The use of an algorithm for determining the thrust of an electric propulsion system during a flight into a geostationary orbit

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Abstract. The article deals with the problem of control during the flight to the geostationary orbit of interorbital transport vehicles with low-thrust electric propulsion systems. The analysis of the influence on the final trajectory parameters of disturbing factors of various kinds, including the presence of a systematic error in the implementation of electric propulsion thrust. An algorithm for narrowing the area of deviation of the final trajectory parameters during the flight to geostationary orbit using low-thrust engines has been developed, which includes an algorithm for forming nominal programs for controlling the thrust vector and an algorithm for refining the amount of thrust created by an electric propulsion system, based on measurements of the actual period of circulation.

1. Introduction
Currently, an urgent problem is to increase the efficiency of transport operations to bring payloads into geostationary orbit.

One of the possible solutions to this problem is the use of electric propulsion engines of low thrust, the high speed of the expiration of the working fluid which provides a significantly lower consumption of the working fluid compared to an engine using chemical fuel. However, low-thrust flights are quite long [1].

To date, there are various projects of interorbital transport vehicles with an electric propulsion system of low thrust, and their multiple use is supposed.
Similar design studies are being carried out at RSC Energia, the research center named after M.V. Keldysh [2 - 5].

Devices of this type have significant dimensions and, accordingly, mass-inertial characteristics, which significantly complicates the process of motion control.
During the flight, various perturbing forces and moments act on the device, which will lead to significant deviations of the actual flight path from the nominal. It is necessary to periodically adjust the motion control program in order to ensure a given accuracy of launching into the target orbit.

2. Formulation of the problem
Let the state vector be characterized by four parameters: the values of the semi-major axis A, eccentricity e, inclination i, longitude λ, which determines the position of the spacecraft in orbit, i.e. $\mathbf{X} = \{A, e, i, \lambda\}$. Then the final state is described by the vector $\mathbf{X}_f = \{A_f, e_f, i_f, \lambda_f\}$. As a result of various
perturbations at the stage of derivation at the end point of the trajectory, we will have a deviation vector of the final state \( \Delta \mathbf{X}_f = \{ \Delta A_f, \Delta e_f, \Delta I_f, \Delta \lambda_f \}^T \), which will be some region \( G \).

The aim of the control at the geostationary orbit site using small thrust engines is to narrow the region \( G \) to the region \( G_A \) - the allowable region of deviations of the state vector parameters that satisfies the given accuracy of the launch [6].

The solution to the control problem is built in two stages:

- the formation of an algorithm that allows you to translate the deviation vector of the final state \( \Delta \mathbf{X}_f \) to a certain region \( G' \) in which one or more components of the vector \( \Delta \mathbf{X}_f \) satisfy a given accuracy (for example, the deviation in inclination \( \Delta I_f \)) (Figure 1);
- the formation of control laws and algorithms for narrowing the domain \( G' \) to the domain \( G \), where all the components of the vector \( \Delta \mathbf{X}_f \) satisfy the given accuracy [7, 8].

This article discusses the solution to the control problem at the first stage.

![Figure 1. Control algorithm with prediction of the final state and identification of disturbing factors.](image)

### 3. Analysis of the influence of disturbing factors

In this work, it was assumed that the direction of the thrust vector is given only by the angle \( \psi \) between the transversal and the thrust vector. In this case, the projections of jet acceleration on the axis of the orbital coordinate system will be equal to:

\[
\begin{align*}
a_T &= \frac{P}{M} \cos \psi, \\
a_S &= 0, \\
a_W &= \frac{P}{M} \sin \psi,
\end{align*}
\]

where \( P \) is the thrust of small thrust engines, \( M \) is the current mass of the spacecraft, \( a_T, a_S, a_W \) are the transverse, radial, and binormal components of jet acceleration, respectively.

The angle control program \( \psi \) when flying to geostationary orbit is defined as follows [9]:

\[
\psi(V_x, u) = \psi_m(V_x) \text{sign}(\cos u)
\]

where \( \psi_m(V_x) \) is the amplitude of the periodic oscillations of the angle \( \psi \).

To simulate changes in the trajectory parameters, equations in equinox elements were used taking into account the action of perturbing accelerations from the non-sphericity of the Earth’s gravitational field, the action of the gravitational fields of the Moon, the Sun and light pressure, and executive errors in the magnitude of the thrust vector were also taken into account.
A series of calculations of the trajectories of the disturbed motion of the interorbital transport vehicle with an electric propulsion system for the optimal control program was carried out (1). In this case, the case was considered when only perturbations of the Earth’s gravitational field, lunar-solar perturbations, and the case when executive errors occur when realizing thrust of an electric propulsion system occur.

The starting mass of the interorbital transport vehicle was taken equal to 20 tons. The velocity of the expiration of the working fluid with was considered equal to 25 km/s. The height of the initial circular orbit was 800 km, the inclination was 51.60, and the initial reactive acceleration $a_0$ was assumed to be 0.0006 m/s$^2$.

Figures 2 and 3 show the deviation regions of the final trajectory parameters in the planes $\Delta A_f - \Delta i_f$ and $\Delta A_f - \Delta e_f$ for the case of perturbations of the Earth's gravitational field and lunar-solar disturbances.

When perturbations from the gravitational fields of the Earth, the Moon, the Sun and light pressure are taken into account, deviations along the semi major axis are about 60 - 70 km, and according to the inclination - up to 0.20. Deviations in eccentricity are up to $3.0 \times 10^3$.

A numerical simulation of the trajectory motion of an interorbital transport vehicle with an electric propulsion system was carried out during a flight to a geostationary orbit, provided that the thrust of the electric propulsion system was realized with some systematic error $\delta P$ (Figures 4 and 5).

![Figure 2](image1.png)  
**Figure 2.** The deviation region of the final trajectory parameters in the plane $\Delta A_f - \Delta i_f$.

![Figure 3](image2.png)  
**Figure 3.** The deviation region of the final trajectory parameters in the plane $\Delta A_f - \Delta e_f$.  

\[\Delta A_f, \text{ km} \]
\[\Delta i_f, \text{ deg} \]
\[\Delta A_f, \text{ km} \]
\[\Delta e_f \times 10^2 \]
Experience in the use of electric propulsion engines in space shows that the deviation of the thrust value (systematic error) relative to the nominal value (form) is from $-2.5\%$ to $+4.5\%$ [10].

Figures 4 and 5 show that in the presence of a systematic error in the implementation of the thrust of an electric propulsion system in the range from $-2.5\%$ to $+4.5\%$, errors in the final value of the average radius of the orbit can be of the order of several thousand kilometres, and errors in the final value of the inclination orbits - of the order of several degrees.

Thus, the calculation results show that actions on an interorbital transport vehicle with an electric propulsion propulsion system of various kinds of disturbances, primarily from inaccurate implementation of the thrust magnitude, lead to significant deviations of the final trajectory parameters.

4. The algorithm for narrowing the region of deviation of the final trajectory parameters during the flight to the geostationary orbit by clarifying the thrust value and correcting the control program

Consider the problem of translating the deviation vector of the final state $\Delta X_f$ into the region $G'$. If it is not possible to build in advance an accurate model of the operating disturbances that change significantly during the flight, it is advisable to use multistep control algorithms with prediction of the final state and identification of disturbing factors (Figure 1).

We introduce the following perturbation model

$$
\nu = V(t, x, \sigma, \xi(t, x)).
$$
Here, $\sigma = \{\sigma_1, \ldots, \sigma_m\}$ is vector of refined parameters of the perturbation model. Symbol $\xi(t, x)$ denotes the set of quantities included in the expression for the vector function $v$ and available for measurement.

The nominal trajectory is calculated according to the given initial conditions and the accepted a priori model of perturbations.

Since perturbations (disturbing accelerations, perturbations of the reactive acceleration vector) that are different from the accepted a priori model act on the spacecraft, the mathematical model of motion should be periodically brought into line with the measured errors of the phase coordinate vector. Since currently quite accurate models of perturbations of the Earth's gravitational field, lunar-solar perturbations have been developed, the value of reactive acceleration is subject to refinement. As the specified parameters, the thrust value of the electric propulsion system is used.

After that, the predicted trajectory is built. If the predicted vector $x_{pr}^{i}(t_f) \notin X_f$, then a new control program $\psi = \psi_i(t)$ should be constructed. The adjustment of the control correction algorithm is made from the condition of the minimum of the predicted final miss, but so that $\chi_{pr}^{i} \in X_f$.

The actual value of traction is determined according to the expression

$$T^i = T_{nom} \left(1 + \frac{\Delta T^i_{calc}}{T_{nom}}\right),$$

where $T^i_{calc}$ is the osculating period of circulation in the measured passive area; $\Delta T^i_{calc}$ - deviation of the osculating period of circulation from the calculated value.

The control program (1) is corrected by updating the modulating function $\psi_m(V_X)$.

5. Simultation of the trajectory movement of an interorbital transport vehicle with an electric propulsion system during a flight into a geostationary orbit with an adjustment of the thrust

Examples of the results of modeling the trajectory motion of an interorbital transport vehicle with an electric propulsion system during a flight into a geostationary orbit with the refinement of the thrust value are given in tables 1 and 2 depending on the number of corrections $N$. Region $G'$ was limited by inclination: $|\Delta_i| \leq 0.1^0$.

| Table 1. The results of the simulation of the flight into the geostationary orbit with a refinement of the thrust ($t_{pas} = 1$ day, $\delta P = 0.005 \ (P_{fact} = 12.06 \ N)$) |
|---|---|---|---|---|---|---|
| $N$ | Traction and duration of the i-th active site | $\sum T^i_{mt}$, days | $T^i_{z}$, days | $\Delta \Delta \delta$, km | $\Delta \delta \delta$, deg | $\Delta e_f$, $V_{X_f}$, km/s |
| i | $P^i$, N | $T^i_{mt}$, days | | | | |
| 1 | 0 | 12 | 63.278 | 125.649 | 126.649 | 135.4 | 0.21 | 0.002 | 7.590 |
| 0 | 1 | 12.114 | 62.371 | | | | | | |
| 2 | 0 | 12 | 63.278 | | | | | | |
| 2 | 1 | 12.114 | 31.186 | 125.889 | 127.889 | 37.4 | 0.07 | 0.002 | 7.607 |
| 2 | 2 | 12.072 | 31.425 | | | | | | |

In tables 1 and 2, the following notations are used: $\sum T^i_{mt}$ - motor time or duration of the i-th active section, $T^i_{z}$ - total flight time, $t_{pas}$ - duration of the passive section, on which the thrust of the electric propulsion system is refined.

6. Conclusion

An algorithm has been developed for narrowing the deviation region of the final trajectory parameters when flying to a geostationary orbit using small thrust engines, which includes an algorithm for
generating nominal thrust vector control programs and an algorithm for refining the thrust generated by an electric propulsion system based on measurements of the actual period of revolution.

The simulation results of the trajectory movement of an interorbital transport vehicle with an electric propulsion system during a flight to a geostationary orbit showed that the algorithm for refining the thrust and correction of the control program allows us to narrow the deviation region of the final trajectory parameters, reducing the error along the major semi axis by an order of magnitude, and transferring the interorbital transport apparatus to near equatorial orbit.

Table 2. The results of the simulation of the flight into the geostationary orbit with a refinement of the thrust (\(t_{\text{pas}} = 1\) day, \(\delta P = 0,035\) (\(P_{\text{fact}} = 12,42\) N))

| N  | Traction and duration of the i-th active site | \(\sum T_{M}^i\), days | \(T_{S}^i\), days | \(\Delta a_{f}\), km | \(\Delta i_{f}\), deg | \(\Delta e_{f}\) | \(V_{Xf}\), km/s |
|----|-------------------------------------------------|------------------------|-----------------|---------------------|------------------|------------|----------------|
| 1  | 0 12                                            | 57.135                 | 870.3           | 1.59                | 0.003            | 7.475     |
| 2  | 0 12                                            | 63.278                 | 121.413         | 121.413             |                  |           |
| 3  | 1 12.825                                        | 28.567                 | 121.995         | 123.995             | 133.2            | 0.30      | 7.589         |
| 4  | 0 12                                            | 63.278                 | 121.995         | 123.995             | 133.2            | 0.30      | 7.589         |
| 5  | 1 12.825                                        | 28.567                 | 122.164         | 125.164             | 60.8             | 0.19      | 7.602         |
| 6  | 2 12.530                                        | 15.075                 | 122.164         | 125.164             | 60.8             | 0.19      | 7.602         |
| 7  | 0 12                                            | 63.278                 | 122.164         | 125.164             | 60.8             | 0.19      | 7.602         |
| 8  | 1 12.825                                        | 28.567                 | 122.221         | 126.221             | 44.4             | 0.08      | 7.606         |
| 9  | 2 12.530                                        | 15.075                 | 122.221         | 126.221             | 44.4             | 0.08      | 7.606         |
| 10 | 3 12.518                                        | 7.622                  | 122.518         | 126.221             | 44.4             | 0.08      | 7.606         |
| 11 | 4 12.514                                        | 7.679                  | 122.518         | 126.221             | 44.4             | 0.08      | 7.606         |

7. References

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