Quantum tunneling, as a non-equilibrium quantum process, is one of the most interesting problems among modern quantum physics. It involves diverse phenomena of nature, such as alpha decay of nucleus [1], quantum cosmology [2], tunneling in Josephson’s junction [3], and biology [4, 5]. However, with the realization of Bose-Einstein condensate (BEC), the macroscopic quantum tunneling (MQT) is now experimentally feasible, such as the Josephson effects in a single BEC Josephson junction [6, 7], Landau-Zener tunneling [8, 9]. The interaction-assisted tunneling of single- and multipoles has been studied experimentally [10, 12] by the escape rate of individual atom in the trap. Such experiments have provided an unique platform to perform many-body physics experiments [13] with isolated environment, controllable potential and inter-atomic interactions.

The dynamics of quantum system in non-equilibrium state [14], has attracted scientists even more attention due to advances in experimental techniques. The recent development of ultrafast laser allows the attosecond angular streaking and attoclock techniques to investigate the temporal momentum distributions of electron, based on the tunneling ionization [15–20]. Quantum tunneling has no counterpart in the classical physics and its detailed process remains mysterious. Particularly, the controversial tunneling time problem is still unsettled [19, 21–23]. The BEC, degenerate as a single macroscopic wave function, is also an excellent platform to study the non-equilibrium dynamics, such as tunneling out of equilibrium [24, 25], because of the directly observable spatial and temporal scales. The tunability of the relevant parameters makes it a quantum simulator for the linear, or non-linear time-dependent Schrödinger equation. In this letter, the matter wave packet was used for the first time to demonstrate the tunneling dynamics in non-equilibrium state, which has only been experimentally studied with optical soliton [26]. Here, BEC is collectively excited by an external perturbation, and the tunneling exhibits as wave packet splitting, rather than the escape of individual particle. Our experiment is analogue to the strong field tunneling ionization experiment and the attosecond streaking technique. The leakages tunneling out of trap potential remain as condensates, and are a chronology of the temporal evolution of a collectively excited condensate, as the temporal imaging of valence electron motion [16, 18].

Our experimental setup is a hybrid trap [27] together with magnetic transport via a linear motor. The $^{87}$Rb BEC of $1 \times 10^5$ atoms was created in the trap with an effective trap depth of 580 nK, and a peak number density of $1.02 \times 10^{13}$ cm$^{-3}$. In our experiment, the location of ODT relative to the center of magnetic quadrupole trap is $(x, y, z) = (0, 0, z_0 = -90 \, \mu m)$. With gravity on the z direction, as shown in Fig. 1(a), the potential well in the vertical z direction can be expressed as:

$$ U(z) = -U_t \exp\left( -2(z - z_0)^2 - \frac{1}{w_0^2} \right) + \beta z \quad (1) $$

where the total effective downward force is $\beta = m_{Rb} g + \mu_R \frac{d \phi}{dz}$, $w_0$ is the ODT beam waist, and $U_t$ is the trap depth. The center of atomic cloud is then located at the stable point $dU/dz = 0$, where is at $z = z_0 - w_0^2 \beta / 4U_t$ by the approximation of the potential well as a quadratic function. The trapping force in the horizontal X-Y plane given by the magnetic quadrupole provides an extra confinement, while the atoms move out of ODT. The 3D illustration of the potential shape is as Fig. 1(c).

After creating BEC in the hybrid trap, a ‘kick’ was then applied to the condensate to perform quantum tunneling. The ‘kick’ is a sudden change in the ODT power. In our system, two different types of kick were employed. The first type is defined as a ‘step’ kick in which ODT power was reduced to a certain value and maintained during out-coupling time as shown in Fig. 1(b). The second type is a ‘pulse’ kick, in which ODT power was increased suddenly for 2 ms and turned back to the previous value. In order to partially compensate gravity, a magnetic gradient was provided by the compensation...
quadrupole field to reduce the downward acceleration as low as 0.77 m/s². The advantage of reducing effective gravitational acceleration is to have a longer observation time by avoiding atom’s quick escape out of the field of view. This quadrupole magnetic field also provided a confinement in X-Y plane.

The sudden change in the ODT power by ΔU is equivalent to shift the stable point (the bottom of the potential well) by $\sim \Delta U \frac{w^2 \beta}{4 \ell^2}$. Because the atomic cloud cannot follow such quick change, it sits on the original position and gains an energy of $\gamma^2 \frac{\beta^2 w^2}{4 \ell^2}$, where $\gamma = \Delta U / U_t$ is the ODT power change ratio. The kicks shifted the location of potential minima and effectively moves the condensate away from the potential minima. Then, a collective mode of BEC was excited due to such a non-adiabatic process [28]. The step-kick also opens a tunneling channel on the bottom of the hybrid trap, where the potential barrier is weakest because of the gravitational field. In our experiment, a typically step-kick is to reduce the trap depth by 30–40%.

A series of periodic matter wave pulses via quantum tunneling was generated as Fig. 2 which shows the time evolution of the BEC dynamics using a step-kick. 4-5 pulses tunnelled out of the BEC reservoir in 90 ms. In some cases with a larger condensate, the out-coupling pulsing can last for 200 ms to generate >10 matter wave pulses. The first pulse came out ~10 ms after the kick. A complete pulse coupled out of the trap in ~30 ms with peak flux $6.34 \times 10^6$ atom/s as shown in Fig. 2(d). The output pulses are with a pulse duration of 10 ms defined by full-width-half-maximum, and a repetition rate of ~50 Hz. The duty cycle is 50%, which implies a symmetrical internal oscillating motion of the BEC reservoir. Considering the X-Y confinement by the magnetic quadrupole trap, the sinusoidal trajectory of the pulses also suggests an accompanied initial transverse momentum. Our observation has been predicated by the numerical simulation under similar scenarios [29–31].

To compare with kicking, the ODT power was ramped down adiabatically in 2 sec to the same final trap depth as the step-kick. In such a case, neither additional kinetic energy, nor collective mode excitation is expected to give. A straight continuous diffusing leakage from the BEC reservoir was observed as Fig. 2(k). It clearly showed that the final trap strength of ODT was still capable of holding the reservoir against the effective gravitational force described earlier. The diffusing leakage goes even above the BEC reservoir, and has no signature of condensate. Conclusively, these are thermalized atoms energized by the ODT light assisted heating, and the condensate can tunnel out only by applying a kick with such a potential barrier.

To characterise the output pulses, the time of flight (TOF) measurement was performed for the first three pulses. The absorption images were taken after 20 ms free fall under only gravity, without the magnetic field, as shown in Fig. 3(a). There were certain amount of thermal atoms in the very beginning of the first output pulse, and the rest of the pulses shows no thermal expansion. That indicates them to be preserved as condensate. Figure 3(c) is a plot of pulse separation expansion with the free fall time. Its linearity indicates that the pulses coupled out of the reservoir are periodic, and the pulse rate showing no slowing down.

In the quantum tunneling, the tunneling efficiency $\eta$ depends on the potential barrier $U(z)$ and the energy of the matter wave $E$. Using the WKB approximation, it
where $N$ is that coupled atom number in the $i$th tunneling. In the case of pulsed output, the out-coupling time that is an important characteristic of quan-
tides of the barrier. With a constant tunneling efficiency from the trap by the kick.

10% of atoms did not acquire sufficient energy to escape
observed residual atom number (the red point), there is number in the trap using Eq.(3). In comparison with the
while, the gray point is the projection of residual atom
dissipate and the oscillation was not damped. Mean-
that the energy of atoms did not
illustrated in Fig. 4 using a pulse-kick. The coupling
values of the barrier. With a constant tunneling efficiency
efficiency decays exponentially as the barrier height ris-
show as exponential saturation and
oscillations versus the time of TOF.

can be expressed as [32]:
\[
\eta = \exp \left( -2 \int_{z_1}^{z_2} \sqrt{\frac{2m}{\hbar^2}} (U(z) - E) dz \right) \quad (2)
\]
where $z_1$ and $z_2$ are the classical turning points on both
sides of the barrier. With a constant tunneling efficiency $\eta$, the transmission probability is then exponentially de-
cay with time that is an important characteristic of quan-
tunneling. In the case of pulsed output, the out-
coupled atom number in the $i^{th}$ pulse is then given by:
\[
N_i = N_0 \eta (1 - \eta)^{i-1} \quad (3)
\]
where $N_i$ is the number of atoms in $i^{th}$ pulse, $N_0$ is total
number of the condensate. The exponential decay then exhibits as a discrete geometric progression. In Fig.
3 shows that the output pulses decay as described by
Eq.(3), i.e., a linear line in logarithmic scale. Thus, the
tunneling coefficient $\eta$ is a constant, and we conclude that $E$ remains constant during the course of the 80 ms
out-coupling time. That is, the energy of atoms did not dissipate and the oscillation was not damped. Mean-
while, the gray point is the projection of residual atom
number in the trap using Eq.(3). In comparison with the
observed residual atom number (the red point), there is
10% of atoms did not acquire sufficient energy to escape
from the trap by the kick.

Controlling the the transmission efficiency $\eta$ by tun-
ing the kick strength $E$ and the barrier height $U$ are
illustrated in Fig. 4 using a pulse-kick. The coupling
efficiency decays exponentially as the barrier height ris-
ing, as shown in Fig. 4(a). While the kick strength was
studied in Fig. 4(b), the potential barrier height was kept
as a constant. They show as exponential saturation and
qualitatively agree with Eq. (2).

Although the trajectory of the tunneling pulse in Fig. 2
has mapped out the internal dynamics of the condensate
reservoir, the conventional TOF method was utilized for
further confirmation. A step-kick was applied with a relatively
low kick strength to prevent any tunneling. After a
holding time, the BEC was then released from ODT. The position of the center of mass (CM) of the atomic cloud
was measured with 10 ms TOF. This was performed for
various holding times, from 10 ms to 30 ms with 2 ms
time resolution. Periodic oscillations of the CM displacements
were found and the variation of the initial velocity
out of trap was derived. The momentum oscillation ins-
side the ODT, the excited collective mode, was mapped
as the CM spatial oscillation after TOF. The result is
as Fig. 5, which shows sinusoidal oscillations for both $V_y$
and $V_z$, but with a $\pi/2$ relative phase shift. The time
period of oscillation is 20 ms that is in good agreement
with the repetition rate of pulse. The amplitudes of the
velocity oscillation are $V_y = 1.5 \mu$m/ms and the maxi-
mum kinetic energy per atom is then $^{87}$Rb calculated to
be 23.5 nK that is lower than the effective trap depth.
It is worthy to note that the created BEC was with a sound velocity of 0.60 $\mu$m/ms, and a chemical potential
of 3.8 nK.

The dynamics of BEC can be well described by the
time-dependent Gross-Pitaevski equation (GPE) that is
a mean-field approximation. While the non-linear inter-
action term in GPE, $a_s|\psi(r, t)|^2 \psi(r, t)$, becomes suffi-
ciently small and negligible in comparison with the ki-
etic and potential energy terms, then it becomes a stan-
ard time-dependent Schrödinger equation. Unlike the
single particle wave function that can only be fully ex-
plored by statistical measurement, the distribution of
BEC that exhibits the probability density by its num-
ber density can be imaged using a CCD camera by a
single picture. We hence used the quantum tunneling of
BEC induced by an energetic kick, which is considered
as a giant wave function with slow motion dynamics, to
resolve the controversal "tunneling time" problem [33].

A series of images were taken with high time resolution
from 0 ms to 40 ms, while the first output pulse is com-
pletely coming out by exerting a step-kick. On the right

\[ \begin{align*}
\psi(r, t) &= \frac{1}{\sqrt{\Omega}} \sum_{n} \sqrt{n} \phi_n(r) \exp\left[i \left(\frac{n^2}{2m} t - \frac{1}{\hbar} \int \int \left(\frac{\delta^2}{\delta \psi^2} - \frac{\hbar^2 \nabla^2}{2m} - V(r) \right) \psi^2 \right) dt \right] \\
\phi_n(r) &= \frac{1}{\sqrt{2^n n!}} (\frac{\hbar}{2m})^{3/4} (\frac{\Omega}{2})^{1/4} e^{-\frac{r^2}{2\Omega}} L_n \left(\frac{\hbar}{\sqrt{2m\Omega}} r\right) \\
L_n(x) &= \frac{1}{n!} \left(\frac{\hbar}{2m\Omega}\right)^{3/4} \left(\frac{2m\Omega}{\hbar}\right)^{1/4} e^{\frac{\hbar}{2m\Omega} r^2} \frac{d^n}{dr^n} \left( e^{-\frac{\hbar}{2m\Omega} r^2} \right) \\
&= \frac{1}{n!} \left(\frac{\hbar}{2m\Omega}\right)^{3/4} \left(\frac{2m\Omega}{\hbar}\right)^{1/4} e^{\frac{\hbar}{2m\Omega} r^2} \frac{d^n}{dr^n} \left( e^{-\frac{\hbar}{2m\Omega} r^2} \right)
\end{align*} \]
FIG. 5. The internal dynamics of BEC by execution of the step-kick. The horizontal velocity $V_y$ and vertical velocity $V_z$ are the initial velocities of the condensate in the trap caused by kick. The velocities are extracted from the 10 ms TOF measurement of the atoms with various holding time after giving a step-kick, but without generating matter wave pulse.

![Graph showing the internal dynamics of BEC](image)

FIG. 6. The time evolution number density along horizontal y-direction presented by false color. The horizontal axis represents out-coupling time of atoms from the reservoir after a kick. There are three stages of the tunneling process: the initial stage, tunneling and final stages.

![Diagram showing the time evolution number density](image)

side of Fig. 6 the typical images at 5 ms, 27 ms and 37 ms show the BEC distribution for the initial, tunneling and final stages, respectively. To observe the time evolution of tunneling, the images with 2 ms time gap were taken and processed by summing each pixel in y-direction to convert them as an one-dimensional (z-direction) distribution, as shown in the left of Fig. 6. A steep change in the velocity of the lower end of the tunneling wave packet appears in the second stage between 20-30 ms. After the completion of tunneling 30-40 ms, the velocity is then slowed down. In contrast to the classical physics, where the potential barrier slows down particle velocity, the wave packet “suddenly” emerges from the other side of the barrier, as wave packet splitting in the quantum tunneling. Although the tunneling wave packet grows up with a finite time period, it may look “instantaneous” and causes the no-tunneling-time-delay measurement in [19]. However, as shown in our experiment, quantum tunneling has no classical analogy and its entire process can not be properly described by the particle transmission [23].

The dynamics of quantum tunneling was demonstrated using BEC by exerting a non-adiabatic kick, which results in a collective rotational motion in the condensate. In our experiment, no energy dissipation was observed during more than 80 ms out-coupling time, and the atomic cloud tunneling out remains in BEC phase. The trace of tunneling leakages is a temporal map to the internal evolution, as a chronology of its dynamics. This methodology was employed in the spatiotemporal motion of valence electron [17, 18]. Our results shed light on the tunneling ionization process. Due to the nature of matter wave in quantum regime, the “tunneling time” that can only be well-defined from the point of view of particle is inadequate [23]. The BEC is a quantum simulator not only for many-body physics, but can also be a giant wave function to simulate the quantum dynamics of single particle.

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