Violations of Locality Beyond Bell’s Theorem

Zeng-Bing Chen, Sixia Yu, and Yong-De Zhang

Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230027, China

Locality and realism are two main assumptions in deriving Bell’s inequalities. Though the experimentally demonstrated violations of Bell’s inequalities rule out local realism, it is, however, not clear what role each of the two assumptions solely plays in the observed violations. Here we show that two testable inequalities for the statistical predictions of two-qubit systems can be derived by assuming either locality or realism. It turns out that quantum mechanics respects a nonlocal classical realism, and it is locality that is incompatible with experimental observations and quantum mechanics.

PACS numbers: 03.65.Ud, 03.65.Ta, 03.67.-a

In their famous paper, Einstein, Podolsky and Rosen (EPR) argued that quantum mechanics (QM) is incomplete according to their criterion of reality based on an implicit assumption of locality. In 1964 Bell derived the celebrated Bell inequalities (BI), enabling quantitative tests of QM versus local realism. The derivation of BI requires mainly the realism and locality assumptions, supplemented also by some other auxiliary assumptions (e.g., no advanced actions). So far, many experiments testing Bell’s theorem completely confirmed QM though certain technical loopholes still exist. Accepting the auxiliary assumptions, the experimental violations of BI necessarily imply that at least one of the two main assumptions underlying BI should be abandoned. Then between locality and realism, which is (or, are both) incompatible with QM?

On the one hand, the locality assumption is, at the first glance, protected by the special theory of relativity. Its correctness can hardly be questioned. On the other hand, realism underlies classical physics as a part of the world view. It implies that there exists a world that is objective and independent of any observations. While EPR’s criterion of physical reality is certainly respected by classical physics, its status in QM seems to be questionable due essentially to the complementarity principle. In quantum mechanical terms, “No elementary phenomenon is a phenomenon until it is a registered (observed) phenomenon”, namely, what is observed on a quantum system is dependent upon the choice of experimental arrangements/contexts.

It should be emphasized that all experiments (the “Bell experiments”) performed so far to test Bell’s theorem (with or without inequalities) always test locality and realism jointly and thus, merely ruled out local realism, but neither locality nor realism alone. Concerning the experimental violations of BI, different attitudes arise in the literature. Some people like to hold the view that QM is nonlocal (though such a nonlocality (“Bell’s nonlocality”) of QM can only be understood in the context of Bell’s theorem). For instance, Stapp argued that QM is a nonlocal theory. This assertion is based on some counterfactual reasonings and arises active controversies. Meanwhile, for other people it seems to be more natural to give up realism. Thus, rejecting locality or realism becomes again one’s philosophical taste, a situation very similar to the time before the publication of Bell’s work, when choosing local realism or QM is a matter of taste. Moreover, while biparticle entangled pure states of any dimensionality always lead to certain violation of BI, the relationship between entanglement and Bell’s nonlocality for the mixed states is very puzzling and remains one of the most important open questions in the field.

In this Letter, we show that, by changing dramatically the usual way we think about the “Bell paradigm”, the separate role of the locality or realism assumption can be tested for statistical predictions of QM by two inequalities, which are derived by only assuming either locality or realism. For the usual Bell experiments with two orthogonal settings per site, locality alone can lead to contradictions with experimental observations and QM, while realistic theories can always reproduce quantum mechanical predictions.

Obviously, to test QM versus realism and versus locality separately, a new falsifiable formulation beyond Bell’s theorem is required. To this end, one needs first to specify the meanings of realism and locality in physical terms. In modern understanding, EPR’s criterion of realism is usually implemented with classical hidden-variable models. Meanwhile, locality means that the experimental results obtained from a physical system at one location should be independent of any observations or actions made at any other spacelike separated locations. Previous prescription on the locality assumption was, unfortunately, considered only within local realistic theories. Recently, we suggested a generic locality condition that is imposed only on probabilities that are observable for localists. The condition is independent upon any theory (realistic or quantum); different theories differ only from their ways of assigning the probabilities appearing in the locality assumption.

The experimental configuration we have in mind is the same as that used in deriving the usual BI. Namely, one considers an ensemble of pairs of two-level systems A and
B, which are sent, respectively, to two spacelike separated observers, Alice and Bob. The two-level systems can be physically implemented by, e.g., spin-1/2 particles or photons with two alternative polarizations. For definiteness, here we consider the spin-1/2 particles. In QM, Alice and Bob need to measure, respectively, \( \mathbf{a} \cdot \sigma^A \equiv \hat{a} \) and \( \mathbf{b} \cdot \sigma^B \equiv \hat{b} \) to obtain the statistical correlations \( E(\mathbf{a}, \mathbf{b}) \) between the two systems. Here \( \sigma^A \) (\( \sigma^B \)) is the Pauli spin operator of Alice’s (Bob’s) particles, and \( \mathbf{a} \) and \( \mathbf{b} \) are two arbitrary unit vectors (experimental settings). QM tells us that the observed values of \( \hat{a} \), \( \hat{b} \) and \( \hat{a} \hat{b} \) can only be \( \pm 1 \) as \( \hat{a}^2 = I^A \), \( \hat{b}^2 = I^B \) and \( (\hat{a}\hat{b})^2 = I^A \otimes I^B \), with \( I \) being the unit operator. QM predicts the correlations

\[
E_{QM}(\mathbf{a}, \mathbf{b}) = \text{Tr}[\rho_{AB}\hat{a}\hat{b}],
\]

where \( \rho_{AB} \) are the states of A and B.

How does a localist interpret the observed correlations (if any)? Obviously, events occurring in the backward light cone of a particle (e.g., particle A or B) may affect the events occurring on the particle. Particularly, events occurring in the backward light cone of the two particles may be “common causes” of the events occurring on A and B, though events occurring on A should not be causes of events occurring on B (vice versa). Denote the joint probability of getting outcomes \( a_i (= \pm 1) \) and \( b_j (= \pm 1) \) as \( P(a_i, b_j) \). Whenever there are correlations in the observed \( P(a_i, b_j) \), the localist may interpret the correlations being solely coming from the common causes. Then the locality assumption reads [18]

\[
P(a_i, b_j) = \sum_{\mu} P(a_i | \mu) P(b_j | \mu) P(\mu),
\]

where the first line is a simple fact in theory of conditional probability. Here the summation may also mean integration, if necessary; \( P(\cdot | \mu) \) are the probabilities conditioned on a given common cause (labelled by \( \mu \)): \( P(\mu) \geq 0 \) are the probabilities for the given cause \( \mu \) to occur, and \( \sum_{\mu} P(\mu) = 1 \). Thus, the given common cause can affect the probabilities with regard to particles A and B; conditioned on the same cause, observable probabilities for A and B must be mutually independent, as required by locality. The correlations predicted by any local theory (LT) are thus

\[
E_{LT}(\mathbf{a}, \mathbf{b}) = \sum_{a_i, b_j} a_i b_j P(a_i, b_j) = \sum_{\mu} P(\mu) \bar{a}_\mu \bar{b}_\mu,
\]

where \( \bar{a}_\mu = \sum_{a_i} a_i P(a_i | \mu) \) with \( |\bar{a}_\mu| \leq 1 \) and \( \bar{b}_\mu = \sum_{b_j} b_j P(b_j | \mu) \) with \( |\bar{b}_\mu| \leq 1 \). If whatever the common causes are, the correlations \( E_{LT}(\mathbf{a}, \mathbf{b}) \) cannot be explained by local predictions [19], then they are nonlocal.

The locality condition [2] imposes constrains merely on observable probabilities. Particularly, localists may reasonably argue that the common causes are not anything that is mysterious or “hidden”; instead, they are experimentally observable and distinguishable (at least in principle) to account for the observed correlations. For instance, they can be random-number generators producing numbers \( \mu \) with probabilities \( P(\mu) \), which are held in a preparing device creating the statistical ensemble under study. In this way, one can exclude any assumption other than locality. A theory has the power of making predictions. Thus, using either QM or RT, one can predict the probabilities in [2]. There are then two other facts supporting [2] as a generic locality condition.

First, we proved [18] recently that for spacelike separated systems, Eq. [2] is obeyed iff the states of the two particles are separable [i.e., entangled states possess quantum nonlocality in the sense of violating the locality condition [2]]. Thus, a local quantum theory (LQT, i.e., QM+locality) predicts

\[
E_{LQT}(\mathbf{a}, \mathbf{b}) = \sum_{\mu} P(\mu) \text{Tr}[\rho_{AB}\hat{a}\hat{b}],
\]

where \( \rho_{AB} \) (\( \rho_{B\mu} \)) are the local density operators conditioned on the common cause \( \mu \) such that \( \bar{a}_\mu = \text{Tr}[\rho_{A\mu}\hat{a}] \) and \( \bar{b}_\mu = \text{Tr}[\rho_{B\mu}\hat{b}] \) (see Eq. [3]).

Second, if the two particles in question are described by a classical realistic theory (RT), Eq. [2] becomes Bell’s locality condition which has been well justified in various aspects in the context of BI [4, 5, 6] and now is widely accepted. To see this, recall that in an RT, the probabilities in [2] are determined by the experimental settings and by a set of hidden variables, denoted collectively by \( \lambda \), and as such \( P(a_i, b_j) = \sum_{\mu} \int d\lambda p(\lambda) P(\lambda | a_i) P(\lambda | b_j) P(\lambda) \), where \( p(\lambda) \geq 0 \) is a normalized probability distribution of \( \lambda \). If one formally identifies the common causes as a part of the hidden variables, then Bell’s locality condition [4, 5, 6] can be obtained. The correlations predicted by local realistic theories (LRT) are then

\[
E_{LRT}(\mathbf{a}, \mathbf{b}) = \sum_{\mu} \int d\lambda p(\lambda) P(\lambda | a_i) A_\mu(\mathbf{a}, \lambda) B_\mu(\mathbf{b}, \lambda),
\]

where \( A_\mu(\mathbf{a}, \lambda) = \sum_{a_i} a_i P(\lambda | a_i) \) with \( |A_\mu(\mathbf{a}, \lambda)| \leq 1 \) and \( B_\mu(\mathbf{b}, \lambda) = \sum_{b_j} b_j P(\lambda | b_j) \) with \( |B_\mu(\mathbf{b}, \lambda)| \leq 1 \). However, an RT without assuming locality predicts

\[
E_{RT}(\mathbf{a}, \mathbf{b}) = \sum_{\mu} \int d\lambda p(\lambda) P(\lambda | \mu) \Gamma_\mu(\mathbf{a}, \mathbf{b}; \lambda),
\]

with \( \Gamma_\mu(\mathbf{a}, \mathbf{b}; \lambda) = \sum_{a_i, b_j} a_i b_j P(\lambda | a_i, b_j | \mu) \) and \( |\Gamma_\mu(\mathbf{a}, \mathbf{b}; \lambda)| \leq 1 \). Clearly, the same locality assumption [2] is underlying both [3] and [4], where QM and realism differ from their distinct ways of assigning probabilities (or measured results) for the same quantities. Now our task is to deduce the consequences for each of the five theories (LT, RT,}
LRT, LQT and QM) in the Bell experiments, to see if there are testable quantitative differences among them.

To obtain the required inequalities, Alice (Bob) needs to measure at another direction \(a_{\perp} \perp a\) (\(b_{\perp} \perp b\)), namely, we are concerned with the inequalities with two orthogonal settings per site. Then consider the following combinations of the correlation functions:

\[
E(a, b_{\perp}) + E(a_{\perp}, b) \equiv X, \quad E(a, b) - E(a_{\perp}, b_{\perp}) \equiv Y.
\]

In terms of \(X\) and \(Y\) QM predicts the following inequality for all two-spin states (entangled or not)

\[
X_{QM}^2 + Y_{QM}^2 \leq 4, \quad (7)
\]

which can, actually, be proved by using the Heisenberg-Robertson uncertainty relation for the composite system. Using the facts that \(|\Gamma_\mu(a, b; \lambda)|\), \(|\Gamma_\mu(a_{\perp}, b_{\perp}; \lambda)|\), \(|\Gamma_\mu(a_{\perp}, b; \lambda)|\), and \(|\Gamma_\mu(a, b_{\perp}; \lambda)|\) \(\leq 1\), it can be proved that \(|X_{RT}| \leq \sum_\mu \int d\rho(\lambda)\rho_\lambda(\mu) |\Gamma_\mu(a, b_{\perp}; \lambda)| \leq 2\), and similarly \(|Y_{RT}| \leq 2\). Thus, the inequality imposed by realism alone is

\[
|X_{RT}| \leq 2, \quad |Y_{RT}| \leq 2. \quad (8)
\]

Note that in a RT, all the four \(\Gamma_\mu\) functions used above can be mutually independent. So no lower bound exists for Eq. \(8\).

From \(9\), one can obtain the “locality inequality” satisfied by any local theory

\[
|X_{LT} \pm Y_{LT}| \leq 2 \quad (9)
\]

due to the fact that \(|(\hat{a}_\mu \pm a_{\perp}\mu)\hat{b}_\mu + (\hat{a}_\mu \mp a_{\perp}\mu)\hat{b}_{\perp}\mu| \leq 2\). Particularly, the BI (due to Clauser, Horne, Shimony and Holt \(8\)) imposed by any local realistic theory reads

\[
|X_{LRT} \pm Y_{LRT}| \leq 2, \quad (10)
\]

for which no tighter bound exists.

However, an LQT predicts an inequality (the “quantum locality inequality”)

\[
X_{LQT}^2 + Y_{LQT}^2 \leq 1, \quad (11)
\]

which is stronger than the locality inequality \(9\). The proof of the inequality \(11\) is easy. Using Eq. \(11\) and the property \(10\) of \(X_{LQT}^2\) and \(Y_{LQT}^2\) being convex functions of local density operators, it suffices to prove the validity of \(11\) for \(\rho_{AB} = \rho_A\rho_B\). Denoting \(\text{Tr}[\rho_A\hat{a}] = \langle \hat{a} \rangle_A\) and \(\text{Tr}[\rho_B\hat{b}] = \langle \hat{b} \rangle_B\), one has \(X_{LQT}^2 + Y_{LQT}^2 = \langle \hat{a} \rangle_A^2 + \langle \hat{a}_{\perp} \rangle_A^2 \langle \hat{b} \rangle_B^2 + \langle \hat{b}_{\perp} \rangle_B^2\) \(\leq 1\), where \(\langle \hat{a} \rangle_A^2 + \langle \hat{a}_{\perp} \rangle_A^2 \leq 1\) and \(\langle \hat{b} \rangle_B^2 + \langle \hat{b}_{\perp} \rangle_B^2 \leq 1\) have been exploited and are direct consequences of the Heisenberg-Robertson uncertainty relation for each subsystem A/B. In the present case the uncertainty relation gives, e.g., for Alice’s particle (1 - \(\langle a \rangle_A^2\)(1 - \(\langle a_{\perp} \rangle_A^2\)) \(\geq \langle (a \cdot a_{\perp}) \cdot \sigma_A \rangle_A^2 + \langle a_{\perp} \rangle_A^2\) yielding \(\langle a_{\perp} \rangle_A^2 \leq \langle a \rangle_A^2 + \langle a_{\perp} \rangle_A^2 \leq \langle a \cdot a_{\perp} \cdot \sigma_A \rangle_A^2 \leq 1\).

![Diagram showing the five inequalities in the X-Y plane.](image)

Since the inequalities \(7\) make their predictions on the same experiments, they can be summarized in a single diagram shown in Fig. 1. Seen from the diagram, there is an interesting relation among the predictions of the five theories

\[
\text{RT} \supset \text{QM} \supset \text{LT}/\text{LRT} \supset \text{LQT}. \quad (12)
\]

Note that the inequalities \(7\) and, thus, the relation \(\text{RT} \supset \text{QM} \supset \text{LT}/\text{LRT}\) are still valid for the cases where the two settings for each site are not orthogonal. The relation \(12\) implies that, e.g., all QM predictions are also predicted by RT though being essentially classical, but the RT predicts something more than QM. Thus, for the system in question all predictions do allow to be interpreted by certain classical hidden-variable model, which must be of a nonlocal nature as a price \(\text{RT}\). In other words, realism (without assuming locality) in itself is not excluded by QM. However, accepting that QM is correct (and very unlikely to be wrong for systems as simple as two-level ones), it is impossible to observe the conflict between RT and QM in the Bell experiments.

The relation \(12\) definitely shows that there are two qualitatively different nonlocality. Any observed correlation that does not satisfy the locality condition \(12\) is nonlocal. When the probabilities in \(12\) are quantum predictions, \(12\) can be reasonably called the “quantum locality condition”, whose violation indicates quantum nonlocality (“quononlocality” for short). It is quononlocality that is proved to be equivalent to entanglement for spacelike separate quantum systems \(13\). The fact that the locality inequality \(9\) and the BI \(10\) take the same form implies that the Bell experiments performed to test \(10\) actually ruled out all local theories (including LRT and LQT) and proved nonlocality of nature for statistical predictions of QM. The distinct trends of locality and realism in Fig. 1
show that realism can mask quononlocality. We think this is the reason why the relation between nonlocality and entanglement is such a notoriously difficult issue when being seen in the context of BI. Indeed, there exist two-partite entangled states (the Werner states) with “hidden nonlocality” (more precisely, hidden quononlocality), hidden by realism so much that it cannot be uncovered by any BI. All states in the region constrained by $|X_{LRT}| + |Y_{LRT}| \leq 2$ and $X_{LQT}^2 + Y_{LQT}^2 > 1$ in Fig. 1 possess hidden quononlocality.

Let us consider the experimental settings under which the locality inequality is violated by QM. It is well known that the bound of BI [10] [and thus, (9)] allowed by QM is $2\sqrt{2}$, known as the Cirel’son bound [20]. However, the 2-setting quantum locality inequality has the bound 4 of maximal violation [see 7] and can reveal a much stronger violation allowed by QM.

Choose the spin singlet state $\langle \psi^- \rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B)$, where $|\uparrow\rangle$ ($|\downarrow\rangle$) is the spin-up (spin-down) state. Then $E_{QM}(a, b) = \langle \psi^- | a b | \psi^- \rangle = -a \cdot b$ gives $X_{QM}^2 + Y_{QM}^2 = (a \cdot b_{⊥} + a_{⊥} \cdot b) + (b_{⊥} - a_{⊥} \cdot b_{⊥})^2$. The maximal bound 4 allowed by QM can easily be attained by choosing the angles from $a_{⊥}$ to $b_{⊥}$ and from $b_{⊥}$ to $a_{⊥}$ to be $\pi/4$. Then for the Werner state [16, 21, 22] $\rho_W = \frac{1}{2} (1 - x) + x |\psi^- \rangle \langle \psi^-|$ (here $1 > x > 0$) and the same settings chosen above, $|X_{QM}| + |Y_{QM}| \leq 2\sqrt{2}x$ implies that when $x > 1/\sqrt{2}$, (9) or (10) is violated by QM. It is already known that for $x = 1/3$ the Werner state $\rho_W$ is entangled [21, 22], i.e., has quononlocality. Therefore, there is hidden quononlocality for $1/\sqrt{2} \geq x > 1/3$ that does not lead to any violation of BI or the locality inequality. Meanwhile, under the same condition $X_{QM}^2 + Y_{QM}^2 = 4x^2$; when $x > 1/2$, the quantum locality inequality is violated by $\rho_W$. Thus, (11) shows a sharper contradiction with QM than (9) or (10).

The fact that the quantum locality inequality cannot fully uncover quononlocality in $\rho_W$ means that its violation is only a sufficient condition for quononlocality, but not a necessary one. Yet, the quantum counterpart of the locality condition is necessary and sufficient for separability of states for spacelike separated systems and thus, all entangled states possess quononlocality. We mention that a necessary and sufficient condition of separability of states for the Bell experiments with three mutually orthogonal settings per site was found recently for two-qubit systems [24].

To summarize, we have established a hierarchy [see Eq. (12) and Fig. 1] of five kinds of theories (RT,QM,LT,LRT and LQT) for the usual Bell experiments with two orthogonal settings per site. The hierarchy enables separate experimental tests of QM versus locality beyond Bell’s theorem. It also sheds new light on the role of locality or realism in the experimental violations of BI and the relationship between entanglement and Bell’s nonlocality. The quantum locality inequality is useful for detecting genuine quononlocality and might find interesting applications in quantum information processing. For instance, for the EPR protocol of quantum cryptography may lead to better test of eavesdropping. Interestingly, violation of locality without inequalities for multiparticle Greenberger-Horne-Zeilinger states can also be proved and will be reported elsewhere. Finally, we stress that (quantum) nonlocality (or, entanglement) cannot be used for superluminal communication.

We thank Jian-Wei Pan and Nai-Le Liu for stimulating discussions. This work was supported by the National NSF of China under Grant No. 10104014, the CAS and the National Fundamental Research Program under Grant No. 2001CB309300.

* Electronic address: zbchen@ustc.edu.cn

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).
[2] J.S. Bell, Physics (Long Island City, N.Y.) 1, 195 (1964).
[3] J.F. Clauser, M.A. Horne, A. Shimony and R.A. Holt, Phys. Rev. Lett. 23, 880 (1969).
[4] J.S. Bell, Speakable and Unspeakable in Quantum Mechanics (Cambridge University Press, Cambridge, 1987).
[5] R.F. Werner and M.M. Wolf, Quantum Inf. Comput. 1 (3), 1 (2001).
[6] A.G. Valdenebro, Eur. J. Phys. 23, 569 (2002).
[7] A. Aspect, Nature (London) 389, 189 (1999).
[8] J.-W. Pan et al., Nature (London) 403, 515 (2000).
[9] P. Grangier, Nature (London) 409, 774 (2001).
[10] J.A. Wheeler and W.H. Zurek, Quantum Theory and Measurement (Princeton University Press, Princeton, New Jersey, 1983).
[11] F. Lalöe, Am. J. Phys. 69, 655 (2001).
[12] H.P. Stapp, Am. J. Phys. 65, 300 (1997).
[13] See, e.g., N.D. Mermin, Am. J. Phys. 66, 920 (1998); A. Shimony and H. Stein, ibid 69, 848 (2001); H.P. Stapp, ibid 69, 854 (2001); W. Unruh, Phys. Rev. A 59, 126 (1999); H.P. Stapp, ibid A 60, 2595 (1999); W. Unruh, ibid A 60, 2599 (1999).
[14] S.L. Braunstein and C.M. Caves, Phys. Rev. Lett. 61, 662 (1988).
[15] N. Gisin and A. Peres, Phys. Lett. A 162, 15 (1992).
[16] R.F. Werner, Phys. Rev. A 40, 4277 (1989).
[17] S. Popescu, Phys. Rev. Lett. 74, 2619 (1995); N. Gisin, Phys. Lett. A 210, 151 (1996); M. Żukowski et al., Phys. Rev. A 58, 1694 (1998); A. Peres, ibid 54, 2685 (1996).
[18] Z.-B. Chen, S. Yu, Y.-D. Zhang, and N.-L. Liu, to be published.
[19] J. Uffink, Phys. Rev. Lett. 88, 230406 (2002); S. Yu, Z.-B. Chen, J.-W. Pan, and Y.-D. Zhang, ibid 90, 080401.
[20] B.S. Cirel’son, Lett. Math. Phys. 4, 93 (1980).
[21] A. Peres, Phys. Rev. Lett. 77, 1413 (1996).
[22] M. Horodecki et al., Phys. Lett. A 223, 1 (1996).
[23] S. Yu, J.-W. Pan, Z.-B. Chen, and Y.-D. Zhang, quant-ph/0301030.
[24] A.K. Ekert, Phys. Rev. Lett. 67, 661 (1991).
[25] D.M. Greenberger, M.A. Horne, A. Shimony, and A. Zeilinger, Am. J. Phys. 58, 1131 (1990).