Estimating star distances with a light bulb

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Abstract
A practical experiment is described to estimate the distance to a star using simple equipment, suitable for secondary or undergraduate students. The brightness of the star is visually matched to a miniature filament lamp, and its distance inferred from the inverse square law. Students gain an appreciation of astronomical units and practice in manipulating powers of 10. By additionally applying knowledge of the H-R diagram and black body radiation, accuracy can be improved to one order of magnitude.

Keywords: astronomy, inverse-square, naked-eye, black body, astrophysics, distance, star

1. Introduction
Measuring the distance to a star with school equipment seems an unlikely challenge, but it can be done quite simply, and with surprising accuracy. Students estimate the brightness of a star relative to the Sun, and then calculate its distance using the inverse square law. Huygens reported using this method in his book Cosmotheoros: The Sun therefore being ... removed so far from us as to make his Diameter but the 27 664 part of that we every day see, will send us still the same Light as the Dog-star now doth. Seeing then Sirius is supposed equal to the Sun, it follows ... that his distance to the distance of the Sun from us is as 27 664 to 1' [1].

2. Method
On a dark clear night, set a small incandescent lamp on a pole and walk away until it looks the same brightness as a star. The lamp filament and the star radiate for the same reason: because they are hot. The power radiated by the lamp is equal to the electrical power input; students measure \( V \) and \( I \) and calculate \( P = V \times I \). Because the Sun is a star, the power of the star will be similar to that of the Sun: \( 4 \times 10^{26} \) W. The star is more powerful than the bulb, but looks the same brightness because it is much further away. Using the inverse square law to take the ratio of the approximately equal fluxes leads to:

\[
\frac{(\text{Star distance})^2}{(\text{Bulb distance})^2} = \frac{\text{Star power}}{\text{Bulb power}} = \frac{4 \times 10^{26} \text{ W}}{P_{\text{bulb}}}
\]

(1)
Typical results are $P_{\text{bulb}} = 0.13 \text{ W}$ and $D_{\text{bulb}} = 70 \text{ m}$, in which case $D_{\text{star}}$ would be estimated to be $4 \times 10^{15} \text{ m}$.

2.1. Ignorance and accuracy

Before working up the readings, express the recorded distance to the bulb as (say) $7 \times 10^{1} \text{ m}$, write the unknown star distance as $x \times 10^{y} \text{ m}$, and discuss the values expected for $x$ and $y$. The point being that really, we have no idea. As Socrates said at his trial:

‘For probably neither of us knows anything noble and good, but he supposes he knows something when he does not know, while I, just as I do not know, do not even suppose that I do.’

This is not a precise measurement of stellar distance, but an estimate, which we might hope to be accurate to within a few orders of magnitude. But it is science (not myth or assertion) because it is based on measurements and mathematics.

We now know that Huygens was out by a factor 20, but he still made a significant step forward. By doing this for themselves, students observe the stars, manipulate powers of ten, and find that one significant figure can sometimes be more than enough. Once the method is understood, it is natural to identify and minimise errors. Students who are aware of the H-R diagram and black body radiation can eliminate the main sources of error, improving the accuracy of the distance estimate to around one order of magnitude.

2.2. Hertzsprung–Russell diagram

Huygens guessed that all stars were of similar brightness, but in fact there are large variations—Sirius, for example, is $25 \times$ brighter than the Sun. Students can see this by observing the Pleiades, which is evidently a cluster of stars at a common distance, but not all the same brightness. The Pleiades were in fact used by Hertzsprung to plot the first H-R diagram (and are still used today as a reference star cluster!). To allow for this variability, students can use the H-R diagram to pick a star similar in temperature and luminosity to the Sun.

In figure 1, the bright northern hemisphere stars which best match the Sun are Procyon and Altair; these are well positioned for observing in autumn and spring. Modern astronomers still use apparent brightness as a way of comparing star distances, matching pairs of stars spectroscopically [2].

2.3. Blackbody radiation

The power of a black body is measured in Watts. The brightness of the black body, its visual magnitude $M_v$, is affected by the spectral sensitivity of the eye—which is aligned to the spectrum of radiation from the Sun. If the temperature of a source is different from the Sun, its emission spectrum will peak at a different wavelength, altering the luminous efficacy of the radiation. Visual magnitudes are obtained by adding a Bolometric Correction (BC) to adjust for this. For most stars the correction is quite small, but it is significant for hot or cool stars like Spica and Antares. The bulb filament is of course even cooler, and requires more correction. Luminous efficacy is calculated by multiplying the normalised spectral radiance at each wavelength by the spectral sensitivity of the eye, and summing the results over the visible range. To normalise to a total radiated power of 1 mW, the radiance values from Planck’s law are divided by the total emittance (Stefan’s law).

Spectral radiance:

$$P_{\lambda,T} = \frac{1}{\lambda^5} \times \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}. \quad (2)$$

Emittance:

$$E_T = \sigma T^4. \quad (3)$$

Normalised spectral radiance:
Estimating star distances with a light bulb

The luminous efficacy of a blackbody with respect to temperature is plotted in figure 2:

\[ \frac{P_{\lambda,T}}{E_T} \]  

(4)

The luminous efficacy of a blackbody with respect to temperature is plotted in figure 2:

Luminous efficacy peaks at 7000 K, at a value of 95 lumens per watt. For miniature filament bulbs, data sheets typically quote efficacies of 2.81 mW^-1 (~2200 K on the chart) [3]. So the ‘Bolometric Correction’ for the bulb is 95/2.8 = 34 times, or 3.8 stellar magnitudes.

2.4. Distance to Procyon

Applying this to determining the distance to Procyon, typical readings are: \( L_{\text{bulb}} \text{(apparent)} = 0.13/34 = 0.0038 \text{ W} \) and \( D_{\text{bulb}} = 70 \text{ m} \), so

\[ \frac{\text{(Star distance)}^2}{\text{(Bulb distance)}^2} = \frac{\text{Star power}}{\text{Bulb power}} = \frac{4 \times 10^{26}}{0.0038}. \]

(5)

⇒ The distance to Procyon is \( 2.27 \times 10^{16} \text{ m} \). This estimate is within an order of magnitude of the accepted value of \( 1.08 \times 10^{17} \text{ m} \). For astronomical estimates, this is a reasonable approximation.

3. Teaching notes

3.1. Observing

The observing site should be appraised in daylight: for safety, and to identify distance markers on the ground. 150 m of open space is needed, with few bright lights and a dark background such as trees. Generalised light pollution is tolerable, because it affects star and bulb equally, but bright lights (or the moon) are more localised. Binoculars can also be used. Comparison is easier if the target star is not too high in the sky, but it should be at least 20 degrees above the horizon to avoid atmospheric effects. If Procyon is not well placed, it is legitimate to use another star of the same brightness as a transfer standard.

The observing method—comparing by eye a star and elevated light bulb—may involve a significant amount of walking backwards in the dark. Great care should be taken not to trip, especially if the experiment is performed in a rural setting, where rabbit holes or other hazards are widespread.

The estimating nature of the experiment embodies a degree of uncertainty, so it is advisable to record the range of distances within which the star and bulb appear to have indistinguishably similar brightness. Distances can be measured by pace counting and comparing with a known landmarks 100 m apart.

3.2. Apparatus

The lamp power needs to be less than 0.3 W to keep \( D_{\text{bulb}} \) at manageable size: e.g. 5 V/20mA, 4.5 V/40mA. Manufacturers include Schiefer and CML; suppliers sometimes use the description ‘grain of wheat’ lamps. A 9 V PP3 battery provides a suitable power source, with a 0.25 W SOT resistor of around 220R to set the operating point, and for current sensing—see figure 3.
Table 1. SI units.

| Star  | Irradiance at observer Wm⁻² | Power L W | Distance m | Luminous efficacy l mW⁻¹ | Apparent brightness Sun = 1 |
|-------|-----------------------------|-----------|------------|--------------------------|----------------------------|
| Sun   | 1.4 × 10⁻²                   | 3.8 × 10²⁶| 1.5 × 10¹¹ | 93                       | 1                          |
| Procyon | 1.8 × 10⁻⁸                 | 2.5 × 10²⁷| 1.0 × 10¹⁷ | 95                       | 1.4 × 10⁻¹¹                 |
| Altair | 1.3 × 10⁻⁸                 | 4.1 × 10²⁷| 1.6 × 10¹⁷ | 91                       | 9.5 × 10⁻¹²                 |
| Bulb  | 2.1 × 10⁻⁶                 | 0.13      | 70         | 2.8                      | 4.7 × 10⁻¹¹                 |

Table 2. Astronomical units.

| Star   | Apparent mag m | Absolute bol. mag Mbol | Distance D pc | Bol. correction BC | Apparent visual mag mv |
|--------|----------------|-----------------------|---------------|-------------------|------------------------|
| Sun    | −26.7          | 4.8                   | 4.8 × 10⁻⁶    | 0.0               | −26.7                  |
| Procyon| 0.38           | 2.7                   | 3.5           | 0.0               | 0.38                   |
| Altair | 0.77           | 2.2                   | 5.1           | 0.0               | 0.77                   |
| Bulb   | −4.9           | 73                    | 2.3 × 10⁻¹⁵   | −3.8              | −1.1                   |

Table 3. Error sources.

| Error source               | Brightness x factor | Distance x factor | Notes                                      |
|----------------------------|---------------------|-------------------|--------------------------------------------|
| Power of Procyon/Altair    | 9                   | 3                 | Difference from the Sun                     |
| Atmospheric transmission   | 1.6                 | 1.3               | 0.5 stellar mags                           |
| Matching brightness by eye | 1.6                 | 1.3               | 0.5 stellar mags, affected by glare        |
| Bulb efficacy (l mW⁻¹)     | 1.3                 | 1.1               | Scatter in data sheet values                |
| Distance to bulb           | —                   | 1.1               | Pace-counting, 50 m measuring tape         |
| Bulb power in watts        | 1.1                 | 1.05              | DVM, resistor, battery droop               |

3.3. Astronomical units

If results are calculated using both SI and astronomical (Vega System) units (see tables 1 and 2), the experiment becomes a useful tool for understanding how these units relate to each other, similar to the approach in [9]:

\[ M_{bol} = -2.5 \times \log \frac{L}{L_0} \] with \( L_0 = 3.0128 \times 10^{26} \) W

\[ m = M + 5 \times \log D - 5 \]

\( \times \) (the inverse square law with \( D \) in parsecs)

\[ M_v = M_{bol} - BC. \]

In the tables, the stellar distances, their magnitudes and luminosity (denoted here as power) are accepted values [5–7]. Bold entries indicate the readings to be recorded by the students, and their ‘given’ starting assumptions.

3.4. Errors

The largest residual error is the actual power of the target star, which is assumed to be the same as the Sun. The H-R diagram figure 1 shows that the match between the ‘nearest neighbour’ stars like Procyon and Altair and the Sun is unlikely to be better than one order of magnitude. Other sources of error are listed in table 3.

To determine the accuracy needed for lamp voltage, students may wish to refer to figure 4.
plotted from data in [4]. Keeping close to the lamp rated voltage ensures the efficacy will be close to the 2.8 lumens per watt on the data sheets. Ensuring the battery keeps the bulb voltage stable during the experiment ensures readings are repeatable.

4. Extensions

4.1. Longer range

Frehinkers may want to extend the experiment by observing say a 100 W lamp at a longer range. The luminous efficacy for this type of lamp is 171 mW−1 but in this case, allowance also needs to be made for atmospheric transmission. Transmission varies with weather and distance according to Allard’s law [8], used in figure 5 to plot the transmission over a distance \( d \) for a meteorological range \( v \) of 10 nm (18.5 km, typical of clear weather).

E.g. for a 100 W lamp at 10 km, the apparent power would be: Apparent power = Power × luminous efficacy × Transmission = \( 100 \times \frac{17}{95} \times 0.2 = 3.6 \) W \( \approx 4 \) W and so:

\[
\frac{(\text{Star distance})^2}{(\text{Bulb distance})^2} = \frac{\text{Star power}}{\text{Bulb power}} \approx \frac{4 \times 10^{26} \text{ W}}{4 \text{ W}}.
\]

(6)

\( D_{\text{star}} \) would then be estimated as 10 km \( \times 10^{13} = 10^{17} \) m—close to the actual stellar distance distances in table 1.

4.2. First principles

Using the power of the Sun as a starting point makes the experimental logic intuitively simple. To work instead from first principles, students could determine the power of the Sun themselves by measuring its irradiance at the surface of the earth. This is the solar constant, \( G_{SC} \approx 1 \text{ kW m}^{-2} \), which can be measured using the equation for heat energy: \( cm \Delta T \); a plastic bottle of water, and a thermometer. The total power radiated by the Sun is then \( L = G_{SC} \times 4\pi \times (1\text{AU})^2 \). The current experiment then estimates star distances in AU (or parsecs): following Huygens.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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David Craig grew up in Aberdeen, Scotland, where he gained a BSc in Natural Philosophy and taught Physics at Robert Gordon’s College. He moved to Edinburgh to become an avionics engineer with Ferranti, retiring 36 years later from what was then called Leonardo in 2016. He now helps encourage young people into Physics and Engineering with the SCDI’s Young Engineers and Science Clubs, and with the IoP in Scotland.

An undergraduate astrophysicist with a love of stars and penchant for daydreaming, Lewis Leslie grew up in the remote outer islands of Orkney, where the dark night skies inspired a passion for astronomy.