Search for new light vector boson using $J/\Psi$ at BESIII and Belle II

Kayoung Ban,1 Yongsoo Jho,1 Youngjoon Kwon,1 Seong Chan Park,1 Seokhee Park,1 and Po-Yan Tseng1,

1Department of Physics and IPAP, Yonsei University, Seoul 03722, Republic of Korea

Abstract

We investigate various search strategies for light vector boson $X$ in $\mathcal{O}(10)$ MeV mass range using $J/\Psi$ associated channels at BESIII and Belle II: (i) $J/\Psi \rightarrow \eta_c X$ with $10^{10} J/\Psi$s at BESIII, (ii) $J/\Psi (\eta_c + X) + \ell \bar{\ell}$ production at Belle II, and (iii) $J/\Psi + X$ with the displaced vertex in $X \rightarrow e^+e^-$ decay are analyzed and the future sensitivities at Belle II with 50 ab$^{-1}$ luminosity are comprehensively studied. By requiring the displaced vertex to be within the beam pipe, the third method results in nearly background-free analysis, and the vector boson-electron coupling and the vector boson mass can be probed in the unprecedented range, $10^{-4} \leq |\varepsilon_e| \leq 10^{-3}$ and $9$ MeV $\leq m_X \leq 100$ MeV with 50 ab$^{-1}$ at Belle II. This covers the favored signal region of $^8\text{Be}^*$ anomaly recently reported by Atomki experiment with $m_X \simeq 17$ MeV.
I. INTRODUCTION

The Standard Model (SM) is a successful theory describing physics at least up to the electroweak scale, having survived more than 40 years of various experimental tests. However, there still remain a handful number of experimental and observational claims that indicate discrepancies from the SM predictions and consequently request extension of the SM: non-zero mass of neutrinos [1], anomalous magnetic moment of muon, \((g - 2)_μ\) [2, 3], existence of dark matter (DM) [4], and baryon vs. antibaryon asymmetry of the universe [6, 7]. There have been discussions of extending the SM by gauging the lepton number, e.g. \(L_μ - L_τ\) or \(L_e - L_τ\) [8], intending to explain DM [9, 10], the muon anomalous magnetic moment \((g - 2)_μ\) [11, 12, 13], and more recently EDGES 21cm anomaly [15]. The extension gives rise to a leptophilic light vector boson, dubbed as \(X\) in this paper. We note that the \(X\) boson may couple to the quarks via interactions with unknown heavy fermions that mix with SM quarks [16]. It may then be responsible for the recent anomaly from the KOTO experiment in \(K_L \rightarrow π^0 ν\bar{ν}\) [17, 18, 19] and also the anomaly from the Atomki experiment in \(^8\)Be*. The preferred mass of \(X\) for these cases is in sub-GeV range; in particular \(m_X \simeq 17\) MeV for Atomki [21, 22].

High luminosity lepton colliders provide ideal environments to test for such light \(X\) boson. Thanks to less severe QCD backgrounds, the lepton colliders have definite advantages over hadron colliders even when the \(X\) boson has feeble couplings with the SM particles. In this paper, we take the lepton colliders, BESIII and Belle II, and study the search strategies of \(X\). In particular, we focus on the channels in association with a \(J/Ψ\) meson, which will be enormously produced at BESIII and also at Belle II, thereby leaving the signals of \(X\) in various channels:

- At BESIII, up to now, \(10^{10}\) \(J/Ψ\) events are collected, thus provide an excellent probe to study the \(J/Ψ\) rare decays to the \(X\) boson.

- At Belle II, even though less number of \(J/Ψ\) are expected, we use the process \(e^+e^- \rightarrow \ell^+\ell^- J/Ψ \rightarrow \ell^+\ell^- η_εX \rightarrow \ell^+\ell^- η_εe^+e^- (\ell = e \text{ or } μ)\), in which \(J/Ψ\) and \(η_ε\) are inferred by the recoil masses of \(\ell^+\ell^-\) and \(\ell^+\ell^- e^+e^-\), respectively.

- At Belle II, we also use the channel \(e^+e^- \rightarrow X + J/Ψ\) where the \(X\) bosons will leave signals with displaced vertices. Due to higher center-of-mass (CM) energy at Belle II, the \(X\) boson will be boosted and travel several millimeters before it decays into \(e^+e^-\).

The rest of this paper is dedicated to studying the sensitivity reach of finding \(X\) boson taking realistic experimental situations into account under the effective field theory framework.

This paper is organized as follows: we first set up our theoretical framework and introduce the effective interactions in Section II. The analysis for BESIII is carried out in Section III.
In Section IV and V, $e^+e^- \rightarrow \ell^+\ell^-J/\Psi$ and $e^+e^- \rightarrow X + J/\Psi$ with displaced vertex signal are analyzed, respectively for Belle II. Finally, our results are summarized in Section VI.

II. EFFECTIVE LAGRANGIAN

The vectorlike interactions of the $X$ boson with the SM fermions, $f$, are introduced by the effective Lagrangian:

$$\mathcal{L} \supset -eX_\mu \sum_f \varepsilon_f \bar{f} \gamma^\mu f,$$

(1)

where we regard the couplings $\varepsilon_f$ as free parameters without knowing the origin. In particular, we will assume four universal couplings, $\varepsilon_u, \varepsilon_d, \varepsilon_e$, and $\varepsilon_\nu$, for up-type quarks, down-type quarks, charged leptons, and neutrinos, respectively, in our analysis below. We also note that the new interactions do not induce any axial anomaly by construction.

If the new boson $X$ is responsible for the recent Atomki anomaly [22] via the process $^{8}\text{Be} + X \rightarrow ^{8}\text{Be} + e^+e^-$, its mass should be $m_X \simeq 17$ MeV and the couplings with the first generation quarks should be in a particular window [23–25]:

$$|\varepsilon_u + \varepsilon_d| \simeq 3.7 \times 10^{-3}. \quad (2)$$

From the NA48/2 experiment for $\pi^0 \rightarrow X\gamma$, we require a protophobic condition [26]:

$$|2\varepsilon_u + \varepsilon_d| < 8 \times 10^{-4}. \quad (3)$$

Taking both relations in Eq. 2 and Eq. 3 into account, we finally get the preferred value for up-type and down-type quark couplings:

$$\varepsilon_u \simeq \pm3.7 \times 10^{-3}, \varepsilon_d \simeq \mp7.4 \times 10^{-3}, \quad (4)$$

which we will rely on below.

The coupling to the leptons, especially to electron and electron-neutrino, are stringently constrained by the beam dump experiment SLAC E141 [27], the anomalous magnetic moment of the electron $g - 2$ [28], and neutrino-electron scattering experiment [29]:

$$4.2 \times 10^{-4} \lesssim |\varepsilon_e| \lesssim 1.4 \times 10^{-3};$$

$$\sqrt{\varepsilon_e \varepsilon_\nu} \lesssim 7 \times 10^{-5}. \quad (6)$$

When a small coupling for neutrino $\varepsilon_\nu \ll 10^{-6}$ is assumed, we do not worry about constraints from neutrinos.
III. SIGNAL AND BACKGROUND FROM \( J/\Psi \to \eta_c e^+ e^- \)

In the SM, the decay \( J/\Psi \to \eta_c \gamma^* \to \eta_c e^+ e^- \) is radiatively allowed \[30\]. Its partial width is expressed with the form factor \( f_{VP}(0) \) for on-shell photon at the vanishing momentum transfer limit \( q^2 = 0 \) \[31\]

\[
\Gamma(J/\Psi \to \eta_c \gamma) = \frac{1}{3} \frac{\alpha_{EM}(m_{J/\Psi}^2 - m_{\eta_c}^2)^3}{8m_{J/\Psi}^3} \left| f_{VP}(0) \right|^2, \tag{7}
\]
from which the form factor \( |f_{VP}(0)| = 0.68949 \text{ GeV}^{-1} \) is determined with the fine structure constant \( \alpha_{EM} \approx 1/128 \), the masses of \( J/\Psi \) and \( \eta_c \), \( m_{J/\Psi} = 3.0969 \text{ GeV} \), and \( m_{\eta_c} = 2.9839 \text{ GeV} \), respectively, and the measured width \( \Gamma(J/\Psi \to \eta_c \gamma) = 1.5793 \text{ keV} \) \[7, 32, 34\]. The form factor \( f_{VP}(q^2) \) for general \( q^2 \neq 0 \) is obtained by \( F_{VP}(q^2) \equiv f_{VP}(q^2)/f_{VP}(0) = 1/(1 - q^2/\Lambda^2) \) from the pole approximation with pole mass \( \Lambda = m_{\eta_c}' = 3.686097 \text{ GeV} \) for \( J/\Psi \).

The normalized differential widths for partial decay widths of \( J/\Psi \to \eta_c \gamma^* \to \eta_c e^+ e^- \) and \( J/\Psi \to \eta_c X^* \to \eta_c e^+ e^- \), respectively for off-shell photon and \( X \) boson are obtained using a common factor \( F_{VP}(q^2) \) \[30\]:

\[
\frac{d\Gamma_{\eta_c \gamma^*}}{dq^2\Gamma_{J/\Psi \to \eta_c \gamma}} = |F_{VP}(q^2)|^2 \times F_{QED}(q^2)
\]
\[
\frac{d\Gamma_{\eta_c X^*}}{dq^2\Gamma_{J/\Psi \to \eta_c \gamma}} = |F_{VP}(q^2)|^2 \times F_X(q^2), \tag{8}
\]
where the kinematic window is given as \( (2m_e)^2 \leq q^2 = m_{e^+e^-}^2 \leq (m_{J/\Psi} - m_{\eta_c})^2 \). The precise expression for the factor \( F_{QED} \) is found in Ref. \[30\] where the factor is found to include the amplitude square and phase space factor for off-shell photon \[30\]. Analogous expression for \( F_X \) is obtained:

\[
F_X(q^2) = \frac{\alpha_{EM}(\varepsilon_c \cdot \varepsilon_e)^2}{3\pi} \left( \frac{q^2}{(q^2 - m_X^2)^2 + m_X^2 \Gamma_X^2} \right)
\times \left[ 1 - \frac{4m_e^2}{q^2} \right]^{1/2} \left( 1 + \frac{2m_e^2}{q^2} \right) \left[ 1 + \frac{q^2}{m_{J/\Psi}^2 - m_{\eta_c}^2} \right] - \frac{4m_{J/\Psi}^2 q^2}{(m_{J/\Psi}^2 - m_{\eta_c}^2)^2} \right]^{3/2}
\]

by replacing the couplings and propagator from \( F_{QED} \).

Assuming that \( \varepsilon_{\nu} \ll 10^{-6} \) and \( \varepsilon_e \approx 10^{-3} \), and the quark channels are kinematically forbidden with \( m_X \leq 2m_{\pi} \), the \( X \) boson dominantly decay to electrons with the width

\[
\Gamma_{X \to e^+ e^-} = \frac{\varepsilon_e^2 \alpha_{EM} m_X}{3} \left( 1 + \frac{2m_e^2}{m_X^2} \right) \sqrt{1 - \frac{4m_e^2}{m_X^2}}, \tag{10}
\]
which is narrow \( \Gamma_X \ll m_X \).

After performing the integration of \( q^2 \), we can obtain the partial decay width \( \Gamma_{\eta_c X^*} \). By inserting favoured coupling values \( \varepsilon_c = \varepsilon_u = 3.7 \times 10^{-3}, \varepsilon_e = 10^{-3} \) and fixing \( m_X = 17 \text{ MeV} \) for \(^8\text{Be}^* \) anomaly, it gives \( \Gamma_{\eta_c X^*} = 9.59 \times 10^{-3} \text{ keV} \) and implies

\[
\text{Br}(J/\Psi \to \eta_c X^* \to \eta_c e^+ e^-) = 1.64 \times 10^{-6} \left( \frac{\varepsilon_c}{10^{-2}} \right)^2,
\]
which is about three orders of magnitude smaller than that of the $\eta_c\gamma^*$ background, 
\[ \text{Br}(J/\Psi \to \eta_c\gamma^* \to \eta_c\gamma^*) = 1.03 \times 10^{-4}. \]

The $e^+e^-$ invariant-mass-squared distributions for signal ($\eta_cX^*$) and background ($\eta_c\gamma^*$) are compared in Fig. 1. The different features are clearly displayed: the signal has a peak at $q^2 = m^2_{X^*}$ and the background is broadly distributed. Therefore our task now is to efficiently extract the signal near the peak and suppress the background.

![Graph](image)

**Fig. 1.** The $e^+e^-$ invariant mass distributions of signal $J/\Psi \to \eta_cX^* \to \eta_c\gamma^*$ and background $J/\Psi \to \eta_c\gamma^* \to \eta_c\gamma^*$, where $q^2 = m^2_{e^+e^-}$. Input parameters are $m_{X^*} = 17$ MeV, $\varepsilon_c = 3.7 \times 10^{-3}$, and $\varepsilon_e = 10^{-3}$.

We first impose a kinematic condition for signal: $M_{e^+e^-} \subset [(m_{X^*} - \sigma_m), (m_{X^*} + \sigma_m)]$, where $\sigma_m$ is the $e^+e^-$ mass resolution which is roughly of the same order of magnitude as the energy resolution $\sigma_E$. The signal yield $S$ and background yield $B$ are now obtained as

\[
S = N_{J/\Psi} \times \int_{(m_{X^*} - \sigma_m)^2}^{(m_{X^*} + \sigma_m)^2} dq^2 \frac{d\Gamma_{\eta_cX^*}}{dq^2}, \\
B = N_{J/\Psi} \times \int_{(m_{X^*} - \sigma_m)^2}^{(m_{X^*} + \sigma_m)^2} dq^2 \frac{d\Gamma_{\eta_c\gamma^*}}{dq^2},
\]

where $N_{J/\Psi}$ is the total number of $J/\Psi$ produced in the collision, and $\Gamma_{J/\Psi} = 92.2$ keV is the total decay width of $J/\Psi$.

The BESIII experiment, which has collected $10^{10}$ $J/\Psi$ events in the resonance process $e^+e^- \to J/\Psi$ [30], plans to increase the size of $J/\Psi$ sample to $10^{11}$ events in the near

| Branching Ratio | Detection efficiency |
|-----------------|----------------------|
| $\text{Br}(\eta_c \to K^+K^-\pi^0)$ | $(1.15 \pm 0.12)\%$ | 18.82% |
| $\text{Br}(\eta_c \to K^0_SK^+\pi^0)$ | $(2.60 \pm 0.21)\%$ | 21.22% |
| $\text{Br}(\eta_c \to 2(\pi^+\pi^-\pi^0))$ | $(15.2 \pm 1.8)\%$ | 3.07% |
TABLE II. For $N_{J/\Psi} = 10^{10}$, and chose favoured parameters $\varepsilon_c = \varepsilon_u = 3.7 \times 10^{-5}, \varepsilon_e = 10^{-3}$, $m_X = 17$ MeV for $^8$Be* anomaly, the significances of signal to background from $J/\Psi \rightarrow \eta_e^+ e^-$ with various energy resolutions of detector and 1.23% $\eta_c$ reconstruction efficiency.

|     | $\sigma_m=1$ MeV | $\sigma_m=2$ MeV | $\sigma_m=5$ MeV | $\sigma_m=10$ MeV | $\sigma_m=15$ MeV |
|-----|------------------|------------------|------------------|------------------|------------------|
| $S$ | 188              | 263              | 277              | 277              | 277              |
| $B$ | 3686             | 7399             | 18989            | 42436            | 87640            |
| $S/\sqrt{B}$ | 3.10 | 3.06 | 2.01 | 1.34 | 0.94 |

The energy resolution of BESIII is $\sigma_E/E \simeq 0.005$ for the final-state electron, which smears the invariant mass distribution of $e^+ e^-$ by $\sigma_m \simeq 1$ MeV. In order to exclusively reconstruct the $J/\Psi \rightarrow \eta_e^+ e^-$ decays, we have to consider $\eta_c$ decay modes that can be fully reconstructed with reasonable background contamination. Table I below lists the branching fractions of a few such $\eta_c$ decay modes along with the corresponding efficiencies [35].

FIG. 2. The significance $S/\sqrt{B}$ on ($m_X, \varepsilon_c$) and ($\varepsilon_c, \varepsilon_e$) planes from $J/\Psi \rightarrow \eta_e^+ e^-$ light X vector boson searches at BESIII. Adopt $N_{J/\Psi} = 10^{10}$ (upper panels) and $N_{J/\Psi} = 10^{11}$ (bottom panels), reconstructed efficiency of $\eta_c$ from Table I and the invariant mass cut $|M_{ee} - m_X| \leq \sigma_m = 2$ MeV. The red boxes indicate the preferred regions for $^8$Be* anomaly.
FIG. 3. The current (left-panel: $N_{J/\Psi} = 10^{10}$) and future (right-panel: $N_{J/\Psi} = 10^{11}$) BESIII sensitivities by assuming the reconstruction efficiencies from Table I and taking the invariant mass cut $|M_{ee} - m_X| \leq \sigma_m = 2$ MeV. Compare with the allowed regions for $^8$Be* anomaly, NA48/2 for $\pi^0$ decay [26] and neutron-nucleus scattering [36].

The overall efficiency $\epsilon$ of the above three $\eta_c$ modes is obtained by adding the individual efficiencies weighted by their corresponding branching fractions: $\epsilon = 1.23\%$. Given these, and taking 17 MeV $X$ boson for $^8$Be* anomaly to be real, we list, in Table II, the expected significances of $J/\Psi \to \eta_c X \to \eta_c e^+e^-$ with $N_{J/\Psi} = 10^{11}$ at BESIII under the assumption of $\epsilon_c = \epsilon_u$, for various $\sigma_m$ values.

For general light vector boson searches through $J/\Psi \to \eta_c e^+e^-$, the variation of the expected significance over $(m_X, \epsilon_c, \epsilon_e)$ are shown in Fig. 2. With the present value of $N_{J/\Psi} = 10^{10}$ at BESIII, the region of sensitivity is $|\epsilon_c| \gtrsim 5 \times 10^{-3}$ at $m_X \simeq 17$ MeV as shown in the upper left panel of Fig. 2 and left panel of Fig. 3. The sensitivity slightly improves as $m_X$ increases, because of the reduction of background (see Fig. 1), and reaches the maximal sensitivity $|\epsilon_c| \gtrsim 3 \times 10^{-3}$ at $m_X \simeq 60$ MeV. But as $m_X$ approaches $m_{J/\Psi} - m_{\eta_c}$, the sensitivity becomes weaker due to the phase space suppression. The two right panels of Fig. 2 show that the significance is independent of the $\epsilon_e$ as we expect from the narrow width approximation. For $N_{J/\Psi} = 10^{11}$ which is expected in the near future, the projected sensitivity becomes $|\epsilon_c| \gtrsim 3 \times 10^{-3}$ at $m_X \simeq 17$ MeV as shown in the bottom-left panel of Fig. 2 and the right panel of Fig. 3 whereby the entire favored region of $^8$Be* anomaly can be probed.

An alternative way to explicitly reconstructing $\eta_c$ in $J/\Psi \to \eta_c e^+e^-$ at BESIII is to use the recoil of $e^+e^-$. As the $e^\pm$ carries low energy around 50 MeV, it gets difficult to distinguish $e^\pm$ tracks from $\pi^\pm$ background. With an improvement of low-energy electron identification in the future, the BESIII with $N_{J/\Psi} = 10^{11}$ can reach the sensitivity of $|\epsilon_c| \simeq 10^{-3}$.
IV. THE $e^+e^- \rightarrow \ell^+\ell^- + J/\Psi \rightarrow \ell^+\ell^-e^+e^-\eta_c$ AT BELLE II

For vector meson $J/\Psi$, the partial width to $e^+e^-$ is given by

$$\Gamma_{J/\Psi \rightarrow e^+e^-} = \frac{g_{J/\Psi ee}^2}{12\pi} m_{J/\Psi} \left( 1 + \frac{2m_e^2}{m_{J/\Psi}^2} \right) \sqrt{1 - \frac{4m_e^2}{m_{J/\Psi}^2}},$$

where $g_{J/\Psi ee} = 8.2048 \times 10^{-3}$ is the coupling strength in the effective interaction $g_{J/\Psi ee} [\bar{e}\gamma^\mu e](J/\Psi)_\mu$ that matches the measured value $\Gamma_{J/\Psi \rightarrow e^+e^-} = 5.53$ keV.

Then the cross sections to $\ell^+\ell^- J/\Psi$ where $\ell = e$, or $\mu$ at Belle II are obtained via $e^+e^- \rightarrow \gamma^* J/\Psi$:

$$\sigma(e^+e^- \rightarrow \gamma^* + J/\Psi \rightarrow e^+e^- J/\Psi) = 286 \text{ fb},$$

$$\sigma(e^+e^- \rightarrow \gamma^* + J/\Psi \rightarrow \mu^+\mu^- J/\Psi) = 58.4 \text{ fb}.$$ (13)

With the design integrated luminosity $L = 50 \text{ ab}^{-1}$, we estimate $N_{J/\Psi} = 1.75 \times 10^7$ events for $e^+e^- \rightarrow \gamma^* + J/\Psi \rightarrow \ell^+\ell^- J/\Psi$ at Belle II. This $N_{J/\Psi}$ is applied to Eq. (11) along with Eq. (12) to give estimates of $S$ and $B$:

$$S = L \times \sigma(e^+e^- \rightarrow \ell^+\ell^- J/\Psi) \times \text{Br}(J/\Psi \rightarrow \eta_c X^* \rightarrow \eta_c e^+e^-) \simeq 28.2 \left( \frac{\varepsilon_c}{10^{-2}} \right)^2,$$

$$B = L \times \sigma(e^+e^- \rightarrow \ell^+\ell^- J/\Psi) \times \text{Br}(J/\Psi \rightarrow \eta_c \gamma^* \rightarrow \eta_c e^+e^-) \simeq 1772.$$ (14)

Therefore, the estimated $S/\sqrt{B}$ is too small at this level so that we will improve the analysis by a more realistic MC study below.

For event generation, we use MG5\textsubscript{aMC}@NLO \cite{37} for both signal and background with 

\texttt{FeynRules v2.0} \cite{38} model for $J/\Psi$, $\eta_c$ mesons and $X$ boson couplings and $X$ couples to the leptons. We generate with the $E_{\text{beam}1,2} = 5.2941$ GeV in the CM frame, which is boosted by $\beta = 0.2732$ with respect to the lab frame. The amplitude of the electromagnetic Dalitz decay, $V \rightarrow Pe^+e^-$ can be written in a Lorentz-invariant form \cite{39},

$$T(V \rightarrow Pe^+e^-) = 4\pi\alpha_{\text{EM}} f_V p e^{\mu\nu\rho\sigma} p_\mu q_\nu \epsilon_\rho \frac{1}{q^2} \bar{u}_1 \gamma_\sigma \gamma_2.$$ (15)
FIG. 5. The $e^+e^−$ invariant mass (left) and $e^+e^−\ell^+\ell^−$ recoil mass (right) distributions for the parton level Monte-Carlo simulation data with the smearing effect. Input parameter is $m_X = 17$ MeV. Here, we normalized to $10^5$ events for each channel.

and we can obtain the interaction Lagrangian as,

$$\mathcal{L} \supset f_V \{ -2\sqrt{\alpha_{\text{EM}}} \partial_\mu P \partial_\nu V_\rho \epsilon^{\mu\nu\rho\sigma} A_\sigma - g_{Xe} \partial_\mu P \partial_\nu V_\rho \epsilon^{\mu\nu\rho\sigma} X_\sigma - 2g e V_\mu \bar{e} \gamma^\mu e - 2g_{Xc} \partial_\mu P \partial_\nu V_\rho \epsilon^{\mu\nu\rho\sigma} X_\sigma \}$$

where $g_{Xc}$, $g_{Xe}$, and $g_{eV}$ are the effective coupling constants, whose numerical values are to be obtained by experiments. In the left panel of Fig. 5, the invariant mass distributions of $e^+e^−$ are plotted, where samples of an equal number of events ($= 10^5$) for each of the signal and background at the parton level have been used before any selection cuts. We give the Gaussian smearing effect with the momentum resolution $\sigma_{p_{\ell\pm}}/p_{\ell\pm} = 0.005$ on the parton level data for our event analysis.

To simulate the effects of the Belle II detector, we apply the following baseline cuts: $|\eta_\gamma^\ast| \leq 1.60$ in the CM frame $^1$ $^3^9$, $|E_{\mu\pm}| \geq 0.6$ GeV, and $|E_{e\pm}| \geq 0.06$ GeV $^2$ in the lab frame $^40$. In addition, we apply kinematic requirements: $|M_{ee} - m_X| \leq 2$ MeV, whereby the dominant background from the process shown in the right panel of Fig. 4 are suppressed. We also require $|M^{\text{rec}}_{e\ell\ell} - m_{\eta_c}| \leq 200$ MeV to eliminate the sub-dominant background $\ell^+\ell^-e^+e^−(\gamma)$. In Table III, we summarize the cumulative effects of the baseline cuts and the kinematic requirements.

| Cuts | B | S |
|------|---|---|
| Processes | $\eta_c\gamma^\ast \rightarrow \eta_cee$ | $\eta_cX \rightarrow \eta_cee$ |
| Baseline Cuts | 100000 | 100000 |
| $|M^{\text{rec}}_{e\ell\ell} - m_{\eta_c}| \leq 200$ MeV | 7170 | 6290 |
| $|M_{ee} - m_X| \leq 2$ MeV | 7071 | 6219 |
| $|M^{\text{rec}}_{e\ell\ell} - m_{\eta_c}| \leq 200$ MeV | 377 | 5880 |

$^2$ Here, we choose unusually small electron energy cut to keep most of the signal, because the $e^+e^−$ from $J/\Psi \rightarrow \eta_c e^+e^−$ are very soft. However, the large background from $\pi^\pm$ might be difficult to be distinguished from our signal.
For the energy-momentum 4-vector of $\eta_c$, we use the energy and momentum recoiling against $e^+e^-\ell^+\ell^-$. The signal and background distributions of the recoil mass $M^{\text{rec}}_{\ell\ell}$, smeared by the charged-track momentum resolution, are displayed in the right panel of Fig. [5]. The signal events clearly show a peak at $M^{\text{rec}}_{\ell\ell} \simeq m_{\eta_c}$, while the background is mostly populated at $M^{\text{rec}}_{\ell\ell} \simeq 0$.

The sensitivities of $\ell^+\ell^-e^+e^-$ search at Belle II are derived from the requirement of $S/\sqrt{B} = 2$. Combining Eq. (14) and Table III, we obtain the corresponding values of $\varepsilon_c$ with respect to luminosities of 50, 100, and 200 ab$^{-1}$ in Table IV. They are about 5 times larger than the estimates from current BESIII sensitivity $|\varepsilon_c| \gtrsim 5 \times 10^{-3}$ in Section III.

| Luminosity | $50 \text{ ab}^{-1}$ | $100 \text{ ab}^{-1}$ | $200 \text{ ab}^{-1}$ |
|------------|-----------------------|-----------------------|-----------------------|
| $|\varepsilon_c|$ | $\gtrsim 1.76 \times 10^{-2}$ | $\gtrsim 1.48 \times 10^{-2}$ | $\gtrsim 1.24 \times 10^{-2}$ |

V. THE $e^+e^- \rightarrow X + J/\Psi \rightarrow e^+e^- + J/\Psi$ DISPLACED VERTEX AT BELLE II

The $X$ boson produced in the process $e^+e^- \rightarrow X + J/\Psi$ travels several millimeters before decaying into $e^+e^-$ in the Belle II detector and leaves displaced vertex. In particular, when the distance of the flight is between 2 mm and 8 mm, which is inside the beam pipe, and outside the interaction region, Belle II has excellent power to reconstruct the displaced vertex.

FIG. 6. (Left Panel) The Feynman diagram of the $e^+e^- \rightarrow X + J/\Psi \rightarrow e^+e^-\ell^+\ell^-$ signal (Right Panel) The schematic picture of the decay process of the long-lived $X$ boson in the Belle II detector.
FIG. 7. Left-panel: The transverse flight distance $d_{xy}$ of 17 MeV $X$ boson for $\varepsilon_e$ in $0.5 \times 10^{-4}$ steps from $3 \times 10^{-4}$ to $8 \times 10^{-4}$. Right-panel: The events pass the baseline cuts and satisfy $2 \text{ mm} \leq d_{xy} \leq 8 \text{ mm}$ from total generated event $10^5$.

and makes the signal almost free from the SM background [41]. Therefore, we propose to use the clean displaced $e^+e^-$ vertex from the $X$ boson decay (with prompt $\ell^+\ell^-$ from $J/\Psi$).

The leading-order Feynman diagram of the relevant process is shown in the left panel of Fig. 6. A typical event with displaced vertex at Belle II detector with the $\ell^+\ell^-$ from $J/\Psi$ decay is schematically shown in the right panel of Fig. 6.

We note that compared with other lighter vector mesons, the heavier mass of $J/\Psi$ helps to induce larger scattering angle in such a way that more events from $X \rightarrow e^+e^-$ will satisfy the cut $|\eta^\pm| \leq 1.60$. Furthermore, the electron and positron from $X \rightarrow e^+e^-$ carry energy above GeV, which make them easier to be distinguished from charged-pion backgrounds.

The other advantage of this channel is that the signal strength only depends on the $\varepsilon_e$ coupling since only $X-e^+e^-$ vertices are involved at tree level. For $0.3 \times 10^{-3} \leq \varepsilon_e \leq 0.8 \times 10^{-3}$, it yields a few mm transverse flight distance $d_{xy}$ which is defined as the distance between the beam axis and the $X$ decay vertex. The left-panel of Fig. 7 shows the distribution of $d_{xy}$ corresponding to several values of $\varepsilon_e$.

TABLE V. The percentage pass baseline cuts mentioned in Section IV and the flight distance $2 \text{ mm} \leq d_{xy} \leq 8 \text{ mm}$ cuts for several values of $\varepsilon_e$ of 17 MeV $X$ boson. $N_S$ is the number of signal events from the $e^+e^-\ell^+\ell^-$ channel at Belle II with 50 ab$^{-1}$ luminosity, and $S_{B=1}$ ($S_{B=0.1}$) is the expected significance assuming 1 (0.1) event of background in the analysis channel after all cuts.

| $\varepsilon_e/10^{-4}$ | 8.0 | 7.0 | 6.0 | 5.0 | 4.5 | 4.0 | 3.0 |
|------------------------|-----|-----|-----|-----|-----|-----|-----|
| Baseline Cuts(%)       | 13.8| 13.8| 13.8| 13.8| 13.8| 13.8| 13.8|
| 2mm < $d_{xy}$ < 8mm (%) | 1.5 | 4.7 | 7.4 | 10.1| 11.0| 10.1| 5.2 |
| $N_S$                  | 1.60| 3.85| 4.42| 4.18| 3.69| 2.69| 0.78|
| $S_{B=0.1}$           | 2.4σ| 4.6σ| 5.0σ| 4.8σ| 4.5σ| 3.6σ| 1.5σ|
| $S_{B=1}$             | 1.6σ| 2.9σ| 3.2σ| 3.1σ| 2.8σ| 2.3σ| 1.2σ|
TABLE VI. The same as Table V but using the $e^+e^-$ channel

| $\varepsilon_e/10^{-4}$ | 8.0 | 7.0 | 5.0 | 4.0 | 3.0 | 2.0 | 1.0 |
|-------------------------|-----|-----|-----|-----|-----|-----|-----|
| Baseline Cuts(%)        | 17.6| 17.6| 17.6| 17.6| 17.6| 17.6| 17.6|
| 2mm $< d_{xy} < 8$ mm(%)| 1.6 | 5.3 | 12.3| 12.9| 7.4 | 2.3 | 0.5 |
| $N_S$                   | 14.6| 35.7| 42.7| 28.7| 9.23| 1.28| 0.07|
| $S_{B=0.1}$             | > 5$\sigma$ | 2.2$\sigma$ | 0.4$\sigma$ |
| $S_{B=1}$               | > 5$\sigma$ | 1.6$\sigma$ | 0.9$\sigma$ |

With the baseline cuts and 2 mm $\leq d_{xy} \leq$ 8 mm, we estimate the signal sensitivity by considering two cases: (i) explicitly reconstructing $J/\Psi \rightarrow \ell^+\ell^-$ (‘$e^+e^-\ell^+\ell^-$ channel’), and (ii) using the recoil mass of $X \rightarrow e^+e^-$ to infer $J/\Psi$ (‘$e^+e^-$ channel’). Tables V and VI show, respectively for the $e^+e^-\ell^+\ell^-$ and $e^+e^-$ channels, the signal efficiencies and expected significances for various assumed values of $\varepsilon_e$, according to the 50 ab$^{-1}$ luminosity at Belle II and the $e^+e^-\rightarrow X+J/\Psi$ cross section

$$\sigma(e^+e^- \rightarrow X + J/\Psi) = 2.77 \times 10^{-2} \times \left(\frac{\varepsilon_e}{10^{-3}}\right)^2 \text{ fb}. \quad (17)$$

The final result of expected sensitivity with Belle II at 50 ab$^{-1}$ luminosity is shown in Fig. 8. Also displayed in Fig. 8 are the expected results with the $\eta_c$-related studies at Belle II and BESIII as discussed in Sections III and IV. The displaced $e^+e^-$ vertex searches can probe the 17 MeV $X$ boson in the region

$$2.5 \times 10^{-4} \leq \varepsilon_e \leq 8.0 \times 10^{-4}$$

with significance larger than 2 by assuming near-zero background, and it covers the $\varepsilon_e$ region preferred by Atomki. While we expect less than one signal event with the currently available Belle data sample of 1 ab$^{-1}$, we can start exploring the Atomki preferred region within a few years once Belle II accumulates data sample of 10 ab$^{-1}$ or more.

Our study with the displaced vertex is extended for wider mass range of $X$-like boson, whereby we determine the region of sensitivity with Belle II at 50 ab$^{-1}$, as displayed in Fig. 8. Two cases for the SM background are considered: $B = 0.1$ and $B = 1$, where $B$ is the number of background events expected. We then use the Poisson probability of the expected background to fluctuate upward, to calculate the $p$-values and the corresponding significances. The 2$\sigma$ significance region with $B = 0.1$ ($B = 1$) is shown by the green (dark-green) contour in Fig. 8. The dark (light)-green-shaded area corresponds to the region with $S_{B=1}$ ($S_{B=0.1}$) $> 2\sigma$, where $S_{B=1}$ ($S_{B=0.1}$) is the expected significance with the background level of 1 (0.1) event. This study can probe the parameter region of $5 \text{ MeV} \leq m_X \leq 100 \text{ MeV}$ and $1.0 \times 10^{-4} \leq \varepsilon_e \leq 3 \times 10^{-3}$, which have not been constrained by any existing experiments. The upper edge of this region is determined by the condition $2 \text{ mm} \leq d_{xy}$, while the lower edge is limited by the statistics. Therefore, with even higher luminosity of Belle II exceeding the target 50 ab$^{-1}$, the lower edge of the sensitivity region can be extended further.
FIG. 8. The green (dark-green) contour corresponds $\geq 2\sigma$ significance assuming SM background $B = 0.1$ ($B = 1$) from $e^+e^-$ channel at Belle II with 50 $ab^{-1}$ luminosity, which probes rest of the favour parameter region of Atomki (red vertical band). The gray-shaded regions are excluded by current experiments relate to $X_{ee}$ coupling. Compare to the sensitivities of $e^+e^- \rightarrow \ell^+\ell^- + J/\Psi \rightarrow \ell^+\ell^- e^+e^- \eta_c$ channel at Belle II and $e^+e^- \rightarrow J/\Psi \rightarrow e^+e^- \eta_c$ channel at BESIII by assuming the kinematic mixing between $X$ boson and photon, bringing the relation $\varepsilon_c = \frac{2}{3}\varepsilon_e$. For the future Belle II experiments 50, 200 $ab^{-1}$ in the 17 MeV mass region, the significance bounds are denoted by magenta, orange dotted lines, respectively. In the case of BESIII for the energy resolutions $\sigma_E = 1$ MeV and 5 MeV, corresponding to section III they are denoted in purple and blue, respectively.

VI. SUMMARY AND CONCLUSION

In summary, we propose several studies using $J/\Psi$ at lepton colliders such as Belle II and BESIII, to search for light vector boson around the mass range suggested by the $^8$Be$^*$ anomaly of the ATOMKI experiment. At BESIII, the $J/\Psi \rightarrow \eta_c X \rightarrow \eta_c e^+e^-$ channel can be used to constrain the vector boson and charm quark coupling, $\varepsilon_c$. With the currently available sample of $N_{J/\Psi} = 10^{10}$ and effective $\eta_c$ reconstruction efficiency of 1.23%, we can exclude the region $|\varepsilon_c| \gtrsim 5 \times 10^{-3}$ for $m_X = 17$ MeV. If $N_{J/\Psi} = 10^{11}$ is produced at BESIII in the near future, exclusion of the region $|\varepsilon_c| \simeq 3 \times 10^{-3}$ might be achieved. If universal coupling to up-type quarks is assumed, we expect that the entire favored signal region from the $^8$Be$^*$ anomaly could then be covered.

On the other hand at Belle II, with higher CM energy of 10.59 GeV, we propose to study the process $e^+e^- \rightarrow \ell^+\ell^- J/\Psi$ followed by $J/\Psi \rightarrow \eta_c X \rightarrow \eta_c e^+e^-$. Using the recoil mass against $\ell^+\ell^- e^+e^-$, we perform MC study and find that the expected production of $J/\Psi$...
events is about three orders of magnitude smaller than that of BESIII, thus yielding the sensitivity of $|\varepsilon_e| \gtrsim 1.8 \times 10^{-2}$ at $m_X = 17$ MeV.

Alternatively, we can study the process $e^+e^- \rightarrow X + J/\Psi \rightarrow e^+e^-\ell^+\ell^-$ at Belle II and directly constrain the $X$ boson-electron coupling. The $X$ boson is boosted by the higher CM energy and heavy mass of $J/\Psi$, producing displaced vertex of $X \rightarrow e^+e^-$ which is longer than several millimeters. Particularly, it is almost background free when the transverse flight distance is $2 \text{ mm} \leq d_{xy} \leq 8 \text{ mm}$. Selecting this window and requiring $> 2\sigma$ significance, it gives the sensitivity $2.0 \times 10^{-4} \leq |\varepsilon_e| \leq 8.0 \times 10^{-4}$ at $m_X = 17$ MeV for 50 ab$^{-1}$ luminosity and covers most of the favored signal region from the claimed $^8\text{Be}^*$ anomaly. Extending the range of the $X$ boson mass, this method can cover the unprecedented parameter space of $9 \text{ MeV} \leq m_X \leq 100 \text{ MeV}$ and $1.0 \times 10^{-4} \leq |\varepsilon_e| \leq 10^{-3}$.

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