Abstract—Collision-free trajectory generation within a shared workspace is fundamental for most multi-robot applications. However, despite of their versatility, many widely-used methods based on model predictive control (MPC) lack theoretical guarantees on the feasibility of underlying optimization. Furthermore, when applied in a distributed manner, deadlocks often occur where several robots block each other indefinitely without resolution. Towards this end, we propose a systematic method called infinite-horizon model predictive control with deadlock resolution (IMPC-DR). It can provably ensure recursive feasibility and effectively resolve deadlocks online in addition to the handling of input and model constraints. The method is based on formulating a convex optimization over the proposed modified buffered Voronoi cells in each planning horizon. Moreover, it is fully distributed and requires only local inter-robot communication. Comprehensive simulation and experiment studies are conducted over large-scale multi-robot systems. Significant improvements of both feasibility and success rate are shown, in comparison with other state-of-the-art methods and especially in crowded and high-speed scenarios.

Index Terms—Multi-robot systems, motion planning, MPC, deadlock resolution, feasibility guarantee.

I. INTRODUCTION

Collision-free trajectory generation is essential for multi-robot systems to perform their missions in a shared environment, such as cooperative inspection, search, rescue and transportation [1]. However, it becomes especially challenging when a large number of robots navigate at high speed in a crowded space as shown in Fig. 1. As summarized in [2], the commonly-seen multi-robot trajectory generation (MRTG) algorithms can be classified into roughly five categories: potential fields [3], [4], [5] that design virtual driving forces induced by artificial potentials, which however often suffer from the problem of being stuck at local minimums; geometric guidance [6], [7], [8], [9] such as reciprocal velocity obstacles (RVO) that analyze the geometric properties based on the position and velocity of the whole team, which mostly neglect the underlying kinodynamic constraints; conflicts resolution [10], [11], [12] that relies on designing heuristic rules to resolve potential collisions, which often lacks guarantees on the collision avoidance; learning-based methods [13], [14], [15], [16] such as reinforcement learning that rely heavily on accurate simulators and thus suffers from the sim-to-real gap, as well as poor generalization to domains different from the training data; and optimization-based methods [17], [18], [19], [20], [21] that have a better performance in versatility, extensibility and numerical stability. However, most of these aforementioned works lack in two critical aspects: feasibility guarantee and deadlock resolution. Consequently, collision-free trajectories are not always ensured to generate.

More specifically, optimization-based methods construct and solve various optimizations to achieve collision avoidance, such as the mixed integer quadratic programming in [18], sequential convex programming in [22], and model predictive control (MPC) in [23]. However, most of the aforementioned methods do not ensure explicitly feasibility of the underlying optimization. To address this problem, the work in [24] proposes to gradually add constraints to the sequence of convex programming when necessary. The authors in [25] further extend this idea to MPC and achieve simultaneous replanning by using the buffered Voronoi cell (BVC) [31]. Another work [26] introduces the notion of control barrier function, which can guarantee recursive collision-free trajectory. Nonetheless, it is overly conservative due to excessive breaking, and often suffers from deadlock. In addition, some other works in [27], [28] and [29] propose to tackle the feasibility problem by...
To summarize, recent works on optimization-based methods assume recursive feasibility without any guarantee, or relies on predefined robot priorities and sequential navigation for deadlock resolution. By contrast, our method IMPC-DR for the first time provably ensures recursive feasibility of MPC-based methods for MRTG and resolves potential deadlocks online without a central coordinator. The effectiveness of the proposed algorithms is verified by numerical simulations and hardware experiments. Compared with other state-of-the-art methods, our method shows a significant increase in both feasibility and success rate, especially for large-scale crowded and high-speed multi-robot systems.

The remaining part of this article is organized as follows. Section II describes the problem statement. The backbone of our method IMPC is presented in Section III. The complete method IMPC-DR with online deadlock resolution is proposed in Section IV. Further details of the proposed algorithm are discussed in Section V. Section VI includes numerical simulations and hardware experiments. Conclusions and future work can be found in Section VII.

II. PROBLEM STATEMENT

This section states formally the MPC-based multi-robot trajectory generation (MRTG) problem, and the exact notion of recursive feasibility and deadlock for MRTG.

A. Robot Dynamics

Consider a team of $N$ robots, where each robot $i \in \mathcal{N} = \{1, 2, \ldots, N\}$ is modeled as an unit mass in $\mathbb{R}^d$, and $d = 2, 3$ is the dimension of the configuration space. Its state $x^i = [p^T, v^T]^T$ includes position $p^i$ and velocity $v^i$, and its acceleration $u^i$ is the control input. Furthermore, its motion is approximated by the commonly-used second-order dynamics:

$$\dot{x}^i = Ax^i + Bu^i,$$

where $A = \begin{bmatrix} 0_d & I_d \end{bmatrix}$, $B = \begin{bmatrix} 0_d \ 0_d \ 0_d \end{bmatrix}$. As often required in practice, both the robot velocity and acceleration are subjected to physical constraints. Specifically, it holds that $\|\Theta_a v^i\|_2 \leq v_{\text{max}}$ and $\|\Theta_a u^i\|_2 \leq u_{\text{max}}$, where $\Theta_a, \Theta_v$ are positive-definite matrices, and $v_{\text{max}}, u_{\text{max}} > 0$ denote the maximum velocity and acceleration, respectively.
Problem 1 (MPC-based MRTG). Design $u^i_k(t)$ for robot $i \in \mathcal{N}$ at time step $t$ for each horizon $k \in \{1, 2, \cdots, K\}$ that solves the following optimization:

$$\min_{u^i_k(t), x^i_k(t)} \mathbf{C}(u^i_k(t), x^i_k(t)) \quad \text{s.t.} \quad \|p^j_k(t) - p^i_k(t)\|_2 \geq r_{\min}, \forall j \neq i, \forall k;$$

$$p^i_k(t) = p^i_{\text{target}};$$

$$x^i_k(t) = Ax^i_{k-1}(t) + Bu_{k-1}(t), \forall k;$$

$$\|\Theta_{a} u^i_{k-1}(t)\|_2 \leq a_{\max}, \forall k;$$

$$\|\Theta_{v} v^i_k(t)\|_2 \leq v_{\max}, \forall k;$$

where $x^i_k(t) = [p^i_k(t)^{T}, v^i_k(t)^{T}]^T$ is the planned state in the $k$-th horizon for robot $i$; $p^i_k(t)$ is the planned position in horizon $k$ of the planned trajectory $P^i(t) = \{p^i_1(t), p^i_2(t), \ldots, p^i_K(t)\}$; $p^i_0(t) = p^i(t)$ and $v^i_0(t) = v^i(t)$ are current state at time step $t$; $\mathbf{C}(\cdot)$ is the cost function to be minimized; $\mathbf{A}, \mathbf{B}$ represent the discretized system matrices under the sampling time $h$, i.e.,

$$\begin{bmatrix} \mathbf{I}_d & h \mathbf{I}_d \\ 0_d & \mathbf{I}_d \end{bmatrix}, \quad \begin{bmatrix} 0_d \\ \mathbf{I}_d \end{bmatrix}. \quad (3)$$

It is worth noting that each robot can only dictate its own trajectory and not other robots’. However, a robot can exchange data with other robots via wireless communication. The objective is to solve the above problem collaboratively in a distributed manner without a central coordinator.

D. Recursive Feasibility and Deadlock

As mentioned previously in Section I, there are two aspects of Problem 1 that are particularly challenging and of particular interest in this work: recursive feasibility and deadlock. Thus, we provide firstly their exact definitions below.

**Definition 1** (Recursive Feasibility). If the optimization in (2) is feasible at time step $t - h$, then the new optimization at time step $t$ is also feasible.

**Definition 2** (Deadlock). Deadlock happens when all robots remain static indefinitely, but at least one robot has not reached its target position.

Recursive feasibility ensures safety of the resulting trajectories, namely no collision will happen. However, for certain configurations such as crowded scenarios, the robots may block each other and cannot make progress towards the targets, commonly known as deadlock [33]. Although no collision happens, this case still prohibits a successful navigation. In the subsequent sections, we will show how the main Problem 1 can be solved, with both recursive feasibility and deadlock resolution ensured.

III. INFINITE HORIZON MODEL PREDICTIVE CONTROL

This section presents the backbone of the proposed solution to Problem 1, which is called the infinite-horizon model predictive control (IMPC). We show that via IMPC the recursive feasibility in Def. 1 is guaranteed.

A. Choice of Horizon

Recursive feasibility of MPC is closely related to the horizon length $K$ in Problem 1. Here, we first analyze this relation and discuss how to choose it.

Finite-horizon MPC is an approximation to infinite-horizon MPC in [37], where the horizon length $K$ ought to be adequately long. Therefore, a goal condition in (2c) is enforced in Problem 1 for the target position. However, this constraint can be overly restrictive in many scenarios, especially when the problem is complex, rendering Problem 1 infeasible. As also proposed in [31], [25], the constraint in (2c) could be deleted and the deviation to the target is penalized in the cost function. However, this would indicate that the state at the terminal horizon, i.e., $x^i_K$, does not have avoidance constraints since

1For the sake of simplicity, the time index will be omitted whenever ambiguity is not caused. For example, $x^i_k(t)$ will be rewritten as $x^i_k$. 
it has no corresponding position from the previous planned trajectory to form them, as shown in Fig. 3. Thus, an over-aggressive trajectory might be generated, causing infeasibility in the next optimization.

To address this problem, we add a terminal constraint to Problem 1 as follows:

\[ x^i_K \in X_e, \text{ where } X_e = \{ x \mid x = Ax + Bu, u \in U \}, \]

where \( X_e \) is a set of state which can have an available input to remain its state. For the system model in (2), the terminal constraint \( x^i_K \in X_e \) can be rewritten as \( v^i_K = 0_d \).

Remark 1. This method is inspired by the braking mechanism in [26], [35], [38], in which safety is guaranteed by braking down. In addition, similar terminal constraints can be found in [35] which however does not consider recursive feasibility. It is worth noting that stationary is a common state in practice, i.e., a ground vehicle being immobile, a quadrotor hovering, or a fixed-wing aircraft flying at constant velocity.

**B. Reformulation of Collision Avoidance Constraint**

In (2), the collision avoidance constraint is enforced explicitly by requiring the inter-robot distance to be more than \( r_{min} \) at all time. However, the future states of other robots are not available at the current time. Thus, we propose to replace them with the predetermined trajectory of other robots. In other words, at each iteration, each robot will inform other robots its predetermined trajectory from the previous iteration.

**Definition 3** (Predetermined Trajectory (PT)). The predetermined trajectory for robot \( i \) at time \( t \) is defined as

\[ \mathcal{P}^i(t) = \{ p^i(1), \ldots, p^i(K) \}, \]

where \( p^i(k) = p^i_{k+1}(t-h) \) for \( k = 1, \ldots, K \).

Based on the predetermined trajectory, we use the spatial separation method to handle inter-robot collision avoidance. This method forming a separating hyperplane between different robots which restricts their corresponding motion space. In this article, inspired by [31], we define the following modified buffered Voronoi cell (BVC) for robot \( i \) and \( j \) that \( j \neq i \):

\[ \mathcal{V}^{ij}_k = \left\{ p \in \mathbb{R}^d \mid (p - \frac{p^i_k + p^j_k}{2})^T (p^i_k - p^j_k) \geq \frac{1}{2} r^{ij}_{min} \right\}, \]

where \( \mathcal{V}^{ij}_k \) is the BVC for robot \( i \) to \( j \) in horizon \( k \) for their given predetermined trajectory positions; and

\[ r^{ij}_{min} = \sqrt{r^{ij}_{min}^2 + h^2 v_{max}^2} \]

is the extended buffer width. The term \( h^2 v_{max}^2 \) is vital to achieving collision avoidance between discretization points. Consequently, the collision avoidance constraints are decoupled and turn to \( p^i_k \in \mathcal{V}^{ij}_k, p^j_k \in \mathcal{V}^{ij}_k \).

re-arrangements, the constraint in (5) can be rewritten as \( a^{ij}_k p^i_k \geq b^{ij}_k, \forall j \neq i \), where the coefficients are given by

\[ a^{ij}_k = \frac{p^i_k - p^j_k}{\|p^i_k - p^j_k\|^2}, \quad b^{ij}_k = a^{ij}_k \frac{p^i_k + p^j_k}{2} + \frac{r^{ij}_{min}}{2}. \]

Note that the above reformulation retains the important property of collision avoidance as stated below.

**Proposition 1.** If \( a^{ij}_k p^i_k \geq b^{ij}_k \) holds, \( \forall (i, j) \) and \( \forall k \), then \( \|p^i_k - p^j_k\|_2 \geq r^{ij}_{min} \) holds, \( \forall (i, j) \) and \( \forall k \). And the planned trajectories are collision-free not only at the sampling points, such as \( p^i_{k-1} \) and \( p^j_k \), but also at any points between them.

**Proof:** Refer to Appendix A for detailed derivations.

**Remark 2.** The MBVC defined in (5) is an extension of the buffered Voronoi cell (BVC) proposed in [31], which is defined as follows:

\[ \mathcal{V}_k^i = \left\{ p \in \mathbb{R}^d \mid (p - \frac{p^i_k + p^i_{k+1}}{2})^T (p^i_k - p^i_{k+1}) \geq \frac{1}{2} r^{i}_{min}, \forall k \neq i \right\}. \]

The main difference is that the predefined trajectory is introduced in MBVC while only the current robot position is used in BVC. The introduction of the predefined trajectory and the extended buffer width dependent of the robot’s maximum velocity in MBVC can ensure safety at all time instants as illustrated in Fig. 4, avoiding the potential scenario that collision may happen between two consecutive sampling points, as pointed out by [35].

![Fig. 4.](image)

**C. Choice of Cost Function**

As mentioned earlier, the cost function in (2a) plays an important role in optimization. The proposed cost function for robot \( i \) is given by

\[ C^i = \frac{1}{2} Q_{tar} (p^i_k - p^i_{target})^2 + \frac{1}{2} \sum_{k=1}^{K-1} Q_{adj} \|p^i_{k+1} - p^i_k\|^2_2, \]

where \( Q_{tar} \) and \( Q_{adj} > 0 \), \( k = 1, 2, \ldots, K - 1 \), are the weighting parameters.
for the current convex optimization, given that the previous
solution in (10), we only need to show one feasible solution
method is ensured by the following theorem.

E. Recursive Feasibility Guarantee

Based on the above discussions, the original optimization
(2) is reformulated as the following convex optimization:

\[
\begin{align*}
\min_{u_i, x_t} & \quad c_i \\
\text{s.t.} & \quad a_{ij}^k p_k \geq b_{ij}^k, \forall j \neq i, \forall k; \quad (10b) \\
& \quad v_k^* = 0_d; \quad (10c) \\
& \quad (2d) - (2f);
\end{align*}
\]

where the cost function in (10a), the collision avoidance
constraints in (10b) and the terminal constraint in (10c) are as
explained earlier; the rest of the constraints remain the same
as in (2).

The complete algorithm is summarized in Alg. 1. First, the
predetermined trajectory is initialized in Line 1 as \(\overline{p}(t_0) = \{p^1(t_0), \ldots, p^d(t_0)\}\). Then, the robots start executing the main
loop in parallel (Line 2). Each robot sends its predetermined
trajectory to others and collects other’s predetermined trajec-
tory as \(\overline{p}(t)\) (Line 4). Thereafter, each robot derives the
collision avoidance constraints \(\text{CONSl}^i\) (Line 5) and their current states \(x^i(t)\) (Line 6). Based on \(\text{CONSl}^i\) and \(x^i(t)\), the convex programming is solved (Line 7) and its solution,
including the planned trajectory, is executed by the lower-level
tracking controller (Line 9). Then, this procedure repeats itself
until all the robots reach their target positions.

D. Overall Algorithm

Based on the above discussions, the original optimization
(2) is reformulated as the following convex optimization:

\[
\begin{align*}
\min_{u_i, x_t} & \quad c_i \\
\text{s.t.} & \quad a_{ij}^k p_k \geq b_{ij}^k, \forall j \neq i, \forall k; \quad (10b) \\
& \quad v_k^* = 0_d; \quad (10c) \\
& \quad (2d) - (2f);
\end{align*}
\]

where the cost function in (10a), the collision avoidance
constraints in (10b) and the terminal constraint in (10c) are as
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including the planned trajectory, is executed by the lower-level
tracking controller (Line 9). Then, this procedure repeats itself
until all the robots reach their target positions.

E. Recursive Feasibility Guarantee

Recursive feasibility, defined as in Def. 1, of the proposed
method is ensured by the following theorem.

**Theorem 1.** The convex optimization in (10) is recursively
feasible under Alg. 1.

**Proof:** To prove the recursive feasibility of the optimization
in (10), we only need to show one feasible solution
for the current convex optimization, given that the previous
one is feasible. More specifically, a sufficient condition is
that if \(x^i_k(t - h)\) and \(u^i_k(t - h)\) is a feasible solution to the
optimization at time \(t - h\), then \(x^i_k(t) = x^i_{k+1}(t - h)\) and
\(u^i_k(t) = u^i_{k+1}(t - h)\) will satisfy all the constraints for the
optimization at time \(t\).

Firstly, it follows that

\[
x^i_k(t) = Ax^i_{k-1}(t) + Bu^i_{k-1}(t); \quad v^i_k(t) = 0_d; \\
\|\Theta_a u^i_{k-1}(t)\|_2 \leq v_{\text{max}}; \quad \|\Theta_p v^i_k(t)\|_2 \leq u_{\text{max}}.
\]

The above constraints hold since \(x^i_k(t - h) = x^i_k(t)\) and
\(u^i_k(t - h) = u^i_k(t)\), as the result of previous optimization,
satisfy all above constraints. Thus, if the constraint in (10b)
also holds, then all the constraints in (10) are satisfied.
Substituting (7) into it yields

\[
\|p^i_{k+1}(t - h) - p^i_k(t - h)\|_2 \geq r^i_{\min}.
\]

As a feasible solution at time \(t - h\), \(p^i_{k+1}(t - h)\) satisfies

\[
v^i_{k+1}(t - h) p^i_{k+1}(t - h) \geq b^i_{k+1}(t - h), \forall j \neq i, \forall k.
\]

In combination with (8), the inequality in (11) holds, i.e.,
all the constraints in (10) are satisfied. Hence, the proof for
recursive feasibility is completed.

**Remark 3.** Compared with [30], the constraints in (10) are
less restrictive as we can easily find the following initially
feasible condition: all robots are static and collision-free.

IV. DEADLOCK RESOLUTION

The previous section proposes a solution to the MPC-
MRTG problem, which ensures the recursive feasibility. In
this section, we further improve this solution by adding a
warning band to MBVC, and thus ensure that its solution is
also deadlock-free.

A. Conditions for Deadlock

Based on Def. 2, when robot \(i\) is stuck at a deadlock, it stays
at its current position and remains static. Namely, the planned
position satisfies \(p^i(t) = p^i_k(t)\), for \(t \geq t_{\text{deadlock}}\), where \(t_{\text{deadlock}}\)
is the starting time of deadlock. A deadlock can happen in the
following two cases:

**Case 1:** for \(t \geq t_{\text{deadlock}}\), there exist \(t\) and horizon \(k = 1, 2, \ldots, K - 1\) such that \(p^i_k(t) \neq p^i_{k+1}(t)\), i.e., not all planned
positions overlap;

**Case 2:** for \(t \geq t_{\text{deadlock}}\), \(p^i_k(t) = p^i_{k+1}(t)\), \forall k = 1, 2, \ldots, K - 1\), i.e., all planned positions overlap.

In Case 1, the robots are blocked and remain static at
horizon \(k = 1\), but not all subsequent horizons, while in
Case 2, the trajectory remains at one point, meaning that no
further progress can be made unless the trajectory is updated.

For Case 1, since there exists a time \(t\) and horizon \(k\)
such that \(p^i_k(t) \neq p^i_{k+1}(t)\) holds, the robots would not
remain static indefinitely if its trajectory is not updated. Thus,
when it occurs, robot \(i\) can remain at its current position
until time \(t + (k + 1)h\) when \(p^i_k(t) \neq p^i_{k+1}(t)\) holds. And
its planned trajectory will be chosen as its predetermined
trajectory. Consequently, the Case 1 deadlock is resolved.
In Section IV-C, it will be shown that this method can still
maintain the recursive feasibility of IMPC.
As to Case 2, all horizon planned positions $p_k^i$ overlap and are equal to $p_i^j$. The following lemma analyzes the condition for such deadlocks.

**Lemma 1** (Deadlock condition for Case 2). Considering a set of robots $N_{case 2} \subseteq N$ at Case 2 deadlock, for each robot $i \in N_{case 2}$, there exist $j \in N_{case 2}$ and $\lambda_K^{ij}$ such that the following:

$$Q_{tar} (p_{target}^i - p_K^i) + \sum_{j} \lambda_K^{ij} = 0$$  \hspace{1cm} (12)

holds, where $a_K^{ij} = \frac{p_i^j - p_i^j}{\|p_i^j - p_i^j\|_2}$. In addition, robot $j$ is the robot that blocks robot $i$.

**Proof:** The proof is provided in Appendix B.

Based on Lemma 1, we define $F^i_A = Q_{tar} (p_{target}^i - p_K^i)$ as the attractive force, which drives the planned position $p_K^i$ to its target position. Similarly, $F^i_K = \lambda_K^{ij} a_K^{ij} \geq 0$ is defined as the repulsive force from robot $j$ for robot $i$ where $a_K^{ij}$ and $\lambda_K^{ij}$ are the direction and magnitude of this repulsive force, respectively. Thus, the deadlock condition (12) is a balance of these forces:

$$F^i_A + \sum_j F^i_K = 0,$$

which is illustrated in Fig. 5. Note that the magnitude of the repulsive force, i.e., $\lambda_K^{ij}$ is only related to (12). Thus, its magnitude can be arbitrarily adjusted to make this equilibrium condition satisfied like robot 1 in Fig. 12, which is the idea behind the deadlock resolution in the subsequent subsection.

![Fig. 5. Deadlock can be treated as a force equilibrium, where the attractive force from the target (in yellow, red, blue) and the repulsive forces from other robots (in green) are balanced. Take robot 1 as an example, the direction of repulsive force is determined by the relative position between its and other robots', the magnitude is adapted to balance the attractive force.](image)

**B. Deadlock Resolution for Case 2**

To ensure that the magnitude of repulsive forces are not arbitrary, we adopt the modified buffered Voronoi with warning band (MBVC-WB) at the terminal horizon $K$, as illustrated in Fig. 6.

![Fig. 6. Illustration of the MBVC-WB, where a warning band is additionally added.](image)

**Definition 4** (MBVC-WB). The MBVC-WB for robot $i \in N$ and $\forall j \neq i$ has a similar form as MBVC but it is only enforced at the terminal horizon $K$ as follows:

$$\psi_K^{ij} = \left\{ p \in \mathbb{R}^d \mid \langle p - \frac{p_K^i + p_K^j}{2}, p_K^i - p_K^j \rangle \geq \frac{r_{min}^i}{2} + w^{ij} \right\}$$

where $w^{ij}$ is an additional variable added to the optimization. It is called the warning distance for robot $i$ from $j \neq i$, which satisfies $0 \leq w^{ij} \leq \epsilon$ and $\epsilon$ is defined as the maximum width of warning band.

Based on the MBVC-WB above, the collision avoidance constraints can be written as $a_K^{ij} p_K^i \geq b_K^{ij} + w^{ij}, \forall j \neq i$, where $a_K^{ij}$ and $b_K^{ij}$ are obtained from (7) as well. Meanwhile, an additional penalty term is added into the cost function:

$$C_w = \sum_{j \neq i} \rho_0 \left( \frac{w^{ij}}{\epsilon} - \ln w^{ij} \right),$$

where $\rho_0 \in \mathbb{R}^+$ is a coefficient and $w^{ij} > 0$. Consequently, the objective function turns to

$$C^i = \frac{1}{2} Q_{tar} \|p_K^{i} - p_{target}^{i}\|^2_2 + \frac{1}{2} \sum_{k=1}^{K} Q_{adj[k]} \|p_{k+1}^{i} - p_{k}^{i}\|^2_2$$

$$+ \sum_{j \neq i} \rho_0 \left( \frac{w^{ij}}{\epsilon} - \ln w^{ij} \right).$$

Since the domain of $w^{ij}$ is set to $w^{ij} > 0$ in $C_w^i$, the constraints $w^{ij} \geq 0$ is omitted.

To summarize, the convex optimization in (10) can be rewritten as follows:

$$\min_{u^i, x^i, w^{ij}} C^i$$

**s.t.**

$$\forall j \neq i, \forall k = 1, 2, \ldots, K - 1;$$

$$a_K^{ij} p_K^i \geq b_K^{ij} + w^{ij}, \forall j \neq i;$$

$$w^{ij} \leq \epsilon;$$

$$(10c), (2d) - (2f);$$

where (14a)-(14e) are different from (10).

Based on the new optimization, the condition in Lemma 1 turns to

$$Q_{tar} (p_{target}^i - p_K^i) + \sum_{j \in K^i} a_K^{ij} \rho_0 \left( \frac{\epsilon - w^{ij}}{\epsilon w^{ij}} \right) = 0,$$

(15)
where \( A' \triangleq \{ j | u^{ij} < \epsilon \} \) and \( w^{ij} = w^{ij} \) holds. The proof of equation (15) can be found in Appendix C. Similar to the analysis in Lemma 1, \( F^i_A = Q_{tar} (\rho^i_{\text{target}} - p^i_K) \) is the attractive force and \( F^j_R = a^{ij}_R \rho_0 e^{-w^{ij}} \) the repulsive force, where \( a^{ij}_R \) and \( \rho_0 e^{-w^{ij}} \) are the direction and magnitude, respectively. The property in (15) can be rewritten as \( F^i_A + \sum_{j \in A'} F^{ij}_R = 0 \). \( \text{(16)} \)

Remark 4. The target positions chosen for the MRTG problem should be feasible. Namely, for any pair of robots \( i \) and \( j \), their target positions should satisfy \( \|p^i_{\text{target}} - p^j_{\text{target}}\|_2 \geq r_{min} + 2e \), which implies no repulsive forces when they reach target positions.

Proposition 2. The necessary condition to form a deadlock is that the sum of all attracting force equal to zero, i.e., \( \sum_i F^i_A = \sum_i (\rho^i_{\text{target}} - p^i_K) = 0 \), \( \text{(17)} \) for robot \( i \) that is trapped in this deadlock.

Proof: Given (16), it holds that \( \sum_i (F^i_A + \sum_{j \in A'} F^{ij}_R) = 0 \). Since \( w^{ij} = w^{ji} \) and \( a^{ij}_R = -a^{ji}_R \), clearly \( F^i_R = -F^i_R \) holds. By definition, \( j \in A' \) implies \( i \in A' \). Then, it follows that \( \sum_i \sum_{j \in A'} F^{ij}_R = 0 \) and thus (17) holds.

Hence, due to this stricter condition, most deadlocks that might be caused by MBVC do not appear, as illustrated in Fig. 7. However, for some cases, the condition for Case 2 deadlock might still hold. To deal with it, we adopt a detection-resolution method. Thus, an indicator to detect a potential Case 2 deadlock is firstly defined in the following.

Definition 5 (Terminal Overlap). A terminal overlap happens when \( p^i_K(t) = p^j_K(t - h) \), \( p^i_K(t) \neq p^i_{\text{target}} \), \( p^j_K(t) = p^j_{\text{target}}(t) \), and \( p^j_{K-1}(t) = p^j_{K-2}(t) \) hold.

It is evident that if Case 2 deadlock happens, the condition for a terminal overlap must hold. A terminal overlap induced by a deadlock has the same necessary condition as specified in (15). The proof is similar to the one in Appendix C.

Remark 5. Different from [26], the proposed algorithm can plan ahead in time by \( K \). Thus, a potential Case 2 deadlock, i.e., terminal overlap, can be detected before a deadlock happens in the future.

After detecting a terminal overlap, a right-hand driving rule is added to the navigation for robots as inspired by [39], i.e., the robot tends to avoid the robots on its left more than its right by generating right-hand forces. More specifically, if a terminal overlap is detected, replace \( \rho_0 \) in (15) by \( \rho^{ij}_0 = \rho_0 e^{(\eta \sin \theta^{ij})} \), where \( \theta^{ij} \) is defined as the angular between the projection in \( x-y \) plane of line terminal predetermined trajectory position of robot \( i \) to its target position and the terminal predetermined trajectory position of robot \( j \), as illustrated in Fig. 8; \( \eta > 0 \) is a coefficient which can adjust the magnitude of right-hand force. As a result, the repulsive force from robot \( j \) to \( i \) is changed and the condition for the current terminal overlap does not hold anymore.

However, a new terminal overlap might happen again even after introducing the right-hand force. When a new terminal overlap is detected, then \( \eta \) is changed incrementally by \( \eta(t) = \eta(t - h) + \Delta \eta \), \( \rho^{ij} = \rho_0 e^{(\eta(t) \sin \theta^{ij})} \), \( \text{(18)} \) where \( \Delta \eta > 0 \) is a design parameter and \( \eta(t_0) = \eta_0 \), where \( \eta_0 \) is a coefficient determining the initial magnitude of the right-hand force. Namely, the right-hand force is increased until the deadlock is resolved radically. Afterwards, \( \eta(t) \) is restored to \( \eta_0 \) if \( w^{ij} = 0 \) holds for \( j \neq i \), i.e., its warning band have no contact with all others'.

C. The Complete Algorithm: IMPC-DR

The complete method that incorporates the infinite-horizon model predictive control and the deadlock resolution (called IMPC-DR) is summarized in Alg. 2 with sub-functions defined in Alg. 4 and 3.

To begin with, several key boolean variables need to be introduced. More specifically, \( b^1_{Case1} \) is true when Case 1 happens; \( b^1_{TO} \) returns true when a terminal overlap is detected; \( b^1_{TOA} \) is true when another terminal overlap is detected again.

The main loop in Alg. 2 runs as follows. Each robot checks if Case 1 has happened. If so, the previously planned trajectory with receding horizon, i.e., predetermined trajectory is followed (Line 7) until some progress is made (Line 8). If a terminal overlap happens, it needs to check whether a terminal overlap happens again or not. If Case 1 does not happen, \( \rho^{ij}_0 \) is returned and Alg. 3 is followed (Line 11). Afterwards, the constraints for collision avoidance are derived (Line 12) as well as its current state (Line 13). Thereafter, the formulated optimization is solved in (10); see Line 14. Based on the results, boolean variables listed in Alg. 4 are computed.
Theorem 2. The complete algorithm IMPC-DR in Alg. 2 is recursive feasible and deadlock-free.

Proof: (Sketch). To begin with, the proof for recursive feasibility is similar to the proof of Theorem 1. Namely, we need to show that all the constraints at the current time $t$ are satisfied when both $x_k^i(t) = x_{k+1}^i(t-h)$ and $u_k^i(t) = u_{k+1}^i(t-h)$ hold. The updated trajectory in Line 7 of Alg. 2 can be treated as the solution to the optimization below:

$$\min_{u_{k-1}^j, x_k^j, w^j} \sum_k \| p_k^j - \tilde{p}_k^j \|_2^2,$$

s.t. (14b) – (14d), (10c), (2d) – (2f), which contains the same constraints as optimization (14). Then, if we can prove that the trajectory derived at time $t-h$ is updated by either optimization (14) or (19), then $\mathcal{P}^i(t)$ is a feasible solution to both optimizations at time $t$.

When both $x_k^i(t) = x_{k+1}^i(t-h)$ and $u_k^i(t) = u_{k+1}^i(t-h)$ hold, constraints in (14b), (10c), (2d)–(2f) are satisfied as proven in Theorem 1. Then, towards the feasibility, not only the constraints (14c) and (14d) but also, for the demand of the domain of objective function (14a), $w^j > 0$ need to be satisfied. If $a_k^{ij}(t)p_{K+1}^i(t-h) > b_k^{ij}(t)$ holds for $\forall j \neq i$, and by choosing

$$w^j = \min\{\varepsilon, a_K^{ij}(t)p_{K+1}^i(t-h) - b_k^{ij}(t)\}$$

then $w^j > 0$ holds as well as the constraints (14c) and (14d). Therefore, we only need to prove that

$$a_k^{ij}(t)p_{K+1}^i(t-h) > b_k^{ij}(t),$$

holds for $\forall j \neq i$. Substituting $a_k^{ij}(t)$ and $b_k^{ij}(t)$ from (7) into (20) yields $\| p_{K+1}^i(t-h) - p_{K+1}^i(t-h) \|_2 > r'_{\min}$. Moreover, since $p_{K+1}^i(t-h) = p_k^i(t-h)$ holds by definition, it yields

$$\| p_{K}^i(t-h) - p_k^i(t-h) \|_2 > r'_{\min}.$$  

As a feasible solution at the previous time step, $p_k^i(t-h)$ satisfies $a_K^{ij}(t-h)p_k^i(t-h) \geq b_K^{ij}(t-h) + w^j(t-h)$, for $\forall j \neq i$. Since $w^j(t-h) > 0$ holds, it follows that $a_K^{ij}(t-h)p_k^i(t-h) > b_K^{ij}(t-h)$, which further implies that both the inequalities (21) and (20) hold. Thus, the recursive feasibility of (14) is retained.

Regarding the deadlock-free property, both the cases in Section IV-A should be discussed. The same analysis for Case 1 still applies, while for Case 2 it can be shown that a terminal overlap never happens. In Alg. 2, the potential equilibrium required by a terminal overlap is always prevented by the right-hand force in (18). This completes the proof.

V. FURTHER DISCUSSIONS

In this section, we discuss some further issues related to the proposed method. Specifically, how the communication range among the robots is designed, and how the aforementioned parameters are chosen for different scenarios.
A. Communication Range

Most related works [16], [18], [22], [23] require a fully-connected network, i.e., each robot is required to communicate with all other robots. This not only results in large communication overhead but also high computation burden. In this work, we propose a distance-based communication strategy that only requires local communication.

Proposition 3 (Communication Range). For any pair of robots \((i, j)\), no communication between them is required, if \(|p^i - p^j|_2 \geq 2v_{\text{max}}K h + r_{\text{min}}^r + 2\epsilon\) holds.

**Proof:** A communication is not necessary, if it affects neither the recursive feasibility of the optimization nor the deadlock resolution.

Clearly, if \(|p^i - p^j|_2 \geq 2v_{\text{max}}K h + r_{\text{min}}^r + 2\epsilon\) holds, then \(|p^k_i - p^k_j|_2 \geq r_{\text{min}}^r + 2\epsilon\) holds for \(k = 1, 2, \ldots, K\). Hence, in the case of finite horizon \(K\), no repulsive force between robots exists. Thus, no potential deadlock can be induced because of the interaction between them.

For robot \(i\), if the predetermined trajectory from robot \(j\) is not received, the constraint (10b) is deleted. Therefore, the following four cases should be considered: (I) the constraint exists at both the previous time’s and the current time’s optimization; (II) the constraint only exists at the previous time; (III) the constraint exists at neither the previous time nor the current time; and (IV) the constraint only exists at the current time.

For Case (I), it is just the ordinary case which has been proved. For Case (II), as the property of recursive feasibility, the optimization without deleting this constraint is feasible. Then, deleting this constraint cannot make this optimization infeasible. For Case (III), obviously a constraint that is not introduced to convex programming will not affect the property of recursive feasibility. Case (IV), different from the other three cases, introduces a new constraint in the optimization at the current time. Firstly, since \(|p^i - p^j|_2 \geq 2v_{\text{max}}K h + r_{\text{min}}^r + 2\epsilon\) holds, it follows that \(|p^k_i - p^k_j|_2 \geq r_{\text{min}}^r + 2\epsilon\), \(\forall k, \epsilon\). This indicates that equation (11) in the proof of Theorem 1 and (21) in the proof of Theorem 2 hold automatically. Then, similar to these proofs, we can show that the predetermined trajectory is a feasible solution as well.

B. Selection of Parameters in Algorithm 2

The parameters in Alg. 2 have significant influence on the performance of the overall algorithm. More specifically, the key parameters are the time step \(h\), the horizon \(K\), the width of warning band \(\epsilon\), in addition to the penalty weights \(Q_{\text{tar}}\) and \(\rho_0\) in the cost function.

The time step \(h\) directly affects the buffer width of MBVC-WB as in (6). A larger \(h\) means a wider buffer, yielding a more conservative collision avoidance. On the other hand, a smaller \(h\) means a larger horizon \(K\) for the same planning time, leading to a larger optimization thus a higher complexity. Moreover, \(K\) should be larger than \(K_{\text{min}}\) which is the minimum time that a robot needs to stop from any allowed velocity. Similarly, for a large maximum velocity \(v_{\text{max}}\), larger \(K\) and smaller \(h\) should be chosen to react faster and plan further, which of course resulting in more computation complexity. A larger width of warning band \(\epsilon\) leads to a better deadlock-resolution performance, but in general a more sensitive reaction to deadlock, thereby longer time to accomplish the navigation task. The position penalty weight \(Q_{\text{tar}}\) and the warning penalty \(\rho_0\) directly influence the magnitude of the attractive and repulsive forces, respectively. A larger \(Q_{\text{tar}}\) results in a closer inter-robot distance and longer time while resolving potential deadlocks. Similarly, larger \(\rho_0\) and \(\Delta\eta\) means a larger right-hand force and thereby a better deadlock resolution, but may lead to a lower-quality trajectory.

VI. SIMULATION AND EXPERIMENT

In this section, the proposed algorithm is validated via numerical simulations and hardware experiments of large-scale multi-robot systems. The algorithms are implemented in Python3, and publicly available at https://github.com/PKU-MACDLab/IMPC-DR. The convex optimizations are formulated by CVXPY [40] and solved by MOSEK [41]. The average time of replanning per one robot is around 0.06s. The numerical simulations include some typical scenarios such as swapping position, narrow passage and symmetry; and random transitions. For evaluation, our methods IMPC and IMPC-DR are compared with another three works: iSCP [24], DMPC [25] and BVC [31].

A. Typical Scenarios

![Fig. 9. The typical scenario of swapping positions. Top: The traditional BVC method may generate a zigzag motion as BVC is induced from the current position and adopted to all horizons. Bottom: By adopting MBVC to each horizon, our method can generate a much smoother trajectory.](image-url)

To begin with, some typical scenarios in MRTG are considered. The maximum velocity \(v_{\text{max}} = 1.0m/s\) and the maximum acceleration \(a_{\text{max}} = 1.5m/s^2\). In the simulation of typical scenarios, the time step \(h\) is chosen as 0.2s, the horizon length \(K = 10\). The width of warning band \(\epsilon\) is chosen as 0.1m. In addition, we set position penalty weights \(Q_{\text{tar}} = 30.0\), \(\rho_0 = 2.0\) and \(\eta_0 = 2.0\).

In our simulation, the heuristically perturbation mechanism that handling deadlock is deleted as it often causes stability issue of the system.
### Table II

**TIME OF RANDOM TRANSITIONS IN 3D**

| Method      | the number of robots |
|-------------|----------------------|
|             | 2       | 4       | 6       | 8       | 10      | 12      | 14      |
| IMPC-DR     | 2.35s   | 2.61s   | 3.13s   | 3.68s   | 4.12s   | 4.94s   | 5.51s   |
| BVC [31]    | 2.86s   | 3.44s   | 3.93s   | 4.51s   | 5.24s   | 5.99s   | 6.80s   |

![Fig. 10](image1.png)

**Fig. 10.** The typical scenario of narrow passage where one robot passes through other two robots at target positions. **Top:** without the proposed warning band, the blue robot is blocked. **Bottom:** the introduction of the warning band to MBVC enables the blue robot to pass through the narrow gate.

1) **Swapping Position:** The first scenario is where the robots swap their positions, as shown in Fig. 9. In our method, MBVC is induced from the predetermined trajectory and adopted to each horizon instead of all horizons. Consequently, the planned trajectories of different robots can intersect, instead of being separated at all horizons in traditional BVC, see \( t = 1.2s \). Thus, compared with BVC [31], our method has a shorter transition time as shown in Table II and smoother trajectories as depicted in Fig. 9.

2) **Narrow Passage:** The second typical scenario is where a robot needs to pass through another two robots who are already at their target positions. This scenario will show the necessity of adding the warning band of MBVC. As shown in Fig. 10, via MBVC-WB, this robot (blue) can squeeze out a way and, during squeezing, its slow movement compels others to get out of the way. The process of squeezing is inevitable since the force equilibrium (16) cannot hold at all in this case and, thus the deadlock can be avoided.

3) **Symmetry:** Lastly, symmetry is one of the common cases for deadlock. Thus, we consider such scenarios. In particular, four robots located in a \( 2m \times 2m \) square transit to their antipodal positions. As shown in Fig. 11, robots approach the center point initially, and then, owing to the right-hand force, they change their direction to the right at around 1.0s. Finally, after this right rotation, they escape this symmetry or, in other words, a potential deadlock at around 3.0s. In comparison, without the deadlock resolution introduced in Section IV, **Case 2** deadlock happens, also shown in Fig. 11. The same phenomenon also appears in the 3D case as shown in Fig. 12, where the designed right-hand force drives the system away from the force equilibrium.

**B. Random Transitions**

To systematically compare our method with other baselines, we consider the scenarios where the initial and target positions are randomly chosen, especially in crowded and high-speed 2D or 3D workspace. 100 tests are generated for each scenario. In each test, we determine it as success if all robots arrive at their target positions within a limited time and does not suffer from any infeasibility of optimization.

1) **Crowded 2D Workspace:** The 2D workspace is set to \( 2m \times 2m \) and the number of robots ranges from 2 to 14. The safety diameter of a robot is chosen as \( 0.3m \). The maximum velocity, maximum acceleration, the warning band width and other parameters are selected the same as before. The only change is that the time step \( h = 0.15 \) and the horizon length \( K = 12 \). The result is summarized in Table III and the process is shown in Fig. 13. It is clear that only IMPC and IMPC-DR do not suffer from infeasibility. Moreover, IMPC-DR can achieve almost 100% success rates in any density. Even for the highly crowded case of 14 robots, the success rate remains 87% for IMPC-DR, much higher than other baselines. It should be noted that the failed tests are caused by the time limitation instead of deadlock. Furthermore, it is worth noting that, after...
choosing $\epsilon$ as 0.1$m$, 14 robots are almost the highest capacity for this finite space. This illustrates that our method is capable of handling extremely crowded scenarios.

2) Crowded 3D Workspace: The 3D workspace is set to $2m \times 2m \times 1m$, where the number of robots ranges from 8 to 32. The robots share the same parameters as in the 2D case. The process is shown in Fig. 13. As summarized in Table IV, similar conclusions can be drawn as in the 2D case. Namely, IMPC and IMPC-DR do not suffer from infeasibility while only IMPC-DR achieves nearly 100% success rate for any number of robots. It is interesting to notice that deadlocks are less likely to appear in the 3D workspace than the 2D counterpart.

3) High-Speed 2D and 3D Workspaces: Last but not least, another key aspect is the high-speed scenario where online adaptation is essential for the successful navigation. In particular, the maximum velocity and acceleration are set to 3$m/s$ and 2$m/s^2$, respectively; the workspace is extended to $10m \times 10m$ and $10m \times 10m \times 5m$ for 2D and 3D respectively. In this scenario, the safety diameter of all robots is set to 1$m$ and the warning band width is extended to 0.2$m$. As summarized in Table V and Table VI, the proposed method IMPC-DR can maintain the same performance in terms of feasibility and the 100% success rate. In contrast, the performance of other baselines such as BVC, iSCP and DMPC degraded significantly, due to mostly over-aggressive trajectories. This highlights the importance of our infinite horizon formulation and the added terminal constraint in (10c).

C. Experiments

To further validate the proposed method, several experiments are performed on a nano quadrotor platform.

1) Hardware Setup: As shown in Fig. 14, the platform consists of several nano quadrotors, based on Bitcraze Crazyfile 2.1. They have 9cm rotor-to-rotor and weight 34g. Their states in the workspace are captured by OptiTrack, an indoor motion capture system, of which the update frequency is 120$Hz$. This information is sent to the main control computer where the proposed trajectory generation algorithm is carried out for all quadrotors. The trajectory of quadrotor is fitted to a 7th-order

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**Fig. 13.** Snapshots of the crowded navigation tasks via the proposed method in 2D by 14 robots (Top) and in 3D by 32 robots (Bottom). The actual trajectories are in solid lines while the predetermined trajectories are in dotted lines.
TABLE V
RANDOM TRANSITION IN HIGH-SPEED 2D SCENARIOS

| Metric | Method   | the number of robots |
|--------|----------|----------------------|
|        |          | 2 4 6 8 10 12 14    |
| Success| IMPC-DR  | 100 100 100 100 100 100 100 |
|        | IMPC     | 100 98 93 77 68 49 37 |
|        | BVC [31] | 95 77 58 42 31 23 11 |
|        | iSCP [24] | 93 57 32 6 6 0 0 |
|        | DMPC [25] | 98 86 65 39 33 10 4 |
| Infeas | IMPC-DR  | 0 0 0 0 0 0 0      |
|        | IMPC     | 0 0 0 0 0 0 0      |
|        | BVC [31] | 5 23 41 52 54 63 70 |
|        | iSCP [24] | 7 43 68 94 94 100 100 |
|        | DMPC [25] | 2 14 35 61 67 90 96 |

TABLE VI
RANDOM TRANSITIONS IN HIGH-SPEED 3D SCENARIOS

| Metric | Method   | the number of robots |
|--------|----------|----------------------|
|        |          | 8 12 16 20 24 28 32 |
| Success| IMPC-DR  | 100 100 100 100 100 100 100 |
|        | IMPC     | 100 100 99 99 100 99 98 |
|        | BVC [31] | 95 77 58 42 31 23 11 |
|        | iSCP [24] | 49 14 5 1 0 0 0 |
|        | DMPC [25] | 83 55 36 23 12 5 0 |
| Infeas | IMPC-DR  | 0 0 0 0 0 0 0      |
|        | IMPC     | 0 0 0 0 0 0 0      |
|        | BVC [31] | 5 23 42 58 69 77 89 |
|        | iSCP [24] | 51 86 95 99 100 100 100 |
|        | DMPC [25] | 17 45 64 77 88 95 100 |

polynomial and then sent to other quadrotors along with its state information via high-frequency radio. After receiving the position information, Kalman filter is used to estimate the current velocity and position. A feedback controller is used to track the updated trajectory based on the work in [43] which ensures high tracking accuracy.

Furthermore, to avoid the inter-quadrotor air turbulence, the minimum distance between quadrotors $r_{min}$ is chosen as 0.3$m$ and the width of warning band $\epsilon$ is chosen as 0.1$m$. The maximum velocity and acceleration of Crazyfile are set to 1$m/s$ and 1$m/s^2$, respectively, to ensure safety. Lastly, the time step $h$ is set to 0.2$s$ and the horizon length 15, to balance the control performance and the computation burden.

2) Results: Similar to the numerical simulations, three typical cases are considered in the hardware experiment: 4 robot antipodal transition in 2D and 3D, see Figs. 11 and 12, and narrow passage in Fig. 10. More specifically, Fig. 15(a) shows one navigation result of 4 robots to their antipodal positions in a square. Different from the simulations, the width of the square is smaller than 1$m$. Similar results in 3D are shown in Fig. 15(b). Collision avoidance is also validated by plotting the relative distance between any pair of robots against the minimum distance 0.38$m$. Fig. 15(c) is the result of the typical deadlock resolution, where the minimum distance is 0.34$m$.

Generally speaking, the results of hardware experiments are consistent with the findings in numerical simulations. However, we notice that due to communication or computation delays, execution of the computed trajectories is not exactly synchronized among the robots. For instance, as shown in Fig. 15(a), the robot in orange updates and executes its trajectory 0.05$s$ later than the robot in green, which is non-negligible compared with the updating frequency 5$Hz$. Although some oscillations might appear, the overall navigation task remains consistent with the numerical simulations in Section VI-A.

Fig. 14. Hardware platform consists of a team of Crazyflies nano quadrotors, a motion capture system, and a control computer.

(a) Four robots transit to their antipodal positions in a square. Left: Trajectory of robots. Right: Distance between robots.

(b) Four robots transit to their antipodal positions in a cube. Left: Trajectory of robots. Right: Distance between robots.

(c) A typical deadlock resolution demo. Left: Trajectories of robots. Right: Distance between robots.

Fig. 15. Hardware experiments of several scenarios, which can be compared with the numerical simulations in Section VI-A.
successful. It shows that our method is robust to asynchronous executions among the robots.

VII. CONCLUSION

This work has proposed a novel and effective navigation algorithm called IMPC-DR for multi-robot systems. It provably ensures recursive feasibility of the underlying optimization, while resolving any potential deadlocks online. Compared with other state-of-the-art baselines, its advantages in crowded and high-speed scenarios are significant, as demonstrated both in simulations and hardware experiments. Future work includes the extension to obstacle-rich environments in combination with search-based planning methods.

APPENDIX A
PROOF OF PROPOSITION 1

To prove Proposition 1, we need the following lemma.

Lemma 2. Consider two line segments in 2D: the line segment from $p_1$ to $p_2$ and the line segment from $q_1$ to $q_2$, where $p_1, p_2, q_1, q_2 \in \mathbb{R}^2$. If

$$||r_1||_2, ||r_2||_2 \geq \sqrt{r_{\min}^2 + \frac{1}{4} ||l_2 - l_1||_2^2}$$

(22)

is satisfied for $r_1 = q_1 - p_1$, $r_2 = q_2 - p_2$, $l_1 = p_2 - p_1$ and $l_2 = q_2 - q_1$, then

$$||p_1 + t(p_2 - p_1) - q_1 - t(q_2 - q_1)||_2 \geq r_{\min}$$

(23)

holds, $\forall t \in [0, 1]$.

Proof of Lemma 2: It is trivial to show that the left-hand side of (23) is equivalent to

$$p_1 + t(p_2 - p_1) - q_1 - t(q_2 - q_1) = r_1 + t(r_2 - r_1).$$

Hence, it suffices to prove that

$$||r_1 + t(r_2 - r_1)||_2 \geq r_{\min}, \forall t \in [0, 1].$$

Let us introduce a function $F(t) = ||r_1 + t(r_2 - r_1)||_2^2$, where $t \in [0, 1]$. Moreover, set $F(t_{\min}) = \min_{t \in [0, 1]} F(t)$, where $t_{\min} \in [0, 1]$ is the time instant where $F(t)$ reaches its minimum.

Consider the following two cases: First, when $r_1 = r_2$ holds, it follows that $F(t) = r_T^2 r_1 = \sqrt{r_{\min}^2 + \frac{1}{4} ||l_2 - l_1||_2^2} \geq r_{\min}^2$; Second, when $r_1 \neq r_2$ holds, we consider further the following three sub-cases: (i) if $t_{\min} = 0$ holds, then $F(0) = r_T^2 r_1 \geq r_{\min}$; (ii) if $t_{\min} = 1$ holds, then $F(1) = r_T^2 r_2 \geq r_{\min}$; (iii) if $0 < t_{\min} < 1$ holds, the minimum of the quadratic function $F(t)$ is given by

$$F(t_{\min}) = \frac{r_T^2 r_1 r_2 r_3 - r_T^2 r_3 r_1 r_2}{(r_2 - r_1)^T (r_2 - r_1)},$$

where $t_{\min}$ is equal to

$$t_{\min} = \frac{r_T^2 (r_2 - r_1)}{(r_2 - r_1)^T (r_2 - r_1)}.$$  (24)

To prove $F(t_{\min}) \geq r_{\min}^2$, it is equivalent to showing that

$$r_T^2 r_1 r_2 r_3 - r_T^2 r_3 r_1 r_2 \geq r_{\min}^2 (r_2 - r_1)^T (r_2 - r_1).$$  (25)

After simple calculations, (25) can be rewritten as

$$(r_T^2 r_1 - r_{\min}^2) (r_T^2 r_2 - r_{\min}^2) \geq (r_T^2 r_2 - r_{\min}^2) (r_T^2 r_1 - r_{\min}^2).$$  (26)

To prove (26), we consider the following two cases:

(i) $r_T^2 r_2 - r_{\min}^2 \geq 0$ holds. Since $t_{\min} \in (0, 1)$ holds in (24), it follows that $r_T^2 r_1 \geq r_T^2 r_2$ and $r_T^2 r_2 \geq r_T^2 r_1$. Thus, we can obtain that

$$r_T^2 r_1 \geq r_T^2 r_2 \geq r_{\min}^2 \geq 0,$$

which proves (26).

(ii) $r_T^2 r_2 - r_{\min}^2 < 0$ holds. Without loss of generality, we can assume that $r_T^2 r_1 \leq r_T^2 r_2$. Then, it is easy to show that $r_T^1 r_1 \geq r_T^2 r_2 + \frac{1}{4} ||l_2 - l_1||_2^2$ given (22). It can be further combined with the simple fact that $l_2 - l_1 = q_2 - q_1 - p_2 + p_1 = r_2 - r_1$, which leads to

$$r_T^1 r_1 \geq r_T^2 r_2 + \frac{1}{4} ||l_2 - l_1||_2^2 = r_T^2 r_2 + \frac{1}{4} (r_T^1 r_1 + r_T^1 r_2 - 2 r_T^2 r_2) \geq r_T^2 r_2 + \frac{1}{4} (2 r_T^1 r_1 - 2 r_T^2 r_2).$$

After re-organizing the terms, we have

$$r_T^2 r_2 \geq r_T^1 r_1 \geq 2 r_{\min} - r_T^1 r_2,$$

and

$$r_T^2 r_2 \geq r_T^1 r_2 \geq r_T^1 r_2 \geq r_T^2 r_2 \geq 0,$$

which proves (26).

Similar to the previous case, it implies that (26) holds.

Now, the proof is completed.

Proof of Proposition 1: For robots $i$ and $j$, if $p_k^i \in V_{ij}^i$ and $p_k^j \in V_{ij}^j$, it follows that $a_k^{ijT} p_k^i \geq b_k^{ij}$ and $a_k^{ijT} p_k^j \geq b_k^{ij}$, respectively. Hence, $a_k^{ijT} p_k^i + a_k^{ijT} p_k^j \geq b_k^{ij} + b_k^{ij}$ holds. Substituting (7) into it, the following holds:

$$a_k^{ijT} (p_k^i - p_k^j) \geq r_{\min}.$$  (27)

Moreover, since $a_k^{ijT} (p_k^i - p_k^j) \leq \|a_k^{ijT}\|_2 \|p_k^i - p_k^j\|_2 \leq \|p_k^i - p_k^j\|_2$, (8) can be easily derived.

For robot $i$, it clearly holds that $p_k^i - 1 \in V_{ij}^i - 1$ and $p_k^i \in V_{ij}^i$. The same applies to robot $j$. Due to (8), it can be shown that

$$\|p_k^i - 1 - p_k^i - 1\|_2 \geq r_{\max} = \sqrt{r_{\min}^2 + h^2 v_{\max}^2}$$

and

$$\|p_k^i - p_k^i\|_2 \geq r_{\max} = \sqrt{r_{\min}^2 + h^2 v_{\max}^2}$$

hold. Then, during the time interval $[t + kh - h, t + kh]$, robot $i$ moves from $p_k^i - 1$ to $p_k^i$ at a constant velocity and robot $j$ from $p_k^j - 1$ to $p_k^j$. If the minimum distance between robots $i$ and $j$ during $[t + kh - h, t + kh]$ is larger than $r_{\min}$, then Proposition 1 holds.
Since the maximum allowable velocity is \( v_{\text{max}} \), it follows that
\[
\|p_k^i - p_{k-1}^i\|_2 \leq h \, v_{\text{max}} \quad \text{and} \quad \|p_k^i - p_{k-1}^j\|_2 \leq h \, v_{\text{max}}.
\]
Consequently, the following holds:
\[
\|p_k^i - p_{k-1}^i\|_2 \geq \sqrt{r_{\text{min}}^2 + h^2 v_{\text{max}}^2}
\]
\[
\geq \sqrt{r_{\text{min}}^2 + \frac{1}{4} (\|p_k^i - p_{k-1}^i\|_2 + \|p_k^j - p_{k-1}^j\|_2)^2}
\]
\[
\geq \sqrt{r_{\text{min}}^2 + \frac{1}{4} \|p_k^i - p_{k-1}^i - p_k^j + p_{k-1}^j\|_2^2}
\]
holds. Similarly, it holds that
\[
\|p_{k-1}^i - p_k^i\|_2 \geq \sqrt{r_{\text{min}}^2 + \frac{1}{4} \|p_{k-1}^i - p_k^i - p_{k-1}^j + p_k^j\|_2^2}
\]
Given these two conditions, it follows directly by Lemma 2 that
\[
\|p_k^i - p_{k-1}^j\|_2 \geq \sqrt{r_{\text{min}}^2 + \frac{1}{4} \|p_k^i - p_{k-1}^j\|_2^2}
\]
That is, the minimum distance between robots \( i \) and \( j \) during the time interval \([t + kh - h, t + kh]\) is larger than \( r_{\text{min}} \). This completes the proof.

**APPENDIX B**

**PROOF OF LEMMA 1**

**Proof:** To begin with, the constraint (2d) in the convex program (10) can be directly expanded as \( x_k^i = A^k x_0^i + A^{k-1} B u_0^i + \cdots + B u_{k-1}^i \). Moreover, the Lagrange function of (10) is given by
\[
\mathcal{L}^i = C^i + \sum_k u_k^i \lambda_k^i (\|\Theta a u_{k-1}^i\|_2 - u_{\text{max}})
\]
\[
+ \sum_k v_k^i \lambda_k^i (\|\Theta b v_{k-1}^i\|_2 - v_{\text{max}})
\]
\[
+ \sum_k \sum_{j \neq i} v_{kj}^i \lambda_{kj}^i (b_{kj}^i - a_{kj}^i T p_k^i)^T + \sum_k v_k^i T (x_k^i - A^k x_0^i - A^{k-1} B u_0^i - \cdots - B u_{k-1}^i)
\]
where \( u_k^i, v_k^i, \lambda_k^i, \lambda_{kj}^i, \nu_k^i = [p_k^i T, v_k^i T]^T \) and \( t \nu^i \) are the Lagrangian multipliers corresponding to the inequality and equality constraints, where \( k = 1, 2, \ldots, K \).

When a deadlock happens, all robots remain static, i.e., \( u_k^i = 0 \) and \( v_k^i = 0 \). It implies that both \( \|\Theta a u_{k-1}^i\|_2 < u_{\text{max}} \) and \( \|v_{k-1}^i\|_2 < v_{\text{max}} \) hold. Hence, according to the complementary slackness condition of Karush-Kuhn-Tucker (KKT) [44], \( u_k^i = 0 \) and \( v_k^i = 0 \) hold at the optimal solution of (10). Furthermore, according to the stationary condition of KKT, the following conditions are satisfied:
\[
\frac{\partial \mathcal{L}^i}{\partial p_k^i} = \frac{\partial C^i}{\partial p_k^i} - \sum_{j \neq i} \lambda_{kj}^i a_{kj}^i + \nu_k^i = 0,
\]
\[
\frac{\partial \mathcal{L}^i}{\partial v_k^j} = \left\{ \begin{array}{ll}
\nu_k^j, & k \neq K \\
\nu_k^j + t \nu^i, & k = K
\end{array} \right.,
\]
\[
\frac{\partial \mathcal{L}^i}{\partial u_{k-1}^j} = -B^T A^{K-k} \nu_K^j - B^T A^{K-k-1} \nu_{k-1}^j
\]
(27c)

Given (3), it follows directly that
\[
A^m B = \begin{bmatrix} m h & I_d \\ I_d & 0 \end{bmatrix}.
\]
Thus, condition (27c) can be re-formulated as
\[

(29)
\]
Furthermore, since \( v_K^T B = 0 \), it follows from (3) that
\[
[p v_K^T, v_K T] \begin{bmatrix} 0 & 1_d \end{bmatrix} = 0,
\]
in which implies \( v_K^T B = 0 \). By condition (27b), it follows that \( v_K^i = 0 \) holds, \( \forall k = 1, 2, \ldots, K \). Then (29) can be further simplified as
\[
(K - k) \, h \, p v_K^i + (K - k - 1) \, h \, p v_{K-1}^i + \cdots + h p v_{k+1}^i = 0.
\]
By setting \( k = K - 1 \), it follows that \( p v_{K-1}^i = 0 \). Hence, the other condition (27a) turns to
\[
\frac{\partial \mathcal{L}^i}{\partial p_k^i} = \frac{\partial C^i}{\partial p_k^i} - \sum_{j \neq i} \lambda_{kj}^i a_{kj}^i = 0.
\]
**APPENDIX C**

**PROOF OF EQUATION (15)**

**Proof:** The Lagrange function of the new convex program is given by
\[
\mathcal{L}' = C^i + \sum_{k=1}^{K} u_k^i \lambda_k^i (\|\Theta a u_{k-1}^i\|_2 - u_{\text{max}})
\]
\[
+ \sum_{k=1}^{K} v_k^i \lambda_k^i (\|\Theta b v_{k-1}^i\|_2 - v_{\text{max}})
\]
\[
+ \sum_{k=1}^{K-1} \sum_{j \neq i} v_{kj}^i \lambda_{kj}^i (b_{kj}^i - a_{kj}^i T p_k^i)^T + \sum_{k=1}^{K} v_k^i T (x_k^i - A^k x_0^i - A^{k-1} B u_0^i - \cdots - B u_{k-1}^i),
\]
where \( u_k^i, v_k^i, \lambda_k^i, \lambda_{kj}^i, \nu_k^i = [p_k^i T, v_k^i T]^T \) and \( t \nu^i \) are the Lagrangian multipliers corresponding to the inequality constraints. Similar to the proof of Lemma 1, it
follows that $w^{ij} = 0$ and $w^{ji} = 0$. Additionally, by the KKT condition, the following conditions:

\[
\frac{\partial C^i}{\partial p_K^i} = \frac{\partial C^i}{p_K^i} - \sum_{j \neq i} \lambda_k^{ij} a_K^{ij} + p'_{ij} = 0, \tag{31a}
\]

\[
\frac{\partial C^i}{\partial w^{ij}} = \frac{\partial C^i}{w^{ij}} + \lambda_k^{ij} + w^{ij} = 0, \tag{31b}
\]

hold. Thus, it can be similarly derived that both $p'_{ij} = 0$ and

\[
\frac{\partial C^i}{\partial p_K^i} = \frac{\partial C^i}{p_K^i} - \sum_{j \neq i} \lambda_k^{ij} a_K^{ij} = 0.
\]

To proceed further, the following fact:

\[
\nu^i = 0, \forall j \neq i \tag{32}
\]

will be proven by contradiction. By substituting (9) into (31b), it follows that

\[
\rho_0(-\frac{1}{w^{ij}} + \frac{1}{\epsilon} + \lambda_k^{ij} + w^{ij} = 0. \tag{33}
\]

If $w^{ij} > 0$ holds, due to the complementary slackness condition of KKT, then $w^{ij} = \epsilon$ holds. Thus, the above equation implies $\lambda_k^{ij} + w^{ij} = 0$. However, the KKT condition implies that both $w^{ij} \geq 0$ and $\lambda_k^{ij} \geq 0$ must hold. Thus, $w^{ij} = 0, \forall j \neq i$.

Therefore, via (13), (31b) and (32), it follows that

\[
\lambda_k^{ij} = -\frac{\partial C^i}{\partial w^{ij}} = -\rho_0 \frac{w^{ij} - \epsilon}{\epsilon w^{ij}}. \tag{34}
\]

By substituting (34) into (31a), it can be derived that

\[
Q_{\text{tar}}(p_{\text{target}}^i - p_{K}^i) + \sum_{j \neq i} a_{K}^{ij} \rho_0 (\epsilon - w^{ij}) = 0.
\]

Lastly, if $j \notin \mathcal{K}_i$, then both $w^{ij} = \epsilon$ and $w^{ij} \epsilon w^{ij}$ hold, which implies (15).

Next, it remains to be shown that $w^{ij} = w^{ji}$ holds. If $w^{ij} < \epsilon$, it clearly follows that $p_0 (-\frac{1}{w^{ij}} + \frac{1}{\epsilon}) < 0$ and $\lambda_k^{ij} > 0$ by the conditions in (32) and (33). This implies that the constraint (14b) is active, i.e.,

\[
a_{K}^{ij} b^{ij}_K = a_{K}^{ij} + w^{ij}. \tag{35}
\]

Note that $p_{K}^i(t) = p_{K}^{j}(t-h) = p_{K}^{i}(t-h) = p_{K}^{i+1}(t-h) = p_{K}^j(t)$ holds for robot $i$ and similar condition holds for robot $j$. Hence, by (7) and (35), it follows that

\[
\frac{||p_{K}^i - p_{j}^f||^2}{2} = \frac{t''_{\min}}{2} + w^{ij}.
\]

Similarly, for robot $j$, it can be derived that

\[
\frac{||p_{K}^j - p_{K}^i||^2}{2} = \frac{t''_{\min}}{2} + w^{ji}.
\]

Consequently, these two equations imply that $w^{ij} = w^{ji}$. This completes the proof.
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