Where is the chiral critical point in 3-flavor QCD?  

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We determine the location of the second order endpoint of the line of first order chiral phase transition in 3-flavor QCD at vanishing chemical potential ($\mu_q$). Using Ferrenberg-Swendsen reweighting for two values of the quark mass we determine the $\mu_q$-dependence of the transition line and locate the chiral critical point. For both quantities we find a significant quark mass dependence.

1. Introduction

Sufficiently hot and dense hadronic matter undergoes a transition to a deconfined, chirally symmetric medium. It is expected that in QCD with the correct, physically realized spectrum of quark masses this transition is a true phase transition, related to non-analyticities of the QCD partition function, only for specific choices of temperature ($T$) and baryon chemical potential ($\mu_B \equiv 3\mu_q$). The transition then is first order for large values of $\mu_B$, ends in a second order endpoint – the chiral critical point – at a certain value $\mu_{\text{crit}}$ and is a crossover transition, not related to singular behavior in thermodynamic observables, for smaller values of the chemical potential (Fig. 1).

The chiral critical point is part of a critical surface in the 3-d parameter space of degenerate u,d-masses ($m_{u,d}$) and a strange ($m_s$) quark mass as well as $\mu_B$. At $\mu_B = 0$ this surface contains the line of second order phase transitions, which separates the region of first order phase transitions for light quarks from a broad crossover region.

At $\mu_B = 0$ we determine the location of the 3-flavor critical point on this transition line. Using reweighting in $\mu_B$ we determine the curvature of the critical surface and estimate the location of the chiral critical point, $\mu_{\text{crit}}$. All calculations have been performed with improved staggered fermions (p4-action) and a Symanzik-improved gauge action [1,2].

2. Critical point at $\mu = 0$

In previous studies with unimproved staggered fermions (see, for instance [3]) the chiral critical point in 3-flavor QCD has been located at a pion mass value of about 300 MeV. Using Binder cumulants constructed from the chiral condensate, $B_4(0) = \langle (\delta \bar{\psi} \psi)^4 \rangle / \langle (\delta \bar{\psi} \psi)^2 \rangle^2$, it has been verified that this point belongs to the Ising universality class [4]. Using the knowledge of the universality class as an input, first calculations with improved staggered fermions (p4-action) on a $12^3 \times 4$ lattice suggested that the critical point shifts to substantially smaller values. In [3] a preliminary value of $m_{\text{crit}} \simeq 190$ MeV has been reported.

Our present analysis with the p4-action is based on simulations on $12^3 \times 4$ and $16^3 \times 4$ with a bare quark mass of $ma = 0.005$ which corre-
the simulation point at $m_a = 0.005$ suggests a critical value of $ma = 0.0007(4)$. At this value of the quark mass the Binder cumulant is consistent with the Ising value and the peak heights in the chiral susceptibility show a significant volume dependence which is consistent with $\chi_{\bar{\psi}\psi}^{\text{peak}} \sim V$. Some results for the reweighted chiral condensate and $\chi_{\bar{\psi}\psi}$ obtained on the $16^3 \times 4$ lattice are shown in Fig. 2. The rather small value found for the critical quark mass translates into a small value of the pion mass, which is only about half its physical value. Such a small value is consistent with the small values typically found in effective model calculations and follows the trend towards a smoother transition found in other calculations with improved staggered fermion actions [5,6]. It differs significantly from the result obtained with standard, unimproved staggered fermions. The actual location of the chiral critical point at $\mu_B = 0$ thus seems to be strongly cut-off dependent. In summary we find,

\[ m_\pi^{\text{crit}} = \begin{cases} 
290(20) \text{ MeV}, \text{ standard action} \\
67(18) \text{ MeV}, \text{ p}_4 \text{ improved action} \\
47 \text{ MeV}, \text{ linear sigma model}
\end{cases} \]

3. Reweighting in $\mu_q$ at $ma = 0.005$

At fixed value of the quark mass we use [1] Taylor expanded reweighting to $O(\mu_q^2)$ of $\chi_{\bar{\psi}\psi}$ and $B_4(0)$ to locate the chiral critical point as that point at which $B_4$ attains the 3d-Ising value. The reweighting is done only in the 2-flavor sector for (u,d)-quarks, i.e. we keep $\mu_{\text{strange}} = 0$. Reweighting was possible only for the data on the $12^3 \times 4$ lattice as reweighting to large enough values of $\mu_q$ was prohibited on the larger lattice due to too large fluctuations in the sign of the fermion determinant. Even on the smaller lattice a tendency of $B_4$ to decrease to the Ising value only sets in for $\mu_q a > 0.05$. A linear extrapolation for $\mu a \geq 0.05$ yields $\mu_{\text{crit}} a = 0.074(13)$ or $\mu_{\text{crit}} / T = 0.296(52)$. An estimate for the transition temperature at the corresponding pion mass is obtained from [7] $T_c / \sqrt{\sigma} = 0.40(1) + 0.039(4)(m_\sigma / \sqrt{\sigma})$. This sets the scale for our estimate of the critical value of the chemical potential in 3-flavor QCD, $\mu_{\text{crit}} = 52(10)$ MeV at...
\[ m_\pi = 170 \text{ MeV}. \] A quadratic interpolation to the physical pion mass value fixes the location of the chiral critical point in 3-flavor QCD,

\[ \mu_c^{\text{crit}} q = 40(9) \text{ MeV}, \quad m_\pi = 140 \text{ MeV}. \quad (3) \]

As expected the value, \( \mu_c^{\text{crit}} B = 120 \text{ MeV} \), found here in 3-flavor QCD is substantially smaller than the value of about 700 MeV estimated for (2+1)-flavor QCD [8].

4. The \( \mu \)-dependence of the transition line

Following the prescription outlined in [4] the \( \mu \)-dependence of the transition temperature has been obtained using Taylor expanded reweighting of susceptibilities to \( \mathcal{O}(\mu^2) \). From the shift of the peak positions one finds the critical couplings, \( \beta_c(\mu_q) \), which to leading order depend linearly on \( \mu^2_q \) and yield an estimate for the slope of \( T_c(\mu_q) \):

\[
\frac{d^2 T_c}{d\mu^2_q} = \frac{1}{N_c^2 T_c(0)} \frac{d^2 \beta_c}{d\mu^2_q} \left( a \frac{d\beta_c}{da} \right) \quad (4)
\]

Comparing calculations in 3-flavor QCD for \( ma = 0.005 \) and 0.1 we find a significant increase in the slope of \( T_c(\mu_q) \) with decreasing \( ma \). Using the asymptotic \( \beta \)-function in (4) gives

\[
\frac{T_c(\mu_q)}{T_c(0)} = \begin{cases} 
1 - 0.025(6) \left( \frac{\mu_q}{T_c(0)} \right)^2, & ma = 0.1 \\
1 - 0.114(46) \left( \frac{\mu_q}{T_c(0)} \right)^2, & ma = 0.005 
\end{cases}
\]

We note that an additional quark mass dependence is hidden here in the transition temperature at \( m_q = 0 \), i.e. \( T_c(0) \), which is used to normalize the transition temperature at \( \mu_q \neq 0 \). Furthermore, taking into account violations of asymptotic scaling in the \( \beta \)-function will lead to a further increase of the curvature of \( T_c(\mu_q) \). At \( ma = 0.01 \) deviations from asymptotic scaling have been analyzed in detail and suggest that the slope increases by almost a factor of 2 [5].

The transition lines for the two different quark masses together with the estimates for the location of the chiral critical points in 3-flavor QCD (left and right boxes) and the (\( \mu_q \)-independent) transition line in the infinite quark mass limit are shown in Fig. 3.

5. Conclusions

At vanishing chemical potential the cut-off dependence of the chiral critical point is large. Calculations with an improved action suggest that the chiral transition in 3-flavor QCD becomes first order at much smaller values of the pion mass than estimated previously. These findings are consistent with other studies of QCD thermodynamics performed with improved staggered fermions [5,6]. A similar influence of cut-off effects is found for the dependence of the chiral critical point on the chemical potential.

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