Some Uncomfortable Thoughts on the Nature of Gravity, Cosmology, and the Early Universe

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Abstract A specific theoretical framework is important for designing and conducting an experiment, and for interpretation of its results. The field of gravitational physics is expanding, and more clarity is needed. It appears that some popular notions, such as ‘inflation’ and ‘gravity is geometry’, have become more like liabilities than assets. A critical analysis is presented and the ways out of the difficulties are proposed.

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1 Introduction

The proximity of the site of this gravitational conference to CERN and its recently completed Large Hadron Collider (LHC) reminds us of the affinity of our ultimate goals in the study of micro- and macro-worlds. Although the LHC will be investigating the minute particles - hadrons, the outreach page of the LHC web-site explains to the wide public that the “aim of the exercise is to smash protons...into each other and so recreate conditions a fraction of a second after the big bang” [1]. One can also see clarifications in other sources of information, according to which the “LHC experiments...will probe matter as it existed at the very beginning of time”, and that this is a “new era of understanding about the origins and evolution of the universe”.

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These cosmological, rather than particle-physical, explanations can perhaps be justified, at least in part, by the alleged difficulties that our colleagues encountered in communication with general public. According to a circulating rumor, at the time when the opening of the LHC was widely covered by TV media, the BBC received numerous messages from angry parents who complained that “you keep talking about all these hardons while children may be watching”. Surely, the descriptive cosmology is safer and easier to convey to the public than the notions of high-energy physics. But in the long run, it is indeed true that the fundamental question of the birth of the Universe is always in the background of our research intentions. This problem fascinates both scientific communities, as well as some part of the rest of population.

Undoubtedly, new important discoveries will be made at LHC and they will bring us closer to answers to very deep issues in physics. However, the record-breaking energies of $10^4$ GeV at our accelerators are still very far away from energies we need to understand in order to tackle the problem of the origins of the Universe. The ability of LHC to answer the questions on the physics of the very early Universe at $(10^{15} - 10^{19})$ GeV can be compared with the ability of a telescope hardly resolving a planetary system to answer the questions on the structure of a human hair. In these matters, our best hopes are associated with cosmic rather than laboratory studies. It is now recognized that there is no better way of trying to unveil the origins of the Universe than by measuring the relic gravitational waves that were generated by a strong variable gravitational field of the emerging Universe. We may be close to discovering relic gravitational waves, and thus answering some of the most fundamental questions. These efforts, as well as some principal issues on the nature of gravity, will be discussed below.

2 Spontaneous Birth of the Universe

The question of origins arises inevitably given the nonstationary character of the world that we observe at large scales. The broad-brush picture of the expanding homogeneous and isotropic distribution of matter and fields is pretty accurate as a zero-order approximation. We believe that this approximation was even better in the past, because the existing deviations from homogeneity and isotropy seem to have been growing in the course of time. It is known from observations that the present size $l_p$ of our patch of approximate large-scale homogeneity and isotropy is at least as big as the present-day Hubble radius $l_H = c/H_0 \approx 10^{28}$ cm. Combining available observations with plausible statistical assumptions one can conclude that the size of this patch is significantly bigger than $l_H$. It was evaluated to be about $500 l_H$ and maybe larger. In the present discussion, we will limit ourselves with $l_p = 10^3 l_H$. It is also known from observations that the present averaged energy density $\rho_p c^2$ of all sorts of matter in our patch is close to $\rho_p = 3 H_0^2 / 8\pi G \approx 10^{-29}$ g/cm$^3$.

Given the observed expansion, one can extrapolate $\rho(t)$ and $l(t)$ back in time by the known laws of physics. Temporarily leaving aside the possible recent interval of accelerated evolution governed by “dark energy” (see Sec.7), we will first have $\rho(t) \propto 1/t^3(t)$ at the matter-dominated stage, up to $\rho_{eq} \approx 10^{-29}$ g/cm$^3$, and then $\rho(t) \propto 1/t^4(t)$ at the radiation-dominated stage. Eventually one encounters a cosmological singularity characterized by the infinitely large energy density. It is reasonable to think that there must exist a smarter answer to the question of initial state of the observed Universe than singularity. Singularity is likely to be only a sign of breakdown of the currently available theories.
It is tempting to begin from the limits of applicability of current theories, that is, from the Planck density $\rho_{Pl} = c^5/G^2h \approx 10^{94}$ g/cm$^3$ and the Planck size $l_{Pl} = (G\hbar/c^3)^{1/2} \approx 10^{-33}$ cm, and imagine that the embryo Universe was somehow created by a quantum-gravity (or by a ‘theory-of-everything’) process in this state and then started to expand (see [3] and references there). Conceptually, it is easier to imagine that the created Universe was spatially closed, which means that its total energy, including gravity, was zero and remains zero. Also, the concept of a closed universe helps avoid the question of environment. But a closed universe is not strictly necessary for our line of argument, so one can think of a small configuration in a possibly larger system. It is also plausible to think that the classical evolution of our patch of the Universe could have started with a natal size somewhat larger than $l_{Pl}$ and a natal energy density, of whatever matter that was there, somewhat lower than $\rho_{Pl}$.

The trouble is, however, that the very natural hypothesis of spontaneous birth of the observed Universe will not bring us anywhere near our present state characterized by $\rho_p$ and $l_p$, unless we make additional assumptions [3].

In Fig. 1 the present state of the accessible Universe is marked by the point P. The hypothesized birth of the Universe is marked by the point B. If we go in the past from the point P according to the laws of matter-dominated and radiation-dominated evolutions (black curve), we totally miss the desired point B. Indeed, the energy density reaches the Planck value $\rho_{Pl}$ when the size of the model universe is 0.3cm instead of the
required $10^{-33}$ cm. On the other hand, if we descend from the point B according to the laws of radiation-dominated and then matter-dominated evolutions (blue curve), we totally miss the point P. Indeed, the present energy density $\rho_p$ is reached while the size of the model universe is 0.03 cm instead of the required $10^{33}$ cm. And the present size $10^{31}$ cm is reached when the density drops to $10^{-126}$ g/cm$^3$ instead of the required $\rho_p$.

If the size $l_p$ of the homogeneous isotropic patch is larger than the assumed $l_p = 10^{34} l_H$, the point P and the black curve in Fig.1 move to the right, and the mis-match of the curves is only exacerbated.

The only way to reach P from B is to assume that the newly born Universe has experienced a ‘primordial kick’ allowing the point of evolution to jump over from the blue curve to the black curve. During the kick, the size of the Universe (or, better to say, the size of the patch of homogeneity and isotropy) should increase by about 33 orders of magnitude, but the energy density may not change too much or may stay constant. In the zero-order approximation, when the homogeneity and isotropy of the patch are maintained, the level of the energy density and the specific points of the start S and finish F of the kick transition are not very important, as we can reach P from B by many ways. However, the actual route becomes extremely important in the first-order approximation, when we have to take into account the quantum-mechanical generation of primordial cosmological perturbations.

If we are on the right track with this whole picture, the observations indicate (see below) that the actual kick trajectory took place at the energy densities around $10^{-10} \rho_{Pl}$ when the Hubble parameter $H$ was around $H_i = 10^{-5} H_{Pl} = 10^{-5} / t_{Pl}$ (horizontal red line in Fig.1), with some possibility of a slight ‘over-kick’ to point $F_{up}$ or ‘under-kick’ to point $F_{down}$, as shown by tilted red lines in Fig.1.

The general relativity allows the required kick trajectories, but only if the properties of the primeval matter were not like those that we deal with in laboratories (even at LHC). The energy density $\rho c^2$ and the effective pressure $p$ should satisfy the condition $\rho c^2 + p = 0$ for the kick to proceed with constant energy density, and $\rho c^2 + p < 0$ for over-kick with increasing energy density or $\rho c^2 + p > 0$ for under-kick with decreasing energy density. The kick Hubble parameter $H_i(t)$ remains constant ($\dot{H} = 0$) for the $\rho c^2 + p = 0$ case, and it is slightly increasing ($\dot{H} > 0$) or decreasing ($\dot{H} < 0$) for over-kick and under-kick trajectories, respectively. If the existing indications (see below) that the initial kick did indeed happen are further supported by observations, the detailed inquiry in the properties of the substance that might have driven the kick will be paramount.

Solutions with $\rho c^2 + p \geq 0$ and $p \approx -\rho c^2$ are allowed by some versions of the scalar field within general relativity. They are usually associated with the notion of inflation (see, for example, [4]). The case with $p = -\rho c^2$ is known as the standard (de Sitter) inflation. The scalar field plus gravity model may not be what we are searching for, but at least it was shown [3] that the required solutions with $p \approx -\rho c^2$ are attractors in the space of all solutions of this dynamical system, so these are typical solutions which can bring us from the quantum boundary S [3] to the end point $F_{down}$. (Scalar field models do not admit the over-kick trajectories.)

It is often stated that inflation was invented by particle physicists in order to solve outstanding cosmological problems. The length of the list of solved problems depends on the enthusiasm of the writing inflationary author. To me personally, most of these problems and solutions smack of the Soviet-style management, when the government creates a problem and then demands the credit for heroically solving it. In any case, inflation was always assumed to have lasted sufficiently long; otherwise the point of
evolution would have fell short of reaching the point $F_{\text{down}}$ on the black curve in Fig.1. Whatever the problems the inflationary hypothesis solves, they are automatically solved by the hypothesis of primordial kick, whose only rationale is to serve as an ‘umbilical cord’ facilitating the joining of the birth event $B$ with the present state of the Universe $P$.

While in the approximation of homogeneity and isotropy the hypothesis of inflation and the hypothesis of initial kick are equal, in the sense that they both solve problems by making plausible assumptions, at the level of cosmological perturbations the inflationary theory got everything wrong, as we shall see below, in Sec.4.

### 3 Primordial Cosmological Perturbations

The essentially classical and highly symmetric, i.e. homogeneous and isotropic, solution describing the initial kick should be augmented by quantum fluctuations of the fields that were present there. It is natural to assume that these quantized fields were initially in their ground (vacuum) states. If the field is properly, superadiabatically, coupled to the strong gravitational field of the kick solution, the quantum-mechanical Schrödinger evolution transforms the initial vacuum state of the field into a multiparticle (strongly squeezed vacuum) state. This is called the superadiabatic, or parametric, amplification; for a recent review of the subject, see [6]. Not all fields couple properly to the highly symmetric solutions under discussion. For example, electromagnetic fields do not. But the gravitational field perturbations do couple superadiabatically to the gravitational field of the kick, and therefore they can be amplified. Apparently, this is how the present-time complexity arises from the past-time simplicity.

The chief gravitational field perturbations are two transverse-traceless degrees of freedom describing gravitational waves and two degrees of freedom, scalar and longitudinal-longitudinal one, which in general relativity exist only if they are accompanied by perturbations of matter. These latter two degrees of freedom describe density perturbations, and usually only one of them is independent. For models of matter such as scalar fields, the equations for density perturbations are almost identical to the equations for gravitational waves. The arising multiparticle states at each frequency are conveniently combined in the power spectra of these quantum-mechanically generated perturbations. The crucial quantity, both for gravitational waves and density perturbations, is the primordial power spectrum of the gravitational field (metric) perturbations. This is because the gravitational field perturbations survive numerous transformations of the matter content of the Universe at the end of the kick and later, and therefore they provide the unambiguous input values for physics and calculations at the radiation-dominated and matter-dominated stages.

The values of the Hubble parameter along the kick trajectory and the shape of the kick trajectory itself are very important, because they define the numerical levels and spectral slopes of the primordial metric spectra. The Hubble parameter $H_i$ determines the amplitude of metric perturbations, whereas the tilt of the red lines in Fig.1 determines the tilt of the spectrum. The primordial metric power spectrum $P(n)$ is a function of the time-independent wavenumber $n$, where the wavenumber $n_{H} = 4\pi$ corresponds to the wavelength which will be equal to $l_H$ today.

Very similar forms of equations for gravitational waves (gw) and density perturbations (dp), and identically the same physics of their superadiabatic amplification, translate into very similar power spectra $P(n)$. In a good approximation, sufficient for
our purposes, the generated gw and dp power spectra can be written in the power-law forms

\[ P(n) \, (gw) = \left( \frac{H_i}{H_{Pl}} \right)^2 \left( \frac{n}{n_H} \right)^{n_t}, \quad P(n) \, (dp) = \left( \frac{H_i}{H_{Pl}} \right)^2 \left( \frac{n}{n_H} \right)^{n_s-1}, \]  

(1)

where the spectral indices \( n_t, n_s \) are constants, and \( n_s - 1 \approx n_t \). For flat kick evolutions, i.e. for horizontal lines like the red line in Fig.1 the generated spectra have flat (Harrison-Zeldovich-Peebles) shape, i.e. \( n_t = n_s - 1 = 0 \). The over-kick or under-kick evolutions tilt the spectrum toward the ‘blue’ or ‘red’ shapes, respectively. Of course, it is simplistic to expect that the red transitions in Fig.1 should be strictly straight lines and the spectral indices \( n_t, n_s \) strict numbers independent of the wavenumber \( n \). In simple models driven by scalar fields, the spectral indices are actually slowly decreasing functions of \( n \).

The derivation of primordial spectra is based only on general relativity and quantum mechanics. The comparison of spectra and their consequences with observations is our best chance to learn whether the kick did indeed take place and how it looked like. In particular, assuming that the observed anisotropies of the cosmic microwave background radiation (CMB) are indeed caused by cosmological perturbations of quantum-mechanical origin, we conclude that the kick Hubble parameter \( H_i \) was close to \( 10^{-5} H_{Pl} \), because this kick has generated the amplitudes of metric perturbations at the level of \( 10^{-5} \), which in their turn produced the observed large-scale CMB anisotropy at the level of \( 10^{-5} \).

The perturbations with wavelengths much shorter than the radius \( l_p \) of our patch were processed in the course of evolution, and therefore the form of the spectrum

\[ h_{rms}(\nu) \]  

Fig. 2 The present-day spectrum of the root-mean-square amplitude \( h_{rms}(\nu) \) of relic gravitational waves. The solid line corresponds to the primordial spectral index \( n_t = 0.2 \, (n=1.2) \), while the dashed line is for \( n_t = 0 \, (n=1.0) \).
in the recombination era (and today) differs from primordial. The spectrum is no
longer a smooth function of the wavenumber (or frequency) but contains maxima and
minima. This is a consequence of the standing-wave pattern of the primordial metric
perturbations - the inevitable feature of the underlying quantum-mechanical squeezing.
The power-law slope of the envelope of the oscillations also changes.

As an example, in Fig. 2 taken from [(6)], we show today’s power spectra as func-
tions of frequency $\nu$ for relic gravitational waves normalized to the observed CMB
anisotropies. Only a few first cycles of oscillations at lowest frequencies are shown.
Two possibilities are depicted in this figure: the dashed line is the resulting spectrum
which originated in the superadiabatic amplification during the horizontal red line
transition of Fig. 1, whereas the solid line is for the resulting spectrum originated in a
slightly over-kick transition. At the highest relevant frequencies, before the spectrum
sharply goes down, one can see a noticeable increase of power in $h_{\text{rms}}$. This is the result
of the after-kick piece of evolution governed by matter with a stiff (Zeldovich) equation
of state $p = \rho c^2$. This piece of evolution was assumed in this calculation, but it is
not guaranteed. We may be not so lucky to have to deal in the future high-frequency
experiments with this increased gravitational-wave power.

4 Cosmological Perturbations in Inflationary Theory

Now we turn to what is broadly addressed as inflation, and specifically to predictions
of inflationary theory on cosmological perturbations. Predictions of inflationary theory are
dramatically different from what was described above, in Sec.3. In attempt of deriving
density perturbations, inflationary authors invariably begin with vacuum fluctuations
of the scalar field (inflaton) in de Sitter space-time. Then, after some jumping between
variables and gauges, they conclude that the amplitudes of scalar metric perturbations
(often denoted by letters $\zeta$ or $R$) should be infinitely large, in the same physical system
and right from the very beginning. Instead of Eq.(1), the inflation-predicted primordial
metric power spectrum for density perturbations reads:

$$P(n) \left( \frac{dp}{d\nu} \right)_{\text{inflation}} = \frac{1}{\epsilon} \left( \frac{H_i}{H_{\text{Pl}}} \right)^2 \left( \frac{n}{n_H} \right)^{ns-1},$$

(2)

where the parameter $\epsilon$ is $\epsilon \equiv -\frac{\dot{H}}{H^2}$. This parameter is almost a constant in cases under
discussion. The standard (de Sitter) inflation is characterized by $\epsilon = 0$ ($p = -\rho c^2$),
and therefore the inflation-predicted standard spectrum blows up to infinity at every
wavelength and for any non-zero value of $H_i$.

Formula (2) is the main contribution of inflationary theory to the subject of cos-
ological perturbations. Having arrived at the incorrect formula with the arbitrarily
small factor $\epsilon$ in the denominator of this formula, inflationary theory elevates the ques-
tion of the "energy scale of inflation" to the status of a major scientific problem, which
inflationists will be happily solving for decades to come. Indeed, one is now free to
start with energy densities $\rho_i$, say, 80 orders of magnitude smaller than $\rho_{\text{Pl}}$ and the
Hubble parameter $H_i$ 40 orders of magnitude smaller than $H_{\text{Pl}}$ (grey line in Fig.1).
One can still claim that (s)he has built a successful inflationary model, because the
required level $10^{-5}$ of primordial metric amplitudes for density perturbations can now
be achieved by simply making the grey line sufficiently horizontal (but not exactly
horizontal) and thus making the denominator $\epsilon$ in Eq. (2) sufficiently small (but not
exactly zero). Astrophysics does not permit one to take \( H_i \) smaller than the value of \( H \) in the era of nucleosynthesis, i.e. below the grey line in Fig. 1. Otherwise, one could have started with the “energy scale of inflation” equal to the energy density of water and still build a successful inflationary cosmology.

Inflationary theory substitutes the predicted divergency of density perturbations at \( \epsilon = 0 \), Eq.(2), by the claim that it is the amount of relic gravitational waves that should be small. This is meant to be represented by the “tensor-to-scalar” ratio \( r \), which is the ratio of the gw power spectrum from Eq.(1) to the dp power spectrum from Eq.(2). In inflationary theory, it is written as

\[
 r = 16\epsilon = -8n_t. \tag{3}
\]

The WMAP (Wilkinson Microwave Anisotropy Probe) team reports the limits on the parameter \( r \) that were found from the observations [(7)]. One can see these limits in the form of the likelihood function for \( r \) in the left panel of Fig. 3 in Ref.(7). The maximum of the likelihood function is at \( r = 0 \), which means that the most likely value of the parameter \( r \) is \( r = 0 \). Then, according to Eq.(3), the most likely value of \( \epsilon \) is \( \epsilon = 0 \). Since \( \epsilon \) is in the denominator of the inflation-predicted power spectrum (2), one concludes that if the inflationary predictions are correct, then the most likely values of density perturbations responsible for the data observed and analyzed by the WMAP team are infinitely large. Nevertheless, this situation is often qualified as the “striking success of inflation” [(8)] demonstrated by the WMAP findings.

For the parameter \( r \), the inflationary theory predicts practically everything what one can possibly imagine, including something like \( r \leq 10^{-24} \) as the inflationary outcome, based on Eq.(2), of the most sophisticated string-inspired models (for a review, see [(8)] and references there). The big problem is not that the seemingly most advanced theories predict \( r \leq 10^{-24} \), thus making the search for relic gravitational waves look ridiculous. The big problem is that when the relic gravitational waves are discovered, the proposers, being misguided by their own derivations, will reject microphysical theories which in fact may be perfectly viable, and will accept theories which in fact may be totally wrong.

5 Discovering Relic Gravitational Waves in the Cosmic Microwave Background Radiation

There are several reasons to believe that the observed CMB anisotropies are caused by cosmological perturbations of quantum-mechanical origin. Probably the major one is the observed oscillations in the CMB power spectra, which are likely to be a reflection of the quantum-mechanical squeezing and the associated standing-wave pattern of the primordial metric perturbations. If cosmological perturbations do indeed have quantum-mechanical origin, Eq.(1) implies that the gw and dp contributions to the lower-order CMB multipoles \( \ell \) should be of the same order of magnitude. Relic gravitational waves have not been scattered or absorbed in any significant amount since the time of their quantum-mechanical generation. The explicit identification of relic gravitational waves in the data would be a monumental step in the study of the primordial kick and the origins of the Universe.

Density perturbations and gravitational waves affect temperature and polarization of the CMB. The polarization is usually characterized by the two components - E and B. The B component is not generated by density perturbations, and therefore it is often
stated that the aim of the exercise is to detect B-modes. This is not so. The aim of the exercise is to detect relic gravitational waves, not to detect B-modes. Gravitational waves are present in all correlation functions of the CMB, and one should be smart enough to distinguish their contribution from competing contributions. The B-mode channel of information has some advantages, but many disadvantages too. For example, the 5-year WMAP data on the BB correlation function are not informative because they are mostly noise. At the same time, the much stronger TE signal is recorded at a large number of data points, which allows one to derive meaningful conclusions about the gw contribution.

The 5-year WMAP TE data were thoroughly analyzed in Ref. (9). The conclusion of this investigation is such that the lower-\(\ell\) TE and TT data do contain a hint of presence of the gravitational wave contribution. In terms of the parameter \(R\), which gives the ratio of contributions of gw and dp to the temperature quadrupole, the best-fit model produced \(R = 0.24\). This means that 20% of the temperature quadrupole is accounted for by gravitational waves and 80% by density perturbations. The residual WMAP noise is high, so the uncertainty of this determination is large, and it easily includes

**Fig. 3** The signal to noise ratio \(S/N\) as a function of \(R\) for the TE (black) and BB (red) observational channels. The points show numerical results, whereas the curves are analytical approximations.
the hypothesis that there is no gw contribution at all. However, the uncertainty will be
much smaller in the forthcoming more sensitive observations, most notably with the
Planck satellite. It is likely that the Planck mission will be capable of strengthening
our belief that the primordial kick did take place, and at the near-Planckian values of
the Hubble parameter, $H_i \approx 10^{-5}H_{Pl}$.

The future TE and BB data from Planck satellite were simulated and analyzed
in [9]. The quality of the future performance of the BB channel is not clear, so the
authors discuss the ‘optimistic’ and ‘realistic’ options. The result of the analysis is
shown in Fig.3 where the signal to noise parameter $S/N$ is plotted as a function of $R$.
It is seen from the graph that if the maximum likelihood value $R = 0.24$ derived from
the WMAP5 TE and TT data is taken as a real signal, it will be present at a better
than 3σ level in the Planck’s TE observational channel, and at a better than 2σ level
in the ‘realistic’ BB channel. The time of discovery of relic gravitational waves may be
nearer than is usually believed.

6 Field-Theoretical Formulation of General Relativity

One of important premises in the above conclusions is the assumption that the general
relativity remains valid up to energy densities approaching Planckian limit. Although
there is no obvious reason to doubt this, any piece of new information on the domain
of applicability of a fundamental theory is useful. The tests of gravitational theories in
various circumstances and conditions is an important part of current research, including
the results and plans discussed at this conference. As usual, the set-up of an experiment
and interpretation of its results partially depend on the accepted theoretical framework,
which in the case of general relativity seems to be obvious and well established: “gravity
is geometry”. I personally feel that the emphasis on the geometrical aspect of gravity
has exhausted its usefulness and has become an impeding rather than a driving factor
in gravitational physics.

There seems to be something odd in the conviction of a large part of our com-
munity that the gravitational waves are “oscillations in the fabric of space-time”, and
that the ongoing and planned gravitational-wave experiments are attempting to mea-
sure the “strain in the fabric of space-time”. This understanding is usually presented
as a consequence of the equivalence principle which forbids, so is believed, such things
as rigorously defined local gravitational energy density and flux of energy. At best, this
understanding allows surrogates, such as the averaged pseudotensors. The argument
seems to be especially appropriate for the current research, wherein the gravitational-
wave detectors are usually small and, in a sense, local. The size of the detector is
much smaller than the wavelength which the detector is most sensitive to, so the
gravitational-wave flux through the detector is supposed to be completely or partially
removable by a coordinate transformation. (In fact, the four pseudotensor components
describing energy density and flux are removable by four coordinate transformation
functions everywhere, not only locally.) Therefore, the argument goes, it is the “squeez-
ing and stretching of space-time itself” that is a proper description of the phenomenon,
and not the absorption of the part of the gravitational wave flux by the detector, which
makes the detector ‘click’ and register the event.

The problem appears to be more than simply linguistic. The concept “gravity is
geometry” can be misleading in the analysis of the detector’s response and in the
interpretation of results. More importantly, it obscures the deeper understanding of
gravity. Namely, the fact that the Einstein’s general relativity is a perfectly consistent non-linear gravitational field theory in flat (Minkowski) space-time, with rigorously defined gravitational energy-momentum tensor, and with no need for geometrical notions of curved space-time.

The field-theoretical approach to general relativity is based on the concept of gravitational field $h^{\mu\nu}(x^\alpha)$ defined in flat space-time with the metric tensor $\gamma^{\mu\nu}(x^\alpha)$. The curvature tensor constructed from $\gamma_{\mu\nu}(x^\alpha)$ is identically zero:

$$\tilde{R}_{\alpha\beta\mu\nu}(\gamma_{\rho\sigma}) = 0. \quad (4)$$

The choice of coordinates $x^\alpha$ is in one’s hands, so the $\gamma^{\mu\nu}(x^\alpha)$ can always be transformed, if necessary, to the Minkowski matrix $\eta^{\mu\nu}$ (class of Lorentzian coordinates).

The gravitational Lagrangian $L^g$ is a quadratic function of first derivatives of the field $h^{\mu\nu}$ (see [10] and references there). The variation of $L^g$ with respect to the field variables $h^{\mu\nu}$ leads to the gravitational equations of motion:

$$\frac{1}{2} \left[ (\gamma^{\mu\nu} + h^{\mu\nu})(\gamma^{\alpha\beta} + h^{\alpha\beta}) - (\gamma^{\mu\alpha} + h^{\mu\alpha})(\gamma^{\nu\beta} + h^{\nu\beta}) \right]_{;\alpha;\beta} = \kappa t^{\mu\nu}. \quad (5)$$

In the right hand side (r.h.s) of these equations stands the gravitational energy-momentum tensor $t^{\mu\nu}$ (and $\kappa \equiv 8\pi G/c^4$). The $t^{\mu\nu}$ is rigorously defined as variational derivative of $L^g$ with respect to variations of the metric tensor $\gamma^{\mu\nu}$, with the constraint $\gamma$ properly taken into account. The $t^{\mu\nu}$ contains squares of first derivatives of the field $h^{\mu\nu}$, but not higher derivatives. The gravitational field equations were deliberately rearranged to the form of Eq.(5), where $t^{\mu\nu}$ is the manifest source for the generalised wave (d’Alembert) operator standing in the left hand side (l.h.s.) of the equations.

In the presence of matter Lagrangian $L^m$, the right hand side of Eq.(5) changes:

$$\frac{1}{2} \left[ (\gamma^{\mu\nu} + h^{\mu\nu})(\gamma^{\alpha\beta} + h^{\alpha\beta}) - (\gamma^{\mu\alpha} + h^{\mu\alpha})(\gamma^{\nu\beta} + h^{\nu\beta}) \right]_{;\alpha;\beta} = \kappa (t^{\mu\nu}|_m + \tau^{\mu\nu}). \quad (6)$$

The $\tau^{\mu\nu}$ is the matter energy-momentum tensor derived as variational derivative of $L^m$ with respect to the metric tensor $\gamma_{\mu\nu}$, whereas $t^{\mu\nu}|_m$ is the modified version of the gravitational energy momentum-tensor $t^{\mu\nu}$. The $t^{\mu\nu}|_m$ includes the term arising from $L^m$ and describing the interaction of gravity with matter.

The divergence (contracted covariant derivative) of the left hand side of equations [5], [6] vanishes identically. This means that the equations of motion contain the differential conservation laws

$$t^{\mu\nu}|_{;\nu} = 0, \quad (t^{\mu\nu}|_m + \tau^{\mu\nu});\nu = 0. \quad (7)$$

The differential conservation laws can be converted into conserved integrals for isolated non-radiating systems.

The geometrical description of gravity can be introduced as follows. The special form of $L^g$, which is the field-theoretical analog of the Hilbert-Einstein Lagrangian, and the universal coupling of gravity to matter assumed in $L^m$, which is a realization of the equivalence principle, allow one to ‘glue together’ the metric tensor $\gamma^{\mu\nu}$ and the field variables $h^{\mu\nu}$ into one tensorial object $g^{\mu\nu}$. The Lagrangians $L^g$ and $L^m$, as well as the gravitational and matter field equations, can now be rewritten in terms of this object and its derivatives alone, plus matter variables.
Specifically, one introduces $g^{\mu\nu}$ according to the rule
\[ \sqrt{-g} g^{\mu\nu} = \sqrt{-\gamma} (\gamma^{\mu\nu} + h^{\mu\nu}) , \] (8)
and $g_{\mu\nu}$ according to the definition $g_{\mu\nu} g^{\alpha\beta} = \delta^\alpha_\nu$. Then, the field equations (5), (6) absorb $\gamma^{\mu\nu}$ and $h^{\mu\nu}$ into $g^{\mu\nu}$ and take the form of Einstein’s geometrical equations
\[ R_{\mu\nu} = 0, \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} , \] (9)
where $R_{\mu\nu}$ is the Ricci curvature tensor constructed from $g_{\mu\nu}$ in the usual manner. The $g_{\mu\nu}$ can be interpreted as a metric tensor of some effective curved space-time. The matter energy-momentum tensor $T_{\mu\nu}$ in Eq.(9) is variational derivative of $L^m$ with respect to $g_{\mu\nu}$, in contrast to $\tau_{\mu\nu}$ which was variational derivative of $L^m$ with respect to $\gamma_{\mu\nu}$.

Note that the field equations as a whole can be rewritten in terms of $g_{\mu\nu}$, but not individual parts of these equations. The gravitational energy-momentum tensor $\tau^{\mu\nu}$ cannot be rewritten as a function of the tensor $g_{\mu\nu}$ and its first derivatives alone. This is something to be expected, as there is no tensor that one could build from $g_{\mu\nu}$ and first derivatives of $g_{\mu\nu}$, apart of $g_{\mu\nu}$ itself. Therefore, there does not exist any meaningful gravitational energy-momentum tensor in the geometrical version of general relativity.

The universal character of the gravitational field, which allows one to ‘glue together’ $\gamma^{\mu\nu}$ and $h^{\mu\nu}$ into a single object $g^{\mu\nu}$ in the total Lagrangian and in the equations of motion, makes the flat space-time ‘non-observable’ in the presence of gravitational fields. This is not surprising. We would have encountered the same ‘non-observability’ of flat space-time in classical electrodynamics had we had access only to test particles with one and the same charge to mass ratio $e/m$. In the absence of neutral particles, which are capable of drawing the lines and angles of the Minkowski world in the region occupied by electromagnetic field, one would be given the option to interpret the motion of charged particles as arising due to the ‘curvature of space-time itself’ rather than due to the external electromagnetic field. This is a possible interpretation, but not particularly illuminating, at least in this case.

The generally covariant theories, including general relativity, admit arbitrary coordinate transformations. A coordinate transformation acting on an object, such for example as the metric tensor or the energy-momentum tensor of the electromagnetic field, can always be rearranged to state that the coordinates are not touched, but the object itself receives an increment in the same point. The rule of changing the numerical values of the object to the new ones involves the technique of Lie derivatives and depends on the transformation properties of the object and a given coordinate transformation. If a dynamical equation is formulated as equality to zero of some tensor composed of the underlying objects, the increments of this tensor vanish when the dynamical equation is satisfied. So, a solution of this dynamical equation translates into a new solution. This procedure of changing the values of objects in the same coordinate frame and generating new solutions can be called a gauge transformation.

The ability to make arbitrary coordinate transformations in the field-theoretical general relativity can be rearranged to state even more. Namely, that the coordinate system $x^\alpha$ and the metric tensor $\gamma_{\mu\nu}(x^\alpha)$ remain untouched, but the set of gravitational field variables $h^{\mu\nu}(x^\alpha)$, as well as matter variables, change to another set of variables in the same coordinate system $x^\alpha$. Under this operation, the gravitational field equations (5), (6), as well as the matter field equations and the conservation laws (7), transform into a combination of themselves, so that a solution of field equations translates into
another solution. A coordinate transformation interpreted this way can be called a ‘true’ gauge transformation, because it looks very much similar to the gauge (gradient) transformation of classical electrodynamics, which changes electromagnetic potentials, but not coordinates and metric. However, the origin of gauge transformations in gravity is different - it is all the same arbitrary coordinate transformations. In general, there is no any other symmetries.

While the equations as a whole are gauge-invariant (transform into a combination of themselves) under the action of ‘true’ gauge transformations, individual parts of the equations are not. For example, in Eq.\(5\) the l.h.s. and r.h.s. receive individual non-zero increments, but such that they precisely cancel each other, leaving Eq.\(5\) gauge-invariant. To demand the gauge-invariance of \(t^{\mu\nu}\) on its own, thus attempting to make \(t^{\mu\nu}\) “physically significant” and not “devoid of physical meaning” \((11)\), would be equivalent to demanding that the field equations should be violated after a gauge transformation.

The same holds true for conservation laws \((7)\). Even if the energy-momentum tensor \(\tau^{\mu\nu}\) represents only a couple of free particles, \(\tau^{\mu\nu}\) changes under the action of ‘true’ gauge transformations. So, the associated change of \(t^{\mu\nu}|_m\) must take care of new positions and new dynamics of the particles. The \(t^{\mu\nu}|_m\) cannot be gauge-invariant on its own, but the gauge-related solutions are observationally equivalent, at least in the classical domain of the theory, and as long as one ignores initial and boundary conditions.

As far as one can see at present, the geometrical and field-theoretical pictures of gravity are representations of one and the same theory of general relativity, not different theories. Each of the viewpoints has its advantages and disadvantages, and we have to become eloquent in both of them. Feynman once remarked \((12)\): “if the peculiar viewpoint taken is truly experimentally equivalent to the usual in the realm of the known there is always a range of applications and problems in this realm for which the special viewpoint gives one a special power and clarity of thought, which is valuable in itself”.

\[\text{Fig. 4 Two famous photos of Einstein. Left: 1905, Bern. Right: 1951, Princeton.}\]
In Fig. 4 one can see two famous photographs of Einstein. He is shown at times of the beginning and the end of his scientific career. In the left photo, the physicist Einstein is here, in Bern, in 1905. He was denouncing the notions of absolute space and time, but was not yet under the influence of geometrical techniques. In the right photo, Einstein is in Princeton, in 1951, when the idea that the space-time is something like a “fabric”, which can curve, wrap, expand, oscillate, etc. was accepted. By that time, Einstein has twice changed his opinion about the reality of gravitational waves. This second picture, is it not addressed to those of us who believe too much literally that “gravity is geometry”?

7 Generalising the General Relativity

The special power and clarity of thought mentioned by Feynman may be especially valuable now, when we are facing the situation that some modifications of general relativity may be required. Hopefully, the recent indications on the accelerated expansion of our approximated homogeneous isotropic Universe can be resolved by more accurate understanding of the limits of applicability of this approximation, and this will be so much for the “dark energy”. But if not, the prospect of modifications of Einstein’s gravity will be looming large. The internal consistency of the candidate modified theory will be a larger hurdle to overcome than agreement with observations.

Without a physical guidance, we are in front of an ocean of possibilities. The geometrical route “gravity is geometry” will definitely lead to undesirable higher-order differential equations in terms of $g_{\mu\nu}$, as there is no way of modifying the second-order dynamical equations (9) except adding a new fundamental constant - the cosmological $\Lambda$-term. In contrast, the very logic of the field-theoretical approach allows natural and consistent generalisations of general relativity without raising the order of differential field equations (10). These generalisations are based on the possibility (and maybe necessity) of adding the ‘mass’-term

$$\sqrt{-\gamma} \left[ k_1 h^{\rho\sigma} h_{\rho\sigma} + k_2 h^2 \right]$$

(10)

to the gravitational Lagrangian $L^g$. In this expression, the constants $k_1$ and $k_2$ have dimensionality of $[\text{length}]^{-2}$, and $h \equiv h^{\mu\nu} \gamma_{\mu\nu}$.

The linearised approximation of this generalised gravity allows one to interpret the introduced constants as masses $m_2$ and $m_0$ of spin $-2$ and spin $-0$ gravitons,

$$\left( \frac{cm_2}{h} \right)^2 = 4k_1, \quad \left( \frac{cm_0}{h} \right)^2 = -2k_1 \frac{k_1 + 4k_2}{k_1 + k_2}.$$

(11)

The interpretation in terms of masses implies $m_0 > 0$ and $m_2 > 0$, but the theory allows also $(m_0)^2 < 0$ and $(m_2)^2 < 0$. When both constants $(m_0)^2$ and $(m_2)^2$ are sent to zero, this finite-range gravitational theory smoothly approaches the equations and observational predictions of the massless general relativity - the property not shared by many other proposed modifications.

It is amazing to see how much the class of allowed solutions broadens and changes under the impact of the seemingly innocent modification (10) of general relativity. The modifications affect the polarization states and propagation of gravitational waves, the event horizons of black holes, the early-time and late-time cosmological evolution. In cosmological applications, the signs and values of $(m_0)^2$ and $(m_2)^2$ are crucial for
The dashed line is a Friedmann solution of general relativity. The upper solid line is a solution of the finite-range gravity for \( \left( \frac{m_0}{m} \right)^2 \equiv \beta^2 < 0 \), while the lower solid line is a solution for \( \beta^2 > 0 \).

the character of changes in the early-time and late-time evolution. In particular, if one allows \((m_0)^2\) to be negative, the late-time evolution of a homogeneous isotropic universe experiences an accelerated expansion, which we may be required to explain if the observations keep demanding this phenomenon. The graph of the modified scale factor \( a \) as a function of time is shown in Fig. 5 taken from the second paper in Ref. [10].

8 Conclusions

The quest for better understanding of gravity is far from over. This conference is certainly not the last one on the topic of nature of gravity. Hopefully, important progress will be reached in the near future, particularly as a result of new experiments, such as Planck and others. Having discovered relic gravitational waves, we will be more confident about the limits of applicability of general relativity and will better understand how cosmic gravity works in concert with quantum mechanics. More excitement is anticipated.

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References

1. LHC web-site: [http://lhc-machine-outreach.web.cern.ch/lhc-machine-outreach/]
2. L. P. Grishchuk, Phys. Rev. D15, 4717 (1992)
3. Ya. B. Zel’dovich, Cosmological field theory for observational astronomers, Sov. Sci. Rev. E Astrophys. Space Phys. Vol. 5, 1986, pp. 1-37 [http://nedwww.ipac.caltech.edu/level5/Zeldovich/Zel_contents.html]
4. S. Weinberg, *Cosmology*, (Oxford University Press, 2008)
5. V. A. Belinsky, L. P. Grishchuk, I. M. Khalatnikov, and Ya. B. Zeldovich, Phys. Lett. B155, 232 (1985); Sov. Phys. JETP 62, 427 (1985)
6. L. P. Grishchuk, Discovering Relic Gravitational Waves in Cosmic Microwave Background Radiation, in "Wheeler Book", eds. I. Ciufolini and R. Matzner (Springer, to be published)
7. E. Komatsu et al. Five-year WMAP Observations: Cosmological Interpretation, arXiv:0803.0547
8. D. Baumann et al. CMBPol Mission Concept Study. Probing Inflation with CMB Polarization, arXiv:0811.3919
9. W. Zhao, D. Baskaran, and L. P. Grishchuk Phys. Rev. D79, 023002 (2009)
10. S. V. Babak and L. P. Grishchuk Phys. Rev. D61, 024038 (1999); Int. J. Modern Physics D 12 (10) 1905 (2003)
11. L. M. Butcher, A. Lasenby, and M. Hobson Phys. Rev. D78, 064034 (2008)
12. R. P. Feynman, in Physics Nobel Prize Lectures (Elsevier, 1965) p.155