High precision and fast solution method for linearized assembly contact point of geometric error surface

Zhihua Liu¹, Zhijing Zhang¹a, Xin Jing¹, Weiming Zhang¹ Zifu Wang¹ and Ke Shang¹

¹Beijing Institute of Technology, No. 5 South Zhong Guan Cun Street, Haidian Beijing 100081, P. R. China

a Corresponding author: zhzhj@bit.edu.cn

Abstract. The geometric error modeling method for assembly[1] proposes a feasible basis for the assembly process considering manufacturing error. However, contacting two mounting surfaces with geometric error and finding their actual contact points are two main difficulties in the study of real three-dimensional parts assembly methods. A high-precision and fast solution method for linearized assembly contact points considering geometric errors is proposed by calculating the point cloud spatial pose of the actual contact surface. This method is suitable for spatial pose calculation in high precision plane assembly of rigid parts, which enables fast, accurate and quantitative solution of the positional relationship of components during assembly and measurement, and improves the accuracy and efficiency of spatial pose prediction and control.

1. Introduction

In terms of true error surface representation about assembly-oriented geometric error modeling, Huang and Ceglarek et al.[2] designed a pattern-based approach to characterize part geometry errors using discrete cosine transform (DCT), which identifies and fabricates the process of error mode. Samper et al.[3] proposed to express shape deviations by establishing trace parameters based on ideal surfaces. This method is a discrete representation method, ignoring the defects of the shape surface in the assembly simulation. Schleich, B et al.[4, 5] and Zhang, Min et al.[6] delved into the fundamental principles of surface models and the effects of such non-ideal surfaces on geometric bias. Wilma and Giovanni.[7] built a manufacturing error model based on the measured data to facilitate the representation of the actual contour of the manufactured surface. Corrado et al.[8] developed a new computer-aided tolerance (CAT) tool based on manufacturing characteristics and assembly conditions, and proposed geometric models with errors to simulate the assembly of components with geometric deviations. With the application of discrete mode decomposition, Cao et al.[9] simulated the surface of multi-scale error and evaluated the error index. Zhang, et al.[1] used b-spline curves for two-dimensional data difference solving, which can express continuous free-form surfaces with few parameters. Louhichi, Borhen et al.[10] Modeling based on tolerance bands in error surface modeling. Cai, N. et al.[11] analyze the stiffness variation based on the difference motion vector and the homogeneous transformation matrix. The local deformation is analyzed based on the influence coefficient method to simulate the actual assembly deformation. Current geometric dimensions and tolerances (GD&T) standards state that the geometry of a cylindrically manufactured part should be characterized by it's roundness, straightness, cylindricity and diameter. However, the standard uses the maximum peak-to-valley to define the formal error - which is a very simple geometric description.
Therefore, measurements based on the GD&T definition of the manufactured part are not valid for identifying and diagnosing sources of error in the manufacturing process. In the expression of cylindrical features, Zhang, X. D et al.[12] used the Fourier polynomial to express the axial hole with error, studied the mechanism of error formation in the machining cylinder, and verified the model with diesel engine parts.

In the assembly process that takes into account the error, each assembly relationship contains two assembly faces, the outer surfaces of the two parts. The non-ideal plane of the part surface has irregular concave-convex features, which leads to that the actual contact form of the two planes is the mixed contact form of point contact and plane contact in microscopic scale. It is these actual contact points and the height and position of the contact surface that result in different assembly poses for each assembly of the same batch, which constitutes part of the assembly error. In the case where the assembly precision is high, the contact pose relationship caused by these topographical features will have a great influence on the assembly accuracy. Moreover, when there are many parts included in the assembly, the assembly errors caused by the contact pose relationship in all assembly relationships will be coupled, or cumulatively increased, or offset reduced, thereby affecting the uncertainty of the assembly accuracy. Further expansion, so the contact pose relationship is a factor that must be considered in precision assembly. However, the calculation of the contact pose relationship also takes into account the calculation speed, because if the calculation is not fast enough, the assembly optimization of the mechanical system cannot be achieved. Therefore, to achieve assembly process simulation and process parameter optimization, a method is needed to accurately and quickly solve the contact pose relationship between components. Therefore, by measuring the assembly plane, parameterizing and describing the topographical features, iteratively solving the contact pose relationship, optimizing the parameters to formulate the assembly scheme, the assembly error can be controlled and the assembly accuracy index can be ensured.

2. Principle of calculation

In order to consider the contact relationship caused by the topography of the assembly surface and solve the pose relationship between the two contact surfaces, the following calculation principle is proposed in this paper:

First, set the face shape of the assembly surface on the parts \( P_n \) and \( x_{i,n}, y_{i,n}, z_{i,n} \) is 3D points for the position on the surface \( n \) to be assembled.

2.1. General principle of pose matching in parts space

Ignoring the contact relationship and the force situation, Taking into consider the spatial plane relationship, there is a formula:

\[
H \cdot P_1 = P_2
\]  

(1)

where \( f_1 \) is the assembly surface 1, \( f_2 \) is the assembly surface 2, \( P_1 \) is the point set on \( f_1 \), \( P_2 \) is the point set on the surface \( f_2 \). The pose relationship between \( f_2 \) and \( f_1 \) is described by a spatial motion transformation matrix, and the point set \( P_1 \) on \( f_1 \) is transformed into a point set \( P_2 \) by spatial transformation \( H \).

\( H \) is a spatial motion matrix, which is a three-dimensional spatial transformation matrix of the spatial plane relative to the assembly coordinate system. \( H = M_x \cdot M_y \cdot M_z \), where \( M_x \), \( M_y \) and \( M_z \) are the rotation and translation of the measurement coordinate system with respect to the x, y and z axes of the assembly coordinate system, respectively, where \( \theta_x \), \( \theta_y \) and \( \theta_z \) represent the rotations in the three directions of the x, y, and z directions, respectively. \( d_x \), \( d_y \) and \( d_z \) represent the amount of translation in the x, y, and z axis directions, respectively.
In real assembly, it is necessary to limit the degree of freedom. Here, the movement in the x direction, the movement in the y direction, and the rotation in the z direction are defined: then the spatial motion matrix \( H \) is simplified as:

\[
M_i = \begin{bmatrix}
1 & 0 & 0 & d_x \\
0 & \cos \theta_x & -\sin \theta_x & 0 \\
0 & \sin \theta_x & \cos \theta_x & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(2)

\[
M_j = \begin{bmatrix}
\cos \theta_y & 0 & \sin \theta_y & 0 \\
0 & 1 & 0 & d_y \\
-\sin \theta_y & 0 & \cos \theta_y & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(3)

\[
M_k = \begin{bmatrix}
\cos \theta_z & -\sin \theta_z & 0 & 0 \\
\sin \theta_z & \cos \theta_z & 0 & 0 \\
0 & 0 & 1 & d_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(4)

For the precision assembly process, because the surface quality of the assembly surface is good, the plane continuous assumption is satisfied, and there is no abrupt change, the assembly angle \( \theta_x \) and \( \theta_y \) is very small, so equation(6), (7) and (8) is satisfied.

\[
\cos x = 1 - \frac{x^2}{2}; x \to 0
\]  

(6)

\[
\sin x = x; x \to 0
\]  

(7)

\[
z_{i,1} \cdot \theta_y \to 0, z_{i,1} \cdot \theta_x \to 0
\]  

(8)

Bring the formula (5) into the formula (1) and ignoring the second order and above infinity, So the equation (1) is expand as equation (9):

\[
x_{i,1} - x_{i,2} = 0 \\
y_{i,1} - y_{i,2} = 0 \\
z_{i,1} - z_{i,2} = x_{i,1} \cdot \theta_y - y_{i,1} \cdot \theta_x - d_z
\]  

(9)

It can be seen from equation (9) that the z-axis error is related not only to the translation distance but also to the rotation angle. There is an error about \( \theta x \) and \( \theta y \). To reduce the error, the origin of the coordinate system should be set at the geometric center of \( f_1 \). In summary, in order to reduce the error, the assembly coordinate system should be set at the geometric center of the assembled moving parts.

2.2. Two-part space surface contact point relationship with geometric error

If both \( f_1 \) and \( f_2 \) have topographical errors, \( f_1 \) and \( f_2 \) are assembled. According to the spatial motion matrix analysis, the contact portion of the two faces \( f_1 \) and \( f_2 \) satisfies the relation of the z component in equation (8);
For the untouched part, the equation (10) relationship is satisfied:

\[
z_{i,1} - z_{i,2} > x_{i,1} \cdot \theta_y - y_{i,1} \theta_x - d_z
\]  \hspace{1cm} (10)

Can meet the conditions when assembling as the equation (11), And there must be at least three equations for i to be established, that is, at least three points are in contact;

\[
z_{i,1} - z_{i,2} \geq x_{i,1} \cdot \theta_y - y_{i,1} \theta_x - d_z
\]  \hspace{1cm} (11)

2.3. Principle of space pose calculation with geometric error under force field

Set the contact form to rigid contact and define \( \rho_i \) as the distribution force at the i-th point. To meet the stability of the contact assembly, the stress balance condition should be met. The support force of the f1 contact surface subject to f2 should be equal to the external force received by f1 (such as gravity, pressure, etc.), and the f1 contact surface should satisfy the moment balance; when the equilibrium condition is satisfied The corresponding potential energy of the upper surface should be converted to a minimum as formula(12); and the non-interference constraint is satisfied as formula(13);

\[
\min \sum \rho_i \left( -x_{i,1} \cdot \theta_y + y_{i,1} \theta_x + d_z \right)
\]  \hspace{1cm} (12)

\[
z_{i,1} - z_{i,2} \geq x_{i,1} \cdot \theta_y - y_{i,1} \theta_x - d_z
\]  \hspace{1cm} (13)

Taking into account the analysis of the above three aspects, the expression that can get an optimization problem can be further converted into formula (14):

\[
\min \sum \rho_i \left( -x_{i,1} \cdot \theta_y + y_{i,1} \theta_x + d_z \right)
\]  \hspace{1cm} (14)

s.t. \( z_{i,1} - z_{i,2} \geq x_{i,1} \cdot \theta_y - y_{i,1} \theta_x - d_z \)

This is a linear programming problem, which is a problem that has been solved for the theory of operations research. The problem is low in complexity and easy to solve. The spatial motion matrix parameter H is obtained by solving the unknown \( \theta_x \), \( \theta_y \) and \( d_z \), where in the constraint condition in the equation (14) is a physical constraint, and the non-interference condition is satisfied.

3. Experiment

This part set the assembly surface for experimental calculation. The original data of 2D and 3D surfaces are shown in Figure 1-2; the assembly result is shown in Figure 3-4; the 2D assembly data is the assembly data of random design, and the 3D data is measured by the CMM(coordinates measuring machine). Actual shape data; the y-axis of the two-dimensional data in the figure is magnified 1000 times; the z-axis ratio of the three-dimensional data is displayed in unequal proportions with the x and y axes.

![Figure 1 Two-dimensional surface to be assembled](image-url)
The actual assembly results calculated by the above algorithm are as follows, and the corresponding assembly parameters are as Table 1 Result of compute.

| parameters | $\theta_x$ (°) | $\theta_y$ (°) | $d_z$ (m) |
|------------|----------------|----------------|------------|
| 2D assembly | -13.637272     | 0              | -82.316735 |
| 3D assembly | -0.036338      | 0.016937       | 0.065879   |

4. Conclusion
The calculation principle and method proposed in this paper can quickly and efficiently calculate the assembly pose relationship of the actual two assembly surfaces during precision assembly, which provides an effective method for virtual assembly of 3D solids with error contact surfaces.
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