Tripartite entanglement detection through tripartite quantum steering in one-sided and two-sided device-independent scenarios

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(Dated: March 1, 2018)

In the present work, we study tripartite quantum steering of quantum correlations arising from two local dichotomic measurements on each side in the two types of partially device-independent scenarios: 1-sided device-independent scenario where one of the parties performs trusted measurements while the other two parties perform trusted measurements and 2-sided device-independent scenario where one of the parties performs trusted measurements while the other two parties perform untrusted measurements. We demonstrate that tripartite steering in the 2-sided device-independent scenario is weaker than tripartite steering in the 1-sided device-independent scenario by using two families of quantum correlations. That is these two families of quantum correlations in the 2-sided device-independent framework detect tripartite entanglement through tripartite steering for a larger region than that in the 1-sided device-independent framework. It is shown that tripartite steering in the 2-sided device-independent scenario implies the presence of genuine tripartite entanglement of $2 \times 2 \times 2$ quantum system, even if the correlation does not exhibit genuine nonlocality or genuine steering.

PACS numbers: 03.65.Ud, 03.67.Mn, 03.65.Ta

I. INTRODUCTION

Multipartite entanglement is a resource for quantum information and computation when quantum networks are considered. Therefore, detecting the presence of multipartite entanglement in quantum networks is an important problem in quantum information science. In particular, a genuinely multipartite entangled state (which is not separable with respect to any partitions) [1] is important not only for quantum foundational research but also in various quantum information processing tasks, for example, in the context of extreme spin squeezing [2], high sensitive metrology tasks [3, 4]. Generation and detection of this kind of resource state is found to be difficult as the detection process deals with tomography and evaluation via constructing entanglement witness which require precise experimental control over the system subjected to measurements. But there is an alternative way to certify the presence of entanglement by observing the violation of Bell inequality [5] as entanglement is necessary ingredient to observe the violation. Motivated by this fact, a number of multipartite Bell type inequalities [6–10, 12] have been proposed to detect the genuine multipartite entanglement. To be specific, if the value of any Bell expression, in a Bell experiment, exceeds the value of the same expression obtained due to measurements on biseparable quantum states, then the presence of genuine entanglement is guaranteed. This kind of research was first initiated in [6, 7] but it took a shape by Bancal et. al. [10] where they have constructed device-independent entanglement witness (DIEW) of genuine multipartite entanglement for such Bell expressions.

The concept of quantum steering was first pointed out by Schrodinger [13] in the context of Einstein-Podolsky-Rosen paradox (EPR) [11], which has no classical analogue. Quantum steering as pointed out by Schrodinger occurs when one of the two spatially separated observers prepares genuinely different ensembles of quantum states for the other distant observer by performing suitable quantum measurements on her/his side. Wiseman et. al. [14] gave the formal definition of quantum steering from the foundational as well as quantum information perspective. Quantum steering is certified by the violation of steering inequalities. A number of steering inequalities have been proposed to observe steering [15]. Violation of such steering inequalities certify the presence of entanglement in a one-sided device-independent way.

In Refs. [17, 18], the notion of steering has been generalized for multipartite scenarios and multipartite steering inequalities have been derived to detect multipartite entanglement in asymmetric networks where some of the parties’ measurements are trusted while the other parties’ measurements are uncharacterized. These studies did not examine genuine multipartite steering, in which the nonlocality, in the form of steering, is necessarily shared among all observers. Genuine multipartite steering has been proposed in [19, 20]. In Refs. [21, 22], genuine tripartite steering inequalities have been derived to detect genuine tripartite entanglement in a partially device-independent way. Characterization of multipartite quantum steering through semidefinite programming has also been performed recently [23].

In the present work, we study tripartite steering (which is
analogous to standard Bell nonlocality) of quantum correlations arising from two local measurements on each side in the two types of partially device-independent scenarios: 1-sided device-independent scenario where one of the parties performs trusted measurements while the other two parties perform trusted measurements and 2-sided device-independent scenario where one of the parties performs trusted measurements while the other two parties perform untrusted measurements.

In the 1-sided device-independent framework, we study tripartite steering of two families of quantum correlations in the context of the following classical simulation scenarios: One of the parties share a hidden variable with the other two parties who share a two-qubit system for each value of the hidden variable and perform incompatible measurements that demonstrate Bell nonlocality [16] in one of the types or perform incompatible measurements that demonstrate EPR steering without Bell nonlocality [17, 24] in the other type. These two families of quantum correlations are called Svetlichny family and Mermin family as they violate a Svetlichny inequality and a Mermin inequality, respectively, in certain range. We demonstrate in which range these two families detect tripartite and genuine tripartite steering in the context of above types of classical simulation scenarios, respectively.

We also explore in which range the Svetlichny family and Mermin family detect tripartite steering and genuine tripartite steering in the 2-sided device-independent framework.

Our study demonstrates that tripartite steering in the 2-sided device-independent framework is weaker than tripartite steering in the 1-sided device-independent framework. In other words, tripartite steering in the context of 2-sided device-independent framework detect tripartite entanglement for a larger region than that in the context of 1-sided device-independent framework. We demonstrate that tripartite steering in the 2-sided device-independent scenario implies the presence of genuine tripartite entanglement of $2 \times 2 \times 2$ quantum system, even if the correlation does not exhibit genuine nonlocality or genuine steering.

The plan of the paper is as follows. In Sections II and III the fundamental ideas of tripartite nonlocality and that of tripartite EPR steering in 1-sided device-independent scenario as well as in 2-sided device-independent scenario, respectively, are presented. In Sections IV and V tripartite steering and genuine tripartite steering in 1-sided device-independent scenario as well as in 2-sided device-independent scenario for Svetlichny family and Mermin family, respectively, are discussed. Certifying genuine tripartite entanglement of $2 \times 2 \times 2$ quantum system through tripartite steering inequality in 2-sided device-independent scenario is also demonstrated in Sections IV and V. Finally, in the concluding Section VI, we discuss summary of the results obtained.

II. TRIPARTITE NONLOCALITY

We consider a tripartite Bell scenario where three spatially separated parties, Alice, Bob and Charlie, perform two dichotomic measurements on their subsystems. The correlation is described by the conditional probability distributions: $P(abc|A_x B_y C_z)$, here $x, y, z \in \{0, 1\}$ and $a, b, c \in \{0, 1\}$. The correlation exhibits standard tripartite nonlocality (i.e., Bell nonlocality) if it cannot be explained by a fully local hidden variable (LHV) model,

$$P(abc|A_x B_y C_z) = \sum_{\lambda} p_\lambda P_A(a|A_x)P_B(b|B_y)P_C(c|C_z),$$

for some hidden variable $\lambda$ with probability distribution $p_\lambda$; $\sum_\lambda p_\lambda = 1$. The Mermin inequality (MI) [25],

$$\langle M \rangle := \langle A_0 B_0 C_1 + A_0 B_1 C_0 + A_1 B_0 C_0 - A_1 B_1 C_1 \rangle_{\text{LHV}} \leq 2,$$

is a Bell-type inequality whose violation implies that the correlation cannot be explained by a fully local hidden variable model as in Eq. (1). Here $\langle A_x B_y C_z \rangle = \sum_{abc} (-1)^{abc} P(abc|A_x B_y C_z)$.

If a correlation violates a MI, it does not necessarily imply that it exhibits genuine tripartite nonlocality [6, 10]. In Ref. [6], Svetlichny introduced the strongest form of genuine tripartite nonlocality (see Ref. [10] for the other two forms of genuine nonlocality). A correlation exhibits Svetlichny nonlocality if it cannot be explained by a hybrid nonlocal-LHV (NLHV) model,

$$P(abc|A_x B_y C_z) = \sum_{\lambda} p_\lambda P_A(a|A_x)P_B(b|B_y)P_C(c|C_z)+ \sum_{\lambda} q_\lambda P_A(ac|A_x C_z)P_B(b|B_y)+ \sum_{\lambda} r_\lambda P_A(ab|A_x B_z)P_C(c|C_z),$$

with $\sum_\lambda p_\lambda + \sum_\lambda q_\lambda + \sum_\lambda r_\lambda = 1$. The bipartite probability distributions in this decomposition can have arbitrary nonlocality.

Svetlichny derived Bell-type inequalities to detect the strongest form of genuine tripartite nonlocality [6]. For instance, one of the Svetlichny inequalities reads,$$
\langle S \rangle := \langle A_0 B_0 C_1 + A_0 B_1 C_0 + A_1 B_0 C_0 - A_1 B_1 C_1 \rangle + \langle A_0 B_1 C_1 + A_1 B_0 C_1 + A_1 B_1 C_0 - A_0 B_0 C_0 \rangle \leq 4.4
$$

Quantum correlations violate the Svetlichny inequality (SI) up to $4 \sqrt{2}$. A Greenberger-Horne-Zeilinger (GHZ) state [26] gives rise to the maximal violation of the SI for a different choice of measurements which do not demonstrate GHZ paradox [27].

In the seminal paper [25], the MI was derived to demonstrate standard tripartite nonlocality of three-qubit correlations arising from the genuinely entangled states. For this purpose, noncommuting measurements that do not demonstrate Svetlichny nonlocality was used. Note that when a Greenberger-Horne-Zeilinger (GHZ) state [26] maximally violates the MI, the measurements that give rise to it exhibit the GHZ paradox [27].

III. DEFINITIONS OF TRIPARTITE EPR STEERING

Before we define tripartite EPR steering, let us review the definition of bipartite EPR steering in the following 1-sided
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**Device-independent scenario.** Two spatially separated parties, Alice (who is the trusted party) and Bob (who is the untrusted party) share an unknown tripartite system described by the density matrix $\rho_{ABC}$ in $\mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B} \otimes \mathbb{C}^{d_C}$ with the dimension of Alice $d_A$ known and the dimension of Bob $d_B$ unknown. On this shared state, Bob performs black-box measurements with the measurement operators $\{M_{b_{xy}}\}_{b_{xy}}$, here $y$ and $b$ denote the measurement choices and measurement outcomes of Bob, respectively, to prepare the set of conditional states on Alice’s side. The above steering scenario is characterized by the set of unnormalized conditional states on Alice’s side $\{\sigma_{b_{xy}}^{A}\}_{b_{xy}}$, which is called an assemblage. Each element in this assemblage is given by $\sigma_{b_{xy}}^{A} = \text{Tr}_B(\mathbb{I} \otimes M_{b_{xy}}\rho_{AB})$.

Wiseman et al. [14] provided an operational definition of steering. According to this definition, Bob’s measurements in the above scenario demonstrates steerability to Alice if the assemblage certifies entanglement. The assemblage which does not certify entanglement, i.e., does not imply steerability from Bob to Alice has a local hidden state (LHS) model as follows: for all $a$, $b$, $y$, each element $\sigma_{b_{xy}}^{A}$ in the assemblage admits the following decomposition:

$$\sigma_{b_{xy}}^{A} = \sum_{\lambda} q_{\lambda} P_{\lambda}(b|B_{x}) \rho_{A}^{\lambda},$$

where $\lambda$ denotes classical random variable which occurs with probability $q_{\lambda}$; $\sum_{\lambda} q_{\lambda} = 1$; $P_{\lambda}(b|B_{x})$ are some conditional probability distributions and the quantum states $\rho_{A}^{\lambda}$ are called local hidden states which satisfy $\rho_{A}^{\lambda} \geq 0$ and $\text{Tr} \rho_{A}^{\lambda} = 1$. Suppose Alice performs projective measurements with measurement operators $\{\Pi_{a_{x}}\}_{a_{x}}$ on the assemblage to detect steerability through the violation of a steering inequality. Then the scenario is characterized by the set of conditional probability distributions,

$$P(ab|A_{x}B_{x}) = \text{Tr} \left( \Pi_{a_{x}} \sigma_{b_{xy}}^{A} \right).$$

The above quantum correlation $P(ab|A_{x}B_{x})$ detects steerability if and only if it cannot be explained by a LHS-LHV model of the form,

$$P(ab|A_{x}B_{x}) = \sum_{\lambda} q_{\lambda} P(\lambda a_{x}, \rho_{A}^{\lambda}) P_{\lambda}(b|B_{x}) \quad \forall a, x, b, y,$$

with $\sum_{\lambda} q_{\lambda} = 1$. Here $P(\lambda a_{x}, \rho_{A}^{\lambda})$ are the distributions arising from the local hidden states $\rho_{A}^{\lambda}$.

On the other hand, the quantum correlation $P(ab|A_{x}B_{x})$ demonstrates Bell nonlocality if and only if it cannot be explained by a LHV-LHV model of the form,

$$P(ab|A_{x}B_{x}) = \sum_{\lambda} q_{\lambda} P_{\lambda}(a|A_{x}) P_{\lambda}(b|B_{x}) \quad \forall a, x, b, y,$$

with $\sum_{\lambda} q_{\lambda} = 1$. The quantum correlation that does not have a LHV-LHV model also implies steering, on the other hand, the quantum correlation that does not have a LHS-LHV model may not imply Bell nonlocality since certain local correlations may also detect steering in the given 1-sided device-independent scenario.

Let us now focus on the definition of tripartite steering. In the tripartite scenario, there are two types of partially device-independent scenarios where one can generalize bipartite EPR steering. These two scenarios are called 1-sided device-independent (1SDI) and 2-sided device-independent (2SDI) scenarios [23].

### A. Tripartite steering in 1SDI scenario

We will consider the following 1-sided device-independent (1SDI) scenario (depicted in FIG. 1): Three spatially separated parties share an unknown tripartite quantum state $\rho_{ABC}$ in $\mathbb{C}^{2} \otimes \mathbb{C}^{2} \otimes \mathbb{C}^{d}$ on which Charlie performs black-box measurements. Suppose $M_{b_{cy}}$ denote the unknown measurement operators of Charlie, then, the scenario is characterized by the set of (unnormalized) conditional two-qubit states on Alice and Bob’s side $\{\sigma_{b_{cy}}^{AB}_{c}c\}_{c}$, each element of which is given as follows:

$$\sigma_{b_{cy}}^{AB}_{c} = \text{Tr}_{C}(\mathbb{I} \otimes \mathbb{I} \otimes M_{b_{cy}}\rho_{ABC}).$$

Alice and Bob can do local state tomography to determine the above assemblage prepared by Charlie.

Analogous to the operational definition of bipartite EPR steering, we will now provide the operational definition of tripartite steering in the above 1SDI scenario. The assemblage $\sigma_{b_{cy}}^{AB}_{c}$ given by Eq. (9) is called steerable if

i) the assemblage prepared on Alice and Bob’s side cannot be reproduced by a fully separable state, in $\mathbb{C}^{2} \otimes \mathbb{C}^{2} \otimes \mathbb{C}^{d}$, of the form,

$$\rho^{ABC} = \sum_{\lambda} p_{\lambda} \rho_{A}^{\lambda} \otimes \rho_{B}^{B} \otimes \rho_{C}^{C},$$

with $\sum_{\lambda} p_{\lambda} = 1$; and

ii) entanglement between Charlie and Alice-Bob is detected.

In the genuine steering scenario, Charlie demonstrates genuine tripartite EPR steering to Alice and Bob if the assemblage prepared on Alice and Bob’s side cannot be reproduced by a biseparable state in $\mathbb{C}^{2} \otimes \mathbb{C}^{2} \otimes \mathbb{C}^{d}$,

$$\rho^{ABC} = \sum_{\lambda} p_{\lambda} \rho_{A}^{\lambda} \otimes \rho_{B}^{B} \otimes \rho_{C}^{C} + \sum_{\lambda} q_{\lambda} \rho_{A}^{\lambda} \otimes \rho_{B}^{B} \otimes \rho_{C}^{C},$$

with $\sum_{\lambda} p_{\lambda} + \sum_{\lambda} q_{\lambda} + \sum_{\lambda} r_{\lambda} = 1$.

Suppose in our tripartite 1SDI scenario, the trusted parties Alice and Bob perform projective measurements $\{\Pi_{a_{x}}\}_{a_{x}}$ and $\{\Pi_{b_{xy}}\}_{b_{xy}}$, respectively, for detecting tripartite steering. Then the scenario is characterized by the set of conditional probability distributions,

$$P(ab|A_{x}B_{x}C_{z}) = \text{Tr} \left( \Pi_{a_{x}} \otimes \Pi_{b_{xy}} \sigma_{b_{cy}}^{AB}_{c} \right),$$

where $\Pi_{a_{x}}$ and $\Pi_{b_{xy}}$ are the measurement operators of Alice and Bob, respectively. Suppose the above quantum correlation...
in Eq. (13); and the correlation quantum correlation does not have a fully LHS-LHV model which are in $C^2$, respectively. It should be noted that if a quantum correlation does not have an fully LHS-LHV model (13), then it does not necessarily imply that it detects tripartite steering from Charlie to Alice-Bob [17]. The correlation $P(abc|A,B,C_z)$ detects tripartite steerability if and only if

i) $P(abc|A,B,C_z)$ detects tripartite steerability if and only if

ii) entanglement between Charlie and Alice-Bob is detected.

The quantum correlation $P(abc|A,B,C_z)$ that detects tripartite steering also detects genuine tripartite steering if it cannot be explained by the following steering LHS-LHV (StLHS) model:

$$P(abc|A,B,C_z) = \sum_\lambda q_\lambda p(ab|A,B,\rho^A_{bc})P_\lambda(c|C_z)$$

with $\sum_\lambda q_\lambda = 1$. Here $p(ab|A,B,\rho^A_{bc})$ are the probability distributions arising from the quantum state $\rho^A_{bc}$ on Alice’s side and Bob’s side, respectively. $P_\lambda(c|C_z)$ is the distribution on Charlie’s side arising from black-box measurements performed on a $d$ dimensional quantum state and $P_\lambda^Q(bc|B,C)$ and $P_\lambda^Q(ac|A,C_z)$ are the distributions that can be produced from a $2 \times d$ quantum state; and $P(ab|A,B,\rho^A_{bc})$ can be reproduced by two-qubit quantum states $\rho^A_{bc}$ shared between Alice and Bob. Note that in the model given in Eq. (14), the bipartite distributions at each $\lambda$ level may have Bell nonlocality or steering without Bell nonlocality [17, 24]. Equivalently, the quantum correlation that detects genuine tripartite steering cannot be reproduced by a biseparable state in $C^2 \otimes C^2 \otimes C^2$. 
B. Tripartite steering in 2SDI scenario

We will consider the following 2-sided device-independent (2SDI) scenario (depicted in FIG. 2): Three spatially separated parties share an unknown tripartite quantum state $\rho_{ABC}$ in $\mathbb{C}^2 \otimes \mathbb{C}^d \otimes \mathbb{C}^d$ on which Bob and Charlie performs local black-box measurements. Suppose $|M_{bly}\rangle_{bc}$ and $|M_{c_{xyz}}\rangle_{xyz}$ denote the unknown measurement operators of Bob and Charlie, respectively. Then, the scenario is characterized by the set of (unnormalized) conditional qubit states on Alice’s side $\{\sigma_{abc}^{\lambda}\}_{\lambda,b,c}$. The each element in this assemblage is given as follows:

$$\sigma_{abc}^{AB} = \text{Tr}_{BC}(1 \otimes M_{bly} \otimes M_{c_{yz}} \rho_{ABC}).$$

(15)

Alice can do local state tomography to determine the above assemblage prepared by Charlie.

We will now provide the operational definition of tripartite steering in the above 2SDI scenario. The assemblage $\{\sigma_{abc}^{\lambda}\}_{\lambda,b,c}$ is called steerable if it cannot be reproduced by a fully separable state in $\mathbb{C}^2 \otimes \mathbb{C}^d \otimes \mathbb{C}^d$ of the form,

$$\rho^{ABC} = \sum_{\lambda} P_{\lambda} \rho_{\lambda}^A \otimes \rho_{\lambda}^B \otimes \rho_{\lambda}^C,$$

(16)

with $\sum_{\lambda} P_{\lambda} = 1$ in the given steering scenario. In our 2SDI scenario, even if entanglement is not certified between Alice and Bob Charlie, tripartite steering can still occur by the presence of Bell nonlocality between Charlie and Bob [17]. When entanglement between Alice and Bob Charlie is detected, our 2SDI scenario demonstrates genuine tripartite steering if the assemblage $\sigma_{abc}^{\lambda}$ cannot be reproduced by a biseparable state as given by Eq. (11) in $\mathbb{C}^2 \otimes \mathbb{C}^d \otimes \mathbb{C}^d$.

Suppose in our tripartite 2SDI scenario, the trusted party Alice performs measurements for detecting tripartite steering. Then the scenario is characterized by the set of conditional probability distributions,

$$P(abc|A_xB_yC_z) = \text{Tr}(M_{abc}\sigma_{abc}^{AB}),$$

(17)

where $M_{abc}$ are the measurement operators of Alice. Suppose the above quantum correlation $P(abc|A_xB_yC_z)$ cannot be explained by a fully LHS-LHV model of the form,

$$P(abc|A_xB_yC_z) = \sum_{\lambda} q_{\lambda} P_{\lambda}(a|A_{x},\rho_{\lambda}^A)P_{\lambda}(b|B_{y})P_{\lambda}(c|C_{z}),$$

(18)

with $\sum_{\lambda} q_{\lambda} = 1$ (Here, $P(a|A_{x},\rho_{\lambda}^A)$ are the distributions arising from the local hidden states $\rho_{\lambda}^A$ which are in $\mathbb{C}^2$). Then, it detects tripartite steerability.

The quantum correlation $P(abc|A_xB_yC_z)$ that detects tripartite steering in our 2SDI scenario also detects genuine tripartite steering if it cannot be explained by the following steering LHS-LHV (StLHS) model:

$$P(abc|A_xB_yC_z) = \sum_{\lambda} r_{\lambda} P_{\lambda}^d(ab|A_{x},B_{y})P_{\lambda}(c|C_{z})$$

$$+ \sum_{\lambda} p_{\lambda} P_{\lambda}(a|A_{x},\rho_{\lambda}^A)P_{\lambda}(bc|B_{y},C_{z})$$

$$+ \sum_{\lambda} q_{\lambda} P_{\lambda}(b|B_{y})P_{\lambda}^d(ac|A_{x},C_{z}),$$

(19)

with $\sum_{\lambda} r_{\lambda} + \sum_{\lambda} q_{\lambda} + \sum_{\lambda} p_{\lambda} = 1$. Here, $P(a|A_{x},\rho_{\lambda}^A)$ are the distributions arising from the qubit states $\rho_{\lambda}^A$ and, $P_{\lambda}(b|B_{y})$ and $P_{\lambda}(c|C_{z})$ are the distribution on Bob’s and Charlie’s sides, respectively, arising from black-box measurements performed on a $d \times d$ quantum state and $P_{\lambda}^d(ab|A_{x},B_{y})$ and $P_{\lambda}^d(ac|A_{x},C_{z})$ are the distribution that can be produced from a $2 \times d$ quantum state; and $P_{\lambda}(bc|B_{y},C_{z})$ can be reproduced by a $d \times d$ quantum state. Note that in the model given in Eq. (19), the bipartite distributions at each $\lambda$ level may have Bell nonlocality or steering without Bell nonlocality [17, 24]. Equivalently, the quantum correlation that detects genuine tripartite steering in our 2SDI cannot be reproduced by a biseparable state in $\mathbb{C}^2 \otimes \mathbb{C}^d \otimes \mathbb{C}^d$.

IV. Svetlichny-type steering

We will study tripartite EPR steering of a family of quantum correlations that belong to the Svetlichny family defined as:

$$P_{S}^{V}((abc|A_xB_yC_z)) = \frac{2 + (-1)^{p_{ab}bly+p_{ac}czy+p_{bc}xyz}}{16} \sqrt{2V},$$

(20)

where $0 \leq V \leq 1$, in the 1SDI and 2SDI scenarios. The Svetlichny family certifies genuine entanglement in a DI way for $V > \frac{1}{\sqrt{2}}$, as it violates the SI in this range. The Svetlichny family has a fully local hidden variable (LHV) model when $V \leq \frac{1}{\sqrt{2}}$ [10]. This implies that in this range, it can also arise from a separable state in the higher dimensional space [28]. We will now give an example of simulation of the Svetlichny family by using three-qubit systems.

**Example 1.** The Svetlichny family can be obtained from noisy three-qubit GHZ state, $\rho = V|\Phi_{GHZ}\rangle\langle\Phi_{GHZ}| + (1 - V)I/8$, where $|\Phi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$, for the measurements that give rise to the maximal violation of the SI; for instance, $A_{0} = \sigma_{x}, A_{1} = \sigma_{y}, B_{0} = (\sigma_{x} - \sigma_{y})/\sqrt{2}, B_{1} = (\sigma_{x} + \sigma_{y})/\sqrt{2}, C_{0} = \sigma_{x}$ and $C_{1} = \sigma_{y}$.

The noisy GHZ state given above is genuinely entangled iff $V > 0.429$ [29].

We consider the following classical simulation scenario for the Svetlichny family to demonstrate in which range it has a steering LHS-LHV model as in Eq. (14) and a fully LHS-LHV model as in Eq. (13) in 1SDI scenario:

**Scenario 1.** Charlie generates his outcomes by using classical variable $\lambda$ which he shares with Alice-Bob. Alice and Bob share a two-qubit system for each value of $\lambda$ and perform pair of incompatible qubit measurements that demonstrate Bell nonlocality of certain two-qubit states; for instance, the singlet state.

**Lemma 1.** In the context of Scenario 1, the Svetlichny family has a steering LHS-LHV model as in Eq. (14) in the range $0 < V \leq \frac{1}{\sqrt{2}}$.

Proof. See Appendix A.

The above Lemma implies the following.
**Proposition 1.** In our 1SDI scenario where Alice and Bob have access to incompatible qubit measurements that demonstrate Bell nonlocality of certain two-qubit states; for instance, the singlet state, while Charlie’s measurements are uncharacterized (i.e., in the context of scenario 1), the Svetlichny family given in Example 1 detects genuine tripartite steering iff \( V > \frac{1}{\sqrt{2}} \).

**Proof.** Since the Svetlichny family violates the Svetlichny inequality for \( V > 1/\sqrt{2} \), it certifies genuine tripartite entanglement in a fully device independent way in that range. Hence, it is followed that the Svetlichny family certifies genuine tripartite entanglement in our 1SDI scenario as well for \( V > 1/\sqrt{2} \). The Svetlichny family, therefore, does not have a steering LHS-LHV model as in Eq. (14) in our 1SDI scenario for \( V > 1/\sqrt{2} \). On the other hand, the steering LHS-LHV model given in the proof of Lemma 1 for the Svetlichny family implies that in the range \( 0 < V \leq \frac{1}{\sqrt{2}} \) the Svetlichny family does not detect genuine tripartite steering. Hence, the Svetlichny family given in Example 1 detects genuine tripartite steering iff \( V > \frac{1}{\sqrt{2}} \). \( \square \)

The steering LHS-LHV model given in the proof of Lemma 1 for the Svetlichny family also implies the following.

**Proposition 2.** In the context of Scenario 1, the Svetlichny family has a fully LHS-LHV model as in Eq. (13) iff \( V \leq \frac{1}{\sqrt{2}} \).

**Proof.** In the steering LHS-LHV model given for the Svetlichny family as in Eq. (A1), the bipartite distributions \( P(ab|A_x,B_y) \) belong to the BB84 family up to local reversible operations (LRO) \(^1\),

\[
P_{BB84}(ab|A_x,B_y) = \frac{1 + (-1)^{a+b}x\cdot y\cdot \delta_{x,y}}{4} W
\]

where \( W = \sqrt{2} V \) is a real number such that \( 0 < W \leq 1 \). In Ref. \([32]\), it has been shown that the BB84 family certifies two-qubit entanglement iff \( W > \frac{1}{2} \). This implies that for \( W \leq \frac{1}{2} \), it can be reproduced by a two-qubit separable state. Therefore, the bipartite distributions \( P(ab|A_x,B_y) \) in Eq. (A1) has a LHS-LHS decomposition iff \( V \leq \frac{1}{\sqrt{2}} \). This implies that the Svetlichny family can be reproduced by an absolute LHS-LHV model,

\[
P_{S^{1SDI}}(abc|A_x,B_y,C_z) = \sum_A q_A P(a|A_x,\rho^A_B)P(b|B_y,\rho^B_C)P(c|C_z,\rho^C_C),
\]

iff \( V \leq \frac{1}{\sqrt{2}} \) in Scenario 1. Here, \( P(b|B_y,\rho^B_C) \) and \( P(c|C_z,\rho^C_C) \) are the distributions arising from the local hidden states \( \rho^B_C \) and \( \rho^C_C \) which are in \( \mathbb{C}^2 \), respectively. \( \square \)

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\(^1\) LRO is designed [31] as follows: Alice may relabel her inputs: \( x \rightarrow x \oplus 1 \), and she may relabel her outputs (conditionally on the input): \( a \rightarrow a \oplus x \oplus \beta \) (\( \alpha, \beta \in \{0, 1\} \)); Bob can perform similar operations.

We now ask the question whether the Svetlichny family detects tripartite steering for \( V > \frac{1}{\sqrt{2}} \) in our 1SDI scenario. To answer this question, we make the following observation.

**Lemma 2.** In our 1SDI scenario, the Svetlichny family detects entanglement between Charlie and Alice-Bob iff \( V > 1/\sqrt{2} \).

**Proof.** The steering LHS-LHV model given in the proof of Lemma 1 for the Svetlichny family implies that for \( V \leq 1/\sqrt{2} \), it can be reproduced by a \( 2 \times 2 \times d \) biseparable state of the form,

\[
\rho^{ABC} = \sum_{\lambda=0}^3 r_\lambda \rho^{A_B}_\lambda \otimes |\lambda\rangle\langle\lambda|,
\]

with \( \sum_\lambda r_\lambda = 1 \). Therefore, the Svetlichny family detects entanglement between Charlie and Alice-Bob iff \( V > 1/\sqrt{2} \) as it violates the Svetlichny inequality in this range. \( \square \)

The above Lemma implies the following.

**Proposition 3.** In our 1SDI scenario where Alice and Bob have access to incompatible qubit measurements that demonstrate Bell nonlocality of certain two-qubit states; for instance, the singlet state, while Charlie’s measurements are uncharacterized (i.e., in the context of scenario 1), the Svetlichny family given in Example 1 detects tripartite steering iff \( V > 1/\sqrt{2} \).

**Proof.** The Svetlichny family detects entanglement between Charlie and Alice-Bob iff \( V > 1/\sqrt{2} \). On the other hand, in the context of Scenario 1, the Svetlichny family does not have a fully LHS-LHV model as in Eq. (13) for \( V > 1/\sqrt{2} \). In other words, in the range \( V > 1/\sqrt{2} \), the Svetlichny family detects entanglement between Charlie and Alice-Bob and does not have a fully LHS-LHV model as in Eq. (13) in 1SDI scenario. Hence, in the context of scenario 1, the Svetlichny family given in Example 1 detects tripartite steering iff \( V > 1/\sqrt{2} \). \( \square \)

The above proposition implies the following.

**Proposition 4.** In our 1SDI scenario, the Svetlichny family does not detect tripartite steering in the range \( 1/2 \sqrt{2} \leq V \leq 1/\sqrt{2} \) despite it does not have a fully LHS-LHV model in this range.

Therefore, the ranges in which the Svetlichny family detects tripartite steering and genuine tripartite steering in our 1SDI scenario are the same.

We now consider the 2SDI scenario described in Sec. III B where the trusted party Charlie performs two mutually untrusted qubit measurements. In Ref. \([17]\), it has been shown that the following steering inequality (Eq. (22) in [17] with \( N \) (Number of parties) = 3 and \( T \) (Number of trusted parties) = 1):

\[
\langle S \rangle^{LHS}_{2SDI} \leq 2 \sqrt{2},
\]

where \( S \) is the Svetlichny operator given in the Svetlichny inequality (4), detects tripartite steering in our 2SDI scenario. Here, \( 2 \times 2 \times d \) indicate that Alice performs qubit measurements while Bob and Charlie perform black-box measurements. Note that the Svetlichny family given in Example 1...
violates the above steering inequality for $V > 1/2$. Thus, the Svetlichny family detects tripartite steering for $V > 1/2$ in the 2SDI scenario.

We are now interested in which range the Svetlichny family detects genuine steering in our 2SDI scenario.

**Proposition 5.** The Svetlichny family detects genuine tripartite steering in our 2SDI scenario iff $V > 1/\sqrt{2}$.

**Proof.** Note that the Svetlichny family can be reproduced by a $2 \times 2 \times d$ dimensional biseparable state of the form given in Eq. (23) for $V \leq 1/\sqrt{2}$. This implies that it does not detect genuine tripartite entanglement in the range $1/2 \leq V \leq 1/\sqrt{2}$ in our 2SDI scenario. On the other hand, the Svetlichny family detects genuine tripartite entanglement for $V > 1/\sqrt{2}$ in the fully device independent scenario. Hence, the Svetlichny family detects genuine tripartite entanglement for $V > 1/\sqrt{2}$ in our 2SDI scenario also. The Svetlichny family, therefore, detects genuine tripartite steering in our 2SDI scenario iff $V > 1/\sqrt{2}$.

Therefore, the ranges in which the Svetlichny family detects tripartite steering and genuine tripartite steering in our 2SDI scenario are different.

We will now demonstrate that tripartite steering as detected by the violation of the steering inequality (24) by using three-qubit systems necessarily due to genuine tripartite entanglement. For this purpose, we now derive a biseparability inequality that detect genuine entanglement of three-qubit systems by using the Svetlichny operator. Suppose the tripartite correlation arising from the scenario where each party performs incompatible qubit measurements. Then it cannot be explained by the nonseparable LHS-LHS (NSLHS) model, with $\sum_{d} p_{d} + \sum_{d} q_{d} + \sum_{d} r_{d} = 1$. Here, $P(a|A_{d},\rho_{A}^{d})$, $P(b|B_{d},\rho_{B}^{d})$, and $P(c|C_{d},\rho_{C}^{d})$ are the distributions which can be reproduced by the qubit states $\rho_{A}^{d}$, $\rho_{B}^{d}$, and $\rho_{C}^{d}$, respectively, and $P_{d}(bc|B_{d},C_{d},\rho_{BC}^{d})$, $P_{d}(ac|A_{d},C_{d},\rho_{AC}^{d})$ and $P_{d}(ab|A_{d},B_{d},\rho_{AB}^{d})$ can be reproduced by the $2 \times 2 \times d$ states $\rho_{BC}^{d}$, $\rho_{AC}^{d}$ and $\rho_{AB}^{d}$, respectively. Note that in the model given in Eq. (25), the bipartite distributions at each $\lambda$ level may have nonseparability. We derive the following biseparability inequality.

**Proposition 6.** If a correlation arising from the scenario where each party performs two qubit incompatible measurements violates the following inequality,

$$\langle S \rangle_{NSLHS}^{2SDI} \leq 2\sqrt{2},$$

then the presence of genuine entanglement of the $2 \times 2 \times 2$ system is certified. Here, $2 \times 2 \times 2$ indicates that Alice, Bob and Charlie have access to qubit measurements.

**Proof.** The Svetlichny operator can be rewritten as follows:

$$S = CHS H_{AB} C_{1} + CHS H'_{AB} C_{0}. \quad (27)$$

Here, $CHS H_{AB} = A_{0} B_{0} + A_{0} B_{1} + A_{1} B_{0} - A_{1} B_{1}$ is the canonical CHSH (Clauser-Horne-Shimony-Holt) operator [16] and $CHS H'_{AB} = -A_{0} B_{0} + A_{0} B_{1} + A_{1} B_{0} + A_{1} B_{1}$ is one of its equivalents. Note that the expectation value of the Svetlichny operator for the correlation which has the nonseparable LHS-LHS model as given in Eq. (25) have the following form:

$$\sum_{d} p_{d} \langle A_{1} \rangle_{\rho_{A}^{d}} \langle CHS H_{BC} \rangle_{\rho_{BC}^{d}} + \sum_{d} p_{d} \langle A_{0} \rangle_{\rho_{A}^{d}} \langle CHS H'_{BC} \rangle_{\rho_{BC}^{d}}$$

$$+ \sum_{d} q_{d} \langle CHS H_{AC} \rangle_{\rho_{AC}^{d}} \langle B_{1} \rangle_{\rho_{B}^{d}} + \sum_{d} q_{d} \langle CHS H'_{AC} \rangle_{\rho_{AC}^{d}} \langle B_{0} \rangle_{\rho_{B}^{d}}$$

$$+ \sum_{d} r_{d} \langle CHS H_{AB} \rangle_{\rho_{AB}^{d}} \langle C_{1} \rangle_{\rho_{C}^{d}} + \sum_{d} r_{d} \langle CHS H'_{AB} \rangle_{\rho_{AB}^{d}} \langle C_{0} \rangle_{\rho_{C}^{d}}. \quad (28)$$

Let us now argue that the above quantity is upper bounded by $2\sqrt{2}$. Consider the first line of the decomposition given in Eq. (28). Suppose Bob and Charlie’s correlation at each $\lambda$ level of this line detects nonseparability. Then $\pm \langle CHS H_{BC} \rangle_{\rho_{BC}^{d}} \pm \langle CHS H'_{BC} \rangle_{\rho_{BC}^{d}} \leq 2\sqrt{2}$. Suppose Bob and Charlie’s correlation at each $\lambda$ level has a LHS-LHS model. Then also $\pm \langle CHS H_{BC} \rangle_{\rho_{BC}^{d}} \pm \langle CHS H'_{BC} \rangle_{\rho_{BC}^{d}} \leq 2\sqrt{2}$. In a similar way, considering the second line of the decomposition given in Eq. (28), one can show that $\pm \langle CHS H_{AC} \rangle_{\rho_{AC}^{d}} \pm \langle CHS H'_{AC} \rangle_{\rho_{AC}^{d}} \leq 2\sqrt{2}$; and considering the third line of the decomposition given in Eq. (28), one can show that $\pm \langle CHS H_{AB} \rangle_{\rho_{AB}^{d}} \pm \langle CHS H'_{AB} \rangle_{\rho_{AB}^{d}} \leq 2\sqrt{2}$. Therefore, any convex combination of the three above mentioned expression should be upper bounded by $2\sqrt{2}$. Hence, we can conclude that in the Scenario where each party performs incompatible qubit measurements, the Svetlichny operator is upper bounded by $2\sqrt{2}$ if the correlation has a nonseparable LHS-LHS model.

Note that the quantum correlation in Example 1 violates the above biseparability inequality for $V > \frac{1}{2}$. Therefore the Svetlichny family certifies genuine entanglement of three-qubit systems for $V > \frac{1}{2}$ in the fully device-dependent scenario.

We can now present the following Theorem.
Observation 1. Quantum violation of the tripartite steering inequality (24) by $2 \times 2 \times 2$ systems certifies genuine entanglement in that $2 \times 2 \times 2$ systems, even if genuine nonlocality or genuine steering is not detected.

Proof. Note that quantum violation of tripartite steering inequality (24) by $2 \times 2 \times 2$ systems implies quantum violation of the biseparability inequality (26) by that $2 \times 2 \times 2$ systems. Because, for both of these two inequalities the upper bounds are the same. \hfill \square

We have illustrated the above theorem with the Svetlichny family given in Example 1 (see Fig. 3).

For a given quantum state, tripartite EPR steering as witnessed by the steering inequality (24) originates from measurement settings that give rise to Svetlichny nonlocality. For this reason, we term this type of tripartite steering as Svetlichny steering.

V. MERMIN-TYPE STEERING

Consider quantum correlations that belong to the Mermin family defined as

$$P_{MF}^V(abc|A_xB_yC_z) = \frac{1 + (-1)^{\lambda_0\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5\lambda_6\lambda_7\lambda_8} \delta_{\lambda_1\lambda_4\lambda_6\lambda_7\lambda_8}}{8},$$

(29)

where $0 < V \leq 1$. The Mermin family is Bell nonlocal for $V > \frac{1}{2}$ as it violates the MI given in Eq. (2). This implies that it certifies tripartite entanglement for $V > \frac{1}{2}$. In that range, the Mermin family is not genuinely nonlocal since it has a NLHV model as in Eq. (3) [33]. However, it certifies genuine tripartite entanglement for $V > \frac{1}{\sqrt{2}}$ in a fully DI way since it violates the Mermin inequality more than $2 \sqrt{2}$ [34]. We will study tripartite steering of the Mermin family in our 1SDI and 2SDI scenarios. We now give an example of the simulation of the Mermin family by using three-qubit system.

Example 2. The Mermin family can be obtained from the noisy 3-qubit GHZ state, $\rho = \mathbb{V}[\Phi_{GHZ}|\Phi_{GHZ}] + (1 - \mathbb{V})4/8$, where $|\Phi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$, for the measurements that give rise to the GHZ paradox; for instance, $A_0 = \sigma_x$, $A_1 = \sigma_y$, $A_0 = \sigma_x$, $B_1 = \sigma_y$, $C_0 = \sigma_x$ and $C_1 = -\sigma_y$.

We consider the following classical simulation scenario for the Mermin family to demonstrate in which range it has a steering LHS-LHV model as in Eq. (14) and a fully LHS-LHV model as in Eq. (13):

Scenario 2. Charlie generates his outcomes by using classical variable $\lambda$ which he shares with Alice-Bob. Alice and Bob share a two-qubit system for each $\lambda$ and perform pair of incompatible qubit measurements that demonstrate EPR steering without Bell nonlocality of certain two-qubit states [17, 24]; for instance, the singlet state.

Lemma 3. In the context of Scenario 2, the Mermin family has a steering LHS-LHV model as in Eq. (14) in the range $0 < V \leq \frac{1}{\sqrt{2}}$.

Proof. See Appendix B for the proof. \hfill \square

The above Lemma implies the following.

Proposition 7. In our 1SDI scenario where Alice and Bob perform pair of incompatible qubit measurements that demonstrate EPR steering without Bell nonlocality of certain two-qubit states [17, 24]; for instance, the singlet state, while Charlie’s measurements are uncharacterized (i.e., in the context of Scenario 2), the Mermin family given in Example 2 detects genuine tripartite steering iff $V > \frac{1}{\sqrt{2}}$.

Proof. Since the Mermin family violates the Mermin inequality more than $2 \sqrt{2}$ for $V > 1/\sqrt{2}$, it certifies genuine tripartite entanglement in the fully device independent scenario in that range [34]. Hence, the Mermin family certifies genuine tripartite entanglement in our 1SDI scenario as well for $V > 1/\sqrt{2}$. This implies that for $V > 1/\sqrt{2}$, it does not have a StLHS model as in Eq. (14). On the other hand, the Mermin family has a StLHS model as in Eq. (14) in the range $0 < V \leq \frac{1}{\sqrt{2}}$ in the context of Scenario 2. The Mermin family given in Example 2, therefore, detects genuine tripartite steering iff $V > \frac{1}{\sqrt{2}}$ in the context of scenario 2. \hfill \square

Proposition 8. In Scenario 2, the Mermin family has a fully LHS-LHV model iff $V \leq \frac{1}{2\sqrt{2}}$.

Proof. In the steering LHS-LHV model given for the Mermin family as in Eq. (B1), the bipartite distributions $P(ab|A_xB_y, \rho_{AB}^\lambda)$ belong to the CHSH family up to local reversible operations [31].

$$P_{CHSH}(ab|A_x) = \frac{2 + (-1)^{\lambda_0\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5\lambda_6\lambda_7\lambda_8} \sqrt{2} W}{8},$$

(30)

where $W = \sqrt{2}V$ is a real number such that $0 < W \leq 1$ and $0 < V \leq \frac{1}{4}$. In Ref. [32], it has been that the CHSH family certifies two-qubit entanglement iff $W > \frac{1}{2}$. This implies that for $W \leq \frac{1}{2}$, it can be reproduced by a two-qubit separable state. Therefore, the bipartite distributions $P(ab|A_xB_y, \rho_{AB}^\lambda)$ in Eq. (B1) has a LHS-LHS decomposition iff $V \leq \frac{1}{2\sqrt{2}}$. This implies that the Mermin family can be reproduced by an absolute LHS-LHV model,

$$P_{MF}^V(abc|A_xB_yC_z) = \sum_{\lambda} q_\lambda P(a|A_x, \rho_{A}^\lambda)P(b|B_y, \rho_{B}^\lambda)P(c|C_z),$$

(31)

if $V \leq \frac{1}{2\sqrt{2}}$ in Scenario 2. Here, $P(b|B_y, \rho_{B}^\lambda)$ and $P(c|C_z, \rho_{C}^\lambda)$ are the distributions arising from the local hidden states $\rho_{B}^\lambda$ and $\rho_{C}^\lambda$ which are in $\mathbb{C}^2$ respectively. \hfill \square

We now ask the question whether the Mermin family detects tripartite steering for $V > \frac{1}{2\sqrt{2}}$ in our 1SDI scenario. To answer this question, we make the following observation.

Lemma 4. In our 1SDI scenario, the Mermin family detects entanglement between Charlie and Alice-Bob iff $V > 1/\sqrt{2}$. 


Proof. The steering LHS-LHV model given in the proof of Lemma 3 for the Mermin family implies that for $V \leq 1/\sqrt{2}$, it can be reproduced by a $2 \times 2 \times d$ biseparable state of the form given by Eq. (11). Therefore, the Mermin family detects entanglement between Charlie and Alice-Bob iff $V > 1/\sqrt{2}$ as it violates the Mermin inequality more than $2\sqrt{2}$ in this range. □

The above Lemma implies the following.

**Proposition 9.** In our 1SDI scenario where Alice and Bob perform pair of incompatible qubit measurements that demonstrate EPR steering without Bell nonlocality of certain two-qubit states [17, 24]: for instance, the singlet state, while Charlie’s measurements are uncharacterized (i.e., in the context of scenario 2), the Mermin family given in Example 2 detects tripartite steering iff $V > \frac{1}{\sqrt{2}}$.

Proof. The Mermin family detects entanglement between Charlie and Alice-Bob iff $V > 1/\sqrt{2}$. On the other hand, in the context of Scenario 2, the Mermin family does not have a fully LHS-LHV model as in Eq. (13) for $V > \frac{1}{2\sqrt{2}}$. In other words, in the range $V > \frac{1}{\sqrt{2}}$, the Mermin family detects entanglement between Charlie and Alice-Bob and does not have a fully LHS-LHV model as in Eq. (13) in our 1SDI scenario. Hence, in the context of scenario 2, the Mermin family given in Example 2 detects tripartite steering iff $V > \frac{1}{\sqrt{2}}$. □

The above proposition implies the following.

**Proposition 10.** In our 1SDI scenario, the Mermin family does not detect tripartite steering in the range $1/2\sqrt{2} \leq V \leq 1/\sqrt{2}$ despite it does not have a fully LHS-LHV model.

Therefore, the ranges in which the Mermin family detects tripartite steering and genuine tripartite steering in our 1SDI scenario are the same.

We will now study tripartite steering of the Mermin family in our 2SDI scenario where the trusted party Charlie performs two mutually unbiased qubit measurements. In Ref. [17], it has been shown that the following steering inequality (Eq. (21) in [17] with $N = 3$ and $T = 1$),

$$\langle M_1^{LHS} \rangle_{2\times 3\times 3} \leq 2,$$

where $M$ is the Mermin operator given in the Mermin inequality (2), detects tripartite steering in our 2SDI scenario. Note that the Mermin family given in Example 2 violates the above steering inequality for $V > 1/2$. Thus, the Mermin family detects tripartite steering for $V > 1/2$ in the 2SDI scenario.

We are now interested in which range the Mermin family detects genuine tripartite steering in our 2SDI scenario.

**Proposition 11.** The Mermin family detects genuine tripartite steering in our 2SDI scenario iff $V > 1/\sqrt{2}$.

Proof. Note that the Mermin family can be reproduced by a $2 \times 2 \times d$ dimensional biseparable state of the form given in Eq. (23) for $V \leq 1/\sqrt{2}$. This implies that it does not detect genuine tripartite entanglement in the range $1/2 \leq V \leq 1/\sqrt{2}$ in our 2SDI scenario. On the other hand, the Mermin family detects genuine tripartite entanglement only for $V > 1/\sqrt{2}$ in the fully device independent scenario. Hence, the Mermin family detects genuine tripartite entanglement in our 2SDI scenario as well for $V > 1/\sqrt{2}$. The Mermin family, therefore, detects genuine tripartite steering in our 2SDI scenario iff $V > 1/\sqrt{2}$. □

Therefore, the ranges in which the Mermin family detects tripartite steering and genuine tripartite steering in our 2SDI scenario are different. In Ref. [37], it was shown that the Mermin inequality detects genuine entanglement of three-qubit systems in the scenario where all three parties perform two mutually unbiased qubit measurements. This implies that the violation of the inequality (32) implies the presence of genuine entanglement if all three parties perform qubit measurements in mutually unbiased bases. Similar to the derivation of the biseparability inequality (26), one can show that the inequality (32) serves as the biseparability inequality if Bob and Charlie also perform incompatible qubit measurements by using the structure of the Mermin inequality. This implies that for $V > \frac{1}{\sqrt{2}}$, the Mermin family detects genuine entanglement of $2 \times 2 \times 2$ in the fully device-dependent scenario where all three parties perform incompatible qubit measurements.

We can now present the following Theorem.

**Observation 2.** Quantum violation of the tripartite steering inequality (32) by $2 \times 2 \times 2$ systems certifies genuine entanglement in that $2 \times 2 \times 2$ systems, even if genuine nonlocality or genuine steering is not detected.

Proof. Note that quantum violation of tripartite steering inequality (32) by $2 \times 2 \times 2$ systems implies quantum violation of the biseparability inequality derived by using the structure of Mermin inequality by that $2 \times 2 \times 2$ systems. Because, for both of these two inequalities the upper bounds are the same. □

We have illustrated the above theorem with the Mermin family given in Example 2 (see Fig. 4).

For a given state, tripartite EPR steering as witnessed by the steering inequality (32) originates from measurement settings that lead to standard nonlocality, which is witnessed only by
the violation of the MI. For this reason, we term this type of tripartite steering as Mermin steering.

VI. CONCLUSION

In this work, we have studied tripartite EPR steering of quantum correlations arising from two local measurements on each side in the two types of partially device-independent scenarios: 1-sided device-independent scenario where one of the parties performs untrusted measurements while the other two parties perform trusted measurements and 2-sided device-independent scenario where one of the parties perform trusted measurements while the other two parties perform untrusted measurements.

We have studied tripartite steering in the 1-sided device-independent framework in the context of the following two types of classical simulation scenarios: One of the parties share a hidden variable with the other two parties who share a two-qubit system for each value of the hidden variable and perform incompatible measurements that demonstrate Bell nonlocality in one of the types or perform incompatible measurements that demonstrate EPR steering without Bell nonlocality in the other type. In the context of these two classical simulation scenarios, we have studied tripartite steering of two families of quantum correlations called Svetlichny family and Mermin family, respectively. We have shown that the ranges in which these families detect tripartite steering and genuine tripartite steering are the same.

On the other hand, in the 2-sided device-independent framework, the ranges in which the Svetlichny family and Mermin family detect tripartite steering and genuine tripartite steering are different. These studies reveal that tripartite steering in the 2-sided device-independent scenario is weaker than tripartite steering in the 1-sided device-independent scenario. That is the Svetlichny family and Mermin family in the 2-sided device-independent framework detect tripartite entanglement for a larger region than that in the 1-sided device-independent framework. Using biseparability inequality, it has been demonstrated that tripartite steering in the 2-sided device-independent framework implies the presence of genuine tripartite entanglement of $2 \times 2 \times 2$ quantum system, even if the correlation does not exhibit genuine nonlocality or genuine steering.

In Ref. [24], the author proposed two inequalities for detecting genuine steering in the Svetlichny-type and Mermin-type one-sided device-independent scenarios. We have demonstrated that these inequalities do not detect genuine steering and they detect tripartite steering of $d \times d \times 2$ systems in the 2-sided device-independent framework. Further, the author argued that the violation of one of these inequalities imply genuine entanglement if one assumes only dimension of the trusted parties to be qubit dimension. However, the present study demonstrates that the violation of these inequalities do not detect genuine entanglement in this context, on the other hand, the violation of those inequalities may imply genuine entanglement in the scenario where the dimensions of all three parties are assumed to be qubit dimension.

ACKNOWLEDGEMENT

Authors are thankful to Prof. Guruprasad Kar and Nirman Ganguly for fruitful discussions. CJ is thankful to Paul Skrzypczyk for useful discussions during the 657.WE-Heraeus Seminar “Quantum Correlations in Space and Time”. DD acknowledges the financial support from University Grants Commission (UGC), Government of India. BB, CJ and DS acknowledge the financial support from project SR/S2/LOP-08/2013 of the Department of Science and Technology (DST), government of India. AM acknowledges support from the CSIR project 09/093(0148)/2012-EMR-I.

Appendix A: Proof for the Lemma 1

For $0 < V \leq \frac{1}{\sqrt{2}}$, the Svetlichny family can be written as

$$P_{SvF}^{r}(abc|A_{i}B_{j}C_{k}) = \sum_{j=0}^{3} r_{j} P(ab|A_{i}B_{j}, \rho_{AB}^{j}) P_{c}(C_{k}) \quad (A1)$$

where $r_{0} = r_{1} = r_{2} = r_{3} = \frac{1}{3}$, and

$$P_{0}(c|C_{k}) = P_{0D}^{0}, \quad P_{1}(c|C_{k}) = P_{0D}^{1}, \quad P_{2}(c|C_{k}) = P_{0D}^{2}, \quad P_{3}(c|C_{k}) = P_{0D}^{3},$$

here,

$$P_{0D}^{c}(c|C_{k}) = \begin{cases} 1, & c = \alpha \sigma_{z} \oplus \beta \\ 0, & \text{otherwise} \end{cases} \quad (A2)$$

Here, $\alpha, \beta \in \{0, 1\}$. The four bipartite distributions $P(ab|A_{i}B_{j}, \rho_{AB}^{j})$ in Eq. (A1) are given as follows:

1. For $A_{i} = 0$, it is given by,

$$P(ab|A_{i}B_{j}, \rho_{AB}^{0}) = \begin{pmatrix}
0 & 1+i\sqrt{2}V & 1-i\sqrt{2}V & 1+i\sqrt{2}V \\
1+i\sqrt{2}V & 0 & 1-i\sqrt{2}V & 1+i\sqrt{2}V \\
1-i\sqrt{2}V & 1+i\sqrt{2}V & 0 & 1-i\sqrt{2}V \\
1+i\sqrt{2}V & 1-i\sqrt{2}V & 1+i\sqrt{2}V & 0
\end{pmatrix},$$

where each row and column corresponds to a fixed measurement $(xy)$ and a fixed outcome $(ab)$ respectively. Throughout the paper we will follow the same convention. Note that, each of the probability distributions must satisfy $0 \leq P(ab|A_{i}B_{j}, \rho_{AB}^{j}) \leq 1$, which implies that $0 < V \leq \frac{1}{\sqrt{2}}$.

This joint probability distribution at Alice and Bob’s side can be reproduced by performing measurements of the observables corresponding to the operators $A_{0} = \sigma_{x}$, $A_{1} = \sigma_{y}$; and $B_{0} = (\sigma_{x} - \sigma_{y})/\sqrt{2}$, $B_{1} = (\sigma_{x} + \sigma_{y})/\sqrt{2}$ on the two-qubit state given by,

$$|\psi_{0}\rangle = \cos \theta |00\rangle + \frac{(1 - i) \sin \theta}{\sqrt{2}} |11\rangle, \quad (A4)$$
with \( \sin 2\theta = \sqrt{2}V; 0 \leq \theta \leq \frac{\pi}{4} \). \(|0\rangle\) and \(|1\rangle\) are the eigenstates of \( \sigma_z \), corresponding to the eigenvalues \(+1\) and \(-1\) respectively.

2. For \( \lambda = 1 \), it is given by,

\[
P(ab|A_xB_y, \rho^{AB}_{\lambda}) = \begin{pmatrix}
\frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} \\
\frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} \\
\frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} \\
\frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V} & \frac{1}{4}\sqrt{2V}
\end{pmatrix}.
\]

Note that, each of the probability distributions must satisfy \( 0 \leq P(ab|A_xB_y, \rho^{AB}_{\lambda}) \leq 1 \), which implies that \( 0 < V \leq \frac{1}{\sqrt{2}} \).

This joint probability distribution at Alice and Bob’s side can be reproduced by performing measurements of the observables corresponding to the operators \( A_0 = \sigma_x, A_1 = \sigma_y; \) and \( B_0 = (\sigma_x - \sigma_y)/\sqrt{2}, B_1 = (\sigma_x + \sigma_y)/\sqrt{2} \) on the two-qubit state given by,

\[
|\psi_{1}\rangle = \cos \theta|00\rangle - \frac{(1-i) \sin \theta}{\sqrt{2}} |11\rangle,
\]

with \( \sin 2\theta = \sqrt{2}V; 0 \leq \theta \leq \frac{\pi}{4} \).

3. For \( \lambda = 2 \), it is given by,

\[
P(ab|A_xB_y, \rho^{AB}_{\lambda}) = \begin{pmatrix}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{pmatrix}.
\]

Note that, each of the probability distributions must satisfy \( 0 \leq P(ab|A_xB_y, \rho^{AB}_{\lambda}) \leq 1 \), which implies that \( 0 < V \leq \frac{1}{\sqrt{2}} \).

This joint probability distribution at Alice and Bob’s side can be reproduced by performing measurements of the observables corresponding to the operators \( A_0 = \sigma_x, A_1 = \sigma_y; \) and \( B_0 = (\sigma_x - \sigma_y)/\sqrt{2}, B_1 = (\sigma_x + \sigma_y)/\sqrt{2} \) used to reproduce the joint probability distributions at Alice and Bob’s side can demonstrate nonlocality of the singlet state given by, \(|\psi^s\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)\). Hence, the Svetlichny family in Example 1 has a steering LHS-LHV model as in Eq. (14) in the range \( 0 < V \leq \frac{1}{\sqrt{2}} \) in Scenario 1.

\[\text{Appendix B: Proof for Lemma 3}\]

Following the steering LHV-LHS model of the Mermin family mentioned in Ref. [35], we can write down the following steering LHS-LHV model of the Mermin family in the range \( 0 < V \leq \frac{1}{\sqrt{2}} \).

\[
P^\lambda_M(abc|A_xB_yC_z) = \sum_{\lambda=0}^{3} r_\lambda P(ab|A_xB_y, \rho^{AB}_{\lambda})P_{\lambda}(c|C_z), \tag{B1}
\]

as it is invariant under the permutations of the parties. Here, \( r_0 = r_1 = r_2 = r_3 = \frac{1}{4} \), and \( P_0(c|C_z) = P^{10}_0, P_1(c|C_z) = P^{01}_0, P_2(c|C_z) = P^{10}_1, P_3(c|C_z) = P^{11}_0 \).

The bipartite distributions in the model (B1) are given as follows:

1. For \( \lambda = 0 \), it is given by

\[
P(ab|A_xB_y, \rho^{AB}_{\lambda}) = \begin{pmatrix}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{pmatrix},
\]

where each row and column corresponds to a fixed measurement \((xy)\) and a fixed outcome \((ab)\) respectively. This joint probability can be reproduced by performing the projective qubit measurements of the observables corresponding to the operators \( A_0 = \sigma_x, A_1 = \sigma_y; \) and \( B_0 = \sigma_x, B_1 = \sigma_y \) on the two-qubit state given by,

\[
|\psi_0\rangle = \cos \theta|00\rangle + \frac{(1+i) \sin \theta}{\sqrt{2}} |11\rangle. \tag{B3}
\]
where, $0 \leq \theta \leq \frac{\pi}{4}$ with $\sin 2\theta = \sqrt{2}V$; $|0\rangle$ and $|1\rangle$ are the eigenstates of $\sigma_z$ corresponding to the eigenvalues $+1$ and $-1$ respectively.

2. For $\lambda = 1$, it is given by

$$P(ab|A_y, B_y, P_{AB}^1) = \begin{pmatrix} \frac{1+V}{4} & \frac{1+V}{4} & \frac{1+V}{4} & \frac{1+V}{4} \\ \frac{1+V}{4} & \frac{1+V}{4} & \frac{1+V}{4} & \frac{1+V}{4} \\ \frac{1+V}{4} & \frac{1+V}{4} & \frac{1+V}{4} & \frac{1+V}{4} \\ \frac{1+V}{4} & \frac{1+V}{4} & \frac{1+V}{4} & \frac{1+V}{4} \end{pmatrix},$$

which can be reproduced by performing the projective qubit measurements of the observables corresponding to the operators $A_0 = \sigma_x$, $A_1 = \sigma_y$, and $B_0 = \sigma_x$, $B_1 = \sigma_y$ on the two-qubit state given by,

$$|\psi_1\rangle = \cos \theta |00\rangle - \frac{(1 + i) \sin \theta}{\sqrt{2}} |11\rangle,$$

where, $0 \leq \theta \leq \frac{\pi}{4}$ with $\sin 2\theta = \sqrt{2}V$.

3. For $\lambda = 2$, it is given by

$$P(ab|A_y, B_y, P_{AB}^2) = \begin{pmatrix} \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} \\ \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} \\ \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} \\ \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} \end{pmatrix},$$

which can be reproduced by performing the projective qubit measurements of the observables corresponding to the operators $A_0 = \sigma_x$, $A_1 = \sigma_y$, and $B_0 = \sigma_x$, $B_1 = \sigma_y$ on the two-qubit state given by

$$|\psi_2\rangle = \cos \theta |00\rangle - \frac{(1 - i) \sin \theta}{\sqrt{2}} |11\rangle,$$

where, $0 \leq \theta \leq \frac{\pi}{4}$ with $\sin 2\theta = \sqrt{2}V$.

4. For $\lambda = 3$, it is given by

$$P(ab|A_y, B_y, P_{AB}^3) = \begin{pmatrix} \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} \\ \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} \\ \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} \\ \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} & \frac{1-V}{4} \end{pmatrix},$$

which can be reproduced by performing the projective qubit measurements of the observables corresponding to the operators $A_0 = \sigma_x$, $A_1 = \sigma_y$, and $B_0 = \sigma_x$, $B_1 = \sigma_y$ on the two-qubit state given by

$$|\psi_3\rangle = \cos \theta |00\rangle + \frac{(1 - i) \sin \theta}{\sqrt{2}} |11\rangle,$$

where, $0 \leq \theta \leq \frac{\pi}{4}$ with $\sin 2\theta = \sqrt{2}V$.

Note that $|\sin 2\theta| \leq 1$ (as $0 \leq \theta \leq \frac{\pi}{4}$), which implies that $V \leq \frac{1}{\sqrt{2}}$. It can be easily checked that the aforementioned observables corresponding to the operators $A_0 = \sigma_x$, $A_1 = \sigma_y$, and $B_0 = \sigma_x$, $B_1 = \sigma_y$ used to reproduce the joint probability distributions at Alice and Bob’s side can demonstrate EPR steering without Bell nonlocality of the singlet state given by, $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Hence, the Mermin family given in Example 2 has a steering LHS-LHV model as in Eq. (14) in the range $0 < V \leq \frac{1}{\sqrt{2}}$ in Scenario 2.

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