Radiation zeros and scalar particles beyond the standard model

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Abstract

Standard radiation zeros arise from the factorization properties of tree-level amplitudes involving a massless photon and can occur when all charged particles in the initial and final state have the same sign. We investigate how several different processes involving new scalar particles beyond the standard model may exhibit radiation zeros and how this structure might be exploited to probe their electromagnetic structure. We focus on (i) unnoticed aspects of angular zeros in the process $e^- + e^- \rightarrow \Delta^{--} + \gamma$ for doubly charged Higgs boson (or any bilepton) production and (ii) the process $\gamma + e^- \rightarrow q + S/V$ for scalar ($S$) or vector ($V$) leptoquarks (LQs). We also discuss how factorized amplitudes and radiation zeros may appear in the gauge boson fusion production of non-conjugate leptoquark pairs via $\gamma + W^\pm \rightarrow S_i + S_j^*$ in high energy $e^\pm e^\pm$ reactions and how the zeros affect the production cross-sections for various types of scalar leptoquarks.
1. Introduction

The factorization properties [1], [2] of tree-level gauge theory amplitudes involving massless gauge quanta can often be used to dramatically simplify the otherwise extremely complex expressions for many cross-sections of physical interest. In some circumstances they can also be seen to encode information on subtle physical effects, such as the destructive interference which leads to radiation zeros [3] in such processes as \( q + q' \rightarrow W + \gamma \), which arise from the interplay between several competing Feynman diagrams. Furthermore, processes with the highly nontrivial angular dependences implied by the existence of radiation zeros are often touted as excellent sources of information on the electromagnetic structure of the produced particles, as non-gauge couplings often destroy the cancellations necessary for the appearance of angular zeros. In this note we make use of both the factorization properties of gauge amplitudes and the possible existence of angular radiation zeros to discuss several processes involving scalar bosons beyond the standard model. We discuss several such processes in which the factorized form of the matrix elements involved can, if nothing else, help to easily explain the magnitude of the total production cross-sections as one varies the electroweak couplings from model to model, while in several cases we see how the presence of angular zeros can be used to probe the electromagnetic couplings of such particles.

2. \( e^- + e^- \rightarrow \Delta^{--} + \gamma \)

While high-energy \( e^+e^- \) or \( \mu^+\mu^- \) colliders offer the potential for a wide variety of discoveries beyond the standard model, similar like-sign collisions, \( e^-e^- \) or \( \mu^-\mu^- \), offer a more specialized set of tests, such as probes of a strongly interacting electroweak
sector \[4\], selectron or chargino pair production \[5\] via \(e^- + e^- \rightarrow \tilde{e}^- + \tilde{e}^-\), \(\chi^- + \chi^-\) via \(t\)-channel neutralino or sneutrino exchange, and searches for exotic Higgs bosons \[6\], \[7\] or more generic doubly charged bileptons, either gauge bosons \[8\] or otherwise \[9\], \[10\]. The resonant production of suitably light, doubly charged Higgs bosons \(\Delta^{--}\) (as, for example, in various versions of left-right models \[11\]) has been discussed \[6\] and given the expected excellent energy resolution of muon colliders, ‘factory-like production’ \[6\] of such particles might be possible, provided the total width is small enough. Once discovered in this way, more detailed studies of processes such as \(e^- + e^- \rightarrow \Delta^{--} + \gamma\) could serve to probe the electromagnetic properties of such a state (or any doubly charged bilepton). Alanakyan \[12\] has recently calculated the cross-section for this process, correcting a much earlier prediction \[13\] by taking all tree-level diagrams into account, and obtains the expression

\[
\frac{d\sigma}{d\cos(\theta)} = \frac{\alpha h_{ee}^2}{s} \left(1 + \frac{2(1 - \beta)}{\beta^2}\right) \beta \cot^2(\theta) \tag{1}
\]

where \(\beta = 1 - M_{\Delta}^2/s\) and \(h_{ee}\) is the appropriate \(ee\) \(\Delta^{--}\) coupling. The author of Ref. \[12\] focuses attention on the collinear singularities (when \(\theta = 0, \pi\)) in the cross-section, which are reminiscent of those in \(e^+ + e^- \rightarrow Z^0 + \gamma\), but does not mention the other dramatic angular dependence, namely the angular zero in the tree-level cross-section when \(\theta = \pi/2\). Given the general arguments of Refs. \[1\] and \[2\], we would expect the cross-section for this process to factorize with a pre-factor containing all of the relevant charge factors. Repeating the calculation for this process with this in mind, we find an entirely equivalent expression given by

\[
\frac{d\sigma}{dt} = \frac{\alpha h_{ee}^2}{s^2} [(Q_1 u - Q_2 t)^2] \left[\frac{s^2 + M_{\Delta}^4}{ut(u+t)^2}\right] \tag{2}
\]
where \( t = -s\beta(1-\cos(\theta))/2 \) and we have chosen to write \( Q_1 = Q_2 \equiv Q_e \) and to separate the pre-factor in square brackets including the initial state charges to emphasize its origin in the factorization of gauge theory amplitudes involving massless photons. The resulting characteristic angular zero at 90° in the center-of-mass frame can obviously be used to probe the electromagnetic structure of the \( \Delta^{--} \) and we will briefly discuss this further below.

This angular zero has already been noticed, in the context of doubly charged vector bileptons, \( V^{--} \), by Cuypers and Raidal [9] who note that its presence depends the standard gauge coupling of the \( V^{--} \) and will be ‘filled in’ if the bilepton has an anomalous magnetic moment coupling given by \( \kappa \neq 1 \). To focus on this in more depth, we note that the cross-section for \( e^- + e^- \rightarrow V^{--} + \gamma \) for a general value of such an anomalous coupling is proportional to

\[
\frac{d\sigma}{dt} \propto \left[ (Q_1u - Q_2t)^2 \right] \frac{u^2 + t^2 + 2sM_V^2}{ut(u + t)^2} + (\kappa - 1)[Q_1u - Q_2t] \left[ \frac{t - u}{(t + u)^2} \right] \\
+ \frac{(\kappa - 1)^2}{2(t + u)^2} \left[ tu + (t^2 + u^2)\frac{s}{4M_V^2} \right]
\]

where we once again keep separate those terms proportional to \( (Q_1u - Q_2t) \) arising from factorization and terms of the form \( (t - u) \) which arise from other kinematical effects. We note that in the very special case of \( e^-e^-, \mu^-\mu^- \) collisions where \( Q_1 = Q_2 \), not only will the standard gauge theory cross-section term vanish at the radiation zero as \( (Q_1u - Q_2t)^2 = (u - t)^2 \), i.e. as two powers of \( \cos(\theta) \), but that the interference term will do so as well, in contrast the most general case where the cross-term will only be suppressed by one power of the \( (Q_1u - Q_2t) \) pre-factor. This implies that the cross-section near the angular location of the radiation zero is additionally sensitive to new physics contained in the \( (\kappa - 1)^2 \) term in the special case of equal initial charges.
Returning to doubly charged Higgs production, we note that if we parameterize the $\Delta^{--}$ electromagnetic coupling with a form-factor given by $F(q^2) \approx 1 - \delta(q^2)$, the angular distribution in Eqn. (2) will now contain terms of order $\delta$ and $\delta^2$. The interference term proportional to $\delta$ turns out to have a kinematic factor which is also proportional to $(t-u)$, just as for the vector case, leaving the radiation zero in $e^- + e^- \rightarrow \Delta^{--} + \gamma$ more sensitive to the presence of non-pointlike electromagnetic couplings through the remaining $\delta^2$ term which does not vanish.

3. $\gamma + e^- \rightarrow q + S$

Resonant leptoquark production via the process $e + q \rightarrow S/V$ for both scalar ($S$) and vector ($V$) leptoquarks has been considered in the context of $ep$ collisions at HERA and beyond any number of times. A logical extension to the process $e + q \rightarrow S/V + \gamma$ has been analyzed in some detail [14] and the presence of the radiation zero and its dependence on the electromagnetic couplings of the leptoquark have been discussed. (We note that various studies of radiation zeros in non-resonant $e + q \rightarrow e + q + \gamma$ scatterings have also appeared [15].) The radiative decays of scalar leptoquarks [16], via a crossed version of this process, namely $S \rightarrow e + q + \gamma$, have also been considered as an extension of processes involving charged scalar particles [17] such as $q\bar{q}' \rightarrow H^\pm \gamma$, $H^\pm \rightarrow q\bar{q}' \gamma$, and $H^\pm \rightarrow \bar{q}(\bar{q}')^* \gamma$ (where $\bar{q}$ are scalar quarks.)

The other crossing of the basic process, namely $\gamma + e \rightarrow S + q$, is of relevance to single scalar leptoquark production in $ee$ and $e\gamma$ collisions and has been discussed by Hewett and Pakvasa [18] (for the special case of $Q_S = -1/3$) and then generalized by Nadeau and London [19] for arbitrary $Q_S$. Neither group presents their results in a
factorized form of the type guaranteed by the arguments of Ref. [1] and the general cross-section is presented in Ref. [19] as

\[
\frac{d\sigma}{d\hat{u}} = -\frac{\pi k\alpha em^2}{2\hat{s}^2} F(\hat{s}, \hat{t}, \hat{u}, M_S^2) \tag{4}
\]

where

\[
F(\hat{s}, \hat{t}, \hat{u}, M_S^2) = \frac{\hat{u} - 2Q^2S\hat{u}(\hat{u} + \hat{s} - M_S^2)}{\hat{s}(\hat{u} + \hat{s})^2} - 2Q^2S\frac{\hat{u}(\hat{u} + \hat{s} - M_S^2)}{\hat{s}(\hat{u} + \hat{s})} + 2(1 + Q^2S)\frac{(\hat{u} - M_S^2)}{\hat{s}}
\]

\[
- (1 + Q^2S)^2 \left[ \frac{\hat{s}}{\hat{u}} + 2 \frac{(\hat{u} - M_S^2)(\hat{u} + \hat{s} - M_S^2)}{\hat{s}\hat{u}} \right]
\]

\[
+ 4Q^2(1 + Q^2S)\frac{(\hat{u} + \hat{s} - M_S^2)(\hat{s}/2 + \hat{u} - M_S^2)}{\hat{s}(\hat{u} + \hat{s})} \tag{5}
\]

In this expression, \( \hat{t} = -\hat{s}\beta(1 - \cos(\theta^*)) / 2, \beta = 1 - M_S^2 / \sqrt{s} \), and \( \theta^* \) is the angle between the produced \( S \) \( q \) and the incident \( \gamma \) \( e^- \) in the center-of-mass frame; the \( l - q - S \) coupling is given as \( g \) and one defines \( g^2 / 4\pi \equiv k\alpha em \).

Using the arguments in Ref. [1] or [2] we know that this expression can be factorized and either by direct calculation or algebraic manipulation of Eqn. [4] we indeed find that the differential cross-section can be written in a very simple form, namely

\[
\frac{d\sigma}{d\hat{u}} = \frac{\pi k\alpha em^2}{2\hat{s}^2} [(1 + Q^2S)\hat{s} + \hat{u}]^2 \left[ \frac{\hat{t}^2 + M_S^4}{\hat{s}\hat{u}(\hat{u} + \hat{s})^2} \right] \tag{6}
\]

(Note the connection to the cross-section in Eqn. [3] which is basically the crossed process, but with different values of the charges of the particles.) The factorization of the entire \( Q_S \) dependence into the term in square brackets helps further explain the relative magnitude of the integrated cross-sections (after appropriate \( p_T \) cuts) seen in Ref. [13] for various values of \( Q_S \). There the authors note that the cross-sections for the two processes involving \( |Q_q| = 2/3 \) quarks (i.e., those with \( Q_S = -5/3, -1/3 \)) are larger than for those involving \( |Q_q| = 1/3 \) quarks (corresponding to \( Q_S = -4/3, -2/3 \))
due to the resulting enhanced coupling of photons to the quarks in the $t$-channel exchange diagram. Using the factorization above, we can also see that the $Q_S = -5/3$ and $Q_S = -1/3$ cross-sections themselves will differ only in the pre-factors given by $[-2\hat{s}/3+\hat{u}]^2$ and $[+2\hat{s}/3+\hat{u}]^2$ and the first term is larger since $\hat{u} < 0$; a similar hierarchy is then present for the $Q_S = -4/3$ and $Q_S = -2/3$ cases.

While the authors of Refs. [18] and [19] present expressions for total cross sections (again, after appropriate cuts), they do not discuss the angular zeros which may be present in these processes. (We note that Cuypers [20] has discussed possible radiation zeros in this process, but does not report the “long analytical forms” for the cross-sections which we have found here to factorize very simply.) We note that the pre-factor in square brackets can vanish when

$$\cos(\theta^*) \equiv y = \frac{2(1+Q_S)}{\beta} - 1 \quad \rightarrow \quad 1 + 2Q_S \quad \text{for } \beta \rightarrow 1 \quad (7)$$

so that there can be angular zeros when $Q_S = -1/3, -2/3$, corresponding to $Q_q = -2/3, -1/3$, since then all charged particles in the initial and final states are of the same sign. In addition, zeros will only be present in the physical region provided that $\beta$ is large enough. The condition that a zero will appear in the observable range of interest, namely $-1 \leq y \leq +1$, is given by

$$\frac{\sqrt{z}}{M_S} \equiv z \geq \frac{1}{\sqrt{-Q_S}} \quad (8)$$

or $\sqrt{z}/M_S \geq 1.73 (1.25)$ for $Q_S = -1/3 (-2/3)$.

Since the angular dependence of the cross-section is determined by the dimensionless term $F(\hat{s}, \hat{t}, \hat{u}, M_S^2)$, we plot this function for four values of $Q_S$ as a function of $y = \cos(\theta^*)$ in Fig. 1 for several different values of $z \equiv \sqrt{z}/M_S$. The differential cross-sections for $y \rightarrow -1$ are larger for cases (a)/(b) compared to (c)/(d) due to the charge
of the exchanged (t-channel) quark ($|Q_q| = 2/3$ compared to $|Q_q| = 1/3$) as mentioned above, while radiation zeros are present in cases (b) and (d) for large enough values of $\beta$ as in Eqn. (8); the angular locations of these zeros are seen to be consistent with Eqn. (7).

Whether the photons in the $\gamma e$ collisions are “effective” (arising from an approximately real, Weizsäcker-Williams photon in $e^+e^-$ colliders) or “real” (arising, for example, from laser backscattering), the angular dependence will not be probed at fixed center-of-mass energy. Either mechanism provides a photon beam with a distribution of energies, and so a range of $\hat{\gamma} = \sqrt{s}/M$ will be probed. Rather than a zero in certain cross sections as seen at the parton level (illustrated in Fig. 1), a broad (in $\cos \theta$) region of reduced cross section will be seen. In Fig. 2, we show the results of a calculation of the lab frame angular distribution at a $\sqrt{s} = 1$ TeV $e^+e^-$ linear collider, utilizing laser backscattering to operate in $e\gamma$ mode. The radiation amplitude zero is, indeed, filled in, but its effect can be seen in the shape of the $\cos \theta$ distribution, much as was noted in Refs. [15] and [20].

The presence of angular zeros is not unique to scalar states in such single leptoquark production processes. Montalvo and Èboli [21] have considered the production of composite vector leptoquark states via the same mechanism, namely $\gamma + e^{-} \rightarrow q + V$. They consider the interaction given by

$$\mathcal{L} = -gV^{ab}_\mu \bar{L}^a \gamma^\mu L^b + H.c.$$  \hspace{1cm} (9)

where $L^a$ are the physical $SU(2)$ left-handed doublets of the standard model and $Q_S = -2/3$ in the specific model considered while we have $g^2/4\pi = k\alpha_{em}$ as before to be consistent with the notation used here. If we ignore the final state quark mass,
one can see that their result for the differential cross-section can be written in a form
totally analogous to Eqn. (6), namely

\[
\frac{d\sigma}{d\hat{t}} = -\frac{12\pi k\alpha_e^2}{\hat{s}^2} \left[(1 + Q_s)\hat{s} + \hat{t}\right]^2 \frac{[\hat{s}^2 + \hat{t}^2 + 2\hat{u}M_V^2]}{\hat{s}\hat{t}(\hat{s} + \hat{t})^2}
\]  

(10)

where the role of \( \hat{t} \) and \( \hat{u} \) are interchanged due to a differing definition of \( \theta^* \) in Ref. [21] compared to the one used here. We note that they have chosen a value of the coupling \( g \) which, along with the standard \( VV\gamma \) vertex gives rise to large energy behavior for 

\( e^+e^- \rightarrow VV^* \)

for which unitarity is maintained at tree-level. The fact that their result in Eqn. (10) is then entirely similar to the crossed version of the standard result for 

\( q + \bar{q} \rightarrow W + \gamma \)  

or that seen in Eqn. (3) (only differing in the factorized term containing the charge dependence) is then easily understood and angular zeros will also be present in the explicit composite model case they consider, subject to the kinematic condition in Eqn. (8).

4. \( \gamma + W^\pm \rightarrow S_i + S_j^* \)

While single production of leptoquarks in \( eq \) or \( \gamma e \) collisions may well be important, the current best limits on leptoquark masses arise from processes involving pair production. Analyses from hadron colliders using \( gg \) and \( q\bar{q} \) fusion processes [23], including appropriate NLO corrections [24], now routinely set limits of order \( M(LQ) > 200 \text{GeV} \) for leptoquarks [25] with branching ratios to charged leptons of \( BR(LQ \rightarrow eq) > 1/2 \). Similarly, production prospects from \( e^+e^- \) [26] and \( \gamma\gamma \) [27] collisions have been examined in great detail. Such processes are important for the extraction of unambiguous mass limits as the production cross-sections depend on the well-defined gauge quantum numbers of the leptoquarks and not on their unknown couplings to \( lq \) pairs. (Such
couplings can, of course, contribute to these pair production processes, such as from the $t$-channel quark exchange diagram in $e^+e^-$ collisions.) In all such cases, one obviously produces pairs of opposite sign, conjugate leptoquarks ($SS^*$ or $VV^*$) and the production cross-sections are essentially independent of the leptoquark generation.

Cuypers, Frampton, and Rückl [28] have noted that it is possible produce pairs of non-conjugate leptoquarks in $e^-e^-$ collisions via $t$-channel quark exchange under very special circumstances, requiring the simultaneous existence of both $|F| = 2$ and $F = 0$ leptoquarks which couple with the appropriate chirality to first-generation leptons. While this is an intriguing possibility, it requires an array of leptoquarks which only appear in very specialized models and relies explicitly on the unknown $LQ - l - q$ couplings which may very well be small, especially for the first generation.

A more standard source of production of two non-conjugate leptoquark pairs in either $e^+e^-$ or $e^-e^-$ collisions arises from the subprocess $\gamma + W \rightarrow S_i + S_j^*$, which is possible provided the leptoquark (of any generation) transforms non-trivially under $SU(2)$. Since many of the standard leptoquark assignments [29], [30] allowed by $SU(3) \times SU(2) \times U(1)$ invariance transform as either doublets or triplets, such processes will be accessible in a much wider variety of models. (In this same context, the analogous $\gamma + W^+ \rightarrow t + \bar{b}$ process [31] has been considered in detail by Kauffman.) In what follows we simply characterize the basic cross-section for this process, indicating how the amplitude factorization can simplify the resulting matrix elements and how the presence of radiation zeros leads to suppression of the cross-section for certain leptoquarks. (A complete discussion of the pair production of leptoquarks via all gauge boson fusion processes will appear elsewhere [32].)
We characterize the basic tree-level process as $\gamma(k) + W^-(p) \to S_i(q_1) + S^*_j(q_2)$ labeling the momenta. The matrix element can be written from the beginning in a factorized form, namely

$$M = 4ieG \left( Q_i - Q_2 \cdot q_1 \right) \left( \frac{q_{1\mu}q_{2\nu}}{l^2_1 - m^2} + \frac{q_{2\mu}q_{1\nu}}{l^2_2 - m^2} + \frac{g_{\mu\nu}}{2} \right) \epsilon^\nu_1(k)\epsilon^\mu_W(p) \tag{11}$$

where $l_1^2 = (p - q_2)^2 = (k - q_1)^2$ and $l_2^2 = (p - q_1)^2 = (k - q_2)^2$, $Q_i = Q(S_i) = Q(S_j) - 1$, and we assume that the masses of the non-conjugate leptoquarks are degenerate, i.e. $M(S_i) = M(S_j) = m$. (Constraints from precision electroweak data, for example the $\rho$ parameter, limit the mass splittings of leptoquarks \cite{33}, especially for $SU(2)$ triplets \cite{34}.) The overall electroweak coupling factor is given by $G = g/\sqrt{2}$, $g$ for $SU(2)$ doublets and triplets respectively. We note that this expression confirms and extends an earlier expression \cite{33} for the amplitude describing the radiative decay of $W$ bosons into massless scalar quark pairs.

The cross-section can then be written in the form

$$\frac{d\sigma}{dt}(\gamma W^- \to S_i S^*_j) = \frac{8\pi\alpha^2 f_W}{s^2 \sin^2(\theta_W)} \left( Q_i + \frac{\tilde{u}}{\tilde{u} + \tilde{t}} \right)^2 G(s, \tilde{t}, \tilde{u}, M^2_W, m^2) \tag{12}$$

where $f_W = 1/2, 1$ for $SU(2)$ doublets and triplets respectively and

$$G(s, \tilde{t}, \tilde{u}, M^2_W, m^2) = \frac{1}{4\tilde{u}^2\tilde{t}^2} \left[ \tilde{u}\tilde{t}[2\tilde{u}\tilde{t} + sM^2_W] - m^2[(s - M^2_W)^2 M^2_W + 4\tilde{u}\tilde{t}s] ight. \right.$$

$$\left. + 4m^4(s - M^2_W)^2 \right] \tag{13}$$

with $\tilde{t} = t - m^2$ and $\tilde{u} = u - m^2$. In this simple form, the electroweak and electromagnetic couplings of different leptoquarks are easily separated into the $f_W$ and pre-factor containing the LQ charge. In contrast to the $\gamma\gamma$ cross-section for production of conjugate pairs, where there is an overall $Q^4_i$ factor which trivially determines the
relative importance of the process for various leptoquark assignments, the interplay between the photon coupling to different diagrams here is slightly more complex, but still encoded in a fairly simple pre-factor. To see what effect varying these parameters has on the LQ production cross-section, we plot, in Fig. 3, the expression

$$f_W \left( Q_i + \frac{1 + y}{2} \right)^2$$

which gives the appropriate combination of these pre-factors in the high energy limit for several types of scalar leptoquarks. We plot this function, which determines the different dependences on charge and electroweak coupling in the angular distributions in the center-of-mass frame for four cases which can appear in $\gamma W^-$ collisions:

| LQ type | $Q(S_i)$ | $Q(S_j)$ | $SU(2)$ rep | $f_W$ |
|---------|-----------|-----------|-------------|-------|
| $R_{2L}, R_{2R}$ | $+2/3$ | $-5/3$ | doublet | $1/2$ |
| $R_{2L}$ | $-1/3$ | $-2/3$ | doublet | $1/2$ |
| $S_{3L}$ | $-2/3$ | $-1/3$ | triplet | $1$ |
| $S_{3L}$ | $+1/3$ | $-4/3$ | triplet | $1$ |

where we use the leptoquark labeling scheme of Ref. [30]. The total center-of-mass cross-section can be written in the form

$$\sigma(s) = \frac{1}{s} \left[ \frac{4\pi\alpha^2}{\sin^2(\theta_W^2)} \right] R(z)$$

where

$$R(z) = \int_{-1}^{+1} \beta \left[ f_W \left( Q_i + \frac{\bar{u}}{\bar{u} + \bar{t}} \right)^2 \right] G(s, \bar{t}, \bar{u}, 0, m^2) \, dy$$

where $\beta \equiv \sqrt{1 - 4m^2/s}$ and $\bar{t} = -s(1 - \beta y)/2$, $\bar{u} = -s(1 + \beta y)/2$, $z \equiv \sqrt{s}/2m$. (Note that given the existing mass limits on leptoquarks, we will ignore $M_W$ as we are interested in the limit where $s \geq 2m >> M_W$.) We plot $R(z)$ versus $z$ in Fig. 4 for the leptoquark charge assignments in Eqn. (15) and Fig. 3 and we see that the differences
in total cross-section are easily explained by the differences in pre-factors. Similar
results for vector leptoquarks (including anomalous couplings) are easily obtained as
generalizations and we are guaranteed that the same electroweak and charge pre-factors
will appear in those processes as well, at least as long as one is restricted to purely
gauge couplings.

5. Acknowledgments

One of us (M.A.D) acknowledges the support of Penn State University through a
Research Development Grant (RDG).
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Figure Captions

Fig. 1. Plots of $F(s, t, u, M_S^2)$ (defined in Eqn. (5)) versus $y = \cos(\theta^*)$ where $\theta^*$ is the angle between the produced $S (q)$ and the incident $\gamma (e^-)$ in the center-of-mass frame. Plots for four possible leptoquark charges are shown and curves for center-of-mass energies described by $z = \sqrt{s}/M = 5.0, 2.0, 1.5, 1.3, 1.1$ are given by the solid, dashed, dot-dash, dotted, and dot-dash-dash lines respectively. Radiation zeros are present only in the case where the charged particles in the final state state (the $S$ and $q/\gamma$) have the same charges as those in the initial state (the $e^-$), consistent with the general theorems of Ref. [2] and are in the physical region for sufficiently large values of $z = \sqrt{s}/M_S$ (Eqn. 8) and their locations are given by Eqn. (7).

Fig. 2. Plots of $d\sigma/d\cos\theta$, Eqn. (6) convoluted with a laser backscattered photon distribution, versus $y = \cos(\theta)$ where $\theta$ is the angle between the produced $S (q)$ and the incident $\gamma (e^-)$ in the lab frame. Plots for four possible leptoquark charges are shown and curves for leptoquark masses described by $z = \sqrt{s}/M = 5.0, 2.0, 1.5, 1.3$ are given by the solid, dashed, dot-dash and dotted lines respectively, for leptoquark production at a $\sqrt{s} = 1\ TeV\ e^+e^-$ collider operating in $e\gamma$ mode. The values of $z$ were chosen for easy comparison with Fig. 1; for $z = \sqrt{s}/M = 1.1$, the mass of the leptoquark is very near the kinematic limit of the collider in $e\gamma$ mode (laser backscattering produces a photon beam with maximum energy slightly lower than the initial electron beam energy), and the cross section is tiny.
Fig. 3. Plot of Eqn. (14) which describes the dependence of non-conjugate leptoquark production on the electroweak and electromagnetic couplings in the high energy limit, namely $F_W(Q_i+(1+y)/2)^2$ versus $y$. The cases we consider are denoted by the values of $(Q_i, f_W)$ given by $(+1/3, 1)$ (solid), $(+2/3, 1/2)$ (dashed), $(-2/3, 1)$ (dot-dash), and $(-1/3, 1/2)$ (dotted).

Fig. 4. Plot of $R(z)$ (which gives the integrated cross-section via Eqn. (17)) versus $z = \sqrt{s}/2m$ for four different cases of non-conjugate leptoquark production. The cases considered are the same as those in Fig. 3, namely $(+1/3, 1)$ (solid), $(+2/3, 1/2)$ (dashed), $(-2/3, 1)$ (dot-dash), and $(-1/3, 1/2)$ (dotted).
(a) $Q_s = -5/3$, $Q_q = +2/3$

(b) $Q_s = -1/3$, $Q_q = -2/3$

(c) $Q_s = -4/3$, $Q_q = +1/3$

(d) $Q_s = -2/3$, $Q_q = -1/3$
(a) $Q_S = -5/3$, $Q_q = +2/3$

(b) $Q_S = -1/3$, $Q_q = -2/3$

(c) $Q_S = -4/3$, $Q_q = +1/3$

(d) $Q_S = -2/3$, $Q_q = -1/3$
\[ f_W(Q_i + (1+y)/2)^2 \]

\[ y = \cos(\theta^*) \]
