Time-dependent approach to many-particle tunneling in one-dimension

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Employing the time-dependent approach, we investigate a quantum tunneling decay of many-particle systems. We apply it to a one-dimensional three-body problem with a heavy core nucleus and two valence protons. We calculate the decay width for two-proton emission from the survival probability, which well obeys the exponential decay-law after a sufficient time. The effect of the correlation between the two emitted protons is also studied by observing the time evolution of the two-particle density distribution. It is shown that the pairing correlation significantly enhances the probability for the simultaneous diproton decay.

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I. INTRODUCTION

The quantum tunneling decay phenomena can be studied either with the time-independent approach [10, 24, 25] or the time-dependent approach [34–39]. In the time-independent approach, one seeks e.g., a Gamow state, which is a purely outgoing wave outside the barrier. The imaginary part of the energy of the Gamow state is related to the decay width, while the real part corresponds to the resonance energy. On the other hand, in the time-dependent approach, one first modifies the potential barrier so that the initial state can be prepared as a bound state of a confining potential. The confinement potential is then suddenly changed to the original barrier, and the initial state evolves in time. The decay width can be obtained from the survival probability of the initial state. An advantage of the time-dependent approach is that the decay width can be calculated with high accuracy even when the decay width is extremely small [40]. An advantage of the time-dependent approach, on the other hand, is that it provides an intuitive way to understand the tunneling decay, even though it may be difficult to apply it to a situation with an extremely small decay width. This approach may provide a useful means to explore the mechanism of many-particle tunneling decay, though it has so far been applied only to two-body decay phenomena, such as α decays and one-proton decays [34–38].

In this paper, we extend the time-dependent approach to two-proton emissions and discuss the dynamics of a two-particle tunneling decay. To this end, we employ the one-dimensional three-body model [41], which consists of a heavy core nucleus and two valence protons. We solve the time-dependent Schrödinger equation by expanding the wave function on a basis with time-dependent expansion coefficients. The decay width is then defined from the decay width and the survival probability. In the case of 2p decay, this corresponds to the pairing correlation between the valence protons. Because of the strong pairing correlation in proton-rich nuclei, 2p emitters are expected to have an even number of protons outside the proton drip-line. Incidentally, it has been well known that the pairing correlation plays an important role in two-neutron transfer reactions [31–33].

Recently, two-proton (2p) radioactivities have been experimentally observed for a few proton-rich nuclei outside the proton drip-line, such as 45Fe [16–18] and 9Be [13–21], and have attracted much attention in connection to e.g., the dinucleon correlations [26–29]. This is a phenomenon of spontaneous emission of two valence protons from the parent proton-rich nuclei, in which the emission of one proton is energetically forbidden. Notice that an analogous process of the two-proton radioactivity, that is, a two-neutron decay has also been observed recently for 16Be [20]. These two-nucleon emission decays may provide a useful testing ground of many-particle tunneling theories.

A primary task for a many-particle tunneling decay is to investigate the effect of interaction or correlation among emitted particles on decay properties such as the

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the survival probability. We shall study the time evolution of the survival probability as well as the density distribution. We shall also discuss the role of pairing correlation in the two-proton decay.

The paper is organized as follows. In Sec. II we detail our formalism for the time-dependent approach to the two-proton decays. We show the results of the calculations in Sec. III. It will be shown that the decay width converges to a constant value after a sufficient time evolution, indicating that the decay rate follows the exponential law. With this method, the time-evolution of a quasi-stationary $2p$ state, namely the density and the flux distributions, can be visualized. Using them, we shall discuss an important role of the pairing interaction between the two protons in the decay process. Finally we summarize the paper in Sec. IV.

II. FORMALISM

A. One-dimensional three-body model

We consider a one-dimensional three-body system with two valence protons and the core-nucleus whose atomic and mass numbers are $Z_c$ and $A_c$, respectively. Neglecting the recoil kinetic energy of the core nucleus, the three-body Hamiltonian reads \[ H = h(x_1) + h(x_2) + v_{pp}(x_1, x_2), \] where $m$ is the nucleon mass, $x_1$ and $x_2$ are the coordinates of the valence protons with respect to the core nucleus. $V(x)$ is the potential between a valence proton and the core, whereas $v_{pp}(x_1, x_2)$ is the interaction between the two valence protons. The core-proton potential, \[ V(x) = V_{\text{nucl}}(x) + V_{\text{coul}}(x), \] consists of the nuclear part $V_{\text{nucl}}$ and the Coulomb part $V_{\text{coul}}$. For the nuclear part, we take a Woods-Saxon form, \[ V_{\text{nucl}}(x) = - \frac{V_0}{1 + e^{(|x|-R)/a}}, \] where \( g \) is the strength of the interaction and $\bar{x} = (x_1 + x_2)/2$. The density dependence is introduced with the Woods-Saxon form ($R$ and $a$ are the same as those in Eq. (4)). Notice that in the limit of $|\bar{x}| \to \infty$, this interaction becomes a pure contact interaction, $-g\delta(x_1 - x_2)$. For simplicity, we neglect the Coulomb interaction between the two protons. We have confirmed that, as long as the one-dimensional three-body system is concerned, its effect on the decay properties can be well taken into account by somewhat reducing the strength $g$ (see also Refs. [23, 44, 45]).

The interaction $v_{pp}(x_1, x_2)$ induces the pairing correlation between the two valence protons. In this work, we adopt a density-dependent contact interaction of the surface type [41], that is,

\[ v_{pp}(x_1, x_2) = -g \left( 1 - \frac{1}{1 + e^{(|\bar{x}|-R)/a}} \right) \delta(x_1 - x_2), \]

Fig. 1: (Color online) The core-proton potential used in our calculations. The initial state at $t = 0$ is constructed with a modified potential $V_{\text{mod}}(x)$ given by Eq. (7), which is shown by the dashed line. For $t > 0$, the potential is changed to the original potential, $V(x)$, given by Eq. (8), as shown by the solid line. We also show the wave function for the bound state of the modified potential at $\epsilon = 2.71$ MeV.

B. Time-dependent method

In order to describe the $2p$ tunneling, we employ the time-dependent method. The first step is to prepare the initial $2p$ state which is confined inside the potential barrier, as in the two-potential method developed by Gurvitz.
In this paper, we assume that all the s.p. states with value of the modified potential at state at $\epsilon$ resembles the resonance state of the original potential. In this paper, we choose the modified potential holds a bound state, which is as close as possible to the resonance energy $E_0$. When this condition is satisfied, the modified potential $V_{\text{mod}}$ holds a bound state, which resembles the resonance state of the original potential. In this paper, we choose $x_c = 10 \text{ fm}$, which yields the bound state at $\epsilon = 2.71 \text{ MeV}$ for the modified potential. The value of the modified potential at $x = x_c$ is $V_{\text{mod}}(x_c) = 4.21 \text{ MeV}$, whereas the barrier height is $V_b = V(x = 8.1 \text{ fm}) = 4.75 \text{ MeV}$.

We solve the single-particle (s.p.) states of the modified Hamiltonian,

$$V_{\text{mod}}(x) = V(x) \begin{cases} |x| \leq |x_c|, & \text{for } x \leq x_c, \\ V(x_c) \begin{cases} |x| > |x_c|, & \text{for } x > x_c. \end{cases} \end{cases}$$

(7)

In Fig. 1, we show the confining potential, $V_{\text{mod}}$, and the original potential, $V$, together with the wave function for the bound state of $V_{\text{mod}}$. The position $x_c$ can be chosen arbitrarily as long as $V(x_c)$ is larger than the resonance energy, although the accuracy will be improved if $x_c$ is chosen so that $V(x_c)$ is as close as possible to the resonance energy $E_0$. When this condition is satisfied, the modified potential $V_{\text{mod}}$ holds a bound state, which resembles the resonance state of the original potential. In this paper, we choose $x_c = 10 \text{ fm}$, which yields the bound state at $\epsilon = 2.71 \text{ MeV}$ for the modified potential. The value of the modified potential at $x = x_c$ is $V_{\text{mod}}(x_c) = 4.21 \text{ MeV}$, whereas the barrier height is $V_b = V(x = 8.1 \text{ fm}) = 4.75 \text{ MeV}$.

We solve the single-particle (s.p.) states of the modified Hamiltonian,

$$h_{\text{mod}}(x) = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V_{\text{mod}}(x),$$

(8)

as

$$h_{\text{mod}}(x) \phi_n(x) = \epsilon_n \phi_n(x).$$

(9)

In this paper, we assume that all the s.p. states with negative energy, that is, $\epsilon_n < 0$, are occupied by the core nucleus. Therefore, there is only one bound state in this potential at $\epsilon = 2.71 \text{ MeV}$. The other positive energy states are in the continuum spectra, which we discretize with a box of $X_{\text{box}} = \pm 120 \text{ fm}$. We have confirmed that the bound state at $\epsilon = 2.71 \text{ MeV}$ corresponds to a resonance state of the original potential, whose energy is stabilized against a variation of $X_{\text{box}}$.

Using these s.p. wave functions, one can obtain the eigenfunctions of the modified three-body Hamiltonian,

$$H_{\text{mod}}(x_1, x_2) = h_{\text{mod}}(x_1) + h_{\text{mod}}(x_2) + v_{\text{pp}}(x_1, x_2),$$

(10)

as

$$\Psi_k(x_1, x_2) = \sum_{n_1 \leq n_2} \delta_{n_1, n_2} \Phi_{n_1, n_2}(x_1, x_2),$$

(11)

where we exclude those s.p. states occupied by the core nucleus. Here, $\Phi_{n_1, n_2}(x_1, x_2)$ is defined as

$$\Phi_{n_1, n_2}(x_1, x_2) = \frac{1}{\sqrt{(1 + \delta_{n_1, n_2})}} \left[ \phi_{n_1}(x_1) \phi_{n_2}(x_2) + \phi_{n_2}(x_1) \phi_{n_1}(x_2) \right] \left| S = 0 \right>.$$

(12)

Because we use spin-independent interactions in the Hamiltonian, the total spin $S$ of the two protons is a good quantum number. We set it to be zero (that is, the spin-singlet state). The spatial part of the $2p$ wave function is therefore symmetric under the exchange of $x_1$ and $x_2$. The coefficients $\delta_{n_1, n_2}$ in Eq. (11) are determined by diagonalizing the Hamiltonian matrix for the modified Hamiltonian, $H_{\text{mod}}$.

The next step is to carry out the time evolution for $t > 0$ starting from the lowest eigenfunction of the modified Hamiltonian, i.e., $\Psi_{k=0}$ in Eq. (11) at the eigenenergy $E_0$. That is,

$$\Psi(t = 0, x_1, x_2) = \Psi_0(x_1, x_2).$$

(13)

Excluding the core-occupied states in the expansion, we have confirmed that there is only one bound state for $H_{\text{mod}}$ in the energy region between 0 and $2V_{\text{mod}}(x_c)$. Similarly to a s.p. resonance state, the lowest state $\Psi_0$ corresponds to the three-body resonance state of the original Hamiltonian $H$. The energy $E_0$ corresponds to the resonance energy of the 3-body system, that is, the $Q$-value for the $2p$ decay.

We then solve the time-dependent Schrödinger equation with the original Hamiltonian $H$,

$$i\hbar \frac{\partial}{\partial t} \Psi(t, x_1, x_2) = H \Psi(t, x_1, x_2),$$

(14)

$$= (H_{\text{mod}} + \Delta V) \Psi(t, x_1, x_2),$$

(15)

where

$$\Delta V = V(x_1) + V(x_2) - V_{\text{mod}}(x_1) - V_{\text{mod}}(x_2),$$

(16)

is the difference between the original and the modified potentials. We expand the time-dependent $2p$ wave func-

![Fig. 2: (Color online) Overlaps between the initial state, $\Psi_0$, and the eigenfunctions $\Psi_k$ of the original three-body Hamiltonian as a function of the corresponding eigenenergies, $E_k$. The solid and the dashed lines correspond to the cases of $g = 20$ and $0 \text{ MeV} \cdot \text{fm}$, respectively. Notice that the initial state is the lowest eigenstate of the modified Hamiltonian, $H_{\text{mod}}$, with the eigenenergy of 4.56 MeV (5.42 MeV) for $g = 20 \text{ MeV} \cdot \text{fm}$ (g=0 MeV · fm).](image-url)
tion with the eigenfunctions of the modified Hamiltonian, that is, \( \Psi_k(x_1, x_2) \) given in Eq. (11), as
\[
\Psi(t, x_1, x_2) = \sum_k c_k(t) \Psi_k(x_1, x_2),
\]
with the initial condition of
\[
c_k(t = 0) = \delta_{k,0}.
\]
Substituting Eq. (17) into (15) and using the orthogonality of \( \Psi_k \), we obtain the differential equation for the expansion coefficients \( c_k(t) \),
\[
i\hbar \frac{dc_k(t)}{dt} = \langle \Psi_i | H | \Psi \rangle = \sum_k c_k(t) (E_i \delta_{i,k} + \langle \Psi_i | \Delta V | \Psi_k \rangle).
\]
Using the wave function so obtained, one can compute the survival probability, \( P_s(t) \), and the decay width, \( \Gamma \), as
\[
P_s(t) \equiv |\langle \Psi_0 | \Psi(t) \rangle|^2 = |c_0(t)|^2,
\]
\[
\Gamma = -\hbar \frac{\dot{P}_s(t)}{P_s(t)}.
\]

When the survival probability is an exponential function of \( t \), the decay width \( \Gamma \) becomes a constant. In the next section, we will show that \( P_s(t) \) indeed has an exponential form after a sufficient time evolution.

III. RESULTS

A. Decay energy and width

Before we numerically solve the time-dependent Schrödinger equation, let us first investigate the overlaps between the initial wave function \( \Psi_0 \) and the eigenfunctions of the original Hamiltonian \( H \). That is,
\[
Q_k \equiv |\langle \tilde{\Psi}_k | \Psi_0 \rangle|^2,
\]
where \( |\tilde{\Psi}_k \rangle \) is the eigenfunctions of the original Hamiltonian satisfying
\[
H |\tilde{\Psi}_k \rangle = \tilde{E}_k |\tilde{\Psi}_k \rangle.
\]
Figure 2 shows the overlaps \( Q_k \) as a function of \( \tilde{E}_k \) for \( g=0 \) and 20 MeV-fm. The initial state is fragmented over several eigenfunctions of the original Hamiltonian, and thus forms a wave packet which evolves in time. As one can see, the fragmentation of the initial state is small, where the energy spreading corresponds to the decay width.

Let us now numerically solve the time-dependent Schrödinger equation. To this end, we use a time mesh of \( \Delta t = 0.01 \text{ fm/c} \). Fig. 3 shows the survival probability and the decay width defined as Eqs. (21) and (22) as a function of time \( t \) for several values of \( g \). One can see that the decay width converges to a constant value after sufficient time-evolution, that indicates the exponential decay-law, \( P_s(t) = e^{-\Gamma t}/\hbar \). Notice that the converged values for the decay width with this model Hamiltonian are in the same order as the experimental width for \(^{13}\text{Be} \) and \(^{16}\text{Ne} \). At shorter period, the decay width shows a transient behaviour \( \tilde{\Delta} \). That is, the survival probability behaves like a parabolic function of \( t \), whereas the decay width increases linearly \( \tilde{\Delta} \).

The dependence of the decay width on the strength of the pairing interaction is shown in Fig. 4 (b). We also show in Fig. 4 (a) the decay energy \( E_0 \) (that is, the eigenenergy of the modified Hamiltonian given by Eq. (10)) and the asymptotic kinetic energy of diproton, \( E_{rel} \), defined as
\[
E_{rel} = E_0 + B_{pp},
\]
where \( B_{pp} = mg^2/4\hbar^2 \) is the binding energy of a diproton. The decay width is estimated at \( t = 1200 \text{ fm/c} \), where it has been well converged (see Fig. 3).

The decay width \( \Gamma \) first decreases as a function of \( g \), despite that the diproton kinetic energy \( E_{rel} \) increases. This should be related to the decrease of the decay energy \( E_0 \), indicating that the sequential two-proton emissions is the main decay mechanism in this region of \( g \) even...
though two protons are bound in this one-dimensional model. For $g \geq 18$ MeV·fm, on the other hand, the decay width increases. This is consistent with the increase of $E_{\text{rel}}$, suggesting that the direct diproton decay, that is the emission of a deeply bound diproton, is the main mechanism in this region.

The transition from the sequential to the diproton decays will be clarified more in the next subsection.

### B. Two-particle density and flux distributions

In order to confirm a transition from a sequential to a simultaneous decays discussed in the previous subsection, we next discuss the time evolution of two-particle density distribution,

$$
\rho(t, x_1, x_2) = \left| \Psi(t, x_1, x_2) \right|^2.
$$

We also analyze the flux distribution defined as,

$$
j_i(t, x_1, x_2) = \frac{\hbar}{2im} \left( \Psi^* \frac{\partial \Psi}{\partial x_i} - \frac{\partial \Psi^*}{\partial x_i} \Psi \right) \quad (j = 1, 2).
$$

Note that the 2p density is normalized as

$$
\int_{-\infty}^{\infty} \rho(t, x_1, x_2) dx_1 dx_2 = 1.
$$

Figs. 5 and 6 show the two-particle density and flux distributions, respectively, for $g = 20$ MeV·fm at $t = 0, 300$ and 600 fm/c. The corresponding quantities for $g = 0$ are also shown in Figs. 7 and 8. The flux distributions are plotted in a form of vector at each value of $(x_1, x_2)$ in the two-dimensional $(x_1, x_2)$ plane, where the core nucleus is located at the origin. Note that there is no flux distribution at $t = 0$ because the initial wave function $\Psi(t = 0)$ can be taken to be real, and we do not show it in the figures.

Figs. 5(a) and 6(a) show the density distribution for...
the initial 2p state, which is confined within the modified potential, $V_{\text{mod}}$. Because of the pairing correlation, the initial density for $g = 20 \text{ MeV} \cdot \text{fm}$ has an asymmetric form, with the peaks along $x_1 = x_2$ being higher than those along $x_1 = -x_2$ \cite{41}. The peaks along the $x_1 = x_2$ line, that is, in the first and third quadrants of these panels, correspond to a compact diproton cluster. On the other hand, the peaks along the $x_1 = -x_2$ line (i.e., in the second and the fourth quadrants) correspond to a configuration in which two protons are located opposite to the core nucleus. If we discard the pairing interaction, the density distribution has four symmetric peaks, as shown in Fig. 4, that is, the probability in the first and third quadrants is the same as that in the second and fourth quadrants \cite{41}.

The effect of pairing correlation is apparent also during the time evolution. In the presence of the pairing correlation, the extension of the two-particle density along the $x_1 = x_2$ line increases significantly, although the extension along the $x_1 = 0$ or $x_2 = 0$ lines is not negligible. This is in marked contrast with the uncorrelated case shown in Fig. 4, in which the two-particle density expands democratically. That is, in the uncorrelated case, the probability of emission of the two protons in opposite directions is equal to that in the same direction. The flux distribution shown in Figs. 6 and 8 also indicate the same behaviour.

In order to investigate the time evolution more quantitatively, we divide the $(x_1, x_2)$ plane into four regions shown in Fig. 9. That is, (i) the region of $x_1 > 16 \text{ fm}$ and $x_2 > 16 \text{ fm}$, as well as the region of $x_1 < -16 \text{ fm}$ and $x_2 < -16 \text{ fm}$, (ii) the region of $x_1 > 16 \text{ fm}$ and $x_2 < -16 \text{ fm}$, as well as the region of $x_1 < -16 \text{ fm}$ and $x_2 > 16 \text{ fm}$, (iii) the region of $-16 \leq x_1 \leq 16 \text{ fm}$ and $|x_2| > 16 \text{ fm}$, as well as the region of $-16 \leq x_2 \leq 16 \text{ fm}$ and $|x_1| > 16 \text{ fm}$, and (iv) the rest in the $(x_1, x_2)$ plane. At each time,
we integrate the two-particle density distribution in each region,
\[ P_k(t) = \int_{\text{region } k} \rho(t, x_1, x_2) dx_1 dx_2, \quad (k = 1 \sim 4). \] (29)

The time evolution of these partial probabilities is shown in Fig.10 for \( g = 0, 20 \) and \( 32 \) MeV · fm. We also show the total decay probability, that is, \( 1 - P_3 = P_1 + P_2 + P_3 \).

We first discuss the behaviour for the uncorrelated case shown in Fig.10(a). In this case, the dominant process is the decay into the third region, \( P_3 \), which corresponds to an emission of one of the valence protons while the other proton remains inside the core-proton potential. As there is no active s.p. bound state in the present core-proton potential, the second proton mainly occupies the s.p. resonance state. This resonance state eventually decays and the second proton is emitted outside the potential after a sufficient time-evolution. This is nothing but the sequential two-proton decay, and can be clearly seen in the probabilities \( P_1 \) and \( P_2 \), that exist only at \( t \gtrsim 600 \) fm/c. Notice that \( P_1 \) and \( P_2 \) are identical to each other, since the second proton is emitted into either the left or the right direction with respect to the core nucleus with an equal probability irrespective to the position of the first proton.

For \( g = 20 \) MeV · fm shown in Fig.10(b), the probability in the region (i) increases considerably due to the pairing correlation, while \( P_3 \) decreases significantly. This partly corresponds to an emission of a bound diproton, that is, the simultaneous two-proton decay. Notice, however, that \( P_3 \) is still larger than \( P_1 \) at \( t \lesssim 800 \) fm/c, and a sequential decay also coexists for this value of \( g \).

As we have shown in Fig.4 the total decay probability, \( P_1 + P_2 + P_3 \), decreases compared to the uncorrelated case.

When the pairing interaction is even stronger, the simultaneous diproton decay becomes dominant. See Fig.10(c) for \( g = 32 \) MeV · fm. In this case, \( P_1 \) is the dominant part of the total decay probability, except for the short time region, at which the high energy components in the initial wave function quickly escape from the potential barrier. The long-time behaviour in this case may correspond to alpha decays in realistic nuclei, for which a tightly bound alpha particle tunnels through the Coulomb barrier of the daughter nucleus.

From these studies, it is evident that the present one-dimensional three-body model nicely describes a transition from an uncorrelated case to a strongly correlated case for many-particle tunneling decays.

**IV. SUMMARY**

We have employed the time-dependent method and investigated many-particle tunneling decays, particularly the two-proton radioactivity. To this end, we have used a one-dimensional three-body model which consists of a core nucleus and two valence protons. In order to describe the decaying process, we first confined the two-proton wave function inside a confining potential. The confining potential was then changed to the original potential in the region (i) increases considerably due to the pairing correlation, while \( P_3 \) decreases significantly. This partly corresponds to an emission of a bound diproton, that is, the simultaneous two-proton decay. Notice, however, that \( P_3 \) is still larger than \( P_1 \) at \( t \lesssim 800 \) fm/c, and a sequential decay also coexists for this value of \( g \). As we have shown in Fig.4 the total decay probability, \( P_1 + P_2 + P_3 \), decreases compared to the uncorrelated case.

When the pairing interaction is even stronger, the simultaneous diproton decay becomes dominant. See Fig.10(c) for \( g = 32 \) MeV · fm. In this case, \( P_1 \) is the dominant part of the total decay probability, except for the short time region, at which the high energy components in the initial wave function quickly escape from the potential barrier. The long-time behaviour in this case may correspond to alpha decays in realistic nuclei, for which a tightly bound alpha particle tunnels through the Coulomb barrier of the daughter nucleus.

From these studies, it is evident that the present one-dimensional three-body model nicely describes a transition from an uncorrelated case to a strongly correlated case for many-particle tunneling decays.
time evolution of the density and flux distributions. We have also analyzed the partial probabilities and discussed the relative importance of the sequential and simultaneous two-proton decays. We have shown that, for the uncorrelated case, the sequential decay is the dominant decay process, while the simultaneous decay plays an essential role in the case of strong pairing correlation. For an intermediate value of the pairing strength, we have shown that both the simultaneous and the sequential two-proton emission coexist.

The one-dimensional three-body model which we employed in this paper is a simple schematic model, with which a deep understanding of many-particle decay process may be obtained. One drawback, however, is that two protons are inevitably bound even with an infinitesimal attraction between the two protons. Even though our model nicely demonstrates the coexistence of the simultaneous and sequential decays for an intermediate pairing interaction, in reality two protons are never bound in vacuum. It will be an important task to extend the present study to realistic two-proton emitters in three-dimension, such as $^6$Be and $^{16}$Ne nuclei. A work towards this direction is in progress, and we will report on it in a separate publication.

Fig. 10: (Color online) The time-evolution of the partial probabilities in the regions defined in Fig. for $g = 0, 20,$ and $32$ MeV-fm. The total decay probability, $P_{\text{tot}} \equiv 1 - P_4 = P_1 + P_2 + P_3$, is also shown by the dot-dashed lines.

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