Self-Organized Criticality Effect on Stability: Magneto-Thermal Oscillations in a Granular YBCO Superconductor

L. Legrand and I. Rosenman
Groupe de Physique des Solides, Unité 17 Associée au CNRS,
Universités Paris 6 et Paris 7, Tour 23, 2 place Jussieu, 75251 Paris Cedex 5, France

R.G. Mints
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences,
Tel Aviv University, Tel Aviv 69978, Israel

G. Collin
Laboratoire Léon Brillouin, CEN-Saclay, 91191 Gif-sur-Yvette Cedex, France

(April 1, 2022)

We show that the self-organized criticality of the Bean’s state in each of the grains of a granular superconductor results in magneto-thermal oscillations preceding a series of subsequent flux jumps. We find that the frequency of these oscillations is proportional to the external magnetic field sweep rate $\dot{H}_e$ and is inversely proportional to the square root of the heat capacity. We demonstrate experimentally and theoretically the universality of this dependence that is mainly influenced by the granularity of the superconductor.

74.60.Ge, 74.72.Bk, 74.80.Bj

The theory of the self-organized criticality explains the dynamics of the nonequilibrium systems similar to sandpiles. The magnetic flux dynamics and, in particular, the magnetization relaxation process in a superconductor with a strong pinning potential for vortices is an interesting and challenging field of application for this theory. In the scope of the ideas of the self-organized criticality the pinned vortices space distribution arises as a result of a subsequent local vortex avalanches, i.e., a series of small local flux jumps is establishing the critical state. The Bean critical state model successfully describes the irreversible magnetization in type-II superconductors by introducing the critical current density $j_c$. In the framework of the Bean model the value of the slope of the stationary magnetic field profile is less or equal to $\mu_0 j_c$. It makes the spatial distribution of vortices in a superconductor with a strong pinning potential similar to the sand particles spatial distribution in a sandpile.

The stationary critical state becomes unstable under certain conditions when the local flux jumps result in a global flux jump driving the system to the normal state. This instability can be preceded by a series of magneto-thermal oscillations. These oscillations have been reported earlier but never have been studied.

Each of the local vortices avalanches establishing the critical state produces a heat pulse and a temperature rise in the superconductor. This temperature rise decreases the critical current density for a certain time interval and, thus, changes the initial conditions for the subsequent vortices avalanches. In other words, the heat pulses produced by the local vortices avalanches result in a correlation mechanism specific for the self-organized criticality of magnetic flux motion in superconductors with a strong pinning potential. In this Letter we demonstrate that this mechanism results in the magneto-thermal oscillations arising close to the threshold of the superconducting state stability. We focus our study on the dependence of the frequency of these oscillations on the temperature and the magnetic field sweep rate in case of a granular superconductor. We point out that the granularity and the randomness of the properties of the superconducting grains result in universality of this dependence.

We begin with the theoretical consideration of the critical state stability and magneto-thermal oscillations. We propose a one-dimensional model of a granular superconductor treating it as a stack of superconducting slabs having the width $2b_i$ ($i = 1, 2, 3, \ldots, N$) randomly distributed with a certain mean value $b$. We assume that there is no electrical contact between the slabs and there is an ideal thermal contact between them. We suppose that the external magnetic field $H_e(t)$ is parallel to the sample surface ($H_e$ is parallel to the $z$-axis) and the magnetic field sweep rate $\dot{H}_e$ is constant. We assume that the critical state arises simultaneously in the entire superconductor and therefore in each of the slabs the background magnetic, $B_i(x,t)$, and electric, $E_i(x,t)$, fields are determined by Maxwell equations

$$\frac{dB_i}{dx} = \pm \mu_0 j_c, \quad \frac{dE_i}{dx} = \pm \dot{H}_e. \quad (1)$$

We suppose also that close to the instability threshold most of the slabs are saturated, i.e., the external magnetic field $H_e$ is higher than the Bean field $B_p = \mu_0 j_c b$.

Magneto-thermal oscillations in the critical state arise as coupled oscillations of small perturbations of the temperature, $\theta$, and the electric field, $\epsilon$. The heat diffusion and Maxwell equations determine the spatial and temporal variations of $\theta$ and $\epsilon$, namely,
\begin{equation}
C \frac{\partial \theta}{\partial t} = \kappa \frac{\partial^2 \theta}{\partial x^2} + j_c \epsilon, \tag{2}
\end{equation}

\begin{equation}
\rho_0 \frac{\partial j}{\partial t} = \frac{\partial^2 \epsilon}{\partial x^2}, \tag{3}
\end{equation}

where \(C\) is the heat capacity, \(\kappa\) is the heat conductivity, and

\begin{equation}
\frac{\partial j}{\partial t} = \frac{\partial j}{\partial E} \frac{\partial E}{\partial t} - \frac{\partial j_c}{\partial T} \frac{\partial \theta}{\partial t}. \tag{4}
\end{equation}

In the critical state the current density, \(j\), is close to \(j_c\). In this region the \(j-E\) curve takes the form

\begin{equation}
j = j_c + j_1 \ln \left( \frac{E}{E_0} \right), \tag{5}
\end{equation}

where \(E_0\) is the voltage criterion at which \(j_c\) is defined, \(j_1\) determines the slope of the \(j-E\) curve, and \(j_1 \ll j_c\).

The relation given by Eq. (5) was first derived in the framework of the Anderson-Kim model, considering the thermally activated uncorrelated hopping of bundles of vortices \(T\). It follows from this model that \(j_1 \propto T\). In the framework of the self-organized criticality the value of \(j_1\) seems to be temperature independent in the range of \(T \ll T_c\). We will consider the later case as it is in a good agreement with numerous experimental data. To be more precise, we suppose that the ratio \(n = j_c/j_1 \gg 1\) is temperature independent at \(T \ll T_c\).

Using Eqs. (3) - (5) we obtain the equations to determine \(\theta(x,t) \propto \exp(\gamma t)\) and \(\epsilon(x,t) \propto \exp(\gamma t)\) in the form

\begin{equation}
\frac{nE_e}{\mu_0 \gamma j_c} \epsilon'' - \epsilon = \frac{nE_i}{j_c} \left| \frac{\partial j_c}{\partial T} \right| \theta, \tag{6}
\end{equation}

\begin{equation}
\kappa \theta'' + nE_i \left| \frac{\partial j_c}{\partial T} \right| \theta - \gamma C \theta = \frac{nE_i}{\mu_0 \gamma} \epsilon''. \tag{7}
\end{equation}

Let us clarify the following calculation qualitatively. Suppose that the initial temperature of the superconductor \(T_0\) increases by a small perturbation \(\theta_0\) arising due to a heat pulse with the energy \(\delta Q_0\). This temperature increase leads to a decrease of the superconducting currents. The reduction of these screening currents results in an additional flux penetration inside the superconductor. This flux motion induces an electric field perturbation \(\epsilon_0\) producing an additional heat release \(\delta Q_1\), an additional temperature rise \(\theta_1\), and, consequently, an additional reduction of the superconducting currents. At certain conditions this process results in an avalanche-type increase of the temperature and magnetic flux in the superconductor, \(i.e.,\) in a critical state instability.

The critical state is stable if the heat release, \(\delta Q\), arising in the process of electric field and temperature perturbations development is less than the maximum heat flux to the coolant. The value of \(\delta Q\) depends on both \(H_e\) and \(H_c\) for the unsaturated grains and only on \(H_c\) for the saturated grains. In our experiments most of the grains are saturated in the magnetic field region corresponding to the magneto-thermal oscillations. Therefore, the heat release in the unsaturated grains, \(\delta Q_u\), is small compared with the heat release in the saturated grains, \(\delta Q_s\). However, the term \(\delta Q_u\) is the only magnetic field dependent term in the heat balance equation and, thus, it determines the value of the global flux jump field \(H_j\). In other words, the relatively small heat release in the unsaturated grains is tuning the superconducting state in a granular superconductor to the instability.

The heat release \(\delta Q\) depends on the frequency of the magneto-thermal oscillations, \(\omega = \text{Im} \gamma\), in both the saturated and unsaturated grains. Thus, to calculate the value of \(\omega\) we take into account only the dominating heat release arising in the saturated grains. In other words, the frequency \(\omega\) is mainly determined by the response of the active media of the saturated grains to the small temperature and electric field perturbations. The vortices avalanches establishing the critical state become strongly correlated in the vicinity of the instability threshold.

Under conditions of our experiments the temperature \(\theta\) is practically uniform over the cross-section of the sample. Therefore, to solve Eq. (8) we consider \(\theta\) to be constant. In a saturated slab the background electric field \(E_i = H_e x\), where \(x = 0\) corresponds to the middle plain. In this case Eq. (8) takes the form

\begin{equation}
\frac{nH_e}{\mu_0 \gamma j_c} x \epsilon'' - \epsilon = \frac{nH_e \theta}{j_c} \left| \frac{\partial j_c}{\partial T} \right| x, \tag{8}
\end{equation}

with the boundary conditions \(\epsilon(\pm b_i) = 0\). Note that the characteristic space scale of Eq. (8) is given by

\begin{equation}
l = \frac{nH_e}{\mu_0 \gamma j_c}. \tag{9}
\end{equation}

The magneto-thermal oscillations exist for low values of \(H_e\), where \(l \ll b_i\). It allows to solve Eq. (8) using the WKB approximation that results in

\begin{equation}
\epsilon = \frac{nH_e \theta}{j_c} \left| \frac{\partial j_c}{\partial T} \right| \left[ x - \sqrt{b_i} \sqrt{\sinh(2 \sqrt{x/l})} \right], \tag{10}
\end{equation}

We integrate now Eq. (8) over the cross-section of the superconductor using Eq. (10) for the electric field \(\epsilon\) and we end up with the frequency of the magneto-thermal oscillations in the form

\begin{equation}
\omega = \frac{\dot{H}_e}{\sqrt{\mu_0 C(T_0) T^*}}, \tag{11}
\end{equation}

where \(T_0\) is the mean temperature of the oscillations and we determine the parameter \(T^*\) as

\begin{equation}
\frac{1}{T^*} = \sum_{i=1}^{N} \frac{n_i^2}{j_c} \left| \frac{\partial j_i}{\partial T} \right| b_i / \sum_{i=1}^{N} b_i. \tag{12}
\end{equation}

Thus, the frequency of the magneto-thermal oscillations in a granular superconductor is proportional to the
magnetic field sweep rate $\dot{H}_e$. The value of the parameter $T_0 \sim T_r/n^2$ is a certain constant in the temperature range $T_0 \ll T_r$. It follows then from Eq. (13) that the ratio $\mu = \omega \sqrt{C(T_0)/H_e}$ is independent on $T_0$ and $\dot{H}_e$, \emph{i.e.}, it is a constant characterizing the properties of the superconductor and its granular structure.

We perform an experimental study of the magneto-thermal oscillations in a textured YBa$_2$Cu$_3$O$_{7-\delta}$ superconductor grown from the melt and heat treated after the preparation. First, we do the measurements using the original sample (S0) with the size of $8 \times 9 \times 5.5$ mm$^3$. Next, we cut this sample in two approximately equal parts (S1 and S2) and we measure the magneto-thermal oscillations in S1 and S2. In this way we check the independence of the oscillations frequency on the size of the sample, \emph{i.e.}, its intrinsic origin influenced mainly by the granular structure of the superconductor.

We perform the magnetic characterization by DC and AC measurements in a Quantum Design SQUID magnetometer using the sample S3 with a size of $0.3 \times 0.5 \times 0.8$ mm$^3$ (the sample S3 is cut from the sample S2). We find the onset temperature at zero magnetic field $T_s \approx 88$ K and the AC susceptibility transition width $\Delta T \approx 5$ K.

We show in Fig. 1 one quarter of the DC magnetization loops measured for the sample S3 for several temperatures in the magnetic field range $0 < H < 5$ T. We find also that in parts of the sample S0 smaller than S3 the Bean penetration field remains the same as for S3. It means that the space scale of the screening current loops is smaller than the dimensions of the sub-samples, \emph{i.e.}, the sample consists of small superconducting grains. We show in the inset in Fig. 1 the temperature variation of the critical current extracted from the saturation value of the magnetization. As can be seen the critical current density decreases linearly in the range $2 < T < 6$ K.

The experimental setup for the magneto-thermal oscillations measurements consists essentially of a sample holder with a thin hollow powdered graphite column with low thermal conductance on which the YBCO sample is maintained. A small size (0.2 mm thick, 2 mm$^2$ surface) carbon thermometer is glued to the sample. The entire sample holder is maintained under a very low pressure of He gas ($2 \times 10^{-6}$ Torr) in order to reduce the heat leak to the coolant. The sample is first cooled down to the starting temperature $T_s$ in zero magnetic field. Then the field is established at a controlled rate and the temperature variation of the sample is measured.

We measure the magneto-thermal oscillations at temperatures in the range $2 < T_0 < 7$ K and the field sweep rates within the interval $15 < H < 60$ G/s. We show in Fig. 2 a typical sample temperature time dependence for $T_0 = 5$ K and $H_e = 20.4$ G/s. At low values of the magnetic field $H_e$ the temperature change due to the small vortices avalanches establishing the critical state. Above a certain magnetic field value, the sample temperature oscillations appear with a period $\tau$ in the range $10 < \tau < 70$ s and an amplitude increasing in time. These magneto-thermal oscillations we mainly explore and analyze. A flux jump occurs close to the Bean field accompanied by a temperature rise up to about 12 K with a characteristic time of the order of 1 s. Then the sample temperature relaxes to the coolant temperature with a rate depending on the heat leak.

We show in Fig. 3 the magneto-thermal oscillations frequency dependence on the field sweep rate, for several temperatures and different samples. As can be seen the value of $\omega$ increases linearly with $H_e$ within the accuracy of our measurements.

We show in Fig. 4 the ratio $\mu = \omega \sqrt{C(T_0)/H_e}$ as a function of the magnetic field sweep rate $\dot{H}_e$ for the samples S0, S1, and S2 for different temperatures $T_0$ from the interval $2 < T_0 < 7$ K. We use the relation:

$$C = 100 T + 5 T^3 \text{ (JK}^{-1}\text{m}^{-3}) \tag{13}$$

to calculate the heat capacity and we normalize the values of $\mu$ by the mean value $\bar{\mu}$ of their distribution. We see in Fig. 4 that the ratio $\mu$ is a constant within the accuracy of our experiments as predicted by Eq. (11). Assuming that $T^e \approx n^2 \left(\frac{dM}{dT}\right)M$ we estimate the value of $n$ as $n \approx 14.5$ which is in a good agreement with the known experimental data.

To verify the uniformity of the sample temperature we estimate the characteristic heat redistribution time $t_h = \frac{Cd^2}{\kappa}$, where $d$ is the sample size. We find that $t_h \approx 0.6$ s using the data $T \approx 5$ K, $C \approx 1200 \text{ JK}^{-1}\text{m}^{-3}$, $\kappa \approx 2 \text{ WK}^{-1}\text{m}^{-1}$, and $d \approx 1$ cm. This time constant is two orders of magnitude less than both the period of the magneto-thermal oscillations and the temperature relaxation time. Thus, the temperature of the sample is uniform as suggested above.

In conclusion, we show theoretically that the self-organized criticality of the Bean's state in each of the grains of a granular superconductor results in magneto-thermal oscillations preceding flux jumping. We study these oscillations experimentally in a granular YBCO samples at temperatures $2 < T < 7$ K and field sweep rates $10 < H_e < 60$ G/s. We find both experimentally and theoretically that the frequency of the magneto-thermal oscillations is proportional to the magnetic field sweep rate and is inversely proportional to the square root of the heat capacity. We demonstrate the universality of this dependence measured for different samples.

1. P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. \textbf{59}, 381 (1987); Phys. Rev. A \textbf{38}, 364 (1988).
2. C. Tang and P. Bak, Phys. Rev. Lett. \textbf{60}, 2347 (1988).
3. C. Tang, Physica \textbf{A194}, 315 (1993).
4. Z. Wang and D. Shi, Solid State Commun. \textbf{90}, 405 (1994).
5. W. Pan and S. Doniach, Phys. Rev. B \textbf{49}, 1192 (1994).
FIG. 1. One quarter of the magnetization loops for the sample S3 for several temperatures. The inset shows the temperature dependence of the critical current density.

FIG. 2. The temperature of the sample S1 as a function of the magnetic field $H_e$ for $\dot{H_e} = 20.4$ G/s and $T_0 = 5.0$ K. The inset shows the magneto-thermal oscillations.

FIG. 3. The dependencies $\omega(\dot{H_e})$ for the samples: S0 at $T_0 = 6$ K (filled circles), S1 at $T_0 = 3.8$ K (filled triangles) and at $T_0 = 5.3$ K (open circles), and sample S2 at $T_0 = 5.2$ K (open squares). The error bars are 8%.

FIG. 4. The ratio $\mu = \frac{\omega(\dot{H_e})}{\omega_0}$ for the samples S0, S1 and S2 as a function of the sweep rate for different temperatures $T_0$ from the range $2 < T_0 < 7$ K. The values of $\mu$ are normalized by the mean value of their distribution. The error bars are 13%.
Fig. 1. L. Legrand, I. Rosenman, R.G. Mints, and G. Collin, "Self-Organized Criticality Effect on Stability: Magneto-Thermal Oscillations in a Granular YBCO Superconductor".
Fig. 2. L. Legrand, I. Rosenman, R.G. Mints, and G. Collin, "Self-Organized Criticality Effect on Stability: Magneto-Thermal Oscillations in a Granular YBCO Superconductor".
Fig. 3. L. Legrand, I. Rosenman, R.G. Mints, and G. Collin, "Self-Organized Criticality Effect on Stability: Magneto-Thermal Oscillations in a Granular YBCO Superconductor".
Fig. 4. L. Legrand, I. Rosenman, R.G. Mints, and G. Collin, "Self-Organized Criticality Effect on Stability: Magneto-Thermal Oscillations in a Granular YBCO Superconductor".