Comment on A.-L. Barabasi, Nature 435 207-211 (2005)

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The purpose of this communication is twofold. First, it clarifies the origin of the power law obtained in the computer simulations presented in A.-L. Barabasi, Nature 435 207-211 (2005) as well as presenting a statistically more sound analysis of the experimental email data used in that paper and in [4]. The second purpose is to clarify the origin of the suggestion of power law probability distribution for the response times of humans presented with a piece of new information either as a news event or through the reception of an email.

In a letter in Nature published May 2005 [1], A.-L. Barabasi presents results from an email experiment[2], which suggest that the time lag $\tau$ between the reception of an email and the following reply for an individual follows a power law probability distribution

$$P(\tau) \sim \tau^{-1}, \quad (1)$$

when averaged over time. Only a single example(user) is shown and the claimed scaling region is extremely limited.

At the conference “Frontier Science 2003 - A Non-Linear World: The Real World” in Pavia, Italy, Sept. 2003, where A.-L. Barabasi also was present and more than one year prior to the submission of [1] to Nature, I showed with high statistical significance, i.e., much higher than that of [1], using the same data set of [2], that when averaged over both time and population the time lag $\tau$ follows a power law probability distribution

$$P(\tau) \sim (\tau + c)^{-1}, \quad (2)$$

where the reason behind the constant $c$ is simply that the computer time lag measured is not the true response time. The prime reason for the “shift” $c$ is that most people do not download and/or read new e-mail messages instantaneously. Not only is the statistical significance substantially higher compared with [1] because of the population averaging, but also because the cumulative distribution of measurements were used thus filtering out much of the high-frequency (Integration effectively result in a low-pass filtering). The deviation between the data and the logarithmic fit for long times can be attributed to limitations in peoples ability to answer a large number of email, i.e., a finite size effect due to a limited amount of time and

\[1\] A quite crude approximation is made here as the time-lag $c$ is taken to be the same for all individuals. This explains the small deviation between the data and the fit for the smallest times.
memory, as well as under-sampling for large time lags due to the limited population considered. One should note that an alternative explanation for the response times of a single individual has been proposed using a log-normal distribution [3]. My main objection to their suggestion, is that I do not know how to interpret the underlying Gaussian distributed log(variable).

These findings was later published in the conference proceedings [4] of Feb. 2004, see fig. 1 for the data analysis. Here it was compared with previous results from another Internet experiment [5], where a portion of the “internaut population”’s response to a forty minute interview with the author on the origin of stock market crashes called “The World (Not) According to GARCH” was published on Friday the 26th of May 2000 on a “radio website” called “Wall Street Uncut” [7]. In this interview, as well as on the website, the URL to the author’s papers [6] was announced making it clear that work on stock market crashes in general and the recent Nasdaq crash in particular could be found on the posted URL. The results was that the response to the interview and URL publication, measured as the number of downloads of papers from the authors homepage as a function of time (days) from the appearance of the interview, also followed a power law probability distribution

\[ P(\tau) \sim \tau^{-1} + k, \]

see fig. 2 for the data analysis. The constant \( k \) is simply a “background” due to downloads from people unaware of the interview as well as “search robots”. Another experiment of the same type can be found in [9], where the sampled time interval is 100 days. Here the exponent was found to be \( \sim -0.6 \) and not \(-1\) suggesting that multiple communication channels might influence the value of the exponent.

Even though the two experiments are not identical, there are a number of similarities which establish a correspondence between the two. At any time \( t \) after the appearance of the interview on [7], the exposed population consists of two groups, namely those who have not downloaded a paper from [6] and those who have. Similarly with respect to the email experiment, at any time \( t \) the population considered consists of two groups, namely those who have an email to answer and those who have not. In both cases, the time lag \( \tau = t - t_0 \), where \( t_0 \) is the time of the appearance of the interview/reception of an email to answer, is the governing variable. The transition from the first state (no action yet) to the second state (have downloaded/answered email) demands the crossing of some threshold specific to each individual. We thus imagine that the announcement of the URL/the reception of e-mails plays the role a “field” to which the exposed population is subjected and study the relaxation process by monitoring the number of downloads/the number of replies as a function of time. Hence, we may view the process of downloading/replying as a diffusion process in a random potential, where the act of downloading/replying is similar to that of barrier-crossings.

In fact, the queuing model proposed by A.-L. Barabasi in [1] is not much more than a reformulation of the Trap-model proposed by myself and co-author in [9] and subsequent papers [5, 4] as an analog to the experiments. Both models use the ad hoc assumption of a power law “trapping time” distribution \( p(\tau) \propto \tau^\gamma \) and introducing a “priority parameter” [1] does not add much new. With respect to his computer simulation, it is well-known that a uniformly random sampling of an exponentially distributed random variable will trivially give a power law with exponent of -1 [10], so it is not obvious what his computer simulations are suppose to prove.
In the conference proceeding [4], I speculate over the origin of such power law response times distributions and specifically whether it is a consequence of the averaging over a population or whether it over time holds on the individual level as well. I list a number of purely qualitative arguments suggesting that this might be the case, but conclude that “it seems a priori a quite formidable task to empirically verify whether these considerations are valid or not” with sufficient statistical significance. In [1], this problem have not been solved at all.

In conclusion, the only difference between the experimental results suggesting a power law distribution of response times presented first in [4] and approximately a year later in [1], is that the former employs an ensemble averaging whereas the latter does not. Considering the quite limited scaling region of figure 2b in [1] as well as the scatter of the points, the author’s conclusion that eq.(1) holds for a single individual is not obvious, but certainly interesting. Compared with this, the scaling region in fig. 1 is over 3 decades. I sincerely hope that A.-L. Barabasi in the future will give due credit to reference [4] and that Nature’s editors and reviewers in the future will follow standard academic procedures for referencing background material. In fact, in 2000, the first experimental results on human response times on a news event, specifically that of an interview published in one of the leading danish newspapers including the author’s URL [9], showing a power law distribution of response times was submitted to Nature and rejected on the grounds of “too many papers on the Internet”.

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References

[1] A.-L. Barabasi, Nature 435 207-211 (2005).
[2] J.P. Eckmann, E. Moses and D. Sergi, Proc. Natl. Acad. Sci. USA 101 14333-14337 (2004).
[3] D. B. Stouffer, R. D. Mahngren, L. A. N. Amaral, http://xyz.lanl.gov physics/0511082.
[4] A. Johansen, Physica A vol 338, no. 1-2, pp. 286-291 (2004).
[5] A. Johansen, Physica A vol 296, no. 3-4, 539 (2001).
[6] Then http://www.nbi.dk/~johansen. Presently http://hjem.get2net.dk/kgs/pub.html.
[7] http://www.wallstreetuncut.com.
[8] http://www.ssrn.com/fen/index.html.
[9] A. Johansen and D. Sornette, Physica A, vol 276, no. 1-2, pp. 338-345 (2000).
[10] W.I. Reed and B. D. Hughes, PRE 66 067103 (2002).
Figure 1: Cumulative distribution of responses as a function of time. The fit is \( N(t) = a \ln \left( \frac{(t + c)}{b} \right) \) with \( a \approx 0.14 \), \( b \approx 0.21 \) hours and \( c \approx 0.25 \) hours. Due to the “wiggles”, the fit has been stabilized by first estimating \( c \) from the data and then fitting \( a \) and \( b \) keeping \( c \) fixed. The origin of the “wiggles” is simply that people send e-mail messages just before leaving their work place. Since people generally share the same working hours (provided that they live in the same time zone), those messages are not answered before the next day.

Figure 2: Cumulative number of downloads \( N(t) \) as a function of time. The fit is \( N(t) = a \ln \left( \frac{t}{b} \right) + kt \) with \( a \approx 583 \), \( b \approx 0.80 \) days and \( k \approx 2.2 \) days\(^{-1} \). The deviation between fit and data after \( \sim 60 \) days is due to another publication of URL on [8].