Designing of Quick Switching Multiple Deferred Sampling System – 3 through Minimum Angle and Minimum Sum of Risk

V. Kaviyarasu¹, G. Sangameshwaran²

¹Assistant Professor, Department of Statistics, Bharathiar University, Coimbatore-641046, Tamilnadu, India
²Research Scholar Department of Statistics, Bharathiar University, Coimbatore-641046, Tamilnadu, India

Abstract: In this paper a new procedures has been developed for Quick Switching Multiple Deferred Sampling System (QSMDSS) – 3. The optimization techniques such as Minimum Angle Method and Minimum Sum of Risks are carried out for the readymade selection of the plan parameter. Further these techniques are illustrated through suitable numerical examples for selection of the system.

Keywords: Quick Switching System, Multiple Deferred Sampling plan, Minimum Angle Method, Minimum Sum of Risk.

1. Introduction

Acceptance sampling is the procedure of randomly inspecting a sample of goods and deciding whether to accept or reject the entire lot based on the sample results. Acceptance sampling method determines whether a batch of goods should be accepted or rejected. Statistical quality control is simply a statistical method for determining the extent to which quality goals are being met without necessarily checking every item produced and for indicating whether or not the variations which occur one exceeding normal expectations.

2. Quick Switching System

Romboski (1969) has presented extensively a system of immediate switching to tightened inspection when the rejection comes under normal inspection. Due to instantaneous switching between normal and tightened plans, this system is referred as Quick Switching System (QSS). Romboski (1969) has studied the QSS by taking single sampling plan as a reference plan. Using the OC function of Markov chain the composite OC function of QSS – 1 has been derived by Romboski (1969) as

\[ P_a(P) = \frac{P_T}{1 - P_N + P_T} \]

Where, \( P_a(P) \) = Probability of acceptance a lot when a system of sampling plans is under operation. \( P_N \) = Proportions of lots expected to be accepted using \((n, c_1)\) and \((n, c_0)\) plan. \( P_T = \) Proportions of lots expected to be accepted using \((n, c_2)\) and \((kn, c_0)\) plan. \([\text{Where } (n, c_1), (n, c_2), (n, c_0), (kn, c_0), (kn, c_0)\) for QSS-1 \((n; c_N, c_T)\) and \((n, c_0), (kn, c_0)\) for QSS-1 \((n, kn, c_0)\)].

3. The condition for application of Quick Switching System

a) The production is steady so that results on current and preceding lots are broadly indicative of a continuing process and submitted lots are expected to be essentially of the same quality.
b) Lots are submitted substantially in their order of production.
c) Inspection by attributes is considered with quality defined as fraction nonconforming \(p\).

4. Multiple Deferred Sampling Plan

The OC function of MDS – 1 \((c_1, c_2)\) plan was given by RambertVaerst (1981a, 1981b). Later Soundararajanet al. (1983). have presented tables for the selection of an MDS – 1 (0, 2) plan for given \(p_1\), \(p_2\), \(\alpha\) and \(\beta\) under the conditions for application of Poisson for OC curve. Sampling plans indexed by \(p_1\) and \(p_2\). Also, they have presented tables in which one can select an MDS – 1 (0, 2) plan. They have made a comparison with conventional sampling plans (such as single and double sampling plans) and it is shown that the MDS-1 \((c_1, c_2)\) type plans required a smaller sample size. Also a special feature of the MDS – 1 (0, 1) plan is highlighted and its design procedure is then indicated. Wortham and Mogg (1970) have introduced a conditional sampling procedure called dependent stage sampling plan.Jayalaxmi (2009) have researched for the QSS – 1, 2 and 3 with Multiple Deferred Sampling (MDS) plan. The Multiple Deferred (dependent) Stage Sampling plans of type MDS – 1 \((c_1, c_2)\) and MDS – 2 \((c_1, c_2)\) are also belonging to the family of conditional sampling procedures.

5. Designing of Quick Switching Multiple Deferred Sampling System

In this section Quick Switching System with Multiple Deferred Sampling plan as reference plan is considered and a new plan is constructed as Quick Switching Multiple Deferred Sampling System – 1 (QSMDSS). Performance measures are indicated. Necessary tables and procedure are given for designing the system. QSMDSS – 1 \((n; u_1, u_2; v_1, v_2; i)\) refers to quick switching system with the normal MDS plan has a sample size \(n\) and acceptance numbers \(u_1, u_2\) \((u_1 < u_2)\) and the tightened MDS sapling plan has a sample of
size ‘n’ and acceptance numbers \( v_1, v_2 \) \((v_1 < v_2, v_2 \leq u_1 \) and \( v_2 < u_2)\).

The operating procedure for this plan is given as follow:
1) From each submitted lot, select a random sample of ‘n’ units and observe the number of non-conforming units, \( d \).
2) If \( d < c_1 \) accept the lot, if \( d > c_2 \) reject the lot.
3) If \( c_1 < d < c_2 \), accept the lot if the future ‘\( n \)’ lots in succession are all accepted.

(Past m lots of Multiple Dependent State Sampling Plan).

The OC function of MDS - \((c_1, c_2)\) plan is given as

\[
P_a(p) = p_a(p, n, c_1) + \{p_a[p(p, n, c_2) - p_a(p, n, c_1)]\} \{p_a(p, n, c_1)\}
\]

Govindaraju (1984) has constructed tables for selection of the MDS – (0, 1) plan using under the conditions for application of Poisson model for the OC curve. These tables can be used as follows.
1) Given sample size and one point on the OC curve like \((p_1, 1-\alpha)\)
2) Given \( p_1, p_2, \alpha \) and \( \beta \)
3) Given \( p_2(\alpha = 0.05) \) and AOQL.

6. Minimum Angle Method

Norman Bush et. al. (1953) has studied different techniques to describe the direction of the OC curve. The methods involve comparison of some portion of OC curve to be evaluated with the corresponding portion of the OC curve. They have taken chord length that is the line joining the AQL and \( P_a \) of 0.05 as

\[
CL = \sqrt{2025 + (P_1 - P_2)^2}
\]

The smaller the chord length, the more nearly the curve approaches ideal one. But in this method the approximation length is poor, so another method is suggested which considers the cosine of chord length.

\[
\cos \theta = \frac{P_2 - P_1}{\sqrt{2025 + (P_1 - P_2)^2}}
\]

Here the small value of cosine \( \theta \) implies the curve approaches to the ideal OC curve.

Further they have considered two points on the OC curve as \((AQL, 1-\alpha)\) and \((AQL, \beta)\) for minimizing the consumer’s risk. But Peach and Littauer (1946), have taken two points on the OC curve as \((p_1, 1-\alpha)\) and \((p_2, \beta)\) for ideal condition to minimize the consumer’s risk. Here another approach of minimization of angle between the lines joining the points \((AQL, \beta), (AQL, 1-\alpha), (LQL, \beta)\) is given due to Singaravelu (1993) using this method one can get a better plan which has an OC curve approaching to the ideal one. The formula for then \( \theta \) is given as

\[
\tan \theta = \frac{Opposites}{Adjacents} = \frac{(p_2-p_1)/(1-\alpha-\beta)}{(p_2-p_1)/[p_a(p_2) - p_a(p_1)]}
\]

Hence for given two points on the OC curve the values of minimum \( \tan \theta \) is obtained.

7. Minimum Sum of Risk

Golub (1953) has introduced a method and tables for finding acceptance number \( c \) for single sampling plan involving minimum sum of producer’s and consumer’s risk for specified \( p_1 \) and \( p_2 \) when sample size is fixed. The Golub’s approach for single sampling plan has been extended by Soundararajan (1981) under model Govindaraju (1984) Poisson have given the tables for the selection of SSP which minimize sum of producer’s and consumer’s risk without specifying sample size under Poisson model. Govindaraju and Subramanian (1990) have studied the selection of single sampling attribute plan involving the minimum sum of risks without fixing the sample size assuming Poisson model.

In acceptance sampling, the producer and the consumer play a dominant role and hence one allows certain levels of risks for producer and consumer, namely \( \alpha = 0.05 \) and \( \beta = 0.10 \).

Subramani (1991) has studied the attributes sampling plans involving minimum sum of procedure’s and consumer’s risk. And their selection and construction of tables based on the Poisson model given \( p_1, p_2, \alpha, \) and \( \beta \) without assuming that the sample size ‘\( n \)’ is known.

Further this approach results in the rounded valued of \( p_2/p_1 \). The expression for the sum of producer’s and consumer’s

\[
\alpha + \beta = [1 - P_a(p_1)] + P_a(p_2)
\]

If the operating ratio \( p_2/p_1 \) and \( n \) is given can be calculated as

\[
np_2 = (p_2/p_1)(np_1).
\]

8. Selection procedure of QSMDSS – 3 plan through Minimum Angle method

Norman Bush et. al. (1953) has stated the approach which involves comparison of some portion of the OC curves. The chord line \( AB \) coincides with that of \( B'B \) and the operating characteristic curve. That is the ideal OC curve passes through \((p_1, 1-\alpha)\) and \((p_2, \beta)\).Singaravelu (1993) has further designed plans involving minimum angle for single and double sampling plans. Here another approach for minimization of angle between the lines joining the points \((AQL, \beta), (AQL, 1-\alpha), (LQL, \beta)\) is given. Applying this method one can get a better plan by minimizing the above mentioned points, through which one can get an ideal OC curve. The expression for \( \tan \theta \) is given as

\[
\tan \theta = \frac{Opposites}{Adjacents}
\]

This can written as

\[
n \tan \theta = \frac{np_2 - np_1}{p_a(p_1) - p_a(p_2)} \quad \text{3.1}
\]

With the help of the stated expression, the angle \( \theta \) is minimized for the given \( np_1 \) and \( np_2 \) values.

Table 3.1 is used to select plans for given AQL \((p_1)\) and LQL \((p_2)\), \( \alpha \) and \( \beta \) involving the minimum tangent angle \( (\tan \theta) \) between the lines formed by the points \((p_1, \beta)\) \((p_1, 1-\alpha)\), \((p_2, \beta)\), \((p_2, 1-\alpha)\). Tables 3.1 given various parameters
minimum $ntan \theta$ against the product of sample size and acceptable quality level ($np_i$) and $p_2/p_1$, with the given values of $p_1$, $p_2$, $\alpha$, and $\beta$. One can find the required sampling plan from Table 3.1. minimum tangent angle ($ntan \theta$) by adopting the following procedure:

1) Compute the operating ratio $p_2/p_1$.
2) For the computed value of $p_2/p_1$, enter the corresponding table in the row headed by $p_2/p_1$ which is equal to or just greater than the computer ratio and obtain $u_1$, $u_2$, $v_1$, $v_2$, $i$, $np_i$ and $np_2$.
3) The sample size is obtained by $n = np_i/p_2$ or $n = np_2/p_2$.
4) Minimum angle can be found as $\theta = \tan^{-1}(ntan \theta/n)$.

**Example:** For a given $p_1 = 0.07$, $p_2 = 0.6$, $\alpha = 0.05$ and $\beta = 0.10$, one can obtain QSMDDSS – 3 as follows:

1) $p_2/p_1 = 0.60/0.07 = 8.57$, thus the nearest tabulated operating ratio is 8.37.
2) Corresponding to operating ratio the parametric values are $u_1 = 0$, $u_2 = 2$, $v_1 = 0$, $v_2 = 1$, $i = 6$ $np_i = 0.275$.
3) $n = np_2/p_1 = 3.9 \pm 4$.
4) $\theta = \tan^{-1}(ntan \theta/n) = \tan^{-1}(2.3847/4) = \tan^{-1}(0.5961) = 30.80$.

Thus the optimum plan is (4, 0, 2, 0, 1, 6) with the minimum angle $\theta = 30.80$.

**9. Selection procedure of QSMDDSS – 3 plan through minimum sum of risks**

Table 3.2 is used to select a minimum risks QSMDDSS – 3 system as reference plan for given $p_1$ and $p_2$. For the system of table of table, the procedure’s risk and the consumer’s risks will be almost 10% each against the fixed value of the operating ratio $p_2/p_1$. Tables give the parameter $u_1$, $u_2$, $v_1$, $v_2$ and $i$, the associated producer’s and consumer’s risks in the body of the table against the product of the sample size and the acceptable quality level ($np_i$).

The following procedure is used for selecting plan for given $p_1$, $p_2$, $\alpha$ and $\beta$.

1) Compute the operating ratio $p_2/p_1$.
2) With the computed value of $p_2/p_1$ enter Table 3.2 in the row headed by $p_2/p_1$, which is equal to or just smaller than the computed ratio.
3) For determining the parameter $u_1$, $u_2$, $v_1$, $v_2$ and $i$, one proceeds from left to right in the row identified in step 2 such that the tabulated producer’s and the consumer’s risks are equal to or just less than the desired values.
4) The sample size $n$ is obtained as $n = np_i/p_1$ values are given in the column heading to the parameter $u_1$, $u_2$, $v_1$, $v_2$ and $i$, identified in step 3.

**Example:** For given $p_1 = 0.008$, $p_2 = 0.18011$ with $\alpha = 0.05$ and $\beta = 0.10$, from the Table 3.2, one find the QSMDDSS – 3 plan involving minimum sum of risk using Acceptance Quality Level:

1) $p_2/p_1 = 22.5138$
2) tabulated $p_2/p_1 = 22.5138$, $np_i = 0.110$
3) Corresponding to $u_1 = 2$, $u_2 = 3$, $v_1 = 1$, $v_2 = 2$ and $i = 2$ given the body of the table 3.2 one obtains $\alpha = 0.006$ and $\beta = 0.008$ against the desired $\alpha = 0.05$ and $\beta = 0.10$.
4) $n = np_i/p_1 = 0.110/0.008 = 13.75 \pm 14$.

Thus optimum plan is (14, 2, 3, 1, 2, 2).

**10. Construction of Tables**

The probability of accepting a lot given the proportion non-conforming under QSMDDSS – 3 as reference plan is given as

$$P_a(P) = \frac{P_a P_3^3 + P_a (1-P_3)(P_4^2 + P_5 + 1)}{P_3^2 + (1-P_3)(P_2^2 + P_5 + 1)}$$

Here $P_a$ is the probability of acceptance for Multiple Deferred Sampling plan. It is well known that for a series of lots from a process, binomial model for the OC curve will be exact in the case of fraction non-conforming. It can be satisfactorily approximated with the Poisson model where $p$ is small, $n$ is large, and $np < 5$ when the quality is measured in terms of non-conformities, the Poisson model is the appropriate one. Under Poisson assumption, the expression for $P_a$ under MDS plan

$$P_a(p) = p_a(p, n, c_1) + \{p_a(p, n, c_2) + p_a(p, n, c_3) + p_a(p, n, c_4)\}$$

For fixed $p_i$, the value of $np_i$ is calculated from QSMDDSS – 3. The parameter $i$, $u_1$, $u_2$, $v_1$, $v_2$ and $i_1$, corresponding to the minimum $|1-P_a(p_i)|$ are obtained. The following Tables 3.2 gives the producer and consumer’s risks are obtained corresponding $u_1$, $u_2$, $v_1$, $v_2$ and $i$, values for which the sum of risks is minimum.

**11. Conclusion**

Acceptance sampling plan have been widely used in industry to determine whether the manufactured item satisfy the specified quality levels or not. At this point, an enterprise must have to take a decision for accepting or rejecting the lots in accordance with randomly chosen units. Quick Switching System plays a dual role with normal and tightened level for the sample size and acceptance number. Here pair of sampling plans was chosen and switching between normal and tightened so this system is named as Quick Switching System. Using Minimum Angle Method some portion of OC curve can be easily evaluated with corresponding person of the ideal OC curve which may protect both the producer as well as consumer which plays a dominant role. Hence one may allowed certain level of risks involved in producer as well as consumer there risk should be minimized at maximum level of using $\alpha$ and $\beta$. For practical utility of the plan, Poisson unity values have been tabulated for a wider range of plan parameters. The present development would be a valuable addition to the literature and useful device to the quality control partitions. Future research can be carried out by selecting Quick Switching System with for various other reference plans for Minimum Angle Method and Minimum Sum of Risks can be carried out. These plans consist of normal and tightened case of two situation of sample may be used for the sample selection.

**References**

[1] DEVARAJ ARUMAINAYAGAM S.D. (1991): “Contribution to the study of Quick Switching System and its Applications”, Ph.D Thesis, Department of
Table 3.1: Parametric values of QSMDSS-3 plan through Minimum angle method

| $u_1$ | $u_2$ | $v_1$ | $v_2$ | $i$ | $n_{p_1}$ | $n_{p_2}$ | $n \tan \theta$ | OR   |
|-------|-------|-------|-------|-----|------------|------------|----------------|------|
| 0     | 2     | 0     | 1     | 2   | 0.514      | 2.356      | 2.1671         | 4.5837|
| 0     | 2     | 0     | 1     | 4   | 0.350      | 2.303      | 2.2976         | 6.5800|
| 0     | 2     | 0     | 1     | 6   | 0.275      | 2.302      | 2.3847         | 8.3709|
| 0     | 4     | 0     | 3     | 6   | 0.276      | 2.302      | 2.3835         | 8.3406|
| 0     | 4     | 0     | 3     | 8   | 0.231      | 2.302      | 2.4365         | 9.9654|
| 0     | 4     | 0     | 3     | 10  | 0.202      | 2.302      | 2.4706         | 11.396|
| 1     | 2     | 0     | 1     | 2   | 1.260      | 2.359      | 1.2929         | 1.8722|
| 1     | 2     | 0     | 1     | 4   | 1.009      | 2.306      | 1.5259         | 2.2854|
| 1     | 2     | 0     | 1     | 6   | 0.875      | 2.305      | 1.6824         | 2.6343|
| 1     | 4     | 0     | 3     | 6   | 0.888      | 2.305      | 1.6671         | 2.5957|
| 1     | 4     | 0     | 3     | 8   | 0.797      | 2.305      | 1.7741         | 2.8921|
| 1     | 4     | 0     | 3     | 10  | 0.732      | 2.305      | 1.8506         | 3.1489|
| 2     | 3     | 1     | 2     | 2   | 2.116      | 3.944      | 2.1506         | 1.8639|
| 2     | 3     | 1     | 2     | 4   | 1.788      | 3.892      | 2.4753         | 2.1767|
| 2     | 3     | 1     | 2     | 6   | 1.607      | 3.892      | 2.6882         | 2.4219|
| 2     | 4     | 1     | 3     | 6   | 1.625      | 3.892      | 2.6671         | 2.3951|
| 2     | 4     | 1     | 3     | 8   | 1.498      | 3.895      | 2.8200         | 2.6001|
| 2     | 4     | 1     | 3     | 10  | 1.407      | 3.892      | 2.9235         | 2.7662|
| 2     | 5     | 1     | 4     | 2   | 2.211      | 3.891      | 1.9765         | 1.7598|
| 2     | 5     | 1     | 4     | 4   | 1.829      | 3.893      | 2.4282         | 2.1285|
| 2     | 5     | 1     | 4     | 6   | 1.630      | 3.892      | 2.6612         | 2.3877|
| 2     | 6     | 1     | 5     | 6   | 1.632      | 3.892      | 2.6588         | 2.3848|
| 2     | 6     | 1     | 5     | 8   | 1.503      | 3.892      | 2.8106         | 2.5895|
| 2     | 6     | 1     | 5     | 10  | 1.410      | 3.892      | 2.9200         | 2.7603|

[10] ROMBOSKI L.D. (1969): “An Investigation of Quick Switching Acceptance Sampling Systems”, Ph.D Thesis, Rutgers- The State University, New Brunswick, New Jersey.

[11] SINGARAVELU N. (1993): “Contribution to the Study of Certain Acceptance Sampling Plans”, Ph.D Thesis, Department of Statistics, Bharathiar University, Coimbatore, Tamilnadu, India.

[12] SUBARAMANIAN C. (1990): “Switching Procedures in Sampling Inspection”, M.Phil Thesis, Department of Statistics, Bharathiar University, Coimbatore, Tamilnadu, India.

[13] SUBRAMANI K. (1991): “Studies on Designing Attribute Acceptance Sampling Plans with Emphasis on Chain Sampling Plans”, Ph.D Thesis, Department of Statistics, Bharathair University, Coimbatore, Tamilnadu, India.

[14] SOUNDARARAJAN V. (1981): “Single Sampling Attributes Plan Indexed by AQL and AOQL”, Journal of Quality Technology, Vol.13, pp.195-200.

[15] VAERST, REMBERT (1981): “A Procedure to Construct Multiple Deferred State Sampling Plan”, Methods of Operation Research, Vol.37, pp.477-485.

[16] WORTHAM A. and MOGG J.M. (1970): “Dependent Stage Sampling Inspection”, The Journal of Production Research, Vol.8, No.4, pp.385-395.

Author Profile

Dr. V. Kaviyarasu is Assistant Professor, Department of Statistics, Bharathair University, Coimbatore, Tamilnadu, India.

G. Sangameshwan is M. Phil Research Scholar, Department of Statistics, Bharathair University, Coimbatore, Tamilnadu, India.
### Table 3.2: Parameters of QSMDS - 3 plan using Minimum Sum of Risks

| # | 0.040 | 0.045 | 0.050 | 0.055 | 0.060 | 0.065 | 0.070 | 0.075 | 0.080 | 0.085 |
|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 94 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 |
| 92 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 |
| 90 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 |
| 88 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 |
| 86 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 |
| 84 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 | 0.2,0.1 |
| 82 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 |
| 80 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 |
| 78 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 |
| 76 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 |
| 74 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 |
| 72 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 | 0.4,0.3 |

**Key:** $u_1, u_2, v_1, v_2$

$\alpha, \beta$