Improved multi-objective sensor optimization method for structural damage identification based on genetic algorithm

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Abstract. An improved multi-objective sensor location optimization method based on genetic algorithm is proposed. According to the analytic relationship between structural response, mode shapes and damage quantities, the sensitivity matrix of structural dynamic response containing both structural modal information and damage information is firstly constructed, and then the objective function of multi-objective sensor location optimization method is established according to the minimum identification error criterion and the principle of ill-posedness in the inverse problem. In addition, a multi-objective genetic algorithm, NSGA-II, is used to solve the optimal location of measurement points. Finally, a simply supported box girder bridge is studied for validating the proposed method. Numerical simulations show that the improved optimization method based on genetic algorithm can achieve the optimization goal better, and the measurement points optimized by the improved method can get better damage identification results at the same level of measurement noise.

1. Introduction
Sensor system is located at the first end of health monitoring system, and its quality directly determines the accuracy and authenticity of health monitoring. Due to the economic factors and the limitations of the structure itself, it is expected that the maximum real damage information of the structure is got from the minimum number of sensors, which is the original intention and ultimate purpose of optimizing sensor layout. In the past two decades, more and more attention has been paid to the sensor location optimization methods based on the identifiability of structural damages. Shi and Law [1] used the damage identification sensitivity matrix as Fisher information matrix to improve the effective independent method proposed by Kammer [2]. This method achieves the optimization of measurement points by gradually deleting the degree of freedom with small contribution to the rank of Fisher information matrix. Liu and Qu [3] determined the damage information contained in each degree of freedom based on the size of the trace of the Fisher information matrix decomposed into each degree of freedom, thus, the calculation of the inverse matrix for the sensitivity Fisher information can be avoided in the optimization process. Sun and Feng [4] constructed a Fisher information matrix containing both damage information and modal information, and considered the ill-posedness of damage identification equation group at the same time, then, realized the multi-objective sensor location optimization by deleting the degree of freedom step-by-step. Li and Ma [5] revised the Fisher information matrix by calculating the contribution of each degree of freedom to modal strain energy on the basis of the original multi-objective sensor optimization method, finally, realized the optimization of sensor locations. The above studies show that the multi-objective sensor location optimization is superior to that with single-objective. However, the local convergence is still a problem needed to be overcome for the original multi-objective optimization method.
In this paper, a genetic algorithm is applied to improve the local convergence of the original multi-objective optimization method, and a numerical example is given to verify the feasibility and effectiveness of the improved method.

2. Fisher information matrix for optimizing sensor location

Assuming that the structural damage is caused by the reduction of structural stiffness, which is independent of other structural characteristics, and that the change of stiffness is so small that it neither affects the continuity of the structure nor has a significant impact on the damping characteristics of the structure, so the change in structural damping can be neglected, then the ith mode shape change $\Delta \Phi_i$ can be expressed as [1]:

$$\Delta \Phi_i = \sum_{k=1}^{n_e} \alpha_k \sum_{r=1}^{n} \frac{-\phi_r^T K_k \phi_i}{\omega_r^2 - \omega_i^2} \Phi_r = S \cdot \alpha$$

(1)

where $K_k$ and $\alpha_k$ represent element stiffness matrix and element stiffness damage coefficient respectively. $n_e$ is the total number of element. $\Phi_i$ and $\omega_i$ represent the ith mode shape vector and the ith mode frequency, respectively.

Sensitivity matrix $S$ and structural element stiffness damage vector $\alpha$ can be expressed in detail as:

$$S = \left( \sum_{r=1}^{n} -\frac{-\phi_r^T K_k \phi_i}{\omega_r^2 - \omega_i^2} \Phi_r \right)$$

(2)

$$\alpha = [\alpha_1 \quad \alpha_2 \quad \ldots \quad \alpha_{n_e}]$$

(3)

In order to establish the optimal objective function containing both modal mode shape information and damage information, Fisher information matrix $A$ is set as [4]:

$$A = (\Phi \quad S)$$

(4)

where $\Phi$ is the mode shape matrix.

3. Improved multi-objective sensor location optimization based on genetic algorithms

3.1. Optimization objective function

In general, it is assumed that the variance of white Gaussian noise on each sensor is $\sigma^2$ and is statistically unrelated. Then the error covariance matrix is [6]:

$$C = \sigma^2/A$$

(5)

In order to reduce errors, it is necessary to maximize the Fisher information matrix $A$. The 2-norm of Fisher information matrix $f_{norm}$ is taken as the comparative parameter:

$$f_{norm} = \frac{1}{\text{norm}_2(A)}$$

(6)

When the minimum value of $f_{norm}$ is obtained, Fisher information matrix $A$ is maximized.

Considering that the ill-posedness of the inverse problem of structural dynamics will affect the results of damage identification in the presence of noise, the condition number of Fisher information matrix is considered to be reduced to decrease the ill-posedness of the inverse problem here. The process can be described as:

$$\text{cond}(A) = \text{cond}(\Phi \quad S)^T \cdot (\Phi \quad S) = [\text{cond}(\Phi \quad S)]^2$$

(7)

$$f_{\text{cond}} = \text{cond}(\Phi \quad S)$$

Minimize: $f_{\text{cond}}$

In order to facilitate the direct comparison between multi-objective functions, it is necessary to standardize each objective function, then the multi-objective optimization function can be described as:

$$F(\theta) = \left[ \frac{f_{\text{norm}}(\theta)}{\text{max}f_{\text{norm}}(\theta)}, \frac{f_{\text{cond}}(\theta)}{\text{max}f_{\text{cond}}(\theta)} \right]$$

(8)

where $\theta$ is the optimal sensor location combination vector. $N_C$ and $n^*$ represent the number of optimal sensors and the number of candidate sensors, respectively.
3.2. Multi-objective sensor location optimization algorithm based on NSGA-II

Sequence optimization algorithm is the most widely used method. It includes cumulative step-by-step method and elimination step-by-step method. This method has high computational efficiency, but it is difficult to obtain the optimal solution and only the near-optimal solution of the objective function can be obtained. Therefore, an optimization algorithm with global convergence is needed to improve the multi-objective sensor optimization algorithm.

The second version of Non-dominated Sorting Genetic Algorithm (NSGA-II) represents the location number of the selected sensor sites as chromosomes, and then forms a population of chromosomes. The population will eventually converge to the globally optimal chromosomes through generation evolution according to the environment determined by the optimization objectives. However, it is essential to ensure that the genes on each chromosome are different since the same location number represents the same sensor. The process of NSGA-II is as following.

Step1: Set basic parameters including population size, length of each chromosome, the maximum number of iterations.

Step2: Generate the random initial group \( P(S)^0 \), calculate the two objective function values corresponding to each chromosome respectively, and store the two values in the newly added two gene positions of the corresponding chromosome for the non dominated sequencing of the elite strategy.

Step3: Let \( P(S)^i \) equal to \( P(S)^{i-1} \), and generate a subpopulation \( Q(S)^i \) by the legacy algorithm (selection, crossover and mutation), calculate the two objective function values of each offspring chromosome and store them in the corresponding gene positions of each chromosome.

Step4: Combine parent and child populations as \( P(S)^i \cup Q(S)^i \), rank the combined populations by elitist strategy, and extract the parent population of the next generation \( P(S)^{i+1} \).

Step5: Repeat Step 3 to Step 4 until the maximum number of iterations is achieved.

4. Numerical Simulation

In this section, a simply supported box girder bridge of 30m is numerically used to verify the feasibility and effectiveness of the improved multi-objective location optimization method based on genetic algorithm. The elastic modulus, the mass density and Poisson's ratio of the bridge is \( 3.45 \times 10^{10} \) N/m², 2500 kg/m³ and 0.2. The section parameters of the bridge include a cross section area of 5.06 m² and a bending moment of inertia of 12.752 m⁴. The bridge is numerically simulated by a finite element model consisting of 31 nodes and 30 two-dimensional beam elements of 1 meter. The first 12 mode shapes of the bridge are selected as model modification parameters and 29 vertical (perpendicular to the deck direction) degrees-of-freedom except that of the two bearing nodes are selected as the set of candidate sensors.

4.1. Parameters Selection in the improved optimization method based on genetic algorithms

In the multi-objective sensor location optimization algorithm based on NSGA-II, the population of genetic algorithm contains 50 chromosomes, the maximum number of evolution is 1000, the selection method is Championship algorithm and the crossover probability and mutation probability are both 0.2.

Multi-objective optimization problem in equation (8) can obtain Pareto solution set by NSGA-II. In order to select the optimal combination of sensor locations in Pareto solution set, a utility function method is adopted and the weight value \( \omega \) is set as 0.5 [7].

\[
I_\omega = \omega f_{norm} + (1 - \omega) f_{cond}
\]  

where, \( I_\omega \) is the utility function. \( f_{norm} \) and \( f_{cond} \) represent the standardized optimization objective functions.

When the weight \( \omega = 0.5 \) is used to calculate the utility function, the identical objective function is applied by both the original method and the improved method in the optimization process. The function values corresponding to the sensor locations combination optimized by the sequential method and the genetic algorithm are 454.3238 and 305.9499, respectively. It can be seen that the improved multi-objective sensor locations optimization method by genetic algorithm can better achieve optimization objectives.
4.2. Comparison of damage identification results

In order to show the effect of the optimized objective function value on the damage identification results, the improved method and the original method (respectively abbreviated as GA and OA) are simultaneously applied to select the nodes for sensors from the set of candidate sensors, and their optimized node sets respectively are (8, 10, 12, 14, 15, 24, 25, 27, 29, 30) for GA and (5, 6, 10, 11, 14, 15, 16, 28, 29, 30) for OA.

Because all the elements are involved in the identification process of the damage identification method based on sensitivity analysis, only one damage scenario is set here, and it includes 10%, 5% and 5% reduction of modulus of elasticity $E$ in elements 10, 17 and 20. The identified structural damage indexes without noise and with 5% are shown in figure 1.

![Figure 1. Comparison of damage identification results](image)

(a) Without noise  
(b) With 5% noise

As shown in figure 1(a), in the case without noise, the identified structural damage indexes are very close to their true values. The 2-norm of error between the identification result of the GA method and the true value is $7.40 \times 10^{-6}$, and that of the OA method is 0.101. Figure 1(b) shows the identified structural parameters deviate from the true ones in the case with 5% noise. The 2-norm of error between the identification result of the GA method and the true value increases to 6.5077, and that of the OA method increase to 10.7251. However, the identified structural damage indexes still can reflect both damage location and damage extent, especially for the GA method. The above results indicate that the measured noise has some negative effect on the results in comparison with results from the noise-free case. However, the damage identification result with fewer errors can be obtained under noisy conditions by selecting sensor locations using the improved multi-objective optimization method based on genetic algorithm. The improved method presents better good noise robustness.

5. Conclusion

In order to make the damage identification algorithm obtain an effective estimation of the damage parameters, the genetic algorithm is used to improve the multi-objective sensor locations optimization method in this paper. A simply supported box girder bridge is used for the study. Numerical simulations with measurement noise show that the improved method can achieve the optimization objectives better, and the damage identification results from the sensor locations selected by using the improved method are more accurate in the case of the same level of measurement noise.

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