Legal requirements for NAWIs: are they good enough for customers’ protection?

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Abstract. This study intends to show a statistical probe based on Bayesian inference to demonstrate if the current legal requirements are enough to ensure customers’ protection.

1. Introduction
The objective of legal metrology for Non Automatic Weighting Instruments (NAWIs) is the protection of the user to avoid financial prejudice and ensure the customer’s protection. Its intention is to ensure correct measurement results over the complete life cycle of the instrument so that the error of any instrument reading is less than the corresponding maximum permitted error according to the accuracy class of the instrument. The application of legal metrology for these instruments is based at international level on the recommendation OIML R 76 [1]. At the European level it is based on the standard EN 45 501 [2], although both documents basically include the same requirements in order to ensure the instruments’ conformity. In this paper the recommendation OIML R 76 [1] will be followed. The conformity assessment is based on three metrological controls: type approval, initial verification, subsequent verification and inspection. In Europe instead of type approval and initial verification a conformity evaluation based on a modular approach has been implemented with the same requirements. In the case of NAWIs the maximum permitted errors for type approval and initial verification are the same, and they are half of the ones demanded for subsequent verification and inspection, which are the tolerance limits for customer’s protection. It is clear that the performance of an instrument may change with time, so the real objective of the conformity assessment is to ensure that the possible errors are below the tolerance limits during the complete life cycle of the instrument. In order to verify compliance with the requirements different measurements are performed in each metrological control. The intention of this paper is to demonstrate how effective the metrological control is in order to ensure its objective.

This study is based on Bayesian inference according to the guide JCGM 106 [3] to determine: the probability of non-conformity for an instrument after the initial verification and after the subsequent verification or inspection just using the information derived from the corresponding requirements fulfilment; the probability of non-conformity of an instrument after the initial verification and the subsequent verification using the information derived from the corresponding requirements fulfilment and also the fulfilment of the type approval requirements; and the risks for, not only the customer, but also the manufacturer. The global customer’s risk will be the probability that a non-conforming instrument will be accepted based on a future measurement result. The global manufacturer’s risk is the probability that a conforming instrument will be rejected based on a future measurement result.

2. Methodology
The idea behind is to use the prior information of the instrument itself before the measurements (type approval requirements) and the measurements themselves (initial or subsequent verification or inspection requirements), both characterized by probability density functions, to provide a “posterior” probability density function used to determine the probability of conformity by means of the application of Bayes’ theorem. In fact we are interested in the probability of non-conformity, which is equal to the unity minus the probability of conformity.

Three cases are studied depending on the knowledge about the prior information of the instrument. In the first case no information is provided, so the assumed probability density function is 1. In the
second case an instrument that fulfills the requirements of type approval is considered. In the third case a full production of the same type of instruments with their corresponding type approval is considered. This is the case when we speak about risk instead of probability of non-conformity. For each of these three cases two other subcases are considered: the instrument passes the initial verification, or the instrument passes the subsequent verification or inspection (requirements are the same for subsequent verification and inspection).

2.1. Determinations of probabilities of non-conformity

These determinations are based on the assumption that the instrument behavior for every load is such that the corresponding measurement error follows a Gaussian (normal) distribution. In this case it can be characterized by an estimate, the mean $y$, and a standard deviation, the corresponding uncertainty $u$.

For this case the guide JCGM 106 [3] provides this formula for the determination of the probability of non-conformity,

$$ 1 - p_e = 1 - \Phi\left(\frac{Tu - y}{u}\right) + \Phi\left(\frac{TL - y}{u}\right) \quad \text{with} \quad \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp(-t^2/2)\,dt $$

where $Tu$ is the upper tolerance limit, $TL$ is the lower tolerance limit.

When no prior information of the instrument is provided, $y = y_0$, is the mean of the measurement results, and $u = u_0$, is the corresponding measurement uncertainty; but, when the prior information of the instrument can be characterized by a normal distribution with mean $y_0$ and standard uncertainty $u_0$ then,

$$ y = y_0 + \frac{y_0}{u_0} \quad \text{and} \quad u = \sqrt{\frac{1}{u_0^2} + 1} $$

This guide also provides the customer’s (consumer’s) risk $R_C$ and the manufacturer’s (producer’s) risk $R_P$ as follows, where $A_U$ is the upper acceptance limit and $A_L$ is the lower acceptance limit,

$$ R_C = \int_{-\infty}^{\frac{Tu - y_0}{u_0}} F(z)\varphi_0(z)\,dz + \int_{\frac{Tu - y_0}{u_0}}^{\infty} F(z)\varphi_0(z)\,dz \quad \text{and} \quad R_P = \int_{\frac{TL - y_0}{u_0}}^{\infty} (1 - F(z))\varphi_0(z)\,dz $$

with

$$ F(z) = \Phi\left(\frac{A_U - y_0 - u_0z}{u_m}\right) + \Phi\left(\frac{A_L - y_0 - u_0z}{u_m}\right) \quad \text{and} \quad \varphi_0(z) = \frac{\exp(-z^2/2)}{\sqrt{2\pi}} $$

2.2. Particularization for the case of NAWIs

For NAWIs the maximum permitted errors and other requirements that have to be fulfilled in the different metrological controls depend basically on the verification scale interval $e$. In fact the maximum permitted error can only be $0.5\,e$, $1\,e$ or $1.5\,e$ for initial verification and type approval and $1\,e$, $2\,e$ or $3\,e$ for subsequent verification and inspection, both cases depending on the instrument class and the load expressed as a multiple of the verification scale interval $e$. This fact allows obtaining results for very general cases if it is assumed that $e$ is equal to the instrument resolution or actual verification interval, which is a basic assumption in this paper.

About the tolerance limits it is clear that they always be $[TL, \, Tu] = [-T, \, T]$, where $T$ is the maximum permitted error for subsequent verification and inspection. On the other hand, the acceptance limits will depend on the considered metrological control; this is, $[A_U, \, A_L] = [mpe, \, mpe]$, where $mpe$ is the maximum permitted error for the corresponding metrological control.

This study is going to performed for all possible results for the measurement errors of the instrument as long as they fulfill the requirements of the metrological controls. It will be clear that $y_0 \subset [mpe, \, mpe]$, where $mpe$ depends on the considered metrological control. When the prior information for the type approval is considered, $y_0 \subset [-T/2, \, T/2]$.

The uncertainty determination is based on the guide EURAMET cg 18 [4] for calibration of NAWIs. Although it is for calibration, the effects that may affect the instrument measurements are basically the same that the ones considered in the metrological controls, which are the ones that have a contribution to the uncertainty. Tables 1 and 2 explain the uncertainty contributions depending on the considered
effect including the sections of OIML R 76 [1] and EURAMET cg 18 [4] documents where the consequences of this effect is discussed. Table 3 includes the obtained uncertainty results.

| Effect                        | Ref. in OIML R 76                  | Ref. in EURAMET cg 18 | Uncertainty contribution |
|-------------------------------|-----------------------------------|-----------------------|--------------------------|
| Resolution (scale interval)   | -                                 | 7.1.1.2               | $e / (2\sqrt{3})$        |
| Accuracy of zero-setting or tare devices | 4.5.2 and 4.6.3 (A.4.2.3 and A.4.6.2) | 7.1.1.1               | $e / (4\sqrt{3})$        |
| Repeatability                | 3.6.1 (A.4.10)                    | 7.1.1.3               | $\text{mpe} / (2\sqrt{3})$ |
| Eccentricity                 | 3.6.2 (A.4.7)                     | 7.1.1.4               | $\text{mpe} / (2\sqrt{3})$ |
| Weights calibration          | 3.7.1                             | 7.1.2.1               | $\text{mpe} / (3\sqrt{3})$ |
| Air buoyancy                 | 3.7.1                             | 7.1.2.2               | $\text{mpe} / (3 \times 4\sqrt{3})$ |
| Weight drift                 | 3.7.1                             | 7.1.2.3               | $\text{mpe} / (3\sqrt{3})$ |

Table 1. Uncertainty contributions considered in initial verification or subsequent verification and inspection. In this table the considered $\text{mpe}$ depends on the metrological control (initial verification, subsequent verification or inspection). The $\text{mpe}$ for subsequent verification and inspection is two times the $\text{mpe}$ for initial verification.

| Effect       | Ref. in OIML R 76               | Ref. in EURAMET cg 18 | Uncertainty contribution |
|--------------|---------------------------------|-----------------------|--------------------------|
| Creep        | 3.9.4.1 (A.4.11.1)              | 7.4.4.2               | $e / (2 \times 2\sqrt{3})$ |
| Zero return  | 3.9.4.2 (A.4.11.2)              | 7.4.4.2               | $e / (2 \times 2\sqrt{3})$ |
| Durability   | 3.9.4.3 (A.6)                   | 7.4.3.3               | $\text{mpe} / (\sqrt{3})$  |
| Temperature  | 3.9.2                           | 7.4.3.1               | $\text{mpe} / (\sqrt{3})$  |

Table 2. Uncertainty contributions to be added to the ones in table 1 for determining the uncertainty considered for type approval. In this table the considered $\text{mpe}$ is always for initial verification.

| $\text{mpe}$ | Initial verification | $\text{mpe}$ | Subsequent verification and inspection |
|--------------|----------------------|--------------|----------------------------------------|
| 0.5 $e$      | 0.41 $e$             | 0.5 $e$      | 0.59 $e$                               |
| 1 $e$        | 0.59 $e$             | 1.5 $e$      | 0.81 $e$                               |
| 1.5 $e$      | 0.59 $e$             | 2 $e$        | 1.04 $e$                               |
| 2 $e$        | 0.61 $e$             | 3 $e$        | 1.51 $e$                               |

Table 3. Standard uncertainties for initial or subsequent verification and inspection as a function of the scale interval $e$, when no type approval information is considered, $u_m$, and when it is considered, $u_0$.

2.3. Calculations
Probabilities of non-conformity have been determined in Microsoft Excel. Risks calculations have been performed with the free software GNU Octave [5].

3. Results
In this section the probabilities of non-conformity and risks have been evaluated as functions of the ratios of the possible measurement results ($y_m$, $y_0$) and their corresponding $\text{mpe}$. This ratios are always going to be in the interval [-100 $\%$, 100 $\%$]. As expected, the probability of non-conformity is higher when the measurement results are assumed to be close to the limits of the interval.

Figure 1. Each colour represents the probability of non-conformity for the each $\text{mpe}$ when only initial verification is considered, 0.5$e$, 1$e$, and 1.5$e$.

Figure 2. Each colour represents the probability of non-conformity for the each $\text{mpe}$ when only subsequent verification or inspection is considered, 0.5$e$, 1$e$, and 1.5$e$. 
Figures 1 and 2 represent the probabilities of non-conformity when no prior information is considered (this is, no type approval is considered). The probabilities of non-conformity are especially high when only subsequent verification or inspection is considered.

Figure 3. Probabilities of non-conformity for $T = e$ when type approval and initial verification are considered.

Figure 4. Probability of non-conformity for $T = e$ when type approval and subsequent verification or inspection are considered.

Figure 3 and 4 represent the probabilities of non-conformity when type approval is considered for the case $T = e$. For $T = 2e$ and $T = 3e$ similar results are obtained when type approval and subsequent verification or inspection are considered. When type approval and initial verification are considered the shape for the result is very similar, but the maximum probability of non-conformity is 3% for $T = 2e$ and 2% for $T = 3e$.

Risks are presented in figures 5 and 6. Customer’s risks are below 1% after initial verification and below 8% after subsequent verification and inspection, but manufacturer’s risk may be as high as 40% after initial verification and 18% after subsequent verification or inspection. This is a very important result, as for the customer there will no more than 8% of the instrument measurements with an error higher than the legal metrology requirements. This is not much compared with the common practice in metrology of defining the expanded uncertainty for a level of confidence of 95%, so that it is expected that 5% of the measurement results will be out of the interval defined by the expanded uncertainty. On the contrary, the manufacturer’s risk is quite high, but this is the price to pay to protect the customer.

Figure 5. Customer’s risk after initial verification in $\circ$ and after subsequent verification and inspection in $\times$.

Figure 6. Manufacturer’s risk after initial verification in $\circ$ and after subsequent verification or inspection in $\times$.

References
[1] OIML R 76:2006 “Non-automatic weighing instruments Part 1: Metrological and technical requirements – Tests”
[2] JCGM 106:2012 “Evaluation of measurement data – The role of measurement uncertainty in conformity assessment”.
[3] EN 45501:2015 “Metrological aspects of non-automatic weighing instruments”.
[4] EURAMET cg 18 “Guidelines on the Calibration of Non-Automatic Weighing. Instruments” Version 4.0 (2015).
[5] GNU octave , version 4.2.1 2017 https://www.gnu.org/software/octave/