Hadronic Axion Model in Gauge-Mediated Supersymmetry Breaking and Cosmology of Saxion

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Abstract

Recently we have proposed a simple hadronic axion model within gauge-mediated supersymmetry breaking. In this paper we discuss various cosmological consequences of the model in great detail. A particular attention is paid to a saxion, a scalar partner of an axion, which is produced as a coherent oscillation in the early universe. We show that our model is cosmologically viable, if the reheating temperature of inflation is sufficiently low. We also discuss the late decay of the saxion which gives a preferable power spectrum of the density fluctuation in the standard cold dark matter model when compared with the observation.
I. INTRODUCTION

The most attractive candidate for the solution of the strong CP problem is the Peccei-Quinn (PQ) mechanism [1]. In this mechanism, there exists an axion which is the Nambu-Goldstone (NG) boson associated with the global $U(1)_{PQ}$ PQ symmetry breaking.

In the framework of gauge mediated supersymmetry (SUSY) breaking theories [2], we have proposed an interesting possibility [3] to dynamically generate the PQ symmetry breaking scale in a so-called hadronic axion model [4]. A gauge singlet PQ multiplet $X$ and colored PQ quark multiplets $Q_P$ and $\overline{Q}_P$ are introduced with the superpotential

$$W = \lambda_P X Q_P \overline{Q}_P,$$

(1)

where $\lambda_P$ is a coupling constant. Here we stress that no mass parameter is introduced. Because in a supersymmetric limit the $U(1)_{PQ}$ symmetry is enhanced to its complex extension, there appears a flat direction $Q_P = \overline{Q}_P = 0$ with $X$ undermined in the same limit. The effects of the SUSY breaking, however, lift the flat direction and fix the vacuum expectation value (vev) of the scalar PQ field $X$. In Ref. [3] we have shown that the balance of the SUSY breaking effects between the gravity mediation and the gauge mediation stabilizes $X$ and gives the nonzero vev $\langle X \rangle = f_{PQ}$ approximately as

$$f_{PQ} \approx \frac{f^2}{m_{\sigma}},$$

(2)

where $m_{\sigma}$ denotes the mass of a saxion field, the real part of the scalar component of $X$. We estimate $m_{\sigma} = \xi m_{3/2}$ with a parameter $\xi$ of order unity. And $f$ is a mass scale fixed by the messenger sector of the gauge mediation mechanism. A current scalar lepton mass limit suggests that $f \gtrsim 10^4$ GeV. In the following analysis, we take the minimum value $f = 10^4$ GeV for a representative value.

\footnote{f is roughly represented as $f^2 \sim \left( \frac{\alpha_s}{\pi} \right) \langle F \rangle$, where $\langle F \rangle$ denotes the F-component vev of the messenger singlet field and $\alpha_s$ is the QCD coupling.}
The axion arises when the $X$ field develops the non-vanishing vev. The decay constant of the axion (the PQ scale) $f_{PQ}$ is constrained by various astrophysical and cosmological considerations. In particular the cooling of the SN 1987A puts a lower bound as $f_{PQ} \gtrsim 10^9$ GeV [3]. On the other hand, the upper bound on $f_{PQ}$ comes from the overclosure limit of the axion abundance. The relic abundance of the axion due to the vacuum misalignment is given by [6,7]

$$\Omega_a h^2 \sim 0.2\theta^2 \left( \frac{f_{PQ}}{10^{12} \text{ GeV}} \right)^{1.18},$$

with $\theta$ being an initial misalignment angle ($|\theta| < \pi$) and $h$ the present Hubble constant in units of 100 km/sec/Mpc. Thus this leads to $f_{PQ} \lesssim 10^{12}$ GeV. Here it has been assumed that no entropy is produced after the QCD phase transition. If extra entropy production mechanism with a reheat temperature $T_R$ is operated after the transition, the estimate of the relic abundance changes as [8,9]

$$\Omega_a h^2 \sim 40\theta^2 \left( \frac{T_R}{10^2 \text{ MeV}} \right) \left( \frac{f_{PQ}}{10^{16} \text{ GeV}} \right)^2.$$

Then the upper bound on the PQ scale becomes $f_{PQ} \lesssim 10^{16}$ GeV for $T_R \simeq 10$ MeV. Note that the big bang nucleosynthesis implies that the reheating temperature should be higher than about 10 MeV.

It follows from Eq. (2) that, in the model we are considering, the allowed region for the PQ scale discussed above

$$10^9 \text{ GeV} \lesssim f_{PQ} \lesssim 10^{12}(10^{16}) \text{ GeV}$$

(5)

can be translated to the allowed region for the saxion mass

$$100 \text{ MeV} \gtrsim m_\sigma \simeq m_{3/2} \gtrsim 100(10^{-2}) \text{ keV}.$$ (6)

Remarkably many models of the GMSB predict a gravitino mass in the above range. Thus the model gives a very simple description of the PQ breaking mechanism solely governed by the physics of the SUSY breaking.
In the model, however, the saxion has a very light mass which is comparable to the gravitino mass and its lifetime may be long. Furthermore the saxion might be produced as a coherent oscillation too much. Therefore, in this paper we will consider the cosmology of the model, especially the saxion cosmology, in detail.

The paper is organized as follows. In the subsequent section, we will discuss cosmological evolution of the saxion and estimate its relic abundance. In Section II, we will consider cosmological constraints on the saxion and show that our model survives the constraints if the reheating temperature of the inflation is sufficiently low. Then in Section IV, we will point out the possibility that the saxion plays a role of a late decaying particle, giving a better fit of the density perturbation in the cold dark matter scenario. Final section is devoted to conclusions.

II. COSMOLOGICAL EVOLUTION OF SAXION

Let us begin by discussing the cosmological evolution of the saxion particle in an inflationary universe, and estimate its relic abundance. Since the saxion has interaction suppressed by the energy scale $f_{PQ}$, it decouples from the thermal bath of the universe when the cosmic temperature becomes about \[ T_{\text{dec}} \sim 10^9 \text{ GeV} \left( \frac{f_{PQ}}{10^{11} \text{ GeV}} \right)^2 \sim 10^9 \text{ GeV} \left( \frac{f}{10^4 \text{ GeV}} \right)^4 \left( \frac{1 \text{ MeV}}{m_{\sigma}} \right)^2. \] (7)

If the temperature achieved after the primordial inflation was higher than $T_{\text{dec}}$, the saxion could be thermalized and the ratio of the energy density of the saxion $\rho_\sigma$ to the entropy density $s$ is estimated as

$$\frac{\rho_\sigma}{s} \sim 10^{-6} \text{ GeV} \left( \frac{m_{\sigma}}{1 \text{ MeV}} \right).$$

(8)

Even if the saxion was not thermalized, the saxion could be produced by scattering processes right after the reheating era. In this case the saxion abundance ($\rho_\sigma/s$) is given by

$$\frac{\rho_\sigma}{s} \sim 10^{-6} \text{ GeV} \left( \frac{m_{\sigma}}{1 \text{ MeV}} \right) \left( \frac{T_R}{T_{\text{dec}}} \right),$$

where $T_R$ is the reheating temperature.
\[ \sim 10^{-17} \text{ GeV} \left( \frac{m_\sigma}{1 \text{ MeV}} \right)^3 \left( \frac{f}{10^4 \text{ GeV}} \right)^{-4} \left( \frac{T_R}{10 \text{ MeV}} \right), \]  

where \( T_R \) is the reheating temperature of the inflation (\( T_R \gtrsim 10 \text{ MeV} \)) and it is highly model dependent [11]. In the gauge-mediated SUSY breaking models, since the longitudinal component of the light gravitino has interaction much stronger than the gravitational one, it would be abundant and overclose the universe unless the mass is lower than about 1 keV. To cure this, the reheating temperature \( T_R \) must be sufficiently low [12,13]

\[ T_R \lesssim 10^6 \text{ GeV} \left( \frac{300 \text{ GeV}}{m_\tilde{g}} \right)^2 \left( \frac{m_{3/2}}{1 \text{ MeV}} \right). \]  

With such a low \( T_R \), the relic abundance of the saxion produced by the scattering in the thermal bath is highly suppressed.

On the other hand, the saxion is also produced as a coherent oscillation. The scalar potential in general receives corrections at an early universe, e.g. during the primordial inflation. Thus the saxion does not sit at the true minimum \( \sigma = f_{PQ} \) but is generically displaced at \( \sigma = \sigma_I \) after the inflation. Knowledge of the initial displacement \( \sigma_I \) is crucial to estimate the abundance of the oscillating saxion, but it depends on the details of supergravity Lagrangian as well as inflation models[9]. Thus in the following, we regard \( \sigma_I \) as an arbitrary parameter which is not larger than the Planck scale \( (M_G = 2.4 \times 10^{18} \text{ GeV}) \) and consider two distinct cases.

2 If one assumes that the universe experienced the late-time entropy production with a sufficiently low reheating temperature, the gravitino is sufficiently diluted and we have no upper bound on the reheating temperature of the primordial inflation \( T_R \) like Eq.(10).

3 \( \sigma_I \sim 0 \) or \( \sigma_I \sim M_G \) may be naturally expected by the supergravity effect.
A. Case 1: Saxion trapped at the origin

Let us first consider the case that the saxion field sits around the origin \((\sigma_I \simeq 0)\) just after the inflation.\(^4\) In this case the PQ quark multiplets \((Q_P\) and \(\overline{Q_P}\)) are in the thermal equilibrium through the rapid QCD interaction, which generates an additional mass term \(\sim T^2|X|^2\) to the saxion potential as a finite temperature effect. Thus the saxion is trapped at the origin till the cosmic temperature becomes below a critical temperature \(T_C\). We expect that \(T_C \sim 100\) GeV, since the saxion obtains a negative soft SUSY breaking mass squared at the origin due to the renormalization group effect of the Yukawa interaction Eq.\((\ref{eq:1})\) and its absolute value is expected to be of the order of the electroweak scale \(\sim 100\) GeV.

Therefore the saxion starts to roll down toward its true minimum at \(T = T_C\) and oscillates around it with the initial amplitude \(\sigma_0 \simeq f_{PQ}\).\(^5\) The ratio of the energy density of this oscillation to the entropy density is estimated as

\[
\frac{\rho_\sigma}{s} \simeq \frac{m_\sigma^2 f_{PQ}^2 \left( \frac{\sigma_0}{f_{PQ}} \right)^2}{\frac{2\pi^2}{45} g_\ast T_C^3},
\tag{11}
\]

where \(g_\ast\) counts the degrees of freedom of the relativistic particles \((g_\ast \sim 100\) at \(T = T_C\)). Here it should be noted that this ratio takes a constant value till the saxion decays if no extra entropy is produced, since \(\rho_\sigma\) and \(s\) are both diluted by \(\rho_\sigma \propto s \propto R^{-3}\) \((R:\) the scale

\(^4\) Thus the \(U(1)_{PQ}\) symmetry is recovered. In this case it should be noted that, since the effective mass which the saxion feels during the primordial inflation is generally comparable to the Hubble parameter during the inflation, the quantum fluctuations for the \(X\) field are strongly suppressed \cite{14}. For special Kähler potential, a massless mode appears in the \(X\) field and a large quantum fluctuation is expected. This would lead to too much axion isocurvature fluctuation. To suppress it, the mass density of the axion or the Hubble parameter during the inflation should be small. We thank M. Kawasaki and T. Yanagida for pointing out this problem.

\(^5\) We are free from the domain wall problem in our model, since \(N_{DW} = 1\).
factor of the universe) as the universe expands. In the model we are discussing, the PQ scale $f_{PQ}$ is given by Eq.(2) and thus the abundance of the saxion oscillation reads

$$\rho_\sigma \simeq 1.1 \times 10^{-2} f_{PQ}^4 \left( \frac{\sigma_0}{f_{PQ}} \right)^2,$$

$$\simeq 1.1 \times 10^8 \text{ GeV} \left( \frac{f}{10^4 \text{ GeV}} \right)^4 \left( \frac{T_C}{100 \text{ GeV}} \right)^{-3} \left( \frac{\sigma_0}{f_{PQ}} \right)^2. \quad (12)$$

Note that it is independent of the saxion mass. Here we have assumed that the reheating process of the primordial inflation completed before the saxion oscillation starts, i.e., $T_R > T_C$. On the other hand, for the case $T_R < T_C$, we expect a dilution of the saxion energy density. The saxion energy density at $T = T_R$ is given by

$$\rho_\sigma(T_R) \simeq \rho_\sigma(T_C) \left( \frac{R(T_C)}{R(T_R)} \right)^3 \simeq \frac{1}{2} m_\sigma f_{PQ}^2 \left( \frac{\sigma_0}{f_{PQ}} \right)^2 \left( \frac{R(T_C)}{R(T_R)} \right)^3. \quad (13)$$

From the fact that the radiation energy density is proportional to $R^{-3/2}$ from the end of the inflation till $T = T_R$, it follows that the ratio of the scale factor $(R(T_C)/R(T_R))$ becomes

$$\left( \frac{R(T_C)}{R(T_R)} \right)^{3/2} = \frac{g_*(T_R)T_R^4}{g_*(T_C)T_C^4}. \quad (14)$$

Thus the saxion abundance at $T = T_R$ is given by

$$\rho_\sigma \simeq 1.2 \times 10^{-3} m_\sigma f_{PQ}^2 \left( \frac{\sigma_0}{f_{PQ}} \right)^2,$$

$$\simeq 1.2 \times 10^{-13} \text{ GeV} \left( \frac{f}{10^4 \text{ GeV}} \right)^4 \left( \frac{T_C}{100 \text{ GeV}} \right)^{-8} \left( \frac{T_R}{10 \text{ MeV}} \right)^5 \left( \frac{\sigma_0}{f_{PQ}} \right)^2. \quad (15)$$

Here note that the reheating temperature $T_R$ should be higher than about 10 MeV in order to maintain the success of the big bang nucleosynthesis. Comparing it with Eq.(12), we find that the relic abundance is diluted significantly by the primordial inflation with a very low $T_R$.

The above discussion does also hold even for the case where the saxion is displaced from the origin $\sigma_I$ by a small amount. This is because the PQ multiplets will be thermalized and generate the potential which traps the saxion at the origin, as far as their effective mass $(\sim \lambda_P \sigma_I)$ is smaller than the maximum temperature achieved after the primordial inflation.

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6 The maximum temperature $T_{MAX}$ achieved after the primordial inflation is given by as
B. Case 2: Saxion displaced far from the origin

If the displacement $\sigma_I$ is large enough, the cosmological evolution of the saxion is completely different. Here, we consider only the case that $(M_G \gtrsim \sigma_I \gtrsim f_{PQ})$. When the Hubble parameter of the universe ($H$) becomes $H \simeq m_\sigma$, the saxion starts to obey the coherent oscillation. The temperature at this time $T_\sigma$ is estimated as

$$T_\sigma \simeq 0.46 \sqrt{m_\sigma M_G},$$

$$\simeq 2.3 \times 10^7 \text{ GeV} \left( \frac{m_\sigma}{1 \text{ MeV}} \right)^{1/2}.$$  \hspace{1cm} (16)

Using the initial amplitude of the oscillation $\sigma_0 = \sigma_I (\gtrsim f_{PQ})$, the saxion abundance of this oscillation is estimated as

$$\frac{\rho_\sigma}{s} \simeq \frac{1}{2} m_\sigma^2 M_G^2 \left( \frac{\sigma_0}{M_G} \right)^2 \left( \frac{2\pi^2 g_\ast T_\sigma^3}{45} \right),$$

$$\simeq 2.8 \times 10^6 \text{ GeV} \left( \frac{m_\sigma}{1 \text{ MeV}} \right)^{1/2} \left( \frac{\sigma_0}{M_G} \right)^2.$$  \hspace{1cm} (17)

Here we have assumed that the saxion oscillation starts after the reheating of the primordial inflation had been completed ($T_R > T_\sigma$). However, such a high $T_R$ leads the gravitino problem for $m_\sigma (\simeq m_{3/2}) \gtrsim 1 \text{ keV}$. Comparison of Eq. \[ (10) \] with the upper bound on $T_R$ [Eq.\[ (10) \]] shows that $T_\sigma$ is always higher than this bound. On the other hand, the inflation with $T_R < T_\sigma$ dilutes the energy density of the saxion as

$$T_{\text{MAX}} \simeq 0.3 T_R^{1/2} V_{\text{i}}^{1/8},$$

where $V_i$ is the vacuum energy of the inflation.

7 Thus the $U(1)_{PQ}$ symmetry is broken and a massless mode, i.e. the axion, appears. See the footnote 4 for an argument on the density fluctuation of the axion field.

8 Note that for the region $\sigma = \sigma_I \gtrsim f_{PQ}$ the potential is governed by the contribution from the gravity mediation effect, and the effective mass of the saxion is almost the same as the gravitino mass, i.e., the saxion mass ($m_{3/2} \simeq m_\sigma$).
\[
\frac{\rho_\sigma}{s} \simeq \frac{1}{8} T_R \left( \frac{\sigma_0}{M_G} \right)^2 \\
\simeq 1.3 \times 10^{-3} \text{ GeV} \left( \frac{T_R}{10 \text{ MeV}} \right) \left( \frac{\sigma_0}{M_G} \right)^2.
\]

(18)

This ratio takes its minimum value for the lowest reheating temperature \( T_R \simeq 10 \text{ MeV} \) and the smallest initial amplitude \( \sigma_0 \simeq f_{PQ} \).

To end this section we estimate the saxion lifetime. When the Hubble parameter becomes comparable to its decay width, the saxion decay occurs. In the present model, the saxion dominantly decays into two axions with the rate\(^9\)

\[
\Gamma(\sigma \rightarrow aa) = \frac{1}{64\pi} \frac{m_\sigma^3}{f_{PQ}^2},
\]

(19)

and the lifetime is given as

\[
\tau_\sigma \simeq 1.3 \times 10^9 \text{ sec} \left( \frac{f}{10^4 \text{ GeV}} \right)^4 \left( \frac{m_\sigma}{1 \text{ MeV}} \right)^{-5}.
\]

(20)

Therefore the saxion becomes stable within the age of the universe (\( \sim 10^{17} \text{ sec} \)) if its mass is smaller than about 10 keV. Note that the value \( \tau_\sigma \) varies very widely because of the strong dependence on the saxion mass.

III. CONSTRAINTS ON COSMIC SAXION

In this section we consider the cosmological constraints on the saxion abundance and examine whether the proposed model is viable or not. Since the energy density of the saxion is not so suppressed compared to the radiation and also it may have a long lifetime, we have to carefully see its cosmological effects.

As described in the previous section, the saxion is produced by the scattering at the reheating era and also as the coherent oscillation. Comparing the both saxion abundances

\(^9\) The saxion may decay into two photons with a very small branching ratio. We will discuss this rare process in the next section.
with each other, one finds that the saxion produced from the thermal bath can be neglected, if one assumes the primordial inflation with a relatively low reheating temperature to avoid the gravitino problem and too much relic abundance of the saxion oscillation. Therefore it should be enough to consider only the saxion abundance coming from the oscillation in the subsequent discussion.

A. Constraints on Saxion Abundance

First of all, the saxion with $\tau_\sigma \gtrsim 1$ sec, i.e., $m_\sigma \lesssim 100$ MeV, may upset the big bang nucleosynthesis. Existence of the extra energy of exotic particle at $T \sim 1$ MeV may speed up the expansion of the universe and increase the number ratio of neutron to proton, resulting in the overproduction of $^4$He. Roughly speaking, the energy of such a particle should be smaller than that of one neutrino species. Thus the saxion abundance should satisfy

$$\frac{\rho_\sigma}{s} \lesssim 10^{-4} \text{ GeV.}$$

(21)

Furthermore, if the saxion lifetime is larger than the age of the universe (i.e., $m_\sigma \lesssim 10$ keV), the saxion oscillation still exists now. Not to overclose the universe, the saxion abundance should be

$$\frac{\rho_\sigma}{s} \lesssim \frac{\rho_c}{s_0} = 3.6 \times 10^{-9} h^2 \text{ GeV},$$

(22)

with the critical density $\rho_c$ and the present entropy density $s_0$.

Next, we turn to the rare decay of the saxion. Since the saxion is light enough in our model, it can not decay into two gluons but decays into two photons via one-loop diagrams if the PQ quark multiplets have QED charges. Thus we expect that the branching ratio of this radiative decay is very suppressed as $B_\gamma \sim (\alpha_{em}/(4\pi))^2 \sim 10^{-7}$. For example, if the PQ quark fields $Q_P$ and $\overline{Q}_P$ are in 5 and 5$^*$ under the SU(5) GUT gauge group, the branching ratio of the radiative decay is given as $B_\gamma \simeq 5 \times 10^{-6}$. Even with such a small $B_\gamma$, extra photons produced at a late time are cosmologically dangerous.
First, the saxion decay is constrained from the diffuse x-ray background spectrum. When the saxion has a lifetime longer than the cosmic time of the recombination \((\tau_\sigma \gtrsim 10^{12} \text{ sec})\), the produced photon directly contributes to the spectrum, and then a very stringent upper bound on \(B_\gamma \times (\rho_\sigma/s)\) comes from the present observation of the spectrum. (The details are found, for example, in Ref. [13].)

If the saxion lifetime is in between \(10^6\) sec and \(10^{12}\) sec, extra radiation energy produced by the saxion decay may alter the cosmic microwave background spectrum from the black-body one. The observation by the COBE satellite [16] gives the following upper bounds on the saxion abundances as

\[
B_\gamma \rho_\sigma/s \lesssim 2.5 \times 10^{-5} T_D \quad \text{for } 10^6 \text{ sec} \lesssim \tau_\sigma \lesssim 10^{10} \text{ sec},
\]

\[
B_\gamma \rho_\sigma/s \lesssim 2.3 \times 10^{-5} T_D \quad \text{for } 10^{10} \text{ sec} \lesssim \tau_\sigma \lesssim 10^{12} \text{ sec},
\]

where \(T_D\) is the cosmic temperature at the saxion decay and estimated as

\[
T_D = 0.60 \left( \frac{\rho_\sigma}{s} \right) \left( \frac{M_G}{(\rho_\sigma/s)^2 \tau_\sigma} \right)^{2/3}.
\]

These constraints on the saxion abundance are summarized in Fig. 1 for the case \(B_\gamma = 5 \times 10^{-6}\). Note that the constraints from the x-ray background spectrum and the CMBR become weaker for the suppressed \(B_\gamma\). We can now compare these cosmological constraints with the saxion abundance estimated in the previous section.

**B. Case 1: Saxion trapped at the origin**

First, we consider the case that the saxion is thermally trapped at the origin after the primordial inflation. For the case that the reheating temperature of the inflation is higher than the critical temperature \((T_R > T_C \sim 100 \text{ GeV})\), the saxion abundance [Eq. (12)] significantly exceeds the upper bound Eq. (21) from the big bang nucleosynthesis. To avoid this the saxion should decay before it, however the lifetime of the saxion is longer than 1
sec if \( m_\sigma \lesssim 100 \text{ MeV} \). Even for the case \( m_\sigma \simeq 100 \text{ MeV} \), the energy of the decay produced axion easily exceeds the bound Eq. (21). Therefore, in this case, we conclude that \( T_R < T_C \).

The saxion abundance [Eq. (15)] for \( T_R < T_C \) takes its minimum value at the lowest \( T_R = 10 \text{ MeV} \), since the dilution of the inflation becomes most effective. In Fig. 1 we show this lower limit of the saxion abundance with various cosmological constraints. Note that now for the case \( T_R = 10 \text{ MeV} \), the PQ scale as high as \( 10^{16} \text{ GeV} \) is allowed in the consideration of the relic abundance of the axion oscillation. It is found that all the region for the saxion mass is cosmologically allowed if we take the sufficiently low reheating temperature \( T_R \simeq 10 \text{ MeV} \). Furthermore the stable saxion with mass \( 10 \text{ eV} \lesssim m_\sigma \lesssim 10 \text{ keV} \) will constitute a dark matter of our universe, since the saxion could achieve \( \Omega_\sigma = (\rho_\sigma/\rho_c)_0 = 1 \) without conflicting with the cosmic x-ray background observation.

In the above result an extremely low reheating temperature is crucial to dilute the saxion (and also the gravitino). The new inflation model [17] gives such a situation. For example, Ref. [18] gives a new inflation model which solves the initial condition problem, where the reheating temperature is given by

\[
T_R \sim m_{3/2}^{9/10} M_P^{1/10},
\]

where the gravitino mass should be larger than about 50 keV since \( T_R \gtrsim 10 \text{ MeV} \). With this reheating temperature the saxion abundance [13] is also shown in Fig. 1. It can be seen that with this new inflation model [18] the saxion with \( m_\sigma \simeq 50 \text{ keV-1 MeV} \) survives the various cosmological constraints.

C. Case 2: Saxion displaced far from the origin

Next we turn to the case that the saxion is displaced far from the origin after the primordial inflation. In order to avoid the overproduction of the gravitino we assume the reheating temperature of the inflation to be lower than \( T_\sigma \) (even for the case that \( m_{3/2} \simeq m_\sigma \lesssim 1 \text{ keV} \)). The saxion abundance for \( T_R \lesssim T_\sigma \) is given as Eq. (18) and takes its minimum
value when the minimum $T_R = 10$ MeV. In Fig. 2 we show the cosmological constraints on the initial amplitude $\sigma_0$ in the saxion abundance Eq. (18) with $T_R = 10$ MeV. Note that in this case the upper bound on the PQ scale is $f_{PQ} \lesssim 10^{16}$ GeV. We find that most of the saxion mass (or the PQ scale) is cosmologically allowed if we take $\sigma_0 \sim f_{PQ}$. And when the saxion has a heavier mass $m_\sigma \simeq 10\sim 100$ MeV, the initial amplitude can take a value from $\sigma_0 \sim f_{PQ}$ to $\sigma_0 \sim M_G$. Similar to the previous case, the saxion with mass $10$ eV $\lesssim m_\sigma \lesssim 10$ keV could be a dark matter of our universe.

To summarize, the hadronic axion model we proposed is cosmologically viable in the both cases as far as the reheating temperature of the inflation is low enough.

To end this section, we emphasize that it has been assumed that no extra entropy production takes place other than the primordial inflation. If such a late-time entropy production like a thermal inflation [19] or an oscillating inflation [13, 20, 21] occurs, tremendous entropy is released at a late-time in the history of the universe (but at least before the big bang nucleosynthesis) and the saxion abundance is diluted by about $10^{10}\sim 10^{20}$ and cosmological constraints on it are extensively relaxed. Thus our model becomes cosmologically viable even with the primordial inflation model with a high reheating temperature.

IV. LATE DECAYING SAXION AND STRUCTURE OF THE UNIVERSE

As discussed in the previous section, the cosmic energy of the saxion oscillation is abundant compared to the radiation and it may decay at very late time of the universe. In this section, we would like to argue that this feature of the saxion particle could explain well the present observed power spectrum of the cosmic structure with some cold dark matter [22], i.e., the saxion can be a candidate for a late decaying massive particle (LDP) [23, 24].

It is well known that the predicted spectrum of the standard cold dark matter (CDM) with $\Omega_0 \simeq 1$ and $h \simeq 0.7$ could not fit well all the observational data simultaneously. The cosmic background explorer (COBE) observed the anisotropy of the temperature of the cosmic microwave background radiation (CMBR) [25] and gave the normalization of the
spectrum at a large scale ($\lambda \sim 10^3 h^{-1}\text{Mpc}$). However, this normalization predicts too much fluctuation at a small scale ($\lambda \sim 10 h^{-1}\text{Mpc}$) than galaxy distribution observations \cite{26}. The late decaying massive particle (LDP), once dominating the energy of the universe before its decay, can delay the epoch of the matter-radiation equality compared to the standard CDM model, which results in the preferred spectrum \cite{23,24}.

Ref. \cite{24} argued that the saxion can be accounted to be the LDP, if the saxion energy density satisfies

$$\frac{\rho_s}{s} > \Omega_0 \times \frac{\rho_c}{s_0},$$

(27)

to cause the saxion domination era, and

$$\tau_s \left( \frac{\rho_s}{s} \right)^2 \simeq xM_G,$$

(28)

to explain the observed power spectrum. Here $x$ is defined as

$$x = 0.23 \left( \frac{\Omega_0 h}{0.3} \right)^2 - 1 \right)^{3/2}.$$

(29)

In the following we will take $x = 2.2$, which corresponds to the case that $\Omega_0 = 1$ and $h = 0.7$. Note that the saxion LDP also survives the big bang nucleosynthesis constraint \cite{21}. In Fig.3 we show the saxion abundance required to become the LDP by using its lifetime [Eq.(20)]. In the same figure, we also show various cosmological constraints. We find that the saxion whose mass is

$$10\text{MeV} \gtrsim m_{s} \gtrsim 100\text{keV}$$

(30)

can be the LDP. However, such a saxion is severely constrained by the CMBR constraints [Eq.(23)].

When the saxion becomes the LDP, the CMBR constraints give the upper bounds on the radiative branching ratio as

$^{10}$Note that the saxion with mass $\sim 10$ MeV is free from the CMBR constraint since the lifetime is short.
\[
B_\gamma \lesssim 8.8 \times 10^{-6}, \quad \text{for} \quad 10^6 \text{sec} \lesssim \tau_\sigma \lesssim 10^{10} \text{sec}, \quad (31)
\]
\[
B_\gamma \lesssim 8.2 \times 10^{-6}, \quad \text{for} \quad 10^{10} \text{sec} \lesssim \tau_\sigma \lesssim 10^{12} \text{sec}. \quad (32)
\]

Therefore \(B_\gamma = 5 \times 10^{-6}\) for the simple case that the PQ quark multiplets \(Q_P\) and \(\overline{Q}_P\) are \(5\) and \(5^*\) of the SU(5) gauge group is marginally allowed. However, \(B_\gamma\) depends very much on their representation. Here we claim that the observational bound of the CMBR at present is comparable to the prediction of the late decaying particle. In other words the late decaying saxion scenario presented here will be tested by more precise experiments of the CMBR in near future (e.g. the experiments by the PLANCK and MAP satellites).

A. Axion CDM and Saxion LDP Scenario

We will explain that the saxion abundance required for the LDP [Eqs.(27) and (28)] can be naturally offered in the present model. First, we consider the case that the saxion is trapped at the origin due to the thermal effects after the primordial inflation. Using the relic abundance of the saxion Eq.(15), the condition for the LDP [Eq.(28)] leads to the required reheating temperature of the inflation as\(^{11}\)

\[
T_R \simeq 2.3 x^{1/10} \left( \frac{\sigma_0}{f_{PQ}} \right)^{-2/5} \frac{M_G^{1/10} m_{\sigma}^{1/2} T_C^{8/5}}{f^{6/5}},
\]

\[
\simeq 130 \text{ MeV} \left( \frac{\sigma_0}{f_{PQ}} \right)^{-2/5} \left( \frac{m_{\sigma}}{1 \text{ MeV}} \right)^{1/2} \left( \frac{T_C}{100 \text{ GeV}} \right)^{8/5} \left( \frac{f}{10^4 \text{ GeV}} \right)^{6/5}, \quad (34)
\]

with the saxion mass Eq.(30).

It is worth noting that if we take the critical temperature as \(T_C > 100 \text{ GeV}\), the reheating of the inflation, which induces the required saxion abundance of the LDP, completes before

\(^{11}\) For example, a new inflation model by Ref. [18] with the reheating temperature (26) leads to the late decaying saxion which mass is

\[
m_\sigma \simeq m_{3/2} \simeq 0.92 \text{ MeV} \left( \frac{\sigma_0}{f_{PQ}} \right)^{-1} \left( \frac{T_C}{100 \text{ GeV}} \right)^4 \left( \frac{f}{10^4 \text{ GeV}} \right)^{-3}, \quad (33)
\]

which just lies in the required mass region [Eq.(30)].
the QCD phase transition. In this case the axion can constitute the dominant component of the dark matter of our universe if \( f_{PQ} \sim 10^{11} - 10^{12} \) GeV. This PQ scale corresponds to the saxion mass \( m_\sigma \sim 100 \) keV–1 MeV. Therefore, in the considering model, if one takes the gravitino mass \( m_{3/2} \simeq m_\sigma \simeq 100 \) keV–1 MeV, the axion can be the CDM of our universe and moreover the saxion can be the late decaying particle to cure the difficulty in the power spectrum of the structure of our universe in the axion CDM model.

Even if the saxion is displaced from the origin after the inflation, a similar argument holds. Using the saxion abundance for \( T_\sigma > T_R \) [Eq.(18)], the saxion which mass in Eq.(30) becomes the LDP when

\[
\frac{\sigma_0}{f_{PQ}} \simeq 0.91 \frac{m_\sigma^{9/4} M_G^{5/4}}{T_R f^3}, \\
\simeq 1.5 \times 10^3 \left( \frac{m_\sigma}{1 \text{ MeV}} \right)^{9/4} \left( \frac{T_R}{100 \text{ GeV}} \right)^{-1/2} \left( \frac{f}{10^4 \text{ GeV}} \right)^{-3}. \tag{35}
\]

Therefore if we take the reheating temperature as \( T_R \gtrsim \Lambda_{QCD} \), the gravitino mass \( m_{3/2} \simeq m_\sigma \) 100 keV–1MeV also leads to the axion CDM. Thus the axion CDM and the saxion LDP scenario is achieved with the moderate \( T_R \) and \( \sigma_0 \).

**B. Moduli CDM and Saxion LDP Scenario**

So far we have assumed that there exists no extra dilution mechanism other than the primordial inflation in the history of the universe. Here we will briefly discuss the case that there exists the late time entropy production such as the thermal inflation \[19\] or the oscillating inflation \[13,20,21\]. The idea of the late time entropy production is that a mini inflation takes place at a late time of the universe (at least before the big-bang nucleosynthesis) and produces tremendous entropy at the reheating epoch, which dilutes the unwanted particles extensively. If one assumes this dilution mechanism, the cosmological constraints on the saxion abundance become much weaker even for the saxion abundance [Eq.(17)] with \( \sigma_0 \sim M_G \). Here we would like to stress that the saxion can play the role of the LDP even in this case.
In the universe with the late time entropy production it is worth noting that the string moduli could be the CDM. The moduli are scalar fields associated with flat directions which characterize classical vacua of the string theory and have only gravitationally suppressed interaction. The effects of the SUSY breaking give the moduli masses $m_\phi$ comparable to the gravitino mass ($m_\phi \simeq m_{3/2}$). In fact if the moduli mass is $m_\phi \lesssim 200$ keV, the condition $\Omega_\phi \sim 1$ could be achieved without conflict to the X($\gamma$)-ray observations after an appropriate amount of late time entropy production is taken into account [15,27,28]. Note that it extensively dilutes various dark matter candidates as well as the unwanted particles and they could not give a sizable contribution to the energy density of the universe. Therefore, the string moduli seem to be a well motivated candidate for the CDM in the universe with the late time entropy production.

When we consider the moduli CDM, we find that the saxion can play a role of the LDP if the saxion is displaced at $\sigma_I \sim M_G$ after the primordial inflation. Note that the moduli $\phi$ is also expected to be displaced from the true vacuum at the order of the Planck scale, i.e., $\phi \sim M_G$. The both fields begin to oscillate at almost the same time, when the Hubble parameter becomes $H \simeq m_\phi \simeq m_\sigma (\simeq m_{3/2})$. Using the initial amplitude of the moduli (saxion) oscillation $\phi_0 (\sigma_0)$, the energy densities of the oscillations are related as

$$\rho_\sigma \simeq \rho_\phi \left( \frac{\sigma_0}{\phi_0} \right)^2.$$  \hspace{1cm} (36)

This equation also holds after some late time entropy production, since the both energy densities are diluted at the same rate. Therefore, the saxion abundance is given by

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12 Cosmology of the string moduli in the gauge-mediated SUSY breaking is discussed by Ref. [13,27,28] in the universe with the thermal inflation, and by Ref. [13,24,29] in the universe with the oscillating inflation.

13 The cosmic axion with relatively high decay constant $f_{PQ} \sim 10^{16}$ GeV may be another candidate for the dark matter [31,88].

14 This initial condition is easily achieved by the effects of the supergravity.
\[
\frac{\rho_\sigma}{s} \simeq \Omega_\phi \frac{\rho_c}{s_0} \left( \frac{\sigma_0}{\phi_0} \right)^2 .
\]

This abundance naturally explain the conditions of the saxion LDP Eqs. (27) and (28). In fact, Eq. (28) leads to the ratio of the initial amplitudes

\[
\frac{\sigma_0}{\phi_0} \simeq 0.32 \left( \frac{m_\sigma}{\Omega_\phi^{1/2}} \left( \frac{\rho_c}{s_0} \right)^{1/2} \right)
\]

\[
\simeq \frac{5.4}{\Omega_\phi^{1/2}} \left( \frac{m_\sigma}{1 \text{ MeV}} \right)^{5/4} \left( \frac{f}{10^4 \text{ GeV}} \right)^{-1} ,
\]

with the saxion mass in the region (30). Here the moduli CDM implies \(\Omega_\phi \simeq 1\). It should be noted that both initial amplitudes is expected to be of the order of the Planck scale, \(\sigma_0 \sim \phi_0 \sim M_G\). Thus for \(\Omega_\phi \simeq 1\) the saxion with the initial amplitude \(\sigma_0\) which is slightly larger than \(\phi_0 \sim M_G\) could be the LDP even with the late time entropy production in the history of the universe, if the saxion has a mass in the region (30). On the other hand the string moduli could be the CDM of our universe (\(\Omega_\phi \simeq 1\)) if its mass is less than about 100 keV. Therefore, if \(m_\sigma \simeq m_\phi \simeq m_{3/2} \sim 100\) keV, the string moduli could be the CDM of our universe while the saxion could plays a role of the LDP so that the present observed power spectrum is well explained.

It is interesting that this scenario will be tested in various cosmological experiments. The moduli CDM with mass \(m_\phi \sim 100\) keV will be tested in the future by the x-ray background observations with high energy resolutions [32]. The line x-ray spectrum from the moduli CDM trapped in the halo of our galaxy will be seen. And the future CMBR observations, as mentioned before, reveal the saxion LDP possibility.

V. CONCLUSIONS

In this paper, we closely investigated the cosmology of the hadronic axion model in the gauge mediated supersymmetry breaking, which we had proposed in a previous paper [3]. In particular we focused on the implications given by the saxion which is the scalar component of the axion supermultiplet. The saxion has a tiny mass comparable to the gravitino mass
and thus its life time tends to be long and decay after the primordial nucleosynthesis. Furthermore since the saxion is a scalar field, it will be abundant in the early universe in the form of the coherent oscillation. Thus the saxion may be cosmologically harmful.

We estimated the energy density of the saxion in two distinctive cases for the cosmological evolution of the saxion coherent mode. Then we examined whether or not the saxion survives the various cosmological constraints from the nucleosynthesis, from the critical density limit, from the diffuse X-ray background spectrum, and from the cosmic microwave background spectrum. We showed in both cases that the model we proposed is cosmologically viable, namely it satisfies the severe cosmological constraints on the saxion field, if the reheating temperature of the primordial inflation is sufficiently low.

Furthermore we argued that the saxion can play a role of the late decaying particle in the structure formation of the universe, giving a better fit of the power spectrum of the density perturbation than the standard cold dark matter model.

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REFERENCES

[1] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett., 38, 1440 (1977); Phys. Rev., D16, 1791 (1977).

[2] For a review, G.F. Giudice and R. Rattazzi, hep-ph/9801271.

[3] T. Asaka and M. Yamaguchi, Phys. Lett., B437, 51 (1998).

[4] J.E. Kim, Phys. Rev. Lett. 43 103 (1979); M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B166 493 (1980).

[5] H.T. Janka, W. Keil, G. Raffelt and D. Seckel, Phys. Rev. Lett., 76 2621 (1996).

[6] M.S. Turner, Phys. Rev., D33, 889 (1986).

[7] See for example, E.W. Kolb and M.S. Turner, The Early Universe, Addison-Wesley (1990).

[8] G. Lazarides, R. Shaefer, D. Seckel and Q. Shafi, Nucl, Phys., B346, 193 (1990).

[9] M. Kawasaki, T. Moroi and T. Yanagida, Phys. Lett., B383, 313 (1996).

[10] K. Rajagopal, M.S. Turner and F. Wilczek, Nucl. Phys. B358 447 (1991).

[11] See for example, A.D. Linde, Particle Physics and Inflationary Cosmology, (Harwood, Chur, Switzerland, 1990).

[12] T. Moroi, H. Murayama and M. Yamaguchi, Phys. Lett., B303, 289 (1993).

[13] A. de Gauvea, T. Moroi and H. Murayama, Phys. Rev., D56, 1281 (1997).

[14] K. Enquvist, K.W. Ng, and K.A. Olive, Nucl. Phys., B303, 713 (1988).

[15] M. Kawasaki and T. Yanagida, Phys. Lett., B399, 45 (1997).

[16] D.J. Fixsen et al., Astrophys. J., 473, 473 (1996).

[17] A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett., 48, 1220 (1982); A.D. Linde, Phys.
[18] K.I. Izawa, M. Kawasaki, and T. Yanagida, Phys. Lett. **B411**, 249 (1997).

[19] D.H. Lyth and E.D. Stewart, Phys. Rev. Lett., **75**, 201 (1995); Phys. Rev., **D53**, 1784 (1996).

[20] T. Moroi, [hep-ph/9807263](http://arxiv.org/abs/hep-ph/9807263).

[21] T. Damour and V.F. Mukhanov, Phys. Rev. Lett. **80**, 3440 (1998);
A.R. Liddle and A. Mazumdar, Phys. Rev. **D58**, 083508 (1998).

[22] S. Chang and H.B. Kim, Phys. Rev. Lett. **77**, 591 (1996).

[23] J.M. Bardeen, J.R. Bond, and G. Efstathiou, Astrophys. J. **321**, 28 (1987).

[24] H.B. Kim and J.E. Kim, Nucl. Phys. **B433**, 421 (1995).

[25] G.F. Smoot et al., Astrophys. J., **396**, L1 (1992).

[26] J.A. Peacock and S.J. Dodds, Mon. Not. R. Astron. Soc., **267**, 1020 (1994).

[27] J. Hashiba, M. Kawasaki, and T. Yanagida, Phys. Rev. Lett., **79**, 4525 (1997).

[28] T. Asaka, J. Hashiba, M. Kawasaki and T. Yanagida, Phys. Rev., **D58**, 083509 (1998).

[29] T. Asaka, M. Kawasaki, and M. Yamaguchi, [hep-ph/9810334](http://arxiv.org/abs/hep-ph/9810334).

[30] B. De Carlos, J.A. Casas, F. Quevedo and E. Roulet, Phys. Lett., **B318**, 447 (1993).

[31] M. Kawasaki and T. Yanagida, Prog. Theor. Phys., **97**, 809 (1997).

[32] T. Asaka, J. Hashiba, M. Kawasaki and T. Yanagida, Phys. Rev., **D58**, 023507, (1998).
FIG. 1. The lower bound of the saxion abundance [Eq.(15)] for the case $T_C = 100$ GeV and $f = 10^4$ GeV (the thick solid line). We take the value of the radiative branching ratio $B_\gamma = 5 \times 10^{-6}$. The upper bounds from the various cosmological constraints are also shown. For the saxion mass larger than about 50 keV, the predicted saxion abundance in the new inflation model is shown with the thin solid line.
FIG. 2. Various cosmological upper bounds on the initial amplitude of the saxion oscillation [Eq. (18)] for the case $T_R = 10$ MeV ($\ll T_\sigma$). We take $f = 10^4$ GeV and $B_\gamma = 5 \times 10^{-6}$. We also show the initial amplitude $\sigma_0 = f_{PQ}$ by the solid line.
FIG. 3. The abundance of the saxion which is required to become the LDP (the solid line). Here we take $f = 10^4$ GeV, $\Omega_0 = 1$ and $h = 0.7$. The upper bound from the big bang nucleosynthesis and the lower bound from the saxion domination are denoted by the dotted lines with shadows. The dot-dashed lines represent the upper bounds of the saxion abundance from the CMBR for the cases that the radiative branching ratio $B_\gamma = 5 \times 10^{-6}$. 