Novel FBG rosette for determining impact location in thin plate-like structure

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Abstract. The paper consists of two interdependent parts. The first part presents numerical simulations of output response of single Fibre Bragg Grating (FBG) sensor which is driven by homogeneous deformation along sensor length. The example of such sensor is FBG sensor glued at only two points. The grating length and modulation depth of the refractive index are the critical parameters contributing to performance of the FBG sensors. Numerical analysis allowed to select an appropriate FBG sensors which will be used in the impact detection problem. In the second part of the paper a novel strain rosette with specific sensor array used in impact localisation problem is discussed and presented. The experiment was carried out on thin composite plate with the use of pulse force excitation. The method is based on estimation of principal direction of perturbed travelling wave initiated at impact point and does not use any information about wave propagation velocity.

Introduction

The ability to locate damage, impact or any other discontinuities in structural materials such as plates, rods, pipes, etc is one of the objectives of structural health monitoring (SHM) systems. The SHM systems are based on various types of sensors and their configurations i.e. PZT and fibre optic sensors or a combination of them. In addition non-contact measurement techniques like laser ultrasound, optical coherence tomography and infrared thermography could be employed. This paper is focused on the application of Fibre Bragg Grating (FBG) sensors for impact localization.

The basic principles and characteristics of FBG sensors with respect to different distribution of parameters along the grating length or under uniform and non-uniform strain fields have been analyzed in: [14], [15], [19], [21], [23]. The outcome of these works is used here to specify appropriate parameters of the FBG sensor (especially minimal grating length) in order to obtain good spectral content of the light reflected from the grating.

The technical progress in the construction engineering requires more advanced materials. The desire to reduce the weight of structure and reduce their manufacturing costs may affect the safety and reliability of designed structure. On the other hand, the structural damage can occur due to many factors which are difficult to predict in advance, e.g.: sudden impact loads. A number of researchers propose the use of FBG sensors for detecting impact loads. For this purpose two methods are used. In the first method impact location is determined by measuring the differences in time-of-flight of
Lamb waves [7], [11]. In the second method, also exploited in this paper, amplitudes of dynamic strain signals are analysed in order to find direction of the impact event [2], [3]. The paper is organised as follows: in Section 2, the parameters describing the grating influence on the reflection spectrum of FBG sensor are analyzed, in Section 3 the strain rosette bonded on the composite plate is used to calculate the angular direction of the impact. Finally, in Section 4 the conclusions are indicated.

The selection of appropriate FBG sensors

1.1. Fibre Bragg Grating sensor

The FBG sensor is an optical device with a permanent periodic modulation of the refractive index in the core of a single-mode fibre optic. The grating pattern is made by exposing the core of the optical fibre to an interference pattern of intense UV laser light [6], [15]. The change in the core refraction index is between $10^{-5}$ and $10^{-2}$ [6], [13]. The periodic perturbation in the core refractive index allows coherent scattering to occur for a narrow wavelength band of incident light travelling within the fibre core. A strong narrowband back reflection of light is generated, centred around the maximum reflecting wavelength $\lambda_B$ when the resonance condition, or phase match, is satisfied:

$$\lambda_B = n_{\text{eff}} \Lambda$$

where $n_{\text{eff}}$ is the effective refractive index and $\Lambda$ is the grating period [20]. The scheme of an FBG sensor is shown in Figure 1.

![Figure 1. A scheme of a Fibre Bragg Grating sensor idea.](image)

The maximum reflecting wavelength value $\lambda_B$ is dependent on the geometrical and physical properties of both the grating and the optical fibre. Longitudinal strains $\varepsilon$ act on the periodicity of the perturbation $\Delta$ (photoelastic effect), while temperature affects both the effective refractive index $n_{\text{eff}}$ (thermo-optic effect) and the periodicity of the perturbation $\Lambda$ (thermoelastic effect). The key point of FBG sensors is their wavelength-encoded nature, which is the absolute parameter providing repeatable measurements [20]. The FBG sensors have many advantages in comparison with electric strain sensors and are promising sensing alternatives for SHM systems. The FBG sensors have small size and weight, multiplexing capabilities, immunity to electromagnetic fields, high corrosion resistance and require no calibration [6], [10], [13]. They can also be mounted onto a structure [1] or can be embedded [3], [18] into the material of a structural element during the manufacturing process.

1.2. The basic equations for numerical simulations of FBG reflection spectrum

There are many parameters which affect the reflection spectrum of the FBG sensor, e.g. the effective refractive index, fibre bending, grating, a temperature, or sensor length [9], [10], [16], [21]. The characteristics of the response from Bragg grating are fully described by [19], [21]: the central Bragg wavelength $\lambda_B$, maximal peak reflectivity $R_{\text{max}}$ of grating which occur at $\lambda_B$, the grating length $L$, the refractive index of the core of optical fibre $n_0$ and modulation depth of refractive index $\Delta n_{\text{eff}}$. 

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Using the coupled-mode theory analytical description of the reflection properties of Bragg gratings may be obtained. The reflection spectrum of a grating \( R \) is given by the following expression:

\[
R(L, \lambda) = \frac{\kappa^2 \sinh(\gamma L)^2}{\Delta \beta^2 \sinh(\gamma L)^2 + \gamma^2 \cosh(\gamma L)^2}
\]  

(2)

where \( \kappa = \frac{\pi \Delta n_{\text{eff}}}{\lambda} \), \( \Delta \beta = \beta - \frac{\pi}{\Lambda} \), \( \beta = \frac{2 \pi n_0}{\lambda} \), \( \gamma = \sqrt{\kappa^2 - \Delta \beta^2} \), \( \lambda \) is wavelength of the reflected light from grating of the sensor and \( \Lambda \) is a grating period.

For light at the central Bragg wavelength, \( \lambda_B \), there is no wave vector detuning and so \( \Delta \beta = 0 \). Then, the reflectivity function becomes

\[
R(L, \lambda) = \tanh(\kappa L)^2
\]  

(3)

A number of numerical simulations shown below have been performed using equation (2) with tacit assumption of no amplitude losses of the signal in the fiber and the sensor is driven by homogeneous deformation along its length. The FBG sensor glued at only two points experiences such deformation.

### 1.2.1. The influence of the modulation depth of the refractive index and the grating length on the sensor spectrum

The parameters used for the simulation were: \( n_0 = 1.47 \), \( \lambda_B = 1550 \text{ nm} \), \( \Lambda = 527.21 \text{ nm} \). The four values for \( \Delta n_{\text{eff}} = 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2} \) were considered. The values of \( R_{\text{max}} \) for grating length varying from \( L=0.25 \text{ mm} \) to \( 40 \text{ mm} \) is shown in Figure 2a. The FBG sensor must provide sufficient reflectivity for most possible variations of \( \Delta n_{\text{eff}} \) because the exact values for \( \Delta n_{\text{eff}} \) are not known for real sensors (manufactures do not provide such data). The longer the sensor is the higher value of \( R_{\text{max}} \) is up to saturation level. The results show that acceptable minimal length of the Bragg grating is about 5 mm provided \( \Delta n_{\text{eff}} \) is not less than \( 10^{-4} \). For more stable results the length of FGB sensor was chosen to be 10 mm for further experimental investigations. The shapes of reflected spectrum for grating length \( L=10 \text{ mm} \) for different \( \Delta n_{\text{eff}} \) are shown in Figure 2b. It is clearly visible that \( R_{\text{max}} \) increases with increasing \( \Delta n_{\text{eff}} \). The smaller \( \Delta n_{\text{eff}} \), the worse band-pass filter properties of the sensor.

Figure 2. The influence of the modulation depth of the refractive index \( \Delta n_{\text{eff}} \) on sensor characteristics: (a) reflectivity \( R \) as a function of \( L \) for \( \lambda_B = 1550 \text{ nm} \), (b) reflectivity \( R \) as a function of \( \lambda_B \) for \( L=10 \text{ mm} \).
1.2.2. The influence of the refractive index of the fibre core on the sensor spectrum

The parameters used for this simulation were: \( \lambda_B = 1550 \text{ nm} \), \( \Delta n_{\text{eff}} = 10^{-4} \). The three values for \( n_0 = 1.4, 1.45, 1.5 \) have been considered which give \( \Lambda = 553.57 \text{ nm}, 534.48 \text{ nm}, 516.67 \text{ nm} \), respectively. The grating length has been varied from \( L = 0.25 \text{ mm} \) to \( 40 \text{ mm} \).

Analysing Figure 3a it is clearly visible that a small change of \( n_0 \) has no influence on the maximum peak reflectivity \( R_{\text{max}} \). Only the width of the spectrum changes slightly, but this has no practical implications. In Figure 3b the influence of modulation depth of the refractive index \( \Delta n_{\text{eff}} \) on the reflection spectrum for particular grating length (\( L = 10 \text{ mm} \)) is presented.

![Figure 3a](image1.png) ![Figure 3b](image2.png)

**Figure 3.** The influence of refractive index of the fibre core \( n_0 \): (a) reflection spectrum \( R \) as a function of \( \lambda_B \) for \( L = 10 \text{ mm} \), (b) influence of \( \Delta n_{\text{eff}} \) on FWHM reflectivity peak for grating length \( L = 10 \text{ mm} \).

Summarizing this part of the ideal case study the following conclusions can be highlighted:

- The reflection spectrum \( R \) strongly depends on \( \Delta n_{\text{eff}} \) and \( L \);
- The rate of \( R_{\text{max}} \) is strongly dependent on the value of \( \Delta n_{\text{eff}} \);
- The changes in \( n_0 \) have no influence on \( R_{\text{max}} \) and slightly affects the full width at half maximum (FWHM) of the sensor;
- For experimental application modulation depth of refractive index \( \Delta n_{\text{eff}} \) should be not smaller than \( 10^{-3} \) and the grating length should be sufficiently long in order to obtain \( R_{\text{max}} \) at least at the level of 50%. Moreover, the grating with saturated reflection spectrum should be also avoided.

Novel FGB rosettes for the impact direction

1.3. Angular localization of the impact using rosette's principal direction

Strain and the corresponding stress state for elastic solid body are described by symmetric, three-dimensional second-order tensors [22]. If the components of these tensors are independent of one of the Cartesian coordinates (usually it is \( z \) coordinate) the general three-dimensional description can be reduced to two-dimensional counterparts: plane strain state or plane stress state. Plane strain state can be described by two-dimensional symmetric second-order strain tensor \( \varepsilon \) whereas plane stress state is defined by two-dimensional symmetric second-order stress tensor \( \sigma \). The Cartesian strain tensor \( \varepsilon \) calculated from rosette measurements are then used to define line of perturbed wave propagation from the impact location. For full impact localization at least two rosettes are needed to determine two principal directions of perturbed waves coming from the impact location. The intersection of the lines determined by principal directions determines locus of mechanical impact, see Figure 4. The idea of the method comes from [6] where Macro Fibre Composite (MFC) rosettes bonded to metal plate were used for the same purpose. The formulas used in the rosette analysis have been derived using
infinitesimal strain theory. The assumptions of this theory are fulfilled if the induced strain perturbation is measured at some distance from the impact location.

For general two-dimensional strain state at the surface of the structural element, at least three strain sensors are needed to calculate principal strains and directions which represent the simplest description of that state. For the purpose of the paper only the principal direction must be calculated. The principal direction is calculated as follows:

- The geometrical configuration of the sensors in rosette must be chosen: the angles $\beta$, $\gamma$ must be determined, see Figure 4 (centre);
- The plane cartesian coordinate system $xy$ need to be oriented with respect to chosen sensor from the rosette: the angle $\alpha$ must be determined;
- The measurement strains $\varepsilon_a$, $\varepsilon_b$, $\varepsilon_c$ from the sensors in rosette configuration must be converted to components $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_{xy}$ of strain tensor $\varepsilon$ defined in cartesian coordinate system $Oxy$. This step can be described by the solution of the corresponding system of equations:

$$
\varepsilon_x = \frac{1}{2}(\varepsilon_x - \varepsilon_y) \cos(2\alpha) + \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \varepsilon_{xy} \sin(2\alpha) \\
\varepsilon_b = \frac{1}{2}(\varepsilon_x - \varepsilon_y) \cos(2(\alpha + \beta)) + \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \varepsilon_{xy} \sin[2(\alpha + \beta)] \\
\varepsilon_c = \frac{1}{2}(\varepsilon_x - \varepsilon_y) \cos[2(\alpha + \beta + \gamma)] + \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \varepsilon_{xy} \sin[2(\alpha + \beta + \gamma)]
$$

- The rotation angle $\phi$ which diagonalizes tensor $R^T \varepsilon R$ using definition of rotation matrix $R$ determines the first principal direction associated with larger principal strain. The second angle is equal to $\phi+90^\circ$ and is associated with the second principal strain. This step can be described by the solution of the corresponding equation:

$$
(\varepsilon_y - \varepsilon_x) \sin(2\phi) + 2 \varepsilon_{xy} \cos(2\phi) = 0
$$

The angle $\phi$ is positive if it increases in counterclockwise direction from $x$ axis. This quantity will be used for angular localization of the impact.

1.4. The experiment for determination of the direction of travelling wave initiated at impact point

The experimental set up consisted of: Carbon Fibre Reinforced Polymer (CFRP) plate (toughened epoxy matrix with carbon fibre Plain 3K fabric), the three FBG sensors, an interrogator and a laptop. The CFRP plate with a geometry as follow: 500 mm x 500 mm x 5 mm was considered. Three FBG sensors (grating length $L=10$ mm), arranged as it is show in Figure 4 were glued to the surface using two-point technique. The FBG rosette was placed in the point marked 4g on the plate (see Figure 4). The particular angles in rosette were: $\alpha=63.1^\circ$, $\beta=26.6^\circ$, $\gamma=25.5^\circ$. The distance between point 4e (point A in Figure 4) and 4g (FBG rosette) was the same as between 4e (point A) and 6e (point B),

\[\begin{align*}
\end{align*}\]
and the same as 6e (point B) and 6g (point C) and was equal 100 mm. In points marked A, B, C the hammer impacts were executed.

Figure 5. Rosette strain measurements (upper row) and changes in principal direction (bottom row) for impact from 0° direction (impact in point C). Shaded areas in the right column show changes caused by impact.

Figure 6. Rosette strain measurements (upper row) and changes in principal direction (bottom row) for impact from 45° direction (impact in point B). Shaded areas in the right column show changes caused by impact.

Rosette strain measurements and calculated principal direction from rosette analysis before, during and after hammer impact executed in three locations are shown in the Figures 5-7. The common feature of principal direction curves for these three impact cases is their stabilization after hammer impact for some time contrary to the regions before impact and after the decay of strain signals. For impacts executed in A and B the line of propagating disturbance is roughly parallel to the sensors and the proposed procedure for angular localization of the impact works very good. For case C
(impact from 0° direction) the calculated angle was between 170° (-10°) and 140° (-40°). The less effective procedure for this impact direction can be partially explained by angular measurement sensitivity of FBG sensors with respect to the incoming wave which is worst when the wave is propagating in perpendicular direction to the sensor.

![Figure 7. Rosette strain measurements (upper row) and changes in principal direction (bottom row) for impact from 90° direction (impact in point A). Shaded areas in the right column show changes caused by impact.](image)

Discussion and conclusion

The novel FGB rosette for the impact direction as well as the proposed method was tested in the experiment.

The reflection spectrum of the FBG sensor strongly depends on $\Delta n_{eff}$ and $L$. For experimental applications modulation depth of refractive index $\Delta n_{eff}$ should not be smaller than $10^{-4}$ and the grating length should be sufficiently long in order to obtain maximal peak reflectivity at least at the level of 50%.

Angular localization case of the impact on composite plate by the use of the notion of principal direction from rosette analysis presented here shows very good results for two impact locations, namely for those with the line of propagating disturbance more or less parallel to sensors. The greater error in calculating line of propagating disturbance is obtained for perpendicularly oriented sensors but even in this case the result is acceptable.

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