Topological Excitations in Doped Spin Ladders

Yujoung Bai

National Creative Research Initiative Center for Superconductivity, Department of Physics,
Pohang University of Science and Technology, Pohang, Kyungbuk 790-784, KOREA

Abstract

We study low-energy magnetic excitations of doped spin-ladders, based on an effective Hamiltonian describing interactions of mobile spin and background spins. The helicity moduli against fluctuations are calculated in terms of the doping concentration, the leg number and interaction strengths in the ladder. The doping ranges for spiral phase are computed and its relation to spin gap structure is discussed. The spiral order can enhance existing antiferromagnetic order at certain doping range or reduce it at very low density of holes. For odd-legged ladders, formation of spin gap at certain narrow doping range becomes possible via coupling of spirals and background spins.

PACS numbers: 74.20.Mn, 75.10.Jm, 71.27.+a, 75.50.Ee
There have been numerous studies on long-range magnetic order in two-dimensional quantum systems, some of which employed mapping to (2+1)D non-linear sigma model (NLsM) for the long-wavelength, low-energy states. Adding the effects of doped holes to 2D antiferromagnets, several works were done of the in-plane spiral modes by mean-field calculation. A couple of studies included the out-of-plane spirals generated in 2D systems, starting with t-J models which incorporated the effects of mobile spin in magnetic background. By spiral modes, we mean vortex rotations formed by background spin phases or long-wavelength excitations arising from fluctuations of staggered magnetization. In lower dimensions, some studies have been done for low-energy excitations in undoped spin ladders. The ground state of even-legged ladders is known to be spin liquid (singlets), given strong to intermediate coupling – the exchange ratio being $J_\perp \geq J_\parallel$. The odd-legged ladders have even-parity channels and odd-parity channel, due to the reflection symmetry. The mean-field theory (MFT) and numerical studies agree that the even-channel makes the spin liquid while the odd-channel makes the Luttinger liquid, before doping. After started doping, the hole spins tend to go into the even-channel, thus keeping the gapped mode just like even-ladders, much more likely than into the odd-channel. It has been mostly accepted that the “topological term” in the NLsM describing a undoped ladder is classified by the parity of leg numbers. The term is shown to be zero for undoped even-legged ladders and $2\pi S n_l$ for odd-legged ladders. As for doped ones, there have been a couple of results which showed clear departure from this type of distinction, by DMRG calculations and by Lanczos calculations along with MFT. In this paper, we focus on the low-energy spin excitations in doped ladders by analytical calculations, discussing the role of the topological term for doped ladders. Doped spin ladders consist of background spins and empty sites with residual spin. The effective interaction between residual spins are antiferromagnetic, if they belong to different sublattices of any bipartite ladder sites. New magnetic excitations arise from coupling of collective polarization and local magnetization, as the effective spin of the hole moves around the ladder sites. In the continuum limit, the distorted spin background induces non-zero “magnetic” flux due to
phase fluctuation. To see the spin response, the helicity modulus is calculated as an explicit function of doping rate, leg number, temperature and interaction strengths in spin ladders. By minimizing the total energy of the system, we find the range of hole concentration which favor the spiral mode, from the helicity modulus. Our results of the doping range for gapped mode in odd-ladders agree well with those existing numerical studies [11] [13], within reasonable range of interaction coupling in our model. Also, we determine how spiral modes enhances/suppresses AF order and the range for these effects.

We begin with an effective Hamiltonian, based on the one made by Shraiman and Siggia for 2D magnets [7]. At half-filling, the low energy state has the commensurate Neel order. For the long-wavelength staggered spin state, the Neel vector is twisted, by $\alpha \frac{y}{L}$ around z-axis at the end of the ladder length $L$ with respect to the first $\hat{n}$ and by $\nu \frac{x}{(n_1 - 1)a_0}$ at the end of the ladder with $n_l$-legs (with the lattice spacing $a_0$). When the mobile spins are introduced upon doping, the Neel vector is allowed to rotate around y-axis as well, to derive all modes generated by collective polarization and phase fluctuation. $\hat{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$.

This is equivalent to twisting the spin quantization axis from site to site via appropriate transformation as done in some existing works [4] [3] [6]. The effective Hamiltonian is

$$H = H_{NLsM} + \frac{1}{2} \left( \frac{\hat{y} \alpha}{L} \right)^2 + \frac{1}{2} \left[ \frac{\hat{x} \nu}{(n_1 - 1)a_0} \right]^2 - \tilde{g} \sum_{a, q} \hat{P}_a(q) \cdot \tilde{j}_a(q) - g' \sum_{a, k} \cos k \tilde{\Psi}_{k + \frac{2}{a}} \tilde{\Psi}_{k + \frac{2}{a}} + \hat{m}(q)$$

(1)

where the magnetization current $\tilde{j}_a(q) = \hat{n} \times \nabla_a \hat{n}$ and the dipole polarization $\tilde{P}_a(q) = \sum_k \sin k_a \tilde{\Psi}_{k - \frac{2}{q}} \tilde{\Psi}_{k + \frac{2}{q}}$. The second and the third term is the rotational kinetic energy of the unimodular $\hat{n}$. The subscript $a$ stands for in-plane(IP) mode and out-of-plane(OP) spirals. The wavevector of the mobile spin is related to the spiral wavenumber $Q_a$ as $\langle \tilde{\Psi}_{k - \frac{2}{q}} \tilde{\Psi}_{k + \frac{2}{q}} \rangle \approx Q_a \sin k_a$. Identifying the components of incommensurability wavevector $\bar{Q} = (Q_x, Q_y)$ with the angular terms in the effective interaction term in $H_{eff}$, the spiral wavevector is $\bar{Q} \rightarrow -\sin \theta \cos \theta (\cos \phi + \sin \phi) \equiv \cos(Q_x/2) - \cos(Q_y/2)$. The spiral wavevector has a symmetry similar to a d-wave. The effective propagation in real space is along the rung direction of the ladder, having arisen from triplet formation along the ladder direction.
This mode is equivalent to the transverse mode [4] or to the stripe phase [3] as termed for 2D systems [2]. The doping range for the spiral order to occur is computed from the helicity modulus against IP twisting. The spiral wavenumber $Q$ is proportional to the doping rate [3] [6]. For the purpose of calculating the helicity modulus, we use $Q \approx \delta t$ for IP mode and $Q \approx \delta$ for OP spirals. For IP mode, the effective Hamiltonian density is given

$$H_{IP} = (\partial_a \psi)^2 + \frac{1}{2} \left( \frac{\dot{y}_\alpha}{L} \right)^2 + \frac{1}{2} \left( \frac{\dot{x}_\nu}{(n_l-1)a_0} \right)^2 - 2\tilde{g}\delta t \{ \cos \phi (\cos \phi + \sin \phi) \} \sin^2 k \pm 2g\delta t \cos \phi \sin k \cos k$$

The free energy per unit area of the ladder plane is

$$F = -\frac{1}{\beta} \ln Tr \exp[-\beta H_{eff}] = -\frac{1}{\beta} \ln \left[ \frac{2\pi(n_l-2)}{a_0(n_l-1)} \right] + \beta \left[ \frac{1}{2} \left( \frac{\dot{y}_\alpha}{L} \right)^2 + \frac{1}{2} \left( \frac{\dot{x}_\nu}{(n_l-1)a_0} \right)^2 \right]$$

$$+ \frac{1}{2\beta} \ln[2\beta^2 \{ 1 + A \cos \phi (\cos \phi + \sin \phi) \}] - \frac{\beta B^2 \cos^2 \phi}{4 \{ 1 + A \cos \phi (\cos \phi + \sin \phi) \}}$$

where $A = -2\tilde{g}\delta t$, $B = -2gt\delta t$ and $\beta = 1/k_BT$. The subscript $\perp$ refers to those along the rung direction while $||$ is for those in the chain direction. The spin wave velocity for doped ladders is derived from mapping the unit ladder cell (repeated regularly as defined by the density of empty sites) to NLsM. Taking only the slow component of the Neel vector and for the coherence length $\gg$ the ladder width$(a_0n_l)$, the coupling now includes the doping effect $g = [(1 - \delta)n_lS]^{-1} \left( 1 + \frac{J_{\perp}}{J_{||}} \right)^{\frac{1}{2}}$. The spin wave velocity is collected from the prefactor, $v_s = 2SJ_{||} \left( \frac{1-\delta}{1-\delta} \right) \left( 1 + \frac{J_{\perp}}{J_{||}} \right)^{\frac{1}{2}}$. As for 2D systems without any hedgehogs, it is known that the topological term $-\frac{\alpha}{4\pi} \int (\partial_a m)^2 d^2r$ is zero, regardless the spin number or the configurations of lattice sites [13] [14] [17]. We discuss the different meaning of this term in doped ladders later. The coupling in the effective interaction($V_{gr}$) is found from relating the hole density to the collective polarization $< P_{gr} >$ and the incommensurability wavenumber $Q$ of the spiral mode. From $< V_{gr} > \approx 2gt Q \ln[2(n_l-1)] < P > \approx gt Q(n_l-1)L/16\pi^2a_0$, we estimate the coupling $gt \approx J_{\perp} Q/ < P > \approx 32a_0\pi^2 \ln[2(n_l-1)]\delta J_{\perp}/L(n_l-1)$, where $L$ is the length
of unit ladder cell, which decreases as hole density increases. The other coupling $\tilde{g}$ can be estimated from renormalized coupling($g_\sigma$) of the NLSM: $1/\tilde{g} = 1/g_\sigma + \text{Tr}[1/(k^2 + V(\tilde{g}))]$. For instance, given the most realistic interaction strengths in cuprate ladders $\frac{J}{J} = 5$, $\tilde{g} = 1$ with $\delta = 0.1$, while $\delta \ll 1$ for much greater $\tilde{g}$. The helicity modulus(HM) against the in-plane fluctuations is computed

$$\mathcal{R}_{IP} = \frac{v_s}{2} \left. \frac{\partial^2 F}{\partial \phi^2} \right|_{\phi=0}$$

$$= \left(1 - 2\delta \right) \frac{(\tilde{g}\delta t_{\perp})^2}{\beta J_{\perp}(1 - 2\tilde{g}\delta J_{\perp}^2)} \left(1 + \frac{J_{\perp}}{2J_{\parallel}}\right)^{\frac{1}{2}} \left[1 - \frac{128\pi^2 \beta \ln[2(n_l - 1)]a_0\delta^2 t_{\perp}}{(n_l - 1)^2(2\tilde{g}\delta J_{\perp}^2 - 1)} \right]$$

This is an explicit function of the doping concentration $\delta$, leg number $n_l$, temperature and the interaction strengths in the ladder $t_{\perp}$, $J_{\perp}$ and $J_{\parallel}$. Fig. 2 is a surface plot of the IP HM versus doping rate and temperature. The common features of HM for all ladders are (i) that the HM diverges at certain doping rate $\delta = 1/(2\tilde{g}J_{\perp})$, (ii) that the sudden jump is followed by sharp decrease in HM to negative values, (iii) that the flat plateau is over wide range of doping and temperature, (iv) that another range for sharp increase of HM is at $0.8 \leq \delta \leq 0.9$. The spin liquid ground state acquires excited triplets upon doping. When the hole density reaches a certain level, intrachain pair formation becomes substantial as well, which makes the whole system very stiff. The diverging modulus followed by drastic decrease as well as flattening over wide range, indicate presence of spin gap, since the HM(stiffness) goes as the inverse of spin susceptibility. In the region of negative HM, the system releases spiral modes to lower the energy. Though HM itself is a local response, the spiral phase (bulkwise) can be formed in the regime of negative HM. As doping increases further, there is another sharp increase in HM at very low temperatures, due to coupling of spirals and intrachain spins. The spiral modes can put a pair of spins into a level in the gap. Thus, the short-range AF order of spin liquids can be lost at certain doping regime. In even-ladders with existing spin gaps, spiral modes can reduce the spin gap sizes. At higher $\tilde{g}$ which is also physically feasible, the spiral phase is absent at low doping, though the other window at much higher doping still remains. In Fig. 2 for 3-leg ladder, the possibility of
generating spin gap in odd-ladders exists around $\delta = 0.1$ with $\tilde{g}=1$. For more attractive $\tilde{g} \geq 1$ which is very likely in physical ladders, the spiral mode and gap formation occur in smaller doping regime of $\delta < 0.1$, at temperatures $T \leq 5K$. The region where the HM becomes negative is commonly $\frac{J_{\perp}}{2g_{\perp}} < \delta < 0.5$ at $T \leq 5K$, producing spiral phases for all ladders. The characteristics are repeated for ladders with any number of legs, though precise values are a bit different, due to the leg-number dependence in $g$. As leg-number increases, the doping regime for IP spiral mode is reduced. Since the even-parity channels in odd-ladders are dominant over the odd-parity channels, the added(effective) spins tend to be in the even-channels. Given the attractive potentials provided by $V_g$’s in our model, binding of the spins in the odd-channel and the spirals(solitons) yield an energy gap. This gives rise to a gapped phase at $\delta \geq \delta_{\text{spiral}}$ in odd-ladders, with all channels participating. Our result of doping rate for the gapped mode in odd-ladders ($\delta_{\text{spiral}} = 0.1$ for $\tilde{g} = 1$) is close to the result by Rice et. al. [11] who found the $\delta_c \geq 0.13$. If the coupling $\tilde{g} > 1$ in physical ladders, which gives our $\delta_{\text{spiral}} < 0.1$, the DMRG result of $\delta_c \geq 0.06$ by White et. al [13] is comparable as well. In our picture, there is interplay between effective interaction strengths and hole concentration, giving the coupling $g$ vary consequently. Existing interpretation of gap generation in odd-ladders by White et.al. uses the idea of domain walls formed by competition between the kinetic energy and the exchange energy. In our picture, domain walls can be interpreted as areas of the spiral vortices with different spiral wavenumber or different chiralities. Alternative understanding has been given with proximity effect to the gapped mode of the even-channel, which enhances the pairing within the odd-channel [11] [18].

For out-of-plane(OP) mode, the effective Hamiltonian for OP spiral is $H_{OP} = (\partial_a \psi)^2 \pm 2gt\delta \sin k \cos k \cos \theta$. The free energy per unit area along the $\hat{z}$ is $F = -\frac{1}{\beta} \left[ \ln \left[ \frac{2\pi}{\beta} \right] + 2(gt \cos \theta \delta \beta)^2 \right]$. The helicity modulus(HM) against the out-of-plane twisting is

$$\Re_{OP} = \frac{\nu_s}{2} \left| \frac{\partial^2 F}{\partial \theta^2} \right|_{\theta=0} = -2\beta \left( \frac{32\pi^2 \ln \left[ 2(n_l-1) \right]}{(n_l-1)} \right)^2 \left( \frac{1-2\delta}{1-\delta} \right) \delta^4 J_{\perp}^3 \left( 1 + \frac{J_{\perp}}{2J_{\parallel}} \right)^{\frac{5}{2}}$$ (5)
For a exchange ratio of $J_\perp = 2J_\parallel$, the result shows formation of OP spiral phase over wide doping range $0 < \delta < 0.5$ for ladders with any number of $n_l > 1$. In all ladders, there are sharp increases in OP HM at $\delta \geq 0.5$ for $T < 15K$, making OP spirals robust. At very high hole density beyond this $\delta$, the peaked modulus indicates forming bound states of OP spirals with opposite chirality(soliton-antisoliton pairs). The spin quantum number would be integers for this new type of condensate. The HM(stiffness) decreases as temperature increases at high doping regime for all ladders. From minimizing the Hamiltonian $H_{eff}$ with respect to a mean-field $Q$ \[7\], the self-consistency condition gives $Q = \frac{\tilde{g}}{\mathcal{R}} \sum_k \sin k (\mu + \varepsilon_k - \tilde{g} Q \sin k) \simeq \frac{2\tilde{g}\beta}{J} \sum_k \sin k (\mu - \varepsilon_k)$. The Fermi distribution function $n^\pm = \exp[\beta(\varepsilon_k \mp \tilde{g} Q \sin k - \mu)] + 1$ gives $n^+ - n^- \approx 2\beta \tilde{g} Q \sin k$ at low $T$. Then the mean effective interaction terms are $< V_{\tilde{g}} > + < V_{g}' > \approx -\frac{\tilde{g}^2}{\mathcal{R}} \sum_k \sin k \sin k (2\beta \tilde{g} Q \sin k)^2 + \frac{1}{2} \mathcal{R}Q^2$. The $< H_{MF} >$ becomes positive when the helicity modulus $\mathcal{R}$ becomes negative, which makes the system unstable. So, the system releases the torsional momenta. That is, the spiral phase with uniform wavenumber throughout the system is favoured when $\mathcal{R} < 0$. From $\mathcal{R}_{IP}$, we find the doping range for spiral phase $\frac{J_\perp}{2\tilde{g}t_{\perp}} \leq \delta < \frac{1}{2}$, which agree with the graphical result. The spin coherence length in the ladder plane is computed when the HM is zero,\[6\]

$$\xi \simeq \frac{128a_0\pi \ln[2(n_l - 1)]\delta \beta J_\perp}{(n_l - 1)[2\delta \tilde{g}t_{\perp}/J_\perp - 1]^{1/2}}$$

When the incommensurate spin state is stable in the regime of positive HM, it is possible to gather the (hole) spins along valleys of the spiral wavevector. This may result in phase separation of hole-rich region and no-hole region \[4\] \[19\], adjacent to the spiral phase. As for spin ladders with even number of legs, the upper limit of doping for Luttinger liquid phase is $\sim J/2\tilde{g}$. The lower limit would be around $J/4\tilde{g}$. In our results, there is no transition like Kosterlitz-Thouless(KT) transition in 2D \[20\], since there is no strong size dependence and no plateau in the stiffness around any particular temperatures. Though there are size dependent terms in the expression of the HM, these are rather weak. Calculated spin coherence suggests that the gap function has a different form from this, due to the doping effect. As far as the helicity modulus, ladders with even/odd-number of legs do not have
differing signs, since the factor \(\ln[2(n_l-1)]\) does not distinguish the parity in total leg number \(n_l\). This is the difference from the result on 1D spin chains which distinguish the parity, but analogous to 2D systems which do not differentiate the spin quantum number being integer or half-odd integer. For ladders with finite number of legs, the boundary condition in the spin wavefunction is given \(\Psi_l(n+1) = \pm \exp[-i\phi]\Psi_1(1)\), where \(l\) is the leg index and \(n\) is the site number in the block of the system \([21]\). The meaningful parity in ladders is that of (leg number plus site number), instead of the leg number alone. The “+” or “−” sign goes as the chirality associated with the winding orientations of spirals, having arisen from the presence of two sublattices (of up or down-spin). So, the ground state in odd – ladders is degenerated for spiral modes with different chiralities at different ladder regions, thus satisfying Lieb-Mattis-Shulz theorem. This means that gapped mode is possible in odd-ladders. This is the same result as found by previous MFT study on odd-ladders \([14]\).

So far, we restricted the spiral winding number \((w)\) to 1, to ensure the spinor wavefunction to be periodic (instead of anti-periodic). Corresponding staggered spin flux (chirality) in continuum limit is \(2\pi w\). In terms of the Neel vector at bipartite ladder sites, the winding number is \(\hat{n}_1 \cdot \hat{n}_2 \times \hat{n}_3 + \hat{n}_1 \cdot \hat{n}_3 \times \hat{n}_4 - \hat{n}_1 \cdot \hat{n}_2 \times \hat{n}_4 - \hat{n}_2 \cdot \hat{n}_3 \times \hat{n}_4\). If the systems were undoped ladders described by NLsM, the \(w\) would be equal to the topological term. This gives the solid angle subtended by the Neel vectors or the Berry phase acquired by the Neel vector while traversing around the path in spin space \([22][23]\). Allowing the spiral modes, the winding number need not be integers. This implies that the topological term in doped ladders does not simply determine their classes by the leg numbers. The induced field acting as twisted boundary would affect the magnetic ordering in ladders. To see whether spiral modes compete with antiferromagnetism in ladders or not, we rewrite the Neel vector including the winding number. \(\hat{n} = (\sin \theta \cos(w\phi), \sin \theta \sin(w\phi), \cos \theta)\). Then, the effective interaction term is \(V_g \sim -\pi^2 w \tilde{g} \delta\), coming from the gradient of \(\hat{n}\). This can provide attractive potential \([24]\) between the polarised spin and the spin current if the \(w\) is positive. To maintain the attractive potential between the spin texture and the spiral (soliton), \(w\) is chosen to be of the chirality “+”. This would result in destroying the AF order of background spins.
However, there is another interaction term $V_{g'}$ in our effective Hamiltonian, which does not depend on the winding number. This contribution can be greater in length scales of a few lattice constant, though it decreases as the doping decreases and/or the leg number increases.

In cases of very low hole density (a few percent) and many number of legs, it is possible that the $V_{g'}$ wins to reduce the AF order. A known result by Arrigoni et. al. on spiral order in 2D Hubbard model shows that the transverse mode enhances AF order. The other spiral mode can enhance ferromagnetic order of the background. Another work by Mori et. al. on 2D $t-J$ model also show two similar spiral phases. As for doped ladders which started with commensurate Neel order at half-filling, our result show that the whether spiral modes reduce or enhance the AF order depends directly on the hole density. Only at very low doping regime (a few percent), spiral order suppresses antiferromagnetism. It is due to that the effective interactions $V_{g'}$ and $\tilde{V}_g$ compete in some doping regime.

In summary, due to the hole motion distorting the spin texture in doped ladders, phase fluctuations of the staggered magnetization yields new type of excitations. In continuum limit, long-range spiral modes arise and give the helicity modulus vary depending on the doping rate, temperature, leg number and interaction strengths. The spiral modes can put magnons into sublevel(s) in the gap. The spin gap is reduced in even-ladders while it can be generated in odd-ladders, depending on hole density. Since the even-parity channels in odd-ladders are dominant over the odd-parity channels, the added spins tend to be in the even-channels. Thus, in the begining of doping, the even-channels give the spin gap as spin liquids of undoped ladders. Given the attractive potentials provided in our model, the spin current in the odd-channels tend to make bound states with the spirals in the even-channel. The enhanced pairing brings out the gapped Luther-Emery phase at $\delta \geq \delta_{\text{spiral}}$ in odd-ladders. Our results of the doping range for gapped mode in odd-ladders agree well with those existing numerical studies. Antiferromagnetism in ladders can be enhanced over substantial doping range via having spiral phase. At very low doping of few percentage, spiral modes promote ferromagnetism. In this paper, we showed analytically, generation of spin gaps in odd-ladders and the explicit relation between magnetic orders and topological
excitation. In our forthcoming study, we are to examine whether the spiral phase enhances superconducting transition or suppresses via tunneling of solitons.

ACKNOWLEDGMENTS

This work is supported by Creative Research Initiatives of the Korean Ministry of Science and Technology. The author is very grateful to Prof. S.I. Lee for the valuable discussions.
REFERENCES

[1] S. Chakravarty, B. Halperin and D. Nelson, Phys. Rev. B\textbf{39}, 2344 (1989)

[2] A. Auerbach and B. E. Brown, Phys. Rev. B\textbf{43}, 7800 (1991)

[3] C. L. Kane \textit{et. al.} Phy. Rev. B\textbf{41}, 2653 (1990)

[4] E. Arrigoni and G. C. Strinati, Phys. Rev. B\textbf{44}, 7455 (1991)

[5] H. Mori and M. Hamada, Phys. Rev. B\textbf{48}, 6242 (1993)

[6] M. Hamada, H. Shimahara and H. Mori, Phys. Rev. B\textbf{51}, 11597 (1995)

[7] B. Shraiman and E. Siggia, Phys. Rev. Lett. \textbf{62}, 1564 (1989)

[8] D. Sénéchal, Phys. Rev. B \textbf{52}, 15319 (1995)

[9] S. Dell’Aringa \textit{et. al.} Phys. Rev. Lett. \textbf{78}, 2457 (1997) \textit{and the references therein}.

[10] B. Normand, J. Kyriakidis and D. Loss, cond-mat/9902104

[11] T. M. Rice \textit{et. al.} Phy. Rev. B\textbf{56}, 14655 (1997)

[12] G. Sierra, J. Math. Phys. A. \textbf{29}, 3289 (1996)

[13] S. R. White and D. J. Scalapino, Phys. Rev. B\textbf{55}, 14701 (1997), Phys. Rev. B\textbf{57}, 3031 (1998)

[14] M. Sigrist and A. Furusaki, J. Phys. Soc. Jpn. \textbf{65}, 2385 (1996)

[15] F. D. M. Haldane, Phys. Rev. Lett. \textbf{61}, 1029 (1988)

[16] E. Fradkin and M. Stone, Phys. Rev. B\textbf{38}, 7215 (1988)

[17] L. B. Ioffe and A. I. Larkin, Int. J. Mod. Phys. B. \textbf{2}, 203 (1988)

[18] V. J. Emery, S. A. Kivelson and O. Zachar, Phys. Rev. B\textbf{56} 6120 (1997)

[19] E. Orignac and T. Giamarchi, Phys. Rev. B\textbf{56}, 7167 (1997)
[20] J. M. Kosterlitz and D.J. Thouless J. Phys. C 6, 1181 (1973)

[21] D. Loss and D. Maslov, Phys. Rev. B74, 178 (1995)

[22] G. Baskaran, Phys. Rev. B63, 2524 (1989)

[23] P. Lee and N. Nagaosa, Phys. Rev. B46, 5621 (1992)

[24] S. John and A. Golubentsev, Phys. Rev. B51, 381 (1995)
FIG. 1. Spiral wavevector $Q_y$ vs. $Q_x$ for $0 < \phi < \pi$. $Q$ reaches the maximum at $Q_x = 0$, which indicates the gap formation along the $y$-direction.

FIG. 2. IP helicity modulus (HM) for 3-leg ladder with $\tilde{g} = 1$, $t_\perp = 0.2$ eV and $J_\perp = 0.04$ eV. The temperature is given in K. When HM becomes negative, spiral phase is formed at $0.1 < \delta < 0.2$. The sharp increase at higher $\delta \geq 0.8$ with subsequent negative region indicates another window of spiral mode. As leg-number increases, the doping regime for IP spirals is shifted to smaller values.

FIG. 3. OP helicity modulus for 3-leg ladder. OP spiral mode is formed at doping range $0 < \delta \leq 0.5$. Overall features are common for ladders with more number of legs, given the weaker dependence of the HM on leg-number than the dependence on doping or temperature.