Stabilization of single-electron pumps by high magnetic fields

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We study the effect of perpendicular magnetic fields on a single-electron system with a strongly time-dependent electrostatic potential. Continuous improvements to the current quantization in these electron pumps are revealed by high-resolution measurements. Simulations show that the sensitivity of tunnel rates to the barrier potential is enhanced, stabilizing particular charge states. Nonadiabatic excitations are also suppressed due to a reduced sensitivity of the Fock-Darwin states to electrostatic potential. The combination of these effects leads to significantly more accurate current quantization.

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Single-electron devices proposed for quantum information technologies [14] and quantum electrical metrology [14–16] feature the capture, manipulation, and release of electrons through a series of gate pulses. To design such devices it is important to understand the electronic response to a rapid time-varying electrostatic potential, often in the presence of externally applied magnetic fields. The effects of magnetic confinement on electronic states [7, 8] and on electron-electron interactions [9] have been studied extensively. However, the effect of magnetic fields on the electron dynamics under time-varying potentials is less established. Semiconductor single-electron pumps in magnetic field are an example of a system which requires a consideration of these effects. It was found that accuracy of the quantization current produced by these devices was strongly enhanced in magnetic fields [10, 11] but the origin of this effect is yet to be explained.

In this Letter we use detailed measurements and numerical models to reveal how perpendicular magnetic fields change the tunneling dynamics of the quasi-bound electrons in a single-electron device—the tunable-barrier electron pump. First we find, through numerical calculations, that magnetic fields enhance the sensitivity of tunnelling rates to the confinement barriers. This is detected through the enhancement in stability of the number of pumped electrons. We then show that magnetic field increases the “stiffness” of the electronic wavefunction against the rapid electrostatic perturbations that arise when operating the pumps at high speed. This protects the trapped electrons from loss through nonadiabatic excitations [12], suppressing this process of quantum spillage. Our results also reveal that accurate current quantization is achievable at high magnetic field in these devices, which is crucial for uses in quantum metrology.

Our pumps use a dynamically formed quantum dot defined in a 2DEG AlGaAs/GaAs heterostructure by two surface gates. The gates cross an etch-defined wire terminated with ohmic electrical contacts, shown schematically in Fig. 1(a) [13]. One of the adjustable energy barriers, the entrance gate, is repeatedly lowered and raised by modulating the potential by $V_{RF}$ about a constant negative value $V_{G1}$ while holding the other exit gate at a constant negative voltage $V_{G2}$ [14]. This process is illustrated in Fig. 1(b)i-iv: Electrons from the source reservoir are loaded into a quantum dot formed in the space between the gates (i) which is increasingly isolated by the rising entrance barrier. Some initially-trapped electrons escape back through this rising barrier before tunneling is cut off completely (ii). These back-tunneling rates $\Gamma_n$ vary dramatically for different charge number $n$ due to the charging energy. This leads to the trapping of the same number of electrons in each cycle, and is the mechanism for quantisation of charge transport in this type of pump (iii). Electrons that remain trapped are forced over the exit barrier into the drain lead (iv), producing a current in an external circuit [15]. The number of electrons pumped by the dot can be changed by adjusting the values of $V_{G1}$ and $V_{G2}$ giving current plateaux at integer multiples of $ef$, where $f$ is the operating frequency. Broadly speaking, $V_{G2}$ controls the number of electrons trapped after stage ii, while $V_{G1}$ determines how many of these electrons are emptied into the drain [15].

Figure 1(c) shows measurements of the output current of a pump operating at a frequency $f = 400$ MHz performed in a $^3$He cryostat with a base temperature of $\sim 300$ mK. Data is shown for magnetic fields up to 14 T in 2 T steps, concentrating on the plateau at $I = ef$ (i.e. $n = 1$) visible over a range of exit gate $V_{G2}$ (data have been offset for clarity). This reveals both a sharpening of the transition region and a lengthening of the current plateaux in magnetic field similar to that seen previously [10, 11], reducing errors in pump operation.

The movement of plateaux boundaries in magnetic field is shown in Fig. 1(d) (this was subtracted in Fig. 1(c) where data are aligned to the edge of the second plateau). This shift in position has been linked to magnetic confinement [17], a reduction in tunnel rate is compensated by a reduction in $V_{G2}$ giving the same pump current. We have performed detailed high resolution measurements [18] on this device as shown in Fig. 1(e). We zoom in on the plateau, expanding the current scale by a factor 4,000. This shows that the magnetic field brings the pumped current continuously closer to the expected value of $ef$ (corresponding to exactly one electron pumped...
per cycle) even at the highest fields accessible with our experimental system. Now we focus upon the relationship between the measured pump current and the electronic wavefunction in the dynamic quantum dot and how these are affected by the introduction of a magnetic field.

The pumped current is determined by the back-tunneling of excess electrons before the dot is isolated from the leads\textsuperscript{[6, 19]} where the entrance barrier is rising rapidly as in Fig. 1(b) ii. The key parameters are the time-dependent tunnel rates $\Gamma_n$ out of the dot, as determined by the confining geometry\textsuperscript{[19]}. The separation of lifetimes between states that differ by one electron $\Gamma_n \ll \Gamma_{n+1}$, combined with the increasing opacity of the tunnel barrier leads to the mean number of electrons captured $\hat{n} \approx n$ an integer. The effect of magnetic field can be investigated by calculating $\Gamma_n(B)$ for a model of the confinement potential.

We have calculated the tunnel coupling for a two-dimensional potential well based on the experimental configuration in a magnetic field. The broadening of the dot levels are calculated by the lattice Green’s function method\textsuperscript{[20, 21]}. Figure 2(b) shows the back-tunnel rate as a function of $V_1$ for a fixed exit barrier height $V_2 = 50 \text{ mV}$. This shows the expected exponential variation at all fields but with a reduction in the overall tunnel coupling at higher fields – the dot is increasingly decoupled from the leads by magnetic confinement. This is expected to shift the operating point of the pump at higher fields as shown above\textsuperscript{[17]}. In addition, the sensitivity of $\Gamma$ to the barrier height (i.e. the slope of the fits in Fig. 2(b)) is strongly enhanced in higher magnetic fields. We
show below that this is responsible for enhanced quantization.

We fit the tunnel rate to the expression $\Gamma = \Gamma_0 \exp[EB/\epsilon(B)]$ where $E_B$ is the difference in energy between the dot level and the barrier $V_1$ and $\epsilon(B)$ is a field-dependent parameter characterizing the barrier/dot geometry. We take $E_B = -eV_1 - E_n$ where $E_n = E_0(V_1, V_2) + (n-1)U_c$, i.e. a single electron energy plus a charging energy $U_c$ for each additional electron. $\epsilon(B)$ can then be extracted from the numerical calculations of $\Gamma(V_1)$ shown above.

We find that $\epsilon(B)$ is reduced by a factor 6 between $B = 0$ T and 10 T, increasing the relative tunnel rates $\delta = \ln(\Gamma_{n+1}/\Gamma_n)$ by the same factor. This is shown schematically in Fig. 2(c) where the $V_1$ ($\approx$ linear in time on the scale of this process) dependence of the decay rates for dots with $n$ (solid lines) and $n+1$ electrons (dotted lines) is plotted. The enhanced sensitivity to barrier height is caused by magnetic confinement of the wavefunction at high field; at fields of $B \approx 10$ T the wavefunction is largely confined to a region of only $\leq 30$ nm across so the electron wavefunction must be forced very close to the barrier to give any appreciable tunnel coupling. In this regime the probability density is so concentrated that small variations in barrier height change the tunnel-coupling very rapidly. In comparison, at low field, the tunnel coupling varies more slowly and the process of decoupling the dot states from those in the leads is more gradual.

Enhancing the ratio of tunnel rates between $n+1$ and $n$ electron states will increase the stability of the number of trapped charges. We show this in Fig. 2(d) where we plot the calculated pump current $I(V_2)$ according to a rate equation model describing the back-tunneling process [19], but including the field dependence of $\epsilon(B)$ found above. This predicts a sharpening of the plateaux transitions like that seen experimentally (e.g. Fig. 1(a)). This model assumes that the dot initially contains several more electrons than the final number pumped but due to the enhanced tunnel rates for higher $n$ all memory of the initial charge state is erased by the rapid decay of these states [19]. The functional form of $I(V_2)$ arises from changes in the dot energy levels as $V_2$ is varied. The electronic wavefunction is “squeezed” against the entrance barrier, tuning the overall tunnel coupling. On plateaux this is set so that the $n+1$ state is fully emptied on the time scale of the changes in barrier height, leaving $n$ electrons trapped. In between plateaux this process is incomplete and the number of electrons trapped fluctuates.

The confinement effect described above increases the range of $V_2$ over which the unstable state is essentially fully emptied. This effect alone gives a longer current plateaux. However, the dependence of tunnel rates on $V_2$ also sensitive to magnetic confinement effects; tunnel rates are more sensitive to $V_2$ at high field giving a re-scaling of $I(V_2)$. The net effect is plateaux of the same length in the control parameter $V_2$ but with much sharper boundaries. Experimentally both sharpening and lengthening of current plateaux in magnetic field have been observed in devices of different design. Increases in the charging energy alone could cause lengthening but we do not expect that the charging energy to change by as much as a factor of two, as observed experimentally. An alternative explanation is that the rescaling effects described above are not identical for $V_{G2}$ and $V_{G1}$. This would naturally give a combination of lengthening and sharpening. Investigating this effects in detail would require a more accurate confinement model, but we believe the gross effect of magnetic field on pump current accuracy is well captured by the above description.

We now address a distinct second effect of magnetic field on this system. In Ref [12] it was shown that the rapid changes in electrostatic confinement potential can populate excited states...
of the dot by nonadiabatic processes [23]. Tunneling from excited states is more likely than from the ground state; electrons "spill" out of the pump and the current plateaux are eroded into a number of unquantised steps. This gives spectroscopic information about the dot states but destroys the accuracy of current quantization. We show here that these excitation features have a non-monotonic field dependence; excitations cannot be detected at low field but are also strongly suppressed at high field. This discovery has consequences for the ultimate accuracy of the quantum dot pump, but also acts to demonstrate the sensitivity of nonadiabatic processes to magnetic confinement.

Figure 3(a) shows \( I(V_{G2}) \) and \( dI/V_{G2} \) for \( f = 1 \text{ GHz} \), which is sufficiently high to induce excitations of the dot via nonadiabatic effects at intermediate fields. Excitation features emerge at modest fields of a few Tesla [12] but as the field is increased above 12 T these features (peaks in the derivative) become significantly weaker. This results in the recovery of accurate current quantization even when operating at higher frequencies. Figures 3(b), (c) shows this effect in more detail. The size of the field window where excitations are seen is of different size in different samples.

The observation of a range of fields where nonadiabatic effects are visible can be explained by a competition between magnetic and electrostatic effects on the electronic wavefunction. Nonadiabatic transition rates depend both on the strength and rapidity of the perturbation of the wavefunction [12, 24]. At high field \( \omega_c \gg \omega_0 \), the relative contribution to the confinement of the electrostatic component is diminished and the magnetic component determines the size of the wavefunction. Changes in the confinement potential during pumping have correspondingly weaker effects on the dot wavefunction and nonadiabatic transition rates will be reduced. Figures 3(d) and 3(e) show, for example, that the probability density \( |\psi|^2 \) and eigenenergy \( E_0 \) of the lowest energy orbital Fock-Darwin state become increasingly insensitive to changes in \( \omega_0 \) at higher field. This is consistent with the disappearance of the excitation features at the highest fields.

A significant increase in the excitation gap between ground and excited states could be responsible for suppressing excitation effects at low field. A value of \( \hbar \omega_0 \approx 8 \text{ meV} \) at \( B = 0 \) has been found in these pumps previously [12] by fitting features in the pumped current to the energy differences \( \Delta_n = E_n - E_0 \) for Fock-Darwin eigenenergies \( E_n(B) \) (here \( n \) is the radial quantum number). At higher field \( \Delta_n \) is reduced but for the above value of \( \omega_0 \) this variation is rather weak, with a drop of only 20% in an applied field of 2 T. If the excitation processes happen earlier in the pumping cycle, when the dot confinement is smaller, the field dependence of \( \Delta_n \) is much stronger (see Fig. 3(f)). In this case the rapid rise in the excitation gap near zero field can cut off the excitation process as seen experimentally.

We have shown that electron dynamics in single-electron tunable-barrier pumps are sensitive to magnetic fields via two mechanisms; the time dependence of tunnel rates and by suppression of nonadiabatic transitions. These effects allow the pumps to operate with error rates smaller than a few parts in \( 10^6 \) at high field. This shows that this is a convenient system in which to study electron dynamics in high magnetic fields.

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