Yukawa Unification in SO(10)

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Abstract

In simple SO(10) SUSY GUTs the top, bottom and tau Yukawa couplings unify at the GUT scale. A naive renormalization group analysis, neglecting weak scale threshold corrections, leads to moderate agreement with the low energy data. However it is known that intrinsically large threshold corrections proportional to $\tan \beta \sim m_t(M_Z)/m_b(M_Z) \sim 50$ can nullify these $t, b, \tau$ mass predictions. In this paper we turn the argument around. Instead of predicting fermion masses, we use the constraint of Yukawa unification and the observed values $M_t, m_b(m_b), M_\tau$ to constrain SUSY parameter space. We find a narrow region survives for $\mu > 0$ with $\mu, M_{1/2} << m_{16}, A_0 \approx -1.9 m_{16}$ and $m_{16} > 1200$ GeV. Demanding Yukawa unification thus makes definite predictions for Higgs and sparticle masses. In particular we find a light higgs with mass $m_{h}^0 = 114 \pm 5 \pm 3$ GeV and a light stop with $(m_{\tilde{t}_1})_{MIN} \sim 450$ GeV and $m_{\tilde{t}_1} << m_{\tilde{b}_1}$. In addition, we find a light chargino and a neutralino LSP. It is also significant that in this region of parameter space the SUSY contribution to the muon anomalous magnetic moment $a_{\mu}^{SUSY} < 16 \times 10^{-10}$. 
1 Introduction

Grand unification with $SU(5)$, $SO(10)$ or even partial unification with $SU(4)_C \times SU(2)_L \times SU(2)_R$ explains the peculiar standard model charge assignments of quarks and leptons and also the observed family structure. Gauge coupling unification at a scale $M_G \sim 3 \times 10^{16}$ GeV in supersymmetric grand unified theories (SUSY GUTs) fits the low energy data well. Moreover $SO(10)$ SUSY GUTs have many profound features:

- All fermions in one family sit in one irreducible 16 dimensional representation.
- The two Higgs doublets, necessary in the minimal SUSY standard model (MSSM), sit in one irreducible 10 dimensional representation.
- Right-handed neutrinos, necessary for a seesaw mechanism for neutrino masses, are naturally included in the 16 dimensional representation.

In addition, in the simplest version of $SO(10)$ the third generation Yukawa couplings are given by a single term in the superpotential $W = \lambda \, 16 \, 10 \, 16$ resulting in Yukawa unification $\lambda_t = \lambda_b = \lambda_{\tau} = \lambda_{\nu} \equiv \lambda$. Hence, like gauge coupling unification, there is a prediction but this time for $M_G = 180 \pm 15$ GeV with large $\tan \beta \sim 50$ (see for example, Anderson et al.); in good agreement with the data. Note, GUT scale threshold corrections to this Yukawa unification boundary condition are naturally small ($\lesssim 1\%$), unlike the corrections to gauge coupling unification which can easily be several percent (see the Appendix for a discussion of perturbative GUT scale threshold corrections to gauge and Yukawa couplings).

This beautiful prediction is however severely weakened by potentially large weak scale threshold corrections proportional to $\tan \beta$. The complete set of one loop corrections is given by

$$m_b(M_Z) = \lambda_b(M_Z) \, \frac{v}{\sqrt{2}} \cos \beta \left( 1 + \Delta m^\tilde{q}_b + \Delta m^{\tilde{c}^+}_b + \Delta m^{\tilde{\chi}^0}_b + \Delta m^{\log}_b + \Delta m^{EW}_b \right). \tag{1}$$

The first three terms are SUSY mass insertion corrections. The dominant contributions

$$\Delta m^\tilde{q}_b \approx \frac{2a_3}{3\pi} \, \frac{\mu m_{\tilde{g}}}{m^2_b} \, \tan \beta$$

and

$$\Delta m^{\tilde{c}^+}_b \approx \frac{\lambda^2}{16\pi^2} \, \frac{\mu A_t}{m^2_t} \, \tan \beta$$

can be as large as 50%. Note in most regions of SUSY parameter space these two terms have opposite sign. $\Delta m^{\tilde{\chi}^0}_b$ is on the other hand small, $O(\%1)$. The log term results from finite wave function renormalization of the bottom quark; it is positive, independent of $\tan \beta$ and the total from all sources is of order 6%. Finally $\Delta m^{EW}_b$, due to Higgs, W and Z exchange, is negligibly small, $O(\%5)$. There are similar corrections to $m_\tau$ and $m_t$. The chargino corrections $m^{\tilde{c}^+}_b$ are also proportional to $\tan \beta$, but are significantly smaller than $\Delta m^{\tilde{c}^+}_b$ since typically we have $m_{\nu_{\tau}} >> m_{\tilde{\tau}}$. Finally the corrections to $m_t$
are not proportional to $\tan \beta$. The complete set of corrections can be found in the papers by Rattazzi and Sarid \cite{8} and by Pierce et al. \cite{10}.

In most regions of SUSY parameter space $\Delta m^2_{\tilde{g}}$ is dominant and in our conventions $\Delta m^2_{\tilde{g}} > 0$ for $\mu > 0$. The sign of $\mu$ is constrained by experiment; in particular $b \to s \gamma$ and $a_{\mu}^{\text{NEW}}$ both favor $\mu > 0$. The same one loop graphs with a photon or gluon insertion and outgoing $b$ replaced with $s$ contributes to $b \to s \gamma$. The chargino term typically dominates and has opposite sign to the SM and charged Higgs contributions, thus reducing the branching ratio for $\mu > 0$. This is necessary to fit the data since the SM contribution is somewhat too big. $\mu < 0$ would on the other hand constructively add to the branching ratio and is problematic. In addition, the recent measurement of the anomalous magnetic moment of the muon $a_{\mu}^{\text{NEW}} = (g - 2)/2 = 43 (16) \times 10^{-10}$ also favors $\mu > 0$ \cite{11}.

In a recent letter \cite{13} we showed that Yukawa unification with $\mu > 0$, including the complete one loop threshold corrections, is only consistent with the data in a narrow region of SUSY parameter space with $\mu, M_{1/2} \sim 100 - 500$ GeV; $A_0 \sim -1.9 m_{16}$; $m_{10} \sim 1.4 m_{16}$ and $m_{16} > 1200$ GeV. The parameters $m_{16}, m_{10}$ denote the soft SUSY breaking mass terms for squark and slepton, Higgs multiplets, respectively. Note the requirement of Yukawa unification thus dramatically constrains the SUSY particle spectrum and Higgs masses.

In this paper we present a more detailed analysis of the SUSY particle spectrum and the allowed parameter range. In addition to fitting electroweak data and the top, bottom and $\tau$ masses, we also include constraints from $b \to s \gamma$ and $B_s \to \mu^+ \mu^-$. The latter constraints increase the predicted stop mass and the mass of the CP odd Higgs $A^0$, the heavy CP even Higgs $H^0$ and charged Higgs $H^\pm$ at the expense of a small increase in $\chi^2$. We find a light CP even Higgs boson with mass $m_{h^0} = 114 \pm 5 \pm 3$ GeV. In addition in the region where $m_{16} < 2000$ GeV, we find a light chargino with mass $m_{\tilde{\chi}^+} \sim 120 - 240$ GeV, a neutralino LSP with mass $m_{\tilde{\chi}^0} \sim 75 - 160$ GeV, a light stop with mass $m_{\tilde{t}_1} \sim 450 - 540$ GeV $<< m_{\tilde{b}_1}$. The first and second generation squark and slepton masses are of order $m_{16}$. It is also significant that in this region of parameter space we find $a_{\mu}^{\text{SUSY}} < 16 \times 10^{-10}$. Note also some recent discussions of Yukawa unification and $a_{\mu}^{\text{SUSY}}$ \cite{14, 15, 16}.

It is well known that electroweak symmetry breaking with large $\tan \beta$ and $m_{16} >> M_{1/2}$ requires Higgs up/down mass splitting \cite{16}. We find however that the fits to third generation fermion masses are sensitive to the mechanism used to split the Higgs masses. In this paper we consider D term and “Just So” Higgs splitting (defined in the text). We study the sensitivity of our results to small GUT scale threshold corrections to Yukawa couplings. Significantly larger threshold corrections are needed for D term splitting versus the Just So case.

The paper is organized as follows. In section 2 we discuss the analysis. We give the results for the case of Just So Higgs splitting in section 3 and D term Higgs splitting in section 4. The constraints of $b \to s \gamma$ and $B_s \to \mu^+ \mu^-$ and additional experimental tests are considered in section 4. For the impatient reader we present detailed results

\footnote{Note, recent theoretical reevaluations of the standard model contribution are now closer to experiment with $a_{\mu}^{\text{NEW}} = 25.6 (16) \times 10^{-10}$ \cite{12}.}
from some typical points in SUSY parameter space in Table 1 (without) and Table 2 (with) the constraints from $b \rightarrow s\gamma$ and $B_s \rightarrow \mu^+\mu^-$ included. Note, we have included the prediction for $a^\text{SU}_\mu$ in Tables 1 and 2; however it has not been included in the $\chi^2$ function when fitting. Finally some theoretical questions are addressed in section 6.

2 Analysis

We use a top - down approach with a global $\chi^2$ analysis [17]. The input parameters are defined by boundary conditions at the GUT scale. The 11 input parameters at $M_G$ are given by — three gauge parameters $M_G$, $\alpha_G(M_G)$, $\epsilon_3$; the Yukawa coupling $\lambda$, and 7 soft SUSY breaking parameters $\mu$, $M_1/2$, $A_0$, $\tan\beta$, $m_{16}^2$, $m_{10}^2$, $\Delta m_H^2$ ($D_X$) for Just So (D term) case.

These are fit in a global $\chi^2$ analysis defined in terms of physical low energy observables. We use two (one)loop renormalization group [RG] running for dimensionless (dimensionful) parameters from $M_G$ to $M_Z$. We require electroweak symmetry breaking using an improved Higgs potential, including $m_4^t$ and $m_4^b$ corrections in an effective 2 Higgs doublet model below $M_{\text{stop}}$ [18]. The $\chi^2$ function includes 9 observables; 6 precision electroweak data $\alpha_{EM}$, $G_\mu$, $\alpha_s(M_Z)$, $M_Z$, $M_W$, $\rho_{\text{NEW}}$ and the 3 fermion masses $M_{\text{top}}$, $m_b(m_b)$, $M_\tau$.

We fit the central values: $M_Z = 91.188$ GeV, $M_W = 80.419$ GeV, $G_\mu \times 10^5 = 1.1664$ GeV$^{-2}$, $\alpha_{EM}^{-1} = 137.04$, $M_\tau = 1.7770$ GeV with 0.1% numerical uncertainties; and the following with the experimental uncertainty in parentheses: $\alpha_s(M_Z) = 0.1180 (0.0020)$, $\rho_{\text{NEW}} \times 10^3 = -0.200 (1.1)$ [20], $M_t = 174.3 (5.1)$ GeV, $m_b(m_b) = 4.20 (0.20)$ GeV.

We include the complete one loop threshold corrections at $M_Z$ to all observables. In addition we use one loop QED and three loop QCD RG running below $M_Z$. Finally, with regards to the calculated Higgs and sparticle masses, the neutral Higgs masses $h$, $H$, $A^0$ are pole masses calculated with the leading top, bottom, stop, sbottom loop contributions; while all other sparticle masses are running masses.

We minimize $\chi^2$ using the CERN subroutine minuit. In order to present our results we typically keep three parameters (such as $\mu$, $M_{1/2}$, $m_{16}$) fixed and minimize $\chi^2$ with respect to the remaining eight parameters. We then plot our results as contours in the two parameter space.

2.1 EWSB and Higgs mass splitting

The first significant constraint derives from electroweak symmetry breaking [EWSB] in the large tan $\beta$ regime. It has been shown that this typically requires $m_{H_u}^2 < m_{H_d}^2$. In fact more general solutions for EWSB exist with Higgs up/down splitting and with less fine-tuning (see [16] and Rattazzi and Sarid [3]). The range of soft SUSY parameters required is consistent with solution (B) of Olechowski and Pokorski [10].

$^2\epsilon_3$, defined in the Appendix, and $\Delta m_H^2$, $D_X$ parametrize GUT scale threshold corrections to gauge coupling unification and Higgs up/down mass splitting, respectively.

$^3$Capital $M$ is used for pole masses and lower case $m$ for $\overline{\text{MS}}$ running masses.

$^4$The error for $m_b(m_b)$ [14] appears to be quite conservative in view of recent claims to much smaller error bars [21].
In our analysis we consider two particular Higgs splitting schemes, we refer to as Just So and D term splitting. In the first case the third generation squark and slepton soft masses are given by the universal mass parameter $m_{16}$, and only Higgs masses are split: Just So Higgs splitting

$$m_{(H_u, H_d)}^2 = m_{10}^2 (1 \mp \Delta m_H^2)$$

(3)

In this case we find $\Delta m_H^2 \sim 13\%$.

In the second case we assume D term splitting, i.e. that the D term for $U(1)_X$ is non-zero, where $U(1)_X$ is obtained in the decomposition of $SO(10) \rightarrow SU(5) \times U(1)_X$. In this second case, we have: D term splitting

$$m_{(H_u, H_d)}^2 = m_{10}^2 \mp 2D_X,$$

$$m_{(Q, u, e)}^2 = m_{16}^2 + D_X,$$

$$m_{(d, L)}^2 = m_{16}^2 - 3D_X.$$

Here we find $\Delta m_H^2 \equiv 2D_X/m_{10}^2 \sim 5\%$.

Just So Higgs splitting does not at first sight appear to be as well motivated as D term splitting. In the Appendix we present two example mechanisms for Just So Higgs splitting. Here we present the most compelling argument. In $SO(10)$, neutrinos necessarily have a Yukawa term coupling active neutrinos to the "sterile" neutrinos present in the 16. In fact for $\nu_\tau$ we have $\lambda_{\nu_\tau} \bar{\nu}_\tau L H_u$ with $\lambda_{\nu_\tau} = \lambda_t = \lambda_b = \lambda_\tau \equiv \lambda$. In order to obtain a tau neutrino mass with $m_{\nu_\tau} \sim 0.06$ eV (consistent with atmospheric neutrino oscillations), the "sterile" $\bar{\nu}_\tau$ must obtain a Majorana mass $M_{\bar{\nu}_\tau} \geq 10^{13}$ GeV. Moreover, since neutrinos couple to $H_u$ (and not to $H_d$) with a fairly large Yukawa coupling (of order 0.7), they naturally distinguish the two Higgs multiplets. With $\lambda = 0.7$ and $M_{\bar{\nu}_\tau} = 10^{13}$ GeV, we obtain a significant GUT threshold correction with $\Delta m_H^2 = .10$, remarkably close to the value needed to fit the data. At the same time, we obtain a small threshold correction to Yukawa unification $< 3\%$. (For more details, see the Appendix.)

3 Results: Just So Higgs splitting

Since the log corrections $\Delta m_b^{log} \sim O(6\%)$ are positive, they must be cancelled in order to obtain $\Delta m_b \leq -2\%$ to fit $m_b$. For $\mu > 0$ the gluino contribution is positive. The chargino contribution is typically opposite in sign to the gluino, since $A_t$ runs to an infrared fixed point, $A_t \propto -M_{1/2}$ (see for example, Carena et al. [8]). Hence in order to cancel the positive contribution of both the log and gluino contributions, a large negative chargino contribution is needed. This can be accomplished for $-A_t > m_\tilde{g}$ and $m_\tilde{\chi} < m_\tilde{b}$. The first condition can be satisfied for $A_0$ large and negative, which helps pull $A_t$ away from its infrared fixed point. The second condition is also aided by large $A_t$. However in order to obtain a large enough splitting between $m_\tilde{\chi}$ and $m_\tilde{b}$, large values of $m_{16}$ are needed. Note, that for universal scalar masses, the lightest stop is typically lighter than the sbottom. On the other hand, D term splitting with $D_X > 0$ gives $m_\tilde{b} \leq m_\tilde{t}$. Recall
$D_X > 0$ is needed for electroweak symmetry breaking. As a result in the case of Just So boundary conditions excellent fits are obtained for the top, bottom and tau masses; while for D term splitting the best fits give $m_b(m_b) \geq 4.59$ GeV. In this section we give the results for the case of Just So Higgs splitting. The results for D term splitting are discussed in section 4.

In Fig. 1, we show $\chi^2$ contours as a function of $\mu$, $M_{1/2}$ for $m_{16} = (1500$ $2000)$ GeV. The shaded region in all the figures is excluded by the experimental bound on the chargino mass, $m_{\tilde{\chi}^+} > 103$ GeV. The $\chi^2 < 2$ contour for $m_{16} = 1500$ GeV is bounded by shaded $< \mu < 220$ GeV, shaded $< M_{1/2} < 380$ GeV. For $m_{16} = 2000$ GeV the region with $\chi^2 < 1(2)$ is contained within the closed curve bounded by 150 (shaded) $< \mu < 250 (380)$ GeV, 220 (shaded) $< M_{1/2} < 450 (550)$ GeV. In Fig. 2, we show $\chi^2$ contours as a function of $M_{1/2}$, $m_{16}$ for $\mu = 150$ GeV; $\chi^2 < 1$ for $m_{16} \geq 2000$ GeV. We see $\chi^2$ continues to decrease as $m_{16}$ increases.

In Table 1 (Fits 1,2) we present the input parameters and output for two representative points with universal squark and slepton masses $m_{16} = 1500$ (2000) GeV with $\mu = 150$ (200) GeV, $M_{1/2} = 250$ (350) GeV. We find reasonable fits ($\chi^2 \leq 3$) only for $m_{16} \geq 1200$ GeV. For $m_{16} < 1200$ GeV, $\chi^2$ increases rapidly.

The bottom quark mass $m_b(m_b)$ is given in Fig. 3 (Left) for $m_{16} = 2000$ GeV. In Fig. 3 (Right) we show that the fits improve with good fits extending to larger values of $M_{1/2}$ as $m_{16}$ increases ($\mu = 150$ GeV is fixed). It should be clear that $m_b$ is the dominant pull on $\chi^2$ as seen by comparing to the $\chi^2$ contours of Fig. 1 (Right).

In Fig. 4 we plot $A_0/m_{16}$ at $M_Z$ as a function of $\mu$, $M_{1/2}$ for fixed $m_{16} = 2000$ GeV. Good fits are obtained for $A_0 \approx -1.9 m_{16}$ for all $m_{16} > 1200$ GeV. Note, even though $A_0$ is very large, the value of $A_t$ at $M_Z$ is significantly smaller since it is driven to an infra-red quasi fixed point. [We come back to this point when we discuss vacuum stability issues later.] Finally reasonable fits require $m_{10} \sim 1.35 m_{16}$ (Fig. 5 Left) and $\Delta m_H^2 \sim 0.13$ (Fig. 5 Right).
Figure 2: $\chi^2$ contours for $\mu = 150$ GeV.

Figure 3: Contours of constant $m_b(m_b)$ [GeV] for $m_{16} = 2000$ GeV (Left) and for $\mu = 150$ GeV (Right).
Figure 4: $A_0/m_{16}$ contours for $m_{16} = 2000$ GeV, with $\chi^2$ contours overlayed.

Figure 5: $m_{10}/m_{16}$ contours (Left) and $\Delta m_H^2$ contours (Right) for $m_{16} = 2000$ GeV, with $\chi^2$ contours overlayed.
Figure 6: Contours of constant mass insertion corrections to $\delta m_b [%]$ from gluino loops (Left), chargino loops (Right) (with $\chi^2$ contours overlayed) and the total one loop correction $\delta m_b [%]$ (lower Center) for $m_{16} = 2000$ GeV.

The significant positive log correction to $m_b$ is the main reason why Yukawa unification is only possible in a narrow region of SUSY parameter space. In order to compensate this, the chargino mass insertion contribution must be significantly larger than the gluino contribution. In Fig. 6 we give the gluino (Left), chargino (Right) mass insertion corrections to $m_b$ and the total weak scale threshold correction $\delta m_b$ (lower Center) for fixed $m_{16} = 2000$ GeV as a function of $\mu$, $M_{1/2}$. Note in the region of $\chi^2 < 1$ the gluino (chargino) mass insertion corrections are large and of order 13 to 26 % (- 23 to -34 %), while the log correction is 5.6 to 6.6 %. These are the dominant corrections. The total SUSY correction to $m_b$ is -3 to - 4%. In the same region, the total SUSY correction to $M_t$ is 7 - 8 and to $M_\tau$ is - 2 to - 4%.

In summary, we have shown that good fits to $b, t$ and $\tau$ masses are only obtained in a narrow region of SUSY parameter space $A_0 \approx -1.9 \ m_{16}$, $m_{10} \sim 1.35 \ m_{16}$ with
Figure 7: Contours of constant $A^0$ mass [GeV] with fixed $m_{16} = 2000$ GeV.

$m_{16} > 1200$ GeV. A Just So Higgs splitting $\Delta m_H^2 \sim 0.13$ is also required.

This has interesting consequences for the Higgs and supersymmetric particle spectrum. In Figs. 7 - 9 we give the $A^0$, $h^0$, $H^0$, $H^\pm$ masses. In Fig. 7, constant $m_{A^0}$ contours are given as a function of $\mu$, $M_{1/2}$ for fixed $m_{16} = 2000$ GeV. We find, for $\chi^2 < 1$, $m_{A^0} \sim 106 - 112$ GeV. We show constant $m_{A^0}$ contours for $\mu$, $M_{1/2}$ for fixed $m_{16} = 1500$ GeV in Fig. 8 (Left), for $m_{16} = 2000$ GeV (Right) and as a function of $M_{1/2}$, $m_{16}$ for $\mu = 150$ GeV in Fig. 8 (lower Center). We find, for $\chi^2 < 1$, $m_{A^0} \sim 118 - 121$ GeV and $m_{H^\pm} \sim 145 - 149$ GeV.

In Figs. 10 - 13 we show constant mass contours for $\tilde{t}_1$, $\tilde{t}_2$, $\tilde{b}_1$, and $\tilde{\tau}_1$ for fixed $m_{16} = 2000$ GeV. We find for $\chi^2 < 1$, $\tilde{t}_1 \sim 175 - 250$ GeV, $\tilde{t}_2 \sim 630 - 800$ GeV, $\tilde{b}_1 \sim 500 - 650$ GeV, $\tilde{\tau}_1 \sim 250 - 500$ GeV. Note, the upper bounds on squark and slepton masses increase as $m_{16}$ increases. Moreover, the first two generation squark and slepton masses are of order $m_{16}$.

Gaugino masses are smooth functions of $\mu$, $M_{1/2}$. The gluino mass is linear in $M_{1/2}$ and satisfies the empirical relation $m_{\tilde{g}} = 2.5 M_{1/2} + 25$ GeV. In Fig. 14 we show constant mass contours for $\chi^{\pm}_1$ and for $\chi^0_1$, the LSP. We find, for $\chi^2 < 1$, $\chi^{\pm}_1 \sim 120 - 240$ GeV and $\chi^0_1 \sim 75 - 160$ GeV. Finally the GUT scale parameters $M_G$, $\alpha_G(M_G)$, $\epsilon_3$, and $\lambda \sim .65 - .7$ and weak scale parameter $\tan \beta \sim 50 - 52$ are weakly dependent on SUSY parameters.

In the following we evaluate the sensitivity of our results to plausible threshold corrections to Yukawa unification at $M_G$. We consider two loop RG running of dimensionful parameters. We also artificially fix the CP odd Higgs mass $m_{A^0}$ by applying appropriate

\footnote{Note in our analysis we have fit $\alpha_s(M_Z) = 0.118 \pm 0.002$. However if the central value for $\alpha_s(M_Z)$ were to decrease to 0.116 we would obtain good fits for $m_b(m_b)$ in a larger region of SUSY parameter space and still have $|\epsilon_3| < 5\%$. This is because both RG running and the gluino correction to the bottom mass are positive and proportional to $\alpha_s(M_Z)$.}
Figure 8: Contours of constant $h^0$ mass [GeV] with fixed $m_{16} = 1500$ GeV (Left); $m_{16} = 2000$ GeV (Right) and with fixed $\mu = 150$ GeV as a function of $M_{1/2}$, $m_{16}$ (lower Center).
Figure 9: Contours of constant $H^0$ mass [GeV] (Left) and $H^\pm$ mass [GeV] (Right) with fixed $m_{16} = 2000$ GeV.

Figure 10: Contours of constant $\tilde{t}_1$ mass [GeV] for fixed $m_{16} = 2000$ GeV (Left) and fixed $\mu = 150$ GeV (Right) with constant $\chi^2$ contours overlayed.
Figure 11: Contours of constant $\tilde{t}_2$ mass [GeV] for fixed $m_{16} = 2000$ GeV (Left) and fixed $\mu = 150$ GeV (Right) with constant $\chi^2$ contours overlayed.

Figure 12: Contours of constant $\tilde{b}_1$ mass [GeV] for fixed $m_{16} = 2000$ GeV (Left) and fixed $\mu = 150$ GeV (Right) with constant $\chi^2$ contours overlayed.
Figure 13: Contours of constant $\tilde{\tau}_1$ mass [GeV] for fixed $m_{16} = 2000$ GeV (Left) and fixed $\mu = 150$ GeV (Right) with constant $\chi^2$ contours overlayed.

Figure 14: Contours of constant $\tilde{\chi}_1^\pm$ mass [GeV] (Left) and constant $\tilde{\chi}_1^0$ mass [GeV] (Right) for fixed $m_{16} = 2000$ GeV with constant $\chi^2$ contours overlayed.
penalties to the $\chi^2$ function. We then discuss the dependence of the Higgs spectrum and $\chi^2$ as a function of $m_{A^0}$. This will become important when considering the decay $B_s \rightarrow \mu^+ \mu^-$ in section 5. In addition in 5 we discuss constraints from the process $b \rightarrow s \gamma$. Both of these latter processes require a description of Yukawa matrices for the heaviest two families.

3.1 Sensitivity to GUT scale threshold corrections

In this section we check the sensitivity of our results to GUT scale threshold corrections to Yukawa unification. We define $\epsilon_b, \epsilon_t$ by

$$\lambda_i = \lambda (1 + \epsilon_i) \quad \text{with} \quad i = b, t \quad \text{and} \quad \lambda_r \equiv \lambda.$$  \hfill (5)

In Fig. 15 we give $\chi^2$ contours as a function of $\epsilon_b, \epsilon_t$ for $M_{1/2} = 300$ GeV, $\mu = 150$ GeV, $m_{16} = 2$ TeV. Just So scalar mass case. On the upper axis we give the equivalent value of $M_{\nu_r} \ll M_G$ (the Majorana mass of the tau neutrino) which contributes to $|\epsilon_b|$ with $|\epsilon_t| = 0$.

Figure 15: $\chi^2$ contours as a function of $\epsilon_b, \epsilon_t$ for $M_{1/2} = 300$ GeV, $\mu = 150$ GeV, $m_{16} = 2$ TeV. Just So scalar mass case. On the upper axis we give the equivalent value of $M_{\nu_r} \ll M_G$ (the Majorana mass of the tau neutrino) which contributes to $|\epsilon_b|$ with $|\epsilon_t| = 0$. The best motivated correction comes from integrating out a heavy tau neutrino with mass $M_{\nu_r}$. Neutrino oscillations consistent with atmospheric neutrino data suggest $M_{\nu_r} \approx 10^{13}$ GeV corresponding to a correction $\epsilon_b = 2.6\%$ with $|\epsilon_t| = 0$. On the upper axis, in Fig. 15, we give the equivalent value of $M_{\nu_r} \ll M_G$ (the Majorana mass...
of the tau neutrino) which contributes to $|\epsilon_b|$ with $|\epsilon_t| = 0$ (see the Appendix). In section we consider D term Higgs splitting where threshold corrections are absolutely essential for reasonable fits.

### 3.2 Two loop vs. one loop RGEs

In the region of parameter space we consider, with $m_{16} > \text{TeV}$, two loop RG running of soft SUSY breaking parameters may have significant consequences for sparticle masses as well as for electroweak symmetry breaking. We have checked however that a two loop RGE analysis for soft SUSY masses does not significantly affect our results. By this we mean that the same low energy results are obtained with small changes in the GUT scale parameters. In Table 1 (Fit 3) we present the comparison for $m_{16} = 2000 \text{ GeV}$, $\mu = 150 \text{ GeV}$, $M_{1/2} = 300 \text{ GeV}$ fixed with all other input parameters varied to minimize $\chi^2$ using one and two loop RGEs. It is clear that the results are not significantly different from Fit 2. A small change in $A_0$ is sufficient to guarantee positive squark masses squared. Of course, when one uses two loop RGEs for soft scalar masses consistency requires including one loop threshold corrections to these masses at the weak scale. We have not included the latter contributions in our calculations; thus we stick with the one loop RGE analysis from $M_G$ to $M_Z$ for dimensionful parameters.

### 3.3 $\chi^2$ dependence on $m_{A^0}$ mass

In the course of our analysis it became clear that there were two minima for $\chi^2$; with a low and high mass solution for the CP odd Higgs mass (see also Ref. [24]). In order to make this behavior explicit we needed a way to choose particular values of $m_{A^0}$ within the $\chi^2$ analysis. We accomplished this by adding a penalty to the $\chi^2$ function for any value of $m_{A^0}$ outside a narrow range. Note, we then found minima of $\chi^2$ for which this penalty vanished.

In Fig. 16 we plot $\chi^2$ as a function of $m_{A^0}$ for fixed $m_{16} = 2000 \text{ GeV}$, $\mu = 150 \text{ GeV}$, $M_{1/2} = 300 \text{ GeV}$. The global minimum is at $m_{A^0} = 110 \text{ GeV}$ with the local minimum at $m_{A^0} \sim 250 \text{ GeV}$ with approximately 35% larger $\chi^2$. The increased pull to $\chi^2$ is mainly due to the $\rho$ parameter. In Fig. 17 we plot the light Higgs mass vs. $m_{A^0}$. Note, at the minimum of $\chi^2$, $m_{h^0}$ is a steeply rising function of $m_{A^0}$. However it quickly reaches a plateau with $m_{h^0} \sim 119 \text{ GeV}$. In Fig. 18 we see that $m_{H^0}$ and $m_{H^\pm}$ increase linearly with $m_{A^0}$. We conclude therefore that the non-SM Higgs masses are not constrained by Yukawa unification. Nevertheless, all the other predictions (the region of SUSY parameter space, the light higgs mass and the sparticle spectrum) remain unchanged, i.e. independent of the non-SM Higgs masses.
Figure 16: $\chi^2$ as a function of the CP odd Higgs mass $m^0_A$.

Figure 17: Light Higgs mass $m^0_h$ as a function of $m^0_A$. 

17
Figure 18: Heavy $m_{H^0}^0$, light $m_h^0$ and charged Higgs $m_{H^±}$ masses as a function of $m_A^0$.

4 Results: D term Higgs splitting

D term splitting for Higgs up/down masses seems natural. We have thus performed a $\chi^2$ analysis with D term splitting. We find that Yukawa unification does not work in this case, i.e. the best $\chi^2$ obtained is $> 5$. It is easy to understand why D term splitting does not work. $D_X > 0$, needed for electroweak symmetry breaking, makes $m_d^2 < m_u^2$ already at $M_G$ (see Eqn. [4]). Therefore, in this case $m_{b_1} < m_{t_1}$ and hence the gluino contribution to $\Delta m_b$ dominates.

It is important to check whether small threshold corrections to Yukawa unification can change this result. In this analysis we take $|\epsilon_b|, |\epsilon_t|$ as large as 30% in order to overlap with the parameter range considered in a recent paper [14]. In Fig. 19 we plot constant $\chi^2$ contours as a function of $\epsilon_b, \epsilon_t$ for $m_{16} = 2000$ GeV, $\mu = 150$ GeV, $M_{1/2} = 300$ GeV fixed and all other input parameters varied to minimize $\chi^2$. Good fits are obtained with $\epsilon_t \approx 0$ and $\epsilon_b \sim -7\%$ or with $\epsilon_t \sim -\epsilon_b \sim 5\%$. These GUT scale corrections to Yukawa unification are significant. They are needed in this case for the RG evolution from $M_G$ to $M_Z$ to drive $m_{b_1} << m_{t_1}$. This is because Yukawa couplings via the RGEs tend to drive squarks lighter, hence $\lambda_t > \lambda_b$ compensates for the unfavorable boundary condition with $m_d^2 < m_u^2$. Good fits are again obtained in the same narrow region of soft SUSY breaking parameters with $A_0 \sim -1.9 \ m_{16}$ and $m_{10} \sim 1.35 \ m_{16}$, as in the case of Just So Higgs splitting and exact Yukawa unification (see Table 2). A few comments are in order. Neutrino threshold corrections give $\epsilon_b > 0$, assuming the sterile neutrinos obtain a Majorana mass $< M_G$ (see Appendix). In addition, although $\epsilon_t \neq 0$ is $SU(5)$ invariant,

6We do not consider threshold corrections to Yukawa unification in excess of 3% to be small. Such significant corrections would require additional physics explanations.
$\epsilon_b \neq 0$ requires $SU(5)$ breaking threshold corrections.

In Table 1 (Fit 4) we present our best fit point with D term splitting and exact Yukawa unification. This best fit has $\chi^2 > 5$ and is thus unacceptable. Fit 5 on the other hand, is one point in parameter space with D term splitting and significant threshold corrections to Yukawa unification, i.e. $\epsilon_b = -0.08$, $\epsilon_t = 0.02$. The results are comparable to Fit 2 with Just So Higgs splitting and exact Yukawa unification. This point with fixed $m_{16} = 2000$ GeV, $\mu = 150$, $M_{1/2} = 300$ is similar to but not completely consistent with the results of Ref. [14]. Besides the fact that we always have larger values for the GUT scale Yukawa coupling than found in Ref. [14], we also require significantly smaller Yukawa threshold corrections.

5 Experimental Tests & More Constraints

The most unexpected result of this analysis is the constraint on the light Higgs mass. We find $m_h^0 \sim 114\pm5\pm3$ GeV where the first uncertainty comes from the range of SUSY parameters with $\chi^2 \leq 1.5$ and the second is an estimate of the theoretical uncertainties in our Higgs mass. Surely this prediction will be tested at either Run III at the Tevatron or at LHC. We have used the analysis of Ref. [18] which is a good approximation for $m_t A_t/M_{(SUSY)}^2 < 0.5$ with $M_{(SUSY)}^2 = (m_{t1}^2 + m_{t2}^2)^2/2$. In Fig. 20 we plot this ratio. Note, due to our large value for $A_t$ we typically find $m_t A_t/M_{(SUSY)}^2 \sim 0.7$ which is somewhat outside the preferred range. We have also compared our results with FeynHiggsFast and find ours to be larger by about 3 GeV. We are not certain of the reason for this difference.

Yukawa unification alone prefers light $A^0$, $H^0$ and $H^\pm$ masses. However in the region

Figure 19: $\chi^2$ contours as a function of $\epsilon_b, \epsilon_t$ for $M_{1/2} = 300$ GeV, $\mu = 150$ GeV, $m_{16} = 2$ TeV. D term splitting case.
of large tan $\beta$ it has been shown that the process $B_s \rightarrow \mu^+ \mu^-$ provides a lower bound on $m_{A^0} \geq 200$ GeV (as pointed out in the recent works Dedes et al. and also Isidori and Retico) so that it is below the experimental upper bound $B(B_s \rightarrow \mu^+ \mu^-) < 2.6 \times 10^{-6}$ (95% CL). This important new constraint is a consequence of the flavor violating quark-quark-$A^0$ couplings which result from the large threshold corrections to CKM angles obtained in the region of large tan $\beta$. This has only a minor impact on $\chi^2$ as discussed above. We find that $\chi^2$ increases by at most 40% for any $m_{A^0}$ less than $\approx$ 350 GeV (Fig. 16). The light Higgs mass $m_h^0$ is rather insensitive to the value of $m_{A^0}$ (Fig. 17); whereas $m_{H^0}, m_{H^+}$ are linearly dependent on $m_{A^0}$ (Fig. 18). We are thus not able to predict the other Higgs masses. Direct observation of $A^0, H^0, H^\pm$ may be difficult at the Tevatron, but should be possible at LHC. On the other hand, $B_s \rightarrow \mu^+ \mu^-$ is a significant constraint and may be discovered at the Tevatron.

We find that the process $b \rightarrow s\gamma$ also provides significant new constraints on SUSY parameter space. In order to calculate $b \rightarrow s\gamma$ we have included second family data ($m_s(2$ GeV) = 110 $\pm$ 35 MeV, $M_b - M_c = 3.4 \pm 0.2$ GeV, $V_{cb} = 0.0402 \pm 0.0019$) in the $\chi^2$ function in order to self-consistently obtain flavor violating SUSY contributions. We have used a parametrization of the Yukawa couplings at $M_G$ which, though not completely general, fits the data well (see Appendix). We find that the coefficient $C^M_{7}\text{MSSM}$ is of order $-C^S_7\text{SM}$ (see for example, Eqn. 9 in Ref. [24]) with the chargino term dominating by a factor of about 5 over all other contributions. This is due to the light stop $\tilde{t}_1$. In fact, $b \rightarrow s\gamma$ is more sensitive to $m_{\tilde{t}_1}$ than $m_b(m_b)$. This is because the amplitude depends on the inverse fourth power of the stop mass while chargino correction to the bottom mass depends only on the inverse second power. Fitting the

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7We thank K.S. Babu and C. Kolda for discussions.

8Note, it has been shown (Ref. [24]) that 2nd - 3rd family flavor mixing can significantly affect the result for $B(b \rightarrow s\gamma)$. 

Figure 20: Contours of constant $m_t A_t/M^2_{(SUSY)}$ with fixed $m_{16} = 2000$ GeV, as a function of $\mu, M_{1/2}$.
central value $B(b \to s\gamma) = 2.96 \times 10^{-4}$ \cite{2} requires a heavier $\tilde{t}_1$ with $(m_{\tilde{t}_1})_{MIN} \sim 450$ GeV; significantly larger than the range which provides the best fits to $m_b$. Nevertheless, $\tilde{t}_1$ is still the lightest squark with significant stop-bottom splitting. The $b_1$, $\tilde{t}_1$ masses also increase significantly. We now find $m_b(m_b)_{MIN} \sim 4.3$.

In Table 2 we present representative points which are consistent with both $B(b \to s\gamma)$ and $B_s \to \mu^+ \mu^-$. Fits 1 and 2 are with Just So Higgs splitting, while Fits 3 and 4 are with D term splitting. Fits 1,3,4 (Table 2) correspond to the same values of $\mu$, $M_{1/2}$, $m_{16}$ as Fits 2,4,5 (Table 1). Fits 1 to 3 have exact Yukawa coupling unification at $M_G$. Fit 4 has $\epsilon_b = -0.08$, $\epsilon_t = 0.02$. $B(b \to s\gamma)$ is the strongest constraint in these fits. Better agreement between Yukawa unification and $B(b \to s\gamma)$ is achieved with increasing $m_{16}$. Also when we fit $B(b \to s\gamma)$ at $+3\sigma$, then we obtain slightly lower squark and slepton masses with the changes indicated in parentheses in Table 2.

We have not reevaluated $\chi^2$ contours including the $B_s \to \mu^+ \mu^-$ and $b \to s\gamma$ constraints. We do not expect the $\chi^2$ contours in the $\mu$, $M_{1/2}$ plane to change significantly, since we can accomodate these new constraints with small changes in the parameter $A_0$ and negligible changes in all others. Thus we expect that the predictions for gaugino masses to be unaffected by the $B_s \to \mu^+ \mu^-$ and $b \to s\gamma$ constraints. Hence the lightest neutralino is the LSP and a dark matter candidate $\tilde{\chi}$\textsubscript{0}. In order to know how observable neutralinos and charginos may be, we encourage the analysis of some new benchmark points consistent with Yukawa unification.

Finally, we recall that proton decay experiments prefer values of $m_{16} > 2000$ GeV and $m_{16} \gg M_{1/2}$ (see Ref. \cite{21}). This is in accord with the range of SUSY parameters found consistent with third generation Yukawa unification. There is however one experimental result which is not consistent with either Yukawa unification or proton decay and that is the anomalous magnetic moment of the muon. Large values of $m_{16} \geq 1200$ GeV lead to very small values for $a_{\mu}^{NEW} \leq 16 \times 10^{-10}$ \cite{22}. Hence a necessary condition for Yukawa unification is that forthcoming BNL data \cite{11} and/or a reanalysis of the strong interaction contributions to $a_{\mu}^{SM}$ will significantly decrease the discrepancy between the data and the standard model value of $a_{\mu}$.

In summary, most of the results of our analysis including only third generation fermions remain intact when incorporating flavor mixing. The light Higgs mass and most sparticle masses receive only small corrections. The lightest stop mass increases, due to $b \to s\gamma$. Nevertheless there is still a significant $\tilde{t}_1 - \tilde{t}_2$ splitting and $m_{\tilde{t}_1} << m_{\tilde{b}_1}$. The $A^0$, $H^0$, $H^+$ masses are necessarily larger in order to be consistent with $B_s \to \mu^+ \mu^-$ \cite{22}, which suggests that this process should be observed soon; possibly at Run III of the Tevatron. Finally, the central value for $a_{\mu}^{NEW}$ must significantly decrease. The “smoking guns” of SO(10) Yukawa unification, presented in this paper, should be observable at Run III of the Tevatron or at LHC. Also, in less than a year we should have more information on $a_{\mu}^{NEW}$.

\footnote{Although $a_{\mu}^{SUSY}$ has not been included in the $\chi^2$ function, we have included the calculated values for $a_{\mu}^{SUSY}$ for the points in Tables 1 and 2.}
6 Discussion

In the previous section we presented some experimental tests of Yukawa unification. Here we consider some open theoretical questions.

Yukawa unification only works in a narrow region of SUSY parameter space with $A_0 \approx -1.9 \ m_{16}$, $m_{10} \approx 1.35 \ m_{16}$ and $m_{16} > 1200 \ GeV$. The question arises, is this boundary condition natural in any SUSY breaking scheme? In mSUGRA, dilaton and anomaly mediated SUSY breaking schemes $A_0 \neq 0$ at the GUT scale. On the other hand, in gauge mediated or gaugino mediated SUSY breaking schemes, $A_0 = 0$ at $M_G$. The latter are thus disfavored by Yukawa unification. In addition, anomaly mediated SUSY breaking has other problems. Slepton masses squared are negative unless other physics is added. More importantly, however, since the gluino and chargino masses have opposite sign, it is difficult to simultaneously fit $b \rightarrow s \gamma$ and $a_\mu^{NEW}$. Finally although $A_0 \neq 0$ at $M_G$ in mSUGRA and dilaton SUSY breaking schemes, this still does not explain why $A_0 \approx -1.9 \ m_{16}$. However, the other relations for Higgs masses, i.e. $m_{10} \approx 1.35 \ m_{16}$ and $\Delta m_{H}^2 = .13$, may be obtained via RG running above $M_G$ or via threshold corrections.

It is an interesting, but not too surprising, result that the region of SUSY parameter space preferred by Yukawa unification is very similar to the region of SUSY parameter space needed to obtain heavy 1st and 2nd generation squarks and sleptons with third generation squarks and sleptons lighter than O(TeV) [28]. First of all, the $SO(10)$ boundary conditions with $m^2_{(Q, \bar{u}, \bar{d}, L, \bar{e})} = m^2_{16}$ and $m^2_{(H_u, H_d)} = m^2_{10}$ are obtained as a result of demanding the inverted scalar hierarchy in Ref. [28], whereas for us they are input. In addition, we need a light stop with large stop-sbottom splitting forcing us to the same region of parameter space with large negative $A_0 \sim -2 \ m_{16}$ and large $m_{16}$ with $m_{10} \sim \sqrt{2} \ m_{16}$.

It would be interesting to see how sensitive our results may be to alternative electroweak symmetry breaking approximations. In this paper we have used the effective 2 Higgs doublet analysis of [28]. This approximation may be particularly well suited to the light Higgs spectrum we obtain in our analysis. The alternate scheme, in which the Higgs tadpoles are evaluated in the MSSM at a scale of order $M_{stop}$ [10], is however more frequently used in the literature.

We have a neutralino LSP and it is important to know if it is consistent with cosmology and possibly a good dark matter candidate. We note that Yukawa unification places us in a region of SUSY parameter space which is markedly different than has been studied in the literature. A preliminary investigation suggests that there are no major problems [27]. Further study in this region of parameter space is now highly motivated by our results.

Finally, let us consider the issue of vacuum stability. Since we have large $A_t$, we may find that the vacuum stability condition $|A_t| < m_{\tilde{t}_L} + m_{\tilde{t}_R}$ [28] in the stop-Higgs sector is violated. Indeed we find only a narrow region in SUSY parameter space with $\chi^2 < 1$ where this constraint is satisfied. However the small change in $A_0$, necessary to

\footnote{Note, this region of parameter space is desirable for suppressing flavor and CP violating SUSY loops.}
fit $B(b \to s\gamma)$, is also sufficient to satisfy this stability constraint.

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A Appendix

In this Appendix we consider the possible GUT scale threshold corrections to gauge and Yukawa unification and to soft SUSY breaking scalar masses.

A.1 Gauge coupling unification

At tree level the three gauge couplings unify at the GUT scale. At one loop all three gauge couplings receive corrections depending logarithmically on an arbitrary scale $\mu$ and the masses of the particles integrated out of the theory. We may choose $\mu \equiv M_G$ such that $\alpha_1(\mu) = \alpha_2(\mu) \equiv \alpha_G$. Then the one loop threshold correction corresponds to a shift given by $\epsilon_3 \equiv (\alpha_3(M_G) - \alpha_G)/\alpha_G$. $\epsilon_3$ obtains contributions from all massive states with SO(10) quantum numbers. We obtain \[ \epsilon_3 = f(\zeta_1, \ldots, \zeta_m) + \frac{3\tilde{\alpha}_G}{5\pi} \log \left| \frac{\det \tilde{M}_t}{M_G \det \tilde{M}_d} \right| + \ldots \] 

where the first term represents the contributions from $W_{sym\, breaking}$. It is only a function of U(1) and R invariant products of powers of vevs $\{\zeta\}$. It is typically large O($\pm 10\%$). The second term comes from the Higgs sector where the color triplet and doublet mass matrices $\tilde{M}_t, \tilde{M}_d$ only include those states, from 5s and $\bar{5}$s of SU(5) contained in $W_{sym\, breaking}$ and $W_{Higgs}$, which mix with the Higgs sector. For further details, see Ref. [30].

A.2 Yukawa coupling unification

The third generation Yukawa couplings are derived from the minimal interaction $W = \lambda \ 16_3 \ 10_H \ 16_3$. At tree level we have $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} \equiv \lambda$. At one loop this relation is corrected [31]. However in this case there are just three sources of corrections: gauge exchange for $10_H$ and $16_3$; Yukawa exchange with color triplet Higgs fields and with heavy right-handed neutrinos in the loop. When one considers a theory of fermion masses for three families then additional Yukawa couplings mixing $16_3$ with other heavy SO(10) states are possible. However even if these new Yukawa couplings are of order one, their contribution to threshold corrections are typically less than 1%. More importantly
however there are corrections which come at tree level in the effective low energy field theory. In a three family model $3 \times 3$ Yukawa matrices are needed in order to obtain both fermion masses and CKM mixing angles. Upon diagonalizing the Yukawa matrices, one effectively obtains tree level corrections to the simple SO(10) relation.

### A.2.1 One loop corrections neglecting $\bar{\nu}_\tau$

Consider first the one loop corrections neglecting $\bar{\nu}_\tau$. We find

$$
\epsilon_b = \frac{\alpha_G}{2\pi} \log\left(\frac{M_5^2}{M_G^2}\right) - 1 - \frac{\lambda^2}{32\pi^2} \log\left(\frac{M_T^2}{M_G^2}\right) - 1
$$

(7)

$$
\epsilon_t = \frac{3\alpha_G}{10\pi} \log\left(\frac{M_{10}^2}{M_G^2}\right) - 1 + \frac{\alpha_G}{4\pi} \log\left(\frac{M_5^2}{M_G^2}\right) - 1
$$

(8)

where $M_{10}$, $M_5$ ($M_T$) is the mass of the SO(10) gauge fields contained in $SO(10)/SU(5)$, $SU(5)/(SU(3) \times SU(2) \times U(1))$ (the mass of the color triplet Higgs fields contained in $10_H$). For $M_5 \approx M_T \sim M_G$ and $M_{10} \leq 10M_G$ we obtain $\epsilon_b \sim -0.7\%$, $\epsilon_t \leq 1.1\%$.

### A.2.2 One loop corrections due to $\bar{\nu}_\tau$ alone

The superpotential for the neutrino sector is given by

$$W = \lambda H_u L_\tau \bar{\nu}_\tau + \frac{1}{2} M_{\bar{\nu}_\tau} \bar{\nu}_\tau \bar{\nu}_\tau
$$

(9)

where $M_{\bar{\nu}_\tau}$ is the effective Majorana mass for the right handed tau neutrino. Neutrino oscillations consistent with atmospheric neutrino data suggest $M_{\bar{\nu}_\tau} \approx m_t^2 / \sqrt{3.5 \times 10^{-3}} \text{ eV}^2 \approx 10^{13} \text{ GeV}$ or $M_{\bar{\nu}_\tau} / M_G \approx 10^{-3}$.

Integrating out the right handed tau neutrino leads to equal finite wave function renormalization of $H_u$, $L_\tau$. Hence

$$
\epsilon_b = \frac{\lambda^2}{32\pi^2} \log\left(\frac{M_G^2}{M_{\bar{\nu}_\tau}^2}\right) + 1, \quad \epsilon_t = 0
$$

(10)

or $\epsilon_b = 2.6\%$ for $\lambda = 0.7$, $M_G = 3 \times 10^{16} \text{ GeV}$, $M_{\bar{\nu}_\tau} = 10^{13} \text{ GeV}$. This is a considerable correction which actually goes in the wrong direction for fitting the third generation masses. We discuss the sensitivity of our results to such a correction in the text.

### A.2.3 Tree level corrections due to more realistic Yukawa matrices

A significant threshold correction to Yukawa unification can come when Yukawa matrices for three families are considered. As an example, in Table 2 the process $b \rightarrow s \gamma$ was calculated using the following ansatze for two family Yukawa matrices, since it has been shown that they provide a good fit for fermion masses and mixing angles \[32\].
\[ Y_u = \begin{pmatrix} \epsilon' & -r \epsilon \\ r \epsilon & 1 \end{pmatrix} \lambda \\
Y_d = \begin{pmatrix} \epsilon & -\sigma r \epsilon \\ r \epsilon & 1 \end{pmatrix} \lambda \\
Y_e = \begin{pmatrix} 3 \epsilon & 3 r \epsilon \\ -3 \sigma r \epsilon & 1 \end{pmatrix} \lambda \]  

(11)

The universal Yukawa coupling \( \lambda \) plus the three new complex parameters \( \epsilon, \epsilon' \) and \( \sigma \) and one real parameter \( r \) were varied to minimize the \( \chi^2 \) function with five additional observables \( M_\mu = 105.66 \pm 1.1 \) MeV, \( M_b - M_c = 3.4 \pm 0.2 \) GeV, \( m_\text{d}(2 \text{ GeV}) = 110 \pm 35 \) MeV, \( V_{cb} = 0.0402 \pm 0.0019 \) and \( B(b \rightarrow s \gamma) \times 10^3 = 0.296 \pm 0.035 \). Clearly there are more parameters than observables, but we are not attempting in this analysis to make any new predictions for fermion masses. We just want to be able to calculate the branching ratio for \( b \rightarrow s \gamma \) self-consistently. With Fit 1 in Table 2 as a guide, we find (upon diagonalizing the \( 2 \times 2 \) Yukawa matrices at the GUT scale) \( \epsilon_b \approx \epsilon_t \approx -0.08 \). Several points should be made here. The first is that this tree level correction is model dependent and much larger than any one loop correction. Secondly, \( \epsilon_b \neq 0 \) is a consequence of the Georgi-Jarlskog like mass matrices distinguishing quarks and leptons. Finally, since \( V_{cb} \approx 0.04 \) is small, we find \( \epsilon_t \approx \epsilon_b \). This latter result is in the wrong direction for obtaining good fits to third generation masses with D term splitting as is evident in Fig. 19 and Fit 4 (Table 2).

### A.3 Higgs mass splitting

D term splitting of the \( H_u, H_d \) masses is quite natural in \( SO(10) \) SUSY GUTs. It can be generated in the process of breaking \( SO(10) \rightarrow SU(5) \) by a mismatch in the vacuum expectation values of the \( 16 \) and \( \overline{16} \) which are needed to break \( SO(10) \rightarrow SU(5) \) and reduce the rank of the group. Once this D term is generated it then gives mass to scalars proportional to their \( U(1)_X \) charge. The Just So case does not at first sight appear to be similarly well motivated. In this Appendix we attempt to rectify this apparent difficulty. It is quite clear that in any SUSY model the Higgs bosons are very special. R parity is used to distinguish Higgs from squarks and sleptons. In addition, a supersymmetric mass term \( \mu \) with value of order the weak scale is needed for the Higgs bosons. Since \( \mu \) is naturally of order \( M_G \), one needs some symmetry argument why it is suppressed. Of course, if the Higgs are special, then perhaps this will help us understand how to obtain splitting of the Higgs up/down while maintaining universal squark and slepton masses. GUT threshold corrections to soft SUSY breaking scalar masses have been considered previously. In Murayama et al. \[7\] it was shown that the necessary condition, \( m_{10} > m_{16} \), can naturally be obtained in \( SO(10) \) with RG running from \( M_{Pl} \) to \( M_G \). In the paper by Polonsky and Pomarol \[16\] the splitting of the soft masses of \( SU(5) \) multiplets within irreducible
SO(10) representations was considered. In the following we consider two novel sources for Higgs up/down splitting in the context of SO(10).

A.3.1 $\nu_\tau$ contribution to Higgs splitting

In the MSSM superpotential below $M_G$ we have the $\nu_\tau$ contribution which distinguishes $H_u$ and $H_d$ (see Eqn. [9]). This leads to a significant threshold correction

$$\Delta m^2_{H_u} \approx \frac{\lambda^2}{16\pi^2} (2m^2_{16} + m^2_{10} + A^2_0) \log(M^2_G/M^2_{10}) + \text{non log terms} \quad (12)$$

Using the values $\lambda = 0.7$, $M_{\nu_\tau} = 10^{13}$ GeV and $M_G = 3 \times 10^{16}$ GeV and the typical boundary conditions $A^2_0 \approx 2m^2_{10} \approx 4m^2_{16}$, we obtain $\Delta m^2_H = \frac{1}{2} \Delta m^2_{H_u}/m^2_{10} = .10$. Note this is remarkably close to the value needed for Just So Higgs splitting (see Fig. 5 (Right) and Fits 1 - 3 (Table 1) and Fits 1,2 (Table 2)).

A.3.2 Another possible source for Higgs splitting

Consider also the possible superpotential for the Higgs sector

$$W = \lambda_A 10 45 10' + X (10')^2 + \psi \psi 10' + M\bar{\psi}\psi + Tr(45^2M^2_{45}) \quad (13)$$

The first two terms are necessary for Higgs doublet/triplet splitting with assumed vevs for the adjoint field $\langle 45 \rangle \sim (B - L) M_G$ and $\langle X \rangle << M_G$. The fields $\psi$, $\bar{\psi}$ are in 16, $\overline{16}$ dimensional representations. Their vevs break SO(10) to SU(5) and split the masses of $5_{10'}$, $5_{10'}$. With this splitting and also an assumed SU(5) invariant splitting in the supersymmetric masses for the 45 we find significant threshold corrections to $\Delta m^2_H$.

Schematically, we find

$$\Delta m^2_H \propto \frac{\lambda_A^2}{16\pi^2} \log(M^2_{1045}/M^2_{2445}) \sin(\theta + \theta')\sin(\theta - \theta') \{r\} + \cdots \quad (14)$$

The factor $r$ represents the ratio of soft scalar masses $m^2_{45}/m^2_{10}$, $m^2_{10'}/m^2_{10}$, $A^2_0/m^2_{10}$ and finally the dots represent the contribution of color triplet states to the loops. The correct sign for $\Delta m^2_H$ can always be obtained. If $\lambda_A^2/4\pi \sim O(1)$ or $r >> 1$, then we can easily obtain $\Delta m^2_H \sim 20 - 30\%$. In the latter case, top, bottom unification will have considerably smaller threshold corrections.

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Table 1: Five representative points of the fits. The first three are with Just So Higgs splitting and the last two are with D term splitting. All fits assume exact Yukawa unification at the GUT scale, except for 5 which has $\epsilon_b = -0.08$, $\epsilon_t = 0.02$. All fits are with one loop RG running from $M_G$ to $M_Z$ for dimensionful parameters, except for 3 which uses two loop running. All entries are in units of GeV to the appropriate power.

| Data points | 1     | 2     | 3     | 4     | 5     |
|-------------|-------|-------|-------|-------|-------|
| Input parameters |       |       |       |       |       |
| $\alpha_G^{-1}$       | 24.46 | 24.66 | 24.66 | 24.73 | 24.58 |
| $M_G \times 10^{-16}$ | 3.36  | 3.07  | 3.07  | 3.13  | 3.16  |
| $\epsilon_3$          | -0.042| -0.040| -0.040| -0.046| -0.039|
| $\lambda$              | 0.70  | 0.67  | 0.67  | 0.80  | 0.63  |
| $m_{16}$               | 1500  | 2000  | 2000  | 2000  | 2000  |
| $m_{10}/m_{16}$        | 1.35  | 1.35  | 1.35  | 1.20  | 1.33  |
| $\Delta m_H^2$        | 0.13  | 0.13  | 0.12  | 0.07  | 0.05  |
| $M_{1/2}$              | 250   | 350   | 350   | 350   | 300   |
| $\mu$                  | 150   | 200   | 200   | 115   | 150   |
| $\tan \beta$          | 51.2  | 50.5  | 50.6  | 54.3  | 51.1  |
| $A_0/m_{16}$           | -1.83 | -1.87 | -1.83 | -0.37 | -1.87 |
| $\chi^2$ observables   |       |       |       |       |       |
| $M_Z$                  | 91.188 (0.091) | 91.13 | 91.14 | 91.14 | 91.15 | 91.15 |
| $M_W$                  | 80.419 (0.080) | 80.45 | 80.45 | 80.44 | 80.44 | 80.44 |
| $G_\mu \times 10^5$   | 1.1664 (0.0012) | 1.166 | 1.166 | 1.166 | 1.166 | 1.166 |
| $\alpha_{EM}^{-1}$    | 137.04 (0.14)  | 137.0 | 137.0 | 137.0 | 137.0 | 137.0 |
| $\alpha_s(M_Z)$       | 0.118 (0.002)  | 0.1175 | 0.1176 | 0.1175 | 0.1161 | 0.1179 |
| $\rho_{new} \times 10^3$ | -0.200 (1.10)  | 0.696 | 0.460 | 0.437 | 0.035 | 0.265 |
| $M_t$                  | 174.3 (5.1)    | 175.5 | 174.6 | 174.4 | 177.9 | 174.1 |
| $m_b(m_b)$             | 4.20 (0.20)    | 4.28  | 4.27  | 4.28  | 4.59  | 4.22  |
| $M_{1/2}$              | 1.7770 (0.0018) | 1.777 | 1.777 | 1.777 | 1.777 | 1.777 |
| TOTAL $\chi^2$         | 1.50 | 0.87 | 0.91 | 5.42 | 0.45 |

| $a_{\mu}^\text{SUSY} \times 10^{10}$ | 25.6 (16) | 9.7  | 5.5  | 5.5  | 6.1  | 6.4  |
Table 2: Four representative points of the fits. The first two are with Just So Higgs splitting and the last two are with D term splitting. All fits assume exact Yukawa unification at the GUT scale, except for 4 which has $\epsilon_b = -0.08, \epsilon_t = 0.02$. All fits are with one loop RG running from $M_G$ to $M_Z$ for dimensionful parameters. For these fits the branching ratio for $b \rightarrow s \gamma$ is included in $\chi^2$. In addition, the CP odd Higgs mass $m_A^0$ is constrained to be 200 GeV. These points are thus consistent with both $B(b \rightarrow s \gamma)$ and $B_s \rightarrow \mu^+ \mu^-$. In Fits 1, 2 we show the change in the $\tilde{t}_1$, $\tilde{b}_1$, $\tilde{\tau}_1$ masses if we fit $b \rightarrow s \gamma$ at the central value $+3\sigma$.

| Input parameters | 1   | 2   | 3   | 4  |
|------------------|-----|-----|-----|----|
| $\alpha_G^4$     | 24.72 | 24.78 | 24.75 | 24.62 |
| $M_G \times 10^{-16}$ | 3.00 | 3.12 | 3.09 | 3.20 |
| $\epsilon_3$     | -0.040 | -0.041 | -0.045 | -0.043 |
| $\lambda$        | 0.63 | 0.61 | 0.80 | 0.60 |
| $m_{16}$         | 2000 | 3200 | 2000 | 2000 |
| $m_{10}/m_{16}$  | 1.32 | 1.30 | 1.19 | 1.30 |
| $\Delta m_H^2$   | 0.13 | 0.13 | 0.07 | 0.05 |
| $M_{1/2}$        | 350  | 350  | 350  | 300  |
| $\mu$            | 200  | 150  | 115  | 150  |
| $\tan \beta$    | 52.5 | 50.6 | 55.0 | 50.7 |
| $A_0/\lambda_{16}$ | -1.71 | -1.83 | -0.05 | -1.75 |

| $\chi^2$ observables | Exp($\sigma$) |
|-----------------------|---------------|
| $M_Z$                 | 91.188 (0.091) | 91.18 | 91.18 | 91.16 | 91.18 |
| $M_W$                 | 80.419 (0.080) | 80.42 | 80.42 | 80.43 | 80.42 |
| $G_\mu \times 10^5$  | 1.1664 (0.0012) | 1.166 | 1.166 | 1.166 | 1.166 |
| $\alpha_{EM}^{-1}$   | 137.04 (0.14)  | 137.0 | 137.0 | 137.0 | 137.0 |
| $\alpha_s(M_Z)$      | 0.118 (0.002)  | 0.1172 | 0.1173 | 0.1162 | 0.1167 |
| $\rho_{new} \times 10^3$ | -0.200 (1.10) | 0.228 | 0.321 | 0.221 | 0.279 |
| $M_t$                 | 174.3 (5.1)    | 173.8 | 172.1 | 178.8 | 172.6 |
| $m_b(m_b)$            | 4.20 (0.20)    | 4.46  | 4.42  | 4.56  | 4.50  |
| $M_r$                 | 1.7770 (0.0018) | 1.777 | 1.777 | 1.777 | 1.777 |
| TOTAL $\chi^2$       | 2.05 | 1.72 | 5.03 | 3.38 |

| $k^0$                 | 118  | 119  | 115  | 118  |
| $H^0$                 | 217  | 217  | 218  | 216  |
| $A^0$                 | 200  | 200  | 200  | 200  |
| $H^+$                 | 229  | 229  | 228  | 228  |
| $\tilde{\chi}_1^0$   | 130  | 110  | 86   | 99   |
| $\tilde{\chi}_2^0$   | 190  | 160  | 126  | 152  |
| $\tilde{\chi}_1^+$   | 178  | 136  | 105  | 131  |
| $\tilde{g}$           | 909  | 904  | 902  | 781  |
| $\tilde{t}_1$         | 509 (-30) | 511 (-27) | 1067 | 443 |
| $\tilde{b}_1$         | 749 (-42) | 903 (-14) | 900  | 550  |
| $\tilde{\tau}_1$     | 459 (+47) | 1001 (-25) | 1173 | 854  |
| $B(b \rightarrow s \gamma) \times 10^3$ | 0.296 (0.035) | 0.297 | 0.297 | 0.297 |
| $a_{\mu}^{SUSY} \times 10^{10}$ | 25.6 (16) | 5.8  | 2.2  | 6.4  | 6.4  |