Correlation methods in cutting arcs

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Abstract The present work applies similarity theory to the plasma emanating from transferred arc, gas-vortex stabilized plasma cutting torches, to analyze the existing correlation between the arc temperature and the physical parameters of such torches. It has been found that the enthalpy number significantly influence the temperature of the electric arc. The obtained correlation shows an average deviation of 3 \% from the temperature data points. Such correlation can be used, for instance, to predict changes in the peak value of the arc temperature at the nozzle exit of a geometrically similar cutting torch due to changes in its operation parameters.

1. Introduction
Plasma cutting is a process of metal cutting at atmospheric pressure by an arc plasma jet, where a transferred arc is generated between a cathode and a work-piece (the metal to be cut) acting as the anode. Small nozzle bore, extremely high enthalpy and operation at relatively low arc current (\(\approx 10 \div 200\)) are a few of the primary features of these torches [1]. The physics involved in such arcs is very complicated. The conversion of electric energy into heat within small volumes causes high temperatures and steep gradients [2]. Dissociation, ionization, large heat transfer rates (including losses by transparent radiation), fluid turbulence and electromagnetic phenomena are all involved. In addition, wide variations of physical properties, such as density, thermal conductivity, electric conductivity and viscosity have to be taken into account.

These factors make hopeless to get an analytical solution for such arcs. Recently, commercial computational fluid dynamic software has become widely available and then numerical plasma modelling has reached a state advanced enough to be of practical use in the study of the cutting arc processes. However, as such programs were not initially developed for the description of thermal plasma process, they need to be complemented with electromagnetic and plasma properties packages [3]. These packages are not easy to obtain. Also the commercial software results somewhat expensive.

On the other hand, experimental data on cutting arcs are hard to obtain due to the harsh ambient conditions. Most of the available experimental data are related to spectroscopic [1] and probe measurements [4-6] in the external plasma region, giving direct information of only part of the variables involved, mainly the temperature.
In this scenario, the knowledge of a proper correlation method to make advantageous use of the experiments already performed on cutting arcs becomes important from the engineering point of view. The present work applies similarity theory [7,8] to the plasma emanating from transferred arc, gas vortex stabilized plasma cutting torches, to analyze the existing correlation between the plasma temperature (a quantity of primary importance in such devices) and the physical parameters of such torches.

2. Arc dimensionless numbers

2.1. Mathematical model
Since a number of complex, interacting, nonlinear processes are simultaneously occurring in cutting arcs (electromagnetic, gas dynamic as well as thermal effects), the conservation equations were used as the starting point for determining dimensionless numbers.

The most frequently used cutting arc models assume the local thermodynamics equilibrium (LTE) approximation [3], so the plasma is characterized by a single temperature \( T \). The fluid part of the thermal plasma model describing such an arc can be expressed as follows.

Total mass conservation

\[
\nabla \cdot (\rho \vec{u}) = 0. \tag{1}
\]

Momentum conservation

\[
\nabla \cdot (\rho \vec{u} \otimes \vec{u} - \vec{\tau}) + \nabla p - \vec{J} \times \vec{B} = 0. \tag{2}
\]

Energy conservation

\[
\nabla \cdot (\rho h + \vec{q}) - \vec{J} \cdot \vec{E} - \nabla p \cdot \vec{u} + \dot{Q}_{\text{rad}} = 0, \tag{3}
\]

where \( \rho \) represents the total mass density, \( \vec{u} \) the fluid velocity, \( p \) the pressure, \( \vec{\tau} \) the stress tensor, \( \vec{J} \) the current density, \( \vec{B} \) the magnetic field, \( h \) the enthalpy, \( \vec{q} \) the total heat flux, \( \vec{E} \) the electric field and \( \dot{Q}_{\text{rad}} \) the power lost by transparent radiation.

Two more equations are required to describe the electromagnetic part of the plasma model. The first is the current continuity equation

\[
\nabla \cdot \vec{J} = 0, \tag{4}
\]

where

\[
\vec{J} = \sigma \vec{E}, \tag{5}
\]

while the second is one of Maxwell’s equations

\[
\nabla \times \vec{B} = \tilde{\mu}_0 \vec{J}, \tag{6}
\]

where \( \sigma \) is the electric conductivity and \( \tilde{\mu}_0 \) the free space magnetic permeability.

The total heat flux in equation (3) describes the heat transported by conduction and the enthalpy transport by mass diffusion, and is defined as [3]

\[
\vec{q} \equiv -\kappa_e \nabla T + \vec{J}_e h_e, \tag{7}
\]

where \( \kappa_e \) is the effective thermal conductivity and \( \vec{J}_e \) is the electron mass diffusion that can be approximated by

\[
\vec{J}_e \approx -\frac{m}{e} \vec{J}, \tag{8}
\]

where \( e \) is the elementary electric charge and \( m \) is the electron mass. Equation (8) neglects the charge transported by ions. In equation (7) \( h_e = 5k_\text{B} T/(2m) \) represents the specific electron enthalpy \( (k_\text{B} \) the Boltzmann’s constant).

The effective viscosity is

\[
\mu_e = \mu_i + \mu_e, \tag{9}
\]

and the effective thermal conductivity is

\[
\kappa_e = \kappa + \frac{\mu_e C_p}{P_e}, \tag{10}
\]
where $C_p$, $\mu$ and $\kappa$ are the constant-pressure plasma specific heat, viscosity and thermal conductivity, respectively. The turbulent viscosity $\mu_t$ and the Prandtl number $P_r$ require extra relationships (which are commonly referred to as the turbulence model) to calculate the turbulent enhanced viscosity and thermal conductivity. Relationships expressing the dependence of gas composition, thermodynamic properties, transport coefficients and radiation power losses on temperature and pressure; must be also added in order to make the set closed. To make the problem definite it is necessary also to specify boundary conditions for all of such variables.

2.2. Dimensionless numbers

In what follows, the set of equations are expressed in dimensionless form by making use of reference values (denoted by the subscript "zero") for the above plasma quantities, and with $L$ as a scale length:

\[
\nabla^* \cdot \left( \rho^* \overline{\mathbf{u}}^* \right) = 0, \\
\nabla^* \cdot \left( \rho^* \overline{\mathbf{u}}^* \otimes \overline{\mathbf{u}}^* \right) = -\frac{\tau_0}{\rho_0 u_0^2} \nabla^* \cdot \overline{\mathbf{F}}^* + \frac{\rho_0}{\rho_0 u_0^2} \nabla^* p^* - \frac{J_0 B_0 L}{\rho_0 u_0^2} \mathbf{J}^* \times \overline{\mathbf{B}}^* = 0, \\
\rho_0 h_0 u_0 \nabla^* \cdot \left( \rho^* \overline{\mathbf{u}}^* h^* \right) + \frac{q_0}{L J_0 E_0} \nabla^* \cdot \overline{\mathbf{J}}^* - \frac{J_0 B_0 L}{L J_0 E_0} \nabla^* p^* \cdot \overline{\mathbf{u}}^* + \frac{\dot{Q}_{rad0}}{J_0 E_0} \dot{\mathbf{Q}}_{rad}^* = 0, \\
\nabla^* \cdot \overline{\mathbf{J}}^* = 0, \\
\overline{\mathbf{J}}^* = \frac{\sigma_0 E^*}{J_0}, \\
\nabla^* \times \overline{\mathbf{B}}^* = \frac{\mu_0 J_0 L}{B_0} \mathbf{J}^*. 
\]

The superscript (*) indicate dimensionless numbers, obtained by dividing dimensional variables by the appropriate scale values ($\rho^* \equiv \rho/\rho_0$, $\overline{\mathbf{u}}^* \equiv \mathbf{u}/u_0$ and so on). The equation (12) gives three dimensionless numbers,

\[
\Pi_{nx} \equiv \frac{\rho_0}{\rho_0 u_0^2}, \quad \Pi_{nx} \equiv \frac{\tau_0}{\rho_0 u_0^2}, \quad \Pi_{em} \equiv \frac{J_0 B_0 L}{\rho_0 u_0^2}. 
\]

The first and the second are the well-known Euler and Reynolds numbers. The third number shows the relative weight between the electromagnetic and inertial forces. The equation (13) gives four dimensionless numbers,

\[
\Pi_h \equiv \frac{\rho_0 u_0 h_0}{J_0 E_0 L}, \quad \Pi_qf \equiv \frac{\rho_0 u_0}{J_0 E_0 L}, \quad \Pi_q \equiv \frac{q_0}{J_0 E_0 L}, \quad \Pi_{rad} \equiv \frac{Q_{rad0}}{J_0 E_0}. 
\]

These numbers reflect the conversion of electric energy into other forms. The enthalpy number $\Pi_h$ takes into account energy which is used for heating the plasma flow, $\Pi_{qf}$ times $\left(\Pi_{nx}\right)^{-1}$ is based on energy used for acceleration, and $\Pi_q$ and $\Pi_{rad}$ involve energy losses by conduction and mass diffusion; and energy losses by electromagnetic radiation, respectively. The Ohm’s and Ampere laws give the numbers,

\[
\Pi_E \equiv \frac{\sigma_0 E^*}{J_0}, \quad \Pi_B \equiv \frac{\mu_0 J_0 L}{B_0}. 
\]

These numbers reflect the relation between self and external magnetic fields (if any), and the process of charged particle transfer in an electric field.

In addition to these numbers, from the boundary conditions various parametric numbers can be obtained. The boundary condition for the arc current ($l$)

\[
l = \int_S \mathbf{J} \cdot d\mathbf{S}, 
\]
(S represents the area of the arc cross section) in dimensionless form becomes
\[ I = J_0 L^2 \int_{S^*} J^* \cdot dS^* , \]
and hence gives the current number \( \Pi_I \)
\[ \Pi_I \equiv \frac{J_0 L^2}{I} . \]
Also, the numbers
\[ \Pi_d \equiv \frac{d}{L} , \quad \Pi_{ip} \equiv \frac{P_{rel}}{P_0} \]
account for the nozzle geometry (diameter \( d \)) and the influence of the inlet gas pressure \( P_{rel} \) on the arc quantities.

The set of dimensionless equations (11-16) gives nine independent dimensionless numbers in agreement with the Buckingham \( \pi \)-theorem, since the number of quantities is thirteen and the number of dimensions four. Besides, the arc boundary conditions introduce two new parametric numbers.

Most of these numbers contain physical properties, such as density and electric conductivity, which vary widely because of the large temperature range, and certain reference values have therefore to be taken. In this work, the scale values of the plasma physical properties were determined for a mean temperature \( T_0 \) which corresponds to an electron concentration of 1% in the plasma at the ambient pressure [8]. These plasma scales (listed in Table 1), were taken from the calculated values in [9].

### Table 1. Scale values of the plasma physical properties.

| Physical variables | Oxygen \( (\times 10^3 \text{ K}) \) | Nitrogen \( (\times 10^3 \text{ K}) \) | Air \( (\times 10^3 \text{ K}) \) |
|-------------------|-----------------|-----------------|-----------------|
| Temperature \((T_0)\) | 9.2 | 9.3 | 9.3 |
| Electrical conductivity \((\sigma_0)\) | \(2.23 \times 10^3\) | \(1.89 \times 10^3\) | \(1.96 \times 10^3\) |
| \( (\Omega^{-1} \text{ m}^{-1}) \) | | | |
| Enthalpy \((h_0)\) | \(2.86 \times 10^7\) | \(4.97 \times 10^7\) | \(4.52 \times 10^7\) |
| \( (J \text{ kg}^{-1}) \) | | | |
| Density \((\rho_0)\) | \(2.1 \times 10^2\) | \(1.83 \times 10^2\) | \(1.88 \times 10^2\) |
| \( (\text{kg m}^{-3}) \) | | | |

3. **Data on cutting arcs**

The experimental data on cutting arcs temperature correspond to the external plasma region (open regions where the pressure has relatively small variations around the atmospheric value). Peak values of the arc temperature close to the nozzle exit for currents in the wide range 12 ÷ 200 A, and for three different working gases (air, nitrogen and oxygen); were taken from the literature.

4. **Generalized arc temperature correlation**

Not all of the obtained dimensionless numbers are of equal importance. Inside the torch the pressure forces are almost balanced by inertial forces (i.e., other processes as viscosity and electromagnetic acceleration are considerably less relevant) and so only the Euler number becomes important in the momentum equation.

On the other hand, since the electrical power delivered in the arc is almost used for heating the plasma flow (i.e., the conversion of electric energy into kinetic energy, while the energy losses by thermal conduction, mass diffusion and radiation are less important); only the enthalpy number becomes relevant in the energy equation. Also from the electromagnetic equations only the electric field number is important. Hence, the three relevant dimensionless numbers are
These numbers can be combined with the parametric numbers to eliminate the unknowns. The following modified form of the enthalpy number can be used as a generalized argument to construct the generalized plasma temperature correlation for the considered type of arc

\[
\Pi_{h}^{(1)} = \Pi_h \Pi_E \left( \Pi_{N_E} \right)^{0.5} \left( \Pi_f \right)^{2} \left( \Pi_{i_p} \right)^{0.5} = \rho_0^{0.5} P_{rel}^{0.5} h_0 \sigma_0 d^{3} I^{-2} .
\] (19)

The generalized function of the peak value of the arc temperature at the nozzle exit may be taken to be the simple ratio

\[
\Pi_T = \frac{T_{exit}}{T_0},
\] (20)

and hence the generalized arc temperature correlation may be expressed as

\[
\Pi_T = f \left( \Pi_h, \Pi_E, \Pi_f, \Pi_{N_E}, \Pi_{i_p} \right) = f \left( \Pi_{h}^{(1)} \right)
\] (21)

![Figure 1. Correlated arc temperature for cutting torches ranging from 12 to 200 A.](image)

The values of the arc temperature predicted using such correlation show a deviation not larger than 10 % with respect to the experimental data (with an average deviation of 3 %); in spite of the fact that such data include different torches ranging from 12 to 200 A.

Note that (22) shows that the centre-line value of the arc temperature strongly depends on both, arc current and nozzle diameter; but slightly on the type of gas and its inlet pressure. The factor \( \rho_0^{1/2} h_0 \sigma_0 \) is a number equal to \( 9.24 \times 10^9 \) for oxygen, \( 1.27 \times 10^{10} \) for nitrogen, and \( 1.21 \times 10^{10} \) kg\(^{1/2}\) s\(^{-1}\) m\(^{-1/2}\) W, for air.

5. Conclusions

The theory of dynamic similarity has been used to construct a generalized arc temperature correlation of a vortex-stabilized cutting plasma torch. The enthalpy number obtained from the energy equation has been found to significantly influence the temperature of the electric arc.

The obtained correlation shows an average deviation of 3 % from the temperature data points. Such correlation can be used, for instance, to predict changes in the peak value of the arc temperature at the nozzle exit of a geometrically similar cutting torch due to changes in its...
operation parameters (electric current, nozzle diameter, working gas specie or/and the inlet gas pressure); at least within the given parameters range. The relation shows that the centre-line value of the arc temperature strongly depends on both, arc current and nozzle diameter; but slightly on the type of gas and its inlet pressure.

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**REFERENCES**

[1] Nemchinsky V A and Severance W S 2006 *J. Phys. D: Appl. Phys.* **39**, R423
[2] Prevosto L, Kelly H and Mancinelli B 2009 *J. Appl. Phys.* **105**, 013309
[3] Gleizes A, González J J and Freton P 2005 *J. Phys. D: Appl. Phys.* **38**, R153
[4] Prevosto L, Kelly H and Mancinelli B 2008 *IEEE Trans. Plasma Sci.* **36**, 263
[5] Prevosto L, Kelly H and Minotti F O 2008 *IEEE Trans. Plasma Sci.* **36**, 271
[6] Prevosto L, Kelly H and Mancinelli B 2009 *IEEE Trans. Plasma Sci.* **37**, 1092
[7] Yas’ko O I 1969 *Brit. J. Appl. Phys. (J. Phys. D)* **2**, 733
[8] Paingankar A M, Das A K, Shirodkar V S, Sreekumar K P and Venkatramani N 1999 *Plasma Sources Sci. Technol.* **8**, 100
[9] Boulos M, Fauchais P, Pfender E 1994 *Thermal Plasmas, Fundamentals and Applications Vol. 1* (Plenum Press)
[10] Colombo V, Concetti A, Ghedini E, Dallavalle S and Vancini M 2008 *IEEE Trans. Plasma Sci.* **36**, 389
[11] Freton P, Gonzalez J J, Gleizes A, Camy Peyret F, Caillibotte G and Delzenne M 2002 *J. Phys. D: Appl. Phys.* **35**, 115
[12] Girard L, Teulet Ph, Razafinimanana M, Gleizes, Camy-Peyret, Baillot E and Richard F 2006 *J. Phys. D: Appl. Phys.* **39**, 1543
[13] Sakuragi S 1995 *Eng. Sci. Reports*, Kyushu University **16**, 383
[14] Pardo Sanjurjo C, González-Aguilar J, Rodríguez Yunta A and Calderón M A G 1999 *J. Phys. D: Appl. Phys.* **32**, 2181
[15] Peters J, Heberlein J and Lindsay J 2007 *J. Phys. D: Appl. Phys.* **40**, 3960