Precise Observables in the MSSM: W mass and the muon magnetic moment

J. Haestier*, S. Heinemeyer†, W. Hollik**, D. Stöckinger ‡, A. M. Weber** and G. Weiglein*

*IPPP, University of Durham, Durham, UK
†Instituto de Física de Cantabria (CSIC-UC), Santander, Spain
**Max-Planck-Institut für Physik, Munich, Germany
‡SUPA, School of Physics, Edinburgh University, UK

Abstract. The precision observables \( M_W \) and \( g - 2 \) of the muon are discussed in the framework of the MSSM. Recent progress in the evaluation of the theoretical predictions is described, and the MSSM predictions are compared with the SM predictions and the experimental values.

INTRODUCTION

Precision observables are a unique laboratory to test the Standard Model (SM) or extensions at the quantum level. Via quantum effects, heavy particles enter the theoretical predictions for such observables, and comparing measurements with predictions leads to valuable information e.g. on the masses of postulated particles such as Higgs bosons or supersymmetric particles.

The power of precision observables is illustrated by comparing the current experimental resolution to the numerical size of quantum effects in the SM and the minimal supersymmetric standard model (MSSM). In the case of the mass of the W boson, \( M_W \), the SM one-loop and two-loop effects amount to about \((-15, -3)\) times the current experimental uncertainty of 0.04\% [1]. The weak SM one-loop and two-loop contributions to the anomalous magnetic moment of the muon \( a_\mu = (g - 2)_\mu / 2 \) are about \((4, -1)\) times as large as the current experimental uncertainty of 0.54 parts per million [2].

The MSSM is a weakly coupled, renormalizable gauge theory [3], and therefore quantum effects are well-defined and calculable. In the MSSM, quantum effects from supersymmetric (SUSY) particles to \( M_W \) and \( a_\mu \) depend on many MSSM parameters, but they can be as large as the corresponding SM quantum effects and thus significantly larger than the experimental uncertainty. Conversely, the experimental measurements significantly constrain the SUSY parameter space (see e.g. the reviews [4, 5]).

In these proceedings we give an update on the current status and recent theoretical developments of the two observables \( M_W \) and \( a_\mu \) in the MSSM.

1 Talk given by D.S. at the SUSY06 Conference on Supersymmetry and the Unification of Fundamental Interactions Irvine, California, USA, 12–17 June 2006
**MW IN THE MSSM**

The mass of the W-boson $M_W$ has been measured at LEP and is being measured at Tevatron. The current experimental value is $M_W^{\text{exp}} = 80.392(29)$ GeV, and the precision could be improved to $\delta_{\text{LHC}} M_W = 15 \text{ MeV}$ at the LHC [6] and to $\delta_{\text{ILC}} M_W = 7 \text{ MeV}$ at a linear $e^+e^-$ collider [7]. On the theoretical side, the SM or MSSM predict a calculable relation between $M_W$ and the muon lifetime $\tau_\mu$ and $M_Z$. Solving this relation for $M_W$ leads to a prediction of $M_W$ in terms of $\tau_\mu$, $M_Z$ and all other model parameters, in particular the masses of the top quark, Higgs bosons, and SUSY particles in the case of the MSSM. For the SM prediction of $M_W$ see [8] and references therein; for a review of previously available MSSM contributions to $M_W$ see [4].

More recently, the Yukawa-enhanced $\mathcal{O}(\alpha_t^2, \alpha_b \alpha_\tau)$ contributions to $M_W$ have been evaluated [9]. This result completes the evaluation of all two-loop MSSM contributions to $M_W$ that enter via the quantity $\Delta \rho$. Detailed estimates for the remaining two-loop contributions, which go beyond $\Delta \rho$, and for unknown higher-order contributions have been derived [9].

In [10], all existing SM and MSSM results have been combined with a new evaluation of the one-loop MSSM contribution that also takes into account complex phases. In this way, a very precise and reliable prediction for $M_W$ in the MSSM has been obtained. This prediction has been implemented in a computer code that will be made publicly available. The remaining theory error of this MSSM prediction of $M_W$, due to the unknown multi-loop contributions estimated in [9] and to the unknown phase dependence beyond the one-loop level [10], has been estimated to $\delta M_W = (4.7 - 10.6) \text{ MeV}$, depending on the SUSY mass scale. Hence the precision of this prediction is better than the current experimental precision and matches the foreseen precision after LHC and a linear $e^+e^-$ collider.

The predictions for $M_W$ in the SM and the MSSM are compared in Figure 1. The possible predictions within the two models as a function of all model parameters give rise to two bands in the $m_t$–$M_W$ plane with only a relatively small overlap region (blue area). For the employed parameter regions see the caption of Figure 1. The MSSM band is divided into two regions. In the very light-shaded green region at least one of the ratios $m_{\tilde{t}_2}/m_{\tilde{t}_1}$ or $m_{\tilde{b}_2}/m_{\tilde{b}_1}$ exceeds 2.5, while in the green region the mass ratios are unconstrained.

The current 68% C.L. experimental results for $m_t$ and $M_W$ slightly favour the MSSM over the SM. More importantly, both within the MSSM and the SM, the precision of the experimental measurements excludes large regions of parameter space.

The prospective accuracies for the LHC and the ILC with GigaZ option are also shown in the plot (using the current central values), indicating the potential for a significant improvement of the sensitivity of the electroweak precision tests.

**A_\mu IN THE MSSM**

The impressive measurement of the anomalous magnetic moment of the muon $a_\mu$ [2] has inspired a lot of progress on the theoretical side. After many refinements (see [11] for recent reviews), the SM prediction deviates by about two standard deviations from
the final experimental value, $\Delta a_\mu (\exp - \SM) = 23.9 (9.9) \times 10^{-10}$ \cite{12}.

The MSSM prediction of $a_\mu$ has been reviewed recently in \cite{5}. The leading contributions from SUSY particles are approximately given by $13 \times 10^{-10} \left( \frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \times \tan \beta \sign(\mu)$ \cite{13,14,15}. Due to the enhancement $\propto \tan \beta$, the ratio of the two Higgs vacuum expectation values, the SUSY contributions could easily be the origin of the deviation $\Delta a_\mu (\exp - \SM)$. Furthermore, the positive value of this deviation implies a preference for a positive $\mu$-parameter in the MSSM.

The status of the MSSM prediction is as follows. The one-loop contributions have been known for a long time (e.g. \cite{13,14,15}; for further references see \cite{5}). The two-loop contributions can be divided into two classes. The class with closed loops of SUSY particles, such as squark or chargino/neutralino loops, is completely known \cite{16,17}; the class without such loops is known in the leading-log approximation \cite{18}. The one-loop and leading two-loop contributions have a very compact analytical form, see \cite{5} and references therein. The remaining theoretical uncertainty due to unknown higher-order contributions has been estimated to be smaller than $3 \times 10^{-10}$ \cite{5}. This is satisfactory at the moment, but it could be significantly improved by a computation of the remaining two-loop contributions.

The current status of $a_\mu$ in the MSSM is summarized in Figure \ref{fig:amu_mssm} which shows the possible MSSM contributions to $a_\mu$ compared with the observed deviation between experiment and the SM prediction. Clearly, the MSSM can accommodate the experimental result for $a_\mu$, and the preferred mass scale is rather low.
FIGURE 2. Allowed values of MSSM contributions to $a_\mu$ as a function of the mass of the lightest observable SUSY particle $M_{\text{LOSP}} = \min(m_{\tilde{\chi}_\pm}, m_{\tilde{\chi}_0}, m_f)$, from an MSSM parameter scan with $\tan\beta = 50$ (see [5] for the employed parameter ranges). The 1σ region corresponding to the deviation between experimental and SM values is indicated. The light yellow region corresponds to all input parameter points that satisfy the experimental constraints from $b$-decays, $M_h$ and $\Delta \rho$. In the red region, smuons and sneutrinos are heavier than 1 TeV. The dashed lines correspond to the contours that arise from ignoring the two-loop corrections from chargino/neutralino- and sfermion-loop diagrams.

REFERENCES

1. [ALEPH, DELPHI, L3, OPAL, SLD Collaborations], Phys. Rept. 427 (2006) 257; [http://lepewwg.web.cern.ch/LEPEWWG/plots/summer2006/]
2. G. W. Bennett et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 89 (2002) 101804 [Erratum-ibid. 89 (2002) 129903]; Phys. Rev. D 73 (2006) 072003.
3. W. Hollik, E. Kraus, M. Roth, C. Rupp, K. Sibold and D. Stöckinger, Nucl. Phys. B 639, 3 (2002).
4. S. Heinemeyer, W. Hollik and G. Weiglein, Phys. Rept. 425 (2006) 265.
5. D. Stöckinger, arXiv:hep-ph/0609168
6. S. Haywood et al., hep-ph/0003275
7. G. Wilson, LC-PHSM-2001-009, see: www.desy.de/~lcnotes/notes.html; U. Baur, R. Clare, J. Erler, S. Heinemeyer, D. Wackeroth, G. Weiglein and D. Wood, hep-ph/0111314
8. M. Awramik, M. Czakon, A. Freitas and G. Weiglein, Phys. Rev. D 69 (2004) 053006.
9. J. Haestier, S. Heinemeyer, D. Stöckinger and G. Weiglein, JHEP 0512 (2005) 027.
10. S. Heinemeyer, W. Hollik, D. Stöckinger, A. M. Weber and G. Weiglein, JHEP 0608 (2006) 052.
11. M. Passera, J. Phys. G 31 (2005) R75.
12. M. Davier and W. J. Marciano, Ann. Rev. Nucl. Part. Sci. 54 (2004) 115.
13. J. L. Lopez, D. V. Nanopoulos and X. Wang, Phys. Rev. D 49, 366 (1994).
14. U. Chattopadhyay and P. Nath, Phys. Rev. D 53, 1648 (1996).
15. T. Moroi, Phys. Rev. D 53 (1996) 6565 [Erratum-ibid. 56 (1997) 4424].
16. S. Heinemeyer, D. Stöckinger and G. Weiglein, Nucl. Phys. B 690 (2004) 62.
17. S. Heinemeyer, D. Stöckinger and G. Weiglein, Nucl. Phys. B 699 (2004) 103.
18. G. Degrassi and G. F. Giudice, Phys. Rev. D 58 (1998) 053007; T. F. Feng, X. Q. Li, L. Lin, J. Maalampi and H. S. Song, Phys. Rev. D 73, 116001 (2006).