Research on the power distribution region and multiple constraint matching of modular multilevel converter

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Summary
This paper focuses on the steady-state power distribution region of the modular multilevel converter (MMC) and the matching among its multiple constraints. First, an equivalent model of the steady-state MMC is established, and its basic power distribution region is solved based on analysis. Second, the influence of the AC current constraint, the AC outlet voltage constraint, the modulation index constraint, the DC current constraint, and the DC voltage constraint on the power distribution region are studied through the phasor analysis method. The power distribution region under the multiple constraints is obtained, and the parameters matching principle among these constraints are studied. Finally, in PSCAD/EMTDC, the simulation results validate the correctness of the obtained power distribution region and the effectiveness of the parameters matching principle among the multiple constraints.

KEYWORDS
constraint, modular multilevel converter, optimization, power distribution region, VSC-HVDC

1 | INTRODUCTION

The modular multilevel converter has excellent operation performance such as good scalability, low switching loss, small output harmonics, and independent control of active and reactive power, which can meet the requirements of the flexible DC transmission system with its increasing voltage level and transmission capacity, thus has become the most preferred converter type for the high-voltage flexible DC transmission system.\(^2,3\)

List of Symbols and Abbreviations:
- AC, alternating current; DC, direct current; EMTDC, electromagnetic transients including DC; HVDC, high-voltage direct current; MMC, modular multilevel converter; PSCAD, power systems computer-aided design; \(u_{ap}\) and \(u_{an}\), the upper and lower arms voltage of MMC; \(i_{ap}\) and \(i_{an}\), the upper and lower arm currents; \(u_{a,b,c}\), the AC outlet voltage of MMC; \(i_{a,b,c}\), AC three-phase currents; \(U_{DC}\), DC voltage; \(I_{DC}\), DC current; \(e_{a,b,c}\), the equivalent voltages of AC grid; \(v_{a,b,c}\), the equivalent output voltage of MMC; \(L_{eq}\), the equivalent inductance; \(L\), the arm inductance of MMC; \(S\), complex power; \(P\), active power; \(Q\), reactive power; \(E\), the RMS value of the AC grid voltage; \(U_{s}\), the RMS value of the AC outlet voltage; \(V_\delta\), the RMS value of the equivalent output voltage; \(\delta\), the phase angle difference between the equivalent output voltage and the AC grid voltage; \(U_N\), rated AC outlet voltage; \(I_N\), rated AC current; \(M\), modulation index; \(V_{sp}\), the peak value of the equivalent output voltage; \(U_{DCN}\), rated DC voltage; \(N_{sub}\), the minimum amount of the submodule put into operation in the upper and lower arms; \(N\), the total amount of submodules of each arm; \(I_{2f}\) and \(\alpha\), the peak value and initial phase of the second harmonic circulating current.

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The structure and operation conditions of MMC are complex and various, and its maximum output capacity of active power and reactive power in steady-state operation are related to a variety of factors. Studying and defining the power distribution region of the converter can help engineers and technicians optimize the overall power distribution of the system, clarify the boundaries of safety operation, and then give full play to the power transmission capacity of the high-voltage flexible DC transmission system with safety guaranteed.

In the available analyses, some research studies have been done on the power distribution region of MMC. The author points out that the maximum apparent power range is determined by the maximum valve current. The dynamic phasor model of MMC in rotating frame is established, and the power distribution region of MMC is obtained through multiple iterations with the Newton-Raphson method. The power distribution region of MMC is obtained by solving the sixth-order dynamic equation of MMC in the rotating frame. An analytical and point-scanning method is proposed to determine the P-Q curve of the converter under the unbalance grid conditions. The relationship between the converter's circulating current and the power output was studied, and the suppression and injection methods of the circulation current are also proposed, which can improve the power output of the converter by reducing the internal loss. The author separately studied the effects of the AC system short circuit ratio, the converter transformer capacity, and the reactive power compensation devices on the operation characteristics of MMC in the steady state and pointed out that the short-circuit ratio is most influential on limiting the power transmission capacity of the converter. The relationship between the submodule capacitor voltage and the output power of the converter was studied, and the author pointed out that as the capacitor voltage increases, the reactive power output capacity of the converter also increases. The author studied the relationship between energy storage and parameters of MMC, and an energy management control is proposed to manage the input and output power of MMC. The influence of submodule capacitor voltage fluctuations on the reactive power output of the converter was studied, and different control strategies and topology structures that can effectively reduce the capacitor voltage ripple were proposed.

It could be seen that the method to determine the power distribution region of MMC in the current research studies is cumbersome and the existing papers do not have a comprehensive consideration about the factors which are affecting the power operation region of MMC. In this paper, the power distribution region of MMC in abc stationary coordinate is studied for presenting a more concise, more intuitive, and clearer result. Then, various factors that have an influence on the power distribution region of MMC are analyzed in detail, and the power distribution region when considering all constraints is obtained. Finally, the parameter matching principle between various constraints is proposed in order to minimize the investment cost of MMC under the required power output/input capability.

2 | EQUIVALENT MATHEMATICAL MODEL OF THE MMC

The basic topology of an MMC is shown in Figure 1.

![Figure 1](image-url)
Because the arm resistance of the converter is very small, the influence of the resistance can be ignored when analyzing the power characteristics of the converter.

The positive direction of each electrical quantity of the converter is shown in Figure 1. \( u_{ap} \) and \( u_{an} \) are the voltages generated by the submodules in the upper and lower arms, respectively. \( i_{ap} \) and \( i_{an} \) are the arm currents, respectively. \( u_{a,b,c} \) respectively represents the AC three-phase voltage of the converter, \( i_{a,b,c} \) are the AC three-phase currents, and \( U_{DC} \) and \( I_{DC} \) are the DC voltage and DC current of the converter.

According to Kirchhoff's voltage law, the upper and lower arms voltages \( u_{ap} \) and \( u_{an} \) can be expressed as

\[
\begin{align*}
  u_{an} &= U_{DC} + u_a + L \frac{di_{an}}{dt} \\
  u_{ap} &= U_{DC} - u_a + L \frac{di_{ap}}{dt}
\end{align*}
\]

The voltage difference between \( u_{ap} \) and \( u_{an} \) can be derived

\[
\begin{align*}
  u_{an} - u_{ap} &= 2u_a - L \frac{d(i_{an} - i_{ap})}{dt}
\end{align*}
\]

According to Kirchhoff's current law

\[
\begin{align*}
  i_a &= i_{an} - i_{ap}
\end{align*}
\]

Substituting (4) into (3)

\[
\begin{align*}
  u_{an} - u_{ap} &= 2u_a - L \frac{di_a}{dt}
\end{align*}
\]

The equivalent output voltage of phase a is defined as

\[
\begin{align*}
  v_a &= \frac{u_{an} - u_{ap}}{2}
\end{align*}
\]

Substituting (5) into (6), the equivalent output voltage can be expressed as

\[
\begin{align*}
  v_a &= u_a - \frac{L}{2} \frac{di_a}{dt}
\end{align*}
\]

By extending the above analysis results to the other two phases, the simplified equivalent circuit of the AC side of the MMC is shown in Figure 2.

In Figure 2, \( e_{a,b,c} \) are the equivalent voltages of the AC grid, \( L_{eq} \) is the equivalent inductance between the equivalent AC grid and the converter, \( L \) is the arm inductance, and \( v_{a,b,c} \) are the equivalent AC output voltage of the converter.
3 | SOLVING OF THE POWER DISTRIBUTION REGION OF MMC

With the power frequency components in the system as the only consideration and from Figure 2, the relationship between the voltage phasors of the AC grid $\dot{E}$ and the equivalent output voltage phasor of the MMC $\dot{V}_s$ can be expressed as

$$
\dot{V}_s = \dot{E} + j\omega L_{eq}\dot{I} + j\frac{\omega L}{2}\dot{I}
$$

(8)

Supposing the initial phase of $\dot{E}$ is 0°, and $\dot{E}$ leads $\dot{V}_s$ by an angle of $\delta$. The phasor relationship among $\dot{E}$, $\dot{V}_s$, and $\dot{I}$ is shown in Figure 3.

Where $U_s$ is the AC outlet voltage of the converter, $j\omega L_{eq}$ is the equivalent reactance between the equivalent AC grid and the converter, $j\omega L/2$ is the equivalent reactance of the arm in the converter.

From (8), the complex power $\ddot{S}$, active power $P$, and reactive power $Q$, which are received by the AC grid can be expressed as

$$
\ddot{S} = 3*\dot{E}^*\dot{I} = 3E\left[\frac{V_s \cos \delta - E}{X} + \frac{V_s \sin \delta}{X}\right] = P + jQ
$$

(9)

$$
P = 3EV_s \sin \delta / X
$$

(10)

$$
Q = 3E(V_s \cos \delta - E)/X
$$

(11)

where $V_s$, $E$ are the RMS value of $\dot{V}_s$ and $\dot{E}$, respectively, $X = \omega L_{eq} + \omega L/2$.

Taking the squares of (10) and (11) and adding them together:

$$
P^2 + \left(Q + \frac{3E^2}{X}\right)^2 = \left(\frac{3EV_s}{X}\right)^2
$$

(12)

According to the topology of half-bridge MMC, assuming that the range of the equivalent output voltage of the converter is

$$
0 \leq V_s \leq V_{\text{max}}
$$

(13)

where $V_{\text{max}}$ is the maximum RMS value of the equivalent output voltage.

From the simultaneous formula of (12) and (13), the basic power distribution region of the MMC can be expressed as

$$
0 \leq P^2 + \left(Q + \frac{3E^2}{X}\right)^2 \leq \left(\frac{3EV_{\text{max}}}{X}\right)^2
$$

(14)

![Figure 3](image-url)  
**Figure 3** The phasor relationship among $\dot{E}$, $\dot{V}_s$, and $\dot{I}$
From formula (14), it is clearly that the power output of the MMC is affected by the AC grid voltage, the converter's equivalent output voltage, and the equivalent impedance between them.

Based on formula (14), the basic power distribution region of the MMC during steady-state operation as shown in Figure 4.

It can be seen from Figure 4 that the basic power distribution region of the converter is a circular area with \((0, -3E^2/X)\) as the center and \(3EV_s/X\) as the radius.

4 | INFLUENCE OF DIFFERENT CONSTRAINTS ON THE POWER DISTRIBUTION REGION

4.1 | AC side constraint

4.1.1 | The influence of the AC outlet voltage on the power distribution region

In order to effectively improve the safety and reliability of the equipment and reduce the adverse impact on the converter and other electrical equipment caused by excessive voltage deviation, a series of regulations has been made for limiting the rated voltage and the allowed deviation of the AC outlet voltage of the converter. Usually, the allowed voltage deviation is within ±2.5% of its rated voltage.

The rated value of the AC outlet voltage is represented by \(U_N\), then the range of \(U_s\) can be expressed as

\[
U_{s\text{min}} \leq U_s \leq U_{s\text{max}} \tag{15}
\]

where \(U_{s\text{min}}\) and \(U_{s\text{max}}\) are the minimum and maximum values of the AC outlet voltage, respectively.

From Figure 3, the relationship among the voltage phasors of the AC grid, the equivalent output voltage phasor of the MMC, and the converter outlet voltage phasor \(\hat{U}_s\) can be expressed as

\[
\hat{V}_s = \left(\frac{\hat{U}_s - \hat{E}}{j\omega L_{eq}}\right) + j\left(\omega L_{eq} + \frac{1}{2}j\omega L\right) + \hat{E} \tag{16}
\]

And the equivalent output voltage \(\hat{V}_s\) can also be expressed as:

![Figure 4](image) The basic power distribution region of the MMC
\[ \dot{V}_s = \sqrt{2}V_s(\cos \delta + j \sin \delta) \]  

(17)

Then, from (16) and (17), the RMS value of the equivalent output voltage and the phase angle difference between the \( \dot{V}_s \) and \( \dot{E} \) can be expressed as

\[ V_s = \sqrt{[(1 + k_m)U_s \cos \theta - k_mE]^2 + [(1 + k_m)U_s \sin \theta]^2} \]  

(18)

\[ \delta = \arctan \frac{(1 + k_m)U_s \sin \theta}{(1 + k_m)U_s \cos \theta - k_mE} \]  

(19)

where \( U_s \) is the RMS value of \( \dot{U}_s \), \( \theta \) is the phase angle difference between \( \dot{V}_s \) and \( \dot{U}_s \), \( k_m = L/2L_{eq}\dot{E} \).

By substituting (18) and (19) into (10) and (11), the active power and reactive power can be expressed as

\[ P = 3EU_s \sin \theta / \omega L_{eq} \]  

(20)

\[ Q = 3E(U_s \cos \theta - E) / \omega L_{eq} \]  

(21)

Taking the squares of (20) and (21) and adding them together:

\[ P^2 + \left( Q + \frac{3E^2}{\omega L_{eq}} \right)^2 = \left( \frac{3EU_s}{\omega L_{eq}} \right)^2 \]  

(22)

The power distribution region of the MMC under the constraint of the AC outlet voltage can be expressed by the simultaneous formula of (15) and (22):

\[ \left( \frac{3EU_{s\min}}{\omega L_{eq}} \right)^2 \leq P^2 + \left( Q + \frac{3E^2}{\omega L_{eq}} \right)^2 \leq \left( \frac{3EU_{s\max}}{\omega L_{eq}} \right)^2 \]  

(23)

From (23), the power distribution region under the AC outlet voltage constrain is shown in Figure 5.
It can be seen from Figure 5 that under the AC outlet voltage constraint, the steady-state power distribution region of the converter is a ring with its center on the negative Q-axis. The width of the ring depends on the maximum and minimum values of the outlet voltage.

4.1.2 | The influence of the AC current range of the converter on the power distribution region

In steady-state operation, the AC current of the converter should be smaller than the rated current under the influence of the AC grid as well as its transmission line and electrical equipment:

\[ I \leq I_N \] (24)

Then, the maximum RMS value of the AC current \( I_{\text{max}} \) satisfies

\[ I_{\text{max}} = I_N \] (25)

According to formulas (8) and (25), the phasor diagram of the converter’s equivalent output voltage is shown in Figure 6.

From Figure 6, the AC current will limit the maximum equivalent output voltage of the MMC. And the relationship between the equivalent output voltage and AC current is

\[ \dot{V}_s = \dot{E} + jXI = \sqrt{2}V_s (\cos \delta + j \sin \delta) \] (26)

where

\[ V_s = \sqrt{[E - XI \sin \phi]^2 + [XI \cos \phi]^2} \] (27)

\[ \delta = \arctan \frac{XI \cos \phi}{E - XI \sin \phi} \] (28)

where \( \phi \) is the phase angle difference between \( \dot{E} \) and \( \dot{I} \).

Formulas (27) and (28) describe the relationship between the equivalent output voltage and the AC phase current of the converter. By substituting (27) and (28) into (10) and (11):
\[ P = 3EI \cos \varphi \]  \hspace{2cm} (29)

\[ Q = -3EI \sin \varphi \]  \hspace{2cm} (30)

Taking the squares of (29) and (30) and adding them together:

\[ P^2 + Q^2 = (3EI)^2 \]  \hspace{2cm} (31)

The power distribution region of the MMC under the AC current constraint can be expressed by the simultaneous formula of (24) and (31):

\[ 0 \leq P^2 + Q^2 \leq (3EI_{\text{max}})^2 \]  \hspace{2cm} (32)

From (32), the power distribution region of the converter under the constraint of the AC current is shown in Figure 7.

From Figure 7, under the AC current constraint, the power distribution region of the converter is a circular area with its center at the origin, and its radius related to the maximum value of the AC current.

### 4.2 Modulation index constraint

The modulation index is the intermediate variable parameter between the DC side and AC side electrical quantities, and the modulation index of the MMC can be defined as

\[ M = \frac{2V_{\text{sp}}}{U_{\text{DCN}}} \]  \hspace{2cm} (33)

where \( V_{\text{sp}} \) is the peak value of the equivalent output voltage of the converter.

Based on the converter topology, the relationship of the modulation index and the amount of the submodule is

\[ M = 1 - \frac{2N_{\text{in}}}{N} \]  \hspace{2cm} (34)

---

**FIGURE 7** The power distribution region of the converter under the AC current constraint
where $N_m$ is the minimum amount of the submodules always put into operation in the upper and lower arms, $N$ is the total amount of submodules of each arm in the MMC.

From (34), the modulation index $M$ will change as the minimum amount of the submodules put into operation in the upper and lower arms changes. When $N_m = 0.5 N$, the modulation index will be equal to 0. And when $N_m = 0$, the modulation index will be equal to 1.

When the modulation index is too small, the converter's equivalent output voltage will have a large distortion and create excessive harmonics.\textsuperscript{16} When the converter's modulation ratio is close to 1, it will over-modulate due to fluctuations of the submodule capacitor voltage.\textsuperscript{17} Therefore, the modulation index should not be too large or too small. Assuming that the minimum value of $M$ is represented by $M_{\text{min}}$ and the maximum value is represented by $M_{\text{max}}$, then the range of the RMS value of the AC equivalent output voltage of converter can be expressed as

$$\frac{M_{\text{min}}U_{\text{DCN}}}{2\sqrt{2}} \leq V_s \leq \frac{M_{\text{max}}U_{\text{DCN}}}{2\sqrt{2}}$$ (35)

From the simultaneous formula of (12) and (35), the power distribution region of the converter under the modulation index constraint can be expressed as

$$\left(\frac{3}{2\sqrt{2}} \frac{EM_{\text{min}}U_{\text{DCN}}}{X}\right)^2 \leq P^2 + \left(Q + \frac{3E^2}{X}\right)^2 \leq \left(\frac{3}{2\sqrt{2}} \frac{EM_{\text{max}}U_{\text{DCN}}}{X}\right)^2$$ (36)

From (36), the power distribution region of the converter under the modulation index constraint can be drawn, as shown in Figure 8.

As for full-bridge MMC or converters with other topology, if the relationship between its equivalent output voltage and its modulation index is determined, the influence of the modulation index on the power distribution region of the converter can be determined by the same analysis method.

4.3 | DC-side constraint

4.3.1 | The influence of the DC voltage range of the converter on the power distribution region

In order to reduce the loss and failures on the DC-side power transmission line of the converter caused by over-voltage and under-voltage, the DC voltage of the MMC-HVDC system should meet relevant regulations and standards. The DC voltage deviation of a multiterminal MMC-HVDC system is usually limited to within ±10% of its rated value:\textsuperscript{18}

$$U_{\text{DCmin}} \leq U_{\text{DC}} \leq U_{\text{DCmax}}$$ (37)

where $U_{\text{DCN}}$ is the rated DC voltage.

Then, the range of the AC equivalent output voltage under the DC voltage constraint can be expressed by the simultaneous formula of (33) and (37):

$$\frac{MU_{\text{DCmin}}}{2\sqrt{2}} \leq V_s \leq \frac{MU_{\text{DCmax}}}{2\sqrt{2}}$$ (38)

The power distribution region of the converter under the DC voltage constraint can be expressed by the simultaneous formula of (12) and (38):
\[
\left( \frac{3}{2\sqrt{2}} \frac{EMU_{DC_{\text{min}}}}{X} \right)^2 \leq P^2 + \left( Q + \frac{3E^2}{X} \right)^2 \leq \left( \frac{3}{2\sqrt{2}} \frac{EMU_{DC_{\text{max}}}}{X} \right)^2
\] (39)

From (39), the power distribution region of the converter changes as the DC voltage changes. When the modulation index is certain, the influence of the DC voltage on the power distribution region is shown in Figure 9.

From Figure 9, under the DC voltage constraint, the power distribution area is a circle with its center located on the negative Q-axis, with its width related to the DC rated voltage and the voltage deviation of it.

### 4.3.2 The influence of the DC current on the power distribution region

In order to prevent the IGBT of the submodules from being burned, restrictions are commonly imposed on the RMS value of the arm current:

**Figure 8** The power distribution region under the modulation index constraint

**Figure 9** The power distribution region under the DC voltage constraint
\[
\begin{align*}
\begin{cases}
I_p \leq I_{N_{\text{max}}} \\
I_n \leq I_{N_{\text{max}}}
\end{cases}
\end{align*}
\]  

(40)

where \(I_p\) and \(I_n\) are the RMS value of the upper and lower arm currents, respectively, and \(I_{N_{\text{max}}}\) is the maximum RMS value of the current allowed in the arm.

When the circulating current is taken into consideration, the currents of the upper and lower arm can be expressed as:

\[
i_p = \frac{1}{3} I_{\text{DC}} - \frac{1}{2} I_{\text{ac}} + I_{2f} \cos (2\omega t - \alpha)
\]  

(41)

\[
i_n = \frac{1}{3} I_{\text{DC}} + \frac{1}{2} I_{\text{ac}} + I_{2f} \cos (2\omega t - \alpha)
\]  

(42)

where \(i_p\) and \(i_n\) are the instantaneous current of the upper and lower arm, respectively. \(I_{\text{DC}}\) and \(I_{\text{AC}}\) are the DC current and AC current of the converter, respectively, \(I_{2f}\) and \(\alpha\) are the peak value and initial phase of the second harmonic circulating current.

The RMS value of the upper and lower arm currents can be expressed as:

\[
I_p = I_n = \sqrt{\left(\frac{1}{3} I_{\text{DC}}\right)^2 + \left(\frac{I_m}{2\sqrt{2}}\right)^2 + \left(\frac{I_{2f}}{\sqrt{2}}\right)^2}
\]  

(43)

where \(I_m\) is the peak value of the AC current.

From (43), the RMS value of the arm current satisfies:

\[
I_p = I_n = \sqrt{\left(\frac{1}{3} I_{\text{DC}}\right)^2 + \left(\frac{I_m}{2\sqrt{2}}\right)^2 + \left(\frac{I_{2f}}{\sqrt{2}}\right)^2} \leq I_{N_{\text{max}}}
\]  

(44)

Then, the AC current and DC current satisfy

\[
\sqrt{\left(\frac{1}{3} I_{\text{DC}}\right)^2 + \left(\frac{I_m}{2\sqrt{2}}\right)^2} \leq \sqrt{I_{N_{\text{max}}}^2 - \left(\frac{I_{2f}}{\sqrt{2}}\right)^2}
\]  

(45)

Therefore, when the restrictions are imposed on the RMS value of the arm current, formula (45) always holds true.

Ignoring the switching loss of the converter, the active power at the AC side and that at the DC side are equal:

\[
U_{\text{DCN}} I_{\text{DC}} = 3V_s \frac{I_m}{\sqrt{2}} \cos (\delta - \varphi)
\]  

(46)

From (33), the equivalent output voltage can be expressed as

\[
V_s = MU_{\text{DCN}}/2\sqrt{2}
\]  

(47)

Substituting (47) into (46), the relationship of the AC current and DC current can be expressed as

\[
\frac{I_{\text{DC}}}{3} = \frac{M I_m}{4} \cos (\delta - \varphi)
\]  

(48)
According to the topology of half-bridge MMC, when no additional assistant control strategies are adopted in the converter, \( \cos(\delta - \varphi) \) and \( M \) satisfy

\[
\begin{align*}
-1 &\leq \cos(\delta - \varphi) \leq 1 \\
0 &< M < 1
\end{align*}
\]  

Then, the AC current and DC current satisfy

\[
\frac{1}{3}|I_{DC}| < \frac{1}{4}I_m
\]  

Because (50) always holds true, (51) always holds true.

\[
\sqrt{\left(\frac{1}{3}I_{DC}\right)^2 + \left(\frac{2}{3\sqrt{2}}I_{DC}\right)^2} \leq \sqrt{I^2_{Nmax} - \left(\frac{I_{2f}}{\sqrt{2}}\right)^2}
\]  

From (51), the DC current of the converter satisfies

\[
|I_{DC}| \leq \sqrt{3\sqrt{I^2_{Nmax} - \left(\frac{I_{2f}}{\sqrt{2}}\right)^2}}
\]  

When the converter is in steady-state operation, the DC current \( I_{DC} \) of the converter should be smaller than its rated current:

\[
|I_{DC}| \leq I_{NDC}
\]  

And when the maximum value of the AC current is \( I_{max} \), from (50), the DC current satisfies

\[
|I_{DC}| < 0.75I_{max}
\]

To sum up, the maximum DC current of the converter should satisfy

\[
|I_{DCmax}| = \min \left[ \frac{3}{4}I_{max}, I_{NDC}, \sqrt{3\sqrt{I^2_{Nmax} - \left(\frac{I_{2f}}{\sqrt{2}}\right)^2}} \right]
\]  

From (55), it is known that the range of the DC current is determined by the maximum value of the AC current, the rated DC current, and the arm current.

Then, the DC current constraint can be expressed as

\[
|I_{DC}| \leq |I_{DCmax}|
\]  

From (56) and the expression of the active power at DC side of the converter, the relationship between the DC current and the equivalent output voltage can be expressed as

\[
P = 3E_{vs} \sin \delta/X = U_{DCN}I_{DC}
\]  

Then, the DC current can be expressed as
\[ I_{DC} = 3E_s \sin \delta / XU_{DCN} \quad (58) \]

Substituting (47) into (58):

\[ I_{DC} = 3ME \sin \delta / 2\sqrt{2X} \quad (59) \]

Because the power of the converter can be transmitted in both directions, the DC current could be positive as well as negative with the positive direction being specified. According to (59), the magnitude of the DC current satisfies the Equation (60):

\[ \frac{3ME}{2\sqrt{2X}} \sin \delta \leq |I_{DC\text{max}}| \quad (60) \]

From (60):

\[ |\sin \delta| \leq \frac{2\sqrt{2I_{DC\text{max}}X}}{3ME} \quad (61) \]

The range of angle \( \delta \) between the AC equivalent output voltage of the converter and the AC grid voltage is related to the maximum DC current and modulation index. And from (61), the range of \( \delta \) can be expressed as:

\[ \delta \in [0, \delta_m] \cup [\pi - \delta_m, \pi + \delta_m] \cup [2\pi - \delta_m, 2\pi] \quad (62) \]

where

\[ \delta_m = \arcsin \left(\frac{2\sqrt{2I_{DC\text{max}}}X}{3ME}\right) \quad (63) \]

According to formulas (10), (11), (62), and (63), when the modulation index is certain, the influence of the DC current constraint on the power distribution region is shown in Figure 10.

The active power corresponding to the converter’s maximum DC current can be expressed as

\[ P_{\text{MAX}} = U_{DCN}I_{DC\text{max}} \quad (64) \]

And, it can be seen from Figure 10 that when the DC voltage and modulation index of the converter are certain, the maximum active power output is affected by the range of the DC current, while the maximum reactive power of the converter does not change.

5 | POWER DISTRIBUTION REGION UNDER THE MULTIPLE CONSTRAINTS

From the above analysis, all constraints affect the power output of the converter through influencing the magnitude or the phase of the equivalent output voltage. Therefore, when the converter is operating in the steady state, it can be considered that the magnitude and phase of the AC equivalent output voltage are the main factors that determine the power distribution of MMC.

From (18), (27), (35), (38), and (62), the phasor diagram of the equivalent output voltage under the multiple constraints can be obtained, as shown in Figure 11.

As shown in Figure 11, the equivalent output voltage phasor of the converter points toward the shadow area from the origin.
Based on Figure 11 and 12, the power distribution region of the converter under the multiple constraints is as shown in Figure 12.

From Figure 12, the active power output and input capacity of the converter are mainly determined by its DC current and AC current, and when the maximum AC current and the maximum DC current of the converter satisfies:

\[ 3EI_{\text{max}} < U_{\text{DCN}}I_{\text{DCmax}} \]  \hspace{1cm} (65)

The AC current constraint becomes the main factor that determines the active power input and output capacity of the converter.

From Figure 12, the input and output capacity of the converter’s reactive power are mainly determined by its modulation index and AC outlet voltage. When the converter can emit reactive power, its maximum reactive power output is determined by its maximum modulation index and maximum AC outlet voltage. When the converter cannot emit reactive power, its minimum reactive power input is determined by its maximum modulation index and maximum AC outlet voltage.
outlet voltage, while its maximum reactive power input is determined by its minimum modulation index and minimum AC outlet voltage.

According to formulas (21) and (36), when the AC outlet voltage of the converter and the modulation index satisfy:

$$\frac{U_{\text{sm}} \cos \theta - E}{\omega L_{\text{eq}}} < \frac{M_{\text{max}} U_{\text{DCN}} - E}{2 \sqrt{2} X}$$

The AC outlet voltage constraint becomes the main factor that determines the reactive power output capacity of the converter. And the maximum reactive power output capacity of the converter can be adjusted by changing its AC outlet voltage.

6 | MATCHING PRINCIPLE OF THE MULTIPLE CONSTRAINTS

It can be seen from Figure 12 that the active and reactive output of the converter cannot reach their maximums at the same time. And in the steady state, different control methods have different requirements for the converter’s active and reactive output capacities. Therefore, when designing the MMC, the parameters of the constraints should be matched to satisfy the requirements for its active and reactive output capacities to avoid meaningless investment and improve the utilization rate of the converter and the electrical equipment on the AC and DC sides.

The converter’s active power output capacity is an important indicator for its performance. When converter does not need to provide reactive power, its power factor could be 1. In this case, the matching between the constraints that have a large impact on the active output of the converter needs to be taken into consideration first.

From Figure 12, the main constraints of the converter’s active power output capacity are the DC current and the AC current. According to (64), the maximum active power output of the converter under the DC current constraint is $U_{\text{DCN}}I_{\text{DCmax}}$, and according to (32), the maximum active power output of the converter under the AC current constraint is $3EI_{\text{max}}$, so when the maximum values of the AC current and the DC current of the converter satisfy

$$P = U_{\text{DCN}}I_{\text{DCmax}} = 3EI_{\text{max}}$$

The converter’s DC current constraint and AC current constraint have the same effect on the converter’s maximum active power output capacity. With the matched current constraints, the power distribution region of the converter is shown in Figure 13.

In addition, some MMCs are required to have a certain reactive power output/input capacity for providing or absorbing constant reactive power in practical applications. In this case, the matching between the constraints that
have a large impact on the reactive power output capacity of the converter should be considered first, based on which the matching between the active power output capacity and the reactive power output capacity can then be examined.

From Figure 12, the main constraints that restrict the reactive power output capacity of the converter are the modulation index and the AC outlet voltage. From (23), the maximum reactive power output of the converter under AC outlet voltage constraint is $3E(U_{\text{max}} - E)/\omega L_{\text{eq}}$. And from (36), under the constraint of the modulation index, the maximum reactive power output of the converter is $3E(M_{\text{max}}U_{\text{DCN}} - E)/2\sqrt{2}X$. When the AC outlet voltage and the modulation index meet

$$3E(U_{\text{max}} - E)/\omega L_{\text{eq}} = 3E(M_{\text{max}}U_{\text{DCN}} - E)/2\sqrt{2}X$$  \hspace{1cm} (68)

The power distribution region of the converter is shown in Figure 14.

As shown in Figure 14, the maximum reactive power output of the converter under the AC voltage and the modulation index constraints is consistent through the matching. In this way, the converter will have the highest utilization rate of reactive power.

After meeting the requirement for the reactive power output capacity, the converter's active power output/input capacity and reactive power output capacity need to be matched as well, and the converter needs to be able to simultaneously provide the required reactive output and active output or input.

Regardless of the active output/input, the converter should have a certain reactive output capacity, and suppose that the required reactive output of the converter is $Q_{\text{Ei}}$, and substitute it into (36), the corresponding active power output under the modulation index constraint is...
Substituting $Q_E$ into (31), the corresponding active power output under the AC current constraint is:

$$P_{E2} = \sqrt{(3EI_{\text{max}})^2 - (Q_E)^2}$$

(70)

The maximum active power output of the converter under the DC current constraint $P_{E3}$ can be obtained by (64).

The power distribution region under the parameter matching principle is shown in Figure 15.

It can be seen from Figure 15 that when all the constraints are properly matched, the converter can provide the required reactive power while outputting the maximum active power. The boundary value of each constraint can be obtained by solving the formula $P_{E1} = P_{E2} = P_{E3}$. By appropriately arranging the rated voltage and current on the AC and DC sides and properly managing the purchase of equipment according to those boundary values, the converter’s equipment can be better utilized, and the investment and construction cost of the converter can be effectively reduced.

In summary, by adjusting the parameters of each constraint, the power distribution region of MMC under different constraints can have similar maximum active or reactive input/output capability. In this way, the difference of power distribution region of MMC under various constraints can be minimized, and the waste of investment for the constraints of high standards can be avoided. This paper only gives the matching principle of the constraints and does not discuss the overall optimization method too much.

## 7 | CASE STUDY

To verify the correctness of the power distribution region, a two-terminal ±250 kV MMC-HVDC system is established on the PSCAD/EMTDC platform as shown in Figure 16, and the main circuit parameters are listed in Table 1.

In the simulation model, in order to make the simulation close to the actual project, nearest level modulation method is adopted for the modulation process. And as shown in Figure 16, MMC1 adopts constant active power and reactive power control strategy, and MMC2 adopts constant DC voltage control strategy. And the block diagram of the MMC controller is shown in Figure 17.

As shown in Figure 17, the controller of MMC consists of an outer and an inner loop controller. And SW1 in the outer loop controller decides the control mode of the converter. For MMC1 in the simulation model, SW1 switches to location 2. And for MMC2, SW1 switches to location 1.

According to the data in Table 1 and formulas (15), (24), (37), and (53), the ranges of the constraints of the test converter are as follows.

The AC outlet voltage constraint:
The AC current constraint:

\[ I_{AC} \leq 1.66 \text{ kA} \]  \hspace{1cm} (72)

The modulation index constraint:

\[ 146.47 \text{ kV} \leq U_s \leq 153.98 \text{ kV} \]  \hspace{1cm} (71)

The AC current constraint:

\[ I_{AC} \leq 1.66 \text{ kA} \]  \hspace{1cm} (72)
0.75 ≤ \( M \) ≤ 0.95 \hspace{1cm} (73)

The DC voltage constraint:

\[
450 \text{kV} \leq U_{\text{DC}} \leq 550 \text{kV}
\] \hspace{1cm} (74)

The DC current constraint:

\[
I_{\text{DC}} \leq 0.98 \text{kA}
\] \hspace{1cm} (75)

Based on the above constraints and the obtaining method of the converter power distribution region, the power distribution region of the MMC under the multiple constraints is shown in Figure 18.

As shown in Figure 18, A (490, 0), B (490, 202.8), C (0, 216.5), D (−490, 202.8), E (−490, −230), F (0, −216.5), and G (490, −230) are operating points on the boundary of the power distribution region. \( A', C', C'', G' \) are points outside the power distribution region.

Case 1. In order to verify the correctness of power distribution region of MMC under all constraints shown in Figure 18, different operation points on the boundary in Figure 18 are selected for simulation verification. First, let the converter’s operating point be point A in normal operation. When \( t = 1.5 \) s, change the operating point to point B. When \( t = 2.5 \) s, change it to point C. When \( t = 3.5 \) s, change it to point F. Lastly, when \( t = 4.5 \) s, change it to point G. The simulation results are shown in Figure 19.

From Figure 19 and (76), (77), (79), and (80), when the operating point of MMC is point B, the modulation index, AC outlet voltage, and DC current reach the maximum values under the corresponding constraints. When the operating point changes to point C, the AC outlet voltage reaches the maximum value under the AC outlet voltage constraint. When the operating point of MMC is F and G, the AC outlet voltage reaches the minimum value under the AC outlet voltage constraint. Therefore, when MMC works on the boundary of the power distribution region, one or more electrical parameters of MMC will reach the critical value of the corresponding constraints, which is consistent with the power distribution region of MMC under multiple constraints shown in Figure 18.

Case 2. In order to verify whether the DC current of MMC exceeds the DC current constraint when it works at point \( A' \) near but outside of the power distribution region, the following simulation verification is carried out. First, let the converter’s operating point be point A in normal operation. When \( t = 1.07 \) s, change the operating point to point \( A' \) by setting the active power output of the converter to 525 MW. The simulation results are shown in Figure 20.

As shown in Figure 20, when the operating point of the MMC changes from A to \( A' \), the DC current rises to 1.05 kA. It can be seen from (75) that when MMC works at point \( A' \) outside the power distribution region, its DC...
current exceeds the upper limit of the DC current constraint, which is 1 kA. Therefore, the correctness of the influence of DC current constraint on the power distribution region of MMC can be verified. However, in order to show the power distribution region under different constraint more clearly, there is no additional control strategy being applied in the simulation system. So, the converter can still operate stably when DC current exceeds its constraint boundary. And the same is true for other variable parameter and case studies. In actual project, the constraint boundary proposed in the paper will be used as the limiting or blocking condition of the converter control system, and once an electrical quantity exceeds its constraint range, the converter will block or still maintain the normal operation state under the limiting controller to ensure the safe operation.

Case 3. In order to verify whether the modulation index of MMC exceeds the modulation index constraint when it works at point C' near but outside of the power distribution region, the following simulation verification is carried out. Suppose that the active output is 0 MW and the reactive output is 215 MVar when the converter operates stably. When \( t = 1.58 \) s, set the converter’s operating point at point C’. When \( t = 1.82 \) s, adjust it to point C''. The simulation results are shown in Figure 21.
As shown in Figure 21, when the operating point of MMC is C, its modulation index is 0.945. From (73), the modulation index is smaller than the maximum critical value of modulation index constraint, which is 0.95. And when MMC works at point C”, its modulation index rises to 0.965. From Case 3, when MMC works at the point C” outside the power distribution region, its modulation index will exceed the range of the modulation index constraint, and the simulation results are consistent with the power distribution region shown in Figure 18, which verifies the correctness of the influence of modulation index constraint on the power distribution region of MMC.

Case 4. In order to verify whether the AC outlet voltage of MMC exceeds the AC outlet voltage constraint when it works at point G’ near but outside of the power distribution region, the following simulation verification is carried out. First, let the converter’s operating point be point A in steady operation. When \( t = 0.83 \) s, adjust the operating point to point G. When \( t = 1.48 \) s, adjust it to point G’. The simulation results are shown in Figure 22.

As shown in Figure 22, when the operating point of MMC is G, its AC outlet voltage reaches the minimum value of the AC outlet voltage constraint. And when MMC works at point G’ outside the power distribution region, its AC outlet voltage will drop to 143.8 kV. From (71), when MMC works at point G’ outside the power distribution region, its AC outlet voltage will exceed the range of the AC outlet voltage constraint, and the simulation results are consistent with the power distribution region shown in Figure 18, which verified the correctness of the influence of AC voltage constraint on the power distribution region of MMC.

From Case 1, 2, 3, 4 and Figure 18, when MMC works on the boundary of the power distribution region under multiple constraints, all the electrical parameters of MMC meet the corresponding constraints. When MMC works outside of the power distribution region under the multiple constraints, one or more electrical parameters of MMC will exceed the corresponding constraints. Therefore, the simulation in PSCAD verifies the correctness of the power distribution region under the multiple constraints shown in Figure 18.

Suppose that the required reactive power output capacity \( Q_E = 180 \) MVar. According to the parameter matching principle, the adjusted constraints are as follows:

The AC outlet voltage constraint:

\[
146.47 \text{ KV} \leq U_s \leq 153.98 \text{ KV} \quad (76)
\]

The AC current constraint:
The modulation index constraint:

\[ 0.765 \leq M \leq 0.935 \] \tag{78} 

The DC voltage constraint:

\[ 450 \text{ KV} \leq U_{DC} \leq 550 \text{ KV} \] \tag{79} 

The DC current constraint:

\[ I_{DC} \leq 0.83 \text{ KA} \] \tag{80} 

Based on the above constraints, the power distribution region under the parameter matching principle is shown in Figure 23.

Case 5. In order to verify the correctness of power distribution region under the parameter matching principle between different constrains shown in Figure 23, different operation points on the boundary and near but outside of the power distribution region are selected to carry the following simulation. When \( t = 1 \text{ s} \), change the operating point from A to A'. When \( t = 1.5 \text{ s} \), adjust it to point B. When \( t = 2 \text{ s} \), adjust it to point B'. When \( t = 2.5 \text{ s} \), adjust it to point F. When \( t = 3 \text{ s} \), adjust it to point F'. When \( t = 3.5 \text{ s} \), adjust it to point G. When \( t = 4 \text{ s} \), adjust it to point H, and when \( t = 4.5 \text{ s} \), adjust it to point H'. The simulation results of this case are shown in Figure 24.

From Figure 24 and (76), (77), (78), (79), and (80), when MMC works on the boundary points of the power distribution region after considering the parameter matching principle, all the electrical parameters meet the corresponding constraints. In addition, when MMC works at the points outside of the power distribution region after considering the parameter matching, one or more electrical parameters of MMC will not meet the corresponding constraints as shown in Figure 23. Therefore, the analysis of Case 5 can verify the correctness of the proposed parameter matching principle.
between different constrains and the comprehensive power distribution region when considering the parameter matching principle shown in Figure 23.

8 | CONCLUSION

In this paper, the influence of various constraints on the distribution region is analyzed comprehensively, and the determining method of the power distribution region under the multiple constraints is proposed. The main results are as follows:

1. An equivalent mathematical model that reflects the relationship among the electrical parameters of MMC is established. Based on that, the basic power distribution region of MMC in abc stationary coordinate is obtained by using the phasor method, and the expression of the power distribution region is more concise, more intuitive, and clearer. The basic power distribution region of MMC is a circle with its center on the
negative half of Q-axis and its area is related to the AC grid voltage, the equivalent output voltage of the converter, and the equivalent impedance between them.

2. The influences of the various constraints such as the modulation index constraint, the AC outlet voltage constraint, the AC current constraint, the DC voltage constraint, and the DC current constraint on the power distribution region of the converter are analyzed separately. Considering the relationship among all the constraints, the power distribution region of MMC under multiple constraints is determined, which is an area symmetrical about the Q-axis, and its left and right boundaries are mainly limited by the current constraints.

3. The parameter matching principle between various constraints is proposed. By adjusting the parameters of each constraint, the power distribution region of MMC under different constraints can have similar maximum active or reactive input/output capability. In this way, the difference of power distribution region of MMC under various constraints can be minimized, and the waste of investment for the constraints of high standards can be avoided.

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CONFLICT OF INTEREST
The authors declare that they have no conflict of interest to this work.

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