Dualities for Loop Amplitudes of $\mathcal{N} = 6$ Chern-Simons Matter Theory

Wei-Ming Chen

NTU

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with Yu-tin Huang(UCLA)
Motivation

- Construct an simple example of odd-loop amplitudes in ABJM theory
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- Construct an simple example of odd-loop amplitudes in ABJM theory
- Get insight for Amplitude/Wilson Loop duality in ABJM theory

\[ \mathcal{N} = 4 \text{ SYM one-loop } 4\text{-point Amplitude:} \]

\[ \mathcal{A}_{\text{tree}}^{4} \frac{ig^{2}N}{8\pi^{2}} \left[ -\frac{(-s/\mu^{2})^{-\epsilon}}{\epsilon^{2}} - \frac{(-t/\mu^{2})^{-\epsilon}}{\epsilon^{2}} + \frac{1}{2} \log^{2} \left( \frac{-s}{-t} \right) + \text{const} + \mathcal{O}(\epsilon) \right] \]

\[ \mathcal{N} = 4 \text{ SYM one-loop } 4\text{-point Wilson loop:} \]

\[ \frac{g^{2}N}{8\pi^{2}} \left[ -\frac{(\tilde{\mu}^{2}x_{13}^{2})^{-\epsilon}}{\epsilon^{2}} - \frac{(\tilde{\mu}^{2}x_{24}^{2})^{-\epsilon}}{\epsilon^{2}} + \frac{1}{2} \log^{2} \left( \frac{x_{13}^{2}}{x_{24}^{2}} \right) + \text{const} + \mathcal{O}(\epsilon) \right] \]

\[ \mathcal{N} = 6 \text{ ABJM two-loop } 4\text{-point Wilson loop:} \]

\[ -\left( \frac{N}{K} \right)^{2} \left[ -\frac{(\mu^{2}x_{13}^{2})^{-2\epsilon}}{(2\epsilon)^{2}} - \frac{(\mu^{2}x_{24}^{2})^{-2\epsilon}}{(2\epsilon)^{2}} + \frac{1}{2} \log^{2} \left( \frac{x_{13}^{2}}{x_{24}^{2}} \right) + \text{const} + \mathcal{O}(\epsilon) \right] \]
Introduction

One-Loop and Two-Loop Amplitudes

Conclusion and Discussion
Outline

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Conclusion and Discussion
Field Content

$\mathcal{N} = 6$ Superconformal Chern-Simon Matter theory (ABJM) 0806.1218

Gauage Fields: $A^a_{\ b} \ A^\hat{a}_{\ \hat{b}}$

Matter Fields: $(\phi^I, \psi^I)^a_{\ \hat{a}} \ (\bar{\phi}^I, \bar{\psi}^I)^a_{\ \hat{a}}$

$U(N) \times U(N)$

$\exists a \ \exists \hat{a}$

$I \in SU(4)$

Dualities for Loop Amplitudes of $\mathcal{N} = 6$ Chern-Simons Matter Theory
On-shell Variables and Superfields

Three-Dimensional Kinematics,

\[ p^{ab} = (\sigma^\mu)^{ab} p_\mu = \lambda^a \lambda^b, \quad \text{if} \quad p^2 = 0 \]

\[ \mathcal{N} = 6 \text{ Superfields, } I \in SU(3) \]

\[ \Phi(\lambda, \eta) = \phi^4 + \eta^I \psi_I + \frac{1}{2} \epsilon_{IJK} \eta^I \eta^J \phi^K + \frac{1}{3!} \epsilon_{IJK} \eta^I \eta^J \eta^K \psi_4 \]

\[ \bar{\Phi}(\lambda, \eta) = \bar{\psi}^4 + \eta^I \bar{\phi}_I + \frac{1}{2} \epsilon_{IJK} \eta^I \eta^J \bar{\psi}_K + \frac{1}{3!} \epsilon_{IJK} \eta^I \eta^J \eta^K \bar{\phi}_4 \]

Wei-Ming Chen

Dualities for Loop Amplitudes of $\mathcal{N} = 6$ Chern-Simons Matter Theory
General n-point Tree Amplitude

- Tree level n-point amplitude, only \( n = \text{even} \) exists.

\[ \hat{A}_n = \hat{A}_n(\Phi(1)_{A_1} \bar{\Phi}(2)_{B_2} \ldots \bar{\Phi}(n)_{B_n}) \]

\[ = \sum_{\sigma \in (S_{n/2} \times S_{n/2})/C_n} A_n(\sigma_1, \ldots \sigma_n) \delta_{B_{\sigma_2}}^{A_{\sigma_1}} \delta_{A_{\sigma_3}}^{B_{\sigma_2}} \ldots \delta_{A_{\sigma_1}}^{B_{\sigma_n}} \]

- Color ordered amplitude \( A_n(\lambda_i, \eta_i) \),

T. Bargheer, F. Loebbert, C. Meneghelli, 1003.6120
General n-point Tree Amplitude

- R-symmetry invariance $\rightarrow \eta^2$
- momentum and momentum supercharge conservation $\rightarrow A_n = \delta^3(P)\delta^3(Q^\alpha I)\delta^3(Q^I_\alpha)f(\lambda), \quad Q^\alpha I \equiv \sum_i \lambda_i^\alpha \eta_i^I$
- Lorentz invariance and Dilatation invariance $\rightarrow f$ with $\lambda$ weight $-4$
- Consistent with field theory computation,

$$A_4 = i\frac{\delta^3(P)\delta^3(Q^\alpha I)\delta^3(Q^I_\alpha)}{\langle 41 \rangle \langle 12 \rangle}$$
Dual Conformal Symmetry

J.M. Drummond, J. Henn, V.A. Smirnov, E. Sokatchev, hep-th/0607160

First observed from one-loop to three loop four-point gluon scattering amplitudes

\[ p_i = x_i - x_{i+1} \]

\[ I[ x_i^\mu ] = \frac{x_i^\mu}{x_i^2}, \quad K = I P I \]

\[ I[ A_4 ] = x_1^2 x_2^2 x_3^2 x_4^2 A_4, \quad \text{integrand level} \]
A Example of Dual Conformal Symmetry

\[ \mathcal{A}_{4}^{1-\text{Loop}} = \mathcal{A}_{4}^{\text{Tree}} L, \]
\[ L = \int \frac{d^4 l}{(2\pi)^4} \frac{l_1^2 (l_1 + p_1)^2 (l_1 + p_1 + p_2)^2 (l_1 - p_4)^2}{2 \pi^4} \]
\[ \rightarrow \int \frac{d^4 x_5}{(2\pi)^4} \frac{x_{13} x_{24}^2}{x_{51}^2 x_{52}^2 x_{53}^2 x_{54}^2} \]
A Example of Dual Conformal Symmetry

- Dual conformal covariant: \( I[A_4^{\text{Tree}}] = x_1^2 x_2^2 x_3^2 x_4^2 A_4^{\text{Tree}} \)
- Dual conformal invariant: \( I[L] = L \)
Dual Superconformal Symmetry

Dongmin Gang, Yu-tin Huang, Eunkyung Koh, Sangmin Lee, Arthur E. Lipstein 1012.5032

Dual superspace is parametrized by $x, \theta, y$:

$$x^{\alpha\beta}_{i,i+1} = x^{\alpha\beta}_{i} - x^{\alpha\beta}_{i+1} = p_{i}^{\alpha\beta} = \lambda_{i}^{\alpha}\lambda_{i}^{\beta}$$

$$\theta^{I\alpha}_{i,i+1} = \theta^{I\alpha}_{i} - \theta^{I\alpha}_{i+1} = q_{i}^{I\alpha} = \lambda_{i}^{\alpha}\eta_{i}^{I}$$

$$y^{IJ}_{i,i+1} = y^{IJ}_{i} - y^{IJ}_{i+1} = r_{i}^{IJ} = \eta_{i}^{I}\eta_{i}^{J}$$

Amplitude transforms covariantly under dual superconformal symmetry:

$$I[\mathcal{A}_{n}] = \prod_{i=1}^{n} \sqrt{x_{i}^{2}\mathcal{A}_{n}} = \prod_{i=1}^{n} \sqrt{x_{i}^{2}\mathcal{A}_{n}^{Tree}} L \quad \text{(only valid in integrand level)}$$

$$I[L] = L$$
Generalized Unitarity Cut

- Assume tree-amplitudes are known
- Unitarity cut, only $L + 1$ propagators at most can be cut.

\[ S = 1 + iT, \quad S^\dagger S = 1 \implies i(T^\dagger - T) = T^\dagger T \]

\[ A|_{\text{cut}} = A_1 A_2 \]

- Generalized unitarity Cut, cut number larger than $L + 1$ is possible.

\[ A|_{\text{cut}} = A_1 A_2 \cdots A_n \]
Procedure to Construct Amplitudes

- Guess all possible dual superconformal invariant integrands
- Cut all possible dual superconformal invariant integrands
- Match cut-integrands with product of tree amplitudes
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One-Loop Amplitude

- Embed 3D into 5D, \(-T^2 - U^2 + V^2 + W^2 + Y^2 = 0\)
- Degree of freedom of 3D, 5 \(- 1\) (light cone condition) \(- 1\) (identification of rescaling \(T \rightarrow \rho T\)) = 3

\[
I_{4}^{1-{\text{loop}}} = \int \mathcal{D}^3 X_5 \frac{4 \epsilon(5, 1, 2, 3, 4)}{X_{51}^2 X_{52}^2 X_{53}^2 X_{54}^2} 
= \int \frac{d^3 x_5}{(2\pi)^3} \frac{2 x_{51}^2 \epsilon_{\mu\nu\rho} x_{21}^\mu x_{31}^\nu x_{41}^\rho + 2 x_{31}^2 \epsilon_{\mu\nu\rho} x_{51}^\mu x_{21}^\nu x_{41}^\rho}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2}
\]
One-Loop Amplitude

\[ i A_4^{Tree}(1, 2, 3, 4) \left|_{\text{cut}} \right. = A_4^{Tree}(1, 2, l_2, -l_1) A_4^{Tree}(-l_2, 3, 4, l_1) \]

\[ I^{1-\text{loop}} = 0 \]
Two-Loop Amplitude

Possible scalar integrals:

One more possible integral:

\[ I_{0s} = \int D^3 x_5 D^3 x_6 \frac{16 \epsilon (5, 1, 2, 3, 4) \epsilon (6, 1, 2, 3, 4)}{X_{51}^2 X_{53}^2 X_{54}^2 X_{56}^2 X_{61}^2 X_{63}^2 X_{62}^2 X_{42}^2} \]

These integrals are not linearly independent:

\[ 2 I_{0s} = I_{1s} - I_{2s} + I_{3s} + I_{3t} + I_{4s}. \]
Two-Loop Amplitude

Match cut

\[
\mathcal{A}_4^{\text{Tree}} \sum_i (c_i l_{ls} + c'_i l_{lt}) \bigg|_{\text{cut}} = \begin{cases} 
\mathcal{A}_4^{\text{Tree}}(1, 2, -l_3, l_2) \mathcal{A}_4^{\text{Tree}}(-l_2, -l_3, l_4, -l_1) \mathcal{A}_4^{\text{Tree}}(-l_1, l_4, 3, 4) \\
0 \end{cases}
\]

\[\Rightarrow \mathcal{A}^{2-\text{Loop}} = \left( \frac{N}{K} \right)^2 \mathcal{A}_4^{\text{Tree}}[-l_0s + l_1s + (s \leftrightarrow t)]\]
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Amplitude/Wilson Loop Duality

- $\mathcal{N} = 6$ ABJM two-loop 4-point Amplitude:

\[-(\frac{N}{k})^2 \mathcal{A}_4^{\text{tree}} \left[ \frac{(s/\mu^2)^{-2\epsilon}}{(2\epsilon)^2} - \frac{(t/\mu^2)^{-2\epsilon}}{(2\epsilon)^2} + \frac{1}{2} \log^2 \left( \frac{-s}{-t} \right) + \text{const} + \mathcal{O}(\epsilon) \right] \]
Amplitude/Wilson Loop Duality

\( \mathcal{N} = 6 \) ABJM two-loop 4-point Amplitude:
\[
- \left(\frac{N}{K}\right)^2 A^\text{tree}_4 \left[ -\frac{(-s/\mu^2)^{-2\epsilon}}{(2\epsilon)^2} - \frac{(-t/\mu^2)^{-2\epsilon}}{(2\epsilon)^2} + \frac{1}{2} \log^2 \left( \frac{-s}{-t} \right) + \text{const} + O(\epsilon) \right]
\]

\( \mathcal{N} = 6 \) ABJM two-loop 4-point Wilson loop:
\[
- \left(\frac{N}{K}\right)^2 \left[ -\frac{(-\mu^2 x_{13}^2)^{-2\epsilon}}{(2\epsilon)^2} - \frac{(-\mu^2 x_{24}^2)^{-2\epsilon}}{(2\epsilon)^2} + \frac{1}{2} \log^2 \left( \frac{x_{13}^2}{x_{24}^2} \right) + \text{const} + O(\epsilon) \right]
\]

\( \mathcal{N} = 4 \) SYM one-loop 4-point Amplitude:
\[
A^\text{tree}_4 \frac{ig^2N}{8\pi^2} \left[ -\frac{(-s/\mu^2)^{-\epsilon}}{\epsilon^2} - \frac{(-t/\mu^2)^{-\epsilon}}{\epsilon^2} + \frac{1}{2} \log^2 \left( \frac{-s}{-t} \right) + \text{const} + O(\epsilon) \right]
\]

\( \mathcal{N} = 4 \) SYM one-loop 4-point Wilson loop:
\[
\frac{g^2N}{8\pi^2} \left[ -\frac{(-\tilde{\mu}^2 x_{13}^2)^{-\epsilon}}{\epsilon^2} - \frac{(-\tilde{\mu}^2 x_{24}^2)^{-\epsilon}}{\epsilon^2} + \frac{1}{2} \log^2 \left( \frac{x_{13}^2}{x_{24}^2} \right) + \text{const} + O(\epsilon) \right]
\]
Amplitude/Wilson Loop Duality in 3D
String Picture for Amplitude/Wilson Loop Duality

\[ \mathcal{N} = 4 \text{SYM} \]
String Picture for Amplitude/Wilson Loop Duality

\[ \mathcal{N} = 6\text{ABJM} \]
Conclusion

- One loop four-point integrand exists and can be integrated to zero.
- Two loop four-point amplitude/Wilson loop duality:

\[ \mathcal{N} = 4 \text{ SYM} \quad A_{4}^{1-\text{Loop}} \quad \langle W_{4} \rangle^{1-\text{Loop}} \]

\[ \mathcal{N} = 6 \text{ ABJM} \quad A_{4}^{2-\text{Loop}} \quad \langle W_{4} \rangle^{2-\text{Loop}} \]
Thank You