Measuring Radial Orbit Migration in the Galactic Disk

Neige Frankel1, Hans-Walter Rix1, Yuan-Sen Ting2,3,4, Melissa Ness5,6, and David W. Hogg1,6,7,8
1 Max Planck Institute for Astronomy, Königstuhl 17, D-69117 Heidelberg, Germany
2 Institute for Advanced Study, Princeton, NJ 08540, USA
3 Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544, USA
4 Observatories of the Carnegie Institution of Washington, 813 Santa Barbara Street, Pasadena, CA 91101, USA
5 Department of Astronomy, Columbia University, 550 W 120th St, New York, NY 10027, USA
6 Flatiron Institute, 162 5th Avenue, New York, NY 10010, USA
7 Center for Cosmology and Particle Physics, Department of Physics, New York University, 726 Broadway, New York, NY 10003, USA
8 Center for Data Science, New York University, 60 5th Avenue, New York, NY 10011, USA

Received 2018 May 22; revised 2018 August 6; accepted 2018 August 18; published 2018 September 26

Abstract

We develop and apply a model to quantify the global efficiency of radial orbit migration among stars in the Milky Way disk. This model parameterizes the possible star formation and enrichment histories and radial birth profiles, and combines them with a migration model that relates present-day orbital radii to birth radii through a Gaussian probability, broadening with age \( \tau \) as \( \sigma_{\text{RMS}} \sqrt{7/8 \, \tau} \). Guided by observations, we assume that stars are born with an initially tight age–metallicity relation at given radius, which becomes subsequently scrambled by radial orbit migration, thereby providing a direct observational constraint on radial orbit migration strength \( \sigma_{\text{RMS}} \). We fit this model with Markov Chain Monte Carlo sampling of the observed age–metallicity distribution of low-\( \alpha \) red clump stars with Galactocentric radii between 5 and 14 kpc from APOGEE DR12, sidestepping the complex spatial selection function and accounting for the considerable age uncertainties. This simple model reproduces the observed data well, and we find a global (in radius and time) radial orbit migration efficiency in the Milky Way of \( \sigma_{\text{RMS}} = 3.6 \pm 0.1 \) kpc when marginalizing over all other aspects of the model. This shows that radial orbit migration in the Milky Way’s main disk is indeed rather strong, in line with theoretical expectations: stars migrate by about a half-mass radius over the age of the disk. The model finds the Sun’s birth radius at \( \sim 5.2 \) kpc. If such strong radial orbit migration is typical, this mechanism indeed plays an important role in setting the structural regularity of disk galaxies.

Key words: Galaxy: abundances – Galaxy: disk – Galaxy: evolution – Galaxy: formation – ISM: abundances – stars: abundances

1. Introduction

To understand how disk galaxies formed and evolved (e.g., Schönrich & Binney 2009b; Mo et al. 2010), we need to understand how our Milky Way, a typical disk galaxy, formed and evolved. In particular, we need to identify and characterize the processes setting the radial and vertical structures of the Galactic stellar disk in terms of stellar ages and abundances.

The present-day structure must at some level reflect both the global initial conditions, such as the total angular momentum and distribution of the cold gas, and the hierarchical merging during the early turbulent phases of the Milky Way’s formation (Brook et al. 2004; Bird et al. 2013; Stinson et al. 2013). The stars’ age distribution obviously reflects the overall star formation history (SFH) of the Galaxy. In addition, the stars’ photospheric element abundances trace the gradual enrichment of the Milky Way, which proceeded differently in different parts of the Galaxy (e.g., Chiappini et al. 2001; Schönrich & Binney 2009b).

But for the last \( \sim 8 \) Gyr, the Milky Way’s dynamical history has been quite quiescent, with the large majority of stars formed since then residing in a thin disk (e.g., Rix & Bovy 2013; Bland-Hawthorn & Gerhard 2016). However, even in this quiescent regime, we cannot expect the stars’ present-day orbits to reflect their birth orbits, as first detailed by Sellwood & Binney (2002, hereafter SB02): there may be a great deal of dynamical evolution on timescales longer than a dynamical time because the Galaxy is not axisymmetric; this is called “secular evolution.”

In particular, radial orbit migration has been recognized as a potentially very important process in both analytic and simulation work (SB02; Roskar et al. 2008a; Minchev & Famaey 2010). Even if a star was born on a circular orbit, its present-day radius may differ from its birth radius for basically two reasons: first, a variety of perturbations in the in-plane or vertical direction may cause increasing epicycles, a process dubbed “blurring” by SB02 to refer to orbital heating. We know from the velocity dispersion in the Galactic disk that for “middle-aged” stars (\( \sim 5 \) Gyr), this leads to radial excursions of about 1 kpc. But SB02 emphasized another process, which they dubbed “churning,” that occurs in the presence of changing, fleeting or complex non-axisymmetric patterns (overdensities) such as spiral arms; these exert torques on stars, and lead to an effective change in a star’s angular momentum or mean orbital radius, without inducing much “blurring.” Here, we focus on the changes in the (instantaneous) orbital radius, and refer to this combined effect of “churning” and “blurring” as “radial orbit migration” or “radius migration” throughout the present analysis.

N-body and cosmological simulations imply that radial orbit migration is very important: the angular momentum (and hence the size of the orbit) of any star may change by of the order of unity over timescales as short as a few Gyr (Grand et al. 2012, 2016; Kubyk et al. 2013), the detail depending on the properties of the non-axisymmetric structures, such as their...
pattern speeds and strengths. But to predict the actual degree of radius migration in any galaxy quantitatively, one would need to have an inventory of all the past speeds and strengths of spiral and bar patterns.

There is well established observational evidence for the relevance of this process. In external galaxies, it makes predictions for the outermost radial density and age profiles of the stellar disk that are in qualitative agreement with observations (e.g., Herpich et al. 2017; Ruiz-Lara et al. 2017). In our Galactic disk, there are the remarkable, long-standing observations that there is no distinct age–metallicity relation of stars in the solar neighborhood (a few hundred parsecs around the Sun), and that there is a wide spread of metallicities at the solar radius (Edvardsson et al. 1993; Haywood 2008; Casagrande et al. 2011). Both observations would be puzzling if stars—at a given time and Galactocentric radius—were born with a very small spread in metallicities. This is expected both from models of chemical evolution (Matteucci & Francois 1989) and from observations of the interstellar gas and young stars in galaxies (e.g., Przybilla et al. 2008).

But if there is an important radial gradient in the metallicities (as observed in the Milky Way, e.g., Genovali et al. 2014) then extensive radius migration, scrambling the orbital radii of stars while keeping their [Fe/H] unchanged, could explain the lack of a relation between age, metallicity, and (present-day) radius at given radius. This has been advocated and worked out by Schönrich & Binney (2009a, 2009b), Roškar et al. (2008b), Minchev et al. (2013), and Hayden et al. (2018). They laid out a picture where three basic ingredients can explain the present-day orbit–age–abundance distribution of Galactic disk stars: (1) disk stars at all epochs and Galactocentric orbital radii were born with a well-determined metallicity ([Fe/H]|R₀), (2) there has always been an evolving outward metallicity gradient, (3) extensive subsequent mixing of orbital radii occurred.

Many of the best observational constraints on such radial orbit migration in the Galaxy stem from very local samples (Nordström et al. 2004; Sanders & Binney 2015), with stars that have estimates of both abundance and age, and indeed these analyses imply very effective radius migration. But if radial orbit migration is a global phenomenon across the Galactic disk, then it calls for a “global” test, i.e., a test with observational data that cover Galactocentric radii that encompass a good fraction of the Galactic disk.

Here we propose measurement of the average, or global, efficiency of radial orbit migration, based on data over a very wide Galactocentric radial range (5 ≤ R ≤ 14 kpc), with age estimates from spectroscopy. APOGEE (Majewski et al. 2017) spectra provide the first large (~20,000) sample with consistent age estimates, τ (Ness et al. 2016), across a large radial range in the Galaxy. Qualitatively, the young stars (τ ≤ 1 Gyr) show a well-defined radial metallicity gradient ([Fe/H] decreasing outward), with a modest scatter in [Fe/H] at any given radius (see Figure 2). “Old” stars (≥10 Gyr) show no discernable metallicity–radius relation, or at least enormous scatter in p([Fe/H]|R). The basic idea (Schönrich & Binney 2009a; Sanders & Binney 2015) is that extensive radius migration has largely erased the original radius–[Fe/H]–age relation.

This approach is related to, but not the same as, “chemical tagging” (Freeman & Bland-Hawthorn 2002; Ting et al. 2015), which aims to identify stars that were born in the same cluster by their near-identical, detailed abundance patterns, even if they are now on widely different orbits. While stars from the same cluster were manifestly born at the same epoch and the same Galactocentric radius, the approach in the present analysis makes a different assumption: that stars born at the same epoch at the same Galactocentric radius have very similar [Fe/H] (e.g., Przybilla et al. 2008).

Any radius migration over the course of a star’s life is best thought of as a combination of diffusion of orbital angular momentum, or guiding radius (churning), and orbital heating (blurring), presuming the star was born on a near-circular orbit. These are two distinct processes of different amplitudes, which can be measured separately using stellar angular momenta and radial action. But here we focus on the stars’ Galactocentric radii R as a proxy, because these quantities are currently available with great fidelity and across a wide range of radii. We also restrict our analysis to stars with ages τ ≤ 8 Gyr, because a model of gradual, secular orbit evolution may not be applicable to the earliest phases of the Galactic (thick) disk formation.

Here we construct a forward model that incorporates the main processes that set the age- and abundance-dependent structure of the Galactic disk: the global SFH, inside-out growth, gradual chemical enrichment, and radial orbit migration. In important aspects, this model draws on the ideas laid out in Sanders & Binney (2015). We then compare this model to APOGEE data, thereby constraining the strength of radial orbit migration from data across the Galaxy. The data are presented in Section 2. The methodology is laid out in Section 3. We then present our results in Section 4, which quantitatively constrain radius migration and affirm how effective it seems to be in the Galaxy. We conclude and comment in Section 5.

2. Data: APOGEE Red Clump Giants

Global constraints on radius migration of stellar orbits call for a sample of stars that covers a wide range in Galactocentric radii at low latitudes and with precise distances, and that has consistent estimates of [Fe/H] and age. The APOGEE (Apache Point Observatory Galactic Evolution Experiment, Majewski et al. 2017) sample of red clump giants (Bovy et al. 2014; Alam et al. 2015) is, by design, very well suited for this purpose. Observing at near-infrared wavelengths for which dust is nearly transparent, the APOGEE spectrograph delivered spectra for giant stars with Galactocentric radii from ~5 to ~14 kpc, as illustrated in Figure 2. Stellar parameters and abundances for this sample (originally from APOGEE DR12, Alam et al. 2015) were re-derived using The Cannon (Ness et al. 2015). Importantly, consistent ages were derived by Ness et al. (2016), using the same data-driven approach to calibrate spectroscopic age estimates to asteroseismic data; the spectroscopic age signature of red clump giants resides in the C and N abundances (at given [Fe/H]), reflecting mass-dependent (and hence age-dependent) dredge-up (Masseron & Gilmore 2015; Martig et al. 2016). Uncertainties in metallicity are about 0.05–0.10 dex, and those in ages (log τ) are 0.2 dex.

Red clump giants are reliable standard candles, see for example Girardi (2016), with photometric distances precise to within 5%. The 3D position in the Galaxy can be obtained from these, assuming the Sun is at a Galactocentric distance R₀ = 8 kpc and height z₀ = 25 pc. Bovy et al. (2014) identified ~20,000 red clump giants in APOGEE with a contamination fraction between ~3% and 10% by red giant branch stars.
The above elements provide us with a set of about 20,000 data \([\text{[Fe/H]}, \tau, R]\) and their uncertainties. For our modeling at hand, it seems sensible to apply a few more cuts to the sample. As we are interested in radial orbit migration as the possibly dominant orbit evolution process in the more quiescent phase of Galactic disk evolution (the last \(\sim8\) Gyr), we eliminate stars with high \([\alpha/\text{Fe}]/\), as illustrated by the gray dots in Figure 1. Additionally, we select stars well in the Galactic plane with altitude \(|z|<1\) kpc.

2.1. Sidestepping the Complex Spatial Selection Function

Given a set of data \([\text{[Fe/H]}, \tau, R]\), the obvious approach would be to construct a parameterized model to predict \(p([\text{[Fe/H]}, \tau, R]|p_m)\), where \(p_m\) are various model parameters describing the possible evolution histories of the Galactic disk (see Section 3) including radial orbit migration. But such direct comparison of model predictions to data requires us to account for the selection function: the probability that any star in the sky enters the survey catalog, given its physical properties. In the case of the APOGEE data at hand, the selection function is (inevitably) complex: stars must (1) belong to the red clump population, and (2) be in the pointing directions of APOGEE and have color and magnitudes to fit the APOGEE survey selection.

First, the number of red clump stars per unit mass of a stellar population is a strong function of age (Girardi & Salaris 2001). Bovy et al. (2014) have calculated with stellar evolutionary models the relative fraction of stars that are in this evolutionary stage as a function of their age for a flat SFH (this is illustrated by the dashed line in Figure 3).

Second, the APOGEE spatial selection function was shown to be a complex function (Bovy et al. 2016). The consequences of spatial distribution for the radial distribution of the APOGEE red clump sample used in our study are visible in Figure 2, where there is, for example, an overdensity of stars observed at the position of the Sun \((\sim8\) kpc). Therefore, we opt not to model this complex distribution. Instead, we work only with the age–metallicity distribution given stellar radii, \(p([\text{[Fe/H]}, \tau]|R)\). The advantage is that the model construction is technically simpler and more robust; but not all of the information contained in the data is used. In particular, we are not exploiting the present-day radial distribution of stars in the Milky Way disk, \(p(R)\).

3. A Model for the Galactic Disk Evolution, Including Radial Orbit Migration

We now lay out a simple parameterized model for the age–abundance–radius structure of the Galactic disk of low-\([\alpha/\text{Fe}]/\) stars. This model specifies different aspects of formation and evolution: when and at what metallicity stars were born, with which radial profile they were born, and how much they migrated, ultimately predicting the joint distribution \(p([\text{[Fe/H]}, \tau, R]|p_m)\), which allows us to calculate the data likelihood for the APOGEE sample given any \(p_m\) and apply Bayes’ theorem to infer the posterior probability function for the model parameters, given the data

\[
P_{\text{post}}(p_m|\{\text{[Fe/H]}, \tau, R\}) = p_C([\text{[Fe/H]}, \tau]|R, p_m) \times p_{\text{pr}}(p_m)/p_{\text{pr}}([\text{[Fe/H]}, \tau]),
\]

with \(p_{\text{post}}\) the posterior probability density function of the model parameters, \(p_C(p_m)\) our prior knowledge on the model parameters, and \(p_{\text{pr}}([\text{[Fe/H]}, \tau])\) the evidence. Such an inference operation requires us to account for data uncertainties. We assume in the present study that the uncertainties in \(R\) and \([\text{[Fe/H]}]\) are negligible (red clump stars have uncertainties of \(\sim5\%) and \(\sim0.05–0.1\) dex in distance and \([\text{[Fe/H]}]\) respectively). We presume that the uncertainties in \(\log \tau\) dominate and are described by a Gaussian with \(\sigma_{\log \tau} = 0.2\) dex (Ness et al. 2016).

3.1. Basic Model Assumptions

In order to describe the evolution of the Galactic disk with a parameterized model, we made several assumptions on the nature and strength of the processes at play. Obviously, the astrophysical inferences from our modeling are only as valid as the assumptions.

1. We assume that the metallicity \([\text{[Fe/H]}]\) of the interstellar medium has negligible variations with azimuth; this is perhaps the strongest assumption involved in the modeling. This assumption is supported by observations of young stars in the Galaxy (e.g., Luck et al. 2006; Przybilla et al. 2008; Genovali et al. 2014). Azimuthal variations in rapidly produced \(\alpha\)-elements have been claimed (Sánchez-Menguiano et al. 2016; Ho et al. 2017), but those in \([\text{[Fe/H]}]\) should be less strong.

2. We do not treat or model explicitly the vertical structure of the Milky Way disk, though there are of course vertical (populations) gradients in it (e.g., Ness et al. 2016), which are affected by radial migration (e.g., Kawata et al.
Such gradients will only appear as scatter at a given radius, interpreted as a consequence of radial orbit migration in the present work.

3. Secular evolution has been the dominant effect of orbit evolution for the past 8 Gyr, which implicitly assumes that the Milky Way had a relatively quiescent life for the past 8 Gyr. We therefore restrict our analysis to stars younger than 8 Gyr, neglecting possible recent external interactions that could be responsible for shaping the Milky Way disk.

It follows from these assumptions that we model radial orbit migration as the only mechanism responsible for the scatter in age–metallicity at given radius. In this work, we interpret all scatter with radius migration, and therefore provide an upper limit on its strength, which should result from radial orbit migration, and therefore provide an upper limit on its strength, which should result from radial orbit migration, and therefore provide an upper limit on its strength.

3.2. Functional Forms for the Different Aspects of the Model

In the following, we use the assumptions stated above and lay out our adopted functional forms for different aspects of the Galactic disk’s formation and evolution: the distributions of (1) the global star formation rate in the disk, (2) birth radii as a function of time, (3) birth metallicities at a given epoch and radius, and (4) the strength of radial orbit migration. We summarize these functional forms in Table 1. These functions are combined to produce Equation (1), from which we can sample the posterior probability distribution function of the parameters $p_m$.

3.2.1. SFH and the Age Distribution of Red Clump Stars

We parameterize the possible age distribution of red clump stars by

$$p(\tau | p_m) \equiv c_1 \text{SFH}(\tau, p_m) f_{\text{RC}}(\tau),$$

where SFH is the SFH of the Milky Way thin disk, $f_{\text{RC}}$ is the relative mass of stars at the red clump stage, and the normalization requires

$$c_1^{-1} \equiv \int_0^{\tau_m} \text{SFH}(\tau, p_m) f_{\text{RC}}(\tau) d\tau.$$

The SFH of the Milky Way thin disk is thought to be extended in time (Bland-Hawthorn & Gerhard 2016) and is manifestly still ongoing. This motivates our choice (Mo et al. 2010) to conventionally parameterize the star formation rate in the Milky Way disk as a slowly decreasing exponential with time, for which we fit the exponential decay timescale $\tau_{\text{SFR}}$. This is a simplification of the model of Sanders & Binney (2015), who go further in detail and include thick-disk star formation. We write the SFH as

$$\text{SFH}(\tau, p_m) = \exp[-(\tau - \tau_m)/\tau_{\text{SFR}}],$$

where $\tau_m$ is the maximum disk age, set to 12 Gyr; $\tau_{\text{SFR}}$ is the model parameter setting the SFH, and is to be fit (i.e., it is an element of $p_m$).

The expected number of red clump stars per unit stellar mass, $f_{\text{RC}}(\tau)$, is a distinct function of age (and a weaker function of metallicity); it has been derived and parameterized in Equation (11) of Bovy et al. (2014). We illustrate $f_{\text{RC}}(\tau)$ in Figure 3 (dashed line) together with one particular choice of SFH (solid line).
Table 1
Summary of the Important Aspects of the Model

| Question Tackled by the Model | Describing Model Parameter | Parameter of $p_\alpha$ to Fit | Relevant Appearance in the Parameterized Equations | Model Aspect, Equation Reference |
|-------------------------------|----------------------------|-------------------------------|---------------------------------------------------|---------------------------------|
| When did stars form?          | Star formation timescale   | $\frac{SFR}{\text{Gyr}}$     | $\text{SFH}(\tau, p_\alpha) = \exp[-(\tau - \tau_0)/\tau_{SFR}]$ | Star formation history, Equation (4) |
| Where did stars form?         | Relative size of the disk at birth and present day | $\alpha_{R_{\exp}}$ | $p(R_0 \mid \tau, p_\alpha) \propto \exp(-R_0/R_{\exp})$ | Inside-out growth, Equation (5) |
| With what [Fe/H] were stars born? | Present-day radius of solar metallicity in ISM | $\frac{R_{\text{Fe/H}}}{\text{kpc}}$ | $\frac{\text{[Fe/H]}}{\text{dex kpc}^{-1}}$ | Chemical enrichment |
|                              | Metallicity gradient in the ISM | $\gamma_{[\text{Fe/H}]}$ | $f(\tau) = \left(1 - \frac{\tau}{\tau_0}\right)^{[\text{Fe/H}]}$ |               |
| How far did stars’ orbits migrate over the disk lifetime? | Diffusion scale length | $\frac{\sigma_{\text{KMS}}}{\text{kpc}}$ | $p(R \mid R_0, \tau, p_\alpha) \propto \exp\left(\frac{(R - R_0)^2}{2 \sigma_{\text{KMS}}^2 \tau/8 \text{ Gyr}}\right)$ | Diffusion in radius, radial migration, Equation (10) |

Figure 3. Model for the global star formation history and the age distribution of red clump stars. We assume that the global star formation history of the (low-\(\alpha\)) Galactic disk can be described (see Equation (4)) by a model family $\text{SFR} \propto \exp\left(-\tau/\tau_{SFR}\right)$, illustrated by the solid line for a star formation timescale $\tau_{SFR} = 6.8 \text{ Gyr}$. The dashed line shows the theoretically expected relative number of red clump stars per unit mass for a constant star formation history. The normalized product of these two functions gives the current age distribution of red clump stars.

3.2.2. Radial Birth Profile and Inside-out Growth

We presume that disk stars are born on near-circular orbits near the mid-plane of the disk. The sizes of their orbits are determined by the angular momentum of the gas from which they formed. We therefore need to parameterize the radial profile of the star-forming gas in the Galactic disk at any time. The Galactic disk is thought to build from inside out, as gas of first low and then higher angular momentum cools and falls into the potential of the dark matter halo (White & Frenk 1991; Mo et al. 1998; Muñoz-Mateos et al. 2007; Fraternali & Tomassetti 2012). This inside-out growth is thought to play a determining role in the metallicity profile of the gas and stars (Schönrich & McMillan 2017), so it is important to incorporate this aspect into our disk model. We parameterize the possible radial birth profile of stars at any given epoch as a decreasing exponential with Galactocentric radius, with a scale length $R_{\exp}$.

$$p(R_0 \mid \tau, p_\alpha) = \exp(-R_0/R_{\exp}(\tau))/R_{\exp}(\tau). \tag{5}$$

We then parameterize inside-out growth by allowing the scale length to increase (linearly) with time,

$$R_{\exp}(\tau) = 3 \text{ kpc} \left(1 - \alpha_{R_{\exp}}\frac{\tau}{8 \text{ Gyr}}\right). \tag{6}$$

The relative size of the disk today and at early times is set by the free parameter to be fit $\alpha_{R_{\exp}}$ (Equation (6)), bound to the interval [0, 1] with the current star-forming disk scale length set to $R_{\exp}(\tau = 0) = 3 \text{ kpc}$. Note that we do not attempt to model the radial profile of the disk beyond 8 Gyr ago, because we deem our secular evolution model inapplicable at such early epochs. The radial scale length of the stellar disk of the Milky Way is not well constrained (see Bland-Hawthorn & Gerhard 2016 for a review). It was shown that such a scale length varies with stellar populations (Bovy et al. 2012). We adopt here the suggested value for the younger stars (in the chemical sense: with low [\(\alpha/\text{Fe}\)]) in the disk of $\sim3$ kpc from Bovy et al. (2012), to model the present-day star-forming gas profile. The possible distributions of stars at birth 8 Gyr ago and today are shown in Figure 4 for a specific choice for $\alpha_{R_{\exp}}$.

3.2.3. Metallicity–Radius–Age Relation

We also need to specify with what [Fe/H] stars were born at time $\tau$ ago at Galactocentric radius $R_0$. At present, disk stars in the Milky Way are born with a tight relation between their birth radii and their metallicities. This is qualitatively seen in data: young subpopulations (e.g., Cepheids, Genovali et al. 2014) of a given [Fe/H] cover a small range of Galactic radii. The spreads in metallicity of open clusters were shown to be about 0.03 dex (Bovy 2016; Ness et al. 2018; Ting et al. 2018). This motivates our assumption that the metallicity profile of the interstellar medium (and hence the metallicity that stars have at birth) can be modeled at any time through a tight relation.
Following the general reasoning of Sanders & Binney (2015), who approximate the output of a simulation of Schönrich & Binney (2009a), we describe the metallicity profile in the star-forming gas disk as the product of a radial profile and a term describing the time dependence of chemical enrichment:

\[
\frac{\text{Fe/H}}{\text{Fe/H}} = F_m - (F_m + \nabla[\text{Fe/H}] R_{\text{now}}^{\text{now}})[\tau])f(\tau) + \nabla[\text{Fe/H}].R. 
\]

(7)

Here, \(F_m\) represents the metallicity of the gas at the center of the disk at \(\tau = 12\) Gyr. We assume it to be fixed at \(-1\) dex, a choice supported by the age–metallicity relation of globular clusters at low Galactocentric radii (e.g., Kruisjes & Minchev et al. 2018). The parameter \(\nabla[\text{Fe/H}]/\text{Fe/H}\) is the metallicity gradient of the interstellar medium in dex kpc\(^{-1}\); it is negative, and is to be fit with the other parameters in \(p_m\). We do not specify the physical mechanisms behind the origin and maintenance of the (birth) gas metallicity gradient. Simulations show such a gradient to be a robust prediction (Grand et al. 2015). It is presumed here to be constant in radius and time, although the metallicity gradient may have evolved over the lifetime of the Galactic disk (Grisoni et al. 2018; Minchev et al. 2018). We discuss the possible impact of this assumed form in Section 4, where different expressions are tested. We expect young stars across the Galactic disk to provide the strongest constraints on this model parameter. Then, \(R_{\text{now}}^{\text{now}}[\tau])\) is the radius at which the present-day (birth) metallicity is solar (\(\text{Fe/H} = 0\)). We expect this parameter to be constrained by the current radii of the youngest red clump stars of solar metallicity. We assume the time dependence of the enrichment to follow the power law

\[
f(\tau) = \left(1 - \frac{\tau}{\tau_m}\right)^{\frac{\text{Fe/H}}{\text{Fe/H}}} \quad \text{with the parameter (to fit) } \gamma_{\text{Fe/H}}, \text{controlling the time dependence of chemical enrichment: linear if } \gamma_{\text{Fe/H}} = 1 \text{, and faster at early times if } \gamma_{\text{Fe/H}} < 1. \text{ Overall, this encapsulates that there is a metallicity gradient in the interstellar medium in the disk, with the disk depleted over time, as illustrated in Figure 5.}

With this parameterization, we now assume that there is an exact birth metallicity at a given stellar age \(\tau\) and birth radius \(R_0\), i.e., \(p(\text{Fe/H} | R_0, \tau, p_m)\) is a \(\delta\)-function at the value of \(\text{Fe/H}\) that satisfies Equation (7). To study radial orbit migration (\(R - R_0\)), we use this functional form of the metallicity profile of the interstellar medium as a function of time to find stellar birth radii, given stellar metallicities and stellar ages. In other words, we invert the age–metallicity relation in Equation (7) and construct the inverse relation \(R_0(\text{Fe/H}, \tau)\), which is a \(\delta\)-function in \(R_0\), centered on

\[
R_0 = \frac{\text{Fe/H} - F_m + (F_m + \nabla[\text{Fe/H}] R_{\text{now}}^{\text{now}})[\tau])f(\tau)}{\nabla[\text{Fe/H}].} .
\]

(8)

Such inversion requires

\[\tau \leq \tau_{\text{max}}(\text{Fe/H}, p_m),\]

for \(R_0\) to be positive. Here, \(\tau_{\text{max}}(\text{Fe/H}, p_m)\) is the maximum stellar age deemed physical by our model evaluated for \(p_m\), given a metallicity \(\text{Fe/H}\). Solving the inequality \(R_0(\text{Fe/H}, \tau) > 0\) for \(\tau\) at a given metallicity in Equation (8),

\[
\tau_{\text{max}}(\text{Fe/H}, p_m) = \tau_m \left(1 - \frac{\text{Fe/H} - F_m}{F_m - \nabla[\text{Fe/H}] R_{\text{now}}^{\text{now}}[\text{Fe/H}] = 0}\right)^{1/\gamma_{\text{Fe/H}}}.
\]

(9)

where we used the assumption that the metallicity gradient in the star-forming gas is always negative; \(\text{Fe/H}\) decreases outward. This inequality can be visualized in Figure 5: combinations of \(\text{Fe/H}\) and \(\tau\) above the 0 kpc line are deemed unphysical. This condition, which is a function of \(p_m\), will therefore provide strong constraints on the parameters to fit in
the age–metallicity—birth radius relation, in particular on $\gamma_{[\text{Fe/H}]}$.

### 3.2.4. Radial Orbit Migration

We now introduce the central part of our model, radial orbit migration, in order to quantify how far stars move from their birth radii as a function of their age. Theoretical and observational arguments suggest that radial orbit migration can be modeled as a diffusion process. Sellwood & Binney (2002) first demonstrated that non-axisymmetric structures such as spiral arms can, through repeated and transient torques on stars co-rotating with them, induce large changes in their angular momenta. Further simulations confirmed this diffusion aspect of radial migration (Schönrich & Binney 2009a; Brunetti et al. 2011). Qualitatively, data show that at a fixed metallicity, a spread in stellar radii increases with stellar ages. This is qualitatively evident in the different [Fe/H]–$R$ spread between the two panels in Figure 2. Motivated by these arguments, we follow Sanders & Binney (2015) and adapt their parameterization to a Galactocentric radius coordinate. In its simplest form, a solution to the diffusion equation in radius gives the following probability for a star to be currently at a Galactocentric radius $R$, given that it was born at $R_0$ a time $\tau$ ago:

$$p(R \mid R_0, \tau, p_m) = c_3 \exp \left( -\frac{(R - R_0)^2}{2 \sigma^2_{RMS} \tau/8 \text{ Gyr}} \right).$$  \hspace{1cm} (10)

where $\sigma_{RMS}$, the radial orbit migration strength (our main astrophysical goal, to fit), represents the extent of radial orbit migration for a star after 8 Gyr (the width of the Gaussian function in Equation (10) at age $\tau = 8$ Gyr). As its age increases, the probability for a star to be on a different orbit than its birth orbit increases, because it has had more time to migrate radially. An illustration of the radial spread of different orbits with, for example, $\sigma_{RMS} = 3.6$ kpc is shown in Figure 6, where the distributions are modulated by the radial birth profile across the disk. Finally, the normalization constant $c_3$ satisfies

$$c_3^{-1} = \sigma_{RMS} \sqrt{\frac{1}{8 \text{ Gyr}}} \frac{\tau}{\sigma_{RMS}} \left( \text{erf} \left( \frac{R}{\sigma_{RMS} \sqrt{2 \sqrt{\tau/8 \text{ Gyr}}} + 1} \right) \right),$$

to ensure that stars do not migrate to negative radii. This parameterization implies a (presumably unphysical) net motion outward, which has a very limited impact on the results as discussed in Section 5.

In this most restricted form, the only free parameter describing radial orbit migration is $\sigma_{RMS}$.

### 3.3. Constructing the Data Likelihood Function

We use the above elements to build a parameterized model that predicts the joint distribution $p([\text{Fe/H}] \mid R, p_m)$ at a given Galactocentric radius $R$ for the low-$\alpha$ Galactic disk. The model consists of two distinct components. The first component is built from the aspects described in the above section for the disk younger than 8 Gyr and is aimed to be informative about the evolution of the Milky Way disk. The model laid out above may not apply to the early phases of the evolution of our Galaxy. As this model for the old disk component is a "nuisance" aspect of the current work, the model is fairly simple and uninformative. However, because age uncertainties grow with age (0.2 dex), one cannot assign stars to a particular component of the model based on their most likely age. We must marginalize over age uncertainties, in Equation (24). As the red clump sample is a fairly young population (with an age distribution that peaks around 2 Gyr, see Figure 3), the total likelihood will be dominated by terms from younger stars.
We start by using Bayes’ rule on

\[
p([\text{Fe/H}], \tau \mid R, \tau_m) = \frac{p([\text{Fe/H}], \tau, R \mid \tau_m) p(R \mid \tau_m)}{p(R \mid \tau_m)} = \frac{p(\tau \mid \tau_m) p([\text{Fe/H}], R \mid \tau, \tau_m)}{p(R \mid \tau_m)}.
\]  

(11)

We will now construct the numerator and the denominator as two distinct models, summarized respectively in Figures 7 and 8. The numerator is the joint distribution of all data given the model parameters \( p([\text{Fe/H}], \tau, R \mid \tau_m) \). But as we do not model the spatial selection function of APOGEE, we should keep the Galactocentric radius \( R \) as given, and hence the ratio with \( p(R \mid \tau_m) \). The first term in the numerator of Equation (11) is the age distribution of red clump stars, given in Equation (2). The second term in the numerator and the denominator are constructed below. We separate stars younger and older than 8 Gyr into two terms, \( p_y \) (young) and \( p_o \) (old), because we believe that the model of secular evolution we have laid out is only applicable to \( \tau < 8 \) Gyr. But, in the presence of significant age uncertainties, we must acknowledge the existence of older stars in the Galactic disk without making assumptions on their possible birth radii, enrichment history, and subsequent radial orbit migration. For those we specify a less informative metallicity–radius distribution

\[
p([\text{Fe/H}], R \mid \tau, \tau_m) = \begin{cases} 
  p_y([\text{Fe/H}], R \mid \tau, \tau_m) & \tau < 8 \text{ Gyr} \\
  p_o([\text{Fe/H}], R \mid \tau, \tau_m) & \tau > 8 \text{ Gyr},
\end{cases}
\]

(12)

where the young term \( p_y([\text{Fe/H}], R \mid \tau, \tau_m) \) is derived by marginalizing the joint distribution of metallicities and birth radii at given time (the age–metallicity–radius relation) \( p([\text{Fe/H}], R_0 \mid \tau, \tau_m) \) over stellar birth radii \( R_0 \). Using \( p([\text{Fe/H}] \mid R_0, \tau, \tau_m) = p([\text{Fe/H}] \mid R_0, \tau, \tau_m) \) i.e., that the metallicity of stars born at a given birth radius \( R_0 \) and time \( \tau \) does not depend on their present-day position \( R \), we marginalize

\[
p_y([\text{Fe/H}], R \mid \tau, \tau_m) = \int_0^\infty p(R \mid R_0, \tau, \tau_m) dR_0,
\]

with the three terms in the integral being the different aspects of the model. The first two terms are radial orbit migration (Equation (10)) and the radial birth profile (Equation (5)), respectively. The third term is the metallicity at birth (a Dirac function due to the tight relation Equation (7), or equivalently Equation (8)), which we express as a probability distribution function for \( R_0 \): \( p([\text{Fe/H}] \mid R_0, \tau, \tau_m) = \delta(R_0 - \bar{R}_0)|R_0\rangle \), with \( \bar{R}_0 \) the analytical solution for the tight relation, defined in

\[\theta_{[\text{Fe/H}]}, \alpha_{\text{Rexp}}, \sigma_{\text{RMS}}\]

\[R_0, i \quad \tau_i \quad \alpha_{\text{Rexp}} \quad \sigma_{\text{RMS}}\]

\[\tau_S F R\]

\[R_0, i \quad \tau_i' \quad \alpha_{\text{Rexp}} \quad \sigma_{\text{RMS}}\]

\[i = 1, \ldots, N\]

Figure 7. Probabilistic graphical model for the joint distribution \( p(\{[\text{Fe/H}], \tau, R \mid \tau_m\}) \) for the <8 Gyr (thin) disk of the Milky Way. Our likelihood is the ratio between this model and the model for \( p(R_0 \mid \tau_m) \) presented in Figure 8. The observed quantities are in gray circles and model parameters are in white circles. The present-day Galactocentric radius \( R_0 \) is in a dashed circle as a reminder that the final likelihood does not predict the present-day observed radial distribution of red clump stars. The filled black dot represents a fixed quantity, here the assumed age errors, from Ness et al. (2016). The \( \theta_{[\text{Fe/H}]} \) circle represents the three enrichment parameters \( \gamma_{[\text{Fe/H}]} \) and \( \gamma_{[\text{Fe/H}]} \). \( R_0 \) is birth radii, \( \tau_i \) are the true ages, and \( \tau_i \) are the measured ages. We infer the parameters that are outside the box; the others are marginalized out.

Figure 8. Sub-model from the model shown in Figure 7, which is used as the denominator \( p(R_0 \mid \tau_m) \) in the ratio of probabilities used as the likelihood in this inference (see Equation (11) and the related text). The nomenclature is the same as in Figure 7.
Equation (8), and \( R_0' \) the Jacobian term relating the distribution in \([\text{Fe/H}]\) and \( \tilde{R}_0' \) \([\text{Fe/H}], \tau \). \( R_0' \) is defined as the inverse of the metallicity gradient

\[
R_0' \equiv \frac{dR_0}{d[\text{Fe/H}]} = \frac{1}{\nabla[\text{Fe/H}]}. 
\]

The Dirac function makes the computation of the integral trivial, simply evaluating the integrand at \( R_0 = \tilde{R}_0 \) defined in Equation (8):

\[
p_y([\text{Fe/H}], R | \tau, p_m) = p_y(R | \tau, p_m) \times p(R | \tau, p_m). \tag{14}
\]

All elements are spelled out to be recast in \( p_y \) of Equation (12), and we can now do the same exercise with \( p_y \). Guided by the data, we presume that the old term \( p_y([\text{Fe/H}], R | \tau, p_m) \) of Equation (12) can be well described by a Gaussian distribution in metallicity and a decreasing exponential in radius. We deem this approximation sufficient for the purpose at hand: this old component is uninformative on radial orbit migration (our interest) and is constructed in order to allow us to treat the large age uncertainties of the data appropriately: with important age uncertainties, we expect a significant number of stars younger than 8 Gyr to have measured ages greater than 8 Gyr, and vice versa. We define

\[
p_y([\text{Fe/H}], R | \tau, p_m) = \frac{1}{R_{\text{old}}} \exp(-R/R_{\text{old}}) \tag{16}
\]

with a scale length \( R_{\text{old}} \), and similarly the metallicity distribution common to all old stars,

\[
p_y([\text{Fe/H}] | \tau, p_m) = \mathcal{N}([\text{Fe/H}], [\text{Fe/H}], \text{std}([\text{Fe/H}])). \tag{17}
\]

The model parameters of \( p_m \) here are the old stars’ scale length \( R_{\text{old}}, \) their mean metallicity \([\text{Fe/H}]\), and their metallicity dispersion \( \text{std}([\text{Fe/H}])\). Now, Equation (12) can be fully written and reintegrated into Equation (11).

Finally, we move on to the denominator in Equation (11), which is the predicted radial distribution of stars, and can be calculated over time from

\[
p(R | p_m) = \int_{0}^{\tau} p(R | \tau, p_m) p(\tau | p_m) d\tau \tag{18}
\]

with the radial distribution of stars being determined by radial orbit migration. Since we presume that conditions at birth are known only for \( \tau \leq 8 \) Gyr, we separate out older stars again:

\[
p(R | \tau, p_m) = \begin{cases} 
   p_y(R | \tau, p_m) & \tau \leq 8 \text{ Gyr} \\
   \tilde{p}_y(R | \tau, p_m) & \sigma_{\text{obs}}^2 < R < 8 \text{ Gyr}.
\end{cases} \tag{19}
\]

The old component \( \tilde{p}_y(R | \tau, p_m) \) is the exponential profile introduced above in Equation (16) with a scale length \( R_{\text{old}} \). The radial distribution of stars of age \( \tau \leq 8 \) Gyr is given by the model described in the above subsection. It is determined by the birth radii of stars of age \( \tau \), and by their further radial orbit migration after a time \( \tau \):

\[
p_y(R | \tau, p_m) = \int_{0}^{R_0} p(R | R_0, \tau, p_m) p(R_0 | \tau, p_m) dR_0. \tag{20}
\]

When this expression is inserted back into Equation (18), it leads to a double integral function (extracted in Equation (21)) of the four variables \((R, \tau, \alpha_{\text{obs}}, \sigma_{\text{rms}})\) .

The evaluation of such a function is computationally expensive: a single evaluation takes of the order of a second, making Markov Chain Monte Carlo (MCMC) sampling on thousands of stars and tens of thousands of MCMC steps rather slow. We therefore precompute the integral

\[
\int_{0}^{R_0} \int_{0}^{\infty} p(R | R_0, \tau, p_m) p(R_0 | \tau, p_m) p(\tau | p_m) dR_0 d\tau \tag{21}
\]

on a large number of points in the 4D space of \( x = (R, \tau, \alpha_{\text{obs}}, \sigma_{\text{rms}}) \) to interpolate it with precision 0.4% using a family of highly flexible nonlinear functions,

\[
f(x) = W_0 \tanh(W_1 \tan(W_2 x + b_2) + b_1) + b_0, \tag{22}
\]

where \( W_i \) and \( b_i \) are matrices of coefficients found by minimizing the difference between Equations (21) and (22) on the pre-computed points, using a regression gradient descent algorithm. The interpolation intervals in the parameter space are chosen to be large enough for our analysis: 3 kpc \(< R < 15 \) kpc, 4 Gyr \(< \tau_{\text{obs}} < 14 \) Gyr, 0 \(< \alpha_{\text{obs}} < 1 \) and 2 kpc \(< \sigma_{\text{rms}} < 8 \) kpc. Possible error propagations during the sum of log-likelihood over all data (Equation (23) just below) are discussed in Section 4.

We can now recast all the elements spelled out above into Equation (11) and build the likelihood function.

The overall likelihood of all the data is given by

\[
\ln p_d([\text{Fe/H}], \tau | \{R_i\}, p_m) = \sum_{i=1}^{N_d} \ln p_d([\text{Fe/H}], \tau | \{R_i\}, p_m), \tag{23}
\]

where \( p_d([\text{Fe/H}], \tau | \{R_i\}, p_m) \) is the likelihood of the data on one object, given the model. Our data \([\text{Fe/H}], \tau, R \) also have uncertainties, dominated by \( \tau \) (we neglect those in metallicity and radius as a first approximation), and therefore we need to marginalize over these uncertainties. This marginalization smoothes out the effects of our 8 Gyr cut: stars will have a non-zero contribution to each component, and this contribution is weighted by its possible age distribution \( p_{\text{age}}(\tau | \tau) \). Therefore, even if an old star had its age underestimated, instead of fully constraining our disk evolution model, it will contribute to both the young and the old component, which prevents net likelihood values of zero being given to the informative (young) model. This is analogous to the role of a model of outliers. Of course, such a non-zero contribution of old stars to the young likelihood component may have an effect on our inference, especially if their enrichment history differs significantly from that of younger stars. However, this effect should be limited by the facts that (1) red clump stars are mainly a young population, reflecting the recent history of the Milky Way disk, and (2) we are
considering only low-\( \alpha \) stars at \( |z| < 1 \) kpc, filtering out a large fraction of the old stars of the red clump sample.

\[
p_{\ell}([\text{Fe/H}, \tau] \mid (R), p_m) = \int p_{\text{obs}}(\tau \mid \tau) p([\text{Fe/H}, \tau] \mid R) d\tau,
\]

where \( \tau_i^{\text{obs}} \) and \( R_i \) are the measured age, metallicity, and Galactocentric radius (the values in our red clump catalog) and \( \tau \) is the potentially true age of the star. Here, \( p_{\text{obs}}(\tau \mid \tau) \) is the error distribution in age: the probability of measuring an age \( \tau_i \) given that the possible true stellar age is \( \tau \) and measurement uncertainties. This distribution is a Gaussian function in log space, such that for \( a = \log_{10}(\tau), a_i = \log_{10}(\tau_i), \) the error in age \( \sigma_a = 0.2 \) dex, we have \( p_{\text{obs}}(a_i \mid a, \sigma_a) = \mathcal{N}(a, a, \sigma_a^2) \). As this noise model may underestimate the errors of very young stars, we apply a different model for stars younger than 0.5 Gyr, where errors are Gaussian in linear space with a standard deviation of \( \sigma_a = 200 \) Myr, \( p_{\text{obs}}(\tau_i \mid \tau, \sigma_a) = \mathcal{N}(\tau_i, \tau, \sigma_a^2) \). Both noise models are normalized to physically plausible age ranges: between the theoretical minimum red clump age (Figure 3) and the age of the universe. Integral (24) gets evaluated separately for each data point (given each \( p_m \)). In practice, we do not need to compute this integral over all the terms in the expression of the distribution \( p([\text{Fe/H}, \tau] \mid R) \) (Equation (11)), but only its numerator because the denominator does not depend on age \( \tau \) (hence, we do not propagate the interpolation errors of the term in Equation (21) along this marginalization over age uncertainties).

### 3.4. Sampling the Parameter PDF

We apply Bayes’ theorem to the likelihood function constructed with the analytical disk evolution model described in Section 3, and APOGEE red clump giants, to express a posterior probability distribution of the global efficiency of radial orbit migration.

The posterior probability distribution of the model parameters is given by

\[
p_{p_\ell}(p_m \mid ([\text{Fe/H}, \tau, R]) = \frac{p_{\ell}([\text{Fe/H}, \tau] \mid (R), p_m) p_{p_m}(p_m)}{p_{p_{\ell}}([\text{Fe/H}, \tau])},
\]

where we presume \( p_{p_{\ell}}([\text{Fe/H}, \tau, R]) \), the evidence term that does not depend on the model parameters, to be a constant. We sample the vector of the nine free parameters \( p_m \equiv \{ \text{SFR}, \alpha_{\text{comp}}, R_{\text{low}}^{\text{FWHM}} = 0, \text{[Fe/H]}, \nabla [\text{Fe/H}], \sigma_{\text{RMS}}, [\text{Fe/H}], \text{std}([\text{Fe/H}], R_{\text{old}}) \} \) by means of Equation (1), and then marginalize over all nuisance parameters \( \{ \text{SFR}, \alpha_{\text{comp}}, R_{\text{low}}^{\text{FWHM}} = 0, \text{[Fe/H]}, \nabla [\text{Fe/H}], \sigma_{\text{RMS}}, [\text{Fe/H}], \text{std}([\text{Fe/H}], R_{\text{old}}) \} \) to extract a posterior distribution for radial orbit migration \( p_{p_\ell}^{\text{RMS}}(\sigma_{\text{RMS}} \mid ([\text{Fe/H}]), \tau, R) \). This is done using the MCMC sampler package Emcee (Foreman-Mackey et al. 2013). In practice, we first perform a maximum likelihood estimation (MLE) of the parameters using the Nelder–Mead method (Nelder & Mead 1965), and sample initial walker positions for 20 Markov chains within small intervals around the best fit results. To compromise between the precision of our results and computational time, we perform several fits on different subsets of stars. For each fit, we use a subset of 1500 stars from our low-\( \alpha /\text{Fe} \) sample, after having selected further those well in the Galactic disk with \( |z| < 1 \) kpc. Each chain is sampled with 7000 iterations. We then marginalize over the nuisance parameters to infer the radial orbit migration strength \( \sigma_{\text{RMS}} \). Our prior on \( \sigma_{\text{RMS}} \) is set by the restricted space where the interpolation of Equation (21) is valid: 2 kpc < \( \sigma_{\text{RMS}} < 8 \) kpc. Other priors are \( 0 < \alpha_{\text{Rexp}} < 1, 3 \) Gyr < \( \tau_{\text{SFR}} < 12 \) Gyr. The priors on other model parameters are also flat; we only constrain distances and durations to be positive. As can be seen in Figure 9, the posteriors are tightly constrained even though we had fairly broad priors.

### 4. Results

We now summarize the results obtained from fitting our disk evolution model to the low-\( \alpha \) APOGEE red clump data, described in Section 2. The maximum likelihood estimates (Equation (23)) for the model parameters are presented in Table 2. All 20 chains of the MCMC procedure converged with 7000 iterations on subsets of 1500 stars out of the 17,500 low-\( \alpha \) available stars of the sample. We show the posterior distributions for the parameters of immediate interest in Figure 9; this shows that all parameters are well constrained by the data, with some covariances but no degeneracies. The full version of the figure, which shows the exploration of the whole parameter space including all nuisance parameters, can be found in Figure 16 in the Appendix.

We first focus on quantifying radial orbit migration, show the model calculation for the best fit parameters, and then comment briefly on the other parameters.

#### 4.1. Radial Orbit Migration

Figure 9 shows that the inferred strength of radial orbit migration is very well constrained. Marginalizing the posterior distribution over the nuisance parameters gives an estimate of \( \sigma_{\text{RMS}} \) of about 3.6 ± 0.1 kpc (see Figure 9). This represents the length scale over which the oldest stars (8 Gyr) have spread around their birth radii. This spread in radius could be caused by either churning or blurring. But we know from the stellar radial velocity dispersion in the solar neighborhood that orbital eccentricities (blurring) have a radial amplitude of about 1 kpc. Therefore, we infer that churning must be the dominant mechanism in explaining the measured value of \( \sigma_{\text{RMS}} \). In Figure 10, our estimate of radial orbit migration \( \sigma (\tau) = 3.6 \) kpc/\( \sqrt{\tau} /8 \) Gyr is illustrated by sampling from the posterior distribution, i.e., the MCMC chains in Figure 9). This result quantifies that the present-day radius is a poor proxy for the birth radius, compared to the metallicity at a given age.

Mathematically, \( \sigma_{\text{RMS}} \) quantifies the distance between the present Galactocentric radius of a star and the birth radius expected from the global model fit. Whenever the quantity of interest is a “scatter” one must explore the extent to which it is attributable to other model shortcomings. We have therefore explored model variants and have found that estimate of this radial orbit migration strength is rather robust. We exercised MCMC estimates holding other model parameters fixed to diverse values, leaving the estimate of radial orbit migration - strength robust.
4.2. Other Parameters

In addition to the radial orbit migration strength, the model also constrains all other aspects: SFH, inside-out growth, and the enrichment history of the Galactic disk. While these are mere nuisance parameters when constraining radial orbit migration strength, they are informative about the evolution of the Galactic disk over the last 8 Gyr.

**SFH:** The data favor a star formation timescale $\tau_{\text{SFR}} = 6 \pm 1$ Gyr for the Galactic low-[α/Fe] disk. This value seems rather low given prior expectations of extended star formation in the thin disk. We find that this estimate depends strongly on the assumed form of the age distribution at young ages ($<1$ Gyr), and it is sensitive to the details of the selection: e.g., we find a longer star formation timescale if we select stars
Radial orbit migration strength inferred in this study with respect to stellar ages.

Figure 10. Radial orbit migration strength inferred in this study with respect to stellar ages.

| $p_{\text{MLE}}$  | Best fit | Description              |
|-------------------|----------|--------------------------|
| $\tau_{\text{SFR}}/\text{Gyr}$ | 6.8      | Star formation timescale |
| $\alpha_{R_{\text{exp}}}$ | 0.3      | Inside-out growth         |
| $R_{\text{exp}}^{\text{now}}/\text{kpc}$ | 8.7      | $R[\text{Fe/H}] = 0, \tau = 0$ |
| $\gamma$ | 0.3      | Enrichment power index    |
| $\nabla[\text{Fe/H}]/\text{dex kpc}^{-1}$ | $-0.075$ | Gradient of $[\text{Fe/H}]_{\text{ISM}}$ |
| $R_{\text{old}}/\text{kpc}$ | 2.5      | Scale length of old disk  |
| $[\text{Fe/H}]/\text{dex}$ | $-0.05$  | Mean metallicity, $\tau > 8$ Gyr |
| std([Fe/H])/dex | 0.15     | std metallicity, $\tau > 8$ Gyr |

Table 2

Best Fit MLE Parameters

with $|z| < 1$ kpc rather than with $|z| < 1.5$ kpc; this should be expected because the proportion of young stars is larger near the mid-plane. Given that the age distribution varies with Galactocentric radius and height above the plane, the uneven APOGEE pointings could induce some $\tau_{\text{SFR}}$ bias for which we do not correct. Additionally, this estimate is degenerate with the old stars’ scale-length parameter $R_{\text{old}}$ (Figure 16 in the Appendix). This is due to the spatial selection function being limited to 5 kpc from the center of the disk: predicting a fast star formation (many old stars) with a small scale length is, according to this model, roughly equivalent to predicting a slow star formation (fewer old stars) but one that is more extended in the disk, preserving the overall observed ratio of young to old stars (the range of Galactocentric radius is 5–14 kpc: we do not see an old stellar population when it is well concentrated in the inner disk). This is because these two scenarios will predict the same number of old stars in the observed regions of the Galactic disk. However, even if our estimate of $\tau_{\text{SFR}}$ is questionable, we note that (1) this does not seem to affect our estimate of radial orbit migration strength, and (2) the observed age distribution of red clump stars is well reproduced, as illustrated in Figure 11, which shows a comparison of the observed age distribution of red clump stars to the one predicted by the model (the details of this procedure are described in Section 4.3.2).

Inside-out growth: $\alpha_{R_{\text{exp}}}$—The growth (i.e., star formation) of the Galactic disk was modeled by the scale-length parameter

$$R_{\text{exp}}(\tau) = 3 \text{kpc} \left(1 - \alpha_{R_{\text{exp}}} \frac{\tau}{8 \text{Gyr}}\right)$$

of newborn stars. We find $\alpha_{R_{\text{exp}}} = 0.42 \pm 0.09$. This implies that the disk was about 40% smaller 8 Gyr ago. This is consistent with observations of high-redshift disk galaxies (e.g., van Dokkum et al. 2013). However, we report that the estimate for this parameter was very sensitive to the assumed functional form for the metallicity profile combined with the age distribution, with covariances with $\tau_{\text{SFR}}$.

Metallicity profile and enrichment history: $[R_{\text{Fe/H}}^{\text{now}}] = 0$, $\gamma[\text{Fe/H}]_{\text{ISM}}$ —The metallicity profile of the cold gas in the disk is described in our model by a simple straight line in radius with a negative gradient. The two model parameters that characterize the metallicity profile are $R_{\text{Fe/H}}^{\text{now}}$, the Galactocentric radius at which the metallicity of the star-forming gas is solar, corresponding to an arbitrary zero-point, and $\gamma[\text{Fe/H}]_{\text{ISM}}$: the present-day metallicity gradient at $R_{\text{Fe/H}}^{\text{now}}$. As these are two “present-day” properties, the youngest stars of our sample are expected to provide the strongest constraints on these parameters. We find the radius of solar metallicity to be about $R_{\text{Fe/H}}^{\text{now}} = 8.8 \pm 0.2$ kpc. The metallicity gradient $\nabla[\text{Fe/H}]$ is found to be $-0.075 \pm 0.002$ dex kpc$^{-1}$. The values of these two parameters are consistent with the left panel of Figure 2, in which we plot the metallicity profile of the young red clump stars. The densest region for $[\text{Fe/H}] = 0$ dex is close to 8 kpc. We note that Sanders & Binney (2015) find different results with their model for the Geneva–Copenhagen Survey data (Nordström et al. 2004), with a radius of solar metallicity of 7.37 kpc and a shallower metallicity gradient of $-0.064$ dex kpc$^{-1}$, and Genovali et al. (2014) measure a gradient of $-0.060 \pm 0.002$ dex kpc$^{-1}$. More recently, Anders et al. (2017) measured the stellar metallicity gradients for red giants in different stellar age bins, and found about $-0.058 \pm 0.008$ dex kpc$^{-1}$ for stars younger than 1 Gyr.

The enrichment history at any radius of the disk is described in our model by a power law of time with index $\gamma[\text{Fe/H}]$. The best MCMC value is $\gamma[\text{Fe/H}] = 0.36 \pm 0.04$. The metallicity of the interstellar medium is plotted with respect to look-back time in Figure 5 (using the MLE results). This result is different from (semi)analytic models used previously in the literature (Schönrich & Binney 2009a, 2009b; Sanders & Binney 2015),
where the enrichment of the interstellar medium generally increases faster at early times and is almost flat at late epochs. Here, we find that the gas metallicity grows continuously at all radii up to the present day. This is, however, very much consistent with Milky Way-like simulations (Grand et al. 2018), and further tests with different forms of metallicity profile would be interesting.

The nuisance model for the disk before 8 Gyr ago—We built a less informative “nuisance” model for the Milky Way disk older than 8 Gyr to avoid sharp age cuts. But these stars contain information on the SFH of the Milky Way: in essence, they help to constrain the $\tau_{\text{SR}}$ parameter only. The other three model parameters that correspond to our model of old stars are $[\text{Fe/H}]$, $\text{std}([\text{Fe/H}])$, and $R_{\text{old}}$. The mean and variance of the metallicity of old stars appear to be robust estimates and do not show degeneracies with other parameters. The MCMC exploration shows $R_{\text{old}}$ to be about 2.4 kpc (see the full corner plot in the Appendix, Figure 16). This value is physically coherent with our prior knowledge of the disk. We note that this $R_{\text{old}}$ parameter, which we model as the “old disk scale length,” is strongly covariant with $\tau_{\text{SR}}$. $\tau_{\text{SR}}$ sets the relative fraction of old and young stars in the Milky Way disk, and the scale length $R_{\text{old}}$ determines the present-day radial distribution of old stars. But the observed relative fraction of old and young stars in the data is fixed. In turn, lower values for $\tau_{\text{SR}}$ (predicting more old stars than observed) lead the model fitting procedure to compensate by placing the overpredicted number of old stars at Galactocentric radii not covered by the survey (e.g., the inner 5 kpc). This results in a small estimate for the old disk scale length $R_{\text{old}}$. Conversely, higher values for $\tau_{\text{SR}}$ (fewer old stars in the disk) need the fitting to compensate by placing more old stars at Galactocentric radii where data are seen (5–14 kpc), and this results in a large estimate for the scale length $R_{\text{old}}$. This is an expected shortcoming when fitting a scale length directly without accounting explicitly for spatial selection effects.

This parameter covariance could also be a manifestation that the scale length of the disk is a function of the ages of the stellar population used to determine it: old stars are more concentrated in the inner disk than young stars (Bovy et al. 2012).

4.3. Tests and Verifications

In this section, we examine some of the model and methodological shortcomings or restrictions that could bias our inferences, such as the approximations made to minimize the computational cost of likelihood evaluations and the convergence of the MCMC procedure. We further address the robustness of the estimate of radial orbit migration strength. Finally, we confront the predictions of our model evaluated at the best MCMC values in the space of the data.

4.3.1. Technical Verifications

The term $p(R \mid p_m)$ in the likelihood function (Equation (21), represented in Figure 8) was interpolated. Interpolation errors on a set of 20,000 test points are less than 0.4%. To see whether interpolation errors have propagated during the overall product of the likelihood over the 1500 stars used for inference, we choose slices in the parameter space at the best MCMC values, and compare the (expensive) true likelihood evaluations to the approximated values. The relative differences are small, as can be seen in Figure 12, which shows three slices of the posterior distribution. Additionally, the generation of mock data and the model itself $p([\text{Fe/H}], R, \tau \mid p_m)$ do not rely on the approximation in $p([\text{Fe/H}], \tau \mid R, p_m)$, so comparisons between the model prediction and the data in Section 4.3.2 also give us confidence that the posterior was approximated well enough for our purpose.

To address the question of the MCMC convergence and the exploration of the parameter space, we have run several more MCMC chains, where the walkers were started in more extended ranges than just the MLE neighborhood. The results remained close to those presented in Figure 9. We have also performed the MCMC procedure on three different random batches of 1500 stars. We found that the radial orbit migration strength $\sigma_{\text{RMS}}$, the present-day metallicity gradient of cold gas at solar radius $\nabla_{[\text{Fe/H}]}$, and the radius of solar metallicity in the interstellar medium $R_{\text{now}}^{[\text{Fe/H}]=0}$ were extremely well constrained. However, the star formation timescale $\tau_{\text{SR}}$ showed some variability (the best $\tau_{\text{SR}}$ varied between 5 and 7.5 Gyr) depending on the sets of stars used, but as discussed in the subsection above, this parameter was found to be sensitive to biases and has an uncertainty of 1 Gyr. Finally, we calculated the potential scale reduction factor, estimating the ratio of variances within single chains and between several chains to $\lessapprox 1.03$ and the autocorrelation time to 180 steps (where the MCMC procedure ran 7000 iterations).

4.3.2. Model Predictions in the Data Space

We generated a mock data set to compare with the APOGEE red clump sample, using rejection sampling on the different aspects of the model evaluated at the best MCMC values. For comparison with APOGEE data, we reproduced the age distribution of red clump stars using the same functional form as in Bovy et al. (2014), and introduced some scatter for the age uncertainties using our noise model (a Gaussian of width 0.2 dex in log$_{10}$ age, and a floor of uncertainties $\sigma_\tau = 200$ Myr for stars younger than 0.5 Gyr). We imitated the possible effect of the radial selection function in our data set using importance sampling (thereby reproducing the radial distribution of stars in our data set). This is a relevant test to do, because inference of the parameters was performed only on a small fraction of the overall catalog: the MCMC procedure was performed (multiple times) on 1500 low-$\alpha$ stars randomly selected in the red clump catalog. Asking whether the model can describe the rest of the 17,500 stars is therefore an interesting test. We show the results in three different plots, allowing data comparison. First, we map the age–metallicity plane $p([\text{Fe/H}], \tau R)$ with contours of both our mock data and the APOGEE red clump sample in different radial bins, see Figure 13. The observed trends are well reproduced in the solar neighborhood and inner disk, but the last panel (13 kpc) shows that the model predicts a distribution broader (in metallicity, so in the vertical direction in Figure 13) than the observed distribution. We suspect that the main differences between our predictions and the data come either from our restrictive model for the evolution of metallicity gradient or from the fact that radial orbit migration strength could depend on radius whereas we fitted a global value. The effect of the metallicity profile will be investigated in subsection 4.3.3.

Second, we integrate the age–metallicity plane $p([\text{Fe/H}], \tau \mid R)$ with respect to age to show the metallicity distribution...
functions at given radii $p(\text{[Fe/H]} \mid R)$, see Figure 14. These distribution functions are well reproduced in most of the disk, showing the expected positive skewness appearing due to radial orbit migration (Hayden et al. 2015; Loebman et al. 2016; Toyouchi & Chiba 2018). The difference between observed and predicted metallicity distribution functions at 13 kpc is more obvious here.

Finally, we compare the model prediction with the data in the radius–metallicity plane, in Figure 15, where the radial spread at fixed metallicity clearly increases with age at similar rates for the observed and mock data. The overall metallicity gradient and the broadening of distributions with time seem to be well reproduced. At young ages, the spread of radii at given metallicity is slightly underestimated by the model. This is because (1) our model assigns any scatter in metallicity at given radius and age to radius migration, and at young ages the probability distribution of a star tends to a Dirac function (Equation (10)), and (2) we neglected measurement errors in metallicity. We note that if star clusters are intrinsically homogeneous but data show additional spread in metallicity at young ages for a given Galactocentric radius, then azimuthal variations of metallicity could be probed by adding one more parameter accounting for scatter in the metallicity–radius–age relation.

4.3.3. Model Variant

The tests presented above showed that (1) the fitting procedure went well for most parameters and the model describes the observations well for most of the Galactic disk, (2) the estimate of radial orbit migration strength is robust, but (3) the model does not reproduce the metallicity distribution functions observed in the outer disk. This can be interpreted in several ways: (a) the functional form of the metallicity profile that we assume is rigid: it describes a straight line for which we fit the gradient $\nabla \text{[Fe/H]}$, the zero-point (after translation of $R_{\text{[Fe/H]}=0}$), and the time evolution of the zero-point $\gamma_{\text{[Fe/H]}}$. But the gradient itself could evolve with time, as pointed out in Minchev et al. (2018), Pilkington et al. (2012), and Wuyts et al. (2016). Or the assumption that the metallicity profile is well described by a straight line could be too simple an extrapolation of the observed gradients. Sanders & Binney (2015) used a different functional form describing a decreasing exponential in radius:

$$\text{[Fe/H]} = F_m \left(1 - \exp \left(-\frac{\nabla \text{[Fe/H]} (R - R_{\text{[Fe/H]}=0})}{F_m} \right) \right) f(\tau).$$

We tested this form with several MCMC procedures, and the estimate of radial orbit migration with this model was $\sigma_{\text{RMS}} = 4.0 \pm 0.1$ kpc, which remains close to our current result and confirms the robustness of the estimate of $\sigma_{\text{RMS}}$ in the present study. Additionally, the outer disk was very well described by mock data from a fit to this model. However, the model predictions in the inner disk were problematic: we systematically overestimated the metallicity of stars born in the inner disk. Sanders & Binney (2015) reported the same high metallicity trend while modeling the solar neighborhood. This gives us confidence that the description of the metallicity profile is a key ingredient in such modeling, and any model-induced rigidity can affect the results significantly (here reproducing the metallicity distribution functions, even though the estimate of radial orbit migration strength was affected by less than 15%). (b) Another interpretation of the disagreement between model predictions and observed metallicity distribution at 13 kpc could be that radial orbit migration occurs differently at different strengths at different radii. We note that we used only one global parameter to describe radial orbit migration over the whole disk, and that outer disk stars are not well described by our global fit, suggesting that $\sigma_{\text{RMS}}$ is a spatial average of a Galactocentric radius-dependent radial orbit migration strength.

9 With a different enrichment prescription $f(\tau)$, which we also tested separately.
5. Discussion and Conclusions

5.1. Summary and Implications

In this study we have quantified the global efficiency of radial orbit migration in the Galactic disk. We have built an analytical model of disk evolution, in good part inspired by Sanders & Binney (2015), which combines the distribution of star formation in radius and time with the chemical enrichment of the ISM, and with subsequent diffusive migration of the stars’ orbital radii. Our model does not attempt to differentiate explicitly whether changes in the orbital radius are to be attributed to churning or blurring.

We have applied this to a set APOGEE red clump stars with age estimates, a large sample with precise distances (covering 5 kpc \( \lesssim R \lesssim 14 \) kpc) and metallicities; this is the first time that such a large and radially extensive data set with consistent estimates of [Fe/H] and \( \tau \) has been available. We sidestepped the complex spatial selection function of this survey and accounted for the 0.2 dex age uncertainties.

This has allowed for the first time an estimate of the overall radial orbit migration efficiency throughout the Galaxy, using \( \langle R, [\text{Fe/H}], \tau \rangle \). Previous studies of radial migration focused on the solar neighborhood (Sanders & Binney 2015, Geneva–Copenhagen Survey data). Other studies of large radial extent in the Galactic disk, using, e.g., APOGEE, had focused mainly on recovering the present-day stellar metallicity distribution functions without the explicit use of stellar ages (Hayden et al. 2015; Toyouchi & Chiba 2018). The model draws its constraints from the mean metallicities at each age and (present-day) radius, and from the spread of these metallicities (growing with age).

Our basic result is that APOGEE data tell us quite directly in this modeling context that radial orbit migration in the Galactic (low-\( \alpha \)) disk is strong, \( \langle |R(\tau) - R_0| \rangle \approx 3.6 \text{ kpc} \sqrt{\tau/8 \text{ Gyr}} \). This means that the characteristic distance over which stars migrate over the age of the disk is comparable to the half-mass radius of the Milky Way disk. Qualitatively, this has of course been implied by a number of earlier studies (e.g., Schönrich & Binney 2009a), and it has been implied by numerical studies of disk dynamics (e.g., Roškar et al. 2008b; Minchev & Famaey 2010). But a stringent and global modeling-based estimate of this efficiency from stellar data across the Galactic disk had not been explored before.

This result has a number of astrophysical implications. First, it tells us that for disk stars older than a few billion years, the combination of age and metallicity should be a better predictor of \( R_0 \). For example, the Sun’s age (4.6 Gyr) and [Fe/H] \( \equiv 0 \) implies in our model context that it was born at \( 5.2 \pm 0.3 \) kpc; it has migrated outward by about 3 kpc since. While it is true that [Fe/H]_\text{birth}(R_\ast) \approx 0 \) dex, the continuous ISM enrichment at all radii does not imply \( R_\ast \approx R_\text{birth} \). This quite precise \( R_\text{birth}(\text{Sun}) \) estimate is in agreement with the broad prediction from chemo-dynamical simulations of Minchev et al. (2013) (between 4.4 and 7.7 kpc), but differs by 2 kpc from the recent results of Minchev et al. (2018) (7.3 \pm 0.6 kpc). Nieva & Przybilla (2012) derived analogous constraints from chemical evolution arguments and data in the solar neighborhood,
inferring a birth radius between 5 and 6 kpc. These chemical evolution arguments are in seeming tension with predictions of the solar birth location based on backward integration of Martínez-Barbosa et al. (2015), finding that the Sun should come from the outer disk rather than from the inner disk.

Further, our results show and confirm that—even in the absence of any significant violent relaxation in the last ∼8 Gyr—the stellar distribution in the Galactic disk experiences significant “dynamical memory loss”; the angular momentum of stars in the disk is not even approximately conserved, though many of these stars may now still be on near-circular orbits. The value of $\sigma_{\text{RMS}}$, when combined with the radial velocity dispersion of the disk, implies that churning is a considerably stronger effect than blurring in the Galactic disk.

We derived these results without having to draw on detailed chemical tagging (Freeman & Bland-Hawthorn 2002). Instead, we relied on the assumption that the spread in birth metallicities among stars born at the same time at the same radius was small over the last 8 Gyr; this is in some sense the most elementary version of chemical tagging.

To the extent that our Galactic disk is typical for large disk galaxies (Rix & Bovy 2013; Bland-Hawthorn & Gerhard 2016), this result helps to explain why the stellar mass density profiles of disks are smooth and approximately exponential. Elmegreen & Struck (2013, 2016) and Herpich et al. (2017) have shown that asymptotically efficient radial migration leads to exponential profiles. Of course, this “thermodynamic limit” of maximal angular momentum entropy would erase all abundance gradients, in conflict with observations. Our analysis here shows that strong radial orbit migration may happen, and still match the radial abundance gradients (at least [Fe/H]) in detail.

5.2. Current Limitations and Future Prospects

In concluding, it may be good to recall some of our main model assumptions and simplifications and possible limitations due to the data set used for inference: (1) we used a very specific description of radial orbit migration, assuming it to be constant across the disk and with a specific time dependence. Describing this as a Gaussian diffusion equation in radius is strictly valid only at large radii in cylindrical coordinates (Brunetti et al. 2011). Here, we instead renormalized the solution for small radii (constant $c_3$ in Equation (10)). We expect our results to be robust against this approximation, because most of our data constraints are more than 5 kpc away from the Galactic center, a distance that is larger than the radial migration strength after 8 Gyr. Further, we deem it plausible that radial orbit migration occurs over a wide range of radii and over much of the disk’s evolution history. Nonetheless, it would be good to explore whether the extensive orbit–abundance data sets allow us to constrain the presumably more complex radial or temporal dependence of radial orbit migration efficiency (Brunetti et al. 2011; Kubryk et al. 2013; Toyouchi & Chiba 2018). For example, a bar could drive enhanced migration near its resonance, but also increase radial motions. To address this question, a modeling context that focuses near the resonances with the bar and that considers diffusion in angular momentum (rather than radius) and

Figure 14. Metallicity distribution functions (MDFs) at six Galactocentric radii: 6, 7, 8, 9, 11, and 13 kpc. The model predictions using the best MCMC values are represented by the red histograms. MDFs of red clump stars are shown in gray. The metallicity distribution functions are well reproduced and a metallicity gradient is visible (shift of the MDFs’ maxima to lower metallicities as $R$ increases), except for the outer disk, where the data show the limitations due to the rigidity of our model.
accounts for blurring would seem more appropriate. (2) At a
basic level, our model explained “scatter” in data with radial
orbit migration. This always begs the question whether other
sources of scatter have been considered exhaustively. For
example, we treated the (dominant) age uncertainties by
explicit marginalization in the model, but did not do the same
for \([\text{Fe}/\text{H}]\) uncertainties to save computational expense. Also,
future work could generalize the assumption that the scatter in
abundance at a given birth radius and epoch was zero, to the
assumption that it was merely “small.” And (3) we restricted
our analysis of radial orbit migration to modeling of Galacto-
centric radius, while angular momentum and radial action
should be modeled to best differentiate churning from blurring.

The arrival of data from Gaia DR2 (Gaia Collaboration et al. 2018)
suggests such a generalized analysis as the next step.

We also eliminated the explicit \(R\) dependence of the model,
to eliminate the model’s dependence on the detailed spatial
selection function. But this approach “to ignore the observed
radius distribution” also eliminates much valuable information.
Future modeling could tackle the spatial selection function head-on (e.g., Bovy et al. 2016).

Our inferences are based on APOGEE DR12 red clump stars
only: while these tracers provide excellent distances, they trace
their underlying population in a very age-biased way, which we
modeled. Nonetheless, the dependence of the SFH \(\tau_{\text{SFH}}\) and
inside-out growth \(\alpha_{\text{Rexp}}\) on the age distribution of the stars
(Section 4) suggests to repeat this analysis with other tracers,
such as red giant branch stars. This is not expected to affect our
findings on radial migration, and \(\tau_{\text{SFH}}\) and \(\alpha_{\text{Rexp}}\) were treated as
nuisance here. But we hope that future data sets can better
constrain the SFH and inside-out growth of the stellar disk.

Finally, we have only considered \([\text{Fe}/\text{H}]\) in this work. The
vast stellar data sets of more detailed measurements of element
abundances must contain much information about where stars
were born and how much they migrated. This, too, bears
detailed modeling.

We wish to thank Wilma Trick, Coryn Bailer-Jones, and
Morgan Fouesneau for valuable conversations. We thank the
anonymous referee for useful comments, which improved the
quality and readability of this work. N.F. acknowledges support
from the International Max Planck Research School for
Astronomy and Cosmic Physics at the University of Heidelberg
(IMPRS-HD). H.-W.R. received support from the European
Research Council under the European Union’s Seventh
Framework Programme (FP 7) ERC Grant Agreement No.
[321035].

The following software was used during this research: Astropy
(Astropy Collaboration et al. 2013), Matplotlib (Hunter 2007),
Pytorch (Paszke et al. 2017), Emcee (Foreman-Mackey et al.
2013). Figures 9 and 16 were produced using the package Corner
(Foreman-Mackey 2016).

Appendix

We include here the full results of the MCMC procedure with
all nuisance parameters, including those describing the old
low-\(\alpha\) stars in the Galactic disk, see Figure 16. This figure

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure15}
\caption{Number density distribution of red clump (RC) stars (top) and mock data (bottom) in the plane of metallicity ([Fe/H]) and Galactocentric radius for two ages bins: young stars (less than 1 Gyr, left) and older stars (measured age between 4 and 6 Gyr, right). The total number of mock stars equals the total number of red clump stars.}
\end{figure}
essentially shows that all nuisance parameters are rather well constrained, but there is a degeneracy between the old disk scale length and the star formation timescale, as explained in Section 4.

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**ORCID iDs**

Neige Frankel 🐝 https://orcid.org/0000-0002-6411-8695
Hans-Walter Rix 🐝 https://orcid.org/0000-0003-4996-9069
Yuan-Sen Ting (丁源森) 🐝 https://orcid.org/0000-0001-5082-9536
David W. Hogg 🐝 https://orcid.org/0000-0003-2866-9403
