Field-reversed bubble in deep plasma channels for high quality electron acceleration

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We study hollow plasma channels with smooth boundaries for laser-driven electron acceleration in the bubble regime. Contrary to the uniform plasma case, the laser forms no optical shock and no etching at the front. This increases the effective bubble phase velocity and energy gain. The longitudinal field has a plateau that allows for mono-energetic acceleration. We observe as low as $10^{-3}$ r.m.s. relative witness beam energy uncertainty in each cross-section and 0.3% total energy spread. By varying plasma density profile inside a deep channel, the bubble fields can be adjusted to balance the laser depletion and dephasing lengths. Bubble scaling laws for the deep channel are derived. Ultra-short pancake-like laser pulses lead to the highest energies of accelerated electrons per Joule of laser pulse energy.

PACS numbers: 52.38.Kd, 52.65.Rr

Plasma wake fields $^1$ provide a feasible path for high gradient particle acceleration $^2$. Especially efficient is the so-called bubble regime of laser-plasma wakefields $^3$ when the laser intensity is high enough to expel all background plasma electrons from the first half of the plasma wave. The advantage of the cavitated region is that it has a transversely uniform accelerating field $^4$ that helps to generate quasi mono-energetic electron bunches readily registered in experiments $^5$. The bubble regime involves rather complicated relativistic dynamics of particles and a nonlinear evolution of the driving laser pulse. Despite various theoretical approaches to the bubble analysis: the phenomenological model of the bubble $^6$, the nonlinear theory of blowout regime $^7$, and the similarity theory $^8$, a self-consistent theoretical description of the bubble regime is still absent. The scalings $^9$ have been extensively tested in 3d PIC simulations $^{10}$.

The bubble theory has been developed for a homogeneous plasma up to now. Straightforward energy conservation arguments have been used to get the “optimal” scaling laws $^9$. It was assumed that the laser energy is converted into the bubble fields first and then harvested by the electron bunch. The leading similarity parameters for the bubble in homogeneous plasmas are the $S$-number $S = n_e / a n_c \ll 1$ and the pulse aspect ratio $\Pi = c\tau / \lambda_0 \leq 1$. Here, $n_e$ is the plasma electron density and $n_c = \pi / r_e \lambda_0^2$ is the critical plasma density for a laser pulse with the wavelength $\lambda_0$, $a = eE_0 / mc \omega_0$ is the relativistically normalized laser field amplitude, $\omega_0 = 2\pi c / \lambda_0$, and $r_e = e^2 / mc^2$ is the classical electron radius.

In this work, we consider the bubble regime of electron acceleration in a deep plasma channel. Normally, plasma channels are used to guide weakly relativistic pulses over distances much larger than the Rayleigh length $Z_R = \pi R^2 / \lambda_0$. These channels are shallow as a rule, i.e., the relative on-axis plasma density depletion is incomplete. Schroeder et al. $^{11}$ suggested recently to use nearly hollow plasma channels to provide independent control over the focusing and accelerating forces. They considered a moderately relativistic laser pulse and a rectangular channel density profile.

A channel is not required for laser guiding in the full developed bubble regime. The laser pulse is self-guided by the cavitated region that is free from low-energy electrons. Thus, it is not expected that a shallow plasma channel changes the bubble dynamics significantly. However, a deep plasma channel that is (nearly) empty on-axis, can strongly modify both the bubble fields, the laser dynamics, and the trapping. Here, we are looking for laser-plasma parameters that maximize energy of the accelerated electron bunch and may improve its quality, particularly, reduce the energy spread.

The technologically important characteristics of a laser pulse is its energy. It is limited, e.g. by size of the active crystal or by compression gratings, etc. The similarity scalings $^9$ for electron bunch energy $\varepsilon_{\text{max}}$ from a bubble in uniform plasma can be expressed in terms of the laser pulse power $P_L$, duration $\tau$ and energy $W_L = P_L \tau$: $\varepsilon_{\text{max}}^{\text{uniform}} \propto \sqrt{P_L e^2 \tau \lambda_0^{-1}} = \lambda_0^{-1} \sqrt{W_L e^2 c \tau}$ (1)

The scaling $^1$ favors longer laser pulses. Phenomenologically, this reflects the mechanism of laser pulse depletion in the standard bubble. The laser pulse interacts with plasma electrons at the very front only. The rest of the pulse propagates freely in the cavitated region. Thus, the pulse tail slowly overtakes its head, where an “optical shock” is formed and the pulse “etches”. This “etching” leads to the pulse shortening $^{12}$ and faster dephasing that in turn lowers the maximum attainable electron energy. The highest electron energies are achieved with “spherical” laser pulses, whose duration $\tau$ equals their radius $R$ $^{10}$. These pulses fill the cavitated region completely.

The bubble in uniform plasma may continuously trap electrons that leads to strong beam loading and large en-
obtained from gauging the scaling laws against PIC simulations only. The final pre-factors can be where is the laser intensity. We omit dimensionless numerical factors like etc. as we are interested in parametric dependencies only. The final pre-factors can be obtained from gauging the scaling laws against PIC simulations.

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arbitrary function of $\omega$ inside the bubble and the electromagnetic field (ion field $\epsilon_\pi$). It is easy to check that the fields $E$, $B$ at the bubble rear part is visible. The straight line gives the analytical solution at the bubble center.

Figure 2: Simulation results for the case $R = 32 \mu m$. a) The cross-section of the plasma electron density and the longitudinal bubble field; b) on-axis accelerating field of the bubble, GV/m. Flattening of $E_z$ at the bubble rear part is visible. The bubble pulses with as large radius $R$ as possible have an advantage. This is the consequence of the new freedom we gain in a deep channel.

A deeper insight in the bubble field configuration provides the quasi-static theory, which implies that the bubble slowly evolves in time and the bubble field depends on $\xi = t - z$ [14][16]. The Maxwell equations for bubble fields in the quasi-static approximation are $(1/r)\partial (rE_r) / \partial r - \partial E_z / \partial t = \rho_{ion}$, $\partial (rB_r) / \partial r - \partial E_r / \partial \xi = 0$, $\partial E_r / \partial \xi - \partial B_\theta / \partial \xi = 0$, $\partial E_\theta / \partial \xi + \partial E_z / \partial r - \partial B_\phi / \partial \xi = 0$, where the cylindrical symmetry is assumed and cylindrical coordinates are used. The coordinates are normalized to $k_{pe} = c/\omega_{pe}$ and time is normalized to $\omega_{pe}^{-1}$, where $\omega_{pe} = 4\pi e^2 n_{ion}(r_b)/m$ is the plasma frequency and $n(r_b)$ is the ion density at the distance $r_b$ from the channel axis. It is easy to check that the fields $E = E_{ion} + E_{EM}$, $B = B_{ion} + B_{EM}$ are the solution of the Maxwell equations, where $E_{ion} = \epsilon_r (1/r) \int_0^{r_0} \rho_{ion} (r) r dr$, $B_{ion} = 0$, $E_{EM} = e_z \times B_{EM} - 2e_z \sigma(\xi)$, $B_{EM} = -\epsilon_\pi r |d\sigma(\xi)/d\xi| \sigma(\xi)$ is an arbitrary function of $\xi$, and $\rho_{ion} = -en_{ion}$.

The bubble field can roughly be presented as a sum of two terms: the ion field $(E_{ion}, B_{ion})$, which depends on ion distribution inside the bubble and the electromagnetic field $(E_{EM}, B_{EM})$, which depends on the bubble geometry and does not depend on the charge distribution.

The function $\sigma(\xi)$ is calculated for a homogeneous plasma in the nonlinear theory of the bubble [6]. The detailed calculation of $\sigma(\xi)$ for a plasma channel will be presented elsewhere. However, in the limit of a large bubble $r_b \omega_{pe}/c \gg 1$, which is relevant for typical experimental conditions, the derivation is simple. It follows from the nonlinear theory of the bubble [6] that at the bubble border ($r = r_b$) in the plane $z = 0$ (i) the velocity of the plasma electrons is close to $-\epsilon_z c$, (ii) the width of the electron sheath around the bubble is much less than the bubble size and (iii) the transverse force from the bubble field on the plasma electrons is close to zero: $F_\perp \approx -E_r - B_\perp \approx 0$. From the last condition $\sigma(\xi)/d\xi$ near the bubble center can be calculated: $\sigma(\xi)/d\xi = S(r_b) \equiv (1/2\xi^2) \int_0^\infty \rho_{ion} (r') r' dr'$. Therefore the bubble field near the bubble center takes the form:

$$E \approx \epsilon_r \left[ \frac{2r_b^2 S(r)}{r} - r S(r_b) \right] - \epsilon_z 2\xi S(r_b), \ B \approx -r S(r_b).$$

For homogeneous plasma $\rho_{ion}(r) = 1$ we recover known expression for the bubble field $E \approx \epsilon_r / (r \epsilon_z c/\omega_{pe} - \epsilon_r) / 2$ and $B \approx -\epsilon_z c/\omega_{pe}$. For the plasma channel with exponential profile discussed above the source function takes a form $S(r) = \{e^{r_0 (r^2 - r^2 + 2s r_s \delta)} / (4\pi r_b^2)\}$, for $r \geq r_0$ and $S(r) = 0$ for $r < r_0$, where $s = \exp(r/r_{ch}) (r - r_{ch}) + \exp(r_0/r_{ch}) (r_{ch} - r_0)$. It follows from the obtained solution that inside the vacuum part of the plasma channel $r < r_0$ the bubble field is purely electromagnetic $B_{ch} = E_r = 2(\xi/r) E_z$. In the plasma walls of the channel the ion field is added and can reverse the sign of the transverse electric field (see Fig.[1]). It follows from Fig.[2](b) and Fig.[2](b) that the electric field defined by Eq. (7) is in a very good agreement with that obtained in the numerical simulation. The field flattening at the bubble end is, however, a more subtle feature. An analytical solution recovering it requires a very careful description of the bubble electron sheath.

To check these scalings, we perform a series of 3d PIC simulations using the code 3DPL. We run the code in laboratory frame for simulations with the smallest pulse radius. The laser pulse is circularly polarized and has the envelope $a(t, r) = a_0 \exp (-r^2/R^2 - t^2/\tau^2)$. The envelope is cut to zero at $t = 2\tau, r = 2\lambda$. We assume laser wavelength $\lambda_0 = 800$ nm. We observe very little or no self-injection in the bubble when an empty on-axis channel is used. Thus, we inject an external witness electron bunch that co-propagates with the laser. It has the length $l_i$, initial energy $E_i$, and the total charge $N_i$. The simulation parameters are collected in the Table[1].

We selected an extremely short laser pulse with the duration $\tau_0 = 4 fs$, radius $R_0 = 8 \mu m$ and energy of $W_1 = 2.2 J$ as a stem for the similarity family. Although such short and energetic pulses do not exist yet, projects are under way to achieve similar parameters. Because this laser pulse is extremely short, just 1.5 cycles short, it is relatively easy to find a matched channel where it can propagate without modulations. The laser pulse
must keep its radius constant over a long distance so that diffraction and relativistic self-focusing stay in balance. According to the similarity rule, we scale the pulse radius and duration simultaneously by a factor 2 as shown in Table I. The most energetic pulse with $\tau_2 = 16$ fs, radius $R_2 = 32 \mu m$ and energy of $W_L = 141 J$ would roughly correspond to parameters of the Apollon laser under construction at École Polytechnique, France.

We scale simultaneously the laser pulse radius and duration with the same factor: $R = \alpha R_0$ and $\tau = \alpha \tau_0$ so that the aspect ratio $\Pi = R/c\tau \approx 6.7$ remains fixed, i.e. the pulses are pancake-like. This allows us to reproduce the wake field (the bubble) exactly without additional search for the optimal plasma channel parameters. The other parameters scale as $R_{ch} = \alpha R_{ch0}$, $n_e = \alpha^{-2} n_{e0}$, $E = \alpha^{-3} E_0$, $L_A = \alpha^3 L_{A0}$, and the particle energy scales as $\varepsilon_{\max} = \alpha^2 \varepsilon_{\max0}$. The acceleration lengths range from $L_A = 2 cm$ for the shortest laser, to $L_A = 1 m$ for the largest laser pulse. These acceleration distances can be simulated only using the Lorentz boost technology [17].

The simulation results for case of the largest laser pulse radius $R = 32 \mu m$, are shown in Fig. 2. The accelerating field, Fig. 2(a), is transversely uniform. It allows for mono-energetic acceleration of wide electron bunches. This transverse field uniformity is also observed in homogeneous plasmas. The on-axis profile of the accelerating field in the channel, Fig. 2(b), however, differs from that in the uniform plasma case. There is a region of flat accelerating field at the very back of the bubble. This region can be used to accelerate reasonably long witness bunches mono-energetically.

We inject a witness bunch that occupies about 10% of the bubble length. The energy spectra of the accelerated bunch is shown in Fig. 3(a) at different times. The bunch stays very monoenergetic for the first 30 cm of acceleration, because it was injected in the flat accelerating field part of the bubble. The relative r.m.s. energy spread $\sigma_E/E$, Fig. 3(b), of the bunch is merely 0.3% when it gains 7.5 GeV energy. This is much better than the ratio of the bunch length to the bubble length that is 10%. Because of dephasing, the bunch slowly leaves this flat $E-$field region and advances into the region with linearly growing $E_z-$field. The bunch gains a positive energy chirp as seen in the bunch longitudinal phase space, Fig. 3(c).

However, the quality of acceleration is best understood if we zoom in at the longitudinal phase space of the witness bunch, Fig. 3(d). The r.m.s relative width of the witness bunch energy uncertainty taken at a particular longitudinal position is merely $10^{-3}$ and is probably defined by numerical resolution of our code. This very narrow energy spread is due to the transverse uniformity of accelerating field in the bubble. The witness bunch experiences no focusing forces in the hollow part of the channel and very soon spreads in the transverse direction up to the field reversal point in the channel walls. Yet, all particles experience the same accelerating force.

Fig. 4 shows the nonlinear evolution of the laser pulse. Very different from the uniform plasma case, we observe no laser pulse shortening and no optical shock formation. The channel parameters were chosen to balance the dephasing length and the laser depletion length: $L_A = L_D = 1 m$.

In conclusion, we have shown that electron acceleration in deep plasma channels is scalable and the energy scalings favor ultra-short pancake-like laser pulses. The accelerating field has a flat field region, where the accelerating field does not depend on the longitudinal coordinate. This allows for monoenergetic acceleration of a witness bunch. We observe 0.3% overall bunch energy spectrum width and $10^{-3}$ r.m.s. relative energy uncertainty at a particular slice of the bunch.
This work has been supported by the Deutsche Forschungsgemeinschaft via GRK 1203 and SFB TR 18, by EU FP7 project EUCARD-2 and by the Government of the Russian Federation (Project No. 14.B25.31.0008) and by the Russian Foundation for Basic Research (Grants No. 13-02-00886, 13-02-97025).

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