Critical network effect induces business oscillations in multi-level marketing systems

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Business-cycle phenomenon has long been regarded as an empirical curiosity in macroeconomics. Regarding its causes, recent evidence suggests that economic oscillations are engendered by fluctuations in the level of entrepreneurial activity. Opportunities promoting such activity are known to be embedded in social network structures. However, predominant understanding of the dynamics of economic oscillations originates from stylised pendulum models on aggregate macroeconomic variables, which overlook the role of social networks to economic activity—echoing the so-called aggregation problem of reconciling macroeconomics with microeconomics. Here I demonstrate how oscillations can arise in a networked economy epitomised by an industry known as multi-level marketing or MLM, the lifeblood of which is the profit-driven interactions among entrepreneurs. Quarterly data (over a decade) which I gathered from public MLMs reveal oscillatory time-series of entrepreneurial activity that display nontrivial scaling and persistence. I found through a stochastic population-dynamic model, which agrees with the notion of profit maximisation as the organising principle of capitalist enterprise, that oscillations exhibiting those characteristics arise at the brink of a critical balance between entrepreneurial activation and inactivation brought about by a homophily-driven network effect. Oscillations develop because of stochastic tunnelling permitted through the destabilisation by noise of an evolutionarily stable state. The results fit altogether as evidence to the Burns-Mitchell conjecture that economic oscillations must be induced by the workings of an underlying “network of free enterprises searching for profit.” I anticipate that the findings, interpreted under a mesoeconomic framework, could open a viable window for scrutinising the nature of business oscillations through the lens of the emerging field of network science. Enquiry along these lines could shed further light into the network origins of the business-cycle phenomenon.

Known widely in literature as network marketing, MLM executes through embedded social networks its essential business functions such as goods distribution, consumption, marketing, and direct selling. That makes MLM a stark microcosm of a networked economy. One salient yet so far overlooked feature of MLM dynamics is the aperiodic oscillations in firm size \( N(t) \), quantified by the number of participating entrepreneurs (Fig. 1). Empirical quarterly firm-size data have been collected from four public MLMs (Supplementary Data; Supplementary Methods, S1): NuSkin Enterprises (NUS), Nature Sunshine (NATR), USANA Health Sciences (USNA), and Mannatech Inc. (MTEX). Publicly-listed firms are chosen because they are required to disclose accurate business data on a regular basis. The average revenue (i.e., total revenue divided by \( N \) for any given quarter) does not proportionately rise with \( N \) (Fig. 1c,d), implying that firm-size expansion does not inevitably translate into revenue growth. Firm size is thus a more reliable quantifier for entrepreneurial activity than is total revenue.

The scaling property of the \( N(t) \) time-series is examined via Hurst analysis (Methods; Supplementary Methods, S1). The Hurst exponents, \( H(1) \) and \( H(2) \), quantify the scaling of the absolute increments and of the power spectrum, respectively. If time series were generated by a Wiener process, such as in the Black-Scholes model, then \( H(1) = 0.5 \). But \( H(1) > 0.5 \) indicates persistence, i.e., changes in one direction usually occur in consecutive periods; whereas \( H(1) < 0.5 \) suggests anti-persistence, i.e., changes in opposite directions usually appear in sequence. Ideally, a single scaling regime means \( H(1) = H(2) \), which applies to time-series generated by unifractal processes such as the Wiener process and the fractal Brownian motion. Table 1 presents Hurst exponents for different MLMs. Generally, \( H(1) > 0.5 \) except for NUS North Asia and NATR with \( H(1) \sim 0.5 \) (within standard deviation); and \( H(1) = H(2) \) within standard deviation. Overall, these features of the time-series suggest that MLM firm dynamics is a non-Wiener unifractal process. Unifractality implies self-similarity such that conclusions drawn at one timescale remains statistically valid at another timescale.

An MLM firm is considered as a population of profit-seeking entrepreneurs. This population exhibits disorder through the presence of two entrepreneur types distinguished by socio-economic status (SES). Let \( A \) and \( B \) denote these types, wherein \( A \) has higher SES than \( B \), and \( N_A \) and \( N_B \) denote their subpopulation size, respectively. The total population at any given time \( t \) is thus \( N(t) = N = N_A + N_B \). Three major processes run the population dynamics: entrepreneurial activation by recruitment; competitive inactivation; and catalytic inactivation due to a network effect. Recruitment is expressed in the following reaction equations:

\[
A \xrightarrow{\mu} 2A, \quad B \xrightarrow{\mu} B + A,
\]

\[
B \xrightarrow{\lambda} 2B, \quad A \xrightarrow{\lambda} A + B,
\]
TABLE I. Hurst exponents $H(1)$ and $H(2)$ for different MLMs estimated using the generalised Hurst method\textsuperscript{[10, 11]}. $H(1) > 0.5$ indicates persistence, whereas $H(1) < 0.5$ indicates anti-persistence. $H(2)$ values, which are closely related to the scaling of the power spectrum, are also shown. Generally, $H(2) = H(1)$ within standard deviation. The standard deviation values are determined from a pre-testing procedure (Supplementary Methods, S1).

| MLM                  | $H(1)$          | $H(2)$          |
|----------------------|-----------------|-----------------|
| MTEX                 | 0.8328 ± 0.1140 | 0.7425 ± 0.1081 |
| USNA United States   | 0.7602 ± 0.1140 | 0.6823 ± 0.1081 |
| USNA Canada          | 0.6619 ± 0.1140 | 0.6013 ± 0.1081 |
| USNA SEA-Pacific     | 0.6383 ± 0.1140 | 0.6139 ± 0.1081 |
| NUS Greater China    | 0.6159 ± 0.1098 | 0.5387 ± 0.1038 |
| NATR                 | 0.5128 ± 0.1032 | 0.5126 ± 0.0876 |
| NUS North Asia       | 0.4780 ± 0.1098 | 0.4789 ± 0.1038 |

Lastly, catalytic inactivation, which denotes the network effect (Methods), is expressed as:

$$Z \xrightarrow{\nu \Phi_{\text{MLM}}} \emptyset, \text{ for } Z \in \{A, B\}. \quad (3)$$

The network structure of the MLM catalyses inactivation of existing participants at the rate $\nu \Phi$, where $\Phi = \sum_{Z \in \{A, B\}} N_Z (N_Z - 1) / N(N - 1)$ is a measure of the probability that two members drawn randomly from the MLM belong to the same type. It has been widely used in literature as a diversity index\textsuperscript{[17]}. Due to interconnectedness and homophily\textsuperscript{[22]}, the inactivation of one could (like a contagion) infect another to follow suit.

Combining equations (1)–(3) results to a Master equation (Supplementary Equation 1) for the state probability density. Perturbation analysis accounts for the fluctuations arising from demographic stochasticity\textsuperscript{[23]}. In terms of the system size $\Omega$ (roughly the size of that part of the overall population considered fit for entrepreneurial activities) the following ansatz is made: $N_A = \Omega + \sqrt{\Omega} \alpha$ and $N_B = \Omega \beta + \sqrt{\Omega} \beta$, where $\alpha$ and $\beta$ are the average concentrations, and $\alpha$ and $\beta$ are the magnitude of the fluctuations of the stochastic variables $N_A$ and $N_B$, respectively (Supplementary Methods, S2). The highest order in the expansion expresses the macroscopic rate equations for $\alpha$ and $\beta$:

$$\dot{\alpha} = \mu (\alpha + \beta) - \nu \Phi (\alpha + \beta) - \delta (\alpha + \beta) \alpha;$$

$$\dot{\beta} = \lambda (\alpha + \beta) - \nu \Phi (\alpha + \beta) - \delta (\alpha + \beta) \beta. \quad (4)$$

Meanwhile, the next highest order term gives the Fokker-Planck equation, or FPE (Supplementary Equation 2), governing the dynamics of the probability density for the magnitude of the fluctuations. From the FPE, the expectation values of the stationary fluctuations are $\langle \alpha \rangle = \langle \beta \rangle = 0$, which supports the interpretation that the deterministic solutions to (4) are the correct average values.

The model is nondimensionalised by setting the characteristic timescale at $t_c = 5$ days (Methods). Consequently, the rates can be squarely related to empirical data by rescaling to appropriate units. Dimensionless rates take on simplified yet meaningful values: $\lambda = 1 - \mu$; $\mu, \nu \in (0, 1)$; and $\Delta = \frac{\mu \delta}{\Omega} \in (10^{-4}, 10^{-3})$. Bifurcation analysis (Supplementary Methods, S4) of the nondimensionalised equation (4) unveils a bifurcation manifold $\mu = \mu_0(\nu)$, where

$$\mu_0(\nu) = \frac{1}{2} + \sqrt{\frac{(1 - \nu)^2}{27 \nu}}, \quad 0 < \nu < 1. \quad (5)$$

Equation (5) coincides with an evolutionarily stable state (ESS) of a population game between the types (Supplementary Methods, S3–S4). The fraction of $A$-entrepreneurs is $x_A = \bar{x} + \Omega^{-1/2} \xi$, where $\bar{x} = \frac{\alpha}{\alpha + \beta}$.

FIG. 1. Empirical data for different publicly-listed MLM firms. $N(t)$ is the firm size, $R(t)$ is average revenue per member, and $t$ is time in quarters. a, USANA Health Sciences (USNA) in Canada, and Southeast Asia and Pacific (including Australia) from March 1998 to March 2012. b, NuSkin (NUS) in Greater China (including Hong Kong) and North Asia (Japan and South Korea) from March 1999 to March 2012. c, Nature Sunshine (NATR) worldwide showing $N$ and $R$ from September 1994 to March 2012. d, Mannatech Inc. (MTEX) worldwide showing $N$ and $R$ from April 1998 to March 2012. Data sets are provided in Supplementary Data.

wherein $\mu$ and $\lambda$ are per-capita rates of recruitment of types $A$ and $B$, respectively, and $\mu > \lambda$ because $A$’s higher SES implies a faster rate of entrepreneurial activation through the support of bigger capital and vaster social resources. Competitive inactivation occurs due to market overlap, or niche overlap\textsuperscript{[16]}, as participants can go head-to-head over the same clientele or market. An encounter rate $\delta$, which can be related to the density of the embedding social network, quantifies the probability of market overlaps. Thus, competitive inactivation is:

$$Z + A \xrightarrow{\delta} Z, \quad Z + B \xrightarrow{\delta} Z \quad \text{for } Z \in \{A, B\}. \quad (2)$$
and \( \xi \) is the fluctuation component. Analysis of the second moments from the FPE shows that the variance diverges as \( \langle \xi^2 \rangle \sim \delta |\mu - \mu_b|^{-1} \) as \( \mu \to \mu_b \) (Supplementary Methods, S5). That is a signature of criticality through which the ESS, where \( x_A = x^* \) and \( N = N^* = 1 - 2\nu \phi(x^*)/\Delta \), is (quite counterintuitively) destabilised as the bifurcation manifold is approached. This mechanism is hereby referred as stochastic tunnelling wherein noise enables the state trajectory to cross a phase barrier that could not have been otherwise traversed without actively tuning the bifurcation parameter (Supplementary Methods, S4).

Stochastic tunnelling drives the business oscillations (Fig. 2). Time series is generated by solving the model using a numerical technique, known as Gillespie’s stochastic simulation algorithm [27], which directly integrates the master equation (Supplementary Methods, S6). Diverging variance indeed allows the solution to wander far enough from the ESS and closer to an unstable point (UEP) which pushes that solution toward the boundary state, where \( x^* = 1 \) and \( N^* = \frac{\mu - \nu \phi}{\Delta} \) (Fig. 2a). That noise also enables the solution to sling back to the ESS consequently forming loops in the phase portrait, hence, oscillations in the time series (Fig. 2b). The time series consist of upswings associated with increasing diversity and downswings with decreasing diversity, i.e., \( \Phi \to 1 \) as \( x_A \to 1 \). The remarkable observation is that recovery from low points of the series coincide with periods when \( A \) is dominant—a case of the fitter entrepreneurs surviving through “recessions” [28].

Profit maximisation is an axiom of capitalist enterprise [28]. MLMs may enhance profitability by maximising the proportion of \( A - \) entrepreneurs (Supplementary Methods, S7). Thus, the time-average value \( \langle x_A \rangle_t \) is examined for various pairs of \( \mu \) and \( \nu \) which consequently depicts the phase diagram of the model (Fig. 3a). Business oscillations come about as a result of stochastic tunnelling through the critical boundary \( \mu = \mu_b \). Phase II, where \( A \) stably dominates (Supplementary Fig. S2b), can be considered Pareto-optimal as the MLM maximises profitability as a whole. But high levels of targeted recruitment, i.e., \( \mu > \frac{4}{7} \), are required. Entrepreneurial activation, however, might in reality be less discriminatory and thus \( \mu \approx \frac{1}{2} \), which denotes higher entropy (Supplementary Discussion). The critical boundary delineates, for any magnitude \( \nu \) of the network effect, the minimum \( \mu \) that promotes long-run dominance of \( A - \) entrepreneurs. Nevertheless, a stronger network effect tends to frustrate that dominance as catalytic inactivation increasingly outpaces activation, leading to degradation of entrepreneurial activity (Supplementary Fig. S2d). The ESS at the III-IV boundary (Fig. 3a) is therefore Pareto-dominated [29].

The Hurst maps (Fig. 3b, c) locate where the real MLMs are on the phase diagram. The Hurst exponents are determined from the same exact method. Clusters appear in the vicinity of the III-IV boundary. On these clusters \( H(1) \) and \( H(2) \) are approximately between 0.5 and 0.8, about the same range of values found in real MLMs (see Table 1). A correlogram plot (Fig. 3d) between \( H(1) \) and \( H(2) \) further confirms not only agreement between model and empirical data, but also their unifractality. Overall, these findings suggest that real-world MLMs are Pareto-dominated economic systems [8], which are operating in an environment characterised by high entropy, i.e., \( \mu \approx \frac{1}{2} \), and by a strong network effect (i.e., \( \nu > \mu \)).

The study paints an illuminating insight about the nature of MLM operations. MLMs have been accused in several instances by discontent participants for ethical violations concerning its business practices [30]. The model justifies such discontent for two reasons. First, that profit is closely associated with recruitment implies less selective entrepreneurial activation. Second, that recruitment proceeds through embedded networks connotes strong network effects. Less-fit entrepreneurs can join the market in droves but are weeded out too soon [28] because of the Pareto-dominated nature of the venture. The feeling of being victimised is thus not at all surprising.

The mesoeconomic framework (i.e., linking microeconomic foundations with macroeconomic phenomena [13]) puts the present study in a broader economic context. A more network-dynamic approach to viewing business cycles is hereby encouraged. Lastly, the mathematical model could be extended or refined, such as by generalising the network effect using the Hölder mean such that \( \Phi(x_A, x_B) = \sqrt[q]{x_A + x_B}, \forall q > 1 \) (Supplementary Discussion, Supplementary Fig. S1); whereas empirical data of higher temporal resolution may become available in the future, to further test the implications that came forth.

**METHODS**

**Network effect.**

The local network effect, which is a relatively new idea in economics [18–21], means that the decision of one entity can influence those by whom that entity is connected to. Particularly, the network effect manifests through inactivation as the value of exiting the enterprise is enhanced through the catalytic action of the connections between participants. Homophily [21, 22] spells that “a contact between similar people occurs at a higher rate than among dissimilar people” [23], and strongly influences contagions that diffuse through social links [24]. MLM participants
Characteristic timescale is chosen at \( t_c = 5 \) days (i.e., 1 month \( \equiv 20 \) days). Assuming that the system-size parameter is of the order, \( \Omega \sim 10^6 \) individuals, and the unit \( u \sim 10^5 \) individuals, then setting \( 10^{-4} < \Delta = \frac{u t_c \delta}{\Omega} < 10^{-3} \) implies an average per-capita encounter rate \( \delta \) between 1 and 10 per month, which is a reasonable estimate.

\[
\begin{align*}
\frac{d\hat{N}_A}{dt} &= (\mu t_c) \hat{N}_A - (\nu t_c) \Phi(\hat{N}_A, \hat{N}_B) \left( \hat{N}_A + \hat{N}_B \right) - (ut_c \delta / \Omega) \left( \hat{N}_A + \hat{N}_B \right) \hat{N}_A, \\
\frac{d\hat{N}_B}{dt} &= (\lambda t_c) \hat{N}_B - (\nu t_c) \Phi(\hat{N}_A, \hat{N}_B) \left( \hat{N}_A + \hat{N}_B \right) - (ut_c \delta / \Omega) \left( \hat{N}_A + \hat{N}_B \right) \hat{N}_B.
\end{align*}
\]

Hurst analysis.

The generalized Hurst method\cite{10, 11} has been coded by one of its authors, T. Aste. The code was downloaded from Matlab File Exchange website, http://www.mathworks.com/matlabcentral/fileexchange/30076 and was used with default settings in the calculation of \( H(1) \) and \( H(2) \).

Nondimensionalisation.

Dimensionless time is defined as \( \hat{t} = t/t_c \), wherein \( t_c = (\mu + \lambda)^{-1} \). Population numbers are in units of \( u \): \( \hat{N}_A = \hat{N}_A u \) and \( \hat{N}_B = \hat{N}_B u \). Consequently, equation \( 1 \) becomes:

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FIG. 3. Phase diagram and Hurst maps of the MLM. a, \(\langle x_A \rangle\), for different pairs of \(\mu\) and \(\nu\) (Resolution: \(\Delta \mu \times \Delta \nu = 0.01 \times 0.02\)). Phase I represents the regime \(0 \leq \nu \leq \frac{1}{4}\) where the critical manifold \(\mu = \mu_0(\nu)\) is inaccessible due to the constraint \(x_A \leq 1\). Phase II denotes the Pareto-optimal region \(\mu > \mu_0(\nu)\) and \(\mu > \nu\) where \(x_A \approx 1\). Phase III, where \(\mu < \mu_0\) is similar to phase I except that here the critical manifold is accessible. Phase IV is where \(\mu > \mu_0\) but \(\mu < \nu\) resulting to degradation of \(N\) due to \(\mu - \nu \Phi < 0\) (Supplementary Fig. 2d). Phase boundaries: I–II,III, \(\nu = \frac{1}{4}\); III–II,IV, \(\mu = \mu_0(\nu)\); and II–IV, \(\mu = \nu\). b, c, Map of the Hurst exponents \(H(1)\) and \(H(2)\). The phase boundaries are superimposed. The data are generated by simulating the model for \(t = 420\) 5–day periods \(\approx 8.75\) years with initial population \(N(0) = 100\) for different pairs of \(\mu\) and \(\nu\) (Resolution: \(\Delta \mu \times \Delta \nu = 0.01 \times 0.02\)). Each data pixel is an average of four stochastic realisations. d, Correlative plot between \(H(1)\) and \(H(2)\). Empirical data are those listed in Table 1. The dashed line \(H(1) = H(2)\) denotes unifractality of the time series.

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