Transverse Momentum Spectra of $J/\psi$ and $\psi'$ Mesons from Quark Gluon Plasma Hadronization in Nuclear Collisions

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Recent results on transverse mass spectra of $J/\psi$ and $\psi'$ mesons in central Pb+Pb collisions at 158 A-GeV are considered. It is shown that those results support a hypothesis of statistical production of charmonia at hadronization and suggest the early thermal freeze–out of $J/\psi$ and $\psi'$ mesons. Based on this approach the collective transverse flow velocity of hadronizing quark gluon plasma is estimated to be $\langle v_T^f \rangle \approx 0.2$. Predictions for transverse mass spectra of hidden and open charm mesons at SPS and RHIC are discussed.

**Key words:** statistical production of charmonia, early freeze–out, transverse flow, inverse slope parameter

The idealized concepts of chemical (hadron multiplicities) and thermal (hadron momentum distributions) freeze-outs were introduced to interpret data on hadron production in relativistic nucleus–nucleus ($A+A$) collisions [1]. The first experimental results on yields and transverse mass ($m_T = \sqrt{p_T^2 + m^2}$) spectra suggested the following scenario: for the most abundant hadron species ($\pi, N, K, \Lambda$) the chemical freeze-out, which seems to coincide with the hadronization of the quark gluon plasma (QGP), is followed by the thermal freeze–out occurring at a rather late stage of the $A+A$ reaction. In this letter we discuss whether the new data of NA50 [2] on transverse mass spectra of $J/\psi$ and $\psi'$ mesons produced in central Pb+Pb collisions at 158 A-GeV are consistent with the above picture. Our consideration is based on a recent hypothesis [3] of statistical production of charmonia at hadronization. We further suggest that thermal freeze–out of charmonia coincides with the hadronization transition. The consequences of this assumptions are in agreement with existing data on $m_T$ spectra. They allow to extract the transverse flow velocity of the hadronizing QGP and lead to new predictions which can be tested by future measurements.

Rescattering among partons and, in the late stage of the reaction process, hadrons created in relativistic $A+A$ collisions should cause local thermalization and the development of transverse collective flow of matter. Thus the final transverse motion of hadrons can be considered as a convolution of the transverse flow velocity of the freezing–out matter element with the thermal motion of the hadrons in the rest frame of this element. The resulting $m_T$ spectrum has approximately exponential shape:

$$\frac{1}{m_T} \cdot \frac{dn}{dm_T} \approx C \cdot e^{-m_T/T^*}, \quad (1)$$

where $C$ and $T^*$ are a normalization and an inverse slope parameters, respectively. The $T^*$ parameter is related to the thermal freeze–out temperature $T_f$ and the mean transverse flow velocity $\langle v_T^f \rangle$. In the nonrelativistic approximation ($m_T < 2m$) contributions from thermal and collective particle motions can be separated and the inverse slope parameter of the observed hadron spectrum can be expressed as [4]:

$$T^* = T_f + \frac{2}{\pi} \cdot m \cdot \langle v_T^f \rangle^2. \quad (2)$$

Note that factor $2/\pi$ appears in Eq. (2) due to the cylindrical geometry expected in high energy $A+A$ collisions (see [4] for details). In the case of the simultaneous thermal freeze–out of all hadrons the inverse slope parameter $T^*$ should follow the linear dependence on $m$ given by Eq. (3). In Fig. 1 we present a compilation of inverse slope parameters measured for various hadron species in central Pb+Pb collisions at 158 A-GeV. It is observed that the simplistic hypothesis of common thermal freeze–out of all hadrons is not supported by the data.

The freeze–out condition for pions can be determined independently of the other hadron species from the results on two pion correlations. This procedure leads to the following values of the pion thermal freeze–out parameters [5]:

$$T_f = 120 \pm 12 \text{ MeV}, \quad \langle v_T^f \rangle = 0.5 \pm 0.12. \quad (3)$$

In Fig. 1 the upper solid line indicates the dependence given by Eq. (2) for the pion freeze–out parameters. The values of $T^*$ for $K^+$ and $p$ are close to those given by this line [4]. The results on $T^*$ for other hadrons are in general below the ‘pion freeze–out line’. This may be
The results for the following hadrons are compiled from: $\pi$ mesons, $K$ mesons and protons [6] (closed triangles up), $\rho$ and $\omega$ mesons [7] (open square), $\phi$ mesons [8] (open triangles up) and [7] (open square), $\Lambda$ hyperons [9] (closed triangle down), $\Xi$ hyperons [10] (open triangles down), $\Omega$ hyperons [11] (closed diamonds), $J/\psi$ mesons [2] (closed square). The filled circles are predictions of the model for $\psi'$ and D mesons. The upper solid line corresponds to Eq. (2) with the freeze–out parameters (3) extracted from the two pion correlation data [5]. The lower solid line is given by Eq. (5) with parameters $T_H = 175$ MeV and $\langle v_T^H \rangle = 0.19$ which correspond to the QGP hadronization.

interpreted in the following way. For hadrons with “low” masses and large interaction cross sections the thermal freeze–out takes place at the late stage of the expansion, i.e., at large $\langle v_T^H \rangle$ and small $T_f$ given by Eq. (3). Heavy hadrons (due to large mass and smaller cross section) are expected to decouple from the system early thus leading to the smaller values of $T^*$ than those expected from the pion freeze–out line. This is seen from the $m_T$-slopes of the hyperons and most clearly can be illustrated by a small $T^*$ value of the $\Omega$ hyperon shown in Fig. 1. This explanation is in the line with the typical hydrodynamical calculations (see, for instance, [12]) – the inverse slope parameter $T^*$ of the hadron is smaller, if it decouples early, i.e., at higher temperatures $T_f$, but smaller velocity $\langle v_T^H \rangle$.

A more careful analysis requires the combination of the hydrodynamic approach at the early stage of the expansion with cascade model calculations at the latest stage [13, 14].

The yields of produced light hadrons are surprisingly well reproduced within the statistical approach to particle production. The temperature parameter extracted from the fit to the multiplicity data is found to be $T_H = 175\pm10$ MeV [15–17]. This chemical freeze–out temperature appears to be quite close to the expected temperature of the transition between the hadron gas and the quark gluon plasma. This fact suggests the possibility to ascribe the observed statistical properties of hadron yield systematics at high energies to the statistical nature of the hadronization process, i.e., the hadrons are “born” at hadronization temperature $T_H$ into the state of chemical equilibrium and their multiplicities are frozen–out afterwards.

Within this approach one can make a rough estimate of a lower limit for the value of the measured inverse slope parameters:

$$T^*_{MIN} = T_H \approx 175 \text{ MeV.}$$

This is done under two extreme assumptions: there is no collective transverse flow of hadronizing matter and there is no rescattering between produced hadrons. The measured values of $T^*$ parameter for all hadrons satisfy the condition $T^* > T^*_{MIN}$ showing that the approach sketched above leads to self-consistent results in the light hadron sector.

A very different approach is traditionally used in modeling the production process of heavy hadrons like quarkonia. It is based on the assumption that quarkonia are created significantly prior the hadronization of QGP. This in general implies that the systematics of quarkonium production should be different than the one established for light hadrons. However recently it was found that multiplicity of the $J/\psi$ mesons in nuclear collisions at high energies follows dependences well known from light hadron data [18, 19] and that it is consistent with the hypothesis of statistical production of $J/\psi$ mesons at hadronization [3]. In particular the data on $J/\psi$ and $\psi'$ yields in central Pb+Pb collisions at 158 A·GeV are consistent with the predictions of statistical models [3, 20, 24] for a typical values of $T_H \cong 175$ MeV extracted from light hadron systematics.

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1For pions, in contrast to other hadrons, the relativistic effects neglected in Eq. (4) are important. This is a main reason why the measured pion inverse slope parameter is above the thermal pion freeze–out line in Fig. 1.
The new hypothesis of statistical $J/\psi$ production at hadronization can be further tested using data on $m_T$ spectra. As it follows from the above discussion one expects for $J/\psi$ mesons an exponential shape of the $m_T$ distribution with the inverse slope parameter $T^* (J/\psi) > T_H$. Recently published experimental results of NA50 on transverse mass spectra of $J/\psi$ mesons in central Pb+Pb collisions at 158 A-GeV confirm this expectation: the shape of the spectrum is approximately exponential with the fitted inverse slope parameter $T^* (J/\psi) = 245 \pm 5$ MeV. The measured $T^* (J/\psi)$ value is significantly smaller than that expected on the base of the pion freeze–out line ($T^* \approx 610$ MeV).

The 'low' value of $T^*$ for $J/\psi$ suggests its rather early thermal freeze–out. One may argue that this is due to the large mass and low interaction cross section of the $J/\psi$ meson. We postulate therefore that the thermal freeze–out of $J/\psi$ coincides with the hadronization of QGP, i.e., that the $J/\psi$ meson does not participate in the hadronic rescattering after hadronization. It is, however, natural to expect that there is a significant collective transverse flow of hadronizing QGP developed at the early stage of partonic rescattering. Consequently the inverse slope parameter of $J/\psi$ meson as well as all other hadrons for which chemical and thermal freeze–outs coincide with hadronization can be expressed as:

$$
T^*_H = T_H + \frac{2}{\pi} \cdot m \cdot (v_T^H)^2,
$$

where $\langle v_T^H \rangle$ is the mean transverse flow velocity of the QGP at the hadronization. Assuming $T_H = 175$ MeV and using measured value of $T^* (J/\psi) = 245$ MeV we find from Eq. (5) $\langle v_T^H \rangle \approx 0.19$. As expected the obtained transverse flow velocity of QGP at hadronization is significantly smaller than the transverse flow velocity of pions ($\approx 0.5$). The linear $m$–dependence of $T^*_H$ is shown in Fig. 1 by the lower solid line. Within the approach discussed here, Eq. (5) can be used to obtain a next estimate of the lower limit of the measured inverse slope parameters for all hadrons. In fact the values of the parameter $T^*$ for all light hadrons are higher than $T^*_H (m)$. The recent results on the $m_T$ spectra of the $\psi'$ meson indicate that $T^* (\psi') \approx T_H (\psi') = 258 \pm 5$ MeV, which suggests that also the $\psi'$ meson (like $J/\psi$) does not participate in the hadronic rescattering.

One may expect that the thermal freeze–out may coincide with hadronization also for the D meson. Under this assumption we calculate the value of the apparent temperature for the D meson: $T^* (D) \approx T^*_H (D) \approx 217$ MeV. Note that this result is significantly lower than the predictions of the model of Ref. [24] although in [24] the same value of the hadronization temperature was used.

Another important issue is the production of open and hidden charm particles in A+A collisions at RHIC energies. One can expect stronger transverse collective flow effects than at the SPS. This will lead to a linear mass dependence of the apparent inverse slope with approximately the same value of $T_H \approx 175$ MeV, but with a larger value of $\langle v_T^H \rangle$. It is necessary to mention that a recent analysis [25] of the particle number ratios of the RHIC data leads to the above value $T_H \approx 175$ MeV of the chemical freeze–out temperature. The preliminary RHIC data of hadron $m_T$ spectra measured by STAR and PHENIX collaborations support the fact of a larger value of the transverse velocity. However, the measurements were made for $\pi^\pm, K^\pm, \bar{p}$ and $p$ only, and, therefore, this corresponds to the late thermal freeze–out stage.

As long as the RHIC data on the inverse slope parameters for the open and hidden charm mesons are absent it is interesting to compare the different model predictions. Assuming $T_H \approx 175$ MeV we need the value of $\langle v_T^H \rangle$ to estimate the inverse slope parameters for charmed hadrons in our approach. The hydrodynamic calculations of Ref. [12] predict the value of $\langle v_T^H \rangle \approx 0.30$ at the hadronization in Au+Au collisions at RHIC. This leads to an increase of the inverse slopes of charmed hadrons at RHIC in comparison to those values at SPS, e.g., $T^* (J/\psi) \approx T^*_H (J/\psi) \approx 350$ MeV and $T^* (D) \approx T^*_H (D) \approx 280$ MeV.

Note that a significantly larger value of $T^* (D) \approx 380$ MeV in Au+Au collisions at RHIC would be obtained, if the thermal freeze-out of D mesons happens at temperature of 130 MeV. On the other hand, the calculations done with the PYTHIA generator for $p + p$ collisions show an even higher value of the inverse slope for D mesons $T^* (D) \approx 440$ MeV [12]. In fact, the situation is even more uncertain as the dynamical transport calculations of the HSD model (Hadron String Dynamics) made for RHIC energy [28] predict a rather small inverse slope of about 225 MeV for all particle species. Therefore, the $T^* (m)$ systematics at RHIC for $m_T - m \leq 1$ GeV will provide an opportunity to find whether the statistical hadronization works, whether a thermal freeze-out of charmed particles happens simultaneously with hadronization and whether the transverse collective flow at hadronization of the QGP is stronger at RHIC than at SPS.

In summary, recent results on transverse mass spectra of $J/\psi$ and $\psi'$ mesons in central Pb+Pb collisions at 158 A-GeV are considered. It is shown that these data support the hypothesis of the statistical production of charmonia at hadronization and suggest a simultaneous hadronization and the thermal freeze–out for $J/\psi$ and $\psi'$ mesons. Based on this approach the collective transverse velocity of hadronizing quark gluon plasma is estimated to be $\langle v_T^H \rangle \approx 0.2$. Prediction for transverse mass spectra of hidden and open charm mesons at SPS and RHIC are discussed.

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