Generic Friedberg-Lee Symmetry of Dirac Neutrinos

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Abstract

We write out the generic Dirac neutrino mass operator which possesses the Friedberg-Lee (FL) symmetry and find that its corresponding neutrino mass matrix is asymmetric. Following a simple way to break the FL symmetry, we calculate the neutrino mass eigenvalues and show that the resultant neutrino mixing pattern is nearly tri-bimaximal. Imposing the Hermitian condition on the neutrino mass matrix, we also show that the simplified ansatz is consistent with current experimental data and favors the normal neutrino mass hierarchy.

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Recent solar, atmospheric, reactor and accelerator neutrino experiments have provided us with very convincing evidence that neutrinos are slightly massive and lepton flavors are significantly mixed. The flavor mixing of three lepton families can be described by a $3 \times 3$ unitary matrix $U$, which is usually parameterized as:

$$
U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},
$$

(1)

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$ (for $ij = 12, 13$ and $23$), and $\delta$ is the CP-violating phase.

If neutrinos are Majorana particles, $U$ should contain two more CP-violating phases, which are referred to as the Majorana phases and have nothing to do with neutrino oscillations. The latest global analysis of current neutrino oscillation data yields $30.9^\circ \leq \theta_{12} \leq 37.8^\circ$, $35.1^\circ \leq \theta_{23} \leq 53.4^\circ$ and $0^\circ \leq \theta_{13} < 12.4^\circ$ with $3\sigma$ uncertainty, but the phase $\delta$ remains entirely unconstrained. While the absolute mass scale of three neutrinos is not yet fixed, their two mass-squared differences have already been determined to a quite good degree of accuracy: $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = (7.14 \cdots 8.19) \times 10^{-5}$ eV$^2$ and $\Delta m_{32}^2 \equiv m_3^2 - m_2^2 = \pm(2.06 \cdots 2.81) \times 10^{-3}$ eV$^2$ with $3\sigma$ uncertainty.

Many theoretical and phenomenological attempts have been made to interpret the smallness of three neutrino masses and the largeness of two neutrino mixing angles. Among them, the flavor symmetry approach is in particular simple and predictive. A new and intriguing flavor symmetry is the one proposed by Friedberg and Lee (FL). In the basis where the mass eigenstates of three charged leptons are identified with their flavor eigenstates, the Dirac neutrino mass operator can be written as

$$
\mathcal{L}_{FL} = \sum_{\alpha,\beta} Y_{\alpha\beta} \left( \overline{\nu}_\alpha - \overline{\nu}_\beta \right) \left( \nu_\alpha - \nu_\beta \right),
$$

(2)

where $\alpha$ and $\beta$ run over $e$, $\mu$ and $\tau$. The FL symmetry means that $\mathcal{L}_{FL}$ is invariant under the translational transformations $\nu_e \to \nu_e + z$, $\nu_\mu \to \nu_\mu + z$ and $\nu_\tau \to \nu_\tau + z$, where $z$ is a constant element of the Grassmann algebra independent of space and time. The corresponding neutrino mass matrix is a symmetric matrix,

$$
M_{FL} = \begin{pmatrix}
b + c & -b & -c \\
-b & a + b & -a \\
-c & -a & a + c
\end{pmatrix},
$$

(3)
where \( a = Y_{\mu \tau} + Y_{\tau \mu} \), \( b = Y_{e \mu} + Y_{\mu e} \) and \( c = Y_{\tau e} + Y_{e \tau} \). Note that the determinant of \( M_{\text{FL}} \) is vanishing (i.e., \( \text{Det}(M_{\text{FL}}) = 0 \)), and thus one of the neutrinos must be massless. One may explicitly break the FL symmetry of \( \mathcal{L}_{\text{FL}} \) to make realistic predictions for both neutrino masses and flavor mixing angles. So far some interesting works have been done to apply the FL symmetry to the Majorana neutrino mass operator [10, 11, 12], to combine the FL symmetry with the seesaw mechanism [13, 14], to extend the FL symmetry to the quark sector [15, 16], and to generalize the FL symmetry in a specific model containing some scalar fields [17].

Here we notice that \( \mathcal{L}_{\text{FL}} \) in Eq. (3) is not the most generic mass operator of Dirac neutrinos which obeys the FL symmetry. The Dirac neutrino mass operator

\[
\mathcal{L}'_{\text{FL}} = \sum_{\alpha,\beta} \sum_{\alpha',\beta'} Y_{\alpha \beta}^{\alpha' \beta'} \left( \nu_{\alpha'} - \nu_{\beta'} \right) \left( \sigma_{\alpha} - \sigma_{\beta} \right),
\]

where the Greek superscripts and subscripts run over \( e, \mu \) and \( \tau \), is more general than \( \mathcal{L}_{\text{FL}} \) and also invariant under the translational transformations \( \nu_e \rightarrow \nu_e + z, \nu_\mu \rightarrow \nu_\mu + z \) and \( \nu_\tau \rightarrow \nu_\tau + z \). Its corresponding neutrino mass matrix \( M'_{\text{FL}} \) takes the form

\[
M'_{\text{FL}} = \begin{pmatrix}
B + C & -B - D & -C + D \\
-B + D & A + B & -A - D \\
-C - D & -A + D & A + C
\end{pmatrix},
\]

where

\[
A = \frac{1}{2} \left[ - \left( Y_{\mu e}^\tau + Y_{e \mu}^\tau - Y_{e \mu}^\tau - Y_{\mu e}^\tau \right) + \left( Y_{e \mu}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) - \left( Y_{\mu e}^{\nu \mu} + Y_{\mu e}^{\nu \mu} - Y_{\mu e}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) \right] + \left( Y_{\mu e}^{\nu \mu} + Y_{\mu e}^{\nu \mu} - Y_{\mu e}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right);
\]

\[
B = \frac{1}{2} \left[ - \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} \right) \right] + \left( Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} \right) + \left( Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} \right);
\]

\[
C = \frac{1}{2} \left[ \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) \right] + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right);
\]

\[
D = \frac{1}{2} \left[ \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) \right] + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right) + \left( Y_{\mu e}^{\nu \mu} + Y_{e \mu}^{\nu \mu} - Y_{e \mu}^{\nu \mu} - Y_{\mu e}^{\nu \mu} \right).\]

\( \text{Eq. (6)} \)
We see that $M'_{FL}$ is an asymmetric matrix and its asymmetry is characterized by non-vanishing $D$. Given $D = 0$, $M'_{FL}$ turns out to be equivalent to $M_{FL}$.

Based on the above observation, we are going to focus our interest on the phenomenological implications of $L'_FL$ for Dirac neutrinos. We shall follow a simple way to break the FL symmetry of $L'_FL$ and obtain the neutrino mass matrix $M_\nu = M'_{FL} + m_0\mathbf{1}$ with $\mathbf{1}$ being the identity matrix. Then we shall show that a nearly tri-bimaximal neutrino mixing pattern, which is favored by current neutrino oscillation data, can always be obtained from $M_\nu$. A simpler and Hermitian form of $M_\nu$ will also be discussed in detail.

It is worth remarking that the nature of massive neutrinos remains unclear, although most theorists believe that they should be Majorana particles. However, there do exist some interesting models in the literature [18], where massive neutrinos are treated as Dirac fermions. Before the nature of neutrinos is experimentally identified, we feel that it makes sense to study the phenomenology of both Dirac and Majorana neutrinos.

Although $M'_{FL}$ in Eq. (5) is asymmetric, it is easy to verify that its determinant vanishes as $M_{FL}$ does. Hence one of the mass eigenvalues of $M'_{FL}$ must be zero. To generate non-vanishing masses for all the three neutrinos, here we follow Ref. [9] to break the FL symmetry of $L'_FL$:

$$L_\nu = L'_FL + m_0 \sum_\alpha \bar{\nu}_\alpha \nu_\alpha. \quad (7)$$

where $m_0$ is in general a complex parameter, and $\alpha$ runs over $e$, $\mu$ and $\tau$. Corresponding to $L_\nu$, the Dirac neutrino mass matrix reads

$$M_\nu = M'_{FL} + m_0\mathbf{1} = \begin{pmatrix} B + C & -B - D & -C + D \\ -B + D & A + B & -A - D \\ -C - D & -A + D & A + C \end{pmatrix} + m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (8)$$

We see that $L_\nu$ or $M_\nu$ can possess the exact $\mu$-$\tau$ symmetry only when both $B = C$ and $D = 0$ are satisfied. To derive the neutrino mass spectrum and the flavor mixing pattern from $M_\nu$, we consider the following unitary transformation:

$$U^\dagger M_\nu M_\nu^\dagger U = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}. \quad (9)$$
where \( m_i \) (for \( i = 1, 2, 3 \)) stand for three neutrino masses. Because we have taken the basis in which the mass and flavor eigenstates of three charged leptons are identical, the unitary matrix \( U \) in Eq. (9) is just the neutrino mixing matrix linking the neutrino mass eigenstates \((\nu_1, \nu_2, \nu_3)\) to the neutrino flavor eigenstates \((\nu_e, \nu_\mu, \nu_\tau)\).

A salient feature of \( M_\nu \) is that the sum of three elements in its any row or column equals \( m_0 \), implying that one of its three eigenvalues must be \( m_0 \). For this reason, the unitary transformation \( U \) used to diagonalize the Hermitian matrix \( M_\nu M_\nu^\dagger \) must have an eigenvector which contains three equal components \( 1/\sqrt{3} \). It is then possible to express \( U \) as a production of the tri-bimaximal mixing matrix \( U_0 \) and a complex rotation matrix \( U_\theta \) in the (1,3) plane:

\[
U = U_0 \otimes U_\theta = \left( \begin{array}{ccc} 2 & 1 & 0 \\ \frac{\sqrt{6}}{1} & \frac{\sqrt{3}}{1} & \frac{1}{1} \\ -\frac{\sqrt{6}}{1} & \frac{\sqrt{3}}{1} & \frac{-\sqrt{2}}{1} \end{array} \right) \otimes \left( \begin{array}{ccc} \cos \theta & 0 & \sin \theta e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin \theta e^{i\delta} & 0 & \cos \theta \end{array} \right),
\]

in which \( \delta \) signifies CP violation and is equivalent to the one defined in Eq. (1). After a straightforward calculation, we obtain

\[
\delta = -\text{arg} (T_{13}) ,
\]

\[
\theta = \frac{1}{2} \arctan \left( \frac{2|T_{13}|}{T_{33} - T_{11}} \right),
\]

where

\[
T_{11} = 3 \left(|B|^2 + |C|^2 + \text{Re}[B^*C] + |D|^2 - \text{Re}[(C - B) D^*] \right) + 3\text{Re}[(B + C) m_0^*] + |m_0|^2 ,
\]

\[
T_{33} = |B|^2 + |C|^2 - \text{Re}[B^*C] + 4|A|^2 + 2\text{Re}[(B + C) A^*] + 3\text{Re}[(C - B) D^*] + 3|D|^2 + 4\text{Re}[A m_0^*] + \text{Re}[(B + C) m_0^*] + |m_0|^2 ,
\]

\[
T_{13} = \sqrt{3} \left(|C|^2 - |B|^2 - i\text{Im}[B^*C] \right) + \sqrt{3}\text{Re}[(B + C) D^*] + 2\sqrt{3}i\text{Im}[(B + C) D^*] + \sqrt{3}(C - B) A^* - 2\sqrt{3}A^*D + \sqrt{3}\text{Re}[(C - B) m_0^*] - 2\sqrt{3}i\text{Im}[D m_0^*] .
\]

Furthermore, three mass eigenvalues of \( M_\nu \) are found to be

\[
m_1 = \sqrt{\frac{1}{2} (T_{11} + T_{33}) - \frac{1}{2} (T_{33} - T_{11}) \cos 2\theta - |T_{13}| \sin 2\theta} ,
\]

\[
m_2 = |m_0| ,
\]

\[
m_3 = \sqrt{\frac{1}{2} (T_{11} + T_{33}) + \frac{1}{2} (T_{33} - T_{11}) \cos 2\theta + |T_{13}| \sin 2\theta} .
\]
Comparing between Eqs. (1) and (10), one may easily arrive at the analytical results of three neutrino mixing angles:

\[
\sin \theta_{12} = \frac{1}{\sqrt{2 + \cos 2\theta}},
\]

\[
\sin \theta_{23} = \frac{\sqrt{2 + \cos 2\theta - \sqrt{3} \sin 2\theta \cos \delta}}{\sqrt{2(2 + \cos 2\theta)}},
\]

\[
\sin \theta_{13} = \frac{2}{\sqrt{6}} |\sin \theta| .
\]  

In addition, we find that the Jarlskog invariant of leptonic CP violation is given by

\[ J = \sin 2\theta \sin \delta/(6\sqrt{3}) \]  

in this phenomenological scenario of Dirac neutrino mixing. Note that \( A, B, C, D \) and \( m_0 \) in \( M_\nu \) can all be complex parameters. Hence it is always possible to find some proper parameter space in which the neutrino mass spectrum obtained in Eq. (13) and the neutrino mixing pattern obtained in Eq. (14) are both compatible with current neutrino oscillation data. In particular, no fine-tuning is needed to make \( U \) consistent with the solar and atmospheric neutrino experiments because \( U \) itself is a nearly tri-bimaximal mixing pattern with small \( \theta \). Instead of carrying out a numerical analysis of \( m_i \) and \( \theta_{ij} \) changing with those model parameters, we shall look at a more specific scenario with \( M_\nu \) being Hermitian in the following.

Given the asymmetric form of \( M_\nu \) in Eq. (8), the Hermitian relation \( M_\nu^\dagger = M_\nu \) can be achieved if and only if \( A, B, C \) and \( m_0 \) are all real and \( D \) is purely imaginary (i.e., \( D^* = -D \)). Let us define \( D = iD' \) and rewrite \( M_\nu \) as

\[
M_\nu = \begin{pmatrix}
B + C & -B - iD' & -C + iD' \\
-B + iD' & A + B & -A - iD' \\
-C - iD' & -A + iD' & A + C
\end{pmatrix}
+ m_0 \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

where \( A, B, C, D' \) and \( m_0 \) are all real. Now \( M_\nu \) is Hermitian and only contains five free parameters. We are going to show that this interesting texture of \( M_\nu \) is actually compatible with current neutrino oscillation data.

With the help of Eqs. (11)—(14), it is straightforward to obtain three neutrino masses and three flavor mixing angles from Hermitian \( M_\nu \) given in Eq. (15). First,

\[
m_1 = \left| (A + B + C + m_0) \mp \sqrt{(A^2 + B^2 + C^2) - (AB + BC + CA) + 3D'^2} \right|,
\]

\[
m_2 = |m_0|,
\]

\[
m_3 = \left| (A + B + C + m_0) \mp \sqrt{(A^2 + B^2 + C^2) - (AB + BC + CA) + 3D'^2} \right|.  
\]  

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Second,

\[
\sin \theta_{12} = \frac{1}{\sqrt{2 + \cos 2\theta}}, \\
\sin \theta_{23} = \frac{\sqrt{2 + \cos 2\theta} - \sqrt{3} \sin 2\theta \cos \delta}{\sqrt{2(2 + \cos 2\theta)}}, \\
\sin \theta_{13} = \frac{2}{\sqrt{6}} |\sin \theta|, \\
\]

where

\[
\delta = \arctan \left( \frac{2D'}{C - B} \right), \\
\theta = \frac{1}{2} \arctan \left( \frac{\sqrt{3} [(C - B)^2 + 4D'^2]}{2A - B - C} \right). \\
\]

Note that \(\delta\) is just the CP-violating phase of \(U\), and \(\theta\) has been restricted to the range \(-\pi/4 \leq \theta \leq \pi/4\). Note also that \(\theta > 0\) and \(\theta < 0\) correspond to the options of “±” signs in the expression of \(m_1\) (or the options of “±” signs in the expression of \(m_3\)) in Eq. (16). Taking account of current experimental constraints on three mixing angles \([7]\), we obtain \(|\theta| < 18^\circ\). The smallness of \(|\theta|\) implies that \(U\) is a nearly tri-bimaximal neutrino mixing pattern.

If both \(B = C\) and \(D' = 0\) hold, then \(M_\nu\) possesses the exact \(\mu\)-\(\tau\) symmetry which gives rise to the exact tri-bimaximal neutrino mixing pattern \(U_0\) (i.e., \(\theta_{12} = \arctan(1/\sqrt{2}) \approx 35.3^\circ\), \(\theta_{13} = 0^\circ\) and \(\theta_{23} = 45^\circ\)). There are two simpler ways to produce the deviation of \(U\) from \(U_0\):

1. \(B \neq C\) and \(D' = 0\). In this special case, we have \(\theta_{13} \neq 0^\circ\) and \(\theta_{23} \neq 45^\circ\) together with \(\delta = 0^\circ\) (CP conservation).

2. \(B = C\) and \(D' \neq 0\). In this special case, we have \(\delta = \pm \pi/2\) (CP violation), \(\theta_{23} = 45^\circ\) and \(\theta_{13} \neq 0^\circ\).

The second possibility is more interesting in the sense that \(|J| = \sin 2\theta/(6\sqrt{3})\) can be as large as a few percent for \(|\theta| \geq 3^\circ\) and may lead to observable CP-violating effects in long-baseline neutrino oscillations.

To illustrate, let us carry out a simple numerical analysis of the parameter space of Hermitian \(M_\nu\) by using current neutrino oscillation data on \((\Delta m_{21}^2, \Delta m_{32}^2)\) and \((\theta_{12}, \theta_{13}, \theta_{23})\) as the inputs. Without loss of generality, we assume \(m_0 > 0\). Our numerical results indicate
that only the normal neutrino mass hierarchy (i.e., $\Delta m_{32}^2 > 0$) is favored in this Hermitian ansatz. The allowed regions of $A$, $B$, $C$, $D'$ and $m_0$ are shown in Fig. 1, where $m_0 \lesssim 0.2$ eV has been taken as a generous upper bound on the absolute neutrino mass scale \cite{21}. Because of $m_0 = m_2$, the lower bound of $m_0$ is $m_0 > \sqrt{\Delta m_{21}^2} \approx 0.09$ eV as one can see from Fig. 1. The Jarlskog invariant $J$ may vary from 0 to 0.057 in the obtained parameter space.

To summarize, we have written out the generic Dirac neutrino mass operator which possesses the FL symmetry and pointed out that its corresponding neutrino mass matrix is actually asymmetric. After introducing a perturbative term to break the FL symmetry, we have calculated the neutrino mass eigenvalues and flavor mixing angles. We find that the resultant neutrino mixing pattern is nearly tri-bimaximal. Imposing the Hermitian condition on the neutrino mass matrix, we have shown that the simplified ansatz is consistent with current experimental data and favors the normal neutrino mass hierarchy.

This work is a simple but useful generalization of the original FL symmetry for Dirac neutrinos. Such a generic FL symmetry can be applied to the quark sector to obtain generic (or Hermitian) quark mass matrices. But it will have no influence on the neutrino mass matrix if massive neutrinos are Majorana particles, because a Majorana neutrino mass matrix must always be symmetric.

In conclusion, the FL symmetry and its breaking mechanism may have a wealth of implications in neutrino phenomenology. The physics behind this interesting flavor symmetry remains unclear to us and deserves a further study, no matter whether massive neutrinos are Dirac fermions or Majorana fermions.

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FIG. 1: The parameter space of $A$, $B$, $C$ and $D'$ versus $m_0$ (all of them in unit of eV) in the scenario of Hermitian $M_\nu$, where only the normal neutrino mass hierarchy (i.e., $\Delta m_{32}^2 > 0$) is allowed.