The ground states of baryoleptonic Q-balls in supersymmetric models

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In supersymmetric generalizations of the Standard Model, all stable Q-balls are associated with some flat directions. We show that, if the flat direction has both the baryon number and the lepton number, the scalar field inside the Q-ball can deviate slightly from the flat direction in the ground state. We identify the true ground states of such nontopological solitons, including the electrically neutral and electrically charged Q-balls.

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I. INTRODUCTION

Nontopological solitons, Q-balls appear in theories with some scalar fields carrying a global conserved quantum number when the scalar potential meets the conditions for Q-ball stability with respect to a decay into scalar particles [1–3]. These conditions are satisfied in supersymmetric generalizations of the Standard Model for the scalar fields carrying the baryon and lepton numbers [4]. In theories with gauge-mediated supersymmetry breaking, Q-balls can be stable with respect to decay into both scalar particles and fermions if the vacuum expectation value (VEV) inside the Q-ball corresponds to a flat direction [5]. These stable objects can form in the early universe from the fragmentation of the Affleck–Dine [6–8] condensate, and they can constitute all or part of cosmological dark matter [9]. The role of supersymmetric Q-balls in cosmology and astrophysics has been the subject of many studies [10–38]. However, it was always assumed that the scalar field that makes up the Q-ball has its VEV aligned exactly along the flat direction. In this paper we will show that, for a class of flat directions, this is not the case, and we will describe the true ground states of the corresponding Q-balls.

What makes the Q-ball stable is a combination of the baryon number conservation and the energy conservation. By construction, a Q-ball has a lower mass than a collection of the scalar particles carrying the same baryon number. However, one should also consider the possibility of a decay into fermions [39]. Very large Q-balls are likely to form from the fragmentation of the AD condensate [9, 36, 40, 41]. For such large Q-balls with the baryon number $Q_B$, the mass per unit baryon number decreases with $Q_B$ as $Q_B^{-1/4}$. When the mass per unit baryon number is below $m_p$, the proton mass, the Q-ball has a lower mass than a collection of protons and neutrons with the same baryon number. Such a Q-ball cannot decay. The same reasoning would not apply to a purely leptonic Q-ball, because the lightest fermions carrying the lepton number, neutrinos, have very small masses, one of which can be arbitrarily close to zero.

However, in supersymmetric models the scalar fields can carry both the baryon number and the lepton number. Many flat directions that admit Q-ball solutions have both the baryon and lepton components related to each other by the requirement that the so-called D-terms in the potential vanish. All the previous analyses of supersymmetric Q-balls focused on the baryonic squark component, assuming that the slepton fields just follow the squarks. This assumption is reasonable, as long as one considers general features of the stable Q-balls. There is an energetic penalty for deviations of the leptonic and baryonic components. However, this energetic penalty may not be sufficient to prevent the emission of a limited number of neutrinos and electrons from a large baryoleptonic Q-ball. Such emission can alter the ground state of the relic Q-balls. In particular, it can give the electric charge to an otherwise neutral Q-ball.\footnote{Electric neutrality is a necessary condition for the cancellation of the D-terms.} Even one unit of electric charge makes a dramatic difference in the experimental signatures of the relic Q-balls [18]. For example, the neutral Q-balls can be detected by Super-Kamiokande [18, 42, 43], while this detector is not well suited for detection of the charged Q-balls. On the other hand, the charged Q-balls could be detected by such detectors as MACRO [18, 44] in some range of parameters. The astrophysical limits on the relic Q-balls may depend on the electric charge [17, 27, 28]. It is, therefore, important to understand the ground states of Q-balls for the purposes of their detection [18, 42–46].

The paper is organized as follows. In section II we review the basic properties of flat-direction Q-balls in the MSSM. In section III we consider the emission of neutrinos from a flat direction Q-ball associated with the $QLd$ flat direction. In section IV we include the contributions of higher-dimensional operators. In section V we calculate the emission of electrons that is energetically allowed. In section VI we show that the relative rates of decay for neutrino and electron emission can favor a positively charged Q-ball.
II. FLAT-DIRECTION Q-BALLS

Minimal Supersymmetric Standard Model (MSSM) has a large number of flat directions [47]. Let us begin by considering a toy model that will capture the main features of Q-balls in the MSSM.

A flat direction in general can be parameterized by a single scalar degree of freedom, although for many flat directions the relation between the gauge eigenstates and the flat direction parameter can be non-trivial [48]. A Q-ball can form along a flat direction parameterized by a squark field $q$ and a slepton field $L$. A Q-ball is a nontopological soliton which minimizes the energy under the constraint of a fixed global charge [1–3]. Thus we minimize along a flat direction parameterized by a squark field $q$ and a slepton field $L$

\[ E = \int d^3x \left[ \frac{1}{2} q^2 + \frac{1}{2} |\nabla q|^2 + \frac{1}{2} \dot{L}^2 + \frac{1}{2} |\nabla L|^2 + U(q, L) \right], \]

while keeping the the baryonic and the leptonic charges separately conserved:

\[ Q_B \equiv \frac{1}{2i} \int d^3x (q \partial_0 q^* - q^* \partial_0 q) \]

\[ Q_L \equiv \frac{1}{2i} \int d^3x (L \partial_0 L^* - L^* \partial_0 L) . \]

One can solve to the above extremization problem by generalizing the method of Lagrange multipliers as used in Ref. [49] to the case of multiple conserved quantum numbers $Q_a$. One looks for a minimum of

\[ E_{\omega_L, \omega_q} = E + \sum_{a=\ell, b} \omega_a \left[ Q_a - \frac{1}{2i} \int d^3x (a \partial_0 a^* - a^* \partial_0 a) \right] \]

We use the same approach as in the single field case [49] to derive the time-dependence of the fields which comprise the Q-ball. Namely, we exploit the fact that we can rewrite eq. (4) as

\[ E_{\omega_L, \omega_q} = \int d^3x \left[ \frac{1}{2} |\dot{L} - i\omega_L L|^2 + \frac{1}{2} |\dot{q} - i\omega_q q|^2 + \frac{1}{2} |\nabla q|^2 \right] \]

\[ + \frac{1}{2} |\nabla L|^2 + \dot{U}(q, L) \right] + \omega_L Q_L + \omega_q Q_B, \]

where $\dot{U} \equiv U(q, L) - (1/2)\omega_L^2 L^2 - (1/2)\omega_q^2 q^2$. Since the first two terms are non-negative definite and no other term has an explicit time dependence, one can set them to zero by choosing

\[ L(r, t) = L(r) e^{i\omega_L t}, \]

\[ q(r, t) = q(r) e^{i\omega_q t}. \]

For definiteness, let us now consider the flat direction for which $L = q$, with the potential given by $U(L, q) = U_0 + \lambda |L|^2 - q^2|^2$, where $U_0$ is a constant. The value of $\lambda$ has no effect on the Q-ball solution with $\omega_L = \omega_q \equiv \omega$ along the flat direction, which, in the thick-wall regime [49], can be approximated as follows:

\[ L(r) = \begin{cases} 
L_0 \sin(\omega r)/|\omega r|, & r \leq R \\
L_1 \exp(-m_L r), & r > R 
\end{cases} \]

\[ q(r) = \begin{cases} 
q_0 \sin(\omega r)/|\omega r|, & r \leq R \\
q_1 \exp(-m_q r), & r > R 
\end{cases} \]

Here the constants $L_1, q_1$ and $R$ are chosen so that they minimize $E_{\omega}$, while the solutions are continuous at $r = R$. We have set the radii of the leptonic and the baryonic components equal to each other, which results in the lower value of $E_{\omega}$. For the flat direction $L_0 = q_0$. The exponential tails in eq. (7) give a negligible contribution to most of the quantities we compute below; in most cases one can assume that the solution vanishes at $r = R$. We will use this approximation in what follows.

Using the profiles of eq. (7) in the expression for the leptonic and baryonic charges (2), we get, approximately,

\[ Q_L = \frac{2\pi^2}{\omega^2} |L_0|^2, \]

\[ Q_B = \frac{2\pi^2}{\omega^2} |q_0|^2, \]
which must be equal by the $L_0 = q_0$ constraint.

The mass of the Q-ball and the frequency $\omega$ can be found from the minimization procedure described above:

$$M(Q_L, Q_B) = \frac{4\sqrt{2\pi}}{3} U_0^{1/4} (Q_L + Q_B)^{3/4},$$  \hspace{1cm} (10)

$$\omega(Q_L, Q_B) = \sqrt{2\pi U_0^{1/4}} (Q_L + Q_B)^{-1/4}. \hspace{1cm} (11)$$

One can also estimate the radius $R$ of the flat-direction Q-ball by neglecting the exponential tail in eq. (7) and setting $L(R) \approx 0$. This yields

$$R(Q_L, Q_B) = \frac{\pi}{\omega(Q_L, Q_B)} = \frac{1}{\sqrt{2}} U_0^{-1/4} (Q_L + Q_B)^{1/4}. \hspace{1cm} (12)$$

eqns. (10)-(12) summarize the general features of the Q-ball formed along a flat direction.

### III. EMISSION OF NEUTRAL LEPTONS

As discussed in the introduction, the flat-direction Q-balls in MSSM are stabilized by the baryon number and energy conservation, but an emission of some small number of leptons is an interesting possibility. Let us now consider a flat-direction Q-ball carrying some lepton number and some baryon number, and decaying into an off-flat-direction Q-ball plus some number of fermionic leptons. In this subsection we consider only the neutrino emission, so that we can ignore Coulomb effects. We label all initial Q-ball values (the FD values) with an $i$ and all final state Q-balls with an $f$.

We assume that the emission of fermions is a small perturbation from the flat direction case, so that the ansatz $\phi(r) = \sin(\omega r)/(\omega r)$ for both squarks and sleptons still holds. Although away from the direction, the value of mass per unit charge in the condensate $\omega$ is different from its flat-direction value, the equality of squark and slepton $\omega$'s still holds for energy reasons. This can be seen by setting $\omega_q \equiv \omega$, $\omega_L \equiv \omega + \delta\omega$ in eq. (5) and minimizing with respect to $\delta\omega$. Then the energy is

$$E_{\omega, \delta\omega} = (\omega + \delta\omega) Q_L + \omega Q_B + \int d^3x \left[ \frac{1}{2} |\nabla q|^2 + \frac{1}{2} |\nabla q|^2 + \hat{U}(q, L) \right]. \hspace{1cm} (13)$$

We note that the gradient term for each field cancels exactly with the quadratic term hidden in $\hat{U}(q, L)$, so that

$$E_{\omega, \delta\omega} = (\omega + \delta\omega) Q_L + \omega Q_B + \lambda \int d^3x |L^2 - q^2|^2. \hspace{1cm} (14)$$

The integral in the above expression is equal

$$I(\delta\omega) = \frac{\lambda}{\pi^3} \int_0^{\pi/\omega} \frac{r^2}{q^2} \left[ \left( \frac{\sin(\omega r)}{\omega r} \right)^2 - L^2 \left( \frac{\sin(\omega + \delta\omega) r}{(\omega + \delta\omega) r} \right)^2 \right]^2 r^2 dr. \hspace{1cm} (15)$$

We use the definition of the U(1) charges in eq. (2) and take $Q_L \approx Q_B$ in the off-flat-direction state to write

$$I(\delta\omega) = \frac{\lambda Q_B^2}{\pi^3} \int_0^{\pi/\omega} \left[ (\sin(\omega r))^2 - (\sin(\omega + \delta\omega) r)^2 \right]^2 \frac{dr}{r^2}. \hspace{1cm} (16)$$

Next, we evaluate $I(\delta\omega)$ using $\sin(\omega + \delta\omega) r \approx (\delta\omega r) \cos \omega r + \sin\omega r$. Keeping terms to $\mathcal{O}(\delta\omega)$ gives

$$I(\delta\omega) = \frac{4\lambda Q_B^2 \delta\omega}{\pi^3} \int_0^{\pi/\omega} \sin^2 \omega r r \cos^2 \omega r dr \hspace{1cm} (17)$$

Minimization of $E_{\omega, \delta\omega}$ with respect to $\delta\omega$ yields

$$\delta\omega = \frac{\pi^2 \omega}{\lambda Q_B}, \hspace{1cm} (18)$$
For realistically large values of $Q_B$ [9], $\delta \omega$ is very small compared to $\omega$ which means $\omega_L \approx \omega_B$. We will neglect the difference between $\omega_L$ and $\omega_B$ in the remainder of the paper.

Let us now evaluate the final state (off-flat-direction) Q-ball mass

$$M_f = \omega_f (Q_f + Q_B) + \frac{4\pi U_0}{3\omega_f^3} + \lambda \int d^3 x |L^2 - q^2|^2$$

Although this is a $(B + L)$-ball, we are interested only in allowing the sleptonic part to change its lepton number. Thus to ensure that $Q_B$ does not change as we vary $L$, we demand that

$$\frac{q_i^2}{\omega_i^2} = \frac{q_f^2}{\omega_f^2}.$$  \hspace{1cm} (20)

We recall that on the flat direction $L_i = q_i$, but this does not hold away from the flat direction. To calculate the potential away from the flat direction, we use

$$|L^2 - q^2|^2 = \left( \frac{\sin \omega r}{\omega r} \right)^4 |L_f^2 - q_f^2|^2. \hspace{1cm} (21)$$

Using the definition of the lepton charge, eq. (2), one can relate final and initial leptonic amplitudes:

$$L_f = \left( \frac{Q_f}{Q_i} \right)^{1/2} \left( \frac{\omega_f}{\omega_i} \right) L_i = \left( \frac{Q_f}{Q_i} \right)^{1/2} q_f.$$  \hspace{1cm} (22)

Using the flat-direction expression for slepton amplitude (2) and rewriting in terms of $N = Q_i - Q_f$, we obtain a simple approximate expression for the potential term:

$$\lambda \int d^3 x |L^2 - q^2|^2 = \frac{\lambda \alpha_4 \omega_f}{\pi^3} N^2,$$ \hspace{1cm} (23)

where $\alpha_4 \approx 0.67$ is a numerical constant defined by

$$\alpha_n \equiv \int_0^\pi \left( \frac{\sin x}{x} \right)^n x^2 dx.$$ \hspace{1cm} (24)

In particular, $\alpha_4 = Si(2\pi) - \frac{\pi}{2} Si(4\pi) \approx 0.67$ and $\alpha_6 \approx 0.39$. Using this expression, we can carry out the minimization of eq. (19) and find that the Lagrange multiplier $\omega$ is

$$\omega_f = \left( \frac{4\pi^7 U_0}{\pi^3 (Q_f + Q_B) + \lambda \alpha_4 N^2} \right)^{1/4} = \omega_i \left( \frac{1}{1 + N/Q_i} \right)^{1/4} \left( \frac{1}{1 + \lambda \alpha_4 N^2/\pi^3 Q_f} \right)^{1/4}.$$ \hspace{1cm} (25)

Thus in the limit $Q_i, Q_f \gg N$, we see that $\omega_f \approx \omega_i$. We therefore take $\omega_f = \omega_i$ in the following. We will verify the self-consistency of this assumption ex post facto. Using the value of $\omega_f$ away from the flat direction, one can compute the final state Q-ball mass as well as the number of neutrinos emitted in the transition from flat-direction to off-flat-direction states. Since the final state Q-ball is lighter than the original flat direction Q-ball the excess energy is released in neutrinos:

$$\Delta M \equiv M_i - M_f = \omega_i N - \frac{\lambda \alpha_4 \omega_i}{\pi^3} N^2$$ \hspace{1cm} (26)

For the emission to be energetically allowed, one must have $\Delta M \geq N m_\nu$. This gives an upper bound on the number of neutrinos that can be emitted as the Q-ball transitions from the initial flat direction state to a state with the VEV that is slightly off the flat direction:

$$N \leq \left( \frac{\pi^3}{\lambda \alpha_4} \right) \left( 1 - \frac{m_\nu}{\omega_i} \right).$$ \hspace{1cm} (27)

For $\lambda \sim 0.1$ and an initial lepton number $Q_i = 10^{24}$ and $\omega_i \sim U_0^{1/4} Q_i^{-1/4} \sim 1$ MeV, as expected from cosmology [7–9], this gives the number of emitted neutrinos $N \lesssim 460$.

We note in passing that $m_\nu$ must be less than the $\omega(Q)$ for emission to be possible. The physical reason for it is clear, since $\omega$ can be thought of as the mass of a scalar particle in the condensate. In most models $\omega(Q) \gg m_\nu$ [7, 8].

The potential we have chosen in eq. (19) is of the form one often encounters for MSSM flat directions in gauge mediated models [5]. In particular, the results of this section apply directly to the flat directions $QdL$ and $QQQL$ [47].
Let us now analyze a realistic model in which we consider Q-ball formation along the $Q_1 \tilde{d}_2 L_1$ direction. This direction has been studied in detail by Dine, Randall and Thomas [50, 51] in the context of a gravity-mediated Affleck-Dine baryogenesis. This direction is useful for our purposes since it carries nonzero baryon and lepton numbers. It is useful in the AD scenario because it carries nonzero $B - L$ (in particular $B - L = -1$), which it must if one would like to have a nonzero baryon number after $B + L$ violating sphaleron processes go out of equilibrium. This flat direction may be parameterized by the complex field $\phi$ which is the VEV given to the squarks and sleptons fields

$$Q_1 = \frac{1}{\sqrt{3}} \left( \begin{array}{c} \phi \\ 0 \end{array} \right), \quad L_1 = \frac{1}{\sqrt{3}} \left( \begin{array}{c} 0 \\ \phi \end{array} \right), \quad \tilde{d}_2 = \frac{1}{\sqrt{3}} \phi,$$  

(28)

where color indices are suppressed and subscripts label generation. The scalar potential on the flat direction is identically zero in the supersymmetric limit, since the $F$-terms and $D$-terms along this direction vanish. This degeneracy is lifted however, by soft SUSY breaking terms, as well as by the higher-dimensional operators.

The SU(2) $\times$ U(1) D-terms are

$$U_D = \frac{g^2}{8} ((|Q_1|^2 - |L_1|^2)^2 + \frac{g^2}{72} (|Q_1|^2 - 3|L_1|^2 + 2|\tilde{d}_2|^2)^2),$$  

(29)

where $g = e/\sin\theta_w$ is the SU(2) coupling and $g' = e/\cos\theta_w$ is the U(1)$_Y$ coupling. When $Q_1^2 = L_1^2 = \tilde{d}_2 = \phi$ the D-terms in the potential vanish as they must.

The non-renormalizable superpotential along a flat direction has the form

$$W_{NR} = \frac{\lambda}{nM^{n-3}} X^n = \frac{\lambda}{nM^{n-3}} \phi^n,$$  

(30)

where $n = mk \geq 4$ and $X = \phi^m$ is a gauge invariant composite operator of the fields that make up the flat direction. For the case at hand $X = Q_1 \tilde{d}_2 L_1 = \phi^3$. $M$ is a large mass scale usually taken to be either the GUT or the Planck scale. This gives a contribution to the scalar potential

$$V_{NR} (\phi) = \frac{|\lambda|^2}{M^{2n-6}} (\phi^* \phi)^{n-1}.$$  

(31)

For the flat direction of interest, the lowest order contribution is for $n = 4$, so

$$V_{NR} (\phi) = \frac{|\lambda|^2}{M^2} (\phi^6).$$  

(32)

The additional contribution to $\Delta M$ can affect the decay of the flat direction Q-ball. The new contribution is

$$\Delta M \supset \int d^3x (V_{NR}(\phi_{FD}) - V_{NR}(\phi_G)),$$  

where $\phi_G$ and $\phi_{FD}$ are the field values in the ground state and flat direction configurations, respectively. Since both squark and slepton field amplitudes in this case are different from their flat-direction values, they both make a contribution to the non-renormalizable part of the potential:

$$\Delta M_{NR} = \frac{|\lambda|^2}{M^2} \int d^3x (|L_i|^6 - |L_f|^6) + (|q_i|^6 - |q_f|^6).$$  

(33)

Using the values for $L_f$ and $Q_f$ found from the previous section, and assuming $N \ll Q_i$, we obtain

$$\Delta M_{NR} = -\frac{|\lambda|^2 \alpha_6 \omega_1^2 Q_i^2 N}{24\pi^6 M^2}.$$  

(34)

This translates into the following bound on $N$

$$N \leq \frac{\pi^3}{\lambda \alpha_4} \left( 1 - \frac{m_{\nu}}{\omega_i} - \frac{|\lambda|^2 \alpha_6 \omega_1^2 Q_i^2}{24\pi^6 M^2} \right).$$  

(35)

Taking $\lambda \approx 0.1, M \approx 10^{18}$ GeV, we see that, with the inclusion of the non-renormalizable term, the neutrino emission becomes energetically forbidden for $Q_i \gtrsim 10^{25}$.

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2 The SU(3) D-terms vanish because the two squark fields, $Q$ and $d$, have the same amplitudes and the same value of $\omega$. 
V. CHARGED LEPTON EMISSION FOR QDL

In addition to the emission of neutrinos, there are, of course, other channels of decay that can alter the Q-ball’s lepton number, while leaving the baryon number unchanged. The transition from the flat direction Q-ball to the true ground state could be accomplished via the charged lepton emission, for example, via emission of electrons, muons, and tau leptons. As before, the emission is only possible as long as \( \omega \) is greater than the mass of the lepton. In most cases, the relic Q-balls have the value of \( \omega \) below the muon mass [7, 8]. However, the emission of electrons remains a possibility. One expects the number of charged leptons emitted from the Q-ball to be smaller than the number of emitted neutral leptons because the emission of a each electron adds electrostatic energy to the final state Q-ball. 

The primary difference in this case is that the final state Q-ball is electrically charged. Now the Q-ball mass with emitted neutral leptons because the emission of a each electron adds electrostatic energy to the final state Q-ball. 

A possibility. One expects the number of charged leptons emitted from the Q-ball to be smaller than the number of cases, the relic Q-balls have the value of \( \omega \) only the electrically charged decay channel. Taking \( \lambda = 0.1 \) and \( \omega_i \gg m_e \), we obtain the bound \( N \lesssim 280 \) on the number of electrons emitted when we consider only the electrically charged decay channel.

As before, one must have \( \omega(Q_i) \geq m_e \) for emission to be kinematically allowed. Since \( \omega_i \sim U_0^{1/4}Q_i^{-1/4} \), this implies \( Q_i \lesssim 10^{28} \). If this bound is not satisfied than electron emission is energetically forbidden. We note that the same constraint for the neutrino case would allow much larger values: \( Q_i \lesssim 10^{56} \) (here we have taken \( m_\nu \sim 0.1 \text{ eV} \)).

VI. COMPETING DECAY CHANNELS

The relic Q-balls can emit both neutrinos and electrons until they reach the true ground state describe above. In each particular case, the history of the AD condensate fragmentation and the subsequent collisions and evaporation of Q-balls should be considered. However, it is instructive to consider a simplified history of Q-ball formation. Let us assume that at some initial time the Q-ball is in its flat-direction state. This state would be the ground state of an infinitely large Q-ball because the mass per unit global charge becomes smaller than the mass of the lightest massive fermion in the limit \( Q \to \infty \). The AD condensate, from which the Q-balls form by fragmentation, can be thought of as a very large, superhorizon-size Q-ball. Hence, it is reasonable to take the flat direction as the initial state of a Q-ball and to consider relaxation of this state into the ground state by emission of fermions, such as neutrinos and electrons. The relative rates of decay determine the final electric charge of the Q-ball. Let us find the lowest energy state achieved when both decay channels are open. The Coulomb forces disfavor the electron emission. With both channels open, the condition for decay \( \Delta M \geq \sum_i N_i m_i \) becomes

\[
\omega \left( (N_e + N_\nu) - \frac{\alpha_4 \lambda (N_\nu + N_e)^4}{\pi^3} - \frac{3e^2 N_e^2}{20\pi^2} \right) \geq N_e m_e + N_\nu m_\nu. \tag{38}
\]

With two lepton decay modes, the energetic constraint does not give us a fixed upper bound on \( N_e \), but instead merely carves out an energetically allowed region in the \( N_e - N_\nu \) plane (See Fig. 1). This region in interesting in that it allows the electron emission of a few hundred. Of course, such a larger emission is not realistic since the rate of electron emission is expected to be much smaller than the rate of neutrino emission due to the Coulomb suppression. It is, therefore, the emission rates that determine the final ground state.

For a sufficiently large Q-ball the rate of decay into fermions is limited by the number of fermionic states that can form inside and cross the boundary per unit time [39]. The maximal rate of fermion emission from the surface of a Q-ball [39] is

\[
\frac{dQ}{dt} = -\frac{A\omega^3}{192\pi^2}, \tag{39}
\]
where $A$ is the Q-ball surface area. This rate is found by calculating the expectation value of the normal component $\langle \mathbf{n} \cdot \mathbf{j} \rangle$ of the lepton current $\mathbf{j}$ in the frequency range $0 \leq k \leq \omega$, as in Ref. [39]:

$$\langle \mathbf{n} \cdot \mathbf{j} \rangle = \frac{1}{(2\pi)^3} \int_0^{k_{\text{max}}} k^2 dk \int_0^1 \cos \theta d\cos \theta \int_0^{2\pi} d\phi = \frac{\omega^3}{192\pi^2}. \quad (40)$$

This calculation only applies to electrically neutral, massless neutrinos. To extend this result to massive, electrically charged leptons (electrons) one must take into account the Coulomb interactions that change the upper bound on the frequency. Let us denote this upper bound $k_{\text{max}}$. For massive, neutral particles $k_{\text{max}} = \sqrt{\omega^2 - m^2}$. Including the effect of Q-ball charging, we obtain

$$\omega = \sqrt{k_{\text{max}}^2 + m^2} + \frac{3e^2 Z_Q^2 \omega}{20\pi^2}, \quad (41)$$

where $Z_Q$ is the electric charge of the Q-ball (this is $N$ in previous notation). For any type of lepton emission the rate of fermion emission from the surface is

$$\frac{dQ}{dt} \sim \frac{A}{24\pi^2} k_{\text{max}}^3. \quad (42)$$

We can use eq. (42) to track the electron and neutrino emission, in order to determine where in the $N_e - N_\nu$ plane a Q-ball will end. We plot this trajectory in the $N_e - N_\nu$ plane as the solid curve in Fig. 1. The intersection of the energetically allowed boundary (dashed curve) and the dynamical path along which the Q-ball evolves (solid curve) gives the maximal electric charge the Q-ball can acquire starting from a (electrically neutral) flat-direction Q-ball. This upper bound is $N_e \leq 25$.

![FIG. 1: The region below the dashed curve contains the allowed parameters for the decays from FD Q-balls. The solid curve shows the trajectory in the $N_\nu-N_e$ plane determined by the respective decay rates into the electrons and neutrinos. Here we show a representative initial Q-ball charge, $Q = 10^{24}$.](image)

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3 Our upper limit is $\omega/2$, rather than $\omega$, since the model of Ref. [39] has a scalar decaying into two fermions.

4 In the massless limit we find only a slightly different value for the number of emitted electrons $N_e \leq \frac{\omega}{e} \sqrt{20/3} \approx 27$. 

VII. CONCLUSION

A large number of flat directions in MSSM and other supersymmetric models have both baryon number and lepton number simultaneously. Q-balls with VEVs along these flat directions can be entirely stable: they owe their stability to the fact that a collection of protons and neutrons with the same baryon number would have a larger mass than the Q-ball mass. However, the true ground state of such Q-balls can be different from the naive ground state used in the literature. In general, the lepton component of a mixed baryoleptonic Q-ball deviates from the flat direction. We have shown this by starting with the naive, flat-direction ground state and following its decay into the true ground state plus some number of leptons. The true ground state can be electrically charged, which can have important ramifications for the experimental search for relic Q-balls.

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