Abstract. This paper studies the stock market return’s volatility in the Eurozone as an input for evaluating the market risk. Stock market returns are endogenously determined by long-term interest rate changes and so is the return’s conditional variance. The conditional variance is the time-dependent variance of the underlying variable. In other words, it is the variance of the returns measured at each moment $t$, so it changes through time depending on the specific market structure at each time observation. Thus, a multivariate EGARCH model is proposed to capture the complex nature of this network. By network, in this context, we mean the chain of stock exchanges that co-move and interact in such a way that a shock in one of them propagates up to the other ones (contagion). Previous studies provide evidence that the Eurozone stock exchanges are deeply integrated. The results indicate that asymmetry and leverage effects exist along with fat tails and endogeneity. In-sample and out-of-sample forecasting tests provide clear evidence that the multivariate EGARCH model performs better than the univariate counterpart to predict the behavior of returns both before and after the 2008 crisis.

1 Introduction

Many studies on stock market volatility use a univariate approach and/or are mostly concerned with modeling the mean parameters of the underlying distribution function. However, understanding the variance effect and its evolution over time is also a major research topic in financial markets, since the variance can provide important information about the asset’s risk and its evolution over time. Moreover, if we can model simultaneously the mean and the variance effects the approach becomes even more informative and it works in a fully (or quasi-fully) endogenous framework. Therefore, an
investor can make more accurate decisions if she knows more deeply how risky is the investment she is about to make.

Higher risk is associated with larger price variations over time and conversely for lower risk. Higher risk is also associated with stock market price declines. This path can move due to changes in other market parameters or to the specific time moment of the measurement. In other words, volatility may depend not only on the behavior of other market variates but it may be also time dependent. Thus, an appropriate environment for studying market volatility may be the use of GARCH-type models. In the next section, we shall discuss these models in more detail. The third section describes the empirical data and results. The original data consist of stock market price indices and long-term interest rates for eleven Eurozone countries over the period 1986-2014. This means that the time window of our analysis covers the pre and the post 2008 crisis period. During the crisis period the long-term interest rates increased as well as market instability. How much was this effect? How long did it take to return to a near equilibrium situation if at all? Section 4 presents the major conclusions of this work.

2 Background and Methodology

2.1 Background

As the globalization process deepens and the economic agents compete in worldwide markets, the internationalization of the financial sources is also intensified. It makes a big difference to compete in a small market with 10 million inhabitants or another one with 500 million. When the price of commodities tends to uniformity across markets it makes sense that financial returns play a greater role in the financial management of firms or other economic agents. Economic agents will tend to search for the most profitable financial sources at the lowest market risk. However, there is a trade-off between financial returns and risk. Higher expected returns are usually associated with higher expected risk and the investor wishes to maximize the former and minimize the latter. The equilibrium point between returns and risk depends on the risk profile of each investor. Assuming that the returns are exogenously and a priori determined, the investor will look, basically, at the risk.
There are many definitions and dimensions of risk. There are also a large number of ways to measure risk. In this study, the volatility in the stock market represents the risk. It is a dispersion measure since its computation derives from the conditional variance obtained out of the estimation of a GARCH-type model. The conditional variance is a time series that represents the estimated variance level of the underlying asset at each moment in time. Low volatility markets are always preferable to high volatility markets, at least if the investor is risk-averse and not a speculator. In addition, the volatility increases when the market prices decrease and vice-versa. Therefore, higher stock market instability is linked with bear stock markets.

Across the whole sample period under analysis (1986-2014), there are two distinct subperiods. The first one ended in 2007/08 with the worldwide economic crisis. Bull stock markets characterize the most part of this relatively tranquil period, with patchy episodes of short-term cracks usually motivated by very specific events. However, none of these cracks had the dimension of the 2008 crisis. Beginning in the US, the global crisis of 2008 affected mainly Europe and in particular the Eurozone economies. It lasted up to 2014 in some cases. Markets turned out to be more turbulent and volatile. Interest rates increased dramatically in many countries due to the sovereign debt crisis that affected mainly the southern European economies. Stock market prices declined very sharply for a couple of years and the bailout-rescued countries (Greece, Portugal, and Ireland) were obviously among the hardest affected. The crisis persisted for a while but in most countries, the stock market started to recover again and volatility went back to lower levels. In this paper, we shall call pre-crisis sample to the period 1986-2008 and post-crisis sample to the period 2008-2014. The breakpoint occurred in September 2008 as was endogenously confirmed by the Lanne-Lütkepohl-Saikkonen [1] unit root test with structural breaks.1

2.2 Conditional Heteroskedasticity

In financial data applications, a popular model of volatility is the Generalized AutoRegressive Conditional Heteroskedasticity or GARCH\((p, q)\). This model, formerly

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1 The results of these tests are available from the authors upon request.
proposed by [2], generalizes the original ARCH($q$) model introduced by [3]. A simple version of the GARCH($p$, $q$) model with $k$ exogenous variables in the mean equation is:\(^2\)

$$y_t = \mu_0 + \sum_{i=1}^{k} \mu_i x_{it} + \varepsilon_t,$$

$$\varepsilon_t = \nu_t \sigma_t,$$

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2,$$  \hspace{1cm} (3)

where $\varepsilon_t \big| I_{t-1} \sim \text{N}(0, \sigma_t^2)$; $\nu_t \sim \text{iid}(0, 1)$; $\alpha_0 > 0$; $\alpha_1, \ldots, \alpha_q, \beta_1, \ldots, \beta_p \geq 0$ and $\alpha_1 + \ldots + \alpha_q + \beta_1 + \ldots + \beta_p < 1$. $I_{t-1}$ denotes the information set available up to the moment $t-1$. The error term $\varepsilon_t$ represents a random disturbance at time $t$ and $\sigma_t^2$ represents its conditional variance. Equation (1) models the mean, that is, the $\mathbb{E}(y_t \mid x_{it})$; equation (2) represents a factorization of the random disturbance into a innovation ($\nu_t$) and the conditional standard deviation of the error term ($\sigma_t$); equation (3) models the conditional variance. The conditional variance is a linear function of its own historical values up to lag $p$ and the historical values of the squared residuals up to lag $q$.

The restriction $\alpha_1 + \ldots + \alpha_q + \beta_1 + \ldots + \beta_p < 1$ is required in order to assure the stability and covariance stationarity of $\varepsilon_t$.\(^3\) If all the $\beta$ parameters in (3) are simultaneously equal to zero then we have the ARCH($q$) model. Notice that the GARCH($p$, $q$) is asymptotically equivalent to the ARCH($\infty$) model and many conditional variance processes can be adequately represented by a GARCH(1, 1) model. In this case the unconditional variance of $\varepsilon_t$ is:

$$\text{var}(\varepsilon_t) = \sigma_\varepsilon^2 = \alpha_0 / (1 - \alpha_1 - \beta_1),$$

with $\alpha_1 + \beta_1 < 1$.

In the GARCH($p$, $q$) model the variance of the error term at the moment $t$, conditional on past errors is given by $\sigma_t^2 = \text{var}(\varepsilon_t \mid \varepsilon_{t-j})$. $\sigma_t^2$ is an increasing function of the magnitude of past errors, independently of its sign, given that the variance equation uses squared errors. Therefore, positive or negative errors of large magnitude tend to follow

\(^2\) Notice that $p$ denotes the number of lags in the conditional variance and $q$ denotes the number of lags in the squared error term.

\(^3\) Otherwise, the regression equation (1) is spurious.
errors of the same kind and likewise for errors of small magnitude. This is known in the literature as volatility clustering and is a common feature of many financial time series based on returns.

Another important feature of many financial return’s time series is leptokurtosis or fat tails. Leptokurtosis occurs when the shape of the distribution is more peaked than the bell-shaped Gaussian distribution. The distribution around the mean value is narrower but the tails are longer and fatter than that in the Gaussian distribution. This means that extreme events are more likely to happen (not so rare) and have a larger magnitude. Therefore, the assumption of random normal movements in the stock market returns (or price changes) does not seem to be adequate. The GARCH-type models allow us to use non-Gaussian distributions to describe the behavior of the errors $\varepsilon_i$. Among them, two popular distributions are the $t$-Student and the Generalized Error Distribution. Both are adequate to deal with leptokurtosis and volatility clustering. In this paper, we shall rely on the $t$-Student distribution of the error term.

Many studies on stock market volatility use GARCH-type models to infer about the time-path behavior of returns. Among them we find [4]-[17]. Other authors analyze the issue of volatility based on statistical physics concepts (see, e.g. [18] among others).

2.3 Asymmetric Volatility

The GARCH($p$, $q$) model gives a symmetric treatment to the conditional variance, that is, positive and negative perturbations of the same magnitude have a similar effect over the estimated conditional variance. However, it is well known that such symmetry may not exist and that in many cases bad news and good news have different impacts on the behavior of volatility. Usually, the volatility is more sensible to bad news than to good news. A commonly cited example is the relationship between the price of oil and the price of gasoline. When the oil price increases the price of gasoline also increases. When the oil price decreases the price of gasoline decreases too but not with the same magnitude.

Nelson [19] suggested the use of an exponential GARCH model to deal with the issue of asymmetry. Nelson’s model is known in the literature as the EGARCH($p$, $q$). In addition to the issue of asymmetry, the EGARCH model offers the advantage of not
imposing non-negativity restrictions on the variance equation parameters in order to secure the non-negativity of $\sigma_t^2$ over its sample domain. As for the GARCH, this model assumes short memory and null persistency. This means that the variables in the model are integrated of order zero [I(0)].

The variance equation of the EGARCH\((p, q)\) model can be written as:

$$
\ln(\sigma_t^2) = \alpha_0 + \sum_{j=1}^{q} \alpha_j \left( \frac{\epsilon_{t-j}}{\sigma_{t-j}} - \sqrt{\frac{2}{\pi}} \right) + \sum_{k=1}^{m} \gamma_k \frac{\epsilon_{t-k}}{\sigma_{t-k}} + \sum_{i=1}^{p} \beta_i \ln(\sigma_{t-i}^2).
$$

The interpretation of $\alpha_j$ and $\beta_i$ is similar to that of the GARCH\((p, q)\) model. The main difference resides in the $\gamma_k$ parameters. These parameters allow us to test both the presence of asymmetric effects and whether there are leverage effects. The effect is asymmetric if $\gamma_k \neq 0$. In addition, if $\gamma_k < 0$ there are leverage effects. In the EGARCH\((p, q)\) model the leverage effects are exponential. Given that the variance equation is specified in order to the logarithm of the conditional variance (and not the conditional variance itself as in the GARCH model), the conditional variance forecasts are always positive.

We already know what the meaning of asymmetric volatility is: different magnitudes of the effect of similar positive and negative shocks in the market. Now, how can we interpret the leverage effect? The leverage effect occurs when the volatility increases more during bear market periods, and vice-versa, after a shock of similar magnitude. It can be seen as a particular case of asymmetry since it occurs when the relationship between returns and volatility is negative. (see, e.g. [20]-[22]).

Indeed, when the stock market share price declines and the firm’s liabilities remain constant in the short-run, the firm’s ratio debt/equity increases. The firm becomes more leveraged and its future more uncertain. Therefore, share prices will become more volatile. Additionally, when volatility increases more in result of negative rather than positive returns, the autocorrelation between squared returns in consecutive periods tends to be high and negative. This will cause asymmetric volatility clustering in the market.

Assuming in (5) that $p = q = 1$, the EGARCH\((1,1)\) variance equation is given by:

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4 The mean equation is the same as in the GARCH model [Eq. (1)].
\[
\ln\left(\sigma_t^2\right) = \alpha_0 + \alpha_1 \left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}}\right) + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \ln\left(\sigma_{t-1}^2\right),
\]  
(6)

where the interpretation of the parameters is similar to that of the EGARCH\((p, q)\) model. Notice that in the above specification exogenous regressors only appear in the mean equation. The variance equation is fully endogenous and depends only on the residual’s series of the mean equation and its estimated conditional variance. Thus, the robustness of the estimates of the variance parameters depends a lot on the correct specification of the mean equation. Yet, is it not possible that the conditional variance itself depends on exogenous regressors, along with endogenous ones? We shall explore this issue in the next subsection.

### 2.4 Multivariate Conditional Volatility

The asymmetric conditional heteroskedasticity model that we propose extends the original EGARCH\((p, q)\) model to a multivariate framework. It includes a set of regressors in the variance equation in order to produce better estimates of the conditional variance time series. The parameters of the exogenous regressors in the variance equation are denoted by the vector \(\theta = (\varphi, \delta)\). The estimation process of this modified EGARCH model poses no serious problems if the mean and variance equation regressors are independent of the error term \(\varepsilon_t\) and of its conditional variance \(\sigma_t^2\). Otherwise, a Feasible Generalized Least Squares estimator can be constructed to estimate the model, such as proposed by [23].

The aim of this study is to analyze the behavior of the conditional variance of the stock market returns of some Eurozone countries over a relatively long time span. Such behavior may shift depending on interest rate changes and other economic and/or financial factors. The occurrence of a large crisis may change the structural behavior of stock market returns, at least while it lasts. Of course, many other economic and behavioral factors may affect the time-path of stock market returns. We shall keep our model as simple as possible and therefore we just model market returns as a function of long-term interest rate changes and a series of dummies denoting the post-crisis years. These
dummies capture the specific effects of each year since the outset of the crisis (2008-2013). The augmented EGARCH(1, 1) that we propose takes the following form:

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \left( \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right) + \gamma_1 \varepsilon_{t-1} + \beta_1 \ln(\sigma_{t-1}^2) + \sum_{i=1}^{l} \varphi_i \varepsilon_{t-1} + \sum_{j=1}^{m} \delta_j D_j,$$

(7)

where the $\varphi_i$ are the parameters of the continuous regressors and the $\delta_j$ are the parameters of the annual dummies. We only included one continuous regressor in (7): the long-run Treasury bill interest rate change (10 years). So $x_{it} = \Delta(TB_t)$ where TB denotes the 10-year Treasury bill interest rate and $\Delta$ is the first difference operator. There are six dummies $D_j$ in (7), one for each year over the time window 2008-2013. The mean equation is the same as shown in (1) where $y_t$ denotes the stock market returns $[\ln(P_t/P_{t-1})]$ and $x_{it}$ is as defined above. Therefore, the specification of our model is multivariate in the mean and in the variance and this is an important improvement relatively to previous studies that adopt a univariate framework. We proceed in the next section with the presentation of the data and main findings.

3 Data and Results

3.1 The Data

The data used in this paper consist of daily stock market price indices and 10-year Treasury bill interest rates for 11 countries that belong to the Eurozone. The sample spans from January 1st 1986 to January 3rd 2014, totalizing a maximum of 7305 data points for each time series. The data points cover five days per week (Monday to Friday). All the countries in this study are included in the developed markets group. The data were collected in the Datastream database. All the daily stock market indices were converted to relative price indices where the base 100 corresponds to the first recorded observation. Stock market returns correspond to the first difference of log relative prices at each moment $t$. Interest rate changes are the first difference of observed interest rates at each moment $t$.

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5 The series do not necessarily start at the same date for all the countries, thus the sample size may vary from series to series.

6 Note that the first difference is given by $\Delta x_t = x_t - x_{t-1}$. 
Figure 1 shows the time series of the daily stock market returns and 10-year interest rate changes for the 11 Eurozone countries over the sample period considered.

![Figure 1](image-url)

**Figure 1.** Daily stock market returns and 10-year interest rate changes for 11 Eurozone countries
Source: Datastream. Sample: 1/06/1986 to 1/03/2014.

The stock market returns panel shows two periods of high and persistent volatility: the first one approximately between 1997 and 2002, and the second one from the 2008 crisis onwards. The 10-year interest rate changes panel exhibits the interest rate instability period following the 2008 crisis with extreme peaks between 2010 and 2012. These extreme peaks correspond to the large rise in the long-term interest rates of the rescued countries. Towards the end of the sample period, the interest rates market becomes a bit more stable.

Table 1 presents some descriptive statistics of the stock market returns and 10-year interest rate changes over the sample period.

Some highlights of these results are: Greece (0.018) and Finland (0.017) have the higher unconditional standard deviation of the 11 stock market returns series analyzed. Greece (0.509) and to a lesser extent Portugal (0.095) have the higher standard deviation of the 10-year interest rate change time series analyzed. All means all close to zero. The Jarque-Bera test rejects the null of normality at the 1% level in all cases. This is consistent with previous findings that stock market returns (and also interest rate chang-
es) should be modeled in a non-Gaussian framework. Volatility clustering and fat tails prevail as the main features of these variates.\footnote{Unit root tests were also carried out and rejected the null of nonstationarity at the 1% level. The results are available from the authors upon request.}

| Country    | Stock Market Returns | 10-year Interest Rate Changes |
|------------|----------------------|------------------------------|
|            | Mean     | SD        | J-B     | N        | Mean     | SD        | J-B     | N        |
| Austria    | 1.84E-04 | 0.011     | 29978   | 7305     | -6.84E-04| 0.040     | 7850    | 7164     |
| Belgium    | 2.51E-04 | 0.010     | 32752   | 7305     | -9.05E-04| 0.047     | 9414    | 6414     |
| Finland    | 2.50E-04 | 0.017     | 18870   | 6725     | -1.51E-03| 0.058     | 130525  | 5829     |
| France     | 2.54E-04 | 0.012     | 10656   | 7305     | -1.01E-03| 0.055     | 64951   | 7284     |
| Germany    | 1.72E-04 | 0.012     | 33911   | 7305     | -6.01E-04| 0.045     | 4773    | 7305     |
| Greece     | 2.42E-04 | 0.018     | 10336   | 6784     | 5.84E-04 | 0.509     | 7.92E+08| 3851     |
| Ireland    | 2.90E-04 | 0.013     | 42020   | 7305     | -8.25E-04| 0.072     | 403673  | 7203     |
| Italy      | 1.09E-04 | 0.013     | 6258    | 7305     | -1.69E-03| 0.071     | 47165   | 5957     |
| Netherlands| 1.97E-04 | 0.012     | 20925   | 7305     | -6.16E-04| 0.042     | 2903    | 7305     |
| Portugal   | 6.50E-05 | 0.010     | 24744   | 6263     | -9.10E-04| 0.095     | 769748  | 5340     |
| Spain      | 2.06E-04 | 0.013     | 9865    | 7004     | -1.41E-03| 0.064     | 43865   | 5936     |

Notes: J-B [24] normality test (H\(_0\): the distribution of \(X\) is Gaussian. Rejection of the null at 1% in all cases). Sample: 1/06/1986 to 1/03/2014.

### 3.2 Multivariate EGARCH Results

Table 2 displays the parameter estimates of the mean and variance equations of the multivariate EGARCH model where the predictor variable is the daily 10-year interest rate change (first difference) and the predicted variable is the daily stock market returns (first difference of the log index). Both variables are I(0).

The mean equation is a static simple linear regression model that relates the predicted with the predictor variable. The variance equation is a dynamic model that relates the log conditional variance (volatility) with its historical values and the historical values of the mean equation residuals. Additionally, the log conditional variance also depends on the 10-year interest rate change and a set of six dummies for the post-crisis years. These variables are used as instruments in order to correctly isolate the true asymmetric effects of the stock market returns.

The interpretation of the mean equation parameter estimates is as usual. The slope coefficient \(\mu_i\) denotes the average impact of a unit variation in the interest rate change on stock market returns. All the estimates are significantly negative at the 1% level ex-
cept the one for Finland. Thus, a positive shock in interest rates implies a decline in returns, which is consistent with theoretical expectations. The magnitude of the impact is larger for France and Italy (0.036) than for the remaining countries. \( \mu_0 \) is the intercept and denotes the expected value of the returns when there is no change in the 10-year interest rate.

### Table 2. Multivariate EGARCH parameter estimates: mean and variance equations

| Country   | Mean Equation | Variance Equation |
|-----------|---------------|-------------------|
|           | \( \mu_0 \)  | \( \alpha_0 \) | \( \alpha_1 \) | \( \gamma_1 \) | \( \beta_1 \) |
| Austria   | 0.001 **      | -0.007 **        | -0.543 **      | 0.247 **      | -0.035 **      | 0.963 **      |
| Belgium   | 0.000 **      | -0.018 **        | -0.395 **      | 0.200 **      | -0.067 **      | 0.975 **      |
| Finland   | 0.001 **      | 0.000            | -0.151 **      | 0.121 **      | -0.034 **      | 0.993 **      |
| France    | 0.001 **      | -0.036 **        | -0.306 **      | 0.148 **      | -0.083 **      | 0.979 **      |
| Germany   | 0.001 **      | -0.025 **        | -0.288 **      | 0.155 **      | -0.070 **      | 0.982 **      |
| Greece    | 0.000         | -0.005 **        | -0.521 **      | 0.194 **      | -0.056 **      | 0.958 **      |
| Ireland   | 0.001 **      | -0.014 **        | -0.346 **      | 0.173 **      | -0.037 **      | 0.977 **      |
| Italy     | 0.000         | * -0.036 **      | -0.317 **      | 0.173 **      | -0.060 **      | 0.980 **      |
| Netherlands | 0.001 **  | -0.011 **        | -0.268 **      | 0.153 **      | -0.070 **      | 0.984 **      |
| Portugal  | 0.001 **      | -0.008 **        | -0.449 **      | 0.247 **      | -0.050 **      | 0.973 **      |
| Spain     | 0.001 **      | -0.018 **        | -0.289 **      | 0.143 **      | -0.071 **      | 0.981 **      |

Notes: Method: ML - ARCH (Marquardt) - Student’s t distribution. ** Significant at 1%. * Significant at 5%. \( \phi_i \) and \( \delta_j \) parameters are not reported but the estimates are available from the authors upon request. Sample: 1/06/1986 to 1/03/2014.

Regarding the variance equation we report the parameter estimates \( \alpha_0 \), \( \alpha_1 \), \( \gamma_1 \) and \( \beta_1 \). All these estimates are significant at the 1% level. \( \alpha_0 \) denotes the intercept which is not constrained to be positive in the EGARCH specification. Note that a negative value of \( \alpha_0 \) means that the expected conditional variance is lower than one when all the variates in the variance equation are equal to zero. \( \alpha_1 \) is the coefficient of the ratio between the absolute value of the mean equation residuals and the conditional standard deviation at time \( t-1 \). \( \gamma_1 \) is the coefficient of the ratio between the residuals and the conditional standard deviation at time \( t-1 \). Finally, \( \beta_1 \) is the first-order autoregression coefficient of the log conditional variance (dependent variable).

As noted in section 2 the parameter \( \gamma_1 \) is the key to assess asymmetric and leverage effects in the volatility caused by past positive and negative shocks. There are asymmetric effects if \( \gamma_1 \neq 0 \). In addition, if \( \gamma_1 < 0 \) we observe exponential leverage effects. This occurs when bad news (\( \varepsilon_{t-1} < 0 \)) imply an increase in volatility. Notice that \( \varepsilon_{t-1} < 0 \) happens when a negative shock at moment \( t-1 \) provokes a decrease of the underlying value.
of the predicted variable at moment $t$, relatively to the long-run equilibrium relationship. This is consistent with the previous findings that volatility (instability) increases when stock market prices decrease. A significant negative value of $\gamma_1$ under the EGARCH model implies that the larger is the absolute value of $\gamma_1$ the smaller is the leverage effect. This is the case of France (-0.083) and Spain (-0.071). Conversely, Finland (-0.034) and Austria (-0.035) exhibit the largest leverage effect over the whole sample period amongst the 11 countries analyzed.

### 3.3 In-sample Forecasts

Based on Eq. (7) one can obtain the estimates of the stock market return’s conditional variance at each moment $t$. These estimates are the in-sample forecasts of the underlying volatility. Using two standard errors as boundaries one can construct confidence intervals around the static forecasts. When the interval of confidence is wider this means that the forecasted conditional variance (volatility) increases. Figure 2 displays the graphs of the in-sample volatility forecasts for the 11 countries analyzed.
The graphs depicted in Figure 2 are elucidative. In all cases except Finland, the boundaries of the confidence interval are close to -0.02 and 0.02 in the pre-crisis period. These boundaries increased dramatically during the crisis period and came back to the pre-crisis values towards the end of the sample window except in Greece, Portugal, and Spain.
Spain (the southern peripheral GPS countries). The peak of high volatility occurred in most cases in the last quarter of 2008 or 2009. A right-side tapered arrow indicates less persistence of the crisis and, therefore, a faster return to the pre-crisis volatility levels. In the case of the rescued countries, Ireland shows a quite fast recovery, which is not the case of Greece and Portugal, at least during the sample window analyzed. Finland exhibits high volatility since the 1990’s and throughout the most part of the sample period.

To conclude, the EGARCH volatility forecasts appear to be consistent with the observed performance of the stock markets, in particular during the crisis and onwards. The distinction between the effect of good and bad news is remarkable and statistically significant. Likewise, the assumption of \( t \)-Student errors rather than Gaussian seems to be more realistic and robust given the leptokurtic feature of the return’s data.

3.4 Out-of-sample Forecasts

The in-sample forecasts presented in the previous section show that the multivariate EGARCH model proposed in this paper is a good option to explain the time-path of the conditional variance even when market turbulence occurs due to exogenous factors. In our case, these exogenous factors include the 10-year interest rate change and the specific effects of the 2008 crisis and subsequent periods. However, the in-sample forecasts are not able to provide information about the forecasting accuracy of the model for future periods. Such information can be obtained from out-of-sample forecasting tests.

Out-of-sample forecasting tests allow us to select the model specification that yields better accuracy forecasts. To this end, we split the whole sample into the subsample used for estimation purposes and the subsample used for forecasting comparisons. We are especially interested in assessing whether the pre-crisis observations are useful to forecast the post-crisis ones. Thus, we re-estimate the models using just the pre-crisis observations. In order to obtain the post-crisis (out-of-sample) forecasts we use the estimated parameters. These forecasts can be compared with the observed values of the underlying forecasted variable. The more accurate forecasts we obtain the better and these will be used to select the best model specification.

The competing model specifications are the multivariate EGARCH and the univariate EGARCH (see, e.g. [25]). Although the Diebold-Mariano [26] forecasting accuracy
test is a popular one, it has some drawbacks that prevent its use. In our case, as the forecasting horizon is long, the Diebold-Mariano test leads to problems of oversizing because the null hypothesis of equal forecasting accuracy tends to widen the confidence interval around $H_0$. An alternative is to test whether a set of forecasts encompasses a rival set ([27]-[28]). These tests allow us to compare the forecasting performance of manifold specifications although in this study we just compare two. We shall use the Harvey et al. [29] methodology extended later to allow for multiple comparisons by [30]. The results are presented in Table 3.

| Country   | F-standard Test | p-value | HLN Test | p-value |
|-----------|-----------------|---------|----------|---------|
| Austria   | 2.166           | (0.151) | 3.416    | (0.066) |
| Belgium   | 3.253           | (0.072) | 1.769    | (0.184) |
| Finland   | 1.981           | (0.178) | 3.528    | (0.061) |
| France    | 2.531           | (0.112) | 3.390    | (0.067) |
| Germany   | 2.578           | (0.109) | 3.243    | (0.074) |
| Greece    | 22.073          | (0.000) | 13.816   | (0.000) |
| Ireland   | 1.350           | (0.246) | 1.799    | (0.180) |
| Italy     | 0.028           | (0.866) | 0.031    | (0.861) |
| Netherlands | 2.718        | (0.100) | 2.796    | (0.095) |
| Portugal  | 4.885           | (0.046) | 3.252    | (0.074) |
| Spain     | 0.128           | (0.720) | 0.115    | (0.735) |

Notes: $H_0$: the multivariate EGARCH encompasses the univariate EGARCH. $H_0$ is rejected if $p < 0.05$. Sample: 1/06/1986 to 1/03/2014.

The null hypothesis postulates that the multivariate EGARCH encompasses the alternative univariate EGARCH model. That is, the multivariate EGARCH has better forecasting accuracy than the univariate one. Having better forecasting accuracy means that the forecasting errors are smaller and we are minimizing the loss function. For each country, we present the statistics and the corresponding $p$-values (in brackets) for the $F$-standard test and the HLN test. The forecasting time series were obtained from one-step ahead static forecasts. The null is only rejected in the case of Greece. Italy and Spain present the most accurate multivariate EGARCH forecasts.

To summarize, volatility plays a central role in forecasting the stock market performance. Reliable forecasts are best obtained in a multivariate framework. This is especially the case when we use data observed over a tranquil period to predict the market
behavior over a turbulent period. Therefore, volatility can be seen as a stock market risk measure. The higher is the volatility the bigger is the risk and the investor can either win or lose much more than in a period of lower volatility. The decision depends on the profile of the investor, that is, on whether she is risk-averse, risk-taker, or risk-neutral. For each profile, there is a ranking of stock markets to invest.

4 Conclusions

This paper analyses the stock market volatility of 11 Eurozone countries over the period 1986-2014. Volatility is modeled in a multivariate framework where interest rate changes and the economic crisis of 2008 help to explain part of the increase of market turbulence at that time. When interest rate changes are used to model the conditional variance of stock returns, the remaining part of the variance equation explains the endogenous behavior of volatility. While most markets adjusted relatively quickly to the pre-crisis levels of volatility, Greece, Portugal, and Spain did not return to their pre-crisis levels during the sample period analyzed. On the other hand, Finland presents high volatility levels at least since the outset of the 1990’s. In-sample forecasts and out-of-sample forecasting tests show that the multivariate EGARCH model performs better than the univariate counterpart in all the cases except Greece. Our approach provides a better way to improve the forecasting ability of stock market returns.

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