Loop Corrections in Very Special Relativity
Standard Model

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Abstract. In this talk we want to study one-loop corrections in VSRSM. In particular, we use the new Sim(2)-invariant dimensional regularization to compute one-loop corrections to the Effective Action in the subsector of the VSRSM that describe the interaction of photons with charged leptons. New stringent bounds for the masses of $\nu_e$ and $\nu_\mu$ are obtained.

1. Introduction
The Standard Model of Particle Physics (SM) is being corroborated at the LHC at a great precision. Moreover, aside from the Higgs particle that was already a part of the SM, neither new particles nor new interactions have been found at the LHC till now [1].

But we know that the SM fails in the description of neutrinos. In the SM neutrinos are massless whereas in Nature at least some of the species of neutrinos must be massive because they exhibit neutrino oscillations [2].

To describe massive neutrinos in a Lorentz-invariant Quantum Field Theory (QFT), new kinds of particles must be postulated, as in the Seesaw Mechanism [3].

If we want to keep the particles and symmetries of the SM and give masses to the neutrinos we are led to Very Special Relativity (VSR) [4]. VSR postulates that the true symmetry of Nature is not the Lorentz group (6 parameters) but a subgroup of it, Sim(2) (4 parameters). Doing so, none of the classical tests of Special Relativity (SR) is affected but it is possible to provide a non-local mass term for the neutrino [5].

Recently, we have used VSR to build the VSRSM [6]. It contains the same particles and symmetries as the SM, but a non-local term provides for the masses as well as neutrino oscillations. New predictions are obtained such as the process $\mu \rightarrow e + \gamma$, which is forbidden in the SM.

In this talk we want to consider loop corrections in the VSRSM. To do that we need to introduce a Sim(2)-invariant dimensional regularization [7], based on a simplification of the Mandelstam-Leibbrandt prescription [8]. Moreover, we choose to compute one loop corrections in the sector of the VSRSM that describe the coupling of photons to charged particles (VSRQED). Since the neutrino and the charged lepton of the same family form a doublet under $SU(2)_L$, they have a common VSR mass. Therefore, by computing processes in VSRQED, we can get information about neutrino masses.
2. The model
In this talk we restrict ourselves to the electron family. \(m\) is the VSR mass of both the electron and the neutrino, producing the second term of (1). After spontaneous symmetry breaking (SSB), the electron acquires a mass \(M = \frac{G_e v}{\sqrt{2}}\), where \(G_e\) is the electron Yukawa coupling and \(v\) is the VEV of the Higgs, i.e. third term of (1). Please see equation (52) of [6]. The neutrino mass is not affected by SSB: \(M_{\nu} = m\).

Restricting the VSRSM after SSB to the interactions between photon and electron alone, we get the VSRQED action:

\[
\mathcal{L} = \bar{\psi} \left( i \left( \partial_\mu + \frac{1}{2} \gamma m^2 (n \cdot D)^{-1} \right) - M \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{(\partial_\mu A_\mu)^2}{4}
\]  (1)

where we denote the electron field by \(\psi\), the photon field by \(A_\mu\) and used the Feynman gauge. In (1) \(n_\mu\) is a fixed null vector, i.e., \(n \cdot n = 0\), while \(D_\mu = \partial_\mu - ieA_\mu\), \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\). We see that the electron mass, determined by the pole of the propagator, is \(M_e = \sqrt{M^2 + m^2}\).

3. Photon Self Energy in VSRQED
In this section we present the computation of the photon self-energy. Applying the Sim(2)-invariant regulator to the addition of the graphs of Figure 2, we get:

\[
i \Pi_{\mu\nu} = \mathcal{A}(\eta_{\mu\nu} q^2 - q_\mu q_\nu) + B \left( -q^2 \frac{n_\mu n_\nu}{(n \cdot q)^2} + \frac{n_\mu q_\nu + n_\nu q_\mu}{n \cdot q} - \eta_{\mu\nu} \right)
\]  (2)
Figure 2. Vacuum polarization one-loop graphs

\[
A = (-ie)^2 \frac{i}{(4\pi)^\omega} \int_0^1 dx \frac{8x(1-x)}{\Gamma(2-\omega)(M_e^2 - (1-x)q^2)^2-\omega}
\]
\[
B = -m^2 e^2 \frac{e^2}{4\pi^2} \int_0^1 dx \frac{1}{(1-x)} \log \left[ 1 - \frac{q^2(1-x)^2}{M_e^2 - q^2(1-x)x} \right].
\]

Here \(-e\) is the electron electric charge, \(m\) the electron-neutrino mass, \(M_e\) the electron mass and \(q_\mu\) is the virtual photon momentum.

We first notice that \(q^\mu \Pi_{\mu\nu} = 0\) as required by \(U(1)\) gauge invariance of the photon field. It is obtained by a straightforward application of the regularized integrals of [7]. Moreover \(B(q^2 = 0) = 0\), therefore the photon remains massless. Also the photon wave-function divergence is the same as in QED.

4. Electron Self Energy in VSRQED

Here we calculate the electron self-energy. Again we have two graphs contributing to the 2-proper vertex, see Figure 3.

\[
-i\Sigma(q) = C \frac{q \cdot q}{n \cdot q} + Dq + E
\]

Figure 3. Electron self-energy one-loop graphs. The second graph vanishes in Feynman gauge.
with:

\[ C = (-ie)^2 m^2 \left[ \frac{i}{16\pi^2} \int_0^1 dx (1-x)^{-1} \ln \left(1 + \frac{q^2(1-x)}{(M^2 - q^2 - i\varepsilon)}\right) \right. \]
\[ + 2i(4\pi)^{-\omega} \int_0^1 dx \frac{\Gamma(2-\omega)}{[\mu^2 x - x(1-x)q^2 + M^2(1-x) - i\varepsilon]^{2-\omega}} \right], \quad (5) \]

\[ D = -2(-ie)^2 \omega (16\pi^2)^{-\omega} \int_0^1 dx \frac{\Gamma(2-\omega)}{[\mu^2 x - x(1-x)q^2 + M^2(1-x) - i\varepsilon]^{2-\omega}}, \quad (6) \]

\[ E = (-ie)^2 2\omega M i(4\pi)^{-\omega} \int_0^1 dx \frac{\Gamma(2-\omega)}{[\mu^2 x - x(1-x)q^2 + M^2(1-x) - i\varepsilon]^{2-\omega}}. \quad (7) \]

### 5. Electron-Electron-Photon Proper vertex (EEP vertex) \( \Gamma^\mu(p + q, p) \)

In this section we discuss the EEP vertex and verify the Ward-Takahashi identity. This is an important test of the gauge invariance of the regulator. The one-loop contribution to \( \Gamma^\mu(p' = p + q, p) \) consists of the addition of three graphs (see Figure 4).

![Figure 4. One-loop contributions to the EEP vertex](image)

As a result of the shift symmetry which is respected by the regulator, \( \int dp f(p_\mu) = \int dp f(p_\mu + q_\mu) \) for arbitrary \( q_\mu \), we can prove the Ward-Takahashi identity. This identity must hold for both the divergent and finite pieces of the Green functions:

\[ -iq_\mu \Gamma^\mu(p + q, p) = S^{-1}(p + q) - S^{-1}(p) \quad (8) \]

Here \( S(p) = \frac{i}{p - M - \Sigma(p)} \) is the full electron propagator and \( \Gamma^\mu(p + q, p) \) is the EEP vertex.

As a simple check, let us verify explicitly that the pole at \( d = 4 \) satisfies (8). For the pole contributions one finds:

\[ P\Sigma(q) = -(-ie)^2 \frac{1}{16\pi^2} \left\{ \frac{2m^2}{n \cdot q} - \frac{1}{2-\omega} \right\} \quad (9) \]

\[ P\Gamma^\mu(p + q, p) = -(-ie)^2 \frac{1}{16\pi^2} \left( \frac{1}{2-\omega} \left( \frac{n_{\mu}}{n \cdot (p + q)} + \frac{\gamma_\mu}{n \cdot q} \right) \right) \quad (10) \]

The divergent piece satisfies the Ward-Takahashi identity (8):

\[ q_\mu P\Gamma^\mu(p + q, p) = P\Sigma(p) - P\Sigma(p + q) = -(-ie)^2 \frac{1}{16\pi^2} \left\{ \frac{1}{2-\omega} \left( \frac{1}{n \cdot (p + q)} - \frac{1}{n \cdot p} \right) \right\}. \]
5.1. Form factors

The on-shell EEP vertex can be written as follows

\[ M_\mu(p,q) = \bar{u}(p+q) \left\{ G_2 \left[ -i\sigma_\mu q_\nu \right] + G_3 \psi Q_\mu + F_3 \phi_\mu q_\nu + \bar{\gamma}_\mu F_1 + F_2 \frac{\sigma_{\mu \nu}}{2M} q_\nu \right\} u(p) \]  

where:

\[ \bar{\gamma}_\mu = \gamma_\mu + \frac{m^2}{2} \frac{\psi n_\mu}{n \cdot p (n \cdot p + n \cdot q)} \]

\[ Q_\mu = q_\mu - q^2 \frac{n_\mu}{n \cdot q} \]

F_1, F_2, F_3, G_2, G_3 are form factors (Lorentz-scalar combinations of n_\mu, p_\mu, q_\mu). Under the Sim(2) scaling n_\mu \rightarrow \lambda n_\mu, F_1, F_2 are invariants, F_3 \rightarrow \lambda^{-2} F_3, G_3 \rightarrow \lambda^{-1} G_3, G_2 \rightarrow \lambda^{-1} G_2.

In the non-relativistic (NR) limit we get Table 1, keeping terms that are at most linear in q_\mu. The operator whose coefficient is the form factor, reduces in this limit to the addition of the terms appearing in the first column of Table 1, corresponding to the same form factor in the second column of Table 1.

**Table 1.** In the right column we list the form factor. In the left column we have the NR limit of the matrix element accompanying the form factor in (11). All form factors are evaluated at q_\mu = 0. Here A_0 is the electric potential and A_i is the vector potential. \( \vec{\varphi}_{s'} \) is a two-dimensional constant vector that corresponds to the NR limit of the Dirac spinors.

| NR limit | Form factor |
|----------|-------------|
| \( 2M e \varphi_1^1 \varphi_s A_0 \) | \( F_1(0) \) |
| \( \frac{3m^2}{4M^2} i \varepsilon_{ijk} \phi_1^i \varphi_s \hat{n}_j q_k A_0 \) | \( F_1(0) \) |
| \( i \varepsilon_{ijk} q_j \phi_1^i \varphi_s A_i \) | \( F_1(0) \) |
| \( -2i n_0 M \varepsilon_{ijk} \varphi_1^i \varphi_4^k \varphi_s q_j A_i \) | \( G_2(0) \) |
| \( -i \varepsilon_{ijk} n_k \frac{m^2}{n^2} \phi_1^i \phi_s \hat{n} \varphi_4 q_j A_i \) | \( G_2(0) \) |
| \( i(2M \varepsilon_{ijk} n_k \varphi_1^i \varphi_s + 2M e \varepsilon_{ijk} \varphi_4^i) A_0 q_j \) | \( G_2(0) \) |
| \( 2M e \varphi_0^1 \varphi_s Q_\mu A_\mu \) | \( G_3(0) \) |
| \( -4M e \varepsilon_{ijk} n_k \varphi_1^i \hat{n} \varphi_s + 4M e \varphi_0^2 \varepsilon_{ijk} \varphi_4^i \varphi_s q_j A_i \) | \( F_3(0) \) |
| \( 4M e n_0 \varepsilon_{ijk} n_j \phi_1^i \varphi_s A_0 q_i \) | \( F_3(0) \) |
| \( i \varepsilon_{ijk} \varphi_4^i \varphi_s A_i q_j \) | \( F_2(0) \) |
| \( -i \frac{m^2}{2M^2} \varepsilon_{ijk} \hat{n}_j \varphi_1^i \varphi_s A_0 q_j \) | \( F_2(0) \) |

To show the power of the Sim(2)-invariant regularization prescription presented in [7], we will compute the one-loop contribution to the (isotropic) anomalous magnetic moment of the electron. It is given by \( F_2(0) - 2n_0 M G_2(0) - 4F_3(0)M e n_0^2 \) (see rows 11, 5 and 9 of Table 1).

Evaluating the integrals according to the Sim(2)-invariant prescription to \( o(m^2) \), we get [7]:

\[ F_2 - 4F_3 M e n_0^2 - 2G_2 n_0 M = \frac{\alpha}{2\pi} \]

where \( \alpha \) is the fine-structure constant. Therefore to this order the QED result holds.

Notice that already at tree level, the model predicts the existence of an anisotropic electric moment of the electron, corresponding to the second line of the list and an anisotropic magnetic
moment of the electron, corresponding to the fourth row of the list, both of the order of \( \frac{m^2}{M^2} \).

The electric dipole moment is:

\[
|\vec{p}| = \frac{3e}{4M_e^2} \left| (\vec{s} \times \hat{n}) \right| \frac{3}{8} \lambda e \frac{m^2}{M_e^2}
\]

where \( \lambda = 2.4 \times 10^{-12} \) m is the Compton wave length of the electron.

Using the best bound on the electric dipole moment of the electron [11], \( |\vec{p}| < 8.7 \times 10^{-29} \) e-cm, we get:

\[
\frac{m^2}{M_e^2} < 9.7 \times 10^{-19}.
\]

For the muon \( \lambda = 1.17 \times 10^{-14} \) m. Using the best bound on the muon electric dipole moment[12], \( |\vec{p}_\mu| < 1.8 \times 10^{-19} \) e-cm, we get:

\[
\frac{m_\mu^2}{M_\mu^2} < 4 \times 10^{-7}.
\]

The Sim(2)-invariant regularization permits to explore the full quantum properties of VSR. They should be systematically tested, in Particle Physics models as well as in Quantum Gravity models.

Acknowledgments
J.A. wants to thank the organizers of the “Workshop on CPT and Lorentz Symmetry in Field theory” for a very stimulating atmosphere. The work of J.A. has been partially financed by Fondecyt 1150390 and Anillo ACT 1417.

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