Kinematic scaling relations of CALIFA galaxies: A dynamical mass proxy for galaxies across the Hubble sequence.

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ABSTRACT
We used ionized gas and stellar kinematics for 667 spatially resolved galaxies publicly available from the Calar Alto Legacy Integral Field Area survey (CALIFA) 3rd Data Release with the aim of studying kinematic scaling relations as the Tully & Fisher (TF) relation using rotation velocity, $V_{\text{rot}}$, the Faber & Jackson (FJ) relation using velocity dispersion, $\sigma$, and also a combination of $V_{\text{rot}}$ and $\sigma$ through the $S_K$ parameter defined as $S_K^2 = K V_{\text{rot}}^2 + \sigma^2$ with constant $K$. Late-type and early-type galaxies reproduce the TF and FJ relations. Some early-type galaxies also follow the TF relation and some late-type galaxies the FJ relation, but always with larger scatter. On the contrary, when we use the $S_K$ parameter, all galaxies, regardless of the morphological type, lie on the same scaling relation, showing a tight correlation with the total stellar mass, $M_\star$. Indeed, we find that the scatter in this relation is smaller or equal to that of the TF and FJ relations. We explore different values of the $K$ parameter without significant differences (slope and scatter) in our final results with respect the case $K = 0.5$ besides than a small change in the zero point. We calibrate the kinematic $S_K$ dynamical mass proxy in order to make it consistent with sophisticated published dynamical models within 0.15 dex. We show that the $S_K$ proxy is able to reproduce the relation between the dynamical mass and the stellar mass in the inner regions of galaxies. Our result may be useful in order to produce fast estimations of the central dynamical mass in galaxies and to study correlations in large galaxy surveys.

Key words: galaxy kinematics – galaxy scaling relations – dynamical mass

1 INTRODUCTION.
Galaxy scaling relations describe trends that are observed between different properties of galaxies. They are assumed to be the consequence of their formation and evolution. Probably the kinematic scaling relation most widely studied for spiral galaxies is the Tully-Fisher relation (hereafter TF); a correlation between luminosity and rotational velocity, firstly reported by Tully & Fisher (1977). It was originally established as a tool to measure distances to spiral galaxies (e.g., Giovanelli et al. 1997). It has been suggested that the slope, zero-point and tightness may have a cosmological origin helping us to understand the formation and evolution of galaxies (e.g Cole et al. 1994; Eisenstein & Loeb 1996; Mo et al. 1998; Avila-Reese et al. 1998; Courteau & Rix 1999; Firmani & Avila-Reese 2000; Navarro & Steinmetz 2000). In the local universe the TF relation is very tight (e.g. Verheijen 2001; Beckeraité et al. 2016; Ponomareva et al. 2017), locating galaxies with rising rotation curves on the low-velocity end and galaxies with declining rotation curve on the high-velocity end (e.g., Persic et al. 1996). The luminosity-based TF is more directly accessible, however, the amount of light

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measured from the stellar population is a function of passband, and therefore different TF relations emerge when observing galaxies at different wavelengths. A physically more fundamental approach instead of luminosity is based on stellar mass, \( M_\star \). The resulting TF relation is well approximated by a single power law with small scatter at least for disk galaxies more massive than \( \sim 10^{10.5} M_\odot \) (e.g. McGaugh et al. 2000; Bell & de Jong 2001; Avila-Reese et al. 2008). A similar correlation between the luminosity (or the stellar mass) of elliptical galaxies and the velocity dispersion in their central regions was established by Faber & Jackson (1976) (hereafter FJ). The shape and scatter of the FJ relation has been less frequently studied because its large residuals show a significant correlation with galaxy size, i.e., a third parameter within the so called fundamental plane (e.g. Djorgovski & Davis 1987; Dressler et al. 1987; Cappellari et al. 2013; Desmond & Wechsler 2017).

It is presumed that galaxy internal kinematics as tracer of the gravitational potential provide the dynamical mass. If spiral and elliptical galaxies were completely dominated by rotation velocity and velocity dispersion, respectively, the TF and FJ relations would provide insights into the connection between galaxies and their dark matter content. However, structural properties, environmental effects or internal physical processes perturb the kinematics of late-type galaxies producing non-circular motions that under/overestimates the circular velocity (e.g. Valenzuela et al. 2007; Randriamampandry et al. 2015; Holmes et al. 2015). On the other hand, elliptical galaxies, although dominated by velocity dispersion, often present some degree of rotation (e.g. Lorenzi et al. 2006; Emsellem et al. 2007, 2011; Cappellari et al. 2011; Rong et al. 2018). Non-circular motions on disk galaxies and rotation on ellipticals may contribute to miss a fraction of the gravitational potential, modifying the scaling relations and precluding them from being directly comparable to theoretical predictions.

Weiner et al. (2006) introduced a new kinematic parameter involving a combination of rotation velocity and velocity dispersion in order to study high redshift galaxies, where in some cases random motions were not negligible. Weiner et al. (2006) showed that such parameter provides a better proxy to the integrated line-width of galaxies emission lines than rotation velocity or velocity dispersion alone, regardless of the galaxy morphology. The parameter is defined as:

\[
S_K^2 = K V_{rot}^2 + \sigma^2, \tag{1}
\]

where \( V_{rot} \) is the rotation velocity, \( \sigma \) is the velocity dispersion and \( K \) a constant that could be extremely complicated function of the formation history, dynamic state and environment of galaxies. Kassin et al. (2007) found that by adopting a value of \( K = 0.5 \), the \( S_0,5 \) parameter presents a tight correlation with the stellar mass for a sample of galaxies at redshift \( z \leq 1.2 \) extracted from the All Wavelength Extended Groth Strip International Survey (AEGIS) and the Deep Extragalactic Evolutionary Probe 2 (DEEP2). This correlation seems to be independent of the morphological type. Other analyses, focused on the evolution of the TF relation at high redshift (\( z \sim 2 \)), explored the \( M_\star - S_0,5 \) relation confirming that turbulent motions might play an important dynamical role (e.g. Cresci et al. 2009; Gnerucci et al. 2011; Vergani et al. 2012; Price et al. 2016; Christensen & Hjorth 2017). Zaritsky et al. (2008) provided a possible explanation of the \( M_\star - S_0,5 \) relation as a virial one, including all galaxy evolution, geometrical and dynamical complications into the \( K \) coefficient.

Cortese et al. (2014, hereafter C14) performed the only systematic study of this relation at low redshift (\( z \leq 0.095 \)). They used the stellar and ionized gas kinematics integrated within one effective radius, \( r_e \), for galaxies observed with the Sydney-AAO Multi-object Integral Field survey (SAMII; Croon et al. 2012). C14 confirmed that all galaxies, regardless of the morphological type, lie on the same kinematic scaling relation \( M_\star - S_0,5 \) with a significant improvement compared with the TF and FJ relations. Although the result is encouraging, the spatial covering of the observations (\( 1r_e \)) and the coarse spatial resolution of the data may contribute to the uncertainties in a similar way as they do it in HI line-width TF estimations (e.g. Ponomareva et al. 2017). Therefore, it is needed to repeat this analysis using data with better spatial resolutions and coverage.

Figure 1. Galaxy Sample distributions in (a) total stellar masses, (b) redshift and (c) morphological type. Blue and red histograms indicate galaxies with ionized gas and stellar kinematics, respectively, whereas the unfilled black histogram indicate galaxies with both, ionized gas and stellar kinematics.
The aim of this paper is to explore and calibrate the TF, \( \lambda / \Delta \lambda \sim 850 \) (FWHM=6Å), and (ii) the V1200 setup, an intermediate resolution mode, that covers the wavelength range between 3700-4800Å, with a resolution of \( \lambda / \Delta \lambda \sim 1650 \) (FWHM=2.7Å). The delivered dataset was reduced using version 2.2 of the CALIFA pipeline, whose modifications with respect to the previous ones (Sánchez et al. 2012b; Husemann et al. 2013; García-Benito et al. 2015) are described in Sánchez et al. (2016d). The final data product of the reduction is a datacube comprising the spatial information in the \( x \) and \( y \) axis, and the spectral one in the \( z \) one. For further details of the adopted data format and the quality of the data consult Sánchez et al. (2016d).

3 ANALYSIS

We here the analysis performed to estimate the stellar mass distribution and the kinematics parameters for the different galaxies included in the current dataset.

3.1 Spectroscopic Analysis

In this paper we use the data-products (ionized gas kinematic maps) derived for the CALIFA V500 setup dataset by Pipe3D pipeline (Sánchez et al. 2016c) based on the Fit3D fitting tool (Sánchez et al. 2016a), together with the stellar line-of-sight velocity and intrinsic dispersion maps for the V1200 setup performed using pPXF by Falcón-Barroso et al. (2017).

Pipe3D models the stellar continuum adopting a multi Single Stellar Population (SSP) template library, taking into account the velocity, dispersion and dust attenuation of the stellar populations. Then, it estimates the main properties of the nebular emission lines. The current implementation of Pipe3D adopted the GSD156 (Cid Fernandes et al. 2013) template library for the analysis of the stellar population properties. This library comprises 156 templates covering 39 stellar ages (from 1Myr to 13Gyr), and 4 metallicities (\( Z/Z_\odot = 0.2, 0.4, 1, \) and 1.5). A spatial binning for the stellar population analysis was applied to reach an homogeneous S/N of 50 across the field of view. The stellar population fitting was applied to the co-added spectra within each spatial bin. Finally, following the procedures described in Cid Fernandes et al. (2013) and Sánchez et al. (2016b), was estimated the stellar-population model for each spaxel by re-scaling the best-fit model within each spatial bin to the continuum flux intensity in the corresponding spaxel. The stellar-population model spectra are then subtracted from the original datacube to create a gas-pure cube comprising only the ionized gas emission lines. For this pure-gas cube, the stronger emission lines were then fitted spaxel by spaxel using single Gaussian models for each emission line in each individual spectrum to derive the corresponding flux intensity and line-of-sight kinematics. In addition, the spatial distribution of the stellar mass densities and the integrated stellar masses at different apertures are recovered from the Pipe3D analysis by taking into account the decomposition in SSPs, the Mass-to-Light ratio of each of them, and the integrated light at each spaxel within the FoV. For this derivation was assumed the Salpeter Initial Mass Function (Salpeter 1955). More details of the fitting procedure, adopted dust atten-

1 Both surveys present a similar projected PSF FWHM of \( \sim 2.5'' \). However CALIFA sample galaxies observed in a considerable lower redshift and narrower redshift range.
uation curve, and uncertainties of the process are given in Sánchez et al. (2016a,c).

Falcón-Barroso et al. (2017) performed a detailed analysis to extract the stellar kinematics for the intermediate resolution CALIFA data (V1200 setup). The data-cubes were spatially binned with the Voronoi 2D binning method of Cappellari & Copin (2003) to achieve an approximately constant signal-to-noise (S/N) of 20 per spaxel taking in account the correlation in the error spectrum of nearby spaxels (see Husemann et al. 2013, for details). This S/N value conserve a good spatial resolution while still being able to reliably estimate the line-of-sight velocity distribution. The stellar kinematics was estimated using the pPXF code of Cappellari & Emsellem (2004). The stellar templates were taken from the Indo-US spectral library (Valdes et al. 2004) with ∼330 selected stars. The stellar rotation velocity, the velocity dispersion and corresponding error were estimated by χ^2 minimization in pixel space as the bi-weight mean and standard deviations of a set of 100 Monte Carlo realizations of the fitting.

3.2 Integrated kinematics.

The original dataset comprises 734 galaxies for the V500 dataset observed within the frame-work of the CALIFA survey Sánchez et al. (2017), and the 300 galaxies for the V1200 dataset described by Falcón-Barroso et al. (2017). From this dataset we perform a selection of the optimal data for the intermediate HI emission profiles in galaxies, i.e., through the width parameter, W (e.g. Mathewson et al. 1992; Vogt et al. 2004). First, a histogram is derived of the velocities estimated for all the good spaxels within the r_e. Then, it is calculated the difference between the 10th and 90th percentile points of this velocity histogram, defined as the width: W = V_{90} − V_{10} (e.g. Catinella et al. 2005). Finally, the rotation velocity is defined as:

\[
V_{rot} = \frac{W}{2(1+z)\sin(i)}, \quad (2)
\]

where z is the redshift and i is the galaxy inclination determined from the observed ellipticity ε as:

\[
\cos(i) = \sqrt{(1-\epsilon^2-q_0^2)/(1-q_0^2)}, \quad (3)
\]

with q_0 being the intrinsic axial ratio of edge-on galaxies. Following Catinella et al. (2012) and C14, we adopted q_0 = 0.2 for all galaxies and set the inclination to 90° edge-on if ε ≥ 0.8.

Integrated rotation velocity estimated by Eq. 2 is a good representation of the maximum rotation velocity, V_{max}, if the kinematics of the galaxy is axi-symmetric (i.e., without non-circular motions). However, this is not always the case. Some galaxies show deviations from a pure rotational pattern due to warps, lopsidedness, bars, outflows, inflows, nuclear activity, etc. (e.g. Bosma 1978; Schoenmakers et al. 1997; Verheijen 2001; Holmes et al. 2015; Kalinova et al. 2017; Sánchez-Menguiano et al. 2017) producing non-circular motions and distorting the velocity profile (i.e., velocity histogram). In the next subsection, we try to quantify these effects in the derivation of V_{max} by performing a more detailed analysis on a limited sample of galaxies and comparing the results.

3.3 Spatially resolved kinematics: V_{max}.

Kinematic maps of spiral galaxies are often treated as being consistent with a purely circular flow pattern. This means that the kinematics of a galactic disk at a certain galactocentric radius can be described by a single tilted ring model defined by three parameters: the rotation velocity and two parameters that describe the local disk orientation with respect to some reference system (e.g. Rogstad et al. 1974). Several routines exists to fit kinematic maps based in this method. The most extensively used is the ROTCUR routine (e.g. Begeman 1989), which fits a set of inclined rings to a velocity field. However, as we mentioned above, the kinematics could be affected by the presence of non-circular motions and in some cases the tilted-ring model is an oversimplification. A more precise kinematic analysis requires tools that consider non-circular motions.

Spekkens & Sellwood (2007) and Sellwood & Sánchez (2010) developed the VELFIT code specifically to characterize the non-circular motions in the kinematics of spiral
galaxies expressed in a Fourier series. We used this code with some improvements (Aquino-Ortíz in prep.) to derive the properties of the velocity maps. This fit provides an estimate of the rotation curve, the kinematic inclination and position angle of the galaxy, together with the amplitude of the non-circular motions as a function of radius. A bootstrap procedure is adopted to estimate the uncertainties on the derived parameters.

The current procedure is not performed over the full dataset, since in many cases the kinematics present clear deviations due to external perturbations or is strongly affected by random motions. We discarded those cases whose kinematics appeared highly disturbed by the presence of large nearby companions or clear indications of being in a merging process. Therefore, we select a control sample, with good quality spatial resolved kinematics, comprising those isolated galaxies with low velocity dispersion, and inclinations between $30^\circ < i < 70^\circ$. This sample of galaxies, that are the best suited for modeling their velocity maps, comprises 42 galaxies for ionized gas kinematics (V500) and 92 galaxies for stellar kinematics (V1200).

The estimated rotation curves for all these galaxies present a great diversity, in agreement with previous results (e.g. Kalinova et al. 2017). For a limited fraction of galaxies ($\sim 10\%$) the spatial coverage was insufficient to measure the maximum velocity, $V_{\text{max}}$. In order to still estimate $V_{\text{max}}$ we follow Bekeraité et al. (2016) and parametrize the rotation curve using the formula proposed by Bertola et al. (1991):

$$v(r) = v_0 + \frac{v_c r}{(r^2 + k^2) \gamma},$$

where: (i) $v_0$ is the systemic velocity of the galaxy; (ii) $v_c$ is a parameter governing the amplitude of the rotation curve; (iii) $k$ describes its sharpness and finally (iv) $\gamma$ allows modeling rising or falling curves, with $\gamma = 1$ for a flat rotation curve.

4.1 TF relation.

The TF relation including early type galaxies based on the integrated analysis are shown in the left panels of Figure 2. These relations show a large scatter, $0.084$ dex in log $V_{\text{opt}}$ for ionized gas kinematics and $0.20$ dex for stellar kinematics. The value for ionized gas kinematics is in agreement with the one reported for the luminosity TF relation estimated by Bekeraité et al. (2016), $\sim 0.09$ dex, despite the fact that their study was based on a detailed analysis of the rotation velocity of a sub-sample of the CALIFA galaxies. In that study they analyzed their velocity within a radius containing $83\%$ of all light, $V_{\text{opt}}$. On the other hand, our scatter for stellar kinematics is lower than the one reported by C14 for SAMI ($\sim 0.25$ dex). The difference with this later study is not surprising because the SAMI sample is dominated by Sc low-mass galaxies, where the rotation curves are still rising at $1r_e$, being far from $V_{\text{max}}$, whereas our sample is dominated by Sa and Sb galaxies (see lower panels of Figure 2).

Figure 3, left-panel, shows the TF relation also including early type galaxies based on the spatially resolved analysis (i.e., using $V_{\text{max}}$). The parameters of the best-fit relation to these data are listed in Table 1. When adopting this improved estimation of the velocity, the scatter decreases to $\sim 0.07$ dex for the ionized gas kinematics, but it increases to $\sim 0.24$ dex for the stellar one. This later value agrees with the one reported by C14. The scatter for our stellar kinematics TF relation increases due to that late-type galaxies move to higher velocities and also for the inclusion of early-type galaxies in the rotation. Such galaxies are undetected in the gas component, being the analyzed sample limited to mostly late-type galaxies.

As a reference we include in Figure 2 and Figure 3 the derivation of the stellar TF relation as presented in Avila-Reese et al. (2008)\(^2\). We use their orthogonal linear fit considering the stellar mass as the independent variable. As expected, there is an offset between this classical derivation and our results for the integrated kinematics. However, for the resolved kinematics, which determines $V_{\text{max}}$, the offset tends to disappear, at least for the spiral galaxies.

4.2 FJ Relation.

Central panels of Figure 2 and Figure 3 show the FJ distributions including late-type galaxies using the integrated kinematics sample and the spatial resolved one, respectively. A reference FJ relation, derived by Gallazzi et al. (2006), has been included for comparison. Our stellar FJ relations show a scatter of $\sim 0.16$ dex ($\sim 0.14$ dex) and $\sim 0.17$ dex ($\sim 0.10$ dex) for gaseous and stellar kinematics, respectively, for the integrated (spatially resolved) subsamples. These dispersions are similar to the ones found by C14 ($\sim 0.16$ dex), but larger than the one reported by Gallazzi et al. (2006) ($\sim 0.07$ dex).

On a parallel situation as the one found for the TF relation, the stellar velocity dispersions and those derived for early-type ones are more in agreement with the FJ relation.

\(^2\) We have increased the stellar mass in Avila-Reese et al. (2008) by $0.15$ dex in order to convert from diet-Salpeter to Salpeter IMF.
than the gaseous dispersions and/or those derived for late-type galaxies.

4.3 $M_\star - S_{0.5}$ relation.

Figure 2, right panels, shows the $M_\star - S_{0.5}$ distribution for the integrated kinematics segregated by gas and stellar kinematics (upper panel) and by morphology (lower panel). As a reference the $S_{0.5}$ relations, derived by C14 and Kassin et al. (2007), have been included, together with the best-fit relation derived with our own data. As in the previous cases, the best-fit parameters for the linear regression have been included in Table 1. The distribution is clearly tighter than those of the FJ relations, with scatter very similar or lower to the one found for the TF relation ($\sim 0.08$ dex).

Figure 3, right panels, shows the same distributions for the resolved kinematics subsample. For this control sample, the scatter decreases significantly to $0.053$ dex and $0.052$ dex for both the ionized gas and stellar kinematics, respectively. As we mentioned above, the slope, zero-point and scatter of the TF and FJ relations could depend on several factors including (i) the morphology of the galaxies, (ii) the adopted shape for the rotational curve and even (iii) the methodology used to measure both the rotational velocity and/or the velocity dispersion (see Colleen et al., for an example of the effects of the uncertainties). For the $S_{0.5}$ parameter, the dependence on morphology and the described offsets between gaseous and stellar kinematics eventually disappear. Thus, galaxies of any morphology lie on the same scaling relation in agreement with previous studies.

C14 found a good agreement in the slope of the $S_{0.5}$ relation derived using integrated kinematics up to $1 r_e$ for the SAMI dataset with that derived by Kassin et al. (2007) for a sample of star-forming galaxies, using the maximum rotational velocities. However, they found larger differences in the zero-point of their relations. In our analysis the behavior is similar. The slope remains unchanged between both the integrated and resolved kinematics, with small differences compared with the ones derived by Cortese et al. (2014) and Kassin et al. (2007). However, our best-fit for the total sample (gas + stars) presents a scatter clearly lower than the one found by previous studies, being $\sim 0.082$ dex for the integrated kinematics and $\sim 0.054$ dex for the spatial resolved one. The reduction in the scatter combining rotation velocity and velocity dispersion in a single parameter, indicate that together they trace the gravitational potential than each one separately. Actually, this latter value is in agreement with the physical interpretation of Zaritsky et al. (2008).

5 DISCUSSION.

We discuss here the implications of the results listed in the previous section, trying to understand how the uncertainties...
may affect them and the physical nature of the described relations.

5.1 Narrowing down the uncertainties.

A critical challenge giving a physical interpretation to galaxy scaling relations are the uncertainties, because they can potentially modify or erase the dependence between the analyzed properties. We have tried to narrow down their effects by performing the analysis twice. Once using the integrated kinematics, following C14, and then, we improved the accuracy by using a spatial resolved kinematic analysis. This second analysis is performed at the expenses of the statistics. We consider that this second dataset is best suited to derive a more accurate $S_{0.5}$ relation.

Table 1 shows there is a clear improvement in the TF and $S_{0.5}$ relations (in most of the cases) when adopting the spatial resolved kinematics. On the other hand, there is only a mild improvement in the FJ relation (since this relation does not involve rotation velocities). To verify the scatter we tried to reproduce the "classical" TF relation using the spatial resolved kinematics. For doing so, we select only the spiral galaxies and compare their distribution in the $M_\ast$-$V_{\text{max}}$ diagram with that of a well established comparison sample: the compilation and homogenization presented in Avila-Reese et al. (2008). Figure 4, left panel, shows this comparison. The parameters derived for the TF relation for both subsamples match pretty well, with very good agreement, in particular for the gas kinematics, as shown in Table 1. Therefore, the spatial resolved kinematic sample seems to be the best one to characterize the scaling relations involving rotation velocities.

Using this new sub-sample we derive the most precise estimation of the $M_\ast$-$S_{0.5}$ relation, shown in Fig. 4, right panel. The parameters of this relation are listed in Table 1. The first result emerging from this analysis is that the scatter is of the order of the $S_{0.5}$ relation found for the complete resolved kinematics ($\sim$0.05 dex). Therefore, to select a better sub-sample in terms of the TF relation does not seem to affect the result. In other words, the inclusion of early-type galaxies affects the TF relation, but it does not affect the $S_{0.5}$ one. Another interesting result is that the scatter in this relation is very similar to that of the TF relation for the same subsample. Therefore, the inclusion of the effects of random motions does not increase the scatter, even for galaxies clearly supported by rotation.

Finally, the slope and zero-point of the $S_{0.5}$ relations found for (i) this particular sub-sample of galaxies that reproduces better the TF relation, (ii) the complete resolved kinematics sample, and (iii) the integrated kinematics sample, when considering both the gaseous and stellar kinematics, agree with each other. Thus, only the precision is increased by performing a detailed resolved kinematics for a TF-compatible subsample, but the general trends are the same. The result of this test suggests that our analysis is not dominated by velocity uncertainties and the early tight correlation presented by C14 and in this paper is real and
Table 1. Orthogonal linear fit parameters to scaling relations. Note. All scatters are estimated from the linear fit as the standard deviation of all residuals, we consider stellar mass, $M_\star$, as independent variable.

| Relation                      | Tully-Fisher | Faber-Jackson | $S_{0.5}$ |
|-------------------------------|--------------|---------------|-----------|
|                               | Galaxies     |               |           |
| Integrated kinematics at $R_e$| scatter      | slope         | zero-point| scatter | slope         | zero-point | scatter | slope         | zero-point |
| Gas                           | 0.084        | 0.27 ± 0.01   | -0.65 ± 0.12 | 0.171    | 0.36 ± 0.02 | -2.03 ± 0.27 | 0.087   | 0.29 ± 0.01 | -1.03 ± 0.12 |
| Stellar                       | 0.200        | 0.16 ± 0.02   | 0.32 ± 0.30  | 0.160    | 0.31 ± 0.03 | -1.37 ± 0.25 | 0.075   | 0.26 ± 0.01 | -0.67 ± 0.10  |
| Total                         | 0.171        | 0.20 ± 0.01   | -0.01 ± 0.18 | 0.165    | 0.34 ± 0.02 | -1.71 ± 0.18 | 0.082   | 0.27 ± 0.01 | -0.79 ± 0.07  |
| Cortese et al. (2014)         | 0.26         | —             | —          | 0.16     | —            | —          | 0.10    | 0.33 ± 0.01 | -1.41 ± 0.08  |
| Kassin et al. (2007)          | —            | —             | —          | —       | —            | —          | —       | 0.34 ± 0.05 | 1.89 ± 0.03   |
| Resolved kinematics, $V_{\text{max}}$ |               |               |           |
| Gas                           | 0.07         | 0.25 ± 0.02   | -0.41 ± 0.17 | 0.10     | 0.31 ± 0.01 | -1.47 ± 0.19 | 0.053   | 0.29 ± 0.01 | -0.92 ± 0.13  |
| Stellar                       | 0.24         | -0.10 ± 0.09  | 3.34 ± 1.07 | 0.14     | 0.53 ± 0.03 | -3.88 ± 0.34 | 0.052   | 0.27 ± 0.01 | -0.72 ± 0.12  |
| Total                         | 0.22         | 0.08 ± 0.04   | 1.37 ± 0.48 | 0.13     | 0.44 ± 0.02 | -2.79 ± 0.22 | 0.054   | 0.27 ± 0.01 | -0.71 ± 0.11  |
| Only spiral galaxies, $V_{\text{max}}$ |               |               |           |
| Gas                           | 0.043        | 0.27 ± 0.01   | -0.63 ± 0.15 | 0.076    | 0.35 ± 0.02 | -1.84 ± 0.21 | 0.043   | 0.29 ± 0.01 | -0.88 ± 0.13  |
| Stellar                       | 0.053        | 0.30 ± 0.02   | -1.00 ± 0.02 | 0.091    | 0.35 ± 0.03 | -1.94 ± 0.34 | 0.052   | 0.28 ± 0.02 | -0.92 ± 0.21  |
| Total                         | 0.052        | 0.28 ± 0.01   | -0.73 ± 0.13 | 0.098    | 0.33 ± 0.02 | -1.69 ± 0.22 | 0.051   | 0.27 ± 0.01 | -0.80 ± 0.13  |
| Avila-Reese et al. (2008)     | 0.045        | 0.27 ± 0.01   | -0.69 ± 0.12 | —       | —            | —          | —       | —            | —          |
A dynamical mass proxy for galaxies across the Hubble sequence.

Figure 4. Scaling relations for our control subsample. Left and right panels shown the TF and $S_{0.5}$ scaling relations, respectively. Blue and red symbols represent gaseous and stellar kinematics for galaxies with inclinations from $30^\circ < i < 70^\circ$. Cyan, magenta and green lines are the best-fit for gas, stellar and total (gas + stellar). In the TF relation we recover in great detail the result of the data compilation from Avila-Reese et al. (2008, their masses were corrected to convert to a Salpeter IMF). It is clear that in galaxies where the random motions are negligible, the $S_{0.5}$ relation tends to be the TF.

not the result of the poorly constrained in velocity for dispersion dominated systems.

5.2 $S_K$ as a proxy of the dynamical mass.

The observed kinematics of a galaxy often is used to infer the total (dynamical) mass enclosed at different radii (e.g. Persic & Salucci 1988; Zavala et al. 2003; Courteau et al. 2014; Ouellette et al. 2017). Assuming that the $M_\star - S_{0.5}$ scaling relation is a consequence of a more physical relation between the dynamical mass and the stellar mass in the inner regions, we suppose that the $S_{0.5}$ parameter traces the dynamical mass as follow:

$$M_{\text{dyn}} \propto S_{0.5}^2 \Rightarrow M_{\text{dyn}} = \eta \frac{r_s S_{0.5}^2}{G} = \eta \frac{r_s (0.5V_{\text{rot}}^2 + \sigma^2)}{G}. \quad (5)$$

where $r_s$ is a characteristic radius of the galaxy, $G$ the gravitational constant and $\eta$ is a structural coefficient which encapsulate information of the shape of the galaxy, projection effects, dynamical structure, etc., in fact it can be included into the $K$ coefficient of the $S_{0.5}$ parameter, however it is useful to introduce $\eta$ in order to compare with former studies. Dynamical models like Jeans Anisotropic Models (JAM's, Cappellari 2008) or Schwarzschild (Schwarzschild 1979) are considered the state-of-the-art inferences of galaxies mass distribution including dynamical enclosed mass. Cappellari et al. (2006) calibrated eq. 5 for a sample of early-type galaxies from the SAURON project (Bacon et al. 2001) using the velocity dispersion instead the $S_{0.5}$ parameter in combination with Schwarzschild dynamical models. They found that the dynamical mass within the effective radius can be robustly recovered using a coefficient $\eta \approx 2.5$, which varies little from galaxy to galaxy.

Figure 5. One to one relation between dynamical masses inferred from dynamical models and kinematic parameter $S_{0.5}$. Blue symbols are the comparison between the Schwarzschild models by Zhu et al. (2018a) with our estimations. Red and green symbols are the comparison between JAMs and Schwarzschild models by Leung et al. (2018) with our estimations, respectively. Both comparisons shown a scatter of $\sim 0.15$ dex. Magenta symbols are the comparison between Schwarzschild and JAMs estimations with a scatter of $0.08$ dex.
Figure 6. Accuracy of the $M_{\text{dyn}} - M_*$ relations based on the $S_{0.5}$ parameter. In top and medium panels we assume that galaxies are rotation or velocity dispersion dominated to estimate the dynamical mass within the effective radius. Red, green and black star symbols represent our CALIFA sample, whereas gray symbols are from the literature compilation. The $S_{0.5}$ dynamical mass estimations perform better than the ones based either only on rotation or dispersion. In the bottom panel we used the $S_{0.5}$ parameter to estimate the dynamical mass using equation (5) and compare them with theoretical predictions based on detailed dynamical models. All the estimated $M_{\text{dyn}} - M_*$ relations are comparable and consistent with observations within the uncertainties. As a reference we also show the semi-empirical predictions of Mancillas et al. (2017) (blue shaded region; see text) which use $\eta = 1$ and are also consistent with our estimations.

Leung et al. (2018) performed a detailed comparison of JAMs and Schwarzschild models for 54 of the CALIFA galaxies included here. We use these dynamical masses, $M_{\text{dyn}}^{JAM\text{rs}}$, to calibrate the eq. 5 based on the $S_{0.5}$ parameter. We found that the enclosed dynamical mass within the effective radius (i.e., using $r_e$ as the characteristic radius in eq. 5) can be robustly recovered using a single coefficient $\eta \approx 1.8$ for all the galaxies, with a narrow dispersion of 0.15 dex. To validate that calibration we compare the estimated dynamical masses by the eq. 5 with those derived using dynamical models for a sample of 300 galaxies analyzed by Zhu et al. (2018a,b), together with the ones by Leung et al. (2018). Figure 5 shows the comparison between the different estimations of the dynamical masses. As expected, the agreement between the values derived using JAMs and Schwarzschild dynamical models for the galaxies studied by Leung et al. (2018) agree one each other with a low scatter of 0.08 dex. Interestingly, we still find a very good agreement using $\eta = 1.8$ between our $S_{0.5}$ derived dynamical masses and sophisticated dynamical mass estimations, with a scatter of ~ 0.15 dex. We may wonder why is the $S_{0.5}$ parameter such a good mass tracer. This is remarkable in view that, we do not systematically study IMF effects (e.g. Martín-Navarro et al. 2015) and kinematic anisotropy (e.g. Zhu et al. 2018a). The enclosed mass within $r_e$ is an integrated quantity weakly sensitive to the specific mass and shape density profile, a similar discussion has been presented by Wolf et al. (2010) for dwarf spheroidal galaxies, only in such grounds the $S_{0.5}$ is a competitive $M_{\text{dyn}}$ proxy.
5.3 The Dynamical-to-stellar mass relation.

We explore the literature in order to compile the most recent state-of-the-art derivations of the dynamical mass in the central regions of galaxies using dynamical models. Cappellari et al. (2013) estimated the dynamical mass within the effective radius for 258 early-type galaxies from the ATLAS3D project (Cappellari et al. 2011) using the JAM’s dynamical models and compared it with the stellar masses. Martinsson et al. (2013) performed a similar study for 24 late-type galaxies extracted from the DiskMass survey (Bershady et al. 2010). Zhu et al. (2018a,b) constructed orbit-superposition Schwarzschild models at different radii that simultaneously fit the observed surface brightness and stellar kinematics for 300 galaxies included in the CALIFA-V1200 resolution subsample studied here. In Zhu et al. (2018b) they constrained the stellar orbit distribution and found that a fraction of stars are within a plane with unperturbed orbits tracing the rotation velocity, while others are out of the plane with perturbed orbits tracing the velocity dispersion. This result implies that the kinematics in galaxies is more complex that just rotation or velocity dispersion: both components are present in all types of galaxies and should be considered to trace the potential. Figure 6, bottom panel, presents the comparison between the distributions of dynamical masses along the stellar ones between this compilation of data extracted from the literature and those ones derived using the equation within the effective radius, based on the $M_{\odot}/L_*$ parameter with $g = 1.8$. In addition we present the dynamical masses derived if we consider only the rotational velocities or the velocity dispersions. All these dynamical masses, derived at $r_e$ are listed in Table A1. We observe a clear offset and a large scatter between our dynamical masses and those derived using detailed models when we use only rotation velocity (mostly for ellipticals) or velocity dispersion (mostly for spirals). However, when we use the dynamical mass proxy based on the $\eta$ parameter, the distribution along the stellar mass is in agreement with the results extracted from the literature. Thus, it seems that the $\eta$ parameter is indeed a good proxy for calculating the dynamical mass.

Our distribution of $M_{\text{dyn}} - M_*$ follows a linear and nearly one-to-one relation for masses in the range $3 \times 10^9 \leq M_*/M_\odot < 5 \times 10^{10}$. The fact that for some galaxies (both from our sample and from other works), the stellar mass seems to be higher than the dynamical one shows the presence of several systematical uncertainties both in the stellar and dynamical mass determinations. Within these uncertainties, what we learn from Fig. 6 is that in the mentioned above mass range luminous matter strongly dominates within $r_e$. Below ~$3 \times 10^9 M_\odot$ there is a clear deviation, with galaxies showing larger dynamical masses than their stellar masses, which indicates that in the low-mass regime galaxies are more dark-matter dominated as less massive they are, even within the effective radius. In the high-mass end, there is some weak evidence of a deviation, with the few E/S0 galaxies at these masses showing again larger dynamical masses than their stellar masses. This difference could be due to more bottom-heavy IMF (e.g. Lyubenova et al. 2016) and/or due to the contribution of dark matter.

Both our data and literature collected ones show similar trends. Indeed, this result is predicted by different theoretical studies, including hydrodynamical cosmological simulations (e.g. Oman et al. 2015) from the Evolution and Assembly of GaLaxies and their Environments (EAGLE) project (Schaye et al. 2015; Crain et al. 2015), and semi-empirical modeling approaches (e.g., Mancillas et al. 2017). We include the latter theoretical predictions for comparison in Figure 6. In Mancillas et al. (2017), a population of galaxies with bulge-to-disk mass ratios lower than ~0.7 was generated by loading the bulge/disc systems into LCDM halos, taking into account the adiabatic contraction of the inner halo by the baryons. The modeled population reproduces well the TF relation, radius-mass, $B/T$-mass, and gas-to-stellar mass relations, and by construction follows the stellar-to-halo ($M_*/M_{\text{vir}}$) relation constrained from a semi-empirical approach for blue galaxies (Rodríguez-Puebla et al. 2015). The predicted inner mass distributions, in particular the stellar-to-dynamical masses within $r_e$, inherit partially the shape of the latter relation, which bends to lower $M_*/M_{\text{vir}}$ ratios both at lower and higher masses. This explains the bends seen for the predictions in Fig. 6 (dashed blue line and shadow region around it). It is encouraging that our observational inferences based on the $\eta$ parameter agrees with these predictions, showing the possibility to attain a connection between the inner galaxy dynamics of the local galaxy population and the properties of the cosmological dark matter halos.

6 CONCLUSIONS

Originally the $S_K$ parameter was introduced as a tool to deal with galaxies difficult to classify or with high amount of velocity dispersion like clumpy high redshift galaxies. The remarkable reduction of scatter in the $S_K$ relation compared with TF and FJ relations found by previous studies (e.g. Cortese et al. 2014) and confirmed with higher accuracy by our study, points toward a more complex internal kinematics in galaxies even in the local Universe: non-circular motions in disk galaxies and some amount of rotation in elliptical galaxies.

In summary, we demonstrate that (i) the $M_*/S_K$ is a tighter correlation than the TF relation or the FJ relation when galaxies of all morphological types are considered, and (ii) this relation is a consequence of the dynamical mass and the relation between this later parameter with the stellar mass. Finally, we propose a simple but competitive procedure to estimate the dynamical mass in galaxies, easier to apply to massive surveys than more detailed analysis, although with lower precision.

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Table A1

| Name         | \(M_\star\) [\(M_\odot\)] | \(M_{\text{dyn}}\) [\(M_\odot\)] | \(r_e\) [arcsec] |
|--------------|-----------------------------|-----------------------------------|------------------|
| IC5376       | 10.16 ± 0.10               | 10.53 ± 0.04                     | 11.62            |
| NGC0036      | 8.67 ± 0.09                | 10.82 ± 0.02                     | 19.34            |
| UGCG0148     | 9.71 ± 0.09                | 10.26 ± 0.06                     | 13.54            |
| MCG-02-02-030| 10.00 ± 0.09               | 10.25 ± 0.03                     | 13.86            |
| NGC00005     | 10.02 ± 0.09               | 10.78 ± 0.01                     | 14.45            |
| NGC00016     | 10.00 ± 0.08               | 10.14 ± 0.03                     | 15.02            |
| NGC00029     | 9.93 ± 0.10                | 11.19 ± 0.04                     | 12.79            |
| IC1528       | 10.04 ± 0.10               | 10.16 ± 0.03                     | 13.95            |
| NGC00132     | 9.75 ± 0.09                | 10.75 ± 0.12                     | 9.64             |
| MCG-02-02-040| 9.44 ± 0.09                | 10.11 ± 0.05                     | 11.62            |

Notes: (1) Galaxy name. (2) Stellar mass within \(r_e\) estimated from PIPE3D. (3) Dynamical mass estimated from the kinematic parameter \(S_{0.5}\). (4) Effective radius \(r_e\).
| Name           | $M_*$ [$M_\odot$] | $M_{\text{dyn}}$ [$M_\odot$] | $r_e$ [arcsec] |
|---------------|------------------|-------------------------------|---------------|
| NGC2410       | 10.49 ± 0.09     | 10.76 ± 0.02                  | 17.91         |
| UGC03944      | 9.57 ± 0.12      | 10.00 ± 0.03                  | 11.79         |
| UGC03969      | 10.34 ± 0.11     | 10.70 ± 0.01                  | 11.16         |
| UGC03995      | 10.64 ± 0.09     | 10.74 ± 0.02                  | 21.78         |
| NGC2449       | 10.30 ± 0.09     | 10.58 ± 0.01                  | 12.86         |
| UGC04029      | 10.09 ± 0.08     | 10.39 ± 0.01                  | 14.97         |
| IC0480        | 9.42 ± 0.10      | 10.19 ± 0.02                  | 11.49         |
| NGC2476       | 10.36 ± 0.11     | 10.37 ± 0.05                  | 7.99          |
| NGC2480       | 8.86 ± 0.06      | 9.69 ± 0.10                   | 10.82         |
| NGC2481       | 9.68 ± 0.10      | 9.97 ± 0.03                   | 7.54          |
| NGC2486       | 10.43 ± 0.09     | 10.51 ± 0.03                  | 12.96         |
| NGC2487       | 10.51 ± 0.08     | 10.36 ± 0.04                  | 18.81         |
| UGC04132      | 10.40 ± 0.10     | 10.70 ± 0.02                  | 13.18         |
| UGC04145      | 10.01 ± 0.11     | 10.34 ± 0.03                  | 7.93          |
| NGC2513       | 10.71 ± 0.08     | 11.21 ± 0.02                  | 19.23         |
| UGC04197      | 9.82 ± 0.10      | 10.57 ± 0.03                  | 14.93         |
| NGC2540       | 10.31 ± 0.10     | 10.48 ± 0.02                  | 15.42         |
| UGC04280      | 9.76 ± 0.09      | 10.11 ± 0.05                  | 11.18         |
| IC2427        | 10.30 ± 0.09     | 10.53 ± 0.02                  | 16.30         |
| UGC04308      | 9.99 ± 0.08      | 10.15 ± 0.03                  | 21.43         |
| NGC2553       | 10.21 ± 0.09     | 10.52 ± 0.03                  | 8.52          |
| NGC2554       | 10.79 ± 0.09     | 10.87 ± 0.01                  | 17.50         |
| NGC2592       | 9.83 ± 0.10      | 10.64 ± 0.01                  | 7.62          |
| NGC2604       | 9.28 ± 0.10      | 9.77 ± 0.08                   | 20.19         |
| NGC2639       | 10.41 ± 0.08     | 10.70 ± 0.01                  | 13.36         |
| UGC04722      | 8.07 ± 0.12      | 9.76 ± 0.05                   | 17.59         |
| NGC2730       | 9.68 ± 0.08      | 9.96 ± 0.03                   | 14.56         |
| NGC2880       | 9.90 ± 0.08      | 9.98 ± 0.01                   | 13.71         |
| IC2487        | 10.00 ± 0.10     | 10.45 ± 0.02                  | 16.77         |
| IC0540        | 9.35 ± 0.10      | 9.74 ± 0.03                   | 14.88         |
| NGC2906       | 9.94 ± 0.08      | 10.10 ± 0.02                  | 15.23         |
| NGC2916       | 10.40 ± 0.08     | 10.55 ± 0.02                  | 20.60         |
| UGC05108      | 10.48 ± 0.10     | 10.81 ± 0.03                  | 9.58          |
| NGC2918       | 10.71 ± 0.10     | 10.93 ± 0.02                  | 9.32          |
| UGC05113      | 10.19 ± 0.10     | 10.71 ± 0.02                  | 8.91          |
| NGC3106       | 10.83 ± 0.08     | 10.77 ± 0.03                  | 17.30         |
| NGC3057       | 8.80 ± 0.09      | 9.26 ± 0.09                   | 18.08         |
| NGC5498NED01  | 9.71 ± 0.11      | 10.67 ± 0.02                  | 10.52         |
| NGC3158       | 11.14 ± 0.10     | 11.64 ± 0.03                  | 22.29         |
| NGC3160       | 10.28 ± 0.10     | 10.76 ± 0.01                  | 12.77         |
| UGC05598      | 9.84 ± 0.10      | 10.15 ± 0.04                  | 11.40         |
| NGC3300       | 10.18 ± 0.10     | 10.81 ± 0.03                  | 13.31         |
| NGC3303       | 10.63 ± 0.10     | 10.75 ± 0.03                  | 9.24          |
| UGC05771      | 10.54 ± 0.10     | 10.81 ± 0.03                  | 8.01          |
| NGC3381       | 9.18 ± 0.07      | 9.18 ± 0.10                   | 14.84         |
| UGC05990      | 8.44 ± 0.12      | 9.24 ± 0.09                   | 9.36          |
| UGC06036      | 10.32 ± 0.10     | 10.94 ± 0.02                  | 11.16         |
| IC0674        | 10.53 ± 0.09     | 10.84 ± 0.02                  | 11.48         |
| UGC06312      | 10.55 ± 0.09     | 10.80 ± 0.02                  | 12.77         |
| NGC3615       | 10.87 ± 0.09     | 11.10 ± 0.03                  | 10.85         |
| NGC3687       | 9.99 ± 0.08      | 9.75 ± 0.04                   | 15.42         |
| NGC3811       | 10.16 ± 0.09     | 10.08 ± 0.04                  | 14.71         |
| NGC3815       | 9.90 ± 0.08      | 10.12 ± 0.03                  | 8.81          |
| NGC3904       | 10.08 ± 0.10     | 10.27 ± 0.05                  | 7.14          |
| NGC4003       | 10.48 ± 0.09     | 10.52 ± 0.05                  | 9.41          |
| NGC4071       | 9.30 ± 0.08      | 9.79 ± 0.10                   | 11.88         |
| NGC4047       | 10.34 ± 0.09     | 10.41 ± 0.11                  | 14.79         |
| Name                      | $M_*$          | $M_{dyn}$ | $r_e$  |
|---------------------------|---------------|-----------|--------|
| (1) [M$_\odot]$           | (2) [M$_\odot$] | [arcsec]  |
| IC4566                    | 10.49±0.09    | 10.59±0.01| 13.16  |
| NGC5987                   | 10.42±0.09    | 10.71±0.02| 22.53  |
| NGC5980                   | 10.39±0.09    | 10.50±0.03| 12.64  |
| NGC6004                   | 10.27±0.07    | 10.21±0.03| 20.41  |
| UGC10097                  | 10.80±0.10    | 10.96±0.02| 10.39  |
| NGC6020                   | 10.38±0.10    | 10.62±0.04| 11.59  |
| NGC6021                   | 10.53±0.09    | 10.51±0.04| 8.47   |
| IC1151                    | 9.49±0.09     | 9.82±0.05 | 19.34  |
| UGC10123                  | 9.88±0.10     | 10.29±0.02| 11.01  |
| NGC6032                   | 9.83±0.10     | 10.16±0.03| 14.79  |
| NGC6060                   | 10.49±0.08    | 10.66±0.01| 20.20  |
| UGC10205                  | 10.69±0.11    | 10.98±0.02| 13.95  |
| NGC6063                   | 9.75±0.10     | 10.05±0.03| 17.78  |
| IC1199                    | 10.33±0.08    | 10.67±0.01| 18.76  |
| UGC10257                  | 9.86±0.09     | 10.28±0.04| 15.25  |
| NGC6081                   | 10.52±0.10    | 10.75±0.01| 10.43  |
| UGC10297                  | 8.62±0.09     | 9.53±0.04 | 10.85  |
| NGC6331                   | 9.50±0.10     | 10.14±0.08| 19.41  |
| NGC6125                   | 10.06±0.09    | 10.93±0.01| 15.38  |
| UGC10337                  | 10.61±0.10    | 11.01±0.01| 14.85  |
| NGC6132                   | 10.00±0.09    | 10.36±0.03| 11.89  |
| NGC6146                   | 11.06±0.09    | 11.26±0.02| 11.00  |
| UGC10380                  | 10.52±0.10    | 10.98±0.03| 12.83  |
| NGC6150                   | 10.65±0.10    | 11.11±0.03| 9.26   |
| UGC10384                  | 9.95±0.11     | 10.35±0.05| 9.27   |
| NGC6159                   | 10.23±0.09    | 10.44±0.02| 10.91  |
| NGC6173                   | 11.18±0.09    | 11.51±0.02| 18.52  |
| NGC6168                   | 9.40±0.10     | 9.85±0.05 | 16.28  |
| NGC6186                   | 9.96±0.09     | 9.94±0.04 | 12.67  |
| UGC10650                  | 8.88±0.09     | 10.48±0.10| 15.29  |
| NGC6278                   | 9.98±0.10     | 10.38±0.01| 9.56   |
| UGC10693                  | 10.77±0.09    | 11.28±0.03| 15.24  |
| NGC6165                   | 10.81±0.09    | 11.12±0.03| 15.73  |
| UGC10710                  | 10.48±0.09    | 10.88±0.03| 12.01  |
| NGC6199                   | 10.02±0.10    | 10.41±0.02| 15.84  |
| NGC6301                   | 10.88±0.08    | 10.94±0.02| 20.01  |
| NGC6314                   | 10.57±0.08    | 10.52±0.03| 8.72   |
| NGC6338                   | 11.06±0.10    | 11.50±0.02| 19.15  |
| UGC10796                  | 9.16±0.12     | 9.78±0.10 | 14.70  |
| IC1256                    | 10.19±0.09    | 10.35±0.02| 14.60  |
| NGC6394                   | 10.32±0.09    | 10.68±0.02| 9.05   |
| UGC10905                  | 10.81±0.09    | 11.04±0.03| 12.37  |
| NGC6411                   | 10.54±0.09    | 10.77±0.03| 17.82  |
| NGC6427                   | 10.00±0.10    | 10.30±0.02| 8.88   |
| UGC10972                  | 10.22±0.10    | 10.51±0.03| 19.31  |
| NGC6478                   | 10.83±0.09    | 10.97±0.02| 17.36  |
| NGC6497                   | 10.52±0.08    | 10.74±0.01| 11.99  |
| NGC6515                   | 10.79±0.09    | 10.97±0.04| 13.07  |
| UGC11228                  | 10.55±0.10    | 10.48±0.03| 7.23   |
| NGC6762                   | 9.91±0.09     | 10.03±0.06| 11.21  |
| MCG-02-51-004             | 10.66±0.09    | 10.75±0.01| 15.80  |
| NGC0941                   | 10.64±0.09    | 10.72±0.01| 15.04  |
| NGC0945                   | 9.11±0.08     | 10.41±0.02| 12.80  |
| NGC0978                   | 10.49±0.10    | 10.73±0.01| 12.85  |
| UGC11649                  | 10.14±0.08    | 10.28±0.02| 14.54  |
| UGC11680NED01             | 10.84±0.09    | 10.94±0.01| 14.56  |
All scatters are estimated from the linear fit as the standard deviation of all residuals, we consider stellar mass, \( M_\star \), as independent variable. \( \log(S_{0.5}) = a + b \log(M_\star) \). \( S_{0.5} \) is given in \([\text{km/s}]\) and \( M_\star \) in \( M_\odot \).

| Fit                          | zero-point(a) | slope(b) | scatter |
|------------------------------|---------------|----------|---------|
| **Integrated kinematics**    |               |          |         |
| *Gas*                        |               |          |         |
| Forward                      | \(-0.95 \pm 0.11\) | 0.29 \pm 0.01 | 0.087   |
| Inverse                      | \(-1.99 \pm 0.18\) | 0.38 \pm 0.01 | 0.100   |
| Bisector                     | \(-1.46 \pm 0.13\) | 0.33 \pm 0.01 | 0.090   |
| Orthogonal                   | \(-1.03 \pm 0.12\) | 0.29 \pm 0.01 | 0.087   |
| *Stellar*                    |               |          |         |
| Forward                      | \(-0.61 \pm 0.10\) | 0.25 \pm 0.01 | 0.075   |
| Inverse                      | \(-1.52 \pm 0.14\) | 0.34 \pm 0.01 | 0.086   |
| Bisector                     | \(-1.06 \pm 0.11\) | 0.29 \pm 0.01 | 0.078   |
| Orthogonal                   | \(-0.67 \pm 0.10\) | 0.26 \pm 0.01 | 0.075   |
| **Total (Gas+Stellar)**      |               |          |         |
| Forward                      | \(-0.72 \pm 0.07\) | 0.26 \pm 0.01 | 0.082   |
| Inverse                      | \(-1.73 \pm 0.11\) | 0.36 \pm 0.01 | 0.096   |
| Bisector                     | \(-1.22 \pm 0.08\) | 0.31 \pm 0.01 | 0.086   |
| Orthogonal                   | \(-0.79 \pm 0.07\) | 0.27 \pm 0.01 | 0.082   |
| **Resolved kinematics**      |               |          |         |
| *Gas*                        |               |          |         |
| Forward                      | \(-0.89 \pm 0.14\) | 0.28 \pm 0.01 | 0.053   |
| Inverse                      | \(-1.29 \pm 0.18\) | 0.32 \pm 0.02 | 0.056   |
| Bisector                     | \(-1.09 \pm 0.15\) | 0.30 \pm 0.01 | 0.054   |
| Orthogonal                   | \(-0.92 \pm 0.13\) | 0.29 \pm 0.01 | 0.053   |
| *Stellar*                    |               |          |         |
| Forward                      | \(-0.67 \pm 0.12\) | 0.26 \pm 0.01 | 0.052   |
| Inverse                      | \(-1.40 \pm 0.17\) | 0.33 \pm 0.01 | 0.056   |
| Bisector                     | \(-1.03 \pm 0.14\) | 0.29 \pm 0.01 | 0.050   |
| Orthogonal                   | \(-0.72 \pm 0.12\) | 0.27 \pm 0.01 | 0.052   |
| **Total (gas+stellar)**      |               |          |         |
| Forward                      | \(-0.66 \pm 0.08\) | 0.26 \pm 0.01 | 0.054   |
| Inverse                      | \(-1.31 \pm 0.13\) | 0.32 \pm 0.01 | 0.059   |
| Bisector                     | \(-0.98 \pm 0.10\) | 0.29 \pm 0.01 | 0.055   |
| Orthogonal                   | \(-0.71 \pm 0.11\) | 0.27 \pm 0.01 | 0.054   |
| **Only spiral galaxies**     |               |          |         |
| *Gas*                        |               |          |         |
| Forward                      | \(-0.87 \pm 0.15\) | 0.28 \pm 0.02 | 0.043   |
| Inverse                      | \(-0.98 \pm 0.11\) | 0.29 \pm 0.01 | 0.044   |
| Bisector                     | \(-0.93 \pm 0.14\) | 0.29 \pm 0.01 | 0.043   |
| Orthogonal                   | \(-0.88 \pm 0.09\) | 0.28 \pm 0.01 | 0.043   |
| *Stellar*                    |               |          |         |
| Forward                      | \(-0.88 \pm 0.10\) | 0.28 \pm 0.01 | 0.051   |
| Inverse                      | \(-1.35 \pm 0.12\) | 0.32 \pm 0.02 | 0.057   |
| Bisector                     | \(-1.12 \pm 0.13\) | 0.30 \pm 0.01 | 0.054   |
| Orthogonal                   | \(-0.92 \pm 0.12\) | 0.28 \pm 0.01 | 0.052   |
| **Total (gas+stellar)**      |               |          |         |
| Forward                      | \(-0.77 \pm 0.12\) | 0.27 \pm 0.01 | 0.051   |
| Inverse                      | \(-1.16 \pm 0.10\) | 0.31 \pm 0.01 | 0.056   |
| Bisector                     | \(-0.96 \pm 0.16\) | 0.29 \pm 0.01 | 0.053   |
| Orthogonal                   | \(-0.80 \pm 0.11\) | 0.27 \pm 0.01 | 0.051   |