Towards the Classification of Non-Marginal Bound States of M-branes and Their Construction Rules

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Abstract

We present a systematic analysis of possible bound states of M-brane solutions (including waves and monopoles) by using the solution generating technique of reduction of M-brane to 10 dimensions, use of T-duality and then lifting back to 11 dimensions. We summarize a list of bound states for one- and two-charge cases including tilted brane solutions. Construction rules for these non-marginal solutions are also discussed.

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1 Introduction

There has been a great advance in our understanding of possible classical solutions of superstrings and supergravities. These classical solutions play important role in strong coupling dynamics of string theories \[1, 2\].

It is now believed that the best candidate for a unified theory underlying all physical phenomena is no longer 10-dimensional string theory but rather 11-dimensional M-theory whose low energy limit is given by the 11-dimensional supergravity. Thus it is expected that these classical solutions can be understood most easily in the 11-dimensional supergravity. In fact, we will see that various different 10-dimensional solutions can be obtained from an 11-dimensional solution. It is thus simpler to consider 11-dimensional solutions, on which we focus in this paper.

It has been known that this theory admits various $p$-brane solutions \[3, 4, 5, 6\], collectively referred to as M-branes. These solutions have been shown to be understood as the intersections of the fundamental $2_M$ and $5_M$ brane solutions \[7, 8\].

Systematic (intersection) rules are given for making various marginal solutions as intersecting M-branes in 11 dimensions \[8, 9, 11, 12, 13, 14, 15\]. However, it is also known that there are a large class of non-marginal solutions typically characterized by the mass formula $m \sim \sqrt{Q_e^2 + Q_m^2}$ in terms of the electric $Q_e$ and magnetic $Q_m$ charges. The rules for constructing these solutions are not explicitly given. It would be quite interesting to try to formulate such rules. For this purpose, it is important to first understand various solutions of this kind.

For 10-dimensional solutions, we can use T-duality \[16\] to generate new solutions from known ones \[17, 18, 19\]. In particular, some non-marginal solutions have been explicitly constructed in this way \[20, 21\], but a unified understanding still seems to be lacking. For the 11-dimensional solutions, there is no analogous technique, but we can make reductions to 10 dimensions and then make various duality transformations to get new 11-dimensional solutions \[16\]. Though several solutions of this kind have been found, they are scattered in various literature \[20, 22, 23, 24, 21, 25, 26, 27, 28, 29, 30, 31\] and no systematic classification has been attempted.

The purpose of this paper is to present a rather systematic analysis of possible “bound
states”, which are typical non-marginal solutions in 11-dimensional supergravity. Some of the solutions have been derived by using sophisticated symmetry in lower dimensions [26]. We will see that all the bound states of this kind can be obtained step by step by using simple duality rules in 10 dimensions without using larger symmetry realized in lower dimensions. We also present many new solutions and discuss the methods how to make these non-marginal solutions, which we call construction rules.

We first summarize fundamental solutions in 11-dimensional supergravity which will be our starting point.

2\(M\)-brane solution:
\[
2_M : \quad ds^2_{11} = H^{1/3} \left[ H^{-1} \left( -dt^2 + dy_1^2 + dy_2^2 \right) + \sum_{i=1}^{8} dx_i^2 \right],
\]
\[
C = \frac{1 - H}{H} dt \wedge dy_1 \wedge dy_2,
\]
(1.1)
where \(H\) is a harmonic function depending on the transverse coordinates \(x_1, \ldots, x_8\).

5\(M\)-brane solution:
\[
5_M : \quad ds^2_{11} = H^{2/3} \left[ H^{-1} \left( -dt^2 + dy_1^2 + \cdots + dy_5^2 \right) + \sum_{i=1}^{5} dx_i^2 \right],
\]
\[
dC = *dH,
\]
(1.2)
where the dual is taken with respect to the transverse coordinates \(x_1, \ldots, x_5\).

Wave solution:
\[
(0_w) : \quad ds^2_{11} = -dt^2 + dy_1^2 + (H - 1)(dt - dy_1)^2 + \sum_{i=1}^{9} dx_i^2,
\]
\[
dC = 0.
\]
(1.3)

Monopole solution:
\[
(0_m) : \quad ds^2_{11} = -dt^2 + \sum_{n=1}^{6} dy_n^2 + H^{-1}(dz + A_i dx_i)^2 + H \sum_{i=1}^{3} dx_i^2,
\]
\[
F_{ij} \equiv \partial_i A_j - \partial_j A_i = \epsilon_{ijk} \partial_k H.
\]
(1.4)

Using the rules summarized in the appendix [16], we can make reduction of (1.1) in the direction of world-volume, say \(y_2\) to get a fundamental string 1\(F\) in 10 dimensions; if
we do this in transverse direction $x_1$, we obtain a D2-brane $2_D$. Let us write this as \[24\]

$$2_M(y_1, y_2) \begin{cases} 
  y_3 \mapsto 1_F(y_1) \\
  x_1 \mapsto 2_D(y_1, y_2)
\end{cases},$$

where we have explicitly written the world-volume coordinates. It is useful to keep track of these coordinates to examine what solutions are possible. Similarly we can derive the rules for other solutions:

$$5_M(y_1, \ldots, y_5) \begin{cases} 
  y_6 \mapsto 4_D(y_1, \ldots, y_4) \\
  x_1 \mapsto 5_S(y_1, \ldots, y_5)
\end{cases},$$

$$0_w(y_1) \begin{cases} 
  y_1 \mapsto 0_D \\
  x_1 \mapsto 0_w(y_1)
\end{cases},$$

$$0_m(y_1, \ldots, y_6, z) \begin{cases} 
  y_6 \mapsto 0_m(y_1, \ldots, y_5, z) \\
  z \mapsto 6_D(y_1, \ldots, y_6)
\end{cases},$$

where $5_S$ is a solitonic 5-brane solution and other solutions are similar ones in type IIA string. These rules can also be used to read off what 11-dimensional solutions are obtained from 10-dimensional ones. Note that the rules give one-to-one correspondence between the solutions in 10 and 11 dimensions.

For T-duality, we can also consider them in various directions. For the world-volume direction, the procedure is clear. For the transverse (space-time) direction, we “delocalize” the solution, i. e. we include the coordinate in the direction of the isometry and suppose that the harmonic functions involved in the solution do not depend on the coordinate, and then use the ordinary duality rules. Using the rules in the appendix, we find \[24\]

$$1_F(y_1) \begin{cases} 
  y_1 \mapsto 0_w(y_1) \\
  x_1 \mapsto 1_F(y_1)
\end{cases},$$

$$5_S(y_1, \ldots, y_5) \begin{cases} 
  y_6 \mapsto 5_S(y_1, \ldots, y_5) \\
  z \mapsto 0_m(y_1, \ldots, y_5, z)
\end{cases}.$$
\[ 0_w(y_1) \begin{cases} y_1 \to 1_F(y_1) \\
 x_1 \to 0_w(y_1) \end{cases}, \quad (1.11) \]

\[ 0_m(y_1, \ldots, y_5, z) \begin{cases} y_5 \to 0_m(y_1, \ldots, y_5, z) \\
 z \to 5_S(y_1, \ldots, y_5) \end{cases}, \quad (1.12) \]

\[ p_D(y_1, \ldots, y_p) \begin{cases} y_p \to (p - 1)_D(y_1, \ldots, y_{p-1}) \\
 x_1 \to (p + 1)_D(y_1, \ldots, y_p, x_1) \end{cases}. \quad (1.13) \]

If we make reductions and T-dualities in mixed directions characterized by an angle, we can get non-marginal bound states of the above solutions.

Thus our procedure to produce new solutions in 11 dimensions are just to (1) make reduction, (2) make T-dualities twice in various directions and then (3) lift it back to 11 dimensions. If we start from various marginal solutions and continue this process, this is enough to derive all possible non-marginal solutions. This is the approach advocated by Tseytlin [19], and we intend to elaborate on this approach in more detail including non-marginal solutions. It is expected that this will lead to rules how to construct solutions in 11 dimensions.

We will see in the next section that all the fundamental solutions (1.1)–(1.4) are connected by duality transformation, and hence starting from just any single solution one can reproduce all other solutions. These solutions are related with more general bound states with angles which characterize how the solutions are interpolated. These solutions are non-marginal and have 1/2 supersymmetry. We will refer to these solutions obtained from the fundamental solutions listed in (1.1)–(1.4) as non-marginal solutions with single charge and summarize them in sect. 2.

If we start from two-charge solutions like \( 2_M \perp 2_M \), we get various bound states with 1/4 supersymmetry. We will derive possible non-marginal bound states obtained from marginal solutions of this kind in sect. 3. Further generalization including boost is discussed in sect. 4.

In sect. 5, we discuss the construction rules for the non-marginal solutions. Finally sect. 6 is devoted to discussions. In particular, we examine the effect of S-duality and the ADM mass for the non-marginal solutions.
2 Non-marginal solutions with single charge

In this section, we present solutions characterized by one independent charge and hence with 1/2 supersymmetry. All the solutions are connected by T-duality and can be obtained from $2_M$-brane solution (or any one of the fundamental solutions).

Let us first discuss what kind of bound states are possible if we start from marginal solutions (1.1)–(1.4) with single charge. Suppose we start from $2_M$ solution (1.1). In the first step of reduction, we get from (1.5) a bound state of $(1_F + 2_D)_A$, where subscript $A$ indicates that it is type IIA solution. If we make T-duality in all possible directions, we find a bound state of $(0_w + 1_F + 1_D + 3_D)_B$, which transforms into $(1_F + 0_w + 0_D + 2_D + 4_D)_A$ bound state in the second T-duality transformation. This in turn can be lifted to 11-dimensional non-marginal solution of two $0_w$, seven $2_M$ and one $5_M$. Instead of lifting there, we can further make third T-duality to get more general bound state of two $0_w$, sixteen $2_M$, fifteen $5_M$ and one monopole. In deriving this result, it is very useful to keep track of world-volume coordinates at each steps.

It is clear from this example that all the fundamental solutions (1.1)–(1.4) are related by the T-duality transformations. We can repeat this procedure to get other possible bound states starting from other fundamental solutions listed in (1.2)–(1.4). Below we discuss all the two-body bound states and some new three-body bound state solutions in order to show explicitly how the above procedure actually works.

For this purpose, we first delocalize the solution (1.1) (include $x_1$ in the isometry direction) and rotate the coordinates $y_2$ and $x_1$:

$$ds_{11}^2 = H^{1/3} \left[ H^{-1} \left( -dt^2 + dy_1^2 + (dy_2 \cos \theta + dx_1 \sin \theta)^2 \right) + (-dy_2 \sin \theta + dx_1 \cos \theta)^2 + \sum_{i=2}^8 dx_i^2 \right],$$

$$C = \frac{1 - H}{H} dt \wedge dy_1 \wedge (dy_2 \cos \theta + dx_1 \sin \theta).$$

We note that, though trivial as 11-dimensional solution, this may be regarded as a bound state of $2_M + 2_M$ lying in the directions of $y_1 - y_2$ for $\theta = 0$ and $y_1 - x_1$ for $\theta = \frac{\pi}{2}$.

Reduction in the $y_2$ direction yields a nontrivial bound state $(1_F + 2_D)_A$ in type IIA
string:
\[
\begin{align*}
\text{ds}_A^2 &= \tilde{H}_\theta^{1/2} \left[ H^{-1}(-dt^2 + dy_1^2) + \tilde{H}_\theta^{-1} dx_1^2 + \sum_{i=2}^{8} dx_i^2 \right], \\
C &= \frac{1 - H}{H} dt \wedge dy_1 \wedge dx_1 \sin \theta, \\
A^{(1)} &= \tilde{H}_\theta^{-1}(1 - H) dx_1 \sin \theta \cos \theta, \\
B^{(1)} &= \frac{1 - H}{H} dt \wedge dy_1 \cos \theta, \\
e^{2\phi} &= H^{-1} \tilde{H}_\theta^{3/2}, \\
\end{align*}
\]
(2.2)

where
\[
\tilde{H}_\theta = \cos^2 \theta + H \sin^2 \theta.
\]
(2.3)

For $\theta = 0$, this is the fundamental string lying in $y_1$; for $\theta = \frac{\pi}{2}$, this becomes D2-brane lying in $y_1 - x_1$.

### 2.1 Bound states of $0_w$ with $2_M$, $5_M$ and monopole ($0_m$)

These solutions can be obtained by the following sequence of reduction, T-duality and lifting:
\[
2_M \xrightarrow{R^2_\theta} (1_F + 2_D)_A \xrightarrow{T_{y_1}} (0_w + 1_D)_B \xrightarrow{T_{y_2}} (0_w + 2_D)_A \xrightarrow{\text{lift}(y_2)} 0_w + 2_M,
\]
(2.4)

where the notation is as follows: $R_\theta$ stands for reduction in (2.2), $T_y$ for a T-duality along $y_1$ and finally lift($y_2$) for lifting the result to 11 dimensions by adding the coordinate $y_2$.

The resulting solution is the bound state $0_w + 2_M$:
\[
\begin{align*}
\text{ds}_{11}^2 &= \tilde{H}_\theta^{1/3} \left[ \tilde{H}_\theta^{-1} \left( -dt^2 + dy_1^2 + (H - 1)(dt \cos \theta + dy_1)^2 + dx_1^2 + dx_2^2 \right) \\
&\quad + dy_2^2 + \sum_{i=3}^8 dx_i^2 \right], \\
C &= \frac{(1 - H) \sin \theta}{\tilde{H}_\theta} (dt + dy_1 \cos \theta) \wedge dx_2 \wedge dx_1.
\end{align*}
\]
(2.5)

This solution was discussed in ref. [20], but it was derived in a different route. Thus there are many routes to give the same solutions.

Instead of lifting, we can further make T-duality to get
\[
(0_w + 2_D)_A \xrightarrow{T_{y_1}} (0_w + 3_D)_B \xrightarrow{T_{y_2}} (0_w + 4_D)_A \xrightarrow{\text{lift}(y_2)} 0_w + 5_M.
\]
(2.6)
The solution now takes the form
\[
\begin{align*}
  ds^2_{11} &= \tilde{H}_\theta^{2/3} \left[ \tilde{H}_\theta^{-1} \left( -dt^2 + dy_1^2 + (H - 1)(dt \cos \theta + dy_1)^2 + dy_2^2 + \sum_{i=1}^{4} dx_i^2 \right) \
  &\quad + \sum_{i=5}^{8} dx_i^2 \right], \\
  dC' &= *dH \wedge (dy_1 + dt \cos \theta) \sin \theta, \tag{2.7}
\end{align*}
\]
again in agreement with ref. [20]. Here \( H \) and \( \tilde{H}_\theta \) depend only on the transverse space \( x_5, \ldots, x_8 \) and \( * \) is a dual in that space.

Again instead of lifting, we can further make T-duality to get the new solution of bound state of wave and monopole:
\[
(0_w + 4D)_A \xrightarrow{T_{y_6}} (0_w + 5D)_B \xrightarrow{T_{y_6}} (0_w + 6D)_A \xrightarrow{\text{lift}^{(z)}} 0_w + 0_m. \tag{2.8}
\]

The metric is given by
\[
\begin{align*}
  ds^2_{11} &= -dt^2 + dy_1^2 + (H - 1)(dt \cos \theta + dy_1)^2 + \sum_{i=1}^{6} dx_i^2 + \tilde{H}_\theta \sum_{i=7}^{8} dx_i^2 \\
  &\quad + \tilde{H}_\theta^{-1}(dz + A_{y_1} \sin \theta dy_1 + A_t \sin \theta \cos \theta dt)^2, \tag{2.9}
\end{align*}
\]
where the gauge fields depend only on \( x_7 \) and \( x_8 \), and satisfy
\[
\begin{align*}
  \partial_{x_8} A_{y_1} &= -\partial_{x_7} H, \quad \partial_{x_7} A_{y_1} = \partial_{x_8} H, \\
  \partial_{x_8} A_t &= \partial_{x_7} H, \quad \partial_{x_7} A_t = -\partial_{x_8} H, \tag{2.10}
\end{align*}
\]
which describe a monopole solution in a special gauge.

### 2.2 Bound states of \( 0_m \) with \( 2_M, 5_M \) and wave \( (0_w) \)

In order to derive bound states of monopole and others, we can continue the duality transformation to the above solutions, but it is easier to start from monopole solution itself.

We rotate \( y_6 \) and \( z \) in the monopole solution (1.4):
\[
\begin{align*}
  ds^2_{11} &= -dt^2 + \sum_{n=1}^{5} dy_n^2 + (dy_6 \sin \theta + dz \cos \theta)^2 \\
  &\quad + H^{-1}(-dy_6 \cos \theta + dz \sin \theta + A_t dx_1)^2 + H \sum_{i=1}^{3} dx_i^2. \tag{2.11}
\end{align*}
\]
Again this is a trivial bound state of $0_m + 0_m$.

Reduction in the direction $y_6$ yields the type IIA solution $0_m + 6D$:

$$\begin{aligned}
    ds_A^2 &= (H\tilde{H}_\theta)^{1/2} \left[ H^{-1} \left( -dt^2 + \sum_{n=1}^{5} dy_n^2 \right) + (H\tilde{H}_\theta)^{-1}(dz + A_i \sin \theta dx_i)^2 + \sum_{i=1}^{3} dx_i^2 \right], \\
    F_{ij} &\equiv \partial_i A_j - \partial_j A_i = \epsilon_{ijk} \partial_k H, \\
    A^{(1)} &= (1 - \tilde{H}_\theta^{-1}) \cot \theta dz - \tilde{H}_\theta^{-1} \cos \theta A_i dx_i, \quad e^{2\phi} = H^{-3/2}\tilde{H}_\theta^{3/2}. \quad (2.12)
\end{aligned}$$

To this solution (2.12), we now make the following sequence of reduction, T-duality and lifting:

$$(0_m + 6D)_A \xrightarrow{T_{y_6}} (0_m + 5D)_B \xrightarrow{T_{y_4}} (0_m + 4D)_A \xrightarrow{\text{lift}(y_0)} 0_m + 5M. \quad (2.13)$$

We find the solution

$$\begin{aligned}
    ds_{11}^2 &= H^{2/3}\tilde{H}_\theta^{1/3} \left[ H^{-1} \left( -dt^2 + \sum_{n=0}^{3} dy_n^2 \right) + \tilde{H}_\theta^{-1}(dy_4^2 + dy_5^2) \\
    &\quad + (\tilde{H}_\theta H)^{-1}(dz + A_i \sin \theta dx_i)^2 + \sum_{i=1}^{3} dx_i^2 \right], \\
    C &= (\tilde{H}_\theta^{-1} - 1)dy_4 \wedge dy_5 \wedge dz \cot \theta + A_i \tilde{H}_\theta^{-1}dy_4 \wedge dy_5 \wedge dx_i \cos \theta, \quad (2.14)
\end{aligned}$$

in agreement with ref. [24].

Instead of lifting, we can further make T-duality to get $0_m + 2M$:

$$(0_m + 4D)_A \xrightarrow{T_{y_4}} (0_m + 3D)_B \xrightarrow{T_{y_2}} (0_m + 2D)_A \xrightarrow{\text{lift}(y_6)} 0_m + 2M. \quad (2.15)$$

The resulting solution is

$$\begin{aligned}
    ds_{11}^2 &= H^{1/3}\tilde{H}_\theta^{2/3} \left[ H^{-1}(-dt^2 + dy_1^2) + \tilde{H}_\theta^{-1}\sum_{n=2}^{6} dy_n^2 + (H\tilde{H}_\theta)^{-1}(dz + A_i \sin \theta dx_i)^2 \\
    &\quad + \sum_{i=1}^{3} dx_i^2 \right], \\
    dC &= dt \wedge dy_1 \wedge d(H^{-1}A_i dx_i) \sin \theta \cos \theta + dH^{-1} \wedge dt \wedge dy_1 \wedge dz \cos \theta. \quad (2.16)
\end{aligned}$$

This is also given in ref. [24].

Again we further make T-duality to get the bound state of wave and monopole:

$$(0_m + 2D)_A \xrightarrow{T} (0_m + 1D)_B \xrightarrow{T} (0_m + 0w)_A \xrightarrow{\text{lift}(y_6)} 0_m + 0w. \quad (2.17)$$

The metric is given in (2.13), showing the consistency of the result.
2.3 2\text{M} and 5\text{M} bound states

The only remaining two-body bound state is that of 2\text{M} and 5\text{M} \cite{22}, which can be obtained by

\[ 2\text{M} \xrightarrow{R(\theta=\pi/2)} (2\text{D})_A \xrightarrow{T_{x_2}} (3\text{D})_B \xrightarrow{T_{x_3}=4} (2\text{D}+4\text{D})_A \xrightarrow{\text{lift}(y_2)} 2\text{M} + 5\text{M}. \tag{2.18} \]

The solution is

\[ ds^2_{11} = \left( H\tilde{H}_\theta \right)^{1/3} \left[ H^{-1}(-dt^2 + dx_2^2 + dx_3^2) + \tilde{H}_\theta^{-1}(dx_1^2 + dy_1^2 + dy^2_2) + \sum_{i=4}^8 dx_i^2 \right], \]

\[ dC = \left( 1 - \frac{H}{\tilde{H}_\theta} \right)^{2/3} \left[ \tilde{H}_\theta^{-1}(dx_2^2 + dx_3^2 + dy_2^2) + \sum_{i=5}^8 dx_i^2 \right], \]

in agreement with ref. \cite{22}.

This exhausts all two-body bound states. One may consider bound states such as 2\text{M} + 2\text{M}, but they are trivial solutions in 11-dimensions in the same sense of (2.1).

We note that it is quite involved to check that (2.19) is really a solution to the field equations of 11-dimensional supergravity, and in particular one has to take into account the Chern-Simon term. Compared with that approach, the method of T-duality is much simpler.

2.4 Three-body bound state: 0\text{w} + 2\text{M} + 5\text{M}

The procedure in the previous subsections can be generalized to include more bound states. For example, let us consider

\[ 2\text{M} \xrightarrow{R(\theta_1)} (1\text{F}+2\text{D})_A \xrightarrow{T_{y_1}} (0\text{w}+1\text{D})_B \xrightarrow{T_{x_2}} (0\text{w}+2\text{D})_A \xrightarrow{T_{x_3}=4} (0\text{w}+1\text{D}+3\text{D})_B \]

\[ \xrightarrow{T_{x_3}} (0\text{w}+2\text{D}+4\text{D})_A \xrightarrow{\text{lift}(y_2)} (0\text{w}+2\text{M} + 5\text{M}). \tag{2.20} \]

By this procedure we get a new bound state solution 0\text{w} + 2\text{M} + 5\text{M}:

\[ ds^2_{11} = \left( \tilde{H}_{\theta_1} \tilde{H}_{\theta_2} \right)^{1/3} \left[ \tilde{H}_\theta^{-1}(dt^2 + dy_3^2 + (H-1)(dt \cos \theta_1 + dy_1^2 + dx_1^2 + dx_2^2)) \right. \]

\[ + \tilde{H}_{\theta_2}^{-1}(dx_2^2 + dx_3^2 + dy_2^2) + \sum_{i=5}^8 dx_i^2 \right], \]

\[ dC = d \left( \frac{1-H}{\tilde{H}_{\theta_1}} \right) \wedge dx_4 \wedge dx_1 \wedge (dt + dy_1 \cos \theta_1) \sin \theta_1 \cos \theta_2 \]
\[ + \ast dH \land (dy_1 + dt \cos \theta_1) \sin \theta_1 \sin \theta_2 \\
+ d\left(\frac{1 - \tilde{H}_{\theta_1}}{\tilde{H}_{\theta_{12}}}\right) \land dx_2 \land dx_3 \land dx_4 \sin \theta_2 \cos \theta_2, \tag{2.21} \]

where $\tilde{H}_{\theta_1}$ is defined as in (2.3) and

\[ \tilde{H}_{\theta_{12}} = \cos^2 \theta_2 + \tilde{H}_{\theta_1} \sin^2 \theta_2. \tag{2.22} \]

Note that the definition of this harmonic function is similar to $\tilde{H}_{\theta}$ in (2.3).

If we put $\theta_1 = 0$, this reduces to wave solution; $\theta_1 = \frac{\pi}{2}$ to $2M + 5M$ solution in (2.19). Notice also that one can introduce wave to the $2M + 5M$ solution in the null isometry direction by the method of ref. [32], and that it gives a similar solution to (2.21) but is actually different. The solution $2M + 5M$ with wave is given below in (3.30).

### 2.5 Three-body bound state: $0_m + 2M + 5M$

Another example of more complicated bound state is obtained by

\[ 0_m \overset{R(\theta_1)}{\rightarrow} (0_m + 6D)_A \overset{T_{y_6}}{\rightarrow} (0_m + 5D)_B \overset{T_{y_4+y_5}}{\rightarrow} (0_m + 4D + 6D)_A \overset{T_{y_3}}{\rightarrow} (0_m + 3D + 5D)_B \]
\[ \overset{T_{y_2}}{\rightarrow} (0_m + 2D + 4D)_A \overset{\text{lift}(y_6)}{\rightarrow} (0_m + 2M + 5M). \tag{2.23} \]

The solution takes the form

\[ ds_{11}^2 = (H \tilde{H}_{\theta_1} \tilde{H}_{\theta_{12}})^{1/3} \left[H^{-1}(dt^2 + dy_1^2) + \tilde{H}_{\theta_1}^{-1}(dy_2^2 + dy_3^2) + \tilde{H}_{\theta_{12}}^{-1}(dy_4^2 + dy_5^2 + dy_6^2) \right] + (H \tilde{H}_{\theta_1})^{-1}(dz + A_i \sin \theta_1 dx_i)^2 + \sum_{i=1}^{3} dx_i^2, \]
\[ dC = \cos \theta_1 \cos \theta_2 dt \land dy_1 \land dz \land dH^{-1} - \cos \theta_1 \sin \theta_1 \cos \theta_2 dt \land dy_1 \land d(H^{-1}A_i dx_i) \]
\[ + \cos \theta_1 \sin \theta_2 dy_2 \land dy_3 \land d(\tilde{H}_{\theta_1}^{-1}A_i dx_i) - \cot \theta_1 \sin \theta_2 dy_2 \land dy_3 \land dz \land d\tilde{H}_{\theta_1}^{-1} \]
\[ + \sin \theta_2 \cos \theta_2 d\left(\frac{\tilde{H}_{\theta_1} - H}{\tilde{H}_{\theta_{12}}}\right) \land dy_4 \land dy_5 \land dy_6, \tag{2.24} \]

where $\tilde{H}_{\theta_1}$ is defined as (2.3) and

\[ \tilde{H}_{\theta_{12}} = \tilde{H}_{\theta_1} \cos^2 \theta_2 + H \sin^2 \theta_2. \tag{2.25} \]

Putting $\theta_1 = 0$ gives the $2M + 5M$ solution in (2.19) while $\theta_1 = \frac{\pi}{2}$ monopole solution. It is also possible to produce a more general bound state describing bound states of all combinations of $2M$, $5M$ and $0_m$ if one includes an additional angle.
2.6 Is boost necessary?

Type IIB bound state of \((1_F + 1_D)_B\) is derived using boost [20], but this can be obtained from (2.2) by T-duality in \(x_1\) direction. The result is

\[
\begin{align*}
\bar{H}^{1/2} & \left[ H^{-1}(-dt^2 + dy_1^2) + \sum_{i=1}^8 dx_i^2 \right], \\
\varphi & = \log H^{-1/2}\bar{H}_{\theta}, \quad \ell = (1 - H)\bar{H}_{\theta}^{-1}\sin \theta \cos \theta, \\
B^{(1)} & = \frac{1 - H}{H}dt \wedge dy_1 \cos \theta, \quad B^{(2)} = \frac{1 - H}{H}dt \wedge dy_1 \sin \theta.
\end{align*}
\]

(2.26)

This agrees with ref. [20]. All possible solutions thus seem to be obtained without using boost for single charge case.

We can continue the procedure to generate more solutions, but we have already generated enough examples to understand the general structure of solutions of this kind.

3 Non-marginal solutions with two charges

In this section, we proceed to the analysis of solutions obtained from marginal solutions with two charges. We can introduce one angle at each step of reduction and T-duality, thus producing general non-marginal bound states of various marginal solutions. However, this produces quite complicated solutions without much physical insight, and so below we present examples of two-body bound states and the procedure how to obtain these. Some of them are known ones, but others are new.

3.1 Solutions obtained from \(2_M \perp 2_M\)

Suppose that we start from \(2_M \perp 2_M\) solution with the world-volume coordinates

\[
\begin{align*}
2_M^{(y_1,y_2)} & \perp 2_M^{(y_3,y_4)} ,
\end{align*}
\]

(3.1)

where we have indicated the world-volume coordinates below each \(2_M\)-brane. Making reduction in the directions \(x_1\) or \(y_4\), we get

\[
\begin{align*}
(2_D^{(y_1,y_2)} & \perp 2_D^{(y_3,y_4)}), \\
(2_D^{(y_1,y_2)} & \perp 1_F^{(y_4)}),
\end{align*}
\]

(3.2)
or their bound state.

Applying the T-duality rule in all possible directions to the first solution, we get

\[
(3_D \perp 3_D)_{B} , \quad (3_D \perp 1_D)_{B} ,
\]

Similarly from the second solution, we get

\[
(3_D \perp 0)_{B} , \quad (1_D \perp 1_F)_{B} , \quad (3_D \perp 1_F)_{B} ,
\]

We now apply T-duality to the rotated directions in all possible way to the first solution in (3.3) to find

\[
T_{x_2-x_3} : (2_D \perp 2_D + 4_D \perp 4_D)_{A} ,
\]

\[
T_{x_2-y_2} : (2_D \perp 2_D + 2_D \perp 4_D)_{A} ,
\]

\[
T_{y_2-y_3} : (2_D \perp 4_D + 4_D \perp 2_D)_{A} ,
\]

\[
T_{y_2-x_3} : (2_D \perp 4_D + 4_D \perp 4_D)_{A} ,
\]

which can be lifted by the rule in the appendix to 11-dimensional non-marginal solutions

\[
(2_M \perp 2_M + 5_M \perp 5_M) ,
\]

\[
(2_M \perp 2_M + 2_M \perp 5_M) ,
\]

\[
(2_M \perp 5_M + 5_M \perp 2_M) ,
\]

\[
(2_M \perp 5_M + 5_M \perp 5_M) .
\]

From the second solution in (3.3), similar procedure yields the solutions

\[
T_{y_3-y_4} : (2_D \perp 2_D + 4_D \perp 0_w)_{A} ,
\]

\[
T_{y_4-x_2} : (4_D \perp 0_D + 4_D \perp 2_D)_{A} ,
\]

which can be lifted to 11-dimensional non-marginal solutions

\[
(2_M \perp 2_M + 5_M \perp 0_w) , \quad (2_M \perp 5_M + 0_w \perp 5_M) .
\]
From the first solution in (3.4), we obtain

\[ T_{y_3-y_2} : (2_D \perp 1_F + 2_D \perp 0_0)_{A}, \]
\[ T_{y_1-x_1} : (2_D \perp 0_0 + 4_D \perp 0_0)_{A}, \]

(3.9)

which can be lifted to 11-dimensional non-marginal solutions

\[ (2_M \perp 2_M + 2_M \perp 0_0), \ (2_M \perp 0_0 + 5_M \perp 0_0). \]

(3.10)

Here we again have not shown other possible bound states which yields already listed 11-dimensional solutions even though they are different solutions in 10 dimensions, because they can be simply obtained from the 11-dimensional solutions.

We have also examined other cases and the only new solutions that can be obtained from the above is that from the second of (3.4) as

\[ T_{y_1-y_3} : (0_D \perp 1_F + 2_D \perp 0_0)_{A}, \]

(3.11)

which can be lifted to 11-dimensional non-marginal solutions

\[ (0_0 \perp 2_M + 2_M \perp 0_0). \]

(3.12)

As a bonus, we find the orthogonal intersection rules for wave and other solutions: Wave can intersects with $2_M$ and $5_M$ over a string (or can propagate on world-volume direction). This actually produces known boosted solutions \[ \text{[8, 32]} \].

As a check, we now summarize the explicit solutions listed above. Of course, there are several routes to obtain same solutions and we show below one possible way for each solutions. Let us start from a rotated $2_M \perp 2_M$ solution

\[
d s_{11}^2 = \left( H_1 H_2 \right)^{1/3} \left[ - (H_1 H_2)^{-1} dt^2 + H_1^{-1} (dy_1^2 + dy_2^2) \right. \\
+ H_2^{-1} \left( dy_3^2 + (dy_4 \cos \theta + dx_1 \sin \theta)^2 \right) + (dy_4 \sin \theta + dx_1 \cos \theta)^2 + \sum_{i=2}^{6} dx_i^2 \right],
\]

\[
C' = \frac{1 - H_1}{H_1} dt \wedge dy_1 \wedge dy_2 + \frac{1 - H_2}{H_2} dt \wedge dy_3 \wedge (dy_4 \cos \theta + dx_1 \sin \theta),
\]

(3.13)

where $H_1, H_2$ are harmonic functions depending on the transverse coordinates $x_2, \cdots, x_6$. 
Upon reduction in $y_4$ direction, we get type IIA $2D \perp 1_F + 2_D \perp 2_D$ solution:

$$ds^2_A = (H_1 \tilde{H}_2)^{1/2} \left[ -(H_1 H_2)^{-1} dt^2 + H_1^{-1}(dy_1^2 + dy_2^2) + H_2^{-1}dy_3^2 + \tilde{H}_2^{-1}dx_1^2 + \sum_{i=2}^6 dx_i^2 \right],$$

$$C = \frac{1 - H_1}{H_1} dt \wedge dy_1 \wedge dy_2 + \frac{1 - H_2}{H_2} dt \wedge dy_3 \wedge dx_1 \sin \theta, \quad e^{2\phi} = H_1^{1/2}H_2^{-1}\tilde{H}_2^{3/2},$$

$$B^{(1)} = \frac{1 - H_2}{H_2} dt \wedge dy_3 \cos \theta, \quad A^{(1)} = \tilde{H}_2^{-1}(1 - H_2)dx_1 \sin \theta \cos \theta. \quad (3.14)$$

### 3.1.1 $2_M \perp 2_M$ and $5_M \perp 5_M$ bound state

We start from (3.14) with $\theta = \frac{\pi}{2}$ and make the following sequence of duality and lifting:

$$2_M \perp 2_M \xrightarrow{R(\theta = \frac{\pi}{2})} (2_D \perp 2_D)_A \xrightarrow{T_x} (3_D \perp 3_D)_B \xrightarrow{T_{x_2-x_3}} (2_D \perp 2_D + 4_D \perp 4_D)_A \xrightarrow{\text{lift}(y_4)} (2_M \perp 2_M + 5_M \perp 5_M). \quad (3.15)$$

The solution thus obtained is

$$ds^2_{11} = (H_1 H_2 \tilde{H}_{12})^{1/3} \left[ -(H_1 H_2)^{-1} dt^2 + \tilde{H}_{12}^{-1}(dx_2^2 + dx_3^2 + dy_1^2) + H_1^{-1}(dy_1^2 + dy_2^2) + H_2^{-1}(dy_3^2 + dx_1^2) + \sum_{i=4}^6 dx_i^2 \right],$$

$$dC = (dH_1^{-1} \wedge dt \wedge dy_1 \wedge dy_2 + dH_2^{-1} \wedge dt \wedge dy_3 \wedge dx_1) \cos \theta$$

$$+ (*dH_1 \wedge dy_3 \wedge dx_1 + *dH_2 \wedge dy_1 \wedge dy_2) \sin \theta$$

$$+ d \left( \frac{1 - H_1 H_2}{H_{12}} \right) \wedge dx_2 \wedge dx_3 \wedge dy_4 \cos \theta \sin \theta, \quad (3.16)$$

where the angle $\theta$ is reintroduced in the angled duality in (3.13) and

$$H_{12} = \cos^2 \theta + H_1 H_2 \sin^2 \theta, \quad (3.17)$$

in agreement with ref. [26]. In that reference, the authors used 8-dimensional large symmetry $SL(2,R)$ to find this solution. However, we do not have to refer to such a special symmetry realized only in lower dimensions, but can derive these solutions step by step. Also this step-by-step method can produce more general bound states.

### 3.1.2 $2_M \perp (2_M + 5_M)$ bound state

We make different T-duality to the solution $(3_D \perp 3_D)_B$ in (3.15):

$$(3_D \perp 3_D)_B \xrightarrow{T_{x_2-x_2}} (2_D \perp 2_D + 2_D \perp 4_D)_A \xrightarrow{\text{lift}(y_4)} (2_M \perp 2_M + 2_M \perp 5_M). \quad (3.18)$$
The result is

\[ ds^2_{11} = (H_1 H_2 \tilde{H}_2)^{1/3} \left[ -(H_1 H_2)^{-1} dt^2 + H_1^{-1} dy_1^2 + (H_1 \tilde{H}_2)^{-1} dy_2^2 + H_2^{-1} (dy_3^2 + dx_1^2) 
+ \tilde{H}_2^{-1} (dy_4^2 + dx_2^2) + \sum_{i=3}^{6} dx_i^2 \right], \]

\[ dC = d \left( \frac{1 - H_2}{H_1} \right) \wedge dx_2 \wedge dy_2 \wedge dy_4 \sin \theta \cos \theta + dH_1^{-1} \wedge dt \wedge dy_1 \wedge dy_2 
+ dH_2^{-1} \wedge dt \wedge dy_3 \wedge dx_1 \cos \theta + *dH_2 \wedge dy_1 \sin \theta, \]  

(3.19)

This solution may be interpreted as an orthogonal intersection of \(2_M\) brane and the non-marginal solution \((2_M + 5_M)\) in (2.19). Viewed this way, this solution is discussed in ref. [23]. There the author derived this solution by empirical rule that the solution should agree with known orthogonal intersecting ones for \(\theta = 0, \frac{\pi}{2}\). This can be automatically generated by T-dualities.

### 3.1.3 \(2_M \perp 5_M\) and \(5_M \perp 2_M\) bound state

Another transformation to \((3D \perp 3D)_B\)

\[ (3D \perp 3D)_B \xrightarrow{T_{y_2 \rightarrow y_1}} (2_D \perp 4_D + 4_D \perp 2_D)_A \xrightarrow{\text{lift}(y_1)} (2_M \perp 5_M + 5_M \perp 2_M), \]  

(3.20)

yields the solution

\[ ds^2_{11} = (H_1 H_2 \tilde{H}_12)^{1/3} \left[ (H_1 H_2)^{-1}(-dt^2 + dx_2^2) + H_1^{-1} dy_1^2 + H_12^{-1}(dy_2^2 + dy_3^2 + dy_4^2) 
+ H_2^{-1} dx_1^2 + \sum_{i=3}^{6} dx_i^2 \right], \]

\[ dC = d \left( \frac{H_2 - H_1}{H_12} \right) \wedge dy_2 \wedge dy_3 \wedge dy_4 \sin \theta \cos \theta 
- \left[ dH_1^{-1} \wedge dt \wedge dy_1 \wedge dx_2 + *dH_2 \wedge dy_1 \right] \cos \theta 
- \left[ dH_2^{-1} \wedge dt \wedge dx_1 \wedge dx_2 + *dH_2 \wedge dx_1 \right] \sin \theta, \]  

(3.21)

where

\[ \tilde{H}_{12} = H_1 \sin^2 \theta + H_2 \cos^2 \theta. \]  

(3.22)

This solution is also derived by using 8-dimensional symmetry [20].

\footnote{Here and in the following, * always means a dual with respect to the transverse space. In the present case of eq. (3.19), the transverse space consists of \(x_i, i = 3, \cdots, 6\).}
3.1.4 \((2M + 5M) \perp 5M\) bound state

Another route is

\[
(3_D \perp 3_D)_B \xrightarrow{T_{y2 \rightarrow y3}} (2_D \perp 4_D + 4_D \perp 4_D)_A \xrightarrow{\text{lift}(y_4)} (2_M \perp 5M + 5M \perp 5M).
\] (3.23)

The resulting solution is

\[
d_{11}^2 = \left( H_1 \tilde{H}_1 \right)^{1/3} \left[ H_2 (H_2)^{-1} \left( -dt^2 + dx_2^2 \right) + H_1^{-1}dy_1^2 + (H_2 \tilde{H}_1)^{-1}(dy_2^2 + dy_4^2) \right. \\
+ \tilde{H}_1^{-1}dx_3^2 + H_2^{-1}(dy_3^2 + dx_1^2) + \sum_{i=4}^6 dx_i^2, \\
\]

\[
dC = d \left( \frac{1 - H_1}{H_1} \right) \wedge dy_2 \wedge dx_3 \wedge dy_4 \sin \theta \cos \theta - dH_1^{-1} \wedge dt \wedge dy_1 \wedge dx_2 \cos \theta \\
+ * dH_1 \wedge dy_3 \wedge dx_1 \sin \theta + * dH_2 \wedge dy_1 \wedge dx_3.
\] (3.24)

This again may be regarded as an orthogonal intersection of \((2M + 5M)\) in \((2.13)\) and \(5M\).

3.1.5 \(2M \perp (0_w + 2M)\) bound state

From \((3.14)\) with \(\theta = 0\), we find a new solution by

\[
(2_D \perp 1_F)_A \xrightarrow{T_{y2 \rightarrow y3}} (3_D \perp 0_w)_B \xrightarrow{T_{y2 \rightarrow y3}} (2_D \perp 0_w + 2D \perp 1_F)_A \xrightarrow{\text{lift}(y_4)} (2_M \perp 0_w + 2M \perp 2M).
\] (3.25)

The solution is

\[
ds_{11}^2 = \left( H_1 \tilde{H}_2 \right)^{1/3} \left[ (H_1 \tilde{H}_2)^{-1} \left( -dt^2 + dx_3^2 + (H_2 - 1)(dt \cos \theta + dy_3)^2 \right) + H_1^{-1}dy_1^2 \right. \\
+ \tilde{H}_2^{-1}(dy_2^2 + dy_4^2) + \sum_{i=1}^6 dx_i^2, \\
\]

\[
C = \frac{1 - H_1}{H_1} dt \wedge dy_3 \wedge dy_1 + \frac{1 - H_2}{H_2} dy_2 \wedge (dt + dy_3 \cos \theta) \wedge dy_4 \sin \theta.
\] (3.26)

This may be regarded as an orthogonal intersection of \(2M\) and \((0_w + 2M)\) in \((2.13)\).

3.1.6 \(2M \perp 2M\) and \(5M \perp 0_w\) bound state

From \((3_D \perp 0_w)_B\) in \((3.25)\), we obtain

\[
(3_D \perp 0_w)_B \xrightarrow{T_{y3 \rightarrow y1}} (2_D \perp 1_F + 4_D \perp 0_w)_A \xrightarrow{\text{lift}(y_4)} (2_M \perp 2M + 5M \perp 0_w).
\] (3.27)
The solution is

\[
\begin{align*}
\text{ds}_{11}^2 &= (H_1 \dot{H}_{12})^{1/3} \left[ - (H_1 \dot{H}_{12})^{-1} \dot{H}_1 dt^2 + H_1^{-1} (dy_1^2 + dy_2^2) \right. \\
&\quad + \dot{H}_{12}^{-1} \left( (H_2 - 1)(dt \sin \theta + dx_1)^2 + dy_3^2 + dy_4^2 + dx_1^2 \right) + \sum_{i=2}^6 dx_i^2 \right], \\
\text{dC} &= d \left( \frac{H_2 - H_1}{H_{12}} \right) \wedge dy_3 \wedge dx_1 \wedge dy_4 \sin \theta + d \left( \frac{1 - H_2}{H_{12}} \right) \wedge dt \wedge dy_3 \wedge dy_4 \cos \theta \\
&\quad + dH_1^{-1} \wedge dt \wedge dy_1 \wedge dy_2 \cos \theta + *dH_1 \sin \theta,
\end{align*}
\]

where \( \dot{H}_{12} \) is defined in (3.22). This solution has a special feature that it contains both \( \dot{H}_{12} \) and \( \tilde{H}_1 \) in the metric, and is not a simple one obtained by introducing a wave in the null isometry direction as in ref. [32].

### 3.1.7 \((2M + 5M) \perp 0_w \) bound state

Another new solution is derived by

\[
(3D \perp 0_w)_B T_{y_3 \rightarrow y_1} \rightarrow (2D \perp 0_w + 4D \perp 0_w)_A \text{lift}(y_4) \rightarrow (2M \perp 0_w + 5M \perp 0_w). \tag{3.29}
\]

The solution is

\[
\begin{align*}
\text{ds}_{11}^2 &= (H_1 \dot{H}_1)^{1/3} \left[ H_1^{-1} \left( - dt^2 + dy_2^2 + dy_3^2 + (H_2 - 1)(dt + dy_3)^2 \right) \right. \\
&\quad + \dot{H}_1^{-1} (dy_1^2 + dy_4^2 + dx_1^2) + \sum_{i=2}^6 dx_i^2 \right], \\
\text{dC} &= -dH_1^{-1} \wedge dt \wedge dy_3 \wedge dy_2 \cos \theta + *dH_1 \sin \theta \\
&\quad - d\dot{H}_1^{-1} \wedge dx_1 \wedge dy_1 \wedge dy_4 \cot \theta.
\end{align*}
\]

This may be considered an orthogonal intersection of \( (2M + 5M) \) in (2.19) and \( 0_w \). In fact, this can also be obtained from (2.19) just by introducing wave in the null isometry direction [32].

### 3.1.8 \( 0_w \perp 2M \) and \( 2M \perp 0_w \) bound state

Again from (3.14) with \( \theta = 0 \), we get

\[
(2D \perp 1F)_A \xrightarrow{T_{y_2 \rightarrow y_1}} (1D \perp 1F)_B \xrightarrow{T_{y_3 \rightarrow y_3}} (0D \perp 1F + 2D \perp 0_w)_A \text{lift}(y_4) \xrightarrow{\rightarrow} (0_w \perp 2M + 2M \perp 0_w). \tag{3.31}
\]
The solution is

\[
    ds^2_{11} = \hat{H}_{12}^{1/3} \left[ \hat{H}_{12}^{-1} \left( -dt^2 + (H_2 - 1)(dt \sin \theta - dy_1)^2 + (H_1 - 1)(dt \cos \theta - dy_4)^2 \\ + dy_1^2 + dy_3^2 + dy_4^2 \right) + dy_2^2 + \sum_{i=1}^{6} dx_i^2 \right],
\]

\[
    C = \frac{H_1 - 1}{H_{12}} (dt - dy_4 \cos \theta) \wedge dy_1 \wedge dy_3 \sin \theta \\
    + \frac{1 - H_2}{H_{12}} (dt - dy_1 \sin \theta) \wedge dy_3 \wedge dy_4 \cos \theta.
\]

(3.32)

3.1.9 \(5_M \perp 0_w\) and \(5_M \perp 2_M\) bound state

From \(2D \perp 1_F\), we also get

\[
    (2D \perp 1_F)_A \xrightarrow{T_{x_1}} (3D \perp 1_F)_B \xrightarrow{T_{y_3 \rightarrow y_4}} (4D \perp 0_w + 4D \perp 1_F)_A \\
    \xrightarrow{\text{lift}(y_4)} (5_M \perp 0_w + 5_M \perp 2_M).
\]

(3.33)

The solution is

\[
    ds^2_{11} = H_1^{2/3} \hat{H}_2^{1/3} \left[ (H_1 \hat{H}_2)^{-1} \left( -dt^2 + (H_2 - 1)(dt \cos \theta + dy_3)^2 + dy_3^2 + dy_4^2 \right) \\ + H_1^{-1}(dy_1^2 + dy_2^2 + dx_1^2) + \hat{H}_2^{-1}dx_2^2 + \sum_{i=3}^{6} dx_i^2 \right],
\]

\[
    dC = d(\hat{H}_2 \sin \theta)^{-1} \wedge (dt + dy_3 \cos \theta) \wedge dx_2 \wedge dy_4 - *dH_1 \wedge dx_2.
\]

(3.34)

This is again an orthogonal intersection of \(5_M\) and \((0_w + 2_M)\) in (2.5).

This completes the two-body new solutions obtained from \(2M \perp 2M\) just by a single step of going through type IIB solutions. We now turn to bound states obtained from other marginal solutions.

3.2 Solutions obtained from \(5_M \perp 5_M\)

If we start from \(5_M \perp 5_M\) solution with the world-volume coordinates

\[
    \begin{array}{c}
        5_M \\
        \mid \mid \\
        (y_1, \ldots, y_5) \\
        \mid \mid \\
        5_M
    \end{array}
    \quad \mid \mid \mid
    \begin{array}{c}
        5_M \\
        \mid \mid \\
        (y_1, \ldots, y_5; y_6, y_7)
    \end{array}
\]

(3.35)

we get upon reduction in the directions \(y_3, y_4\) or \(x_1\)

\[
    (4D \perp 4D)_A, \quad (4D \perp 5_S)_A, \quad (5_S \perp 5_S)_A,
\]

(3.36)
or their bound state.

Applying the T-duality rule in all possible directions, we get

\[(3D \perp 3D)_B, \ (3D \perp 5D)_B, \ (5D \perp 5D)_B, \ (3D \perp 5S)_B, \ (3D \perp 0m)_B, \]
\[(5D \perp 5S)_B, \ (5D \perp 0m)_B, \ (5S \perp 5S)_B, \ (5S \perp 0m)_B, \ (0m \perp 0m)_B, \]

and their bound states. From these, after T-duality we find new solutions

\[(2D \perp 4D + 2D \perp 6D)_A, \ (2D \perp 6D + 4D \perp 6D)_A, \]
\[(2D \perp 6D + 4D \perp 6D)_A, \ (2D \perp 6D + 4D \perp 6D)_A, \]
\[(4D \perp 4D + 4D \perp 6D)_A, \ (4D \perp 4D + 6D \perp 6D)_A, \]
\[(4D \perp 6D + 6D \perp 4D)_A, \ (4D \perp 6D + 6D \perp 6D)_A, \]

which yield the following new solutions:

\[(2M \perp 5M + 2M \perp 0m), \ (2M \perp 5M + 5M \perp 0m), \]
\[(2M \perp 0m + 5M \perp 5M), \ (2M \perp 0m + 5M \perp 0m), \]
\[(5M \perp 5M + 5M \perp 0m), \ (5M \perp 5M + 0m \perp 0m), \]
\[(5M \perp 0m + 0m \perp 5M), \ (5M \perp 0m + 0m \perp 0m). \]

As a by-product of this procedure, we can read off the orthogonal intersection rules for monopole and other solutions and itself, which are discussed in a recent paper [12]. We note in particular that there are two possible intersections of two monopoles arising from that through \((6D \perp 6D)_A\) and that obtained from \((5S \perp 5S)_A\).\(^7\) For the orthogonal intersection of monopole and 2\(M\) and 5\(M\) branes, there are also other intersections that can be derived through different route from \(2M + 5M\) [12]. The explicit form of the solutions will be given below.

Solutions discussed in this subsection are all new bound states.

### 3.2.1 Solution \((5M + 0m) \perp 2M\)

We make the following transformation:

\[
(5M \perp 5M) \xrightarrow{R_{\gamma^3}} (4D \perp 4D)_A \xrightarrow{T_{\psi^6}} (5D \perp 3D)_B \xrightarrow{T_{\gamma^2-\psi^7}} (4D \perp 2D + 6D \perp 2D)_A
\]

\(^7\) Both are over 4-branes, but how the gauge fields are introduced is different.
The solution is

\[
\begin{align*}
\text{ds}_{11}^2 &= H_1^{2/3}(\tilde{H}_1 H_2)^{1/3} \left[ (H_1 H_2)^{-1}(-dt^2 + dy_1^2) + \tilde{H}_1^{-1}dy_2^2 + H_1^{-1}(dy_4^2 + dy_5^2 + dy_6^2) \\
&\quad + (\tilde{H}_1 H_2)^{-1}dy_7^2 + (H_1 \tilde{H}_1)^{-1}(dz + A_1 \sin \theta dx_1) + \sum_{i=1}^{3} dx_i^2 \right], \\
dC &= dH_2^{-1} \wedge dt \wedge dy_1 \wedge dy_7 + d(\tilde{H}_1^{-1}A_1 dx_1) \wedge dy_2 \wedge dy_7 \cos \theta \\
&\quad + d\tilde{H}_1^{-1} \wedge dy_2 \wedge dy_7 \wedge dz \cot \theta, \\
dA &= *dH_1,
\end{align*}
\]

(3.41)

where \(\tilde{H}_1\) is as defined in (2.13) with \(H\) replaced by \(H_1\). This solution may be understood as the orthogonal intersection of the bound state \((5_M + 0_m)\) given in (2.14) and \(2_M\).

### 3.2.2 Solution 5\(_M\) ⊥ 2\(_M\) + 0\(_m\) ⊥ 5\(_M\)

From \((5_D \perp 3_D)_B\) in (3.40), we proceed as

\[
(5_D \perp 3_D)_B \overset{T_{y_2}^{-x_1}}{\rightarrow} (4_D \perp 2_D + 6_D \perp 4_D)_A \overset{\text{lift}(z)}{\rightarrow} (5_M \perp 2_M + 0_m \perp 5_M).
\]

(3.42)

to find

\[
\begin{align*}
ds_{11}^2 &= H_1^{2/3}(\tilde{H}_1 H_2)^{1/3} \left[ (H_1 H_2)^{-1}(-dt^2 + dy_1^2) + \tilde{H}_1^{-1}dy_2^2 + H_1^{-1}(dy_4^2 + dy_5^2 + dy_6^2) \\
&\quad + dy_7^2 + H_2^{-1}dy_7^2 + (H_1 \tilde{H}_1)^{-1}(dz + A_1 \sin \theta dy_7) + \sum_{i=2}^{3} dx_i^2 \right], \\
dC &= \left[ d(\tilde{H}_1^{-1}A_1) \wedge dy_2 \wedge dy_7 \wedge dx_1 + dH_2^{-1} \wedge dt \wedge dy_1 \wedge dy_7 \right] \cos \theta \\
&\quad + *dH_1 \wedge dy_7, \\
dA &= - *dH_1 \wedge dy_2,
\end{align*}
\]

(3.43)

where \(\tilde{H}_1\) is defined in (3.17).

### 3.2.3 Solution 5\(_M\) ⊥ 5\(_M\) + 0\(_m\) ⊥ 2\(_M\)

Another transformation on \((5_D \perp 3_D)_B\) yields

\[
(5_D \perp 3_D)_B \overset{T_{y_2}^{-x_1}}{\rightarrow} (4_D \perp 4_D + 6_D \perp 2_D)_A \overset{\text{lift}(z)}{\rightarrow} (5_M \perp 5_M + 0_m \perp 2_M).
\]

(3.44)

\(^8\) In the last line of (3.41), \(A\) is a 1-form \(A_i dx_i\). The same notation is used in the following as well.
The solution is
\[ ds^2_{11} = H_1^{2/3} (\hat{H}_{12} H_2)^{1/3} \left[ (H_1 H_2)^{-1} (-dt^2 + dy_1^2 + dy_2^2) + H_1^{-1} (dy_4^2 + dy_5^2) + \hat{H}_2^{-1} (dy_6^2 + dy_7^2) + (H_1 \hat{H}_{12})^{-1} (dz + A_i \sin \theta dx_i)^2 + \sum_{i=1}^{3} dx_i^2 \right], \]
\[ dC = \left[ d(A, dx_1 H_2 \hat{H}_{12}^{-1}) \wedge dy_6 \wedge dy_7 + *dH_2 \wedge dy_4 \wedge dy_5 \right] \cos \theta \nonumber \]
\[ + dH_2^{-1} \wedge dt \wedge dy_1 \wedge dy_2 \sin \theta + d \left( \frac{H_2 - H_1}{H_{12}} \right) \wedge dy_6 \wedge dy_7 \wedge dz \sin \theta \cos \theta, \]
\[ dA = *dH_1, \] (3.45)

where \( \hat{H}_{12} \) is given in (3.22).

### 3.2.4 Solution \( 0_m \perp (2_M + 5_M) \)

Another solution is obtained by
\[ (5_D \perp 3_D) \xrightarrow{T_{g_3^{-x_1}}} (6_D \perp 2_D + 6_D \perp 4_D)_A \xrightarrow{\text{lift}(z)} (0_m \perp 2_M + 0_m \perp 5_M). \] (3.46)

The result is
\[ ds^2_{11} = (\hat{H}_2 H_2)^{1/3} \left[ H_2^{-1} (-dt^2 + dy_1^2 + dy_2^2) + dy_4^2 + dy_5^2 + dy_6^2 \right. \]
\[ + \hat{H}_2^{-1} dy_7^2 + H_1 \hat{H}_2^{-1} dx_1^2 + (H_1 \hat{H}_2)^{-1} (dz + A_i dx_i)^2 + H_1 \sum_{i=2}^{3} dx_i^2 \left. \right], \]
\[ dC = *dH_2 \wedge dy_4 \wedge dy_5 \wedge dy_6 \sin \theta + dH_2^{-1} \wedge dt \wedge dy_1 \wedge dy_2 \cos \theta \nonumber \]
\[ + d\hat{H}_2^{-1} \wedge dy_7 \wedge dx_1 \wedge dz \cot \theta, \]
\[ dA = - *dH_1 \wedge dx_1. \] (3.47)

This may be understood as an orthogonal intersection of \( 0_m \) and \( (2_M + 5_M) \) in (2.19).

From the solutions discussed so far, we can read off the orthogonal intersection rules for the monopole with \( 5_M \) and \( 2_M \); the intersections are over 2- and 5-branes, respectively, in the above solutions. (There are also other cases obtained from \( 2_M \perp 5_M \).)

### 3.2.5 Solution \( (5_M + 0_m) \perp 5_M \)

The last one from \( (5_D \perp 3_D)_B \) is
\[ (5_D \perp 3_D)_B \xrightarrow{T_{g_3^{-x_1}}} (4_D \perp 4_D + 6_D \perp 4_D)_A \xrightarrow{\text{lift}(z)} (5_M \perp 5_M + 0_m \perp 5_M). \] (3.48)
The solution is
\[ ds_{11}^2 = (H_1 H_2)^{2/3} \bar{H}_1^{1/3} \left[ (H_1 H_2)^{-1}(-dt^2 + dy_1^2 + dy_2^2) + H_1^{-1}(dy_4^2 + dy_5^2) + (\bar{H}_1 H_2)^{-1}dy_6^2 \right. \\
+ H_2^{-1}dy_7^2 + \bar{H}_1^{-1}dx_1^2 + (H_1 H_2 \bar{H}_1)^{-1}(dz + A_{y_7} \sin \theta dy_7)^2 + \sum_{i=2}^3 dx_i^2 \right], \]
\[ dC = d(A_4 dx \bar{H}_1^{-1}) \wedge dy_6 \wedge dy_7 \wedge dx_1 \cos \theta + *dH_2 \wedge dy_4 \wedge dy_5 \wedge dx_1 \]
\[ + d\bar{H}_1^{-1} \wedge dy_6 \wedge dx_1 \wedge dz \cot \theta, \]
\[ dA = -*dH_1 \wedge dy_7. \]
\[ (3.49) \]

This may be regarded as an orthogonal intersection of \((5_M + 0_m)\) in \((2.14)\) and \(5_M\).

3.2.6 Solution \(5_M \perp 5_M + 0_M \perp 0_m\)

From \((4_D \perp 4_D)_A\) in \((3.40)\), we take the route
\[ (4_D \perp 4_D)_A \xrightarrow{T_{x_1}^x} (5_D \perp 5_D)_B \xrightarrow{T_{x_1}^x} (4_D \perp 4_D + 6_D \perp 6_D)_A \xrightarrow{\text{lift}(z)} (5_M \perp 5_M + 0_M \perp 0_m). \]
\[ (3.50) \]

The solution is
\[ ds_{11}^2 = (H_1 H_2)^{2/3} \bar{H}_{12}^{1/3} \left[ (H_1 H_2)^{-1}(-dt^2 + dy_1^2 + dy_2^2) + \bar{H}_{12}^{-1}(dx_1^2 + dx_2^2) + H_1^{-1}(dy_4^2 + dy_5^2) + H_2^{-1}(dy_6^2 + dy_7^2) \right. \\
+ (H_1 H_2 \bar{H}_{12})^{-1} \{ dz + \sin \theta (A_{y_5} dy_5 + B_{y_7} dy_7) \}^2 + dx_3^2 \right], \]
\[ dC = d\left[ (A_{y_5} dy_5 + B_{y_7} dy_7) \bar{H}_{12}^{-1} \right] \wedge dx_1 \wedge dx_2 \cos \theta \]
\[ + d \left( \frac{1 - H_1 H_2}{\bar{H}_{12}} \right) \wedge dx_1 \wedge dx_2 \wedge dz \sin \theta \cos \theta, \]
\[ dA_{y_5} = \partial_{x_3} H_2 dy_4, \quad dB_{y_7} = \partial_{x_3} H_1 dy_6, \]
\[ (3.51) \]

where \(\bar{H}_{12}\) is defined in \((3.17)\) and the monopole gauge field 1-forms \(A\) and \(B\) live in the spaces \((x_3, y_1, y_5)\) and \((x_3, y_6, y_7)\), respectively. Here we have chosen the gauge \(A_{x_3} = A_{y_6} = B_{x_3} = B_{y_4} = 0\) (this is possible because the harmonic functions only depend on the coordinate \(x_3\)). For \(\theta = \frac{\pi}{2}\), this agrees with the orthogonal intersection for monopoles given in \([12]\):
\[ ds_{11}^2 = -dt^2 + dy_1^2 + dy_2^2 + dx_1^2 + dx_2^2 + H_2(dy_4^2 + dy_5^2) + H_1(dy_6^2 + dy_7^2) \]
\[ + (H_1 H_2)^{-1}(dz + A_{y_5} dy_5 + B_{y_7} dy_7)^2 + H_1 H_2 dx_3^2. \]
\[ (3.52) \]
For completeness, we also record another possible intersection obtained from $5_S + 5_S$:

$$ds_{11}^2 = -dt^2 + dy_1^2 + \cdots + dy_6^2 + H_1 dy_7^2 + H_2 dy_6^2 + H_1^{-1} (dz + \sum_{i=1}^{3} A_i dy_i)^2$$

$$+ H_2^{-1} (dz + \sum_{i=1}^{3} B_i dy_i)^2 + H_1 H_2 (dx_1^2 + dx_2^2),$$

where the harmonic functions depend on $x_1$ and $x_2$.

### 3.2.7 Solution $5_M \perp 0_m + 0_m \perp 5_M$

Another from $(5_D \perp 5_D)_B$ in (3.50) is

$$(5_D \perp 5_D)_B \xrightarrow{T_{y_2}^{-y_1}} (4_D \perp 6_D + 6_D \perp 4_D)_A \xrightarrow{\text{lift}(z)} (5_M \perp 0_m + 0_m \perp 5_M).$$

The solution is

$$ds_{11}^2 = (H_1 H_2)^{2/3} \hat{H}_{12}^{1/3} \left[ (H_1 H_2)^{-1} (-dt^2 + dy_1^2 + dy_2^2 + dx_1^2) + H_1^{-1} dy_4^2 
+ H_2^{-1} dy_6^2 + \hat{H}_{12}^{-1} (dy_5^2 + dy_7^2) 
+ (H_1 H_2 \hat{H}_{12})^{-1} (dz - \cos \theta A_{y_4} dy_4 + \sin \theta B_{y_6} dy_6)^2 + \sum_{i=2}^{3} dx_i^2 \right],$$

$$dC = d \left[ \sin \theta H_1 (A_{y_4} dy_4 - dz \cos \theta) + \cos \theta H_2 (B_{y_6} dy_6 + dz \sin \theta) \right] \hat{H}_{12}^{-1} \wedge dy_5 \wedge dy_7,$$

$$dA_{y_4} = -* dH_2, \quad dB_{y_6} = -* dH_1,$$

where we have made a gauge choice similar to the above case.

### 3.2.8 Solution $0_m \perp (5_M + 0_m)$

The last solution in this series is obtained by

$$(5_D \perp 5_D)_B \xrightarrow{T_{y_2}^{-y_1}} (6_D \perp 4_D + 6_D \perp 4_D)_A \xrightarrow{\text{lift}(z)} (0_m \perp 5_M + 0_m \perp 0_m).$$

The solution is

$$ds_{11}^2 = H_2^{2/3} \hat{H}_{2}^{1/3} \left[ H_2^{-1} (-dt^2 + dy_1^2 + dy_2^2 + dx_1^2) + dy_4^2 + dy_5^2 + H_1 H_2^{-1} dy_6^2 
+ \hat{H}_2^{-1} dy_7^2 + H_1 \hat{H}_{2}^{-1} dx_2^2 + (H_1 H_2 \hat{H}_{2})^{-1} (dz + \sin \theta A_{y_4} dy_4 + B_{y_6} dy_6)^2 + H_1 dx_3^2 \right],$$

$$dC = d \left[ \sin \theta A_{y_4} dy_4 + B_{y_6} dy_6 + dz \hat{H}_2^{-1} \right] \wedge dy_7 \wedge dx_2 \cot \theta,$$

$$dA_{y_4} = -\partial_{x_3} H_2 dy_5, \quad dB_{y_6} = -\partial_{x_3} H_1 dx_2.$$
This can be understood as an orthogonal intersection of $0_m$ and $(5_M + 0_m)$ in (2.14).

### 3.3 Solutions obtained from $2_M \perp 5_M$

We summarize solutions obtained from $2_M \perp 5_M$. Consider

\[ 2_M \perp 5_M \quad \text{with} \quad (y_{1,2}) \quad \text{in} \quad (2.14), \]

and we get upon reduction in the directions $y_1, y_2, y_3$ or $x_1$

\[ (1_F \perp 4_D)_{A(1)}, (1_F \perp 5_S)_{A(1)}, (2_D \perp 4_D)_{A(2)}, (2_D \perp 5_S)_{A(2)}, \]

or their bound state.

Applying the T-duality rule in all possible directions, we get

\[ (0_w \perp 5_D)_{B(1)}, (1_F \perp 5_D)_{B(1)}, (0_w \perp 5_S)_{B(2)}, (1_F \perp 5_S)_{B(2)}, \]
\[ (1_F \perp 0_m)_{B(3)}, (1_D \perp 5_D)_{B(3)}, (1_D \perp 5_S)_{B(4)}, (1_D \perp 0_m)_{B(4)}, \]

and their bound states. From these, after T-duality we find new solutions

\[ (1_F \perp 4_D + 0_w \perp 6_D)_{A(5)}, (0_w \perp 4_D + 0_w \perp 6_D)_{A(6)}, \]
\[ (0_w \perp 6_D + 1_F \perp 6_D)_{A(7)}, (0_w \perp 6_S + 1_F \perp 0_m)_{A(8)}, \]

which yield the following new solutions in 11 dimensions:

\[ (2_M \perp 5_M + 0_w \perp 0_m), (0_w \perp 5_M + 0_w \perp 0_m), \]
\[ (0_w \perp 0_m + 2_M \perp 0_m), (0_w \perp 5_M + 2_M \perp 0_m). \]

We refrain from giving explicit metrics since it is straightforward to derive them once the routes and possible solutions are given.

### 3.4 Other bound state solutions

It is clear that we can obtain many bound states by repeating the above procedure starting from other possible solutions. The bound states thus obtained include any two combinations of the solutions

\[ 0_w \perp 2_M, \ 0_w \perp 5_M, \ 0_w \perp 0_m, \ 2_M \perp 2_M, \ 2_M \perp 5_M, \]
\[ 2_M \perp 0_m, \ 5_M \perp 5_M, \ 5_M \perp 0_m, \ 0_m \perp 0_m. \]
If we also include angles at each steps of reductions and dualities, we can have more general non-marginal solutions interpolating these solutions.

In the above examples, we have not continued making dualities to produce more solutions. However, it is not our purpose here to exhaust these bound states but to present examples of typical solutions and a systematic method of producing general non-marginal solutions which we believe are useful in searching for the construction rules for how to construct non-marginal solutions directly in 11 dimensions. We will make an attempt to formulate it in sect. 5.

Starting from three-charge solutions like $2_M \perp 2_M \perp 2_M$, we can similarly construct bound states of these solutions involving orthogonal intersections of all fundamental solutions summarized in the introduction. Since the technique is now fairly clear, we do not give explicit examples of these cases.

4 Solutions with tilted branes

We have considered only reductions and T-dualities with rotations among space coordinates. It is then natural to consider more general reductions including boost which mixes time and space coordinates. Let us discuss the effect of boost in some detail since there is not much discussion. We will show that the resulting solutions are bound states with waves, some of which can also be obtained by the T-duality transformations in previous sections. However, we have not considered further T-duality transformations of these solutions. We now show that T-duality on such solutions introduces titling of branes.

4.1 Single brane

We consider the $2_M$ brane and introduce the boost along one of the transverse direction

$$
t \rightarrow t \cosh \beta - x_1 \sinh \beta, \quad x_1 \rightarrow -t \sinh \beta + x_1 \cosh \beta.
$$

There are also solutions given in ref. 34, but they reduce to orthogonal intersections for single center case, and are not discussed here.
(Boost on the world-volume direction is trivial for single brane.) The metric $g_{x_1 x_1}$ becomes

$$g_{x_1 x_1} = H^{-2/3}(- \sinh^2 \beta + H \cosh^2 \beta). \tag{4.2}$$

This is well-defined for $\beta = 0$, but not for $\beta = \infty$. To make both limits well-defined, we should keep $Q \cosh^2 \beta$ finite. A convenient way to do this is to replace the harmonic function $H$ in our solution by $\tilde{H}$ and also put

$$\cosh \beta = \frac{1}{\sin \theta}, \quad \sinh \beta = \frac{\cos \theta}{\sin \theta}. \tag{4.3}$$

Namely we reduce the charge according to the boost. One then finds

$$ds_{11}^2 = \tilde{H}_\theta^{1/3} \left[ \tilde{H}_\theta^{-1} \left( -dt^2 + dx_1^2 + (H - 1)(dt \cos \theta + dx_1)^2 + dy_1^2 + dy_2^2 \right) \right. $$

$$+ \left. \sum_{i=2}^8 dx_i^2 \right], \quad C = \frac{(1 - H) \sin \theta}{\tilde{H}_\theta} (dt - dx_1 \cos \theta) \wedge dx_2 \wedge dx_1, \tag{4.4}$$

basically the same solution as (2.5), a bound state of wave and $2_M$-brane. This is the general feature and one always gets bound states with wave if one starts from other solutions. In fact, from $5_M$-brane, one finds $(0_w + 5_M)$ in (2.7), and from wave one has $(0_w + 0_w)$:

$$ds_{11}^2 = -dt^2 + dy_1^2 + dy_2^2 + (H - 1)(dt + \cos \theta dy_1 + \sin \theta dy_2)^2 + \sum_{i=1}^8 dx_i^2. \tag{4.5}$$

Since such bound states can also be obtained by angled T-duality and lifting discussed in sect. 2, we do not need to consider such a boosted reduction.

We can consider further T-duality of these solutions. For example, we can make reduction to (4.4) in $x_1$ to get a type IIA solution, which can be interpreted as a bound state of $(0_D + 2_D)_A$ [24]. For general angle $\theta$ we have background $B^{(1)}$ which will produce off-diagonal metrics after T-duality. However, we find that such off-diagonal metrics can be removed by space rotations in 11 dimensions, and the solution is equivalent to a trivial rotated $2_M + 2_M$ bound state in (2.1). This is to be expected since the boost has the effect of rotating objects lying in transverse direction with respect to it, but such a rotation of a single brane can be removed by a coordinate rotation.
We can repeat the same analysis for 5M brane (1.2). After the reduction, one gets a bound state of \((0_D + 5S)_A\) [24]. One finds in this case again similar bound state solutions with apparent off-diagonal metrics which can be removed.

4.2 Tilted \((2_M + 2_M \perp 2_M)\) brane

Starting from the \(2_M \perp 2_M\) solution

\[
ds^2_{11} = (\tilde{H}_1 \tilde{H}_2)^{1/3} \left[ -(\tilde{H}_1 \tilde{H}_2)^{-1} dt^2 + \tilde{H}_1^{-1} (dy_1^2 + dy_2^2) + \tilde{H}_2^{-1} (dy_3^2 + dy_4^2) + \sum_{i=1}^{6} dx_i^2 \right],
\]

\[
C = \frac{1 - \tilde{H}_1}{\tilde{H}_1} dt \wedge dy_1 \wedge dy_2 + \frac{1 - \tilde{H}_2}{\tilde{H}_2} dt \wedge dy_3 \wedge dy_4,
\]

there are two possibilities to introduce boost in 11 dimensions. The first one is to do it in the transverse direction \(x_1\) which is discussed in ref. [26], and this again gives a bound state of \((0_w + 2_M \perp 2_M)\). Making reduction in \(x_1\), T-duality twice along \(y_1\) and \(y_3\), one gets a tilted \((2_D + 2_D \perp 2_D)\) solution. We lift it to 11-dimensions to find tilted \((2_M + 2_M \perp 2_M)\) solution

\[
ds^2_{11} = H_1^{1/3} \left[ -H_1^{-1} dt^2 + (H_1 H_2)^{-1} \left( \tilde{H}_1 dy_1 + (H_1 - 1) \sin \theta \cos \theta dy_2 \right)^2 + \tilde{H}_1^{-1} dy_2^2 \\
+ (H_1 H_2)^{-1} \left( \tilde{H}_2 dy_3 + (H_2 - 1) \sin \theta \cos \theta dy_4 \right)^2 + \tilde{H}_2^{-1} dy_4^2 + \sum_{i=1}^{6} dx_i^2 \right],
\]

\[
C = \left( \frac{(H_1 - 1) \tilde{H}_2}{H_{12}} dt \wedge dy_2 \wedge dy_3 + \frac{(H_2 - 1) \tilde{H}_1}{H_{12}} dt \wedge dy_1 \wedge dy_4 \right) \sin \theta \\
+ \frac{1 - H_1}{H_{12}} dt \wedge dy_1 \wedge dy_3 \cos \theta + \frac{1}{2} \frac{(1 - H_1)(1 - H_2)}{H_{12}} dt \wedge dy_2 \wedge dy_4 \sin^2 \theta \cos \theta,
\]

where we have defined \(H_{12}\) by

\[
H_{12} = \frac{\tilde{H}_1 \tilde{H}_2 - \cos^2 \theta}{\sin^2 \theta}.
\]

For generic \(\theta\), this describes two \(2_M\)-branes, with one lying along \(y_3\) and the direction making an angle \(\frac{\pi}{2} + \theta\) with \(y_1\) and another along \(y_1\) and the direction making an angle \(\frac{\pi}{2} + \theta\) with \(y_3\). For \(\theta = 0\), \(H_{12} = H_1 + H_2 - 1\) is a harmonic function and \(\tilde{H}_1 = \tilde{H}_2 = 1\), and we have a \(2_M\)-brane with 1/2 supersymmetry. For \(\theta = \frac{\pi}{2}\), \(H_{12} = H_1 H_2\), \(\tilde{H}_1 = H_1, \tilde{H}_2 = H_2\), and we have a \(2_M \perp 2_M\), which has 1/4 supersymmetry. The reason why the number of
remaining supersymmetry changes is that we adopted the convention that the charge is sent to zero in the infinite boost limit (see (1.3)).

The second possibility is to make boost in \( y_4 \) direction to produce a bound state with wave. Then making reduction in \( y_4 \), T-duality twice along \( y_1 \) and \( x_1 \), we get a slightly different tilted \((2_D + 2_D \perp 2_D)\) solution, which can be lifted to 11 dimensions:

\[
ds_{11}^2 = (H_1 \tilde{H}_2)^{1/3} \left[ -(H_1 \tilde{H}_2)^{-1} dt^2 + (H_1 \tilde{H}_1)^{-1} \left( \tilde{H}_1 dy_1 + (H_1 - 1) \sin \theta \cos \theta dy_2 \right)^2 \right. \\
+ \tilde{H}_1^{-1} dy_2^2 + \tilde{H}_2^{-1} (dy_3^2 + dy_4^2) + H_1^{-1} dx_i^2 + \sum_{i=2}^{6} dx_i^2 \left] , \\
C = \frac{1 - H_1}{H_1} dt \wedge dx_1 \wedge (-dy_1 \cos \theta + dy_2 \sin \theta) + \frac{1 - H_2}{H_2} \sin^2 \theta dt \wedge dy_3 \wedge dy_4. \tag{4.9}
\]

Compared with the above boost, this is a solution in which one of the \( 2_M \) brane is not tilted.

In this solution, instead of replacing \( H_2 \) by \( \tilde{H}_2 \), we could have also kept \( H_2 \). The resulting solution is then tilted \((2_D \perp 2_D)\) solution obtained by replacing \( \tilde{H}_2 \) in (4.9) by \( H_2 \).

### 4.3 General features of the solutions

We have seen the effect of boost is just to produce bound states of wave and the original solutions. This feature is valid not only in 11 dimensions but also in 10 dimensions. This means that T-duality after boost is equivalent to making T-duality to bound states with waves. Then making reductions and T-dualities yields solutions in which branes are tilted.

These solutions can be understood from the fact that the boost generally introduces rotations to objects lying in transverse directions with respect to it. In fact, we have made boost in the direction transverse to both the \( 2_M \)-branes in the solution (4.7), and we get both branes tilted. In the second solution (4.9), we have made it along one of the \( 2_M \)-brane, and that \( 2_M \)-brane in the resulting solution is not tilted whereas the other is.

We can continue to make similar boost, reduction and dualities with other configurations such as \( 2_M \perp 5_M \) or \( 5_M \perp 5_M \). This has been discussed in ref. [23] for \( 2_M \perp 5_M \) and in ref. [26] for \((4_D \perp 4_D)_A\) (directly connected to \( 5_M \perp 5_M \)). The procedure described
in the previous section is general, and similar tilted solutions can be obtained from more
general orthogonal intersections of M-branes like $2_M \perp 2_M \perp 2_M$ by including boost
(or wave). In all these cases, again the basic structure of the resulting solutions are the
same as the solutions (4.7) and (4.9) discussed in the previous subsection, with angles
between the branes, and how they are tilted are determined by which boost (or wave)
one introduces.

Though we did not continue T-dualities on solutions with waves in sect. 3, we expect
that it produces further tilted solutions.

5 Construction Rules

In this section, we discuss the construction rules for 11-dimensional solutions. From all
the examples we have discussed in this paper, we find that the following rules for the
non-marginal non-tilted solutions are valid (the rules for tilted ones are not discussed
here):

1. To each fundamental $p_i$-brane solution, we assign a harmonic function $H_i$ depending
   on the transverse coordinates, and multiply its inverse to the metric of the coordi-
   nates belonging to the $p_i$-brane in the conformal frame in which the transverse part
   $\sum_i dx_i^2$ is free. The rules for other solutions of wave and monopole are similar.

2. $p$-brane can intersect orthogonally with $p$-brane over $(p-2)$-brane: wave can inter-
   sect with others over a string: A 2-brane can intersect orthogonally with 5-brane
   over string, and with monopole over 2-brane or 0-brane: A 5-brane can intersect
   orthogonally with monopole over 5-brane or 3-brane: A monopole can intersect
   orthogonally with monopole over 4-brane. These rules can be read off from the
   solutions we discussed and are also given in ref. [12].

3. To each non-marginal solution with one charge we discussed in sect. 2, we assign a
   modified harmonic function $\tilde{H}_i$ defined in (2.3). If the solution is among intersect-
   ing constituents, we multiply its inverse to the metric of the relevant coordinates
   such that putting the solutions to the fundamental ones reduces the configuration
compatible with the above rules. To construct further bound states, we use further modified one \((2.22)\).

4. For the non-marginal solutions with two charges, the rules are as follows: Let us call \(0_w \rightarrow 2_M \rightarrow 5_M \rightarrow 0_m\) proper order. Suppose that the solution is of the type \(A_1 \perp B_1 + A_2 \perp B_2\). If the orders of the solutions \((A_1, A_2)\) and \((B_1, B_2)\) are both in the same order, proper or its opposite, we use the combination \(\tilde{H}_{12}\) defined as in \((3.17)\); if the two orders are opposite, we use the combination \(\hat{H}_{12}\) defined in \((3.22)\). The solutions must be consistent with the orthogonal rules if one puts the solutions to the orthogonal ones, and also should agree with the bound states discussed in sect. 2 if one puts one of the charges to zero.

Rules 1 and 2 are the ordinary orthogonal intersection rules. Rule 3 is a generalization of that given in ref. [23] for intersections involving the non-marginal one-charge bound state \((2_M + 5_M)\). Though rules 3 and 4 appear to be ambiguous, the rules are actually useful enough to derive the solutions we discussed in sects. 2 and 3. We now illustrate how to use these rules by several examples.

Consider the solution \(2_M \perp 2_M + 5_M \perp 5_M\). For \(\theta = 0\), we know from the orthogonal intersection rules that we should assign the coordinates to each brane as follows:

\[
\begin{align*}
2_M & \quad \perp \quad 2_M \\
(y_1, y_2) & \quad \perp \quad (y_3, y_4).
\end{align*}
\]

\((5.1)\)

For \(\theta = \frac{\pi}{2}\), this must be \(5_M \perp 5_M\) with coordinates

\[
\begin{align*}
5_M & \quad \perp \quad 5_M \\
(y_1, y_2, y_5, y_6, y_7) & \quad \perp \quad (y_3, y_4, y_5, y_6, y_7),
\end{align*}
\]

\((5.2)\)

intersecting over 3-brane. These are the only possible configurations to make the number of world-volume coordinates minimum. For the first case, the metric for each coordinates must change as

\[
\begin{align*}
H_1^{-1}(dy_1^2 + dy_2^2) + H_2^{-1}(dy_3^2 + dy_4^2) + dy_5^2 + dy_6^2 + dy_7^2 \\
\rightarrow \quad H_1^{-1}(dy_1^2 + dy_2^2) + H_2^{-1}(dy_3^2 + dy_4^2) + (H_1 H_2)^{-1}(dy_5^2 + dy_6^2 + dy_7^2).
\end{align*}
\]

\((5.3)\)

\(^{10}\)Here and below, we discuss metrics up to an overall conformal factor because it is easily determined by a similar reasoning.
According to the rule 4 above, we can use $\tilde{H}_{12}$ to reproduce this metric and this gives the solution given in (3.16). The second possibility is excluded because putting $H_1 = 1$ reduces the solution to $(2_M + 5_M)$ state but incompatible with (2.19). Our rule does forbid this because we cannot reproduce the necessary metric change by $\tilde{H}_{12}$.

Another example is $2_M \perp 5_M + 5_M \perp 2M$. Let us assign the coordinates as

$$2_M \perp 5_M \perp 0_w \rightarrow 5_M \perp 0_w , \text{ or } 5_M \perp 0_w . \quad (5.4)$$

According to the rule 4, we should use $\hat{H}_{12}$ to reproduce the appropriate metric change

$$(H_1 H_2)^{-1} dy_1^2 + H_1^{-1} dy_2^2 + H_2^{-1} (dy_3^2 + \cdots + dy_6^2)$$

$$\rightarrow (H_1 H_2)^{-1} dy_1^2 + H_1^{-1} dy_2^2 + H_2^{-1} dy_3^2 + H_1^{-1} (dy_4^2 + \cdots + dy_6^2), \quad (5.5)$$

and we are lead to the solution given in (3.21).

A more subtle case is the solution $2_M \perp 2M + 5_M \perp 0_w$. In this case, we can consider two possible choices for the coordinates:

$$2_M \perp 2_M \rightarrow 5_M \perp 0_w , \text{ or } 5_M \perp 0_w . \quad (5.6)$$

The first case is (3.28), but we have not encountered the second. For the first one, the metric should change as

$$-(H_1 H_2)^{-1} dt^2 + H_1^{-1} (dy_1^2 + dy_2^2) + H_2^{-1} (dy_3^2 + dy_4^2) + dy_5^2$$

$$\rightarrow H_1^{-1} \left[ -dt^2 + dy_1^2 + \cdots + dy_5^2 + (H_2 - 1)(dt + dy_5)^2 \right] . \quad (5.7)$$

According to our rule 4, we should use $\hat{H}_{12}$ for these solutions. However, the metric for the time coordinate can be reproduced only if we use $\tilde{H}_1$ as well in the numerator as $(H_1 \tilde{H}_{12})^{-1} H_1$. Thus the precise rule seem to be that only the inverse of the functions listed in rule 4 should be multiplied to the metric, as in rule 1. By this rule we precisely obtain the solution (3.28). Moreover, the second possibility is excluded by considering the metric for the coordinate $y_5$, since its metric changes as $1 \rightarrow H_1^{-1}$ which can be reproduced only if one uses tilde type function in the denominator, in contradiction to the rule. Again such a solution is excluded by the consistency with the $(0_w + 2_M)$ bound state in (2.3) for $H_1 = 1$. 

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Examples of the rule 3 above are (3.19), (3.24), (3.30), (3.34), (3.41), (3.47), (3.49) and (3.57), and these solutions can be easily reproduced by the above rules. Other solutions are examples of the rule 4 above.

The generalization of these rules to solutions with more charges is straightforward. For example, it is easy to make $A_1 \perp B_1 \perp C_1 + A_2 \perp B_2 \perp C_2 + A_3 \perp B_3 \perp C_3$ type of bound states by an obvious generalization of these rules. One can also obtain orthogonal intersections of these non-marginal and marginal solutions by similar rules.

6 Discussions

We have examined various solutions produced by using the solution generating technique of reduction, T-duality and lifting. This produces in general non-marginal as well as tilted brane solutions. On this basis, we have also presented construction rules for these non-marginal solutions. We have not included S-duality in these discussions. Let us now discuss what happens if the S-duality is also taken into account.

It is easy to see how the S-duality transforms 10-dimensional solutions into others by using the rules given in [16]. For type IIB solutions, this is almost trivial. We find

$$1_D \leftrightarrow 1_F, \quad 5_D \leftrightarrow 5_S,$$

and other solutions $3_D, 7_D, 0_m$ and wave are invariant. For type IIA solutions, we find

$$\begin{align*}
1_F & \quad \rightarrow & 1_F & \quad \rightarrow & 1_F & \quad \rightarrow & 1_F & \quad \rightarrow & 1_F & \quad \rightarrow & 1_F & \quad \rightarrow & 1_F & \quad \rightarrow & 1_F \\
2_D & \quad \rightarrow & 2_D & \quad \rightarrow & 2_D & \quad \rightarrow & 2_D & \quad \rightarrow & 2_D & \quad \rightarrow & 2_D & \quad \rightarrow & 2_D & \quad \rightarrow & 2_D \\
4_D & \quad \rightarrow & 4_D & \quad \rightarrow & 4_D & \quad \rightarrow & 4_D & \quad \rightarrow & 4_D & \quad \rightarrow & 4_D & \quad \rightarrow & 4_D & \quad \rightarrow & 4_D \\
5_S & \quad \rightarrow & 5_S & \quad \rightarrow & 5_S & \quad \rightarrow & 5_S & \quad \rightarrow & 5_S & \quad \rightarrow & 5_S & \quad \rightarrow & 5_S & \quad \rightarrow & 5_S \\
6_D & \quad \rightarrow & 6_D & \quad \rightarrow & 6_D & \quad \rightarrow & 6_D & \quad \rightarrow & 6_D & \quad \rightarrow & 6_D & \quad \rightarrow & 6_D & \quad \rightarrow & 6_D \\
0_w & \quad \rightarrow & 0_w & \quad \rightarrow & 0_w & \quad \rightarrow & 0_w & \quad \rightarrow & 0_w & \quad \rightarrow & 0_w & \quad \rightarrow & 0_w & \quad \rightarrow & 0_w \\
0_m & \quad \rightarrow & 0_m & \quad \rightarrow & 0_m & \quad \rightarrow & 0_m & \quad \rightarrow & 0_m & \quad \rightarrow & 0_m & \quad \rightarrow & 0_m & \quad \rightarrow & 0_m \\
0_D & \quad \rightarrow & 0_D & \quad \rightarrow & 0_D & \quad \rightarrow & 0_D & \quad \rightarrow & 0_D & \quad \rightarrow & 0_D & \quad \rightarrow & 0_D & \quad \rightarrow & 0_D
\end{align*}$$

(6.1)

where $y(x)$ stands for world-volume (transverse) direction. We thus see that the effect of S-duality is the same as the reduction and lifting discussed in the introduction (1.5)–(1.8) and hence at least part of the effect of S-duality is incorporated if we find solutions in 11-dimensional theory.

\[\text{\textsuperscript{11}}\text{From the 11-dimensional perspective, S-duality can be understood as the interchange of the 10-th and 11-th coordinates [14], and thus this is a simple consequence of this fact.}\]
Let us discuss the general feature of the ADM mass formula for these non-tilted non-marginal solutions with unbroken supersymmetries. It is not difficult to show that the non-marginal solutions with one charge typically have the mass formula with single term like

\[ m \sim \sqrt{Q_1^2 + Q_2^2 + \cdots}, \]  

(6.3)

where the number of charges depends on how many electric and magnetic charges are involved in the solution. For example, the three-body solution (2.21) has

\[ Q_e = Q \sin \theta_1 \cos \theta_2, \quad Q_m = Q \sin \theta_1 \sin \theta_2, \quad Q_3 = Q \cos \theta_1, \]  

(6.4)

and

\[ m \sim \sqrt{Q_e^2 + Q_m^2 + Q_3^2}. \]  

(6.5)

For the non-marginal solutions with two charges, the formula typically takes the sum of two terms

\[ m \sim \sqrt{Q_1^2 + Q_2^2 + \cdots} + \sqrt{Q_1'^2 + Q_2'^2 + \cdots}, \]  

(6.6)

For example, we get from (3.16)

\[ Q_{e1} = Q_1 \cos \theta, \quad Q_{e2} = Q_2 \cos \theta, \quad Q_{m1} = Q_1 \sin \theta, \quad Q_{m2} = Q_2 \sin \theta, \]  

(6.7)

so that the mass is given by

\[ m \sim \sqrt{Q_{e1}^2 + Q_{m1}^2 + Q_{e2}^2 + Q_{m2}^2}. \]  

(6.8)

We can continue this analysis for three-charge solutions and so on. Given such a large number of classical non-marginal solutions with unbroken supersymmetries, it would be interesting to examine their quantum properties. In this connection, we point out that all the non-marginal solutions found in this paper have an interesting property. If we calculate the determinant of the metrics for the nine-space part, which are relevant to the nine-area calculation, we find that tilde or hatted functions all cancel out from the expression for non-marginal solutions discussed in sects. 2 and 3. This means that
the nine-area or entropy for these non-marginal solutions are independent of the angles which characterize the non-marginality. This is equivalent to T-duality invariance and we expect the same property is valid for other non-marginal solutions involving more than two charges.

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**A Appendix: Reduction, T-duality and lifting rules**

In this appendix, we summarize the reduction, T-duality and lifting rules which are heavily used in the text. We use the same convention as ref. 16.

11 D SUGRA → IIA:

From 11-dimensional supergravity with the metric $\hat{g}_{\hat{\mu}\hat{\nu}}$ and a 3-form $\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}$, upon dimensional reduction in $y$, we get type IIA supergravity with a metric, 3-, 2- and 1-forms and dilaton:

\[
\begin{align*}
\hat{g}_{\hat{\mu}\hat{\nu}} &= (\hat{g}_{yy})^{\frac{1}{2}} (\hat{g}_{\hat{\mu}\hat{\nu}} - \hat{g}_{\hat{\mu}y}\hat{g}_{\hat{\nu}y}/\hat{g}_{yy}) , \\
\hat{A}^{(1)}_{\hat{\mu}} &= \hat{g}_{\hat{\mu}y}/\hat{g}_{yy} , \\
\hat{\phi} &= \frac{3}{4} \log (\hat{g}_{yy}) .
\end{align*}
\]

(A.1)

IIA → 11 D SUGRA:

Conversely 11-dimensional supergravity is recovered by the formula

\[
\begin{align*}
\hat{g}_{\hat{\mu}\hat{\nu}} &= e^{-\frac{2}{3}\hat{\phi}} \hat{g}_{\hat{\mu}\hat{\nu}} + e^{\frac{4}{3}\hat{\phi}} \hat{A}^{(1)}_{\hat{\mu}} \hat{A}^{(1)}_{\hat{\nu}} , \\
\hat{g}_{\hat{\mu}y} &= e^{\frac{4}{3}\hat{\phi}} \hat{A}^{(1)}_{\hat{\mu}} , \\
\hat{g}_{yy} &= e^{\frac{4}{3}\hat{\phi}} .
\end{align*}
\]

(A.2)

**T-duality (type–IIB → type–IIA):**

\(^{12}\)S-duality is manifest in 11 dimensions, so that the entropy is U-invariant.
The T-duality rules from type–IIB to type–IIA are

\begin{align}
\hat{g}_{\mu\nu} &= \hat{j}_{\mu\nu} - \left(\hat{j}_{\mu\rho} \hat{j}_{\nu\rho} - \hat{B}_{\mu}^{(1)} \hat{B}_{\nu}^{(1)}\right) / \hat{j}_{xx}, \\
\hat{j}_{\mu\nu} &= \hat{B}_{\mu}^{(1)} / \hat{j}_{xx}, \quad \hat{g}_{xx} = 1 / \hat{j}_{xx}, \\
\hat{C}_{\mu\nu} &= \frac{2}{3} \left[\hat{B}_{\mu}^{(2)} + 2 \hat{B}_{\mu\nu} \hat{j}_{\nu\rho} / \hat{j}_{xx}\right], \\
\hat{C}_{\mu\nu\rho} &= \frac{8}{3} \hat{D}_{\mu\nu\rho} + \varepsilon^{\mu\nu\rho} \hat{B}_{\mu}^{(1)} \hat{B}_{\nu}^{(1)} + \varepsilon^{\mu\nu\rho} \hat{B}_{\mu}^{(1)} \hat{B}_{\nu}^{(1)} \hat{j}_{\nu\rho} / \hat{j}_{xx}, \\
\hat{B}_{\mu}^{(1)} &= \hat{B}_{\mu}^{(1)} + 2 \hat{B}_{\mu\nu} \hat{j}_{\nu\rho} / \hat{j}_{xx}, \quad \hat{B}_{\mu}^{(1)} = \hat{j}_{\mu} / \hat{j}_{xx}, \\
\hat{A}_{\mu}^{(1)} &= -\hat{B}^{(2)}_{\mu} + \hat{j} \hat{B}_{\mu}^{(1)}, \quad \hat{A}_{\mu}^{(1)} = \hat{\ell}, \\
\hat{\phi} &= \hat{\phi} - \frac{1}{2} \log (\hat{j}_{xx}). \quad (A.3)
\end{align}

T-duality (type–IIA → type–IIB):

The converse rules are

\begin{align}
\hat{g}_{\mu\nu} &= \hat{g}_{\mu\nu} - \left(\hat{g}_{\mu\rho} \hat{g}_{\nu\rho} - \hat{B}_{\mu}^{(1)} \hat{B}_{\nu}^{(1)}\right) / \hat{g}_{xx}, \\
\hat{g}_{\mu\nu} &= \hat{B}_{\mu}^{(1)} / \hat{g}_{xx}, \quad \hat{g}_{xx} = 1 / \hat{g}_{xx}, \\
\hat{D}_{\mu\nu\rho} &= \frac{8}{3} \left[\hat{C}_{\mu\nu\rho} - \hat{A}_{\mu \nu} \hat{B}_{\nu}^{(1)} + \hat{g}_{\mu\nu} \hat{B}_{\nu}^{(1)} \hat{A}_{\mu}^{(1)} / \hat{g}_{xx} - \frac{3}{2} \hat{g}_{\mu\nu} \hat{C}_{\nu\rho} / \hat{g}_{xx}\right], \\
\hat{B}_{\mu}^{(1)} &= \hat{B}_{\mu}^{(1)} + 2 \hat{g}_{\mu\nu} \hat{B}_{\nu}^{(1)} / \hat{g}_{xx}, \quad \hat{B}_{\mu}^{(1)} = \hat{g}_{\mu} / \hat{g}_{xx}, \\
\hat{B}_{\mu}^{(2)} &= \frac{2}{3} \hat{C}_{\mu\nu\rho} - 2 \hat{A}_{\mu \nu} \hat{B}_{\nu}^{(1)} + 2 \hat{g}_{\mu\nu} \hat{B}_{\nu}^{(1)} \hat{A}_{\mu}^{(1)} / \hat{g}_{xx}, \\
\hat{B}_{\mu}^{(2)} &= -\hat{A}_{\mu}^{(1)} + \hat{A}_{\mu}^{(1)} \hat{g}_{\mu} / \hat{g}_{xx}, \\
\hat{\phi} &= \hat{\phi} - \frac{1}{2} \log (\hat{g}_{xx}), \quad \hat{\ell} = \hat{A}_{\mu}^{(1)}. \quad (A.4)
\end{align}

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