Perturbations in the upper layers of $\alpha$ Centauri A

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Abstract. The emerging field of asteroseismology allows the direct study of stellar interiors with an incredibly high precision. We used a seismic parameter based on the phase shift as a diagnostic tool to infer the presence of a new layer of rapid variation in the external layers of the primary component of the stellar system Alpha Centauri AB. This layer is, apparently, a thin region where the acoustic modes suffer a strong scattering. Our tests indicate that this layer should be located at an acoustical depth of approximately 1400 s (0.939 R), which corresponds to a depth of 6% below the surface of the star. This is somehow unexpected since the internal structure of this sun-like star is predicted to be similar to the Sun.

1. Introduction

Like in the Sun, many sun-like stars have a complex surface structure where it occurs some of the most complex fluid dynamics due to the interaction between magnetic fields, turbulent convection and stellar pulsations. In this work, we present a method to study the structure of the upper layers of a sun-like star as it is possible to be inferred from the most complete observational table of frequencies. In particular we apply the acoustic phase shift method to probe the physics mechanisms operating in these upper layers. We have found that in this sun-like star the existence of a rapid variation (RV) layer in the external region of the star will produce a significant amount of scattering of acoustic waves. This relatively thin layer is very likely produced by a rapid variation in the sound speed similar to variations of sound speed that can occur in transition layers like the known examples in the Sun of the bottom of the convection zone and the He II ionization zone. The diagnostic is done by means of a seismic parameter which allows to infer the scattering of acoustic waves in the surface of the star using the available table of observational frequencies.

2. Determination of the acoustic phase shift

In this work, we determine the seismic observable $\beta$, a proxy of the phase shift of the acoustic modes occurring in the envelope of the sun-like stars. We show that a small variation in the structure (typically smaller than $\sim 1\%$ in the sound speed) produces a glitch in the acoustic potential that could explain the origin of the seismic observable $\beta$. The fourth-order system of ordinary differential equations describing the linear adiabatic oscillations in the Cowling approximation (the gravitacional effects are small in the outer layers) can be reduced to a second order Schrödinger-type equation considering that the trajectories of the acoustic waves...
in the surface layers are vertical [1]. It follows that the equation of acoustic oscillations reads

$$\frac{d^2 \psi}{d\tau^2} + (\omega^2 - U^2) \psi = 0$$

(1)

where \(\tau\) is the acoustic depth, \(U\) the acoustic potential, \(\omega (\equiv 2\pi \nu)\) the cyclic frequency and \(\psi\) the wave function. The acoustic potential \(U\) is defined by

$$U^2(\tau) = \frac{g}{c} \left( \frac{g}{c} - \frac{d \ln h}{d\tau} \right) + \frac{1}{4} \left( \frac{d \ln \zeta}{d\tau} \right)^2 - \frac{1}{2} \frac{d^2 \ln \zeta}{d\tau^2}$$

(2)

with \(h(r) = \rho^{-1} \exp \left(-2 \int_0^r \frac{g c^2}{\Delta \rho} dr\right)\) and \(\zeta(r) = \frac{r^2 h}{c}\), where the variables have their usual meaning [2].

**Figure 1.** Left panel: The acoustic potential obtained from the stellar envelope of \(\alpha\) Centauri A. We can notice the superadiabatic region at \(\tau \approx 80 \text{ s} \approx 0.9931R\), the zone of partial ionization of helium at \(\tau \approx 750 \text{ s} \approx 0.9840R\) and the base of the convective zone at \(\tau \approx 2500 \text{ s} \approx 0.7759R\). The horizontal dashed lines correspond to waves with frequencies \(\nu = 1.5 \text{ mHz}\) and \(\nu = 3.0 \text{ mHz}\). The vertical bar L1 located around \(\tau = 750 \text{ s}\) (blue bar) indicates the position of one of our hypothesis for the existence of a rapid variation layer whereas the vertical bar L2 located around \(\tau = 1400 \text{ s}\) (red bar) indicates the variation layer that better describe the observations. Right panel: The seismic parameter \(\beta(\nu)\) in three cases: \(\beta\) obtained from the ”unperturbed” acoustic potential (bell shape - dark line); \(\beta\) with a perturbation in the zone of the partial ionization of helium (oscillatory character and longer period - blue line) and finally \(\beta\) with a perturbation located around the 1400 s (oscillatory character and shorter period - red line).

The outer phase shift of acoustic waves \(\alpha\) is computed from the stellar envelope, by numerically solving the following equation

$$\frac{d\theta}{d\tau} = -\omega + \omega^{-1} U^2(\tau) \cos^2 \theta$$

(3)

where \(\theta = \alpha \pi + \pi/4 - \omega \tau\). This equation is solved numerically with the proper initial boundary condition at \(\tau = 0\). The details of the numerical procedure can be found in [3]. \(\beta(\nu)\) is obtained from numerical differentiation of \(\alpha/\nu\) as

$$\beta(\omega) = \beta(\nu) = -\nu^2 \frac{d}{d\nu} \left( \frac{\alpha}{\nu} \right).$$

(4)
This method has been applied in the past to determine the outer phase shift by several authors [4], [5], [6] and [7]. The properties of the observable $\beta(\nu)$ were largely discussed among others by [8]. The acoustic potential $U$ used in this calculation was determined from a detailed stellar structure model of $\alpha$ Centauri A specifically computed for this analysis. Fig. 1 shows the acoustic potential and the $\beta$ calculated for the envelope of the star. This is identical but not equal to the acoustic potential and the $\beta$ calculated for the present standard solar model [10], [11].

The $\beta(\nu)$ seismic parameter is the best proxy of the phase shift of acoustic waves, that occurs as a consequence of the scattering of these waves in the upper layers of the star. $\beta(\nu)$, as previously mentioned, can be computed from a table (theoretical or observational) of frequencies. The algorithm reads

$\beta_{l,n}(\nu) = \frac{\nu_{l,n} - L \frac{\partial \nu}{\partial L} - n \frac{\partial \nu}{\partial n}}{\partial \nu/\partial n}$,

where $L = l + 1/2$ and $l$ and $n$ are the degree and radial order of the oscillation mode. The partial derivatives in $\beta_{l,n}(\nu)$ were treated carefully due to the small number of points available in the observational frequency table. This algorithm is based upon the formula proposed by [12], to study the acoustic phase shift in the Sun. $\beta_{l,n}(\nu)$ computed as indicated in equation (5) allows a direct comparison between the star and its stellar model, specifically, highlighting the differences of the physical processes occurring in the upper layers of the star.

![Figure 2](image-url)

**Figure 2.** Theoretical $\beta$ computed from an evolution model of the star $\alpha$ Centauri A: the solid smooth curve corresponds to $\beta$ obtained from the surface acoustic potential (fig. 1a). We have also calculated $\beta$ from a theoretical frequency table for the models with degrees $l = 1$ (diamonds) and $l = 2$ (circles), these points are connected with solid black lines. With dashed colored lines we represented the observational $\beta$ for modes also with the degrees $l = 1$ (magenta) and $l = 2$ (green).

### 3. Test case: $\alpha$ Centauri A

In what concerns the observational part, we used the frequency table of [13] to compute $\beta$. Several tests were performed to verify the quality of the algorithm to compute $\beta$ from this observational frequency table. Looking at Figure 2 we can notice a dispersion in the observational modes that is not present in the theoretical modes.

A detailed analysis of the observed $\beta$ suggests that the real star could have a small rapid variation zone that produces a glitch in the acoustic potential located in the external layers of the star. In Figure 3 we compare the red and the blue (modified; shorter period and larger period respectively) with the black (unmodified) scenarios. These tests with the modified
acoustic potential are able to reproduce the oscillatory behaviour of $\beta$ better than the unmodified potential.

![Graphs showing $\beta(\nu)$ for different $l$ values]

Figure 3. In this figure we compare the two modified scenarios of Fig.1 (blue and red) with the observational signature $\beta$ (dashed lines) for the modes with the degrees $l = 1$ and $l = 2$. We noticed that the modified acoustic potential can reproduce the oscillatory character of the observational $\beta$, and the location the better explains the period of the oscillation is around 1400 s (red line).

4. Summary and conclusion

We study a Sun-like star from the point of view of the acoustic phase shift. Specifically we wanted to know the impact of a small variation of the sound speed in the acoustic potential and consequently in the seismic parameter $\beta$. We derived $\beta(\nu)$ from an observational frequency table, of the star $\alpha$ Centauri A and noticed that there is a disagreement between the theoretical $\beta$ (obtained numerically by integrating the wave equation through the external layers and from a theoretical frequency table) and the $\beta$ calculated directly from an observational table. This disagreement is characterized by an oscillatory feature that is present in the observational $\beta$ but is absent in the theoretical model. This method allows to determine with precision the location and thickness of a glitch in a star. A preliminary analysis of $\alpha$ Centauri A suggests a glitch located at $\sim 0.94 R$ ($\sim 1400$ s) and a thickness $< 0.2 \% R$. This technique can also be used together with another different methods of analysis [14] to improve the identification of the location of acoustic glitches in the interiors of sun-like stars.

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