Absorption by Nonextremal D3-branes

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ABSTRACT

We calculate the absorption probabilities for a class of massless fields whose linear perturbations leave the near-extremal D3-brane background metric unperturbed. It has previously been found that, for extremal D3-branes, these fields share the same absorption probability as that of the dilaton-axion. We find that these absorption probabilities diverge from each other as we move away from extremality. The form of the corresponding effective Schrödinger potentials leads us to conjecture that the absorption of various fields by nonextremal D3-branes depends on the polarization of angular momentum.

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1 Introduction

Over the past few years, there has been much work done on the scattering of fields due to the curved backgrounds of p-brane configurations of M theory and string theory \[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\]. Most of this research has focused on the scattering of minimally-coupled massless scalar fields, though there has been some work on fermions \[12\], two-form fields \[13\] and massive scalars \[14\]. The absorption probabilities of various linear field perturbations for which the ten-dimensional metric is left unperturbed have been calculated for extremal D3-branes \[15\]. It has been shown, through the analysis of wave equations in Schrödinger form, that certain half-integer and integer spin massless modes of the extremal D3-brane have identical absorption probabilities \[16\], which implies that such fields couple on the dual field theory to operators forming supermultiplets of strongly coupled gauge theory. In an effort to enhance our understanding of non-BPS states, the present paper continues this study for non-extremal D3-branes, which have already been probed in the near-extremal case by minimally-coupled massless scalars \[17, 18\].

Our discussion is organized as follows. In section II, we present the wave equations for various linear field perturbations in a non-extremal D3-brane background, for which the ten-dimensional metric is left unperturbed. In section III, we calculate the low-energy absorption probabilities for these field perturbations in the background of near-extremal D3-branes, using a method that was initially applied to a minimally-coupled scalar probe \[17, 18\]. In section IV, we provide concluding remarks.

2 Effective potentials

The D3-brane of type IIB supergravity is given by

\[
ds_{10}^2 = H^{-1/2}(-fdt^2 + dx_1^2 + dx_2^2 + dx_3^2) + H^{1/2}(f^{-1}dr^2 + r^2d\Omega_5^2),
\]

\[
G(5) = d^4x \wedge dH^{-1} + *(d^4x \wedge dH^{-1}).
\]

where \( H = 1 + \frac{R^4}{r^4} \) and \( f = 1 - \frac{r_o}{R} \). \( R \) specifies the D3-brane charge and \( r_o \) is the non-extremality parameter. The field equations are

\[
R_{\mu\nu} = -\frac{1}{6}F_{\mu\rho\sigma\tau\kappa}F^{\rho\sigma\tau\kappa},
\]

\[
F_{\mu\nu\rho\sigma\tau} = \frac{1}{5!}\epsilon_{\mu\nu\rho\sigma\tau\mu'\nu'\rho'\sigma'\tau'}F^{\mu'\nu'\rho'\sigma'\tau'},
\]

\[
D^\mu \partial_{[\mu} A_{\nu]} = -\frac{2i}{3}F_{\nu\rho\sigma\tau\kappa}D^\rho A^{\tau\kappa},
\]

\[
D^\mu \partial_{\mu} B = 0,
\]
where a dot above a symbol for a field denotes its background value. We study the wave equations for linear perturbations that leave the ten-dimensional background metric unperturbed. Deriving the radial wave equations in the background of non-extremal D3-branes is a straightforward generalization of what is done in [15] for the extremal case.

2.1 Dilaton-axion

The dilaton and axion are decoupled from the D3-brane in type IIB theory, satisfying the minimally-coupled scalar wave equation

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} \sqrt{-g} g^{\mu \nu} \partial_{\nu} \phi = 0. \tag{2.3}
\]

Thus, the radial wave equation of a dilaton-axion in the spacetime of a non-extremal D3-brane is given by

\[
\left( \frac{1}{r^5} \frac{\partial}{\partial r} f r^5 \frac{\partial}{\partial r} + \omega^2 \frac{H}{f} - \frac{\ell (\ell + 4)}{r^2} \right) \phi = 0, \tag{2.4}
\]

where \( \ell = 0, 1, \ldots \).

We shall express the wave equation in Schrödinger form, and study the characteristics of the Schrödinger effective potential. By the substitution

\[
\phi = r^{-5/2} f^{-1/2} \psi, \tag{2.5}
\]

we render (2.4) in the Schrödinger form

\[
\left( \frac{\partial^2}{\partial r^2} - V_{\text{eff}} \right) \psi = 0, \tag{2.6}
\]

where

\[
V_{\text{eff}}(\ell) = -\frac{\omega^2}{f^2} H + \frac{(\ell + 3/2)(\ell + 5/2)}{4 r^4} + \frac{(f - 1)(-f + 16)}{4 f^2 r^2}. \tag{2.7}
\]

Factors that are shared by the incident and outgoing parts of the wave function cancel out when calculating the absorption probability. Thus, the absorption probability of \( \phi \) and \( \psi \) are the same.

Technically, \( V_{\text{eff}} \) cannot be interpreted as an effective potential, since it is dependent on the particle’s incoming energy. However, this is of no consequence for our analysis of the form of the wave equations for various particles.

2.2 Scalar from the two-form

For the free indices of the two-form taken to lie along the \( S^5 \), the radial wave equation is

\[
\left( \frac{H}{r} \frac{\partial}{\partial r} f r \frac{\partial}{\partial r} + \frac{H}{f} \omega^2 - \frac{(\ell + 2)^2}{r^2} + \frac{4 R^4}{H r^6} (\ell + 2) \right) a_{(\alpha \beta)} = 0, \tag{2.8}
\]
where $\ell = 1, 2, \ldots$ The sign ± corresponds to the sign in the spherical harmonic involved in the partial wave expansion.

By the substitution

$$a_{(\alpha\beta)} = \left(\frac{H}{f}\right)^{1/2} \psi_{(\alpha\beta)},$$

we render (2.8) into the Schrödinger form:

$$V_{\text{eff}}(\ell) = -\frac{\omega^2 H}{f^2} + \frac{(\ell + 3/2)(\ell + 5/2)}{f r^2} + \frac{(f - 1)(15 f + 16)}{4 f^2 r^2} + \frac{(H - 1)[-f^2 (15 H + 49) + f(\pm 16(\ell + 2)H + 30H + 2) + H - 1]}{4 r^2 f^2 H^2}.$$

(2.10)

2.3 Vector from the two-form

We now consider one free index of the two-form along $S^5$ and one free index in the remaining 5 directions. For the tangential components of the vector, the radial wave equation is

$$\left(\frac{1}{r^3} \frac{\partial}{\partial r} r^3 f \frac{\partial}{\partial r} + \frac{\omega^2 H}{f} - \frac{(\ell + 1)(\ell + 3)}{r f^2}\right) a_1 = 0,$$

(2.11)

where $\ell = 1, 2, \ldots$

By the substitution

$$a_1 = (r^3 f)^{-1/2} \psi,$$

(2.12)

we render (2.11) in the Schrödinger form with

$$V_{\text{eff}}(\ell) = -\frac{\omega^2 H}{f^2} + \frac{(\ell + 3/2)(\ell + 5/2)}{f r^2} + \frac{(f - 1)(3 f + 16)}{4 f^2 r^2}.$$

(2.13)

The radial and time-like components of the vector can be determined from each other by

$$\frac{\partial}{\partial r} a_o = \frac{1}{i \omega} \left[\omega^2 - \frac{(\ell + 1)(\ell + 3)}{r^2 H} f\right] a_r,$$

(2.14)

and

$$\frac{1}{r^3} \frac{\partial}{\partial r} r^3 \left[\frac{\partial}{\partial r} a_0 + i \omega a_r\right] - \frac{(\ell + 1)(\ell + 3)}{f r^2} a_0 = 0,$$

(2.15)

where $\ell = 1, 2, \ldots$

(2.14) and (2.15) can be decoupled. The wave equation for $a_r$ is

$$\left(\frac{H \partial}{\partial r} f \frac{\partial}{\partial r} \frac{r f}{H} + \omega^2 H - \frac{(\ell + 1)(\ell + 3)}{r^2} f\right) a_r = 0.$$

(2.16)

By the substitution

$$a_r = (rf^3)^{-1/2} H \psi,$$

(2.17)
we render (2.16) into the Schrödinger form with
\[
V_{\text{eff}}(\ell) = -\frac{\omega^2 H}{f^2} + \left(\frac{\ell + 3/2}{f r^2} + (f - 1)\frac{35 f + 16}{4 f^2 r^2}\right). \tag{2.18}
\]
The wave equation for \(a_0\) is
\[
\left(\frac{f}{r} \frac{\partial}{\partial r} \frac{r^3 f}{\omega^2 r^2 H} - (\ell + 1)(\ell + 3) f \frac{\partial}{\partial r} + 1\right) a_0 = 0. \tag{2.19}
\]
By the substitution
\[
a_0 = (\frac{r^3 f}{\omega^2 r^2 H - (\ell + 1)(\ell + 3)f})^{-1/2} \psi, \tag{2.20}
\]
(2.19) can be rendered into the Schrödinger form. However, the corresponding potential is singular. Instead, we determine \(a_0\) directly from \(a_r\) via
\[
a_0 = \frac{i f}{\omega r} \frac{\partial}{\partial r} \left(\frac{r f}{H} a_r\right). \tag{2.21}
\]

2.4 Antisymmetric tensor from 4-form

For two free indices of the 4-form along \(S^5\) and two free indices in the remaining 5 directions, the coupled radial wave equations for the components of the antisymmetric tensor derived from the 4-form are
\[
b_{3r} = \mp \frac{\omega r H}{\ell + 2 f} b_{12}, \tag{2.22}
\]
\[
\frac{\partial}{\partial r} b_{03} - i \omega b_{3r} = \pm \frac{i}{r} (\ell + 1) b_{12} \tag{2.23}
\]
and
\[
\frac{\partial}{\partial r} b_{12} = \pm \frac{i}{r f} (\ell + 2) b_{03}, \tag{2.24}
\]
where \(\ell = 1, 2, ..\) Eliminating \(b_{03}\) and \(b_{3r}\) we get
\[
\left(\frac{1}{r} \frac{\partial}{\partial r} r f \frac{\partial}{\partial r} + \frac{\omega^2 H}{f} - \frac{(\ell + 2)^2}{r^2}\right) b_{12} = 0. \tag{2.25}
\]
By the substitution
\[
b_{12} = \left(\frac{r f}{H}\right)^{-1/2} \psi, \tag{2.26}
\]
we render (2.25) into Schrödinger form with
\[
V_{\text{eff}}(\ell) = -\frac{\omega^2 H}{f^2} + \left(\frac{\ell + 3/2}{f r^2} + (f - 1)\frac{15 f + 16}{4 f^2 r^2}\right). \tag{2.27}
\]
### 2.5 Two-form from the antisymmetric tensor

The equations for two-form perturbations polarized along the D3-brane are coupled. For s-wave perturbations, they can be decoupled \[13, 15\], and the radial wave equation is

\[
\left( \frac{1}{r^5 H} \frac{\partial}{\partial r} r^5 f H \frac{\partial}{\partial r} + \frac{\omega^2 H}{f} - \frac{16 R^8}{r^{10} H^2} \right) \phi = 0, \tag{2.28}
\]

where \( \ell = 0, 1, \ldots \)

By the substitution

\[
\phi = (r^5 f H)^{-1/2} \psi, \tag{2.29}
\]

we render (2.28) into Schrödinger form with

\[
V_{\text{eff}}(\ell) = -\frac{\omega^2 H}{f^2} + \frac{15}{4fr^2} + \frac{12(H - 1)^2}{r^2 f H^2} + \frac{(f - 1)(15f + 16)}{4r^2 f^2}. \tag{2.30}
\]

### 2.6 General form of effective potential

For an extremal D3-brane, the effective potentials for the dilaton-axion, vector from the two-form and the antisymmetric tensor from the 4-form all reduce to

\[
V_{\text{eff}}(\ell) = -\omega^2 H + \frac{(\ell + 3/2)(\ell + 5/2)}{r^2}. \tag{2.31}
\]

Thus, the above fields have identical absorption probabilities. The effective potentials for the scalar from the two-form and the two-form from the antisymmetric tensor include additional terms, which merely have the effect of changing the partial wave number by \( \ell \rightarrow \ell \pm 1 \) \[16\].

For a non-extremal D3-brane, the effective potentials for the dilaton-axion, vector from the two-form and the antisymmetric tensor from the 4-form are of the form

\[
V_{\text{eff}}(\ell) = -\frac{\omega^2 H}{f^2} + \frac{(\ell + 3/2)(\ell + 5/2)}{fr^2} + \frac{(f - 1)(a - 1)(a + 1)f + 16}{4f^2 r^2}, \tag{2.32}
\]

where \( a = 0 \) for the dilaton-axion, \( a = 2 \) for the tangential components of the vector from the two-form, \( a = 4 \) for the antisymmetric tensor from the 4-form, and \( a = 6 \) for the radial component of the vector from the two-form. For the scalar from the two-form and the two-form from the antisymmetric tensor, the effective potential has additional terms that are proportional to the charge of the D3-brane. However, for a chargeless D3-brane, the scalar and two-form fields fit into the above scheme with \( a = 4 \). It is interesting to note the change of grouping of absorption probabilities between the cases of extremal and chargeless D3-branes. The effective potential for the time-component of the vector from the two-form does not appear to fit into this scheme, though certain potential terms may merely have the effect of changing the partial wave number.
We conjecture that $a$ is a parameter that depends on the polarization of the angular momentum. Note that this parameter only plays a role in absorption away from extremality.

3 Low-energy absorption probabilities for a near-extremal D3-brane

3.1 Dilaton-axion, vector from two-form, and antisymmetric tensor from 4-form

We will solve the wave equations in the approximation that

$$r_o \ll R \ll 1/\omega. \quad (3.1)$$

We work in the limit in which the frequency is large compared to the temperature, which means that $r_o \leq R^2 \omega$. We render the wave function in a form that will be convenient for isolating the singularity at $r = r_o$:

$$\psi = (r^5 f)^{+1/2} \tilde{\phi}. \quad (3.2)$$

Substituting (3.2) into (2.6) together with (2.32) yields

$$\left( \frac{1}{r^5} \partial_r f r^5 \partial_r + \omega^2 H f - \frac{\ell (\ell + 4)}{r^2} + \frac{a^2}{4r^2} (1 - f) \right) \tilde{\phi} = 0. \quad (3.3)$$

Note that $\phi = F(r) \tilde{\phi}$, where $F(r)$ is a nonsingular function of $r$ for $r > r_o$.

We will now use the same approximation and procedure as that by Siopsis in the case of a minimally-coupled scalar [17, 18]. We use three matching regions. In the outer region defined by $r \gg R^2 \omega$, (3.3) becomes

$$\left( r^2 \partial_r^2 + 5r \partial_r + \omega^2 r^2 - \ell (\ell + 4) \right) \tilde{\phi} = 0. \quad (3.4)$$

The solution is

$$\tilde{\phi} = \frac{1}{r^2} J_{\ell+2} (\omega r). \quad (3.5)$$

In the intermediate region defined by $r - r_o \gg r_o$ and $r \ll R$, expressed in terms of the dimensionless quantity $z \equiv (r - r_o)/r_o$,

$$\left( \frac{1}{z^2} \partial_z z^5 \partial_z + \frac{16\kappa^2}{z^2} - \ell (\ell + 4) \right) \tilde{\phi} = 0, \quad (3.6)$$

where

$$\kappa = \frac{\omega r_o}{4} (1 + \frac{R^4}{r_o^4})^{1/2} \approx \frac{R^2 \omega}{4r_o} = \frac{\omega}{4\pi T_H}. \quad (3.7)$$
and \( T_H = \frac{r_o^{-2}}{\pi R^2} \) is the Hawking temperature. The corresponding wave function solution is

\[
\tilde{\phi} = \frac{A}{r_o^{2}z^{2}}H^{(1)}_{\ell+2}(\frac{4\kappa}{z}),
\]

where we consider the purely incoming solution. Matching asymptotic form of the intermediate solution (3.8) for large \( z \) with the asymptotic form of the outer solution (3.5) for small \( \omega r \) yields the amplitude

\[
A = \frac{i\pi}{(\ell + 2)!} (\frac{\omega R}{2})^{2\ell + 4}.
\]

For \( z \ll \kappa \), we expand the intermediate wave function solution (3.8):

\[
\tilde{\phi} = \frac{(-i)^{\ell+5/2}}{r_o^{2}\sqrt{2\pi}\kappa} z^{-3/2}e^{4i\kappa/z} \left( 1 + \frac{i(\ell + 3/2)(\ell + 5/2)z}{8\kappa} + O(\kappa^{-2}) \right).
\]

The inner region is defined by \( r - r_o \ll R^2\omega \). The inner and intermediate regions overlap, since \( r_o \ll R^2\omega \). There is a singularity in the wave function at \( r = r_o \), which can be isolated by taking the wave function to be \( f^{i\kappa}\varphi \).

Isolating this singularity in the wavefunction enables us to calculate the dominant term in the near-extremal absorption probability. For the inner region, we substitute (3.11) into (3.3) and express the result in terms of the dimensionless parameter \( x \equiv r/r_o \):

\[
x^2f(x)\partial_x^2\varphi + x[5 - (1 - 8i\kappa)x^{-4}]\partial_x\varphi + \left( 16\kappa^2h(x) - \frac{a^2}{4}(1 - f(x)) \right)\varphi = 0,
\]

where

\[
f(x) = 1 - x^{-4},
\]

and

\[
h(x) \equiv \frac{1 + x^2 + x^4}{(1 + x^2)x^4}.
\]

Since \( x \ll \kappa \), we shall expand the inner wave function solution in \( \kappa^{-1} \):

\[
\varphi = AB e^{i\kappa\alpha(x)}\beta(x)(1 + \frac{i}{\kappa}\gamma(x) + O(\kappa^{-2})).
\]

We will solve for \( B \) such that \( \varphi(1) = B \), which implies that \( \alpha(1) = \gamma(1) = 0 \) and \( \beta(1) = 1 \).

We solve for the functions \( \alpha, \beta \) and \( \gamma \) by plugging (3.15) into (3.12):

\[
\alpha(x) = -4 \int_1^x dy \frac{1 + \sqrt{1 + h(y)y^4(y^4 - 1)}}{y(y^4 - 1)}.
\]
\[ \beta(x) = \exp \left( -\frac{1}{2} \int_1^x dy \frac{y(y^4 - 1)\alpha'' + (5y^4 - 1)\alpha'}{4 + y(y^4 - 1)\alpha'} \right). \] (3.17)

\[ \gamma(x) = -\frac{1}{2} \int_1^x dy \frac{y^4(\ell + 4) - y(y^4 - 1)\beta'' - (5y^4 - 1)\beta'}{y(y^4 - 1)\alpha' + 4}. \] (3.18)

We match the expressions for \( \alpha, \beta \) and \( \gamma \) in the large \( x \) limit with the asymptotic form of the intermediate solution (3.10) and solve for \( B \):

\[ B = \frac{(-i)^{\ell+5/2}}{r_o^2 \sqrt{2\pi \kappa}} \left( 1 - \frac{i(\ell + 3/2)(\ell + 5/2)}{8\kappa} \right). \] (3.19)

The absorption probability is

\[ P = 8\pi\kappa r_o^4 |A|^2 |B|^2. \] (3.20)

Plugging in the amplitudes \( A \) and \( B \), we find that

\[ P_{\text{near-extremal}} = \left( 1 + \left( \frac{4(\ell + 2)^2 - 1}{32\kappa} \right)^2 \right) P_{\text{extremal}}, \] (3.21)

where

\[ P_{\text{extremal}} = \frac{4\pi^2}{((\ell + 1)!((\ell + 2)!)^2} \left( \frac{\omega R}{2} \right)^{4\ell + 8}. \] (3.22)

Our result agrees with [17, 18] for the minimally-coupled scalar. As we have shown, for near-extremal D3-branes, the dilaton-axion, tangential and radial components of the vector from the two-form and the antisymmetric tensor from the 4-form have identical absorption probabilities, as in the extremal case. As can be seen, as \( \kappa \to \infty \), we recover the result previously obtained for an extremal D3-brane.

### 3.2 Time-like component of vector from two-form

Using (2.17), (2.21) and (3.2) we find the wave function for the longitudinal component of the vector from the two-form directly in terms of the radial component wave function:

\[ a_o = \frac{if}{\omega \tau} \partial_r (r^3 \tilde{\varphi}). \] (3.23)

We can find the wave function solutions for \( a_o \) in the three regions directly from the dilaton wave functions (3.3), (3.8) and (3.13). We must be careful to expand to \( O(\kappa^{-3}) \) in the dilaton wave functions in order for \( a_o \) to be of order \( O(\kappa^{-2}) \). Matching \( a_o \) in the three regions yields the same absorption probability as for the dilaton given by (3.21). Thus, for near-extremal D3-branes, the longitudinal component of the vector from the two-form and dilaton-axion have identical absorption probabilities, as in the extremal case.
3.3 Two-form from the antisymmetric tensor

Substituting (3.2) into (2.6) together with (2.30) yields

\[
\left( \frac{1}{r^5} \partial_r r^5 \partial_r + \omega^2 \frac{H}{f} + \frac{4}{r^2} (1 - f) - \frac{12(H - 1)^2}{r^2 H^2} \right) \tilde{\phi} = 0. \tag{3.24}
\]

We require four regions for matching. Consider \( r_1 \) and \( r_2 \) such that \( r_2 \ll R \ll r_1 \). The outer region is given by \( r > r_1 \), where \( \omega r_1 \ll 1 \). In the outer region, the wave equation and solution are given by (3.4) and (3.5), respectively, with \( \ell = 0 \).

The outer intermediate region is given by \( r_1 > r > r_2 \) and \( \omega R^2 \ll r \), so that we can ignore the \( \omega^2 \) term in the wave equation:

\[
\left( \frac{1}{r^5} \partial_r r^5 \partial_r - \frac{12(H - 1)^2}{r^2 H^2} \right) \tilde{\phi} = 0. \tag{3.25}
\]

The corresponding wave function solution is

\[
\tilde{\phi} = CH^{3/2} + DH^{-1/2}. \tag{3.26}
\]

Matching the outer and outer intermediate regions yields

\[
C + D = \frac{\omega^2}{8}. \tag{3.27}
\]

The inner intermediate region is defined by \( r - r_o \gg r_o \) and \( r \ll R \). Expressed in terms of the dimensionless quantity \( z \equiv (r - r_o)/r_o \), the wave equation in this region is

\[
\left( \frac{1}{z^3} \partial_z z^5 \partial_z + \frac{16\kappa^2}{z^2} - 12 \right) \tilde{\phi} = 0. \tag{3.28}
\]

The corresponding wave function solution is

\[
\tilde{\phi} = A \frac{1}{r_o^2 z^2} H_4^{(1)} \left( \frac{4\kappa}{z} \right), \tag{3.29}
\]

where we take the purely incoming solution. Matching the wave function across the two intermediate regions yields

\[
A = \frac{i\pi}{12} \left( \frac{\omega R}{2} \right)^6, \quad C = 0. \tag{3.30}
\]

For \( z \ll \kappa \), we expand the inner intermediate wave function solution (3.29):

\[
\tilde{\phi} = \frac{(-i)^{1/2} A}{r_o^2 \sqrt{2\pi\kappa}} z^{-3/2} e^{4ik/z} (1 + \frac{i(7/2)(9/2)z}{8\kappa} + O(\kappa^{-2})). \tag{3.31}
\]

For the inner region, we substitute (3.11) into (3.24) and express the result in terms of the dimensionless parameter \( x \):

\[
x^2 f(x) \partial_x^2 \varphi + x[5 - (1 - 8i\kappa)x^{-4}] \partial_x \varphi + \left( 16\kappa^2 h(x) + 4(1 - f(x)) - 12 \right) \varphi = 0. \tag{3.32}
\]
where $f(x)$ and $h(x)$ are given by (3.13) and (3.14), respectively.

Following the case for the dilaton, we plug the ansatz for the inner wave function solution, given by (3.15), into (3.32) and solve for the amplitude $B$ by matching the inner solution with the inner intermediate solution. The result is that

$$B = \left(-i\right)^{1/2} \frac{1}{r_0^2 \sqrt{2\pi \kappa}} \left(1 - \frac{i(7/2)(9/2)}{8\kappa}\right).$$  \hspace{1cm} (3.33)

Plugging amplitudes $A$ and $B$ into (3.20), we find that the absorption probability is

$$P_{\text{near-extremal}} = \left(1 + \left(\frac{64}{32\kappa}\right)^2\right) P_{\text{extremal}},$$  \hspace{1cm} (3.34)

where $P_{\text{extremal}}$ is the same as that given for the dilaton in (3.22) with $\ell \rightarrow \ell + 1$. As with previous cases, as $\kappa \rightarrow \infty$, we recover the result previously obtained for an extremal D3-brane. We have shown that, for near-extremal D3-branes, the two-form from the anti-symmetric tensor does not have the same absorption probability as the dilaton-axion with $\ell \rightarrow \ell + 1$, while it does in the extremal case.

### 3.4 Scalar from two-form

Substituting (3.2) into (2.6) together with (2.8) yields

$$\left(\frac{1}{r^5} \frac{\partial}{\partial r} r^5 \frac{\partial}{\partial r} + \omega^2 \frac{H}{f} - \frac{\ell(\ell + 4)}{r^2} + \frac{4}{r^2} (1 - f) - \frac{(H - 1)[4(1 + (\ell + 2)H + 30H + 2) + H - 1]}{4r^2 f H^2}\right) \tilde{\phi} = 0.$$  \hspace{1cm} (3.35)

We require four regions for matching, as defined in the previous section for the two-form. In the outer region, the wave equation and solution are given by (3.4) and (3.5), respectively. In the outer intermediate region, we may ignore the $\omega^2$ term in the wave equation:

$$\left(\frac{1}{r^3} \frac{\partial}{\partial r} r^5 \frac{\partial}{\partial r} - \frac{\ell(\ell + 4)}{r^2} - \frac{(H - 1)[4(1 + (\ell + 2)H + 30H + 2) + H - 1]}{r^2 H^2}\right) \tilde{\phi} = 0.$$  \hspace{1cm} (3.36)

The corresponding wave function solution is

$$\tilde{\phi} = C r^{\pm(\ell + 2) - 2} + D r^{\mp(\ell + 2) - 2} \left(1 + \frac{\ell + 2}{\ell + 2 \pm 2} \frac{R^4}{r^4}\right).$$  \hspace{1cm} (3.37)

Matching the outer and outer intermediate regions yields

$$C = \frac{1}{(\ell + 2)!} \left(\frac{\omega}{2}\right)^{\ell + 2}, \quad D = 0,$$  \hspace{1cm} (3.38)

for the positive sign and

$$D = \frac{1}{(\ell + 2)!} \left(\frac{\omega}{2}\right)^{\ell + 2}, \quad C = 0.$$  \hspace{1cm} (3.39)
for the negative sign.

Expressed in terms of the dimensionless quantity $z \equiv (r - r_o)/r_o$, the wave equation in the inner intermediate region is

$$
\left( \frac{1}{z^3} \partial_z z^5 \partial_z + \frac{16\kappa^2}{z^2} \right) - \ell(\ell + 4) \mp 4(\ell + 2) - 4) \tilde{\phi}.
$$

(3.40)

The corresponding wave function solution is

$$
\tilde{\phi} = A \frac{1}{r_o^2 z^2} H_{\ell+2 \pm 2}^{(1)} \left( \frac{4\kappa}{z} \right),
$$

(3.41)

where we take the purely incoming solution.

Matching the wave function across the two intermediate regions yields

$$
A_{\pm} = \frac{i\pi}{(\ell + 2 \pm 1)!(\ell + 1 \pm 1)!} \left( \frac{\omega R}{2} \right)^{2\ell+4 \pm 2}.
$$

(3.42)

For $z \ll \kappa$, we expand the inner intermediate wave function solution (3.41):

$$
\tilde{\phi} = \frac{(-i)^{\ell+1/2}}{r_o^2 \sqrt{2\pi\kappa}} z^{-3/2} e^{4i\kappa/z} \left( 1 + \frac{i(\ell + 3/2 \pm 2)(\ell + 5/2 \pm 2)}{8\kappa} z + O(\kappa^{-2}) \right).
$$

(3.43)

For the inner region, we substitute (3.11) into (3.36) and express the result in terms of the dimensionless parameter $x$:

$$
x^2 f(x) \partial_x^2 \varphi + x [5 - (1 - 8i\kappa)x^{-1}] \partial_x \varphi + \left( 16\kappa^2 h(x) + 4(1 - f(x)) - \ell(\ell + 4) \mp 4(\ell + 2) + \frac{15}{4} f(x) - \frac{15}{2} - \frac{1}{f(x)} \right) \varphi = 0.
$$

(3.44)

where $f(x)$ and $h(x)$ are given by (3.13) and (3.14), respectively.

Following the case for the dilaton, we plug the ansatz for the inner wave function solution, given by (3.15), into (3.44) and solve for the amplitude $B$ by matching the inner solution with the inner intermediate solution. The result is that

$$
B = \frac{(-i)^{\ell+1/2}}{r_o^2 \sqrt{2\pi\kappa}} \left( 1 - \frac{i(\ell + 3/2 \pm 2)(\ell + 5/2 \pm 2)}{8\kappa} \right).
$$

(3.45)

Plugging amplitudes $A$ and $B$ into (3.20), we find that the absorption probability is

$$
P_{\text{near-extremal}} = \left( 1 + \left( \frac{(\ell + 3/2 \pm 2)(\ell + 5/2 \pm 2)}{8\kappa} \right)^2 \right) P_{\text{extremal}},
$$

(3.46)

where $P_{\text{extremal}}$ is the same as that given for the dilaton in (3.22) with $\ell \to \ell \pm 1$. As with previous cases, as $\kappa \to \infty$, we recover the result previously obtained for an extremal D3-brane.

We have shown that, for near-extremal D3-branes, the scalar from the two-form does not have the same absorption probability as the dilaton-axion with $\ell \to \ell \pm 1$, while it does in the extremal case.
4 Discussion

We have obtained a simple relation between the extremal and near-extremal absorption probabilities of a D3-brane:

\[ P_{\text{near-extremal}} = \left(1 + \left(\frac{4\nu^2 - 1}{32\kappa}\right)^2\right) P_{\text{extremal}}, \]  

(4.1)

where \( \nu = \ell + 2 \) for the dilaton-axion, the vector from the two-form and the antisymmetric tensor from the 4-form, \( \nu = 4 \) for the two-form from the antisymmetric tensor, and \( \nu = \ell + 4 \) for the scalar from the two-form. Note that (4.1) has the same form as that of various fields scattered by a four-dimensional \( \mathcal{N} = 4 \) supergravity equal-charge black hole [19].

As we would expect, a perturbation from extremality increases the absorption probability. At near-extremality, the absorption probability of the dilaton-axion, vector from the two-form and antisymmetric tensor from the 4-form are identical, just as they are at extremality [11]. At extremality, the absorption probability of the scalar from the two-form and the two-form from the antisymmetric tensor are simply related to that of the dilaton-axion by a change in the partial wave number \( \ell \to \ell \pm 1 \). For near-extremality, this relation breaks down, and the scalar and two-form fields are absorbed more than if this were the case.

We have found numerically that, further away from extremality, the dilaton-axion, vector from the two-form and antisymmetric tensor from the 4-form no longer share the same absorption probability [20]. As can be seen from the effective potential (2.32), this is due to a parameter \( a \) which we conjecture depends on the angular momentum polarization. \( a = 0 \) for the dilaton-axion, \( a = 2 \) for the tangential components of the vector from the two-form, \( a = 4 \) for the antisymmetric tensor from the 4-form, and \( a = 6 \) for the radial component of the vector from the two-form. There are additional potential terms for the scalar from the two-form and the two-form from the antisymmetric tensor. However, these terms vanish for a chargeless D3-brane, enabling the scalar and two-form fields to fit into the above scheme with \( a = 4 \). It is rather curious how the grouping of absorption probabilities changes between the cases of extremal and chargeless D3-branes. At first look, it appears that the time-component of the vector from the two-form does not fit into this scheme, though certain potential terms may merely be equivalent to a change in the partial wave number. We conjecture that the parameter \( a \), which only plays a role away from extremality, depends on the polarization of the angular momentum.
The absorption probability can be expressed in terms of Gamma functions:

\[ P \approx \frac{8\pi^3\kappa}{\nu!(\nu - 1)!} \left| \frac{\Gamma(\nu/2 + 1/4 + 2i\kappa)\Gamma(\nu/2 + 3/4 + 2i\kappa)}{\Gamma(1 + 4i\kappa)} \right|^2 \left( \frac{\omega r_o}{2} \right)^{2\nu}, \tag{4.2} \]

so long as one keeps in mind that the formula holds only to order \( \kappa^{-2} \). For \( \nu = \ell + 2 \), this absorption probability is the same as that derived in \([17, 18]\) for a minimally-coupled scalar.

Siopsis expressed the absorption probability in the form (4.2) in order to interpret factors in terms of left and right-moving temperatures \( T_L \) and \( T_R \). In particular, the general form of a black hole grey-body factor is

\[ P_{b.h.} \sim \left| \frac{\Gamma(\ell + 2 + i\frac{\omega}{4\pi T_L})\Gamma(\ell + 2 + i\frac{\omega}{4\pi T_R})}{\Gamma(1 + \frac{\omega}{2\pi T_H})} \right|^2. \tag{4.3} \]

Thus, by comparing (4.2) with (4.3), we are led to the conclusion that both left and right-moving modes contribute to the absorption probability at temperatures

\[ T_L = T_R = T_H \tag{4.4} \]

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