Reasoning and Facts Explanation in Valuation Based Systems

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Abstract In the literature, the optimization problem to identify a set of composite hypotheses $H$, which will yield the $k$ largest $P(H|S_e)$ where a composite hypothesis is an instantiation of all the nodes in the network except the evidence nodes [14] is of significant interest. This problem is called "finding the $k$ Most Plausible Explanation (MPE) of a given evidence $S_e$ in a Bayesian belief network". The problem of finding $k$ most probable hypotheses is generally NP-hard [2]. Therefore in the past various simplifications of the task by restricting $k$ (to 1 or 2), restricting the structure (e.g. to singly connected networks), or shifting the complexity to spatial domain have been investigated.

A genetic algorithm is proposed in this paper to overcome some of these restrictions while stepping out from probabilistic domain onto the general Valuation based System (VBS) framework is also proposed by generalizing the genetic algorithm approach to the realm of Dempster-Shafer belief calculus.

Keywords: Genetic algorithm, most plausible explanation, graphoidal expert systems.

1 Introduction

Bayesian network, BN for brevity, is a powerful model for probabilistic reasoning [9]. From a formal standpoint the BN is a triplet $BN = (X^*, E^*, P^*)$ where $G = (X^*,E^*)$ is a directed acyclic graph, and $P^*$ is a set of conditional probabilities $p(x—Pa(x))$ where $Pa(x)$ stands for the set of parents of $x \in X^*$ in the graph $G$. The graph $G$ represents qualitative interrelationships among the variables specified in the set $X^*$, while $P^*$ gives a quantitative description of these interrelationships. Knowing the conditionals $P^*$ we can express the joint probability distribution, $p(X^*) = p(x_1,x_2,...,x_n)$, as the product of all the probabilities $p(x|Pa(x))$. Note that the symbol $p(x|Pa(x))$ is a shorthand of the next formula: $p(x = a|x_1 = b_1,...x_k = b_k)$, where $x_1,...,x_k = Pa(x)$ and defined for all the values $a, b_1,...,b_k$ ranging over the discrete domains of the corresponding variables.

Now, the problem of the probabilistic reasoning can be stated as follows. Assume that $E \subset X^*$ is a set of clamped variables, i.e. the variables with known values. Our task is to find the conditional probability

$$p^*(x|E) = p(X^*)/p(E)$$

for a variable $x \in (X^* - E)$.

On the other hand, knowing the values of the variables specified in the set $E$ we may be interested in the values of the remaining variables that provide the maximal value of the joint probability distribution. That is, if $E = x_1, x_2, ..., x_k$ and $x_i = a_i^*$, $i = 1..k$, we are searching for such values $x_j = b_j^*$ , $j = (k+1)..<n$, that
The problem (2) is referred to as the facts explanation as the values \( x_j = b^*_j, j = (k+1), \ldots, n \) together with \( x_i = a^*_i, i = 1, k \) form the most probable configuration. More precisely the equation (2) defines so-called first most probable explanation, or \( 1-\text{MPE} \) for short. In quite similar way we can define \( k-\text{MPE} \) yielding \( k \)-th largest value of the \( p \ast (x|E) \).

It appears that both the problems (1) and (2) are extremely difficult from the numerical standpoint. To be illustrative assume that \( X^* \) contains 56 binary variables. In this case the joint probability distribution consists of \( 2^{56} \) values. If our computer can calculate the terms of the probability distribution for one million values per second, then it will only take our computer 2283 years to come up with the whole distribution function!

Fortunately both the problems can be solved in a reasonable time and without computing the joint probability distribution. The first solution, restricted to the case when \( G \) is a tree was proposed by J. Pearl [9], and next it was extended to the case when \( G \) is a graph by S.L. Lauritzen and D.J. Spiegelhalter [4]. Further improvement to this problem was proposed by F.V. Jensen. Almost at the same time G. Shafer and P.P. Shenoy [10] have considered similar problem of computing a belief function (i.e. a generalization of a probability function) for a variable \( x \in X^* \) given a set of clamped variables. Later results of A. Kong led P.P. Shenoy to the notion of so-called Valuation Based System, or VBS for brevity [12]. Within such a system we can represent knowledge in different domains including probability theory, Dempster-Shafer theory, possibility theory and so on. More recent studies show that the framework of VBS is also appropriate for representing and solving Bayesian decision problems and optimization problems [13]. The graphical representation of a problem is called a valuation network, and the method for solving problems is called the fusion algorithm [8].

In this paper we briefly describe the idea of a VBS and we show how it can be used in the reasoning process and in the fact explanation problem.

Up to this moment we have no satisfactory algorithms to verify the "summarized expert’s experience" against a detailed knowledge (about a population) that is stored in a database. Apart already mentioned papers, the practical attempt to solve this problem was developed by our research group in the form of the SEAD-1 system reported in [5]. The system incorporates various tools supporting a researcher in different parts of the cycle observation-generalization-theory-exertion. The main functions of the system are: storing observations and results of experiments (data base), storing acquired theoretical generalizations (knowledge base) and tools supporting knowledge acquisition (statistical data analysis system, learning from examples system, cf. [6] and knowledge verification (against new incoming experimental results in the database or against new single cases). The main feature of our solution is the implementation of all system components into one organism who’s heart is database pumping and absorbing data. The next, just finished, version of the system relies upon introducing different model of knowledge base. The system SEAD-2 is the experimental computer aided decision-making system based on mixed, probabilistic/Dempster-Shafer, knowledge representation, and it fulfills the next requirements:

- representation of joint belief function as a belief network (knowledge base),
- representation of joint belief function as a random sample (data base),
- calculation of marginal belief function in relation to observed facts (consultation session),
- automatic extraction of belief function from random sample (knowledge generation)

Although we implement quite effectively so-called message passing algorithm [11] the system was oriented towards computing marginals for the nodes of the network only. Frequently we are interested
in finding joint probability functions (or valuations if we use the language of VBS's) for different subsets of the set of variables. This problem was firstly solved by Xu \[17\], but it is possible to find more general and more effective solution. In this paper we present this general idea, and next we focus on the problem of finding so-called most probable explanation of a given set of observations.

2 Valuation Based Systems

Network-based approaches are at present the widely accepted approaches for uncertainty processing and reasoning. Among them Bayesian networks, \[9\], designed for the case of probabilistic reasoning, and more general valuation based systems, or VBS for short, \[12\], are most popular. From a graphical point of view Bayesian networks are directed acyclic graphs, while VBS’s are hypergraphs. Since any Bayesian network can be translated to an equivalent VBS assume that we start from a given Bayesian network which can be regarded as a triplet \((X^*, E^*, P^*)\) where

\[
X^* = \{x_1, x_2, ..., x_m\} \text{ is a set of variables. With each variable } x_i \text{ we associate its discrete frame } \Theta_i.
\]

In the sequel we shall write \(\Theta\) to denote the Cartesian product \(\Theta_1 \times \Theta_2 \times ... \times \Theta_m\) and we shall write \(\Theta(h) = \{\Theta_i | i \in h\}\). Thus \(\Theta = \Theta(X^*)\).

\(E^*\) is a set of directed edges over \(X^*\) such that \((X^*, E^*)\) is a directed acyclic graph (DAG). Its nodes represent the variables and the edges represent conditional dependency relationship among variables,

\(P^*\) is a set of conditional probabilities \(p(X_i|Pa(X_i))\), \(i = 1..m\), where \(Pa(x_i)\) stands for the set of parents of \(i\)-th variable in the DAG. When \(x_i\) has no parents then \(P(x_i)\) represents the unconditional probability of \(x_i\).

The joint probability of the set of variables \(X^*\) is defined as

\[
P(x_1, x_2, ..., x_m) = \prod_{i=1}^{m} p(x_i|Pa(x_i))
\]

The computation of full probability distribution is a time consuming problem. For instance if we have 56 binary variables then the total space of configurations, \(\Theta\) consists of \(2^{56}\) elements. If our computer can calculate the probability for one million configurations per second, then it will take almost 2,283 years to come up with the whole joint probability function.

To overcome this difficulty we transform the Bayesian network into a Valuation Based System which can be represented by the triplet \((X^*, H^*, V^*)\) where \(H^*\) is a family of subsets of the set of variables \(X^*\) and \(V^*\) is a set of valuations. In general valuations are primitives in the VBS framework and, as such, require no definition. Intuitively they represent some knowledge about the variables included in the sets \(h_i\) in \(H^*\), called local universes. Under probabilistic case the local universes are defined as:

\(h_i = x_i \cup Pa(x_i)\). With such a universe, \(h_i\), we associate the valuation \(v_i = p(x_i|Pa(x_i))\). To perform inferences with the set of valuations we define two operations: combination (\(\otimes\)) that corresponds to the aggregation of knowledge and marginalization (\(\downarrow\)) that corresponds to the coarsening of the knowledge. The join valuation \(v\) for the set \(X^*\) is computed due to the equation

\[
v = \otimes_{i=1}^{m} v_i
\]

while the marginal valuation for some subset \(h\) of \(X^*\) is computed from \(v\) according to the equation

\[
v(h) = v_{\downarrow h}
\]

It is easy to observe that under probabilistic context \(\otimes\) is equivalent to the product operator while \(\downarrow\) is equivalent to the summation. In other words \((\otimes, \downarrow) = (\cdot, \sum)\). Thus if \(h = \{x_j, ..., x_k\}\) then
\[ v(h) = \sum_{x \in (X^* - h)} \prod_{i=1}^{m} v_i = \bigotimes_{i=1}^{m} v_i^{j_h} \]

In the Dempster-Shafer calculus, [13], instead of the probability distribution we use so-called basic
probability function, \( m \). If \( h \) is a subset of \( X^* \) then as \( m \) we regard a set-function \( m : 2^{\Theta(h)} \to [0,1] \)
such that

\begin{align*}
(m1) \quad m(A) \geq 0 & \text{ for all } A \in 2^{\Theta(h)}, \\
(m2) \quad \sum \{ m(A) | A \in 2^{\Theta(h)} \} = 1. \\
\end{align*}

A belief function \( Bel \) is defined as \( Bel : 2^{\Theta(h)} \to [0,1] \) so that \( Bel(A) = \sum_{B \subseteq A} m(B) \). A plausibility function be \( Pl : 2^{\Theta(h)} \to [0,1] \) with \( \forall A \in 2^{\Theta(h)} \) \( Pl(A) = 1 - Bel(\Theta(h) - A) \).

Another very useful set function is so-called commonality function, \( Q \), that is computed from \( m \) due
to the equation \( Q(A) = \sum \{ m(B) | B \supseteq A \} \). If \( Q_1 \) and \( Q_2 \) are two commonality function, they are combined due to so-called Dempster’s Rule of Combination

\[ (Q_1 \otimes Q_2)(A) = k \cdot Q_1(A^{h_1}) \cdot Q_2(A^{h_2}), A \subseteq (\Theta(h_1) \cup \Theta(h_2)) \]

where \( k^{-1} = \sum \{( -1)^{|A|+1} Q_1(A^{h_1}) \cdot Q_2(A^{h_2}) | A \subseteq (\Theta(h_1) \cup \Theta(h_2)) \} \) is the normalizing constant such that \( k \neq 0 \). Here e.g. \( A^{h_1} \) stands for the projection of the set \( A \) to the set \( h_1 \) of variables. Similarly, if \( Q \) is a commonality function over the set of variables \( h \) and \( g \) is a subset of \( h \) then the marginalization is defined as follows

\[ Q^{j_{g}}(A) = \sum \{( -1)^{|B|+1} Q(B) | B \subseteq \Theta(h) \} \]

s.t. \( C^{j_{g}} \supseteq A \), and \( B \supseteq C \).

Assuming that \( \otimes \) is a commutative and associative operation, and that \( \downarrow \) is distributive with respect
to \( \otimes \), consult [12], we are able to define so-called Message Passing Algorithm that is a universal
tool for making inferences under several uncertainty formalisms. To apply this algorithm we must
convert first the hypergraph \( (X^*, H^*) \) into a secondary structure called Markov tree (i.e. an acyclic
hypergraph that covers the original hypergraph). Intuitively the Message Passing Algorithm tells the
nodes of a Markov tree in what sequence to send their messages to propagate the local information
throughout the tree. The algorithm is defined by two parts: a fusion rule, which describes how
incoming messages are combined to make marginal valuations and outgoing messages for each node;
and a propagation algorithm, which describes how messages are passed from node to node so that
all of the local information is globally distributed. Just as propagation takes place along the edges
of the tree, fusion takes place within the nodes. More formally, to compute the marginal \( v(h) \) for a
subset \( h \) of \( X^* \) (where \( h \) is a member of \( H^* \) or a subset of some set from \( H^* \)) we must combine the
original valuation \( v_h \) assigned to this subset and the messages sent by all its neighbors in the tree:

\[ v(h) = v_h \otimes (\otimes \{ M_{g \to h} | g \in N(h) \}) \]

where \( N(h) \) stands for the set of neighbors of \( h \) in the tree, and \( M_{g \to h} \) is the message sent from \( g \)
to its neighbor \( h \), which is computed by projecting on the set \( g \cap h \) the combination of \( v_g \) and the
messages sent by the neighbors \( g \) except \( h \):

\[ M_{g \to h} = (v_g \otimes (\otimes \{ M_{k \to g} | k \in (N(g) - h) \}))^{j_{(g \cap h)}} \]

This algorithm can be easily adopted to solving the 1-MPE problem. Under probabilistic and also
in Dempster-Shafer calculus (under assumptions that are listed in subsection 3.2.) context we take
the pair of operators $(\otimes, \downarrow) = (\cdot, \max)$. However, we are sometimes interested in flexibility of this solution, so would like to know the subsequent 2nd,...,kth best solutions. But it is hard to extend this pathway of solution to the general k-MPE problem. Therefore we decided to try out the genetic approach.

3 Genetic Algorithm for Finding MPE.

Following e.g. [7] we define general genetic algorithm as a kind of a probabilistic algorithm which maintains a population of individuals $P(t) = \{x_1^t, x_2^t, ..., x_n^t\}$ for iteration $t$. Each individual represents a potential solution to the problem at hand, and, in any genetic program, is implemented as some data structure $S$. Each solution $x_i(t)$ is evaluated to give some measure of ”fitness”. Then, a new population - iteration $(t+1)$ - is formed by selecting the more fit individuals. Some members of the new population are recombined, i.e., transformed by means of two ”genetic” operators to a new form. These operators are: (1) the unary mutation operator that create new individuals by a small changes in a single individual, and (2) the n-ary, where $n \geq 2$, crossover operator, which creates new individual by combining parts on the n individuals. After some number of iterations the program converges, and the best individual represents the optimum solution. The general implementation of this idea can be summarized in the form of the next pseudocode (see [7] for details):

```
Procedure GeneticAlgorithm
begin t:=0;
    Initialise $P(t)$;
    Evaluate $P(t)$;
    while (not termination-condition) do
    begin
        $t:=t+1$;
        Select $P(t)$ from $P(t-1)$;
        Recombine $P(t)$;
        Evaluate $P(t)$;
    end;
end;
```

This idea quite interestingly translates into a program for finding most probable explanations both in probabilistic and generalized, Dempster-Shafer, case. Below we present details concerning the implementation.

3.1 Probabilistic Case

Here the situation is quite simple. The aim is to find such a configuration $\theta^* = \{\theta_1^*, \theta_2^*, ..., \theta^*_m\}$ that

$$p(x_1 = \theta_1^*, x_2 = \theta_2^*, ..., x_m = \theta_m^*) = \max_{\theta \in \Theta} \prod_{i=1}^{k} p(x_i = \theta_i | Pa(x_i)) = \theta(Pa(x_i))$$

where $\theta(Pa(x_i))$ stands for the projection of the configuration $\theta$ onto the set of variables $Pa(x_i)$. Hence as an individual we take simply a vector $x = \theta$, where each member $\theta_i$ of $\theta$ takes its values from the domain of the variable $x_i$. Hence we depart from the ”standard” binary representation of the individuals. This however fastens computations.

The mutation and crossover operators are implemented in almost standard way. There is a nuance, however. Frequently we ask for an MPE already knowing values of some variables. If $C$ stands for the set of clamped variables, then, after the recombination phase the values at positions corresponding to the variables form the $C$ must remain unchanged. There is a number of strategies to attain this
goal. But in our implementation we used the next one. In case of mutation denote \( p_m \) the probability of mutation. Then for each element of the individual \( x \) and such that it does not correspond to a variable from the \( C \) we generate a random real number \( r \) from the unit interval. If \( r < p_m \) we replace this element by another from the domain of the corresponding variable. The crossover is obvious also, that is we choose two parent individuals and the crossing point, and finally we replace the parents by a pair of their offsprings.

The fitness of each individual is computed by means of the maximized function \( p(\cdot) \). The values of a conditional probability \( P(x_i|Pa(x_i)) \) are stored in the structure \( \text{universe} \) defined below

```plaintext
universe = record
variables: SetOfVariables; (*i.e. \( \text{variables} = \{X_i \cup Pa(X_i)\} \)*)
card: byte; (*cardinality of the set \( \text{variables} \)*)
valuations: array [1..k] of real;
end;
```

In our implementation we assume that the variables are numbered consecutively from 1 to \( \text{variables} \) (number of variables). Further the variables stored in the field \( \text{variables} \) are written in the next order: first is given he conditioned variable and next the conditioning variables.

Assuming that the domains of all the variables are stored in the table referred to as \( \text{domains} \) and that individuals are represented as arrays called \( \text{config} \) the value of the conditional probability \( P(x_i|Pa(x_i)) \) for a given individual \( x \) are computed by means of the next function

```plaintext
Function FindValuation(x: config; sp: universe): real;
var j,position,size: integer;
begin
  with sp do
  begin
    position := x[variables[card]];
    size := domains[variables[card]];
    for j:= card-1 downto 1 do
    begin
      position := position + size*(x[variables[j]]-1);
      size := size*domains[variables[j]];
    end;
    FindValuation := valuations[position];
  end;
end; {FindValuation}
```

The set of all universes is stored in the array referred to as \( \text{universes} \). Now it is easy to write down the function that computes fitness of an individual \( x \). Besides the already mentioned function \( \text{FindValuation} \) we need a boolean function \( \text{Blocked} \). It is needed when we compute \( k \)-MPE. That is a solution to an \( i \)-MPE problem, \( i < k \), is removed on a stack of blocked configurations. Now, looking for the \( k \)-th explanation we check is the individual \( x \) belongs to this stack. If so, we mark it as blocked and we search for another individual. Below we present the pseudocode of the function.

```plaintext
Function ObjectiveFunction(x: config; pro: cuniverses): real;
var j: integer; temp: real;
begin

The set of all universes is stored in the array referred to as \( \text{universes} \). Now it is easy to write down the function that computes fitness of an individual \( x \). Besides the already mentioned function \( \text{FindValuation} \) we need a boolean function \( \text{Blocked} \). It is needed when we compute \( k \)-MPE. That is a solution to an \( i \)-MPE problem, \( i < k \), is removed on a stack of blocked configurations. Now, looking for the \( k \)-th explanation we check is the individual \( x \) belongs to this stack. If so, we mark it as blocked and we search for another individual. Below we present the pseudocode of the function.

```plaintext
Function ObjectiveFunction(x: config; pro: cuniverses): real;
var j: integer; temp: real;
begin

```
if Blocked(x) then ObjectiveFunction := 0 else
begin
  temp := 1;
  for j := 1 to nvaluations do
    temp := temp*FindValuation(x,pro[j]);
  ObjectiveFunction := temp;
end;
end; {ObjectiveFunction}

3.2 Case of Dempster-Shafer Calculus

Extension of this approach to Dempster-Shafer theory of evidence is not straightforward. First of all we do not have the general concept of Bayesian network decomposition of the joint belief distribution but rather a decomposition in terms of a hypergraph [11]. Though a Bayesian network-like decomposition has been proposed by Cano et al [1], but most belief functions cannot be decomposed that way.

Contrary to probabilistic case, valuations are not defined exclusively for elements of $\Theta$, but rather for elements of the power set of $\Theta$.

Further, the observed variables do not need to be clamped to a single value, but they can represent a set of values.

Then we have the difficult choice which configuration evaluation function to optimize: mass function, belief function, plausibility function and the commonality function. They represent various aspects of partial ignorance of reasoner’s knowledge and it is hard to discard any of them a priori.

If $f(x) = (\bigoplus_i Bel_i)(x)$ should be our target function, then in general $f(x)$ is not a function of $Bel_1(x)$, $Bel_2(x)$, ... because of Dempster rule of evidence combination, where evidence from supersets contributes to subsets of values upon combination of evidence. Last not least we have a difficult problem of how to understand a configuration: as a cross product of singleton values of all the variables, as a cross product of subsets of domains of all variables, or as a subset of the cross product of all the domains of all variables.

We had to make some choices and we can only justify them, not to prove their validity in mathematical sense.

- First we decided that we want to find $k$ most optimal singleton configurations. We just understand that the belief function represents an imprecise information on some singleton configurations.
- We assumed that we want to find most plausible configurations that is ones that cannot be rejected. Just we find an explanation good if it is hard to prove it wrong rather then being easy to be proved right.

These two choices greatly simplified our task. One can prove that then the following target function gets its maximum at the configuration solving the $k$-MPE problem:

$$f(\mathbf{x}) = (\bigoplus_i Q_i)(\mathbf{x}) = \prod_{i=1}^{n} Q_i(\mathbf{x}^{Space(Q_i)})$$

($Space(Q_i)$ - the set of variables for which $Q_i$ is defined) subject eventually to constraints that

1. the set of observed variables $E$ has values restricted to the observed sets of values
2. when we are looking for $i$th most probable solution, we punish $f()$ so that $f(\mathbf{x})=0$ whenever $\mathbf{x}$ is one of already found 1st, 2nd, (i-1)st most probable configurations configurations.
Table 1: m-functions of components in the decomposition of Bel

| m_1 in vars | A,B       |
|-------------|-----------|
| {(b1,a1)}   | 0.20      |
| {(b2,a1),(b1,a2),} | 0.10    |
| (b1,a1), (b2,a3), (b1,a2), (b2,a2) | 0.45 |
| {{(b1,a1), (b2,a3), (b1,a3)} | 0.25 |
| m_2 in var  | A         |
| {a1}        | 0.40      |
| {a2}        | 0.05      |
| {a3}        | 0.15      |
| {a1,a2}     | 0.35      |
| {a1,a3}     | 0.05      |
| m_3 in vars | C,B       |
| {(c1,b1)}   | 0.10      |
| {(c1,b2),(c1,b1)} | 0.10    |
| (c1,b2), (c2,b1) | 0.70 |
| {(c1,b1), (c1,b2), (c2,b2)} | 0.10 |
| m_4 in vars | F,B       |
| {(f1,b1)}   | 0.05      |
| {(f2,b1)}   | 0.10      |
| {(f2,b2)}   | 0.32      |
| {(f1,b2)}   | 0.03      |
| {(f2,b1), (f2,b2), (f1,b2)} | 0.15 |
| {(f1,b1) (f2,b1) (f2,b2)} | 0.35 |

The configurations are represented and evaluated essentially in the same way as in probabilistic case, because they were designed for hypergraph representation of joint probability distribution. The genetic algorithm procedures are also the same except that mutation and cross-over operations are designed in such a way as to keep values of observed ("clamped") variables within the observed value sets (rather than restricting them to a single value). That is, compared to probabilistic case, mutation is not forbidden at clamped variables but rather the scope of mutation is restricted to the subset of the values of the given variable.

Notice, however the difference that in probabilistic case f() was in fact the sought ith most probable configuration probability itself, but in DST case f() is only proportional to the sought ith most plausible configuration plausibility.

If we would like to find the most optimal configuration not restricting ourselves to the singleton set, we would fall into several triviality traps (not to mention explosion of complexity).

- If maximization of plausibility function would be the criterion of optimality, then always the universe set Θ would win because $P_l(Θ) = 1$ and this is the maximum value of Pl-function.
- The same effect is achieved if belief function is maximized because $Bel(Θ) = 1$ and this is the maximum value of Bel-function.
- If we maximize the commonality function then for every non-singleton configuration $C$, any configuration $C' \subset C$ is at least as optimal as $C$.
- Finding an optimal configuration for the mass function would be a complex task because of the Dempster-rule of evidence combination.

EXAMPLE: Let us consider the joint belief distribution Bel (derived from the probabilistic example
| $m_7$ in vars | G,F |
|---------------|-----|
| \{(g1,f1),(g2,f1),(g1,f2)\} | 0.40 |
| \{(g1,f2)\} | 0.08 |
| \{(g2,f1),(g1,f2)\} | 0.20 |
| \{(g2,f2)\} | 0.32 |

| $m_8$ in vars | J |
|--------------|---|
| \{j1\} | 0.46 |
| \{j2\} | 0.54 |

| $m_9$ in vars | D |
|--------------|---|
| \{d1\} | 0.40 |
| \{d2\} | 0.60 |

| $m_{10}$ in vars | H,G |
|-----------------|-----|
| \{(h1,g1)\} | 0.10 |
| \{(h1,g2)\} | 0.18 |
| \{(h2,g1)\} | 0.10 |
| \{(h2,g2)\} | 0.02 |
| \{(h1,g1), (h1,g2), (h2,g1), (h2,g2)\} | 0.20 |
| \{(h1,g1), (h1,g2)\} | 0.40 |

| $m_5$ in vars | E,C,D |
|---------------|------|
| \{(e1,c1,d1)\} | 0.20 |
| \{(e1,c2,d1), (e2,c2,d2)\} | 0.05 |
| \{(e2,c1,d1)\} | 0.10 |
| \{(e1,c1,d1), (e1,c2,d1), (e2,c1,d1), (e2,c2,d1), (e2,c2,d2)\} | 0.05 |
| \{(e1,c2,d1), (e1,c2,d2), (e2,c1,d2)\} | 0.30 |
| \{(e1,c1,d1), (e1,c2,d1), (e1,c2,d2), (e2,c1,d1), (e2,c2,d1), (e2,c2,d2)\} | 0.30 |

| $m_6$ in vars | I,F,J |
|---------------|------|
| \{(i1,f1,j1)\} | 0.10 |
| \{(i1,f2,j2)\} | 0.10 |
| \{(i1,f1,j1), (i1,f2,j1)\} | 0.20 |
| \{(i1,f1,j1), (i1,f2,j1), (i1,f2,j2), (i2,f1,j2)\} | 0.20 |
| \{(i1,f1,j1), (i1,f2,j1), (i1,f2,j2), (i2,f1,j2), (i2,f1,j1), (i1,f1,j2)\} | 0.10 |
| \{(i1,f1,j1), (i1,f2,j1), (i1,f2,j2), (i2,f1,j2), (i2,f1,j1), (i2,f2,j2)\} | 0.20 |
| \{(i1,f1,j1), (i1,f1,j2), (i1,f2,j1), (i1,f2,j2), (i2,f1,j1), (i2,f2,j2)\} | 0.10 |
Table 2: $Q$-functions of components in the decomposition of $Bel$ - listed only for singleton subsets of respective domains.

| $Q_1$ in vars | A,B | $Q_5$ in vars | E,C,D | $Q_7$ in vars | G,F |
|---------------|-----|---------------|-------|---------------|-----|
| \{(b1,a1)\}   | 0.90| \{(e1,c1,d1)\}| 0.55  | \{(g1,f1)\}  | 0.40|
| \{(b1,a2)\}   | 0.55| \{(e1,c1,d2)\}| 0.35  | \{(g1,f2)\}  | 0.68|
| \{(b1,a3)\}   | 0.25| \{(e1,c2,d1)\}| 0.70  | \{(g2,f1)\}  | 0.60|
| \{(b2,a1)\}   | 0.10| \{(e1,c2,d2)\}| 0.60  | \{(g2,f2)\}  | 0.32|
| \{(b2,a2)\}   | 0.45| \{(e2,c1,d1)\}| 0.45  |             |     |
| \{(b2,a3)\}   | 0.70| \{(e2,c1,d2)\}| 0.65  |             |     |
| $Q_2$ in var A | \{(e2,c2,d1)\}| 0.30  | \{(j1)\} | 0.46 |
| \{a1\}        | 0.80| \{(e2,c2,d2)\}| 0.40  | \{(j2)\} | 0.54 |
| \{a2\}        | 0.40| $Q_6$ in vars | 1,F,J  |             |     |
| \{a3\}        | 0.20| \{(i1,f1,j1)\}| 0.80  | \{d1\} | 0.40 |
| \{a3\}        | 0.20| \{(i1,f1,j2)\}| 0.40  | \{d2\} | 0.60 |
| $Q_3$ in vars C,B | \{(i1,f2,j1)\}| 0.90  | \{(h1,g1)\}| 0.70 |
| \{(c1,b1)\}   | 0.30| \{(i1,f2,j2)\}| 0.70  | \{(h1,g2)\}| 0.78 |
| \{(c1,b2)\}   | 0.90| \{(i2,f1,j1)\}| 0.20  | \{(h2,g1)\}| 0.30 |
| \{(c2,b1)\}   | 0.70| \{(i2,f1,j2)\}| 0.60  | \{(h2,g2)\}| 0.22 |
| $Q_4$ in vars F,B | \{(i2,f2,j1)\}| 0.10  |     |     |
| \{(f1,b1)\}   | 0.40| \{(i2,f2,j2)\}| 0.30  |     |     |
| \{(f1,b2)\}   | 0.18|     |     |     |
| \{(f2,b1)\}   | 0.60|     |     |     |
| \{(f2,b2)\}   | 0.82|     |     |     |

of Sy\([14]\)) decomposed in belief functions $Bel_i$, $i=1,...,10$ such that

$$Bel = \bigoplus_{i=1}^{10} Bel_i$$

where the $m$ functions are given in Table 1.

To find the most plausible configuration explaining the observation that H equals $h_1$, F equals $f_2$, G equals $g_2$, J equals $j_2$ and A is either $a_1$ or $a_3$, we first have to identify the $Q$ functions values of all singleton sets appearing in belief functions $Bel_i$, $i=1,...,10$. The resulting $Q$-functions are listed in Table 2.

Then the genetic algorithm can be started, with e.g. variable clamped: H to $h_1$, F to $f_2$, G to $g_2$ and J to $j_2$, A restricted to \{a1,a3\}.

Final solution gave ObjectiveFunction equal 0.01100484 was reached for the chromosome (1,1,2,2,1,1,2,1,2,2) that is the most plausible configuration is:

$a_1,b_1,c_2,d_2,e_1,f_1,g_2,h_1,i_2,j_2$.

4 Concluding Remarks

Our genetic approach to solving the problem of finding the $k$ Most Plausible Explanation (MPE) of a given evidence $S_e$ within the general framework of Valuation Based Systems (VBS) consists generally in a special design of the target maximized function as:

$$f(x) = \prod_{i=1}^{n} P(x_i | Pa(x_i))$$
in probabilistic case and
\[ f(\mathbf{x}) = \prod_{i=1}^{n} Q_i \left( \mathbf{x}_{\downarrow \text{Space}(Q_i)} \right) \]
in case of DST, where \( P(x_i|Pa(x_i)) \) and \( Q_i \left( \mathbf{x}_{\downarrow \text{Space}(Q_i)} \right) \) are component valuation in a graphoidal decomposition of joint belief distribution, subject eventually to constraints that (1) the set of observed variables \( \mathbf{E} \) has clamped values or value sets (2) when we are looking for \( i \)th most probable/plausible solution, we punish \( f() \) so that \( f(\mathbf{x}) = 0 \) whenever \( \mathbf{x} \) is one of already found 1st, 2nd, \( (i-1) \)st most probable/plausible configurations. Mutation and cross-over operations are designed in such a way as to keep clamping of observed variables intact. We tested this approach against results presented in the literature e.g. for single connected networks of Sy [14].

The essential gain from using genetic algorithm approach, compared to other approaches, consists however in the possibility of finding \( k \) most probable/plausible explanations not only for singly connected networks [14, 3], or biparite graphs[15], or hypertrees, but also for multiply connected bayesian networks and for general hypergraph representations of joint belief distributions, both in probabilistic and DST case. The complex task of transforming bayesian network representation or general hypergraph representation into a hypertree or another singly connected network, advocated by some authors (e.g. [9, 14]), is not necessary.

Further research is needed to exploit the full potential of this technique. On the one hand the performance of the algorithm for lager networks, with 100 and more nodes needs to be tested. On the other hand an interesting question is the possibility of modifying cross-over and/or mutation operations to take advantage from (conditional) independence information contained in the graphoidal structures of decompositions.

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