Tests of Lorentz and CPT violation with Neutrinos

Teppei Katori
Massachusetts Institute of Technology
ICHEP2012, Melbourne, Australia, July 10, 2012
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Outline
1. Spontaneous Lorentz symmetry breaking
2. What is Lorentz and CPT violation?
3. Test for Lorentz violation with neutrinos
4. Conclusion

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1. Spontaneous Lorentz symmetry breaking

2. What is Lorentz and CPT violation?

3. Test for Lorentz violation with Neutrinos

4. Conclusion
Every fundamental symmetry needs to be tested, including Lorentz symmetry.

After the recognition of the theoretical processes that create Lorentz violation, testing Lorentz invariance became very exciting.

Lorentz and CPT violation has been shown to occur in Planck-scale theories, including:
- string theory
- noncommutative field theory
- quantum loop gravity
- extra dimensions
- etc

However, it is very difficult to build a self-consistent theory with Lorentz violation...

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After the recognition of the theoretical processes that create Lorentz violation, testing Lorentz invariance became very exciting.

Lorentz and CPT violation has been shown to occur in Planck-scale theories, including:
- string theory
- noncommutative field theory
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- etc

However, it is very difficult to build a self-consistent theory with Lorentz violation...

Y. Nambu
(Nobel Prize winner 2008),
picture taken from CPT04 at Bloomington, IN
1. Spontaneous Lorentz symmetry breaking (SLSB)

vacuum Lagrangian for fermion $L = i\gamma^\mu \partial_\mu \gamma^\nu$

e.g.) SSB of scalar field in Standard Model (SM)
- If the scalar field has Mexican hat potential

$$L = \frac{1}{2}(\phi^*)^2 - \frac{1}{2}m^2 \phi^* \phi - \frac{1}{4}\lambda(\phi^* \phi)^2$$

$M(\phi) = \phi^* \phi < 0$
1. Spontaneous Lorentz symmetry breaking (SLSB)

vacuum Lagrangian for fermion \( L = i \bar{m} \)

- e.g. SSB of scalar field in Standard Model (SM)
- If the scalar field has Mexican hat potential

\[
L = \frac{1}{2} \left( \frac{1}{2} \phi^2 \right) - \frac{1}{4} \phi^2
\]

\[
M(\phi) = \phi^2 < 0
\]

Particle acquires mass term!
1. Spontaneous Lorentz symmetry breaking (SLSB)

vacuum Lagrangian for fermion \[ L = i\bar{m} \]

- e.g. SSB of scalar field in Standard Model (SM)
  - If the scalar field has Mexican hat potential
    \[ L = \frac{1}{2} \phi^2 - \frac{1}{2} \lambda \phi^4 \]
    \[ M(\phi) = \lambda \phi^2 < 0 \]

- e.g. SLSB in string field theory
  - There are many Lorentz vector fields
  - If any of vector field has Mexican hat potential
    \[ M(a^\mu) = \lambda a^\mu_a < 0 \]

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1. Spontaneous Lorentz symmetry breaking (SLSB)

Vacuum Lagrangian for fermion

\[ L = \bar{i} \gamma^m \partial_m \gamma^a \bar{a} \]

E.g.) SSB of scalar field in Standard Model (SM)
- If the scalar field has Mexican hat potential

\[ L = \frac{1}{2} \left( \phi^2 \right)^2 - \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \]

\[ M(\phi) = \phi^2 < 0 \]

E.g.) SLSB in string field theory
- There are many Lorentz vector fields
- If any of vector field has Mexican hat potential

\[ M(a^\mu) = \phi^2 < 0 \]

Lorentz symmetry is spontaneously broken!
1. Test of Lorentz violation

Test of Lorentz violation is to find the coupling of these background fields and ordinary fields (electrons, muons, neutrinos, etc); then the physical quantities may depend on the rotation of the earth (sidereal time dependence).

vacuum Lagrangian for fermion

\[ \mathcal{L} = i \bar{Y} \gamma^m \partial_m Y - m Y \gamma^5 + \bar{Y} \gamma^m \gamma^a \gamma^m Y + \bar{Y} \gamma^m \gamma^c \gamma^m \partial_n Y \ldots \]

...background fields of the universe...

solar time: 24h 00m 00.0s
sidereal time: 23h 56m 04.1s (Earth rotation period)
1. Spontaneous Lorentz symmetry breaking

2. What is Lorentz and CPT violation?

3. Test for Lorentz violation with neutrinos

4. Conclusion
2. What is Lorentz violation?

\[ -\langle x \rangle \neq \langle x \rangle \]

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2. What is Lorentz violation?

\[ a(x) \]
2. What is Lorentz violation?

Under the particle Lorentz transformation:

\[ U^{-1}(x) \ a \ (x) U^{1} \]
2. What is Lorentz violation?

Under the particle Lorentz transformation:

\[ \Psi(x) \gamma_\mu a^\mu \Psi(x) \rightarrow U[\Psi(x) \gamma_\mu a^\mu \Psi(x)]U^{-1} \]
\[ \neq \Psi(\Lambda x) \gamma_\mu a^\mu \Psi(\Lambda x) \]

Lorentz violation is observable when a particle is moving in the fixed coordinate space.
2. What is Lorentz violation?

Under the particle Lorentz transformation:

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\[ \neq \Psi(\Lambda x)\gamma_\mu a^\mu \Psi(\Lambda x) \]

Lorentz violation is observable when a particle is moving in the fixed coordinate space.

Under the observer Lorentz transformation:

\[ \overline{\Psi}(x) a \ (x) \]

\[ x \rightarrow \Lambda^{-1}x \]
2. What is Lorentz violation?

Under the particle Lorentz transformation:

\[ \Psi(x) \gamma_\mu a^\mu \Psi(x) \rightarrow U[\Psi(x) \gamma_\mu a^\mu \Psi(x)] U^{-1} \]

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Lorentz violation is observable when a particle is moving in the fixed coordinate space.

Under the observer Lorentz transformation:

\[ \Psi(x) \gamma_\mu a^\mu \Psi(x) \rightarrow \Psi(\Lambda^{-1} x) \gamma_\mu a^\mu \Psi(\Lambda^{-1} x) \]

Lorentz violation cannot be generated by observers motion (coordinate transformation is unbroken).

All observers agree for all observations.
1. Spontaneous Lorentz symmetry breaking

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3. Test of Lorentz violation with neutrino oscillation experiments

Lorentz violation is realized as a coupling of particle fields and background fields, so the basic strategy to find Lorentz violation is:

(1) choose the coordinate system
(2) write down the Lagrangian, including Lorentz-violating terms under the formalism
(3) write down the observables using this Lagrangian
3. Test of Lorentz violation with neutrino oscillation experiments

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- Neutrino beamline is described in Sun-centred coordinates

![Diagram of Earth, Sun, and coordinate system](image)
Lorentz violation is realized as a coupling of particle fields and background fields, so the basic strategy to find Lorentz violation is:

1. choose the coordinate system
2. write down the Lagrangian, including Lorentz-violating terms under the formalism
3. write down the observables using this Lagrangian

Standard Model Extension (SME) is the standard formalism for the general search for Lorentz violation. SME is a minimum extension of QFT with Particle Lorentz violation

SME Lagrangian in neutrino sector

\[
L = \frac{1}{2} i \gamma_{AB} \bar{\nu}_A \gamma^0 \nu_B - M_{AB} \bar{\nu}_A \gamma^0 \nu_B + h.c.
\]

SME coefficients

\[
\begin{align*}
A_B &= A_B + c_{AB} + d_{AB} + e_{AB} + if_{AB} + \frac{1}{2} g_{AB} + \cdots \\
M_{AB} &= m_{AB} + im_{5AB} + a_{AB} + b_{AB} + \frac{1}{2} H_{AB} + \cdots
\end{align*}
\]
3. Test of Lorentz violation with neutrino oscillation experiments

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(1) choose the coordinate system
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Various physics are predicted under SME, but among them, the smoking gun of Lorentz violation is the sidereal time dependence of the observables

solar time: 24h 00m 00.0s
sidereal time: 23h 56m 04.1s

Lorentz-violating neutrino oscillation probability for short-baseline experiments

\[ P \rightarrow e = \left( \frac{L}{\hbar c} \right)^2 \left| (C)_{e} + (A_s)_{e} \sin \Theta_{\odot} + (A_c)_{e} \cos \Theta_{\odot} + (B_s)_{e} \sin 2 \Theta_{\odot} + (B_c)_{e} \cos 2 \Theta_{\odot} \right|^2 \]
3. Test of Lorentz violation with neutrino oscillation experiments

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Lorentz-violating neutrino oscillation probability for short-baseline experiments

\[ P_{\nu} = \left( \frac{L}{hc} \right)^2 \left| (C)_e \sin \Theta_{\oplus} + (A_s)_e \sin \Theta_{\oplus} + (A_c)_e \cos \Theta_{\oplus} + (B_s)_e \sin 2 \Theta_{\oplus} + (B_c)_e \cos 2 \Theta_{\oplus} \right|^2 \]

Sidereal variation analysis for short baseline neutrino oscillation is 5-parameter fitting problem
3. LSND experiment

LSND is a short-baseline neutrino oscillation experiment at Los Alamos.

$$\bar{\nu}_\mu \xrightarrow{oscillation} \bar{\nu}_e + p \rightarrow e^+ + n$$

$$n + p \rightarrow d + \gamma$$

LSND saw the 3.8\(\sigma\) excess of electron antineutrinos from muon antineutrino beam; since this excess is not understood by neutrino Standard Model, it might be new physics.
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Data is consistent with flat solution, but sidereal time solution is not excluded.

Small Lorentz violation could be the solution of LSND excess.
3. Tandem Model

Small Lorentz violation could be the solution of LSND excess. But can such solution be allowed by other experiments?
→ It is possible to construct a phenomenological neutrino oscillation model, based on Lorentz violation, using only 3 free parameters (tandem model).

Tandem model can reproduce:
- solar neutrino oscillation
- atmospheric neutrino oscillation
- reactor neutrino oscillation
- LSND neutrino oscillation

Tandem model also predicts small excess at the low energy region for MiniBooNE

Recent development of Lorentz violating neutrino oscillation models, see for example, Diaz and Kostelecký, PRD85(2012)016013
3. MiniBooNE experiment

MiniBooNE is a short-baseline neutrino oscillation experiment at Fermilab.

\[ \text{oscillation} \rightarrow e + n \rightarrow e^- + p \]

\[ \text{oscillation} \rightarrow e^- + p \rightarrow e^+ + n \]

Neutrino mode analysis: MiniBooNE saw the 3.0\(\sigma\) excess at low energy region

Antineutrino mode analysis: MiniBooNE saw the 1.4\(\sigma\) excess at low and high energy region

(However MiniBooNE low energy excesses are much bigger than tandem model prediction)
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\[ \text{oscillation} \rightarrow e + p \rightarrow e^+ + n \]

Neutrino mode analysis: MiniBooNE saw the $3.0\sigma$ excess at low energy region.

Antineutrino mode analysis: MiniBooNE saw the $1.4\sigma$ excess at low and high energy region.

Electron neutrino candidate data prefer sidereal time independent solution (flat).

Electron antineutrino candidate data prefer sidereal time dependent solution, but statistical significance is marginal.

We find no evidence of Lorentz violation.

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3. MiniBooNE experiment

MiniBooNE is a short-baseline neutrino oscillation experiment at Fermilab.

\[ \nu + n \rightarrow e^- + p \]

\[ \bar{\nu} + p \rightarrow e^+ + n \]

Neutrino mode analysis: MiniBooNE saw the 3.0\(\sigma\) excess at low energy region.

Antineutrino mode analysis: MiniBooNE saw the 1.4\(\sigma\) excess at low and high energy region.

Since we find no evidence of Lorentz violation, we set limits on the SME coefficients.

These limits exclude SME values to explain LSND data, therefore there is no simple Lorentz violation motivated scenario to accommodate LSND and MiniBooNE results simultaneously.

| Coefficient | \(\epsilon \mu (\nu \text{ mode low energy region})\) | \(\epsilon \mu (\bar{\nu} \text{ mode combined region})\) |
|-------------|---------------------------------|---------------------------------|
| Re\((a_L)^T\) or Im\((a_L)^T\) | 4.2 \times 10^{-20} GeV | 2.6 \times 10^{-20} GeV |
| Re\((a_L)^X\) or Im\((a_L)^X\) | 6.0 \times 10^{-20} GeV | 5.6 \times 10^{-20} GeV |
| Re\((a_L)^Y\) or Im\((a_L)^Y\) | 5.0 \times 10^{-20} GeV | 5.9 \times 10^{-20} GeV |
| Re\((a_L)^Z\) or Im\((a_L)^Z\) | 5.6 \times 10^{-20} GeV | 3.5 \times 10^{-20} GeV |
| Re\((e_L)^{XY}\) or Im\((e_L)^{XY}\) | — | — |
| Re\((e_L)^{XZ}\) or Im\((e_L)^{XZ}\) | 1.1 \times 10^{-19} | 6.2 \times 10^{-20} |
| Re\((e_L)^{YZ}\) or Im\((e_L)^{YZ}\) | 9.2 \times 10^{-20} | 6.5 \times 10^{-20} |
| Re\((e_L)^{XX}\) or Im\((e_L)^{XX}\) | — | — |
| Re\((e_L)^{YY}\) or Im\((e_L)^{YY}\) | — | — |
| Re\((e_L)^{ZZ}\) or Im\((e_L)^{ZZ}\) | 3.4 \times 10^{-19} | 1.3 \times 10^{-19} |
| Re\((e_L)^{TT}\) or Im\((e_L)^{TT}\) | 9.6 \times 10^{-20} | 3.6 \times 10^{-20} |
| Re\((e_L)^{TX}\) or Im\((e_L)^{TX}\) | 8.4 \times 10^{-20} | 4.6 \times 10^{-20} |
| Re\((e_L)^{TY}\) or Im\((e_L)^{TY}\) | 6.9 \times 10^{-20} | 4.9 \times 10^{-20} |
| Re\((e_L)^{TZ}\) or Im\((e_L)^{TZ}\) | 7.8 \times 10^{-20} | 2.9 \times 10^{-20} |
3. Double Chooz experiment

So far, we have set limits on

1. $\nu_e \leftrightarrow \nu_\mu$ channel: LSND, MiniBooNE, MINOS (<10^{-20} \text{ GeV})
2. $\nu_\mu \leftrightarrow \nu_\tau$ channel: MINOS, IceCube (<10^{-23} \text{ GeV})

The last untested channel is $\nu_e \leftrightarrow \nu_\tau$

It is possible to limit $\nu_e \leftrightarrow \nu_\tau$ channel from reactor $\nu_e$ disappearance experiment

$$P(\nu_e \leftrightarrow \nu_e) = 1 - P(\nu_e \leftrightarrow \nu_\mu) - P(\nu_e \leftrightarrow \nu_\tau) \approx 1 - P(\nu_e \leftrightarrow \nu_\tau)$$

Double Chooz observed the $3.1\sigma$ disappearance signal of electron antineutrinos from the reactor
3. Double Chooz experiment

So far, we have set limits on
1. $\nu_e \leftrightarrow \nu_\mu$ channel: LSND, MiniBooNE, MINOS ($<10^{-20}$ GeV)
2. $\nu_\mu \leftrightarrow \nu_\tau$ channel: MINOS, IceCube ($<10^{-23}$ GeV)

The last untested channel is $\nu_e \leftrightarrow \nu_\tau$

It is possible to limit $\nu_e \leftrightarrow \nu_\tau$ channel from reactor $\nu_e$ disappearance experiment

\[
P(\nu_e \leftrightarrow \nu_e) = 1 - P(\nu_e \leftrightarrow \nu_\mu) - P(\nu_e \leftrightarrow \nu_\tau) \sim 1 - P(\nu_e \leftrightarrow \nu_\tau)
\]

Double Chooz observed the $3.1\sigma$ disappearance signal of electron antineutrinos from the reactor

Preliminary result shows small disappearance signal prefers sidereal time independent solution (flat)

We will be able to set limits in the $e-\tau$ sector for the first time; $\nu_e \leftrightarrow \nu_\tau$ ($<10^{-21}$ GeV)

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Conclusion

Lorentz and CPT violation has been shown to occur in Planck-scale physics.

There is a world wide effort to test Lorentz violation with various state-of-the-art technologies.

LSND and MiniBooNE data suggest Lorentz violation is an interesting solution to neutrino oscillation.

MiniBooNE antineutrino mode data prefer sidereal time dependent solution, although statistical significance is not high. Limits from MiniBooNE exclude simple Lorentz violation motivated scenario for LSND.

MiniBooNE, LSND, MINOS, IceCube, and Double Chooz set stringent limits on Lorentz violation in neutrino sector in terrestrial level.
Thank you for your attention!
Backup
3. MiniBooNE oscillation analysis results

Neutrino mode low energy excess
MiniBooNE see the excess in low energy region.

Antineutrino mode excess
MiniBooNE see the excess in combined region.

These excesses are not predicted by neutrino Standard Model ($\nu$SM).
Oscillation candidate events may have sidereal time dependence.
2. What is CPT violation?

CPT symmetry is the invariance under CPT transformation

\[ L \xrightarrow{\text{CPT}} \Theta L \Theta^{-1} = L' = L, \quad \Theta = \text{CPT} \]

CPT is the perfect symmetry of the Standard Model, due to CPT theorem.

**CPT-even**

- QED
- Weak
- QCD

**CPT-odd**
2. What is CPT violation?

CPT symmetry is the invariance under CPT transformation

\[ \mathcal{L} \xrightarrow{\text{CPT}} \Theta \mathcal{L} \Theta^{-1} = \mathcal{L}' = \mathcal{L}, \quad \Theta = \text{CPT} \]

CPT is the perfect symmetry of the Standard Model, due to CPT theorem

- CPT-even Lorentz violating coefficients (even number Lorentz indices, ex., \( a^\mu \), \( g^{\lambda \mu \nu} \))
- CPT-odd Lorentz violating coefficients (odd number Lorentz indices, ex., \( c^{\mu \nu} \), \( \kappa^{\alpha \beta \mu \nu} \))
2. Modern tests of Lorentz violation

http://www.physics.indiana.edu/~kostelec/faq.html

The last meeting of Lorentz and CPT violation was in summer 2010.
Next meeting will be in summer 2013
2. Modern tests of Lorentz violation

http://www.physics.indiana.edu/~kostelec/faq.html

Topics:
* searches for CPT and Lorentz violations involving
  birefringence and dispersion from cosmological sources
  clock-comparison measurements
  CMB polarization
  collider experiments
  electromagnetic resonant cavities
  equivalence principle
  gauge and Higgs particles
  high-energy astrophysical observations
  laboratory and gravimetric tests of gravity
  matter interferometry
  neutrino oscillations
  oscillations and decays of K, B, D mesons
  particle-antiparticle comparisons
  post-newtonian gravity in the solar system and beyond
  second- and third-generation particles
  space-based missions
  spectroscopy of hydrogen and antihydrogen
  spin-polarized matter
* theoretical studies of CPT and Lorentz violation involving
  physical effects at the level of the Standard Model, General Relativity, and beyond
  origins and mechanisms for violations
  classical and quantum issues in field theory, particle physics, gravity, and strings
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- origins and mechanisms for violations
- classical and quantum issues in field theory, particle physics, gravity, and strings

Test of Lorentz invariance with neutrinos is very interesting, because neutrinos are the least known standard model particles!
2. Neutrino oscillations, natural interferometers

Neutrino oscillation is an interference experiment (e.g. double slit experiment)

For double slit experiment, if path $\nu_1$ and path $\nu_2$ have different lengths, they have different phase rotations and it causes interference.

In terms of neutrinos, if Hamiltonian eigenstates $\nu_1$ and $\nu_2$ are different, that can be the source of neutrino oscillations.
2. Lorentz violation with neutrino oscillation

Neutrino oscillation is an interference experiment (e.g. double slit experiment)

If 2 neutrino Hamiltonian eigenstates, \( \nu_1 \) and \( \nu_2 \), have different phase rotations, they cause quantum interference.
2. Lorentz violation with neutrino oscillation

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If 2 neutrino Hamiltonian eigenstates, $\nu_1$ and $\nu_2$, have different phase rotations, they cause quantum interference.

If $\nu_1$ and $\nu_2$ have different couplings with Lorentz-violating field, that can be the source of neutrino oscillations.
2. Lorentz violation with neutrino oscillation

Neutrino oscillation is an interference experiment (e.g. double slit experiment)

If 2 neutrino Hamiltonian eigenstates, $\nu_1$ and $\nu_2$, have different phase rotations, they cause quantum interference.

If $\nu_1$ and $\nu_2$ have different couplings with Lorentz-violating field, that can be the source of neutrino oscillations.

Interference fringe (oscillation pattern) depends on the sidereal motion. The measured scale of neutrino eigenvalue difference is comparable the target scale of Lorentz violation ($<10^{-19}$GeV).
3. Test of Lorentz violation with neutrino oscillation experiments

Lorentz violation is realized as a coupling of particle fields and background fields, so the basic strategy to find Lorentz violation is:

(1) fix the coordinate system
(2) write down the Lagrangian, including Lorentz-violating terms under the formalism
(3) write down the observables using this Lagrangian

Standard Model Extension (SME) is the standard formalism for the general search for Lorentz violation. SME is a minimum extension of QFT with Particle Lorentz violation.

Modified Dirac Equation (MDE) of neutrinos

\[ i \begin{pmatrix} A_B \\ M_{AB} \end{pmatrix} B = 0 \]

SME coefficients

\[
\begin{align*}
A_B &= A_B + c_{AB} + d_{AB} \\
M_{AB} &= m_{AB} + i m_{5AB} + a_{AB} + b_{AB} + \frac{1}{2} H_{AB} + \frac{1}{2} g_{AB} + i f_{AB} + e_{AB} + f_{AB} 
\end{align*}
\]