Finding the Optimal Bus-Overtaking Rules for Bus Stops with Two Tandem Berths

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Abstract: Overtaking rule is a key factor for the estimation of bus discharge flow and bus delay at stops. In general, there are four kinds of overtaking rules, namely no-overtaking, enter-overtaking, exit-overtaking and free-overtaking. This paper studies a two-berth tandem bus stop in a saturated state and proposes calculation models for the maximum bus discharge flow and average berth blocking time under different overtaking rules. Cellular automata simulation is applied to verify the model’s reliability. Then the influence of bus dwell time characteristics and overtaking rules are analyzed. Results show that overtaking has a positive impact on the maximum bus discharge flow and average berth blocking time to a certain extent. If only one overtaking behavior is allowed, the exit-overtaking rule is recommended. The study reveals that overtaking behavior plays an important role in bus service level and operational efficiency. Bus-overtaking rules are suggested to be changed with different bus flow states to obtain the optimal berth effectiveness.

Keywords: overtaking rules; maximum bus discharge flow; average berth blocking time; cellular automata simulation

1. Introduction

Bus queuing and overtaking are common at bus stops with multiple bus lines, especially on high frequency routes. A key feature of an unreliable service is the irregular operating time of buses at stops. Unstable operating time dynamics are the critical factor that causes buses to wait and overtake casually. This will also increase real bus dwell time and passenger waiting time, causing more additional operating costs for the transit agency. One way for transit agencies to stop the progression of instability among dwell time is to find the optimal overtaking rules and provide control methods for buses.

It is assumed that the curbside bus stop with two tandem berths is isolated from traffic signals and other traffic. There are four different overtaking rules for buses at stops, namely, no-overtaking, enter-overtaking, exit-overtaking, and free-overtaking. The details of these four rules are as follows:

No-overtaking: Buses are not allowed to change lanes and overtake other buses in the vicinity of bus stops.

Enter-overtaking: When the two berths are vacant, queuing buses can directly enter the stop one by one. If the upstream berth (Berth 2) is occupied and the downstream berth (Berth 1) is vacant, the queuing bus outside the stop can change lanes and enter Berth 1 directly. However, buses cannot change lanes and overtake out of the stop. As shown in Figure 1 below.

Exit-overtaking: If the bus in Berth 2 completes service first, but there is still a bus in Berth 1, the bus can change lanes and leave the stop. However, queuing buses outside the stop cannot change lanes to enter the stop.

Free-overtaking: Buses can always change lanes and overtake to enter and exit in the vicinity of bus stops.
The maximum bus discharge flow is defined as the maximum number of buses, which complete the process of entering, dwelling and exiting within one hour. The average berth blocking time is divided into the entering berth blocking time and the exiting berth blocking time. The entering berth blocking time is the duration of time that Berth 2 is occupied and Berth 1 is vacant, but the queuing bus cannot enter the stop because enter-overtaking is prohibited. The exiting berth blocking time is the duration of time that the bus in Berth 2 is leaving, but there is a bus dwelling in Berth 1, and the leaving bus is blocked because exit-overtaking is prohibited.

This paper aims to estimate the maximum bus discharge flow under four overtaking rules, analyze the influence of the average dwell time and the dwell time variation coefficient, and calculate the average berth blocking time to find the optimal overtaking rules for different conditions. Moreover, cellular automata models are applied to verify the model’s reliability. The rest of the paper is organized as follows. Section 2 is the literature review. Section 3 presents the analytical maximum bus discharge flow models. Section 4 discusses four overtaking rules. The maximum bus discharge flow and the average berth blocking time for different overtaking rules are compared and evaluated. Finally, the optimal bus-overtaking rules are recommended. Section 5 is the conclusion.

2. Literature Review

Bus-stop capacity and waiting delay have been studied over the past few decades. One commonly used bus-stop capacity calculation model is proposed by the Highway Capacity Manual (HCM). The model only reports the empirical number of effective berths and omits the real-word influence of bus-overtaking rules on stop capacity.

Some studies have related bus-stop capacity to berth number and other operational indicators [1–3]. Wang et al. [4] find that added berths produced diminishing returns in capacity and the returns in capacity are influenced by failure rate, and variation coefficients of bus arrival headway and service time. Fernández [5] concludes that a stop cannot operate near its absolute capacity, for upstream queues will develop even for low degrees of saturation. Gu et al. [6] study bus-stop capacity for isolated and curbside bus stops and describe the disruptive bus interactions that occurred at multi-berth stops by numerical results. Gu et al. [7] formulate analytical models to argue why certain overtaking rules can enhance bus-stop discharge flow when the time to serve boarding and alighting passengers are highly varied across buses. Gu et al. [8] reveal the variations in bus service time and discuss how to choose the suitable number of bus berths on the special delay target. The present models can also be applied to other serial queueing systems. Shen and Gu et al. [9] develop capacity models for near- and far-side stops with multiple berths in a dedicated bus lane. The key operating factors and the characteristic of bus traffic are considered to replace the original capacity formulas of curbside bus stops in professional handbooks. In general, Gu proposes that buses are allowed to exit berths by overtaking other buses downstream and this rule can diminish bus-stop capacity in some special conditions. The bus-stop capacity of no-overtaking and exit-overtaking are evaluated, but the impact of various overtaking rules on the berth block effect has not been discussed further and the applicable conditions for different berth layouts are still not clear.
Other studies have related bus-stop delay to berth layout and special operational indicators [10–13]. Alonso et al. [14] analyze the quantified significant operational delays suffered by users and operators due to consecutive bus arrival at stops. The model can be applied to multi berths linearly arranged following first-in first-out discipline. Johari et al. [15] discuss the impacts of bus stop location and berth number on urban network traffic performance. The simulation experiments show that adding berth number increases the network capacity and decrease the network average delay median for both car traffic and public transport system. Bunker [16] explains the relationship between bus-stop upstream average waiting time and loading area utilization ratio. This demonstrates that general on-street bus stops are less productive than BRT stations. Wu et al. [17] optimize the bus departure times and dwell time to improve the flexibility and operational efficiency of shuttle bus systems. And the total system costs including passenger waiting times, passenger in-vehicle times, and bus operating costs are minimized. Tan et al. [18] introduce the concept of berth assignment redesign to better evaluate the performance of different berth layouts and makes an appropriate trade-off between stability and efficiency. Traffic volume, bus volume, and the available exit area length are all considered. Results show the performance improvement is limited in non-rush hours and optimal berth assignment could help reduce the average bus delay in rush hours. Li et al. [19] analyze the temporal characteristics of failure duration rate to evaluate bus waiting time and capacity drop at stops.

About overtaking models, Gibson et al. [20] consider the influence of bus passengers and other traffic and recommend the implementation of overtaking maneuvers to improve the bus operational efficiency. Sun and Schmöcker [21] study the impact of allowing and not allowing overtaking on passenger behavior and find that overtaking is a favorable countermeasure to bunching if the front-bus preference is high or the arriving-to-loading burden is heavy. When the front-bus preference exceeds 0.5, the higher the front-bus preference is, the more improvement could be obtained by allowing for overtaking. Zhao et al. [22] propose a bus-loading area effectiveness model to reveal the impact of bus driving behavior. It is noted that for driving behavior parameters, the critical overtaking distance gap and the dwelling space gap between two adjacent buses can be tested based on field data survey. Bian et al. [23] present a bus service time estimation model to evaluate the impact of overtaking. Results indicate that overtaking weakens the influence of queueing and decreases the service time at bus stops. Bian et al. [24] consider four different overtaking rules at roadside bus stops, and analyze the bus waiting delay under low, medium and high bus frequencies. The approximation results and simulation results are compared. Some practical factors that impede the implementation of overtaking are discussed. Bian believes that overtaking has a positive impact on the bus-stop capacity and the expected average delay. And the work reveals inherent reasons why exit-overtaking is a better choice if only one kind of overtaking is allowed. Mozzoni et al. [25] use automatic vehicle location data to check the transfer reliability at the operational level and distinguish bus stops where the transfer synchronization is not maintained. Cortés et al. [26] review some microsimulators that consider vehicles trying to overtake dwelling buses, dwelling buses at stops and vehicles in adjacent lanes to understand the effect of bus-stop operation on general traffic and replicate the real entering and leaving paths of operating buses in different stop types. Cellular automata models have been widely used in capacity calculation, delay evaluation and lane change models. Cremer and Ludwig [27] apply cellular automata to the dynamic process of traffic flow through urban networks. Liu et al. [28] analyze the utilization of each berth at bus stops based on cellular automata simulation and determine the effective number of berths at bus stops. Guzman et al. [29] consider different types of vehicles, propose an asymmetric two-lane cellular automata traffic flow model, and analyze the risk of lane changing. In different areas and situations inside public transport systems, cellular automata models can handle complex traffic behaviors by changing cell update rules and the dynamic parameter models have advantages in the saturated state. Xue et al. [30] use improved cellular automata models to study the behavioral dynamics
of bus passengers’ boarding and alighting actions. These crowded traffic models can be applied to other complex scenarios. Research shows that cellular automata models have practical value and can provide theoretical references for public transport systems. Therefore, this paper constructs cellular automata models to simulate bus-overtaking actions at stops. Table 1 shows the review of the literature on the use of analytical models and simulation models.

### Table 1. Review of the literature on the use of analytical models and simulation models.

| Methods | Types | References |
|---------|-------|------------|
| Analytical models | Bus-stop capacity model | [2–7,9,16] |
| | Bus queueing model | [3,8,24] |
| | Bus dwell time model | [10,11,23] |
| | Bus delay model | [12–14,16,25,31] |
| | Optimization model | [1,17,18] |
| | Failure rate model | [8,16,19] |
| | Bus berth model | [6,15,18,22] |
| | Overtaking model | [7,20–22,24] |
| Simulation models | PASSION | [2,5,26] |
| | AIMSUN | [3,26] |
| | other microscopic simulation model | [16,18,26] |
| | Cellular automata model | [28,30] |

### 3. Maximum Bus Discharge Flow Models

Set the time period from the empty state of all berths to the next occurrence of this state as a cycle. The number of buses served in a cycle is no less than the number of berths. Then time-space diagrams are used to describe the dynamic state of bus stops. Moreover, the Monte Carlo simulation is applied to calculate the repeated service time and the maximum bus discharge flow. In general, bus dwell time refers to the serving time for passengers and the opening and closing time of bus doors. Assuming that the dwell time of all buses $S_k (k = 1, 2, \ldots, n)$ are independent random variables that follow a normal distribution, $k$ denotes the $k$-th bus that enters the stop, and the dwell time variation coefficient $C_s$ is the quotient of bus dwell time standard deviation and expected value.

The length of one cycle is:

$$E(T) = t_e + n \cdot E(s) - E(T_s)$$  \hspace{1cm} (1)

Here, $t_e$ represents the time that buses need to enter the berth. $n$ indicates the number of buses served in one cycle. $E(s)$ represents the average bus service time. $E(T_s)$ represents the repeated service time in one cycle. When overtaking is prohibited, the repeated service time in one cycle is the minimum service time of two sequence buses. Similar to the no-overtaking rule, the cycle time under the enter-overtaking rule and the exit-overtaking rule is calculated. For the free-overtaking rule, the empty state of all berths is nonexistent, then a judgment algorithm considering bus running conflicts is constructed.

For no-overtaking rules, there is both the entering berth blocking time of queuing buses and the exiting berth blocking time of buses in Berth 2.

$$W = W_q + W_b$$  \hspace{1cm} (2)
The entering berth blocking time refers to the additional waiting time for the queuing bus outside the stop in cases that Berth 1 is vacant but Berth 2 is occupied, that is, the difference between the service time of two buses in the same cycle.

\[ W_q = \Pr\{S_1 < S_2\} \cdot E(S_2 - S_1) \]  \hspace{1cm} (3)

The exiting berth blocking time refers to the additional waiting time for buses to leave from Berth 2 in cases that Berth 1 is still occupied. Additionally, the outside queuing bus has to wait more time. That is,

\[ W_b = \Pr\{S_1 > S_2\} \cdot \frac{[E(S_1 - S_2) + (E(S_1 - S_2) + t_i)]}{2} \]  \hspace{1cm} (4)

3.1. Enter-Overtaking Rule

The number of buses served in a cycle under the enter-overtaking rule may be more than two. If bus 2 completes the service first, it has to wait for bus 1 to complete the service and leave together. If bus 1 completes service and leaves first, the first queuing bus outside the stop can overtake and enter Berth 1 for service. At this time, the number of buses served in a cycle is no less than three. Figure 2 shows two situations.

![Figure 2. Time-space trajectory diagram under the enter-overtaking rule. (a) Three buses served in a cycle (b) Four buses served in a cycle.](image)

Here, the probability of the number of buses served in a cycle is calculated by the Monte Carlo experiment. A random count experiment is performed and the average value is obtained. The main procedure is as follows:

a. Generate a random matrix for bus dwell time \( S_i \) that obeys the normal distribution;

b. When deciding the number of buses served in a cycle is equal to \( x \), it is necessary to judge whether it meets the following conditions. Table 2 shows the judgment conditions for the number of buses served in a cycle.

c. If yes, count once, that is \( m = m + 1 \);

d. Loop the above steps to get the probability \( p \).

| \( x \) | Judgment Conditions |
|-------|---------------------|
| 2     | \( s_1 > s_2 \)     |
| 3     | \( (s_1 + t_{gap}) < s_2 < (s_1 + t_{gap}) + s_3 \) |
| 4     | \( (s_1 + t_{gap}) + (s_3 + t_{gap}) < s_2 < (s_1 + t_{gap}) + (s_3 + t_{gap}) + s_4 \) |
| 5     | \( (s_1 + t_{gap}) + (s_3 + t_{gap}) + (s_4 + t_{gap}) < s_2 < (s_1 + t_{gap}) + (s_3 + t_{gap}) + (s_4 + t_{gap}) + s_5 \) |

Table 2. Judgment conditions for the number of buses served in a cycle.
Correspondingly, 
\[ E(T_n) = \begin{cases} E(\min\{S_1, S_2\}) & n = 2 \\ E(\max\{S_1, S_2, \ldots, S_n\}) - (n - 2) \cdot t_{gap} & n > 2 \end{cases} \] (5)

Here, the interval time is the sum of time that the bus leaves the stop and the time that the queuing bus overtakes in the stop.

\[ t_{gap} = t_l + t_{oe} \] (6)

The estimation model of the maximum bus discharge flow is as follows

\[ B_{max} = \sum \left[ p(n) \cdot \left( n \cdot \frac{3600}{E(T = n)} \right) \right] \] (7)

The exiting berth blocking time is one cycle time \( T \) minus the time wherein buses enter the bus stop and the longest bus dwell time in this cycle.

\[ W_q = \begin{cases} \frac{E|S_1 - S_2| + E|S_1 - S_2| + t_l}{2} & n = 2 \\ \frac{E(T = n) - t_e - E(\max\{S_1, S_2, \ldots, S_n\})}{2} & n > 2 \end{cases} \] (8)

For the exit-overtaking rule, the entering berth blocking time is 0, which is. Therefore, the average berth blocking time is equal to the exiting berth blocking time.

\[ W = \sum [p(n) \cdot W_q(n)] \] (9)

### 3.2. Exit-Overtaking Rule

Similar to the enter-overtaking rule, if bus 2 completes the service first, it can leave the stop immediately. At this time, the number of buses served in a cycle may be three or more. If bus 1 completes the service first, the queuing bus has to wait until bus 2 completes service and leaves the stop. Then two buses in the queue enter the stop at the same time and the number of buses served in a cycle is two.

The interval time and the probability of the number of buses served in a cycle under the exit-overtaking rule are different from the enter-overtaking rule, as the time required for buses to overtake in and overtake out is different. Here, \( t_{gap} = t_l + t_{oe} \), other calculation methods are the same as in Section 3.1.

Under the exit-overtaking rule, if there are only two served buses in a cycle, the entering berth blocking time for queuing buses is \( |S_1 - S_2| \) if there are more than two served buses in a cycle, the entering berth blocking time is one cycle time \( T \) minus the time wherein buses enter the bus stop \( t_e \) and the longest dwell time \( S_1 \) in this cycle.

\[ W_q = \begin{cases} |S_1 - S_2| & n = 2 \\ E(T = n) - t_e - E(\max\{S_1, S_2, \ldots, S_n\}) & n > 2 \end{cases} \] (10)

For the exit-overtaking rule, the exiting berth blocking time is 0, which is \( W_b = 0 \). Therefore, the average berth blocking time is equal to the entering berth blocking time.

### 3.3. Free-Overtaking Rule

For the free-overtaking rule, the whole process can be simplified as two different bus flow \( b_1 \) and \( b_2 \) stopping at fixed Berth 1 and Berth 2. The number of buses served in a cycle cannot be counted, and the above method cannot be used to calculate the maximum bus discharge flow. However, when the bus in Berth 2 is preparing to leave the stop, and the queuing bus wants to overtake and enter Berth 1, a bus running conflict happens. This conflict may affect the maximum bus discharge flow and the average berth blocking time, as shown in Figure 3 below.
Conflict may affect the maximum bus discharge flow and the average berth blocking time, as shown in Figure 3 below.

**Figure 3. Schematic diagram of running conflicts under the free-overtaking rule.**

The necessary condition for the conflict is that the bus flow $b_1$ of Berth 1 and the bus flow $b_2$ of Berth 2 arrive at the conflict area at the same time, that is, the leaving action of the bus in Berth 2 happens after the instant time that the bus in Berth 1 leaves the stop and the queuing bus overtakes into Berth 1. As shown in Figure 4 below.

**Figure 4. Time-space trajectory diagram under free-overtaking rule.**

Therefore, this paper constructs an algorithm based on the judgment of bus conflicts. By judging whether there is a conflict in the stop area, the maximum bus discharge flow can be obtained. The algorithm flowchart is shown in Figure 5. Through the following judging procedures, the maximum bus discharge flow $b_1$ of Berth 1 and $b_2$ of Berth 2 are calculated. The sum of $b_1$ and $b_2$ is the maximum bus discharge flow for free-overtaking rules, the average berth blocking time is equal to zero.
4. Model Evaluation and Result Analysis

4.1. Model Evaluation

Figures 6–8 show the analytical results and the simulation results of the maximum bus discharge flow under overtaking rules. The solid line represents the analytical results, and the solid point represents the cellular automata simulation results. Figures 6a–8a show the relationship between the bus dwell time variation coefficient and the maximum discharge flow when the average bus dwell time is 25 s, and Figures 6b, 7b and 8b show the relationship between the average bus dwell time and the maximum discharge flow when the bus dwell time variation coefficient is 0.6. The simulation results fluctuate up and down with the analytical results. It is obvious that there is no big difference between the simulation results and the analytical results, indicating that the model can effectively evaluate the maximum bus discharge flow of bus stops.

![Algorithm flowchart under free-overtaking rule.](image)

**Figure 5.** Algorithm flowchart under free-overtaking rule.

![Comparison of simulation and analytical results under enter-overtaking rule.](image)

**Figure 6.** Comparison of simulation and analytical results under enter-overtaking rule. (a) \( E(S) = 25 \) s (b) \( Cs = 0.6 \).
In order to evaluate the performance of the maximum bus discharge flow model, three error measurement values are calculated: average absolute error (MAE), root mean square error (RMSE), and average absolute percentage error (MAPE). The error measurement of the maximum bus discharge flow is shown in Table 3, which shows that the analytical models have good reliability.

| E(S) = 25 s  | Cs = 0.6 |
|--------------|----------|
| MAE          | RMSE     | MAPE(%)  | MAE  | RMSE  | MAPE(%)  |
| No-overtaking| 3.62     | 5.34     | 1.96 | 1.96  | 2.35     | 1.06     |
| Enter-overtaking | 4.31 | 5.94     | 2.15 | 5.45  | 6.26     | 2.08     |
| Exit-overtaking | 4.70 | 6.24     | 2.33 | 2.38  | 2.98     | 1.19     |
| Free-overtaking | 1.86 | 2.59     | 0.84 | 3.40  | 4.17     | 1.37     |

4.2. Maximum Bus Discharge Flow Comparison

Figure 9 compares the maximum bus discharge flow under different overtaking rules for two-berth bus stops. The maximum bus discharge flow under the exit-overtaking rule is slightly larger than the enter-overtaking. This is because queuing buses have to change lanes to overtake and enter the stop under the enter-overtaking rule, while buses under the...
exit-overtaking rule only need to change lanes once to leave the stop, and queuing buses can enter the stop directly, so the enter-overtaking cycle time is slightly longer, causing the advantage of exit-overtaking to be slightly more than enter-overtaking.

Figure 9a shows the maximum bus discharge flow decreases with the bus dwell time variation coefficient increase. The advantage of free-overtaking is most obvious. When $Cs$ is small, free-overtaking can help keep the bus-stop discharge flow in a stable state. However, when $Cs$ is close to 1, the maximum bus discharge flow shows a rapid downward trend. This is because when the volatility of bus dwell time is in an extreme condition, the chaotic enter and exit driving behaviors caused by free-overtaking will lead to the loss of maximum bus discharge flow.

Secondly, compared with enter-overtaking and exit-overtaking, the no-overtaking rule is recommended when $Cs$ is less than 0.4. When $Cs$ is more than 0.43, the advantage of enter-overtaking and exit-overtaking becomes more and more obvious, because when the bus dwell time fluctuates greatly, the time waste caused by no-overtaking is also more obvious.

Figure 9b shows the maximum bus discharge flow decreases with the average bus dwell time increase. The maximum bus discharge flow of free-overtaking is greater than the other three rules. This is because free-overtaking brings the largest berth utilization. In comparison, the maximum bus discharge flow of no-overtaking is the smallest in this case.

4.3. Average Berth Blocking Time Comparison

Figure 10 is the average berth blocking time comparison of different overtaking rules. It can be seen that the average berth blocking time of exit-overtaking is less than enter-overtaking. This is because under the exit-overtaking rule, only the queuing bus caused by Berth 1 is blocked, and under the enter-overtaking rule, if the bus in Berth 2 cannot leave the stop, then the upstream bus bunching caused by this situation will be worse. That means the loss of bus-stop efficiency. Therefore, the average berth blocking time of exit-overtaking is shorter than enter-overtaking. Considering that the maximum bus discharge flow under the exit-overtaking rule is slightly larger than the enter-overtaking if only one overtaking behavior is allowed, the exit-overtaking rule is recommended.
Based on probability theory and queuing theory, Bian uses arrival rate and service rate to estimate bus stop capacity and bus waiting delay, assuming that bus dwell time follows an exponential distribution. Second, our work focuses on the maximum bus discharge flow and the average berth blocking time, which is consistent with Gu’s work [7] and Bian’s work [24]. This reveals the inherent reasons why the no-overtaking rule is not recommended and demonstrates the significance of flexible bus-overtaking rules. If traffic

Figure 10. Average berth blocking time comparison of different overtaking rules. (a) \( E(S) = 25 \) s (b) \( Cs = 0.6 \).

Figure 10a shows the average berth blocking time increases with the bus dwell time variation coefficient increase. The average berth blocking time of no-overtaking is most obvious. However, when \( Cs \) is less than 0.2, the average berth blocking time of no-overtaking is rather small. This is because when the variability of the bus dwell time is small, the two buses within one cycle can be regarded as a unit. When \( Cs \) is more than 0.2, the bus service time under the no-overtaking rule will increase with the increase of the variation coefficient, resulting in an obvious increase in the average berth blocking time.

Figure 10b shows the average berth blocking time increases with the average bus dwell time increase. As the average bus dwell time increases, the average berth blocking time also increases. Moreover, the differences between the three bus-overtaking rules are becoming more and more obvious. This is because both the exiting berth blocking time and entering berth blocking time increase with the increase of the average bus dwell time simultaneously, so the average berth blocking time of no-overtaking is also more obvious. This indicates that no-overtaking brings more bus delays compared with other overtaking rules.

5. Discussion

Based on the proposed model, the maximum bus discharge flow and the average berth blocking time for different overtaking rules can be computed, and the influence of bus dwell time characteristics is analyzed by numerical results. The trend that the average berth blocking time increases with the bus dwell time variation coefficient is the same as Niu’s work [31].

Our results partially overlap with that of Bian [24] because we are all studying the operation of bus stops under different overtaking behaviors. However, our work is different. First, the model calculation in this paper uses bus dwell time as an explanatory variable to estimate the maximum bus discharge flow and the average berth blocking time under a saturation state. It is assumed that bus dwell time follows a normal distribution. Based on probability theory and queuing theory, Bian uses arrival rate and service rate to estimate bus stop capacity and bus waiting delay, assuming that bus dwell time follows an exponential distribution. Second, our work focuses on the maximum bus discharge flow and the average berth blocking time, while Bian’s work focuses on the waiting delay of the bus queueing system for different bus frequencies. In addition, we establish cellular automata models to analyze and verify the reliability of analytical models.

Results show that the no-overtaking rule has no benefits for both the maximum bus discharge flow and the average berth blocking time, which is consistent with Gu’s work [7] and Bian’s work [24]. This reveals the inherent reasons why the no-overtaking rule is not recommended and demonstrates the significance of flexible bus-overtaking rules. If traffic
6. Conclusions and Future Work

6.1. Conclusions

This research studies a two-berth bus stop in a saturated state and proposes calculation models for the maximum bus discharge flow and average berth blocking time under different overtaking rules and analyzes the influence of bus dwell time. Cellular automaton models are constructed to verify the reliability of analytical models. Results show that bus delay caused by the block effect between tandem berths is obvious and bus-overtaking rules are effective measures to weaken the block effect caused by berth layout.

Numerical experiments unveil insights that have practical implications. The current bus-overtaking rule set by the transit agency is no-overtaking, especially in peak hours. Bus drivers can neither overtake in or out of the stop in compliance with driving rules. Our models indicate that this will bring more serious congestion to bus stops.

The benefits of free-overtaking are obvious for tandem berths, both in terms of bus discharge flow and berth blocking time. That is, free-overtaking is almost unaffected by the block effect between berths. For a curbside bus stop totally isolated from traffic signals and other traffic, free-overtaking should be encouraged to adapt to the unstable dwell time dynamics. If traffic conditions are limited, the exit-overtaking rule can be considered to decrease bus delays and improve the performance of the bus queueing system at stops.

6.2. Future Work

The main purpose of this research is to examine the viability and potential benefits of bus-overtaking rules for tandem berths. The limitation of this study is that the considered scenarios are extreme cases in the bus-stop area. However, on a real urban road, the impact of motor vehicles and non-motor vehicles cannot be ignored. In addition, it is worth noticing that bus passengers and their interactions with buses and car traffic are not simulated in this study. Some simplifications can be further relaxed via modifications and extensions of our work.

The present models ignore the effects of passengers and other traffic on bus-overtaking rules. It shapes the future research direction which is finding the optimal combination of buses and other traffic flow in the bus-stop area to optimize the system performance. Researchers and practitioners might also be interested in how to evaluate bus delays more precisely to provide better real-time control maneuvers. The method built upon time-space diagrams can be used to develop bus delay models under different bus-overtaking rules.

The proposed method can also be applied to investigate the effects of other operational characteristics of bus berths. Another potential research direction can be the effect of berth number and berth layout on bus-overtaking efficiency. Ongoing work is directed at refining the present models to accommodate different berth layouts, where the effect of bus passengers and other traffic are discussed in more detail.

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Nomenclature

| Variables and Parameters | Definitions |
|--------------------------|-------------|
| $S_k$                    | The bus dwell time of the $k$-th bus |
| $C_S$                    | The bus dwell time variation coefficient |
| $E(T)$                   | The time of one cycle |
| $t_e$                    | The time of buses entering the bus stop |
| $n$                      | The number of buses served in one cycle |
| $E(s)$                   | The average bus dwell time |
| $E(T_s)$                 | The repeated service time in one cycle |
| $B_{max}$                | The maximum bus discharge flow of the bus stop |
| $W$                      | The average berth blocking time |
| $W_q$                    | The entering berth blocking time |
| $W_b$                    | The exiting berth blocking time |
| $t_{gap}$                | The interval time, that is, the sum of time of bus leaving the stop and time of the queuing bus overtaking in the stop |
| $t_l$                    | The time of bus leaving the stop |
| $t_{ol}$                 | The time of bus overtaking in the stop |
| $p(n)$                   | The probability that the number of buses served in a cycle is $n$ |
| $t_{ol}$                 | The time of bus overtaking out the stop |
| $W_b(n)$                 | The exiting berth blocking time that the number of buses served in a cycle is $n$ |
| $W_q(n)$                 | The entering berth blocking time that the number of buses served in a cycle is $n$ |
| $b_1$                    | The maximum bus discharge flow of Berth 1 |
| $b_2$                    | The maximum bus discharge flow of Berth 2 |
| $i$                      | The number of bus conflicts at the stop in one hour |

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