Chiral partner structure of light nucleons in an extended parity doublet model

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We study chiral partner structure of four light nucleons, $N(939)$, $N(1440)$, $N(1535)$ and $N(1650)$ using an effective chiral model based on the parity doublet structure. In our model we introduce four chiral representations, $\{1, 2\}$, $\{2, 1\}$, $\{2, 3\}$ and $\{3, 2\}$ under $SU(2)_L \otimes SU(2)_R$ symmetry. We determine the model parameters by fitting them to available experimental values of masses, widths and the axial charge of $N(939)$ together with the axial charges of $N(1535)$ and $N(1650)$ by lattice analyses. We find five groups of solutions: In a group the chiral partner to $N(939)$ is $N(1440)$ having small chiral invariant mass. In another group, the chiral partner is a mixture of $N(1535)$ and $N(1650)$ having a large chiral invariant mass. We claim that off-diagonal elements of axial-charge matrix can be used for distinguishing these groups. We also discuss changes of masses associated with chiral symmetry restoration, which could emerge in high density matter.

I. INTRODUCTION

One of the most important features of QCD relevant to the low-energy hadron physics is the chiral symmetry and its spontaneous breaking. The spontaneous symmetry breaking generates mass differences between chiral partners as well as the mixing among different chiral representations. It is interesting to study the role of the chiral symmetry breaking to determine the properties and structures of baryons such as amount of the masses of baryons generated by the chiral symmetry breaking and the chiral partner of ground state nucleon.

In a hadronic model for light nucleons based on the parity doublet structure,\textsuperscript{15} the chiral partner of $N(939)$. One of the important feature of the model is the existence of the chiral invariant mass denoted by $m_0$, which is not originated from the spontaneous chiral symmetry breaking. In other word, the mass splitting between $N(939)$ and its chiral partner $N(1535)$ denoted by $m_0$, which implies that the masses of $N(939)$ and its chiral partner tend to $m_0$ when the chiral symmetry is restored and their mass splitting is given by spontaneous chiral symmetry breaking.

This parity doublet structure is extended to include hyperons and/or excited nucleons such as $N(1440)$ and $N(1650)$ (see, e.g. Refs. \textsuperscript{16,17}).

Particularly, in Ref. \textsuperscript{16}, two $\{2, 1\}$ and $\{2, 3\}$ representations under the chiral $SU(2)_L \otimes SU(2)_R$ are introduced to study $N(939)$, $N(1440)$, $N(1535)$ and $N(1650)$. Two types of solutions were found: In one type, the chiral partner of $N(939)$ is a mixture of $N(1535)$ and $N(1650)$ and two chiral invariant masses are almost 200 MeV. In another type, $N(939)$ and $N(1535)$ are chiral partners to each other and two chiral invariant masses are about 1000 MeV. On the other hand, in Ref. \textsuperscript{17}, a model is constructed in the three-flavor framework to further introduce the chiral $\{2, 3\}$ representations under the chiral $SU(3)_L \otimes SU(3)_R$ symmetry, which correspond to the chiral $\{2, 3\}$ representations under the chiral $SU(2)_L \otimes SU(2)_R$ symmetry, and that the chiral invariant mass of the $N(939)$ is about 800 MeV. The large value of the chiral invariant mass of $N(939)$ seems consistent with the results by the lattice QCD analysis in Ref. \textsuperscript{18}, which shows that is almost constant even if temperature is increased.

In this paper, we make a general analysis using a two-flavor parity doublet model including chiral $\{1, 2\}$ and $\{2, 3\}$ representations under the chiral $SU(2)_L \otimes SU(2)_R$ symmetry, with including derivative interactions to the pion fields. We will show that there exist five groups of solutions distinguishable by chiral inavariant masses and mixing rates of nucleons. In a group of solutions, the chiral partner of $N(939)$ is $N(1440)$ having small chiral invariant mass of about 100 MeV and $N(939)$ is dominated by $\{2, 3\}$ representation. In another group, on the other hand, $N(939)$ belongs dominantly $\{1, 2\}$ representation having a large chiral invariant mass, and chiral partner of $N(939)$ is a mixture of $N(1535)$ and $N(1650)$. Furthermore, we give predictions of off-diagonal elements of axial-charge matrix, which could be checked in future lattice analysis. We also show changes of nucleon masses when the vacuum expectation value of $\sigma$, which is an order parameter of the spontaneous chiral symmetry breaking, is changed.

This paper is organized as follows: In section \textsuperscript{IV} we construct an extended model with parity doublet structure. Section \textsuperscript{IV} is a main part, where we show the numerical results of fitting on the chiral invariant masses and chiral partner structure. In sections \textsuperscript{V} and \textsuperscript{VI} we study off-diagonal components of axial-charge matrix and change of nucleons mass as predictions. Finally we will give a brief summary and discussions in section \textsuperscript{VI}.
II. AN EXTENDED PARITY DOUBLET MODEL

In this section we introduce four baryon fields with parity doublet structure and construct a Lagrangian for baryons and scalar and pseudoscalar mesons based on the SU(2)_L⊗SU(2)_R chiral symmetry.

A. Model construction

The chiral representations of quarks under SU(2)_L⊗SU(2)_R are written as

\[ q_L \sim (2, 1), \quad q_R \sim (1, 2), \]  \tag{1} \]

where the 2 and 1 in above bracket express doublet and singlet, respectively. Since baryons are expressed as direct products of three quarks, we have following possibilities for the chiral representations of baryons:

\[ q \otimes q \otimes q \sim [(2, 1) \oplus (1, 2)]^3 \]

\[ \sim 3[(2, 1) \oplus (1, 2)] \oplus 3[(3, 2) \oplus (2, 3)] \]

\[ \oplus [(4, 1) \oplus (1, 4)]. \]  \tag{2} \]

After the chiral symmetry is spontaneously broken down to the flavor symmetry, nucleons appear from the representations of (2, 1) ⊕ (1, 2) and (3, 2) ⊕ (2, 3). In this paper we introduce two baryon fields corresponding to these two representations:

\[ \psi_{1l} \sim (2, 1), \quad \psi_{1r} \sim (1, 2), \]
\[ \psi_{2l} \sim (1, 2), \quad \psi_{2r} \sim (2, 1), \]
\[ \eta_{1l} \sim (2, 3), \quad \eta_{1r} \sim (3, 2), \]
\[ \eta_{2l} \sim (3, 2), \quad \eta_{2r} \sim (2, 3). \]  \tag{3} \]

Here the subscripts \( l \) and \( r \) express the chirality:

\[ \gamma_5 \psi_{il} = -\psi_{il}, \quad \gamma_5 \psi_{ir} = +\psi_{ir}, \]
\[ \gamma_5 \eta_{il} = -\eta_{il}, \quad \gamma_5 \eta_{ir} = +\eta_{ir}. \]  \tag{4} \]

for \( i = 1, 2. \)

For clarifying the representations under the chiral symmetry, we explicitly write the superscripts of the baryon fields as

\[ (\psi_{1l})^a, (\psi_{1r})^a, (\psi_{2l})^a, (\psi_{2r})^a, \]
\[ \eta_{1l}^{(a,\alpha)}, \eta_{1r}^{(ab,\alpha)}, \eta_{2l}^{(ab,\alpha)}, \eta_{2r}^{(a,\alpha)}. \]  \tag{5} \]

where \( a, b = 1, 2 \) are for \( SU(2)_L \) and \( \alpha, \beta = 1, 2 \) for \( SU(2)_R \). Note that the superscripts \( ab \) and \( \alpha \beta \) of \( \eta \) fields are symmetrized to express 3 representation: e.g. \( \eta_{1l}^{(a,\alpha)} = \eta_{1l}^{(a,\alpha)}. \) The transformation properties under the parity and the charge conjugation are defined as

\[ \Psi_{1l,1r} \rightarrow \gamma_0 \Psi_{1r,1l} , \quad \Psi_{2l,2r} \rightarrow -\gamma_0 \Psi_{2r,2l}, \]  \tag{6} \]
\[ \Psi_{1l,1r} \rightarrow C(\Psi_{1r,1l})^T , \quad \Psi_{2l,2r} \rightarrow -C(\Psi_{2r,2l})^T, \]  \tag{7} \]

where \( C = i\gamma^2\gamma^0 \) and \( \Psi = \psi, \eta. \) The covariant derivatives for the fields are expressed as

\[ D_\mu \psi_{1l,2l} = (\partial_\mu - i \varphi_{\mu}) \psi_{1l,2l}, \]
\[ D_\mu \psi_{1r,2r} = (\partial_\mu - i \varphi_{\mu}) \psi_{1r,2r}, \]  \tag{8} \]

and

\[ (D_\mu \eta_{1l,2l})^{(a,\alpha)} = \partial_\mu \eta_{1l,2l}^{(a,\alpha)} - i(\varphi_{\mu})^{(a,\alpha)} \]
\[ -i[(\varphi_{\mu})^{(a,\alpha)} - (\varphi_{\mu})^{(a,\alpha)}], \]
\[ (D_\mu \eta_{1r,2r})^{(a,\alpha)} = \partial_\mu \eta_{1r,2r}^{(a,\alpha)} - i(\varphi_{\mu})^{(a,\alpha)} \]
\[ -i[(\varphi_{\mu})^{(a,\alpha)} - (\varphi_{\mu})^{(a,\alpha)}]. \]  \tag{9} \]

where \( \varphi_{\mu} \) and \( \varphi_{\mu}^{(a,\alpha)} \) are the external gauge fields introduced by gauging the chiral \( SU(2)_L \otimes SU(2)_R \) symmetry.

Next we introduce a 2 × 2 matrix field \( M \) expressing a nonet of scalar and pseudoscalar mesons made of a quark and an antiquark. The representation under \( SU(2)_L \otimes SU(2)_R \) of the \( M \) is

\[ M = \frac{\sigma}{2} + i\pi . \tilde{T} \sim (2, 2). \]  \tag{10} \]

The transformation properties under the parity and the charge conjugation are given by

\[ M \rightarrow M^T, \quad M \rightarrow M^T. \]  \tag{11} \]

The covariant derivative for \( M \) is expressed as

\[ (D_\mu M)^a = \partial_\mu M^a - i(\varphi_{\mu})^a b M^b + i M_a^b (\varphi_{\mu})^a \]  \tag{12} \]

Using the fields introduced above we construct a Lagrangian invariant under the chiral \( SU(2)_L \otimes SU(2)_R \) symmetry.

Let us first consider terms including only \( \psi_1 \) and \( \psi_2 \) and their Yukawa interaction to \( M \). In the present
A part including $\eta_1$ and $\eta_2$ with $M$ is

$$L^{(2)} = (\bar{\eta}_1)(\bar{\eta}_2)(\bar{\eta}_1)(\bar{\eta}_2)/(a,\alpha,\beta) + (\bar{\eta}_1)(\bar{\eta}_2)/(a,\alpha,\beta) + (\bar{\eta}_1)(\bar{\eta}_2)/(a,\alpha,\beta) + (\bar{\eta}_1)(\bar{\eta}_2)/(a,\alpha,\beta)$$

Yukawa interaction terms connecting $\psi$ fields to $\eta$ fields are expressed as

$$L^{(3)} = -y_1[\epsilon_\alpha(\bar{\psi}_1\eta_1\alpha)(M_1^\beta)\beta\eta_1\alpha + \epsilon_\beta(\bar{\psi}_1\eta_1\beta)(M_1^\alpha)\alpha\eta_1\alpha + \epsilon_\beta(\bar{\psi}_1\eta_1\alpha)(M_1^\beta)\beta\eta_1\alpha + \epsilon_\beta(\bar{\psi}_1\eta_1\beta)(M_1^\alpha)\alpha\eta_1\alpha]$$

In addition to the non-derivative interactions shown above, we need to include derivative interactions. Possible interaction terms including one derivative are given by

$$L^{(4)} = \frac{i}{f_\pi}(-a_1[\bar{\psi}_1\eta_1\alpha)(M_1^\beta)\beta\eta_1\alpha + \epsilon_\beta(\bar{\psi}_1\eta_1\beta)(M_1^\alpha)\alpha\eta_1\alpha + \epsilon_\beta(\bar{\psi}_1\eta_1\alpha)(M_1^\beta)\beta\eta_1\alpha + \epsilon_\beta(\bar{\psi}_1\eta_1\beta)(M_1^\alpha)\alpha\eta_1\alpha]$$

Combining the above terms together, the Lagrangian in the present analysis is given by

$$L = L^{(1)} + L^{(2)} + L^{(3)} + L^{(4)} + L_{\text{meson}}$$

where the mesonic part $L_{\text{meson}}$ is written as

$$L_{\text{meson}} = \text{Tr}[D_\mu M \cdot D^\mu M^\dagger] - V(M)$$

where $V(M)$ is a meson potential term. In this paper, we do not specify the form of the potential, but we assume that this potential provides the vacuum expectation value (VEV) of $M$ as $\langle M \rangle = \text{diag}(f_\pi/2, f_\pi/2)$, where $f_\pi$ is the pion decay constant.

**B. Mass matrix**

We have constructed the Lagrangian by requiring the chiral $SU(2)_L \otimes SU(2)_R$ invariance. To study the properties of nucleons, we decompose baryons in the chiral representation to irreducible representations of the flavor symmetry as

$$\psi_{1,2} = N^{(1)}_{1,2}, \quad \psi_{1,2} = N^{(1)}_{1,2},$$

$$\eta^{(a,\alpha,\beta)} = \Delta^{a,b} \eta^{(a,\alpha,\beta)} + \sqrt{\lambda} (\epsilon_\alpha \delta^a_\delta + \epsilon_\alpha \delta^a_\delta) (\eta^{(1,2)}_{1,2})^k,$$

$$\eta^{(a,\alpha,\beta)} = \Delta^{a,b} \eta^{(a,\alpha,\beta)} + \lambda (\epsilon_\alpha \delta^a_\delta + \epsilon_\alpha \delta^a_\delta) (\eta^{(1,2)}_{1,2})^k.$$
The mass eigenstates denoted by
\[ N_{\text{phys}}^T = (N_0^{(1)}, N_+^{(1)}, N_-^{(1)}, N_0^{(2)}) \]
are obtained by diagonalizing the above mass matrix \( M_N \). We note that parities of all fields in \( N' \) are due to the redefinition given in Eq. (22). Two eigenvalues of the mass matrix \( M_N \) are negative in our analysis, and we regard the parities of these state as negative.

### C. One pion interactions and axial charges

The interaction terms of nucleons to one pion are given from the Lagrangian as
\[ \bar{N}'C'_{\mu N}i\gamma_5\pi N' + N'C_{\mu N}i\gamma_5\partial_{\mu}\gamma_5N', \]  
where \( \pi = \bar{\pi} \cdot \tilde{r} \) and
\[ C'_{\mu N} = \left( \begin{array}{cccc} -\frac{g_1}{\sqrt{2}} & -\frac{g_1^T}{\sqrt{2}} & 0 & 0 \\ -g_1 & g_1^T & 0 & 0 \\ 0 & 0 & -\frac{g_2}{\sqrt{2}} & -\frac{g_2}{\sqrt{2}} \\ 0 & 0 & -g_2 & -g_2 \\ \end{array} \right), \]  
\[ C_{\mu N} = \left( \begin{array}{cccc} 0 & 0 & -\frac{a_1}{2f_{\pi}} & -\frac{a_3}{2f_{\pi}} \\ 0 & 0 & -\frac{a_1}{2f_{\pi}} & -\frac{a_3}{2f_{\pi}} \\ -\frac{a_1}{2f_{\pi}} & -\frac{a_1}{2f_{\pi}} & 0 & 0 \\ -\frac{a_3}{2f_{\pi}} & -\frac{a_3}{2f_{\pi}} & 0 & 0 \\ \end{array} \right). \]

Axial-vector charge matrix is determined as
\[ \bar{N}'G_A'\gamma_{\mu}A'_{\mu}\gamma_5N', \]
where
\[ G'_A = \left( \begin{array}{cccc} -1 & 0 & -a_1\sigma_0 & a_3\sigma_0 \\ 0 & 0 & -a_1\sigma_0 & a_3\sigma_0 \\ -a_1\sigma_0 & -a_1\sigma_0 & 1 & 0 \\ a_3\sigma_0 & a_3\sigma_0 & 0 & -\frac{3}{2} \end{array} \right). \]  

In the present analysis, we identify the mass eigenstates as
\[ N_{\text{phys}}^T = (N(939), N(1440), N(1535), N(1650)). \]

### III. CHIRAL INVARIANT MASSES AND PARTNER STRUCTURE

In this section we determine the values of model parameters and study the mixing structure of relevant baryons.

As we said in the previous section, we set the VEV of \( \sigma \) to be the pion decay constant:
\[ \sigma_0 = f_\pi = 92.4\text{MeV} \]  

\begin{table}[h]  
| \(P\) | Mass | Width | \(\Gamma_{N^* \rightarrow N\pi}\) | Axial charge |
|---|---|---|---|---|
| N(939) | 939.0±1.3 | - | 1.272±0.002 |
| N(1440) | 1430±20 | 228±74 | - |
| N(1535) | 1535±10 | 68±19 | O(0.1)[lat] |
| N(1650) | 1655±15 | 84±23[to N(939)] 22±15[to N(1440)] | 0.55[lat] |

**TABLE I.** Experimental values of masses and partial decay widths of baryons listed in Ref. [19]. The column indicated by \( P \) = ± shows the parity of the nucleon. Unit of masses and widths is MeV. The error of \( m_{N(939)} \) expresses the mass difference between the proton and neutron. [lat] indicates that the value is obtained by the lattice analysis in Ref. [20].

Besides this parameter, there are twelve parameters in this model:
\[ m_0^{(1)}, m_0^{(2)}, g_1, g_2, g_3, g_4, y_1, y_2, a_1, a_2, a_3, a_4 \]  

We list values of relevant physical quantities determined from experiments and lattice analyses in Table I. Among them, we use the following ten physical values as inputs: nucleon masses:
\[ m_{N(939)} = 939\text{MeV} \]  
\[ m_{N(1440)} = 1430\text{MeV} \]  
\[ m_{N(1535)} = 1535\text{MeV} \]  
\[ m_{N(1650)} = 1650\text{MeV} \]  

partial decay widths:
\[ \Gamma(N(1440) \rightarrow N(939) + \pi) = 228\text{MeV} \]  
\[ \Gamma(N(1535) \rightarrow N(939) + \pi) = 68\text{MeV} \]  
\[ \Gamma(N(1650) \rightarrow N(939) + \pi) = 84\text{MeV} \]  
\[ \Gamma(N(1650) \rightarrow N(1440) + \pi) = 22\text{MeV} \]  

and axial charges:
\[ g_A(N(939)) = 1.272 \]  
\[ g_A(N(1650)) = 0.55 \]  

In addition to the above inputs, we use the following range of \( g_A(N(1535)) \) to restrict the parameters:
\[ -0.25 \leq g_A(N(1535)) \leq 0.25. \]  

Furthermore, we restrict the parameters by requiring all the components of axial-charge matrix on the physical base are no larger than 5.

In this analysis, we first fix the chiral invariant masses \( m_0^{(1)} \) and \( m_0^{(2)} \) to certain values of every 5 MeV from 0 MeV to 1500 MeV, and determine other ten parameters from the values shown in Eqs. (35)-(37). Here, \( m_0^{(1)} \)  

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1 In calculating the decay widths, we use the pion mass as \( m_\pi = 137\text{MeV} \).
and $m_0^{(2)}$ are chiral invariant masses of $[(1, 2) \oplus (2, 1)]$ and $[(2, 3) \oplus (3, 2)]$ representations, respectively.

We find that solutions are categorized into five groups as shown in Fig. 1. In Group 1 indicated by purple + symbols, both chiral invariant masses are less than 100 MeV. In the range where $m_0^{(1)}$ is between 200 MeV and 900 MeV, three lumps exist: Group 2 indicated by blue □ symbols; Group 3 by light green x symbols; and Group 4 light blue * symbols. They are characterized by

$$
\begin{align*}
&m_0^{(1)} \sim m_0^{(2)} \text{ [Group 2]}, \\
&m_0^{(1)} \geq m_0^{(2)} \text{ [Group 3]}, \\
&m_0^{(1)} < m_0^{(2)} \text{ [Group 4]}, \\
&m_0^{(1)} \geq m_0^{(2)} \text{ [Group 5]}. 
\end{align*}
$$

In Group 5 indicated by yellow □, the chiral invariant mass $m_0^{(1)}$ takes large value of about 1000 MeV.

In Fig. 2, we show the mixing structure of nucleons: $N(939)$, $N(1440)$, $N(1535)$ and $N(1650)$ for Group 1 to Group 5. Here the horizontal axis shows the value of axial-charge of $N(1535)$ and the vertical axis shows the percentages of $\psi_1$ indicated by magenta ∨ symbols, $\eta_1$ by brown ○ symbols, $\psi_2$ by green ◦ symbols and $\eta_2$ by navy Δ symbols. In Table II, we summarize features of mixing rates for each group. The first row in Fig. 2 shows that the dominant component of $N(939)$ is $\eta_1$ indicated by brown ○ belonging to $[(2, 3) \oplus (3, 2)]$ representation. We note that we cannot find any solutions for $g_A(N(1535)) \geq -0.1$ in the Group 1. One can easily see that $N(1440)$ is dominated by $\eta_2$ (navy Δ), $N(1535)$ by $\psi_1$ (magenta ∨) belonging to $[(1, 2) \oplus (2, 1)]$ representation and $N(1650)$ by $\eta_2$ (green ◦). Since $\eta_1$ and $\eta_2$ are chiral partners to each other, we conclude that $N(1440)$ dominated by $\eta_1$ and $N(1650)$ by $\eta_2$, the chiral partner to $N(939)$ dominated by $\eta_1$. We would like to stress that this partner structure can be realized when the chiral invariant masses of $N(939)$ and $N(1440)$ are small.

In Group 2 (the second row of Fig. 2), $\eta_1$ (brown ○) belonging to $[(2, 3) \oplus (3, 2)]$ representation is a dominant component in the $N(939)$ and $\psi_1$ (magenta ∨) almost occupies $N(1535)$, similarly to Group 1. A difference between Group 1 and Group 2 appears in the rate of $\psi_2$ (green ◦) in $N(1440)$. In Group 1, the mixing rate of $\psi_2$ in $N(1440)$ is smaller than 0.1 as can be seen in the first row of Fig. 2. On the other hand, the rate of $\psi_2$ is larger than 0.2 and $\psi_2$ is included in $N(1440)$ dominantly as shown in the second row of Fig. 2. Here the rate of $\eta_2$ component (navy Δ) included in $N(1650)$ is high, but $N(1440)$ and $N(1535)$ include a certain amount of the $\eta_2$ component. So, it is difficult to identify the chiral partner of $N(939)$ in Group 2.

In Group 3, Group 4 and Group 5, $N(939)$ is composed of $\psi_1$ (magenta ∨) or $\eta_2$ (navy Δ) dominantly and negative parity nucleons, $N(1535)$ and $N(1650)$ have $\psi_2$ (green ◦) or $\eta_2$ (brown ○) mainly, as can be seen in the third, fourth and fifth rows in Fig. 2. This indicates that the chiral partner of $N(939)$ is a mixture of two negative parity nucleons in these groups, differently from Group 1 and Group 2. Table II summarizes these results. Table II shows that Group 5 is distinguished from Group 3 by the mixing rate of $\eta_2$ component in $N(1650)$: The rate is larger than 0.1 in Group 3, while it is no greater than 0.1 in Group 5. On the other hand, it is difficult to distinguish Group 4 with Group 5 and Group 3 with Group 4 by mixing rates. In the present work, we use the values of chiral invariant masses in addition to the mixing rates to separate these Groups: The chiral invariant masses satisfy $m_0^{(1)} < m_0^{(2)}$ in Group 4; Group 3 is characterized by $m_0^{(1)} \geq m_0^{(2)}$ and $\eta_2 > 0.1$ in $N(1650)$, while Group 5 by $m_0^{(1)} \geq m_0^{(2)}$ and $\eta_2 \leq 0.1$ in $N(1650)$.

| Group color | Group 1 purple | Group 2 blue | Group 3 light green | Group 4 sky | Group 5 yellow |
|-------------|----------------|--------------|--------------------|-------------|---------------|
| N(939)      | $\eta_1 > 0.8$| $\eta_1 > 0.35$| $\eta_1 < 0.45$   | $\eta_1 < 0.35$| $\eta_1 < 0.35$|
| N(1440)     | $\eta_1 < 0.01$| $\eta_1 < 0.025$| $\eta_1 < 0.25$   | $\eta_1 < 0.75$| $\eta_1 < 0.6$  |
| N(1650)     | $\eta_2 < 0.1$ | $\eta_2 > 0.1$  | $\eta_2 > 0.1$    | $\eta_2 < 0.3$ | $\eta_2 \leq 0.1$ |

TABLE II. Features of mixing rates. For example, in the column for Group 1, $\eta_1 > 0.8$ in the row of $N(939)$ implies that the percentage of $\eta_1$ component in $N(939)$ is always larger than 0.8.
In Ref. [15], two \([1, 2] \oplus (2, 1)\) representations are used to study \(N(939), N(1440), N(1535)\) and \(N(1650)\). The authors found two types of solutions. In one type (Type-A), one is that the chiral partner of \(N(939)\) is a mixture of \(N(1535)\) and \(N(1650)\) in vacuum and both chiral invariant masses are almost 200 MeV. In another type (Type-B) \(N(939)\) and \(N(1535)\) are chiral partners to each other and both chiral invariant masses are about 1000 MeV. In the present analysis, we find solutions for which \(N(939)\) is dominated by \([1, 2] \oplus (2, 1)\) representation in Groups 3, 4 and 5. Comparing mixing structure and the chiral invariant masses, we think that the Type-A is consistent with Group 3 and Type-B with Group 4 or 5.

In Ref. [14], \([3, \bar{3}] \oplus (3, \bar{3})\) and \([3, 6] \oplus (6, 3)\) representations under the chiral \(U(3)_L \otimes U(3)_R\) symmetry are introduced to study six nucleons including \(N(939), N(1440), N(1535)\) and \(N(1650)\). This analysis indicates that \(N(939)\) have \([3, 6] \oplus (6, 3)\] dominantly and the chiral invariant mass is \(500 \text{ MeV} \sim 800 \text{ MeV}\). This is consistent with Group 2 in our analysis.

### TABLE III. Typical values of mixing rates of nucleons.

| Nucleon | rep | Group 1 | Group 2 | Group 3 | Group 4 | Group 5 |
|---------|-----|---------|---------|---------|---------|---------|
| \(N(939)\) | \(\psi_1\) | 0.113 | 0.21 | 0.518 | 0.677 | 0.768 |
| | \(\eta_1\) | 0.856 | 0.735 | 0.098 | 0.032 | 0.001 |
| | \(\psi_2\) | 0.002 | 0.052 | 0.133 | 0.163 | 0.227 |
| | \(\eta_2\) | 0.029 | 0.002 | 0.251 | 0.128 | 0.004 |
| \(N(1440)\) | \(\psi_1\) | 0.002 | 0.003 | 0.355 | 0.092 | 0.002 |
| | \(\eta_1\) | 0.029 | 0.043 | 0.021 | 0.223 | 0.041 |
| | \(\psi_2\) | 0.033 | 0.627 | 0.368 | 0.01 | 0.001 |
| | \(\eta_2\) | 0.936 | 0.327 | 0.256 | 0.675 | 0.956 |
| \(N(1535)\) | \(\psi_1\) | 0.819 | 0.641 | 0.126 | 0.012 | 0.137 |
| | \(\eta_1\) | 0.109 | 0.222 | 0.801 | 0.212 | 0.322 |
| | \(\psi_2\) | 0.068 | 0.007 | 0.057 | 0.675 | 0.528 |
| | \(\eta_2\) | 0.004 | 0.130 | 0.015 | 0.161 | 0.013 |
| \(N(1650)\) | \(\psi_1\) | 0.066 | 0.146 | 0.001 | 0.218 | 0.093 |
| | \(\eta_1\) | 0.006 | 0.0 | 0.08 | 0.533 | 0.636 |
| | \(\psi_2\) | 0.897 | 0.341 | 0.442 | 0.212 | 0.244 |
| | \(\eta_2\) | 0.031 | 0.540 | 0.477 | 0.037 | 0.027 |

### IV. AXIAL CHARGES

In the previous section, we discussed the mixing structure of nucleons together with their chiral invariant masses. In this section, we study axial charges in detail.

We define transition axial charge as off-diagonal elements of following axial-charge matrix on physical base.

\[
N'_{\text{phys}}G'_{\text{Aphys}}\gamma^\muA_{\mu}\gamma^5N'_{\text{phys}}
\]

where

\[
G'_{\text{Aphys}} = \begin{pmatrix}
    g_A(N_1) & g_A(N_1N_2) & g_A(N_1N_3) & g_A(N_1N_4) \\
    g_A(N_1N_2) & g_A(N_2) & g_A(N_2N_3) & g_A(N_2N_4) \\
    g_A(N_1N_3) & g_A(N_2N_3) & g_A(N_3) & g_A(N_3N_4) \\
    g_A(N_1N_4) & g_A(N_2N_4) & g_A(N_3N_4) & g_A(N_4)
\end{pmatrix}
\]
and

\[(N_1, N_2, N_3, N_4) \equiv (N_+^{(1)}, N_+^{(2)}, N_-^{(1)}, N_-^{(2)})_{\text{phys}} \] (42)

\(\equiv (N_1', N_2', N_3', N_4') \) phys (42)

In the present model, the following relation the diagonal axial charges is satisfied:

\[ \sum_{i=1}^{4} g_A(N_i) = 0 \] (43)

Now we use axial charge: \( g_A(N_1) = g_A(N(939)) = 1.272 \) and \( g_A(N_4) = g_A(N(1650)) = 0.55 \) as input. So this relation is

\[ g_A(N_2) + g_A(N_3) = -1.822. \] (44)

We plot this relation in Fig. 3. This plot shows that, when the axial charge of \( N(1535) \) is in the range consistent with the lattice analysis, the axial charge of \( N(1440) \) is negative.

In Fig. 4 we plot predicted values of \( g_A(N_2N_3) \) and \( g_A(N_2N_4) \). This shows that \( |g_A(N_2N_3)| \) of Group 1 and Group 2 is always larger than 2 and 0.5, respectively. On the other hand, \( |g_A(N_2N_4)| \) of Group 1 is smaller than 2 and that of Group 2 is above 1. We see that Group 1 is able to be distinguished from other groups.

In Fig. 5 we plot predicted values of \( g_A(N_3N_4) \) and \( g_A(N_1N_3) \). Predicted values of \( g_A(N_3N_4) \) and \( g_A(N_1N_3) \) are plotted in Fig. 5. We note that \( |g_A(N_1N_3)| \) belonging to Group 1 and Group 2 are larger than 1, and that \( |g_A(N_3N_4)| \) of Group 2 and Group 5 lies between 1 and 3 and that of Group 1 is above 1. In particular, \( |g_A(N_3N_4)| \sim 2 \) in Group 5.

We summarize typical predicted values of transition axial-charges and the range of transition axial-charges in Table. IV.

V. CHANGE OF NUCLEON MASSES

In this section we study the change of nucleon masses when the VEV of \( \sigma \) changed. We plot the dependences
TABLE IV. Typical predicted values of axial charges. Rows indicated by #: (j = 1, 2, or 3) show that the absolute value of axial charge lie in the range shown in the rows.

FIG. 7. Dependences of nucleon masses on the value of the VEV of σ for Groups 1-4 and those for Group 5 in Fig. 8 for some choices of the chiral invariant masses, \( m_0^{(1)} \) and \( m_0^{(2)} \).

FIG. 8. Dependences of nucleon masses on the VEV of σ in Group 5 for \( (m_0^{(1)}, m_0^{(2)}) = (1070\text{MeV, 50MeV}) \) (left figure) and \( (1210\text{MeV, 1050MeV}) \) (right figure).

In the case of Group 5 shown in Fig. 8, the value of \( m_0^{(1)} \) is about 1000 MeV, while \( m_0^{(2)} \) takes values in wide range. Left panel of Fig. 8 \( [(m_0^{(1)}, m_0^{(2)}) = (1070\text{MeV, 50MeV})] \) shows that the mass of the ground state is stable for \( \sigma_0 > 60 \text{MeV} \) and it decreased towards \( m_0^{(2)} \) as \( \sigma_0 \) is decreased from 60 MeV. On the other hand, the right panel \( [(1210\text{MeV, 1050MeV})] \) shows that all the masses are stable against the change or \( \sigma_0 \).

Since \( \sigma_0 \) is an order parameter of chiral symmetry, Figs. 7 and 8 show that nucleon masses are degenerated to chiral invariant masses when the chiral symmetry is restored in e.g., high temperature and/or density.

**VI. SUMMARY AND DISCUSSIONS**

We introduced two types of nucleons belonging to \( [(2, 1) \oplus (1, 2)] \) and \( [(2, 3) \oplus (3, 2)] \) representations of the chiral SU(2)\(_L\) \( \otimes \) SU(2)\(_R\) group together with their parity partners. We constructed an effective chiral Lagrangian based on the parity doublet structure. We fitted model parameters to the masses, decay widths and axial-charges of \( N(939), N(1440), N(1535) \) and \( N(1650) \). Our results show that there are five groups of solutions which are separated by chiral invariant masses and mixing structure of nucleons. In Group 1, both the chiral invariant masses, \( m_0^{(1)} \) for \( [(2, 1) \oplus (1, 2)] \) and \( m_0^{(2)} \) for \( [(2, 3) \oplus (3, 2)] \), are small as seen from Fig. 1. In this group, the ground state \( N(939) \) is dominated by \( [(2, 3) \oplus (3, 2)] \) component and its chiral partner is \( N(1440) \). In Group 5, on the other hand, the dominant component of \( N(939) \) is \( [(2, 1) \oplus (1, 2)] \), whose chiral invariant mass is about 1000 MeV, and the chiral partner of \( N(939) \) is a mixture of negative parity nucleons.

We gave predictions off-diagonal elements of axial-charge matrix called transition axial-charges, which shows that some groups can be excluded when some of them are determined by e.g., lattice analysis in future. We also study the change of nucleon masses when the VEV of \( \sigma, \sigma_0 \), is changed. In Groups 1-4, all nucleons...
masses are decreased with decreasing $\sigma_0$ if two chiral invariant masses are smaller than mass of $N(939)$ as shown in Fig. [7]. In Group 5, on the other hand, the behavior depends on the value of $m_0^{(2)}$. For small $m_0^{(2)}$ the mass of $N(939)$ is stable for $\sigma_0 > 60$ MeV and it decreases toward $m_0^{(2)}$, while for large $m_0^{(2)}$ it is stable for all $\sigma_0$. This seems consistent with the lattice analysis in Ref. [17, 18], which shows that, with increasing temperature, the mass of the positive parity nucleon mass is stable, while that of the negative parity nucleon mass decreased.

The chiral representation of $[(2, 3) \oplus (3, 2)]$ includes $\Delta$ baryon in addition to nucleons. For studying $\Delta$, we need to include $[(4, 1) \oplus (1, 4)]$ representations which do not include nucleons. It is interesting to study $\Delta$ baryons by constructing a model including $[(4, 1) \oplus (1, 4)]$ representations. [Study of $\Delta$ baryon based on the parity doublet structure are done in e.g., Refs. [8, 21, 24].]

We can extend the model to three flavor case based on the SU(3)$_L \otimes$SU(3)$_R$ symmetry to study hyperons as done in Refs. [6, 7, 10, 16]. The parity doublet structure can also be extended to the baryons including heavy quarks as done in Refs. [25, 26]. We leave this for future work.

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[1] C. E. Detar and T. Kunihiro, “Linear $\sigma$ Model With Parity Doubling,” Phys. Rev. D 39, 2805 (1989). doi:10.1103/PhysRevD.39.2805
[2] D. Jido, Y. Nemoto, M. Oka and A. Hosaka, Nucl. Phys. A 671, 471 (2000) doi:10.1016/S0375-9474(99)00844-1 [hep-ph/9803306].
[3] D. Jido, M. Oka and A. Hosaka, “Chiral symmetry of baryons,” Prog. Theor. Phys. 106, 873 (2001) doi:10.1143/PTP.106.873 [hep-ph/0101005].
[4] S. Gallas, F. Giacosa and D. H. Rischke, “Vacuum phenomenology of the chiral partner of the nucleon in a linear sigma model with vector mesons,” Phys. Rev. D 82, 014004 (2010) doi:10.1103/PhysRevD.82.014004 [arXiv:0907.5084 [hep-ph]].
[5] S. Gallas and F. Giacosa, “Mirror versus naive assignment in chiral models for the nucleon,” Int. J. Mod. Phys. A 29, no. 17, 1450098 (2014) doi:10.1142/S0217751X14500985 [arXiv:1308.4817 [hep-ph]].
[6] Y. Nemoto, D. Jido, M. Oka and A. Hosaka, “Decays of 1/2− baryons in chiral effective theory,” Phys. Rev. D 57, 4124 (1998) doi:10.1103/PhysRevD.57.4124 [hep-ph/9710445].
[7] H. X. Chen, V. Dmitrasinovic, A. Hosaka, K. Nagata and S. L. Zhu, “Chiral Properties of Baryon Fields with Flavor SU(3) Symmetry,” Phys. Rev. D 78, 054021 (2008) doi:10.1103/PhysRevD.78.054021 [arXiv:0806.1997 [hep-ph]].
[8] V. Dmitrasinovic, A. Hosaka and K. Nagata, “Nucleon axial couplings and $[(1/2, 0) + (1/2, 1)]$ chiral multiplet mixing,” Mod. Phys. Lett. A 25, 233 (2010) doi:10.1142/S0217732310032494 [arXiv:0912.2372 [hep-ph]].
[9] V. Dmitrasinovic, A. Hosaka and K. Nagata, Int. J. Mod. Phys. E 19, 91 (2010) doi:10.1142/S0218310110016506 [arXiv:0912.2396 [hep-ph]].
[10] H. X. Chen, V. Dmitrasinovic and A. Hosaka, “Baryon fields with U(L)3 X U(R)3 chiral symmetry II: Axial currents of nucleons and hyperons,” Phys. Rev. D 81, 054002 (2010) doi:10.1103/PhysRevD.81.054002 [arXiv:0912.4338 [hep-ph]].
[11] H. X. Chen, V. Dmitrasinovic and A. Hosaka, “Baryon Fields with $U_L(3) \times U_R(3)$ Chiral Symmetry III: Interactions with Chiral (3, 3) + (3, 3) Spinless Mesons,” Phys. Rev. D 83, 014015 (2011) doi:10.1103/PhysRevD.83.014015 [arXiv:1009.2422 [hep-ph]].
[12] H. X. Chen, V. Dmitrasinovic and A. Hosaka, “Baryons with $U_L(3) \times U_R(3)$ Chiral Symmetry IV: Interactions with Chiral (8, 1) + (1, 8) Vector and Axial-vector Mesons and Anomalous Magnetic Moments,” Phys. Rev. C 85, 055205 (2012) doi:10.1103/PhysRevC.85.055205 [arXiv:1109.3130 [hep-ph]].
[13] V. Dmitrasinovic, H. X. Chen and A. Hosaka, “Baryon fields with UL(3)UR(3) chiral symmetry, V. Pion-nucleon and kaon-nucleon terms,” Phys. Rev. C 93, no. 6, 065208 (2016) doi:10.1103/PhysRevC.93.065208.
[14] H. Nishihara and M. Harada, “Extended Goldberger-Treiman relation in a three-flavor parity doublet model,” Phys. Rev. D 92, no. 5, 054022 (2015) doi:10.1103/PhysRevD.92.054022 [arXiv:1506.07956 [hep-ph]].
[15] L. Olbrich, M. Zetényi, F. Giacosa and D. H. Rischke, “Three-flavor chiral effective model with four baryonic multiplets within the mirror assignment,” Phys. Rev. D 93, no. 3, 034021 (2016) doi:10.1103/PhysRevD.93.034021 [arXiv:1511.05035 [hep-ph]].
[16] L. Olbrich, M. Zetényi, F. Giacosa and D. H. Rischke, Phys. Rev. D 97, no. 1, 014007 (2018) doi:10.1103/PhysRevD.97.014007 [arXiv:1708.01061 [hep-ph]].
[17] G. Aarts, C. Alton, S. Hands, B. Jäger, C. Praki and J. I. Skullerud, “Nucleons and parity doubling across the deconfinement transition,” Phys. Rev. D 92, no. 1, 014503 (2015) doi:10.1103/PhysRevD.92.014503 [arXiv:1502.03603 [hep-lat]].
[18] G. Aarts, C. Alton, D. De Boni, S. Hands, B. Jäger, C. Praki and J. I. Skullerud, “Light baryons below and above the deconfinement transition: medium effects and parity doubling,” JHEP 1706, 034 (2017) doi:10.1007/JHEP06(2017)034 [arXiv:1703.09246 [hep-ph]].
Particle Data Group Collaboration M. Tanabashi et al., “Review of Particle Physics,” Phys. Rev. D 98, no. 3, 030001 (2018). doi:10.1103/PhysRevD.98.030001

T. T. Takahashi and T. Kunihiro, Phys. Rev. D 78, 011503 (2008) doi:10.1103/PhysRevD.78.011503 [arXiv:0801.4707 [hep-lat]].

V. Dexheimer, S. Schramm and D. Zschiesche, “Nuclear matter and neutron stars in a parity doublet model,” Phys. Rev. C 77, 025803 (2008) doi:10.1103/PhysRevC.77.025803 [arXiv:0710.4192 [nucl-th]].

D. Jido, T. Hatsuda and T. Kunihiro, “Chiral symmetry realization for even parity and odd parity baryon resonances,” Phys. Rev. Lett. 84, 3252 (2000) doi:10.1103/PhysRevLett.84.3252 [hep-ph/9910375].

C. Sasaki and I. Mishustin, “Thermodynamics of dense hadronic matter in a parity doublet model,” Phys. Rev. C 82, 035204 (2010) doi:10.1103/PhysRevC.82.035204 [arXiv:1005.4811 [hep-ph]].

S. Gallas, F. Giacosa and G. Pagliara, “Nuclear matter within a dilatation-invariant parity doublet model: the role of the tetraquark at nonzero density,” Nucl. Phys. A 872, 13 (2011) doi:10.1016/j.nuclphysa.2011.09.008 [arXiv:1105.5003 [hep-ph]].

J. Steinheimer, S. Schramm and H. Stocker, “The hadronic SU(3) Parity Doublet Model for Dense Matter, its extension to quarks and the strange equation of state,” Phys. Rev. C 84, 045208 (2011) doi:10.1103/PhysRevC.84.045208 [arXiv:1108.2596 [hep-ph]].

V. Dexheimer, J. Steinheimer, R. Negreiros and S. Schramm, Phys. Rev. C 87, no. 1, 015804 (2013) doi:10.1103/PhysRevC.87.015804 [arXiv:1206.3086 [nucl-th]].

S. Benic, I. Mishustin and C. Sasaki, Phys. Rev. D 91, no. 12, 125034 (2015) doi:10.1103/PhysRevD.91.125034 [arXiv:1502.05373 [nucl-th]].

Y. Motohiro, Y. Kim and M. Harada, “Asymmetric nuclear matter in a parity doublet model with hidden local symmetry,” Phys. Rev. C 92, no. 2, 025201 (2015) Erratum: [Phys. Rev. C 95, no. 5, 059903 (2017)] doi:10.1103/PhysRevC.95.059903 [arXiv:1505.00988 [nucl-th]].

D. Suenaga, Phys. Rev. C 97, no. 4, 045203 (2018) doi:10.1103/PhysRevC.97.045203 [arXiv:1704.03630 [nucl-th]].

Y. Takeda, H. Abuki and M. Harada, “Novel dual chiral density wave in nuclear matter based on a chiral partner structure,” Phys. Rev. C 82, 035204 (2010) doi:10.1103/PhysRevC.82.035204 [arXiv:1005.4811 [hep-ph]].

D. Suenaga, “Spectral function for $\bar{D}^0_s$ $(0^+)$ meson in isospin asymmetric nuclear matter with chiral partner structure,” [arXiv:1805.01709 [nucl-th]].

I. J. Shin, W. G. Paeng, M. Harada and Y. Kim, “Nuclear structure in Parity Doublet Model,” [arXiv:1805.03402 [nucl-th]].