Constrained model based control for minimum-time start of hydraulic turbines

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Abstract. This paper introduces a simplified model of hydraulic turbines including the hydraulic nonlinear hill-chart and a first order model of the penstock. Based on the resulting reduced model, a graphical representation of the vector fields of the resulting controlled system is obtained under band unlimited actuator. This ideal 2D-graphical representation enables an exact evaluation of the lower bound on the minimum achievable start-time as well as the time structure of the control profile. The consequences of the use of a band-limited actuator is also analyzed enabling a close estimation of the lower bound on the start time in practical situations.

Introduction

In a pump storage plant (PSP), a sole runner enables both turbining water from an upstream tank to a downstream reservoir and pumping water from the downstream reservoir to the upstream tank. Therefore PSPs provide an excellent energy storage solution that is necessary for widespread use of renewable intermittent sources. The use of hydraulic storage induces however some new paradigms. The first that is widely addressed by hydraulic engineers, is the efficiency enhancement of conversion during steady flow operations, like in [1], [2]. The second is the need for fast transient when switching between its different modes (see Figure 1). The frequency of these switches increases when the hydraulic is used to compensate for intermittency [3]. This especially concerns the operations of start-up in turbine and pump modes.

This paper is focused on the start-up in turbine mode. This is the time necessary to drive the turbine from rest towards the connection-to-grid rotational speed. In our case of study, this sequence is critical because of the use of synchronous machines – without electric transformation – in the PSP. This topology is more often used for hydroelectric production because it needs less structural work (is therefore cheaper) and provides best efficiency than with an asynchronous machine. But the design of synchronous machines prevents decoupling between output voltage frequency and rotational speed. Consequently, the coupling to the grid must be done carefully as any discrepancy between the grid voltage and output voltage of the machine at connection time can result in strong current that may be harmful to the machine. At General Electric (GE), and for such big machines, three criteria must be respected before coupling the machine to the grid:
Figure 1. Sequences and operating modes of a Pump Storage Plant. In this paper, the optimization of the start-up in turbine mode sequence (in red) is addressed.

- The rotational speed $N$, that is proportional to the output voltage frequency, is in a $\pm 0.2\%$ band around its final desired value.
- The output voltage amplitude is the same as the grid voltage amplitude.
- The phase between the output voltage and grid voltage, and its evolution, are contained.

The voltage and phase equality being done on shorter time scales by an independent controller, only the first above mentioned issue is considered in this paper.

The start-up optimization method developed in this paper is related to model control. In order to provide accurate results, a work on the modelisation of the plant has been done. Unlike in [4], [5], and [6], that reuse the model derived by Kundur in [7], we express a more realistic dynamic of the penstock with a first order transfer function. We also provide the full expression of the runner’s dynamic through the use of its hill-chart. We can then fully address its non-linearity at every moment whereas [5] and [8] use a linearization of this characteristic at some operating points in order to provide robust control or PID gain scheduling. The method developed in this paper holds its strength from two points: first, the original modelisation of the penstock dynamic and secondly, the use of the complete hill-chart characteristic that describe the precise behavior of the runner at any time. We aim at providing a good accuracy modelisation in order to elaborate constrained-optimal start-up control.

This paper is organized as follows: The dynamic model of the plant that is first derived in section 1. The control problem is stated in section 2. The control design is proposed in section 3 first in the case of ideal actuator and then in the presence of limitation on the actuators. Section 4 summarizes the paper and gives hints for further investigation.

1. Mathematical Modeling

We consider the PSP depicted in Figure 1. It is assumed that the hydroelectric machinery is located along a linear and constant section penstock. In most cases, this approximation can
be made true by calculating an equivalent linear and constant section penstock. The full hill-
chart characteristic described as in [9] is used to provide the hydro-turbine behavior. The
whole plant works under a constant head $H_b$ and the electric machine which is linked
to the runner through a rigid shaft is only taken into account for its inertia. Indeed, the
start-up sequence occurs off-grid. The only actuating body is the guide vanes opening $\gamma$, that
enables the control of the hydraulic flow in the runner and in the pipes. In what follows, the
elementary models of each part of the PSP are introduced.

1.1. Penstock dynamic

Let us denote by $h(x, t)$ and $Q(x, t)$ the water-hammer pressure and the flow respectively at
instant $t$ and abscissa $x$ along the penstock. According to [10], these two quantities are linked
through the following nonlinear transfer function:

$$\frac{\partial h}{\partial Q} = -\frac{H_0}{Q_0} \cdot \frac{t_w}{t_e} \cdot \tanh(s \cdot t_e)$$ (1)

Where $t_w = \frac{Q_0}{H_0} \cdot \frac{L}{a} s$ is the water starting time (s), $t_e = \frac{L}{a}$ is the elastic time (s) in which, $L$ denotes
the length of the pipe (m) while $a$ refers to the wave velocity in the pipe (m/s). $S$ is the constant
cross-section of the pipe ($m^2$). $Q_0$ is the standard flow ($m^3/s$) while $s$ is the Laplace variable.
Note that $Q_0$ is used to introduce the time $t_w$ although it plays no role in the computation.

In this paper, the first order Taylor expansion of (1) is used to represent the penstock dynamic,
that is:
\[
\frac{\partial h \cdot \text{approx}}{\partial Q} \quad (s) = -\frac{H_0}{Q_0} \cdot \frac{t_w}{t_e} \cdot \frac{t_e \cdot s}{1 + t_e \cdot s}
\] (2)

Figure 1.1 shows how this approximation fits to the dynamic of the water-hammer effect (1) in the low frequencies. We note that the commonly used relation giving the derivative of the flow rate \( Q \) as a function of \( h \) ([4], [5], [6], [7], [8], [11]) is used when approximating \( \tanh(s \cdot t_e) \approx s \cdot t_e \): our method should provide better results as it uses a richer approximation of the \( \tanh \). We add that the rationale behind the use of this approximation to evaluate the lower bound on the start-up time is that any controller that would take into account the true rich dynamic of the penstock is likely to be more cautious than a minimum-time control law based on the high-pass first order filter. Consequently, the minimum-time controller based on (2) can be considered as a valid lower bound of start-up time for the system in which the original expression (1) is used.

Now using the variable \( x_1 = t_e \cdot h + \frac{H_0}{Q_0} \cdot t_w \cdot Q \), the dynamic (2) becomes:
\[
x_1 = -\frac{1}{t_e} \cdot x_1 + \frac{t_w}{t_e} \cdot \frac{H_0}{Q_0} \cdot Q
\] (3)
\[
h = \frac{1}{t_e} \cdot x_1 - \frac{t_w}{t_e} \cdot \frac{H_0}{Q_0} \cdot Q
\] (4)

1.2. Turbine’s dynamic
The more accurate practice in hydraulic turbines description is to use the hill-chart \( w_1(.) \) and \( w_2(.) \) that describe the torque \( T \) \((N, m)\) and the algebraic constraint involving the flow rate \( Q \):
\[
T = w_1(H_n, Q, N, \gamma)
\] (5)
\[
Q = w_2(H_n, Q, N, \gamma)
\] (6)

Where \( H_n \) is the neat head at the runner’s boundary \((m)\), \( N \) is the rotational speed and \( \gamma \) is the guide vane’s position \((\%\)\). The neat head \( H_n \) is linked to the static head \( H_b \) and the water-hammer effect pressure \( h \) described in (2) through (neglecting head losses):
\[
H_n(t) = H_b + h(t)
\] (7)

Considering a fluid friction coefficient \( f \), the dynamic equation is given by:
\[
\dot{N} = \frac{1}{J} \cdot (T - f \cdot N)
\] (8)

Where \( J \) denotes the inertia of the rotating parts. Concatenating (3), (6), and (8), the global dynamic that is used hereafter becomes:
\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{N}
\end{pmatrix} =
\begin{pmatrix}
\frac{1}{t_e} & 0 \\
0 & -\frac{f}{J}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
N
\end{pmatrix} +
\begin{pmatrix}
0 & \frac{t_w}{t_e} \cdot \frac{H_0}{Q_0} \\
\frac{1}{J} & 0
\end{pmatrix}
\begin{pmatrix}
\dot{x}_1 \\
\dot{N}
\end{pmatrix} +
\begin{pmatrix}
w_1^*(x_1, N, \gamma) \\
w_2^*(x_1, N, \gamma)
\end{pmatrix}
\] (9)

Where \( w_1^* \) and \( w_2^* \) are obtained from \( w_1 \) and \( w_2 \) by solving the algebraic equation (6) in the unknown \( Q \).

2. The minimum time start control problem
Let us denote by \( x := (x_1, N)^T \in \mathbb{R}^2 \) the state vector of (9). Denote also by \( x^d \in \mathbb{R}^2 \) a given desired steady state associated to some desired rotational speed \( N^d \). This paper aims at designing a state-feedback law of the form:
\[
\gamma = K(x, x^d)
\] (10)

That steers the system from the initial state \( x = 0 \) to the desired steady state \( x^d \) in a minimum time while meeting the following control and state constraints
\[
\forall i \in \{1, 2\}, \quad x_i \in [0, x_i^{\text{max}}], \quad \gamma \in [0, \gamma^{\text{max}}]
\] (11)
From a practical point of view, the problem is to find the time-optimal trajectory of $\gamma$ that steers our system from the initial state $(0,0)$ to a final steady state corresponding to $N$ stabilized in the band $N^d \pm 0.2\%$. As mentioned in the introduction, fast start of the system is a crucial condition for the success of the hydraulic storage solution in smoothing the intermittency-induced drop in the grid voltage.

3. Derivation of the control law
The problem is solved in the sequel in two successive steps:

(i) First, the case of ideal (infinitely fast) actuator is considered. The advantage of this step is twofold: firstly it delivers the minimum achievable start time regardless of the actuator. It is shown also that the ideal assumption has to be extended to the measurement rate also (very short measurement sampling time would be required). Secondly, this ideal study enables easier understanding of the mechanism involved in the solution before the realistic case is addressed.

(ii) In the second step, the realistic case of limited bandwidth actuator is considered. The solution is then obtained by proposing a model predictive approach following the guideline of the ideal case.

These two steps are explained in the following two sections.

3.1. Ideal Case: Infinite bandwidth Actuator
The advantage of having a reduced two dimensional model is that the corresponding behavior (under different values of $\gamma$) can be studied in the phase plan $(x_1,N)$. From such analysis, control design can be derived regardless of the degree of non-linearity of the system. Figure 4 shows the evolution of the system with constant gates opening $\gamma$ and for several initializations. The stationary states corresponding to different desired speed values $N^d$ can also be viewed. It comes out clearly that the system is not always stable and for high openings ($\gamma \geq 20\%$) that would be suitable for high $N^d$, an oscillation occurs around the stationary domain − hydro-turbine experts will recognize the “S” phenomena. Figure 5 shows two facts. On the left graph, we see that the bigger $x_1$ is, the bigger $\dot{N}$ and this is true for every opening $\gamma$. The plot on the right of Figure 5 shows vector fields of the dynamic system for different values of $\gamma$. This plots clearly shows that there is a boundary (dotted line of the right plot) beyond which it becomes impossible to increase $x_1$.

These observations suggest the following control strategy:

(i) Define an upper bound $x_1^{\text{max}}$ that lies to left of the right boundary depicted in dotted line on the Right plot of Figure 5.

(ii) As far as $x_1 < x_1^{\text{max}}$ use the maximum value $\gamma^{\text{max}}$ of $\gamma$:

$$\gamma^{(1)} = \gamma^{\text{max}}$$

(iii) When $x_1 = x_1^{\text{max}}$, use the control law that keeps $x_1 - x_1^{\text{max}} = 0$ as far as $N < N^d$, that is:

$$\gamma^{(2)} \text{ such that } x_1 = -\lambda x_1 \cdot (x_1 - x_1^{\text{max}})$$

where $\lambda > 0$. This strategy keeps $x_1$ maximal to maximize $\dot{N}$. The feasibility of this tracking is guaranteed by the fact that $x_1^{\text{max}}$ is lower than the boundary defined above.

(iv) When $N^d$ is reached, the control is used to keep $N - N^d = 0$

$$\gamma^{(3)} \text{ such that } \dot{N} = -\lambda_N \cdot (N - N^d)$$

Where $\lambda_N > 0$. Note that using $\gamma = 0$ steers the state from the right to the left horizontally (see figure 5).
Figure 4. Free evolution of (9) under constant values of $\gamma$, and for several initialization.

Figure 5. (Left): evolution of $\dot{N}$ as a function of $x_1$ and $N$, for several values of guide vanes’ opening $\gamma$. (Right): Vector fields of the dynamic system for different values of $\gamma$.
Figure 6. Evolution of the closed-loop system with ideal actuator and the feedback law given by (12)-(15). $N^d = 300$.

(v) When the state $x$ is in a sufficiently small neighborhood of the desired state $x^d$, the control is switched to a linear optimal regulator of the form:

$$\gamma^{(4)} = \gamma^d - K \cdot (x - x^d)$$  \hspace{1cm} (15)

Where the gain $K$ can be computed by solving the corresponding Riccati equations using the matrices of the linearized system around the desired steady state.

The behavior of the closed-loop system under the control law (12)-(15) is depicted in Figure 6 for different values of $x_1^{\max}$. The corresponding Start-up times are given in Figure 7. Note that, when switching between phases, guide vanes’ opening is subject to abrupt changes. Obviously this strategy could not be used with a band-limited actuator. As an example, the time needed by the actuator to close would cause an unacceptable overshoot of $N$. However, despite the unrealistic requirements over sampling period and actuator dynamic, the above computations enable a lower bound on the minimum start-time to be computed which is regardless of the actuator being used. This is important in the sense that if the resulting lower bound is still too long for the mission that is assigned to the turbine in its environment, then the hydro-mechanical design of the installation has to be changed, neither actuator nor control strategy can recover the situation.

3.2. Realistic Design under Bandwidth-Limited Actuators – MPC approach

Let us first define the limitations on the actuators. The actuators used in a hydro-electric plant that controls the position of the guide vanes are oleo-hydraulic actuators. They come with their internal control loop and can be seen as second order with settable coefficients. Typical
Figure 7. Evolution of $t_r^{99.8\%}$ as a function of $x_1^{max}$ with an ideal actuator.

Figure 8. Profile of $\tilde{\gamma}_c(t \rightarrow t + N_p \cdot \tau)$ and predicted future evolution $\tilde{\gamma}(t \rightarrow t + N_p \cdot \tau)$.

limitations on actuators involve both constraints on amplitude ($\gamma \in [\gamma^{\min}, \gamma^{\max}]$) and on rate of change ($|\dot{\gamma}| \leq \dot{\gamma}^{\max}$). Let $\xi \in \mathbb{R}^2$ the state vector of the actuator model:

$$
\dot{\xi} = A_\xi \cdot \text{Sat}(\xi) + B_\xi \cdot \gamma_c \quad (16)
$$

$$
\gamma = C_\xi \cdot \text{Sat}(\xi) \quad (17)
$$

$\gamma_c$ is the set-point values while Sat implements the saturation. Recall that one seeks a control law that maximizes $x_1$ throughout the start-up path. However, the system must keep the ability to fall back towards the stationary target point and without overshoot of $N$ nor $x_1$. The proposed solution is based on the use of Model Predictive Control (MPC). This strategy is based on the prediction of the behavior of the system under a set of given input profiles $\tilde{\gamma}_c(t \rightarrow t + N_p \cdot \tau)$ with $\tau$ the control sampling period while $N_p$ stands for the horizon of prediction. In order to reduce the number of decision variable, the parametrized profile for $\tilde{\gamma}_c(t \rightarrow t + N_p \cdot \tau)$ that is depicted on figure 8 is used. In this parametrization, the only decision variable if the set-point $\gamma^*$ to be imposed over the next sampling period $[t, t + \tau]$. After $t + \tau$ the set-point is reset to 0. Once the best admissible $\gamma^*$ is found (see below), it is applied during the interval $[t, t + \tau]$ and at instant $t + \tau$ the whole procedure is repeated in a receding-horizon way (see [12] for more details on MPC design and stability issues). In order to define the best $\gamma^*$, a cost function $J(\gamma^*)$ is defined by:

$$
J(\gamma^*) = -\tilde{x}_1(t + \tau) + C(\max(\tilde{x}_1) - x_1^{\max}) + C(\max(N) - N^{\max}) \quad (18)
$$

Where $C(\cdot)$ penalizes constraints violation according to:

$$
C(r) = \begin{cases} 
10^8 & \text{if } r \geq 0 \\
0 & \text{else}
\end{cases} \quad (19)
$$
Figure 9. Closed-loop behavior under the control law (12)-(15) for \( N_d = 300 \), \( \dot{\gamma}_{\text{max}} = \{10, 20, 200\}\%/\text{sec} \), and \( x_{1\text{max}} = 150 \).

The rationale behind the definition of \( J(\gamma^*) \) is to focus on the maximization of \( x_1 \) at time \( t + \tau \) while being able to satisfy the constraints if needed over the whole prediction horizon. At each time step we apply \( \gamma^* \) between \( t \) and \( t + \tau \). Once \( t + \tau \) is reached, the best value of \( J(\gamma^*) \) is recomputed based on the new value of the state of the system and so one. As in (15), when a sufficiently closed neighborhood of the desired state \( x^d \) is reached, the linear optimal quadratic regulator feedback (linear gain \( K \)) is again used.

The behavior of the closed-loop system under this control design is depicted in the figure 9 and table 1. This suggests that near optimality is achieved as one succeeds, for a given system, a given actuator performances, to maximize \( \dot{N} \) along the start-up sequence with the specified constraints (11).

4. Conclusion
In this paper a simple and computationally cheap state feedback is proposed for constrained minimum-time start-up of hydraulic turbines. The feedback is based on the analysis of the vector fields of a reduced two-dimensional model of the hydro-mechanical system. This enables to fully address the model non-linearity and hence provides a relevant feedback over a wide domain of operations which is a key condition induced by the integration of renewable context. The validity of such algorithm is intimately related to the good characterization of the dynamic of the system. More precisely, figure 1.1 shows that the elastic time \( t_e = \frac{L}{c} \) is a key quantity that determine the validity of the proposed model and the bigger it goes, the more questionable the model is. However, even in this case, we believe that the estimation of the lower bound on the start time remains relevant for any \( t_e \).
Table 1. Results of the MPC modification of the control (12)-(15). The presented time are minimal.

| $N^d = 300$ | $\gamma^{\text{max}}$ (%/s) | $t^95$% (s) | $t^{99.8}$% (s) |
|-------------|-----------------------------|-------------|-----------------|
| $\infty$    | 8.37                        | 8.92        |                 |
| 200         | 8.45                        | 8.99        |                 |
| 20          | 9.22                        | 10.17       |                 |
| 10          | 10.37                       | 11.73       |                 |

| $N^d = 320$ | $\gamma^{\text{max}}$ (%/s) | $t^95$% (s) | $t^{99.8}$% (s) |
|-------------|-----------------------------|-------------|-----------------|
| $\infty$    | 9.35                        | 9.98        |                 |
| 200         | 9.42                        | 10.1        |                 |
| 20          | 10.17                       | 11.34       |                 |
| 10          | 11.24                       | 12.67       |                 |

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