Numerical Study of Radiation Pattern from Intrinsic Josephson Junctions Attached to Finite Size Substrates

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Abstract. In this study, we have investigated numerically three dimensional radiation patterns from mesa-structured intrinsic Josephson junctions (IJJs) attached to finite size substrates. We have calculated electromagnetic fields inside and outside of IJJs simultaneously using three dimensional calculation model. The radiation patterns emitted by the mesa have been calculated for three different substrates. We have found that the radiation patterns reflect the existence of dual radiation source, that is, uniform part of $ac$ Josephson current and non-uniform part of $ac$ Josephson current corresponding to the cavity modes. Moreover, we have found that the radiation pattern changes dramatically with the width of the substrates. The radiation patterns are strongly affected by the diffraction at substrate edges.

1. Introduction

Recently, the radiation of coherent THz waves from high $T_c$ superconductor Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCCO) single crystal has been reported. In BSCCO samples, $ac$ Josephson currents flow under dc bias voltages between intrinsic Josephson junctions (IJJs) which consists of stacking of superconducting CuO$_2$ layers and insulating Bi-Sr-O layers. The intense radiation has been reported in both experimental and theoretical studies under the conditions that the $ac$ Josephson frequencies coincide with the frequencies of electromagnetic standing waves inside the BSCCO mesa. However, the precise nature of radiation mechanism is not fully understood.

The analysis of the radiation pattern from samples is a one of effective approaches to clarify the radiation mechanism. Thus, the radiation pattern has been investigated experimentally and theoretically. However, the theoretical studies have considered “infinite” or “two-dimensional” substrates and not investigated in detail the effect the geometry of substrates on radiation pattern. In experiment, BSCCO mesas have been attached to substrates whose sizes are comparable to the wave length of the radiation wave, hence the sizes and shapes of mesas could strongly affect the radiation patterns. In this paper, we present calculations of three dimensional radiation patterns from the mesa-structured IJJs attached to finite size substrates. The radiation patterns emitted by the mesa have been calculated for three different substrates. We have found that the radiation patterns reflect the existence of two types of radiation source. Moreover, we have found that the radiation pattern changes dramatically with the width of the substrates.
2. Model & Method

We consider a rectangular mesa sample whose IJJs stack along $z$ axis. The mesa is put on the substrate and sandwiched by the substrate and the electrode whose width is same as the mesa. Figure 1 shows the schematic figure of three dimensional calculation system. We assume the substrate and the electrode are perfect electric conductor (PEC). For the outer boundary of the calculation region, we use the perfectly matched layer absorbing boundary condition. It should be noted that the BSCCO single crystals have been mostly used as the substrates of the IJJs in the experiments. Although the BSCCO is not a usual conductor, the $ab$ plane of the BSCCO has high reflectivity like good conductor in THz region[8]. Thus, the radiation pattern calculated on the PEC substrate is similar to that on the BSCCO crystal. In this study, we focus on the radiation in the outer branch where $I$-$V$ curves are reversible and assume in-phase mode of the phase differences. In this approximation, the phase differences and electromagnetic field are uniform in the $z$ direction, and time evolution of electromagnetic fields and the phase differences in the IJJs are described by the following equations[7]

\[
\begin{align*}
\frac{\partial}{\partial t'} P' &= E'_z, \\
B'_x &= - \frac{\partial}{\partial y'} P' , \\
B'_y &= \frac{\partial}{\partial x'} P , \\
\frac{\partial}{\partial t'} E'_z &= \frac{1}{\epsilon_c} \left( \frac{\partial}{\partial x'} B'_y - \frac{\partial}{\partial y'} B'_x \right) + j_{ext} \sin P - \beta E'_z ,
\end{align*}
\]

where, $P$ is the phase difference between IJJs, $E_z$ is electric field, $B_x$, $B_y$ is oscillating part of magnetic field, $j_{ext}$ is the homogeneous external current injected into IJJs. The parameter $\beta = 4\pi \lambda_c \sigma_c / c \sqrt{\epsilon_c}$ is normalized conductivity and $\epsilon_c$ is the dielectric constant of the junctions. In this study, we take $\epsilon_c = 16$, $\beta = 0.075$. Here, we assume the electric field parallel to the IJJ plane is negligibly small ($E_x = E_y = 0$) and the external magnetic field is zero. In the above equations, we use non dimensional quantities as follows: length $x' = x / \sqrt{\epsilon_c} \lambda_c$, time $t' = \omega_p t$ where $\omega_p = c / \sqrt{\epsilon_c} \lambda_c$, and electromagnetic field $E' = (2eD / \hbar \omega_p) E$, $B' = (2eD / \hbar \omega_p) B$ where $D$ is the thickness of insulating layers of IJJs and $\lambda_c$ is the magnetic penetration depth along the IJJ plane. In the outside region of the IJJs, we solve the three dimensional Maxwell’s equation. The far field radiation patterns are calculated from the equivalent electric/magnetic current along the surface of calculation region.

Figure 1. A schematic view of three dimensional calculation model.

Figure 2. Radiation intensity - Voltage curve

3. Results

First, we calculate the radiation intensity-voltage ($R$-$V$) curves of the mesa. In this paper, we consider the mesa whose width $w$ and length $l$ are $0.48 \lambda_c$ and $0.72 \lambda_c$. The width and length of the upper electrode is the same as those of the mesa, and the thickness of the electrode is $0.02 \lambda_c$. In Fig. 2, we show the $R$-$V$ curve of this mesa. In this figure, we can see sharp peaks
at $V = 8.7V_p$ and $V = 12.7V_p$, where $V_p = \hbar \omega_p / 2e$. The ac Josephson frequency $f_J = 2eV/h$ at these voltages satisfy the cavity resonance condition $f_J = (c/\sqrt{\epsilon})\sqrt{((n/2w)^2 + (m/2l)^2}$, where $n$ and $m$ is an integer. In this calculation, $(n,m) = (0,2)$ at $V = 8.7V_p$ and $(n,m) = (2,0)$ at $12.7V_p$. Then we investigate the internal mode of the mesa at $V = 8.7V_p$ and $V = 12.7V_p$. In Figs. 3(a) and (b), we show the amplitude of oscillating part of electric field in the mesa at $V = 8.7V_p$ and $V = 12.7V_p$. These figures indicate the appearance of the standing waves corresponding to the cavity modes $(0,2)$ and $(2,0)$. It should be noted that the amplitudes of electric field at the position of the nodes have finite values. These results indicate that the uniform oscillating currents exist in the mesa in addition to the non uniform oscillating currents corresponding to the cavity modes. Such kind of internal mode have already been proposed as “dual source mode” in previous theoretical study[6].

Next, we have calculated the three dimensional radiation patterns of the mesa. Here, we focus on the radiation pattern at $V = 12.7V_p$ where the strongest emission is observed in our calculation. We consider three substrates whose widths $L_x$ are $2.88\lambda_c$, $5.76\lambda_c$ and $8.64\lambda_c$. We assume the length $L_y$ and thickness $L_z$ of these substrates are same, and we take $L_y = 5.76\lambda_c$ and $L_z = 0.2\lambda_c$. Figures 4(a) shows three dimensional plot of radiation intensity $I(\theta, \phi)$ for $L_x = 5.76\lambda_c$, and Fig. 4 (b) shows polar plot of $I(\theta, 90^\circ)$ in the $x$-$z$ plane and $I(\theta, 0^\circ)$ in the $y$-$z$ plane. The radiation intensity decreases to zero at $\theta = 0^\circ$ and becomes strong in the $x$ direction which is parallel to the wave vector of the electromagnetic standing wave. These properties agree well with the radiation pattern emitted from usual patch antennas satisfying $(2,0)$ cavity resonance condition. However, as shown in Fig 4 (b), the radiation intensity in the $y$ direction which is normal to the wave vector of the standing wave is finite, while the radiation from the patch antenna in this direction is zero. Thus, the radiation in the $y$ direction comes from the uniform oscillating mode and reflect the existence of dual radiation source in the mesa. Moreover, the intensity peaks around $\pm 60^\circ$ shown in Fig. 4 (a) and (b) has also been reported in the recent experimental study for BSCCO mesa whose substrate size is comparable to our study[9].

In Fig. 5 (a), we show the intensity $I(\theta, 90^\circ)$ vs $\theta$ for three different substrate whose $L_x$ are $2.88\lambda_c$, $5.76\lambda_c$ and $8.64\lambda_c$. The angles of two large peaks changes from $\theta = \pm 45^\circ$ to $\pm 60^\circ$. Moreover small peaks around $\theta = \pm 30^\circ$ appear for $L_x = 8.64\lambda_c$. In Fig. 5 (b), we show the
intensity $I(\theta, 0^\circ)$ vs $\theta$ for three substrates. In contrast to $I(\theta, 90^\circ)$, the angles of the peaks do not change with $L_x$.

The change of these radiation patterns with substrate size can be explained by the effect of the diffraction at substrate edges. According to the geometrical theory of diffraction [10], a total electromagnetic wave is described by the linear combination of the rays emitted from the mesa and diffracted by the substrate edges. These rays interfere with each other, and thus, the far field radiation patterns change with the width of the substrate which is related to the distances between the rays. In this study, we consider three substrates whose $L_x$ are different. The $x$ component of the distances between the rays change with $L_x$ whereas the $y$ component of those do not change. In consequence, as shown in Figs. 5 (a) and (b), the peak structures of radiation intensity in the $x$-$z$ plane $I(\theta, 90^\circ)$ drastically change, while those in the $y$-$z$ plane $I(\theta, 0^\circ)$ do not greatly change.

4. Conclusion

In this study, we have studied numerically the three dimensional radiation patterns from mesa-structured intrinsic Josephson junctions (IJJs) attached to finite size substrates. We have calculated the electromagnetic fields inside and outside of IJJs simultaneously using three dimensional calculation model. The radiation patterns emitted by the mesa have been calculated for three different substrates. These radiation patterns reflect the existence of two types of radiation source, that is, the uniform part of $ac$ Josephson current and the non-uniform part of $ac$ Josephson current corresponding to the cavity modes. Moreover, we have found that the radiation pattern changes dramatically with the width of the substrates. The radiation patterns are strongly affected by the diffraction at the substrate edges.

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