Eddy current interactions in a Ferromagnet-Normal metal bilayer structure, and its impact on ferromagnetic resonance lineshapes

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We investigate the effect of eddy currents on ferromagnetic resonance (FMR) in ferromagnet-normal metal (FM/NM) bilayer structures. Eddy-current effects are usually neglected for NM layer thicknesses below the microwave (MW) skin depth ($\approx 800$ nm for Au at 10 GHz). However, we show that in much thinner NM layers (10-100 nm of Au or Cu) they induce a phase shift in the FMR excitation that results in a strong asymmetry of the measured absorption lines. In contrast to typical eddy-current effects, the asymmetry is larger for thinner NM layer and is tunable through changing the sample geometry and the NM layer thickness.

INTRODUCTION

Eddy currents are induced currents in conductors by changing magnetic fields. These currents flow in closed loops perpendicular to the driving fields, and produce additional Oersted fields that partially compensate the external driving fields. The effects of eddy currents on FMR in conducting films is well known in the limit of film thickness approaching their electro-magnetic skin depth ($\approx 800$ nm for bulk Au at 10 GHz). In those cases, eddy-current effects can lead to linewidth broadening and give rise to spin-wave excitations due to inhomogenous microwave fields [1, 2].

The microwave frequency spin dynamics in nanostructures usually involves stacks of layers combining FM and NM at the nm scale [3, 4]. Although eddy-current effects are usually neglected in metals with thicknesses below their skin depth, some studies have shown that this may be important also for normal metal (NM) films far below their skin depth [5, 10]. In these studies it was predominantly microwave-screening effects that were considered, and little attention was paid to how the induced Oersted fields can affect the magnetization dynamics in an adjacent ferromagnetic thin film.

Ferromagnetic resonance (FMR) spectroscopy experiments probe static and dynamic properties of magnetic materials. The technique relies on measuring the microwave absorption associated to the precession of the magnetization. In FMR experiments, position and width of absorption lines carry valuable information about material parameters such as anisotropy fields and magnetic damping [11]. Further, recent experiments have used differences in symmetry of the FMR lines to study the spin pumping from a magnetic material to a normal metal [12-14]. Hence, to correctly interpret experimental data involving FMR it is important to understand how eddy currents—even in very thin films—can cause modifications in the measured FMR lineshape.

In this study we investigate the contribution of eddy currents to the FMR absorption lineshapes in ferromagnet-normal metal (FM/NM) bilayer structures. We have systematically studied how the sample geometry and NM thickness affects the coupling between microwave (MW) fields and eddy-current-induced fields, and we show that this coupling is tunable through changing the sample geometry and the NM layer thickness.

THEORETICAL MODEL FOR THE OBSERVED LINESHAPES

The ferromagnetic resonance is usually driven directly by the MW field from a cavity or from a coplanar waveguide/microstrip line. However, capping a FM sample with a NM layer leads to circulating eddy currents in the NM, and additional Oersted fields in the FM. These Oersted fields have a different phase with respect to the MW fields—there is a relative phase lag between the MW fields and the Oersted fields from the induced currents—, and this results in a distortion of the FMR lineshape. A sketch of the FM/NM bilayer geometry and the path of the induced eddy currents is shown in Fig. 1. The induced currents flow in closed loops in the sample plane, with highest current density along the sample edges [15-16]. Fig. 1 and c compares two representative FMR lineshapes for a 10 nm Py sample before and after capping it with 10 nm Au; although resonance frequency and linewidth stay constant, the lineshape changes considerably.

To understand the origin of the distorted lineshapes due to the induced eddy currents, we consider a model describing the magnetization dynamics of the FM, start-
The effective magnetic field, \(H^\text{M}\), where \(H^\text{M} = \gamma M_0\), includes the external field \(H_0\), the anisotropy field, \(H^\text{A}\), composed of MW fields and fields from the eddy currents. The phase of the microwave excitation in any FMR experiment is arbitrary and can depend on many factors. However, as we are only interested in relative phase differences, we can set the reference phase of the MW field to zero. The combined driving field can thus be written in the form:

\[
\mathbf{h}_\text{ac}(t) = \mathbf{h}_\text{MW} e^{i \omega t} + \mathbf{h}_\text{ind} e^{i (\omega t + \phi)} = \mathbf{h} e^{i \omega t} (1 + \beta i). \tag{2}
\]

Where \(\phi\) is the relative phase difference between the MW field and the induced field, and the parameter \(\beta\) accounts for the relative magnitude of the two fields and their phases.

We assume the external applied field, \(H_0\), is along \(\mathbf{z}\) in the film plane (see Fig. 1) and that the perturbations of the oscillatory field, \(\mathbf{h}_\text{ac}\), are perpendicular to the external applied field (the components in the direction of the applied field do not directly perturb the dynamics of \(\mathbf{M}\)). The magnetization \(\mathbf{M}(t) = M_0 \mathbf{z} + \mathbf{m} e^{i \omega t}\), where \(\mathbf{m} \perp \mathbf{z}\). The magnetic response to small excitation fields, \(\mathbf{m} = \chi \mathbf{h}\), is determined by the dynamic susceptibility tensor \(\chi\).

The elements of the susceptibility tensor \(\chi\) was determined by solving eq. 1, and discarding higher order terms. Setting \(\mathbf{n} = \chi \mathbf{h}\) and introducing \(\omega_0 = \gamma H\) and \(\omega_M = \gamma M_0\), this gives:

\[
\chi = \begin{pmatrix} \chi_{xx} & i \chi_{xy} \\ -i \chi_{yx} & \chi_{yy} \end{pmatrix}, \tag{3}
\]

where the matrix elements are given by

\[
\chi_{xx} = \chi_{yy} = \frac{(1 + \beta i) \omega_M (\omega_0 + i \alpha \omega)}{\omega_0^2 - \omega^2 (1 + \alpha^2) + 2 i \alpha \omega \omega_0}, \tag{4}
\]

\[
\chi_{xy} = \chi_{yx} = \frac{(1 + \beta i) \omega_M}{\omega_0^2 - \omega^2 (1 + \alpha^2) + 2 i \alpha \omega \omega_0}. \tag{5}
\]

The observable quantity in our FMR experiments is the MW power absorption, which is proportional to the imaginary part of the diagonal elements, \(\Im(\chi_{xx})\). One can then separate the real and imaginary part of the diagonal elements \(\chi_{xx}\) and \(\chi_{yy}\). Assuming low damping \((\alpha^2 \approx 0)\), this gives the following:

\[
\Re(\chi_{xx}) = \frac{\omega_0 \omega_M (\omega_0^2 - \omega^2) + \beta \omega \omega_M (\omega_0^2 + \omega^2)}{(\omega_0^2 - \omega^2)^2 + (2 \alpha \omega \omega_0)^2}, \tag{6}
\]

\[
\Im(\chi_{xx}) = \frac{-\alpha \omega \omega_M (\omega_0^2 + \omega^2) + \beta \omega_0 \omega_M (\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (2 \alpha \omega \omega_0)^2}. \tag{7}
\]

As the FMR linewidth for permalloy films is small compared to the resonance frequency, one can assume that one does not need to deviate far from the resonance in order to observe the shape of the curve. That being the case, \(\omega_0^2 + \omega^2 \approx 2 \omega_0^2\), and

\[
(\omega_0^2 - \omega^2)^2 = (\omega_0 + \omega)^2 (\omega_0 - \omega)^2 \approx 4 \omega_0^4 (\omega_0 - \omega)^2. \tag{8}
\]

Hence, for narrow linewidths, eq. 7 is well approximated by:

\[
\Im(\chi_{xx}) \approx \left( \frac{\omega_0 M \Gamma}{4} \right) - 1 + \beta (\omega_0 - \omega)/\Gamma \left( \frac{\omega_0 - \omega)^2 + (\Gamma/2)^2 \right), \tag{9}
\]

where the parameter \(\Gamma = 2 \alpha \omega\) has been introduced to describe the linewidth.

In our set-up the microwave frequency is fixed at 9.4 GHz, and the magnetic field \(H_0\) is then swept to locate the Ferromagnetic resonance at the resonance field, \(H_0 = H_{R}\), satisfying the condition for the resonance frequency,
\[ \omega_R = \gamma \sqrt{H_R (H_R + 4\pi M_s)} \]

To extract the parameter \( \beta \) from our experiments we thus rewrite eq. (9), and neglect an unimportant proportionality factor:

\[ \chi \propto \frac{1 + \beta (H_0 - H_R)}{(H_0 - H_R)^2 + (\Gamma/2)^2} \]  (10)

This expression consists of two components: a symmetric Lorentzian absorption line arising from the in-phase driving fields, and an antisymmetric line proportional to \( \beta \) arising from out-of-phase driving fields. Through the \( \beta \) parameter, FMR lineshapes in an otherwise unperturbed system is thus a direct measure of the phase shift effects due to eddy currents.

**RESULTS AND DISCUSSION**

**Effect of sample geometry**

We first focus on the effect of the sample geometry. A full in-plane 360 degrees rotation of a sample of dimensions \( 1 \times L \text{ mm} \) with a thickness of 10 nm Py capped with 10 nm Au is shown in Fig. 3, where \( \theta = 0 \) corresponds to an applied field, \( H_0 \), parallel to the short side of the sample. We note that although capping the sample with a thin NM layer affects the asymmetry considerably, the resonance field \( H_R \) and linewidth \( \Gamma \) is unaffected. Thicker NM layers of materials with considerable spin orbit coupling would lead to a linewidth broadening due to loss of spin angular momentum through spin pumping effects, but for thin Cu/Au layers this effect is negligible [17, 18].

The microwave field in the cavity can be considered uniform on the length scale of the sample, and rotationally symmetric due to its cylindrical shape. If the microwave excitaton is inhomogenous when rotating a long sample, it could be possible to excite magnetostatic modes in the FM film [19]. However, if this was the case in our experiments one should observe the same asymmetry for the FM without the NM capping layer. To conclusively rule out this as a cause of the asymmetry, we performed control experiments where we re-positioned the sample with an offset from the centre of the cavity (offset of the same order as the sample dimension). This did not affect the asymmetry of the lineshape, indicating that an inhomogenous MW in the cavity field could not be the cause of the observed effect.

To investigate the effect of sample geometry further, a set of samples of dimensions \( 1 \times L \text{ mm} \), where \( L \) ranged from 0.5 to 4 mm was studied. The samples were again of 10 nm Py capped with 10 nm Cu and 5 nm Ta to prevent oxidation. We notice that the sample length in the direction parallel to the applied field is the main parameter that determines the asymmetry of the FMR lineshapes, given by the parameter \( \beta \). Fig. 3 shows the dependence of sample length when the varying dimension is parallel to the applied field. We see that the asymmetry increases with the sample's length and reaches a value where \( \beta \) appears to diverge at a length of about 3.3 mm. Samples with a length below 1 mm have lineshapes almost identical to samples with no NM capping.
We consider now the basic physics to describe the above results. The induced eddy currents flow in closed loops in planes perpendicular to the MW magnetic field, which is perpendicular to the film plane in our experiment. Thus, to obtain circulating eddy currents as shown in Fig. 1, it depends on having the MW field perpendicular to the film plane. We have conducted control experiments where the MW fields were applied in the film plane and we observed that the FMR lineshapes were always symmetric, indicating there were no observable effect of the eddy currents.

In our experimental geometry, the induced eddy currents flow mainly in circulating paths, with higher current density along the sample edges [15][16]. The induced Oe fields have a component in the film plane and another perpendicular to the film plane. As indicated in Fig. 1, for the sample edges that are parallel to the applied field, the Oe fields will have the main in-plane component perpendicular to the applied field and could thus affect the FMR of the Py film. On the other hand, currents along sample edges perpendicular to the applied field will give rise to an in-plane Oersted field that is parallel to the applied field, and should not affect the FMR response. The observed strong rotational dependence (See, Fig. 3a), suggest that the effective driving field has an important contribution in the sample plane; the contribution from the component perpendicular to the sample plane should not depend on the direction of the sample edges with respect to the applied field, \( H_0 \).

We now compare the length series with the rotational measurements by using a simple geometric approximation: we consider that the length of the sample parallel to the applied field is given by \( L(\theta) = l \sin(\theta) + w \cos(\theta) \), where \( l \) and \( w \) are the length (3 mm) and width (1 mm) of the sample, and \( \theta = 0 \) corresponds to the applied field parallel to the short side of the sample. We have plotted in Fig. 3b the rotational measurements following this approach and we can see that the resulting curve is almost identical to the length series.

To investigate the effect of sample size closer, we designed a control experiment that consisted of taking a large sample of dimensions 1×3 mm and dividing it into electrically isolated regions of 1×1 mm with an automated scriber that scratched the sample without breaking it—we limited the size of the possible current loops. This is illustrated in figure 4.

![Figure 3](image1)

**FIG. 3.** (a) Angular dependence of the \( \beta \) parameter describing the FMR lineshape for a sample of 10 nm Py capped with 10 nm Au with dimensions 1×3 mm; the applied field is rotated 360 degrees in the film plane. (b) Sample length dependence of \( \beta \) for samples of 10 nm Py capped with 10 nm Cu (and 5 nm Ta to prevent oxidation) of dimension 1×L mm, i.e each datapoint in the “Length series” corresponds to a separate sample of length L. We also plotted in (b) the rotational measurements shown in (a) for a single sample, considering that the effective length in the direction of the applied field, \( H_0 \), is approximated by \( L(\theta) = l \sin(\theta) + w \cos(\theta) \), \( l = 3 \) mm and \( w = 1 \) mm.

![Figure 4](image2)

**FIG. 4.** Scratching the sample limits the size of the possible current loops, reducing the magnitude of the induced fields.

We tested this for samples with no NM capping, and the FMR signal was not affected. However, the same procedure on a sample capped with 10 nm Au presented a remarkable effect: the lineshape before scratching the sample was strongly asymmetric, but after scratching the film it returned to being symmetric again and matched the lineshapes for a sample of dimensions 1×1 mm.

The asymptotic behavior of \( \beta \) as the sample length increases can be understood by considering how the sample size effects the magnitude of the induced currents. We estimate the current path as a rectangular loop of length \( l \), width \( w \), and cross section \( \sigma \). The induced electromagnetic force (EMF) is then given by the rate of change of magnetic flux through the area enclosed by the loop; its
absolute value is given by $|\epsilon| = lw \left| \frac{\partial h}{\partial t} \right| \propto lw \ h_{\text{MW}} 2\pi f$, where $f$ and $h_{\text{MW}}$ are the MW frequency and amplitude respectively. The resistance of such a loop is given by $R = 2\rho(l + w)/\sigma$, where $\rho$ is the resistivity of the NM and $\sigma$ is the cross section of the current path. The induced current is finally given by $I = \epsilon/R$. We consider as an approximation that the magnetic field resulting from a current $I$ in such a plane is given by $h_{\text{ind}} = \mu_0 I/2\varsigma$, where $\varsigma$ is the width of the current path and $\mu_0$ is the vacuum permeability. The expression for the induced field is thus proportional to the sample size. We calculated the induced field for a square sample, $w = l$, of 10 nm thickness. Using the resistivity data for thin metal films from [6], and in the presence of a MW field of 9.4 GHz and 6 $\mu$T, we obtained that at a dimension of about $2 \times 2$ mm the induced field equals the MW field. Our estimated value corresponds well with what we see in the experiments: when the sample size approaches the mm scale, the effects become increasingly important.

**Thickness of NM layer**

Next, we focus on another important parameter that governs the effect of eddy currents: the thickness of the NM layer. We prepared samples with a NM (both with Au and Cu) thickness ranging from 10 to 100 nm. The experiments presented here were also performed in two more samples, where we obtained the same results. The measured asymmetry parameter $\beta$ as a function of NM thickness is shown in Fig. 5 for a sample of dimensions $1 \times 3$ mm, with the applied field parallel to the long side of the sample. Replacing the Au layer by Cu shows a similar behavior; the thicker the NM, the more symmetric the FMR lineshapes. For a thickness above 50 nm, one reaches a regime where the lineshapes are almost independent of layer thickness.

To explain the thickness dependence, we use a simplified model where we assume that induced eddy currents circulate in two dimensional planes in the NM layer. The Oersted fields originated by the eddy currents have a relative phase, $\phi$, compared to the external MW field, which in the ideal case of no inductance is expected to be $\phi = \pi/2$ ($I_{\text{Eddy}} \propto \frac{\partial h}{\partial t}$). However, due to the inductance and resistance of the NM film, there will be an additional phase between the MW field and the induced field that depends strongly on the NM thickness. We can then write the relative phase $\phi$ as:

$$\phi = \frac{\pi}{2} + \tan^{-1}\left(\frac{\omega L}{R}\right), \quad (11)$$

where $\omega$ is the microwave frequency, $L$ and $R$ are the inductance and resistance of the film.

We take an approximate value for the inductance $L \approx 10^{-8}$ H [29] considering a rectangular current path along the edges of the NM layer, and sample dimensions of $w=1$ mm and $l=3$ mm. We estimate the film resistance, $R$, as a function of the thickness by introducing a correction factor, $\eta$, that describes the correction to the film conductivity compared to the bulk value for a thickness below the electron mean free path [6].

We have computed the phase shift between the MW field and the induced field accounting for the sample size and material parameters (we considered $\eta$ to be a linear function of the film thickness compared to the mean free path, $l_{\text{mfp}}$: $\eta = \min(1, \delta l_{\text{mfp}})$, where $\delta$ is the NM thickness. The inset of Fig. 5 shows the thickness dependence of the phase shift, $\phi$; for a thick NM layer the phase difference approaches an asymptotic value of $\pi$, which corresponds to $\beta = 0$ (i.e., the asymmetric lineshapes disappear quickly as the NM thickness increases). These calculations agree well with the experimental data in Fig. 5, where the asymmetry drops off quickly with the thickness.

We investigated thinner NM layers of 5 nm and the FMR lineshapes were similar to single Py films. We believe this is because we had non-continuous metal films for these thicknesses; Au films tend to be granular and the Cu films might have oxidized. We also investigated the thick film limit, and deposited NM layers of 150, 200, 500, 750 and 1000 nm Au. For a thickness above approx 500 nm, the lineshapes appeared slightly asymmetric again. However, the model we have used to in-
vestigate the effects of eddy currents assumes that the current flows in two dimensional planes in the NM and this is no longer valid for these thicknesses.

**SUMMARY**

To summarize, we have shown that induced eddy currents can play an important role in FM/NM bilayer structures for certain sample geometries. In FMR measurements, the influence on lineshape asymmetries has to be taken into account for NM layers below 50nm and sample dimensions above approx. 1mm² when the MW field has a significant component perpendicular to the film plane.

The dynamics of the system is determined by the interplay of the MW fields and induced fields by eddy currents, and we have shown that this coupling is tunable through changing the sample geometry and the NM layer thickness. The tunability of the coupling opens up possibilities to use patterned NM structures to tailor the local field geometry and phase of the induced microwave fields, which could be of importance for magnonics applications.

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