Confined transverse-electric graphene plasmons in negative refractive-index systems

Xinyan Zhang1,2, Hao Hu3, Xiao Lin1,4, Lian Shen1,2, Baile Zhang4,5, and Hongsheng Chen1,2,6

Transverse electric graphene plasmons are generally weakly confined in the direction perpendicular to the graphene plane. They are featured by a skin depth $\delta$, namely the penetration depth of their evanescent fields into the surrounding environment, much larger than the wavelength $\lambda$ in free space (e.g., $\delta > 10\lambda$). The weak spatial confinement of transverse electric graphene plasmons is now the key drawback that limits their practical applications. Here we report the skin depth of TE graphene plasmons can be largely decreased down to the subwavelength scale (e.g., $\delta < \lambda/10$) in negative refractive-index environments. The underlying mechanism originates from the different existence conditions for TE graphene plasmons in negative and positive refractive-index environments. To be specific, their existence in negative (positive) refractive-index environments requires $\text{Im}(\sigma) > 0$ ($\text{Im}(\sigma) < 0$) and lies in the frequency range of $\hbar\omega/\mu_c < 1.667$ ($\hbar\omega/\mu_c > 1.667$), where $\sigma$, and $\mu_c$ are the surface conductivity and chemical potential of graphene, respectively.

INTRODUCTION

In their seminal work in 2007, Mikhailov S. A. and Ziegler K.1 proposed an exotic electromagnetic mode in the monolayer graphene, namely the transverse electric (TE, or $s$-polarized) graphene plasmons. The TE graphene plasmons lie in the frequency range of $\hbar\omega/\mu_c > 1.667$, since their existence requires $\text{Im}(\sigma) < 0$, where $\sigma$ and $\mu_c$ are the surface conductivity and chemical potential of graphene, respectively. Another key feature for TE graphene plasmons is the spatial confinement. Note that highly confined surface plasmons2–4, such as the transverse magnetic (TM, or $p$-polarized) graphene plasmons5–7, can enable the flexible control of light flow in the subwavelength scale and even the extreme nanoscale; as such, they can enable many promising applications, including the on-chip terahertz to X-ray radiation sources8,9, miniaturized modulators10, subwavelength guidance11–13, deep-subwavelength imaging14–17, and light energy harvesting and scattering18–20. The spatial confinement of graphene plasmons in the direction perpendicular to the graphene plane can be quantitatively characterized by the skin depth $\delta$. Here the skin depth $\delta$ is defined as the penetration depth of the evanescent fields carried by graphene plasmons into the surrounding environment. For TE graphene plasmons, their skin depth is inversely proportional to $|\text{Im}(\sigma)|$. Due to the small achievable negative value of $\text{Im}(\sigma)$ (i.e., $\text{max}(|\text{Im}(\sigma)|) \sim G_0$ (Fig. 1)), TE graphene plasmons are featured by a skin depth at least in the wavelength scale, where $G_0 = e^2/4\hbar$ is the universal optical conductivity. To be specific, we generally have $\delta > 10\lambda$ (Supplementary Fig. 1) for TE plasmons in monolayer graphene, where $\lambda$ is the wavelength in free space. As severely limited by the weak spatial confinement, only several potential applications of TE graphene plasmons have been reported, such as Brewster effects22, polarizers23, optical sensors24, waveguide phase, and amplitude modulators25. On the other hand, rapid progress in nano-photronics has fueled a quest for highly confined TE graphene plasmons, in addition to the highly confined TM graphene plasmons. This way, highly confined graphene plasmons can be achieved without stringent requirement on the polarization of light and can benefit more practical applications based on TE waves. Such a quest still remains elusive, although many researches of TE polaritons in graphene26–29 and other 2D materials30–34 have been ignited by the pioneering work in 2007. Here we theoretically reveal a viable way to largely enhance the spatial confinement of TE graphene plasmons by using the environment with the negative permeability or refractive index. As firstly proposed by Veselago in 196835, the negative refractive-index materials simultaneously have negative permittivity and negative permeability; they have triggered tremendous researches both on fundamental science and practical applications36–40 exemplified by the well-known negative refraction41, the perfect lens/superlens37,42, the inverse Doppler effect43 and the backward Cherenkov radiation44–46. In principle, the environment with the negative permeability or refractive index can be effectively constructed, for example, by metamaterials36,41,51 and photonic crystals52–55. We find the existence condition of TE graphene plasmons in negative refractive-index environments is drastically different from that in positive refractive-index environments. In negative refractive-index environments, the existence of TE graphene plasmons become to require $\text{Im}(\sigma) > 0$; as a result, TE graphene plasmons now lie in the frequency range of $\hbar\omega/\mu_c < 1.667$ at room temperature. Moreover, due to the availability of the large positive value of $\text{Im}(\sigma)$ ($\text{max}(|\text{Im}(\sigma)|) \sim 10^3 G_0$), the skin depth of TE graphene plasmons can be largely decreased down to the subwavelength scale (e.g., $\delta < \lambda/10$).
where \( k_z = \sqrt{\frac{\varepsilon_z}{\varepsilon_r}} \mu_\| - q^2 \) is the out-of-plane (perpendicular to the graphene plane) component of wavevector; \( \mu_\| \) is the permeability in free space.

Equation (2) explicitly indicates the existence condition for TE graphene plasmons, as briefly summarized in Fig. 1b, c. To be specific, for the environment with \( \mu_r > 0 \) (e.g., positive refractive-index environments), Eq. (2) has solutions only if \( \text{Im}(\sigma_\|) < 0 \), where the negative \( \text{Im}(\sigma_\|) \) lies in the frequency range of \( \omega \mu_\| > 1.667 \) (Fig. 1c). In contrast, for the environment with \( \mu_r < 0 \) (e.g., negative refractive-index environments), the existence of TE graphene plasmons becomes to require \( \text{Im}(\sigma_\|) > 0 \) from Eq. (2) (Fig. 1b) and lies in the range of \( \omega \mu_\| < 1.667 \) at room temperature (Fig. 1c).

On the other hand, Eq. (2) also indicates that the skin depth of TE graphene plasmons is proportional to 1/|\( \text{Im}(\sigma_\|) \)| for arbitrary \( \mu_r \), since the skin depth is mathematically defined as \( \delta = 1/|\text{Im}(\mu_\|)| \). In other words, in the frequency range where TE graphene plasmons could exist, a larger value of |\( \text{Im}(\sigma_\|) \)| would lead to a smaller skin depth and thus a larger spatial confinement. Note that the value of \( \text{Im}(\sigma_\|) \) for the monolayer graphene is dominantly determined by \( \sigma_{\text{tra}} \) especially in the frequency range of \( \omega \mu_\| < 1.667 \), while it is mainly determined by \( \sigma_{\text{inter}} \) if \( \omega \mu_\| > 1.667 \). As a result, the maximum value of |\( \text{Im}(\sigma_\|) \)| can reach \( \sim 40 \) at \( \omega \mu_\| < 1.667 \) due to the contribution of \( \sigma_{\text{tra}} \) (Fig. 1c); in contrast, it is only \( \sim 0.3 \) at \( \omega \mu_\| > 1.667 \) with the negligible contribution from \( \sigma_{\text{inter}} \) (Fig. 1c). This way, the minimum skin depth of TE graphene plasmons in negative permeability environments would be much smaller than that in positive permeability environments.

Moreover, if \( \omega \mu_\| < 1.667, \sigma_{\text{tra}} \) in Eq. (1) is dependent on the temperature \( T \), the relaxation time \( \tau \), and the chemical potential \( \mu_\| \), besides the angular frequency \( \omega \). These parameters \( (T, \tau, \mu_\|) \) provide us extra degrees of freedom to achieve the large value of |\( \text{Im}(\sigma_\|) \)|; see for example in Fig. 2. Then in negative permeability environments, these parameters could enable us the capability to flexibly modulate the basic features of TE graphene plasmons (Figs. 2 and 3), including their spatial confinement. Below the influence of these parameters on TE graphene plasmons in negative refractive-index environments is analyzed in detail, where the negative refractive-index environment (\( \mu_r < 0 \) and \( \varepsilon_r < 0 \)) is a typical negative permeability environment (\( \mu_r < 0 \)). In addition, the loss in the surrounding environment is artificially neglected, since the reasonable amount of loss will not have a drastic influence on the confined TE graphene plasmons.

### Influence of the relaxation time on TE graphene plasmons

Figure 2 shows the influence of relaxation time on TE graphene plasmons in negative refractive-index environments, from the perspective of the in-plane wavevector. According to Eq. (2), it is straightforward to derive the expression for the in-plane wavevector \( q \) that is

\[
q = \sqrt{k_z^2 \varepsilon_r \mu_\| - \frac{1}{4} \varepsilon_\| \mu_\| \omega^2 \mu_\|^2 \sigma_\|^2},
\]

where \( k_z = \sqrt{\frac{\varepsilon_z}{\varepsilon_r}} \mu_\| - q^2 \) is the out-of-plane (perpendicular to the graphene plane) component of wavevector; \( \mu_\| \) is the permeability in free space.

To facilitate the discussion, the effective refractive index of TE graphene plasmons is denoted as \( n_{\text{eff,lo}} = \text{Re}(q)/k_0 \) and plotted in Fig. 2; see the information of \( \text{Im}(q)/k_0 \) in Supplementary Fig. 2. In addition, since the quality factor \( \text{Re}(q)/\text{Im}(q) \) is oftentimes regarded as a key parameter to characterize the basic feature of surface plasmons, the quality factor of TE graphene plasmons is also briefly discussed in Supplementary Figs. 3 and 4 in Supplementary Note 4. For the monolayer graphene, the maximum positive value of \( \text{Im}(\sigma_\|) \) appears in the frequency range of \( \omega \mu_\| > 0.1 \) (Fig. 2a). If the relaxation time increases (i.e., the loss in graphene decreases), max(|\( \text{Im}(\sigma_\|) \)|) in the interested frequency range increases and can be up to \( \sim 100 \) (Fig. 2a). Due to the availability of large positive \( \text{Im}(\sigma_\|) \),
maximum \( n_{\text{eff},0} \) in negative refractive-index environment is much larger than 1 (e.g., up to \( n_{\text{eff},0} \approx 1.4 \) in Fig. 2b, c; also see Supplementary Note 5), and the maximum value of \( n_{\text{eff},0} \) would increase if the relaxation time increases. Such a large value of \( n_{\text{eff},0} \) is favored for the practical application of TE graphene plasmons. We emphasize that in positive refractive-index environment, \( n_{\text{eff},0} \) is generally very close to 1, such as \( n_{\text{eff},0} \approx 1.00006 \) in Supplementary Fig. 5 in Supplementary Note 6.

Influence of \( T \) and \( \mu_c \) on TE graphene plasmons

Figure 3 shows the influence of the temperature \( T \) and the chemical potential \( \mu_c \) on TE graphene plasmons in negative refractive-index environments.
Refractive-index environments. In short, if the temperature or the chemical potential increases, the achievable maximum value of \(n_{\text{eff}, 0}\) for TE graphene plasmons would increase, e.g., up to \(n_{\text{eff}, 0} = 1.65\) in Fig. 3a. We note that in Fig. 3a, the achievable maximum value of \(n_{\text{eff}, 0}\) at high temperatures is more sensitive to the temperature variation than that at low temperatures. This phenomenon is caused by the fact that in Eq. (1), the temperature-insensitive term of \(\frac{\mu_c k_0^2 T}{n^\prime} \frac{\partial}{\partial n^\prime} \left( \frac{k_T}{n^\prime} \right)\) plays a dominant role at low temperatures for \(\sigma_{\text{intra}}\) while the temperature-sensitive term of \(\frac{\mu_c k_0^2 T}{n^\prime} \frac{\partial}{\partial n^\prime} \left( \frac{k_T}{n^\prime} \right)\). Hence, 2 \ \ln \left( e^{-\frac{\mu c}{k_B T}} + 1 \right) becomes important at high temperatures.

Correspondingly, if the temperature or the chemical potential increases, the minimum skin depth of TE graphene plasmons would decrease (Fig. 3b). To be specific, the minimum skin depth of TE graphene plasmons in negative refractive-index environments can readily become subwavelength, such as \(\delta/\lambda < 1\) for the case with \(\mu_c = 0.2\) eV at \(T = 300\) K in Fig. 3b. Furthermore, the skin depth can even be decreased down to the deep-subwavelength scale, such as \(\delta/\lambda < 0.1\) for the case with \(\mu_c = 0.5\) eV at \(T = 300\) K in Fig. 3b. As such, the usage of negative refractive-index environments can largely decrease the minimum skin depth of TE graphene plasmons by at least two orders of magnitude, compared to positive refractive-index environments in which \(\delta/\lambda > 10\) (Supplementary Fig. 1). We emphasize that the enticing subwavelength skin depth of TE graphene plasmons can already be achieved at room temperature, although the temperature’s influence in Fig. 3 is studied in a relatively wide range of temperature and the high temperature such as 3000 K in practical scenarios might lead to the instability of negative refractive-index materials. Moreover, it is worthy to highlight that the phenomenon of the temperature-induced large enhancement of the spatial confinement for TE graphene plasmons is only exists in negative refractive-index environments (Fig. 3) and will not happen for positive refractive-index environments (Supplementary Fig. 5). In addition, TE graphene plasmons in negative refractive-index environments generally have a relatively small quality factor \(\text{Re}(q)/\text{Im}(q)\) (Fig. 3c), due to their high spatial confinement and the large material loss of graphene at the studied frequency range.

Influence of \(\mu_c\) and \(\varepsilon_f\) on TE graphene plasmons

Figure 4 shows the drastic difference of TE graphene plasmons in positive and negative refractive-index environments from another perspective of view, that is, the influence of \(\mu_c\) and \(\varepsilon_f\) on \(\text{Re}(q)/|k|\) of TE graphene plasmons, where \(k = k_0 \sqrt{\varepsilon_r \mu_r}\) is the wavevector of light in the surrounding environment. Physically, \(\text{Re}(q)/|k|\) is equivalent to the ratio between the wavelength of light in the surrounding environment \(\lambda_{\text{environ}}\) and the wavelength of TE graphene plasmons \(\lambda_{\text{plasmon}}\), namely \(\text{Re}(q)/|k| = \lambda_{\text{environ}}/\lambda_{\text{plasmon}}\). That is, a large \(\text{Re}(q)/|k|\) indicates a larger contrast between \(\lambda_{\text{environ}}\) and \(\lambda_{\text{plasmon}}\). Note that \(k = k_0\) where \(k_0\) is the wavevector of light in free space, and thus \(\text{Re}(q)/|k|\) is not the effective refractive index of TE graphene plasmons. \(n_{\text{eff}, 0}\) is the relative refractive index of TE graphene plasmons discussed in Figs. 2 and 3. From Fig. 4, the variation of both \(\mu_c\) and \(\varepsilon_f\) for negative refractive-index environments would have a large impact on \(\text{Re}(q)/|k|\) than that for positive refractive-index environments. To be specific, in negative refractive-index environments, \(\text{Re}(q)/|k| > 10\) is achievable if we increase \(\mu_c\) and decrease \(\varepsilon_f\). As such, a larger contrast between \(\lambda_{\text{environ}}\) and \(\lambda_{\text{plasmon}}\) exists in negative refractive-index environments. In contrast, in positive refractive-index environments, \(\text{Re}(q)/|k|\) is insensitive to the variation of \(\mu_c\) and \(\varepsilon_f\), and it is always very close to 1. Therefore, there is the negligible contrast between \(\lambda_{\text{environ}}\) and \(\lambda_{\text{plasmon}}\) in positive refractive-index environments. Note that the large value of \(\text{Re}(q)/|k|\) is favored in practical applications, which can be used, for example, to achieve the extraordinarily large scattering cross section from tiny objects in low-index environments. More discussion on the influence of \(\mu_c\) and \(\varepsilon_f\) can be obtained in Supplementary Figs. 6 and 7 in Supplementary Note 7.

In conclusion, we have theoretically revealed some emerging features of TE graphene plasmons in negative refractive-index environments, including their existence condition of \(\text{Im}(\sigma_{\text{eff}}) > 0\) and their existing frequency range of \(\hbar \omega/\mu_c < 1.667\). Importantly, these TE graphene plasmons can become highly confined in the direction perpendicular to the graphene plane. To be specific, their skin depth can decrease down to the deep-subwavelength scale (e.g., \(\delta/\lambda < 10\)). Then the existence of these highly confined TE graphene plasmons should be robust to various surrounding environments (i.e., the permittivity and/or permeability of the substrate and superstrate can be largely different). Such a feature is drastically different from the weakly confined TE graphene plasmons in the positive refractive-index environment, which exist mainly in the almost symmetric environments (the substrate and superstrate should have the negligible difference in their permittivity or permeability). Our findings in this work further indicate that the negative refractive-index materials might serve as a versatile platform to enable more practical applications of TE graphene plasmons, such as subwavelength guidance, some exotic scattering phenomena of light, and the exploration of TE plasmons in controlling the free electron radiation (e.g., Chernovkov radiation).

METHODS

Dispersion of TE graphene plasmons

Without loss of generality, the monolayer graphene is located at the interface between region 1 and region 2 (Fig. 1a), where region 1 with \(z < 0\) (region 2 with \(z > 0\)) has the relative permittivity \(\varepsilon_1\) (\(\varepsilon_2\)) and the relative permeability \(\mu_1\) (\(\mu_2\)). For TE graphene plasmons, their electric fields are along the \(z\) direction. According to the electromagnetic theory, one can
set the electric fields in each region as

\[ E_1 = yE_1 e^{ik_1 z - \omega t} \]

\[ E_2 = yE_2 e^{ik_2 z + \omega t} \]

Accordingly, the relationship between \( k_{1,2} \) and \( q \) is

\[ k_{1,2} = \frac{\omega}{c} \sqrt{\varepsilon_1,2 \mu_1,2 - q^2} \]

In the above equations, \( q \) is the component of the wavevector parallel to the interface, \( k_1 \) and \( k_2 \) are the component of the wavevector perpendicular to the interface in region 1 and region 2, respectively, and \( \omega \) is the angular frequency. The magnetic field in each region can be obtained according to \( \nabla \times E = i\omega \mu_0 H \). By enforcing the boundary conditions:

\[ \hat{n} \times (E_1 - E_2) = 0 \]

\[ \hat{n} \times (H_1 - H_2) = \alpha \varepsilon_{\text{boundary}} \]

we can obtain the dispersion of TE graphene plasmons as

\[ \frac{k^2_{1,2}}{\mu_1,2} + \omega_0 \mu_0 = 0 \]

where \( \mu_0 \) is the permeability in free space.

**DATA AVAILABILITY**

The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

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AUTHOR CONTRIBUTIONS
X.L. conceived the idea. X.Z. performed the calculations. H. H., X.L., L.S., B.Z., and H.C. contributed insight and discussion on the results. X.Z. and X.L. wrote the paper. X.L., B.Z. and H.C., supervised the project.

COMPETING INTERESTS
The authors declare no competing interests.

ADDITIONAL INFORMATION
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Correspondence and requests for materials should be addressed to X.L., B.Z. or H.C.

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