Sensitivity to the pion-nucleon coupling constant in partial-wave analyses of
\[ \pi N \rightarrow \pi N, \; NN \rightarrow NN, \; \text{and} \; \gamma N \rightarrow \pi N \]

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Abstract

We summarize results obtained in our studies of the pion-nucleon coupling constant. Several different techniques have been applied to \( \pi N \) and \( NN \) elastic-scattering data, and the existing database for single-pion photoproduction. The most reliable determination comes from \( \pi N \) elastic scattering. The sensitivity in this reaction was found to be greater, by at least a factor of 3, when compared with analyses of \( NN \) elastic scattering or single-pion photoproduction.

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1. Introduction

A number of groups have recently extracted the pion-nucleon coupling constant \( \frac{g^2}{4\pi} \), mainly from analyses of \( \pi N \) and \( NN \) elastic scattering. In the \( \pi N \) case, results have been obtained via dispersion relations \([1]\), through the fitted couplings of a tree-level model \([2]\), and from the long-range part of a potential approach \([3]\). Values of \( \frac{g^2}{4\pi} \) have been determined from the one-pion-exchange part of the \( NN \) interaction in fits \([4,5]\) to either the “full” low-energy database or restricted kinematic regions expected \([6]\) to be more sensitive to this coupling.

In order to determine which data are most sensitive to \( \frac{g^2}{4\pi} \), we map \( \chi^2 \) versus \( \frac{g^2}{4\pi} \) for a variety of energy-dependent partial-wave analyses. The parabolic “widths”, near the resulting minima, are taken as a measure of the “uncertainty” in the determinations. In Section 2, we give results from our analysis of \( \pi N \) elastic scattering to \( T_\pi=2.1 \) GeV. In Section 3, we discuss a number of global fits of \( NN \) elastic scattering data to \( T_{lab}=400 \) MeV. In Section 4, we consider our current analysis of pion-photoproduction taken to \( E_\gamma=500 \) MeV, as well as mappings extended to \( E_\gamma=2 \) GeV. This reaction also shows a sensitivity to the value of \( \frac{g^2}{4\pi} \). Finally, in Section 5, we give a summary of our conclusions.

2. \( \pi N \) elastic analysis to 2.1 GeV

In our most recent analysis of this reaction, we have added fixed-t dispersion-relation constraints on the \( C^+ \) amplitude and forward dispersion-relation constraints on the \( E^\pm \) amplitudes in order to facilitate an extraction of the \( \sigma \)-term. A mapping of \( \chi^2 \), as a function of \( \frac{g^2}{4\pi} \), reveals a very deep minimum at 13.73(0.01). This is, of course, the charged \( \pi NN \) coupling, and we shall refer to the (0.01) as a mapping error.

The coupling constant must be extracted through the evaluation of a dispersion relation and this leads to an extraction error. The Hamilton dispersion relation for the \( B^+ \) amplitude was used by Carter, Bugg and Carter \([7]\) to extract the long-standing value of \( \frac{g^2}{4\pi}=14.28 \). This is the value used in the Karlsruhe-Helsinki \([8]\) and Carnegie-Mellon—Berkeley analyses \([9]\) of the late 1970’s. If we use the Karlsruhe solution, KA84, in the Hamilton dispersion relation, for energies between 100 and 600 MeV (Tlab), we obtain a value of \( \frac{g^2}{4\pi}=14.40(0.23) \), where (0.23) is the RMS deviation from a mean value of 14.40. Evaluating this dispersion relation for our current solution, SP99, results in a value of \( \frac{g^2}{4\pi}=13.73(0.03) \). The (0.03) we shall refer to as an extraction error and, it is evident that this is the primary source of uncertainty. A more complete description of our solution, SP99, is given by Marcello Pavan in a separate contribution to these proceedings.

3. \( NN \) elastic analysis

Sensitivity to the coupling constant is built into our representation for \( NN \) elastic scattering in the following fashion. The scattering amplitude (suppressing spin) is given by:

\[
f = f_{\text{coul}} + B_p + \sum_{l=0}^{8} (2l+1)P_l(Z)f(l, E)e^{2i\phi_l},
\]

where \( B_p \) gives the peripheral Born amplitude (the Born term minus the partial waves between \( l = 0 \) and \( l = 8 \)), and

\[
f(l, E) = K(l, E)/(1 - iK(l, E)),
\]

where
\[ K(l, E) = \text{Born}_l + \sum_i \alpha_i A_{li}. \]  

(3)

The Born term contains a t-channel pole proportional to \( g_0^2 / 4\pi \) and a u-channel pole proportional to \( g_c^2 / 4\pi \). The expansion bases, \( A_{li} \), are chosen to have a left-hand cut that starts at the \( 2\pi \) threshold and have the correct threshold behavior \(^2\).

The sensitivity of the fitting scheme to \( g^2 / 4\pi \) clearly depends upon the amount of phenomenology (number of \( \alpha_i \) terms used). In order to estimate the effect of phenomenological contributions, we considered two different solutions fitted to the current GW data base below \( 400 \) MeV. (The upper limit was chosen to keep the analyses essentially elastic.) Solution SP40 was obtained by fitting 57 parameters for waves with \( J < 7 \), while SG40 utilized 48 parameters for waves with \( J < 5 \). In all of our mappings, we assumed that there was no charge splitting; \( g_0 = g_c \).

Tables I and II summarize our results from the \( \chi^2 \)-mapping of these solutions. Uppsala data at 96 and 162 MeV have been used in conjunction with the GW data base. The columns labeled PPx and NPx are mappings which exclude the Uppsala cross sections. Solution SG40 shows a greater sensitivity, as expected, with a consequent increase in \( \chi^2 \) of around 350 for the 9 eliminated search parameters. This might raise questions concerning the actual value of \( g^2 / 4\pi \) extracted in such analyses. However, our focus is on sensitivity which increases by about 50% as we go from SP40 to SG40. We make no attempt to address the issue of possible charge-splitting, but it should be pointed out that \( pp \) data depend only upon \( g_0^2 / 4\pi \) whereas \( np \) data depend upon both coupling constants.

The focus of the workshop has been on \( np \) charge-exchange cross-sections, where there is an obvious and strong dependence upon the charged coupling constant, but it is important to realize which other data are sensitive to the coupling constant. In order to partially resolve that issue, we used solution SG40 to map a data base which excluded ALL \( np \) cross sections on the assumption that total cross sections would set the scale of the interaction, while the angular shape would be constrained by the abundant spin data in the \( np \) data base. The resultant solution, renamed SX40, was used to generate the results in Table III.

While this is certainly an inappropriate determination of the coupling constant, it does reveal a substantial sensitivity in the spin data, and suggests that their inclusion, in any extraction of \( g^2 / 4\pi \) from \( np \) elastic scattering data, will alter the final result.

4. Analysis of pion-photoproduction data

Sensitivity to \( g^2 / 4\pi \) is built into the representation used to parameterize pion photoproduction in manner similar to \( NN \) elastic scattering. The scattering amplitude \( M \) is basically taken to be

\[ M = B_p + \sum_{l=0}^{l_{\text{max}}} m_l a_l(\theta), \]  

(4)

where

\[ B_p = \text{Born} - \sum_{l=0}^{l_{\text{max}}} b_l a_l(\theta). \]  

(5)

The contribution, \( B_p \), is the residual Born contribution not included in the fitted multipole \( (m_l) \) amplitudes, \( b_l \) is the projected Born term, and \( a_l(\theta) \) represents the expansion basis. The amplitudes, \( m_l \), are then expressed as
\[ m_l = (b_l + c_l) (1 + iT_{\pi N}) + d_l T_{\pi N}, \]  

(6)

c_l and d_l being structure functions of energy which are fitted to the available data and \( T_{\pi N} \) being the elastic \( \pi N \) scattering amplitude. This form ensures that Watson’s theorem is satisfied below the 2-pion production threshold. Dependence upon the coupling constant is contained in \( T_{\pi N} \), as described in Section 2, and in the Born terms, \( B_p \) and \( b_l \).

Table 4 characterizes a fit to pion-photoproduction data from threshold (\( E_\gamma \approx 145 \text{ MeV} \)) to 500 MeV. It reveals a sensitivity about 50\% weaker than that obtained from the \( np \) mapping of solution SG40 (an error of \( \pm 0.13 \) versus \( \pm 0.09 \)). A more extensive study of the full data base, which extends from threshold to about 2 GeV, is summarized in Fig. 1, where the maximum value of \( E_\gamma \) was varied from 500 MeV to 2 GeV. An average of the extracted \( g^2/4\pi \) values reveals a sensitivity of about \( \pm 0.14 \).

5. Conclusions

We have looked at three fundamental scattering reactions having strong signatures for the pion-nucleon coupling constant, \( g^2/4\pi \). The most sensitive determination, by far, is based upon \( \pi N \) elastic analysis, with fixed-\( t \) and forward dispersion relation constraints. This yields uncertainties of \( (\pm 0.01) \) (mapping error) + \( (0.03) \) (extraction error). Next in sensitivity are the combined \( NN \) analyses to 500 MeV, where the uncertainty is estimated to be \( (\pm 0.13) \) (for SP40) and \( (\pm 0.09) \) (for SG40). Least in sensitivity was pion-photoproduction, where a fit to 500 MeV gave an uncertainty of \( (\pm 0.13) \) and where an average taken over various maximum energies gave only a slightly different sensitivity \( (\pm 0.14) \).

Clearly the \( \pi N \) elastic reaction shows a far greater sensitivity and, in our opinion, provides the least model-dependent determination.

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REFERENCES

[1] R. A. Arndt, I. I. Strakovsky, R. L. Workman, and M. M. Pavan, Phys. Rev. C 52, 2120 (1995); an updated analysis is available in nucl-th/9807087. Determinations via the GMO sum rule also fit into this category and are discussed in separate contributions to these proceedings.

[2] E. Matsinos, Phys. Rev. C 56, 3014 (1997).

[3] R.G.E. Timmermans, πN Newsletter 13, 80 (1997).

[4] R.A. Arndt, I.I. Strakovsky, and R.L. Workman, Phys. Rev. C 52, 2246 (1997).

[5] V. Stoks, R. Timmermans, and J.J. de Swart, Phys. Rev. C 47, 512 (1993).

[6] T.E.O. Ericson et al., Phys. Rev. Lett. 81, 5254 (1998); R.A. Arndt, I.I. Strakovsky, and R.L. Workman, Phys. Rev. C 52, 2246 (1995).

[7] D.V. Bugg, A.A. Carter, and J.R. Carter, Phys. Lett. 44B, 278 (1973).

[8] The Karlsruhe solution, KA84, was provided in a subroutine from R. Koch (private communication); R. Koch, Z. Phys. C 29, 597 (1985).

[9] R.L. Kelly, R.E. Cutkosky, Phys. Rev. D 20, 2782 (1979); R.E. Cutkosky et al., ibid. 20, 2804 (1979); 20, 2839 (1979).
Figure captions

Figure 1. Extracted values of $g^2/4\pi$ using different upper limits for the lab photon energy ($E_{\gamma}$). See text.
Table I. Fit SP40 from 0-400 MeV (57 parameters, $J_{\text{max}}=6$) including Uppsala data. Results are for $pp$ and $np$ data, Uppsala measurements at 96 MeV (U96) and 162 MeV (U162). The mapping in $\chi^2$ is calculated with and without (columns PPx and NPx) the $\chi^2$ contribution from Uppsala data. The fit incorporates 3421 $pp$ data and 3856 $np$ data (U96: 53 data, U162: 54 data).

| $g^2/4\pi$ | $\chi_{pp}^2$ | $\chi_{np}^2$ | $\chi_{u96}^2$ | $\chi_{u162}^2$ | $\chi_{PPx}^2$ | $\chi_{NPx}^2$
|---|---|---|---|---|---|---|
| 13.50 | 4377 | 5440 | 92 | 255 | 4371 | 5000
| 13.75 | 4375 | 5415 | 89 | 252 | 4368 | 4980
| 14.00 | 4386 | 5399 | 87 | 249 | 4379 | 4968
| 14.25 | 4411 | 5291 | 85 | 246 | 4404 | 4963
| 14.50 | 4450 | 5391 | 83 | 243 | 4443 | 4966

$g^2/4\pi_{\text{min}}$ 13.67(10) 14.37(13) — — 13.68(10) 14.28(13)
Table II. Fit SG40 from 0-400 MeV (49 parameters, $J_{\text{max}}=4$) including Uppsala data. Notation as in Table I.

| $g^2/4\pi$ | $\chi^2_{pp}$ | $\chi^2_{np}$ | $\chi^2_{u96}$ | $\chi^2_{u162}$ | $\chi^2_{PPx}$ | $\chi^2_{NPx}$ |
|------------|----------------|----------------|----------------|-----------------|----------------|----------------|
| 13.50      | 4510           | 5475           | 89             | 251             | 4512           | 5038           |
| 13.75      | 4513           | 5470           | 85             | 249             | 4515           | 5037           |
| 14.00      | 4533           | 5482           | 82             | 246             | 4535           | 5052           |
| 14.25      | 4569           | 5512           | 80             | 244             | 4572           | 5085           |
| 14.50      | 4622           | 5559           | 78             | 242             | 4626           | 5134           |

$g^2/4\pi_{\text{min}}$ 13.56(9) 13.69(9) — — 13.58(9) 13.64(9)
Table III. Fit SX40 from 0-400 MeV (48 parameters, $J_{\text{max}}=4$) excluding $np$ charge-exchange cross section data. Fit includes 3421 $pp$ data and 1683 $np$ data (see text).

| $g^2/4\pi$ | $\chi^2_{pp}$ | $\chi^2_{np}$ |
|------------|----------------|----------------|
| 13.50      | 4507           | 2216           |
| 13.75      | 4511           | 2200           |
| 14.00      | 4530           | 2192           |
| 14.25      | 4566           | 2188           |
| 14.50      | 4618           | 2190           |

$g^2/4\pi_{\text{min}}$ 13.57(9) 14.28(15)
Table IV. Fit to pion-photoproduction data (threshold to 500 MeV) for different values of $g^2/4\pi$ (see text). Fit includes 3441 $\pi^0p$ data, 2416 $\pi^+n$ data, and 974 $\pi^-p$ data.

| $g^2/4\pi$ | $\chi^2_{\text{all}}$ | $\chi^2_{\pi^0p}$ | $\chi^2_{\pi^+n}$ | $\chi^2_{\pi^-p}$ |
|------------|-----------------|----------------|----------------|----------------|
| 13.50      | 13084           | 7457           | 3723           | 1865           |
| 13.75      | 13074           | 7465           | 3727           | 1843           |
| 14.00      | 13071           | 7473           | 3735           | 1824           |
| 14.25      | 13075           | 7482           | 3745           | 1808           |
| 14.50      | 13088           | 7493           | 3760           | 1795           |

$g^2/4\pi_{\text{min}}$ 13.97(13) — — —
\[ \frac{g_{\pi}^2}{4\pi} = 13.91 \pm 0.14 \]