Bottom-Strange Associated Production at High Energy $e^+e^-$ Colliders in Standard Model

Chao-Shang Huang $^a$, Xiao-Hong Wu $^a$ and Shou-Hua Zhu $^{b,a}$

$^a$ Institute of Theoretical Physics, Academia Sinica, P. O. Box 2735, Beijing 100080, P. R. China

$^b$ CCAST (World Lab), P.O. Box 8730, Beijing 100080, P.R. China

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Abstract

We investigate the flavor changing neutral current $bsV(V=\gamma,Z)$ couplings in the production vertex for the process $e^+e^- \rightarrow bs$ or $\bar{b}s$ in the standard model. The precise calculations keeping all quark masses non-zero are carried out. Production cross sections are found to be the order of $10^{-3}$ fb at LEP II and the order of $10^{-1}$ fb when center-of-mass energy is near the mass of neutral gauge boson Z.
I. INTRODUCTION

There are no flavor changing neutral currents (FCNC) at tree-level in the standard model (SM). FCNC appear at loop-levels and consequently offer a good place to test quantum effects of the fundamental quantum field theory on which SM based. Furthermore, they are very small at one loop-level due to the unitarity of Cabbibo-Kobayashi-Maskawa (CKM) matrix. In models beyond SM new particles beyond the particles in SM may appear in the loop and have significant contributions to flavor changing transitions. Therefore, FCNC interactions give an ideal place to search for new physics. Any positive observation of FCNC couplings deviated from that in SM would unambiguously signal the presence of new physics. Searching for FCNC is clearly one of important goals of the next generation of high energy colliders [1].

The flavor changing transitions involving external up-type quarks which are due to FCNC couplings are much more suppressed than those involving external down-type quarks in SM. The effects for external up-type quarks are derived by virtual exchanges of down-type quarks in a loop for which GIM mechanism [2] is much more effective because the mass splitting between down-type quarks are much less than those between up-type quarks. Therefore, for example, the $b_s$ transition which is studied in the paper has larger probability to be observed than that for the $t_c$ transition.

The $b$-hadron system promises to give a fertile ground to test the SM and probe new physics. The FCNC vertices $b_sV(V=\gamma, Z)$ have been extensively examined in rare decays of $b$-hadron system [3, 4, 5]. The observation of FCNC processes in both the exclusive $B \to K^*\gamma$ and inclusive $B \to X_s\gamma$ channels has placed the rare B decays on a new footing and has put a stringent constraint on classes of models [6]. Analyses of the inclusive decay $B \to X_s l^+ l^-$ show that in the minimal supergravity model (SUGRA) there are regions in the parameter space where the branching ratio of $b \to sl^+ l^-(l = e, \mu)$ is enhanced by about 50% compared to the SM [7] and the first distinct signals of SUSY could come from the observation of $B \to X_s \mu^+ \mu^-$ if $\tan \beta$ is large ($\geq 30$) and the mass of the lightest neutral Higgs boson $m_h$.
is not too large (say, less than 150 Gev) [3]. The B factories presently under construction will collect some $10^7-10^8$ B mesons per year which can be used to obtain good precision on low branching fraction modes.

The FCNC vertices $b s V(V=\gamma, Z)$ can also be investigated via bottom-strange associated production. In the paper we shall investigate the process

$$e^+e^- \rightarrow b\bar{s} \text{ or } \bar{b}s.$$  

Comparing $b$ quark rare decays where the momentum transfer $q^2$ is limited, i.e., it should be less or equal to mass square of $b$ quark $m_b^2$, the production process (1) allows the large (time-like) momentum transfer, which is actually determined by the energies available at $e^+e^-$ colliders. The reaction (1) has some advantages because of the ability to probe higher dimension operators at large momenta and striking kinematic signatures which are straightforward to detect in the clean environment of $e^+e^-$ collisions. In particular, in some extensions of SM which induce FCNC there are large underlying mass scales and large momentum transfer so that these models are more naturally probed via $b\bar{s}$ associated production than $b$ quark rare decays.

It has been shown that the cross sections of $e^+e^- \rightarrow t\bar{c}$ in SM are too small to be observed at LEP or NLC [5]. As pointed above, in SM the cross sections of $e^+e^- \rightarrow b\bar{s}$ should be much larger than those of $t\bar{c}$ final states. Are they large enough to be seen at LEP or NLC? In the paper we would like to address the problem by calculating cross sections and backward-forward asymmetry of the process (1) in SM.

II. ANALYTIC CALCULATIONS

In SM for the process (1) there are three kinds of Feynman diagram at one loop, self energy-type, triangle and box diagram, which are shown in Fig.1. We carry out calculations in the Feynman-t’Hooft gauge. The contributions of the neutral Higgs $H$ and Goldstone bosons $G^{0,\pm}$ which couple to electrons are neglected since they are proportional to the electron mass and we have put the mass of electron to zero.
We do the reduction using FeynCalc \cite{13} and keep all masses non-zero except for the mass of electron. To control the ultraviolet divergence, the dimensional regularization is used. As a consistent check, we found that all divergences are canceled in the sum of contributions of all Feynman diagrams. The calculations are carried out in the frame of the center of mass system (CMS) and Mandelstam variables have been employed:

\[ s = (p_1 + p_2)^2 = (k_1 + k_2)^2 \quad t = (p_1 - k_1)^2 \quad u = (p_1 - k_2)^2, \]

where \( p_1, p_2 \) are the momentum of electron and positron respectively, and \( k_1, k_2 \) are the momentum of bottom quark \( b \), and anti-strange quark \( \bar{s} \) respectively.

The amplitude of process \( e^+e^- \to \bar{b}s \) can be expressed as

\[
M = \sum_{j=u,c,t} 16\pi^2 \alpha^2 V_{jb}^* V_{js} \left[ g_1 \bar{u}_b \gamma^{\mu} P_R v_s \bar{v}_e \gamma_\mu P_R u_e + g_2 \bar{u}_b \gamma^{\mu} P_L v_s \bar{v}_e \gamma_\mu P_R u_e + g_3 \bar{u}_b \gamma^{\mu} P_R v_s \bar{v}_e \gamma_\mu P_L u_e + g_4 \bar{u}_b \gamma^{\mu} P_L v_s \bar{v}_e \gamma_\mu P_R u_e + g_5 \bar{u}_b \gamma^{\mu} P_R v_s \bar{v}_e \gamma_\mu P_L u_e + \right. \\
\left. g_6 \bar{u}_b \gamma^{\mu} P_L v_s \bar{v}_e \gamma_\mu P_R u_e + g_7 \bar{u}_b \gamma^{\mu} P_R v_s \bar{v}_e \gamma_\mu P_L u_e + g_8 \bar{u}_b \gamma^{\mu} P_L v_s \bar{v}_e \gamma_\mu P_R u_e + g_9 \bar{u}_b \gamma^{\mu} P_R v_s \bar{v}_e \gamma_\mu P_L u_e + \right. \\
\left. g_{10} \bar{u}_b \gamma^{\mu} P_L v_s \bar{v}_e \gamma_\mu P_R u_e + g_{11} \bar{u}_b \gamma^{\mu} P_R v_s \bar{v}_e \gamma_\mu P_L u_e \right]
\]

where \( \alpha \) is fine structure constant, \( V_{ij} \) is CKM matrix element, \( P_L \) is defined as \((1 - \gamma^5)/2\), and \( P_R \) is defined as \((1 + \gamma^5)/2\). The expressions of the coefficients \( g_j (j = 1, 2, \ldots 11) \) can be found in Appendix.

Having the amplitude \( M \), it is straightforward to obtain the differential cross section by

\[
\frac{d\sigma}{d\cos\theta} = \frac{N_c |k_1|}{16\pi s^{\frac{3}{2}}} \frac{1}{4} \sum_{spins} |M|^2
\]

where \( N_c \) is the color factor and \( \theta \) is the angle between incoming electron \( e^- \) and outgoing bottom quark \( b \).

### III. NUMERICAL RESULTS

In the numerical calculations the following values of the parameters have been used \cite{11}:

\[
3
\]
\[ m_e = 0, \quad m_u = 0.005\text{GeV}, \quad m_c = 1.4\text{GeV}, \quad m_t = 175\text{GeV}, \quad m_s = 0.17\text{GeV}, \]
\[ m_b = 4.4\text{GeV}, \quad m_w = 80.41\text{GeV}, \quad m_z = 91.187\text{GeV}, \quad \Gamma_z = 2.5\text{GeV}, \quad \alpha = \frac{1}{128} \]

In order to keep the unitary condition of CKM matrix exactly, we employ the standard parameterization and took the values \[ [11,12] \]
\[ s_{12} = 0.220, \quad s_{23} = 0.039, \quad s_{13} = 0.0031, \quad \delta_{13} = 70^\circ \]

Numerical results are shown in Figs. 2, 3, 4.

In Fig.2, we show the total cross section \( \sigma_{\text{tot}} \) of the process \( e^+e^- \rightarrow b\bar{s} \) as a function of the center-of-mass energy \( \sqrt{s} \). There are three peaks, corresponding to the pole of neutral gauge boson \( Z^0 \), a pair of charged gauge boson \( W \) threshold, and a pair of top quark \( t\bar{t} \) threshold respectively. In most of high energy region, total cross section is the order of \( 10^{-3} \) fb, which is too small to be seen at LEP II or planning NLC colliders. Therefore, even a small number of \( b\bar{s} \) events, detected at LEP II or NLC, will unambiguously indicate new FCNC couplings beyond SM. Smallness of the total cross section can easily be understood. One has

\[
\sum_{\text{spins}} |M|^2 = e^8 | \sum_{j=u,c,t} V_{jt}^* V_{jc} f(x_j, y_j)|^2
\]
\[
= e^8 | V_{tb}^* V_{ts} \left( \frac{m_t^2 - m_c^2}{m_w^2} \frac{\partial f}{\partial x_j} |_{x_j, y_j = 0} + \ldots \right) |^2,
\]

(5)
due to GIM mechanism, where \( x_j = m_j^2/m_w^2, y_j = m_j^2/s \), and "..." denote the less important terms for \( \sqrt{s} \geq 200 \text{ Gev} \). Assuming \( \frac{\partial f}{\partial x_j} |_{x_j, y_j = 0} = O(1) \), one obtains from eqs. (4), (5)

\[ \sigma \sim 10^{-3} \text{fb} \]
at \( \sqrt{s} = 200 \text{ Gev} \).

We fixed the center-of-mass energy \( \sqrt{s} \) at 200 Gev. Differential cross section of the process at the energy as a function of \( \cos \theta \) is shown in Fig.3.

The Fig.4 is devoted to the backward-forward asymmetry
A_{FB} = \frac{\int_0^{\pi/2} \frac{d\sigma}{d\theta} d\theta - \int_{\pi/2}^{\pi} \frac{d\sigma}{d\theta} d\theta}{\int_0^{\pi/2} \frac{d\sigma}{d\theta} d\theta + \int_{\pi/2}^{\pi} \frac{d\sigma}{d\theta} d\theta} \tag{6}

as a function of \sqrt{s}.

To summarize, we have calculated the process $e^+e^- \to b\bar{s}$ in SM. We found that the total cross section is of the order of $10^{-3} fb$ in the high energy region which is still too small to be seen at LEP II or planning NLC. However, it is worth to note that the total cross section at Z resonance may reach as large as $10^{-1} fb$. Therefore, it is possible to see the process if a luminosity reaches 100-1000 $fb^{-1}$. In addition to that, the process is of a good place to search for new physics.

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APPENDIX

\begin{align*}
g_1 &= m_s (B_0^b (m_b^2 - m_s^2) (m_j^2 - m_w^2) (m_j^2 + 2m_w^2) + B_0^d m_w^2 (m_b^2 - 2m_b^2 m_j^2 + m_j^2 + m_s^2 m_w^2 + m_j^2 m_w^2 - 2m_j^2)) - B_0^c m_j^2 (m_b^2 - 2m_b^2 m_j^2 + m_j^2 + m_s^2 m_w^2 + m_j^2 m_w^2 - 2m_j^2)(a_1 - 4a_2 s_w^4) + 2m_b m_s (2C_{10}^c + C_{11}^c m_b^2 + C_{22}^c m_j^2 + C_1^c (m_b^2 + m_j^2 - 2m_w^2) + C_{22}^c s + C_{12}^c (m_b^2 - m_s^2 + s))(a_3 + 6a_4 s_w^2 - 8a_4 s_w^4) - 6C_{00}^d m_b m_s (a_3 + 4a_4 s_w^2 (c_w^2 - s_w^2)) + 12m_b m_s m_w^2 (C_{11}^d + C_{12}^d) (a_3 + 6a_4 c_w^2 s_w^2 - 2a_4 s_w^4)
\end{align*}

\begin{align*}
g_2 &= -a_3 m_j^2 + 8a_4 m_j^2 s_w^4 - 6m_w^2 (C_{11}^d m_b^2 + C_{22}^d s + C_{12}^d (m_b^2 - m_s^2 + s))(a_3 + 8a_4 c_w^2 s_w^2) + m_b m_s^2 (B_0^b (m_b^2 - m_s^2) (m_j^2 - m_w^2) + (m_b^2 m_s^2 - m_b^2 m_j^2 + m_s^2 m_j^2 + m_j^2 m_w^2 - 2m_j^2)) (C_{00}^d - B_0^c) + m_w^2 (B_0^b (2m_b^2 - m_s^2) - B_0^c (2m_s^2 - m_b^2))(a_1 + 6a_2 s_w^2 - 4a_2 s_w^4) - 6C_{12}^d m_w^2 (a_3 (m_b^2 - m_s^2 + s) + 4a_4 s_w^2 ((m_b^2 - m_s^2) (c_w^2 - s_w^2) + 2s_w^2)) - 6C_{00}^d (a_3 (m_j^2 + 6m_w^2) + &
\[4a_4 s_w^2 (c_w^2 m_j^2 + 12c_w^2 m_w^2 - m_j^2 s_w^2)) + 2(2C_{00}^c + C_{11}^c m_b + C_{22}^c s + C_{12}^c (m_b^2 - m_s^2 + s))
\]
\[(a_3 m_j^2 + 2m_w^2) + 4a_4 s_w^2 (3m_w^2 - 2m_j^2 s_w^2 - 4m_w^2 s_w^2)) + 2C_0^c m_j^2 (a_3 (m_b^2 + m_s^2 - m_j^2 - 2m_w^2) +
\]
\[2a_4 s_w^2 (3m_s^2 - 3m_j^2 - 4m_s^2 s_w^2 - 4m_s^2 s_w^2 + 8m_w^2 s_w^2)) + 2C_0^c (a_3 s(m_j^2 + 2m_w^2) -
\]
\[2a_4 s_w^2 (3m_b^2 m_j^2 - 3m_b^2 m_j^2 - 6m_b^2 s + 4m_j^2 s_w^2 + 8m_w^2 s_w^2)) - 6C_1^d m_w^2 (a_3 (m_b^2 - m_s^2 + s) +
\]
\[4a_4 s_w^2 (m_b^2 c_w^2 - m_b^2 s_w^2 - 2c_w^2 s_w^2 + 2c_w^2 s_w^2)) - 6C_0^d m_w^2 (a_3 (s - m_s^2 - m_j^2) +
\]
\[4a_4 s_w^2 (2s_w^2 m_b^2 + 2m_j^2 m_s^2 - 2c_w^2 m_s^2)) + 2C_1^c (a_3 (2s_w^2 m_b^2 + m_b^2 m_s^2 + m_b^2 m_j^2 - 2m_w^2 m_s^2) +
\]
\[2a_4 s_w^2 (3m_b^2 m_s^2 - 3m_b^2 m_j^2 + 6m_b^2 m_s^2 m_w^2 - 4m_j^2 m_s^2 s_w^2 - 8m_w^2 s_w^2 -
\]
\[6m_w^2 m_s^2 + 8m_w^2 m_s^2 s_w^2))
\]

\[g_3 = -a_5 m_b m_s (2D_{23}^f + D_{3}^f) + m_s (B_0^m (m_b^2 - m_s^2) (m_b^2 - m_c^2) (m_b^2 + 2m_w^2) + B_0^m m_s^2 (m_b^2 - m_b^2 m_j^2 +
\]
\[m_j^2 + m_b^2 m_w^2 + 2m_j^2 m_w^2 - 2m_j^2) - B_0^m m_s^2 (m_b^2 - m_b^2 m_j^2 + m_j^2 + m_b^2 m_w^2 + m_j^2 m_w^2 - 2m_j^2)) (a_1 +
\]
\[2a_2 s_w^2 - 4a_2 s_w^2 + 2m_b m_s (2C_{00}^e + C_{11}^e m_b^2 + C_{00}^e m_j^2 + C_{11}^e m_j^2 + C_{00}^e m_j^2 + 2m_w^2) + C_{22}^e s + C_{00}^c +
\]
\[C_{12}^e (m_b^2 - m_s^2 + s)) (a_3 - 3a_4 + 10a_4 s_w^2 - 8a_4 s_w^2) - 6C_{00}^e m_b m_s (a_3 - 2a_4 (1 - 2s_w^2)) +
\]
\[12m_b m_s m_w^2 (C_0^d + C_{12}^d) (a_3 - a_4 (3 - 4s_w^2)(1 - 2s_w^2))
\]

\[g_4 = -a_5 (2D_{00}^f - (2D_{13}^f + D_{1}^f)(m_b^2 - t) + 2D_{23}^f t + D_{3}^f t) - a_3 m_j^2 - 4a_4 m_j^2 s_w^2 (1 - 2s_w^2) -
\]
\[6m_w^2 (C_{11}^d m_b^2 + C_{22}^d s + C_{12}^d (m_b^2 - m_s^2 + s)) (a_3 - 4a_4 s_w^2 (1 - 2s_w^2) +
\]
\[m_b m_s (B_0^m (m_b^2 - m_s^2) (m_b^2 - m_c^2) (m_b^2 + 2m_w^2) + B_0^m m_s^2 (m_b^2 - m_b^2 m_j^2 + m_j^2 + m_b^2 m_w^2 +
\]
\[m_j^2 m_w^2 - 2m_j^2) - B_0^m m_s^2 (m_b^2 m_s^2 - m_b^2 m_j^2 - m_b^2 m_s^2 m_w^2 + m_j^2 m_w^2 - m_j^2 m_w^2 - 2m_j^2)) (a_1 -
\]
\[3a_2 + 8a_2 s_w^2 - 4a_2 s_w^2) - 6C_{00}^e m_w^2 (a_3 (m_b^2 - m_s^2 + s) + 2a_4 (s_w^2 m_b^2 - c_w^2 m_b^2 - 2c_w^2 s +
\]
\[m_w^2 s_w^2 + 2c_w^2 m_b^2 s_w^2 - m_s^2 s_w^2 - 2c_w^2 m_s^2 s_w^2 + 4c_w^2 s_w^2 - 2m_b^2 s_w^2 + 4m_s^2 s_w^2) - 6C_{00}^e (a_3 (m_j^2 + 6m_w^2) +
\]
\[2a_4 (m_j^2 s_w^2 - c_w^2 m_j^2 - 12c_w^2 m_j^2 + 2c_w^2 m_j^2 s_w^2 + 24c_w^2 m_j^2 s_w^2 - 2m_j^2 s_w^2) +
\]
\[2(2C_{00}^e + C_{11}^e m_b^2 + C_{22}^e s)(a_3 (m_j^2 + 2m^2) + 2a_4 (2m_j^2 s_w^2 - 3m_s^2 + 10m_w^2 s_w^2 - 4m_s^2 s_w^2 - 8m_w^2 s_w^2) +
\]
\[2C_{00}^e m_j^2 (a_3 (m_b^2 + m_s^2 - m_j^2 - 2m_w^2) + a_4 (3m_j^2 - 3m_s^2 + 4m_w^2 s_w^2 + 10m_s^2 s_w^2 - 10m_j^2 s_w^2 - 8m_s^2 s_w^2 -
\]
\[8m_b^2 s_w^2 + 8m_j^2 s_w^2 + 16m_w^2 s_w^2)) + 2C_{00}^e (a_3 s(m_j^2 + 2m_w^2) + a_4 (3m_b^2 m_j^2 - 3m_b^2 m_j^2 -
\]
\[6m_j^2 s_w^2 - 6m_b^2 m_j^2 s_w^2 + 6m_b^2 m_j^2 s_w^2 + 4m_j^2 s_w^2 + 20m_j^2 s_w^2 - 8m_j^2 s_w^2 + 16m_j^2 s_w^2) +}
\[2C_{12}^e(m_b^2 - m_s^2 + s)(a_3(m_j^2 + 2m_w^2) - 2a_4(3m_w^2 - 2m_j^2s_w^2 - 10m_w^2s_w^2 + 4m_j^2s_w^4 + 8m_w^2s_w^4)) -
6C_{12}^d(m_b^2 - m_s^2 + s) + 2a_4(m_b^2s_w^2 - 3c_2w^2m_b^2 + 6c_2w^2m_b^2s_w^2 - 2m_b^2s_w^4 + 2c_2w^2t - 4c_2w^2s_w^2t +
2c_2w^4 - 4c_2w^4s_w^2u)) - 6C_{12}^d(m_b^2 - m_s^2 - m_j^2) + 2a_4(m_b^2s_w^2 + c_w^2m_b^2 - 2c_2w^2m_b^2s_w^2 - 2m_b^2s_w^4 -
2m_b^2s_w^4 + 4m_j^2s_w^4 + 2m_b^2s_w^4 - 4m_b^2s_w^2s_w^2 - 2sc_w^2 + 4sc_w^2s_w^2)) + 2C_{1}^e(a_3(m_b^2m_s^2 + m_j^2m_s^2 +
2m_b^2m_s^2 - 2m_j^2m_s^2) -
20m_b^2m_s^2s_w^2 - 8m_b^2s_w^2s_w^4 - 8m_b^2m_s^2s_w^4 + 16m_b^2m_s^2s_w^4 - 6m_b^2 - 20sm_b^2s_w^2 - 16sm_b^2s_w^4) \tag{10}
\]

\[g_5 = -24a_4m_b^2s_w^2(C_{2}^e m_j^2 - 2C_{2}^d m_w^2) - 4m_s(C_{1}^c m_b^2 + C_{0}^e m_j^2 + C_{1}^e(m_b^2 + m_j^2 - 2m_w^2))(a_3 + 6a_4s_w^2 -
8a_4s_w^4 - 6m_s(C_{1}^d m_b^2 + 2C_{1}^d m_w^2 + C_{1}^d(m_b^2 - m_j^2 + 2m_w^2))(a_3 + 4a_4m_w^2s_w^2 - 4a_4s_w^4) -
6C_{12}^d m_s(a_3(m_b^2 - m_j^2 - 2m_w^2) + 4a_4s_w^2(m_b^2s_w^2 - c_w^2m_j^2 - 4c_2w^2m_j^2 - m_b^2s_w^2 + m_j^2s_w^2)) -
4C_{12}^m(m_b^2 - m_j^2 - 2m_w^2) + 2a_4s_w^2(3m_b^2 - 6m_w^2 - 4m_b^2s_w^2 + 4m_j^2s_w^2 + 8m_w^2s_w^2)) \tag{11}
\]

\[g_6 = 24a_4m_b^2s_w^2(C_{2}^e m_j^2 - 2C_{2}^d m_w^2) - 12m_b^2m_w^2(C_{0}^d + 2C_{1}^d)(a_3 + 8a_4m_w^2s_w^2) -
4m_b^2(C_{0}^d + 2C_{1}^d)(a_3 - 8a_4s_w^4) - 6C_{11}^d m_b(a_3(m_j^2 + 2m_w^2) + 4a_4s_w^2(c_w^2m_j^2 + 4c_2w^2m_j^2 - m_j^2s_w^2)) +
6C_{12}^d m_b(a_3(m_b^2 - m_j^2 - 2m_w^2) + 4a_4s_w^2(c_w^2m_b^2 - c_w^2m_j^2 - 4c_2w^2m_w^2 - m_b^2s_w^2 + m_j^2s_w^2)) -
4C_{11}^d m_b(a_3(m_j^2 + 2m_w^2) + 4a_4s_w^2(3m_b^2 - 2m_j^2s_w^2 - 4m_w^2s_w^2)) + 4C_{12}^d m_b(a_3(m_j^2 - m_j^2 - 2m_w^2) +
2a_4s_w^2(3m_b^2 - 6m_b^2 - 4m_b^2s_w^2 + 4m_j^2s_w^2 + 8m_w^2s_w^2)) \tag{12}
\]

\[g_7 = 2a_5m_s(D_{12}^f + D_{21}^f) + 12a_4m_s(C_{2}^e m_j^2 - 2C_{2}^d m_w^2)(1 - 2s_w^2) - 4m_s(C_{11}^d m_b^2 + C_{0}^d m_j^2 + C_{1}^e(m_b^2 +
m_j^2 - 2m_w^2))(a_3 - 3a_4 + 10a_4s_w^2 - 8a_4s_w^4) - 6m_s(C_{11}^d m_b^2 + 2C_{0}^d m_w^2 +
C_{1}^d(m_b^2 - m_j^2 + 2m_j^2))(a_3 - 2a_4(s_w^2 - c_w^2)^2) - 6C_{12}^d m_s(a_3(m_b^2 - m_j^2 - 2m_w^2) + 2a_4(c_w^2m_j^2 -
c_w^2m_b^2 + 4c_2w^2m_b^2 + m_b^2s_w^2 + 2c_w^2m_b^2s_w^2 - m_b^2s_w^2 - 2c_w^2m_j^2s_w^2 - 8c_w^2m_b^2s_w^2 - 2m_b^2s_w^4 + 2m_j^2s_w^4)) -
4C_{12}^d m_s(a_3(m_b^2 - m_j^2 - 2m_w^2) + a_4(6m_b^2 - 3m_b^2 + 10m_b^2s_w^2 - 4m_j^2s_w^2 - 20m_b^2s_w^2 - 8m_b^2s_w^4 +
8m_j^2s_w^4 + 16m_b^2s_w^4)) \tag{13}
\]

\[g_8 = -2a_5m_b(D_{12}^f - D_{21}^f + D_{2}^f) - 12a_4m_b(C_{2}^e m_j^2 - 2C_{2}^d m_w^2)(1 - 2s_w^2) - 12m_b^2m_w^2(C_{0}^d + 2C_{1}^d)(a_3 -
\]
\[
4a_4w^2(1 - 2s_w^2)) - 4m_bm_2^2(C_0^e + 2C_1^e)(a_3 + 4a_4s_w^2(1 - 2s_w^2)) - 6C_{11}^d m_b(a_3(m_j^2 + 2m_w^2) - 2a_4(c_w^2m_j^2 + 4c_w^2m_w^2 - m_j^2s_w^2 - 2c_w^2m_j^2s_w^2 - 8c_w^2m_w^2s_w^2 + 2m_j^2s_w^4)) + 6C_{12}^d m_b(a_3(m_j^2 - m_j^2 - 2m_w^2) - 2a_4(c_w^2m_s^2 - c_w^2m_w^2 - m_s^2m_w^2 - 2c_w^2m_s^2m_w^2 + m_s^2s_w^2 + 2c_w^2m_j^2s_w^2 + 8c_w^2m_w^2s_w^2 + 2m_j^2s_w^4 - 2m_j^2s_w^4) - 4C_{11}^e m_b(a_3(m_j^2 + 2m_w^2) + 2a_4(2m_j^2s_w^2 - 3m_w^2 + 10m_j^2s_w^4 - 8m_j^2s_w^4)) + 4C_{12}^e m_b(a_3(m_j^2 - m_j^2 - 2m_w^2) + a_4(6m_w^2 - 3m_j^2 + 10m_j^2s_w^2 - 4m_j^2s_w^2 - 20m_j^2s_w^4 + 8m_j^2s_w^4 + 16m_j^2s_w^4))
\]

\[
g_9 = 2a_5(D_{12}^f - 2D_{13}^f + D_{22}^f - D_{23}^f + D_2^f)
\]

\[
g_{10} = -a_5 m_s(2D_{13}^f + 2D_{23}^f + D_3^f)
\]

\[
g_{11} = a_5 m_b(2D_{13}^f + D_1^f)
\]

where \(a_i\) is defined as
\[
a_1 = \frac{1}{192\pi^2 s_m b m_s^2 m_w^2 (m_b^2 - m_s^2 s_w^2)}, \quad a_2 = \frac{1}{768\pi^2 s_m b m_s^2 m_w^2 (m_b^2 - m_s^2)^2 (m_s^2 - m_z^2) \Gamma_z - s) c_w^2 s_w^2},
\]
\[
a_3 = \frac{1}{268 s_m b m_s^2 s_w^2}, \quad a_4 = \frac{1}{768\pi^2 s_m b m_s^2 (m_s^2 - m_z^2) \Gamma_z - s) c_w^2 s_w^2}, \quad a_5 = \frac{1}{32\pi^2 s_w^2}
\]

where \(c_w = \cos \theta_w\) and \(s_w = \sin \theta_w\). In the presentation of \(g_j\) above, we have used the definition of scalar integrals \(B_s, C_s,\) and \(D_s\) [3], and these functions, \(B_s, C_s,\) and \(D_s\), with superscripts a,b,...,f have the arguments
\[
(0, m_j^2, m_w^2), \quad (m_b^2, m_j^2, m_w^2), \quad (m_j^2, m_j^2, m_w^2), \quad (m_b^2, m_b^2, s, m_w^2, m_j^2, m_w^2) \]
\[
(m_b^2, m_b^2, s, m_j^2, m_j^2, m_j^2), \quad (0, m_b^2, m_b^2, 0, t, s, 0, m_w^2, m_j^2, m_w^2)
\]
respectively. Here \(m_j\) denotes the mass of up-type quark \(u, c, t\).

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FIG. 1. Typical Feynman diagram of process $e^+ e^- \rightarrow b\bar{s}$
FIG. 2. Cross section of the process $e^+ e^- \rightarrow b\bar{s}$ as a function of $\sqrt{s}$. 
FIG. 3. Differential cross section of the process $e^+ e^- \rightarrow b \bar{s}$, where $\sqrt{s} = 200$ GeV.
FIG. 4. $A_{FB}$ of the process $e^+ e^- \rightarrow b\bar{s}$ as a function of $\sqrt{s}$. 