Improvement the scheduling based on the reducing of the dimensionality of the system

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Abstract. In this paper a full set of system parameters “building site + resources” has been determined. Methods of discrete description of the closed system dynamics and dynamics of the open system, interacting with external objects, have been formulated. In the four-dimensional phase-space the step function, quantitatively describing the construction production calendar plan, has been defined. It is shown that the stored resources do not determine the duration of construction project implementation, but rather limit it. On the other hand, the non-stock resources determine the possibility of construction project implementation only, when the upper boarder-line of the construction project duration is fixed. The method of reducing the dimension of the phase space by taking into account the spatial and technological relationships has been formulated. It is shown that the problem of reducing the dimension of the proposed algorithm can significantly extend the class of NP-solvable problem of optimizing scheduling and allow visualization of the algorithm to optimize construction production calendar-plan, enabling a significant expansion of the area of its practical application. The possibilities of the use of orthogonal sections of the graphical representation of construction calendar plan in three-dimensional configuration space in strategic and operational management of building production practice, has been demonstrated.

1. Introduction

Development of the methods of construction scheduling has a centuries-old history. However, at the first stages of development, they were presented in the form of empirical set of optima control rules of collective work [1]. Only at the turn of XIX and XX centuries, F. Taylor, in his classic work “Principles of Scientific Management”, implemented the scientific formulation of the optimal manufacturing process management. Within this approach, L. Gant had developed the first scientific methods of schedule plan (SP) and formats of its visualization (the so-called Gantt charts/diagrams). From 1930-s, the study of the flow-line method of work organization, which allows avoiding the
discrepancies, attributed to the linear and parallel methods of construction work implementation [2] has begun. The flow-line method can be characterized by the following features:

- splitting the scope and processes of work for creation of the most favorable conditions for individual workers, based on their specialization
- the maximum combination of work processes in time

The flow-line method of scheduling provides uniformity of resource consumption and the stable rhythm of production of finished products, creating favorable conditions for work of allied organizations: contractors, factories, and transport companies. However, the use of this method is limited by the need to combine equally rhythmic and multi rhythmic components of flows, which is rarely achievable in practice [3]. The practical use of such method is also limited by instability of the flow-line methods to external stochastic effects [4, 5]. Another significant disadvantage of the flow-line method, which does not allow performing the system optimization, is the iterative nature of the resource constraints accounted. These and other disadvantages of traditional methods of construction scheduling have stimulated the search for alternative methods of calendar planning [6].

New perspectives for SP improvement appeared in connection with the introduction of computer technology, thus a possibility to apply methods of operations research (linear and dynamic programming, multi-criteria optimization, game theory, and others) has arisen [7]. Application of these methods has allowed formulating and implementing a simple computer algorithm of description of SP of the construction project implementation, later called the Critical Path Method (CPM) [8].

Choice of critical paths is key moment of planning in this approach because it defines technological, financial and managerial risks [9]. However, the splitting of the set of operation sequences into critical and non-critical subsets is not uniquely determined [9]. As a consequence several critical paths are possible for one method of project implementation [10]. In addition, for large projects, The CPM analysis leads to NP-unsolvable problem and does not allow to make an unambiguous decision about optimal method of implementation [11].

The development of this approach allowed creating (Program (Project) Evaluation and Review Technique - PERT) [12], which is based on optimization of the process logical scheme. Recording of stochastic external influences, uncertainty and ambiguity of system dynamics is executed with the use of Graphical Evaluation and Review Technique - GERT) [13].

Currently used methods and algorithms of rational ordering in time and space of technological operations in the course of various technical and economic processes are very diverse [14]. These methods have been successfully applied in the planning of works at industrial factories [15]; for optimizing logistics processes [16]; for planning military operations [17]; for implementing state contracts with a large number of participants [18].

In domestic practice, computer technology for solving scheduling problems is used to visualize the solutions obtained by empirical methods as part of the standard software packages [19]. Meanwhile, the improvement of construction scheduling and optimization of construction project are possible with the use of deterministic finite automata [20]. One of the main components defining the deterministic automaton is a finite set of system conditions [21]. This paper is devoted to the formation of methods that can fully describe and visualize the conditions of the system "building site + resources".

2. Description of the dynamics of "building site + resources" system

Description of the dynamics of "building site + resources" system requires specification of sets of the discrete states of individual subsystems and dependence of the components of these sets from the time aspect. Building site can be quantitatively characterized by the following set of elements:

- \( W \) – types of work at the entire construction building site
- \( S \) – spatial elements (number of rooms, plots of land, etc.), in each of which all construction works are carried out with the workload volume \( v_{k,j} \) \( (k = 1,2,\ldots,S; j = 1,2,\ldots,W) \)

In the equation (1) is shown summing up the volume of work in all the spatial elements, we get a full scope of work of types following:
By this, let’s assume that the capacity of non-stock resources and the specific consumption of the stored resource do not depend on the type of room.

For the construction project implementation the $R_s$ types of stored and $R_n$ non-stock types of resources are in need. At the beginning of construction project realization stored resources (financial, material) are described by their total volume $R_j$ ($i = 1, 2, ..., R_s$). In contrast, the non-stock resources (machines, equipment, personnel) are described not by their quantity, but by the capacity $P_{j,i}$ ($i = 1, 2, ..., R_n; j = 1, 2, ..., W$) for each type of work. For example, working personnel can have different qualifications and skills to perform various technological processes. Moreover, the productivity of machines and mechanisms is determined not only by their technical characteristics, but also by the work performed.

The system dynamics is determined by the time interval $T$ (depending on the detailing level of construction scheduling - hours, shifts, days, weeks, etc., which are considered further to be the same, normalized to a single value). The duration of construction project measured by $T$, in contrast to the characteristics of the construction building site $W, S,$ and to characteristics of its performers $R_s, R_n$, is determined dynamically during the planning process itself.

Dynamics of the closed system of the stored resource is described by the non-increasing function, because such resource can only be spent out or remain unused at certain stages of the construction project. In contrast, the non-stock resource may increase (i.e. workers can improve their skills and performance.

Dynamics of resources in the open system depends not only on the system properties, but also on its interaction with external objects. In particular, the financial resources can be replenished in the course of the project. It may also be necessary to lease or purchase equipment or hiring additional working personnel. The reverse processes are also possible (i.e. some machines and equipment are sold out, rented out, or got destroyed, some personnel is dismissed, etc.). The change in volume of stored resources in the $T$ time period can be described by the function of $\Delta R_{j,t}$. For non-stock resource $\Delta P_{j,i,t}$ ($k = 1, 2, ..., R_n; i = 1, 2, ..., W$) describes the change in capacity. These functions, depending on the sign, describe the increase or decrease in $j$-th power resource or $T$-th time interval. Therefore, accumulated due to external sources resources and capacity in the time interval $t_0 + 1$ can be define by the following equations:

$$R_{j,t_0+1} = R_j + \sum_{t=1}^{t_0} \Delta R_{j,t_0+1}$$

$$P_{j,i,t_0+1} = P_{j,i} + \sum_{t=1}^{t_0} \Delta P_{j,i,t}$$

For the determination of resources consumption, during the execution of construction works, it is necessary to set specific consumption costs of resources stored per unit of work performed $r_{j,i}$ and specific capacity of use of the non-stock resources $p_{j,i}$.

The task of the calendar planning of construction process consists of allocation of resources on the premises, according to the types of rooms, types of activities, and time intervals, with the fulfilling of condition of nonnegative balance of resources for any $t$. Quantitatively calendar plan is described by the step function $\delta_{t,k,i,j}$ that takes a value equal to one, if in the $t$ time interval in the $k$ room for the performance of $i$ type of work the $j$ resource is used. In all other cases, the function $\delta_{t,k,i,j}$ takes zero value.
The duration of the project realization $T$ is limited only by the capacity of non-stock resource and determined by the condition of completion of all works in all rooms/premises, with full respect of technological conditions, which leads to the equalities:

$$T = \max(T_{ki}); k = 1, 2, ..., S; i = 1, 2, ..., W$$

Equation (4) with any parameters has a formal solution, which might be unsatisfactory in practice according to the implementation duration criterion. Thus non-stock resources determine the feasibility of the project realization, only when upper border of the project duration is fixed.

On the other hand, storable resources do not determine the duration of completion, but only the possibility of its implementation. The total use of the stored resources should not exceed their availability at all times, that is quantitatively described by the condition

$$\sum_{t=1}^{T_{ki}} \sum_{j=1}^{R_x} \delta_{t,k,i,j} P_{j,i,t} = v_{k,i}$$

$$T = \max(T_{ki}); k = 1, 2, ..., S; i = 1, 2, ..., W$$

In particular, for the closed system the left side of the equation (5) does not depend on the time value and equals to the original volume of stored resources. In this case, the condition (5) is simplified and takes the form of balance of stored resources during the implementation of the entire project:

$$\sum_{t=1}^{t_0} \sum_{k=1}^{S} \sum_{j=1}^{P_{j,i,t_0}} \delta_{t,k,i,j} P_{j,i,t_0} \leq P_{j,i}$$

$$\sum_{t=1}^{t_0} \sum_{k=1}^{S} \sum_{j=1}^{P_{j,i}} \delta_{t,k,i,j} P_{j,i} \leq P_{j,i}$$

3. Visualization of the set of states "building site + resources"

Description of the dynamics of "building site + resources" system requires specification of sets of the four-dimensional character of the calendar-planning problem (room, work, resources, and time) does not allow displaying information about the dynamics and current state of the system in three-dimensional space in full. An even greater loss of information occurs, when the dynamics of the system is reflected on the plane. Therefore, any two-dimensional techniques of visualization of the calendar plan (such as, most commonly used in practice of planning, line diagrams, network diagrams, charts, organizational analysis, and others) inevitably reflect only part of the necessary information and require further explanations. These limitations are in act for any two-dimensional analytical building sites (matrices, tables, etc.).

The reduction of the system dimensionality by taking into account the spatial and technological relationships allows overcoming this contradiction, solving the problem by introducing the concept of planning unit, which combines the spatial and technological information about the building site’s state. Let us call the planning unit (hereinafter - PU) the collection of information about the quantity of rooms at the building site premises, amount of the work and workload volumes that need to be performed for entire project implementation. In such formulation, the problem of calendar-planning becomes three-dimensional and can be described by the step function $\delta_{t,n,j}$ that takes a value equal to one, if in $t$ time interval in the $n$PU the $j$ resource is used.

Since the capacity of the non-stock resource and the specific consumption of the stored resource do not depend on the type of room in the right sides of equations (4) - (6), then the collection of indices $i$ and $k$ is simply replaced by an index $n$, taking values $n = 1, 2, ..., U$ in the interval (hereinafter $U$-
number of planning units building site). Therefore, in such problem formulation the equation, describing the calendar-planning scheduling for the time interval of project implementation with resource constraints take the following form:

\[
\sum_{t=1}^{T_n} \sum_{j=1}^{R_j} \delta_{t,n,j} P_{j,i,t} = u_{k,n} \quad T = \max(T_n); n = 1, 2, ..., U
\]  

(7)

\[
\sum_{t=1}^{t_0} \delta_{t,n,j} P_{j,n} \leq P_{j,n,t_0}
\]  

(8)

\[
\sum_{t=1}^{T} \sum_{n=1}^{U} \delta_{t,n,j} P_{j,n} \leq \sum_{n} P_{j,n} = P_j
\]  

(9)

In these equations \(P_j\)-is the total number of \(j\) storable resources.

Recording of spatial and technological schedule plan linkages within the PU allows to significantly expand the class of NP -solvable [22] problems of scheduling optimization. This is due to the fact that technologically impossible system states are excluded from consideration at the stage of task setting and do not contribute to a set of states of "building site + resources" system.

The main practical criterion making such problems belong to NP-solvable class is the possibility of creating the exact algorithm, which is not limited to a complete processing of all (or a priori to an undefined part) of the variants of studied system. For the NP-unsolvable problems, it was shown that even an exponential increase in the speed of computers does not significantly expand the class of practically solvable problems.

In addition, the decrease of the problem dimension can not only make the problems of optimization of more complex systems NP-solvable, but also determine the three-dimensional configuration space, the elements of which will fully describe the dynamics of the construction project. This dimension of the configuration space allows visualizing the algorithm of optimization scheduling that enables a significant expansion of the area of its practical application.

Lets define the discrete Cartesian configuration space, in which the x-axis represents the number of non-stock resource \((j)\); the ordinate represents the number of the time interval \((t)\); Z-axis represents the planning unit number \(PU(n)\). Thus, the elementary space unit is represented by a cube with the edges of unit length. In such a space, the function \(\delta_{t,k,j}\) describes the position of \(j\) non-stock resource activated in the \(t\) time interval in \(n\)-unit of schedule plan. The example of graphical representation of three-dimensional schedule plan is shown in figure 1.
The implementation of graphical presentation (figure 1) requires the use of five types of non-stock resources. To aid the visualization, two layers for time periods are shifted along the Y-axis. Highlighted are the elementary cells of configuration space, reflecting the PU, on which the work is done, along with time resources used (from the analytical point of view, the elementary cells of configuration space are those in which $\delta_{t,n,j}$). For example, in the first time interval $t_1$ only two types of resources are activated for the implementation of two PUs. In the second time interval $t_2$ the scope of work has not changed and the same two PU are activated, but construction works are intensified and the number of resource types used has increased up to four.

Two-dimensional cross-section of three-dimensional graphical representation (GP) of schedule with planes perpendicular to the coordinate axes allows to get comfortable in the practice of the construction interface. Firstly, a set of front sections planes $i = \text{const}$ (Figure 2) reflect PUs, in which in a fixed time interval $t_i$ the construction work performed and non-stock resources are used. This information is a useful contribution to the practice of operational construction management, as it allows formulating the current parameters for each planning period (shift, day, week, etc.) the flow of vehicles and machinery, manpower and other resources.

The sections of the GP planes perpendicular to the PU axes ($n = \text{const}$) reflect the dynamics of the use of non-stock resource (figure 3). Analysis of this information allows to identify the dynamics of resources use and quickly redirect the non-stock resources (the use intensity of which in the analyzed period of time is small) to the other building sites. On the other hand, the high (close to 100% resource utilization) triggers the risks processes occurrence due to stochastic impact on the system. The analysis of sections with GP planes $n = \text{const}$ allows the prediction of such risk processes. In the operational management practice, this information can also help to determine the times of the beginning of use and the complete release of each resource, which in its turn will optimize their use.

The sections with the GP planes perpendicular to the resources axis ($j = \text{const}$), reflect the dynamics of work in all areas of the building site (figure 4). The analysis of this information allows determining the intervals of high intensity work on the premises. This situation (as is by the utilization of high intensity of the resources use) is unstable with respect to stochastic effects.

To aid the visualization, the figures reflect also non-activated PUs and resources, i.e. those for which the function $\delta_{t,n,j}$ takes a zero value for all non-fixed indexes.
On the graphical presentation in figure 2 there are seven units of scheduling are implemented by using thirteen non-stock resources. Highlighted are the resources used by the implementation of PU. On this time axis the non-activated PUs and the resources are not displayed.

The figure 2 shows that the first PU uses the resources 1 - 3 and 5; the resource 4, described in the time period, is used for the implementation of all PUs except of the first one; implementation of the third PU needs resources 6 - 13, with the exception of 8; resource 8 is only necessary for the implementation of the fifth PU.

In the figure 3 describes that at the first stage of realization only recourses № 1 – 4 are used. Therefore, at this stage, the remaining resources can be promptly forwarded to other construction sites. Resource number 4 is used very intensively and constantly needed. By the stochastic effects on the fourth resource the risk of default work schedule occurs. Therefore, it is appropriate to create the operational reserve of this resource. The same is true for the resource 8, which is not used only in the first scheduling stage. In contrast, the № 5 resource is not used and in the described period of time can be promptly redirected to the other construction sites; wherein the respective column section GP can be omitted. Maximum demand for non-stock resources can be observed in the third time phase, and the minimum - the second, and fourth, fifth and sixth.

Figure 2. Example of GP (i =const) front section for the PU realization in the time period.

Figure 3. Example of GP section for the PU realization in six time periods, for which twelve resources are necessary.
Figure 4. Example of GP section for the resource needed during the six time intervals of schedule planning for the implementation of the nine PUs.

In the figure 4 is shown, that four Planning Units 1 - 3 and 5 are not implemented. In contrast, the number PU 4 is implemented in all intervals except the third one. This situation requires an analysis of the possible risks of stochastic impacts. The fourth period of planning is very busy at this building site, in which the work is carried out at more than half of the PU of construction site. This situation also calls for a special analysis.

4. Conclusion
Thus, algorithms of schedule plan of construction production plan, based on the reducing of the dimensionality of the system described by taking into account the spatial and technological relationships allow us to solve important practical problems. First of all to construct the algorithm to improve NP-solvability of the scheduling problem. In addition, the reduction of dimension makes it possible to determine the three-dimensional configuration space, the elementary cell of which allows to completely lossless display the dynamics of the "building site + resources" system.

Orthogonal section of the configuration space allow us to formulate a method of visualization of schedule plan, which has a number of practical advantages in comparison with applicable previously (Gantt charts, diagrams of the PERT, network diagrams, and so on). It allows to graphically display information such as the dynamics of the project, including the information about the state of the building site and the use of resources at fixed intervals. In practice, such planning visualization allows us to formulate the current planning period for each flow of vehicles, machinery, manpower, and other non-stock resources. In addition the method makes it possible to predict the risk in relation to stochastic effects of the situation and plan preventive measures for their abolishment.

References
[1] Kustova T N, Kamakin O B 2001 Economy history: Education guidance (Rybinsk: RGATA)
[2] Dikman L G 2012 The Construction Management (Moscow: ASV Publishing House)
[3] Vasilyev V M 2001 Management in construction (Moscow: ASV Publishing House)
[4] Afanasyev A V 1982 Spasmodic streams with continuous performance of peer works (in Improvement of the organization and management of construction) (Leningrad: LISI) pp 13-22
[5] Gusakov A A 1994 Organizational and engineering reliability of construction (Moscow: SvR - Argus Publ)
[6] Larichev O, Sternin M 1992 Knowledge-based approach for solving the multicriteria assignment problem ed. Linster M (Arbeits papiere der GMD 630 Sisyphus Models of problem solving)

[7] Uskov V V 2016 Innovations in construction: organization and management (Moscow: Infra-Inzheneriya)

[8] Krüger, Wilfried 2006 Excellence in Change - Wege zur strategischen Erneuerung 3. Auflage, (Wiesbaden: Gabler Verlag) pp 212-213

[9] O’Brien J J 1965 CPM in Construction Management and Scheduling by the Critical Path Method (New York: McGraw-Hill)

[10] Shaffer L R, Ritter J B, Meyer W L 1965 The Critical Path Method (New York: McGraw-Hill)

[11] Smith S F, Fox M S, Ow P S 1986 Constructing and Maintaining Detailed Production Plans: Investigations into the Development of Knowledge-Based Factory Scheduling Systems AI Magazine vol 7 pp 45–61

[12] Project Management Institute 2013 A Guide to the Project Management Body of Knowledge 5th ed. Project Management Institute

[13] MacCrimmon K R, Ryavec C A 1964 An Analytical Study of the PERT Assumption vol 12 no 1 (in Opt. Res) pp 16–38

[14] Pinedo M 1995 Scheduling: Theory, Algorithms and Systems (New Jersey: Prentice Hall Englewood Cliffs)

[15] Muth J F, Thompson G L 1963 Industrial Scheduling (New York: Wiley) 577 p

[16] Meyer W 1992 Geometrische Methoden zur Lösung von Job-Shop Problemen und deren Verallgemeinerungen PhD thesis (Universität Osnabrück Fachbereich Mathematik/Informatik)

[17] Baker R 1974 Introduction to Sequencing and Scheduling (New York: JohnWiley & Sons)

[18] Lawler E L 1982 Preemptive scheduling of precedence-constrained jobs on parallel machines ed. Dempster M A H, Lenstra J H, Rinnooy Kan A H G, Deterministic and stochastic scheduling (Proceedings of the NATO Advanced Study and Research Institute on Theoretical Approaches to Scheduling Problems held in Durham, July 6–17, 1981) vol 84 of NATO Advanced Study Institute Series C: Mathematical and Physical Sciences, Dordrecht (D. Reidel Publishing Co.) pp 101–123

[19] Uskov V V 2011 Computer Technologies in the Preparation and Management of Construction Projects (Vologda: Infra-Inzheneriya Publ)

[20] John E Hopcroft, Rajeev Motwani, Jeffrey D Ullman 2006 Introduction to Automata Theory, Languages, and Computation 3 ed. (Massachusetts: Addison Wesley)

[21] Jacques Sakarovitch 2009 Elements of automata theory (Cambridge: Cambridge University Press)

[22] Mishchenko V YA, Dobrosotskikh M G 2016 NP Solvable task of scheduling of construction, reconstruction and repair of objects Proceedings of higher education institutions. Textile industry technology vol. 6(366) (Ivanovo: IVSPU) pp 13-20