Field theories with (2,0) AdS supersymmetry in $\mathcal{N} = 1$ AdS superspace

Jessica Hutomo and Sergei M. Kuzenko

Department of Physics M013, The University of Western Australia
35 Stirling Highway, Crawley W.A. 6009, Australia

jessica.hutomo@research.uwa.edu.au, sergei.kuzenko@uwa.edu.au

Abstract

In three dimensions, it is known that field theories possessing extended $(p,q)$ anti-de Sitter (AdS) supersymmetry with $\mathcal{N} = p + q \geq 3$ can be realised in (2,0) AdS superspace. Here we present a formalism to reduce every field theory with (2,0) AdS supersymmetry to $\mathcal{N} = 1$ AdS superspace. As nontrivial examples, we consider supersymmetric nonlinear sigma models formulated in terms of $\mathcal{N} = 2$ chiral and linear supermultiplets. The (2,0) $\rightarrow$ (1,0) AdS reduction technique is then applied to the off-shell massless higher-spin supermultiplets in (2,0) AdS superspace constructed in [1]. As a result, for each superspin value $\hat{s}$, integer ($\hat{s} = s$) or half-integer ($\hat{s} = s + \frac{1}{2}$), with $s = 1, 2, \ldots$, we obtain two off-shell formulations for a massless $\mathcal{N} = 1$ superspin-$\hat{s}$ multiplet in AdS$_3$. These models prove to be related to each other by a superfield Legendre transformation in the flat superspace limit, but the duality is not lifted to the AdS case. Two out of the four series of $\mathcal{N} = 1$ supersymmetric higher-spin models thus derived are new. The constructed massless $\mathcal{N} = 1$ supersymmetric higher-spin actions in AdS$_3$ are used to formulate (i) higher-spin supercurrent multiplets in $\mathcal{N} = 1$ AdS superspace; and (ii) new topologically massive higher-spin off-shell supermultiplets. Examples of $\mathcal{N} = 1$ higher-spin supercurrents are given for models of a complex scalar supermultiplet. We also present two new off-shell formulations for a massive $\mathcal{N} = 1$ gravitino supermultiplet in AdS$_3$. 
Contents

1 Introduction 2

2 (2,0) \rightarrow (1,0) AdS superspace reduction 4
   2.1 Geometry of (2,0) AdS superspace: Complex basis . . . . . . . . . . . . . . 4
   2.2 Geometry of (2,0) AdS superspace: Real basis . . . . . . . . . . . . . . 6
   2.3 Reduction from (2,0) to \mathcal{N} = 1 AdS superspace . . . . . . . . . 8
   2.4 The (2,0) AdS supersymmetric actions in AdS^{3|2} . . . . . . . . . . . 11
   2.5 Supersymmetric nonlinear sigma models . . . . . . . . . . . . . . . . . . 13

3 Massless higher-spin models: Type II series 15
   3.1 The type II theory . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 15
   3.2 Reduction of the gauge prepotentials to AdS^{3|2} . . . . . . . . . . . . . . 17
   3.3 Massless higher-spin \mathcal{N} = 1 supermultiplets in AdS_3 . . . . . . . 20
      3.3.1 Longitudinal formulation for massless superspin-(s + \frac{1}{2}) multiplet . 20
      3.3.2 Transverse formulation for massless superspin-s multiplet . . . . . . 21

4 Massless higher-spin models: Type III series 23
   4.1 The type III theory . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 23
   4.2 Reduction of the gauge prepotentials to AdS^{3|2} . . . . . . . . . . . . . . 24
   4.3 Massless higher-spin \mathcal{N} = 1 supermultiplets in AdS_3 . . . . . . . 27
      4.3.1 Longitudinal formulation for massless superspin-s multiplet . . . . . 27
      4.3.2 Transverse formulation for massless superspin-(s + \frac{1}{2}) multiplet . 28

5 Analysis of the results 29

6 Non-conformal higher spin supercurrents 31
   6.1 \mathcal{N} = 1 supercurrents: Half-integer superspin case . . . . . . . . . . 31
   6.2 \mathcal{N} = 1 supercurrents: Integer superspin case . . . . . . . . . . . . . 32
   6.3 From \mathcal{N} = 2 supercurrents to \mathcal{N} = 1 supercurrents . . . . . . . 32
1 Introduction

In three spacetime dimensions, the AdS group is a product of two simple groups,

$$\text{SO}(2, 2) \cong \left( \text{SL}(2, \mathbb{R}) \times \text{SL}(2, \mathbb{R}) \right) / \mathbb{Z}_2,$$

and so are its supersymmetric extensions $\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})$.\(^{1}\) This implies that $\mathcal{N}$-extended AdS supergravity exists in several incarnations\(^{2}\), which are known as the $(p, q)$ AdS supergravity theories, where the integers $p \geq q \geq 0$ are such that $\mathcal{N} = p + q$. The so-called $(p, q)$ AdS superspace\(^{3}\)

$$\text{AdS}_{(3|p,q)} = \frac{\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})}{\text{SL}(2, \mathbb{R}) \times \text{SO}(p) \times \text{SO}(q)}$$

may be interpreted as a maximally symmetric solution of $(p, q)$ AdS supergravity\(^{2}\). Within the off-shell formulation for $\mathcal{N}$-extended conformal supergravity which was first sketched in\(^{4}\) and then fully developed in\(^{5}\), $\text{AdS}_{(3|p,q)}$ originates as a maximally symmetric supergeometry with covariantly constant torsion and curvature generated by a symmetric torsion $S^{IJ} = S^{JI}$, with the structure-group indices $I, J$ taking values from 1 to $\mathcal{N}$. It turns out that $S^{IJ}$ is nonsingular and can be brought to the form

$$S^{IJ} = S \text{ diag}(\underbrace{+1, \cdots, +1}_p, \underbrace{-1, \cdots, -1}_{q = \mathcal{N} - p}),$$

\(^{1}\)More general AdS supergroups exist for $\mathcal{N} \geq 4$.

\(^{2}\)In the case of $\mathcal{N} = 1$ AdS supersymmetry, both notations $(1, 0)$ and $\mathcal{N} = 1$ are used in the literature. We will often use the notation $\text{AdS}^{12}$ for $\mathcal{N} = 1$ AdS superspace.
for some positive parameter $S$ of unit dimension. For $p = \mathcal{N} \geq 4$ and $q = 0$, there exist more general AdS superspaces \[3\] than the conformally flat ones defined by \[1,2\].

In the extended $\mathcal{N} = p + q \geq 3$ case, general $(p, q)$ supersymmetric field theories in AdS$_3$ can be realised in $(2,0)$ AdS superspace, AdS$_{(3|2,0)}$ \[3,6\]. Such realisations are often useful for applications, for instance, in order to study the target space geometry of supersymmetric nonlinear $\sigma$-models in AdS$_3$ \[6\]. It is worth elaborating on the $\sigma$-model story in more detail. For the $\mathcal{N} = 3$ and $\mathcal{N} = 4$ choices, manifestly $(p, q)$ supersymmetric formulations have been constructed \[3\] for the most general nonlinear $\sigma$-models in AdS$_3$ (these formulations make use of the curved superspace techniques developed in \[5\]). This manifestly supersymmetric setting is very powerful since it allows one to generate arbitrary nonlinear $\sigma$-models with $(p, q)$ AdS supersymmetry. However, it also has a drawback that the hyperkähler geometry of the $\sigma$-model target space is hidden. In order to uncover this geometry, the formulation of the nonlinear $\sigma$-model in $(2,0)$ AdS superspace becomes truly indispensable \[6\].

This work is somewhat similar in spirit to \[6,13\], however our goals are quite different. Specifically, we develop a formalism to reduce every field theory with $(2,0)$ AdS supersymmetry to $\mathcal{N} = 1$ AdS superspace. This formalism is then applied to carry out the $(2,0) \rightarrow (1,0)$ AdS reduction of the off-shell massless higher-spin supermultiplets in AdS$_{(3|2,0)}$ constructed in \[1\]. There are at least two motivations for pursuing such an application. Firstly, certain theoretical arguments imply that there exist more general off-shell massless higher-spin $\mathcal{N} = 1$ supermultiplets in AdS$_3$ than those described in \[14\]. Secondly, $\mathcal{N} = 1$ supermultiplets of conserved higher-spin currents have never been constructed in AdS$_3$ (except for the superconformal multiplets of conserved currents in Minkowski superspace \[15\] which can readily be lifted to AdS$_3$). Both issues will be addressed below. In particular, we will derive new off-shell higher-spin $\mathcal{N} = 1$ supermultiplets in AdS$_3$, which will be used to construct new topologically massive higher-spin supermultiplets.

---

3 General aspects of $(2,0)$ supersymmetric field theory in AdS$_3$ were studied in \[7\].

4 Analogous results exist in four dimensions. The most general $\mathcal{N} = 2$ supersymmetric $\sigma$-model in AdS$_4$ was constructed \[8,9\] using a formulation in terms of $\mathcal{N} = 1$ covariantly chiral superfields, as an extension of the earlier analysis in the super-Poincaré case \[10,11\]. One of the main virtues of the $\mathcal{N} = 1$ formulation \[8,9\] is its geometric character, however the second supersymmetry is hidden. General off-shell $\mathcal{N} = 2$ supersymmetric $\sigma$-models in AdS$_4$ were actually formulated a few years earlier \[12\] in $\mathcal{N} = 2$ AdS superspace. The latter approach makes $\mathcal{N} = 2$ supersymmetry manifest, but the hyperkähler geometry of the $\sigma$-model target space is hidden. The two $\sigma$-model formulations are related via the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ AdS superspace reduction \[13\].
The table of contents reflects the structure of the paper. Our notation and conventions follow [5].

2 \((2,0) \rightarrow (1,0)\) AdS superspace reduction

The aim of this section is to elaborate on the details of procedure for reducing the field theories in \((2,0)\) AdS superspace to \(\mathcal{N} = 1\) AdS superspace. Explicit examples of such a reduction are given by considering supersymmetric nonlinear \(\sigma\)-models.

2.1 Geometry of \((2,0)\) AdS superspace: Complex basis

We begin by briefly reviewing the key results concerning \((2,0)\) AdS superspace, see \([6,7]\) for the details. There are two ways to describe the geometry of \((2,0)\) AdS superspace, which correspond to making use of either a real or a complex basis for the spinor covariant derivatives. We first consider the formulation in the complex basis.

The covariant derivatives of \((2,0)\) AdS superspace are

\[
\mathcal{D}_A = (D_a, D_\alpha, \bar{D}^\alpha) = E_A + \Omega_A + i\Phi_A J, \quad E_A = E_A^M \frac{\partial}{\partial z^M}.
\]

(2.1)

where \(z^M = (x^m, \theta^\mu, \bar{\theta}^\mu)\) are local superspace coordinates, and \(J\) is the generator of the \(R\)-symmetry group, \(U(1)_R\). The generator \(J\) is defined to act on the covariant derivatives as follows:

\[
[J, D_\alpha] = D_\alpha, \quad [J, \bar{D}^\alpha] = -\bar{D}^\alpha, \quad [J, D_a] = 0.
\]

(2.2)

The Lorentz connection, \(\Omega_A\), can be written in several equivalent forms, which are

\[
\Omega_A = \frac{1}{2} \Omega_A^{bc} M_{bc} = -\Omega_A^b M_b = \frac{1}{2} \Omega_A^{\beta\gamma} M_{\beta\gamma}.
\]

(2.3)

The relations between the Lorentz generators with two vector indices \((M_{ab} = -M_{ba})\), one vector index \((M_a)\) and two spinor indices \((M_{\alpha\beta} = M_{\beta\alpha})\) are given in Appendix [A]

The covariant derivatives of \((2,0)\) AdS superspace obey the following graded commutation relations:

\[
\{D_\alpha, D_\beta\} = 0, \quad \{\bar{D}^\alpha, \bar{D}^\beta\} = 0, \quad (2.4a)
\]

\[
\{D_a, \bar{D}^\beta\} = -2i (D_{a\beta} - 2S M_{a\beta}) - 4i \varepsilon_{\alpha\beta} S J, \quad (2.4b)
\]
\[ [\mathcal{D}_a, \mathcal{D}_\beta] = (\gamma_a)_\beta^\gamma SD_\gamma , \quad [\mathcal{D}_a, \bar{\mathcal{D}}_\beta] = (\gamma_a)_\beta^\gamma S\bar{D}_\gamma , \quad (2.4c) \]
\[ [\mathcal{D}_a, \mathcal{D}_b] = -4S^2M_{ab} . \quad (2.4d) \]

Here the parameter \( S \) is related to the AdS scalar curvature as \( R = -24S^2 \).

There exists a universal formalism to determine isometries of curved superspace backgrounds in diverse dimensions [16, 17]. This formalism was used in [7] to compute the isometries of (2,0) AdS superspace (as well as supersymmetric backgrounds in off-shell \( \mathcal{N} = 2 \) supergravity theories [18]). The isometries of (2,0) AdS superspace are generated by the Killing supervector fields \( \zeta^A E_A \), which are defined to solve the master equation
\[ \left[ \zeta + \frac{1}{2} l^{bc} M_{bc} + i\tau J, \mathcal{D}_A \right] = 0 , \quad (2.5a) \]
where
\[ \zeta = \zeta^B \mathcal{D}_B = \zeta^b \mathcal{D}_b + \zeta_\beta \mathcal{D}_\beta + \bar{\zeta}^\beta \bar{\mathcal{D}}^\beta , \quad \bar{\zeta}^b = \zeta^b , \quad (2.5b) \]
and \( \tau \) and \( l^{bc} \) are some real \( U(1)_R \) and Lorentz superfield parameters, respectively. It follows from eq. (2.5) that the parameters \( \zeta_\alpha, \tau \) and \( l_{\alpha\beta} \) are uniquely expressed in terms of the vector parameter \( \zeta_{\alpha\beta} \) as follows:
\[ \zeta_\alpha = i \frac{1}{6} \bar{D}^\beta \zeta_{\beta\alpha} , \quad \tau = i \frac{1}{2} D^\alpha \zeta_\alpha , \quad l_{\alpha\beta} = 2 (\mathcal{D}_{(\alpha} \zeta_{\beta)} - S\zeta_{\alpha\beta}) . \quad (2.6) \]
The vector parameter \( \zeta_{\alpha\beta} \) satisfies the equation
\[ \mathcal{D}_{(\alpha} \zeta_{\beta\gamma)} = 0 . \quad (2.7) \]
This implies the standard Killing equation,
\[ \mathcal{D}_a \zeta_b + \mathcal{D}_b \zeta_a = 0 . \quad (2.8) \]
One may also prove the following relations
\[ \mathcal{D}_a \tau = \frac{1}{3} \bar{D}^\beta l_{\alpha\beta} = 4S\zeta_\alpha , \quad \bar{\mathcal{D}}_a \zeta_\beta = 0 , \quad \mathcal{D}_{(\alpha} l_{\beta\gamma)} = 0 . \quad (2.9) \]
The Killing supervector fields prove to generate the supergroup \( \text{OSp}(2|2; \mathbb{R}) \times \text{Sp}(2, \mathbb{R}) \), the isometry group of (2,0) AdS superspace. Rigid supersymmetric field theories in (2,0) AdS superspace are required to be invariant under the isometry transformations. An infinitesimal isometry transformation acts on a tensor superfield \( U \) (with suppressed indices) by the rule
\[ \delta \zeta U = (\zeta + \frac{1}{2} l^{bc} M_{bc} + i\tau J) U . \quad (2.10) \]
2.2 Geometry of (2,0) AdS superspace: Real basis

Instead of dealing with the complex basis for the (2,0) AdS spinor covariant derivatives, eq. (2.1), it is more convenient to switch to a real basis in order to carry out reduction to \( \mathcal{N} = 1 \) AdS superspace AdS\(^{3|2} \). Following [3], such a basis is introduced by replacing the complex operators \( D_\alpha \) and \( \bar{D}_\alpha \) with \( \nabla^I_\alpha = (\nabla^1_\alpha, \nabla^2_\alpha) \) defined as follows:

\[
D_\alpha = \frac{1}{\sqrt{2}} (\nabla^1_\alpha - i \nabla^2_\alpha) \quad \text{and} \quad \bar{D}_\alpha = -\frac{1}{\sqrt{2}} (\nabla^1_\alpha + i \nabla^2_\alpha).
\]

(2.11)

In a similar way we introduce real coordinates, \( z^M = (x^m, \theta^I_\mu) \), to parametrise (2,0) AdS superspace. Defining \( \nabla_a = D_a \), the algebra of (2,0) AdS covariant derivatives (2.4) turns into

\[
\{\nabla^I_\alpha, \nabla^J_\beta\} = 2i \delta^{IJ} \nabla_{\alpha\beta} - 4i \delta^{IJ} S M_{\alpha\beta} + 4 \varepsilon_{\alpha\beta} \varepsilon^{IJ} S J ,
\]

(2.12a)

\[
[\nabla_a, \nabla^I_\beta] = S (\gamma_a)^\beta_\gamma \nabla^I_\gamma , \quad [\nabla_a, \nabla_b] = -4 S^2 M_{ab} ,
\]

(2.12b)

The action of the U(1)\(_R\) generator on the spinor covariant derivatives is given by

\[
[J, \nabla^I_\alpha] = -i \varepsilon_{IJ} \nabla^J_\alpha .
\]

(2.13)

As may be seen from (2.12), the graded commutation relations for the operators \( \nabla_a \) and \( \nabla^\bot_\alpha \) have the following properties:

1. These (anti-)commutation relations do not involve \( \nabla^2_\alpha \),

\[
\{\nabla^1_\alpha, \nabla^\bot_\beta\} = 2i \nabla_{\alpha\beta} - 4i S M_{\alpha\beta} ,
\]

(2.14a)

\[
[\nabla_a, \nabla^\bot_\beta] = S (\gamma_a)_{\beta}^\gamma \nabla^\bot_\gamma , \quad [\nabla_a, \nabla_b] = -4 S^2 M_{ab} .
\]

(2.14b)

2. Relations (2.14) are identical to the algebra of the covariant derivatives of AdS\(^{3|2} \), see [2.4].

We thus see that AdS\(^{3|2} \) is naturally embedded in (2,0) AdS superspace as a subspace. The real Grassmann variables of (2,0) AdS superspace, \( \theta^I_\mu = (\theta^I_1, \theta^I_2) \), may be chosen in such a way that AdS\(^{3|2} \) corresponds to the surface defined by \( \theta^\mu_2 = 0 \). We also note that no U(1)\(_R\) curvature is present in the algebra of \( \mathcal{N} = 1 \) AdS covariant derivatives. These properties make possible a consistent (2,0) \( \rightarrow \) (1,0) AdS superspace reduction.

\[5\] The antisymmetric tensors \( \varepsilon^{IJ} \) and \( \varepsilon_{IJ} \) are normalised as \( \varepsilon^{12} = \varepsilon_{12} = 1 \).
Now we will recast the fundamental properties of the \((2,0)\) AdS Killing supervector fields in the real representation \((2.11)\). The isometries of \((2,0)\) AdS superspace are described in terms of those first-order operators

\[
\zeta := \zeta^B \nabla_B = \zeta^b \nabla_b + \zeta^\beta \nabla^J_\beta , \quad J = 1,2 ,
\]

which solve the equation

\[
[\zeta + \frac{1}{2} l^{bc} M_{bc} + i \tau J, \nabla_A] = 0 ,
\]

for some real parameters \(\tau\) and \(l^{ab} = -l^{ba}\). Equation \((2.15b)\) is equivalent to

\[
\nabla^I_a \zeta^J_\beta = -\varepsilon_{\alpha \beta} \varepsilon^{IJ} \tau + S \delta^{IJ} \zeta_\alpha \beta + \frac{1}{2} \delta^{IJ} l_{\alpha \beta} , \quad \nabla^I_a \sigma_b = 2i \zeta^\beta I (\gamma_b)_{\alpha \beta} , \quad \nabla^I_a \tau = -4i S \varepsilon^{IJ} \zeta_\alpha J , \quad \nabla^I_a l_{\beta \gamma} = 8i S \varepsilon_{\alpha (\beta \gamma)} ,
\]

and

\[
\nabla_a \zeta_b = l_{ab} = -l_{ba} , \quad \nabla_a \zeta^\beta I = -S \zeta^\alpha I (\gamma_a)_{\alpha \beta} , \quad \nabla_a \tau = 0 , \quad \nabla_a l^{bc} = 4S^2 (\delta^b_a \zeta^c - \delta^c_a \zeta^b) .
\]

Some nontrivial implications of the above equations which will be important for our subsequent consideration are:

\[
\nabla^I_{(\alpha} \zeta_{\beta \gamma)} = 0 , \quad \nabla^I_{(\alpha} l_{\beta \gamma)} = 0 , \quad \nabla^I_{(\alpha} \sigma_{\beta \gamma)} = 2S \delta^{IJ} \zeta_\alpha \beta , \quad \nabla^I_{(\alpha} \tau_{\beta \gamma)} = 0 , \quad \nabla^I_{(\alpha} \zeta^J_\beta) = \frac{1}{6} \nabla^I_{\beta} \varepsilon^{\alpha \beta} - \frac{i}{12S} \nabla^I_{\beta} \sigma^\alpha = -\frac{i}{4S^2} \varepsilon^{IJ} \nabla^I_{J \tau} , \quad \tau = -\frac{1}{4} \varepsilon_{IJ} \nabla^I \zeta^J_\gamma .
\]

Equation \((2.17a)\) implies that \(\zeta_a\) is a Killing vector field,

\[
\nabla_a \zeta_b + \nabla_b \zeta_a = 0 ,
\]

while \((2.17b)\) is a Killing spinor equation. The real parameter \(\tau\) is constrained by

\[
(\nabla^2)^2 \tau = (\nabla^1)^2 \tau = 8i S \tau , \quad \nabla_a \tau = 0 .
\]
2.3 Reduction from (2,0) to $\mathcal{N} = 1$ AdS superspace

Given a tensor superfield $U(x, \theta_I)$ on (2,0) AdS superspace, its $\mathcal{N} = 1$ projection (or bar-projection) is defined by

$$U| := U(x, \theta_I)|_{\theta_2 = 0}$$

(2.21)
in a special coordinate system to be specified below. By definition, $U|$ depends on the real coordinates $z^M = (x^m, \theta^\mu)$, with $\theta^\mu := \theta_\mu^1$, which will be used to parametrise $\mathcal{N} = 1$ AdS superspace $\text{AdS}^{3|2}$. For the (2,0) AdS covariant derivative

$$\nabla_A = (\nabla_a, \nabla_\alpha) = E_A^M \frac{\partial}{\partial z^M} + \frac{1}{2} \Omega_A^{bc} M_{bc} + i \Phi_A J,$$

(2.22)
its bar-projection is defined as

$$\nabla_A| = E_A^M | \frac{\partial}{\partial z^M} + \frac{1}{2} \Omega_A^{bc} | M_{bc} + i \Phi_A | J.$$

(2.23)

We use the freedom to perform general coordinate, local Lorentz and $U(1)_R$ transformations to choose the following gauge condition

$$\nabla_a| = \nabla_a, \quad \nabla_\alpha| = \nabla_\alpha,$$

(2.24)
where

$$\nabla_A = (\nabla_a, \nabla_\alpha) = E_A^M \frac{\partial}{\partial z^M} + \frac{1}{2} \omega_A^{bc} M_{bc}$$

(2.25)
denotes the set of covariant derivatives for $\text{AdS}^{3|2}$, which obey the following graded commutation relations:

$$\{\nabla_\alpha, \nabla_\beta\} = 2i \nabla_{\alpha \beta} - 4i S M_{\alpha \beta},$$

(2.26a)
$$[\nabla_\alpha, \nabla_\beta] = S (\gamma_\alpha)_{\beta \gamma} \nabla_\gamma,$$

$$[\nabla_a, \nabla_\beta] = -4 S^2 M_{ab}.$$

(2.26b)

In such a coordinate system, the operator $\nabla_\alpha|$ contains no partial derivative with respect to $\theta_2$. As a consequence, $(\nabla_{\alpha_1}^1 \cdots \nabla_{\alpha_k}^1 U)| = \nabla_{\alpha_1} \cdots \nabla_{\alpha_k} U|$, for any positive integer $k$, where $U$ is a tensor superfield on (2,0) AdS superspace. Let us study how the $\mathcal{N} = 1$ descendants of $U$ defined by $U_{\alpha_1 \cdots \alpha_k} := (\nabla_{\alpha_1}^2 \cdots \nabla_{\alpha_k}^2 U)|$ transform under the (2,0) AdS isometries, with $k$ a non-negative integer.

We introduce the $\mathcal{N} = 1$ projection of the (2,0) AdS Killing supervector field (2.15)

$$\zeta| = \xi^b \nabla_b + \xi^\beta \nabla_\beta + \epsilon^\beta \nabla_\beta|,$$

$$\xi^b := \zeta^b|, \quad \xi^\beta := \zeta^\beta|, \quad \epsilon^\beta := \zeta_\beta^2|.$$
We also introduce the $\mathcal{N} = 1$ projections of the Lorentz and U(1)$_R$ parameters in (2.15):

$$\lambda^{bc} := l^{bc} , \quad \epsilon := \tau .$$ (2.28)

It follows from (2.15) that the $\mathcal{N} = 1$ parameters $\xi^B = (\xi^b, \xi^\beta)$ and $\lambda^{bc}$ obey the equation

$$\left[ \xi + \frac{1}{2} \lambda^{bc} M_{bc}, \nabla_A \right] = 0 , \quad \xi = \xi^B \nabla_B = \xi^b \nabla_b + \xi^\beta \nabla_\beta ,$$ (2.29)

which tells us that $\xi^B$ is a Killing supervector field of $\mathcal{N} = 1$ AdS superspace [3]. This equation is equivalent to

$$\nabla_{(\alpha} \xi_{\beta \gamma)} = 0 , \quad \nabla_\beta \xi^{\beta \alpha} = -6i \xi^\alpha ,$$ (2.30a)

$$\nabla_\alpha \xi_\beta = \frac{1}{2} \lambda_{\alpha \beta} + S \xi_{\alpha \beta} ,$$ (2.30b)

$$\nabla_{(\alpha} \lambda_{\beta \gamma)} = 0 , \quad \nabla_\beta \lambda^{\beta \alpha} = -12i S \xi^\alpha .$$ (2.30c)

These relations automatically follow from the (2,0) AdS Killing equations, eqs. (2.16a) – (2.16d), upon $\mathcal{N} = 1$ projection. Thus $(\xi^a, \xi^\alpha, \lambda^{ab})$ parametrise the infinitesimal isometries of AdS$_3$ [3] (see also [14]).

The remaining parameters $\epsilon^\alpha$ and $\epsilon$ generate the second supersymmetry and U(1)$_R$ transformations, respectively. Using the Killing equations (2.18), it can be shown that they satisfy the following properties

$$\epsilon_\alpha = \frac{i}{4S} \nabla_\alpha \epsilon , \quad \epsilon = -\frac{1}{2} \nabla^\alpha \epsilon_\alpha ,$$ (2.31a)

$$(i \nabla^2 + 8S) \epsilon = 0 , \quad \nabla_a \epsilon = 0 .$$ (2.31b)

These imply that the only independent components of $\epsilon$ are $\epsilon|_{\theta=0}$ and $\nabla_\alpha \epsilon|_{\theta=0}$. They correspond to the U(1)$_R$ and second supersymmetry transformations, respectively.

Given a matter tensor superfield $U$, its (2,0) AdS transformation law

$$\delta_\xi U = \left( \xi + \frac{1}{2} l^{bc} M_{bc} + i \tau J \right) U$$ (2.32)

turns into

$$\delta_\xi U| = \delta_\xi U| + \delta_\xi U| ,$$ (2.33a)

$$\delta_\xi U| = \left( \xi^b \nabla_b + \xi^\beta \nabla_\beta + \frac{1}{2} \lambda^{bc} M_{bc} \right) U| ,$$ (2.33b)

$$\delta_\epsilon U| = \left( \epsilon^\beta \left( \nabla_\beta^2 U| \right) + i \epsilon J U| \right) .$$ (2.33c)
It follows from (2.15) and (2.33) that every $N = 1$ descendant $U_{\alpha_1...\alpha_k} := (\nabla^2_{\alpha_1} \cdots \nabla^2_{\alpha_k} U)$ is a tensor superfield on AdS$^{3|2}$,

$$\delta_\zeta U_{\alpha_1...\alpha_k} = \left( \zeta^b \nabla_b + \zeta^\beta \nabla_\beta + \frac{1}{2} \lambda^{bc} M_{bc} \right) U_{\alpha_1...\alpha_k}. \quad (2.34)$$

For the $\epsilon$-transformation we get

$$\delta_\epsilon U_{\alpha_1...\alpha_k} = e^\beta (\nabla^2_{\beta} U_{\alpha_1} \cdots \nabla^2_{\alpha_k} U) \bigg| + ie (J \nabla^2_{\alpha_1} \cdots \nabla^2_{\alpha_k} U) \bigg| \quad (2.35)$$

$$= e^\beta U_{\beta \alpha_1...\alpha_k} - \epsilon \sum_{l=1}^{k} \nabla^2_{\alpha_1} \cdots \nabla^2_{\alpha_{l-1}} \nabla^1_{\alpha_l} \nabla^2_{\alpha_{l+1}} \cdots \nabla^2_{\alpha_k} U \bigg| + iqeU_{\alpha_1...\alpha_k},$$

where $q$ is the U$(1)_R$ charge of $U$ defined by $J U = qU$. In the second term on the right, we have to push $\nabla^1_{\alpha_l}$ to the far left through the $(l-1)$ factors of $\nabla^2_{\alpha_l}$'s by making use of the relation $\{\nabla^1_{\alpha_l}, \nabla^2_{\beta}\} = 4\epsilon_{\alpha_l\beta}S J$ and taking into account the relation

$$(\nabla^1_{\alpha_l} \nabla^2_{\alpha_1} \cdots \nabla^2_{\alpha_{l-1}} \nabla^2_{\alpha_{l+1}} \cdots \nabla^2_{\alpha_k} U) \bigg| = \nabla^1_{\alpha_l} U_{\alpha_1...\alpha_{l-1} \alpha_{l+1}...\alpha_k}. \quad (2.36)$$

As the next step, the U$(1)_R$ generator $J$ should be pushed to the right until it hits $U$ producing on the way insertions of $\nabla^1$. Then the procedure should be repeated. As a result, the variation $\delta_\epsilon U_{\alpha_1...\alpha_k}$ is expressed in terms of the superfields $U_{\alpha_1...\alpha_k+1}$, $U_{\alpha_1...\alpha_k}$, $\cdots$ $U_{\alpha_1}$,$U$.

So far we have been completely general and discussed infinitely many descendants $U_{\alpha_1...\alpha_k}$ of $U$. However only a few of them are functionally independent. Indeed, eq. (2.12a) tells us that

$$\{\nabla^2_{\alpha}, \nabla^2_{\beta}\} = 2i\nabla_{\alpha\beta} - 4iSM_{\alpha\beta}, \quad (2.37)$$

and thus every $U_{\alpha_1...\alpha_k}$ for $k > 2$ can be expressed in terms of $U$, $U_a$ and $U_{\alpha_1\alpha_2}$. Therefore, it suffices to consider $k \leq 2$.

Let us give two examples of matter superfields on (2,0) AdS superspace. We first consider a covariantly chiral scalar superfield $\phi$, $D_\alpha \phi = 0$, with an arbitrary U$(1)_R$ charge $q$ defined by $J \phi = q\phi$. It transforms under the (2,0) AdS isometries as

$$\delta_\zeta \phi = (\zeta + iq\tau) \phi. \quad (2.38)$$

When expressed in the real basis (2.11), the chirality constraint on $\phi$ means

$$\nabla^2_{\alpha} \phi = i \nabla^1_{\alpha} \phi, \quad (2.39)$$

As a result, there is only one independent $\mathcal{N} = 1$ superfield upon reduction,

$$\phi := \phi \bigg|.$$

10
We then get the following relations

\[ \nabla_\alpha^2 \phi | = i \nabla_\alpha \varphi , \]  
(2.41a)  
\[ (\nabla_\alpha^2)^2 \phi | = -\nabla^2 \varphi - 8i q S \varphi . \]  
(2.41b)

The \( \epsilon \)-transformation (2.35) is given by

\[ \delta_\epsilon \varphi = i \epsilon^\beta \nabla_\beta \varphi + i q \epsilon \varphi . \]  
(2.42)

Our second example is a real linear superfield \( \mathbb{L} = \bar{\mathbb{L}} \), \( \bar{\mathcal{D}}^2 \mathbb{L} = 0 \). The real linearity constraint relates the \( \mathcal{N} = 1 \) descendants of \( \mathbb{L} \) as follows:

\[ (\nabla_\alpha^2)^2 \mathbb{L} = (\nabla_\alpha^{1\dagger})^2 \mathbb{L} , \]  
(2.43a)  
\[ \nabla_\alpha^{1\dagger} \nabla_\alpha^{2\dagger} \mathbb{L} = 0 . \]  
(2.43b)

Thus, \( \mathbb{L} \) is equivalent to two independent, real \( \mathcal{N} = 1 \) superfields:

\[ X := \mathbb{L}| , \quad W_\alpha := i \nabla_\alpha^2 \mathbb{L}| . \]  
(2.44)

Here \( X \) is unconstrained, while \( W_\alpha \) obeys the constraint (2.43b)

\[ \nabla_\alpha W_\alpha = 0 , \]  
(2.45)

which means that \( W_\alpha \) is the field strength of an \( \mathcal{N} = 1 \) vector multiplet. Since \( \mathbb{L} \) is neutral under the \( R \)-symmetry group \( U(1)_R \), \( J \mathbb{L} = 0 \), the second SUSY and \( U(1)_R \) transformation laws of the \( \mathcal{N} = 1 \) descendants of \( \mathbb{L} \) are as follows:

\[ \delta_\epsilon X = \delta_\epsilon \mathbb{L}| = \epsilon^\beta (\nabla_\beta^2 \mathbb{L}|) = -i \epsilon^\beta W_\beta , \]  
(2.46a)  
\[ \delta_\epsilon W_\alpha = i (\nabla_\alpha^2 \delta_\epsilon \mathbb{L}|) = \epsilon^\beta (\nabla_\beta^2 \nabla_\alpha \mathbb{L}|) - \epsilon [J, \nabla_\alpha^2 \mathbb{L}|] 
= -\epsilon^\beta \nabla_\alpha \nabla_\beta X - \frac{i}{2} \epsilon_\alpha \nabla \beta X - i \epsilon \nabla_\alpha X . \]  
(2.46b)

### 2.4 The (2,0) AdS supersymmetric actions in AdS\(^{3|2}\)

Every rigid supersymmetric field theory in (2,0) AdS superspace may be reduced to \( \mathcal{N} = 1 \) AdS superspace. Here we provide the key technical details of the reduction.

In accordance with [5–7,18], there are two ways of constructing supersymmetric actions in (2,0) AdS superspace: (i) either by integrating a real scalar \( \mathcal{L} \) over the full (2,0) AdS superspace \( \bar{\mathcal{D}}^2 \mathcal{E} \mathcal{L} \)

\[ S = \int d^3x d^2 \theta d^2 \bar{\theta} \mathcal{E} \mathcal{L} = \frac{1}{16} \int d^3x e D^2 D^2 \mathcal{L} \bigg|_{\theta = 0} = \frac{1}{16} \int d^3x e \bar{\mathcal{D}}^2 \mathcal{D}^2 \mathcal{L} \bigg|_{\bar{\theta} = 0} \]  
(2.47)

\( \bar{\mathcal{D}}^2 \mathcal{D}^2 \mathcal{L} \) is the component inverse vierbein is defined as usual, \( e_\alpha^m(x) = E_\alpha^m|_{\theta = 0} \), with \( e^{-1} = \text{det}(e_\alpha^m) \).
\[ \int d^3 x e \left( \frac{1}{16} D^\alpha \bar{D}^2 D_\alpha + i S \bar{D}^\alpha D_\alpha \right) L \bigg|_{\theta=0} = \int d^3 x e \left( \frac{1}{16} \bar{D}_\alpha D^2 \bar{D}^\alpha + i S \bar{D}^\alpha D_\alpha \right) L \bigg|_{\theta=0}, \]

with \( E^{-1} = \text{Ber}(E_A M) \); or (ii) by integrating a covariantly chiral scalar \( L_c \) over the chiral subspace of the \( (2,0) \) AdS superspace,

\[ S_c = \int d^3 x d^2 \theta \mathcal{E} L_c = -\frac{1}{4} \int d^3 x e \bar{D}^2 L_c \bigg|_{\theta=0}, \quad \bar{D}^\alpha L_c = 0, \quad (2.48) \]

with \( \mathcal{E} \) being the chiral density. The superfield Lagrangians \( L \) and \( L_c \) are neutral and charged, respectively with respect to the group \( U(1)_R \):

\[ J_L = 0, \quad J_{L_c} = -2 L_c. \quad (2.49) \]

The two types of supersymmetric actions are related to each other by the rule

\[ \int d^3 x d^2 \theta d^2 \bar{\theta} \mathcal{E} L = \int d^3 x d^2 \theta \mathcal{E} L_c, \quad L_c := -\frac{1}{4} \bar{D}^2 L. \quad (2.50) \]

Instead of reducing the above actions to components, in this paper we need their reduction to \( \mathcal{N} = 1 \) AdS superspace. We remind the reader that the supersymmetric action in AdS\(^{3|2} \) makes use of a real scalar Lagrangian \( L \). The superspace and component forms of the action are:

\[ S = \int d^3 z E L = \frac{1}{4} \int d^3 x e \left( i \nabla^2 + 8 S \right) L \bigg|_{\theta=0}. \quad (2.51) \]

For the action \((2.47)\) we get

\[ S = \int d^3 x d^2 \theta d^2 \bar{\theta} E L = -\frac{i}{4} \int d^3 z E (\nabla^2)^2 L \bigg|, \quad (2.52) \]

with \( E^{-1} = \text{Ber}(E_A M) \). The chiral action \((2.48)\) reduces to AdS\(^{3|2} \) as follows:

\[ S_c = \int d^3 x d^2 \theta \mathcal{E} L_c = 2i \int d^3 z E L_c \bigg| \quad (2.53) \]

Making use of the \((2,0) \) AdS transformation law \( \delta L = \zeta L, \delta L_c = (\zeta - 2i \tau) L_c \), and the Killing equation \((2.15b)\), it can be checked explicitly that the \( \mathcal{N} = 1 \) action defined by the right-hand side of \((2.52)\), or \((2.53)\) are invariant under the \((2,0) \) AdS isometry transformations.

12
2.5 Supersymmetric nonlinear sigma models

To illustrate the \((2,0) \to (1,0)\) AdS superspace reduction described above, here we discuss two interesting examples.

Our first example is a general nonlinear \(\sigma\)-model with \((2,0)\) AdS supersymmetry \([6,7]\). It is described by the action

\[
S = \int d^3x d^2\theta d^2\bar{\theta} \, E \, K(\phi^i, \bar{\phi}^\bar{j}) + \left\{ \int d^3x d^2\theta \, E \, W(\phi^i) + \text{c.c.} \right\}, \quad \mathcal{D}_\alpha \phi^i = 0 ,
\]

where \(K(\phi^i, \bar{\phi}^\bar{j})\) is the Kähler potential of a Kähler manifold and \(W(\phi^i)\) is a superpotential. The \(U(1)_R\) generator is realised on the dynamical superfields \(\phi^i\) and \(\bar{\phi}^\bar{j}\) as

\[
iJ^i(\phi)\partial_\alpha + \bar{J}^\bar{j}(\bar{\phi})\partial_\bar{\alpha},
\]

where \(J^i(\phi)\) is a holomorphic Killing vector field such that

\[
J^i(\phi)\partial_\alpha K = -\frac{i}{2} \mathcal{D}(\phi, \bar{\phi}), \quad \bar{\mathcal{D}} = \mathcal{D},
\]

for some Killing potential \(\mathcal{D}(\phi, \bar{\phi})\). The superpotential has to obey the condition

\[
J^i(\phi)\partial_\alpha W = -2iW
\]

in order for the action \((2.54)\) to be invariant under the \((2,0)\) AdS isometry transformations

\[
\delta \phi^i = (\zeta + i\tau J) \phi^i .
\]

In the real representation \((2.11)\), the chirality condition on \(\phi^i\) turns into

\[
\nabla_\alpha^2 \phi^i = i \nabla_{\overline{\alpha}}^1 \phi^i .
\]

It follows that upon \(\mathcal{N} = 1\) reduction, \(\phi^i\) leads to just one superfield,

\[
\phi^i := \phi^i | .
\]

In particular, we have the following relations

\[
\nabla_\alpha^2 \phi^i = i \nabla_{\overline{\alpha}} \phi^i ,
\]

\[
(\nabla_\alpha^2)^2 \phi^i = -\nabla_\alpha^2 \phi^i - 8S \mathcal{J}^i(\varphi) .
\]

Using the reduction rules \((2.52)\) and \((2.53)\), we obtain

\[
S = \int d^3z E \left\{ -iK_{ij}(\varphi, \bar{\varphi}) \nabla_\alpha \varphi^i \nabla_{\overline{\alpha}} \bar{\varphi}^\bar{j} + S \mathcal{D}(\varphi, \bar{\varphi}) + (2iW(\varphi) + \text{c.c.}) \right\} ,
\]

13
where we have made use of the standard notation

\[ K_{i_1 \ldots i_p \bar{j}_1 \ldots \bar{j}_q} := \frac{\partial^{p+q} K(\phi, \bar{\phi})}{\partial \phi^{i_1} \cdots \partial \phi^{i_p} \partial \bar{\phi}^{\bar{j}_1} \cdots \partial \bar{\phi}^{\bar{j}_q}}. \]  

(2.63)

The action \[ (2.62) \] is manifestly \( \mathcal{N} = 1 \) supersymmetric. One may explicitly check that it is also invariant under the second supersymmetry and \( R \)-symmetry transformations generated by a real scalar parameter \( \epsilon \) subject to the constraints \[ (2.31) \], which are:

\[ \delta \epsilon \phi^i = i \epsilon \alpha \nabla^\alpha \phi^i + \epsilon \mathcal{J}^i(\phi) . \]  

(2.64)

The family of supersymmetric \( \sigma \)-models \[ (2.54) \] includes a special subclass which is specified by the two conditions: (ii) all \( \phi \)'s are neutral, \( J\phi^i = 0 \); and (ii) no superpotential is present, \( W(\phi) = 0 \). In this case no restriction on the Kähler potential is imposed by eq. \[ (2.56) \], and the action \[ (2.54) \] is invariant under arbitrary Kähler transformations

\[ K \rightarrow K + \Lambda + \bar{\Lambda}, \]  

(2.65)

with \( \Lambda(\phi^i) \) a holomorphic function. The corresponding action in \( \mathcal{N} = 1 \) AdS superspace is obtained from \[ (2.62) \] by setting \( \mathcal{D}(\phi, \bar{\phi}) = 0 \) and \( W(\phi) = 0 \), and thus the action is manifestly Kähler invariant.

Let us also consider a supersymmetric nonlinear \( \sigma \)-model formulated in terms of several Abelian vector multiplets with action \[ [7] \]

\[ S = -2 \int d^3x \, d^2 \theta \, d^2 \bar{\theta} \, E F(\mathbb{L}^i) , \quad \mathcal{D}^2 \mathbb{L}^i = 0 , \quad \mathbb{L}^i = \mathbb{L}^i , \]  

(2.66)

where \( F(x^i) \) is a real analytic function of several variables, which is defined modulo linear inhomogeneous shifts

\[ F(x) \rightarrow F(x) + b_i x^i + c , \]  

(2.67)

with real parameters \( b_i \) and \( c \). The real linear scalar \( \mathbb{L}^i \) is the field strength of a vector multiplet. Upon reduction to \( \mathcal{N} = 1 \) AdS superspace, \( \mathbb{L}^i \) generates two different \( \mathcal{N} = 1 \) superfields:

\[ X^i := \mathbb{L}^i , \quad W_{\alpha}^i := i \nabla^2_{\alpha} \mathbb{L}^i . \]  

(2.68)

Here the real scalar \( X^i \) is unconstrained, while the real spinor \( W_{\alpha}^i \) obeys the constraint

\[ \nabla^\alpha W_{\alpha}^i = 0 , \]  

(2.69)
which means that $W_i^a$ is the field strength of an $\mathcal{N} = 1$ vector multiplet. Reducing the action (2.60) to $\mathcal{N} = 1$ AdS superspace gives

$$S = -\frac{i}{2} \int d^{3,2}z \, E g_{ij}(X) \left\{ \nabla^\alpha X^i \nabla_\alpha X^j + W^{ai} W_{aj}^i \right\},$$

(2.70)

where we have introduced the target-space metric

$$g_{ij}(X) = \frac{\partial^2 F(X)}{\partial X^i \partial X^j}.$$  

(2.71)

The vector multiplets in (2.70) can be dualised into scalar ones, which gives

$$S_{\text{dual}} = -\frac{i}{2} \int d^{3,2}z \, E \left\{ g_{ij}(X) \nabla^\alpha X^i \nabla_\alpha X^j + g^{ij}(X) \nabla^\alpha Y^i \nabla_\alpha Y^j \right\},$$

(2.72)

with $g^{ij}(X)$ being the inverse metric. Riemannian metrics of the type (2.71) appeared in the literature twenty years ago in the context of $\mathcal{N} = 4$ supersymmetric quantum mechanics [19] and $\mathcal{N} = 4$ superconformal mechanics [20].

3 Massless higher-spin models: Type II series

There exist two off-shell formulations for a massless multiplet of half-integer superspin $(s + \frac{1}{2})$ in $(2,0)$ AdS superspace [1], with $s = 2, 3, \ldots$, which are called the type II and type III series [7] by analogy with the terminology used in [7] for the linearised off-shell formulations for $\mathcal{N} = 2$ supergravity ($s = 1$). In this section we describe the $(2,0) \rightarrow (1,0)$ AdS superspace reduction of the type II theory. The reduction of the type III theory will be given in section 4.

3.1 The type II theory

We fix an integer $s > 1$. In accordance with [1], the massless type II multiplet of superspin $(s + \frac{1}{2})$ is described in terms of two unconstrained real tensor superfields

$$\mathcal{V}^{(II)}_{(s+\frac{1}{2})} = \left\{ S_{\alpha(2s)}, \mathbf{L}_{\alpha(2s-2)} \right\},$$

(3.1)

Type I series will be referred to the longitudinal formulation for the gauge massless half-integer superspin multiplets in $(1,1)$ AdS superspace [21] and Minkowski superspace [22]. The type I series and its dual are naturally related to the off-shell formulations for massless higher-spin $\mathcal{N} = 1$ supermultiplets in four dimensions [23–25]. The type II and type III series have no four-dimensional counterpart.
where $\mathcal{F}_{\alpha(2s)} = \mathcal{F}_{(\alpha_1...\alpha_{2s})}$ and $\mathcal{L}_{\alpha(2s-2)} = \mathcal{L}_{(\alpha_1...\alpha_{2s-2})}$ are symmetric in their spinor indices.

The dynamical superfields are defined modulo gauge transformations of the form

$$
\delta_\lambda \mathcal{F}_{\alpha(2s)} = \bar{D}_{(\alpha_1} \lambda_{\alpha_2...\alpha_{2s})} - D_{(\alpha_1} \bar{\lambda}_{\alpha_2...\alpha_{2s})} \equiv g_{\alpha(2s)} + \bar{g}_{\alpha(2s)} ,
$$

(3.2a)

$$
\delta_\xi \mathcal{L}_{\alpha(2s-2)} = -\frac{i}{2} (\bar{D}^\beta \lambda_{\beta(2s-2)} + D^\beta \bar{\lambda}_{\beta(2s-2)}) ,
$$

(3.2b)

where the gauge parameter $\lambda_{\alpha(2s-1)}$ is unconstrained complex. Eq. (3.2a) implies that the complex gauge parameter $g_{\alpha(2s)}$ is a covariantly longitudinal linear superfield,

$$
g_{\alpha(2s)} := D_{(\alpha_1} \lambda_{\alpha_2...\alpha_{2s})} , \quad \bar{D}_{(\alpha_1} g_{\alpha_2...\alpha_{2s+1})} = 0 .
$$

(3.3)

The gauge transformation of $\mathcal{F}_{\alpha(2s)}$, eq. (3.2a), corresponds to the superconformal gauge prepotential [21][22]. The prepotential $\mathcal{L}_{\alpha(2s-2)}$ is a compensating multiplet. In addition to (3.2b), the compensator $\mathcal{L}_{\alpha(2s-2)}$ also possesses its own gauge freedom of the form

$$
\delta_\xi \mathcal{L}_{\alpha(2s-2)} = \xi_{\alpha(2s-2)} + \bar{\xi}_{\alpha(2s-2)} , \quad \bar{D}_\beta \xi_{\alpha(2s-2)} = 0 ,
$$

(3.4)

with the gauge parameter $\xi_{\alpha(2s-2)}$ being covariantly chiral.

Associated with $\mathcal{L}_{\alpha(2s-2)}$ is the real field strength

$$
\mathbb{L}_{\alpha(2s-2)} = i D^\beta \bar{D}_\beta \mathcal{L}_{\alpha(2s-2)} , \quad \bar{\mathbb{L}}_{\alpha(2s-2)} = \bar{\mathbb{L}}_{\alpha(2s-2)} ,
$$

(3.5)

which is a covariantly linear superfield,

$$
D^2 \mathbb{L}_{\alpha(2s-2)} = 0 \iff \bar{D}^2 \bar{\mathbb{L}}_{\alpha(2s-2)} = 0 .
$$

(3.6)

It is inert under the gauge transformation (3.4), $\delta_\xi \mathbb{L}_{\alpha(2s-2)} = 0$. From (3.2b) we can read off the $\lambda$-gauge transformation of the field strength:

$$
\delta_\lambda \mathbb{L}_{\alpha(2s-2)} = \frac{1}{4} (D^\beta \bar{D}^2 \lambda_{\beta(2s-2)} - \bar{D}^\beta D^2 \bar{\lambda}_{\beta(2s-2)}) .
$$

$$
= -\frac{s}{2s+1} D^\beta D^\gamma (g_{\beta\gamma(2s-2)} + \bar{g}_{\beta\gamma(2s-2)}) - \frac{2is}{2s+1} D^\beta \bar{g}_{\beta\gamma(2s-2)} .
$$

(3.7)

The type II theory is described by the action

$$
S^{(II)}_{(s+\frac{1}{2})} = \left( -\frac{1}{2} \right)^s \int \mathcal{L}_{\alpha(2s)} \mathcal{L}_{\alpha(2s-2)} \left[ \frac{1}{8} \mathcal{F}_{\alpha(2s)} D^\beta \bar{D}^2 \delta_{\beta(2s)} + \frac{s}{8} \{ D_{\beta}, D_{\gamma} \} \mathcal{F}_{\delta(2s-2)} [D^\delta, D^\mu] \mathcal{L}_{\delta(2s-2)} + \frac{s}{2} \{ D_{\beta}, \mathcal{L}_{\delta(2s-2)} \} D^\beta \bar{D}^\mu \mathcal{L}_{\delta(2s-2)} + 2is \bar{S}_{\alpha(2s)} D^\beta \bar{D}^\beta \mathcal{F}_{\alpha(2s)} \right] .
$$
\[-\frac{2s-1}{2}\left(\mathbb{L}^{\alpha(2s-2)}[\mathcal{D}^\beta, \bar{\mathcal{D}}^\gamma]\mathcal{H}_{\beta\gamma\alpha(2s-2)} + 2\mathbb{L}^{\alpha(2s-2)}\mathbb{L}_{\alpha(2s-2)}\right)
- \frac{(s-1)(2s-1)}{4s}\left(\mathcal{D}_\beta\mathcal{L}^{\beta\alpha(2s-3)}\mathcal{D}^2\mathcal{L}^\gamma\mathcal{L}_{\gamma\alpha(2s-3)} + \text{c.c.}\right)
- 4(2s-1)\mathcal{S}\mathcal{L}^{\alpha(2s-2)}\mathbb{L}_{\alpha(2s-2)}\right\}.

(3.8)

It is invariant under the gauge transformations (3.2) and (3.4).

The structure \(\mathcal{D}_\beta\mathcal{L}^{\beta\alpha(2s-3)}\mathcal{D}^2\mathcal{L}^\gamma\mathcal{L}_{\gamma\alpha(2s-3)}\) in (3.8) is not defined for \(s = 1\). However, it comes with the factor \((s - 1)\) and therefore drops out from (3.8) for \(s = 1\). The action (3.8) for \(s = 1\) coincides with the linearised action for (2,0) AdS supergravity, which was originally derived in section 10.1 of [7].

### 3.2 Reduction of the gauge prepotentials to AdS\(^3\)/2

Let us turn to reducing the gauge prepotentials (3.1) to \(\mathcal{N} = 1\) AdS superspace\(^8\). Our first task is to work out such a reduction for the superconformal gauge multiplet \(\mathcal{H}_{\alpha(2s)}\).

In the real representation (2.11), the longitudinal linear constraint (3.3) takes the form
\[
\nabla_\alpha^2(\alpha_1 g_{\alpha_2...\alpha_{2s+1}}) = i\nabla_\alpha^1(\alpha_1 g_{\alpha_2...\alpha_{2s+1}}) .
\]

(3.9)

It follows that \(g_{\alpha(2s)}\) has two independent \(\theta_2\)-components, which are
\[
g_{\alpha(2s)} = \nabla^2_{\alpha_1} g_{\alpha_2...\alpha_{2s+1}} .
\]

(3.10)

The gauge transformation of \(\mathcal{H}_{\alpha(2s)}\), eq. (3.2a), allows us to choose two gauge conditions
\[
\mathcal{H}_{\alpha(2s)} = 0 , \quad \nabla^2_{\alpha} \mathcal{H}_{\alpha(2s-1)} = 0 .
\]

(3.11)

In this gauge we stay with the following unconstrained real \(\mathcal{N} = 1\) superfields:
\[
H_{\alpha(2s+1)} := i\nabla^2_{\alpha_1} \mathcal{H}_{\alpha_2...\alpha_{2s+1}} ,
\]
\[
H_{\alpha(2s)} := \frac{i}{4}(\nabla^2)^2 \mathcal{H}_{\alpha(2s)} .
\]

(3.12a, b)

There exists a residual gauge freedom which preserves the gauge conditions (3.11). It is described by unconstrained real \(\mathcal{N} = 1\) superfields \(\zeta_{\alpha(2s)}\) and \(\bar{\zeta}_{\alpha(2s-1)}\) defined by
\[
g_{\alpha(2s)} = -\frac{i}{2} \zeta_{\alpha(2s)} , \quad \bar{\zeta}_{\alpha(2s)} = \zeta_{\alpha(2s)} .
\]

(3.13a)

\(^8\)In the super-Poincaré case, the \(\mathcal{N} = 2 \rightarrow \mathcal{N} = 1\) reduction of \(\mathcal{H}_{\alpha(2s)}\) has been carried out in [26].
\( \nabla^2 g_{\alpha(2s-1)\beta} | = \frac{2s + 1}{2s} \zeta_{\alpha(2s-1)} \), \hspace{1cm} \bar{\zeta}_{\alpha(2s-1)} = \zeta_{\alpha(2s-1)} \).

The gauge transformation laws of the superfields (3.12) are given by

\[
\delta H_{\alpha(2s+1)} = i \nabla_{(\alpha_1} \zeta_{\alpha_2...\alpha_{2s+1})} \),
\]

\[
\delta H_{\alpha(2s)} = \nabla_{(\alpha_1} \zeta_{\alpha_2...\alpha_{2s})} \).
\]

Our next step is to reduce the compensator \( \mathcal{L}_{\alpha(2s-2)} \) to \( N = 1 \) AdS superspace. Making use of the representation (2.11), we observe that the chirality condition (3.4) reads

\[
\nabla^2 \xi_{\alpha(2s-2)} = i \nabla_{\beta} \bar{\xi}_{\alpha(2s-2)} \).
\]

The gauge transformation (3.4) allows us to impose a gauge condition

\[
\mathcal{L}_{\alpha(2s-2)} | = 0 \).
\]

Thus, upon reduction to \( N = 1 \) superspace, we have the following real superfields

\[
\Psi_{\beta; \alpha(2s-2)} := i \nabla^2 \mathcal{L}_{\alpha(2s-2)} | ,
\]

\[
L_{\alpha(2s-2)} := i \frac{1}{4} (\nabla^2)^2 \mathcal{L}_{\alpha(2s-2)} | .
\]

Here \( \Psi_{\beta; \alpha(2s-2)} \) is a reducible superfield which belongs to the representation \( 2 \otimes (2s - 1) \) of \( SL(2, \mathbb{R}) \), \( \Psi_{\beta; \alpha_1...\alpha_{2s-2}} = \Psi_{\beta; (\alpha_1...\alpha_{2s-2})} \). The condition (3.16) is preserved by the residual gauge freedom generated by a real unconstrained \( N = 1 \) superfield \( \eta_{\alpha(2s-2)} \) defined by

\[
\xi_{\alpha(2s-2)} | = - \frac{i}{2} \eta_{\alpha(2s-2)} \), \hspace{1cm} \bar{\eta}_{\alpha(2s-2)} = \eta_{\alpha(2s-2)} \).
\]

We may now determine how the \( \eta \)-transformation acts the superfields (3.17a) and (3.17b). We obtain

\[
\delta_{\eta} \Psi_{\beta; \alpha(2s-2)} = i \nabla_{\beta} \eta_{\alpha(2s-2)} \),
\]

\[
\delta_{\eta} L_{\alpha(2s-2)} = 0 \),
\]

where we have used the chirality constraint (3.15) and the expression (3.18) for the residual gauge transformation.

Next, we analyse the \( \lambda \)-gauge transformation and reduce the \( N = 2 \) field strength \( \mathbb{L}_{\alpha(2s-2)} \) to AdS\(^3\). In the real basis for the covariant derivatives, the real linearity constraint (3.6) is equivalent to two constraints:

\[
(\nabla^2)^2 \mathbb{L}_{\alpha(2s-2)} = (\nabla^2)^2 \mathbb{L}_{\alpha(2s-2)} \),
\]
\[ \nabla_1^\beta \nabla_2^\beta \mathcal{L}_\alpha(2s-2) = 0 . \] (3.20b)

These constraints imply that the resulting \( \mathcal{N} = 1 \) components of \( \mathcal{L}_\alpha(2s-2) \) are given by
\[ \left. \mathcal{L}_\alpha(2s-2) \right|, \quad \left. i \nabla_2^\beta \mathcal{L}_\alpha(2s-2) \right|, \] (3.21)
of which the former is unconstrained and the latter is a constrained \( \mathcal{N} = 1 \) superfield that proves to be a gauge-invariant field strength, as we shall see below. The relation between \( \mathcal{L}_\alpha(2s-2) \) and the prepotential \( \mathcal{L}_\alpha(2s-2) \) is given by (3.2), which can be expressed as
\[ \mathcal{L}_\alpha(2s-2) = - \frac{i}{2} \left\{ (\nabla_1^\beta)^2 + (\nabla_2^\beta)^2 \right\} \mathcal{L}_\alpha(2s-2) . \] (3.22)

We now compute the bar-projection of (3.22) in the gauge (3.16) and make use of the definition (3.17b) to obtain
\[ \left. \mathcal{L}_\alpha(2s-2) \right| = -2L_\alpha(2s-2) . \] (3.23)

Making use of (3.22) and (3.17a), the bar-projection of \( i \nabla_2^\beta \mathcal{L}_\alpha(2s-2) \) leads to the \( \mathcal{N} = 1 \) field strength
\[ \mathcal{W}_\beta;\alpha(2s-2) := i \nabla_2^\beta \mathcal{L}_\alpha(2s-2) \] (3.24)
\[ \mathcal{W}_\beta;\alpha(2s-2) = \bar{\mathcal{W}}_\beta;\alpha(2s-2) \], and is a descendant of the real unconstrained prepotential \( \Psi_\beta;\alpha(2s-2) \) defined modulo gauge transformation (3.19a). The field strength proves to be gauge invariant under (3.19a), and it satisfies the condition
\[ \nabla^\beta \mathcal{W}_\beta;\alpha(2s-2) = 0 , \] (3.25)
as a consequence of (3.20b) and the identity (A.7b). Let us express the gauge transformation of \( \mathcal{L}_\alpha(2s-2) \), eq. (3.7) in terms of the real basis for the covariant derivatives. This leads to
\[ \delta \mathcal{L}_\alpha(2s-2) = \frac{is}{2s + 1} \left\{ \nabla_1^\beta \nabla_2^\gamma \left( g_{\beta\gamma\alpha}(2s-2) + \bar{g}_{\beta\gamma\alpha}(2s-2) \right) + \nabla_\beta^\gamma \left( g_{\beta\gamma\alpha}(2s-2) - \bar{g}_{\beta\gamma\alpha}(2s-2) \right) \right\} , \] (3.26)

In a similar way, one should also rewrite \( \nabla_2^\beta \delta \mathcal{L}_\alpha(2s-2) \) in the real basis. This allows us to derive the gauge transformations for \( L_\alpha(2s-2) \) and \( \mathcal{W}_\beta;\alpha(2s-2) \)
\[ \delta L_\alpha(2s-2) = - \frac{s}{2(2s + 1)} \nabla_\beta^\gamma \zeta_{\beta\gamma\alpha}(2s-2) , \] (3.27a)
\[ \delta \mathcal{W}_\beta;\alpha(2s-2) = i \left( \nabla^\gamma \nabla_\beta - 4iS^\gamma_\beta \right) \zeta_{\gamma\alpha}(2s-2) . \] (3.27b)
We can then read off the transformation law for the prepotential $\Psi_{\beta;\alpha(2s-2)}$

$$\delta \Psi_{\beta;\alpha(2s-2)} = -\zeta_{\beta\alpha(2s-2)} + i\nabla_{\beta} \eta_{\alpha(2s-2)} ,$$

(3.27c)

where we have also taken into account the $\eta$-gauge freedom (3.19a).

Applying the $\mathcal{N}=1$ reduction rule (2.52) to the type II action (3.8), we find that it becomes a sum of two actions,

$$S_{(s+\frac{1}{2})}[\mathcal{D}_{\alpha(2s-2)}, \mathcal{L}_{\alpha(2s-2)}] = S_{(s+\frac{1}{2})}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}] + S_{(s)}[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}] .$$

(3.28)

Explicit expressions for these $\mathcal{N}=1$ actions will be given in the next subsection.

### 3.3 Massless higher-spin $\mathcal{N}=1$ supermultiplets in AdS$_3$

The gauge transformations (3.14a), (3.14b), (3.27a) and (3.27c) tell us that in fact we are dealing with two different $\mathcal{N}=1$ supersymmetric higher-spin gauge theories.

Given a positive integer $n > 0$, we say that a supersymmetric gauge theory describes a multiplet of superspin $n/2$ if it is formulated in terms of a superconformal gauge prepotential $H_{\alpha(n)}$ and possibly a compensating multiplet. The gauge freedom of the real tensor superfield $H_{\alpha(n)}$ is

$$\delta \zeta H_{\alpha(n)} = i^{n}(-1)^{[n/2]}\nabla_{(\alpha_1 \zeta_{\alpha_2 ... \alpha_n)} , (3.29)$$

with the gauge parameter $\zeta_{\alpha(n-1)}$ being real but otherwise unconstrained.

#### 3.3.1 Longitudinal formulation for massless superspin-$(s + \frac{1}{2})$ multiplet

One of the two $\mathcal{N}=1$ theories provides an off-shell formulation for the massless superspin-$(s + \frac{1}{2})$ multiplet. It is formulated in terms of the real unconstrained gauge superfields

$$\mathcal{Y}_{(s+\frac{1}{2})} = \left\{ H_{\alpha(2s+1)}, L_{\alpha(2s-2)} \right\} ,$$

(3.30)

which are defined modulo gauge transformations

$$\delta H_{\alpha(2s+1)} = i\nabla_{(\alpha_1 \zeta_{\alpha_2 ... \alpha_{2s+1)} , (3.31a)$$

$$\delta L_{\alpha(2s-2)} = -\frac{s}{2(2s+1)}\nabla^{\beta\gamma} \zeta_{\beta\gamma\alpha(2s-2)} ,$$

(3.31b)
where the parameter \( \zeta_{\alpha(2s)} \) is unconstrained real. The gauge-invariant action is
\[
S_{\parallel(s+\frac{1}{2})}(H_{\alpha(2s+1)}, L_{\alpha(2s-2)}) = \left( -\frac{1}{2} \right)^s \int d^3 z \ E \left\{ -\frac{i}{2} H^{\alpha(2s+1)} Q H_{\alpha(2s+1)} - \frac{i}{8} \nabla_\beta H^{\beta(2s)} \nabla^2 \nabla^\gamma H_{\gamma\alpha(2s)} + \frac{i}{8} \nabla_\beta H^{\beta\gamma\alpha(2s-1)} \nabla^\rho H_{\rho\delta\alpha(2s-1)} \right.
\]
\[
+ (2s - 1) L^{\alpha(2s-2)} \nabla^\beta \nabla^\gamma H_{\beta\gamma\delta(2s-2)}
\]
\[
+ 2(2s - 1) \left( L^{\alpha(2s-2)} (i \nabla^2 - 4S) L_{\alpha(2s-2)} - \frac{i}{s} (s - 1) \nabla_\beta L^{\beta\alpha(2s-3)} \nabla^\gamma L_{\gamma\alpha(2s-3)} \right)
\]
\[
+ S \left( s \nabla_\beta H^{\beta\alpha(2s)} \nabla^\gamma H_{\gamma\alpha(2s)} + \frac{1}{2} (2s + 1) H^{\alpha(2s+1)} (\nabla^2 - 4iS) H_{\alpha(2s+1)} \right) \right\},
\] (3.32)

where \( Q \) is the quadratic Casimir operator of the 3D \( \mathcal{N} = 1 \) AdS supergroup (A.9). The action (3.32) coincides with the off-shell \( \mathcal{N} = 1 \) supersymmetric action for massless half-integer superspin in AdS in the form given in [14]. This supersymmetric gauge theory in AdS\(^{3|2}\) was described in [14]. Its flat-superspace limit was presented earlier in [26]. In what follows, we will refer to the above theory as the longitudinal formulation for the massless superspin-(\( s + \frac{1}{2} \)) multiplet.

The structure \( \nabla_\beta L^{\beta\alpha(2s-3)} \nabla^\gamma L_{\gamma\alpha(2s-3)} \) in (3.32) is not defined for \( s = 1 \). However it comes with the factor \( (s - 1) \) and drops out from (3.32) for \( s = 1 \). The resulting action
\[
S_{\parallel\left(\frac{3}{2}\right)}(H_{\alpha(3)}, L) = -\frac{1}{2} \int d^3 z \ E \left\{ -\frac{i}{2} H^{\alpha(3)} Q H_{\alpha(3)} - \frac{i}{8} \nabla_\beta H^{\beta\alpha(2)} \nabla^2 \nabla^\gamma H_{\gamma\alpha(2)}
\]
\[
+ \frac{i}{4} \nabla_\beta H^{\beta\gamma\alpha} \nabla^\rho H_{\rho\delta\alpha} + L \nabla^\beta H^{\gamma\delta} H_{\gamma\delta} + 2L (i \nabla^2 - 4S) L
\] (3.33)
\[
+ S \left( \nabla_\beta H^{\beta\alpha(2)} \nabla^\gamma H_{\gamma\alpha(2)} + \frac{3}{2} H^{\alpha(3)} (\nabla^2 - 4iS) H_{\alpha(3)} \right) \right\}
\]
is the linearised action for \( \mathcal{N} = 1 \) AdS supergravity. In the flat-superspace limit, the action is equivalent to the one given in [27].

### 3.3.2 Transverse formulation for massless superspin-s multiplet

The other \( \mathcal{N} = 1 \) theory provides a formulation for the massless superspin-s multiplet. It is described by the unconstrained real superfields
\[
\mathcal{V}_{\perp(s)} = \left\{ H_{\alpha(2s)}, \Psi_{\beta, \alpha(2s-2)} \right\},
\] (3.34)

which are defined modulo gauge transformations of the form
\[
\delta H_{\alpha(2s)} = \nabla_{(a_1} \zeta_{a_2...a_{2s})},
\] (3.35a)
\[ \delta \Psi_{\beta;\alpha(2s-2)} = -\zeta_{\alpha(2s-2)} + i\nabla_\beta \eta_{\alpha(2s-2)} , \]  

where the gauge parameters \( \zeta_{\alpha(2s-1)} \) and \( \eta_{\alpha(2s-2)} \) are unconstrained real. The gauge-invariant action is given by

\[
S^a_{(s)}[H_{\alpha(2s)}; \Psi_{\beta;\alpha(2s-2)}] = \left( -\frac{1}{2} \right)^s \int d^{3|2}z \ E \left\{ \frac{1}{2} H^{\alpha(2s)} (i\nabla^2 + 8sS) H_{\alpha(2s)} - i s \nabla^\alpha H^\beta H_{\alpha(2s-1)} - (2s - 1) W^\beta;\alpha(2s-2) \nabla^\gamma H_{\gamma\beta\alpha(2s-2)} - \frac{i}{2} (2s - 1) \left( W^\beta;\alpha(2s-2) W_{\beta;\alpha(2s-2)} + \frac{s-1}{s} W^\beta;\alpha(2s-3) W^\gamma;\gamma\alpha(2s-3) \right) - 2i(2s - 1) S \Psi^\beta;\alpha(2s-2) W_{\beta;\alpha(2s-2)} \right\} , \tag{3.36a}
\]

where \( W^\beta;\alpha(2s-2) \) denotes the field strength

\[ W^\beta;\alpha(2s-2) = -i \left( \nabla^\gamma \nabla_\beta - 4i S^\gamma_\beta \right) \Psi^\gamma;\alpha(2s-2) , \quad \nabla^\beta W^\gamma;\gamma\alpha(2s-3) = 0 . \tag{3.36b} \]

The action (3.36) defines a new \( \mathcal{N} = 1 \) supersymmetric higher-spin theory which did not appear in [14,21,26] even in the super-Poincaré case.

The structure \( W^\beta;\beta\alpha(2s-3) W^\gamma;\gamma\alpha(2s-3) \) in (3.36a) is not defined for \( s = 1 \). However it comes with the factor \((s - 1)\) and drops out from (3.36a) for \( s = 1 \). The resulting gauge-invariant action

\[
S^a_{(1)}[H_{\alpha(2)}; \Psi_\beta] = -\frac{1}{2} \int d^{3|2}z \ E \left\{ \frac{1}{2} H^{\alpha(2)} (i\nabla^2 + 8S) H_{\alpha(2)} - i \nabla^\alpha H^\beta \nabla^\gamma H_{\gamma\alpha} - W^\beta \nabla^\gamma H_{\gamma\beta} - \frac{i}{2} W^\beta W_\beta - 2i S \Psi^\beta W_\beta \right\} \tag{3.37}
\]

provides an off-shell realisation for a massless gravitino multiplet in AdS3. In the flat-superspace limit, this model reduces to the one described in [26].

In the \( s > 1 \) case, the gauge freedom of the prepotential \( \Psi^\beta;\alpha(2s-2) \) (3.35) allows us to impose a gauge condition

\[ \Psi^\beta;\alpha(2s-2) = 0 \iff \Psi^\beta;\alpha(2s-2) = \sum_{k=1}^{2s-2} \varepsilon^\beta_{\alpha k} \varphi_{\alpha_1\dot{\alpha}_k...\alpha_{2s-2}} , \tag{3.38} \]

for some field \( \varphi_{\alpha(2s-3)} \). Since we gauge away the symmetric part of \( \Psi^\beta;\alpha(2s-2) \), the two gauge parameters \( \zeta_{\alpha(2s-1)} \) and \( \eta_{\alpha(2s-2)} \) are related. The theory is now realised in terms of the following dynamical variables

\[
\left\{ H_{\alpha(2s)} , \varphi_{\alpha(2s-3)} \right\} , \tag{3.39}
\]
with the gauge freedom
\[
\delta H_{\alpha(2s)} = -\nabla_{(\alpha_1\alpha_2}\eta_{\alpha_3...\alpha_{2s})} ,
\]
\[
\delta \varphi_{\alpha(2s-3)} = i\nabla^\beta \eta_{\beta\alpha(2s-3)} .
\]
(3.40a)
(3.40b)

It follows that in the flat-superspace limit, \( S = 0 \), and in the gauge (3.38), the action (3.36) coincides with eq. (B.25) of [21]. The component structure of this model will be discussed in Appendix B.1.

4 Massless higher-spin models: Type III series

In this section we carry out the \( \mathcal{N} = 1 \) AdS superspace reduction of the type III theory [1] following the procedure employed in section 3.

4.1 The type III theory

We fix a positive integer \( s > 1 \). In accordance with [1], the massless type III multiplet of superspin \( (s + \frac{1}{2}) \) is described in terms of two unconstrained real tensor superfields

\[
\mathcal{V}^{(III)}_{(s+\frac{1}{2})} = \left\{ \mathcal{H}_{\alpha(2s)} , \mathcal{V}_{\alpha(2s-2)} \right\} ,
\]
(4.1)

which are symmetric in their spinor indices, \( \mathcal{H}_{\alpha(2s)} = \mathcal{H}_{(\alpha_1...\alpha_{2s})} \) and \( \mathcal{V}_{\alpha(2s-2)} = \mathcal{V}_{(\alpha_1...\alpha_{2s-2})} \).

The dynamical superfields are defined modulo gauge transformations of the form

\[
\delta_\lambda \mathcal{H}_{\alpha(2s)} = \mathcal{D}_{(\alpha_1\lambda_{\alpha_2...\alpha_{2s}})} - \mathcal{D}_{(\alpha_1\bar{\lambda}_{\alpha_2...\alpha_{2s}})} = g_{\alpha(2s)} + \bar{g}_{\alpha(2s)} ,
\]
(4.2a)
\[
\delta_\lambda \mathcal{V}_{\alpha(2s-2)} = \frac{1}{2s} \left( \bar{\mathcal{D}}^\beta \lambda_{\beta\alpha(2s-2)} - \mathcal{D}^\beta \bar{\lambda}_{\beta\alpha(2s-2)} \right) ,
\]
(4.2b)

where the gauge parameter \( \lambda_{\alpha(2s-1)} \) is unconstrained complex, and the longitudinal linear parameter \( g_{\alpha(2s)} \) is defined as in (3.3). As in the type II case, \( \mathcal{H}_{\alpha(2s)} \) is the superconformal gauge multiplet, while \( \mathcal{V}_{\alpha(2s-2)} \) is a compensating multiplet. The only difference from the type II case occurs in the gauge transformation law for the compensator \( \mathcal{V}_{\alpha(2s-2)} \).

The compensator \( \mathcal{V}_{\alpha(2s-2)} \) also possesses its own gauge freedom of the form

\[
\delta_\xi \mathcal{V}_{\alpha(2s-2)} = \xi_{\alpha(2s-2)} + \bar{\xi}_{\alpha(2s-2)} , \quad \bar{\mathcal{D}}_{\beta} \xi_{\alpha(2s-2)} = 0 ,
\]
(4.3)

with the gauge parameter \( \xi_{\alpha(2s-2)} \) being covariantly chiral, but otherwise arbitrary.
Associated with $\mathfrak{V}_{\alpha(2s-2)}$ is the real field strength

$$V_{\alpha(2s-2)} = iD^\beta \bar{D}_\beta \mathfrak{V}_{\alpha(2s-2)} \ , \quad \bar{V}_{\alpha(2s-2)} = \bar{V}_{\alpha(2s-2)} \ ,$$

(4.4)

which is inert under (4.3), $\delta_\xi V_{\alpha(2s-2)} = 0$. It is not difficult to see that $V_{\alpha(2s-2)}$ is covariantly linear,

$$\mathcal{D}^2 V_{\alpha(2s-2)} = 0 \iff \bar{\mathcal{D}}^2 \bar{V}_{\alpha(2s-2)} = 0 \ .$$

(4.5)

It varies under the $\lambda$-gauge transformation as

$$\delta_\lambda V_{\alpha(2s-2)} = \frac{i}{4s} \left( D^\beta \bar{D}^\gamma \bar{\lambda}^{\beta\gamma\alpha(2s-2)} + \bar{D}^\beta \bar{D}^\gamma \lambda^{\beta\gamma\alpha(2s-2)} \right) \ .$$

(4.6)

Modulo normalisation, there exists a unique action being invariant under the gauge transformations (4.2) and (4.3). It is given by

$$S^{(III)}_{s+\frac{1}{2}}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{V}_{\alpha(2s-2)}] = \left( -\frac{1}{2} \right)^s \int d^3x d^2\theta d^2\bar{\theta} \ E \left\{ \frac{1}{8} \mathfrak{H}_{\alpha(2s)}^\alpha \mathcal{D}^\beta \bar{D}^\gamma \mathfrak{V}^{\beta\gamma\alpha(2s-2)} \right. + \left. \frac{1}{16} \left( [\mathcal{D}_{\beta}, \bar{\mathcal{D}}_{\gamma}] \mathfrak{H}_{\beta\gamma(2s-2)} \right) \left[ [\mathcal{D}^\delta, \bar{\mathcal{D}}^{\rho}] \mathfrak{H}_{\delta\rho(2s-2)} \right) \right. \ 
+ \frac{1}{4} \left( \mathcal{D}_{\beta\gamma} \mathfrak{H}_{\beta\gamma(2s-2)} \right) \left( \mathfrak{H}_{\delta\rho(2s-2)} + \frac{1}{2} \mathfrak{H}_{\delta\rho(2s-2)} \mathfrak{V}_{\alpha(2s-2)} \right) \ 
- \frac{2s-1}{2} \left( \mathfrak{V}_{\alpha(2s-2)} \mathfrak{H}_{\beta\gamma(2s-2)} + \frac{1}{2} \mathfrak{V}_{\alpha(2s-2)} \mathfrak{V}_{\alpha(2s-2)} \right) \ 
+ \frac{1}{8} (s-1)(2s-1) \left( \mathcal{D}_{\beta} \mathfrak{V}_{\alpha(2s-3)} \right) \left( \mathfrak{H}_{\beta\gamma(2s-3)} \mathfrak{V}_{\gamma\alpha(2s-3)} + \text{c.c.} \right) \ 
+ 2s(2s-1)S \mathfrak{V}_{\alpha(2s-2)} \mathfrak{V}_{\alpha(2s-2)} \right\} \ .$$

(4.7)

Although the structure $\mathcal{D}_{\beta} \mathfrak{V}_{\alpha(2s-3)} \mathcal{D}^\gamma \mathfrak{V}_{\gamma\alpha(2s-3)}$ in (4.7) is not defined for $s = 1$, it comes with the factor $(s-1)$ and drops out from (4.7) for the $s = 1$ case. In this case the action coincides with the type III supergravity action in (2,0) AdS superspace, which was originally derived in section 10.2 of [7].

4.2 Reduction of the gauge prepotentials to AdS$^{3|2}$

The reduction of the superconformal gauge multiplet $\mathfrak{H}_{\alpha(2s)}$ to AdS$^{3|2}$ has been carried out in the previous subsection. We saw that in the gauge (3.11), $\mathfrak{H}_{\alpha(2s)}$ is described by the two unconstrained real superfields $H_{\alpha(2s+1)}$ and $H_{\alpha(2s)}$ defined according to (3.12), with
their gauge transformation laws given by eqs. (3.14a) and (3.14b), respectively. Now it remains to reduce the prepotential \( \mathcal{W}_{\alpha(2s-2)} \) to \( \mathcal{N} = 1 \) AdS superspace, following the same approach as outlined in the type II series. The gauge transformation (4.3) allows us to choose a gauge condition

\[
\mathcal{W}_{\alpha(2s-2)} = 0 .
\]  

(4.8)

The compensator \( \mathcal{W}_{\alpha(2s-2)} \) is then equivalent to the following real \( \mathcal{N} = 1 \) superfields, which we define as follows:

\[
\Upsilon_{\beta;\alpha(2s-2)} := i \nabla^2_{\beta} \mathcal{W}_{\alpha(2s-2)} | ,
\]

(4.9a)

\[
V_{\alpha(2s-2)} := i \frac{1}{4} (\nabla^2)^2 \mathcal{W}_{\alpha(2s-2)} | .
\]

(4.9b)

The residual gauge freedom, which preserves the gauge condition (4.8) is described by a real unconstrained \( \mathcal{N} = 1 \) superfield \( \eta_{\alpha(2s-2)} \) defined by

\[
\xi_{\alpha(2s-2)} | = -\frac{i}{2} \eta_{\alpha(2s-2)} , \quad \bar{\eta}_{\alpha(2s-2)} = \eta_{\alpha(2s-2)} .
\]

(4.10)

As a result, we may determine how (4.9a) and (4.9b) vary under \( \eta \)-transformation

\[
\delta_\eta \Upsilon_{\beta;\alpha(2s-2)} = i \nabla_{\beta} \eta_{\alpha(2s-2)} ,
\]

(4.11a)

\[
\delta_\eta V_{\alpha(2s-2)} = 0 .
\]

(4.11b)

Next, we analyse the \( \lambda \)-gauge transformation and reduce the field strength \( V_{\alpha(2s-2)} \) to AdS\(^{3|2} \). In the real basis for the covariant derivatives, the real linearity constraint (4.5) turns into:

\[
(\nabla^2)^2 V_{\alpha(2s-2)} = (\nabla^1)^2 V_{\alpha(2s-2)} ,
\]

(4.12a)

\[
\nabla^1 \beta \nabla^2 \beta V_{\alpha(2s-2)} = 0 .
\]

(4.12b)

This tells us that \( V_{\alpha(2s-2)} \) is equivalent to two real \( \mathcal{N} = 1 \) superfields

\[
V_{\alpha(2s-2)} | , \quad i \nabla^2_{\beta} V_{\alpha(2s-2)} | .
\]

(4.13)

The relation between the field strength \( V_{\alpha(2s-2)} \) and the prepotential \( \mathcal{W}_{\alpha(2s-2)} \) is given by (4.14), which can be expressed as

\[
V_{\alpha(2s-2)} = -\frac{i}{2} \left\{ (\nabla^1)^2 + (\nabla^2)^2 \right\} \mathcal{W}_{\alpha(2s-2)} .
\]

(4.14)
We now compute the bar-projection of (4.14) in the gauge (4.8) and make use of the definition (4.9b) to obtain
\[
\mathbb V_{\alpha(2s-2)} = -2V_{\alpha(2s-2)} .
\]
(4.15)

The bar-projection of \(i\nabla^2_{\beta}V_{\alpha(2s-2)}\) leads to the \(\mathcal N = 1\) field-strength
\[
\Omega_{\beta;\alpha(2s-2)} := i\nabla^2_{\beta}V_{\alpha(2s-2)}| = -i\left(\nabla^{\gamma}\nabla_{\beta} - 4iS\delta^\gamma_{\beta}\right)\Upsilon_{\gamma;\alpha(2s-2)} ,
\]
(4.16)

which is a real superfield, \(\Omega_{\beta;\alpha(2s-2)} = \bar{\Omega}_{\beta;\alpha(2s-2)}\), and is a descendant of the real unconstrained prepotential \(\Upsilon_{\beta;\alpha(2s-2)}\) defined modulo gauge transformation (4.11a). One may check that the field strength is invariant under (4.11a) and obeys the condition
\[
\nabla_{\beta}\Omega_{\beta;\alpha(2s-2)} = 0 .
\]
(4.17)

Let us express the gauge transformation of \(V_{\alpha(2s-2)}\), eq. (4.6) in terms of the real basis for the covariant derivatives. This leads to
\[
\delta V_{\alpha(2s-2)} = -\frac{1}{2s+1}\left\{\nabla_{\gamma}^1 \nabla_{\delta}^2 \left(g_{\beta\gamma\alpha(2s-2)} - \bar{g}_{\beta\gamma\alpha(2s-2)}\right)
+ \nabla^\beta \left(g_{\beta\gamma\alpha(2s-2)} + \bar{g}_{\beta\gamma\alpha(2s-2)}\right)\right\} ,
\]
(4.18)

One should also express its corollary \(\nabla^2_{\beta}\delta V_{\alpha(2s-2)}\) in the real basis for the covariant derivatives. We determine the gauge transformations law for \(V_{\alpha(2s-2)}\) and \(\Omega_{\beta;\alpha(2s-2)}\) to be
\[
\delta V_{\alpha(2s-2)} = \frac{1}{2s+1} \nabla^\beta \zeta_{\beta\alpha(2s-2)} ,
\]
(4.19a)
\[
\delta \Omega_{\beta;\alpha(2s-2)} = \frac{1}{2s+1} \left(\nabla^{\gamma} \nabla_{\beta} \nabla^{\delta} - 4iS\nabla^{\delta}\delta^\beta_{\delta}\right)\zeta_{\delta\gamma\alpha(2s-2)} .
\]
(4.19b)

From (4.19b) we read off the transformation law for the prepotential \(\Upsilon_{\beta;\alpha(2s-2)}\):
\[
\delta \Upsilon_{\beta;\alpha(2s-2)} = \frac{i}{2s+1} \left(\nabla^{\gamma} \zeta_{\beta\alpha(2s-2)} + (2s+1)\nabla_{\beta} \eta_{\alpha(2s-2)}\right) ,
\]
(4.20)

where we have also taken into account the \(\eta\)-gauge freedom (4.11a).

Performing \(\mathcal N = 1\) reduction to the original type III action (4.7), we arrive at two decoupled \(\mathcal N = 1\) actions
\[
S^{(III)}_{(s+\frac{1}{2})}[\mathcal H_{\alpha(2s)}, \Upsilon_{\alpha(2s-2)}] = S^+_{(s+\frac{1}{2})}[H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}] + S^0_{(s)}[H_{\alpha(2s)}, V_{\alpha(2s-2)}] .
\]
(4.21)

We will present the exact form of these actions in the next subsection.
4.3 Massless higher-spin $\mathcal{N} = 1$ supermultiplets in AdS$_3$

Upon reduction to $\mathcal{N} = 1$ superspace, the type III theory leads to two $\mathcal{N} = 1$ supersymmetric gauge theories.

4.3.1 Longitudinal formulation for massless superspin-$s$ multiplet

One of the two $\mathcal{N} = 1$ theories provides an off-shell realisation for massless superspin-$s$ multiplet described in terms of the real unconstrained superfields

$$\mathcal{V}'_{(s)} = \left\{ H_{\alpha(2s)}, V_{\alpha(2s-2)} \right\},$$

which are defined modulo gauge transformations of the form

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1 \zeta_{\alpha_2 \ldots \alpha_{2s}})};$$

$$\delta V_{\alpha(2s-2)} = \frac{1}{2s} \nabla^\beta \zeta_{\beta \alpha(2s-2)},$$

where the gauge parameter $\zeta_{\alpha(2s-1)}$ is unconstrained real. The gauge-invariant action is given by

$$S_{(s)}'_{(s)}[H_{\alpha(2s)}, V_{\alpha(2s-2)}] = \left( -\frac{1}{2} \right)^s \int d^3z \ E \left\{ \frac{1}{2} H_{\alpha(2s)} (i\nabla^2 + 4S) H_{\alpha(2s)} - \frac{i}{2} \nabla_{\beta} H^{\alpha(2s-1)} \nabla^\gamma H_{\gamma \alpha(2s-1)} - (2s-1) V^{\alpha(2s-2)} \nabla^\beta H_{\beta \gamma \alpha(2s-2)} \right. $$

$$+ (2s-1) \left( \frac{1}{2} V^{\alpha(2s-2)} (i\nabla^2 + 8sS) V_{\alpha(2s-2)} + (s-1) \nabla_{\beta} V^{\beta \alpha(2s-3)} \nabla^\gamma V_{\gamma \alpha(2s-3)} \right) \right\}. $$

Modulo an overall normalisation factor, (4.24) coincides with the off-shell $\mathcal{N} = 1$ supersymmetric action for massless superspin-$s$ multiplet in the form given in [14]. In the flat-superspace limit it reduces to the action derived in [26].

Although the structure $\nabla_{\beta} V^{\beta \alpha(2s-3)} \nabla^\gamma V_{\gamma \alpha(2s-3)}$ in (4.24) is not defined for $s = 1$, it comes with the factor $(s-1)$ and thus drops out from (4.24) for $s = 1$. The resulting gauge-invariant action

$$S_{(1)}'_{(1)}[H_{\alpha(2)}, V] = -\frac{1}{2} \int d^3z \ E \left\{ \frac{1}{2} H^{\alpha(2)} (i\nabla^2 + 4S) H_{\alpha(2)} - \frac{i}{2} \nabla_{\beta} H^{\alpha \beta} \nabla^\gamma H_{\gamma \alpha} - V \nabla^\beta H_{\beta \gamma} + \frac{1}{2} V (i\nabla^2 + 8S) V \right\} $$

(4.25)

describes an off-shell massless gravitino multiplet in AdS$_3$. In the flat-superspace limit, it reduces to the gravitino multiplet model described in [28] (see also [26]).
4.3.2 Transverse formulation for massless superspin-\((s + \frac{1}{2})\) multiplet

The other theory provides an off-shell formulation for massless superspin-\((s + \frac{1}{2})\) multiplet. It is described by the unconstrained superfields
\[
\mathcal{V}_{(s+\frac{1}{2})}^\perp = \left\{ H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)} \right\},
\]
which are defined modulo gauge transformations of the form
\[
\begin{align*}
\delta H_{\alpha(2s+1)} &= i\nabla_\alpha \zeta_{\alpha_2...\alpha_{2s+1}}; \\
\delta \Upsilon_{\beta;\alpha(2s-2)} &= \frac{i}{2s+1} (\nabla^\gamma \zeta_{\beta\alpha(2s-2)} + (2s + 1) \nabla_\beta \eta_{\alpha(2s-2)}) .
\end{align*}
\]

The gauge-invariant action is
\[
S^\perp_{(s+\frac{1}{2})}[H_{(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}] = \left( -\frac{1}{2} \right)^s \int d^{3\frac{1}{2}} z E \left\{ -\frac{i}{2} H^{\alpha(2s+1)} Q H_{\alpha(2s+1)} + \frac{i}{8} \nabla_\beta H^{\alpha(2s)} \nabla^2 \nabla^\gamma H_{\gamma\alpha(2s)} + \frac{i}{8} \nabla_\beta H^{\gamma\alpha(2s-1)} \nabla^\rho H_{\rho\delta\alpha(2s-1)} - \frac{i}{4} (2s - 1) \Omega^{\beta;\alpha(2s-2)} \nabla_\gamma \Omega_{\gamma\alpha(2s-2)} - \frac{i}{8} (2s - 1) \left( \Omega^{\beta;\alpha(2s-2)} \Omega_{\beta;\alpha(2s-2)} - 2(s - 1) \Omega_{\beta;\alpha(2s-3)} \Omega_{\gamma\alpha(2s-3)} \right) \right. \\
&\left. + S \left( H^{\alpha(2s+1)} \left( \nabla^2 - 4iS \right) H_{\alpha(2s+1)} + \frac{1}{2} \nabla_\beta H^{\alpha(2s)} \nabla^\gamma H_{\gamma\alpha(2s)} \right) + i s (2s - 1) \nabla_\beta H^{\alpha(2s-2)} \nabla^\gamma \Omega_{\gamma\alpha(2s-2)} \right\} ,
\]
where \(\Omega^{\beta;\alpha(2s-2)}\) denotes the real field strength
\[
\Omega^{\beta;\alpha(2s-2)} = -i \left( \nabla^\gamma \nabla_\beta - 4iS \delta^\gamma_\beta \right) \Upsilon_{\gamma;\alpha(2s-2)} , \quad \nabla_\beta \Omega^{\beta;\alpha(2s-2)} = 0 .
\]

This action defines a new \(\mathcal{N} = 1\) supersymmetric higher-spin theory which did not appear in \([14, 21, 26]\).

The structure \(\Omega^{\beta;\alpha(2s-3)} \Omega_{\gamma\alpha(2s-3)}\) in (4.28a) is not defined for \(s = 1\). However it comes with the factor \((s - 1)\) and hence drops out from (4.28a) for \(s = 1\). The resulting gauge-invariant action
\[
S^\perp_{(\frac{1}{2})}[H_{(3)}, \Upsilon_{\beta}] = -\frac{1}{2} \int d^{3\frac{1}{2}} z E \left\{ -\frac{i}{2} H^{\alpha(3)} Q H_{\alpha(3)} - \frac{i}{8} \nabla_\beta H^{\alpha(2)} \nabla^2 \nabla^\gamma H_{\gamma\alpha(2)} + \frac{i}{8} \nabla_\beta H^{\gamma\alpha} \nabla^\rho H_{\rho\delta\alpha} - \frac{i}{4} \Omega^{\beta\gamma} \nabla_\beta H_{\gamma\delta\alpha} \right\}
\]
provides an off-shell formulation for a linearised supergravity multiplet in AdS$_3$. In the flat-superspace limit, it reduces to the linearised supergravity model proposed in [26].

5 Analysis of the results

Let $s > 0$ be a positive integer. For each superspin value, integer $(s)$ or half-integer $(s + \frac{1}{2})$, we have constructed two off-shell formulations which have been called longitudinal and transverse. Now we have to explain this terminology.

Consider a field theory in AdS$_3$ that is described in terms of a real tensor superfield $V_{\alpha(n)}$. We assume the action to have the form

$$S^\parallel[V_{\alpha(n)}] = \int d^{3|2}z \mathcal{E} \left( \mathcal{L}(i^{n+1}\nabla_{\beta}V_{\alpha(n)}) \right).$$

(5.1)

It is natural to call $\nabla_{\beta}V_{\alpha(n)}$ a longitudinal superfield, by analogy with a longitudinal vector field. This theory possesses a dual formulation that is obtained by introducing a first-order action

$$S_{\text{first-order}} = \int d^{3|2}z \mathcal{E} \left\{ \mathcal{L}(\Sigma_{\beta;\alpha(n)}) + i^{n+1}W_{\beta;\alpha(n)}\Sigma_{\beta;\alpha(n)} \right\},$$

(5.2)

where $\Sigma_{\beta;\alpha(n)}$ is unconstrained and the Lagrange multiplier is

$$W_{\beta;\alpha(n)} = i^{n+1}\left( \nabla_{\gamma}\nabla_{\beta} - 4iS_{\delta_{\beta}}^\gamma \right)\Psi_{\gamma;\alpha(n)} , \quad \nabla_{\beta}W_{\beta;\alpha(n)} = 0 ,$$

(5.3)

for some unconstrained prepotential $\Psi_{\gamma;\alpha(n)}$. Varying (5.2) with respect to $\Psi_{\gamma;\alpha(n)}$ gives

$$\nabla_{\beta}\nabla_{\gamma}\Sigma_{\beta;\alpha(n)} - 4iS\Sigma_{\gamma;\alpha(n)} = 0 \quad \Rightarrow \quad \Sigma_{\beta;\alpha(n)} = i^{n+1}\nabla_{\beta}V_{\alpha(n)} ,$$

(5.4)

and then $S_{\text{first-order}}$ reduces to the original action (5.1). On the other hand, we may start from $S_{\text{first-order}}$ and integrate $\Sigma_{\beta;\alpha(n)}$ out. This will lead to a dual action of the form

$$S^\perp[\Psi_{\gamma;\alpha(n)}] = \int d^{3|2}z \mathcal{E}_{\text{dual}}(W_{\beta;\alpha(n)}) .$$

(5.5)

This is a gauge theory since the action is invariant under gauge transformations

$$\delta\Psi_{\gamma;\alpha(n)} = i^{n+1}\nabla_{\gamma}\eta_{\alpha(n)} .$$

(5.6)
The gauge-invariant field strength $W_{\beta;\alpha(n)}$ can be called a transverse superfield, due to the constraint (5.3) it obeys.

It is natural to call the dual formulations (5.1) and (5.5) as longitudinal and transverse, respectively.

Now, let us consider the transverse and longitudinal formulations for the massless superspin-$s$ models, which are given by eqs. (3.36) and (4.24), respectively. These actions depend parametrically on $S$, the curvature of AdS superspace. We denote by $S^\perp_{(s)}[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}]_{FS}$ and $S^\parallel_{(s)}[H_{\alpha(2s)}, V_{\alpha(2s-2)}]_{FS}$ these actions in the limit $S = 0$, which corresponds to a flat superspace. The dynamical systems $S^\perp_{(s)}[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}]_{FS}$ and $S^\parallel_{(s)}[H_{\alpha(2s)}, V_{\alpha(2s-2)}]_{FS}$ prove to be related to each other by the Legendre transformation described above. Thus $S^\perp_{(s)}[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}]_{FS}$ and $S^\parallel_{(s)}[H_{\alpha(2s)}, V_{\alpha(2s-2)}]_{FS}$ are dual formulations of the same theory. This duality does survive if $S$ is non-vanishing.

The same feature characterises the longitudinal and transverse formulations for the massless superspin-$(s + \frac{1}{2})$ multiplet, which are described by the actions (3.32) and (4.28), respectively. The flat-superspace counterparts of these higher-spin models, which we denote by $S^\parallel_{(s+\frac{1}{2})}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}]_{FS}$ and $S^\perp_{(s+\frac{1}{2})}[H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}]_{FS}$, are dual to each other. However, this duality does not survive if we turn on a non-vanishing AdS curvature.

The above discussion can be illustrated by considering the model for linearised gravity in AdS$_3$. It is described by the action

$$S_{\text{gravity}} = \frac{1}{8} \int d^3 x \ e \left\{ h^{\alpha(4)} \nabla_\beta h_{\alpha(4)} \nabla^\beta h_{\alpha(4)} - \nabla_\beta h^{\beta(2)\alpha(2)} \nabla^\gamma h_{\alpha(2)\gamma(2)} + \frac{1}{2} \nabla^\alpha \eta \nabla_\beta \nabla^{\beta(2)} h_{\alpha(2)\beta(2)} - \frac{1}{4} \nabla^\alpha \eta \nabla_\alpha \eta \nabla^{\alpha(2)\beta(2)} + 8 S^2 h^{\alpha(4)} h_{\alpha(4)} + 6 S^2 \eta^2 \right\}, \quad (5.7)$$

which is invariant under gauge transformations

$$\delta_\xi h_{\alpha(4)} = \nabla_{(\alpha_1\alpha_2} \xi_{\alpha_3\alpha_4)}, \quad \delta_\xi \eta = \frac{2}{3} \nabla^{\alpha(2)} \xi_{\alpha(2)}. \quad (5.8)$$

In the flat-space limit, $S = 0$, the model possesses a dual formulation in which the scalar compensator $\eta$ is replaced with a gauge one-form $\hbar$. However, such a duality transformation cannot be lifted to AdS$_3$.

---

9There is another dual realisation in which $h_{\alpha(4)}$ turns into a gauge one-form $h_{\hbar;\alpha(4)}$ with an additional gauge freedom.
6 Non-conformal higher spin supercurrents

In the previous sections, we have shown that there exist two different off-shell formulations for the massless higher-spin $\mathcal{N} = 1$ supermultiplets. Massless half-integer superspin theory can be realised in terms of the dynamical variables (3.30) and (4.26), while the models (3.34) and (4.22) define massless multiplet of integer superspin $s$, with $s > 1$. These models lead to different $\mathcal{N} = 1$ higher-spin supercurrent multiplets. Our aim in this section is to describe the general structure of $\mathcal{N} = 1$ supercurrent multiplets in AdS.

6.1 $\mathcal{N} = 1$ supercurrents: Half-integer superspin case

Our half-integer supermultiplet in the longitudinal formulation (3.30) can be coupled to external sources

$$S_{\text{source}}^{(s+\frac{1}{2})} = \int d^3z E \left\{ iH^{(2s+1)}J_{\alpha(2s+1)} + 4L^{\alpha(2s-2)}S_{\alpha(2s-2)} \right\} . \quad (6.1)$$

The condition that the above action is invariant under the gauge transformations (3.31) gives the conservation equation

$$\nabla^\beta J_{\beta\alpha(2s)} = -\frac{2s}{(2s+1)} \nabla_{(\alpha_1\alpha_2S_{\alpha_3\ldots\alpha_{2s}})} . \quad (6.2)$$

For the transverse theory (4.26) described by the prepotentials $\{H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}\}$, we construct an action functional of the form

$$S_{\text{source}}^{(s+\frac{1}{2})} = \int d^3z E \left\{ iH^{(2s+1)}J_{\alpha(2s+1)} + 2i\Upsilon^{\beta;\alpha(2s-2)}U_{\beta;\alpha(2s-2)} \right\} . \quad (6.3)$$

Requiring that the action is invariant under the gauge transformations (4.27) leads to

$$\nabla^\beta J_{\beta\alpha(2s)} = \frac{2s}{2s+1} \nabla_{(\alpha_1U_{\alpha_2\ldots\alpha_{2s}})} , \quad \nabla^\beta U_{\beta;\alpha(2s-2)} = 0 . \quad (6.4)$$

From the above consideration, it follows that the most general conservation equation in the half-integer superspin case takes the form

$$\nabla^\beta J_{\beta\alpha(2s)} = \frac{2s}{2s+1} \left( \nabla_{(\alpha_1U_{\alpha_2\ldots\alpha_{2s}})} - \nabla_{(\alpha_1\alpha_2S_{\alpha_3\ldots\alpha_{2s}})} \right) , \quad (6.5a)$$

$$\nabla^\beta U_{\beta;\alpha(2s-2)} = 0 . \quad (6.5b)$$
6.2 $\mathcal{N} = 1$ supercurrents: Integer superspin case

In complete analogy with the half-integer superspin case, we couple the prepotentials (4.22) in terms of which the integer superspin $s$ is described, to external sources

$$S^{(s)}_{\text{source}} = \int d^3 z E \left\{ H^{(2s)}_{\alpha} J_{\alpha(2s)} + 2s V^{(2s-2)}_{\alpha} R_{\alpha(2s-2)} \right\} .$$  (6.6)

For such an action to be invariant under the gauge freedom (4.23), the sources must be conserved

$$\nabla^\beta J_{\beta \alpha(2s-1)} = \nabla(\alpha_1 R_{\alpha_2 \cdots \alpha_{2s-1}}) .$$  (6.7)

Next, we turn to the transverse formulation (3.34) characterised by the prepotentials \{ $H^{(2s+1)}_{\alpha(2s+1)}$, $\Psi^{(2s-2)}_{\beta; \alpha(2s-2)}$ \} and construct an action functional

$$S^{(s)}_{\text{source}} = \int d^3 z E \left\{ H^{(2s)}_{\alpha(2s)} + i\Psi^{(2s-2)}_{\beta; \alpha(2s-2)} T_{\beta; \alpha(2s-2)} \right\} .$$  (6.8)

Demanding that the action be invariant under the gauge transformations (3.35), we derive the following conditions

$$\nabla^\beta J_{\beta \alpha(2s-1)} = iT_{\alpha(2s-1)} , \quad \nabla^\beta T_{\beta; \alpha(2s-2)} = 0 .$$  (6.9)

From the above consideration, the most general conservation equation for the multiplet of currents in the integer superspin case is given by

$$\nabla^\beta J_{\beta \alpha(2s-1)} = \nabla(\alpha_1 R_{\alpha_2 \cdots \alpha_{2s-1}} + iT_{\alpha(2s-1)} , \quad \nabla^\beta T_{\beta; \alpha(2s-2)} = 0 .$$  (6.10a)

$$\nabla^\beta T_{\beta; \alpha(2s-2)} = 0 .$$  (6.10b)

6.3 From $\mathcal{N} = 2$ supercurrents to $\mathcal{N} = 1$ supercurrents

In our recent paper [1], we constructed the general conservation equation for the $\mathcal{N} = 2$ higher-spin supercurrent multiplets in (2,0) AdS superspace, which takes the form

$$\mathbf{D}^\beta J_{\beta \alpha(2s-1)} = \mathbf{D}(\alpha_1 (\Psi^{(2s-2)}_{\alpha_2 \cdots \alpha_{2s-1}} + iT_{\alpha(2s-1)})) .$$  (6.11)

Here $J_{\alpha(2s)}$ denotes the higher-spin supercurrent, while the trace supermultiplets $\Psi^{(2s-2)}_{\alpha(2s-2)}$ and $T_{\alpha(2s-2)}$ are covariantly linear. The explicit form of this multiplet of currents was presented by considering simple $\mathcal{N} = 2$ supersymmetric models for a chiral scalar superfield. Unlike in 4D $\mathcal{N} = 1$ supergravity where every supersymmetric matter theory can
be coupled to only one of the off-shell supergravity formulations (either old-minimal or new-minimal), here in the (2,0) AdS case our trace multiplets require both type II and type III compensators to couple to.

The general conservation equation (6.11) naturally gives rise to the \( N = 1 \) higher-spin supercurrent multiplets discussed in the previous subsection. One may show that in the real basis, (6.11) turns into:

\[
\nabla^1 \beta J^\beta \alpha(2s-1) = \nabla^1_{(\alpha_1} Y_{\alpha_2\cdots\alpha_{2s-1})} - \nabla^2_{(\alpha_1} Z_{\alpha_2\cdots\alpha_{2s-1})} , \tag{6.12a}
\]

\[
\nabla^2 \beta J^\beta \alpha(2s-1) = \nabla^1_{(\alpha_1} Z_{\alpha_2\cdots\alpha_{2s-1})} + \nabla^2_{(\alpha_1} Y_{\alpha_2\cdots\alpha_{2s-1})} , \tag{6.12b}
\]

The real linearity constraints on the trace supermultiplets are equivalent to

\[
(\nabla^2)^2 Y_{\alpha(2s-2)} = (\nabla^1)^2 Y_{\alpha(2s-2)} , \quad \nabla^1 \beta \nabla^2 \beta Y_{\alpha(2s-2)} = 0 \tag{6.13a}
\]

\[
(\nabla^2)^2 Z_{\alpha(2s-2)} = (\nabla^1)^2 Z_{\alpha(2s-2)} , \quad \nabla^1 \beta \nabla^2 \beta Z_{\alpha(2s-2)} = 0 . \tag{6.13b}
\]

It follows from (6.12) and (6.13) that \( J_{\alpha(2s)} \) contains two independent real \( N = 1 \) supermultiplets:

\[
J_{\alpha(2s)} := J_{\alpha(2s)}| , \quad J_{\alpha(2s+1)} := i \nabla^2_{(\alpha_1} Y_{\alpha_2\cdots\alpha_{2s+1})} | , \tag{6.14a}
\]

while the independent real \( N = 1 \) components of \( Y_{\alpha(2s-2)} \) and \( Z_{\alpha(2s-2)} \) are defined by

\[
R_{\alpha(2s-2)} := Y_{\alpha(2s-2)}| , \quad U_{\beta;\alpha(2s-2)} := i \nabla^2_{\beta} Y_{\alpha(2s-2)}| , \tag{6.15a}
\]

\[
S_{\alpha(2s-2)} := Z_{\alpha(2s-2)}| , \quad T_{\beta;\alpha(2s-2)} := i \nabla^2_{\beta} Z_{\alpha(2s-2)}| . \tag{6.15b}
\]

Making use of (6.13), one may readily show that

\[
\nabla^\beta U_{\beta;\alpha(2s-2)} = 0 , \tag{6.16a}
\]

\[
\nabla^\beta T_{\beta;\alpha(2s-2)} = 0 . \tag{6.16b}
\]

On the other hand, eq. (6.12) implies that the \( N = 1 \) superfields obey the following conditions

\[
\nabla^\beta J_{\beta\alpha(2s)} = \frac{2s}{2s+1} \left( \nabla_{(\alpha_1} U_{\alpha_2\cdots\alpha_{2s})} - \nabla_{(\alpha_1} S_{\alpha_2\cdots\alpha_{2s})} \right) , \tag{6.17a}
\]

\[
\nabla^\beta J_{\beta\alpha(2s-1)} = \nabla_{(\alpha_1} R_{\alpha_2\cdots\alpha_{2s-1})} + iT_{\alpha(2s-1)} . \tag{6.17b}
\]

Indeed, the right-hand side of eq. (6.17a) coincides with (6.5a). Therefore, eqs. (6.16a) and (6.17a) define the \( N = 1 \) higher-spin current multiplets associated with the massless half-integer superspin formulations (3.30) and (4.26). In a similar way, it can be observed that eqs. (6.16b) and (6.17b) correspond to the \( N = 1 \) higher-spin supercurrents for the two integer superspin models (3.34) and (4.22).
7 Examples of $\mathcal{N} = 1$ higher-spin supercurrents

In this section we give an explicit realisation of the $\mathcal{N} = 1$ multiplet of higher-spin supercurrent introduced earlier.

Consider a massless chiral scalar multiplet in (2,0) AdS superspace with action

$$S = \int d^3x \, d^2\theta \, d^2\bar{\theta} \, E \, \Phi \Phi , \quad \bar{D}_a \Phi = 0 .$$ (7.1)

The chiral superfield is charged under the $R$-symmetry group $U(1)_R$,

$$J\Phi = -r \Phi .$$ (7.2)

This action is a special case of the supersymmetric nonlinear sigma model studied in subsection (2.4) with a vanishing superpotential, $W(\Phi) = 0$. Making use of (2.62), the reduction of the action (7.1) to $\mathcal{N} = 1$ AdS superspace is

$$S = \int d^{3/2}z \, E \left\{ -i\nabla^a \phi \nabla_a \phi + 4r \mathcal{S} \phi \phi \right\} ,$$ (7.3)

where we have denoted $\phi := \Phi \big|$. This action is manifestly $\mathcal{N} = 1$ supersymmetric. It also possesses hidden second supersymmetry and $U(1)_R$ invariance. These transformations are

$$\delta_{\bar{\epsilon}} \phi = i\epsilon^a \nabla_a \phi - i\epsilon r \phi , \quad \delta_{\epsilon} \bar{\phi} = i\epsilon^a \nabla_a \bar{\phi} + i\epsilon r \bar{\phi} ,$$ (7.4)

where $\epsilon^a$ is given in terms of $\epsilon$ according to (2.31a), and the real parameter $\epsilon$ is constrained by (2.31b). It can be shown that on the mass shell it holds that

$$(i\nabla^2 + 4r\mathcal{S})\phi = 0 , \quad (i\nabla^2 + 4r\mathcal{S})\bar{\phi} = 0 .$$ (7.5)

It was shown in [1] that by using the massless equations of motion, $\bar{D}^2 \Phi = 0$, the $\mathcal{N} = 2$ higher-spin supercurrent multiplet associated with the theory (7.1) is described by the conservation equation

$$\bar{D}_{(-1)(2s)} = \bar{D}_{(1)(2s-2)} .$$ (7.6a)

Here the real supercurrent $\bar{J}_{(2s)} = \bar{J}_{(2s)}$ is given by

$$\bar{J}_{(2s)} = \sum_{k=0}^{s} (-1)^k \left\{ \frac{1}{2} \left( \begin{array}{c} 2s \\ 2k + 1 \end{array} \right) \bar{D}_{(2)}^{k} \bar{D}_{(1)} \Phi \bar{D}_{(2)}^{s-k-1} \bar{D}_{(1)} \Phi + \left( \begin{array}{c} 2s \\ 2k \end{array} \right) \bar{D}_{(2)}^{k} \bar{\Phi} \bar{D}_{(2)}^{s-k} \bar{\Phi} \right\} ,$$ (7.6b)
while the trace multiplet $T_{(2s-2)}$ has the form

$$T_{(2s-2)} = 2iS(1 - 2r)(2s + 1)(s + 1) \sum_{k=0}^{s-1} \frac{1}{2s - 2k + 1} (-1)^k \left( \frac{2s}{2k + 1} \right) \times D^{k}_{(2)} \bar{\Phi} D^{s-k-1}_{(2)} \Phi.$$  

(7.6c)

One may check that $T_{(2s-2)}$ is covariantly linear,

$$\bar{\mathcal{D}}^2 T_{(2s-2)} = 0, \quad \mathcal{D}^2 T_{(2s-2)} = 0.$$  

(7.6d)

As is seen from (7.6c), $T_{(2s-2)}$ vanishes for $r = 1/2$, in which case $\Phi$ is an $\mathcal{N} = 2$ superconformal multiplet.

The complex trace multiplet $T_{(2s-2)}$ may split it into its real and imaginary parts:

$$T_{(2s-2)} = Y_{(2s-2)} - iZ_{(2s-2)},$$  

(7.7a)

with

$$Y_{(2s-2)} = 2iS(1 - 2r)(2s + 1)(s + 1) \sum_{k=0}^{s-1} \frac{2k - s + 1}{(2k + 3)(2s - 2k + 1)} \times (-1)^k \left( \frac{2s}{2k + 1} \right) D^{k}_{(2)} \bar{\Phi} D^{s-k-1}_{(2)} \Phi,$$  

(7.7b)

$$Z_{(2s-2)} = -2S(1 - 2r)(2s + 1)(s + 1)(s + 2) \sum_{k=0}^{s-1} \frac{1}{(2k + 3)(2s - 2k + 1)} \times (-1)^k \left( \frac{2s}{2k + 1} \right) D^{k}_{(2)} \bar{\Phi} D^{s-k-1}_{(2)} \Phi.$$  

(7.7c)

Note that we make use of a condensed notation [1] throughout this section. In this notation, we introduce auxiliary real variables $\zeta^\alpha \in \mathbb{R}^2$, such that any tensor superfield $U_{\alpha(m)}$ can be associated with the following field

$$U_{(m)}(\zeta) := \zeta^\alpha_1 \ldots \zeta^\alpha_m U_{\alpha_1 \ldots \alpha_m},$$  

(7.8)

which is a homogeneous polynomial of degree $m$ in $\zeta^\alpha$. Let us introduce operators which increase the degree of homogeneity in $\zeta^\alpha$:

$$\mathcal{D}_{(1)} := \zeta^\alpha \mathcal{D}_\alpha, \quad \bar{\mathcal{D}}_{(1)} := \zeta^\alpha \bar{\mathcal{D}}_\alpha, \quad \mathcal{D}_{(2)} := i\zeta^\alpha \zeta^\beta \mathcal{D}_{\alpha \beta}.$$  

(7.9)

In addition, we also have two operators that decrease the degree of homogeneity in $\zeta^\alpha$

$$\mathcal{D}_{(-1)} := \mathcal{D}_\alpha \frac{\partial}{\partial \zeta^\alpha}, \quad \bar{\mathcal{D}}_{(-1)} := \bar{\mathcal{D}}_\alpha \frac{\partial}{\partial \zeta^\alpha}.$$  

(7.10)
The operators associated with the real spinor covariant derivatives may be defined in a similar way:

\[ \nabla_{(1)}^1 := \zeta^\alpha \nabla_{\alpha}^1, \quad \nabla_{(1)}^2 := \zeta^\beta \nabla_{\alpha}^2, \quad \nabla_{(2)} := i\zeta^\alpha \zeta^\beta \nabla_{\alpha\beta}, \]  
(7.11)

\[ \nabla_{(-1)}^1 := \nabla_{\alpha} \frac{\partial}{\partial \zeta^\alpha}. \]  
(7.12)

In accordance with (6.14), the supercurrent \( J_{(2s)} \) reduces to two different multiplets upon projection to \( \mathcal{N} = 1 \) superspace:

\[ J_{(2s)} := \left| J_{(2s)} \right| \]

\[ = \sum_{k=0}^{s} (-1)^{k+1} \left\{ \left( \frac{2s}{2k+1} \right) \nabla_{(2)}^k \nabla_{(1)} \bar{\varphi} \nabla_{(2)}^{s-k-1} \nabla_{(1)} \varphi \right. \]

\[ \left. - \left( \frac{2s}{2k} \right) \nabla_{(2)}^k \varphi \nabla_{(2)}^{k-s} \right\}, \]  
(7.13a)

\[ J_{(2s+1)} := i \sqrt{2} \left| J_{(2s+1)} \right| = -\frac{1}{\sqrt{2}} \left( D_{(1)} + \bar{D}_{(1)} \right) \left| J_{(2s)} \right|, \]

\[ = (2s+1) \sum_{k=0}^{s} \frac{1}{2s-2k+1} \frac{1}{(-1)^{k+1} \left( \frac{2s}{2k} \right) \left\{ \nabla_{(2)}^k \varphi \nabla_{(2)}^{s-k} \nabla_{(1)} \varphi \right. \}

\[ \left. + (-1)^{s-1} \nabla_{(2)}^k \varphi \nabla_{(2)}^{s-k} \nabla_{(1)} \bar{\varphi} \right\}, \]  
(7.13b)

of which the former corresponds to the integer superspin current and the latter half-integer superspin current.

In the case of half-integer superspin, the conservation equation (6.5) is satisfied provided we impose (7.5):

\[ \nabla_{(-1)} J_{(2s+1)} = \frac{2s}{2s+1} \left( \nabla_{(1)} U_{(2s-1)} + i \nabla_{(2)} S_{(2s-2)} \right), \quad \nabla^\beta U_{\beta; (2s-2)} = 0, \]  
(7.14a)

with

\[ S_{(2s-2)} := \left| Z_{(2s-2)} \right| \]

\[ = -2s(1-2r)(2s+1)(s+1)(s+2) \sum_{k=0}^{s-1} \frac{1}{(2k+3)(2s-2k+1)} \]

\[ \times (-1)^k \left( \frac{2s}{2k+1} \right) \nabla_{(2)}^k \varphi \nabla_{(2)}^{s-k-1} \varphi, \]  
(7.14b)

\[ U_{\beta; (2s-2)} := -\frac{1}{\sqrt{2}} \left( D_{\beta} + \bar{D}_{\beta} \right) \left| Y_{(2s-2)} \right|, \]
\[ = -2iS(1 - 2r)(2s + 1)(s + 1) \sum_{k=0}^{s-1} \frac{2k - s + 1}{(2k + 3)(2s - 2k + 1)}(-1)^k \left( \begin{array}{c} 2s \\ 2k + 1 \end{array} \right) \]

\[
\times \left\{ \nabla^k \tilde{\varphi} \nabla^{s-k-1} \nabla_\beta \varphi + (-1)^s \nabla^k \nabla^{s-k-1} \nabla_\beta \tilde{\varphi} + 2iS(s - k - 1)\zeta_\beta \left( \nabla^k \tilde{\varphi} \nabla^{s-k-2} \nabla_\beta \varphi \right) + (-1)^s \nabla^k \tilde{\varphi} \nabla^{s-k-2} \nabla_\beta \tilde{\varphi} \right\}. \tag{7.14c} \]

It may also be verified that the \( \mathcal{N} = 1 \) supercurrent multiplet for integer superspin obeys the conditions (6.10) on-shell:

\[
\nabla_{(1)} J_{(2s)} = \nabla (2s - 2) + iT_{(2s-1)} , \quad \nabla^\beta T_{\beta; (2s-2)} = 0 . \tag{7.15a} \]

with

\[
R_{(2s-2)} := \left| Y_{(2s-2)} \right| \]

\[
= 2iS(1 - 2r)(2s + 1)(s + 1) \sum_{k=0}^{s-1} \frac{2k - s + 1}{(2k + 3)(2s - 2k + 1)} \left( \begin{array}{c} 2s \\ 2k + 1 \end{array} \right) \]

\[
\times (-1)^k \left( \begin{array}{c} 2s \\ 2k + 1 \end{array} \right) \nabla^k \tilde{\varphi} \nabla^{s-k-1} \varphi , \tag{7.15b} \]

\[
T_{\beta; (2s-2)} := - \frac{1}{\sqrt{2}} (D_\beta + \bar{D}_\beta) Y_{(2s-2)} \right| , \]

\[
= 2S(1 - 2r)(2s + 1)(s + 1)(s + 2) \sum_{k=0}^{s-1} \frac{1}{(2k + 3)(2s - 2k + 1)}(-1)^k \left( \begin{array}{c} 2s \\ 2k + 1 \end{array} \right) \]

\[
\times \left\{ \nabla^k \tilde{\varphi} \nabla^{s-k-1} \nabla_\beta \varphi + (-1)^s \nabla^k \tilde{\varphi} \nabla^{s-k-1} \nabla_\beta \tilde{\varphi} + 2iS(s - k - 1)\zeta_\beta \left( \nabla^k \tilde{\varphi} \nabla^{s-k-2} \nabla_\beta \varphi \right) + (-1)^s \nabla^k \tilde{\varphi} \nabla^{s-k-2} \nabla_\beta \tilde{\varphi} \right\} . \tag{7.15c} \]

The above technique can also be used to construct \( \mathcal{N} = 1 \) higher-spin supercurrents for the Abelian vector multiplets model described by the action (2.66). We will not elaborate on a construction in the present work.

In four dimensions, various aspects of the higher-spin supercurrent multiplets were studied in [29–32] in the \( \mathcal{N} = 1 \) super-Poincaré case, and in [33] for \( \mathcal{N} = 1 \) AdS supersymmetry.
8 Applications and open problems

Let us briefly summarise the main results obtained in this paper. In section 2 we
developed a formalism to reduce every field theory with (2,0) AdS supersymmetry to
$\mathcal{N} = 1$ AdS superspace. In sections 3 and 4 we applied this reduction procedure to
the off-shell massless higher-spin supermultiplets in AdS$_{(3|2,0)}$ constructed in [1]. For each
superspin value, integer ($s$) or half-integer ($s + \frac{1}{2}$), we constructed two off-shell gauge
formulations for massless $\mathcal{N} = 1$ supermultiplets in AdS$_3$. In section 5, we proved that
these two theories, known as the longitudinal and transverse, are dually equivalent only
in the flat superspace limit. In section 6 we formulated the non-conformal higher-spin
supercurrent in $\mathcal{N} = 1$ AdS superspace. In 7 we provided the explicit examples of these
supercurrents for models of a chiral scalar superfield.

There are several interesting applications of the results obtained in this paper. In
particular, the massless higher-spin $\mathcal{N} = 1$ supermultiplets in AdS$_3$, which were derived in
sections 3 and 4, can be used to construct off-shell topologically massive supermultiplets in
AdS$_3$ by extending the approaches advocated in [14,22,26]. Such a massive supermultiplet
is described by a gauge-invariant action being the sum of massless and superconformal
higher-spin actions, following the philosophy of topologically massive theories [34–37].

Given a positive integer $n$, the conformal superspin-$\frac{n}{2}$ action [14,26,38] is

$$S_{SCS}^{(n/2)}[H_{\alpha(n)}] = -\frac{i^n}{2^{(n/2)+1}} \int d^3z E H_{\alpha(n)}^\alpha W_{\alpha(n)}(H),$$  \hspace{1cm} (8.1)

where $W_{\alpha(n)}(H)$ denotes the higher-spin super-Cotton tensor. The latter is a unique
descendant of $H_{\alpha(n)}$ with the properties

$$W_{\alpha(n)}(\delta_\zeta H) = 0,$$  \hspace{1cm} (8.2a)

$$\nabla^\beta W_{\beta\alpha(n-1)} = 0,$$  \hspace{1cm} (8.2b)

where $\delta_\zeta H_{\alpha(n)}$ is the gauge transformation [3,29]. These properties imply the gauge
invariance of (8.1). In a flat superspace, $W_{\alpha(n)}$ has the form [38]

$$S = 0 \quad \implies \quad W_{\alpha_1...\alpha_n} = \left(-\frac{i}{2}\right)^n \nabla^{\beta_1} \nabla_{\alpha_1} \cdots \nabla^{\beta_n} \nabla_{\alpha_n} H_{\beta_1...\beta_n}.$$  \hspace{1cm} (8.3)

The construction of $W_{\alpha(n)}$ in arbitrary conformally flat backgrounds is described in [39].

Given a positive integer $s$, there are two off-shell gauge-invariant formulations for a
topologically massive superspin-$s$ multiplet in AdS$_3$. The corresponding actions are:

$$S^{(s)}_{(\parallel)}[H_{\alpha(2s)}, V_{\alpha(2s-2)}; \mu] = S_{SCS}^{(s)}[H_{\alpha(2s)}] + \mu^{2s-1} S^{(s)}_{(\|)}[H_{\alpha(2s)}, V_{\alpha(2s-2)}],$$  \hspace{1cm} (8.4a)
The dynamical system (8.4a) was introduced in [14], while its flat-superspace counterpart appeared earlier in [26]. The other theory, eq. (8.4b), is a new formulation for massive superspin-$s$ multiplet in AdS$_3$.

In the Minkowski superspace limit, the dynamical systems (8.4a) and (8.4b) are equivalent, since they are related to each other by the superfield Legendre transformation described in section 5. On the mass shell, dynamics can be recast in terms of the gauge-invariant field strength $W_{\alpha(2s)}$ which obeys the equations [26]

\[ D^3W_{\beta_1\ldots\alpha_{2s-1}} = 0, \quad -\frac{i}{2}D^2W_{\alpha(2s)} = m\sigma W_{\alpha(2s)}, \quad \sigma = \pm 1, \quad (8.5) \]

where the mass $m$ and helicity parameter $\sigma$ are determined by $\mu$.

It is an interesting open problem to understand whether the AdS models (8.4a) and (8.4b) lead to equivalent dynamics, modulo a redefinition of the mass parameter $\mu$.

There are two off-shell gauge-invariant formulations for a topologically massive superspin-$(s + \frac{1}{2})$ multiplet in AdS$_3$. The corresponding actions are:

\begin{align*}
S_{(s+\frac{1}{2})}^{\parallel}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}]|\mu| = & \ S_{SCS}^{(s+\frac{1}{2})}[H_{\alpha(2s+1)}] + \mu^{2s-1}S_{(s)}^{\perp}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}] , \quad (8.6a) \\
S_{(s+\frac{1}{2})}^{\perp}[H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}]|\mu| = & \ S_{SCS}^{(s+\frac{1}{2})}[H_{\alpha(2s+1)}] \\
& + \mu^{2s-1}S_{(s+\frac{1}{2})}^{\perp}[H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}] . \quad (8.6b)
\end{align*}

The theory defined by (8.6a) was introduced in [14], while its flat-superspace counterpart appeared earlier in [26]. The other model, eq. (8.6b), is a new formulation for a massive superspin-$(s + \frac{1}{2})$ multiplet in AdS$_3$.

In the Minkowski superspace limit, the dynamical systems (8.6a) and (8.6b) are equivalent, since they are related to each other by the superfield Legendre transformation described in section 5. It is also an interesting open problem to understand whether the models (8.6a) and (8.6b) in AdS$_3$ generate equivalent dynamics.

We now present two off-shell formulations for the massive $\mathcal{N} = 1$ gravitino supermultiplet in AdS$_3$ and analyse the corresponding equations of motion. The equations (8.5) describe the irreducible massive multiplet of superhelicity $\kappa = (s + \frac{1}{4})\sigma$ [40], with the $\mathcal{N} = 1$ superhelicity operator being defined according to [41].

The construction of the models (8.7) and (8.16) is similar to those used to derive the off-shell formulations for massive superspin-1 and superspin-3/2 multiplets in four dimensions [42–52].
of the longitudinal theory (4.25) is described by the action

\[
S_{(1),\mu}^{||} = -\frac{1}{2} \int d^{3}z E \left\{ \frac{i}{2} H^{\alpha \beta} \nabla^{2} H_{\alpha \beta} - \frac{i}{2} \nabla_{\beta} H^{\alpha \beta} \nabla^{\gamma} H_{\gamma \alpha} - V \nabla^{\alpha \beta} H_{\alpha \beta} + \frac{i}{2} V \nabla^{2} V + (\mu + 2S) H^{\alpha \beta} H_{\alpha \beta} - 2(\mu - 2S) V^{2} \right\},
\]  

(8.7)

with \(\mu\) a real mass parameter. The massive gravitino action is thus constructed from the massless one by adding mass-like terms. In the limit \(\mu \to 0\), the action reduces to (4.25).

The equations of motion for the dynamical superfields \(H^{\alpha \beta}\) and \(V\) are

\[
2 \nabla^{\gamma} (\alpha H_{\beta})_{\gamma} - i \nabla^{2} H_{\alpha \beta} - 2 \nabla_{\alpha \beta} V - 4 \mu H_{\alpha \beta} = 0 ,
\]

(8.8a)

\[
\nabla^{\alpha \beta} H_{\alpha \beta} = (i \nabla^{2} + 8S - 4\mu) V .
\]

(8.8b)

Multiplying (8.8a) by \(\nabla^{\alpha \beta}\) and noting that \([\nabla_{\alpha \beta}, \nabla^{2}] = 0\) yields

\[
-i \nabla^{2} \nabla^{\alpha \beta} H_{\alpha \beta} + 4 \Box V - 4 \mu \nabla^{\alpha \beta} H_{\alpha \beta} = 0 .
\]

(8.9)

Substituting (8.8b) into (8.9) leads to

\[
V = 0 .
\]

(8.10)

Now that \(V = 0\) on-shell, eq. (8.8b) turns into

\[
\nabla^{\alpha \beta} H_{\alpha \beta} = 0 ,
\]

(8.11)

while (8.8a) can equivalently be written as

\[
-i \nabla^{\gamma} \nabla_{\alpha} H_{\beta \gamma} - (2\mu + 4S) H_{\alpha \beta} = 0 .
\]

(8.12)

Making use of the identity (A.7b), it immediately follows from (8.12) that

\[
\nabla^{\alpha} H_{\alpha \beta} = 0 ,
\]

(8.13)

and then (8.12) is equivalent to

\[
-\frac{i}{2} \nabla^{2} H_{\alpha \beta} = (\mu + 2S) H_{\alpha \beta} .
\]

(8.14)

Therefore, we have demonstrated that the model (8.7) leads to the following conditions on the mass shell:

\[
V = 0 ,
\]

(8.15a)
\[ \nabla^\alpha H_{\alpha\beta} = 0 \quad \Rightarrow \quad \nabla^{\alpha\beta} H_{\alpha\beta} = 0 , \quad (8.15b) \]
\[ -\frac{i}{2} \nabla^2 H_{\alpha\beta} = (\mu + 2S)H_{\alpha\beta} . \quad (8.15c) \]

Such conditions are required to describe an irreducible on-shell massive gravitino multiplet in 3D \( \mathcal{N} = 1 \) AdS superspace \[40\].

In the transverse formulation \( (3.37) \), the action for a massive gravitino multiplet is defined by
\[
S_{(1),\mu}^\perp = -\frac{1}{2} \int d^3z \ E \left\{ \frac{i}{2} H^{\alpha\beta} \nabla^2 H_{\alpha\beta} - i\nabla_\beta H^{\alpha\beta} \nabla^\gamma H_{\gamma\alpha} - H^{\alpha\beta} \nabla_\alpha \Psi_\beta - \frac{i}{2} \Psi^\alpha \Psi_\alpha \right. \\
+ \left( \mu + 4S \right) H^{\alpha\beta} H_{\alpha\beta} - i(\mu + 2S) \left( \Psi^\alpha \Psi_\alpha + 2\mu \Psi^\alpha \Psi_\alpha \right) \right\} . \quad (8.16)
\]

In the limit \( \mu \to 0 \), the action reduces to \( (3.37) \). One may check that the equations of motion for this model imply that
\[
\Psi_\alpha = 0 , \quad (8.17a) \\
\nabla^\alpha H_{\alpha\beta} = 0 \quad \Rightarrow \quad \nabla^{\alpha\beta} H_{\alpha\beta} = 0 , \quad (8.17b) \\
-\frac{i}{2} \nabla^2 H_{\alpha\beta} = (\mu + 4S)H_{\alpha\beta} . \quad (8.17c)
\]

The actions \( (8.7) \) and \( (8.16) \) can be made into gauge-invariant ones using the Stueckelberg construction.

In the Minkowski superspace limit, the massive models \( (8.7) \) and \( (8.16) \) lead to the identical equations of motion described in terms of \( H_{\alpha\beta} \):
\[
D^\alpha H_{\alpha\beta} = 0 , \quad -\frac{i}{2} D^2 H_{\alpha\beta} = \mu H_{\alpha\beta} . \quad (8.18)
\]

In the AdS case, the equations \( (8.15) \) and \( (8.17) \) lead to equivalent dynamics modulo a redefinition of \( \mu \). It is an interesting open problem to understand whether there exists a duality transformation relating these models.

It should be emphasised that there is an alternative construction of gauge-invariant Lagrangian formulations for massive higher-spin supermultiplets in \( \mathbb{R}^{2,1} \) and AdS\(_3\), which were developed in \[53\,54\]. It is obtained by combining the massive bosonic and fermionic higher-spin actions \[55\,56\], and therefore this construction is intrinsically on-shell. The formulations given in \[53\,56\] are based on the gauge-invariant approaches to the dynamics of massive higher-spin fields, which were advocated by Zinoviev \[57\] and Metsaev \[58\]. It is an interesting open problem to understand whether there exists an off-shell uplift of these models.
All off-shell higher-spin $\mathcal{N} = 2$ supermultiplets in AdS$_3$, both with (2,0) and (1,1) AdS supersymmetry [1,21], are reducible gauge theories (in the terminology of the Batalin-Vilkovisky quantisation [59]), similar to the massless higher-spin supermultiplets in AdS$_4$ [25]. The Lagrangian quantisation of such theories is nontrivial. In the four-dimensional case, the quantisation of the theories proposed in [25] was carried out in [60]. All off-shell higher-spin $\mathcal{N} = 1$ supermultiplets in AdS$_3$, which we have constructed in this paper, are irreducible gauge theories that can be quantised using the Faddeev-Popov procedure [61] as in the non-supersymmetric case, see e.g. [62,63]. This opens the possibility to develop heat kernel techniques for higher-spin theories in AdS$_3^{3/2}$, as an extension of the four-dimensional results [16,64,65].

Acknowledgements:
We are very grateful to Dmitri Sorokin for fruitful discussions and to Darren Grasso for comments on the manuscript. The work of JH is supported by an Australian Government Research Training Program (RTP) Scholarship. The work of SMK is supported in part by the Australian Research Council, project No. DP160103633.

A Notation, conventions and $\mathcal{N} = 1$ AdS identities

We summarise our notation and conventions which follow [5]. The Minkowski metric is $\eta_{ab} = \text{diag}(-1, 1, 1)$. The spinor indices are raised and lowered by the rule

$$
\psi^\alpha = \varepsilon^{\alpha\beta} \psi_\beta , \quad \psi_\alpha = \varepsilon_{\alpha\beta} \psi^\beta .
$$

(A.1)

Here the antisymmetric SL(2, $\mathbb{R}$) invariant tensors $\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}$ and $\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}$ are normalised as $\varepsilon_{12} = -1 , \varepsilon^{12} = 1$.

We make use of real Dirac $\gamma$-matrices, $\gamma_a := ((\gamma_a)_\alpha^\beta)$ defined by

$$
(\gamma_a)_\alpha^\beta := \varepsilon^{\beta\gamma}(\gamma_a)_{\alpha\gamma} = (-i\sigma_2, \sigma_3, \sigma_1) .
$$

(A.2)

They obey the algebra

$$
\gamma_a \gamma_b = \eta_{ab} \mathbb{1} + \varepsilon_{abc} \gamma^c ,
$$

(A.3)

where the Levi-Civita tensor is normalised as $\varepsilon^{012} = -\varepsilon_{012} = 1$. Some useful relations involving $\gamma$-matrices are

$$
(\gamma^a)_{\alpha\beta}(\gamma_a)^{\rho\sigma} = -(\delta^a_\rho \delta^\sigma_{\beta} + \delta^\sigma_\rho \delta^a_{\beta}) ,
$$

(A.4a)
\[ \varepsilon_{abc}(\gamma^b)_{\alpha\beta}(\gamma^c)_{\gamma\delta} = \varepsilon_{\gamma}(\gamma^a)_{\beta\delta} + \varepsilon_{\delta}(\gamma^a)_{\alpha\gamma} ; \quad (A.4b) \]

\[ \text{tr}[\gamma_a \gamma_b \gamma_c \gamma_d] = 2 \eta_{ac} \eta_{bd} - 2 \eta_{ac} \eta_{db} + 2 \eta_{ad} \eta_{bc} . \quad (A.4c) \]

Given a three-vector \( A_a \), it can equivalently be described as a symmetric rank-2 spinor \( A_{\alpha\beta} = A_{\beta\alpha} \),

\[ A_{\alpha\beta} := (\gamma^a)_{\alpha\beta} A_a , \quad A_a = -\frac{1}{2}(\gamma^a)^{\alpha\beta} A_{\alpha\beta} . \quad (A.5) \]

The relationship between the Lorentz generators with two vector indices \( (M_{ab} = -M_{ba}) \), one vector index \( (M_a) \) and two spinor indices \( (M_{\alpha\beta} = M_{\beta\alpha}) \) is as follows: \( M_a = \frac{1}{2} \varepsilon_{abc} M^{bc} \) and \( M_{\alpha\beta} = (\gamma^a)_{\alpha\beta} M_a \). These generators act on a vector \( V_c \) and a spinor \( \Psi_\gamma \) by the rules:

\[ M_{ab} V_c = 2 \eta_{c[a} V_{b]} , \quad M_{\alpha\beta} \Psi_\gamma = \varepsilon_{\gamma(a}(\Psi_{\beta)} . \quad (A.6) \]

We collect some useful identities for \( \mathcal{N} = 1 \) AdS covariant derivatives, which we denote by \( \nabla_A = (\nabla_a, \nabla_\alpha) \). Making use of the (anti)-commutation relation \((2.12a)\) and \((2.12b)\), we obtain the following identities

\[ \nabla_\alpha \nabla_\beta = \frac{1}{2} \varepsilon_{\alpha\beta} \nabla^2 + i \nabla_{\alpha\beta} - 2iS M_{\alpha\beta} , \quad (A.7a) \]

\[ \nabla^2 \nabla_\alpha = 4iS \nabla_\alpha , \quad (A.7b) \]

\[ \nabla^2 \nabla_\alpha = -\nabla_\alpha \nabla^2 + 4iS \nabla_\alpha = 2i \nabla_{\alpha\beta} \nabla^\beta + 2iS \nabla_\alpha - 4iS \nabla^\beta M_{\alpha\beta} , \quad (A.7c) \]

\[ -\frac{1}{4} \nabla^2 \nabla^2 = \Box - 2iS \nabla^2 + 2S \nabla^\alpha \nabla_\alpha M_{\alpha\beta} - 2S^2 M_{\alpha\beta} M_{\alpha\beta} , \quad (A.7d) \]

where \( \nabla^2 = \nabla_\alpha \nabla_\alpha \) and \( \Box = \nabla_\alpha \nabla_\alpha = -\frac{1}{2} \nabla_\alpha \nabla_\alpha \). An important corollary of \((A.7a)\) and \((A.7d)\) is

\[ [\nabla_\alpha \nabla_\beta, \nabla^2] = 0 \implies [\nabla_{\alpha\beta}, \nabla^2] = 0 . \quad (A.8) \]

The left-hand side of \((A.7d)\) can be expressed in terms of the quadratic Casimir operator of the 3D \( \mathcal{N} = 1 \) AdS supergroup \([14]\):

\[ Q = -\frac{1}{4} \nabla^2 \nabla^2 + iS \nabla^2 , \quad [Q, \nabla_A] = 0 . \quad (A.9) \]

We also note the following commutation relation

\[ [(\nabla^\perp)^2, (\nabla^\perp)^2] - 4iS (\nabla^\perp)^2, \nabla^2_{\alpha\beta} = 16S \nabla_{\alpha\beta} \nabla^2_{\gamma\delta} - 16S^2 \nabla^2_{\alpha\beta} - 32S^2 \nabla^2_{\alpha\beta} M_{\alpha\beta} - 32iS^2 \nabla^2_{\alpha\beta} J . \quad (A.10) \]
Given an arbitrary superfield $F$ and its complex conjugate $\bar{F}$, the following relation holds
\[ \nabla_\alpha F = -(-1)^{\epsilon(F)} \nabla_\alpha \bar{F}, \] (A.11)
where $\epsilon(F)$ denotes the Grassmann parity of $F$.

B Component structure of $\mathcal{N} = 1$ higher-spin actions

In this appendix we will discuss the component structure of the two new off-shell $\mathcal{N} = 1$ supersymmetric higher-spin theories: the transverse massless superspin-$s$ multiplet (3.36), and the transverse massless superspin-$(s + \frac{1}{2})$ multiplet (4.28a). For simplicity we will carry out our analysis in flat Minkowski superspace. In accordance with (2.51), the component form of an $\mathcal{N} = 1$ supersymmetric action is computed by the rule
\[ S = \int d^{3/2}z \left| L = \frac{i}{4} \int d^3x \, D^2 L \right|_{\theta = 0}, \quad L = \bar{L}. \] (B.1)

B.1 Massless superspin-$s$ action

Let us first work out the component structure of the massless integer superspin model (3.36). In the flat-superspace limit, the transverse action (3.36) takes the form
\[ S^\perp_s[H^\alpha(2s), \psi; \alpha(2s-2)] = \left( -\frac{1}{2} \right)^s \int d^{3/2}z \left\{ \frac{i}{2} H^\alpha(2s) D^2 H^\alpha(2s) - \frac{i}{s} (2s - 1) W^\beta(2s-2) D^\gamma H^\gamma(2s-2) - \frac{i}{2} (2s - 1) \left( W^\beta(2s-2) W^\gamma(2s-2) + \frac{s - 1}{s} W^\beta(2s-3) W^\gamma(2s-3) \right) \right\}. \] (B.2)

As described in (3.38), it is possible to choose a gauge condition $\psi(\alpha_1; \alpha_2; \ldots; \alpha_{2s-1}) = 0$, such that the above action turns into
\[ S^\perp_s[H^\alpha(2s), \psi; \alpha(2s-2)] = \left( -\frac{1}{2} \right)^s \int d^{3/2}z \left\{ \frac{i}{2} H^\alpha(2s) D^2 H^\alpha(2s) - \frac{i}{s} (2s - 1) \left( W^\beta(2s-2) D^\gamma H^\gamma(2s-2) - \frac{i}{2} \left( \phi(2s-3) \Delta^\varphi(2s-3) - \frac{i}{s} (s - 1)(s - 2)(2s - 3) \frac{s}{2s - 1} \right) \right) \right\}. \] (B.3)
It is invariant under the following gauge transformations

\[ \delta H_{\alpha(2s)} = -\partial_{\alpha_1 \alpha_2} \eta_{\alpha_3 \ldots \alpha_{2s}} , \]  
\[ \delta \varphi_{\alpha(2s-3)} = iD^\beta \eta_{\beta \alpha(2s-3)} , \]  

where the gauge parameter \( \eta_{\alpha(2s-2)} \) is a real unconstrained superfield.

The gauge freedom (B.4) can be used to impose a Wess-Zumino gauge

\[ \varphi_{\alpha(2s-3)} \big| = 0 , \quad D_{(\alpha_1 \varphi_{\alpha_2 \ldots \alpha_{2s-2}})} \big| = 0 . \]  

In order to preserve these gauge conditions, the residual gauge freedom has to be constrained by

\[ D^\beta \eta_{\beta \alpha(2s-3)} \big| = 0 , \quad D^2 \eta_{\alpha(2s-2)} \big| = 2i \partial^\beta (\alpha_1 \eta_{\alpha_2 \ldots \alpha_{2s-2}}) \beta \big| . \]  

These imply that there remain two independent, real components of \( \eta_{\alpha(2s-2)} \):

\[ \xi_{\alpha(2s-2)} := \eta_{\alpha(2s-2)} \big| , \quad \lambda_{\alpha(2s-1)} := iD_{(\alpha_1 \eta_{\alpha_2 \ldots \alpha_{2s-1}})} \big| . \]  

In the gauge (B.5), the independent component fields of \( \varphi_{\alpha(2s-3)} \) can be chosen as

\[ y_{\alpha(2s-4)} := -\frac{2s - 2}{2s - 1} D^\beta \varphi_{\alpha_1 \ldots \alpha_{2s-4}} \big| , \quad y_{\alpha(2s-3)} := \frac{i}{2} D^2 \varphi_{\alpha(2s-3)} \big| . \]  

We define the component fields of \( H_{\alpha(2s)} \) as

\[ h_{\alpha(2s)} := -H_{\alpha(2s)} \big| , \quad \]  
\[ h_{\alpha(2s+1)} := i\frac{s}{2s + 1} D_{(\alpha_1 H_{\alpha_2 \ldots \alpha_{2s+1}})} \big| , \quad y_{\alpha(2s-1)} := iD^\beta H_{\beta \alpha_1 \ldots \alpha_{2s-1}} \big| , \]  
\[ F_{\alpha(2s)} := \frac{i}{4} D^2 H_{\alpha(2s)} \big| . \]  

Applying the reduction rule (B.1) to the \( \mathcal{N} = 1 \) action (B.3), we find that it splits into bosonic and fermionic parts:

\[ S_{\perp(2s)}^\perp[H_{\alpha(2s)}; \Psi_{\beta; \alpha(2s-2)}] = S_{\text{bos}} + S_{\text{ferm}} . \]  

The bosonic action is given by

\[ S_{\text{bos}} = \left( -\frac{1}{2} \right)^s \int d^3 x \left\{ 2(1 - s) F^{\alpha(2s)} F_{\alpha(2s)} + 2s F^{\alpha(2s-1)} \partial^\gamma \beta h_{\alpha(2s-1)\gamma} \right. \]
\[ \left. -\frac{1}{2} (s - 1) h^{\alpha(2s)} \Box h_{\alpha(2s)} - \frac{(2s - 1)(2s - 3)}{2s(s - 1)} y^{\alpha(2s-4)} \Box y_{\alpha(2s-4)} \right\} . \]
\[-\frac{(2s-1)(2s-3)}{4(s-1)}g^{\alpha(2s-4)}\partial^\beta\gamma\partial^\delta\lambda h_{\beta\gamma\delta\lambda}(2s-4)\]
\[-\frac{(s-2)(2s-1)(2s-3)(2s-5)}{16s(s-1)^2}\partial_{\delta\lambda}y^{\delta\lambda}(2s-6)\partial^\beta\gamma y_{\beta\gamma}(2s-6)\right\}.
\tag{B.13}

Integrating out the auxiliary field \(F_{\alpha(2s-2)}\) leads to
\[S_{\text{bos}} = \left(-\frac{1}{2}\right)^s \frac{2s-1}{2s-2} \int d^3 x \left\{ h^{\alpha(2s)} \Box h_{\alpha(2s)} - \frac{s}{2} \partial_{\delta\lambda} h^{\delta\lambda}(2s-2) \partial^\beta\gamma h_{\beta\gamma}(2s-2) \right. \]
\[-\frac{2s-3}{2s} \left[ sy^{\alpha(2s-4)}\partial^\beta\gamma\partial^\delta\lambda h_{\beta\gamma\delta\lambda}(2s-4) + 2y^{\alpha(2s-4)} \Box y_{\alpha(2s-4)} \right] \]
\[+ \frac{(s-2)(2s-5)}{4(s-1)} \partial_{\delta\lambda}y^{\delta\lambda}(2s-6)\partial^\beta\gamma y_{\beta\gamma}(2s-6)\right\}.
\tag{B.14}

This action is invariant under the gauge transformations
\[\delta_\xi h_{\alpha(2s)} = \partial_{(\alpha_1\alpha_2\xi\alpha_{s-2}),}\]
\[\delta_\xi y_{\alpha(2s-4)} = \frac{2s-2}{2s-1} \partial^\beta\gamma \xi_{\beta\gamma(1)\alpha_{s-4}}.
\tag{B.15, B.16}

The gauge transformations for the fields \(h_{\alpha(2s)}\) and \(y_{\alpha(2s-4)}\) can be easily read off from the gauge transformations of the superfields \(H_{\alpha(2s)}\) and \(\varphi_{\alpha(2s-3)}\), respectively. Modulo an overall normalisation factor, \([B.14]\) corresponds to the massless Fronsdal spin-\(s\) action \(S^{(2s)}_F\) described in [14].

The fermionic sector of the component action is described by the real dynamical fields \(h_{\alpha(2s+1)}, y_{\alpha(2s-1)}, y_{\alpha(2s-3)}\), defined modulo gauge transformations of the form
\[\delta_\lambda h_{\alpha(2s+1)} = \partial_{(\alpha_1\alpha_2\lambda_{s-2}\alpha_{s+1})}\]
\[\delta_\lambda y_{\alpha(2s-1)} = \frac{1}{2s+1} \partial^\beta(\alpha_1, \lambda_{s-2}\alpha_{s-1})_{\alpha\beta}\]
\[\delta_\lambda y_{\alpha(2s-3)} = \partial^\beta\gamma \lambda_{\beta\gamma(1)\alpha_{s-3}}.
\tag{B.17, B.18, B.19}

The gauge-invariant action is
\[S_{\text{ferm}} = \left( -\frac{1}{2} \right)^s \frac{i}{2} \int d^3 x \left\{ h^{\alpha(2s)} \partial^\beta h_{\alpha(2s)} + 2(2s-1)y^{\alpha(2s-1)}\partial^\beta h_{\gamma\alpha}(2s-1) \right. \]
\[+ 4(2s-1)y^{\alpha(2s-2)}\partial^\gamma h_{\alpha(2s-2)} + \frac{2}{s} (2s+1)(s-1)y^{\alpha(2s-3)}\partial^\gamma y_{\beta\gamma(2s-3)} \]
\[- \frac{(s-1)(2s-3)}{s(2s-1)} y^{\alpha(2s-4)}\partial^\gamma y_{\alpha(2s-4)} \right\}.
\tag{B.20}

It may be shown that \(S_{\text{ferm}}\) coincides with the Fang-Fronsdal spin-\(s + \frac{1}{2}\) action, \(S_{FF}^{(2s+1)}\) [14].
We have thus proved that at the component level and upon elimination of the auxiliary field, the transverse theory \((\text{B.3})\) is equivalent to a sum of two massless models: the bosonic Fronsdal spin-\(s\) model and the fermionic Fang-Fronsdal spin-(\(s + \frac{1}{2}\)) model.

### B.2 Massless superspin-(\(s + \frac{1}{2}\)) action

We will now elaborate on the component structure of the massless half-integer superspin model in the transverse formulation \((\text{4.28a})\). The theory is described in terms of the real unconstrained prepotentials \(H_{\alpha(2s+1)}\) and \(\Upsilon_{\beta;\alpha(2s-2)}\). In Minkowski superspace, the action \((\text{4.28a})\) simplifies into

\[
S_{(s+\frac{1}{2})}^{\perp} [H_{(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}] = \left( -\frac{1}{2} \right)^s \int d^3z z \left\{ -\frac{i}{2} H_{\alpha(2s+1)} \Box H_{\alpha(2s+1)} 
- \frac{i}{8} D_\beta H^{\beta\alpha(2s)} D^2 D^\gamma H_{\gamma\alpha(2s)} + \frac{i}{8} \partial_\gamma H^{\beta\gamma\alpha(2s-1)} \partial^\delta H_{\beta\delta\alpha(2s-1)} 
- \frac{i}{4} (2s - 1) \Omega^{\beta;\alpha(2s-2)} \partial^\gamma H_{\beta\gamma\delta\alpha(2s-2)} 
- \frac{i}{8} (2s - 1) \left( \Omega^{\beta;\alpha(2s-2)} \Omega_{\beta;\alpha(2s-2)} - 2(s - 1) \Omega_{\beta;\alpha(2s-3)} \Omega^{\gamma\gamma\alpha(2s-3)} \right) \right\}, \quad (\text{B.21})
\]

with the following gauge symmetry

\[
\delta H_{\alpha(2s+1)} = i D_{(\alpha_1 \zeta_{\alpha_2...\alpha_{2s+1}}} , \quad (\text{B.22a}) \\
\delta \Upsilon_{\beta;\alpha(2s-2)} = \frac{i}{2s + 1} \left( D_\gamma \Omega^{\beta;\alpha(2s-2)} + (2s + 1) D_\beta \eta_{\alpha(2s-2)} \right) , \quad (\text{B.22b})
\]

The action \((\text{B.21})\) involves the real field strength \(\Omega_{\beta;\alpha(2s-2)}\)

\[
\Omega_{\beta;\alpha(2s-2)} = -i D^\gamma D_\beta \Upsilon_{\gamma;\alpha(2s-2)} , \quad D^\beta \Omega_{\beta;\alpha(2s-2)} = 0 . \quad (\text{B.23})
\]

The gauge transformations \((\text{B.22})\) allow us to impose a Wess-Zumino gauge on the prepotentials:

\[
H_{\alpha(2s+1)}| = 0 , \quad D^\beta H_{\beta\alpha_1...\alpha_{2s}} = 0 , \quad \Upsilon_{\beta;\alpha(2s-2)}| = 0 , \quad D^\beta \Upsilon_{\beta;\alpha(2s-2)}| = 0 . \quad (\text{B.24})
\]

The residual gauge symmetry preserving the gauge conditions \((\text{B.24})\) is characterised by

\[
D_{(\alpha_1 \zeta_{\alpha_2...\alpha_{2s+1}}} | = 0 , \quad D^2 \zeta_{\alpha(2s)}| = -\frac{2s}{s + 1} \partial^\beta (\alpha_1 \zeta_{\alpha_2...\alpha_{2s}})\beta , \quad (\text{B.25a}) \\
D_\beta \eta_{\alpha(2s-2)}| = D_{(\beta \eta_{\alpha(2s-2)}}| = -\frac{1}{2s + 1} D^\gamma \zeta_{\gamma\beta\alpha(2s-2)}| , \quad (\text{B.25b})
\]

47
\[ D^2 \eta_{\alpha(2s-2)} = -\frac{i}{2s+1} \partial^\beta \zeta_{\beta \gamma \alpha(2s-2)} . \] 

As a result, there are three independent, real gauge parameters at the component level, which we define as

\[ \xi_{\alpha(2s)} := \zeta_{\alpha(2s)} , \quad \lambda_{\alpha(2s-1)} := -i \frac{s}{2s+1} D^\beta \zeta_{\beta \alpha(2s-1)} , \quad \rho_{\alpha(2s-2)} := \eta_{\alpha(2s-2)} . \] (B.26)

Let us now represent the prepotential \( \Upsilon_{\beta; \alpha(2s-2)} \) in terms of its irreducible components,

\[ \Upsilon_{\beta; \alpha(2s-2)} = \Upsilon_{\beta \alpha_1 ... \alpha_{2s-2}} + \sum_{k=1}^{2s-2} \varepsilon_{\beta \alpha_k} Z_{\alpha_1 ... \alpha_k ... \alpha_{2s-2}} , \] (B.27)

where we have introduced the two irreducible components of \( \Upsilon_{\beta; \alpha(2s-2)} \) by the rule

\[ Y_{\beta \alpha_1 ... \alpha_{2s-2}} := \Upsilon_{(\beta; \alpha_1 ... \alpha_{2s-2})} \quad \text{and} \quad Z_{\alpha_1 ... \alpha_{2s-3}} := \frac{1}{2s-1} \Upsilon_{\beta; \alpha_1 ... \alpha_{2s-3}} . \] (B.28)

The next step is to determine the remaining independent component fields of \( H_{\alpha(2s+1)} \) and \( \Upsilon_{\beta; \alpha(2s-2)} \) in the Wess-Zumino gauge (B.24).

In the bosonic sector, we have the following set of fields:

\[ h_{\alpha(2s+2)} := -D_{(\alpha_1 H_{\alpha_2 ... \alpha_{2s+2})} \quad \text{and} \quad \Upsilon_{\beta; \alpha(2s-2)} \] (B.29a)

\[ y_{\alpha(2s)} := D_{(\alpha Y_{\alpha_2 ... \alpha_{2s}}) \quad \text{and} \quad \Upsilon_{\beta; \alpha(2s-2)} \] (B.29b)

\[ z_{\alpha(2s-2)} := -\frac{1}{s} (2s-1) D_{(\alpha Z_{\alpha_2 ... \alpha_{2s-2})} \quad \text{and} \quad \Upsilon_{\beta; \alpha(2s-2)} \] (B.29c)

\[ z_{\alpha(2s-4)} := -(s-1) D^\beta Z_{\beta \alpha(2s-4)} \quad \text{and} \quad \Upsilon_{\beta; \alpha(2s-2)} \] (B.29d)

Reduction of the action (B.21) to components leads to the following bosonic action:

\[ S_{\text{bos}} = \left( -\frac{1}{2} \right)^s \int d^3 x \left\{ -\frac{1}{4} h_{\alpha(2s+2)} D_{(\alpha \lambda \alpha(2s)} \partial^\beta \zeta_{\beta \gamma \alpha(2s)} + \frac{3}{16} \partial_{\delta \lambda} h_{\delta \lambda \alpha(2s)} \partial^\beta \zeta_{\beta \gamma \alpha(2s)} \right. \]

\[ + \frac{1}{4} (2s-1) \partial_{\delta \lambda} h_{\delta \lambda \alpha(2s)} \partial^\beta (\alpha_1 y_{\alpha_2 ... \alpha_{2s}}) + \frac{1}{4} (2s-1) (s-1) z_{\alpha(2s-2)} \partial^\beta \zeta_{\beta \gamma \alpha(2s-2)} \]

\[ - \frac{1}{4} (2s-1) y_{\alpha(2s)} D_{(\alpha \lambda \alpha(2s)} \partial^\beta \zeta_{\beta \gamma \alpha(2s-2)} \]

\[ - (s-1) (2s-1) z_{\alpha(2s)} D_{(\alpha \lambda \alpha(2s)} \partial^\beta \zeta_{\beta \gamma \alpha(2s-2)} \]

\[ - \frac{1}{4} (s-1) (s+2) (2s-1) (2s-3) \partial_{\delta \lambda} z_{\delta \lambda \alpha(2s-4)} \partial^\beta \zeta_{\beta \gamma \alpha(2s-4)} \]

\[ + (s-1) (2s-1) \partial_{\beta \gamma} y_{\beta \gamma \alpha(2s-2)} D_{(\alpha_1 z_{\alpha_2 ... \alpha_{2s-2})} \delta} \]

\[ - \frac{s}{4} (2s-3) (4s^2 - 12s + 11) z_{\alpha(2s-4)} D_{(\alpha \lambda \alpha(2s-4)} \partial^\beta \zeta_{\beta \gamma \alpha(2s-4)} \]

\[ \right\} . \]
\[\begin{align*}
&\frac{3s}{8(s-1)(2s-1)}(s-2)(2s-3)(2s-5)\partial_{\delta\lambda}z^{\delta\lambda\alpha(2s-6)}\partial^{\beta\gamma}z^{\beta\gamma\alpha(2s-6)} \\
&+\frac{1}{4}(s+1)(2s-3)x^{\alpha(2s-4)}\partial^{\beta\gamma}\partial_{\delta\lambda}y_{\beta\gamma\delta\lambda(2s-4)} \\
&+\frac{1}{2}(s-2)(2s+1)(2s-3)\partial_{\beta\gamma}z^{\beta\gamma\alpha(2s-4)}\partial^{\delta}_{(\alpha_{1}z_{\alpha_{2}...\alpha_{2s-4}})}
\end{align*}\], \((B.30)\)

which proves to be invariant under gauge transformations of the form

\[\begin{align*}
\delta_{\xi}h_{\alpha(2s+2)} &= \partial_{(\alpha_{1}\alpha_{2}\xi_{\alpha_{3}...\alpha_{2s+2}})} \\
\delta_{\xi,\rho}y_{\alpha(2s)} &= -\frac{1}{s+1}\partial^{\beta}_{(\alpha_{1}\xi_{\alpha_{2}...\alpha_{2s}})}\beta - \partial_{(\alpha_{1}\alpha_{2}\rho_{\alpha_{3}...\alpha_{2s}})} \\
\delta_{\xi,\rho}z_{\alpha(2s-2)} &= \frac{1}{2s(2s+1)}\partial^{\beta\gamma}\xi_{\beta\gamma\alpha(2s-2)} + \frac{1}{s}\partial^{\beta}_{(\alpha_{1}\rho_{\alpha_{2}...\alpha_{2s-2}})}\beta \\
\delta_{\rho}z_{\alpha(2s-4)} &= \partial^{\beta\gamma}\rho_{\beta\gamma\alpha(2s-4)}
\end{align*}\], \((B.31a-d)\)

Let us consider the fermionic sector. We find that the independent fermionic fields are:

\[\begin{align*}
h_{\alpha(2s+1)} &:= \frac{i}{4}D^{2}H_{\alpha(2s+1)} \\
y_{\alpha(2s-1)} &:= \frac{i}{8}D^{2}Y_{\alpha(2s-1)} \\
y_{\alpha(2s-3)} &:= \frac{i}{2}(2s-1)D^{2}Z_{\alpha(2s-3)}
\end{align*}\], \((B.32a-c)\)

and their gauge transformation laws are given by

\[\begin{align*}
\delta_{\lambda}h_{\alpha(2s+1)} &= \partial_{(\alpha_{1}\alpha_{2}\lambda_{\alpha_{3}...\alpha_{2s+2}})} \\
\delta_{\lambda}y_{\alpha(2s-1)} &= \frac{1}{2s+1}\partial^{\beta}_{(\alpha_{1}\lambda_{\alpha_{2}...\alpha_{2s-1}})}\beta \\
\delta_{\lambda}y_{\alpha(2s-3)} &= \partial^{\beta\gamma}_{\lambda_{\beta\gamma\alpha(2s-3)}}
\end{align*}\], \((B.33a-c)\)

The above fermionic fields correspond to the dynamical variables of the Fang-Fronsdal spin-\((s+\frac{1}{2})\) model. As follows from \((B.33a), (B.33b)\) and \((B.33c)\), their gauge freedom is equivalent to that of the massless spin-\((s+\frac{1}{2})\) gauge field. Indeed, direct calculations of the component action give the standard massless gauge-invariant spin-\((s+\frac{1}{2})\) action \(S_{FF}^{(2s+1)}\).

The component structure of the obtained supermultiplets is a three-dimensional counterpart of so-called (reducible) higher-spin triplet systems. In AdS\(_{D}\) an action for bosonic higher-spin triplets was constructed in [66] and for fermionic triplets in [67, 68]. Our superfield construction provides a manifestly off-shell supersymmetric generalisation of these systems. It might be of interest to extend it to AdS\(_{4}\).
References

[1] J. Hutomo and S. M. Kuzenko, “Higher spin supermultiplets in three dimensions: (2,0) AdS supersymmetry,” Phys. Lett. B 787, 175 (2018) [arXiv:1809.00802 [hep-th]].

[2] A. Achúcarro and P. K. Townsend, “A Chern-Simons action for three-dimensional anti-de Sitter supergravity theories,” Phys. Lett. B 180, 89 (1986).

[3] S. M. Kuzenko, U. Lindstrøm and G. Tartaglino-Mazzucchelli, “Three-dimensional (p,q) AdS superspaces and matter couplings,” JHEP 1208, 024 (2012) [arXiv:1205.4622 [hep-th]].

[4] P. S. Howe, J. M. Izquierdo, G. Papadopoulos and P. K. Townsend, “New supergravities with central charges and Killing spinors in 2+1 dimensions,” Nucl. Phys. B 467, 183 (1996) [arXiv:hep-th/9505032].

[5] S. M. Kuzenko, U. Lindstrøm and G. Tartaglino-Mazzucchelli, “Off-shell supergravity-matter couplings in three dimensions,” JHEP 1103, 120 (2011) [arXiv:1101.4013 [hep-th]].

[6] D. Butter, S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Nonlinear sigma models with AdS supersymmetry in three dimensions,” JHEP 1302, 121 (2013) [arXiv:1210.5906 [hep-th]].

[7] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Three-dimensional N=2 (AdS) supergravity and associated supercurrents,” JHEP 1112, 052 (2011) [arXiv:1109.0496 [hep-th]].

[8] D. Butter and S. M. Kuzenko, “N=2 supersymmetric sigma-models in AdS,” Phys. Lett. B 703, 620 (2011) [arXiv:1105.3111 [hep-th]].

[9] D. Butter and S. M. Kuzenko, “The structure of N=2 supersymmetric nonlinear σ-models in AdS_4,” JHEP 1111, 080 (2011) [arXiv:1108.5290 [hep-th]].

[10] U. Lindstrøm and M. Roček, “Scalar tensor duality and N = 1, 2 nonlinear sigma models,” Nucl. Phys. B 222, 285 (1983).

[11] C. M. Hull, A. Karlhede, U. Lindstrøm and M. Roček, “Nonlinear sigma models and their gauging in and out of superspace,” Nucl. Phys. B 266, 1 (1986).

[12] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Field theory in 4D N=2 conformally flat superspace,” JHEP 0810, 001 (2008) [arXiv:0807.3368 [hep-th]].

[13] D. Butter, S. M. Kuzenko, U. Lindstrom and G. Tartaglino-Mazzucchelli, “Extended supersymmetric sigma models in AdS_4 from projective superspace,” JHEP 1205, 138 (2012) [arXiv:1203.5001 [hep-th]].

[14] S. M. Kuzenko and M. Ponds, “Topologically massive higher spin gauge theories,” JHEP 1810, 160 (2018) [arXiv:1806.06643 [hep-th]].

[15] A. A. Nizami, T. Sharma and V. Umesh, “Superspace formulation and correlation functions of 3d superconformal field theories,” JHEP 1407, 022 (2014) [arXiv:1308.4778 [hep-th]].

[16] I. L. Buchbinder and S. M. Kuzenko, Ideas and Methods of Supersymmetry and Supergravity or a Walk Through Superspace, IOP, Bristol, 1995 (Revised Edition: 1998).
[17] S. M. Kuzenko, “Supersymmetric spacetimes from curved superspace,” PoS CORFU 2014, 140 (2015) [arXiv:1504.08114 [hep-th]].

[18] S. M. Kuzenko, U. Lindström, M. Roček, I. Sachs and G. Tartaglino-Mazzucchelli, “Three-dimensional N=2 supergravity theories: From superspace to components,” Phys. Rev. D 89, 085028 (2014) [arXiv:1312.4267 [hep-th]].

[19] E. E. Donets, A. Pashnev, J. J. Rosales and M. M. Tsulaia, “N=4 supersymmetric multidimensional quantum mechanics, partial SUSY breaking and superconformal quantum mechanics,” Phys. Rev. D 61, 043512 (2000) [hep-th/9907224].

[20] E. E. Donets, A. Pashnev, V. O. Rivelles, D. P. Sorokin and M. Tsulaia, “N=4 superconformal mechanics and the potential structure of AdS spaces,” Phys. Lett. B 484, 337 (2000) [hep-th/0004019].

[21] J. Hutomo, S. M. Kuzenko and D. Ogburn, “N= 2 supersymmetric higher spin gauge theories and current multiplets in three dimensions,” arXiv:1807.09098 [hep-th].

[22] S. M. Kuzenko and D. X. Ogburn, “Off-shell higher spin N=2 supermultiplets in three dimensions,” Phys. Rev. D 94, no. 10, 106010 (2016) [arXiv:1603.04668 [hep-th]].

[23] S. M. Kuzenko, V. V. Postnikov and A. G. Sibiryakov, “Massless gauge superfields of higher half-integer superspins,” JETP Lett. 57, 534 (1993) [Pisma Zh. Eksp. Teor. Fiz. 57, 521 (1993)].

[24] S. M. Kuzenko and A. G. Sibiryakov, “Massless gauge superfields of higher integer superspins,” JETP Lett. 57, 539 (1993) [Pisma Zh. Eksp. Teor. Fiz. 57, 526 (1993)].

[25] S. M. Kuzenko and A. G. Sibiryakov, “Free massless higher-superspin superfields on the anti-de Sitter superspace” Phys. Atom. Nucl. 57, 1257 (1994) [Yad. Fiz. 57, 1326 (1994)] [arXiv:1112.4612 [hep-th]].

[26] S. M. Kuzenko and M. Tsulaia, “Off-shell massive N=1 supermultiplets in three dimensions,” Nucl. Phys. B 914, 160 (2017) [arXiv:1609.06910 [hep-th]].

[27] S. J. Gates Jr., M. T. Grisaru, M. Roček and W. Siegel, Superspace, or One Thousand and One Lessons in Supersymmetry, Benjamin/Cummings (Reading, MA), 1983, [hep-th/0108200].

[28] W. Siegel, “Unextended superfields in extended supersymmetry,” Nucl. Phys. B 156, 135 (1979).

[29] S. M. Kuzenko, R. Manvelyan and S. Theisen, “Off-shell superconformal higher spin multiplets in four dimensions,” JHEP 1707, 034 (2017) [arXiv:1701.00682 [hep-th]].

[30] I. L. Buchbinder, S. J. Gates Jr. and K. Koutrolikos, “Higher spin superfield interactions with the chiral supermultiplet: conserved supercurrents and cubic vertices,” Universe 4, no. 1, 6 (2018) [arXiv:1708.06262 [hep-th]].

[31] J. Hutomo and S. M. Kuzenko, “Non-conformal higher spin supercurrents,” Phys. Lett. B 778, 242 (2018) [arXiv:1710.10837 [hep-th]].

[32] K. Koutrolikos, P. Kočić and R. von Unge, “Higher spin superfield interactions with complex linear supermultiplet: conserved supercurrents and cubic vertices,” JHEP 1803, 119 (2018) [arXiv:1712.05150 [hep-th]].
[33] E. I. Buchbinder, J. Hutomo and S. M. Kuzenko, “Higher spin supercurrents in anti-de Sitter space,” JHEP 1809, 027 (2018) [arXiv:1805.08055 [hep-th]].

[34] W. Siegel, “Unextended superfields in extended supersymmetry,” Nucl. Phys. B 156, 135 (1979).

[35] J. F. Schonfeld, “A mass term for three-dimensional gauge fields,” Nucl. Phys. B 185, 157 (1981).

[36] S. Deser, R. Jackiw and S. Templeton, “Three-dimensional massive gauge theories,” Phys. Rev. Lett. 48, 975 (1982).

[37] S. Deser, R. Jackiw and S. Templeton, “Topologically massive gauge theories,” Annals Phys. 140, 372 (1982) [Erratum-ibid. 185, 406 (1988)].

[38] S. M. Kuzenko, “Higher spin super-Cotton tensors and generalisations of the linear-chiral duality in three dimensions,” Phys. Lett. B 763, 308 (2016) [arXiv:1606.08624 [hep-th]].

[39] S. M. Kuzenko and M. Ponds, “Conformal geometry and (super)conformal higher-spin gauge theories.” [arXiv:1902.08010 [hep-th]].

[40] S. M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, “Higher derivative couplings and massive supergravity in three dimensions,” JHEP 1509, 081 (2015) [arXiv:1506.09063 [hep-th]].

[41] L. Mezincescu and P. K. Townsend, “Quantum 3D superstrings,” Phys. Rev. D 84, 106006 (2011) [arXiv:1106.1374 [hep-th]].

[42] V. I. Ogievetsky and E. Sokatchev, “Superfield equations of motion,” J. Phys. A 10, 2021 (1977).

[43] N. Berkovits and M. M. Leite, “First massive state of the superstring in superspace,” Phys. Lett. B 415, 144 (1997) [hep-th/9709148].

[44] N. Berkovits and M. M. Leite, “Superspace action for the first massive states of the superstring,” Phys. Lett. B 454, 38 (1999) [hep-th/9812153].

[45] R. Altendorfer and J. Bagger, “Dual supersymmetry algebras from partial supersymmetry breaking,” Phys. Lett. B 460, 127 (1999) [hep-th/9904213].

[46] R. Altendorfer and J. Bagger, “Dual anti-de Sitter superalgebras from partial supersymmetry breaking,” Phys. Rev. D 61, 104004 (2000) [hep-th/9908084].

[47] I. L. Buchbinder, S. J. Gates Jr., W. D. Linch and J. Phillips, “New 4D, N = 1 superfield theory: Model of free massive superspin-3/2 multiplet,” Phys. Lett. B 535, 280 (2002) [hep-th/0201096].

[48] I. L. Buchbinder, S. J. Gates Jr., W. D. Linch and J. Phillips, “Dynamical superfield theory of free massive superspin-1 multiplet,” Phys. Lett. B 549, 229 (2002) [hep-th/0207243].

[49] T. Gregoire, M. D. Schwartz and Y. Shadini, “Massive supergravity and deconstruction,” JHEP 0407, 029 (2004) [hep-th/0403224].

[50] I. L. Buchbinder, S. J. Gates Jr., S. M. Kuzenko and J. Phillips, “Massive 4D, N = 1 superspin 1 and 3/2 multiplets and dualities,” JHEP 0502, 056 (2005) [hep-th/0501199].

[51] S. J. Gates Jr. and S. M. Kuzenko, “4D, N = 1 higher spin gauge superfields and quantized twistors,” JHEP 0510, 008 (2005) [hep-th/0506255].
[52] S. J. Gates Jr., S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “New massive supergravity multiplets,” JHEP 0702, 052 (2007) [hep-th/0610333].

[53] I. L. Buchbinder, T. V. Snegirev and Y. M. Zinoviev, “Lagrangian formulation of the massive higher spin supermultiplets in three dimensional space-time,” JHEP 1510, 148 (2015) [arXiv:1508.02829 [hep-th]].

[54] I. L. Buchbinder, T. V. Snegirev and Y. M. Zinoviev, “Lagrangian description of massive higher spin supermultiplets in AdS$_3$ space,” JHEP 1708, 021 (2017) [arXiv:1705.06163 [hep-th]].

[55] I. L. Buchbinder, T. V. Snegirev and Y. M. Zinoviev, “Gauge invariant Lagrangian formulation of massive higher spin fields in (A)dS space,” Phys. Lett. B 716, 243 (2012) [arXiv:1207.1213 [hep-th]].

[56] I. L. Buchbinder, T. V. Snegirev and Y. M. Zinoviev, “Frame-like gauge invariant Lagrangian formulation of massive fermionic higher spin fields in AdS$_3$ space,” Phys. Lett. B 738, 258 (2014) [arXiv:1407.3918 [hep-th]].

[57] Y. M. Zinoviev, “On massive high spin particles in AdS,” hep-th/0108192.

[58] R. R. Metsaev, “Gauge invariant formulation of massive totally symmetric fermionic fields in (A)dS space,” Phys. Lett. B 643, 205 (2006) [hep-th/0609029].

[59] I. A. Batalin and G. A. Vilkovisky, “Quantization of gauge theories with linearly dependent generators,” Phys. Rev. D28, 2567 (1983).

[60] I. L. Buchbinder, S. M. Kuzenko and A. G. Sibiryakov, “Quantization of higher spin superfields in the anti-de Sitter superspace,” Phys. Lett. B 352, 29 (1995) [hep-th/9502148].

[61] L. D. Faddeev and V. N. Popov, “Feynman diagrams for the Yang-Mills field,” Phys. Lett. B 25, 29 (1967).

[62] C. Fronsdal and H. Hata, “Quantization of massless fields with arbitrary spin,” Nucl. Phys. B 162, 487 (1980).

[63] M. R. Gaberdiel, R. Gopakumar and A. Saha, “Quantum W-symmetry in AdS$_3$,” JHEP 1102, 004 (2011) [arXiv:1009.6087 [hep-th]].

[64] I. N. McArthur, “Super b(4) coefficients,” Phys. Lett. B 128, 194 (1983); “Super b(4) coefficients in supergravity,” Class. Quant. Grav. 1, 245 (1984).

[65] I. L. Buchbinder and S. M. Kuzenko, “Matter superfields in external supergravity: Green functions, effective action and superconformal anomalies,” Nucl. Phys. B 274, 653 (1986).

[66] A. Sagnotti and M. Tsulaia, “On higher spins and the tensionless limit of string theory,” Nucl. Phys. B 682, 83 (2004) [hep-th/0311257].

[67] D. P. Sorokin and M. A. Vasiliev, “Reducible higher-spin multiplets in flat and AdS spaces and their geometric frame-like formulation,” Nucl. Phys. B 809, 110 (2009) [arXiv:0807.0206 [hep-th]].

[68] A. Agugliaro, F. Azzurli and D. Sorokin, “Fermionic higher-spin triplets in AdS,” Nucl. Phys. B 907, 633 (2016) [arXiv:1603.02251 [hep-th]].