System for Uncertainty Factors Accounting When Optimizing and Choosing Effective Options for Network Work Schedules on a Dynamic Model with Dead-End Controls

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Abstract. This article proposes accounting for uncertainty factors in a dynamic optimization model. Accounting for uncertainty factors in equipment performance is represented by a decreasing piecewise-constant function, the domain of which is the expert ordinal measurement scale. The basis of the optimization of the execution time on the example of creating a helicopter technology is the dead-end control method which refers to such an admissible Boolean control in which the replacement of an arbitrary zero component by one leads to a violation of the resource constraint. To ensure stable work of a manufacturer, several options of network schedules are usually calculated based on various initial data. The final choice of the effective option is made according to a vector criterion, which can include the time of completion of a complex of works, the degree of intensity of a plant's work plans, the uniformity of the equipment load (work teams), etc.

1. Introduction

When planning the design of helicopter equipment, a plant's planning and dispatch service faces the problem of recording organizational and technical uncertainty factors, the lack of which leads to overtime and unplanned labor costs. In this regard, at the stage of design of helicopter technology, the plant needs to solve the problem of rational distribution of the complex of works for various types of equipment in general, taking into account limited resources and uncertainty factors. In this case, the production lines form a service system with group service technology [1].

Generally, the task of work planning, the sequence of which is usually represented in the form of a directed graph (network), in conditions of limited resources and uncertainty factors, belongs to the class of problems of multi-criteria selection and high-dimensional schedule theory [2]. In works [3-6], the problem of planning interrelated work presented as directed graphs on limited resources is narrowed down to the problem of optimal control of a dynamic system. Work [6] generalizes the results obtained in work [3] and some results of the optimal control theory.

For models with discrete time, the most widely used methods are dynamic programming [7], and only in the particular case for Markov dynamic problems [8] discrete optimization methods are used [8, 9].
Within the system for accounting for uncertainty factors, the process of optimizing network work schedules when creating helicopter equipment on a dynamic model with dead-end controls is reduced to the following stages:

1. The stage of preparing the initial data. At this stage, the scope of the upcoming work, the list of required equipment, the choice of manufacturing technology, material resources (technical, labor, etc.) are specified.

2. The stage of generating options for work schedules for various sets of uncertainty factors and resource data, which are considered labor costs (per person-hour), equipment (labor teams), consumables, etc. At this stage, for each option, an optimal solution is found according to the criterion of minimizing the execution time on a dynamic model with dead-peak controls.

3. The stage of multi-criteria selection of a single or several effective options for schedules and implementation of the decision made in the form of orders and tasks.

The criteria for a multi-criteria selection can be the time of a complex of works, the degree of intensity of a plant’s work plans, taking into account the expert accounting for uncertainty factors, the uniformity of the equipment load (work teams), etc.

The calculation of several options of schedules, taking into account the predicted risks associated with uncertainty factors, allows for more stable work of the manufacturer.

2. Accounting for uncertainty factors in work planning

To formalize the optimization problem and select an effective option of the network work schedule, we introduce the following notations:

- \( R = \{r_i: l = 1, n_R\} \) – a complex of partially ordered activities;
- \( w_l \) – the volume of \( l \) work, measured in the corresponding units;
- \( M = \{s_m: m = 1, n_M\} \) – a service system consisting of equipment (technical means, devices, requirements, teams);
- \( P_m \) – standard power of \( m \) equipment;
- \( \sigma = [t_0, t_f] \) – planning interval (month, quarter).

Based on the volume (labor intensity) \( w_l \) and power the execution time of the \( l \) work on the \( m \) tool can be represented as:

\[
\tau_{lm}^m = \frac{w_l}{P_m}.
\]

(1)

Let us introduce a decreasing piecewise-constant function \( n_\phi \) of uncertainty factors:

\[
\psi(\phi_1, \phi_2, ..., \phi_{n_\phi}) \in [0, 1],
\]

(2)

leading to a decrease in equipment performance, which will amount to:

\[
\hat{P}_m = \psi(\phi_1, \phi_2, ..., \phi_{n_\phi}) \cdot P_m,
\]

(3)

whence the performing time on the \( m \) tool, with accounting for uncertainty factors, will increase by the value:

\[
\Delta \tau_{lm}^m = \hat{\tau}_{lm}^m - \tau_{lm}^m,
\]

(4)

where \( \hat{\tau}_{lm}^m = \frac{w_l}{\hat{P}_m}; \tau_{lm}^m = \frac{w_l}{P_m}; \ n_\phi \) – the number of uncertainty factors.

An example of a graph of a piecewise-constant uncertainty function from five factors is shown in Fig. 1.
Let us introduce the degree of intensity of the monthly (quarterly) work plans of a plant, taking into account the number of uncertainty factors in the form

$$\eta(n_{\phi}) = \frac{Z_{n_{\phi},t} - Z_{n,t}}{Z_{n,t}} \times 100 \%, \ t \in \sigma = [t_0, t_f],$$

(5)

where $Z_{n,t}$ – is the normative workload in standard hours (labor costs), taken as 100% at a given time interval without taking into account uncertainty factors ($n_{\phi} = 0$);

$Z_{n_{\phi},t}$ – is the predicted workload, taking into account uncertainty factors ($n_{\phi} \geq 1$);

$n_{\phi}$ – the number of uncertainty factors.

Accounting for uncertainty in the network schedules for the implementation of a complex of works is based on expert data, namely: the number of uncertainty factors in the planning and implementation of projects for the manufacture of helicopter equipment; expert assessments of changes in planned labor costs; subjective probabilities of plan fulfillment.

Table 1 shows the values of the degree of intensity of the planned work in labor costs, depending on the uncertainty factors.

**Table 1.** Intensity degree of the work plan based on the uncertainty factors (fragment).

| № | Uncertainty factor                                                   | Scale (in points) | Intensity of the plan $\eta(n_{\phi})$, % | Subjective probability of execution of the plan, % |
|---|---------------------------------------------------------------------|-------------------|-------------------------------------------|-----------------------------------------------|
| 1 | No uncertainty factors                                              | 0                 | 0                                         | 100                                           |
|   | Change of priorities of execution in time or directive inclusion of new projects in the production plan | 1                 |                                           |                                               |
| 2 | Delays in delivery times. Incomplete amount of information in the TOR for work, for equipment and units Changes in harmonization of documentation. Change in actual power over time. Test failures | 10-25             | 80                                        |                                               |
|   | Delays in harmonization of documentation. Change in actual power over time. The requirement to maintain resource loading at 100% | 25-50             | 60                                        |                                               |
| 3 | Change in actual power over time. Test failures                      | 50-70             | 40                                        |                                               |
| 4 | Change in actual power over time. Test failures                      | 70-100            | 20                                        |                                               |
3. Dynamic model of work planning taking into account uncertainty factors

The optimization problem will be calculated using a dynamic interpretation of the work execution control process in the planning interval \( \sigma = \{ t_0, t_f \} \). The control process will be carried out using a Boolean variable \( u_{m_1} \in \{0, 1\} \), which we will give the following interpretation:

\[
u_{m_1}(t) = \begin{cases} 
1, & \text{if the activity } \zeta_i \text{ at time } t \in \sigma \\
0, & \text{otherwise}
\end{cases}
\]  

(6)

For the sake of simplicity, we will assume that each activity can be performed on one of the equipment from a given list. Then the change in the state of \( \zeta_1 \) on the \( m \) equipment is determined by the finite-difference equation

\[
x_{l,m}^m(t_{k+1}) = x_{l,m}^m(t_k) + \frac{\Delta t_k}{\sigma l_{m}} u_{m_1}(t_k),
\]

(7)

where \( x_{l,m}^m(t_k) \in \{0, 1\} \) – is the variable of the state of work on \( m \)-equipment, which characterizes the share of the work done by the \( m \)-equipment, \( n_{m}(t_k) \) – is the number of activities simultaneously performed on the \( m \)-equipment, that meet the specified constraints;

\[
\Delta t_k = t_{k+1} - t_k
\]

– is the value of the sampling interval of the work execution control process.

In this regard, we will characterize the boundary conditions for the task of optimizing the execution time of a complex of partially ordered works by vectors:

\[
\vec{x}(t_0) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0); \vec{x}(t_f) = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1).
\]

(8)

The order of execution of partially ordered works is represented as a constraint

\[
\sum_{m=1}^{M} \sum_{q \in R_l} u_{m_1}(t_k) \{ x_q^m(t_{q_1}) - x_q^m(t_{q_0}) \right) = 0 \quad \forall t_k \in \sigma.
\]

(9)

where \( R_l = \{ r_q \in R: r_q < r_l \} \) is a set of activities immediately preceding the activity \( r_l \).

In constraint (9) the activity \( r_l \) cannot start \( u_{m_1}(t_k) = 1 \) until the activity from \( R_l^- \) is finished, i.e. it reaches the state \( x_q^m(t_{q_1}) = 1 \).

In accordance with the content interpretation of the task, no more than one activity can be performed on \( m \) equipment at any given time

\[
\sum_{m=1}^{M} \sum_{l=1}^{n_q} u_{m_1}(t_k) \leq 1, \forall t_k \in \sigma.
\]

(10)

Generally the rate of change in the work state variable is a non-linear quantity, so the time to complete the work depends on the number of activities simultaneously performed on \( m \) equipment. The dynamics of changes in the speed of work at the time of connection or termination of parallel work is shown in Fig. 2.

![Figure 2](image-url)  

**Figure 2.** Dynamics of changes in the speed of work execution at the moment of connection or termination of the parallel activity.
The resource constraints at the a $t_k$ time point can be represented as:
\[ b_{1}^{m}u_{m1}(t_k) + \cdots + b_{l}^{m}u_{ml}(t_k) + \cdots + b_{n_{m}}^{m}u_{mn_{m}}(t_k) \leq B_{m}(t_k), \quad (11) \]
where $b_{l}^{m}$ is the volume of the allocated resource for the $l$ activity at time $t_k$ on $s_{m}$ equipment; $B_{m}(t_k)$ is the total allocated resource for the complex of works at time $t_k$ on $s_{m}$ equipment, $m \in M$.

Along with the state of work (7), we introduce the equations
\[ y_{m}(u_{ml}) = \sum_{l=1}^{n_{R}} \int_{t_{0}}^{t_{f}} u_{ml}(t) dt, \quad \forall m \in M, \quad (12) \]
that may be used to estimate the total volume of equipment activity.

The vector of control variables is represented as
\[ \vec{u}_{m}(t) = (u_{m1}(t), ..., u_{ml}(t), ..., u_{mn_{r}}(t)), \quad \forall m \in M, \quad \forall t \in \sigma. \quad (13) \]

The problem of optimization and selection of an effective option of the network schedule with varying input data can be presented as a two-criterion problem of minimizing the completion time of a complex of works
\[ F_{1} = \sum_{m=1}^{n_{M}} \sum_{l=1}^{n_{R}} t_{lf}^{m}(u_{ml}(t)) \rightarrow \min \quad \text{under } u_{ml}(t) \quad (14) \]
and uniform equipment loading
\[ F_{2} = \sum_{l=1}^{n_{M}-1} \sum_{j=l+1}^{n_{M}} |y_{l}(t_{f}) - y_{j}(t_{f})| \rightarrow \min. \quad (15) \]

Thus, the problem of optimization of network graphs taking into account uncertainty factors is narrowed down to finding an acceptable control $\vec{u}(t)$, $\forall t \in \sigma$ which provides for equations (7), (12) the fulfillment of the specified boundary conditions (8), constraints (9) – (11) and delivers a minimum of criteria (14) and (15). Finding control variables in the form of piecewise constant functions of time is equivalent to constructing a work schedule in an interval form:
\[ T(\vec{u}([t_{0}, t_{f}]) = \left\{ [t_{l}^{m}, t_{f}^{m}]: l = 1, n_{R}; m = 1, n_{M} \right\}, \quad (16) \]
where $t_{l}^{m}$ is the moment of the beginning of service $r_{l}$ to the activity on the $m$ equipment; $t_{f}^{m}$ is the moment of completion of service $r_{l}$ to the activity on the $m$ equipment.

4. Algorithm for solving the problem by the dead-end control method in the interval planning procedure

The state of the work execution program at the current time $t_k \in \sigma$ on the $m$ equipment will be characterized by a set of vectors:
\[ X_{m}(t_k) = (\vec{x}_{m}(t_k), \vec{u}_{m}(t_k)), \quad m = 1, n_{M}, \quad (17) \]
where $\vec{x}_{m}(t_k) = (x_{1}^{m}(t_k), ..., x_{l}^{m}(t_k), ..., x_{n_{R}}^{m}(t_k))$ is the vector of the state of work performed on $m \in M$ equipment;
\[ x_{l}^{m}(t_k) \in [0, 1] \] is the set of state values accepted by the work in the course of its execution.

The initial state of the program on $m \in M$ equipment corresponds to the zero vector $\vec{x}_{m}(t_{0}) = (0, 0, 0, ..., 0)$, and the final state corresponds to the vector $\vec{x}_{m}(t_{f}) = (1, ..., 1)$.

Step 1. Forming the work front
\[ R_{m}(t_k) = \{ r_{l} \in R: l = 1, n_{R_{m}} \} \quad (18) \]
on $m \in M$ equipment, that meet the constraints (logical, resource, time).

Step 2. Construction of dead-end controls. The construction of dead-end controls begins with ordering the work numbers in descending order of weights (volumes) from the front $R_{m}(t)$ (hereinafter, the equipment number index is omitted):
\[ b_{l}^{m} \geq \cdots \geq b_{l}^{m} \geq \cdots \geq b_{n_{R_{m}}}^{m}. \quad (19) \]
In the first dead-end control at the time $t \in \sigma$ we include the first activity, which corresponds to the control $u_{m1}^{(1)}(t_k) = 1$, if it does not violate the constraints (19), i.e. there is the inequality $b_{l}^{m}u_{ml}^{(1)}(t_k) \leq B_{m}(t_k)$, otherwise we assume $u_{m1}^{(1)}(t_k) = 0$.

We do the same with the second, third and $l$ activities in accordance with the formula


\[ u_{ml}^{(1)}(t_k) = \begin{cases} 1, & \text{if} \sum_{j=1}^{i} b_j^m \leq B_m^i \\ 0, & \text{if} \sum_{j=1}^{i} b_j^m > B_m^i \end{cases} \quad (20) \]

consistently for \( i = 1, 2, \ldots, n_{R_m} \).

As a result, we get the first dead-end control consisting of zeros and ones:

\[ \bar{u}_m^{(1)}(t_k) = (u_{ml}^{(1)}(t_k), \ldots, u_{m_{n_{R_m}}}^{(1)}(t_k)), \quad (21) \]

satisfying the constraint (19).

Using the first dead-end control, we will build the second one. To do this, we find the lowest digit in (21), which contains zero. In all digits to the right of it, we write zeros instead of ones. In the resulting binary number, the first unit on the right is transferred one digit to the right. If the received control is inadmissible by constraint (19), then this unit is shifted one more digit to the right until the control is admissible.

Next, in the digits to the right of this unit, we place units according to the same rule (20). As a result we get a multiple of dead-end controls

\[ U_m(t_k) = \left\{ \bar{u}_m^{(q)}(t_k) \mid q = 1, n_{U(t_k)} \right\}, \quad (22) \]

where \( \bar{u}_m^{(q)}(t_k) = (u_{ml}^{(q)}(t_k), \ldots, u_{m_{n_{R_m}}}^{(q)}(t_k)) \).

The step-by-step (local) criterion of the control work process depending on the dead-end controls \( \bar{u}_m^{(q)}(t_k) \in U_m(t_k) \), has the form

\[ L(\bar{u}_m^{(q)}(t_k)) = \sum_{k=1}^{n_1} \bar{u}_m^{(q)}(t_k) \rightarrow \min_{\bar{u}_m^{(q)}(t_k) \in U_m(t_k)}, \quad (23) \]

provided that

\[ \sum_{l=1}^{n_{R_m}} b_l^m u_{ml}^{(q)}(t_k) \leq B_m^i, u_{ml}^{(q)}(t_k) \in \{0, 1\}, l = 1, 2, \ldots, n_{R_m}. \quad (24) \]

Step 3. Calculation of the optimal dead-end control. For each dead-end control \( \bar{u}_m^{(q)}(t_k) \in U(t_k), q = 1, n_{U(t_k)} \) we find the value with the objective function. As the optimal, we take the dead-end one that provides the minimum value to the local criterion (23):

\[ \bar{u}_m^{(*)}(t_k) = \arg \min L(\bar{u}_m^{(q)}(t_k)), \forall \bar{u}_m^{(q)}(t_k) \in U(t_k). \quad (25) \]

Step 4. Calculation of variable quantization step for dead-end control. From the recurrent equation (7), the time step of quantization is found from the condition

\[ \Delta t_k = \min_{r_i \in R_m(t_k)} \left\{ \frac{\bar{u}_m^{(*)}(t_k)}{t_l^m} \times n_m(t_k) \times [1 - x_l^m(t_k)] \times u_{ml}^{(q)}(t_k) \right\}, \quad (26) \]

whence the next discrete moment in time:

\[ t_{k+1} = t_k + \Delta t_k. \quad (27) \]

Step 5. Calculation of the program state variables using the recurrent formula (7):

\[ x_l^m(t_{k+1}) = x_l^m(t_k) + \frac{\Delta t_k}{t_l^m - n_m(t_k)} u_{ml}(t_k). \quad (28) \]

Step 6. Checking the end condition for the amount of work performed. If the condition is met

\[ x_l^m(t_f) = 1, \forall l = 1, n_R. \quad (29) \]

then the process of forming the program is over and each activity is given an interval for its execution: \( T(\bar{u}_m(t_f)) \) (16).

The justification of the stated dead-end control algorithm is based on the following theorem.

Theorem 1 (on the existence of an optimal solution). Among all the dead-end controls in the set \( U_m \) (22), there is at least one dead-end control that meets the local criterion (23) when condition (24) is met.

Proof. Let \( \bar{u}_m^{(*)}(t_k) \) be the optimal dead-end control that provides the minimum of criterion (23). Let us assume the opposite that \( \bar{u}_m^{(*)}(t_k) \) is not optimal and dead-end control, i.e. \( \bar{u}_m^{(*)}(t_k) \notin U_m(t_k) \) (22). Let us expand it to the dead-end control \( \bar{u}_m^{(q)}(t_k) \in U_m(t_k) \) (22).
This means that there is an activity \( r_i \in R_m(t_k) \), which will be included for servicing and the value of the criterion \( L(\bar{u}^{(q)}_m(t_k)) \) will increase by the amount of time \( \bar{t}^m_i \) to complete this activity. Then we have inequality

\[
(\bar{u}^{(0)}_m(t_k)) > L(\bar{u}^{(r)}_m(t_k)).
\]

(30)

Therefore, the assumption of suboptimality \( \bar{u}^{(r)}_m(t_k) \) is not true, which was to be proved.

It is easy to see that the activities assigned to service in accordance with the criter-on \( L(\bar{u}^{(q)}_m(t_k)) \), are partially ordered in ascending order of the given execution time \( \bar{t}^m_i \) which in turn minimizes the global criterion \( F_1 \) (14) [2, 10].

5. Choosing work plans based on various data and uncertainty factors

Currently, making effective management decisions when choosing projects or a strategic development program is an urgent task. The problem of multi-criteria assessment and selection of the most effective variant of the work plan arises [11] due to the fact that it is necessary to consider various options for using resources (economic, labor, financial), loading equipment, accounting for uncertainty factors when planning work.

We will present many options for plans for performing complexes of works in the form of a tuple

\[
P = (T_g(R,M,\Phi), F = \{F_j:j = \overline{1,n}\}, g = \overline{1,n}),
\]

(31)

where \( T_g(R, M, \Phi) \) are options of schedules for performing partially ordered complexes of works \( R \) service system \( M \);

\( F \) – many indicators of the effectiveness of plans;

\( \Phi = \{\phi_i:i = \overline{1,n}\} \) – many factors of uncertainty.

Formally, the problem of multi-criteria selection and evaluation of plans \( T_g \in P \) is represented as

\[
F_0 = (F_1(T_g), F_2(T_g), \ldots, F_{n_p}(T_g)) \rightarrow \max (\min),
\]

(32)

where \( \gamma^{(g)}_\Sigma = F_0(T_g) \) – are generalized estimates of \( T_g \) plans.

The choice of an effective option or a set of plans \( T_g \in P \) is carried out according to the best generalized estimates, which can be presented in the form of ordering:

\[
T_g_1 \geq T_g_2 \geq \ldots \geq T_g_n \iff \gamma^{(g_1)}_\Sigma \geq \gamma^{(g_2)}_\Sigma \geq \ldots \geq \gamma^{(g_n)}_\Sigma.
\]

(33)

In order for the criteria to meet the requirement of uniformity when constructing generalized assessments of objects, that is, to have a common scale each gradation of which reflects the same level of preferences for each object (plan), it is necessary to go to the resulting canonical point or normalized scale. If the generalized (aggregated) assessments of objects are presented in the ordinal (point) canonical scale, then they are comparable in the scale of difference, and if in quantitative, then in the scale of relations [12].

6. Example of solving the problem of interval planning of a complex of works by the dead-end control method

Table 2 shows model (conditional) data for calculating intervals for 9 activities performed on the same type of equipment (the equipment index is omitted). The maximum amount of allocated resource is set to 176 units, i.e. \( B(t) = 176 \) for any \( t \in T = \{0, t_f\} \).

The boundary conditions for this problem are written as:

\[
\bar{x}(0) = (0,0,0,0,0,0,0,0,0) ; \bar{x}(t_f) = (1,1,1,1,1,1,1,1,1).
\]

(34)

| №  | Technological work \( r_t \) | Labor capacity at maximum allocated equipment resource \( b_t \) | Execution time |
|----|-----------------------------|---------------------------------|----------------|
|    |                             |                                 |                |

Table 2. Initial data \( \text{P}=8 \) [item/time unit] – productivity.
For clarity, we present logical (technological) constraints in the form of a graph, in which the vertices are the activities, and the edges characterize the relations of the previous execution between them (Fig. 3).

\[ \sum_{t=0}^{10} b_i u_i(t) \leq 176, \forall t \in T = [0, t_f). \] (35)

Having solved the optimization problem of performing a complex of works on a discrete model with dead-end controls, we obtain intermediate results of calculations in Table 3.

Step-by-step calculations of work execution intervals using the dead-end control algorithm are presented in the Annex.

### Table 3. The results of the calculations.

| \( n_e \) | \( t_k \) | Optimal dead-end control \( u(t_k) \) | Operation status vector \( \hat{x}(t_j) \) |
|---|---|---|---|
| 4 | 12 | \( \bar{u}(12) = (0, 0, 0, 0, 1, 1, 0, 0, 0) \) | \( \hat{x}(12) = (1, 1, 1, 0, \frac{3}{4}, 1, 0, 0, 0) \) |
| \( n_e \) | \( t_k \) | \( u(0) = (1, 0, 0, 0, 0, 0, 0, 0) \) | \( \hat{x}(0) = (0, 0, 0, 0, 0, 0, 0, 0) \) |
| 2 | 2 | \( \bar{u}(0) = (1, 0, 0, 0, 0, 0, 0, 0, 0) \) | \( \hat{x}(0) = (0, 0, 0, 0, 0, 0, 0, 0) \) |
| 3 | 6 | \( \bar{u}(6) = (0, 0, 0, 0, 1, 1, 0, 0, 0) \) | \( \hat{x}(6) = (1, 1, 1, 0, 0, 0, 0, 0, 0) \) |
The time intervals of work execution are presented in Table 4.

| № | Work name                      | Start of work | End of work | Labor intensity |
|---|-------------------------------|---------------|-------------|---------------|
| 1 | Manufacturing F1              | 0             | 2           | 8             |
| 2 | Manufacturing F                | 0             | 6           | 24            |
| 3 | Control systems for general    | 2             | 6           | 12            |
|    | helicopter equipment -B1-X    |               |             |               |
| 4 | Complex of on-board           | 12            | 14          | 40            |
|    | equipment -17-X               |               |             |               |
| 5 | Service station manufacturing  | 6             | 14          | 32            |
| 6 | DTM F-2                       | 6             | 12          | 24            |
| 7 | DTM F-3                       | 22            | 26          | 16            |
| 8 | Fairing                       | 14            | 26          | 48            |
| 9 | Docking F-1, F-2              | 26            | 28          | 16            |

### 7. Conclusion

This approach allows for accounting for uncertainty factors that are taken into account in reducing the capacity (productivity) of equipment and labor resources. In addition, the dynamic model presented in this article allows solving the problem of reducing the dimension of the task of calculating work schedules. At the same time, the duration of the calculated work intervals assumes a margin of time to smooth arising emergency situations.

Thus, the solution of the task of planning complexes of works is effective only within the framework of a dynamic optimization model, taking into account uncertainty factors.

As a planning recommendation, the initial plan, taken as 100%, should include additional work that may appear due to uncertainty factors.

#### Annex. Calculation of work execution intervals by the dead-end control algorithm

**Iteration 1.**

At the first step, for \( t_0 = 0 \) we have the work front \( R(t_0) = \{r_1, r_2\} \), that satisfies the logical constraints with which we identify the work state variables:

\[
\bar{x}(t_0) = (x_1(0), x_2(0)) = (0, 0). \tag{36}
\]

At steps 2 and 3, we have control: \( \bar{u}(t_0) = (u_1(0), u_2(0)) = (1, 1) \), that satisfies the resource constraint:

\[
48u_1(0) + 128u_2(0) = 176 \leq B(t_0). \tag{37}
\]

At step 4, we calculate the variable quantization step for \( n(t_0) = 2 \) activities:

\[
\Delta t_0 = \min_{i \in \{1, 2\}} \{ t \in n(t_0) \} \{ 1 - x_i(0) \} = \min_{i \in \{1, 2\}} \{ 1 \cdot 2 \cdot [1 - 0] \} = \min_{i \in \{1, 2\}} \{ 2 \} = 2, \tag{38}
\]

then the discrete moment in time: \( t_1 = t_0 + \Delta t_0 = 2 \). Next, at step 5, we calculate the values of the program state variables using the formula:

\[
x_i(t_{k+1}) = x_i(t_k) + \frac{\Delta t_k}{t_k n(t_k)} \cdot u_i(t_k). \tag{39}
\]
We have
\[ \tilde{x}(t_1) = (x_1(2), x_2(2)) = \left( 1, \frac{1}{3} \right), \] (40)
where \( x_1(2) = 0 + \frac{2}{1 - 2} \cdot 1 = 1; \) \( x_2(2) = 0 + \frac{2}{2 - 3} \cdot 1 = \frac{1}{3}. \)
Since the stopping criterion is not met, we proceed to step 1.

Iteration 2.

At the first step, for \( t_1 = 2 \) we have the work front \( R(t_1) = \{ r_2, r_3, r_5, r_6 \} \) that satisfy the logical constraints with which we identify the work state variables
\[ \tilde{x}(t_1) = (x_1(2), x_3(2), x_5(2), x_6(2)) = \left( \frac{1}{3}, 0, 0, 0 \right). \] (41)

At step 2, the construction of dead-end directions begins by ordering the work numbers in descending order of weights
\[
\begin{align*}
\beta_2 & \geq \beta_3 \geq \beta_5 \geq \beta_6 \iff 98 > 64 > 60 > 56, \quad (42) \\
\end{align*}
\]
as a result, we get six dead-end controls for activity \( r_2, r_3, r_5, r_6: \)
\[
\begin{align*}
\tilde{u}_1(2) &= (1, 1, 0, 0); \quad \tilde{u}_2(2) = (1, 0, 1, 0); \quad \tilde{u}_3(2) = (0, 1, 0, 1); \\
\tilde{u}_4(2) &= (0, 1, 0, 1); \quad \tilde{u}_5(2) = (0, 0, 1, 0); \quad \tilde{u}_6(2) = (0, 0, 1, 1). \\
\end{align*}
\]
At step 3, we take as optimal the dead-end \( \tilde{u}_1(2) = (1, 1, 0, 0), \) providing the minimum value for the step-by-step criterion \( L(\tilde{u}_i(t_k)) \) (see Table 5).

**Table 5. The results of the calculations**

| \( \bar{u}_i(t_1) \) | \( \sum_{i=1}^{n_{\bar{u}_i}} b_i u_i^{(q)}(t_k) \leq B \) | \( L(\tilde{u}_i(t_k)) \) |
|-----------------------|-----------------------------------------------|------------------|
| \( \bar{u}_1(2) \)   | 166                                           | 5                |
| \( \bar{u}_2(2) \)   | 158                                           | 7                |
| \( \bar{u}_3(2) \)   | 154                                           | 6                |
| \( \bar{u}_4(2) \)   | 128                                           | 6                |
| \( \bar{u}_5(2) \)   | 124                                           | 5                |
| \( \bar{u}_6(2) \)   | 116                                           | 7                |

At step 4, we calculate the variable quantization step for \( n(t_1) = 2 \) activities \( r_2, r_3, \):
\[
\Delta t_1 = \min_{i \in \{2, 3\}} \left\{ \frac{\tilde{t}_i \cdot n(t_2) \cdot (1 - x_i(2))}{2 \cdot 2 \cdot [1 - 0]} \right\} = 4, \quad (43)
\]
then the discrete moment of time: \( t_2 = t_1 + \Delta t_1 = 6. \) Next, at step 5, we calculate the values of the program state variables using the formula (40).

We have
\[
\tilde{x}(t_2) = (x_2(6), x_3(6), x_5(6), x_6(6)) = (1, 1, 0, 0), \quad (44)
\]
where \( x_2(6) = \frac{1}{3} + \frac{4}{2 - 2} \cdot 1 = 1; \) \( x_3(6) = 0 + \frac{4}{2 - 2} \cdot 1 = 1. \)
Since the stopping criterion is not met, we proceed to step 1.

Iteration 3.

In the first step for \( t_2 = 6 \) we have the work front \( R(t_2) = \{ r_4, r_5, r_6 \}, \) that satisfy the logical constraints with which we identify the state variables of the activities
\[
\tilde{x}(t_2) = (x_4(6), x_5(6), x_6(6)) = (0, 0, 0). \quad (45)
\]

At step 2, the construction of dead-end controls begins with ordering the work numbers in descending order of weights
\[
\begin{align*}
\beta_4 & \geq \beta_5 \geq \beta_6 \iff 64 > 60 > 56, \quad (46) \\
\end{align*}
\]
as a result, we get three dead-end controls for activities \( r_4, r_5 \) and \( r_6: \)
\[
\begin{align*}
\tilde{u}_4(4) &= (1, 1, 0); \quad \tilde{u}_2(4) = (1, 0, 1); \quad \tilde{u}_3(4) = (0, 1, 1). \quad (47)
\end{align*}
\]
At step 3, we take as optimal the dead-end $\bar{u}_3(4) = (1,0,1)$, providing the minimum value for the step-by-step criterion $L(\bar{u}_l(t_k))$ (see table 6).

| №  | Dead-end controls | $\sum_{i=1}^{n_k} b_i u_l^{(q)}(t_k) \leq B,$ | $L(\bar{u}_l(t_k)) = \sum_{k=1}^{n} \tilde{t}_k u_l^{(q)}(t_k)$ |
|-----|-------------------|---------------------------------------------|--------------------------------------------------|
| 1   | $\bar{u}_1(4) = (1,0,0)$ | 124                                         | 9                                               |
| 2   | $\bar{u}_2(4) = (1,0,1)$ | 120                                         | 8                                               |
| 3   | $\bar{u}_3(4) = (0,1,1)$ | 116                                         | 7                                               |

At step 4, we calculate the variable quantization step for $n(t_2) = 2$ activities $r_5, r_6$:

$$\Delta t_2 = \min_{l \in \{5,6\}} \{ \tilde{t}_l n(t_2) [1 - x_l(6)] \} = \min_{l \in \{5,6\}} \{ 4 \cdot 2 \cdot [1 - 0] \} = 6, \quad (48)$$

then the discrete moment in time: $t_3 = t_2 + \Delta t_2 = 12$. Next, at step 5, we calculate the values of the program state variables using the formula (40).

We have

$$\bar{x}(t_3) = (x_4(12), x_5(12), x_6(12)) = (0, 0, 0) = \left(0, \frac{3}{4}, 1\right), \quad (49)$$

where $x_5(12) = 0 + \frac{6}{4} \cdot 1 = \frac{3}{4}$; $x_6(12) = 0 + \frac{6}{3} \cdot 1 = 1$.

Since the stopping criterion is not met, we proceed to step 1.

Iteration 4.

At the first step for $t_3 = 12$ have the work front $R(t_3) = \{ r_4, r_5 \}$, that satisfy the logical constraints with which we identify the work state variables

$$\bar{x}(t_3) = (x_4(12), x_5(12)) = \left(0, \frac{3}{4}\right). \quad (50)$$

At step 2 and 3, we have the control: $\bar{u}(t_3) = (u_4(12), u_5(12)) = (1,1)$, satisfying the resource constraint:

$$64 u_4(12) + 60 u_5(12) = 124 \leq B(t_3). \quad (51)$$

At step 4, we calculate the variable quantization step for $n(t_3) = 2$ activities $r_5, r_6$:

$$\Delta t_3 = \min_{l \in \{4,5\}} \{ \tilde{t}_l n(t_3) [1 - x_l(12)] \} = \min_{l \in \{4,5\}} \{ 5 \cdot 2 \cdot [1 - 1] \} = 2, \quad (52)$$

then the discrete moment of time: $t_4 = t_3 + \Delta t_3 = 14$. Next, at step 5, we calculate the values of the program state variables using the formula (40).

We have

$$\bar{x}(t_4) = (x_4(14), x_5(14)) = \left(\frac{1}{3}, 1\right), \quad (53)$$

where $x_4(14) = 0 + \frac{2}{3} \cdot 1 = \frac{2}{3}$; $x_5(12) = \frac{3}{4} + \frac{2}{4} \cdot 1 = 1$.

Since the stopping criterion is not met, we proceed to step 1.

Iteration 5.

At the first step for $t_4 = 14$ have the work front $R(t_4) = \{ r_4, r_5 \}$, that satisfy the logical constraints with which we identify the work state variables

$$\bar{x}(t_4) = (x_4(14), x_5(14)) = \left(\frac{1}{3}, 0\right). \quad (54)$$

At step 2 and 3, we have the control: $\bar{u}(t_4) = (u_4(14), u_5(14)) = (1,1)$, satisfying the resource constraint:

$$64 u_4(14) + 60 u_5(14) = 124 \leq B(t_4). \quad (55)$$

At step 4, we calculate the variable quantization step for $n(t_4) = 2$ activities $r_5, r_6$:
\[ \Delta t_4 = \min_{i \in \{4,6\}} \{ n(t_i) \hat{r}_i [1-x_i(22)] \} = \min_{i \in \{4,6\}} \left\{ \frac{5 \cdot 2 \cdot \left\lfloor \frac{1}{5} \right\rfloor}{6 \cdot 2 \cdot \left\lfloor \frac{1}{3} \right\rfloor} \right\} = 8, \] (56)

then the discrete moment of time: \( t_5 = t_4 + \Delta t_4 = 22. \) Next, at step 5, we calculate the values of the program state variables using the formula (40).

We have
\[ \vec{x}(t_5) = (x_7(22), x_8(22)) = \left( \frac{1}{5}, 0 \right), \] (57)

where
\[ x_4(22) = \frac{1}{5} + \frac{8}{5 \cdot 2} = \frac{1}{5}; \quad x_8(22) = 0 + \frac{8}{6 \cdot 2} = \frac{2}{3}. \]

Since the stopping criterion is not met, we proceed to step 1.

Iteration 6.

At the first step for \( t_5 = 22 \) have the work front \( R(t_5) = \{ r_7, r_8 \} \), that satisfy the logical constraints with which we identify the work state variables
\[ \vec{x}(t_5) = (x_7(22), x_8(22)) = \left( \frac{0}{5}, \frac{2}{3} \right). \] (58)

At step 2 and 3, we have the control: \( \vec{u}(t_5) = (u_7(22), u_8(22)) = (1,1) \), satisfying the resource constraint:
\[ 84u_7(22) + 60u_8(22) = 144 \leq B(t_5). \] (59)

At step 4, we calculate the variable quantization step for \( n(t_5) = 2 \) activities \( r_5, r_6 \):
\[ \Delta t_5 = \min_{i \in \{7,8\}} \{ \hat{r}_i n(t_5) [1-x_i(22)] \} = \min_{i \in \{7,8\}} \left\{ \frac{2 \cdot 2 \cdot \left\lfloor 1 - 0 \right\rfloor}{6 \cdot 2 \cdot \left\lfloor 1 - \frac{2}{3} \right\rfloor} \right\} = 4, \] (60)

then the discrete moment of time: \( t_6 = t_5 + \Delta t_5 = 26. \) Next, at step 5, we calculate the values of the program state variables using the formula (40).

We have
\[ \vec{x}(t_6) = (x_7(26), x_8(26)) = \left( \frac{0}{5}, \frac{2}{3} \right), \] (61)

where
\[ x_7(26) = 0 + \frac{4}{2 \cdot 2} \cdot 1 = 1; \quad x_8(26) = \frac{2}{3} + \frac{4}{6 \cdot 2} \cdot 1 = 1. \]

Since the stopping criterion is not met, we proceed to step 1.

Iteration 7.

At the first step for \( t_6 = 26 \) we have one work \( R(t_6) = \{ r_9 \} \), that satisfy the logical constraints with which we identify the work state variables \( x_9(26) = 0 \).
\[ \vec{x}(t_5) = (x_7(22), x_8(22)) = \left( \frac{0}{5}, \frac{2}{3} \right). \] (62)

At step 2 and 3, we have the control: \( u_9(26) = 1 \), satisfying the constraint:
\[ 32u_9(26) = 32 \leq B(t_6). \] (63)

At step 4, we calculate the variable quantization step for \( n(t_6) = 1 \) works, which will be the value \( \Delta t_6 = 2 \), then the discrete moment of time:
\[ t_7 = t_6 + \Delta t_6 = 28. \] (64)

Then we have \( x_9(28) = 0 + \frac{2}{2 \cdot 1} \cdot 1 = 1. \)

Since the boundary condition is met
\[ \vec{x}(t_f) = (1,1,1,1,1,1,1,1,1,1), \] (65)

for \( t_f = 28. \)

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