Defragmentation of Preservice Teacher’s Thinking Structures in Solving Higher Order Mathematics Problem

S Wulandari*, M U Gusteti

Department of Mathematics Education, Sekolah Tinggi Keguruan dan Ilmu Pendidikan Adzkia, Padang, Indonesia, Jl.Taratak Paneh No.7, Korong Gadang, Kalumbuk

*suci.w@stkipadzkia.ac.id

Abstract. Demands to have higher order thinking skills is not only aimed for students. A teachers must have that skills first. The problem now is the lack of preservice teacher’s competence in solving mathematics problem with high order thinking skills. This study aims to find out the construction errors and problem solving by preservice teacher when solving problem with high order thinking skills, especially on trigonometry material. In order to repair these errors, a defragmentation process is carried out on their structure of thought. The qualitative research methods applied to describe thinking structures and the form of defragmentation given. The research data were analyzed using Miles and Huberman’s model of data analysis, namely reducing data, presenting data and making conclusions. The research found that there are three forms of concept’s construction errors and solving problem, they are wrong pseudo, mislogical construction and hole of construction. Based on the findings of errors, the researcher performs a defragmentation process of thinking structure by providing scaffolding. The form of scaffolding given were giving a recommendation to rechecking, recall existing schemes for assimilation, and explaining the actual logics.

1. Introduction

A teacher must be able to solve various problems, both routine problems and non-routine problems. “For problems that require higher-order thinking, the solution strategy is not immediately apparent. Problems that require higher-order thinking are nonroutine problems” [1]. Problems that contain higher-order thinking demands are related to the realm of cognitive reasoning, which includes the ability to find conjectures, analysis, generalizations, connections, synthesis, irregular problem solving, and justification or proof [2]. One of the popular ones today is the problem of using Higher-Order Thinking Skills (HOTS). HOTS are thinking skills in which there is the ability to solve problems, the ability to think critically and creatively, and the ability to link existing schemes to a person's cognitive structure. HOTS is the ability to connect, manipulate and change knowledge and experiences that are already owned critically and creatively in determining decisions to solve problems in new situations [3].

The current problem is the lack of competence of a teacher or teacher in solving problems with HOTS. Siti Rohmi Yuliati and Ika Lestari also expressed the need to have the ability to complete HOTS [4]. In this century, an international benchmarking that periodically measure and compare the progress of mathematics education in some countries were PISA (Program for International Student Assessment) and TIMSS (The International Mathematics and Science Survey) [5]. PISA focuses on assessing mathematical skills on the ability to solve mathematical problems about daily life, while
TIMSS can give important information about factors that influence student achievement such as student background, student attitudes toward subjects, teachers and class characteristics. So, this TIMSS’s result can be a benchmark by the government in determining their policy about the education. This needs to be considered by the teacher. Minimizing students' errors means correcting misunderstanding of the material and rearranging the wrong thinking structures of students. Thus, this study has the aim of knowing the form of concept construction errors and problem solving for further defragmentation in order to correct these errors.

The defragmentation of thinking structures is the knitting back of incorrect schemas in a person's cognitive structure. Defragmentation of thinking structures can be interpreted as cognitive restructuring in individuals. With the defragmentation method, it is hoped that it can minimize the errors of students in solving problems. Understanding concepts in mathematics is essential for solving mathematical problems. The schema or information that is in a person's cognitive structure is involved in this. Misconceptions allow procedural errors to solve a mathematical problem. Mental activities that are carried out by involving schemas in cognitive structures to solve problems are called thinking processes. To avoid errors in solving these mathematical problems, someone must have the correct thinking structure. The thinking structure is a representation of the thinking process in the form of a problem solving flow that is carried out by a person when he or she resolves a problem.

Subanji's problem solving errors are conceptual construction errors and problem solving which are reviewed based on the connection schemes in the structure of students' thinking. There are five construction errors made by students, namely pseudo construction, construction holes, mis-analogical construction, mis-connection and mis-logical construction. Pseudo construction is a form of construction that a person does as if he were right but he cannot provide justification or construction as if he is wrong but he can correct his errors after reflection. This construction error or pseudo construction occurs when students answer a problem but it seems as if the answer is correct even though it is not in accordance with the substance of the concept or seems to be wrong even though students can actually convey it correctly after reflection. Student errors can be pseudo true and pseudo false. True pseudo occurs when the answer given is correct, but after reflection it turns out that the reasoning process is wrong. Pseudo error occurs when the answer given is wrong, but after reflection it turns out that students are actually able to reason correctly to give the correct answer. Hole construction is a concept construction or problem solving construction where the scheme formed in the construction process is incomplete. Construction holes occur because of certain unconstructed schemes in the thinking structure of students. When experiencing construction holes, students do not know or do not pay attention to the terms of a concept. So that in solving problems students only construct the procedure, not yet construct the concept. Mis-analogical construction is a concept construction or problem-solving construction where an analogy error occurs in the construction process. Mis-logical construction or analogical thinking mistakes are based on students' errors in constructing problem solving in giving assumptions based on analogy. Here students have the assumption that some concepts are considered analogies or equated. The connection hole occurs when students are unable to see the relationship between one concept and another. Such holes also occur when students do not know what concepts to use in solving a problem. In solving problems, students are not able to relate the concepts they have to the problems they solve, so that the connections that occur cannot work well. Mis-logical constructions or logical thinking errors occur when students make assumptions which they think are true even though they are actually wrong in substance and are not logical in concept. Mis-logical constructs are conceptual constructions or problem-solving constructs where in the construction process errors occur in logical thinking. The logic that students do in solving problems is not correct.

Defragmentation is the solution to the problems described earlier. In defragmentation, the structure of a person's thinking that is not properly re-woven. This knitting will correct the errors made. Cognitive restructuring is a method that is carried out with the aim of rearranging thoughts, eliminating irrational beliefs that cause tension and anxiety for a person which has been affecting their
emotions and behavior [6]. The thinking structure can be reorganized in terms of overcoming problems in the thinking structure of students [8].

2. Research Methods
This research method is quantitative research, with the type of research that is descriptive exploratory research. Datas in this research analyzed using Milles and Huberman. There are three steps of data analysis techniques: reducing, presenting, and concluding [9]. This study describes the thinking structure of students and the form of defragmentation. This research will be carried out in the study program of Mathematics Education, Sekolah Tinggi Keguruan dan Ilmu Pendidikan Adzkia for students of 2018 (4’th semester) and 2019 (2’nd semester) who have studied trigonometry since senior high school and are re-working in college. The subjects of this research are three students, each of them as the culprit who is categorized as very crucial, crucial and not crucial in making construction errors and problem solving according to Subanji [7].

In the implementation of this research, the research subject was asked to provide an explanation of how he solved the problem given. After that, the researcher classified the mistakes made by the research subjects into the types of errors that have been discussed previously. The research instrument is the researcher with the help of supporting instruments, namely the problem sheet.

3. Results and Discussion
Research begins with conducting preliminary research. Following are the problems given in the preliminary research.

![Figure 1. Trigonometric Problems](image)

Based on the preliminary study, there were 75% testees made errors in problem solving. They were categorized into 3 groups based on the level of seriousness of the error, namely the high, medium and low groups. The high group is testee who made very critical errors (S3). The medium group is the testee who made a critical errors (S2). The low group is the testee who made errors that are not too critical (S1).

3.1. Error and Thinking Structure Description of S1 and Its Defragmentation
Thinking process of S1 in solving a given problem begins with reading the problem, then writing down what is known from the problem. S1 did not write down in detail what is known, he only expresses his understanding of the problem by making illustrations according to his understanding. S1 solved the problem by paying attention to a right triangle with sharp angles of 24 degrees and 66 degrees. Furthermore, to determine one side of the triangle, S1 used tangent. Thus the length of the base was obtained. The same was done with the next right triangle with acute angles, namely angles of 36 degrees and 54 degrees. From this triangle S1 also get the side of the base. After the lengths of the two bases of these two right triangles had been obtained, then S1 calculated the difference to get what
S1 wanted. In the procedure and structure of the S1 thinking structure there are no errors. Even though the S1 thought process was correct, some problem-solving algorithms have errors. This is the first error that S1 made.

Figure 2. Errors in determining the angle

Problem solving presented by S1 indicates a pseudo-construction. What happens to S1 is pseudo-error \[10\]. After the researcher asked S1 to explain again what he thought when solving the problem. The researcher only asks to explain the problem-solving process that S1 has done. Subanji revealed that in order to for defragmentation to occur, it requires scaffolding, analysis of the construction process, cognitive conflict, and disequilibration \[8\][11]. Not much scaffolding has been provided yet. It turned out that after S1 read the answer again by himself, S1 realized his mistake. S1 indicates that the magnitude of the angle, that is, the angle of 24 degrees, is incorrect. S1 thinks to himself why he can write 24 in the answer. S1 thought about it so in the end he said he doesn't remember why. Then S1 continued to explain the reasons why other angles such as the 66 degree angle, and the 54 degree angle he got. S1 knows that the sum of the three angles in a triangle is 180 degrees. With an explanation like this, it can be seen that S1 actually understands the concept of the triangle well and correctly. Thus this supposed error at an angle of 24 degrees should not have occurred. Because if the concept of the angle in a right triangle was been understood, then the concept of a right angle should not be wrong. This happens because of carelessness, because such mistakes are made without reason, while the concepts and reasoning they have are correct. With a little intervention in the form of rechecking, S1 was able to realize his mistakes and provide the correct answers. Therefore, the process of restructuring or defragmentation of the thinking structure carried out on S1 is not too many and complicated. Here is the second error that S1 makes.

Figure 3.1 S1 Thinking Structure Before Defragmenting
3.2. Error and Thinking Structure Description of S2 and Its Defragmentation

Thinking process of S2 begun with reading the problem and understanding it. S2 wrote it in the form of known points and a picture with several sizes in it as illustration. Next, S2 recreated the observed triangle from the previous illustration. This image is made with the aim to make it easier for S2 to get the side length of the triangle. S2 determined the unknown angles. S2 used the sine principle to determine one side of the triangle it creates. S2 was not able to immediately get the side of the base of the triangle, but first S2 determined the hypothesis and then used it in the Pythagorean Theorem to find the base side. The same was done with the second triangle which S2 also redraws. After getting the two sides of the bases of the two right triangles, S2 proceed with calculating the difference to get what S1 wanted. In the procedure and structure of the S1 thinking structure there were no errors. Even though the S2 thought process was correct, some problem solving procedures or algorithms were faulty. Here's the first mistake S2 made.

S2 made a pseudo wrong because the wrong answer he wrote could be corrected after reflecting. S2 reflection was carried out through interviews regarding this error. S2 said she didn't remember or forgot how she got the 41 degree angle. However, after the researcher gave the intervention in the form of assimilation of concepts that he already understood, S2 was able to provide the correct answer. When interviewed, S2 seemed to be able to understand the concept of opposite angles. If a 27 degree angle was obtained by utilizing an understanding of two opposite angles, then this 41 degree angle should not be wrong. The researcher tried to assimilate the schematic of the opposite angle that S2
already had. The scaffolding given was in the form of picking up the existing schemes in the S2 cognitive structure. Giving question, hints, reminder, direction, or encouragement to someone who make a mistake can help them solve the problem [11]. Researchers provided scaffolding in the form of questions related to the concept of opposite angles that S2 previously mentioned. By providing this scaffolding, it turns out that S2 was able to apply the concept of opposite angles to the same conditions, namely for an angle of 51 degrees. Thus the pseudo-error experienced by S2 could be corrected by scaffolding in the form of a recall of the existing scheme so that the assimilation process was successful. Here's the second error S2 made.

Figure 5. Error in determining an angle’s amount

Error that S2 made here was in determining the angle’s amount of 68 degrees. This was a miss-logical construction. S2 added an angle of 41 degrees with an angle of 27 degrees. Miss-logical construction or logical thinking errors can occur when assumptions are not logical [8]. The miss-logical construction that occurs here turns out to be because S2 assumes that the larger the triangle shape, the bigger the angle must be. The logic presented by S2 was incorrect. S2 asked to reflect and rethink his assumptions. The researcher intervened by providing scaffolding in the form of an actual logical explanation. Thus S2 understands the wrong assumption. So, the mis-logical construction carried out by S2 could be corrected by scaffolding in the form of explaining the actual logic or describing the correct logic.

Figure 6.1 Thinking Structure S2 Before Defragmenting
3.3. Error and Thinking Structure Description of S3 and Its Defragmentation

S3 understood this problem. However, there were several concepts that have not been constructed by S3 when solving the problem, namely related to the concept of the sides in a right triangle. Construction hole occur when there are still schemes that have not formed properly. S3 undergoes a construction hole on the right triangle concept. This results in errors in the use of the tangent comparison principle. First, S3 had not been able to determine the correct angular position in the use of tangents. Second, S3 had not been able to determine the front side of an angle in a right triangle. Third, S3 had not been able to determine the side of the angle in a right triangle. The following describes the error.

Figure 7. Misconception of the side of a right triangle

Based on the results of the interview, S3 had not been able to determine the correct angle location when using the tangent. S3 felt so confident in how it determines the tangent of the 27 degree angle. S3 was able to explain that the tangent is the ratio of the front of the corner to the side of the corner. However, to get the length of one of the desired triangle sides S3 did not use the right triangle it should be. Researchers provided questions to generate schemes that S3 already has. The concept of an angled triangle and the principle of comparative trigonometry had been studied since high school and has also been repeated when taking Trigonometry courses in college. With scaffolding in the form of questions, schemes which regarding to right triangles, the sides of right triangles and their relation to the tangent principle could be knitted back as they should.
Defragmentation is one of the solution to correct errors in solving math problems. By making defragmenting diagrams, the wrong structure of student thinking can be seen. An advanced interviews can help detect the form of mistakes that were made. After that, the form of scaffolding which given also can be determined. Scaffolding is given in stages. Students were given assistance little by little. For example by giving a question. The questions given should be able to generate understanding of wrong concepts, to correct wrong assumptions that have been formed in students' thinking structures, as well as to create connections between missing conceptions. Scaffolding must be reduced until students become independent \cite{12,13}.

4. Conclusion
Based on the results of the research and discussion above, it can be concluded that in solving high-level mathematical problems, especially in trigonometric material, there are three forms of concept
Construction errors and problem-solving found, namely pseudo-wrong, mis-logical constructions and construction holes. These errors were corrected by the process of restructuring the thinking structure (restructuring). One way of restructuring the thinking structure (defragmentation) can be done through scaffolding. Defragmentation through scaffolding aims to correct errors that occur in a person's thinking structure. The form of scaffolding given depends on the seriousness of the error and the subject who did it. Several forms of scaffolding that are suggested to correct these errors include: providing recommendations for rechecking, recall of existing schemes to be assimilated, and explaining or explaining the actual logic.

Therefore, researchers want to provide some suggestions to educators and other researchers. First, there is a need for research to find other forms of errors in solving high-level mathematical problems in other materials. Second, it is necessary to carry out further research related to anxiety in dealing with high-level math problems. Third, educators should be accustomed to giving advice to students to increase caution in solving math problems, especially math problems that require higher order thinking skills. It is hoped that this research can serve as an illustration of the structure of thinking in solving high-level mathematical problems and can be used as a reference in determining approaches and strategies in learning.

Acknowledgement
The authors would like to thank Direktorat Riset dan Pengabdian Masyarakat (DRPM) from Kementerian Riset dan Teknologi/ Badan Riset dan Inovasi Nasional for funding this research project (Hibah Penelitian Dosen Pemula 2019).

References
[1] Susan M Brookhart 2010 How to Assess Higher-Order Thinking Skills In Your Classroom (USA: ASCD)
[2] Budiman A Jailani 2014 Pengembangan Instrumen Asesmen Higer Order Thinking Skill (HOTS) pada Mata Pelajaran Matematika SMP Kelas VII Semester 1 Jurnal Riset Pendidikan Matematika 139-151
[3] Dinni H N 2018 HOTS (High Order Thinking Skills) dan Kaitannya dengan Kemampuan Literasi Matematika PRISMA 170-176
[4] Yuliati S H, and Lestari I 2018 Higher-Order Thinking Skills (HOTS) Analysis Of Students In Solving HOTS Question In Higher Education PERSPEKTIF Ilmu Pendidikan 181-188
[5] Pratama G S and Retnawati H 2018 Urgency of Higher Order Thinking Skills (HOTS) Content Analysis in Mathematics Textbook IOP Conf. Series: Journal of Physics 1-8
[6] Kumalasari F, Nusantara T, and Sa’dijah C 2016 Defragmenting Struktur Berpikir Siswa dalam Menyelesaikan Masalah Pertidaksamaan Eksponen Jurnal Pendidikan 246-255
[7] Subanji 2015 Teori Kesalahan Konstruksi Konsep dan Pemecahan Masalah Matematika (Malang: UM Press)
[8] Hidayanto T, Subanji, and Hidayanto E 2017 Deskripsi Kesalahan Struktur Berpikir Siswa SMP dalam Menyelesaikan Masalah Geometri Serta Defragmentingnya: Suatu Studi Kasus Jurnal Kajian Pembelajaran Matematika 72-81
[9] Sugiyono 2011 Metode Penelitian Kuantitatif, Kualitatif, dan R&D (Bandung: Alfabeta)
[10] Subanji 2015 Teori Kesalahan Konstruksi Konsep dan Pemecahan Masalah Matematika (Malang: Universitas Negeri Malang)
[11] S. Haryanti 2018 “Pemecahan Masalah Matematika melalui Metode Defragmenting JKPM (Jurnal Kaji. Pendidik. Mat.) 3(2) 199-204
[12] Damayanti N W 2015 Praktik Pemberian Scaffolding oleh Mahasiswa Pendidikan Matematika pada Mata Kuliah Strategi Belajar Mengajar (SBM) Matematika Likhitaprajna 18: 1 (87 - 97)
[13] Anghileri J 2006 Scaffolding Practices that Enhance Mathematics Learning Journal of Mathematics Teacher Education (9) 33–52