Hypothesis-driven Online Video Stream Learning with Augmented Memory

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Abstract

The ability to continuously acquire new knowledge without forgetting previous tasks remains a challenging problem for computer vision systems. Standard continual learning benchmarks focus on learning from static iid images in an offline setting. Here, we examine a more challenging and realistic online continual learning problem called online stream learning. Like humans, some AI agents have to learn incrementally from a continuous temporal stream of non-repeating data. We propose a novel model, Hypotheses-driven Augmented Memory Network (HAMN), which efficiently consolidates previous knowledge using an augmented memory matrix of “hypotheses” and replays reconstructed image features to avoid catastrophic forgetting. Compared with pixel-level and generative replay approaches, the advantages of HAMN are two-fold. First, hypothesis-based knowledge consolidation avoids redundant information in the image pixel space and makes memory usage far more efficient. Second, hypotheses in the augmented memory can be re-used for learning new tasks, improving generalization and transfer learning ability. Given a lack of online incremental class learning datasets on video streams, we introduce and adapt two additional video datasets, Toybox and iLab, for online stream learning. We also evaluate our method on the CORe50 and online CIFAR100 datasets. Our method performs significantly better than all-state-of-the-art methods, while offering much more efficient memory usage. All source code and data are publicly available at https://github.com/kreimanlab/AugMem

1. Introduction

The world is not stationary. To thrive in evolving environments, humans are capable of continual acquisition and transfer of new knowledge, from a stream of highly temporally correlated visual stimuli over multiple tasks, while retaining previously learnt experiences [15, 45]. In standard incremental class continual learning problems, neural networks are presented with stationary images that are independent and identically distributed (iid), with multiple presentations of each image within a task [12, 33, 37]. Mimicking human learning experiences, here we tackle a more challenging and realistic variation of incremental class learning, named online stream learning, with two salient characteristics: (a) the input is in the form of video streams where the data are temporally highly correlated; and (b) during online learning, data are presented only once and no repeated visits over the old data are allowed within a task.

In online stream learning, AI systems tend to fail to retain good performance across previously learnt tasks [16, 21, 32]. Numerous methods for alleviating catastrophic forgetting have been proposed, the simplest being to jointly train models on both old and new tasks, which demands a large amount of resources to store previous training data and hinders learning of novel data in real time.

Inspired by memory-augmented networks in one-shot image classification [41], memory retrieval [13], and video prediction [14], we propose the Hypothesis-driven Augmented Memory Network (HAMN) for online stream learning in image classification tasks. The network learns a set of hypotheses in the augmented memory, such that the latent representation of an image can be constructed by a linear combination of the hypotheses. Multi-head content-based attention allows HAMN to share the augmented memory over multiple image features, further compressing representations in memory and allowing features to be reconstructed from hypotheses in parallel. Hypothesis-sharing in the augmented memory also enables transfer learning and generalization to new tasks. The hypotheses
learned in previous tasks are stored in the augmented memory, enabling HAMN to classify new objects.

To prevent catastrophic forgetting, we impose three constraints: (i) To preserve encoded hypotheses in augmented memory, we regularize the network to write new hypotheses into rarely-used locations in memory. (ii) We store the memory indices activated by samples from previous tasks, reconstruct these samples from the corresponding hypotheses, and replay the constructed samples during training. (iii) We apply logit matching [38] during replay to maintain the learnt class distributions from all previous tasks.

We evaluated our method under two typical online stream learning protocols (Fig. 1), incremental class iid and incremental class instance on the CORe50 dataset [30]. Because there is a lack of online video stream datasets, we introduce two additional video stream datasets for object recognition, Toybox [48] and iLab [5], by adapting them to the online stream learning setting. In addition, to evaluate the online incremental class setting over long-range tasks, we also include results on the standard online CIFAR100 dataset [24] consisting of 20 tasks. HAMN is better than state-of-the-art (SOTA) methods across all four datasets, ranging from short-range to long-range tasks, and from static images to video streams, while being more memory-efficient than other methods. HAMN demonstrates memory retention and adaptation to new tasks, even while going through continuous presentation of training examples only once.

2. Related Works

Online stream learning approaches can be categorized into (1) weight regularization, and (2) replay methods. Most weight regularization methods store weights trained on previous tasks and impose constraints on weight updates for new tasks [8, 17, 22, 25, 26, 52]. For example, Elastic Weight Consolidation (EWC) [22] stores parameters, estimates their importance for previous tasks, and penalizes changes on new tasks. Selecting “important” parameters for previous tasks complicates implementation by necessitating exhaustive hyper-parameter tuning. Besides, storing the importance of the millions of parameters required by SOTA recognition models across all previous tasks is costly [49]. Learning without Forgetting (LwF) mitigates this problem by storing only logits from previous tasks, but its performance is inferior to many replay methods. Stable SGD [34] studied the effect of dropout, learning rate decay, and batch sizes on widening the tasks’ local minima in previous tasks and minimizing catastrophic forgetting.

In replay methods, a subset of images or features from a previous task are stored or generated and later shown to the model to prevent forgetting [2, 38, 50]. Compared with weight regularization, replay methods generally lead to enhanced performance [35, 38] and reduced memory storage [8, 31]. Following prototypical contrastive learning [44], subsequent replay methods regularize network parameters with evolving prototypes and constrain their relations with inter and intra class samples [10, 53].

Raw-pixel replay, where images from previous tasks are stored and replayed, involves redundant information and is memory-inefficient. Recent works [2, 4] enhance sample selection in the replay buffer by maximizing diversity or uncertainty and approximating feasible regions in old tasks. Relying on limited sets of replay images can also lead to overfitting. The Bias Correction Method [50] applies a linear model with validation set replays from old tasks to avoid the imbalanced data distribution between old and new tasks. The adaptive aggregation network [28] introduces two types of residual blocks to balance plasticity and stability dynamically. Our method improves generalization by learning discriminative, information-rich hypotheses in a latent space, which can then be re-used to construct multiple new samples for replay and knowledge transfer to new tasks.

To limit storage, generative replay systems complement new tasks with “pseudo-data” that resemble historical data. [29, 39, 43]. Deep Generative Replay [43] is a Generative Adversarial Network that synthesizes training data over all previously learnt tasks. Generative approaches have succeeded with simple and artificial datasets but have failed with more complex inputs [3]. Later work uses bi-level optimization to generate mnemonics for replay [29]. All of these generative models work at the pixel-level. Subsequent works propose to generate examples at the feature lev-
els [27, 42, 46]. However, the generative models needed to create adequate synthetic data tend to be large and memory-intensive [49].

Previous work (REMINd) [16] showed that replay of latent features using memory indexing is efficient at preventing catastrophic forgetting. However, REMIND relies on Product Quantization [20], where a codebook for constructing the memory is pre-computed and fixed. This hinders adaptability to new tasks. Instead of fixed codebooks, our method learns a set of discrete hypotheses on the fly and replays constructed samples from the learnt hypotheses based on a memory indexing mechanism learnt through new tasks.

3. Online Stream Learning Setups

[Protocols] We consider two incremental class settings for the online stream learning protocols [16] (Fig. 1):

Class-iid: Images are randomly shuffled within each task but not interspersed among tasks, and are shown only once.

Class-instance: Each class contains short video clips of different objects. In the CORRe50 dataset, the clips are presented one after the other in random order. This introduces a strong biasing effect of seeing large numbers of images from the same class in sequence, making stream learning highly challenging. Thus, in the Toybox and iLab datasets, we provide a complementary test of stream learning capabilities, finding a mid-point of difficulty between class-iid and class-instance. On these datasets, the image order alternates on each frame between two clips from different classes, while preserving the ordering of each clip.

[Datasets] In each video dataset, we used different data/task orderings for each training run. A global holdout test set of frame sequences was used for all training runs.

The CORRe50 dataset [30] contains images of 50 objects in 10 classes. Each object has 11 instances, which are 15 second video clips of the object under particular conditions and poses. We followed [16] in the ordering, training and testing data splits.

Due to a lack of video datasets for object recognition in online stream learning, we introduce 2 extra video datasets, originally used for studying object transformations.

The Toybox dataset [48] contains videos of toy objects from 12 classes. We used a subset of the dataset containing 348 toy objects with 10 instances per object, each containing a spatial transformation of that object’s pose. We sampled each instance at 1 frame per second resulting in 15 images per instance per object. We chose 3 of the 10 instances for our test set, leaving 7 instances for training.

The iLab dataset [5] contains videos of toy vehicles. We used a subset of the dataset containing 392 vehicle objects in 14 classes, with 8 backgrounds (instances) per object and 15 images per instance. We chose 2 of the 8 instances per object for our test set.

To increase number of classes in online incremental class learning, we included the standard image dataset Online-CIFAR100 [24]. The dataset is split up into 20 tasks with 5 classes each. All training images within a task are randomly shuffled and presented only once to all models.

4. Our proposed model (HAMN)

4.1. Overview

HAMN consists of a 2D convolutional neural network (2D-CNN) coupled with an augmented memory bank (Fig. 2). The memory matrix stores a fixed number of hypotheses. HAMN extracts feature maps from an image at an intermediate layer of the 2D-CNN. The reading attention mechanism guides the heads to select a linear combination of hypotheses, with higher attention assigned to memory slots with content more similar to the feature maps. To further compress the hypotheses in the augmented memory while maintaining rich representations, we use multi-head reading attention to interact with the memory bank. The number of plausible feature maps increases exponentially with the number of reading heads and hypotheses, enabling a diverse representation space. The latter part of the 2D-CNN combines the constructed feature maps with the hypotheses in the memory for classification. To ensure the memory bank learns useful hypotheses for replay in later tasks, in addition to a classification loss, we imposed a logistic loss, commonly used in knowledge distillation [18], based on the soft class distribution predicted from the constructed and the extracted feature maps, respectively.

4.2. Feature Extraction and Classification

The 2D-CNN contains two nested functions: $F(\cdot)$ for feature extraction, with parameters $\theta_F$, consisting of the first few convolution layers; and $P_t(\cdot)$ for classification, with parameters $\theta_{P_t}$ at task $t$, consisting of the last few convolution layers. Since early convolution layers in 2D-CNNs are highly transferable [51], $\theta_F$ is trained for object recognition using ImageNet [11] and then fixed over all tasks. The parameters $\theta_{P_t}$ in $P_t(\cdot)$ depend on task $t$. After pre-training on ImageNet, we fine-tune $P_t(\cdot)$ such that HAMN learns new sets of hypotheses and decision boundaries incrementally to classify new object classes.

Up to task $t$, HAMN has seen a total of $C_t$ classes. Given an image $I_t$ shown during task $t$, we define the output tensor $Z_t$ from the feature extraction step as $Z_t = F(I_t)$. $Z_t$ is of size $S \times W \times H$ for channels, width, and height respectively. The output feature maps $Z_t$ can then be used to produce a logit vector $q_t = P_t(Z_t)$. The logit vector $q_t \in \mathbb{R}^{C_t}$ contains the activation values over all $C_t$ seen classes and is passed as input to the softmax function, which outputs the predicted probability vector $p_t \in \mathbb{R}^{C_t}$ over all $C_t$ classes.

We used $p_t(c)$ and $q_t(c)$ to denote the predicted probability and logit value for class $c$ respectively. We define
Figure 2. Schematic illustration of the proposed hypotheses-driven Augmented Memory Network (HAMN) for online stream learning tasks. HAMN model consists of a 2D-CNN and an augmented memory containing a set of hypotheses. The feature extractor in the 2D-CNN acts as a controller and decides which hypotheses to retrieve from the memory based on a content-similarity addressing mechanism. To further compress the memory and increase representation variety, multi-head reading attention is used. The re-constructed feature maps from the augmented memory (dashed line box) can be used for classification loss $L_{\text{new}}$. To ensure these hypothesis-driven feature maps are as discriminative as the output of the CNN feature extractor, a logit distillation loss $L_{\text{distill}}^{\text{new}}$ is used between the two. To avoid catastrophic forgetting, we stored the memory indices of retrieved hypotheses along with the logits from previous tasks. During replay (shaded area), we used the stored memory indices to re-construct feature maps for classification loss.

$L_{\text{classi}}(\cdot)$ as the cross-entropy loss between $p_t$ and its corresponding ground truth class label $y_c$:

$$L_{\text{classi}}(p_t, y_c) = - \sum_{i=1}^{C_t} \delta(i - y_c) \log(p_t(i))$$

where $\delta(x) = 1$ if $x = 0$ and $\delta(x) = 0$ otherwise.

4.3. Augmented Memory

We define $M_t$ as the contents of the $N \times M$ memory bank at task $t$. There are $N$ memory slots, each storing a hypothesis vector of dimension $M$. Thus, we have $M_t = [M_t(1), M_t(2), \ldots, M_t(i), \ldots, M_t(N)]$ where $M_t(i)$ is a hypothesis vector indexed by $i$.

**Multi-head content-based reading attention:** The feature extractor $F(\cdot)$ serves as a controller and outputs the tensor $Z_t$ as the reading key of image $I_t$. Based on the similarity between $Z_t$ and each hypothesis in the memory $M_t$, a content-based memory addressing mechanism draws a hypothesis from $M_t$. One could store the individual $Z_t$ of each image $I_t$ as a hypothesis in the memory bank and replay each hypothesis to prevent catastrophic forgetting, but this requires extensive memory resources. To further compress and remove redundancies in $Z_t$, we used a multi-head reading attention mechanism. For each image, the feature tensor $Z_t$ of size $S \times W \times H$ is split into groups, each of which interacts with the shared $M_t$. We partition $Z_t$ along the $S$ channels into $D$ groups with each group $d$ of size $S/D \times W \times H$. A hypothesis should represent a local concept and be location-invariant. Thus, $Z_t$ consists of multiple reading keys $z_{t,d,l}$ indexed by group $d$ and spatial location $l$ on $Z_t$ and $l \in \{1, 2, \ldots, L = W \times H\}$:

$$z_{t,d,l} = \{z_{t,1,1}, \ldots, z_{t,d,1}, \ldots, z_{t,d,L}\}, \quad z_{t,d,l} \in \mathbb{R}^{S/D}$$

(1)

For content-based addressing, each reading head compares its reading key $z_{t,d,l}$ with each hypothesis $M_t(i) \in \mathbb{R}^{S/D}$ by a similarity measure $K[\cdot, \cdot]$. To discourage attention blurring over all hypotheses, each reading head bundles with a hyperparameter of a positive constant value $\beta$. $\beta$ denotes the attention sharpening strength and it can amplify or attenuate the precision of hypothesis selection. This mechanism produces a reading attention vector $w_{t,d,l}$ over $N$ locations based on the content similarity, defined as the inner product $K[u, v] = \langle u, v \rangle$.

$$w_{t,d,l}(i) = \frac{e^{\beta K[z_{t,d,l}, M_t(i)]}}{\sum_{j} e^{\beta K[z_{t,d,l}, M_t(j)]}}$$

(2)

The overall reading attention $w_t$ for all reading heads can then be written as $w_t = \{w_{t,1,1}, \ldots, w_{t,d,1}, \ldots, w_{t,d,L}\}$.

**Construction from multiple hypotheses:** The retrieved content vector $r_{t,d,l}$ for its reading key $z_{t,d,l}$, of size $M$, is computed as the expectation of sampled hypotheses modulated by the reading attention vector $w_{t,d,l}$.
We define the final feature tensor $\tilde{Z}_t$, from a set of hypotheses in the memory bank $M_t$, as the content retrieved using each reading key $\tilde{Z}_t = \{r_{t,1,1}, ..., r_{t,d,1}, ..., r_{t,d,D,L}\}$.

To enforce that the hypothesis-driven tensor $\tilde{Z}_t$ has discriminative representations as $Z_t$, we perform classification on $\tilde{Z}_t$ using the classifier $P_t(\cdot)$ attached with softmax. In addition, we introduce the distillation loss $L_{\text{distill}}$ on the logits $q_t$ and $\tilde{q}_t$ of $Z_t$ and $\tilde{Z}_t$ respectively:

$$L_{\text{distill}}(q_t, \tilde{q}_t) = \sum_{i=1}^{C_t} q_t(i) \log(\tilde{q}_t(i))$$

$$(1 - q_t(i)) \log(1 - \tilde{q}_t(i))$$

Note that although HAMN makes class predictions based on both $Z_t$ and $\tilde{Z}_t$, we use the predictions from $Z_t$ as the final predicted result during testing.

### 4.4. Avoiding catastrophic forgetting

In the incremental class setting (see Protocols), HAMN is presented with images $I_t$ from new classes $c^{new}$ in task $t$ where $c^{new}$ belongs to the complement set of $\{1, ..., c^{old}, ..., C_t-1\}$ in $\{1, ..., c^{old}, ..., c^{new}, ..., C_t\}$. We take three measures below to avoid catastrophic forgetting.

**Rarely-used memory locations:** Every component in HAMN is differentiable, enabling hypothesis learning via gradient descent. For each new image $I_t$ in task $t$, HAMN learns a new set of hypotheses in $M_t$. The update of hypotheses could undesirably occur in recently-used memory locations from previous tasks. To emphasize accurate encoding of new hypotheses and preserve the old hypotheses in most-used memory locations learnt from the previous task $t-1$, we keep track of the top $k$ most-used memory locations for each reading attention vector and aggregate those locations over all reading heads:

$$A_{t-1} = \bigcup_{l=1}^{D} \bigcup_{D} \bigcup_{L} m(w_{t-1,d,l,1:k})$$

where $m(\cdot)$ returns the top $k$ indices with largest attention values in $w_{t-1,d,l}$. Thus, we can define the memory usage loss $L_{\text{mem}} = \delta(A_{t-1})(M_{t-1} - M_t)^2$. An indicator function $\delta(\cdot)$ returns a binary vector of size $N$ where the vector element at index $i$ is 1 if $i \in A_{t-1}$ and 0 otherwise.

**Hypothesis replay:** Replay using memory indexing is more efficient in preventing catastrophic forgetting than using pixels [16]. We store the reading attention $w_{t-1}$ from the previous task $t-1$ and re-construct the feature tensor $\tilde{Z}_{t-1}$ from $M_t$, which is replayed in the current task $t$. To further compress memory usage, we store the index with the largest attention value in $w_{t-1,d,l}$ for each reading head:

$$r_{t,d,l} = \sum_i w_{t,d,l}(i) M_t(i)$$

We can approximate $w_{t-1}$ with a one-hot vector generated using $\delta(w_{t-1}^{old})$ and construct $\tilde{Z}_{t-1}$ using $M_t$ for replay. Storing only the top-1 index in each reading head greatly reduces its memory usage from $N \times D \times W \times H$ to $D \times W \times H$.

**Distillation across tasks:** Similar to the work [38], we use a distillation loss to transfer knowledge from the same neural network between different tasks to ensure that the discriminative information learnt previously is not lost in the new task $t$. Thus, given the constructed feature tensor $\tilde{Z}_{t-1}$ and its stored logits vector $q_{t-1}^{new}$, we compute its distillation loss $L_{\text{distill}}(q_{t-1}^{new}, p_{t}(\tilde{Z}_{t-1}))$ using Eq. 4.

**Sampling strategy for replays:** Some algorithms select representative image examples to store and replay based on different scoring functions [6, 9, 23]. However, random sampling uniformly across classes yields outstanding performance in continual learning tasks [49]. Hence, we adopt a random sampling strategy and store $X$ logits and top-1 reading attention tensor pairs $(q_{t-1}^{new}, w_{t-1}^{old})$ corresponding to images $I_{t-1}$ from old classes $c^{old}$ of previous tasks. Depending on the number of seen $C_{t-1}$ classes, the storage for each old class contains $X/C_{t-1}$ pairs. To prevent catastrophic forgetting, we define the total loss:

$$L_{\text{total}} = \gamma L_{\text{mem}} + \alpha \sum_{l=1}^{I_{t-1}} (L_{\text{classi}}(p_{l}^{old}, y_{c}^{old})$$

$$+ L_{\text{distill}}(q_{t-1}^{new}, P_{t}(\tilde{Z}_{t-1})))$$

$$+ \sum_{i=1}^{I_{t-1}} L_{\text{classi}}(p_{i}^{new}, y_{c}^{new}) + L_{\text{distill}}(q_{t}, \tilde{q}_{t})$$

where $\alpha$ and $\gamma$ are regularization hyperparameters.

### 4.5. Implementation and training details

We used the SqueezeNet [19] architecture as the backbone. Following PyTorch [36] conventions for SqueezeNet,
the feature extractor $F(\cdot)$, as the controller, includes convolutional blocks up to layer 12 of SqueezeNet and the classification network $P_t(\cdot)$ includes the remaining layers. The tensor $Z_t$ after layer 12 is of size $S = 512 \times (W = 13) \times (H = 13)$. We split $Z_t$ along the $S$ axis into $D = 512/M$ groups, resulting in $13 \times 13 \times (512/M)$ reading keys of length $M$. Depending on the number of tasks to learn in each dataset, we defined $N = 100, 100, 1000, 1000$ and $M = 8, 8, 128, 128$ respectively for CORE50, iLab, Toybox, and CIFAR100. We initialized the memory bank randomly before training. We tried initializing the memory bank with normalized k-means clustered centers trained on CIFAR100 [24] but there were no improvements. In video datasets, given that each reading head has 100 hypotheses to read from, HAMN could construct 100$^{13\times13\times(512/M)}$ feature maps, creating a rich latent space. We chose $k = 1$, taking the top-1 attention index in each reading head, to compute the aggregated attention vector $A_{t-1}$ for regularizing memory using $L_{mem}$. HAMN stores $X = 200$ old samples for CORE50, iLab, and Toybox, and $X = 2000$ for CIFAR100, which has a higher number of classes. Empirically, we set the following hyperparameter values: $\beta = 5$, $\gamma = 1000$, $\alpha = 5$. Pseudocode is shown in Algorithm 1.

5. Experimental Details

5.1. Baselines

We compared HAMN with continual learning methods in several categories. To control for the effect of the network architecture on performance, we used SqueezeNet [19] pre-trained on ImageNet [11] as the backbone for the majority of the stream learning algorithms that we used as baselines, except in cases where algorithm designs caused the substitution of SqueezeNet to require extensive modifications. Unlike the other methods, HAMN introduces an augmented memory bank between intermediate layers of SqueezeNet. Due to the additional, randomly-initialized parameters introduced by the augmented memory, we trained HAMN for multiple epochs on only the first task to allow HAMN to achieve similar performance to other methods on the first task, enabling a fair comparison on subsequent tasks.

Parameter Regularization Methods: We compared against Elastic Weight Consolidation (EWC) [22], Synaptic Intelligence (SI) [52], Memory Aware Synapses (MAS) [1], Learning without Forgetting (LwF) [26], Stable SGD [34], and naive L2 regularization (L2) where the L2 distance indicating parameter changes between tasks is added to the loss [22]. Due to varying source code availability, some algorithms were re-adapted for online stream learning.

Memory Distillation and Replay Methods: We compared against Gradient Episodic Memory (GEM) [31], Averaged GEM (AGEM) [8], Incremental Classifier and Representation Learner (iCARL) [38], Bias Correction method (BIC) [50], Gradient sample selection (GSS) [2], Continual Prototype Evolution (CoPE) [10], Adaptive Aggregation Network (AAN) [28], REMIND (REM) [16], and Rainbow Memory (RM) [4].

Lower bound is trained sequentially over all tasks without any measures to avoid catastrophic forgetting.

Upper bound is trained on shuffled images from both the current task and all previous tasks over multiple epochs.

Chance predicts class labels by randomly choosing 1 out of the total of $C_t$ classes up to current task $t$.

We report Top-1 classification accuracy on the current task for all seen $C_t$ classes after 10 runs per protocol. See average top-1 accuracy over all tasks in Supp. material.

5.2. Memory Footprint Comparison

SqueezeNet has $J \approx 1.2 \times 10^6$ parameters. Weight regularization methods other than LwF and StableSGD have to store importance values for $J$ parameters for each task, causing memory requirements to grow linearly with the number of tasks. In more challenging classification tasks, the network size and memory usage tend to increase. To provide a fair comparison with the weight regularization methods, we allocated a comparable amount of memory to the replay methods for storing examples to replay in subsequent tasks. Since image replay methods require storing at least one image from each previous class, we store 15 images for all classes for all raw pixel replay methods over all 3 video stream datasets and 100 images for CIFAR100.

To illustrate memory usage, consider CORE50 in the class instance protocol. Weight regularization methods require memory of size $5J$, i.e., about 6 million parameters for the 5 tasks. The dimension of the logit vector changes over tasks (i.e., 2, 4, etc.). For simplicity, we treat any HAMN logit vector $q_{t-1}$ of constant dimension 10 for all classes in the 5 tasks. The top-1 attention index tensor $w_t^{i-1}$ is of constant size $D \times W \times H = 10,816$ parameters. We store $X = 200$ old pairs of logit and attention indices, resulting in 2 million parameters - equivalent to storing $\approx 13$ images of size $3 \times 224 \times 224$ - which is 2 images fewer than raw pixel replay methods. In CIFAR100, our memory usage is even more dramatically reduced to 9 times smaller than the replay methods. Compared with parameter regularization methods, HAMN uses only one third as much memory in CORE50 and 11 times less memory in CIFAR100.

6. Results and Discussion

6.1. Online Stream Learning Performance

An ideal online stream learning method should avoid catastrophic forgetting while adapting to new tasks. A trivial algorithm that learns the first task and stops learning thereafter could have perfect memory for Task 1, but fail to classify new objects in subsequent tasks. Fig. S2 shows the results of continual learning methods on the CORE50 dataset over 5 tasks across two stream learning protocols (See Supp.
material for results in individual tasks on the Toybox and iLab datasets). We also report the averaged top-1 classification accuracy over all learnt classes across all tasks on Toybox and iLab datasets in the summary Table 2. HAMN generally performed better than other continual learning methods on CORe50 (Fig. S2). From Table 2, HAMN outperforms next best method (L2) by 4.8% and 1.7% on average in class iid and instance scenarios, respectively, on iLab (also see Supp. Fig S1c, S2c, and S3c). HAMN also surpasses the next best baseline (iCARL) with an improvement of 6% and 3.2% on class iid and instance, respectively, on the Toybox dataset (also see Supp. Fig S1b, S2b, and S3b).

Methods generally performed similarly on task 1 after one epoch, with the upper bound performing slightly better due to training on multiple epochs. As more tasks are added, task 1 performance decreased due to forgetting (Supp. Fig S5). For example, on task 5 of CoRE50, many baseline methods were barely above chance and were comparable to the lower bound. The best baseline methods were iCARL, L2, and REMIND. HAMN achieved the highest accuracy overall (see also Supp. Fig S3a). HAMN consistently outperformed the best previous method, iCARL, across all tasks by 7% on average. Thus, given comparable memory usage with iCARL, HAMN can re-use learnt hypotheses and transfer knowledge from previous tasks more effectively. Similarly, the feature maps reconstructed from learnt hypotheses used for replay carry discriminative information. REMIND heavily relies on a pre-computed and fixed codebook for constructing the memory based on first tasks. This hinders adaptability to new tasks. REMIND underperformed when there is limited feature diversity with only 2 classes in the first task in all 3 video datasets. Its performance remained inferior even in the online CIFAR100 image dataset. Finally, HAMN was three-fold more efficient in memory usage than parameter regularization methods. HAMN’s improvement in memory usage efficiency was even more pronounced for CIFAR100 (Sec 5.2).

The class instance protocol is more challenging than class iid, as shown by the performance differences of each method between the two protocols (compare Fig. S2a and Fig. S2b). Compared with its own performance in the class iid setting, one reason HAMN is worse in the class instance setting is overfitting of the learnt hypotheses to particular tasks. One way to eliminate hypothesis overfitting is to decrease the hyperparameter controlling the attention strength $\beta$, encouraging the network to use many hypotheses drawn from different tasks simultaneously. In an ablation study described below, we show the importance of controlling the attention strength $\beta$ in the two protocols.

Figure S4 provides visualizations of the predicted logit vectors after hypothesis replays during the class instance protocol. We randomly choose 50 logit vectors from each class and plot them using t-sne [47]. In this example, Task 1 involves separating two classes: cups and scissors. As expected, the representation of those two classes is quite distinct, which leads to a high classification accuracy. In Tasks 3 and 5, the plots show the representation of the added classes. Remarkably, the two initial classes remain distinctly separated despite no training on those classes after the first task. This shows that HAMN accommodates new classes while maintaining the clustering of previous ones. We also provide visualization of the learnt augmented memory (see Supp. Sec.S2), demonstrating that HAMN learns meaningful and consistent object representations over tasks.

Moreover, we tested all models in the standard benchmark online-CIFAR100 dataset with 20 tasks. Table 2 shows that HAMN consistently outperforms other competitive models and leads the second best model by a huge margin of 11.5% (see also Supp. Fig S1d, S2d, and S3d).

### 6.2. Ablation Study

We assessed the importance of design choices by evaluating ablated versions of the model from the predictions based on $Z_t$ on CORe50 (Table S5). First, the multi-head reading mechanism reduces memory usage by compressing
We introduced two distillation losses in HAMN, during replay and training. There was no significant effect of removing the distillation loss for the class iid protocol. However, accuracy dropped by 5% in the class instance protocol.

Sixth, in MemUsageLoss, we removed the least used memory loss $L_{mem}$, which was introduced to prevent useful old hypotheses from being over-written. Removing $L_{mem}$ results in a 3% accuracy drop for both protocols, demonstrating that preventing interference between old and new hypotheses helps prevent catastrophic forgetting.

Seventh, we removed replay from the HAMN model (NoReplay). As expected, $L_{mem}$ alone is not sufficient to avoid catastrophic forgetting. The large decrease in accuracy of 15-20% for both protocols emphasizes that replay is essential and demonstrates that the learnt hypotheses capture representative information for old classes.

Lastly, rather than storing random samples in replay buffers from previous tasks, we tested the herding sampling strategy [38]. We did not observe any performance increase. It is possible that herding only selects essential components in object representations in each class and these representations are already present with the same minimal number of memory indices in HAMN. In contrast, random sampling might increase the chance of introducing more diversity to object representations per object class.

### 7. Conclusions

We addressed the problem of online stream learning for classification tasks and proposed a novel method of memory index replay using hypotheses stored in augmented mem-

| HAMN (ours) | EWC [22] | GEM [31] | AGEM [8] | rCARL [35] | L2 [22] | MAS [1] | Naive [52] | Stable [35] | GSS [2] | BC [50] | LwF [16] | REM [28] | AAN [4] | RM |
|------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| iid        | 39.1    | 39.4    | 39.2    | 40.6    | **50.1**| 44.1    | 41.7    | 39.2    | 39.2    | 34.1    | 35.5    | 32.9    | 42.9    | 44.9    | 39.5    | 37.3    | 33.3    |
| inst.      | 53.7    | 39.4    | 38.8    | 37.1    | 50.5    | 43.6    | 42.0    | 38.9    | 38.9    | 35.3    | 35.1    | 32.5    | 41.9    | 45.3    | 39.5    | 31.1    | 31.6    |
| iid        | 46.8    | 35.0    | 35.1    | 32.4    | 41.0    | **42.0**| 41.3    | 35.2    | 35.1    | 26.7    | 34.1    | 32.3    | 38.0    | 34.5    | 34.1    | 32.1    | 28.4    |
| inst.      | 44.1    | 35.7    | 34.5    | 29.2    | 42.1    | **42.5**| 40.5    | 34.5    | 34.8    | 29.8    | 34.5    | 32.0    | 37.4    | 32.1    | 34.5    | 28.9    | 33.3    |
| online CIFAR100 dataset [23] |       |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| iid        | 45.1    | 15.7    | -       | 21.1    | **33.6**| 13.6    | 18.4    | 15.2    | 15.2    | 17.9    | 12.8    | 16.3    | 18.4    | 16.6    | 16.4    | 15.4    | 12.1    |

Table 2. Averaged Top-1 classification accuracy over all tasks in class iid and instance stream learning protocols on Toybox, iLab datasets and online class incremental iid protocol on CIFAR100 dataset. See Sec. 3 for experiments and datasets, Sec. 5.1 for baselines, and Supp. Fig S3 for individual plots over all datasets. The best and second best results are bolded.

Figure 4. Clusters of class embeddings learnt in task 1 remain separated in subsequent tasks after hypothesis replays.

We provide t-sne [47] plots of logit vectors from each class after hypothesis replays within a class instance training run on the CORe50 dataset. Task 1 is a binary classification problem. Tasks 3 and 5 are 6-choose-1 and 10-choose-1 classification problems respectively. Colors correspond with the object classes in the legend. We use the shaded blobs to emphasize the clusters of cups and scissors from the 1st task and their corresponding locations in subsequent tasks.

feature maps into a minimal number of hypotheses needed for reconstruction. Instead of using SqueezeNet as a backbone, we tested MobileNet v2 [40] with fewer channels in the feature extractor. Compared with SqueezeNet (with $S = 512$ channels in the feature maps), in MobileNet v2 layer 7, the feature maps are of size $96 \times 14 \times 14$. A hypothesis vector of dim. 8 would result in $12 \times 14 \times 14$ reading keys. Assuming a fixed memory size, the fewer reading keys than SqueezeNet would reduce the reconstruction error onto the original feature maps. As expected, we observed stronger continual learning performance for MobileNet v2.

Second, we tested the importance of the number of memory slots $N$ by reducing it from 100 to 50. There was a drop in accuracy on the final task of almost 10% (class iid) and 2% (class instance). This implies that we need a sufficient number of hypotheses to discriminate between classes.

Third, we reduced the number of stored samples by half (pairs of top-1 attention memory indices and their corresponding logits) from $X = 200$ to $X = 100$. For class iid, accuracy dropped by $\approx 5\%$, indicating that we need to store multiple samples per class to represent class diversity. Reducing $X$ is not as harmful as reducing $N$, suggesting that the hypotheses in the augmented memory are highly discriminative and that there exist regularities among representations within the same class that reduce the need to store as many replay samples as in pixel-based replay. In the class instance protocol, accuracy improved by 4% in the class instance protocol, suggesting that models more easily overfit in the class instance setting.

Fourth, in MemSparseness, we decreased the reading attention strength $\beta$ (Eq. 2) from 5 to 1. A higher $\beta$ forces the model to attend to fewer memory slots. Setting $\beta$ to 1 produces a more blurred distribution over all hypotheses, causing an accuracy drop of $\approx 1\%$ in the class iid protocol. Surprisingly, the lower sparsity helped the model in the class instance protocol. One conjecture is that HAMN more easily overfits in the class instance protocol, so a blurred hypothesis distribution could help alleviate the strong preference for a single hypothesis during predictions.

Fifth, distillation loss is important for knowledge transfer between networks [18] and across sequential tasks [38].
ory. In addition to ameliorating catastrophic forgetting on benchmark datasets, our method uses memory more efficiently than other methods, while generalizing to novel concepts even when trained only once on a continuous stream.

Despite the promising results of HAMN in online stream learning, several future directions should be explored. First, to test long-range online stream learning abilities, there is a lack of large video datasets for object recognition with hundreds of classes. Second, HAMN requires a feature extractor pre-trained on ImageNet, hindering its ability of generalization to other domains (e.g., X-ray scans). For continual learning in new domains, one could first train the feature extractor via unsupervised learning in the new domain, e.g., via generative adversarial networks [7], and then use it with HAMN for online stream learning tasks in the new domain.

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S8. Appendix

List of supplementary figures

1. Top-1 classification accuracy in each task in the class iid stream learning setting on the CORe50, Toybox, iLab and CIFAR100 datasets, Fig S1
2. Top-1 classification accuracy in each task in the class instance stream learning setting on the CORe50, Toybox and iLab datasets, Fig S2
3. Average top-1 classification accuracy over all tasks in the class iid and class instance stream learning settings on the CORe50, Toybox, iLab and CIFAR100 datasets, Fig S3
4. Embedding clusters of classed learned by upper bound over 5 tasks (a) and confusion matrix between actual class labels and predicted class labels after the 5th task in class instance setting on CORe50 dataset, Fig S4
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S9. Preventing catastrophic forgetting

A proficient stream learning method should not only show good memory retention and avoid catastrophic forgetting, but should also be able to adapt to new tasks. In the main text, we described the average classification accuracy over all previous tasks. To evaluate the ability to prevent catastrophic forgetting, here we report the classification accuracy only on the first task (first two classes seen by the model) after each continual learning algorithm learns a new task. If the algorithm completely forgets what has been learnt in the first task and overfits to the current task (i.e. current 2 new classes in CORe50, iLab, and Toybox, or current 5 new classes in CIFAR100), the classification accuracy would be 0% for the older classes. For example, on CORe50, an algorithm with catastrophic forgetting would achieve high accuracy (e.g. 95%) in the current task, and 0% accuracy on each previous task due to overfitting.

Conversely, if an algorithm learns the first task perfectly (100%) but completely fails to learn additional new tasks, the algorithm would achieve 100% accuracy on all images in Task 1 after training on all subsequent tasks; but 0% of accuracy on all images from other tasks.

Figure S5a shows the average classification accuracy on the first two classes in Task 1 throughout all the tasks of CORe50. Most of the algorithms have an average classification accuracy of around 20% on Task 1 while our method achieves 76.9% in class iid and 75.1% in class instance settings. Although REMIND achieves the highest accuracy on Task 1, the average classification accuracy over all tasks is lower than that of our method across all four datasets. This indicates that REMIND does not adapt to new tasks as well as our method, possibly due to its reliance on a fixed, pre-computed codebook for encoding and decoding memories.

Overall, after comparison with all baselines in terms of both classification accuracy on all images from all tasks or only from the first task, our HAMN model not only generalizes to learn new tasks, but also demonstrates excellent memory retention capabilities. Similar conclusions can be drawn from the Toybox, iLab, and CIFAR100 datasets (Fig S3b, S3c, S3d). Our method achieves the highest average classification accuracy over all tasks on all four datasets in both class iid and class instance settings.

S10. Memory Interpretation

To interpret what the learnt hypotheses in the augmented memory represent, we provide a visualization of 2D probabilistic hypothesis activation maps for three hypotheses on an image from each class within a class instance training run on CORe50 (Fig S6). The brighter regions denote locations where the hypothesis of interest is more likely to get activated. We find consistent activation patterns over all classes. For example, some hypotheses focus on object parts while others focus on the background and hand features. We also provide a side-by-side comparison of the hypotheses from the same memory index between tasks 1 and 5. We do not observe a change of hypothesis patterns with new tasks, implying that the learnt hypotheses are not overwritten by new ones thanks to the least used memory loss $L_{\text{mem}}$. This is useful to avoid catastrophic forgetting and to assist transfer learning on new tasks. We also verified the importance of $L_{\text{mem}}$ in the ablation study.

S11. Ablation Study using $\tilde{Z}_t$

In the main paper, we assessed the importance of design choices by evaluating ablated versions of our model from the predictions based on $Z_t$ on CORe50 (Table 1). In contrast, here we provide the ablation results based on the predictions from $\tilde{Z}_t$. 

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| Accuracy (%) | class | instance |
|--------------|-------|----------|
| HAMN (SqueezeNet) | 45.0 ± 5 | 39.7 ± 6 |
| HAMN (MobileNet) | 59.6 ± 1 | 48.8 ± 3 |
| NumMemSlot | 46.7 ± 3 | 34.2 ± 2 |
| NumIndexReplays | 41.0 ± 4 | 34.5 ± 5 |
| MemSparseness | 47.2 ± 4 | 39.2 ± 3 |
| DistillationLoss | 50.6 ± 4 | 42.8 ± 5 |
| MemUsageLoss | 28.1 ± 3 | 30.2 ± 4 |
| NoReplay | 15.0 ± 1 | 15.4 ± 1 |
| Replay with Herding | 31.3 ± 3 | 33.7 ± 3 |

Table S3. **Top-1 classification accuracy in the 5th task for the ablated HAMN models after 10 runs on CORe50 in the class iid and instance protocols.** Only runs with accuracy above 80% on the first task were used in the calculations. NumMemSlot class instance required 20 initial runs to yield more than one run with task 1 accuracy above 80%. See Sec 6.2 for each ablated method. ± values denote root mean squared error (RMSE).
Figure S1. **Top-1 classification accuracy over all tasks in class iid stream learning settings on CORe50 (a), Toybox (b), iLab (c), and CIFAR100 (d) datasets.** Each color shows a different model (see main text for abbreviations and model definitions). The dashed line denotes chance level. See main paper for experimental protocols and datasets as well as baselines. The error bars denote root mean standard deviation (RMSD) over total 10 runs. Part a is the same as Fig.3a in the main text and is reproduced here for comparison purposes.
Figure S2. **Top-1 classification accuracy over all tasks in class instance stream learning settings on CORe50 (a), Toybox (b) and iLab (c) datasets.** The figure format and conventions follow Figure S1. Part a is the same as Fig.3b in the main text and is reproduced here for comparison purposes.
Figure S3. Average top-1 classification accuracy over all tasks in class iid (white) and instance stream learning (black) settings on CORe50 (a), Toybox (b), iLab (c), and CIFAR100 (d) datasets. Dashed line is chance.
Figure S4. Embedding clusters of classes learned by upper bound over 5 tasks (a) and confusion matrix between actual class labels and predicted class labels after the 5th task in class instance setting on CORe50 dataset. (a). t-sne plots of logit vectors from each class. The first task (Task = 1) is a binary classification problem. The 3rd task is a 1-choose-6 classification problem. The 5th task is a 1-choose-10 classification problem. Colors correspond with the object classes in the legend. Note that the representation of the first two classes (scissors and cups) remains distinct throughout the different tasks, showing that the model does not completely forget the initial classes learnt during Task 1. (b). Confusion matrix after the 5th task. The sequence of actual labels (top to bottom) follows the class presentation order across tasks, i.e. scissors and cup in 1st task, remote and can in the second task and so on. See the scale bar on the right for probability values. See Fig 4 in the main paper to compare with our HAMN method.
Figure S5. Average top-1 classification accuracy on Task 1 over all tasks in class iid (white) and instance (black) stream learning settings on CORe50 (a), Toybox (b), iLab (c), and CIFAR100 (d) datasets.
Figure S6. **Each hypothesis represents a concept and each learnt concept remains the same across tasks.** To visualize what activates each hypothesis in the augmented memory, given any image $I_t$, we used its top-1 attention index tensor $w_s$ of size $D \times W \times H$, selected the hypothesis index of interest, and computed its activation probability over all locations, resulting in a 2D probabilistic hypothesis activation map. We overlaid this map onto the original image, with lower activation probabilities corresponding to higher opacity (darker regions). The brighter regions denote locations where the selected hypotheses get activated with higher probability.