A Causal Source which Mimics Inflation

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(22/7/96)

How unique are the inflationary predictions for the cosmic microwave anisotropy pattern? In this paper, it is asked whether an arbitrary causal source for perturbations in the standard hot big bang could effectively mimic the predictions of the simplest inflationary models. A surprisingly simple example of a ‘scaling’ causal source is found to closely reproduce the inflationary predictions. This Letter extends the work of a previous paper (ref. 6) to a full computation of the anisotropy pattern, including the Sachs Wolfe integral. I speculate on the possible physics behind such a source.

The prospect of mapping the cosmic microwave background (CMB) anisotropies to high resolution has raised the exciting possibility of confirming fundamental theories of the origin of structure in the universe. The inflationary theory, being the simplest and most complete, is the present front-runner, and the latest CMB measurements do even seem to hint at the presence of first Doppler peak in the angular power spectrum, at $l \sim 200$ as predicted by the simplest, spatially flat inflationary models. These spectra are distinct from those predicted by cosmic defect or baryon isocurvature models, and it is an important question whether spectra of this form are really a unique prediction of inflation. Or could a non-inflationary mechanism somehow replicate them?

The fundamental difference between inflationary and non-inflationary mechanisms of structure formation is that inflation alters the causal structure of the early universe, adding on a prior epoch during which correlations are established on scales much larger than the Hubble radius. This is of course how the standard ‘horizon puzzle’ is solved. The perturbations produced during inflation are susceptible to very detailed test, and one could hope that the same feature of ‘super-horizon’ scale correlations (quotes indicate a standard big bang definition) could be used as a signature. If ‘super-horizon’ perturbations were shown to exist, this would strongly confirm inflationary structure formation, for no causal mechanism could have produced them within the standard big bang.

Since COBE observed large scale perturbations on the CMB sky, one might think this already showed that ‘super-horizon’ perturbations were present at last scattering. But these large angle anisotropies could have been produced causally within the standard big bang, through a time dependent gravitational potential along the line of sight. Cosmic defects, as well as open universe or $\Lambda$ dominated models provide explicit examples of theories where this actually happens.

The smaller angle anisotropies are a more promising probe because they are due to local effects on the surface of last scattering, which are strongly constrained by causality (Figure 1). In particular, the pattern of Doppler peaks caused by phase coherent oscillations in the photon-baryon fluid provides a possible signature of ‘super-horizon’ curvature perturbations. Early papers exploring this include refs. 3, 4 and 5. This Letter is a follow-up of 6, where an inflation-mimicking model was proposed. A recent paper by Hu, Spergel and White 7 criticising that model is now undergoing revision.

In this Letter I ask whether a causal source acting purely via gravity within the standard hot big bang could generate CMB anisotropies similar to those in flat inflationary models. I further restrict consideration to ‘scaling’ sources, mainly for simplicity. This restricted set of models turns out to provide a surprisingly simple inflationary mimic. I emphasise that the mimic is not a theory, but simply an ‘ansatz’ constructed by hand to provide a consistent solution to the Einstein equations. As such it provides a counterexample to some arguments in the literature (see e.g. 5). Furthermore it has a sufficiently simple form that it is not out of the question that it might actually be realised in a future theory of structure formation - in that sense the counterexample may turn out to be constructive.

FIG. 1. The causality constraint on the microwave background anisotropy. The picture is in comoving coordinates: the outer circle represents our causal horizon in the standard big bang. The inner circle is the surface of last scattering, on which photons are ‘set free’ from the hot plasma on their path to us. Circles show the domains of influence on these photons - the radius is the light travel distance since the hot big bang $\tau_{LS}$. This subtends an angle $\Theta_{LS} \sim 1.1^\circ$ in a flat universe with standard recombination. No causal physics operating within the standard big bang could have generated correlations between photons on the last scattering surface at points separated by more than $2\Theta_{LS}$ on the sky.
I deal with the linearised Einstein equations in the ‘stiff’ approximation, in which the induced perturbations are assumed to have negligible effect on the source. In this approximation, the source stress energy tensor $\Theta_{\mu\nu}$ is covariantly conserved with respect to the background metric:

$$\dot{\Theta}_{00} + \frac{\dot{a}}{a}(\Theta_{00} + \Theta) = \Pi; \quad \Pi + 2\frac{\dot{a}}{a} = \partial_i \partial_j \Theta_{ij} \quad (1)$$

where $\Pi = \partial_i \Theta_{0i}$. Dots denote derivatives with respect to conformal time $\tau$, and $a(\tau)$ is the scale factor.

A formalism for dealing with such sources was proposed in [3]. Here I shall consider only ‘coherent’ sources, which are representable in terms of a single set of ‘master functions’. These are ordinary functions, which obey [3]. The unequal time correlator $<\Theta_{\mu\nu}(r, \tau)\Theta_{\rho\lambda}(0, \tau')>$ equals the spatial convolution of the master function for $\Theta_{\mu\nu}$ with that for $\Theta_{\rho\lambda}$.

As argued in [3], the causality constraint implies that the master functions $\Theta_{\mu\nu}(r, \tau)$ are zero for all $r > \tau$. If we ignore vector and tensor perturbations, they are spherically symmetric, and we can represent them as $\Theta_{i0}(r, \tau) = x_i J_i(r, \tau)$, $\Theta_{ij}(r, \tau)$ with $\Theta_{ii}(r, \tau)$ the pressure. In situations where matter is being accelerated, we can expand in a Taylor series. The leading term $\Theta = \Pi$ is the pressure $P$, and $\Theta^A$ the anisotropic stress. In situations where matter is being actively moved, as it will be here, the anisotropic stresses are generally of the same order as the pressure.

Some general properties can now be seen. The Fourier transforms (assumed to exist) are analytic about $k = 0$, and can be expanded in a Taylor series. The leading terms are then (by isotropy) $\Theta_{i0} \sim k^0$, $\Theta_{ii} \sim k_i$ (from which $\Pi \sim k^2$ follows) and $\Theta_{ij} \sim \delta_{ij}$. In Fourier space we write $\Theta_{ij}(k) \equiv 4\delta_{ij} \Theta + (k_i k_j - \delta_{ij}) \Theta^S$, where $\Theta^S = k^{-1} d\Theta^A / dk - \partial^2 \Theta^A / dk^2$. It follows that $\Theta^S \sim k^2$.

I now specialise to ‘scaling’ sources, in which one assumes that the source $a_i$ involves a number with dimensions of the inverse of Newton’s constant $G$ and $b_i$ involves no other length scale apart from the horizon scale $\tau$. If these conditions are met, the source-perturbation equations are scale-invariant, apart from the violation of scaling caused by the radiation-matter transition. Scaling and dimensional analysis imply that (see e.g. [3]) $\Theta_{i0}(k, \tau) \sim \tau^{-\frac{4}{3}} f_1(k \tau)$, $\Theta_{ij}(k, \tau) \sim \tau^{-\frac{2}{3}} f_2(k \tau)$, $\Pi(k, \tau) \sim \tau^{-\frac{2}{3}} f_3(k \tau)$, $\Theta^S(k, \tau) \sim \tau^{-\frac{4}{3}} f_4(k \tau)$, where the $f_i$ have power series expansions in $k^2$ obeying the restrictions noted above. These four $f_i$s are are related by the two energy momentum conservation equations [4]. So for example, the $k^0$ term in the first equation relates the leading terms in $f_1$ and $f_2$, and the $k^2$ term in the second equation relates the leading terms in $f_2$ and $f_3$. But even after applying these equations, we still have essentially two free functions remaining. We also have some freedom in how to incorporate the matter-radiation transition into the source.

I assume the background spacetime is flat, and has metric $ds^2 = a^2(\tau)(d\tau^2 + (\delta_{ij} + h_{ij}(x, \tau))dx^i dx^j)$, with $\tau$ conformal time and $a(\tau)$ the scale factor. I work in initially unperturbed synchronous gauge, which is especially suitable for causal theories since the Einstein equations are manifestly causal (in contrast, in Newtonian gauge the influence of anisotropic stresses propagates acausally [9]). This gauge is also straightforward to interpret - for example the radiation density contrast $\delta R$ is just that measured in the rest frame of freely falling particles like cold dark matter particles.

We are interested in computing the CMB temperature distortion on the sky in a direction $\mathbf{n}$:

$$\frac{\delta T}{T}(\mathbf{n}) = \frac{1}{4} \delta R(i) - \mathbf{n} \cdot \mathbf{v}_R(i) - \frac{1}{2} \int d\tau J(x, \tau) n^i n^j \quad (2)$$

where $\delta R$ is the density contrast, and $\mathbf{v}_R$ the velocity, of the photon fluid on the surface of last scattering. The last term is the Sachs-Wolfe integral, representing the change in the proper path length field along the line of sight. This expression holds in the ‘instantaneous recombination’ approximation, which I shall use throughout.

The first two terms are local effects on the surface of last scattering. They are determined from:

$$\delta C + \frac{\dot{a}}{a} \delta C = 4\pi G \left( \sum \rho N \delta_N + \Theta_{00} + \Theta \right), \quad (3)$$

$$\dot{\delta R} = \frac{4}{3} \dot{\Theta} C - \frac{4}{3} \nabla \cdot \mathbf{v} R \cdot \mathbf{v}_R = -(1 - 3c_s^2) \frac{\dot{a}}{a} \mathbf{v}_R + \frac{3}{4} c_s^2 \nabla \delta R \quad (4)$$

where $c_s$ is the speed of sound in the photon-baryon fluid. The only component of the stress energy tensor that enters is $\Theta_{00} + \Theta$, i.e. $\rho + 3P$. Thus prior to last scattering, only one of the two free functions in $\Theta_{\mu\nu}$ contributes - the other is literally ‘invisible’ in the CMB anisotropy.

In the simplest inflationary theory, the surface term $\frac{\dot{a}}{a} \dot{\delta R}$ captures most of the relevant physics determining the location of the Doppler peaks. In ref. [3], I focussed only on this term, and found a causal source $\Theta_{00} + \Theta$ which mimicked the same term in the inflationary theory. The idea was simply to choose

$$\Theta_{00} + \Theta \propto f_1(r) + f_2(r) \propto \delta(r - A \tau) \quad 0 < A \leq 1 \quad (5)$$

representing a spherical shell expanding at some fraction of the speed of light. I found that for $A$ close to unity, the $\delta R$ surface term closely matched the inflationary one. Such a shell of matter is similar in form to a supernova explosion - for a spherical shell of neutrinos, one has $\Theta_{ij} \sim \Sigma p'p' \propto x^2 x^j / r^2$. In that case, $\Theta^S$ and $\Theta$ are comparable in magnitude, and the same is true here.

Here I extend the computation to the entire expression [3]. The Sachs Wolfe integral introduces some dependence on the anisotropic part of the metric perturbation. To compute this it is necessary to further specify $\Theta_{\mu\nu}$.
by for example specifying another of the $f_i$ functions. The simplest choice leaving $f_1 + f_2$ fixed is to specify $f_3$. Then equations (11) are used as follows: the energy equation is integrated to determine $\Theta_{00}$, and the momentum equation is differentiated to determine $\Theta^S$. Of course this must be done consistently with the matching of the leading terms as discussed above.

In Fourier space, the choices I make for the source are:

$$\Theta_{00} + \Theta = \frac{a \sin Ak\tau}{\dot{a}} Ak\tau^2$$

(6)

as in [3], with the prefactor incorporating the radiation-matter transition in a simple way. For $\Pi$, we must satisfy $\Pi(k) \sim k^2$ at small $k$. Equivalently, the integral $\int_0^\tau r^2 dr \Pi(r, \tau) = 0$. This is most easily satisfied by taking $\Pi(r, \tau)$ to be the sum of two delta functions, of equal weight but opposite sign. Their Fourier transform produces:

$$\Pi = -\frac{E(\tau)}{k^2 + B^2 - C^2} \left(\frac{\sin B k\tau}{B} - \frac{\sin C k\tau}{C}\right)$$

(7)

where $E(\tau)$ is a messy function obtained by analytically solving for the coefficient of $k^2$ term in the momentum equation [4]. It equals $\frac{2}{3\pi}$ in the radiation era and $\frac{2}{3\pi}$ in the matter era. A set of values which I find leaves the Sachs Wolfe integral sub-dominant is $B = 1.0$ and $C = 0.5$.

The anisotropic metric perturbation $h^S(k)$ (defined analogously to $\Theta^S$) is now given by $h^S = h = -24\pi G(\Pi + \Sigma_N(N + \rho_N) a^2i k \cdot v_N)/k^2$. Finally, I model the free streaming of photons and neutrinos after last scattering following ref. [3]. The initial conditions for the $\Theta_{00}$ and the perturbations are set up deep in the radiation era $\tau < \tau_{EQ}$, and well outside the horizon $k\tau < 1$: they are read off from the $k^0$ terms in the energy conservation, and perturbation equations:

$$\Theta_{00} = 2\tau - \frac{1}{3} \delta_R = \delta_\nu = \frac{4}{3} \delta_C = D \tau^2 \quad v_R = 0$$

(8)

with the constant $D$ determined by setting the total pseudoenergy $\tau_{00} = k^2(h - h^S)/(24\pi G) = \Theta_{00} + \sum_N N a^2 \delta_N + (\dot{a}/a) \delta_C/(4\pi G)$ to zero. With these choices there are no superhorizon perturbations in the photon-to-CDM, photon-to-baryon or photon-to-neutrino ratios. The pseudoenergy is just the Ricci scalar of the spatial slices, so setting it zero means there are no curvature perturbations either. These initial conditions are thus both ‘adiabatic’ and ‘isocurvature’ - there are simply no perturbations in the universe on superhorizon scales. The complete $C_l$ spectrum of the causal model defined in equations (6-8) is shown in Figure 2.

What about the sub-horizon behaviour of the source? In the construction above, where I integrate the energy equation to determine $\Theta_{00}$, there is no reason it should tend to zero well inside the horizon. However, because I explicitly turn off $\Theta_{00} + \Theta$ inside the horizon, the source ceases to have any effect on the fluid perturbations and the trace part of the metric $h = -2\delta_C$. The effect on the anisotropic part $h^S$ is similarly turned off because $\Pi$ goes to zero. In effect, I have turned off all the ‘gravitationally active’ components of the source, but there is no reason for the energy $\Theta_{00}$, the pressure $\Theta$ or the anisotropic stresses $\Theta^S$ to separately vanish - they only need to satisfy the relations $\Theta_{00} + \Theta = 0$ and $\Theta + 2\Theta^S = 0$. This is reminiscent of the behaviour of a straight cosmic string - it carries energy but generates no gravitational field. In any case, the sub-horizon source is removed if one adds a term $c_1 k^2 (\dot{a}/a) \Theta_{00}$ to $\Theta_{00} + \Theta$, or a term $-c_2 k^2 \Theta_{00}$ to $\Pi$. Either of these makes $\Theta_{00}$, $\Theta$, and $\Theta^S$ go to zero inside the horizon as $\exp(-\text{const.}k^2 \tau^2)$. Figure 3 shows the evolution of the components of $\Theta_{\mu\nu}$ with and without these modifications, and Figure 4 shows the corresponding $C_l$ spectra. The moral is that there is a lot of freedom inside the horizon to make very large alterations in the source without significantly affecting the CMB anisotropies.

How sensitive to the particular choice of Ansatz is the result? Figure 4 compares the $A = 1$ model with the cases $A = 0.7$, and $A = 0.1$. The leftward shift in the first peak was explained in ref. [3]. As shown there, it is also easy to arrange for a shift to the right. So while the $A = 1$ model is on the causality limit, there is a substantial region of parameter space around it with similar $C_l$ spectra. It would be very difficult to observationally separate these models from inflation, especially in the realistic case where we are unlikely to know all the rele-
vant cosmological parameters ($\Omega, \Omega_B, h, \Omega_\Lambda$ and so on) in advance. I have so far explored only a very tiny part of model space, and preliminary investigations of more general Ansätze indicate that a very wide range of $C_l$ spectra are possible, especially when the Sachs Wolfe integral becomes a dominant effect.

I have given myself a great deal of freedom in constructing this source, and it is far from clear that realistic physics could produce it. Nevertheless, the reader may tolerate some speculations. As mentioned, the form of $\rho + 3P$ required is similar to that resulting from a supernova explosion, but whereas in the latter case the energy redshifts away as $a(t)^{-1}$, here, scaling evolution requires that the energy in the shell increases - by dimensions, $E \sim G^{-1}t$. This requires some positive feedback mechanism, which one could conceivably arrange with unstable dark matter, decaying via stimulated emission of Goldstone bosons, or even gravity waves. Another issue is the Gaussianity or otherwise of the perturbations. If the source is made up of a very large number of ‘explosions’ which are allowed to superpose, then it can be made arbitrarily Gaussian. But if the exploding shells interact, there would be a limit to their number density, and the perturbations would then be nonGaussian.

The conclusion of this paper is that causality alone is insufficient to distinguish the inflationary $C_l$ predictions from those of non-inflationary models. Of course the observational confirmation of one of these spectra would be a tremendous success for inflation, but the door would nevertheless still be left open to other possible explanations of cosmic structure formation.

I thank A. Albrecht and J. Magueijo for discussions, R. Crittenden for collaboration on the codes used here, and W. Hu, D. Spergel and M. White for helpful correspondence. This work was supported by a grant from Cambridge University and PPARC, UK.

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