A Review on Structural Behavior, Analysis and Design of RC Dapped End Beams

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Abstract. This paper represents a historical review of the theoretical and experimental studies carried out previously to study the behavior of reinforced concrete dapped end beams. Prestressing is one of the techniques that have been adopted to enhance the performance of such members. Furthermore, a dapped end needs to be strengthened due to various reasons to improve its performance. Consequently, the review has been extended to include studied such aspects. It was concluded that the ultimate capacity enhanced by reducing the shear-slenderness ratio and horizontal tension reaction and increasing the strength of concrete and fiber content. In addition, nib main reinforcement affects significantly the mode of failure, cracking and ultimate load. However, it was observed that the inclined hanger stirrups or bent performed better than that of vertical alignment and that the prestressing level and strand profile affect the shear capacity and performance significantly. On the other hand, it was found that the external bonding CFRP is the most adopted strengthening material and that the STM (Strut-and-Tie Model) approach results in better predictions of the ultimate load than the shear-friction method. Moreover, directions for future work are proposed, several conclusions have been drawn and a comprehensive list of references is provided.

Keywords: Dapped end beams; Hanger zone; Re-entrant corner; Prestressed dapped end; Strengthening

1. Introduction

Precast concrete (PC) was widely used beyond the 1950s, and its application spread dramatically around the world. PC structures have several features including rapid construction, more strength, and durability, energy-efficient, excellent appearance, noise minimizing, high durability. In general, such structures are segmented and joined using different types of linking elements. One connection detailing that is widely used in precast buildings and parking garages is the dapped end (DE). A dapped end is created when the web or stem of a beam is notched at the bottom corner; the bearing location is moved higher in the cross-section. The notch is termed as the “dap” and the portion of concrete being above the dap is called as the "nib". The dapped end allowed reducing the overall depth of a precast floor or roof structure by recessing the supporting corbel or L-edge into the supported beam. Thus, reducing substantially the
overall height of a building, providing better lateral stability of the beams and providing an adequate and economical construction system (Mattock and Chan, 1979).

The flow of stress at the dapped ends is interrupted by the sudden change in geometry, Figure 1. Thus, zones of disturbance in stress distribution are created in the vicinity of the re-entrant corner and within the nib. Such regions of discontinuity are referred to as disturbed regions (D-regions). Due to this complicated problem, traditional methods are not appropriate to design DEBs. Two methods have been used: shear-friction theory (Mattock, 1976) and STM models (ACI 318-19). The designers frequently ignore the aspects of proper details which may result in poor serviceability, severe cracking in the D-region, spalling of concrete cover, or even cause premature brittle failure. Figure 2 shows the most common failure modes of dapped ends.

2. Shear friction Approach versus Strut and Tie Models

Several approaches and techniques have been proposed to study the behavior of RC dapped end beams (DEBs). One of the earliest studies was by Mattock and Chan (1979). Eight DEs with a shear span/depth (a/d) ratio of less than 1.0 were studied. The design was based on the procedure of Mattock (1976) with considering the DE as an inverted corbel and substituting the compressive force in a column that joined with corbel by a hanger reinforcement (A_{vh}) as shown in (Fig. 3). The DE support of some specimens was subjected to a horizontal force in addition to the vertical reaction. The first version of “shear friction” design procedure for DEs was proposed which include firstly checking the ultimate shear stress in nib not to be more than:

\[ \nu_u = \text{Min}(\frac{\nu_u}{\phi b d}; 0.2 f'_c) \]  

Then, hanger (A_{vh}), reinforcement to be calculated from:

\[ A_{vh} = \frac{\nu_u}{\phi f_y} \]  

The main nib (A_s) can be obtained from the condensed form based on Mattock and Chan (1979):

\[ A_s = \text{Max}\{\text{Max}\{\frac{\nu_u}{\phi b d f_y}; 0.2 f'_c\frac{b d}{f_y}\}; \frac{N_u}{\phi f_y} \frac{2}{3} A_{vh}\} \]  

The horizontal nib (A_h) can be obtained from:

\[ A_h = 0.5(A_s - \frac{N_u}{\phi f_y}) \]  

In which Vu and Vn: ultimate and nominal shear capacities; b and d are the cross-section dimensions; fy and f': yield stress and compressive strength of steel and concrete respectively; \(\phi=0.85\) and K= 0.5, 0.25 and 0.31 for normal weight, all-lightweight and sanded lightweight concrete respectively

The main findings were:
1- The DE may be considered as an inverted corbel providing “the shear span (a) is equivalent to the distance from the dap-vertical reaction to the center of the hanger stirrups \((A_{\text{h}})\).”

2- Cracking initiated firstly at the internal corner of the nib (re-entrant corner) within the load level of 20% of the maximum capacity. However, cracks propagated at nearly 45° from the horizontal and penetrate a depth of about two-thirds the depth of the DE. However, in the final stage of loading; (70%-95% of the maximum load); the main nib reinforcement yielded accompanied with some compression spalling at the top face close to concentrated point load.

In 1983, Liem adopted Mattock and Chan approach (1979) to predict the capacity of DEBs with inclined hanger reinforcement (HR). Eight full-scale DEBs tested with inclined bars instead of the horizontal bars. Various amounts of HR and several geometries of DEs were considered. It was reported that inclined reinforcement is \((\sqrt{2})\) times more than the horizontal reinforcement in terms of the amount of steel required and that the geometry of the dapped end has no effect on the ultimate shear strength. A design procedure was proposed for DEs with \(a/d < 1\) based on equilibrium conditions and considering the inclined HR. The theoretical results yielded that the failure load of a DE with reinforcement of \((45^\circ)\) inclination is twice the strength of that with horizontal or vertical reinforcement.

In 1991, Barton et. al. verified the validity of the STM model to simulate DEBs with different reinforcement schemes. Four details of DEBs beam investigated. Two were based on STM models. Whereas, the other two DEs have detailing based on the PCI method and the Menon/Furlong (1977) design methods. The major finding reached by:

1- STM model seemed to be satisfactory to qualify the detailing of the DEB. However, all methods underestimated capacity of the tested specimens.

2- Failure of specimens was by a ductile mode (steel yielding occurred before concrete crushing). Moreover, a slightly more shear reinforcement is needed if STM model is adopted compared to the other design procedures.

Yang et. al. (2011), proposed a mechanism analysis based on upper-bound theorem (the energy principle) to estimate the critical plane of failure plane and the corresponding shear strength of RC DEBs., Figure. 4. Results were compared with the failure load for 47 specimens obtained in literature. In addition, two techniques have been utilized to analyze the DEBs, which were; PCI design method and the simplified STM model of ACI 318-05. Results revealed that:

1- The two techniques highly underestimated the shear strength of DEBs. In contrast, adequate prediction was found by the proposed mechanism analysis.

2- The PCI design method insufficiently treated effects of \(a/d\) ratio, amount of HR and value of horizontal load on the shear strength of DEBs. Whereas, such effects may be included by the mechanism analysis.

3- Both the developed mechanism analysis and STM model obviously determine the critical plane and similar results were obtained from the two methods.
3. Grade of Concrete Effect

In 2003, Lin et al. tested 24 DEB specimens with concrete strengths of (34-69) MPa. The amount of main DE steel, and (a/d) ratio were studied besides the grade of concrete. Results indicated that the shear strength of DEs enhanced with increasing the main nib reinforcement and adopting higher strengths of concrete. Moreover, it relates inversely with a/d ratio. A simplified procedure to predict the shear capacity of RC DEs was proposed and verified against the approach of the PCI Design Handbook. It was found that the proposed method could predict efficiently the shear strength of RC DEBs if compared with the shear friction method.

In 2003, Lu et al. tested twelve RC DEBs considering the same variables of Lin et.al. 2003. A new system of STM elements was proposed to predict the shear strength of the DEBs based on the relationship between vertical and horizontal shears in terms of the ratios of the vertical shear carried by the diagonal and horizontal mechanisms. Several recommendations have been addressed:

1. Small values of shear slenderness ratio, higher amounts of main nib reinforcement may result in a significant enhancement in the shear capacity of DEBs. Furthermore, some improvement in shear resistance was observed when adopting higher grades of concrete
2. The proposed procedure was more convergent than the PCI approach that underestimates the shear capacity.

In 2012, Lu et al. tested 24 RC DEs considering the grade of concrete, a/d ratio and the horizontal load value. Results revealed that the stiffness and the failure load increases with reducing shear slenderness the ratio and ratio of a horizontal load. The softened strut-and-tie model for estimating the shear capacity of DEBs with a shear slenderness ratio greater than unity was extended and calibrated with the STM model of ACI318 code and the PCI method. The result showed that the proposed model and STM model could assess the shear capacity of the RC DEBs more adequately than the PCI approach.

In 2015, Aswin, et al. considered the concrete strength as the main parameter of their study involved testing four large-scale RC-DEB specimens. Three were (NSC) of F’c=27MPa, while the forth was HSC with PVC fibers (79 MPa) at DE only. A value of (a/d) of 0.91 was used to satisfy the PCI requirements. Other variables studied were ratios of the nib and main beam reinforcement. Comparison have been made with the results obtained using three codes PCI, ACI STM model, RELIM (2000). It was reported that:

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**Figure 4** failure mechanism of dapped-end beams (Yang et al. 2011)
1- Using HSC or ECC (Engineering Cementitious concrete) with polyvinyl alcohol (PVA) fibers in the dapped area improved the capacity by (51.9%).
2- Increasing the amount of nib and main flexural reinforcements led to an enhancement in the failure load by (62.2%) and (46.7%) respectively.
3- No single code is suitable for analyzing all the beams; Analysis using PCI code was found to become accurate than ACI code and RILEM provision in predicting the failure load of RC-DEBs.

In 2019, Mohammed et al. tested experimentally twenty-seven full-scale DEs with two grades of engineered cementitious composites (ECC), namely were 85 Mpa (ECC-85) and 105 Mpa (ECC-105). The control specimen was designed according to the requirement of standard code and of normal strength concrete of 28 Mpa. In some specimens, engineered cementitious composites (ECC-85&ECC-105) have been incorporated in the DE region. Results revealed that the reinforced ECC- DEB showed better performance compared to the NSC-DEBs and higher ECC strength resulting in higher failure capacity. The main points of the study were:
1- The First crack emanated from the re-entrant corner for all DEBs. Moreover, using Diagonal Reinforcement (DR) in DE- zone improved the capacity by 40.79% and deflection of 24.29% compared to the case with vertical HR.
2- It was recommended not to use the diagonal reinforcement as a full substitution for the HR. Because the bottom corner of the full-depth beam experienced severe damage due to the loss in confinement at an un-dapped zone.
3- There was an average enhancement in the ultimate load of 59.97% and maximum deflection of 46.67% for DEB with ECC-85 relative to the RC-DEBs. Meanwhile, DEB utilized ECC-105 exhibited an average improvement on the ultimate load of 11.26% and the ultimate deflection of 11.88% relative to DEB utilized ECC-85.

4. Effect of steel fiber

In 2008, Mohamed and Elliott studied reducing the effect of disturbance of stresses within the dapped end region by incorporating (1%) hooked-end steel fibers as a partial substituent for vertical HR. Twenty self-compacting SFRC DEBs were tested experimentally. The variables that investigated were the detailing of the reinforcement, the grade of concrete as well as the depth of the extended end. A softened STM model was proposed to predict the shear capacity. The proposed model was verified against the PCI method. Several conclusions have been reported:
1. The STM concept is an efficient method to treat the disturbed regions of the DEs and the "compressive diagonal strut" carried all shear forces. In addition, the proposed method yielded results higher than the PCI procedure.
2. The mode of failure for most of the tested beams was the "shear-compression" type, which occurred along the diagonal strut and the strength of steel bars has a small influence on the shear capacity of the DEBs.
3. Using steel fibers yielded positive results, in either full addition or partial substitution for HR.

5. Effect of a/d or depth of nib end

In 2005, Wang et al. tested 24 DEB with several configurations of inclined HR. The main variables were the depth of nib end, the type and effective distance to provide the stirrups and the bent detail of the main steel. Furthermore, a Truss Mechanism for shear strength prediction has been proposed. Several conclusions were reported:
1- First cracking initiated at the re-entrant corner of the DE with the angle in range (40-60) degrees from the horizontal. With progress in loading, the diagonal crack developed towards the point of application of the concentrated load of about 30 degrees. The failure occurred whenever the diagonal cracks penetrated through the most thickness of the compression consequently a failure of the shear-compression type occurred.
2- The depths of beams and extended end play a dominant role in developing the diagonal cracks and shear strength. Thus, the depth of DE must not be less than 0.45h (where h: full depth of the beam).
3- The inclined stirrups and longitudinal bent reinforcements had a significant impact on the shear strength than the closed vertical stirrups. Also, The HR should be distributed within a distance of (half
to full depth) of the extended end, provided that the cover not less than 40 mm.

In 2013, Ahmad, et al. studied the effect of nib depth on strength of DEBs. Four DEBs were tested experimentally. Specimens were divided into two groups, one with (nib depth/beam depth) of (279 mm/457 mm) whereas the second has relative values of (178 mm/305 mm). It was concluded that the shear capacity was effected by a/d ratio rather than the overall depth of the beam. In addition, the results revealed that the STM model underestimated results for DEBs with high dap depths only, and the design using the STM model was influenced by the strut inclination. Furthermore, it was obtained that DEBs have the ability to resist loads beyond the formation of the diagonal crack due to arch action.

In 2014, Aswin, et al. proposed a simplified STM model considering the study of (Wang et al., 2005). The results were calibrated against the PCI method and STM models of (ACI-318-08, Euro Code 2 and BS 8110) codes. Grade of concrete, nib geometry, a/d ratio, types and space of distribution of the hanger stirrups, the bent shape of the longitudinal steel, amount of the reinforcements of the extended end and hanger were considered as influential parameters. It was found that Euro Code 2 and BS 8110 were less accurate in comparison with the PCI method and ACI-318-08. Moreover, it was concluded that using inclined stirrups, bent reinforcement less than 90 degrees, higher nib depth, closer and higher number of stirrups yielded higher failure loads.

In 2015, Lu, et al. tested 24-DEB with (a/d) ratio of (1.19-1.51). Concrete strengths of (32.5-70) MPa have been adopted. Parameters that studied included (a/d) ratio, grade of concrete, nib longitudinal reinforcement and hanger stirrups of the DEBs. Results revealed that the shear capacity might be improved when adopting higher strengths of concrete and smaller values of (a/d) ratio. Moreover, a model has been proposed to predict the shear capacity of the dapped ends based on linear bending theory. Such a model was calibrated against the results of 32 specimens available in literature in addition to the specimens tested throughout the study. It was concluded that the proposed model adequately estimated the shear strength of DEs. However, it was reported that the prediction of the STM of the ACI Code is noticeably conservative.

6. Internal detailing of dapped ends

In 2009, Peng studied the effect of HR detailing of the DE- region. Two DEB with four different reinforcing details which are; C-Shaped, closely distributed HR, conventional HR and Two-sided stirrups HR. Such detailing was studied in terms of the anchorage of the HR and the main steel at the nib. It was found that:
1- STM models adopted by ACI318-02 resulted in conservative results. Whereas, using the early version of PCI 1971 and PCI 1999 Design Hand Book resulted in poor detailing requirements. Most of the specimens failed in brittle failure modes without yielding of the entire HR.
2- Proper anchorage reinforcement details have a great effect on ductility and affected the shear capacity of about 44%. However, placing the HR close to the dapped ends developed higher shear capacity.

In 2016, Desnerck, et al. studied the effect of the internal defects in reinforcement detailing within the dapped zone on the capacity of the DEBs. Four specimens with different reinforcements detailing. The control specimen was designed adopting STM model; whereas the other DEBs included intentionally one defect either missing diagonal bars, missing nib steel bars or a reduced HR. Results revealed that: 1-STM models underestimates the failure load and would provide a safe design results. In all beams, cracking initiated at the re-entrant corner with a load level of about 20–33% and angle of about (40–45) degrees except the beam without a U-bar where the angle was more than 75 degrees.
2-Elimination the diagonal or U-bar nib steel bars, a mode of nib failure occurred. A reduction in the failure load has been noticed when omitting the diagonal reinforcement bars by about (39%). In contrast, the reduction of the HR resulted in a shear failure in the full-depth section of the beam with a reduction in capacity of about (10%).

In 2017, the same authors studied the influence caused by local defects in hanger and beam main reinforcement. Large-scale tests on nine DEBs were carried out with low (LSC) and normal strength concrete (NSC) considering four types of defects which were; local corrosion, anchorage cracking, small shear reinforcement ratio, and inconvenient internal reinforcement detailing. It was reported that:
1- A considerable drop in capacity of 35% caused by a local reduction of 50% of the amount of the diagonal, U and vertical steel bars at the extended end of the DEB.

2- The loss of capacity due to inappropriate anchorage of the diagonal bars, a deficiently shear reinforcement or existence of cracking in concrete around the tension steel located at the CTT zone when exist separately, did not exceed 10%. However, simultaneous occurrence of several defects led to drop in capacity in a range of (40%-60%).

In 2018, Desnerck et al. extended the previous study to determine the effects of single and combined defects when exist close to the bottom corner of the full-depth beam (CTT, compression – Tension- Tension nodal point). Twelve DEB were tested with various deficiencies. STM modelling was adopted to design the DEB and simulate deteriorations such as corrosion, insufficient anchorage lengths, and crack formation. It was addressed that:

1- Corrosion in steel can be included by reducing cross-sectional area of the reinforcing bar, and then, the capacity of the tension tie, can be included. Furthermore, insufficient anchorage lengths can be considered using some reduction factor in the tie capacity or by reducing the residual bond capacity. In addition, those STM simulations yielded conservative predictions of loss in capacity within a range of 16-57%.

2- To incorporate the effect of longitudinal cracking around steel into the capacity, it was suggested to use some reduction factor ranging (0.85 to 0.3 or in some cases 0.0) for the bond strength of the concrete.

In 2019, Falcon et al. tested experimental 28 DEBs in which 15 HR reinforcement arrangements, with and without diagonal reinforcement were considered. The ratio and the scheme of reinforcement were discussed. HR either installed as one layer or distributed within hanger zone. Based on the results of the experimental tests, a simplified approach to determine the geometry of the STM models for DEB with one layer- HR was proposed. The nodal point on top of the HR in DEB, which is called compression-compression-tension (CCT) Node, was considered in the study. The main findings that were addressed:

1– The deformation capacity of the DEB arrangements is efficient to produce the full strength of the mechanisms with orthogonal and inclined alignment.

2- The failure mode within the nodal region at the top end of the vertical tie at hanger reinforcement was of "concrete spalling" which controls the failure load of DEB, except for very few amount of reinforcement.

3– The proposed procedure, allows the capacity of the CCT node to be developed including the concrete cover. Furthermore, the HR distribution over wide distance reduces the member’s strength due to increasing (a/d) ratio

In (2020), the impact of introducing openings in DEBs without any additional reinforcement was monitored by Shakir et al. Nine specimens were investigated in the experimental study. Different shapes, sizes and locations for opening were examined. It was found that a significant improvement in failure load with range of 23 to 25% was gained when the main reinforcement of the nib (extended end) reinforcement was substituted by inclined reinforcement. In addition, the results revealed that the best location of opening was within regions of high positive bending moment. Moreover, if opening reinforcement was not provided, unexpected modes of failure may occur.

7. Prestressed dapped ends

A new approach to improve the response of the DEB was adopted by making use of prestressing concept. In which prestressing steel is extended to the extended end. Thus flexural and shear stress will be reduced, cracking initiation will be delayed corresponding to non-prestressed DEBs. In 1973, Werner and Dilger tested five posttensioned DEBs. One included no special reinforcement at DE region. Some of the other four DEBs had inclined support at one end to introduce horizontal tensile forces of 50% of the vertical reaction. Some specimens included conventional stirrups at DE-region, others included inclined bars as shown in Figure 5. The configuration and quantity of DE reinforcement, horizontal tensile force and load level to initiate cracking at the RE were discussed. The main conclusions were:

1- For some specimens, premature failures occurred at the anchorage regions of the post-tensioning bars and it was concluded that there is a need for external reinforcement in such regions. One of the beams failed by a flexure mode caused by concrete crushing with the mid-span zone.
2- It was recommended that the main steel bars at extended end to be extended a distance not less that beam depth after the re-entrant corner and to be provided with hooks to provide full bond with concrete.

In 1974, Hamondi and Phang investigated several variables that affect the response of PC DEBs through testing six prototype-flanged beams. Such variables were shape of the DEs; detailing of reinforcement in D-region; (a/d) ratio; and prestressing level. It was concluded that:
1- Reinforcing the DE zone with prestressed inclined bars can be considered as a good method to satisfy strength requirements thereby develop the required performance.
2- Prestressing may restrict development of shear cracks within a DE at working load stage.

In 1975, Hamoudi et al. tested eight PC T-beams with DEs to check the adequacy of different detailing proposals for shear reinforcement which were the form of Post-tensioned HS rods, U-shaped stirrups, and inclined bars with bent. The design was based on stress functions obtained from the analysis of the DE using the theory of elasticity. The main conclusions were:
1- Shear capacity of PC beams with DEs can be estimated with acceptable accuracy according to the elastic analysis. In addition, shear capacity for DEB enhanced with increasing prestress force and reducing shear slenderness ratio.
2- For DEs reinforced with post-tensioning rods, the load of shear crack was the maximum capacity of the beam. Whereas for DEs included mild steel, the web reinforcement carried the loads beyond shear cracking.
3- In beams with small amount of reinforcement and high prestressing load, flexural cracks initiated first and spread in a high rate causing yielding of the tension steel and overall failure. For beam with high steel area and low prestressing load, shear cracks will develop first and failure may be caused by crushing of concrete at the DE.

In 1986, Mattock and Theryo tested 12- full scale PC T-beams subjected to a combined vertical and horizontal loads with several reinforcement arrangements for the DEs. Inclined loading setup was used to introduce a horizontal force of 20% of the support reaction. Some of the finding of this study were:
1- The horizontal extensions of the inverted U-looped HR need not to be less than 1.7 times the steel development length($l_d$). Furthermore, better results were obtained, in terms of crack development control, by using inclined alignment of hanger bars in comparison with the vertical hanger stirrups.
2- Making half of the prestressing strands of the draped profile through the dapped end contributes in significant improvement in crack restraining.

In 1989, So K. M. P. tested two PC T-beams with DEs under combined vertical and horizontal loads. Each specimen represented one-half of a standard double-T-section and the four dapped ends have different DE detailing of reinforcement. Noticeable enhancement in ductility of the nib end when using two layers of wire fabric, which used as a shear reinforcement in the hanger zone. Also, it was addressed that introducing some inclination in the bottom of a DE yield more adequate than the rectangular one.
This is due to the small stress intensity compared with the rectangular nib end.

Nanni and Huang (2002), proposed a new updated alternative arrangements of HR called as" a Z-bar" that satisfied design requirements of DEs based on the PCI Design approach. Two PC double T- beams with DEs specimens were tested. Three prestressing strands located below the nib and four strands located above the nib. It was reported that:
1- The anchorage of the steel reinforcement is necessary and the bars fillet welded to a two- piece end plate appears to perform satisfactorily”.
2- The failure pattern for both beams was of the "shear-flexure" type at the full depth of the beam, due to the small value of (a/d) which was not more than 1.0.

In 2013, Brinkley examined the efficiency of various dapped-end reinforcement arrangements in four full-scale prestressed single tee beams PC T-beams. The loading set-up included the application of vertical and horizontal forces in DE, with eight HR configurations. The study was focused on the detailing of the bend of HR, the use of a heavy U- WWF (Welded Wire Fabric), extending prestressing strands through the nib end. Several conclusions have been reported:
1- The first initiation of cracking occurred at the re-entrant corner and propagated diagonally (at about 45 degrees) towards the top compression fibers of the beam. Meanwhile, the angle of cracks became less steep with the progress in loading and at failure, the angle become close to 30 degrees. Furthermore, it is found that adopting inclined bars as HR significantly restricted widening, rate of propagation of cracks, and contribute in capacity enhancement by about 30% relative to the conventional HR detailing.
2- All the tested DEBs cracked at service loading stage and failed by "diagonal-shear" within the full-depth section type that developed independently from initial cracks emanated from the re-entrant corner.
3- Passing one strand within the extended end resulted in good enhancement in cracking performance if compared with specimens with stands passed below the nib end.

In 2014, Martinez and Meli tested four-PC DEB models at a 1:3.6 scale under vertical loads. Study focused on the efficiency of adopting diagonal reinforcement in lieu vertical stirrups within the DEs with or without some prestressing in the nib, to control the diagonal crack emanating at the RE corner. The important findings were:
1- Prestressing of the dapped end is an adequate method to reduce the re-entrant cracks and can be used in practices in case where very strict measure of corrosion control is necessary.
2- The PCI method predictions were conservatives to determine the loading capacity for the critical failures modes. The STM model that proposed by Mattock has the best method to evaluate the load capacity. Moreover, failure of all the specimens was governed by the diagonal tension at the re-entrant corners.

In 2014, Al- Khazraji studied experimentally and numerically (by ANSYS 9) the response of PC DEBs under static and impact loads. Seventeen SCC (Self-Consolidating Concrete) specimens have been tested. The study was focused on several variables including; the prestress effect, strand profile, and the grade of concrete. Design of specimens was based on PCI method. Results showed an enhancement in the static failure load by (35%) and (97%) in case of straight and draped strand when using prestressing stress of (0.62*ultimate strength of prestressing steel). Furthermore, an enhancement in failure load by 15% and 10 % for the two strand profiles when the grade of concrete increased from (40 MPa) to (60 MPa) respectively was obtained and that the pre-stress reduced deflection by (23%) with negligible difference between the strand profiles.

In 2015, Botros, then in 2017, Botros et al., conducted an extensive studies considering several variable including prestressing force, grade of concrete, detailing of HR, depth of the nib end, length of horizontal extension of HR. Many full scale PC T-beams have been tested with including six different HR configurations. Four of vertical orientation, which were L-, CZ, C- and Z-shapes. The two others were inclined L-shape and Welded Wire Reinforcement (WWR). It was concluded that:
1- The first crack Initiated at the RE-entrant corner of most DEs. Cracking at service load could be restrained by passing the strand within the nib end. Moreover, It was reported that the suggested configurations can be adopted in the practice, and PCI method yielded a conservative solution for capacity by (35% to 74%). Furthermore, the inclined L configuration resulted in a good improvement
in strength and serviceability compared with other arrangements because the HR is nearly perpendicular to the direction of the diagonal crack.

2- Adopting higher concrete strengths contribute in improving the strength of the DE region but not directly proportioned with the square root of the concrete strength. Furthermore, it was concluded that increasing amount of HR increases strength and ductility after cracking; however, it does not prevent the sudden nature of failure.

8. Studies on Strengthening of RC Dapped End Beams

Two main techniques were considered by researchers for strengthening DEBs, which are: utilizing Fiber Reinforced Polymers (FRP) and external prestressing. In 2000, Huang, et al., used externally bonded (EB) FRP composites in strengthening PC double Tee DEBs with/without end anchors. Two different strengthening systems were investigated using orthogonal wrapping technique with end-anchors to insure fiber rupture in lieu delamination. It was found that:

1-Using EB FRP arrangements enhanced ductility and strength. Specimens failed by "shear-flexure" rather than "shear at the re-entrant corner". Specimens which were designed by PCI approach yielded very conservative results for the conventional DEBs.

2-The end U-anchor configuration improved the failure capacity of a DEBs, and attain fiber rupture in lieu of delamination of the FRP sheets.

In 2001, Tan investigated experimentally several strengthening techniques with different FRP composites on DEs. Upgrading was done with carbon FRP plates (CP), carbon fiber sheets (CS), or glass fiber fabrics (GS). The FRP system was fixed with several anchorage schemes on both ends. Beyond failure, the weaker side was upgraded with additional reinforcement and the beam was tested again to determine the activity of the FRP system at the other end. It was addressed that:

1-An enhancement in failure load of 43%, 75% and 80% for the CP, CS and GS schemes respectively was obtained and STM model was accurate enough to estimate the shear strength of the DEs.

2-Using an anchor bolt in the CP system yielded enhancement in capacity by16 %. Moreover, using of bonded transverse fabric resulted in further increase in strength of 33% and 41% for the CS and GS systems respectively.

In 2005, Taher studied efficiency of strengthening DEs including various defects in steel detailing of the dapped ends. 52 DEs were strengthened with different techniques including bonding of steel angle at the recess corner, unbounded inclined steel bolt anchoring in pre-drilled hole, external steel plate jacketing, exterior carbon fiber wrapping within the beam stem, exterior CFRP stripping and combination of carbon fiber wrapping and strapping. A comparison was conducted with the STM model. It was concluded that the mode of failure was mainly influenced by the introduced defect in the recess zone (extended end zone). EB FRP strengthening systems constituted a viable solution for repairing applications and to provide easy fabrication, horizontal carbon fiber wrapping with inclined CFRP stripping is recommended.

In 2006 Huang and Nanni, studied strengthening of DEBs with two techniques, internal steel reinforcement and external bonding (CFRP) laminates. Three full scale beams with dapped ends have been considered with five tests with several reinforcement arrangements including; two 1-layer CFRP strengthened DEs (one with U-anchor) and one steel reinforced DE with loading at 2.4 m from far support, and one 2-layers FRP strengthened DE and one steel reinforced one at the loading distance of 1.5 m. Two failure scenarios were observed: delamination and rupture failure of CFRP. Some of the findings of this study were:

1- Using the U-anchor systems to the external debonding (CFRP) laminates increased the ultimate capacity and ensured fiber rupture instead of debonding of the FRP sheets.

2- ACI 440.2R-02 predictions for the shear strength of FRP strengthened DEs are found to be highly conservative.

In 2012, Nagy-Gyorgy et al., discussed the adequacy of upgrading RC DEBs using EB CFRPs. Due to errors during assembly stage of several PPC (precast prestressed concrete) beams, some diagonal cracks propagated in the DEs. Thus, on-site retrofitting was done using CFRP plates to control further crack widening. In the empirical phase of the study, four similar specimens were considered: one control
specimen, two retrofitted with HS CFRP plates, and one with high-modulus CFRP sheets with layouts 45/90, 0/90 and 0/45/90 respectively. The specimens upgraded with plates had slightly higher failure load than the control specimen, but damaged by debonding, while the specimens strengthened with sheets exhibited no enhancement in capacity and failed by the rupture of fibers. Nonlinear FEA of the specimens demonstrated that debonding is more likely to occur at the inner end of DEs and the capacity have been enhanced by 20% if the plates had been fixed with end anchors.

In 2013, Dăescu extended the previous study (Nagy-Gyorgy et al) by assessing the activity of upgrading RC DEBs using CFRP sheets using non-linear finite element (FEA). The same strengthening configurations were investigated in terms of ultimate capacities and the failure modes. Same parameters have been considered. It was observed that:
1- The strengthening systems with “vertical fibers have been provided marginal increasing for some specimens of (2%)”, for others have provided increasing capacities about (6.9% to 37.7%).
2- The optimum solution for strengthening is the fabrics with fibers at horizontal and at (45°) with increment in capacity of (37.7%).

Sas et al., 2014 used nonlinear FEA modeling to investigate the most appropriate arrangement of (CFRP) composites to upgrade RC DEBs. 24 external bonding and NSM (Near Surface Mounted) reinforcement schemes have been investigated. Parameters that considered included the characteristics of the CFRP, the strengthening scheme and the inclination of the sheets with respect to the longitudinal axis. Two failure modes were observed: rupture and debonding of the CFRP. The results demonstrated that HS (High Strength) NSM FRPs can significantly improve the capacity of DEBs and yielding strains in reinforcement can be effectively reduced by using high modulus CFRPs.

In 2014, Atta and Al-Shafeiy studied experimentally the applications of (FRP) sheets, (FRP) laminates and external prestressing steels under torsional moments on behavior of the DEs. The main conclusions were: using of external prestressing increase cracking loads, failure loads and ductility compared to other techniques. Also, strengthening by CFRP laminates following the external bonding technique produced a severe drop in rotation capacity within the final stages of loading due to debonding failure between concrete surface and CFRP sheets.

In 2016, Atta and Taman tested seven RC DEB that strengthened with different shapes of external prestressing technique directions: horizontal, vertical and inclined, as shown in Figure 6. Results showed that:
1- Vertical external prestressing techniques are a very effective to increase the capacity of DEB up to (82%).
2- Strengthening of dapped-end beams using vertical external prestressing techniques is the best technique from a ductility point of view. Horizontal external prestressing techniques lead to compression failures at the compression zone. The use of two layers of external vertical prestressing strengthening techniques at the undapped zone leads to sudden brittle failure at the extended end.
3- The best technique for strengthening dapped-end beams from both the ultimate load and ductility points of view is using two external prestressing layers, one at the dapped end and the other at the undapped end with two upper separate rigid plates.

![Figure. 6 Configuration of strengthening techniques of Atta and Taman, 2016](image-url)
Shakir and Abd (2019) conducted an experimental investigation related the response of self-consolidating HSC DEBs retrofitted with CFRP fabrics. A total of 14-specimens have been tested, amongst 12-specimens included intentionally various defects in either nib or HR. Two values of (a/d) ratio namely, 1.5 and 1.0 are considered. It was concluded that using half of the design hanger reinforcements, results in a reduction of capacity by (13%) for both values of (a/d). Moreover, it was noticed that fixing the CFRP sheets with 45 degrees yielded the best performance in comparison with other arrangements.

Shakir and Abd (2020) Investigated experimental the extent of efficiency of using the NSM steel bars in strengthening the SCC RC dapped end regions under short-term static loading with (a/dd) values of 1.0 and 1.5. It was concluded that upgrading the nib region with horizontally aligned NSM steel bars resulted in increasing the load capacity by about 28.7% and 24% for a/d =1 and 1.5 respectively. Furthermore, the enhancement in failure load for strengthened hanger regions vertical and inclined (with 45 degrees) configurations for a/d =1 was 21.4% and 14.2% respectively.

It can be seen that there is no study considered repairing and rehabilitation of damaged dapped ends, behavior of dapped ends under elevated temperature, seismic, cyclic and dynamic loading. Furthermore, incorporating rolled steel sections within the nib end and to be extended to the full depth part. More investigations are needed to study the behavior of DE beams having opening that accommodate cross beam or dapped end beams.

9. Conclusions

Based on the extensive review on dapped end, the following conclusions can be drawn:

1- It was mostly reported that shear friction (SF) method yield less accurate results if compared with the STM models because does not adequately characterize the effect of (a/d) ratio, horizontal load effects, hanger reinforcement. However, STM models of Euro Code 2 and BS 8110 were less accurate in comparison with PCI method and ACI-318-08(STM models).

2- It was recommended the zone of hanger stirrups to be half to full depth (not more than nib depth), cover of the first hanger stirrup of 40 mm, Moreover, it recommended that (nib depth/ depth of beam) not to be less than 0.45.

3- Increasing amount of HR increases strength and ductility after cracking; however, it does not prevent the sudden nature of failure. Furthermore, solutions obtained by ACI318 models seemed to be conservative i.e., the reduction of capacity is less influenced by HR reduction of no more than 50%. However, significant enhancement in capacity can be obtained if the nib main reinforcement made inclined to cross the diagonal cracking from the re-entrant corner.

4- The first cracks usually initiate from the re-entrant corner at a load level of 20-33%, and propagated towards the compression face with an angle (40-60) degrees from the horizontal. Then, the failure crack follows mostly angle of 30 degrees. Thus, the inclination of the strengthening sheets to be orthogonal with the expected path of cracking.

5- The closed vertical stirrups are less effective than of inclined orientation and bent bar configuration. Using inclined hanger or steel bars result in better serviceability expressed by minimizing crack width and crack development rate. An improvement in capacity by about 30% was recorded compared with vertical hanger detail.

6- The concrete strength is not a dominant factor to control the behavior of DEBs as most of the DE zone is under tension. However, incorporating some fibers with suitable aspect ratio may improve tensile strength. but from the economical view, it is recommended to use fibers within DE only.

7- The size, shape and location of such openings affect the response of dapped ends. The optimum location of opening was within regions of high positive bending moment. However, introducing non-preplanned may result in unexpected modes of failure openings.
8-Different schemes used to strengthen the dapped end region, (L-, CZ, C- , Z-, inclined L-shapes and welded Wire Reinforcement (WWR)) However, the inclined L form yielded good performance in comparison to other forms.

9- Prestressing the DE zone with inclined bars of dropped profile may result in significant development in the performance, enhancing shear capacity and restrict propagation (and widening) of diagonal shear cracks at working load stage.

10- Using EB FRP configurations with carbon FRP plates (or sheets) or glass fiber fabrics for strengthening the DE zone improved both ductility and strength and shifted the failure to be as "shear-flexure" rather than "shear at the reentrant corner". Moreover, using the U-anchor systems with the externally bonded laminates improved the strength and resulted in fiber rupture in lieu of deboning of the laminates.

11- The optimum orientation sheet (or plates) used for strengthening the dapped end zone (DE) is 45 degrees from the horizontal.

12-The FEA method can be adopted adequately to study the effect of different variables that may influence the behavior of conventional and prestressed RC dapped ends. Also, It can be utilized to determine the optimum material, quantity and configuration of strengthening (or rehabilitation) need to be used. Some studies showed that the effects of internal defects could be studied effectively.

13- For most specimens, the predicted load carrying capacity by STM was governed by the yielding of the reinforcing bars at the range of (70-95) % for inadequate anchorage lengths, rather than exceeding the capacity of the concrete struts. This was the case even for the low concrete strength specimens. There are three failure modes in DEBs with a /d < 1: flexure failure, diagonal compression failure and tensile failure initiated by the yielding of hanger bars. The failures modes of DEBs with a /d > 1 is dominated by flexure failure

14- Some specimens exhibited ductile behavior in which the steel yielded before the concrete failed in compression. Some studies addressed four failure modes were observed which are diagonal tension cracks within the nib or the full-depth of the beam. Flexure-shear cracking in the full-depth precipitated by longitudinal splitting of the web and strand bond failure, crushing of the diagonal strut near the bottom corner of the stem.

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