Comparing underwater vehicle positioning algorithms using single-beacon navigation

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Abstract. Comparative analysis of two algorithms for underwater vehicle positioning using measurements of range to a drifting buoy and radial velocity is carried out. The algorithms based on building location contours and polynomial filtering are considered. Invalid data rate and radial RMS position error are analyzed to compare the algorithms. Based on the analysis made, the polynomial filtering algorithm is accepted as preferable.

1. Introduction
Positioning of autonomous underwater vehicle (AUV) using drifting buoy with no need to surface is a perspective trend [1-5]. A buoy should have communication equipment to receive signals of satellite navigation system (SNS) and hydroacoustic equipment to transmit its coordinates to AUV. Let us assume that AUV equipment can also determine the distance to the buoy and approach speed as a time sample. At the same time AUV performs dead reckoning using heading and speed data. Earlier [6,7] two algorithms for AUV positioning using buoy data were proposed by the authors. The first one [6] is based on constructing location contours, and the second one [7] utilizes polynomial filtering. This paper brings the presented algorithms into comparison based on simulation close to the full-scale experiment.

2. Problem statement
Let us introduce AUV Cartesian coordinates in the reference frame $O\xi\eta\varsigma$ with the origin at the buoy’s radiating hydroacoustic antenna position (under SNS receiver antenna). The axis $O\eta$ points North; axis $O\xi$ points East, and axis $O\varsigma$ points downwards (Fig. 1). AUV is located at the point $M$ and moving with speed $V$, heading $\psi$ at constant depth $H$ relative to the beam. We get measurements of the range $OM = \rho$ and the rate of its change (range rate) $V_\rho = \dot{\rho}$. The coordinates $\xi$, $\eta$ should be determined while minimizing the estimation root-mean-square errors (RMS) as much as possible. Motion
and measurement equations for time $k$ can be written as

$$
\begin{align*}
\xi_k &= \xi_{k-1} + V_k \sin \psi_k + \Delta \xi_k + w_{1k}, \\
\eta_k &= \eta_{k-1} + V_k \cos \psi_k + \Delta \eta_k + w_{2k}, \\
\rho_k &= \rho_k + v_{pk} = \sqrt{\xi_k^2 + \eta_k^2 + H^2} + \nu_{pk}, \\
\dot{\rho}_k &= \dot{\rho}_k + v_{\dot{p}k} = \frac{\xi_k V_k \sin \psi_k + \eta_k V_k \cos \psi_k}{\sqrt{\xi_k^2 + \eta_k^2 + H^2}} + \nu_{\dot{p}k},
\end{align*}
$$

(1)

where $\Delta \xi_k$, $\Delta \eta_k$ are determined components conditioned by the buoy drift and computed as increments of its coordinates by SNS data; $w_{1k}, w_{2k}$ are generating white noise random sequences with covariance $Q$ assumed known; $v_{pk}, v_{\dot{p}k}$ are the range and range rate noises, respectively, which are approximated by zero-mean Gaussian white noise random sequences independent from $w_{1k}$ and from each other, with known RMS $\sigma_{\Delta \rho}$ and $\sigma_{\Delta \rho \dot{}}$. The matrix $Q$ is formed taking into account that

$$
\begin{align*}
W_{1k} &\approx \Delta V_k \sin \psi_k + V_k \Delta \psi_k \cos \psi_k, \\
W_{2k} &\approx \Delta V_k \cos \psi_k - V_k \Delta \psi_k \sin \psi_k,
\end{align*}
$$

(3)

where $V_k, \psi_k$ are current values of specified AUV speed and heading, respectively; $\Delta V_k, \Delta \psi_k$ are their fluctuations (in particular due to buoy drift), which are assumed to be zero-mean white noise random sequences.

The right sides of (2) are nonlinear functions of the desired coordinates $\xi_k, \eta_k$. The problem is also complicated by the presence of errors in navigation parameters, and the characteristics of these errors can be known only approximately. These factors suggest that the estimation algorithm should be rather simple and robust to the changes in motion and disturbance models. Below we will consider two options for constructing such an algorithm.

3. Algorithm Based on Building of Location Contours (BLC)

This algorithm relies on deriving coordinates of intersection points of location contours corresponding to the fixed values of $\rho$ and $\dot{\rho}$. These contours are, respectively, the circumference with the origin $O'$ and two rays set in this point (Fig. 1, dotted lines). So we can derive AUV coordinates as the functions of rate $\rho$ and range rate $\dot{\rho}$ and form the observation model as

$$
\begin{align*}
\xi_{1k}^{BLC} &= \frac{\rho_k^*}{V_k} \sin \psi_k \pm \rho_k^* \cos \psi_k = \xi_k + v_{1k}^{BLC}, \\
\xi_{2k}^{BLC} &= \frac{\rho_k^*}{V_k} \cos \psi_k \mp \rho_k^* \sin \psi_k = \eta_k + v_{2k}^{BLC},
\end{align*}
$$

(4)

where $\rho_k^* = \sqrt{\rho_k^2 - H^2 - \rho_k^2 \rho_k^2 / V_k^2}$. Here we approximate measurement noises $v_{1k}^{BLC}, v_{2k}^{BLC}$ by zero-mean white noise random sequences with variances specified by the following equations:

$$
\begin{align*}
\sigma^2_{v_{1k}^{BLC}} &= \left( \rho_k^* \sin \psi_k \pm \rho_k^* \left(1 - \rho_k^2 / V_k^2\right) \cos \psi_k \right)^2 \sigma_{\Delta \rho}^2 + \left( \rho_k^* \sin \psi_k \mp \rho_k^* \rho_k^* / V_k \sin \psi_k \right)^2 \sigma_{\Delta \psi}^2, \\
\sigma^2_{v_{2k}^{BLC}} &= \left( \rho_k^* \cos \psi_k \mp \rho_k^* \left(1 - \rho_k^2 / V_k^2\right) \sin \psi_k \right)^2 \sigma_{\Delta \rho}^2 + \left( \rho_k^* \cos \psi_k \pm \rho_k^* \rho_k^* / V_k \cos \psi_k \right)^2 \sigma_{\Delta \psi}^2.
\end{align*}
$$

(5)

Uncertainties in (4) and (5) can be eliminated in the presence of a priori data on relative positions of the buoy and AUV trajectory.
The processing of observations (4) is implemented by the Kalman filter, which is set up to the AUV motion model (1) with current measured parameters $V$, $\psi$, $H$ and no maneuvering supposed. The initial filter parameters can be set using the first data set $\rho$, $\dot{\rho}$.

### 4. Polynomial Filtering (PF) Algorithm

This algorithm deals with observations formed as computed values

$$
\rho^2 - H^2 = \xi^2 + \eta^2,
$$

$$
\rho \dot{\rho} = \xi \dot{\xi} + \eta \dot{\eta},
$$

using measured values of $\rho$, $\dot{\rho}$ and $H$. Based on (6) the observation model can be constructed as

$$
y_{1k}^{PF} = \xi_k^2 + \eta_k^2 + \rho_{1k}^{PF},
$$

$$
y_{2k}^{PF} = \xi_k V_k \sin \psi_k + \eta_k V_k \cos \psi_k + \nu_{2k}^{PF},
$$

where

$$
\nu_{1k}^{PF} = 2\rho_k \nu_{pk} + \nu_{\rho k},
$$

$$
\nu_{2k}^{PF} = \dot{\rho}_k \nu_{pk} + \rho_k \nu_{\dot{\rho} k} + \nu_{pk} \nu_{\dot{\rho} k}.
$$

The right sides of Eqs. (7) are the polynomial functions of the required coordinates $\xi$, $\eta$. The observations formed are processed using polynomial filter [8] set to the motion model (1). To form the initial conditions for filtering, a priori data on current AUV position can be used; as an alternative initial coordinates $\xi$, $\eta$ can be calculated using Eqs. (4) with no measurement noise assumed.

Taking into account Eqs. (8) and assuming that the measurement errors $\nu_{pk}$, $\nu_{\dot{\rho} k}$ are small as compared with the corresponding measured parameters, one can neglect non-zero mean of $\nu_{1k}^{PF}$ and form the covariance matrix of errors $\nu_{1k}^{PF}$, $\nu_{2k}^{PF}$ as

$$
R_k = \begin{pmatrix}
4\rho_k^2 \sigma^2_{\Delta \rho} & 2\rho_k \dot{\rho}_k \sigma^2_{\Delta \rho} \\
2\rho_k \dot{\rho}_k \sigma^2_{\Delta \rho} & \rho_k^2 \sigma^2_{\Delta \rho} + \dot{\rho}_k^2 \sigma^2_{\Delta \rho}
\end{pmatrix}.
$$

Note that the algorithms share a common feature, namely, they ignore the range rate measurements while distance measurements are absent.

### 5. Example

Let us compare the considered algorithms using computer simulation of AUV motion at a speed of 17 kts, heading 346°, depth 10 m towards the drifting buoy. Fluctuations of AUV motion parameters are simulated as the output of shaping filter matched with the full-scale experiment data. The buoy drift is simulated similarly. RMS errors of initial coordinates are assumed to be 200 m.

Figure 2 shows the true AUV trajectories (bold line) and 1500 sample estimates given by two algorithms for different measurement implementations. Initial AUV coordinates are set so that the traverse to the buoy does not exceed 600 m. The buoy drift during the estimation process is shown in Fig. 2 as a navy-blue line.

Figure 2 demonstrates admissible quality of both algorithms in terms of AUV positioning. Sampling distance RMS errors (DRMS) at the end of estimation interval are found to be 25 m for BLC algorithm and 8 m for PF algorithm and coincide with the corresponding calculated values.

At the same time traverse decreasing leads to raising significance of the second intersection point of location contours that corresponds to the second (false) extremum of a posteriori probability density function. Figure 3 demonstrates simulation results for the case of 100 m traverse.
The rate of invalid data corresponding to false extremum is found to be 1.5% for BLC algorithm and 7.5% for PF algorithm. True DRMS errors at the end of estimation interval are found to be 75 m and 160 m, respectively; the relevant calculated values are 15 m and 11 m.

To reduce the rate of invalid data AUV heading maneuver is proposed; corresponding AUV trajectory is shown in Fig. 4. Maneuvering efficiency is confirmed only for PF algorithm (rate of invalid data drops to 0.4 %); BLC algorithm assuming AUV straight motion does not provide valid estimates (a rate of invalid data of 4%). True DRMS errors at the end of estimation interval in this case are found to be 135 m (BLC) and 42 m (PF); the corresponding calculated values are 15 m and 11 m, respectively.

To improve the robustness of BLC algorithm it is proposed to process data received during maneuvering only for correction of a priori data on relative positions of the buoy and AUV trajectory.

![Fig. 2. True AUV trajectory (bold line) and its estimates using: a) BLC algorithm; b) PF algorithm (traverse of 600 m)](image1)

![Fig. 3. True AUV trajectory (bold line) and its estimates using: a) BLC algorithm; b) PF algorithm (traverse of 100 m)](image2)

The rate of invalid data corresponding to false extremum is found to be 1.5% for BLC algorithm and 7.5% for PF algorithm. True DRMS errors at the end of estimation interval are found to be 75 m and 160 m, respectively; the relevant calculated values are 15 m and 11 m.

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To improve the robustness of BLC algorithm it is proposed to process data received during maneuvering only for correction of a priori data on relative positions of the buoy and AUV trajectory.
6. Conclusions
The comparison of two AUV positioning algorithms (building of location contours and polynomial filtering algorithm) using data from a drifting buoy is carried out under conditions close to the full-scale experiment. It is established that generally PF algorithm provides rather accurate estimates of AUV coordinates and demonstrates better quality of estimates on out-of-straight trajectories. At the same time PF is outperformed by BLC algorithm on straight trajectories when false extremum occurs in the domain of uncertainty. AUV heading maneuver improves PF estimation quality, however, the question of selecting the optimal maneuver is left open.

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