Model for neutrino mixing based on SO(10)

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Abstract

Assuming grand unified theory (GUT) and supersymmetry, we propose a simple model which can consistently accommodate the masses and mixings for quarks and leptons. The grand unified group is SO(10), and $\mathbf{10}$, $\mathbf{120}$, and $\mathbf{126}$ representations are introduced for the Higgs superfields which give masses to the quarks and leptons. The differences of masses and mixings between the quarks and the leptons are attributed to the Higgs boson structure. Below the GUT energy scale, the model is the same as the minimal supersymmetric standard model except its inclusion of dimension-5 operators for the small neutrino masses. The renormalization group equations of the independent parameters for the Higgs couplings with the quarks and leptons are given explicitly to connect the two energy scales of GUT and electroweak theory.

Key words: neutrinos, SO(10), Higgs
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1 Introduction

Implications of non-vanishing neutrino masses are accumulating from the experiments for solar [1] and atmospheric [2] neutrinos which show neutrino oscillations. This is the first experimental evidence that suggests physics beyond the standard model (SM). The possibility of oscillation has also been examined by the neutrinos from nuclear reactors. The negative result of CHOOZ [3] gives constraints on the masses and mixing allowed for the neutrinos. The result of KamLAND [4] confirms the oscillations of the solar neutrinos. It is a task for the extension of the SM to accommodate these experimental results. In particular, the extreme smallness of the masses and the largeness of the generation mixing angles, both of which are observed in the experiments, should be accounted for naturally.
From a theoretical point of view, it is reasonable for the SM to be extended under grand unified theory (GUT). Furthermore, supersymmetry may have to be introduced in order that the SM be consistently embedded in GUT models. On the other hand, some of the GUT models, such as those based on SO(10) [5], automatically contain right-handed neutrinos. If they have large Majorana masses, the lightness of the neutrinos could be naturally understood. The plausibility of supersymmetric GUT models as the extension of the SM are then further strengthened.

One tough obstacle confronts GUT models, concerning the generation mixing of the leptons. The lepton mixing is described by the Maki-Nakagawa-Sakata (MNS) matrix as the Cabibbo-Kobayashi-Maskawa (CKM) matrix does the quark mixing. Contrary to the CKM matrix, the observed neutrino oscillations show that the off-diagonal elements of the MNS matrix are not suppressed. However, since the leptons and the quarks are contained in the same representations of the grand unified group, similar magnitudes are deduced, by simple consideration, for corresponding elements of the two matrices. A successful GUT model is required to naturally implement the difference of mixing between the quarks and the leptons. Although there are various solutions advocated for the problem [6], rather contrived schemes have been invoked.

Aiming at a natural explanation for the generation mixings of the leptons and the quarks, we present a model base on SO(10) and supersymmetry in which the difference is simply attributed to the Higgs boson structure [7]. This model includes $\mathbf{10}$, $\mathbf{120}$, and $\mathbf{126}$ representations of the SO(10) group for the Higgs bosons which give masses to the quarks and leptons. The $\mathbf{120}$ representation becomes the origin of the different generation mixings. Large Majorana masses for the right-handed neutrinos are generated by a vacuum expectation value (VEV) of the $\mathbf{126}$ representation. Below the GUT energy scale, this model is described as the minimal supersymmetric standard model (MSSM) with dimension-5 operators composed of the SU(2) doublet superfields for the left-handed leptons and the Higgs bosons. The coefficients of the Higgs couplings with the quarks and leptons evolve between the energy scales of GUT and electroweak theory. We obtain the renormalization group equations for the eigenvalues of the coefficient matrices and the independent arguments of the MNS and CKM matrices. Quantitative analyses are then made on the neutrino oscillations. It is shown that the lepton mixing observed is compatible with the quark mixing in certain ranges of the model parameters.

In SO(10) GUT models the Higgs bosons for the quark and lepton masses must be contained in $\mathbf{10}$, $\mathbf{120}$, or $\mathbf{126}$ representations. The direct product for one of these three representations with two spinor $\mathbf{16}$ representations for the quarks and leptons becomes an SO(10) singlet. The model with two Higgs fields of $\mathbf{10}$ and $\mathbf{126}$ has been studied extensively [8]. However, all the experimental results for the neutrino oscillations are only marginally accommodated. The
10 and 126 representations make the Higgs couplings symmetric on the generation indices, and their SU(2)-doublet components couple to both the quarks and the leptons. On the other hand, the 120 representation leads to antisymmetric Higgs couplings, and the SU(2)-doublet components which couple to the quarks may be different from those to the leptons. By including 120, the correlation of the masses and mixings between the quarks and the leptons becomes more flexible.

In sect. 2 we summarize the interpretation of neutrino oscillations in the framework of the model with three generations for leptons. In sect. 3 the renormalization group equations for the MSSM with the dimension-5 operators are obtained explicitly in terms of independent parameters for the coefficients of Higgs couplings with quarks and leptons. In sect. 4 the GUT based on SO(10) and supersymmetry is discussed. After deriving in general the relations of Higgs couplings for the quarks and the leptons, a plausible model is specified. In sect. 5 we make numerical analyses of the masses and mixing of the neutrinos. Conclusions are given in sect. 6.

2 Neutrino Oscillations

The deficit of neutrinos from the sun and the anomaly for neutrinos from the atmosphere can be explained by neutrino oscillations. The atmospheric neutrino observation gives that the mass-squared difference and the mixing angle between the $\mu$-neutrino and a certain neutrino are given by

$$\Delta m_{\text{atm}}^2 = (1 - 8) \times 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{\text{atm}} > 0.85. \quad (1)$$

There are several oscillation scenarios for the solar neutrino problem. However, a most likely solution is given by a large mixing angle between the $e$-neutrino and a certain neutrino under the Mikheyev-Smirnov-Wolfenstein effect, suggesting the ranges

$$\Delta m_{\text{sol}}^2 = (2 - 10) \times 10^{-5} \text{eV}^2, \quad \sin^2 2\theta_{\text{sol}} = 0.5 - 0.9. \quad (2)$$

The measurements by KamLAND are consistent with these results. On the other hand, by the experiment of CHOOZ the oscillation has not been observed for the $e$-neutrino, which excludes the combined region

$$\Delta m_{\text{chooz}}^2 > 1 \times 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{\text{chooz}} > 0.2. \quad (3)$$

These experimental measurements are taken into consideration in constructing our model.
We interpret the above results in the model with three generations of massive neutrinos. In this framework the strength of the $W$-boson interaction for a neutrino and a charged lepton depends on the generations. The mass eigenstate $\nu_i$ for the neutrino of the $i$-th generation is related to the eigenstate $\tilde{\nu}_j$ of interaction with the charged lepton of the $j$-th generation by

$$\nu_i = (V_{MNS})_{ij} \tilde{\nu}_j,$$  \hspace{1cm} (4)

where $V_{MNS}$ denotes the MNS matrix. The survival probability for the interaction eigenstate $\tilde{\nu}_i$ after run of distance $L$ with energy $E$ is given by

$$P(\tilde{\nu}_i \rightarrow \tilde{\nu}_i) = \left| \sum_{k=1}^{3} |(V_{MNS})_{ki}|^2 \exp(-i \frac{m_{\nu k}^2}{2E} L) \right|^2,$$  \hspace{1cm} (5)

where $m_{\nu k}$ represents the mass eigenvalue for the neutrino of the $k$-th generation. The mass-squared difference is hereafter written as

$$\Delta m_{ij}^2 = m_{\nu i}^2 - m_{\nu j}^2.$$  \hspace{1cm} (6)

We assume that the neutrino masses are hierarchical, $m_{\nu 1} < m_{\nu 2} < m_{\nu 3}$, and not degenerated, similarly to the quarks and charged leptons.

In the CHOOZ experiment, the phases in Eq. (5) are found to satisfy the relations $(\Delta m_{21}^2/2E)L \ll (\Delta m_{31}^2/2E)L \sim 1$. The survival probability of the $e$-neutrino is expressed by

$$P(\tilde{\nu}_e \rightarrow \tilde{\nu}_e) = 1 - 4 |(V_{MNS})_{31}|^2 (1 - |(V_{MNS})_{31}|^2) \sin^2 \frac{\Delta m_{31}^2}{4E} L.$$  \hspace{1cm} (7)

Thus, the measured quantities are translated as

$$\Delta m_{\text{chooz}}^2 = \Delta m_{31}^2, \quad \sin^2 2\theta_{\text{chooz}} = 4 |(V_{MNS})_{31}|^2 (1 - |(V_{MNS})_{31}|^2).$$  \hspace{1cm} (8)

For the atmospheric neutrino oscillation, the phases in Eq. (5) satisfy the relations $(\Delta m_{21}^2/2E)L \ll (\Delta m_{32}^2/2E)L$ and $(\Delta m_{21}^2/2E)L \ll 1$. The survival probability of the $\mu$-neutrino is given by

$$P(\tilde{\nu}_\mu \rightarrow \tilde{\nu}_\mu) = 1 - 4 |(V_{MNS})_{32}|^2 (1 - |(V_{MNS})_{32}|^2) \sin^2 \frac{\Delta m_{32}^2}{4E} L.$$  \hspace{1cm} (9)

The experimental results are expressed by

$$\Delta m_{\text{atm}}^2 = \Delta m_{32}^2, \quad \sin^2 2\theta_{\text{atm}} = 4 |(V_{MNS})_{32}|^2 (1 - |(V_{MNS})_{32}|^2).$$  \hspace{1cm} (10)
For the solar neutrino oscillation, the relations $|V_{MNS}^{31}|^2 \ll |V_{MNS}^{21}|^2$ and $|V_{MNS}^{31}|^2 \ll |V_{MNS}^{11}|^2$ could hold from Eqs. (1), (3), (8), and (10). These constraints make it possible to evaluate the survival probability of the $e$-neutrino by

$$P(\tilde{\nu}_e \rightarrow \tilde{\nu}_e) = 1 - 4|V_{MNS}^{21}|^2 (1 - |V_{MNS}^{21}|^2) \sin^2 \frac{\Delta m^2_{21}}{4E}. \quad (11)$$

Therefore, we obtain the equations

$$\Delta m^2_{sol} = \Delta m^2_{21}, \quad \sin^2 2\theta_{sol} = 4|V_{MNS}^{21}|^2 (1 - |V_{MNS}^{21}|^2). \quad (12)$$

These evaluations are used to discuss the predictions of the model.

3 Energy evolution

We assume that physics below the GUT energy scale is described by the MSSM with the dimension-5 operators composed of the superfields for left-handed leptons and Higgs bosons. This assumption is fulfilled in the model which is presented afterward. The superpotential relevant to the quark and lepton masses is given by

$$W = W_1 + W_2,$$
$$W_1 = \eta^{ij}_{d} H_1 \times Q^i D^c j + \eta^{ij}_{u} H_2 \times Q^i U^c j + \eta^{ij}_e H_1 \times L^j E^c j + \text{H.c.},$$
$$W_2 = \frac{1}{2} \kappa^{ij} H_2 \times L^i H_2 \times E^j + \text{H.c.}, \quad (13)$$

where $H_1$ and $H_2$ stand for the superfields for the Higgs bosons with hypercharges $-1/2$ and $1/2$, respectively. Superfields are denoted by $Q^i$, $U^c i$, and $D^c i$ for the quarks and $L^i$ and $E^c i$ for the leptons, in a self-explanatory notation, with $i$ being the generation index. The SU(3) group indices are understood. The dimension-5 superpotential $W_2$ yields the operators

$$L = -\frac{1}{2} \kappa^{ij} \left( \phi^+_{H_2} \right) \left( \phi^0_{H_2} \right) \left( \nu^c_\tilde{L} \right) \left( \phi^0_{H_2} \nu^c_\tilde{L} \right) + \text{H.c.}, \quad (15)$$

where $\phi^+_{H_2}$ and $\phi^0_{H_2}$ represent the scalar components of the superfield $H_2$. At the electroweak energy scale where the SU(2)-doublet Higgs bosons have non-vanishing VEVs, these dimension-5 operators become tiny Majorana mass terms for the left-handed neutrinos. The other operators arising from $W_2$ cause negligible effects.
The coefficient matrices $\eta_d$, $\eta_u$, $\eta_e$, and $\kappa$ are diagonalized by unitary matrices $U_L$'s and $U_R$'s. The CKM matrix for the quarks and the MNS matrix for the leptons are defined by

\begin{align}
V_{\text{CKM}} &= U_L^{d\dagger}U_L^d, \quad U_L^{d\dagger}U_L^{d*} = \eta_d^D, \quad U_L^{d\dagger}U_R^{d*} = \eta_d^D, \quad (16) \\
V_{\text{MNS}} &= U_L^{e\dagger}U_L^e, \quad U_L^{e\dagger}U_R^{e*} = \eta_e^D, \quad U_R^{e\dagger}U_R^{e*} = \eta_e^D, \quad (17)
\end{align}

where $\eta_d^D$, $\eta_u^D$, $\eta_e^D$, and $\kappa^D$ stand for diagonalized matrices. Taking the VEVs of electroweak symmetry breaking for positive, the diagonal elements of $\eta_d^D$, $\eta_u^D$, and $\kappa^D$ should be positive, while those of $\eta_e^D$ should be negative. The quarks and leptons then have positive masses. Those diagonal elements are expressed by $\eta_{di}$, $\eta_{ui}$, $\eta_{ei}$, and $\kappa_i$.

A 3×3 unitary matrix has nine independent parameters. For the explicit expression of $V_{\text{CKM}}$ or $V_{\text{MNS}}$, we adopt the following parametrization [9]:

\begin{align}
V &= P_+ V_0 P_-, \\
V_0 &= \begin{pmatrix}
s_1 s_2 c_3 + e^{i\delta} c_1 c_2 & c_1 s_2 c_3 - e^{i\delta} s_1 c_2 & s_2 s_3 \\
-s_1 c_2 c_3 - e^{i\delta} c_1 s_2 & c_1 c_2 c_3 + e^{i\delta} s_1 s_2 & c_2 s_3 \\
-s_1 s_3 & -c_1 s_3 & c_3
\end{pmatrix}
\end{align}

where $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$ ($i = 1, 2, 3$). Without loss of generality, the angles $\theta_1$, $\theta_2$, and $\theta_3$ can be taken to lie in the first quadrant. At the electroweak energy scale, the numbers of the physical parameters for the CKM matrix and the MNS matrix are four and six, respectively. However, we need the general form of parametrization in Eq. (18) to discuss the energy dependencies of the CKM and MNS matrices.

The values of $\eta_d^D$, $\eta_u^D$, $\eta_e^D$, $\kappa^D$, $V_{\text{CKM}}$, and $V_{\text{MNS}}$ evolve depending on the energy scale. The renormalization group equations for these parameters and the gauge coupling constants of SU(3)×SU(2)×U(1) close on themselves at the one-loop level. We give the equations of the independent parameters, where the inequalities $\eta_{d\alpha}^2 \ll \eta_{d\alpha}^2$, $\eta_{u\alpha}^2 \ll \eta_{u\alpha}^2$, and $\eta_{e\alpha}^2 \ll \eta_{e\alpha}^2$ ($\alpha = 1, 2$) are taken into account.

The gauge coupling constants:

\begin{align}
\mu \frac{d g_3^2}{d \mu} &= -3 \frac{g_3^4}{8\pi^2}, \quad \mu \frac{d g_2^2}{d \mu} = \frac{g_2^4}{8\pi^2}, \quad \mu \frac{d g_1^2}{d \mu} = \frac{33 g_1^4}{5 8\pi^2}. \quad (19)
\end{align}

The diagonalized Higgs coupling coefficients:
The CKM matrix:

\[ \mu \frac{d \eta_{d i}}{d \mu} = -\frac{\eta_{d i}^2}{8\pi^2} \left[ \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{7}{15} g_1^2 - 3 \eta_{d i}^2 - \eta_{e i}^2 \right] - |(V_{CKM})_{3i}|^2 \eta_{u i}^2, \]

\[ \mu \frac{d \eta_{u i}}{d \mu} = -\frac{\eta_{u i}^2}{8\pi^2} \left[ \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{13}{15} g_1^2 - 3 \eta_{u i}^2 - \eta_{u i}^2 \right] - |(V_{CKM})_{3i}|^2 \eta_{d i}^2, \]

\[ \mu \frac{d \eta_{e i}}{d \mu} = -\frac{\eta_{e i}^2}{8\pi^2} \left[ 3 g_2^2 + \frac{9}{5} g_1^2 - \eta_{e i}^2 - 3 \eta_{e i}^2 - 3 \eta_{d i}^2 \right], \]

\[ \mu \frac{d \kappa_{i}}{d \mu} = -\frac{\kappa_{i}}{8\pi^2} \left[ 3 g_2^2 + \frac{3}{5} g_1^2 - 3 \eta_{u i}^2 - |(V_{MNS})_{3i}|^2 \eta_{e i}^2 \right]. \]

The CKM matrix:

\[ \mu \frac{d \theta_{q1}}{d \mu} = -\frac{\eta_{u i}^2}{16\pi^2} s_1 c_1 s_3^2, \]

\[ \mu \frac{d \theta_{q2}}{d \mu} = -\frac{\eta_{d i}^2}{16\pi^2} s_2 c_2 s_3^2, \]

\[ \mu \frac{d \theta_{q3}}{d \mu} = -\frac{\eta_{u i}^2 + \eta_{d i}^2}{16\pi^2} s_3 c_3, \]

\[ \mu \frac{d \delta_1}{d \mu} = 0, \]

\[ \mu \frac{d \rho_{qa}}{d \mu} = 0 \quad (a = 1 - 5), \]

where \( \theta_{q_i} \) and \( \delta_q \) stand for the arguments in \( V_0 \) for the CKM matrix, with \( c_i = \cos \theta_{q_i} \) and \( s_i = \sin \theta_{q_i} \), and \( \rho_{qa} \) represents an additional phase parameter in \( P_+ \) and \( P_- \).

The MNS matrix:

\[ \mu \frac{d \theta_{l1}}{d \mu} = -\frac{\eta_{e i}^2}{16\pi^2} s_2 c_2 c_3 \left[ (B_R + C_R)c_6 - (B_I - C_I)s_6 \right], \]

\[ \mu \frac{d \theta_{l2}}{d \mu} = -\frac{\eta_{e i}^2}{16\pi^2} s_2 c_2 \left[ -A_R s_3^2 + (B_R + C_R)c_3^2 \right], \]

\[ \mu \frac{d \theta_{l3}}{d \mu} = -\frac{\eta_{e i}^2}{16\pi^2} s_3 c_3 \left[ -B_R c_2^2 + C_R s_2^2 \right], \]

\[ \mu \frac{d \delta_i}{d \mu} = -\frac{\eta_{e i}^2}{16\pi^2} \left[ -A_I s_3^2 - (B_R + C_R)s_6 \frac{c_1^2 - s_1^2}{s_1 c_1} s_2 c_2 c_3 \right. \]

\[ \left. - B_I \left( c_6 \frac{c_1^2 - s_1^2}{s_1 c_1} s_2 c_2 c_3 + s_2 c_3^2 - c_2^2 \right) \right]. \]
contains all the superfields for quarks and leptons of one generation, for both

\[ + C_I \left( c_3 \frac{c_1^2 - s_1^2}{s_1 c_1} s_2 c_2 c_3 - c_2^2 c_3^2 + s_2^2 \right), \]  

(32)

\[ \mu \frac{d\rho_{11}}{d\mu} = -\frac{\eta_{c_3}}{16\pi^2} \left[ (A_I - B_I) c_2 s_2^2 + C_I (c_3^2 - s_2^2 s_3^2) \right], \]  

(33)

\[ \mu \frac{d\rho_{13}}{d\mu} = -\frac{\eta_{c_3}}{16\pi^2} \left[ (A_I - C_I) s_2 s_3^3 + B_I (c_3^2 - s_2^2 s_3^2) \right], \]  

(34)

\[ \mu \frac{d\rho_{31}}{d\mu} = \frac{\eta_{c_3}}{16\pi^2} c_3 \left[ (B_R + C_R) s_6 c_1 \frac{c_1}{s_1} s_2 c_2 \right. \]

\[ + B_I \left( c_5 \frac{c_1}{s_1} s_2 c_2 - c_2^2 c_3 \right) - C_I \left( c_5 \frac{c_1}{s_1} s_2 c_2 + s_2^2 c_3 \right), \]  

(35)

\[ \mu \frac{d\rho_{15}}{d\mu} = \frac{\eta_{c_3}}{16\pi^2} c_3^2 \left[ B_I c_2^2 + C_I s_2^2 \right], \]  

(36)

\[ \mu \frac{d\rho_{15}}{d\mu} = \frac{\eta_{c_3}}{16\pi^2} c_3^2 \left[ B_I c_2^2 + C_I s_2^2 \right], \]  

(37)

\[ A_R = \frac{2\kappa_1 \kappa_2}{\kappa_1^2 - \kappa_2^2} \cos 2(\rho_1 - \rho_2) + \frac{\kappa_1^2 + \kappa_2^2}{\kappa_1^2 - \kappa_2^2}, \]

\[ A_I = \frac{2\kappa_1 \kappa_2}{\kappa_1^2 - \kappa_2^2} \sin 2(\rho_1 - \rho_2), \]

\[ B_R = \frac{2\kappa_2 \kappa_3}{\kappa_2^2 - \kappa_3^2} \cos 2\rho_2 + \frac{\kappa_2^2 + \kappa_3^2}{\kappa_2^2 - \kappa_3^2}, \]

\[ B_I = \frac{2\kappa_2 \kappa_3}{\kappa_2^2 - \kappa_3^2} \sin 2\rho_2, \]

\[ C_R = \frac{2\kappa_3 \kappa_1}{\kappa_3^2 - \kappa_1^2} \cos 2\rho_1 + \frac{\kappa_3^2 + \kappa_1^2}{\kappa_3^2 - \kappa_1^2}, \]

\[ C_I = -\frac{2\kappa_3 \kappa_1}{\kappa_3^2 - \kappa_1^2} \sin 2\rho_1, \]

where \( \theta_{li} \) and \( \delta_l \) stand for the arguments in \( V_0 \) for the MNS matrix, with \( c_i = \cos \theta_{li} \), \( s_i = \sin \theta_{li} \), \( c_\delta = \cos \delta_l \), and \( s_\delta = \sin \delta_l \); and \( \rho_{+a} \) represents an additional phase parameter in \( P_+ \) and \( P_- \). The mutual dependencies of the independent parameters in energy evolution are manifestly seen, thanks to the explicit expressions in terms of the parameters themselves. For instance, the MNS matrix receives large quantum corrections if some of the neutrino mass coefficients \( \kappa_i \) are roughly degenerated [10].

4 Model

The grand unified group of our model is SO(10). Its spinor 16 representation contains all the superfields for quarks and leptons of one generation, for both
left-handed and right-handed components. The right-handed neutrinos are therefore naturally incorporated. The decomposition of the direct product for two 16's is given by $16 \times 16 = 10 + 120 + 126$. The Higgs superfields which give masses to the quarks and leptons must be assigned to 10, 120, or 126 representations. We introduce one superfield for each representation.

The superpotential relevant to the quark and lepton masses are given, in the framework of $SU(3) \times SU(2) \times U(1)$, by

$$W = \eta^{ij} \left[ H_{10}^5 \times \left( Q^i D^{cj} + L^i E^{cj} \right) + H_{10}^5 \times \left( Q^i U^{cj} + L^i N^{cj} \right) \right]$$

$$+ \epsilon^{ij} \left[ H_{120}^5 \times \left( Q^i D^{cj} + L^i E^{cj} \right) + \frac{1}{\sqrt{3}} H_{120}^{45} \times \left( Q^i D^{cj} - 3L^i E^{cj} \right) \right]$$

$$+ 2H_{120}^5 \times L^i N^{cj} + \frac{2}{\sqrt{3}} H_{120}^{45} \times Q^j U^{cj} + \zeta^{ij} \left[ H_{120}^{45} \times \left( Q^i D^{cj} - 3L^i E^{cj} \right) + H_{120}^5 \times \left( Q^i U^{cj} - 3L^i N^{cj} \right) \right]$$

$$+ \sqrt{6} L^{iT} H_{126}^{15} L^i + \sqrt{6} H_{126}^4 N^i N^{cj} \right] + H.c.,$$

where superfields $N^{ci}$ for the right-handed neutrinos appear in addition to the superfields $Q^i$, $U^{ci}$, $D^{ci}$, $L^i$, and $E^{ci}$ for the quarks and leptons. Higgs superfields are denoted by $H$'s with upper and lower indices showing transformation properties under $SU(5)$ and $SO(10)$, respectively: $H_{10}^5$, $H_{120}^5$, $H_{120}^{45}$, and $H_{126}^{45}$ are $SU(2)$ doublets with hypercharge $-1/2$; $H_{10}^5$, $H_{120}^5$, $H_{120}^{45}$, and $H_{126}^{45}$ are $SU(2)$ doublets with hypercharge $1/2$; $H_{120}^{45}$ is an $SU(2)$ triplet; and $H_{126}^5$ is an $SU(2) \times U(1)$ singlet. Each superfield has been normalized. Owing to the antisymmetric property of 120, the coupling $H_{120}^5 \times Q^j U^{cj}$ does not appear. The coupling constants $\eta^{ij}$ and $\zeta^{ij}$ are symmetric for the generation indices, while $\epsilon^{ij}$ are antisymmetric.

We can see from Eq. (38) the characteristic of the 120 representation. Any $SU(2)$ doublet in 10 or 126 couples both the quark and the lepton superfields. As a result, in Eqs. (13) and (14), the Higgs coupling coefficients for the leptons become related to those for the quarks. On the other hand, for 120, four $SU(2)$ doublets ($\sqrt{3}H_{120}^5 + H_{120}^{45})/2$, $(H_{120}^5 - \sqrt{3}H_{120}^{45})/2$, $H_{120}^5$, and $H_{120}^{45}$ couple to $Q^i D^{cj}$, $L^i E^{cj}$, $L^i N^{cj}$, and $Q^j U^{cj}$, respectively. The Higgs coupling coefficients could be less constrained.

The $SU(2)$-doublet Higgs superfields for electroweak symmetry breaking are composed of the superfields in 10, 120, 126, and some other representations. Among the possible linear combinations of $SU(2)$ doublets with the same hypercharge, only one doublet should be kept light to satisfy the unification of the gauge coupling constants for $SU(3) \times SU(2) \times U(1)$. The two doublets with hypercharges $-1/2$ and $1/2$ assume the role of the Higgs superfields $H_1$ and $H_2$ in Eqs. (13) and (14). The other linear combinations must have large
masses and decouple from theory below the GUT energy scale. We express the Higgs superfields by

\[ H_1 = (C_1^d)_{11} H_{10}^5 + (C_1^d)_{12} H_{120}^5 + (C_1^d)_{13} H_{126}^{45} + \ldots, \]

\[ H_2 = (C_2^d)_{11} H_{10}^5 + (C_2^d)_{12} H_{120}^5 + (C_2^d)_{13} H_{126}^{45} + \ldots, \]

where \( C_1 \) and \( C_2 \) represent unitary matrices. Some components of \( H_1 \) and \( H_2 \) belong to the representations different from 10, 120, and 126, which are denoted by the ellipses. For instance, one superfield of 126 is included in the model. Its SU(5)-singlet scalar component has a large VEV to cancel the VEV of \( H_1^{126} \), and the VEVs of the auxiliary D fields for SO(10) are kept small. This 126 representation contains SU(2) doublets, which may become the components of \( H_1 \) and \( H_2 \).

The matrices \( C_1 \) and \( C_2 \) should be determined by the Higgs potential at the GUT energy scale. However, we take them as independent parameters in this paper, assuming that an appropriate Higgs potential could be constructed. Since the Higgs potential contains also the fields which are to break SO(10) correctly down to SU(3) × SU(2) × U(1), it is very complicated to analyze the whole potential. The Higgs potential is also supposed to induce the well-known split between the light SU(2) doublets and the heavy SU(3) triplets, as well as the split between the light SU(2) doublets, \( H_1 \) and \( H_2 \), and the other heavy SU(2) doublets.

The superpotential in Eq. (13) is now determined. The coefficient matrices are given by

\[ \eta_d = \eta (C_1^d)_{11} + \epsilon [(C_1^d)_{21} + \frac{1}{\sqrt{3}} (C_1^d)_{31}] + \zeta (C_1^d)_{41}, \]

\[ \eta_u = \eta (C_2^d)_{11} + \frac{2}{\sqrt{3}} \epsilon (C_2^d)_{31} + \zeta (C_2^d)_{41}, \]

\[ \eta_e = \eta (C_1^d)_{11} + \epsilon [(C_1^d)_{21} - \sqrt{3} (C_1^d)_{31}] - 3 \zeta (C_1^d)_{41}. \]

The dimension-5 superpotential in Eq. (14) is induced by the interactions of \( N^{ci} \) in Eq. (38),

\[ W = \eta_u^{ij} H_2^i L^j N^{ci} + \sqrt{6} \epsilon^{ij} H_{126}^{45} N^{ci} N^{cj} + \text{H.c.}, \]

\[ \eta_v = \eta (C_2^d)_{11} + 2 \epsilon (C_2^d)_{21} - 3 \zeta (C_2^d)_{41}. \]

If the scalar component of \( H_{126}^{126} \) has a large VEV, the right-handed neutrinos and sneutrinos receive large masses. The mass matrix of the neutrinos is given
by

\[ M_{\nu_R} = 2\sqrt{3}v_S\zeta, \]

(46)

with \(< H_{126}^\dagger > = v_S/\sqrt{2}\). The mass-squared matrix of the sneutrinos is expressed as \( M_{\nu_R}M_{\nu_R}^\dagger \). Exchanges of these particles lead to an effective superpotential given in Eq. (14),

\[ \kappa = -\eta_\nu (M_{\nu_R})^{-1} \eta_\nu^T. \]

(47)

The coupling \( L^T H_{126}^\dagger L^j \) in Eq. (38) could give Majorana masses to the left-handed neutrinos, if the neutral scalar component of \( H_{126}^\dagger \) has a non-vanishing VEV. However, this VEV has to be as small as the neutrino masses measured in experiments. Then, an extreme fine-tuning of the Higgs potential would become inevitable. We therefore discard this possibility, assuming that \( H_{126}^\dagger \) is heavy enough not to develop a non-vanishing VEV.

The coefficient matrices \( \eta_d, \eta_u, \) and \( \eta_e \) in Eq. (13) and \( \kappa \) in Eq. (14) at the GUT energy scale are related to each other through Eqs. (41), (42), (43), and (47). For independent parameters we can take \( \eta_u + \eta_u^T, \eta_d + \eta_d^T, \epsilon, C_1, \) and \( C_2. \) Then the symmetric parts of the coefficient matrices for the leptons are given by

\[ \eta_e + \eta_e^T = \frac{3r_1 + r_4}{r_1 - r_4} (\eta_d + \eta_d^T) + \frac{4}{r_1 - r_4} (\eta_u + \eta_u^T), \]

(48)

\[ \eta_\nu + \eta_\nu^T = \frac{4r_1 r_4}{r_1 - r_4} (\eta_d + \eta_d^T) + \frac{1 + 3r_4}{r_1 - r_4} (\eta_u + \eta_u^T), \]

(49)

\[ M_{\nu_R} = \sqrt{3}v_S \left[ \frac{r_1}{r_1 - r_4} (\eta_d + \eta_d^T) - \frac{1}{r_1 - r_4} (\eta_u + \eta_u^T) \right], \]

(50)

\[ r_1 = \frac{(C_2)_{11}}{(C_1)_{11}}, \quad r_4 = \frac{(C_2)_{41}}{(C_1)_{41}}. \]

The mass matrix \( M_{\nu_R} \) is symmetric. The antisymmetric parts of the coefficient matrices are given by

\[ \eta_d - \eta_d^T = 2 \left[ (C_1)_{21} + \frac{1}{\sqrt{3}} (C_1)_{31} \right] \epsilon, \]

(51)

\[ \eta_u - \eta_u^T = \frac{4}{\sqrt{3}} (C_2)_{31} \epsilon, \]

(52)

\[ \eta_e - \eta_e^T = 2 \left[ (C_1)_{21} - \sqrt{3} (C_1)_{31} \right] \epsilon, \]

(53)

\[ \eta_\nu - \eta_\nu^T = 4 (C_2)_{21} \epsilon. \]

(54)
Depending on the structures of Eqs. (39) and (40), the 120 representation could contribute exclusively to any one of \( \eta_d, \eta_u, \eta_e, \) and \( \eta_\nu. \)

We now make an assumption that the \((\sqrt{3}H_{120}^5 + H_{120}^{75})/2\) and \(H_{120}^{45}\) components in the Higgs superfields \(H_1\) and \(H_2\), respectively, can be neglected, taking the equations \((C_1)_{21} + (1/\sqrt{3})(C_1)_{31} = (C_2)_{31} = 0.\) Then, the coefficient matrices for the quarks become symmetric, \(\eta_d = \eta_d^T, \eta_u = \eta_u^T.\) Adopting a generation basis in which the coefficient matrix for the up-type quarks is diagonal, we obtain the equations

\[
\eta_d = V_{CKM}^* V_{Dd}^T, \quad \eta_u = \eta_u^-.
\]

The matrices \(\eta_d\) and \(\eta_u\) are determined by their eigenvalues and the CKM matrix. On the other hand, the coefficient matrices for the leptons are given by

\[
\eta_e = -\frac{3r_1 + r_4}{r_1 - r_4} \eta_d + \frac{4}{r_1 - r_4} \eta_u + 4\bar{\epsilon},
\]

\[
\eta_\nu = -\frac{4r_1 r_4}{r_1 - r_4} \eta_d + \frac{r_1 + 3r_4}{r_1 - r_4} \eta_u + 2r_2 \bar{\epsilon},
\]

\[
M_{\nu R} = \frac{2\sqrt{3}v_S}{(C_1)_{41}} \left[ \frac{r_1}{r_1 - r_4} \eta_d - \frac{1}{r_1 - r_4} \eta_u \right],
\]

\[
\bar{\epsilon} = (C_1)_{21} \epsilon, \quad r_2 = \frac{(C_2)_{21}}{(C_1)_{21}}.
\]

The matrices \(\eta_e, \eta_\nu,\) and \(M_{\nu R}\) are expressed as linear combinations of \(\eta_d, \eta_u,\) and \(\bar{\epsilon}.\) With \(\eta_u\) being diagonal, the matrix \(\eta_d\) is roughly diagonal simultaneously. However, for the matrix \(\bar{\epsilon},\) only off-diagonal elements have non-vanishing values. The contribution of 120 may make the off-diagonal elements of \(\eta_e\) and/or \(\eta_\nu\) non-negligible, which could enhance the generation mixing for the leptons.

The independent model parameters at the GUT energy scale are given by the diagonal matrices \(\eta_d^D\) and \(\eta_u^D,\) the CKM matrix \(V_{CKM},\) the ratio \(r_1, r_2,\) and \(r_4,\) the antisymmetric matrix \(\bar{\epsilon},\) and the right-handed neutrino mass scale \(v_S/(C_1)_{41}.\) At the electroweak energy scale, the eigenvalues of \(\eta_d, \eta_u,\) and \(\eta_e\) are known experimentally, if the ratio \(\tan \beta\) of the VEVs for \(H_1\) and \(H_2\) is given. The CKM matrix has been measured. The quantities obtained experimentally for the neutrinos are the mass-squared differences and the mixing angles. These observed quantities have to be accommodated by suitable values of the model parameters at the GUT energy scale.
5 Numerical analyses

We show numerically that the observed neutrino oscillations can be described by our model. Since the number of the model parameters at the GUT energy scale is large, a systematical analysis in the whole parameter space is complicated. Instead, we demonstrate the viability of the model by presenting two numerical examples within the ranges of real values for \( r_1, r_2, r_4, \) and \( \tilde{\epsilon} \).

In Figs. 1 and 2 the mixing parameters \( \sin^2 2\theta_{\text{atm}}, \sin^2 2\theta_{\text{sol}}, \sin^2 2\theta_{\text{chooz}}, \) and the ratio of mass-squared differences \( \Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2 \) at the electroweak energy scale are shown as functions of the model parameter \( r_2 \). The values of \( \tan \beta, r_1, r_4, \) and \( \tilde{\epsilon} \) are listed in Table 1, with the sets (A) and (B) corresponding to Figs. 1 and 2, respectively. The other model parameters \( \eta_D^u, \eta_D^d, \) and \( V_{\text{CKM}} \) at the GUT energy scale are tuned to give the quark and charged lepton masses and the CKM matrix at the electroweak energy scale shown in Table 2. The CKM matrix can be expressed by the standard parametrization [11] and the \( CP \)-violating phase of this parametrization is denoted by \( \delta_{SP} \). These masses at the electroweak energy scale for the quarks and charged leptons, as well as the magnitudes of the CKM matrix elements, are consistent with the experiments [12]. The \( CP \)-violating phase \( \delta_{SP} \) lies in the range allowed by all the \( CP \) violation phenomena observed in the \( K^0-\bar{K}^0 \) and \( B^0-\bar{B}^0 \) systems [13]. The outcomes in Table 2 do not vary with \( r_2 \). The value of \( v_S/(C_{141})_4 \) deter-
Fig. 2. The mixing parameters and the ratio of mass-squared differences for the neutrinos at the electroweak energy scale for the parameter set (B): i) \( \sin^2 2\theta_{\text{atm}} \), ii) \( \sin^2 2\theta_{\text{sol}} \), iii) \( \sin^2 2\theta_{\text{chooz}} \), iv) \( \Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}} \).

Table 1
The parameter sets (A) and (B) for Figs. 1 and 2, respectively.

|       | (A)       | (B)       |
|-------|-----------|-----------|
| \( \tan \beta \) | 20        | 30        |
| \( r_1 \)   | -3.9      | -1.9      |
| \( r_4 \)   | -7.3      | -5.0      |
| \( \tilde{\epsilon}_{12} \) | \(-4.0 \times 10^{-4}\) | \(-9.0 \times 10^{-4}\) |
| \( \tilde{\epsilon}_{13} \) | \(3.6 \times 10^{-3}\) | \(4.9 \times 10^{-3}\) |
| \( \tilde{\epsilon}_{23} \) | \(9.2 \times 10^{-3}\) | \(7.6 \times 10^{-3}\) |

mines the scale of \( \kappa \) and does not affect the presented four observables for the neutrino oscillations.

We can see from Figs. 1 and 2 that the atmospheric and solar neutrino oscillations, under the constraints from the CHOOZ experiment, are realized simultaneously for certain parameter values. In Table 3 the resultant values of \( \sin^2 2\theta_{\text{atm}}, \sin^2 2\theta_{\text{sol}}, \sin^2 2\theta_{\text{chooz}}, \) and \( \Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}} \) are explicitly given for the set (A) with \( r_2 = 12.5 \) and for the set (B) with \( r_2 = 7.5 \). If the mass scales of the right-handed neutrinos are put at \( v_S/(C_{14}) = 1.0 \times 10^{15} \) GeV in the set (A) and at \( v_S/(C_{14}) = 6.0 \times 10^{14} \) GeV in the set (B), the mass-squared differences are given by \( \Delta m^2_{\text{sol}} = 2.5 \times 10^{-5} \) GeV\(^2\), \( \Delta m^2_{\text{atm}} = 5.5 \times 10^{-3} \) GeV\(^2\).
Table 2
The masses of the quarks and charged leptons (in unit of GeV) and the CKM matrix (its elements and \(CP\)-violating phase in the standard parametrization) at the electroweak energy scale.

\[
\begin{array}{ccc}
m_t & 1.8 \times 10^2 & |(V_{CKM})_{12}| = 0.22 \\
m_c & 6.8 \times 10^{-1} & |(V_{CKM})_{13}| = 0.0036 \\
m_u & 2 \times 10^{-3} & |(V_{CKM})_{23}| = 0.041 \\
m_b & 3.0 & \delta_{SP} = 1.3 \\
m_s & 9.3 \times 10^{-2} & \\
m_d & 5 \times 10^{-3} & \\
m_{\tau} & 1.8 & \\
m_{\mu} & 1.0 \times 10^{-1} & \\
m_e & 5 \times 10^{-4} &
\end{array}
\]

Table 3
The mixing parameters and the ratio of mass-squared differences for the neutrinos at the electroweak energy scale with \(r_2 = 12.5\) and \(r_2 = 7.5\) for the sets (A) and (B), respectively.

|                  | (A)       | (B)       |
|------------------|-----------|-----------|
| \(\sin^2 2\theta_{atm}\) | 0.95      | 0.94      |
| \(\sin^2 2\theta_{sol}\)  | 0.59      | 0.86      |
| \(\sin^2 2\theta_{chooz}\) | 0.16      | 0.0026    |
| \(\Delta m_{sol}^2/\Delta m_{atm}^2\) | 0.0045    | 0.055     |

and by \(\Delta m_{sol}^2 = 8.7 \times 10^{-5} \text{ GeV}^2\), \(\Delta m_{atm}^2 = 1.6 \times 10^{-3} \text{ GeV}^2\), respectively. Note that the mixing angle \(\sin^2 2\theta_{chooz}\) is predicted to be around the present experimental bound for the case (A), while to be much smaller than it for the case (B).

6 Conclusions

We have discussed the masses and mixings of quarks and leptons within the framework of GUT coupled to supersymmetry. In GUT models the Higgs couplings for quarks and leptons are closely related to each other. The large generation mixing for the leptons, which is observed experimentally through the neutrino oscillations, cannot coexist trivially with the small generation mixing for the quarks. Some natural explanation for the difference of mixing between the quarks and the leptons is sought.
To solve the problem of generation mixing, we have proposed a model with SO(10) and supersymmetry. This model is a simple extension of the minimal SO(10) model, and includes a Higgs superfield of $120$ representation, as well as two Higgs superfields of $10$ and $T_{26}$ representations. The energy evolution of the Higgs couplings with quarks and leptons have also been taken into account between the energy scales of GUT and electroweak theory.

By the simple enlargement of the Higgs sector, the masses and mixings of the quarks and leptons are consistently accommodated without invoking much contrived schemes. The $120$ representation makes the Higgs couplings different between the quarks and the leptons. The small neutrino masses are traced back to large masses for the right-handed neutrinos and sneutrinos generated by $T_{26}$. The model parameters are constrained to correctly give the quark and charged lepton masses and the CKM matrix. In spite of these constraints, the experimental results for the neutrino masses and the MNS matrix can be described well in certain regions of the parameter space.

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References

[1] B.T. Cleveland et al., Astrophys. J. 496 (1998) 505; W. Hampel et al. (GALLEX Collaboration), Phys. Lett. B447 (1999) 127; J.N. Abdurashitov et al. (SAGE Collaboration), Phys. Rev. C60 (1999) 055801; M. Altmann et al. (GNO Collaboration), Phys. Lett. B490 (2000) 16; S. Fukuda et al. (Super-Kamiokande Collaboration), Phys. Rev. Lett. 86 (2001) 5656; Q.R. Ahmad et al. (SNO Collaboration), Phys. Rev. Lett. 89 (2002) 011301.

[2] S. Fukuda et al. (Super-Kamiokande Collaboration), Phys. Rev. Lett. 85 (2000) 3999.

[3] M. Apollonio et al., Phys. Lett. B466 (1999) 415.

[4] K. Eguchi et al. (KamLAND Collaboration), Phys. Rev. Lett. 90 (2003) 021802.

[5] G. Anderson, S. Dimopoulos, L.J. Hall, S. Raby, and G.D. Starkman, Phys. Rev. D49 (1994) 3660; D.-G. Lee and R.N. Mohapatra, Phys. Rev. D51 (1995) 1353;
L.J. Hall and S. Raby, *Phys. Rev.* D51 (1995) 6524;
Y. Achiman and T. Greiner, *Nucl. Phys.* B443 (1995) 3;
K.S. Babu and S.M. Barr, *Phys. Rev. Lett.* 75 (1995) 2088;
C.H. Albright and S. Nandi, *Phys. Rev.* D53 (1996) 2699;
Z. Berezhiani and Z. Tavartkiladze, *Phys. Lett.* B409 (1997) 220;
C.H. Albright, K.S. Babu, and S.M. Barr, *Phys. Rev. Lett.* 81 (1998) 1167;
K. Matsuda, Y. Koide, and T. Fukuyama, *Phys. Rev.* D64 (2001) 053015;
M. Bando and M. Obara, OCHA-PP-202, hep-ph/0302034 (2003).

[6] I. Dorsner and S.M. Barr, *Nucl. Phys.* B617 (2001) 493, and references therein.

[7] N. Oshimo, *Phys. Rev.* D66 (2002) 095010.

[8] K.S. Babu and R.N. Mohapatra, *Phys. Rev. Lett.* 70 (1993) 2845;
    B. Brahmachari and R.N. Mohapatra, *Phys. Rev.* D58 (1998) 015001;
    T. Fukuyama and N. Okada, JHEP 11 (2002) 011.

[9] S.G. Naculich, *Phys. Rev.* D48 (1993) 5293.

[10] N. Haba and N. Okamura, *Eur. Phys. J.* C14 (2000) 347;
    J.A. Casas, J.R. Espinosa, A. Ibarra, and I. Navarro, *Nucl. Phys.* B573 (2000)
    652.

[11] K. Hagiwara *et al.*, *Phys. Rev.* D66 (2002) 010001.

[12] See, e.g., H. Fusaoka and Y. Koide, *Phys. Rev.* D57 (1998) 3986.

[13] G.C. Branco, G.C. Cho, Y. Kizukuri, and N. Oshimo, *Nucl. Phys.* B449 (1995)
    483.