Operational Metrics for an Offshore Wind Farm & Their Relation to Turbine Access Restrictions and Position in the Array

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Abstract. This study explores operations & maintenance requirements for offshore wind turbines. It does so by calculating performance, reliability and maintenance metrics from an operational database provided by a large offshore wind farm. Distributions of number of repairs and repair times per turbine are shared, as well as number of visits. A focus is placed on the effect of tidal access restrictions and position in the array by comparing clusters of turbines within the wind farm. It was found that tidal access restrictions lead to an increase in mean time to repair of 16%, and 0.22% decrease in technical availability. Turbines in the first few rows with reference to the prominent wind direction experience more minor failures on average, while those constantly operating in the wake of others are characterised by more major failures, and therefore a higher mean time to repair.

1. Introduction

Offshore wind power is expected to make up an increasingly significant proportion of the energy mix within the UK in future years [1]. This is driven by ambitious renewable energy targets and continuous innovation within the wind industry. Despite remarkable progress in recent years in lowering the cost of offshore wind power, the competitive nature of the energy marketplace continues to place pressure on offshore wind assets to reduce their cost of energy. In particular, costs associated with Operations and Maintenance (O&M) activities are seeing more scrutiny as wind farms begin to move further offshore into deeper waters. Not only does this present a financial issue, with O&M costs estimated to account for up to one third of the cost of energy [2]; but more critically a safety issue, in seeking to minimise risk of injury to technicians.

Research into operational decision making is therefore an active field in academia and industry, with numerous decision making tools having been developed [3]. Some of these are O&M cost models which are employed to assess broad strategy and sensitivity to various input factors [4]. Others are concerned with day-to-day operational decisions, performing optimisation of maintenance scheduling and vessel routing [5–9]. However, there are few studies which explore O&M requirements and effectiveness directly by a retrospective analysis of operational data from an offshore wind farm. This is what this study aims to achieve, with particular scrutiny on two aspects: (i) the effect of tidal access restrictions and (ii) location in the array on turbine maintainability. Tidal access restrictions might arise where a wind farm is situated in shallow water depths, and maintenance cannot be undertaken during low tide at the site. Location in
the array may be assumed to effect maintainability as it has been noted that turbine reliability has a dependence on wind speed and turbulence intensity. Those turbines facing the prominent wind direction compared to those constantly operation in the wakes of other turbines see varying wind conditions, and may therefore have different reliability characteristics.

The following reports the measurement of operational performance metrics for a large offshore wind farm and explores the variation of these metrics due to the two factors mentioned above. The metrics chosen were: technical availability, failure rate, mean time to repair, active repair time, lost production per repair, crew visits and technician man hours. This analysis is valuable to the research community for three reasons. First, there is little information pertaining to offshore wind turbine reliability/maintainability currently in the literature. Second, to the authors’ best knowledge there are no studies which attempt to quantify the effect of tidal access restrictions on wind turbine performance. Such a study may be informative for potential future sites in very shallow water depths. Third, it provides evidence for the effect of location in the array on turbine reliability.

2. Data Audit
The analysis is based on operational data provided by a large offshore wind farm, consisting of a fleet of modern, geared wind turbines with a multi-MW power rating. The database available for analysis consists of two years of SCADA data, maintenance activity logs and weather data, spanning the time period from July 2018 to July 2020. It is assumed that this is beyond the ‘early failures’ period on the bathtub curve commonly considered to represent WT failure intensity through time [10]. The data used in this analysis is summarised by table 1. Of the factors influencing the planning and cost of maintenance of offshore wind farms as listed in [3], the metadata presented can be used to represent: power production, inspections, repairs, failures of turbines, repair time, wave height, wind speed, weather windows, travel time, environmental conditions (dependent on time and season), types of maintenance and distance from shore.

As with all data sources, there are inevitable limitations. First, the completeness of transfer of control data - the data cataloguing technician movements from vessel to turbine and vice-versa - is heavily dependent on compliance of the technicians. It is difficult to quantify the completeness of this data, but two indicative figures are that: 6.8% of all ‘drop-offs’ have no corresponding ‘pick-up’ and 5% of turbine downtime is unaccounted for by cross-referencing SCADA data with operational data (operational data referring here specifically to maintenance task information and technician/vessel movements). While this isn’t ideal, inaccuracies introduced by human error are highly likely in work orders and the data in question is judged to be of relatively high quality.

Second, the data table ‘tasks’ is useful for categorising maintenance into broad groups, but it lacks the detail of more traditional long-text work orders. Most corrective tasks are assigned an alarm code as a descriptor, so it is difficult to categorise turbine failures by component, as is typical for wind turbine reliability analyses. Reliability statistics are therefore restricted to the turbine as a whole for this analysis. It is also difficult to categorise repairs as ‘manual restarts’ - i.e. where no material is consumed, as there is no detail about the resources/spare parts used. Manual restarts are therefore included in the failure rates - they will naturally fall under the heading of ‘minor repairs’.

3. Definitions of Metrics and Their Prevalence in the Literature
There are many indicators which are used to assess the characteristics of wind turbines in terms of performance, reliability, maintenance, finance and safety. This section gives a brief overview of those metrics used in this analysis, providing a definition used and an overview of similar
| Data Table Name                      | Relevant Column Headings                                      | Comments                                                                 |
|-------------------------------------|----------------------------------------------------------------|--------------------------------------------------------------------------|
| Operations Planned Movements        | person ID, manual acknowledge time, tetra acknowledge time,   | Transfer of control times is a mix of technicians radioing in to          |
|                                     | card swipe time                                                 | acknowledge transfer & a card swipe system                                |
| Tasks/Task Types                    | description, name (task types)                                 | Use this information to categorise maintenance tasks                      |
| Operations Shift Tasks              | task ID, shift ID, drop off vessel shift ID                     | Connects task, shift & vessel information                                 |
| Vessel Stops                        | location ID, type (drop-off/pick-up), stop order                | More info on transfer of control                                          |
| Turbines                            | name, longitude, latitude, water depth, restrictions           | Contains info relevant to access restrictions & distance to shore        |
| SCADA                               | active power, wind speed                                        | Typical SCADA data - a mix of 10 minute and 2Hz resolution               |
| Weather                             | met mast wind speed 80m, Hsig1, Hsig2, met mast tide level     | Met mast measurements & readings from two wave buoys                     |

Table 1: Metadata table containing the parts of the operational database used here.

measurements in the literature. For a more detailed overview of their respective advantages and drawbacks see [11].

3.1. Performance Indicators
With reference to an offshore wind farm, performance typically equates to efficiency. Most pertinent to the operators is the question: *is the wind farm producing as much energy as it could?* [11]. As catalogued by [12, 13], variants on time- and energy-based availability are common, as well as capacity factor. In this analysis technical availability, $A_{tech}$ is used, as suggested by [14]. It is defined:

$$A_{tech} = \frac{t_{available}}{t_{available} + t_{unavailable}}, \quad (1)$$

where $t_{available}$ represents time of full or partial turbine performance, technical standby, requested shutdowns, and downtime due to environment and grid, and $t_{unavailable}$ represents only time of corrective actions and forced outage.

Despite being most indicative of turbine reliability and maintainability, the technical availability is less informative about energy capture than energetic availability. Therefore, lost production is also used as a measure so that the effect of different maintenance actions can be compared via an ‘opportunity cost’. This is estimated by building a linear regression between the active power of neighbouring turbines, then using the nearest fully operational turbine at the time of an outage to infer a lost production. An example of the relationship between neighbouring turbines is shown in figure 1. Lost production is calculated in MWhs.

3.2. Failures, Failure Rate and Mean Time To Repair
Wind turbine reliability is concerned with two questions: (i) *how often does a turbine fail?* And (ii) *what are the impacts of those failures in terms of downtime and resources?*. What researchers mean when they say that a turbine *fails* varies, as there is no standardised definition of a failure within the wind industry. Here it is defined as a downtime event accompanied by a recorded unscheduled visit to a turbine (defined as a ‘corrective’ work order in the database).
This includes faults which are resolved by manual restarts but excludes those resolved by remote or automatic restarts. After cataloguing all of the failures over the 2 years of data, failure rate is presented in a per turbine per year format.

Mean time to repair (MTTR) is the average time to return a wind turbine to its functional state. It is calculated here by averaging the downtime from downtime events associated with corrective maintenance tasks. We also include repair time as a metric, which differs from MTTR in that it only measures the amount of active maintenance time on the turbine (i.e. excluding lead times, transportation and logistical delays).

While there are a number of initiatives which provide reliability information for onshore sites, statistics for offshore wind turbines remains sparse in the literature. There is one notable study [15] which provides failure rates by component for offshore wind turbines, as well as repair times and resource requirements for unscheduled maintenance. The SPARTA initiative provides benchmarking studies for the industry within the UK - notably one of their publications presents repair rate per turbine throughout the year [16]. However, they do not state how this is defined.

3.3. Visits and Man-Hours
This analysis also uses number of turbine visits and man hours as metrics to track maintenance requirements. A ‘visit’ is classified as the displacement of a maintenance crew from a vessel onto a turbine. Man hours measure the collective time spent on turbines by technicians. Less visits should be needed in the case of higher reliability and/or in the case of a well optimised maintenance strategy. A larger ratio of number of visits to man-hours implies poorer turbine maintainability, i.e. it implies a shorter maintenance window and a given maintenance action is less likely to be effective.

In the literature, this has been explored a little by the SPARTA initiative [17] which reports a trend of decreasing technician transfers in the industry, reaching 6.8 transfers per turbine per month as of 2018. This seems high and they provide no information about how ‘transfer’ is defined, so it is assumed that it is different from how ‘visit’ is used here - perhaps a displacement of one technician to a turbine rather than a full team.

4. Methodology
The methodology involved four main stages, as summarised in figure 2. First, an ‘operations’ data frame was constructed out of work orders, shift rotas, transfer of control data and turbine
information via SQL. The first step of this was to retrieve timestamps for manually acknowledged movements from turbine to vessel, or those confirmed with a card swipe. The next was to attach technician, vessel, location and task information to that transfer. Task information was used to categorise the transfer under one of the following maintenance types: annual service, inspection, defects & tasks (D&Ts), or corrective. Actions which didn’t fit into these categories are not considered henceforth.

Second, a ‘downtime catalogue’ data frame was constructed by identifying turbine downtime events from SCADA data and cross-referencing these with the operations data frame. This resulted in a timeline of downtimes for each turbine with an associated maintenance type and description. Further queries were made to calculate number of visits, repair time and lost production, as well as categorise corrective tasks into ‘minor repair’, ‘major repair’ and ‘major replacement’ based on the man-hours required. Repairs taking 30 man-hours or less to remedy are classed as minor repairs, major repairs have between 30 and 120 man-hours and major replacements more than 120.

Third, outcomes of the downtime catalogue were aggregated by turbine. Turbines were categorised into 5 groups based on tidal access restrictions and wind exposure. Refer to figure 3 for a schematic of the arrangement of these groups, as specified by the following:

- Group 1 (G1) consisted of the turbines nearest to the operational base of the wind farm, which are mostly on the edge of the array facing the predominant wind directions from the south-west.
- Group 2 (G2) were the turbines in the middle of the farm, which constantly operated downwind of other turbines.
- Group 3 (G3) were the turbines farthest from the operational base, which were exposed to the second-most prominent wind direction from the north-east.
- Tidally restricted turbines (T) are turbines situated in shallow water depths, meaning that they are inaccessible at low tide levels. T is separate from G1, G2 and G3 - i.e. G1, G2, and G3 contain no tidally-restricted turbines. Were they to be grouped by location, T would be split almost equally between G2 and G3.
- Non-tidally restricted turbines (NT) is a combination of G2 and G3, so that it can be compared to T.

Fourth, mean values and standard error of the mean (SEM) for each of the selected metrics are presented on a farm-wide basis, and means were compared between the different groups of
turbines. Tidally restricted turbines are compared directly against not-tidally restricted, and comparisons are made between groups G1, G2, and G3. The Kolmogorov-Smirnov test [18] was used to identify significant differences between the distributions of the disparate groups. Probability distributions were also fit to key metrics to provide an insight into their uncertainty. Best-fit distributions were selected by: (i) assuming that each metric can be modelled as a continuous random variable, (ii) binning the data by percentiles and fitting a set of candidate distributions and (iii) performing a chi-squared goodness of fit test on each of the candidate distributions and selecting the best fitting.

5. Results and Discussion

The operational metrics explored are summarised in table 2. The column 'farm-wide' presents values averaged over the entire farm, while columns G1, G2 and G3 present averages within those respective groups. The column 'TvNT' presents the percentage difference between mean values of tidally-restricted turbines and non-tidally restricted turbines. Each percentage difference is preceded by a +/- signifying whether the metric is greater/lesser for the tidally-restricted group. For example, in the 'All - MTTR' row +16.12% signifies that failures for tidally restricted turbines have a greater MTTR by 16.12% on average. Farm-wide metrics and their uncertainty are explored further in figure 4. The parameters for these best fit distributions are detailed in the appendix.

As observed in table 2, the failure rate is 6.75 failures per year, which is lower than that recorded in both [15] and [16] (8.27 and 15.84 respectively). This is surprising, given that the definition of failures included here includes manual restarts, which [15] does not. It is difficult to compare these figures directly though, due to the different definitions used. The majority of these failures (84.6%) are minor repairs, however this majority accounts for only 40% of the downtime. We see that the majority of the downtime for repairs is attributable to logistical issues, transportation or weather delays rather than the repair itself. For minor repairs the ratio of active repair time to downtime is 0.27 - dropping to 0.18 for major repairs and 0.11 for major replacements.

5.1. Tidal Access Restrictions

It can be seen in Figure 5a that tidal access restrictions lead to shorter visit durations when compared to their non-tidally-restricted counterparts. It can be seen in Figure 5b that this
Table 2: Summary of the operational metrics measured, averaged for all turbines in the farm, along with standard error of the mean, displayed in brackets. Also showing average values for groups G1, G2 and G3. TvNT is a direct comparison of the means of the T and NT groups. Entries marked with an * in the TvNT column produced a significant Kolmogorov-Smirnov statistic and p-values < 0.05, implying statistically significant differences in their probability distributions. Those marked with a † in the G1 and G2 columns represent significant differences to G3 turbines under the same criteria. Those marked with a ‡ in the G2 column represent significant differences between G2 and G1 turbines.

![Graphical summary of some of the farm-wide metrics tracked in this study.](image-url)
produces a marginal shift in the MTTR probability density to the right. On average, this leads to 16.12% more downtime from failures for tidally-restricted turbines. From table 2 we see that this effect becomes more prominent as the type of repair becomes more intensive to resolve. However, the results of the Kolmogorov-Smirnov test imply that the distributions of MTTR can only be safely assumed to differ in the Mr failure category. The restrictions also entail more visits per repair, in an environment where it is desirable to reduce the number of technician transfers and the consequent safety risk. Again, this effect is amplified with the severity of the fault. We also see generally less time is available per trip for works undertaken in the remit of 'annual services', resulting in higher downtimes and more visits per service.

All of these statistics point to a reduced maintainability for tidally restricted turbines. This is important for wind farms which may be deployed in regions characterised by shallow water depths. For instance, currently operational offshore wind farms in Vietnam, Japan, Finland and Sweden, have an average water depth of just 2m, 6m, 8.5m and 8.5m respectively [19]. The implication for potential future sites situated in shallow water depths is that energy production may be less efficient than would be expected otherwise. For this particular site, it amounts to a 0.22% reduction in technical availability, despite there being a slightly smaller repair rate - and half as many major replacements - for tidally restricted turbines. Failures also become more costly - as evidenced in this study by an increase in lost production per failure of 27% for minor repairs and 29% for major repairs, as is documented in table 2. It would also be interesting to scrutinise the progression of these turbines’ reliability over time, i.e. whether the comparative lack of opportunity for preventative works has a greater impact towards the end of their lifetime.

5.2. Location in the Array

There are two aspects to consider when comparing positions in the array. The first is that the prominent wind speed at the site, as shown by figure 6a, is from the south westerly direction. This leads to greater average wind speeds for G1 compared to the other groups. The other is that the operational base is located to the south-west of the wind farm, meaning that those turbines of the south-west facing edge also take less time to travel to and are generally lower down the drop-off order list.

Figure 6b shows the effect that this has on failures. There is a clear shift in the probability density function for turbines in G1 to those in G2 and G3 to the right, and a marginal shift
between G2 and G3. On average this amounts to 7.1 repairs per year on G1, 6.98 on G2 and 6.19 on G3. Note that by the results of the Kolmogorov-Smirnov test, there is only a statistically significant difference in the failure rate distributions between G1 and G3 turbines. It is also interesting to see that G1, which has the highest average wind speed, has a greater MTTR on average than G3, despite it having a higher percentage of minor repairs (86.4% compared to 82.9%) and shorter travel times. This might imply that the effect of higher failure rates for the G1 turbines is exacerbated in times of high wind speed, when access conditions are less favourable. This is consistent across all failure categories. The ‘middle’ group of turbines - those constantly affected by the wakes of G1 and G3 - have the greatest MTTR due to the fact that a higher percentage of their failures are major repairs or replacements. It is interesting that minor failures on the farm are dominated by the turbines seeing the highest wind speed, whereas more serious failures are dominated by turbines constantly operating in the wakes of others.

These two factors - the percentage of minor to major repairs and the effect of seasonality on repair times - appear to be highly influential on a turbine’s MTTR. Considering the different reliability behaviour for disparate groups of turbines which are exposed to different wind conditions may lead to more accurate cost models for offshore wind farms. For example, as opposed to modelling all failures from (e.g.) the same Poisson process or Weibull distribution with no variation, models could be explored utilising different failure time distributions for different locations in the farm. Seasonality is typically already accounted for within the offshore logistics sub-module, however its effect may be compounded for turbine groups with higher failure rates.

6. Conclusion
This study explores the the variation of operational metrics for turbines within an offshore wind farm, focusing on effect that tidal access restrictions and location in the array have on offshore wind turbine maintainability, and in turn how this impacts performance. The analysis was based on an operational database provided by a large operational wind farm. Operational metrics were measured and compared for turbines subject to tidal access restrictions and at
different positions in the array. Technical availability, failure rate, lost production, mean time to repair, repair time, number of visits and man hours were the metrics used.

Tidally restricted turbines are characterised by generally shorter visits, resulting in differences in maintainability. This disparity was evident by a higher MTTR, lost production per failure and number of visits for tidally restricted turbines compared to non-tidally restricted. There was no significant effect in technical availability however, in part due to different failure rates between the different groups of turbines. Minor failure rates were higher for turbines nearest to the south-westerly edge of the array, which is the prominent wind direction at the site. However, those turbines almost constantly operating in the wakes of the others showed higher major repair and major replacement rates. This resulted in a higher time to repair and lower availability on average.

7. Acknowledgements
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Appendix: Summary of the best fit distributions.
A summary of the best-fit distributions used to model the data are detailed here. Table 3 provides parameters for those best fit distributions.
Variable | Distribution | Parameters
--- | --- | ---
Number of failures | Exponential Modified Normal | $K = 1.74$, $\text{loc} = 3.42$, $\text{scale} = 1.87$
Number of failures - G1 | Exponential Modified Normal | $K = 1.30$, $\text{loc} = 4.40$, $\text{scale} = 2.08$
Number of failures - G2 | Exponential Modified Normal | $K = 2.45$, $\text{loc} = 2.83$, $\text{scale} = 1.70$
Number of failures - G3 | Alpha | $c = 6.32$, $\text{loc} = -12.84$, $\text{scale} = 117.25$
Repair Time - All | Alpha | $c = 2.92$, $\text{loc} = -6.66$, $\text{scale} = 32.81$
Repair Time - minor repairs | Beta | $A = 3.81$, $B = 852744$, $\text{loc} = -1.52$, $\text{scale} = 1319765$
Repair Time - major repairs | Exponential Modified Normal | $K = 7.89$, $\text{loc} = 6.03$, $\text{scale} = 1.11$
Repair Time - Major Replacements | Alpha | $c = 3.65$, $\text{loc} = -19.42$, $\text{scale} = 247.06$
Repair Time - Annual Services | Exponentiated Weibull | $a = 0.72$, $b = 4.47$, $\text{loc} = -4.87$, $\text{scale} = 34.61$
Downtime - Annual Services | Exponential Modified Normal | $K = 2.36$, $\text{loc} = 13.74$, $\text{scale} = 6.36$
Number of Visits | Exponential Modified Normal | $K = 1.13$, $\text{loc} = 16.02$, $\text{scale} = 3.41$

Table 3: Summary of best fit distributions for the metrics measured in this study.

**Exponential Modified Normal**

A can be thought of as the sum of a standard normal random variable and an independent exponentially distributed random variable with rate $1/K$:

$$f(x, K) = \frac{1}{2K} \exp \left( \frac{1}{2K^2} - x/K \right) \text{erfc} \left( -\frac{x - 1/K}{\sqrt{2}} \right),$$  \hspace{1cm} (2)

and the loc and scale parameters are used to shift and scale the distribution respectively. This holds true for all distributions.

**Exponentiated Weibull**

Similarly, an exponentiated Weibull describes a Weibull distribution with shape parameter $a$, modified by exponentiation parameter $b$:

$$f(x, a, b) = ab(1 - \exp(-x^b))^{a-1} \exp(-b)x^{b-1}.$$  \hspace{1cm} (3)

**Alpha**

The probability density function of the alpha distribution is given by:

$$f(x, c) = \frac{1}{x^2\Phi(c)\sqrt{2\pi}} \exp \left( -\frac{1}{2}(c - 1/x) \right),$$  \hspace{1cm} (4)

where $\Phi$ is the normal cumulative density function and $c$ a shape parameter.

**Beta**

The probability density function of the beta distribution is given by:

$$f(x, A, B) = \frac{\Gamma(A + B)x^{A-1}(B - x)^{B-1}}{\Gamma(A)\Gamma(B)},$$  \hspace{1cm} (5)

where $\Gamma$ is the Gamma function, and $A$ and $B$ shape parameters.