Bloch-wall phase transition in the spherical model

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Abstract. The temperature-induced second-order phase transition from Bloch to linear (Ising-like) domain walls in uniaxial ferromagnets is investigated for the model of $D$-component classical spin vectors in the limit $D \to \infty$. This exactly soluble model is equivalent to the standard spherical model in the homogeneous case, but deviates from it and is free from unphysical behavior in a general inhomogeneous situation. It is shown that the thermal fluctuations of the transverse magnetization in the wall (the Bloch-wall order parameter) result in the diminishing of the wall transition temperature $T_B$ in comparison to its mean-field value, thus favoring the existence of linear walls. For finite values of $T_B$ an additional anisotropy in the basis plane $x, y$ is required; in purely uniaxial ferromagnets a domain wall behaves like a 2-dimensional system with a continuous spin symmetry and does not order into the Bloch one.

1. Introduction

The spherical model proposed in 1952 by Berlin and Kac [1] (see also [2, 3]) is extensively used up to now as the only exactly soluble model describing the phase transition in 3-dimensional magnetic systems. In contrast to the mean field approximation (MFA), the spherical model describes, in a simplified manner, the thermal fluctuations of spins, which can be taken into account exactly due to their Gaussian nature. The technique for the consideration of inhomogeneous systems described by the spherical model was developed by Barber and Fisher [4] for the surface-induced inhomogeneity in layered magnets and elaborated by Abraham and Robert [5] for the problem of phase separation (i.e., the domain wall (DW) formation) in the spherical model. Later the inhomogeneous states of the bounded spherical model induced by antiperiodic [6] and twisted [7] boundary conditions were investigated.

The results obtained for inhomogeneous states of the spherical model possess some unexpected features. The phase transition temperature $T_c$ of a 4-dimensional ferromagnetic slab consisting of $N \gg 1$ layers turns out to be higher than that of the bulk in the case of free-edge boundary conditions [8]. The two-domain state induced by the magnetic field $\pm H$ in two half-spaces is characterized by the DW width diverging and the domain magnetization vanishing in the limit $H \to 0$, i.e., in contrast to the underlying Ising model the spherical model does not exhibit the phase separation [9]. As was argued already in [8], such features are the consequence of the global spin constraint [1], which obviously becomes not so good in the inhomogeneous case.

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Another version of the spherical model — the model of isotropic $D$-component classical spin vectors in the limit $D \to \infty$ — was proposed by Stanley [8, 9], who showed that in the homogeneous case this model is equivalent to the spherical model by Berlin and Kac [1]. The normalization condition for a spin on a lattice site $i$, $|m_i| = 1$, becomes, in the case $D \to \infty$, very similar to the global spin constraint, which is the reason of the equivalence of the two models. However, since the spin normalization condition does not mix the spin variables on different lattice sites, the $D$-vector model by Stanley with $D \to \infty$ is more physically appealing than the original spherical model. Moreover, the two models become non-equivalent in a general inhomogeneous situation, where, as was shown by Knops [10], the $D = \infty$ model corresponds to some generalization of the spherical model using a local spin constraint. This idea was substantiated in the work by Costache et al. [11], who calculated the Curie temperature $T_c(N)$ of a ferromagnetic film of $N \gg 1$ layers with free boundary conditions in 4 dimensions using a set of independent spin constraints in each layer. The numerically calculated values of $T_c(N)$ monotonically increase with $N$ to the bulk value $T_c(\infty)$, which is physically expected and differs from the result of Barber and Fisher [4] for the standard spherical model.

The further advantages of the $D$-vector model are the possibilities of consideration of finite-$D$ and anisotropic systems. The latter is important, in particular, for the calculation of finite-size corrections to $T_c$ of ferromagnetic films mentioned above in the actual 3-dimensional case. Since such a film with $N < \infty$ is a 2-dimensional system, the $T_c$-corrections are finite only in the presence of the stabilizing uniaxial anisotropy [13]. In spite of its advantages in comparison to the standard spherical model, the $D$-vector model with $D \gg 1$ was much less used than mentioned. As exclusions one can cite the works by Abe and Hikami [13, 14] and by Okabe and Masutani [15] dealing with the $1/D$ expansion for 3-dimensional systems and the work by Okamoto [16] where the uniaxial spherical model with a transverse field was considered.

It should be noted that practically all the researchers dealing with the isotropic $D$-vector model used the designations $N$ or $n$ instead of the original $D$. Such a modification is, however, not justified in a general anisotropic case, where the number $n$ of the relevant order parameter components determining the symmetry and thus the universality class of a system can be different from $D$. As an example one can consider the rather general “$n$-$D$ model” [17, 18] having the first $n \leq D$ components coupled by the exchange interaction with the equal strength and the rest $D - n$ components “free”. Among realizations of the $n$-$D$ model are, in particular, the $x,y$ model ($D = 3$, $n = 2$) and the plane rotator one ($n = D = 2$) belonging to the same universality class determined by $n$ but having different values of $T_c$ depending on both $n$ and $D$. Correspondingly, in a general case the $1/n$ expansion of the critical indices is not the same as the $1/D$ expansion of non-universal quantities.

The general qualitative result of [1], the absence of the phase separation in the spherical model (but not the disappearance of the domain magnetization !), can be explained by the fact that this model behaves in the bulk like the isotropic $D = \infty$ model [8, 17], which obviously exhibits no phase separation. For the $D$-vector model the separation of a specimen into domains with opposite magnetizations by domain walls of a finite width requires an easy-axis anisotropy, which makes the intermediate orientation of the magnetization in the wall energetically unfavourable in comparison to that in domains. Clearly, this actual situation cannot be treated either with the help of the spherical model in its standard formulation [1], or with the improved version [14, 19], which is equivalent to the isotropic $D = \infty$ model in the general inhomogeneous case.

The problem arising here — the study of the influence of thermal fluctuations on the domain wall structure — is not only important for comparing the properties
of different versions of the spherical model. The physics of domain walls at elevated
temperatures is itself rather interesting and unexplored area, whereas since the seminal
work by Landau and Lifshitz [20], the majority of researchers have addressed
the zero-temperature statics and dynamics of DWs based on the assumption of the constant magnitude of the magnetization in the wall.

The first theoretical investigation of the temperature variation of the structure
of domain walls is due to Bulaevskii and Ginzburg [21], who with the help of the
phenomenological version of MFA, using a macroscopic Landau free energy in the
vicinity of $T_c$, predicted a phase transition from Bloch to linear DWs in uniaxial
ferromagnets at some $T_B < T_c$. Qualitatively this phase transition can be explained
by the fact that the spins in the center of a Bloch wall, which are forced to lie
perpendicular to the easy axis, experience a molecular field smaller than in domains
and hence order at some temperature $T_B$ less than $T_c$, which leads to linear (Ising-like)
walls (LW) in the region $T_B < T < T_c$. For ferromagnets whose anisotropy energy is
much smaller than the exchange interaction, the LW temperature region is narrow.

Complementary, the transition from Bloch to linear walls at $T = 0$ depending on
the anisotropy was investigated by van den Broek and Zijlstra [22] with numerical
methods. It was found that LWs are realized if the ratio of the anisotropy energy
to the exchange one exceeds 2/3; the DW width $\delta$, is in this case, comparable with
the lattice spacing $a_0$. Later this transition was discovered by Sarker, Trullinger and
Bishop [23] in the framework of a formal soliton theory independently of Bulaevskii
and Ginzburg. The problem was addressed also by Niez and Lajzerowicz [24, 25],
where the factor 2/3 mentioned above was calculated analytically.

The first indirect experimental evidence for the transition from Bloch to linear do-
main walls was obtained from the optical observations of the temperature dependence
of the period of the domain structure in YFeO$_3$ just below $T_c$ [26]. Lately the LWs
were observed in the dynamical susceptibility experiments on the low-temperature
ferromagnets GdCl$_3$ [27] and LiTbF$_4$ [28]. In [29, 30] the DW mobility was calcu-
lated in the whole temperature range, which exhibited a deep minimum at $T_B$. Such
a minimum was observed recently in the dynamical susceptibility experiments on the
high-temperature Ba and Sr hexaferrites [31, 32].

Recent experiments also provided evidence of strong fluctuational effects about
the DW phase transition. The transition temperature $T_B$ was substantially lower than
its mean-field estimate, and the critical index $\beta_B$ of the Bloch-wall order parameter
(the transverse magnetization in the centre of the wall) was about 0.1 in contrast
to the MFA value 1/2. Such strong fluctuations are actually not surprising since a domain wall is a 2-dimensional object. The analysis by Lawrie and Lowe [33]
making use of renormalization-group arguments has led to the clear result that a
domain wall in a biaxial ferromagnetic model having an additional anisotropy in the
x, y plane (which is usually the effective one due to the magnetostatic field [20])
belongs to the universality class of the 2-dimensional Ising model, and hence one can
expect $\beta_B = 1/8$. In contrast, in a purely uniaxial model without the dipole-dipole
interaction a domain wall behaves like a 2-dimensional plane rotator model and can
show only the Kosterlitz-Thouless phase transition without ordering to a Bloch wall.

The absence of the long-range order (i.e., the transverse magnetization compo-
ment) in a domain wall in a purely uniaxial ferromagnet can be demonstrated with
the help of the linear spin-wave theory. The thermal disordering of Bloch walls is due
to the so-called Winter magnons [34], the excitations localized on the domain wall
with the dispersion law $\varepsilon_q^2 \propto Aq^2(Aq^2 + K_{\perp})$ ($A$ is the inhomogeneous exchange constant,
$K_{\perp}$ is the in-plane anisotropy constant and $q$ is a 2-dimensional wavevector).
The first factor in $\varepsilon_q^2$ corresponds to the free translational motion of the wall and the
second describes the rotation of the magnetization in the center of the wall in the x, y
plane. It can be seen that the number of Winter magnons, which in the classical case
is proportional to \( \int d^2q/\varepsilon_q \), diverges logarithmically at small \( q \), if \( K_{\perp} \) tends to zero. Thus, in a purely uniaxial ferromagnetic model the linear walls cannot order to the Bloch ones at any non-zero temperature. For a small non-zero in-plane anisotropy \( K_{\perp} \) the shift of \( T_B \) from its MFA value due to fluctuations should be very essential.

The aim of this work is to find an exact solution for the domain wall magnetization profile in biaxial ferromagnets at non-zero temperatures and their transition temperature \( T_B \) in the framework of the spherical model in its \( D \)-vector version. Instead of modifying the approach of [4, 5] based on the calculation of the partition function with the steepest descent method, we will use the diagram technique for classical spin systems [13, 18]. This diagram technique, which is a generalization of the “Ising part” of the spin operator diagram technique of Vaks et al [36, 37], makes it possible to locate and sum up all the diagrams surviving in the limit \( D \to \infty \) and can be reformulated for the present purposes for inhomogeneous situations.

The approximation obtained by summing up such diagrams (without going to the limit \( D \to \infty \)) is the so-called self-consistent Gaussian approximation (SCGA), which was first formulated by Horwitz and Callen [38] for the Ising model (\( D = 1 \)). SCGA yields rather good results for the thermodynamic quantities of the Ising [33] and the classical Heisenberg (\( D = 3 \)) [40] models on 3-dimensional lattices in the whole temperature range and can be of importance for a possible improvement of the presently obtained results for the domain wall structure in the spherical limit with regard to systems with finite \( D \). A detailed description of the classical spin diagram technique and SCGA can be also found in the recent publication [13].

The rest of the paper is organized as follows. In Section 2 the diagram technique for classical spin systems and the construction of SCGA in the inhomogeneous case is described. In Section 3 SCGA is simplified for \( D \to \infty \), and a closed system of equations for magnetization and spin-spin correlation function describing the domain wall in the spherical limit is derived. In Section 4 the magnetization profile of a fluctuating domain wall is calculated and the dependence of the transition temperature \( T_B \) on the anisotropy parameters is analyzed. In Section 5 further problems of the DW statics and dynamics at elevated temperatures are discussed.

### 2. Classical spin diagram technique and SCGA

The appropriate classical \( D \)-vector Hamiltonian with biaxially anisotropic ferromagnetic exchange interaction can be written in the form

\[
\mathcal{H} = -\frac{1}{2} \sum_{ij} J_{ij} \left( m_{zi} m_{zj} + \eta m_{yi} m_{yj} + \sum_{\alpha = 3}^{D} \eta_{\alpha} m_{\alpha i} m_{\alpha j} \right),
\]

where \( i, j \) are the lattice sites, \( \mathbf{m}_i \) is the normalized \( D \)-component vector, \( |\mathbf{m}_i| = 1 \), the dimensionless anisotropy factors satisfy \( \eta_{\alpha} \leq \eta \leq 1 \) and all \( \eta_{\alpha} \) are, for simplicity, taken to be equal to each other. For \( D = 3 \) equation (2.1) reduces to the anisotropic classical Heisenberg model. In the chosen geometry the average magnetization in the bulk is directed parallel or antiparallel to the easy axis \( z \), in the centre of a Bloch wall it takes on one of two possible orientations along the second easy axis \( y \). All variables describing the magnetization profile of a plane DW are functions of the coordinate \( x \) only. The temperature-normalized molecular field \( \xi_i \) acting on a spin on the site \( i \) from its neighbours is given by

\[
\xi_i = -\frac{\partial (\beta \mathcal{H})}{\partial \mathbf{m}_i} = \beta \sum_j J_{ij} \left( m_{zj} \mathbf{e}_z + \eta m_{yj} \mathbf{e}_y + \sum_{\alpha = 3}^{D} \eta_{\alpha} m_{\alpha j} \mathbf{e}_{\alpha} \right),
\]

where \( \beta \equiv 1/T \) and \( \mathbf{e} \) are unit vectors in appropriate directions. The mean field approximation consists in neglecting fluctuations of the molecular field \( \xi_i \); replacing
m_{zj} \Rightarrow \langle m_{zj} \rangle, \ m_{yj} \Rightarrow \langle m_{yj} \rangle \text{ and } m_{\alpha j} \Rightarrow 0 \text{ in (2.2), one arrives at the inhomogeneous Curie-Weiss equation}
\langle m_i \rangle = B(\xi_i) \xi_i/\xi_i, \quad (2.3)

where \( B(\xi) \) is the Langevin function. For small anisotropy \((1 - \eta \ll 1)\) the magnetization varies slowly on the scale of the lattice spacing, and the continuous approximation can be applied to (2.3). In this case the zero-temperature results of Landau and Lifshitz [20] and the finite-temperature ones of Bulaevskii and Ginzburg [21] for the DW magnetization profile are recovered (see below).

Fluctuations of the molecular field (2.2) can be taken into account within the framework of a perturbative scheme based on the diagram technique for classical spin systems [17, 35, 18]. The perturbative expansion of the thermal average of any quantity \( A \) characterizing a classical spin system (e.g., \( A = m_i \)) can be obtained by rewriting (2.1) as \( H = H_0 + V_{\text{int}} \), where \( H_0 \) is the MFA Hamiltonian with the averaged molecular field determined by (2.3), and expanding the expression
\[ \langle A \rangle = \frac{1}{Z} \int \prod_{j=1}^N dm_j A \exp(-\beta H), \quad \|m_j\| = 1 \quad (2.4) \]
in powers of \( V_{\text{int}} \). The averages of various spin vector components \( \alpha, \beta, \gamma, \ldots = 1, \ldots, D \) on various lattice sites \( i, j, k, \ldots \) with the Hamiltonian \( H_0 \) can be expressed through spin cumulants, \( \Lambda_{\ldots} \), (see below) in the following way:
\[ \langle m_{\alpha i} \rangle_0 = \Lambda_{\alpha i}, \quad \langle m_{\alpha i} m_{\beta j} \rangle_0 = \Lambda_{\alpha \beta i} \delta_{ij} + \Lambda_{\alpha i} \Lambda_{\beta j}, \quad \langle m_{\alpha i} m_{\beta j} m_{\gamma k} \rangle_0 = \Lambda_{\alpha \beta \gamma i} \delta_{ijk} + \Lambda_{\alpha \beta i} \Lambda_{\gamma k} \delta_{ij} + \Lambda_{\alpha \gamma i} \Lambda_{\beta j} \delta_{ki} + \Lambda_{\alpha i} \Lambda_{\beta j} \Lambda_{\gamma k}, \quad (2.5) \]
etc., \( \delta_{ij}, \delta_{ijk}, \text{etc.} \), are the site Kronecker symbols equal to 1 for all site indices coinciding with each other and to zero in all other cases. For one-site averages \( (i = j = k = \ldots) \) equation (2.5) reduces to the well-known representation of moments through cumulants (semi-invariants), generalized for the multi-component case. In the graphical language (see, e.g., Fig. 1) the decomposition (2.5) corresponds to all possible groupings of small circles (spin components) into oval blocks (cumulant averages). The circles coming from \( V_{\text{int}} \) (the “inner” circles) are connected pairwise by the wavy interaction lines representing the quantity \( \eta_{\alpha \beta} J_{ij} \). In diagram expressions the summation over the coordinates \( i \) and component indices \( \alpha \) of inner circles is carried out. One should not take into account disconnected diagrams (i.e., those containing disconnected parts with no “outer” circles belonging to \( A \) in (2.4)), since these diagrams are compensated for by the expansion of the partition function \( Z \) in the denominator of (2.4). Consideration of numerical factors shows that each diagram contains the factor \( 1/n_s \), where \( n_s \) is the number of the symmetry group elements of a diagram (see (2.9) and (2.10), the symmetry operations do not concern outer circles). In the homogeneous case it is more convenient for practical calculations to use the Fourier representation and calculate integrals over the Brillouin zone rather than lattice sums. As the lattice sums are subject to the constraint that the coordinates of the circles belonging to the same block coincide with each other (due to the Kronecker symbols in (2.3), in the Fourier representation the sum of wavevectors coming to or going out of any block along interaction lines is zero. The cumulant spin averages in (2.3) can be obtained by differentiating the generating function \( \Lambda(\xi) = \ln Z_0(\xi) \) over appropriate components of the dimensionless molecular field \( \xi \):
\[ \Lambda_{\alpha_1 \alpha_2 \ldots \alpha_p}(\xi) = \frac{\partial^p \Lambda(\xi)}{\partial \xi_{\alpha_1} \partial \xi_{\alpha_2} \ldots \partial \xi_{\alpha_p}}, \quad (2.6) \]
Fig. 1: Self-consistent Gaussian approximation (SCGA) for classical spin systems. (a) and (c): block summations for the renormalized magnetization and pair spin cumulant averages; (b): Dyson equation for the spin-spin correlation function.

where $Z_0(\xi) = \text{const} \times \xi^{-(D/2-1)}I_{D/2-1}(\xi)$ is the partition function of a $D$-component classical spin and $I_\nu(\xi)$ is the modified Bessel function. The two lowest-order cumulants which will be needed below can be written explicitly as

$$\Lambda_\alpha(\xi) = B(\xi)\frac{\xi_\alpha}{\xi}, \quad (2.7)$$

$$\Lambda_{\alpha\beta}(\xi) = \frac{B(\xi)}{\xi} \left( \delta_{\alpha\beta} - \frac{\xi_\alpha \xi_\beta}{\xi^2} \right) + B'(\xi)\frac{\xi_\alpha \xi_\beta}{\xi^2},$$

where $\delta_{\alpha\beta}$ is the spin component Kronecker symbol, $B(\xi) = d\Lambda(\xi)/d\xi$ is the Langevin function for $D$-component spins and $B'(\xi) = dB(\xi)/d\xi$. The expressions for the 3- and 4-spin cumulants can be found in [35]. It should be stressed that the spin cumulants (2.7) appearing in the diagrams generated originally by the expansion of (2.4) in powers of $V_{\text{int}}$ (the unrenormalized diagrams) simplify, since there is only few non-zero components of the molecular field $\xi$ (for a domain wall in the chosen geometry $\xi_z$ and $\xi_y$). The complete form of spin cumulants (2.7) is needed, however, for the construction of SCGA allowing for the fluctuations of other components of the molecular field. For Ising systems the classical spin diagram technique coincides with the “Ising part” of the standard SDT [35, 37] and can be used with Brillouin functions $B_S$ of a general spin $S$. In the book [37] more technical details concerning the construction of SDT for Ising systems can be found, which play the same role in the present classical SDT.

The next step beyond MFA is the self-consistent Gaussian approximation taking into account pair correlations of the molecular field acting on a given spin from its neighbors, which implies the Gaussian statistics of the molecular field fluctuations (see Fig. 1). Since subsequently we are going to take the limit $D \rightarrow \infty$, only fluctuations of the molecular field components with $\alpha = 3, \ldots, D$ should be taken into account, because their total contribution exceeds that of the fluctuations of $z$ and $y$ components by a factor of the order of $D$. The diagram sequence represented in Fig. 1 is equivalent
to a closed system of nonlinear equations for the averaged magnetization $\langle m_i \rangle$ and the correlation function $S^{\alpha\alpha}_{jj'} \equiv \langle m_\alpha m_{\alpha j} \rangle$ of the spin components with $\alpha = 3, \ldots, D$. The diagrammatic equation in Fig. 1a is the generalization of the Curie-Weiss equation (2.3) for the magnetization (the angle brackets are dropped):

$$ m_i = \partial \tilde{\Lambda}(\xi_i, l_{\alpha i}) / \partial \xi_i = \tilde{\Lambda}_z(\xi_i, l_{\alpha i}) e_z + \tilde{\Lambda}_y(\xi_i, l_{\alpha i}) e_y $$  \hspace{1cm} (2.8)

where the (averaged) molecular field $\xi_i$ is given by the expression (2.2) without the last term and $l_{\alpha i}$ is related to the dispersion of the molecular field fluctuations on the site $i$:

$$ l_{\alpha i} \equiv \frac{1}{2} \langle \xi_{\alpha i} \xi_{\alpha i} \rangle = \frac{1}{2} ! \eta_{\alpha \beta}^{2} \sum_{j j'} J_{ij} J_{i j'} S^{\alpha \alpha}_{jj'} . $$  \hspace{1cm} (2.9)

The spin cumulant averages $\tilde{\Lambda}_{\ldots}$ on a site $i$ (see Fig. 1a,c) renormalized by Gaussian fluctuations of the molecular field are given by

$$ \tilde{\Lambda}_{\ldots} = \Lambda_{\ldots} + D \sum_{\alpha=3}^{D} \Lambda_{\ldots \alpha} l_{\alpha} + D \sum_{\alpha, \beta=3}^{D} (1 - \delta_{\alpha \beta} + \frac{1}{2} ! \delta_{\alpha \beta}) \Lambda_{\ldots \alpha \beta} l_{\alpha} l_{\beta} + \ldots, $$  \hspace{1cm} (2.10)

where taking into account only the first term corresponds to MFA. These series, describing the influence of pair-correlated fluctuations of different components of the molecular field, can be rewritten as

$$ \tilde{\Lambda}_{\ldots} = \prod_{\alpha=3}^{D} \sum_{n_{\alpha}=0}^{\infty} \frac{1}{n_{\alpha}!} \left( l_{\alpha} \frac{\partial^2}{\partial \xi_{\alpha}^2} \right)^{n_{\alpha}} \Lambda_{\ldots}(\xi_i) = \exp \left[ \sum_{\alpha=3}^{D} l_{\alpha} \frac{\partial^2}{\partial \xi_{\alpha}^2} \right] \Lambda_{\ldots}(\xi_i). $$  \hspace{1cm} (2.11)

Such exponential differential operators were considered by Horwitz and Callen [38] for the Ising model. Generalization of their results for the multi-component case yields a closed formula

$$ \tilde{\Lambda}_{\ldots} = \frac{1}{\pi(D-2)/2} \int_{D-2}^{\infty} e^{-\zeta_i^2} \Lambda_{\ldots}(\zeta_i), $$  \hspace{1cm} (2.12)

where $\zeta_i$ is the spread molecular field given by

$$ \zeta_i \equiv \xi_i + 2 \sum_{\alpha=3}^{D} l_{\alpha 1/2} r_{\alpha} e_{\alpha}, $$  \hspace{1cm} (2.13)

and the integration in (2.12) is performed with respect to the $(D - 2)$-component vector variable $r \equiv \{r_{\alpha}\}$.

The Dyson equation for the spin-spin correlation function $S^{\alpha \alpha}_{jj'}$ entering (2.9) is represented in Fig. 1b and has the analytical form

$$ S^{\alpha \alpha}_{jj'} = \tilde{\Lambda}_{\alpha \alpha j} \delta_{jj'} + \tilde{\Lambda}_{\alpha \alpha i} \eta_{\alpha \beta} \sum_{j j'} J_{ij} J_{i j'} S^{\alpha \alpha}_{jj'}. $$  \hspace{1cm} (2.14)

Applying this equation two times, one can simplify the expression for the quantity $l_{\alpha i}$ to

$$ l_{\alpha i} = \frac{1}{2 \tilde{\Lambda}_{\alpha \alpha i}} \left( \frac{S^{\alpha \alpha}_{ii}}{\tilde{\Lambda}_{\alpha \alpha i}} - 1 \right) . $$  \hspace{1cm} (2.15)

The system of SCGA equations for a domain wall in a biaxial ferromagnet stated above simplifies for a small anisotropy $(1 - \eta \ll 1)$ and in the limit $D \to \infty$, which will be pursued in the next section. For $\eta_{\alpha} = 0$ the molecular field fluctuations measured by the quantity $l_{\alpha i}$ vanish according to (2.14) and (2.15), and the magnetization equation (2.8) reduces to the MFA result (2.3).
3. The spherical limit

In the spherical limit $D \to \infty$ the Langevin function $B(\xi)$ (see (2.3), (2.7)) simplifies to

$$B(\xi) \equiv \frac{x}{1 + \sqrt{1 + x^2}},$$

(3.1)

where the scaled variable $x$ (which should not be confused with the coordinate $x$ appearing below) is given by $x \equiv 2\xi/D$. Correspondingly, the spin cumulant averages (2.6) considered as functions of $x$ can be estimated as

$$\Lambda_{\alpha_1\alpha_2...\alpha_k} \propto D^{1-k}.$$  

(3.2)

With the help of this estimate it can be shown that in the limit $D \to \infty$ SCGA becomes exact, since all other more complicated diagrams die out, at least as $1/D$ [35]. Indeed, a unification of two oval blocks into a larger one, which leads to a more complicated diagram (e.g., $\Lambda_{\alpha\alpha} \times \Lambda_{\beta\beta} \Rightarrow \Lambda_{\alpha\alpha\beta\beta}$ in Fig. 1), leads to the appearance of an additional factor $1/D$, since

$$\Lambda_{\alpha_1\alpha_2...\alpha_{m+n}} \propto \Lambda_{\alpha_1\alpha_2...\alpha_m} \times \Lambda_{\alpha_{m+1}...\alpha_{m+n}}/D.$$  

(3.3)

In [35] some of such higher-order diagrams were considered in the framework of the $1/D$ expansion for low-dimensional classical ferro- and antiferromagnets.

For the consideration of the limit $D \to \infty$ it is convenient to introduce the well behaved dimensionless temperature variable

$$\theta \equiv T/T_{\text{MFA}},$$

where $T_{\text{MFA}} = J_0/D$ and $J_0$ is the zero component of the exchange interaction, as well as the further $D$-independent variables:

$$G_{i} \equiv (D/\theta)\tilde{\Lambda}_{\alpha\alpha i}, \quad \tilde{l}_i \equiv l_{\alpha i}/D, \quad s_{i\alpha} \equiv D S_{i\alpha}^{\alpha\alpha}.$$  

(3.4)

Expression (2.15) can now be rewritten as

$$\tilde{l}_i = \frac{1}{2\theta G_{i}} \left( \frac{s_{i\alpha}}{\theta G_{i}} - 1 \right),$$

(3.5)

and the expression for the square of the spread value of the argument $x$ in (2.12) reads

$$x_{i}^2 \equiv \left( \frac{2\xi_{i}}{D} \right)^2 = \left( \frac{2\theta}{\theta} \sum_{j} \lambda_{ij} m_{xj} \right)^2 + \left( \frac{2\eta}{\theta} \sum_{j} \lambda_{ij} m_{yj} \right)^2 + \frac{16\tilde{\xi}_{i}}{D} \sum_{r=3}^{D} r_{r}^2,$$

(3.6)

where $\lambda_{ij} = J_{ij}/J_0$. It can be seen that the contributions of the fluctuations of the $\alpha$-components of the molecular field to (2.6), each of them being small as $1/D$, are essential due to their large number. Now for $D \gg 1$ the Gaussian integrals (2.12) can be easily calculated by applying the identity

$$\frac{1}{\pi^{1/2}} \int_{-\infty}^{\infty} dx \, e^{-x^2} f(ax^2) \simeq f(a/2), \quad a \ll 1$$

(3.7)

successively $D - 2$ times. Thus, the integration leads simply to the replacement $r_{a}^2 \Rightarrow 1/2$ in (2.6). Now for the quantity $G_{i}$ of (3.4) with the use of the second of the formulae (2.7) and of the asymptotic expression (3.1) one gets

$$G_{i} = \frac{2}{\theta} \frac{1}{1 + \sqrt{1 + x_{i}^2}}.$$  

(3.8)
The magnetization equation (2.8) can be simplified by using the first of the formulae (2.7), as well as (3.1), to

\[ m_{zi} = G_i \sum_j \lambda_{ij} m_{zj}, \quad m_{yi} = \eta G_i \sum_j \lambda_{ij} m_{yj}. \]  

(3.9)

Finally, determining \( x^2 \) from (3.8) as a function of \( G_i \), equating it to (3.6) with \( r_a^2 = 1/2 \) and using (3.5) and (3.9), one arrives at the equation

\[ s_{ii} + m_i^2 = 1, \]  

(3.10)

which is nothing but the kinematic identity \( m_i^2 = 1 \) in the limit \( D \to \infty \). The normalized correlation function \( s_{ii}' \) determined by (3.4) satisfies the linear equation following from (2.14),

\[ s_{ii}' = \theta G_i \delta_{ii} + \eta \alpha G_i \sum_j \lambda_{ij} s_{ji}' \]  

(3.11)

with the variable coefficient \( G_i \).

Equations (3.9)–(3.11) constitute the closed system of equations, which can be applied for the calculation of the domain wall magnetization profile in the spherical limit.

In the homogeneous case (or in one of the domains) \( m_y = 0 \) and \( m_z \) and \( G \) are constants. In this case equation (3.11) can be easily solved with the help of the Fourier transformation, which results in

\[ s_{ii} = v_0 \int \frac{d\mathbf{q}}{(2\pi)^3} s_{i} = \theta G P(\eta \alpha G), \quad P(X) \equiv v_0 \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{1}{1 - X \lambda_{\mathbf{q}}}, \]  

(3.12)

where \( v_0 \) is the unit cell volume and \( \lambda_{\mathbf{q}} \equiv J_{\mathbf{q}}/J_0 \). In the long-wavelength limit \( \lambda_{\mathbf{q}} \approx 1 - \alpha q^2 \), where \( \alpha \sim a_0^2 \) and \( a_0 \) is the lattice spacing. The lattice integral \( P(X) \) has the following properties:

\[ P(X) \approx \begin{cases} 1 + X^2/z, & X \ll 1 \\ W - c_0 (1 - X)^{1/2}, & 1 - X \ll 1 \end{cases}, \]  

(3.13)

where \( z \) coincides with the number of the nearest neighbors for the nn interactions and \( W \) (the Watson integral) and \( c_0 = v_0/(4\pi \alpha^{3/2}) \) are lattice-dependent constants. For a simple cubic (sc) lattice \( v_0 = a_0^3 \) and \( \alpha = a_0^2/6 \), hence \( c_0 = (2/\pi)(3/2)^{3/2} \).

Since the sum in the right-hand part of the first of the equations (3.9) equals \( m_z \), this equation is satisfied only if \( m_z = 0 \) (above \( T_c \)) or \( G = 1 \) (below \( T_c \)). In these cases from equation (3.10) one gets the temperature-dependent magnetization \( m \equiv m_z \):

\[ m = (1 - \theta/\theta_c)^{1/2}, \quad \theta \leq \theta_c \equiv 1/P(\eta \alpha). \]  

(3.14)

It can be seen that in the fully isotropic case (\( \eta = \eta_a = 1 \)) the value of the phase transition temperature in the bulk \( \theta_c = 1/P(\eta \alpha) \) reduces to the well-known result \( \theta_c = 1/W \).

The width of a Bloch wall \( \delta_B \) in a uniaxial ferromagnet is determined by the balance between the anisotropy and inhomogeneous exchange energies. For small anisotropy \( (1 - \eta \ll 1) \) the condition \( \delta_B \gg a_0 \) is fulfilled. In this case the change of the domain wall magnetization at a distance of \( a_0 \) is small, and for systems with nn interactions one can rewrite the lattice sum in (3.3) about some point \( r \equiv r_i \) as

\[ \sum_j \lambda_{ij} \mathbf{m}_j \approx \mathbf{m}(r) + \alpha \Delta \mathbf{m}(r), \]  

(3.15)
where the second term with the Laplace-operator $\Delta$ is small in comparison to the first one. The quantity $G_i$ in (3.4) determined by (3.4) is in the small-anisotropy case also a continuous function of the coordinate with the scale $\delta$. Moreover, as can be seen from (3.9) and (3.15), the deviation of $G$ from its bulk-value 1 is small, i.e.,

\[ G(r) = 1 + G_1(r), \quad G_1 \ll 1. \] (3.16)

Now one can rewrite the equations (3.9) in terms of the normalized magnetization $n \equiv m/m_e$, where $m_e$ is the equilibrium bulk magnetization given by (3.14). The result in the chosen geometry is of the form

\[ \alpha n''(x) = -G_1(x)n_z(x), \quad \alpha n''(y) = -G_1(x)n_y(x) + (1 - \eta)n_y(x). \] (3.17)

The kinematic equation (3.10) can be represented in terms of $n$ as

\[ \epsilon(1 - n^2) = \theta G_i/\theta - 1, \quad \epsilon \equiv \theta_c/\theta - 1. \] (3.18)

Unlike the magnetization equation (3.9), the equation for the correlation function (3.11) cannot, in general, be written in a continuous form of the type (3.17). In the general case we are going to consider, where $\eta$ is not necessarily close to 1, the correlation length of the $\alpha$ spin components below $\theta_c$, \( \xi_{\alpha} = \sqrt{\alpha/\eta_c/(1 - \eta)} \) (3.19)

(not to be confused with components of the normalized molecular field $\xi$), which can be determined from $s_\mathbf{q}$ in (3.12), can be comparable with the lattice spacing $a_0$. Moreover, even in the case $1 - \eta_c \ll 1$ the continuous approximation for $s_i\mathbf{v}$ does not yield the correct bulk result (3.12) which is formed by the integration over the whole Brillouin zone and not only over the logwavelength region $q \ll 1$. But it can, nevertheless, be applied for the calculation of the wall properties, as we shall see below.

### 4. Domain wall magnetization profile

Before proceeding to the solution of equations (3.17), (3.18) and (3.11) in the general situation, let us consider at first the case $\eta = 0$, where the spin fluctuations play no role and the situation is described exactly by MFA. Here the solution of (3.11) yields $s_{ii} = \theta G_i$, and with the help of (3.18) one gets $G_1(x) = \epsilon[1 - n^2(x)]$. Adopting it in (3.17) and solving the resulting equations, one arrives at the magnetization profile \[ n_z = \pm \tanh(x/\delta), \quad n_y = \pm \rho/\cosh(x/\delta), \] (4.1)

where

\[ \rho = \begin{cases} \sqrt{1 - \tau}, & \tau \equiv 2(1 - \eta)/\epsilon \leq 1, \\ 0, & \tau \geq 1, \end{cases} \quad \delta = \begin{cases} \delta_B = \sqrt{\alpha/(1 - \eta)}, & \tau \leq 1, \\ \delta_L = \sqrt{2\alpha/\epsilon} = \delta_B \tau^{1/2}, & \tau \geq 1. \end{cases} \] (4.2)

It can be seen that the crossover from the Bloch ($\rho = 1$) to the linear ($\rho = 0$) wall proceeds with increasing temperature through the elliptic one having $n_z^2 + n_y^2/\rho^2 = 1$ with $0 < \rho < 1$, and the transverse component of the magnetization in the wall, $n_y$, disappears through a second-order phase transition. The temperature of the DW phase transition can be written in three forms:

\[ \tau_B = 1, \quad \epsilon_B = 2(1 - \eta) \ll 1, \quad \theta_B = 1/(1 + \epsilon_B) \equiv 1 - \epsilon_B, \] (4.3)
the quantity \( \tau \) playing the same role for a DW as the “absolute” temperature \( \theta \) for the bulk (cf. (3.14)). The temperature-dependent factor \( \rho \) in (4.1) can be interpreted as the DW order parameter \( m_B \). Whereas the Bloch-wall width \( \delta_B \) is temperature-independent, the width of the linear wall \( \delta_L \) is determined by the balance of the homogeneous and inhomogeneous exchange energies and is diverging at \( \theta_c \). Considering the first of equations (3.1) for \( x \gg \delta_L \), one can identify

\[
1 - \tanh(x/\delta_L) \cong \exp(-x/\xi_{cz}) \implies \delta_L = 2\xi_{cz},
\]

where \( \xi_{cz} \) is the correlation length of the \( z \) spin components. One should also note the analogy between the Bloch-wall width \( \delta_B \) and the transverse correlation length \( \xi_{co} \), which coinside for a purely uniaxial (\( \eta_\alpha = \eta \)) model with small anisotropy. The function \( G_1 \) entering equations (3.17) can be written as

\[
G_1(x) = \frac{\epsilon(1 - \rho^2)}{\cosh^2(x/\delta)} = \frac{2\alpha}{\delta^2} \frac{1}{\cosh^2(x/\delta)}.
\]

Since \( \alpha \sim a_0^2 \), in the small-anisotropy case \( G_1 \sim (a_0/\delta)^2 \ll 1 \) in the whole temperature interval.

Now we proceed to the solution of the magnetic interface problem described by equations (3.11), (3.18), and (3.14) in the general case \( \eta_\alpha \neq 0 \). The solution of the Dyson equation for the correlation function (3.11) depends on the relation between the correlation length \( \xi_{co} \) of (3.18) and the other length scales, \( a_0 \) and \( \delta \). If \( \xi_{co} \gg a_0 \), which is satisfied for \( 1 - \eta_\alpha \ll 1 \), the continuous approximation to equation (3.11) can be applied. If \( \xi_{co} \ll \delta \) (i.e., \( 1 - \eta_\alpha \gg 1 - \eta \), see (4.2)), the correlation function \( s_{ij} \) can be easily calculated locally with respect to the slowly changing magnetization profile (or, more exactly, the profile of \( G \)) in the wall. For uniaxial ferromagnets with a small anisotropy \((1 - \eta \ll 1)\) considered throughout this paper these limiting cases overlap in the region \( 1 - \eta \ll 1 - \eta_\alpha \ll 1 \). Let us consider at first the case \( \xi_{co} \ll \delta \). Here one can make a replacement \( G_i \to G'_i \) in (3.11), after which this equation can be solved like in the homogeneous case. With the use of (3.12) and (3.16) one gets

\[
s_{ii} = \theta G_i P(\eta_\alpha G_i) \cong \theta P(\eta_\alpha)[1 + I(\eta_\alpha)G_1(x)],
\]

where

\[
I(\eta_\alpha) = 1 + \frac{\eta_\alpha P'(\eta_\alpha)}{P(\eta_\alpha)} \cong \begin{cases} 1 + 2\eta_\alpha^2/z, & \eta_\alpha \ll 1, \\ \frac{c_0}{2P(\eta_\alpha)} \sqrt[4]{1 - \eta_\alpha}, & 1 - \eta_\alpha \ll 1 \end{cases}
\]

and \( c_0 \) is determined by (3.13). Now with the use of (4.6) and (3.18) one can express \( G_1 \) through the magnetization profile \( n(x) \):

\[
G_1(x) = \epsilon[1 - n^2(x)]I^{-1}(\eta_\alpha).
\]

This expression differs only by a constant from that used in the beginning of this section in the MFA limit \( \eta_\alpha = 0 \). Solving now the magnetization equations (3.17) like above, one gets the same DW magnetization profile (4.1), where the parameters \( \rho \) and \( \delta \) are given by (4.2) with the renormalized DW temperature: \( \tau \to \tilde{\tau} = \tau I(\eta_\alpha) \).

The critical values of the three temperature variables (cf. (1.3)) read now

\[
\tau_B = I^{-1}(\eta_\alpha) < 1, \quad \epsilon_B = 2(1 - \eta)I(\eta_\alpha), \quad \theta_B = \theta_c/(1 + \epsilon_B),
\]

where \( \theta_c \) is given by (3.14). One can see from (3.19) and (4.7) that for \( 1 - \eta_\alpha \ll 1 \) the effective transition temperature \( \tau_B \) becomes small. On the other hand, due to the validity condition \( 1 - \eta_\alpha \gg 1 - \eta \) the absolute temperature \( \theta_B \) of the DW phase transition remains in the limiting case under consideration close to \( \theta_c \) (\( \epsilon_B \ll 1 \)).
1). It can be seen that in this case the domain wall does not demonstrate any 2-
dimensional behaviour, and its phase transition at $\theta_B < \theta_c$ can still be described
qualitatively by the effective diminishing of the ordering interaction for the wall spins
forced perpendicularly to the easy axis $z$, as was said in the Introduction. The effect
in the case $1 - \eta_0 \ll 1$ is much larger than according to the MFA estimates because
of the divergence of the function $I(\eta_0)$ of (4.7). Accordingly, the width of the linear
wall can considerably exceed its mean-field value (4.2):

$$
\delta_L = \sqrt{\frac{2\alpha}{\mathcal{C}}} \cdot I^{1/2}(\eta_0) \approx \sqrt{\frac{\alpha \mathcal{C}_0}{\mathcal{C}P(\eta_0)}} \frac{1}{(1 - \eta_0)^{1/4}} 
$$

(4.10)
in the case $1 - \eta_0 \ll 1$. Since $\delta_L$ is related to the longitudinal correlation length
(see (4.4)), this result shows a non-trivial influence of fluctuations of transverse spin
components on the longitudinal spin correlations in the anisotropic spherical model.

One can also check that the function $G_1(x)$ of (3.18) is still given by the expression
with the changed value of the DW width $\delta$.

In the other limiting case, $\xi_{\alpha \alpha} \gg a_0$, a continuous approximation of the type (3.16)
can be applied to the Dyson equation (3.11). Making the Fourier transformation with
respect to the coordinates $y$ and $z$ and using the conditions $1 - \eta_0 \ll 1$ and (3.16),
one arrives at the differential equation for the correlation function

$$
\alpha s''(x) - [1 - \eta_\alpha + \alpha q^2 - G_1(x)] s(x) = -\theta \delta(x - x'),
$$

(4.11)
where $\alpha q^2 \equiv q_y^2 + q_z^2$ and the “mute” argument $x'$ of $s$ was dropped. This equation
should be solved to yield $s$ with $x = x'$ as a function or functional of $G_1$, and $s_ii$
(cf. (4.3)) should be obtained by the integration of $s$ over $q_y$ and $q_z$. Then, as above,
$G_1$ should be found from (3.18) and used in the magnetization equations (3.17). All
this seems to be too complicated since equation (4.11) cannot be solved analytically
for the arbitrary function $G_1(x)$. But the expected result that the DW transition
temperature $\theta_B$ turns to zero in the purely uniaxial case $\eta_\alpha = \eta$ signals that there
should be an exact solution to the problem. We can try to find it assuming that
$G_1(x)$ has the same functional form as above, (4.5), with some renormalized value of
the DW width $\delta$ as a parameter. Then using a new variable $u \equiv \tanh(x/\delta)$, equation
(4.11) can be rewritten as

$$
\frac{d}{du} (1 - u^2) \frac{d}{du} + \left(2 - \frac{\mu^2}{1 - u^2}\right) s(u) = -\frac{\delta \theta}{\alpha} \delta(u - u'), \quad \mu^2 \equiv \frac{\delta^2}{\alpha} (1 - \eta_\alpha + \alpha q^2) 
$$

(4.12)
and solved in terms of the adjoined Legendre functions

$$
P^\pm \mu(u) = \frac{u \pm \mu}{\Gamma(2 \pm \mu)} \left(1 + u\right)^{\pm \mu/2}
$$

(4.13)
which leads to

$$
s(x, x', q) = \frac{\theta \delta}{2\alpha \mu} \exp\left(-\frac{\mu}{\delta}(x - x')\right) \left[1 + \frac{1 - \tanh\frac{\mu}{\delta} \tanh\frac{x'}{\delta}}{\mu^2 - 1} \left(1 + \mu \tanh\frac{|x - x'|}{\delta}\right) \right].
$$

(4.14)

This (not translationally invariant) expression can be reduced in the case $x = x'$ with
the help of (4.5) to the form

$$
s(x, x, q) = \frac{\theta}{2\alpha^{1/2} \sqrt{1 - \eta_\alpha + \alpha q^2}} \left[1 + \frac{G_1(x)}{2[1 - \eta_\alpha - (1 - \eta)/\delta^2 + \alpha q^2]} \right],
$$

(4.15)

where $\tilde{\delta} \equiv \delta/\delta_B$ (see (4.2)). Since the structure of this expression is analogous to that
of (4.6), it is now clear that the choice of $G_1$ in the form (4.5) was right. Integrating
over the 2-dimensional wavevector $\mathbf{q}$ to get $s_{ii}$ and proceeding as above, one gets functionally the same results (4.1) and (4.2) with a new renormalized DW temperature

$$
\tilde{\tau} = \tau I(\eta_{\alpha}, \eta, \delta),
$$

where

$$
I(\eta_{\alpha}, \eta, \delta) = \frac{c_0}{4P(\eta_{\alpha})} \frac{\delta}{\sqrt{1 - \eta}} \ln \left( \frac{\delta}{\sqrt{1 - \eta_{\alpha}}} \frac{\sqrt{1 - \eta_{\alpha}} + \sqrt{1 - \eta}}{\sqrt{1 - \eta_{\alpha}} - \sqrt{1 - \eta}} \right). \tag{4.17}
$$

The latter simplifies in the limit $1 - \eta_{\alpha} \gg 1 - \eta$ to the second limiting expression in (4.7). It can be seen from (4.15) that in the case under consideration, $1 - \eta_{\alpha} \ll 1$, the integral (4.17) is determined by the long-wavelength region, which justifies using the continuous approximation for the transverse correlation function $s$. In the region of elliptic walls ($\tilde{\tau} < 1$) one has $\delta = \delta_B$ and hence in (4.17) one has $\delta = 1$. The critical values of both the DW temperature $\tau$ and the absolute temperature $\theta$ (4.9) determined now by $I(\eta_{\alpha}, \eta, 1)$ go to zero in the uniaxial limit:

$$
\tau_B \propto \theta_B \propto 1 / \ln \frac{1}{\eta - \eta_{\alpha}}, \quad \eta_{\alpha} \to \eta, \tag{4.18}
$$

which corresponds to the 2-dimensional nature of domain walls. One can see that the 2-dimensional effects lead to a further decrease of the DW phase transition temperature: $\tau_B \equiv I^{-1}(\eta_{\alpha}, \eta, 1) < \tau_B^{(0)} \equiv I^{-1}(\eta_{\alpha})$, where $I(\eta_{\alpha})$ is given by (4.7).

In the linear-wall region ($\tilde{\tau} > 1$) the normalized wall width $\delta$ is given by the solution of the transcendental equation $\delta^2 = \tau I(\eta_{\alpha}, \eta, \delta)$ following from (4.2) and (4.16). The latter can be rewritten in the natural units in the form

$$
\delta_L = \frac{\sqrt{\alpha}}{2\epsilon} P(\eta_{\alpha}) \ln \left( \frac{\delta_L + \xi_{ca}}{\delta_L - \xi_{ca}} \right), \tag{4.19}
$$

where $\xi_{ca}$ is the transverse correlation length given by (3.19). Far from $\tau_B$, where $\delta_L \gg \xi_{ca}$, the solution of (4.19) leads to the formula (4.10). This asymptotic dependence can also be represented in the form $\delta = \sqrt{\tau I(\eta_{\alpha})} = \sqrt{\tau / \tau_B^{(0)}}$, which is the analogue of Curie-Weiss asymptote for the bulk susceptibility $\chi(T)$ of a ferromagnet far above $T_c$. In the purely isotropic model, $\eta_{\alpha} = \eta$, the linear DW structure is realized in the whole temperature range. The DW width $\delta_L$ determined by the solution of (4.19) shows a crossover to the Bloch-wall width $\delta_B$ at low temperatures:

$$
\delta_L \cong \delta_B \left[ 1 + 2 \exp \left( \frac{2\epsilon}{\sqrt{1 - \eta}} P(\eta) c_0 \right) \right], \quad \epsilon \equiv \theta \theta_c / \theta - 1 \gg \sqrt{1 - \eta}. \tag{4.20}
$$

It is worth to note that in this limiting case the longitudinal correlation length, $\xi_{cz} = \delta_L/2$, is determined by the transverse one, $\xi_{ca} = \delta_B$.

The temperature dependences of the DW order parameter $m_B \equiv \rho$ and the normalized DW width $\delta$ are represented in Fig. 2 for different values of $\eta_{\alpha}$ and $1 - \eta = 10^{-3}$. One can see that for $1 - \eta_{\alpha} = 10^{-2}$ the fluctuational decrease of the DW transition temperature is essential, although 2-dimensional effects are still negligible in this case. In contrast, for $1 - \eta_{\alpha} = 1.2 \cdot 10^{-3}$ they come into play, the corresponding additional diminishing of the transition temperature becomes essential, and the temperature dependence of $\delta$ in the logarithmic scale is no longer a straight line.

5. Discussion

In the main part of this paper the structure of domain walls in the biaxial ferromagnetic model described by the classical-vector Hamiltonian (2.4) was calculated exactly
in the whole temperature range $T < T_c$ in the spherical limit $D \to \infty$. The main qualitative result is that in the purely uniaxial model ($\eta_{\alpha} = \eta$) the Bloch walls do not exist at any non-zero temperature (see (4.18)) being disordered by thermal fluctuations to the linear (Ising-like) ones. This result complements the well-known fact that the Bloch walls in the purely uniaxial model (considered within the phenomenological micromagnetic approach which is equivalent to MFA) cannot move since their maximal velocity (the Walker velocity) is equal to zero. In the opposite limit, $\eta_{\alpha} = 0$, the model with $D \to \infty$ total spin components and a finite number (here 2) of interacting ones is realized. In this case spin fluctuations die out and MFA becomes exact. The temperature of the phase transition from Bloch to linear walls $T_B$ changes as function of $\eta_{\alpha}$ from its MFA value $T_B = (1 - \epsilon_B)T_c$, $\epsilon_B = 2(1 - \eta) \ll 1$ at $\eta_{\alpha} = 0$ to 0 at $\eta_{\alpha} = \eta$ (see (4.3) and (4.9)).

In the actual case of a small anisotropy, $1 - \eta \ll 1$, the behavior of a domain wall is more complicated than that of a purely 2-dimensional object, since the DW width $\delta$ is much larger than the lattice spacing $a_0$. In the spherical limit this leads to the existence of two different mechanisms of the DW ordering, depending on the value of $\eta_{\alpha}$. For the strong anisotropy in the basis plane $x, y$, i.e., $1 - \eta_{\alpha} \gg 1 - \eta$, the correlation length $\xi_{\alpha\alpha}$ of (3.19) for all temperatures is much shorter than the wall width $\delta$ in (4.2), and the DW phase transition at $T_B$ can be interpreted as the locally shifted bulk one (the “perturbed 3-dimensional” case, see (4.6)–(4.9)). In contrast, in the case of the two anisotropies comparable with each other, $1 - \eta_{\alpha} \sim 1 - \eta$, the true 2-dimensional situation is realized (see (4.17) and (4.18)). Such a separation does not, however, persist for models with finite values of $D$ (e.g., for the Heisenberg model, $D = 3$), where the DW phase transition should always be of a 2-dimensional character. For such models the fluctuations of the ordering spin components, $m_z$ and $m_y$, also play a role, and in the temperature interval about $T_B$, where the diverging correlation length $\xi_{cy}$ exceeds the DW width $\delta$, a 2-dimensional behavior is realized. Since $\delta \gg a_0$, this temperature interval should be much narrower than for a purely 2-dimensional system. The asymptotic critical behavior of the DW order parameter...
$m_B = \rho$ in (4.1) is described by the critical index $\beta_B = 1/8$ of the 2-dimensional Ising model, as was confirmed experimentally in [31, 32].

As a subject of future investigations, the temperature dependence of the DW magnetization profile in the self-consistent Gaussian approximation, without going to the limit $D \to \infty$, can be considered. Although it can be connected with more complicated numerical calculations, one can expect to obtain, with the help of SCGA, essentially more accurate results for domain walls in comparison with the spherical approximation, as was demonstrated for the bulk properties [14, 15]. The other problem is to formulate dynamic equations for fluctuating domain walls and to calculate their mobility. Such equations can, in principle, be obtained with the help of some dynamical generalization of the classical spin diagram technique [4]. Unfortunately, in the dynamical case one cannot make use of going to the limit $D \to \infty$ with all related simplifications, and only SCGA for the Heisenberg model can be used as the underlying static approach.

A promising field for the application of the methods of this paper is the surface effects in finite and semi-infinite magnetic systems, which are very sensitive to the anisotropy. This problem was addressed recently in [12].

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