Random vibration endurance test of automotive component using virtual prototyping

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Abstract. Automotive component manufacturers are required to validate their product reliability by assessing their mechanical endurance through predefined by the car manufacturer test procedures. A possible way to avoid (or to decrease their number) failures is to use Virtual Prototyping (VP) techniques. This study aims to present a methodology for random vibration endurance test of automotive component using virtual prototyping, based on common engineering analysis tools. There is a certain number of specialized tools for this type of testing but they require directed investment that are not accessible for every automotive industry subcontractor. This is demonstrated by a case study from the automotive industry – a cluster assembly (dashboard).

1. Introduction
The automotive industry aims to ensure certain reliability and safety parameters that are highly dependent on endurance assessment of car structures. Vehicles are subjected to complex random variable amplitude loadings, leading to faults in the components [1]. Automotive component manufacturers are required to validate their products reliability by assessing their mechanical endurance through predefined by the car manufacturer test procedures. These test procedures define vibration profiles (Power Spectrum Density of acceleration) along each axis and mono-axial dynamic load is applied subsequently for all axes, in predefined work environment (temperature, humidity, etc.). Existing studies of vibration testing in the automotive industry [2, 3] propose the following classification of the types of vibration signals used:

1) Fixed sine: sinusoidal cycle loadings, applied at a given frequency and at a certain acceleration value.
2) Swept sine: sine function that gradually changes frequency and/or amplitude over time.
3) Shocks: pulsed signals, based on an excitation of single type half-sine to represent a shock input on both directions along tested axis.
4) Random (PSD): very close to real component loads. It simulates loads by PSD, or represents random vibration environment, typical for driving conditions. This type of testing is the most realistic one for automotive component validation [2, 3].

The random vibration endurance test is quite typical and commonly used method. This test evaluates the function and strength of a particular component when vibration is applied by simulating vehicle vibration conditions, and also checks for deterioration or predicts failure in its parts. All parts and materials must be functional without any permanent deformation. Mechanical damage such as deformation or cracks is forbidden. This test is performed over a physical prototype of the product.
Thus, it is performed, usually, when the product is in the stage of prototyping and any more serious problem that is found during testing means there is the necessity for design change and new physical prototype. Failures found during testing means expenses – in terms of time and money – and they should be avoided.

A possible way to avoid (or to decrease their number) failures is to use Virtual Prototyping (VP) techniques. Under fierce development globally, approach of incorporating VP in Product Development Process (PDP) to validate products is becoming common practice. It has enhanced capabilities, offered to the engineers, for reducing the design cycle times of product and costly prototypes [4-6]. The virtual prototype is based on simulation models, but taking into account the connectivity and mutual influence of the effects and induced effects in studying the behavior of the object, thus getting as close as possible to the needed final physical object in terms of shape, size, functions and elements of behavior. It is possible to “reproduce” physical testing also, and provides opportunities to explore the product behavior in detail by analyzing its “internal” parameters (stress, deformations, etc.) and to understand the process and even the complex reasons for certain failure.

Virtual Prototyping is a promising technology that could decrease time-to-market and expenses, but requires an appropriate approach (methodology) for a modeled physical process. Usually, it is based on an existing hypothesis or theory.

Damages and failures, caused by random vibration endurance testing, are typically caused by fatigue. Fatigue assessment uses SN curves of critical structural details, in addition to the spectra, i.e. it represents material behaviour locally. This requires a known hypothesis for damage accumulation. There is a wide variety of hypotheses that examine this type of calculations - cumulative damage. A widely used method for calculation of cumulative damage is the Palmgren-Miner-Hypothesis. It gives certain simplicity, and it is based on SN curve. This hypothesis could use a modification according to Haibach, where there is a change after knee-point (k' = 2k-i; k' = k for materials with corrosion) in order to also include the damage by stress amplitudes below the knee-point (refer to figure 1) [7-9].

![Figure 1. Most commonly used cumulative damage calculation methods [5].](image)

This study aims to present a methodology for random vibration endurance test of automotive component using virtual prototyping, based on common engineering analysis tools. There is a certain number of specialized tools for this type of testing but they require directed investment that are not accessible for every automotive industry subcontractor.

2. Methodology
The methodology is based on Palmgren-Miner-Hypothesis. It uses data for base excitations (Power Spectrum Density – PSD) along each of the three axes, as specified by car manufacturer. The methodology is step-based, and uses available types of analyses in commercial conventional Finite
Element Analysis (FEA) software – modal and random vibration (with applied PSD). Further, the results are additionally combined and calculated to assess fatigue of the examined component.

The methodology is presented graphically in figure 2 below.

Random vibration analysis is usually performed over a large range of frequencies — from 5 Hz to 1000 Hz. This is repeated in all 3 directions of application of the base excitation and is combined in a final result. The steps, as shown in figure 2, are described below:

**Figure 2.** Step-based methodology of random vibration endurance test using virtual prototyping.

- **S0: Static analysis:** Thermal loads are applied (temperature of environment) to determine prestressed condition of the entire structure and to be used in the next step;
- **S1: Modal analysis:** to obtain modal frequencies and use them as a basis for subsequent PSD analyses;
- **S2: Random Vibration (PSD) analyses:**
  - 3 analyses – each for base excitation at X, Y and Z. Excitation is by acceleration PSD, measured during test (defined by car manufacturer) with values as shown in table 1.
  - S2.1: Extracted values for equivalent stress values at standard deviations $1\sigma$, $2\sigma$, $3\sigma$ – for each of the above mentioned PSD analyses.
  - S2.2: Determination of Response Power Spectral Density (RPSD) for zone with highest equivalent stress – to determine dominant frequency (natural) for the proper direction: $f_{d(X/Y/Z)}$.
- **S3: Fatigue Analysis:** root mean square (RMS) stress quantities are used in conjunction with the standard fatigue analysis procedure – Three-Band Technique using Miner’s Cumulative Damage Ratio:
  - S3.1: Determination of number of stress cycles needed to produce a fatigue failure.
  - S3.2: The actual number of fatigue cycles (n) accumulated during 32 hours of vibration testing.
  - S3.3: Miner’s cumulative fatigue damage ratios.
  - S3.4: Total reliability (used life) of the structure, based on T time for testing.

The approximate number of stress cycles $N_1$ required to produce a fatigue failure for the $1\sigma$, $2\sigma$ and $3\sigma$ stresses can be obtained from the following equation:

$$N_{1(1\sigma/2\sigma/3\sigma)} = N_2 \left( \frac{\sigma_2}{\sigma_{1(1\sigma/2\sigma/3\sigma)}} \right)^{b},$$

where: $N_{1(1\sigma/2\sigma/3\sigma)}$ – number of cycles to fail @ dynamic stresses of $1\sigma$, $2\sigma$ and $3\sigma$; $\sigma_{1(1\sigma/2\sigma/3\sigma)}$ – RPSD stress of $1\sigma$, $2\sigma$ and $3\sigma$.

Needed material data – $N_2$(material), $\sigma_2$(material) and $b$(material), are derived from material characteristics.
Table 1. PSD acceleration data.

| f (Hz) | PSD G acc. X g²/Hz | f (Hz) | PSD G acc. Y g²/Hz | f (Hz) | PSD G acc. Z g²/Hz |
|--------|---------------------|--------|---------------------|--------|---------------------|
| 5      | 6.1×10⁻³           | 5      | 2.43×10⁻²           | 5      | 3.13×10⁻²           |
| 10     | 6.1×10⁻³           | 10     | 2.43×10⁻²           | 35     | 3.13×10⁻²           |
| 11     | 1.22×10⁻²          | 11     | 1.38×10⁻²           | 36     | 9.15×10⁻²           |
| 15     | 1.22×10⁻²          | 30     | 1.38×10⁻²           | 40     | 9.15×10⁻²           |
| 16     | 2.31×10⁻²          | 31     | 1.14×10⁻²           | 41     | 5.33×10⁻²           |
| 25     | 2.31×10⁻²          | 40     | 1.14×10⁻²           | 45     | 5.33×10⁻²           |
| 26     | 4.22×10⁻²          | 41     | 1.81×10⁻²           | 46     | 2.41×10⁻³           |
| 45     | 4.22×10⁻²          | 50     | 1.81×10⁻²           | 52     | 2.41×10⁻³           |
| 46     | 6.62×10⁻²          | 51     | 9.77×10⁻³           | 53     | 4.76×10⁻²           |
| 54     | 6.62×10⁻²          | 60     | 9.77×10⁻³           | 60     | 4.76×10⁻²           |
| 55     | 1.92×10⁻²          | 61     | 1.34×10⁻²           | 61     | 1.64×10⁻²           |
| 70     | 1.92×10⁻²          | 70     | 1.34×10⁻²           | 80     | 1.64×10⁻²           |
| 71     | 8.4×10⁻³           | 71     | 8.65×10⁻³           | 81     | 6.31×10⁻³           |
| 85     | 8.4×10⁻³           | 80     | 8.65×10⁻³           | 90     | 6.31×10⁻³           |
| 86     | 3.06×10⁻³          | 81     | 2.47×10⁻³           | 91     | 4.91×10⁻³           |
| 100    | 3.06×10⁻³          | 100    | 2.47×10⁻³           | 100    | 4.91×10⁻³           |
| 101    | 6.1×10⁻⁴           | 101    | 7.44×10⁻⁴           | 101    | 2.88×10⁻³           |
| 120    | 6.1×10⁻⁴           | 170    | 7.44×10⁻⁴           | 170    | 2.88×10⁻³           |
| 121    | 9.12×10⁻⁴          | 171    | 7.56×10⁻⁴           | 171    | 1.62×10⁻³           |
| 250    | 9.12×10⁻⁴          | 250    | 7.56×10⁻⁴           | 250    | 1.62×10⁻³           |

S3.2: The actual number of fatigue cycles (n) accumulated during 24 hours of vibration testing for one direction, @ one of the examined temperatures (t_{max} = 85°C, t_{min} = 20°C and t_{min} = -40°C) can be obtained from the percentage of time exposure for the 1σ, 2σ and 3σ values:

\[ n_{(1σ/2σ/3σ)}(85°C/20°C/-40°C) = t_d(X/Y/Z) \times T_{temp(85°C/20°C/-40°C)} \times P(1σ/2σ/3σ) \]

where: \( T_{temp(85°C/20°C/-40°C)} = T \times \Delta \) and \( T = 24h = 86400 \) sec – time for testing at each of directions; \( \Delta_{85°C} = 0.603 \); \( \Delta_{20°C} = 0.087 \); \( \Delta_{-40°C} = 0.310 \) – share of test duration at proper temperature (refer to figure 3); \( P_{(1σ)} = 0.683 \); \( P_{(2σ)} = 0.273 \); \( P_{(3σ)} = 0.042 \) (refer to figure 4);

S3.3: Miner’s cumulative fatigue damage ratios (every stress cycle uses up part of the fatigue life of a structure) for each direction, @ each of examined three temperatures are calculated, based on \( T_{temp(85°C/20°C/-40°C)} \) time for testing:

\[ R_{(85°C/20°C/-40°C)} = \sum_{1σ}^{3σ} n_{(1σ/2σ/3σ)} = \frac{n_{(1σ)}}{N_{(1σ)}} + \frac{n_{(2σ)}}{N_{(2σ)}} + \frac{n_{(3σ)}}{N_{(3σ)}} \]

- reliability of the structure for \( T_{temp(85°C/20°C/-40°C)} \) testing with excitation along the proper axis (X or Y or Z) at proper temperature – 3 values in total for each direction of base excitation;

S3.4: Reliability (used life) of the structure, in the proper direction is calculated as follows:

\[ R_X = R_{X,85°C} + R_{X,20°C} + R_{X,-40°C} \]
\[ R_Y = R_{Y,85°C} + R_{Y,20°C} + R_{Y,-40°C} \]
\[ R_Z = R_{Z,85°C} + R_{Z,20°C} + R_{Z,-40°C} \]

S3.4: Total reliability (used life) of the structure, based on 3*T = 3*24h = 72h time for testing:

\[ R_L = R_X + R_Y + R_Z \]
3. Virtual prototyping demonstration by use case

The herein presented methodology is illustrated through a demonstration of a case case of an automotive component – dashboard (cluster assembly). It is a relatively complex assembly and it is shown in figure 5, together with materials specification for each of modelled part.

A meshed structure is built, based on the above mentioned virtual prototype. This numerical model contains 833 000 nodes and 433 000 elements and is shown in figure 6 to present the density of the mesh. All contacts between both modelled parts are presented as “Bonded” type.

3.1. Material properties

Six different materials are used in the examined model and their parameters are based on data defined by material suppliers and additional internet available resources. All major parameters used in subsequent analyses are shown in table 2 and in figure 7.

Table 2. Material properties for components of the numerical model (material designations according to figure 5).

| Parameter                        | ABS   | PC    | PMMA  | PP T30 | PP T40 | FR4   |
|----------------------------------|-------|-------|-------|--------|--------|-------|
| Elasticity modulus, E, MPa       | defined as function of temperature – see figure 7 |
| Poisson’s ratio, $\mu$           | 0.35  | 0.37  | 0.38  | 0.36   | 0.36   | 0.2   |
| Density, $\rho$, kg/m$^3$         | 1050  | 1200  | 1190  | 1140   | 1250   | 2800  |
| Coef. of therm. exp., $\alpha$, $10^6$m/m$^o$K | 95    | 65    | 80    | 68     | 55     | 11    |

1 - Used data source – common value for plastics
Figure 5. Virtual prototype of a cluster assembly, used for demonstration.

Figure 6. Meshed model of examined cluster assembly.

Figure 7. Elasticity module values versus temperature – for all used materials.

3.2. Boundary conditions
The applied boundary conditions – as base excitations – application surfaces (supports) and PSD used – are shown in figure 8. They represent mounting conditions of the examined cluster assembly. Three different temperatures: -40ºC, 20ºC and 85ºC – are used for static analyses and subsequent modal analyses. The steps for each analysis include initial steady state analysis, analysis under thermal loads, and subsequent prestressed modal analysis that uses static analysis results. Next, random vibration analysis is performed.

Figure 8. Applied boundary conditions.
3.3. S0 and S1: Modal analyses results
The first three natural frequencies for each load case (-40°C, 20°C and 85°C) are compared graphically in figure 9.

![Figure 9. S1: Modal analysis results.](image)

3.4. S2: Random Vibration (PSD) analyses – S2.1: Extracted values for equivalent stress values at standard deviations 1σ, 2σ, 3σ
The results from step S2, at three different temperatures (3 separate analyses), and three directions of applied base excitation – are shown by samples of equivalent (von Mises) distribution fields for 1σ standard deviation in figures 10a, 10b and 10c below.

![Figure 10. Equivalent (von Mises) distribution fields at different excitations and temperatures, MPa.](image)

a) excitation along X axis, @85°C  
b) excitation along Y axis, @20°C  
c) excitation along Z axis, @-40°C
All needed results, for each temperature range and each excitation direction, are summarized in table 3 below by percentage of occurrence (1s, 2s, 3s) extracted from the performed analyses – for each component.

**Table 3.** Random vibration analyses results – extracted values for equivalent stress values at standard deviations 1σ, 2 σ, 3 σ.

| Max eq stress, MPa | Components | Back cover | Light box | LCD support | LCD cover | Bezel | Lens | Dial | PCB |
|-------------------|------------|------------|-----------|-------------|-----------|-------|------|------|-----|
| 1σ (68.3%)        | 85°C       | 0.071      | 0.330     | 0.180       | 0.093     | 0.180 | 0.360 | 0.120 | 0.350|
|                   | 20°C       | 0.050      | 0.500     | 1.340       | 0.770     | 0.350 | 0.920 | 0.160 | 0.290|
|                   | -40°C      | 0.002      | 0.280     | 0.290       | 0.130     | 0.060 | 0.008 | 0.420 | 0.040|
|                   | 85°C       | 0.142      | 0.660     | 0.360       | 0.186     | 0.360 | 0.720 | 0.240 | 0.700|
| 2σ (27.3%)        | 20°C       | 0.100      | 1.000     | 2.680       | 1.540     | 0.700 | 1.840 | 0.320 | 0.580|
|                   | -40°C      | 0.004      | 0.560     | 0.580       | 0.260     | 0.120 | 0.016 | 0.840 | 0.080|
|                   | 85°C       | 0.213      | 0.990     | 0.540       | 0.279     | 0.540 | 0.108 | 0.360 | 1.050|
| 3σ (4.2%)         | 20°C       | 0.150      | 1.500     | 4.020       | 2.310     | 1.050 | 2.760 | 0.480 | 0.870|
|                   | -40°C      | 0.006      | 0.840     | 0.870       | 0.390     | 0.180 | 0.024 | 1.260 | 0.120|
| 1σ (68.3%)        | 85°C       | 0.071      | 0.330     | 0.180       | 0.093     | 0.180 | 0.360 | 0.120 | 0.350|
|                   | 20°C       | 0.050      | 0.500     | 1.340       | 0.770     | 0.350 | 0.920 | 0.160 | 0.290|
|                   | -40°C      | 0.002      | 0.280     | 0.290       | 0.130     | 0.060 | 0.008 | 0.420 | 0.040|
|                   | 85°C       | 0.142      | 0.660     | 0.360       | 0.186     | 0.360 | 0.720 | 0.240 | 0.700|
| 2σ (27.3%)        | 20°C       | 0.100      | 1.000     | 2.680       | 1.540     | 0.700 | 1.840 | 0.320 | 0.580|
|                   | -40°C      | 0.004      | 0.560     | 0.580       | 0.260     | 0.120 | 0.016 | 0.840 | 0.080|
|                   | 85°C       | 0.213      | 0.990     | 0.540       | 0.279     | 0.540 | 0.108 | 0.360 | 1.050|
| 3σ (4.2%)         | 20°C       | 0.150      | 1.500     | 4.020       | 2.310     | 1.050 | 2.760 | 0.480 | 0.870|
|                   | -40°C      | 0.006      | 0.840     | 0.870       | 0.390     | 0.180 | 0.024 | 1.260 | 0.120|
| 1σ (68.3%)        | 85°C       | 0.071      | 0.330     | 0.180       | 0.093     | 0.180 | 0.360 | 0.120 | 0.350|
|                   | 20°C       | 0.050      | 0.500     | 1.340       | 0.770     | 0.350 | 0.920 | 0.160 | 0.290|
|                   | -40°C      | 0.002      | 0.280     | 0.290       | 0.130     | 0.060 | 0.008 | 0.420 | 0.040|
|                   | 85°C       | 0.142      | 0.660     | 0.360       | 0.186     | 0.360 | 0.720 | 0.240 | 0.700|
| 2σ (27.3%)        | 20°C       | 0.100      | 1.000     | 2.680       | 1.540     | 0.700 | 1.840 | 0.320 | 0.580|
|                   | -40°C      | 0.004      | 0.560     | 0.580       | 0.260     | 0.120 | 0.016 | 0.840 | 0.080|
|                   | 85°C       | 0.213      | 0.990     | 0.540       | 0.279     | 0.540 | 0.108 | 0.360 | 1.050|
| 3σ (4.2%)         | 20°C       | 0.150      | 1.500     | 4.020       | 2.310     | 1.050 | 2.760 | 0.480 | 0.870|
|                   | -40°C      | 0.006      | 0.840     | 0.870       | 0.390     | 0.180 | 0.024 | 1.260 | 0.120|

3.5. Random Vibration (PSD) analyses – S2.2: Determination of Response Power Spectral Density (RPSD) for zone with highest equivalent stress

The results from the analyses are used also to determine the dominant frequencies by direction and are listed below:

- along X axis: $f_{x(85^\circ)} = 88\text{Hz}$, $f_{x(20^\circ)} = 108\text{Hz}$ and $f_{x(40^\circ)} = 115\text{Hz}$.
- along Y axis: $f_{y(85^\circ)} = 134\text{Hz}$, $f_{y(20^\circ)} = 170\text{Hz}$ and $f_{y(40^\circ)} = 119\text{Hz}$.
- along Z axis: $f_{z(85^\circ)} = 130\text{Hz}$, $f_{z(20^\circ)} = 174\text{Hz}$ and $f_{z(40^\circ)} = 120\text{Hz}$.

3.6. S3: Fatigue Analysis

Next, fatigue calculations are performed, using the formula shown in the previous chapter. The results are shown in table 4 below – for all examined directions of support excitation.

The maximal time used for testing of each component is less than 0.01% of its resource. There is no component with reliability parameter for life prediction less than $1 \times 106\text{h}$. Nevertheless, the component results are compared in figure 11 below by reciprocal value of their used time ($R_e$) during tests. Thus, it is seen that most loaded components (with less R that corresponds to less remaining life) are LCD support, LCD cover and the dial.

Nevertheless, there are no critical components and total life prediction for the examined component – cluster assembly – fully corresponds to the car manufacturer requirements. This result could be used for design approval and reason for proceeding further with the required physical prototyping.
### Table 4. Fatigue analysis calculations.

| Parameters / Components | Back cover | Light box | LCD support | LCD cover | Bezel | Lens | Dial | PCB |
|-------------------------|------------|-----------|-------------|-----------|-------|------|------|-----|
| N_{fatigue} (68.3%)     | 85°C       | 20°C      | 40°C        | 85°C      | 20°C  | 40°C | 85°C | 20°C |
|                         | 3.1E+06    | 4.0E+06   | 4.0E+06     | 4.0E+06   | 4.0E+06 | 4.0E+06 | 4.0E+06 | 4.0E+06 |
|                         | 1.0E+06    | 1.0E+06   | 1.0E+06     | 1.0E+06   | 1.0E+06 | 1.0E+06 | 1.0E+06 | 1.0E+06 |
|                         | 1.5E+06    | 1.3E+06   | 1.5E+06     | 1.5E+06   | 1.3E+06 | 1.5E+06 | 1.5E+06 | 1.3E+06 |
|                         | 5.0E+06    | 5.0E+06   | 5.0E+06     | 5.0E+06   | 5.0E+06 | 5.0E+06 | 5.0E+06 | 5.0E+06 |
|                         | 3.0E+06    | 3.0E+06   | 3.0E+06     | 3.0E+06   | 3.0E+06 | 3.0E+06 | 3.0E+06 | 3.0E+06 |
|                         | 1.0E+06    | 1.0E+06   | 1.0E+06     | 1.0E+06   | 1.0E+06 | 1.0E+06 | 1.0E+06 | 1.0E+06 |
|                         | 1.5E+06    | 1.5E+06   | 1.5E+06     | 1.5E+06   | 1.5E+06 | 1.5E+06 | 1.5E+06 | 1.5E+06 |
|                         | 5.0E+06    | 5.0E+06   | 5.0E+06     | 5.0E+06   | 5.0E+06 | 5.0E+06 | 5.0E+06 | 5.0E+06 |

**Note:** The table continues with similar entries for other parameters and components.
4. Conclusions

The presented methodology facilitates random vibration endurance testing using virtual prototyping and moreover – it uses conventional software tools for numerical modeling. The described methodology requires software for modal and random vibration analyses that is available in the most widely used FEA programs. It is also based on the widely used in automotive industry hypothesis of Palmgren-Miner.

This is demonstrated by a case study from the automotive industry – a cluster assembly (dashboard). Modelled assembly components are assessed as rigidity and general dynamic behavior prior to performing physical prototyping that saves time and expenses. Another important drawback is the option to investigate in detail the stress distributions inside the examined parts and – if it is needed – easily change design and test new modifications.

Generally, the study presents a methodology for a random vibration endurance test of automotive components using virtual prototyping based on common engineering analysis tools.

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