Vacuum properties of high quality value tuning fork in high magnetic field up to 8 Tesla and at mK temperatures

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Abstract. Tuning forks are very popular experimental tools widely applied in low and ultra low temperature physics as mechanical resonators and cantilevers in the study of quantum liquids, STM and AFM techniques, etc. As an added benefit, these forks being cooled, have very high Q-value, typically $10^6$ and their properties seems to be magnetic field independent. We present preliminary vacuum measurements of a commercial tuning fork oscillating at frequency 32 kHz conducted in magnetic fields up to 8 T and at temperature $\sim 10$ mK. We found an additional weak damping of the tuning fork motion depending on magnetic field magnitude and we discuss physical nature of the observed phenomena.

1. Introduction
Quartz tuning forks are versatile mechanical resonators with really high Q-values responsible for their superior sensitivity. No wonder that they are applied in all sub-fields of low temperature physics in the study of quantum liquids, STM and AFM techniques, etc. [1, 2, 3, 4, 5, 6, 7, 8] and even as the important part in scanning probe techniques like AFM, STM, etc. [9, 10, 11]. In the latter case the fork measurements are often carried out in vacuum and at low temperatures including strong magnetic fields applied. Despite its obvious importance, the influence of strong magnetic fields on the vacuum resonant properties of such tuning fork at mK temperature range (to our knowledge) has not been studied yet. Therefore, the main aim of our work was to investigate the response of tuning fork under such conditions.

2. Experimental details
To perform our measurements a commercially available quartz tuning fork resonating at $\sim 32$ kHz was used. In order to be able to measure the tuning fork in strong magnetic fields a few modifications have been made. Firstly, the metal can was removed and magnetic leads were replaced with non-magnetic ones (twisted thin copper wire pair in our case). Once out of the can, the dimensions of the tuning fork were measured by an optical microscope (see fig.1). Finally, the bare tuning fork was fixed on a specially designed copper holder and this whole setup was mounted on the cold finger of our cryogen-free dilution refrigerator Oxford Triton 200, which is capable to cool our samples down to 10 mK in the magnetic fields up to 8 T. The orientation of tuning fork’s prongs was parallel to the applied magnetic field.
The response of tuning fork in the form of piezoelectric current on driving force was measured by verified technique [2, 12]. The driving force i.e. an AC-voltage slowly swept in frequency was provided by the function generator Agilent 33521A. An additional attenuator attenuated this excitation voltage by 40 dB. When driven with AC voltage at frequency close to the fork resonant frequency, the quartz crystal starts to oscillate and the piezoelectric current flows as a direct consequence of periodic changes of the crystal lattice polarization in time. This resulting piezoelectric current was detected and converted to voltage by home-made current-to-voltage (I/V) converter with gain $10^5$ V/A [13]. The output voltage signal from I/V converter was measured by the phase-sensitive (lock-in) amplifier SR 830, which splits the measured signal into two phase components: in phase (absorption) component and quadrature (dispersion) component relative to the reference signal provided by above mentioned function generator.

The typical resonant curves are shown on the right side of fig. 1. Experimental data can be fitted using well known Lorentz relations

$$I_{\text{abs}} = I_0 \frac{(f\Delta f)^2}{(f_0^2 - f^2)^2 + (f\Delta f)^2}$$

$$I_{\text{disp}} = I_0 \frac{f\Delta f(f_0^2 - f^2)^2}{(f_0^2 - f^2)^2 + (f\Delta f)^2}$$

(1) (2)

to obtain the resonant frequency $f_0$, the frequency linewidth $\Delta f$ and the amplitude of piezoelectric current $I_0$. There are still some constant and linear backgrounds in actual measurements present, which were taken into account as well [12]. The piezoelectric current $I_0$ is directly proportional to the velocity $v$ of prong tip of tuning fork. The constant of proportionality is the tuning fork constant $\alpha$ and can be determined experimentally using following expression [2]:

$$\alpha = \sqrt{\frac{2 m_{\text{vac}} \Delta \omega}{R}}.$$  

(3)

Here $\Delta \omega = 2\pi \Delta f$ is the angular linewidth, $R$ is the resistance of tuning fork at resonance, that models the damping of the fork motion and can be obtained from the fit of $I_0$ vs. the amplitude of driving AC voltage $U_{\text{exc}}$. Finally, $m_{\text{vac}} = 0.25 \rho T W L$ is the effective mass of one
3. Results and discussion

All measurements were performed in vacuum at ~ 10 mK. Figure 2 shows dependencies of the piezoelectric current amplitude $I_0$ on excitation voltage $V_{\text{exc}}$ and magnetic field applied. The dependence of current amplitude $I_0$ on excitation voltage amplitude $V_{\text{exc}}$ suggests that the tuning fork resistance $R$ (i.e. the damping of the fork motion) is rising with the increase of magnetic field (the corresponding slope - the conductance showed in fig 2 is decreasing). Similarly, the frequency linewidth $\Delta f$ is increasing as well. As measurements were carried out in vacuum, there is no influence of an external environment on the tuning fork motion and the tuning fork resistance in zero magnetic field $R_0$ reflects only an intrinsic damping processes. This intrinsic process of energy dissipation can be associated with shear friction as consequence of periodically bending crystal lattice of quartz.

The linear fits to the experimental data show a non-zero offset and there is a slight deviation of frequency linewidth $\Delta f$ observed at small excitation voltages $U_{\text{exc}} \leq 30 \, \mu V$. Both these phenomena can be attributed to the contribution of the TTL logic (used as a reference signal for lock-in amplifier) to the excitation voltage $U_{\text{exc}}$ due to the presence of capacitive coupling [12]. Thus, a non-zero piezoelectric current $I_0$ can also be detected even for zero excitation voltage amplitude $U_{\text{exc}}$. However, the corresponding signal measured by lock-in amplifier is continuously shifted from 0° to 90° in phase as $U_{\text{exc}} \to 0$ V. This also explains deviation of the measured linewidths $\Delta f$ from constant value for low excitation voltage amplitudes.

The increase of the fork resistance $R$ with the rising magnitude of magnetic field indicates the presence of additional damping mechanism acting on tuning fork motion. Assuming that intrinsic damping process in zero field ($R_0$) i.e. the shear friction is magnetic field independent, a magnetic contribution to the tuning fork resistance $R_{\text{mag}}$ can be simply estimated as $R_{\text{mag}} = R_B - R_0$. The fig. 3 illustrates the resulting dependence of $R_{\text{mag}}$ on applied magnetic field. What could be an origin of the additional, the field depended damping in quartz tuning fork?
The forced motion of one prong of the tuning fork pointing in the direction of z axis in zero magnetic field is described by the Euler-Bernoulli equation

$$\rho A \frac{\partial^2 x}{\partial t^2} + 2\gamma \rho A \frac{\partial x}{\partial t} + E_m J_y \frac{\partial^4 x}{\partial z^4} = F(z) \cos(\omega t),$$  \hspace{1cm} (4)$$

where $A$ is the arm cross-section, $\rho$ is the density, $E_m$ is the Young’s modulus of the material, $J_y$ is the moment of inertia of the prong cross-section and $\gamma$ characterizes the damping process. The external force $F(z)$ is provided by the AC-voltage applied on electrodes of the tuning fork and resulting harmonic electric field $E$ creates the time dependent charge polarization $p(t)$ of the crystal lattice of the tuning fork. This time dependent charge polarization $p(t)$ i.e. the piezoelectric current carries information about fork motion.

Once the magnetic field is applied on tuning fork, in general, there are two effects acting simultaneously on dynamics of the tuning fork motion. The first, in our geometry - magnetic field applied in parallel with prongs of the tuning fork, magnetic field affects the excitation force provided by electric field $E$ by adding a force acting perpendicularly to the former one $(qE + \mathbf{p} \times \mathbf{B})$. This additional Lorentz force has a tendency to twist the oscillating dipole moments inside quartz crystal from the direction of electric field $E$. This effect effectively reduces the magnitude of the piezoelectric current and thus increases the fork’s resistance. The ratio $(\mathbf{p} \times \mathbf{B})/qE$ defines a tangent of an angle ($\theta$), by which the oscillating dipoles are twisted. Then, using this simplest physical picture and assuming the smallness of $\theta$, the magnetic contribution to the tuning fork resistance can be expressed in form $R_{mag} = a_1 B + a_2 B^3$, where $a_1$ and $a_2$ are constants. The second effect is a damping associated with generation of the eddy currents in fork’s metallic electrodes of during its oscillating motion. An EDAX analysis of the electrodes showed that electrodes consist of Cr, Ag and Sn. The damping caused by eddy currents is proportional to $B^2$.

The lines in fig. 3 show the fits to the experimental data considering each mechanism (dipole twist and eddy currents) separately and acting together (without $B^3$ term). As follows from data fits the contribution due to the eddy currents does not fit experimental data properly, so additional contribution originating in dipole twist needs to be considered as well. Moreover, the dipole twist mechanism is capable to describe our experimental data by itself. Both above mentioned mechanisms should depend on relative orientation of the tuning fork and magnetic field, which could help to discriminate these two effects. However, the most important fact which comes out from data analysis is that in our configuration (tuning fork’s prongs orientated in parallel with magnetic field), the magnitude of the $\alpha$ constant is almost independent on magnetic field within error of $\sim 5\%$ (see Table 1.).
4. Conclusions
We have presented that the fork constant $\alpha$ of our 32 kHz quartz tuning fork is almost independent on the magnetic field applied. This result can be of large importance for measurements performed in high magnetic fields, e.g. in various scanning probe techniques utilizing quartz tuning forks as probes. The origin of additional damping of tuning fork motion due to the presence of magnetic field is still an open question. To elucidate this problem more experiments with different relative orientation of tuning fork’s prongs with respect to magnetic field are needed.

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Table 1. Values (and their errors) determined from experiment for frequency linewidth $\Delta f$, tuning fork resistance $R$ and calculated values (errors) of fork constant $\alpha$ according to relation (3) for different magnetic fields applied.

| $B$ (T) | $\Delta f$ (Hz) | $\Delta f_{err}$ (Hz) | $R_{err}$ ($\Omega$) | $\alpha_{err}$ (A.s.m$^{-1}$) |
|--------|----------------|----------------------|----------------------|-------------------------------|
| 0      | 0.06894        | ±0.00225             | 4370.59876           | 57.15386 $\pm 6.14709 \times 10^{-6}$ |
| 1      | 0.07905        | ±0.00237             | 5011.08829           | 76.42912 $\pm 6.14755 \times 10^{-6}$ |
| 2      | 0.08885        | ±0.00271             | 5523.22733           | 84.74155 $\pm 6.20787 \times 10^{-6}$ |
| 3      | 0.09695        | ±0.00261             | 5871.39784           | 39.92771 $\pm 6.28943 \times 10^{-6}$ |
| 4      | 0.10649        | ±0.00151             | 6369.90292           | 40.71070 $\pm 6.32847 \times 10^{-6}$ |
| 6      | 0.13341        | ±0.00165             | 8185.47428           | 85.27016 $\pm 6.24880 \times 10^{-6}$ |
| 8      | 0.17520        | ±0.00448             | 10164.13935          | 94.14669 $\pm 6.42605 \times 10^{-6}$ |

References
[1] Clubb D O, Buu O V L, Bowley R M, Nyman R, Owens-Bradley J R 2004 *J. Low Temp. Phys.* **136** 1
[2] Blaauwgeers R, Blažková M, Človečko M, Eltsov V B, de Graaf R, Hosio J, Krusius M, Schmoranzer D, Schoewe W, Skrbek L, Skyba P, Soltntsev R E and Zmeev D E 2007 *J. Low Temp. Phys.* **146** 537
[3] Blažková M, Človečko M, Gažo E, Skrbek L, Skyba P 2007 *J. Low Temp. Phys.* **148** 305
[4] Blažková M, Človečko M, Eltsov V B, Gažo E, de Graaf R, Hosio J J, Krusius M, Schmoranzer D, Schoewe W, Skrbek L, Skyba P, Soltntsev R E and Vinen W F 2000 *J. Low Temp. Phys.* **150** 525
[5] Pentti E M, Tuorinniemi J T, Salmela A J, Sebedash A P 2008 *Phys. Rev. B* **78** 064509
[6] Bradley D I, Fear M J, Fisher S N, Guenault A M, Haley R P, Lawson C R, McClintock P V E, Pickett G R, Schanen R, Tsepelin V, Wheatland L A 2009 *J. Low Temp. Phys.* **156** 116
[7] Schmoranzer D, La Mantia M, Sheshin G, Gritsenko I, Zadorozhko A, Rotter M, Skrbek L 2011 *J. Low Temp. Phys.* **163** 317
[8] Ahlström S L, Bradley D I, Človečko M, Fisher S N, Guenault A M, Guise E A, et al 2014 *J. Low Temp. Phys.* **175** 140
[9] Gunther P, Fischer U Ch, Dransfeld K 1989 *Appl. Phys. B* **48** 89
[10] Rychen J, Ihn T, Studerus P, Hermann A, Ensslin K 1999 *Rev. Sci. Instrum.* **70** 2765
[11] Rychen J, Ihn T, Studerus P, Hermann A, Ennslin K, Hug H J, van Schendel P J A and Guntherod H J 2000 *Rev. Sci. Instrum.* **71** 1695
[12] Skyba P 2010 *J. Low Temp. Phys.* **160** 219
[13] Holt S, Skyba P 2012 *Rev. Sci. Instrum.* **83** 064703