Open Strings in a B-field Background as Electric Dipoles

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Abstract

Studying dynamics of open strings attached to a D2-brane in a NS two form field background, we find that these open strings act as dipoles of $U(1)$ gauge field of the brane. This provides an string theoretic description of the flux modifications needed for the DBI action on noncommutative torus.

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Introduction

After the interesting work of A. Connes, M. Douglas and A. Schwarz (CDS) [1], noncommutative geometry and especially the noncommutative torus is shown to play a crucial role in the M-theory compactifications [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. In [1], CDS conjectured that the Super Yang-Mills (SYM) formulated on a noncommutative torus describes the discrete light cone quantization of M-theory when we have a non-zero three form background of 11 dimensional supergravity. In that paper, CDS used the SYM action with an additional topological term proportional to the field strength. This topological term in the case of noncommutative compact spaces leads to some modification in BPS spectrum, necessitated by the U-duality.

Since the eleven dimensional three-form is related to the NSNS two form field of string theory, the CDS conjecture means that the low energy dynamics of D0-branes or more generally any D-brane, in a B field background is described by a gauge theory on noncommutative torus, where \( \theta \), the deformation parameter of the torus is determined by the B field background [2, 3, 4, 5, 6, 7, 8, 12, 14, 15]. It was argued in [8] that, there should be an additional term in the Matrix model action proportional to \( \dot{X} \), the time derivative of the M(atrix)-valued collective coordinates of D0-branes, \( X \). It is easy to show that this term can be obtained from the topological term discussed earlier by CDS, but as they mentioned, they had no physical reasoning for adding this term. Also there were argued that the coefficient of this term is proportional to the winding number of longitudinal membranes.

Generalizing the ideas of [2], it was shown in [14] that the scattering amplitudes for open strings attached to a D2-brane in the B field background in the low energy limit, is properly described by the SYM defined on a noncommutative two torus with deformation parameter identified with the background B field. It was also discussed in [14] that higher \( \alpha' \) corrections to the open string scattering amplitude, are given by the DBI action defined on noncommutative torus.

In this paper, studying the open string dynamics, in a non-zero electric field of the D2-brane, we show that in the case of non-zero B-field background these open strings act as an electric dipole. Hence in a non-vanishing electric field one should modify the SYM action, the action describing the open strings dynamics at low energies, by adding a term proportional to dipole moment of these open strings, which shows the interaction of these dipoles with electric field. We will argue that, in the noncommutative gauge theory, as argued by Hofman and Verlinde [13], generalizing the usual \( Tr \) of non-Abelian gauge theory to \( Tr_\theta \), defined by CDS, the \( SL(3,Z) \times SL(2,Z) \) symmetry still remains and it automatically reproduces the
term, we find it as dipole interactions in string theory. However if we do the quantization procedure correctly [16], unlike the works of [1,8], there is no need to add an additional term proportional to $\dot{X}$ to the M(atrix)-model action compactified on the noncommutative torus. In this way we provide a more intuitive description of using noncommutative geometry methods.

Elaborating on this point we will show that, considering the dipole-background electric interactions, assures the full U-duality group, from the string theoretic calculations, without using the noncommutative geometry. From the M-theory point of view, the dipole moment which are conserved quantities give the winding of longitudinal membrane [1,8]. This winding number is a new degree of freedom should be added to the usual Matrix model, when we have a non-vanishing three form field of the 11 dimensional supergravity.

Open String Dipoles

It was shown and discussed by Witten [17], that the massless states of open strings attached to a $D_p$-brane, form a vector multiplet of a $N=1 D=10 U(1)$ gauge theory dimensionally reduced to $(p+1)$\footnote{Unless it is mentioned explicitly by gauge theory here we mean SYM or DBI where either of which are gauge invariant.}. There, it was also discussed that the end point of any open string attached to a D-brane carries the unit charge of that $U(1)$ gauge theory. This point was explicitly worked out by studying the BPS excitation of the $U(1)$ SYM or DBI [18]. Let us consider an open string having its both ends on the same brane. Since these open strings are oriented, ends of them, from the gauge theory point of view, look like plus-minus unit charges. So at the first sight, it seems that they form an electric dipole and not a gauge particle of that $U(1)$ theory. This problem is easily resolved if we look at the open strings more carefully. The bosonic part of the mode expansion of such vector states are:

$$X^\mu = x^\mu + p^\mu \tau + \sum_{n \neq 0} a_n e^{-i n\tau} \cos n\sigma \quad \mu = 0, \ldots, p.$$ (1)

Since the vector state is described by $b_{-1/2}^\mu |VAC>$\footnote{$b_{-1/2}^\mu$ is the NS sector creation operator.}, $X^\mu(0, \tau)$ and $X^\mu(\pi, \tau)$ for such states have the same value. Hence the two end points, or the plus-minus charges, are really on top of each other and there is no dipole moment, and these open strings simply give the gauge multiplet of $U(1)$ gauge theory. In order to discuss the same issue in the case with non-zero B-field, first one should build the mode expansion of corresponding open strings. Here after for simplicity, we only consider a D2-brane with non-zero B-field on it, however
our discussion can easily be generalized to the case of other D-branes. The open strings attached to such a D2-brane are described by [18]

\[
\begin{align*}
\partial_\sigma X^0 &= 0 \\
\partial_\sigma X^i + B^i_j \partial_\tau X^j &= 0 & i, j &= 1, 2 \\
\partial_\tau X^a &= 0 & a &= 3, ..., 9.
\end{align*}
\] (2)

Hence the mode expansion of these open strings are [14]

\[
\begin{align*}
X^0 &= x^0 + p^0 \tau + \sum_{n \neq 0} a^0_n e^{-i n \pi} \cos n \sigma \\
X^i &= x^i + (p^i \tau - B^i_j p^j \sigma) + \sum_{n \neq 0} \frac{e^{-i n \pi}}{n} (ia^i_n \cos n \sigma + B^i_j a^j_n \sin n \sigma) & i, j &= 1, 2 \\
X^a &= x^a + \sum_{n \neq 0} a^a_n e^{-i n \pi} \sin n \sigma,
\end{align*}
\] (3)

where \(x^i\) show an arbitrary point on D2-brane.

Let us again study these open strings from the gauge theory point of view. In this case, unlike the previous case \((B = 0)\) the plus-minus charges locating at the end points of open strings are not coincident any more, hence these open strings look like electric dipoles with the moment \(P^i:\)

\[
P^i = X^i(0, \tau) - X^i(\pi, \tau) = \pi B^i_j p^j.
\] (4)

As we see the dipole moment is proportional to \(B\) field and open string momenta, and is always perpendicular to the momentum vector, \(p^i\).

As it is discussed by many people, the gauge theory governing the D-brane or open strings dynamics in a non-zero B field background, is not a usual DBI action but, it is the DBI defined on a noncommutative torus. In the noncommutative case again we can talk about the electric charges of the \(U(1)\) theory defined by the zero momentum sector of open strings. In the \(B = 0\) case (or more generally any rational B), since our theory enjoys the \((2+1)\) Lorentz invariance, we can always make these dipoles to be zero. In contrast, for \(B \neq 0\) our theory suffers from the lack of Lorentz symmetry \([1, 12]\) then, especially when the brane is compact, these dipoles can not be removed and they are the intuitive origin of the noncommutativity of torus and the Moyal bracket structure. Form the gauge theory point of view, these dipoles are really the gauge particles of the noncommutative gauge theory, where they interact through the Moyal bracket terms of the action. And one can also understand the noncommutative SYM, either by the open string dynamics [14], or by these dipole-dipole interactions.
Here we briefly discuss some of the issues of the dipole description of noncommutative gauge theory in (2+1) dimensions. A more extensive work will appear [19].

i) Dipole moment conservation; Since the dipole moment is proportional to the open string momentum, the momentum conservation in each vertex will immediately result in the dipole moment conservation.

ii) Dipoles always move so that their dipole moment are normal to their momentum.

iii) Because of Moyal bracket structure, parallel dipoles are non-interacting.

iv) The high energy dipole-dipole scattering is suppressed by the Moyal bracket structure.

Interaction of Dipoles with Electric Background

Besides the dipole-dipole interactions, which are described by the Moyal gauge theory, i.e. the gauge theory defined on a noncommutative torus, there are dipole-electric background interactions, which are not present in the noncommutative gauge theory and we should add them. In other words, to have a theory fully invariant under U-duality, the $Tr \rightarrow Tr_{ \theta}$ substitution should be done not only for free fields, but also for currents and fluxes (or BPS charges) [13].

To work out the form of the dipole-background interactions, first one should discuss the explicit form of dipole moment. So let us consider a (D2-D0)-brane system, winding around the two torus defined by

$$\tau = \frac{R_2}{R_1} e^{i\alpha}, \quad \rho = iR_1R_2 \sin \alpha + B. \quad (5)$$

Along the calculations of [12], mode expansion of open strings attached to such a brane system is given by eq. (3), where $p^i$ in the usual complex notation of the torus, is

$$p = \frac{(r_1 + q_1 \tau)(n + m \rho)}{|n + m \rho|^2} \sqrt{\frac{\rho_2}{\tau_2}}. \quad (6)$$

with $r_1, q_1$ being two arbitrary integers, $(n, m)$ two integers that their greatest common divisor shows the winding number of D2-brane around the torus and their ratio, $\frac{m}{n}$ gives the density of D0-branes distributed on the D2-brane. Hence according to eq. (4):

$$P = \pi i B \frac{(r_1 + q_1 \tau)(n + m \rho)}{|n + m \rho|^2} \sqrt{\frac{\rho_2}{\tau_2}}. \quad (7)$$

To include the dipole-background interactions, we should add the proper term to the gauge theory action. Since we finally want to find the BPS spectrum of the the (D2-D0)-brane system, we use the DBI action:

$$S = S_0 + \int d^3 x P^i F_{0i} = \int L_{mod}.dt. \quad (8)$$
where $S_0$ is the usual DBI action:

$$S_0 = -\frac{1}{g_s} \int d^3x \sqrt{\text{det}(g + F)} = \int L_0 dt.$$  \hfill (9)

Here we have put $l_s = 1$ but at last will reintroduce it. There could also be a WZ term, $\int C \wedge F$, since it does not alter our arguments much, we will not consider it at this stage, but will come back to it in our final results. To check the BPS spectrum, we build the Hamiltonian:

$$H = \mathcal{F}_{0i} \frac{\partial L_{\text{mod.}}}{\partial \mathcal{F}_{0i}} - L_{\text{mod.}}.$$  \hfill (10)

with

$$\Pi_0 = \frac{\partial L_0}{\partial \mathcal{F}_{0i}} = \frac{\partial L_{\text{mod.}}}{\partial \mathcal{F}_{0i}} - \mathcal{P}.$$  \hfill (11)

The conjugate momenta of $\mathcal{F}_{0i}$, $\frac{\partial L_{\text{mod.}}}{\partial \mathcal{F}_{0i}}$, should be quantized as dual torus vector. Hence comparing eq. (11) with the results of [12], everything is the same except for the shift by $\mathcal{P}$ in the conjugate momenta:

$$r_2, q_2 \rightarrow r_2 + r_1 \theta, q_2 + q_1 \theta.$$  \hfill (12)

Finally putting all of these together, we find

$$H = \frac{1}{l_s g_s} \frac{|n + m \rho|}{\sqrt{\rho^2}} \left( 1 + g_s^2 \frac{\frac{1}{\tau_2}|(r_2 + r_1 \rho) + \tau(q_2 + q_1 \rho)|^2}{|n + m \rho|^2} \right)^{1/2}.$$  \hfill (13)

If we had also considered the WZ term along the lines of [13], would end up with:

$$H = \frac{1}{l_s g_s} \frac{|n + m \rho|}{\sqrt{\rho^2}} \left( 1 + g_s^2 \frac{\frac{1}{\tau_2}|(r_2 + r_1 \rho) + \tau(q_2 + q_1 \rho)|^2}{|n + m \rho|^2} + (C_2 + C_1 \tau)^2 \right)^{1/2}.$$  

This spectrum as discussed in [13] has both $\text{SL}(3,\mathbb{Z})$ and $\text{SL}(2,\mathbb{Z})_N$ symmetries. In order to compare our results with those of Matrix model [1,8], one should take the $l_s, g_s \rightarrow 0$ limit, which again will give the results of [12], with:

$$r_2, q_2 \rightarrow r_2 - r_1 \theta, q_2 - q_1 \theta.$$  \hfill (14)

**M-theory Interpretations**

As we see, considering the missing dipole-background interactions, modifies conjugate momentum of the electric fields living on the two torus by adding a term proportional to open strings momentum, eq. (4). In the M-theory side, the open strings momenta and m, number
of the D0-branes, are conserved charges related to $C_{\mu ij}$ components of 11 dimensional three-form, where $\mu = 0, \ldots, 7$ and $i, j$ denoting the directions of $T^3$ M-theory compactified on. And $r_2, q_2, n$ (roughly speaking, the number of D2-branes), are related to $g_{\mu i}$ components of metric. More precisely $(r_1, q_1, m)$ and $(r_2, q_2, n)$ form two $Sl(3, Z)$ vectors, and $(r_2, r_1), (q_2, q_1), (n, m)$ act as $Sl(2, Z)_N$ doublets. In terms of M2-branes these conserved charges are the KK momenta and winding modes of M2-brane compactified on $T^3$. Generalizing the ideas of [21], to the non-zero $C_{ijk}$, we find that the momenta should be modified by windings [1], similar to what we have found here from the string theory arguments.

Concluding Remarks

In this work completing the results of [12,14], we provide a string theoretic justification of the $Tr \rightarrow Tr_\theta$ substitution, when we have a NSNS two-form background field. It was argued by Hofman and Verlinde [13] that a proper Matrix model treatment of M-theory with the three-form background is given by Born-Infeld on noncommutative torus. It was also discussed [13] that, going from $T^2$ to $T^2_\theta$, one should modify the related fluxes too. Here we studied the open strings attached to D2-brane with a non-zero B field more carefully and in this way provide an string theoretic description of the flux modification, and observed that these open strings look like electric dipoles of $U(1)$ gauge theory. Considering these dipoles sheds light on the noncommutative geometry methods and shows an explicit way to study the renomalizability of noncommutative gauge theories [20].

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