PRIMORDIAL NON-GAUSSIANITY AS A SIGNATURE OF PRE-INFLATIONARY RADIATION ERA

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Primordial non-Gaussianity generated in an inflationary model where inflation is preceded by a radiation era is discussed. It is shown that both bispectrum and trispectrum non-Gaussianities are enhanced due to the presence of pre-inflationary radiation era. One distinguishing feature of such a scenario is that the trispectrum non-Gaussianity is larger than the bispectrum one.

Keywords: Inflation, Radiation era, Primordial non-Gaussianity, CMBR

1. Introduction

In a scenario where inflation is preceded by a radiation era, the inflaton field can initially be in thermal equilibrium with the radiation field before decoupling and thus can retain its thermal distribution at the onset of inflation. Due to this initial thermal distribution of inflaton field the power spectrum of the primordial perturbations is modified by an extra temperature dependent factor \( \text{coth}(k/2T) \) which enhances the power of the \( TT \) angular power spectrum of CMBR at largest scales. This angular power spectrum of CMBR is in accordance with the observations of WMAP if the comoving temperature \( T \) of the primordial perturbations is less than \( 10^{-3} \) Mpc\(^{-1} \) and thus cannot discriminate this scenario with a generic super-cool inflationary scenario at the level of power spectrum.

On the other hand, primordial non-Gaussianity (NG) which is a measure of departure from a Gaussian distribution of primordial fluctuations generated during inflation, has become very crucial for both theoretical and observational cosmology as measuring or simply constraining NG can discriminate between several degenerate inflationary models which produce the same scale-invariant power spectrum and thus can quantify the dynamics of very early universe. The non-Gaussian parameter \( f_{NL} \) arising from three-point correlation function, known as Bispectrum, is constrained by WMAP 5yr data as \( -151 < f_{NL} < 253 \) whereas PLANCK can probe up to \( f_{NL} \sim 5 \). Another non-Gaussian parameter \( g_{NL} \) arising from four-point correlation function, known as Trispectrum, is loosely constrained by WMAP as \( |\tau_{NL}| < 10^8 \) whereas PLANCK can probe \( \tau_{NL} \) up to 560. But measurement of primordial non-Gaussianities imprinted in 21 cm background radiation in future experiments can constrain the bispectrum as \( f_{NL} < 1 \) and the trispectrum as \( \tau_{NL} \sim 10 \).

We will discuss here the non-Gaussian features of such a scenario where inflation is preceded by a radiation era and will show that at the level of trispectrum this scenario differs from a super-cool inflation scenario which can be considered as a signature of pre-inflationary radiation era.
2. Enhancement of Bispectrum

In a generic inflationary scenario a non-Gaussian feature arises due to non-linear evolution of comoving curvature perturbation \( R(t, x) \) as

\[
R_{NL}(t, x) = \frac{\Delta}{3} \delta \phi_L(t, x) + \frac{1}{2} \frac{\partial}{\partial \phi} \left( \frac{\Delta}{3} \delta \phi_L^2(t, x) + \mathcal{O}(\delta \phi_L^3), \right)
\]

though the inflaton fluctuations \( \delta \phi_L(t, x) \) are Gaussian in nature. This kind of non-linear evolution of comoving curvature perturbations yields a non-vanishing three-point correlation function or the Bispectrum which can be quantified as

\[
\langle R(k_1) R(k_2) R(k_3) \rangle = \left( \frac{3}{(2\pi)^3} \right) \int_{NL} \frac{P_R(k)}{k^3} \left( \frac{P_R(k)}{k'^3} + 2 \text{ perms} \right),
\]

where the non-linear parameter in a generic inflationary scenario turns out to be of the order of slow-roll parameters \( f_{NL} = \frac{5}{6} (\delta - \epsilon)^{10} \) and thus too small \( (\mathcal{O}(10^{-2})) \) to be detected by any present or forthcoming experiments. \( P_R \) in the above equation represents the two-point correlation function, called the power spectrum, and the delta function ensures that the three momentum form a triangle due to which \( f_{NL} \) is quantified in several triangle configurations namely (i) Squeezed configuration \( (|k_1| \approx |k_2| \approx k \gg |k_3|) \), (ii) Equilateral configuration \( (|k_1| = |k_2| = |k_3| = k) \) and (iii) Folded configuration \( (|k_1| = |k_2| = 1/k_3 = k) \).

If inflation is preceded by a radiation era then

- the inflaton field will have an initial thermal distribution
- the thermal vacuum \( |\Omega \rangle \) will have finite occupation \( N_k |\Omega \rangle = n_k |\Omega \rangle \)
- the probability of the system to be found in an energy state \( \epsilon_r \) would be

\[
p(k_1, k_2, k_3, k_4) = \left( \frac{1}{12} \right) \left( \epsilon_r \right)^2 \left( \frac{1}{n_1 n_2 n_3 n_4} \right) \left( \frac{1}{\epsilon_r} \right) \left( \frac{1}{\epsilon_r} \right)
\]

Due to these above mentioned points the correlation functions (two-point, three-point and four-point) now have to be thermally averaged. The thermally averaged two-point correlation function yields the extra \( \coth(k/2T) \) factor\(^{11}\) in the power spectrum as has been mentioned before. Thermal averaging of the three-point correlation function enhances the bispectrum \( \mathcal{O}^{12} \). The enhanced \( f_{NL} \) has been computed in different triangle configuration and it is most enhanced in the Equilateral configuration where it is enhanced by a factor of 91 with respected to that of in generic inflationary scenarios. Thus it would be in the range of observation of future experiments of 21 cm background radiation\(^{13}\).

3. Enhancement of Trispectrum

The trispectrum is known as the connected part of the four-point correlation function of primordial perturbation and can be written as

\[
\langle R(k_1) R(k_2) R(k_3) R(k_4) \rangle_c = \frac{1}{2} \left( \langle R(k_1) R(k_2) R(k_3) R(k_4) \rangle - \langle R_L(k_1) R_L(k_2) \rangle \langle R_L(k_3) R_L(k_4) \rangle + 2 \text{ perm} \right).
\]

In a generic inflationary scenario the non-Gaussianity arising from trispectrum, quantified by the parameter \( \tau_{NL} \), turns out to be \( \tau_{NL} = \left( \frac{\delta}{f_{NL}} \right)^{14} \) i.e. \( \mathcal{O}(10^{-4}) \).
Thus a generic scenario of inflation yields a trispectrum non-Gaussianity which is much smaller than a bispectrum non-Gaussianity.

In a presence of pre-inflationary radiation era one has to thermal average over the four-point correlation function as has been discussed before. It turns out that after thermal averaging the four-point correlation function is not equal to the square of thermal average of two-point correlation function. Thus a non-vanishing four-point correlation function exists even without non-linear terms in the comoving curvature perturbations. This shows that the $|\tau_{NL}|$ will not depend upon the slow-roll parameters and can be as large as $4^{29}$ and which is within the detection range of future 21 cm background radiation experiments.

4. Conclusion

Presence of pre-inflationary radiation era yields an initial thermal distribution of the inflaton fluctuations. If the initial temperature of these fluctuations is less than $10^{-3}$ Mpc$^{-1}$ then the power spectrum is in accordance with the observation and does remain indistinguishable from the generic case at the level of power spectrum. On the other hand, such a pre-inflationary radiation era enhances primordial non-Gaussianity in both bispectrum and trispectrum level and these non-Gaussianities are within the observable range of the future experiments of primordial non-Gaussianities imprinted in the 21 cm background radiation. Interestingly we found out the trispectrum in this case does not depend upon slow-roll parameters due to thermal averaging and is orders of magnitude larger than the bispectrum non-Gaussianity. Thus this very feature of primordial non-Gaussianities can be considered as a signature of pre-inflationary radiation era.

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