Occurrence probability and conductance contribution of necklace states: in support of a new scenario of Anderson phase transition

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Abstract. The occurrence probability $P_n$ of necklace states (NS) and the logarithm-conductance distribution $P_{\ln G}$ in one-dimensional random configurations are investigated. Theoretically and numerically, we find that both $P_n$ and $P_{\ln G}$ do not follow ‘single parameter scaling’ theory generally, since they depend on localization length $\xi$ even with the same $L/\xi$, where $L$ is system length. Based on theoretical $P_n$, our prediction of the behavior of typical conductance, dominated by NS, agrees very well with numerical results. Our theoretical and numerical results support the critical role of NS in Anderson phase transition.
1. Introduction

Anderson localization [1] is one of the most important achievements in condensed-matter physics in the last decades, and the study of one-dimensional (1D) random systems has always been fruitful in understanding it. Surprisingly, even in strong localized regimes, e.g. in 1D random systems, not all states are localized. As predicted by Pendry [2] and Tartakovskii et al [3], there are ‘necklace states’ (NS), formed by chains of nearly degenerate localized states. The wave can hop from one localized state to another and propagate through some random configurations. The most special character of NS is that, although they are very rare in long configurations, in ensemble average they dominate the conductance $G = \frac{1}{\Delta\omega} \int_{\Delta\omega} T(\omega)d\omega$ since they can generate mini-bands in the transmission spectra [4]. Recently, the NS were observed in optical systems [5, 6] and induced wide research interests [5–13]. Properties such as ‘quasi-extended field’ [5, 6], ‘transmission mini-band’ [5, 6] and ‘multi-resonance response’ [5, 7, 8, 12] are studied. Some statistical studies are also carried out [7, 9], such as the optimized length for different dominant orders of NS. The existence of NS in 2D systems is also reported [13].

Very recently, in [14] a new scenario of Anderson phase transition (APT), with NS having a critical role, has been proposed. In the scenario, when the randomness strength is reduced to the critical value, although the decay length of a single localized state is still finite, the wave could percolate through random configurations by NS, so that the localization length $\xi$ diverges. In other words, the NS are relevant in APT. This scenario is very interesting since the NS are ‘seeds’ of APT in localized regimes. Since there is no direct observation or convincing numerical illustration for such a scenario, more evidence and studies are needed. An obvious weakness of the scenario is that the relevant ‘seeds’ of APT must widely exist, but NS are very rare generally, especially for the high-order ones. Even more, this weakness is seemingly sealed by the famous single parameter scaling (SPS) hypothesis [15–17]. SPS implies that the effects of increasing or decreasing randomness strength (with shorter or longer $\xi$) may be completely compensated by varying system length $L$ to keep $L/\xi$ as a constant [18]. SPS is confirmed by the study of logarithm-transmission distribution $P_{\ln T}$ [16, 17]. It is found that, at a certain frequency, the $\ln T$ of 1D configurations is Gaussian distributed, whose mean value and variance can be fully characterized by a single parameter $L/\xi$ [16–18]. But, whether NS properties follow SPS has not been studied. In particular, the scaling property of NS occurrence probability, which is very essential since it determines the tendency of how NS population changes when approaching the APT, has not been reported. So, further studies are needed for the new scenario and NS role in APT.

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Although it is a difficult topic even for modern researchers, the study of NS starts from the \( n \)th-order NS occurrence probability \( P_n \) and their contribution to the ensemble average of conductance \([2, 4]\). Because of the complexity of coupling between localized states in the random systems, the strict prediction of \( P_n \) is almost impossible; hence some assumptions are needed to make the topic simpler. In previous works, \( P_n \) has been progressively studied by Pendry \([2, 4]\) and Bertolotti \textit{et al} \([5, 7]\). In Pendry’s pioneering works \([2, 4]\), \( P_n \) was estimated with three assumptions: (i) the nearly degenerate localized states are exactly evenly distributed inside the system, (ii) the frequency width of the NS mini-band is proportional to the spatial overlap integral of nearly degenerate states, which means that the mode repulsion by coupling between localized states is the main origin of NS mini-bandwidth; (iii) fixed \( \xi \) and density of states (DOS) \( \rho(\omega) \). From \( P_n \) in these works, the NS dominant order \( n_{\text{dom}} \propto \sqrt{L} \) is obtained; therefore, the fractional dimension \( D_{\text{NS}} = 1/2 \) of NS is predicted. In \([5, 7]\), \( P_n \) is calculated again with the DOS from experimental measurements and the frequency width of the NS mini-band referred to that of Pendry. But their theoretically predicted transmission only qualitatively agrees with measurement. After a detailed summary of the results of previous studies, we find that the scaling properties of \( P_n \) have not been systematically studied, especially its scaling property with the localization length \( \xi \), which is the most important length scale in random systems. Furthermore, after carefully analyzing the assumptions and the physical reasoning behind these assumptions in previous studies, we think that some of them could be improved. With \( P_n \) and the mini-bandwidth, it is easy to calculate the contribution of NS to the total conductance of random systems. In Pendry’s works, instead of conductance, based on the generalized transfer matrix formalisms he calculated the correlation of transmission at different frequencies to show the contribution of NS. In this work, numerically and theoretically we will directly calculate the conductance \( G = \frac{1}{\Delta \omega} \int_{\Delta \omega} T(\omega) d\omega \) to quantitatively check the NS contribution. Such defined conductance is also utilized in previous NS studies \([8, 9]\) to study the contribution of dominant order NS, but scaling properties of NS are not mentioned\(^4\).

In this work, we will carry out the scaling study of NS occurrence probability \( P_n \) and logarithm-conductance distribution \( P_{\ln G} \). Different from previous studies, we firstly abandon Pendry’s assumption (iii) and start from a new value, ‘the DOS per unit length’ \( \rho_{\perp, \omega} = \frac{\partial^2 N}{\partial x \partial \omega} \), where \( N \) is the number of states in a certain frequency range in a certain length of the system. Our substituting assumption is that \( \rho_{\perp, \omega} \) is almost a constant when we change the system length \( L \) or change the randomness strength \( W \) (also the localization length \( \xi \)). The physical reason for this assumption will be explained later. From this new assumption, and the ‘weakened’ Pendry’s assumptions (i) and (ii), we can derive \( P_n \) in a self-consistent way by an integration over a certain frequency range and certain space range, where both frequency and space ranges are functions of \( n, L \) and \( \xi \). Our theory can be reduced to the theories of Pendry and Bertolotti in certain conditions. The main results of our study are concluded as follows. For the first time, theoretically and numerically we find that \( P_n \) and \( P_{\ln G} \) do not follow SPS, because they are \( \xi \) dependent even with a constant \( L/\xi \). The detailed form of \( P_n \propto \xi^{n-1}e^{-L/\xi} \) is derived, so, even with the same \( L/\xi \), the occurring probability of NS becomes much higher for larger \( \xi \) configurations, especially for high-order NS. From our theoretical form of \( P_n \), we can predict the non-SPS statistical behavior of logarithm conductance \( \ln G \), whose mean value \( \langle \ln G \rangle \) corresponds to the typical conductance \( G_{\text{typ}} = e^{\langle \ln G \rangle} \) of random configurations. Our

\(^4\) The conductance, defined as \( G = \frac{1}{\Delta \omega} \int_{\Delta \omega} T(\omega) d\omega \), is the real observable value in most experiments. It is also used in \([8, 9]\) to study NS contribution, although called ‘transmission’ in those works.
Figure 1. (a) A spectrum of one random configuration over a large frequency range. The range $\Delta \omega$ of our interest is marked by two red dashed lines, which is also shown in (b). The inset is the schematic diagram of our random binary-layered model. (b) $\xi$ is almost a constant in the frequency range $\Delta \omega$.

Theoretical prediction from $P_n$ quantitatively agrees very well with numerical results, i.e. when the randomness becomes weaker (with larger $\xi$), even with a constant $L/\xi$ by increasing $L$, the typical conductance $G_{\text{typ}}$ which is dominated by NS contribution becomes larger. Comparing with SPS behavior of $\ln T$, we also discuss the physical meaning of non-SPS behavior of $\ln G$, which reveals the local correlation of spectra of random systems. Such correlation helps us have a deeper insight into the complex field coupling inside random systems and their spectra. Since our methods are not limited to 1D systems, we have carried out similar theoretical derivations in 2D and 3D systems too and find that the main conclusions of 1D systems are also fit for 2D and 3D random systems where APT could occur under certain conditions. Our results imply that, when approaching the Anderson transition point from the localized regime (with increasing $\xi$), the NS number could increase dramatically and this effect is not ruled by the SPS parameter $L/\xi$. Hence, our study is a strong support of the new APT scenario with NS having a critical role.

2. Model

Our 1D optical random model is composed of binary-dielectric layers, as shown in the inset of figure 1(a). The thicknesses of two kinds of layers are $d_A = d_B = 500$ nm while refractive indices are $n_A = 1$ and $n_B = n_{B0} + W \times \gamma$, where $n_{B0} = 3$ is the average index of layer $B$, $W$ is randomness strength and $\gamma$ is a random number evenly distributed in the range $[-0.5, 0.5]$. Transmission and field distribution are calculated by the standard transfer matrix method [4].
Figure 2. (a) A typical transmission spectrum of one configuration with $L = 5\xi$ and $W = 0.4$. The blue dashed line shows the phase change of the transmission coefficient. (b) Field intensity versus position and frequency of the NS in figure 2(a), which shows that the positions of two localized states are nearly at $L/4$ and $3L/4$.

A transmission spectrum of one random configuration is shown in figure 1(a). Our study will focus on the frequency range $\Delta\omega$ between (100, 103.18 THz), which is at the center of the second photonic band when $W = 0$ and marked by the red dashed lines in figures 1(a) and (b). The localization length $\xi$ is calculated by $\xi = -2L/\langle \ln T \rangle$ over an ensemble average of $10^5$ configurations. Figure 1(b) shows $\xi$ versus $\omega$ for different $W$. In our chosen frequency range $\Delta\omega$, $\xi(\omega)$ varies by less than 1%, so that $\xi$ is taken as a constant in the frequency range for a certain $W$.

A typical transmission spectrum and the phase change of transmission coefficient for a configuration with $W = 0.4$ and $L = 5\xi$ are shown in figure 2(a). There are sharp peaks of localized states with $\pi$ phase jumps, and a ‘mini-band’ (or coupled peaks) of second-order NS with a $2\pi$ phase jump that is marked in figure 2(a). Figure 2(b) shows the electric field intensity $|E(x)|^2$ versus position and frequency of the marked NS, which clearly shows that NS are formed by two coupled nearly degenerate localized states. Between the coupled peaks of NS, generally there are valleys, and we define the minimum transmission value of valleys as $T_v$, which is determined by the coupling strength and the frequency difference of localized states. In our numerical calculation, we judge a NS on the spectrum by two criteria: (i) an $n-\pi$ phase jump for the $n$th NS and (ii) $T_v$ larger than a critical value. We note that the critical value of $T_v$ is a freely chosen parameter, but it does not influence the main conclusions of this work when we choose different critical values in the range $[0.1, 0.5]$. 

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3. Theory

We first study the scaling property of occurrence probability \( P_n \), especially its relation with localization length \( \xi \). Before the theoretical derivation, from a simple scaling argument, we easily obtain that the probability \( P_n \) should depend on \( \xi \) even with the same \( L/\xi \). The original works [2–4] tell us that the NS are from the nearly degenerate and spatially overlapping localized states. Assume that two configurations \( A \) and \( B \) have very different randomness strength \( W \), so that the localization lengths are quite different, e.g. \( \xi_A \gg \xi_B \). Thus a localized state I inside \( A \) is spatially much more extended than a localized state II inside \( B \). Since state I can spatially overlap with many more other localized states than state II, the probability of finding a nearly degenerate neighbor (then, form NS) for state I is much higher than for state II. It should be emphasized that this comparison is still available even with \( L_A/\xi_A = L_B/\xi_B \). Actually, this picture is a local one (in scale of \( \xi \)), totally independent of system length.

Next, we will try to theoretically obtain the quantitative results of \( P_n \) based on a stricter derivation. First we abandon assumption (iii) of Pendry’s works and start from a new point. Our starting point is the ‘DOS per unit length’, \( \rho_{\omega,x} = \frac{\delta^2 N}{\delta x \delta \omega} \), where \( N \) is the number of states. \( \rho_{\omega,x} \) is very useful in our study since its value is almost a constant for systems with different length \( L \) and randomness strength \( W \). Let us first study the \( \rho_{\omega,x} \) of a periodic system with \( W = 0 \). The value of \( \rho_{\omega,x} \) of the periodic system can be estimated in the following way. The total state number of the system in a band is \( N = 2L/(d_a + d_b) \), where \( (d_a + d_b) \) is the lattice constant, and the bandwidth is about \( \Omega_B \sim \pi c/\bar{n}(d_a + d_b) \), where \( \bar{n} = (n_a d_a + n_b d_b)/(d_a + d_b) \) is the average index of a cell and \( c \) is light speed; then, \( \rho_{\omega,x} = N/\Omega_B L \sim 2n/\pi c \). When randomness is introduced, \( N \) does not change, but some of the states originally inside the band will fall into gaps, so that \( \rho_{\omega,x} \) could be a little smaller. However, for the frequency range (at band center) of our interest, \( \rho_{\omega,x} \) changes very little for different randomness; hence \( \rho_{\omega,x} \) is almost a constant for our systems with different \( L \) and \( W \). So we substitute assumption (iii) of previous works by the assumption of ‘constant \( \rho_{\omega,x} \)’ which is more general for cases with varying system length \( L \) and varying localization length \( \xi \). We note that a similar definition of \( \rho_{\omega,x} \) is also seen in [19] for other topics.

With an almost constant \( \rho_{\omega,x} \), we can estimate \( P_n \) by integration over a certain spatial range and frequency range as follows:

\[
P_n = (P_1)^{n-1} = \left( \rho_{\omega,x} \int_{x_n-\Delta x_n}^{x_n+\Delta x_n} \int_{-\Delta \Omega/2}^{\Delta \Omega/2} \frac{d\omega dx}{2} \right)^{n-1},
\]

where \( P_1 \) is the probability of finding a neighboring nearly degenerate localized state in frequency range \([-\Delta \Omega/2, \Delta \Omega/2]\) and spatial range \([x_n-\Delta x_n, x_n+\Delta x_n]\), and \( x_n = L/n \) is the average distance between the coupled localized states. As in the case of as [2, 4], the frequency integral range \( \Delta \Omega \) is supposed to be determined by the spatial overlap of localized states \( \Delta \Omega = \Omega_0 \exp(-x/\xi) \), where \( x \) is the distance between two localized states, \( \exp(-x/\xi) \) is proportional to the spatial overlapping integral, and \( \Omega_0 \) is a parameter that shows the repulsive effect of two coupled nearly degenerate states (and in our model it is approximately a constant, \( \Omega_0 \sim 101 \text{THz} \)). Besides \( \rho_{\omega,x} \), another tricky point of our theory is that we have weakened Pendry’s assumption (i) that all localized states are exactly evenly distributed, i.e. \( x = x_n = L/n \). In our theory, we assume that \( x \) is in a range \([x_n-\Delta x_n, x_n+\Delta x_n]\), where \( \Delta x_n \) can be obtained from the confinement of different orders of NS. We know that the average distances between localized states of \((n-1)\)-th, \(n\)th- and \((n+1)\)-th-order NS are \( x_{n-1} = L/(n-1), x_n = L/n \) and

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$x_{n+1} = L/(n+1)$, respectively. Now we calculate the probability of $n$th-order NS and assume that all localized states are almost (not exactly) even distributed in a random system; therefore, it is reasonable to assume that our integration range must be closer to $x_n$ than to $x_{n-1}$ or $x_{n+1}$, so that $\Delta x_- = (x_n - x_{n+1})/2$, $\Delta x_+ = (x_{n-1} - x_n)/2$. Then we obtain the probability $P_n$ of the $n$th-order NS as

$$P_n \approx (\rho_{\omega,n} \Omega_n \xi)^{n-1} \cdot e^{-L/\xi} \cdot F(n),$$

where $F(n) = [e^{2L/(\xi^2-1)} - 1]^{n-1}$ is a slowly varying function for large $n$. Equation (2) is the main theoretical result of this work, which explicitly shows that $P_n$ depends not only on $L/\xi$ but also on $\xi$, as expected in the scaling argument.

It is interesting to compare our theory with previous work. First, we note that, for a fixed system with certain length $L$ and $\xi$ (or $W$), both ‘DOS per unit length’ $\rho_{\omega,x}$ and DOS $\rho_\omega = \int \rho_{\omega,x} \mathrm{d}x$ are constant and our equation (1) is reduced to that of Pendry’s model [2, 4] and later work [5, 7]. However, for the scaling study of NS, we generally need to consider cases with varying $L$ and $\xi$. Now, $\rho_{\omega,x}$ is still nearly a constant while the DOS $\rho_\omega$ is a variable. Since our theory is from a ‘good’ value $\rho_{\omega,x}$ in random systems, it is not surprising that we can reveal more detailed information than previous works. Even more, in our theory, we suppose that the distance $x$ between the coupled localized states has a tolerance, keeping the averaged value $\langle x \rangle$ the same as $x_n$. With $x$ in a range, we can obtain the NS mini-bandwidth, which shows up as the integration frequency range $\Delta \Omega = \Omega_0 \exp(-x/\xi)$ in our theory, also a variable. Obviously, our theory is more general. Actually, all these considerations are made to describe the complexity of coupled states in random systems and obtain reasonable $P_n$ with self-consistency.

4. Numerical results

The $\xi$ dependence of $P_n$ is checked by our numerical calculation. With $L/\xi = 15$, for different $\xi$, we count the occurring probability of second, third and fourth orders of NS over 40 000 configurations which can guarantee the statistical stability very well. We count NS based on criteria such as $T_v > 0.3$, and a phase jump of $2\pi$, $3\pi$, $4\pi$ for $P_2$, $P_3$, $P_4$, respectively. The results of $P_2$, $P_3$ and $P_4$ versus $\xi$ are shown in figure 3. According to equation (2), $P_2$ should increase linearly with $\xi$, while $P_3 \propto \xi^2$ and $P_4 \propto \xi^3$. Our numerical results agree with our theoretical prediction very well, i.e. $P_2$ is a linear function, and the values of $P_2$ and $P_3$ are on the fitting curves from our theory. We have also numerically calculated other cases, such as with different critical values of $T_v$, with different $L/\xi$ and even with different refractive index contrasts. All results are consistent with our theoretical ones.

Next, we will study the statistical properties of logarithm conductance $\ln G$ since the minibands of NS dominate the statistics of $G$ [2–4]. Firstly, we will show some basic properties of $\ln G$ statistics numerically, comparing with that of $\ln T$. Then we will try to theoretically predict the behavior of $\ln G$, and check our prediction numerically. Every statistical distribution is obtained from $9 \times 10^3$ random configurations, with different $\xi$ but the same $L/\xi = 15$. The statistical distributions of $\ln T$ at a certain frequency are shown in the inset of figure 4(a), which follow SPS very well, as expected. But the probability distributions of $\ln G$, as shown in figure 4(a), are quite different. Two basic conclusions of $\ln G$ distribution can be made from figure 4(a). Firstly, similar to $\ln T$ [16], $\ln G$ is almost normally distributed. Secondly, quite different from $\ln T$, even with the same $L/\xi$, the mean value and variance of $\ln G$ distribution are $\xi$ dependent. Obviously, $\ln G$ statistics does not follow SPS too. The mean value of $\ln G$...
Figure 3. Occurrence probability of different order NS versus $\xi$ for $L/\xi = 15$ systems. Black triangle is for $P_2$, blue square for $10 \times P_3$ and red circle for $60 \times P_4$. The dashed lines are the fitting curves based on $\alpha + \beta \xi^{(n-1)}$.

Figure 4. (a) Statistical distribution of $\ln G$ with $L/\xi = 15$ for systems with different $\xi$. (b) Numerical results of $G_{\text{typ}}$ versus $\xi$ for $L/\xi$ as 16.9 and 38, respectively.

corresponds to the *typical* value of $G$. From figure 4(a), we can see that when $\xi$ is longer (weaker randomness), the typical value of $G$ increases even with the same $L/\xi$, and this would be intriguing if our study was near APT. The different scaling behavior of $\ln T$ and $\ln G$ enables us to have a deeper insight into the transmission spectra of random systems. The $\ln T$ statistics
is from many examples of a ‘single point’ (at a certain frequency) in the spectra, while the \( \ln G \) statistics is from examples of ‘multi-neighboring points’ (the integral over a frequency range) in the spectra, so that \( \ln G \) statistics includes the local correlation information of the spectra (such as the ‘mini-bands’ of NS). Actually, the original NS study by Pendry [2, 4] is based on the correlation function of spectra too.

After numerical study of \( \ln G \) statistics, we will show that from our theoretically derived \( P_n \), we can correctly predict the statistical behavior of \( \ln G \). We carry out the prediction and the verification as follows. Firstly, from \( P_n \), we can approximately predict the length \( L \) (in units of \( \xi \)) of the configurations in which a certain order of NS will be dominant. Then we can calculate the typical conductance value \( G_{\text{typ}} = e^{(\ln G)} \) versus \( \xi \) by equation (2) since \( G_{\text{typ}} \) is dominated by mini-bands of NS. Finally, we can compare our prediction with our numerical results. The dominant order \( n_d \) of NS is determined by the maximum contribution of conductance, which is calculated by the frequency range times \( P_n \) [4]

\[
\frac{d}{dn} [\Omega_0 e^{-L/n\xi} \cdot P_n]_{n=n_d} = 0. 
\]

With equation (2) and \( \rho_{a,x} \sim \frac{1}{L n a} \), where \( a \) is the average cell length, this directly results in

\[
n_d^2 \approx \frac{L/\xi}{\ln(\xi/a)}. \tag{4}
\]

From equation (4), for a certain \( n_d \), we can predict the approximate value of \( L/\xi \) since \( \ln(\xi/a) \) is a slowly varying function. If \( \xi \) is in the range from 34.1 to 137.5, supposing \( n_d = 2 \), then from equation (4) \( L/\xi \) should be in the range [14.1, 19.7] where the middle value is 16.9; or supposing \( n_d = 3 \), then \( L/\xi \) is in the range [31.8, 44.3] where the middle value is 38.05. On the other hand, from equation (2), if the second (or third) order NS dominate \( G \), then the typical value of conductance \( G_{\text{typ}} = \exp(\ln G) \propto P_n \) should increase with \( \xi \) (or \( \xi^2 \)). All our predictions are based on equations (2) and (4). Then, \( G_{\text{typ}} \) versus \( \xi \) can be obtained numerically for a certain \( L/\xi \) value: such as, for \( L/\xi = 16.9 \) and \( L/\xi = 38.05 \), according to our theoretical prediction, \( G_{\text{typ}} \) should be \( \propto \xi \) and \( \propto \xi^2 \), respectively. We have done the calculation over 25 000 configurations for every numerical value, and the results are shown in figure 4(b).

For the \( L/\xi = 16.9 \) case, exactly as theoretically predicted, \( G_{\text{typ}} \) increases linearly, and for the \( L/\xi = 38.05 \) case, \( G_{\text{typ}} \) is approximately \( \propto \xi^2 \). Actually, this is the first statistical demonstration of the famous conclusion that ‘the conductance \( G \) is dominated by NS’ [2–4]. We want to mention that from equation (4) the fractal dimension \( D_{\text{NS}} = 1/2 \) of dominant NS can also be obtained, the same as [4]. Finally, it is emphasized that our derivations and discussions are based on the 1D model, but they can be used for 2D and 3D systems too, with only small differences, such as the form of the overlap integral.

5. Discussion and conclusion

Now, we can discuss the relation between our results and the new APT scenario. Since NS properties do not follow SPS, when randomness strength \( W \) decreases, the occurrence probability of NS and their contribution to \( G \) cannot be statistically compensated by varying \( L \); hence, the role of NS is liberated from SPS confinement and could be critical at APT. The non-SPS properties of our study can also present more detailed information for the scenario if it is correct. Suppose that the system approaches the APT point from the localized regime with weaker and weaker randomness. The localized states in the system become so extended that...
the occurrence probability of dominant order NS becomes larger until it approaches unity; then statistically the wave can percolate through the system by the NS and APT occurs. Of course, we are still far from the demonstration of the scenario, but the non-SPS properties make it possible for NS to bridge the localized and delocalized phases.

In conclusion, the NS occurrence probability $P_n$ and the logarithm-conductance distribution $P_{\ln G}$ of random configurations are theoretically and numerically investigated, with emphasis on their scaling properties. For the first time, we find that both $P_n$ and $P_{\ln G}$ are $\xi$ dependent even with the same $L/\xi$, so they do not follow SPS generally. From our new theoretical form of $P_n$, we can predict the behavior of typical conductance $G_{\text{typ}}$, which agrees very well with numerical results. The importance of our study is to support NS as having a critical role in APT. Based on our scaling study, further topics, such as the percolation-like wave propagation through NS and the correlation properties of spectra, could be studied, especially near the APT point.

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