Model Independent Tests of Cosmic Gravity

By Eric V. Linder

Berkeley Center for Cosmological Physics & Berkeley Lab, University of California, Berkeley, CA 94720, USA
Institute for the Early Universe, Ewha Womans University, Seoul, Korea

Gravitation governs the expansion and fate of the universe, and the growth of large scale structure within it, but has not been tested in detail on these cosmic scales. The observed acceleration of the expansion may provide signs of gravitational laws beyond general relativity. Since the form of any such extension is not clear, from either theory or data, we adopt a model independent approach to parametrising deviations to the Einstein framework. We explore the phase space dynamics of two key post-GR functions and derive a classification scheme and an absolute criterion on accuracy necessary for distinguishing classes of gravity models. Future surveys will be able to constrain the post-GR functions’ amplitudes and forms to the required precision, and hence reveal new aspects of gravitation.

Keywords: gravitation, general relativity, cosmic expansion, cosmic growth, galaxy surveys

1. Introduction

Gravitation is the force that dominates the universe, from setting the overall expansion rate to forming the large scale structures of matter. Yet our first precision tests of gravitation on large scales indicate our understanding of gravity acting on the components we see and expect – baryonic and dark matter – is insufficient. The cosmic expansion is not decelerating as it should given these ingredients, but accelerating, pointing to either an exotic component with negative active gravitational mass (the sum of the energy density plus three times the pressure) or new aspects of gravity. The growth and clustering of galaxies likewise do not agree with a universe possessing only gravitationally attractive matter within general relativity.

Having been surprised in our first two tests of cosmic gravity, we naturally look to explore expansions of the classical framework. Canonical general relativity (GR) with an exotic dark energy component, whether Einstein’s cosmological constant or otherwise, is certainly one possibility. Extending our knowledge of gravity is another, and is what this article concentrates on.

One question is how to systematically analyse extensions to the known framework. This can be done by working within an alternate, fully formed theory, but such first principles theories are scarce to nil (but see other articles within these Transactions for some possible guiding principles). Instead we take a phenomenological, or (ideally) model independent, approach in analogy to the manner in which compact source gravity uses the parameterised post-Newtonian formalism. Such model inde-
Table 1. Translation between several different parameterisations of extended gravity and the light/growth functions $G$ and $V$.

| Functions | Parameterisation | Reference |
|-----------|-----------------|-----------|
| $G, V$ | Bins in $k, z$ | Daniel & Linder 2010 |
| $\mu = 2G - V$, $\omega = \frac{G - V}{\sqrt{G}}$ | $\mu, \omega \sim a^s$ or bins in $z$ | See translation table in Daniel et al. 2010 |
| PPF: $f_G = G^{-1} - 1$, $g = \frac{2G}{G - V}$ | model dependent | Hu & Sawicki 2007, Hu 2008 |
| MGCAMB: $\gamma = \frac{2G}{V} - 1$, $\mu = V$ | $\gamma, \mu = \frac{1 + \beta \nu^2 \lambda^2_{\nu} \kappa^2 a^s}{1 + \lambda^2_{\nu} \kappa^2 a^s}$ | Zhao et al. 2009, Bertschinger & Zukin 2008 |
| $\Sigma = G$, $\mu = V$ | $\Sigma = 1 + \Sigma_s, \mu = 1 + \mu_s a^s$ | Song et al. 2010, Song et al. 2010b |
| PCA | Zhao et al. 2010 |

Building on a robust framework of the conservation and continuity equations, we parameterise functions from the equations of motion most closely tied to photon and matter density perturbation observables. This is equivalent to starting from the metric potentials themselves, although not directly from an action. In Sec. 2 we describe the parameterisation, and consistency structure of the system of equations, in more detail. Discussing the types and reach of data enabled by future surveys such as BigBOSS, Euclid, and WFIRST, in Sec. 3 we explore the degree of constraints that may be placed on these extended gravity, or post-GR, quantities in comparison to our knowledge today. We draw connections between this model independent approach and representatives of strong coupling and dimensional reduction classes of gravity ($f(R)$ and DGP, respectively) in Sec. 4, as well as exploring a different tack to distinguishing between gravity models through a phase space analysis similar to that used for dark energy.

2. Framing Gravity

In the equations of motion for cosmological perturbations in a homogeneous, isotropic background four quantities enter: the time-time and space-space metric potentials (equal to each other within GR), the mass density perturbation field, and the velocity perturbation field (taking a perfect fluid, e.g. ignoring pressure and anisotropic stress). Conservation of stress-energy gives the continuity equation relating the density and velocity fields, and the Euler equation relating the velocity and time-time metric potential, so we are left with two free connecting equations. These can be chosen, for example, to be the gravitational slip between the two metric potentials and a modified Poisson equation relating the space-space potential to the density field, or they could be two Poisson-like equations relating one potential to the density field and the sum of the potentials to the density. The latter choice turns out to give greater complementarity between the parameterised functions, with one closely tied to the growth of matter structures and the other closely related to photon perturbations such as gravitational lensing deflection and the integrated Sachs-Wolfe (ISW) effect.

Table 1 shows several different forms of model independent parameterisation, and how they translate one into the other, adapted and extended from Daniel et al. 2010.

Recently, several groups (Song et al. 2010, Daniel et al. 2010, Zhao et al. 2010, Daniel & Linder 2010) have advocated the “light/growth” forms we will use here,
due to their close relations with observables and their near orthogonality. The defining equations are

\begin{align}
-k^2(\phi + \psi) &= 8\pi G_N a^2 \bar{\rho}_m \Delta_m \times \mathcal{G} \\
-k^2\psi &= 4\pi G_N a^2 \bar{\rho}_m \Delta_m \times \mathcal{V}.
\end{align}

where $\psi$ is the time-time metric potential, $\phi$ is the space-space metric potential (in conformal Newtonian gauge), $G_N$ is Newton’s constant, $\bar{\rho}_m$ is the homogeneous matter density, $\Delta_m$ the perturbed matter density (gauge invariant), $k$ is the wavenumber, and $a$ is the scale factor. The functions $\mathcal{G}(k,a)$ and $\mathcal{V}(k,a)$ generically are length scale and time dependent. Arising from the sum of potentials, $\mathcal{G}$ (meant to evoke an effective Newton’s constant) predominantly governs photon perturbations through light deflection and the ISW effect. Coming from the velocity equation, $\mathcal{V}$ predominantly governs growth and motion of structure. They thus probe reasonably distinct areas of extension to standard gravity.

These functions also map onto cosmological observations in somewhat orthogonal ways. Cosmic microwave background (CMB) data is mostly sensitive to the ISW effect and hence $\mathcal{G}$ (since we want the gravitational modifications to be responsible for current acceleration, we assume at high redshift, e.g. at CMB last scattering, the theory acts like general relativity without modifications). Weak gravitational lensing involves $\mathcal{G}$ through the light deflection law, and $\mathcal{V}$ to some extent through the growth of structure. Galaxy distributions and motions are most sensitive to $\mathcal{V}$.

Note that the phenomenological gravitational growth index parameter $\gamma$ of Linder 2005, Linder & Cahn 2007 is directly related to $\mathcal{V}$ (Daniel et al. 2010).

Cosmological data, now or in the near future, will not have sufficient leverage to reconstruct the general functions $\mathcal{G}(k,a)$ and $\mathcal{V}(k,a)$. Just as with the dark energy equation of state $w(a)$ describing the cosmic expansion, one can only constrain a very limited number of parameters describing the functions. Some assume a particular time dependence, and possibly neglect scale dependence (e.g. Song et al. 2010b). More generally, one can use principal component analysis to determine the best constrained eigenmodes of the functions (Zhao et al. 2009b). Here we will use a similar but simpler approach of dividing the functions into bins of redshift and wavenumber, since this provides a more direct physical interpretation of the results: gravity is modified in a certain way at low/high redshift and smaller/larger scales.

We find that two bins in redshift and two in wavenumber, for each of the two post-GR functions (we call this the $2 \times 2 \times 2$ gravity model), is the extent of the leverage that next generation surveys will provide, so more complex parameterisations are not useful.

### 3. Constraining Gravity

To effectively constrain the post-GR parameters of $\mathcal{G}(k_i, z_i)$ and $\mathcal{V}(k_i, z_i)$, where $i = 1, 2$ represent the two bins, we need observational data that is sensitive to both the effects on the photons (for $\mathcal{G}$) and the matter growth (for $\mathcal{V}$). Distance measures such as Type Ia supernova distances or baryon acoustic oscillation scales are useful for determining background quantities such as the matter density $\Omega_m$ that might have covariance with the post-GR parameters.

For the current state of the art data we can consider CMB photon perturbation spectra (WMAP: Jarosik et al. 2010), supernova distances (Amanullah et al. 2010),
galaxy clustering (Reid et al. 2010), weak gravitational lensing (CFHTLS: Fu et al. 2008, COSMOS: Massey et al. 2007), and CMB temperature-galaxy crosscorrelation (Ho et al. 2008, Hirata et al. 2008). The results, discussed in detail in Daniel et al. 2010, Daniel & Linder 2010 (also see Bean & Tangmatitham 2010, and Lombriser et al. 2009, Thomas, Abdalla, & Weller 2009 for DGP constraints, Lombriser et al. 2010 for $f(R)$ constraints), show that while $G$ is currently bounded to lie within 10-20% of the GR value for each of the four combinations of low/high wavenumber and low/high redshift, $V$ is only weakly limited to within $\pm 1$ of the GR value. This indicates that current growth probes do not have sufficient leverage to look for deviations from general relativity on cosmic scales. Moreover, the two weak lensing data sets do not agree with each other, with COSMOS showing consistency with GR while CFHTLS gives up to 99% cl deviations at high wavenumbers and low redshift. This may be due to difficulties in extracting accurate weak lensing shear measurements on small angular scales where the density field is more nonlinear.

To place significant limits on $V$ and growth, future data sets including galaxy clustering and weak lensing surveys covering much more sky area, accurately, and ideally to greater depth are required. (Another, nearer term probe will be measurement of the CMB lensing deflection field.) Surveys such as BigBOSS (Schlegel et al. 2009, Stril, Cahn, & Linder 2010), Euclid (Refregier et al. 2010, Martinelli et al. 2010), KDUST (Zhao et al. 2010b), LSST (LSST 2009), and WFIRST (Gehrels 2010) offer great potential gains. Clear understanding of redshift space distortions would enable probes of the matter velocity field in addition to the density field and serve as a method to measure the growth rate (see below), giving further windows on extensions to gravitation theory.

Figure 1 illustrates one example of future leverage possible on the $2 \times 2 \times 2$ post-GR model-independent parameterisation testing gravitation on cosmic scales. The constraints on the growth and $V$ tighten to the few–10% precision level, giving tests of 8 different post-GR variables (and all their crosscorrelations) to better than 10%. That could deliver strong guidance on the nature of cosmic gravity.

To compactify all the information on testing gravity into a single variable (which is not always desirable), we can also examine the gravitational growth index parameterisation approach and the constraints that future data will be able to place on $\gamma$. Remember that this is only a partial characterisation of extensions to gravity, but can serve as an alert to deviations from general relativity (or to matter coupling) if the derived value of $\gamma$ shows time or scale dependence or is inconsistent with $\gamma_{GR} \approx 0.55$.

One promising method to measure $\gamma$ is redshift space distortions in the galaxy power spectrum (Linder 2008, Guzzo et al. 2008). This depends on the growth rate $f \equiv d \ln \Delta_m / d \ln a \approx \Omega_m(a)^{\gamma}$, as well as the growth itself $\Delta_m(a)$ and the galaxy bias $b(a)$. One needs strong knowledge of the bias and modeling of the redshift distortion form (beyond linear theory), or excellent data (clear angular dependence maps or higher order correlations) to separate out $\gamma$ without assuming a form for the bias. If the shape of the bias is fixed, keeping the amplitude as a fit parameter, then next generation galaxy surveys such as BigBOSS, Euclid, or WFIRST can measure $\gamma$ to $\sim 7\%$, simultaneously with fitting the expansion history and neutrino mass effects on growth (Stril, Cahn, & Linder 2010).

Another probe is weak gravitational lensing, either by itself (in which case the mass is measured and galaxy bias does not enter) or in crosscorrelation with the
Figure 1. Filled contours show 68% and 95% cl constraints on $V^{-1}$ and $G^{-1}$ for the two redshift and two wavenumber bins using mock future BigBOSS, Planck, and WFIRST supernova data. The dotted contours recreate the 95% cl contours from Figs. 8 of Daniel & Linder 2010 using current data, to show the expected improvement in constraints. The x’s denote the fiducial GR values (note the offset of current contours may be from systematics within the CFHTLS weak lensing data). Adapted from Daniel & Linder 2010.

galaxy density field (where one can form ratios of observables to separate out the galaxy bias). Recall that extensions to gravity act on weak lensing in multiple ways: the growth of the matter power spectrum alters (which can be phrased in terms of $\gamma$), but also the light deflection law changes, involving the post-GR parameter $G$ separate from $\gamma$. This must also be included in the fit, except that many classes of extended gravity (such as DGP and $f(R)$ gravity) actually have $G = 1$ on cosmological scales. One other subtlety is that the relation of the matter power spectrum to the photon temperature power spectrum changes with the altered growth, modifying the mapping between the primordial photon perturbation amplitude $A_s$ and the present mass amplitude $\sigma_8$. When both weak lensing and cosmic microwave background data are included in the constraints, the treatment of $A_s$ and $\sigma_8$ must be made consistent.

Figure 2 illustrates the effects of fitting the gravitational growth index $\gamma$ simultaneously with determining the effective dark energy equation of state $w(a) = w_0 + w_a(1 - a)$, i.e. the expansion history. The area figure of merit in the $w_0$-$w_a$ plane decreases by 45%, but as Huterer & Linder 2007 pointed out the consequences of neglecting to fit for the growth index are worse. For an assumption mistaken by
Figure 2. Weak gravitational lensing probes both the growth history and expansion history of the universe, so failure to account for possible extended gravity effects on the growth overestimates the tightness of constraints on the expansion history parameters $w_0$ and $w_a$. Determination of $w_0$ and $w_a$ simultaneously with fitting for the growth index $\gamma$ is nearly independent of the value of $\gamma$, here shown with typical fiducial values for GR (0.55), DGP gravity (0.68), and $f(R)$ gravity (0.42). The 68% confidence level constraints on $\gamma$ are about 0.11 for the case shown of space-based weak gravitational lensing alone over 4000 deg$^2$. For the impact of other growth effects (e.g. neutrino mass, spatial curvature) on weak lensing see Das et al. 2011.

$\Delta \gamma$ and weak lensing alone as a probe the derived value of $w_a$ would be biased as $\Delta w_a \approx 8 \Delta \gamma$. When fitting for $\gamma$ simultaneously, however, the estimation of $w_0$ and $w_a$ do not depend strongly on the fiducial $\gamma$. This is a reflection of $\gamma$ being defined specifically to separate the expansion history influence on growth from any “beyond general relativity” effects (Linder 2005), making it a key method to test gravity.
4. Paths of Gravity

While the model independent approach allows exploration without assuming a particular theory, it is of interest as well to consider some specific models and their mapping into the post-GR parameters we have described. One can talk about three broad classes of extensions to gravity in terms of the physics resting them to general relativity in solar system conditions, as observations require (Jain & Khoury 2010, also see Durrer 2011, Maartens 2011, Uzan 2011 in this volume): dimensional reduction where below a Vainshtein radius the theory acts like GR (e.g. DGP or cascading gravity), strong coupling where extra degrees of freedom freeze out through gaining a large mass in a chameleon mechanism (e.g. \( f(R) \) or scalar-tensor gravity), or screening where the extra degrees of freedom decouple and vanish through symmetry restoration (symmetron gravity). On cosmic scales, the first and third classes behave similarly, so we examine two representative cases: DGP gravity and \( f(R) \) gravity.

In both of these cases, the light variable \( \mathcal{G} \) is simply equal to unity, the GR value. However the growth variable \( \mathcal{V} \) is affected. The expressions become

\[
\mathcal{V}_{\text{DGP}} = \frac{2 + 4\Omega_m^2(a)}{3 + 3\Omega_m^2(a)}
\]

\[
\mathcal{V}_{f(R)} = \frac{3 + 4\kappa^2(k,a)}{3 + 3\kappa^2(k,a)}
\]

where \( \Omega_m(a) = 8\pi G_N\bar{\rho}_m(a)/[3H^2(a)] \) is the dimensionless matter density, \( H = \dot{a}/a \) is the Hubble expansion rate, and \( \kappa = k/[aM(a)] \) where \( M(a) \approx (3d^2 f/dR^2)^{-1/2} \) is the effective scalar field mass. Note that DGP gravity does not have scale dependence on cosmic scales above the Vainshtein scale; gravity is scale free (e.g. the force is a power law with distance) on both the large scale (5-dimensional gravity) and small scale (4-dimensional gravity) limits and only the Vainshtein scale defined from the 5-d to 4-d crossover breaks this. On the other hand, \( f(R) \) gravity has both scale and time dependence, though tied together in a specific manner.

Recall that the gravitational growth index \( \gamma \) is related to \( \mathcal{V} \). For DGP gravity, \( \gamma = 0.68 \) (Lue, Scoccimarro, & Starkman 2004, Linder 2005, Linder & Cahn 2007) is an excellent approximation to use for calculating the matter density linear growth factor as a function of redshift, good to 0.2%. A mild time dependence can be incorporated into \( \gamma \) (though this is not necessary) through Eq. (27) of Linder & Cahn 2007. For \( f(R) \) gravity, \( \gamma \) is not generally as well approximated by a constant in time, and has non-negligible scale dependence at redshifts \( z \approx 1–3 \) (e.g. Tsujikawa et al. 2009, Motohashi, Starobinsky, & Yokoyama 2010, Appleby & Weller 2010) although the details depend on the specific \( f(R) \) model and parameters.

From Eqs. (4.1)-(4.2) we can create phase plane diagrams of the evolution of the post-GR function \( \mathcal{V} \). This is analogous to the dark energy phase plane \( w \sim w' \) for the dark energy equation of state and its time variation, where prime denotes \( d/d\ln a \). For dark energy, such diagrams led to clear distinction of certain physical classes (Caldwell & Linder 2005) as well as calibration of the compact and accurate parameterisation \( w(a) = w_0 + w_a(1 - a) \) (de Putter & Linder 2008). (Also see Song et al. 2010 for comparison of \( \mathcal{G} \) and \( \mathcal{V} \) at a fixed time in extended gravity vs interacting dark energy.)
Figure 3 shows the results in the $V-V'$ plane for DGP and $f(R)$ gravity. For DGP gravity the equation for the phase space trajectory is

$$V' = \frac{-4\Omega_m^2(1 - \Omega_m)}{(1 + \Omega_m)(1 + \Omega_m^2)^2}$$

(4.3)

$$= -18(1 - V) \left( V - \frac{2}{3} \right) \left[ 1 + \sqrt{\frac{3V - 2}{4 - 3V}} \right]^2,$$

(4.4)

where we used the relations

$$\Omega'_m = 3w\Omega_m(1 - \Omega_m) = -3\Omega_m \frac{1 - \Omega_m}{1 + \Omega_m},$$

(4.5)

and $\Omega_m$ is the time dependent matter density.

For $f(R)$ gravity, the phase space trajectory is

$$V' = \frac{2\kappa\kappa'}{3(1 + \kappa^2)^2}$$

(4.6)

$$\rightarrow \frac{2(s - 1)\kappa^2}{3(1 + \kappa^2)^2} = -6(s - 1)(V - 1)(V - 4/3),$$

(4.7)

where in the second line we parameterise $M(a) = M_0a^{-s}$. If we wanted to look at the phase space evolution with respect to inverse length $k$ rather than time $a$, then the equation still holds, with $s = 2$.

Both classes of gravity theory act as thawing cases in the nomenclature of dark energy phase space: they are frozen in the general relativity state $(V, V') = (1, 0)$ in the past, then as the Hubble parameter or Ricci scalar curvature drop from the cosmic expansion the theory thaws and moves away from GR. The theories eventually freeze to an asymptotic attractor with $V' = 0$ in the future, with $V = 2/3$ in the case of DGP gravity (weaker gravity) and $V = 4/3$ in the case of $f(R)$ gravity (stronger gravity). Note that interpreting $f(R)$ as a scalar-tensor theory agrees with the expectation that gravity should strengthen, since forces carried by a scalar field are attractive. The clear separation of phase space territory for the two classes, as for thawing and freezing fields of dark energy, shows how observations could distinguish the nature of gravity. This also defines a science requirement that $V$ should be measurable to an accuracy better than 0.1 for a $3\sigma$ distinction of gravity theories.

However, there is one further point regarding $f(R)$. While the form $M(a) \sim a^{-s}$ is a reasonable description for the past behavior of the scalar field mass (see, e.g., Bertschinger & Zukin 2008, Zhao et al. 2009, Appleby & Weller 2010), in the future we expect $M$ to freeze to a constant (e.g. as $R$ itself does when the theory goes to the de Sitter attractor state; thanks to Stephen Appleby for pointing this out). If instead we parameterise the scalar field mass as $M(a) = M_1a^{-s} + M_\ast$ (accurate to $\sim 1\%$ for at least some models), then the phase space trajectory does not asymptote to $(4/3, 0)$ but rather returns to the GR limit of $(1, 0)$ as $a \gg 1$ and $\kappa = k/(aM) \rightarrow 0$ in the future.

Figure 4 illustrates this behavior. As $M_\ast$ increases (dotted curve) or for wavenumbers near the mass scale $M$ (dot-dashed curve), the cosmic version of the chameleon mechanism begins to operate and the trajectory heads back toward GR. Also note
Figure 3. The phase space trajectories of DGP gravity and \( f(R) \) gravity resemble thawing dark energy, except here the theories move away from general relativity in the post-GR growth parameter plane, instead of thawing from a cosmological constant. Unlike canonical dark energy, the theories thaw in opposite directions: DGP moves to weaker gravity than GR while \( f(R) \) moves to stronger gravity. The phase space location today for each of the theories is shown by the squares (for \( \Omega_m = 0.27 \)).

that because \( M \) is no longer a power law in \( a \), different wavenumbers \( k \) do not simply correspond to a rescaling of \( a \) and so the single \( f(R) \) trajectory in Fig. 3 breaks up into varied paths for different \( k \). The evolution along a path varies as well. This can be seen by the different values of \( V \) at \( a = 0.5 \) for \( k/M_1 = 10 \) (blue square near the peak of the figure) vs \( k/M_1 = 100 \) (red square near the right side). However, by the present (and for several e-folds to the future) this \( k \) dependence has vanished, with \( V(a = 1) \) shown with stars for the two cases agreeing to within 0.5%. This holds for all \( k/M_1 \gg 1 \) (note that the scalar mass \( M_1 \) is likely to be of order the Hubble constant for observationally allowed models). Only for \( k/M_1 \approx 1 \) does scale dependence today enter (and the trajectory as a whole deviate). Since
In the $f(R)$ case with a de Sitter future the scalar field mass $M$ freezes at a non-zero constant, causing the theory to restore GR on cosmic scales just like with the chameleon mechanism on small scales. The heavy black curve recreates the $M_*=0$, vanishing mass case of Fig. 4 that freezes at $V=4/3$, however the finite mass cases return to $V=1$. While the wavenumber dependence of the post-GR parameter at $z=1$ (squares) is appreciable, by $z=0$ the $k$-dependence is negligible (stars). Since the gravitational growth index $\gamma$ is directly related to $V$, this explains the scale independence of $\gamma$ at the present in such $f(R)$ models.

the gravitational growth index $\gamma$ is directly related to $V$, this explains the scale independence of $\gamma$ in such $f(R)$ models at the present (see, e.g., Tsujikawa et al. 2009, Appleby & Weller 2010).

5. Conclusions

Cosmic acceleration may be a sign that gravitation deviates from general relativity on large scales, pointing the way to a deeper theory of gravity and perhaps the
nature of spacetime and fundamental physics. Even apart from this, cosmological observations now have the capability to test gravity on scales barely probed, and we should certainly do so.

In the absence of a compelling specific model, and to remain receptive to surprises in how gravity behaves, a model independent approach in terms of parameterising the relations between the metric potentials and matter density and velocity observables has advantages. We have explored here the “\(2 \times 2\times 2\)” approach parameterising light/growth functions \(G\), \(V\), motivated by observable effects on photon and matter perturbations. These are divided into bins sensitive to both scale and time dependence.

We find strong complementarity between \(G\) and \(V\), with each probing specific aspects of extensions to gravity, and both capable of being constrained by a variety of cosmological methods. Current surveys are making some inroads on determining \(G\), but constraining \(V\) requires future large surveys such as BigBOSS, Euclid, or WFIRST. Crosscorrelations between different probes will be valuable as well, and geometric measures such as supernova and BAO distances will be essential to fit simultaneously the cosmic expansion history. Conversely, fitting extended gravity growth parameters such as \(\gamma\) when using probes such as weak lensing to measure the equation of state is necessary to avoid bias.

Viewing the post-GR parameters in a dynamical phase plane yields insights similar to its use for dark energy. Models such as DGP and \(f(R)\) gravity act as thawing fields, although evolving in opposite directions away from GR. Phase diagrams also can illustrate the scale dependence of \(\gamma\) at various epochs, and most importantly deliver a science requirement on distinguishing classes of gravity: future surveys should aim to determine \(V\) to better than 0.1. Planned next generation experiments such as BigBOSS, Euclid, and WFIRST can indeed potentially reach this level, with the caveat that expansion history should be tested as well, such as through supernova distances immune to gravitational modifications.

Gravity can and should be tested on all scales, on laboratory, solar system, compact object, galactic, cluster, cosmological, and horizon scales. The approach discussed here is designed for model independence on cosmological scales, smaller than the horizon. On horizon scales the light/growth functions become more complicated or insufficient as other terms in the equations become important (see, e.g., Bertschinger & Zukin 2008, Hu & Sawicki 2007, Ferreira & Skordis 2010).

Another model independent approach is to check consistency relations within general relativity (see, e.g., Zhang et al. 2007, Acquaviva et al. 2008, Reyes et al. 2010). These can provide an alert to deviations, and then one must adopt specific models or more detailed parameterisations such as discussed here to characterise the physics.

Acknowledgments

I gratefully acknowledge Stephen Appleby, Scott Daniel, and Tristan Smith for valuable discussions and collaborations. I thank the Centro de Ciencias Pedro Pascual in Benasque, Spain and the Kavli Royal Society International Centre for hospitality. This work has been supported in part by the Director, Office of Science, Office of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231, and the World Class University grant R32-2009-000-10130-
0 through the National Research Foundation, Ministry of Education, Science and Technology of Korea.

References

Acquaviva, V., Hajian, A., Spergel, D.N., Das, S. 2008 Next Generation Redshift Surveys and the Origin of Cosmic Acceleration. *Phys. Rev. D* **78**, 043514.

Amanullah, R. *et al.* 2010 Spectra and Light Curves of Six Type Ia Supernovae at $0.511 < z < 1.12$ and the Union2 Compilation. *Astrophys. J.* **716**, 712.

Appleby, S.A. & Weller, J. 2010 Parameterizing scalar-tensor theories for cosmological probes. *J. Cosmol. Astropart. Phys.* **1012**, 006.

Bean, R. & Tangmatitham, M. 2010 Current constraints on the cosmic growth history. *Phys. Rev. D* **81**, 083534.

Bertschinger, E. & Zukin, P. 2008 Distinguishing Modified Gravity from Dark Energy. *Phys. Rev. D* **78**, 024015.

Caldwell, R.R. & Linder, E.V. 2005 The Limits of Quintessence. *Phys. Rev. Lett.* **95**, 141301.

Daniel, S.F., Linder, E.V., Smith, T.L., Caldwell, R.R., Cooray, A., Leauthaud, A., Lombriser, L. 2010 Testing general relativity with current cosmological data. *Phys. Rev. D* **81**, 123508.

Daniel, S.F. & Linder, E.V. 2010 Confronting general relativity with further cosmological data. *Phys. Rev. D* **82**, 103523.

Das, S., de Putter, R., Linder, E.V., Nakajima, R. 2011 Weak lensing science, surveys, and systematics. [arXiv:1102.5090](http://arxiv.org/abs/1102.5090)

de Putter, R. & Linder, E.V. 2008 Calibrating Dark Energy. *J. Cosmol. Astropart. Phys.* **0810**, 042.

Durrer, R. 2011 this volume.

Ferreira, P.G. & Skordis, C. 2010 The linear growth rate of structure in Parametrized Post-Friedmann Universes. *Phys. Rev. D* **81**, 104020.

Fu, L. *et al.* 2008 Very weak lensing in the CFHTLS Wide: Cosmology from cosmic shear in the linear regime. *Astron. Astrophys.* **479**, 9.

Gehrels, N. 2010 The Joint Dark Energy Mission (JDEM) Omega. [arXiv:1008.4936](http://arxiv.org/abs/1008.4936);

[http://wfirst.gsfc.nasa.gov](http://wfirst.gsfc.nasa.gov)

Guzzo, L. *et al.* 2008 A test of the nature of cosmic acceleration using galaxy redshift distortions. *Nature* **451**, 541-545.

Hirata, C.M., Ho, S., Padmanabhan, N., Seljak, U., Bahcall, N.A. 2008. Correlation of CMB with large-scale structure: II. Weak lensing. *Phys. Rev. D.*, **78**, 045320.

Ho, S., Hirata, C., Padmanabhan, N., Seljak, U., Bahcall, N. 2008 Correlation of CMB with large-scale structure: I. ISW Tomography and Cosmological Implications. *Phys. Rev. D.*, **78**, 043519.

Hu, W. 2008 Parametrized Post-Friedmann Signatures of Acceleration in the CMB. *Phys. Rev. D.*, **77**, 103524.

Hu, W. & Sawicki, I. 2007 A Parameterized Post-Friedmann Framework for Modified Gravity. *Phys. Rev. D.*, **76**, 104043.

Huterer, D. & Linder, E.V. 2007 Separating Dark Physics from Physical Darkness: Minimalist Modified Gravity vs. Dark Energy. *Phys. Rev. D.*, **75**, 023519.

Jain, B. & Khoury, J. 2010 Cosmological Tests of Gravity. *Annals Phys.* **325**, 1479-1516.

Jarosik, N. *et al.* 2011 Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Sky Maps, Systematic Errors, and Basic Results. *Astrophys. J. Suppl.* **192**, 14.
Linder, E.V. 2005 Cosmic growth history and expansion history. *Phys. Rev. D* **72**, 043529.
Linder, E.V. 2008 Redshift Distortions as a Probe of Gravity. *Astropart. Phys.* **29**, 336-339.
Linder, E.V. & Cahn, R.N. 2007 Parameterized beyond-Einstein growth. *Astropart. Phys.* **28**, 481-488.
Lombriser, L., Hu, W., Fang, W., Seljak, U. 2009 Cosmological Constraints on DGP Braneworld Gravity with Brane Tension. *Phys. Rev. D* **80**, 063536.
Lombriser, L., Slosar, A., Seljak, U., Hu, W. 2010 Constraints on $f(R)$ gravity from probing the large-scale structure. arXiv:1003.3009
LSST Science Collaborations 2009 LSST Science Book, Version 2.0. arXiv:0912.0201 ; http://www.lsst.org/lsst.
Lue, A., Scoccimarro, R., Starkman, G.D. 2004 Probing Newton’s Constant on Vast Scales: DGP Gravity, Cosmic Acceleration and Large Scale Structure. *Phys. Rev. D* **69**, 124015.
Maartens, R. 2011 this volume.
Martinelli, M., Calabrese, E., De Bernardis, F., Melchiorri, A., Pagana, L., Scaramella, R. 2011 Constraining Modified Gravity Theories by Weak Lensing with Euclid. *Phys. Rev. D* **83**, 023012.
Massey, R. et al. 2007 COSMOS: 3D weak lensing and the growth of structure. *Astrophys. J. Suppl.* **172**, 239.
Motohashi, H., Starobinsky, A.A., Yokoyama, J. 2010 Phantom Boundary Crossing and Anomalous Growth Index of Fluctuations in Viable $f(R)$ Models of Cosmic Acceleration. *Prog. Theor. Phys.* **123**, 887-902.
Refregier, A. et al. 2010 Euclid Imaging Consortium Science Book. arXiv:1001.0061 ; http://sci.esa.int/euclid.
Reid, B.A. et al. 2010 Cosmological Constraints from the Clustering of the Sloan Digital Sky Survey DR7 Luminous Red Galaxies. *Mon. Not. Roy. Astron. Soc.* **404**, 60.
Reyes, R., Mandelbaum, R., Seljak, U., Baldauf, T., Gunn, J.E., Lombriser, L., Smith, R.E. 2010 Confirmation of general relativity on large scales from weak lensing and galaxy velocities. *Nature* **464**, 256-258.
Schlegel, D.J. et al. 2009 BigBOSS: The Ground-Based Stage IV Dark Energy Experiment. arXiv:0904.0468
Song, Y-S., Hollenstein, L., Caldera-Cabral, G., Koyama, K. 2010 Theoretical Priors On Modified Growth Parametrisations. *J. Cosmol. Astropart. Phys.* **1004**, 018.
Song, Y-S., Zhao, G-B., Bacon, D., Koyama, K., Nichol, R.C., Pogosian, L. 2010b Complementarity of Weak Lensing and Peculiar Velocity Measurements in Testing General Relativity. arXiv:1011.2106
Stril, A., Cahn, R.N., Linder, E.V. 2010 Testing Standard Cosmology with Large Scale Structure. *Mon. Not. Roy. Astron. Soc.* **404**, 239-246.
Thomas, S.A., Abdalla, F.B., Weller, J. 2009 Constraining Modified Gravity and Growth with Weak Lensing Mon. Not. Roy. Astron. Soc. **395**, 197-209.
Tsujikawa, S., Gannouji, R., Moraes, B., Polarski, D. 2009 The dispersion of growth of matter perturbations in $f(R)$ gravity. *Phys. Rev. D* **80**, 084044.
Uzan, J-P. 2011 this volume.
Zhang, P., Liguori, M., Bean, R., Dodelson, S. 2007 A discriminating probe of gravity at cosmological scales. *Phys. Rev. Lett.* **99**, 141302.
Zhao, G-B., Pogosian, L., Silvestri, A., Zylberberg, J. 2009 Searching for modified growth patterns with tomographic surveys. *Phys. Rev. D* **79**, 083513.
Zhao, G-B., Pogosian, L., Silvestri, A., Zylberberg, J. 2009b Cosmological Tests of General Relativity with Future Tomographic Surveys. *Phys. Rev. Lett.* **103**, 241301.
Zhao, G-B. et al. 2010 Probing modifications of General Relativity using current cosmological observations. *Phys. Rev. D* **81**, 103510.
Zhao, G-B., Zhan, H., Wang, L., Fan, Z., Zhang, X. 2010b Probing Dark Energy with the Kunlun Dark Universe Survey Telescope. arXiv:1005.3810