Abstract—In this work, we consider a remote monitoring scenario in which multiple sensors share a wireless channel to deliver their status updates to a process monitor via an access point (AP). Moreover, we consider that the sensors randomly arrive and depart from the network as they become active and inactive. The goal of the sensors is to devise a medium access strategy to collectively minimize the long-term mean network Age of Information (AoI) of their respective processes at the remote monitor. For this purpose, we propose specific modifications to ALOHA-QT algorithm, a distributed medium access algorithm that employs a Policy Tree (PT) and Reinforcement Learning (RL) to achieve high throughput. We provide the upper bound on the mean network AoI for the proposed algorithm along with pointers for selecting its key parameter. The results reveal that the proposed algorithm reduces mean network AoI by more than 50 percent for state of the art stationary randomized policies while successfully adjusting to a changing number of active users in the network. The algorithm needs less memory and computation than ALOHA-QT while performing better in terms of AoI.

I. INTRODUCTION

Applications involving Internet of Things (IoT) have emerged across many industries to make up industry 4.0. In the near future, connected robotics and autonomous systems will be a significant driving force behind the design of 5G and beyond [1]. Monitoring the states of the robotic machinery and its environment via multiple sensors will result in a large amount of Machine Type Communication (MTC) data. This data is characterized by periodic traffic generation, and short packet duration [2]. Many industry 4.0 applications such as factory robots, automated forklifts, and conveyor belts need not be active at all times. As individual tasks arise sporadically and are completed by the machines, the number of active users transmitting MTC data will be dynamic.

AoI is a performance metric especially suitable for real-time monitoring applications because it measures the freshness of information coming from a remote source [3]. AoI depends on different aspects of the overall system, such as sampling rate of sensors, queue management, etc. From a medium access and control (MAC) design perspective, improving AoI requires us to jointly optimize the transmission rate, delay, and the probability of successful reception of the information. Hence, adapting and designing special MAC protocols with the goal of reducing AoI have to be considered.

One way to optimize AoI in wireless IoT networks is to use grant-based channel access protocols where a centralized scheduler keeps track of all the active users in the network and distributes the network resources efficiently to all the systems. However, they are typically inefficient for MTC applications and complex to implement due to the overhead in signaling, and coordination [4]. This reason compels us to look at simpler distributed grant-free random access (RA) protocols for MTC applications which are descendants of the well-known ALOHA [5] and Slotted ALOHA (SA) [6]. The simplicity in implementation of RA protocols comes with the trade-off of poor performance in terms of AoI when compared to the grant-based solutions due to the frequent collision of packets [7]. In fact, it was shown in [8] that the mean network AoI gap between grant based policies and grant free policies is $O(n)$ where $n$ is the number of active users in the network. In order to bridge this gap, the users must overcome the collision problem by learning to coordinate and select transmission times in a way that the chances of packets colliding is reduced.

Stationary randomized policies to reduce collisions have been suggested to improve the performance of RA schemes. They rely on knowing or estimating the active number of users in the network [6], [9]–[11]. This value is difficult for the users to evaluate in a decentralized setup [12]. One such scheme [10], relies on Poisson distributed packet arrivals at the users to estimate the number of active users in the network in order to optimize an AoI threshold. It however, cannot be applied to the generate-at-will [13] model addressed in this paper. In [11] the authors present Age-Dependent Random Access (ADRA), an extension of [9] where the users access the channel only if a predefined AoI threshold is exceeded. The channel access probability and the AoI threshold are both a function of the number of active users in the network. The DRR algorithm [14] achieves optimal AoI by requiring the AP to establish the number of active users in the network and relaying this information to the users via a feedback. This scheme therefore requires a more complex feedback as well as offloads some complexity to the AP and moves in the direction of centralized scheduling. ALOHA-Q [15] and ALOHA-QT [16] require neither the users nor the AP to ascertain the number of active users. Both the algorithms maintain the simplicity of classical RA and achieve better channel utilization using RL to coordinate with each other over the feedback of the AP. The high utilization and flexibility of the ALOHA-QT algorithm piques our interest to investigate its performance in terms of AoI. ALOHA-QT employs a PT [17] to divide the transmission slots in a frame into non-conflicting
This paper proposes a better performing and computationally cheaper version of ALOHA-QT and calls it modified ALOHA-QT or mAQT. The modifications are better suited for remote monitoring in MTC applications where the number of active users is changing over time. We obtain an upper bound for the mean network AoI by using properties of a well-known abstract data structure called the full binary tree (FBT).

II. SYSTEM MODEL

We consider a remote process monitor connected to an AP, receiving the status update packets of $M$ physical processes over a wireless network. Each process has a sensor and transmitter associated with it, and we call this subsystem a user to be consistent with the terminology used in MAC protocols. Time is divided into slots of equal duration. The length of a status update packet is assumed to be constant, and the transmitter takes the duration of the entire slot to transmit a single packet. Throughout the paper, we express all time-related quantities in terms of slots.

We consider that every user has two states - active and inactive. There are $n[t]$ active users in the network at time $t$. The number of slots a user spends in a state follows a geometric distribution with a transition probability of $p$ on every slot. Therefore, the average number of slots before a state transition for every user is $1/p = k$. On average, there is an activation or a deactivation once every $k$ slots in the entire network.

To generate a packet, the sensor accurately samples the state of the process, and the transmitter encapsulates it into a status update packet ready for transmission. This packet generation process of a user is assumed to be generate-at-will \(^1\), where an active user generates a new status update packet at the beginning of only those slots where it has decided to transmit. Hence, the user always has the freshest state encapsulated in any transmitted packet. The state of the physical process needs to be monitored only if the user is active. Such a system model can be imagined in a factory-like scenario shown in Figure 1 where the state of the machinery needs to be monitored. Individual machines are not occupied at all times and therefore only need to be monitored when they are performing a task.

A decision to transmit $d_i[t] \in \{1, 0\}$ is made at the start of slot $t$ by each active user $i$ according to its policy where $d_i[t] = 1$ if the user decides to transmit and $d_i[t] = 0$ if it decides to abstain from transmitting. At the end of the slot, the AP broadcasts the slot outcome as feedback $F[t] \in \{1, e, 0\}$ to all the users. If only one user transmits on the channel in a slot, the AP is able to receive the packet successfully, and we call this a success slot, i.e. $F[t] = 1$. When more than one user transmits in a slot, their signals interfere, and the AP can neither decode any of the packets nor extract the number of users who transmitted on the channel. This scenario is called a collision, i.e. $F[t] = e$. We assume only an interference-limited channel such that transmission by any user fails only in the case of a collision. When no users transmit on the channel in a given time slot, then we say it is an idle slot, i.e. $F[t] = 0$. The feedback is assumed to be immediate and perfect i.e., all active users receive the feedback at the end of the slot.

At the beginning of a slot, the AP sends the successfully received packet (if any) from the previous slot to the process monitor. If no user transmits on the channel in a given time slot, then we say it is a collision, i.e. $F[t] = e$. The feedback is assumed to be immediate and perfect i.e., all active users receive the feedback at the end of the slot.

![Figure 1. Example Scenario: Remote monitoring of factory robots who need to transmit their status updates only when they have a task at hand.](image1)

To the monitor, which updates the state of the respective process received packet (if any) from the previous slot to the process.

The feedback is assumed to be immediate and perfect i.e., all active users receive the feedback at the end of the slot.

![Figure 2. Linear AoI progression: AoI of a user grows linearly until the user successfully transmits its status update packet at $t = 1$ and $t = 5$. The AoI becomes 1 after a successful transmission.](image2)

![Figure 3. Binary Policy Tree: An example of the evolution of AoI for a particular user $i$ is shown in Figure 3. Here, $d_i[1] = 1$, $F[1] = 1$, $d_i[5] = 1$ and $F[5] = 1$. For time slots other than 1 and 5, the user $i$ either refrained from transmitting or experienced collisions.](image3)

III. POLICY TREE BASED ALGORITHM

A binary policy tree is shown in Figure 3. Each node in the tree is represented by a tuple $(c, 2^l)$, where $l \in \mathbb{N}$. The call is made with a schedule with level $l$. Every active user in the network keeps a time slot counter $t$. The schedule $(c, 2^l)$ prescribes transmission if $t \mod 2^l = c$. For example, the schedule $(3, 4)$ prescribes a transmission when $t = 3, 7, 11, 15, 19, 23, \ldots$. The tree is arranged in a way that the children of a parent schedule $(c, 2^l)$ are $(c, 2^{l+1})$ and $(c+2^{l+1}-1, 2^l)$.

\[^1\]Henceforth, we only talk about binary trees.
Table I. Symbols for the different parameters used in ALOHA-QT and mAQT. The optimal values given in the last column were obtained after running a gridsearch. These were used or the evaluation in Section V.

\[ (c + 2^l, 2^{l+1}) \] and hence all slots that prescribe transmission for both the children are present in the parent. For example, while one child of the schedule \((3, 4)\), e.g., \((3, 8)\) prescribes transmission when \(t = 3, 11, 19, \ldots\), the other child \((7, 8)\) prescribes transmission when \(t = 7, 15, 23, \ldots\). The schedules at the same level \(l\) prescribe transmission at the same rate \(2^l\) slots) but with different offsets. In MAC algorithms employing PT, every user transmits according to one or more schedules. As long as users do not select ancestors or descendants of schedules selected by other users, they will have selected non-conflicting transmission slots. The number of schedules in the PT is determined by the depth \(J\) (maximum level) of the tree. A PT with depth \(J\) has \(2^{J+1} - 1\) schedules.

A. ALOHA-QT Algorithm

ALOHA-QT (Algorithm 1) is a distributed expert-based RL algorithm using which, every user selects non-conflicting schedules in a PT. Notation of parameters and state variables used in this algorithm are given in Tables I and II respectively. The algorithm iteratively assigns a weight \(w_{(c, 2^l)}\) to every schedule \((c, 2^l)\) in the PT according to its potency to achieve a non-conflicting transmission. First (in step 0), every user initializes the weights of all the schedules in the PT such that higher schedules (closer to the root node of the PT) have higher weights. This ensures that the users explore transmitting at higher rates before moving down the PT. Small noise is added to each weight to reduce the probability of two schedules at the same level being initiated with the same weight. The noise also ensures that the initial behavior of all the users is not the same. The rest of the steps are then performed by the users once every slot.

1. **Initialization:**
   \[ t_i \leftarrow 0. \]
   \[ S \leftarrow \{(c, 2^l) \mid 0 \leq c < 2^l, 0 \leq l \leq J\}. \]
   \[ \forall w_{(c, 2^l)} \in S : w_{(c, 2^l)} \leftarrow w_{(c, 2^l)} \cdot \left(1 - \gamma_0 \cdot U(0, 1)\right). \]
   \[ \forall \sigma \in S : \{w_\sigma\} \sigma \in S \]
   \[\text{At every slot, do}\]

2. **Active Schedule Update:**
   \[ A \leftarrow \{(c, 2^l) \in S \mid t_i \mod 2^l = c\}. \]

3. **Schedule Selection:**
   \[ I \leftarrow \{\text{arg max}_{\sigma \in S} W_\sigma \cap \{\sigma \in S \mid w_\sigma > \eta\}. \]

4. **Decision:**
   - If \( A \cap I = \emptyset \) and \( \kappa = 1 \) then \( d_i[t_i] = 1 \)
   - else \( d_i[t_i] = 0. \)

5. **Reward Selection:**
   - If \((F[t_i], d_i[t_i]) = (0, 0)\) or \((1, 1)\) then \( \alpha \leftarrow \alpha^+ \)
   - else \( \alpha \leftarrow \alpha^- \).

6. **Weight Update:**
   \[ \forall \sigma \in A : w'_\sigma \leftarrow w_\sigma \cdot e^{\alpha \cdot U(0, 1)}. \]

7. **Voluntary Relinquishment:**
   - If \( U(0, 1) < \epsilon \) then \( \forall \sigma \in A : w'_\sigma \leftarrow 0. \)

8. **Weight Normalization:**
   \[ W \leftarrow \sum_{\sigma \in \rho} w_\sigma, \quad W' \leftarrow \sum_{\sigma \in \rho} w'_\sigma. \]
   \[ \delta \leftarrow W - W'. \]
   - If \( \delta > 0 \) and \( W' < w_{\text{init}} \cdot |S| \)
   - then \( \forall \sigma \in S : X_\sigma \leftarrow \{0, 1\}. \)
   - else \( \forall \sigma \in S : w_\sigma \leftarrow w'_\sigma + \delta \cdot (X_\sigma / \sum_{\sigma} X_\sigma). \)

9. **Time Increment:**
   \[ t_i \leftarrow t_i + 1. \]

Every instance of \( U(0, 1) \) in the above algorithm is an independent random sample drawn from the uniform distribution in the interval \([0, 1]\).

Algorithm 1 The ALOHA-QT Algorithm

\[
\begin{align*}
0: & \quad \text{Initialization:} \\
& \quad t_i \leftarrow 0. \\
& \quad S \leftarrow \{(c, 2^l) \mid 0 \leq c < 2^l, 0 \leq l \leq J\}. \\
& \quad \forall w_{(c, 2^l)} \in S : w_{(c, 2^l)} \leftarrow w_{(c, 2^l)} \cdot \left(1 - \gamma_0 \cdot U(0, 1)\right). \\
& \quad \forall \sigma \in S : \{w_\sigma\} \sigma \in S \\
& \quad \text{At every slot, do} \\
1: & \quad \text{Active Schedule Update:} \\
& \quad A \leftarrow \{(c, 2^l) \in S \mid t_i \mod 2^l = c\}. \\
2: & \quad \text{Schedule Selection:} \\
& \quad I \leftarrow \{\text{arg max}_{\sigma \in S} W_\sigma \cap \{\sigma \in S \mid w_\sigma > \eta\}. \\
3: & \quad \text{Decision:} \\
& \quad \text{if } A \cap I = \emptyset \text{ and } \kappa = 1 \text{ then } d_i[t_i] = 1 \\
& \quad \text{else } d_i[t_i] = 0. \\
4: & \quad \text{Reward Selection:} \\
& \quad \text{if } (F[t_i], d_i[t_i]) = (0, 0) \text{ or } (1, 1) \text{ then } \alpha \leftarrow \alpha^+ \\
& \quad \text{else } \alpha \leftarrow \alpha^- \).
5: & \quad \text{Weight Update:} \\
& \quad \forall \sigma \in A : w'_\sigma \leftarrow w_\sigma \cdot e^{\alpha \cdot U(0, 1)}. \\
6: & \quad \text{Voluntary Relinquishment:} \\
& \quad \text{if } U(0, 1) < \epsilon \text{ then } \forall \sigma \in A : w'_\sigma \leftarrow 0. \\
7: & \quad \text{Weight Normalization:} \\
& \quad W \leftarrow \sum_{\sigma \in \rho} w_\sigma, \quad W' \leftarrow \sum_{\sigma \in \rho} w'_\sigma. \\
& \quad \delta \leftarrow W - W'. \\
& \quad \text{if } \delta > 0 \text{ and } W' < w_{\text{init}} \cdot |S| \quad \forall \sigma \in S : X_\sigma \leftarrow \{0, 1\}. \\
& \quad \text{else } \forall \sigma \in S : w_\sigma \leftarrow w'_\sigma + \delta \cdot (X_\sigma / \sum_{\sigma} X_\sigma). \\
8: & \quad \text{Bound Enforcement:} \\
& \quad \forall \sigma \in S : w_\sigma \leftarrow \min(1, w_\sigma). \\
9: & \quad \text{Time Increment:} \\
& \quad t_i \leftarrow t_i + 1. 
\end{align*}
\]

Table II. State variables used in ALOHA-QT and mAQT. The cardinality of the sets of variables are shown in the last column. They essentially
in section\textsuperscript{[II]} is used to select a positive or negative reward by the user.
5) \textbf{Step 5}: The weights of all $J + 1$ active schedules are updated at every time slot. A negative reward selection in step 4 will decrease the weights while a positive reward selection will increase the weights in this step. As users become active and inactive, schedules become unfavourable and promising respectively. Therefore, a multiplicative update strategy is used for facilitating the quick adaptation of weights in such a dynamic environment. Small noise is added to all the updated weights to break ties between schedules that might have the same value of weight. From a classical RL sense, this is the reward function of the algorithm.

6) \textbf{Step 6}: With a small constant probability $\epsilon$, the user sets the weights of all active schedules to 0. This happens randomly once every few hundred slots to make sure that the users do not hold higher schedules indefinitely.

7) \textbf{Step 7}: If the weights of active schedules were reduced (either due to negative feedback or relinquishment) and if the sum of the weights in the PT falls below a value of $w_{\text{init}} \cdot |S|$, the lost weights in this step are redistributed across all the schedules in the PT. This allows the users to quickly explore alternative schedules if their selected schedule starts to give negative feedback \textsuperscript{[18]}.

8) \textbf{Step 8}: The users make sure that the weights of all schedules remain at most 1. This way, the positive multiplicative update from a good schedule does not increase indefinitely.

9) \textbf{Step 9}: The time counter is updated so that the user can process the next slot.

In this manner, the users explore schedules in the PT and learn to coordinate over time in order to select non-conflicting schedules in a distributed manner. This coordination is achieved only via the broadcast feedback at the end of the slot. It is important to note that each user $i$ selects schedules in a distributed manner and the time slot counter $t_i$ does not need to be the same for all users in the network: a schedule $(c, 2^l)$ for a user $a$ with time slot counter $t_a$ is the same as a schedule $((c + s) \mod 2^l, 2^l)$ for a user $b$ with time slot counter $t_b = t_a + s$. Thus, the users in the network need not synchronize their time slot counters. This property is especially useful if we need to change the number of users $M$. A new user can be introduced in the network and it needs to only synchronize the start of a time slot and does not need to obtain any additional information from other users or the AP.

\textbf{B. Application specific changes to ALOHA-QT}

The ALOHA-QT algorithm \textsuperscript{[16]} is designed for a system to optimize throughput in a fair manner by avoiding collisions via implicit coordination over the feedback. The authors show its applicability in a system model where all the users are active, when the number of active users is slowly increasing or slowly decreasing as well as when there is frequent activation and deactivation at the start of every 100th slot. The system model makes no assumptions on the maximum number of users that the scheme can accommodate.

This differs from the model which we have defined in section\textsuperscript{[II]} In this model, the activation and deactivation are less frequent but random and do not need to occur at the beginning of a slot batch. The system designer knows the maximum number of users $M$ in the network. We applied the ALOHA-QT algorithm to our system model and made two key observations in terms of AoI. These observations lead us to suggest to following changes to the ALOHA-QT algorithm to better suit the system model presented in section\textsuperscript{[II]}:

1) \textit{Skip voluntary relinquishment in step 6:}

This step was designed to make sure that no user holds a higher schedule for a long time. If any user relinquishes selected schedules in this step, other users compete to grab these schedules, causing collisions. At the same time, the user who relinquished the schedules begins transmitting in some other users selected schedule producing further collisions. This causes the throughput to drop temporarily. While this trade-off might be useful in maintaining fairness with a static number of users over a long time, it was found that it does not help when the users spend a random amount of time in the network before becoming inactive. Therefore, we suggest skipping this step for our application.

2) \textit{Select only one schedule in step 2:}

The possibility of allowing the users to select more than one schedule in ALOHA-QT was designed to allow a flexible throughput for all active users. Firstly, it was observed that in most cases, the secondary schedules were either children or siblings of the primary selected schedule i.e. the one with maximum weight. Secondly, it was also observed that once a user selects one or more schedules, their weights quickly rise up to become 1. This is caused by the reinforcing effect of the multiplicative update on receiving positive feedback. A newly active node entering the network needs to compete with the selected schedules of many users in order to find a new collision-free schedule in the PT. If we allow the users to select more than one schedule, the newly active user is likely to face competition from more schedules and hence take more time to find a new collision-free schedule in the PT.

3) \textit{When PT is settled, only run Step 3:}

With the suggested modifications, the users select only one unique schedule in the PT which is neither an ancestor nor a descendent of the schedules selected by other users. When we establish the network with all active users running the algorithm with the mentioned modifications, they take some time (we call this settling time) to select their unique schedule in the PT. This results in the network achieving full channel utilization. We call this condition a “settled PT”. However the tree does not remain settled forever, as there will be arrivals and departures of users in the network as they become active and inactive. These events unsettle the tree for a certain amount of time (we call this resettling time) before the tree settles once again. We propose that the users in a settled tree do not change their selected schedules unless there is an arrival or departure of a user in the network. Therefore, they do not need to update
any state variables when they are in a settled tree. If there are no collisions or idles detected in the last $2^j$ slots, the users deem that the PT is settled. In a settled state, the users perform only step 3 of the algorithm. Thus doing minimum work while still avoiding collisions and idle slots entirely.

The comparison results between pure ALOHA-QT and the modifications (modified ALOHA-QT or mAQT) are shown in section [V]

IV. Analysis

We assume that the average time between events that disturb a settled tree $\frac{k}{a}$, is less than the average resetting time. Hence the users spend more time in a settled tree rather than in an unsettled one. In this section, we obtain insights into the AoI performance of the mAQT when the tree is settled. In such a case, the number of active users $n[t]$ for all time slots $t$ in this period is constant. Therefore, we drop the time index for the ease of notation and refer to the number of active users as $n$ in this section.

One realization of a settled tree for $n = 5$ is shown in Figure 4(a). The $n$ leaf nodes filled with color are the selected schedules by each of the 5 users in the system. The resulting AoI of each user over 20 time slots after the tree is settled is shown in Figure 4(b). It can be seen that there is a successful transmission at every slot, resulting in the network achieving full channel utilization.

The $\Delta_i$ for every user is cyclic with period $2^{l_i}$. The long-term ($t \to \infty$) mean AoI per slot of a user $i$, $\bar{\Delta}_i$ in a settled tree is the mean AoI of the period,

$$\bar{\Delta}_i = \frac{1}{2^{l_i}} \sum_{t=t_0+1}^{t_0+2^{l_i}} \Delta_i[t] = \frac{\sum_{t=t_0+1}^{t_0+2^{l_i}} t}{2^{l_i}} = \frac{2^{l_i} + 1}{2}. \quad (2)$$

The mean network AoI $\bar{\Delta}$ is the mean AoI per slot of all active users in the network,

$$\bar{\Delta} = \frac{1}{n} \sum_{i=1}^{n} \bar{\Delta}_i = \frac{1}{2} (1 + \frac{1}{n} \sum_{i=1}^{n} 2^{l_i}). \quad (3)$$

Even though the channel is fully utilized when the tree is settled, the fraction of the channel resources utilized by each user in the system can be different. Every user obtains $2^{-l_i}$-th of the channel resources and $\sum_{i=1}^{n} 2^{-l_i} = 1$. The mAQT algorithm has a degree of randomness, which may lead to different realizations of the PT for the same number of users $n$. Thus the users may obtain different values of $\Delta$ even for the same number of users $n$ depending on the manner in which they settle. To analyze this further we make use of the balance property of a FBT [19], an abstract data structure commonly used in computer science.

A. Settled Trees as Full Binary Trees

A FBT is defined as a tree structure where every node has either two or no children. A settled PT looks exactly like a FBT with a node in the FBT representing a schedule in the PT. The leaf nodes (ones without children) represent the selected schedules of the $n$ users. Any sub-tree of a FBT is also a FBT.

We define the set of selected levels (of schedules) by the users of particular realization $r$ of a settled PT with $n$ users as $\mathcal{T}_r = \{l_1, l_2, ..., l_n\}$. The height of a FBT $h$ is defined as the number of edges between the root node and the farthest leaf node. For realization $r$ of settled tree the height is $l_{\max}^r = \max(\mathcal{T}_r)$.

A fully balanced FBT is where the difference between heights of the two principle sub-tree of any sub-tree is at most 1. The tree in Figure 4(b) is an example of a fully balanced FBT. The closer the values of $l_i$ in a particular realization of a settled tree are to each other, the more balanced that PT is.

Theorem 1. The mean AoI per user $\bar{\Delta}$ of a settled policy tree depends on how balanced the tree is. For a given $n$, a fully balanced FBT is $\Delta$ optimal.

Proof. A FBT has at least two sibling leaf nodes at height $h$. Let the leaf node(s) for realization $a$ at a higher level be at $l_a^m$ such that $l_a^m > l_a^m$. Now, consider the two siblings at $l_a^m$ and one leaf node at $l_a^m$. We explicitly write these three values in last sum in equation (3).

$$\bar{\Delta}_a = \frac{1}{2} \left(1 + \frac{1}{n} \sum_{i=4}^{n} 2^{l_i} + \frac{1}{n} (2^{l_{\max}^m} + 2^{l_{\max}^m} + 2^{l_{\min}^m}) \right), \quad (4)$$

where $\bar{\Delta}_a$ stands for the $\bar{\Delta}$ for realization $a$. Now, we perform balancing operation on this tree to produce a new realization $b$ with the same number of leaf nodes $n$. This can be done by removing the sibling pair at level $l_{\min}^m$ and giving the leaf at $l_{\min}^m$ a pair of children. Hence the tree:

1. Loses one leaf at level $l_{\min}^m$.
2. Adds two leaves at level $l_{\min}^m$ + 1.
3. Loses two leaves at level $l_{\max}^m$.
4. Adds one leaf at level $l_{\max}^m$ - 1.

Therefore,$$
\bar{\Delta}_b = \frac{1}{2} \left(1 + \frac{1}{n} \sum_{i=4}^{n} 2^{l_i} + \frac{1}{n} (2^{l_{\max}^m} + 2^{l_{\min}^m+1} + 2^{l_{\min}^m+1}) \right). \quad (5)
$$

Subtracting $\bar{\Delta}_b$ from $\bar{\Delta}_a$ and simplifying it further we get,$$
\bar{\Delta}_a - \bar{\Delta}_b = \frac{3}{2n} (2^{l_{\max}^m} - 1 - 2^{l_{\min}^m}) \geq 0. \quad (6)
$$

With equality holding if $l_{\min}^m = l_{\max}^m - 1$, which is the case for a fully balanced FBT. Hence, as long as the tree is not fully balanced, this balancing operation results in a lower $\Delta$. \hfill \square

We are interested in finding the least balanced realization that will provide the upper bound of $\bar{\Delta}$ for a given number of users $n$. A fully unbalanced tree or a skewed tree, has two leaf nodes at $n-1$th level and one leaf node in all the levels between $n-1$th level and root node. Putting these values of $l_i$ in equation (3) and using the expression for the sum a geometric series, we get the mean AoI for a skewed tree,$$
\bar{\Delta}_{skew} = \frac{1}{2} (1 + \frac{3 \cdot 2^n - 1}{2n}). \quad (7)
$$

B. Optimal selection of parameter $J$

We can force the tree to settle in a way that more unbalanced realizations are possible to materialize. This can be done by...
selecting the parameter tree depth \( J \) to be lower than \( n - 1 \). As we reduce the value of \( J \) further, we eliminate the possibility of the PT settling into the more unbalanced realizations. Figure 5 shows how the upper bound of the mean AoI per user per slot can be decreased by decreasing \( J \). However, in order to have at least one schedule for each user, the tree depth must be greater than or equal to the height of a fully balanced realization, i.e., \( J \geq \lceil \log_2 n \rceil \).

The selection of \( J \) should take into account the maximum number of active users the system designer would like to provision for when the number of users is time-varying. In our system model presented in section II, the number of active users will never exceed \( M \). Therefore, we can safely select \( J = \lceil \log_2 M \rceil \). In general, the activation/deactivation model and its properties and total number of users in the network should be taken into consideration when selecting \( J \).

V. Evaluation

The box plots in Figure 6 show the distribution of resettling time for a given \( n \) = \{13, 18, 23, 28\} under an activation or deactivation over 50 simulation runs. The upper and lower whisker of the boxplot encapsulate the entire range of the obtained data i.e. the maximum and minimum resettling time. It is seen that the resettling time never exceeded 1100 slots and the maximum mean resettling time (center line of box plot) is 300 slots. Next, we simulate random geometrically distributed activations and deactivations for a system with \( M = 32 \) users, \( k = 50,000 \) and \( n[0] = 16 \). The parameters for the algorithm shown in Table I were obtained using the gridsearch method. The channel utilization (fraction of successful slots) for 50 simulation runs of 50,000 slots each is shown in Figure 7. The maximum and minimum utilization over the 50 runs is shaded in the region around the mean. The number of users \( n[t] \) is measured at the beginning of each batch of 100 slots. The seed of the random number generator which produced the activations and deactivations was kept the same to show the variation in the resettling time and to make meaningful comparisons between different schemes. The key demonstration from this figure is that the PT is unsettled (observed by a drop in utilization) by a change in the number of active users. However, it then manages to always settle and attain near full utilization after it is given enough time to resettle. Only the selected schedules of some users are disturbed during the resettling period which can be seen from the observation that the utilization in Figure 7(a) never drops below 0.8. For comparison, SA with the optimum access probability of \( \frac{1}{n^2} \) achieves utilization of only 0.4. SA is a totally random medium access scheme and therefore suffers from collisions due to lack of coordination between users.
TABLE III. The number of weights $|V|$ required for the three expert-based RL algorithms. These weights are updated frequently and specify the amount of memory needed by each of the algorithms.

| Algorithm          | Weights                  |
|--------------------|---------------------------|
| ALOHA-Q            | $2^n$                     |
| ALOHA-Qt           | $2^{(2^n+1)} - 1$         |
| mAQT               | $2^{(2^n+1)} - 1$         |

TABLE IV. The worst case complexity ($O$), of each step for the three expert-based RL algorithms. The proposed mAQT has half the run time in step 2 and skips step 6 entirely compared to ALOHA-Qt. Its performance is the nonetheless better for our system model.

| Step   | ALOHA-Q | ALOHA-Qt | mAQT   |
|--------|---------|----------|--------|
| 2      | $2^J$   | $2^{(2^J+1) - 1}$ | $2^{(2^J+1) - 1}$ |
| 5      | $J + 1$ | $J + 1$   |         |
| 6      | $J + 1$ | Skipped   |         |
| 7      | $2^J$   | $2^{(2^J+1) - 1}$ | $2^{(2^J+1) - 1}$ |
| 8      | $2^J$   | $2^{(2^J+1) - 1}$ | $2^{(2^J+1) - 1}$ |

Figure 7(b) shows $\Delta$ for each batch for the same two cases. The shaded color marks the region between 10th and 90th percentile over 50 simulation runs. Here the $\Delta$ for SA is greater than a factor of 4 as compared to mAQT. We also compare the analytical $\Delta$ of ADRA [11], where the users jointly optimise the channel access probability and an AoI of a remote monitoring network with a time-varying AoI of a remote monitoring network with a time-varying AoI. Here, the $\Delta$ for ADRA is greater than a factor of 2 as compared to mAQT. Note that it was assumed that the users in SA and ADRA have a priori knowledge of $n[t]$ which was not the case for mAQT. This is an unrealistic advantage given to the users in SA to demonstrate the power of our proposed method. Stationary random policies of SA and ADRA is far outperformed by mAQT due to the implicit coordination achieved between users over the feedback instead of relying on only on random chance.

In Figure 7(c) we zoom in on the mAQT region and compare it with ALOHA-Q [15], which is another algorithm that employs RL to achieve collision-free transmissions. In ALOHA-Q, each user selects a unique slot in a frame of fixed size $F$. A necessary condition for ALOHA-Q to settle is $n[t] \leq F$. We set the frame size to $2^J = F$, since that is the maximum number of users our setting for mAQT can accommodate. It is trivial to see that the upper bound for mAQT will always be smaller than that of ALOHA-Q since there will always be empty slots in ALOHA-Q unless $n[t] = 2^J$. From Tables III and IV we see that the better performance of mAQT compared to ALOHA-Q comes at the cost of needing more memory and computation. The availability of different transmission rates in mAQT makes sure that no channel resources are wasted on idle slots, which is an important advantage over ALOHA-Q. The best case for our network setup would be for the users to transmit in a round-robin fashion [8]. This can be achieved via a centralized scheduling scheme or a partially centralized scheme with a complex feedback such as the DRR algorithm [14]. We show this RR plot as a baseline reference for the best case. To the best of out knowledge, no distributed MAC algorithm gets closer the this baseline than mAQT.

In Figure 7(d) we see that mAQT performs better than ALOHA-Qt which justifies the modifications made to it, mentioned in section III-A. Table IV shows the worst case run-

TABLE V. The mean network AoI per user per slot over all 50,000 slots. The proposed algorithm shows 15 percent reduction in $\Delta$ from ALOHA-Qt and 50 percent reduction in the same from state of the art ADRA algorithm.

| Algorithm          | $\Delta$ |
|--------------------|----------|
| RR                 | 8.93     |
| mAQT               | 13.07    |
| ALOHA-Qt           | 15.32    |
| ALOHA-Q            | 16.55    |
| ADRA               | 26.71    |
| SA                 | 52.48    |

VI. Conclusions

In this paper, we ponder the goal of minimizing the mean AoI of a remote monitoring network with a time-varying number of users without a centralized scheduler. We make application-specific changes to the distributed RL algorithm ALOHA-Qt, which employs a policy tree (PT) to facilitate the coordination between users in a network so that they can select non-conflicting transmission slots. The users collectively obtain nearly full channel utilization when the PT is settled. This settled PT resembles a full binary tree, and the analysis of its properties shows that the balance of the settled tree affects the mean network AoI. We also show how the selection of a design parameter in the algorithm, namely the tree depth $J$, can be used to improve the mean network AoI by eliminating the possibility of the tree settling into more unbalanced realizations. Simulation results show that the suggested algorithm reduces mean network AoI by 50 percent for state of the art age-dependent random access (ADRA) protocol, without the need for any interference cancellation or out-of-band communication. With this paper, we show that use of PT to improve AoI is promising in a decentralized MAC setup. Some assumptions made in this work are not representative of real-life scenarios. For example, channel conditions other than interference, like noise, might cause a transmission or feedback to be lost. Hence, our future work will include implementing mAQT on a hardware testbed such as [20] to investigate its performance outside of simulations.

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Fig. 7. Evaluation Results: The same arrival and departure pattern is applied
for 30 simulation runs. (a) mAQT always settles and achieves two times
higher utilization than SA. (b) The mean AoI comparison between mAQT,
SA and ADRA. (c) When provisioning for the same number of users, mAQT
performs better than ALOHA-Q. (d) The modifications suggested to ALOHA-
QT improve the performance for the given system model.