Theoretical analysis of cylindrical roller bearing with flexible rings mounted in groove elastic support

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Abstract
Based on the structural mechanics and rolling bearing dynamics, this paper presented a dynamics model of cylindrical roller bearing with flexible outer ring and a novel elastic support named groove elastic support (GES), and then the load distribution and cage slip ratio of cylindrical roller bearing with GES (BES) were analyzed. The findings are as following: (1) The number of loaded rollers of BES is more than that of cylindrical roller bearing with rigid support (BRS), and the largest contact load between roller and ring of BES is lower than that of BRS. (2) The number of loaded rollers of BES decreases with the groove number, arc beam thickness and outer ring’s thickness. The smaller radial load, ranging from 500N to 1400N in this paper, has no obvious effect on the number of loaded roller for BES and BRS; (3) Under the same condition, cage slip ratio of BES is much lower than that of BRS. Cage slip ratio of BES increases with the groove number, arc beam thickness and outer ring’s thickness. All results reveal that the GES has a large impact on the load distribution and cage slip ratio in cylindrical roller bearing, and will be beneficial to extend the bearing service life under the condition of high-speed and light-load.

Keywords: Cylindrical roller bearing, Groove elastic support, Flexible outer ring, Load distribution, Cage slip ratio

Nomenclature

| Symbol | Description |
|--------|-------------|
| Q_j   | contact force of roller j; |
| T_j   | traction force between roller j and raceways; |
| M_Qj  | additional moment due to Q_j; |
| M_Tj  | additional moment due to T_j; |
| m     | mass of each components; |
| D_w   | roller diameter; |
| D_m   | pitch diameter of bearing; |
| Omega | rotation angular velocity of the jth roller; |
| Omega_m | rotate speed of cage; |
| RN    | roller number; |
| P_d   | radial clearance of cylindrical roller bearing; |
| Omega_i | angular velocity of inner ring; |
| phi_j | azimuth angle of the jth roller; |
| F_r   | radial load; |
| phi_p | azimuth angle of point in outer ring; |
| delta | outer ring’s out-of-roundness at the phi_p; |

| Symbol | Description |
|--------|-------------|
| GES   | groove elastic support; |
| D_i   | inner diameter of GES; |
| D_o   | outer diameter of GES; |
| n     | arc beam number or groove number of GES; |
| H     | arc beam thickness of GES; |
| H'    | groove gap of GES; |
| alpha | transition angular of GES; |
| L     | width of GES; |
| delta | radial displacement of GES center; |
| H_o   | thickness of outer ring; |

Superscripts /Subscripts

i : inner ring;  
o : outer ring;  
c : cage;  
r : roller.
1. Introduction

As a crucial support part in aero-engine rotor, cylindrical roller bearing usually works under the condition of high-speed and light-load. At present, in order to reduce the critical speed and realize the rotor to operate over its critical speed safely, the elastic support with lower stiffness is usually used in the rotor system combined with cylindrical roller bearing. However, whether the elastic support is contributed to the performances of cylindrical roller bearing, it has not been paid much attention, right now.

Harris and Kotzalas(2006) pointed out that when bearing is mounted in the elastic support, the flexible deformation of ring and elastic support would evidently affect the load distribution and dynamic performance of bearing. Jones and Harris(1963) adopted classic elastic mechanics to calculate outer ring deformation of rolling idler gear bearing and studied the effect of outer ring flexibility on the fatigue life based on rolling bearing static theory. Yao, Chi and Huang(2012) used the equivalent rigid elements and curved Timoshenko beam elements to build the flexibility and three-dimensional multi-body model of rolling bearing rings, and then analyzed the natural frequencies, deformation of outer ring and the load distribution. Based on elastic mechanics and rolling bearing dynamics, Chen, Liu and Cheng(2013) established angular contact ball bearing dynamic stiffness model which considered the deformation of inner ring fitted on the hollow axis, and analyzed the effect of radial deflection of inner ring on bearing’s dynamic stiffness. The simulation and test results indicated that the radial deflection of inner ring is directly proportional to bearing axial load and inversely proportional to rotation speed. Cheng(1996) built the finite element method(FEM) model of bearing and housing, and found that the fit clearance between bearing and house has obvious effect on the load distribution of bearing and the rational design of house as an effective measure can improve the life of bearing. The models of shaft, bearing inner ring, outer ring and pillow block were built by FEM, which used nonlinear spring element to represent bearing ball(Tibbits,2004). This study found that the maximum load acted on the ball decreased due to a bearing installed in a pillow block, and the life increased. Ni, Liu and Deng (2010) and Shi et al. (2010) respectively investigated the axial stiffness of thin-wall angular contact ball bearing by FEM and found that the outer ring deflection, due to clearance fit between bearing and housing, has a great effect on the axial stiffness of bearing and the stiffness is much closer to the test result than that of the model which only considers the Hertz deformation between roller and ring of bearing. To reduce the calculation time of FEM model, Cavallaro, Nelia and Bon(2005) proposed an efficient method which is based on the Roark formula(Young et al. 2002), FEM and bearing quasi-dynamics to analyze the circular degree of outer ring, and then discussed the heat dissipation, load distribution, and the contact pressure in radial cylindrical roller bearing. The results show that the internal load distributions were significantly affected by the flexibility of housing and bearing ring, which induce an extension of the loading zone and a lower contact pressure. Based on Cavallaro’s research, Defaye(2008) built a global model of cylindrical roller bearing with flexible rings mounted in a squeeze film damper (SFD) to accurately calculate the deformation of outer ring, and he found that a SFD generates an asymmetry of the roller load distribution with respect to the roller bearing’s radial load. Considering the uneven stiffness of the rings, Olave et al. (2010) proposed a procedure to obtain the load distribution in a four contact point slewing bearing, which result has a very good correlation with that obtained in ABAQUS. Shu et al. (2015) and Mao et al. (2016,2018) respectively presented a thin-wall, integrated, squirrel-cage flexible support ball and cylindrical bearing, quasi-dynamic iterative FEM model to calculate the structural deformation of the thin-wall rings and support structures and analyze the static and dynamic characteristics of roller bearing. The results show that the load distribution of roller bearing on the squirrel-cage flexible support is more uniform than that on rigid support. Liu, Wu and Shao(2017) presented a finite element model of roller bearing and found the raceway thickness has a great influence on the stress and the contact force, but the paper did not clearly state support form of the roller bearing.

The above-mentioned works have focused on the load distribution and fatigue life of cylindrical roller bearing with flexible outer ring or housing based on the static theory of rolling bearing, and did not involve the ring’s integral deformation and elastic support at the condition of bearing dynamics theory. Considering load distribution and contact stress relate to roller bearing fatigue life(Fujiwara and Yamauchi (2010), Aschenbrenner et al. (2020), Zheng et al. (2019)), as well, cage slip ratio is as one of the most important indicator of roller bearing’s dynamic characteristics(Deng et al. (2018), Hou and Wang(2020),) and directly affected bearing life (Zhan et al. (2020)), this paper mainly analyzed and compared the contact load and cage slip of roller bearing with groove elastic support(GES). GES, as a new-type elastic support with a smaller volume than squirrel-cage elastic support, can reduce size and weight of aero-engine rotor system. Based on mechanics of material theory, Sun et al.(2017) gave the analytical formulas of GES’s radial stiffness, and found
GES has the homogeneous radial stiffness which is vital to improve rotation accuracy and reliability of aero-engine rotor. Subsequently, based on the dynamic analysis of rolling bearings and assuming rigid ring of roller bearing, Sun et al. (2018) established the dynamic differential equations for high-speed cylindrical roller bearings with GES, and found GES can reduce radial vibration effectively. To further investigate the characteristics of roller bearing with GES, especially considering the deformation of outer ring of cylindrical roller bearing with GES(BES), this paper proposed an analysis method of BES. Firstly, based on the structural mechanics and rolling bearing dynamics, the dynamic differential equations of BES were presented, which involves the flexibility of bearing’s outer ring and GES. And then, the impact of structural parameters of GES, bearing’s outer ring, working conditions on the load distribution and cage slip ratio were studied. All the findings in this paper can provide a theoretical basis for the designing of cylindrical roller bearing with the GES.

2. Dynamics model of BES with flexible outer ring

The GES is an atypical elastic support, which is mounted between housing and outer ring with an interference fit. The schematic model of cylindrical roller bearing with GES are shown in Fig.1.

2.1 Governing equation Coupled model for flexible outer ring-groove elastic support

Under the radial load, outer ring of bearing with elastic support would have elastic deformation. So, the interference $\delta_o(\epsilon_p)$ between outer ring and GES would occur at different location (in Fig.2), which is the out-of-roundness of outer ring. Because of flexibility and interaction force between outer ring and the GES, that would reach to a new balance status. To obtain the new balance status, a coupled model should be established for flexibility of outer ring and GES. Based on the paper by Nelias(2008), the coupled model was established to determine a local equilibrium of deformation between the outer ring and GES, which is a function of their respective stiffness. This local equilibrium at the $\epsilon_p$ angle is depicted in Fig.2. And then, the new out-of-roundness of outer ring $\delta'_o(\epsilon_p)$ at the $\epsilon_p$ angle can be written as:

$$\delta'_o(\epsilon_p) = \frac{K_o(\epsilon_p)\delta_o(\epsilon_p)}{K_o(\epsilon_p) + K_{GES}(\epsilon_p)}$$

(1)

In Eq.(1), $K_{GES}(\epsilon_p)$ and $K_o(\epsilon_p)$ are the radial stiffness of GES and outer ring at the $\epsilon_p$ angle respectively, the detailed expresses of $K_{GES}(\epsilon_p)$ and $K_o(\epsilon_p)$ were shown in Section.2.1.1 and Section.2.1.2. According to the paper by Nelias(2008), $K_{GES}(\epsilon_p)$ and $K_o(\epsilon_p)$ are assumed as the linear stiffness.
2.1.1 Radial stiffness of GES

Figure 3 is the structure of GES and deformation of an arc beam.

In Fig.3(c), A point and B point are the two ends of the \( p \)th arc beam; \( \varepsilon_p \) is the azimuth angle of the \( p \)th arc beam; the initial position of coordinate system of GES center coincides with bearing center \( \{O;X,Y,Z\} \), and GES center does not rotate but just translate in YOZ plane; \( \gamma \) is the angle between GES center and \( \{O;X,Y,Z\} \); \( \Delta(\varepsilon_p) \) is the radial displacement of the \( p \)th arc beam end B at azimuth of \( \varepsilon_p \) which is vector sum of \( \delta'(\varepsilon_p) \) and \( \delta \).

According to the structural mechanics, the reacted forces \( F_y \) and \( F_z \) between GES and outer ring are shown in Eq.(2). And the radial stiffness of GES at the azimuth of \( \varepsilon_p \) is shown in Eq.(3).

\[
\begin{align*}
F_y &= -\sum_{p=1}^{n} F_{yp} \\
F_z &= -\sum_{p=1}^{n} F_{zp} \\
K_{GES}(\varepsilon_p) &= \frac{F_z \sin \varepsilon_p - F_y \cos \varepsilon_p}{\Delta(\varepsilon_p)}
\end{align*}
\]  

In Eq.(2) and Eq.(3), the detailed derivation procedures of Eq.(2) and Eq.(3) are in the Appendix A.
2.1.2 Radial stiffness of flexible outer ring

Due to the flexibility of outer ring, outer ring can be regarded as a thin ring. And then, the structural deformation of outer ring caused by the contact forces between rollers and outer ring was shown in Fig.4.

![Fig.4 Structural deformation of outer ring](image)

The out-of-roundness $\delta_o(\varepsilon)$ of thin outer ring caused by the $RN$ equidistant loads $Q_j$ can be expressed as Eq.(4) (Liu and Chiu, 1974).

$$\delta_o(\varepsilon) = \frac{R^3}{4\pi E_o I_o} \sum_{j=1}^{RN} Q_j f_o(\phi_j)$$ (4)

Where $R$ is the radius of bearing outer ring; $E_o$ is material elastic modulus; $I_o$ is the inertia moment of cross-section; $\psi_j = \phi_j - \varepsilon$; $f_o(\psi_j) = -\psi_j \sin \psi_j + (\phi_j / 2 - \pi \psi_j + \pi) \cos \psi_j - 2$ (Liu and Chiu, 1974); $\phi_j$ is the azimuth of $Q_j$.

According to the paper by Nelias (2008), the local radial stiffness of outer ring at the azimuth of $\varepsilon$ can be deduced from Eq.(4).

$$K_o(\varepsilon) = K_o(\psi_j) = \frac{Q_j}{\delta_o(\phi_j)} = \frac{Q_j}{\frac{R^3}{4\pi E_o I_o} \sum_{j=1}^{RN} Q_j f_o(\phi_j)}$$ (5)

As $\psi_j < \varepsilon < \psi_{j+1}$, the radial stiffness at $\varepsilon$ is calculated by linear interpolation method.

2.2 Dynamic models of cylindrical roller bearing with GES

Because the forces of roller, cage and inner ring are same as the paper by Cui et al. (2018), this part only gives the outer ring dynamic models and differential equations and the others refer to the paper by Cui et al. (2018).

When bearing is working, outer ring is simultaneously acted by the normal forces $Q_j$, traction forces $T_j$, addition moments $M_j$ of rollers, cage guiding force and the supporting force of GES. The forces acted on outer ring are shown in Fig.5. However, in practical, outer ring only have a displacement in radial plane but no rotation, so that the addition moments are overlooked.

![Fig.5 Schematic diagram of outer ring forces](image)
The dynamic differential equations of roller can be written in Eq.(6).

\[
\begin{align*}
\ddot{y}_o &= \sum_{j=1}^{\text{EN}} \left( -Q_j^o \cos \varphi_j - T_j^o \sin \varphi_j \right) - F_{y,\text{c}}^o \cos \varphi_c - F_{z,\text{c}}^o \sin \varphi_c + F_y \\
\ddot{z}_o &= \sum_{j=1}^{\text{EN}} \left( Q_j^o \sin \varphi_j - T_j^o \cos \varphi_j \right) + F_{y,\text{c}}^o \sin \varphi_c - F_{z,\text{c}}^o \cos \varphi_c + F_z
\end{align*}
\]

(6)

In Eq.(6), \( \ddot{y}_o, \ddot{z}_o \) are the displacement accelerations of outer ring in \( \{O;X,Y,Z\} \); \( F_y,F_z \) are the forces of GES, the detailed expresses shown in Eq.(2); \( \varphi_c \) is the included angle between \( \{o;\text{x}_o,\text{y}_o,\text{z}_o\} \) and \( \{o;\text{x}_c,\text{y}_c,\text{z}_c\} \).

2.3 Solution procedure of dynamics model

The dynamics differential equations of cylindrical roller bearing with GES considering outer ring’s flexibility are solved by GSTIFF (Gear Stiff) integer algorithm with variable step (Zhang et al. 2016). The solution procedure is shown in Fig.6.

Fig.6 Solution procedure of model between roller bearing and couple model

(1) The basic parameters are inputted. The contact forces between rollers and outer ring and the radial stiffness of groove elastic support in Eq.(3) are obtained through solving dynamics equations in which the outer ring is treated as rigid ring.

(2) The contact forces used as the initial values are substituted into Eq.(5) to calculate the out-of-roundness of the flexible outer ring.

(3) Based on the coupled model of groove elastic support and flexible ring in Eq.(1), a new out-of-roundness of outer ring can be gotten under a new equilibrium status.

(4) The new out-of-roundness of outer ring is re-injected in the cylindrical roller bearing dynamic differential equations, which are solved by GSTIFF integer algorithm with variable step, and then the new contact forces are obtained.

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If contact forces meet the convergence error, the iteration is completed, and then the circular deformation of outer ring, load distribution and cage slip ratio of BES are obtained. Otherwise, the program returns to Step(2).

3. Results and Discussion

Given that the flexibility of outer ring and GES, the impact of structural parameters and working conditions on the load distribution and cage slip ratio was analyzed. The main parameters are shown in Table 1.

| Item                                | Value |
|-------------------------------------|-------|
| Elastic support inner diameter      |       |
| \(D_i/\text{mm}\)                   | 140   |
| Groove width \(H'/\text{mm}\)       | 0.5   |
| Arc beam thickness \(H/\text{mm}\)   | 1.0   |
| Translation angular \(a'^\circ\)     | 2     |
| Elastic support width \(L/\text{mm}\) | 19    |
| Bearing outer diameter \(d_o/\text{mm}\) | 140  |
| Bearing inner diameter \(d_i/\text{mm}\) | 110  |
| Roller diameter \(D_w/\text{mm}\)    | 8     |
| Roller length \(l/\text{mm}\)       | 10    |
| Roller number \(RN\)                | 34    |

3.1 Load distribution of cylindrical roller bearing with GES
3.1.1 Impact of structural parameters on load distribution

The impact of structural parameters on load distribution were shown in Fig.7, Fig.8 and Fig.9, respectively. The azimuth of the largest contact force is set to 0°. Considering the load-carrying capacity of the support, the least groove number \(n\) is 18.

In Fig.7, assuming that bearing speed is 6000r/min, the number of loaded rollers increases from 7 to 13 with groove number \(n\) from 42 to 18. When the radial loads are 500N, 1000N and 1500N, the maximum contact loads of BES are decreased by 55.6%, 48.9% and 44.2% respectively compared with that of BRS. This means that the number of loaded rollers increases with the groove number \(n\), and the GES with less groove number \(n\) is beneficial to reduce the maximum contact force of cylindrical roller bearing.
In Fig.8, assuming that bearing speed is 6000r/min, and the radial loads acted on inner ring is 500N, the number of loaded rollers increases from 7 to 13 with the decreasing of arc beam thickness \( H \), and when the groove numbers \( n \) are 18, 30 and 42, the maximum roller contact forces of BES are decreased by 55.8\%, 45.2\% and 30.2\% respectively compared with that of BRS. This means that the GES with the thinner arc beam and the less groove number is beneficial to reduce the maximum contact force of cylindrical roller bearing.

Assuming that bearing speed is 6000r/min, and the radial forces acted on inner ring is 500N in Fig.9, the number of loaded rollers increased from 7 to 13 with decreasing of outer ring thickness, and when the groove numbers \( n \) is 18, 30 and 42, the maximum contact forces of BES decrease 55.8\%, 45.2\% and 30.2\% respectively compared with that of BRS. This means that the thinner outer ring is more beneficial to reduce the maximum contact force of BES with less groove number.

### 3.1.2 Impact of radial force on load distribution

Assuming that bearing speed is 6000r/min, and the groove number \( n \) is 18, 30 and 42, respectively. The impact of radial force on load distribution was shown in Fig.10.
In Fig.10, the maximum contact load increases with the radial load. When groove number is 18, the number of loaded rollers of BES has no obvious change. When groove numbers $n$ are 30 and 42, the number of loaded roller is slightly added. In addition, more number of loaded rollers appeared in BES with less groove number, which equals to bearing with the smaller clearance. It is well-known that as radial force is light and changes in a small range, the number of loaded roller of roller bearing with small clearance will not change significantly. So, the number of loaded roller in BES has no obvious change in Fig.10. If bearing bears a heavy load, the number of loaded rollers would change obviously.

3.2 Cage slip ratio of cylindrical roller bearing with GES

Cage slip ratio $S_c$ is defined as:

$$S_c = \left(1 - \frac{\Omega_m}{\omega_m}\right) \times 100\% \quad (7)$$

In Eq.(7), $\Omega_m$ is practical rotation speed of cage which is obtained through solving the differential equations which refer to the paper by Cui et al.(2018) and are explained in the Appendix B; $\omega_m$ is cage theory rotation speed, the detailed express refers to the book by Gupta(2012).

3.2.1 Impact of structural parameter on cage slip ratio

The impact of structural parameters on cage slip ratio were shown in Fig.11, Fig.12 and Fig.13, respectively.

In Fig.11, cage slip ratio of BES is obviously lower than that of BRS. Cage slip ratio increases with the groove number $n$. So, the GES with less groove number is more efficient to reduce cage slip ratio. For instance, in Fig.11(c), when the bearing speed is 18000r/min, the groove numbers $n$ are 18, 26, 34 and 42, cage slip ratios of BES are decreased by 53.8%, 40.6%, 29.1% and 15.1% respectively compared with that of BRS.

In Fig.12, cage slip ratio of BES increases with the arc beam thickness of GES. The GES with the thinner arc beam is more efficient to reduce cage slip ratio. For instance, in Fig.12(c), when the groove numbers $n$ is 18 and arc beam
thickness $H$ changes from 1mm to 3 mm, the cage slip ratios of BES are decreased by 60.2%, 44.1%, 33.8%, 22.4% and 10.5% respectively compared with that of BRS.

![Graphs showing the impact of outer ring thickness on cage slip ratio.](image1)

In Fig.13, cage slip ratio of BES increases with outer ring thickness. The GES with the thinner outer ring is more efficient to reduce cage slip ratio. For instance, in Fig.13(c), when the groove number $n$ is 18 and outer ring thickness changes from 3.5mm to 19.5 mm, cage slip ratios of BES are decreased by 60.2%, 46.8%, 34.3%, 14.3% and 4.3% respectively compared with that of BRS.

3.2.2 Impact of working conditions on cage slip ratio

Fig.14 and Fig.15 show the rotation speed and radial force affect cage slip ratio.

![Graphs showing the impact of rotation speed on cage slip ratio.](image2)

In Fig.14, cage slip radio of BES is obviously lower than that of BRS. Cage slip ratio increases with the rotation speed. The higher rotation speed, the GES is more efficient to reduce cage slip ratio. For instance, when bearing is running at 14000 r/min and 18000 r/min, cage slip ratios of BES with 18 grooves are reduced by 71% and 53.8% respectively, compared with that of BRS.

![Graphs showing the impact of radial load’s effects on cage slip ratio.](image3)
In Fig.15, cage slip ratio increases with decreasing of radial load. And when bearing is running under 600N radial load, cage slip ratios of BES with 18 groove number are reduced by 74.1% and 68.5% at 8000r/min and 10000r/min respectively compared with that of BRS. It means that when bearing is working at the lighter radial load, the GES has more efficient on reducing cage slip ratio.

4. Test verification

To verify the simulation results of the model in this paper, the rig in Fig.16 was established to test the cage dynamic characteristics of aircraft engines bearing. Fig.17 shows detail of cage’s speed measuring equipment, in which Hall element as the sensor mounted in end cover in Fig.17(a) can induce the speed of magnetic patch inset in the cage shown in Fig.17(b). The parameters of testing bearing were shown in Table 1. Because the designed rotation speed of the test shaft is below 12000r/min, so the test rotation speed is smaller than 12000r/min to protect the test rig.

![Fig.16 Test rig of cylindrical roller bearing with GES](image)

![Fig.17 Cage speed measuring equipment](image)

(a) Installation diagram of Hall element  
(b) Location of magnetic patch inset in the cage

4.1. Impact of groove number \( n \) on cage slip ratio

Fig.18 shows the impact of groove number \( n \) on cage slip ratio, in which \( T \) denotes the theory results, and \( E \) denotes the experiment results. The radial force acted on inner ring of cylindrical roller bearing is 500N.
In Fig.18, the test results of cage slip ratio and theory computed results have the same varying trend with the increase of groove number, and they are respectively approximate. The cage slip radios of BES and BRS in test are slightly larger than that in theory computation, which is caused by assumption of line stiffness of elastic support and outer ring in the coupled model of this paper and the model of BRS without considering thermic effect on the radial clearance of cylindrical roller bearing.

4.2. Impact of working conditions on cage slip ratio

Fig.19 and Fig.20 show the impact of rotation speed of inner ring and radial force on cage slip ratio. In Fig.19, the radial force acted on inner ring of cylindrical roller bearing is 800N. In Fig.20, the rotation speed of inner ring is 8000r/min.

Fig.19 and Fig.20 show that cage slip ratios increase with the rotation speed and decrease with the radial force both by test and theory computation methods. The cage radio of BES in test is larger than that in theory computation. The reason of these phenomena is same to Section.4.1.
5. Conclusions

(1) In this paper, a new theory analysis method of BES, which considers the integral deformation of flexible outer ring and GES, was presented. It has shown that the GES has a large impact on the load distribution and cage slip ratio in cylindrical roller bearing, which indicates the GES can increase the fatigue life of cylindrical roller bearing at the condition of high-speed and light-load.

(2) Under the same working conditions, the number of loaded rollers of BES increases significantly but the largest contact force decreases obviously, compared with that of BRS. The number of loaded rollers decreases with groove number, arc beam thickness and outer ring thickness, but the largest contact force presents a contrary tendency. When the radial force is smaller, it has no obvious impact on the number of loaded rollers, no matter bearing is supported by GES or rigid support.

(3) Cage slip ratio of BES is obviously lower than that of BRS. Under the same working condition, cage slip ratio of BES increases with the groove number, arc beam thickness and outer ring thickness. When BES is working at the higher speed and lighter radial load, GES shows more potent inhibition on cage slip ratio, compared with BRS.

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Appendix A

In order to establish the model of GES’s radial stiffness, the following hypotheses are proposed:

(1) Outer ring of the cylindrical bearing just has a displacement in OYZ plane.

(2) When the arc beam of GES bends with the radial force, the ends of arc beam A and B have no angular displacement.

(3) The bending deformation of arc beam belongs to the local small deformation, which has no influence on the global dimension of GES.

The force and deformation on the end B of the $p$th arc beam in OYZ plane are shown in Fig. A1. Because of interference fit between GES and bearing housing, the end A of arc beam is considered to be fixed. The other end B of arc beam moves to a point due to a radial force. Because the displacement of point B, $\Delta p$ is the vectorial sum of the displacement of outer ring mass center $\delta$ and the out-of-roundness of outer ring $\delta'_p$, the $p$th arc beam deformation and forces in local coordinate system are shown in Fig. A2.

In Fig. A1, $\{z_p, c_p, \eta_p\}$ is a local coordinate system of the $p$th arc beam; $C_p$ point is the midpoint of arc beam; $z_p$ axis is the tangential direction of the arc beam; $\eta_p$ axis is perpendicular to $z_p$ axis; $R$ is the radius of the arc beam, $R=D_i/2+3\times H/2+H'$; $\Phi$ is the span angle of arc beam; $\varphi$ is the included angle between the tangent of an arbitrary point on arc beam and the chord of arc beam; $\beta_p$ is the included angle between the line BB’($\Delta_p$) and the $\eta_p$ axis; $a_p$ and $b_p$ are the components along $z_p$ axis and $\eta_p$ axis. $a_p = \Delta_p \sin \beta_p$, $b_p = \Delta_p \cos \beta_p$, $f_{\varphi p}$, $f_{\eta p}$ and $M_p$ are the internal forces and moment of B point; $\eta_s$ is the elastic center of arc beam.
Due to the hypotheses without angular displacement at the end B, the moment $M_p$ is neglected, and the inner forces at the end B can be written as follows, according to the elastic center method (Hjelmstad 2005).

\[
\begin{align*}
\sigma_p &= -\frac{a_p}{2} \left( \frac{\eta - \eta_p}{E} \right) ds + \int \frac{\xi}{2} \cos \varphi ds - \int \frac{\mu}{2} \sin^2 \varphi ds \\
\sigma_p &= -\frac{b_p}{2} \left( \frac{\eta - \eta_p}{E} \right) ds + \int \frac{\xi}{2} \sin \varphi ds - \int \frac{\mu}{2} \cos^2 \varphi ds
\end{align*}
\]

(A.1)

In Eq (A.1), $\mu$ is material shear factor, for the rectangular cross section $\mu$ is 1.2; $E$ is material elastic modulus; $G$ is material shear modulus; $I$ is the inertia moment of cross-section, $I = \frac{LH^3}{12}$; ($\xi$, $\eta$) is the coordinate of arbitrary point on the arc beam in \{O_p, C_p, n_p\}; $\xi$ = $R$ sin $\theta$, $\eta$ = $R$ (1 - cos $\theta$); $ds$ = $R$ d($\Phi/2$ - $\theta$) = $R$ d$\theta$; $A_s$ is the area of cross-section, $\eta = \int \frac{1}{2} (\eta/E) ds / \int \frac{1}{2} (1/E) ds$.

(a) Deformation (b) Forces of arc beam end

Fig. A2. Deformation and forces of arc beam

In Fig. A2, the internal forces of point B are decomposed in \{O, X, Y, Z\}; $\varepsilon_p$ is the azimuth angle of the $p$th arc beam; $\zeta_p$ is the included angle between Y axis and the chord direction of the $p$th arc beam; $z_0$ and $y_0$ are the components of $\delta$ and can be calculated by Eq. (A.2).

\[
\begin{align*}
& z_0 = -\delta \sin \gamma \\
& y_0 = -\delta \cos \gamma
\end{align*}
\]

(A.2)

Supposing $\varepsilon_1$ is the azimuth angle of the first arc beam, span angle $\varphi$ can be written as $\varphi = 2\pi/n - \alpha/2$, $n$ is the number of arc beam. The relationships of the geometric parameters are shown as Eq. (A.3).

\[
\begin{align*}
\varepsilon_p &= \varepsilon_1 + 2(p - 1) \pi/n \\
\zeta_p &= \frac{\pi}{2} \left( \frac{\varphi}{2} + \varepsilon_p \right) \\
\gamma &= \arctan \frac{z_0}{y_0} \\
\Delta_p &= \sqrt{\delta^2 + (\delta_p)^2} - 2\delta \delta_p \cos(\pi - \gamma - \varepsilon_p) \\
\beta_1 &= \pi - \gamma - \frac{\varphi}{2} - \varepsilon_p \\
\beta_p &= \arcsin \frac{\delta_p - \sin(\pi - \varepsilon_p)}{\Delta_p} \\
\beta_p &= \gamma - \zeta_p - \beta_2 \\
\beta_p &= \beta_1 + \beta_2
\end{align*}
\]

(A.3)
The components of the internal forces at B point in \( \{O,X,Y,Z\} \) are shown as Eq. (A.4).

\[
\begin{align*}
F_{yB} &= f_{yP} \sin \xi_p - f_{xP} \cos \xi_p \\
F_{zB} &= -f_{yP} \cos \xi_p - f_{xP} \sin \xi_p
\end{align*}
\]  
(A.4)

Due to outer ring without rotation, the moment needn’t to be calculated, and then the force components \( F_y, F_z \) (in Eq. 2) of outer ring through GES can be calculated.

The radial stiffness of the GES at the azimuth of \( \xi_p \) (point B) can be written as Eq. (A.5).

\[
K_{GES}(\xi_p) = \frac{F_z \sin \delta_p - F_y \cos \delta_p}{\Delta_p \cos (\beta_p + \xi_p + \zeta_p)} = \frac{F_z \sin \delta_p - F_y \cos \delta_p}{\Delta}
\]  
(A.5)

Appendix B

When roller bearing is working, cage is simultaneously acted by collision forces and friction forces of rollers, guiding force of outer ring and combined resistance of oil/air mixture to cage’s surfaces. Schematic diagrams of cage force and force expressions refer to the paper by Cui et al.(2018). According the forces on cage, the dynamic equilibrium equation of cage(Cui et al.(2018)) can be obtained as Eq.(B.1) – Eq.(B.5).

\[
m_c \frac{d^2 \gamma_c}{dt^2} = \sum_{j=1}^{2} \left( F_{yB(j)} \sin \phi_j - F_{yB(j)} \cos \phi_j \right) + F_{cy} \cos \phi_c - F_{cz} \sin \phi_c - G_{cage}
\]  
(B.1)

\[
m_c \frac{d^2 z_c}{dt^2} = \sum_{j=1}^{2} \left( F_{zB(j)} \sin \phi_j - F_{zB(j)} \cos \phi_j \right) + F_{cz} \sin \phi_c + F_{cy} \cos \phi_c
\]  
(B.2)

\[
J_{cx} \frac{d \omega_{cx}}{dt} = \sum_{j=1}^{2} \left( 0.5 F_{zB(j)} D_w \right) - M_{cx} - T_{CDO} - T_{CDS}
\]  
(B.3)

\[
J_{cy} \frac{d \omega_{cy}}{dt} = \sum_{j=1}^{2} -M_{zj} \cos \phi_j
\]  
(B.4)

\[
J_{cz} \frac{d \omega_{cz}}{dt} = \sum_{j=1}^{2} -M_{zj} \sin \phi_j
\]  
(B.5)

In Eq. (B.1) – Eq. (B.5), \( m_c \) is the cage mass. \( F_{cy}, F_{cz} \) and \( M_{cx} \) are forces and moment caused by hydrodynamic action between the cage centering surface and the outer ring guiding surface, respectively. \( F_{yB(j)}, F_{zB(j)} \) and \( M_{zj} \) are the interaction force, the friction force and skew torque between the roller and the cage pocket respectively. \( G_{cage} \) is cage gravity. \( T_{CDO} \) and \( T_{CDS} \) are the resistance torque on the cage ends and faces caused by the oil/air. \( J_{cx}, J_{cy} \) and \( J_{cz} \) are the moment inertias of the cage, respectively; \( \gamma_c \) and \( z_c \) are position coordinates of the cage in the inertia coordinate system \( \{O; X,Y,Z\} \), respectively; \( \omega_{cx}, \omega_{cy} \) and \( \omega_{cz} \) are the angular velocity of the cage, respectively. \( \Omega_m \) is equal to \( \Omega_{m0} \) in Eq.7.

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