infty) = 0 \) without loss of generality. After integration by part expression \[ \text{[21]} \] has the form

\[ N_B(B) = \frac{g^2}{4\pi^2} \Omega_\theta \int_{m_Q + m_L}^\infty d\omega \int_{-\infty}^{\infty} d\tau \chi(\tau) e^{\pm 2i\omega \tau}, \tag{21} \]

where the \( \Omega_\theta \) is the volume containing the phase value \( \theta_i \). Here the surface terms appear to be zero at \( t = \infty \) due to asymptotic of field \( \chi \) and at \( t = -\infty \) due to Feynman radiation conditions.

For our estimations it is enough to accept that the phase changes as

\[ \theta(t) \approx \theta_i (1 - m_\theta t) \tag{22} \]

during first oscillation. We also put \( m_Q = m_L = 0 \) that is reasonable for estimations. Substituting \[ \text{[22]} \] and \[ \text{[2]} \] into \[ \text{[21]} \] we come to

\[ N_B(B) \approx \frac{g^2 f^2 m_\theta}{8\pi^2} \Omega_\theta \theta_i^2 \int_{\mp \frac{\pi}{\omega}}^\infty d\omega \frac{\sin^2 \omega}{\omega^2}, \tag{23} \]

where the sign in the lower limit of integral corresponds to baryon or antibaryon net excess generation respectively. The reasonability of our approximation follows from comparison of \[ \text{[22]} \] at small \( \theta_i << 1 \)

\[ N_B - N_B^{\pm} = \frac{g^2 f^2 m_\theta}{8\pi^2} \Omega_\theta \theta_i^3 \tag{24} \]

with the result of \[ \text{[10]} \].

Using for spatially homogeneous field \( \chi = \frac{f}{\sqrt{2}} e^{i\theta} \) the expression for baryon charge

\[ Q = i(\chi^* d\chi/dt - d\chi^*/dt\chi) = -f d\theta/dt, \tag{25} \]

one can easily conclude that \( Q > 0 \) if \( \theta > 0 \) during classical movement of phase \( \theta \) to zero. Thus the antclockwise rotation gives rise to antibaryon excess while the clockwise rotation to the baryon excess one.

During reheating, the inflaton energy converts into the radiation. It is assumed that reheating takes place when the Mexican–hat potential is not sensitive to the PNG tilt yet. This implies that the total decay width of inflaton \( \Gamma_{\text{tot}} \) into light degrees of freedom exceeds the mass \( m_\phi \). In the other words the reheating is going on under the condition \[ \text{[19]} \]. The relaxation of \( \theta \) field starts when \( H \approx m_\theta \) and converts to the baryons or antibaryons. Baryonic charge is converted inside a comoving volume after reheating owing to very effective decay during the cosmological time. This means that the baryon–to–entropy ratio in \( n_B(B)/s = \text{Const} \) in the course of expansion. The entropy density after thermalization is given by

\[ s = \frac{2\pi^2}{45} g_\ast T^3 \tag{26} \]

where \( g_\ast \) is the total effective massless degrees of freedom. Here we concern with the temperature above the electroweak symmetry breaking scale. At this temperature all the degrees of freedom of the standard model are in equilibrium and \( g_\ast \) is at least equal to 106.75. The temperature is connected with expansion rate as follow

\[ T = \left( \frac{m_p H}{1.66 g_\ast^{1/2}} \right)^{1/2} = \left( \frac{m_p m_\phi}{g_\ast^{1/4}} \right)^{1/2} \tag{27} \]

The last part of expression \[ \text{[27]} \] takes into account that the relaxation starts under the condition \( H \approx m_\phi \). Using the formulas \[ \text{[23]}, \text{[20]}, \text{[27]} \] we are able to get the baryon/antibaryon asymmetry

\[ \frac{n_B(B)}{s} = \frac{45g^2}{16\pi^4 g_\ast^{1/2}} \left( \frac{f}{m_p} \right)^{3/2} \frac{f}{\Delta} Y(\theta_i) \tag{28} \]

The function \( Y(\theta) = \theta^2 \int_{-\theta/2}^{\theta/2} d\omega \frac{\sin^2 \omega}{\omega^2} \) takes into account the dependence of amplitude of baryon asymmetry and its sign on the initial phase value in the different space regions during inflation.

The expression \[ \text{[28]} \] allows us to get the observable baryon asymmetry of the Universe as a whole \( n_B/s \approx 3 \cdot 10^{-10} \). In the model under consideration we have supposed initially that \( f \geq H \simeq 10^{-6} m_p \). The natural value of coupling constant is \( g \leq 10^{-2} \). We are coming to observable baryon asymmetry at quite reasonable condition \( f/\Lambda \geq 10^3 \) (see e.g. \[ \text{[30]} \]).

IV. DISCUSSION

In this paper we have proposed a model for inhomogeneous baryogenesis on the base of the spontaneous baryogenesis mechanism \[ \text{[1]} \]. The model predicts the generation of antizillas with sizes exceeding the critical surviving size. The antibaryon number density relative to background baryon density in the resulting antizillas
and its number depends on the value of phase established at the beginning and on the parameters of PNG field potential. It is possible to have one or several antizillas the volume box corresponding to every galaxy depending on the parameter values. The observational consequences of existence of antizillas and the restrictions on their number and sizes have been analyzed in papers [37, 38].

Of course we may in general expect that some region with size exceeding \( l_c \) would contain antibaryon excess after the annihilation of small primordial domains and antidomains contained in this region is completed. However the probability to have such region is suppressed exponentially. Therefore to have observational acceptable number of antimatter regions [37] with the size exceeding the critical survival size, a superluminous cosmological expansion in the formation of primordial antimatter proto-domain seems necessary.

As we have mentioned, the additional problem for the most models of inhomogeneous baryogenesis invoking phase transitions at the inflation epoch is prediction of the large scale unwanted topological defects. Our scheme contains the premise for existence of domain walls too. Such walls are not formed when the only minimum of PNG potential exists, what corresponds in the considered model to the fluctuations around \( \theta = 0 \), when the North pole (\( \theta = \pi \)) is not crossed. But in the case, when such crossing takes place the multiple degeneracy of vacua appears (e.g. vacua with \( \theta = 0 \) and \( \theta = 2\pi \)). The equation of motion that corresponds to potential (17) admits kink-like, domain wall solution, which interpolates between two adjacent vacua. Thus, when the PNG tilt is significant, domain wall is formed along the closed surface (e.g. \( \theta = \pi \)). In the other words every antizilla with high relative antibaryon density will be encompassed by domain wall bag. The wall stress energy \( \Delta \approx 8 f \Lambda^2 \) leads to the oscillation of wall bag after the whole bag enters the cosmological horizon. During the oscillations the energy stored in the walls is released in the form of quanta of NG field and gravitational waves. As we are taken \( 0 < \theta_0 < \pi \), the wall’s bag will have the scale of the order of modern horizon, if the dispersion \( \sigma_{\text{max}} \) is large as \( \pi - \theta_0 \). Owing to very large oscillation period such big wall bag would presumably survive to the present time, which would be cosmological disaster. Thus the upper limit on the dispersion will be \( \sigma_{\text{dis}} < \pi \). From the other hand this condition should be valued if we want to have parameters of antizilla population that do not contradict to direct and indirect observational constraints. It means that we will have wall bags with the sizes less then cosmological horizon and that walls had to decay until present time. The mechanisms of their decay is a subject of separate paper, in which we also plan to obtain additional constraints on the model, which follow from the condition that walls do not dominate within the cosmological horizon before the bag decays. If the energy density of walls is sufficiently high to give local wall dominance in the border region before the bag enters the horizon, it is of interest to analyze the role of superluminous expansion in the border regions in the bag evolution (see e.g. [11]). The interesting question on the wall interaction with baryons in the course of wall contraction and decay will be also studied separately.

In general all baryogenesis models that are able to generate some amount of antimatter regions look like radical limit of models with local baryon number density fluctuations so called isocurvature fluctuations [32, 42]. It is known that the contribution of isocurvature fluctuations to the cosmic microwave background (CMB) anisotropy obeys to \( \Delta T / T = -\frac{\alpha}{4} \phi \delta B_1 \), where \( \delta B_1 \) is the amplitude of initial baryon number fluctuations and \( \Omega_B (\Omega_B) \) are the total (baryon) density (in units of critical density). As it follows from our numerical illustration (see [1] and expression (23) we must have quite large amplitude of initial baryon number fluctuations \( \delta B_1 \sim h/\theta_0 \sim 10^{-2} \) at the biggest cosmological scales, and consequently we would have large amplitude of isocurvature fluctuations at large scales that contradicts with COBE measurements [12].

To be keeping away of the problem of large-scale isocurvature fluctuations, we can, for example, prevent the fluctuations of phase at largest cosmological scales. The point is that to have antizilla with size exceeding few kpc we do not need to start phase fluctuations at the e-folds that correspond to the biggest cosmological scales. It is sufficiently to start fluctuations of phase from the moment, for instance, when the scale \( 8h^{-1}Mpc \) leaves Hubble horizon during inflation, namely after the 6.2 e-folds from the beginning of inflation. We took this scale, because it is known that at the scale less then \( 8h^{-1}Mpc \) we could be generated initial baryon number fluctuations at the level \( \delta B_1 \approx 10^{-2} \) without any contradictions with observations.

One of the natural way to prevent the phase fluctuations at the early inflation is to keep \( U(1) \) symmetry restored during first 7 e-folds. The mechanism that is able to restore symmetry during inflation has been consider in the works [24, 27, 28, 43]. According to that works we can introduce interaction between inflaton field \( \phi \) and field \( \chi \). The simple potential of such kind may be chosen as \( V(\phi, \chi) = \frac{1}{4} \lambda_4 \phi^4 + V(\chi) + \nu \phi^2 \chi^2 \chi, \) where \( \nu = m_\chi^2 / c M_p^2 \), and \( c \approx 1 \). The effective mass of the field \( \chi \) depends on \( \phi \) directly \( m_\chi^2 (\phi) = m_\chi^2 + \nu \phi^2 \). One considers here for simplicity the case \( \nu = m_\chi^2 / c M_p^2 \). This implies that the effective value of mass \( m_\chi^2 (\phi) \) during inflation is given by \( \nu (\phi^2 - c M_p^2) \) and is positive because of very large value of the inflation field. It means that our \( U(1) \) symmetry is restored during the period when the amplitude of the inflaton field exceeds \( \phi_c = \sqrt{c} M_p \), and the field \( \chi \) settles into the minimum of its symmetric potential. During this period there was no NG boson valley and phase fluctuations. After the moment that inflaton field turns to be less then \( \phi_c \) the symmetry breaking takes place and the NG potential has the radius \( f_{cif} = \sqrt{\nu (c M_p^2 - \phi^2) / \lambda_4} \) and fluctuations are started. To keep symmetry restored
during first 7 e–folds we should have $\phi_c = 4M_p$. After the moment of symmetry breaking it is allowed to start the fluctuations of phase with appropriate dispersion to create antizillahs, without any contradictions with observed CMB anisotropy. Of course to evaluate the distribution of antizillahs by sizes we have to take another parameters then we have used in our numerical example, but it does not change the main result of this paper.

Another story will take place if we would like to consider the AD baryogenesis as a basis for generation of antizillahs.

As it was discussed in the introduction the dynamics of the AD field is more complicated that in the case of spontaneous baryogenesis. Moreover it depends on the fact, D– or F– term inflation takes place. Also some details depend on the dimension ($d = 4,6...$) of non–renormalizable term lifting the flat direction [10,13], but it is enough for the brief discussion to circumscribe ourself with the minimal AD baryogenesis [13], where $d = 4$. Thus in the case of D– term inflation, when the coherent slow rolling of AD field and inflaton are already established, the maximal radius $f_{eff}^{AD(D)} \approx 10^{16}GeV$ of effectively massless angular direction can be obtained from the requirement that radial de Sitter fluctuations of AD field would not disturb significantly the spectral index of primordial adiabatic density perturbations $\delta$ measured by COBE. Thereby, it is possible to get dispersion of phase fluctuations at the level $h \approx 10^{-2}$ that is required for successful generation of antizillahs. The similar situation we could have in the case of $F$– term inflation [10,13] because the AD potential gets an order of $H^2$ negative mass squared term during inflation, which causes the effective minimum at $f_{eff}^{AD(F)} \approx C_F \sqrt{Hm_p} \approx 10^{16}GeV$ (the $C_F$ is a constant of order of one).

The isocurvature fluctuations in the model of inhomogeneous AD baryogenesis with dispersion of phase fluctuations appropriate for antizillahs generation should be already observed by COBE [14]. Moreover this fluctuations can get some amplification owing to possible transformation of fluctuations of AD condensate into the isocurvature fluctuations of neutralinos [13]. The exact solution of the problem of isocurvature fluctuations for the AD based antimatter generation is the subject of separate investigation. Here we can only present some speculations, how to avoid the large isocurvature fluctuations at large cosmological scales, which are based on the similar strategy that has been chosen in the case of spontaneous baryogenesis.

As it has been mentioned in the Introduction, to organize the angular effectively massless direction in the AD potential we should accept the condition of the absence of order $H$ correction to the A– term both during and after inflation [16]. This condition gets automatically satisfied in the case of D– term inflation [17], while it is not true if the inflation is F– term dominated (see for example [13]). According to this observation we can hope to find the such kind of trajectory of inflaton in field space that corresponds to the F– term dominated inflation in the beginning and then goes into D– term dominated regime. It implies that during the F– term dominated inflation the angular direction gets a mass of order $H$ and imaginary component of AD field is dumped and exponentially close to the minimum caused by this effective mass term. In such situation there are no de Sitter fluctuations of the phase. The fluctuations start only at the moment when the inflation goes to the D– term dominated regime and the angular direction turns to be effectively massless, because there is no correction of order $H$ to the A– term anymore. As we estimated before, to put the maximal scale of isocurvature fluctuations far below the modern cosmological horizon the transition from F– term to D– term inflation should take place 5–10 e–folds after the beginning of inflation. How to organize such transition is the subject of separate publication, but it seems that it could appear, for example, in the context of a realistic supergravity theory derivin from the weak coupled supestring [44], which is already beyond the MSSM. There is some possibility to generate the F– term from a Fayet–Iliopoulos D– term [10]. It could preserve the flatness of F– term direction during the first 5–10 e–folds of inflation causing the F– term domination firstly and subsequent trasformation of the vacuum energy into the D– term domination mode when it is allowed to begin phase fluctuations of AD field with dispersion appropriate for generation of antizillahs and without contradictions with COBE measurements.

We would like to notice in conclusion that the regions with antimatter in matter–dominated Universe could arise naturally in the variety of models. The main issue, that is needed, is a valley of potential. It is the山谷s that are responsible for formation of causally separated regions with different values of field which in its turn give rise to antimatter domains. Many extensions of standard model based on supersymmetry possess this property, what strongly extends the physical basis for cosmic antimatter searches.

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