MIRROR WORLD AND AXION: RELAXING COSMOLOGICAL BOUNDS

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The cosmological (upper) limit on the Peccei-Quinn constant, related to the primordial oscillations of the axion field, can be relaxed for a mirror axion model. The simple reason is that the mirror world is colder and so the behavior of the axion temperature-dependent mass is dominated by the contribution from the mirror sector. So the coherent oscillations start earlier and correspondingly the axion mass density $\Omega_a h^2$ is reduced.

Keywords: Axion; mirror world; cosmology.

The CP violating term $L_\Theta = \Theta(\alpha_s/8\pi)G_{\mu\nu}\tilde{G}^{\mu\nu}$, in the QCD Lagrangian, leads to observable effects (e.g. non zero neutron electric dipole moment), experimentally not observed. This fact is referred to as the strong CP problem (for a general reference see Ref. 1).

In the Peccei-Quinn (PQ) mechanism, which is the most appealing solution of this problem, the parameter $\Theta$ becomes a dynamical field, the axion $a = (f_{\text{PQ}}/N)\Theta$, whose potential is minimized for the CP even configuration $\langle a \rangle = 0$. The axion emerges as the pseudo-Goldstone mode of a spontaneously broken global axial symmetry $U(1)_{\text{PQ}}$. Here $f_{\text{PQ}}$ is a constant, with dimension of energy, called the PQ constant, and $N$ stands for the color anomaly of $U(1)_{\text{PQ}}$ current. In the following, we will usually refer to the constant $f_a = f_{\text{PQ}}/N$, which characterizes the axion phenomenology. Astrophysical considerations exclude all the value of $f_a$ up to $10^{10}$ GeV\cite{118}. On the other hand, the cosmological limit, related to the primordial oscillations of the axion field, demands the upper bound $f_a < 10^{12}$ GeV\cite{119}. Let us briefly review the origin of this last limit.

In most axion models, the PQ symmetry breaking occurs when a complex scalar field, $\phi$, which carries PQ charge, acquires a vacuum expectation value (VEV) $\langle \phi \rangle \sim f_{\text{PQ}}$. This occurs as the temperature of the universe cools down below the value of the PQ temperature. As said, the axion is identified with the angular degree of freedom $a = f_a\Theta$. Today $\Theta$ is settled in the CP-conserving minimum $\Theta = 0$. 

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However, after the PQ symmetry breaking, as the temperature is of the order of the PQ scale, the axion is massless and the initial value of the phase $\Theta$ is chosen stochastically. We indicate this initial value with $\Theta_i$.

When the universe is cold enough for the QCD phase transition, instanton effects curve the potential so that the axion acquires a non zero mass $m_a(T) = m Q(T)$, where $m$ is the zero temperature limit.$^4$

$$m \simeq 0.5 \frac{f_a m_{\pi}}{f_a} \simeq 0.62 \text{ eV} \frac{10^7 \text{GeV}}{f_a}. \quad (1)$$

The function $Q(T)$ for temperature $T \gg \Lambda$ ($\Lambda \sim 200 \text{ MeV}$ is the QCD scale) was calculated in Ref. 5 and can be approximated as:

$$Q(T) = A \left( \frac{\Lambda}{T} \right)^b, \quad (2)$$

where $A = 0.1 \times 10^{0.5}$ and $b = 3.7 \pm 0.1$. This power law is valid when $T > \Lambda$ while for $T \ll \Lambda$ we have $Q = 1$ and the mass becomes independent of the temperature. We use this relation until $Q(T) < 1$, that is until $T > T_c = A^{1/b} \Lambda \simeq 100 \text{ MeV}$, and assume that $Q(T) = 1$ for $T < T_c$.

Then $\Theta$ starts to roll down, slowed by the Hubble expansion, following the equation:

$$\ddot{\Theta} + 3H(T)\dot{\Theta} + m_a^2(T)\Theta = 0, \quad (3)$$

where $H(T) = 1.66g^* T^2 / M_{Pl}$ is the Hubble expansion rate. When the curvature term dominates on the (Hubble) friction term, $\Theta$ begins to oscillate with the frequency $m_a(T_i)$. This happens at the temperature value $T = T_i$ defined by the equation $m_a(T_i) = 3H(T_i) \approx 5g_{*s_i} T^2_i / M_{Pl}$, where $M_{Pl} = 1.22 \times 10^{19} \text{ GeV}$ is the Plank mass and $g_{*s_i} = g_{*s}(T_i)$ is the effective number of the particle degrees of freedom at $T = T_i$.

For temperatures $T < T_i$, when the coherent oscillations commence, eq. 3 can be considerably simplified, and leads to the quite simple result $n_a R^3 = \text{const}$, where $n_a(T) = \rho_a(T)/m_a(T)$ is the axion number density. In other words, the number of axions in a comoving volume remains in fact constant during the universe expansion. We further assume that no entropy production takes place after the moment when the axion field begins to oscillate. Then the axion number density to the entropy density ratio, $\eta_a = n_a(T)/s(T)$, remains constant at $T < T_i$, and thus the present energy density of axions is $\rho_a^0 = \eta_a s_0 m_a$, where $s_0 = (2\pi^2/45)g_{*s} T_0^3$ is the present entropy density, $g_{*s} = 3.91$ and $T_0 = 2.725 \text{ K}$ is the CMB temperature. At the initial moment $t_i$, the axion number density is $n_a(T_i) \simeq \rho_a(T_i)/m_a(T_i) \simeq \frac{1}{2} m_a(T_i) \Theta_i^2 f_a^2$.

$^a$For the standard particle content, $g_{*s}(T) = 110.75$ for $T > 100 \text{ GeV}$, 86.25 for $100 \text{ GeV} > T > 5 \text{ GeV}$, 75.75 for $5 \text{ GeV} > T > 2 \text{ GeV}$, 61.75 for $2 \text{ GeV} > T > \Lambda$, 17.25 for $\Lambda > T > 100 \text{ MeV}$, 10.75 for $100 \text{ MeV} > T > 1 \text{ MeV}$.
Comparing to the critical density \( \rho_{cr} = 8.1 h^2 \cdot 10^{-47} \text{ GeV}^4 \), we obtain:

\[
\Omega_a h^2 = 0.77 \frac{\Theta_i^2 f_{12}}{g_{1/2}^i T_{11}},
\]

where \( T_{11} = (T_i/1 \text{ GeV}) \) and \( f_{12} = f_a/10^{12} \text{ GeV} \).

Substituting (2) in the expression \( m_a(T_i) = 3H(T_i) \approx 5g_{1/2} T_i^2 / M_{Pl} \), which defines the temperature \( T_i \), we obtain:

\[
T_i = \Lambda \left( \frac{A_{0} M_{Pl}}{5g_{1/2}^i \Lambda^2} \right)^{1/2} \approx f_{12}^{-0.175} \Lambda_{200}^{0.65} \times (0.9 \pm 0.2) \text{ GeV},
\]

where \( \Lambda_{200} = (\Lambda/200 \text{ MeV}) \) and the uncertainties of about 20 percent are related to uncertainties in parameters \( A \) and \( b \). This estimation very weakly depends on \( f_{12} \) and thus for the Peccei-Quinn scales of the cosmological relevance, starting from the astrophysical bound \( f_a \geq 10^{10} \text{ GeV} \) to \( f_a \text{ order GUT scale} 10^{15-16} \text{ GeV} \), \( T_i \) changes within interval 2 GeV – 200 MeV and hence \( g_{1/2} \approx 8 \). This estimation is valid, however, until \( T_i > T_c = A/\Lambda \approx 100 \text{ MeV} \).

Therefore, we obtain:

\[
\Omega_a h^2 \approx 0.10 \Lambda_{200}^{-0.65} \Theta_i^{2} f_{12}^{1.175}.
\]

Demanding \( \Omega_a h^2 < 0.15 \), the natural choice \( \Theta_i \sim 1 \), leads to \( f_{12} \lesssim 1.5 \).

It is worth underlining that the only upper limit on the PQ scale comes from cosmology. This limit is a few order of magnitude less than the typical GUT scale. Thus, due to cosmological considerations, it seems quite improbable to insert the axion in any GUT model. The obvious question is then: how universal is relation (6)? We will show that for a mirror axion model this can change and, consequently, the cosmological bound can be relaxed.

The idea of mirror world was introduced many years ago and has been successfully applied in different sectors of astrophysics and cosmology. The mirror world consists of another, parallel, sector of ”mirror” particles and interactions with the Lagrangian completely similar to that of the ordinary particles. In other words, it has the same gauge group and coupling constants as the ordinary world, so that the Lagrangian of the whole theory is invariant with respect to the Mirror parity (M-parity) which interchanges the two sectors.

The possibility to implement the PQ mechanism in the mirror world scenario is discussed in Ref. 8. The general feature is the following: the total Lagrangian must be of the form \( L + L' + \lambda L_{int} \), where \( L \) represents the ordinary Lagrangian,

\(^a\)For \( T < T_c \) instead we have \( m_a(T) = m, g_a(T)^{1/2} \approx 4 \) and hence \( T_i \approx f_{12}^{-1/2} \times 60 \text{ GeV} \). This condition holds for \( f_a > 3.2 \cdot 10^{17} \Lambda_{200}^{-2} \text{ GeV} \).

\(^b\)If the axion phase transition took place after inflation, then for the initial value \( \Theta_i \) one implies the rms average from a uniform distribution of initial values from \( -\pi \) to \( \pi \), \( \Theta_i = \pi / \sqrt{3} \). Thus, in this case we obtain an upper limit on \( f_{12} < 0.5 \). However, if the PQ symmetry were broken before or during inflation, then the value of \( \Theta_i \) is the same in the whole Universe within the single inflationary patch, and in fact it is an arbitrary parameter selected by some random choice.
the mirror one and $L_{\text{int}}$ is an interaction term. For $\lambda = 0$, the total Lagrangian contains two identical $U(1)_{\text{axial}}$ symmetries, while the $L_{\text{int}}$ term breaks these in just the usual $U(1)_{\text{PQ}}$. Thus only one axion field results, which is defined in both sectors.

As long as the M-parity is an exact symmetry, the particle physics is exactly the same in the two worlds, and so the strong CP problem is simultaneously solved in both sectors. In particular, the axion couples to both sectors in the same way and their non-perturbative QCD dynamics produce the same contribution to the axion effective potential. This situation does not bring drastic changes of the axion properties; just the axion zero temperature mass is increased by a factor of $\sqrt{2}$ with respect to the standard expression. On the other hand, on a cosmological ground, the physics is less trivial. Cosmologically the mirror sector cannot be identical to ours. If so, the energy density due to the extra (mirror) degrees of freedom would be much bigger than what is allowed in the standard cosmology scenario. The mirror sector must then be colder than ours by a factor $x = T'/T < 0.64$. As a consequence, the energy density of the whole universe is essentially the ordinary one, and the Hubble expansion is completely driven by our sector: $H(T) \sim 1.66g_*^{1/2}T^2/M_{\text{Pl}}$, where $g_*$ counts only the ordinary degrees of freedom.

Let us consider, now, the primordial oscillations of the axion field for a generic mirror axion model. For this case, we have $m_a = \sqrt{2}m$ in the zero temperature limit. However, at finite temperatures the behavior of the axion mass is dominated by the contribution from the mirror sector, which has a lower temperature than the ordinary one, $T' \ll T$, and thus we have $m_a(T) \approx mA(\Lambda/xT)^{b}$. Therefore, the axion mass grows faster with the temperature, and the temperature $T_i$ at which the axion field starts to oscillate scales as $x^{-0.65}$. Correspondingly, the axion mass density $\Omega_a h^2$ roughly scales as $\sqrt{2}x^{0.65}$. E.g., for $x = 0.1$, we obtain $\Omega_a h^2$ about 3 times smaller than in the estimate.

However, this situation is valid until the condition $T_i(x) > T_c(x)$, where $T_c(x)$ is defined as the temperature at which $m_a(T) = m$. This condition roughly translates in $x > 0.01f_{12}^{1/2}$. In this case we obtain:

$$T_i = \sqrt{\frac{mM_{\text{Pl}}}{5g_*^{1/2}}} \approx 123 \text{ GeV} \quad \frac{1}{g_*^{1/4}f_{12}^{1/2}},$$

and hence:

$$\Omega_a h^2 = 2.7 \cdot 10^{-3}\Theta_i^2 f_{12}^{3/2},$$

which, for $\Theta_i \sim 1$, leads to the upper limit $f_{12} < 15$.

More precisely, all the mirror degrees of freedom are equivalent to $\Delta N_\nu \simeq 6.14$ extra neutrinos, in evident contradiction with the observation of the Helium abundance predicted by Big Bang Nucleosynthesis (BBN). The value $x = 0.64$ corresponds to $\Delta N_\nu = 1$. For more details see Ref. 9. For a complete discussion on the limit on $\Delta N$ allowed by BBN, e.g. see Ref. 10 and references therein.
In conclusion, we have considered here a very simple model for mirror axion. In fact, in this, the axion properties are essentially the same as for the standard invisible axion, and only the mass is increased by a factor $\sqrt{2}$. On the other hand, we have shown that its cosmology is non trivial, because of the different temperature of the mirror sector. This leads to a weaker cosmological bound on the PQ scale. In particular it can be closer to the GUT scale.

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