Random Asymmetric Carrier PWM Method for PMSM Vibration Reduction

JIAQUN XU* AND HONGQIANG ZHANG
Faculty of Information Technology, Beijing University of Technology, Beijing 100124, China
Corresponding author: Jiaqun Xu (xjq@bjut.edu.cn)

ABSTRACT The underwater noise can be caused by the radial electromagnetic vibration of permanent magnet synchronous motor (PMSM) for electric propulsion. The difficulty in decreasing the underwater noise is not only the high-frequency but also low-frequency vibration reduction. This paper proposes a novel random asymmetric carrier pulse width modulation (RACPWM) method for PMSM vibration reduction in the whole frequency range. Instead of the conventional symmetric triangular carrier PWM, the asymmetric carrier PWM (ACPWM) is used to decrease the low-frequency phase current harmonics and the associated low-frequency vibration. Meanwhile, in order to suppress the high-frequency vibration, the RACPWM control based on the amplitude randomization of the sawtooth carrier is further presented to reduce the high-frequency phase current harmonics. Moreover, the RACPWM is equivalent to the random slope ACPWM in view of the output signals. Furthermore, without the need of any additional circuits in the drive, the presented method is simple to implement by software. Simulation and experimental results show that the proposed RACPWM method can effectively reduce both the low-frequency and the high-frequency vibration.

INDEX TERMS Permanent magnet synchronous motor (PMSM), asymmetric carrier, sawtooth carrier, vibration reduction, underwater noise, current harmonics.

I. INTRODUCTION
Due to the advantages of high efficiency and high power density, permanent magnet synchronous motor (PMSM) can be directly coupled with the propeller [1], which is suitable for electric propulsion [2], [3], special for naval applications [4], [5]. The underwater noise can be caused by the radial electromagnetic vibration associated with the phase current harmonics of the propulsion motor [6]–[8], which will not only affect the marine environment and the sensitive underwater acoustics equipment, but also go against the invisibility and survivability of military ships and underwater vehicles [9]. The difficulty in reducing the underwater noise is the vibration suppression of the propulsion motor in both high-frequency and low-frequency range [9].

The noise and the vibration is closely related to both the motor structure and the control method. The analysis of the radial force of PMSM is investigated [10], the permanent magnet structure, airgap length and stator core geometry are optimized [11]. Compared with the analysis and optimization of the motor structure, the improved control method for vibration reduction can be easier to implement. Space vector pulse width modulation (SVPWM) is widely used in PMSM drives, and the advanced control method based on SVPWM is also applied to improve the performance of PMSM drives [12]. However, the SVPWM in the PMSM drive will generate the obvious phase current harmonics nearby the carrier frequency and its multiples, which can result in the high-frequency vibration and noise [9]. Random PWM (RPWM) methods, including random pulse position (RPP) [13], random centered distribution (RCD) [14], variable delay random (VDR) [9], random zero voltage vector distribution (RZD) and random switching frequency (RSF) PWM [15], [16], have been extensively studied and are appropriate to spread the harmonics in a wide frequency range. Among the abovementioned conventional RPWM methods, RSFPWM has a significant effect on high-frequency harmonic diffusion and vibration reduction [9], [16]. The RSFPWM method for naval propulsion drive is evaluated in [9]. In [16], five RPWM strategies, including RZD, RCD, RPP, RSF and VDR, are compared, which shows that RSFPWM has the good performance and lower current total harmonic distortion (THD). In [17], a hybrid dual random SVPWM strategy with the combination of RZD and RSF is

The associate editor coordinating the review of this manuscript and approving it for publication was Shihong Ding.

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applied to a five-phase voltage source inverter. Apart from
the above conventional RPWM methods, recent researches
present other strategies to deal with high-frequency vibra-
tion and noise. In [18]–[20], the SVPWM with modified
switching sequence is used to reduce the noise in the range
of human hearing, but the high-frequency harmonics cannot
be eliminated. In application of two-segment three-phase
PMSM, the interleaved technique with magnetically coupled
inductors [21] and phase shift PWM technology [22], [23],
are proposed, but two inverters are necessary. Additionally,
some strategies involving carrier are reported to reduce high-
frequency harmonics. In [24], the triangular carrier and its
inverted version are used to synthesise the random carrier via
the hardware circuit. In [25], the multiple triangular carriers
with different frequency from hardware circuits are used for
the motor drive. In [26], the sinusoidal triangular carrier is applied to
the motor. In [27], two different values are used for the period
register of the microcontroller to generate the asymmetrical
carrier wave with an unchanged slope. Besides, the random
slope PWM is presented, and the slope of the triangular wave
changes according to the random signal from a hardware
circuit [28].

The aforementioned researches typically focus on the
high-frequency noise, which are not applicable to the low-
frequency harmonics, and thus cannot completely meet
the requirement of underwater noise reduction. In fact, for the
longer transmission distance, the low-frequency noise caused
by the low-frequency vibration should need more attention.
Different from the existing studies, this paper proposes a
novel random asymmetric carrier PWM (RACPWM) method
to reduce the phase current harmonics and the radical vibra-
tion in the whole frequency range. The low-frequency har-
monics reduction by the asymmetric carrier PWM (ACPWM)
and the high-frequency harmonics reduction by the random
slope carrier are investigated. Moreover, based on the saw-
tooth carrier, this paper presents an easy implementation
scheme by software for the RACPWM. The effectiveness of
the proposed method is verified well by the simulation and
experimental results.

This paper is organized as follows. Section II presents the
analytical method and the characteristics of the phase current
harmonics of ACPWM. In Section III, the new RACPWM
method based on the ACPWM and the sawtooth carrier is
proposed. In Section IV, the proposed scheme is verified
by the simulation results. In Section V, the experimental
validation is presented. Section VII gives the conclusion.

II. PHASE CURRENT HARMONICS OF ACPWM
A. ANALYTICAL METHOD OF PHASE CURRENT
HARMONICS
The PWM signals can be generated by the modulation wave
and the carrier. Instead of the conventional triangular carrier,
Fig. 1 shows the ACPWM principle with the asymmetric
carrier.

In Fig. 1, \( T \) is the constant asymmetric carrier period;
\( \beta_m T \) is the rise time for the ramp from point C to A in the
m-th carrier period; \( \beta_m \) is the asymmetric factor in the range
of \([0, 1]\). When \( \beta_m = 0.5 \), the conventional triangular carrier
can be obtained.

The modulation wave of phase-A \( r_a \) defined in the \([-1, 1]\)
can be converted to \( r'_a \) in the \([0, 1]\) as (2) [29], where \( M \) is the
modulation index, \( \omega \) is the frequency of the modulation wave

\[
r_a = \frac{2}{\sqrt{3}} M \left[ \sin(\omega t) + \frac{1}{6} \sin(3\omega t) \right]  \tag{1}
\]

\[
r'_a = (1 + r_a) / 2  \tag{2}
\]

Assuming that the amplitude of \( r'_a \) is constant in one carrier
period, moreover, triangles ABC and ADE, AGC and AFE,
are similar, respectively, and then the relationship among
the length of the different line segments in Fig. 1(a) can be derived as

\[
\begin{align*}
\frac{DE}{BC} &= \frac{AF}{AG} \\
\frac{DE}{BC} &= \frac{EF}{CG} \tag{3}
\end{align*}
\]

From (3), (4) can be listed, where \( d_{am} T \) is the high level
duration of phase-A between point E and D; \( \delta_{am} T \) is the rise
time of phase-A for the ramp from point C to E

\[
\begin{align*}
(d_{am} T) / T &= (1 - r'_a) / 1 \\
(d_{am} T) / T &= (\beta_m T - \delta_{am} T) / (\beta_m T) \tag{4}
\end{align*}
\]

From (2) and (4), \( \delta_{am} \) and \( d_{am} \) can be deduced as

\[
\begin{align*}
\delta_{am} &= \beta_m (1 - d_{am}) \tag{5} \\
d_{am} &= (1 - r_a) / 2 \tag{6}
\end{align*}
\]
Extending the PWM voltage shown in Fig. 1(b) to multiple periods, then, the voltage $u_{kn}$ [Fig. 1(c)] can be described as

$$u_{kn} = \begin{cases} 
0, & T_m < t < (T_m + \delta_{km} T) \\
U_{dc}, & (T_m + \delta_{km} T) < t < (T_m + \delta_{km} T + d_{km} T) \\
0, & (T_m + \delta_{km} T + d_{km} T) < t < T_{m+1}
\end{cases}$$

(7)

where $\delta$ denotes $a$, $b$, $c$, respectively.

The phase voltage of PMSM can be written as

$$u_k = R_i k + L_s \frac{di_k}{dt} + e_k$$

(8)

where $u_k$, $i_k$, $e_k$ are the phase voltage, current and back-EMF, respectively; $R$ is the phase resistance; $L_s$ is the inductance.

The voltage $u_{ab}$ and $u_{ac}$ can be obtained from (7) and (8) as

$$\begin{align*}
  u_{ab} &= u_{an} - u_{bn} = R_i a - R_i b + L_s \frac{di_a}{dt} - L_s \frac{di_b}{dt} + e_a - e_b \\
  u_{ac} &= u_{an} - u_{cn} = R_i a - R_i c + L_s \frac{di_a}{dt} - L_s \frac{di_c}{dt} + e_a - e_c
\end{align*}$$

(9)

The phase current relationship can be expressed as

$$i_a + i_b + i_c = 0$$

(10)

Based on (9) and (10), the phase current can be deduced as

$$\frac{di_k}{dt} + \frac{R}{L_s} i_k = \frac{1}{3L_s} \left[ u_{ab} + u_{ac} + (-2e_a + e_b + e_c) \right]$$

(11)

Subsequently, based on (9) and (11), the phase current can be obtained as

$$i_a = e^{-\frac{R}{L_s} dt} \int \frac{1}{3L_s} \left( 2u_{an} - u_{bn} - u_{cn} - 2e_a + e_b + e_c \right) e^{\frac{R}{L_s} dt} dt$$

(12)

From (12), the phase current $i_a$ can be calculated, moreover, the current harmonics can be analyzed via the fast Fourier transform (FFT) technique.

Table 1 shows the parameters of the PMSM, which are applicable to the analysis, simulation and experiment in the paper. To verify the proposed ACPWM and RACPWM effectively, two conventional methods, including SVPWM and RSFPWM, are also analyzed. In this paper, the carrier frequency is 10 kHz in SVPWM, ACPWM and RACPWM, and the range of the switching frequency is from 9 kHz to 11 kHz in RSFPWM.

### TABLE 1. Parameters of PMSM.

| Parameter             | Value  |
|-----------------------|--------|
| Rated power (kW)      | 1.1    |
| Rated speed (r/min)   | 500    |
| Rated torque (N·m)    | 22     |
| Back-EMF (V)          | 75     |
| Phase inductance (mH) | 1      |
| Phase resistance (Ω)  | 0.32   |
| Pole pairs            | 4      |

### B. PHASE CURRENT HARMONIC CHARACTERISTICS OF ACPWM

Fig. 2 shows the analytical results of high-frequency current harmonics of SVPWM and ACPWM. The results show that, similar to SVPWM, ACPWM also has the obvious harmonics at the integer multiples of the switching frequency, which will inevitably result in the obvious high-frequency vibration and the associated high-frequency noise.

![Fig. 2. Analytical results of high-frequency phase current harmonics.
(a) SVPWM, $\beta_m = 0.5$. (b) ACPWM, $\beta_m = 0.8$.](image)

Fig. 3 shows the analytical results of low-frequency current harmonics of SVPWM and ACPWM, and the fundamental frequency $f_0$ is 33.3 Hz. As shown in Fig. 3, compared with those of SVPWM, the 2th and 4th harmonic amplitudes of ACPWM increase slightly, however, the 5th and 7th harmonic amplitudes decrease significantly. Moreover, the calculation results indicate that ACPWM can decrease the amplitudes of both 5th and 7th harmonics by nearly 54%. In addition, the low-frequency current harmonics of ACPWM are generally more dispersed and lower, which is helpful to reduce the low-frequency vibration.

To further clarify the effect of the carrier asymmetry, the low-frequency current harmonic amplitudes with different $\beta_m$ are calculated. The result in Fig. 4 shows that, the fundamental amplitudes are nearly invariable, and the harmonic amplitudes are symmetrical distribution, which almost remain unchanged when $\beta_m \geq 0.8$ or $\beta_m \leq 0.2$. When $0.2 < \beta_m < 0.8$, with the increase of the degree of carrier asymmetry, the 5th and 7th harmonic amplitudes decrease significantly, and the 2th and 4th harmonic amplitudes increase slightly.

The vibration of the motor is determined by the radial electromagnetic force from the current harmonics. The above analysis indicates that, ACPWM can effectively reduce the amplitudes of the low-frequency current harmonics, which is
beneficial to suppress the low-frequency vibration and the associated underwater noise. However, the high-frequency harmonics and noise cannot be decreased by using ACPWM.

### III. RACPWM BASED ON SAWTOOTH CARRIER

It can be seen from above analysis that, ACPWM has a good low-frequency harmonic characteristic, but the high-frequency harmonics are unpleasant. To further reduce the high-frequency harmonics, a novel RACPWM method is presented, which is also simple to implement by software.

#### A. RANDOM SLOPE ACPWM WITH TRIANGULAR CARRIER

Fig. 5 shows the random slope ACPWM. \( k_1 \) and \( k_2 \) are the slope of the asymmetric carrier, respectively. \( R_1 \) and \( T \) are the constant peak value and the carrier period, respectively. \( r_1, r_2 \) and \( r_3 \) are the amplitude of the respective modulation wave in the \([0, R_1]\). \( \beta_m T \) represents the time to reach \( R_1 \). Assuming that the synthetic voltage is in the first sector, the vectors involved are \( U_1 \) (100), \( U_2 \) (110), \( U_0 \) (000) and \( U_7 \) (111).
The slope of the triangular carrier can be expressed as

$$k_1 = R_1/(\beta_m T)$$  \hspace{1cm} (13)
$$k_2 = k_1 \beta_m / (1 - \beta_m)$$  \hspace{1cm} (14)

When $k_1$ and $k_2$ vary with the random $\beta_m$, it can be seen from Fig. 5 that, the voltage vectors will change accordingly, that means the random pulse position can be acquired by varying rising and falling slope of the triangular carrier simultaneously. Moreover, the maximum range of output pulse signals can be obtained when $\beta_m$ changes randomly in the range of $[0, 1]$, which is beneficial to the high-frequency current harmonic diffusion and the vibration reduction.

### B. NOVEL RACPWM BASED ON SAWTOOTH CARRIER

The above random slope ACPWM will help to reduce high-frequency harmonics, however, it is difficult to implement by software, for the reason that the microcontroller can only generate a limited number of carrier waves with the constant slope. In view of the same output pulse signals as the random slope ACPWM, a new RACPWM method based on the sawtooth carrier is proposed, which can be directly implemented by software. The RACPWM principle based on the sawtooth carrier is shown in Fig. 6. In the figure, $T$ is the carrier period; $R$ is the random amplitude of the sawtooth carrier, which corresponds to the ramp rise time $\beta_m T$ of the asymmetric carrier; $R_2$ is the peak value of $R$, where $R_2 = 2R_1$, $0 \leq R \leq R_2$.

The core of the RACPWM method is the $R$ randomization of the sawtooth carrier, which can be equivalent to the $\beta_m$ randomization of the asymmetric triangular carrier.

It can be seen from Fig. 6 that $R$ can be expressed as

$$R = R_2 \times \beta_m$$  \hspace{1cm} (15)

Thus, the $\beta_m$ randomization of the triangular carrier can be converted to the $R$ randomization of the sawtooth carrier. Besides, to output the same PWM signals as those of the asymmetric triangular carrier, the six compare values are necessary for the sawtooth carrier. Based on the asymmetric carrier and the modulation wave, the compare values can be expressed as

$$a_1 = R[(r_a + 1)/2]$$
$$a_2 = R[(r_b + 1)/2]$$
$$a_3 = R[(r_c + 1)/2]$$
$$b_1 = R_2 - (R_2 - R)[(r_a + 1)/2]$$
$$b_2 = R_2 - (R_2 - R)[(r_b + 1)/2]$$
$$b_3 = R_2 - (R_2 - R)[(r_c + 1)/2]$$  \hspace{1cm} (16) (17)

where $r_a$, $r_b$ and $r_c$ are the saddle-shaped modulation waves in the $[-1, 1]$; $a_1$, $a_2$ and $a_3$ are the compare values of the sawtooth carrier in the $[0, R]$, which correspond to the asymmetric carrier in the $[0, \beta_m T]$; $b_1$, $b_2$ and $b_3$ are the compare value of the sawtooth carrier in the $[R, R_2]$, which correspond to the asymmetric carrier in the $[\beta_m T, T]$.

The derivation of (16) and (17) is shown in the Appendix.

### C. IMPLEMENTATION OF RACPWM WITH SAWTOOTH CARRIER

Fig. 7 shows the implementation of the proposed RACPWM method with the sawtooth carrier. In Fig. 7, the modulation waves $r_a$, $r_b$ and $r_c$ are directly from the conventional SVPWM based on the field oriented control, and the difference between the RACPWM and the SVPWM is the modulation link. In the modulation link of the RACPWM, the random $R$ for the sawtooth carrier is calculated based on $R_2$ and the random $\beta_m$ by (15), then by (16) and (17), the six compare values for sawtooth carrier are calculated.
The six compare values are directly used to the compare register of the microcontroller. In this way, the PWM output signals equivalent to those of the random slope ACPWM can be obtained.

From the implementation process, it can be seen that the RACPWM has some distinctive advantages. With the constant switching frequency, the slope of the equivalent asymmetric carrier can vary randomly in a large range, which is beneficial to the high-frequency harmonic diffusion and vibration reduction. Moreover, without the need of any additional hardware circuit to generate PWM signals, the RACPWM with the sawtooth carrier is easy to implement by software. Most important of all, the equivalent asymmetric carrier is helpful to decrease the low-frequency vibration and underwater noise.

IV. SIMULATION RESULTS

The simulation model is shown in Fig. 8. The PMSM model, inverter model and field oriented control model are realized by software of ANSYS Maxwell, ANSYS Simplorer and Matlab Simulink, respectively. The four control methods, including SVPWM, ACPWM, RSFPWM and RACPWM, are implemented.

The simulation results of the low-frequency current harmonics are shown in Fig. 9(a) and (b). Compared with...
the conventional SVPWM, it is clear that ACPWM can generally decrease the low-frequency current harmonics, which is helpful to reduce the low-frequency vibration and underwater noise. The simulation results of high-frequency current harmonics without random control are shown in Fig. 9(c) and (d). Both ACPWM and SVPWM have the obvious current harmonics at the integer multiples of switching frequency, which will cause the high-frequency vibration and noise.

The simulation results of the high-frequency current harmonics with random control are shown in Fig. 9(e) and (f). It can be seen that, compared with the conventional RSFPWM, the RACPWM can also spread the harmonics to a wide frequency range far from the integer times of switching frequency, which is beneficial for reducing high-frequency vibration and noise.

It should be noted that the simulation results are in good accordance with the previous analytical results.

V. EXPERIMENTAL VERIFICATION
The proposed RACPWM control method is validated on the experimental platform in Fig. 10. The digital signal processor (DSP) of TMS320F28335 is used in the motor controller. The phase current is measured by the oscilloscope and then
analyzed by MATLAB software. The vibration is measured by the acceleration sensor and data acquisition instrument. The main parameters of the PMSM are listed in Table 1.

A. PHASE CURRENT TEST
The test results of phase current are shown in Fig. 11.

Fig. 11(a) and (b) show the experimental waveforms of the phase current corresponding to the conventional SVPWM and the presented ACPWM, which are similar and sinusoidal.

As shown in Fig. 11(c) and (d), there is an obvious difference between the two groups of low-frequency current harmonics. In comparison with SVPWM, ACPWM has the more dispersed and generally lower amplitude harmonics, which is beneficial to reduce the low-frequency vibration and underwater noise.

From Fig. 11(e) and (f), it can be seen that both SVPWM and ACPWM have large high-frequency current harmonics at the integer multiples of switching frequency. Therefore, ACPWM will also lead to the obvious high-frequency vibration. Fig. 11(g) and (h) show that, compared with SVPWM and ACPWM, both RSFPWM and RACPWM can disperse the high-frequency current harmonics, which is helpful to suppress the high-frequency vibration and noise.

To compare the two methods clearly, Table 2 presents the amplitudes of the phase current harmonics under different conditions. It can be seen from Table 2 that, ACPWM is helpful for the low-frequency harmonics mitigation, and the analytical and simulation results are in good agreement with the experimental data for both low-frequency and high-frequency harmonics.

B. VIBRATION TEST
To further prove the presented RACPWM method, Fig. 12 shows the experimental results of the vibration acceleration. The low-frequency vibration results are shown in Fig. 12(a) and (b). It is clear that ACPWM can significantly reduce the low-frequency vibration caused by the low-frequency current harmonics, e.g., the calculation result shows that ACPWM can reduce the vibration by nearly 57% at 266.4 Hz. The high-frequency vibration are shown...
TABLE 2. Amplitudes of phase current harmonics.

| Control Method | Frequency (Hz) | Analysis (%) | Simulation (%) | Experiment (%) |
|----------------|---------------|--------------|----------------|----------------|
| SVPWM (βm=0.5) | 166.5         | 0.37         | 0.43           | 0.45           |
|                | 233.1         | 0.26         | 0.3            | 0.36           |
|                | 10 k          | 0.59         | 0.45           | 0.51           |
|                | 20 k          | 0.79         | 0.92           | 0.95           |
|                | 30 k          | 0.12         | 0.16           | 0.28           |
| ACPWM (βm=0.8) | 66.6          | 0.10         | 0.11           | 0.13           |
|                | 133.2         | 0.06         | 0.07           | 0.09           |
|                | 166.5         | 0.17         | 0.2            | 0.25           |
|                | 233.1         | 0.12         | 0.14           | 0.18           |
|                | 10 k          | 1.08         | 1.31           | 1.26           |
|                | 20 k          | 0.17         | 0.14           | 0.18           |
|                | 30 k          | 0.10         | 0.09           | 0.12           |

TABLE 3. HSF of different control methods.

| Control method | Phase current | Vibration |
|----------------|---------------|-----------|
|                | Simulation    | Experiment |
| SVPWM          | 0.263         | 0.284     | 0.069     |
| ACPWM          | 0.261         | 0.273     | 0.068     |
| RSFPWM         | 0.091         | 0.107     | 0.027     |
| RACPWM         | 0.064         | 0.075     | 0.018     |

in Fig. 12(c)-(f). The results show that both ACPWM and SVPWM have the obvious high-frequency vibration related to the switching frequency. Similar to RSFPWM, RACPWM can also disperse the high-frequency vibration in a wide frequency range, which is beneficial to suppress the noise. It should be noted that, the vibration test results coincide with the phase current harmonic characteristics, whether in low-frequency or high-frequency range.

C. HSF ANALYSIS

To evaluate the high-frequency harmonic spreading degree, the harmonic spread factor (HSF) is introduced, and the smaller HSF value means the better effect of randomization [17]. Table 3 shows HSF of different control methods. It is clear that, HSF of the phase current is coincident with that of the vibration, and the results between experiment and simulation are consistent. Moreover, HSF of ACPWM and SVPWM is much higher than that of RSFPWM and RACPWM, and HSF of RACPWM is the smallest among the four methods, which indicates that RACPWM can spread the high-frequency harmonics to a wide frequency range.

VI. CONCLUSION

This paper proposed a novel RACPWM method based on the sawtooth carrier to reduce the electromagnetic vibration of PMSM in the whole frequency range. The asymmetric triangular carrier is used to decrease the low-frequency phase current harmonics, and the asymmetric carrier with the random slope in a large range can further disperse the high-frequency current harmonics. The analytical and simulation results are in good accordance with the experimental results about the phase current harmonics, and the vibration test results coincide with the current harmonic characteristics. The results indicates that, compared with the conventional SVPWM, the presented ACPWM can reduce the low-frequency current harmonics and the associated low-frequency vibration, but cannot decrease the high-frequency harmonics. With the smallest HSF, the proposed RACPWM based on the ACPWM can both decrease the low-frequency current harmonics and spread the high-frequency current harmonics to a wide frequency range. Thus, compared with the conventional PWM, the proposed RACPWM method has a significant suppression effect on not only the high-frequency but also the low-frequency vibration, which is beneficial to reduce the underwater noise caused by the propulsion motor. In addition, based on the amplitude randomization of the sawtooth carrier, the proposed RACPWM scheme with the constant switching frequency is easy to implement by software. Both the simulation and experimental results verify the effectiveness of the proposed method. The random PWM for PMSM is the active control method with the random control variables, which is different from the randomly occurring uncertain disturbance problem [30]. The future work can integrate the proposed RACPWM method with the optimization of motor structure and the related advanced scheme, e.g., the second-order sliding mode control with output constraint [31], to improve the performance of the closed-loop system.

APPENDIX

The derivation of (16) and (17) is shown below.

\[ b_3 = \frac{R_2 - (R_2 - R) [(r_c + 1)/2]}{(R_2 - R - (r_c + 1)/2)} \] (A7)
Substituting (A1) and (A3) into (A2), $a_3$ can be obtained as

$$a_3 = R \left[ (r_c + 1)/2 \right]$$  \hspace{1cm} (A8)

Similarly, $a_1$, $a_2$, $b_1$ and $b_2$ can be obtained.

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