Constraints on single-field inflation with WMAP, SPT and ACT data— A last-minute stand before Planck

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ABSTRACT: We constrain models of single field inflation with the pre-Planck CMB data. The data used here is the 9-year Wilkinson Microwave Anisotropy Probe (WMAP) data, South Pole Telescope (SPT) data and Atacama Cosmology Telescope (ACT) data. By adding in running of spectral index parameter, we find that the $\chi^2$ is improved by a factor of $\Delta \chi^2 = 8.44$, which strongly indicates the preference of this parameter from current data. In addition, we find that the running of spectral index $\alpha_s$ does not change very much even if we switch to different pivot scales, which suggests that the power law expansion of power spectrum is accurate enough till the 1st order term. Furthermore, we find that the joint constraints on $r - n_s$ give very tight constraints on single-field inflation models, and the models with power law potential $\phi^p$ can only survive if $0.9 \lesssim p \lesssim 2.1$, so a large class of inflation models have already been ruled out before Planck data. Finally, we use the $f_{NL}$ data to constrain the non-trivial sound speed $c_s$. We find that the current constraint is dominated by the power spectrum constraints which have some inconsistency with the constraints from $f_{NL}$. This poses important questions of consistency between power spectrum and bispectrum of WMAP data.

KEYWORDS: CMB, inflation.

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1. Introduction

The inflationary model \cite{1, 2, 3} has achieved a great success in modern cosmology, and it has been confirmed by many high precision CMB and Large scale structure experiments \cite{4, 5, 6}. It provides a good explanation to a series problems such as flatness problem, horizon problem, and monopole problem in the standard cosmology scenario. In addition, inflation paradigm provides a natural explanation for the origin of primordial perturbations which constitute the seeds for the large scale structure we can see today. Therefore, identifying the realistic inflation model becomes an important task in observational cosmology.

Astronomical observations provide a large mount of data to constrain the cosmological parameters, especially inflation models. The default cosmology model people always use is the “six-parameter” \( \Lambda \)CDM cosmology model, in which the canonical single-field slow-roll inflation (sound speed \( c_s = 1 \)) is assumed in the model. However, the class of slow-roll inflation models already have some weak tension with the observational data. In Ref. \cite{4}, it is shown that the generic \( \phi^p \) inflation model cannot provide consistent \( r - n_s \) values within reasonable range of number of e-folds. In addition, WMAP 9-year data \cite{7} suggests that the local non-Gaussianity has a large positive value, while the orthogonal non-Gaussianity is a large negative value, and these values are hardly to be produced in the canonical single-field slow-roll inflation models. Given these interesting tension between the canonical single-field slow-roll inflation model and the current observational data, we would like to explore the possibilities of non-trivial sound speed \( c_s \neq 1 \) as well as non-zero running of spectral index \( dn_s/d\ln k \) to test their consistency with current combination of WMAP 9-year data \cite{4}, 
ACT data \cite{ACT} and SPT data \cite{SPT}. We intend to finish this work right before Planck data release (expected on 21st March, 2013) in order to make an immediate comparison before and after the Planck data. We hope that our work will motivate theorists to explore more phenomena in the general single-field slow-roll inflation model given the tight constraints on single field inflation models.

Figure 1: WMAP9 temperature data with lensed ACT and SPT data. WMAP9, ACT and SPT data are mainly in the range $2 \leq l \lesssim 1000$, $300 \lesssim l \lesssim 3000$, and $700 \lesssim l \lesssim 3000$ respectively. The theoretical curve is the lensed CMB power spectrum with WMAP 9-year cosmological parameters and the light blue band is the cosmic variance. The Planck data will further tighten up the error-bars in the middle regime. This figure is reprinted with permission from Mark Halpern.

This paper is organized as follows. In Sec. 2, we will discuss the model we are focusing on, and the data we will use to constrain the models. In Sec. 3, we will present our results of fitting. The concluding remarks will be presented in the last section.

2. Methodology

2.1 The Model

We will use standard 6–parameter ΛCDM model as our basic model\footnote{The free running parameters are \{Ω_bh^2, Ω_c h^2, Ω_Λ, τ, n_s, A_s\}, which are fractional baryon, cold dark matter, and dark energy density, optical depth, spectral index and amplitude of primordial scalar perturbation respectively.}. We then allow $r$ (tensor-to-scalar ratio), $dn_s/d\ln k$ (running of spectral index), $c_s$ (sound speed for the curvature perturbation modes) to be varied since we want to explore the level of constraints from these parameters. The sound speed is related to the tilt of tensor power spectrum in
the general single-field inflation model through \[8, 9\]

\[ n_t = \frac{-r}{3c_s}, \]  

(2.1)

where the tensor power spectrum is parameterized as

\[ P_t(k) = A_t(k_0) \left( \frac{k}{k_0} \right)^{n_t}, \]  

(2.2)

Here \( k_0 \) is the pivot scale. Thus the tensor to scalar ratio is defined as

\[ r = \frac{A_t(k_0)}{A_s(k_0)}. \]  

(2.3)

Since tensor power spectrum also contribute to CMB angular power spectrum \( C_{TT} \) on very large scales, we will use the CMB temperature angular power spectrum to constrain \( r \) and \( c_s \). For more discussion on how the sound speed \( c_s \) changes the data fitting is given in \([10]\).

In addition, we add the “running of running” parameter which characterizes the running of running of spectral index, i.e.

\[ \beta_s = \frac{d\alpha_s}{d\ln k} = \frac{d^2 n_s}{d\ln k^2}. \]  

(2.4)

Thus the scalar power spectrum is parameterized as

\[ P_s(k) = A_s(k_0) \left( \frac{k}{k_0} \right)^{n_s(k_0)-1+\frac{1}{2}\alpha_s(k_0)\ln\left(\frac{k}{k_0}\right)+\frac{1}{2}\beta_s\ln^2\left(\frac{k}{k_0}\right)}. \]  

(2.5)

Note that once the “running of running” \( (\beta_s) \) is introduced into the model, the running of spectral index \( \alpha_s \) becomes a scale-dependent quantity. To remove any ambiguity, we need to specify the pivot scale in the power law expansion (Eq. (2.4)), this is why the \( \alpha_s \) is related to \( k_0 \). However, if \( \alpha_s \) turns out to be less dependent on \( k_0 \), it means that the truncation till \( \alpha_s \) is enough (1st order), and there is no need to introduce a higher order truncation \( (\beta_s) \).

The reason we want to release \( \beta_s \) as the running of running parameter is that SPT data \([5]\) gives a detection of a negative value of the running of spectral index \( \alpha_s = dn_s/d\ln k \) at \( k_0 = 0.025\text{Mpc}^{-1} \). So we would like to add this parameter as a higher order effect to monitor any possible “running of running”. Even though it has not been detected, it is expected to be significantly constrained and is useful for the reconstruction of canonical single-field slow-roll inflation \([11, 12]\).

\[ \text{Here we assume } c_s \text{ as a constant. The perturbation mode with sound speed } c_s \text{ crosses horizon during inflation when } c_s k = aH. \]  

Considering \( n_s - 1 = -2\epsilon - \eta - s \sim \mathcal{O}(10^{-2}) \) and both slow-roll parameters \( \epsilon \) and \( \eta \) are far less than 1, we conclude that \( s \sim \mathcal{O}(10^{-2}) \), where \( s \equiv \frac{\dot{c}_s}{Hc_s} \). On the other hand, since \( d\ln k \simeq Hdt \),

\[ d\ln c_s/d\ln k = s \]  

and then \( c_s(k) = c_s(k_0) \left( \frac{k}{k_0} \right)^s \) which shows that \( c_s \) is roughly scale independent.
2.2 The data

We will use the most precise class of CMB data up-to-date, which is the combination of WMAP 9-year data [4], SPT data [5] and ACT data [6]. The temperature angular power spectrum from three data sets is shown in Fig. 1. The combined data is named as “CMB data” in the following discussion. We set the maximum $l$-range of scalar model to be 7000 ($l_{max}^s = 7000$), and maximal tensor $l$-range to be 3000 ($l_{max}^t = 3000$) in the running of MCMC chains. In addition, we add Baryon Acoustic Oscillation data [13] as well as $H_0$ prior from HST (Hubble-Space-Telescope) project [14] into our data source. In order to explore the variation of sound speed, we add constrained $f_{NL}$ data provided by WMAP 9-year bispectrum into our likelihood. The sound speed $c_s$ is related to the equilateral and orthogonal type of non-Gaussianity $f_{NL}$ through Eq.(57) in [7]. So according to [7] we assign $f_i = (f_{NL}^{eq}, f_{NL}^{orth})$ as the data vector, which is

$$f_{NL}^{eq} = 51 \pm 136 \quad (-221 < f_{NL}^{eq} < 323 \text{ at } 95\%CL), \quad (2.6)$$

$$f_{NL}^{orth} = -245 \pm 100 \quad (-445 < f_{NL}^{orth} < -45 \text{ at } 95\%CL). \quad (2.7)$$

Then we use the $\chi^2$ function (Eq.(58) in [7]) to calculate the best-fit value of $c_s^3$, i.e.

$$\chi^2 = \sum_{ij} f_i F_{ij} f_j - 2 \sum_i F_{ii} f_i \hat{f}_i + \sum_{ij} \hat{f}_i F_{ij} F_{jj}^{-1} \hat{f}_j \quad (2.8)$$

where $F_{ij}$ is the lower right four elements of the Fisher Matrix

$$F = \begin{pmatrix} 25.25 & 1.06 & -2.39 \\ 1.06 & 0.54 & 0.20 \\ -2.39 & 2.20 & 1.00 \end{pmatrix} \times 10^{-4}, \quad (2.9)$$

and $\hat{f}_i = (51, -245)$.

For the extended model of $\Lambda$CDM, we will release $r$ and $\alpha_s$ in the CAMB code [15] and further modify the code to incorporate running of running parameter ($\beta_s$). We run CosmoMC [16, 17] to generate MCMC samples. We will express our results in term of best-fit value of marginalized likelihood, as well as 1$\sigma$ and 2$\sigma$ confidence level (CL) (68.3% and 95.4% CL).

3. Results

3.1 Canonical single-field slow-roll inflation model ($c_s = 1$)

3.1.1 $\Lambda$CDM cosmology model

We first fix $c_s = 1$ and investigate the constraints on parameters $r$ and $\alpha_s$. The data sets we use here are CMB data, BAO and $H_0$. Here we consider “6-parameter model”, “6-parameter+$r$ model”, “6-parameter+$\alpha_s$” model and “6-parameter+$r$+$\alpha_s$” model which are expressed as “$\Lambda$CDM”, “$\Lambda$CDM+$r$”, “$\Lambda$CDM+$\alpha_s$” and “$\Lambda$CDM+$r$+$\alpha_s$” models respectively.

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3The parameter $A$ in Eq.(57) in [6] is running as a free parameter.
Figure 2: Likelihood of $r$ in case of $\alpha_s$ fixed and $\alpha_s$ as free parameter.

In Fig. 2, we can see that the likelihood of $r$ shifts a little if we switch $\alpha_s$ on and off. The solid line is $\Lambda$CDM+$r$ model, and the dotted line is $\Lambda$CDM+$r+\alpha_s$ model. In addition, the likelihood becomes broader in $\Lambda$CDM+$r+\alpha_s$ model, and the upper limit is also higher. This indicates that without the direct polarization power spectrum, it is hard to draw concrete upper limit on the amplitude of tensor mode $r$, since adding a single extra-parameter can greatly broaden the constraint on $r$.

Figure 3: Likelihood of $\alpha_s$ in case of $r$ fixed and $r$ free.

Similar thing exists in Fig. 3. The solid line is the $\Lambda$CDM+$\alpha_s$, and the dotted one is $\Lambda$CDM+$\alpha_s+r$. One can see that the likelihood of $\alpha_s$ is broader if $r$ is released as a free
parameter. This means that the two parameters have some level of degeneracy, which is potentially able to be broken if the future polarization data is added.

The Fig. 4 shows the likelihoods of $n_s$ for three models. Here we consider all of the three models, i.e. $\Lambda$CDM+$r$, $\Lambda$CDM+$\alpha_s$, $\Lambda$CDM+$r+\alpha_s$. We can see that not only the peak of distribution shift, but also the range of confidence level of $n_s$ changes quite a lot in three different model: if we add $r$, the spectral index still prefers a “red” spectrum as $n_s < 1$, but such situation does not exist anymore in the case of $\Lambda$CDM+$\alpha_s$ and $\Lambda$CDM+$\alpha_s+r$.

![Figure 4: Likelihood of $n_s$ in different models.](image)

Fig. 5 shows the contours of joint constraints on $\alpha_s$ and $n_s$ in $\Lambda$CDM+$\alpha_s$ model (the blue solid curves) and $\Lambda$CDM+$\alpha_s+r$ model (the red dashed curves). We can see adding $r$ leads to the shift of $n_s$ towards bluer region, and the constraints become broader.

![Figure 5: Joint constraint of $n_s$ and $\alpha_s$ in $\Lambda$CDM+$\alpha_s$ (blue solid contours) and $\Lambda$CDM+$\alpha_s+r$ model (red dashed contours). The contours show 1$\sigma$ and 2$\sigma$ constraints.](image)
In Table 1, we list the results of fitting by fixing \( c_s = 1 \). One can see that by introducing \( \alpha_s \) parameter, the \( \chi^2 \) really improve significantly \( (\Delta \chi^2 = 4.22) \), indicating that the current data prefer the inflation model with running of the spectral index.

In the left panel of Fig. 6, we compare our joint constraints on \( r \) and \( n_s \) with the results from WMAP 9-year paper [4]. WMAP 9-year results used WMAP 9-year data, combined with old SPT data, old ACT data, BAO data and \( H_0 \) data and obtain the black contours (1\( \sigma \) and 2\( \sigma \) CL). We used the similar combination, except that our SPT and ACT data are the corresponding new data sets [5, 6]. By updating the new data of ACT and SPT, one can see that the constraints are tightened up to some extent. This suggests that the new SPT and ACT data really provide a large level arm for WMAP9 data, which offer more constraining power on small scales CMB angular power spectrum.

We use our results of joint constraints on plane of \( r - n_s \) to discuss its implication for inflation models (Right panel of Fig. 6).

- Chaotic inflation model [18] whose potential is given by \( V(\phi) \propto \phi^p \). This model predicts \( r = \frac{4p}{N}, \ n_s = 1 - \frac{p+2}{2N} \), where \( N \) is the number of e-folds before the end of inflation. Given the current constraints on the amplitude of inflation and the “slow-roll” parameter, \( N \) is around 60 but with some uncertainty of reheating process. Here we take the range of 50-60 as the reasonable range of number of e-folds. The region
Table 1: Results of fitting by fixing $c_s = 1$. We set $k_0 = 0.002 \text{Mpc}^{-1}$, $r_{\text{max}} = 7000$, and $l_{\text{max}} = 3000$ in the running of MCMC chains.

|                | $\Lambda$CDM | $\Lambda$CDM+$r$ | $\Lambda$CDM+$\alpha_s$ | $\Lambda$CDM+$r+\alpha_s$ | $\Lambda$CDM+$r+\alpha_s+\beta_s$ |
|----------------|--------------|-----------------|--------------------------|----------------------------|----------------------------------|
| $n_s$          | 0.961 ± 0.007| 0.959 ± 0.006   | 1.018 ± 0.027            | 1.066 ± 0.040              | 1.089 ± 0.080                    |
| $r_{(95\%\text{CL})}$ | $\leq 0.12$  | $\leq 0.021$    | $\leq 0.035$             | $\leq 0.050$               | $0.005 \pm 0.021$               |
| $\alpha_s$     | $\leq 0.021$ | $\leq 0.035$    | $\leq 0.050$             | $0.005 \pm 0.021$          |                                  |
| $\beta_s$      | $\leq 0.021$ | $\leq 0.035$    | $\leq 0.050$             | $0.005 \pm 0.021$          |                                  |
| Best fit -ln(Like) | 4921.52      | 4921.15         | 4917.30                  | 4916.91                    | 4917.54                          |
| $\Delta \chi^2$| 0            | $-0.74$         | $-8.44$                  | $-9.22$                    | $-7.96$                          |

between two dashed lines in Fig. 3 indicates the prediction of chaotic inflation. One can see that the models with $p = 2$ [18] and $p = 2/3$ [19] are disfavored at around $2\sigma$ level, and only the models with $p \in [0.9, 1.8]$ for $N = 50$ or $p \in [1.5, 2.1]$ for $N = 60$ are still consistent with data within $95\%$ CL.

- Spontaneously broken SUSY (SBS) inflation model [20] with potential $V(\phi) = V_0 \left(1 + c \ln \frac{\phi}{Q}\right)$, where the potential is assumed to be dominated by $V_0$ and $c \ll 1$. This model predicts $r = 0$ and $n_s = 1 - \frac{1}{N}$. The spectral index in this model is quite large and it is disfavored at more than $95\%$ CL.

- Mass term (MT) inflation model [21] with potential $V(\phi) = V_0 - \frac{1}{2}m^2\phi^2$ where the mass term is assumed to be subdominant. The tensor-to-scalar ratio and spectral index in this model are respectively given by $r = 0$ and $n_s = 1 + 2\eta$ where $\eta = -m^2M_p^2/V_0$. This model can fit the data very well if $\eta = -0.02$.

3.1.2 Comparison of different pivot scale and the influence of running of runing of spectral index ($\beta_s$)

In the former sections, all the fittings are done at pivot scale $k_0 = 0.002 \text{Mpc}^{-1}$ and the running of spectral index is preferred at more than $2\sigma$ level. In this section, we investigate the distributions of $\alpha_s$ at different pivot scales. We use the model $\Lambda$CDM$+r+\alpha_s$. The solid line is $k = 0.002 \text{Mpc}^{-1}$, and the dotted line is $k = 0.025 \text{Mpc}^{-1}$ in Fig. 4. It shows that when the pivot scale change, the distribution of $\alpha_s$ almost does not change at all. This means that the constraints on $\alpha_s$ is not sensitive to the pivot scale you choose, which indicates that the truncation of power index expansion (Eq. (2.5)) is accurate enough till 1st order.
Considering the higher order power effect of the primordial power spectrum, we introduce a new parameter \( \beta_s \) to characterize the “running of running” (Eqs. (2.4) and (2.5)). Left panel of Fig. 7 shows the joint constraint on \( \alpha_s \) and \( \beta_s \), and the right panel shows the marginalized distribution of \( \beta_s \) with a flat prior. We can see that the peak of \( \beta_s \) slightly deviates from 0, but is perfectly consistent with zero within 1\( \sigma \) CL. This means that the current data do not support the “running of running of spectral index”, and therefore the power law expansion of the scalar power spectrum (Eq. (2.5)) is accurate enough till the \( \alpha_s \) term. This is consistent with what we find in Sec. 3.1.2. The fitting results are shown in Table 1.

Figure 7: The marginalized distribution of running of spectral index \( \alpha_s \) at different pivot scales.

![Figure 7](image)

Figure 8: Left: Joint constraints on \( \alpha_s - \beta_s \). Right: Marginalized distribution of \( \beta_s \) with 1\( \sigma \) CL 0.005 \( \pm \) 0.021.

![Figure 8](image)
3.2 General single-field inflation Model ($c_s$ free)

Figure 9: Likelihood of $c_s$ in different datasets.

In this section, we release $c_s$ as a free parameter which is constrained by the $f_{NL}$ data from WMAP 9-year results $^4$. Table 2 shows the best fit (-log(Like)) and confidence level of $n_s$, $r$ and $\alpha_s$ of two models. It can be seen that adding $\alpha_s$ significantly reduces the best fit -log(Like), but enlarge both the confidence interval of $r$ and $n_s$.

Fig. 9 shows the distribution of $c_s$. The dotted line is the likelihood from $f_{NL}$ data while the solid line is marginalized probabilities from the CMB+BAO+$H_0+f_{NL}$ data. We can see that the $f_{NL}$ prefers a very low value of $c_s$ while the CMB data sets prefer a larger value, which indicates some tension between each other. In addition, the combined constraints are dominated by CMB power spectrum simply because the number of CMB power spectrum data is far greater than the $f_{NL}$ data. The tension between the $f_{NL}$ data and the CMB power spectrum data may have a variety of indications:

1) it may indicate that the power spectrum and bispectrum data are not consistent with each other, which suggests that there are some uncleaned systematics in the data sets;

2) it may also indicate that the underlying model, i.e. single-field inflation cannot work at all when we confront it with CMB power spectrum and bispectrum data.

In any case, we need to develop a method which can direct relate $c_s$ with power spectrum and bispectrum (not just $f_{NL}$ data) and globally fit this parameter by using full spectrum of CMB data. Such work is in progress.

4. Conclusion

In this paper, we combine the most recent pre-Planck CMB data to constrain the inflation

$^4$A similar constraint from WMAP 7-year data and Background Imaging of Cosmic Extragalactic Polarization (BICEP) experiment was given in [10].
Table 2: Results of fitting with $c_s$ as a free parameter. Here we use the same $k_0$ and $l_{max}$ as Table 1.

|                       | $\Lambda$CDM+$r$ | $\Lambda$CDM+$r+c_s$ | $\Lambda$CDM+$r+c_s+\alpha_s$ |
|-----------------------|------------------|-----------------------|---------------------------------|
| $n_s$                 | $0.959 \pm 0.006$| $0.958 \pm 0.006$    | $1.064 \pm 0.040$               |
| $r(95\%CL)$           | $<0.12$          | $<0.11$              | $<0.40$                         |
| $\alpha_s$            | –                | –                    | $-0.034 \pm 0.012$              |
| Best fit -ln(Like)    | 4921.15          | 4922.14              | 4918.17                         |
| $\Delta \chi^2$      | 0                | 1.98                 | $-5.96$                         |

model parameters. Our data consists of WMAP 9-year data [4], ACT data [6], SPT data [5], Baryon Acoustic Oscillation data [13] as well as $H_0$ prior [14]. We mainly find four interesting results from our numerical fitting:

- if we add in the running of spectral index $\alpha_s = dn_s/d\ln k$, the $\chi^2$ value reduces a lot, which indicates that it improves the fit to data very much.

- By adding in a ‘3rd-order’ parameter, i.e. running of running $\beta_s$, we find that current data do not support non-zero detection of $\beta_s$. In addition, by switching to different pivot scales, the constraints on $\alpha_s$ do not vary a lot. These two tests strongly suggest that the expansion of the power spectrum is accurate enough till the 1st order ($\alpha_s$ term), and there is no observational hint for the higher order scale-dependent terms.

- Due to the new ACT and SPT data we used, our constraints on $r - n_s$ is tighter than the WMAP 9-year results [4]. Our constraints is already able to rule out a large class of single-field inflation model even before Planck data. We show that the single field inflation with power law $\phi^p$ can only survive if $p$ is in between 0.9 and 2.1, and Spontaneously broken SUSY (SBS) inflation is ruled out firmly by current observational data.

- We release sound speed $c_s$ as a free parameter, and find that the constraints on $c_s$ from $f_{NL}$ data and CMB power spectrum are not consistent with each other. This strongly indicates that either there is some unaccounted systematics in the bispectrum data that may incur extra-error in the $f_{NL}$ estimation, or the model of varying $c_s$ cannot work at all given these two datasets. In any case, this motivates us to explore a set of formulism that directly compute power spectrum and bispectrum given a $c_s$ value.

In conclusion, we find that pre-Planck data have already been able to set tight constraints on single field inflation model. But current observational data still leave many open questions to be solved. We hope such issues will be resolved when the Planck data becomes available in a few days.

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