Oblique parameters of BSM models with three CP-even neutral scalars

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Abstract

We express the oblique parameters $S$, $T$, $U$, $V$, $W$, and $X$ in terms of the corresponding mixing matrices in the framework of three BSM models with three CP-even neutral scalars. We consider three types of the extension of the scalar sector of the SM with non-standard (i) two real singlet scalars, (ii) one complex doublet and one real singlet scalar, and (iii) two complex doublets. We present the expressions such that one can use these when all the neutral CP-even scalars have VEV, or one of them does not have any VEV. The principal benefit of presenting the oblique parameters in this way is, the sole knowledge of the mixing matrices in the corresponding scalar sector is enough to extract the expression of the oblique parameter of that particular BSM model.

1 Introduction

To parametrise the new physics (NP) effects on the Electroweak (EW) precision observables, the oblique parameters play an important role \cite{1}. This requires to satisfy some criteria, such that, (i) The EW gauge group has to be the standard $SU(2)_L \times U(1)_Y$, with no non-standard gauge bosons, (ii) NP particles couple to the light fermions suppresively, whereas they couple mainly to the SM gauge bosons, (iii) The energy scales of the NP are at $q^2 \approx 0$, $q^2 = m_Z^2$, and $q^2 = m_W^2$. The relevant six quantities, viz, $S$, $T$, $U$, $V$, $W$, $X$ are defined and expressed in \cite{1–4}. In general, the SM contribution to an oblique parameter ($O_{SM}$) is subtracted from the NP contribution ($O_{NP}$) to define the oblique parameter ($O$), i.e.

$$O = O_{NP} - O_{SM}.$$  \hspace{1cm} (1)

In the SM, the $\rho$ parameter at tree level remains equal to one, but in BSM models, its value differ from unity. Hence, one can constrain any BSM model in this way. The oblique parameter $T$, is related to the $\rho$ parameter as,

$$\Delta \rho = \alpha T,$$ \hspace{1cm} (2)

where, $\Delta \rho$ is the NP part of the quantity, and $\alpha = e^2/(4\pi)$ is the fine structure constant.

Among various types of BSM models, addition of non-standard particles under certain symmetries are of special interest. Extension of the scalar sector by one or more, real or complex, charged or neutral, singlet, doublet, and triplet are well studied in literature, for example \cite{5–9}, and many more.

In this paper, we consider three types of extension of the scalar sector of the SM, viz, (i) SM extended with two real singlet scalars ($Rx2SM$) \cite{10}, (ii) SM extended with scalars, one complex doublet and one real singlet ($N2HDM$), and (iii) SM extended with two complex doublet scalars ($3HDM$). All of these BSM models possess three CP-even neutral scalars. A general expression of the oblique parameters for BSM models with arbitrary number of $SU(2)_L$ singlet and doublet are given in \cite{11,12}. We use these results in our paper to provide the expressions of the oblique parameters in term of the rotation matrices and some pre-defined functions.

The expressions of the oblique parameters $S$ and $U$ for BSM models with extra non-standard multiplets, larger than $SU(2)_L$ doublets, are prescribed in \cite{13}. For aligned $2HDM$ and $3HDM$, oblique parameters are given in \cite{15}. Expressions of some of the oblique parameters in complex singlet scalar extended SM \cite{16–19}, Two Higgs Doublet Model \cite{8,20–29,29,30}, Three Higgs Doublet Model \cite{31–33},

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Two real singlet scalars extended SM [34], real singlet scalar extended two Higgs doublet model [35] are already in literature, yet all the six oblique parameters ($S, T, U, V, W, X$) in terms of the elements of the rotation matrices, for the two real singlet scalar extended SM, one real singlet scalar extended 2HDM, and three Higgs doublet model, are not expressed in the literature. Our main goal for this paper is to provide the complete list of all the six oblique parameters for these three BSM models with three CP-even neutral scalars.

This paper is arranged as follows. In Section 2, we give the brief description of the models. In Section 3, we enlist the oblique parameters for these three models. We conclude in Section 4. Mixing matrices, and the calculation in detail may be found in the appendices.

2 The models

In this section, we briefly recollect the scalar sectors of the models relevant to our paper. Here, for each model, we consider the most general scalar sector, where, all neutral scalars have VEVs, and hence all of them mix with each other. We present the models accordingly. For the cases, where all the neutral scalars do not possess VEV, the mixing matrices get simplified.

2.1 The Two Real Singlet Scalar extended Standard Model

The scalar sector of Two Real Singlet Scalar extended SM (Rx2SM) consists of two real $SU(2)_L$ singlets, in terms of the component fields,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \Phi_S = \rho_s, \quad \Phi_X = \rho_x,$$

where $\Phi_S$ and $\Phi_X$ are the singlets and $\Phi$ is the SM doublet. After spontaneous symmetry breaking (SSB), the vacuum expectation value (VEV) for the singlets are $<\rho_s> = v_s$ and $<\rho_x> = v_x$, and that for the neutral field of the doublet is $<\phi> = v/\sqrt{2}$, where, $v$ is the Electroweak VEV. After expanding the neutral fields about their VEVs,

$$\phi^0 = \frac{1}{\sqrt{2}} (v + \phi^0) \quad \text{with} \quad \phi^{0 \nu} = \rho + i \eta,$$

$$\Phi_S = v_s + \chi^{0}_1 \quad \text{with} \quad \chi^{0}_1 = \rho_s,$$

$$\Phi_X = v_x + \chi^{0}_2 \quad \text{with} \quad \chi^{0}_2 = \rho_x.$$  \hspace{1cm} (4)

One can easily obtain the mass eigenstates from the unphysical fields, via the rotation matrix, as,

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = O_\alpha \begin{pmatrix} \rho \\ \rho_s \\ \rho_x \end{pmatrix},$$  \hspace{1cm} (5)

where, $O_\alpha$ is a $3 \times 3$ orthogonal matrix.

We assign the physical charged and neutral scalars respectively, as,

$$S^+ = G^+, \quad S_1^0 = G^0, \quad S_2^0 = H_1, \quad S_3^0 = H_2, \quad S_4^0 = H_3.$$  \hspace{1cm} (6)

We have $n = 1$ charged scalar field, $S^+$, which is Goldstone boson, and $m = 4$ real neutral scalar fields, $S_{1,2,3,4}^0$, out of which $S_1^0$ is the Goldstone boson. The charged scalar $S^+$ is connected to the unphysical scalar $\phi^+$ through the matrix $U$. The matrices $V$ and $R$ connect the neutral component of
the SM doublet $\Phi$ and that of the BSM singlets $\Phi_{S,X}$ to the physical neutral scalars $S^0_{1,2,3,4}$ respectively. Following the procedures of [11,12],

$$
\begin{align*}
\phi^+ &= U S^+, \\
\phi^0 &= \sum_{b=1}^4 V_{1b} S^0_b, \\
\chi^0_l &= \sum_{b=1}^4 R_{lb} S^0_b, \quad (l = 1, 2),
\end{align*}
$$

(7)

where the matrices $U$, $V$ and $R$ of dimensions $1 \times 1$, $1 \times 4$ and $2 \times 4$ respectively, are given by,

$$
\begin{align*}
U &= \mathbb{1}, \\
V &= \begin{pmatrix}
 i & O_{\alpha 11} & O_{\alpha 21} & O_{\alpha 31}
\end{pmatrix}, \\
R &= \begin{pmatrix}
 0 & O_{\alpha 12} & O_{\alpha 22} & O_{\alpha 32} \\
 0 & O_{\alpha 13} & O_{\alpha 23} & O_{\alpha 33}
\end{pmatrix}.
\end{align*}
$$

(8)

2.2 The Real Singlet Scalar extended Two Higgs Doublet Model

The scalar sector of Next-to-Two Higgs Doublet Model ($N^2HDM$) consists of two complex $SU(2)_L$ doublets and one real $SU(2)_L$ singlet, in terms of the component fields,

$$
\Phi_k = \begin{pmatrix} \phi^+_k \\ \phi^0_k \end{pmatrix}, \quad (k = 1, 2), \quad \Phi_S = \rho_s,
$$

(9)

where $\Phi_S$ is the singlet and $\Phi_k$ is the $k$-th doublet. After spontaneous symmetry breaking (SSB), the vacuum expectation value (VEV) for the singlet is $\langle \rho_s \rangle = v_s$, and that for $k$-th neutral field is $\langle \phi_k \rangle = v_k/\sqrt{2}$, such that, the total Electroweak VEV, $v$, can be expressed as,

$$
v^2 = \sum_{k=1}^2 v_k^2 = (246 \text{ GeV})^2.
$$

(10)

After expanding the neutral fields about their VEVs,

$$
\phi^0_k = \frac{1}{\sqrt{2}}(v_k + \phi^0_k), \quad \text{with} \quad \phi^0_k = \rho_k + i \eta_k, \\
\Phi_S = v_s + \chi^0 = \rho_s.
$$

(11)

One can easily obtain the mass eigenstates from the unphysical fields, via the rotation matrix, as follows.

For CP-even scalar sector,

$$
\begin{pmatrix}
 H_1 \\
 H_2 \\
 H_3
\end{pmatrix} = O_{\alpha} \begin{pmatrix}
 \rho_1 \\
 \rho_2 \\
 \rho_s
\end{pmatrix},
$$

(12)

where, $O_{\alpha}$ is a $3 \times 3$ orthogonal matrix.

For CP-odd and charged scalar sectors,

$$
\begin{pmatrix}
 G^0 \\
 A
\end{pmatrix} = O_{\beta} \begin{pmatrix}
 m_1 \\
 m_2
\end{pmatrix}, \quad \begin{pmatrix}
 G^\pm \\
 H^\pm
\end{pmatrix} = O_{\beta} \begin{pmatrix}
 \phi^\pm_1 \\
 \phi^\pm_2
\end{pmatrix},
$$

(13)

where, $O_{\beta}$ is a $2 \times 2$ orthogonal matrix.
We assign the physical charged and neutral scalars respectively, as,

\[
S_1^+ = G^+, \quad S_2^+ = H^+, \\
S_1^0 = G^0, \quad S_2^0 = H_1, \quad S_3^0 = A, \quad S_4^0 = H_2, \quad S_5^0 = H_3.
\] (14)

We have \( n = 2 \) charged scalar fields, \( S_{1,2}^+ \), out of which \( S_1^+ \) is the Goldstone boson, and \( m = 5 \) real neutral scalar fields, \( S_{1,2,3,4,5}^0 \), out of which \( S_1^0 \) is the Goldstone boson, and the rest are the neutral physical scalars. The charged scalars \( S_{1,2}^+ \) are connected to the unphysical scalars \( \phi_{1,2}^+ \) through the matrix \( U \). The matrices \( V \) and \( R \) connect the neutral components of the doublets \( \Phi_{1,2} \) and that of the singlet \( \Phi_S \) to the physical neutral scalars \( S_{1,2,3,4,5}^0 \) respectively. Following the procedures of [11,12],

\[
\phi_l^+ = \sum_{a=1}^{2} U_{la} S_a^+, \\
\phi_l^0 = \sum_{b=1}^{5} V_{lb} S_b^0, \\
\chi_l^0 = \sum_{b=1}^{5} R_{lb} S_b^0,
\] (15)

with \( l = 1, 2 \).

The matrices \( U, V \) and \( R \) of dimensions \( 2 \times 2, 2 \times 5 \) and \( 1 \times 5 \) respectively, are given by,

\[
U = (O_\beta)^T, \\
V = \begin{pmatrix} 0 & O_{\alpha_{11}} & 0 & O_{\alpha_{21}} & O_{\alpha_{31}} \\ 0 & O_{\alpha_{12}} & 0 & O_{\alpha_{22}} & O_{\alpha_{32}} \end{pmatrix} + i \begin{pmatrix} O_{\beta_{11}} & 0 & O_{\beta_{21}} & 0 & 0 \\ O_{\beta_{12}} & 0 & O_{\beta_{22}} & 0 & 0 \end{pmatrix}, \\
R = \begin{pmatrix} 0 & O_{\alpha_{13}} & 0 & O_{\alpha_{23}} & O_{\alpha_{33}} \end{pmatrix}.
\] (16)

2.3 The Three Higgs Doublet Model

The scalar sector of the Three Higgs Doublet Model (3HDM) consists of three complex \( SU(2)_L \) doublets, where \( k \)-th doublet can be expanded in terms of its component fields, as,

\[
\Phi_k = \begin{pmatrix} \phi_k^+ \\ \phi_k^0 \end{pmatrix}, \quad (k = 1, 2, 3).
\] (17)

After spontaneous symmetry breaking (SSB), the vacuum expectation value (VEV) for \( k \)-th neutral field is \( < \phi_k > = v_k/\sqrt{2} \), such that, the total Electroweak VEV, \( v \), can be expressed as,

\[
v^2 = \sum_{k=1}^{3} v_k^2 = (246 \text{ GeV})^2.
\] (18)

After expanding the neutral fields about their VEVs,

\[
\phi_k^0 = \frac{1}{\sqrt{2}}(v_k + \phi_k^0), \quad \text{with} \quad \phi_k^0 = \rho_k + i \eta_k.
\] (19)

One can easily obtain the mass eigenstates from the unphysical fields, via the rotation matrices, as follows.

For CP-even scalar sector,

\[
\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = O_\alpha \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix},
\] (20)
for CP-odd scalar sector,
\[
\begin{pmatrix}
G_0 \\
A_1 \\
A_2
\end{pmatrix} = O_{\beta\gamma_1} \begin{pmatrix}
\eta_1 \\
\eta_2 \\
\eta_3
\end{pmatrix},
\]
and for charged scalar sector,
\[
\begin{pmatrix}
G^\pm \\
H_1^\pm \\
H_2^\pm
\end{pmatrix} = O_{\beta\gamma_2} \begin{pmatrix}
\phi_1^\pm \\
\phi_2^\pm \\
\phi_3^\pm
\end{pmatrix},
\]
where, \(O_{\alpha}, O_{\beta\gamma_1,2}\) are 3 \(\times\) 3 orthogonal matrices.

We assign the physical charged and neutral scalars respectively, as,
\[
S_1^+ = G^+, S_2^+ = H_1^+, S_3^+ = H_2^+, \\
S_1^0 = G^0, S_2^0 = H_1, S_3^0 = A_1, S_4^0 = H_2, S_5^0 = A_2, S_6^0 = H_3.
\]

We have \(n = 3\) charged scalar fields, \(S_{1,2,3}^+\), out of which \(S_1^+\) is the Goldstone boson, and \(m = 6\) real neutral scalar fields \(S_{1,2,3,4,5,6}^0\), out of which \(S_1^0\) is the Goldstone boson, and the rest are the neutral physical scalars. The charged scalars \(S_{1,2,3}^+\) are connected to the unphysical scalars \(\phi_{1,2,3}^+\) through the matrix \(U\). The matrix \(V\) connects the neutral components of the doublets \(\Phi_{1,2,3}\) to the physical neutral scalars \(S_{1,2,3,4,5,6}^0\). Following the procedures of \([11,12]\),
\[
\phi_l^+ = \sum_{a=1}^3 U_{la} S_a^+, \\
\phi_l^0 = \sum_{b=1}^6 V_{lb} S_b^0,
\]
with \(l = 1,2,3\).

The matrices \(U\) and \(V\) of dimensions 3 \(\times\) 3 and 3 \(\times\) 6 respectively, are given by,
\[
U = (O_{\beta\gamma_2})^T, \\
V = \begin{pmatrix}
0 & O_{\alpha_{11}} & 0 & 0 & O_{\alpha_{21}} & 0 & O_{\alpha_{31}} \\
0 & O_{\alpha_{12}} & 0 & 0 & O_{\alpha_{22}} & 0 & O_{\alpha_{32}} \\
0 & O_{\alpha_{13}} & 0 & 0 & O_{\alpha_{23}} & 0 & O_{\alpha_{33}}
\end{pmatrix} + i \begin{pmatrix}
(O_{\beta\gamma_1})_{11} & 0 & (O_{\beta\gamma_1})_{21} & 0 & (O_{\beta\gamma_1})_{31} & 0 \\
(O_{\beta\gamma_1})_{12} & 0 & (O_{\beta\gamma_1})_{22} & 0 & (O_{\beta\gamma_1})_{32} & 0 \\
(O_{\beta\gamma_1})_{13} & 0 & (O_{\beta\gamma_1})_{23} & 0 & (O_{\beta\gamma_1})_{33} & 0
\end{pmatrix}.
\]

One must not confuse the matrices \(U\), and \(V\) in the Eqns. \([8, 16, 25]\) with the oblique parameters.

### 3 Results

In this section, we enlist the expressions of the oblique parameters in terms of the components of the rotation matrices of the scalar sectors of the three models. For this, we also need the definitions of some well known functions \([11,12,14]\), as,
The oblique parameters for different BSM models in terms of the rotation matrices have a key benefit as the knowledge of the components of the corresponding rotation matrices is enough to perceive the expression of the oblique parameters. In this paper, to give the expressions of the oblique parameters, we consider the scalar sectors of the models, such that, all the neutral scalars have VEVs, and all of them mix with each other. But, for the special cases, where all the neutral scalars do not have VEVs, then also these expressions for oblique parameters are valid, provided, the rotation matrices for the scalar sector of the corresponding BSM model are changed accordingly.

We adopt the convenient way to express the oblique parameter $O$ as $\mathcal{O}$, where different oblique parameters are actually multiplied by different prefactors to yield,

$$ S \equiv \frac{\alpha}{4s_W^2 c_W^2} S, \quad T \equiv \alpha T, \quad U \equiv \frac{\alpha}{4s_W^2} U, \quad V \equiv \alpha V, \quad W \equiv \alpha W, \quad X \equiv \frac{\alpha}{s_W c_W} X, $$

where, $s_W$ and $c_W$ are sine and cosine of the weak mixing angle $\theta_W$ respectively.

Following [1–4], here we summarise the definitions of these oblique parameters, as :
In this subsection, we enlist the oblique parameter expressions for the three models in the next three subsections. For the models, where all the neutral scalars mix with each other, all the components of the mixing matrix become simplified, four of its components become zero, and one becomes unity.

3.1 The Two Real Singlet Scalar extended Standard Model (Rx2SM)

In this subsection, we enlist the oblique parameters for the Two Real Singlet scalars extended SM (Rx2SM), in terms of the rotation matrix $O_\alpha$.

$$\mathcal{S} = \frac{g^2}{384\pi^2 c_W^2} \sum_{j=1}^{3} O_{\alpha j}^2 \left( \frac{m_{h_j}^2}{m_Z^2} + 3 \frac{m_{h_j}^2}{m_W^2} \right) \left( \ln m_{h_j}^2 + \hat{G}(m_{h_j}^2, m_Z^2) - \hat{G}(m_{h_j}^2, m_W^2) \right) \left( \ln m_{h_j}^2 + \hat{G}(m_{h_j}^2, m_Z^2) - \hat{G}(m_{h_j}^2, m_W^2) \right),$$

$$\mathcal{T} = \frac{3g^2}{64\pi^2 m_W^2} \sum_{j=1}^{3} O_{\alpha j}^2 \left( F(m_Z^2, m_{h_j}^2) - F(m_W^2, m_{h_j}^2) \right)$$

$$- \left( F(m_Z^2, m_{h_{SM}}^2) - F(m_W^2, m_{h_{SM}}^2) \right),$$

$$\mathcal{U} = \frac{g^2}{384\pi^2} \sum_{j=1}^{3} O_{\alpha j}^2 \left( \hat{G}(m_{h_j}^2, m_W^2) - \hat{G}(m_{h_{SM}}^2, m_Z^2) \right),$$
In this subsection, we enlist the oblique parameters for One Real Singlet scalar extended 2HDM.

3.2 The Real Singlet Scalar extended Two Higgs Doublet Model (N2HDM)
In this subsection, we enlist the oblique parameters for One Real Singlet scalar extended 2HDM (N2HDM), in terms of the rotation matrices \( O_\alpha \) and \( O_\beta \).

\[
\mathcal{V} = \frac{g^2}{384\pi^2 c_W^2} \left[ \sum_{j=1}^{3} (O_{\alpha j})^2 \hat{H} \left( m_{h_j}^2, m_Z^2 \right) - \hat{H} \left( m_{h_{SM}}^2, m_Z^2 \right) \right],
\]

\[
\mathcal{W} = \frac{g^2}{384\pi^2} \left[ \sum_{j=1}^{3} (O_{\alpha j})^2 \hat{H} \left( m_{h_j}^2, m_W^2 \right) - \hat{H} \left( m_{h_{SM}}^2, m_W^2 \right) \right].
\]

\[
\mathcal{X} = \frac{g^2}{384\pi^2 c_W^2} \left[ \left( 2s_W^2 - 1 \right)^2 G \left( m_{H^+}, m_{h_j}^2, m_Z^2 \right) + \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{i1}} \right)^2 G \left( m_{h_1}^2, m_A^2, m_Z^2 \right)
\]

\[
+ \sum_{r=2}^{3} \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{ri}} \right)^2 G \left( m_A^2, m_{h_r}^2, m_Z^2 \right) - 2 \ln m_{H^+} + \sum_{j=1}^{3} \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{j1}} \right) \ln m_{h_j}^2
\]

\[
+ \ln m_A^2 + \sum_{j=1}^{3} \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{j1}} \right)^2 \hat{G} \left( m_{h_j}^2, m_Z^2 \right) - \left( \ln m_{h_{SM}}^2 + \hat{G} \left( m_{h_{SM}}^2, m_Z^2 \right) \right) \right],
\]

\[
\mathcal{T} = \frac{g^2}{384\pi^2 m_W^2} \left[ \sum_{j=1}^{3} \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{j1}} \right)^2 F \left( m_{H^+}, m_{h_j}^2 \right) + F \left( m_{H^+}, m_A^2 \right)
\]

\[
- \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{j1}} \right)^2 F \left( m_{h_1}^2, m_A^2 \right) - \sum_{r=2}^{3} \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{ri}} \right)^2 F \left( m_A^2, m_{h_r}^2 \right)
\]

\[
+ 3 \sum_{j=1}^{3} \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{j1}} \right)^2 \left( F \left( m_Z^2, m_{h_j}^2 \right) - F \left( m_W^2, m_{h_j}^2 \right) \right)
\]

\[
- 3 \left( F \left( m_Z^2, m_{h_{SM}}^2 \right) - F \left( m_W^2, m_{h_{SM}}^2 \right) \right) \right],
\]

\[
\mathcal{U} = \frac{g^2}{384\pi^2} \left[ \sum_{j=1}^{3} \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{j1}} \right)^2 G \left( m_{H^+}, m_{h_j}^2, m_W^2 \right) + G \left( m_{H^+}, m_A^2, m_W^2 \right)
\]

\[
- \left( 2s_W^2 - 1 \right)^2 G \left( m_{H^+}, m_{h_j}^2, m_Z^2 \right) - \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{j1}} \right)^2 G \left( m_{h_1}^2, m_A^2, m_Z^2 \right)
\]

\[
- \sum_{r=2}^{3} \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{ri}} \right)^2 G \left( m_A^2, m_{h_r}^2, m_Z^2 \right)
\]

\[
+ \sum_{j=1}^{3} \left( \sum_{i=1}^{2} O_{\beta_{2i} \alpha_{j1}} \right)^2 \left( \hat{G} \left( m_{h_j}^2, m_W^2 \right) - \hat{G} \left( m_{h_j}^2, m_Z^2 \right) \right)
\]

\[
- \left( \hat{G} \left( m_{h_{SM}}^2, m_W^2 \right) - \hat{G} \left( m_{h_{SM}}^2, m_Z^2 \right) \right) \right].
\]
\[ \mathcal{V} = \frac{g^2}{384\pi^2 c_W} \left[ (2s_W^2 - 1)^2 H \left( m_{H^+}^2, m_{H^+}^2, m_Z^2 \right) + \left( \sum_{i=1}^{2} O_{\beta_2i} O_{\alpha_{1i}} \right)^2 H \left( m_{h_1}^2, m_A^2, m_Z^2 \right) \right. \\
+ \sum_{r=2}^{3} \left( \sum_{i=1}^{2} O_{\beta_2i} O_{\alpha_{ri}} \right)^2 H \left( m_{A}^2, m_{h_i}^2, m_Z^2 \right) + \left( \sum_{i=1}^{2} O_{\beta_1i} O_{\alpha_{ji}} \right)^2 \hat{H} \left( m_{h_j}^2, m_Z^2 \right) \\
- \hat{H} \left( m_{h_{SM}}^2, m_Z^2 \right) \right], \quad (41) \]

\[ \mathcal{W} = \frac{g^2}{384\pi^2} \left[ \sum_{j=1}^{3} \left( \sum_{i=1}^{2} O_{\beta_2i} O_{\alpha_{ji}} \right)^2 H \left( m_{H^+}^2, m_{h_j}^2, m_W^2 \right) + H \left( m_{H^+}^2, m_A^2, m_W^2 \right) \right. \\
+ \sum_{j=1}^{3} \left( \sum_{i=1}^{2} O_{\beta_1i} O_{\alpha_{ji}} \right)^2 \hat{H} \left( m_{h_j}^2, m_W^2 \right) - \hat{H} \left( m_{h_{SM}}^2, m_W^2 \right) \right], \quad (42) \]

\[ \mathcal{X} = -\frac{g^2 s_W}{192\pi^2 c_W} \left[ (2s_W^2 - 1) G \left( m_{H^+}^2, m_{H^+}^2, m_Z^2 \right) \right]. \quad (43) \]

### 3.3 The Three Higgs Doublet Model (3HDM)

In this subsection, we enlist the oblique parameters for the Three Higgs Doublet Model (3HDM), in terms of the rotation matrices \( O_\alpha, O_{\beta\gamma_1}, \) and \( O_{\beta\gamma_2} \).

\[ \mathcal{S} = \frac{g^2}{384\pi^2 c_W} \left[ (2s_W^2 - 1)^2 \sum_{i=1}^{2} G \left( m_{H_i^+}^2, m_{H_i^+}^2, m_Z^2 \right) \right. \\
+ \sum_{r=2}^{3} \sum_{i=1}^{3} O_{\beta\gamma_{1rj}} O_{\alpha_{ij}} G \left( m_{h_i}^2, m_{A_{r-1}}^2, m_Z^2 \right) \right. \\
+ \sum_{r=2}^{3} \sum_{i=1}^{3} O_{\beta\gamma_{1rj}} O_{\alpha_{ij}} G \left( m_{A_{r-1}}^2, m_{h_i}^2, m_Z^2 \right) \\
- 2 \sum_{i=1}^{2} \ln m_{H_i^+}^2 + \sum_{j=1}^{3} \left( \sum_{k=1}^{3} O_{\beta_{1jk}} O_{\alpha_{jk}} \right) \ln m_{h_j}^2 + \sum_{i=1}^{2} \ln m_{A_i}^2 \\
+ \sum_{j=1}^{3} \left( \sum_{k=1}^{3} O_{\beta_{1jk}} O_{\alpha_{jk}} \right) \hat{G} \left( m_{h_j}^2, m_Z^2 \right) - \left( \ln m_{h_{SM}}^2 + \hat{G} \left( m_{h_{SM}}^2, m_Z^2 \right) \right) \right], \quad (44) \]
\[
\bar{T} = \frac{g^2}{64\pi^2m_W^2} \left[ \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{3} O_{\gamma2k,i+1} O_{\alpha j} \right] F \left( m_{H_1^+}^2, m_{h_1}^2 \right) \\
+ \sum_{i=1}^{2} \sum_{r=2}^{3} \sum_{k=1}^{3} O_{\gamma r k} O_{\gamma2k,i+1} \left( 2F \left( m_{H_1^+}^2, m_{A_{r-1}}^2 \right) \right) \\
- \sum_{r=2}^{3} \sum_{i=1}^{r-1} \sum_{j=1}^{3} O_{\gamma r j} O_{\alpha ij} \left( F \left( m_{h_1}^2, m_{A_{r-1}}^2 \right) \right) \\
+ \sum_{r=2}^{3} \sum_{i=1}^{r-1} \sum_{j=1}^{3} O_{\gamma r j} O_{\alpha ij} \left( F \left( m_{A_{r-1}}^2, m_{h_i}^2 \right) \right) \\
+ \sum_{j=1}^{3} \sum_{k=1}^{3} O_{\gamma1k} O_{\alpha jk} \left( F \left( m_{Z}^2, m_{h_j}^2 \right) \right. \\
- \left. \left( m_{h_{SM}}^2, m_{h_{SM}}^2 \right) \right) \\
- \left( m_{W}^2, m_{h_{SM}}^2 \right) \right],
\] (45)

\[
\bar{U} = \frac{g^2}{384\pi^2} \left[ \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{3} O_{\gamma2k,i+1} O_{\alpha j} \right] G \left( m_{H_1^+}^2, m_{h_1}^2, m_{W}^2 \right) \\
+ \sum_{i=1}^{2} \sum_{r=2}^{3} \sum_{k=1}^{3} O_{\gamma r k} O_{\gamma2k,i+1} \left( 2G \left( m_{H_1^+}^2, m_{A_{r-1}}^2, m_{W}^2 \right) \right) \\
- \left( 2s_W^2 - 1 \right) \sum_{i=1}^{2} G \left( m_{H_1^+}^2, m_{H_1^+}^2, m_{Z}^2 \right) \\
- \sum_{r=2}^{3} \sum_{i=1}^{r-1} \sum_{j=1}^{3} O_{\gamma r j} O_{\alpha ij} \left( G \left( m_{h_1}^2, m_{A_{r-1}}^2, m_{Z}^2 \right) \right) \\
- \sum_{r=2}^{3} \sum_{i=1}^{r-1} \sum_{j=1}^{3} O_{\gamma r j} O_{\alpha ij} \left( G \left( m_{A_{r-1}}^2, m_{h_i}^2, m_{Z}^2 \right) \right) \\
+ \sum_{r=2}^{3} \sum_{i=1}^{r-1} \sum_{j=1}^{3} O_{\gamma r j} O_{\alpha ij} \left( \hat{G} \left( m_{h_j}^2, m_{W}^2 \right) \right. \\
- \left. \left( m_{h_j}^2, m_{Z}^2 \right) \right) \\
- \left( \hat{G} \left( m_{h_{SM}}^2, m_{W}^2 \right) \right. \\
- \left. \left( m_{h_{SM}}^2, m_{Z}^2 \right) \right) \right],
\] (46)
\[ \mathcal{V} = \frac{g^2}{384\pi^2 c_W^2} \left[ (2s_W^2 - 1)^2 \sum_{i=1}^{2} H \left( m_{H_i^+}^2, m_{H_i^+}^2, m_Z^2 \right) \right. \\
+ \sum_{r=2}^{3} \left( \sum_{j=1}^{r-1} \sum_{i=1}^{3} \left( \sum_{j=1}^{r} O_{\beta\gamma_{rj}} O_{\alpha_{ij}} \right)^2 \right) H \left( m_{H_{r-1}}^2, m_{A_{r-1}}^2, m_Z^2 \right) \\
+ \sum_{j=1}^{3} \left( \sum_{k=1}^{3} O_{\beta\gamma_{11k}} O_{\alpha_{jk}} \right)^2 \bar{H} \left( m_{h_j}^2, m_Z^2 \right) - \bar{H} \left( m_{h_{SM}}^2, m_Z^2 \right) \right], \tag{47} \]

\[ \mathcal{W} = \frac{g^2}{384\pi^2} \left[ \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{3} O_{\beta\gamma_{rk}} O_{\alpha_{jk}} \right] \left. H \left( m_{H_i^+}^2, m_{h_j}^2, m_W^2 \right) \right. \\
+ \sum_{i=1}^{2} \sum_{j=1}^{3} \left( \sum_{k=1}^{3} O_{\beta\gamma_{rk}} O_{\alpha_{jk}} \right)^2 \left. H \left( m_{H_i^+}^2, m_{A_{r-1}}^2, m_W^2 \right) \right. \\
+ \sum_{j=1}^{3} \left( \sum_{k=1}^{3} O_{\beta\gamma_{11k}} O_{\alpha_{jk}} \right)^2 \left. \bar{H} \left( m_{h_j}^2, m_W^2 \right) - \bar{H} \left( m_{h_{SM}}^2, m_W^2 \right) \right], \tag{48} \]

\[ X = -\frac{g^2 s_W}{192\pi^2 c_W} \left[ (2s_W^2 - 1)^2 \sum_{i=1}^{2} G \left( m_{H_i^+}^2, m_{H_i^+}^2, m_Z^2 \right) \right]. \tag{49} \]

4 Conclusions

Considering three BSM scenarios, each having three CP-even neutral scalars, we calculate the oblique parameters viz., S, T, U, V, W, and X. We consider the extension of the SM with two real singlet scalars, with one complex doublet and one real singlet scalar, and finally with two complex doublets. We give the expressions, considering all the CP-even neutral scalars having VEVs. As we have already discussed, we present the expressions of the oblique parameters in such a way that, the knowledge of the mixing matrices for each model is enough to extract the full expression of the oblique parameters of that particular model. These expressions are valid for the cases, where all the CP-even neutral scalars have VEVs and mix with each other, as well as for the special cases, when one of the three CP-even neutral scalars does not possess VEV and the mixing between them is restricted.

The Rx2SM, having no BSM charged scalars, does not contribute to the oblique parameter X. One thing is to be noted that, the expressions of the oblique parameters are quite simpler for the extension of the SM with singlets than that with doublets, as expected. It is reflected in our result, as one can see that, the expressions are simplest for Two Real Singlet scalars extended SM, a little complicated for N2HDM, and toughest for 3HDM, among the three BSM models, we considered.

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A  Mixing Matrices

Below we list down the most general case in all the three models, where all the neutral CP-even scalars have VEVs.

The CP-even neutral scalar mixing matrix $O_{\alpha}$ is given by,

$$O_{\alpha} = \begin{pmatrix}
    c_{\alpha_1}c_{\alpha_2} & s_{\alpha_1}c_{\alpha_2} & s_{\alpha_2} \\
    -s_{\alpha_1}c_{\alpha_3} - c_{\alpha_1}s_{\alpha_2}s_{\alpha_3} & c_{\alpha_1}c_{\alpha_3} - s_{\alpha_1}s_{\alpha_2}s_{\alpha_3} & c_{\alpha_2}s_{\alpha_3} \\
    -c_{\alpha_1}s_{\alpha_2}c_{\alpha_3} + s_{\alpha_1}s_{\alpha_3} & -c_{\alpha_1}s_{\alpha_3} - s_{\alpha_1}s_{\alpha_2}c_{\alpha_3} & c_{\alpha_2}c_{\alpha_3}
\end{pmatrix}. \quad (A.1)$$

The mixing matrix for CP-odd and charged scalars in the N2HDM is given by,

$$O_{\beta} = \begin{pmatrix}
    c_{\beta} & s_{\beta} \\
    -s_{\beta} & c_{\beta}
\end{pmatrix}. \quad (A.2)$$

In 3HDM, the mixing matrices for CP-odd and charged scalar sector are $O_{\beta_{\gamma_1}}$ and $O_{\beta_{\gamma_2}}$ respectively, with

$$O_{\beta_{\gamma_i}} = \begin{pmatrix}
    c_{\beta_1}c_{\beta_2} & s_{\beta_1}c_{\beta_2} & s_{\beta_2} \\
    -s_{\beta_1}c_{\gamma_i} - c_{\beta_1}s_{\beta_2}s_{\gamma_i} & c_{\beta_1}c_{\gamma_i} - s_{\beta_1}s_{\beta_2}s_{\gamma_i} & s_{\beta_2}s_{\gamma_i} \\
    -c_{\beta_1}s_{\beta_2}c_{\gamma_i} + s_{\beta_1}s_{\gamma_i} & -s_{\beta_1}s_{\beta_2}c_{\gamma_i} - c_{\beta_1}s_{\gamma_i} & c_{\beta_2}c_{\gamma_i}
\end{pmatrix}, \quad (i = 1, 2). \quad (A.3)$$

For special cases, where,

- either (i) $v_S = 0$ in $Rx2SM$, or (ii) $v_2 = 0$ in $N2HDM$, or (iii) $v_2 = 0$ in $3HDM$, the mixing matrix for CP-even neutral scalar sector $O_{\alpha}$ as given in the Eq. (A.1) is modified to,

$$O_{\alpha} = \begin{pmatrix}
    c_{\alpha} & 0 & s_{\alpha} \\
    0 & 1 & 0 \\
    -s_{\alpha} & 0 & c_{\alpha}
\end{pmatrix}. \quad (A.4)$$

- either (i) $v_X = 0$ in $Rx2SM$, or (ii) $v_S = 0$ in $N2HDM$, or (iii) $v_3 = 0$ in $3HDM$, the mixing matrix for CP-even neutral scalar sector $O_{\alpha}$ as given in the Eq. (A.1) is modified to,

$$O_{\alpha} = \begin{pmatrix}
    c_{\alpha} & s_{\alpha} & 0 \\
    -s_{\alpha} & c_{\alpha} & 0 \\
    0 & 0 & 1
\end{pmatrix}. \quad (A.5)$$

B  Oblique parameter calculation in detail

To calculate the oblique parameters, we require some matrix multiplications, such as, $Im[(V^\dagger V)]$, $U^\dagger V$. Here, using Eqns. [8,16,25], we enlist such matrix components, useful to derive the results given in the Section (3).

- **Rx2SM**

  $$Im[V^\dagger V] = \begin{pmatrix}
    0 & -O_{\alpha_{11}} & -O_{\alpha_{21}} & -O_{\alpha_{31}} \\
    O_{\alpha_{11}}^2 & 0 & 0 & 0 \\
    O_{\alpha_{11}}O_{\alpha_{21}} & 0 & 0 & 0 \\
    O_{\alpha_{11}}O_{\alpha_{31}} & 0 & 0 & 0
\end{pmatrix}, \quad (B.1)$$

  $$U^\dagger V = V = (i \quad O_{\alpha_{11}} \quad O_{\alpha_{21}} \quad O_{\alpha_{31}}), \quad (B.2)$$
\[ (V^\dagger V)_{11} = 1, \quad (V^\dagger V)_{jj} = O_{\alpha k}^2, \quad \text{with } k = (j - 1). \] (B.3)

\[ \text{N2HDM} \]

\[ \text{Im}[V^\dagger V] = \sum_{l=1}^{2} \begin{pmatrix} 0 & -O_{\beta_1 l}O_{\alpha_1 l} & 0 & -O_{\beta_1 l}O_{\alpha_2 l} & -O_{\beta_1 l}O_{\alpha_3 l} \\
O_{\beta_1 l}O_{\alpha_1 l} & 0 & O_{\beta_2 l}O_{\alpha_1 l} & 0 & 0 \\
0 & -O_{\beta_2 l}O_{\alpha_1 l} & 0 & -O_{\beta_2 l}O_{\alpha_2 l} & 0 \\
O_{\beta_1 l}O_{\alpha_2 l} & 0 & O_{\beta_2 l}O_{\alpha_2 l} & 0 & 0 \\
O_{\beta_1 l}O_{\alpha_3 l} & 0 & 0 & 0 & 0 \end{pmatrix}, \] (B.4)

\[ U^\dagger V = \sum_{l=1}^{2} \begin{pmatrix} i & O_{\beta_1 l}O_{\alpha_1 l} & 0 & O_{\beta_1 l}O_{\alpha_2 l} & O_{\beta_1 l}O_{\alpha_3 l} \\
0 & 0 & i & O_{\beta_2 l}O_{\alpha_2 l} & O_{\beta_2 l}O_{\alpha_3 l} \end{pmatrix}, \] (B.5)

\[ (V^\dagger V)_{11} = 1, \quad (V^\dagger V)_{22} = \sum_{l=1}^{2} (O_{\alpha_1 l})^2, \quad (V^\dagger V)_{33} = 1, \]

\[ (V^\dagger V)_{44} = \sum_{l=1}^{2} (O_{\alpha_2 l})^2, \quad (V^\dagger V)_{55} = \sum_{l=1}^{2} (O_{\alpha_3 l})^2. \] (B.6)

\[ \text{3HDM} \]

\[ \text{Im}[V^\dagger V] = \sum_{j=1}^{3} \begin{pmatrix} 0 & -O_{\alpha_1 j}O_{\beta_1 1 j} & 0 & -O_{\alpha_2 j}O_{\beta_1 1 j} & -O_{\alpha_3 j}O_{\beta_1 1 j} \\
O_{\alpha_1 j}O_{\beta_1 1 j} & 0 & O_{\alpha_1 j}O_{\beta_1 2 j} & 0 & 0 \\
0 & -O_{\alpha_2 j}O_{\beta_1 1 j} & 0 & -O_{\alpha_2 j}O_{\beta_1 2 j} & 0 \\
O_{\alpha_2 j}O_{\beta_1 1 j} & 0 & O_{\alpha_2 j}O_{\beta_1 2 j} & 0 & -O_{\alpha_3 j}O_{\beta_1 1 j} \\
O_{\alpha_3 j}O_{\beta_1 1 j} & 0 & 0 & 0 & 0 \end{pmatrix}, \] (B.7)

\[ U^\dagger V = \begin{pmatrix} 0 & O_{\beta_1 1 j}O_{\alpha_1 j} & 0 & O_{\beta_1 1 j}O_{\alpha_2 j} & O_{\beta_1 1 j}O_{\alpha_3 j} \\
O_{\beta_1 1 j}O_{\alpha_1 j} & 0 & O_{\beta_1 1 j}O_{\alpha_2 j} & 0 & 0 \\
0 & O_{\beta_1 1 j}O_{\alpha_2 j} & 0 & 0 & 0 \\
O_{\beta_1 1 j}O_{\alpha_3 j} & 0 & O_{\beta_1 1 j}O_{\alpha_3 j} & 0 & 0 \\
O_{\beta_1 1 j}O_{\alpha_3 j} & 0 & O_{\beta_1 1 j}O_{\alpha_3 j} & 0 & 0 \end{pmatrix} + i \begin{pmatrix} O_{\beta_1 1 j}O_{\beta_1 2 j} & 0 & O_{\beta_1 1 j}O_{\beta_1 1 j} & 0 & 0 \\
0 & O_{\beta_1 1 j}O_{\beta_1 1 j} & 0 & 0 & 0 \\
o_{\beta_1 1 j}O_{\beta_1 1 j} & 0 & 0 & 0 & 0 \\
o_{\beta_1 1 j}O_{\beta_1 1 j} & 0 & 0 & 0 & 0 \end{pmatrix} \] (B.8)

\[ (V^\dagger V)_{jj} = 1, \quad \text{with } j = (1, 3, 5), \]

\[ (V^\dagger V)_{jj} = \sum_{k=1}^{3} (O_{\alpha k})^2, \quad \text{with } j = (2, 4, 6), \quad l = j/2. \] (B.9)
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