Simulating matched filter detection of ionized bubble around a quasar in the epoch of reionization

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Abstract. The recent discovery of the $z = 7.085$ quasar has opened up a new window to peep into the inter galactic medium during the epoch of reionization (EoR). The detection of the ionized region around such a high redshift quasar is in principle capable of constraining the neutral fraction of the IGM and the quasar’s age. Here we present the possibility of detection of such an ionized region around a quasar at $z = 8$ using redshifted 21-cm observations of the neutral hydrogen and possible ways to constrain IGM neutral fraction and quasar’s age through this detection.

1. Introduction
The epoch of reionization is one of the least known periods in the history of our Universe. According to the present understanding, reionization of neutral hydrogen (H I) is an extended process spanning over the redshift range $6 \lesssim z \lesssim 15$. The intergalactic medium (IGM) during this period is characterised by bubbles of ionized hydrogen (H II), centred around luminous sources (galaxies and quasars). Though very rare but the high redshift quasars can ionize very large volumes of the IGM around them with respect to their stellar counterpart. Detection of these H II bubbles will directly probe the state of the local IGM around the ionizing sources. It will also constrain the properties of the quasar, such as its luminosity and age.

The highest redshift at which a quasar has been detected till date is $z = 7.085$ (Mortlock et al., 2011). The Ly-$\alpha$ absorption spectrum of this object reveals a highly ionized near zone of radius $2.1 \pm 0.1$ Mpc (physical). (Bolton et al., 2011) have constrained the $\tau_Q$ and $\langle x_{HI} \rangle$ using simulated Ly-$\alpha$ absorption spectra of this quasar. This study shows that there are several combinations of the $\tau_Q$ and $\langle x_{HI} \rangle$ all of which can reproduce the observed data. The constrains thus obtained over $\langle x_{HI} \rangle$ and $\tau_Q$ are very wide. Ly-$\alpha$ absorption spectra are pencil beam observation along a line of sight (LoS) and they get saturated for a $x_{HI} > 10^{-4}$, because of these reasons they are limited in their ability to determine the age of the quasar and the local neutral fraction.

The redshifted 21-cm emission from neutral hydrogen in the epoch of reionization is believed to be a powerful tool to detect H II bubbles around quasars. The 21-cm intensity is directly proportional to the H I density, and it is in principle possible to probe the entire ionization profile of the H II bubble. This is particularly motivated by the fact that the presently functioning
GMRT, LOFAR and upcoming MWA\(^1\) are all sensitive to the HI signal from the EoR. However, redshifted HI signal is extremely small relative to the sensitivities of the present and upcoming telescopes and it will be buried deep in foregrounds which are a few orders of magnitude larger than the signal.

Datta, Bharadwaj & Choudhury (2007) (hitherto Paper I) have proposed a matched filter technique to detect ionized bubbles in radio-interferometric observations of redshifted 21-cm emission. The matched filter optimally combines the entire signal of an ionized bubble while minimizing the noise and the foreground contributions. This technique assumes the H II bubble to be spherical. However, a growing spherical bubble will appear anisotropic along the LoS to a present day observer due to the finite light travel time (FLTT) (Shapiro & Giroux, 1987; Wyithe & Loeb, 2004; Yu, 2005; Wyithe, Loeb & Barnes, 2005; Sethi & Haiman, 2008) and also due to the evolution of global ionization fraction (Geil et al., 2008). In an earlier work (Majumdar et al. (2011); hitherto Paper II), we have analytically quantified this anisotropy and studied the possibility of detecting such a bubble in a targeted search around a known quasar.

In this work we investigate the possibility of using such an observation to constrain the properties of the quasar and the IGM. For this we improve upon the spherical filter by calculating the apparent, anisotropic shape of the quasar bubble and using this as a template for the filter. We expect this to give a better match to the bubble that is actually present in the data. We also use a semi-numerical technique to simulate the quasar bubble while incorporating the effect of other H II bubbles due to stellar sources and density fluctuations in the IGM.

2. Simulation

For a isotropic and constant photon emission rate \((\dot{N}_{\text{phs}})\) of a quasar the solution to the growth model for the H II bubble around it takes the form

\[
r(\tau) = r_S \left[1 - \exp\left(-\frac{\tau}{\tau_{\text{rec}}}\right)\right]^{\frac{4}{3}} \tag{1}
\]

[eq. (2) of Paper II] where \(\tau_{\text{rec}} = x_H \sqrt{C \langle n_H \rangle \alpha_B}^{-1}\) and \(r_S = \left(3\dot{N}_{\text{phs}}\tau_{\text{rec}}/(4\pi x_H \sqrt{C \langle n_H \rangle})\right)^{1/3}\)

The age of the quasar \(\tau_Q\) is further related to the angle with the LoS \(\phi\) by

\[
\tau_Q = \tau + \frac{r(\tau)}{c}(1 - \cos \phi) \tag{2}
\]

[eq. (7) of Paper II]. The simultaneous solution of these two equations provide the observed apparent shape of the bubble. The apparent anisotropy in shape is quantified by a parameter \(\eta\) (eq. (9) of Paper II). A value \(\eta > 0\) indicates that the bubble is elongated along the LoS and a value \(\eta < 0\), indicates that it is compressed along the LoS.

We have implemented the semi-numerical formalism proposed by Choudhury, Haehnelt & Regan (2009) for generating the ionization field at a given redshift. We have generated the dark matter distribution at \(z = 8\) using a Particle Mesh Nbody code. The volume of the simulation is constrained by the 16 Gigabytes of memory available in our computer. We perform our simulation in a periodic box of size 85.12 Mpc (comoving) with 1216\(^3\) grid points and 608\(^3\) particles, with a mass resolution \(M_{\text{part}} = 7.275 \times 10^7 \ h^{-1} M_\odot\).

We identify halos within the simulation box using a standard Friend-of-Friend algorithm, with a fixed linking length \(b = 0.2\) (in units of mean inter particle distance) and minimum halo mass = 10\(M_{\text{part}}\). We assume that the ionizing luminosity from a galaxy is proportional to the mass of it’s halo. In the semi-numerical formalism, a region is said to be ionized if the average

\(^1\) http://www.gmrt.ncra.tifr.res.in; http://www.lofar.org/ and http://www.haystack.mit.edu/ast/arrays/mwa/
number of photons reaching there exceeds average neutral hydrogen density at that point. We have used a 256$^3$ grid with a grid spacing of 0.3325 Mpc for simulating the ionization maps. The quasar is assumed to be located at the most massive halo and snapshots of H I maps at different stages of growth of its bubble is generated. We then choose sections of the simulation box at different stages of the bubble’s growth and stack them together to generate the apparent shape of the H II bubble as seen by a present day observer. We consider both uniform and non-uniform density dependent recombinations in our simulations. We choose three distinct stages of anisotropy to generate the quasar bubble’s apparent shape and $\dot{N}_{phs}$ and $\tau_Q$ are adjusted so that the bubble can be accommodated within the simulation box. We choose the background mass averaged neutral fraction of the IGM to be $\langle x_{HI} \rangle_M = 0.5$ at $z = 8$ and to remain constant during the growth of the bubble. Figure 1 shows the simulated quasar bubble at very early ($\eta = 0.05$), middle ($\eta = -0.13$) and very late ($\eta = -0.02$) stages of its growth.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** This shows the simulated quasar bubble at different stages of its growth in one realization of our simulations. Top and bottom panels show the shapes considering uniform and non-uniform recombinations respectively. Red square shows the location of the quasar in each panel.

### 3. Bubble detection and parameter estimation

HI maps are then converted into visibilities to calculate the estimator for matched filter search as in Paper I.

$$\hat{E} = \sum_{a,b} S_f(U_a, \nu_b) \hat{V}(U_a, \nu_b)$$

(3)

Here the filter $S_f(U, \nu)$ (see Paper I for details) is the signal expected from the bubble that we are searching for and $\hat{V}(U, \nu)$ are the measured visibilities. All the contributions to $\hat{V}$, except the signal $S(U, \nu)$, are assumed to be random variables of zero mean, uncorrelated to the filter. In a targeted search the sky plane position of the quasar will be known. The parameters which control the bubble as well as the anisotropic filter’s shape are $\dot{N}_{phs}/C$, $x_{HI}/C$ and $\tau_Q$. Among them, $\dot{N}_{phs}$ can be estimated from the Ly-$$\alpha$$ absorption spectra of the quasar and the clumping factor $C$ from simulations. In the matched filter technique we expect the signal to noise ratio (SNR)

$$\text{SNR} = \frac{\langle \hat{E} \rangle}{\sqrt{\langle (\Delta \hat{E})^2 \rangle}}$$

(4)
to peak when the filter parameters \((x_{HI} \text{ and } \tau_Q)\) exactly match with that of the bubble. We use eq. (19) of Paper I to calculate the noise contribution to the variance of the estimator. We propose that the best matched filter parameters will give an estimate of \(x_{HI}\) and \(\tau_Q\). Figure 2 shows the estimates of \(x_{HI}\) and \(\tau_Q\) obtained from six different realizations of our simulations considering both uniform and non-uniform recombinations in the IGM. We observe that in most of the realizations the estimates are well within the uncertainties predicted for 9000 hrs of observations with GMRT. At the early stage of growth \((\eta = +ve)\) when the bubble appears elongated along the LoS, one can only obtain a lower limit on \(x_{HI}\) even for 9000 hrs of observation but precise lower and upper bounds on \(\tau_Q\) can be marked. On the contrary at very late stages of growth \((\eta \sim 0)\) one can precisely measure \(x_{HI}\) but can only obtain a lower limit on \(\tau_Q\). The middle stage of growth \((\eta = -ve)\) when the bubble appears compressed along LoS appears to be the most suitable for parameter estimation, and lower and upper bounds on both \(x_{HI}\) and \(\tau_Q\) can be obtained at this stage. We also observe that in almost all realizations the \(x_{HI}\) is slightly underestimated from its actual value. Clustering of stellar sources around the quasar is the main reason for this underestimation.

**Figure 2.** This shows the estimated quasar and IGM parameters from six different realizations of our simulations. Circles and stars represent search results from uniform and non-uniform recombination scenarios respectively. Pink square shows actual values of input parameters. Blue and green shaded regions show the uncertainties \((\Delta SNR = 1)\) in the estimated parameters for 4000 and 9000 hrs of observations with GMRT.

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