Calibration procedure of measuring system for vehicle wheel load estimation

M Kluziewicz\textsuperscript{1} and M Maniowski\textsuperscript{2}

\textsuperscript{1, 2} Cracow University of Technology, Poland
E-mail: mmaniowski@pk.edu.pl

Abstract. The calibration procedure of wheel load measuring system is presented. Designed method allows estimation of selected wheel load components while the vehicle is in motion. Mentioned system is developed to determine friction forces between tire and road surface, basing on measured internal reaction forces in wheel suspension mechanism. Three strain gauge bridges and three-component piezoelectric load cell are responsible for internal force measurement in suspension components, two wire sensors are measuring displacements. External load is calculated via kinematic model of suspension mechanism implemented in Matlab environment. In the described calibration procedure, internal reactions are measured on a test stand while the system is loaded by a force of known direction and value.

1. Introduction and goals
Modelling of vehicle motion requires implementing characteristics of examined tires \cite{1}. Among known methods of tire characteristics estimation the measurements can be conducted either on test benches or in the real road conditions using a test vehicle \cite{2, 8}. The author’s research is focused on a development of a tire-to-loose-surface interaction model which can be applied in a rally car dynamics analysis \cite{3, 4}. In a view of the best representation of tire work conditions, it is crucial to conduct appropriate road tests on a real gravel surface (figure 1).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.jpg}
\caption{Test vehicle side-slipping on a gravel surface.}
\end{figure}
The most commonly used devices for road and track testing are measuring hubs. Multi-component wheel hubs for a direct load measuring are commercially accessible, but extremely expensive, not suitable for tough working conditions and result in a change of suspension system parameters. Having regard to the disadvantages of available solutions, the authorial measuring system enabling indirect tire characteristics estimation has been designed [3]. This system exploits existing front wheel suspension of Macpherson strut type. One mechanism performs various roles: wheel guiding, shock absorbing and measuring reaction forces in suspension joints (Fig. 2).

Described approach bases on a measurement of internal forces (using strain gauges and load cells) and suspension mechanism configuration coordinates (mechanical and optical displacement sensors) [Estimation of wheel]. Relations between input and output data are described by kinetostatic model of the mechanism by using jacobian matrix and spatial transformations. Computer algorithm was implemented in Matlab environment.

Featured test vehicle is a front wheel drive, fully rally-prepared Citroen Saxo VTS (Fig.1). The main advantages of using rally car are: increased crew safety, ease of sensors assembly and simplicity of setup adjustments.

This paper is focused on a calibration method and bench tests of designed measuring system. Data calculated by estimation algorithm are compared to the measured ones.

\[ \text{Figure 2. Suspension mechanism of Citroen Saxo VTS front left wheel along with mounted sensors (left) and corresponding kinematic model (right).} \]

2. Estimation of tire model parameters
Tire model generates forces (F) and torques (M) in a contact patch between the tire tread and road surface in relation to different parameters, what can be expressed by the diagram (Fig.3).
There are various tire models describing aforementioned relation by using physical and/or empirical approaches [1]. According to the model complexity, number of its parameters varies from several to hundreds. Process of model parameters estimation can be carried out when a set of inputs and outputs (Fig. 3) is known from measurements, which ought to be conducted in a most similar to real work conditions.

External loads characteristics \( (F_x, F_y, M_z) \) in a contact patch between tire tread and road surface are assumed as in a following equation:

\[
F_x, F_y, M_z = f(F_z, \gamma, \alpha, S_x, \beta, T_b, r_d, \text{surface parameters})
\]  

(1)

where:
- \( F_x, F_y, M_z \) – Tire load components estimated by internal loads of suspension mechanism in a specific suspension travel position \( (s) \) and steering rack setting \( (p) \).
- \( F_z \) – Tire vertical load, measured indirectly through suspension travel and force in top mount.
- \( \gamma \) – Wheel camber angle, estimated by: known suspension dimensions, measured suspension travel \( (s) \), steering rack setting \( (p) \) and chassis roll angle.
- \( \alpha \) – Tire slip angle, estimated via longitudinal and lateral vehicle speed (Correvit QL) and steering wheel rotation angle (cable sensor).
- \( S_x \) – Longitudinal tire slip, estimated via vehicle speed, wheel rotation speed, and dynamic wheel radius.
- \( \beta \) – Vehicle sideslip angle, calculated from longitudinal and lateral vehicle speed.
- \( T_b \) – Tire tread temperature, measured with pyrometer gauge.
- \( r_d \) – Dynamic wheel radius, estimated via axial force in a damper module.

**Surface parameters** such as loose layer depth, size distribution, moisture content, soil volume density.

For a specified mechanism setting, defined by suspension travel \( (s) \) and steering rack position \( (p) \), static task can be described by the following equation:

\[
W = J^T R
\]

(2)

where:
- \( W = [F_x, F_y, M_s, M_y, M_z]^T \) – complex load vector translated to the wheel’s centre of rotation,
\[ R = [R_1 \ R_2 \ R_3 \ R_4 \ R_5 \ R_6]^T \] – internal loads vector: \( R_1 \) – reaction in a front arm rod joint; \( R_2 \) – reaction in a rear arm rod joint; \( R_3 \) – reaction in a steering rod; \( R_4, R_5, R_6 \) – longitudinal, lateral and vertical reactions in a strut top mount,

\( J \) – jacobian matrix for static analysis of parallel mechanism [5].

Equation (2) describes linear relations between unknown external load (W) and measured internal load (R) in mechanism. Forces and torques in a centre of wheel rotation are related with joints reactions by the jacobian matrix. Jacobian matrix is determined for a specific suspension travel position (s) and steering rack setting (p).

In order to measure a tire load the wheel suspension mechanism was adapted. A kinematic model of MacPherson strut suspension (Fig.4) with rack-and-pinion steering system was formulated according to the following assumptions:

- the wheel knuckle is joint by spherical pairs with: lower wishbone (5, 6), tie rod (7) and top mount (8) in the strut end;
- two degrees of the wheel knuckle mobility correspond to the wheel bounce (controlled by the strut height \( h \)) and turn motions (controlled by the steering rack displacement \( p \));
- the lower wishbone, due to a special design, can be treated as a composition of two rods (5, 6) with spherical pairs at its ends;
- direct kinematic task is solved using a vector method [5];
- joints and links are ideal;
- internal reactions in the suspension are determined under static conditions;
- following vector transformations are written instantaneously with respect to the wheel reference system.

Cartesian coordinates \( (k) \) of the wheel reference system are the following:

\[
\begin{bmatrix} b_x \\ b_y \\ b_z \\ \beta_x \\ \beta_y \end{bmatrix} \]

where:

- \( b \) – position vector of the wheel centre [m];
- \( \beta \) – vector of the wheel orientation angles [rad].

According to the considered suspension kinematic structure (Fig.2), the wheel knuckle is constrained by the following coordinates:

\[
q = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ a_{4x} \\ a_{4y} \end{bmatrix} \] (4)

These coordinates (4) describe blocked motions, due to: constant lengths of the three rods \( (d_i, i=1..3) \) and unchangeable position of the top mount spherical pair \( (a_{4x}, a_{4y}, a_{4z}) \). Each rod \( (i=1..3) \) of the wheel suspension is described by the following vectors:
\[ d_i = b_i - a_i \]  
(5)

with the length:
\[ d_i = \|d_i\| \text{[m]} \]
(6)

and the unit vector:
\[ \hat{a}_i = \frac{d_i}{d_i} \]
(7)

Kinematic relation between virtual changes of Cartesian (3) and constraints (4) coordinates, can be written as [5]:
\[ \partial q = J \partial k \]
(8)

Jacobian matrix \( J \) of parallel mechanism can be defined as twist coordinates (each row) of the constraints, as follows:
\[
J = \begin{bmatrix}
[\hat{d}_1]^T & [b_1 \times \hat{d}_1]^T \\
[\hat{d}_2]^T & [b_2 \times \hat{d}_2]^T \\
[\hat{d}_3]^T & [b_3 \times \hat{d}_3]^T \\
1 & -\varepsilon_z & \varepsilon_y \\
\varepsilon_z & 1 & -\varepsilon_x \\
-\varepsilon_y & \varepsilon_x & 1
\end{bmatrix}_{6x6}
\]
(9)

where:
\[ \bar{a}_4 = \begin{bmatrix}
0 & -a_{4,z} & a_{4,y} \\
a_{4,z} & 0 & -a_{4,x} \\
-a_{4,y} & a_{4,x} & 0
\end{bmatrix} \text{(skew symmetric matrix)} \]

\( \varepsilon_x, \varepsilon_y, \varepsilon_z \) – small deviation angles [rad] of force sensor orientation in the top mount (around \( x, y \) and \( z \) axes accordingly).

Jacobian matrix (9) depends on the mechanism geometry and is determined for a specific suspension travel (\( b_z \)) and steering rack position (\( p \)).

Forces and torques generated in the tire tread patch are collected in a spatial load vector:
\[ W_t = [F_x F_y F_z M_x M_y M_z]^T \]
(10)

This load (8) for quasi-static conditions can be transferred to the wheel center, by the formula:
\[ W = H W_t \]
(11)

\[ H = \begin{bmatrix}
[1]_{3x3} & [0]_{3x3} \\
[0]_{3x3} & [1]_{3x3}
\end{bmatrix} \]
(12)

\[ \hat{r} = \begin{bmatrix}
0 \\
0 \\
-r_d
\end{bmatrix} \]
(13)

\( r_d \) – dynamic tire radius [m].
According to the assumed constraints (4) in the suspension mechanism, there are 6 reaction forces (Fig.5), which are grouped in the vector:

\[ \mathbf{R} = [R_1, R_2, R_3, R_4, R_5, R_6]^T \]  \hspace{1cm} (14)

where:

- \( R_1 \) – force in the front rod [N];
- \( R_2 \) – force in the rear rod [N];
- \( R_3 \) – force in the steering rod [N];
- \( R_4, R_5, R_6 \) – longitudinal, lateral and vertical reactions in strut top mount [N].

Applying the virtual work principle, which describes balance of works done by an external and internal loads, it can be stated:

\[ \mathbf{W}^T \partial \mathbf{k} = \mathbf{R}^T \partial \mathbf{q} \]  \hspace{1cm} (15)

Transforming equation (15) by applying kinematic relations (8), the static task can be written as:

\[ \mathbf{W} = \mathbf{f}^T \mathbf{R} \]  \hspace{1cm} (16)

Equation (16) describes linear relations between unknown external wheel load (\( \mathbf{W} \)) and the mechanism internal load (\( \mathbf{R} \)), that can be measured.

### 3. Measuring system calibration

First step in calibration procedure was to obtain sensitivity factors for strain gages stuck to suspension arm rods and steering rod. For this purpose mentioned suspension components were coupled with hydraulic cylinder and force transducer HBM U9B inside of steel frame. While axial tensile or compression force was applied, the relation between voltage of strain gauge bridge diagonal and force measured by reference sensor was recorded. Basing on this data, slope and y-intercept of the regression line was calculated. Calibration process of three-component piezoelectric force sensor (McPherson strut top-mount) was presented in previous paper [6].
Figure 4. Scheme of test bench for side load application.

After assembly of complete measuring system vehicle was set on a test bench presented on drawing 2. Front left wheel was standing on a wheel alignment turntable, placed on an electronic scale. Turntable was attached by a force transducer to a hydraulic cylinder directed orthogonally to vehicle longitudinal axis. Relation between reaction forces in suspension mechanism \((R_1, R_2, R_3, R_4, R_5, R_6)\) and extortion force \((F_y)\) combined with derived torque \((M_x)\) at a determined vertical load \((F_z)\) was measured. Friction force of turntable movement was modelled in accordance with Coulomb friction law. Graphical plot of reaction forces as a function of variable side load is presented on drawing 5. Thin dashed lines represent the results of three tests, whereas data obtained from calculation model are plotted as a solid lines.

Most significant values of reaction forces were measured in: McPherson strut topmount (green colour, result of the vehicle mass), front arm rod (red) and rear arm rod (orange). Relations between reaction forces and extortion force in examined range are linear.
Substantial accordance in functions graph can be a proof of proper formulation of mathematical task and valid kinematic model. Hysteresis loops are the effect of dry friction occurring in turntable under the wheel.

Similar tests were conducted with longitudinal load on a front wheel. In this case hydraulic cylinder was placed in a front of tire contact patch, parallel to vehicle longitudinal axis. Measured reaction forces ($R_1 \ R_2 \ R_3 \ R_4 \ R_5 \ R_6$) are result of longitudinal extortion force ($F_x$) combined with derived torque ($M_y$) at a determined vertical load ($F_z$).
Figure 7. Plot of internal reactions as a functions of external longitudinal load.

Most significant reactions were measured in: rear arm rod (orange), front arm rod (red), McPherson strut topmount (green). Other components ($R_3$, $R_4$, $R_5$) are changing slightly (Fig. 7). Calculated reactions are close to the real ones. Relations between external load and internal forces are linear in the examined range.

4. Conclusions

The innovative method of measuring wheel load by using suspension mechanism is described. Forces and torques acting on a rotating wheel are estimated basing on internal forces and configuration coordinates of suspension mechanism. The relation of aforementioned input and output data featuring jacobian matrix is presented along with equation derivation and evaluation procedure.

Good correspondence between calculated and measured reactions proves the usefulness of designed system. Relations between input and output data are linear as it was expected. Thorough calibration of all measuring devices and introduction of exact mechanism dimensions to the kinematic model are crucial for the proper functioning of whole system.

An accuracy of developed estimation method is desired to be at level of 5%. Thereby system can be successfully used for tire characteristics measurement and tire model development. According to carried uncertainty analysis [7] and results of calibration, specified accuracy is very probable to be achieved.

Presented actions are just a single part of system verification. With the aim of estimating wheel load on the basis of measured internal reactions, exactly the opposite task should be solved, which requires inversion of jacobian matrix. Results of full estimation during road test will be presented in next paper.
References

[1] Pacejka H B 2002 Tyre and Vehicle Dynamics, Butterworth-Heinemann, (Oxford)

[2] Kluziewicz M and Maniowski M 2012 Porównanie efektywności wyścigowej i rajdowej techniki pokonania luku samochodem przednionapędowym, Czasopismo techniczne 5-M/2012, Zeszyt 10, Rok 109.

[3] Kluziewicz M and Maniowski M 2014 Estimation of wheel load state by using suspension joints reactions, Badania Pojazdów, praca zbiorowa pod redakcją Władysława Mitiańca, Opracowanie Monograficzne, p 61 (Cracow)

[4] Kluziewicz M and Maniowski M 2010 Stany nadsterowności podsterownego samochodu z przednim napędem Zeszyty Naukowe Instytutu Pojazdów Politechniki Warszawskiej, Vol. 1(77), pp. 169-177

[5] Knapczyk J and Maniowski M 2006 Elastokinematic Modeling and Study of Five-Rod Suspension with Subframe, Mechanism and Machine Theory, Vol. 41, pp. 1031–1047,

[6] Kluziewicz M 2015 Experimental verification of kinetostatic model of strut suspension mechanism, Technical Transactions i. 7. Mechanics i. 2-M, Wydawnictwo PK (Cracow)

[7] Kluziewicz M and Maniowski M 2016 Uncertainty analysis of innovative method for wheel load measurement, Mechatronics: Ideas, Challenges, Solutions and Applications, Vol. 414, Springer

[8] Luty W 2014 Badania eksperymentalne ogumienia w nieustalonych warunkach znoszenia bocznego, The Archives of Automotive Engineering, Vol. 66, nr 4, p 133