Empirical Shear Strength Criterion for Artificial Joint

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Abstract. An accurate understanding of shear strength in joints is critical to effective geotechnical engineering. The complete rock shear test continues to shear after shear strength reaches it speak value, resulting in the appearance of a softening stage and a residual stage. Existing research is mainly aimed at the development of dilatancy and shear models of peak strength; few consider post peak processes. This paper attempted to develop a better understanding of changes to shear strength during shear. Direct shear tests of artificial rock joints with the same shape were conducted under the action of 5 normal stresses. By determining a shear dilatancy angle model for the shear process, shear strength can be determined using the Mohr-Coulomb law. The values calculated using our new method agree well with experimental results, indicating that the criterion can be used to accurately estimate the shear strength of rock joints. The parameters required by the proposed criterion can be easily determined in test.

Keywords: Rock joint, Direct shear test, Shear strength, Artificial joint

1. Introduction
The study of fracturing under normal stress is a mature field. However, in nature, rock mass failures are primarily caused by shear action. When the shear strength of a rock mass is higher than the peak shear strength, shear displacement increases rapidly, allowing for extension of fractures and resulting massed struction. Because this process is one of the main factors leading to many geotechnical engineering accidents, it is a topic of concern in engineering and academic circles.

For the analysis of the shear strength of the rock mass, the scholars mainly focus on the determination of the shear strength model. Patton [1], who first introduced the basic friction angle and dilatancy angle into Coulomb's theory in the field of rock mechanics, put forward the famous Patton model. The model makes two assumptions: (1) that the joint surface consists of an undulating toothed pattern with undulant angle i, and (2) that the adhesive strength of the joint surface is 0. The Patton model is a linear model with a simple for mand clear physical meaning. A number of similar regular tooth models have since been proposed. Jaeger [2] proposed an empirical formula considering the effect of cohesion on peak shear strength, and Shen et al. [3] introduced the internal friction angle correction factor and the comprehensive cohesive force correction factor to the established empirical equation.

Note that the above models are all linear. This is problematic because the test data show that the linear shear strength model cannot satisfy all joint planes. To address this short coming, Ladanyi and Archambault [4, 5] developed an on linear shear strength model based on the results of direct shear tests conducted on saw tooth triangular joints. Their model assumes that the dilatancy angle changes with normal stress, and that when the shear failure occurs, the error will be larger.
Using Patton’s model, Barton [6, 7] made the next breakthrough. By analyzing the relationship between the JRC, the normal stress, and the dilatancy angle, Barton developed a change law relating to the dilatancy angle and the JRC, and put forward a shear strength model containing the JRC. This model is the most widely used at present. On the basis of elastic-plastic theory, Plesha [8] and Jing [9] expresses the evolution of the dilatancy angle by the exponential function of cumulative dissipative work, and over comes the unreasonableness of the dilatancy angle with the reversible change during shear. In fact, when the joint plane is destroyed, the main reason is that the asperities were destroyed by tensile strength. Therefore, research on shear strength should consider the asperities tensile strength [10, 11].

The main challenge of making Patton’s theoretical model closer to the actual situation lies in how to accurately describe the changing law of the dilatancy angle and establish an accurate dilatancy model. The key factors affecting the dilatancy angle are the distribution of joint plane asperities in the shear direction and the strength of the joint plane. The above shear strength models either simplify the joint plane or describe the roughness of the joint plane in only two dimensions; both models do not correspond to the actual situation. By conducting many direct shear tests based on a statistical-mathematical model of effective shear dip and the corresponding contact area of the joint plane, Grasselli [12, 13] first proposed a fitting algorithm for the 3D roughness of the joint plane, establishing a true 3D peak shear strength model. In addition to the tensile strength of the rock mass, the remaining parameters were determined using the joint topography function. The primary deficiency of this model calculates peak dilatancy angle by determining the residual internal friction angle and introducing a magnifying factor. In short, the model does not conform to the form of the Coulomb’s theory. To address this shortcoming, Tang [14, 15] considered the relationship between the peak shear dilatancy angle and the 3D morphologies based on Grasselli’s model, and proposed an improved peak shear strength model conforming to Coulomb’s theory. Finally, Yang [16] established the peak shear strength model by introducing the maximum effective shear dip angle and roughness parameter.

By establishing the peak dilatancy model and introducing it into the Patton’s formula, the peak shear strength model can also be established. The existing peak dilatancy model is mainly as follows. Barton [6] determined the peak dilatancy angle using JRC, JCS, and normal stress, and put the model in exponential form. Schneider [17] described the relationship between the dilatancy angle and normal stress and shape parameters in negative exponential form and put forward the corresponding calculation model. Jing [9] established the calculation mode by founding the relationship between dilatancy angle and normal stress.

Several criterions have been put forward over the years [18-20] for the prediction of the peak shear strength standard of rock joints; most are empirical. Despite this, there is no shear strength model for softening shear strength and residual shear strength after peak shear strength. Moreover, when the strength model is applied, the parameters related to the joint plane need to be obtained. However, when the joint plane is destroyed, this becomes difficult. This paper provides a method for determining the shear strength of the joint plane at any time during shear. By determining the shear dilatancy angle model for the shear process, the shear strength can be determined using the Mohr-Coulomb law.

Direct shear tests are usually used to determine the shears trength values of rock joints under different loading conditions. Unfortunately, in practice it is difficult to find natural rocks with similar shapes and surface properties [21]. Based on the above research, a series of experiments were conducted on 10 sets of artificial joints with the same shape.

2. Direct Shear Test

2.1. Test System
A TJXW-600 Microprocessor control coupled shear-seepage test system was used to conduct direct shear tests (Fig. 1). The normal and shear loads are provided by a servo oil source. The maximum load for normal and shear stresses is 600 kN, and the accuracy of the load measurement is ±1%. The testing
apparatus provides a constant normal load (CNL). CNL is mainly used for rock slope stability analysis. Displacement was measured by wire displacement sensor. The normal displacement precision is 0.003 mm, and the shear displacement precision is 0.04 mm. The stress, strain, displacement, and other factors are controlled by an EDC closed-loop controller. The hydraulic loading system is composed of a water tank and nitrogen cylinder, and the maximum water pressure is 3 MPa. The shear box is divided into an upper part and a lower part. The lower specimen is fixed during shear, and the upper specimen is moved. The maximum shear displacement is 35 mm.

**Figure 1.** TJXW-600 Microprocessor-control coupled shear-seepage test system

2.2. Preparation of Specimen

The fabricated rock specimen is cylindrical, with a basal diameter of 200mm. and a height of 75mm. The test material is extremely strong gypsum. The mass ratio of gypsum to water to retarder is 1: 0.25: 0.005.

There are two kinds of structure plane in the specimen. One is smooth gypsum surface without undulant angles, and the other is regularly toothed. The cross-section of each tooth is a right triangle, and the undulant angle is 30°. The structure of the specimen is shown in Fig. 2. The physical parameters of the specimens are listed in Table 1.
### Table 1. Parameters of the Specimens

| Density $\rho$/g·cm$^{-3}$ | Compressive strength $\sigma$/MPa | Elastic modulus $E$/GPa | Poisson ratio $\mu$ | Cohesion $c$/MPa | Internal friction angle $\phi$ |
|---------------------------|---------------------------------|------------------------|---------------------|------------------|--------------------------|
| 2.066                     | 38.8                            | 28.7                   | 0.23                | 5.3              | 60                       |

2.3. Test Process

Among the many specimen combinations, two types were selected as research subjects. In combination I, the upper and lower specimens are smooth specimens. In combination II, the upper and lower specimens are regular toothed specimens, and the upper and lower surfaces of the regular teeth are matching. The two combinations are shown in Figure 3 (the pink arrow is the shear stress direction).

Figure 3. Specimen Combinations

Barton [6] pointed out that when engineering problems related to rock mass happen, the effective normal stresses are commonly between 0.1 MPa and 2.0 MPa. Therefore, direct shear tests were conducted under normal stresses of 1.0–3.0 MPa. The test scheme is listed in Table 2. The first 5 cases were combination I, and the latter 5 cases were combination II. The normal stress increased by 5 levels,
and the shear rate was 0.25 mm/s. Since only the strength is studied, these tests were carried out without water.

| Case | Vertical pressure (Mpa) | Shear velocity (mm/s) | Type of specimen combination |
|------|------------------------|-----------------------|------------------------------|
| 1    | 1.27                   |                       |                               |
| 2    | 1.59                   |                       | I                            |
| 3    | 1.91                   |                       | I                            |
| 4    | 2.23                   |                       | I                            |
| 5    | 2.55                   | 0.25                  | I                            |
| 6    | 1.27                   |                       | I                            |
| 7    | 1.59                   |                       | I                            |
| 8    | 1.91                   |                       | I                            |
| 9    | 2.23                   |                       | I                            |
| 10   | 2.55                   |                       | I                            |

2.4. Test Results

Our test scheme allows us to obtain the curves of shear stress and normal displacement increment with shear displacement.

As can be seen from Fig. 4, shear stress increases with increasing normal stress. Increases in normal stress reduce the dilatancy angle of asperity [21]. Under complex stress, the area of the failure surface increases during shearing, the friction between the joint planes increases, and the shear resistance increases. After failure of the joint planes, the normal stress continues to be applied, and the experiment enters the softening stage and the residual shear stage. The main source of the shear stress is the resistance produced by the grinding debris fillings, as well as the friction between two joint planes, the joint planes and the debris fillings, the debris fillings themselves. The magnitudes of these resistance and friction values depend on the volume, distribution and accumulation form of debris fillings.

When the normal stress $\sigma_n$ is large and the dilatancy angle is small, the initial stage normal displacement deformation $\Delta u$ is smaller (Fig. 5). After failure of the joint plane, the larger normal stress reduces the volume of the debris fillings, leading to a trend of decreasing $\Delta u$ with increasing $\sigma_n$. The stresses change the distribution and accumulation form of debris fillings, which also affect the magnitude of $\Delta u$. 
3. Empirical Strength Criterion

For the regular teeth of the joint plane, the peak shear strength can be determined by Patton’s criteria. Patton introduces the basic friction angle $\phi_b$ and the dilatancy angle $i$ into the Coulomb’s theory.

$$\tau = \sigma \tan(\phi_b + i)$$  \hspace{1cm} (1)

Where $\phi_b$ is the friction angle of the smooth-surfaced specimen; when the peak strength is determined, $i$ is the peak dilatancy angle $i_p$, for regular teeth and $i$ is undulant angle.

The peak shear strength $\tau_s$ under different normal stress $\sigma_n$ of smooth specimens is shown in Figure 6. From the fitting data, the value of $\phi_b$ is about 46.4°.

Figure 6. Relationship between of normal stress and shear stress for the smooth joint plane

By substitute the calculated $\phi_b$ and undulant angle $i_p$ into Eq.1, the peak shear strength of the rough specimen under different normal stresses can be calculated. The calculated values are compared with the test values in Fig. 7.
As can be seen in Fig. 7, Patton’s formula provides a good fit with the test results when used to calculate peak shear strength. However, Patton’s formula only describes the simple linear relationship between $\sigma_n$ and $\tau$ before the peak shear strength and does not reflect the softening and residual phase after the peak shear strength. $i$ is constantly changing during shear; the specific value of $i$ during the test process can be determined using the following formula:

$$i = \arctan \frac{\Delta u}{\Delta \delta}$$

(2)

where $\Delta \delta$ is shear the displacement variation of the corresponding $\Delta u$.

The change of $i$ with $\delta$ under different normal stresses is shown in Figure 8. For the unified dimension, the transverse coordinates are set as the ratio of the shear displacement value $\delta_i$ to the total shear of displacement $\delta_t$.

![Figure 7. Calculated and experimental values of peak shear stress](image1)

![Figure 8. Relation of the dilatancy angle to shear displacement for different schemes](image2)
From Fig. 8, the average initial dilatancy angle values for Cases 1 through 5 are 32.1°, 29.88°, 31.34°, 32.84°, and 30.71°, respectively. These values are close to the regular teeth’s fluctuation angle 30° of the joint plane. There are two main reasons for the errors between the initial dilatancy angles. First, the occlusal degree of the specimens in the initial state of does not guarantee complete consistency. Second, the tooth undergoes some plastic deformation during shear. From curve $i - \delta / \delta_t$, it can be seen that before $t$ reaches peak shear strength, $i$ remains unchanged. After the failure of the joint plane, $i$ rapidly decreases in nonlinear form and eventually approaches 0. The value of $\sigma_n$ has an effect on the trend of the curve change. The larger the value of $\sigma_n$, the less the degree of curvature. After the failure of the joint plane, when $i$ begins to decrease, the relationship between $i$ and $\delta / \delta_t$ is a power function:

$$i = A \left( \frac{\delta_t}{\delta} \right)^B$$

(3)

Where $A$ and $B$ are fitting parameters.

The fitting parameters obtained for each case are shown in the Table 3.

| Cases | A   | B   | R²  |
|-------|-----|-----|-----|
| Case6 | 0.294 | -2.097 | 0.971 |
| Case7 | 0.229 | -2.185 | 0.988 |
| Case8 | 0.132 | -2.265 | 0.992 |
| Case9 | 0.083 | -2.339 | 0.973 |
| Case10 | 0.029 | -2.426 | 0.987 |

The data in Table 3 show that the test data in each case are in good agreement with the fitting data. This shows that the dilatancy angle has obvious related with the change of shear displacement. The main factors that affect the fitting parameters are the joint plane compressive strength $JCS$ and the normal stress $\sigma_n$. The parameters $A$ and $B$ with the curve of $JCS/\sigma_n$ are shown in Figure 9.

(a) Evolution of parameter $A$ with changes in normal stresses during shear
Figure 9. Evolution of parameters $A$ and $B$ of curve $i$-$\delta$ under different normal stresses during shear

Fig. 9 shows that $A$, $B$ and $JCS/\sigma_n$ have linear relationships to each other. However, the fitting results are not ideal; the volatility of $A$ and $B$ is larger when $\sigma_n$ is not constant. The reason for this is that the specific values of $A$ and $B$ depend on the form, size and accumulation of the detrital filling. Unfortunately, it is difficult to ensure that the physical and mechanical parameters of the specimens are completely uniform. As a result, the results for $A$ and $B$ exhibit some discreteness. The following results can be obtained from Fig. 10:

$$
A = 0.0158 \frac{\sigma_n}{JCS} - 0.1927 \\
B = 0.0187 \frac{\sigma_n}{JCS} - 2.6688
$$

To substitute Eq. (4) into Eq. (3), the relationship between $i$ and $\sigma_n$, $\delta$, as shown in Eq. (5):

$$
i = (0.0158 \frac{\sigma_n}{JCS} - 0.1927) \delta (0.0187 \frac{\sigma_n}{JCS} - 2.6688)
$$

Similarly, based on the Mohr-Coulomb theory, the softening shear strength and residual shear strength can be determined according to the changing law of the dilatancy angle during shear.

After the failure of the joint planes, the primary source of shear strength is the resistance that accompanies grinding detrital fillers the friction between the detrital filler and the joint surface. If the shear strength is still calculated with $\phi_b$ in this case, the shear strengths of the softening stage and the residual phase will be underestimated. In this paper, modified parameter $f$ (value 1.24) is used to modify $\phi_b$, that is:

$$
\tau = \sigma \tan(f \phi_b + i)
$$

To bring $i$ into Eq. (6), the shear strength of different shear displacement after the peak shear strength were calculated. It can be compared the calculated results with the measured results in Figure 10. Fig. 10 shows a comparison between the calculated values and the test values of Cases 6 through 10. Because the second peak shear strength cannot be reflected by the change in $i$, it is ignored. As can be seen in Fig. 10, the post-peak shear strength obtained by Eq. (6) is in good agreement with the test data.
Figure 10. Calculated and experimental value of the shear strength after peak value of different schemes during shear

Combined with the above equation, the empirical strength criterion for regular teeth can be obtained

$$\tau = \begin{cases} 
\sigma_n \tan (\phi_b + i_p), & \delta = \delta_p \\
\sigma_n \tan \left(1.24\phi_b + (0.0158 \frac{\sigma_n}{JCS} - 0.1927)\delta\right)^{0.0185 \frac{\sigma_n}{JCS} - 2.6688}, & \delta > \delta_p 
\end{cases}$$

(7)

where $\sigma_n$ is the normal strength, $i_p$ is the peak dilatancy angle, $\phi_b$ is the basic friction angle, $JCS$ is the joint plane compressive strength, $\delta$ is the shear displacement, and $\delta_p$ is the peak shear displacement.
4. Conclusions

Direct shear tests were carried out under radiation flow on 10 groups of artificial joints. It concluded that Patton’s formula can well approximate with the measured peak shear strength. Using Patton’s formula and combined with the shear dilatancy model, it is established an empirical model of shear strength after the failure of the joint plane (during the softening stage and the residual stage). The results of the experiment are well anastomosed. Based on the two-stage strength model, it is established a shear strength empirical model for the entire shear process.

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