Orientation Dependence of Confinement-Deconfinement Phase Transition in Anisotropic Media

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Abstract
We study the temporal Wilson loops with arbitrary orientation in anisotropic holographic QCD. Anisotropic QCD is relevant to describe quark-gluon plasma (QGP) in heavy-ions-collisions (HIC). We use an anisotropic black brane solutions for a bottom-up anisotropic QCD approach in 5-dim Einstein-dilaton-two-Maxwell model constructed in previous work. We calculate the minimal surfaces of the corresponding probing open string world-sheet in anisotropic backgrounds with various temperatures and chemical potentials. The dynamical wall (DW) locations, providing the quark confinement, depend on the orientation of the quark pairs, that gives a crossover transition between confinement/deconfinement phases in the dual gauge theory.

Keywords: AdS/QCD, holography, phase transition

1. Introduction
Study of the phase diagram in the temperature and chemical potential \((\mu, T)\)-plane is one of the most important questions in QCD. It is known that perturbative methods are inapplicable to study this problem. The lattice QCD still has difficulties with the study of theories with non-zero chemical potential. The gravity/gauge duality provides an alternative tool to study this problem [1][2][3].

The phase diagram has been experimentally studied only for small \(\mu\) and large \(T\) values (RHIC, LHC) on the one hand and for low energies (small \(T\)) and finite chemical potential values (SPS) on the other hand. One of the FAIR and NICA tasks is the experimental study of this diagram in between these two particular cases. For this purpose the results of the beam scanning in HIC are supposed to be analyzed. In this context noting that there is an obvious anisotropy — the nonequivalence of the longitudinal and transverse directions — in HIC, one can say that the phase diagram is tried under anisotropic conditions. In fact it is believed, that QGP formed in HIC is initially in an anisotropic state. Isotropisation occurs approximately in \(0.5−2\text{ fm/c}\) after a collision [4]. Therefore it seems natural to assume that the results of the beam scanning give indications of the phase transition in an anisotropic QCD. This anisotropy can be taken into account holographically. An additional argument for plasma anisotropy in HIC is the estimation of multiplicity, which is holographically supported by an anisotropic model [5]. Anisotropic lattice QCD is a subject of studies in [6][7][8][9].

Anisotropy in the gravity side can be taken into account by anisotropic metric, that can be provided by adding magnetic ansatz of Maxwell field to dilaton gravity action. Non-zero chemical potential is introduced via electric ansatz for the second Maxwell field [9]. Thereby the 5-dimensional dilaton gravity with two Maxwell fields turns out to be the most suitable model. Such model was considered in [10][11]. The simplest anisotropic model, characterized by anisotropic parameter \(\nu\), has been investigated in [5].

In this paper we consider a 5-dim metric defined by anisotropic parameter \(\nu\), non-trivial warp-factor, non-zero time component of the first Maxwell field and non-zero longitudinal magnetic component of the second Maxwell field. We take the warp-factor in the simplest form \(b(z) = e^{\frac{\nu z}{2}}\), as this particular case allows to construct explicit solution [10]. We study the confinement/deconfinement phase transition line for the pair of quarks in the anisotropic QGP. It is natural to expect that the phase transition depends on the orientation of the quark pair relative to the anisotropy axis. Anisotropy axis in QGP created in HIC is defined by the axes of ions collisions. We show that the confinement/deconfinement phase transition line depends on the angle \(\theta\) between quarks line and heavy ions collisions line. We calculate the expectation values of the rectangular temporal Wilson loop \(\langle W_{t\theta} \rangle\) for different orientation of the spacial part of the Wilson loop and find the conditions of the confinement/deconfinement phase transition for this line. For this purpose we introduce the effective potential \(V(z)\) that depends on the angle \(\theta\) and describes the interquark interaction. The confinement takes place when the effective potential \(V\) has a critical point. We find conditions, under which the critical point exists, and study the dependence of the confinement/deconfinement phase transition temperature on chemical potential \(\mu\) and angle \(\theta\).

The specific feature of the holographic description of the confinement/deconfinement is the position of the phase diagram associated with the Wilson loop behavior relative to the line of the Hawking-Page phase transition, characterized by the 5-dim background metric. It is
evident, that unlike the confinement/deconfinement transition line, the Hawking-Page transition line’s position on the phase diagram doesn’t depend on the angle $\theta$. As a result the change of this angle leads to changing of the mutual arrangement of the confinement/deconfinement transition line and Hawking-Page transition line on the phase diagram. We find the critical value $\theta_{cr}$, for which the top of the Hawking-Page transition line and the top of the confinement/deconfinement transition line coincide.

The paper is organized as follows. In Sect. 2.1.1 we describe the 5-dim black brane solution in the anisotropic background. In Sect. 2.1.2 we calculate the expectation value of the temporal Wilson loop. In Sect. 3 we find the condition of the confinement-deconfinement phase transition for zero and non-zero temperature and perform the phase diagrams depending on the angle $\theta$.

2. Setup

2.1. The model

We consider a 5-dimensional Einstein-dilaton-two-Maxwell system. In the Einstein frame the action of the system is specified as

$$S = \int d^5x \sqrt{-\det(g_{\mu\nu})} \left[ R - \frac{f_1(\phi)}{4} F_{\mu\nu}^1 - \frac{f_2(\phi)}{4} F_{\mu\nu}^2 - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) \right],$$

(1)

where $F_{\mu\nu}^1$ and $F_{\mu\nu}^2$ are the squares of the Maxwell fields $F_{\mu\nu}^1 = \partial_\mu A_\nu^{(1)} - \partial_\nu A_\mu^{(1)}$ and $F_{\mu\nu}^2 = q d\mu \wedge d\nu$. $f_1(\phi)$ and $f_2(\phi)$ are the gauge kinetic functions associated with the corresponding Maxwell fields, $V(\phi)$ is the potential of the scalar field $\phi$.

To find the black brane solution in the anisotropic background we used the metric ansatz in the following form:

$$ds^2 = G_{\mu\nu} dx^\mu dx^\nu = \frac{L^2 b(z)}{z^2} \left[ -g(z) dt^2 + dx^2 + z^2 \left( dy_1^2 + dy_2^2 \right) + \frac{dz^2}{g(z)} \right],$$

(2)

$$\phi = \phi(z), \quad A_\mu^{(1)} = A_\mu(z) \delta_\mu^0,$n

(3)

$$F_{\mu\nu}^{(2)} = q dy_1 \wedge dy_2,$n

(4)

where $b(z)$ is the warp factor and $g(z)$ is the blackening function; we set the AdS radius $L = 1$. All the quantities in formulas and figures are presented in dimensionless units.

Note that in [10] the following strategy to study holographic model is used. First, one takes the warp-factor suitable for phenomenological application, in particular one can take $b = e^{\frac{\phi}{2}}$. Second, the anisotropic multiplier $z^{2-\nu}$ is also fixed by phenomenological reasons [5]. Third, one takes a specific function $f_1$ by reasons of simplicity. And finally, using E.O.M. following from (2), one finds coupling function $f_2$ (Fig. 1), potential $V$ (Fig. 1), Maxwell field potential $A_\mu$ and blackening function $g$. The last one has the form:

$$g = 1 - \frac{z^{2+\nu}}{z_{h}^{2+\nu}} \frac{6(z_{h}^{2+\nu})}{\delta_6(z_{h}^{2+\nu})} - \frac{\mu^2 c z^{2+\nu} e^{\frac{c^2}{\mu^2}}}{4 \left( 1 - e^{\frac{c^2}{\mu^2}} \right)^2} \delta_6(c z^{2+\nu}) + \frac{\mu^2 c z^{2+\nu} e^{\frac{c^2}{\mu^2}}}{4 \left( 1 - e^{\frac{c^2}{\mu^2}} \right)^2} \frac{6(z_{h}^{2+\nu})}{\delta_6(z_{h}^{2+\nu})} \delta_6(c z_{h}^{2+\nu}),$$

and the function $\delta_6(x)$ has the following expansion [12]:

$$\delta_6(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n! (1 + n + \frac{1}{2})}.$$

The potential $V$ can be approximated by a sum of two exponents and a negative constant:

$$V_{2EM}(\phi, \mu, \nu) = V_0(\nu) - C_\gamma(\mu, \nu) e^{K_1(\nu) \phi} + C_8(\mu, \nu) e^{K_2(\nu) \phi}.$$

(6)

For $c = -1$ and $\nu = 4.5$

$$V_0 = -0.5778, \quad K_1 = 0.7897, \quad K_2 = 2.0995$$

(7)

with the coefficients depending on the chemical potential $\mu$:

$$C_\gamma(\mu) = 23.0779 + 2.4236 \mu^2, \quad C_8(\mu) = 0.0575 + 4.9919 \mu^2.$$

(8)

Note, that in [13] an explicit isotropic solution for the dilaton potential as a sum of two exponents and zero chemical potential has been constructed. It would be interesting to generalize this construction to the anisotropic and non-zero chemical potential cases.
2.2. The Wilson loop

The purpose of our consideration is to calculate the expectation value of the temporal Wilson loop

\[ W[C_\vartheta] = e^{-S_{\vartheta}}, \quad (9) \]

oriented along vector \( \vec{n} \), such that \( n_x = \cos \vartheta, \ n_y = \sin \vartheta \).

Following the holographic approach \[14, 15, 16\] we have to calculate the value of the Nambu-Goto action for the test string in our background:

\[ S = -\frac{1}{2\pi\alpha'} \int dt d\xi \ e^{\sqrt{\frac{2}{3} \phi(z)}} \sqrt{-\det G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu}, \quad (10) \]

where \( G_{\mu\nu} \) is given by (2). The world sheet presented in Fig. 2 is parameterized as

\[ X^0 = t, \quad X^1 = \xi \cos \vartheta, \quad X^2 = \xi \sin \vartheta, \quad X^3 \equiv y_2 = \text{const}, \quad X^4 \equiv z = z(\xi). \]

The action (10) can be rewritten:

\[ S = -\frac{\tau}{2\pi\alpha'} \int d\xi \ M(z(\xi)) \sqrt{F(z(\xi)) + (z'(\xi))^2}, \quad \tau = \int dt, \quad (11) \]

\[ M(z(\xi)) = \frac{b(z(\xi))}{z(\xi)^2} \ e^{\sqrt{\frac{2}{3} \phi(z)}}, \quad F(z(\xi)) = g(z(\xi)) \left(z(\xi)^2 - 2 \sin^2(\theta) + \cos^2(\theta)\right). \quad (12) \]

Let us introduce the effective potential:

\[ \mathcal{V}(z) \equiv M(z) \sqrt{F(z)}. \quad (13) \]
From (11) we have representations for the character length of the string and the action:

\[
\frac{\ell}{2} = \int_0^\infty \frac{dz}{\sqrt{F(z) - 1}},
\]

\[
\frac{S}{2} = \int_0^\infty \frac{V(z) M(z) dz}{\sqrt{\frac{V(z)}{M(z)} - 1}},
\]

where \(z_*\) is a top point. Here we introduce the UV cut-off \(\epsilon\), since \(M\) has singular behaviour near \(z \to 0\):

\[
M(z) \sim \frac{M_0}{z^k}, \quad k > 1, \quad F(0) = 1.
\]

From (14) and (15) we see that \(S\) and \(\ell\) make sense if the potential is a decreasing function in the interval \(0 < z < z_*\).

\[
V(z) > V(z_*), \quad 0 < z_* < z_{\text{min}},
\]

where \(z_{\text{min}}\) is the local minimum of \(V(z)\). We are interested in studying the asymptotics of \(S\) at large \(\ell\). To get \(\ell \to \infty\) and \(S \to \infty\) we have to take \(z_* = z_{\text{min}}\). Indeed, substituting

\[
\frac{V^2(z)}{V^2(z_{\text{min}})} = 1 + \frac{V_2(z - z_{\text{min}})^2 + o((z - z_{\text{min}})^2)}{V_{\text{min}}} = \frac{V''(z_{\text{min}})}{V(z_{\text{min}})},
\]

into (14) and (15), we get

\[
\ell = 2 \int_0^{z_{\text{min}}} \frac{dz}{\sqrt{F(z) - 1}} \approx \sqrt{\frac{V(z_{\text{min}})}{F(z_{\text{min}})}} \log(z_{\text{min}} - z),
\]

\[
S = 2 \int_0^{z_{\text{min}}} \frac{V(z) M(z) dz}{\sqrt{V(z_{\text{min}})}} \approx M(z_{\text{min}}) \sqrt{\frac{V(z_{\text{min}})}{F(z_{\text{min}})}} \log(z_{\text{min}} - z).
\]

so that \(\ell \to \infty\) as \(z \to z_{\text{min}} - 0\) and \(S \to \infty\) as \(z \to z_{\text{min}} - 0\).

The stationary point, \(V|_{z=z_{\text{DW}}} = 0\), is usually called a dynamical domain wall (DW) point, that satisfies the equation:

\[
z = z_{\text{DW}}: \quad \frac{M'(z)}{M(z)} + \frac{1}{2} \frac{F'(z)}{F(z)} = 0.
\]

Taking the top point \(z_* = z_{\text{DW}}\), we get

\[
S \sim \sigma_{\text{DW}} \ell, \quad \sigma_{\text{DW}} = M(z_{\text{DW}}) \sqrt{F(z_{\text{DW}})}.
\]

3. Confinement/deconfinement phase transition

In our case the effective potential depends on the warp factor, the scalar field and the angle. To find stationary points of \(V(z)\) we solve the equation (21) for the potential (13) with arbitrary angle:

\[
z = z_{\text{DW}}: \quad \mathcal{D}W_\theta \equiv \sigma(z, \nu, \psi) - \frac{2}{z} - \frac{1 - \frac{1}{\nu}}{\cos^2(\theta) + \frac{1 - \frac{1}{\nu}}{\cos^2(\theta)}} + \frac{g'}{2g} = 0,
\]

\[
\sigma(z, \nu, \psi) = cz + \frac{1}{\nu \psi} \sqrt{\frac{2}{3}} \sqrt{3c \nu \psi z^2 \left(\frac{\nu \psi}{2} - 3\right) + 4 \nu - 4}.
\]

It is possible to obtain particular cases for \(\theta = 0, \pi/2\) from the expression (23).

Let us first consider the case of the zero temperature, i.e. \(g = 1\), and deal with equations (Fig. 3):

\[
z = z_{\text{DW}}: \quad \sigma_z(z, c, \nu) - \frac{2}{z} = 0,
\]

\[
z = z_{\text{DW}}: \quad \sigma_v(z, c, \nu) - \nu \psi + 1 = 0,
\]

\[
z = z_{\text{DW}}: \quad \sigma_\theta(z, c, \nu, \theta) - \frac{1 - \frac{1}{\nu}}{\cos^2(\theta) + \frac{1 - \frac{1}{\nu}}{\cos^2(\theta)}} = 0.
\]
For the non-zero temperature we can rewrite the equation (23) (Fig. 3):

\[ z = z_{DW}; \quad DW_x = \Sigma(z, z_0, \mu, c, \nu) - \Theta(z, \nu, \theta) = 0, \]  
\[ \Sigma(z, z_0, \mu, c, \nu) = cz + \frac{1}{\nu z} \sqrt{2} \left( \frac{3c^2 v^2 z^2}{2} - 3 \right) + 4\nu - 4 - \frac{g'_{\nu}}{2g}, \]
\[ \Theta(z, \nu, \theta) = \frac{2}{z} \left( 1 - \frac{1}{2} \right) z^{1/2} \frac{\sin^2(\theta)}{\cos^2(\theta) + z^{1/2} \sin^2(\theta)}. \]

This expression leads to the particular cases \( \theta = 0, \pi/2 \):

\[ \theta = 0 \Leftrightarrow z = z_{DW}: \quad DW_x = \Sigma(z, z_0, \mu, c, \nu) - \frac{2}{z} = 0, \]
\[ \theta = \pi/2 \Leftrightarrow z = z_{DW}: \quad DW_y = \Sigma(z, z_0, \mu, c, \nu) - \frac{\nu + 1}{\nu z} = 0. \]

The expression for the temperature \( T(z_0, \mu, c, \nu) \) is (10):

\[ T(z_0, \mu, c, \nu) = \frac{g'_{\nu}(z_0)}{4\pi} = e^{-\frac{g'_{\nu}}{2cz_0}} \left( 1 - e^{-\frac{\mu^2 c z_0^2}{6(c z_0^2 - \nu^2)}} \right) \left( 1 - e^{-\frac{\mu^2 c z_0^2}{6(c z_0^2 - \nu^2)}} \right). \]

Let us remind that in (10) we have also studied the thermodynamical properties of the constructed black hole background and found the large/small black hole phase transitions (BB-transitions) at the temperature \( T_{BB}(\mu) \). Hawking-Page phase transition takes place at \( z_{BB,HP} \), where the free energy equals zero. The particular value of \( z_{BB,HP} \) depends on parameters \( c \) and \( \nu \) and is larger for larger negative \( c \), i.e. \( z_{BB,HP}(c_1, \nu) < z_{BB,HP}(c_2, \nu) \) for \( c_1 < c_2 < 0 \). For the anisotropic background the Hawking-Page horizon is less than for the isotropic one with the same \( c < 0 \). For \( \mu = 0 \) and \( T < T_{HP}(0) \) the black hole dissolves to thermodynamically stable thermal gas. If the system cools down with the non-zero chemical potential less than some critical value \( \mu_{cr} \), the background undergoes the phase transition from a large to a small black hole. This is a generalization of the corresponding effect in the isotropic case [17-19,20,21]. We have found that in anisotropic case the temperature of the large/small black hole phase transition is less than in the isotropic one, i.e. \( T_{BB}^{(\nu)}(\mu) < T_{BB}^{(iso)}(\mu) \). The value of the critical chemical potential, up to which this phase transition exists, in the anisotropic case is larger compared to the isotropic one, \( \mu_{cr}^{(\nu)} > \mu_{cr}^{(iso)} \). Also, we have found that the point \( (\mu_{cr}^{(\nu)}, T_{cr}^{(\nu)}) \) for \( \nu \to 1 \) goes smoothly to \( (\mu_{cr}^{(iso)}, T_{cr}^{(iso)}) \).

In Fig. 4(6) we can see the angle dependence of the confinement/deconfinement phase transition on the Wilson loop orientation. We choose the intermediate angle values \( \theta = 10^\circ, 45^\circ, 60^\circ \). In the boundary cases the graphs coincide with the curves for \( W_{FF} \) (blue solid line) and \( W_{FT} \) (magenta solid line) from [10]. In our consideration we take into account the Hawking-Page phase transition (dashed pink line). For the critical angle \( \theta_{c1} = 22^\circ \) (green dashed line) the Wilson loop phase transition line intersects the Hawking-Page phase transition line.

Figure 3: A) Functions \( \sigma_x, \sigma_y, \sigma_\theta \) (blue lines), \( \sigma_x(c, c, c, \nu) \) (magenta lines), \( \sigma_\theta(c, c, c, \nu, \theta) \) (cyan dashed lines) for the angle \( \theta = 54^\circ, \nu = 4.5 \) and for different \( c \). B) DWs’ positions, corresponding to the Wilson loops \( W_{FF}, W_{FT}, W_{FT} \) in the anisotropic case \( \nu = 4.5 \) are given by intersections of gray lines representing \( \Sigma(\theta) \) and the blue, cyan, magenta lines representing another part of the equation (28) for angles \( \theta = 0^\circ, 54^\circ, 90^\circ \). Here we vary \( z_0 \) and \( c \). In all cases to get the DW position we take the minimal intersection point.
at its very end point from above. For the critical angle $\theta_{c2} = 54^\circ$ (dashed cyan line) the top point $\mu = 0$ for the confinement/deconfinement phase transition of the Hawking-Page line and the Wilson loop line is chosen to be the same. On Fig. 5 we see, that in this case the Hawking-Page phase transition line coincides with the Wilson loop line along the entire length. From this angle the Wilson loop line fully determines the confinement/deconfinement phase transition. This situation preserves up to the next critical angle $\theta_{c3} = 78^\circ$ (light pink dashed line). Here the Wilson loop phase transition line intersects the Hawking-Page phase transition line at its very end point from below.

4. Results and conclusions

We have found the dependence of the confinement/deconfinement phase transition line on the orientation of the quark pair. For this purpose we have studied the behavior of the temporal Wilson loops in the particular 5-dimensional anisotropic background supported by dilaton and two-Maxwell field constructed in [10]. The diagram is defined in $(\mu, T)$-plane for arbitrary angles. In this model we have determined the critical angles $\theta_{c1} = 22^\circ$, $\theta_{c2} = 54^\circ$, $\theta_{c3} = 78^\circ$. For the critical angle $\theta_{c1} = 22^\circ$ the Hawking-Page phase transition line and the phase transition line determined by the Wilson loop have only one common point. In this case the whole Hawking-Page line determines the confinement/deconfinement phase transition. For the angle $\theta_{c2} = 54^\circ$ the top point of the Hawking-Page phase transition coincides with the top point of the confinement/deconfinement phase transition. For $\theta_{c3} = 78^\circ$ the Hawking-Page phase transition line and phase transition line determined by the Wilson loop have only one common point again. In this case the whole confinement/deconfinement phase transition line is determined by the Wilson loop. In all likelihood these calculations are relevant in the context of the future NICA and FAIR projects.

We have studied the dependence of behavior of the temporal Wilson loops on the orientation specified by the arbitrary angle $\theta$ in the background (3). This result is the generalization of the two particular cases of orientation, considered in [10], that can be associated with boundary values $\theta = 0, \pi/2$. We demonstrated that the phase diagram depends on the orientation [11]. Taking into account the instability zones of the anisotropic background, we have found more complicated confinement/deconfinement phase diagrams for differently oriented temporal Wilson loops and the details are the following.

- For $0^\circ \leq \theta < \theta_{c1} = 22^\circ$ parts of regions near zero values of the chemical potential enter the instability regions of our background, where the small black holes collapse to large ones. Here the horizon suddenly blows up to pass, so that the confinement phase transforms to the deconfinement one by a Hawking-Page phase transition. After the chemical potential exceeds some critical value, the confinement/deconfinement phase transition is no longer determined by the background and the influence on the Wilson loop starts to dominate, analogous to the longitudinal orientation case, presented as $W_{xT}$ in [10] and associated with $\theta = 0^\circ$. The isotropic case can be regarded as the reduction to the described scenario as well.

- For $\theta_{c1} = 22^\circ$ we have only one common point of the Hawking-Page phase transition line and the line corresponding to the Wilson loop contribution.

Figure 4: Phase transition diagrams for $\theta = 10^\circ$ (upper plots, brown dashed line) and $\theta_{c1} = 22^\circ$ (lower plots, green dashed line), the intersection region (left plots) and the general picture (right plots).
• For $\theta_{c1} = 22^\circ \leq \theta < \theta_{c2} = 54^\circ$ we have no intersections of the Hawking-Page phase transition line and the line corresponding to the Wilson loop contribution, so there is a jump from the Hawking-Page to the Wilson loop line.

• For $\theta_{c2} = 54^\circ$ the Wilson loop line on the phase diagram shows rather good coincidence with the Hawking-Page phase transition line, so the difference between two scenarios disappears as well as the instability region.

• For $\theta_{c2} = 54^\circ \leq \theta < \theta_{c3} = 78^\circ$ we have no intersections of the Hawking-Page phase transition line and the line corresponding to the Wilson loop contribution. In this case the whole confinement/deconfinement phase transition line is determined by the Wilson loop.

• For $\theta_{c3} = 78^\circ$ we have only one common point of the Hawking-Page phase transition line and the line corresponding to the Wilson loop contribution.
For $\theta_{\perp} = 78^\circ \leq \theta < 90^\circ$ the confinement/deconfinement phase transition is determined by the Wilson loop starting from the zero values of chemical potential and meets the instability of the background in the small region of non-zero chemical potential only. In this small region the Hawking-Page phase transition takes place and afterwards the influence of the Wilson loop becomes dominant again. This picture is analogous to the case of the transversal orientation case, presented as $W_{TF}$ in [10] and associated with $\theta = 90^\circ$.

In isotropic background such a case is not implemented.

We should also note that all these considerations are applicable to the energies high enough, as for $T \to 0$ effects of anisotropy disappear. As to the future investigations, the following natural questions to static and non-static properties of our model are worth noting. As has been mentioned, the anisotropic background constructed in [10] can be generalized to provide a more realistic model. In this case the solution can be given in terms of quadratures only and we suppose to generalize the Wilson loop calculations to this more realistic case. As to static properties, it is natural to

- investigate $\theta$-oriented Wilson loops based on more complicated factor $b(z)$, in particular such that in the isotropic limit it fits the Cornell potential known by lattice QCD;
- study the Regge spectrum for mesons, adding the probe gauge fields to the backgrounds and find its dependence on $\theta$;
- consider estimations for direct photons and find dependence on orientation [11];
- evaluate transport coefficients and their dependence on the anisotropy;
- estimate the holographic entanglement entropy and find its dependence on $\theta$; note that this has been done in [5] for zero chemical potential and $\theta = 0, \pi/2$; the isotropic case for non-zero chemical potential has been considered recently in [22].

As to the thermalization processes, which are the main motivations of our consideration of the anisotropic background (see details in [11, 23]), it would be interesting to investigate the behavior of the temporal Wilson loop during thermalization. This problem for zero chemical potential has been studied in [23]. It is also interesting to generalize the result of paper [25] and consider thermalization of the spacial Wilson loops for non-zero chemical potential. This will give the dependence of the drag-forces on the chemical potential.

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