Vibration reduction using autoparametric resonance in a high-\( T_c \) superconducting levitation system

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Abstract. High-\( T_c \) superconducting levitation systems have very small damping and enable stable levitation without control. Therefore, they can be applied to various kinds of application. However, there are some problems that small damping produces large vibration and nonlinearity of magnetic force can generate complicated phenomena. Accordingly, analysis of these phenomena and reduction of vibration occurring in the system are important. In this study, we examined reduction of vibration without using any absorbers, but utilizing autoparametric resonance caused by nonlinear coupling between vertical oscillation and horizontal oscillation. We conducted numerical analysis and experiments in order to investigate motions of a rigid bar levitated by the electromagnetic force from high-\( T_c \) superconductors. As a result, if the ratio of the natural frequency of vertical oscillation and that of horizontal oscillation is two to one, the vertical oscillation decreases while the horizontal oscillation is excited. Thus, it was confirmed that the amplitude of a primary resonance can be reduced by occurrence of autoparametric resonance without using any absorbers.

1. Introduction
High-\( T_c \) (High-Critical-Temperature) superconducting levitation systems have very small damping and stable levitation without control. Therefore, they can be applied to flywheels for energy storage, conveyors in clean room and so on [1]. However, in such small damping systems, there is a problem that vibration amplitude may increase easily. Too large vibration may cause destruction of the system, hence it is important to suppress vibration in a high-\( T_c \) superconducting levitation system.

In previous study, Takazakura et al. confirmed that parametrically excited oscillation in a high-\( T_c \) superconducting levitation system can be reduced by using a vibration absorber [2]. Besides, H. Yabuno et al. confirmed the validity of an autoparametric absorber to stabilize 1/3-order subharmonic resonance [3]. However, there are few studies of vibration reduction by using nonlinearity of a system itself.

The purpose of this study is reduction of the amplitude of the primary resonance of a levitated body by utilizing autoparametric resonance, one of nonlinear interaction between plural vibration modes, without using any absorbers. In general, if their natural frequency ratio is 2 to 1, the autoparametric resonance can occur in a multiple degree-of-freedom coupled nonlinear system [4]. Here we deal with a system which consists of a rigid body and two permanent magnets levitated above two high-\( T_c \) superconducting bulks. In this system, vertical motion and horizontal motion are coupled nonlinearly by magnetic force. We aim at transferring kinetic
energy from vertical motion to horizontal motion through autoparametric resonance. In order to utilize the autoparametric resonance, we examined numerically and experimentally how it affects the vibration amplitude of the levitated body in the vertical direction when the primary resonance occurs.

2. Analytical model and equations
An analytical model is shown in Figure 1. The main system of the model is a symmetric rigid bar supported at both its ends by electromagnetic forces between permanent magnets and superconductors. Using this model, we considered the two degrees of freedom defined by the motion of the levitated bar. The coordinates are shown in Figure 1. The initial position of the center of the levitated body before field cooling is \((x, Z) = (0, z_0)\). After field cooling, the levitated body balances with the gravity at the equilibrium position \((0, z_0 - z_s)\). We further define the relative coordinate \(z\) in the vertical direction from the equilibrium position. The superconductors are excited in the vertical direction on the amplitude \(a\) and the frequency \(N\).

The magnetic forces exerted by superconductors to magnets, \(F_z\) and \(F_x\), are evaluated here by using the frozen mirror image method \([5]\). Further, the Taylor expansion is applied them up to the third order terms. The equations of motion of the levitated body, nondimensionalized by the natural period in the vertical direction \(\Omega_z\) and \(z_s\), are derived as follows,

\[
\ddot{z} + \mu_z \dot{z} + z - \alpha_{zz} z^2 - \alpha_{xx} x^2 + \alpha_{zzz} z^3 + \alpha_{xxx} x^2 z = b \nu^2 \cos \nu t \tag{1}
\]

\[
\ddot{x} + \mu_x \dot{x} + \omega_x^2 x - \alpha_{xx} x z - \alpha_{xxx} x^3 + \alpha_{xxz} x^2 z = 0 \tag{2}
\]

where \(\mu_\_\) are damping coefficients, \(\alpha_\_\) are coefficients of the Taylor series of the magnetic forces \((F_z\) and \(F_x\) up to the third order terms, \(b = \frac{a}{|z_s|}\) is the excitation amplitude, \(\nu = \frac{N}{\Omega_z}\) is the excitation frequency and \(\omega_x\) is the natural frequency in the \(x\) direction.

![Figure 1. Analytical model.](image)

3. Numerical calculation
We carried out numerical calculation of (1) and (2) by using the Runge-Kutta 4th order method (RK4). The length of a time step of calculation was one-128th of the period of excitation, and the number of steps was \(2^{16}\). We calculated on condition that the natural frequency of the levitated body in the vertical direction is twice as much as that in the horizontal direction, and also on condition that it is not twice. The parameters of the two cases are shown in Table 1. We increased the excitation frequency \(\nu\) around \(\nu = 1.0\). Figure 2 and Figure 3 show the numerical results of frequency responses of \(z\) and \(x\), respectively, where solid circles show the results under \(\omega_z : \omega_x = 2 : 1\) and open circles show those under \(\omega_z : \omega_x \neq 2 : 1\). It can be found that if \(\omega_z : \omega_x = 2 : 1\), vibration whose frequency is \(\nu/2(\simeq \omega_x)\) occurs in the horizontal direction under the excitation frequency \(\nu\) in the neighborhood of \(\omega_z(=1)\). Correspondingly, the amplitude of
vibration in the vertical direction is lowered compared to that under $\omega_z: \omega_x \neq 2:1$. These numerical results show that the vertical resonance can be effectively reduced by occurrence of autoparametric resonance.

| Table 1. Values of dimensionless parameters used in numerical calculation. |
|---------------------------------|--------|--------|--------|--------|--------|--------|
| $\omega_z : \omega_x$ | $\alpha_{zz}$ | $\alpha_{xx}$ | $\alpha_{zzz}$ | $\alpha_{xxz}$ | $\alpha_{zz}$ | $\alpha_{xxx}$ |
| = 2 : 1 | $5.24 \times 10^{-1}$ | $4.83 \times 10^{-2}$ | $1.90 \times 10^{-1}$ | $2.20 \times 10^{-2}$ | $9.65 \times 10^{-2}$ | $5.50 \times 10^{-3}$ |
| $\neq 2 : 1$ | $9.30 \times 10^{-2}$ | $4.90 \times 10^{-3}$ | $4.83 \times 10^{-1}$ | $4.67 \times 10^{-2}$ | $1.60 \times 10^{-1}$ | $1.96 \times 10^{-2}$ |

Figure 2. Numerically obtained frequency responses of $z$.

Figure 3. Numerically obtained frequency responses of $x$.

4. Experiments
We carried out experiments using an experimental setup shown in Figures 4 and 5. Two cylindrical shaped GdBCO high-$T_c$ superconducting bulks of melt-quench type ($\phi$45, 20H) were used here. They were cooled by LN$_2$ (77 K). Two cylindrical permanent magnets ($\phi$20, 10H), consisting of Nd, Fe and B, were used. Residual magnetization of each magnet was 1.45 T. Each magnet, placed above the center of each superconducting bulk, was embedded in a hole made at each end of an aluminum bar shaped a rectangular parallelepiped (300W, 40D, 10H). The distance between the magnets was 230 mm. Two different initial cooling positions of the bar supported by acrylic spacers, were tried: $z_0 = 16.15$ mm and $z_0 = 16.10$ mm. They correspond to the case of $\omega_z: \omega_x = 2 : 1$ and the case of $\omega_z: \omega_x \neq 2 : 1$, respectively. By using a vibration generator, periodic vertical excitation was given to the superconductors. We measured the displacement of excitation together with the displacements in the vertical and horizontal directions by laser displacement sensors, while we changed the excitation frequency $\nu$ around the neighborhood of the natural frequency of $z$. Figures 6 and 7 show experimental results of frequency responses in the vertical and horizontal directions, respectively, where solid circles show results under $\omega_z: \omega_x = 2 : 1$ and open circles show those under $\omega_z: \omega_x \neq 2 : 1$. The experimental results were qualitatively in good agreement with the numerical ones.
Figure 4. Sketch of our experimental setup.

Figure 5. Photograph of our experimental setup.

Figure 6. Experimentally obtained frequency responses of $z$.

Figure 7. Experimentally obtained frequency responses of $x$.

5. Conclusion
We investigated reduction of vibration amplitude in a high-$T_c$ superconducting levitation system by utilizing autoparametric resonance without using any absorbers. We conducted numerical calculations and experiments on a high-$T_c$ superconducting levitation system. Both results show that the vibration amplitude of the primary resonance reduced with horizontal vibration resonating. The horizontal vibration resonated on condition that the natural frequency of the levitated body in the vertical direction is twice as much as that in the horizontal direction. Therefore we conclude that autoparametric resonance, caused by coupling between the vertical and horizontal motions due to nonlinearity of electromagnetic force, can occur if the natural frequency in the vertical direction is twice as much as that in the horizontal direction. Occurrence of autoparametric resonance can reduce the amplitude of the primary resonance in the vertical direction in exchange for occurrence of horizontal vibration.

References
[1] Hull J R and Murakami M 2004 Proc. of the IEEE 92 1705-18
[2] Takazakraura T, Sakaguchi R and Sugiura T 2013 IEEE Transactions on Appl. Superconductivity 23 3600204
[3] Yabuno H, Endo Y and Aoshima N 1999 J. Vibration and Acoustics 121 309-15
[4] Tondl A, Ruigrok T, Verhulst F and Nabergoj R 2000 Autoparametric Resonance in Mechanical Systems (Cambridge: Cambridge University Press) pp 44-63
[5] Kordyuk A A 1998 J. Appl. Phys. 83 610-2