Toward a Formal Model of the Shifting Relationship between Concepts and Contexts during Associative Thought

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Abstract. The quantum inspired State Context Property (SCOP) theory of concepts is unique amongst theories of concepts in offering a means of incorporating that for each concept in each different context there are an unlimited number of exemplars, or states, of varying degrees of typicality. Working with data from a study in which participants were asked to rate the typicality of exemplars of a concept for different contexts, and introducing a state-transition threshold, we built a SCOP model of how states of a concept arise differently in associative versus analytic (or divergent and convergent) modes of thought. Introducing measures of expected typicality for both states and contexts, we show that by varying the threshold, the expected typicality of different contexts changes, and seemingly atypical states can become typical. The formalism provides a pivotal step toward a formal explanation of creative thought processes.

Keywords: Associative thought; concepts; context dependence; contextual focus; creativity; divergent thinking; dual processing; SCOP

1 Introduction

This paper unites two well-established psychological phenomena using a quantum-inspired mathematical theory of concepts, the State-C0ntext-Property (SCOP) theory of concepts. The first phenomenon is that the meaning of concepts shifts, sometimes radically, depending on the context in which they appear [19, 13, 9]. It is this phenomenon that SCOP was developed to account for [3–5]. Here we use SCOP to model a different though related psychological phenomenon. This second psychological phenomenon was hinted at in the writings of a number of the pioneers of psychology, including Freud [17], Piaget [10], and William James [20]. They and others have suggested that all humans possess two distinct ways of thinking. The first, sometimes referred to as divergent or associative thought, is thought to be automatic, intuitive, diffuse, unconstrained, and conducive to unearthing remote or subtle associations between items that share features, or that are correlated but not necessarily causally related. This may yield a promising idea or solution though perhaps in a vague, unpolished form. There is evidence that associative thinking involves controlled access to, and integration of, affect-laden material, or what Freud referred to as “primary process” content [17,
Associative thought is contrasted with a more controlled, logical, rule-based, convergent, or analytic mode of thought that is conducive to analyzing relationships of cause and effect between items already believed to be related. Analytic thought is believed to be related to what Freud termed “secondary process” material.

A growing body of experimental and theoretical evidence for these two modes of thought, associative and analytic, led to hypothesis that thought varies along a continuum between these two extremes depending on the situation we are in [7, 15, 17, 3, 11, 13, 14, 20]. The capacity to shift between the two modes is sometimes referred to as contextual focus, since a change from one mode of thought to the other is brought about by the context, through the focusing or defocusing of attention [11, 12]. Contextual focus is closely related to the dual-process theory of human cognition, the idea that human thought employs both implicit and explicit ways of learning and processing information [16, 8]. It is not just the existence of two modes of thought but the cognitive consequences of shifting between them, that we use SCOP to model in this paper.

2 The SCOP Theory of Concepts

The SCOP formalism is an operational approach in the foundations of quantum mechanics in which a physical system is determined by the mathematical structure of its set of states, set of properties, the possible (measurement) contexts which can be applied to this entity, and the relations between these sets. The SCOP formalism is part of a longstanding effort to develop an operational approach to quantum mechanics known as the Geneva-Brussels approach [1]. If a suitable set of quantum axioms is satisfied by the set of properties, one recovers via the Piron-Šolér representation theorem the standard description of quantum mechanics in Hilbert space [1]. The SCOP formalism permits one to describe not only physical entities, but also potential entities [2], which means that SCOP aims at a very general description of how the interaction between context and the state of an entity plays a fundamental role in its evolution. In this work we make use of the SCOP formalism to model concepts, continuing the research reported in [4, 5, 3, 6].

Formally a conceptual SCOP entity consists of three sets $\Sigma$, $\mathcal{M}$, and $\mathcal{L}$: the set of states, the set of contexts and the set of properties, and two additional functions $\mu$ and $\nu$. The function $\mu$ is a probability function that describes how state $p$ under the influence of context $e$ changes to state $q$. Mathematically, this means that $\mu$ is a function from the set $\Sigma \times \mathcal{M} \times \Sigma$ to the interval $[0, 1]$, where $\mu(q, e, p)$ is the probability that state $p$ under the influence of context $e$ changes to state $q$. We write

$$\mu : \Sigma \times \mathcal{M} \times \Sigma \rightarrow [0, 1]$$

$$(q, e, p) \mapsto \mu(q, e, p) \quad (1)$$

The function $\nu$ describes the weight, which is the renormalization of the applicability, of a certain property given a specific state. This means that $\nu$ is a function from the set $\Sigma \times \mathcal{L}$ to the interval $[0, 1]$, where $\nu(p, a)$ is the weight of property $a$ for the concept in state $p$. We write

$$\nu : \Sigma \times \mathcal{L} \rightarrow [0, 1]$$

$$(p, a) \mapsto \nu(p, a) \quad (2)$$
Thus the SCOP is defined by the five elements $(\Sigma, M, L, \mu, \nu)$. States of a concept are denoted by means of the letters $p, q, r, \ldots$ or $p_1, p_2, \ldots$, and contexts by means of the letters $e, f, g, \ldots$ or $e_1, e_2, \ldots$. When a concept is not influenced by any context, we say it is in its ground state, and we denote the ground state by $\hat{p}$. The unit context, denoted $1$, is the absence of a specific context. Hence context $1$ leaves the ground state $\hat{p}$ unchanged. Exemplars of a concept are states of this concept in the SCOP formalism.

Note that in SCOP, concepts exist in what we refer to as a state of potentiality until they are evoked or actualized by some context. To avoid misunderstanding we mention that $\mu(p, e, q)$ is not a conditional probability of transitioning from state $p$ to $q$ given that the context is $e$. Contexts in SCOP are not just conditions, but active elements that alter the state of the concept, analogous to the observer phenomenon of quantum physics, where measurements affect the state of the observed entity. Indeed, a SCOP concept can be represented in a complex Hilbert space $\mathcal{H}$. Each state $p$ is modelled as a unitary vector (pure state) $|p\rangle \in \mathcal{H}$, or a trace-one density operator (density state) $\rho_p$. A context $e$ is generally represented by a linear operator of the Hilbert space $\mathcal{H}$, that provokes a probabilistic collapse by a set of orthogonal projections $\{P_e^i\}$. A property $a$ is always represented by an orthogonal projector $P_a$ in $\mathcal{H}$ respectively. The contextual influence of a context on a concept is modelled by the application of the context operator on the concept's state. A more detailed explanation can be found in [4, 5].

3 The Study

Our application of SCOP made use of data obtained in a psychological study of the effect of context on the typicality of exemplars of a concept. We now describe the study.

3.1 Participants and Method

Ninety-eight University of British Columbia undergraduates who were taking a first-year psychology course participated in the experiment. They received credit for their participation.

The study was carried out in a classroom setting. The participants were given questionnaires that listed eight exemplars (states) of the concept HAT. The exemplars are: state $p_1$: ‘Cowboy hat’, state $p_2$: ‘Baseball cap’, state $p_3$: ‘Helmet’, state $p_4$: ‘Top hat’, state $p_5$: ‘Coonskin cap’, state $p_6$: ‘Toque’, state $p_7$: ‘Pylon’, and state $p_8$: ‘Medicine Hat’. They were also given five different contexts. The contexts are: the default or unit context $e_1$: The hat, context $e_2$: Worn to be funny, context $e_3$: Worn for protection, context $e_4$: Worn in the south, and context $e_5$: Not worn by a person.

The participants were asked to rate the typicality of each exemplar on a 7-point Likert scale, where 0 points represents “not at all typical” and 7 points represents “extremely typical”. Note that all the contexts except $e_1$ make reference to the verb “wear”, which is relevant to the concept HAT. The context $e_1$ is included to measure the typicality of the concept in a context that simulates the pure meaning of a HAT, i.e. having no contextual influence, hence what in SCOP is meant by “the unit context”.
3.2 Results

A summary of the participants’ ratings of the typicality of each exemplar of the concept HAT for each context is presented in Table 1. The contexts are shown across the top, and exemplars are given in the left-most column. For each state and context in the table there is a pair of numbers \((a; b)\). \(a\) represents the averaged sum of the Likert points across all participants (average typicality). \(b\) is the context dependent state-transition probability. The bottom row gives the normalization constant of each transition probability function. Grey boxes have transition probability below the threshold \(\alpha = 0.16\).

| Exp. Data      | \(e_1\)   | \(e_2\)   | \(e_3\)   | \(e_4\)   | \(e_5\)   |
|----------------|-----------|-----------|-----------|-----------|-----------|
| \(p_1\) Cowboy hat | (5.44;0.18) | (3.57;0.14) | (3.06;0.13) | (6.24;0.28) | (0.69;0.05) |
| \(p_2\) Baseball cap | (6.32;0.21) | (1.67;0.06) | (3.16;0.13) | (4.83;0.21) | (0.64;0.04) |
| \(p_3\) Helmet | (3.45;0.11) | (2.19;0.08) | (6.85;0.28) | (2.85;0.13) | (0.86;0.06) |
| \(p_4\) Top hat | (5.12;0.17) | (4.52;0.17) | (2.00;0.08) | (2.81;0.12) | (0.92;0.06) |
| \(p_5\) Coonskincap | (3.55;0.11) | (5.10;0.19) | (2.57;0.10) | (2.70;0.12) | (1.38;0.1) |
| \(p_6\) Toque | (4.96;0.16) | (2.31;0.09) | (4.11;0.17) | (1.52;0.07) | (0.77;0.05) |
| \(p_7\) Pylon | (0.56;0.02) | (5.46;0.21) | (1.36;0.05) | (0.68;0.03) | (3.95;0.29) |
| \(p_8\) Medicine Hat | (0.86;0.02) | (1.14;0.04) | (0.67;0.03) | (0.56;0.02) | (4.25;0.31) |
| \(N(e)\) | 30.30      | 25.98      | 23.80      | 22.22      | 13.51      |

Table 1. Summary of the participants’ ratings of the typicality of the different exemplars of the concept HAT for different contexts. See text for detailed explanation.

4 Analysis of Experimental Data and Application to the Model

In this section we use SCOP to analyze the data collected in the experiment, and apply it to the development of a tentative formal model of how concepts are used differently in analytic and associative thought.

4.1 Assumptions and Goals

We model the concept HAT by the SCOP \((\Sigma, \mathcal{M}, \mathcal{L}, \mu, \nu)\) where \(\Sigma = \{p_1, \ldots, p_8\}\) and \(\mathcal{M} = \{e_1, \ldots, e_5\}\) are the sets of exemplars and contexts considered in the experiment (see table 1). We did not consider properties of the concept HAT, and hence \(\mathcal{L}\) and \(\nu\) are not specified. This is a small and idealized SCOP model, since only one experiment with a fairly limited number of states and contexts is considered, but it turned out to be sufficient to carry out the qualitative analysis we now present. Moreover, it will be clear that the approach can be extended in a straightforward way to the construction of more extended SCOP models that include the applicabilities of properties. Note also that the Hilbert space model of this SCOP can be constructed following the procedure explained in [5].

Recall how the participants estimated the typicality of a particular exemplar \(p_i\), \(i \in \{1, \ldots, 8\}\) under a specified context \(e_j\), \(j \in \{1, \ldots, 5\}\) by rating this typicality from
0 to 7 on a Likert scale. Since these ratings play a key role in the analysis, we introduce the Likert function $L$:

$$L : \Sigma \times \mathcal{M} \to [0,7]$$

$$(p,e) \mapsto L(p,e)$$

where $L(p,e)$ is the Likert score averaged over all subjects given to state $p$ under context $e$.

We also introduce the total Likert function $N$ which gives the total Likert score for a given context:

$$N : \mathcal{M} \to [0,56]$$

$$e \mapsto N(e) = \sum_{p \in \Sigma} L(p,e),$$

The Likert score $L(p,e)$ is not directly connected to the transition probability $\mu(p,e,\hat{p})$ from the ground state of a concept to the state $p$ under context $e$. However, the renormalized value of $L(p,e)$ to the interval $[0,1]$ provides a reasonable estimate of the transition probability $\mu(p,e,\hat{p})$. Hence we introduce the hypothesis that the renormalized Likert scores correspond to the transition probabilities from the ground state, or

$$\mu(p,e,\hat{p}) = \frac{L(p,e)}{N(e)}$$

This is an idealization since the transition probabilities are independent although correlated to this renormalized Likert scores. In future work we plan experiments to directly measure the transition probabilities.

Let us pause briefly to explain why these functions have been introduced. If we consider the unit context, it would be natural to link the typicality to just the Likert number. For example, for the unit context, exemplar $p_1$: ‘Cowboy hat’ is more typical than $p_6$: ‘Toque’ because $L(p_6,e_1) < L(p_1,e_1)$ (see table 1). If one examines more than one context, however, such a conclusion cannot easily be drawn. For example, consider the exemplar $p_7$: ‘Pylon’, under both the context $e_2$: ‘Worn to be funny’ and context $e_5$: ‘Not worn by a person’, we have that $L(p_7,e_5) < L(p_7,e_2)$, but $p_7$ is more typical under context $e_5$ than under $e_2$. This is because $N(e_5) < N(e_2)$, i.e. the number of Likert points given in total for context $e_2$ is much higher than the number of Likert points given in total for the context $e_5$. This is primarily due to the fact that Likert points have been attributed by participant per context.

Note that $\frac{N(e)}{8}$ is the average typicality of exemplars under context $e$, and the average transition probability (renormalized typicality) is $\mu^* = \frac{1}{8}$ for all the contexts. We want to identify the internal structure of state transitions of a concept making use of the typicality data. Therefore we define a transition probability threshold $\alpha \in [0,1]$. We say that $p \in \Sigma$ is improbable for context $e \in \mathcal{M}$ if and only if $\mu(p,e,\hat{p}) < \alpha$, meaning that it is improbable that a transition will happen under this context to states with transition probability lower than the threshold. By means of this transition threshold we can also express the idea that for a given concept, there are only a limited number of possible...
transitions from the ground state to other states. We express this mathematically by introducing a new collection of transition probabilities, such that for this new collection the transition probability is equal to zero when it is below this threshold, thereby prohibiting transitions from a specific context to states that we called improbable for this context for the original collection of transition probabilities we started with. Since the sum of all transition probabilities over all possible states that can be transitioned to needs to be equal to 1 for any set of transition probabilities corresponding to an experimental situation, next to equaling to zero the transition probabilities below the threshold, we need to renormalize the remaining transition probabilities. Hence, if we denote \( \mu_\alpha \) the new collection of transition probabilities, we have

\[
\mu_\alpha(p,e,\hat{p}) = \begin{cases} 
0 & \text{if } \mu(p,e,\hat{p}) \leq \alpha, \text{else} \\
\frac{\mu(p,e,\hat{p})}{\sum_{p \in \Sigma, \alpha < \mu(p,e,\hat{p})} \mu(p,e,\hat{p})} & \end{cases}
\]

Thus, after imposing a threshold, a concept becomes a more constrained structure. At first glance this may appear to be an artificial bias in our analysis. However, we do not introduce the threshold to arbitrarily eliminate some exemplars, but to study the evolution of this \textit{biased structure} as the threshold changes. This leads to the next step, which is to model what happens to the exemplars and contexts when there is a shift between associative and analytic thought modes of thought.

For each exemplar \( p \) and context \( e \) such that \( \mu(p,e,\hat{p}) > \alpha \) we have that \( \mu_\alpha(p,e,\hat{p}) > \mu(p,e,\hat{p}) \). The new collection of transition probabilities induced by \( \alpha \) corresponds to the fact that in an associative mode we gain access to remote meanings while in an analytic mode of thought we lose them. Hence, the transition probability to an unusual exemplar \( p \), which is zero for a high setting of transition probabilities (and thus considered a \textit{strange exemplar} for the concept within this setting) could rise above zero for the new \( \alpha \)-induced setting of transition probabilities. This occurs when the strange exemplar \( p \) is typical \textit{compared to} other exemplars under context \( e \), i.e., \( \mu(p,e,\hat{p}) \) is high enough. Thus, one shifts to a more associative mode of thought by decreasing the threshold, thereby enabling unusual exemplars to come into play. We propose that this is the mechanism that underlies contextual focus [3, 11, 12].

5 Analysis of the States and Contexts

5.1 Expected Context Typicality

Since the SCOP model is a probabilistic model, the typicalities estimated by the participants in the experiment by numbers on the Likert scale are not the expected typicalities, because the transition probabilities must also be taken into account. This expresses the potentiality (and corresponding probability), which is fundamental to the SCOP approach. Indeed, it makes only sense to speak of the “potential typicality” of a certain exemplar, and this potentiality is expressed by the value of the transition probability to this exemplar, which means that this “potential typicality” is the “expected typicality” which equals to the product of the Likert value with the transition probability,
The hat and Not worn by a person when $\alpha = 0$, the horizontal line at $\mu(:,:,\hat{p}) = 0.16$ shows the transition threshold used to identify atypical exemplars in table 1.

This provides now also a means of introducing a genuine measure of context typicality, using the state transition probability model, and the mode of thought determined by the threshold $\alpha$. For a given context $e$ and a given threshold $\alpha$ the “expected typicality $T(e, \alpha)$ of this context $e$” is given by

$$T(e, \alpha) = \sum_{p \in \Sigma} L(p, e) \cdot \mu_\alpha(p, e, \hat{p})$$

For example, consider the context $e_5$: Not worn by a person and the unit context $e_1$: The hat. We have $2.87 = T(e_5, 0) < T(e_1, 0) = 4.82$. But most of the contributions to $T(e_5, 0)$ come from the exemplars $p_7$: ‘Toque’ and $p_8$: ‘Medicine Hat’. Indeed, $L(p_7, e_5)\mu(p_7, e_5, \hat{p}) + L(p_8, e_5)\mu(p_8, e_5, \hat{p}) = 2.46$. On the other hand, $e_1$ is the most typical context at zero threshold because many exemplars have a high Likert score. Thus, the values of its transitions probabilities $\mu(:,:,\hat{p})$ are spread more homogeneously among the exemplars, leading to a flatter distribution with smaller probability values than the more typical exemplars of the $e_5$ distribution (see figure 1). If the threshold $\alpha$ is sufficiently high ($\alpha \geq 0.21$ in this case), $\mu_\alpha(:,:,\hat{p})$ becomes the zero function because all the states in context $e_1$ are improbable for the threshold $\alpha$, but context $e_5$ maintain their most probable states ($p_7$: ‘Toque’ and $p_8$: ‘Medicine Hat’), because the transition probabilities of the states $p_7$ and $p_8$ are higher than $\alpha$. Furthermore, the transition probabilities are amplified in the renormalized transition function $\mu_\alpha(:,:,\hat{p})$ because $p_1, ..., p_6$ are improbable in context $e_5$ for the threshold $\alpha = 0.21$. This observation makes it possible to explain how we can use the transition threshold to gain a clearer picture of what is going on here.

These results reveal a dependency relationship between the threshold and the expected context typicality $T$. Figure 2 shows the function $T(e, \alpha)$ for different values of $\alpha$ for each context. What actually comes to mind depends both on alpha and on the context you are in, and Figure 2 expresses both of these. The top bar of each graph shows the relevance of the context for the corresponding value of alpha. The different coloured bars indicate which exemplars are available to transition to for the given value of the threshold $\alpha$. We posit that the more different coloured bars there are, the greater the potential for entanglement of the different exemplars. The area of the filled box for a particular exemplar represents the transition probability with re-
Fig. 2. Relevance of the contexts considered in the experiment, with respect to the threshold $\alpha$.

spect to the total size of the bar for the corresponding alpha. We considered the values of $\alpha \in \{0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35\}$ to show how different exemplars remain able to be activated for different contexts, and how the probability distribution is affected by the renormalization. First, note that the expected typicality is an increasing function with respect to $\alpha$ until it reaches a maximal value that deactivates all exemplars. This is because the threshold is imposed to deactivate exemplars for which the transition probability is not sufficiently high, thus the remaining exemplars after imposing the threshold are those with higher transition probabilities. This implies that these remaining exemplars have comparatively higher typicality. Thus for the renormalized probability distribution, their expected typicality increases. Secondly, note that contexts $\{e_1, e_2, e_3, e_4\}$ are qualitatively similar for small values of $\alpha$, i.e. all the exemplars can be activated with small probability values. However, the differences among the contexts are amplified as the threshold increases. This implies that in our model, an associative mode of thought permits activation of more exemplars at the cost of losing the meaningful specificity of the context. In contrast, in an analytic mode of thought, fewer exemplars are activated and they have higher transition probabilities due to the amplification of their probability values induced by the renormalization $\mu_{\alpha}$. Thus one is able to clearly differentiate the meaning of each context, at the cost of having less exemplars available for transition to.

Note that the threshold that makes no transition possible (all exemplars deactivated) varies with the context. The value required to deactivate all exemplars reflects the flatness of the probability distribution at $\alpha = 0$. The flatter the distribution, the smaller the value of $\alpha$ required to deactivate all exemplars. Indeed, in our model, context $e_1 =$The hat requires the smallest threshold. This is because as $e_1$ gets flatter, the transition probabilities at $\alpha = 0$ have values close to the average probability $\mu^* = \frac{1}{8}$. For context $e_5$, the qualitative behavior with respect to $\alpha$, i.e. the deactivation of certain exemplars as the threshold $\alpha$ increases, is the same as in the other contexts. However, context $e_5$ differs from other contexts in two important respects. First, $e_5$ is the only context that remains activated for exemplar $p_8$ : ‘Medicine Hat’ for $\alpha > 0$, and is the only context that deactivates the exemplars $p_1$ : ‘Cowboy hat’ and $p_2$ : ‘Baseball cap’ for small val-
ues of $\alpha$. Secondly, $e_5$ is the context that requires the largest threshold to deactivate all its exemplars. This is because $e_5$ has the most rugged distribution at $\alpha = 0$. Indeed, most of the transition probability at $\alpha = 0$ is concentrated on exemplars $p_7$ ‘Pylon’ and $p_8$ ‘Medicine Hat’. These differences between $e_5$ and the rest of the contexts reflect the semantic opposition that context $e_5=\text{Not worn by a person}$ has with the other contexts that state circumstances in which the concept HAT is elicited in a common-sense meaningful way.

| $T(e)$ | # typical exemplars | Context relevance at $\alpha = 0$ | Type of exemplar |
|--------|---------------------|----------------------------------|------------------|
| Large  | Large               | High                             | Very Representative |
| Medium | Large               | Medium                           | Poorly representative |
| Medium | Small               | Low                              | Unexpected        |
| Small  | Small               | Low                              | Non-representative |

Table 2. Types of contexts and the type of exemplars they have.

6 Discussion and Future Directions

This paper builds on previous work that uses, SCOP, a quantum-inspired theory of concepts, and psychological data, to model conceptual structure, and specifically semantic relations between the different contexts that can influence a concept. Here we focus on how these contexts come into play in analytic versus associative thought. It is suggested that the notion of a transition threshold that shifts depending on the mode of thought, as well as newly defined notions of state and context expected typicality, are building blocks of a formal theory of creative thinking based on state transition probabilities in concepts. We posit that the more exemplars come to mind given a particular context and mode of thought, the greater the potential for entanglement of the different exemplars. The model is consistent with the occasional finding of unexpected meanings or interpretations of concepts. We propose that these new associations occur when a new context creates an unlikely set of new exemplars, which may potentially they exert quantum-like effects on one another. The paper also strengthens previous evidence that in order to account for the multiple meanings and flexible properties that concepts can assume, it is necessary to incorporate context into the concept definition.

The model developed here is small and idealized. In future research we plan to extend and generalize it. An interesting parameter that we have not yet explored is the sum of the expected typicality of a single exemplar with respect to the set of contexts. We believe that this can be interpreted as a measure of the exemplar representativeness given in Table 2. Much as the expected typicality of any given context is subject to change, unexpected exemplars could become more or less representative if the transition threshold changes. Further analysis could provide a richer description of this. Another interesting development is to study the structure of the transition probabilities when applying successive renormalizations induced by sequences of thresholds imposed to the concept structure. We could establish, straight from the data, a threshold-dependent hierarchy of pairs $(p, e)$, that gives an account of the context-dependent semantic distance between
exemplars. This could be used to model the characteristic, revealing, and sometimes surprising ways in which people make associations.

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