Relevant OTOC operators: footprints of the classical dynamics

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The out-of-time order correlator (OTOC) has recently become relevant in different areas where it has been linked to scrambling of quantum information and entanglement. It has also been proposed as a good indicator of quantum complexity. In this sense, the OTOC-RE theorem relates the OTOCs summed over a complete base of operators to the second Renyi entropy. Here we have studied the OTOC-RE correspondence on physically meaningful bases like the ones constructed with the Pauli, translation and reflection operators on the torus [25]. In this work we determine that not all of the OTOCs are good indicators of quantum complexity, but we are able to classify them in terms of the information they provide on the dynamical features. Our system consists of two perturbed and coupled Arnold cat maps with different dynamics. The three possible cases were considered, i.e. both maps being hyperbolic (chaotic) (HH), both elliptic (regular) (EE) and a mixed scenario where one map is hyperbolic and the other is elliptic (HE.EH) [17]. Also, we have considered three different bases constructed with Pauli or $SU(N)$, translation and reflection operators on the torus [25]. In all cases we have taken the non evolving density operator as localized pure states. Our results show that performing the summation in Eq.2 with a set of only 35% or less of the operators, in any of the chosen bases, 80% of

I. INTRODUCTION

There is a great interest in the OTOC nowadays, coming from different areas like high energy and gravity, condensed matter, many-body systems, quantum information, and quantum chaos. This measure has been introduced in the superconductivity context [1] where the exponential growth as a function of time has been associated with chaotic behavior. The OTOC is usually defined as a 4-point out-of-time order correlator,

$$C(t) = \langle \hat{M}(t) \hat{V}(0) \hat{M}(t) \hat{V}(0) \rangle$$

where $\langle \cdot \rangle = Tr[\cdot]/N$ is the thermal average and $M(t)$ is an operator evolved in the Heisenberg picture. The establishment of an upper limit to the growth rate of the OTOC in black hole models [2] has led to an interest surge on this measure. Examples of this can be found in many-body physics [3-10], quantum chaos [11-13], high energy physics [14], and the link between topological gravity and quantum chaos [15]. Recently, the OTOC behavior has been studied for bi-partite systems. In [16] it was found that for the chaotic case the scrambling process has two phases, one in which the exponential growth is within the subsystem and a second one which depends only on the interaction. In [17] the OTOC has proven to be a very good indicator of quantum complexity [18-19] when considering all possible dynamical scenarios.

The OTOC is conceptually related to scrambling of quantum information [20,22] and entanglement [17]. It is in this respect that the OTOC-RE theorem [23,24] establishes the equivalence of the linear entropy $S_L$ with the 4-point OTOC averaged over a complete operator basis of some arbitrary partition of the system. Following the scheme presented in [24], we can summarize the

$$S_L = 1-e^{-S^{(2)}_A} = 1 - \sum_{\hat{M} \in B} Tr[\hat{M}(t)\hat{\rho}(0)\hat{M}^\dagger(t)\hat{\rho}(0)]$$

where $A$ and $B$ are two partitions of our system, $\hat{\rho}(0)$ is the initial (non evolving) density operator of the whole system, $S^{(2)}_A = -logTr[\hat{\rho}_A^2]$ is the second Renyi entropy and $S_L$ is the linear entropy. The $\hat{M}$ operators act on the subsystem $B$ and define a complete basis normalized according to $\sum_{\hat{M} \in B} M_{ij} (M_{lm})^\dagger = \delta_{im}\delta_{lj}$. In Eq.2 we have taken the second evolved operator as $\hat{M}^\dagger(t)$, being the transpose and conjugate of the first one in such a way to extend the validity of the theorem to unitary operators. This result prescribes an average behavior for different OTOCs in a given basis, but it is important to ask ourselves how meaningful this is. As a matter of fact, is each one of the terms appearing in 2 equally relevant, making the same contribution to the linear entropy? In this work we determine that not all of the OTOCs are good indicators of quantum complexity, but we are able to classify them in terms of the information they provide on the dynamical features.

Our system consists of two perturbed and coupled Arnold cat maps with different dynamics. The three possible cases were considered, i.e. both maps being hyperbolic (chaotic) (HH), both elliptic (regular) (EE) and a mixed scenario where one map is hyperbolic and the other is elliptic (HE.EH) [17]. Also, we have considered three different bases constructed with Pauli or $SU(N)$, translation and reflection operators on the torus [25]. In all cases we have taken the non evolving density operator as localized pure states. Our results show that performing the summation in Eq.2 with a set of only 35% or less of the operators, in any of the chosen bases, 80% of
is recovered. On the other hand, this set of relevant operators is given by those that best capture the dynamics of the system, being suitable for complexity measures. For reflection and translation bases, they show clear footprints of the underlying classical dynamics in phase space.

This paper is organized as follows: in Section II we present our system with a brief description of the properties of the Hilbert space on the torus. We also describe the operator bases that we use for the OTOC-RE properties of the Hilbert space on the torus. We also describe the values of $\chi_p, \chi_q$ that can be chosen in the range $[0,1]$. These bases have the following normalization with $2\pi i \chi_p$ and $2\pi i \chi_q$ arbitrary Floquet angles that determine the so called prequantization. The values of $\chi_p, \chi_q$ can be chosen in the range $[0,1]$, we take $\chi_p = \chi_q = 0$. The previous boundary conditions can be satisfied if there is an integer $N$, so that

$$\hbar = \frac{1}{2\pi N}. \quad (3)$$

This implies a Hilbert space $\mathcal{H}_N$ of finite dimension $N$. We take $|q_n\rangle$ and $|p_m\rangle$ with $n, m = 0, 1, \ldots, N-1$ as bases of $\mathcal{H}_N$. The states $|q_j\rangle$ are periodic Dirac delta distributions at positions $j = n/N \mod (1)$, with $n$ an integer in $[0, N-1]$. These bases have the following normalization conditions,

$$\langle q_m | q_n \rangle = \langle p_m | p_n \rangle = \delta^{(N)}_{m,n}$$

with $\delta^{(N)}_{i,j}$ the N-periodic Kronecker delta defined as

$$\delta^{(N)}_{i,j} = \sum_{k=-\infty}^{\infty} \delta_{i+j+kN}$$

The bases are exchanged with the transformation kernel,

$$\langle p_m | q_n \rangle = \frac{1}{\sqrt{N}} e^{\pm i \pi m n / N}.$$ 

Position and momenta are then points in a discrete lattice on the torus with separation $1/N$, i.e. the quantum phase space [27].

The quantization of the cat map [28] which is one of the most simple paradigmatic models of chaotic dynamics, has helped to elucidate many questions in the quantum chaos area [28–31]. Here we consider the behavior of two coupled perturbed cat maps, a two degrees of freedom example, which can have different types of dynamics. For each degree of freedom, the map is defined on the 2-Torus as

$$\begin{pmatrix} q_{t+1} \\ p_{t+1} \end{pmatrix} = M \begin{pmatrix} q_t \\ p_t \end{pmatrix} + \epsilon(q_t)$$

with $q$ and $p$ taken modulo 1, and the perturbation

$$\epsilon(q_t) = -\frac{K}{2\pi} \sin(2\pi q_t).$$

The matrix $M$ defines the dynamics. For the chaotic case we have chosen the hyperbolic map

$$M_h = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix},$$

while for the regular behavior we have taken the elliptic map

$$M_c = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$ 

The propagator in position representation is given by the $N \times N$ unitary matrix

$$U_{jk} = A \exp \left[ -i \pi \frac{(M_{11} j^2 - 2 j k + M_{22} k^2) + F}{N M_{12}} \right]$$

where

$$A = [1/(iN M_{12})]^{1/2}$$

and,

$$F = [iK N/(2\pi)] \cos(2\pi j / N).$$

We can extend it to two degrees of freedom defined in a four-dimensional phase space of coordinates $(q^1, q^2, p^1, p^2)$ [19] as

$$\begin{pmatrix} q^1_{t+1} \\ p^1_{t+1} \end{pmatrix} = M_1 \begin{pmatrix} q^1_t + \epsilon(q^1_t) + \kappa(q^1_t, q^2_t) \\ p_t^1 \end{pmatrix}$$

and

$$\begin{pmatrix} q^2_{t+1} \\ p^2_{t+1} \end{pmatrix} = M_2 \begin{pmatrix} q^2_t + \epsilon(q^2_t) + \kappa(q^1_t, q^2_t) \\ p_t^2 \end{pmatrix},$$

where the coupling between both maps is given by $\kappa(q^1_t, q^2_t)$. Hence, the quantum evolution for this case is given by the tensor product of the one degree of freedom maps

$$U_{2}^{2P}_{j_1, k_1, k_2} = U_{j_1 k_1} U_{j_2 k_2} C_{j_1, j_2},$$

with coupling matrix,

$$C_{j_1 j_2} = \exp \left\{ -i \frac{K_N}{2\pi} \cos \left[ \frac{2\pi}{N} (j_1 + j_2) \right] \right\}.$$
where \(j_1, j_2, k_1, k_2 \in \{0, \ldots, N-1\}\). We fix \(K = 0.25\) and \(K_e = 0.5\) (Anosov condition \([29]\) ), and \(N = 2^6\) throughout this work.

For the complete set of operators spanning one of the subsystems in Eq. \([2]\) we have chosen three different sets. They are the so called computational or Pauli base, the translation base, and the reflection base \([25]\). The first one is relevant for multi-qubit systems (canonical in quantum computation and information), while the translation and reflection ones define the chord and Wigner (or center) functions \([32]\) respectively, allowing for a more direct comparison with classical counterparts.

**Pauli base.** For qubit systems, the typical base chosen is \(\{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}\) where \(\sigma_0 = \mathbb{1}\) and the rest of the \(\sigma_i\)'s are \(2 \times 2\) Pauli matrices. For dimensions \(N = 2^k\), we can extend this basis by taking the complete system as a direct product of \(k\) single qubits,

\[
\left\{ \bigotimes_{i=1}^{k} \sigma_{j_i} \right\}
\]

**Translation and reflection bases.** In \([25]\) translation operators \(\hat{T}_x\) on the torus are described by their chord \(\xi = (\xi, \xi_q) = (r/N, s/N)\) with \(r\) and \(s\) integer indices. A complete basis of \(N^2\) independent operators is obtained for chords performing up to one loop on the torus, that is for \(r\) and \(s\) belonging to the interval \([0, N-1]\). The matrix elements of the translation operators \(\hat{T}_x\) in the position representation are given by,

\[
\langle q_i | \hat{T}_{x(r,s)} | q_j \rangle = e^{i \frac{2\pi r}{N}(j-i)+x_s} \delta_{N,j+i+s} e^{-i \frac{2\pi}{N}(\xi+\chi_p)(j-i-s)}.
\]

(9)

For the case of reflection operators \(\hat{R}_x\), they are described by their center point \(x = (x_p, x_q) = (a/N, b/N)\) with half-integer indices \(a\) and \(b\) with values in \([0, N-1]\) in order to complete a basis of \(N^2\) independent operators. That is, a quarter of the torus contains the complete information for the reflection basis. The matrix elements in the position representation are

\[
\langle q_i | \hat{R}_{x(a,b)} | q_j \rangle = e^{i \frac{2\pi}{N}(j-i)(a+x_s)} \delta_{N,j+2b-i} e^{-i \frac{2\pi}{N}s(2b-i-j)}.
\]

(10)

We recall that in both cases we have chosen the Floquet angles \((\chi_q, \chi_p)\) as zero.

### III. RESULTS

For completeness, we first check the validity of the OTOC-RE theorem (Eq.\([2]\)) for all the dynamical scenarios and all the operator bases described in Sec. II. Fig. \([1\ a]\) corresponds to \(S_L\) and all OTOCs sums as a function of the time \(t\) (map steps), for both dynamics being hyperbolic (HH), while in Fig. \([1\ b]\) and \(c\) we show the HE and EE cases, respectively. We consider a coherent state located at the fixed point \((q, p) = (0.5, 0.5)\) on each tori. Fig. \([1\ d]\) displays the EE case where the coherent state is located at \((q, p) = (\pi/4, \pi/4)\) (not a fixed point). The theorem clearly holds regardless of the dynamics or the chosen base.

![Figure 1](image_url)

We have classified each OTOC in Eq. \([2]\) according to its contribution to the sum. In fact, their relevance is essentially given by the corresponding area under the curve up to a time \(t_0\). We proceed in the following way: for each operator \(M\) we have calculated the area \(A_M(t_0)\) as

\[
A_M(t_0) = \int_0^{t_0} C_M(t) dt.
\]

(11)

where \(C_M(t)\) is the OTOC

\[
C_M(t) = Tr[\hat{M}(t)\hat{\rho}(0)\hat{M}^\dagger(t)\hat{\rho}(0)].
\]

(12)

Then, we have ordered the operators using \(A_M(t_0)\) which reflects their contribution to the total area under \(1 - S_L(t)\), given by \(A_S(t_0) = \int_0^{t_0} 1 - S_L(t) dt\). Finally we determine a cutoff criterion which consists of reaching the value \(0.8A_S(t_0)\) by simply adding the areas contributed by each operator’s OTOC like \(\sum_R A_M(t_0)\), where \(R\) means that the sum only runs from the most up to the least relevant one. This provides us with the number of OTOCs necessary to reach what we will refer to as the effective \(S_L\) behavior.

We first consider the HH case with coherent states at \((q, p) = (0.5, 0.5)\). Due to the chaotic nature of the dynamics the OTOCs and \(S_L\) both grow exponentially \([14]\) at an early stage, hence we only look up to \(t_0 = 10\). In Fig. \([2\ a]\) we show \(S_L(t_0)\) (black lines) and the partial sum obtained with the most relevant OTOCs (filled symbols) for each operator base. For the Pauli base, only 263 from
a total of 4096 terms were needed in order to reach the effective $S_L$ behavior. Meanwhile, for the translation and reflection bases 166 and 697 terms were needed, respectively. The effective entropy behavior is recovered with less than 20% of the operators. In addition, in Fig. 2 we also show the contribution of the remaining OTOCs (empty symbols) which is markedly lower than that of the most relevant ones. We notice that in all Figures we display $1 - \sum_R C_M(t)$ which is directly compared to $S_L$, then the values corresponding to the empty symbols are to be subtracted from the filled ones to recover the entropy.

Figure 2. (Color online) $S_L$ ((black) solid line with filled circles) given by the l.h.s of Eq. (2) for the HH case with coherent states taken at $(q,p) = (0.5,0.5)$. Pauli ((red) solid line with filled squares) operators sum considering the 263 most relevant terms in the r.h.s of Eq. (2). Translation ((blue) solid line with filled diamonds) and reflection ((green) solid line with filled down triangles) bases with 166 and 697 terms respectively. Empty symbols with dotted lines show the contribution of the remaining terms for the Pauli ((red) dashed line with empty squares), translation ((blue) dashed line with empty diamonds) and reflection ((green) dashed line with empty down triangles) bases. $t_0 = 10$.

Next we look into the HE map where the operator base is taken for the regular subsystem and coherent states are placed at $(q,p) = (0.5,0.5)$. In Fig. 3 we see that $S_L$ grows slower than in the previous case (until saturation) due to the mixed character of the dynamics, leading us to a longer integration time ($t_0 = 40$). To recover the effective $S_L$ behavior this time we needed 199 Pauli, 110 translation and 1285 reflection operators, i.e. less than 30% of the total operators in the worst case.

Finally, we take the EE map with coherent states at $(q,p) = (0.5,0.5)$ and then at $(q,p) = (\pi/4,\pi/4)$. The first case is shown in Fig. 4 ($t_0 = 10$), where the effective $S_L$ behavior is recovered by just 81 operators in the Pauli, 101 in the translation, and 117 in the reflection bases. In this dynamical scenario the coherent state does not explore the entire phase space but just rotates around the fixed point, giving a hint to explain this clear reduction in the number of relevant operators. In this case we have re-scaled the partial sum of the most relevant OTOCs for a better comparison with $S_L$ (the sum of the remaining ones is left unchanged). In Fig. 5 we display the results when placing the coherent states at $(q,p) = (\pi/4,\pi/4)$. Since this is not at a fixed point the entropy grows up to saturation at a slower rate than in the HH case, leading us to consider $t_0 = 30$. We recover the effective $S_L$ behavior with 413 operators in the Pauli base, 103 in the translation and 1450 in the reflection one, i.e. about 35% of the operators in the worst case.

We mention that not only the sum but each one of the quantities $1 - C_{M_R}(t)$ (where $M_R$ stands for the relevant operators) approximate the linear entropy very well (up to normalization), i.e. we claim that $S_L = 1 - e^{-S_L^{(2)}}(t) \approx 1 - C_{M_R}(t)$. The remaining operators have a different behavior.

On the other hand, it is interesting to investigate if the amount of relevant operators changes as a function of the integration time $t_0$, and eventually how this change is. In Figures 6, 7, and 8 we show the number of relevant operators for Pauli, translation and reflection bases respectively for each dynamics and different integration times, needed to achieve the effective $S_L$ behavior. For all bases, we notice that if the system has at least one hyperbolic degree of freedom, the number of operators grows steeply with the integration time. If the system is completely elliptic and the coherent states are located at the fixed point, the number of operators is essentially constant, while if they are not at a periodic orbit, the number of operators grows with a rate much slower than
Figure 4. (Color online) $S_L$ ((black) solid line with filled circles) given by the l.h.s of Eq.2 for the EE case with coherent states taken at $(q, p) = (0.5, 0.5)$. Pauli ((red) solid line with filled squares) operators sum considering the 81 most relevant terms in the r.h.s of Eq.2. Translation ((blue) solid line with filled diamonds) and reflection ((green) solid line with filled down triangles) bases with 101 and 117 terms respectively. Empty symbols with dotted lines show the contribution of the remaining terms for the Pauli ((red) dashed line with empty squares), translation ((blue) dashed line with empty diamonds) and reflection ((green) dashed line with empty down triangles) bases. $t_0 = 10$.

Figure 5. (Color online) $S_L$ ((black) solid line with filled circles) given by the l.h.s of Eq.2 for the EE case with coherent states taken at $(q, p) = (\pi/4, \pi/4)$. Pauli ((red) solid line with filled squares) operators sum considering the 413 most relevant terms in the r.h.s of Eq.2. Translation ((blue) solid line with filled diamonds) and reflection ((green) solid line with filled down triangles) bases with 103 and 1450 terms respectively. Empty symbols with dotted lines show the contribution of the remaining terms for the Pauli ((red) dashed line with empty squares), translation ((blue) dashed line with empty diamonds) and reflection ((green) dashed line with empty down triangles) bases. $t_0 = 30$.

Figure 6. (Color online) Number of relevant Pauli operators, for different integration times $t_0$ and dynamics. (Blue) line with circles stands for the HH case, (green) line with squares for EH, (orange) line with down triangles for HE, and finally (red) line with diamonds and (black) line with crosses for the EE cases with coherent states at $(0.5, 0.5)$ and $(\pi/4, \pi/4)$, respectively. $N = 2^6$.

Figure 7. Number of relevant translation operators, for different integration times $t_0$ and dynamics. Same color code as in Fig. 6. $N = 2^6$.

in the mixed (HE and EH; we have taken both points of view in order to better look into dynamical properties) or totally hyperbolic (HH) cases, specially for the reflection base (see Fig. 8). For the HH case, if we take long integration times, we will have that almost all operators (not all since we only require the effective $S_L$ behavior) are relevant and equivalent reminding us of the underlying classical ergodicity in this scenario. Growth in the number of relevant operators gives us more hints on the OTOCs sensitivity for quantum complexity, providing with an alternative natural indicator of it. As a final remark, from Figs. 6-7 and 8 we see that the
number of relevant operators can be dependent on the base. An extreme example is given by the Kirkwood one whose operators are defined by

\[ K_{(i,j)} = |q_i\rangle \langle p_j|, \]

(13)

and for which there is a clear association with phase space representations, having a direct classical meaning [11]. For any of the operators in this basis it is straightforward to show that

\[ C_{K_{(i,j)}}(t) = \rho_A^2(t), \]

(14)

hence all of them are equally relevant in sensing the dynamics, so special care must be taken at the time of selecting the base if one wants to profit from the OTOCs ability to characterize quantum complexity.

All the previous analysis has led us to look for an explanation on the physical meaning of the operator relevance at the time to describe the \( S_L \) behavior or the quantum complexity in general. In order to proceed we restrict ourselves to translation and reflection operators since they can be represented in chord and center phase space. In Eqs. 9 and 10, we identified each one of these operators with a couple of indexes, \( (r,s) \) for translations and \( (a,b) \) for reflections, which are related to the chord of translation and the reflection center, respectively. These indices can be represented in a 2D plot, allowing to visualize the different operators. Figures 9 and 10 show the most relevant translation and reflection operators for each dynamics and different integration times \( t_0 \). For reflections, we have chosen an odd Hilbert space dimension of \( N = 65 \) in order to deploy the complete basis from the quarter torus with half integer indices into the full one with integer indices. This allows a clearer visualization of the classical structures in phase space [33].
In the HH case, we see that the relevance of translation and reflection operators grows along the unstable manifold of our map, indicated in Figures 9 and 10 with a black solid line. The number of relevant operators grows with $t_0$ and finally extends to the entire phase space. In the HE scenario, the relevant translation operators (see Fig. 9) are grouped around the identity operator (the chord is null) because we are looking at the elliptic degree of freedom. However, the number of them increases with $t_0$, reflecting the spreading of the coherent state due to the influence of the hyperbolic map. Relevant reflections are concentrated in the center of the phase space where the coherent state is (see Fig. 10) and, when $t_0$ increases more operators are needed to describe the dynamics enlarging the corresponding distribution. A similar situation arises in the EH case (now we observe the hyperbolic subsystem), i.e. the number of relevant operators grows and its distribution spreads along the unstable direction for both translations and reflections. Finally, in the EE case for the coherent state at the periodic point, the distributions remain localized and again the translation operators which better capture the dynamics are the ones closer to the identity, while the corresponding reflection operators are those at the center of the phase space. In both cases the number of relevant operators does not change. Finally, if we locate the coherent states out of the fixed point, the relevant translation operators are still closer to the identity but the reflection ones follow the evolution of the distributions as in [18].

In all these cases, we can observe that the set of relevant operators follow the footprints of the classical dynamical evolution and this provides with a clear interpretation of the relevance criterion developed in this work. However, we underline that some bases of operators are more sensitive than others, following the footprints closer and allowing to reveal the classical structures and the quantum complexity in a clearer way. As we have previously seen in Eq. [14], for the Kirkwood base all of the OTOCs are equivalent in following the linear entropy behavior. For a pure state $\rho$ with the translation operators basis, we can see that the OTOC can be expressed in terms of

$$\rho_\xi(t) = Tr(\hat{T}_\xi \hat{\rho}(t)), $$

the chord representation of the evolved density $\hat{\rho}(t)$, as

$$C_{T\xi}(t) = \rho_\xi(t)\rho_{-\xi}(t). \tag{15}$$

Meanwhile, for the reflection basis, the OTOC can be expressed in terms of the Wigner function

$$W_x(t) = (2\pi\hbar) \ Tr(\hat{R}_x \hat{\rho}(t)), $$

as

$$C_{Rx}(t) = \frac{1}{(2\pi\hbar)^2} W_x^2(t). \tag{16}$$

This makes the OTOC in the reflection basis remarkably sensitive to the classical structures in phase space, providing with a very clear link to complexity measures [19].

IV. CONCLUSIONS

Recently quantum chaos and high energy physics have become closely related through a chaoticity measure, the OTOC. An interesting bridge towards a more general interpretation as a complexity measure has been provided from the quantum information perspective via the OTOC-RE theorem which relates it to the second Renyi entropy [17, 23, 24]. In this work we have deepen on the study of this relation for a paradigmatic bipartite system covering the main kinds of dynamics, i.e. two coupled and perturbed Arnold cat maps. We have studied the behavior of three different bases of operators, namely the Pauli, translation and reflection ones.

We have defined a criterion of relevance for each operator from these bases relying on their corresponding OTOC contribution to the linear entropy $S_L$ up to time $t_0$. Armed with this tool we have found that less than 35% of the operators of these widely used bases are enough to reach the effective $S_L$ behavior. This means that to characterize the system in terms of its complexity the whole basis of operators is not needed in general but a much lower fraction instead (we underline that this is basis dependent though). The least relevant operators revealed as poor indicators of the dynamical complexity of the system. Moreover, the scaling of the number of relevant operators as a function of the time $t_0$ proved to be an alternative indicator of complexity, much in the same sense as the scaling of the number of operations is a measure for algorithmic complexity.

Finally, for the translation and reflection operators which can be directly represented in phase space our concept of relevance turns out to have an easy interpretation. The set of relevant operators follows the quantum footprints of the corresponding classical evolution (more or less closely depending on the basis). In the future we will investigate this relation even more deeply taking into account generic density operators.

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