The $B_s \to K$ Form Factor in The Whole Kinematically Accessible Range

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Abstract

A systematic analysis is presented of the $B_s \to K$ form factor $f(q^2)$ in the whole range of momentum transfer $q^2$, which would be useful to analyzing the future data on $B_s \to K$ decays and extracting $|V_{ub}|$. With a modified QCD light cone sum rule (LCSR) approach, in which the contributions cancel out from the twist 3 wavefunctions of $K$ meson, we investigate in detail the behavior of $f(q^2)$ at small and intermediate $q^2$ and the nonperturbative quantity $f_{B^*}g_{B^*B_sK}$ ($f_{B^*}$ is the decay constant of $B^*$ meson and $g_{B^*B_sK}$ the $B^*B_sK$ strong coupling), whose numerical result is used to study $q^2$ dependence of $f(q^2)$ at large $q^2$ in the single pole approximation. Based on these findings, a pole model from the best fit is formulated, which applies to the calculation on $f(q^2)$ in the whole kinematically accessible range. Also, a comparison is made with the standard LCSR predictions.

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A study on heavy-to-light exclusive processes plays a complementary role in the determination of fundamental parameters of the standard model (SM) and in the development of QCD theory. At present, an important task in the SM is to extract the $|V_{ub}|$ parameter. Recently, a new QCD factorization formula [1] has been proposed for nonleptonic $B$ decays and has been applied to discuss phenomenology of $B \to \pi\pi, \pi K$ and $\pi D$. This approach, however, is not adequate to extracting $|V_{ub}|$ from the relevant data, for the effects of the long distance QCD are anyway difficult to control in exclusive nonleptonic processes. Semileptonic $B$ decays into a light meson, induced by $b \to u$ transition, are regarded as the most promising processes suitable for such a purpose. Nevertheless, in this case precision extraction of $|V_{ub}|$ requires a rigorous estimate of the relevant hadronic matrix elements. It is a great challenge, because of our inability to deal with nonperturbative QCD effects from the first principle. Heavy quark symmetry is less predictive, and also lattice QCD calculation is restricted to a certain kinematical region, for heavy-to-light decays. Although QCD sum rule method has been being an effective QCD-based approach to nonperturbative dynamics, with its extensive uses in phenomenology, the resulting form factors behave very badly in the heavy quark limit $m_Q \to \infty$, the reason being that one omits the effects of the finite correlation length between the quarks in the physical vacuum. In order to overcome the disadvantage, an alternative to the ”classical” sum rule method, QCD light-cone sum rules (LCSR)[2], has been developed and has proven to be an advanced tool to deal with heavy-to-light transitions. There exist a lot of its applications in the literature. For a detailed description of this method, see [3]. However, a problem to need solving is how to control the pollution by higher twist, especially twist 3 wavefunctions, which are known very poorly and whose influence on the sum rules is considerable in most cases. In [4, 5], an improved LCSR approach has been worked out, to eliminate twist 3 wavefunctions and enhance the reliability of sum rule calculations, and has been applied to reexamine heavy-to-light form factors in the region of momentum transfer $0 \leq q^2 \leq m_b^2 - 2m_b\Lambda_{QCD}$, where the operator product expansion (OPE) goes effectively in powers of small light-cone distance $x^2$.

Most of previous works [3, 4, 5, 6] are devoted to discussing $B \to \pi, \rho$ semileptonic transitions within the context of LCSR, with the aim to extract $|V_{ub}|$. A study on $B_s \to K$ semileptonic processes is equally important. As compared with the case $B \to \pi, \rho$, however, the $B_s \to K$ form factors are more difficult to evaluate, for SU(3) breaking corrections to the twist 3 wave functions of $K$ meson have not been investigated completely in the literature. Explicitly, this problem can be avoided in our approach [4, 5]. On the other hand, to calculate the semileptonic widths one must find another way to estimate the form factors at the large momentum transfer $m_b^2 - 2m_b\Lambda_{QCD} \leq q^2 \leq (m_{B_s} - m_K)^2$. In the letter, we investigate the $B_s \to K$ form factor $f(q^2)$ at the total momentum transfer with the improved LCSR and a
pole model.

Let us start with the following definition of the $B_s \rightarrow K$ form factors $f(q^2)$ and $\bar{f}(q^2)$:

$$\langle K(p)|\bar{\pi}\gamma_\mu b|B(p+q)\rangle = 2f(q^2)p_\mu + \bar{f}(q^2)q_\mu.$$  \hspace{1cm} (1)

For $B_s \rightarrow K\ell\bar{\nu}_\ell$ transitions, as $l = e, \mu$ we can neglect the contributions from $\bar{f}(q^2)$ due to the smallness of $m_{e,\mu}$ and therefore only the form factor $f(q^2)$ is relevant. It can precisely be represented as

$$f(q^2) = \frac{f_{B^*}g_{B^*B_sK}}{2m_{B^*}(1-q^2/m_{B^*}^2)} + \int_0^\infty \rho(\sigma)\frac{d\sigma}{1-q^2/\sigma}$$

$$= F_G(q^2) + F_H(q^2), \hspace{1cm} (2)$$

with $f_{B^*}$ being the decay constant of $B^*$ meson, $m_{B^*}$ the $B^*$ meson mass, $g_{B^*B_sK}$ the strong coupling defined by

$$\langle B^*(q, e)K(p)|B_s(p+q)\rangle = -g_{B^*B_sK}(p\cdot e), \hspace{1cm} (3)$$

and $\rho(\sigma)$ a spectral function with the threshold $\sigma_0$. Obviously, $F_G(q^2)$ stands for the contribution from the ground state $B^*$ meson, which describes the principal behavior of $f(q^2)$ around $q^2 = q_{\text{max}}^2$, and $F_H(q^2)$ parametrizes the higher state effects in the $B^*$ channel. As we have known, the form factor $f(q^2)$ may be estimated for the small and intermediate momentum transfers by means of LCSR, and also the nonperturbative parameter $f_{B^*}g_{B^*B_sK}$ is accessible within the same framework. Accordingly, modelling the higher state contributions by a certain assumption and then fitting (2) to its LCSR result $f_{\text{LC}}(q^2)$ in the region accessible to the light cone OPE, we might derive the form factor $f(q^2)$ in the total kinematical range to a better accuracy. For this purpose, we follow the procedure in [4, 5] and consider a chiral current correlator used for a LCSR sum rule calculation on $f_{\text{LC}}(q^2)$ and $f_{B^*}g_{B^*B_sK}$,

$$\Pi_\mu(p, q) = i \int d^4x e^{ix\tau} \langle K(p)|T\{\bar{\pi}(x)\gamma_\mu(1+\gamma_5)b(x), \bar{b}(0)(1+\gamma_5)s(0)\}|0\rangle$$

$$= F(q^2, (p+q)^2)p_\mu + \bar{F}(q^2, (p+q)^2)q_\mu \hspace{1cm} (4)$$

Inserting complete sets of the relevant intermediate states $|B^H\rangle$ in (4) and using the definition $\langle 0|\bar{s}\gamma_\mu b|B_s\rangle = \frac{m_{B_s}^2}{m_B+m_s}f_{B_s}$ and $\langle 0|\bar{\mu}\gamma_\mu b|B^*\rangle = m_{B^*}f_{B^*}e_\mu$, we have the two hadronic representations of the invariant function $F(q^2, (p+q)^2)$,

$$F_1^H(q^2, (p+q)^2) = \frac{2f_{\text{LC}}(q^2)m_{B_s}^2f_{B_s}}{(m_B+m_s)(m_{B_s}^2 - (p+q)^2)} + \int_0^\infty \frac{\rho_1^H(s)}{s -(p+q)^2}ds$$ 

$$\hspace{1cm} (5)$$
\[ F_2^H(q^2, (p + q)^2) = \frac{m_B^2m_B^* f_{B_s}f_{B^*}g_{B^*B,K}}{(m_b + m_s)(m_{B_s}^2 - q^2)(m_{B_s}^2 - (p + q)^2)} \]
\[ + \int \int \frac{\rho^H(s_1, s_2)\Theta(s_1 - s_0')\Theta(s_2 - s_0)}{(s_1 - q^2)(s_2 - (p + q)^2)} ds_1 ds_2. \]  

Several definite interpretations for (5) and (6) are in order. The two dispersion integrals include, in addition to the contributions of the resonances carrying the same quantum numbers as the corresponding ground states in the pole terms, the effects due to the relevant orbit-excited \( B \) mesons. Taking it into account that these orbit-excited states are far from the lowest \( B_s \) and \( B^* \) mesons, and the lowest two of them are slightly below the first excited \( B_s \) and \( B^* \) mesons in mass, their contributions can effectively absorbed into a dispersion integral so that thresholds \( s_0 \) and \( s_0' \) should correspond to the squared masses of the lowest \( 0^+ \) \( B_s \) and \( 1^+ \) \( B \) mesons respectively. On the other hand, the vector current \( \bar{b} \gamma^\mu b \) and axial-vector current \( \bar{b} \gamma^\mu \gamma_5 b \) couple also to \( 0^+ \) and \( 0^- \) \( B \) mesons, respectively, which should be considered in (6). The invariant function, however, does not receive such a contribution as we have checked. Therefore, it is safe to separate the hadronic expression \( F_2^H(q^2, (p + q)^2) \) into a pole term and a dispersion integral.

The task left is to calculate the correlator in QCD theory in order to obtain the desired sum rules. To this end, we work in the large space-like momentum regions: \( (p + q)^2 \ll 0 \) for the \( f_{LC}(q^2) \) case and \( q^2 \ll 0 \), \( (p + q)^2 \ll 0 \) for the \( f_{B_s}f_{B^*}g_{B^*B,K} \) case, so that the light cone OPE can be used for the correlator under consideration. After contracting the \( b \) quark operators, we encounter some nonlocal matrix elements, which can systematically be expanded in powers of the deviation from the light cone \( x^2 = 0 \) by defining the relevant light cone wavefunctions of \( K \) meson classified in terms of twist. The explicit parametrizations of all those can be found in the literature, and here are no more given for simplicity. A long but straightforward calculation yields the light-cone QCD form of the invariant function \( F(q^2, (p + q)^2) \),

\[ F_{QCD}^QCD(q^2, (p + q)^2) = 2f_Km_b \left\{ \int_0^1 du \left[ \frac{\varphi_K(u)}{m_b^2 - (q + up)^2} - \frac{8m_b^2[g_1(u) - G_2(u)]}{[m_b^2 - (q + up)^2]^3} + \frac{2ug_2(u)}{m_b^2 - (q + up)^2} \right] \right. \]
\[ + \left. \int_0^1 d\alpha \int D\alpha_i \frac{2\varphi_{\perp}(\alpha_i) + 2\tilde{\varphi}_{\perp}(\alpha_i) - \varphi_{\parallel}(\alpha_i) - \tilde{\varphi}_{\parallel}(\alpha_i)}{[m_b^2 - (q + \beta p)^2]^2} \right\}, \]  

(7)

to twist 4 accuracy. Here \( \varphi_K(u) \) is the twist 2 wavefunction, while the others have twist 4; the parameter \( \beta = \alpha_1 + \alpha_\alpha_3 \) and \( D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3) \). At this point, we put once again an emphasis on that differently from the existing LCSR calculations, the twist 3 wavefunctions make precisely a vanishing contribution to the correlator we choose. This is essentially important to enhancing precision of the LCSR calculation.
As usual, we need to make the Borel improvements on the theoretical expression: \( F^{QCD}(q^2, p^2 + q^2) \rightarrow \bar{F}_1^{QCD}(q^2, M^2) \), \( F^{QCD}(q^2, p^2 + q^2) \rightarrow \bar{F}_2^{QCD}(M_1^2, M_2^2) \), and then match them onto the individual Borel improved hadronic forms. Invoking the quark-hadron duality ansatz, the final sum rules for \( f_{LC}(q^2) \) and \( g_{B^*B_sK} \) read respectively:

\[
f(q^2) = \frac{m_b(m_b + m_s)}{m_{B_s}^2} f_K e^{m_{B_s}^2/M^2} \left\{ \left[ \Delta \varphi_K (u) \right] - \frac{2m_b^2}{uM^2} \right\} \int_0^1 \frac{du}{u} e^{-\frac{m_b^2}{uM^2}} \left[ \varphi_K (u) \right]
\]

\[
- \frac{4m_b^2}{uM^2} g_2(v) + \frac{2}{uM^2} \int_0^u g_2(v)dv \left( 1 + \frac{m_b^2 + q^2}{uM^2} \right)
\]

\[
+ \sum_{\alpha=1}^3 \int_0^1 d\alpha \Theta(\beta - \Delta) e^{-\frac{m_b^2}{\beta^2 M^2}} \left[ 2\varphi_\perp (\alpha_i) + 2\varphi_\parallel (\alpha_i) - \varphi_\parallel (\alpha_i) - \varphi_\parallel (\alpha_i) \right]
\]

\[
- \frac{4m_b^2 e^{-\frac{s_0}{\beta^2 M^2}}}{(m_b^2 - q^2)^2} \left[ 1 + \frac{s_0 - q^2}{M^2} \right] g_1(\Delta) - \frac{1}{(s_0 - q^2)(m_b^2 - q^2) du} \frac{dg_1(\Delta)}{du}
\]

\[
- 2e^{-\frac{s_0}{\beta^2 M^2}} \left[ \frac{m_b^2 + q^2}{(s_0 - q^2)(m_b^2 - q^2)} g_2(\Delta) - \frac{1}{(m_b^2 - q^2)} \left( 1 + \frac{m_b^2 + q^2}{m_b^2 - q^2} \left( 1 + \frac{s_0 - q^2}{M^2} \right) \right) \right] \int_0^{\Delta} \frac{dg_2(v)dv}{dv}
\]

\[
(8)
\]

\[
f_{B_s} f_{B^*} g_{B^*B_sK} = \frac{2m_b(m_b + m_s)f_K e^{m_{B_s}^2/M^2}}{m_{B_s}^2 m_{B^*}} \left\{ \left[ \Delta \varphi_K (1/2) \right] - \frac{m_b^2 + m_{B^*}^2}{M^2} \right\} \varphi_K (1/2)
\]

\[
+ e^{-\frac{m_b^2 + m_{B^*}^2}{\beta^2 M^2}} \left[ g_2(1/2) - \frac{4m_b^2}{M^2} \right] g_1(1/2) - \int_0^{1/2} g_2(v)dv \right\]

\[
+ \sum_{\alpha=1}^3 \int_0^{1/2 - \alpha_1} \frac{1}{1/2 - \alpha_1} \frac{d\alpha_1}{\alpha_1} \left[ 2\varphi_\perp (\alpha_i) + 2\varphi_\parallel (\alpha_i) - \varphi_\parallel (\alpha_i) - \varphi_\parallel (\alpha_i) \right]
\]

\[
(9)
\]

where \( \Delta = \frac{m_b^2 - q^2}{s_0 - q^2 - m_K^2} \) and \( M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \). In the derivation of (9), we have taken \( M_1^2 = M_2^2 \) due to the fact that \( B_s \) and \( B^* \) mesons are nearly degenerate in mass, which renders the continuum subtraction reduce to a simple replacement \( e^{-\frac{m_b^2 + m_{B^*}^2}{M^2}} \rightarrow e^{-\frac{m_b^2 + m_{B^*}^2}{\beta^2 M^2}} - e^{-\frac{s_0}{\beta^2 M^2}} \) for the leading twist 2 term.

Turning now to the numerical discussions on the sum rules. \( B \) channel parameters entering the sum rules are the \( b \) quark mass \( m_b \), \( B \) meson masses \( m_{B_s} \) and \( m_{B^*} \), decay constants \( f_{B_s} \) and \( f_{B^*} \), and threshold parameter \( s_0 \). We take \( m_{B_s} = 5.369 \text{ GeV} \), \( m_{B^*} = 5.325 \text{ GeV} \) and \( m_b = 4.8 \text{ GeV} \). As for the decay constants \( f_{B_s} \) and \( f_{B^*} \), we have to reanalyze them in the two-point QCD sum rule approaches \[4, 5\] with chiral current correlators, to keep a consistency with the sum rules in question. The results are found to be \( f_{B_s} = 0.142 \text{ GeV} \) and \( f_{B^*} = 0.132 \text{ GeV} \), as the threshold parameter \( s_0 = 34 \text{ GeV}^2 \) corresponding to the mean value of squared masses of the lowest \( 0^+ \) \( B_s \) and \( 1^+ \) \( B \) mesons. For the decay constant of \( K \) meson and mass of \( s \)
quark, we use $f_K = 0.16 \text{ GeV}$ and $m_s = 0.15 \text{ GeV}$. The important point is to specify the
set of the light-cone wave functions of $K$ meson. Unlike the case of $\pi$ meson, SU(3) breaking
effects need considering for the distribution amplitudes of $K$ meson. In the work, we use the
model presented in [9] for the leading twist wavefunction, which is based on an expansion over
orthogonal Gegenbauer polynomials with coefficients determined by means of QCD sum rules.
The explicit expression is

$$
\varphi_K(u) = 6u(1-u) \left\{ 1 + 1.8 \left[ (2u - 1)^2 - \frac{1}{5} \right] - 0.5(2u - 1) \left[ 1 + 1.2[(2u - 1)^2 - \frac{3}{7}] \right] \right\}, \quad (10)
$$

at the scale $\mu_b = \sqrt{m_{B_s}^2 - m_b^2}$, measuring the mean virtuality of the $b$ quark. For the twist-4
wave functions, we neglect the SU(3) breaking effects and utilize the same forms as those of $\pi$
meson investigated in [10].

Having fixed the input parameters, one must look for a reliable range of the Borel parameters
$M^2$ and $\overline{M}^2$, which can be determined by the standard procedure. The fiducial intervals are
found to be $8 \text{ GeV}^2 \leq M^2 \leq 17 \text{ GeV}^2$, depending slightly on $q^2$, for $q^2 = 0 - 17 \text{ GeV}^2$ and
$5 \text{ GeV}^2 \leq \overline{M}^2 \leq 10 \text{ GeV}^2$. In the two "windows", the twist 4 wavefunctions contribute less
than 9% and 7%, and the continuum states at the levels lower than 25% and 22%, respectively.
The sum rule results for $f_{\text{LC}}(q^2)$ show a weak dependence on $M^2$ up to $q^2 = 17 \text{ GeV}^2$, varying
between ±3%−±5% relative to their central values. For the product $f_{B_s}f_{B^*}g_{B^*B_sK}$, the resulting
sum rule is $f_{B_s}f_{B^*}g_{B^*B_sK} = 0.55 \text{ GeV}^2$, the uncertainty due to $\overline{M}^2$ being ±4%. Taking its
central value, we get $g_{B^*B_sK} = 29$. To evaluate better the $B$ pole contribution in (2), however,
we would give a direct sum rule result for $f_{B^*}g_{B^*B_sK}$, which can be obtained utilizing the
analytic form instead of the numerical result for the two-point sum rule for $f_{B_s}$ in (9). The result is
$f_{B^*}g_{B^*B_sK} = 3.57 - 4.19 \text{ GeV}$, depending on the Borel parameters. The sum rule
prediction $f_{\text{LC}}(q^2)$, together with that from the $B^*$ pole approximation, is illustrated in Fig. 1.
It is explicitly demonstrated that a perfect match between them appears at $q^2 \approx 15 - 20 \text{ GeV}^2$

The influence on the sum rules should be investigated in detail from several important
sources of uncertainty: the twist 2 distribution amplitude $\varphi_K(u)$, $b$ quark mass $m_b$, decay
constants $f_{B_s}$ and $f_{B^*}$, and threshold parameter $s_0$. Concerning the light cone wavefunction
$\varphi_K(u)$, there are some determinations other than that in (10) in the literature. To investigate
the sensitivity of the sum rules to the choice of the non-asymptotic coefficients in $\varphi_K(u)$, we
consider the two models suggested in [7] and [8], and confront the resulting sum rules with our
ones. If adopting $\varphi_K(u)$ in [7], the resulting changes amount to $-8\% - -9\%$ for the $f_{\text{LC}}(q^2)$
case and to ±5% for the $f_{B^*}g_{B^*B_sK}$ case. The almost same situation exists for that used in [8].
Therefore, the uncertainties caused by $\varphi_K(u)$ may be estimated at a considerably small level.
As for the $B$ channel parameters $m_b$, $f_{B_s}$, $f_{B^*}$ and $s_0$, considering a correlated variation in the individually allowed ranges would give sufficient information on the uncertainties induced by them. This can be done in such a way where letting $m_b$ vary from 4.7 to 4.9 GeV, we observe the behavior of $f_{LC}(q^2)$ and $f_{B^*}g_{B^*B_sK}$ by requiring that the relevant decay constants take only the best fitting values. We find that such an effect amounts to 6% and 5%, respectively. At present, the total uncertainties in $f_{LC}(q^2)$ and $f_{B^*}g_{B^*B_sK}$ can respectively be estimated to be 20% and 18%, by adding linearly up all the considered errors.

It is important and interesting to make a comparison of our sum rule results and those from the standard LCSR based on the corrector of vector and pseudoscalar currents, which are easy to obtain using the twist-3 wavefunctions suggested in , leaving the twist-4 distribution amplitudes unchanged and making a corresponding replacement of the other relevant input parameters in (79) and (44) of the second reference in [6]. We observe that the standard approach gives the same matching range as in our case and the resulting deviations from our predictions turn out to be between $-10\% - -15\%$, depending on $q^2$, in the total kinematically accessible region. This denotes that both approaches are essentially compatible with each other within the available errors.

With the yielded findings we would give a specific parametrization for $f(q^2)$ applicable to the whole kinematical region, which is helpful for the future practical application. Assuming the higher state contribution in (2) to obey $F_H(q^2) = a/(1 - bq^2/m_{B^*}^2 - cq^4/m_{B^*}^4)$, we have a pole model for $f(q^2)$,

$$f(q^2) = \frac{f_{B^*}g_{B^*B_sK}}{2m_{B^*}(1 - q^2/m_{B^*}^2)} + \frac{a}{1 - bq^2/m_{B^*}^2 - cq^4/m_{B^*}^4}. \quad (11)$$

The parameter $a$ can easily be fixed at $-0.07$, using the central values of $f_{LC}(0)$ and $f_{B^*}g_{B^*B_sK}$. In the region $q^2 = 0 - 18 GeV^2$, the best fit of (11) to $f_{LC}(q^2)$ yields $b = 1.11$, and $c = -8.33$. The resulting $q^2$ dependence of $f(q^2)$ is demonstrated in Fig.1 too, for a comparison. It turns out that the fitting results reproduce precisely the LCSR prediction up to $q^2 = 18 GeV^2$ and support considerably the single pole description of the $B_s \to K$ form factor $f(q^2)$ at large $q^2$.

Also, it is worthwhile to look roughly into SU(3) breaking effects in heavy-to-light decays by considering the ratio of the derived $B_s \to K$ form factor over the corresponding $B \to \pi$ one. The $B \to \pi$ form factor has already been obtained for small and intermediate $q^2$ in the improved LCSR approach in [6]. Using all the same method as in present case, we can understand its behavior at large $q^2$ and further get a parametrization holding for the total kinematical range. For the common kinematical region to the two processes, the resulting ratios, a comparable result $1.05 - 1.15$ with that from the standard approach, favor a small SU(3) breaking effect.

We have given a detailed discussion on the $B_s \to K$ form factor $f(q^2)$ in the whole kine-
matical region. To avoid the contamination with the twist 3 wavefunctions, in which SU(3)
breaking corrections have not been analyzed systematically, an improved LCSR approach with
some kind of chiral current correlator has been applied to estimate the form factor $f_{LC}(q^2)$ at
small and intermediate $q^2$. The nonperturbative quantity $f_{B^*}\cdot g_{B^*B_sK}$, an important input in the
$B^*$ pole model for $f(q^2)$, has also been calculated within the same framework and the sum rule
result has been adopted to study the behavior of $f(q^2)$ at large $q^2$. We find that the resulting
$f_{LC}(q^2)$ matches quite well with the estimate from the $B^*$ pole model at $q^2 = 15 - 20$ GeV$^2$.
A comparison shows that our predictions are in basic agreement with those from the standard
LCSR. Based on our findings, a pole model for $f(q^2)$ has been worked out, which is applicable
to the total kinematically accessible region. The results presented here would be used as ana-
lyzing the future data on $B_s \to K$ decays and extracting $|V_{ub}|$. A future lattice calculation
of the $B_s \to K$ form factors, which is available for large $q^2$, will provide a direct test of our
predictions. The same approach applies also to discuss other heavy-to-light processes.

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Figure 1: The $B_s \to K$ form factor $f(q^2)$ in the total kinematical range. The solid line denotes the LCSR result $f_{LC}(q^2)$, which is reliable for $0 \leq q^2 \leq 17 \text{ GeV}^2$. The dotted line expresses the $B^*$ pole prediction suitable for large $q^2$. The best fit of Eq. (11) to $f_{LC}(q^2)$ is illustrated by the dashed line. It should be understood that the plotted curves correspond to the central values of all the relevant parameters.