Mapping Markov: On effective master equations for two accelerated qubits

Greg Kaplanek\textsuperscript{1,2,3,*} and Erickson Tjoa\textsuperscript{4,5,†}

\textsuperscript{1}Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, L8S 4M1, Canada
\textsuperscript{2}Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada
\textsuperscript{3}Theoretical Physics, Blackett Laboratory, Imperial College, London, SW7 2AZ, UK
\textsuperscript{4}Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada
\textsuperscript{5}Institute for Quantum Computing, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada

We revisit the problem involving two constantly accelerating Unruh-DeWitt detectors using Open Effective Field Theory methods. We study the time evolution of the joint detector state using a Markovian approximation which differs from the standard one taken in the literature. We show that this Markovian limit already implies complete positivity of the dynamical evolution map without the rotating wave approximation (RWA), albeit in a narrower region of parameter space than normally considered. This shows that the standard microscopic derivation of the Gorini-Kossakowski-Sudarshan-Lindblad master equation (which usually invokes the RWA) is at best valid over a much smaller parameter space than what the Markovian regime allows for. We provide explicit validity bounds for when the Markovian approximation holds, which show that two well-known special cases studied in the literature, namely the “stacked trajectory” limit (when detector trajectories are taken to be on top of one another) and large gap-to-acceleration ratio violate the validity of Markovian approximation. We also show that Markovian dynamics without RWA can lead to rather different qualitative predictions for entanglement dynamics compared to RWA-based solutions, thus emphasizing further the need to track the regime of validity of all approximations.

I. MOTIVATION

It is by now well-known that vacuum states in quantum field theory (QFT) contain correlations between disjoint spacetime regions \cite{48,49}. In particular, vacuum entanglement is responsible for various physical phenomena such as the Unruh and Hawking effects. However, due to the ultraviolet properties of field-theoretic entanglement and the lack of direct measurement theory in QFT, one often resorts to the use of an external probe as a way to study the entanglement structure of the quantum field. In relativistic quantum information, the paradigmatic example is the Unruh-DeWitt (UDW) detector model \cite{1,2}, where a two-level quantum system (qubit) is coupled locally to a quantum scalar field.

From the perspective of relativistic quantum information, the study of entanglement dynamics between two UDW detectors typically falls into two classes: (i) via the entanglement harvesting protocol (see, e.g., \cite{45,46}), or (ii) through open quantum systems methods (see, e.g., \cite{28,32}). Although the objectives may vary, at the technical level the main difference between these two approaches is the interaction timescale. On the one hand, the entanglement harvesting protocol is by construction restricted to relatively short timescales, since one wants to extract entanglement from the field (and not through signalling between the two detectors \cite{32}). On the other hand, the open quantum systems framework often aims to obtain late-time dynamics, which is useful when one wishes to understand long-time processes such as thermalization. For example, open system methods allow one to directly determine if (and when) a single detector approaches a Gibbs state (see, e.g., \cite{33,41}), instead of stopping at the detailed balance condition or Planckian transition rate \cite{41} (see, e.g., \cite{45,46}).

In this work we revisit the open system framework between two UDW detectors undergoing uniform accelerations. Our motivation is based on two important considerations:

1. Making late-time predictions in perturbation theory reliably is notoriously difficult (see, e.g., \cite{17,50}). The issue is that the strength of the detector-field coupling imposes a natural timescale for which the perturbation series at any given order is valid. In essence, perturbative expansions of quantities like $e^{-igH\text{int}t} \approx 1 - igH\text{int}t + \ldots$ generically become suspect at late times when $gH\text{int}t$ becomes too large. This is also true for detectors that are switched on and off adiabatically and smoothly (“carefully” \cite{51,53}). In particular, one needs to have control over how long is “long times” when studying weakly-coupled systems — one virtue of the open quantum system approach is that it provides a way in which to resum late-time breakdowns of perturbation theory so as to more reliably gain access to late-time behaviour.

2. The use of semigroup master equations, which captures the reduced open system dynamics in the presence of a large environment, is often done without careful or explicit analysis of the validity of the

\textsuperscript{1} These are necessary but not sufficient conditions for thermalization as they do not capture what happens to density matrix coherence.
approximations that go into it. One of the most common examples is the invocation of the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) master equation, among many others (e.g. [28][42]).

In more detail, the usual approach to a microscopic derivation of a GKSL master equation for the reduced density matrix $\rho_{\text{sys}}(\tau)$ of the detector (“system”) at time $\tau$ typically involves three distinct approximations:

1. First, one perturbs the underlying Liouville-von Neumann equation for the full density matrix using the Born approximation. This is justified when the environment is large compared to the system. However, the resulting master equation for $\rho_{\text{sys}}(\tau)$ is intractable because it depends on its entire history of evolution (i.e., the evolution has memory).

2. This is where the second approximation — the Markovian approximation — is employed, by working in a regime of parameter space where the evolution is “memoryless” (meaning here that the evolution equation for $\rho_{\text{sys}}(\tau)$ is time-local). At this point, the resulting evolution equation is infamous of the “Redfield-type”: the differential equation for $\rho_{\text{sys}}(\tau)$ induces a dynamical evolution map $\Phi_{\tau} : \rho_{\text{sys}}(0) \mapsto \rho_{\text{sys}}(\tau)$ that is believed to be not completely positive (CP) [58][59].

3. The third approximation, known as the secular approximation or rotating wave approximation (RWA), is then used in order to make $\Phi_{\tau}$, a completely positive and trace-preserving (CPTP) map, i.e., a quantum channel. The inspiration for this approximation is from quantum optics in which rapidly oscillating terms in the master equation can be neglected when the system is near resonance with an oscillating environment (like a laser tuned to a specific frequency), and formally described by Davies in [60][61].

Notably, once the three approximations are taken, the resulting master equation can be treated like a black box and it is often used liberally, at the risk of not tracking the validity of its usage.

In this work we argue that the lack of CP-property mentioned above in Step (2) and (3) arises because the Markovian limit is not carefully taken. Our point of view is that Step (2) is valid in a specific region of parameter space where the bath correlation time (in this work, the reciprocal of the Unruh temperature $\sim 1/\alpha$) is the shortest timescale in the problem — including timescales associated with the system operators (such as $\sim 1/\Omega$, where $\Omega$ are the qubit energy gaps). As we show here, carefully performed Markovian limits amount to approximating both the system state $\rho_{\text{sys}}(\tau)$ in the interaction picture and the system observables (the monopole operator of the qubit detector) as memoryless.

When this is done, the so-called Born-Markov approximations (Steps (1) and (2) in the above) produce an evolution map that is CP-preserved according to the GKSL theorem [62][63]. This implies that the RWA is not necessary (see also [64], and therefore, the standard approach in employing the RWA-based GKSL master equation (see, e.g., [58], for standard reference) is at best valid on much smaller parameter space than what the Born-Markov approximations allow for. We provide explicit bounds (“validity relations”) for which the Born-Markov approximation is valid. The derived bounds show that two well-known special cases studied in the literature, (i) the “stacked trajectory” limit (when detector trajectories are taken to be equal with proper separation $L = 0$, see e.g., [28][32][33][35][39]), and (ii) large gap-to-acceleration ratio $\Omega/\alpha \gtrsim 1$ (see, e.g., [30][33][35][47][40][42]), and (iii) inertial qubits interacting with vacuum state $|0\rangle |0\rangle [31][40][41]$, all of which violate the validity of the Markovian approximation.

In this work we also compare the late-time state and entanglement dynamics between the two detectors between the RWA-based evolution vs the one without RWA. In both cases the fixed point of the evolution is a maximally mixed state (understood as a high-temperature Gibbs state), so one cannot use the late-time state to check the validity of RWA. It is also worth noticing that early-time dynamics around $g^2\alpha \tau \ll 1$ calculated using Markovian master equation is not reliable, since at early times non-Markovian effects are important. The takeaway is that just because the Markovian solution exists for early and late times does not mean the full dynamics agree with Markovian one at all times. Also importantly, within the Born-Markov approximations one cannot infer the Unruh temperature $T_U = \alpha/(2\pi)$ from its late-time density matrix: the reason is because we are in the high-temperature limit and the asymptotic state is effectively the zeroth-order expansion in $\Omega/\alpha \propto \Omega/T_U$ of the Gibbs state. The information about the Unruh temperature must be obtained elsewhere, e.g., by computing the decay rate or Fisher information (for recent examples of the latter using the RWA see [65][69]).

This paper is organized as follows. In §III we discuss the UDW setup for two uniformly accelerated detectors. From there we develop the Nakajima-Zwanzig master equation for the joint detector state in §III and then further take its Markovian limit in §IV obtaining the late-time asymptote for the state and compute the

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2 That is, one can end up with predictions of negative probabilities with the computed reduced density matrix. This is sometimes known as “slippage of initial conditions” [54][57].

3 This is already ignoring the possibility of other problems appearing in RWA: for example, some form of RWA known as pre-trace RWA can lead to causality violation [65] or even lack of Markovian limit [66].

4 Sometimes compared to the two-atom Dicke model [67].
timescales for the approach to this fixed point. In [7] we calculate the validity relations that constrain the parameters in order for the Born-Markov approximations to be valid. Finally, in [7] we compare our results without the RWA with those that are derived using the RWA. We use the mostly-plus signature for the metric and set $c = \hbar = 1$.

II. QUBITS IN SPACE

In this section we formulate the detector-field interaction based on the Unruh-DeWitt model, sending two qubits (two-level systems) along parallel accelerated trajectories while interacting with a free massless scalar field $\phi$ in (3+1)-dimensional Minkowski space. We work in perturbation theory to order $O(g^2)$ where the coupling strength of the detector-field interaction is $g \ll 1$.

Consider two observers Alice and Bob, each carrying a pointlike two-level Unruh-DeWitt (UDW) detector, put along parallel accelerated trajectories in flat spacetime with worldlines (in Minkowski coordinates)

\begin{align}
\gamma_a(\tau) &= (t(\tau), x(\tau), 0, 0), \\
\gamma_b(\tau) &= (t(\tau), x(\tau), L, 0),
\end{align}

with acceleration $a > 0$, where $t(\tau) = \frac{1}{a} \sinh(\alpha \tau)$, $x(\tau) = \frac{1}{a} \cosh(\alpha \tau)$ and $L > 0$ is the proper separation along the $y$-direction. As they are parallel in the transverse direction, the two observers’ worldlines can be parametrized by the same proper time and we set the initial proper times along each trajectory $\tau = 0$ to align with $t = 0$.

The massless scalar field $\phi$ in Minkowski spacetime satisfies the Klein-Gordon equation $\partial_\mu \partial^\mu \phi = 0$ with mode decomposition in the inertial quantization frame $(t, x)$ given by (in the interaction picture)

\begin{equation}
\phi(t, x) = \int \frac{d^3 k}{\sqrt{2(2\pi)^3 \omega_k}} [a_k e^{-i \omega_k t + ik \cdot x} + \text{h.c.}],
\end{equation}

where $\omega_k = |k|$ is the relativistic dispersion for massless fields. The canonical commutation relation is given by $[a_k, a_k^\dagger] = \delta^3(k - k') \mathbb{1}$ (with $\mathbb{1}$ the identity on the field Hilbert space) and all other commutators vanish. The free Hamiltonian for the scalar field reads

\begin{equation}
\mathcal{H}_\phi = \frac{1}{2} \int_{\Sigma_t} d^3 \mathbf{x} \left[ \partial_\tau^2 (t, \mathbf{x}) + \partial_\mathbf{x} \phi(t, \mathbf{x}) \partial_\mathbf{x} \phi(t, \mathbf{x}) \right],
\end{equation}

where $\tau = \partial_\mathbf{x} \phi$ and $\Sigma_t$ is a constant-$t$ Cauchy surface in Minkowski spacetime. This free Hamiltonian generates time translations with respect to Minkowski time $t$.

For the $4 \times 4$ matrices acting on the joint qubit subspace of the Hilbert space we use the notation

\begin{equation}
\sigma_\alpha^a := \sigma_\alpha \otimes \mathbb{1} \quad \text{and} \quad \sigma_\alpha^b := \mathbb{1} \otimes \sigma_\alpha
\end{equation}

where $\mathbb{1}$ is the $2 \times 2$ identity matrix, $\sigma_\alpha$ are the standard Pauli matrices for $\alpha \in \{1, 2, 3, \pm\}$, and in particular $\sigma_\pm = \frac{1}{2} (\sigma_1 \pm i \sigma_2)$.

For simplicity, we consider both detectors to be identical with energy gap $\Omega$, so that the free Hamiltonian for Alice and Bob’s detectors reads

\begin{equation}
\mathcal{H} := \mathcal{H}_\alpha + \mathcal{H}_b \quad \text{with} \quad \mathcal{H}_j := \frac{1}{2} \sigma_3^j .
\end{equation}

The ground and excited states of each detector are denoted by $|\downarrow\rangle$ and $|\uparrow\rangle$ respectively, with the action of the $su(2)$ ladder operators given by $\sigma_+ |\downarrow\rangle = |\uparrow\rangle$ and $\sigma_- |\uparrow\rangle = |\downarrow\rangle$. The free Hamiltonian for the detector-field system reads

\begin{equation}
\mathcal{H}_0 = \frac{d \tau}{dt} (\mathcal{H} \otimes \mathbb{1} + \mathbb{1} \otimes \mathcal{H}_\phi) .
\end{equation}

In the interaction picture, two detectors interact with the field via the interaction Hamiltonian

\begin{equation}
\mathcal{H}_\text{int}^i(t) = g \frac{d \tau}{dt} \sum_{j = a, b} \mu_j^i (\tau(t)) \otimes \phi [y_j(\tau(t))] ,
\end{equation}

with the superscript $i$ denoting the interaction picture. The monopole operator of each detector appearing in $\mathcal{H}_\text{int}$ is given by

\begin{equation}
\mu_j^i (\tau) := e^{-i \Omega \tau} \mu_j e^{i \Omega \tau} = \sigma_3^j e^{i \Omega \tau} + \sigma_3^j e^{-i \Omega \tau} ,
\end{equation}

where $\mu_j := \sigma_3^j + \sigma_3^j$ is the monopole operator in the Schrödinger picture. For simplicity we have chosen each qubit to interact with the field with the same coupling strength $g \ll 1$.

With this setup, the interaction picture density matrix $\rho^i(t)$ of the detector-field system evolves according to the Liouville-von Neumann equation

\begin{equation}
\frac{d \rho^i(t)}{dt} = -i [\mathcal{H}_\text{int}^i(t), \rho^i(t)] .
\end{equation}

The aim of this work is to determine the evolution of the joint qubit state i.e. the reduced density matrix obtained by tracing over the field’s degrees of freedom where

\begin{equation}
\rho^i_{\text{det}}(t) := \text{tr}_\phi [\rho^i(t)] .
\end{equation}

In this work we assume, as is standard in the literature, that the initial state of the detector-field system at $t = 0$ is uncorrelated such that

\begin{equation}
\rho^i(0) = \rho^i_{\text{det}}(0) \otimes |0\rangle |0\rangle ,
\end{equation}

where $|0\rangle$ is the Minkowski vacuum defined by $a_k |0\rangle = 0$ for all $k$, and take the initial joint detector state $\rho^i_{\text{det}}(0)$ to be arbitrary $\mathbb{1}$.

For later use, the joint qubit state in the Schrödinger picture, denoted by $\rho^i(t)$, is related to the interaction picture version by

\begin{equation}
\rho^i_{\text{det}}(\tau) = e^{-i \tau} \rho^i(t) e^{i \tau} .
\end{equation}

An often unstated fact is that pointlike UDW detector model is incompatible with arbitrary initial state of the detector, in that it will lead to ultraviolet (UV) divergences. One has to impose (a) UV cutoff or (b) spatial smearing to regulate the UV divergence. In open quantum systems problems, a hard UV cutoff is usually imposed instead of giving a finite size to the detector due to its mathematical tractability.
III. LATE TIMES AND MASTER EQUATIONS

In this section we begin with the Liouville-von Neumann equation (9) and connect its standard perturbative expansion to master equations which are better suited for studying late times. Using the Nakajima-Zwanzig master equation (equivalent to the Born approximate expansion to master equations which are better suited for studying late times. Using the Nakajima-Zwanzig master equation (equivalent to the Born approximate expansion to master equations which are better suited for studying late times. Using the Nakajima-Zwanzig master equation (equivalent to the Born approximate expansion to master equations which are better suited for studying late times. Using the Nakajima-Zwanzig

A. From perturbation theory to master equations

We begin by noting an equivalent formulation of the Liouville-von Neumann equation (9),

\[ \rho'(t) = \rho'(0) - i g \int_0^t dt' [H_{\text{int}}(t') \rho'(0)] \]

\[ -g^2 \int_0^t dt' \int_0^{t'} dt'' [H_{\text{int}}(t'), [H_{\text{int}}(t''), \rho'(t'')]] + \mathcal{O}(g^3) , \]

which lends itself useful to perturbative calculations. Invoking the standard perturbative (Dyson) series expansion on (13) yields to second-order in the qubit-field coupling

\[ \rho'(t) = \rho'(0) - i g \int_0^t dt' [H_{\text{int}}(t') \rho'(0)] \]

\[ -g^2 \int_0^t dt' \int_0^{t'} dt'' [H_{\text{int}}(t'), [H_{\text{int}}(t''), \rho'(t'')]] + \mathcal{O}(g^3) . \]

After partial trace over the field, the second term vanishes due to the vanishing of the one-point function \( \langle 0 | \phi(t, x) | 0 \rangle = 0 \), and the joint state of the detectors up to \( \mathcal{O}(g^2) \) reads

\[ \rho'_{\text{AB}}(t) \approx \rho_{\text{AB}}(0) \]

\[ -g^2 \int_0^t dt' \int_0^{t'} dt'' \text{tr}_{\phi} [H_{\text{int}}(t'), [H_{\text{int}}(t''), \rho'(0)]] . \]

In many contexts such as entanglement harvesting protocol [4,17], this is the order in which the final state of the detectors is often worked out.

In this context, the standard perturbative approach applies in a regime where \( 1 \ll \sigma \tau \ll 1/g^2 \), and begins to breakdown when \( g^2 \sigma \tau \sim \mathcal{O}(1) \). The utility of the open system approach is that there exists a Markovian regime truncated at the same order as [14] that allows us to study the same problem in the late-time regime \( g^2 \sigma \tau \sim \mathcal{O}(1) \) by “resummation” of terms to all orders in \( g^2 \sigma \tau \). The way this works is to note that perturbative series like [14] gives time evolution of the system from \( \tau_0 \) to \( \tau \) so long as \( g^2 \sigma (\tau - \tau_0) \ll 1 \) holds. Within this window, one can differentiate the perturbative expression to yield a differential equation for \( \rho(\tau) \). If this differential equation is time-local in \( \rho(\tau) \) i.e. not depending on its entire integrated history of evolution from \( \tau_0 \) to \( \tau \), then the same differential equation applies in any other perturbatively small window from any \( \tau_j \) to \( \tau \), so long as \( g^2 \sigma (\tau - \tau_j) \ll 1 \). The master equation then applies over much larger timescales, since it can be trusted over the union of such perturbatively small but overlapping time domains allowing for integration out to late times where \( g^2 \sigma \tau \sim \mathcal{O}(1) \) (but \( g^4 \sigma \tau \ll 1 \).

Indeed the time-local nature of Markovian master equations is the essential property that we need to resum the late-time breakdown of [14] to all orders in \( g^2 \sigma \tau \) while neglecting \( \mathcal{O}(g^4 \sigma \tau) \) effects. This resummation argument is, in essence, a renormalization group argument familiar from particle physics (e.g. see [70] as well as [47, 48, 71] for a late-time analogy using particle decays).

Our task is now clear — what remains is to:

1. turn Eq. (14) into a time-local Markovian equation that is valid up to late times as specified above using suitable approximations;
2. find an explicit, late-time resummed solution to the Markovian regime;
3. find the domain of validity of the approximations that go into (1) and (2), and show that the resulting equation defines a completely-positive evolution.

Note that Step (3) is often neglected, which may lead to unphysical results. We discuss this further in §V.

To gain access to late times, we apply the Born approximation to Eq. (13),

\[ \rho'(t) \approx \rho_{\text{AB}}(t) \otimes |0\rangle \langle 0| \]

which neglects correlations between the joint qubit state and the field. The resulting state at \( \mathcal{O}(g^2) \) reads

\[ \rho'_{\text{AB}}(t) \approx \rho_{\text{AB}}(0) \]

\[ -g^2 \int_0^t dt' \int_0^{t'} dt'' \text{tr}_{\phi} [H_{\text{int}}(t'), [H_{\text{int}}(t''), \rho_{\text{AB}}(t'') \otimes |0\rangle \langle 0|]] . \]

By taking the derivative with respect to \( t \), we obtain the integro-differential equation

\[ \frac{d \rho'_{\text{AB}}}{dt} \approx -g^2 \int_0^t dt' \text{tr}_{\phi} [H_{\text{int}}(t'), [H_{\text{int}}(t'), \rho'_{\text{AB}}(t') \otimes |0\rangle \langle 0|]] . \]

\[ ^{6} \text{In this case, temporal smooth switching functions multiplying the interaction Hamiltonian are sometimes used to turn off the interaction before perturbative breakdown occurs.}\]

\[ ^{7} \text{The Born approximation is a weaker requirement compared to the standard Dyson series truncation [15] because we can have } \rho_{\text{AB}}(t'') \neq \rho_{\text{AB}}(0) \text{ for long interactions without significantly changing the bath state.}\]

\[ ^{8} \text{These correlations can be shown to be } \mathcal{O}(g^4) \text{ [72] so it only contributes as } \mathcal{O}(g^4) \text{ effect. For any significant backreaction onto the the field state, the Born approximation is not valid.} \]
This equation is useful in that it only depends on \( \rho'_{ab} \), unlike \([14]\). Note however that Eq. \([18]\) is not time-local because it depends on the entire history (“memory”) of its evolution.

It is worth emphasizing that the equation of motion of the form Eq. \([18]\) is precisely the \( \mathcal{O}(g^2) \)-truncation of the Nakajima-Zwanzig equation \([73,74]\) (see e.g.\([58,69]\) for introductory material). The basic idea behind the Nakajima-Zwanzig equation is that we can define a projection \( \mathcal{P} \) such that

\[
\mathcal{P}[\rho'(t)] := \rho'_{ab}(t) \otimes |0\rangle\langle 0| .
\]

This splits the total state into “relevant” part projected by \( \mathcal{P} \) (the system) and “irrelevant” part projected by its complement \( 1 - \mathcal{P} \) (the environment). Since the Liouville-von Neumann equation \([9]\) is linear one can use it to derive an exact equation of motion for \( \mathcal{P}[\rho'(t)] \) alone which is of the form

\[
\frac{\partial \mathcal{P}[\rho'(t)]}{\partial t} = \int_0^t dt' \mathcal{K}(t, t') \mathcal{P}[\rho'(t')] ,
\]

where \( \mathcal{K}(t, t') \) is a “memory kernel” that measures information backflow from the detector to the field. The nice feature of the Nakajima-Zwanzig equation is that what we called the “Born approximation” in Eq. \([18]\) is naturally built-in as the leading-order expansion of the memory kernel \( \mathcal{K}(t, t') \), thus it has the natural interpretation that indeed we are neglecting memory effect due to backreaction to the field. The Nakajima-Zwanzig formalism provides a very natural organizing principle for perturbative expansion in a way that makes clear the information flow between the “relevant part” (the system) and “irrelevant part” (the environment) of the total system. Following \([69]\), we refer to Eq. \([18]\) as the Nakajima-Zwanzig master equation at second order (NZ-ME2).

**B. Nakajima-Zwanzig equation for two accelerated qubits**

After tracing out the field degrees of freedom, NZ-ME2 \([18]\) results in

\[
\frac{d\rho_{ab}'(t)}{dt} = g^2 \sum_{j,k \in \{A,B\}} \int_0^t dt' \frac{d\tau(t)}{dt} \frac{d\tau(t')}{dt'} \times \left( W_{jk}(\tau(t), \tau(t')) [\mu_k'(\tau(t'))\rho_{ab}'(t'), \mu_j'(\tau(t))] + \text{H.c.} \right) ,
\]

where H.c. denotes Hermitian conjugate and the sum runs over the labels \( j, k \in \{A,B\} \). Here \( t, t' \) are Minkowski time variables, \( \tau \) is the common proper time for the detectors. The pullback of the vacuum Wightman two-point function along the trajectories \( y_j, y_k \) denoted \( W_{jk} \), is given by

\[
W_{jk}(\tau, \tau') := \langle 0|\phi(y_j(\tau))\phi(y_k(\tau'))|0\rangle .
\]

For the parallel accelerated trajectories \([21]\), \( W_{jk} \) simplifies greatly: we get the “self-correlations”

\[
W_{ss}(\Delta \tau) := W_{\Delta \lambda}(\tau, \tau') = W_{ab}(\tau, \tau')
\]

\[
= -\frac{a^2}{16\pi^2 \sinh^2 \left[ \frac{\Delta \tau}{2} \left( \Delta \tau - i\epsilon \right) \right]} ,
\]

and “cross-correlations”

\[
W_{xs}(\Delta \tau) := W_{ab}(\tau, \tau') = W_{\Delta \lambda}(\tau, \tau')
\]

\[
= -\frac{a^2}{16\pi^2 \sinh^2 \left[ \frac{\Delta \tau}{2} \left( \Delta \tau - i\epsilon \right) \right]} - \left( \frac{a^2}{4} \right)^2
\]

where \( \Delta \tau = \tau - \tau' \).

By performing a change of variable \( s = \Delta \tau \), the resulting NZ-ME2 can be re-expressed in terms of proper time \( \tau \) such that

\[
\frac{d\rho_{ab}'(\tau)}{d\tau} \simeq g^2 \sum_{j,k \in \{A,B\}} \int_0^\tau ds \times \left( W_{jk}(s) [\mu_j'(\tau-s)\rho_{ab}'(\tau-s), \mu_k'(\tau)] + \text{H.c.} \right) .
\]

Using the uncoupled basis \( \{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\} \), the integro-differential equation \([25]\) for \( \rho_{ab}'(\tau) \) has two decoupled components: that is, we can split the density matrix into

\[
\rho_{ab}'(\tau) = \rho_{ab,X}'(\tau) + \rho_{ab,O}'(\tau)
\]

where

\[
\rho_{ab,X}'(\tau) = \begin{bmatrix} \rho_{11}'(\tau) & 0 & 0 & \rho_{44}'(\tau) \\ 0 & \rho_{22}'(\tau) & \rho_{23}'(\tau) & 0 \\ 0 & \rho_{32}'(\tau) & \rho_{33}'(\tau) & 0 \\ \rho_{41}'(\tau) & 0 & 0 & \rho_{44}'(\tau) \end{bmatrix},
\]

\[
\rho_{ab,O}'(\tau) = \begin{bmatrix} 0 & 0 & \rho_{21}'(\tau) & 0 \\ 0 & \rho_{12}'(\tau) & \rho_{13}'(\tau) & 0 \\ 0 & \rho_{31}'(\tau) & 0 & \rho_{34}'(\tau) \\ 0 & \rho_{42}'(\tau) & \rho_{43}'(\tau) & 0 \end{bmatrix}.
\]

We call these decoupled pieces the X-block and O-block respectively (due to the positions of the nonzero matrix elements). Note that \( \rho_{ab,X}'(\tau) \) is known as an X-state and some of its properties have been investigated in the literature (see e.g. \([75]\) ). The components of Eq. \([27]\) are not all independent since we can use

\[
\rho_{44}'(\tau) = 1 - \rho_{11}'(\tau) - \rho_{22}'(\tau) - \rho_{33}'(\tau) ,
\]

\[
\rho_{nm}'(\tau) = \rho_{nm}^*(\tau) \quad \text{for} \quad n \neq m \in \{1,2,3,4\} .
\]

Therefore, for the X-block we have a system of seven coupled ordinary differential equations (ODE), while for O-block we have eight coupled ODEs with eight variables. The full explicit expressions for the ODEs for X-block and O-block are given in Appendix A and Appendix B respectively.

Before we solve these equations, let us remark on the choice of initial state \( \rho_{ab}(0) \). Interestingly, many existing studies involving two UDW detectors in open system framework restrict their attention to an X-state as
the initial state $\rho_{0}$ because the time evolution preserves the $X$-block \cite{76}. Here we see that this restriction is unnecessary since $X$-block completely decouples from $O$-block, so one can evolve the $O$-block independently anyway. We see in the next section (for nonzero energy gap $\Omega > 0$) that the $O$-block tends towards zero at late-times, so only the $X$-block survives in the long time limit.

IV. TWO-QUBIT ASYMMETRIC STATE IN THE MARKOVIAN ASYMMETRIC REGIME

As mentioned earlier, the main obstruction to solving Eq. (25) (or equivalently the ODEs given in Appendices A and B) and obtaining late-time results is that it is not time-local: the integrals on the RHS have memory over their entire history of the evolution integrating over functions of $\tau - s$. To gain access to late time, we must enter a Markovian regime where the dynamics is memoryless. As we will shall see, there are subtleties involved in taking Markovian limit of Eq. (25).

A. A different Markovian approximation

Let us start with a Markovian approximation of Eq. (25) that is different from the one usually taken in the literature. We see how this compares with the standard approach in §IVB.

The physical essence of the Markovian limit is the observation that the environment correlators $W_{s,s}(\tau)$ are sharply peaked about $s = 0$: this implies that there exists a regime in which the timescales associated with the system evolve much slower than the timescales set by the environment. This timescale is set by $1/a$ as can be seen from Eqs. (23) and (24) where the environment correlators fall off exponentially fast:

\begin{align*}
W_{s}(\tau) & \approx \frac{a^2}{2\pi^2}e^{-as} \quad as \gg 1, \\
W_{s}(\tau) & \approx \frac{a^2}{2\pi^2}e^{-as} \quad as \gg \max \{1, \frac{1}{2}\log(\frac{a}{2})\}. \quad (29b)
\end{align*}

This means that when the system evolves slower than the environment, the history/memory dependence in the RHS of (25) becomes negligible\footnote{It is sometimes colloquially stated that fast (Markovian) environment dynamics means that $W_{s,s}(\tau) \sim \delta(\tau)$ underneath the integral sign in (25) — from this point of view it also makes sense that one removes the history of integration as done in Eq. (31).} and we can perform Taylor series about $s = 0$ where

\begin{align*}
\mu_{j}^{+}(\tau - s)\rho_{ab}^{+}(\tau - s) & \approx \mu_{j}^{+}(\tau)\rho_{ab}^{+}(\tau) - s(\mu_{j}^{+}(\tau)\rho_{ab}^{+}(\tau) + \mu_{j}^{+}(\tau)\rho_{ab}^{+}(\tau)) + O(s^2), \\
\mu_{j}^{-}(\tau - s)\rho_{ab}^{-}(\tau - s) & \approx \mu_{j}^{-}(\tau)\rho_{ab}^{-}(\tau) - s(\mu_{j}^{-}(\tau)\rho_{ab}^{-}(\tau) + \mu_{j}^{-}(\tau)\rho_{ab}^{-}(\tau)) + O(s^2), \quad (30)
\end{align*}

and a similar Taylor series for the opposite ordering of operators $\rho_{ab}^{+}(\tau - s)\mu_{j}^{+}(\tau - s)$ in (25). Therefore, the leading order of the series expansion is memoryless and reads

\[ \frac{d\rho_{ab}}{d\tau} \approx g^2 \sum_{j,k} \int_{0}^{\tau} ds \left( W_{jk}(s) \mu_{j}^{+}(\tau)\mu_{k}^{+}(\tau) + H.c. \right), \]

and only the correlators $W_{s,s}$ depend on $s$.

Physically, the series expansion leading to (31) essentially makes $1/a$ the shortest timescale of the problem, i.e. the Markovian limit is where the environment dynamics are extremely rapid as compared to the system dynamics. In this regime, the environment erases the history of integration in Eq. (25), including the integration over the (system) monopole operators $\mu_{j}^{+}$. This is a very important point that we stress from time to time in this work: since we are in the interaction picture, memoryless Markovian limit requires that the memory is neglected from both the monopole operators and the states, otherwise the residual memory leads to problems as we see later on.

Another noteworthy point that is often neglected is that the Taylor approximation step from (25) to (31) provides a means of quantifying when the Markovian approximation applies: indeed, it is now clear that the Markovian limit begins to fail when the subleading derivative terms in (30) become too large. Bounding the next-to-leading-order terms in (20) to be small relative to the leading-order terms maps out the parameter space where the Markovian approximation applies. We explore such Markovian validity bounds in full detail in §V so as to determine the parameter ranges for which we can trust the Markovian evolution predicted by Eq. (31).

Time-dependent coefficients and late times

Finally, while Eq. (31) is already memoryless (or time-local), it turns out the upper limit of $\tau$ on the integrals prevents straightforward integration out to very late times for the problem at hand. To see why, note that Eq. (31) can be recast into a system of ordinary differential equations (ODEs) for a vector $u(\tau)$ whose components are built out of the entries of the density matrix $\rho_{ab}^{+}(\tau)$: schematically, it takes the general form

\[ \frac{du(\tau)}{d\tau} = A(\tau)u(\tau) + v(\tau), \quad (32) \]

where $v(\tau)$ is some possibly nonzero vector and $A(\tau)$ is a square matrix, both of whose entries contain the integral transforms of environment correlators $W_{s,s}(\tau)$ appearing in Eq. (31) (with upper limits $\tau$ on the integrals). Even for the case when $v(\tau) = 0$, the matrix ODE above cannot be solved in closed form unless $A(\tau)$ obeys very specific properties\footnote{Such as when $[A(s), A(s')] = 0$ for all $s, s' \in (0, \tau)$.}. Formal solutions to
generally involve time-ordered exponentials which in turn requires perturbative treatments that the late-time resummation was meant to avoid.

Another important point that is often missed is that although Eq. (32) can be organized into a Lindblad-like form (see Eq. 18), the resulting coefficients are time-dependent and therefore this master equation does not obey the assumptions of the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) theorems stated in [62, 77]. In particular, if one tries to put Eq. (32) into “Lindblad form”, the corresponding Kossakowski matrix of Lindblad coefficients can be apparently negative-definite especially at early times, which is usually taken to be a violation of the complete-positivity (CP) property for the evolution. However, Lindblad’s theorem only applies for master equations in Lindblad form with time-independent Kossakowski matrix (see [64] for more details). Consequently, one cannot make claims about the CP property of the evolution (or lack thereof) by invoking Lindblad’s theorem at this stage. In fact, since we cannot perform late-time resummation yet with time-dependent \( \Lambda(\tau) \), it is pointless to check the CP properties though the GKSL theorems at this juncture.

The upshot is that the practical calculation of late-time resummation requires more than just removing memory effect: we need to find a regime where the time-local equation (31) can be approximated as a matrix ODE with constant coefficients, which is exactly solvable in closed form. That is, we need to work in the regime where Eq. (32) reduces to

\[
\frac{d}{d\tau} u(\tau) = \Lambda u(\tau) + v, \tag{33}
\]

where \( \Lambda \) is now a constant matrix and \( v \) is also a constant vector. Indeed, this is the case in later sections when we consider late-time dynamics of the detectors.

To this end, given the falloff of the environment correlators given in (29a) and (29b), we assume that

\[ a \tau \gg \max \left\{ 1, \frac{1}{2} \log \left( \frac{a^2}{t^2} \right) \right\} \tag{34} \]

so that we can approximate \( \sim \infty \) on the upper limit of the integrals in our Markovian equation of motion (31) giving

\[
\frac{d\rho^{(\tau)}_{ab}}{d\tau} \simeq g^2 \sum_{j,k} \int_0^\infty ds \left( \mathcal{W}_{jk}(s)[\mu_j^*(\tau)\rho_{ab}(\tau), \mu_k^*(\tau)] + \text{H.c.} \right). \tag{35}
\]

This is the master equation whose late-time resummed solution is amenable to explicit computation.

We solve this equation in §IV.C but first we take a small detour to discuss why we use the stated Markovian approximation instead of the standard one usually taken in the literature.

In the majority of the literature, the “standard” Markovian approximation is usually taken as the approximation

\[
\rho^{(\tau)}_{ab}(\tau - s) \simeq \rho^{(\tau)}_{ab}(\tau) - s \dot{\rho}^{(\tau)}_{ab}(\tau) + \ldots \tag{36}
\]

as opposed to (30) taken above. We now argue that the only self-consistent way of taking the standard approximation (36) for the setup at hand is exactly the one considered in this work.

The usual logic for keeping only the leading-order term of the state \( \rho^{(\tau)}_{ab}(\tau - s) \simeq \rho^{(\tau)}_{ab}(\tau) - s \dot{\rho}^{(\tau)}_{ab}(\tau) + \ldots \) in any integro-differential master equations, such as those that arise from Nakajima-Zwanzig formalism, is that we are guaranteed that \( \dot{\rho}^{(\tau)}_{ab}(\tau) \sim \mathcal{O}(g^2) \) from the master equation we start from. Since these derivative terms in the approximation are suppressed by two extra powers of the coupling strength, this seems to imply that all the subleading terms in the Taylor series can be safely ignored. What this argument fails to account for is that the derivative terms can become dangerously large if energy scales associated with the system become too large (compared to the energy scales of the environment).

In order to demonstrate where the problem lies, it is most easily illustrated for the single detector case where \( \Omega/a \gtrsim 1 \) alone causes the Markovian approximation to break down (for two detectors there are more conditions, as we see in §V). By ignoring Bob’s detector entirely and only focusing on Alice’s detector, one gets the analog of the Nakajima-Zwanzig equation (25) for Alice’s reduced density matrix \( \rho^{(\tau)}_a := \text{tr}_b [\rho^{(\tau)}_{ab}] \) where (see §3.2 of [48] and also [49, 50, 78] for more details)

\[
\frac{d\rho^{(\tau)}_a}{d\tau} \simeq g^2 \int_0^\infty ds \left( \mathcal{W}_a(s)[\mu^*(\tau - s)\rho^{(\tau)}_a(\tau, \mu^*(\tau)] + \text{h.c.} \right) \tag{37}
\]

where \( \mu^* := \text{tr}_b[\mu^*_b] \). The failure of the Markovian approximation in the case of \( \Omega/a \gtrsim 1 \) is most easily appreciated by studying the evolution of the off-diagonal components \( \rho^{(\tau)}_{a,12} \) of Alice’s reduced density matrix

\[
\frac{d\rho^{(\tau)}_{a,12}}{d\tau} \simeq -2g^2 \int_0^\tau ds \text{Re} [\mathcal{W}_a(s)] e^{i\Omega s} \rho^{(\tau)}_{a,12}(\tau - s) + 2g^2 e^{i2\Omega \tau} \int_0^\tau ds \text{Re} [\mathcal{W}_a(s)] e^{-i\Omega s} \rho^{(\tau)}_{a,12}(\tau - s). \tag{38}
\]

If one uses (30), as is usually done in the literature, then the resulting Markovian equation of motion for the off-diagonal components yields

\[
\frac{d\rho^{(\tau)}_{a,12}}{d\tau} \simeq -2g^2 \int_0^\infty ds \text{Re} [\mathcal{W}_a(s)] e^{i\Omega s} \rho^{(\tau)}_{a,12}(\tau) + 2g^2 e^{i2\Omega \tau} \int_0^\infty ds \text{Re} [\mathcal{W}_a(s)] e^{-i\Omega s} \rho^{(\tau)}_{a,12}(\tau). \tag{39}
\]

In Eq. (38) and those that follow, one should include a renormalization of Alice’s detector gap in order to get the Markovian solution (30) — we omit this detail here, see Appendix C.
where we have also assumed $\alpha \tau \gg 1$ so the upper limit on the integral can be approximated by $\sim \infty$. The time-local differential equation [(39)] turns out to have the solution in the non-degenerate regime $g^2C \ll \Omega$ 

$$\rho_\alpha^{\prime}\!(\tau) \simeq A e^{-g^2C\tau} + B e^{-(g^2C+2\Omega)\tau}$$  \hspace{1cm} (40)$$

where $C$ is given by 

$$C := 2 \int_0^\infty ds \text{Re}[W_s(s)] \cos(\Omega s) = \frac{\Omega}{4\tau} \coth \left( \frac{\tau}{a} \right) , \hspace{1cm} (41)$$

and $A$ and $B$ are constant coefficients which can be explicitly computed but whose form is not important in what follows (see Appendix B).

To check when the Markovian approximation is valid, one can insert the solution [(39)] and see when [(38)] is well-approximated by [(39)]. Although somewhat tedious (see Appendix C), it can be shown that 

$$\int_0^\tau ds \text{Re}[W_s(s)] e^{+(\tau-s)C} e^{+g^2Cs} \simeq \int_0^\infty ds \text{Re}[W_s(s)] e^{-\Omega s} .$$  \hspace{1cm} (42)$$

Noting that $\text{Re}[W_s(s)] \propto e^{-\mu s}$ for $a \gg 1$, the above approximation can only work when 

$$g^2C \ll a, \hspace{1cm} (43a)$$

$$e^{+(	au-s)C} e^{+g^2Cs} \simeq e^{-\Omega s} \simeq 1. \hspace{1cm} (43b)$$

The main point here is that while the first condition [(43a)] can be satisfied by making $g$ sufficiently small, the second condition [(43b)] can hold underneath the integral sign in [(42)] only when $\Omega \ll \alpha$. In other words, for $\Omega/a \geq 1$ it is impossible to satisfy the approximation [(42)], and so the Markovian approximation cannot be consistently applied. The argument given here is related to the diagnostic for the failure of the Markovian approximation described in [45] where it was explicitly shown that derivative terms in the Taylor series [(36)] become too large when $\Omega/a \geq 1$. This extra requirement that $\Omega/a \ll 1$ is often missed in the literature.

What we have seen above is that the correct way of taking the Markovian approximation necessarily accounts for the requirement that $\Omega/a \ll 1$. We now claim that our version of Markovian approximation in §IVA (also used in [72]) does account for this. Following §IVA, the right way of taking the approximation for Alice’s qubit is to perform Taylor series expansion about $\tau$ on both the monopole and the state:

$$\mu(\tau) - s = \sum_{n=0}^{\infty} \mu^n(\tau) s^n \simeq \mu^n(\tau) / s^n.$$

Note that by approximating $\mu^n(\tau) / s^n$ we are automatically requiring that the energy scales associated with the system (in this case $\Omega$ appearing in the detector’s monopole operator) are small compared to those associated with the environment (encoded in the falloff of the environment correlators). This amounts to erasing any history-dependence in the system operators as well as the state itself. Furthermore, by bounding the derivative terms in [(44)] relative to the leading-order term, it can be checked explicitly for a single detector that indeed the Markovian approximation is valid only when $\Omega/a \ll 1$.

In fact, what we believe to be the correct Markovian approximation also solves other problems that otherwise require further renormalization of divergences or other sleights of hands. Using the approximation [(44)] on [(38)] gives us a different Markovian equation of motion (c.f. Eq. [(39)]) 

$$\frac{d\rho_\alpha^{\prime}\!(\tau)}{d\tau} \simeq -2g^2 \int_0^\infty ds \text{Re}[W_s(s)] \rho_\alpha^{\prime}\!(\tau)$$

$$+ 2g^2 e^{2\Omega \tau} \int_0^\infty ds \text{Re}[W_s(s)] \rho_\alpha^{\prime\prime}\!(\tau).$$

which has the solution (c.f. Eq. [(40)]) 

$$\rho_\alpha^{\prime}\!(\tau) \simeq A_s e^{-g^2C\tau} + B_s e^{-(g^2C+2\Omega)\tau}, \hspace{1cm} (46)$$

$$A_s = \rho_\alpha^{\prime\prime}\!(0) + \frac{g^2C_s}{2\Omega} \rho_\alpha^{\prime\prime}\!(0), \hspace{1cm} B_s = -\frac{g^2C_s}{2\Omega} \rho_\alpha^{\prime\prime}\!(0),$$

and the constant $C_s$ reads 

$$C_s := \lim_{\Omega \to 0^+} C = \int_0^\infty ds \text{Re}[W_s(s)] = \frac{a}{4\pi^2} .$$  \hspace{1cm} (47)$$

This solution has three crucial features: (1) it is much simpler than Eq. [(40)], (2) it is independent of any UV divergences, and most importantly (3) contrary to what is often said in the literature, it preserves complete positivity (CP) property for the entirety of its evolution without further approximations (including RWA) [48]. The requirement $\Omega/a \ll 1$ is encoded in the definition of $C_s$, since $C/a = C_s + \mathcal{O}(\Omega^2/a^2)$. It is important to note that the Markovian solution still has dependence on $\Omega$: it just cannot appear in the “matrix coefficients” of the Markovian equation of motion.

In the remainder of this paper, we use this logic in applying the analog of the Markovian approximation [(44)], namely Eq. [(39)], to our more complicated two-detector problem. We see later that the “correct” Markovian approximation also circumvents the need for tracking any UV divergences associated with the “standard” Markovian approximation and will preserve complete positivity without RWA in the regime where Markovian limit is valid. Before doing so, we first show in the next subsection that the results in a Lindblad equation which makes manifest the CP-preserving property and hence why RWA is not necessary.

C. Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) form

The best way to show why the Markovian approximation [(44)] results in CP-preserving dynamics (without further approximation) is to cast our master equation into a Schrödinger picture equation of the form

$$\frac{\partial \rho_{AB}}{\partial \tau} = -i[H_{\text{eff}}, \rho_{AB}(\tau)] + \mathcal{D}[\rho_{AB}(\tau)].$$  \hspace{1cm} (48)$$
Eq. [48] takes the form analogous to the the Liouville-von Neumann equation with an effective Hamiltonian $\hbar_{\text{eff}}$ (sometimes called the Lamb shift) and with an extra term involving the dissipation superoperator $D$ capturing the non-unitary open dynamics of the time evolution generated by the master equation. For bipartite qubits, Eq. [48] is said to be in Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) form if the dissipator has the form [62, 63]:

$$D[\rho_{AB}] = \sum_{\alpha, \beta = 1}^{3} \sum_{j,k=A,B} \gamma_{jk} \left( \sigma_{j}^{k} \rho_{AB} \sigma_{\alpha}^{\beta} - \frac{1}{2} \left\{ \sigma_{\alpha}^{\beta} \sigma_{j}^{k}, \rho_{AB} \right\} \right)$$

where the matrix $\gamma := [\gamma_{jk}^{\alpha\beta}]$ is known as the Kossakowski matrix. The time evolution of the joint qubit state $\rho_{AB}$ is said to be completely positive (CP) if the linear superoperator $\Phi_{\alpha} : \rho_{AB}(0) \mapsto \rho_{AB}(\tau)$ is a quantum channel, i.e. $\Phi_{\alpha}$ is completely positive and trace-preserving (CPTP) map. It is known that the dynamical evolution is CP if and only if the Kossakowski matrix $\gamma$ is positive semidefinite, $\hbar_{\text{eff}}$ is Hermitian, and the “jump operators” (here $\gamma_{jk}^{\alpha\beta}$) form an orthonormal basis for the Hilbert-Schmidt operators on the joint detector Hilbert space [58, 62, 63, 64, 77, 79, 80].

Let us now recast the Markovian dynamics encoded in equation [48] into GKSL form. Converting [48] to the Schrödinger picture using [12] yields

$$\frac{\partial \rho_{AB}}{\partial \tau} \simeq -i[\hbar_\Lambda + \hbar_b, \rho_{AB}] + g^2 \sum_{j,k} \int_{0}^{\infty} ds \left( W_{jk}(s) [\mu_k(0) \rho_{AB}(\tau), \mu_j(0)] + \text{h.c.} \right) ,$$

with the sum running over $j,k \in \{A,B\}$ as before. We stress again that a crucial difference that distinguishes the usual procedure in the literature from ours is the way the Markovian approximation is implemented.

In addition to the constant $C_s$ defined in [47], we also define two other constants that depend on separation $L$:

$$C_x := 2 \int_{0}^{\infty} ds \text{ Re } |W_x(s)| = \frac{\sin^{-1}(aL/2)}{2\pi^2 L \sqrt{1 + (aL/2)^2}} ,$$

$$K_x := 2 \int_{0}^{\infty} ds \text{ Im } |W_x(s)| = -\frac{1}{4\pi L \sqrt{1 + (aL/2)^2}} .$$

The Markovian master equation [50] can be reorganized into GKSL form [48] with

$$\hbar_{\text{eff}} = \hbar_\Lambda + \hbar_b + g^2 K_x \sigma_1^a \sigma_1^a .$$

The effective Hamiltonian operator $\hbar_{\text{eff}}$ is here the joint free Hamiltonian of the detectors $\hbar_\Lambda + \hbar_b$ together with an extra degeneracy-lifting environment-induced interaction term $g^2 K_x \sigma_1^a \sigma_1^a$. Notably, this so-called “Lamb shift” term is finite and the degeneracy in the joint free Hamiltonian is lifted because the spectrum of $\hbar_{\text{eff}}$ is $\{-\Omega, -g^2 K_x, +g^2 K_x, \Omega\}$ while the spectrum of $\hbar_\Lambda + \hbar_b$ is $\{-\Omega, 0, 0, \Omega\}$.

We now claim that the dynamics described by $\hbar_{\text{eff}}$ and the $6 \times 6$ Kossakowski matrix,

$$\gamma = \begin{bmatrix} \gamma_{\alpha\beta}^{\alpha\beta} & \gamma_{\alpha\beta}^{\alpha\beta} \\ \gamma_{\alpha\beta}^{\alpha\beta} & \gamma_{\alpha\beta}^{\alpha\beta} \end{bmatrix}$$

defines a CP-preserving evolution map without further approximation. This follows directly from the fact that $\hbar_{\text{eff}}$ is Hermitian and the Kossakowski matrix $\gamma$ is indeed positive-semidefinite, where the only two nonzero eigenvalues are

$$\lambda_1[\gamma] = g^2 (C_s + C_x) , \quad \lambda_2[\gamma] = g^2 (C_s - C_x) ,$$

which are non-negative since $C_s \geq C_x > 0$ for any $L \geq 0$. In other words, the Markovian master equation [50] studied in this work using the correct Markovian approximation [30] already defines CP-preserving evolution without the need for any additional approximations such as the RWA.

---

[13] The “standard” Markovian approximation would have given

$$\frac{\partial \rho_{AB}}{\partial \tau} \simeq -i[\hbar_\Lambda + \hbar_b, \rho_{AB}(\tau)] + g^2 \sum_{j,k} \int_{0}^{\infty} ds \left( W_{jk}(s) [\mu_k(-s) \rho_{AB}(\tau), \mu_j(0)] + \text{h.c.} \right) .$$
D. Markovian limit of the \(X\)-block

We are now ready to perform the late-time resummation for the late-time dynamics of the detectors. Recall that the evolution decouples\(^{14}\) into two sets of matrix ODE for the \(X\)- and \(O\)-blocks as stated in \((26)\). In what follows we find it more convenient to work in the Schrödinger picture.

Starting from the Markovian dynamics \((50)\), the \(X\)-block component can be rearranged into a matrix ODE with constant coefficient of the form \((53)\): this reads

\[
\frac{d\mathbf{x}}{d\tau} = (\mathbf{M}_0 + g^2\mathbf{M}_2)^T \mathbf{x}(\tau) + g^2 \mathbf{b},
\]

where we define the vector of \(X\)-block components \(\mathbf{x}(\tau)\) and the constant vector \(\mathbf{b}\) to be

\[
\mathbf{x}(\tau) := \begin{bmatrix} \rho_{11}(\tau) & \rho_{22}(\tau) & \rho_{33}(\tau) & \text{Re}[\rho_{14}(\tau)] & \text{Im}[\rho_{14}(\tau)] & \text{Re}[\rho_{23}(\tau)] & \text{Im}[\rho_{23}(\tau)] \end{bmatrix}^T,
\]

\[
\mathbf{b} := \begin{bmatrix} 0 & C_s & C_s & -C_s & -K_s & C_s & 0 \end{bmatrix}^T,
\]

and \(\mathbf{M}_0, \mathbf{M}_2\) are constant matrices

\[
\mathbf{M}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2\Omega & 0 \\
0 & 0 & -2\Omega & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
\mathbf{M}_2 = \begin{bmatrix} -2C_s & C_s & C_s & -2C_s & -2K_s & 2C_s & 0 \\
0 & -3C_s & -C_s & 2C_s & 0 & -2C_s & -2K_s \\
0 & -C_s & -3C_s & 2C_s & 0 & -2C_s & 2K_s \\
0 & 2C_s & 2C_s & -2C_s & 0 & 2C_s & 0 \\
2K_s & K_s & K_s & 0 & -2C_s & 0 & 0 \\
0 & -2C_s & -2C_s & 2C_s & 0 & -2C_s & 0 \\
0 & K_s & -K_s & 0 & 0 & 0 & -2C_s \end{bmatrix}.
\]

The determinant of the matrix is given by

\[
\det(\mathbf{M}_0 + g^2\mathbf{M}_2) = -256g^{10}\Omega^2 \mathbf{C}_s \left(\mathbf{C}_s^2 - \mathbf{C}_x^2\right) \left(\mathbf{C}_s^2 + K_s^2\right)
\]

which is nonzero so long as \(\Omega > 0\) and \(L > 0\) (since \(C_s \leq C_s\) with \(C_x \rightarrow C_s\) in the limit \(L \rightarrow 0^+\)). Remarkably, this alone already implies that any inferences made for the dynamics immediately become suspect in the limit that the trajectories are “stacked” on top of one another with \(L \rightarrow 0^+\) (considered, e.g., in \((28)\)).

The eigenvalues for \(\mathbf{M}_0 + g^2\mathbf{M}_2\) are given by\(^{15}\)

\[
\lambda_1^X \simeq -2g^2C_s,
\lambda_2^X \simeq -3g^2C_s - g^2\sqrt{C_s^2 + 8C_x^2},
\lambda_3^X \simeq -3g^2C_s + g^2\sqrt{C_s^2 + 8C_x^2},
\lambda_4^X = -2g^2\left(C_s - iK_x\right),
\lambda_5^X = -2g^2\left(C_s + iK_x\right),
\lambda_6^X \simeq -2i\Omega - 2g^2C_s,
\lambda_7^X \simeq +2i\Omega - 2g^2C_s.
\]

Since the determinant \((59)\) is nonzero this means that the matrix is invertible (so long as \(\Omega, L > 0\)). This is sufficient for computing the steady state solution

\[
\frac{d\mathbf{x}}{d\tau} = 0 \implies \mathbf{x}_* = -(\mathbf{M}_0 + g^2\mathbf{M}_2)^{-1}g^2\mathbf{b}.
\]

Furthermore since \(\text{Re}[\lambda_j^X] < 0\) for all \(j\) at \(\mathcal{O}(g^2)\), we can conclude that the dynamics sink towards the above steady-state \(\mathbf{x}_*\), for any initial state \(\mathbf{x}(0)\). The stationary solution \(\mathbf{x}_*\) reads

\[
\mathbf{x}_* \simeq \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}^T,
\]

and more generally, the solution to \((56)\) for arbitrary \(\tau\) is given by

\[
\mathbf{x}(\tau) = e^{(\mathbf{M}_0 + g^2\mathbf{M}_2)t}(\mathbf{x}(0) - \mathbf{x}_*) + \mathbf{x}_*.
\]

This is the late-time resummed solution for the \(X\)-block since it is valid for \(g^2\tau \sim 1\).

We can evaluate this general solution \((63)\) numerically or perturbatively along the lines of \((48)\) (e.g. see Eq. \((105)\)). The numerical calculation of \((63)\) will be relevant, for instance, when we want to calculate the amount of entanglement that the two detectors acquire via interaction with the quantum field.

E. Markovian limit for \(O\)-block

For the \(O\)-block, we perform the same procedure as before (see Appendix \(B\) for the NZ-ME2 before taking...
the Markovian approximation) which leads to

$$\frac{d \mathbf{y}(\tau)}{d\tau} = (N_0 + g^2 N_2) \mathbf{y}(\tau),$$

(64)

where we define

$$\mathbf{y}(\tau) := [\rho_{12}(\tau) \rho_{13}(\tau) \rho_{24}(\tau) \rho_{34}(\tau) \rho_{12}^*(\tau) \rho_{13}^*(\tau) \rho_{24}^*(\tau) \rho_{34}^*(\tau)]^T,$$

(65)

and where $N_0, N_2$ are constant matrices

$$N_0 = \begin{bmatrix}
-i\Omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -i\Omega & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -i\Omega & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & i\Omega & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & i\Omega & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & i\Omega \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad N_2 = \begin{bmatrix}
2C_s & -\alpha & C_x & C_s & C_s & C_x & -\alpha^* & 0 \\
-\alpha & -2C_s & C_s & C_x & C_s & 0 & -\alpha^* & 0 \\
C_s & C_x & -\alpha^* & -2C_s & 0 & -\alpha & C_s & C_s \\
C_s & C_s & 0 & -\alpha & -\alpha^* & -2C_s & C_s & C_s \\
-\alpha^* & 0 & C_s & C_x & C_s & C_s & -2C_s & -\alpha \\
0 & -\alpha^* & C_x & C_s & C_s & C_x & -\alpha & -2C_s
\end{bmatrix},$$

(66)

with the shorthand $\alpha := C_x - iK_x$. Furthermore we find the determinant

$$\text{det}(N_0 + g^2 N_2) = \Omega^8 + 16g^4\Omega^6 C_s^2 + 64g^8\Omega^4 C_s^2 (C_s^2 - C_x^2)$$

(67)

which is always nonzero in the perturbative limit for nonzero gap ($\Omega > 0$). The matrix eigenvalues at $O(g^2)$ are given by

$$\lambda_0^2 \approx -i\Omega + g^2(-2C_s + C_x - \sqrt{(C_s - C_x)^2 - K_x^2})$$

$$\lambda_2^2 \approx -i\Omega + g^2(-2C_s + C_x + \sqrt{(C_s - C_x)^2 - K_x^2})$$

$$\lambda_3^2 \approx -i\Omega + g^2(-2C_s - C_x - \sqrt{(C_s + C_x)^2 - K_x^2})$$

$$\lambda_4^2 \approx -i\Omega + g^2(-2C_s - C_x + \sqrt{(C_s + C_x)^2 - K_x^2})$$

(68a)

with the corresponding conjugate pairs

$$\lambda_3^* = \lambda_2^2, \quad \lambda_0^* = \lambda_2^2, \quad \lambda_3^* = \lambda_3^2, \quad \lambda_4^* = \lambda_4^2.$$ 

(68b)

Since the matrix $N_0 + g^2 N_2$ is invertible (for $\Omega > 0$), we have the general late-time resumed solution

$$\mathbf{y}(\tau) = e^{(N_0 + g^2 N_2)\tau} \mathbf{y}(0).$$

(69)

At large $\tau \to \infty$, we have $\mathbf{y}(\tau) \to 0$, i.e. the $O$-block decays to zero. This follows from the fact that the stationary solution $\mathbf{y}_s$ is given by

$$\frac{d \mathbf{y}_s}{d\tau} = 0 \implies \mathbf{y}_s = 0,$$

(70)

because all the eigenvalues of $N_0 + g^2 N_2$ have negative real parts (so long as $\Omega > 0$). This result implies that regardless of the initial state chosen for the two detectors, only the $X$-block contribution survives in the late-time limit.

**F. Thermalization and (lack of) entanglement**

The late-time steady-state solution for the NZ-ME2 equation follows by combining the steady-state solutions $\mathbf{x}_s$ and $\mathbf{y}_s$ for both $X$-block and $O$-block we obtained earlier. The result is that the late-time stationary state is maximally mixed for any initial joint state of both qubits:

$$\rho_{\text{An}}(\tau) = \frac{1}{2} \otimes \frac{1}{2} + O(g^4),$$

(71)

which is clearly separable. Observe that the steady state solution (71) is independent of $\Omega, a, L$ and any UV cutoffs (like $\epsilon$ used to regulate coincident limit divergences). In other words, calculations in the Markovian regime at leading order are unable to probe the temperature of the field even if both detectors do thermalize. This is because the Markovian regime where we need $\Omega/a \ll 1$ corresponds to the high-temperature limit, hence the steady state solution only picks out the zeroth-order expansion in $\Omega/a$. Any dependence of the state on $\Omega/a$ can only appear at finite $\tau$.

Furthermore, the fact that the asymptotic final state is independent of $L$ means that the Markovian regime washes out the effect of field-mediated communication between the two detectors. Any $L$-dependent corrections to the steady-state solution can only appear at finite $\tau$ at this order in perturbation theory. It is worth emphasizing that the lack of $L$-dependence at late times on its own is not very surprising: already in other contexts such as entanglement harvesting, detectors are unable to get entangled by the quantum field vacuum when the energy gap is too small compared to other scales of the problem [9, 12, 81, 82]. Similarly, in perturbative short-time Dyson series expansions, accelerated detectors suffer entanglement degradation [19]. These older results already suggest (in hindsight) that one should
not expect any entanglement in the late-time Markovian regime when both detectors are in their ground states.

That said, we should still be able to infer the Unruh temperature from the late-time dynamics indirectly. Indeed, since the solutions to the ODEs are given by Eqs. (63) and (69) (we repeat here for convenience)

\[ x(\tau) = e^{(\mathcal{M}_0 + g^2 \mathcal{M}_2)T}(x(0) - x_*) + x_* , \quad (72) \]

\[ y(\tau) = e^{(\mathcal{N}_0 + g^2 \mathcal{N}_2)T}y(0) , \]

the eigenvalues of \( \mathcal{M}_0 + g^2 \mathcal{M}_2 \) and \( \mathcal{N}_0 + g^2 \mathcal{N}_2 \) set the scale for the thermalization process. In the limit of large separation \( (aL \gg 1) \), we have \( C_x/C_s \ll 1 \) and \( K_x/C_s \ll 1 \), so that the maximum of the real part of the eigenvalues is given by

\[ \max_j \text{Re}(\lambda_j^\text{X}) \approx -2g^2C_s = \frac{g^2a}{2\pi^2} \equiv \frac{g^2T_0}{\pi} . \quad (73) \]

This is equal to the decay rate of a single accelerating detector experiencing thermal bath at Unruh temperature \( T_0 = a/(2\pi) \) found in the literature (see, e.g., [44]). Therefore, while the asymptotic final state cannot tell us about the Unruh temperature, we can still learn about the Unruh effect from its decay rates so long as the detectors are far enough apart.

In contrast, for \( aL \ll 1 \) is not tractable in the Markovian regime since \( \lambda_j^\text{X} \) approaches zero as \( aL \) gets smaller. This can be attributed to field-mediated communication (from the commutator) between the two detectors at small separation which dominates. In effect, what is happening is that for \( aL \ll 1 \), the detectors can exchange information with one another via the field commutator, so one detector “stores” the memory of the other detector. Consequently, the decay process becomes much slower at small separation \( (\text{Re}[\lambda_j^\text{X}] \approx 0^-) \).

V. VALIDITY RELATIONS FOR MARKOVIAN LIMIT

In §IV we obtained the Markovian solution for the two-detector dynamics and showed that the evolution is CP without the need for RWA. This issue is largely ignored in the literature, especially so when two detectors are considered. In the majority of past literature we are aware of, it is assumed that the Markovian limit can be taken, without specifying when it is valid in terms of the relevant scales for the problem at hand. We now find explicitly the requirements for the Markovian limit to be valid, and we show that similar two-detector calculations in the past can actually violate these requirements.

Recall that the Markovian approximation can be viewed in terms of the Taylor series [30], repeated here,

\[ \mu_j^\tau(\tau - s)\rho_{ab}^\tau(\tau - s) \]

\[ \approx \mu_j^\tau(\tau)\rho_{ab}^\tau(\tau) - s(\mu_j^\tau(\tau)\rho_{ab}^\tau(\tau) + \mu_j^\tau(\tau)\rho_{ab}^\tau(\tau)) + \ldots . \quad (74) \]

This is physically motivated by the fact that the environment correlators \( W_{s,x} \) are strongly peaked about \( s = 0 \) in the master equation. Inserting (74) into (25), we get

\[ \frac{\text{d}}{\text{d}\tau} \rho_{ab}^\tau \simeq g^2 \sum_{j,k \in \{A,B\}} \int_0^\infty \text{d}s \left( W_{jk}(s)[\mu_j^\tau(\tau)\rho_{ab}^\tau(\tau),\mu_k^\tau(\tau)] + \text{H.c.} \right) \]

\[ - g^2 \sum_{j,k \in \{A,B\}} \int_0^\infty \text{d}s \left( sW_{jk}(s)[\mu_j^\tau(\tau)\rho_{ab}^\tau(\tau) + \mu_j^\tau(\tau)\rho_{ab}^\tau(\tau),\mu_k^\tau(\tau)] + \text{H.c.} \right) + \ldots . \]

Using \( \mu_j^\tau(\tau) = -\Omega(-i\sigma_+^j e^{i\Omega\tau} + i\sigma_-^j e^{-i\Omega\tau}) \) we can put Eq. (75) into the Lindblad-like form, which in the Schrödinger picture reads

\[ \frac{\text{d}}{\text{d}\tau} \rho_{ab}^\tau \simeq -i[h_{\text{eff}},\rho_{ab}^\tau(\tau)] + \mathcal{D}_{\gamma}[\rho_{ab}^\tau(\tau)] - i[J_{\text{eff}},\rho_{ab}^\tau(\tau)] - \mathcal{D}_{\eta}[h_{\text{eff}},\rho_{ab}^\tau(\tau)] + \mathcal{D}_{\zeta}[\rho_{ab}^\tau(\tau)] . \quad (76) \]

The first two terms of Eq. (76) constitute the Markovian part: the effective Hamiltonian \( h_{\text{eff}} \) is given in Eq. (52), while the dissipator \( \mathcal{D}_{\gamma} \) is given by Eq. (49), with Kossakowski matrix \( \gamma \) computed in Eqs. (53a,53b). The next three terms contain the leading non-Markovian corrections: first, we have \( J_{\text{eff}} \) defined by

\[ J_{\text{eff}} := -\frac{g^2\Omega^2}{2}(\sigma_+^3 + \sigma_+^1) - \frac{g^2\Omega^2}{2}(\sigma_2^1\sigma_1^3 + \sigma_1^2\sigma_2^3) . \quad (77) \]

16 One exception is [23] for a single detector, but the method is not very portable for two detectors.
The last two terms are extra “dissipation terms” $D_{\eta}, D_{\zeta}$ with the corresponding “Kossakowski matrices” $\eta$ and $\zeta$ given by

\[
[\eta^\alpha^\beta_{AA}] = [\eta^\alpha^\beta_{AB}] = \begin{bmatrix}
D'_s & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\quad \text{and} \quad
[\zeta^\alpha^\beta_{AA}] = [\zeta^\alpha^\beta_{AB}] = \begin{bmatrix}
0 & \frac{1}{2} (D'_s - iS'_x) & 0 \\
\frac{1}{2} (D'_s + iS'_x) & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

The constant coefficients $D'_{s,x}$ and $S'_{s,x}$ are given by

\[
D'_{s,x} := 2 \int_0^\infty ds \, \text{Re} \left[ W_{s,x}(s) \right] s \quad \text{(79a)}
\]
\[
S'_{s,x} := 2 \int_0^\infty ds \, \text{Im} \left[ W_{s,x}(s) \right] s \quad \text{(79b)}
\]

which evaluate to (see Appendix III)

\[
D'_s = \frac{\log(ac)}{2\pi^2}, \quad D'_x = \frac{\text{Re} \left[ Li_2(\ell^2) - Li_2(\ell^2) \right]}{4\pi^2 aL \sqrt{1 + (aL/2)^2}} \quad \text{(80a)}
\]
\[
S'_s = -\frac{1}{4\pi}, \quad S'_x = \frac{\sinh^{-1}(aL/2)}{2\pi aL \sqrt{1 + (aL/2)^2}} \quad \text{(80b)}
\]

where $Li_2(z)$ is the polylogarithm of order 2 [83] and we used the shorthand

\[
\ell_\pm := \frac{aL}{2} \pm \sqrt{1 + \left(\frac{aL}{2}\right)^2}. \quad \text{(81)}
\]

Note that $D'_s$ is a UV-regulated function with the infinitesimal $\epsilon > 0$ appearing in the $\epsilon$-prescription of the environment correlators $W_{s,x}$. It is worth stressing that the UV regulator is needed because sharp switching is not compatible with pointlike limits in (3+1) dimensions: the UV regulator is given the interpretation of a position-space cutoff on the size of the detector below which we cannot resolve.

### A. Matrix ODE derivation of validity bounds

The idea of finding where the Markovian approximation applies is to bound the last three terms in (76) involving $\beta$, $\eta$ and $\zeta$ to be parametrically small compared to the first two terms involving $\eta_{\text{eff}}$ and $\gamma$. Whenever in parameter space this is true, the Lindbladian dynamics studied in this work are valid — this results in a set of bounds which must be satisfied involving functions of $g$, $a$, $\Omega$ and $L$ (as well as a UV cutoff $\epsilon$) which we call validity bounds.

The simplest way of doing this is to again split (76) into the $X$- and $O$-blocks, keeping the subleading non-Markovian correction as in (76) and find the analogous matrix ODE to Eq. (33): the result is an equation of the form

\[
\frac{du}{d\tau} \approx \begin{cases}
\mathbb{A}u + \mathbb{Y}v & \text{Markov,} \\
-\mathbb{Y}u - \mathbb{Z}u & \text{lead. non-Markov.}
\end{cases}
\]

The matrix $\mathbb{Y}$ can be obtained from $\mathbb{A}$ by the replacement $C_{s,x} \to D'_{s,x}$ and $K_{s,x} \to S'_{s,x}$ (with the rest of the entries zero), while the matrix $\mathbb{Z}$ depends only on combinations of $\Omega D'_s$ and $\Omega S'_s$. Iteratively plugging (33) into the RHS of (82) gives us

\[
\frac{du}{d\tau} \approx (\mathbb{1} - \mathbb{Y})(\mathbb{1} - \mathbb{Z})^{-1}\mathbb{A}u + (\mathbb{1} - \mathbb{Y})v, \quad \text{(83)}
\]

where we have used the fact that $\mathbb{A}$ is invertible (for both the $X$- and $O$-blocks, so long as $\Omega, L > 0$). Eq. (83) tells us that for the Markovian approximation to be valid, we need

\[
\mathbb{1} - \mathbb{Y} \simeq \mathbb{1}, \quad \mathbb{1} - \mathbb{Z}^{-1} \simeq \mathbb{1}, \quad \text{i.e., each matrix element satisfies}
\]

\[
|Y_{am}| \ll 1 \quad \text{and} \quad |(ZA^{-1})_{nm}| \ll 1. \quad \text{(84)}
\]

Eq. (85) provides us with a very compact way of stating the constraints required for validity of Markovian approximation.

What remains to be done is to write down the information contained in (85) in terms of physical parameters $g$, $a$, $\Omega$ and $L$. Studying the $X$-block in the above manner results in entries of the matrix $\mathbb{Y}$ which from $|Y_{am}| \ll 1$ yield

\[
g^2 |D'_s| = g^2 \frac{\log(ac)}{2\pi^2} \ll 1, \quad \text{(86a)}
\]
\[
g^2 |D'_x| = g^2 \frac{\text{Re} \left[ Li_2(\ell^2) - Li_2(\ell^2) \right]}{4\pi^2 aL \sqrt{1 + (aL/2)^2}} \ll 1, \quad \text{(86b)}
\]
\[
g^2 |S'_{s,x}| = g^2 \frac{\sinh^{-1}(aL/2)}{2\pi aL \sqrt{1 + (aL/2)^2}} \ll 1 \quad \text{(86c)}
\]

where $\ell_\pm$ is defined in (81). It is straightforward to see that $|S'_{s,x}| \ll 1$ for all $aL \geq 0$, hence the third bound

\[
|S'_{s,x}| \ll 1.
\]

\footnote{Note that in other contexts such as entanglement harvesting, pointlike detector models may still work because some initial states are insensitive to these UV issues. In the validity relations derived here, the UV regulators will appear explicitly as we consider arbitrary qubit initial states.}
is automatically satisfied. The first bound a single-detector bound (also encountered in [48]) and ensures the smallness of the coupling $g$ must compensate for the largeness of the UV cutoff $\epsilon$. The non-trivial bound ends being the second one, since it depends on the detector separation: for $aL \gtrsim 1$ we always have $|D'_X| \ll 1$, however

$$g^2 |D'_X| \simeq \frac{g^2(1 - \log(aL))}{2\pi^2} \quad \text{for } aL \ll 1. \quad (87)$$

What this means is that small $aL$ must be compensated by weaker coupling $g$, and so one cannot make $aL$ arbitrarily small. This bound is distinct from the UV cutoff requirement that demands $\epsilon \ll L$.

The condition $|(ZA^{-1})_{nm}| \ll 1$ for the $X$-block introduces more bounds, which are generally very complicated due to the matrix inverse. However, all the bounds involving $aL$ are only non-trivial when $aL \ll 1$ (i.e., they can be easily satisfied for $aL \gtrsim 1$), so below we restrict our attention only for $aL \ll 1$. The $g$-dependent bounds are

$$g^2 |\log(ac)| \ll 1, \quad \frac{g^2}{aL} \ll 1, \quad \frac{3\pi\Omega/a - g^2aL}{(aL)^2} \ll 1, \quad \frac{g^2}{4aL} + g^2|\log(ac)| \ll 1, \quad (88)$$

while the $g$-independent bounds are

$$\frac{\pi\Omega}{a} \ll 1, \quad \frac{\pi\Omega}{a(aL)^2} \ll 1, \quad \frac{\Omega|\log(a^2\epsilon L)|}{a(aL)^2} \ll 1. \quad (89)$$

One of the main takeaways from this analysis is that $aL$, which measures detector separation in units of $a$, cannot be arbitrarily small: it is bounded below by all other parameters involving $ac, \Omega/a$ and $g^2$. Very small $aL$ amounts to very closely-spaced detectors and the field-mediated communication makes memoryless approximation harder to satisfy.

We can perform the same kind of (tedious) analysis for the $O$-block and it turns out that up to irrelevant numerical factors and linear combinations of the above conditions, the $O$-block does not contain any new information about Markovian validity.

### B. Summary: when is Markovian approximation valid?

The short story is that the Markovian validity favours weak coupling $g \ll 1$ (due to perturbation theory), as well as small detector gaps $\Omega \ll a$ and large separation $aL \gtrsim 1$. Overall, we can summarize the validity relations for Markovian limit as follows:

(i) By default, the Markovian approximation requires that $\Omega/a \ll 1$. This requirement is often implicit or ignored in the literature.

(ii) In general the pointlike limit is ill-defined for arbitrary qubit initial states when one considers sharp switching, which is the usual approach in open quantum systems (with rare exceptions such as [43]). Consequently, the UV divergences encountered are to be interpreted as ignorance about the detectors finite spatial extent. The validity bounds require

$$\frac{g^2|\log(ac)|}{2\pi^2} \ll 1. \quad (90)$$

That is, $ac$ cannot be arbitrarily small: either we probe the “high temperature” regime ([large $a$](large effective size of the detector prescribed by the UV cutoff $\epsilon$), or that detector size cannot be arbitrarily small.

(iii) In the presence of two detectors, we require that $g^2|D'_X| \ll 1$, where $D'_X$ depends on the dimensionless detector separation $aL$. For large $aL$ this is automatically satisfied if (i) and (ii) are properly satisfied; however, for small $aL \ll 1$ we require that $g, a, \Omega, L$ and $\epsilon$ work together to obey

$$\frac{\pi\Omega}{a(aL)^2} \ll 1, \quad \frac{\Omega|\log(a^2\epsilon L)|}{a(aL)^2} \ll 1, \quad g^2 \ll aL \quad (91)$$

which are simplified versions of the $aL$-dependent validity bounds given in (88) and (89). The crucial point is that $aL$ cannot be arbitrarily small: it is bounded below by quantities involving $g, \Omega/a, ac$. Thus this condition favours $aL \gtrsim 1$; for $aL \ll 1$ one has to more carefully tune $\Omega/a, ac$ and $g$ in order to compensate for the non-Markovianity this introduces.

(iv) Finally, the matrices governing the evolution for the $X$- and $O$-blocks are treated as perturbative in the coupling $g$ (for nonzero $\Omega > 0$ where matrix determinants are nonzero), this means that the matrices $M_0, N_0$ are large compared to the perturbations $g^2M_2, g^2N_2$ in the matrix ODEs [56 and 66]. What this amounts to is remaining in a regime where the eigenvalues $\lambda^{X,O}$ given in [60] and [68] are perturbative in the coupling — this means that the oscillation scale $\Omega > 0$ must be large compared to the $O(g^2)$ corrections. At the end of this day this enforces

$$g^2 \ll \frac{\Omega}{a} \quad \text{and} \quad g^2 \ll \Omega L \quad (92)$$

which roughly speaking ensures that $g^2$ is the

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18 At the same time, no finite-sized realistic detector can maintain its rigid shape for arbitrarily large accelerations, e.g., due to ionization. So in practice high-temperature regime is highly non-trivial.
slowly-varying (as described in (36)), these works would
approximation where only the reduced density matrix is
used all apply an additional approximation relative to
that the microscopically derived Lindblad equations
sometimes the two detectors can end up entangled.
this work (see, e.g., [28–33, 38–42]), most notably that
the literature which seem to conflict with the results of
dresses the fact that there are several different results in
an arbitrary initial joint detector state. This section ad-
bear expected to scramble any information contained in
prising since a fast/hot environment should generically
separable and maximally mixed is not in itself very sur-
VI. COMPARISON WITH ROTATING-WAVE
approximation
The fact that the steady state at late times (71) is
separable and maximally mixed is not in itself very sur-
prising since a fast/hot environment should generically
be expected to scramble any information contained in
an arbitrary initial joint detector state. This section ad-
resses the fact that there are several different results in
the literature which seem to conflict with the results of
this work (see, e.g., [28, 33, 35, 32]), most notably that
times the two detectors can end up entangled.
What makes comparison to these works difficult is
that the microscopically derived Lindblad equations
used all apply an additional approximation relative to
our work: the RWA. Beginning with a Born–Markov
approximation where only the reduced density matrix is
slowly-varying (as described in (36)), these works would
find a GKSL equation whose Kossakowski matrix has in
general some negative eigenvalues (hence spoiling com-
plete positivity) as well as explicit dependence on diver-
gences which cannot be renormalized into an effective
Hamiltonian. The RWA is then applied to rectify this
apparent CP-violation, dropping certain terms in the
GKSL equation under the guise that they should not
be important when the system oscillates quickly. After
applying the RWA, the resulting Kossakowski matrix
then ends up having non-negative eigenvalues for any
sizes of parameters in the problem, including large de-
tector gaps \( \Omega \gtrsim a \), and the expression is free of any
non-renormalizable divergences.
As we have already argued in the preceding sections
(motivated by an Effective Field Theory (EFT) way of
thinking), the Markovian approximation has a domain
of validity that restricts the parameter space that we can
use. Once this is recognized and we strictly remain in
this subset of the parameter space, the resulting Marko-
vian dynamics is CP without further approximations.
Applying RWA at best will only restrict the domain
of validity even further, making the resulting master
equation even more restrictive in its use. Our results
demonstrate that not only is RWA unnecessary, but in
general one should always track the regime of validity
of all approximations involved, otherwise one risks ob-
taining nonsensical master equations and output states
that do not reflect the physical problem at hand.
In this last section, we investigate what happens if we
apply the RWA anyway, within the domain of validity of
the (“correct”) Markovian approximation, so as to make
an easier point of comparison to the aforementioned lit-
erature. The point of this exercise is two-fold: first to
emphasize that both the dynamics studied in this pa-
er and the RWA yields late-time states which are not
entangled (when constrained to be within the regime
of validity of Markovianity). Furthermore, we show that
the dynamics between the two approaches can differ not-
tably, which means that carelessly applying the RWA
alters physical predictions significantly.
A. The RWA-based solution
Let us now check what happens if we were to per-
form the RWA and see if the differences between this
and the dynamics reported in this work are significant.
Following for example [58], taking the RWA amounts to
dropping all terms coming from
\[
\sigma^j_+ \sigma^k_\pm \rho_{AB}(\tau), \quad \sigma^j_\pm \rho_{AB}(\tau) \sigma^k_\pm.
\] (94)
In the interaction picture, these terms arise from prod-
ucts of the monopole operators with a phase \( e^{\pm i(E+\bar{E})\tau} \)
that correspond to fast “counter-rotating” terms (here
\( E, \bar{E} \in \{\Omega, 0, 0, -\Omega\} \) denotes the spectrum of the sys-
tem Hamiltonian \( \mathbf{h} = \mathbf{h}_\lambda + \mathbf{h}_0 \)). The standard lore
is that these terms oscillate much more quickly com-
pared to the slow “co-rotating” terms that come with
\[\text{\textsuperscript{19}}\text{Strictly speaking, enforcing the validity bounds [22] is about re-
mainning in the perturbative/non-degenerate regime as opposed to just being Markovian.}\]
\[\text{\textsuperscript{20}}\text{The fact that (iii) implies that } L \neq 0 \text{ is disallowed is unsurpris-
ing firstly because the environment-induced interaction diverges in this limit, so the density matrix does not gener-
cally decohere properly. This is not a real problem
because for gapless regime we can fully solve the
dynamics non-perturbatively (see [84]); (2) since
we also have } \Omega/a \ll 1, \text{ it must mean that
}\]
\[\Omega/a \sim g \] (93)
so that } \Omega/a \text{ is small enough for Markovianity to
hold, but large enough for the perturbative calcula-
tion to work.}
\[ e^{i(E-E')\tau}, \text{ and so should be neglected. This procedure,} \]
more rigorously described by Davies [60] [61], yields a Lindblad equation of the form [45] with Kossakowski matrix \( \gamma^{(\text{RWA})} \), with components

\[
[ \gamma_{\alpha\beta}^{(\text{RWA})} ] = \begin{bmatrix} \frac{g^2 C_{\beta}}{2} & 0 & 0 \\ 0 & \frac{g^2 C_{\alpha}}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix},
\] (95a)

\[
[ \gamma_{\alpha\beta}^{(\text{RWA})} ] = \begin{bmatrix} \frac{g^2 C_{\beta}}{2} & 0 & 0 \\ 0 & \frac{g^2 C_{\alpha}}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix},
\] (95b)

\[ \mathcal{L} \mathcal{T} \] (96)

\[ c.f. \text{ Eqs. (56) and (66). The matrix } \mathcal{L} \mathcal{T} + g^2 \mathcal{L} \mathcal{T} \text{ can be inverted and the general solution for the } X \text{-block is} \]

\[ x^{(\text{RWA})}(\tau) = e^{(\mathcal{L} \mathcal{T} + g^2 \mathcal{L} \mathcal{T}) \tau} (x(0) - x_s) + x_s, \] (100)

where \( x_s \) ends up evaluating to be \( x_s \) the same steady-state solution as given earlier in Eq. (62) without applying RWA.

For the \( O \)-block one can check that the block-diagonal matrix \( \mathcal{N}_0 + g^2 \mathcal{N}_2^{(\text{RWA})} \) is invertible with all eigenvalues having negative real parts. This proves that at late times the \( O \)-block decays to zero for arbitrary initial states of the field. Together, we have shown that for any initial state of the field, the steady state solution at late times is \textit{exactly} the same as without RWA: it is also a separable mixed state at leading order in perturbation theory:

\[ \rho_{\alpha\beta}^{(\text{RWA})}(\infty) = \frac{1}{2} \otimes \frac{1}{2} + \mathcal{O}(g^4) = \rho_{\alpha\beta}(\infty). \] (101)

Being explicit about the dynamics in the RWA case, it turns out that at \( \mathcal{O}(g^2) \) the matrices \( \mathcal{L} \mathcal{T} + g^2 \mathcal{L} \mathcal{T} \) and \( \mathcal{N}_0 + g^2 \mathcal{N}_2^{(\text{RWA})} \) have the same eigenvalues as the earlier (non-RWA) eigenvalues listed in Eqs. (60) and (68). Since the real part of these eigenvalues are all negative this confirms that RWA also sinks towards a separable mixed state.

The main difference between applying RWA and the earlier Markovian description arises when one tracks the finite-time dependence of the components of the density matrix. For example, let us consider \( \rho_{14}^{(\text{RWA})}(\tau) \), which can be obtained from two of the components of \( x^{(\text{RWA})}(\tau) \) in Eq. (100). We find that

\[ \rho_{14}^{(\text{RWA})}(\tau) \approx \rho_{14}(0) e^{-2i\Omega - 2\gamma C_s} \tau. \] (102)

In contrast, for the non-RWA version the answer is somewhat more complicated: one way to proceed is to compute the (right) eigenvectors \( x^X \) of the matrix \( \mathcal{L} \mathcal{T} + g^2 \mathcal{L} \mathcal{T} \) (as a series in \( g \)), so that the solution (63) is equivalent to the ansatz

\[ x(\tau) = \sum_{j=1}^{7} c_j e^{X_j^X \tau} r_j^X + x_s, \] (103)

where the coefficients \( c_j \) may also be computed as a series in \( g \). This gives
\[ \rho_{14}(\tau) \simeq \left( \rho_{14}(0) + \frac{igC_x \text{Re}[\rho_{14}(0)]}{\Omega} + \frac{ig^2C_x [2\rho_{22}(0) + 2\rho_{33}(0) - 1]}{2\Omega} \right) e^{-2i\Omega - 2g^2C_x \tau} + \frac{g^2K_x [2\rho_{11}(0) + \rho_{22}(0) + \rho_{33}(0) - 1]}{2\Omega} e^{-2g^2C_x \tau} \]

\[ - \frac{ig^2C_x \left( - \frac{C_x}{\sqrt{2}} + \sqrt{8 + \frac{C_x}{\sqrt{2}}} \right) \left( 2\rho_{22}(0) + 2\rho_{33}(0) - 1 - \frac{C_x}{\sqrt{2}} + \sqrt{8 + \frac{C_x}{\sqrt{2}}} \right) \right) e^{g^2 \left( -3C_x - \sqrt{C_x^2 + 8C_x^2} \right) \tau} \]

\[ - \frac{ig^2C_x \left( \frac{C_x}{\sqrt{2}} + \sqrt{8 + \frac{C_x}{\sqrt{2}}} \right) \left( 2\rho_{22}(0) + 2\rho_{33}(0) - 1 - \frac{C_x}{\sqrt{2}} + \sqrt{8 + \frac{C_x}{\sqrt{2}}} \right) \right) e^{g^2 \left( -3C_x + \sqrt{C_x^2 + 8C_x^2} \right) \tau} \]

where \( O(g^4) \) effects have been neglected. In both cases, for any given initial data \( \rho_{14}(0) \) we have \( \rho_{14}(\tau) \rightarrow 0 \) and \( \rho_{14}^{(\text{RWA})} \rightarrow 0 \) at very late times. In other words, both non-RWA and RWA solutions have the same late-time behaviour for \( \rho_{14} \), which we already know since the stationary state at late time is diagonal in the uncoupled energy eigenbasis.

The main difference between Eqs. (105) and (102) can be understood in the context of the theorems outlined by Davies [60, 61], and amounts to the simultaneous limit \( g^2 \rightarrow 0 \) and \( \tau \rightarrow \infty \) while keeping \( g^2a\tau \sim O(1) \). What is important to note is that Davies’ limit in fact includes taking RWA: while the result of Davies is of course mathematically sound, the master equation obtained via Davies’ approach cannot be used if we insist on not applying RWA. To put it another way, Davies’ theorem does not account for the free parameters in the microscopic Hamiltonian which will vary from problem to problem. Since we started from a UDW interaction (which is the microscopic description of the setup), we are obliged to restrict our attention to a subset of parameter space where Born-Markov approximations apply. This in turn requires us to restrict to “high-temperature” regime \( \Omega/a \ll 1 \) (and all the complicated validity relations found earlier). We only get the same result as Davies’ approach if we also apply RWA, and from an EFT perspective this means that we have to add more constraints to the parameter space in addition to the validity relations we have found earlier. These restrictions are not given by Davies’ theorem and depends on the system under consideration.

### B. Entanglement generation and degradation: with RWA vs without RWA

As shown in the previous subsection, at finite times the two procedures yield different predictions since the density matrices \( \rho_{14}^{\text{RWA}}(\tau) \neq \rho_{14}(\tau) \) (compare for example [105] and [102]). In particular, it can be shown that the RWA result approaches the full result when \( \Omega L \) is large. This is consistent with the fact that RWA may lead to superluminal signalling in relativistic settings, yet large \( \Omega L \) is precisely the regime where the detectors are so widely separated that the superluminal (but finite-time) propagation becomes negligible. This means that RWA and the full result particularly disagrees precisely when causal relations between the two detectors matter.

In this subsection we underline this point by studying the finite-time entanglement between the two detectors, showing RWA evolution can in general differ significantly from the non-RWA effective Markovian evolution studied in this work. We choose to study negativity [85] as the entanglement monotone, defined by

\[ \mathcal{N}[\rho_{AB}] = \frac{||\rho_{AB}^\text{tr}||_1 - 1}{2}, \]

where \( \Gamma_A \) denotes the partial transpose with respect to subsystem A and \( || \cdot ||_1 \) is the trace norm.

We now show what happens to the entanglement generation and degradation by plotting \( \mathcal{N}[\rho_{14}(\tau)] \) for various initial states \( \rho_{14}(0) \) by comparing RWA-based vs non-RWA based solutions. We do not attempt to reproduce the plots in [20] since they are outside the domain of validity of the Born-Markov approximations (namely \( \Omega/a \ll 1 \) must always be enforced in what follows).

In Figure 1 we show the case when the detectors begin the evolution in their ground states \( \mid \downarrow \downarrow \rangle \). The common parameter choices are \( \Omega/a = 0.01 \), \( g = 0.01 \) and we consider both smaller \( (aL = 0.25) \) and larger \( (aL = 2) \) separations. We see a stark difference between the RWA-based solution and the one without RWA: at all times the RWA-based solution cannot generate entanglement, while the full Born-Markov solution studied in this work (without RWA) can get temporarily entangled at later times. The smaller the detector separation, the longer the entanglement persists for non-RWA solutions. Above \( aL \sim 2 \) both solutions remain separable at all times.

We emphasize that our results are completely within the domain of validity of the approximations (see Section 7 for how to check its validity). This can be compared with Figure 2 where we show the case when the detectors are initially prepared in states \( \mid \downarrow \uparrow \rangle \), based on the choice of states in [20] with or without RWA for the same set of parameters. This time, the amount of entanglement is larger, however the differences between RWA and non-RWA are very slight: the negativity are
FIG. 1. Initially prepared in ground state \(|\downarrow\downarrow\rangle\) with or without RWA. The common parameter choices are \(\Omega/a = 0.01\) and \(g = 0.01\). Left: \(aL = 0.25\). Right: \(aL = 2\). Note that for RWA-based solution the detectors cannot get entangled from the ground state even though the non-RWA solution could. This plot is reliable for the part where \(g^2 \tau \gg 1\).

FIG. 2. Initially prepared in states \(|\downarrow\uparrow\rangle\) based on the choice of states in [30], with or without RWA. The common parameter choices are \(\Omega/a = 0.01\) and \(g = 0.01\). Left: \(|\downarrow\uparrow\rangle\), \(aL = 0.25\). Right: \(|\downarrow\uparrow\rangle\), \(aL = 2\). The difference between the negativities with or without RWA is of order \(10^{-9}\) to \(10^{-7}\). Very similar, and one can check that their differences \(\mathcal{N} - \mathcal{N}^{(\text{RWA})}\) is of order \(10^{-9}\) to \(10^{-7}\) (i.e., way below \(\sim g^2\)).

In Figure 3, we show the case when the detectors are in the Bell state \(|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)\) with the same parameter choices. Since the Bell state is maximally entangled, there is no surprise in having entanglement degradation especially after interacting with the environment. This scenario is particularly interesting because it highlights what RWA actually does to entanglement dynamics: it “smoothens out” oscillatory behaviour of negativity obtained with only Born-Markov approximation, as can be seen from the zoomed-in version in the middle figure of Figure 3. The oscillatory behaviour also is modulated by a decaying function at late times (the rightmost figure) and the differences in negativity is \(O(g^2)\). Therefore, we see that the non-RWA solution approaches RWA result at very late times. Figure 3 gives an explicit demonstration of how RWA is a form of averaging/coarse-graining, essentially by removing the oscillatory components and get the overall large-timescale behaviour right. It is worth noting that Figure 3 has similar feature as the one shown in Figure 1 of [35], even though there the parameters chosen are strictly speaking outside the validity Markovian approximation (due to large \(\Omega/a \gg 1\)).

To summarize, the results we obtained so far show that indeed analyses obtained with or without RWA can drastically differ depending on the physical setups and initial data at hand. In Figure 1, the non-RWA solution can generate entanglement while the RWA solution cannot; in Figure 3, we see the opposite where the entanglement dynamics is hardly distinguishable. On a closer look, which is best demonstrated using Bell states, we see that the RWA indeed applies averaging/coarse-graining procedure so that any oscillatory behaviour at short timescales are smoothened out. We have made sure that all our plots are within the regime of validity in which the dynamics is manifestly CP and Markovian even without RWA.

VII. CONCLUSION

In this work we argue that the lack of CP property after performing the Born-Markov approximation in the standard open systems approach is problematic, as there is residual memory in the system operators that was
not removed when performing the Markovian approximation. This leads to spurious divergences and lack of complete positivity, which typically ends up being resolved by ad hoc procedures such as the RWA. We show this explicitly in the context of the oft-studied model of two accelerating detectors interacting with a quantized massless scalar field in flat spacetime. The fact that the bath (quantum field) is relativistic is important as it is well-known that rotating wave approximations can lead to important causality violations [65]: this has to do with the fact that for fields in their vacuum states, both the co-rotating and counter-rotating terms are important and there is no single “rotating frame” that can counter all frequencies of the bath modes (i.e. the field is a continuum of infinitely many modes, as opposed to quantum optics settings where the bath is a laser tuned to a single particular frequency).

Our work has a wider implication to generic open system framework: more specifically, it suggests that the “infamous” property of being non-CP for Redfield-type equations seen in standard literature (see, e.g., [59]) is not quite correct. The problem is that when one performs the Born-Markov approximations, one has to restrict the parameter spaces for which the resulting equation is valid. Our example shows that the CP property is already guaranteed simply by faithfully staying in the regime where the Markovian approximation is valid. Applying RWA to fix the non-CP problem amounts in some sense to “shifting the goalpost”: while the resulting GKSL-RWA equation is manifestly CP even for large $\Omega/a \gtrsim 1$, we are not allowed to do so because we needed $\Omega/a \ll 1$ to even arrive at this step. Therefore any calculation for $\Omega/a \gtrsim 1$ is automatically not reliable.

This problem of “misusing” certain open systems approximations is actually worse than it appears because there is a general expectation that accelerating detectors should thermalize and entangle in some form or another. While both expectations are fine, showing that they do hold properly is rather tricky. For example, we here show that both the RWA-based and non-RWA-based dynamics have the same stationary solutions given by the maximally mixed state: this is to be understood as a high-temperature thermal state since we are in the regime where $\Omega/a \ll 1$. Therefore, the asymptotic state cannot easily be used to test the validity of approximations taken. This is exacerbated by the fact that in the RWA-based approach, one can obtain an actual thermal Gibbs state with arbitrarily-sized Unruh temperature without properly removing the memory effect when taking the Markovian limit. This is a very tempting outcome, but we view this as an example of getting an expected result via incorrect reasoning.

It is interesting to see that there is merit in approaching problems in open quantum systems by treating the problem as an EFT (i.e. in the Open EFT framework [72, 86–108] typically used for studying quantum fields themselves as opposed to qubits). From an EFT perspective, when a hierarchy of scales can be utilized (in our case between the timescales of the environment and system), relative “effective” simplicity can arise. Furthermore, every EFT has its domain of validity and every approximation shrinks this domain; everything works so long as one remains strictly within the regime where the approximation is expected to work. Conversely, the breakdown of any approximate equation necessarily arises due to being outside the domain of validity of that approximation. Following this EFT line of thought, applying the RWA/ secular approximation to fix CP-violations is an odd thing to do because it amounts to moving the target by changing the original problem to another (possibly unrelated) problem.

We suspect that there are broader classes of problems within the open system framework that have these sort of spurious issues arising from apparent non-CP properties of approximate master equations. One line of investigation we are pursuing is to identify whether such problems arise in more general open systems settings simply due to forcing the resulting equation to work outside the domain of applicability of approximations taken. If these issues can be solved by properly accounting for memory effects then this would make the open system framework more reliable, robust and more widely applicable for making physical predictions.
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Appendix A: Nakajima-Zwanzig evolution of the $X$-block

In this Appendix we give the explicit evolution equations for the $X$-block $\rho_{AB,X}^I(\tau)$ defined in (25) for the Nakajima-Zwanzig master equation (26) at $\mathcal{O}(g^2)$ given in §IV. The $X$-block contains five independent matrix elements. The coupled integro-differential equations for these matrix elements are given by:

\[
\frac{\partial \rho_{11}}{\partial \tau} = 2g^2 \int_0^\tau ds \text{Re}[\mathcal{W}_s(s)] \cos(\Omega s) \left( -2\rho_{11}(\tau - s) + \rho_{22}(\tau - s) + \rho_{33}(\tau - s) \right) \\
+ 2g^2 \int_0^\tau ds \text{Im}[\mathcal{W}_s(s)] \sin(\Omega s) \left( 2\rho_{11}(\tau - s) + \rho_{22}(\tau - s) + \rho_{33}(\tau - s) \right) \\
- 2g^2 e^{-2i\Omega \tau} \int_0^\tau ds \ e^{i\Omega s} \mathcal{W}_s(s) \rho_{14}^I(\tau - s) - 2g^2 e^{2i\Omega \tau} \int_0^\tau ds \ e^{-i\Omega s} \mathcal{W}_s(s) \rho_{14}^I(\tau - s) \\
+ 4g^2 \int_0^\tau ds \left( \text{Re}[\mathcal{W}_s(s)] \cos(\Omega s) + \text{Im}[\mathcal{W}_s(s)] \sin(\Omega s) \right) \text{Re}[\rho_{23}^I(\tau - s)] \\
(A1)
\]

\[
\frac{\partial \rho_{22}}{\partial \tau} = 2g^2 \int_0^\tau ds \text{Re}[\mathcal{W}_s(s)] \cos(\Omega s) \left( 1 - 3\rho_{22}(\tau - s) - \rho_{33}(\tau - s) \right) \\
+ 2g^2 \int_0^\tau ds \text{Im}[\mathcal{W}_s(s)] \sin(\Omega s) \left( 1 - 2\rho_{11}(\tau - s) - \rho_{22}(\tau - s) - \rho_{33}(\tau - s) \right) \\
+ 2g^2 e^{-2i\Omega \tau} \int_0^\tau ds \ e^{i\Omega s} \text{Re}[\mathcal{W}_s(s)] \rho_{14}^I(\tau - s) + 2g^2 e^{2i\Omega \tau} \int_0^\tau ds \ e^{-i\Omega s} \text{Re}[\mathcal{W}_s(s)] \rho_{14}^I(\tau - s) \\
- 4g^2 \int_0^\tau ds \left( \text{Re}[\mathcal{W}_s(s)] \text{Re}[\rho_{23}^I(\tau - s)] + \text{Im}[\mathcal{W}_s(s)] \text{Im}[\rho_{23}^I(\tau - s)] \right) \cos(\Omega s) \\
(A2)
\]

\[
\frac{\partial \rho_{33}}{\partial \tau} = 2g^2 \int_0^\tau ds \text{Re}[\mathcal{W}_s(s)] \cos(\Omega s) \left( 1 - \rho_{22}(\tau - s) - 3\rho_{33}(\tau - s) \right) \\
+ 2g^2 \int_0^\tau ds \text{Im}[\mathcal{W}_s(s)] \sin(\Omega s) \left( 1 - 2\rho_{11}(\tau - s) - \rho_{22}(\tau - s) - \rho_{33}(\tau - s) \right) \\
+ 2g^2 e^{-2i\Omega \tau} \int_0^\tau ds \ e^{i\Omega s} \text{Re}[\mathcal{W}_s(s)] \rho_{14}^I(\tau - s) + 2g^2 e^{2i\Omega \tau} \int_0^\tau ds \ e^{-i\Omega s} \text{Re}[\mathcal{W}_s(s)] \rho_{14}^I(\tau - s) \\
- 4g^2 \int_0^\tau ds \left( \text{Re}[\mathcal{W}_s(s)] \text{Re}[\rho_{23}^I(\tau - s)] - \text{Im}[\mathcal{W}_s(s)] \text{Im}[\rho_{23}^I(\tau - s)] \right) \cos(\Omega s) \\
(A3)
\]
\[ \frac{\partial \rho_{14}'}{\partial \tau} = -4g^2 \int_0^\tau ds \ e^{i\Omega s} \text{Re}[\mathcal{W}_s] \rho_{14}'(\tau - s) + 4g^2 e^{2\Omega s} \int_0^\tau ds \ e^{-i\Omega s} \text{Re}[\mathcal{W}_s] \text{Re}[\rho_{23}'](\tau - s) \]
\[ -2g^2 e^{2\Omega s} \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] e^{-i\Omega s} \left( 1 - 2\rho_{23}'(\tau - s) - 2\rho_{33}'(\tau - s) \right) \]
\[ -2i g^2 e^{2\Omega s} \int_0^\tau ds \ \text{Im}[\mathcal{W}_s] e^{-i\Omega s} \left( 1 - 2\rho_{11}'(\tau - s) - \rho_{12}'(\tau - s) - \rho_{33}'(\tau - s) \right) \]

(A4)

\[ \frac{\partial \rho_{13}'}{\partial \tau} = 2g^2 e^{-2\Omega s} \int_0^\tau ds \ e^{i\Omega s} \text{Re}[\mathcal{W}_s] \rho_{13}'(\tau - s) + 2g^2 e^{2\Omega s} \int_0^\tau ds \ e^{-i\Omega s} \text{Re}[\mathcal{W}_s] \rho_{13}'(\tau - s) \]
\[ -4g^2 \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] \cos(\Omega s) \rho_{23}'(\tau - s) + 2g^2 \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] \cos(\Omega s) \left( 1 - 2\rho_{22}'(\tau - s) - 2\rho_{33}'(\tau - s) \right) \]
\[ + 2g^2 \int_0^\tau ds \ \text{Im}[\mathcal{W}_s] \sin(\Omega s) \left( 1 - 2\rho_{11}'(\tau - s) - 2\rho_{22}'(\tau - s) \right) \]

(A5)

Appendix B: Nakajima-Zwanzig evolution of the O-block

In this Appendix we give the explicit evolution equations for the O-block \( \rho_{1i1O}''(\tau) \) defined in (24) for the Nakajima-Zwanzig master equation (25) at \( \mathcal{O}(g^2) \) given in (14) The O-block contains four independent matrix elements. The coupled integro-differential equations for these matrix elements are given by:

\[ \frac{\partial \rho_{12}''}{\partial \tau} = -2g^2 \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] \cos(\Omega s) \left( \rho_{12}'(\tau - s) - \rho_{34}'(\tau - s) \right) - \text{Im}[\mathcal{W}_s] \left( \rho_{12}'(\tau - s) + \rho_{34}'(\tau - s) \right) \]
\[ -2g^2 \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] e^{i\Omega s} \rho_{12}'(\tau - s) + 2g^2 e^{2\Omega s} \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] e^{-i\Omega s} \rho_{12}'(\tau - s) \]
\[ -2g^2 e^{2\Omega s} \int_0^\tau ds \ e^{-i\Omega s} \mathcal{W}_s(\tau - s) \rho_{24}'e^{i\Omega s} + 2g^2 \int_0^\tau ds \ \rho_{24}'(\tau - s) \left( \text{Re}[\mathcal{W}_s] \cos(\Omega s) + \text{Im}[\mathcal{W}_s] \sin(\Omega s) \right) \]
\[ + 2g^2 e^{2\Omega s} \int_0^\tau ds \ e^{-i\Omega s} \text{Re}[\mathcal{W}_s] \rho_{12}'(\tau - s) - 2g^2 \int_0^\tau ds \ \mathcal{W}_s(\tau) \cos(\Omega s) \rho_{13}'(\tau - s) \]

(B1)

\[ \frac{\partial \rho_{13}''}{\partial \tau} = -4g^2 \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] e^{i\Omega s} \rho_{13}(\tau - s) + 2i g^2 \int_0^\tau ds \ \mathcal{W}_s(\tau) \sin(\Omega s) \rho_{13}(\tau - s) \]
\[ + 2g^2 \int_0^\tau ds \ \rho_{24}'(\tau - s) \left( \text{Re}[\mathcal{W}_s] \cos(\Omega s) + \text{Im}[\mathcal{W}_s] \sin(\Omega s) \right) + 2g^2 e^{2\Omega s} \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] e^{-i\Omega s} \rho_{13}'(\tau - s) \]
\[ -2g^2 e^{2\Omega s} \int_0^\tau ds \ \mathcal{W}_s(\tau) e^{-i\Omega s} \rho_{34}'(\tau - s) + 2g^2 \int_0^\tau ds \ \rho_{34}'(\tau - s) \left( \text{Re}[\mathcal{W}_s] \cos(\Omega s) + \text{Im}[\mathcal{W}_s] \sin(\Omega s) \right) \]
\[ + 2g^2 e^{2\Omega s} \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] \left( e^{-i\Omega s} \rho_{13}'(\tau - s) - 2g^2 \int_0^\tau ds \ \mathcal{W}_s(\tau) \cos(\Omega s) \rho_{12}'(\tau - s) \right) \]

(B2)

\[ \frac{\partial \rho_{24}''}{\partial \tau} = -2g^2 \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] e^{i\Omega s} \rho_{24}'(\tau - s) - 2g^2 \int_0^\tau ds \ \left( \text{Re}[\mathcal{W}_s] \cos(\Omega s) + \text{Im}[\mathcal{W}_s] \sin(\Omega s) \right) \rho_{24}'(\tau - s) \]
\[ + 2g^2 \int_0^\tau ds \ \left( \text{Re}[\mathcal{W}_s] \cos(\Omega s) - \text{Im}[\mathcal{W}_s] \sin(\Omega s) \right) \rho_{13}'(\tau - s) + 2g^2 e^{2\Omega s} \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] e^{-i\Omega s} \rho_{24}'(\tau - s) \]
\[ -2g^2 e^{2\Omega s} \int_0^\tau ds \ \mathcal{W}_s(\tau) e^{-i\Omega s} \rho_{12}'(\tau - s) + 2g^2 \int_0^\tau ds \ \rho_{12}'(\tau - s) \left( \text{Re}[\mathcal{W}_s] \cos(\Omega s) - \text{Im}[\mathcal{W}_s] \sin(\Omega s) \right) \]
\[ + 2g^2 e^{2\Omega s} \int_0^\tau ds \ e^{-i\Omega s} \text{Re}[\mathcal{W}_s] \rho_{34}'(\tau - s) - 2g^2 \int_0^\tau ds \ \mathcal{W}_s(\tau) \cos(\Omega s) \rho_{34}'(\tau - s) \]

(B3)

\[ \frac{\partial \rho_{34}''}{\partial \tau} = -2g^2 \int_0^\tau ds \ \left( \text{Re}[\mathcal{W}_s] \cos(\Omega s) + \text{Im}[\mathcal{W}_s] \sin(\Omega s) \right) \rho_{34}'(\tau - s) - 2g^2 \int_0^\tau ds \ \text{Re}[\mathcal{W}_s] e^{i\Omega s} \rho_{34}'(\tau - s) \]
where the constant coefficients $\rho_{12}(\tau - s) + 2g^2 e^{2i\Omega \tau} \int_0^\tau ds \text{Re} [\mathcal{W}_s(s)] e^{-i\Omega_s} \rho_{24}^*(\tau - s)$

$+ 2g^2 \int_0^\tau ds \text{Re} [\mathcal{W}_s(s)] e^{-i\Omega_s} \rho_{13}^*(\tau - s) - 2g^2 e^{2i\Omega \tau} \int_0^\tau ds \text{Re} [\mathcal{W}_s(s)] e^{-i\Omega_s} \rho_{12}^*(\tau - s)$

$- 2g^2 \int_0^\tau ds \text{Re} [\mathcal{W}_s(s)] e^{-i\Omega_s} \rho_{24}^*(\tau - s) + 2g^2 e^{2i\Omega \tau} \int_0^\tau ds \text{Re} [\mathcal{W}_s(s)] e^{-i\Omega_s} \rho_{12}^*(\tau - s)$

(B4)

Appendix C: When is Alice's detector Markovian?

In this section we expand the argument leading to Eq. (42), which shows that Alice’s detector is only Markovian when $\Omega / a \ll 1$ even in the “standard” Markovian approach. Recall that the Nakajima-Zwanzig equation for the off-diagonal components of Alice’s detector is (38), repeated here for convenience:

$$\frac{d\rho_{12}}{d\tau} \simeq ig^2 D \rho_{12} + F_{NZ}(\tau) ,$$

where the counter-term $g^2 D \rho_{12}^*(\tau)$ (see the definition (C3) below) included to ensure that the Markovian solution oscillates at the physical detector gap [83]. Applying the “standard” Markovian approximation $\rho_\alpha(\tau - s) \simeq \rho_\alpha(\tau)$ commonly employed in the literature yields Eq. (39), we get

$$\frac{d\rho_{12}}{d\tau} \simeq ig^2 D \rho_{12} + F_M(\tau) ,$$

with a counter-term $g^2 D \rho_{12}^*(\tau)$ included to ensure that the Markovian solution oscillates at the physical detector gap [83]. Applying the “standard” Markovian approximation $g_{\alpha}(\tau - s) \simeq g_\alpha(\tau)$ commonly employed in the literature yields Eq. (39), we get

$$\frac{d\rho_{12}}{d\tau} \simeq ig^2 D \rho_{12} + F_M(\tau) ,$$

with a counter-term also included. Here we have defined a constant $D$ that is UV-regulated (i.e. UV-divergent in the limit $\epsilon \to 0^+$):

$$D := 2 \int_0^\tau ds \text{Re} [\mathcal{W}_s(\tau)] \sin(\Omega_s) = \Omega / 2\pi^2 \log(\epsilon^2 a) + \Omega / 2\pi^2 \text{Re} \left[ \psi(0) - i/\alpha \right] ,$$

(C3)

where $\psi(0)$ is the digamma function [83], and $\epsilon$ the finite UV cutoff associated with the $i\epsilon$-prescription of the correlator $\mathcal{W}_s(s)$ defined in [23]. Using the definition of constant $C$ from [11], Eq. (C2) simplifies to

$$\frac{d\rho_{12}}{d\tau} \simeq -g^2 C \rho_{12}(\tau + g^2 (C - iD)e^{2i\Omega \tau} \rho_{12}^*(\tau).$$

(C4)

The Markovian solution of Eq. (C2) is given by

$$\rho_{12}(\tau) \simeq A e^{-g^2 C \tau} + B e^{-g^2 (C + 2iD) \tau}$$

(C5)

where the constant coefficients $A$ and $B$ are given by

$$A = \rho_{12}(0) + \rho_{12}^*(0) \frac{g^2 (D + iC)}{2 \Omega} \quad \text{and} \quad B = -\rho_{12}^*(0) \frac{g^2 (D + iC)}{2 \Omega} .$$

(C6)

We are interested in the conditions under which the RHS of (C1) is approximately equal to the RHS of (C2), i.e., when $F_{NZ}(\tau) \simeq F_M(\tau)$. To self-consistently answer this question we here insert the Markovian solution (C5) into both $F_{NZ}(\tau)$ and $F_M(\tau)$ and explore constraints on the parameters that yield $F_{NZ}(\tau) \simeq F_M(\tau)$. Substituting the Markovian solution (C5) into $F_{NZ}(\tau)$ and $F_M(\tau)$ we get

$$F_{NZ}(\tau) = 2g^2 e^{-g^2 C \tau} \int_0^\tau ds \text{Re} [\mathcal{W}_s(s)] e^{+i\Omega_s} (B^* - A) e^{+g^2 C s} + 2g^2 e^{-g^2 C + 2iD \tau} \int_0^\tau ds \text{Re} [\mathcal{W}_s(s)] e^{-i\Omega_s} (-B + A^*) e^{+g^2 C s} ,$$

(C7)

$$F_M(\tau) = 2g^2 e^{-g^2 C \tau} \int_0^\tau ds \text{Re} [\mathcal{W}_s(s)] e^{-i\Omega_s} (B^* - e^{+i\Omega_s} A) + 2g^2 e^{-g^2 C + 2iD \tau} \int_0^\tau ds \text{Re} [\mathcal{W}_s(s)] (-e^{+i\Omega_s} B + A e^{-i\Omega_s}) .$$

(C8)
Now, if we assume that \( g^2 C \ll a \) and \( \alpha \gg 1 \) then (C7) becomes approximately

\[
F_{NZ}(\tau) \approx 2g^2 e^{-g^2 \tau^2} \int_0^\infty ds \Re \mathcal{W}(s) e^{i\Omega s} e^{-A + B^*} + 2g^2 e^{(-g^2C + 2i\Omega)\tau} \int_0^\tau ds \Re \mathcal{W}(s) e^{i\Omega s} e^{-A + B^*} .
\] (C9)

However, for arbitrary \( \Omega > 0 \) this is in general not approximately equal to \( F_M(\tau) \), since

\[
F_{NZ}(\tau) - F_M(\tau) \approx 2g^2 e^{-g^2 \tau^2} B^* \int_0^\infty ds \Re \mathcal{W}(s) (e^{i\Omega s} - e^{-i\Omega s}) + 2g^2 e^{(-g^2C + 2i\Omega)\tau} B \int_0^\tau ds \Re \mathcal{W}(s) (e^{i\Omega s} - e^{-i\Omega s}) .
\] (C10)

The only way we can have \( F_{NZ}(\tau) - F_M(\tau) \approx 0 \) in the above is if we additionally assume that \( \Omega/a \ll 1 \) so that \( e^{i\Omega s} \approx e^{-i\Omega s} \approx 1 \) under the integral sign, as claimed in the main text surrounding Eq. (42).

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**Appendix D: Some useful integrals**

In this appendix we compute the functions \( C_\times \) and \( K_\times \) defined in (51a) and (51b), as well as \( D'_\times \) and \( S'_\times \) defined in (79b) and (79a) (where we note that \( C_\times, D'_\times \) and \( S'_\times \) have all been computed in [18] [20]).

First we compute \( C_\times \) and \( S_\times \), and to this end we define the related Fourier cosine and sine transforms

\[
C_\times(\Omega) := 2 \int_0^\infty ds \Re \mathcal{W}_\times(s) \cos(\Omega s) ,
\]

\[
S_\times(\Omega) := 2 \int_0^\infty ds \Im \mathcal{W}_\times(s) \sin(\Omega s) .
\]

The values of these integrals depend on the proper acceleration \( a \), detector separation \( L \) as well as the energy gap \( \Omega \). These integrals have the properties

\[
C_\times = \lim_{\Omega \to 0^+} C_\times(\Omega) , \quad S_\times = \lim_{\Omega \to 0^+} \frac{dS_\times(\Omega)}{d\Omega} ,
\]

and are useful definitions because the thermality encoded in the correlator \( \mathcal{W}_\times \) implies that \( C_\times(\Omega) \) and \( S_\times(\Omega) \) are related for arbitrary \( \Omega > 0 \). In particular, \( \mathcal{W}_\times \) satisfies the KMS condition,

\[
\mathcal{W}_\times(\tau - i\beta) = \mathcal{W}_\times^*(\tau) ,
\]

with \( \beta = 2\pi/a = T_0^{-1} \) the inverse Unruh temperature, which in turn implies the detailed balance relationship

\[
C_\times(\Omega) = -\coth \left( \frac{\pi \Omega}{a} \right) S_\times(\Omega) .
\] (D5)

Since the imaginary part of the Wightman function is precisely half the expectation value of the field commutator, we have (see [12])

\[
\Im\langle 0 | \phi(t, \mathbf{x}) \phi(t', \mathbf{x'}) | 0 \rangle = \frac{i}{4\pi^2 |\Delta \mathbf{x}|} \left[ \delta(\Delta \mathbf{t} + |\Delta \mathbf{x}|) - \delta(\Delta \mathbf{t} - |\Delta \mathbf{x}|) \right] ,
\]

where \( \Delta \mathbf{x} = \mathbf{x} - \mathbf{x'} \) and \( \Delta \mathbf{t} = \mathbf{t} - \mathbf{t'} \). Substituting the trajectories (1) and setting \( \tau = \tau' \) gives

\[
\Im\langle 0 | \mathcal{W}_\times(s) | 0 \rangle = -\frac{a\delta \left( s - \frac{2}{a} \sinh^{-1}(aL/2) \right)}{8\pi L\sqrt{1 + (aL/2)^2}} .
\] (D7)
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