The High Energy Neutrino Nuisance at a Medium Baseline Reactor Experiment

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Abstract

10 years from now medium baseline reactor experiments will attempt to determine the neutrino mass hierarchy from the differences $(RL + PV)$ between the extrema of the Fourier transformed neutrino spectra. Recently Qian et al. have claimed that this goal may be impeded by the strong dependence of the difference parameter $RL + PV$ on the reactor neutrino flux and on slight variations of $|\Delta M^2_{32}|$. We demonstrate that this effect results from a spurious dependence of the difference parameter on the very high energy (8+ MeV) tail of the reactor neutrino spectrum. This dependence is spurious because the high energy tail depends upon decays of exotic isotopes and is insensitive to the mass hierarchy. An energy-dependent weight in the Fourier transform not only eliminates this spurious dependence but in fact increases the chance of correctly determining the hierarchy.

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This year the Daya Bay [1, 2] and RENO [3] experiments have demonstrated beyond any reasonable doubt that $\theta_{13}$ is as much as an order of magnitude larger than had been suspected several years ago. This large value of $\theta_{13}$ implies that 1-3 neutrino oscillations may be observed at medium baselines, which we define to be 40-80 km. The medium baseline neutrino spectrum may then be used to determine the neutrino mass hierarchy [4]. Such experiments are now not only practical but indeed they will be performed within the next decade [5, 6, 7].

However at these baselines, due to a degeneracy in the high energy neutrino spectrum [8, 9, 10], a determination of the hierarchy requires a measurement of 1-3 oscillations at low neutrino energies $E$. As a result of the finite energy resolution of the detector and various interference effects [10] these low energy peaks are difficult to identify individually at medium baselines $L$. Nonetheless if the nonlinear energy response of a detector is well understood then one may measure the sum of the peaks by studying the $k \sim |\Delta M^2_{31}|/2$ region of the $L/E$-Fourier transform of the neutrino spectrum [11]. The most popular variables for such a determination are the fractional difference $RL$ between the deepest minima of the Fourier cosine transform and the difference $PV$ between the deepest minimum and the highest peak of the Fourier sine transform [12, 13]. Although these two variables are somewhat degenerate, an improvement may be obtained by considering their sum $RL + PV$.

A serious obstruction to this analysis, and thus to plans to measure the neutrino mass hierarchy at medium baselines, has been described in Ref. [9]. The authors observed that the combination $RL + PV$ is very sensitive to the choice of model of the reactor neutrino flux $\Phi(E)$ and to variations of $|\Delta M^2_{32}|$ which are smaller than the precision to which this mass difference has been determined by MINOS [14]. While the observed shift appears to depend upon both $|\Delta M^2_{32}|$ and the hierarchy, from Fig. 4 of Ref. [9] it can be seen that the shift depends only upon the effective mass difference [15, 10]

$$\Delta M^2_{\text{eff}} = \cos^2(\theta_{12})|\Delta M^2_{31}| + \sin^2(\theta_{12})|\Delta M^2_{32}|.$$

(1)

The neutrino flux from reactors is known poorly. The theoretical normalization has recently increased by about 3% [16] and the 6+ MeV flux has increased by an additional 3% [17]. The flux beyond about 8 MeV is not known at all due to its strong dependence upon decays of exotic isotopes [16]. Even worse, all of these theoretical fluxes are about 6% above the observed fluxes at very short [18] and 1 km [19] baselines. Thus the large sensitivity of $RL + PV$ upon the poorly known fluxes and $\Delta M^2_{\text{eff}}$ appreciably reduces the probability that a medium baseline reactor experiment can correctly determine the neutrino mass hierarchy.

We will now explain the cause of this strong dependence. As the dependence of $RL$,
$PV$ and $RL + PV$ upon these parameters is virtually indistinguishable, for brevity we will consider only

$$RL = \frac{R - L}{R + L}$$

which is the fractional difference between two minima $R$ and $L$ of the Fourier cosine transform of the neutrino spectrum

$$F_c(k) = \int d\left(\frac{L}{E}\right) \frac{E^2 \Phi(E)\sigma(E)}{4\pi L^2} P_{ee} \left(\frac{L}{E}\right) \cos\left(\frac{kL}{E}\right)$$

where the tree level neutrino inverse $\beta$ decay cross section is [20]

$$\sigma(E) = 0.0952 \times 10^{-42} \text{cm}^2 \frac{E_e \sqrt{E_e^2 - m_e^2}}{\text{MeV}^2}, \quad E_e = E - m_n + m_p$$

and the electron neutrino survival probability is

$$P_{ee} = \sin^4(\theta_{13}) + \cos^4(\theta_{12})\cos^4(\theta_{13}) + \sin^4(\theta_{12})\cos^4(\theta_{13}) + \frac{1}{2}(P_{12} + P_{13} + P_{23})$$

$$P_{12} = \sin^2(2\theta_{12})\cos^2(\theta_{13})\cos\left(\frac{\Delta M_{31}^2 L}{2E}\right), \quad P_{13} = \cos^2(\theta_{12})\sin^2(2\theta_{13})\cos\left(\frac{\Delta M_{31}^2 L}{2E}\right)$$

$$P_{23} = \sin^2(\theta_{12})\sin^2(2\theta_{13})\cos\left(\frac{\Delta M_{32}^2 L}{2E}\right).$$

Following Ref. [12], a $3%/\sqrt{E}$ energy resolution is included by convoluting the observed energy spectrum with

$$\exp\left(-\frac{(E - E')^2}{0.0018(E_e + m_e)\text{MeV}}\right).$$

We use the neutrino mass matrix parameters of Ref. [5].

As was demonstrated in Ref. [10], the minima whose difference defines $RL$ lie just on either side of $k = |\Delta M_{31}^2|/2$. These minima arise from the Fourier transform of $P_{13}$ which is independent of the hierarchy, but the contribution of $P_{23}$ provides a perturbation which makes the right (left) minimum deeper for the normal (inverted) hierarchy.

The problem observed in Ref. [9] is that, depending upon the reactor flux model used, the transform of the unoscillated reactor flux $\Phi(E)\sigma(E)E^2/L^3$ itself may contribute peaks near $k = |\Delta M_{31}^2|/2$ which interfere with those of $P_{13} + P_{23}$ and so affect $RL$. While the cosine transform of the unoscillated flux $\Phi(E)\sigma(E)E^2/L^3$ is independent of the neutrino mass splittings, the locations of the peaks of $P_{13} + P_{23}$ are proportional to $\Delta M_{31}^2$. This means that the relative phase between the Fourier transform of the unoscillated spectrum and that of $P_{13} + P_{23}$ depends on the precise value of $\Delta M_{31}^2$. As a result the oscillations in the Fourier transform of $\Phi(E)\sigma(E)$ lead to an $\Delta M_{31}^2$-dependence in the quantity $RL$ just
Figure 1: The cosine transforms of the unoscillated flux (black dashed curve) and the full $P_{13} + P_{23}$ oscillated flux (red solid curve) are shown. As $\Delta M_{\text{eff}}^2$ varies, the $P_{13} + P_{23}$ peaks move and so the interference between the two contributions to the cosine transform of the neutrino spectrum varies. The reactor fluxes used are those of the 1980’s.

of the kind observed in Ref. [9] using old reactor flux models. In fact, using the $^{235}$U flux from Ref. [21], the $^{239}$Pu and $^{241}$Pu fluxes from [22] and the Gaussian approximated $^{238}$U flux from Ref. [23] with the isotope ratios of Ref. [12] we find an oscillation in the unoscillated spectrum term in Eq. (3). Using this old model of the reactor flux, in Fig. 1 we compare the Fourier transform of the unoscillated term with that of the $P_{13} + P_{23}$ term, which is sensitive to the hierarchy. One can see that the unoscillated term is periodic with the same wavelength as was observed in Fig. 4 of Ref. [9], and thus the interference between these two terms oscillates as $\Delta M_{\text{eff}}^2$ varies, shifting the $P_{13} + P_{23}$ peaks and so reproducing the effect reported in that note.

Ref. [9] concludes that this strong dependence of $RL$ upon the reactor flux means that a precise knowledge of this flux is desirable to determine the neutrino mass hierarchy at a short baseline experiment. Our conclusions differ. This difference arises from the observation that, as can be seen in Fig. 2, near $k = |\Delta M_{\text{eff}}^2|/2$ the oscillations of the cosine transform of the reactor spectrum are governed by the highest energy neutrinos (8+ MeV) which, according to Refs. [8, 10], are not sensitive to the hierarchy. On the other hand in Fig. 2 we see that the effect is present in the case of both old and new fluxes if the spectrum is cut off at 8.5 MeV while it is negligible if the spectrum is cut off at 12.8 MeV. However since the quadratic and quintic fits to these fluxes are only reliable below 6.5 MeV and marginally reliable below 8 MeV, there is no reason to trust a naive extrapolation to 12.8 MeV.

As $RL$ depends strongly on the spectrum between 8.5 and 12.8 MeV, which in turn is independent of the hierarchy, this high energy spectrum provides a nuisance parameter for the
Figure 2: The cosine transforms of the unoscillated flux is shown for numerically interpolated fluxes from the 1980’s [21, 22, 23] (black dotted curve), for a quadratic fit to fluxes from the 1980’s [23] and for quintic fits of the new fluxes of Ref. [17]. The latter two are shown with cutoffs of 8.5 MeV (dashed curves) and 12.8 MeV (solid curves). The blue dashed curve corresponds to the quadratic fit flux. The red and green solid curves, corresponding to 12.8 MeV cutoffs, are close to zero. Therefore the interference effect is present if the cutoff is at 8.5 MeV and but not if the fits are naively extrapolated to 12.8 MeV. This demonstrates that $RL$ is sensitive to the neutrino spectrum above 8.5 MeV.

**determination of the hierarchy using $RL$.** The solution suggested in Ref. [9] is to determine the spectrum precisely, however so few neutrinos are observed in this range that such a determination would be difficult, indeed the spectrum is not understood at the required precision even at the energies with high fluxes [18]. Even if such a measurement were possible, then $RL$ would still be likely to depend upon $\Delta M^2_{31}$ with a higher sensitivity than the mass determination at MINOS and probably at NO$\nu$A, making a determination of the hierarchy at a medium baseline more challenging.

Our solution is to replace $RL$ and $PV$ with indicators that are insensitive to the high energy neutrino spectrum, by providing an energy-dependent weight $w(E)$ on the neutrino spectrum in the Fourier transform. As we saw in Fig. 2, a simple cutoff in the Fourier transform will amplify the spurious dependence. The weight needs to cut off the high energies gradually, with derivative scales much longer than $|\Delta M^2_{31}|$, so as to not itself introduce spurious peaks in the critical part of the Fourier transforms. One such choice of weight which we have found works quite well is a Gaussian

$$F_c(k) = \int d \left( \frac{L}{E} \right) e^{-\frac{0.08 \text{MeV}^2}{E^2} \frac{E^2 \Phi(E) \sigma(E)}{L^4 \pi L^2} P_{ee} \left( L \frac{L}{E} \right) \cos \left( kL \frac{L}{E} \right)}.$$

The same weight serves well in both the sine transform and also the nonlinear transforms of
Figure 3: Here we see the weighted Fourier transform of the spectrum without oscillations (blue solid curve) and with oscillations in the case of the normal (black solid curve) and inverted (red dashed curve) hierarchies. One can see that the solid, blue unoscillated curve is very close to zero. We have checked that this curve is essentially independent of the cutoff and so the reactor spectrum no longer affects $R_L$. Comparing with Fig. 1 one can see that the difference $R_L$ between the depths of the minima is even greater in this weighted case, allowing for a better determination of the hierarchy than was possible with an unweighted Fourier transform.

Ref. [10] which determine the hierarchy more reliably than $R_L + PV$ at baselines below about 55 km [24]. As can be seen by comparing the unweighted and weighted cosine transforms in Figs. 1 and 3, not only does the weighting procedure preserve $R_L$, but it actually increases the difference in the peak sizes between the normal and inverted hierarchies. Thus this solution to the dependence upon the high energy neutrino tail not only removes the spurious dependence, for any high energy reactor spectrum, but it increases the chance of success of the determination of the hierarchy. In simulations [24] we will show that when the weight function is optimized with a neural network this increase is of order 2%.

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