Super Picard-Fuchs Equation and Monodromies for Supermanifolds

Payal Kaura\textsuperscript{1}, Aalok Misra\textsuperscript{2} and Pramod Shukla\textsuperscript{3}
Indian Institute of Technology Roorkee,
Roorkee - 247 667, Uttaranchal, India

Abstract

Following [1] and [2], we discuss the Picard-Fuchs equation for the super Landau-Ginsburg mirror to the super-Calabi-Yau in WCP\((3|2)[1,1,1,3][1,5]\), (using techniques of [3, 4]) Meijer basis of solutions and monodromies (at 0,1 and \(\infty\)) in the large and small complex structure limits, as well as obtain the mirror hypersurface, which in the large Kähler limit, turns out to be either a bidegree-(6,6) hypersurface in WCP\((3|1)[1,1,1,2] \times WCP(1|1)[1,1,6]\) or a \((\mathbb{Z}_2\text{-singular})\) bidegree-(6,12) hypersurface in WCP\((3|1)[1,1,2,6][6] \times WCP(1|1)[1,1,6]\).

Introductions

The periods are the building blocks, e.g., for getting the prepotential (and hence the Kähler potential and the Yukawa coupling) in \(\mathcal{N} = 2\) type II theories compactified on a Calabi-Yau, and the superpotential for \(\mathcal{N} = 1\) type II compactifications in the presence of (RR) fluxes. It is in this regard that the Picard-Fuchs equation satisfied by the periods, become quite important. Also, traversing non-trivial loops in the complex structure moduli space of type IIB on a Calabi-Yau mirror to the one on the type IIA side, corresponds to shifting of the Kähler moduli in the Kähler moduli space on the type IIA side. This results in mixing of flux numbers corresponding to RR fluxes on the type IIA side, implying thereby that dimensions of cycles on the type IIA side, loose their meaning. The mixing matrix for the flux numbers is given by the monodromy matrix. It hence becomes important to evaluate the same.

In the context of generalizing mirror symmetry to include rigid manifolds, it was conjectured in [6] that the mirrors for the same are given by supermanifolds. Further, in the past couple of years, supermanifolds have been shown to be relevant to open/closed string dualities [5].

In this paper, we study some algebraic geometric aspects of a supermanifold in a super weighted complex projective space free of any potential orbifold singularities. In section 1, based on techniques developed in [1] in the context of supermanifolds and [3, 4] regarding evaluation of Meijer basis for the periods as solutions to the Picard-Fuchs equation, we obtain the Super Picard-Fuchs equation for the mirror to a super Calabi-Yau and the periods for the same, both in the large and small complex structure limits. We further discuss the

\textsuperscript{1}email: pa123dph@iitr.ernet.in
\textsuperscript{2}e-mail: aalokfph@iitr.ernet.in
\textsuperscript{3}email: pmathpph@iitr.ernet.in
monodromies at 0, 1 and \(\infty\) (using again techniques developed in \([3, 4]\)). In section 2, we obtain the mirror hypersurface involving supermanifolds which are not super Calabi-Yau’s.

## 2 Super Picard-Fuchs Equation, Meijer basis for periods and Monodromies

In this section, we discuss the super Picard-Fuchs equation for the super Landau-Ginzburg mirror to the super-Calabi-Yau in \(\text{WCP}^{(3|2)}[1, 1, 1, 3][1, 5]\), (using techniques of \([3, 4]\)) Meijer basis of solutions and monodromies in the large and small complex structure limits.

### 2.1 The Periods

As shown in \([4]\), the gauged linear sigma model corresponding to the supermanifold \(\text{WCP}^{(3|2)}[1, 1, 1, 3][1, 5]\) consists of four chiral superfields \(\Phi^{I=0,1,2,3}\) and two fermionic superfields \(\Theta^{I=0,1}\) satisfying the D-term constraint: 
\[
|\Phi^0|^2 + |\Phi^1|^2 + |\Phi^2|^2 + 3|\Phi^3|^2 + |\Theta^0|^2 + 5|\Theta^1|^2 = r.
\]

The mirror Super Landau-Ginsburg model is given in terms of four twisted chiral superfields \(Y^{I=0,1,2,3}\) (mirror to the four \(\Phi^I\)’s), two more twisted chiral superfields \(X^{I=0,1}\) (mirror to the two \(\Theta^I\)’s), and two sets of fermionic superfields \(\eta^{I=0,1}\) and \(\chi^{I=0,1}\) satisfying the mirror constraint: 
\[
Y^0 + Y^1 + Y^2 + 3Y^3 - X^0 - 5X^1 = t.
\]

The periods can be expressed as:
\[
\Pi(t) = \int \prod_{I=0}^{3} dY^I \prod_{J=0}^{1} dX^J d\eta^J d\chi^J e^{\sum_{I=0}^{3} e^{-Y^I} + \sum_{J=0}^{1} e^{-X^J} (1 + \eta^J \chi^J)} \delta(\sum_{I=0}^{3} Q^I Y^I - \sum_{J=0}^{1} q^J X^J - t),
\]
where \(Q^{0,1,2,3} \equiv (1, 1, 1, 3)\) and \(q^{0,1} \equiv (1, 5)\). In the spirit of \([2]\), consider now:

\[
\bar{\Pi}(t) = \int \prod_{I=0}^{3} dY^I \prod_{J=0}^{1} dX^J d\eta^J d\chi^J e^{\sum_{I=0}^{3} \mu^I e^{-Y^I} + \sum_{J=0}^{1} \nu^J e^{-X^J} + \sum_{J=0}^{1} e^{-X^J} \eta^J \chi^J} \delta(\sum_{I=0}^{3} Q^I Y^I - \sum_{J=0}^{1} q^J X^J - t).
\]

The deformations \(\mu^I\)’s and \(\nu^J\)’s can be absorbed into shifting the Kähler parameter \(t\) to \(t' = t - \ln \left(\frac{\mu_0 \mu_1 \mu_2 \mu_3}{\nu_0 \nu_1}\right)\).

One can then see that one gets the following Super Picard-Fuchs equation:
\[
\frac{\partial^6 \bar{\Pi}(t')}{\partial \mu_0 \partial \mu_1 \partial \mu_2 \partial \mu_3} = e^{-t'} \frac{\partial^6 \bar{\Pi}(t')}{\partial \nu_0 \partial \nu_1}.
\]

By noticing:
\[
\frac{\partial}{\partial \mu_i^{Q^I}} = \prod_{i=0}^{Q^I-1}(-Q^I \frac{d}{dt'} - i), \quad \frac{\partial}{\partial \nu_j^{q^J}} = \prod_{i=0}^{q^J-1}(q^J \frac{d}{dt'} - i),
\]

one gets the following:
\[
\left(-\frac{d}{dt'}\right)^3 \prod_{i=0}^{2} \left(-3 \frac{d}{dt'} - i\right) \bar{\Pi}(t') = e^{-t'} \frac{d}{dt'} \prod_{i=0}^{4} \left(5 \frac{d}{dt'} - i\right) \bar{\Pi}(t').
\]
Further, setting $e^{-t} \equiv z$, and by replacing $\frac{5z}{3}$ by $z$ (noticing that $\Delta_z \equiv z \frac{d}{dz}$ is scale invariant), one gets the final form of the Picard-Fuchs equation:

$$
\Delta_z^4(\Delta_z - \frac{1}{3})(\Delta_z - \frac{2}{3})\Pi(z) = z\Delta_z^2 \prod_{i=1}^{4}(\Delta_z + \frac{i}{3})\tilde{\Pi}(z).
$$

(6)

Comparing 6 with the following equation for the generalized hypergeometric functions:

$$
[\Delta_z \prod_{i=1}^{q}(\Delta_z + \beta_i - 1) - z \prod_{j=1}^{p}(\Delta_z + \alpha_j)]\tilde{\Pi}(z) = 0,
$$

(7)

the solution to which is given by $\ _6F_5\left( \begin{array}{c}
\alpha_1 \alpha_2 \alpha_3 \ldots \alpha_p \\
\beta_1 \beta_2 \beta_3 \ldots \beta_q
\end{array} \right) (z)$.

From the above solution, following [3], the Meijer basis of solutions is obtained using properties of $\ _pF_q$ and the Meijer function $I$:

$$
\ _pF_q\left( \begin{array}{c}
\alpha_1 \alpha_2 \alpha_3 \ldots \alpha_p \\
\beta_1 \beta_2 \beta_3 \ldots \beta_q
\end{array} \right) (z) = \frac{\prod_{i=1}^{p}\Gamma(\beta_i)}{\prod_{j=1}^{q}\Gamma(\alpha_j)} I\left( \begin{array}{c}
0 \alpha_1 \ldots \alpha_p \\
0 \beta_1 \ldots \beta_q
\end{array} \right) (-z)
$$

where

$$
I\left( \begin{array}{c}
a_1 \ldots a_A \\
c_1 \ldots c_C
\end{array} \right) \left( \begin{array}{c}
b_1 \ldots b_B \\
d_1 \ldots d_D
\end{array} \right) (z), I\left( \begin{array}{c}
a_1 \ldots (1 - d_t) \ldots a_A \\
c_1 \ldots c_C
\end{array} \right) \left( \begin{array}{c}
b_1 \ldots b_B \\
d_1 \ldots d_t \ldots d_D
\end{array} \right) (-z),
$$

(8)

where a hat implies that the corresponding entry is missing, satisfy the same equation. Now, one would mimic the symplectic structure for bosonic manifolds, for supermanifolds as well, and construct the following column period vector:

$$
\Pi(z) = \left( \begin{array}{c}
F_0 \\
F_1 \\
F_2 \\
Z^0 \\
Z^1 \\
Z^2
\end{array} \right).
$$

(9)

Now, to get an infinite series expansion in $z$ for $|z| < 1$ as well as $|z| > 1$, one uses the following

$$
I\left( \begin{array}{c}
a_1 \ldots a_A \\
c_1 \ldots c_C
\end{array} \right) \left( \begin{array}{c}
b_1 \ldots b_B \\
d_1 \ldots d_D
\end{array} \right) (z) = \frac{1}{2\pi i} \int_{\gamma} ds \prod_{i=1}^{A}\Gamma(a_i - s) \prod_{j=1}^{B}\Gamma(b_j + s) \prod_{k=1}^{C}\Gamma(c_k - s) \prod_{l=1}^{D}\Gamma(d_l + s) z^s,
$$

(10)

where the contour $\gamma$ lies to the right of: $s + b_j = -m \in \mathbb{Z}^- \cup \{0\}$ and to the left of: $a_i - s = -m \in \mathbb{Z}^- \cup \{0\}$.

This, $|z| < 1$ and $|z| > 1$ can be dealt with equal ease by suitable deformations of the contour (See [3, 4]). Additionally, instead of performing parametric differentiation of infinite series to get the $ln$-terms, one get the same (for the large complex structure limit: $|z| < 1$) by evaluation of the residue at $s = 0$ in the Mellin-Barnes contour integral in (10) as is done explicitly to evaluate the six integrals.
The guiding principle is that of the six solutions to \( \Pi \), one should generate solutions in which one gets \((lnz)^P\), \(P = 0, ..., 3\) and one can then identify terms independent of \(lnz\) with \(Z^0\), three \((lnz)\) terms with \(Z^{1,2,3}\), three \((lnz)^{P\leq2}\) terms with \(F_{1,2,3} \equiv \frac{\partial F}{\partial z^i}\), and finally \((lnz)^{P\leq3}\) term with \(F_0 \equiv \frac{\partial F}{\partial z^0} \).

One (non-unique) choice of solutions for \(\Pi(z)\) is given below:

\[
\begin{pmatrix}
I \left( \begin{array}{c|c c c c c c c}
0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 \\
. & . & . & . & 1 & 1 & 1 & 1 \\
\end{array} \right) (z) \\
I \left( \begin{array}{c|c c c c c c c}
0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 \\
. & . & . & . & 1 & 1 & 1 & 1 \\
\end{array} \right) (-z) \\
I \left( \begin{array}{c|c c c c c c c}
0 & 0 & 2 & 3 & 4 & . & . & . \\
. & . & . & . & 1 & 1 & 1 & 1 \\
\end{array} \right) (z) \\
I \left( \begin{array}{c|c c c c c c c}
0 & 1 & 2 & 3 & 4 & . & . & . \\
. & . & . & . & 1 & 1 & 1 & 1 \\
\end{array} \right) (-z) \\
I \left( \begin{array}{c|c c c c c c c}
0 & 0 & 2 & 3 & 4 & . & . & . \\
. & . & . & . & 1 & 1 & 1 & 1 \\
\end{array} \right) (-z) \\
I \left( \begin{array}{c|c c c c c c c}
0 & 0 & . & 3 & 4 & . & . & . \\
. & . & . & . & 1 & 1 & 1 & 1 \\
\end{array} \right) (-z) \\
\end{pmatrix} = 
\begin{pmatrix}
F_0 \\
F_1 \\
F_2 \\
Z^0 \\
Z^1 \\
Z^2 \\
\end{pmatrix}
\]

(11)

Using techniques of [3, 4], and defining (the polygamma function) \(\psi(z) \equiv \frac{\Gamma'(z)}{\Gamma(z)}\), one gets the following results:

\[
F_0 = \frac{\Gamma(-\frac{1}{9})^2 \Gamma(\frac{5}{15})^3 \Gamma(\frac{3}{5}) \Gamma(\frac{7}{5}) \Gamma(\frac{3}{5})}{z^\frac{1}{2} \Gamma(-\frac{1}{15}) \Gamma(\frac{7}{15}) \Gamma(\frac{3}{5})^2} + \frac{\Gamma(-\frac{2}{9})^2 \Gamma(-\frac{2}{9}) \Gamma(-\frac{2}{9}) \Gamma(\frac{7}{5}) \Gamma(\frac{3}{5})^2}{z^\frac{1}{2} \Gamma(-\frac{2}{15}) \Gamma(\frac{2}{15}) \Gamma(\frac{3}{5})^2} + \frac{\Gamma(-\frac{3}{9})^2 \Gamma(-\frac{3}{9}) \Gamma(-\frac{3}{9}) \Gamma(\frac{7}{5}) \Gamma(\frac{3}{5})^2}{z^\frac{1}{2} \Gamma(-\frac{3}{15}) \Gamma(\frac{1}{15}) \Gamma(\frac{3}{5})^2} \\
- \frac{\Gamma(-\frac{4}{9})^2 \Gamma(-\frac{4}{9}) \Gamma(-\frac{4}{9}) \Gamma(\frac{7}{5}) \Gamma(\frac{3}{5})^2}{z^\frac{1}{2} \Gamma(-\frac{4}{15}) \Gamma(\frac{0}{15}) \Gamma(\frac{3}{5})^2} + \frac{\Gamma(-\frac{5}{9})^2 \Gamma(-\frac{5}{9}) \Gamma(-\frac{5}{9}) \Gamma(\frac{7}{5}) \Gamma(\frac{3}{5})^2}{z^\frac{1}{2} \Gamma(-\frac{5}{15}) \Gamma(\frac{-1}{15}) \Gamma(\frac{3}{5})^2} \\
- \frac{\Gamma(-\frac{6}{9})^2 \Gamma(-\frac{6}{9}) \Gamma(-\frac{6}{9}) \Gamma(\frac{7}{5}) \Gamma(\frac{3}{5})^2}{z^\frac{1}{2} \Gamma(-\frac{6}{15}) \Gamma(\frac{-2}{15}) \Gamma(\frac{3}{5})^2} + \frac{\Gamma(-\frac{7}{9})^2 \Gamma(-\frac{7}{9}) \Gamma(-\frac{7}{9}) \Gamma(\frac{7}{5}) \Gamma(\frac{3}{5})^2}{z^\frac{1}{2} \Gamma(-\frac{7}{15}) \Gamma(\frac{-3}{15}) \Gamma(\frac{3}{5})^2} \\
- \frac{\Gamma(-\frac{8}{9})^2 \Gamma(-\frac{8}{9}) \Gamma(-\frac{8}{9}) \Gamma(\frac{7}{5}) \Gamma(\frac{3}{5})^2}{z^\frac{1}{2} \Gamma(-\frac{8}{15}) \Gamma(\frac{-4}{15}) \Gamma(\frac{3}{5})^2} + \frac{\Gamma(-\frac{9}{9})^2 \Gamma(-\frac{9}{9}) \Gamma(-\frac{9}{9}) \Gamma(\frac{7}{5}) \Gamma(\frac{3}{5})^2}{z^\frac{1}{2} \Gamma(-\frac{9}{15}) \Gamma(\frac{-5}{15}) \Gamma(\frac{3}{5})^2} + ... \theta(|z| - 1)
\]
\[ + \theta(1 - |z|) \left[ \frac{\mathcal{X}}{6 \Gamma(\frac{5}{3}) \Gamma(\frac{5}{3})} + \frac{z \Gamma(\frac{2}{3}) \Gamma(\frac{2}{3}) \Gamma(\frac{2}{3}) (-4 + 2 \gamma + \log(z) + \psi(\frac{6}{5}) - \psi(\frac{3}{5}) + \psi(\frac{5}{3}) - \psi(\frac{5}{3}) + \psi(\frac{5}{3}))}{16 \Gamma(\frac{3}{5}) \Gamma(\frac{3}{5})} \right. \\
\left. - \frac{z^2 \Gamma(\frac{11}{5}) \Gamma(\frac{11}{5}) \Gamma(\frac{11}{5}) \Gamma(\frac{11}{5}) (-4 + 2 \gamma + \log(z) + \psi(\frac{6}{5}) - \psi(\frac{3}{5}) + \psi(\frac{5}{3}) - \psi(\frac{5}{3}) + \psi(\frac{5}{3}))}{16 \Gamma(\frac{3}{5}) \Gamma(\frac{3}{5})} \right] + \ldots, \]

where

\[ \mathcal{X} \equiv \Gamma(\frac{1}{5}) \Gamma(\frac{2}{3}) \Gamma(\frac{3}{5}) \Gamma(\frac{4}{5}) (8 \gamma^3 + (\log(z))^3 + \pi^2 \psi(\frac{1}{5}) + \psi(\frac{1}{5})^3 - \pi^2 \psi(\frac{1}{3}) - 3 \psi(\frac{1}{5})^2 \psi(\frac{1}{3}) + 3 \psi(\frac{1}{5}) \psi(\frac{1}{3})^2 - \psi(\frac{1}{3})^3 \\
+ \pi^2 \psi(\frac{1}{3}) + 3 \psi(\frac{1}{5})^2 \psi(\frac{1}{3}) - 6 \psi(\frac{1}{5}) \psi(\frac{1}{3})^3 + 3 \psi(\frac{1}{3})^2 \psi(\frac{1}{5}) - 3 \psi(\frac{1}{5}) \psi(\frac{1}{3})^2 - 2 \psi(\frac{1}{5})^2 - 2 \psi(\frac{1}{3})^2 + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5})^2 - 2 \psi(\frac{1}{5}) \psi(\frac{1}{3})^2 + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) - 2 \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{5}) + \pi^2 \psi(\frac{1}{3})^2 + 3 \psi(\frac{1}{3})^2 \psi(\frac{1}{5}) - \psi(\frac{1}{3})^3 \\
- 3 \psi(\frac{1}{3})^2 \psi(\frac{1}{5}) + 3 \psi(\frac{1}{5})^2 \psi(\frac{1}{3}) - 3 \psi(\frac{1}{3})^2 \psi(\frac{1}{5}) + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5})^2 - 3 \psi(\frac{1}{5}) \psi(\frac{1}{3})^2 + \pi^2 \psi(\frac{1}{3})^2 + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) - \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{3}) - \psi(\frac{1}{3})^3 \\
+ 6 \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5})^2 \psi(\frac{1}{3}) - \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{3}) - \psi(\frac{1}{3})^3 \\
+ 6 \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5})^2 \psi(\frac{1}{3}) - \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{3}) - \psi(\frac{1}{3})^3 \\
+ 6 \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5})^2 \psi(\frac{1}{3}) - \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{3}) - \psi(\frac{1}{3})^3 \\
+ 6 \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5})^2 \psi(\frac{1}{3}) - \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{3}) - \psi(\frac{1}{3})^3 \\
+ 6 \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5})^2 \psi(\frac{1}{3}) - \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{3}) - \psi(\frac{1}{3})^3 \\
+ 6 \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5})^2 \psi(\frac{1}{3}) - \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{3}) - \psi(\frac{1}{3})^3 \\
+ 6 \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + 3 \psi(\frac{1}{3}) \psi(\frac{1}{5})^2 \psi(\frac{1}{3}) - \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) + \psi(\frac{1}{3}) \psi(\frac{1}{5}) \psi(\frac{1}{3}) \psi(\frac{1}{3}) - \psi(\frac{1}{3})^3} \]

(12)
\[ +2 \psi\left(\frac{2}{5}\right) \psi\left(\frac{3}{5}\right) + \psi\left(\frac{3}{5}\right)^2 - 2 \psi\left(\frac{2}{5}\right) \psi\left(\frac{2}{3}\right) - 2 \psi\left(\frac{3}{5}\right) \psi\left(\frac{2}{3}\right) + \psi\left(\frac{2}{3}\right)^2 - 2 \psi\left(\frac{1}{3}\right) (\psi\left(\frac{1}{3}\right) - \psi\left(\frac{2}{3}\right) - \psi\left(\frac{3}{5}\right) + \psi\left(\frac{2}{3}\right) - \psi\left(\frac{4}{5}\right)) + \psi\left(\frac{1}{3}\right) - \psi\left(\frac{1}{3}\right) - \psi\left(\frac{2}{3}\right) + \psi\left(\frac{4}{5}\right)\]

\[ + \psi\left(\frac{2}{5}\right) + \psi\left(\frac{3}{5}\right) - \psi\left(\frac{4}{5}\right) + \psi\left(\frac{2}{3}\right) + \psi\left(\frac{1}{3}\right) + \psi\left(\frac{3}{5}\right) - \psi\left(\frac{2}{3}\right) + \psi\left(\frac{4}{5}\right) + 3 (\psi\left(\frac{1}{3}\right) + \psi\left(\frac{2}{3}\right))\]

\[ + 2 \psi\left(\frac{2}{5}\right) \psi\left(\frac{4}{5}\right) + 2 \psi\left(\frac{3}{5}\right) \psi\left(\frac{3}{5}\right) - 2 \psi\left(\frac{2}{5}\right) \psi\left(\frac{2}{3}\right) - 2 \psi\left(\frac{3}{5}\right) \psi\left(\frac{2}{3}\right) + \psi\left(\frac{2}{3}\right)^2 - 2 \psi\left(\frac{1}{3}\right) (\psi\left(\frac{1}{3}\right) - \psi\left(\frac{2}{3}\right) - \psi\left(\frac{3}{5}\right) + \psi\left(\frac{2}{3}\right) - \psi\left(\frac{4}{5}\right)) + \psi\left(\frac{1}{3}\right) \]

\[ - \psi\left(\frac{1}{3}\right) + \psi\left(\frac{2}{3}\right) + \psi\left(\frac{3}{5}\right) - \psi\left(\frac{2}{3}\right) + \psi\left(\frac{4}{5}\right) + \psi\left(\frac{1}{3}\right) - \psi\left(\frac{2}{3}\right) + \psi\left(\frac{4}{5}\right) - 2 \psi\left(1\right); \]

It would be beneficial to understand how the above result (which is the most involved among all the periods) has been obtained. From the Mellin-Barnes integral representation of the function relevant to the \( F_0 \)'s evaluation, one sees that one has to evaluate the following contour integral:

\[ F_0 = \frac{1}{2\pi i} \int_{\gamma} \frac{[\Gamma(-s)]^2[\Gamma(s)]^2 \prod_{j=1}^{5} \Gamma\left(\frac{j}{5} + s\right)}{[\Gamma(1+s)]^2 \prod_{j=1}^{5} \Gamma\left(\frac{j}{5} + s\right)} z^s ds. \tag{13} \]

The above contour integral can be evaluated using the method of residues as follows. One notices that the integrand has poles of order 4 at \( s = 0 \) (relevant to \( |z| < 1 \)), of order 2 at \( s = m \in \mathbb{Z}^{+} \) and of order 1 at \( s + \frac{i}{5} = -m \in \mathbb{Z}^{-} \) (relevant to \( |z| >> 1 \)).

To evaluate the residue at \( s = 0 \), define:

\[ \Omega_1(s) \equiv s^4 \frac{[\Gamma(-s)]^2[\Gamma(s)]^2 \prod_{j=1}^{5} \Gamma\left(\frac{j}{5} + s\right)}{[\Gamma(1+s)]^2 \prod_{j=1}^{5} \Gamma\left(\frac{j}{5} + s\right)} z^s. \tag{14} \]

To evaluate the residue, one needs to evaluate \( \frac{d^{4}}{ds^{4}} \Omega_1(s)|_{s=0} \). Taking the derivative of the logarithm of \( \Omega_1(s) \), one gets:

\[ \Omega'(s) = \Omega_1(s) \Omega_2(s), \tag{15} \]

where \( \Omega_2(s) \equiv -\Psi(1-s) + \sum_{j=1}^{5} \Psi\left(\frac{j}{5} + s\right) - \sum_{i=1}^{2} \Psi\left(\frac{i}{5} + s\right) + \ln z. \) Similarly, using (15),

\[ \Omega''(s) = \Omega_1(s) \Omega_2(s)^2 + \Omega_1(s) \Omega_2'(s), \tag{16} \]

where \( \Omega_2'(s) \equiv \Psi'(1-s) + \sum_{j=1}^{5} \Psi'(\frac{j}{5} + s) - \sum_{i=1}^{2} \Psi'(\frac{i}{5} + s). \) Finally, using (15), one gets:

\[ \Omega_3'' = \Omega_1(s) \Omega_2'(s)^3 + 3 \Omega_1(s) \Omega_2(s) \Omega_2'(s) + \Omega_1(s) \Omega_2'(s), \tag{17} \]

where \( \Omega_2''(s) = \Psi''(1-s) + \sum_{j=1}^{5} \Psi''\left(\frac{j}{5} + s\right) - \sum_{i=1}^{2} \Psi''\left(\frac{i}{5} + s\right). \) Putting everything together and expanding out the terms, one gets (12).
One can similarly evaluate the other components of the period vector, which are given below:

\[ F_1 = \left[ \frac{\Gamma(-\frac{4}{5})^2 \Gamma\left(\frac{1}{5}\right) \Gamma\left(\frac{3}{5}\right) \Gamma\left(\frac{4}{5}\right)}{z^{\frac{4}{5}} \Gamma\left(-\frac{2}{5}\right) \Gamma\left(-\frac{1}{5}\right) \Gamma\left(\frac{3}{5}\right) \Gamma\left(\frac{4}{5}\right)} - \left( \frac{\Gamma\left(\frac{1}{5}\right)^2 \Gamma\left(-\frac{4}{5}\right) \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{2}{5}\right)}{z^{\frac{1}{5}} \Gamma\left(-\frac{1}{5}\right) \Gamma\left(-\frac{2}{5}\right) \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{4}{5}\right)} \right) + \frac{\Gamma\left(-\frac{4}{5}\right)^2 \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{2}{5}\right) \Gamma\left(-\frac{1}{5}\right)}{z^{\frac{1}{5}} \Gamma\left(-\frac{2}{5}\right) \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{4}{5}\right) \Gamma\left(-\frac{5}{5}\right)} \right] \theta(|z| - 1) \]

\[ + \left[ \frac{\gamma}{2 \Gamma\left(\frac{4}{5}\right) \Gamma\left(\frac{3}{5}\right)} - \left( \frac{z^2 \Gamma\left(\frac{5}{5}\right) \Gamma\left(\frac{6}{5}\right) \Gamma\left(\frac{7}{5}\right)}{\Gamma\left(\frac{5}{5}\right) \Gamma\left(\frac{6}{5}\right) \Gamma\left(\frac{7}{5}\right)} \right) \right] \theta(1 - |z|), \tag{18} \]

where

\[ \gamma \equiv -\left( \frac{2}{5} \right)^2 \Gamma\left(\frac{2}{5}\right) \Gamma\left(\frac{3}{5}\right) \Gamma\left(\frac{4}{5}\right) \Gamma\left(\frac{5}{5}\right) \left( 4 \gamma^2 + \log(-z))^2 + \psi\left(\frac{1}{5}\right)^2 - 2 \psi\left(\frac{1}{3}\right) \psi\left(\frac{1}{3}\right) + \psi\left(\frac{1}{3}\right)^2 + \psi\left(\frac{1}{2}\right) \psi\left(\frac{1}{2}\right) - 2 \psi\left(\frac{1}{3}\right) \psi\left(\frac{1}{3}\right) \right) \]

\[ + \psi\left(\frac{2}{5}\right)^2 + 2 \psi\left(\frac{1}{5}\right) \psi\left(\frac{1}{3}\right) - 2 \psi\left(\frac{1}{3}\right) \psi\left(\frac{1}{3}\right) + 2 \psi\left(\frac{2}{5}\right) \psi\left(\frac{3}{5}\right) + \psi\left(\frac{2}{3}\right)^2 - 2 \psi\left(\frac{1}{5}\right) \psi\left(\frac{2}{3}\right) + 2 \psi\left(\frac{1}{3}\right) \psi\left(\frac{2}{3}\right) - 2 \psi\left(\frac{2}{5}\right) \psi\left(\frac{2}{3}\right) - 2 \psi\left(\frac{1}{5}\right) \psi\left(\frac{4}{5}\right) + 2 \psi\left(\frac{1}{3}\right) \psi\left(\frac{4}{5}\right) + 2 \psi\left(\frac{2}{5}\right) \psi\left(\frac{4}{5}\right) + 2 \psi\left(\frac{3}{5}\right) \psi\left(\frac{4}{5}\right) - 2 \psi\left(\frac{2}{3}\right) \psi\left(\frac{4}{5}\right) + \psi\left(\frac{4}{5}\right) \right]

\[ + 4 \gamma \left( \psi\left(\frac{1}{5}\right) - \psi\left(\frac{1}{3}\right) + \psi\left(\frac{2}{5}\right) + \psi\left(\frac{3}{5}\right) - \psi\left(\frac{2}{3}\right) + \psi\left(\frac{4}{5}\right) \right) \]

\[ + \psi\left(\frac{4}{5}\right) + \psi\left(\frac{1}{5}\right) - \psi\left(\frac{1}{3}\right) + \psi\left(\frac{2}{5}\right) + \psi\left(\frac{3}{5}\right) - \psi\left(\frac{2}{3}\right) + \psi\left(\frac{4}{5}\right) \right); \]

\[ F_2 = \left[ \frac{\Gamma\left(-\frac{3}{5}\right)^2 \Gamma\left(\frac{1}{5}\right) \Gamma\left(\frac{2}{5}\right) \Gamma\left(\frac{3}{5}\right)}{z^{\frac{3}{5}} \Gamma\left(-\frac{4}{5}\right) \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{2}{5}\right) \Gamma\left(-\frac{1}{5}\right)} - \left( \frac{\Gamma\left(-\frac{2}{5}\right)^2 \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{4}{5}\right) \Gamma\left(-\frac{5}{5}\right)}{z^{\frac{2}{5}} \Gamma\left(-\frac{1}{5}\right) \Gamma\left(-\frac{2}{5}\right) \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{4}{5}\right)} - \left( \frac{\Gamma\left(-\frac{1}{5}\right)^2 \Gamma\left(-\frac{2}{5}\right) \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{4}{5}\right)}{z^{\frac{1}{5}} \Gamma\left(-\frac{1}{5}\right) \Gamma\left(-\frac{2}{5}\right) \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{4}{5}\right)} \right) \right] \theta(|z| - 1) \]

\[ + \left[ \frac{\gamma}{2 \Gamma\left(\frac{4}{5}\right) \Gamma\left(\frac{3}{5}\right)} + \left( \frac{\psi\left(\frac{5}{5}\right) \psi\left(\frac{6}{5}\right) \psi\left(\frac{7}{5}\right)}{\Gamma\left(\frac{5}{5}\right) \Gamma\left(\frac{6}{5}\right) \Gamma\left(\frac{7}{5}\right)} \right) \right] \theta(1 - |z|), \tag{19} \]

where

\[ \gamma \equiv -\left( \frac{2}{5} \right)^2 \Gamma\left(\frac{3}{5}\right) \Gamma\left(\frac{4}{5}\right) \left( 4 \gamma^2 + \log(z))^2 + \psi\left(\frac{1}{5}\right)^2 - 2 \psi\left(\frac{1}{5}\right) \psi\left(\frac{2}{5}\right) + \psi\left(\frac{2}{5}\right)^2 - 2 \psi\left(\frac{1}{5}\right) \psi\left(\frac{3}{5}\right) + 2 \psi\left(\frac{2}{5}\right) \psi\left(\frac{3}{5}\right) - 2 \psi\left(\frac{1}{5}\right) \psi\left(\frac{3}{5}\right) \right) \]
\[+\psi(\frac{3}{5})^2 + 2 \psi(\frac{1}{3}) \psi(\frac{2}{3}) - 2 \psi(\frac{2}{5}) \psi(\frac{2}{3}) - 2 \psi(\frac{3}{5})^2 - 4 \gamma(\psi(\frac{1}{3}) - \psi(\frac{2}{5}) - \psi(\frac{3}{5}) + 2 \psi(\frac{1}{3})) - 4 \psi(\frac{1}{3}) \psi(\frac{4}{5}) + 4 \psi(\frac{2}{5}) \psi(\frac{4}{5}) + 4 \psi(\frac{3}{5}) \psi(\frac{4}{5}) - 4 \psi(\frac{2}{5}) \psi(\frac{4}{5}) + 4 \psi(\frac{4}{5})^2 + 2 \log(z) (2 \gamma - \psi(\frac{1}{3}) + \psi(\frac{2}{5}) + \psi(\frac{3}{5}) - \psi(\frac{2}{3}) + 2 \psi(\frac{4}{5}) - \psi'(\frac{1}{3}) + \psi'(\frac{2}{5}) + \psi'(\frac{3}{5}) - \psi'(\frac{2}{3}))\];

\[Z_0 = -\left(\frac{\Gamma(\frac{1}{5}) \Gamma(\frac{2}{5}) \Gamma(\frac{3}{5}) \Gamma(\frac{4}{5})}{\Gamma(\frac{1}{3}) \Gamma(\frac{2}{3})}\right) \theta(1 - |z|) + \left[\frac{\Gamma(\frac{1}{5})^2 \Gamma(\frac{2}{5}) \Gamma(\frac{3}{5})}{(-z)^\frac{3}{5} \Gamma(\frac{2}{5}) \Gamma(\frac{3}{5})^3 \Gamma(\frac{6}{5})^2}\right] \theta(|z| - 1)\]

\[Z_1 = -\left(\frac{\Gamma(\frac{2}{5}) \Gamma(\frac{3}{5})}{\Gamma(\frac{2}{5}) \Gamma(\frac{3}{5})}\right) (2 \gamma + \log(-z) - \psi(\frac{1}{3}) + \psi(\frac{2}{5}) + \psi(\frac{3}{5}) - \psi(\frac{2}{3}) + 2 \psi(\frac{4}{5})) \theta(1 - |z|) + \left[\frac{\Gamma(\frac{2}{5}) \Gamma(\frac{3}{5})}{(-z)^\frac{3}{5} \Gamma(\frac{2}{5}) \Gamma(\frac{3}{5})^3 \Gamma(\frac{6}{5})^2}\right] \theta(|z| - 1)\]

\[Z_2 = -2 \gamma - \log(-z) + \psi(\frac{1}{3}) - 2 \psi(\frac{2}{5}) + \psi(\frac{3}{5}) - 2 \psi(\frac{4}{5}) \theta(1 - |z|) + \left[\frac{\Gamma(\frac{2}{5}) \Gamma(\frac{3}{5})}{(-z)^\frac{3}{5} \Gamma(\frac{2}{5}) \Gamma(\frac{3}{5})^3 \Gamma(\frac{6}{5})^2}\right] \theta(|z| - 1)\]
Analogous to bosonic manifolds, one can make predictions about the world-sheet instanton contributions to the periods on the super gauged linear sigma model using the above results for the mirror Landau-Ginsburg model.

2.2 Monodromies

We now discuss monodromy for the mirror super Landau-Ginsburg model corresponding to the supermanifold considered in this paper. To discuss the same, the Picard-Fuchs equation can be written in the form[3, 4]:

\[
\left( \Delta^6_z + \sum_{i=1}^{5} B_i(z) \Delta^i_z \right) \Pi(z) = 0. \tag{23}
\]

The Picard-Fuchs equation in the form written in (23) can alternatively be expressed as the following system of six linear differential equations:

\[
\Delta_z \begin{pmatrix}
\tilde{\Pi}(z) \\
\Delta_z \tilde{\Pi}(z) \\
(\Delta_z)^2 \tilde{\Pi}(z) \\
& \ddots \\
& & (\Delta_z)^5 \tilde{\Pi}(z)
\end{pmatrix} = \begin{pmatrix}
0 & 1 & 0 & \ldots & 0 \\
0 & 0 & 1 & \ldots & 0 \\
\vdots & & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & 1 \\
0 & -B_1(z) & -B_2(z) & \ldots & -B_5(z)
\end{pmatrix} \begin{pmatrix}
\tilde{\Pi}(z) \\
\Delta_z \tilde{\Pi}(z) \\
(\Delta_z)^2 \tilde{\Pi}(z) \\
& \ddots \\
& & (\Delta_z)^5 \tilde{\Pi}(z)
\end{pmatrix}. \tag{24}
\]

The matrix on the RHS of (24) is usually denoted by \( A(z) \).

If the six solutions, \( \Pi_{i=1,\ldots,6} \), are collected as a column vector \( \Pi(z) \), then the constant monodromy matrix \( T \) for \( |z| << 1 \) is defined by:

\[
\Pi(e^{2\pi i}z) = T\Pi(z). \tag{25}
\]

The basis for the space of solutions can be collected as the columns of the “fundamental matrix” \( \Phi(z) \) given by:

\[
\Phi(z) = S_6(z)z^{R_6}, \tag{26}
\]

where \( S_6(z) \) and \( R_6 \) are 6×6 matrices that single and multiple-valued respectively. Note that \( B_i(0) \neq 0 \), which
influences the monodromy properties. Also,
\[
\Phi(z) = \begin{pmatrix}
\tilde{\Pi}_1(z) & \cdots & \tilde{\Pi}_9(z) \\
\Delta^2_{\tilde{\Pi}_1}(z) & \cdots & \Delta^2_{\tilde{\Pi}_9}(z) \\
\Delta^3_{\tilde{\Pi}_1}(z) & \cdots & \Delta^3_{\tilde{\Pi}_9}(z) \\
\cdots & \cdots & \cdots \\
\Delta^5_{\tilde{\Pi}_1}(z) & \cdots & \Delta^5_{\tilde{\Pi}_9}(z)
\end{pmatrix}, \tag{27}
\]

implying that
\[
T = e^{2\pi i R^t}. \tag{28}
\]

Now, writing \(z^R = e^{R\ln z} = 1 + R\ln z + R^2(\ln z)^2 + \ldots\), and further noting that there are no terms of order higher than \((\ln z)^4\) in \(\tilde{\Pi}(z)\) obtained above, implies that the matrix \(R\) must satisfy the property:
\[
R^m = 0, \ m = 4, \ldots, \infty.
\]
Hence, \(T = e^{2\pi i R^t} = 1 + 2\pi i R^t + \frac{(2\pi i)^2}{2}(R^t)^2 + \frac{(2\pi i)^3}{6}(R^t)^3\). Irrespective of whether or not the distinct eigenvalues of \(A(0)\) differ by integers, one has to evaluate \(e^{2\pi i A(0)}\). The eigenvalues of \(A(0)\) of (32), are 0, \(\frac{1}{3}, \frac{2}{3}\), and hence four of the six eigenvalues differ by an integer (0).

Now, the Picard-Fuchs equation (6) can be rewritten in the form (23), with the following values of \(B_i\)'s:
\[
B_1 = 0, \quad B_2 = -\frac{24z}{625(1 - z)}, \quad B_3 = -\frac{2z}{5(1 - z)}, \quad B_4 = \frac{\left(\frac{2}{3} - \frac{7z}{9}\right)}{1 - z}, \quad B_5 = \frac{-1 - 2z}{1 - z}. \tag{29}
\]

Under the change of basis \(\tilde{\Pi}(z) \rightarrow \tilde{\Pi}'(z) = M^{-1}\tilde{\Pi}(z)\), and writing \(\tilde{\Pi}_i(z) = \sum_{i=0}^3 (ln z)^i q_{ij}(z)\) (See [3]), one sees that
\[
\tilde{\Pi}'_i(z) = \sum_{i=0}^3 (ln z)^i q'_{ij}(z), \\
q'(z) = q(z)(M^{-1})^t, \\
\Phi'(z) = \Phi(z)(M^{-1})^t, \quad S'(z) = S(z)(M^{-1})^t, \quad R' = M^t R(M^{-1})^t. \tag{30}
\]

By choosing \(M\) such that \(S'(0) = 1_6\), one gets
\[
T(0) = M(e^{2\pi i A(0)})^t M^{-1}. \tag{31}
\]

The matrix \(A(0)\) is given by:
\[
A(0) = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-2}{9} \\
0 & 0 & 0 & 0 & \frac{-2}{9} & 1
\end{pmatrix} \tag{32}
\]

One can show, using Mathematica, that:
\[
e^{2\pi i A(0)} =
\]
Writing the solution vector $\tilde{\Pi}_i$ as $\tilde{\Pi}_i = \sum_{j=0}^{4}(lnz)^j q_{ji}$ (following the notation of [3]), one notes:

$$ (\Phi')_{0i} = (\tilde{\Pi}')_{i} = \left(S' z^{A(0)} \right)_{0i} = (lnz)^i q_{ji}.$$  \hspace{1cm} (33)

From (33), one gets the following:

$$ (q'(0))_{ji} = \frac{\delta_{ji}}{j!}, \quad 0 \leq (i, j) \leq 3. \hspace{1cm} (34)$$

Now, using Mathematica, one gets:

$$ z^{A(0)} =$$

$$ \begin{pmatrix} 1 \log(z) & \log(z)^2 \frac{\log(z)^3}{2} & \log(z)^3 \frac{\log(z)^3}{6} \end{pmatrix}$$

$$ \begin{pmatrix} -3 \left(31-32 z^{\frac{1}{5}}+z^{\frac{1}{3}}\right)+270 \log(z)+42 \log(z)^2+4 \log(z)^3 \hspace{1cm} 3 \left(81-156 \log(z)+378 \log(z)+54 \log(z)^2+4 \log(z)^3\right) \end{pmatrix}$$

$$ \begin{pmatrix} 0 \log(z) & \log(z)^2 \frac{\log(z)^3}{2} & \log(z)^3 \frac{\log(z)^3}{6} \end{pmatrix}$$

$$ \begin{pmatrix} -9 \left(45-48 z^{\frac{1}{6}}+3 z^{\frac{1}{5}}+14 \log(z)+21 \log(z)^2\right) \hspace{1cm} 9 \left(3 \left(3-4 z^{\frac{1}{8}}+z^{\frac{1}{5}}\right)+2 \log(z)\right) \end{pmatrix}$$

$$ \begin{pmatrix} 0 \log(z) & \log(z)^2 \frac{\log(z)^3}{2} & \log(z)^3 \frac{\log(z)^3}{6} \end{pmatrix}$$

$$ \begin{pmatrix} -9 \left(7-8 z^{\frac{1}{7}}+z^{\frac{1}{6}}\right) \hspace{1cm} 9 \left(3 \left(-1+z^{\frac{1}{8}}\right)+2 \log(z)\right) \end{pmatrix}$$

$$ \begin{pmatrix} 0 \log(z) & \log(z)^2 \frac{\log(z)^3}{2} & \log(z)^3 \frac{\log(z)^3}{6} \end{pmatrix}$$

$$ \begin{pmatrix} -3 \left(3-4 z^{\frac{1}{8}}+z^{\frac{1}{5}}\right) \hspace{1cm} 9 \left(-1+z^{\frac{1}{8}}\right)^2 \end{pmatrix}$$

$$ \begin{pmatrix} 0 \log(z) & \log(z)^2 \frac{\log(z)^3}{2} & \log(z)^3 \frac{\log(z)^3}{6} \end{pmatrix}$$

$$ \begin{pmatrix} 0 \log(z) \end{pmatrix}$$

$$ \begin{pmatrix} 2 \log(z) \end{pmatrix}$$

$$ \begin{pmatrix} 0 \log(z) \end{pmatrix}$$

$$ \begin{pmatrix} 0 \log(z) \end{pmatrix}$$

Then from the expression for $z^{A(0)}$ above, writing

$$ (e^{A(0)})_{0i} = f_{0i}(z^{\frac{1}{5}}) + \sum_{n=1}^{3} c_n^{(0i)}(lnz)^n, \hspace{1cm} (35)$$

where $0 \leq i \leq 5$, one gets:

$$ (q'(0))_{0i} = f_{0i}(0),$$

$$ (q'(0))_{ij} = c_i^{0j}, \quad 1 \leq i \leq 3, \quad 4 \leq j \leq 5. \hspace{1cm} (36)$$
One can show that the matrices $q$ and $q'$ are given as:

\[
q = \begin{pmatrix}
q_{00} & q_{01} & q_{02} & q_{03} & q_{04} & q_{05} \\
q_{10} & q_{11} & q_{12} & 0 & q_{14} & q_{15} \\
q_{20} & q_{21} & q_{22} & 0 & 0 & 0 \\
q_{30} & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
\]

and

\[
q' = \begin{pmatrix}
1 & 0 & 0 & 0 & -\left(\frac{81}{16}\right) & \frac{243}{16} \\
0 & 1 & 0 & 0 & -\left(\frac{405}{8}\right) & \frac{567}{8} \\
0 & 0 & 1 & 0 & -\left(\frac{63}{8}\right) & \frac{81}{8} \\
0 & 0 & 0 & 1 & -\left(\frac{2}{4}\right) & \frac{3}{4}
\end{pmatrix},
\]

Finally, using $q' = q(M^{-1})$, one gets 24 equations in 36 elements of $M$. Further constraints on the 36-24=12 elements are expected to come by imposing the requirement $(T^n - 1)^m = 0$ for some $n, m \in \mathbb{Z}^+$ (See [3] and references therein).

For $|z| >> 1$, the period vector can be written as $\Pi_i = (A_{ij}(\infty))\pi_j$, where $\pi_j \equiv z^{-\frac{1}{4}}$. One thus sees that the monodromy for $u_j$ is given by the matrix

\[
T_\pi(\infty) = \begin{pmatrix}
e^{-\frac{2\pi i}{5}} & 0 & 0 & 0 \\
0 & e^{-\frac{4\pi i}{5}} & 0 & 0 \\
0 & 0 & e^{-\frac{6\pi i}{5}} & 0 \\
0 & 0 & 0 & e^{-\frac{8\pi i}{5}}
\end{pmatrix},
\]

using which the monodromy at $\infty$, $T_\infty$, of the period vector can be determined from the equation:

\[
A(\infty)T_\pi(\infty) = T(\infty)A(\infty). \quad (37)
\]

The matrix $A(\infty)$ is given by:

\[
A(\infty) = \begin{pmatrix}
A_{11} & A_{12} & A_{13} & A_{14} \\
A_{21} & A_{22} & A_{23} & A_{24} \\
0 & A_{32} & A_{33} & A_{34} \\
A_{41} & A_{42} & A_{43} & A_{44} \\
0 & A_{52} & A_{53} & A_{54} \\
0 & 0 & A_{63} & A_{64}
\end{pmatrix},
\]

where

\[
A_{11} = \frac{\Gamma(-\frac{1}{5})^2 \Gamma(\frac{1}{5})^3 \Gamma(\frac{2}{5}) \Gamma(\frac{3}{5})}{\Gamma(\frac{2}{15}) \Gamma(\frac{7}{15}) \Gamma(\frac{8}{5})^2}, \quad A_{12} = \frac{\Gamma(-\frac{2}{5})^2 \Gamma(-\frac{3}{5}) \Gamma(\frac{1}{5}) \Gamma(\frac{2}{5})^3}{\Gamma(-\frac{1}{15}) \Gamma(\frac{1}{15}) \Gamma(\frac{3}{5})^2},
\]

\[
A_{13} = \frac{\Gamma(-\frac{3}{5})^2 \Gamma(-\frac{4}{5}) \Gamma(-\frac{1}{5}) \Gamma(\frac{1}{5}) \Gamma(\frac{2}{5})^2}{\Gamma(-\frac{4}{15}) \Gamma(\frac{4}{15}) \Gamma(\frac{3}{5})^2}, \quad A_{14} = \frac{\Gamma(-\frac{4}{5})^2 \Gamma(-\frac{5}{5}) \Gamma(-\frac{2}{5}) \Gamma(-\frac{1}{5}) \Gamma(\frac{1}{5})^2}{\Gamma(-\frac{5}{15}) \Gamma(-\frac{7}{15}) \Gamma(\frac{1}{5})^2},
\]
\[
\begin{align*}
A_{21} &= \frac{\Gamma(-\frac{3}{5})^2 \Gamma\left(-\frac{1}{5}\right) \Gamma\left(-\frac{1}{5}\right)}{\Gamma\left(-\frac{7}{15}\right) \Gamma\left(-\frac{1}{15}\right) \Gamma\left(-\frac{1}{15}\right) \Gamma\left(-\frac{1}{15}\right)}, \\
A_{22} &= -\frac{\Gamma(-\frac{3}{5})^2 \Gamma\left(-\frac{1}{5}\right) \Gamma\left(-\frac{1}{5}\right)}{\Gamma\left(-\frac{7}{15}\right) \Gamma\left(-\frac{1}{15}\right) \Gamma\left(-\frac{1}{15}\right) \Gamma\left(-\frac{1}{15}\right)}, \\
A_{23} &= \frac{\Gamma\left(-\frac{3}{5}\right)^2 \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{3}{5}\right)}{\Gamma\left(-\frac{7}{15}\right) \Gamma\left(-\frac{1}{15}\right) \Gamma\left(-\frac{1}{15}\right) \Gamma\left(-\frac{1}{15}\right)}, \\
A_{24} &= \frac{\Gamma\left(-\frac{3}{5}\right)^2 \Gamma\left(-\frac{3}{5}\right) \Gamma\left(-\frac{3}{5}\right)}{\Gamma\left(-\frac{7}{15}\right) \Gamma\left(-\frac{1}{15}\right) \Gamma\left(-\frac{1}{15}\right) \Gamma\left(-\frac{1}{15}\right)}.
\end{align*}
\]

Using the argument of [3], one sees that the monodromy at 1 is related to the same at 0 and \(\infty\) by the relation:

\[
T(1) = \left(T(0)\right)^{-1} T(\infty).
\]

### 3 The Mirror Hypersurface

In this section, following [4], we obtain the mirror hypersurface to the super Calabi-Yau in \(\text{WCP}^{(3/2)}[1, 1, 1, 3]1, 5\). To do so, we first integrate out \(X^0\) to yield:

\[
\int \prod_{i=1}^{3} dY^i dX^1 d\eta^1 d\chi^1 dY^0 d\eta^0 d\chi^0 e^{-Y^1 + \sum_{l=1}^{3} Q^I \gamma^I + 5X^1 + \eta^0 \chi^0 + e^{-X^1} (1 + \eta^I \chi^I)},
\]

which after redefining \(e^t \chi^0\) as \(\chi^0\) and introducing \(\hat{Y}^I\) and \(\hat{X}^1\) via the definitions: \(Y^I = \hat{Y}^I + Y^0\) and \(X^1 = \hat{X}^1 + Y^0\), and integrating out \(\eta^0\) and \(\chi^0\):

\[
\int \prod_{i=1}^{3} dY^i dX^1 d\eta^1 d\chi^1 dY^0 e^{-Y^0 - \sum_{i=1}^{3} Q^I \hat{Y}^I + 5X^1} \text{Exp}[e^{-Y^0} (1 + e^{-\sum_{i=1}^{3} Q^I \hat{Y}^I + 5X^1} + \sum_{i=1}^{3} e^{-Y^I} + e^{-X^1} + e^{-X^1} \eta^1 \chi^1)].
\]
Defining $\Lambda \equiv e^{-y^0}$, one gets:

$$
\int \prod_{l=1}^3 dY^l dX^1 d\eta^1 d\chi^1 e^{\Lambda(1+e^{-\sum_{l=1}^3 q^l Y^l + x^1 + t} + \sum_{l=1}^3 e^{-y^l + e^{-x^1 + e^{-x^1 \eta^1 \chi^1}}}),}
$$

(41)

which after the redefinitions:

$$
e^{-\hat{Y}_1} \equiv (x_1 y_1)^6, \quad e^{-\hat{Y}_2} \equiv y_2^6, \quad e^{-\hat{Y}_3} \equiv y_3^2, \quad e^{-\hat{X}_1} \equiv \hat{x}_1^n,
$$

(42)

gives the following mirror hypersurface after performing the $\Lambda$ integral:

$$
1 + e^\hat{Y}_1 \hat{y}_1^6 \hat{y}_2^6 \hat{y}_3^6 + (x_1 y_1)^6 + y_2^6 + y_3^2 + \hat{x}_1^n + \hat{x}_1^n \eta^1 \chi^1 = 0.
$$

(43)

One can rewrite (43) by defining $x_1 \equiv u^5$ and $x_1^n \eta^1$ as $\eta^1$ as:

$$
1 + e^\hat{Y}_1 \hat{y}_1^6 \hat{y}_2^6 \hat{y}_3^6 + y_2^6 u^30 + y_3^2 + u^5 + \eta^1 \chi^1 = 0,
$$

(44)

which in the limit $t \to -\infty$, and appropriately shifting $y_1$, gives:

$$
1 + y_1^6 u^30 + y_2^6 + y_3^2 + \eta^1 \chi^1 = 0,
$$

(45)

or

$$
1 + y_1^6 x_1^6 + y_2^6 + y_3^2 + \eta^1 \chi^1 = 0.
$$

(46)

Interestingly, the mirror hypersurface (46) can be viewed either as

(a) (assuming that the inhomogeneous coordinates $x_1, \chi_1$ and $y_1, y_2, y_3, \eta_1$ are to be thought of as the following ratio $\frac{x_1}{x_0}, \frac{\chi_1}{y_0}; \frac{y_1}{y_0}, \frac{y_2}{y_0}, \frac{y_3}{y_0}$ of homogeneous coordinates $x_0, x_1, \chi_1; y_0, y_1, y_2, y_3$) as a bidegree-(6,6) hypersurface in the $x^0 = y^0 = 1$ coordinate patch of the non-singular supermanifold $WCP^{(3|1)}[1, 1, 1, 2|6](y^{i=0,1,2,3}, \eta^1) \times WCP^{(1|1)}[1, 1|6](x^{i=0,1, \chi_1})$:

$$
x_0^6 y_0^6 + x_1^6 y_1^6 + x_2^6 y_2^6 + x_3^6 y_3^2 + y_1 \chi_1 = 0;
$$

(47)

as the sum of the weights for the bosonic and Grassmanian coordinates do not match, neither of the WCP’s corresponds to a super Calabi-Yau (See [6]),

or

(b) (assuming that the inhomogeneous coordinates $x_1, \chi_1$ and $y_1, y_2, y_3, \eta_1$ are to be thought of as the following ratio $\frac{x_1, \chi_1, \eta_1}{x_0, y_0, y_0, y_0}$ of homogeneous coordinates $x_0, x_1, \chi_1; y_0, y_1, y_2, y_3$) as a bidegree-(6,12) hypersurface in the $x^0 = y^0 = 1$ coordinate patch of the ($Z_2$) singular supermanifold $WCP^{(3|1)}[1, 1, 2|6] \times WCP^{(1|1)}[1, 1|6]$: $y_0^{12} x_0^6 + y_0^{12} y_1^6 x_1^6 + x_0^6 y_0^6 + x_0^6 y_3^2 + y_0^6 \eta_1 \chi_1 = 0.$

(48)

Once again neither of the supermanifolds that are multiplied, are super Calabi-Yau’s for the same reason as given above.

As part of future work, it will be interesting to verify by calculations of correlation functions to see either which of the two LG models actually correspond to the mirror GLSM or whether both are the LG duals but in different corners of the moduli space - given that the hypersurface in the GLSM side was non-singular, perhaps it is the non-singular GL dual that will be chosen. The results of this paper can be readily generalized to other super weighted complex projective spaces.
References

[1] M. Aganagic and C. Vafa, *Mirror symmetry and supermanifolds*, arXiv:hep-th/0403192.

[2] K. Hori and C. Vafa, *Mirror symmetry*, arXiv:hep-th/0002222.

[3] B. R. Greene and C. I. Lazaroiu, *Collapsing D-branes in Calabi-Yau moduli space. I*, Nucl. Phys. B 604, 181 (2001) [arXiv:hep-th/0001025].

[4] A. Misra, *Type IIA on a compact Calabi-Yau and D = 11 supergravity uplift of its orientifold*, Fortsch. Phys. 52, 831 (2004) [arXiv:hep-th/0311186].

[5] E. Witten, *Perturbative gauge theory as a string theory in twistor space*, Commun. Math. Phys. 252, 189 (2004) [arXiv:hep-th/0312171].

[6] S. Sethi, *Supermanifolds, rigid manifolds and mirror symmetry*, Nucl. Phys. B 430, 31 (1994) [arXiv:hep-th/9404186].