Dynamical study of the light scalar mesons below 1 GeV in a flux-tube model

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The light scalar mesons below 1 GeV as tetraquark states are studied in the framework of the flux-tube model, the multi-body confinement instead of the additive two-body confinement is used. From the calculated results, we find that the light scalar mesons, \( \sigma \), \( \kappa \) could be well accommodated in the diquark-antidiquark tetraquark picture in the flux-tube model and they could be color confinement resonances. The mass of the first radial excited state of \([ud][\bar{u}\bar{d}]\) is 1019 MeV, which is close to the mass of \( f_0(980) \). Whereas \( a_0(980) \) can not be fitted in this interpretation.

Keywords: tetraquark, multi-body confinement, flux-tube model

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1. Introduction

The charged \( \kappa \), a scalar meson, was recently observed by BES collaboration.\(^{[1]}\) The Breit-Wigner mass and the decay width are obtained to be \( 826 \pm 49^{+49}_{-40} \) MeV and \( 449 \pm 156^{+144}_{-146} \) MeV, and the pole position is determined to be \( (764 \pm 63^{+71}_{-54}) - i(306 \pm 149^{+143}_{-134}) \) MeV/c\(^2\). They are in good agreement with those of the neutral \( \kappa \): the mass and the decay width are \( 878 \pm 23^{+54}_{-55} \) MeV and \( 499 \pm 53^{+55}_{-87} \) MeV, respectively, observed by the BES and other collaborations.\(^{[2]}\)

The understanding of scalar mesons, which have the same quantum numbers as the vacuum, is a crucial problem in low-energy quantum chromodynamics (QCD) since they could shed light on the chiral symmetry breaking mechanism and presumably also on confinement in QCD. Although many properties of scalar mesons have been studied for decades, it is still a puzzle for the understanding of the internal structure of the scalar mesons. Their masses do not fit into the quark model predictions.\(^{[3,4]}\) The flavor structures of these light scalar mesons below 1 GeV,
$a_0(980)$, $f_0(980)$, $\sigma$ and $\kappa$, are still open questions. In the $q\bar{q}$ configuration, the $p$-wave relative motion between $q$ and $\bar{q}$ has to be invoked to account for the spin and parity of the scalar mesons. This leads to much higher masses for them. Another possible configuration for scalar mesons is tetraquark state. In tetraquark configuration, the light scalar mesons could be classified into an SU(3) flavor nonet if the diquark picture is used. They are expressed as

\[ \sigma = [ud][\bar{u}\bar{d}], f_0^0 = \frac{[su][\bar{s}\bar{u}] + [sd][\bar{s}\bar{d}]}{\sqrt{2}}; \]
\[ \kappa^+ = [ud][d\bar{s}], \kappa^0 = [ud][\bar{u}\bar{s}], \kappa^0 = [us][\bar{u}\bar{d}]; \]
\[ a_0^+ = [su][\bar{s}\bar{d}], a_0^0 = \frac{[su][\bar{s}\bar{d}] - [sd][\bar{s}\bar{u}]}{\sqrt{2}}, a_0^- = [sd][\bar{s}\bar{u}]. \]

Jaffe et al. interpreted light scalar mesons as tetraquark states with all the relative orbital angular momenta assumed to be zeros. Weinstein et al. described light scalar mesons as hadronic molecular states due to strong meson-meson interaction. The spectrum of light scalar mesons below 1.0 GeV were studied in the $q\bar{q}$ picture by including instanton interaction. Bhavyashri et al. studied the instanton-induced interaction in light meson spectrum on the basis of the phenomenological harmonic models for quarks. Vijande et al. studied the scalar mesons in terms of the mixing of a chiral nonet of tetraquarks with conventional $q\bar{q}$ states.

A multi-quark state as a multi-body system is quite different from ordinary hadrons ($q\bar{q}$ meson and $qqq$ baryon) because the multi-quark state has more color structures than that of ordinary hadrons. The color structures of a multi-quark state is no longer trivial and the properties of the multi-quark states may be sensitive to the hidden color structure. A tetraquark state, if its existence is confirmed, may provide important information about the low-energy QCD interaction which is absent from the ordinary hadrons. Some authors had studied the tetraquark system with the three-body $qq\bar{q}$ and $q\bar{q}q$ interaction. The exotic hadrons were also studied as multiquark states in the flux tube model in our previous work. These studies suggest that the multi-body confinement, instead of the additive two-body confinement, might be more suitable in the quark model study of multiquark states. The newly updated experimental data might shed more light on the possibility of existence of tetraquark states and QCD interaction for multi-quark states.

The aim of this paper is to study the properties of scalar mesons below 1 GeV and their quark contents in the flux tube model with multi-body confinement potential. The powerful method for few-body systems with high precision, Gaussian expansion method (GEM) is used here. The paper is organized as follows: in Sec. 2, the flux-tube model with multi-body interaction are introduced. A brief introduction of GEM and the construction of the wave functions of tetra-quark states are given in Sec. 3. The numerical results and discussions are presented in Sec. 4 and a brief summary is given in the last section.
2. Quark model and multi-body confinement potential

Long term studies in the past several decades on hadrons indicate that ordinary hadrons (\(q\bar{q}\) meson and \(qqq\) baryon) can be well described by QCD-inspired quark models. Low energy QCD phenomena are dominated by two well known quark correlations: confinement and chiral symmetry breaking. The perturbative, effective one gluon exchange, properties of QCD should also be included. Hence, the main ingredients of the quark model are: constituent quarks with a few hundred MeV effective mass, phenomenological confinement potential, effective Goldstone bosons and one-gluon exchanged between these constituent quarks.

For ordinary hadrons, the color structures of them are unique and trivial, naive models based on two-body color confinement interactions proportional to the color charges (\(\lambda_i \cdot \lambda_j\)) can describe the properties of ordinary hadrons well. However, the structures of a multiquark state are abundant, which include important QCD informations that is absent from ordinary hadrons. There is no any theoretical reason to extend directly the two-body confinement in naive quark model to multi-quark system. Furthermore, the direct application of the two body confinement to multi-quark system induces many serious problems, such as anti-confinement and color Van der Waals force. Many theoretical works has been done to try to amend those serious drawbacks. The string flip model for multi-quark system was proposed by M. Oka to avoid the pathological Van der Waals force. Three quark confinement is explored by introducing strings which connect quarks according to a certain configuration rule.

Recent lattice QCD studies show that the confinement of multi-quark states is multi-body interaction and is proportional to the minimum of the total length of strings which connect the quarks to form a multiquark state. Based on these studies, a naive flux-tube or string model with multi-body confinement has been proposed for multiquark systems. The harmonic interaction approximation, i.e., the total length of strings is replaced by the sum of the square of string lengths, is assumed to simplify the numerical calculation.

The diquark-antidiquark picture of tetraquark states has been discussed by several authors. In the present work, the scalar mesons below 1 GeV are studied as diquark-antidiquark systems in the flux-tube model. The color structure is shown in Fig.1, where a solid dot represents a quark while a hollow dot represents an anti-quark, \(r_i\) is quark’s position, \(y_i\) represents a junction where three strings (flux tubes) meet. A thin line connecting a quark and a junction represents a fundamental representation, i.e. color triplet. A thick line connecting two junctions is for a color sextet or other representations, namely a compound string. The different types of string may have different stiffness. Color couplings satisfying overall color singlet of tetra-quark are \([qq][\bar{q}\bar{q}]_1\) and \([qq][\bar{q}\bar{q}]_2\), the subscripts represent the dimensions of color representations.

In the flux-tube model with quadratic confinement, the confinement potential, which is believed to be flavor independent, of the tetraquark state has the following
$V^C = k \left[ (r_1 - y_1)^2 + (r_2 - y_1)^2 + (r_3 - y_2)^2 + (r_4 - y_2)^2 + \kappa_d (y_1 - y_2)^2 \right]$,  \hspace{1cm} (1)

where $k$ is the stiffness of the string with the fundamental representation which is determined by meson spectrum, $k\kappa_d$ is the compound string stiffness. The compound string stiffness parameter $\kappa_d$ depends on the color representation, $d$, of the string,

$$\kappa_d = \frac{C_d}{C_3},$$  \hspace{1cm} (2)

where $C_d$ is the eigenvalue of the Casimir operator associated with the $SU(3)$ color representation $d$ of the string. $C_3 = \frac{1}{4}$, $C_6 = \frac{10}{3}$ and $C_8 = 3$.

Therefore, the minimum of the confinement interaction has the following form,

$$V^C_{\text{min}} = k \left[ R_1^2 + R_2^2 + \frac{\kappa_d}{1 + \kappa_d} R_3^2 \right].$$  \hspace{1cm} (3)

Taking into account potential energy shift, the confinement potential $V^C_{\text{min}}$ used in the present calculation has the following form

$$V^C_{\text{min}} = k \left[ (R_1^2 - \Delta) + (R_2^2 - \Delta) + \frac{\kappa_d}{1 + \kappa_d} (R_3^2 - \Delta) \right].$$  \hspace{1cm} (4)

Carlson and Pandharipande also considered similar flux tube energy shift which they assumed to be proportional to the number of quarks $N$. Obviously, the confinement potential $V^C$ is a multi-body interaction rather than a two-body interaction. It should be emphasized here that our approach is different from that in
Iwasaki’s work \cite{12} where the four-body problem is simplified to two-body one by treating diquark as a antiquark and antidiquark as a quark.

The other parts of the Hamiltonian are rest masses, kinetic energies, one-gluon-exchange potential and Goldstone-boson-exchange potentials, \cite{29}

\[
H = \sum_{i=1}^{4} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + V^C + \sum_{i>j}^4 (V^G_{ij} + V^B_{ij}),
\]

\[
V^B_{ij} = v^B_{ij} \sum_{a=1}^{3} F^a_i F^a_j + v^K_{ij} \sum_{a=4}^{7} F^a_i F^a_j + v^\eta_{ij} (F^\eta_i F^\eta_j \cos \theta_p - F^\eta_i F^\eta_j \sin \theta_p),
\]

\[
v^G_{ij} = \frac{\alpha_s}{r_{ij}} \left[ \frac{1}{r_{ij}} - \frac{3}{2} \delta(r_{ij}) \left( \frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{3m_im_j} \sigma_i \cdot \sigma_j \right) \right],
\]

where \( T_{CM} \) is the center-of-mass kinetic energy, \( F_i, \lambda_i \) are flavor, color SU3 Gellmann matrices. \( Y(x) \) is the standard Yukawa function and all other symbols have their usual meaning. The \( \delta \)-function should be regularized \cite{46,47}

\[
\delta(r_{ij}) = \frac{1}{4\pi} e^{-r_{ij}/r_0(\mu)}
\]

where \( \mu \) is the reduced mass of \( q_i \) and \( q_j \) and \( r_0(\mu) = \bar{r}_0/\mu \). The effective scale-dependent strong coupling constant is given by \cite{47}

\[
\alpha_s(\mu) = \frac{\alpha_0}{\ln \left( \frac{\mu^2 + \mu_0^2}{\Lambda_0^2} \right)}.
\]

3. Wave functions and gaussian expansion method

The total wave function of a tetra-quark state can be written as a sum of the following direct products of color, isospin and spatial-spin terms,

\[
\Phi_{I,M_1,J_T,M_T} = \sum_{l,s,i,c,L} \xi_{l,s,i,c,L}^{I,J_T} \left[ \left[ \phi_{l,m_1}^{G}(r) \Psi_{s_1,m_1} \right]_{J_1,M_1} \left[ \psi_{l_2m_2}^{G}(R) \Psi_{s_2,m_2} \right]_{J_2,M_2} \right]_{J_{12},M_{12}} \chi_{L,M}^{G}(X),
\]

Here \( I \) and \( J_T \) are total isospin and total angular momentum respectively. \( \Psi_{s_1,m_1} \) (\( \Psi_{s_2,m_2} \)), \( \eta_{l_1,m_1} \eta_{l_2,m_2} \) and \( \chi_{l_1,m_1} \chi_{l_2,m_2} \) are spin, flavor and color wave functions of, diquark (anti-diquark), respectively. \( \xi_{l,s,i,c,L}^{I,J_T} \)'s denote Clebsh-Gordan coefficients coupling. The coefficient \( \xi_{l,s,i,c,L}^{I,J_T} \) is determined by diagonalizing the Hamiltonian, subscripts \( l, s, i, c, L \) represent all possible intermediate quantum numbers, therefore our calculations are multi-channel coupling calculations. The Jacobi coordinates of
the tetraquark are defined as
\[ X = m_1 r_1 + m_2 r_2 - m_3 r_3 - m_4 r_4 / (m_1 + m_2), \]
\[ R = r_3 - r_4 \]
where particles 1 and 2 are two quarks and particle 3 and 4 are two anti-quarks.

\[ \mathbf{L}, l_1 \text{ and } l_2 \text{ are the orbital angular momenta associated with coordinates } X, r \text{ and } R, \text{ respectively.} \]

The calculation is done in the center-of-mass coordinate system \((R_{CM} = 0)\). The tetra-quark state is an overall color singlet with well defined parity \(P = (-1)^{l_1 + l_2 + L}\), isospin \(I\) and total angular momentum \(J_T\). For scalar mesons, we set the angular momentum \(L, l_1 \text{ and } l_2\) to be zero.

For color part, the color singlet is constructed in the following two ways,
\[ \chi^1_c = \bar{3}_1 12 \otimes \bar{3}_3 34, \]
\[ \chi^2_c = 6_1 12 \otimes \bar{6}_3 34, \]
both "good" diquark and "bad" diquark are included.

With respect to the flavor part, the flavor wave function read as
\[ \eta_I = \eta_1 12 \otimes \eta_3 34. \]

Gaussian size parameters are taken as the following geometric progression numbers
\[ \nu_n = \frac{1}{r_n^2}, \ r_n = r_1 a^{n-1}, \ a = \left( \frac{r_{n_{max}}}{r_1} \right)^{1/n_{max}-1}. \]

### 4. Numerical results and discussions

Now we turn to the tetraquark state calculation. The model parameters are fixed by reproducing the ordinary meson spectrum and are listed in Table 1. The meson spectrum can be reproduced very well. Because the flux-tube model is reduced to the ordinary quark model for \(q \bar{q}\) system, the obtained meson spectra (from light to heavy) are similar to that of other work, e.g. Ref.47. Parts of the calculated meson spectrum are shown in Table 2. The experimental values are taken from PDG compilation\cite{48}. 

Table 1. The model parameters (set I). The masses of $\pi$, $K$, $\eta$ take the experimental values.

| $m$ | $m_0$ | $k$ | $r_0$ | $\Lambda_0$ | $\mu_0$ | $\Delta$ | $\alpha_0$ | $\Lambda_\pi$ | $\Lambda_K$ | $\Lambda_\eta$ | $\theta_F$ |
|-----|-------|-----|-------|-----------|--------|--------|--------|-----------|-----------|-----------|--------|
| MeV | MeV   | MeV fm$^{-2}$ | MeV fm | fm$^{-1}$ | fm$^{-1}$ | fm$^{-1}$ | fm$^{-1}$ | fm$^{-1}$ | fm$^{-1}$ | fm$^{-1}$ |       |
| 313 | 520   | 213.3 | 30.85 | 0.187     | 0.113  | 0.5    | 4.25   | 4.2       | 5.2       | 15       |        |

Table 2. The meson spectrum (unit: MeV).

| Meson | $\pi$ | $K$ | $\rho$ | $K^*$ | $\omega$ | $\phi$ | $\sqrt{\langle r^2 \rangle}$ (fm) |
|-------|-------|-----|-------|-------|---------|-------|-------------------------------|
| Cal.  | 139   | 502 | 761   | 897   | 735     | 1023  | 0.57                          |
| Exp.  | 139   | 496 | 770   | 898   | 780     | 1020  | 0.60                          |

The energies of scalar meson states can be obtained by solving the four-body Schrödinger equation

$$(H - E) \Phi_{I M_I J M_J} = 0$$

with Rayleigh-Ritz variational principle. In GEM the calculated results are converged with $n_{1\text{max}}=6$, $n_{2\text{max}}=6$ and $n_{3\text{max}}=6$. Minimum and maximum ranges of the bases are 0.1 fm and 2.0 fm for coordinates $r$, $R$ and $X$, respectively.

Quark contents and the corresponding masses in the three different quark models for the light scalar mesons as tetra-quark states are shown in Table 3, where $n$ stands for a non-strange quark ($u$ or $d$) while $s$ stands for a strange quark, $I$ and $N$ denote total isospin and principal quantum number of total radial excitation, $S$, $L$ and $J$ have their usual meaning. “Naive” stands for naive quark model, where only one-gluon-exchange potential is taken into account in addition to the additive two-body confinement. “Chiral” stands for chiral quark model, where one-gluon-exchange and one-Goldstone-boson-exchange are included besides the additive two-body confinement. The masses in the naive and chiral model are much higher (several hundreds MeV) than those in flux-tube model, the origin of this discrepancy mainly comes from the different type of confinement interaction, a two-body confinement potential is applied in the naive and chiral model, whereas a multi-body interaction confinement is used in the flux-tube mode. Zou et al. studied scalar mesons in the quark model by introducing three-body confinement interaction. Their study also indicate that the multi-body confinement potential, instead of two-body interaction, should be applied in the study of multi-quark states. The naive quark model gives the highest masses, due to the absence of Goldstone boson exchange, which induces additional attraction for the tetraquark system.

In the framework of the flux tube model, it can be seen from Table 3 that the lowest masses of $nn\bar{n}$ and $nn\bar{s}$ system are 587 MeV and 948 MeV, which are close to the masses of $\sigma$ and $\kappa$. If the existence of $\sigma$ and $\kappa$ is further confirmed, the tetraquark state is a possible interpretation. This interpretation is in agreement with many other studies. Predojevic et al. recently studied light scalar mesons $\sigma$ and $\kappa$ by lattice QCD simulation, they also found that $\sigma$ and $\kappa$ have sizable tetra-quark components $nn\bar{n}$ and $nn\bar{s}$, respectively. In the case of $f_0(980)$ and
Table 3. Numerical results for three models (unit: MeV).

| Flavor | $nn\bar{n}$ | $nn\bar{n}$ | $nn\bar{n}$ | $nn\bar{s}$ | $n\bar{n}s$ | $n\bar{s}s$ |
|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| $IJ^P$ | 00$^+$      | 00$^+$      | 10$^+$      | $\frac{1}{2}0^+$ | 00$^+$        | 10$^+$      |
| $N^{2S+1}L_J$ | 0$^1$S$0_0$ | 1$^1$S$0_0$ | 0$^1$S$0_0$ | 0$^3$S$0_0$ | 0$^1$S$0_0$ | 0$^3$S$0_0$ |
| Naive  | 938         | 1431        | 1431        | 1216        | 1456        | 1456        |
| Chiral | 666         | 1237        | 1406        | 1122        | 1454        | 1454        |
| Flux-tube | 587       | 1019        | 1210        | 948         | 1314        | 1318        |

| Candidate | $\sigma$ | $f_0(980)$? | $\kappa$ | — | — |
| mass      | 541 ± 39  | 980 ± 10    | 826 ± 49+49 | | |

Table 4. The parameter set II.

|        | $k$ | $\nu_0$ | $\Delta$ | $\alpha_0$ |
|--------|-----|---------|----------|------------|
|        | MeV fm$^{-2}$ | MeV fm | fm$^2$- | -          |
|        | 267  | 30.0    | 0.6      | 4.09       |

Table 5. Numerical results in flux-tube model (unit: MeV).

| Flavor | $nn\bar{n}$ | $nn\bar{n}$ | $nn\bar{n}$ | $nn\bar{s}$ | $n\bar{n}s$ | $n\bar{s}s$ |
|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| $IJ^P$ | 00$^+$      | 00$^+$      | 10$^+$      | $\frac{1}{2}0^+$ | 00$^+$        | 10$^+$      |
| $N^{2S+1}L_J$ | 0$^1$S$0_0$ | 1$^1$S$0_0$ | 0$^1$S$0_0$ | 0$^3$S$0_0$ | 0$^1$S$0_0$ | 0$^3$S$0_0$ |
| Flux-tube | 531         | 969         | 1180        | 908         | 1270        | 1275        |

$a_0(980)$, many theoretical studies assumed them as tetra-quark states with quark content $n\bar{s}s$ with isospin $I = 0$ and $I = 1$, respectively. The masses for $n\bar{s}s$ states are much higher than experimental values even in the flux tube model. Thus their main components seem to be not the $n\bar{s}s$ in the quark model. Instead, the mass of the radial excited state of $\sigma$ is 1019 MeV, which is close to the mass of $f_0(980)$. Taking $f_0(980)$ as $nn\bar{n}$ state is consistent with Vijande’s work.\cite{Vijande} The observed $f_0(980) \to K\bar{K}$ process can be explained by $n\bar{s}s$ components mixing and the calculation is on going. Peláez also suggested that three scalar mesons, $\sigma$, $\kappa$ and $f_0(980)$, have dominant tetraquark component, whereas $a_0(980)$ might be a more complicated systems\cite{Pelaez} which is also consistent with our results.

In order to check the dependence of numerical results on model parameters, we make the same calculations on scalar mesons with another set of parameters which listed in Table 4 (the unchanged parameters are not listed). The almost same meson spectrum is obtained. The results for tetraquark states are shown in Table 5. Comparing Table 4 and 5, our results are rather stable against the variation of model parameters.

5. Summary

The scalar mesons below 1 GeV are studied as diquark-antidiquark system in the flux-tube model including multi-body confinement interaction on the basis of the lattice QCD calculations and other two models involving two-body confinement po-
tential. In the naive and chiral quark models, it is difficult to explain the observed scalar mesons as $q\bar{q}$ or tetra-quark states. In the flux-tube model, rather low masses of tetraquark states are obtained. It is possible to assign the low-lying scalar mesons as tetraquark states, at least tetraquark is the main component. For example, $\sigma$ and $\kappa$ can be assigned as $nn\bar{n}\bar{n}$ and $nn\bar{n}\bar{s}$ with $J^P = 0^+$, if their existence is further confirmed. To assign $f_0(980)$ and $a_0(980)$ as the tetraquark states $qs\bar{q}\bar{s}$ is questionable since the theoretical masses are much higher than their experimental values. The interpretation of the $a_0(980)$ as a lowest $qq\bar{q}\bar{q}$ state would give a much higher mass (about 200 MeV higher) than experiment data, which is still a mysterious system for the time being. However, in our calculation the mass of the radial excitation of the $\sigma$ is close to the mass of $f_0(980)$. The problem of this assignment, the small decay width of $f_0(980) \to K\bar{K}$, can be accounted for by the mixing of $ns\bar{n}\bar{s}$ component with $nn\bar{n}\bar{n}$. Our results are rather stable against the variation of the model parameters.

At now the nature of scalar meson is still an open question. The interpretation of the scalar mesons as tetraquark states is a possibility. In fact, the scalar mesons should be the superpositions of $q\bar{q}$, $qq\bar{q}\bar{q}$ and other components in a Fock space expansion approach and the dominant one is determined by quark dynamics. The mixing between two-body and four-body configurations would require the knowledge of the quark-antiquark pair creation-annihilation interaction, which is calculating in our group by using a $^3P_0$ model tentatively. The crucial test of the quark structure of the scalar mesons should be determined by the systematic study of these scalar mesons’ decay pattern. Of course, the final answer could only be obtained through a combined effort of the further experimental measurement and theoretical efforts.

As mentioned before, the flavor structures of scalar mesons are complicated, even limiting to four-quark system. At the same time, their color structures are also complicated, such as $3 \otimes 3$, $6 \otimes 6$, $1 \otimes 1$ and $8 \otimes 8$. The mixing of different color structures should be determined by the transition interaction between different color structures. Unfortunately, at present we do not have any reliable information about these transition interactions.

The comparative studies of three models reported here indicate that the flux-tube model with a multi-body confinement potential on the basis of lattice QCD can give lower masses of scalar mesons as tetra-quark states than other two models with two-body confinement interaction. The multi-body interaction plays an important role in reducing the energy of the system. We therefore strongly suggest that a multi-body confinement potential, instead of two-body interaction, should be employed in the study of multi-quark system. In this way, multi-quark systems can form multi-body flux-tubes connecting quarks into a color singlet multi-quark states. The states can not decay into two colorful hadrons directly due to color confinement. They must transform into color singlet mesons before decaying. This decay mechanism is similar to compound nucleus formation and therefore should induce a resonance which is named as a “color confined, multi-quark resonance” state before. It is different from all of those microscopic resonances discussed by S. Weinberg. Currently, an
urgent task is to obtain the information about the transition interaction between different Fock space configurations and different color structures which are crucial both in the structure and decay width calculations.

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