Signal timing model of urban intersection based on double restraint gravitational model

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Abstract: The assumption of independent arrival process in a polling model might be unrealistic for many intersections that are part of an arterial system, because an output process of the first intersection contributes to the input process for the next intersection. The double restraint gravitational model is used to present the relationship among the traffic flow between the neighbour intersections and is mapped to the arrive rate in the polling signal timing model. In order to test the optimisation effect of the model, the basic-gated service signal timing model with historical average rate is used for comparison, and the data of the regional urban road network in Kunming are used for validation. The result shows that the model can adapt to the traffic dynamics better and achieve a lower mean vehicle delay.

1 Introduction

Polling systems are frequently applied in road traffic to model a situation where queues arise due to the fact that multiple flows of traffic have to share one signalised traffic intersection [1, 2]. When a polling system is applied in intersections, queues are formed by the lines of cars waiting before a red traffic signal. The time that is required for one vehicle to pass the stop line can be viewed as service time, while the times that yellow light, i.e. the clearance times of the intersection, can be considered as switch-over times. One of the most influential papers on traffic signal light queue in [3]; in which traffic light with fixed green and red time has been viewed as a fixed-cycle traffic-light (FCFL) queue system. Although the FCFL is well studied in theoretical analysis in [4–7], it is limited in applications. The desire for fairness (more capacity for busy intersections) leads to many dynamic traffic signal control algorithms [8–10]. The mean delays and mean queue length for intersections with independent Poisson arrivals are achieved. However, there are several aspects that make polling models for signalised intersections different from the classical polling models. One of the most obvious differences is the assumption of independent arrival process might be realistic for an isolated traffic intersection, but many intersections are part of an arterial system, which means that an output process of the first intersection contributes to the input process for the next intersection. So it is essential to take the fuzzy uncertainty of the traffic distribution into account, when design the signal control algorithms [11].

The gravity model is well-used in the analysis of urban traffic demand, to predict the travel distribution of residents and reflect the relationship among the production-attraction and traffic impedance values [12, 13]. In the micro-traffic environment, in order to forecast the traffic relationship between the two neighbour intersections, we seem them as the communities in the double restraint gravitational model [14]. Traffic flow statistics at the intersections are used to parameter calibration. The origin-destination (OD) matrix is mapped to the vehicle arrival rate of each phase, when configure signal timing under the polling model with a certain service scheme.

2 Related models

2.1 Threshold service polling model

The threshold service polling model is an intelligent traffic signal timing design method based on the independent intersection-based vehicle driving threshold control. The traffic signal switch and timing method is performed according to the following strategy:

(i) In the time period when one of the signal lights enters the green light release period, all the vehicles waiting in line in the lane arrived before (the prerequisite does not exceed the intersection saturation value) are released, and then the phase of the switching signal is performed;

(ii) Follow the rules of threshold service when the vehicle is released, that is to say, the vehicle arriving within the current green light time period must wait one cycle and wait until the next green light time period to release it;

(iii) For the saturated or oversaturated state, actually considering the road restrictions and the equality of the vehicles, the algorithm sets the maximum value of the number of cars allowed to pass through each green light duration \(L_0\). When the traffic volume at the intersection is about to reach saturation, There is a certain phase because the queue length of the vehicle is too long and it will occupy the green time for a long time. This will lead to unfairness caused by waiting for the red lights in other phases to be too long. The algorithm sets the number of vehicles whose phase arrives before the green light is greater than \(L_0\). When it is equal to \(L_0\), only \(L_0\) cars will be released during the green time. After that, the lights will be switched to other phases for service. For the setting of \(L_0\), it can largely avoid the unfairness caused by a certain phase occupying the green light for a long time.

According to the above process, the average queue length, average cycle time, and timing matching formula obtained are as shown in (1), (2), and (3):

The average queue captain:

\[
g(i) = \alpha_i \lambda_i \sum_{k=1}^{N_i} \gamma_k \left(1 - \sum_{k=1}^{N_i} \rho_k \right) \tag{1}
\]

Average cycle time:

\[
\bar{\theta} = g(i) / \alpha_i \lambda_i = \sum_{k=1}^{N_i} \gamma_k \left(1 - \sum_{k=1}^{N_i} \rho_k \right) \tag{2}
\]

Timing matching formula:

\[
T_i = \beta_i \arg \min \left( g(i), L_0 \right)
\]
\[ C = \sum_{i=1}^{N} (q_i + T_i) \]  (3)

\( T_i \) is the effective green time of phase \( i \) and \( C \) is the timing signal period. \( a \) is the batch average (vehicle). \( \beta \) is the average through time of the vehicle, different kind of vehicle might pass the intersection with a different time, all types of vehicles are converted into cars by a coefficient. \( \gamma \) is the yellow light time, and \( L_{\text{th}} \) is the maximum number of cars allowed to pass during each green light period.

It is obvious that in actual environment, as the road infrastructure could not be reconstruct easily, the arriving rate of vehicles and the traffic speed in the intersection for per phase become the key parameters in signal timing, especially in the above polling model. Most of the researches on signal timing algorithm focus on the model and pay less attention on parameter calibration. Arrival rate and passing time always derive from real-time acquisition, i.e. use the statistical average values in a certain segment as the parameters in the following phase. This involved frequently fluctuate when the time segment is short or stagnation when it is too long. In addition, it could not take the fuzzy uncertainty of the traffic distribution into account. Hence, a double restraint gravitational model is employed to realise parameter calibration in polling timing algorithm.

### 2.2 Gravity model

To determine the arrival rate and service time in the signal timing algorithm on polling, it is necessary to forecast the traffic flow come from different vanes into the intersection. The gravity model can play a role in the distribution and prediction of traffic flow. The idea is the traffic volume distribution between two target areas that is proportional to the product of the population density of the two target areas, and the impedance between the traffic area and the traffic area becomes inverse.

When using the gravity method for traffic prediction, the intersection-to-intersection origin-destination (OD) matrix, be calibrated in polling timing algorithm.

#### Step 1: Let \( K_j \) = 1, find \( K_jq_j = 1/\sum_j K_jA_j/C_j \) and find \( K_j + 1 \), \( K_j+1 = 1/\sum_j K_jP_j/C_j \),

#### Step 2: Convergence judgment, if \( 1 - \epsilon < K_j+1 < 1 + \epsilon \) and \( 1 - \epsilon < K_j+1 < 1 + \epsilon \),

#### Step 3: Complete the calculation, otherwise let \( n + 1 = n \), and return to the fourth step to repeat.

### 3 Traffic time control based on road information

#### 3.1 model flow prediction

A junction, which can be divided into four traffic areas, can be seen as a cell in the gravity model. The current traffic volume and the amount of attraction \( P \) per minute is shown in the OD matrix of Table 1, where the cells are located. The amount of traffic generated and the amount of attraction are within one minute.

| Table 1 | Current OD matrix |
|---------|-------------------|
| A P     | 1 2 ... j         |
| 1       | \( P_1 \) \( \ldots \) \( P_{ij} \) |
| 2       | \( \ldots \) \( \ldots \) \( \ldots \) \( \ldots \) |
| i       | \( P_i \) \( \ldots \) \( \ldots \) \( \ldots \) |

| Table 2 | Impedance OD matrix |
|---------|---------------------|
| A P     | 1 2 ... j          |
| 1       | \( q_1 \) \( \ldots \) \( q_{ij} \) |
| 2       | \( \ldots \) \( \ldots \) \( \ldots \) \( \ldots \) |
| i       | \( \ldots \) \( \ldots \) \( \ldots \) \( \ldots \) |

traffic flow the occurrence zone has \( K \) correspondence, and each attraction zone has \( K_j \) correspondence, so that the amount of occurrence and attraction can be guaranteed. \( q_j \) is the travel generalised cost of cell \( i \) to cell \( j \), i.e. the travel impedance; \( f(q_j) \) is the generalised cost function of cell \( i \) to cell \( j \), i.e. the impedance function. The impedance function describes the impedance between two traffic areas, which can be calculated by the distance of the traffic zone, travel time, travel costs, and so on. Several frequently used impedance functions such as exponential function \( f(q_j) = ae^{-\beta} \), power function \( f(q_j) = q_j^{\alpha} \) and combination function \( f(q_j) = ae^{-\beta} + q_j^{\alpha} \). In the impedance function, the parameters \( a \) of the trip features are often obtained through data fitting. Under normal circumstances, this series of parameters is a constant.

Among them, there are many ways to calibrate the parameters used to determine the function between the impedance function and the trip cost in the traffic state of a time zone. In this case, the least square algorithm is more commonly used. The example of the calibration of the power constraint double constraint gravity model:

\[ \bar{Q}_{ij} = K_j^{\alpha}P_jA_j/q_j^{\alpha} \]  (5)

\[ K_i = \left( \sum_j K_j^{\alpha}/q_j^{\alpha} \right)^{-1} \]  (6)

\[ K_j = \left( \sum_i K_iA_i/C_i \right)^{-1} \]  (7)

First let \( n = 0, n \) is the number of calculations, \( \alpha \) is given,

Step 1: Let \( K_i^0 = 1 \), find \( K_i^0/K_i = 1/\sum_j K_jA_j/C_j \) and find \( K_i^{1+1} \), \( K_i^{1+1}/K_i^{1+1} = 1/\sum_j K_jP_j/C_j \);

Step 2: Convergence judgment, if \( 1 - \epsilon < K_i^{1+1} < 1 + \epsilon \),

Step 3: Complete the calculation, otherwise let \( n + 1 = n \), and return to the fourth step to repeat.
less than the 3% limit, it can accept the maximum value of the flow in it, that is, obtain the prediction signal timing design table. The above steps can be repeated to obtain the formulas (5), (6) and (7) to obtain the signal timing design algorithm, the average traffic impedance is $\bar{R}$, where $P_{ij}$ denotes the value of travel matrix and $q_{ij}$ is the impedance factor between intersection $i$ and $j$. After the gravity model algorithm, the average traffic impedance is $\bar{R}$ ($P$ is the value of the predicted travel matrix). When the error $\frac{|\bar{R} - R/R|}{R} \times 100\%$ is far less than the 3% limit, it can accept $\gamma = 1$, otherwise adjust $\gamma$.

3.2 Signal timing based on threshold service polling model

Convert $P_{ij}$ to arrival rate $\lambda$, $P_{ij}$ is the amount of traffic from $i$ to $j$ in a certain period of time in the gravity model. To facilitate the calculation here, the same $P_{ij}$ is regarded as the traffic volume of $i$ to $j$ in one minute. The arrival rate refers to the number of vehicles arriving at the intersection per second, is, therefore, $\lambda = P_{ij}/60$. This directly translates traffic into vehicle arrival rates that can be used for timing calculations. Substituting the gravity model into the threshold service polling model will allow a new round of signal timing.

The intersection phase distribution is shown in Fig. 1. Each phase should take the maximum value of the flow in it, that is, East-West and West-East use the same phase and the same green time. Therefore, when the timing is selected, the traffic is selected to match the signal time.

Since the statistical vehicle number in the above OD matrix is counted in one minute, the arrival rate is $\lambda_i = P_{ij}/60$, and the arrival rates $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are obtained. Through the threshold service polling model, the semaphore under this flow is timed and designed, that is, the parameters $\lambda, \alpha, \beta, \gamma, L_\text{th}$ are substituted into the formulas (5), (6) and (7) to obtain the signal timing design table. The current OD matrix is used to calculate the predicted OD matrix as described above. The above steps can be repeated to obtain the prediction signal timing design table.

4 Traffic simulation results and analysis

Taking the intersection of Changyuan South Road and Kunzhou Road in Kunming City as an example, the control strategy of the downwind model service flow rotation training model for gravity model prediction are tested based on the VISSIM simulation platform.

According to the analysis of the actual traffic conditions at the intersection of Kunzhou Road and Changyuan South Road, the width of each roadway is designed to be 3.5 m according to the size of the map and 1:1. The size of the vehicle uses the default size of the system, and the slope factor is not taken into consideration. The vehicle speed distribution function is set as an s-shaped distribution around the median value and the vehicle speed is distributed between 30 km/h and 60 km/h. Wiedemann 74 follow-up vehicle model is selected in traffic simulation vehicles.

As shown in Fig. 1, the intersection is assigned into four signal phase, where vehicle lane No.1 and No.5 are signed as phase 1, No.2 and No.6 are seemed as phase 2, No.3 and No.7 are seemed as phase 3, and No.4 and No.8 are assigned as phase 4. Threshold method polling model in (7) is used to calculate the green time for each phase. In the first way, we use the on-spot traffic flow monitoring data in a certain hour to allocate the traffic signal for the successive hour. In the second way, together with the monitoring data from the neighbour intersections, data set are used for the traffic flow forecasting based on the double restraint gravitational model.

Tables 3 and 4 illustrate the signal timing with original traffic data and the traffic predicted data based on the gravity model, respectively. The normalised original traffic flow of 4 phases are normalised as $\lambda_1 = 0.083, \lambda_2 = 0.133, \lambda_3 = 0.083, \lambda_4 = 0.167$ and the number of vehicle vane $\alpha = 2$, through time parameter $\beta = 2$, yellow time $\gamma = 4$, intersection width $L_{\text{th}} = 20$.

In order to compare the effect of the timing scheme, we simulated the intersection traffic flow and analyse the mean queue length and mean delay under following two situations: Situation 1: Signal timing based on the original traffic volume; Situation 2: Signal timing based on predicted traffic volume.

As shown in Figs. 2 and 3, in predict data signal timing environment (situation 2) the traffic delay is better than situation 1, even though the parking times in some vanes are slightly more, such as vane 7 and vane 8.

In Fig. 4, the traffic mitigation effect achieved by the polling timing scheme under situation 2 in total vanes are better than the scenes under situation 1. That means if the traffic volume changes according to the link information, while the signal timing plan is not adjusted in time, congestion will be caused by traffic delay to a large extent. It also verified the rationality and superiority of the timing plan for traffic flow prediction and threshold service polling model based on link information.

5 Conclusion

In order to analyse complex traffic behaviours and improve intersection traffic conditions, scholars have proposed different ways of controlling intelligent traffic lights and obtained some important conclusions, but most of them can only complete...
This paper revolves around traffic signal timing control, summarises and analyses the control parameters and control methods of the threshold service polling model for signal control. Furthermore, fuzzy uncertainty of the traffic distribution is taken into account, when design the signal control algorithms. Double restraint gravitational model is used to predict the traffic flow based on the on-spot information, thus obtaining a road section–based information. The follow-up work will further study in the following aspects:

(i) The parameter setting can be more reasonable. Many of the parameter settings here are based on the actual road segment information, and the parameters in the gravity model are just to determine whether the convergence still need to be further introduced to the algorithm parameters such as vehicle energy consumption and pollutant emissions in control.

(ii) Traffic pressure can only be relieved at a certain limit. If traffic saturation is achieved, the role of this model is greatly reduced; when the traffic volume reaches saturation in the road section, signal timing alone cannot be solved well, it can be solved by studying the improvement of the infrastructure and the improvement of the vehicle technology.

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7 References

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