From Open Set to Closed Set: Supervised Spatial Divide-and-Conquer for Object Counting

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Abstract—Visual counting, a task that aims to estimate the number of objects from an image/video, is an open-set problem by nature, i.e., the number of population can vary in $[0, +\infty)$ in theory. However, collected data and labeled instances are limited in reality, which means that only a small closed set is observed. Existing methods typically model this task in a regression manner, while they are prone to suffer from an unseen scene with counts out of the scope of the closed set. In fact, counting has an interesting and exclusive property—spatially decomposable. A dense region can always be divided until sub-region counts are within the previously observed closed set. We therefore introduce the idea of spatial divide-and-conquer (S-DC) that transforms open-set counting into a closed-set problem. This idea is implemented by a novel Supervised Spatial Divide-and-Conquer Network (SS-DCNet). Thus, SS-DCNet can only learn from a closed set but generalize well to open-set scenarios via S-DC. SS-DCNet is also efficient. To avoid repeatedly computing sub-region convolutional features, S-DC is executed on the feature map instead of on the input image. We provide theoretical analyses as well as a controlled experiment on toy data, demonstrating why closed-set modeling makes sense. Extensive experiments show that SS-DCNet achieves the state-of-the-art performance on three crowd counting datasets (ShanghaiTech, UCF_CC_50 and UCF-QNRF), a vehicle counting dataset (TRANSCOS) and a plant counting dataset (MTC), with a 7.77% relative improvement on the UCF-QNRF, 33.1% on the TRANSCOS, and 26.4% on the MTC. SS-DCNet also reports the state-of-the-art cross-domain performance on crowd counting datasets. Particularly in the task from UCF-QNRF to ShanghaiTech Part_A, SS-DCNet even beats most existing models trained directly on the target domain. Code and models have been made available at: https://tinyurl.com/SS-DCNet.

Index Terms—Object Counting, Open Set, Close Set, Spatial Divide-and-Conquer, Local Count Models

1 INTRODUCTION

Counting is an open-set problem by nature as a count value can range from 0 to $+\infty$ in theory. It is therefore typically modeled in a regression manner. Benefiting from the success of convolutional neural networks (CNNs), state-of-the-art deep counting networks often adopt a multi-branch architecture to enhance the feature robustness to dense regions [2], [4], [33]. However, the observed patterns in datasets are limited in practice, which means that networks can only learn from a closed set. Are these counting networks still able to generate accurate predictions when the number of objects is out of the scope of the closed set? According to Figure 2, local counts observed in the closed set exhibit a long-tailed distribution. Extremely dense patches are rare while sparse patches take up the majority. As what can be observed, increased local density leads to significantly increased relative mean absolute error (rMAE). Is it necessary to set the working range of CNN-based counter to the maximum count value observed, even with a majority of samples are sparse such that the counter works poorly in this range?

In fact, counting has an interesting and exclusive property—spatially decomposable. The above problem can be largely alleviated with the idea of spatial divide-and-conquer (S-DC). Suppose that a network has been trained to accurately predict a closed set of counts, say $0 \sim 20$. When facing an image with extremely dense objects, one can keep dividing the image into sub-images until all sub-region counts are less than 20. The network can then count these sub-images and sum over all local counts to obtain the global image count. Figure 1 graphically depicts the idea of S-DC. A follow-up question is how to spatially divide the count. A naive way is to upsample the input image, divide it into sub-images and process sub-images with the same network. This approach, however, is likely to blur the image and lead to significantly increased computation cost and memory consumption. Inspired by fully convolutional...
networks and RoI pooling [12], we show that it is feasible to achieve S-DC on feature maps, as conceptually illustrated in Figure 3. By decoding and upsampling the feature map, the later prediction layers can focus on the feature of local areas and predict sub-region counts accordingly.

To implement the above idea, we propose a simple yet effective Supervised Spatial Divide-and-Conquer Network (SS-DCNet). SS-DCNet learns from a closed set of count values but is able to generalize to open-set scenarios. Specifically, SS-DCNet adopts a VGG16 [39]-based encoder and an UNet [35]-like decoder to generate multi-resolution feature maps. All feature maps share the same counter. The counter can be designed by following the standard local count regression paradigm [25] or by discretizing continuous count values into a set of intervals as a classifier following [19], [23]. Furthermore, a division decision module is designed to decide which sub-region should be divided and to merge different levels of sub-region counts into the global image count.

We provide theoretical analyses to shed light on why the transition from the open set to the closed set makes sense for counting. We also show through a controlled toy experiment that, even given a closed training set, SS-DCNet effectively generalizes to the open test set. The effectiveness of SS-DCNet is further demonstrated on three crowd counting datasets: (ShanghaiTech [53], UCF_CC_50 [14] and UCF-QNRF [15]), a vehicle counting dataset (TRANCOS [13]), and a plant counting dataset (MTC [25]). Results show that SS-DCNet indicates a clear advantage over other competitors and sets the new state of the art. In addition, we remark that the closed set of SS-DCNet executes an implicit transfer in the output space, which is backed by state-of-the-art performance under the cross-domain evaluations. In particular, SS-DCNet even beats most state-of-the-art counting models that are trained directly on the target domain in the task from UCF-QNRF to ShanghaiTech Part_A.

Overall, the main contributions of this work are as follows.

- We investigate the explicit supervision for S-DC, which leads to a novel SS-DCNet. SS-DCNet is applicable to both regression-based and classification-based counters and can produce visually clear spatial divisions;
- We report state-of-the-art counting performance over 5 challenging datasets with remarkable relative improvements. We also show good transferability of SS-DCNet via cross-dataset evaluations on crowd counting datasets.

A preliminary conference version of this work appeared in [49] where S-DCNet, the first version of SS-DCNet, was developed. Here we extend [49] in the following aspects: i) we provide theoretical analyses why closed set modeling makes sense; ii) we further enhance S-DCNet at the methodology level by investigating further a regression-based closed-set counter, by integrating a count-orientated upsampling operator and by improving the model training with explicit supervision of spatial divisions; iii) we provide more ablative studies and qualitative analyses to highlight the role of S-DC; and iv) we give an insight of SS-DCNet w.r.t. its good transferability in the output space and report state-of-the-art performance under the cross-domain setup.

## 2 RELATED WORK

Current CNN-based counting approaches are mainly built upon the framework of local regression. According to their regression targets, they can be categorized into two categories: density map regression and local count regression. We first review these two regression paradigms. Since SS-DCNet works not only in regression counts but also in classification, some works that reformulate the regression problem are also discussed.

### 2.1 Density Map Regression

The concept of density map was introduced in [18]. The density map contains the spatial distribution of objects, thus can be smoothly regressed. Zhang et al. [52] may be the first to adopt a CNN to regress local density maps. Then almost all subsequent counting networks followed this idea.
Among them, a typical network architecture is multi-branch. MCNN [53] and Switching-CNN [2] used three columns of CNNs with varying receptive fields to depict objects of different scales. SANet [4] adopted Inception [44]-liked modules to integrate extra branches. CP-CNN [41] added two extra density-level prediction branches to combine global and local contextual information. ACSCP [37] inserted a child branch to match cross-scale consistency and an adversarial branch to attenuate the blurring effect of the density map. iCCNN [34] incorporated two branches to generate high-quality density maps in a coarse-to-fine manner. IG-CNN [1] and D-ConvNet [38] drew inspirations from ensemble learning and trained a series of networks or regressors to tackle different scenes. DecideNet [22] attempted to selectively fuse the results of density map estimation and object detection for different scenes. Unlike multi-branch approaches, Idrees et al. [15] employed a composition loss and simultaneously solved several counting-related tasks to assist counting. CSRNet [21] benefited from dilated convolution which effectively expanded the receptive field to capture contextual information.

Existing deep counting networks aim to generate high-quality density maps. However, density maps are actually in the open set as well. For a single point, different kernel sizes lead to different density values. When multiple objects exist and are close, density patterns are even much diverse. Since observed samples are limited, density maps are certainly in an open set. In addition, density maps do not have the physical property of spatial decomposition. We therefore cannot apply S-DC to density maps.

### 2.2 Local Count Regression

Local count regression directly predicts count values of local image patches. This idea first appeared in [7] where a multi-output regression model was used to regress region-wise local counts simultaneously. Authors of [10] and [25] introduced such an idea into deep counting. Local patches were first densely sampled in a sliding-window manner with overlaps, and a local count was then assigned to each patch by the network. Inferring redundant local counts were finally normalized and fused to the global count. Stahl et al. [43] regressed the counts for object proposals generated by Selective Search [47] and combined local counts using an inclusion-exclusion principle. Inspired by subitizing, the ability for a human to quickly counting a few objects at a glance, Chattopadhyay et al. [5] transferred their focus to the problem of counting objects in everyday scenes. The main challenge thus shifted to large intra-class variances rather than the occlusions and perspective distortions in crowded scenes.

While some methods above [5], [43] leverage the idea of spatial divisions, they still regress the open-set counts. Despite the fact that local region patterns are easier to be modelled than the whole image, the observed local patches are still limited. Since only finite local patterns (a closed set) can be observed, new scenes in reality have a high probability including objects out of the range (an open set). Moreover, dense regions with large count values are rare (Figure 2) and the networks may suffer from sample imbalance. In this paper, we show that a counting network is able to learn from a closed set with a certain range of counts, say $0 \sim 20$, and then generalizes to an open set (including counts $> 20$) via S-DC.

### 2.3 Beyond Simple Regression

Regression is a natural approach to estimate continuous variables, such as age, depth, and counts. However, some works suggest that regression is encouraged to be reformulated as an ordinal regression problem or a classification problem, which often enhances performance and benefits optimization [6], [11], [20], [28], [23] for many vision tasks. Ordinal regression is usually implemented by modifying well-studied classification algorithms and has been applied to the problem of age estimation [28] and monocular depth prediction [11]. Li et al. [20] further showed that directly reformulating regression to classification was also a good choice. In counting, the idea of blockwise classification is also investigated [23]. All these attempts motivate us to devise a classification-based closed-set counter. In this work, in addition to the standard regression-based modeling as in [25], SS-DCNet also follows [20] and [23] to discretize local counts and classify count intervals. Indeed, we observe in experiments that classification with S-DC typically works better than direct regression.

### 2.4 Open-Set Problems in Computer Vision

Many vision tasks are open-set by nature, such as depth prediction [11], [20], age estimation [6], [28], object recognition [36], visual domain adaptation [31], etc. While the sense of the open set may be different, they generally suffer from poor generalization as object counting. However, we currently find that, only the learning target of counting, i.e., the count value, can be easily transformed into a closed set (via spatial division).

### 3 Supervised Spatial Divide-and-Conquer Network

In this section, we describe how to construct a closed-set counter. We also explain our proposed SS-DCNet in detail.

#### 3.1 Closed-Set Counter

In local count modeling, there are two ways to define a counter in the closed set $[0, C_{\text{max}}]$, i.e., counting by regression [10], [25] and counting by classification [49], [23]. In practice, $C_{\text{max}}$ should be not greater than the maximum local count observed in the training set. It is clear that treating $C_{\text{max}}$ as the maximum prediction will cause a systematic error, but the error can be mitigated via S-DC, discussed in Section 6.

**Regression-Based Counter (R-Counter):** R-Counter directly regresses count values within the closed set. If predicted count values are greater than $C_{\text{max}}$, the predictions will simply be truncated to $C_{\text{max}}$.

**Classification-Based Counter (C-Counter):** Instead of regressing open-set count values, C-Counter discretizes local counts and classifies count intervals as in [23]. Specifically, we define an interval partition of $[0, +\infty)$ as $[0, \{0, C_1\})$, $\{C_2, C_3\}$, ..., $\{C_{M-1}, C_{\text{max}}\}$ and $(C_{\text{max}}, +\infty)$. These $M + 1$ sub-intervals...
are labeled to the 0-th to the M-th classes, respectively. For example, if a count value falls into \((C_2, C_3)\), it is labeled as the 2-nd class. The median of each sub-interval can be adopted when recovering the count from the interval. Notice that, for the last sub-interval \((C_{\text{max}}, +\infty]\), \(C_{\text{max}}\) will be used as the count value if a region is classified into this interval.

In what follows, we term the network as SS-DCNet (reg) when R-Counter is adopted, and SS-DCNet (cls) when C-Counter is used.

### 3.2 Single-Stage Spatial Divide-and-Conquer

As shown in Figure 4, SS-DCNet includes a VGG16 [39] feature encoder, an UNet [35]-like decoder, a closed-set counter, a division decider and an up-sampler. The structure of the counter, the division decider and the up-sampler are shown in Table 1. Note that, the average pooling layer in the counter has a stride of 2, so the final prediction has an output stride of 64.

The feature encoder removes fully-connected layers from the pre-trained VGG16. Suppose that the input patch is of size 64 \(\times\) 64. Given the feature map \(F_0\) (extracted from the Conv5 layer) with \(\frac{1}{16}\) stride of the input image, the counter predicts the local count value \(C_0\) conditioned on \(F_0\). Note that \(C_0\) is the local count without S-DC, which is also the final output of previous approaches [5], [10], [25].

We execute the first-stage S-DC on the fused feature map \(F_1\). \(F_1\) is divided and sent to the shared counter to produce the division count \(C_1 \in \mathbb{R}^{2 \times 2}\). Concretely, \(F_0\) is upsampled by \(\times 2\) in an UNet-like manner to \(F_1\). Given \(F_1\), the counter fetches the local features that correspond to spatially divided sub-regions, and predicts the first-level division counts \(C_1\). Each of the \(2 \times 2\) elements in \(C_1\) denotes a sub-count of the corresponding \(32 \times 32\) sub-region.

With local counts \(C_0\) and \(C_1\), the next question is to decide where to divide. We learn such decisions with another network module, division decider, as shown in the right part of Figure 4. At the first stage of S-DC, the division decider generates a soft division mask \(W_1\) of the same size as \(C_1\) conditioned on \(F_1\) such that for any \(w \in W_1, w \in [0, 1]\), \(w = 0\) means no division is required at this position, and the value in \(C_0\) is used. \(w = 1\) implies that here the initial prediction should be replaced with the division count in \(C_1\). Since both \(W_1\) and \(C_1\) are 2 times larger than \(C_0\), \(C_0\) is required to be upsampled by \(\times 2\) to \(C_0\). Notice that, since \(C_0\) denotes the local count of a \(64 \times 64\) region, the sum of \(C_0\) should equal to \(C_0\). The upscaling of \(C_0\) is therefore a re-distribution operator that assigns \(C_0\) to each sub-region. We compute the re-distribution map \(U_1\) from the upsampler conditioned on \(F_1\), and the sum of \(U_1\) equals to 1. We then upsample \(C_0\) to \(\hat{C}_0\) by

\[
\hat{C}_0 = (C_0 \odot 1_{2 \times 2}) \circ U_1,
\]

where “\(\circ\)” denotes Kronecker product and \(1_{2 \times 2}\) denotes a \(2 \times 2\) matrix filled with 1. Finally, the first-stage division result \(DIV_1\) takes the form

\[
DIV_1 = (1 - W_1) \circ \hat{C}_0 + W_1 \circ C_1,
\]

where 1 denotes a matrix filled with 1 and is with the same size of \(W_1\), and “\(\circ\)” denotes the Hadamard product.

### 3.3 Multi-Stage Spatial Divide-and-Conquer

SS-DCNet can execute multi-stage S-DC by further decoding, dividing the feature map until reaching the output of the first convolutional block. In this sense, the maximum division time is 4 in VGG16 for example. Actually we show later in experiments that a two-stage division is sufficient to guarantee satisfactory performance. In multi-stage S-DC, \(DIV_{i−1}(i \geq 2)\) is first upsampled as per

\[
DIV_{i-1} = (DIV_{i-1} \odot 1_{2 \times 2}) \circ U_i,
\]

and then merged according to

\[
DIV_i = (1 - W_i) \circ DIV_{i-1} + W_i \circ C_i,
\]

in a recursive manner. Multi-stage SS-DCNet is summarized in Algorithm 1.

### 3.4 Loss Functions

Here we elaborate the loss functions used in an N-stage SS-DCNet.

**Counter Loss:** As mentioned in Section 3.1, both R-Counter and C-Counter can be chosen. We adopt \(\ell_1\) loss, denoted by \(L_{R_i}, i = 0, 1, 2, ..., N\), for each level of output \(C_i\) when R-Counter is used, and cross-entropy loss, denoted by \(L_{C_i}, i = 0, 1, 2, ..., N\), when C-Counter is chosen. Note that, both ground-truth local counts and predicted counts are truncated.

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### Table 1

| Counter | Division decider/UpSampler |
|---------|---------------------------|
| 2 \times 2 AvgPool, s 2 | 2 \times 2 AvgPool, s 2 |
| 1 \times 1 Conv, 512, s 1 | 1 \times 1 Conv, 512, s 1 |
| 1 \times 1 Conv, \(1/(\text{class num.})\), s 1 | 1 \times 1 Conv, 1, s 1 |
| \_ | Sigmoid/Spatial Softmax |

*AvgPool denotes Average Pooling. Convolutional layers are defined in the format: kernel size Conv, output channel, s stride. Each convolutional layer is followed by ReLU except the last layer. In particular, a Sigmoid function is attached at the end of division decider to generate soft division masks. A Spatial Softmax function is applied at the End of Upsampler, which constrains the sum of upsampling weights in each \(2 \times 2\) adjacent regions to be 1 and ensures consistent local count values in the same image area after upsampling. The final output channel is 1 for R-Counter and class num for C-Counter.*
Division Loss: $L_i$ for $i \in \{1, 2, \ldots N\}$, for $W_i$ can be deduced as
\[
L_i = -\sum_{j=1}^{H_i} \sum_{k=1}^{W_i} \mathbb{1}\{C_{i-1}^{gt}[j,k] > C_{max}\} \
\times \log(\max(W_i[2j-1 : 2j, 2k-1 : 2k]))
\]
where $C_{i-1}^{gt}[j,k]$ denotes the element of the $j$-th row and the $k$-th column of $C_{i-1}$, and $W_i[2j-1 : 2j, 2k-1 : 2k]$ the elements lying in the $(2j-1)$-th to $2j$-th row, $(2k-1)$-th to $2k$-th column of $W_i$. The overall division loss $L_{div} = \sum_{i=1}^{N} L_i$.

Division Consistency Loss: When R-Counter is used, we can further constrain the consistency between different $C_i$s when $C_i$s are in the range of the closed set $[0, C_{max}]$. For $C_0$ and $C_1$, the division consistency loss is defined by
\[
L_{eq} = \mathbb{1}\{C_0^{gt} \leq C_{max}\} \times (C_0 - \text{sum}(C_1)),
\]
where ‘sum’ is the operator that returns the sum of all elements. Following Eq. 8, the consistency loss between $C_{i-1}$ and $C_i$, $i = 1, 2, \ldots N$, is
\[
L_{eq} = \sum_{j=1}^{H_{i-1}} \sum_{k=1}^{W_{i-1}} \mathbb{1}\{C_{i-1}^{gt}[j,k] \leq C_{max}\} \
\times |C_{i-1}[j,k] - \text{sum}(C_i[2j-1 : 2j, 2k-1 : 2k])|
\]
The overall division consistency loss $L_{eq} = \sum_{i=1}^{N} L_{eq}$. Notice that, when C-Counter is adopted, the gradient of the consistency loss cannot be backpropagated because count values are discretized into count intervals and represented by class labels. We simply drop the consistency loss in this case.
As a summary, for SS-DCNet (reg) the final loss \( L_{reg} \) is
\[
L_{reg} = L_R + L_m + L_{up} + L_{div} + L_{reg},
\]
and for SS-DCNet (cls) the final loss \( L_{cls} \) is
\[
L_{cls} = L_C + L_m + L_{up} + L_{div}.
\]

4 OPEN SET OR CLOSED SET? A THEORETICAL ANALYSIS

How does SS-DCNet benefit from transforming count values from open set to closed set? Here we attempt to provide some theoretical insights of the benefits of closed set transformed by S-DC.

**Definition 1** (Spatial Division of an Image). Given an image \( I \in \mathbb{R}^{H \times W \times K} \), where \( H \), \( W \) and \( K \) denote the height, width and channel dimensions, respectively, the spatial division of \( I \) leads to a group of sub-images \( \{ I_i \in \mathbb{R}^{H_i \times W_i \times K} \}_{i=1,2,...,M} \) that satisfy:

i) \( I_i \subset I \);

ii) \( I_i \cap I_j = \phi \), for \( i \neq j \);

iii) \( I_i \cup I_j \cup ... \cup I_M = I \).

**Definition 2** (Open Set and Closed Set). Given a positive number \( C_{max} \) for all \( x \geq 0 \) and \( x \in \mathbb{R} \), we can define a closed set \( \mathcal{S}_O \) by \( \{ x | 0 \leq x \leq C_{max} \} \) and an open set \( \mathcal{S}_C \) by \( \{ x | x > C, C \leq C_{max} \} \).

In object counting, \( C_{max} \) is the maximum count value observed in the training set.

**Lemma 1.** Given an image \( I \), let \( \boxminus \) be the spatial dichotomy division operator such that \( \boxminus(I) = \{ I_i \}_{i=1,2,3,4} \). Let \( \boxminus^N \) further denote \( \boxminus \) is applied for \( N \) times. We have \( \boxminus^N(I) = \{ I_j \}_{j=1,2,...,4^N} \).

**Proof.** Suppose that \( M \) is the number of divided sub-images after \( N \) divisions.

i) For \( N = 1 \), according to the definition of \( \boxminus \),
\[
\boxminus(I) = \{ I_i \}_{i=1,2,3,4},
\]
which means \( M = 4 \);

ii) For \( N = t \), assume \( M = 4^t \), we have
\[
\boxminus^t(I) = \{ I_k \}_{k=1,2,...,4^t},
\]
then when \( N = t + 1 \), for each \( I_k \in \boxminus^t(I) \),
\[
\boxminus^{t+1}(I_k) = \boxminus(\boxminus^t(I)) = \{ \boxminus(I_k), k = 1, 2, ... 4^t \},
\]
so \( M = 4 \times 4^t = 4^{t+1} \) holds for \( N = t + 1 \).

Since both i) and ii) hold, by mathematical induction, we can deduce \( M = 4^N \) after \( N \) divisions.

According to Lemma 1, we know that, an image \( I \) will be divided into \( 4^N \) sub-images at most after \( N \) divisions. A subsequent question of interest is that, how many spatial divisions are required to transfer count values from \( \mathcal{S}_O \) to \( \mathcal{S}_C \)? This leads to our following proposition.

**Proposition 1** (Minimum Division Times). Assume an image \( I \) with a count value \( C^* > C_{max} \), \( C^* \in \mathcal{S}_O \), is divided by the \( \boxminus \) operator, then at least \([ \log_4 \frac{C^*}{C_{max}} ]\) division times are required to transfer \( C^* \) into \( \mathcal{S}_C \).

**Proof.** Suppose after \( N \) division times, \( I \) is divided into \( M \) sub-images \( \{ I_i \}_{i=1,2,...,M} \) with local count values \( c_i \) such that \( 0 \leq c_i \leq C_{max} \) and \( \sum_{i=1}^{M} c_i = C^* \). Since
\[
c_i \leq \max_{i=1,2,...,M} c_i \leq C_{max},
\]
we have
\[
C^* = \sum_{i=1}^{M} c_i \leq M \times \max_{i=1,2,...,M} c_i \leq M \times C_{max}.
\]
Hence, \( M \geq \frac{C^*}{C_{max}} \). With Lemma 1, we know \( M = 4^N \).

By Definition 3, it is clear that the expectation of \( |\epsilon_r| \) varies as the ground truth \( C \) changes. We thus have

**Definition 3.** Let \( C (C > 0) \) be the ground-truth value, and \( \hat{C} \) the inferred value. We define the relative error by \( \epsilon_r = \frac{\hat{C} - C}{C} \) and the absolute error by \( \epsilon_a = |\hat{C} - C| = C \times |\epsilon_r| \).

By Definition 3, \( \epsilon_r \) is continuous. Before the presentation of our main results, we further need the conclusion of the following theorem.

**Theorem 1** (Extreme Value Theorem [33]). If \( f(x) \) is a continuous function defined in the closed interval \([a, b]\), then \( \exists c \in [a, b] \) that satisfies \( f(c) = \max_{a \leq x \leq b} f(x) \).

According to Definition 3, Definition 4 and Theorem 1, we arrive at our final proposition.

**Proposition 2.** Let \( \epsilon_a^o \) denote the absolute counting error on \( \mathcal{S}_O \), \( \epsilon_r^o \) the counting error on \( \mathcal{S}_C \) (after spatial divisions), \( f(x) = \mathbb{E}_{C=x} |\epsilon_r| \), and \( C_{max} \) a predefined positive number. Given a count value \( C^* \), if \( C^* > C_{max} \) and \( f(C^*) = \max_{0 \leq x \leq C_{max}} f(x) \), then
\[
\mathbb{E}_{C=C^*} [\epsilon_a^o] < \mathbb{E}_{C=C^*} [\epsilon_r^o].
\]

**Proof.** For an image \( I \) with a count value \( C^* \), a spatial division of \( I \), i.e., \( \{ I_i \}_{i=1,2,...,M} \), could be found as per Definition 1. Their corresponding local counts \( \{ c_i \}_{i=1,2,...,M} \) satisfy
\[
i) \ 0 \leq c_i \leq C_{max}, \ i = 1, 2, ..., M, \ 	ext{and}

ii) \( \sum_{i=1}^{M} c_i = C^* \).

Let the \( \epsilon_r^i \) denote the relative counting error on \( \mathcal{S}_O \), and \( \epsilon_a^i \) the relative counting error of each \( I_i \) on \( \mathcal{S}_C \). By Definition 3, we have
\[
\epsilon_a^i = |C^* \times \epsilon_r^i| = C^* \times |\epsilon_r^i|,
\]
and
\[
\epsilon_r^i = \left| \sum_{i=1}^{M} c_i \times \epsilon_r^i \right|.
\]

By Definition 4,
\[
\mathbb{E}_{C=C^*} [\epsilon_a^o] = \mathbb{E}_{C=C^*} [C^* \times |\epsilon_r^o|] = C^* \times \mathbb{E}_{C=C^*} [\epsilon_r^o] = C^* \times f(C^*).
\]
With generalized triangle inequality [17], we have
\[
e^a_x = \sum_{i=1}^{M} C_i \times e^i_x \leq \sum_{i=1}^{M} |C_i| \times e^i_x \leq \sum_{i=1}^{M} |C_i| \times |e^i_x|. \tag{20}
\]
By taking the expectation of both sides of Eq. (20), it amounts to
\[
\mathbb{E}_{C=C^*} [e^a_x] \leq \sum_{i=1}^{M} C_i \times \mathbb{E}_{C=C^*} [e^i_x] = \sum_{i=1}^{M} C_i \times f(c_i), \tag{21}
\]
According to Theorem 1, \( \exists \zeta \in [0, C_{max}] \) such that
\[
f(\zeta) = \max_{0 \leq x \leq C_{max}} f(x). \]
Hence, \( f(c_i) \leq f(\zeta) \), for \( i = 1, 2, \ldots, M \), so
\[
\mathbb{E}_{C=C^*} [e^a_x] \leq \sum_{i=1}^{M} C_i \times f(c_i) \leq \sum_{i=1}^{M} c_i \times f(\zeta) = f(\zeta) \times \sum_{i=1}^{M} c_i = f(\zeta) \times C^*.
\]
If \( f(C^*) > f(\zeta) = \max_{0 \leq x \leq C_{max}} f(x) \), with Eq. (19), we have
\[
\mathbb{E}_{C=C^*} [e^a_x] \leq f(\zeta) \times C^* < f(C^*) \times C^* = \mathbb{E}_{C=C^*} [e^a_x]. \tag{23}
\]
Proof completes.

Corollary 1. Let \( P \) be the learning target and is spatially divisible. Let \( e^a_x \) denote the absolute error of \( P \) on \( S_O \), \( e^a_x \) the absolute error of \( P \) on \( S_C \) (after spatial divisions), \( f(x) = \mathbb{E}_{p=x} [e^a_x] \), and \( P_{max} \) a predefined positive number. Given a positive number \( P^* \), if \( P^* > P_{max} \) and \( f(P^*) > \max_{0 \leq x \leq P_{max}} f(x) \), then
\[
\mathbb{E}_{P>P^*} [e^a_x] < \mathbb{E}_{P=P^*} [e^a_x].
\]

5 OPEN SET OR CLOSED SET? A TOY-EXAMPLE JUSTIFICATION
As aforementioned, counting is an open-set problem, while the model is learned in a closed set. Can a closed-set counting model really generalize to open-set scenarios? Here we show through a controlled toy experiment that, the answer is negative. In addition, in this experiment we illustrate that SS-DCNet indeed works better than that without S-DC, which supports our Proposition 2. Inspired by [18], we synthesize a cell counting dataset to explore the counting performance outside a closed training set.

5.1 Synthesized Cell Counting Dataset
We first generate 500 256 \( \times \) 256 images with 64 \( \times \) 64 sub-regions containing only 0 \~ 10 cells to construct the training set (a closed set). To generate an open testing set, we further synthesize 500 images with sub-region counts evenly distributed in the range of \([0, 20]\).

5.2 Baselines and Protocols
We adopt three approaches for comparisons, they are: i) a density regression baseline CSRNet [21]; ii) a regression baseline with pretrained VGG16 as the backbone and the R-Counter used in SS-DCNet as the backend, without S-DC; iii) a classification baseline with the same VGG16 and the C-Counter, without S-DC; iv) our proposed SS-DCNet, which learns from a closed set but adapts to the open set via S-DC. According to Proposition 1, at least 1-time division is required for SS-DCNet to transform count values from the open set to the closed set. We adopt both SS-DCNet (reg) and SS-DCNet (cls) with 1-time division for comparison.

Regarding the discretization of count intervals, we choose 0.5 as the step because cells may be partially presented in local patches. As a consequence, we have a partition of \([0], (0.0, 0.5], (0.5, 1], ..., (9.5, 10]\) and \((10, +\infty)\). All approaches are trained with standard stochastic gradient descent (SGD). The learning rate is initially set to 0.0001 and is decreased by \(\times 10\) when the training error stagnates.

5.3 Observations
According to Fig. 5 (b), it can be observed that both regression and classification baselines work well in the range of the closed set (0 \~ 10), but the counting error increases rapidly when counts are larger than 10. This suggests a conventional counting model learned in a closed set cannot generalize to the open set. However, SS-DCNet can achieve accurate predictions even on the open set, which confirms the advantage of S-DC.
They are defined as performance to verify the generalization ability of SS-DCNet. We have the following discussions:

- As shown in Fig. 5(c), \( f(C_0) > \max_{0 \leq x \leq C_{max}} f(x) \) satisfies for the classification baseline when the patch count \( C_0 \geq 12 \). According to Proposition 2, under the condition above, SS-DCNet (cls) will show lower MAE than the classification baseline without S-DC. When \( C_0 \geq 16 \), the same conclusion can be drawn between SS-DCNet (reg) and its open-set regression baseline. When \( C_0 \in [10, 12] \), \( f(C_0) > \max_{0 \leq x \leq C_{max}} f(x) \) is no longer true for the classification baseline. As shown in Fig. 5(b), SS-DCNet (cls) only reports comparable results against the classification baseline. When \( C_0 \in [10, 16] \), the same observation can be made between SS-DCNet (reg) and its open-set regression counterpart.

In general, our toy experiment verifies Proposition 2 to a certain extent. It is encouraged to transform count values from the open set to the closed set.

5.4 Analyses

Relative mean absolute error (rMAE) is an estimation of \( f(x) \) as per Definition 4, and the mean absolute error (MAE) is \( \mathbb{E}_{C=C_0} \{e_a\} \). Fig. 5(c) and (b) report how these two metrics vary, respectively. We have the following discussions:

- When \( C_0 \in [0, 25] \), \( f(C_0) > \max_{0 \leq x \leq C_{max}} f(x) \) is no longer true for the classification baseline. As shown in Fig. 5(b), SS-DCNet (cls) only reports comparable results against the classification baseline.

6 EXPERIMENTS ON REAL-WORLD DATASETS

Extensive experiments are further conducted to demonstrate the effectiveness of SS-DCNet on real-world datasets. We first describe some essential implementation details. After that, ablation studies are conducted on the ShanghaiTech Part_A [53] dataset to highlight the benefit of S-DC. We then compare SS-DCNet against current state-of-the-art methods on five public datasets. Finally, we also report cross-domain performance to verify the generalization ability of SS-DCNet.

Mean Absolute Error (MAE) and Root Mean Squared Error (MSE) are chosen to quantify the counting performance. They are defined as

\[
MAE = \frac{1}{Z} \sum_{i=1}^{Z} |C_i^{pre} - C_i^{gt}|, \tag{24}
\]

\[
MSE = \frac{1}{Z} \sum_{i=1}^{Z} (C_i^{pre} - C_i^{gt})^2, \tag{25}
\]

where \( Z \) denotes the number of images, \( C_i^{pre} \) denotes the predicted count of the \( i \)-th image, and \( C_i^{gt} \) denotes the corresponding ground-truth count. MAE measures the accuracy of counting, and MSE measures the stability. Lower MAE and MSE imply better counting performance.

6.1 Implementation Details

6.1.1 Interval Partition for C-Counter

We generate ground-truth counts of local patches by integrating over the density maps. The counts are usually not integers, because objects can partly present in cropped local patches. We evaluate two different partition strategies. In the first partition, we choose 0.5 as the step and generate partitions as \{0\}, \{0, 0.5\}, \{0.5, 1\}, ..., \{C_{max} - 0.5, C_{max}\} and \{C_{max}, +\infty\}, where \( C_{max} \) denotes the maximum count of the closed set. This partition is named as One-Linear Partition.

In the second partition, we further finely divide the sub-interval \((0, 0.5]\), because this interval contains a sudden change from no object to part of an object, and a large proportion of objects lie in this sub-interval. A small step of 0.05 is further used to divide the sub-interval \((0, 0.5]\), i.e., \((0, 0.05]\), \((0.05, 0.1]\), ..., \((0.45, 0.5]\). Other intervals remain the same as One-Linear Partition. We call this partition Two-Linear Partition.

6.1.2 Data Preprocessing

We follow the same data augmentation used in [21], except for the UCF-QNRF dataset [15] where we adopt two data augmentation strategies. In particular, 9 sub-images of \( \frac{1}{4} \) resolution are cropped from the original image. The first 4 sub-images are from four corners, and the remaining 5 are randomly cropped. Random scaling and flipping are also executed.

6.1.3 Training Details

SS-DCNet is implemented with PyTorch [32]. We train SS-DCNet using SGD. The encoder in SS-DCNet is directly adopted from convolutional layers of VGG16 [39] pretrained on ImageNet, and the other layers employ random Gaussian
6.2 Ablation Study on the ShanghaiTech Part_A

6.2.1 Is SS-DCNet Robust to \( C_{\text{max}} \) ?

When reformulating the counting problem into classification, a critical issue is how to choose \( C_{\text{max}} \), which defines the closed set. Hence, it is important that SS-DCNet is robust to the choice of \( C_{\text{max}} \).

We conduct a statistical analysis on count values of local patches in the training set, and then set \( C_{\text{max}} \) with the quantiles ranging from 100% to 80% (decreased by 5%). Two-stage SS-DCNet is evaluated. Another baseline of classification without S-DC is also used to explore whether counting can be simply modeled in a closed-set classification manner. To be specific, we reserve the VGG16 encoder and the C-Counter in this classification baseline.

Results are presented in Figure 6. It can be observed that the MAE of the classification baseline increases rapidly with decreased \( C_{\text{max}} \). This result is not surprising, because the model is constrained to be visible to count values not greater than \( C_{\text{max}} \). This suggests that counting cannot be simply transformed into closed-set classification. However, with the help of S-DC, SS-DCNet exhibits strong robustness to the changes of \( C_{\text{max}} \). It seems the systematic error brought by \( C_{\text{max}} \) can somewhat be alleviated with S-DC. Regarding how to choose concrete \( C_{\text{max}} \), the maximum count of the training set seems not the best choice, while some small quantities even deliver better performance. Perhaps a model is only able to count objects accurately within a certain degree of denseness. We also notice Two-Linear Partition is slightly better than One-Linear Partition, which indicates that the fine division to the \((0, 0.5]\) sub-interval has a positive effect.

According to the above results, SS-DCNet is robust to \( C_{\text{max}} \) in a wide range of values, and \( C_{\text{max}} \) is generally encouraged to be set less than the maximum count value observed. In addition, there is no significant difference between two kinds of partitions. For simplicity, we set \( C_{\text{max}} \) to be the 95% quantile and adopt Two-Linear Partition in the following experiments.

6.2.2 How Many Times to Divide?

SS-DCNet can apply S-DC up to 4 times, but how many times are sufficient? Here we evaluate SS-DCNet with different division stages. The maximum count value of \( 64 \times 64 \) image patches in the test set is 136.50 and \( C_{\text{max}} = 22 \). With Proposition 1, we know 2-time division is required at least. Quantitative results are listed in Table 2. It can be observed that when the division time \( N \) varies from 0 to 2, the counting error MAE significantly decreases for both SS-DCNet (reg) and SS-DCNet (cls). However, counting accuracy saturates when \( N \) continues increasing. In general, two-stage S-DC seems sufficient. We use this setup in the following experiments.

### Table 2

| Division time | SS-DCNet (cls) MAE | SS-DCNet (cls) MSE | SS-DCNet (reg) MAE | SS-DCNet (reg) MSE |
|--------------|-------------------|-------------------|-------------------|-------------------|
| 0            | 76.0              | 132.5             | 76.7              | 144.6             |
| 1            | 57.8              | 92.0              | 61.0              | 98.1              |
| 2            | 56.1              | 88.9              | 59.5              | 95.0              |
| 3            | 57.0              | 92.7              | 60.1              | 97.2              |
| 4            | 59.1              | 100.0             | 62.8              | 99.1              |

6.2.3 The Effect of S-DC

To highlight the effect of S-DC, we compare SS-DCNet against several regression and classification baselines. These baselines adopt the same architecture of VGG16 encoder and the counter in SS-DCNet. Classification is the result of \( C_0 \) adopting C-Counter without S-DC, and \( C_{\text{max}} \) is set to be the 95% quantile \( (C_{\text{max}} = 22) \). For regression baselines, we employ R-counter to obtain the prediction \( C_0 \) without S-DC. We create two regression baselines. open-set regression + S-DC is straightforward. We do not limit the output range, and it can vary from 0 to \(+\infty\). \( S_0 \) regression indicates that the output range is constrained within \([0, C_{\text{max}}]\). \( C_{\text{max}} \) also set to 22 for a fair comparison). Any large outputs will be clipped to \( C_{\text{max}} \).

Results are shown in Table 3. We can see that counting by classification without S-DC suffers from the limitation of \( C_{\text{max}} \) and performs even worse than \( S_0 \) regression. \( S_0 \) regression also suffers from the same problem. However, with S-DC, SS-DCNet (reg/cls) significantly reduces the counting error and outperforms both their regression/classification baseline by a large margin. It suggests that a counting model can learn from a closed set and generalize well to an open set via S-DC. We notice that SS-DCNet (cls) performs better than SS-DCNet (reg). It seems that reformulating counting in classification is more effective than in regression. One plausible reason is that the optimization is easier and less sensitive to sample imbalance in classification than in regression.
We further analyze the counting error of 64 × 64 local patches in detail. As shown in Figure 7, we observe that the direct prediction \( C_0 \) without S-DC performs worse than the \( S_r \) regression baseline and CSRNet, which can be attributed to the limited \( C_{\text{max}} \) of the C-Counter. After embedding S-DC, the counting errors of \( \text{DIV}_1 \) and \( \text{DIV}_2 \) significantly reduce and outperform open-set regression and CSRNet. Such a benefit is even much clearer in dense patches with local counts greater than 100. It justifies our argument that, instead of regressing a large count value directly, it is more accurate to count dense patches through S-DC.

### 6.2.4 Choices of Loss Functions

Here we validate the effect of different loss functions used in SS-DCNet. Results are reported in Table 4. As analyzed in S-DCNet [49], \( L_C \) provides supervision to \( C_i \)'s, and \( L_m \) implicitly supervises the division weights \( W_i \)'s. With only \( L_C \) and \( L_m \), S-DCNet (cls) can achieve good division results. However, S-DCNet (reg) cannot report competitive results as S-DCNet (cls) with \( L_R \) and \( L_m \). After incorporating the upsampling loss \( L_u \), in the SS-DCNet (reg/cls), MAE reduces by 3.2/0.5. Such an improvement can be attributed to the replacement of the average upsampling in S-DCNet with learned upsampling in SS-DCNet. \( L_{\text{div}} \) provides explicit supervision for spatial division weight \( W_i \)'s. One can see that, \( L_{\text{div}} \) can further improve the counting performance of SS-DCNet (reg/cls), and clear division results can be observed as shown in Fig. 8. Moreover, division consistency loss \( L_{\text{eq}} \) is also effective for SS-DCNet (reg), with 1.1 improvement in MAE. Overall, SS-DCNet (reg/cls) shows a clear advantage over its previous version S-DCNet [49] with the help of additional supervision \( L_u \), \( L_{\text{div}} \) and \( L_{\text{eq}} \).

### 6.2.5 Spatial Divide-and-Conquer Versus Spatial Attention

To highlight the difference between S-DC and spatial attention (SA), we remove the division decider, generate a 3-channel output conditioned on \( F_2 \), then normalize it with softmax to obtain \( W_{\text{att}}^0 \), \( W_{\text{att}}^1 \) and \( W_{\text{att}}^2 \). The final count is merged as \( W_{\text{att}}^0 \ast \text{upsample}(C_0) + W_{\text{att}}^1 \ast \text{upsample}(C_1) + W_{\text{att}}^2 \ast C_2 \). In SHTech PartA, SA achieves 64.1 MAE and 109.9 MSE, worse than SS-DCNet. As per the visualization of \( W_{\text{att}} \) in Fig. 8, we find SA only focuses on the highest resolution, and no effect of division is observed. Instead, SS-DCNet learns to divide local patches when local counts are greater than \( C_{\text{max}} \). In addition, SS-DCNet executes fusion recursively, while SA fuses the prediction in a single step.

### 6.3 Comparison with State of the Art

According to the ablation study, the final configurations of SS-DCNet are summarized in Table 5.

#### 6.3.1 The ShanghaiTech Dataset

The ShanghaiTech crowd counting dataset [53] includes two parts: Part_A and Part_B. Part_A has 300 images for training and 182 for testing. This part represents highly congested scenes. Part_B contains 716 images in relatively sparse scenes, where 400 images are used for training and 316 for testing. Quantitative results are listed in Table 6. The improvements of SS-DCNet are two-fold. First, with the explicit supervision of S-DC, SS-DCNet (cls) performs better than our previous S-DCNet. Second, our method outperforms the previous state-of-the-art PGCNet [51] in Part_A and competitive results (6.6 MAE) as SPANet [9] (6.5 MAE) in Part_B, respectively. These results suggest SS-DCNet is able to adapt to both sparse and crowded scenes.
6.3.2 The UCF_CC_50 Dataset

UCF_CC_50 [14] is a tiny crowd counting dataset with 50 images in extremely crowded scenes. The number of people within an image varies from 96 to 4633. We follow the 5-fold cross-validation as in [14]. Results are shown in Table 7. Our method surpasses S-DCNet and the previous best method, PaDNet [45], with 12.2% and 3.4% relative improvements in MAE, respectively.

6.3.3 The UCF-QNRF Dataset

UCF-QNRF [15] is a relatively large crowd counting dataset with 1535 high-resolution images and 1.25 million head annotations. There are 1201 training images and 334 test images. It contains extremely congested scenes where the maximum count of an image can reach 12865. Some images in the UCF-QNRF dataset are too large, with the longer side equals to 10000, to process the whole image. There are two ways to solve this problem: i) cropping the original image into $224 \times 224$ sub-images following [15]; ii) resizing the original image to make the longer side no larger than 1920 as in [50], [27], then 9 sub-images of $\frac{1}{4}$ resolution are cropped from the original image for data augmentation as described in Section 6.1.2. Results are reported in Table 8. We can make following observations:

• For S-DCNet, it works better with strategy ii) than i). This means that resizing is a better choice than cropping. We think the reasons are two-fold. First, the receptive field of a CNN is limited, thus it cannot cover over-size images. Second, if cropping over-size images into $224 \times 224$ sub-images, the surrounding pixels of sub-images, termed 'local visual context', are invisible to the CNN. However, the local visual context can provide support information.
to distinguish overlapped objects as demonstrated in [48], and CNNs tend to perform poorly when local context is lost.

- With the explicit supervision of S-DC, SS-DCNet (cls) brings a significant improvement over S-DCNet by 15.8 in MAE, and SS-DCNet (reg) shows by 5.3 in MAE.
- SS-DCNet (reg) reports competitive results against the current state-of-the-art BAYESIAN+ [27], while SS-DCNet (cls) outperforms BAYESIAN+ by 6.8 in MAE and 11 in MSE.
- It is worth noting that, SS-DCNet only learns from a closed set with \( C_{\text{max}} = 8.0 \), which is only 6% of the maximum count 131.5 according to Table 5. SS-DCNet, however, generalizes to large counts effectively and predicts accurate counts.

6.3.4 The TRANCOS Dataset

Aside from crowd counting, we also evaluate SS-DCNet on a vehicle counting dataset, TRANCOS [13], to demonstrate the generality of SS-DCNet. TRANCOS contains 1244 images of congested traffic scenes in various perspectives. It adopts the Grid Average Mean Absolute Error (GAME) [13] as the evaluation metric. \( GAME(L) \) divides an image into \( 2^L \times 2^L \) non-overlapping sub-regions and accumulates the MAE over sub-regions. Larger \( L \) implies more accurate local predictions. In particular, \( GAME(0) \) downgrades to MAE. The GAME is defined by

\[
GAME(L) = \frac{1}{N} \sum_{n=1}^{N} \left( \sum_{l=1}^{L} (C_{\text{pre}}^l - C_{\text{gt}}^l) \right),
\]

where \( N \) denotes the number of images, \( C_{\text{pre}}^l \) and \( C_{\text{gt}}^l \) are the predicted and ground-truth count of the \( L \)-th sub-region, respectively. Results are listed in Table 9. SS-DCNet surpasses other methods under all \( GAME(L) \) metrics, and particularly, delivers a 33.1% relative improvement than SPN [8] on \( GAME(3) \). This suggests SS-DCNet not only achieves accurate global predictions but also behaves well in local regions.

6.3.5 The MTC Dataset

We further evaluate our method on a plant counting dataset, i.e., the MTC dataset [25]. The MTC dataset contains 361 high-resolution images of maize tassels collected from 2010 to 2015 in the wild field. In contrast to pedestrians or vehicles that have similar physical sizes, maize tassels are with heterogeneous physical sizes and are self-changing over time. We believe that this dataset is suitable for justifying the robustness of SS-DCNet to object-size variations. We follow the same setting as in [25] and report quantitative results in Table 10. Although the previous best method, TasselNetv2† [48], already exhibits accurate results, SS-DCNet still shows a substantial degree of improvement (26.4% on MAE and 29.8% on MSE).

Qualitative results are shown in Fig 9. We can observe that SS-DCNet produces accurate predictions for various objects from sparse to dense scenes.

6.4 Cross Dataset Generalization

We further conduct cross-dataset experiments on the ShanghaiTech [53] (A and B) and UCF-QNRF (QNRF) [15] datasets to show the transfer ability of SS-DCNet. Quantitative results are shown in Table 11. The ‘regression’ and ‘classification’ methods are baselines for SS-DCNet (reg) and S-DCNet (cls), respectively, which adopt the VGG16 [39] as the feature encoder and R-Counter/C-Counter in SS-DCNet but do not apply S-DC. We can make following observations:

- Consistent improvements in MAE are observed when comparing SS-DCNet (reg/cls) to its baselines. Especially in SS-DCNet (cls) vs. baseline cls, the MAE of \( B \rightarrow QNRF \) shows a 40.89% relative improvement when S-DC is added.
- Two types of SS-DCNet report superior or at least competitive results than other state-of-the-art methods under all cross-dataset tasks, which suggest SS-DCNet has strong transferring ability.
- SS-DCNet (cls) transferred from the QNRF dataset reports even better results (61.8 MAE) than most state-of-the-arts methods (e.g., 68.2 MAE for CSRNet and 67.0 MAE for SANet) trained on the ShanghaiTech dataset.
- All methods trained on the ShanghaiTech [53] dataset report worse cross-dataset results than trained directly on the target datasets. By contrast, all methods trained on the QNRF [15] dataset exhibits at least competitive transferring results against state-of-the-art methods trained on the target dataset. This may be attributed to the fact that the ShanghaiTech dataset is too small, with only 300 training samples in the Part_A and 400 in the Part_B, to train a robust model, while the QNRF dataset provides sufficient training samples.

Overall, SS-DCNet demonstrates state-of-the-art results in all cross-dataset experiments. The good performance of SS-DCNet may be explained from its implicit transferring ability in the output space, which shares the same spirit with [46]. To justify this, we analyze the case of \( QNRF \rightarrow A \) and visualize the distribution of count values with and without S-DC in Fig. 10. It can be observed that, the distribution of count values significantly shift between SHA and QNRF without S-DC, but the divergence of the distribution narrows down after count values are transformed into a closed set [0, 8]. We further compute the Jensen-Shannon divergence [30] \( J_s \) to quantify the divergence of the distribution, and find that \( J_s = 0.0178 \) between SHA and QNRF without S-DC and \( J_s = 0.0133 \) with S-DC. The smaller \( J_s \) is, the smaller divergence between two distributions shows. This means the divergence of the output (count value) space reduces after closed-set transformation. We think this is the main reason why SS-DCNet reports remarkable performance in the task of \( QNRF \rightarrow A \). In short, SS-DCNet improves the cross-dataset generalization via the implicit adaptation of the output (count values) space.

7 Conclusion

Counting is an open-set problem in theory, but only a finite closed set can be observed in reality. This is particularly true because any dataset is always a sampling of the real world. Inspired by the decomposable property of counting, we
propositional logic problem and implement this transformation with the idea of S-DC. We propose supervised S-DC in a deep counting network termed SS-DCNet. We provide a theoretical analysis showing why the transformation from the open set to closed set makes sense for counting. A toy experiment and extensive evaluations on standard benchmarks are also conducted to show that, even given a closed training set, SS-DCNet can effectively generalize to open-set scenarios. Furthermore, SS-DCNet shows its good generalization ability via state-of-the-art cross-dataset performance.

Many vision tasks are open-set by nature, depth estimation for example, while it is not immediately clear on how to transform them into a closed set like counting. It would be interesting to explore how to transform other vision tasks into a closed set setting.

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