Isoscaling behavior in the Isospin dependent Quantum Molecular Dynamics model

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(Dated: March 30, 2022)

The isoscaling behavior is investigated in the frame of Isospin dependent Quantum Molecular Dynamics (IQMD) models. The isotopic yields ratio $Y_2/N$ for reactions $^{48}$Ca+$^{40}$Ca and $^{40}$Ca+$^{48}$Ca at different entrance channels are simulated and presented, the relationship between the isoscaling parameter and the entrance channel is analyzed, the results show that $\alpha$ and $\beta$ reduce with the rise of incident energies and increase with the impact parameter $b$, which can be attributed to the temperature varying of the pre-fragments in different entrance channels. The relation of $\alpha$ and $\beta$ symmetry-term coefficient $C_{sym}$ reveals that the chemical potential difference $\Delta\mu$ is sensitive to the symmetry-term coefficient $C_{sym}$.

PACS numbers: 25.70. Pq, 24.10 Nz

Experimental investigations of the properties of isospin asymmetric nuclear matter are receiving increased attention due to the possibility of creating nuclear material with appreciable isospin asymmetry using neutron-rich beams. This is expected to enable one to extrapolate the present understanding regarding the effective nucleon-nucleon interaction to the unknown domain of large isospin asymmetry. The isospin asymmetry of reaction parts plays an important role in the evolution of nucleon exchange processes which form the basis of many nuclear transport models. It is an effective indicator of the degree of chemical equilibrium reached in heavy ion reactions.

To minimize undesirable complications stemming from the sequential decays of primary unstable fragments, it has been proposed that isospin effects can be best studied by comparing the same observable in two similar reactions that differ mainly in isospin asymmetry. If two reactions, 1 and 2, have the same temperature but different isospin asymmetry, for example, the ratio of a specific isotope yields with neutron and proton number $N$ and $Z$ obtained from system 2 and system 1 have been observed to exhibit isoscaling, i.e., exponential dependence of the form:

$$R_{21}(N, Z) = \frac{Y_2(N, Z)}{Y_1(N, Z)} = C \exp(\alpha N + \beta Z). \quad (1)$$

where $\alpha$ and $\beta$ are the scaling parameters and $C$ is an overall normalization constant, with the convention of that the neutron and proton composition of reaction 2 is more neutron-rich than that of reaction 1. The systematization of the experimental data in form (1) has been conformed to variety of reactions over wide energy range and reaction systems. This scaling law occurs naturally within various reaction processes, such as the evaporation, deep inelastic scattering, fission, and multifragmentation. In the framework of different models, it has been revealed that the isotope yields ratio are not affected very much by the sequential decays following the fast stage of the reaction, so $R_{21}(N, Z)$ deduced from the detected isotope fragments can reflect the isotope yields ratio of the primary fragments. The isoscaling parameters $\alpha$ and $\beta$ can be used to study the isospin dependent properties in nuclear collisions, and this method allows one to study the early reaction stage of the decay fragmenting system, to understand the dependence of measured isotope distribution on properties of hot emitter, the nuclear asymmetry term in nuclear matter equation of state (EOS), and allows the direct comparison between experimental measurement with theoretical simulation.

To explore the isospin properties and the fragmentation in dynamical nuclear collisions, we study the isoscaling phenomenon by the IQMD model, which is based on the general QMD model to include explicitly isospin-degrees of freedom. The QMD model is classical in essence because the time evolution of the system is determined by classical canonical equation of motion, however, many important quantum features are included in this prescription. It is well known that the dynamics in heavy ion collision at intermediate energies is mainly governed by three components: the mean fields, two-body collision and Pauli blocking, therefore, for isospin-dependent reaction dynamics model it is important for these three components to include isospin degrees of freedom. In addition, in initialization of projectile and target nuclei, the samples of neutrons and protons in phase space are also treated separately since there

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* Supported by the Major State Basic Research Development Program under Contract No G2000774004, the National Natural Science Foundation of China (NNSFC) under Grant No 10328259 and 10135030, and the Chinese Academy of Sciences Grant for the National Distinguished Young Scholars of NNSFC.

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exists a large difference between neutron and proton density distributions for nuclei far from the \( \beta \)-stability line \[10,17\].

In the IQMD model, the nuclear mean field is given by

\[ U(\rho, \tau_z) = U^{dd} + U^{Yuk} + U^{Coul} + U^{Sym} + U^{MDI} \]  

(2)

where \( U^{dd} \) is the density-dependent (Skyrme) potential, \( U^{Yuk} \) the Yukawa (surface) potential, \( U^{Coul} \) the Coulomb energy, \( U^{Sym} \) the symmetry energy term and \( U^{MDI} \) the momentum dependent interaction, but in our present calculation, \( U^{MDI} \) was not included. The \( U^{dd} \) can be written as

\[ U^{dd} = a\left( \frac{\rho}{\rho_0} \right) + b\left( \frac{\rho}{\rho_0} \right)^\gamma \]  

(3)

with \( \rho_0 = 0.16 \text{fm}^{-3} \) (the normal nuclear matter density), \( a = -356 \text{MeV}, b = 303 \text{MeV} \), and \( \gamma = 1.17 \) (corresponding to the Soft EOS).

\[ U^{Yuk} = \frac{V_y}{2} \sum_{i \neq j} \frac{1}{r_{ij}} \exp(Lm^2) \times \left[ \exp(mr_{ij}) \text{erf}(\sqrt{Lm} - r_{ij}/\sqrt{4L}) - \exp(mr_{ij}) \text{erf}(\sqrt{Lm} + r_{ij}/\sqrt{4L}) \right] \]  

(4)

\[ U^{Coul} = \frac{e^2}{4} \sum_{i \neq j} \frac{1}{r_{ij}} (1 + \tau_{iz})(1 - \tau_{zj}) \text{erfc}(r_{ij}/\sqrt{4L}) \]  

(5)

\[ U^{Sym} = \frac{C_{sym}}{2\rho_0} \sum_{i \neq j} \frac{1}{(4\pi L)^{3/2}} \exp(-\frac{(r_i - r_j)^2}{4L}) \]  

(6)

with \( V_y = -0.0024 \text{GeV} \), \( m = 0.83 \), and \( L \) is the so-called Gaussian wave-packet width (here \( L = 2.0 \text{fm}^2 \)). The relative distance \( r_{ij} = |r_i - r_j| \). The \( \tau_{iz} \) is the \( z \)-th component of the isospin degree of freedom for the \( i \)-th nucleon, which is equal to 1 and -1 for proton and neutron, respectively. \( C_{sym} \) is symmetry energy strength. More details on QMD and IQMD models are discussed in Ref \([10,17,18,20]\).

We first perform the reaction simulations for \( ^{40}\text{Ca} + ^{40}\text{Ca} \) and \( ^{48}\text{Ca} + ^{48}\text{Ca} \) collisions with impact parameter \( b = 0 \) at several different incident energies \( E/A = 25, 35, 50 \) and 70 MeV, the dynamical process was simulated until \( t = 200 \text{fm}/c \). In central collisions, many fragments are formed, which can be basically divided into projectile-like, target-like and neck region emitted products. In the neck region of central collision, it is more reliable that the thermal and chemical equilibrium could be achieved, so we select the products with the parallel velocity condition of \( |v_{ij}/v_{in}| < 0.2 \), where \( v_{ij} \) means the parallel velocity of the products, and \( v_{in} \) is the incident velocity, by which determined the fragments emitted from the neck region parts. In the following discussion all simulation results are under such selection.

\[ \text{FIG. 1: The fragment yield ratio of the IQMD simulations for central collisions } ^{48}\text{Ca} + ^{48}\text{Ca} \text{ and } ^{40}\text{Ca} + ^{40}\text{Ca} \text{ at } 35\text{MeV}/A, \alpha \text{ and } \beta \text{ are printed inside the figure.} \]

While extracting the isoscaling parameters \( \alpha \) and \( \beta \), we use \( R_{21}(N) = C'\exp(\alpha N) \) and \( R_{21}(Z) = C'\exp(\beta Z) \), respectively. Fig. 2 plots the variation of \( \alpha \) and \( \beta \) with the incident energies for central collisions \( ^{40}\text{Ca} + ^{40}\text{Ca} \) and \( ^{48}\text{Ca} + ^{48}\text{Ca} \), one can find that the isoscaling parameters \( \alpha \) decreases with the increasing of the incident energy as well as \( \beta \). In the grand-canonical approximation \[8,9,18\], \( \alpha = \Delta\mu_n/T \) and \( \beta = \Delta\mu_p/T \), provided a common temperature \( T \) for both systems exist \[8,9,18\]. In two similar reactions \( ^{40}\text{Ca} + ^{40}\text{Ca} \) and \( ^{48}\text{Ca} + ^{48}\text{Ca} \), \( \Delta\mu_n \) and \( \Delta\mu_p \) should not show any significant changes with the temperature for light charged particles as the model calculations \[11\], therefor the reduction of \( \alpha \) and \( \beta \) with the increasing of incident energies can be mainly attributed to the rise of
mass of the decaying prefragment, is the excitation energy of the prefragment, and $A$ is less violent from central to peripheral collision, then systems generally be defined as $T = \frac{k(E_0)}{\langle A_0 \rangle}$, where $E_0$ is the excitation energy of the prefragment, $k$ denotes the inverse level density parameter, the initial temperature $T$ determined by the excitation of the decaying prefragment. The peripheral collision have less excitation energy comparing with the central collision, which means low temperature the emitting source can be reached. The free

neutron and proton chemical potential difference $\Delta \mu_n$ and $\Delta \mu_p$ does not change with the varying of the temperature very much [11], the reduction of isoscaling parameters $\alpha$ and $|\beta|$ can be caused by the temperature difference.

Furthermore, to explore the aspect of equilibrium in fragment emission in IQMD simulation, we study the relationship between the isoscaling and the fragment isospin asymmetry in IQMD simulation. The following relation (7) was found relating the isoscaling parameter $\alpha$, the $(Z/A)^2$ of fragments, and the symmetry energy coefficient $C(Z)$ by the form [12]

$$\alpha = 4C_{\text{sym}}[(Z_1/A_1)^2 - (Z_2/A_2)^2]/T$$

(7)

where $C_{\text{sym}}$ is the symmetry energy coefficient and $T$ is the system temperature, 1 and 2 denote the neutron-deficient and neutron-rich reaction system, respectively. In IQMD simulation, three reactions of $^{40}\text{Ca} + ^{40}\text{Ca}$, $^{48}\text{Ca} + ^{48}\text{Ca}$ and $^{60}\text{Ca} + ^{60}\text{Ca}$ are simulated with different symmetry-term potential coefficient $C_{\text{sym}}$, at the incident energy $E/A = 35$MeV, the relation between the $(Z/A)^2$, symmetry potential coefficient $C_{\text{sym}}$ and the isoscaling parameter $\alpha$ from three simulations are plotted in Fig. 4, where $Z/A$ is calculated from the emitted fragments, averaged over the fragment $5 \leq Z \leq 8$. One can find that linear relation is generally kept between $\alpha$ and $(Z/A)^2$ for different $C_{\text{sym}}$, this linear function can prove the chemical potential equilibration in reactions $^{40}\text{Ca} + ^{40}\text{Ca}$, $^{48}\text{Ca} + ^{48}\text{Ca}$, and $^{60}\text{Ca} + ^{60}\text{Ca}$ though different symmetry potential coefficients $C_{\text{sym}}$ are used in the simulations, since equation (7) is satisfied only in thermal and chemical potential equilibration.
The insert figure shows that the isoscaling parameter $\alpha$ increases with the increasing of symmetry-term coefficient $C_{sym}$, in both $Y(^{48}\text{Ca}+^{48}\text{Ca})/Y(^{40}\text{Ca}+^{40}\text{Ca})$ and $Y(^{60}\text{Ca}+^{60}\text{Ca})/Y(^{40}\text{Ca}+^{40}\text{Ca})$. Up to now no evidence show asymmetry-term coefficient $C_{sym}$ have impact on the system temperature, since the temperature of the system, as we mentioned above, mainly determined by the incident energy, also means the excitation energy of the reaction. Other models like isospin dependent lattice gas model (LGM) \cite{15} and statistical multifragmentation (SMM) model \cite{11} predicted that the chemical potential differences between two similar reaction $\Delta \mu_n$ varies little in different temperature. But the chemical potential difference $\Delta \mu_n$ changes a lot with different symmetry-term coefficients $C_{sym}$, the increasing of $\alpha$ mainly comes from the contribution of chemical potential difference $\Delta \mu_n$.

In summary, with help of the IQMD simulation, the isotope yield ratio in the multi-fragmentation process in intermediate energies heavy ion collision, shows the isoscaling behavior in different entrance channels. The isoscaling parameters $\alpha$ and $\beta$ drop with the incident energies and raise with the impact parameters, which can be basically attributed to the different temperature of the emitting source. The isoscaling law also presents a linear relation between $\alpha$ and $(Z/A)^2$, but the $\alpha$ varies with symmetry-term coefficient $C_{sym}$ which indicates $\alpha$ can be served as a sensitive probe to the symmetry potential as LGM shows \cite{15}.

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