Radiation and Mass Transfer Effects on MHD Oscillatory Flow for Carreau Fluid through an Inclined Porous Channel

Dheia G. Salih Al-Khafajy

Department of Mathematics, College of Science, University of Al-Qadisiyah, Diwaniya, Iraq.

Received: 28/7/2019 Accepted: 22/10/2019

Abstract

This paper aims to study a mathematical model showing the effects of mass transfer on MHD oscillatory flow for Carreau fluid through an inclined porous channel under the influence of temperature and concentration at a slant angle on the centre of the flow with the effect of gravity. We discussed the effects of several parameters that are effective on fluid movement by analyzing the graphs obtained after we reached the momentum equation solution using the perturbation series method and the MATHEMATICA program to find the numerical results and illustrations. We observed an increased fluid movement by increasing radiation and heat generation while fluid movement decreased by increasing the chemical reaction parameter and Froude number.

Keywords: Carreau fluid, MHD, oscillatory flow, thermal radiation, inclined porous channel.

1. Introduction

The flow of electrically oriented liquids across porous parallel plates has become an important problem because of their important applications in the sciences that affect human life. This appears in the extraction of crude oil from the earth as well as in food industry and the study of the movement of blood and other liquids in the body of the organism. Many researchers studied the oscillator flow to transfer liquids between two parallel plates under the influence of the magnetic field under different conditions. The ongoing flow through two parallel horizontal plates of an electrolytic conductive, viscous and incompressible fluid was examined by Attia and Kotb [1]. Makinde and Mhone [2] studied the combined effect of a random magnetic field and thermal transfer of radiation to an unstable
flow of high-optical fluid connected through a channel filled with irregular porous walls under saturated temperature conditions. Khudair and Al-Khafajy [3] suggested a model of heat transfer on MHD oscillatory flow for Williamson fluid through the porous plate for two types of flow (Couette flow and Poiseuille flow). Recently, Ahmed and Al-Khafajy [4] suggested a model of MHD peristaltic transport for Jeffrey fluid with varying temperature and concentration through a porous medium. Al-Khafajy [5] studied the effects of MHD oscillatory slip flow for Jeffrey fluid with variable viscosity through a porous plate with varying temperature and concentration.

The heat transfer and flow of fluids in an inclined channel are of special importance in the petroleum extraction and transport problems. This fact motivated scientists to explore the flows confined in an inclined channel [6-8].

Our objective here is to study the mathematical model for the influence of MHD oscillatory slip flow for Carreau fluid through an inclined channel with varying temperature and concentration. The perturbation technique series was used to solve the problem. The results of the physical parameters problem were discussed using graphs.

2. Mathematical Formulation

Let us consider the flow of a non-Newtonian (Carreau) fluid under the effects of radioactive heat transfer and electrically-applied magnetic field as depicted through an inclined porous channel with a width of \( d \) (Figure-1). Fluids are supposed to have very small electromagnetic power produced with a low electrical conductivity. We think of the system of Cartesian coordinates so that \((u(y,t), 0, 0)\) is the velocity vector.

The basic equations governing are given by:

The continuity equation is given by:
\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0
\]  

The momentum equations are:

In the \( x \)-direction:
\[
\rho \left( \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right) = -\frac{\partial \bar{p}}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho g \beta_T \sin(\theta) (T - T_0) + \rho g \beta_c \sin(\theta) (C - C_0) - \sigma B_0^2 \sin^2(\theta) \bar{u} - \frac{\mu}{\kappa} \bar{u} + \rho g \sin(\theta)
\]  

In the \( y \)-direction:
\[
\rho \left( \frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} \right) = -\frac{\partial \bar{p}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} - \frac{\mu}{\kappa} \bar{v} - \rho g \cos(\theta)
\]

The temperature equation is given by:
\[
\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} + \frac{H_g}{\rho c_p} (T - T_0)
\]

The concentration equation is given by:
\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} - K'_r (C - C_0) + \frac{\partial K_T}{\partial T} \frac{\partial^2 T}{\partial y^2}
\]

where \( \bar{u} \) is the axial velocity, \( \rho \) is the density of the fluid, \( p \) is the pressure, \( \sigma \) is the electrical conductivity, \( B_0 \) is the strength of the magnetic field, \( g \) is the acceleration due to gravity, \( T \) is the temperature, \( C \) is the concentration, \( C_0 \) is the specific heat at constant pressure, \( q \) is the radiation heat flux, \( K \) is the thermal conductivity, \( H_g \) is the heat generation, \( D \) is the coefficient of mass diffusivity, \( (0 \leq \varepsilon \leq \pi) \) is the angle between velocity field and magnetic field strength, \( (0 \leq \theta \leq \pi) \) is the angle between the centre channel and the ground acceleration, and \( K_T \) is the thermal diffusion ratio. The corresponding boundary conditions are given by:
\[
u = 0, T = T_0, C = C_0 \text{ at } y = 0 \quad \text{and} \quad u = 0, T = T_d, C = C_d \text{ at } y = d.
\]

The radioactive heat flux [9] is given by:

\[ T = T_d, C = C_d \text{ at } y = d. \]
The radiation absorption is denoted by $\eta$.

The basic equation for the Carreau fluid is given as:

$$ S = -\bar{p}I + \tau $$

(8)

$$ \bar{\tau} = \left[ \mu_c + (\mu - \mu_c)(1 + (\bar{Y} \bar{\gamma})^{n-1}/2) \right] \bar{E} $$

(9)

where $\bar{p}$ is the pressure, $I$ is the unit tensor, $\bar{\tau}$ is the extra stress tensor, $Y$ is the time constant, $\mu_c$ is the infinite shear rate viscosity, the case for which $Y < 1$, and $\mu_c = 0$. We can write the component of extra stress tensor according to the following:

$$ \bar{\tau} = \mu \left[ 1 + \left( \frac{n-1}{2} \right) Y^2 \bar{\gamma}^2 \right] \bar{E} $$

(10)

The Rivlin-Ericksen tensors are given as:

$$ \bar{T} \text{ and } \bar{S} $$

(11)

The stress component is given by:

$$ \bar{\tau}_{xy} = \bar{\tau}_{yx} = \mu \left[ \frac{\partial u}{\partial y} + \left( \frac{n-1}{2} \right) Y^2 \left( \frac{\partial u}{\partial y} \right)^3 \right] $$

(12)

3. Method of Solution

The governing equations for the non-dimensional conditions are:

$$ x = \frac{x}{h}, \ y = \frac{y}{h}, \ u = \frac{u}{U}, \ T = \frac{T-T_d}{T-T_0}, \ \bar{p} = \frac{\bar{p}h}{\mu U}, \ Pe = \frac{\rho h U c_p}{K}, \ Fr = \frac{h K_r^2}{U}, $$

$$ We = \frac{\bar{W}U}{h}, \ \bar{r}_{xy} = \frac{h}{U}, \ C = \frac{C_c - C}{C_d - C}, \ \bar{\gamma} = \frac{4n^2h^2}{K}, $$

$$ \bar{T} = \frac{t}{h}, \ Re = \frac{\rho h U}{\mu U}, \ Da = \frac{k}{h}, \ Gr = \frac{\rho \beta_h h^2 (T-T_0)}{\mu U}, \ Sr = \frac{\bar{D} R (T_d-T_0)}{U T_m (C_d-C)} $$

(13)

$$ \bar{M}^2 = \frac{\rho C_p h^2}{\mu U}, \ Sc = \frac{\mu h}{D}, \ \bar{H} = \frac{h g h^2}{K}, \ \bar{G} = \frac{\rho \beta_h h^2 (T-T_0)}{\mu U}, \ Fr = \frac{h K_r^2}{U} $$

where $U$ is the mean flow velocity, $Da$ is Darcy number, $Re$ is Reynolds number, $M$ is the magnetic parameter, $Pe$ is the Peclet number, $K$ is the radiation parameter, $Sc$ is the Schmidt number, $Sr$ is the Soret number, $\bar{H}$ is the heat generation parameter, $T_m$ is the mean temperature, $Gr$ is the thermal Grashof number, $Gc$ is the solutal Grashof number, $K_r$ is the chemical reaction parameter and $Fr$ is the Froude number.

By substituting (12) into equations (1)-(6) and (11), we have the following of non-dimensional equations:

$$ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 $$

(14)

$$ Re \frac{\partial u}{\partial x} = - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + +Gr \sin(\epsilon) T + Gc \sin(\epsilon) C - \left( M^2_2 + \frac{1}{Da} \right) u + \frac{Re}{Fr} \sin(\theta) $$

(15)

$$ Pe \frac{\partial v}{\partial t} = \frac{\partial^2 \tau_{xy}}{\partial y^2} + (K + H) T $$

(16)

$$ \frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - \frac{K_r C}{Sr} \frac{\partial^2 \tau_{xy}}{\partial y^2} $$

(17)

and $\tau_{xy} = \frac{\partial u}{\partial y} + \left( \frac{n-1}{2} \right) \bar{W} e^2 \left( \frac{\partial u}{\partial y} \right)^3$

4. Solution of the Problem

The solution of the heat equation (15) with boundary conditions $T'(0) = 0, T'(1) = 1$, (after offsetting the non-dimensional parameters (12) into equation (6)) is as follows [5]

$$ T(y,t) = \frac{\bar{T} \bar{C} \sin(\bar{A}y)}{\bar{A} \sin(\bar{A}y)} e^{i \omega t}. $$

(19)

The solution of the concentration equation (16) with boundary conditions $C(0) = 0, C(1) = 1$, (after offsetting the non-dimensional parameters (12) into equations (6)) is [5]

$$ C(y,t) = \frac{\left( \frac{\beta h^2}{(A^2 + B^2)(-1 + e^{2\bar{h}})} \right) e^B - e^{-B}}{A^2 + B^2} \bar{A} \sin(\bar{A}y) e^{i \omega t}. $$

(20)
where \( A = \sqrt{\mathcal{K} + \mathcal{H}} - i \omega \mathcal{P} \), \( B = \sqrt{\mathcal{S} \mathcal{C}(K_r + i \omega)} \) and \( \omega \) is the frequency of the oscillation.

To solve the motion equation (18), let
\[
\frac{d \phi}{d \tau} = -\lambda e^{\omega t}, \quad u(y, t) = u_1(y) e^{\omega t}
\]
and \( \omega \) is the frequency of the oscillation.

By substituting equation (21) into equation (18), we obtain
\[
\Re \left( \frac{\partial}{\partial \tau} u_1(y) e^{\omega t} \right) = \lambda e^{\omega t} + \frac{\partial^2}{\partial y^2} u_1(y) e^{\omega t} + \frac{3(n-1)}{2} \mathcal{W} e^2 \left( \frac{\partial}{\partial y} u_1(y) e^{\omega t} \right)^2 \frac{\partial^2}{\partial y^2} u_1(y) e^{\omega t} + \mathcal{G} \sin(\xi) \mathcal{T} + \mathcal{G} \sin(\xi) \mathcal{C} - \left( \mathcal{M}_2^2 + \frac{1}{\mathcal{D}_a} \right) u_1(y) e^{\omega t} + \frac{\mathcal{R} \mathcal{E}}{\mathcal{F} \mathcal{R}} \sin(\theta),
\]
with boundary conditions \( u_1(0) = u_1(1) = 0 \), (after offsetting the non-dimensional parameters (12) into equation (6))

After simplifying, we obtain
\[
\omega \Re u_1(y) = \lambda + \frac{\partial^2}{\partial y^2} u_1(y) + \frac{3(n-1)}{2} \mathcal{W} e^2 \left( \frac{\partial}{\partial y} u_1(y) \right)^2 \frac{\partial^2}{\partial y^2} u_1(y) + \mathcal{G} \sin(\xi) \mathcal{T}_1(y) + \mathcal{G} \sin(\xi) \mathcal{C}_1(y) - \left( \mathcal{M}_2^2 + \frac{1}{\mathcal{D}_a} \right) u_1(y) + \frac{\mathcal{R} \mathcal{E}}{\mathcal{F} \mathcal{R}} e^{-\omega t} \sin(\theta).
\]
where \( \mathcal{T}_1(y) = \csc(\mathcal{A}) \sin(\mathcal{A}y) \)
and \( \mathcal{C}_1(y) = \left( e^{\left( A^2 + B^2 + 2 \mathcal{S} \mathcal{C} A^2 \right)} \right)^{\left( e^{B y} - e^{-B y} \right)} - \mathcal{S} \mathcal{C} A^2 \mathcal{C} \mathcal{S} \mathcal{C} \mathcal{A} \mathcal{S} \mathcal{C} \mathcal{A} \mathcal{Y} \left( e^{A y} - e^{-A y} \right).

It is difficult to solve the nonlinear differential equation (22) and, thus, we propose a perturbation technique to solve this equation by taking a small value for \( \mathcal{W} \). Accordingly, we write:
\[
u_1 = u_{10} + \mathcal{W} e^{2} u_{11} + O(\mathcal{W} e^{4})
\]

By substituting equation (23) into equation (22), with boundary conditions \( u_1(0) = u_1(1) = 0 \), then equating the like powers of \( \mathcal{W} \), we obtain the following results presented in the forthcoming subsections:

### 4.1. Zeros-Order System (\( \mathcal{W} = 0 \))
\[
\frac{\partial^2 u_{10}}{\partial y^2} = \left( \omega \Re + \mathcal{M}_2^2 + \frac{1}{\mathcal{D}_a} \right) u_{10} = -\left( \lambda + \mathcal{G} \sin(\xi) \mathcal{T}_1 + \mathcal{G} \sin(\xi) \mathcal{C}_1 + \frac{\mathcal{R} \mathcal{E}}{\mathcal{F} \mathcal{R}} e^{-\omega t} \sin(\theta) \right)
\]
The associated boundary conditions are: \( u_{10}(0) = u_{10}(1) = 0 \).

### 4.2. First-Order System (\( \mathcal{W} = 1 \))
\[
\frac{\partial^2 u_{11}}{\partial y^2} = \left( \omega \Re + \mathcal{M}_2^2 + \frac{1}{\mathcal{D}_a} \right) u_{11} = -\frac{3(n-1)}{2} \mathcal{W} e^2 \left( \frac{\partial}{\partial y} u_{10} \right)^2 \frac{\partial^2 u_{10}}{\partial y^2}
\]
The associated boundary conditions are: \( u_{11}(0) = u_{11}(1) = 0 \).

### 4.3. Zeros-Order Solution
The solution of equation (24) subset to the associate boundary conditions is:
\[
u_{10} = \frac{\mathcal{F}}{\mathcal{D} \left( 1 + e^{\mathcal{Y}} \right)} \left( e^{\mathcal{Y} \mathcal{D} y} + e^{\mathcal{Y} \mathcal{D} (1-y)} \right)
\]

### 4.4. First-Order Solution
The solution of equation (25) subset to the associate boundary conditions is:
\[
u_{11} = \left( \frac{3 e^{2 \omega t} F^2 (3 n - 1)}{16 D^2 \left( 1 + e^{\mathcal{Y}} \right)^3} \right) \left( e^{-3 \mathcal{Y} \mathcal{D} y} \left( e^{\mathcal{Y} \mathcal{D} y} + e^{3 \mathcal{Y} \mathcal{D}} + 2(1 - 2 \mathcal{Y} \sqrt{D}) e^{\mathcal{Y} \mathcal{D} (1+y)} + 2(1 + 2 \mathcal{Y} \sqrt{D}) e^{3 \mathcal{Y} \mathcal{D}} + e^{3 \mathcal{Y} \mathcal{D}} e^{\mathcal{Y} \mathcal{D} (1-y)} \right) \right.
\]
where \( D = i \omega \Re + \mathcal{M}_2^2 + \frac{1}{\mathcal{D}_a} \) and \( \mathcal{F} = \lambda + \mathcal{G} \sin(\xi) \mathcal{T}_1 + \mathcal{G} \sin(\xi) \mathcal{C}_1 + \frac{\mathcal{R} \mathcal{E}}{\mathcal{F} \mathcal{R}} e^{-\omega t} \sin(\theta) \)

### 5. Results and Discussion
This section discusses the effects of varying "temperature and concentration" on MHD oscillation slip flow for Carreau fluid through an inclined permeable channel. The perturbation technique is applied to calculate convergent chain solutions’ results obtained for non-dimensional distribution and displayed graphically. We use the "MATHEMATICA" program to find numerical results and
illustrations. Numerical assessments of analytical results and some of the graphically significant results are presented in Figures-(2-9).

Figure-2 shows the influences of the radiation parameter (\(\mathcal{K}\)) and heat generation parameter (\(\mathcal{H}\)) on the velocity profiles function \(u\) vs. \(y\). It was found that \(u\) increases with increasing both \(\mathcal{K}\) and \(\mathcal{H}\). Figure- 3 shows that the velocity profile \(u\) goes down by increasing the influences of the Soret number (\(Sr\)) and frequency of the oscillation (\(\omega\)). Figure- 4 demonstrates that the velocity profile \(u\) goes down with increasing the magnetic parameter (\(M\)) and that \(u\) rises up with increasing Darcy number (\(Da\)). Figure-5 shows the influences of Reynolds number (\(Re\)) and Froude number (\(Fr\)) on the velocity profiles function \(u\) vs. \(y\). We noted that the increase of \(Re\) causes an increased velocity of the fluid, while the velocity of the fluid is decreased with increasing \(Fr\). Figure- 6 shows that the velocity profile \(u\) rises with increasing both solutal Grashof number (\(Gr_c\)) and thermal Grashof number (\(Gr\)). Figure-7 shows the influence of Weissenberg number (\(We\)) and pressure parameter (\(\lambda\)) on the velocity profiles function \(u\) vs. \(y\). We noted that the increase of \(We\) decreases the velocity of the fluid, while the velocity of the fluid is increased with increasing \(\lambda\). Figure-8 shows the influences of the inclined angle of the magnetic field (\(\epsilon\)) and the inclined angle of the ground acceleration (\(\theta\)) on the velocity profiles function \(u\) vs. \(y\). We observed that the velocity profile \(u\) rises with increasing both \(\epsilon\) and \(\theta\). The last Figure-9 shows the influences of chemical reaction parameter (\(K_r\)) and Schmidt number (\(Sc\)) on the velocity profiles function \(u\) vs. \(y\). We noted that the increase of \(K_r\) and \(Sc\) gives increases the velocity of the fluid.
Figure 5- Velocity profile for various values of $Re$ and $Fr$ with $n = 3, \omega = 1, \kappa = 2, \mathcal{H} = 2, Gr = 1, Pe = 0.7, M = 1, Sc = 0.6, Sr = 0.1, Gc = 1, Gr = 1, Ka = 0.5, Da = 0.5, \lambda = 1, \epsilon = \frac{\pi}{4}, \theta = \frac{\pi}{4}$.

Figure 6- Velocity profile for various values of $Gc$ and $Gr$ with $n = 3, \omega = 1, \kappa = 2, \mathcal{H} = 2, Re = 2, Pe = 0.7, M = 1, Sc = 0.6, Sr = 0.1, Gc = 1, Ka = 0.5, Da = 0.5, Fr = 1, \lambda = 1, \epsilon = \frac{\pi}{4}, \theta = \frac{\pi}{4}$.

Figure 7- Velocity profile for various values of $We$ and $\lambda$ with $n = 3, \omega = 1, \kappa = 2, \mathcal{H} = 2, Gr = 1, Re = 2, Pe = 0.7, M = 1, Sc = 0.6, Sr = 0.1, Gc = 1, Gr = 1, Ka = 0.5, Da = 0.5, Fr = 1, \epsilon = \frac{\pi}{4}, \theta = \frac{\pi}{4}$.

Figure 8- Velocity profile for various values of $\epsilon$ and $\theta$ with $n = 3, \omega = 1, \kappa = 2, \mathcal{H} = 2, Gr = 1, Re = 2, Pe = 0.7, M = 1, Sc = 0.6, Sr = 0.1, Gc = 1, Gr = 1, Ka = 0.5, Da = 0.5, Fr = 1, \lambda = 1$. 
Figure 9- Velocity profile for various values of $Sc$ and $K_r$ with $n = 3, \omega = 1, \mathcal{K} = 2, \mathcal{H} = 2, Gr = 1, Re = 2, Pe = 0.7, M = 1, Sr = 0.1, Gc = 1, Gr = 1, Da = 0.5, Fr = 1, \lambda = 1, \varepsilon = \frac{\pi}{4}, \theta = \frac{\pi}{4}$.

6. Conclusions
We reached interesting results through studying the effects of varying "temperature and concentration" on MHD oscillatory flow of the Carreau fluid through an inclined permeable channel. We discussed the effects of several parameters that are effective on fluid movement by analyzing the graphs obtained after we reached the momentum equation solution using the perturbation series method and the MATHEMATICA program to find the numerical results and illustrations. A summary of the results obtained is provided as follow:

The velocity profiles rise up by increasing the parameters of permeability, radiation, heat generation, pressure, Reynolds number, solutal Grashof number, thermal Grashof number, the inclined angle of the magnetic field, and the inclined angle of the ground acceleration, while the velocity profiles go down by increasing the parameters of magnetic, chemical reaction, frequency of the oscillation, Soret number, Froude number, Weissenberg number and Schmidt number.

References
1. Attia HA and Kotb NA. 1996. MHD flow between two parallel plates with heat transfer, Acta Mechanica, 117(1-4): 215-20.
2. Makinde, O.D. and Mhone, P.Y. 2005. Heat transfer to MHD oscillatory flow in a canal filled with porous medium, Rom. Jour. Phys., 50: 931-938.
3. W. S. Khudair and Dheia G.S. Al-khafajy. 2018. Influence of heat transfer on Magnetohydrodynamics oscillatory flow for Williamson fluid through a porous medium, Iraqi J. Sci., 59(1B): 389–397.
4. Ahmed A. Hussein Al-Aridhee and Dheia G. Salih Al-Khafajy. 2019. Influence of MHD Peristaltic Transport for Jeffrey Fluid with Varying Temperature and Concentration through Porous Medium, IOP publishing. Journal of Physics, 2nd International Scientific Conference, College of Science, University of Al-Qadisiyah, April 24-25, Iraq.(to appear in Journal of Physics)
5. Dheia G. Salih Al-Khafajy. 2019. Radiation and Mass Transfer Effects on MHD Oscillatory Flow for Jeffery Fluid with Variable Viscosity Through Porous Channel in the Presence of Chemical Reaction. Sci. Int. (Lahore), 31(2), 2nd SICAPM, College of Science, University of Baghdad, February 27-28, Iraq.
6. J.Srinivas, J.V.Ramana Murthy, Ali J. Chamkha. 2015. Analysis of entropy generation in an inclined channel flow containing two immiscible micropolar fluids using HAM, International Journal of Numerical Methods for Heat & Fluid Flow, 26: 1027-1049.
7. Hayat T., Sadia Ayub, Ahmed Alsaeedi, Bashir Ahmad. 2018. Numerical simulation of buoyancy peristaltic flow of Johnson-Segalman nanofluid in an inclined channel, Results in Physics, 9: 906-915.
8. Hanumesh Vaidya, C. Rajashekhara, G. Manjunatha, K. V. Prasad. 2019. Peristaltic mechanism of a Rabinowitsch fluid in an inclined channel with compliant wall and variable liquid properties, Journal of the Brazilian Society of Mechanical Sciences and Engineering, Springer, 41(1), doi:10.1007/s40430-018-1543-4.
9. Cogley, A. C. L.,Vinvent, W.G. and Giles, E.S. 1968. Differential approximation for radiative transfer in a non-gray near equilibrium. American Institute of Aeronautics and Astronautics, 6: 551-553.