Investigation of the stable and unstable states of seismographs using poles and zeros pattern

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Abstract

In this study, a simple Simulink model was designed and presented which can describe the overall operation of seismograph based on the desired input and output response.

By converting Laplace to seismograph equations and by defining Numerator and Denominator fractions to Num. and Den. (Numerator and Denominator) transfer WWSSN (World-Wide Standard Seismographic Network) functions, the changeability of pole-zero was taken into consideration which indicates the stability and instability of the system layout with respect to pole - zero layout. In fact, the poles and zeros are placed in the seismograph response based on frequency, amplitude and phase (FAP) and are analyzed well. Through this method can also be used to study seismograph transfer function to predict the seismograph output in environments with different seismic noise and to select the most appropriate value for the parameters controlling the output of the device.

Keywords: Geophysics, Geoscience, Applied mathematics, Computational mathematics, Electrical engineering
1. Introduction

There are two types of seismograph, namely mechanical (mass-spring) and electromagnetic. Important parameters are taken into account for construction of these machines. The equation of mechanical motion is 
\[ \ddot{x} + 2\xi \dot{x} + \omega^2 x = 0 \]
and the equation of electromagnetic seismograph motion is 
\[ H + 2\beta \Omega_0 H + \Omega_0^2 H = 0 \] [1].

By comparing the equation of electromagnetic and mechanical seismograph motion, some corresponding parameters like \( \beta, \Omega_0, \epsilon, \) and \( \omega_s \) are found that play a similar role in the functioning of seismograph.

Therefore, it seems that some issues like performance (mechanical systems, differential equation, convertors), determination of important parameters in the design and construction (damping, natural frequency, critical damping resistance, sensitivity) and transfer functions (high pass \( H(s) \), Low-pass \( L(s) \), band pass \( B(s) \)) are important in seismographs [2].

In this study, Laplace transformation (conversion) and its application in Seismographs differential equation are applied using MATLAB-SIMULINK on these parameters and by changing the above parameters, the output is studied and interpreted [1].

This output includes drawing and application of developer Step, bode, Root locus, Nyquist, impulse, polar, etc. diagrams using fixed numerical values of long period seismographs of standard global seismic network (WWSSN-LP), including \( c = E\gamma \omega_g^2 = 383.6/\text{sec}, \ 2\omega_i h_i = 0.5027/\text{sec}, \ \omega_s^2 = 0.1755/\text{sec}^2, \ 2\omega_k h_g = 0.1257/\text{sec} \) and \( \omega_k^2 = 0.00487/\text{sec}^2 \) based on the change in the output response diagram [1].

Each of the main issues in designing and manufacturing the seismograph device are briefly described.

2. Background

2.1. The structure of the device seismograph

The general structure of the seismograph is briefly described.

2.2. Principles of operation

As it was noted, seismograph is a device through which the movement of the earth can be observed and recorded. Generally, this devise involves Inertia system, spring connected to the oscillator section (moving mass), fixed frame, and attenuation device. The transducer section of the seismograph generates output signal proportional to the relative motion of frame to the mass [2].
2.3. Mechanical seismograph

A qualitative evaluation of the behavior of mechanical seismograph and harmonic motion of the earth (frame) can yield the following result:

- For earth movements with very high frequencies, the suspended mass tends to maintain its absolute position and the relative movement of frame shows the movement of the earth [2].
- For earth movements with very low frequencies, the mass tends to follow the movement of the frame and the relative movement approaches zero [2].

The qualitative description of high pass characteristic relates the relative displacement of the mass to the earth’s displacement, especially between the very high and very low frequency, it can result in the balance of all internal and external forces effective on mass leading to second differential equation as following [2]:

\[-m \left[ x''(t) + w''(t) \right] - d \dot{x}(t) - c x(t) = 0 \quad [1, 2] \]

Where, displacement is $x(t)$, velocity is $\dot{x}(t)$, acceleration is $x'(t)$ and earth’s acceleration (frame) is $w''(t)$ [2].

2.4. Differential equation

If Eq. (1) is divided into m, we will have:

\[x''(t) + 2 \alpha \dot{x}(t) + \omega_0^2 x(t) = -w''(t) \quad [1, 2] \]

Where, $\omega_0^2 = c/m$ and $\omega_0 = 2\pi f_0$ & $d/m = 2\alpha \omega_0$.

Introducing the natural frequency $f_0 = \sqrt{c/m/2\pi}$ (in Hertz or HZ), and dimensionless damping constant ($\alpha = 1$, resulting in critical damping), $\alpha = d/2m\omega_0$, a solution for differential equation is obtained by the application of LAPLACE transfer [2].

2.5. Converters (Transducers)

- Converters are the displacement of most capacitors or poles inductions. In the beginning of the development and expansion of seismography, this was by a mechanical lever, a movable pen, or deviation of light rays by a moving mirror and recording it on a photography paper [2].
- Speed converters in most cases are a cylindrical coil that a permanent magnet was inside the cylindrical gap moving along its axis producing a voltage proportional to the change in flux within the coil [2].
2.6. Free movement of seismograph mass

A common method for testing or calibration of seismograph is placing mass in \( t = 0 \), from a fixed position and observing and recording its free movement (For more information refer to [2]).

3. Design

3.1. Seismograph transfer function

Let's use Laplace transfer to solve the seismograph equation. Laplace transfer of equation is [3]:

\[
\ddot{x}_r(t) + 2\varepsilon \dot{x}_r(t) + \omega_0^2 x_r(t) = -u_g(t) \text{ or}
\]

\[
S^2 X_r(S) + 2\varepsilon S X_r(S) + \omega_0^2 X_r(S) = -S^2 U_g(S) \quad [1,5]
\]

Therefore:

\[
(S^2 + 2\varepsilon S + \omega_0^2) X_r(S) = -S^2 U_g(S) \quad [1,5]
\]

So, the transfer function is:

\[
T(S) = \frac{X_r(S)}{U_g(S)} = \frac{-S^2}{S^2 + 2\varepsilon S + \omega_0^2} \quad [1,5]
\]

It is noted that the frequency response function of \( A_0 \) \( A_1 \) \( = \frac{-u_2}{u_2 + \omega_0} \) can be obtained from transfer function Eq. (5) by replacing \( j\omega \) by \( S \). Relations such as (5), are transfer functions of seismograph.

Since the second-degree equation \( x^2 + bx + c = 0 \) has \( x_{1,2} = -b/2 \pm \sqrt{b^2/4 - c} \) roots, for the place of \( P_{1,2} \) poles we will have (roots of polynomial denominator in (5)):

\[
P_{1,2} = -\varepsilon \pm \sqrt{\varepsilon^2 - \omega_0^2} = -h\omega_0 \pm \omega_0 \sqrt{h^2 - 1} = \left( h \pm \sqrt{h^2 - 1} \right) \omega_0 \quad [1,4]
\]

For the low damping mode (Sub damping) \( [h < 1] \), location of poles is:

\[
P_{1,2} = -\left( h \pm j\sqrt{1 - h^2} \right) \omega_0 \quad [1,4]
\]

With distance of pole from origin we will have:

\[
|P_{1,2}| = \left| \left( h \pm j\sqrt{1 - h^2} \right) \right| \omega_0 = \sqrt{h^2 + (1 - h^2)} \cdot |\omega_0| = |\omega_0| \quad [1,4]
\]
However, the poles of a low damping seismograph are located in the left half-page (first quarter), and the size of the source are $|\omega_0|$. Value $h \cdot |\omega_0|$, distance from the imaginary axis $j\omega$ shows [3].

### 3.2. High pass function $H(s)$

The transfer function $H(s)$ relates the movement of mass $x(t)$ to the earth’s movement $w(t)$, and the speed of the mass $x'(t)$ to the earth’s speed $w''(t)$ and corresponds to the normalized second degree high pass characteristic [2]:

$$H(s) = \frac{s^2}{s^2 + 2\alpha \omega_0 s + \omega_0^2} = \frac{\text{num}}{\text{den}} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2\alpha \omega_0 & \omega_0^2 \end{bmatrix} \quad [1, 5] \quad (9)$$

Now by arranging the required blocks, applying Numerator and denominator fractions (num, den) and choosing appropriate parameters, different system responses (Bode, Nyquist, polar, step, Root Locus, ...) can be analyzed (for more information refer to [3]). However, in this study the focus is more on Root Locus diagrams.

By changing and selecting the appropriate $\omega_0$, we have in fact studied sensitivity coefficient or spring hardness (sensor) which has a very important role in recording various ground motions (slow and fast) [1].

In some applications, designers want to design a circuit whose frequency response size has a sharp peak in a special frequency so that the reinforcement will be in a frequency-selecting manner, and only the frequencies located in the narrow band are reinforced [4].

In the Fig. 1, overall performance of seismograph; fast, accurate, easy and error-free drawing of different diagrams (step, impulse, slope, Nyquist, Bode, Root Locus, etc.), the system’s response to different values of attenuation and other parameters are some of the advantages of this system using MATLAB software. These diagrams, investigating systems are used in control, electrical, mechanics, etc., engineering, in different frequency domains, system stability, transient response, etc. and have a special place [1].

![Fig. 1. Simple Simulink model designed (by Abrehdari [1]) and presented to describe overall performance of seismograph based upon the desired inputs (step, impulse, slope, Nyquist, Bode, Root Locus ...) and output response.](https://doi.org/10.1016/j.heliyon.2018.e00823)
3.3. Low-pass function $L(s)$

The transfer function $L(s)$ relates mass displacement $x(t)$ to the earth acceleration $\ddot{w}(t)$ and corresponds to the normalized second-degree low-pass characteristic [2]:

$$L(s) = \frac{-\omega_0^2}{s^2 + 2\alpha \omega_0 s + \omega_0^2} \equiv \frac{\text{num}}{\text{den}} \equiv \frac{0}{\begin{bmatrix} 0 & 0 & -\omega_0^2 \\ 1 & 2\alpha \omega_0 & \omega_0^2 \end{bmatrix}} \left[ \frac{1}{2}, \frac{1}{5} \right]$$  \hspace{1cm} (10)

Here, we talk about a low pass filter which is more important for removing high frequencies, i.e. the filter that passes low frequencies around $\omega = 0$ and weakens or eliminates high frequencies, or the system poles get closer from the negative region of the real axis to the imaginary and virtual $j\omega$ axis, and move $\omega_0$ in the opposite direction of clock hands ($-\omega_0$), hence negative values belong to $\omega_0$ which is the case in the interpretation of high pass and band pass, etc. filters. We have the same discussions and diagrams of $H(s)$ for $L(s)$, but in this section, we try to show the place of poles and zeros on real and mixed axes and analyze the system from filter perspective [4].

By increasing $\omega$, the length of polarization vectors with respect to point $s = j\omega$ increases and the angle of each vector changes from 0 to $\pi/2$. By increasing $\alpha$ one pole gets closer to the $j\omega$ axis, showing that there is a slow sentence in response, the other pole gets far from the $j\omega$ axis which shows that the other sentence of response is rapidly damped. Therefore, per a very large $\alpha$, there is a pole near to $j\omega$ axis, which determines the dominant behavior of the system response in large $t$ [4].

3.4. Band pass function $B(s)$

The transfer function $B(s)$ relates mass speed to earth’s acceleration (equivalent to $p(t)/m$, dependent upon external force $p(t)$), and corresponds to the normalized second-degree bands pass characteristic [2]:

$$B(s) = \frac{-\omega_0}{s^2 + 2\alpha \omega_0 s + \omega_0^2} \equiv \frac{\text{num}}{\text{den}} \equiv \frac{0}{\begin{bmatrix} 0 & -\omega_0 \\ 1 & 2\alpha \omega_0 \\ \omega_0^2 \end{bmatrix}} \left[ \frac{1}{2}, \frac{1}{5} \right]$$  \hspace{1cm} (11)

Band pass systems are a filter which passes a frequency band, weakens higher and lower frequencies of that band. In this filter cut frequencies, are the boundary frequencies between pass and eliminate bands. Figs. 2, 3, 4, 5, 6, 7 and 8 show the location of poles, zeros and $\omega$ changes [4].
Fig. 2. Root Locus diagram of the system $H_d(s)$ [1].

Fig. 3. Root Locus diagram of system response $H_d(s)$ of relation (15) for attenuation values 0 and 0.6, (the displacement of poles should be considered, System is still stable (respectively from a to b)) [1].

Fig. 4. Root Locus diagram of system response $H_d(s)$ of relation (15) for the attenuation value of 0.9, Note gradual system instability (note the displacement of the poles location) [1].
4. Analysis

4.1. Long period seismograph transfer functions of worldwide standardized network — (WWSSN-LP)

WWSSN-LP Seismographs include normal electrodynamic seismometer, set by 15 seconds free period and a mirror long period galvanometer with about 90 seconds free period [5].

In summary and ignoring other required steps, the transfer function of an electromagnetic seismograph (input: displacement and output: voltage), is:

$$H(s) = \frac{E_s^3}{(s^2 + 2\omega_s h_s + \omega_s^2)} \equiv \frac{\text{num}}{\text{den}} = \begin{bmatrix} E & 0 & 0 \\ 1 & 2\omega_s h_s & -\omega_s^2 \end{bmatrix} \begin{bmatrix} 1.5 \end{bmatrix}$$

Fixed numerical values are $c = E\gamma\omega_g^2 = 383.6/\text{sec}$, $2\omega_s h_s = 0.5027/\text{sec}$, $\omega_s^2 = 0.1755/\text{sec}^2$, $2\omega_s h_g = 0.1257/\text{sec}$ and $\omega_g^2 = 0.00487/\text{sec}^2$. When using the numerical values in Simulink model, attention should be paid to indexes g (for galvanometer) and S (for seismograph) [5].

By placing these values in the Numerator and denominator of fractions and applying them in Simulink and setting the relevant parameters, system response and practical importance of each parameter can be explained using different charts (Root Locus, impulse, step, Bode, Nyquist), as well as the discussions related to filters.

Fig. 5. Root Locus diagram of the system. Root Locus is drawn to the left and thereby increasing the stability and speed response carefully or vice versa to reduce system stability and slow response carefully (See Figs. 5, 6, 7 and 8) [1].
Since the electromagnetic seismograph includes couples (pair) seismometer-galvanometer system, we start to examine the system using formula (13).

Galvanometer is a low pass second class filter and includes the following transfer function [5]:

Fig. 6. Roots Locus (zeros and poles) of system (14) [1].

Fig. 7. Root Locus system response diagram $H_d(s)$ of relation (15) with zero intervention time and attenuation [1].
Here, the same method of mechanical seismographs is applied and it can be understood that both systems function quite similarly and corresponding parameters play the same role. By changing the parameters in the system and relevant Simulink model, a change in the shape of diagrams is observed and this demonstrates the important and effective role of the parameters.

In the model of any system, when a parameter is changed, in fact we have changed the system, and can use the model of that change together with relevant figures, but

$$H_s(s) = \frac{\gamma \omega_s^2}{s^2 + 2\omega_s h_s + \omega_s^2} \equiv \begin{bmatrix} 0 & 0 & \gamma \omega_s^2 \\ 1 & 2\omega_s h_s & \omega_s^2 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

(13)

Fig. 8. Root locus system response diagram $H_s(s)$ of relation (15) with greater time and zero attenuation [1].

Fig. 9. Diagram of the output response of galvanometer to displacement with damping constant (the vertical axis is amplitude ranges and the horizontal axis is time (sec)) [1].
in order to avoid a long discussion, only the basic model is presents. Figs. 9, 10, 11, 12, and 13 show the effect of different damping constant is galvanometer:

Usually, it is better that the designed system in response to input signals has less errors as much as possible, hence the logical system attenuation is considered. The graphs show output response for various damping constants and the basic changes of the graphs with gradual increase or decrease of the damping constant shows the effectiveness of these parameters. Factor $c = E \gamma \omega_g^2 = 383.6/\text{sec}$ in the numerator has no effect on the output response. When the transfer function is non-sensitive to the analysis of desired factors, the device might be out of sensitivity, and engineers believe that the rules of obtaining the transfer function of the device should be carefully considered. For example, understanding the transfer functions of WWSSN seismograph with the galvanometer having 15 seconds period and 90 seconds seismometer is difficult [5].

Impulse responses of the seismometer and galvanometer of WWSSN system is shown in the Figures. We can choose system input as a function of slope, acceleration, step or calibration current, etc. To display a complete shock or pulse of input $\delta$, we should choose it to non-widescreen [5].

When the parameter $\omega_g$ takes different values, it leads to basic changes in the graphs of output response and shows the effectiveness of that parameter in system. The graph of output response of frequency change is the opposite to response attenuation [5].

Output signal of seismograph response (Fig. 13) is the convolution of galvanometer input signal (Figs. 10 and 12) the response of broadband shock convolution is the applied seismograph response and Figs. 12 and 13 are almost seismograph response (For more information refer to [1]), [5].

With different values of attenuation parameters, the system output will completely change which shows the system’s great dependence on this parameter.

Fig. 10. Graph of the output response of galvanometer to displacement with damping constant 0.12 (the vertical axis is amplitude ranges and the horizontal axis is time (sec)) [1].
If you refer to [5] Reference, you will see the similarity between Figs. 9, 10, 11, 12, and 13 of the system designed in this study and the WWSSN seismographs response. This proves the correct performance of Simulink system designed.

4.2. Root locus diagrams of transfer functions of seismographs

Specifying the place of num and den roots and convergence region in the S plane is an appropriate graphic method for describing Laplace transformation. In sum, the transfer function $H_d$ of seismograph, defined and simplified, is obtained by multiplying factors of relations (12) and (14).
\[ H_d(s) = \frac{cs^3}{(s^2 + 2\omega_s h_s + \omega_s^2)(s^2 + 2\omega_s h_g + \omega_g^2)} \quad [1, 5] \] (14)

Since input and output signals are based on the displacement, the absolute value of transfer function \(|H_d(s)|\), totally depends on magnification frequency of seismograph. Gain factor (gain/improvement) \(C\) has physical dimension of inverse seconds (1/sec) (however it is not a magnification!!). To obtain the magnification on the basis of angular frequency \(\omega\), we have \(M(\omega) = |H_d(i\omega)|\) estimate and therefore:

\[ M(\omega) = \frac{C\omega^3}{\sqrt{(\omega_s^2 - \omega^2)^2 + 4\omega^2 \omega_s^2 h_s^2} \sqrt{(\omega_g^2 - \omega^2)^2 + 4\omega^2 \omega_g^2 h_g^2}} \quad [1, 5] \] (15)

Relationship (14) is the analysis to elements of transfer function which is accepted as system sub-branches. Numerical values method is used and denominator of fraction is expanded to fourth degree polynomials (Fourth-order polynomial). By placing numerical values, we have:

\[ H_d(s) = \frac{383.6s^3}{(s^4 + 0.6283s^3 + 0.2435s^2 + 0.0245s + 0.000855)} \equiv \frac{num}{den} \equiv \begin{bmatrix} 0 & 383.6 & 0 & 0 \\ 1 & 0.6283 & 0.2435 & 0.0245 & 0.000855 \end{bmatrix} \quad [1, 5] \] (16)

Using Simulink MATLAB, we will have similar performance to relationship (14) and only the numerical values have changed. The advantage of formula (16) is its shortness and the poles and zeros of this function are easily determined from Eq. (14), for example, we immediately understand that there are three zeroes in \(s = 0\).

Rapid and accurate drawing of different diagrams (Nyquist, polar, step, impulse, Bode, logarithmic, Root Locus, etc.), the system response for different values of attenuation and other parameters are some of the other advantages of this system using MATLAB software. The figures are used in control, electrical, mechanic, etc. engineering in different frequency areas, system stability and transient response, etc. and have a special place.

By specifying certain points and asymptotes, calculating exit angle of mixed poles and enter angle into mixed zeros, the root locus of a complex system can be drawn with many poles and zeros, although roots locus method is basically a trial and error method.

In these figures locus branches start from open-loop poles and end in open-loop zeros (limited or unlimited zeros). Since poles and zeros are always in pairs, their root locus is symmetrical with the real axis.
If the degree of Denominator polynomial is larger than that of Numerator polynomial, when $S$ tends to infinity, $X(s)$ becomes zero, and conversely, if the degree of Numerator polynomial is larger than that of Denominator, when $S$ tends to infinity $X(s)$ becomes unbounded which can be treated as infinite zero and pole (Refer to [4]).

With the help of Bode and Nyquist diagrams system stability can be estimated. For example, in Fig. 14 it can be seen that a right turn is around point -1.

Therefore, we have unstable poles and we need to inject a positive phase for stabilization of the system to have rotation in the counterclockwise direction of time.

To achieve this goal, we add a zero and place it in the -1.

Now we will try to improve system response, and to this aim, we should correct compensator poles. It should be noted that poles close to the source affect the response in the low frequencies and poles away from source affect the response in high frequencies. Therefore, frequency response (Bode diagram) should be smooth shape and Nyquist diagram should be elliptical. Figs. 2, 3, 4, 5, 6, 7, 8, 15 and 16 show the effect of different damping constant to lead the system toward stability and instability.

Fig. 17 shows the effect of different damping constant to lead the system toward modifications of amplitude and phase.

If the Numerator and the Denominator have the same degree, $X(s)$ has $k$ poles in infinity. In this case, $X(s)$ in the infinity has neither zero nor pole, and generally, the number of repetitions of a zero or a pole in a location is called pole and zero degrees.

If we had several poles, the speed of response corresponding to each pole depends on its distance from the source. The closer the pole to the source, its corresponding sentence in the impulse response becomes damped very quickly, and the sitting time of step response becomes shorter.
Such diagrams clearly describe the effect of each open loop pole or zero on the closed loop poles of the system and show the locus of roots of the equation per change of interest from zero to infinity. Also, the graphs show the way open-loop poles and zeros are corrected for the desired response and are appropriate for rapid access to approximate results.

Fig. 15. Graph of system Nyquist response $H_d(s)$ of relation (15) for attenuation values 0 and 0.6, (respectively from (a) to (b)) [1].

Fig. 16. Clockwise rotation around point -1 but with little attention, we find that by adding gain and shifting Nyquist curve to left we can change turn around point -1 counterclockwise and lead the system toward stability [1].

Fig. 17. Bode diagrams of system response $H_d(s)$ of relation (15) for damping values 0, 0.6 and 0.9 (respectively from a to f) [1].
The Roots Locus is also called root path. This method is a very powerful graphical method for investigating the effect of changing a system parameter on the locus of closed loop poles.

Graphs of response Step, Nyquist, Bode and etc. of the System for different frequencies and damping can be seen in the reference [1].

All figures can be shown in three-dimensional (3D) and animated (refer to [1]).

Declarations

Author contribution statement

Seyed Hossein Abrehdari: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Additional information

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