Formation of color-singlet gluon-clusters and inelastic diffractive scattering

Part I. Theoretical arguments and experimental indications for self-organized criticality (SOC) in systems of interacting soft gluons

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Abstract

It is proposed, that color-singlet gluon-clusters can be formed in hadrons as a consequence of self-organized criticality (SOC) in systems of interacting soft gluons, and that the properties of such spatiotemporal complexities can be probed experimentally by examining inelastic diffractive scattering. Theoretical arguments and experimental evidences supporting the proposed picture are presented — together with the result of a systematic analysis of the existing data for inelastic diffractive scattering processes performed at different incident energies, and/or by using different beam-particles. It is shown that the size- and the lifetime-distributions of such gluon-clusters can be directly extracted from the data, and the obtained results exhibit universal power-law behaviors — in accordance with the expected SOC-fingerprints.

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1. Interacting soft gluons in the small-\(x_B\) region of DIS

A number of striking phenomena have been observed in recent deep-inelastic electron-proton scattering (DIS) experiments in the small-\(x_B\) region. In particular it is seen, that the contribution of the gluons dominates, and that large-rapidity-gap (LRG) events exist. The latter shows that the virtual photons in such processes may encounter “colorless objects” originating from the proton.

The existence of LRG events in these and other scattering processes have attracted much attention, and there has been much discussion on problems associated with the origin and/or the properties of such “colorless objects”. Reactions in which “exchange” of such “colorless objects” dominate are known in the literature as “diffractive scattering processes”. While the concepts and methods used by different authors in describing such processes are in general very much different from one another, all the authors (experimentalists as well as theorists) seem to agree on the following (see also Refs. [2–7, 9–11]):

(a) Interacting soft gluons play a dominating role in understanding the phenomena in the small-\(x_B\) region of DIS in general, and in describing the properties of LRG events in particular. (b) Perturbative QCD should be, and can be, used to describe the LRG events associated with high transverse-momentum (\(p_\perp\)) jets which have been observed at HERA and at the Tevatron. Such events are, however, rather rare. For the description of the bulk of LRG events, concepts and methods beyond the perturbative QCD, for example Pomeron Models based on Regge Phenomenology, are needed. It has been suggested a long time ago (see the first two papers in Ref. [7]) that, in the QCD language, “Pomeron-exchange” can be interpreted as “exchange of two or more gluons” and that such results can be obtained by calculating the corresponding Feynman diagrams. It is generally felt that non-perturbative methods should be useful in understanding “the small-x phenomena”, but the question, whether or how perturbative QCD plays a role in such non-perturbative approaches does not have an unique answer.

In a recent Letter, we proposed that the “colorless objects” which play the dominating
role in LRG events are color-singlet gluon-clusters due to self-organized criticality, and that optical-geometrical concepts and methods are useful in examining the space-time properties of such objects.

The proposed picture is based on the following observation: In a system of soft gluons whose interactions are not negligible, gluons can be emitted and/or absorbed at any time and everywhere in the system due to color-interactions between the members of the system as well as due to color-interactions of the members with gluons and/or quarks and antiquarks outside the system. In this connection it is important to keep in mind that gluons interact directly with gluons and that the number of gluons in a system is not a conserved quantity. Furthermore, since in systems of interacting soft-gluons the “running-coupling-constant” is in general greater than unity, non-perturbation methods are needed to describe the local interactions associated with such systems. That is, such systems are in general extremely complicated, they are not only too complicated (at least for us) to take the details of local interactions into account (for example by describing the reaction mechanisms in terms of Feynman diagrams), but also too complicated to apply well-known concepts and methods in conventional Equilibrium Statistical Mechanics. In fact, the accumulated empirical facts about LRG events and the basic properties of gluons prescribed by the QCD are forcing us to accept the following picture for such systems:

A system of interacting soft gluons can be, and should be considered as an open dynamical complex system with many degrees of freedom, which is in general far from equilibrium.

In our search for an appropriate method to deal with such complex systems, we are led to the following questions: Do we see comparable complex systems in Nature? If yes, what are the characteristic features of such systems, and what can we learn by studying such systems?
2. Characteristic features of open dynamical complex systems

Open, dynamical, complex systems which are in general far from equilibrium are not difficult to find in Nature — at least not in the macroscopic world! Such systems have been studied, and in particular the following have been observed by Bak, Tang and Wiesenfeld (BTW) some time ago\textsuperscript{12}: This kind of complex systems may evolve to self-organized critical states which lead to fluctuations extending over all length- and time-scales, and that such fluctuations manifest themselves in form of spatial and temporal power-law scaling behaviors showing properties associated with fractal structure and flicker noise respectively.

To be more precise, BTW\textsuperscript{12} and many other authors\textsuperscript{13} proposed, and demonstrated by numerical simulations, the following: Open dynamical complex systems of locally interacting objects which are in general far from equilibrium can evolve into self-organized structures of states which are barely stable. A local perturbation of a critical state may “propagate”, in the sense that it spreads to (some) nearest neighbors, and than to the next-nearest neighbors, and so on in a “domino effect” over all length scales, the size of such an “avalanche” can be as large as the entire system. Such a “domino effect” eventually terminates after a total time $T$, having reached a final amount of dissipative energy and having effected a total spatial extension $S$. The quantity $S$ is called by BTW the “size”, and the quantity $T$ the “lifetime” of the avalanche — named by BTW a “cluster” (hereafter referred to as BTW-cluster or BTW-avalanche). As we shall see in more details later on, it is of considerable importance to note that a BTW-cluster cannot, and should not be identified with a cluster in the usual sense. It is an avalanche, not a static object with a fixed structure which remains unchanged until it decays after a time-interval (known as the lifetime in the usual sense).

In fact, it has been shown\textsuperscript{12,13} that the distribution ($D_S$) of the “size” (which is a measure of the dissipative energy, $S$) and the distribution ($D_T$) of the lifetime ($T$) of BTW-clusters in such open dynamical complex systems obey power-laws:

$$D_S(S) \sim S^{-\mu},$$

(1)
$D_T(T) \sim T^{-\nu}, \quad (2)$

where $\mu$ and $\nu$ are positive real constants. Such spatial and temporal power-law scaling behaviors can be, and have been, considered as the universal signals — the “fingerprints” — of the locally perturbed self-organized critical states in such systems. It is expected that the general concept of self-organized criticality (SOC), which is complementary to chaos, may be the underlying concept for temporal and spatial scaling in a wide class of open non-equilibrium complex systems — although it is not yet known how the exponents of such power laws can be calculated analytically.

SOC has been observed in a large number of open dynamical complex systems in non-equilibrium among which the following examples are of particular interest, because they illuminate several aspects of SOC which are relevant for the discussion in this paper.

First, the well known Gutenberg-Richter law for earthquakes as a special case of Eq.(1): In this case, earthquakes are BTW-clusters due to SOC. Here, $S$ stands for the released energy (the magnitude) of the observed earthquakes. $D_S(S)$ is the number of earthquakes at which an energy $S$ is released. Such a simple law is known to be valid for all earthquakes, large (up to 8 or 9 in Richter scale) or small! We note, the power-law behavior given by the Gutenberg-Richter law implies in particular the following. The question “How large is a typical earthquake?” does not make sense!

Second, the sandpile experiments which show the simple regularities mentioned in Eqs.(1) and (2): In this example, we see how local perturbation can be caused by the addition of one grain of sand (note that we are dealing with an open system!). Here, we can also see how the propagation of perturbation in form of “domino effect” takes place, and develops into BTW-clusters/avalanches of all possible sizes and durations. The size- and duration-distributions are given by Eqs.(1) and (2) respectively. This example is indeed a very attractive one, not only because such experiments can be, and have been performed in laboratories, but also because they can be readily simulated on a PC.

Furthermore, it has been pointed out, and demonstrated by simple models, that
the concept of SOC can also be applied to Biological Sciences. It is amazing to see how phenomena as complicated as Life and Evolution can be simulated by simple models such as the “Game of Life” and the “Evolution Model”.

Having seen that systems of interacting soft-gluons are open dynamical complex systems, and that a wide class of open systems with many degrees of freedom in the macroscopic world evolve to self-organized critical states which lead to fluctuations extending over all length- and time-scales, it seems natural to ask the following: Can such states and such fluctuations also exist in the microscopic world — on the level of quarks and gluons? In particular: Can SOC be the dynamical origin of color-singlet gluon-clusters which play the dominating role in inelastic diffractive scattering processes?

3. SOC in inelastic diffractive scattering processes?

Because of the special role played by “the colorless objects” in inelastic diffractive scattering, and the possible relations between such objects and color-singlet gluon-clusters which can be formed in systems of interacting soft gluons, it should be of considerable interest to study the questions mentioned at the end of the last section, as well as in the title of this section. A simple and effective way of answering them, is to check whether the characteristic properties of SOC, in particular the SOC-“fingerprints” mentioned in Eqs. (1) and (2) show up in the relevant experiments. In order to perform such a comparison, we need to extract the spatial and the temporal distributions of the gluon-clusters.

What are such “colorless objects”? Is it possible that the colorless objects which are associated with the proton-target and which play the dominating role in inelastic diffractive scattering processes are BTW-clusters which exist due to SOC in systems of interacting soft gluons? Can we examine the properties of such colorless objects by studying the final states of the above-mentioned scattering processes?

To answer these questions, it is useful to recall the following: As color-singlets, such colorless objects can exist inside and/or outside the proton, and the interactions between
such color-singlets as well as those between such objects and “the mother proton” should be of Van der Waals type. Hence it is expected that such a colorless object can be readily separated as an entire object from the mother proton in scattering processes in which the momentum-transfer is sufficient to overcome the binding energy due to the Van der Waals type of interactions. This means, in inelastic diffractive scattering the beam-particle (which is the virtual photon $\gamma^*$ in DIS) should have a chance to encounter one of the color-singlet gluon-clusters. For the reasons mentioned above, the struck colorless object can simply be “knocked out” and/or “carried away” by the beam-particle in such a collision event. Hence, it seems that the question whether “the colorless objects” are indeed BTW-clusters is something that can be answered experimentally. In this connection we recall that, according to the general theory of SOC, the size of a BTW-cluster is characterized by its dissipative energy, and in case of systems of interacting soft gluons associated with the proton, the dissipative energy carried by the BTW-cluster should be proportional to the energy fraction $x_P$ carried by the colorless object. Hence, if the colorless object can indeed be considered as a BTW-cluster due to SOC, we should be able to obtain information about the size-distribution of such color-singlet gluon-clusters by examining the $x_P$-distributions of LRG events in the small-$x_B$ region of DIS.

Having this in mind, we now take a closer look at the measured “diffractive structure function” $F_2^{D(3)}(\beta, Q^2; x_P) \equiv \int dt F_2^{D(4)}(\beta, Q^2; x_P, t)$. Here, we note that $F_2^{D(4)}(\beta, Q^2; x_P, t)$ is related to the differential cross-section for large-rapidity-gap events

$$\frac{d^4\sigma^D}{d\beta dQ^2 dx_P dt} = \frac{4\pi\alpha^2}{\beta Q^4} (1 - y + \frac{y^2}{2}) F_2^{D(4)}(\beta, Q^2; x_P, t),$$

in analogy to the relationship between the corresponding quantities [namely $d^2\sigma/(dx_B dQ^2)$ and $F_2(x_B, Q^2)$] for normal deep-inelastic electron-proton scattering events

$$\frac{d^2\sigma}{dx_B dQ^2} = \frac{4\pi\alpha^2}{x_B Q^4} (1 - y + \frac{y^2}{2}) F_2(x_B, Q^2).$$

The kinematical variables, in particular $\beta$, $Q^2$, $x_P$ and $x_B$ (in both cases) are directly measurable quantities, the definitions of which are shown in Fig.1 together with the corresponding diagrams of the scattering processes. We note that, although these variables are
Lorentz-invariants, it is sometimes convenient to interpret them in a “fast moving frame”,
for example the electron-proton center-of-mass frame where the proton’s 3-momentum $\vec{P}$ is
large (i.e. its magnitude $|\vec{P}|$ and thus the energy $P^0 \equiv (|\vec{P}|^2 + M^2)^{1/2}$ is much larger than
the proton mass $M$). While $Q^2$ characterizes the virtuality of the space-like photon $\gamma^*$, $x_B$
can be interpreted, in such a “fast moving frame” (in the framework of the celebrated parton
model), as the fraction of proton’s energy $P^0$ (or longitudinal momentum $|\vec{P}|$) carried by
the struck charged constituent.

We recall, in the framework of the parton model, $F_2(x_B, Q^2)/x_B$ for “normal events” can
be interpreted as the sum of the probability densities for the above-mentioned $\gamma^*$ to interact
with a charged constituent of the proton. In analogy to this, the quantity $F_2^{D(3)}(\beta, Q^2; x_P)/\beta$
for LRG events can be interpreted as the sum of the probability densities for $\gamma^*$ to interact
with a charged constituent which carries a fraction $\beta \equiv x_B/x_P$ of the energy (or longitudinal
momentum) of the colorless object, under the condition that the colorless object (which we
associate with a system of interacting soft gluons) carries a fraction $x_P$ of proton’s energy
(or longitudinal momentum). We hereafter denote this charged-neutral and color-neutral
gluon-system by $c_0^*$ (in Regge pole models this object is known as the “pomeron”). Hence,
by comparing Eq. (3) with Eq. (4) and by comparing the two diagrams shown in Fig. 1(a)
and Fig. 1(b), it is tempting to draw the following conclusions:

The diffractive process is nothing else but a process in which the virtual photon $\gamma^*$
encounters a $c_0^*$, and $\beta$ is nothing else but the Bjorken-variable with respect to $c_0^*$ (this is
why it is called $x_{BC}$ in Ref. [10]). This means, a diffractive $e^- p$ scattering event can be
envisaged as an event in which the virtual photon $\gamma^*$ collides with “a $c_0^*$-target” instead of
“the proton-target”. Furthermore, since $c_0^*$ is charge-neutral, and a photon can only directly
interact with an object which has electric charges and/or magnetic moments, it is tempting
to assign $c_0^*$ an electro-magnetic structure function $F_2^c(\beta, Q^2)$, and study the interactions
between the virtual photon and the quark(s) and antiquark(s) inside $c_0^*$. In such a picture
(which should be formally the same as that of Regge pole models, if we would replace the
$c_0^*$‘s by “pomeron”) we are confronted with the following two questions:
First, is it possible and meaningful to discuss the $x_P$-distributions of the $c_0^*$'s without knowing the intrinsic properties, in particular the electromagnetic structures, of such objects?

Second, are gluon-clusters hadron-like, such that their electromagnetic structures can be studied in the same way as those for ordinary hadrons?

Since we wish to begin the quantitative discussion with something familiar to most of the readers in this community, and we wish to differentiate between the conventional-approach and the SOC-approach, we would like to discuss the second question here, and leave the first question to the next section. We recall (see in particular the last two papers in Ref.[7], in order to see whether the second question can be answered in the affirmative, we need to know whether $F_2^{D(3)}(\beta, Q^2; x_P)$ can be factorized in the form

$$F_2^{D(3)}(\beta, Q^2; x_P) = f_c(x_P) F_2^c(\beta, Q^2).$$

(5)

Here, $f_c(x_P)$ plays the role of a “kinematical factor” associated with the “target $c_0^*$”, and $x_P$ is the fraction of proton’s energy (or longitudinal momentum) carried by $c_0^*$. [We could call $f_c(x_P)$ “the $c_0^*$-flux” — in exactly the same manner as in Regge pole models[7], where it is called “the pomeron flux”.] $F_2^c(\beta, Q^2)$ is “the electro-magnetic structure function of $c_0^*$” [the counterpart of $F_2(x_B, Q^2)$ of the proton] which — in analogy to proton (or any other hadron) — can be expressed as

$$\frac{F_2^c(\beta, Q^2)}{\beta} = \sum_i e_i^2 [q_i^c(\beta, Q^2) + \bar{q}_i^c(\beta, Q^2)],$$

(6)

where $q_i^c(\bar{q}_i^c)$ stands for the probability density for $\gamma^*$ to interact with a quark (antiquark) of flavor $i$ and electric charge $e_i$ which carries a fraction $\beta$ of the energy (or longitudinal momentum) of $c_0^*$. It is clear that Eq.(6) should be valid for all $x_P$-values in this kinematical region, that is, both the right- and the left-hand-side of Eq.(6) should be independent of the energy (momentum) carried by the “hadron” $c_0^*$.

Hence, to find out experimentally whether the second question can be answered in the affirmative, we only need to check whether the data are in agreement with the assumption
that $F_2^{c}(\beta, Q^2)$ prescribed by Eqs.(5) and (6) exists. For such a test, we take the existing data and plot $\log\left[F_2^{D(3)}(\beta, Q^2; x_P)/\beta\right]$ against $\log\beta$ for different $x_P$-values. We note, under the assumption that the factorization shown in Eq.(5) is valid, the $\beta$-dependence for a given $Q^2$ in such a plot should have exactly the same form as that in the corresponding $\log\left[F_2^{c}(\beta, Q^2)/\beta\right]$ vs $\log\beta$ plot; and that the latter is the analog of $\log\left[F_2(x_B, Q^2)/x_B\right]$ vs $\log x_B$ plot for normal events. In Fig.2 we show the result of such plots for three fixed $Q^2$-values (3.5, 20 and 65 GeV$^2$, as representatives of three different ranges in $Q^2$). Our goal is to examine whether or how the $\beta$-dependence of the function given in Eq.(6) changes with $x_P$. In principle, if there were enough data points, we should, and we could, do such a plot for the data-sets associated with every $x_P$-value. But, unfortunately there are not so much data at present. What we can do, however, is to consider the $\beta$-distributions in different $x_P$-bins, and to vary the bin-size of $x_P$, so that we can explicitly see whether/how the shapes of the $\beta$-distributions change. The results are shown in Fig.2. The $\beta$-distribution in the first row, corresponds to the integrated value $\tilde{F}_2^{D}(\beta, Q^2)$ shown in the literature. Those in the second and in the third row are obtained by considering different bins and/or by varying the sizes of the bins. By joining the points associated with a given $x_P$-interval in a plot for a given $Q^2$, we obtain the $\beta$-distribution for a $c_0^*$ carrying approximately the amount of energy $x_P P^0$, encountered by a photon of virtuality $Q^2$. Taken together with Eq.(6) we can then extract the distributions $q_i^{c_0^*}(\beta, Q^2)$ and $\bar{q}_i^{c_0^*}(\beta, Q^2)$ for this $Q^2$-value, provided that $F_2^c(\beta, Q^2)/\beta$ is independent of $x_P$. But, as we can see in Fig.2, the existing data show that the $x_P$-dependence of this function is far from being negligible! Note in particular that according to Eq.(5), by choosing a suitable $f_P(x_P)$ we can shift the curves for different $x_P$-values in the vertical direction (in this log-log plot); but we can never change the shapes of the $\beta$-distributions which are different for different $x_P$-values!

In order to see, and to realize, the meaning of the $x_P$-dependence of the distributions of the charged constituents of $c_0^*$ expressed in terms of $F_2^c(\beta, Q^2)/\beta$ in LRG events [see Eqs.(5) and (6)], let us, for a moment, consider normal deep-inelastic scattering events in the $x_B$-region where quarks dominate ($x_B > 0.1$, say). Here we can plot the data for
log\(F_2(x_B,Q^2)/x_B\) as a function of \(x_B\) obtained at different incident energies \((P^0's)\) of the proton. Suppose we see, that at a given \(Q^2\), the data for \(x_B\)-distributions taken at different values of \(P^0\) are very much different. Would it still be possible to introduce \(F_2(x_B,Q^2)\) as “the electro-magnetic structure function” of the proton, from which we can extract the \(x_B\)-distribution of the quarks \(q_i(x_B,Q^2)\) at a given \(Q^2\)? The fact that it is not possible to assign an \(x_P\)-independent structure function \(F_2^c(\beta,Q^2)/\beta\) to \(c_0^*\) which stands for the “pomeron”, and whose “flux” \(f_c(x_P)\) is expected to be independent of \(\beta\) and \(Q^2\), deserves to be taken seriously. It strongly suggest that the following picture cannot be true: “There exists a universal colorless object (call it pomeron or \(c_0^*\) or something else) the exchange of which describes diffractive scattering in general and DIS off proton in particular. This object is hadron-like in the sense that it has not only a typical size and a typical lifetime, but also a typical electromagnetic structure which can e.g. be measured and described by an “electromagnetic structure function”.

In summary of this section, we note that the empirical facts mentioned above show that no energy-independent electromagnetic structure function can be assigned to the expected universal colorless object \(c_0^*\). This piece of experimental fact is of considerable importance, because it is the first indication that, if there is a universal “colorless object”, this object cannot be considered as an ordinary hadron. In other words, it has to be something else! In fact, as we shall see below, this property is closely related to the observation that such an object cannot have a typical size, or a typical lifetime. The final answer to the question mentioned in the title of this section will be presented in Section 7.

4. Distributions of the gluon-clusters

After having seen that the existing data does not allow us to assign an energy-independent electromagnetic structure function to “the colorless object” such that the universal colorless object \((c_0^*)\) can be treated as an ordinary hadron, let us now come back to the first question in Section 3, and try to find out whether it is never-the-less possible, and meaningful, to
talk about the $x_P$-distribution of $c^*_0$. As we shall see in this section, the answer to this question is Yes! Furthermore, we shall also see, in order to answer this question in the affirmative, we do not need the factorization mentioned in Eq.(5), and we do not need to know whether the gluon-clusters are hadron-like. But, as we have already mentioned above, it is of considerable importance to discuss the second question so that we can understand the origin and the nature of the $c^*_0$'s.

In view of the fact that we do use the concept “distributions of gluons” in deep-inelastic lepton-hadron scattering, although the gluons do not directly interact with the virtual photons, we shall try to introduce the notion “distribution of gluon-clusters” in a similar manner. In order to see what we should do for the introduction of such distributions, let us recall the following:

For normal deep-inelastic $e^-p$ collision events, the structure function $F_2(x_B,Q^2)$ can be expressed in term of the distributions of partons, where the partons are not only quarks and antiquarks, but also gluons which can contribute to the structure function by quark-antiquark pair creation and annihilation. In fact, in order to satisfy energy-momentum-conservation (in the electron-proton system), the contribution of the gluons $x_gg(x_g,Q^2)$ has to be taken into account in the energy-momentum sum rule for all measured $Q^2$-values. Here, we denote by $g(x_g,Q^2)$ the probability density for the virtual photon $\gamma^*$ (with virtuality $Q^2$) to meet a gluon which carries the energy (momentum) fraction $x_g$ of the proton, analogous to $q_i(x_B,Q^2)$ [or $\bar{q}_i(x_B,Q^2)$] which stands for the probability density for this $\gamma^*$ to interact with a quark (or an antiquark) of flavor $i$ and electric charge $e_i$ which carries the energy (momentum) fraction $x_B$ of the proton. We note, while both $x_B$ and $x_g$ stand for energy (or longitudinal momentum) fractions carried by partons, the former can be, but the latter cannot be directly measured.

Having these, in particular the energy-momentum sum rule in mind, we immediately see the following: In a given kinematical region in which the contributions of only one category of partons (for example quarks for $x_B > 0.1$ or gluons for $x_B < 10^{-2}$) dominate, the structure function $F_2(x_B,Q^2)$ can approximately be related to the distributions of that particular kind
of partons in a very simply manner. In fact, the expressions below can be, and have been, interpreted as the probability-densities for the virtual photon $\gamma^*$ (with virtuality $Q^2$) to meet a quark or a gluon which carries the energy (momentum) fraction $x_B$ or $x_g$ respectively.

$$
\frac{F_2(x_B, Q^2)}{x_B} \approx \sum_i e_i^2 q_i(x_B, Q^2) \quad \text{or} \quad \frac{F_2(x_B, Q^2)}{x_g} \approx g(x_g, Q^2).
$$

The relationship between $q_i(x_B, Q^2)$, $g(x_g, Q^2)$ and $F_2(x_B, Q^2)$ as they stand in Eq.(7) are general and formal (this is the case especially for that between $g$ and $F_2$) in the following sense: Both $q_i(x_B, Q^2)$ and $g(x_g, Q^2)$ contribute to the energy-momentum sum rule and both of them are in accordance with the assumption that partons of a given category (quarks or gluons) dominate a given kinematical region (here $x_B > 0.1$ and $x_B < 10^{-2}$ respectively). But, neither the dynamics which leads to the observed $Q^2$-dependence nor the relationship between $x_g$ and $x_B$ are given. This means, without further theoretical inputs, the simple expression for $g(x_g, Q^2)$ as given by Eq.(7) is practically useless!

Having learned this, we now discuss what happens if we assume, in diffractive lepton-nucleon scattering, the colorless gluon-clusters ($c_0^*$’s) dominate the small-$x_B$ region ($x_B < 10^{-2}$, say). In this simple picture, we are assuming that the following is approximately true: The gluons in this region appear predominately in form of gluon clusters. The interaction between the struck $c_0^*$ and the rest of the proton can be neglected during the $\gamma^*c_0^*$ collision such that we can apply impuls-approximation to the $c_0^*$’s in this kinematical region. That is, here we can introduce — in the same manner as we do for other partons (see Eq.[7]), a probability density $D_S(x_P|\beta, Q^2)$ for $\gamma^*$ in the diffractive scattering process to “meet” a $c_0^*$ which carries the fraction $x_P$ of the proton’s energy $P^0 = (|\vec{P}|^2 + M^2)^{1/2} \approx |\vec{P}|$ (where $\vec{P}$ is the momentum and $M$ is the mass of the proton). In other words, in diffractive scattering events for processes in the kinematical region $x_B < 10^{-2}$, we should have, instead of $g(x_g, Q^2)$, the following:

$$
\frac{F_2^{D(3)}(\beta, Q^2; x_P)}{x_P} \approx D_S(x_P|\beta, Q^2).
$$
Here, $x_P P^0$ is the energy carried by $c_0^*$, and $\beta$ indicates the corresponding fraction carried by the struck charged constituent in $c_0^*$. In connection with the similarities and the differences between $q_i(x_B, Q^2), g(x_B, Q^2)$ in (4) and $D_S(x_P|x, Q^2)$ in (8), it is useful to note in particular the significant difference between $x_g$ and $x_P$, and thus that between the $x_g$-distribution $g(x_g, Q^2)$ of the gluons and the $x_P$-distribution $D_S(x_P|x, Q^2)$ of the $c_0^*$’s: Both $x_g$ and $x_P$ are energy (longitudinal momentum) fractions of charge-neutral objects, with which $\gamma^*$ cannot directly interact. But, in contrast to $x_g$, $x_P$ can be directly measured in experiments, namely by making use of the kinematical relation

$$x_p \approx \frac{Q^2 + M^2_x}{Q^2 + W^2},$$

and by measuring the quantities $Q^2, M^2_x$ and $W^2$ in every collision event. Here, $Q, M_x$ and $W$ stand respectively for the invariant momentum-transfer from the incident electron, the invariant-mass of the final hadronic state after the $\gamma^* - c_0^*$ collision, and the invariant mass of the entire hadronic system in the collision between $\gamma^*$ and the proton. Note that $x_B \equiv \beta x_p$, hence $\beta$ is also measurable. This means, in sharp contrast to $g(x_g, Q^2)$, experimental information on $D_S(x_P|x, Q^2)$ in particular its $x_P$-dependence can be obtained — without further theoretical inputs!

5. The first SOC-fingerprint: Spatial scaling

We mentioned at the beginning of Section 3 that in order to find out whether the concept of SOC indeed plays a role in diffractive DIS we need to check the fingerprints of SOC shown in Section 2, and that such tests can be made by examining the corresponding cluster-distributions obtained from experimental data. We are now ready to do this, because we have learned in Sections 3 and 4 that it is not only meaningful but also possible to extract $x_P$-distributions from the measured diffractive structure functions, although the gluon-clusters cannot be treated as hadrons. In fact, as we can explicitly see in Eqs.(8) and (9), in order to extract the $x_P$-dependence of the gluon-clusters from the data, detailed knowledge about the intrinsic structure of the clusters are not necessary.
Having these in mind, we now consider $D_S$ as a function of $x_P$ for given values of $\beta$ and $Q^2$, and plot $F_2^{D(3)}(\beta, Q^2; x_P)/x_P$ against $x_P$ for different sets of $\beta$ and $Q^2$. The results of such log-log plots are shown in Fig. 3. As we can see, the data suggest that the probability-density for the virtual photon $\gamma^*$ to meet a color-neutral and charged-neutral object $c_0^*$ with energy (longitudinal momentum) fraction $x_P$ has a power-law behavior in $x_P$, and the exponent of this power-law depends very little on $Q^2$ and $\beta$. This is to be compared with $D_S(S)$ in Eq.(1), where $S$, the dissipative energy (the size of the BTW-cluster) corresponds to the energy of the system $c_0^*$. The latter is $x_PP^0$, where $P^0$ is the total energy of the proton.

It means, the existing data show that $D_S(x_P|\beta, Q^2)$ exhibits the same kind of power-law behavior as the size-distribution of BTW-clusters. This result is in accordance with the expectation that self-organized critical phenomena may exist in the colorless systems of interacting soft gluons in diffractive deep-inelastic electron-proton scattering processes.

We note, up to now, we have only argued (in Section 1) that such gluon-systems are open, dynamical, complex systems in which SOC may occur, and we have mentioned (in Section 2) the ubiquitousness of SOC in Nature. Having seen the experimental evidence that one of the “SOC-fingerprints” (which are necessary conditions for the existence of SOC) indeed exists, let us now take a second look at the colorless gluon-systems from a theoretical standpoint. Viewed from a “fast moving frame” which can for example be the electron-proton c.m.s. frame, such colorless systems of interacting soft gluons are part of the proton (although, as color-singlets, they can also be outside the confinement region). Soft gluons can be intermittently emitted or absorbed by gluons in such a system, as well as by gluons, quarks and antiquarks outside the system. The emission- and absorption-processes are due to local interactions prescribed by the well-known QCD-Lagrangian (here “the running coupling constants” are in general large, because the distances between the interacting colored objects cannot be considered as “short”; remember that the spatial dimension of a $c_0^*$ can be much larger than that of a hadron!). In this connection, it is useful to keep the following in mind: Due to the complexity of the system, details about
the local interactions may be relatively unimportant, while general and/or global features — for example energy-flow between different parts (neighbors and neighbor’s neighbors . . .) of the system — may play an important role.

How far can one go in neglecting dynamical details when one deals with such open complex systems? In order to see this, let us recall how Bak and Snellen17 succeeded in modeling some of the essential aspects of The Evolution in Nature. They consider the “fitness” of different “species”, related to one another through a “food chain”, and assumed that the species with the lowest fitness is most likely to disappear or mutate at the next time-step in their computer simulations. The crucial step in their simulations that drives evolution is the adaption of the individual species to its present environment (neighborhood) through mutation and selection of a fitter variant. Other interacting species form part of the environment. This means, the neighbors will be influenced by every time-step. The result these authors obtained strongly suggests that the process of evolution is a self-organized critical phenomenon. One of the essential simplifications they made in their evolution models17,18 is the following: Instead of the explicit connection between the fitness and the configuration of the genetic codes, they use random numbers for the fitness of the species. Furthermore, as they have pointed out in their papers, they could in principle have chosen to model evolution on a less coarse-grained scale by considering mutations at the individual level rather than on the level of species, but that would make the computation prohibitively difficult.

Having these in mind, we are naturally led to the questions: Can we consider the creation and annihilation processes of colorless systems of interacting soft gluons associated with a proton as “evolution” in a microscopic world? Before we try to build models for a quantitative description of the data, can we simply apply the existing evolution models17,18 to such open, dynamical, complex systems of interacting soft-gluons, and check whether some of the essential features of such systems can be reproduced?

To answer these questions, we now report on the result of our first trial in this direction: Based on the fact that we know very little about the detailed reaction mechanisms in such gluon-systems and practically nothing about their structures, we simply ignore them, and
assume that they are self-similar in space (this means, colorless gluon-clusters can be considered as clusters of colorless gluon-clusters and so on). Next, we divide them in an arbitrary given number of subsystems $s_i$ (which may or may not have the same size). Such a system is open, in the sense that neither its energy $\varepsilon_i$, nor its gluon-number $n_i$ has a fixed value. Since we do not know, in particular, how large the $\varepsilon_i$’s are, we use random numbers. As far the $n_i$’s are concerned, since we do not know how these numbers are associated with the energies in the subsystems $s_i$, except that they are not conserved quantities, we just ignore them, and consider only the $\varepsilon_i$’s. As in Ref.[17] or in Ref.[18], the random number of this subsystem as well as those of the fixed or random (see the first paper of Ref.[18]) neighbors will be changed at every time-step. Note, this is how we simulate the processes of energy flow due to exchange of gluons between the subsystems, as well as those with gluons/quarks/antiquarks outside the system. In other words, in the spirit of Bak and Sneppen[17] we neglecting the dynamical details totally. Having in mind that, in such systems, the gluons as well as the subsystems ($s_i$’s) of gluons are virtual (space-like), we can ask: “How long can such a colorless subsystem $s_i$ of interacting soft gluons exist, which carries energy $\varepsilon_i$?” According to the uncertainty principle, the answer should be: “The time interval in which the subsystem $s_i$ can exist is proportional to $1/\varepsilon_i$, and this quantity can be considered as the lifetime $\tau_i$ of $s_i$.” In this sense, the subsystems of colorless gluons are expected to have larger probabilities to mutate because they are associated with higher energies, and thus shorter “lifetimes”. Note that the basic local interaction in this self-organized evolution process is the emission (or absorption) of gluons by gluons prescribed by the QCD-Lagrangian — although the detailed mechanisms (which can in principle be explicitly written down by using the QCD-Lagrangian) do not play a significant role.

In terms of the evolution model[17,18], we now call $s_i$ the “species” and identify the corresponding lifetime $\tau_i$ as the “fitness of $s_i$”. Because of the one-to-one correspondence between $\tau_i$ and $\varepsilon_i$, where the latter is a random number, we can also directly assign random numbers to the $\tau_i$’s instead. From now we can adopt the evolution model[17,18] and note that, at the start of such a process (a simulation), the fitness on average grow, because the least fit are
always eliminated. Eventually the fitness do not grow any further on average. All gluons have a fitness above some threshold. At the next step, the least fit species (i.e. the most energetic subsystem $s_i$ of interacting soft gluons), which would be right at the threshold, will be “replaced” and starts an avalanche (or punctuation of mutation events), which is causally connected with this triggering “replacement”. After a while, the avalanche will stop, when all the fitnesses again will be over that threshold. In this sense, the evolution goes on, and on, and on. As in Refs.\cite{17} and \cite{18}, we can monitor the duration of every avalanche, that is the total number of mutation events in everyone of them, and count how many avalanches of each size are observed. The avalanches mentioned here are special cases of those discussed in Section 2. Their size- and lifetime-distributions are given by Eq.(1) and Eq.(2) respectively. Note in particular that the avalanches in the Bak-Sneppen model correspond to sets of subsystems $s_i$, the energies ($\epsilon_i$) of which are too high “to be fit for the colorless systems of low-energy gluons”. It means, in the proposed picture, what the virtual photon in deep-inelastic electron-proton scattering “meet” are those “less fit” one — those who carry “too much” energy. In a geometrical picture this means, it is more probable for such “relatively energetic” colorless gluons-clusters to be spatially further away from the (confinement region of) the proton.

There exists, in the mean time, already several versions of evolution models\cite{3,4,5,6} based on the original idea of Bak and Sneppen\cite{17}. Although SOC phenomena have been observed in all these cases\cite{3,4,5,6}, the slopes of the power-law distributions for the avalanches are different in different models — depending on the rules applied to the mutations. The values range from approximately $-1$ to approximately $-2$. Furthermore, these models\cite{3,4,5,6} seem to show that neither the size nor the dimension of the system used for the computer simulation plays a significant role.

Hence, if we identify the colorless charge-neutral object $c_0^*$ encountered by the virtual photon $\gamma^*$ with such an avalanche, we are identifying the lifetime of $c_0^*$ with $T$, and the “size” (that is the total amount of dissipative energy in this “avalanche”) with the total amount of energy of $c_0^*$. Note that the latter is nothing else but $x_P P^0$, where $P^0$ is the total energy
of the proton. This is how and why the $S$-distribution in Eq. (1) and the $x_P$-distribution of $D_S(x_P|\beta, Q^2)$ in Eq.(8) are related to each other.

6. The second fingerprint: Temporal scaling

In this section we discuss in more detail the effects associated with the time-degree-of-freedom. In this connection, some of the concepts and methods related to the two questions raised in Section 3 are of great interest. In particular, one may wish to know why the parton-picture does not work equally well for hadrons and for gluon-clusters. The answer is very simple: The time-degree of freedom cannot be ignored when we apply impulse-approximation, and the applicability of the latter is the basis of the parton-model. We recall that, when we apply the parton model to stable hadrons, the quarks, antiquarks and gluons are considered as free and stable objects, while the virtual photon $\gamma^*$ is associated with a given interaction-time $\tau_{\text{int}}(Q^2, x_B)$ characterized by the values $Q^2$ and $x_B$ of such scattering processes. We note however that, this is possible only when the interaction-time $\tau_{\text{int}}$ is much shorter than the corresponding time-scales related to hadron-structure (in particular the average propagation-time of color-interactions in hadron). Having these in mind, we see that, we are confronted with the following questions when we deal with gluon-clusters which have finite lifetimes: Can we consider the $c^*_0$'s as "free" and "stable" particles when their lifetimes are shorter than the interaction-time $\tau_{\text{int}}(Q^2, x_B)$? Can we say that a $\gamma^*-c^*_0$ collision process takes place, in which the incident $\gamma^*$ is absorbed by one a or a system of the charged constituents of $c^*_0$, when the lifetime $T$ of $c^*_0$ is shorter than $\tau_{\text{int}}(Q^2, x_B)$?

Since the notion "stable objects" or "unstable objects" depends on the scale which is used in the measurement, the question whether a $c^*_0$ can be considered as a parton (in the sense that it can be considered as a free "stable object" during the $\gamma^*-c^*_0$ interaction) depends very much on on the interaction-time $\tau_{\text{int}}(Q^2, x_B)$. Here, for given values of $Q^2$, $x_B$, and thus $\tau_{\text{int}}(Q^2, x_B)$, only those $c^*_0$'s whose lifetime (T's) are greater than $\tau_{\text{int}}(Q^2, x_B)$ can absorb the corresponding $\gamma^*$. That is to say, when we consider diffractive electron-proton scattering
in kinematical regions in which $c_0^*$'s dominate, we must keep in mind that the measured cross-sections (and thus the diffractive structure function $F_{2}^{D(3)}$) only include contributions from collision-events in which the condition $T > \tau_{\text{int}}(Q^2, x_B)$ is satisfied!

We note that $\tau_{\text{int}}$ can be estimated by making use of the uncertainty principle. In fact, by calculating $1/q^0$ in the above-mentioned reference frame, we obtain

$$\tau_{\text{int}} = \frac{4|\vec{P}|}{Q^2} \frac{x_B}{1 - x_B},$$

which implies that, for given $|\vec{P}|$ and $Q^2$ values,

$$\tau_{\text{int}} \propto x_B, \quad \text{for } x_B \ll 1.$$

This means, for diffractive $e^-p$ scattering events in the small-$x_B$ region at given $|\vec{P}|$ and $Q^2$ values, $x_B$ is directly proportional to the interaction time $\tau_{\text{int}}$. Taken together with the relationship between $\tau_{\text{int}}$ and the minimum lifetime $T(\text{min})$ of the $c_0^*$'s mentioned above, we reach the following conclusion: The distribution of this minimum value, $T(\text{min})$ of the $c_0^*$'s which dominate the small-$x_B$ ($x_B < 10^{-2}$, say) region can be obtained by examining the $x_B$-dependence of $F_{2}^{D(3)}(\beta, Q^2; x_P)/\beta$ discussed in Eqs. (5), (6) and in Fig. 2. This is because, due to the fact that this function is proportional to the quark (antiquark) distributions $q_i^c(\bar{q}_i^c)$ which can be directly probed by the incident virtual photon $\gamma^*$, by measuring $F_{2}^{D(3)}(\beta, Q^2, x_P)/\beta$ as a function of $x_B \equiv \beta x_P$, we are in fact asking the following questions: Do the distributions of the charged constituents of $c_0^*$ depend on the interaction time $\tau_{\text{int}}$, and thus on the minimum lifetime $T(\text{min})$ of the to be detected gluon-clusters? We use the identity $x_B \equiv \beta x_P$ and plot the quantity $F_{2}^{D(3)}(\beta, Q^2; x_P)/\beta$ against the variable $x_B$ for fixed values of $\beta$ and $Q^2$. The result of such a log-log plot is given in Fig. 4. It shows not only how the dependence on the time-degree-of-freedom can be extracted from the existing data, but also that, for all the measured values of $\beta$ and $Q^2$, the quantity

$$p(x_B|\beta, Q^2) \equiv \frac{F_{2}^{D(3)}(\beta, Q^2; x_B/\beta)}{\beta}$$

is approximately independent of $\beta$, and independent of $Q^2$. This strongly suggests that the quantity given in Eq. (12) is associated with some global features of $c_0^*$ — consistent with the
observation made in Section 3 which shows that it cannot be used to describe the structure of $c^*_0$. This piece of empirical fact can be expressed by setting $p(x_B|\beta, Q^2) \approx p(x_B)$. By taking a closer look at this log-log plot, as well as the corresponding plots for different sets of fixed $\beta$- and $Q^2$-values (such plots are not shown here, they are similar to those in Fig.3), we see that they are straight lines indicating that $p(x_B)$ obeys a power-law. What does this piece of experimental fact tell us? What can we learn from the distribution of the lower limit of the lifetimes (of the gluon-systems $c^*_0$’s)?

In order to answer these questions, let us, for a moment, assume that we know the lifetime-distribution $D_T(T)$ of the $c^*_0$’s. In such a case, we can readily evaluate the integral

$$I[\tau_{int}(x_B)] \equiv \int_{\tau_{int}(x_B)}^{\infty} D_T(T) dT,$$

and thus obtain the number density of all those clusters which live longer than the interaction time $\tau_{int}(x_B)$. Hence, under the statistical assumption that the chance for a $\gamma^*$ to be absorbed by one of those $c^*_0$’s of lifetime $T$ is proportional to $D_T(T)$ (provided that $\tau_{int}(Q^2, x_B) \leq T$, otherwise this chance is zero), we can then interpret the integral in Eq.(13) as follows:

$I[\tau_{int}(Q^2, x_B)] \propto p(x_B)$ is the probability density for $\gamma^*$ [associated with the interaction-time $\tau_{int}(x_B)$] to be absorbed by $c^*_0$’s. Hence,

$$D_T(x_B) \propto \frac{d}{dx_B} p(x_B).$$

This means in particular, the fact that $p(x_B)$ obeys a power-law in $x_B$ implies that $D_T(T)$ obeys a power-law in $T$. Such a behavior is similar to that shown in Eq.(2). In order to see the quality of this power-law behavior of $D_T$, and the quality of its independence of $Q^2$ and $\beta$, we compare the above-mentioned behavior with the existing data. In Fig.5, we show the log-log plots of $d/dx_B[p(x_B)]$ against $x_B$. We note that $d/dx_B[p(x_B)]$ is approximately $F_2^{D(3)}(\beta, Q^2; x_B/\beta)/(\beta x_B)$. The quality of the power-law behavior of $D_T$ is explicitly shown in Fig.5.
7. $Q^2$-dependent exponents in the power-laws?

We have seen, in Sections 5 and 6, that in diffractive deep-inelastic electron-proton scattering, the size- and the lifetime-distributions of the gluon-clusters obey power-laws, and that the exponents depend very little on the variables $\beta$ and $Q^2$. We interpreted the power-law behaviors as the fingerprints of SOC in the formation processes of such clusters in form of BTW-avalanches. Can such approximately independence (or weak dependence) of the exponents on $Q^2$ and $\beta$ be understood in a physical picture based on SOC? In particular, what do we expect to see in photoproduction processes where the associated value for $Q^2$ is zero?

In order to answer these questions, let us recall the space-time aspects of the collision processes which are closely related to the above-mentioned power-law behaviors. Viewed in a fast moving frame (e.g. the c.m.s. of the colliding electron and proton), the states of the interacting soft gluons originating from the proton are self-organized. The colorless gluon-clusters caused by local perturbations and developed through “domino effects” are BTW-avalanches (see Sections 1 and 5), the size-distribution of which [see Eqs.(8) and (1)] are given by Fig.3. This explicitly shows that there are gluon-clusters of all sizes, because a power-law size-distribution implies that there is no scale in size. Recall that, since such clusters are color-singlets, their spatial extensions can be much larger than that of the proton, and thus they can be “seen” also outside the proton by a virtual photon originating from the electron. In other words, what the virtual photon encounters is a cloud of colorless gluon-clusters everyone of which is in general partly inside and partly outside the proton.

The virtual photon, when it encounters a colorless gluon-cluster, will be absorbed by the charged constituents (quarks and antiquarks due to fluctuation of the gluons) of the gluon-system. Here it is useful to recall that in such a space-time picture, $Q^2$ is inversely proportional to the transverse size, and $x_B$ is a measure of the interaction time [See Eqs. (10) and (11) in Section 6] of the virtual photon. It is conceivable, that the values for the cross-sections for virtual photons (associated with a given $Q^2$ and a given $x_B$) to collide with
gluon-clusters (of a given size and a given lifetime) may depend on these variables. But, since the processes of self-organization (which produce such gluon-clusters) take place independent of the virtual photon (which originates from the incident electron and enters “the cloud” to look for suitable partners), the power-law behaviors of the size- and lifetime-distributions of the gluon-clusters are expected to be independent of the properties associated with the virtual photon. This means, by using γ∗’s associated with different values of Q² to detect clusters of various sizes, we are moving up or down on the straight lines in the log-log plots for the size- and lifetime distributions, the slopes of which do not change. In other words, the approximative Q²-independence of the slope is a natural consequence of the SOC picture.

As far as the β-dependence is concerned, we recall the results obtained in Sections 3 and 4, which explicitly show the following: The gluon-clusters (c⁰∗’s) cannot be considered as hadrons. In particular, it is neither possible nor meaningful to talk about “the electromagnetic structure of the gluon-cluster”. This suggests, by studying the β-dependence of the “diffractive structure functions” we cannot expect to gain further information about the structure of the gluon-clusters or further insight about the reaction mechanisms.

Having seen these, we try to look for measurable quantities in which the integrations over β have already been carried out. A suitable candidate for this purpose is the differential cross-section

\[
\frac{1}{x_P} \frac{d^2\sigma^D}{dQ^2 dx_P} =
\int d\beta \frac{4\pi\alpha^2}{\beta Q^4} \left( 1 - y + \frac{y^2}{2} \right) \frac{F_2^{D(3)}(\beta, Q^2; x_P)}{x_P}
\approx \int d\beta \frac{4\pi\alpha^2}{\beta Q^4} \left( 1 - y + \frac{y^2}{2} \right) D_S(x_P|\beta, Q^2)
\] (15)

Together with Eqs.(3) and (8), we see that this cross-section is nothing else but the effective β-weighted x_P-distribution D_S(x_P|Q^2, β) of the gluon-clusters. Note that the weighting factors shown on the right-hand-side of Eq.(15) are simply results of QED! Next, we use the data for F_2^{D(3)} which are available at present, to do a log-log plot for the integrand of the expression in Eq.(15) as a function of x_P for different values of β and Q². This is shown
in Fig.6a. Since the absolute values of this quantity depend very much, but the slope of the curves very little on $\beta$, we carry out the integration as follows: We first fit every set of the data separately. Having obtained the slopes and the intersection points, we use the obtained fits to perform the integration over $\beta$. The results are shown in the 

$$\log \left( \frac{1}{x_P} \frac{d^2 \sigma^D}{dQ^2 dx_P} \right) \text{ versus } \log (x_P)$$

plots of Fig.6b. These results show the $Q^2$-dependence of the slopes is practically negligible, and that the slope is approximately $-1.95$ for all values of $Q^2$.

Furthermore, in order to see whether the quantity introduced in Eq.(15) is indeed useful, and in order to perform a decisive test of the $Q^2$-independence of the slope in the power-law behavior of the above-mentioned size-distributions, we now compare the results in deep-inelastic scattering with those obtained in photoproduction, where LRG events have also been observed. This means, as in diffractive deep-inelastic scattering, we again associate the observed effects with colorless objects which are interpreted as system of interacting soft gluons originating from the proton. In order to find out whether it is the same kind of gluon-clusters as in deep-inelastic scattering, and whether they “look” very much different when we probe them with real ($Q^2 = 0$) photons, we replot the existing $d\sigma/dM_x^2$ data for photoproduction experiments performed at different total energies, and note the kinematical relationship between $M_x^2$, $W^2$ and $x_P$ (for $Q^2 \ll M^2$ and $|t| \ll M_x^2$):

$$x_P \approx \frac{M_x^2}{W^2}$$ (16)

The result of the corresponding 

$$\log \left( \frac{1}{x_P} \frac{d\sigma}{dx_P} \right) \text{ versus } \log (x_P)$$

plot is shown in Fig.7. The slope obtained from a least-square fit to the existing data is $-1.98 \pm 0.07$.

The results obtained in diffractive deep-inelastic electron-proton scattering and that for diffractive photoproduction strongly suggest the following: The formation processes of gluon-clusters in the proton is due to self-organized criticality, and thus the spatial distributions
of such clusters — represented by the $x_P$-distribution — obey power-laws. The exponents of such power-laws are independent of $Q^2$. Since $1/Q^2$ can be interpreted as a measure for the transverse size of the incident virtual photon, the observed $Q^2$-independence of the exponents can be considered as further evidence for SOC — in the sense that the self-organized gluon-cluster formation processes take place independent of the photon which is “sent in” to detect the clusters.

Having these results, and the close relationship between real photon and hadron in mind, we are immediately led to the following questions: What shall we see, when we replace the (virtual or real) photon by a hadron — a proton or an antiproton? (See in this connection Fig.8, for the notations and the kinematical relations for the description of such scattering processes.) Should we not see similar behaviors, if SOC in gluon-systems is indeed the reason for the occurrence of colorless gluon-clusters which can be probed experimentally in inelastic diffractive scattering processes? To answer these questions, we took a closer look at the available single diffractive proton-proton and proton-antiproton scattering data, and in order to make quantitative comparisons, we plot the quantities which correspond to those shown in Fig.9a and Fig.9b. These plots are shown in Fig.9a and Fig.9b. In Fig.9a, we see the double differential cross-section $(1/x_P)d^2\sigma/(dtdx_P)$ at four different $t$-values. In Fig.9b, we see the integrated differential cross-section $(1/x_P)d\sigma/dx_P$. Note that, here

$$x_P \approx M^2/s,$$  

(17)

where $\sqrt{s}$ is the total c.m.s. energy of the colliding proton-proton or antiproton-proton system. Here, the integrations of the double differential cross-section over $t$ are in the ranges in which the corresponding experiments have been performed. (The extremely weak energy-dependence has been ignored in the integration.) The dashed lines in all the plots in Figs.9a and 9b stand for the slope $-1.97$ which is the average of the slope obtained from the plots shown in Figs.9b and 9. This means, the result shows exactly what we expect to see: The fingerprints of SOC can be clearly seen also in proton- and antiproton-induced inelastic diffractive scattering processes, showing that such characteristic features are indeed
universal and robust!

We are thus led to the following conclusions. Color-singlet gluon-clusters can be formed in hadrons as a consequence of self-organized criticality (SOC) in systems of interacting soft gluons. In other words, “the colorless objects” which dominate the inelastic diffractive scattering processes are BTW-avalanches (BTW-clusters). Such color-singlet gluon-clusters are in general distributed partly inside and partly outside the confinement region of the “mother-hadron”. Since the interactions between the color-singlet gluon-clusters and other color singlet objects (including the target proton) should be of Van der Waals type, it is expected that such an object can be readily driven out of the above-mentioned confinement region by the beam-particle in geometrically more peripheral collisions. This is why we examined inelastic single-diffractive scattering processes at high energies in which virtual photon, real photon, proton, and antiproton are used as beam particles. This is also why we can extract the universal distributions of such color-singlet gluon-clusters directly from the data. In particular, the fact that $x_P$ is the energy fraction carried by the struck colorless gluon-cluster, and the fact that the $x_P$-distributions are universal, it is tempting to regard such $x_P$-distributions as “the parton-distributions” for diffractive scattering processes. Can we make use of such “parton-distributions” to describe and/or to predict the measurable cross-sections in inelastic diffractive scattering processes? This and other related questions will be discussed in Part II of the present paper.

Acknowledgement

We thank P. Bak, X. Cai, D. H. E. Gross, C. S. Lam, Z. Liang, K. D. Schotte, K. Tabelow and E. Yen for helpful discussions, R. C. Hwa, C. S. Lam and J. Pan for correspondence, and FNK der FU-Berlin for financial support. Y. Zhang also thanks Alexander von Humboldt Stiftung for the fellowship granted to him.
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FIGURES

Fig. 1. The well-known Feynman diagrams (a) for diffractive and (b) for normal deep-inelastic electron-proton scattering are shown together with the relevant kinematical variables which describe such processes.

Fig. 2. $F_D^D(\beta, Q^2; x_P)/\beta$ is plotted as a function of $\beta$ for given $x_P$-intervals and for fixed $Q^2$-values. The data are taken from Ref. [3]. The lines are only to guide the eye.

Fig. 3. $F_D^D(\beta, Q^2; x_P)/x_P$ is plotted as a function of $x_P$ for different values of $\beta$ and $Q^2$. The data are taken from Ref. [3].

Fig. 4. $F_D^D(\beta, Q^2; x_B)/\beta$ is plotted as a function of $x_B$ in the indicated $\beta$- and $Q^2$-ranges. The data are taken from Ref. [3].

Fig. 5. $F_D^D(\beta, Q^2; x_B/\beta)/(\beta x_B)$ is plotted as a function of $x_B$ for fixed $\beta$- and $Q^2$-values. The data are taken from Ref. [3].

Fig. 6. (a) $(1/x_P)d^3\sigma^D/d\beta dQ^2 dx_P$ in units of GeV$^{-4}$ is plotted as a function of $x_P$ in different bins of $\beta$ and $Q^2$. The data are taken from Ref. [3]. (b) $(1/x_P)d^2\sigma^D/dQ^2 dx_P$ in units of GeV$^{-4}$ is plotted as a function of $x_P$ in different bins of $Q^2$. The data are taken from Ref. [3].

Fig. 7. $(1/x_P)d\sigma/dx_P$ for photoproduction $\gamma + p \rightarrow X + p$ is plotted as a function of $x_P$. The data are taken from Ref. [19]. Note that the data in the second paper are given in terms of relative cross sections. Note also that the slopes of the straight-lines are the same. The two dashed lines indicate the lower and the upper limits of the results obtained by multiplying the lower solid line by $\sigma_{tot} = 154 \pm 16(\text{stat.}) \pm 32(\text{syst.}) \mu b$. This value is taken from the third paper in Ref. [19].

Fig. 8. Diagrams for different single diffractive reactions, together with the definitions of the relevant kinematic variables.
Fig. 9. a) \((1/x_P)d^2\sigma/dx_Pdt\) for single diffractive \(p + p \rightarrow p + X\) and \(p + \bar{p} \rightarrow p + X\) reactions is plotted as a function of \(x_P\) at different values of \(t\) and \(\sqrt{s}\). The data are taken from Refs. [4,5].

b) The integrated (with respect to two different \(|t|\)-ranges) differential cross section \((1/x_P)d\sigma/dx_P\) for single diffractive \(p + p \rightarrow p + X\) and \(p + \bar{p} \rightarrow p + X\) reactions is plotted as a function of \(x_P\).

\[
\begin{align*}
Q^2 &= -q^2 & x_B &= \frac{-q^2}{2Pq} \\
y &= \frac{qP}{kP} & W^2 &= (q+P)^2 \\
t &= q_c^2 & \beta &= \frac{-q^2}{2q_c q} \\
x_P &= \frac{qq_c}{qP} &= \frac{x_B}{\beta}
\end{align*}
\]

Fig. 1 (a)
\[ q = k - k' \]

Fig. 1 (b)

\[ Q^2 = -q^2 \quad x_B = \frac{-q^2}{2Pq} \]

\[ y = \frac{qP}{kP} \quad W^2 = (q + P)^2 \]
Fig. 3
$F_2^{D(3)}(\beta, Q^2, x_P) / \beta$

- H1 (Data 1994)
- H1 (Data 1993)
- ZEUS (Paper 1996)
- ZEUS (Paper 1995)

$0.01 \leq \beta \leq 0.9$

$2.5 \text{ GeV}^2 \leq Q^2 \leq 65 \text{ GeV}^2$

**Fig. 4**
Fig. 5
Fits to the data shown in Fig. 6a integrated over $\beta$ for fixed $Q^2$

slope: -1.954

Fig. 6b
Fig. 7
\[ \begin{align*}
\gamma^* \text{ or } \gamma(q) & \quad X(p_X) \\
Q^2 &= -q^2, \quad W^2 = (q+P)^2 \\
p^2_X &= (q+q_c^r)^2 = M^2_X \\
t &= q^2_c (\leq 0) \\
x_P &= \frac{q q_c^r}{q P} \approx \frac{M^2_X + Q^2}{W^2 + Q^2} \\
P^2 &= P^{'2} = M^2 \\
\gamma^* + p \rightarrow X + p \quad \text{or} \\
\gamma + p \rightarrow X + p \quad \text{for } |t| (\leq 4M^2, \text{ say}) \ll W^2
\end{align*} \]

\[ \begin{align*}
\bar{p} \text{ or } p(k) & \quad X(p_X) \\
k^2 = M^2, \quad s = (k+P)^2 \\
p^2_X &= (k+q_c^r)^2 = M^2_X \\
t &= q^2_c (\leq 0) \\
x_P &= \frac{k q_c^r}{k P} \approx \frac{M^2_X}{s} \\
P^2 &= P^{'2} = M^2 \\
\bar{p} + p \rightarrow X + p \quad \text{or} \\
p + \bar{p} \rightarrow X + p \quad \text{or} \\
p + p \rightarrow X + \bar{p} \quad \text{for } |t| (\leq 4M^2, \text{ say}) \ll s
\end{align*} \]

**Fig. 8**
\[
\frac{d^2\sigma}{dx dt} (\text{mb} \cdot \text{GeV}^{-2})
\]

- \(t = 0.05 \text{ GeV}^2\)
  - \(\sqrt{s} = 1800 \text{ GeV (pp)}\)
  - \(\sqrt{s} = 546 \text{ GeV (pp)}\)
  - \(\sqrt{s} = 20 \text{ GeV (pp)}\)
  - \(\sqrt{s} = 14 \text{ GeV (pp)}\)

- \(t = 0.15 \text{ GeV}^2\)
  - \(\sqrt{s} = 27 \text{ GeV (pp)}\)
  - \(\sqrt{s} = 23 \text{ GeV (pp)}\)

- \(t = 0.40 \text{ GeV}^2\)
  - \(\sqrt{s} = 27 \text{ GeV (pp)}\)
  - \(\sqrt{s} = 23 \text{ GeV (pp)}\)

- \(t = 2.95 \text{ GeV}^2\)
  - \(\sqrt{s} = 45 \text{ GeV (pp)}\)

Fig. 9a
Fig. 9b