A Novel Approach for the Solution of Multi Objective Interval Transportation Problem

G Ramesh¹, G Sudha² and K Ganesan³

Department of Mathematics, Faculty of Engineering and Technology
SRM Institute of Science and Technology, Kattankulathur, Chennai – 603203, India.
Email: rameshapg@gmail.com, sudha.g@ktr.srmuniv.ac.in, gansan_k@yahoo.com

Abstract: In this paper, we propose a Multi-objective transportation problem where the source and destination parameters are chosen as interval numbers. The main objective of the problem is to minimize the total transportation cost and total delivery time. A new algorithm is proposed to determine initial allocation to the basic cells and the corresponding Multi-objective interval transportation cost without converting the problem to its classical version. The initial interval transportation cost may vary during optimality test when allocations are changed. Numerical examples are presented to illustrate the proposed method.

1. Introduction

In today’s highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers becomes stronger. How and when to send the product to the customers in the quantities. They want in a cost effective manner, become more challenging. Transportation models provide a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods. We develop the separation method with using the midpoint and width of the interval in the Multi-objective interval transportation problem.

Most of the models developed for solving the transportation problem are with the assumption that the supply, demand and the cost per unit values are exactly known. But in real world applications, the supply, the demand and the cost per unit of the quantities are generally not specified precisely i.e. the parameters are interval in nature. Impreciseness in the parameters means the information for these parameters are not complete. But even with incomplete information, the model user is normally able to give a realistic interval for the parameters. Therefore the use of interval transportation problems is more appropriate to model and solve the real world problems.

Many researchers proposed different methods for the solution of Multi-objective interval transportation problems. Ishibuchi and Tanaka [8] studied Multiobjective programming in optimization of the interval objective function. Hitchcock [7] originally developed the transportation problem in 1941 with his research paper. Arsham and Khan [1] developed a simplex type algorithm to solve the general transportation problem. Bit et al. [2] applied fuzzy programming technique with linear membership function to solve the multi-objective transportation problem. G. Ramesh and K. Ganesan [14] a new approach to interval transportation problems and we propose a new method for the solution of fully interval transportation problem without converting them to classical transportation problem. Deepika Rani [4] studied Fuzzy Programming Technique for Solving Different Types of Multi-objective Transportation Problem.
Kasana [10] developed different approaches to generate the set of efficient solutions for multiobjective transportation problems. The solution procedure of this method depends on determining the set of efficient solutions and, finally, the decision maker is responsible for selecting the preferred solution out of this set. Liang [12] proposed an interactive fuzzy multi-objective linear programming model for solving an integrated production-transportation planning problem in supply chains. In this method, author has applied the max-min approach of Zimmermann to solve the auxiliary single-objective mode. But, it is well-known that the solution yielded by max-min operator might not be unique nor efficient. There are many approaches in the literature that can be used for solving stochastic programming problems. Moreover Wang et al. [16] some basic works about uncertain random variables were conducted by. In mathematical interval programming models deal with uncertainty and interval coefficients. Yu, Kuo-Jen Hu and An-Yuan Chang [16] an interactive approach for the multi-objective transportation problem with interval parameters, Taylor & Francis, international journal of production research. Das at al. [5] used fuzzy programming technique to solve MOITP in which cost coefficient, destination and source parameters are in interval form. Dal binder Kaur, Sathi Mukherjee and Kajla Basu [11] present the solution of a multi-objective and multi-index real-life transportation problem by applying an exponential membership function in fuzzy programming technique. Jignasha G. Patel, Jayesh M. Dhodiya [9] Solving Multi-Objective Interval Transportation Problem Using Grey Situation Decision-Making Theory Based On Grey Numbers.

In general, most of the existing techniques provide only crisp solutions for the interval transportation problem. In this paper we propose a simple method, for the interval optimal solution of interval transportation problems without converting them to classical transportation problems and the results obtained are discussed.

The rest of this paper is organized as follows: In section 2, we recall the basic concepts of interval numbers and related results. In section 3, we define interval transportation problem and prove related theorems. In section 4, we propose interval Version of Vogel’s Approximation Algorithm (IVAM). In section 5, numerical examples are provided to illustrate the methods proposed in this paper.

2. Preliminaries
The aim of this section is to present some notations, notions and results which are of useful in our further consideration.

2.1. Interval numbers
Let $\mathcal{A} = [a_1, a_2] = \{x : a_1 \leq x \leq a_2, x \in \mathbb{R}\}$. If $\mathcal{A} = a_1 = a_2 = a$, then $\mathcal{A} = [a, a] = a$ is a real number (or a degenerate interval). Let $\mathbb{I}_R = \{\mathcal{A} = [a_1, a_2] : a_1 \leq a_2 \}$ and $\mathbb{I}_I = \{\mathcal{A} = [a_1, a_2] : a_1 > a_2 \}$ be the set of all proper intervals and be the set of all improper intervals on the real line $\mathbb{R}$. We shall use the terms interval and interval number interchangeably. The midpoint and width (or half-width) of an interval number $\mathcal{A} = [a_1, a_2]$ are defined as $m(\mathcal{A}) = \frac{a_1 + a_2}{2}$ and $w(\mathcal{A}) = \frac{a_2 - a_1}{2}$. The interval number $\mathcal{A}$ can also be expressed in terms of its midpoint and width as $\mathcal{A} = [a_1, a_2] = (m(\mathcal{A}), w(\mathcal{A}))$.

2.2. Ranking of Interval Numbers
Sengupta and Pal [3] proposed a simple and efficient index for comparing any two intervals on $\mathbb{I}_R$ through decision maker’s satisfaction.

2.1 Definition. Let $\mathcal{A}_1 = [a_1, a_2]$ and $\mathcal{A}_2 = [b_1, b_2]$ in $\mathbb{I}_R$, then for $m(\mathcal{A}_1) < m(\mathcal{A}_2)$, we construct a premise $(\mathcal{A}_1 < \mathcal{A}_2)$ which implies that $\mathcal{A}_1$ is inferior to $\mathcal{A}_2$ (or $\mathcal{A}_2$ is superior to $\mathcal{A}_1$).

An acceptability function $\mathcal{A}_\mathcal{L} : \mathbb{I}_R \times \mathbb{I}_R \rightarrow [0, \infty)$ is defined as:

$$\mathcal{A}_\mathcal{L}(\mathcal{A}_1, \mathcal{A}_2) = \frac{m(\mathcal{A}_2) - m(\mathcal{A}_1)}{w(\mathcal{A}_2) + w(\mathcal{A}_1)}$$

where $w(\mathcal{A}_2) + w(\mathcal{A}_1) \neq 0$. 

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A_p may be interpreted as the grade of acceptability of the “the first interval number to be inferior to the second interval number”. For any two interval numbers $\vec{a}$ and $\vec{b}$ in $\mathbb{IR}$ either $A(\vec{a} \preceq \vec{b}) \geq 0$ (or) $A(\vec{b} \preceq \vec{a}) = 0$ (or) $A(\vec{b} \preceq \vec{a}) + A(\vec{a} \preceq \vec{b}) = 0$.

If $A(\vec{a} \preceq \vec{b}) = 0$ and $A(\vec{b} \preceq \vec{a}) = 0$, then we say that the interval Numbers $\vec{a}$ and $\vec{b}$ are equivalent (non-inferior to each other) and we denote it by $\vec{a} \equiv \vec{b}$. Also if $A(\vec{a} \preceq \vec{b}) \geq 0$, then $\vec{a} \preceq \vec{b}$ and if $A(\vec{b} \preceq \vec{a}) \geq 0$, then $\vec{b} \preceq \vec{a}$.

A New Interval Arithmetic

Ming Ma et al. [13] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index function. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which are the least upper bound and Greatest lower bound in the lattice $L$. That is for $a, b \in L$ we define $a \wedge b = \min\{a, b\}$ and $a \vee b = \max\{a, b\}$.

For any two intervals $\vec{a} = [a_1, a_2], \vec{b} = [b_1, b_2] \in \mathbb{IR}$ and for $* \in \{+, \cdot, \div\}$, the arithmetic operations on $\vec{a}$ and $\vec{b}$ are defined as:

$\vec{a} * \vec{b} = [a_1, a_2] * [b_1, b_2] = \langle m(\vec{a}), w(\vec{a}) \rangle * \langle m(\vec{b}), w(\vec{b}) \rangle = \langle m(\vec{a}) * m(\vec{b}), \max\{w(\vec{a}), w(\vec{b})\} \rangle$.

In particular

(i). Addition : $\vec{a} + \vec{b} = \langle m(\vec{a}), w(\vec{a}) \rangle + \langle m(\vec{b}), w(\vec{b}) \rangle = \langle m(\vec{a}) + m(\vec{b}), \max\{w(\vec{a}), w(\vec{b})\} \rangle$.

(ii). Subtraction : $\vec{a} - \vec{b} = \langle m(\vec{a}), w(\vec{a}) \rangle - \langle m(\vec{b}), w(\vec{b}) \rangle = \langle m(\vec{a}) - m(\vec{b}), \max\{w(\vec{a}), w(\vec{b})\} \rangle$.

(iii). Multiplication : $\vec{a} \times \vec{b} = \langle m(\vec{a}), w(\vec{a}) \rangle \times \langle m(\vec{b}), w(\vec{b}) \rangle = \langle m(\vec{a}) \times m(\vec{b}), \max\{w(\vec{a}), w(\vec{b})\} \rangle$.

(iv). Division : $\vec{a} \div \vec{b} = \langle m(\vec{a}), w(\vec{a}) \rangle \div \langle m(\vec{b}), w(\vec{b}) \rangle = \langle m(\vec{a}) \div m(\vec{b}), \max\{w(\vec{a}), w(\vec{b})\} \rangle$,

provided $m(\vec{b}) \neq 0$.

3. Main Results

Consider a fully interval transportation problem with $m$ sources and $n$ destinations involving numbers. Let $\vec{a}_i$ be the availability at source $i$ and $\vec{b}_j$ be the requirement at destination $j$. Let $\vec{c}_{ij}$ be the unit interval transportation cost from source $i$ to destination $j$. Let $\vec{x}_{ij}$ denote the number of interval units to be transported from source $i$ to destination $j$. Now the problem is to find a feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total interval transportation cost is minimized.

3.1. Mathematical formulation of interval transportation problem

The mathematical model of fully interval transportation problem is as follows
Minimize $\mathcal{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

subject to $\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, 3, ..., m$

$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, 3, ..., n$

$\mathcal{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$, where $i = 1, 2, 3, ..., m; \quad j = 1, 2, 3, ..., n$ and $\mathcal{Z}_{ij} \in [\ell, r]$ for all $i$ and $j$

and $\ell_i, r_i, \ell_j, r_j$ in IR, where $\ell_{ij}$ is the interval unit transportation cost from $i^{th}$ source to $j^{th}$ destination. The objective is to minimize the total Multi-objective interval transportation cost, in this paper the Multi-objective interval transportation problem is solved by interval version of Vogel’s method. This Multi-objective interval transportation problem is explicitly represented by the following Multi-objective interval transportation table.

### 3.1 Definition
A set of non-negative allocations $x_{ij}$ which satisfies (in the sense equivalent) the row and the column restrictions is known as fuzzy feasible solution.

### 3.2 Definition
An interval feasible solution to an interval transportation problem with $m$ sources and $n$ destinations is said to be an interval basic feasible solution if the number of positive allocations are $(m+n-1)$. If the number allocations in an interval basic solution is less than $(m+n-1)$, it is called degenerate interval basic feasible solution.

### 3.3 Definition
An interval feasible solution is said to be interval optimal solution if it minimizes the total interval transportation cost.

### 3.2 General Form of Interval Transportation Problem with Multiple Objectives

The formulation of MITP is the problem of minimizing $k$ interval valued objective functions with interval supply and interval destination parameters is given and an efficient algorithm is presented to find the optimal solution of MITP. The mathematical model of MITP when all the cost coefficient, supply and demand are interval-valued is given by:

Minimize $\mathcal{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} [\ell_{ij}^k, r_{ij}^k] x_{ij}$ where $k = 1, 2, ..., K$

subject to $\sum_{j=1}^{n} x_{ij} = [\ell_i^1, r_i^1], i = 1, 2, 3, ..., m$

$\sum_{i=1}^{m} x_{ij} = [\ell_j^1, r_j^1], j = 1, 2, 3, ..., n$

$\sum_{i=1}^{m} \ell_{ij}^k = \sum_{j=1}^{n} \ell_{ij}^k$, and $\sum_{j=1}^{n} \ell_{ij}^k = \sum_{i=1}^{m} \ell_{ij}^k$ where $i = 1, 2, 3, ..., m; \quad j = 1, 2, 3, ..., n$ and $\mathcal{Z}_{ij} \in [\ell, r]$ for all $i$ and $j$

Where the source parameter lies between left limit $\ell_{ij}$ and right limit $r_{ij}$. Similarly, destination parameter lies between left limit $\ell_{ij}$ and right limit $r_{ij}$ and $[\ell_{ij}, r_{ij}]$, $(k = 1, 2, ..., K)$ is an interval indicating the uncertain cost for the transportation problem; it can exemplify delivery time, quantity of goods delivered.

### 4. Proposed Algorithms

#### 4.1 Interval Version of Vogel’s Approximation Method (IVAM)

**Step 1.** Express the Multi-objective interval transportation problem in the transportation table.
Step 2. Express the all interval parameters supply, demand and unit transportation cost in the transportation problem in terms of midpoint and half width. That is in the form of $\bar{a} = [a_1, a_2] = \langle m(\bar{a}), w(\bar{a}) \rangle$.

Step 3. In the Multi-objective interval transportation table, determine the row penalties for each row by subtracting the lowest cost from the next lowest cost of that row. In the similar way calculate the column penalties for each column. Write down the row penalties aside each row and the column penalties below each column.

Step 4. Identify the column or row with largest interval penalty. In case of tie, break the tie arbitrarily. Select a cell with minimum interval cost in the selected column (or row, as the case may be) and assign the maximum units possible by considering the demand and supply position corresponding to the selected cell.

Step 5. Delete the column/row for which the supply and demand requirements are met.

Step 6. Continue steps 1 to 3 for the resulting Multi-objective interval transportation table until the supply and demand of all sources and destinations have been met.

4. Numerical examples

Example 5.1. A company has three production facilities (origins) A1, A2 and A3 with production capacity of 8, 19 and 17 units of a product respectively. These units are to be shipped to four warehouses B1, B2, B3 and B4 with requirement of 11, 3, 14 and 16 units respectively. The transportation cost and transportation time between companies to warehouses are given below.

|       | B1    | B2    | B3    | B4    |
|-------|-------|-------|-------|-------|
| A1    | [1,2] | [1,3] | [5,6] | [4,8] |
| A2    | [1,2] | [7,10]| [2,6] | [3,5] |
| A3    | [7,9] | [7,11]| [3,5] | [5,7] |

|       | B1    | B2    | B3    | B4    |
|-------|-------|-------|-------|-------|
| A1    | [3,5] | [2,6] | [2,4] | [1,5] |
| A2    | [4,6] | [7,9] | [7,10]| [9,11]|  
| A3    | [4,8] | [1,3] | [3,6] | [1,7] |

Let us solve the same problem by our method. Express all the interval parameters $\bar{a} = [a_1, a_2]$ in terms of midpoint and width as $\bar{a} = [a_1, a_2] = \langle m(\bar{a}), w(\bar{a}) \rangle$. Now the given Multi-objective interval transportation problem becomes.
Table-5.1.a Multi-objective Interval Transportation table

|   | B1       | B2       | B3       | B4       | Supply   |
|---|----------|----------|----------|----------|----------|
| A1| <1.5 , 0.5> | <2 , 1>  | <7 , 2>  | <6 , 2>  | <8 , 0>  |
| A2| <1.5 , 0.5> | <8.5 , 1.5> | <4 , 2>  | <4 , 1>  | <19 , 0> |
| A3| <8 , 1>  | <9 , 2>  | <4 , 1>  | <6 , 1>  | <17 , 0> |
| Demand | <11 , 0> | <3 , 0>  | <14 , 0> | <16 , 0> |          |

Hence the initial basic feasible solution for the Multi-objective interval transportation problem using interval Vogel’s approximation method is:

Table-5.1.a Multi-objective Interval Transportation table

|   | B1       | B2       | B3       | B4       | Supply   |
|---|----------|----------|----------|----------|----------|
| A1| [5 , 0]  | [3 , 0]  | [5 , 6]  | [4 , 8]  | [8 , 0]  |
|    | [1 , 2]  | [1 , 3]  |          |          |          |
| A2| [6 , 0]  | [7 , 10] | [2 , 6]  | [13 , 0] | [19 , 0] |
|    | [1 , 2]  |          | [3 , 5]  |          |          |
| A3| [7 , 9]  | [7 , 11] | [14 , 0] | [3 , 0]  | [17 , 0] |
|    |          | [3 , 5]  | [5 , 7]  |          |          |
| Demand | [11 , 0] | [3 , 0]  | [14 , 0] | [16 , 0] |          |

The initial Multi-objective interval transportation cost
\[= \langle 7 , 0.5 \rangle + \langle 6 , 1 \rangle + \langle 52 , 1 \rangle + \langle 9 , 0.5 \rangle + \langle 56 , 1 \rangle + \langle 18 , 1 \rangle\]
\[= \langle 148.5 , 1 \rangle\]

we are considering from the above Table: 5.1.b

|   | B1       | B2       | B3       | B4       | Supply   |
|---|----------|----------|----------|----------|----------|
| A1| <4 , 1>  | <4 , 2>  | <3 , 1>  | <3 , 2>  | <8 , 0>  |
| A2| <5 , 1>  | <8 , 1>  | <8.5 , 1.5> | <10 , 1> | <19 , 0> |
| A3| <6 , 2>  | <2 , 1>  | <4.5 , 1.5> | <1.5 , 0.5> | <17 , 0> |
| Demand | <11 , 0> | <3 , 0>  | <14 , 0> | <16 , 0> |          |

Hence the initial basic feasible solution for the Multi-objective interval transportation problem using interval Vogel’s approximation method is:
Table-5.1.b . Multi-objective Interval Transportation table

| B1      | B2      | B3      | B4      | supply |
|---------|---------|---------|---------|--------|
| A1      | [3, 5]  | [2, 6]  | [6, 0]  | [2, 0] |
|         |         |         | [2, 4]  | [1, 5] |
| A2      | [11, 0] | [7, 9]  | [8, 0]  | [9, 11]| [19, 19]|
|         | [4, 6]  |         | [7, 10] |        |
| A3      | [4, 8]  | [3, 0]  | [3, 6]  | [14, 0]|
|         |         | [1, 3]  |         | [1, 7] |
| Demand  | [11, 11]| [3, 3]  | [14, 14]| [16, 16]|

The initial Multi-objective interval transportation cost
= 18, 1> + <6, 2> + <55, 1> + <68, 1.5> + <6, 1> + <21, 0.5>
= 174, 2>
= 172, 176

Comparison:

| Deepika Rani [4] (linear membership function) | Deepika Rani [4] (Exponential membership function) | Deepika Rani [4] (Hyperbolic membership function) | S.K.Das, A.Goswami and S.S.Alam [6] | J.G.Patel and J.M.Dhodyya | Developed Method |
|------------------------------------------------|--------------------------------------------------|-------------------------------------------------|-----------------------------------|------------------------|------------------|
| [172.2, 222.55] | [171.50,221.63] | [172.2,222.55] | [119.14,214.42] | [146, 241] | [147.5, 149.5] |
| [206.1,252.75] | [207.54,254.36] | [206.1,252.75] | [180.64,241.1] | [133, 222] | [172, 176] |

**Example 5.2:** A company has three production facilities (origins) A1, A2 and A3 with production capacity of [7, 9], [17, 21] and [16, 18] units of a product respectively. These units are to be shipped to four warehouses B1, B2, B3 and B4 with requirement of [10, 12], [2, 4], [13, 15] and [15, 17] units respectively. The transportation cost and transportation time between companies to warehouses are given below

Table:5.2.a . Multi-objective Interval Transportation table

| [1, 2] | [1, 3] | [5, 9] | [4, 8] |
|--------|--------|--------|--------|
| [1, 2] | [7, 10]| [2, 6] | [3, 5] |
| [7, 9] | [7, 11]| [3, 5] | [5, 7] |

Table:5.2.b. Multi-objective Interval Transportation table

| [3, 5] | [2, 6] | [2, 4] | [1, 5] |
|--------|--------|--------|--------|
| [4, 6] | [7, 9] | [7, 10]| [9, 11]|
| [4, 8] | [1, 3] | [3, 6] | [1, 2] |

Hence the initial basic feasible solution for the Multi-objective interval transportation problem using interval Vogel’s approximation method is:
Table: 5.2.a. Multi-objective Interval Transportation table

|     | B1     | B2     | B3     | B4     | supply |
|-----|--------|--------|--------|--------|--------|
| A1  | [4, 6] | [2, 4] | [5, 9] | [4, 8] | [7, 9] |
|     | [1, 2] | [1, 3] |        |        |        |
| A2  | [4, 8] | [7, 10]| [2, 6] | [11, 15]| [17, 21]|[
|     | [1, 2] |        |        | [3, 5] |        |
| A3  | [7, 9] | [7, 11]| [14, 1]| [1, 5] | [16, 18]|[
|     |        |        | [3, 5] |        |        |
| Demand | [10, 12]| [2, 4] | [13, 15]| [15, 17]|        |

The initial Multi-objective interval transportation cost is [146.5, 150.5]
Hence the initial basic feasible solution for the Multi-objective interval transportation problem using interval Vogel’s approximation method is:

Table: 5.2.b. Multi-objective Interval Transportation table

|     | B1     | B2     | B3     | B4     | supply |
|-----|--------|--------|--------|--------|--------|
| A1  | [3, 5] | [2, 6] | [5, 7] | [1, 3] | [7, 9] |
|     |        |        | [2, 4] | [1, 5] |        |
| A2  | [11, 2]| [7, 9] | [6, 10]| [9, 11]| [17, 21]|[
|     | [4, 6] |        | [7, 10]|        |        |
| A3  | [4, 8] | [2, 4] | [3, 6] | [13, 15]| [16, 18]|[
|     |        | [1, 3] |        | [1, 2] |        |
| Demand | [10, 12]| [2, 4] | [13, 15]| [15, 17]|        |

The initial Multi-objective interval transportation cost is [172, 176].
Comparison:

| Deepika Rani [4] (linear membership function) | Deepika Rani[4] (Exponential membership function) | Deepika Rani[4] (Hyperbolic membership function) | J.G.Patel and J.M.Dhodiya | Developed Method |
|----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------|-----------------|
| [159.02, 205.03]                            | [158.95, 204.90]                               | [159.02, 205.03]                               | [137, 225]                 | [146.5, 150.5]  |
| [176.54, 227.95]                            | [186.48, 227.95]                               | [176.54, 227.95]                               | [119, 202]                 | [172, 176]      |

5. Conclusion
In this paper, we have proposed interval versions of VAM method for solving the Multi-objective interval transportation problems without converting them to classical Multi-objective interval transportation problems. To illustrate the proposed method, numerical examples are solved and the results obtained are discussed. Examples are proposed to show that our method works successfully.
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