Abstract—Generation of realistic topologies plays an important role in determining the accuracy and validity of simulation studies. This study presents a discussion to justify why, and how often randomly generated adjacency matrices may not not conform to wireless topologies in the physical world. Specifically, it shows through analysis and random trials that, more than 90% of times, a randomly generated adjacency matrix will not conform to a valid wireless topology, when it has more than 3 nodes. By showing that node triplets in the adjacency graph need to adhere to rules of a geometric vector space, the study shows that the number of randomly chosen node triplets failing consistency checks grow at the order of \(O(\text{base}^3)\), where base is the granularity of the distance metric. Further, the study models and presents a probability estimate with which any randomly generated adjacency matrix would fail realization. This information could be used to design simpler algorithms for generating \(k\)-connected wireless topologies.

I. INTRODUCTION

Simulation studies can be easily setup for wired networks by generating a random adjacency matrix for modeling a random topology. As long as finite non-negative entries are chosen for the adjacency matrix, it could be used to represent a valid wired topology. However, in this paper we discuss how, and why this may not hold true in the case of wireless topologies. Specifically, this study addresses the following questions:

1) Correctness: Are randomly generated topologies always valid, if not under what conditions.
2) Frequency of Failure: What percentage of randomly generated matrices are invalid?
3) Dominant Failure Factor: What feature of the matrix decides the probability of the topology being invalid?
4) Implication: Using this understanding, we propose designing algorithms with a relatively direct approach for generating \(k\)-connected graphs.

Rest of the paper is organized as follows. Section II shows an example where a randomly generated adjacency matrix does not represent a wireless topology. Section III describes the problem statement, and present our approach for determination of valid matrices. Section IV presents a comparison of results from random trials with an approximation generated by our probability function. Finally, we present a brief conclusion.

II. DISCUSSION

A. Example Of An Invalid Wireless Topology

We first address the question of whether a randomly generated adjacency matrix can result in a non-realizable wireless topology. Figure 1 shows the positions of three previously mapped points A, B, and C in a one dimensional metric space (\(\mathbb{R}^1\)). For this problem, we consider that all nodes have similar radio capabilities and can communicate with each other only if they are within unit distance of each other. As per this condition, we have node B connected to nodes A and C.

Now consider a case where the adjacency matrix generating the topology in Figure 1 has an additional entry for a fourth node D, which has links to A and C but is not within coverage of node B. Such a wireless topology is physically not possible in the one dimensional metric space (\(\mathbb{R}^1\)). Note that this problem cannot be solved by using a different channel, since all the nodes will need to be on the same frequency to be connected\(^1\). It is also important to observe that this failure occurs even when we do not have any planarity constraints like requiring non-intersecting graph edges. Non-uniform radio coverage for nodes D and B also fails to solve the problem. This is because both nodes D and B need to be on the line, and a non-uniform radio coverage in either directions (left or right) will result in disconnection from nodes A and/or C.

This problem can be extended to all higher dimensions in metric spaces \(\mathbb{R}^n\), \(n > 0\), which could result in invalid physical topologies. The only factor that varies across these dimensions is the nature of the wireless coverage. In \(\mathbb{R}^3\), we consider a line of unit (manhattan) distance on each side of the node, in \(\mathbb{R}^2\), the coverage can be assumed in the form of a unit circle in the plane (euclidean distance), similarly, a unit sphere in

\(^1\)We refer to a connection between any nodes 1 and 2, as a general term to signify that 1 and 2 have a significant SNR to communicate with each other. This connectivity is at the layer-1 and is independent of any access control mechanisms used at a higher layer in the network stack.
A. Wireless Topologies

Consider a network graph G which is generated by a random adjacency matrix \( A_{adj} \in \mathbb{R}^{n \times n} \), where \( n \) denotes the number of nodes in the graph G. The individual entries in \( A_{adj} \) will denote the link conditions between corresponding wireless nodes. In this study, given a specific combination of three values \( P, Q, R \), we observe that they satisfy the triangle inequality requirements in the \( \mathbb{R}^3 \) metric space. Let \( \| \cdot \| \) represent an arbitrary distance norm, Triangle inequality requirement states that the sum of the lengths of any two sides (say \( \| x \| + \| y \| \)) has to be greater than the third side (\( \| x + y \| \)). While mapping the fourth node D, with the requirement \( A_{adj}(A, D) = 1 \), \( A_{adj}(D, C) = 1 \), and \( A_{adj}(B, D) = 0 \), we observe that the triangle inequality fails for the node sets \( \{B, D, A\} \) and \( \{B, D, C\} \). Thus using simple triangle inequality as a test for the function \( F \), i.e by determining if the generated wireless topology fits in a geometric vector space, we can classify random matrices as representing valid or invalid wireless topologies.

B. Problem Statement

Now that we have shown an example of an invalid wireless topology generation, we will explicitly define the problem. Consider a network graph G which is generated by a random adjacency matrix \( A_{adj} \in \mathbb{R}^{n \times n} \). In this study, given a specific combination of three values \( A, B, C \), we observe that they satisfy the triangle inequality requirements in the \( \mathbb{R}^3 \) metric space. Let \( \| \cdot \| \) denote the link conditions between corresponding wireless nodes. In this study, given a specific adjacency matrix \( A_{adj} \), we define a function \( F() \) to tell us whether the given adjacency matrix represents a valid or invalid wireless topology.

\[
F : G(A_{adj} \in \mathbb{R}^{n \times n}) \rightarrow \{Valid, Invalid\} \tag{1}
\]

Once determined, \( F() \) can be used as a test for incrementally adding neighbor nodes to an adjacency matrix for generating \( k \)-connected graphs. We also calculate the probability \( P_F \) with which \( F() \) will fail, which could be used as a metric for determining the average number of trials that would be required for valid wireless topology generation.

III. MODELING

A. Wireless Topologies & Vector Spaces

To determine \( F() \) defined above, we briefly discuss why the random adjacency matrix used for Figure 1 fails. If we consider, the first three nodes A, B, C, we observe that they satisfy the triangle inequality requirements in the \( \mathbb{R}^3 \) metric space. Let \( \| \cdot \| \) represent an arbitrary distance norm, Triangle inequality requirement states that the sum of the lengths of any two sides (say \( \| x \| + \| y \| \)) has to be greater than the third side (\( \| x + y \| \)). While mapping the fourth node D, with the requirement \( A_{adj}(A, D) = 1 \), \( A_{adj}(D, C) = 1 \), and \( A_{adj}(B, D) = 0 \), we observe that the triangle inequality fails for the node sets \( \{B, D, A\} \) and \( \{B, D, C\} \). Thus using simple triangle inequality as a test for the function \( F \), i.e by determining if the generated wireless topology fits in a geometric vector space, we can classify random matrices as representing valid or invalid wireless topologies.

B. Estimating Adjacency Matrix Failure Probability \( (P_F) \)

A randomly generated adjacency matrix \( A_{adj} \) for a wireless topology with \( n \) nodes will fail when any one combination of three links fails the triangle inequality check. Hence, the probability of at least one failure is: \( P_F = 1 - P_NF \), where the \( P_NF \) is the probability that no combination in the adjacency graphs fails the triangle inequality check. Thus, \( P_F \) can be calculated as:

\[
P_F = 1 - (1 - P_\Delta)^{N_{pairs}}, \tag{2}
\]

where \( N_{pairs} \) denotes the number of combinations of nodes checked in a randomly generated matrix, and \( P_\Delta \) denotes the probability of failure of the triangle inequality on any random adjacency triplet. We define an adjacency triplet as any single combination of three values \( A_{adj}(P, Q) \), \( A_{adj}(Q, R) \) and \( A_{adj}(P, R) \) that describe link conditions between any three nodes P, Q and R. The \( N_{pairs} \) are determined by the number of non-diagonal entries in the adjacency matrix. Since the adjacency matrix is representing a wireless topology, it should be symmetric akin to a metric space distance matrix. Hence, \( N_{pairs} = \binom{n^2 - n}{3} \times (\frac{n^2 - n}{2} - 1) \).

C. Determining \( P_\Delta \)

To estimate \( P_\Delta \), we use the complete set of adjacency triplets \( S_3 \) described as:

\[
S_3 = \{A_{adj}(P, Q), A_{adj}(Q, R), A_{adj}(P, R)\}, \tag{3}
\]

defined \( \forall P, Q, R \in A_{adj} \). To determine \( P_\Delta \) we can either use combinations or permutations on \( S_3 \) to determine failure probability of combinations. For all such possible permutations and combinations over \( S_3 \), we determine \( P_\Delta \) by calculating the fraction of adjacency triplets that fail strict \((\leq)\) or non-strict \((<)\) triangle inequality checks. While evaluating, we vary the base, which denotes the maximum number of discretized values that can be used to represent the link between two points. E.g. when we chose the base as 1, the link represented in the adjacency matrix can either take the values as 0 (off) or 1(on). If the base is 2, the possible values are 0,1,2 and so on. Results for this model are as described in Figure 2.

We observe that the fraction of permutations or combinations resulting in the triangle inequality failure remain fairly constant, irrespective of the increasing number base. Also, we note that the number of combinations being evaluated are growing as \( O(base^3) \). Hence, we conclude that, the number of unique combinations failing are also increasing at \( O(base^3) \) to keep the ratio constant.

IV. MONTE CARLO TESTS

In this section we estimate and compare the probability with which a randomly generated adjacency matrix will fail when mapped as a wireless topology. We compare our failure probability estimate \( (P_F) \) with the failure probability obtained through randomized trials. In these comparisons, we use the discreteness of link qualities (or the discreteness of distance) and the size of wireless topologies as the two varying parameters.
generated adjacency matrices for every distance failure probability obtained from trials of stabilizes for higher values of the distance coarsely described (E.g. on or off). This result is a direct topology. (Fig. 3. Probability of failure of a randomly generated adjacency matrix in function of varying topology size.

A. Discreteness Of Link Connectivity Representation

In this test, as with the estimation of \( P_\Delta \), we vary the maximum number of discretized values \( \text{(base)} \) that can be used to represent the link between two points. Results from our estimates and those from monte-carlo tests are correspondingly marked as Estimate: \(*\) and MTC: \(*\) in the Figure 3. The results show that our estimate of \( P_F \) is able to closely match the failure probability obtained from trials of 1000 randomly generated adjacency matrices for every distance \( \text{base} \). For all topology sizes: 2, 3, 4 (nodes each), we observe that the probability of the matrix failing to conform to a wireless topology \( (P_F) \) can be high when the link connectivity is coarsely described (E.g. on or off). This result is a direct consequence of: \( P_F \propto P_\Delta \). Hence, we observe that as \( P_\Delta \) stabilizes for higher values of the distance \( \text{base} \), \( P_F \) stabilizes too.

B. Impact Of Varying Topology Size

The size of a topology can be changed by varying the number of nodes. Edges are not explicitly used as a factor for changing topology size since the number and type of edges are randomly decided. In this experiment, we vary the size of the wireless topology from 1 to 10 nodes. For every topology, an edge can have a value uniformly distributed among the number of discretized distance values given by the \( \text{base} \). We generate 1000 random matrices for each topology size.

As shown in the results in Figure 4 the estimated failure probability \( P_F \) (denoted by Estimate: \(*\)) closely matches that obtained from random trials (MTC: \(*\)). Further, we observe that failure probability quickly approaches 1. This matches with our estimate since, \( P_F \propto N_{\text{pairs}} \), and \( N_{\text{pairs}} \) increase at least as \( O(n^2) \). An important implication of this result is that as the size of the wireless topology goes beyond 3 nodes, it is almost certain that a randomly generated adjacency matrix will not conform to a wireless topology.

V. RELATED WORK

A class of studies has focussed on enumerating the characteristics of wired networks \([1, 2]\) that need to be taken into consideration while generating topologies from random graphs. Consequently, a parallel area of research is focussed on efficient generation \([3]\) and improvement of the features of random graphs to model real wired networks \([4]\).

With concerns to wireless networks, \([5]\) investigates the impact of spatial distribution of nodes on the minimum node degree, and the k-connectivity in random network graphs. We take a completely opposite view of the problem in determining if a randomly generated adjacency could be used for faithfully representing a realistic wireless topology.

VI. CONCLUSIONS

This study describes an approach for determining if randomly generated adjacency matrices can conform to wireless topologies in the physical world. It is shown that these random matrices are prone to failure, specially for topologies with more than 3 nodes. Using this information, an alternative approach can be taken for random wireless topology creation. Instead of designing simulation studies based on random placement of nodes and then generating k-connected graphs, algorithms could be designed that would iteratively add nodes to the adjacency graph based on k-connectivity requirements as long as they do not violate constraints of the geometric space.

REFERENCES

[1] M. B. Doar and A. Nexion, “A better model for generating test networks,” in IEEE Global Telecommunications Conf. (Globecom), 1996, pp. 86–93.
[2] E. Zegura, K. Calvert, and S. Bhattacharjee. “How to model an internetwork,” in In Proceedings of IEEE INFOCOM, 1996, pp. 594–602.
[3] J. Leskovec, D. Chakrabarti, J. Kleinberg, and C. Faloutsos. “Realistic, mathematically tractable graph generation and evolution, using kneckrone multiplication,” Lecture Notes in Computer Science: Knowledge Discovery In Databases, vol. 3721, pp. 133–145, 2005.
[4] A. Rodionov and H. Choo. “On generating random network structures: Connected graphs,” Lecture Notes in Computer Science: Information Networking, vol. 3090, pp. 483–491, 2004.
[5] C. Bettstetter. “On the minimum node degree and connectivity of a wireless multihop network,” in MobiHoc ’02: Proceedings of the 3rd ACM international symposium on Mobile ad hoc networking & computing, New York, NY, USA: ACM, 2002, pp. 80–91.