Bousso’s Covariant Entropy Bound 
and 
Padmanabhan’s Emergent Universe

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Abstract

We study the Padmanabhan’s emergent Universe in the context of Bousso’s covariant entropy conjecture. We find that for a flat Universe, this conjecture can be applied for the system of Padmanabhan’s emergent Universe. It turns out that the maximum “bulk entropy” of Padmanabhan’s emergent Universe coincides with the upper bound of Bousso’s covariant entropy on the null surface defined by Hubble horizon, provided that the Universe is just filled by the cosmological constant or radiation field which represent maximal entropy during inflation and subsequent radiation dominant era. This maximal entropy is lost by the appearance of matter system in the Universe at matter dominant era. Applying D-bound on the matter system in the Padmanabhan’s emergent Universe, we find that the apparent cosmological horizon of a flat Universe in matter dominant era has less area and entropy than those (maximal) of apparent cosmological horizon of an empty de-Sitter space, in complete agreement with our conclusion. The maximal area and entropy in the Padmanabhan’s emergent Universe are recovered “as soon as possible” by transition from matter dominant to cosmological constant eras, provided that the matter inside the Universe is moved completely outward the apparent cosmological horizon in “an accelerating way” at late times.

Keywords: Covariant entropy bound, Emergent Universe.

1 Introduction

The idea that gravity behaves as an emergent phenomenon is referred to the proposal made by Sakharov in 1967 [2]. In this proposal which is named as the induced gravity, the spacetime background emerges as a mean field approximation of some underlying microscopic degrees of freedom similar to hydrodynamics or continuum elasticity theory from molecular physics [3]. Current research works on the relation between gravitational dynamics and thermodynamics support such a point of view [1]. In this line of activity, the major attention is focused on how the gravitational field equations can be obtained from the thermodynamical point of view. In 1995, the Einstein field equations are obtained in the pioneer work by Jacobson by using the equivalence principle and Clausius relation \(dQ = TdS\) where \(Q\), \(T\) and \(S\) are the heat, temperature and entropy, respectively [5]. The key point is to demand that the Clausius relation holds for the all local Rindler causal horizons with \(Q\) and \(T\) interpreted as the energy flux and Unruh temperature, as seen by an accelerated observer located inside the horizon. In this regard, the Einstein filed equations are nothing but the equations of state of spacetime. The Clausius relation also arises when one treats the gravitational field equations as an entropy balance law across a null surface, i.e \(S_m = S_{grav}\) [6].

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Moreover, another viewpoint that the gravity is not a fundamental interaction has been advocated by Verlinde [7]. In this viewpoint, gravity appears as an entropic force resulted from the changes in the information associated with the positions of bodies. He derived the Newton’s law of gravitation with the assumption of the entropic force together with the equipartition law of energy and the holographic principle. In a cosmological setup, considering the holographic principle, the energy equipartition law, the Unruh temperature with the Komar mass as the source to produce the gravitational field, one can obtain the Friedmann equations of the FRW Universe [8].

A similar approach was also implemented by Padmanabhan [9]. He obtained the Newton’s law of gravitation by combining the equipartition law of energy for the horizon degrees of freedom with the thermodynamical relation $S = \frac{E}{2T}$ such that $S$, $T$ and $E$ are the entropy and the horizon temperature and the active gravitational mass, respectively [10]. He also argued that the current accelerated expansion of the Universe can be derived from the discrepancy between the surface and bulk degrees of freedom through the relation $\Delta V/\Delta t = N_{\text{sur}} - N_{\text{bulk}}$ such that $N_{\text{bulk}}$ and $N_{\text{sur}}$ are the degrees of freedom related to matter and energy content inside the bulk and surface area, respectively [11]. Note that in this way, the existence of a spacetime manifold, its metric and curvature is presumed, primarily.

These studies magnifies the importance of thermodynamics and the corresponding thermodynamical quantities even for cosmological systems. In this regard, the entropy and its bounds for thermodynamical systems are vastly investigated by various physicists. The existence of a universal bound on the entropy $S$ of any thermodynamic system with the total energy $M$ is proposed by Bekenstein as [12]

$$S \leq 2\pi RM, \tag{1}$$

where $R$ is defined as the circumferential radius of the system as

$$R = \sqrt{\frac{A}{4\pi}}, \tag{2}$$

such that $A$ is the area of the smallest sphere circumscribing the system. For a system contained in a spherical volume, the gravitational stability requires the condition $M \leq R/2$. Then, the equation (1) can be rewritten as

$$S \leq \frac{A}{4}. \tag{3}$$

The derivation of equation (1) involves a gedanken experiment in which a thermodynamical system is dropped into a much larger size Schwarzschild black hole. Based on the generalized second law of thermodynamics, the entropy of the system should not exceed the entropy of the radiation emitted by the black hole while relaxing to its original size [13] [14] [15] [16]. The corresponding entropy to this radiation is estimated in the works [17] [18]. This entropy bound was shown to hold in wide classes of thermodynamically equilibrium systems, independent of the fundamental derivation of the bound [19]. In order to keep the validity of these bounds, Bekenstein imposed some conditions. One can refer to these conditions as: $i)$ the system must have constant and finite size, $ii)$ the system must have limited self-gravity, $iii)$ the matter components with negative energy density should not be allowed. In this regard, the so-called Bekenstein system is a thermodynamical system which satisfies all of the mentioned conditions for the application of Bekenstein’s bound. When these conditions are not satisfied, some entropic bounds can easily violate the Bekenstein’s bound. The simplest example is a homogeneous spacelike hypersurface in a flat Friedmann-Robertson-Walker Universe. The entropy of a sufficiently large spherical volume will exceed the boundary area [20]. This is because space is infinite, the entropy density is constant, and volume grows faster than surface area. As another example, consider a system undergoing gravitational collapse. Before it is collapsed towards the black hole singularity, its surface area becomes arbitrarily small. Since the entropy cannot decrease, the Bekenstein’s bound is violated. From the point of view of semi-classical gravity and thermodynamics, there is no reason to expect that any entropy bound applies to such systems.

1Here, the limited self gravity means that the gravity must not be the dominant force in the system. Consequently, one has to omit gravitationally collapsing objects and sufficiently large regions of cosmological space-times by this condition.

2The reason is that the bound relies on the gravitational collapse of systems with excessive entropy and is intimately connected with the idea that information requires energy. With allowing the matter with negative energy, one is able to add entropy to a system without increasing the mass, by adding entropic matter with positive mass as well as an appropriate amount of negative mass.
Some counterexamples had been proposed in [21, 22, 23, 24]. These candidates for counterexamples to the Bekenstein’s bound are clarified and refuted by Bekenstein in [25] by stressing that the energies of all essential parts of the system must be included in the energy which is imposed by the bound. Also in Ref. [26] he refuted the two counterexamples reported in [21], while in Ref. [27] he was successful in showing that the Page’s proposed bound [21] as the alternative of the bound [1] is also violated.

The holographic conjecture [28, 29], was a good starting point for Fischler and Susskind [20] to suggest some kind of entropy bounds which hold even for large regions of cosmological solutions, for which Bekenstein’s conditions are not satisfied. The Fischler-Susskind bound [20], is not a general proposal. This is because, for example, it applies to the universes which are not closed or re-collapsing, while for sufficiently small surfaces in a wide class of cosmological solutions one can find other prescriptions [30, 31, 32, 33, 34].

A general proposal by Bousso [1] for entropy bound, was a successful project in imposing covariant entropy conjecture. The contribution of Bekenstein’s seminal paper [12] in completing this project is undeniable. Also, the proposal of Fischler and Susskind [20], in using light-like hypersurfaces to relate entropy and area is very important. Using of light-rays, this proposal can formulate the holographic principle [28, 29, 35]. Indeed, Corley and Jacobson [35] were pioneer to take a space-time point of view in locating the entropy related to an area. The concept of “past and future screen-maps” and the suggestion of choosing only one of the two in different regions of cosmological solutions were their achievement. Moreover, they recognized the importance of caustics of the light-rays leaving a surface. The application of Bekenstein’s bound to sufficiently small regions of the Universe can be found in the investigation of authors [20, 30, 31, 32, 33, 34]. They have carefully exposed the difficulties that arise when such rules are pushed beyond their range of validity [20, 30, 33]. These insights are invaluable in the search for a general prescription.

In a complement view by Bousso, in the way of proposing an entropy bound for a system, he conjectures the following entropy bound which is valid in all spacetimes admitted by Einstein’s equation: Let $A$ be the area of any two-dimensional surface. Let $L$ be a hypersurface generated by surface-orthogonal null geodesics with non-positive expansion. Let $S$ be the entropy on $L$. Then $S \leq A/4$. [1]. In this paper, we will apply the covariant entropy bound on spatial cosmological region and compare it with entropy bound coming from the Padmanabhan’s Emergent Paradigm. In doing so, first of all we consider the covariant entropy conjecture in the first section. In section two, we find an entropy bound which comes by means of Padmanabhan’s Emergent Paradigm. In section three, we attempt to identify the maximum entropy bound coming from covariant entropy conjecture, with the entropy of Padmanabhan’s Emergent Paradigm.

## 2 Cosmological Entropy Bounds

### 2.1 Covariant Entropy Conjecture

Covariant Entropy Conjecture

\[ \text{""Let } M \text{ be a } 4\text{-dimensional space-time manifold on which Einstein’s equation is satisfied subject to the dominant energy condition for the matter. Let } A \text{ be the area of a connected } 2\text{-dimensional spatial surface } B \text{ contained in } M. \text{ Let } L \text{ be a hypersurface bounded by } B \text{ and generated by one of the four null congruences orthogonal to } B. \text{ Let } S \text{ be the total entropy contained in } L. \text{ If the expansion of congruence is non-positive at every point on } L (\text{measured in the direction away from } B), \text{ then } S \leq A/4". \] [1]

Since the conjecture is manifestly $T$- invariant (time reversal invariant), the covariant entropy bound does not even refer to “future” and “past.” This is the most significant property of covariant entropy conjecture. The other point is that, the thermodynamic entropy and the generalized second law of thermodynamics, which underlies Bekenstein’s bound, are not $T$-invariant. So one can say that the spacelike projection theorem is not $T$-invariant. This means that it refers to past and future explicitly. This property persuaded some people to conclude that the origin of the covariant bound is statistical rather than thermodynamics [1].

The covariant entropy conjecture is a correct law, which results in an entropy bound for spatial regions in cosmology. An interesting application of the spacelike projection theorem is for the normal regions [1]. It shows that in which situations we can treat the interior of the apparent horizon as a Bekenstein system. Let $A$ be the area of a sphere $B$. It can be on or inside of the apparent horizon (the word “inside” has a natural meaning in normal regions). In this condition the future-directed ingoing light-sheet $L$ exists, so we can assume that $B$ is complete ($B$ is the only boundary of apparent horizon $\tilde{B}$). Let $V$ be a region inside...
of and bounded by \( B \), on any spacelike hypersurface containing \( B \). If no black holes are produced, \( V \) will be in the causal past of \( L \), and the conditions for the spacelike projection theorem are satisfied. Therefore, the entropy on \( V \) will not exceed \( A/4 \). In particular, we may choose \( B \) to be on the apparent horizon, and \( V \) to be on the spacelike slice preferred by the homogeneity of the FRW cosmologies. We summarize these considerations in the following corollary:

**Cosmological Corollary**  “Let \( V \) be a spatial region inside the apparent horizon of an observer. If the future-directed ingoing light-sheet \( L \) of the apparent horizon has no other boundaries and if \( V \) is entirely contained in the causal past of the light-sheet \( L \), the entropy on the spatial region \( V \) cannot exceed a quarter of the area of apparent horizon” [1].

According to above explanations, the spheres beyond the apparent horizon are anti-trapped and do not possess future-directed light-sheets. So the spacelike projection theorem cannot be applied, and the statement about the entropy enclosed in the spatial volumes cannot be made. Thus, the covariant entropy conjecture considers the apparent horizon as a particular surface. It indicates the largest surface to which the spacelike projection theorem can possibly apply, and hence the largest region one can apply as the Bekenstein’s system is the inside of it.

The de Sitter space is an example in which the conditions of above corollary are satisfied by its apparent horizon. Here, the cosmological horizon is the same as apparent one at \( r = (3/\Lambda)^{1/2} \). So, with the above explanations the entropy within the cosmological horizon cannot exceed \( 3\pi/\Lambda \).

Bekenstein’s bound can be applied to spatial regions in cosmological solutions with the use of this corollary. It can be derived by the covariant entropy bound but it is not equivalent to it. This corollary is a statement of limited scope like as the spacelike projection theorem [1]. It does not give any information about relation of entropy with the area of trapped or anti-trapped surfaces in the Universe. Even for the surfaces contained in the apparent horizon, a “spacelike” bound applies only under some certain conditions. Thus, this corollary is used to define the range of validity of Bekenstein’s entropy/area bound in cosmological solutions. Moreover, the covariant entropy conjecture associates at least two hypersurfaces with any surface in any space-time, and bounds the entropy on those hypersurfaces. For this reason, the corollary is far less general than the covariant entropy conjecture.

### 3 The Entropy Bound of Emergent Paradigm

According to Padmanabhan’s proposal, the difference between the surface degrees of freedom and the bulk degrees of freedom in a region of space may result in the accelerated expansion of the Friedmann-Robertson-Walker (FRW) Universe through the relation \( \Delta V/\Delta t = N_{\text{sur}} - N_{\text{bulk}} \) where \( N_{\text{bulk}} \) and \( N_{\text{sur}} \) are referred to the degrees of freedom related to matter and energy content inside the bulk and surface area, respectively [11].

For an expanding Universe, we have the following condition for the Padmanabhan’s formula

\[
\frac{\Delta V}{\Delta t} \geq 0,
\]

which demands

\[
N_{\text{sur}} - N_{\text{bulk}} \geq 0.
\]

On the other hand, we know that the relation between surface entropy \( S_{\text{sur}} \) and surface degrees of freedom is as follows

\[
4S_{\text{sur}} = N_{\text{sur}},
\]

where the entropy of the surface is \( \frac{A}{4} \), \( A \) being the area of the surface enclosed by the Hubble horizon \( r_H \).

One can also write the bulk degrees of freedom in terms of its energy \( E \) and temperature \( T \) as

\[
N_{\text{bulk}} = \frac{2E}{T},
\]
where the thermodynamic temperature of our cosmological system is \( H/2\pi \). So, one can rewrite the equations \(5\), \(6\) and \(7\) as follows

\[
\pi r_H E \leq S_{\text{sur}},
\]

which can be interpreted as a definition of emergent “lower entropy bound”. The reason why we call \(8\) as the emergent lower entropy bound is that it is a trivial rewriting of the Friedmann equation in terms of nonstandard variables \(r_H, E\) and \(S\), and has no independent content. For example, unlike \(S\) in the covariant bound, Padmanabhan’s \(N_{\text{bulk}}\) is not defined as the von Neumann entropy or the thermodynamic entropy of an actual bulk matter system, rather it is just a suggestive name given to a quantity that is directly defined in terms of quantities like \(H, \rho\), and \(p\) that appear in the Friedmann equation. So, there is no nontrivial content to the statement that the Friedmann equation can be expressed in terms of such quantities. That is why the relation \(8\) cannot be considered as a definition of a “lower entropy bound” for the surface entropy \(S_{\text{sur}}\), so it merely can be interpreted as a definition of emergent “lower entropy bound” of a cosmological system in the framework of emergent Universe scenario. Therefore, it is meaningless to compare the covariant upper entropy bound with \(\pi r_H E\) as the emergent lower bound of \(S_{\text{sur}}\), unless some specific conditions are provided in order for this comparison becomes meaningful.

### 4 Covariant Entropy Bound in Emergent Model

Here, we consider a flat universe \(k = 0\) such that the Hubble horizon in Padmanabhan’s paradigm becomes exactly the same as apparent horizon. If this condition is provided, then the Hubble horizon plays the role of null surface enclosing the Universe. Hence, the Bousso’s covariant entropy bound becomes applicable to the system of Universe in Padmanabhan’s paradigm and one can compare the Bousso’s covariant upper entropy bound with \(\pi r_H E\) as the emergent lower bound of \(S_{\text{sur}}\).

#### 4.1 Misner-Sharp Energy

Let us start with the Misner-Sharp energy. One can calculate the total Misner-Sharp energy inside the Hubble horizon as

\[
M(r_H) = \int_0^{r_H} 4\pi r^2 \rho dr = \frac{4\pi}{3} r_H^3 \rho,
\]

where \(r_H\) is the Hubble horizon radius and \(M = E\). Moreover, for the apparent horizon we have \(r = 2M(r)\) in which for our cosmological case with a flat spatial geometry the apparent and Hubble horizons coincides and consequently this formula takes the form of \(r_H = 2M(r_H)\). Also, using the Friedmann equations for \(k = 0\), we have \(r_H = \sqrt{\frac{3}{8\pi \rho}}\). Then, using \(8\) and \(9\), we obtain

\[
\frac{\pi r_H^2}{2} \leq S_{\text{sur}}.
\]

The maximum of Bousso’s covariant entropy bound for \(k = 0\) and the null surface defined by \(r_H\) is given by

\[
S = \frac{A}{4} = \pi r_H^2.
\]

On the other hand, we demand the inequality \(8\) to be saturated (for \(k = 0\) and the null surface defined by \(r_H\)) as

\[
S_{\text{sur}} = \pi r_H E,
\]

such that it can be compared with \(11\) on the null surface defined by \(r_H\). In doing so, if we put the Misner-Sharp energy \(E = \frac{r_H^2}{2}\) in \(12\), we arrive at the result that the Misner-Sharp Energy has no capability for having the equal values of entropy bounds \(11\) and \(12\) on this null surface.
4.2 Komar Energy

In this subsection, we repeat the calculation of the previous subsection for the Komar energy and try to remove the above inconsistency. To begin with, we consider the Komar energy as the total energy in the bulk enclosed by the surface of the Hubble horizon, as

\[
E(r_H) = \int_{r_H}^{r_H} 4\pi r^2|\rho + 3p|dr = \frac{4\pi}{3} r_H^3 |\rho| (1 + 3\omega),
\]

where we have considered the barotropic equation \( p = \omega \rho \). Then, from the inequality \( 8 \), we obtain

\[
4\pi r_H \left( \frac{4\pi}{3} r_H^3 |\rho| \right) (1 + 3\omega) \leq A = 4S,
\]

where the L.H.S becomes maximum (equality case) at \( r_H \) as

\[
A = 4S_{\text{sur}} = 2\pi r_H (2M(r_H)) |1 + 3\omega|.
\]

Using \( 15 \), we obtain

\[
S_{\text{sur}} = \frac{\pi r_H^2 |1 + 3\omega|}{2}.
\]

This shows that, unlike the Misner-Sharp Energy, the Komar Energy has capability for having the equal values of entropy bounds \( 11 \) and \( 16 \) on the null surface for two specific values of \( \omega \) which will be discussed in the following.

5 Discussion and concluding remarks

By applying Bousso’s covariant entropy conjecture for the cosmological spatial region in one hand, and the entropy bound which comes from the Padmanabhan’s Emergent Paradigm, on the other hand, we have shown that these two entropy bounds are comparable just for the flat \((k = 0)\) FRW Universe and are equal on the null surface defined by Hubble horizon \( r_H \), provided that:

- inside of the apparent horizon be filled by the radiation, namely \( \omega = \frac{1}{3} \),

or

- inside of the apparent horizon be pure de Sitter space subject to the cosmological constant, namely \( \omega = -1 \).

In other words, the maximal entropy inside the apparent horizon of the flat FRW universe occurs when it is filled completely by the radiation field or cosmological constant. The fact that both radiation and cosmological constant correspond to the maximal entropy on the apparent horizon, may represent a symmetry between the radiation and cosmological constant. The origin of this symmetry may be a “one to one” correspondence between the number of degrees of freedom in the bulk and on the surface of apparent horizon. At early Universe, dominated by the cosmological constant, the number of degrees of freedom in the bulk and on the surface of apparent horizon are equal. At the subsequent radiation dominant era, the correspondence between the number of degrees of freedom in the bulk and on the surface of apparent horizon still holds. In other words, the transition from cosmological constant to radiation dominant eras does not alter the maximal entropy property of the Universe. However, at matter dominant era \( \omega = 0 \) the maximal entropy property is lost. Therefore, one may conclude that the current acceleration of the Universe is nothing but a tendency of the system of Universe to transit from matter dominant era to cosmological constant era with \( \omega = -1 \) to recover the maximal entropy property at late time.

This conclusion may be based on the Bousso’s \( D \)-bound on matter entropy in de Sitter space \( 37 \). \( D \)-bound is derived by supposing a matter system within the apparent cosmological horizon of an observer in a universe with a future de-Sitter asymptotic. Such observer is a witness of thermodynamical process by which the matter system is moved outward the cosmological horizon. Therefore, after the matter system is moved outward the horizon, the observer will find himself in the space-time that has been converted to empty pure
de-Sitter space. The initial thermodynamical system, namely the asymptotic de Sitter space including the matter system, has the total entropy

$$S = S_m + \frac{A_c}{4}, \quad (17)$$

where $S_m$ is the entropy of the matter system inside the cosmological horizon and $A_c/4$ is the Bekenstein-Hawking entropy associated with the enclosing apparent cosmological horizon. At the end of thermodynamical process by which the matter is moved outward the cosmological horizon, the final entropy of the system will be $S_0 = A_0/4$ in which $A_0$ is the area of cosmological horizon of de Sitter space devoid of matter system. Regarding the generalized second law $S \leq S_0$, we find

$$S_m \leq \frac{1}{4}(A_0 - A_c), \quad (18)$$

which is the so-called $D$-bound on the matter systems in an asymptotically de Sitter space. Using the fact $S_m \geq 0$, one realizes that $A_c \leq A_0$ which indicates that a cosmological horizon enclosing a matter system has smaller area and entropy than those of cosmological horizon of an empty de-Sitter space.

Applying $D$-bound on the matter system in the Padmanabhan’s emergent Universe, we find that when a flat FRW Universe is in matter dominant era with $\omega = 0$, the corresponding apparent cosmological horizon has less area and entropy than the maximal area and entropy of apparent cosmological horizon of an empty de-Sitter space with $\omega = -1$, in complete agreement with our conclusion in the first paragraph. Therefore, in order for the maximal horizon area and entropy be reached “as soon as possible” for Padmanabhan’s emergent Universe, the matter inside the Universe is moved outward the apparent cosmological horizon in “an accelerating way”.

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References

[1] R. Bousso. A covariant entropy conjecture, JHEP 9907, 004 (1999).

[2] A. D. Sakharov, Vacuum quantum fluctuations in curved space and the theory of gravitation, Sov. Phys. Dokl. 12, 1040 (1968) [Dokl. Akad. Nauk Ser. Fiz. 177, 70 (1967); Sov. Phys. Usp. 34, 394 (1991); Gen. Rel. Grav. 32, 365 (2000)].

[3] M. Visser, Sakharov’s induced gravity: A Modern perspective, Mod. Phys. Lett. A 17, 977 (2002).

[4] T. Padmanabhan, Thermodynamical Aspects of Gravity: New insights, Rept. Prog. Phys. 73, 046901 (2010).

[5] T. Jacobson, Thermodynamics of space-time: The Einstein equation of state, Phys. Rev. Lett. 75, 1260 (1995).

[6] T. Padmanabhan, A Physical Interpretation of Gravitational Field Equations, AIP Conf. Proc. 1241, 93 (2010); T. Padmanabhan, Entropy density of spacetime and thermodynamic interpretation of field equations of gravity in any diffeomorphism invariant theory, arXiv: hep-th/0903.1254.

[7] E. P. Verlinde, On the Origin of Gravity and the Laws of Newton, JHEP 1104, 029 (2011).

[8] R. -G. Cai, L. -M. Cao and N. Ohta, Friedmann Equations from Entropic Force, Phys. Rev. D 81, 061501 (2010).

[9] T. Padmanabhan, Equipartition of energy in the horizon degrees of freedom and the emergence of gravity, Mod. Phys. Lett. A 25, 1129 (2010).
[10] T. Padmanabhan, Gravitational entropy of static space-times and microscopic density of states, Class. Quant. Grav. 21, 4485 (2004).

[11] T. Padmanabhan, Emergence and Expansion of Cosmic Space as due to the Quest for Holographic Equipartition, arXiv: hep-th/1206.4916v1.

[12] J. D. Bekenstein: A universal upper bound on the entropy to energy ratio for bounded systems. Phys. Rev. D 23, 287 (1981).

[13] J. D. Bekenstein: Black holes and the second law. Nuovo Cim. Lett. 4, 737 (1972).

[14] J. D. Bekenstein: Black holes and entropy. Phys. Rev. D 7, 2333 (1973).

[15] J. D. Bekenstein: Generalized second law of thermodynamics in black hole physics. Phys. Rev. D 9, 3292 (1974).

[16] S. W. Hawking: Particle creation by black holes. Commun. Math. Phys. 43, 199 (1974).

[17] J. D. Bekenstein: Do we understand black hole entropy? arXiv: gr-qc/9409015

[18] D. N. Page: Particle emission rates from a black hole. II. Massless particles from a rotating hole. Phys. Rev. D 14, 3260 (1976).

[19] M. Schiffer and J. D. Bekenstein: Proof of the quantum bound on specific entropy for free fields. Phys. Rev. D 39, 1109 (1989).

[20] W. Fischler and L. Susskind: Holography and cosmology, arXiv: hep-th/9806039.

[21] D. N. Page, Comment on a universal upper bound on the entropy-to-energy ratio for bounded systems, Phys. Rev. D 26, 947 (1982).

[22] D. N. Page, Huge violations of Bekenstein’s entropy bound, arXiv: gr-qc/0005111.

[23] D. N. Page, Subsystem entropy exceeding Bekenstein’s bound, arXiv: hep-th/0007237.

[24] D. N. Page, Defining entropy bounds, JHEP 0810, 007 (2008).

[25] Jacob D. Bekenstein, On Page’s examples challenging the entropy bound, arXiv: gr-qc/0006003v3.

[26] J. D. Bekenstein, Specific entropy and the sign of the energy, Phys. Rev. D 26, 950 (1982).

[27] J. D. Bekenstein, Entropy bounds and the second law for black holes, Phys. Rev. D 27, 2262 (1983).

[28] G. ’t Hooft: Dimensional reduction in quantum gravity, arXiv: gr-qc/9310026.

[29] L. Susskind: The world as a hologram. J. Math. Phys. 36, 6377 (1995).

[30] R. Easther and D. A. Lowe: Holography, cosmology and the second law of thermodynamics, Phys. Rev. Lett. 82, 4967-4970 (1999).

[31] G. Veneziano: Pre-bangian origin of our entropy and time arrow, Phys. Lett. B 454, 22-26 (1999).

[32] D. Bak and S.-J. Rey: Cosmic holography, Class. Quant. Grav. 17, L83 (2000).

[33] N. Kaloper and A. Linde: Cosmology vs. holography, Phys.Rev.D 60, 103509 (1999).

[34] R. Brustein: The generalized second law of thermodynamics in cosmology, Phys. Rev. Lett. 84, 2072 (2000).

[35] S. Corley and T. Jacobson: Focusing and the holographic hypothesis. Phys. Rev. D 53, 6720 (1996).

[36] R. Bousso and L. Susskind, Multiverse interpretation of quantum mechanics, Phys. Rev. D 85, 045007 (2012).

[37] R. Bousso, JHEP 01, 04 (2001).