New numerical model of heat and moisture transfer in the wet brick

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Abstract The subject of this study is numerical modelling of the fully coupled heat and moisture transfer in the building material, i.e., in a brick. The new mathematical model of equilibrium transport of heat and moisture in the form of continuous liquid medium (funicular region), discontinuous liquid medium (pendular region) and gaseous phase with dry air is proposed. As independent variables, the total mass moisture content and temperature are chosen. Both energy and moisture balance equations are mutually coupled. It is due to dependence of capillary and vapor pressures on the moisture content and temperature. Additionally, vaporization and condensation phenomena are also accounted for in the energy balance equation. Both transport equations are supplemented with boundary conditions expressing the coupled heat and moisture transfer at the dried wall or heat transfer at impermeable walls. Balance equations are discretized on the rectangular control volume grid in the framework of the in-house computational code. A grid-based method of generation of the control volume mesh is proposed, with ‘half’ control volumes related to boundary grid nodes created at domain walls. This approach enables deeper insight into implemented code as well as simplifies analysis of influence of selected terms in balance equations and implementation of complex hygro-thermal boundary conditions.

1. Introduction
There are several sources of moisture in building materials. One can mention capillary suction from foundations, rain infiltrating roofing, water leakages due to water installation failure, etc., so it is a common problem. Water can appear in such materials in three phases, namely as vapor, liquid and solid. Presence of water in building materials is generally detrimental. It increases heat losses, can lead to destruction of the building structure due to icing or its degradation and can intensify growth of fungi and molds.

In recent years the considerable focus on experimental and numerical investigation of moisture processes in building materials is observed. The first theoretical and experimental analysis of simultaneous energy and moisture transfer was carried out by Luikov [1]. The macroscale description of transport processes based on the volume averaging of microscale transport balance equations was derived by Whittaker [2]. Wide spectrum of problems related to heat and mass transport across porous materials was presented in the monograph by Kaviany [3]. Coupled heat and mass transfer processes accompanying condensation and transport of liquid in pendular state was investigated by Ogniewicz and Tien [4]. Teriku at al. [5] developed computational model of transient transport of coupled heat, air and moisture through a multilayered porous media. As the independent variables, temperature, relative humidity and total humid air pressure were chosen. The model was successfully benchmarked using...
analytical, numerical and experimental cases. The role of uncertainty in heat and moisture transport properties, related to measurement errors and heterogeneity of material, on results of numerical simulation was investigated numerically by Defraeye et al. [6]. The fully equilibrium model of heat, dry air and water transport was considered. The Monte-Carlo method was used to analyze several cases corresponding to many randomly disturbed physical properties of building material. The results indicated that uncertainties in the taken heat and moisture transport properties resulted in significant differences in drying behavior properties. The fully equilibrium model of heat, water and moist air transport was presented by Van Belleghem et al. [7] for fully funicular liquid water flow. Authors investigated the role of boundary conditions for the 2D drying case and compared results of simulations with experimental data.

Presented model is a continuation of earlier works performed by authors of this paper (e.g., [8-10]). In these papers computational models were developed twofold, as a fully equilibrium [8] or nonequilibrium one [9, 10]. In this submission the equilibrium model of heat and water transport is presented and further developed. The model is capable to capture two forms of liquid water flow, pendular, when liquid water is continuous and convective transport is present, and funicular, when liquid water is discontinuous, and diffusive transport of gases prevails. Transport of water is inseparably related to transport of heat. The temperature variation across the building material changes conditions of evaporation and condensation of water. Evaporation of water at the surface of building material decreases its temperature and this, in turn retards the moisture transport to surroundings. So, the heat and mass transfer equations should be taken into account in the reliable model of moisture transfer across the building material. Additionally, the total pressure of the humid air can be variable across the porous building material, due to changes in temperature and related evaporation/condensation processes, but this effect is assumed here to be negligible, so it is neglected here. In this paper the equilibrium numerical model of heat and moisture transport is presented, in the simple 1D problem statement. Performed simulations show the impact of the initial volumetric water (moisture) content and relative humidity of the ambient air on dynamics of the drying process, namely the temporal changes of temperature and water content at the surface of the wet brick. Additionally, the role of value of the limiting level of volumetric water content, \( W_{\text{min}} \), on progress of drying process is investigated.

2. Model of moisture and energy transport

A brick has a very complex porous structure. Its pores are open and connected, so gases and fluids present in it can be freely transported across. Three phases are being identified: (a) solid structure of the brick, (b) liquid phase (water), and (c) vapor phase (gaseous mixture of dry air and vapor).

Their volumetric fractions are \( \varepsilon_s, \varepsilon_l \) and \( \varepsilon_g \), respectively. They satisfy the following condition:

\[
\varepsilon_s + \varepsilon_l + \varepsilon_g = 1
\]  

(1)

where \( \varepsilon_p \) is the porosity of the porous material.

In the proposed model the volumetric water content, namely total mass of water related to volume of material, is considered. It is the sum of volumetric water content in liquid and vapor phases:

\[
W = W_l + W_g = \rho_l \varepsilon_l + \rho_g \varepsilon_g
\]  

(2)

where \( \rho_l \) and \( \rho_g \) are densities of water vapor and liquid, respectively.

The sum of volumes of liquid water and vapor are equal to volume of pores, so:

\[
\varepsilon_p = \frac{W_l}{\rho_l} + \frac{W_g}{\rho_g}
\]  

(3)

Due to condensation and evaporation processes mass of liquid and vapor phase changes. Perfect equilibrium between liquid and vapor phases is assumed, so volumetric moisture contents in liquid and vapor phases is simply related to total moisture volumetric content, \( W \):

\[
\]
Liquid water can be present in pores of the brick in continuous (pendular) or discontinuous (funicular) form. It means that if the content of liquid water is sufficiently high, it can form a continuous structure and transport of water is driven mainly by the gradient of the capillary pressure. Otherwise, the only mode of water transport is the diffusive transport of gases. The funicular form liquid water is attained if the total water content is lower than some predefined value, \( W_{min} \), i.e., \( W < W_{min} \). Otherwise, the pendular form is present.

2.1. Transport of moisture

Mass transport equations for vapor and liquid water are following:

\[
\frac{\partial}{\partial t}(\rho_v \varepsilon_v) + \nabla \cdot \mathbf{j}_v = -\mathbf{m}_v \quad \text{and} \quad \frac{\partial}{\partial t}(\rho_g \varepsilon_g) + \nabla \cdot \mathbf{j}_g = \mathbf{m}_g
\]

(5)

where \( \mathbf{j}_v \) and \( \mathbf{j}_l \) are mass fluxes of vapor and liquid water, respectively. These fluxes, for the uniform total pressure of gas in pores, \( p_g \), and neglecting the impact of gravity, takes the following forms:

\[
\mathbf{j}_v = \frac{\varepsilon_g}{\mu_l} \mathbf{K}_l \nabla p_v \quad \text{and} \quad \mathbf{j}_g = -\frac{\varepsilon_g}{\rho_g} D_{v,\text{ef}} \nabla \left( \frac{\rho_g}{\rho_v} \right)
\]

(6)

where: \( D_{v,\text{ef}} \) is the effective diffusion coefficient of water vapor in dry air, \( K_l \) is the permeability of the porous medium for liquid water, \( \mu_l \) is the liquid water viscosity and \( \varepsilon \) accounts for mutual interactions of liquid and gaseous phases. Parameter \( \varepsilon \) varies between 0 for the fully funicular and 1 for fully pendular liquid water transport. The relation between \( \varepsilon \) and \( W \) in the range \([W_{min} - 1, W_{min}]\) is given with a sine-like function, which takes in these points values 0 and 1, respectively.

Both dry air and water vapor are treated as perfect gases, so their densities can be expressed from the following Clapeyron equation of state:

\[
\frac{\rho_g}{\rho_v} = \left( \frac{M_v}{M_g} \right) \frac{B}{T}, \quad \frac{\rho_g}{\rho_v} = \frac{M_v}{M_g} \frac{B}{T}, \quad \rho_g = \frac{\rho_v}{\rho_v} + \frac{\rho_v}{\rho_v}
\]

(7)

where \( B \) is the universal gas constant and \( M_g \) and \( M_v \) are molar masses of dry air and water, respectively.

Partial pressure of vapor is related to actual relative humidity:

\[
\varphi = \frac{p_v}{p_{sat}(T)}
\]

(8)

and, according to the Kelvin’s law, is related to the capillary pressure by:

\[
p_v = -\rho_v \frac{B T}{M_v} \ln \varphi
\]

(9)

Saturation pressure is a function of temperature only and is given with the following relation:

\[
p_{sat}(T) = 611 \cdot \exp \left[ \frac{17.08(T - 273.15)}{T - 38.97} \right]
\]

(10)

After summation of eq. (5) the total moisture transport takes the following form:

\[
\frac{\partial W}{\partial t} + \nabla \cdot (\mathbf{j}_v + \mathbf{j}_l) = 0
\]

(11)
All parameters present in the mass fluxes of moisture in eq. (6) are dependent on temperature, \( T \), and total water content, \( W \), so they can be presented as follows:

\[
\mathbf{j}_l = D_l^T \nabla T + D_l^W \nabla W \quad \text{and} \quad \mathbf{j}_v = D_v^T \nabla T + D_v^W \nabla W
\]

(12)

where the respective coefficients are defined as:

\[
D_l^T = \zeta K_i \frac{\partial p}{\partial T}, \quad D_l^W = \zeta K_i \frac{\partial p}{\partial W}, \quad D_v^T = \frac{M_s M_v}{M_g BT} D_{v,a,ef} \frac{\partial p}{\partial T}, \quad D_v^W = \frac{M_s M_v}{M_g BT} D_{v,a,ef} \frac{\partial p}{\partial W}
\]

(13)

The total water transport equation takes the following final form:

\[
\frac{\partial W}{\partial t} + \nabla \left( \left( D_l^T + D_l^W \right) \nabla T + \left( D_v^W + D_v^W \right) \nabla W \right) = 0
\]

(14)

2.2. Energy transport in the moist material

The thermal equilibrium between all phases is assumed, so single energy balance equation can be expressed in the form:

\[
\frac{\partial H}{\partial t} + \nabla \cdot \left( \mathbf{j}_l h_l + \mathbf{j}_v h_v \right) = \nabla \cdot \left( k_{ef} \nabla T \right)
\]

(15)

where \( k_{ef} \) is the effective thermal conductivity of the porous medium.

\( H \) is the total enthalpy per unit volume of the multiphase medium, defined as:

\[
H = \rho_l \varepsilon_l h_l + \rho_v \varepsilon_v h_v + \overline{\rho}_g \varepsilon_g h_g + \overline{\rho}_a \varepsilon_a h_a
\]

(16)

Specific enthalpies are related to the triple point temperature as the reference one, and can be expressed as:

\[
h_l = c_s \left( T - T_{ref} \right), \quad h_i = c_i \left( T - T_{ref} \right), \quad h_a = c_{p,a} \left( T_{ref} - T_{ref} \right), \quad h_v = \Delta h_v + c_{p,v} \left( T - T_{ref} \right)
\]

(17)

where \( c_s, c_i, c_{p,a} \) and \( c_{p,v} \) are specific heats of components and \( \Delta h_v \) is the latent heat of evaporation. The reference temperature is equal to \( T_{ref} = 273.15 \text{K} \).

Substituting definitions of specific enthalpies, eq. (17), to the energy balance, eq. (15), it can be expressed using the temperature as independent variable. Then using liquid water and vapor mass fluxes expressed with eq. (12), the energy equation reduces to the following form

\[
\left( \rho c \right)_{ef} \frac{\partial T}{\partial t} + \left( \left( D_l^T c_l + D_l^W c_{p,v} + D_v^T c_{p,a} \right) \nabla T + \left( D_v^W c_l + D_v^W c_{p,v} + D_v^W c_{p,a} \right) \nabla W \right) \cdot \nabla T =
\]

\[
= \nabla \cdot \left( k_{ef} \nabla T \right) - m_v \Delta h_v
\]

(18)

where \( (\rho c)_{ef} \) is the effective volumetric specific heat given by:

\[
(\rho c)_{ef} = \varepsilon_s \rho_s c_s + \varepsilon_i \rho_i c_i + \varepsilon_g \overline{\rho}_g c_g + \varepsilon_a \overline{\rho}_a c_a
\]

(19)

and \( m_v \) is the intensity of evaporation and condensation calculated with formula:

\[
m_v = \frac{dW_v}{dt}
\]

(20)

2.3. Boundary conditions and solution procedure

Two types of boundary conditions are considered. Across the surface of the brick, which is in contact with ambient air, both heat and moisture transport are possible. Fluxes of moisture and heat satisfy the following relations:
\[ (\mathbf{j}_l + \mathbf{j}_v) \cdot \mathbf{n} = \mathbf{j}_{l,\text{amb}} = h_m \left[ \bar{\rho}_v^e \left( T_e \right) - \bar{\rho}_v^e \left( T_{\text{amb}} \right) \right] \]
\[ -k_{ef} T + \mathbf{j}_h + \mathbf{j}_v \cdot \mathbf{n} = h_r \left( T_b - T_{\text{amb}} \right) \]

where \( h_r \) and \( h_m \) are heat and mass transfer coefficients, respectively, while \( T_e \) and \( T_{\text{amb}} \) are temperature of the wall and ambient air, respectively. If the wall is assumed insulated and additionally no mass transfer is possible, the right sides of eq. (21) are set zero.

The procedure of solution of coupled transport equations of moisture and energy is following. Primarily the moisture transport equation, eq. (14) is solved. Next, the moisture content in liquid and gaseous phases is calculated with eq. (4). Finally, the energy transport equation, eq. (18), is solved. This procedure is repeated until the convergence criteria are met.

### 2.4. Material properties and closure relations

Solution of the presented above equations requires information related to material properties and closure relations, e.g., isotherm of sorption. Simulations were performed for the ceramic brick. Its material properties refer to those used in [7] and are listed in table 1.

| Property                  | Symbol | Value       |
|---------------------------|--------|-------------|
| Density of water          | \( \rho_l \) (kg/m\(^3\)) | 1000.0      |
| Density of brick          | \( \rho_s \) (kg/m\(^3\)) | 2087.0      |
| Specific heat of dry air  | \( c_a \) (J/kg/K)     | 1005.0      |
| Specific heat of water    | \( c_l \) (J/kg/K)     | 4192.1      |
| Specific heat of brick    | \( c_s \) (J/kg/K)     | 840.0       |
| Specific heat of vapor    | \( c_v \) (J/kg/K)     | 1875.2      |
| Molecular mass of dry air | \( M_a \) (kg/kmol)    | 28.86       |
| Molecular mass of vapor   | \( M_v \) (kg/kmol)    | 18.0        |
| Capillary moisture content| \( W_{\text{cap}} \) (kg/m\(^3\)) | 130.0      |
| Porosity of brick         | \( \varepsilon_p \)   | 0.13        |
| Latent heat of evaporation| \( \Delta h_{lv} \) (J/kg) | 2.6\( \cdot 10^6 \) |
| Universal gas constant    | \( B \) (J/kmol/K)     | 8314        |

Closure relationships for the system of equations are the same, as were used in [7]:

- Vapor diffusivity in pores:
  \[ D_{v-a,\text{ef}} = 1.053 \cdot 10^{-6} \frac{1 - W/W_{\text{cap}}}{0.503 \left(1 - W/W_{\text{cap}}\right)^2 + 0.497} \] (22)

- Effective thermal conductivity of the moist brick:
  \[ k_{ef} = 1 + 0.0047 W \] (23)

- Water permeability in the brick:
  \[ K_i = \frac{1.1437 \cdot 10^{-9}}{\left[1 + \left(1.76 \cdot 10^{-8} \rho_l\right)^{4.3}\right]^{0.6}} \] (24)
Retention curve:

\[
W = W_{\text{cap}} \left\{ 0.846 \left[ 1 + \left( 1.394 \times 10^{-5} p_{e} \right)^{4} \right]^{-0.75} + 0.154 \left[ 1 + \left( 0.9011 \times 10^{-5} p_{e} \right)^{1.69} \right]^{-0.408} \right\}
\]  

(25)

3. Problem statement

Presented balance equations of the total moisture content, eq. (14), and energy, eq. (18), are discretized in the 1D domain (figure 1). The representation of a 1D control volume grid is based on the grid points, so outmost nodes are located on left and right walls and so two ‘half’ control volumes are generated at boundaries. The left boundary is adiabatic and impermeable, and the mass and energy transfer are possible across the right boundary, so boundary conditions given by eq. (21) are imposed there.

![Figure 1. Schematic of the considered domain and boundary conditions.](image)

Temperature of the ambient air, \(T_{\text{amb}}\), and initial temperature, \(T_{0}\), are equal to 23.8°C for all considered cases. The relative humidity, \(\varphi_{\text{amb}}\), can be equal to 50 or 60%. The initial volumetric fraction of water, \(W_{0}\), is equal to 80% or 90% of \(W_{\text{cap}}\). The heat transfer coefficient, \(h_{T}\), is equal to 25 W/(m²K) for all cases. The mass transfer coefficient, \(h_{m}\), is calculated according to the Chilton-Colburn analogy, for Lewis number, \(\text{Le} = 0.808\), and is equal to 0.0287 m/s.

4. Results and discussion

The impact of initial water content in a brick and relative humidity of drying air on the drying process is investigated. Temporal variations of temperature and volumetric water content at the surface of the dried brick are presented in figure 2. The characteristic shape of the temperature curve is visible in figure 2A. Temperature drops in short time to its lowest value and is kept at constant value over some time period. Its length depends on both, \(W_{0}\) and \(\varphi_{\text{amb}}\). Obviously, the higher value of \(W_{0}\) or \(\varphi_{\text{amb}}\) leads to longer the first period of drying, when temperature is at its lowest level. The value of the lowest temperature depends strongly on \(\varphi_{\text{amb}}\) and is practically \(W_{0}\) independent (figure 2A). During the first period of drying, considerable changes in total water content are observed (figure 2B). After that time, temperature rises abruptly and variations in water content start to be much slower. It is due to change of the mechanism of water transport, from convective transport of liquid water to diffusion of water vapor. The value of the water content, at which water removal slows down is the same for all cases, equal to 15.4 kg/m³, which corresponds to 11.8% of \(W_{\text{cap}}\).
Figure 2. Temporal variation of: A) temperature and B) volumetric water content at the surface of the brick for initial water content of $W_0 = 80\% \cdot W_{\text{cap}}$ (continuous line) and $W_0 = 90\% \cdot W_{\text{cap}}$ (dashed line), for $\varphi_{\text{amb}} = 50\%$ (blue line) and $\varphi_{\text{amb}} = 60\%$ (red line).

The role of the existence of the funicular transport region, given with the limiting value of the volumetric water content, $W_{\text{min}}$, was performed for two values of $W_{\text{min}}$, namely 20 kg/m$^3$ and 25 kg/m$^3$. Below that value the convective transport of liquid water is suppressed. Temporal variations of temperature and volumetric water content at the surface of the dried brick, predicted numerically for two values of $W_{\text{min}}$, and related to the case where liquid water transport is fully pendular is presented in figure 3. After the volumetric water content attains the $W_{\text{min}}$, it is kept approximately at the same level, so drying is considerably retarded. With increasing value of $W_{\text{min}}$, the rate of temperature increase is higher.

Figure 3. Temporal variation of: A) temperature and B) volumetric water content at the surface of the brick for initial water content of $W_0 = 80\% \cdot W_{\text{cap}}$ and $\varphi_{\text{amb}}=50\%$ for fully pendular transport (blue line) and for partially funicular transport; with $W_{\text{min}} = 20$ kg/m$^3$ (red line) and $W_{\text{min}} = 25$ kg/m$^3$ (yellow line).

5. Conclusions
In this paper the equilibrium model of heat and moisture transport was presented. It was applied to simulation of drying of uniform material, porous brick, in simple 1D geometry. The role if initial water content as well as relative humidity of drying air on variations of temperature and water content at the dried surface were investigated numerically. The temporal changes of temperature reveal characteristic shape (e.g., similar to these shown in [7]), which reflects two stages of drying. During the former, the
convective transport of liquid water is dominant, temperature considerably drops, and is maintained at the constant level. Concurrently, the amount of moisture rapidly decreases. The second stage starts when the temperature starts to growth. The dominant mechanism of moisture transport changes, it is now driven mainly by diffusive transport of vapor. It results with rapid decrease of the rate of moisture changes at the surface. If the existence of funicular transport is taken into account, the higher values of water content are observed, corresponding to \( W_{\text{min}} \) values, and temperature growth rate is higher.

In this paper the simple model of heat and water (moisture) transport was presented. It will be generalized in future for cases involving changes in total pressure of the moist air and presence of nonequilibrium mass transfer between liquid and vapor phases. Numerical results will be also validated with experimental results, predicted with the facility presented in papers [11, 12].

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References
[1] Luikov A V 1966 *Heat and Mass Transfer in Capillary Porous Bodies* (Oxford: Pergamon)
[2] Whitaker S 1977 Simultaneous heat, mass and momentum transfer in porous media: Theory of drying *Advances in Heat Transfer* **13** 119–203
[3] Kaviany M 1995 *Principles of Heat Transfer in Porous Media* (New York: Springer)
[4] Ogniewicz V and Tien C L 1981 Analysis of condensation in porous insulations *Int. J. Heat Mass Transf.* **24** 421–29
[5] Tariku F, Kumaran K and Fazio P 2010 Transient model for coupled heat, air and moisture transfer through multilayered porous media *Int. J. Heat Mass Transf.* **53** 3035–44
[6] DeFraeye T, Blocken B and Carmeliet J 2013 Influence of uncertainty in heat-moisture transport properties on convective drying of porous materials by numerical modelling *Chem. Eng. Res. Des.* **91** 36–42
[7] Van Belleghem M, Ameel B, Janssens A and De Paepe M 2011 Modeling heat and moisture transport in porous materials with CFD for building applications *HEFAT2011, 8th Int. Conf. on Heat Transfer, Fluid Mechanics and Thermodynamics* (Pointe Aux Piments) 455–63
[8] Furmański P, Seredyński M, Łapka P, Wiśniewski T S, Pietrak K, Cieślikiewicz Ł, Kubiś M and Wasik M 2018 A model of heat and moisture transfer in a building material during drying based on phase equilibrium in the moist region *Proc. 5th Int. Conf. on Contemporary Problems of Thermal Engineering CPOTE 2018* (Gliwice) pp 663–73
[9] Łapka P, Wasik M, Furmański P, Seredyński M, Cieślikiewicz Ł, Pietrak K, Kubiś M, Wiśniewski T S and Jaworski M 2018 Preliminary mathematical and numerical transient models of convective heating and drying of a brick *MATEC Web Conf.* **240** 01022
[10] Wasik M, Cieślikiewicz Ł, Łapka P, Furmański P, Kubiś M, Seredyński M, Pietrak K, Wiśniewski T S and Jaworski M 2019 Initial credibility analysis of a numerical model of heat and moisture transfer in porous building materials *AIP Conf. Proc.* **2078** 020106
[11] Cieślikiewicz Ł, Łapka P, Wasik M, Kubiś M, Pietrak K, Wiśniewski T S, Furmański P and Seredyński M 2018 Development of the experimental stand for investigation of heating and drying phenomena in the porous building materials with one surface of the sample exposed to the flowing air *E3S Web Conf.* **70** 03003
[12] Cieślikiewicz Ł, Wasik M, Kubiś M, Łapka P, Bugaj M, Pietrak K, Wiśniewski T S, Furmański P and Seredyński M 2019 Development of the experimental stand with centrally located specimen for the investigation of heat and moisture phenomena in porous building materials *Civ. Environ. Eng. Reports* **29** 53–65