A Dynamical $\eta'$–Mass from an Infrared Enhanced Gluon Exchange

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Abstract. The pseudo–scalar flavor–singlet meson mixes with two gluons. A dimensional argument by Kogut and Susskind shows that this can screen the Goldstone pole of the chiral limit in this channel, if the gluon correlations are infrared enhanced. Using a gluon propagator as singular as $\sigma/k^4$ for $k^2 \to 0$ we relate the screening mass to the string tension $\sigma$. In the Witten–Veneziano action to describe the $\eta$–$\eta'$ mixing this relation yields masses of about 810MeV for the $\eta'$, 430MeV for the $\eta$ and a mixing angle of about $-30^\circ$ from the phenomenological value $\sigma \approx 0.18\text{GeV}^2$. The very weak temperature dependence of the string tension should make this mechanism experimentally distinguishable from exponentially temperature dependent instanton model predictions.

More than twenty years ago Kogut and Susskind pointed out that for dimensional reasons a non–vanishing contribution to the mass of the pseudo–scalar flavor–singlet meson in the chiral limit can result from its mixing with two non–perturbatively infrared enhanced gluons corresponding to a momentum space propagator $D(k) \sim \sigma/k^4$ for $k^2 \to 0$ [1]. Such infrared enhanced gluon correlations are known to lead to an area law in analogy to the Schwinger model in two dimensions. The identification of the string tension $\sigma$ shows that effects due to infrared enhanced gluons can be expected to be complementary to instanton models. In particular, a description of the $\eta$–$\eta'$ mixing driven by the string tension [2], provides an interesting alternative to the standard solution of the $U_A(1)$ problem by instantons.

Phenomenologically, this mixing is described by the $\eta_8$–$\eta_0$ mass matrix [3],

$$
\frac{1}{2} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} \begin{pmatrix} \frac{4}{3}m_K^2 - \frac{1}{3}m_{\pi}^2 & \frac{2}{3}\sqrt{2}(m_{\pi}^2 - m_K^2) \\ \frac{2}{3}\sqrt{2}(m_{\pi}^2 - m_K^2) & \frac{2}{3}m_K^2 + \frac{1}{3}m_{\pi}^2 + \frac{2N_f}{f_0^2}\chi^2 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}
$$

(1)

where the screening mass in the flavor–singlet component, $m_0^2 := 2N_f \chi^2/f_0^2$, is given by a non–vanishing topological susceptibility,
\[ \chi^2 := \frac{g^2}{(32\pi^2)^2} \int d^4 x \, (\bar{G}G(x) \bar{G}G(0)) \quad \text{with} \quad (2) \]

\[ \bar{G}G = e^{\mu\nu\rho\sigma} \partial_\mu \text{tr}(A_\nu \partial_\rho A_\sigma - ig\frac{2}{3} A_\nu A_\rho A_\sigma) . \]

In the Instanton Liquid Model the topological susceptibility, given by the density of instantons, is \( \chi^2 \approx 1 \text{fm}^{-4} \), and the mass eigenvalues are \( m_\eta \approx 530 \text{MeV} \), \( m_{\eta'} \approx 1170 \text{MeV} \) together with a mixing angle of \( \theta \approx -11.5^\circ [4] \).

Here, we concentrate on the mixing of the flavor–singlet pseudo–scalar with two uncorrelated gluons. According to the Kogut–Susskind argument, for infrared enhanced gluons \( \sim \sigma/k^4 \), the corresponding diagram, see fig. 1, can contribute to the topological susceptibility for the meson momentum \( P \to 0 \). To explore this conjecture and its quantitative consequences, we use the following model interaction for quarks in the Landau gauge,

\[ g^2 D_{\mu\nu}(k) = P_{\mu\nu}(k) \left( \frac{8\pi \sigma}{k^4} + \frac{16\pi^2/9}{k^2 \ln(e + k^2/\Lambda^2)} \right) . \quad (3) \]

The second term, subdominant in the infrared, was added to simulate the effect of the leading logarithmic contribution of perturbative QCD for \( N_f = 3 \). Strictly speaking, a quark interaction of the form (3) cannot arise from gluons alone in Landau gauge, since the product \( g^2 D_{\mu\nu} \) is not renormalization group invariant for any finite number of flavors or colors. Even though this is assumed in the Abelian approximation, ghost contributions do implicitly enter in the RG invariant interaction (by the dressing of the quark–gluon vertex function). In fact, three quite different approaches to the pure gauge theory are available at present to suggest that the strong infrared enhancement of the interaction might be generated by ghost contributions in Landau gauge [5].

From the axial anomaly, the quark triangle \( \Gamma^{ab}_{\mu\nu} \) in fig. 1 has the limit, \( P \to 0, k^2 = 0 \) :

\[ \Gamma^{ab}_{\mu\nu} \to \delta^{ab} \epsilon_{\mu\nu\rho\sigma} k^\rho P^\sigma \sqrt{N_f f_0^{-1} g^2/(8\pi^2)} \]
This model independent form, determining the coupling of two gluons to the pseudo–scalar flavor–singlet bound state in the infrared, is particularly suited for the present calculation, since the contribution to $\chi^2$ is obtained from $P \to 0$, and since the gluon interaction (3) weights the integrand so strongly in the infrared ($\sim \sigma/(k \pm P/2)^4$). With this, all contributions containing ultraviolet dominant terms of the interaction (3) vanish for $P \to 0$, and we obtain [2],

$$m_0^2 = \lim_{P^2 \to 0} \Pi(P^2) = \frac{2N_f}{f_0^2} \chi^2 = \frac{3N_f}{f_0^2} \frac{\sigma}{\pi^2}. \quad (4)$$

The phenomenological string tension $\sigma = 0.18\text{GeV}^2$ and $f_0 \approx f_\pi = 93\text{MeV}$ thus yield $m_0^2 \approx 0.346\text{GeV}^2$, and the physical mass eigenstates are, $m_{\eta'} \approx 810\text{MeV}$ and $m_\eta \approx 430\text{MeV}$, with a corresponding mixing angle $\theta \approx -30^\circ$.

Using free constituent quarks of a mass of about $300\text{MeV}$ in the triangle to suppress spurious ultraviolet contributions, from $f_0^2 \approx f_\pi^2 (1 + \Pi'(P^2)|_{P^2 \to 0})$ with $\Lambda \approx 500\text{MeV}$ in (3), we obtain an additional contribution to the decay constant of the flavor–singlet of about 30% as compared to the pion [2].

As these values are reasonably close to experiment, we conclude that the $U_A(1)$–anomaly might be encoded in the infrared behavior of QCD Green’s functions. Whether the Kogut–Susskind mechanism or the instanton based solution to the $U_A(1)$ problem is realized in nature, can be assessed from their respective temperature dependences. If the origin of the $\eta'$ mass is predominantly due to instantons, the $\eta - \eta'$ mixing angle is expected to vary exponentially with temperature, leading to a significant change of $\eta$ and $\eta'$ production rates in relativistic heavy ion collisions [6]. On the other hand, lattice calculations indicate that the string tension is almost temperature independent up to the deconfinement transition. This offers the possibility to study the physics of the $U_A(1)$ anomaly experimentally.

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