KGEval: Accuracy Estimation of Automatically Constructed Knowledge Graphs

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Abstract

Automatic construction of large knowledge graphs (KG) by mining web-scale text datasets has received considerable attention recently. Estimating accuracy of such automatically constructed KGs is a challenging problem due to their size and diversity and has largely been ignored in prior research. In this work, we try to fill this gap by proposing KGEval. KGEval uses coupling constraints to bind facts and crowdsource those few that can infer large parts of the graph. We demonstrate that the objective optimized by KGEval is submodular and NP-hard, allowing guarantees for our approximation algorithm. Through experiments on real-world datasets, we demonstrate that KGEval best estimates KG accuracy compared to other baselines, while requiring significantly lesser number of human evaluations.

1 Introduction

Automatic construction of Knowledge Graphs (KGs) from Web documents has received significant interest over the last few years, resulting in the development of several large KGs consisting of hundreds of predicates (e.g., isCity, stadiumLocatedInCity(Stadium, City)) and millions of their instances called beliefs (e.g., (Joe Luis Arena, stadiumLocatedInCity, Detroit)). Examples of such KGs include NELL (Mitchell et al., 2015), Knowledge-Vault (Dong et al., 2014) etc.

Due to imperfections in the automatic KG construction process, many incorrect beliefs are also found in these KGs. Knowing accuracy for each predicate in the KG can provide targeted feedback for improvement and highlight its strengths from weaknesses, whereas overall accuracy of a KG can quantify the effectiveness of its construction-process. Knowing accuracy at predicate-level granularity is immensely helpful for Question-Answering (QA) systems that integrate opinions from multiple KGs (Samadi et al., 2015). For such systems, being aware that a particular KG is more accurate than others in a certain domain, say sports, helps in restricting the search over relevant and accurate subsets of KGs, thereby improving QA-precision and response time. In comparison to the large body of recent work focused on construction of KGs, the important problem of accuracy estimation of such large KGs is unexplored – we address this gap in this paper.

True accuracy of a predicate may be estimated by aggregating human judgments on correctness of each and every belief in the predicate. Even though crowdsourcing marketplaces such as Amazon Mechanical Turk (AMT) provide a convenient way to collect human judgments, accumulating such judgments at the scale of larges KGs is prohibitively expensive. We shall refer to the task of manually classifying a single belief as true or false as a Belief Evaluation Task (BET). Thus, the crucial problem is: How can we select a subset of beliefs to evaluate which will best estimate the true (but unknown) accuracy of KG and its predicates?

A naive and popular approach is to evaluate randomly sampled subset of beliefs from the KG. Since random sampling ignores relational-couplings present among the beliefs, it usually results in oversampling and poor accuracy estimates. Let us motivate this through an example.

1Note that belief evaluation can not be completely automated and will require human-judgment. If an algorithm could accurately predict correctness of a belief, then it may as well be used during KG construction rather than during evaluation.
Motivating example: We motivate efficient accuracy estimation through the KG fragment shown in Figure 1. Here, each belief is an edge-triple in the graph, for example (RedWings, isA, Sportsteam). There are six correct and two incorrect beliefs (the two incident on Taj Mahal), resulting in an overall accuracy of 75% (= 6/8) which we would like to estimate. Additionally, we would also like to estimate accuracies of the predicates: homeStadiumOf, homeCity, stadiumLocatedInCity, cityInState and isA.

We now demonstrate how evaluation labels of beliefs are constrained by each other. Type consistency is one such coupling constraint. For instance, we know from KG ontology that the homeStadiumOf predicate connects an entity from Stadium category to another entity in Sports Team category. Now, if (Joe Louis Arena, homeStadiumOf, Red Wings) is evaluated to be correct, then from these type constraints we can infer that (Joe Louis Arena, isA, Stadium) and (Red Wings, isA, Sports Team) are also correct. Similarly, by evaluating (Taj Mahal, isA, State) as false, we can infer that (Detroit, cityInState, Taj Mahal) is incorrect.

Additionally, we have Horn-clause coupling constraints (Mitchell et al., 2015; Lao et al., 2011), such as homeStadiumOf(x, y) ∧ homeCity(y, z) → stadiumLocatedInCity(x, z). By evaluating (Red Wings, homeCity, Detroit) and applying this horn-clause to the already evaluated facts mentioned above, we infer that (Joe Louis Arena, stadiumLocatedInCity, Detroit) is also correct. We explore generalized forms of these constraints in Section 3.1.

Thus, evaluating only three beliefs, and exploiting constraints among them, we exactly estimate the overall true accuracy as 75% and also cover all predicates. In contrast, the empirical accuracy by randomly evaluating three beliefs, averaged over 5 trials, is 66.7%.

Our contributions in this paper are the following: (1) Systematic study into the important problem of evaluation of automatically constructed Knowledge Graphs. (2) A novel crowdsourcing-based system KGEval to estimate accuracy of large knowledge graphs (KGs) by exploiting dependencies among beliefs for more accurate and faster KG accuracy estimation. (3) Extensive experiments on real-world KGs to demonstrate KGEval’s effectiveness and also evaluate its robustness and scalability.

All the data and code used in the paper are available at http://talukdar.net/mall-lab/KGEval

2 Overview and Problem Statement

2.1 KGEval: Overview

We try to estimate correctness of as many beliefs as possible while evaluating only a subset of them through crowdsourcing. KGEval achieves this goal using an iterative algorithm which alternates between the following two stages:

- **Control Mechanism** (Section 3.4): In this step, KGEval selects the belief which is to be evaluated next using crowdsourcing.

- **Inference Mechanism** (Section 3.3): Coupling constraints are applied over evaluated beliefs to automatically estimate correctness of additional beliefs.

This iterative process is repeated until there are no more beliefs to be evaluated. Single iteration of KGEval over the KG fragment from Figure 1 is shown in Figure 2 where, belief (John Louis Arena, homeStadiumOf, Red Wings) is selected and evaluated by crowdsourcing. Subsequently, the inference mechanism uses type coupling constraints to infer (JL Arena, isA, Stadium) and (R. Wings, isA, Team) also as true. Next, we formalize the notations used in this paper.

2.2 Notations and Problem Statement

We are given a KG with n beliefs. Evaluating a single belief as true or false forms a Belief Evaluation Task (BET). Coupling constraints are derived by determining relationships among BETs, which we further discuss in Section 3.1. Notations are also summarized in Table 1.
Inference algorithm helps us work out evaluation labels of other BETs using constraints \( C \). For a set of already evaluated BETs \( Q \subseteq \mathcal{H} \), we define inferable set \( \mathcal{I}(G, Q) \subseteq \mathcal{H} \) as BETs whose evaluation labels can be deduced by the inference algorithm. We calculate the average true accuracy of a given set of evaluated BETs \( \mathcal{Q} \subseteq \mathcal{I}(G, Q) \subseteq \mathcal{H} \) by \( \Phi(\mathcal{Q}) = \frac{1}{|\mathcal{Q}|} \sum_{h \in \mathcal{Q}} t(h) \).

KGEval aims to sample and crowdsource a BET set \( \mathcal{Q} \) with the largest inferable set, and solves the optimization problem:

\[
\arg \max_{\mathcal{Q} \subseteq \mathcal{H}} |\mathcal{I}(G, \mathcal{Q})|
\]  

### 3 KGEval: Method Details

In this section, we describe various components of KGEval.

#### 3.1 Coupling Constraints

The evaluation labels of beliefs are often dependent on each other due to rich relational structure of KGs. In this work, we derive coupling constraints \( C \) from the KG ontology and link-prediction algorithms, such as PRA (Lao et al., 2011) over NELL and AMIE (Galarraga et al., 2013) over Yago. These rules are jointly learned over entire KG with millions of facts and are assumed true.

We use conjunction-form first-order-logic rules and refer to them as Horn clauses. Examples of a few such coupling constraints are shown below.

\begin{align*}
C_2: & (x, \text{homeStadiumOf}, y) \rightarrow (y, \text{isA}, \text{sportsTeam}) \\
C_5: & (x, \text{homeStadiumOf}, y) \land (y, \text{homeCity}, z) \rightarrow (x, \text{stadiumLocatedInCity}, z)
\end{align*}

Each coupling constraint \( C_i \) operates over \( \mathcal{H}_i \subseteq \mathcal{H} \) to the left of its arrow and infers label of the BET on the right of its arrow. \( C_2 \) enforces type consistency and \( C_5 \) is an instance of PRA path. These constraints have also been successfully employed earlier during knowledge extraction (Mitchell et al., 2015) and integration (Pujara et al., 2013). Note that the constraints are directional and inference propagates in forward direction.

#### 3.2 Evaluation Coupling Graph (ECG)

To combine all beliefs and constraints at a common place, for all \( \mathcal{H} \) and \( C \), we construct a graph with two types of nodes: (1) a node for each BET \( h \in \mathcal{H} \), and (2) a node for each constraint \( C_i \in \mathcal{C} \). Each \( C_i \) is connected to all \( h \) nodes that participate in it. We call this graph the Evaluation Coupling Graph (ECG), represented as \( G = (\mathcal{H} \cup \mathcal{C}, \mathcal{E}) \) with set of edges \( \mathcal{E} = \{(C_i, h) \mid h \in \text{Dom}(C_i), \forall C_i \in \mathcal{C}\} \). Note that ECG is a bipartite factor graph (Kschischang et al., 2001) with \( h \) as variable-nodes and \( C_i \) as factor-nodes.
Figure 3 shows ECG constructed out of the motivating example in Figure 1 with $|C| = 8$ and separate nodes for each of the edges (beliefs or BETs) in KG. We pose the KG evaluation problem as classification of BET nodes in the ECG by allotting them a label of 1 or 0 to represent true or false respectively.

### 3.3 Inference Mechanism

Inference mechanism helps propagate true/false labels of evaluated beliefs to other non-evaluated beliefs using available coupling constraints (Bragg et al., 2013). We use Probabilistic Soft Logic (PSL), (Broecheler et al., 2010) as our inference engine. Below we briefly describe the internal workings of our inference engine for accuracy estimation problem.

Potential function $\psi_j$ is defined for each $C_j$ using Lukaseiwicz t-norm and it depicts how satisfactorily constraint $C_j$ is satisfied. For example, $C_5$ mentioned earlier is transformed from first-order logical form to a real valued number by

$$\psi_j(C_5) = \left( \max\{0, h_x + h_y - 1 - h_w\} \right)^2 \tag{2}$$

where $C_5 = h_x \land h_y \rightarrow h_w$, where $h_x$ denotes the evaluation score $\in [0, 1]$ associated with the BETs.

The probability distribution over label assignment is so structured such that labels which satisfy more coupling constraints are more probable. Probability of any label assignment $\Omega(\mathcal{H}) \in \{0, 1\}^{\mathcal{H}}$ over BETs in $G$ is given by

$$P(\Omega(\mathcal{H})) = \frac{1}{Z} \exp \left[ - \sum_{j=1}^{|C|} \theta_j \psi_j(C_j) \right] \tag{3}$$

where $Z$ is the normalizing constant and $\psi_j$ corresponds to potential function acting over BETs $h \in \text{Dom}(C_j)$. Final assignment of $\Omega(\mathcal{H})_{PSL} \in \{0, 1\}^{\mathcal{H}}$ labels is obtained by solving the maximum a-posteriori (MAP) optimization problem

$$\Omega(\mathcal{H})_{PSL} = \arg \max_{\Omega(\mathcal{H})} P(\Omega(\mathcal{H}))$$

We denote by $M_{PSL}(h, \gamma) \in [0, 1]$ the PSL-estimated score for label $\gamma \in \{1, 0\}$ on BET $h$ in the optimization above.

**Inferable Set using PSL:** We define estimated label for each BET $h$ as shown below.

$$l(h) = \begin{cases} 1 & \text{if } M_{PSL}(h, 1) \geq \tau \\ 0 & \text{if } M_{PSL}(h, 0) \geq \tau \\ \emptyset & \text{otherwise} \end{cases}$$

where threshold $\tau$ is system hyper-parameter. Inferable set is composed of BETs for which inference algorithm (PSL) confidently propagates labels.

$$\mathcal{I}(G, Q) = \{ h \in \mathcal{H} \mid l(h) \neq \emptyset \}$$

Note that two BET nodes from ECG can interact with varying strengths through different constraint nodes; this multi-relational structure requires soft probabilistic propagation.

### 3.4 Control Mechanism

Control mechanism selects the BET to be crowd-evaluated at every iteration. We first present the following two theorems involving KGEval’s optimization in Equation (1). Please refer Appendix for proofs of both theorems.

**Theorem 1. [Submodularity]** The function optimized by KGEval (Equation (1)) using the PSL-based inference mechanism is submodular (Lovász, 1983).

The proof follows from the fact that all pairs of BETs satisfy the regularity condition (Jegelka and Bilmes, 2011; Kolmogorov and Zabih, 2004), further used by a proven conjecture (Mossel and Roch, 2007).

**Theorem 2. [NP-Hardness]** The problem of selecting optimal solution in KGEval’s optimization (Equation (1)) is NP-Hard.

Proof follows by reducing NP-complete Set-cover Problem (SCP) to selecting $Q$ which covers $\mathcal{I}(G, Q)$.

**Justification for Greedy Strategy:** From Theorem 1 and 2, we observe that the function optimized by KGEval is NP-hard and submodular.
Algorithm 1 KGEval: Accuracy Estimation of Knowledge Graphs

Require: $H$: BETs, $C$: coupling constraints, $B$: assigned budget, $S$: seed set, $c(h)$: BET cost
1: $G = \text{BuildECG}(H, C)$
2: $B_r = \emptyset$
3: $Q_0 = S$, $t = 1$
4: repeat
5: $h^* = \arg\max_{h \in H \cup Q} |I(G, Q_{t-1} \cup \{h\})|
6: \text{CrowdEvaluate}(h^*)
7: $Q_t = I(G, Q_{t-1} \cup \{h^*\})$
8: $B_r = B_r - c(h^*)$
9: $Q = Q \cup Q_t$
10: if $Q \equiv H$ then
11: \text{Exit}
12: end if
13: Acc$_t = \frac{i}{|Q|} \sum_{h \in Q} l(h)$
14: $t = t + 1$
15: until CONVERGENCE
16: return Acc$_t$

Results from (Nemhauser et al., 1978) prove that greedy hill-climbing algorithms solve such maximization problem within an approximation factor of $(1 - 1/e) \approx 63\%$ of the optimal solution. Hence we iteratively select the next BET which gives the greatest increase in size of inferable set.

We acknowledge the importance of crowdsourcing aspects such as label-aggregation, worker’s quality estimation etc. Appendix A.1 presents a mechanism to handle noisy crowd workers under limited budget.

3.5 Bringing it All Together

Algorithm 1 presents KGEval. In Lines 1-3, we build the Evaluation Coupling Graph $G = (H \cup C, E)$ and use the labels of seed set $S$ to initialize $G$. In lines 4-16, we repetitively run our inference mechanism, until either the accuracy estimates have converged, or all the BETs are covered. In each iteration, the BET with the largest inferable set is identified and evaluated using crowdsourcing (Lines 5-6). The new inferable set $Q_t$ is estimated. These automatically annotated nodes are added to $Q$ (Lines 7-10).

Convergence: In this paper, we define convergence whenever the variance of sequence of accuracy estimates $[\text{Acc}_{t-1}, \ldots, \text{Acc}_{t-1}, \text{Acc}_t]$ is less than $\alpha$. We set $k = 9$ and $\alpha = 0.002$ for our experiments.

4 Experiments

To assess the effectiveness of KGEval, we ask the following questions: (1). How effective is KGEval in estimating KG accuracy, both at predicate-level and at overall KG-level? (Section 4.3). (2). What is the importance of coupling constraints on its performance? (Section 4.4). (3). And lastly, how robust is KGEval to estimating accuracy of KGs with varying quality? (Section 4.5).

4.1 Model Description

| Evaluation set | $H_N$ | $H_Y$ |
|----------------|-------|-------|
| #BETs          | 1860  | 1386  |
| #Constraints   | $|C_N| = 130$ | $|C_Y| = 28$ |
| #Predicates    | 18    | 16    |
| Gold Acc.      | 91.34% | 99.20% |

Table 2: Details of BET subsets used for accuracy evaluation. (Section 4.1.2).

4.1.1 Setup

Datasets: For experiments, we consider two KGs: NELL and Yago2. From NELL (NELL), we choose a sub-graph of sports related beliefs NELL-sports, mostly pertaining to athletes, coaches, teams, leagues, stadiums etc. We construct coupling constraints set $C_N$ using top-ranked PRA inference rules for available predicate-relations (Lao et al., 2011). The confidence score returned by PRA are used as weights $\theta_i$. We use NELL-ontology’s predicate-signatures to get information for type constraints. Please note that PSL is capable of handling weighted constraints and also learn their weights (relative importance). So, it is not critical to provide absolutely correct constraints. We also select YAGO2-sample (YAGO), unlike NELL-sports, is not domain specific. We use AMIE horn clauses (Galárraga et al., 2013) to construct multi-relational coupling constraints $C_Y$. For each $C_i$, the score returned by AMIE is used as rule weight $\theta_i$. Table 2 reports the statistics of datasets used, their true accuracy and number of coupling constraints. Obtaining gold-labels for millions of facts is non-trivial and expensive as crowdsourcing over full KG incurs significant cost.

Size of evaluation set: NELL-sport consists of 23,422 beliefs with 13,290 unique entities and 53 unique predicates. Whereas YAGO-sample has 31,720 beliefs, with unique 32,103 entities and 17 predicates. In order to calculate accuracy, we
require gold evaluation of all beliefs in the evaluation set. Since obtaining gold evaluation of the entire (or large subsets of) NELL and Yago2 KGs will be prohibitively expensive, we take subset of these KGs for evaluation. (KGEval) consists of datasets used, their crowdsourced labels, coupling constraints and code for inference/control.

Initialization: Algorithm 1 requires initial seed set \( S \) which we generate by randomly evaluating \( |S| = 50 \) beliefs from \( \mathcal{H} \). To maintain fairness, all baselines start from \( S \). For asserting true (or false) value for beliefs, we set a high soft label confidence threshold at \( \tau = 0.8 \) (see Section 3.3).

4.1.2 Crowdsourcing of BETs

To compare KGEval predictions against human evaluations, we evaluate all BETs \( \{ \mathcal{H}_N \cup \mathcal{H}_Y \} \) on AMT. For the ease of workers, we translate each entity-relation-entity belief into human readable format before posting to crowd.

We published BETs on AMT under ‘classification project’ category. We hired AMT recognized master workers for high quality labels and paid \$0.01\) per BET. To compare between ‘master’ and ‘noisy’ workers, we correlated their labels individually to expert labels on random subset and observed that master workers were better correlated (93\%) as compared to three non-masters (89\%). Consequently we consider votes of master workers for \( \{ \mathcal{H}_N \cup \mathcal{H}_Y \} \) as gold labels, which we would like our inference algorithm to be able to predict. As the average turnaround time for AMT tasks runs into a few minutes (Dupuis et al., 2013), KGEval is effectively real-time within such turnaround time range.

4.1.3 Performance Evaluation Metrics

Performance of various methods are evaluated using the following two metrics. To capture accuracy at the predicate level, we define \( \Delta_{\text{predicate}} \) as the average of difference between gold and estimated accuracy of each of the \( R \) predicates in KG.

\[
\Delta_{\text{predicate}} = \frac{1}{|\mathcal{H}|} \left( \sum_{r \in R} \left( \Phi(\mathcal{H}_r) - \frac{1}{|\mathcal{H}_r|} \sum_{h \in \mathcal{H}_r} l(h) \right) \right)
\]

We define \( \Delta_{\text{overall}} \) as the difference between gold and estimated accuracy over the entire evaluation set.

\[
\Delta_{\text{overall}} = \Phi(\mathcal{H}) - \frac{1}{|\mathcal{H}|} \sum_{h \in \mathcal{H}} l(h)
\]

Above, \( \Phi(\mathcal{H}) \) is the overall gold accuracy, \( \Phi(\mathcal{H}_r) \) is the gold accuracy of predicate \( r \) and \( l(h) \) is the label assigned by the currently evaluated method. \( \Delta_{\text{overall}} \) treats entire KG as a single bag of BETs whereas \( \Delta_{\text{predicate}} \) segregates beliefs based on their type of predicate-relation. For both metrics, lower is better.

4.2 Baseline Methods

Since accuracy estimation of large multi-relational KGs is a relatively unexplored problem, there are no well established baselines for this task (apart from random sampling). We present below the baselines which we compared against KGEval.

Random: Randomly sample a BET \( h \in \mathcal{H} \) without replacement and crowdsource for its correctness. Selection of every subsequent BET is independent of previous selections.

Max-Degree: Sort the BETs in decreasing order of their degrees in ECG and select them from top for evaluation; this method favors selection of more centrally connected BETs first.

Independent Cascade: This method is based on contagion transmission model where nodes only infect their immediate neighbors (Kempe et al., 2003). At every time iteration \( t \), we choose a BET which is not evaluated yet, crowdsource for its label and let it propagate its evaluation label in ECG.

KGEval: Method proposed in Algorithm 1.

4.3 Effectiveness of KGEval

Experimental results of all methods comparing \( \Delta_{\text{overall}} \) and \( \Delta_{\text{predicate}} \) at convergence, are presented in Table 3. We observe that KGEval is able to achieve the best estimate across both datasets and metrics. Due to the significant positive bias in \( \mathcal{H}_Y \) (see Table 2), all methods do fairly well as per \( \Delta_{\text{overall}} \) on this dataset, even though KGEval still outperforms others. Also, KGEval is able to estimate KG accuracy most closely while utilizing least number of crowd-evaluated queries. This clearly demonstrates KGEval’s effectiveness.

Nodes in coupling graph with higher degrees are those which participate in large number of constraints. In real KGs, such facts tend to be correct as they interact with several other facts. Hence, MaxDegree overestimates the accuracy by propagating True label. In contrast, Random samples True and False labels in unbiased fashion.

Predicate-level Analysis: Here, we consider the top two baselines from Table 3, viz., Random and
Robustness to Noise: In order to test robustness of the methods in estimating accuracies (See Section 4.3) of various methods with number of crowd-evaluated queries (BET evaluations) to reach the \( \Delta_{\text{overall}} \) converged estimate. (See Section 4.3)

KGEval, and compare performance on the \( \mathcal{H}_N \) dataset. We use \( 1 - \frac{\Delta_{\text{overall}}}{\Delta_{\text{predicate}}} \) as the metric, which means \( \Delta_{\text{overall}} \) computed over individual predicates. Here, we are interested in evaluating how well the two methods have estimated per-predicate accuracy when KGEval’s \( \Delta_{\text{overall}} \) has converged. Comparison of per-predicate performances of the two methods is shown in Figure 4. Observe that KGEval significantly outperforms Random baseline. Its advantage lies in exploiting the coupling constraints among beliefs, where evaluating a belief from certain predicate helps infer beliefs from other predicates.

### 4.4 Importance of Coupling Constraints

This paper is largely motivated by the thesis – exploiting richer relational couple minimize costs while improving accuracy, we replaced KGEval’s greedy control mechanism in Line 5 of Algorithm 1 with these two control mechanisms. In our experiments, we observed that both Random+inference and Max-degree+inference are able to estimate accuracy more accurately than their control-only variants. Secondly, even though the accuracies estimated by Random+inference and Max-degree+inference were comparable to that of KGEval, they required larger number of crowd-evulation queries – 1.2x and 1.35x more, respectively. This shows effectiveness of greedy mechanism.

**Rate of Coverage:** In case of large KGs with scarce budget, it is imperative to have a mechanism which covers greater parts of KG with lesser number of crowdsource queries. Figure 5 shows the fraction of total beliefs whose evaluations were automatically inferred by different methods as a function of number of crowd-evaluated beliefs. We observe that KGEval infers evaluation for the largest number of BETs at each supervision level.

**Robustness to Noise:** In order to test robustness of the methods in estimating accuracies

| Constraint Set | Iterations to Convergence | \( \Delta_{\text{overall}} \) (%)
|----------------|---------------------------|-------------------|
| C             | 140                       | 0.5               |
| C - C_{b3}    | 250                       | 0.9               |
| C - C_{b3} - C_{b2} | 335               | 1.1               |

Table 4: Performance of KGEval with ablated constraint sets. Additional constraints help in better estimation with lesser iterations (see Section 4.4)
of KGs with different gold accuracies, we artificially added noise to $\mathcal{H}_N$ by flipping a fixed fraction of edges, otherwise following the same evaluation procedure as in Section 3.5. We analyze $\Delta_{\text{overall}}$ (and not $\Delta_{\text{predicate}}$) because flipping edges in KG distorts predicate-relations dependencies and present in Table 5. We evaluated all the methods and observed that while performance of other methods degraded significantly with diminishing KG quality (more noise), KGEval was significantly robust to noise.

**Scalability comparisons with MLN:** Markov Logic Networks (Richardson and Domingos, 2006) can serve as a candidate for Inference Mechanism. We compared the runtime performance of KGEval with PSL and MLN as inference engines. While PSL took 320 seconds to complete one iteration, the MLN implementation (PyMLN) could not finish grounding the rules even after 7 hours. This justifies our choice of PSL as the inference engine for KGEval.

### Table 5: Accuracy estimate (higher is better) over entire KG by various baselines in the presence of noise.

| Method       | $\Delta_{\text{overall}}$ (%) | # Queries |
|--------------|-------------------------------|-----------|
| Random       | 1.8                           | 563       |
| Max-Degree   | 4.2                           | 1249      |
| Ind-Cascade  | 1.2                           | 370       |
| KGEval       | 0.8                           | 106       |
| Random       | 1.8                           | 728       |
| Max-Degree   | 6.2                           | 1501      |
| Ind-Cascade  | 1.2                           | 406       |
| KGEval       | 0.2                           | 115       |

**5 Related Work**

Even though Knowledge Graph (KG) construction is an active area of research, we are not aware of any previous research which systematically studies the important problem of estimating accuracy of such automatically constructed KGs. Random sampling has traditionally been the most preferred way for large-scale KG accuracy estimation (Mitchell et al., 2015).

Traditional crowdsourcing research has typically considered atomic allocation of tasks where the requester posts them independently. KGEval operates in a rather novel crowdsourcing setting as it exploits dependencies among its tasks (BETs or belief evaluations). Our notion of interdependence (coupling constraints) among tasks is more general and different than related ideas explored in the crowdsourcing literature before (Kolobov et al., 2013; Bragg et al., 2013; Sun et al., 2015). Even though coupling constraints have been used for KG construction (Nakashole et al., 2011; Galárraga et al., 2013; Mitchell et al., 2015), they have so far not been exploited for KG evaluation. We address this gap in this paper.

The task of knowledge corroboration (Kasneci et al., 2010) proposes probabilistic model to utilize a fixed set of basic first-order logic rules for label propagation and is closely aligned with our motivations. However, unlike KGEval, it does not try to reduce the number of queries to crowdsource or maximize coverage.

### 6 Conclusion

In this paper, we have initiated a systematic study into the important problem of evaluation of automatically constructed Knowledge Graphs. In order to address this challenge, we have proposed KGEval, an instance of a novel crowdsourcing paradigm where dependencies among tasks presented to humans (BETs) are exploited. To the best of our knowledge, this is the first method of its kind. We demonstrated that the objective optimized by KGEval is in fact NP-Hard and submodular, and hence allows for the application of simple greedy algorithms with guarantees. Through extensive experiments on real datasets, we demonstrated effectiveness of KGEval. We hope to extend KGEval to incorporate varying evaluation cost, and also explore more sophisticated evaluation aggregation.

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A Appendix

Submodularity: A real valued function \( f \), which acts over subsets of any finite set \( \mathcal{H} \), is said to be submodular if
\[
\forall R, S \subset \mathcal{H} \text{ it fulfills } f(R) + f(S) \geq f(R \cup S) + f(R \cap S).
\]

We call potential function \( \psi \) as pairwise regular if for all pairs of BETs \( \{p, q\} \in \mathcal{H} \) it satisfies
\[
\psi(0, 1) + \psi(1, 0) \geq \psi(0, 0) + \psi(1, 1)
\]
where \( \{0, 1\} \) represent true/false. Equivalence of submodular and regular properties are established (Kolmogorov and Zabih, 2004; Jegelka and Bilmes, 2011). Using non-negative summation property (Lovász, 1983), \( \sum_{j \in C} \theta_j \psi_j \) is submodular for positive weights \( \theta_j \geq 0 \).

We consider a BET \( h \) to be confidently inferred when the soft score of its label assignment in \( (G, Q) \) is greater than threshold \( \tau_n \in [0, 1] \). From above we know that \( \mathbb{P}(l(h)) \) is submodular with respect to fixed initial set \( Q \). Although max or min of submodular functions are not submodular in general, but (Kempe et al., 2003) conjectured that global function of Equation (1) is submodular if local threshold function \( \mathbb{P}(h|Q) \geq \tau_n \) respected submodularity, which holds good in our case of Equation (3). This conjecture was further proved in (Mossel and Roch, 2007) and thus making our global optimization function of Equation (1) submodular.

Proof. (for Theorem 2) We reduce KGEval to NP-complete Set-cover Problem (SCP) so as to select \( Q \) which covers \( \mathcal{I}(G, Q) \). For the proof to remain consistent with earlier notations, we define SCP by collection of subsets \( \mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_m \) from set \( \mathcal{H} = \{ h_1, h_2, \ldots, h_n \} \) and we want to determine if there exist \( k \) subsets whose union equals \( \mathcal{H} \). We define a bipartite graph with \( m + n \) nodes corresponding to \( \mathcal{I}_i \)’s and \( h_j \)’s respectively and construct edge \((\mathcal{I}_i, h_j)\) if \( h_j \in \mathcal{I}_i \). We need to find a set \( Q \), with cardinality \( k \), such that \( |\mathcal{I}(G, Q)| \geq n + k \).

Choosing our BET-set \( Q \) from SCP solution and further inferring evaluations of other remaining BETs using PSL will solve the problem in hand. \( \square \)

A.1 Noisy Crowd Workers and Budget

Here, we provide a scheme to allot crowd workers so as to remain within specified budget and upper bound total error on accuracy estimate. We have not integrated this mechanism with Algorithm 1 to maintain its simplicity.

We resort to majority voting in our analysis and assume that crowd workers are not adversarial. So expectation over responses \( r_h(u) \) for a task \( h \) with respect to multiple workers \( u \) is close to its true label \( l(h) \) (Tran-Thanh et al., 2013), i.e.,
\[
\mathbb{E}_{u \sim \mathcal{D}(u, h)}[r_h(u)] - l(h) \leq \frac{1}{2}
\]
where \( \mathcal{D} \) is joint probability distribution of workers \( u \) and tasks \( h \).

Our key idea is that we want to be more confident about BETs \( h \) with larger inferable set (as they impact larger parts of KG) and hence allocate them more budget to post to more workers. We determine the number of workers \( \{w_{h_1}, \ldots, w_{h_n}\} \) for each task such that \( h_t \) with larger inference set have higher \( w_{h_t} \). For total budget \( B \), we allocate
\[
w_{h_t} = \left\lfloor \frac{w_{h_t} \cdot \left(1 - \frac{1}{e^{c_{\max}}}\right)}{c_{\max}} \right\rfloor
\]
where \( i_t \) denotes the cardinality of inferable set \( \mathcal{I}(G, Q \cup h_t) \), \( c \) the cost of querying crowd worker, \( i_{\max} \) the size of largest inferable set and \( \gamma \in [0, 1] \) constant.

This allocation mechanism easily integrates with Algorithm 1; in (Line 8) we determine size of inferable set \( i_t = |Q_t| \) for task \( h_t \) and allocate \( w_{h_t} \) crowd workers. Budget depletion (Line 9) is modified to \( B_t = B_t - w_{h_t} c(h_t) \). The following theorem bounds the error with such allocation scheme.

Theorem 3. [Error bounds] The allocation scheme of redundantly posing \( h_t \) to \( w_{h_t} \) workers does not exceed the total budget \( B \) and its expected estimation error is upper bounded by \( e^{-O(n)} \), keeping other parameters fixed. The expected estimation error over all tasks is upper bounded by \( e^{-O(B)} \).

Proof. Let \( \gamma \in [0, 1] \) control the reduction in size of inferable set by \( i_{t+1} = \gamma i_t \). By allocating \( w_{h_t} \) redundant workers for each task \( h_t, \forall t \in \{1 \ldots n\} \) with size of inferable set \( i_t \), we incur total cost of
\[
\sum_{t=1}^{n} c \cdot w_{h_t} = \sum_{t=1}^{n} \frac{B i_t (1 - \gamma)}{c i_{\max}} \cdot c
\]
\[
= \left( \sum_{t=1}^{n} i_t \right) \cdot \frac{B (1 - \gamma)}{i_{\max}}
\]
\[
= \left( i_{\max} (1 - \gamma T) \right) \cdot \frac{B (1 - \gamma)}{i_{\max}}
\]
\[
\leq B
\]
Note that the above geometric approximation helps in estimating summation \( \sum_{t=1}^{n} i_t \) at iteration \( t \leq n \).

Error Bounds: Here we show that the expected error of estimating of \( h_t \) for any time \( t \) decreases exponentially in the size of inferable set \( i_t \). We use majority voting to aggregate \( w_{h_t} \) worker responses for \( h_t \), denoted by \( \hat{r}_{h_t} \in \{0, 1\} \)
\[
\hat{r}_{h_t} = \left\lfloor \frac{1}{w_{h_t}} \sum_{k=1}^{w_{h_t}} r_{h_t}(u_k) - \frac{1}{2} \right\rfloor + 1
\]
where \( r_{h_t}(u_k) \) is the response by kth worker for \( h_t \). The error from aggregated response can be given by \( \Delta(h_t) = |\hat{r}_{h_t} - l(h_t)| \), where \( l(h_t) \) is its true label. From Equation (5) and Hoeffding-Azuma bounds over \( w_{h_t} \), i.i.d responses and error margin \( \varepsilon_t \), we have
\[
\Delta(h_t) = \mathbb{E}\left\{ \left| \frac{1}{w_{h_t}} \sum_{k=1}^{w_{h_t}} r_{h_t}(u_k) - \mathbb{E}(r_{h_t}(u_k)) \right| \geq \varepsilon_t \right\}
\]
\[
= 2 \exp \left( -2 \frac{B i_t (1 - \gamma)}{c i_{\max}} \varepsilon_t^2 \right)
\]
For fixed budget \( B \) and given error margin \( \varepsilon_t \), we have \( \Delta(h_t) = e^{-O(n)} \). Summing up over all tasks \( t \), by union bounds we get the total expected error from absolute truth as \( \Delta(B) = \sum_{t=1}^{n} \Delta(h_t) \).
\[
\Delta(B) \leq n \cdot 2 \exp \left( -2 \frac{B i_{\max} (1 - \gamma)}{c i_{\max}} \varepsilon_{\min}^2 \right)
\]
The accuracy estimation error will decay exponentially with increase in total budget for fixed parameters. \( \square \)
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