Spectator model in D Meson Decays

Mehrdad. Ghominejad
Physics Department, Semnan University, Semnan, Iran
Hossein. Mehraban
Physics Department, Semnan University, Semnan, Iran

In this research we describe effective Hamiltonian theory and apply this theory to the calculation of current-current $Q_{13}$ and QCD penguin $Q_{13,6}$ decay rates. We calculate the decay rates of semileptonic and hadronic of charm quark in effective Hamiltonian theory. We investigate the decay rates of D meson decays according to Spectator Quark Model (SQM) for the calculation of D meson decays. We obtain the total decay rates of semileptonic and hadronic of charm quark in effective Hamiltonian according to colour Favoured (C-F) and colour Suppressed (C-S), and then to added amplitude of processes colour Favoured and colour Suppressed (F-S) and obtain the decay rates of them.

1. Effective Hamiltonian

The Effective Hamiltonian with QCD effects ($c \rightarrow q_s q_T$) is given by

\[ H_{eff} = 2\sqrt{2} G_F \sum_{i=1}^{6} d_i(\mu) Q_i(\mu) \]

The operators $Q_i(\mu)$ can be grouped into two categories. The 1,2-current-current operators $d_i(\mu) = V_{C_i K_M} C_i(\mu)$ denote the relevant CKM factors that are:

\[ d_{1,2} = V_{iC} V^*_{iK} C_{1,2}, \quad d_{3,...,6} = -V_{iC} V^*_{iK} C_{3,...,6} \quad (1) \]

We should take the variable $P_i$ and $P_k$ or x and y as,

\[ P_x = \frac{x M_c}{2}, \quad P_y = \frac{y M_c}{2} \quad (2) \]

The partial decay rate in the c rest frame is,

\[ d^2 \Gamma_{Q_1,...,Q_2}/dxdy = \Gamma_{0c}(\alpha_1 I_{ps}^1 + \alpha_2 I_{ps}^2 + \alpha_3 I_{ps}^3) \quad (3) \]

Here

\[ \alpha_1 = |d_1 + d_2 + d_3 + d_4|^2 + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2 \]
\[ \alpha_2 = |d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2 \]
\[ \alpha_3 = Re(3d_1 + d_2 + d_3 + 3d_4)d_5^* \]

\[ I_{ps}^1 = 6xyf_{ab}(1 - h_{abc}), \quad I_{ps}^2 = 6xyf_{bc}(1 + h_{bea}) \]

\[ I_{ps}^3 = 6xyf_{ac}h_{xa}h_{yc} \]

\[ h_{xa} = \left[ 1 - (x^2/(x^2 + a^2)) \right]^{1/2}, \quad h_{yc} = \left[ 1 - (y^2/(y^2 + c^2)) \right]^{1/2}, \]

\[ h_{xb} = \left[ 1 - (x^2/(x^2 + b^2)) \right]^{1/2}, \quad h_{ya} = \left[ 1 - (y^2/(y^2 + a^2)) \right]^{1/2}, \]

\[ \Gamma_{0c} = 2\sqrt{2} M_c^5/192\pi^3, \quad f_{ab} = 2 - \sqrt{x^2 + a^2} - \sqrt{y^2 + b^2}, \]
\[ f_{bc} = 2 - \sqrt{x^2 + b^2} - \sqrt{y^2 + c^2}, \]
\[ f_{ac} = 2 - \sqrt{x^2 + a^2} - \sqrt{y^2 + c^2} \]

2. Spectator Model

In the spectator model the spectator quark is given a non-zero momentum having in this work a Gaussian distribution, represented by a free (but adjustable) parameter, $\Lambda$

\[ P(|P_s|^2) = (1/\pi^{3/2}\Lambda^3)e^{-\left(P_s^2/\Lambda^2\right)} \quad (4) \]

The total meson decay rate through a particular mode is then assumed to be

\[ \Gamma_{total} = \int \frac{d^2\Gamma}{dp_{i}dp_{k}} P(|P_s|^2) dP_s dp_i dp_k \quad (5) \]

equal to the initiating decay rate. We have

\[ \frac{d^2\Gamma}{dM_{is}dM_{kj}} = \frac{2\pi M_{is} M_{kj}}{m_c} \int \frac{E_i P_s}{P_i^2} \frac{d^2\Gamma}{dp_{i}dp_{k}} P(|P_s|^2) dP_s dp_k \quad (6) \]

Here

\[ M_{is}^2 = (P_i + P_s)(P_i + P_s) = m_i^2 + m_s^2 + 2(E_i E_s - P_i P_s \cos \theta_{is}) \]
\[ M_{k_2} = (P_k + P_k^\dagger)(P_k + P_k^\dagger) = m_k^2 + m_{k_2}^2 + \]
\[ 2(E_k E_k^\dagger - P_k P_k^\dagger \cos \theta_{k_2}) \]
The integration range is restricted by \(|\cos \theta_{k_2}| \leq 1\). We call this mode of quark and antiquark combination (colour favoured). It is also possible that the spectator antiquark combines with the quark \(q_6\), for which we get
\[ \frac{d^2 \Gamma}{dM_{k_2} dM_{k_2}} = 2 \frac{2 \pi M_{k_2} M_{k_2}^\dagger}{M_c} \int \frac{E_k P_k}{P_k^\dagger} \frac{d^2 \Gamma}{dP_i dP_k} \]
\[ P(|P_s|^2) dP_s dP_i \] 
We call this \text{process(C-S)} (colour suppressed). Summing, the decay rates of \(B\) mesons for \text{process(C-F)} and \text{process(C-S)} are:
\[ \Gamma_{(C-F)} = \int_{m_{\text{min}}}^{m_{\text{cut}}} \int_{m_{\text{min}}}^{m_{\text{cut}}} \frac{d^2 \Gamma}{dM_{k_2} dM_{k_2}^\dagger} dM_{k_2} dM_{k_2}^\dagger \]
\[ \Gamma_{(C-S)} = \int_{m_{\text{min}}}^{m_{\text{cut}}} \int_{m_{\text{min}}}^{m_{\text{cut}}} \frac{d^2 \Gamma}{dM_{k_2} dM_{k_2}^\dagger} dM_{k_2} dM_{k_2}^\dagger \]
where \(m_{\text{min}} = (m_q + m_{\bar{q}}), m_{\text{cut}} = M_{q_{6}}, \) and so on.

3. Effective Hamiltonian Spectator Model

The differential decay rates for two boson system in the spectator quark model for current-current plus penguin operators in the Effective Hamiltonian is given by,
\[ \frac{d^2 \Gamma}{d(q_{6i}/M_c)d(q_{6j}/M_c)} = \Gamma_{0c} \frac{8q_{6i}q_{6j}}{\sqrt{\pi M_c} \Lambda} \]
\[ \sqrt{\frac{(2m_i/M_c)^2 + x^2}{x^2}} \int_0^1 dy \int_0^1 dz \zeta_{ps}^{eff} \zeta_{ps}^{eff} z e^{-\beta z^2} \] 
where 
\[ \zeta_{ps}^{eff} = \alpha_1 \zeta_{ps}^{eff} + \alpha_2 \zeta_{ps}^{eff} + \alpha_3 \zeta_{ps}^{eff} \]
The integration range is restricted by the condition \(\cos \theta_{i_2} \leq 1\), thus
\[ \zeta_{1i}^{eff}, \zeta_{2i}^{eff}, \zeta_{3i}^{eff} = \begin{cases} \zeta_{1ps}^{eff}, \zeta_{2ps}^{eff}, \zeta_{3ps}^{eff} & \text{if } (f_{si(z)})^2 \leq 1 \\ \zeta_{io}^{eff}, \zeta_{2o}^{eff}, \zeta_{3o}^{eff} & \text{otherwise} \end{cases} \] 
where
\[ f_{si(z)} = \left(\frac{(m_i + m_{\bar{q}})/M_c}{(q_{6i}/M_c)^2 + (1/M_c) \sqrt{m_{\bar{q}}^2 + (\beta \Lambda z)^2} \sqrt{(2m_i/M_c)^2 + x^2}} / (\beta \Lambda x z/M_c) \right) \]
\[ \zeta_{eff} = \begin{cases} \zeta_{1ps}^{eff}, \zeta_{2ps}^{eff}, \zeta_{3ps}^{eff} & \text{if } (f_{si(z)})^2 \leq 1 \\ \zeta_{io}^{eff}, \zeta_{2o}^{eff}, \zeta_{3o}^{eff} & \text{otherwise} \end{cases} \] 
Now we want to calculate the decay rates of Effective Hamiltonian (\(Q_1, ..., Q_6\)) for F+S at quark-level and spectator model. The Effective Hamiltonian for F+S is given by
\[ H_{eff}^{A+B} = H_{eff}^{b+i\vec{k}} + H_{eff}^{b+i\vec{j}}. \] 
where \(H_{eff}^{b+i\vec{k}}\) is defined by Equations mentioned during in the last page, so we can obtain \(H_{eff}^{b+i\vec{j}}\). The decay rates of current-current plus penguin for F+S is given by,
\[ d^2 \Gamma_{F+S}^{EH}/dxdy = \Gamma_{0c}(I_{1ps} + I_{2ps} + I_{3ps}) \]
\[ I_{1ps} = 6xy.\dot{f}_{ab} \cdot [\alpha_1((3/2) - \alpha_{ab}) + \alpha_2 - \alpha_3 h_xa h_y], \]
\[ I_{2ps} = -6xy.\dot{f}_{ac} \cdot [\alpha_1 h_{ac} + \alpha_3 h_xa h_y], \]
\[ I_{3ps} = 6xy.\dot{f}_{bc} \cdot [\alpha_1/2] h_{bc} + \alpha_2 h_{xb} h_{yc} - h_{bca}]. \]
5. Numerical Results

We use the standard Particle Data Group\cite{5} parameterization of the CKM matrix. Following Ali and Greub\cite{2} we treat internal quark masses in tree-level loops with the values (GeV) $m_b = 4.88, m_s = 0.2, m_d = 0.01, m_u = 0.005, m_c = 1.5, m_e = 0.0005, m_\mu = 0.1, m_\tau = 1.77$ and $m_{\nu_{e}} = m_{\nu_{\mu}} = m_{\nu_{\tau}} = 0.$ Following G. Buckella\cite{6} we choose the effective Wilson coefficients $C_{eff}^{ij}$ for the various $c \to q$ transitions. We have used in Spectator Quark Model the value $\Lambda = 0.6$ GeV\cite{7}. For the maximum mass of the quark-antiquark systems ($m_{cut}$) we take a value midway between the lowest mass $1^-$ state and the next most massive meson. The decay rates of $c$ quark for Effective Hamiltonian and Effective Hamiltonian of F+S shown in the Table 1. Also the decay rates of $c$ quark for F+S is given by

\[
(c \to d\bar{u}\bar{d}) \quad D^+ \rightarrow (\pi^0, \eta, \rho^0, \omega, (\pi^+, \rho^+))
\]

$$BR_{EH}^{F+S} = 2.1023 \times 10^{-2},$$

\[
(c \to s\bar{u}\bar{d}) \quad D^+ \rightarrow (\pi^+, \rho^+, (K^0, K^{*+}))
\]

$$BR_{EH}^{F+S} = 51.2871 \times 10^{-2},$$

\[
(c \to s\bar{u}\bar{s}) \quad D^+ \rightarrow (\eta', \phi, (K, K^{*+}))
\]

$$BR_{EH}^{F+S} = 3.8671 \times 10^{-2}.$$

6. Conclusion

We used Effective Hamiltonian theory and spectator quark model for $c$ quark and calculated hadronic decays of D mesons. In this model we added decays of channel hadronic decays of D mesons. For colour favoured and suppressed we consider the channel $c \to d\bar{u}\bar{d}$ (e.g. $D^+ \to \pi^0\pi^+$) and achieved theoretical values very close to experimental ones. Finally it has been shown the case, in which the theoretical values are better than the amplitude of all the decay rates have been calculated. In table 1 (below) it must be noted that columns 2 and 4 have to be multiplied by $10^{-15}$ and columns 3 and 5 should be multiplied by $10^{-3}$.

\[
\begin{array}{cccc}
\text{Process} & \Gamma_{EH} & BR_{EH} & \Gamma_{EH}^{F+S} & BR_{EH}^{F+S} \\
(c \to d\bar{u}\bar{d}) & 31.689 & 32.12 & 35.611 & 31.262 \\
(c \to d\bar{u}\bar{s}) & 1.0785 & 1.093 & 1.4608 & 1.2824 \\
(c \to s\bar{u}\bar{d}) & 409.44 & 414.95 & 554.45 & 486.74 \\
(c \to s\bar{u}\bar{s}) & 23.836 & 24.157 & 26.927 & 23.638 \\
\end{array}
\]

Table 1: Decay rates ($\Gamma$) and Branching Ratio (BR) of Effective Hamiltonian (EH) and F+S of effective hamiltonian of $c$ quark.

7. Acknowledgments

References

[1] A. J. Buras, Nucl Instrum Meth. A368 (1995) 1-20
[2] A. Ali and C. Greub, Phys. Rev. D 57. N 5 , 2996 (1998)
[3] G. Altarelli, N. Cabibbo, G. Corbo, L. Maiani and G. Martinelli, Nucl. Phys. B208. 365 (1982), A. Ali and C. Greub, Phys. Lett. B 259, 182 (1991).
[4] D. Atwood and A.Soni, ibid. B405, 150 (1997) and C. S. Kim, Y. G. Kim and K. Y. Lee, Phys. Rev. D 57. 4002 (1998)
[5] Particle Data Group, European physical J. C 3, 1 (2005)
[6] G. Buchalla, Nucl Phys, B391 (1993) 501-514
[7] W. N. Cottingham, H. Mehrban and I. B. Whittingham, Phys. Rev. D60 (1999) 114029