Different regimes of the uniaxial elongation of electrically charged viscoelastic jets due to dissipative air drag

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Abstract

We investigate the effects of dissipative air drag on the dynamics of electrified jets in the early stage of the electrospinning process. The main idea is to use a Brownian noise to model air drag effects on the uniaxial elongation of the jets. The numerical model is used to probe the dynamics of electrified polymer jets at different conditions of air drag force. Numerical simulations show that the elongation of the charged jet filament is significantly affected by the presence of air drag force, providing prospective beneficial implications for the optimal design of future electrospinning experiments.

1 Introduction

In the recent years, organic nanofibers have gained a broad fundamental and industrial interest, due to their peculiar physical properties and to their numerous potential applications, such as tissue engineering, air and water filtration, drug delivery and regenerative medicine. Flexible fibers can be used on a micro- down to nano-scale in electrical, mechanical and optical systems. In particular, the small cross-section of electrospun nanofibers in combination with their extreme length (in principle up to km when polymer solutions with high degree of molecular entanglement are used) provides a high surface-area ratio which offers intriguing perspectives for practical applications. As a consequence, several studies have been focused on the characterization and production of such 1D organic nanostructures. Many reviews[1, 2, 3, 4, 5, 6] and books[7, 8, 9] concerning electrospun nanofibers have been published in the last two decades.

Following the pioneering works of Rayleigh[10] and, later, Zeleny[11], electrospun nanofibers are synthesized at laboratory scale by the uniaxial elongation of a jet, which is ejected at the nozzle from the surface of a charged polymer solution (see Fig 1). This elongation of a jet is produced by applying an intense external electrostatic field[12, 13, 14] (typically 10⁵ – 10⁶ V · m⁻¹) between the spinneret and a conductive collector. During the jet path from the nozzle to the collector, the stream cross-section can decrease up to six orders of magnitude, providing a nano-sized jet, which is the prime goal of electrospinning experiments. Electrospinning involves mainly two sequential stages in the uniaxial elongation of the extruded polymer jet: an initial quasi-steady stage, in which the electric field stretches the jet in a straight path away from the nozzle, and a second stage characterized by a bending instability induced by small perturbations, which misalign the jet from its initial axis of elongation. These small disturbances can originate from mechanical vibrations at the nozzle or hydrodynamic-aerodynamic perturbations along the jet path. According to the Earnshaw’s theorem[15], a misalignment provides an electrostatic-driven bending instability before the jet reaches the conductive collector, where the fibers are finally deposited. As a consequence, the jet path length between the nozzle and the collector increases, and the stream cross-section undergoes a further decrease. The prime goal of electrospinning experiments is to minimize the radius of the collected fibers. By a simple argument of mass conservation, this is tantamount to maximizing the jet length by the time it reaches the collecting plane. By the same argument, it is therefore of interest to minimize the length of the initial stable jet region. Consequently, the bending instability is a desirable effect, as it produces a higher surface-area-to-volume ratio of the jet, which is transferred to the resulting nanofibers.

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Notwithstanding its major interest, a comprehensive investigation to understand the transition between different regimes of the jet dynamics is not complete, and the relation between dissipative-perturbing forces and the first quasi-steady stage of the electrospinning process is still in need of further clarification. In particular, an uniaxial dissipative-perturbing force, such as the air drag force, can reduce the initial straight path, so that the distance covered by the jet between the nozzle and the collector increases, and the stream cross-section can further decrease.

Simulation models can be a useful tool to elucidate the phenomenon and provide information which might be used for the design of future electrospinning experiments. Numerical simulations can be used to improve the capability of predicting the key processing parameters and exert a better control on the resulting nanofiber morphologies. In the last years, many modeling papers were published[16] [17] [15] [13] [20] [21]. These models usually treat the jet filament as obeying the equations of continuum mechanics[22] [23] [24] [18] [25], or as a series of discrete elements obeying the equations of Newtonian mechanics[16] [17], as it is the case in this work.

Here we investigate the uniaxial elongation of an electrified polymer jet in the early stage of its dynamics at the nozzle of the ejecting apparatus and under stationary dissipative-perturbing force. We model the perturbation effect by a simple Brownian term. In particular, we assume that a Brownian term can efficiently reproduce a stationary perturbation related to multiple simultaneous tiny impacts along the direction of the jet elongation axis, as in the case of an air drag force generated by the movement of a polymer jet through a gaseous medium. Relations between air drag force and Brownian motion were already proposed in literature starting from experimental observations[26] [27] [28] [29]. In the present work, we extend the unidimensional bead-spring model, developed by Pontrelli et al. [30], to include a dissipative-perturbing force which models the effects of the air drag force. This is realised by adding two force terms to the set of equations of motion (EOM): a random term and a dissipative term. Further, we assume that the two terms obey the fluctuation-dissipation Langevin relation, so that the system is described by a Langevin-like stochastic differential equation. Anticipating the conclusions, we observe that a second quasi steady stage appears by applying different magnitude of dissipative-perturbing force. Further, the proposed approach provides a useful starting point to develop stochastic three-dimensional models of electrospinning process.

The article is organized as follows. In Sec. 2 we present our model for the uniaxial elongation of a viscoelastic jet, and we introduce the corresponding set of stochastic EOM, which governs the dynamics of system. Hence, the time integrator of the stochastic EOM is also reported in Sec. 2. Results are reported and discussed in Sec. 3. Finally, the Conclusions are outlined in Sec. 4.

2 Model and time integrator

Let us consider a rectilinear electrified viscoelastic jet in the electrospinning experiment. In order to model the stretching, we represent the filament by a viscoelastic dumbbell with two charged beads of mass \(m\) and same charge \(e\) (Fig. 1). We assume one of the two beads to be fixed, denoted by symbol \(a\), and the other, denoted \(b\), free to move. The beads are located at mutual distance \(l\). The dumbbell, \(ab\), models a viscoelastic jet, so that a viscoelastic force pulls \(b\) back to \(a\). We denote \(h\) the distance of the collector plate from the injection point, and \(V_0\) the applied voltage between the two beads. The bead \(b\) is subject to three different forces: the force due to the external electrical field \(V_0/h\), the Coulomb repulsive force between the two beads, and the viscoelastic force. Anticipating the results in Sec. 4, in this scheme the competition between electrostatics and viscoelastic forces characterizes the first stage of the elongation process, while the second stage is governed by the force due to the external electrical field, which drives the jet towards the collector.

The time evolution of viscoelastic materials can be properly described by the following ordinary differential equations proposed by Reneker et. al [16] . The combined action of the three introduced forces (gravity and surface tension are neglected) govern the deformation of the jet by the following equation:

\[
m \frac{dv}{dt} = - \frac{e^2}{l^2} + \frac{eV_0}{h} + \pi r^2 \sigma,
\]

where \(v\) is the velocity of the bead \(b\), \(t\) the time, \(r\) the cross-sectional radius of the filament, \(\pi r^2 \sigma\) the force pulling the bead \(b\) back to \(a\) given by the viscoelasticity of the jet (assumed positive), and \(\sigma\) the stress of the viscoelastic force. Assuming a viscoelastic Maxwellian liquid jet, the time evolution of the stress \(\sigma\) related to the viscoelastic force is provided by the equation:

\[
\frac{d\sigma}{dt} = G \frac{dl}{dt} - \frac{G}{\mu} \sigma,
\]

where \(G\) is the elastic modulus, and \(\mu\) the viscosity of the fluid jet. The velocity \(v\) satisfies the kinematic equation:

\[
\frac{dl}{dt} = -v.
\]
In order to adopt a nondimensional form of these equations as is customary in fluid mechanics, we define a length scale \( L = (\epsilon^2 / \pi r^2 G)^{1/2} \) with \( r_0 = r \) \((t = 0)\), and the relaxation time \( \tau = \mu / G \). Then we define \( \bar{t} = t / L \) in units of the equilibrium length \( L \) at which Coulomb repulsion matches the reference viscoelastic stress \( G \). The time \( \bar{t} = t / \tau \) is the time \( t \) rescaled in \( \tau \) units. We define \( W = -v \) and \( \bar{W} = W \cdot \tau / L \). Applying the condition that the volume of the jet is conserved, \( \pi r^2 \bar{t} = \pi r_0^2 L \), we write the set of equations of motion:

\[
\frac{d\bar{t}}{dt} = \bar{W} \tag{4a}
\]

\[
\frac{d\bar{\sigma}}{dt} = \frac{\bar{W}}{\bar{t}} - \bar{\sigma} \tag{4b}
\]

\[
\frac{d\bar{W}}{d\bar{t}} = \bar{V} - F_{ve} \bar{\sigma} + \frac{Q}{\bar{t}^2} \tag{4c}
\]

where the parameters denoted by bars are dimensionless. The dimensionless groups are given by:

\[
Q = \frac{\epsilon^2 \mu^2}{L^3 m G^2} \]
\[
V = \frac{e V_0 \mu^2}{h L m G^2} \]
\[
F_{ve} = \frac{\pi r^2 \mu^2}{m L G} \tag{5}
\]

A reminder for the definitions of the dimensionless parameters is reported in Tab 1. We now extend this model to include air drag effects on the dynamics of electrified jets. We add two force terms to Eq. 4c: a random term and a dissipative term. Denoted \( \bar{D}_a \) a generic diffusion coefficient in velocity space and \( \alpha \) a dissipative term, we assume that the dissipative term has the form \( \alpha W \), while the random force term the form \( \sqrt{2D_v} \eta(t) \), where \( \eta(t) \) is a stochastic process that is nowhere differentiable with \( < \eta(t_1) \eta(t_2) > = \delta \left( |t_2 - t_1| \right) \), and \( < \eta(t) > = 0 \). Note that the dissipative term \( \alpha W \) is usually dependent on the geometry of the jet, which changes in time. In particular, based on experimental results, the dissipative air drag force was proposed equal to \( f_{air} = 0.65 \pi r_0 \mu_0 / (2r_0 / \nu_a)^{-0.81} W^{1.21} \), where \( \mu_0 \) and \( \nu_a \) are the air density and kinematic viscosity, respectively. Assuming a constant volume of the jet, so that \( \bar{t} = t_0 \sqrt{L / \bar{t}} \), we obtain

\[
\frac{dW}{dt} = \frac{e^2}{\bar{t}^3} + \frac{e V_0}{h} - \pi r^2 \sigma - m \alpha W + \sqrt{2m^2 \bar{D}_v} \eta(t) \tag{6}
\]

In order to be consistent with the adopted description, we introduce:

\[
\bar{\alpha} = \alpha \tau \]
\[
\bar{D}_v = D_v \cdot \tau^3 / \bar{t}^2 \tag{7}
\]

where \( \bar{\alpha} \) and \( \bar{D}_v \) are the dimensionless version of \( \alpha \) and \( D_v \). Using these definitions and the dimensionless groups, we rewrite the Eq. 6 as:

\[
\frac{d\bar{W}}{d\bar{t}} = \bar{V} - F_{ve} \bar{\sigma} + \frac{Q}{\bar{t}^2} - \bar{\alpha} W + \sqrt{2\bar{D}_v} \eta(\bar{t}) \tag{8}
\]

We highlight that \( \bar{D}_v \) sets the width of the fluctuations. In particular, it is possible to demonstrate that the variance \( \sigma_{\bar{W}} (t)^2 \) of the velocity \( \bar{W} \) due to only the random and dissipative terms at time \( \bar{t} \), computed over an ensemble of stochastic trajectories, is equal to \( \sigma_{\bar{W}} (t)^2 = \eta \left( \bar{W} (t) \right)^2 \leq \left( \bar{D}_v / \bar{\alpha} \right) \left( 1 - e^{-2\alpha \tau} \right) \), and, consequently, \( \lim_{t \to \infty} \sigma_{\bar{W}} (t)^2 = \bar{D}_v / \bar{\alpha} \), which is sometimes called fluctuation-dissipation Langevin relation.

In order to integrate the differential equations of motion we discretise \( t \) as \( t_i = t_0 + i \Delta t \) with \( i = 1, \ldots, n_{steps} \) where \( n_{steps} \) denotes the number of sub-intervals. In this work we use the explicit strong order scheme proposed by E. Platen, whereof the order of strong convergence was evaluated in literature equal to 1.5. It is worth to underline that for the specific case under investigation the diffusion coefficient vector has only one non-zero component equal to \( \bar{D}_v \), which is constant. As consequence, the original integration scheme considerably simplifies. This scheme avoids the use of derivatives by corresponding finite differences in the same way as Runge-Kutta schemes do for ODEs in a deterministic setting, and it is briefly summarised as follows.
Let us consider a Brownian motion vector process \( \mathbf{X} = \{X_t, t\} \) of \( d \)-dimensional satisfying the stochastic differential equation
\[
\frac{d\mathbf{X}}{dt} = \mathbf{a}(t, X^1, \ldots, X^d) + \mathbf{b}d\Omega
\]
where \( \mathbf{a} \) and \( \mathbf{b} \) are vectors of \( d \)-dimensional usually called drift and diffusion vector coefficients, and \( \Omega(t) \) denotes a Wiener process. Denoted \( Y^k_t \) the approximation for the \( k \)-th component of the vector \( \mathbf{X} \) at time \( t \) the form of the integrator is:
\[
Y^k_{t+\Delta t} = Y^k_t + b^k \Delta \Omega + \frac{1}{2\sqrt{\Delta t}} \left[ a^k \left( \tilde{Y}_+ - \tilde{Y}_- \right) \right] \Delta \Psi + \frac{1}{4} \left[ a^k \left( \tilde{Y}_+ - \tilde{Y}_- \right)^2 + 2a^k + a^k \left( \tilde{Y}_- \right) \right] \Delta t
\]
with the vector supporting values
\[
\tilde{Y}_+ = Y_t + a\Delta t \pm b\sqrt{\Delta t} \quad \tilde{\Phi}_+ = \tilde{Y}_+ \pm b \left( \tilde{Y}_+ \right) \sqrt{\Delta t}.
\]
Here, \( \Delta \Omega \) and \( \Delta \Psi \) are normally distributed random variables constructed from two independent \( N(0,1) \) standard Gaussian distributed random variables \( U_1 \) and \( U_2 \) by means of the linear transformation
\[
\Delta \Omega = U_1 \sqrt{\Delta t}, \quad \Delta \Psi = \frac{1}{2} \Delta t^{3/2} \left( U_1 + \frac{1}{\sqrt{3}} U_2 \right).
\]

3 Results and Discussion

In the case \( \bar{a} = 0 \) and \( D_\alpha = 0 \), Eq. 8 reduces to Eq. 3. Consequently, the integration scheme described by Eq. 10 can be used also to integrate Eqs. 4 for the deterministic case. We intend now to exploit the time reversibility to assess a suitable time step \( \Delta t \) for the specific case under investigation. To this purpose, we integrate in time forward and backward Eqs. 4 in the interval \( \ell_0 = 0 \) and \( \ell_5 = 5 \) at different values of timestep \( \Delta t \), and we compute the average absolute error \( \Delta Y = |Y_{2n,steps} - Y_0| \), where \( Y \) is a generic variable of Eqs. 4 (for instance, \( \ell \)). We noted in preliminary test that the average absolute error \( \Delta Y \) is lower than \( 10^{-12} \) for time step \( \Delta t = 10^{-2} \). Thus, we conclude that a time step \( \Delta t \) equal to \( 10^{-3} \) is a suitable and conservative choice for the specific case under investigation.

We next discuss some metastable and asymptotic regimes of electrified jets, associated with the Eq. 8. As a reference case, we consider the typical values of \( Q = F = 12 \) and \( V = 2 \), already investigated in previous works. First, we study the deterministic case, given by imposing \( \bar{a} = 0 \) and \( D_\alpha = 0 \). All the simulations start at the same initial conditions: \( \ell = 1 \), \( \bar{a} = 0 \) and \( W = 0 \). In Fig 2 we report the time evolution of the velocity \( W(\ell) \). We identify two sequential stages in the elongation process (denoted \( A \) and \( B \) in Figure). In the first regime, we observe a little increase of \( W(\ell) \) which rises up to achieve a quasi stationary point denoted \( \ell_* \), where the viscoelastic force \( F_{ve}\bar{a}(\ell_*) \) balances the sum of the two force terms \( \frac{Q}{\ell(\ell_a)^2} \) and \( V \), providing an merely zero value of the total force. Then, in the second stage the velocity trend comes to a near linearly increasing regime. Note that the point \( \ell_* \) corresponds to the lower limit of the derivative \( \partial W/\partial \ell \), and it discerns the two stages of the dynamics. In Fig. 3 we show the three force terms \( \frac{Q}{\ell(t)^2} \), \( V \) and \( F_{ve}\bar{a}(\ell) \). In the first stage we observe an early transient, characterized by the build-up of the term \( \frac{F_{ve}\bar{a}(\ell)}{\ell(t)^2} \), which peaks around \( \ell = 0.5 \) under the Coulomb force \( \frac{Q}{\ell(t)^2} \) and the external electric field \( V \). In the second stage, the term \( \frac{F_{ve}\bar{a}(\ell)}{\ell(t)} \) and \( \frac{Q}{\ell(t)^2} \) start to decay, and the dynamic tends asymptotically to be governed only by the term \( V \).

We point out that a smaller \( \ell_* \) corresponds to a shorter straight path of the electrified jet, so that the distance covered by the jet between the nozzle and the collector increases, and the stream cross-section decreases, which is a main goal of electrospinning experiments. In Fig 2 we report also the time evolution of the length \( \ell(\ell) \). Note that after \( \ell_* \) the velocity comes to a near linearly increasing regime, as expected by the mentioned relation \( \lim_{\ell \to \infty} \partial W(\ell) / \partial \ell = V \). Similarly, we observe that \( \ell(\ell) \) increases as a quadratic term, since the limit of its second derivative is constant, \( \lim_{\ell \to \infty} \partial^2 \ell(\ell) / \partial \ell^2 = V \).

We now investigate the elongation of jet under stochastic perturbation. In particular, it is our interest to explore how the position of \( \ell_* \) is altered by the dissipative-perturbing force \( -\bar{a}W + \sqrt{2D_\eta/q} \ell \) in Eq. 8. We investigate different magnitudes of \( \bar{a} \) keeping constant the ratio \( D_\eta/\bar{a} \). We stress that the magnitude of
the parameters $\tilde{\alpha}$ and $\tilde{D}_\nu / \tilde{\alpha}$ is depending on the amplitude of the modeled perturbation. As example, let us consider the aforementioned experimental formula $m \alpha = 0.65 \pi r^3 \rho_a (2 r / \nu_a)^{-0.81}$. Taking $\rho_a = 1210$ g/m$^3$, $\nu_a = 0.15$ m$^2$/sec, $r = 2 \cdot 10^{-5}$ m, $l = 3.19 \cdot 10^{-3}$ m (which corresponds to $l (l_\ast)$ for the deterministic case) we obtain $m \alpha = 7.12 \cdot 10^{-5}$ g/s. For a typical value of relaxation time $\tau = 10^{-2}$ s, and density of the liquid jet $\rho_l = 1000$ kg/m$^3$, we obtain a value of $\tilde{\alpha}$ equal to 0.18. In order to properly represent the statistical process, we run 10000 independent trajectories for each different value of $\tilde{\alpha}$. Hence, we compute the time dependent mean value of our observables along the dynamics. We also assess the statistical dispersion as interquartile range (IQR). All the trajectories were carried out at the reference values of $Q = F = 12$ and $V = 2$.

In Fig. 4 we show the time evolution of the velocity $\tilde{W} (\tilde{t})$ for the different values of friction coefficient $\tilde{\alpha}$. First of all, we note, in all the cases with $\tilde{\alpha} \neq 0$ the presence of two quasi-stationary points denoted $\tilde{t}_\ast$ and $\tilde{t}_{\ast\ast}$, instead of only one, like in the deterministic system. Here, $\tilde{t}_\ast$ is the point of coordinates $(\tilde{t}_\ast, \tilde{W} (\tilde{t}_\ast))$, where the system reaches the condition $\partial \tilde{W} (\tilde{t}) / \partial \tilde{t} = 0$ for the first time, and similarly $\tilde{t}_{\ast\ast}$ is the point $(\tilde{t}_{\ast\ast}, \tilde{W} (\tilde{t}_{\ast\ast}))$ where the system reaches the condition $\partial \tilde{W} (\tilde{t}) / \partial \tilde{t} = 0$ for the second time. Hence, in Fig. 4 we identify three sequential stages of the uniaxial elongation process (denoted in the Figure as $A$, $B$ and $C$, respectively) which are delimited by the points $\tilde{t}_\ast$ and $\tilde{t}_{\ast\ast}$. In order to explain the three different stages, we need to examine the force terms. First of all, the term $\frac{Q}{\tilde{t} (\tilde{t})^2}$ decays rapidly so that it plays a secondary role. The first stage is characterized by the terms $\frac{F_{ve} \tilde{\alpha} (\tilde{t})}{\tilde{t} (\tilde{t})}$ and $\tilde{\alpha} \tilde{W}$, which increase as consequence of the jet stretching due to the external electric field $V$. This initial stage comes to the first quasi stationary point $\tilde{t}_\ast$, where the terms $\frac{F_{ve} \tilde{\alpha} (\tilde{t})}{\tilde{t} (\tilde{t})}$ and $\tilde{\alpha} \tilde{W}$ balance the Coulomb term $V$. We report in Fig. 6 all the force terms for the case $\tilde{\alpha} = 1$. In the second stage, the terms $\frac{F_{ve} \tilde{\alpha} (\tilde{t})}{\tilde{t} (\tilde{t})}$ and $\tilde{\alpha} \tilde{W}$ become larger in modulus than the opposite Coulomb term $V$, so that we observe a decrease of the velocity $\tilde{W}$ due to the two terms. At the same time, the term $\frac{F_{ve} \tilde{\alpha} (\tilde{t})}{\tilde{t} (\tilde{t})}$ starts to decay, and the sum of the two terms $\frac{F_{ve} \tilde{\alpha} (\tilde{t})}{\tilde{t} (\tilde{t})}$ and $\tilde{\alpha} \tilde{W}$ becomes insufficient to balance the opposite Coulomb term $V$. Hence, we observe the second quasi stationary point $\tilde{t}_{\ast\ast}$. In the third stage, after $\tilde{t}_{\ast\ast}$, the jet dynamics is governed only by the remaining opposite terms $V$ and $\tilde{\alpha} \tilde{W}$, since the term $\frac{F_{ve} \tilde{\alpha} (\tilde{t})}{\tilde{t} (\tilde{t})}$ tends to zero.

As consequence, the velocity $\tilde{W}$ rises again under the external electrical force $V$, to achieve asymptotically a final stationary regime where the dissipative force $\tilde{\alpha} \tilde{W}$ balances completely $V$. This final stationary regime is clearly evident in Fig. 6 where the trend of $\frac{F_{ve} \tilde{\alpha} (\tilde{t})}{\tilde{t} (\tilde{t})}$, $V$, $\frac{Q}{\tilde{t} (\tilde{t})^2}$ and $\tilde{\alpha} \tilde{W} (\tilde{t})$ is represented for the case $\tilde{\alpha} = 1$.

Tab. 2 summarizes the computed $\tilde{t}_\ast$ and $\tilde{t}_{\ast\ast}$ for all three values of $\tilde{\alpha}$. We note that the gap between $\tilde{t}_\ast$ and $\tilde{t}_{\ast\ast}$ becomes larger by increasing $\tilde{\alpha}$, and the path length through the second stage increases (see Fig. 4). Note that the straight path of the electrified jet is described by the observable $\tilde{l} (\tilde{t}_\ast)$. We observe a decrease of the length $\tilde{l} (\tilde{t}_{\ast\ast})$ by increasing $\tilde{\alpha}$ (Fig. 5), provided that the initial stage of the elongation process is reduced as a consequence of the uniaxial perturbation. Advantages for electrospinning processes coming from unveiling and rationalizing in depth these effects can be numerous, including the possibility of better controlling the dynamics of electrified jets, and consequently the diameter and morphology of collected nanostructures, as well as the assembly and positions of nanofibers impinging onto the collector. A route to better control these nanofabrication processes could therefore entail the identification and tailoring of air drag mechanisms, which can be eventually performed by means of suitable gas-injecting systems nearby the spinneret.

4 Conclusions

Summarizing, we have investigated the flow of charged viscoelastic fluids in the presence of stationary stochastic perturbations. A Brownian term has been used to model the effects of a stationary dissipative-perturbing force on the stretching properties of electrically charged jets, providing significant qualitative new insights. The main finding is that perturbation forces, such as air drag force, change significantly the dynamics of the electrospinning process, leading to the presence of a second quasi stationary point. Further, we observe a reduction of the linear extension of the jet in the initial stage of the electrospinning process by increasing the dissipative force term. These conclusions may provide benefits to set up the experimental conditions which might enhance the efficiency of the electrospinning process. These may include, in particular, environmental vibrations and air flows in the process atmosphere.
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### Tables

| Physical Parameter                          | Dimensional Symbol (units) | Dimensionless Symbol | Dimensionless Symbol | Definition                        |
|--------------------------------------------|----------------------------|----------------------|----------------------|-----------------------------------|
| Time                                       | t (s)                      | l                    | l/τ                  |                                   |
| Length of the rectilinear jet part         | l (m)                      | l̃                   | l/L                  |                                   |
| Velocity                                   | u (m/s)                    | u̇                   | u · τ/L              |                                   |
| Absolute velocity                          | W (m/s)                    | W̃                   | W · τ/L              |                                   |
| Stress                                     | σ (g/(cm s³))              | σ̃                   | σ/G                  |                                   |
| Friction coefficient                       | α (s⁻¹)                   | α̃                   | ατ                   |                                   |
| Velocity diffusion coefficient             | Duv (m²/s³)                | Duṽ                 | Duv · τ³/L²          |                                   |

Table 1: Definitions of the dimensionless parameters employed in the text. For each dimensional parameter the corresponding unit of measurement is reported in parentheses. We remind that G is the elastic modulus, μ the viscosity of jet, τ = μ/G the relaxation time, and L the equilibrium length at which Coulomb repulsion matches the reference viscoelastic stress G.
| $\bar{\alpha}$ | $D_{\alpha}/\bar{\alpha}$ | $t_*$ | $l(t_*)$ | $\bar{\sigma}(t_*)$ | $W(t_*)$ | $l(t_{**})$ | $\bar{\sigma}(t_{**})$ | $W(t_{**})$ |
|-------------|-----------------|-------|----------|----------------|--------|----------|----------------|--------|
| 0           | 0               | 0.86  | 3.39     | 0.81           | 3.52   | ...      | ...            | ...    |
| 0.1         | 1               | 0.68  | 2.73     | 0.74           | 3.34   | 1.12     | 4.19           | 0.82   | 3.31 |
| 0.5         | 1               | 0.49  | 2.04     | 0.58           | 2.94   | 1.86     | 5.46           | 0.60   | 2.23 |
| 1           | 1               | 0.42  | 1.79     | 0.48           | 2.61   | 2.20     | 4.98           | 0.44   | 1.42 |

Table 2: Values of the adimensional variables length $\bar{l}$, stress $\bar{\sigma}$ and velocity $\bar{W}$ at the quasi-stationary points $t_*$ and $t_{**}$ computed for different values of $\bar{\alpha}$. For the deterministic case we report the data of the unique quasi-stationary point $t_*$. Note that we observe a larger gap between $t_*$ and $t_{**}$ by increasing $\bar{\alpha}$. Furthermore, we stress the decrease of the length $l(t_*)$ (see Fig. 5), providing that the initial stage of the elongation process is reduced as a consequence of the uniaxial perturbation.

**Figures**

Figure 1: Schematic drawing of the electrospinning process (not in scale), showing $h$ the distance between the collector plate and the injection point (nozzle), $V_0$ the applied voltage between these two elements, and the $z$ reference axis whose origin is fixed at the injection point.
Figure 3: Time evolution for the deterministic case of the three force terms: $-\frac{F_{ve}\bar{\sigma}}{l}$ (dashed line), $V$ (dotted, horizontal line), and $\frac{Q}{l^2}$ (dashed-dotted line). Note that the viscoelastic force $-\frac{F_{ve}\bar{\sigma}}{l}$ peaks at about $\bar{t} = 0.5$. Subsequently, all the force terms $-\frac{F_{ve}\bar{\sigma}}{l}$ and $\frac{Q}{l^2}$ decay to zero with the exception of $V$, which is due to the external electrical field.

Figure 2: Time evolution of the velocity $\bar{W}(\bar{t})$ (continuous line). Two stages of the elongation process are observed: an early transient (labeled A), which comes to a quasi stationary point (denoted by a star symbol). The quasi-stationary point $\bar{t}_* \approx$ corresponds to the lower limit of the derivative $\partial \bar{W}(\bar{t}) / \partial \bar{t}$. We also report the length $\bar{l}(\bar{t})$ rescaled by a factor $1/5$ (dotted line).
Figure 4: Time evolution of the velocity $\bar{W}(\bar{t})$ for different values of $\bar{\alpha}$. From top to bottom curves: $\bar{\alpha} = 0$ (continuous line), 0.1 (dashed-dotted line), 0.5 (dashed line), and 1 (dotted line), keeping $\bar{D}_c/\bar{\alpha} = 1$ for all the cases. The quasi-stationary points are depicted as symbol for all the $\bar{\alpha}$ values. The error bars are computed as IQR. On top the initial trend $\bar{W}(\bar{t})$ is enlarged. Here the three sequential stages of the uniaxial elongation process are labeled $A$, $B$ and $C$, and drawn in blue, red and green, respectively.

Figure 5: Time evolution of the term $\bar{l}(\bar{t})$ for different values of $\bar{\alpha}$ equal to 0 (continuous line), 0.1 (dashed-dotted line), 0.5 (dashed line), and 1 (dotted line), keeping $\bar{D}_c/\bar{\alpha} = 1$ for all the cases. The quasi-stationary points are depicted as in Fig. 4. The error bars are computed as IQR.
Figure 6: Time evolution of the four force terms: \(-\bar{\alpha}\bar{W}\) (continuous line), \(-\frac{F_{ve}\bar{\sigma}}{\bar{t}}\) (dashed line), \(V\) (dotted line), and \(\frac{Q}{\bar{t}^2}\) (dashed-sotted line) for the stochastic case \(\bar{\alpha} = 1\) and \(\bar{D}_v/\bar{\alpha} = 1\). The quasi-stationary points are depicted as star symbols. The viscoelastic force \(-\frac{F_{ve}\bar{\sigma}}{\bar{t}}\) peaks at about \(\bar{t} = 0.5\). Subsequently, the force terms \(-\frac{F_{ve}\bar{\sigma}}{\bar{t}}\) and \(\frac{Q}{\bar{t}^2}\) decay to zero, while the term \(-\bar{\alpha}\bar{W}\) tends asymptotically to a stationary regime, and the external electrical field \(V\) remains constant.