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MATRIX METHOD OF RECEIVING THE FULL COMPOSITION
OF THE GROUPS OF RELATIVITY OF BOOLEAN FUNCTIONS

The article describes a matrix method for obtaining the full composition of the groups of relativity
of Boolean functions on the basis of a universal permutation matrix.

This method makes it possible to obtain the full composition of the group of relativity on the ba-
sis of one Boolean function of its composition, the name of the group of relativity (the smallest binary
number of Boolean function in the group), to construct the minimal form for any of Boolean functions
of the group without the process of minimization if at least one function from the group of relativity is
already minimized.

The phenomenon of the groups of relativity in symbolic logic is due to the problem of numerology.
It is due to the fact that all arguments of Boolean function are absolutely equal, but when constructing a
truth table, columns must be put in a certain order. As a result, there are large groups of functions hav-
ing the same properties, because they have the same internal structure. The advantage of group data is
that they completely cover the full range of Boolean functions without overlapping one another. This
makes it possible to significantly reduce the number of objects studied within the complete set L(n) of all
Boolean functions f(n) by examining only one Boolean function from the whole group.

The full composition of the group of relativity based on the truth table of the function can be
formed by performing two equivalence operations – by rearranging columns of arguments in places or
by replacing the arguments columns with their inverses, without changing in both cases the values in
the column of the result. It is these actions that underlie the implementation of the method. To simplify
the implementation of the method, recursive procedures are replaced by cyclic ones.

This method is developed as a working tool for studying the relationships between the groups of
relativity in terms of the decomposition of Boolean functions in order to find new effective methods of
minimization.

Keywords: Boolean function, groups of relativity, universal matrix of permutations.

Relevance of research. The problem of minimizing Boolean functions (BFs) with a large
number of arguments is one of the most laborious stages in the process of synthesizing digital units
(CBs) of promising computer systems.

A key step in the development of new methods for minimizing BF is the study and re-
search of patterns of the structure and internal organization of BF, as well as factors facilitating
the process of their minimization.

In the research of BF whole groups of Boolean functions – groups of relativity (GR) [1] –
are found, combining BF with the same proper-
ties (see Table 1). As a result, BF data inside the
group have the same circuitry [2].

The main advantage of GR is the fact that
the groups of relativity cover the complete set of
BFs. Therefore, the study of only one BF from the
GR instead of all BFs allows for a significant ac-
celeration of the study of a complete set of BFs.

Taking into account the importance of the
groups of relativity in the systematic study of the
complete set of BFs, the actual problem con-
sists in the development of an effective method
for obtaining the full composition of GR based
on one of its BFs.
Table 1

| №  | Number of arguments in BF | Number of BF in L(n) | Number of relational groups in L(n) |
|-----|--------------------------|----------------------|------------------------------------|
| 1   | 2                        | $2^2=16$             | 5                                  |
| 2   | 3                        | $2^3=56$             | 24                                 |
| 3   | 4                        | $2^4=65536$          | 402                                |
| 4   | 5                        | $2^5=4294967296$     | $1.228.158$                        |
| 5   | 6                        | $2^6 \approx 1.84467 \cdot 10^{19}$ | $400.507.806.843.7$               |

Analysis of recent research and publications. Belarusian group of scientists under the guidance of Zakrevsky [3] pays considerable attention to the study of the properties and characteristics of BF structure. The laboratory of logical design, organized by him in 1971, gave a whole galaxy of scientists of this school: Bibilo P. N. [6], Pottosin Yu. V. [7], Cheremisinova L. D. [8], Toropov N. R. [9] and others.

Considerable attention to the research problem is given to this topic at Omsk State University [4]. Researches are conducted also in Ukraine [5].

Formulating the goals of the article. Solving the problem of minimizing BF with a large number of arguments based on unrepeatable Boolean functions enables to construct high-performance computer systems in positional binary numerical systems that perform arithmetic calculations similar to systems of residual classes (SOK), where each digit of the number is calculated separately without taking into account junior grades. For systematic research in this direction, there is a need to develop a simple method for the rapid acquisition of the full composition of BF in a specific GR based on one of its BFs.

The purpose of the article is to describe a developed matrix method for obtaining the full membership of relational groups of Boolean functions.

Presenting main material. Boolean functions are a subset of logical functions that can be described using a truth table (TT BF) (Table 2).

The result column $y$ specifies the binary number of a specific BF:

$$y = y_{n-1}y_{n-2}\ldots y_3y_2y_1y_0,$$

where $y_i$ is either 0 or 1.

Obviously, one BF corresponds to a specific binary number. Binary numbers allow to see all BF states, to sort and group them by properties.

In [1] it is shown that the full GR composition on the basis of TT BF can be formed by performing two equivalence operations – by rearranging columns of arguments or by replacing the columns of arguments with their inverses, without changing in both cases the values in the result of column TT BF.

The subset of the permutation of arguments contains in the general case $n!$ variants of Boolean functions, and the subset of the substitution of arguments by their inversions – $2^n$ variants of BF. Therefore, the maximum amount of BF within one GR may be, if not taking into account the mutual overlapping of BF:

$$N \leq n!2^n.$$  \hspace{1cm} (1)

As it follows from the formula, the number of permutations, depending on the number of arguments, grows in an avalanche way (Table 3).

The matrix method for obtaining the full membership of the groups of the relativity of Boolean functions is based on the use of a universal permutation matrix. The versatile matrix of permutations is a table whose rows are all variants of permutations obtained as a result of permutations of columns and inversion of arguments in TT BF. Elements of the matrix are the index numbers of the binary number BF.

The method of obtaining the full membership of GR consists of two stages:

1. Preparatory stage – the stage of obtaining a universal matrix of permutations. The universal matrix is unique to BF that contain a specific number of arguments. Therefore, it is built once and used in the future for the full GR.
The size of the permutations matrix for BF containing n arguments

| №  | Number of arguments | Matrix size | Number of characters per line | Number of rows |
|-----|---------------------|-------------|-------------------------------|----------------|
| 1   | 2                   |             | 4                             | 8              |
| 2   | 3                   |             | 8                             | 48             |
| 3   | 4                   |             | 16                            | 384            |
| 4   | 5                   |             | 32                            | 3 840          |
| 5   | 6                   |             | 64                            | 46 080         |
| 6   | 7                   |             | 128                           | 645 120        |
| 7   | 8                   |             | 256                           | 10 321 920     |
| 8   | 9                   |             | 512                           | 185 794 560    |
| 9   | 10                  |             | 1024                          | 3 715 891 200  |
| 10  | 11                  |             | 2048                          | 81 749 606 400 |
| 11  | 12                  |             | 4096                          | 1 961 990 553 600 |
| 12  | 3                   |             | 8192                          | 51 011 754 393 |

At the preparatory stage, one must first build a complete list of permutations of columns of arguments in places and inversions of columns of arguments in TI BF. Then, for each combination of arguments, the corresponding binary index number BF should be constructed.

The algorithm for obtaining a universal matrix of permutations has the form (Fig. 1).

2. The main stage – Stage of getting the full membership of GR.
To obtain the full membership of GR in the zero line of the universal permutation matrix, one must place the binary number of BF from the desired GR (Table 5) and, by index, construct all the BFs from the given GR.

Application of a universal matrix of permutations for building a complete GR

| №  | BF Binary Number | Result |
|-----|------------------|--------|
| 0   | 0 1 0 1          |        |
| 1   | 3 2 1 0          | 0101   |
| 2   | 3 1 2 0          | 0011   |
| 3   | 2 3 0 1          | 1010   |
| 4   | 1 3 0 2          | 1100   |
| 5   | 1 0 3 2          | 0101   |
| 6   | 2 0 3 1          | 0011   |
| 7   | 0 1 2 3          | 1010   |
| 8   | 0 2 1 3          | 1100   |

As a result, for the given BF 0101 the GR has been received with four BFs: 0011, 0101, 1010, 1100. The GR has the smallest number of BFs – 0011.
Conclusions:
1. The phenomenon of the groups of relativity in symbolic logic is due to the problem of numerology. It is due to the fact that all arguments in BF are absolutely equal, but when constructing TI BF, they need to be put in a certain order.
2. A matrix method for obtaining a full composition of the groups of relativity of Boolean functions is developed in the article on the basis of a universal permutation matrix.
3. This method makes it possible to obtain the name of GR (the smallest binary number BF in GR), the complete GR composition based on one BF of its composition, to construct a minimal form for any of BFs from the GR without the minimization process, if at least one BF is already minimized from GR.
4. This method is a working tool for investigating the interrelationships between GRs in terms of BF decomposition. This makes it possible to significantly reduce the number of objects studied within the complete set $L(n)$ of all BFs $f(n)$ by examining only one BF from the entire group.

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МАТРИЧНИЙ МЕТОД ОТРИМАННЯ ПОВНОГО СКЛАДУ ГРУП РЕЛЯТИВНОСТІ БУЛЕВИХ ФУНКЦІЙ

У статті описано матричний метод отримання повного складу груп релятивності буле-вих функцій на основі універсальної матриці перестановок. Цей метод дає можливість отри-мати повний склад групи релятивності на основі однієї булеової функції із її складу, назву груп
релятивності (найменший бінарний номер булєвої функції в групі), побудувати мінімальну фо-
рму для будь-якої булєвої функції зі складу групи без виконання процесу мінімізації, якщо вже
мінімізована хоча б одна функція зі складу групи релятивності.

Це дає можливість суттєво зменшити кількість досліджуваних об'єктів у межах повної
множини L(n) всіх булевих функцій f(n), досліджуючи лише одну булеву функцію з усієї групи.
Для спрощення реалізації методу рекурсивні процедури замінено на циклічні.

Цей метод розроблено як робочий інструмент для дослідження взаємозв'язків між групами релятивності з точки зору декомпозиції булевих функцій для пошуку нових ефективних
методів мінімізації.

Ключові слова: булева функція, групи релятивності, універсальна матриця перестановок.

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