Neutrino distributions for a rotating core-collapse supernova with a Boltzmann-neutrino-transport

Akira Harada
Institute for Cosmic Ray Research, The University of Tokyo, 5-1-5 Kashiwa-no-ha, Kashiwa, Chiba, 277-8582, Japan
E-mail: harada@icrr.u-tokyo.ac.jp

Abstract. We simulate the collapse of a rotating core of the progenitor with 11.2 solar mass with the Boltzmann-radiation-hydrodynamics code, which solves the Boltzmann equations for neutrino transfer directly. We pay particular attention to the neutrino distribution in phase space, which is affected by the rotation. By solving the Boltzmann equations directly, we can assess the rotation-induced distortion of the angular distribution in momentum space, which gives rise to the rotational component of the neutrino flux. We compare the Eddington tensors calculated both from the raw data and from the M1-closure approximation. We find that the difference in the Eddington factors reaches \( \sim 20\% \) in our simulation. This is due to the different dependence of the Eddington and flux factors on the angular profile of the neutrino distribution function, and hence modification to the closure relation is needed.

1. Introduction

The core-collapse supernova (CCSN) is the stellar explosive death. Its explosion energy is \( \sim 10^{51} \) erg. The energy source is the released gravitational energy when the stellar core collapse to form a neutron star (NS). The neutrino heating mechanism, the leading scenario of the explosion, is as follows: The massive star forms an iron core at its center. Eventually, this iron core collapses gravitationally. This collapse is stopped by the rapid rise of the matter pressure originating from the nuclear strong interaction. At that time, the bounce shock is launched. This bounce shock stalls because the nuclei of the accreting matter are photodissociated using the energy of the shock. Soon after the bounce, the proto-neutron star (PNS) is formed at the center. The PNS contains many protons and much thermal energy. The neutrinos emitted from the PNS carries the lepton number and thermal energy away, and the PNS evolves into the NS. A fraction of these neutrinos are absorbed by the matter behind the shock and heat them. Thanks to this energy supply, the shock revives and the CCSN successfully explodes.

A lot of works are devoted to proof that this mechanism certainly works. However, no simulations successfully reproduced the observables of explosions currently. First, the spherically symmetric simulations were conducted, but it is concluded that the CCSN never explode under spherical symmetry except for the lightest progenitor. This is because even the most sophisticated simulations failed to revive the shock. They are the Boltzmann-radiation-hydrodynamics simulations in which the Boltzmann equations for neutrino transport are directly solved. Afterward, the multidimensional CCSN simulations have been performed. In these multi-D simulations, the shock revival is obtained thanks to the help of turbulence. On the
other hand, the explosion energy in the simulations is $\sim 10^{50}$ erg and much smaller than the observed explosion energy.

Nowadays the supernova modelers are trying to perform the 3D simulations without any spatial symmetry. However, the conclusion like what was obtained by the spherically symmetric Boltzmann-radiation-hydrodynamics simulations has not been obtained. Since we think the Boltzmann simulations under axisymmetry is necessary to conclude the 2D simulations, we developed the Boltzmann-radiation-hydrodynamics code for the CCSN under axisymmetry. In this article, we report the results of the stellar core-collapse simulations with rotating progenitor using the code [1]. The rotation has both positive and negative impacts on the CCSN, and hence it should be examined via simulations. Besides, the rotation distorts the neutrino distributions. Thanks to the distortion, nontrivial distribution of neutrinos is obtained. One important mission of the Boltzmann-radiation-hydrodynamics simulations is to evaluate the accuracy of the approximation methods utilized by other works, and such a nontrivial distribution provides a severe test for them.

2. Numerical Setup

The basic equations of our simulation are the Boltzmann equation for neutrino transport, the hydrodynamics equations including composition or the electron fraction, and the Poisson equation for the gravitational potential. A lot of special techniques to solve these equations are implemented in the code, and they are explained in [2, 3, 4]. The progenitor star is the $11.2 M_\odot$ model taken from [5]. Although it experiences the non-rotating evolution, we impose the shellular rotational velocity of $\Omega(r) = 1 \text{ rad s}^{-1}/(1 + (r/1,000 \text{ km}))$ at the onset of the collapse. The nuclear equation of state (EOS) employed here is the Furusawa-Shen EOS [6]. This EOS is based on the relativistic mean-field theory for the nuclear interaction and the Thomas-Fermi approximation for the non-uniform matter. Furthermore, the nuclear statistical equilibrium is considered for the nuclear composition at sub-nuclear densities. The neutrino interaction is based on Bruenn’s standard set. Besides, some updates are included: the nucleon-nucleon bremsstrahlung is considered; the electron capture rate on heavy and light nuclei is set to be consistent with the composition in the EOS.

3. Simulation Results

Figure 1 shows the evolution of the shock. For comparison, the shock evolution of the non-rotating model taken from [7] is also shown. The shock evolution is very similar for rotating and non-rotating models. Since the shock in the non-rotating model did not show a successful revival, we conclude that the rotating model also fails to revive the shock.

The more interesting features are seen in the momentum space of neutrinos. Figure 2 shows the angular distributions of neutrinos at the points inside and outside of the shock. For the neutrinos inside the shock, the distribution is almost isotropic for higher-energy neutrinos while it is forward-peaked for lower-energy neutrinos. Since the neutrino opacity is lower for lower-energy neutrinos, they start to escape from the supernova core while the higher-energy neutrinos are still trapped there. At outside the shock, all neutrinos have forward-peaked distributions since they are almost transparent to the supernova matter. Besides, higher-energy neutrinos have more tilted distributions to the rotational (or $\phi$-) direction. This is because the higher-energy neutrinos decouple from the matter at larger radii, and hence the rotationally-dragged distribution remains. This dragged distribution results in the existence of a $\phi$-component of the neutrino flux.

Finally, Figure 3 shows the radial profiles of the eigenvalues of the Eddington tensor of neutrinos to evaluate the accuracy of the M1-closure approximation. The Eddington tensor is the stress tensor divided by the energy density. The M1-closure approximation gives the Eddington tensor by the fitting formula [8]. Specifically, the Eddington factor (the largest eigenvalue of the
**Figure 1.** The shock evolution of the rotating and non-rotating model. The red and blue lines are for the rotating and non-rotating models, respectively. The thick solid lines are the angular-averaged shock radii, while the thin dashed lines represent the maximum and minimum shock radii.

**Figure 2.** Angular distributions of neutrinos. The distance from the center is proportional to the value of the distribution function. The different colors correspond to the neutrino energies: red, green, and blue are for 1, 4, and 19 MeV, respectively. Note that the distributions are normalized by their maximum values. The left panel shows the distributions at \( \sim 60 \) km (inside the shock), and the right panel shows the point at \( \sim 170 \) km (outside the shock). Both of them are on the equator.

Eddington tensor) is determined by the flux factor (the absolute value of the flux divided by the energy density), and in turn, it determines the Eddington tensor. However, the Eddington factor is nothing but the quadrupole component of the angular distribution while the flux factor is the dipole component. The fractional error of M1-approximation reaches \( \sim 20\% \) as seen in the figure, and this is because the approximation tries to determine the quadrupole component from the dipole component. In order to improve the M1-approximation, some information related to the quadrupole component may be required.
Figure 3. The radial profiles of the eigenvalues of the Eddington tensors. The solid and dashed lines are the eigenvalues calculated from the distribution function and the M1-closure prescription, respectively. The blue lines correspond to the Eddington factor, while the red and green lines represent the other two eigenvalues. The positions of the shock are indicated by the vertical dash-dotted lines. The left, middle, and right panels show the radial profiles along the equatorial, north-east, and north directions. The lower panels show the fractional error between the dashed and solid lines in the upper panels.

4. Conclusions
We have performed the rotational core-collapse of $11.2 M_\odot$ star by using the Boltzmann-radiation-hydrodynamics code. The hydrodynamic evolution is very similar to the non-rotating counterpart which fails to explode. The momentum space distributions are examined and found to be determined by the energy-dependent opacity and the rotation of matter. The tilted distribution gives rise to the rotational flux. Finally, the accuracy of the M1-closure approximation is investigated by the eigenvalues of the Eddington tensor. It is found that the Eddington factor of the M1-closure method has an error of $\sim 20\%$.

As the future prospects, we are developing the 3D version and the general relativistic version of our code. Indeed, the 3D simulation using the code is conducted up to $\sim 20$ ms after the core bounce, and the turnover of the prompt convection is correctly captured. Besides, the numerical relativity module to connect to the code is under development. We would like to elucidate the explosion mechanism of the CCSNe by using the fully developed version of our code.

References
[1] Harada A, Nagakura H, Iwakami W, Okawa H, Furusawa S, Matsufuru H, Sumiyoshi K and Yamada S 2019 Astrophys. J. 872, 181
[2] Sumiyoshi K and Yamada S 2012 Astrophys. J. Suppl. Ser. 199, 17
[3] Nagakura H, Sumiyoshi K and Yamada S 2014 Astrophys. J. Suppl. Ser. 214, 16
[4] Nagakura H, Iwakami W, Furusawa S, Sumiyoshi K, Yamada S, Matsufuru H and Imakura A 2017 Astrophys. J. Suppl. Ser. 229, 42
[5] Woosley S E, Heger A and Weaver T A 2002 Rev. of Mod. Phys. 74, 1015
[6] Furusawa S, Sumiyoshi K, Yamada S and Suzuki H, Astrophys. J. 772, 95
[7] Nagakura H, Iwakami W, Furusawa S, Okawa H, Harada A, Sumiyoshi K, Yamada S, Matsufuru H and Imakura A 2018 Astrophys. J. 854, 136
[8] Shibata M, Kiuchi K, Sekiguchi Y and Suwa Y 2011 Prog. of Theo. Phys. 125, 1255