Gravitational-Wave Implications for the Parity Symmetry of Gravity at GeV Scale

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Einstein’s general relativity, as the most successful theory of gravity, is one of the cornerstones of modern physics. However, the experimental tests for gravity in the high energy region are limited. The emerging gravitational-wave astronomy has opened an avenue for probing the fundamental properties of gravity in strong and dynamical field, and in particular, high energy regime. In this work, we focus on the parity symmetry of gravity. For broken parity, the left- and right-handed modes of gravitational waves would follow different equations of motion, dubbed as birefringence. We perform the first full Bayesian inference of the parity conservation of gravity by comparing the state-of-the-art waveform with the compact binary coalescence data released by LIGO and Virgo collaboration. We do not find any violations of general relativity, thus obtain the lower bound of the parity-violating energy scale to be 0.09 GeV through the velocity birefringence of gravitational waves. This provides the most stringent experimental test of gravitational parity symmetry up to date, and for the first time, in the high energy region, which ushers in a new era of using gravitational waves to test the ultraviolet behavior of gravity. We also find third-generation gravitational-wave detectors can enhance this bound to \(\mathcal{O}(10^2)\) GeV if there is still no violation, comparable to the current Large Hadron Collider (LHC) energy scale in particle physics.

Symmetry is an essential characteristic of the fundamental theories of modern physics and thus must be tested experimentally. In this work, we focus on testing the parity symmetry which indicates the invariance of physical laws under reversed spatial coordinates. It is well-known that parity is conserved for strong and electromagnetic interactions but is broken in the weak interaction as firstly confirmed by the beta-decay experiment in cobalt-60\(^{\text{[3]}}\). Gravitational parity is conserved in Einstein’s general relativity (GR). Nevertheless, various parity-violating gravity models, including Chern-Simons gravity\(^{\text{[4]}}\), ghost-free scalar-tensor gravity\(^{\text{[5]}}\), the symmetric teleparallel equivalence of GR theory\(^{\text{[6]}}\) and Hořava-Lifshitz gravity\(^{\text{[7]}}\) have been proposed for different motivations such as gravity quantization. In particular, in some fundamental theories of gravity, such as string theory and loop quantum gravity, the parity violation in the high energy regime is inevitable\(^{\text{[8]}}\). However, the observational evidence for gravity in the high energy scale is limited, which leaves the gravitational wave (GW) observations as a last resort \(^{\text{[9]}}\). In contrast to the tests of the Solar system or binary pulsars, GW reflects the wave behavior of the gravitational field. Therefore, the tiny deviation from GR, if exists, could be complemented and magnified due to the GW observation, which demonstrates the feasibility of probing the high energy behavior of gravity through GWs.

We first construct the generalized GW waveform generated by compact binary coalescence (CBC) with parity violation within the effective field theory (EFT) formalism. EFT provides a systematic framework to encode all kinds of modifications to an existing theory that could arise given certain new physics, thus simultaneously testing a range of modified gravity theories at once. To investigate the possible propagation effect due to parity violation, we consider the perturbation theory of gravitational field. EFT suggests that the leading-order modification to the GR linearization action comes from two terms with three derivatives, i.e., \(\epsilon^{ijk}h_{i\alpha}\partial_\alpha h_{k\beta}\) and \(\epsilon^{ijk}\partial^2h_{i\alpha}\partial_\alpha h_{k\beta}\) with \(\epsilon^{ijk}\) the antisymmetric symbol and \(h_{ij}\) the tensor perturbation of metric, \(\partial_j\) and \(a\) dot denote the derivatives with respect to spatial coordinates and time, respectively.\(^{\text{[10]}}\) \(\dot{h}_{ij}\) is the Laplacian, \(i,j,... = 1,2,3\) refer to spatial coordinate. Both terms are parity-violating. Dimensional analysis dictates that these new terms are each suppressed by an energy scale. We expect the two energy scales are of the same order, and denote collectively by \(M_{\text{PV}}\). Otherwise, if the two energy scales differ by orders of magnitude, only the term with lower energy scale dominates, thus we can neglect another term and our result for \(M_{\text{PV}}\) will not change (see Methods). This EFT with leading-order extensions to GR can be mapped to all the existing specific parity-violating modified gravity models in the market\(^{\text{[11]}}\). Also note that since parity violation emerges at the leading order modification, we expect the most stringent test on gravity from the propagation of GW, at least from the viewpoint of EFT, is on the gravitational parity symmetry.

As demonstrated by Ref.\(^{\text{[12]}}\), the modifications to GR-based GW waveform only arise from the propagation effect given the leading order parity violation modification discussed above, because the generation effect occurs on a radiation-reaction timescale much smaller than the GW time of flight and its impact on the evolution of the GW waveform is negligible. Given the parity-violating terms, the equation of motion for the GW circular polarization mode \(h_A\) in a Friedmann-Robertson-Walker (FRW) universe is

\[
h''_A + (2 + \nu_A)A h'_A + (1 + \mu_A)k^2 h_A = 0,
\]

where \(A = L\) or \(R\) stands for the left- and right-hand modes, \(H\) is the conformal Hubble parameter, \(k\) is the wave-number, a prime denotes the derivative with respect to the conformal cosmic time. Note that \(\mu_A = \nu_A = 0\) would reduce Eq. (1) to GR. Dimensional analysis indicates that both terms relate to the energy scale \(M_{\text{PV}}\) by \(\mu_A \propto \rho h_{\text{M}_{\text{PV}}}\) and \(\nu_A \propto \rho h_{\text{M}_{\text{PV}}}\) with \(pr = 1\) and \(pl = -1\). The broken parity leads to asymmetry between the left- and right-hand circular polarization modes of GW during propagation. In particular, the opposite sign of \(\mu_A\) (as well as \(\nu_A\)) for different modes leads to the birefringence effect of GWs, which is a characterized phenomenon for GW propagation in the parity-violating gravity. We find that the propagation of GWs can be affected in two ways. The term \(\mu_A\) modifies the conventional dispersion relation of GWs. As a result, the velocities of left- and right-hand circular polarization of GWs are different, dubbed as the velocity birefringence of GW\(^{\text{[13]}}\). On the other hand, the term \(\nu_A\) induces the different damping rates for two polarization modes when...
Theories, both effects exist. For each circular polarization mode, the
former effect exactly induces the phase modifications of the GW wave-
form, and the latter one induces the amplitude modifications. Con-
straints on modification of equation of motion with form Eq. (1) are
also obtained by LIGO and Virgo collaboration in Ref. [1], but only
parity conserving terms are considered. In contrast, our work focus
on the parity-violating effect. Note that, in EFT, the next-to-leading
order modification terms in the action, which are leading-order parity-
conserving terms, also follow the Eq. (1) with $\nu_\alpha \propto (k/M_{PV})^2$ and
$\mu_\alpha \propto (k/M_{PV})^2$ [1]. Therefore, their effects on the GW waveform
are much smaller than the parity-violating effects.

From the equation of motion, one can derive the parity-violating
GW waveform in the frequency domain, which is

$$h_{\alpha}^{PV}(f) = h_{A}^{GR}(f)(1 + \rho_\alpha \delta h) e^{i \Omega_\alpha \delta \Psi},$$

where $f$ is the frequency, and the amplitude modification $\delta h$ and
the phase modification $\delta \Psi$ are induced by $\nu_\alpha$ and $\mu_\alpha$, respectively. It
should also be noted that since $\delta \Psi$ is larger than $\delta h$ by about 20 or-
der of magnitude (see Methods), it is safe to only take into account the
contribution of $\delta \Psi$. But we also consider a special scenario where the
velocity birefringence is forbidden, i.e., $\delta \Psi = 0$ as is the case for, e.g.,
Chern-Simons gravity.

With the waveform in Eq. (3), we can perform a direct comparison
with the GW data using Bayesian inference to test the parity violation.
Up to date, LIGO and Virgo collaborations have released the data of
confident CBC events from the first and second observation run. The
catalog GWTC-1 include ten binary black hole (BBH) events and a
binary neutron star (BNS) event GW170817 [13]. Recently, a second
BNS event, GW190425, has been released [14]. We analyze the open
data of the twelve events with the inference module of the open-
source software PyCBC [15] developed for GW astronomy, which in turn
has dependency on LALSuite [16].

For all the GW events, we do not find any signatures of parity viola-
tion. We thus put the lower limit of $M_{PV}$ to be 0.09 GeV in the general
parity-violating gravity. This is tighter than the Solar system tests and
the binary pulsar observation by 17 orders of magnitude. The reason
for this dramatic improvement is that the velocity birefringence effect
in the modification of GW waveform is accumulated during the prop-
gagation of GW signals, and can be greatly amplified for distant GW
events. We note that this result has direct application on constraining a
range of specific parity-violating gravity models with velocity birefrin-
gence, including the ghost-free scalar-tensor gravity [3], the symmetric
teleparallel equivalence of GR theory [4] and Hořava-Lifshitz gravity [41].
The detailed correspondence between the above modified gravity mod-
els and the EFT formalism can be found in Ref. [17]. We have made
our inference results open to facilitate mapping the constraint to any
specific parity-violating gravity theories that one is interested in.

By similar analysis, the constraint by only considering the amplti-
dude birefringence modification is $M_{PV} > 1 \times 10^{-22}$ GeV. This result
is directly compared to and is consistent with Refs. [18,19] which
focuses on testing Chern-Simons gravity with GW. Compared to the
constraint from velocity birefringence, the loose result for amplitude
birefringence is because $\delta h$ is negligibly small compared to $\delta \Psi$ and the
GW detection is less sensitive to amplitude modification than phase.

Our results of the constraint on $M_{PV}$ are shown in Fig. 1 where
we have combined the results from the twelve CBC events to give an
overall constraint. Other existing constraints including the Solar system
tests and the binary pulsar observation are also plotted for comparison.

With the continuing sensitivity upgrade during the advanced LIGO
and Virgo runs, the future GW astronomy is even more powerful to
test the parity symmetry of gravity. The KAGRA detector has joined
the global network very recently. The advanced LIGO and Virgo de-
tectors are expected to achieve the design sensitivity in a few years [20].

The third generation ground-based GW detectors, including the Ein-
stein Telescope and the Cosmic Explorer, are under projection currently
[21]. We investigate the ability of future GW astronomy to constrain the
lower limit of $M_{PV}$ by simulations. With one-year observation time,
$M_{PV}$ is expected to be constrained to be no less than $O(1)$ GeV with the
second generation detectors with designed sensitivity and $O(10^2)$
GeV with the projected third generation detectors. This is compar-
able with the existing high energy experiments in particle physics from
LHC. These results indicate that, through GW observations, for the first
time it becomes feasible to test the ultraviolet behavior of gravity in the
high energy region experimentally.

We note that other constraints on the violation of parity symme-
try of gravity have also been derived in various analysis using GW or
non-GW observations. In the following, we briefly summarize these
results to describe the status-of-the-art of the constraint on $M_{PV}$. The
results are also presented in Fig. 1 as a comparison of this work. Com-
pared to the following constraints which also use GW, one major dif-
fERENCE is that our work makes use of the inspiral-merger-ringdown
parity-violating GW waveform (Eq. (2)) which represents the maxi-
mum information that can be extracted from the signal. In addition, the
Bayesian nature of our method allows for combining the results from a
catalog of events.

Waveform-independent constraint from GWs: Our previous work
Ref. [19] develops a waveform-independent way to decompose the left-
and right-hand polarization modes from the observed GW data if the
sky position of the event can be fixed by the observation of its elec-
tromagnetic counterparts. From the frequency-time representations we
can read out the arrival times of GW in each frequency band. Ac-
cording to the velocity birefringence effect, for a specific frequency
f, the arrival time difference between the two modes is $|t_{L} - t_{R}| = (2\pi f/M_{PV})^2 \int_{t_{0}}^{t_{f}} a^{-2}dt$ [19], where $a$ is the scale factor of the Universe,
t0 and tf are the cosmic time of the arrival and the emission of the GW
event respectively. This formula gives a direct relation between the ar-
ival time difference and the energy scale of parity violation $M_{PV}$. Ap-
plying to the D3100 data, we find that if a nearly edge-on BNS at 40 Mpc
is detected by the second-generation detector network consisting of ad-
vanced LIGO, advanced Virgo, KAGRA, and LIGO-India, one could

![Figure 1](image-url)
derive the waveform-independent bound $M_{\text{PV}} > 1.4 \times 10^4$ eV. At this writing, the BNS signal GW170817 is the unique GW event with observed electromagnetic counterparts. However, this event is nearly face-on with the inclination angle $\iota > 152^\circ$ [31], thus the right-hand mode of this event is completely dominant. Therefore, the difference of the arrival time between the two modes cannot be determined from GW170817.

**Constraint from GW speed:** The velocity birefringence effect of GW can be constrained from the speed of GW, which in turn can be obtained by comparing the arrival time of GW signal and that of the electromagnetic signal. For the event GW170817/GRB170817A, the arrival time difference between GW and gamma-ray burst is 1.7 seconds. Assuming the difference of their emission time is less than 10 seconds, the speed of the right-hand circular mode of GW is in the range $-7 \times 10^{-16} < 1 - v_R < 3 \times 10^{-15}$ [32]. According to the relation $|v_R - 1| = \pi f/M_{\text{PV}}$ [33], one obtains the constraint $M_{\text{PV}} \gtrsim 10^4$ eV. This is consistent with the result in Ref. [33] where the authors focus on the ghost-free scalar-tensor gravity models with parity violation. Bounds on GW birefringence were also obtained in Ref. [34] with GW150914 for testing local Lorentz invariance by identifying the speed difference between two polarization modes. We note that our EFT formulation can be mapped to the formulation proposed in Ref. [35], more detailed analysis is beyond the scope of this work and thus leave as a future work.

**Constraint from Solar system tests:** In the parity-violating gravity, an important feature is that it leads to a change of frame-dragging effects around rotating objects, which can be used to test the theory. Focusing on the Chern-Simons gravity, Ref. [36] calculates the linearized metric of the spacetime around a non-relativistic, constant-density spinning body. It is found that the gravitomagnetic field in the parity-violating gravity differs from that in GR, which induces the modifications in the precession of orbits of gyroscopes moving in this spacetime. Using the measurement of Lense-Thirring drag around the Earth by the LAGEOS satellites, Ref. [36] sets the constraint on the characteristic Chern-Simons length scale, $k_{\text{CS}}^{-1} \lesssim 1000$ km, which corresponds to $M_{\text{PV}} \gtrsim 2 \times 10^{-13}$ eV.

**Constraint from binary pulsar:** In the parity-violating gravity, the theory selects a preferred direction in spacetime that corrects the precession of the orbital plane. Thus, observations of gravitomagnetic precession can be used to test the validity of the theory. In Refs. [37,38], the authors focus on the Chern-Simons modified gravity with a time-like Chern-Simons coupling scalar field, and calculate the leading-order correction to the post-Keplerian parameters of binary systems. They find that the precession of the periastron is corrected by the parity-violating term. Using the measurements of the rate of periastron precession in the double pulsar system PSR J0737-3039 A/B, they obtain a constraint of $k_{\text{CS}}^{-1} \lesssim 0.4$ km, which corresponds to $M_{\text{PV}} \gtrsim 5 \times 10^{-10}$ eV. Note that the constraints of $M_{\text{PV}}$ from the measurements of the Solar system and binary pulsar are based on the effect of frame-dragging modifications in the theory, which is the local effect rather than the propagation effect. Since the frame-dragging modification around rotating objects is a common feature for all parity-violating gravity models, although these constraints are derived for a specific parity-violating theory, i.e. Chern-Simons gravity, we expect that the conclusions are applicable for other theories of parity-violating gravity.

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**Methods**

In what follows we present our methods for inferring the constraints on parity violation in gravity from GW measurements. We first introduce the construction of the parity-violating GW waveform in the EFT framework, then discuss the Bayesian inference for obtaining the constraints and forecast the constraints with future GW astronomy.

**GW waveform with parity violation** In this part we follow Refs. [39,40] to briefly review the derivation of the GW waveform with parity-violation in the EFT framework. In the FRW universe, choosing the unitary gauge, the equation of motion of GW is determined by the following quadratic action [39]

$$S = \frac{1}{16\pi G} \int dt d^3 x \left[ \frac{1}{4} h_{ij}^2 - \frac{1}{4a^2} (\partial_i h_{jk})^2 + \frac{1}{4} \left( \frac{c_1}{a M_{\text{PV}}} \epsilon^{ijk} \partial_i h_{jk} + \frac{c_2}{a^2 M_{\text{PV}}} \epsilon^{ijk} \partial^2 \partial_j h_{kl} \right) \right],$$

(3)

where the last two terms with three derivatives correspond to the contribution from parity violation. $c_1$ and $c_2$ are dimensionless coefficients, which are functions respective to cosmic time in general. Note that, in EFT, the leading-order modifications from parity-conserving terms in action are suppressed by $M_{\text{PV}}^0$ or $M_{\text{PV}}^1$, which indicates the much looser constraint (or no constraint) of $M_{\text{PV}}$ by GW observing these terms in [39,40]. From the action, we can derive the equation of motion of GW in the vacuum as shown in Eq. (1). The exact forms for $\mu_A$ and $\nu_A$ are [39]

$$\nu_A = \left( \rho_A \Omega_\nu \tau (k/a M_{\text{PV}})^{-1}/H, \right.$$

$$\mu_A = \rho_A \alpha_\nu \tau (k/a M_{\text{PV}})^{-1},$$

(4)

where $\tau$ is the cosmic conformal time. The functions $\alpha_\nu \equiv -c_1$ and $\alpha_\mu \equiv c_1 - c_2$ are two arbitrary functions of time which can only be determined given a specific model of modified gravity. For the GW events at the local Universe, these two functions can be approximately treated as constant, i.e. ignoring their time-dependence. Note that we consider $\alpha_\nu$ and $\alpha_\mu$ to be $\sim O(1)$ by absorbing the order of magnitude into $M_{\text{PV}}$. But a special case when $\alpha_\nu = 0$ (i.e., $c_1 = c_2 = 0$) for Chern-Simons gravity is also considered, the corresponding result is plotted in Fig. [41] as amplitude birefringence.

The explicit parity-violating GW waveform can be derived from solving the equation of motion, as schematically shown by Eq. (2). The amplitude and phase modifications to the GR-based waveform due to birefringence take the following parametrized form

$$\delta h(f) = -A_v \pi f, \quad \delta \phi(f) = A_\mu (\pi f)^2/H_0,$$

(5)

where $H_0$ is the Hubble constant. The coefficients $A_v$ and $A_\mu$ are given by

$$A_v = M_{\text{PV}}^2 \left( \alpha_\nu (z = 0) - \alpha_\nu (z = 1) \right),$$

$$A_\mu = M_{\text{PV}}^2 \int_0^{\pi f} \alpha_\mu(z)(1 + z^2) dz' \sqrt{1 + \Omega_\mu (1 + z^2)^3 + \Omega_A},$$

(6)
where $z$ is the redshift of the GW event. In analysis we adopt a Planck cosmology ($\Omega_{M} = 0.315, \Omega_{\Lambda} = 0.685, H_0 = 67.4 \text{ km/s/Mpc}$). We also convert the left- and right-hand GW polarization modes into the plus and cross modes which are used more often in GW data analysis. The relation between the parity-violating modes and the GR modes is
\[
\begin{align*}
    h_{+}^{\nu}(f) &= h_{x}^{GR}(f) - h_{y}^{GR}(f)(\delta\delta h - \delta\psi), \\
    h_{x}^{\nu}(f) &= h_{x}^{GR}(f) + h_{y}^{GR}(f)(\delta\delta h - \delta\psi).
\end{align*}
\]
(7)
The above expression represents the waveform we employ to compare with the GW data. Let us assume the GW is emitted at the redshift $z \sim O(0.1)$, and $\alpha_v$ and $\alpha_a$ are expected to be the same order constant. We find the ratio $\delta\delta h/\psi \sim \pi f / H_0 \gtrsim 10^{20}$, where $f \sim 100$ Hz for the ground-based GW detectors. Therefore, in the general parity-violating gravity, except for the case with only amplitude birefringence (e.g. Chern-Simons modified gravity), the corrections of GW waveform $h_{+}^{\nu}(f)$ mainly come from the contribution of velocity birefringence rather than that of amplitude birefringence.

**Bayesian inference for GW events** Bayesian inference framework is broadly employed in GW astronomy for estimating the source parameters and selecting the preferred model from observation. Given the data $d$ of GW signal and a waveform model $H$, Bayes theorem claims
\[
P(\theta | d, H, I) = \frac{P(d | \theta, H, I) P(\theta | H, I)}{P(d | H, I)},
\]
(8)
where $\theta$ are the parameters characterizing $H$, $I$ is any other background information. $P(\theta | H, I)$ is the prior distribution for $\theta$ and $P(d | \theta, H, I)$ is the likelihood for obtaining the data given a specific set of model parameters. The posterior $P(\theta | d, H, I)$ contains all the information about the results of parameter estimation.

For the Gaussian and stationary noise from GW detectors, the likelihood function is
\[
P(d | \tilde{H}, H, I) \propto \exp \left[ -\frac{1}{2} \sum_i (d_i - h_i(\tilde{H}))^2 \right],
\]
where $h_i(\tilde{H})$ is the GW waveform template in model $H$, and $i$ represents the $i$-th GW detector. The inner product $\langle a | b \rangle$ is defined to be
\[
\langle a | b \rangle = 4\Re \int \frac{a(f)}{S_n(f)} \frac{b^*(f)}{S_h(f)} df,
\]
(10)
where $S_n(f)$ is the one-side noise power spectral density (PSD) of the GW detector.

To select the model favored by observation, normalizing both sides of Eq. [5] and we can obtain the Bayes evidence
\[
P(d | H, I) = \int d\tilde{H} P(d | \tilde{H}, H, I) P(\tilde{H} | H, I).
\]
(11)
Bayesian ratio is defined as the ratio of evidence between two competitive models $H_1$ and $H_2$ which are GR and parity-violating gravity within this work,
\[
B_2^1 = \frac{P(d | H_2, I)}{P(d | H_2, I)}.
\]
(12)
The odds ratio between model $H_1$ and model $H_2$ can be expressed by
\[
O_2^1 = \frac{P(H_1 | d, I)}{P(H_2 | d, I)} = \frac{P(d | H_1, I) P(H_1 | I)}{P(d | H_2, I) P(H_2 | I)} = B_2^1 \frac{P(H_1 | I)}{P(H_2 | I)}.
\]
(13)
If the competitive models are assumed to be equally likely before any measurement, then odds ratio is equal to Bayesian ratio. Odds ratio quantitatively reflects the preference of data for competitive models.

We employ the open-source software PyCBC with the open data from [1] to perform the Bayesian inference. For the GR waveform $h^{GR}(f)$, we use the spin precessing waveform IMRPhenomPv2 [14, 15] when analyzing BBH events and spin aligned waveform with tidal deformaibility added IMRPhenomNRTidal [16] for BNS events. The parity-violating waveform is constructed based on the above template through Eq. [7]. We perform parameter estimation by selecting $16s$ data for BBH and $200s$ data for BNS events to account for the relatively long signal. The data is sampled at $2048$ Hz and the likelihood is evaluated between $20$ Hz and $1024$ Hz. The PSD is generated from $1000s$ data using the median estimation with $8s$ Hann-windowed segments and overlapped by $4s$. The prior is chosen to be consistent with that of Ref. [1] and uniformly distributed for $\alpha_v$ and $\alpha_a$. The posterior distribution is sampled by the nest sampling algorithm dynesty [3] over the fiducial BBH and BNS source parameters plus the parity-violating parameters $\alpha_v$ for velocity birefringence or $A_\mu$ for amplitude birefringence. Also note that, since we do not find any violation of GR and aim for putting a lower limit to $M_{PV}$, we ignore the calibration error from GW detectors, thus our lower limit should be interpreted as a conservative result for constraining $M_{PV}$. All the settings and parameter estimation results can be found in our released posterior files.

Fig. 2 shows the marginalized posterior distribution for $A_\mu$ for velocity birefringence. From the figure, we can observe that the GR value $A_\mu = 0$ is within the $90\%$ confidence level for every event. We also report that the natural logarithm of the Bayes ratio between GR and the parity-violating gravity is distributed in the range $[1.6, 5.8]$ for all the events, confirming no parity violation for gravity.

The relatively low-mass BBH events, such as GW151226, GW170608, and the two BNS events give tighter constraints on $A_\mu$. This is because the velocity birefringence contribution corresponds to a $5.5$ post-Newtonian (PN) order modification to the GR waveform which has larger impact on higher frequency, in line with the expectation that the low-mass events with higher cut-off frequency and longer signal yield better constraints.
respectively, and constrain by the third generation detectors, and the constraint with $200$ events is $6.09$ GeV. LIGO and Virgo [2] it is expected that there are $0(10^5)$ BBH coalescence events within $5000$ Mpc in one year. Therefore, assuming the constraint on $M_{PV}$ is inversely proportional to the square root of event number, the resultant constraint can reach $0(10^2)$ GeV with a one-year observation with the third generation detectors. This demonstrates the promising future of GW astronomy to probe the ultraviolet property of gravity in the high energy region, which could shed light on deviations from GR, if exists, arising from the $100$ GeV region.

Testing parity of gravity in the high energy region with future GW detectors We have shown that the current GW detection is capable of probing the sub GeV parity-violating energy scale. With the continuing upgrade for GW detectors, we also forecast the ability of GW astronomy to constrain $M_{PV}$. We consider four sets of detector configurations based on technologies currently available or under investigation, and simulate $200$ BBH events from GR for each set. For the first set, we choose the advanced LIGO, advanced Virgo and KAGRA network, all running with designed sensitivity. The second and third sets substitute the two LIGO detectors with the $2.5$ generation detector $A+$ and the Voyager configuration, respectively. The last set uses the third generation detectors including the Einstein Telescope and Cosmic Explorer. As the number of detections increasing, the constraints for the lower limit of $M_{PV}$ becomes tighter. In particular, the third generation detector can detect all the BBH coalescence signals within $5000$ Mpc and can constrain $M_{PV} > 0(10)$ GeV with $200$ events. With a one-year observation run, the third generation detectors are expected to improve the constraint to $0(10^2)$ GeV.

Data availability The posterior files for velocity and amplitude birefringence of the twelve GW events are released in https://yi-fan-wang.github.io/ParityWithGW/

Code availability This work used the open-source software PyCBC which is available from https://github.com/gwastro/pycbc.

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Figure 3 | The posterior distributions for $M_{PV}^{-1}$. The inference results for the parity-violating energy scale for velocity birefringence are plotted. The results from individual GW event and the combination of all the events are considered. The abscissa value of the “x” marker represents the 90% upper limit for $M_{PV}^{-1}$, or equivalently, the 90% lower limit for $M_{PV}$, which is $0.09$ GeV.

Figure 4 | The projected constraints for $M_{PV}$ for future GW detectors. Using $200$ BBH signals from GR, the constraints for the lower limit of the parity-violating energy scale for velocity birefringence are plotted. We consider four sets of global GW detectors network which are (1) the second generation detectors including advanced LIGO, advanced Virgo, and KAGRA with design sensitivity (2) the $2.5$ generation detector $A+$ (3) the $2.5$ generation detector Voyager (4) the third generation detector with the Einstein Telescope and Cosmic Explorer. As the number of detections increasing, the constraints for the lower limit of $M_{PV}$ becomes tighter. In particular, the third generation detector can detect all the BBH coalescence signals within $5000$ Mpc and can constrain $M_{PV} > 0(10)$ GeV with $200$ events. With a one-year observation run, the third generation detectors are expected to improve the constraint to $0(10^2)$ GeV.

\[
p(M_{PV} | \{d_i\}, H, I) \propto \prod_{i=1}^{N} p(M_{PV} | d_i, H, I),
\]

where $d_i$ denotes the $i$-th GW event. The combined result show that the 90% lower limit for $M_{PV}$ is $0.09$ GeV. This is the first observational evidence of gravitational parity conservation from dynamical and strong-field observation and the tightest constraint on $M_{PV}$ up to date.

![Image](https://example.com/image3.png)
