For years, there has been a lot of evidence about the missing matter in the Universe. It is known that the components of the Universe are radiation, baryons, neutrinos, etc. but observations show that their contribution is less than 5% of the total mass of the Cosmos, in agreement with Big Bang Nucleosynthesis predictions. This suggests that there must exist a non-baryonic type of matter in galaxies and clusters of galaxies [13]. Recently, the observations of Ia-type supernovae [3,4] showed that there must exist another component that accelerates the expansion of the Universe. This new component must have a negative equation of state 

\[ p = -\omega \rho \]

The observations point out into a flat Universe filled with radiation, plus baryons, plus neutrinos, etc. contributing with \(~5\%\), a dark matter component with \(~25\%\) and the so called dark energy contributing with \(~70\%\) to the total mass of the Cosmos [1]. One of the most successful models until now is the \(\Lambda\) Cold Dark Matter (\(\Lambda\)CDM) model, where the dark energy is a cosmological constant [3]. However, some problems of this model has not been solved yet. First of all, if a cosmological constant exists, why is its contribution to the total matter of the same order of magnitude as baryons and cold dark matter? This is the cosmic coincidence problem. Also, the suggested value for the cosmological constant appears well below the values predicted by particle physics. On the other hand, the existence of a cosmological constant leads to a strong fine tuning problem over the initial conditions of the Universe.

These last facts open the possibility for the scalar fields as strong candidates to be the missing matter of the Universe [3,15]. A reliable model for dark energy is a fluctuating, inhomogeneous scalar field, rolling down a scalar potential, called Quintessence (Q) [13]. For this case, great effort has been done to determine the appropriate scalar potential that could explain current cosmological observations [11,14]. One example, is the pure exponential potential [10,14]. It has the advantages that it mimics the dominant density background and it appears naturally as a solution for a completely scalar dominated Universe [1]. But nucleosynthesis constraints require that the scalar field contribution be \(\Omega_\Phi \leq 0.2\), which indicates that the scalar field would never dominate the Universe [1]. However, a special group of scalar potentials has been proposed in order to avoid the fine tuning and coincidence problem: the tracker solutions [11], where the cosmology at late times is extremely insensitive to initial conditions. A typical potential is the pure inverse power-law one, \(V(\Phi) \sim \Phi^{-\alpha} (\alpha > 0)\) [11,10]. Although it reduces the fine tuning and the cosmic coincidence problem, the predicted value for the current equation of state for the quintessence is not in good agreement with supernovae results [11].

In this letter we use a potential cosh, in order to mimic a standard cold dark matter with a quintessential dark energy. Then we will investigate the scalar field fluctuations and the implications in structure formation in directions suggested by some authors. We obtain that the scalar field is a ultra-light particle which behaves just as cold dark matter. Using previous works [8,19], it is then possible that a scalar field fluctuation could explain the formation of the galaxy halos.

In a recent paper [12], we showed that the potential

\[ V(\Psi) = \tilde{V}_0 [\sinh (\alpha \sqrt{\kappa_o} \Psi)]^\beta \]

(1)
a good candidate for the dark energy. Its asymptotic behavior at early (late) times is the attractive inverse power-law (exponential) one. Its parameters are given by

\begin{equation}
\begin{aligned}
\alpha &= \frac{-3\omega_\Psi}{2\sqrt{3(1+\omega_\Psi)}}, \\
\beta &= \frac{2(1+\omega_\Psi)}{\omega_\Psi}, \\
\rho_\varphi &= \left( \frac{2V_o}{1-\omega_\Psi} \right)^{\frac{1}{1-\beta/2}} \rho_{oCDM}^{\beta/2},
\end{aligned}
\end{equation}

where \(\rho_{oCDM}\) and \(\rho_\varphi\) are the current energy densities of cold dark matter and dark energy, respectively, and \(\omega_\Psi\) is the current equation of state for the dark energy. It eliminates the fine tuning problem and dominates only at late times, driving the Universe to a power-law inflationary stage (for which the scale factor \(a \sim t^p\), with \(p > 1\)). Thus, again, we will take it as our model for the dark energy.

At the same time, there exists strong evidence for the scalar fields to be the dark matter at galactic level. If the dark matter component is the scalar field, then it was demonstrated in [18] that a scalar field fluctuation could behave in exactly the same way as the halo of a galaxy. The halos of galaxies (the scalar field fluctuations) could be axial symmetric [18] or spherically symmetric [19], in both cases, the geodesics of exact solutions fit the rotation curves of galaxies quite well. Besides, the \(\Lambda\)CDM model over predicts subgalactic structure and singular cores of the halos of galaxies [20]. In order to solve these problems, some authors have proposed power-law and power-law-like scalar potentials [17,21,22] to be the dark matter in the Universe, and it is worth to mention that some of them could be tracker solutions by themselves [23]. All attention has been put on the quadratic potential \(\Phi^2\), because of the well known fact that it behaves as pressureless matter due to its oscillations [20], implying that \(\omega_\varphi \simeq 0\), for \(<\rho_\varphi >\approx \omega_\varphi \rho_\gamma < \rho_\varphi >\). A reliable model for the dark matter can then be the potential (8 and references therein)

\begin{equation}
V(\Phi) = V_o \left[ \cosh(\sqrt{\kappa_o} \Phi) - 1 \right]
\end{equation}

because this potential joins together the attractive properties of an exponential potential and the already mentioned quadratic potential, as it can be seen from its asymptotic behavior.

We consider a flat, homogenous and isotropic Universe. Thus we use the flat Friedmann-Robertson-Walker (FRW) metric

\begin{equation}
ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2 (d\theta^2 + \sin^2(\phi) d\phi^2) \right].
\end{equation}

The components of the Universe are baryons, radiation, three species of light neutrinos, etc., and two minimally coupled and homogenous scalar fields \(\Phi\) and \(\Psi\), which represent the dark matter and the dark energy, respectively. Thus, the evolution equations for this Universe are

\begin{equation}
\begin{aligned}
\dot{H} &= \frac{\kappa_o}{3} (\rho + \rho_\Phi + \rho_\varphi), \\
\dot{\rho} + 3H (\rho + p) &= 0, \\
\dot{\Phi} + 3H \Phi + \frac{dV(\Phi)}{d\Phi} &= 0, \\
\dot{\Psi} + 3H \Psi + \frac{dV(\Psi)}{d\Psi} &= 0,
\end{aligned}
\end{equation}

being \(\kappa_o \equiv 8\pi G\), \(\rho\) the energy density of radiation, plus baryons, plus neutrinos, etc., \(\rho_\Phi = \frac{1}{2} \dot{\Phi}^2 + V(\Phi)\) and \(\rho_\varphi = \frac{1}{2} \dot{\varphi}^2 + \dot{V}(\Psi)\).

We start the evolution of the Universe in the radiation dominated era (RD), with large (small) and negative (positive) values for the scalar field \(\Phi\) (\(\Psi\)). Taking the initial condition \(\rho_\varphi < \rho_\gamma\), the energy density \(\rho_\varphi\) is subdominant and behaves as a cosmological constant. The tracker solution [24] will be reached only until the matter dominated era (MD), that is, until the background equation of state becomes \(\omega_\Phi = 0\). Since then, it will evolve with a constant equation of state \(\omega_\Psi\) and will dominate the current evolution of the Universe as an effective exponential potential. The Universe would then be in a power-law inflationary stage. More details can be found in [24]. Now, we will focus our attention in potential (3).

During RD, the scalar field energy density \(\rho_\Phi\) tracks the radiation energy density. More, the ratio of \(\rho_\Phi\) to the total energy density is constant and equals to (see [10,25] and references therein):

\begin{equation}
\frac{\rho_\Phi}{\rho_\gamma + \rho_\varphi} = \frac{4}{3\lambda^2},
\end{equation}

with \(\rho_\gamma\) the contribution due to radiation. In order to recover the success of the CDM model, we will make the scalar energy follow the standard cold dark matter at the epoch of its oscillations. Thus, we investigate the behavior of the scalar field \(\Phi\) near to the transition point \(|\sqrt{\kappa_o} \lambda \Phi| = 1\) (see eqs. (3)), i.e., the point when the scalar field is leaving the radiation solution (exponential-like potential) and entering the dust solution (quadratic-like potential). Taking \(a_*\) as the value for the scale factor when this transition occurred (the scale factor has been normalized to \(a = 1\) today), we find that it can be approximately given by
\[ a_* \approx \frac{4}{\lambda^2 - 4} \left( \frac{\Omega_{\text{CDM}}}{\Omega_{\gamma}} \right) . \]  

(7)

From this, it can be shown that for potential \( \Phi \) it follows

\[ \frac{\kappa_0 V_0}{(\lambda^2 - 4)^3} \approx \frac{1.7}{3} \left[ \left( \frac{\Omega_{\text{CDM}}}{\Omega_{\gamma}} \right)^3 \Omega_{\gamma} \right] H_0^2. \]

(8)

being \( \Omega_{\text{CDM}} \) and \( \Omega_{\gamma} \) the current measured values for the densities of dark matter and radiation, respectively, and \( H_0 \) the current Hubble parameter. The restriction from nucleosynthesis for the early exponential behavior of the potential requires

\[ \frac{\rho_\Phi}{\rho_\gamma} = \frac{4}{\lambda^2 - 4} < 0.2 \]

at the radiation dominated era \[10\]. Then we have that \( \lambda > 2\sqrt{6} \). A numerical solution for the density parameters \( \Omega_X = (\kappa_0 \rho_X)/(3H^2) \) is shown in fig. (1), and the time when oscillations start is well given by eq. (7), and with the values from eq. (8) the solution mimics quite well the standard CDM model until today (see for example \[12\]). Note that the change of \( \rho_\Phi \) to a dust solution occurred before the radiation-matter equality for the values given by eq. (8). This allows the scalar field \( \Phi \) to dominate the evolution of the Universe later, and to provoke a MD era \[20\].

Now we will investigate the fluctuations in the scalar dark matter component. Using an amended version of CMBFAST \[27\] and taking adiabatic initial conditions \[10\], we observe that the scalar fluctuations of \( \Phi \) make the scalar density contrast \( \delta_\Phi = (\delta \rho_\Phi/\rho_\Phi) \) follow the standard dark matter density contrast (see fig. \[3\]) \[28\]. We have then a kind of tracker solution for the fluctuations of the scalar dark matter, too. This last fact makes the potential \( \Phi \) a reliable dark matter one.

\[ \text{FIG. 1. Evolution of the dimensionless density parameters vs the scale factor } a \text{ with } \Omega_{\text{M}} = 0.30: \Lambda \text{CDM (black) and } \Psi \Phi DM \text{ for two values of } \lambda \text{ (red), } \lambda = 8 \text{ (green). } \text{The equation of state for the dark energy is } \omega_\Psi = -0.8. \]

\[ \text{FIG. 2. Evolution of the density contrasts } \delta_b \text{ (baryons), } \delta_{\text{CDM}} \text{ (standard cold dark matter) and } \delta_\Phi \text{ (scalar dark matter) vs the scale factor } a \text{ for the models given in } \text{fig. (1). The modes shown are } k = 0.1 \text{ Mpc}^{-1} \text{ (top) and } k = 10 \times 10^{-5} \text{ Mpc}^{-1} \text{ (bottom). } \]

The mass for the scalar field \( \Phi \) is \( m_\Phi^2 = V''(0) = \kappa_0 V_0 \lambda^2 \). Observe that from eq. (9) we have a minimal value for the mass of the field. Using eq. (8) we get

\[ m_{\Phi, \text{min}}^2 \approx 1.08 \times 10^5 \left[ \left( \frac{\Omega_{\text{CDM}}}{\Omega_{\gamma}} \right)^3 \Omega_\gamma \right] H_0^2. \]

(10)

implying that \( m_\Phi > 3 \times 10^{-26} \text{ eV} \); thus, we are dealing with an ultra-light particle as dark matter. Since the Compton length is related to the mass by \( \lambda_C = m_\Phi^{-1} \), there will be a maximum value for \( \lambda_C \) given by

\[ \lambda_{C, \text{max}} \approx 3.0 \times 10^{-3} \left[ \left( \frac{\Omega_{\text{CDM}}}{\Omega_{\gamma}} \right)^3 \Omega_\gamma \right]^{-1/2} H_0^{-1}, \]

(11)

and then \( \lambda_C < 200 \text{ pc} \).

From eq. (3), we can see that there is a degeneracy because of the infinite pairs \( (V_0, \lambda) \) that are available for the same values of \( \Omega_{\text{CDM}} \) and \( \Omega_\gamma \), and that we recover the standard dark matter model if \( \lambda \to \infty \). In fact, observe that we have a one-parameter theory where we can chose \( V_0, \lambda \) or \( m_\Phi \) as a free parameter. Then, we need another observational constraint to fix completely the parameters of the potential. In \[17\], it was suggested that the Compton length could be a cutoff for structure
formation, but its value is not big enough to be useful. In [22], it is studied a similar model but here the scalar particles behave like a relativistic gas before the time of radiation-matter equality, the gas being non-relativistic at the current epoch. This last fact ensures that the minimal scale for dark matter halos is of order of kpc. The potential used in [22] is

$$V(\Phi) = \frac{m_\Phi^2}{2}\Phi^2 + \kappa \Phi^4$$

(12)

being $\kappa$ the dimensionless free parameter of the model. For the potential (3), $\kappa$ is no longer a free parameter, but $\kappa = \kappa_0 \lambda^2 m_\Phi^2/4!$. Then, in our case the minimal radius for compact equilibrium now reads [22]

$$r_c = (3/2) (\kappa_0 V_o)^{-1/2}.$$  

(13)

Taking the minimal value for $\lambda$ allowed by nucleosynthesis and using eq. (8), the available values for $r_c$ are

$$r_c \leq 2 \times 10^{-2} \left( \frac{\Omega_{CDM}}{\Omega_c} \right)^{3/2} \Omega_{CDM}^{-1/2} H_o^{-1}.$$  

(14)

thus $r_c \leq 6\lambda C \simeq 1.2kpc$. This value can be useful in order to explain the suppression of galactic substructure and could give us the new constraint we need to fix all the parameters of the model.

Summarizing, a model for the Universe where 95% of the energy density is of scalar nature can be possible. This would have strong consequences in structure formation, like the suppression of subgalactic objects due to the dark matter composed of a ultra-light particle.

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