Non-equilibrium Model for Nanofluid Free Convection Inside a Porous Cavity Considering Lorentz Forces

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In current article, transportation of CuO nanoparticles through a porous enclosure is demonstrated. The enclosure has complex shaped hot wall. Porous media has been simulated via two temperature equations. Magnetic force impact on nanofluid treatment was considered. Control volume based finite element method has been described to solve current article in vorticity stream function form. Single phase model was chosen for nanofluid. Nanofluid characteristics are predicted via KKL model. Roles of solid-nanofluid interface heat transfer parameter (Nhs), porosity, Hartmann and Rayleigh numbers have been illustrated. Outputs illustrated that conduction mode reduces with augment of Ra. Increasing magnetic forces make nanofluid motion to decrease. Temperature gradient of nanofluid decreases with augment of Nhs. Reducing porosity leads to enhance in Nusselt number.

Solar power collectors and drying technologies are two common uses for porous enclosure. In some application, researchers should consider two- temperature model. Alsabery et al.¹ demonstrated nanoparticle transportation in a tilted porous cavity. They indicated that convective flow is significantly influenced by the permeable layer augmentation. Lu et al.² investigated about convective flow in a composite porous media containing gyrotactic microorganism with anisotropic slip. They considered the effect of activation energy. Zaimi et al.³ illustrated nanofluid boundary layer movement on a porous plate. Sheikholeslami and Shehzad⁴ showed the two temperature model for nanoparticle migration inside a permeable medium. They revealed that Nu increases with decrease of porosity. Khan et al.⁵ reported the nanofluid mixed convection over an oscillating vertical plate. They utilized Laplace transform method to solve the governing equations.

Sheikholeslami and Shehzad⁶ investigated the role of radiation on nanoparticle treatment. They found that Nu decreases with reduce of radiation parameter. Haq et al.⁷ used carbon nanotubes to improve convective heat transfer over plate with slip flow. Carbon Nanotubes has been dispersed in to engine oil by Haq et al.⁸ is examined of magnetic forces. Tripathi et al.⁹ illustrated viscous dissipation and Hall effects on nanofluid rotating flow. Selimefendigil and Oztop¹⁰ depicted impact of inclination on hydrothermal behavior. They found that tilted angle can be used as control parameter.

Promvonge et al.¹¹ applied new way in a duct to improve the thermal characteristics. Sheikholeslami¹² described the impact of electric filed on nanofluid free convection. He proved that Nusselt number enhances by adding electric field. Aman et al.¹³ illustrated the nanofluid thermal improvement in migration of CNTs nanoparticles. Sheikholeslami and Seyyednezhad¹⁴ illustrated nanofluid Electrohydrodynamic flow in a permeable enclosure. Different applications of Fe3O4-water nanofluid were categorized by Sheikholeslami and Rokni¹⁵. Akbar et al.¹⁶ showed the role of Hartmann flow on nanoparticles migration in a duct. Najib et al.¹⁷ demonstrated the impact of chemical factor on flow style. They found that the Nu augments with augment of curvature. Various articles were been available about nanoparticle migration through porous media¹⁸⁻²¹.

Current publication is about nanoparticle migration in a porous enclosure with two temperature model via CVFEM considering magnetic force. Results illustrate the roles of significant parameters on contours.

Explanation of Geometry

Figure 1 illustrates the details of current geometry. The hot wall can formulate by:

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Figure 1. (a) Geometry and the boundary conditions with (b). A sample triangular element and its corresponding control volume.

| Coefficient values | CuO—Water               |
|--------------------|-------------------------|
| $a_1$              | $-26.593310846$         |
| $a_2$              | $-0.403818333$          |
| $a_3$              | $-33.356805$            |
| $a_4$              | $-1.915825591$          |
| $a_5$              | $6.42185846558E-02$     |
| $a_6$              | $48.40336955$           |
| $a_7$              | $-9.78756683$           |
| $a_8$              | $190.24561009$          |
| $a_9$              | $10.928586565$          |
| $a_{10}$           | $-0.72069983664$        |

Table 1. The coefficient values of CuO—Water nanofluid.

| Material | $\rho \text{ (kg/m}^3\text{)}$ | $C_p \text{ (J/kg K)}$ | $k \text{ (W/m K)}$ | $\beta \times 10^6 \text{ (K}^{-1}\text{)}$ | $d_p \text{ (nm)}$ | $\sigma \text{ (m}^{-1}\text{)}$ |
|----------|---------------------------------|-----------------------|---------------------|---------------------------------|-------------------|-----------------|
| Water    | 997.1                           | 4179                  | 0.613               | 21                             | $-$              | 0.05            |
| CuO      | 6500                            | 540                   | 18                  | 29                             | 29               | $10^{-10}$      |

Table 2. Thermo physical properties of water and nanoparticles.
\[ r_{\text{out}} = r_{\text{in}} + A \cos(N(\zeta - \zeta_0)) \quad (1) \]

\[ r_{\text{out}}, r_{\text{in}}, A, N \] are outer and inner radius, amplitude, number of undulation. The porous enclosure is full of nanofluid and influenced by magnetic force.

**CVFEM and Explanation**

**Formulation.** According to existence of magnetic force and two temperature model for porous medium the basic formulas are:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2) \]

\[-\frac{\partial P}{\partial x} + \sigma_{ij} B_0^2 \left[ v (\sin \gamma)(\cos \gamma) - u(\sin \gamma)^2 \right] - \frac{\mu_{nf}}{K} u = 0 \quad (3)\]
Figure 3. Streamlines ($\Psi$), isotherms for the nanofluid ($\theta_{nf}$) and the solid ($\theta_s$) at $Ra = 100$, $Ha = 0$, $\phi = 0.04$.

Table 4. $Nu_{ave}$ for various $Gr$ and $Ha$ at $Pr = 0.733$.
\[
\frac{h_{nf}}{K} v = -\frac{\partial P}{\partial y} + (T - T_0)g_{by} \beta_{nf}
\]
\[
+ \sigma_{nf} B_0^2 [\mu (\cos \gamma) (\sin \gamma) - \nu (\cos \gamma)^2]
\]  
(4)

\[
\frac{1}{\varepsilon} \left\{ \frac{\partial T_{nf}}{\partial x} + \nu \frac{\partial T_{nf}}{\partial y} \right\} = \frac{h_{nf} \varepsilon}{(\varepsilon) (\rho C_p)_{nf}} (-T_{nf} + T_0) + \frac{k_{nf}}{(\rho C_p)_{nf}} \left( \frac{\partial^2 T_{nf}}{\partial y^2} + \frac{\partial^2 T_{nf}}{\partial x^2} \right)
\]  
(5)

Figure 4. Streamlines (Ψ), isotherms for the nanofluid (\(\theta_{nf}\)) and the solid (\(\theta\)) at Ra = 100, Ha = 20, \(\phi = 0.04\).
\[
\frac{k_f}{(\rho C_p)_f} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{k_{nf}}{(1 - \phi)(\rho C_p)_s} (T_{nf} - T_i) = 0
\]

(\(\rho C_p\), \(\rho\beta\), \(\rho\), \(\sigma\), and \(k_{nf}, \mu_{nf}\) can define as: \(22\).

\[
(\rho C_p)_{nf} = \phi(\rho C_p)_p + (1 - \phi)(\rho C_p)_f
\]
Equation 8:
\[
(\rho\beta)_nf = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_p
\]

Equation 9:
\[
\rho_{nf} = \rho_f(1 - \phi) + \rho_p\phi
\]

Equation 10:
\[
\frac{\sigma_{nf}}{\sigma_f} = \frac{(MM - 1)3\phi}{(MM + 2) + \phi(1 - MM)} + 1, \quad MM = \frac{\sigma_p}{\sigma_f}
\]

Figure 6. Streamlines (Ψ), isotherms for the nanofluid (θ_{nf}) and the solid (θ_{s}) at Ra = 500, Ha = 20, φ = 0.04.
Figure 7. Streamlines ($\Psi$), isotherms for the nanofluid ($\theta_{nf}$) and the solid ($\theta_s$) at $Ra = 1000$, $Ha = 0$, $\phi = 0.04$.

\[
\frac{k_{nf}}{k_f} = 1 - 3 \left( 1 - \frac{k_f}{k_f} \right)^{-\phi} + \frac{\nu_f}{\rho_f \alpha_{nf}} c_{p_f} (5 \times 10^4) g'(d_p, T, \phi) \rho_f \phi
\]

\[
g'(d_p, T, \phi) = (a_2 \ln(d_p) + a_3 + a_3 \ln(\phi) + a_3 \ln(d_p)^2 + a_4 \ln(d_p) \ln(\phi)) \ln(T) + (a_5 \ln(d_p) \ln(\phi) + a_6 \ln(d_p^2) + a_7 \ln(d_p) + a_8 + a_9 \ln(\phi))
\]

(11)
\[ \mu_f = \frac{\mu_f}{(1 - \phi)^2} + \frac{\mu_f}{k_f} (k_{\text{Brownian}}/Pr) \]

(12)

where \( \phi \) is nanofluid volume fraction. Required characteristics and parameters are illustrated in Tables 1 and 2. Considering following definitions:

Figure 8. Streamlines (\( \Psi \)), isotherms for the nanofluid (\( \theta_{nf} \)) and the solid (\( \theta_s \)) at \( Ra = 1000, Ha = 20, \phi = 0.04 \).
\[ \nu = -\frac{\partial \psi}{\partial x}, \quad (X, Y) = (x, y)/L, \quad \nu = \frac{\partial \psi}{\partial y} \]

\[ \Theta_{ef} = (T_{ef} - T_e)/(T_h - T_e), \quad \Psi = \psi/\alpha_{nf}, \quad \Theta_f = (T_f - T_e)/(T_h - T_e). \]

Final equations are:

\[ \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = \frac{A_1}{A_2} \frac{Ha}{A_3} \left[ \frac{\partial^2 \Psi}{\partial Y^2} (\sin^2 \gamma) + \frac{\partial^2 \Psi}{\partial X^2} (\cos^2 \gamma) + 2 \frac{\partial^2 \Psi}{\partial X \partial Y} (\sin \gamma)(\cos \gamma) \right] \]

\[ \frac{\partial^2 \Theta_{ef}}{\partial Y^2} + \frac{\partial^2 \Theta_{ef}}{\partial X^2} + Nhs(\Theta_f - \Theta_{ef}) = -\frac{\partial \Theta_{ef}}{\partial Y} \frac{\partial \Psi}{\partial X} + \frac{\partial \Psi}{\partial Y} \frac{\partial \Theta_{ef}}{\partial X}. \]
\[ \varepsilon \left( \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial X^2} \right) + Nhs \delta(\theta_{nf} - \theta_s) = 0 \] (16)

where:

A_1 = \frac{\rho_{nf}}{\rho_f}, \quad Ra = (\rho\beta_f) \frac{k_g L \Delta T}{\alpha_f \mu_f}, \quad A_5 = \frac{\mu_{nf}}{\mu_f}

A_2 = \frac{(\rho C_p)_{nf}}{(\rho C_p)_f}, \quad \delta_s = k_{nf}/[k_f(1 - \varepsilon)], A_4 = \frac{k_{nf}}{k_f}

A_3 = \frac{(\rho \beta)_nf}{(\rho \beta)_f}, \quad Ha = \frac{\sigma_f K B_0^2}{\mu_f}

Nhs = h_{nf} L^2/k_{nf}, \quad A_6 = \frac{\sigma_{nf}}{\sigma_f} \tag{17}

Boundary conditions are:
\[ \theta_{of} = \theta_{i} = 0.0 \] on outer wall
\[ \Psi = 0.0 \] on all walls
\[ \theta_{of} = \theta_{i} = 1.0 \] on inner wall

\[ \begin{align*}
\text{Nu}_{loc} \text{ and } \text{Nu}_{ave} \text{ are:} \\
\text{Nu}_{loc} &= \frac{k_{of}}{k_f} \frac{\partial \theta_{of}}{\partial r} \\
\text{Nu}_{ave} &= \frac{1}{2\pi} \int_{0}^{2\pi} \text{Nu}_{loc} \, dr
\end{align*} \tag{19} \]

CVFEM. The innovative in which triangular element is used and upwind method is applied for advection term is CVFEM (Fig. 1(b)). Gauss-Seidel is the name of the method which is used for final step as mentioned in ref.\textsuperscript{23}.

Mesh Independent Test and Validation

Obviously, the final outputs should not alter by changing mesh size. So, this test should be done for various cases as illustrated Table 3. Also, we should be sure about accuracy of written code by applying this code for previous published problem. Table 4 and Fig. 2 illustrates nice accuracy\textsuperscript{24–26}.

Results and Discussion

In current research, migration of CuO nanoparticles is simulated via Non-equilibrium. Innovative method is applied to show the impacts of porosity, buoyancy, magnetic forces and the Nhs. Results show that \text{Nu}_{ave} reduces with augment of Ha, ε, Nhs. Convective flow enhances with increase of Nhs and ε while it reduces with enhance of magnetic force. When Ra = 1000, ε = 0.3, increasing Nhs leads to 17.06 percent decrement of \text{Nu}_{ave} in absence of magnetic field. Impact of Nhs is negligible in existence of Lorentz force. Also \text{Nu}_{ave} decreases about 53% with augment of Hartmann number.

Conclusions

In current research, migration of CuO nanoparticles is simulated via Non-equilibrium. Innovative method is applied to show the impacts of porosity, buoyancy, magnetic forces and the Nhs. Results show that \text{Nu}_{ave} reduces with augment of Ha, ε, Nhs. Convective flow enhances with increase of Nhs and ε while it reduces with enhance of magnetic force. When Ra = 1000, ε = 0.3, increasing Nhs leads to 17.06 percent decrement of \text{Nu}_{ave} in absence of magnetic field. Impact of Nhs is negligible in existence of Lorentz force. Also \text{Nu}_{ave} decreases about 53% with augment of Hartmann number.

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Author Contributions
I.K. formulated the problem. M.S. computed the numerical results. I.K. and M.S. wrote the main manuscript text.

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