Phantom and inflation scenarios from a 5D vacuum through form-invariance transformations of the Einstein equations

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Abstract

We study phantom and inflationary cosmologies using form-invariance transformations of the Einstein equations with respect to ρ, H, a and p, from a 5D vacuum. Equations of state and squared fluctuations of the inflaton and phantom fields are examined.

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I. INTRODUCTION

Currently the universe undergoing to a period of accelerating expansion. The implications for cosmology should be that the cosmological fluid is dominated by some sort of fantastic energy density, which has negative pressure and plays an important role today. Experimental evidence suggests that the present values of the dark energy and matter components, in terms of the critical density, are approximately $\Omega_\chi \simeq 0.7$ and $\Omega_M \simeq 0.3$[1]. The most conservative assumption is that $\Omega_\chi$ corresponds to a cosmological parameter which is constant and the equation of state is given by a constant $\omega_\chi = p/\rho = -1$, describing a vacuum dominated universe with pressure $p$ and energy density $\rho$. Such exotic fluids may be framed in theories with matter fields that violate the weak energy condition[2]. These models were called phantom cosmologies, and their study represents a currently active area of research in theoretical cosmology[3]. The motivation for phantom matter is provided by string theory[4].

The possibility that our world may be embedded in a $(4+d)$-dimensional universe with more than four large dimensions has attracted the attention of a great number of physics. One of these higher-dimensional theories, where the cylinder condition of the Kaluza-Klein theory[5] is replaced by the conjecture that the ordinary matter and fields are confined to a 4D subspace, usually referred to as a brane is the Randall and Sundrum model[6]. One important physical problem in higher-dimensional theories is to develop a full understanding of implications in 4D. Therefore, it is essential to compare and contrast the effective pictures generated in 4D by different versions of 5D relativity, where the extra dimension is not assumed to be compactified. Extensions of General Relativity to five and more dimensions seem to provide a possible route to unification of gravity with interactions of particle physics[7]. The Campbell-Magaard[8] theorem serves as a ladder to go between manifolds whose dimensionality differs by one. This theorem, implies that every solution of the 4D Einstein equations with arbitrary energy-momentum tensor can be embedded, at least locally, in a solution of the 5D Einstein field equations in vacuum.

In this work we explore phantom and inflationary cosmologies using form-invariance transformations in the Einstein equations. These transformations were previously studied in this context in[9], but on an extended 5D de Sitter expansion. In this work we focus our study on an universe which is governed by a decaying cosmological function. The dynamics of
such a universe was studied previously in [10], but in another framework. Our main interest in this work relies in the study of the dynamics of the quantum fluctuations of the phantom and the inflaton fields.

II. FORM-INVARIANCE TRANSFORMATIONS: INFLATION AND PHANTOM COSMOLOGIES

Several cosmological models have been pondered in terms of the form-invariance of their dynamical equations under a group of symmetry transformations that preserve the form of the Einstein equations. A form-invariance transformation is a prescription to relate the quantities $a, H, \rho$ and $p$ of our original model, with the quantities $\bar{a}, \bar{H}, \bar{\rho}$ and $\bar{p}$ of what we will call the transformed model, so that it satisfies

$$3\bar{H}^2 = \bar{\rho},$$

$$\dot{\bar{\rho}} + 3\bar{H}(\bar{\rho} + \bar{p}) = 0,$$

and are given by

$$\bar{\rho} = \bar{\rho}(\rho),$$

$$\bar{H} = \left(\frac{\bar{\rho}}{\rho}\right)^{1/2} H,$$

$$\bar{p} = -\bar{\rho} + \left(\frac{\rho}{\bar{\rho}}\right)^{1/2} (\rho + p) \frac{d\bar{\rho}}{d\rho},$$

where $\bar{\rho} = \bar{\rho}(\rho)$ is an invertible arbitrary function. For a barotropic equation of state of a perfect fluid $p = (\gamma - 1)\rho$, one obtains

$$\bar{\gamma} = \left(\frac{\rho}{\bar{\rho}}\right)^{3/2} \frac{d\bar{\rho}}{d\rho} \gamma.$$
leaves invariant the form of the Friedmann dynamical equations of both models. As it was shown in [13], under the transformation $\bar{\rho} = n^2 \rho$ with $n$ being a constant, the scalar field and its potential transform as

$$\dot{\phi}^2 = n \dot{\phi}^2,$$

$$\bar{V} = n^2 \left( \frac{1}{2} \dot{\phi}^2 + V \right) - \frac{n}{2} \dot{\phi}^2,$$

where $\bar{V}$ is the phantom potential. Thus it can be easily proved that when $n = \pm 1$, the (+) branch corresponds to an identical transformation whereas the (−) branch leads to a phantom cosmology. In that case the previous expressions become

$$\dot{\phi}^2 = -\dot{\phi}^2,$$

$$\bar{V}(\bar{\phi}) = \dot{\phi}^2 + V(\phi).$$

Clearly the energy density of the transformed model $\bar{\rho} = (1/2) \dot{\phi}^2 + \bar{V}(\bar{\phi}) = -(1/2) \dot{\phi}^2 + \bar{V}(\bar{\phi})$ corresponds to a phantom scalar field $\bar{\phi}$ with the relation $\bar{\phi} = i\phi$ being valid. In this letter we shall consider inflation as the initial cosmological model and phantom cosmology as the transformed one. Hence, by $p$, $\rho$, $a$ and $H$ ($\bar{p}$, $\bar{\rho}$, $\bar{a}$ and $\bar{H}$) we shall denote respectively pressure, energy density, the scale factor and the Hubble parameter, during inflation (phantom cosmology).

In the following section we shall explore the dynamics of the inflaton and phantom fields through the form-invariance transformations, but from the 5D vacuum state.

### III. DYNAMICS OF THE INFLATON AND PHANTOM FIELDS

We consider the 5D action

$$I = \int d^5x \sqrt{|(5)g_{0}|} \left( \frac{R}{16\pi G} + \mathcal{L}^{(5)}_{\varphi} \right),$$

(7)

for a massless scalar field $\varphi$, which is free of any interactions

$$\mathcal{L}^{(5)}(\varphi, \varphi, B)_{\varphi} = \frac{1}{2} g^{AB} \varphi_{,A} \varphi_{,B}.$$ 

(8)
Here, \((5)g\) is the determinant of the covariant tensor metric \(g_{AB}\), such that the Riemann-flat metric \(R_{ABCD}=0\) is described by line element
\[
ds^2 = \psi^2 \frac{\Lambda(t)}{3} dt^2 - \psi^2 e^{2 \int \sqrt{\Lambda(t)/3} dt} dr^2 - d\psi^2,
\]
where \(dr^2 = dx^2 + dy^2 + dz^2\), \(t\) is the cosmic time, \(\Lambda(t)\) is a decreasing cosmological function and \(\psi\) a noncompact extra dimension.

A. The 5D inflaton field dynamics

The equation of motion for the inflaton field is
\[
\ddot{\phi} + \left(3 \sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{2\Lambda}\right) \dot{\phi} - \frac{\Lambda}{3} e^{-2 \int \sqrt{\Lambda(t)/3} dt} \nabla^2_{\phi} \phi - \frac{\Lambda}{3} \left(4 \psi \frac{d\phi}{d\psi} + \psi^2 \frac{d^2\phi}{d\psi^2}\right) = 0,
\]
which describes the dynamics on the 5D metric (9).

B. The 5D phantom field dynamics

Now we can make the Wick transformation \(\bar{\phi} = i\phi\), so that action (7) can be written as
\[
I = \int d^5 x \sqrt{\left| \frac{(5)g}{g_0} \right| \left( \frac{R}{16\pi G} + \mathcal{L}_{\bar{\phi}}^{(5)} \right)},
\]
where the density Lagrangian written in terms of the massless scalar field \(\bar{\phi}\), is
\[
\mathcal{L}_{\bar{\phi}}^{(5)}(\phi, \bar{\phi}, \dot{\phi}, \xi) = -\frac{1}{2} g^{AB} \bar{\phi}_{,A} \bar{\phi}_{,B}.
\]
The dynamics for the phantom field is given by
\[
\ddot{\bar{\phi}} - \left(3 \sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{2\Lambda}\right) \dot{\bar{\phi}} + \frac{\Lambda}{3} e^{-2 \int \sqrt{\Lambda(t)/3} dt} \nabla^2_{\bar{\phi}} \bar{\phi} + \frac{\Lambda}{3} \left(4 \psi \frac{d\bar{\phi}}{d\psi} + \psi^2 \frac{d^2\bar{\phi}}{d\psi^2}\right) = 0.
\]
This equation describes the dynamics of the phantom field \(\bar{\phi}\) on the metric (9).

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1 Latin indices \(A, B\) run from 0 to 4.
IV. EFFECTIVE 4D DYNAMICS: STATIC FOLIATION

We consider the equation of motion for the inflaton field \( \psi = \psi_0 \). This equation can be evaluated on the foliation \( \psi = \psi_0 \). The effective 4D dynamics

\[
\ddot{\phi} + \left( 3 \sqrt{\frac{\Lambda}{3} - \frac{\dot{\Lambda}}{2 \Lambda}} \right) \dot{\phi} - \frac{\Lambda}{3} e^{-2 f \sqrt{\Lambda(t)/3 \psi^2}} \nabla_r \dot{\phi} - \frac{\Lambda}{3} \left( 4 \psi \frac{d\phi}{d\psi} + \psi^2 \frac{d^2 \phi}{d\psi^2} \right) \bigg|_{\psi = \psi_0} = 0, \tag{14}
\]

where, after making separation of variables, we can perform the identification \([14]\):

\[
- \frac{\Lambda}{3} \left( 4 \psi \frac{d\phi}{d\psi} + \psi^2 \frac{d^2 \phi}{d\psi^2} \right) \bigg|_{\psi = \psi_0} = \frac{\Lambda m^2}{3} \varphi(t, \vec{r}, \psi_0), \tag{15}
\]

\( m^2 \geq 0 \) being a constant of separation. Hence, the dynamics on the effective 4D hypersurface

\[
ds^2 = \psi^2_0 \frac{\Lambda(t)}{3} dt^2 - \psi^2_0 e^{2 f \sqrt{\Lambda(t)/3 \psi^2}} dr^2,
\]

will be

\[
\ddot{\phi} + \left( 3 \sqrt{\frac{\Lambda}{3} - \frac{\dot{\Lambda}}{2 \Lambda}} \right) \dot{\phi} - \frac{\Lambda}{3} e^{-2 f \sqrt{\Lambda(t)/3 \psi^2}} \nabla_r \dot{\phi} + \frac{\Lambda m^2}{3} \varphi = 0. \tag{16}
\]

In the same manner we can obtain the effective equation of motion for \( \bar{\phi} \) on the foliation \( \psi = \psi_0 \), by using separation of variables on the metric \([13]\), so that we obtain

\[
\ddot{\bar{\phi}} - \left( 3 \sqrt{\frac{\Lambda}{3} - \frac{\dot{\Lambda}}{2 \Lambda}} \right) \dot{\bar{\phi}} + \frac{\Lambda}{3} e^{-2 f \sqrt{\Lambda(t)/3 \psi^2}} \nabla_r \bar{\phi} - \frac{\Lambda m^2}{3} \bar{\phi} = 0. \tag{17}
\]

In order to describe the dynamics of the fields \( \phi \) and \( \bar{\phi} \) we shall consider a semiclassical expansion for both fields: \( \phi = \phi_c(t, \psi_0) + \delta \phi(t, \vec{r}, \psi_0) \) and \( \bar{\phi} = \bar{\phi}_c(t, \psi_0) + \bar{\delta} \phi(t, \vec{r}, \psi_0) \).

A. Classical dynamics and equations of state

We shall consider the case where

\[
\Lambda(t) = \frac{3n^2}{t^2}, \tag{18}
\]

so that \( \dot{\Lambda}(t) = -\frac{6n^2}{t^3} \), where \( n \gg 1 \) is a constant. The effective scalar potential \( V(\phi) \) is given by

\[
V_{\text{eff}}(\phi) = \frac{\Lambda \psi^2}{3} \left( \frac{d\phi}{d\psi} \right)^2 \bigg|_{\psi = \psi_0} = \frac{M_{\text{eff}}^2(\psi_0)}{2} \phi^2(t, \vec{r}, \psi_0), \tag{19}
\]

where the effective mass is

\[
M_{\text{eff}}^2(\psi_0) = \frac{\Lambda m^2}{3}. \tag{20}
\]
The equation of motion for $\varphi_c(t, \psi_0)$ is

$$\ddot{\varphi}_c + \left(3\sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{2\Lambda}\right)\dot{\varphi}_c + \frac{\Lambda m^2}{3} \varphi_c = 0.$$  \hspace{1cm} (21)

A particular solution for this equation is

$$\varphi_c(t, \psi_0) = \varphi_0 t^c,$$  \hspace{1cm} (22)

where $c$ is given by

$$c = n \left[ -\frac{3}{2} + \sqrt{\frac{9}{4} - m^2} \right].$$  \hspace{1cm} (23)

The kinetic component of energy density $T_{\varphi_c}$, and the potential $V_{\varphi_c}$, are

$$T_{\varphi_c} = \frac{1}{2} \left( \frac{3}{\psi_0^2 \Lambda} \dot{\varphi}_c^2 \right), \quad V_{\varphi_c} = \frac{M^2(\psi_0)}{2} \varphi_c^2,$$  \hspace{1cm} (24)

so that the pressure, $p_{\varphi_c} = T_{\varphi_c} - V_{\varphi_c}$, and the energy density, $\rho_{\varphi_c} = T_{\varphi_c} + V_{\varphi_c}$, give us the equation of state for inflation: $\omega = p_{\varphi_c}/\rho_{\varphi_c}$, which in this case is given by

$$\omega = \frac{\left(-\frac{3}{2} + \sqrt{\frac{9}{4} - m^2}\right)^2 - m^2}{\left(-\frac{3}{2} + \sqrt{\frac{9}{4} - m^2}\right)^2 + m^2}.$$  \hspace{1cm} (25)

On the other hand, the effective mass for the phantom field is

$$\bar{M}_{\text{eff}}^2(\psi_0) = -\frac{\Lambda \psi_0^2}{3} \left( \frac{2c^2}{t^2} + \frac{2m^2}{\psi_0^2} \right).$$  \hspace{1cm} (26)

Notice that $\lim_{t \to \infty} \bar{M}_{\text{eff}}^2(\psi_0) \to -2M_{\text{eff}}^2$. The equation of state for the phantom system $\bar{\omega}_{\varphi_c} = \bar{p}_{\varphi_c}/\bar{\rho}_{\varphi_c}$, is

$$\bar{\omega} = \frac{3 \left(-\frac{3}{2} + \sqrt{\frac{9}{4} - m^2}\right)^2 + m^2}{\left(-\frac{3}{2} + \sqrt{\frac{9}{4} - m^2}\right)^2 + m^2},$$  \hspace{1cm} (27)

where we have taken into account that $\bar{\varphi}_c = i \varphi_c$, and that

$$\bar{p}_{\varphi_c} = \frac{\varphi_c^2}{2} - V(\varphi_c), \quad \bar{\rho}_{\varphi_c} = -\frac{3\varphi_c^2}{2} - V(\varphi_c).$$  \hspace{1cm} (28)

To slow-roll conditions to be fulfilled, we would need that $m^2 \ll 1$, so that $\omega \gtrsim -1$ and $\bar{\omega} < -1$, for inflation and phantom cosmology, respectively. It is interesting to notice that in both cases $\lim_{m \to 0} \omega = \lim_{m \to 0} \bar{\omega} = -1$. Finally, values of $\bar{\omega}$ (pointed line) and $\omega$ (continuous line) were plotted for small values of $m$ in the figure (1). Notice that both are very symmetric with respect to $-1$, and they are very sensitive to $m$. 

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B. Quantum fluctuations

Both equations (16) and (17) are linear, so that can be expanded in terms of Fourier modes. The dynamics of the time-dependent modes of quantum fluctuations in both cases, are respectively given by

\[ \ddot{\varphi}_k + \left( 3 \sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{2\Lambda} \right) \dot{\varphi}_k + \left[ \frac{k^2 \Lambda}{3} e^{-2 \int \sqrt{\Lambda(t)/3} dt} + \frac{\Lambda m^2}{3} \right] \varphi_k = 0, \quad (29) \]

and

\[ \ddot{\varphi}_k - \left( 3 \sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{2\Lambda} \right) \dot{\varphi}_k + \left[ \frac{k^2 \Lambda}{3} e^{-2 \int \sqrt{\Lambda(t)/3} dt} - \frac{\Lambda m^2}{3} \right] \varphi_k = 0. \quad (30) \]

If we take into account the case (18), the general solutions of the equations (29) and (30), are respectively

\[ \varphi_k(t) = \left( \frac{t}{t_0} \right)^{-3n/2} \left[ C_1 H^{(1)}(\nu)(x(t)) + C_2 H^{(2)}(\nu)(x(t)) \right], \quad (31) \]

\[ \bar{\varphi}_k(t) = \left( \frac{t}{t_0} \right)^{3n/2+1} \left[ \bar{C}_1 H^{(1)}(\mu)(x(t)) + \bar{C}_2 H^{(2)}(\mu)(x(t)) \right], \quad (32) \]

such that \( H^{(1,2)} \) are the first and second kind Hankel functions, \( \nu = \frac{3}{2} \sqrt{1 - 4m^2/9}, \quad \mu = \frac{3}{2} \sqrt{1 + \frac{4m^2}{9} + \frac{12}{9n} + \frac{4}{9n^2}} \) and \( x(t) = k(t/t_0)^{-n} \). Notice that, since \( m^2 \ll 1 \) and \( n \gg 1 \), we obtain that \( \nu < 3/2 \) and \( \mu > 3/2 \).

In order to calculate the squared \( \varphi \)-fluctuations on cosmological scales \( k \ll k_0(t) \), we shall need the asymptotic expressions of the Hankel functions for \( x \ll 1 \): \( H^{(1,2)}(\nu)(x(t)) \bigg|_{x < 1} \simeq -\frac{i}{\pi} \Gamma(\nu) \left( \frac{x}{2} \right)^{-\nu} \). These fluctuations describe the infrared sector, which during inflation is given by super Hubble fluctuations, and are given by

\[ \langle \delta \varphi^2 \rangle_{IR} = \frac{H^3}{\pi^3} \frac{BB^* 2^{2\nu-2}}{(3-2\nu)} \left[ \epsilon \left( \frac{n^2}{3} \left( \frac{9}{4} - \frac{1}{4n^2} + m^2 \right) \right)^{1/2} \right]^{3-2\nu}, \quad (33) \]

where \( B = \frac{i}{2} \sqrt{\frac{\pi}{11}} = C_2/H^{3/2} \) and \( H = n/t_0, \ t_0 \) being the time at the end of inflation. The modes are normalized by choosing the Bunch-Davies vacuum. Notice that for \( k \gg k_0(t) \)

\[ k_0^2 = \frac{n^2}{3} \left( \frac{9}{4} - \frac{1}{4n^2} + m^2 \right) t^{2n}, \quad (34) \]

the solutions become unstables. This function is the time dependent wavenumber related to the size of the horizon.
The $\varphi$-fluctuations in phantom cosmology hold
\[
\left\langle \delta \varphi^2 \right\rangle_{IR} = \frac{H^3 \tilde{B} \tilde{B}^* 2^{2\mu-2}}{\pi^3 (3 - 2\mu)} \left[ \epsilon \left( \left( \frac{(3n + 1)(3n + 3)}{4n^2} + m^2 \right) \right)^{1/2} \right]^{3-2\nu} \left( \frac{t}{t_0} \right)^{6n+2}, \tag{35}
\]
which increases with time. Here, $\tilde{B} = \frac{i}{2} \sqrt{\frac{2}{H}} = \tilde{C}_2/H^{3/2}$ which is determined by normalization of the $\delta \varphi_k(t)$ modes.

The relevant wavenumber $\bar{k}_0$ is
\[
\bar{k}_0^2 = \left( \frac{(3n + 1)(3n + 3)}{4n^2} + m^2 \right) t^{2n}. \tag{36}
\]
This case is similar to whose of the inflaton field, because in both cases the modes are unstables on cosmological scales and oscillate on sub Hubble scales. Finally, the squared $\dot{\delta} \varphi$-fluctuations increase as $\left\langle \dot{\delta} \varphi^2 \right\rangle_{IR} \sim t^{6n}$, so that they can be neglected with respect to $\left\langle \delta \varphi^2 \right\rangle_{IR}$, which remains dominant at the end of the phantom epoch.

V. FINAL COMMENTS

We have revisited phantom and inflationary scenarios through the form-invariance transformations of the Einstein equations with respect to $\rho$, $H$, $a$ and $p$, from a 5D vacuum state. In particular, the dynamics of quantum fluctuations of the phantom and the inflaton fields was studied. One of the interesting results here obtained is that the behavior of phantom field fluctuations $\langle \varphi^2 \rangle$ is different to the inflaton ones; the large-scale (super Hubble) amplitude of these fluctuations is not freezed and increases dramatically with time. Hence, phantom fields should be not a good candidate to describe inflation, at least under the conditions here studied. However, we believe that this field is a good candidate to describe the evolution of a asymptotic future of the universe. Finally, we have studied the equation of state for inflation and phantom scenarios. The results plotted in figure (1) show that the big rip observed in phantom cosmology is sensitive to the parameter $m$.

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FIG. 1: Values of $\bar{\omega}$ (pointed line) and $\omega$ (continuous line) plotted for small values of $m$. 