Connecting Link Between Leptogenesis and Oscillations

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Abstract

It is shown how, in a class of models, the sign of the baryon number of the universe can be related to CP violation in neutrino oscillation experiments.

1 Introduction

In this talk I will describe mainly the content of the recent work in [1].

One of the most profound ideas is [2] that baryon number asymmetry arises in the early universe because of processes which violate CP symmetry and that terrestrial experiments on CP violation could therefore inform us of the details of such cosmological baryogenesis.

The early discussions of baryogenesis focused on the violation of baryon number and its possible relation to proton decay. In the light of present evidence for neutrino masses and oscillations it is more fruitful to associate the baryon number of the universe with violation of lepton number [3]. In the present Letter we shall show how, in one class of models, the sign of the baryon number of the universe correlates with the results of CP violation in neutrino oscillation experiments which will be performed in the foreseeable future.

Present data on atmospheric and solar neutrinos suggest that there are respective squared mass differences $\Delta_{a} \simeq 3 \times 10^{-3} eV^{2}$ and $\Delta_{s} \simeq 5 \times 10^{-5} eV^{2}$. The corresponding mixing angles $\theta_{1}$ and $\theta_{3}$ satisfy $\tan^{2}\theta_{1} \simeq 1$ and $0.6 \leq \sin^{2}2\theta_{3} \leq 0.96$ with $\sin^{2}\theta_{3} = 0.8$ as the best fit. The third mixing angle is much smaller than the other two, since the data require $\sin^{2}2\theta_{2} \leq 0.1$.

A first requirement is that our model [1] accommodate these experimental facts at low energy.
2 The Model

In the minimal standard model, neutrinos are massless. The most economical addition to the standard model which accommodates both neutrino masses and allows the violation of lepton number to underly the cosmological baryon asymmetry is two right-handed neutrinos $N_{1,2}$.

These lead to new terms in the lagrangian:

$$\mathcal{L} = \frac{1}{2}(N_1, N_2) \left( \begin{array}{cc} M_1 & 0 \\ 0 & M_2 \end{array} \right) \left( \begin{array}{c} N_1 \\ N_2 \end{array} \right) +$$

$$+ (N_1, N_2) \left( \begin{array}{ccc} a & a' & 0 \\ 0 & b & b' \end{array} \right) \left( \begin{array}{c} l_1 \\ l_2 \\ l_3 \end{array} \right) H + \text{h.c.} \quad (1)$$

where we shall denote the rectangular Dirac mass matrix by $D_{ij}$. We have assumed a texture for $D_{ij}$ in which the upper right and lower left entries vanish. The remaining parameters in our model are both necessary and sufficient to account for the data.

For the light neutrinos, the see-saw mechanism leads to the mass matrix:

$$\hat{\mathcal{L}} = D^T M^{-1} D$$

$$= \left( \begin{array}{ccc} a^2 \frac{M_1}{M_1} + (a')^2 & 0 & \frac{b}{M_2} \frac{(b')^2}{M_2} \\ a^2 \frac{M_1}{M_1} & 0 & \frac{b}{M_2} \frac{b'}{M_2} \\ \frac{b}{M_2} & \frac{b'}{M_2} & 0 \end{array} \right) \quad (2)$$

We take a basis where $a, b, b'$ are real and where $a'$ is complex $a' \equiv |a'| e^{i\delta}$. To check consistency with low-energy phenomenology we temporarily take the specific values (these will be loosened later) $b' = b$ and $a' = \sqrt{2}a$ and all parameters real. In that case:

$$\hat{\mathcal{L}} = \left( \begin{array}{ccc} (a^2) \frac{M_1}{M_1} & \frac{2a^2}{M_2} + \frac{k^2}{M_2} & 0 \\ 2a^2 \frac{M_1}{M_1} & \frac{k^2}{M_2} & 0 \\ 0 & 0 & \frac{k^2}{M_2} \end{array} \right) \quad (3)$$

We now diagonalize to the mass basis by writing:

$$\mathcal{L} = \frac{1}{2} \nu^T \hat{\mathcal{L}} \nu = \frac{1}{2} \nu^T U^T \tilde{\mathcal{L}} U \nu' \quad (4)$$

where

$$U = \left( \begin{array}{ccc} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{array} \right) \times$$

$$\times \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{array} \right) \quad (5)$$
We deduce that the mass eigenvalues and $\theta$ are given by

$$m(\nu'_3) \simeq \frac{2b^2}{M_2}; \quad m(\nu'_2) \simeq \frac{2a^2}{M_1}; \quad m(\nu'_1) = 0 \quad (6)$$

and

$$\theta \simeq \frac{m(\nu'_2)}{\sqrt{2m(\nu'_3)}} \quad (7)$$

in which it was assumed that $a^2/M_1 \ll b^2/M_2$.

By examining the relation between the three mass eigenstates and the corresponding flavor eigenstates we find that for the unitary matrix relevant to neutrino oscillations that

$$U_{e3} \simeq \frac{\sin \theta}{\sqrt{2}} \simeq \frac{m(\nu_2)}{\langle 2m(\nu_3) \rangle} \quad (8)$$

Thus the assumptions $a' = \sqrt{2}a$, $b' = b$ adequately fit the experimental data, but $a$ and $b$ could be varied around $\sqrt{2}a$ and $b$ respectively to achieve better fits.

But we may conclude that

$$2b^2/M_2 \simeq 0.05\text{eV} = \sqrt{\Delta a} \quad 2a^2/M_1 \simeq 7 \times 10^{-3}\text{eV} = \sqrt{\Delta s} \quad (9)$$

It follows from these values that $N_1$ decay satisfies the out-of-equilibrium condition for leptogenesis (the absolute requirement is $m < 10^{-2}\text{eV}$) while $N_2$ decay does not. This fact enables us to predict the sign of CP violation in neutrino oscillations without ambiguity.

### 3 Connecting Link

Let us now come to the main result. Having a model consistent with all low-energy data and with adequate texture zeros in $\hat{L}$ and equivalently $D$ we can compute the sign both of the high-energy CP violating parameter ($\xi_H$) appearing in leptogenesis and of the CP violation parameter which will be measured in low-energy $\nu$ oscillations ($\xi_L$).

We find the baryon number $B$ of the universe produced by $N_1$ decay proportional to

$$B \propto \xi_H = (ImDD^\dagger)_{12}^2 = Im(a'b)^2 = +Y^2a^2b^2 \sin 2\delta \quad (10)$$

in which $B$ is positive by observation of the universe. Here we have loosened our assumption about $a'$ to $a' = Yae^{i\delta}$. 

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At low energy the CP violation in neutrino oscillations is governed by the quantity
\[ \xi_L = Im(h_{12}h_{23}h_{31}) \] (11)
where \( h = \hat{L}\hat{L}^\dagger \).

Using Eq. (2) we find:
\[
\begin{align*}
    h_{12} &= \left( \frac{a^3 a^*}{M_1^2} + \frac{a|a'|^2 a^*}{M_2^2} \right) + \frac{aa' b^2}{M_1 M_2} \\
    h_{23} &= \left( \frac{b b' a'^2}{M_1 M_2} \right) + \left( \frac{b^3 b'}{M_2^2} + \frac{bb'}{M_1 M_2} \right) \\
    h_{31} &= \left( \frac{aa'bb'}{M_1 M_2} \right)
\end{align*}
\] (12)
from which it follows that
\[ \xi_L = -\frac{a^6 b^6}{M_1^2 M_2^2} \sin 2\delta |Y^2(2 + Y^2)| \] (13)

Here we have taken \( b = b' \) because the mixing for the atmospheric neutrinos is almost maximal.

Neutrinoless double beta decay (\( \beta\beta_0 \)) is predicted at a rate corresponding to \( \hat{L}_{ee} \approx 3 \times 10^{-3} \text{eV} \).

The comparison between Eq. (10) and Eq. (13) now gives a unique relation between the signs of \( \xi_L \) and \( \xi_H \).

As a check of this assertion we consider the equally viable alternative model
\[
D = \begin{pmatrix}
    a & 0 & a' \\
    0 & b & b'
\end{pmatrix}
\] (14)
in Eq. (1) where \( \xi_L \) reverses sign but the signs of \( \xi_H \) and \( \xi_L \) are still uniquely correlated once the \( \hat{L} \) textures arising from the \( D \) textures of Eq. (1) and Eq. (14) are distinguished by low-energy phenomenology. Note that such models have five parameters including a phase and that cases B1 and B2 in [6] can be regarded as (unphysical) limits of (1) and (14) respectively.

This fulfils in such a class of models the idea of [2] with only the small change that baryon number violation is replaced by lepton number violation.

4 Further Properties

The model of [1] has additional properties which we allude to here briefly:
1) It is important that the zeroes occurring in Eq. (1) can be associated with a global symmetry and hence are not infinitely renormalized. This can be achieved.

2) The model has four parameters in the texture of Eq. (2) and leads to a prediction of $\theta_{13}$ in terms of the other four parameters $\Delta_a, \Delta_S, \theta_{12}$, and $\theta_{23}$. The result is that $\theta_{13}$ is predicted to be non-zero with magnitude related to the smallness of $\Delta_S/\Delta_a$.

Details of these properties are currently under further investigation.

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