Geometry of the Divergences Problem in QFT

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Abstract

The divergences problem in QFT should be overcame presumably due to the unification of the fundamental interactions [1]. We evidently cannot to achieve this goal now. Together with this there are divergences in problems where the high-energy processes simply cannot be involved (say electron self-energy, Lamb's shift, etc.). Last ones reflect the formal character of perturbation theory applied to interacting secondly quantized amplitudes of the pointwise particles. These difficulties are borrowed partly from the classical theory. I would like to establish some general framework for the future unified theory avoiding to use the method of classical analogy.

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1 Introduction

The classical material points moving in space-time are main original objects of the constructive quantum mechanics (QM) and quantum field theory (QFT). This approach bring a lot of conceptual and technical problems. All progress of the theoretical physics demonstrates that the pointwise in space-time interaction is approximate: the mechanical interaction of solid bodies is the photon exchanges, nuclear interaction provided by pions, and the pointwise Fermi interaction realized by the $W^\pm, Z^0$ exchange in electro-weak theory. We may assume that all attempts to localize matter in space-time, i.e. increasing of resolution by higher energy of collision leads in fact to the delocalization by defreezing new internal degrees of freedom. On the other hand, mathematically, this locality leads to the singular functions (the most serious artifact of the local QFT).

The modern nonlocal objects like strings and membranes arose due to attempts to avoid these difficulties. Since physical status these objects is not clear up to now and their logic leads far away from ordinary physical paradigm [2], it is worth while do find a different solution of the divergences problematic. I show here another method of introduction of nonlocal objects arising in dynamical space-time which is built as specific section in the tangent fibre bundle over the "vacuum landscape".

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2 Geometry of the Divergences Problem

I start with simple Dirac’s example of the divergences problem \[^{[1]}\]. Dirac took the fermionic model Hamiltonian \( \hat{H} = \frac{1}{2}(a_{nm}\eta_m\eta_n - \bar{a}_{nm}\bar{\eta}_m\bar{\eta}_n) \). It is assumed that \( \bar{\eta}_n|S > = 0 \) for all \( 1 \leq n \leq \infty \), where \( |S > \) is a “standard” vector. The matrix \( a_{mn} \) is defined by Dirac as follows: \( a_{mn} = \delta_{m+1,n} - \delta_{m,n+1} \). It is easy to see \( \hat{H}^2|S > = -\frac{1}{2}Tr(\bar{a}a)|S > = \infty|S > \). It should be noted that the ‘deformation’ of the standard state vector \( |S > \) in this artificial example has only ‘longitudinal’ character, i.e. only phase of the state vector \( |S > \) is changed, not its direction. Formally one can find Schrödinger solution with this Hamiltonian \( |\Psi(t) > = \cos(\frac{\hat{H}t}{\hbar})|S > \), corresponding to the infinitely fast phase oscillation. Definitely physicist has some aversion to such behavior, but from the point of view of the projective geometry (and postulate of the ordinal quantum mechanics) ‘deformed’ vector \( |\Psi(t) > \) belongs to the same ray as \( |S > \), i.e. this is the same quantum state. This gives us the main idea to avoid the divergences problem. Namely, the orthogonal projection along of the vacuum state is the subtraction the ‘longitudinal’ component of the variation velocity of the action state \( |\Psi(t) > \). Let put \( \eta_n \) to be creation operator and \( |\xi_n > = \eta_n|S >= \alpha_n|S > + |n > \) is deformed standard vector, so that \( (n|S >= 0 \), therefore, \( < S|\xi_n >= < S|\eta_n|S >= \alpha_n < S|S > \) and, hence, \( \alpha = \frac{< S|\eta_n|S >}{< S|S >} \). Now we can express the ‘transversal’ part of the standard vector deformation

\[
|n >= |\xi_n > - \frac{< S|\xi_n >}{< S|S >}|S >, \quad (2.1)
\]

so that it is orthogonal to \( < S > \). Let me calculate only the ‘transversal’ components of the \( |\Psi(t) > = \exp(i\hat{H}t)|S > \) which I will define as follows:

\[
|\Psi(t) > = |\Psi(t) > - \frac{< S|\Psi(t) >}{< S|S >}|S > = \exp(i\frac{\hat{H}t}{\hbar})|S > - \frac{|S > < S|}{< S|S >} \exp(i\frac{\hat{H}t}{\hbar})|S > \quad (2.2)
\]

Now I apply this definition to calculation of all orders of \( |\Psi(t) > : \)

\[
|\Psi(t)_0 > = |S > - \frac{< S|\hat{H}|S >}{< S|S >}|S > = |S > - \frac{|S > < S|}{< S|S >}|S > = 0, \quad (2.3)
\]

\[
|\Psi(t)_1 > = t(\hat{H}|S > - \frac{< S|\hat{H}|S >}{< S|S >}|S >) = t(\hat{H}|S > - \frac{|S > < S|}{< S|S >} \hat{H}|S >) = t\hat{H}|S >, \quad (2.4)
\]

\[
|\Psi(t)_2 > = \frac{t^2}{2!}(\hat{H}^2|S > - \frac{< S|\hat{H}^2|S >}{< S|S >}|S >) = \frac{t^2}{2!}(\hat{H}^2|S > - \frac{|S > < S|}{< S|S >} \hat{H}^2|S >)
= \frac{t^2}{2!(\frac{1}{4})}(a_{mn}a_{pq}\eta_m\eta_n\eta_p\eta_q - \bar{a}_{mn}a_{pq}\bar{\eta}_m\bar{\eta}_n\bar{\eta}_p\bar{\eta}_q)|S > \n- \frac{|S > < S|}{< S|S >} \frac{1}{4}(a_{mn}a_{pq}\bar{\eta}_m\bar{\eta}_n\eta_p\eta_q - \bar{a}_{mn}a_{pq}\bar{\eta}_m\bar{\eta}_n\eta_p\eta_q)|S >
= \frac{t^2}{2!(\frac{1}{4})}(a_{mn}a_{pq}\eta_m\eta_n\eta_p\eta_q - 2Tr(\bar{a}a)|S >)
\]
where terms, giving nil contribution were omitted. We can see that divergent second order term was canceled out. Let now see what happen in the third order. It is easy to see that

\[
\hat{H}^3|S> = \frac{1}{8}(a_{mn}\eta_m\eta_n - \bar{a}_{mn}\bar{\eta}_m\bar{\eta}_n)(a_{pq}\eta_p\eta_q - \bar{a}_{pq}\bar{\eta}_p\bar{\eta}_q)(a_{rs}\eta_r\eta_s - \bar{a}_{rs}\bar{\eta}_r\bar{\eta}_s)|S>
\]

\[
= \frac{1}{8}(a_{mn}\eta_m\eta_n a_{pq}\eta_p\eta_q a_{rs}\eta_r\eta_s - a_{mn}\eta_m\eta_n a_{pq}\eta_p\eta_q \bar{a}_{rs}\bar{\eta}_r\bar{\eta}_s - a_{mn}\eta_m\eta_n \bar{a}_{pq}\bar{\eta}_p\bar{\eta}_q a_{rs}\eta_r\eta_s + a_{mn}\eta_m\eta_n \bar{a}_{pq}\bar{\eta}_p\bar{\eta}_q \bar{a}_{rs}\bar{\eta}_r\bar{\eta}_s + \bar{a}_{mn}\bar{\eta}_m\bar{\eta}_n a_{pq}\eta_p\eta_q a_{rs}\eta_r\eta_s - \bar{a}_{mn}\bar{\eta}_m\bar{\eta}_n a_{pq}\eta_p\eta_q \bar{a}_{rs}\bar{\eta}_r\bar{\eta}_s - \bar{a}_{mn}\bar{\eta}_m\bar{\eta}_n \bar{a}_{pq}\bar{\eta}_p\bar{\eta}_q a_{rs}\eta_r\eta_s + \bar{a}_{mn}\bar{\eta}_m\bar{\eta}_n \bar{a}_{pq}\bar{\eta}_p\bar{\eta}_q \bar{a}_{rs}\bar{\eta}_r\bar{\eta}_s)|S>
\]

\[
= \frac{1}{8}(a_{mn}\eta_m\eta_n a_{pq}\eta_p\eta_q a_{rs}\eta_r\eta_s - 2T^{\hat{a}\hat{a}}(a_{mn}\eta_m\eta_n)|S> - 2T^{\hat{a}\hat{a}} a_{mn}\eta_m\eta_n)|S>. \tag{2.6}
\]

and, therefore,

\[
|\Psi(t)_3> = \frac{t^3}{3!}(\hat{H}^3|S> - \frac{<S|\hat{H}^3|S>}{<S|S>}|S>)
\]

\[
= \frac{t^3}{3!}\frac{1}{8}(a_{mn}a_{pq}a_{rs} \eta_m\eta_n \eta_p\eta_q \eta_r\eta_s - 2T^{\hat{a}\hat{a}}(a_{mn}\eta_m\eta_n)|S>. \tag{2.7}
\]

One may see that the divergences alive in the third order. Since the indefinite trace \(T^{\hat{a}\hat{a}} = -\infty\) is a coefficient before the transversal to the vacuum two-fermionic term, the compensation projective term does not help. Nevertheless, we can extract the useful hint: the vacuum vector (the standard vector in Dirac’s example) should be smoothly changed, and, furthermore, the transversal component should be reduced during the “smooth” evolution. One may image some a smooth surface with a normal vector, taking the place of the vacuum vector. Then the orthogonal projection acting continuously is in fact the covariant differentiation of the tangent Hamiltonian vector field \(H\). One has in fact the modification of the creation-annihilation operators of quantum particles. Let me recall that the main technical result of Dirac approach \(H\) is the calculations of the coefficients \(Y_n\) and \(Z_n\), modifying the initial creation-annihilation operators.

Of course, the trivial example of Dirac has only pedagogical sense and one should take some more realistic model. Dirac undertook attempts to find out the better approximation to the vacuum state vector of the QED \(H\). The main idea was to safe physical meaning of the Schrödinger picture in the QED. The approximate state vector found by Dirac comprises of ‘longitudinal’ and ‘transversal’ components corresponding creations electron-positron pears and the two electron-positron pears. Thereby ‘deformed’ vacuum vector has not only trivial modulus variation, but the variation of a direction too. Both of these approximate vectors \(|V_1>\) and \(|V_2>\) suffer on divergences and Dirac came to the conclusion, that the Schrödinger vector in QED does not exist at all, because it does not lie in any separable Hilbert space \(H\). In spite of this extreme point of view, I will assume that a tangent vector of state ‘creeps’ along the projective Hilbert space \(CP(N-1)\) from one to another tangent Hilbert spaces \(T_CP(N-1) = C^{N-1}\) at different generalized coherent
states (GCS) serving for the parametrization of quantum setup \cite{5}. Hereafter I will use finite dimension case $SU(N)$, $CP(N-1)$, etc., but where it is necessity, the limit $N \to \infty$ is keeping in mind. This modification requires, of course, a deep reconstruction of the QFT \cite{6,7,8,9,10}. Physically main content of this program is to design all physical entities in the terms of the pure quantum state space geometry. Many inequivalent representations realized now in the tangent fibre bundle over $CP(\infty)$.

3 Action projective state space

Hereafter (beside the paragraph 4) I will use the indices as follows: $0 \leq a \leq N$, and $1 \leq i, k, m, n, s \leq N - 1$

One of the most serious modification concerns the scheme of the “second quantization” procedure.

First. In the second quantization method one has formally given particles whose properties are defined by some commutation relations between creation-annihilation operators. Note, that the commutation relations are only the simplest consequence of the curvature of the dynamical group manifold in the vicinity of the group’s unit (in algebra). Dynamical processes require, however, finite group transformations and, hence, the global group structure. The main technical idea is to use vector fields over group manifold instead of indefinite Dirac’s q-numbers. This scheme therefore looking for the dynamical nature of the creation and annihilation processes of quantum particles.

Second. The quantum particles (energy bundles) should gravitate. Hence, strictly speaking, their behavior cannot be described as a linear superposition. Therefore the ordinary second quantization method (creation-annihilation of free particles) is merely a good approximate scheme due to the weakness of gravity. Thereby the creation and annihilation of particles are time consuming dynamical non-linear processes. So, linear operators of creation and annihilation (in Dirac sense) do exist as approximate quantities.

Third. For sure there is an energy quantization but the dynamical nature of this process is unknown. Avoiding the vacuum stability problem, its self-energy, etc., we primary quantize, however, the action, not energy. The relative (local) vacuum of some problem is not the state with minimal energy, it is a state with an extremal of some action functional.

POSTULATE 1.

I assume that there are elementary quantum states (EQS) $| \bar{a} \rangle$, $a = 0, 1, \ldots$ of abstract Planck’s oscillator whose states correspond to the quantum motions with given Planck’s action quanta. Thereby only action subject to primary quantization but the quantization of dynamical variables such as energy, spin, etc., postponed to dynamical stage. Physical oscillators are distributed due to space-time dependence of frequencies obeying field equations which should established due to a new variation problem.

Presumably there are some non-linear field equations whose soliton-like solution provides the quantization of the dynamical variables but their field carriers are smeared in dynamical space-time. Therefore, quantum “particles”, and, hence, their numbers should arise as some countable solutions of non-linear wave equations. In order to establish acceptable field equation capable intrinsically describe all possible degrees of freedom defreezing under intensive interaction we should to build some universal ambient Hilbert
state space $\mathcal{H}$. I will use the universality of the action whose variation capable generate any dynamical variable. Vectors of action state space $\mathcal{H}$ I will call action amplitude (AA). Some of them will be EQS’s of motion corresponding to entire numbers of Planck’s quanta $|\hbar a >$. Generally (AA) are their coherent superposition

$$|G> = \sum_{a=0}^{\infty} g^a |\hbar a >. \quad (3.1)$$

may represented of the ground state of some quantum system. In order to avoid the misleading reminiscence about Schrödinger state vector I will use $|G>, |S>$, instead of $|\Psi>$. Since the action in itself does not create gravity, it is legible to create such linear superposition of $|\hbar a> = (a)^{-1/2}(\pi ^{\dagger })^a |0>$ constituting $SU(\infty)$ multiplete of the Planck’s action quanta operator $\hat{S} = \hbar \pi^{\dagger}\pi$ with the spectrum $S_a = \hbar a$ in the separable Hilbert space $\mathcal{H}$. The standard basis $\{|\hbar a >\}_{a=0}^{\infty}$ will be used with the ‘principle’ quantum number $a = 0, 1, 2, ...$ assigned by Planck’s quanta counting.

Since any ray AA has isotropy group $H = U(1) \times U(N)$, in $\mathcal{H}$ effectively act only coset transformations $G/H = SU(N)/S[U(1) \times U(N-1)] = CP(N-1)$. Therefore the ray representation of $SU(N)$ in $C^N$, in particular, the embedding of $H$ and $G/H$ in $G$, is the state-dependent parametrization. Therefore, there is a diffeomorphism between space of the rays marked by the local coordinates in the map $U_j : \{|G>, |g^j| \neq 0\}, j > 0$

$$\pi^1 = \frac{g^0}{g^j}, \ldots, \pi^j = \frac{g^{j-1}}{g^j}, \pi^{j+1} = \frac{g^{j+1}}{g^j}, \ldots, \pi^{N-1} = \frac{g^{N-1}}{g^j}, ... \quad (3.2)$$

and the group manifold of the coset transformations $G/H = SU(N)/S[U(1) \times U(N-1)] = CP(N-1)$ [14], where $N \to \infty$. This diffeomorphism provided by the coefficient functions $\Phi_a^i$ of the local generators (see below). The choice of the map $U_j$ means, that the comparison of quantum amplitudes refers to the amplitude with the action $\hbar j$. The breakdown of $SU(\infty)$ symmetry on each AA to the isotropy group $H = U(1) \times U(\infty)$ contracts full dynamics down to $CP(\infty)$. The physical interpretation of these transformations is given by the

**POSTULATE 2.**

Super-equivalence principle: the unitary transformations of the AA may be identified with the physical unitary fields. The coset transformation $G/H = SU(\infty)/S[U(1) \times U(\infty)] = CP(\infty)$ is the quantum analog of classical force: its action is equivalent to some physically distinguishable variation of AA in $CP(\infty)$.

I will assume that all “vacua” solutions belong to single separable projective Hilbert space $CP(N-1)$. The vacuum is now merely the stationary point of some action functional, not solution with the minimal energy. Energy will be associated with tangent vector field to $CP(N-1)$ giving velocity of the action variation in respect with a “second time” [15] close to the notion of Newton-Stueckelberg-Horwitz-Piron (NSHP) time [16]. Dynamical space-time will be built at any vacuum (see below). Therefore Minkowskian space-time is functionally local in $CP(N-1)$ and the space-time motion dictated by the field equations connected with two infinitesimally close “vacua”. The connection between these local space-times may be physically established by the measurement given in terms of geometry of the base manifold $CP(N-1)$. It seems like the Everett’s idea about
“parallel words”, but has of course different physical sense. Now we are evidences of the Multiverse concept \[2\]. I think there is only one Universe but there exists continuum of dynamical space-times each of them related to one point of the “vacuum landscape” \(CP(N - 1)\). The standard approach, identifying Universe with space-time, is too strong assumption from this point of view.

4 Local dynamical variables during NSHP evolution

When quantum setup traverses in \(CP(\infty)\), the evolution curve may be associated with trace of the vacuum extremal in \(CP(\infty)\) under the action variation \(S + \delta S\). The length of the evolution curve in \(CP(\infty)\) may be measured in seconds. Then the length of the evolution curve may be identified with the NSHP time. The velocities (rates of change the action against NSHP time) of transition from one superposition state to another is a measure of system energy. These velocities are tangent vectors to \(CP(N - 1)\). On the other hand they are operators of differentiation (variation) of some functionals (mathematical scalar fields) \(D S = F^i(\Omega)\frac{\partial S}{\partial \pi^i} + c.c.\) or operators (mathematical vector fields) \(D V^n = F^i(\Omega)(\frac{\partial V^n}{\partial \pi^i} + \Gamma_{ik}^n V^k) + c.c.\) over \(CP(\infty)\). The distribution of frequencies (energies) will be established by some non-linear field equation which arise as the condition of the parallel transport of the Hamiltonian vector field \[5, 6\].

In order to build Hamiltonian vector field one needs a convenient representation of the hermitian Hamiltonian. It is well known that each hermitian matrix may be represented as the linear combination of Habbard matrix \(\hat{B}_k^i\)

\[
\hat{H} = E_i^k \hat{B}_k^i = E_i^k
\]

where \(E_i^k = E_i^k^*\). Avoiding two-index numeration let me use \(SU(N)\) traceless generators like Pauli, Gell-Mann, etc., matrices. The problem is to find \(\hat{\Lambda}_\alpha\) for given \(\hat{B}_k^i\) and the inverse problem. For any \(N \geq 2\) one has the follows numeration scheme

for \(n = 2; \hat{\Lambda}_1 = \hat{B}_2^1 + \hat{B}_1^2; \hat{\Lambda}_2 = i(\hat{B}_2^2 - \hat{B}_1^1); \hat{\Lambda}_3 = \frac{1}{\sqrt{1}}(\hat{B}_1^1 - \hat{B}_2^2)\);

for \(n = 3; \hat{\Lambda}_4 = \hat{B}_3^1 + \hat{B}_1^3; \hat{\Lambda}_5 = i(\hat{B}_3^3 - \hat{B}_1^1); \hat{\Lambda}_6 = \hat{B}_3^2 + \hat{B}_2^3; \hat{\Lambda}_7 = i(\hat{B}_3^3 - \hat{B}_2^2); \hat{\Lambda}_8 = \frac{1}{\sqrt{3}}(\hat{B}_1^1 + \hat{B}_2^2 - 2\hat{B}_3^3)\);

for \(n = 4; \hat{\Lambda}_9 = \hat{B}_4^1 + \hat{B}_1^4; \hat{\Lambda}_{10} = i(\hat{B}_4^4 - \hat{B}_1^1); \hat{\Lambda}_{11} = \frac{1}{\sqrt{6}}(\hat{B}_1^1 + \hat{B}_2^2 - \hat{B}_3^3 - \hat{B}_4^4)\);
\[
\begin{align*}
\hat{\Lambda}_{11} &= \hat{B}_1^2 + \hat{B}_2^2; \hat{\Lambda}_{12} = i(\hat{B}_2^3 - \hat{B}_1^3); \\
\hat{\Lambda}_{13} &= (\hat{B}_1^3 + \hat{B}_3^3); \hat{\Lambda}_{14} = i(\hat{B}_3^4 - \hat{B}_1^4); \\
\hat{\Lambda}_{15} &= \frac{1}{\sqrt{6}}(\hat{B}_1^4 + \hat{B}_2^4 + \hat{B}_3^4 - 3\hat{B}_4^4);
\end{align*}
\]

(4.2)

Let me introduce \(m = \min(i, k) \geq 1\) and \(M = \max(i, k) \geq 2\). Then it is clear that \(\hat{\Lambda}_\alpha\) belongs to the set of the \(2(M - 1) + 1\) matrices

\[
\text{for } n = M; \hat{\Lambda}_{M^2-2M+1} = \hat{B}_{M-1}^2 + \hat{B}_1^{M-1}; \hat{\Lambda}_{M^2-2M+2} = i(\hat{B}_{M-1}^1 - \hat{B}_1^{M-1});
\]

\[
\hat{\Lambda}_{M^2-2M+2m-1} = \hat{B}_{M-1}^m + \hat{B}_1^{M-1}; \hat{\Lambda}_{M^2-2M+2m} = i(\hat{B}_{M-1}^m - \hat{B}_1^{M-1});
\]

\[
\hat{\Lambda}_{M^2-1} = \sqrt{\frac{2}{M(M-1)}}(\hat{B}_1^1 + \hat{B}_2^2 + \hat{B}_3^3 + ... + \hat{B}_{M-1}^{M-1} - (M-1)\hat{B}_M^M);
\]

(4.3)

Therefore two matrices \(\hat{B}^k_i\) and \(\hat{B}^k_i\) if \(i \neq k\) define the matrix \(\hat{\Lambda}_{M^2-2M+2m-1} = \hat{B}_{M-1}^m + \hat{B}_1^{M-1}\) and matrix \(\hat{\Lambda}_{M^2-2M+2m} = i(\hat{B}_{M-1}^m - \hat{B}_1^{M-1})\), but if \(i = k\), then \(\hat{B}_M^M\) corresponds to the diagonal matrix \(\hat{\Lambda}_{M^2-1} = \sqrt{\frac{2}{M(M-1)}}(\hat{B}_1^1 + \hat{B}_2^2 + \hat{B}_3^3 + ... + \hat{B}_{M-1}^{M-1} - (M-1)\hat{B}_M^M)\).

In order to solve the inverse problem, namely, for given \(\hat{\Lambda}_\alpha\) to find the correspond matrix \(\hat{B}_k^k\) one should to find maximal solution of the inequality \(M^2 \leq \alpha + 1\). It means that \(\hat{\Lambda}_\alpha\) belongs to the set \([1, M]\). If some entire \(M\) is such that \(M^2 = \alpha + 1\), then one has the diagonal matrix \(\hat{B}_M^M\). But if \(M^2 < \alpha + 1\) then it is easy to find the position of given \(\hat{\Lambda}_\alpha\) from the set \([1, M]\). Namely, the quantity of rows \(M-1\) in the set gives us the index \(i = M - 1\), and the number of row containing \(\hat{\Lambda}_\alpha\) gives us the index \(k = m\) from one of the equations

\[
m = \frac{\alpha - M^2 + 2M - 1}{2} \quad \text{or} \quad m = \frac{\alpha - M^2 + 2M}{2},
\]

(4.4)

depending on which nominator is even. Therefore any traceless hermitian operator having matrix representation may be represented as a single index linear combination of the \(\hat{\Lambda}_\alpha\) matrices. For example, the operator of harmonic oscillator coordinate is as follows

\[
\hat{x} = \sqrt{\frac{\hbar}{2m\omega}} \sum_{n=1}^{\infty} \sqrt{n} \hat{\Lambda}_{(n+1)^2-3};
\]

(4.5)

and for the momentum operator one has

\[
\hat{p} = \sqrt{\frac{2m\omega}{\hbar}} \sum_{n=1}^{\infty} \sqrt{n} \hat{\Lambda}_{(n+1)^2-2};
\]

(4.6)
Since commutators and anti-commutators of $\hat{\Lambda}_\alpha$ matrices give linear combination of the unit matrix and $\hat{\Lambda}_\alpha$ matrices, such algebraic operations as summation and multiplication lead to algebra of the unitary group $AlgU(\infty)$. Hence many Hermitian reasonable Hamiltonian $\hat{H}$ may be represented as follows:

$$\hat{H} = \epsilon\hat{\Delta} + \hbar \sum_{n=1}^{\infty} \Omega^n \hat{\Lambda}_\alpha,$$

(4.7)

where $\hat{\Delta}$ is unit matrix. Then $\hbar \sum_{n=1}^{\infty} \Omega^n \hat{\Lambda}_\alpha = \hat{H} - \epsilon\hat{\Delta}$. Multiplying both sides by $\hat{\Lambda}_\beta$ and taking into account $Tr(\hat{\Lambda}_\beta \hat{\Lambda}_\alpha) = 2\delta_{\alpha\beta}$, we get

$$Tr(\hat{\Lambda}_\beta (\hat{H} - \epsilon\hat{\Delta})) = \hbar \sum_{n=1}^{\infty} \Omega^n Tr(\hat{\Lambda}_\beta \hat{\Lambda}_\alpha) = 2\hbar \Omega^\beta,$$

(4.8)

and, therefore, the “multipole” coefficient functions may be extracted

$$\Omega^\alpha = \frac{1}{2\hbar} Tr(\hat{\Lambda}_\alpha (\hat{H} - \epsilon\hat{\Delta})) = \frac{1}{2\hbar} Tr(\hat{\Lambda}_\alpha \hat{H}).$$

(4.9)

The coefficients $\Omega^\alpha$ for the Hamiltonian of the harmonic oscillator

$$\hat{H}_{HO} = \hbar \omega \begin{pmatrix} 1/2 & 0 & \ldots & 0 & \ldots \\ 0 & 3/2 & \ldots & 0 & \ldots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & n+1/2 \end{pmatrix},$$

(4.10)

is for off diagonal matrices as follows:

$$\Omega^\alpha = \frac{1}{2\hbar} Tr(\hat{\Lambda}_\alpha \hat{H}_{HO}) = \omega(m - 1/2 + M - 3/2)/2 = \omega(m + M - 2)/2$$

(4.11)

and for the diagonal matrices $\hat{\Lambda}_\alpha$ they are given by

$$\Omega^\alpha = \frac{1}{2\hbar} Tr(\hat{\Lambda}_\alpha \hat{H}_{HO}) = \omega \sqrt{\frac{1}{8M(M-1)} \left( \sum_{k=1}^{M-1} k - M + 1 \right)}.$$

(4.12)

Hermitian Hamiltonian in the form (4.7) used in wide spectrum of physical problems [14]. In these problems coefficient functions $\epsilon$ and $\Omega^\alpha$ usually arise from the classical space-time model. In our case we should find equations for these functions starting from conditions applied to the LDV represented by vector fields in functional space. In other words one should find “field shell” of quantum particles in dynamical space-time. Now we should introduce the dynamical space-time based on the quantum measurements in terms of $CP(N - 1)$ geometry [5, 6].
Hereafter I will use the representation of the Hamiltonian vector fields as follows:

$$\frac{d\pi^i}{d\tau} = H^i = \frac{1}{2} \Phi^i_{\alpha} \Omega^\alpha = \frac{1}{2} \Phi^i_{\alpha} \Omega^\alpha \hat{\Theta}(\hat{\lambda}_\alpha \hat{H}),$$

(4.13)

where

$$\Phi^i_{\alpha} = \lim_{\epsilon \to 0} \epsilon^{-1} \left\{ \frac{\exp(i\epsilon\lambda^h)_{m}^{i}}{\exp(i\epsilon\lambda^b)_{m}^{i}} \frac{g^m}{g^3} \right\} = \lim_{\epsilon \to 0} \epsilon^{-1} \{ \pi^i(\epsilon\lambda^h) - \pi^i \},$$

(4.14)

Perfectly isolated quantum systems such as “elementary” particles do not exist. But it is natural to assume that the elementary excitations of the ground (vacuum) should exist as some entity, invariant relative the choice of different physical field constituting a setup. On the other hand one should avoid the artificial distinction between the bare “elementary” particle and its “surrounding” field.

In the framework of $SU(N)$ symmetry I use a model of the quantum field theory based on a local Hamiltonian of interaction. It consists of the sum of $N^2 - 1$ the energies of the ‘elementary systems’ (particle plus fields) is equal to the excitation energy of the GCS, and the local Hamiltonian $\hat{H}$ is linear against the partial derivatives

$$\hat{H} = h\Omega^\alpha \Phi^i_{\alpha} \frac{\partial}{\partial \pi^i} + \text{c.c.}$$

$$= \hat{T}_h + \hat{U}_b = h\Omega^b \Phi^i_{b} \frac{\partial}{\partial \pi^i} + h\Omega^h \Phi^i_{h} \frac{\partial}{\partial \pi^i} + \text{c.c.}$$

(1.15)

This Hamiltonian describes the interaction between quantum system, given be the local analog of the spin-operators (Pauli, Gell-Mann, etc.) expressed in coordinates $(\pi^1, ..., \pi^i, ...)$, and its adjoint unitary “field shell” $\Omega^\alpha$ actually transforming states of the quantum system. We have in fact a self-consistent problem but under some reasonable assumptions this may be reduced to pure non-linear field equations for the “field shell”.

The dynamical variables corresponding symmetries of the GCS and their breakdown should be expressed now in terms of the local coordinates $\pi^k$. Hence the internal dynamical variables and their norms should be state-dependent, i.e. local in the state space $[5, 6, 7, 8]$. These local dynamical variables realize a non-linear representation of the unitary global $SU(N)$ group in the Hilbert state space $C^N$. Namely, $N^2 - 1$ generators of $G = SU(N)$ may be divided in accordance with Cartan decomposition: $[B, B] \in H, [B, H] \in B, [B, B] \in H$. The $(N - 1)^2$ generators

$$\Phi^i_{h} \frac{\partial}{\partial \pi^i} + \text{c.c.} \in H, \quad 1 \leq h \leq (N - 1)^2$$

(4.16)

of the isotropy group $H = U(1) \times U(N - 1)$ of the ray (Cartan sub-algebra) and $2(N - 1)$ generators

$$\Phi^i_{b} \frac{\partial}{\partial \pi^i} + \text{c.c.} \in B, \quad 1 \leq b \leq 2(N - 1)$$

(4.17)

are the coset $G/H = SU(N)/S[U(1) \times U(N - 1)]$ generators realizing the breakdown of the $G = SU(N)$ symmetry of the GCS. Furthermore, $(N - 1)^2$ generators of the Cartan
sub-algebra may be divided into the two sets of operators: $1 \leq c \leq N - 1$ ($N - 1$ is the rank of $\text{AlgSU}(N)$) Abelian operators, and $1 \leq q \leq (N - 1)(N - 2)$ non-Abelian operators corresponding to the non-commutative part of the Cartan sub-algebra of the isotropy (gauge) group. Here $\Phi_{\sigma}^i$, $1 \leq \sigma \leq N^2 - 1$ are the coefficient functions of the generators of the non-linear $SU(N)$ realization. They give the infinitesimal shift of $i$-component of the coherent state driven by the $\sigma$-component of the unitary multipole field rotating the generators of $\text{AlgSU}(N)$ and they are defined by the formulas (4.14). These show how look $SU(N)$ generators in the vicinity of the origin of the map $U_j : |G >, |g^1| \neq 0$ being expressed in the local ray's coordinates ($\pi^1, ..., \pi^i, ..., \pi^{N-1}$). This representation is useful since embedding Cartan sub-group, coset manifold, etc., are state-dependent, i.e. requires local coordinates.

Now I will introduce the Lagrangian $\mathcal{L}$ and the canonical momentum $P_i$. Let me put

$$\mathcal{L} = ||E|| = \hbar \sqrt{G_{ik}^* (\Omega^\alpha \Phi^i_\alpha)(\Omega^\beta \Phi^k_\beta)^*} = \hbar \sqrt{G_{ik}^* V^i V^k}$$

(4.18)

since

$$V^i = \frac{d\pi^i}{d\tau} = \Omega^\alpha \Phi^i_\alpha.$$  

(4.19)

Then the canonical momentum is as follows

$$P_i = \frac{\partial \mathcal{L}}{\partial V^i} = \frac{\hbar^2}{2||E||} G_{ik}^* V^k$$

(4.20)

and its Hermitian conjugated value is

$$P^i = \frac{\hbar^2}{2||E||} G^{ik} V^k,$$

(4.21)

Therefore,

$$P_i = \frac{\hbar^2}{2||E||} V^i,$$

(4.22)

and then the generator of the momentum (tangent vector) is as follows

$$\vec{P} = P^i \frac{\partial}{\partial \pi^i} + c.c. = \frac{\hbar^2}{4||E||} V^i \frac{\partial}{\partial \pi^i} + c.c.$$  

(4.23)

Since the velocity $V^i$ has physical dimension of frequency, the momentum $P_i$ has the physical dimension of the action. The contraction of corresponding LDV's has the square of modulus

$$P^i P_i = \frac{\hbar^4}{4||E||^2} G^{ik} V^i V^k = \frac{\hbar^2}{4}$$

(4.24)

equal to the minimal uncertainty.
5 Dynamical quantum space-time

The internal hidden dynamics of the quantum configuration given by AA should be somehow reflected in physical space-time. Therefore we should solve the “inverse representation problem”: to find locally unitary representation of dynamical group SU\((N)\) in the dynamical space-time where acts the induced realization of the coherence group SU\((2)\) of the Qubit spinor [5, 6]. Its components subjected to the “quantum Lorentz transformations” [15]. We should build the local spinor basis invariantly related to the ground states manifold \(CP(N − 1)\). First of all we have to have the local reference frame (LRF) as some analog of the “representation” of \(SU\((N)\)”. Each LRF and, hence, \(SU\((N)\)” “representation” may be marked by the local coordinates (3.2) of the “vacuum landscape”. Now we should almost literally repeat differential geometry of a smooth manifold embedded in flat ambient Hilbert space. The geometry of this smooth manifold is the projective Hilbert space equipped with the Fubini-Study metric that may be expressed in the local coordinates as follows

\[
G_{ik}^* = \frac{(1 + \sum |\pi_s|^2)\delta_{ik} - \pi^i\pi^k}{(1 + \sum |\pi_s|^2)^2},
\]

(5.1)

and with the affine connection

\[
\Gamma_{mn}^i = \frac{1}{2} G^{ip} \left( \frac{\partial G_{mp}^r}{\partial \pi^n} + \frac{\partial G_{pn}^r}{\partial \pi^m} - \frac{\partial G_{mn}^r}{\partial \pi^p} \right) = -\frac{\delta_{mn} \pi^r + \delta_{nm} \pi^r}{1 + \sum |\pi_s|^2}.
\]

(5.2)

The velocity of ground state evolution relative NSHP time is given by the formula

\[
|H\rangle = \frac{d|G\rangle}{d\tau} = \frac{\partial g^a}{\partial \pi^i} \frac{d\pi^i}{d\tau} |ah\rangle = |T_i\rangle = \frac{d\pi^i}{d\tau} = H^i|T_i\rangle,
\]

(5.3)

is the tangent vector to the evolution curve \(\pi^i = \pi^i(\tau)\), where

\[
|T_i\rangle = \frac{\partial g^a}{\partial \pi^i} |ah\rangle = T^a_i |ah\rangle.
\]

(5.4)

Then the “acceleration” is as follows

\[
|A\rangle = \frac{d^2|G\rangle}{d\tau^2} = |g_{ik}\rangle \frac{d\pi^i}{d\tau} \frac{d\pi^k}{d\tau} + |T_i\rangle \frac{d^2\pi^i}{d\tau^2} = |N\rangle \frac{d\pi^i}{d\tau} \frac{d\pi^k}{d\tau} + \left( \frac{d^2\pi_s}{d\tau^2} + \Gamma_{ik}^s \frac{d\pi^i}{d\tau} \frac{d\pi^k}{d\tau} \right)|T_s\rangle,
\]

(5.5)

where

\[
|g_{ik}\rangle = \frac{\partial^2 g^a}{\partial \pi^i \partial \pi^k} |ah\rangle = |N_{ik}\rangle + \Gamma_{ik}^s |T_s\rangle,
\]

(5.6)

and the normal state

\[
|N\rangle = N^a |ah\rangle = \left( \frac{\partial^2 g^a}{\partial \pi^i \partial \pi^k} - \Gamma_{ik}^s \frac{\partial g^a}{\partial \pi^s} \right) \frac{d\pi^i}{d\tau} \frac{d\pi^k}{d\tau} |ah\rangle.
\]

(5.7)
to the “hypersurface” of the ground states. Then the minimization of this “acceleration” under the transition from point \( \tau \) to \( \tau + d\tau \) may be achieved by the annihilation of the tangential component

\[
\left( \frac{d^2 \pi^s}{d\tau^2} + \Gamma_{ik}^s \frac{d\pi^i}{d\tau} \frac{d\pi^k}{d\tau} \right) T_s > 0 \tag{5.8}
\]

i.e. under the condition of the affine parallel transport of the Hamiltonian vector field

\[
dH^s + \Gamma_{ik}^s H^i d\pi^k = 0. \tag{5.9}
\]

The Gauss-Codazzi equations

\[
\frac{\partial N^a}{\partial \pi^i} = B^a_s T_s^a \\
\frac{\partial T^a_k}{\partial \pi^i} - \Gamma_{ik}^s T_s^a = B_{ik} N^a \tag{5.10}
\]

I used here instead of the anthropic principle \cite{2}. These give us dynamics of the vacuum (normal) vector and the tangent vectors, i.e. one has the LRF dynamics modeling the “moving representation” or moving quantum setup

\[
\frac{dN^a}{d\tau} = \frac{\partial N^a}{\partial \pi^i} \frac{d\pi^i}{d\tau} + \text{c.c.} = B^a_s T_s^a \frac{d\pi^i}{d\tau} + \text{c.c.} = B^a_s T_s^a H^i + \text{c.c.}
\]

\[
\frac{dT^a_k}{d\tau} = \frac{\partial T^a_k}{\partial \pi^i} \frac{d\pi^i}{d\tau} + \text{c.c.} = (B_{ik} N^a + \Gamma_{ik} T_s^a) \frac{d\pi^i}{d\tau} + \text{c.c.} = (B_{ik} N^a + \Gamma_{ik} T_s^a) H^i + \text{c.c.}(5.11)
\]

Please, remember that \( 0 \leq a \leq N \), but \( 1 \leq i, k, m, n, s \leq N - 1 \). In order to find the matrix \( B_s^a \) of the second quadratic form of the ground states “hypersurface” we shall use the self-adjoint Weingarten mapping \( L_{(\pi^1, ..., \pi^{N-1})}(\vec{D}) = -\nabla_{\vec{D}}|N > \) \cite{3} where \( \vec{D} \) refers to some local dynamical variable (LDV) \cite{3}.

If we would like to have some embedding of the “Hilbert (quantum) dynamics” in space-time we should to formalize the quantum observation (or measurement of some dynamical variable).

The measurement, i.e. attributing a number to some dynamical variable or observable has in physics subjective as well as objective sense. Namely: the numeric value of some observable depends as a rule on a setup (the character of motion of laboratory, type of the measuring device, field strength, etc.). However the relationships between numeric values of dynamical variables and numeric characteristics of laboratory motion, field strength, etc., should be formulated as invariant, since they reflect the objective character of the physical interaction used in the measurement process. The numbers obtained due to the measurements carry information which does not exist a priori, i.e. before the measurement process. But the information comprised of subjective as well as objective invariant part reflects the physics of interaction. The last is one of the main topics of quantum field theory. Since each measurement reducible (even if it is unconscious) to the answer of the question ”yes” or ”no”, it is possible to introduce formally a quantum dynamical variable ”logical spin 1/2” \cite{7} whose coherent states represent the quantum bit of information ”Qubit”.
POSTULATE 3

I assume that the invariant i.e. physically essential part of information represented by the coherent states of the "logical spin 1/2" is related to the space-time structure.

Such assumption is based on the observation that on one side the space-time is the manifold of points artificially depleted of all physical characteristics (material points without reference to masses). In principle arbitrary local coordinates may be attributed to these points. On the other hand as we know from general relativity the metric structure depends on the matter distribution and the zero approximation of the metric tensor $g_{\mu\nu} = \eta_{\mu\nu} + \cdots$ gives the Lorentz invariant interval. The spinor structure Lorentz transformations represents the transformations of the coherent states of the "logical spin 1/2" or "Qubit". Thereby we can assume the measurement of the quantum dynamical variables expressed by the "Qubit" spinor "creates" the local space-time coordinates. I will formulate non-linear field equations in this local space-time due to a variational principle referring to the generator of the quantum state deformation.

Now one should build the Qubit spinor in the local basis $(|N>, |D>)$ for the quantum question with “yes” or “no” spectrum in respect with the measurement of some local dynamical variable $D$. I will assume that there is natural state $|D> of the quantum system in the LRF representation equal to the lift of $LDV \in T_\pi CP(N-1)$ into the environmental Hilbert space $\mathcal{H}$, and there is expectation state $|D_{\text{expect}}>=\alpha_0|N> + \beta_0|\bar{D}>$, associated with the “measuring device” tuning. This notional measuring device is associate with the local unitary projector along the normal $|N>$ and onto the natural state $|\bar{D}>$. In fact it defines the covariant derivative in $CP(N-1)$. The lift-vectors $|N>, |D>$ are given by the solutions of (5.11) arising under the measurement of the LDV $\bar{D}$. In general $|D>$ it is not a tangent vector to $CP(N-1)$. But renormalized vector defined as the covariant derivative $|\bar{D}> = |D> - <\text{Norm}|D>|\text{Norm}>$ is a tangent vector to $CP(N-1)$ if $|\text{Norm}>=\sqrt{|N>|N>}. The operation of the velocity renormalization is the orthogonal (unitary) projector. Indeed,

$$|\bar{D}> = |D> - <\text{Norm}|D>|\text{Norm}>$$

Then at the point $(\pi^1, ..., \pi^{N-1})$ one has two components of the Qubit spinor

$$\alpha(\pi^1, ..., \pi^{N-1}) = \frac{<N'|D_{\text{expect}}>}{<N'|N>},$$

$$\beta(\pi^1, ..., \pi^{N-1}) = \frac{<\bar{D}'|D_{\text{expect}}>}{<\bar{D}'|\bar{D}>}$$

(5.13)

then at the infinitesimally close point $(\pi^1 + \delta^1, ..., \pi^{N-1} + \delta^{N-1})$ one has new Qubit spinor

$$\alpha(\pi^1 + \delta^1, ..., \pi^{N-1} + \delta^{N-1}) = \frac{<N'|D_{\text{expect}}>}{<N'|N'>},$$

$$\beta(\pi^1 + \delta^1, ..., \pi^{N-1} + \delta^{N-1}) = \frac{<\bar{D}'|D_{\text{expect}}>}{<\bar{D}'|\bar{D}>}$$

(5.14)
where the basis \( |N'>, |\tilde{D}' > \) is the lift of the parallel transported \( |N >, |\tilde{D} > \) from the infinitesimally close \( (\pi^1 + \delta^1, ..., \pi^{N-1} + \delta^{N-1}) \) back to \( (\pi^1, ..., \pi^{N-1}) \).

These two spinors being connected with infinitesimal “Lorentz spin transformations matrix” \( L \) create the local dynamical space-time \([5, 6]\). The coordinates \( x^\mu \) of points in this space-time serve in fact merely for parametrization of deformations of the “field shell” arising under its motion according to non-linear field equations.

6 Nonlinear gauge field equations for transition from infinitesimally close dynamical quantum space-times

“Particle” now associated with its “field shell” in the dynamical space-time. At each GCS \( (\pi^1, ..., \pi^{N-1}) \) of the \( CP(N-1) \) one has an “expectation value” of LDV’s defined by a measuring device. But this GCS may by reached along of one of continuum pathes. Therefore the comparison of two vector fields and their “expectation values” in neighborhood points requires some natural rule. The “natural” in our case means that the comparison has sense only for same “field shell”. For this reason one should have a “self-identification” procedure. The affine parallel transport in \( CP(N-1) \) of vector fields is a natural and the simplest rule for the comparison of generators corresponding “field shells”. Physically the self-identification of “particle” at different GCS, i.e. setups, literally means that vector fields corresponding to its LDV’s are covariant constants relative the Fubini-Study metric. Since we have only the unitary fields as parameters of GCS transformations I assume that in accordance with the super-equivalence principle under the infinitesimal shift of the unitary field \( \delta \Omega^\alpha \) in the dynamical space-time, the shifted Hamiltonian field should coincide with the infinitesimal shift of tangent Hamiltonian field generated by the parallel transport in \( CP(N-1) \) during NSHP time \( \delta \tau \). Hence, under small specific variation of the unitary fields \( \Omega^\alpha \), obeying field equations given below, two dynamical variables

\[
\mathcal{D}_1 = F^i_1(\pi, \Omega) \frac{\partial}{\partial \pi^i} + c.c., \\
\mathcal{D}_2 = F^i_2(\pi, \Omega) \frac{\partial}{\partial \pi^i} + c.c. 
\]  

such as Hamiltonian \( H^i \) and momentum \( P^i \)

\[
F^i_1(\pi, \Omega) = H^i = \hbar \Omega^\alpha \Phi^i_\alpha = \hbar V^i \\
F^i_2(\pi, \Omega) = P^i = \frac{\hbar^2}{2||E||} V^i, 
\]

subjected to the infinitesimal parallel transport. Then assuming that the affine parallel transport of each dynamical variable generated by the Hamiltonian along the evolution curve should be accompanied with specific variations of \( \Omega^\alpha \) one has

\[
F^i_{1,2}(\pi, \Omega + \delta \Omega) = F^i_{1,2}(\pi, \Omega) - \Gamma^i_{mn} F^m_{1,2}(\pi, \Omega) V^n \delta \tau, 
\]

or separately for \( H^i \) and \( P^i \):

\[
\hbar (\Omega^\alpha + \delta \Omega^\alpha) \Phi^k_\alpha = \hbar \Omega^\alpha (\Phi^k_\alpha - \Gamma^k_{mn} \Phi^m_\alpha V^n \delta \tau), 
\]
Then taking into account

\[ P^i(\Omega^\alpha + \delta \Omega^\alpha) = P^i(\Omega^\alpha) - \Gamma_{mn}^k P^m(\Omega^\alpha) V^n \delta \tau. \]  

(6.4)

Then taking into account

\[ P^i(\Omega + \delta \Omega, \Omega^* + \delta \Omega^*) = P^i(\Omega, \Omega^*) + \frac{\partial P^i(\Omega, \Omega^*)}{\partial \Omega^\sigma} \delta \Omega^\sigma + \frac{\partial P^i(\Omega, \Omega^*)}{\partial \Omega^\tau} \delta \Omega^\tau + \ldots \]  

(6.5)

where complex partial derivatives are as follow \( \frac{\partial}{\partial \Omega^\sigma} = \frac{1}{2} \left( \frac{\partial}{\partial \Omega^\sigma} - i \frac{\partial}{\partial \Omega^\tau} \right) \) and \( \frac{\partial}{\partial \Omega^\tau} = \frac{1}{2} \left( \frac{\partial}{\partial \Omega^\sigma} + i \frac{\partial}{\partial \Omega^\tau} \right) \) one has the differentials

\[ \delta \Omega^\alpha = - \Omega^\alpha \Gamma_{mn}^i V^n \delta \tau, \]

\[ \delta P^i = \frac{\partial P^i(\Omega, \Omega^*)}{\partial \Omega^\sigma} \delta \Omega^\sigma \]

\[ = \frac{\hbar}{4} (\Phi^i(G_{jk}, V^iV^k) - 1) - \frac{1}{2} V^i G_{jk} \Phi^j \Phi^k (G_{jk}, V^iV^k)^{-3/2} \delta \Omega^\sigma. \]  

(6.6)

Assuming that infinitesimal coordinates variation is generated by the Poincaré group transformations \( \delta x^\mu = \Lambda_\mu^\nu x^\nu \delta \tau + u^\mu \delta s \), where \( \delta s = \epsilon \delta t \) is “proper time”, one has “4-velocity” of evolution relative NSHP time

\[ U^\mu = \frac{\delta x^\mu}{\delta \tau} = \Lambda_\mu^\nu x^\nu + \left( \frac{1}{\sqrt{1 - v^2/c^2}} \frac{\mathbf{v}}{c \sqrt{1 - v^2/c^2}} \right) \frac{\delta s}{\delta \tau} = \Lambda_\mu^\nu x^\nu + cu^\mu \frac{\delta t}{\delta \tau}. \]  

(6.7)

where \( \Lambda_\mu^\nu \) corresponds to the \( \bar{L} = 1 + \frac{1}{2} \delta \tau \mathbf{d}(\mathbf{d} - i\mathbf{\omega}) \) connecting infinitesimally close Qubit spinors [5,14]. Taking into account

\[ \frac{\partial F_{1,2}(\pi, \Omega)}{\partial \Omega^\alpha} \delta \Omega^\alpha = \frac{\partial F_{1,2}(\pi, \Omega)}{\partial \Omega^\tau} \delta x^\mu \frac{\partial x^\mu}{\partial \tau} \]  

(6.8)

one has the two sets of field equations in the local dynamical space-time

\[ \Lambda_\mu^\nu x^\nu \frac{\partial F^i_{1,2}(\pi, \Omega)}{\partial \Omega^\nu} \frac{\partial \Omega^\mu}{\partial x^\mu} = - \Gamma^i_{mn} F^m_{1,2}(\pi, \Omega) V^m, \]

\[ cu^\mu \frac{\partial x^\mu}{\partial \tau} \frac{\partial F^i_{2}(\pi, \Omega)}{\partial \Omega^\nu} \frac{\partial \Omega^\mu}{\partial x^\mu} = - \Gamma^i_{mn} F^m_{2}(\pi, \Omega) V^m. \]  

(6.9)

The homogenous part of “4-velocity” is given by \( y^\mu = \Lambda_\mu^\nu x^\nu \) and, therefore, the field equations generated by the Lorentz transformations read now as follows:

\[ y^\nu \frac{\partial \Omega^\mu}{\partial x^\nu} = - \Omega^\mu \Gamma^m_{mn} V^m. \]  

(6.10)

Hence, the first set of \( 1 \leq b \leq 2(N - 1) \) equations gives the potential energy expressed by the coset components \( \Omega^b \) and compensated by homogeneous Lorentz transformations.

Let me suppose that local space-time shift (non-homogeneous part of Poincaré group) should be represented by the variation of the “field shell” leading to the affine parallel transported LDV of momentum. Then a straightforward calculation yields the field quasi-linear equations in partial derivatives

\[ u^\mu \frac{\partial \Omega^b}{\partial x^\mu} \frac{\delta t}{\delta \tau} = \frac{\mathsf{c} \hbar}{2} (\Phi^i_h(G_{ik}, V^iV^k)^{-1/2} - \frac{1}{2} V^i G_{jk} \Phi^j_h \Phi^k_h (G_{ik}, V^iV^k)^{-3/2} \]  

...
These equations generated by LDV’s of Hamiltonian \(N\) for the Hamiltonian flow (4.13). Some solutions for the simplest case \(\Omega\) have the solution \(\Omega\) for \(f\) but they concern the same “field shell” represented by the commutation relations of the space-time transformations accompanying the infinitesimal field representing homogeneous part of Lorentz group solutions. We can build two field equations like (6.11), (6.10). One of them for gauge unitary transformations when one tries to determine arbitrary functions arising in PDE equations of characteristics may be written as follows over-determined system of PDE’s

\[
E_1 : C_{1t}(\rho, \Theta, \Phi) \frac{\partial \rho}{\partial t} + C_{1r}(\rho, \Theta, \Phi) \frac{\partial \rho}{\partial r} = R_1(\rho, \Theta, \Phi), \\
E_2 : C_{2t}(\rho, \Theta, \Phi) \frac{\partial \rho}{\partial t} + C_{2r}(\rho, \Theta, \Phi) \frac{\partial \rho}{\partial r} = R_2(\rho, \Theta, \Phi), \\
E_3 : C_{3t}(\rho, \Theta, \Phi, u^\mu) \frac{\partial \rho}{\partial t} + C_{3r}(\rho, \Theta, \Phi, u^\mu) \frac{\partial \rho}{\partial r} = R_3(\rho, \Theta, \Phi, u^\mu). (6.13)
\]

These field equations representing the natural “corpuscular-wave duality”, since their equations of characteristics

\[
\begin{align*}
\frac{dt}{C_{1t}(\rho, \Theta, \Phi)} &= \frac{dr}{C_{1r}(\rho, \Theta, \Phi)} = \frac{d\rho}{R_1(\rho, \Theta, \Phi)}, \\
\frac{dt}{C_{2t}(\rho, \Theta, \Phi)} &= \frac{dr}{C_{2r}(\rho, \Theta, \Phi)} = \frac{d\rho}{R_2(\rho, \Theta, \Phi)}, \\
\frac{dt}{C_{3t}(\rho, \Theta, \Phi, u^\mu)} &= \frac{dr}{C_{3r}(\rho, \Theta, \Phi, u^\mu)} = \frac{d\rho}{R_3(\rho, \Theta, \Phi, u^\mu)}. (6.14)
\end{align*}
\]

give “corpuscular-like” motions along trajectories.

Let me assume that one has two LDV’s, say, Hamiltonian \(\bar{H}\) and momentum \(\bar{P}\). Since \(F_{1,2}(\pi, \Omega)\) consist in general all components of \(\Omega^\alpha\), one should take into account the commutation relations of the space-time transformations accompanying the infinitesimal unitary transformations when one tries to determine arbitrary functions arising in PDE solutions. We can build two field equations like (6.11), (6.10). One of them for gauge field representing homogeneous part of Lorentz group \(E_1(\Omega) = 0\) with the solution \(\Omega_{s1} = \Omega_{s1}(f_1(c^2 t^2 - r^2))\), containing an arbitrary function \(f_1(c^2 t^2 - r^2)\). The third one \(E_3(\Omega) = 0\) has the solution \(\Omega_{s3} = \Omega_{s3}(f_3(r - l(u_x + u_y + u_z)))\), containing an arbitrary function \(f_3(r - l(u_x + u_y + u_z))\). Two nonequivalent equations should have different solutions, but they concern the same “field shell” represented by the \(\rho(t, r)\)! Let me apply the equation \(E_3\) to the solution \(\Omega_{s1}\). This yields \(E_3(\Omega_{s1}(f_1(c^2 t^2 - r^2))) = 0\), then one will have a solution for \(f_{s1} = f_1(c^2 t^2 - r^2)\). Now let me apply the equation \(E_1\) to the solution \(\Omega_{s3}\). This yields \(E_1(\Omega_{s3}(f_3(r - l(u_x + u_y + u_z))) = 0\), then one will have a solution for \(f_3 = f_3(r - l(u_x + u_y + u_z))\), where \(u\) is rotated \(v\). One should find the condition of the self-consistency of these equations generated by LDV’s of Hamiltonian \(\bar{H}\) and momentum \(\bar{P}\). I will discuss this problem elsewhere.

Field equations (6.11), (6.13) has relativistic solutions \(\Omega^\alpha\) for the coefficient functions for the Hamiltonian flow (4.13). Some solutions for the simplest case \(N = 2\) have been

\[
= -\Gamma^m_{nn} P^m(\Omega) V^n. (6.11)
\]
The choice of the dimension $N > 2$ gives different adjoint representation with dimension $N^2 - 1$. The limit $N \to \infty$ is now interesting for our aim. This is a difficult task to analyze the infinite set of so complicated non-linear PDE system. But the increment of the dimensionality sharply decreases amplitudes of all processes that effectively leads to finite dimension problem. Nevertheless, some estimation is encouraging.

The Fubini-Study metric gives us the possibility to measure distance between adjacent rays corresponding original and deformed states in $CP(N - 1)$ or in $CP(\infty)$. In the last case, however, should be sure that the ‘transversal’ part of the deformed vector belongs to the tangent Hilbert space too. Then the natural measure is the length of a curve. The derivative of a tangent vector $\frac{dT^i(\pi)}{d\pi}$ generally is not tangent vector even in finite dimension case, but the covariant derivative $\frac{dT^i(\pi)}{d\pi} + \Gamma^i_{jk} k \frac{dk^j(\pi)}{d\pi}$ is the tangent to $CP(N - 1)$, where $\Gamma^i_{jk}$ is the affine connection. One can treat this behavior of the tangent state vector as a finite dimension non-linear model of the “beat out” of the state vector due to the ‘divergences problem’ [4] from one tangent Hilbert space to another (Dirac thought that it is “beating out” to nowhere). Then we can assume that the real divergence problem may be resolved in similar manner: in the infinite dimension there is smooth manifold $CP(\infty)$ and dynamical quantum state (tangent vector) creeps from one tangent Hilbert space to another. The convergence is required now only locally in the specified tangent Hilbert space (in the sense of the Fubini-Study metric). It should be noted that if $\sum_{n=0}^{\infty} |g^n|^2 < \infty$, then each coherent state $(\pi^1, \ldots, \pi^k, \ldots, \pi^{N-1}, \ldots)$ belongs to the Hilbert space $l^2$ since from the inequality, for example in the map $U_0 : \{|G >, |g^n| \neq 0\}$, follows that $\sum_{n=0}^{\infty} |g^n|^2 = |g^0|^2 (1 + \sum_n |\pi^n|^2) < \infty$ and, consequently, $\sum_{n=0}^{\infty} |\pi^n|^2 < \infty$.

Let us assume that tangent vector field $T^i(\pi)$ belongs to $l^2$, i.e. $\sum_{i=1}^{\infty} |T^i|^2 < \infty$. Let me put $\beta = \sum_k |T^k|^2 \sum_s |\pi^n|^2 - \sum_k |T^k \pi^n|^2 > 0$. Since for some $K < \infty$ one has $0 \leq L = \sum_k |T^k|^2 \sum_s |\pi^n|^2 < K$ and $0 \leq M = \sum_k |T^k \pi^n|^2 < K$, and, $L - M > 0$ one has $\beta = L - M < L + M < 2K < \infty$, and, therefore, $G_{ik} T^i T^k = \sum_k |T^k|^2 (1 + \sum_s |\pi^n|^2)^{-2} (1 + \sum_k |T^k|^2) < \infty$.

The requirement of the covariant derivative of the vector field $T^i(\pi)$ elimination leads to the parallel transport equation $\frac{d\xi}{d\tau} = \frac{d T^i(\pi)}{d\pi} \frac{\pi^m}{1 + \sum_{n=1}^{\infty} |\pi^n|^2}$, where $\xi^m = \frac{d\xi}{d\pi}$.

Because the operation of the vector summation in $l^2$ is correct, i.e. $\sum_{n=1}^{\infty} |T^n + \xi^n|^2 < \infty$, if $T$, $\xi \in l^2$, and because $\alpha_m = \frac{\sum_{n=1}^{\infty} |\pi^n|^2}{1 + \sum_{n=1}^{\infty} |\pi^n|^2} < 1$, one has the inequality $\sum_{n=1}^{\infty} |T^n + \xi^n|^2 < \infty$. Nevertheless, one has not guaranty that any sequence of $\Lambda^i$ belongs to the $l^2$. But there is a possibility to find such $\delta_1, \Delta_1$ that for $\sum_{s=1}^{\infty} |\pi^s|^2 < \delta_1$ or for $\sum_{s=1}^{\infty} |\pi^s|^2 > \Delta_1$ we will get $\sum_{s=1}^{\infty} |\Lambda^s|^2 < \infty$.

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References

[1] P.A.M. Dirac, *Lectures on quantum field theory*, New York, Yeshiva University, 1967.
[2] S.Weinberg, arXive: hep-th/0511037
[3] J.A. Thorp, *Elementary Topics in Differential Geometry*, Springer-Verlag, New-York, Heidelberg, Berlin, (1979).

[4] P.A.M. Dirac, Nuovo Cimento, Suppl. 6, 322 (1957).

[5] P.Leifer, arXive: gr-qc/0503083

[6] P.Leifer, arXive: gr-qc/0505051

[7] P.Leifer, Found. Phys. 27, (2) 261 (1997).

[8] P.Leifer, Found. Phys. Lett., 11, (3) 233 (1998).

[9] P.Leifer, JETP Letters, 80, (5) 367 (2004).

[10] P.Leifer, Found. Phys. Lett., 18, (2) 195 (2005).

[11] A.L. Besse, *Manifolds all of whose Geodesics are Closed*, Springer-Verlag, Berlin, Heidelberg, New-York, 1978.

[12] I. Prigogin, *From being to becoming*, San Francisco, W.H. Freeman and Company, 1980.

[13] L.P. Horwitz, hep-ph/9606330

[14] A.M. Perelomov, *Generalized coherent states and their applications*, Moskow, “Nauka”, 1987.

[15] C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation*, W.H. Freeman and Company, San Francisco, 1973.