Exact Spectral Functions of a Non Fermi Liquid in 1 Dimension

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Abstract

We study the exact one electron propagator and spectral function of a solvable model of interacting electrons due to Schulz and Shastry. The solution previously found for the energies and wave functions is extended to give the spectral functions, which turn out to be computable, interesting and non trivial. They provide one of the few examples of cases where the spectral functions are known asymptotically as well as exactly.

I. INTRODUCTION

Schulz and Shastry\textsuperscript{3} have introduced a new class of gauge coupled one-dimensional (1D) Fermi systems that are non Fermi liquid in the sense that the momentum distribution function has a cusp at the Fermi momentum $k_F$ rather than a jump as in a Fermi liquid.\textsuperscript{2} This behavior is of the sort first found by Luttinger in the context of his study of a one dimensional model that is popularly known as the Luttinger model.\textsuperscript{3} The model introduced by Schulz and Shastry (SS) is in fact intimately connected to the Luttinger model, and is best viewed as a reinterpretation of Luttinger's original model as a gauge theory. Particles of different species exert a mutual gauge potential on each other, and this is sufficient to destroy the Fermi liquid. This model has the added property that the charge and spin correlations are unaffected by the interaction, owing to the “gauge” nature of interaction. The fermionic Green's functions, however, are non trivial, and have characteristic singularities that are popularly known as the Tomonaga-Luttinger liquid behavior.\textsuperscript{4} The asymptotic long distance behavior of the one electron correlation function is known (see below) by one of several arguments, including Luttinger’s original one.

Our motivation in the present work is to compute the exact one electron Green’s function for the SS model, utilizing our knowledge of the complete spectrum of the same, and using techniques familiar from Anderson’s treatment of the Orthogonality Catastrophe issue in the X-ray edge problem.\textsuperscript{6} This is of great interest since usually one does not have access to the exact Green’s function even in 1D, and one has to be content with the asymptotic behavior. For interpreting experiments, such as those on photoemission, one wants to know more than just the asymptotics, and this possibility is realized here for the particular model of SS.
We first write down the basic lattice Fermi model in 1D, outline the pseudo unitary transformation that eliminates the gauge interactions in favor of a twisted boundary condition. Using this transformation we formulate the problem of calculating the one electron Green’s function.

II. THE MODEL

Let us write the model for two component electrons hopping and interacting via the Hamiltonian

$$\mathcal{H} = -t \sum_{j=0}^{L-1} \sum_{\sigma} \exp(i\sigma \alpha [\hat{n}_{j,\uparrow} + \hat{n}_{j+1,\downarrow}]) c_{j,\sigma}^\dagger c_{j+1,\sigma} + \text{H.c.},$$

where for concreteness we have simplified the original model presented in Ref. 1. The unitary transformation

$$U_1 = \exp\left(i \sum_{l>m} \alpha [\hat{n}_{l,\uparrow} - \hat{n}_{m,\downarrow}]\right)$$

transforms the Eq. (1) into a simple hopping Hamiltonian with twisted boundary conditions. To regain a translational invariant Hamiltonian we apply a second unitary transformation

$$U_2 = \prod_{l=0}^{L-1} \exp \frac{2i\alpha l (\hat{N}_{\uparrow} - \hat{N}_{\downarrow})}{L}.$$ (3)

The combined transformation $U = U_2 U_1$ commutes with $\mathcal{T}$, where $\mathcal{T}$ is the translational operator which shifts one site to the right (e.g. $\mathcal{T} \hat{n}_{j} \mathcal{T}^\dagger = \hat{n}_{j+1}$). The effect of $U$ on the fermion operators is

$$U c_{j,\sigma}^\dagger U^\dagger = e^{-i\alpha N_{\sigma}} e^{i\alpha \hat{n}_{j,\sigma}} c_{j,\sigma}^\dagger \prod_{l=0}^{j-1} \exp(2i\alpha \hat{n}_{l,\sigma}) \prod_{l=0}^{L-1} \exp \frac{2i\alpha (l-j) \alpha \hat{n}_{l,\sigma}}{L},$$

$$U c_{j,\sigma} U^\dagger = e^{i\alpha N_{\sigma}} e^{-i\alpha \hat{n}_{j,\sigma}} c_{j,\sigma} \prod_{l=0}^{j-1} \exp(-2i\alpha \hat{n}_{l,\sigma}) \prod_{l=0}^{L-1} \exp \frac{-2i\alpha (l-j) \alpha \hat{n}_{l,\sigma}}{L},$$

while the density operators are invariant, $U \hat{n}_{j,\sigma} U^\dagger = \hat{n}_{j,\sigma}$, and the transformed Hamiltonian $\tilde{\mathcal{H}} = U \mathcal{H} U^\dagger$ reads

$$\tilde{\mathcal{H}} = -t \sum_{j=0}^{L-1} \sum_{\sigma} \left( e^{2i\alpha n_{\sigma}} c_{j,\sigma}^\dagger c_{j+1,\sigma} + \text{H.c.} \right),$$

where $n_{\sigma} = N_{\sigma}/L$ is the density of $\sigma$ spin fermions. Thus we see that the transformed hopping has a “dynamically generated” gauge field. In the eigenvalue problem

$$\tilde{\mathcal{H}} |\tilde{\phi}\rangle = E |\tilde{\phi}\rangle$$

the eigenstates $|\tilde{\phi}\rangle$ are products of noninteracting one-particle states with momenta $k$ created with $c_{k,\sigma}^\dagger = L^{-1/2} \sum_t e^{ikt} c_{k,\sigma}^\dagger$ operator, $|\tilde{\phi}\rangle = \prod_{k,\sigma} c_{k,\sigma}^\dagger |0\rangle$. The momenta are quantized as $L k_{j,\sigma} = 2\pi I_{j,\sigma}$ with $I_{j,\sigma}$ integers. The total energy and momentum of the states is
\[ E = \sum_{\sigma} \sum_{j=1}^{N_\sigma} \varepsilon_\sigma(k_{j,\sigma}), \quad P = \sum_{\sigma} \sum_{j=1}^{N_\sigma} k_{j,\sigma}, \]  

and the one-particle energy is

\[ \varepsilon_\sigma(k) = -2t \cos(k + 2\sigma\alpha n_\sigma). \]  

Thus we must have the eigenstates of \( \mathcal{H} \)

\[ |\phi\rangle = U^\dagger |\tilde{\phi}\rangle, \]

with the energy and momentum given also by Eq. (7). In the ground state the \( k \) states between the Fermi momenta \( k_{F,\sigma}^{-} \) and \( k_{F,\sigma}^{+} \) are filled \((k_{F,\sigma}^{\pm} = \pm \pi n_\sigma - 2\alpha n_\sigma)\). In the thermodynamic limit the energy \( E \) does not depend on \( \alpha \) and is equal with the energy of the noninteracting \( \alpha = 0 \) case. For finite size systems \( \alpha \) enters only through the \( O(1/L) \) corrections.

## III. SPECTRAL FUNCTIONS

Our goal is to calculate the spectral functions, which we define as

\[
A_\sigma(k, \omega) = \sum_f |\langle f | c_{k,\sigma}^\dagger |\text{GS}\rangle|^2 \delta(\omega - E_{f,\sigma}^{N+1} + E_{\text{GS}}),
\]

\[
B_\sigma(k, \omega) = \sum_f |\langle f | c_{k,\sigma} |\text{GS}\rangle|^2 \delta(\omega - E_{\text{GS}} + E_{f,\sigma}^{N-1}).
\]

The local \( (k \) averaged) spectral functions are defined as

\[
A_\sigma(\omega) = \frac{1}{L} \sum_k A_\sigma(k, \omega),
\]

\[
B_\sigma(\omega) = \frac{1}{L} \sum_k B_\sigma(k, \omega).
\]

We concentrate on \( A_\uparrow(k, \omega) \), since \( B_\sigma(k, \omega) \) is calculated analogously.

As we mentioned in the introduction, 1D interacting fermions behave as Luttinger-liquids, which is characterized, among others, by the power-law behavior of correlation function for small energies. In our case, as we will see later, the main contribution for \( 0 < \alpha < \pi \) comes from

\[
A_\uparrow(k, \omega) \approx c_1 \frac{[(\omega - \varepsilon_F)^2 - u^2(k - k_\uparrow^{(1)})^2]^{(\alpha/\pi)^2}}{\omega - \varepsilon_F - u(k - k_\uparrow^{(1)})} + c_1 \frac{[(\omega - \varepsilon_F)^2 - u^2(k - k_\uparrow^{(1)})^2]^{(\alpha/\pi)^2}}{\omega - \varepsilon_F + u(k - k_\uparrow^{(1)})} + c_2 \frac{[(\omega - \varepsilon_F)^2 - u^2(k - k_\uparrow^{(3)})^2]^{(\alpha/\pi-1)^2}}{\omega - \varepsilon_F - u(k - k_\uparrow^{(3)})} + c_2 \frac{[(\omega - \varepsilon_F)^2 - u^2(k - k_\uparrow^{(3)})^2]^{(\alpha/\pi-1)^2}}{\omega - \varepsilon_F + u(k - k_\uparrow^{(3)})},
\]

where \( k_\sigma^{(\nu)} = \nu \pi n_\sigma - 2\sigma\alpha n_{-\sigma} \) are the (Fermi) momenta of the singularities, \( c_1 \) and \( c_2 \) are constants and \( u \) is the velocity of the excitations. Usually in Luttinger-liquids the velocities
of the spin and charge excitations are different and they both appear in spectral functions. In our case, however, due to the gauge origin of the interaction, the spin and charge velocities are equal to the fermi velocity $v_F$. The spectral function has a nonanalytical, branch cut structure near the fermi momenta, and finite weight appears for higher multiples of the fermi momenta ($k^{(3)}_\uparrow$). The local density of states near Fermi energy reads

$$A(\omega) \approx c_1(\omega - \varepsilon_F)^2(\alpha/\pi)^2 + c_2(\omega - \varepsilon_F)^2(\alpha/\pi-1)^2,$$

which for the noninteracting $\alpha = 0$ reproduces the Fermi-liquid step function.

We now consider the exact evaluation of the spectral functions. As a preliminary to the discussion for general $\alpha$, let us note the special cases of $\alpha = 0$ and $\alpha = \pi$, where the spectral functions can be calculated more or less trivially.

(i) The $\alpha = 0$ case is nothing else but the usual tight binding Hamiltonian

$$\mathcal{H} = -t \sum_{j,\sigma} (c_{j,\sigma}^\dagger c_{j+1,\sigma} + \text{H.c.})$$

of noninteracting electrons, as $e^{i\alpha \hat{n}_j} = 1$ in Eq. (16). For the spectral functions we recover the familiar

$$A^{(0)}_\sigma(k, \omega) = \delta(\omega + 2t \cos k)\Theta(\omega - \varepsilon_F), \quad B^{(0)}_\sigma(k, \omega) = \delta(\omega + 2t \cos k)\Theta(\varepsilon_F - \omega),$$

i.e. a Dirac-delta peak following the cosine-like dispersion of the free fermions.

(ii) When $\alpha = \pi$ the model actually corresponds to the electron-hole symmetric correlated hopping model with $t_{AA} = t_{BB} = -t$ and $t_{AB} = t$ (the hopping amplitudes $t_{AA}$, $t_{BB}$ and $t_{AB}$ are defined in Ref. [7]):

$$\mathcal{H} = -t \sum_{j,\sigma} (1-2\hat{n}_{j,\sigma})(1-2\hat{n}_{j+1,\sigma})c_{j,\sigma}^\dagger c_{j+1,\sigma} + \text{H.c.} .$$

The Hamiltonian (19) can be diagonalized with the help of a unitary transformation

$$\tilde{U} = \prod_{j=1}^{L} (-1)^{\hat{n}_j,\uparrow \hat{n}_j,\downarrow},$$

which is simpler than $\tilde{U} = \tilde{U}_1\tilde{U}_2$ given by Eqs. (2) and (3), and it transforms the fermionic operators as

$$\tilde{U}c_{j,\sigma}^\dagger \tilde{U}^\dagger = (1-2\hat{n}_{j,\sigma})c_{j,\sigma}^\dagger, \quad \tilde{U}c_{j,\sigma} \tilde{U}^\dagger = (1-2\hat{n}_{j,\sigma})c_{j,\sigma} .$$

so the transformed fermionic operators remain “local”. Furthermore, this transformation is not any more restricted to the 1D case. The evaluation of the matrix elements is now convenient for operators in site representation, and the matrix element in Eq. (11) becomes

$$|\langle f | c_{k,\uparrow}^\dagger | \text{GS} \rangle|^2 = L|\langle f | c_{0,\uparrow}^\dagger | \text{GS} \rangle|^2 \delta_{k,P_f-P_{\text{GS}}},$$

where the $c_{0,\uparrow}^\dagger$ creates fermion on site 0. Next, we apply the canonical transformation to formulate the problem using the transformed wave function [the analog of Eq. (3)], and for the spectral function we get
\[ A_t(k, \omega) = L \sum_f |\langle f | (1 - 2\hat{n}_{j,\uparrow}) c^\dagger_{j,\uparrow} |\widehat{GS}\rangle|^2 \delta(\omega - E_f^{N+1} + E_{GS}) \delta_{k, P_f^{N+1} - P_{GS}}. \]  

(23)

The wave functions are product of the spin-up and spin-down part, the evaluation is straightforward and leads to

\[ A_t(k, \omega) = (1 - 2n_\uparrow)^2 A_t^{(0)}(k, \omega) + \frac{4}{L^2} \sum_{q \in FS_\uparrow} \sum_{q' \in FS_\downarrow} \sum_{k' \not\in FS_\uparrow} \delta(\omega - \varepsilon_\downarrow(q') + \varepsilon_\downarrow(q) - \varepsilon_\uparrow(k')) \delta_{k, q' - q + k'}, \]

(24)

and a similar equation gives \( B_t(k, \omega) \). In the spectral function we can identify the following two distinct features: (a) a Dirac-delta contribution following the cosine-like dispersion, which is the reminder of the noninteracting spectral function [Eq. (17)] suppressed by a factor of \((1 - 2n_\uparrow)^2\); (b) a broader continuum coming from the propagator dressed with a single loop. As we increase the filling, the weight of the Fermi jump for zero magnetization \( n_\uparrow = n_\downarrow = n/2 \) decreases as \((1 - n)^2\), and will disappear at half filling, leaving us with an \( A(\omega) \propto \omega^2 \) density of states [the \( c_2 \sim (1 - n)^2 \) in Eq. (13) for \( \alpha = \pi \)]. To illustrate this behavior, we present the evolution of the local spectral function with density in Fig. (1).

(iii) General case: Like in the previous case, in evaluating the matrix elements we use the site representation, given by Eq. (22). Next, we apply the canonical transformation to formulate the problem using the transformed wave functions:

\[ \langle f | c^\dagger_{0,\uparrow} | GS\rangle = \langle \tilde{f} | c^\dagger_{0,\uparrow} e^{i\alpha_0 \hat{n}_{\uparrow}} \hat{R} | GS\rangle e^{-i\alpha N_{\uparrow}}, \]

(25)

where \( \hat{R} = \prod_k e^{2i\alpha \hat{n}_{k,\downarrow}/L} \) [see Eq. (9)]. As in the transformed basis the wave functions are product of the spin up and down free fermion wave functions, \(|GS\rangle = |GS_{\uparrow}\rangle |GS_{\downarrow}\rangle\) and \(|\tilde{f}\rangle = |\tilde{f}_{\uparrow}\rangle |\tilde{f}_{\downarrow}\rangle\), the matrix element factorizes, and we get

\[ A_t(k, \omega) = L \sum_f |\langle \tilde{f}_\uparrow | c^\dagger_{0,\uparrow} | GS_{\uparrow}\rangle|^2 \times |\langle f_{\downarrow} | e^{i\alpha_0 \hat{n}_{\downarrow}} \hat{R} | GS_{\downarrow}\rangle|^2 \times \delta(\omega - E_{f,\uparrow} + E_{GS,\uparrow} - E_{f,\downarrow} + E_{GS,\downarrow}) \delta_{k, P_f,\downarrow - P_{GS,\downarrow} + P_{f,\uparrow} - P_{GS,\uparrow}}. \]

In the equation above the \( c^\dagger_{0,\uparrow} \) creates a fermion with energy \( \varepsilon_\uparrow(k') \) and momentum \( k' \not\in FS_\uparrow \), in which case the matrix element is \(|\langle \tilde{f}_\uparrow | c^\dagger_{0,\uparrow} | GS_{\uparrow}\rangle|^2 = 1/L\). This allows us to write the spectral function as a convolution

\[ A_t(k, \omega) = \frac{1}{L} \sum_{k' \not\in FS_\uparrow} A'_t(k - k', \omega - \varepsilon_\uparrow(k')), \]

(26)

with

\[ A'_t(\omega, k) = L \sum_{f_{\downarrow}} |\langle \tilde{f}_{\downarrow} | e^{i\alpha_0 \hat{n}_{\downarrow}} \hat{R} | GS_{\downarrow}\rangle|^2 \delta(\omega - E_{f,\downarrow} + E_{GS,\downarrow}) \delta_{k, P_{f,\downarrow} - P_{GS,\downarrow}}. \]

(27)

The interesting and nontrivial part of the calculation comes from the \( |\langle \tilde{f}_\downarrow | e^{i\alpha_0 \hat{n}_{\downarrow}} \hat{R} | GS_{\downarrow}\rangle|^2 \) matrix element. In the next and crucial step, we eliminate the \( e^{i\alpha_0 \hat{n}_{\downarrow}} \). This can be easily
accomplished after the observation that translating the operator $\hat{R}$ a similar factor appears: $T\hat{R}T^\dagger = e^{2ia(\hat{n}_{0,1}-\hat{n}_1)}\hat{R}$. So
\[
\langle \tilde{f}_i | e^{2ia\hat{\eta}_{0,1}} \hat{R} | \overline{G_{S_1}} \rangle = e^{i(2\alpha n_i - P_{\hat{f}_{i,1}} + P_{\overline{G_{S_1}}})} \langle \tilde{f}_i | \hat{R} | \overline{G_{S_1}} \rangle.
\] (28)

Next, we note that $e^{ia\hat{\eta}_{0,1}} = (e^{i\alpha} + e^{2ii\alpha_{0,1}})/(1 + e^{i\alpha})$, and we end up with
\[
\langle \tilde{f}_i | e^{ia\hat{\eta}_{0,1}} \hat{R} | \overline{G_{S_1}} \rangle = \frac{e^{i\alpha} + e^{i(P_{\overline{G_{S_1}}} - P_{\hat{f}_{i,1}} + 2\alpha n_i)}}{1 + e^{i\alpha}} \langle \tilde{f}_i | \hat{R} | \overline{G_{S_1}} \rangle.
\] (29)

To evaluate $\langle \tilde{f}_i | \hat{R} | \overline{G_{S_1}} \rangle$, we replace $| \overline{G_{S_1}} \rangle = \prod_j c_{k_j,\uparrow}^\dagger |0\rangle$ and $| \tilde{f}_i \rangle = \prod_i c_{k_i,\uparrow}^\dagger |0\rangle$. Then we move $\hat{R}$ to the right across $c_{k_i,\uparrow}^\dagger$'s so that it acts to the vacuum, $\hat{R}|0\rangle = |0\rangle$. However, as $\hat{R}c_{k_i,\uparrow}^\dagger = c_{k_i+2\alpha/L,\downarrow}^\dagger \hat{R}$, the $k$ momenta are shifted by $2\alpha/L$ (this is equivalent to twisting the boundary conditions):
\[
\langle \tilde{f}_i | \hat{R} | \overline{G_{S_1}} \rangle = \langle 0 | \prod_{i=1}^{N_i} c_{k_i,\uparrow}^\dagger \hat{R} \prod_{j=1}^{N_i} c_{k_j,\downarrow}^\dagger |0\rangle = \langle 0 | \prod_{i=1}^{N_i} c_{k_i,\uparrow}^\dagger \prod_{j=1}^{N_i} c_{k_j,\downarrow}^\dagger |0\rangle.
\] (30)

Here we have to calculate overlap of free fermion wave functions with different phase shifts due to the removal of a $\uparrow$-spin fermion. This problem arises e.g. in the X-ray edge problem (Anderson’s orthogonality catastrophe), and the one-dimensional analog was discussed in Ref. 8. For the reader’s convenience, we repeat here the main points. The anticommutation relation between the operators with different phase shifts reads
\[
A_{ij} = \left\{ c_{k_i,\uparrow}^\dagger, c_{k_j,\downarrow}^\dagger \right\} = e^{\pm i\alpha} \frac{\sin \alpha}{\sin \left( \frac{k_i - k_j}{2} + \frac{\alpha}{\ell} \right)}.
\] (31)

The overlap of the wave functions can be further calculated as $|\langle \tilde{f}_i | \hat{R} | \overline{G_{S_1}} \rangle|^2 = |\det A_{ij}|^2$:
\[
|\langle \tilde{f}_i | \hat{R} | \overline{G_{S_1}} \rangle|^2 = \left| \begin{array}{cccc}
\{ c_{k_1+2\alpha/L,\downarrow}^\dagger \} & \cdots & \{ c_{k_1+2\alpha/L,\downarrow}^\dagger \}
\vdots & \ddots & \vdots
\{ c_{k_{N_i},\uparrow}^\dagger \} & \cdots & \{ c_{k_{N_i},\uparrow}^\dagger \}
\end{array} \right|^2
= \frac{\sin^{2N_i} \alpha}{L^{2N_i}} \left| \begin{array}{ccc}
\frac{1}{\sin \left( \frac{k_1 - k_1'}{2} + \frac{\alpha}{\ell} \right)} & \cdots & \frac{1}{\sin \left( \frac{k_1 - k_{N_i}'}{2} + \frac{\alpha}{\ell} \right)}
\vdots & \ddots & \vdots
\frac{1}{\sin \left( \frac{k_{N_i} - k_1'}{2} + \frac{\alpha}{\ell} \right)} & \cdots & \frac{1}{\sin \left( \frac{k_{N_i} - k_{N_i}'}{2} + \frac{\alpha}{\ell} \right)}
\end{array} \right|^2.
\]

This determinant is actually a Cauchy determinant and can be expressed as a product, so we end up with
\[
|\langle \tilde{f}_i | \hat{R} | \overline{G_{S_1}} \rangle|^2 = \frac{\sin^{2N_i} \alpha \prod_{j>i} \sin^2 \frac{k_1 - k_j}{2} \prod_{j>i} \sin^2 \frac{k_j' - k_1'}{2}}{\prod_{j>i} \sin^2 \left( \frac{k_j' - k_j}{2} + \frac{\alpha}{\ell} \right)}.
\] (32)
For the special $\alpha = 0$ [where $A'_\uparrow(\omega, k) = L\delta(\omega)\delta_{k,0}$] and $\alpha = \pi$ cases, taking the suitable limits, we recover the the results of Eqs. (17) and (24), respectively. In the $\alpha = \pi$ case the phase shift equals $2\pi/L$, which is exactly the spacing between two adjacent $k$ value. thus the orthogonality catastrophe is absent.

Following the same approach, for the photoemission part we get

$$B'_\uparrow(k, \omega) = \frac{1}{L} \sum_{k' \in \text{FS}_\uparrow} B'_\uparrow(\omega - \varepsilon_\uparrow(k'), k - k'),$$

with

$$B'_\uparrow(\omega, k) = L \sum_f |\langle \tilde{f}_\downarrow | e^{-i\alpha\hat{n}_0.\hat{R}} | \tilde{G}_{\downarrow} \rangle|^2 \delta(\omega - E_{\text{GS},\downarrow} + E_f) \delta_{k,P_{\text{GS},\downarrow},-P_{\downarrow}}.$$  

The product in Eq. (32) can be evaluated numerically and spectral functions for relatively large system can be obtained. The numerical result is presented in Fig. 2 for some large size systems. Starting from $\alpha = 0$, we observe that there is an overall shift in momentum proportional to $-2\alpha n_\downarrow$ (which we compensated for in the figure), and that apart of the main contribution, which follows the cosine-like dispersion, additional continuum-like features appear. Finally, for even larger values of $\alpha$ another cosine-like feature appears with a considerable weight.

Alternatively, for the low energy part further analytical considerations can be applied. Starting from Eq. (32), the weights of the peaks can be expressed via $\Gamma$ functions in the $L \to \infty$ limit, leading to the power-law behavior of the Luttinger liquid spectral function, and the exponents can be associated with the phase shift. We find singularities where the momenta of the final state are closely packed. These happen at

$$k^{(\nu)}_\uparrow = \nu\pi n_\uparrow - 2\alpha n_\downarrow,$$

with $\nu$ an odd integer. The most important ones for small $\alpha$ are with $\nu = \pm 1$, which coincides with the Fermi momenta $k^{\pm}_{F,\uparrow}$. As we can follow in Fig. 3, increasing $\alpha$ we get weight for the tower at $k^{(3)}_\uparrow$, which eventually becomes symmetric with $k^{(1)}_\uparrow$ for $\alpha = \pi$, while the weight of the tower at $k^{(-1)}_\uparrow$ disappears at the same time. The behavior of the primed spectral functions in Eq. (26) has a simple behavior near $k = 0$:

$$A'_\uparrow(k, \omega) \propto [(\omega - \varepsilon_F)^2 - u^2 k^2]^{(\alpha/\pi)^2 - 1},$$

while near $k = 2\pi n_\downarrow$:

$$A'_\uparrow(k, \omega) \propto [(\omega - \varepsilon_F)^2 - u^2 (k - 2\pi n_\downarrow)^2]^{(\alpha/\pi-1)^2 - 1}.$$  

This leads to the power law behavior of the $A'_\uparrow(k, \omega)$ as presented in Eq. (14). The values of the exponents are tabulated for some selected $\alpha$ in Table. [1].

The weight transfer can be quantified by observing the sum rules. While the zeroth momentum is constant,
\[
\begin{align*}
\int_{-\infty}^{\varepsilon_F} B_\uparrow(\omega) d\omega &= n_\uparrow \\
\int_{\varepsilon_F}^{+\infty} A_\uparrow(\omega) d\omega &= 1 - n_\uparrow,
\end{align*}
\] (38)

the first already shows the large weight transferred to energies far from the Fermi energy:

\[
\begin{align*}
\int_{-\infty}^{\varepsilon_F} \omega B_\uparrow(\omega) d\omega &= \sum_i \langle \text{GS} | c_{i,\uparrow}^{\dagger} [H, c_{i,\uparrow}] | \text{GS} \rangle \\
&= -\frac{2t}{\pi} \sin(\pi n_\uparrow) - \frac{4t}{\pi} n_\uparrow \sin(\pi n_\downarrow)(1 - \cos \alpha), \\
\int_{\varepsilon_F}^{+\infty} \omega A_\uparrow(\omega) d\omega &= \sum_i \langle \text{GS} | c_{i,\uparrow}^{\dagger} [H, c_{i,\uparrow}] | \text{GS} \rangle \\
&= \frac{2t}{\pi} \sin(\pi n_\uparrow) + \frac{4t}{\pi} (1 - n_\uparrow) \sin(\pi n_\downarrow)(1 - \cos \alpha).
\end{align*}
\] (39)

We can see that the weight transfer to higher energies is the largest for \( \alpha = \pi \) and at half-filling.

IV. CONCLUSIONS

We have presented the exact one electron Green’s function for a model fermi system in 1D with a non Fermi liquid behavior for essentially any value of the interaction strength. A few exact analytical calculations of the spectral function for model systems, such as the 1/r^2 exchange t-J model, with a projection to single occupancy are available in literature.\(^8\)\(^9\)\(^10\)\n
The Green’s function for this system obtained here does require some numerics, and is not totally analytical. However unlike the situation in projected models, such as the t-J model, it satisfies the sum rules familiar from text books for weakly interacting Fermi liquids (e.g. the complete electron sum rule with large \( \omega \) behavior of \( G \) as \( 1/\omega \)). This feature makes the present model particularly interesting in the context of the programme of reconstruction of the spectral function from its moments (e.g. see Ref. \(^11\)).

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FIG. 1. The local spectral functions $B(\omega)$ (darker) and $A(\omega)$ (lighter shading) for $\alpha = \pi$. The filling increases from $n = 0$ (top curve) to $n = 1$ (bottom plot) in increments of $1/3$. 
FIG. 2. The evolution of the $\omega$ and $k$ dependent spectral function as a function of $\alpha$ for $n = 1/2$ (left) and $n = 1$ (right plots). The shading is proportional to $A(k, \omega)$ and $B(k, \omega)$, the dashed line denotes the Fermi energy. The shift of the Fermi momenta [Eq. (35)] is compensated for by introducing $q = k + \alpha n$ in the plot. We omitted the trivial $\alpha = 0$ case.
FIG. 3. The spectral function for $\alpha = 3\pi/4$ and $n = 1$. Here the Fermi energy is at $\omega = 0$.

FIG. 4. The local spectral functions $B(\omega)$ (darker) and $A(\omega)$ (lighter shading) for $n = 2/3$. The $\alpha$ changes from 0 (noninteracting case, top curve) to $\pi$ (bottom plot) in increments of $\pi/4$. To minimize finite size effects, the curves show the average of $L = 303, 279, 255, 231, 207$ and 183.
FIG. 5. To illustrate the weight transfer for small $\alpha$, we compare the local spectral function for $\alpha = \pi/4$ (solid line) to the $\alpha = 0$ case (dashed). The $\alpha = \pi/4$ case behaves as $A(\omega) \sim |\omega - \varepsilon_F|^{1/8}$ near the Fermi energy.
| $\alpha$ | 0 | $\pi/4$ | $\pi/2$ | $3\pi/4$ | $\pi$ |
|---------|---|--------|--------|--------|-----|
| $2(\alpha/\pi)^2$ | 0 | $1/8$ | $1/2$ | $9/8$ | 2 |
| $2(\alpha/\pi - 1)^2$ | 2 | $9/8$ | $1/2$ | $1/8$ | 0 |

TABLE I. The exponents in the local spectral function [Eq. (15)].