Statistical analysis of vapor distribution in a cavitation flow based on an ensemble of instantaneous liquid velocity fields

A S Severin$^{1,2}$, M V Timoshevskiy$^1$, B B Ilyushin$^1$, K S Pervunin$^1$

$^1$Institute of Thermophysics SB RAS, Novosibirsk 630090, Russia
$^2$Novosibirsk State University, Novosibirsk 630090, Russia

E-mail: pervunin@itp.nsc.ru (K S Pervunin)

Abstract. A new method was developed for statistical analysis of ensembles of instantaneous velocity fields measured by PIV in liquid (continuous phase) to determine the distribution of the vapor phase in cavitating flow. The method is based on two main principles: the absence of tracers used for PIV measurements in vapor, and the statistical independence of individual measurements. This allowed establishing an exponential dependence of repeatability of the vapor phase at a certain point of a cavitating flow. Compliance with this theoretical law was verified using the Pearson chi-square test. All theoretical distributions were divided into several groups depending on the time-averaged local vapor content calculated over the entire ensemble of realizations and the probability of a single event. As a result, dimensions of the stationary part of an attached cavity and the place of detachments of cloud cavities from the hydrofoil surface were determined using the new method of statistical analysis for an unsteady cloud cavitation regime.

1. Introduction

The phenomenon of cavitation negatively affects the operation of hydraulic equipment, leading to erosion of the surface material, unsteadiness of the flow, increased noise levels, and vibrations of structures. A comprehensive and detailed study of cavitation flows requires using modern panoramic measurement and visualization methods with the high spatial and temporal resolution, for example, the particle image velocimetry (PIV) technique, as well as advanced methods for analyzing experimental data.

In the last three decades, the PIV method has been actively used for the diagnosis of cavitating flows. It was first tested to study the hydrodynamics of cavitating flows at microscales, near the boundaries of individual bubbles, and around incipient cavities [1]. The PIV method was first used to measure the flow velocity field near the cavitating cylinder and hydrofoil in combination with the laser-induced fluorescence (LIF) approach [2–5], when fluorescent particles are added to the working liquid, and an optical filter is installed on the camera lens to suppress laser light. Thus, the authors were able to measure the flow velocity near the interphase boundaries of cavitation structures for the first time, which is necessary for analyzing the interaction of continuous and dispersed phases.

In early works, the distributions of average and turbulent characteristics were calculated from an extremely limited number of instantaneous PIV realizations. For example, the sample volume in [6] was only 72 instantaneous velocity fields, and in [4] even less – 50 realizations. It is obvious that such sampling is not enough to calculate the turbulent characteristics with high accuracy. Given this, the authors of these works make only qualitative conclusions, despite quantitative measurements. Such a
small volume of statistics in the first works on cavitation is associated with the insufficient level of
development of computing and measuring technology at that time.

With the improvement of the technologies, the PIV method has also been further developed. So, in
[7] the authors used a PIV with a high spatial resolution to study the flow in the vicinity of the tip of a
three-dimensional blade, which made it possible to capture slender tip vortices, in the cores of which
favorable conditions for the inception of cavitation occurs. In [8] the flow dynamics near vapor
formations for fully developed sheet and cloud flow regimes were studied using time-resolved PIV. In
this configuration, the PIV method allows both dynamically tracing the change in the instantaneous
distribution of the dispersed phase in the flow and calculating the frequency spectra of velocity
fluctuations, which is most relevant for non-stationary cavitation regimes.

Despite significant progress in the application of panoramic methods for diagnosing two-phase
dispersed flows, including a huge database of various experimental data obtained by the PIV method,
an in-depth statistical analysis of PIV data has not yet been carried out. At the same time, experimental
studies in the vast majority of cases are limited to measurements in a liquid and visualization – these
methods do not provide quantitative information about the distribution and concentration of the vapor
dispersed phase in a two-phase flow. In the current work, a new method for analyzing two-phase flows
based on the statistical analysis of ensembles of fields of the instantaneous velocity of a continuous
liquid phase obtained by the PIV method is presented.

2. Experimental setup

The experiments were carried out using the cavitation rig at the Institute of Thermophysics SB RAS,
described in detail in [9]. The test object was a reduced model of the Francis turbine guide vane with a
chord length \( C = 100 \) mm and a maximum thickness \( H_{\text{max}} = 0.2149C \), described in detail in [10]. The
hydrofoil was installed at an angle of attack \( \alpha = 9^\circ \). Distilled water was used as the working liquid and its temperature was 30°C. The main dimensionless parameters that determine the cavitation flow
regimes are the cavitation number \( \sigma \) and the Reynolds number \( \text{Re}_C \): 
\[ \sigma = \frac{2(P - P_{\text{sat}})}{\rho v^2}, \quad \text{Re}_C = \frac{v \bar{C}}{\nu}, \]

where \( P \) is the hydrostatic pressure at the inlet to the working channel, \( P_{\text{sat}} \) is the pressure of saturated water vapor, \( \rho \) is the density of water, \( \nu \) is the velocity of the incoming flow measured by the PIV method upstream relative to the location of the hydrofoil, \( \nu \) is the kinematic viscosity of water. In this
research, we considered the developed non-stationary cloud regime of cavitation flow around the
hydrofoil with \( \text{Re}_C = 1.32 \times 10^6, \sigma = 1.86 \).

3. Statistical method for detecting the vapor phase

3.1. The principle of the method

At the stage of cross-correlation processing of the original PIV images (i.e., when calculating the
instantaneous velocity vector field), each vector is assigned a certain status. This status contains
information about the local flow conditions at a particular time, and, therefore, can be used for
statistical analysis of the local structure of the cavitation flow for all implementations. So, the status 0
is attributed to masked vectors. Status 2 is assigned to incorrectly calculated velocity vectors filtered
out at the stage of cross-correlation processing. Status 3 corresponds to the correct vectors corresponding to the continuous phase (liquid). Vectors that are filtered out by the minimum number of tracers (when the number of tracers for the current calculated cell is below the specified threshold) have the status 5.

Having information about the statuses of all vectors for each instantaneous velocity field, and
assuming that the velocity vectors with status 5 correspond to the dispersed (vapor) phase in the
cavitation flow, it is possible to conduct a statistical analysis of the distribution of the vapor phase in
the cavitation flow according to PIV data for the liquid phase. The assumption made about the vector
statuses for the dispersed phase follows directly from the qualitative analysis of the PIV image, which
clearly shows that the tracers are only in the liquid phase and are absent in cavitation structures. Thus,
the idea of the proposed method of statistical analysis is to study the regularity of the repeatability of
the appearance of vectors with a certain status at each specific point of the flow from one realization to another over the entire statistical sequence.

Further, having made all the necessary assumptions and analyzing only certain vectors with a given status, it is possible to construct a histogram of the repeatability of events (finding the vapor phase at a specific point of the cavitation flow) over the entire data sample. Since the frequency of PIV measurements (~4 Hz) is significantly less than the characteristic frequencies of cloud cavity shedding in the non-stationary cloud cavitation regime (~40 Hz), such events can be considered statistically independent. The probability of a repeat of statistically independent events is equal to the product of the probabilities for each individual event. Therefore, the probability function will have the exponential form: \( P(n) \sim q^n = e^{n \ln(q)} = e^{-\lambda n} \), where \( q \) is the probability of a single event, \( A = \ln(1/q) \) is a constant.

### 3.2. Implementation of the method

Compliance with this theoretical law was verified using the Pearson chi-square test. To assess the correspondence of the exponential probability function to the theoretical distribution, an expression of the form \( P(n) = Be^{-\lambda n} \) was taken, where the parameters \( A \) and \( B \) were estimated using the least-squares method. To test the null hypothesis, the chi-square test statistic was determined by the ratio:

\[
\chi^2 = \sum_{i=1}^{r} \left( \frac{(y_i - mP(x_i))^2}{mP(x_i)} \right); \quad m = \sum_{i=1}^{r} y_i; \quad \sum_{i=1}^{r} P(x_i) = 1,
\]

where \( y_i \) is the height of the \( i \)-th column of the experimental histogram, \( P(x) \) is the probability function of the theoretical distribution, \( r \) is the number of bins of the histogram for which the test statistic was calculated.

When the hypothesis of a theoretical distribution for \( m \to \infty \) is fulfilled, this statistic converges to the chi-square distribution with \( r - 1 - s \) degrees of freedom, where \( s \) is the number of parameters of the theoretical distribution. In this paper, \( s = 2 \) (two parameters \( A \) and \( B \)), the parameter \( r \) was selected empirically equal to 5, that is, the correspondence of the experimental histogram to the exponential function was checked by the first five columns. This allowed finding the p-value using the quantiles of the chi-square distribution. In this paper, the null hypothesis was accepted at the level of 0.05. This means that if the p-value exceeds this value, the null hypothesis is accepted, otherwise, it is rejected.

### 4. Results

#### 4.1. Zonal division of the velocity field

A preliminary analysis of the histograms constructed from experimental data showed that when approaching the region of the attached cavity, the nature of the distributions smoothly transforms from exponential to uniform. This means that in the cavity region, the events under consideration are no longer statistically independent. At the same time, there are other types of distributions. To facilitate further data analysis, the entire set of various types of theoretical distributions is divided into several groups according to the following criteria. The points at which the percentage of the dispersed phase in the entire sample does not exceed ten percent (\( \beta \leq 10\% \)) belong to the "liquid" group (continuous phase). If \( \beta > 95\% \), then such a vector falls into the group "\( \beta = 0.95 \)" (vapor). At that, the case when \( \beta = 100\% \) ("\( \beta = 1.00\)") is also isolated separately. These two groups represent the stationary part of the attached cavity.

For the remaining types of histograms at \( 10\% < \beta \leq 95\% \), the null hypothesis is checked using the Pearson's chi-square test for the first \( r \) columns of the histogram, provided that all of them are non-zero. Further dividing takes place according to the boundary values of the probability of a single event \( q \), which for clarity and simplicity are taken to be 0.25, 0.5, 0.75 (Figure 1). If the number of non-zero bins of the histogram is less than \( r \), or if there are zero bins among the first \( r \) bins, then such statistics are considered insufficiently reliable, and the corresponding vector falls into the "bubbles" group. This category includes points where the vapor phase appears from time to time. Vectors that have not passed the test of the null hypothesis fall into the category of "undefined".
4.2. Statistical analysis of the flow around the guide vane

This classification allows dividing the entire studied flow area into several zones, of which each has its own distribution law of the events repeatability. As can be seen in Figure 2, the obtained field separates the region of the stationary cavity (red dots), in which $\beta > 95\%$. It should be noted that the cavity area is divided into two parts by a small green area, for which $q \geq 0.75$. This means that tracers are regularly registered in this zone, that is, the liquid appears, whereas this does not occur in other parts of the cavity. Obviously, this is due to the periodic shedding of large-scale clouds in the non-stationary cloud cavitation regime, so that at this point there is a break of the interphase boundary of the cavity.

Conclusions

The suggested method of statistical analysis was developed and applied to a cavitating flow around a 2D hydrofoil to investigate the distribution of the dispersed (vapor) phase based on PIV velocity measurements in liquid (continuous phase). The following two principles provide the ground for this method: the absence of tracers in vapor structures, which are used in PIV measurements, and the statistical independence of events that are appearances of the vapor phase in a given point of the measurement area. This allowed assuming that the probability density function of repeatability of these events has an exponential form. To verify this assumption, Pearson's chi-square test was used.

This approach made it possible to divide all theoretical distributions into several groups depending on the probability of a single event and conduct accordingly a zonal fragmentation of the entire measurement area, considering time-averaged local vapor content and the type of the theoretical distribution. As applied to the cavitating flow, the implemented method makes it possible to reveal the
presence and location of the stationary and pulsating parts of a cavitation sheet, determine its dimensions and shape and find the place of detachments of cavitation clouds. Finally, this approach allows significantly optimizing data analysis, reducing the processing time, and saving computational resources by limiting experiments solely to standard PIV measurements.

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