Role of \( N^*(1650) \) in the near threshold \( pp \rightarrow p\Lambda K^+ \) and \( pp \rightarrow p\Sigma^0 K^+ \) reactions*  

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Abstract  

We investigate the \( pp \rightarrow p\Lambda K^+ \) and \( pp \rightarrow p\Sigma^0 K^+ \) reactions at beam energies near their thresholds within an effective Lagrangian model, where the strangeness production proceeds via the excitation of \( N^*(1650) \), \( N^*(1710) \), and \( N^*(1720) \) baryonic resonances. It is found that the \( N^*(1650) \) resonance dominates both these reactions at near threshold energies. The contributions from this resonance together with the final state interaction among the outgoing particles are able to explain the observed beam energy dependence of the ratio of the cross sections of the two reactions in the near threshold region.  
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Recently, at the Cooler Synchrotron (COSY) facility in Jülich measurements have been performed \cite{1,2} for the associated strangeness production in proton-proton (pp) collisions at near threshold beam energies. A very interesting result of these studies is that the ratio (R) of the total cross sections for the \(pp \rightarrow p\Lambda K^+\) and \(pp \rightarrow p\Sigma^0 K^+\) reactions (to be referred as \(\Lambda K^+\) and \(\Sigma^0 K^+\) reactions respectively) at the same excess energy (defined as \(\epsilon = \sqrt{s} - m_p - m_Y - m_K\), with \(m_p\), \(m_Y\), and \(m_K\) being the masses of proton, hyperon, and kaon respectively and \(s\) the invariant mass of the collision), is about 28\(^{+6}_{-9}\) for \(\epsilon < 13\) MeV. This result is very intriguing because at higher beam energies \cite{3} this ratio is only around 2.5.

Assuming that the hyperon production proceeds solely due to the kaon(K)-exchange mechanism and that the final state interaction (FSI) effects among the outgoing particles are absent, R is given essentially by the ratio \((g_{\Lambda NK}^2/g_{\Sigma NK}^2)\) of the squares of coupling constants at the vertices from which the \(K^+\) meson emerges. Although, values of \(g_{\Lambda NK}\) and \(g_{\Sigma NK}\) are not known with certainty \cite{4}, yet the SU(6) prediction of this ratio \cite{5} is 27 which would nearly explain the observed value of R. However, \(\pi\)-exchange mechanism is shown \cite{4,6–9} to be important for these reactions. The two mechanisms taken together lead to a considerably lower value \cite{4}(\(\sim 3.6\)) for R. Another qualitative explanation \cite{2} of these data suggests that the dominant \(\Sigma - p\) final state interaction, which includes the \(\Sigma N \rightarrow \Lambda N\) conversion process, suppresses the \(\Sigma^0\) production. Although some support in favor of this conversion does exist \cite{10}, it is not evident that the whole of the observed enhancement is really due to the produced \(\Sigma\) particle being converted to \(\Lambda\) by the FSI effects.

Recently, a few quantitative calculations have been reported to explain this result. Assuming that the \(\pi\)- and \(K\)-exchange processes are the only mechanism leading to the strangeness production, the authors of Ref. \cite{11} show within a (non-relativistic) distorted wave Born approximation (DWBA) model that while the \(\Lambda K^+\) reaction is dominated by the \(K\)-exchange only, both \(K\)- and \(\pi\)-exchange processes play an important role in the case of \(\Sigma^0 K^+\) reaction. Therefore, if the amplitudes corresponding to the two exchanges in the latter case interfere destructively, the production of \(\Sigma^0\) is suppressed as compared to that of \(\Lambda\). It is also shown in Ref. \cite{11} that FSI effects, although important, can not explain the large value of R on their own. However, a conclusive evidence in support of the relative signs of \(\pi\)- and \(K\)-exchange amplitudes being opposite to each other is still lacking. Furthermore, other mechanisms like excitation, propagation, and decay of intermediate baryonic resonances play (see eg. \cite{3,12}) an important role in the strangeness production, which may change the scenario of Ref. \cite{11}. It is also not clear if this model can simultaneously explain the relatively smaller value of R at larger beam energies (i.e. for \(\epsilon \sim 1\) GeV).

In Ref. \cite{13} two types of boson exchange models have been used to calculate the ratio R. In one of them, the strangeness production proceeds solely via \(\pi\)- and \(K\)-exchange mechanisms. Neglecting the interference between the corresponding amplitudes and making corrections for the FSI effects via the Jost function method of the Watson-Migdal theory \cite{14}, the predictions of this model are found to be in agreement with the observed ratio within a factor of 2 in the near threshold region. However, these authors find \(K\)- and \(\pi\)-exchange amplitudes to be of similar magnitudes for both \(\Lambda K^+\) and \(\Sigma^0 K^+\) reactions in the near threshold region, which in disagreement with the results of Ref. \cite{11}. Moreover, form of their Watson-Migdal FSI amplitude is at variance with that given in Ref. \cite{14} and by other authors \cite{1,4,17}.
In the second model used in Ref. [13] (called as resonance model in [8]), the strangeness production proceeds via $\pi$-, $\eta$-, and $\rho$-exchange processes and the excitation of intermediate baryonic resonant states of $N^*(1650)$, $N^*(1710)$, $N^*(1720)$, and $\Delta(1920)$. In this case too, with FSI effects included, they get the similar result for $R$. However, the excitation of the $N^*(1650)$ baryonic resonance has not been included in the calculations of the cross sections for the $\Sigma^0K^+$ reaction in [13]. In the near threshold region, the $\Lambda K^+$ reaction has been shown [6,12] to be dominated by this resonance. There is no a priori reason to believe that it will not be the same for the $\Sigma^0K^+$ reaction.

In this paper, we investigate the $\Lambda K^+$ and $\Sigma^0K^+$ reactions at near threshold as well as higher beam energies in the framework of an effective Lagrangian approach (ELA) [6,15,18]. In this model, the initial interaction between two incoming nucleons is modeled by an effective Lagrangian which is based on the exchange of the $\pi$-, $\rho$-, $\omega$-, and $\sigma$- mesons. The coupling constants at the nucleon-nucleon-meson vertices are determined by directly fitting the T-matrices of the nucleon-nucleon (NN) scattering in the relevant energy region [19]. The ELA uses the pseudo-vector (PV) coupling for the nucleon-nucleon-pion vertex (unlike the resonance model) and thus incorporates the low energy theorems of current algebra and the hypothesis of partially conserved axial-vector current (PCAC). In contrast with the resonance model, both the $\Lambda K^+$ and $\Sigma^0K^+$ reactions proceed via excitation of the $N^*(1650)$, $N^*(1710)$, and $N^*(1720)$ intermediate baryonic resonance states. The interference terms between the amplitudes of various resonances (which are ignored in [13]) are retained. To describe the near threshold data, the FSI effects in the final channel are included within the framework of the Watson-Migdal theory [14,15]. ELA has been used earlier to describe rather successfully the $pp \rightarrow pp\pi^0$ and $pp \rightarrow pn\pi^+$ [6,15,18] as well as $pp \rightarrow p\Lambda K^+$ [6] reactions at both near threshold and higher beam energies.

In the present form of the ELA the energy dependence of the cross section due to FSI is separated from that of the primary production amplitude and the total amplitude is written as,

$$A_{fi} = M_{fi}(pp \rightarrow pYK^+) \cdot T_{ff}, \quad (1)$$

where $M_{fi}(pp \rightarrow pYK^+)$ is the primary associated hyperon $YK^+$ production amplitude, while $T_{ff}$ describes the re-scattering among the final particles which goes to unity in the limit of no FSI. The latter is taken to be the coherent sum of the two-body on-mass-shell elastic scattering amplitudes $t_i$ (with $i$ going from 1 to 3), of the interacting particle pairs $j - k$ in the final channel. This type of approach has been used earlier to describe the pion [13,16,20], $\eta$-meson [17,21], $\Lambda K^+$ [1] and $\phi$-meson [22] production in $pp$ collisions.

An assumption inherent in Eq. (1) is that the reaction takes place over a small region of space (which is fulfilled rather well in near threshold reactions involving heavy mesons). Under this condition the amplitudes $t_i$ can be expressed in terms of the inverse of the Jost function $J_{ti}(q_i)$ [14,15]. Assuming the relative orbital angular momentum between pairs $j - k$ to be zero and using a (Coulomb) modified formula [23] for the effective range expansion of the phase-shift of the relevant pair, we can write [14],

$$t_i(q_i) = (J_0(q_i))^{-1} = \frac{(q_i^2 + \alpha_i^2)r_{0i}^c/2}{1/\alpha_i^c + (r_{0i}^c/2)q_i^2 - iq_i}, \quad (2)$$

where $\alpha$ is defined as

$$\alpha = \frac{1}{2}(1 - \eta)$$
\[ \alpha = \left( \frac{1}{r_{0i}} \right) \left[ 1 + \left( 1 + 2r_{0i}/a_{0i} \right)^{1/2} \right] \]  

(3)

with \( a_{0i}^c \) and \( r_{0i}^c \) being the Coulomb modified [3] effective range \( (r_{0i}) \) and scattering length \( (a_{0i}) \) parameters respectively and \( q_i \) the relative momentum for the \( j - k \) interacting pair. It is clear that for large \( q_i \), the amplitude \( t_i \) goes to unity. It should be noted that the form of the Jost function given in [13] does not lead to Eq. (2). Even though the square of the absolute value of Eq. (2) agrees with that of the corresponding function given in [13], the two forms lead to different results if the FSI corrections in more than one final channel are considered.

The validity of the factorization method (Eq. (1)) for applications to the near threshold meson production in \( pp \) collisions has recently been investigated in Refs. [24–27]. It has been shown in Ref. [25] that cross sections for the \( pp \rightarrow pp\pi^0 \) reaction calculated using Eq. (1) are very similar to those obtained by treating \( M_{fi} \) as an effective operator acting on the nucleon wave functions calculated with realistic \( NN \) interactions. Furthermore, it is noted in Ref. [26] that for terms where \( \pi^0 \) production proceeds via exchange of mesons between the colliding nucleons, the results of the factorization approximation are quite similar to those obtained by the DWBA calculations. It is only for the direct (bremsstrahlung) terms (which are not included in \( M_{fi} \)), that the results of the two calculations differ from each other appreciably.

On the other hand, it is argued in Refs [24,27] that although the energy dependence of the production process may be described correctly by Eq. (1) (particularly for the production of heavier mesons), its absolute magnitude could be uncertain because of the off-shell effects at the production vertices. We have accounted for these effects by an off-shell extrapolation of the on-shell FSI amplitude by multiplying it by a monopole form factor with a cut-off parameter of 0.2 GeV, as suggested in [7,21]. In this method, both absolute magnitude as well as shape of the FSI factor are affected by the off-shell corrections. In our calculations, the difference between the off-shell and on-shell FSI factors is similar to that seen in Ref. [27] for the case of Yamaguchi potential calculations of the \( \eta \)-meson. The form factor approach for the off-shell effects used here is based essentially on the Yamaguchi type of separable \( YN \) potential. It may be improved by using the off-shell structure of some more realistic interaction. However, this will imply going beyond the factorization approach of Eq. (1), which is beyond the scope of this paper.

In our calculation of the FSI amplitudes, the values of the parameters \( a_0 \) and \( r_0 \) for the \( \Lambda-p \) and the \( \Sigma-p \) systems were taken from the \( \tilde{A} \) model of the \( YN \) interaction of the Jülich-Bonn group [28]. The values for these parameters for the \( \Lambda-p \) system were the same as those given in Ref. [3], while for the \( \Sigma-p \) system, \( a_0 \) and \( r_0 \) were 2.28 fm and 4.96 fm for the singlet state, and 0.76 fm and 2.50 fm for the triplet states respectively. We have also considered the FSI interaction in the \( K^+-Y \) channel, which is possible only within the factorization approach that has the additional advantage of making it possible to account for the FSI effects among all the three particles in the outgoing channel. Since different two-body FSI amplitudes in the final channel contribute coherently, the baryon-meson interactions, although weaker on their own, may still be influential through the interference terms. The values [29,30] of \( a_0 \) and \( r_0 \) were -0.065-i0.040 and -15.930-i8.252 respectively for the \( K^+-\Lambda \) system, and -0.201-i0.131 and -1.757-i0.0835 respectively for the \( K^+-\Sigma^0 \) system.

The amplitude \( M_{fi} \) for the two reactions has been calculated in a way similar to that described in Ref. [3] using the same set of parameters. However, we additionally require
the coupling constants for the $N^*\Sigma^0 K^+$ vertices in the calculation of the cross sections for the $\Sigma^0 K^+$ reaction. For $N^*(1710)$ and $N^*(1720)$ resonances, these were determined from the corresponding branching ratios (adopted from Ref. [31]) for their decay to the $\Sigma K$ channel. While choosing their values, we ensured that the sum of the branching ratios of all the relevant channels does not exceed unity. The resulting coupling constants are given by $g_{N^*\Sigma^0 K^+}/4\pi = 8.242$ and 0.220 for $N^*(1710)$ and $N^*(1720)$ resonances respectively with their signs being negative and positive respectively.

However, such a procedure can not be used to determine $g_{N^*\Sigma^0 K^+}$ for $N^*(1650)$, as the on-shell decay of this resonance to the $\Sigma K$ channel is inhibited. Instead, we tried to determine this coupling constant by fitting the available data on the $\pi^+ p \rightarrow \Sigma^+ K^+$, $\pi^- p \rightarrow \Sigma^0 K^0$, and $\pi^- p \rightarrow \Sigma^- K^+$ reactions in an effective Lagrangian coupled channels approach [29,30], where all the available data for the transitions from $\pi N$ to five meson-baryon final states, $\pi N$, $\pi \pi N$, $\eta N$, $K\Lambda$, and $K\Sigma$ are simultaneously analyzed for center of mass energies ranging from threshold to 2 GeV. In this analysis all the baryonic resonances with spin $\leq \frac{3}{2}$ up to excitation energies of 2 GeV are included as intermediate states. The best fit resulted in a value of 0.233 for the $N^*(1650)K\Sigma$ coupling, but due to very few data points available for the $\pi^- p \rightarrow K^+ \Sigma^-$ channel, it may still be premature to attach much significance to this value. On the other hand, a value of 0.450 provides a very nice agreement with the data of the $\Sigma^0 K^+$ reaction. Furthermore, since fitting with a fixed value of 0.450 to the available $\pi p \rightarrow K\Sigma$ data with the model mentioned above resulted in a comparable overall $\chi^2$ (although the former value provides a somewhat lower $\chi^2$ for $\pi^- p \rightarrow K^+ \Sigma^-$, cf. Fig. 1), the latter value has been used in all the results shown in this paper. The shapes of the form factors and the values of the cut-off parameters appearing therein were taken to be the same as those used in the case of $\Lambda K^+$ reaction [3].

In Fig. 2 we show the individual contributions of various nucleon resonances to the total cross section of the $\Sigma^0 K^+$ reaction near the production threshold as a function of $\epsilon$. We see that, as in the case of the $\Lambda K^+$ reaction, the cross section for this reaction too is dominated by the $N^*(1650)$ resonance excitation. Thus, at the near threshold energies, both these reactions proceed preferentially via excitation of this resonance. Looking only at the values of coupling constants, one might expect the dominance of $N^*(1710)$ resonance for both the reactions even at these energies. This is particularly so for the $\Sigma^0 K^+$ reaction, where the threshold energy is very close to the excitation energy of this resonance. However, in the near threshold region the relative dominance of various resonances is determined by the dynamics of the reaction where the difference of about 60 MeV in excitation energies of $N^*(1650)$ and $N^*(1710)$ resonances plays a crucial role. Yet, some differences in the relative contributions of $N^*(1710)$ resonance in the two reactions at these energies are noteworthy. For $\Sigma^0 K^+$ reaction the contribution of this resonance is about a factor of 3-4 larger as compared to that in the case of $\Lambda K^+$ reaction. This is the reason for the interference effects among the resonances being relatively larger in Fig. 2 as compared to that in the $\Lambda K^+$ case [3]. It may be remarked here that in both cases one-pion exchange between the incident protons gives maximal contribution to the cross sections as compared to $\rho$-, $\omega$-, and $\sigma$-meson exchanges.

The total cross sections for the $\Lambda K^+$ and $\Sigma^0 K^+$ reactions as a function of $\epsilon$ are shown in Fig. 3. The calculations are the coherent sum of all resonances and meson exchange processes as described earlier. The $\Lambda K^+$ results are the same as those shown in [3]. For the $\Sigma^0 K^+$ reaction, there is a reasonable agreement between theory and the data except for
very small values of $\epsilon$ where our calculations underpredict the experimental cross sections by a factor of about 1.5. Keeping in mind the fact that all parameters of the model, except for those of $N^*Yp$ vertices and the FSI, were the same in the two calculations and that no parameter was freely varied, this agreement is quite satisfactory. It should be noted that unlike Ref. [11], we do not require to introduce arbitrary normalization constants to get the agreement between calculations and the data.

In Fig. 4, we compare our calculations with the data for the ratio $R$ as a function of $\epsilon$. We have shown here the results for excess energies up to 1 GeV, where the first high energy data is available. It is clear that our calculations are able to describe the strong fall-off of $R$ between low and high energies even though they somewhat overestimate the effects at the lowest points. It is interesting to note that at the near threshold energies, calculations done without FSI effects can already explain the data up to 40-50%. Therefore, all of the observed value of $R$ at these beam energies can not be accounted for by the FSI alone, which is in agreement with the observation made in [11]. It should again be emphasized that without considering the contributions of the $N^*(1650)$ resonance for the $\Sigma^0K^+$ reactions the calculated ratio would be at least an order of magnitude larger. Therefore, these data are indeed sensitive to the details of the reaction mechanism. At higher beam energies ($\epsilon > 300$ MeV), values of $R$ obtained with and without FSI effects are almost identical. In this region the reaction mechanism is different; here the $N^*(1710)$ resonance makes the dominant contribution [6] and FSI related effects are unimportant. This is the most likely cause for the difference in the values of $R$ in the two regions.

In summary, we have studied the $pp \rightarrow p\Lambda K^+$ and $pp \rightarrow p\Sigma^0K^+$ reactions within an effective Lagrangian model. Most of the parameters of the model are fixed by fitting the $NN$ T-matrix, which restricts the freedom of varying them freely in order to fit the data. The reactions proceed via the excitation of the $N^*(1650)$, $N^*(1710)$, and $N^*(1720)$ intermediate baryonic resonant states. An important result of our study is that in the near threshold region both these reactions proceed predominantly via excitation of the $N^*(1650)$ intermediate baryonic resonant state. To the extent that the final state interaction effects in the exit channel can be accounted for by the Watson-Migdal theory, our model is able to explain the experimentally observed large ratio of the total cross sections of the two reactions in the near threshold region. It can also explain the relatively smaller value of this ratio at higher beam energies where the reactions are dominated by the $N^*(1710)$ resonance and the FSI related effects are negligible.

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Figure Captions

Fig. 1 The total cross section for the $\pi^- p \rightarrow \Sigma^0 K^0$ (upper part) and $\pi^- p \rightarrow \Sigma^- K^+$ (lower part) reactions as a function of invariant mass $s$. The solid and dashed lines show the results of a coupled channels K-matrix calculation [29] with the values of the coupling constant for $N^*(1650)\Sigma K$ vertex of 0.233 and 0.450 respectively.

Fig. 2 Contributions of $N^*(1650)$ (dotted line), $N^*(1710)$ (dashed line) and $N^*(1720)$ (dashed-dotted line) baryonic resonances to the total cross section for the $pp \rightarrow p\Sigma^0 K^+$ reaction as a function of the excess energy. Their coherent sum is shown by the solid line.

Fig. 3 Comparison of the calculated and the experimental total cross section for the $pp \rightarrow p\Lambda K^+$ (solid line and solid squares) and $pp \rightarrow p\Sigma^0 K^+$ (dashed line and solid circles) as a function of the excess energy. The experimental data are from Refs. [1] (solid squares) and [2] (solid circles).

Fig. 4 Ratio of the total cross sections for $pp \rightarrow p\Lambda K^+$ and $pp \rightarrow p\Sigma^0 K^+$ reaction as a function of the excess energy. The solid and dashed lines show the results of our calculations with and without FSI effects respectively. The data are from [2,3].
FIG. 1. The total cross section for the $\pi^- p \to \Sigma^0 K^0$ (upper part) and $\pi^- p \to \Sigma^- K^+$ (lower part) reactions as a function of invariant mass $s$. The solid and dashed lines show the results of a coupled channels K-matrix calculation \cite{29} with the values of the coupling constant for $N^*(1650)\Sigma K$ vertex of 0.233 and 0.450 respectively.
FIG. 2. Contributions of $N^*(1650)$ (dotted line), $N^*(1710)$ (dashed line) and $N^*(1720)$ (dashed-dotted line) baryonic resonances to the total cross section for the $pp \rightarrow p\Sigma^0K^+$ reaction as a function of the excess energy. Their coherent sum is shown by the solid line.
FIG. 3. Comparison of the calculated and the experimental total cross section for the $pp \rightarrow p\Lambda K^+$ (solid line and solid squares) and $pp \rightarrow p\Sigma^0 K^+$ (dashed line and solid circles) as a function of the excess energy. The experimental data are from Refs. [1] (solid squares) and [2] (solid circles).
FIG. 4. Ratio of the total cross sections for \( pp \rightarrow p\Lambda K^+ \) and \( pp \rightarrow p\Sigma^0 K^+ \) reaction as a function of the excess energy. The solid and dashed lines show the results of our calculations with and without FSI effects respectively. The data are from [2,3].