PERFORMANCE EVALUATION AND OPTIMIZATION OF COGNITIVE RADIO NETWORKS WITH ADJUSTABLE ACCESS CONTROL FOR MULTIPLE SECONDARY USERS

YUAN ZHAO*
School of Computer and Communication Engineering
Northeastern University at Qinhuangdao, Qinhuangdao 066004, China

WUYI YUE
Department of Intelligence and Informatics
Konan University, Kobe 658-8501, Japan

Abstract. In this paper, we consider a cognitive radio network with multiple secondary users (SUs). The SU packets in the system can be divided into two categories: SU1 packets and SU2 packets, where SU1 packets have transmission priority over SU2 packets. Considering the absolute priority of the primary users (PUs), the PU packets have the highest priority in the system to transmit. In order to guarantee the Quality of Service (QoS) of the network users, as well as reduce the average delay of the SU2 packets, we propose an adjustable access control scheme for the SU2 packets. A newly arriving SU2 packet can access the system with an access probability related to the total number of packets in the system. A variable factor is also introduced to adjust the access probability dynamically. Based on the working principle of the adjustable access control scheme, we build a discrete-time queueing model with a finite waiting room and an adjustable joining rate. With a steady-state analysis of the queueing model, using a three-dimensional Markov chain, we derive some performance measures, such as the total channel utilization, the interruption rate, the throughput, and the average delay of the SU2 packets. Moreover, we show the influence of the adjustment factor on different system performance measures by using numerical results. Finally, considering the trade-off between the throughput and the average delay of the SU2 packets with respect to the adjustment factor, we build a net benefit function and show an optimal algorithm to optimize the adjustment factor.

1. Introduction. An increasing demand for spectrum resources is posing new challenges for modern communication. As a means of resolving the problem of a scarcity of spectrum resources, new technology in the form of cognitive radio networks has emerged [9, 10]. In cognitive radio networks, the secondary users (SUs) can make opportunistic use of the spectrum when the spectrum is not occupied by any primary users (PUs) [12, 14]. This kind of opportunistic spectrum occupying scheme can effectively enhance spectrum utilization.

2010 Mathematics Subject Classification. Primary: 68M10, 68M20; Secondary: 60J10.
Key words and phrases. Cognitive radio networks, multiple secondary users, access control, adjustment factor, optimization.

The reviewing process of this paper was handled by Yutaka Takahashi.

*Corresponding author: Yuan Zhao.
In cognitive radio networks, PU packets have a higher priority than SU packets. However, large numbers of SU packets in the system may influence the transmission of the PU packets in some cases [1]. In [1], the authors analyzed the transmission service outage and degradation of the PU packets caused by the packet access of the SUs with non-ideal spectrum sensing. Therefore, in order to guarantee the Quality of Service (QoS) of the PU packets, it is necessary to control the system access of the SU packets. On the other hand, to reduce the average delay of the SU packets, it is also necessary to limit the access of large numbers of SU packets into the system. In [11], the authors have presented a survey of perspectives related to the design of medium access control protocols for the SU packets in cognitive radio networks.

In [8], in order to control the access of the SU packets, a threshold for the newly arriving SU packets was set in a cognitive radio network. When the number of SU packets in the system was above the threshold, the newly arriving SU packets would leave. The authors also optimized the threshold by using Nash equilibrium theory, and showed the numerical results for the optimal threshold.

In [6], the authors proposed an adaptive access control scheme for the SU packets in a cognitive radio network by using a single channel. They assumed that the SU packets would be admitted to join the system with a certain probability, which was inversely proportional to the total number of packets in the system. By building a two-dimensional Markov chain, they derived the throughput and the average delay of the SU packets.

However, as mentioned above, most research relating to controlling the access of SU packets in cognitive radio networks did not attribute different priority levels to SUs. We note that there are different types of transmission requests in practical communication networks. To be consistent with real communication networks, it is necessary to consider the different prioritization among SUs. Recently, only a few studies have focused on the performance analysis of cognitive radio networks with prioritized SUs.

In [7], the authors divided the SU calls into SU1 calls and SU2 calls in a cognitive radio network with multiple channels. They assumed the SU1 calls have a higher priority than the SU2 calls. By building a three-dimensional Markov chain, they derived some performance measures, such as the forced termination probability of the SU1 call and the SU2 call, respectively.

In [16], the authors proposed a non-preemptive priority scheme in a cognitive radio network with multiple SUs. By constructing a three-dimensional Markov chain, they obtained the steady-state distribution of the system model. Accordingly, they derived the interruption rate of the SU1 packets and the SU2 packets, respectively.

However, the above research relating to cognitive radio networks with multiple SUs did not consider introducing an access control scheme for the SU packets with a lower priority.

In this paper, we consider three types of packets to access a single channel in a cognitive radio network. In order to control the system access of the SU2 packets and reduce the average delay of the SU2 packets adaptively, we propose an adjustable access control scheme for the SU2 packets in cognitive radio networks with multiple SUs. A newly arriving SU2 packet can access the system with an access probability that is related to the number of packets in the system. Also, we specifically introduce an adjustment factor to dynamically adjust the access probability. By building a queueing model with a finite waiting room and an adjustable joining rate, we derive the expressions of some important system performance measures. With numerical
results, we show the change trends of the performance measures with different parameter settings.

We note that this paper is an extended version of our conference paper presented in [17] and is substantially different from [17] in that gives some new expressions for many more performance measures, such as the total channel utilization, the blocking rate, the throughput and the average delay of the SU2 packets with additional analysis of the system performances. Specifically, in this paper, we add optimization analysis to show how to set the adjustment factor to balance the throughput and the average delay of the SU2 packets.

The remainder of this paper is organized as follows. The system model and model analysis are demonstrated in Section 2. In Section 3, different performance measures, such as the total channel utilization, and the blocking rate, the interruption rate, the throughput and the average delay of the SU2 packets are derived. In Section 4, numerical results for different performance measures are presented to show the influence of the adjustment factor. In Section 5, the optimization for the adjustment factor is given. Finally, conclusions are drawn in Section 6.

2. System model and model analysis.

2.1. System model. A cognitive radio network with a single channel is considered in this paper. There are three types of packets in the system, they are PU packets, SU1 packets and SU2 packets. The PU packets have the highest priority and the SU2 packets have the lowest priority. In practical real applications, the SU packets with real-time transmission can be considered to be the SU1 packets, and the other SU packets (such as non-real-time data) can be considered to be the SU2 packets.

A buffer with finite capacity $K$ ($K > 0$) is allocated for the SU2 packets. In order to reduce the average delay of the PU packets and the SU1 packets, there are no buffer settings for the PU packets and the SU1 packets. In cognitive radio networks, if a PU packet accesses a channel but the channel is being used by an SU packet, in order to avoid interference to the PU packet, the SU packet will release the channel and the transmission of this SU packet will be interrupted [9, 15]. In this model, we assume that a newly arriving PU packet can interrupt the transmission of an SU1 packet or an SU2 packet, even if the SU1 or SU2 packet is occupying the channel. However, when a PU packet arrives at the system, if the channel is being occupied by another PU packet, the newly arriving PU packet will be blocked and leave the system. In parallel, a newly arriving SU1 packet can interrupt the transmission of an SU2 packet that is occupying the channel.

In order to control the system access of the SU2 packets adaptively, we propose an adjustable access control scheme for the SU2 packets. The working principle of the proposed adjustable access control scheme can be depicted in Fig. 1.

Just as shown in Fig. 1, in this adjustable access control scheme, a newly arriving SU2 packet will be admitted access to the system with an access probability related to the total number of packets in the system. Moreover, an adjustment factor is also introduced to control the weight of the number of packets in the access probability. We can denote the access probability as $q_i = 1/(i\tau + 1)$, where $i$ is the total number of packets in the system, and $\tau$ is the adjustment factor. We note that if an SU2 packet is admitted access to the system but finds the buffer is full, this SU2 packet will be blocked by the system.

Moreover, considering the higher priority of the PU packets and the SU1 packets, the transmission of the SU2 packets may be interrupted. In order to reduce the
adverse influence arising from any interrupted SU2 packets, we assume that the interrupted SU2 packets will be forced to leave the system.

In queueing theory, given the model assumptions outlined above, the three types of packets can be considered to be three types of customers with different priorities in queueing theory. Based on the working principle of the proposed cognitive radio network, we can build a priority queueing model with a finite waiting room and an adjustable joining rate.

We assume an early arrival system with a slotted timing structure, and a time axis ordered by \( t = 1, 2, \ldots \). In order to avoid complexity without loss of generality, we assume that whether an SU2 packet arriving during the interval \((n, n^+)\) can be admitted access to the system or not is dependent on the number of packets in the system at the instant \( t = n^- \).

We assume that the arrival intervals of the PU packets, the SU1 packets and the SU2 packets follow geometric distributions with parameters \( \lambda_1, \lambda_{21} \) and \( \lambda_{22} \), respectively. Similarly, we assume that the transmission time of a PU packet, an SU1 packet and an SU2 packet follow geometric distributions with parameters \( \mu_1, \mu_{21} \) and \( \mu_{22} \), respectively.

We denote \( L_n \) as the total number of packets in the system at the instant \( t = n^- \). We also denote \( L_n^{(1)} \) and \( L_n^{(21)} \) as the number of PU packets and SU1 packets in the system at the instant \( t = n^- \), respectively. \( \{L_n, L_n^{(21)}, L_n^{(1)}\} \) constitutes a three-dimensional discrete-time Markov chain with the state space \( \Omega \) as follows:

\[
\Omega = (0, 0, 0) \cup \{(i, 0, 0) \cup (i, 1, 0) \cup (i, 0, 1) : 1 \leq i \leq K + 1\}.
\] (1)

2.2. **Model analysis.** We define \( P \) as the state transition probability matrix of \( \{L_n, L_n^{(21)}, L_n^{(1)}\} \), and \( P \) can be given in a block-structure form as follows:

\[
P = \begin{pmatrix}
P_{0,0} & P_{0,1} & P_{0,2} & \cdots & P_{0,K-1} & P_{0,K} & P_{0,K+1} \\
P_{1,0} & P_{1,1} & P_{1,2} & \cdots & P_{1,K-1} & P_{1,K} & P_{1,K+1} \\
P_{2,0} & P_{2,1} & P_{2,2} & \cdots & P_{2,K-1} & P_{2,K} & P_{2,K+1} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
P_{K-1,0} & P_{K-1,1} & P_{K-1,2} & \cdots & P_{K-1,K-1} & P_{K-1,K} & P_{K-1,K+1} \\
P_{K,0} & P_{K,1} & P_{K,2} & \cdots & P_{K,K-1} & P_{K,K} & P_{K,K+1} \\
P_{K+1,0} & P_{K+1,1} & P_{K+1,2} & \cdots & P_{K+1,K-1} & P_{K+1,K} & P_{K+1,K+1}
\end{pmatrix}
\] (2)
where $P_{u,v}$ is the transition probability matrix for the total number of packets changing from $u$ to $v$, $u = 0, 1, \ldots, K + 1$, $v = 0, 1, \ldots, K + 1$. We note here that the structure of $P$ is similar to that presented in [16]. However, considering the adjustable access control scheme proposed in this paper, each non-zero block in $P$ is much more complicated than that presented in [16].

In following paper, for compactness of presentation, we introduce a notation
\[
\alpha_u = \frac{\lambda_2}{(\tau_u + 1)}, \quad u = 0, 1, 2, \ldots, K + 1.
\]
Moreover, we also introduce the overbar notation to denote the probability of a complement event, for instance, $\bar{\alpha}_u = 1 - \alpha_u$.

We discuss each non-zero block in $P$ as follows.

1. $P_{0,0}$ is the one-step transition probability for the total number of packets in the system being fixed at 0. When the total number of packets in the system is 0, the only one possibility is that there are no PU packets, SU1 packets or SU2 packets in the system. Therefore $P_{0,0}$ is a vector with a scalar value given as follows:
\[
P_{0,0} = \bar{\lambda}_1 \bar{\lambda}_2 \lambda_2.
\]

2. $P_{0,1}$ is the one-step transition probability matrix for the total number of packets in the system being increased from 0 to 1. When the total number of packets in the system is 1, there are three possibilities for the packets in the system. Firstly, there is only one SU2 packet in the system. Secondly, there is only one SU1 packet in the system. Thirdly, there is only one PU packet in the system. Therefore $P_{0,1}$ is a $1 \times 3$ matrix given as follows:
\[
P_{0,1} = \begin{pmatrix}
\bar{\lambda}_1 \bar{\lambda}_2 \lambda_2 \\
\bar{\lambda}_1 \lambda_2 \lambda_2 \\
\bar{\lambda}_1 \lambda_2 \lambda_2
\end{pmatrix}.
\]

3. $P_{0,2}$ is the one-step transition probability matrix for the total number of packets in the system being increased from 0 to 2. When the total number of packets in the system is 2, there are three possibilities for the packets in the system. Firstly, there are two SU2 packets in the system. Secondly, there is an SU1 packet and an SU2 packet in the system. Thirdly, there is a PU packet and an SU2 packet in the system. Therefore, $P_{0,2}$ is a $1 \times 3$ matrix given as follows:
\[
P_{0,2} = \begin{pmatrix}
\bar{\lambda}_1 \lambda_2 \\
\lambda_1 \lambda_2 \lambda_2 \\
\lambda_1 \lambda_2 \lambda_2
\end{pmatrix}.
\]

4. $P_{1,0}$ is the one-step transition probability matrix for the total number of packets in the system being decreased from 1 to 0. As stated in (1)-(3), $P_{1,0}$ is a $3 \times 1$ matrix given as follows:
\[
P_{1,0} = \begin{pmatrix}
\lambda_1 \lambda_2 \bar{\alpha}_u \mu_2 \\
\lambda_1 \lambda_2 \bar{\alpha}_u \mu_2 \\
\lambda_1 \lambda_2 \bar{\alpha}_u \mu_2
\end{pmatrix}.
\]

5. $P_{u,u-1}$ $(2 \leq u \leq K + 1)$ is the one-step transition probability matrix for the total number of packets in the system being decreased from $u$ to $u - 1$. When the total number of packets in the system is $u$, there are three possibilities for the packets in the system. Firstly, there are $u$ SU2 packets in the system. Secondly, there is an SU1 packet and $u - 1$ SU2 packets in the system. Thirdly, there is a PU packet and $u - 1$ SU2 packets in the system. Therefore, $P_{u,u-1}$ is a $3 \times 3$ matrix given as follows:
\[
P_{u,u-1} = \begin{pmatrix}
\lambda_1 \lambda_2 \bar{\alpha}_u \mu_2 \\
\lambda_1 \lambda_2 \bar{\alpha}_u \mu_2 \\
\lambda_1 \lambda_2 \bar{\alpha}_u \mu_2
\end{pmatrix}.
\]
influence of both the PU packets and the SU1 packets. When there is no packet in the system, the channel will be used by any packet. When there is no packet in the system, the channel will

Therefore, in this section, we focus on analyzing the performance measures of the SU2 packets. We firstly derive the expression for the total channel utilization, then we show the expressions of some performance measures of the SU2 packets, such as the average queue length, the access rate, the blocking rate, the interruption rate, the throughput, and the average delay.

The total channel utilization $\delta$ is defined as the probability that the channel is being used by any packet. When there is no packet in the system, the channel will

3. Performance measures. In this section, with the steady-state distribution obtained in Section 2, we derive the expressions of some important system performance measures. We note that the admission control scheme proposed in this paper is focused on controlling the access of the SU2 packets. On the other hand, in the proposed adjustable access control scheme, the performance of the SU2 packets will be influenced by both the PU packets and the SU1 packets.

Therefore, in this section, we focus on analyzing the performance measures of the SU2 packets. We firstly derive the expression for the total channel utilization, then we show the expressions of some performance measures of the SU2 packets, such as the average queue length, the access rate, the blocking rate, the interruption rate, the throughput, and the average delay.

The total channel utilization $\delta$ is defined as the probability that the channel is being used by any packet. When there is no packet in the system, the channel will
be idle. Therefore, we can give the expression for the total channel utilization $\delta$ as follows:

$$\delta = 1 - \pi_{0,0,0}. \quad (12)$$

The average queue length $E_{SU2}$ of the SU2 packets is defined as the average number of SU2 packets in the system in the steady state. With the steady-state distribution $\pi_{i,j,k}$, we can give the expression for the average queue length $E_{SU2}$ of the SU2 packets as follows:

$$E_{SU2} = \sum_{i=0}^{K+1} i \pi_{i,0,0} + \sum_{i=1}^{K+1} (i-1)(\pi_{i,1,0} + \pi_{i,0,1}). \quad (13)$$

The access rate $\phi$ of the SU2 packets is defined as the number of newly arriving SU2 packets that are admitted access to the system per slot. We can give the expression for the access rate $\phi$ of the SU2 packets as follows:

$$\phi = \lambda_{22} \pi_{0,0,0} + \sum_{i=1}^{K+1} \frac{\lambda_{22}}{\tau_{i} + 1} (\pi_{i,0,0} + \pi_{i,1,0} + \pi_{i,0,1}). \quad (14)$$

The blocking rate $\beta$ of the SU2 packets is defined as the number of SU2 packets that are admitted access to the system but are blocked by the system per slot. We can give the expression for the blocking rate $\beta$ of the SU2 packets as follows:

$$\beta = \frac{\lambda_{22}}{\tau(K+1) + 1} (\bar{\mu}_{22} + \mu_{22}(1 - \tilde{\lambda}_{1}\tilde{\lambda}_{21}))\pi_{K+1,0,0}$$

$$+ \frac{\lambda_{22}}{\tau(K+1) + 1} (\bar{\mu}_{21} + \mu_{21}(1 - \tilde{\lambda}_{1}\tilde{\lambda}_{21}))\pi_{K+1,1,0}$$

$$+ \frac{\lambda_{22}}{\tau(K+1) + 1} (\bar{\mu}_{1} + \mu_{1}(1 - \tilde{\lambda}_{1}\tilde{\lambda}_{21}))\pi_{K+1,0,1}. \quad (15)$$

The interruption rate $\gamma$ of the SU2 packets is defined as the number of SU2 packets whose transmissions are interrupted by the PU packets or the SU1 packets per slot. We can give the expression for the interruption rate $\gamma$ of the SU2 packets as follows:

$$\gamma = \sum_{i=1}^{K+1} \bar{\mu}_{22}(1 - \tilde{\lambda}_{1}\tilde{\lambda}_{21})\pi_{i,0,0}. \quad (16)$$

The throughput $\theta$ of the SU2 packets is defined as the number of SU2 packets that are transmitted completely per slot. We can give the expression for the throughput $\theta$ of the SU2 packets as follows:

$$\theta = \phi - \beta - \gamma. \quad (17)$$

The average delay $\sigma$ of the SU2 packets is defined as the average time length for an SU2 packet sojourning in the system. By using Little’s law [13], we can give the expression for the average delay $\sigma$ of the SU2 packets as follows:

$$\sigma = \frac{E_{SU2}}{\phi - \beta} \quad (18)$$

where $E_{SU2}$ is the average queue length of the SU2 packets defined in Eq. (13).
4. **Numerical results.** In this section, we show the change trends for different performance measures of the SU2 packets with numerical results. In the following numerical results, in order to verify the effectiveness of the proposed adjustable admission control scheme compared to the available access scheme without admission control [16], we reference the parameter settings in [16]. In the following numerical results, some common parameters can be set as follows: \(K = 5, \mu_1 = \mu_{21} = \mu_{22} = 0.5\).

Figure 2 shows how the total channel utilization \(\delta\) changes with respect to the adjustment factor \(\tau\).

![Figure 2. Total channel utilization \(\delta\) vs. adjustment factor \(\tau\)](image)

From Fig. 2, we find that as the adjustment factor \(\tau\) increases, the total channel utilization \(\delta\) will decrease. This is because as the adjustment factor increases, the possibility for an SU2 packet being admitted to join the system will decrease. This means the possibility of the channel being occupied by an SU2 packet will decrease, leading to a decrease in the total channel utilization.

On the other hand, we find that as the arrival rate of any type of packet (PU packet, SU1 packet, or SU2 packet) increases, the total channel utilization \(\delta\) will increase. The reason may be that as the arrival rate of any packet increases, the number of packets in the system will increase, and then the possibility for the channel being occupied will also increase. This will lead to an increase in the total channel utilization.

Figures 3-5 show how the interruption rate \(\gamma\), the throughput \(\theta\) and the average delay \(\sigma\) of the SU2 packets changes with respect to the adjustment factor \(\tau\).

From Figs. 3-5, we can find that as the adjustment factor \(\tau\) increases, the interruption rate \(\gamma\), the throughput \(\theta\) and the average delay \(\sigma\) of the SU2 packets all show decreasing tendencies. This is because as the adjustment factor increases, the access probability for the SU2 packets will decrease, resulting in greater number of SU2 packets being refused access to the system. As a result, the number of SU2 packets in the system will be decreased, and the number of SU2 packets being
interrupted or being transmitted completely will also be decreased. Therefore, the interruption rate, the throughput and the average delay of the SU2 packets will decrease as the adjustment factor increases.

From Figs. 3-5, we conclude that as the arrival rate $\lambda_1$ of the PU packets or the arrival rate $\lambda_{21}$ of the SU1 packets increases, the interruption rate $\gamma$ and the average delay $\sigma$ of the SU2 packets will increase and the throughput $\theta$ of the SU2 packets will decrease. The reason for these change trends is that as the arrival rate of the PU packets or the arrival rate of the SU1 packets increases, more PU packets
or more SU1 packets will arrive at the system. This means the transmission of more SU2 packets will be interrupted, and at the same time, more SU2 packets will have to wait in the buffer. Therefore, the interruption rate and the average delay of the SU2 packets will increase. Furthermore, as the arrival rate of the PU packets or the arrival rate of the SU1 packets increases, the possibility of the SU2 packets being transmitted will decrease, and the throughput of the SU2 packets will also decrease.

Moreover, from Figs. 3-5, we conclude that as the arrival rate $\lambda_{22}$ of the SU2 packets increases, the interruption rate $\gamma$, the throughput $\theta$ and the average delay $\sigma$ of the SU2 packets all show a tendency to increase. The reason for these change trends is that the higher the arrival rate of the SU2 packets is, the more SU2 packets will gain access to the system, the more SU2 packets will have to wait in the buffer, and the greater the average delay of the SU2 packets will be. On the other hand, as the number of SU2 packets in the system increases, the possibility of the SU2 packets being transmitted will increase. However, the possibility for the transmission of SU2 packets being interrupted will also increase. Therefore, both the throughput and the interruption rate of the SU2 packets will increase as the arrival rate of the SU2 packets increases.

Notably, when the adjustment factor $\tau$ is equal to 0 in the numerical results, the system performance of the conventional access scheme without access control [16] can be obtained. Compared to conventional access scheme, the admission control scheme proposed in this paper can decrease the interruption rate and the average delay of the SU2 packets effectively. However, the throughput of the SU2 packets will also decrease. Therefore, in practical network management, we should set the adjustment factor based on different network running environments. For example, for networks with a higher throughput requirement, the adjustment factor should be set lower. While for networks that are sensitive to average delay of packets, the adjustment factor should be set higher.
5. Optimization for adjustment factor. Based on the numerical results shown in Section 4, we find that as the adjustment factor increases, the average delay of the SU2 packets will decrease. This is what we want to see. However, adversely, as the adjustment factor increases, the throughput of the SU2 packets will decrease. This is what we don’t want to see. In cognitive radio networks, the throughput and the average delay are the most important performance measures when evaluating the system performance. In this section, considering the trade-off between the throughput and the average delay of the SU2 packets, we firstly build a net benefit function $B(\tau)$ with respect to the adjustment factor $\tau$ as follows:

$$B(\tau) = C_1\theta - C_2\sigma$$

(19)

where $\theta$ is the throughput of the SU2 packets, $\sigma$ is the average delay of the SU2 packets. $C_1$ and $C_2$ are the impact factors for the net benefit function $B(\tau)$. $C_1$ can be seen as the reward for the throughput of the SU2 packets, and $C_2$ can be seen as the cost for the SU2 packets sojourning in the system. We note that $C_1$ and $C_2$ can be set based on different network requirements in practice. For example, for the networks with a higher need for throughput, $C_1$ can be set higher, while for the networks with a lower tolerance for delay, $C_2$ should be set higher.

From Eq. (19), the optimal adjustment factor $\tau^*$ can be given as follows:

$$\tau^* = \arg \max_{0 \leq \tau \leq 1} \{B(\tau)\}$$

(20)

where “arg max” stands for the argument of the maximum [4].

We note that it is difficult to prove the convex property of the net benefit function $B(\tau)$ and then derive the analytical results for the optimal adjustment factor $\tau^*$ because of the high complexity of the throughput $\theta$ and average delay $\sigma$ of the SU2 packets in the net benefit function $B(\tau)$. Due to the fact the adjustment factor $\tau$ is a continuous quantity, we attempt to obtain an approximate (local) optimal solution $\tau^*$ by using a steepest descent method [3] for different parameter settings. We note that the steepest descent method is an unconstrained minimization technique. However, the value range of the adjustment factor $\tau$ is $0 \leq \tau \leq 1$ and the optimization objective is maximizing the net benefit function. In order to implement the steepest descent method in our optimization algorithm, by referencing the penalty function method [3], we firstly construct a penalty function $F(\tau)$ as follows:

$$F(\tau) = -B(\tau) + \eta\omega(\tau).$$

(21)

In Eq. (21), $\eta > 0$ is called a penalty factor and $\omega(\tau)$ is called a penalty term. Considering the value range $0 \leq \tau \leq 1$ of the adjustment factor $\tau$, we can give the penalty term $\omega(\tau)$ as follows:

$$\omega(\tau) = 1/(\tau - 0) + 1/(1 - \tau).$$

(22)

With the penalty function $F(\tau)$, we give an optimal algorithm to obtain the optimal adjustment factor $\tau^*$ as follows:

In Algorithm 1 mentioned above, the step size $\psi$, the decline coefficient $\vartheta$ and the tolerance $\epsilon$ in line numbers 3-5 can be set based on the accuracy requirement of the optimal algorithm. Moreover, considering the complexity of the net benefit function, the differential operation for $F(\tau)$ in line number 3 can be approximated numerically as follows:

$$\frac{dF(\tau)}{d\tau} \approx \frac{F(\tau + \varsigma) - F(\tau)}{\varsigma}$$

(23)

where $\varsigma$ is an arbitrary small number (for example, $\varsigma = 10^{-6}$).
Algorithm 1 Optimal algorithm to obtain the optimal adjustment factor $\tau^*$.

Input: $C_1, C_2, K, \lambda_1, \lambda_{21}, \lambda_{22}, \mu_1, \mu_{21}, \mu_{22}$

Output: $\tau^*$

1: Begin
2: Set the initial value $\tau_0$ of the adjustment factor $\tau$ with $n = 0$;
3: Calculate $\tau_{n+1} = \tau_n - \psi \frac{dF(\tau)}{d\tau}|_{\tau = \tau_n}$, where $\psi$ is called the step size;
4: Calculate $|F(\tau_{n+1}) - F(\tau_n)|$ and $|\tau_{n+1} - \tau_n|$. Go to line number 5 if $|F(\tau_{n+1}) - F(\tau_n)| < \epsilon$ as well as $|\tau_{n+1} - \tau_n| < \epsilon$, where $\epsilon$ is tolerance; Otherwise, set $n = n + 1$ and repeat line number 3;
5: Calculate $\eta\omega(\tau_n)$. Go to line number 6 if $\eta\omega(\tau_n) < \epsilon$; Otherwise, go back to line number 2 by setting $\tau_0 = \tau_n$, $\eta = \eta\vartheta$, where $\vartheta$ is the decline coefficient of the penalty factor $\eta$;
6: Obtain $\tau^* = \tau_n$, and compute the corresponding $B(\tau^*)$;
7: Return $\tau^*$.
8: End

Moreover, the convergence for Algorithm 1 can reference the analysis and proof for the steepest descent method and the penalty function method given in [3].

In order to verify the feasibility of Algorithm 1, with the same parameter settings in Section 4, by setting $C_1 = 113$, $C_2 = 3$, $K = 5, 10$ in the optimization algorithm as an example, we obtain the optimal adjustment factor $\tau^*$ and the maximum net benefit $B(\tau^*)$ for different parameter settings in Table 1.

**Table 1. Optimal adjustment factor $\tau^*$ and the maximum net benefit $B(\tau^*)$**

| Buffer capacity | Arrival rates of packets | Optimal adjustment factor $\tau^*$ | Maximum net benefit $B(\tau^*)$ |
|-----------------|--------------------------|-----------------------------------|---------------------------------|
| $K$             | $\lambda_1, \lambda_{21}, \lambda_{22}$ |                                    |                                 |
| 5               | $\lambda_1 = 0.1, \lambda_{21} = 0.1, \lambda_{22} = 0.2$ | 0.0006                            | 7.4084                          |
| 5               | $\lambda_1 = 0.2, \lambda_{21} = 0.1, \lambda_{22} = 0.2$ | 0.1178                            | 3.5230                          |
| 5               | $\lambda_1 = 0.2, \lambda_{21} = 0.2, \lambda_{22} = 0.2$ | 0.3232                            | 0.0113                          |
| 5               | $\lambda_1 = 0.2, \lambda_{21} = 0.2, \lambda_{22} = 0.3$ | 0.5887                            | 2.4589                          |
| 10              | $\lambda_1 = 0.1, \lambda_{21} = 0.1, \lambda_{22} = 0.2$ | 0.0252                            | 7.3280                          |
| 10              | $\lambda_1 = 0.2, \lambda_{21} = 0.1, \lambda_{22} = 0.2$ | 0.1341                            | 3.4942                          |
| 10              | $\lambda_1 = 0.2, \lambda_{21} = 0.2, \lambda_{22} = 0.2$ | 0.3342                            | 0.0031                          |
| 10              | $\lambda_1 = 0.2, \lambda_{21} = 0.2, \lambda_{22} = 0.3$ | 0.6012                            | 2.4494                          |

From Table 1, we find that as the buffer capacity $K$ of the SU2 packets changes from 5 to 10, the optimal adjustment factor $\tau^*$ shows an obvious increasing change trend. The reason is that as the buffer capacity increases, more SU2 packets can access the buffer and wait in the system, then the average delay of the SU2 packets will increase. In order to reduce the average delay of the SU2 packets, we should increase the adjustment factor to decrease the access probability of the SU2 packets.

On the other hand, looking at Table 1, we find that as the arrival rate $\lambda_1$ of the PU packets or the arrival rate $\lambda_{21}$ of the SU1 packets increases, the optimal adjustment factor $\tau^*$ will also increase. The reason may be that as the arrival rate
of the PU packets or the arrival rate of the SU1 packets increases, the possibility of the SU2 packets being transmitted will decrease, and a large number of SU2 packets have to wait in the system. In order to reduce the average delay of the SU2 packets, the adjustment factor should be set higher.

Moreover, from Table 1, we conclude that as the arrival rate $\lambda$ of the SU2 packets increases, the optimal adjustment factor $\tau^*$ will increase. This is because as the arrival rate of the SU2 packets increases, more SU2 packets will gain access to and wait in the system. In order to control the access of the SU2 packets, the adjustment factor should be set higher.

6. Conclusions. In this paper, in order to guarantee the QoS of the network users in the system, we proposed an adjustable access control scheme for the SU2 packets in cognitive radio networks with multiple SUs. We built a discrete-time pre-emptive priority queuing model with a finite waiting room and an adjustable joining rate to capture the working principle of the proposed adjustable access control scheme. With steady-state analysis, we derived the analytical solution for the total channel utilization, the interruption rate, the throughput, and the average delay of the SU2 packets. We also presented numerical results to show the influence of the adjustment factor on the system performance. From the numerical results, we concluded that the adjustment factor had an important effect on the system performance. Moreover, we concluded that compared to the conventional access scheme without admission control, the adjustable admission control scheme proposed in this paper can decrease the interruption rate and the average delay of the SU2 packets effectively. Finally, considering the trade-off between the throughput and the average delay of the SU2 packets, we built a net benefit function and put forward an optimal algorithm to optimize the adjustment factor.

In this paper, we focused on proposing an access control scheme in a cognitive radio network with a single channel. When multiple channels are considered, the system model and model analysis will be more complicated. As a future work, we will expand our research by analyzing a system with multiple channels.

Acknowledgments. This work was supported in part by the Natural Science Foundation of Hebei Province (F2016501073), the Doctoral Scientific Research Foundation of Liaoning Province (201601016), the National Natural Science Foundation of China (61701097, 61472342), the Fundamental Research Funds for the Central Universities (N152303007), the Scientific Research Fund of Hebei Education Department (QN2016307), and the Doctoral Foundation of Northeastern University at Qinhuangdao (XNB201606), China and was supported in part by MEXT, Japan.

REFERENCES

[1] S. Aghajeri, A. Sharafat and K. Navaie, Primary service outage degradation in dynamic spectrum sharing with non-ideal spectrum sensing, IET Communications, 6 (2012), 1252–1261.

[2] A. Alfa, Queueing Theory for Telecommunications: Discrete Time Modelling of a Single Node System, Springer, New York, 2010.

[3] E. P. Chong and S. Zaks, An Introduction to Optimization, Third Edition, Wiley, Hoboken, 2008.

[4] C. Ding, K. Wang and S. Lai, Channel coordination mechanism with retailers having fairness preference-An improved quantity discount mechanism, Journal of Industrial and Management Optimization, 9 (2013), 967–982.

[5] A. Greenbaum, Iterative Methods for Solving Linear Systems, Society for Industrial and Applied Mathematics, Philadelphia, 1997.
[6] S. Jin, Y. Zhao, W. Yue and Z. Saffer, Performance analysis and optimization of an adaptive admission control scheme in cognitive radio networks, Mathematical Problems in Engineering, 2013 (2013), Article ID 727310, 10 pages.

[7] Y. Lee, C. Park and D. Sim, Cognitive radio spectrum access with prioritized secondary users, Applied Mathematics & Information Sciences, 6 (2012), 595S–601S.

[8] H. Li and Z. Han, Socially optimal queuing control in cognitive radio networks subject to service interruptions: To queue or not to queue?, IEEE Transactions on Wireless Communications, 10 (2011), 1656–1666.

[9] J. Marinho and E. Monteiro, Cognitive radio: Survey on communication protocols, spectrum decision issues, and future research directions, Wireless Networks, 18 (2012), 147–164.

[10] M. Naeem, A. Anpalagan, M. Jaseemuddin and D. Lee, Resource allocation techniques in cooperative cognitive radio networks, IEEE Communications Surveys & Tutorials, 16 (2014), 729–744.

[11] N. Nguyen-Thanh, A. Pham and V. T. Nguyen, Medium access control design for cognitive radio networks: A survey, IEICE Transactions on Communications, E97-B (2014), 359–374.

[12] S. Sharma, T. Bogale, S. Chatzinotas, B. Ottersten, L. Le and X. Wang, Cognitive radio techniques under practical imperfections: A survey, IEEE Communications Surveys & Tutorials, 17 (2015), 1858–1884.

[13] N. Tian and Z. Zhang, Vacation Queueing Models: Theory and Applications, Springer, New York, 2006.

[14] E. Tragos, S. Zeadally, A. Fragkiadakis and V. Siris, Spectrum assignment in cognitive radio networks: A comprehensive survey, IEEE Communications Surveys & Tutorials, 15 (2013), 1108–1135.

[15] D. Willkomm and A. Wolisz, Efficient QoS support for secondary users in cognitive radio systems, IEEE Wireless Communications, 17 (2010), 16–23.

[16] Y. Zhao and W. Yue, Cognitive radio networks with multiple secondary users under two kinds of priority schemes: Performance comparison and optimization, Journal of Industrial and Management Optimization, 13 (2017), 1449–1466.

[17] Y. Zhao and W. Yue, An adjustable access control scheme in cognitive radio networks with multiple secondary users, in Proceedings of 11th International Conference on Queueing Theory and Network Applications, ACM, (2016), Article No. 10, 5 pages.

Received February 2017; revised July 2017.

E-mail address: yuanzh85@163.com
E-mail address: yue@konan-u.ac.jp