Bending Fatigue Life Prediction of Spur Gear in Axial Misalignment Condition

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Abstract. Misalignment between pinions to wheel gears induces bending stresses continued influences the fatigue life of the spur gear system. Lack of research projected to the issues causes uncertain in their useful lifetime estimation. In this paper, the bending fatigue life of the spur gear in axial misalignments condition was predicted. A three teeth FEM model of spur gear with the same geometrical profiles was constructed according to ISO 6336-1:2006. The gear was subjected with five misalignment from 0.1mm to 0.5mm at different torque loadings from 50Nm to 300 Nm. Exerted with quasi-static approach; the stress variant was analysed to stress life model as well as the damage accumulation model for fatigue life prediction in ANSYS V19. Results showed that the application of axial misalignment had a small impact significantly reduce the life of the gear, in percentage difference between 0.1 - 2%. This suggested that the bending fatigue life did not reflect to the existence of axial misalignments as the effect is very small that it is possible to be neglected.

1. Introduction
The design of spur gears was dictated by various system-level requirements that were established for the power transmission system. These requirements are set to achieve increased levels of durability and reliability of the gear design. Although performance, speed, torque, cost, noise level, etc., are clearly the important factors, nonetheless several other considerations would also need to be reviewed. One of the important key factors in gear design is the failure cause analysis. This analysis is critical in order to determining the possible cause of failures, and to the extent of preventing the gears design from “premature failing” during their operating life time. Polak [1] had categorized spur gear failures into 15 types, of which only one was distinguished here known as misalignments between the gear teeth.

Misalignment implies the shifting of theoretical pinion-wheel gear position to the actual happened during the gear in action [2]. It occur due to shaft deflection, manufacturing errors, assemblies or variation of the other transmission parameters [3]. The occurrence of this action typically alters the location of the active contact pinion- face width directly to the tooth flank, which in turn leads to
undistributed large stresses. Hence, it may also reduce the bending fatigue life of the gear system. Misalignment between spur gear teeth can occur in many forms and variants, but basically, it may be divided into four categories known as axial, radial, yaw and yawing pitch misalignments. While all misalignment may affected the gears, axial misalignments was observed severely damage that reduce life of the gear system.

Axial Misalignment \((A)\) was represents the deviation of the pinion offset from its nominal value. The offset happened at exactly in line of action \(LOA\) that allows a pinion to deviate in its nominal position to the wheel in global \(z\)-axis \((ZG)\). A positive \(A\) is the offset for pinion in \(-z\) direction while negative \(A\) is in \(+z\) direction (figure 1). This error is also referred to in the literature as the parallel offset misalignments [4]. The reducing the effective contact width of the pinion across the tooth face had caused high intensity of bending stress. The nature cyclic loading of the gear function, constantly effected the strength continued to reduce the bending fatigue life of the gear. In this paper, the bending fatigue life of the spur gear in axial misalignments condition was predicted. A three teeth FEM model of spur gear with five axil misalignment from 0.1mm to 0.5mm was study according to quasi-static approach.

![Figure 1. Definition of axial misalignment in this research](image)

2. Methodology
In essence, methodology use in this research is divided into three main phases. The first phase was 3D CAD modelling of gear pair engagement in aligned and misaligned conditions. A standard 3D CAD model of spur gear compliance with design guide from ISO-6336[5] was used to generate the basic sketch of gear teeth. The concept of involute curve generation was adopted. Two gears with similar profiles as table 2.0 was created. The next step requires them to be positioned in an assembly. The procedure to bring the two gears into perfect alignment and axial misalignment condition with five different value from 0.1 - 0.5 mm was describe by [6]. General properties of the gear used for this studies can be referred to table 1.
2.1 Finite Element Modelling

In order to reduce the computational time, only three teeth gear model are used in FEM Modelling. Three teeth model has been used by several researchers for gear studies [2,7]. Figure 2 showed the loads and boundary conditions applied to both gears through their respective local centres, where the local centres are coupled to the rest of each gear body (pinion and wheel) using a kinematic link inside of the gear-to-shaft hole. A kinematic link is an imaginary relatively linkage that connect between pinion-wheel in their relative motion. Reference centre nodes M1 and M2, are defined on the pinion and wheel for the circular motion from the centre hole to the pinion tooth and from the wheel tooth to the wheel centre holes through their respective rigid surfaces.

![Figure 2. Load and boundary condition for FEM Modelling](image)

Table 1 General properties of the gear used in this studies

| Parameter                     | Symbol | Unit | Pinion | Gear |
|-------------------------------|--------|------|--------|------|
| Geometrical properties        |        |      |        |      |
| Normal Module                 | m      | mm   | 5      |      |
| Normal Pressure Angle         | α₀     | degree | 20°   |      |
| Number of Teeth               | a      |       | 18     | 18   |
| Pitch Diameter                | PM     | mm   | 90     | 90   |
| Centre Distance               | C      | mm   | 90     |      |
| Face width                    | b      |      | 12     |      |
| Material properties ANSI XE1045 (113) | | | | |
| Modulus of Elasticity         | E      | GPa  | 206    |      |
| Poisson’s Ratio               | µ      |      | 0.300  |      |
| Density                       | ρ      | kg/m³ | 7830   |      |
| Material fatigue strength properties (113) | | | | |
| Ultimate Tensile Strength    | σₚₖ   | MPa  | 621    |      |
| Yield Strength                | Sy     |      | 580    |      |

Both gears’ central control points are restricted to allow only rotation about their local z-axes, where no rigid body motions are allowed. At this point, the finite element solution is ready to begin. To simulate one meshing period of the gear engagements, the magnitudes of loads (Torques) is applied constantly through the pinion side at stiffness portion A. This load is set with time steps to meet the quasi-static analysis requirements (slow motion) such as describe in [6]. The rotation is then blocked.
at B with the same stiffness value at each of the steps. All 0.6s time steps with 30° of pinion engagements position are used in this model.

2.2 Bending Fatigue Life Prediction

Every fatigue analysis requires material properties. Material properties are normally compiled from a series of experimental fatigue test results, usually in controlled laboratory environment. For this research, material fatigue properties are retrieved from experimental plot data provided from gear design book [8]. The equation that correlates the material fatigue data to the bending stress of a pinion or a mating gear tooth may be estimated by using a simplified Basquin equation, which describes the high-cycle, low-strain regime, where the nominal strains are elastic. From the data, Basquin's equation to relate stress amplitude and reversals to failure is given by the formula in [8].

\[
S_f = \sigma_f' (2N_f)^b
\]

Where \( S_f \) is HCF stress, \( \sigma_f' \) is the fatigue strength coefficient (for most metals \( \approx \sigma_f \), the true fracture strength), \( b \) is the fatigue strength exponent or Basquin’s exponent (\( \approx -0.05 \) to \(-0.12\)), and \( 2N_f \) is the number of reversals to failure.

Figure 3 Gear material fatigue properties [8]

The material fatigue data is then converted into material database files and embedded in a process flow connected through ANSYS material database. This material database is use for fatigue analysis using FATIGUE module in ANSYS workbench. By practice, selection of the gear material is dependent on many circumstances. However, due to limitation of the research only one gear material was selected. This material is considered as linear and homogeneous isotropic material. With these assumptions, a linear reaction of the fatigue life cycle prediction was used, where the stress ratio (R) fluctuating at mean stress = 0 such as figure 4 can be expected [8]. For these reason the model will focus only to the final fracture life of the pinion. It was estimated base on the Miner damage rule that cause in each cycle’s rotation of the gear. The estimation of bending fatigue life was based on the estimation life of the damage tooth in each cycles where the equation 2 applied;

\[
L=1/D
\]

Where, \( L \) is the estimated life and \( D \) is the amount of damage calculated using Miner’s rule.
3. Results and Discussion

The effect of the axial misalignment on bending fatigue life (BFL) under different loading conditions is depicted in figure 5 below. The result was compared to the aligned model condition under different torque loadings from 50Nm to 300 Nm. The percentage difference between the gears with misalignments to the gear in alignment was calculated using equation 3:

\[
\% \text{ difference} = \left| \frac{\ln(BFL_{\text{misalignment}}) - \ln(BFL_{\text{align}})}{\ln(BFL_{\text{align}})} \right| \times 100\%
\]  

(3)

where, \(BFL_{\text{misalignment}}\) is a BFL measurement for the gear in axial misalignment and \(BFL_{\text{align}}\) was the BFL measure for the gear in alignment. The symbol \(\left| \right|\) represents absolute number whereby any negative symbol is neglected. In refer to figure 5, it was observed that when the loading torque below 100Nm, axial misalignment had no influence to the BFL. This was the state where the endurance life of the gears were infinite; i.e., probability to fail is almost zero. Theoretically, this state was the main goal in gear design. The endurance limit is the highest value of the tooth root bending stress (TRBS) in a symmetrical cycle of mechanical load variation, or the maximum stress of an asymmetrical cycle, to which a material can be subjected for an unlimited number of cycles without failure [8].

However, the result changes when the gear operates at torque 150-300 Nm. At this range, the impact of axial misalignment can be seen as the BFL reduces. It was known that the fatigue life of the gears depends on material, loading and damage model used, hence, for gears in axial misalignments, two kinds of loadings affect the BFL, i.e., torque and the additional forces, which become significant from axial misalignment loading. The increasing of torque increases bending load in the nominal direction of the line of action (LOA). Hence, with the effect of axial misalignment, a tilting of the contact pressure creates a momentary bending loading that will add more bending loading. This additional loading significantly reduce the BFL of the gear, however, in small percentage difference between 0.1 - 2% only. This suggested that the BFL does not reflect to the existence of axial misalignments as the effect is very small that it is possible to be neglected.
Figure 5. Bending fatigue life (BFL) prediction for the gear in axial misalignment under different loading conditions

Figure 6. BFL prediction of the gear in five different axial misalignments, plotted with SN fatigue curve

The prediction of BFL for the gears was known based on a theory that every load cycle is damaging to the gear tooth [9]. The amount of damage depends on the actual stress levels accumulating at the tooth root, whereby anticipating fatigue life at that stress or load levels may be done by referring to material fatigue properties. Figure 6 shows BFL prediction of the gear in five different axial misalignments, plotted with SN fatigue curves. It is observed that prediction for gears in axial misalignments happens below the material fatigue stress curve. The fact that fatigue may occur before the endurance limit (red line) was reasonable if considering the gear operating condition subjecting with axial misalignments. As evidence, the momentary bending force increases when the gear is subjected to axial misalignments. These forces increase TRBS value, thus reducing the BFL of the gear. However, the amount correlate with 01-0.5mm of axial misalignment – was insignificant to justify that the axial misalignment reduces the BFL prediction of the gear.

4. Conclusions
In this paper, the BFL prediction of the spur gear in axial misalignment condition was study and discussed. It can be seen that axial misalignment have limited effects on the strength and BFL of the gear. Slight decrease/increase in axial misalignments was found influence very slight in small
percentage difference between 0.1 - 2% as such the effects on the TRBS, loading significantly reduces the BFL of the gear; Under various loads, BFL of the gear in axial misalignment also shows a small percentage difference to the aligned gears. This suggested that the bending fatigue life did not reflect to the existence of axial misalignments as the effect is very small that it is possible to be neglected.

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