Numerical Investigations of Haline-Convective Flows of Saline Groundwater

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Abstract. Numerical investigations of saline water flows in a horizontal porous layer resulted from water evaporation at the upper boundary and salt accumulation nearby are carried out. The mathematical model includes the continuity, Darcy’s and contaminant transport equations. Over the threshold of saline profile stability, different regimes of haline convection in low- and high-permeable layers are simulated. As found, steady regular convection occupying a whole domain is induced in the case of weak evaporation. Stochastic convection with a small-scale structure of salt "drops" and salt precipitation are developed in the case of intensive evaporation. The maps of regimes are plotted and boundaries between regimes are determined.

1. Introduction

Points of groundwater motion and salt transport are closely related to the problem of amount and quality of water resources demanded for social and industrial supplies. An information on water circulation in a subsoil of earth is also useful in case of extraction of oil, gas and coal [1]. Water motion and accumulation of contaminants can occur due to evaporation of water from the surface of the soil. In this case, groundwater moves up and, evaporating, disappears in the atmosphere so that salt accumulates near the front of phase transition. As a result, more concentrated and, respectively, heavier fluid is located over less concentrated and light fluid that can lead to development of haline convection. Instability of salinity profile was investigated analytically [2], convective flows and mass transfer in a high-permeable porous layer were simulated numerically [3-5]. We aim to determine conditions on stability of the formed saline profile in a low-permeable layer and study universal features of haline-convective flows in porous formations with the different permeability.

2. Mathematical formulation and numerical method

We consider a porous layer at length $H_x$ and height $H_y$ filled initially with pure water, moving up at the constant velocity $v_0$ (figure 1). At the lower boundary, the concentration of dissolved salt $c$ increases linearly from zero during the time $t_{s0}$ and is fixed reaching the value $c_0$. Values at the lower boundary are denoted by the subscript “0”. Salt enters the layer together with the upward flow and due to diffusion. At the front of phase transition coinciding with the upper boundary, water evaporates. It means that water passes through the upper boundary but salt remains in the domain. The boundary condition at the top of layer sets the amount of salt coming to the boundary with the upflow to be equal to the amount of salt diffusing down. Salt accumulation and redistribution near the upper
boundary forms the salinity profile with the maximum of concentration at the top. As fluid near the upper boundary is denser than below, haline gravity-driven convection can develop.

Haline convection is characterised by variables $v$, $\rho$, $\rho_c$, $P$ which are the fluid velocity, fluid density (mass of salt and water per the volume), contaminant density (mass of dissolved salt per the volume) and pressure. The concentration $c$ is defined by $\rho$ and $\rho_c$ as $c = \rho_c / \rho$. The mathematical model includes the continuity, Darcy’s and contaminant transport equations [4-6]. The set of equations is transformed into a dimensionless from using the dimensionless variables $W = (U, V) = \nabla v_0$, $S = (\rho - \rho_0)(\rho_{sat} - \rho_0)^{-1}$, $\Pi = (P - P_0 - (P_0 - P_a)y / H_y)(H_y g(\rho_{sat} - \rho_0))^{-1}$. The height of layer $H_y$, velocity of fluid upflow $v_0$, time $H_y / v_0$, density $(\rho_{sat} - \rho_0)$, pressure $H_y g(\rho_{sat} - \rho_0)$ are used as scales. Here, $\rho_0$ and $\rho_{sat}$ are the densities of pure water and saturated solution, $P_a$ is the pressure at $y = H_y$. The set of dimensionless equations is as follows

$$\nabla \cdot W = 0, \quad W = W_0 - Pa(\nabla \Pi - Se), \quad \frac{\partial S}{\partial t} + W \cdot \nabla S = Pe^{-1} \Delta S$$

(1)

The initial and boundary conditions have the form

$$t = 0: \quad W = W_0, \quad S = 0, \quad \Pi = 0$$

$$y = 0: \quad W = W_0, \quad \Pi = 0, \quad S = S_0 \quad \text{at} \quad t < t_s, \quad S = S_0 \quad \text{at} \quad t \geq t_s$$

$$y = 1: \quad W = W_0, \quad SW_0 - Pe^{-1} \nabla S = 0, \quad \Pi = 0$$

$$x = 0, \quad x = A: \quad n \cdot W = 0, \quad n \cdot \nabla S = 0, \quad n \cdot \nabla \Pi = 0$$

(2)

Here, $A = H_x / H_y$ is the aspect ratio, $W_0 = (0, 1)$ is the velocity at $y = 0$, and $n$ is the unit vector normal to boundaries, $e = (0; -1)$. Note that the density $S$ relates to the concentration $c$ by the relation $c = S c_{sat}(\alpha c_{sat}(S - 1) + 1)^{-1}$ which includes concentration in saturated solution $c_{sat}$.

The dimensionless parameters are

$$Pa = k g(\rho_{sat} - \rho_0)(\phi \mu_s v_0)^{-1}, \quad Pe = H_y v_0 D_c^{-1}, \quad S_0$$

(3)

Here, $g$ is the Earth’s gravity force acceleration, $\phi$ and $k$ are the soil porosity and permeability, $\mu_s$ is the solution viscosity, $D_c$ is the salt diffusivity. In (3), $Pa$ is similar to the Rayleigh-Darcy

![Figure 1. Sketch of the problem.](image-url)
number (based on the width of concentration boundary layer \( \delta = D_c / v_0 \)) and \( Pe \) is the Peclet number.

Numerical simulations are carried out at \( H_y = 10 \) m, \( \rho^0 = 10^3 \) kg m\(^{-3}\), \( \mu_x = 10^3 \) Pa s, \( D_c = 1.6 \times 10^{-9} \) m\(^2\) s\(^{-1}\), \( \phi = 0.03 \), \( c_{sat} = 0.265 \), \( A = 6, 10 \). The time \( t_s / t_0 \) normalized by the scale \( H_y / v_0 \) is equal to 1. Two magnitudes of permeability \( k = 10^{-15}, 10^{-13} \) m\(^2\) are set. The upflow velocity \( v_0 \) and concentration at the bottom \( c_0 \) are varied. We employ the finite-difference numerical code [6]. Computations with the use of 4-core processor Core i7-920 are fulfilled. Space grid from 501x51 to 2001x201 and the time step \( \tau = 10^{-5}, 10^{-2} \) are used.

3. Results and discussion

We obtain three main regimes. The first (I) corresponds to the stable salinity profile which becomes steady in time because supply of salt with water upflow to the top is equilibrated by its diffusion to the bottom. Convective motions do not develop in this case. Second (II) and third (III) regimes occur if the salinity profile is unstable. In the regime II, steady haline convection is induced. Convective motions occupy all considered domain. In a low-permeable formation at \( k = 10^{-15} \) m\(^2\), the regime II is realised rarely. We obtain a single calculation of steady convection taking place at \( v_0 = 3 \times 10^{-10} \) m s\(^{-1}\) and \( c_0 = 0.03 \) which is shown in figure 2. The stream function \( \Psi \) is defined numerically from the equation \( \partial^2 \Psi / \partial x^2 + \partial^2 \Psi / \partial y^2 = -\partial V / \partial x + \partial U / \partial y \) with the boundary conditions: \( \Psi = -x \) at \( y = 0, 1 \); \( \Psi = 0 \) at \( x = 0 \) and \( \Psi = -A \) at \( x = A \). As obvious, upflow is only distorted (figure 2 (a)). The stream function \( \Psi \) corresponding to the motion relative to initial nondisurbed vertical flow is defined similar to \( \Psi \) but in terms of the velocity \( \mathbf{W} - \mathbf{W}_0 = (U, V - V_0) \). The condition \( \psi = 0 \) is applied at all boundaries. Isolines of \( \psi \) are closed (figure 2 (b)). Fluid motions are with the module of maximal and minimal fluid velocity 1.17 and 0.752 (in dimensionless units), respectively. As haline convection are not intensive in comparison with upward flow, isolines of density \( S \) are only slightly curved (figure 2 (c)).

![Figure 2](image_url)  
**Figure 2.** Isolines of stream function \( \Psi \) (a), stream function relative to initial motion \( \psi \) (b) and salt concentration \( c \) (c) in steady convection at \( k = 10^{-15} \) m\(^2\). Ranges: \( \Psi \in [-10.0; 0] \), \( \psi \in [-0.090; 0.090] \), \( c \in [0.03; 0.192] \).
Increasing in \( v_0 \) or \( c_0 \) leads to a break of salinity profile and salt precipitation that corresponds to regime III. As obtained at \( k = 10^{-15} \text{ m}^2 \), a lot of salt precipitates before haline convection to develop. For example, at \( v_0 = 10^{-8} \text{ m s}^{-1} \) and \( c_0 = 10^{-3} \), about a half of all salt amount transits into the solid form during the time \( t = 20 \) while the module of maximal fluid velocity almost does not increase being 1.0000024. We are not able to simulate such process because our model does not include the porosity variable in time which is resulted from decreasing in void space due to precipitated solid salt. At \( k = 10^{-13} \text{ m}^2 \), fluid before salt precipitation can be involved in motion near the upper boundary forming unsteady salt “drops” [5]. We can simulate haline convection only with little salt precipitation until solid salt occupies a negligible void space and to obtain convective patterns in a long time. Such calculation is given in figure 3 demonstrating haline convection at \( k = 10^{-13} \text{ m}^2 \), \( v_0 = 2 \times 10^{-8} \text{ m s}^{-1} \), and \( c_0 = 10^{-4} \). Figures 3 (a), (b) show an upper half of layer. Only about 0.003 \% of salt mass precipitates during the time \( t = 18 \), after that process of precipitation stops. As obvious, small saline “drops” arise near the upper boundary initially (figure 3 (a)). Fluid motions develop near the upper boundary so that “drops” take place into a semi-infinite medium. “Drops” grow in time and merge together (figure 3 (b)). They transform into oblong balls which grow and approach the lower boundary (figure 3 (c)). The fluid velocity varies from -4.22 to 4.33 (a), from -2.98 to 3.27 (b), from -3.42 to 3.42 (c). After 130 time units from the beginning, when oblong balls reach the lower boundary, salt starts to precipitate again and this in large quantities. Computation is stopped as our model is not applicable to the case of variable porosity.

We have fulfilled a set of calculations at \( k = 10^{-15} \text{ m}^2 \) and combined our results with the data at \( k = 10^{-13} \text{ m}^2 \) form [5] to obtain maps of regimes shown in figure 4. If upflow has a small velocity \( v_0 \) corresponding to a weak evaporation of water at the upper boundary, a formed saline profile remains stable both at low and high permeability (regime I). In this case, salt diffusion to the bottom is equal to the salt flux with water upflow. If evaporation is more intensive and velocity \( v_0 \) becomes higher, salt supply to the top increases so that diffusion is not sufficient to equilibrate salt flux. As a result, convection in regime II or III develops. The regime II in a low-permeable layer is obtained in a single calculation (figure 4 (a)) as discussed early. However, in a high-permeable layer, there is a relatively wide range of \( v_0 \) and \( c_0 \) ensuring the regime II to be realized (figure 4 (b)). In the last case, both steady distorted upflow and circulated flow with a regular vortex structure can occur [5]. At the most

**Figure 3.** Isolines of salt concentration \( c \) at \( k = 10^{-13} \text{ m}^2 \). The time moments are \( t = 9 \) (a), 21 (b), 75 (c). Range: \( c \in [0.0001; 0.20] \).
high $v_0$, salt supply to the top is so large that only stochastic motions and supersaturation of solution with salt precipitation are observed (regime III).

Let’s consider dimensionless parameters to bring together the data from figure 4. The Rayleigh-Darcy number characterizing haline convection in a whole domain includes the layer height $H_y$ and the density difference reaching the maximal value $(\rho_{sat} - \rho^0)$. One can write the maximal Rayleigh-Darcy number as follows

$$Ra = \frac{kgH_y}{\phi \mu_s D_c} (\rho_{sat} - \rho^0)$$ (4)

Note, $Ra$ and the dimensionless parameter $Pa$ in the set of equations (1) are related as $Ra = Pa \cdot Pe$. However, the typical density difference in our calculations is $(\rho^* - \rho^0)$, where $\rho^*$ is the real fluid density at the upper boundary. One can obtain $\rho^*$ from numerical solutions as the fluid density average over the upper boundary at the moment of haline convection onset. If convection does not manage to develop before salt precipitation, we take $\rho_{sat}$ as $\rho^*$. The Rayleigh-Darcy number including $(\rho^* - \rho^0)$ is denoted by superscript “*” and written as

**Figure 4.** Map of regimes in $(v_0, c_0)$ at $k = 10^{-15}$ (a), $10^{-13}$ m$^2$ (b).

**Figure 5.** Map of regimes in $(Pe, Ra^*)$ at $k = 10^{-15}$ (red), $10^{-13}$ m$^2$ (blue).
\[ Ra^* = \frac{k g H_y}{\phi \mu_1 D_c} (\rho^* - \rho^0) \]  

(5)

Following from expressions above, we have \( Ra^* = Ra(\rho^* - \rho^0)(\rho_{sat} - \rho^0)^{-1} = Pa Pe K \) with the coefficient \( K = (\rho^* - \rho^0)(\rho_{sat} - \rho^0)^{-1} \). One can reduce the expression of \( K \) to the form including the salt concentration \( c \). We use the linear equation of state \( \rho = \rho_0 + \alpha \rho_c \) at \( \alpha = 0.64 \) and obtain the final expression

\[ K = \frac{(c^* - c_0)(1 - \alpha c_{sat})}{c_{sat}(1 - \alpha c^*)(1 - \alpha c_0)} \]  

(6)

The data from figure 4 are transformed into a dimensionless form and, with the use of (6), reduced to the universal \( Ra^*(Pe) \) relation shown in figure 5. We see that there is a boundary between stable (I) and unstable (II, III) regimes which is nearly independent of \( Pe \). One can determine that this boundary is described by the threshold value \( Ra_{sh}^* = 25 \) both in low- and high-permeable layers.

4. Conclusions

We investigated stability of salinity profile in a low-permeable porous layer resulted from evaporation of water at the upper boundary (coinciding with the front of phase transition) and salt accumulation nearby. The results obtained are compared with available data for the case of high-permeable porous layer. If evaporation is weak and, consequently, the upflow velocity is small, salinity profile is stable both at low and high permeability. With increasing in the velocity, steady haline convection develops. Convective motions distort the initial upward flow at low permeability. If upflow velocity increases more, fluid motions become unsteady. A saline profile breaks leading to precipitation of enough amount of salt in a low-permeable layer so that the model of uniform porosity becomes inapplicable. In a high-permeable layer, stochastic convection formed by saline “drops” can occur before intensive salt precipitation. “Drops” grow and merge together transformed into oblong balls which reach in time the lower boundary. All data are presented as a universal dependence of the Rayleigh-Darcy number \( Ra^* \) on the Peclet number \( Pe \) in the map of regimes. The boundary between stable and unstable regimes is determined as \( Ra^* = Ra_{sh}^* \) including the threshold value \( Ra_{sh}^* = 25 \).

Acknowledgments

The author would like to thank G.G. Tsypkin for many fruitful discussions. This work has been supported by the Russian Foundation for Basic Research (grant № 15-08-01365).

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