Simulation of EHD lubrication for heavy-duty journal bearing based on flexible multi-body dynamics

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Abstract. For heavy-duty journal bearing, the structural flexibility of the shaft and the bearing block has an important influence on the dynamic characteristics of bearing behaviour. Therefore, the deformation and modal resonance generated at work must be considered. In this paper a computational method for solving Elastohydrodynamic lubrication based on flexible multi-body dynamics is introduced. The dynamic substructure condensation method is used to reduce the shaft and the bearing block, obtain the flexible body model of the crankshaft and the body, reduce the computational solution scale, and improve the computational efficiency.

1. Introduction
With the improving of the rotating machinery, journal bearing wear failure to become one of the main factors affecting the service life of the rotating machinery, so it is necessary to form the effective analysis of the journal bearing lubrication method, which can accurate obtain the lubrication and friction behaviour of the journal bearing. The results can be used to optimize the main bearing structure and lubricating system. In this paper a computational method for solving Elastohydrodynamic lubrication based on flexible multi-body dynamics is introduced. Because of the complex shaft structure and many parts, if the traditional finite element method is adopted to obtain its mass matrix and stiffness matrix, the calculation scale will be too large. In order to realize effective engineering application, the computational efficiency need to be improved. The dynamic substructure condensation method is used to reduce the shaft and the bearing block, obtain the flexible body model of the crankshaft and the body, and reduce the computational solution scale.

2. Flexible multi-body dynamics method

2.1. Substructure condensation
For the computation of dynamic behaviour of non-linear components, the equations should consider nonlinear loads and non-linear inertia forces. Meanwhile the non-linear forces and moments caused by contact have to be represented. The accuracy of the computation and the total amount of time that is needed for the computation are significantly related to the number of degrees of freedom, therefore, it is necessary to find a balance between calculation speed and calculation accuracy. The number of degrees of freedom for the computational model should be reduced based on FEA model [1].

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The modal synthesis method is a typical dynamic substructure analysis method, the method by the modal transformation, will represent the structure of the sports in physical coordinate equation expressed in modal coordinates transformation, because of the mode number of the degrees of freedom is much less than the physical degrees of freedom, which can make the structure motion equation was simplified. For the undamped system, the equation of motion can be expressed when there is no load in the degree of freedom[2]:

\[
\begin{bmatrix}
M_{mm} & M_{ms} \\
M_{ms} & M_{ss}
\end{bmatrix}
\begin{bmatrix}
x_m \\
x_s
\end{bmatrix}
+
\begin{bmatrix}
K_{mm} & K_{ms} \\
K_{ms} & K_{ss}
\end{bmatrix}
\begin{bmatrix}
x_m \\
x_s
\end{bmatrix}
=
\begin{bmatrix}
f_m \\
0
\end{bmatrix}
\]  

(1)

\(M\) and \(K\) are the mass matrices and stiffness matrices of the model, \(x\) and \(f\) are the displacement and force vectors of the model, \(m\) and \(s\) are respectively the coordinates of the degrees of freedom and the degrees of freedom. The following equation can be obtained from equation (2):

\[
(M_{mm}\ddot{x}_m + M_{ms}\ddot{x}_s) + (K_{mm}x_m + K_{ms}x_s) = 0
\]

(2)

The solution of this equation is:

\[
x_s = -(K_{ss} - \lambda^2 M_{ss})^{-1} K_{ms} x_m
\]

(3)

The eigenvalue \(\lambda\) is calculated by taking a value that is close to the value of order \(i\) eigenvalue \(\lambda_i\), and the equation (2) can be expressed as:

\[
\begin{bmatrix}
x_m \\
x_s
\end{bmatrix}
\begin{bmatrix}
I \\
-(K_{ss} - \lambda^2 M_{ss})^{-1} K_{ms}
\end{bmatrix}
\begin{bmatrix}
x_m \\
x_s
\end{bmatrix}
= T_s x_m
\]

(4)

According to equation (4), equation (1) can be condensed after the transformation:

\[
M_R \ddot{x}_m + K_R x_m = f_R
\]

(5)

And the resulting mass matrix, stiffness matrix and force vector are expressed as follow:

\[
M_R = T_s^T M T_s, \quad K_R = T_s^T K T_s, \quad f_R = T_s^T f
\]

(6)

2.2. Flexible body kinematics

The dynamic analysis of the flexible body is based on the theory of rigid body dynamics. The flexible body dynamic can be described as the motion of rigid body is determined by a set of local coordinates and local elastic deformation of local structure. Therefore, for the movement of the flexible body, it is necessary not only to use local coordinates to determine the position of its centre of mass, but also to use the deformation vector to determine the deformation of the node on the flexible body relative to the centre of mass and inertia. The particle \(B\) of the flexible body reaches the space position with the global motion and the elastic deformation to the space position, and the space position can be expressed as the sum of the three vectors.

\[
r_p = \bar{x} + \bar{s}_p + \bar{u}_p
\]

(7)

\(\bar{x}\) is the vector of the global coordinate system origin \(G\) to the local coordinate system origin \(B\) of the flexible body. \(\bar{s}_p\) is the vector in local coordinate \(B\) when the particle is not deformed. \(\bar{u}_p\) is the vector of the position relative to the position before the deformation. The above expression can be expressed as the matrix form in the whole coordinate system:

\[
r_p = \mathbf{x} + ^{p}\mathbf{A}(\mathbf{s}_p + \mathbf{u}_p)
\]

(8)

\(\mathbf{x}\) is the coordinate vector of the local coordinate origin of the flexible body in the global coordinate system relative to the global coordinate origin \(G\), expressed as \(\mathbf{x}, y, z\), \(\mathbf{s}_p\) is the coordinate vector of the
relative position between particle $P$ and local coordinate origin $B$. Since the centre of the flexible body is usually set as the origin of local coordinates, the $P$ is constant. \( A \) is the transformation matrix for the transformation of coordinate vectors in the local coordinate system to the global coordinate system. The direction of the local coordinate system relative to the global coordinate system can be expressed as $\varphi$, $\theta$, $\phi$. \( u_p \) is the linear deformation vector of particle $P$ in the local coordinate system. The deformation vector is usually expressed as the form of the mode superposition:

$$ u_p = \Phi_p \cdot q $$  \(9\)

\( \Phi_p \) is the translational degree of translational degree of particle $P$, which is separated from the modal matrix. The specification of this matrix is $3 \times m$, $m$ is the modal quantity. The modal coordinate $q_{i,(i=1,m)}$ is the generalized coordinate of the flexible body. Therefore, the generalized coordinates of the flexible body can be expressed as:

$$ \xi = \begin{bmatrix} x & y & z & \varphi & \theta & \phi & q_{i,(i=1,m)} \end{bmatrix} $$ \(10\)

$x, y, z, \varphi, \theta, \phi$ represent the position and Angle of the flexible body; $q_{i,(i=1,m)}$ is the modal coordinate vectors, which is used to represent the deformation of the flexible body. The kinematics quantities of flexible body points can be obtained by means of generalized coordinate and mode superposition method.

2.3. Flexible body dynamics

The dynamic equation of the flexible body is based on the generalized coordinates, which can reflect the large scale rigid body displacement and the elastic deformation of the small scale line of the flexible body. The generalized coordinate vector $\xi$ can be represented as the control equation of the Lagrange equation [3]:

$$ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\xi}} \right) - \frac{\partial L}{\partial \xi} + \frac{\partial \Psi}{\partial \xi} \cdot \lambda = \Delta $$ \(11\)

$\xi$ is the generalized coordinate of the flexible body, $\dot{\xi}$ is the derivative of $\xi$ with respect to time. $L$ is the Lagrange equation, which is defined as $L = T - V$, and $T$ are kinetic energy and $V$ are potential energy. $\Psi$ is the constraint equation, $\lambda$ is the Lagrange constraint factor, $\Delta$ is the generalized force caused by external forces. Under the generalized coordinates control differential equation based on Lagrange equation at the end of the form as follows:

$$ M \cdot \ddot{\xi} + \dot{M} \cdot \dot{\xi} - \frac{1}{2} \partial \dot{M} \partial \xi \cdot \dot{\xi} + K \cdot \xi + f_g + D \cdot \dddot{\xi} + \frac{\partial f_p}{\partial \xi} \cdot \lambda = \Delta $$ \(12\)

$\dddot{\xi}$ is the second derivative of the generalized coordinates with respect to time, $M$ is the mass matrix of the flexible body; $\dot{M}$ is the first order matrix for the mass matrix of the flexible body. $\partial \dot{M} / \partial \xi$ is the partial derivative of mass matrix with respect to the generalized coordinates. $K$ is the generalized stiffness matrix; $f_g$ is generalized mass force; $D$ is the modal damping matrix.

2.4. Flexible body dynamics

The large end bearing of the connecting rod is used to connect the crank pin and the connecting rod. Under the action of shear and radial compression and convergence clearance, the oil film of the bearing can produce radial pressure distribution, thus providing support and torque to the bearing. With the decrease of oil film thickness in the convergence gap, the support stiffness and damping have a nonlinear function relationship. The nonlinear characteristics of oil film are simulated by nonlinear spring damping in the model of flexible multibody dynamics. The supporting force of the bearing can be expressed as follow:
\[ f = k_{\text{Joint}} \cdot \Delta x + d_{\text{Joint}} \cdot \Delta \dot{x} \quad (13) \]

\( \Delta x \) is the distance between the main degrees of freedom of the connecting rod and the main degrees of freedom of the crank pin. \( \Delta \dot{x} \) is the first derivative of the distance. \( k_{\text{Joint}} \) and \( d_{\text{Joint}} \) can be obtained through the following nonlinear equations:

\[ k_{\text{Joint}} = k_0 \left( \frac{k_B}{k_0} \right)^{\frac{1}{2}} \quad (14) \]

\[ d_{\text{Joint}} = d_0 \left( \frac{d_B}{d_0} \right)^{\frac{1}{2}} \quad (15) \]

Nonlinear spring-damping elements can be defined by parameters \( k_0, k_B, d_0, d_B \) and reference length. In this paper, the spring damping model used in the whole model is NONL model.

3. Elastohydrodynamic lubrication model

In order to obtain the flexible dynamic characteristics of the crankshaft and the elastic fluid lubrication characteristics of the main bearing, an EHD lubrication model was used to simulate the main bearing oil film to obtain the oil film pressure distribution in the cycle of the engine [4].

3.1. Reynolds equation

The pressure distribution of oil film is determined by the stiffness of the bearing oil film and the oil film damping, which are generally obtained by solving the Reynolds equation of bearing oil film under unsteady state.

Assuming that the fluid has the same viscosity and density in the direction of oil film thickness, the cylindrical coordinate expression of Reynolds equation in two-dimensional generalized coordinate system can be expressed as follows [5]:

\[ \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( \frac{\rho h^3}{\eta} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial x} \left( \frac{\rho h^3}{\eta} \frac{\partial p}{\partial x} \right) = 6V \frac{\partial (\rho h)}{r \partial \theta} + 12 \frac{\partial (\rho h)}{\partial t} \quad (16) \]

3.2. Boundary condition

The following Reynolds boundary conditions are used in the solution process.

\[
\begin{align*}
 p(\theta, \pm B/2) &= p(\theta_2, \pm B/2) = p_a \\
p(\theta_2, x) &= p_s \\
\frac{\partial p}{\partial \theta} &= 0 \\
p(\theta_1, x) &= p_s
\end{align*}
\]

(17)

Where \( \theta_1 \) is the inlet position of the bearing; \( \theta_2 \) is the oil position; \( p_a \) is environmental pressure; \( p_s \) is for oil supply pressure. Assuming that the fluid is an incompressible flow, \( \rho \) is a constant, which simplifies the equation to:

\[ \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial x} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial x} \right) = 6V \frac{\partial (h)}{r \partial \theta} + 12 \frac{\partial (h)}{\partial t} \quad (18) \]

The distribution of oil film thickness is as the centre shaft rotary motion and journal bearing center along the radial extrusion motion synthesis results. In the above equation, \( V = r \Omega \), \( \Omega \) is the effective angular velocity of moving bearing, which can be obtained by the following formula:

\[ \Omega = \omega_b + \omega_e - 2 \dot{\delta} \quad (19) \]

Where \( \omega_e \) is the axial angular velocity; \( \omega_b \) is the bearing angular velocity; \( \dot{\delta} \) is the rate of change of the eccentric Angle. For the engine main bearing, \( \omega_b = 0 \), equation (19) can be simplified as:
\[ \Omega = \omega_j - 2\dot{\varepsilon} \]  

(20)

Because the thickness of the oil film can be expressed as \( h = c(1 + \varepsilon \cos \theta) \), and \( c = R - r \), \( \varepsilon = \frac{e}{c} \) is eccentricity. Therefore, equation (16) can be expressed as:

\[
\frac{1}{r^2} \frac{\partial}{\partial \theta} \left[ c^\prime (1 + \varepsilon \cos \theta) \frac{\partial p}{\partial \theta} \right] + \frac{1}{r} \frac{\partial}{\partial \theta} \left[ c^\prime (1 + \varepsilon \cos \theta) \frac{\partial p}{\partial \theta} \right] = -6\Omega \eta c \varepsilon \sin \theta + 12\eta c \dot{\varepsilon} \cos \theta
\]  

(21)

\( \dot{\varepsilon} \) is the rate of change of the eccentricity of the eccentricity, which reflects the velocity of the rotation of the journal. Equation (21) is the Reynolds equation for lubrication calculation of motor sliding bearing. In order to distinguish it from the generalized Reynolds equation, this formula is called the special Reynolds equation.

3.3. Equation for bearing lubrication

By introducing the filling rate \( \gamma \) of oil in the equation, the generalized Reynolds equation can be extended to consider the Reynolds equation of oil filling rate. The radial sliding bearing Reynolds equation is extended as follows:

\[
\frac{\partial}{\partial y} \left( \frac{1}{12\eta} \gamma h^3 \frac{\partial p}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{1}{12\eta} \gamma h^3 \frac{\partial p}{\partial x} \right) = \gamma \left( \frac{V_1 + V_2}{2} \right) \frac{\partial (h)}{\partial y} + \left( \frac{V_1 + V_2}{2} \right) h \frac{\partial^2 y}{\partial y^2} + \frac{\partial (\gamma h)}{\partial t}
\]  

(22)

Since the main bearing axis is still in the engine working process, the shaft neck is rotated, so in the equation (22), \( V_1 = 0 \), \( V_2 = \dot{V} \), the formula can be changed to:

\[
\frac{\partial}{\partial y} \left( \frac{1}{12\eta} \gamma h^3 \frac{\partial p}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{1}{12\eta} \gamma h^3 \frac{\partial p}{\partial x} \right) = \gamma \frac{V}{2} \frac{\partial (h)}{\partial y} + \gamma \frac{V}{2} \frac{h \partial^2 y}{\partial y^2} + \frac{\partial (\gamma h)}{\partial t}
\]  

(23)

The boundary conditions of the quality conservation boundary of JFO are used as the boundary conditions of equation (23), including axial boundary conditions, circumferential boundary conditions, boundary conditions of oil supply regions and cavitation boundary conditions, as shown in Table 1.

**Table 1. JFO mass conserved boundary conditions.**

| Boundary Conditions                      | Equation                                      |
|------------------------------------------|-----------------------------------------------|
| Axial boundary conditions                | \( p = p_a \left( x = \pm \frac{B}{2} \right) \) |
| Circumferential boundary conditions      | \( \theta = 0 \)                             |
|                                          | \( |x| \leq \frac{l}{2}, \ p = p_a \)          |
|                                          | \( \frac{l}{2} < |x| \leq \frac{B}{2}, \ p = p_a \left( \frac{B - 2|x|}{B - l} \right) \) |
|                                          | \( \frac{\partial p}{\partial \theta} = 0, \ p = p_a \left( \theta = \frac{y}{r} \right) \) |
| Boundary conditions of oil supply region | \( p = p_a \)                                |
| Cavitation boundary condition            | \( p = p_r, \ \gamma = 1 \)                 |
|                                          | \( p < p_r, \ \gamma < 1 \)                 |

4. Heavy-duty journal bearing simulation

4.1. Dynamic substructure condensation

The 3D mesh model of the body was established by Hypermesh finite element pre-processing software. The cylinder sleeve, bearing and bearing backing by 8 node hexahedron unit, and the rest of the 10
node tetrahedral two unit. The width direction of the bearing shell divided into 4 equal parts, and the main journal consistent. The total number of nodes in the finite element model is 341391, and the number of elements is 188492. 8 nodes hexahedron element is 20352, and 10 node tetrahedron element is 168140. The flexible multi-body dynamics modelling is shown in figure 1.

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4.2. Lubrication model of the journal bearing
An elastic hydrodynamic lubrication bearing model (EHD2) is used to connect the main bearing pairs. The lubrication model can describe the axle neck and bearing elastic deformation, bearing clearance, axis of mass conservation (non-cavitation model), the supply of the lubricating oil (oil tank, oil hole and line etc.) and other factors, is a more accurate to obtain the bearing pressure distribution model. The lubrication model of the journal bearing is shown in figure 2.

4.3. EHD lubrication simulation
The flexible multi-body dynamics model of journal bearing is driven by the explosion pressure curve of the cylinder. The burst pressure curve in this paper is obtained through the actual engine cylinder pressure measurement experiment, which is shown in figure 3.

Figure 1. Flexible multi-body dynamics model of bearing system including 4 journal bearing.

Figure 2. The lubrication model of the journal bearing.

Figure 3. The explosion pressure curve of the cylinder of engine.

Figure 4 shows that when the crankshaft Angle at 4.71 ° and 604.36 °, main bearing of load is bigger, and the oil film pressure distribution along the z direction of the local cylindrical coordinates is
relatively uniform. When the crankshaft Angle at 362.88 ° and 482.93 °, main bearing oil film pressure is low, and distribution is uneven, main bearing on the load is small.

Figure 4. The oil film thickness of the second main bearing in the explosion of each cylinder.

5. Conclusions
For heavy-duty journal bearing, the structural flexibility of the shaft and the bearing block has an important influence on the dynamic characteristics of bearing behaviour. The following conclusions can be obtained through the above analysis:

- The dynamic substructure condensation method is useful to reduce the computational solution scale, and improve the computational efficiency.
- The main bearing is mainly affected by adjacent explosion pressure of the cylinder, not adjacent to each cylinder explosion pressure on the main bearing force effect is small.
- With the increase of engine outbreak of peak stress, peak bearing force of the bearing is also increasing, of which the second and third main bearing of the bearing force was bigger.

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