The Unruh effect and oscillating neutrinos

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Abstract. We give an overview of the issues and ambiguities associated with the Unruh effect, and argue that, as well as a very interesting phenomenon, it can also be used as a probe of fundamental physics. In particular, We point out that, because the detectable neutrino is not a mass Eigenstate, the Unruh effect works in a qualitatively different way than for any inertial process. For inertial processes, neutrinoes are produced as charged eigenstates, rather than as mass Eigenstates as in the comoving frame. This makes the Unruh effect detectable in microscopic processes, via, for example, $p \rightarrow n l^+ \bar{\nu}_l$ decays. Such an experiment would be invaluable both as a tool to measure neutrino masses and mixing angles, and to investigate the fundamental quantization of fields.

1. Introduction
Quantum field theory has justifiably been at the forefront of fundamental physics for the last few decades. The physical world in this picture is represented by a Lorentz invariant manifold (spacetime) on which point-like quantum particles, described by irreducible representations of the Lorentz group, propagate and interact locally [1]. While mathematically this description is subject to some ambiguities, on a phenomenological level this language has described all microscopic phenomena ever observed at a fantastic precision. Its big drawback is that it seems profoundly incompatible with gravity. As well as technical difficulties [2], combining quantum field theory with the curved space formalism of gravitational physics presents problems with such fundamental issues of quantum mechanics as unitarity [4].

Such effects are particularly puzzling because they do not require gravitational fields of the order of the Planck scale to be observed, but manifest themselves in the arbitrarily weak field limit. Black hole evaporation, occurring outside the event horizon (where the field is considered weak) is a famous example.

Indeed, given the equivalence principle, black hole evaporation has a counterpart in “inertial” non-gravitational physics: As shown in [10], an \textit{accelerated} detector in vacuum sees a thermal bath of particles with a temperature proportional, in natural units, to acceleration[11]. The origin of the particles seen by the detector is exactly the same as in Hawking radiation: the mismatch of the creation operator $a$ in the inertial frame (Which, for a Spin zero particle, creates a momentum Eigenstate solving the Klein-Gordon energy operator, $\partial^2 - m^2$ whose Eigenfunctions are $\sim e^{ikx}$) and in the accelerated (Rindler) frame, where the creatin operator...
\[ P(r) = \left| \int d^3 x_R < p | a_p | 0 >_M \right|^2 \propto f_{FD/BE}(p, \frac{a}{2\pi}) \]
2. Neutrino oscillations and gravity

While a phenomenological description of them has been known for decades [5], this model has not been incorporated into a rigorous field theory defined beyond one-particle propagators [6]. This is in contrast to the superficially similar meson mixing, successfully incorporated into the standard model [1]. This is however not surprising: In quark mixing via the CKM matrix, the mixing is due to the non-commutativity of the strong and electroweak charged currents. In neutrinos, the mixing is between mass states and weak flavor states.

This makes neutrinos the only interaction where there is no superselection rule forbidding mass mixings (meson Eigenstates subject to mixing, such as $K_S, K_L, K_0$ and $\bar{K}_0$ are still mass degenerate). Consider neutrinos interacting both gravitationally and via the electroweak interaction, modeled here for simplicity as a point-like four-fermion interaction. The Lagrangian for our theory is then

$$L = \frac{1}{2\kappa} \left( \partial_\mu \partial_\nu h^{\mu\nu} - g^{\mu\alpha} \partial_\nu \partial_\rho h^{\alpha\rho} \right) + G T_{\mu\nu} h^{\mu\nu} +$$

$$+ \sum_{l=e,\mu,\tau} G_F \tilde{J}_{\mu l} \tilde{J}^l_{\mu} + \sum_{i=1,3} \bar{\psi}_i (\gamma_\mu \partial^\mu - m_i) \psi_i + O(h^2, \psi^2)$$

The fact that GR is spin 2, and Lorentz invariance force [1] the $h_{\mu\nu}$ to couple to conserved quantities only, and the only available tensor quantity is the energy-momentum tensor $T_{\mu\nu}$.

Gauge symmetry, underlying electroweak interactions between neutrinos, also ensure $\langle \tilde{J}_{\mu l} \rangle$ to be conserved.

The Pontecorvo mechanism makes these conservation laws impossible to satisfy simultaneously since the energy-momentum tensor $\tilde{T}_{\mu\nu}$ and the weak currents $\tilde{J}_{\mu}$ do not commute even in the linearized limit of Eq. 2 (for other particles such problems come beyond the linear order [4]).

To see this explicitly, neglecting kinematic mass differences we can express

$$\tilde{J}^l_{\mu} \simeq \sum_{j=1,3} U_{ij} \left( \bar{\psi}_j \gamma^\mu \psi_j \right)$$

and $U_{ij}$ is a unitary $3 \times 3$ matrix. Considering the weak interaction as much weaker than the rest mass or the neutrino momentum, the energy momentum tensor is the one of the usual free fermionic field

$$\tilde{T}_{\mu\nu} \simeq \langle T_{\mu\nu} \rangle = \sum_{i=1,3} \left[ \bar{\psi}_i \gamma_{\mu} \partial_{\nu} \psi_i - g_{\mu\nu} \left( \bar{\psi}_i (\gamma_\alpha \partial^\alpha - m_i) \psi_i \right) \right] + O(h\psi)$$

by inspection an Eigenvalue of the operator defined in Eq. 4 is not an eigenvalue of Eq. 3 and viceversa. The mixing coefficients are the coefficients of $U_{ij}$.

What does this mean? Normally, in the weakly interacting limit, the only measurement that does not commute with $T_{\mu\nu}$ is the position. This, however, is no threat to the equivalence principle: To modify gravitational dynamics perceptively by a quantum measurement one would have to measure position by a precision comparable to the inverse of the curvature of space in a given frame. This, by definition, is not a local measurement (this is how Planck unit of $10^{-19} GeV^{-1}$ described in the introduction is defined). Thus, in the weak field small curvature limit the equivalence principle arises as a good effective description.

Not so for neutrinoes. If we consider $\langle G_F J_{\mu l} \rangle$ point-like the last term in Eq. 2 can be used to modify the energy momentum tensor with arbitrary soft momentum exchange, $T_{\mu\nu}(\psi_i) \rightarrow \sum_j U_{ij} T_{\mu\nu}(\psi_j)$ which, for a neutrino-self gravitating system will have instantaneous (and in the semiclassical limit) non-causal effects on geometry.

3
The gravitational interaction of neutrino matter is not a feasible measurement in the foreseeable future (Indeed neutrino oscillations were originally proposed as a test for Lorentz invariance [8, 9]). However, the counter-part of this in accelerating rather than gravitational frames, the Unruh effect [10], yields the theoretical ambiguity of basic quantum field theory which could be investigated in the laboratory.

![Figure 1](https://example.com/fig1.png)

**Figure 1.** What an accelerating $p \rightarrow n\nu\bar{\nu}$ process looks like in a comoving frame (panel (b)) and an inertial frame (panel (a)). In the semiclassical limit the $\gamma$ line implies an Euler-Heisenberg Lagrangian rather than perturbation theory.

### 3. Inertial and comoving frame pictures

In the Minkowski frame (Fig. 1 left panel), the calculation amounts to electron and neutrino production by a classical source [21], because the Fermi theory current-current interaction is treated with a classical hadronic current $J_{Ld}^\mu \tilde{J}_{Lh,\mu} \rightarrow \tilde{J}_{Ld}^\mu \tilde{J}_{Lh,\mu}$ [13]. The calculation is “semiclassical” in that quantized lepton fields are produced by a classical source. The time dependence of the source $J_{Ld}^{(d)}$ includes an oscillating phase $e^{i(m_\nu - m_p)t}$ contributing to the energy of the outgoing particles. To understand better what the process can reveal about neutrino oscillations and the Unruh effect, we examine the physics conditions necessary to reduce the hadronic current operator to a classical current, and this begins with an expansion in the small ratios of the acceleration to the other momentum scales $a/m_W$, $a/m_p$, $a/\Delta m$, where $\Delta m = m_\nu - m_p$.

To consider the proton motion prescribed, the acceleration must be small $a \ll m_p$, so that radiation reaction and backreaction on the external field is negligible. In fact, the proton’s compositeness gives a more stringent constraint since electromagnetic fields $eB, eE \simeq m_\pi^2$ affect its structure, so we require $a \ll m_\pi^2/m_p \sim 19$ MeV. This implies also $a \ll m_W$ so that the Fermi effective theory is applicable. The limitation to $a/\Delta m \ll 1$ will become clear later. The perturbative scale is $a$ and hence the integration volume is $d^4 x \sim a^{-4}$, consistent with being the physical scale of the saddlepoint in the tunneling potential for the leptons, which is localized (at the scale $a^{-1}$) to the region around the turning point. For the same reason, the acceleration should be constant over $\sim a^{-1}$ in order for the semiclassical approximation to apply for the leptons, which have Broglie wavelengths of order $\Delta m$.

The $p \rightarrow n\nu\bar{\nu}$ operator from the lagrangian interaction in Eq. (2) $\mathcal{O} = 4G_F^2 |\Psi_\nu\gamma^\mu P_L\bar{\Psi}_p)(\bar{\nu}_\gamma\gamma^\mu P_L\bar{\nu}) |^2$ is a perturbative interaction, and the Fermi theory suffices since the momenta involved are all $\ll m_W$. To match the Rindler frame calculation which involves a correlation function restricted to a classical trajectory, we must reduce the proton to a point-like classical particle. This differs from the semiclassical approximation in field theory, which entails solving the Dirac or Klein-Gordon equation in the presence of the classical accelerating potential [22, 25]. To reduce to a classical trajectory, the anti-particle components of the proton wavefunction are integrated out first by using translational invariance of the quasi-constant external field to focus on the region around the turning point in the trajectory where the proton is nonrelativistic. Since $a \ll m_\pi$, the hadron is nonrelativistic for a time of order $a^{-1}\ln m_\pi/m_\pi > a^{-1}$, found by solving for the time for the proton to gain $1m_p$ of energy. In this
frame, the proton and neutron are heavy fermions, allowing us to extract the mass as the large part of the phase and integrate out the antiparticle components of the spinor. Corrections to the dynamics of the $h_\nu$ fields as well as antiparticle components are suppressed by $1/m_p$. Also at leading order in $1/m_p$, we can neglect the recoil, which means we drop the residual nucleon momenta and set $v = v'$. Considering the “coherence time” of the process to be $dt \sim 1/m_W$, the velocity change during the interaction is $dv = a dt \sim a/m_W$, also a subleading correction.

As the nucleons are nonrelativistic particles with negligible recoil, the matrix element of the conversion reaction in its proper time (be it the characteristic time of spontaneous inverse $\beta$ decay of the hadron) will vanish under the phase space integral. Since we have used $dt/u_0(\tau) = d\tau$ and one now inserts the accelerated trajectory

$$\xi(\tau) = a^{-1}(\sinh a\tau, 0, 0, \cosh a\tau), \quad u_\mu = \frac{d\xi_\mu}{d\tau}$$  

(7)

The reason for the finite result for a process seemingly violating energy conservation is in the noninertial trajectory: The process is kinematically allowed due to the acceleration of the proton, because the external potential provides the rest energy difference between initial and final states. As long as the potential is “weak” $a \ll \Delta m$, the process is exponentially suppressed, just as is spontaneous pair production in quasi-constant electric fields [22, 25].

The phase factor depends on the coordinates $x^\mu$ with spatial coordinates replaced by $\xi^i(\tau)$ of the trajectory. Expanding around the origin where the hadron is instantaneously at rest, $\tau = t - \Delta x \simeq t(1 - at/8)$ to leading order in $at$. Since the phase factor enforces energy-momentum conservation, $t \sim \Delta m^{-1}$ and we have $t - t' = \tau - \tau'$ to leading (zeroth) order in $a/\Delta m$.

Making the substitution $t - t' \rightarrow \tau - \tau'$, the integrand has no dependence on $\tau = \tau + \tau'$, and one can define a rate per unit (proper) time. Defining $T = \int d\tau$ we have

$$\frac{1}{T} \frac{dW}{d^4 k d^4 k'} = \frac{dW}{d^4 k d^4 k'} = G_F^2 \int d\sigma \ e^{i2a^{-1}(k_0 + k'_0)\sinh \sigma a} \times (k_0 k'_0 + k_z k'_z + k_\perp k'_\perp \cosh 2\sigma a)$$  

(8)

where the rescaled variables are $\tilde{k}_0 = k^0 \cosh \tau a - k_z \sinh \tau a$, $\tilde{k}_z = k^z \cosh \tau a - k_0 \sinh \tau a$ and $k_\perp = k_\perp'$ We note that $k$ and $k'$ are related by a Lorentz transformation and, provided the phase space is Lorentz invariant, the “rate” in Eq. 8 is also Lorentz invariant, defined in terms of the conversion reaction in its proper time (be it the characteristic time of spontaneous inverse $\beta$ decay or the characteristic interaction time with the Unruh bath).

The simplest experimental observable is the number of observed electron-neutrino pairs. To compare the inertial frame rate with the Rindler frame rate Eq. (16), we must integrate Eq. (8) over the electron and neutrino momenta in the final state. Usually we would ensure the 4-momentum integrals converge by putting the electron and neutrino on mass shell. Since $\Delta m \gg m_\nu$, it is a good approximation to the neutrino phase space to treat it as massless,

$$W = \int \frac{d^2 k d^2 k'}{d^2 k d^2 k'} \frac{dW}{d^2 k d^2 k'} \delta(k^2 - m_\nu^2) \delta((k')^2)$$  

(9)
One may worry that including the neutrino mass in the phase is “more correct”, and doing so would lead to the inclusion of PMNS factors for the conversion to neutrino mass eigenstates. However, we recall that a neutrino is created in a flavor eigenstate, oscillates over macroscopic distances and is detected in an experiment which again projects a flavor eigenstate. Since the detection mechanism for neutrinos does not involve a projection onto a momentum eigenstate, it would be inappropriate to restrict the neutrino phase space to mass shell.

Performing the integrals as in Eq. (9), the result converges to the \( m_e \to 0 \) limit given in [18]

\[
W = \frac{G_F^2 m_e^2 a^2}{32 \pi^7} e^{\pi \Delta m / a} H\left(\frac{\omega}{a}, \frac{m_e}{a}, 0\right)
\]

where

\[
H\left(\frac{\omega}{a}, \frac{m_e}{a}, \frac{m_\nu}{a}\right) = 6 |K_{\omega, +}^{(m_e/a)}|^2 |K_{\omega, -}^{(m_\nu/a)}|^2 + \text{Re} \left[ K_{\omega, +}^{(m_e/a)} K_{\omega, -}^{(m_\nu/a)}^* \right]
\]

and \( \omega' = \omega - \Delta m \). The \( m_\nu \) dependence of the integrand prevents factoring out the PMNS matrix and using \( \sum_i U_{ei} U_{ei} = 1 \).

Given the definition of \( H \) in Eq. eq 11 and the formula, used in [18], linking the integrals used here to the Meijer-G function [13]

### 3.1. Comoving frame calculation

The inertial frame calculation above is compared to the comoving frame reactions \( p,e \to n,\nu,p\bar{\nu} \to n \) and \( p\bar{\nu} \to en \) corresponding to the absorption by a static proton of, respectively, an electron, a neutrino and both an electron and a neutrino from the Unruh thermal bath (Fig. 1 right panel).

The coordinates in the comoving frame are given by the Rindler metric, which in the right wedge \( z > |t| \) has the form \( ds^2 = g_{\mu\nu} dx^\mu dx^\nu = u^2 du^2 - dv^2 \). The spacelike coordinate \( u \) is related to the Minkowski coordinates by \( u^2 = z^2 - t^2 \) \((0 < u < \infty)\) and determines the magnitude of the acceleration of the trajectory relative to the Minkowski space. The eigenstates are determined by the Dirac Hamiltonian [27] for each lepton \( \ell = e, \nu_i \). The \( \tilde{\gamma} \) are the Rindler frame Dirac matrices, satisfying \( \{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2 g^{\mu\nu} \).

The eigenmodes are

\[
\chi_{\omega, +} = N^{1/2} \begin{pmatrix} i\Phi^+_{\omega} & \Phi^+_{\omega} \\ 0 & 0 \end{pmatrix}, \quad \chi_{\omega, -} = i\gamma^0 \gamma^2 \chi_{\omega, +}, \quad \Phi^\pm_{\omega} = K_{\omega, +\pm}^{(m_\ell u)}
\]

The Rindler frequency takes all real values with no mass gap, and the mass appears in the wavefunction. It is the intrinsic dependence of the wavefunction on the mass that makes the sum over mass eigenstates not factorizable. For each field, the electron and each neutrino mass eigenstate, a corresponding complete set of solutions covers the right spacelike wedge.

Forming the matrix elements for the processes, we take the Minkowski in and out states, bringing in the Bogoliubov coefficients relative to the Minkowski particle \( \psi^+ \) and antiparticle \( \psi^- \) modes \( \chi_\omega = \alpha_\omega \psi^+ + \beta_\omega \psi^- \),

\[
\alpha_\omega = \frac{e^{i\omega/2a}}{(2 \cosh(\pi \omega / a))^{1/2}}, \quad \beta_\omega = e^{-i\omega/2a} \alpha_\omega,
\]
Additionally, the neutrino mass eigenstates in the initial or final states requires introducing a factor of $U_{i\ell}$ to rotate to the electron field appearing in the operator. The matrix elements are

$$i \mathcal{M}_I = J^\mu_I(x) \sum_i U^\dagger_{ei}(|\chi^{(\ell i)}\tilde{\gamma}_\mu \tilde{P}_a \chi\rangle(x)|e\rangle)$$

$$i \mathcal{M}_{II} = J^\mu_{II}(x) \sum_i U_{ei}(|\epsilon^{(\ell i)}\tilde{\gamma}_\mu \tilde{P}_a \epsilon\rangle(x)|\bar{\nu}_i\rangle)$$

$$i \mathcal{M}_{III} = J^\mu_{III}(x) \sum_i U_{ei}(0|\chi^{(\ell i)}\tilde{\gamma}_\mu \tilde{P}_a \chi\rangle(x)|e\rangle)$$

where $J^\mu_I(x) = \delta^\mu_\| e^{i\Delta M I B^2}(x_\perp)\delta(u - a^{-1})$ in Rindler coordinates. Note that $\tilde{\gamma}^5 = \tilde{\gamma}_\mu \gamma^\mu \gamma^5 = \gamma^5$.

Summing the processes $I, II, III$ and integrating, the result has an analytic form similar to [13], with the additional sum over neutrino mass eigenstates,

$$W_{\text{tot}} = \frac{G^2_\text{F} a}{8\pi^3} e^{-\frac{\Delta m_a}{a}} \sum_i U^\dagger_{ei} U_{ei} \int_{-\infty}^{\infty} d\omega H\left(\frac{\omega}{a}, \frac{m_a}{a}, \frac{m_a}{a}\right)$$

4. Interpretation

As we pointed out in the introduction, this discrepancy is expected because of the ambiguity inherent in combining non-mass Eigenstate fields with non inertial frames. Deforming to a nonflat metric deforms the mass eigenstate creation and annihilation operators. In the infrared limit, such operators define on-shell states, yet these are not selected by inertial detectors. Thus, one cannot expect, as is generally true for interacting field theories, for descriptions in different frames to give the same scalar observables. To interpret this result, it must be remembered that by writing the lagrangian Eq. (2), we consider the neutrino mass a tree-level operator, unmodified by the interaction producing the acceleration. This is consistent with the neutrino mass being either a “fundamental” operator or generated by other interactions that have been integrated out, such as coupling to the Higgs or high scale beyond Standard Model physics. Measuring rates in agreement with the inertial frame calculation would provide evidence that the neutrino mass is an effective operator of one of these types. Specifically, this treatment in the inertial frame is appropriate as long as the cutoff scale for the (effective) mass operator is $\gg a$. Matching the comoving frame calculation to the observed rate in this case requires deeper understanding how the Higgs condensate transforms under the noninertial coordinate change or how effective operators with finite cutoff scales are represented in the curved space.

There is a possibility that the next generation lasers will allow us to investigate this phenomenon experimentally. The challenges in observing this effect are (1) achieving sufficiently high $a$ to overcome exponential suppression, and (2) maintaining that acceleration for sufficiently long that the quasi-constant approximation applies. The first may be partially offset by very large number of experiments. The second may be partially offset by studying more general non-constant accelerated trajectories; however, in all cases the duration of the acceleration must be much larger than the “equilibration time” to the noninertial vacuum [28]. As a conservative estimate $\Delta t < (\alpha m_a)^{-1} \sim 6 \mu m = 2$ fs, though there is significant dependence on the acceleration profile. The field must also do work on the accelerated particles seeing that the final state rest energy is $\approx \Delta M$ greater than the initial state.

Only electromagnetic fields can provide high accelerations for durations comparable to $\Delta t$. In the lab, these are provided by high intensity lasers such as the Texas Petawatt [29] or ELI [25, 30]. These create $L \sim 20 - 40\lambda \simeq 20 - 40\mu m$ pulses and achieve fields of $eE = (10^{-4} - 10^{-5})m_e^2$ written in terms of the QED critical field $m_e^2/e$ where QED effects in classical external fields are important. A proton in such a field of this strength is accelerated by $a \simeq 10^{-6}$ MeV. By comparison, other long wavelength fields are: (1) the mean electrostatic field inside an LHC bunch, estimated $eE \sim (keV)^2$, using a cylinder with dimensions of the bunch ($N = 10^{11}$, bunch radius $10^{-5}$ cm and length $L \sim 10$ cm); (2) mean fields inside fixed targets at hadron colliders.
$eE \sim (100\text{eV})^2$ again with $L \sim 10\text{ cm}$; and (3) plasma wakefield and crystal accelerator concepts $eE < 10^{-6}m_e^2$ over $L \sim \mu\text{m}$. Larger $L$ in the LHC bunch is not sufficient to overcome the relative $e^{-103}$ suppression.

The proposed experimental program explores a physics domain orthogonal to that usually considered for beyond Standard Model physics: instead of one scattering event with large momentum transfers we consider a uniform acceleration, which means a “large” number of interactions with “soft” quanta. Since the effective potential giving rise to most, if not all mass terms is thought to be the Higgs mechanism involving a condensate of zero momentum quanta, it is reasonable to suppose new physics will show up in this regime provided the scale of the relevant term of the effective lagrangian is comparable to the acceleration, as it certainly is here.

In conclusion, we have argued that the conversion of accelerated protons into neutrons is a promising laboratory to study the origin of neutrino masses and oscillations. If the inertial frame calculation is correct, we do not have sensitivity to the neutrino mass absolute values, unless our choice of the phase space is incorrect. Admittedly, technologically such experiments are still somewhat out of reach, but we hope that the ingenuity of the QED and intense laser community will make such experiments feasible in our lifetimes.

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