Coherent multi-mode conversion from microwave to optical wave via a magnon-cavity hybrid system

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Coherent conversion from microwave to optical wave opens new research avenues towards long distance quantum network covering quantum communication, computing, and sensing out of the laboratory. Especially multi-mode enabled system is essential for practical applications. Here we experimentally demonstrate coherent multi-mode conversion from the microwave to optical wave via collective spin excitation in a single crystal yttrium iron garnet (YIG, Y₃Fe₅O₁₂) which is strongly coupled to a microwave cavity mode in a three-dimensional rectangular cavity. Expanding collective spin excitation mode of our magnon-cavity hybrid system from Kittel to multi magneto-static modes, we verify that the size of YIG sphere predominantly plays a crucial role for the microwave-to-optical multi-mode conversion efficiency at resonant conditions. We also find that the coupling strength between multi magneto-static modes and a cavity mode is manipulated by the position of a YIG inside the cavity. It is expected to be valuable for designing a magnon-hybrid system that can be used for coherent conversion between microwave and optical photons.

I. INTRODUCTION

Strong coupling induced by resonant light-matter interaction can give rise to coherent information transfer between distinct physical systems in quantum and classical information processing. The coherent transfer of quantum state is a key role in realizing large-scale quantum optical networks and long distance quantum sensing and imaging, since it allows quantum information to be exchanged between different systems that operate at different energy scales. A platform for transferring multi-mode states will be attractive to the practical application of quantum-enhance metrology and communication. After the first demonstration of optical frequency conversion, the photon frequency conversion has been implemented with crystals in optical domain, and with superconducting circuits in the microwave domain. Since it has great potential in realizing large-scale quantum optical networks with superconducting qubits, recently, the microwave to optical field conversion has been intensively attracted and experimentally demonstrated by using intermediate systems, such as optomechanical systems, electro-optical systems, atomic ensembles, and magnon. So far, the maximum microwave-to-optical (MO) conversion efficiency reached 47% at low temperature. A ferromagnetic material, an yttrium iron garnet (YIG), in a microwave cavity offers strong interaction between magnon and microwave cavity modes at both low and room temperatures since YIG has a Curie temperature of 560 K and a net spin density of $2.1 \times 10^{22} \mu_B \text{cm}^{-3}$ (Bohr magneton) that is five orders of magnitude higher than a net spin density of $10^{16} - 10^{18} \mu_B \text{cm}^{-3}$ in paramagnetic materials. High Verdet constant with sharp linewidth of electron spin resonance in microwave domain also makes YIG noticeable in Faraday effect. Recently, YIG based materials have been studied on a novel concept for ultrafast magneto-optic polarization modulation with frequencies up to THz orders. Longer spin excitation time than paramagnetic spin system is another advantage of YIG and its hybrid system. In this hybrid system, the MO conversion is achieved through the Faraday effect and Purcell effect. The magnetization oscillation induced and amplified by the Purcell effect of a microwave cavity mode creates the sidebands to the incidental optical wave, resulting in coherent conversion between microwave and optical wave. So far, the MO conversion via YIG-cavity system has been focused only for the Kittel mode.

In this paper, we report coherent multi-mode conversion from the microwave to optical wave fields, which is based on a hybrid system consisting of a sphere of YIG and a three-dimensional rectangular microwave cavity. We experimentally demonstrate and verify that the size of YIG is a dominant factor of the coherent multi-mode conversion efficiency. The conversion efficiency is theoretically derived by using the interaction Hamiltonian with the ferromagnetic-resonance (FRM or Kittel mode, KM) and a higher magneto-static mode (MSM) and experimentally characterized by normal-mode splitting, coupling strengths of the ferromagnetic-resonance, and magneto-static modes. For the near-uniform microwave cavity field, all the spins in the YIG sphere precess in phase and it is coupled to a superconducting microwave resonator. Its conversion efficiency reached 47% at low temperature. A ferromagnetic material, an yttrium iron garnet (YIG), in a microwave cavity offers strong interaction between magnon and microwave cavity modes at both low and room temperatures since YIG has a Curie temperature of 560 K and a net spin density of $2.1 \times 10^{22} \mu_B \text{cm}^{-3}$ (Bohr magneton) that is five orders of magnitude higher than a net spin density of $10^{16} - 10^{18} \mu_B \text{cm}^{-3}$ in paramagnetic materials. High Verdet constant with sharp linewidth of electron spin resonance in microwave domain also makes YIG noticeable in Faraday effect. Recently, YIG based materials have been studied on a novel concept for ultrafast magneto-optic polarization modulation with frequencies up to THz orders. Longer spin excitation time than paramagnetic spin system is another advantage of YIG and its hybrid system. In this hybrid system, the MO conversion is achieved through the Faraday effect and Purcell effect. The magnetization oscillation induced and amplified by the Purcell effect of a microwave cavity mode creates the sidebands to the incidental optical wave, resulting in coherent conversion between microwave and optical wave. So far, the MO conversion via YIG-cavity system has been focused only for the Kittel mode.

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version from microwave photons to optical photons. The total Hamiltonian \( \hat{H}_t \) describing the conversion process including the KM and a higher MSM can be given by

\[
\hat{H}_t = \hat{H}_c + \hat{H}_s + \hat{H}_o,
\]

\[
\hat{H}_c = -i\sqrt{2}\kappa_c \left[ \hat{a}^\dagger \hat{a} e^{i\chi} - \hat{a} \hat{a}^\dagger e^{-i\chi} \right],
\]

\[
\hat{H}_s = \omega_c \hat{a}^\dagger \hat{a} + \omega_K \hat{s}_K^\dagger \hat{s}_K + \omega_M \hat{s}_M^\dagger \hat{s}_M + g_K \left( \hat{a}^\dagger \hat{s}_K + \hat{a} \hat{s}_K^\dagger \right) + g_M \left( \hat{a}^\dagger \hat{s}_M + \hat{a} \hat{s}_M^\dagger \right),
\]

\[
\hat{H}_o = -i\sqrt{2}\delta_K \left( \hat{s}_K + \hat{s}_K^\dagger \right) \left[ \hat{b}_1(t) e^{i\Omega_0 t} - \hat{b}_1^\dagger(t) e^{-i\Omega_0 t} \right]
\]

\[
- i\sqrt{2}\delta_M \left( \hat{s}_M + \hat{s}_M^\dagger \right) \left[ \hat{b}_1(t) e^{i\Omega_0 t} - \hat{b}_1^\dagger(t) e^{-i\Omega_0 t} \right],
\]

(1)

where \( \hbar = 1 \). The subscripts \( K \) and \( M \) stand for the KM and a higher MSM, respectively. The MO conversion proceeds with the following steps in the Hamiltonians: \( \hat{H}_c \rightarrow \hat{H}_s \rightarrow \hat{H}_o \). \( \hat{H}_c \) describes the interaction Hamiltonian between an itinerant microwave mode \( \hat{a}_i \) and the microwave cavity mode \( \hat{a} \) with external coupling rate \( \kappa_c \), which results from the rotating-wave approximation. \( \hat{H}_s \) including the system Hamiltonian describes the interaction Hamiltonian between cavity and magnon modes. \( g_K \) and \( g_M \) represent the magnon-microwave photon coupling strengths for KM and MSM, respectively. Here, we note that \( g_K \) and \( g_M \) include the overlapping coefficient \( \xi \), which is related to the space variation effect between the magnetic field of the cavity mode and magnon modes. \( \hat{a}^\dagger (\hat{a}) \) is the creation (annihilation) operator for the microwave photon at the angular frequency \( \omega_c \). \( \hat{s}_K^\dagger (\hat{s}_K) \) and \( \hat{s}_M^\dagger (\hat{s}_M) \) represent the collective spin excitations for KM and MSM at angular frequency \( \omega_K \) and \( \omega_M \), respectively (see Appendix A). The number of spins in a YIG sphere can contribute to both KM and MSM. \( \hat{H}_o \) describes the interaction Hamiltonian between magnon modes \( \hat{s} \) and a traveling optical photon mode \( \hat{b}_1 \) with angular frequency \( \Omega_0 \). \( \delta_K \) and \( \delta_M \) refer to the optical photon-magnon coupling rate for KM and MSM, respectively. Since it includes both Stokes and anti-Stokes processes that are involved in the MO conversion, we leaves the Hamiltonian \( \hat{H}_s \) without the rotating-wave approximation. The MO conversion indicates that the itinerant microwave photon mode \( \hat{a}_i \) is converted to the traveling optical photon mode \( \hat{b}_0 \).

In our experiment, the strongly coupled magnons and cavity microwave photon mode can be determined by normal mode splittings in transmission spectra which are measured from the input and output channels (see Fig. 2(c)). According to the input-output relations, equations of motions for a cavity mode and magnon modes can be obtained from quantum Langevin equation (see Appendix A). As a result, the transmission for multimagnon modes can be given by

\[
S_{21}(\omega) = -i \frac{2\kappa_c}{\omega - \omega_c + i\kappa_1 - \sum_m g_m^2 \chi_m},
\]

(2)
where

\[ \chi_m(\omega) = \left[ \omega - \omega_m + i\gamma_m \right]^{-1}. \]  

(3)

Here, \( \kappa_t = \kappa_e + \kappa_i \) is the total loss rate which includes both external coupling rate \( \kappa_e \) and internal loss of the cavity \( \kappa_i \).

For the conversion process from the microwave to optical wave, the conversion coefficients with amplification factor \( \kappa_m \) for the KM and a MSM are obtained as (see Appendix A)

\[ S_{31,K}(\omega) = -2\sqrt{\beta_K\delta_K\kappa_e} \frac{g_K\chi_K\chi_e(1 + g_M^2\chi_M\chi_c T_M)}{1 - g_K^2\chi_K\chi_e(1 + g_M^2\chi_M\chi_c T_M)}, \]

(4)

and

\[ S_{31,M}(\omega) = -2\sqrt{\beta_M\delta_M\kappa_e} \frac{g_M\chi_M\chi_e(1 + g_K^2\chi_K\chi_c T_K)}{1 - g_M^2\chi_M\chi_e(1 + g_K^2\chi_K\chi_c T_K)}, \]

(5)

where

\[ \chi_e(\omega) = \left[ \omega - \omega_e + i\kappa_i \right]^{-1}, \]

\[ T_m(\omega) = \left[ 1 - g_m^2\chi_m\chi_e \right]^{-1}. \]

(6)

Therefore, the MO conversion efficiency including both KM and a MSM modes can be given by

\[ \eta_t(\omega) = |S_{31,K}(\omega)|^2 + |S_{31,M}(\omega)|^2. \]

(7)

Here, at the resonant condition \( \omega - \omega_c = \omega - \omega_K = \omega - \omega_M = 0 \), the two-mode conversion efficiency can be represented in terms of cooperativities for \( C_K = \frac{\delta_K}{\kappa_e\gamma_K} \) and \( C_M = \frac{\delta_M}{\kappa_e\gamma_M} \),

\[ \eta_t = \frac{4\kappa_e}{(1 + C_K + C_M)^2} \left[ \delta_K\beta_K\frac{C_K^2}{g_K} + \delta_M\beta_M\frac{C_M^2}{g_M} \right]. \]

(8)

In this work, the two-mode conversion efficiency given in Eq 8 is well matched to the experimental results. If we expand the interaction Hamiltonian to possess higher order terms, the multi-mode MO conversion efficiency can be obtained by

\[ \eta_t = \frac{4\kappa_e}{(1 + \sum_m C_m)^2} \sum_m \left[ \delta_m\beta_m\frac{C_m^2}{g_m} \right], \]

(9)

where \( m \) is a mode index for each MSM. Since so far the MO conversion for multi-modes has not been reported in magnon-cavity system, our theoretical result can be applied to multi-mode conversion based on ferromagnetic material-hybrid systems.

### III. EXPERIMENTS AND RESULTS

As a ferromagnetic material, we use commercial YIG spheres of diameter 0.45, 0.75, and 1 mm from Ferrisphere and Microsphere. A 3D rectangular cavity is made of oxygen-free copper with the volume \( V_c \) of \( 20 \times 20 \times 4 \) mm\(^3\) and the fundamental mode TE\(_{101}\) is used for magnetic-dipole coupling. Fig. 2(a) shows the microwave magnetic field distribution of the fundamental mode TE\(_{101}\) at the resonant frequency \( \omega_c/2\pi \) of 10.598 GHz, simulated by COMSOL Multiphysics\textsuperscript{®}. The YIG...
sphere mounted on the alumina rod along the crystal axis \langle 110 \rangle is placed near the maximum of the magnetic field in order to get the largest coupling strength and the uniformity of the field as shown in Fig. 2(a). Fig. 2(b) presents measured transmission magnitude and phase without a YIG sphere through the cavity. As a result, the frequency of TE\textsubscript{101} mode (\(\omega_c/2\pi\)) is 10.632 GHz, and the external cavity loss rate (\(\kappa_e/2\pi\)) and the internal cavity loss (\(\kappa_i/2\pi\)) are 2.1 MHz and 0.6 MHz, respectively.

In order to manipulate the magnetic field, a set of neodymium-iron-boron magnets applies a static magnetic field of around 380 mT to the YIG sphere. The magnetic components of the microwave field perpendicular to the bias field induce the spin flip, and excite the magnon mode in YIG. The magnetic circuit consists of a set of permanent magnets and a pair of Helmholtz coils with 800 turns of wires for each. The cavity is placed at the center of a pair of Helmholtz coils, so a static magnetic field along the z-axis is applied to the crystal axis \langle 100 \rangle of the YIG sphere across the cavity. Helmholtz coils driven by a bipolar current supplier combine with the permanent magnets and tunes the resonance frequencies of magnon modes. The magnetic field measured by a flux gate sensor (3MTS) provides the field-to-current conversion rate of \(dB/dI = 70\) Gauss/A. Fig. 2(c) shows the experimental set-up for the microwave to light conversion. We use temperature controlled butterfly diode laser to deliver 1550-nm cw input power of 5 mW before the YIG and get the transmission of 80 %. We also use some of optics and microwave components such as polarizer and HWP to define the linear polarization, lens to focus the laser into the YIG, fast photo detector to receive the transmitted laser with optical side band, low noise microwave amplifiers with 30 dB amplification and isolators to increase the signal, and a vector network analyzer by probing the transmission through the hybrid system.

Fig. 3(a) shows the measured microwave transmission spectrum, \(|S_{21}(\omega)|^2\), of the hybrid system with the YIG diameter of 0.45 mm as a function of the frequency and the static magnetic field. The horizontal and diagonal dashed lines (yellow) show the frequency of the fundamental mode TE\textsubscript{101} and the Kittel mode frequency, respectively. The white-dashed line describes the dispersion of the resonance frequency obtained by diagonalizing \(\hat{H}\) as given in Eq. (1). (b) Cross sections of \(|S_{21}(\omega)|^2\) at static magnetic fields corresponding to \(B = 0.3797\), 0.3804, 0.3811, and 0.3818 T. Solid lines are theoretical curves given by Eq. (2) for the data (solid dots). The individual data sets are vertically offset for clarity. (c) The phase \(S_{21}(\omega)\) with theoretical hand-fits at the static magnetic fields. (d) Measured MO conversion spectrum, \(|S_{31,K}(\omega)|^2\), of the same system. Here, the MO conversion spectrum is obtained from the raw data which is amplified by a microwave amplifier (30 dB) and detected by a fast photodiode. (e) Cross sections of \(|S_{31,K}(\omega)|^2\) at static magnetic fields corresponding to \(B = 0.3797\), 0.3804, 0.3811, and 0.3818 T. Solid lines are theoretical curves given by Eq. (7) for the data (solid dots). (f) The phase \(S_{31,K}(\omega)\) with theoretical hand-fits at the static magnetic fields. The individual data sets are vertically offset for clarity.
fundamental mode frequency of the cavity and the diagonal dashed line presents the Kittel mode frequency, \( f_{11} = \omega_{11}/2\pi = \mu_0 \gamma H_{01}/2\pi \) (see Appendix B). White-dashed lines are the dispersion curves of the resonance frequency, \( \omega_{k, \pm} = \omega + \frac{g K}{2} \pm \sqrt{4 g^2 K^2 + (\omega_k - \omega_K)^2} \), which are obtained from the diagonalization of the interaction Hamiltonian \( \hat{H}_s \) in Eq. (1) without the last term. In order to quantify the coupling strength and the damping rate of ferromagnetic resonance frequency, the experimental data at some of magnetic fields are hand-fitted into the theoretical transmission coefficient \( S_{31}(\omega) \) given in Eq. (2) (see Fig. 3(b)). As a result, the total cavity linewidth \( \gamma = \frac{1}{e} \frac{d\delta\theta}{d\omega} \), the frequency \( \gamma \) and polarization matching conditions between the microwave field is coherently converted to the optical wave and the phase of the cavity mode is proportional to the square root of the exchange field. Fig. 3(c) and 3(f) also show that the total cavity linewidth \( \gamma = \frac{1}{e} \frac{d\delta\theta}{d\omega} \), the frequency \( \gamma \) and polarization matching conditions between the microwave field is coherently converted to the optical wave field. Fig. 3(c) shows the cross-sectional experimental data at some of magnetic fields in Fig. 3(d) that are hand-fitted into the theoretical transmission coefficient \( |S_{31}(\omega)|^2 \) given in Eq. (7). As a result, \( g_K/2\pi \) and \( \gamma_K/2\pi \) are 28.5 MHz and 2.4 MHz, respectively that are quite close to the result of \( |S_{21}(\omega)|^2 \). In order to evaluate the MO conversion efficiency, we first estimate the optical photon-magnon coupling rate \( \delta_K \) which is given by \( \delta_K = \frac{g_{K}^{2}}{\zeta_{\mu} K_{\mu} \hbar} \). With \( l = 0.45 \) mm being the length of the YIG sample, \( N_{K} = 3.16 \times 10^{27} \) m\(^{-3} \) and \( V_{m} = \frac{4}{3} \pi \times 0.225 \) mm\(^3\) being the spin density and the spatial volume of the magnetostatic mode, \( V = 3.8 \) radians/cm at 1.55 \( \mu m \) that result in \( G_{K} = 4\gamma_{/n} K_{/2} \) for the Kittel mode reaches \( \frac{\delta_{K}}{2\pi} = 0.0036 \) Hz. Therefore, under the near resonant conditions at \( \omega - \omega_{c} = \omega - \omega_{K} = 0 \), the total conversion efficiency \( \eta_{c} = |S_{31,K}|^2 \) can be approximated in terms of the coupling strength \( g_{K} \).

With all obtained parameters for the YIG sphere with 0.45 mm diameter, we attain \( \eta_{c} = 8.45 \times 10^{-11} \). This low conversion efficiency results from the small light-magnon coupling rate. Actually, the maximum conversion efficiency is occured at particular detunings from the cavity resonance and the Kittel mode frequency. However, in this experiment, we are interested in the multi-mode conversion efficiency at the degenerate point.

In order to examine the YIG size dependence of system parameters, we also measured the transmission spectra of YIG spheres with diameters of 0.75 and 1 mm, as shown in Fig. 4(a) and 4(e). As a result, \( |S_{21}(\omega)|^2 \) and a strong coupling regime is reached for the Kittel mode reaches up to about 3.5 \( \times 10^{3} \) as shown in Table 1. In addition to larger coupling strengths, another avoided level crossing feature is observed because of the strong coupling corresponding to the nonuniform MSMs which can be also coupled to the cavity mode. The coupling strengths of MSM are \( g_{M}/2\pi = 4 \) and 12 MHz, and the decay rates are \( \gamma_{M}/2\pi = 1.1 \) and 0.9 MHz for YIG spheres with 0.75 and 1.0 mm diameter, respectively. Based on the fitting parameters, 2D spectra of \( |S_{21}(\omega)|^2 \) for each case are simulated in Fig. 4(b) and (f). Here, the red-dashed line describes the nonuniform MSM which is identified by magnetostatic theory. In general, the relation between MSM frequencies and the external magnetic field is linear for \( i - |j| = 0 \) or 1 as mentioned in Appendix B. When the YIG sphere is subjected to an oscillating magnetic field at \( \omega_{ij} \) and a strong coupling regime is reached.
at $H_s$, avoided level crossings appear at the regions where the resonance frequencies of two subsystems are matched, that make it possible to distinguish a MSM with $i$ and $j$ associated to an avoided level crossing. In our case, additional avoided level crossing is placed at the (2,0) mode which can be given by $\omega_{20}$.

$$\omega_{20} = \mu_0 \gamma M_s \sqrt{\left(\frac{H_o}{M_s} - \frac{1}{3}\right) \left(\frac{H_o}{M_s} + \frac{7}{15}\right)}.$$  \hspace{1cm} (11)

Fig. 4(c) and (g) show the MO conversion spectra, $\eta$, measured for the YIG diameter of 0.75 and 1.0 mm, respectively. These MO conversion spectra present raw data which are amplified and detected by using a microwave amplifier and a fast photodiode. One can find the same avoided level crossing features, as shown in Fig. 4(a) and 4(c), which clearly demonstrates the coherent conversion from microwave to optical photons. Based on the fitting parameters, 2D spectra of $\eta$ for each case are simulated in Fig. 4(d) and 4(h). As a result, when we take into account both KM and MSM contributions, the total conversion efficiency $\eta$ are $5.12 \times 10^{-12}$ and $3.46 \times 10^{-12}$ for 0.75 and 1.0 mm YIG spheres, respectively. We summarize system parameters for each YIG sphere in Table 1 that are obtained from the two-mode MO conversion process.

To examine the size dependence of a YIG sphere for MO conversion efficiency, we first evaluate the volume dependence of parameters used for ML conversion efficiency at resonant conditions as shown in Fig. 5. According to the Ref. [30], it demonstrated that the coupling strength $g_K$ of the Kittel mode is proportional to the square root of the volume (or the number of spins) of YIG spheres. $g_K$ is linear-fitted to $f(x) = 130.97 x$, where $x$ is the square root of volume $V^{1/2}$. For the higher MSM, $g_M$ is not proportional to the linear function, but rather the quadratic function which is $f(x) = 22.19 x^2$. This might be due to the fact that the spin excitations induced by non-uniform field do not linearly contribute to a higher mode. According to the relation of $C_m = \frac{g^2}{\kappa_t \gamma_m}$, the cooperativity $C_K$ is fitted to $f(x) = 6569.43 x^2$ and for $C_M$, $f(x) = 228.79 x^4$ is used.

In addition, $\delta_K$ and $\delta_M$ also have the dependence of the number of spins. $\delta_K$ is fitted to $f(x) = 0.693 + 0.625/x$ and $\delta_M$ is fitted to $f(x) = 0.139/x^2$ as shown in Fig. 5(b). By using these fitting values of system parameters, we obtain the theoretical fit curve for the MO conversion...
TABLE I. System parameters for two-mode MO conversion

| Parameter | 0.45-mm dia. | 0.75-mm dia. | 1.0-mm dia. |
|-----------|--------------|--------------|-------------|
| $g_K$ [MHz] | $2\pi \times 28.6$ | $2\pi \times 67.3$ | $2\pi \times 91.0$ |
| $\gamma_K$ [MHz] | $2\pi \times 2.3$ | $2\pi \times 1.1$ | $2\pi \times 0.95$ |
| $g_M$ [MHz] | $< 2\pi \times 1.0$ | $2\pi \times 4.0$ | $2\pi \times 12.0$ |
| $\gamma_M$ [MHz] | $> 2\pi \times 2.0$ | $2\pi \times 1.5$ | $2\pi \times 0.9$ |
| $C_K$ | 132 | 1373 | 3487 |
| $C_M$ | 0.19 | 3.6 | 64 |
| $N_K$ | $1.51 \times 10^{17}$ | $8.36 \times 10^{17}$ | $1.53 \times 10^{18}$ |
| $N_M$ | $< 1.84 \times 10^{14}$ | $2.95 \times 10^{15}$ | $2.66 \times 10^{16}$ |
| $V_m$ [m$^3$] | $4.77 \times 10^{-11}$ | $2.21 \times 10^{-10}$ | $5.24 \times 10^{-10}$ |
| $n_K$ [m$^{-3}$] | $3.16 \times 10^{27}$ | $3.79 \times 10^{27}$ | $2.92 \times 10^{27}$ |
| $n_M$ [m$^{-3}$] | $< 3.87 \times 10^{24}$ | $1.34 \times 10^{25}$ | $5.07 \times 10^{25}$ |
| $G_K$ [m$^2$] | $4.81 \times 10^{-25}$ | $4.02 \times 10^{-25}$ | $5.21 \times 10^{-25}$ |
| $G_M$ [m$^2$] | $> 1.14 \times 10^{-22}$ | $2.99 \times 10^{-23}$ | $3.93 \times 10^{-22}$ |
| $\delta_K$ [mHz] | $2\pi \times 3.61$ | $2\pi \times 1.80$ | $2\pi \times 1.75$ |
| $\delta_M$ [Hz] | $2\pi \times 2.95$ | $2\pi \times 0.512$ | $2\pi \times 0.101$ |
| $\eta_t$ | $8.45 \times 10^{-11}$ | $5.12 \times 10^{-12}$ | $3.46 \times 10^{-12}$ |

Subscripts $K$ and $M$ denote the Kittel mode and MSM, respectively.

Efficiency based on Eq. (8) as presented in Fig. (b). As a YIG size increases, the total MO conversion efficiency at the resonant condition decreases since the increments of coupling strength and cooperativity lead to the drop in the MO conversion efficiency as given by Eq. (8). In our system, the conversion efficiency at the resonant condition is limited to $10^{-11}$ order. This mainly comes from the small coupling rate $\delta_K$ and $\delta_M$ between the optical photons and magnons although it depends on the experimental conditions such as the quality of the sample and proper alignment. Therefore, we need to improve the coupling rate $\delta_n$ to enhance coherent quantum conversion efficiency between microwave and optical photons. There are several suggestions as mentioned in ref. [26]. One possible way was to use the optical whispering gallery modes (WGMs) of an YIG sphere. No one has achieved a significant improvement, however, supposedly due to the small overlap between the Kittel mode and WGMs. Other suggestions are to utilize a magnetic material with a large Verdet constant such as CrBr$_3$ and iron garnet based on rare-earth atoms.

IV. DISCUSSION

We clearly observe not only the YIG size dependence of the MO conversion but also the coupling strength between the multi-magnetostatic mode and a cavity. But note that the multi-mode MO conversion features are not prominent compared to the single-mode MO conversion since the most of spins are involved in the KM mode that makes $g_M$ and $C_M$ much lower than the values of $g_K$ and $C_K$. In order to make the dominant contribution of spins to higher MSM, we carefully position a 1.0 mm YIG sphere off the uniform microwave mode region, so that a non-uniform MSM also appears at the degenerate point as shown in Fig. (a). In this configuration, the anti-crossings due to the higher MSM become larger since the number of spins contributing to the higher mode increases. Fig. (b) shows the simulation result of $|S_{21}(\omega)|^2$ based on Eq. (2). As a result, $g_K/2\pi$ and $g_M/2\pi$ are 83.4 and 25 MHz, respectively, which are few orders larger than decay rates of $\gamma_K/2\pi = 1.1$ and $\gamma_M/2\pi = 0.5$ MHz, that indicates the strong couplings between the cavity mode and the KM and MSM. Fig. (c) presents the 2D spectrum of $\eta_t$. Based on the measured data, $\eta_t$ is simulated by using Eq. (1) as shown in Fig. (d). We find out that the theoretical model is well matched with the experimental results. Here, we ignore higher modes in the tail of the spectrum because their contributions are very small in the MO conversion efficiency. The total multi-mode MO conversion efficiency is $1.02 \times 10^{-11}$ at the resonant condition. To date, adjustable MO conversion for multi-modes has not been reported in magnon-cavity system.
FIG. 6. (a) Measured $|S_{21}(\omega)|^2$ through the 1.0-mm YIG-cavity hybrid system while YIG position adjusted to enhance the MSMs. (b) The simulated spectrum of $|S_{21}(\omega)|^2$ from Eq. (2). (c) Measured $\eta_t$ of the same system. The measured spectrum is based on the raw data which is amplified by a microwave amplifier (30 dB) and detected by a fast photodiode. (d) The simulated spectrum of $\eta_t$ from Eq. (7).

V. CONCLUSION

We have experimentally demonstrated coherent multi-mode conversion from microwave to optical fields via a YIG sphere in a rectangular microwave cavity. A large number of spins in ferromagnetic materials easily couple the collective excitation to cavity photons, that makes it possible to hybridize the microwave photon modes and magnetostatic modes. A traveling optical field is coupled to a microwave field through this hybrid system. We first observed YIG size dependence of conversion efficiency by measuring the normal-mode splitting between the magnetostatic modes and the microwave cavity modes, where the coupling strength is in the order of magnitude larger than the decay rates. Based on our multi-mode MO conversion model, we analyzed all the system parameters with experimental data, confirming that the theoretical model is consistent with the experimental results. The total multi-mode conversion efficiency at the resonant condition reaches $1.02 \times 10^{-11}$ for 1.0 mm-YIG sphere. We also evaluate the multi-mode MO conversion efficiency by manipulating position of the YIG sphere inside the cavity. These sharp and adjustable multi-mode conversion shows the possibility of coherent conversion of multi-mode quantum states while keeping coherence time. This work will also provide optimal design conditions of a cavity magnon-microwave photon system that can be used for coherent conversion between microwave and optical photons.

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Appendix A: The interaction Hamiltonian for the multi-mode microwave-to-optical wave conversion

The Hamiltonian for the magnon-cavity system can be written as

\[ H_s = \omega_c \hat{a}^\dagger \hat{a} + \sum_{m=K,M} \left[ g_{BM} B_z^m \hat{S}_z^m + g_m (\hat{a} \hat{S}_m^+ + \hat{a}^\dagger \hat{S}_m^-) \right], \]

(A1)

where \( \omega_c \) is the angular frequency of the cavity mode TE\(_{101} \), \( \hat{a}^\dagger \) (\( \hat{a} \)) is the microwave photon creation (annihilation) operator, and \( m = K, M \) denotes the Kittel mode (KM) and magneto static modes (MSM), respectively, \( g \) is the electron g-factor, \( \mu_B \) is the Bohr magneton, and \( B_z^m \) is the effective magnetic field affected by the magnon modes of the YIG sphere. The exchange interaction between electron spins can be ignored because of the long-wavelength discrete modes of spins in the YIG sphere. Since the frequency of the corresponding magnon mode is different from each other, the Hamiltonian for each magnon mode can be written separately. Here, \( \hat{S}_m^\dagger \), \( \hat{S}_m^- \) by using the Holstein-Primakoff transformation,

\[ \hat{S}_m^\dagger = \hat{S}_m^+ + i \hat{S}_m^- = \sqrt{2 S_m} \hat{S}_m^\dagger S_m^\dagger, \hat{S}_m^- = \sqrt{2 S_m} - i \hat{S}_m^\dagger S_m^- \]

\[ \left( \sqrt{2 S_m} - i \hat{S}_m^- \hat{S}_m^\dagger \right) \hat{S}_m^- = \sqrt{2 S_m} = \left( \sqrt{2 S_m} - i \hat{S}_m^- \hat{S}_m^\dagger \right) \hat{S}_m^- = S_m \]

\[ \hat{S}_m^\dagger = \hat{S}_m^+ + i \hat{S}_m^- \]

For the Kittel mode, since the magnetic dipole coupling between the spins engenders a uniform demagnetization field parallel to the magnetization in a sphere, the demagnetizing field plays no role in the magnetization dynamics for the Kittel mode. For the non-uniform profile for MSM, the variation in space plays a crucial role not only in the frequency calculation but also in the coupling with the exciting field as well as the light.

Therefore, the interaction Hamiltonian of the multi-mode MO conversion can be given by Eq. (A1) which consists of the magnon, microwave photon, optical photon, and their interactions. According to the input-output relation, equations of motions for a cavity mode and magnon modes can be obtained from quantum Langevin equation. For the cavity mode \( \dot{a} \),

\[ \dot{a}(t) = -i \[ a, \hat{H}_s \] - \kappa \dot{a}(t) - \sqrt{2 \kappa} a(t) \]

\[ = -i \omega_c \dot{a}(t) - i (g K \hat{s}_K(t) + g M \hat{s}_M(t)) - \kappa \dot{a}(t) \]

\[ - \sqrt{2 \kappa} a(t), \]  \hspace{1cm} (A3)

where the total loss rate \( \kappa \) includes both external and internal losses of the cavity. The cavity mode \( \dot{a} \) can be given by

\[ \dot{a}(t) = \chi_c(\omega) \left[ g K \hat{s}_K(t) + g M \hat{s}_M(t) - i \sqrt{2 \kappa} a(t) \right], \]  \hspace{1cm} (A4)

where

\[ \chi_c(\omega) = [\omega - \omega_c + i \kappa]^{-1}. \]  \hspace{1cm} (A5)

Since magnons do not couple directly to the cavity, no additional input and output magnons are involved. Therefore, the equation of motion of \( \hat{s}_K \) can be given by

\[ \dot{s}_K(t) = \chi_K(\omega) g \dot{a}(t), \]

where

\[ \chi_K(\omega) = [\omega - \omega_K + i \gamma]^\dagger \]  \hspace{1cm} (A6)

In the same manner, \( \hat{s}_M \) has the similar form which is

\[ \dot{s}_M(t) = \chi_M(\omega) g \dot{a}(t), \]

where

\[ \chi_M(\omega) = [\omega - \omega_M + i \gamma]^\dagger \]  \hspace{1cm} (A7)

Substituting Eq. (A6) and Eq. (A8) into Eq. (A4) and applying the Fourier transform of the cavity mode \( \dot{a} \), we can derive

\[ \dot{a}(\omega) = -i \sqrt{2 \kappa} T^{-1} \dot{a}(\omega), \]

(A10)

where

\[ T = \omega - \omega_c + i \kappa - (g_K^2 \chi_K + g_M^2 \chi_M). \]

(A11)

In our experiment, we obtain the transmission spectrum which can be determined by measuring the output port 2 from the input port 1. For no input in port 2 and the same external coupling rate at both ports, the boundary condition becomes \( \dot{a}_{0,2}(\omega) = \sqrt{2 \kappa} \dot{a}(\omega) \) that results in the transmission,

\[ S_{21} \equiv \frac{\dot{a}_{2,2}}{\dot{a}_{1,1}} = -i 2 \kappa T^{-1}. \]

(A12)

For the transmission for multi modes, Eq. (A12) can be extended to \( T = \omega - \omega_c + i \kappa - \sum_m g_m^2 \chi_m \).
In the conversion process from microwave to optical wave, we can obtain the equation of motions for magnon modes which are given by

$$\dot{s}_K(t) = -i[H_s, \hat{s}_K] - \gamma_K \hat{s}_K(t) - \sqrt{2\omega_K} \left( \hat{b}_1(t)e^{i\omega t} - \hat{b}_1^*(t)e^{-i\omega t} \right)$$

$$\dot{s}_M(t) = -i[H_s, \hat{s}_M] - \gamma_M \hat{s}_M(t) - \sqrt{2\omega_M} \left( \hat{b}_1(t)e^{i\omega t} - \hat{b}_1^*(t)e^{-i\omega t} \right).$$  \hspace{1cm} (A13)

As a result, magnon modes $\hat{s}_K$ and $\hat{s}_M$ are written as

$$\dot{s}_K(t) = \chi_K \left[ g_K \hat{a}(t) - i\sqrt{2g_K} \hat{b}_1(t) \right]$$

$$\dot{s}_M(t) = \chi_M \left[ g_M \hat{a}(t) - i\sqrt{2g_M} \hat{b}_1(t) \right].$$  \hspace{1cm} (A14)

After substituting Eq. (A14) into Eq. (A13) and applying the Fourier transform, we can obtain magnon modes for KM and MSM which are given by

$$\hat{s}_K(\omega) = g_K g_M \chi_K \chi_c T_K \hat{s}_M(\omega) - i\sqrt{2\omega} g_K \chi_K \chi_c T_K \hat{a}_1(\omega) - i\sqrt{2\omega} g_K \chi_K T_K \hat{b}_1(\omega)$$

$$\hat{s}_M(\omega) = g_K g_M \chi_K \chi_c T_M \hat{s}_M(\omega) - i\sqrt{2\omega} g_M \chi_M \chi_c T_M \hat{a}_1(\omega) - i\sqrt{2\omega} g_M \chi_M T_M \hat{b}_1(\omega),$$  \hspace{1cm} (A15)

where

$$T_K(\omega) = \left[ 1 - g_K^2 \chi_K \chi_c \right]^{-1}$$

$$T_M(\omega) = \left[ 1 - g_M^2 \chi_M \chi_c \right]^{-1}.$$  \hspace{1cm} (A16)

If we substitute $\hat{s}_K(\hat{s}_M)$ into $\hat{s}_M(\hat{s}_K)$ in Eq. (A15), we can obtain the MO conversion coefficients for KM and MSM. For the KM, by considering the Stokes ($\Omega = \Omega_0 - \omega$) and anti-Stokes ($\Omega = \Omega_0 + \omega$) processes and the boundary conditions $\hat{b}_1(\Omega_0 - \omega) = \hat{b}_1(\Omega_0 + \omega) + \sqrt{2\omega} K \hat{c}_K(\omega)$ and $\hat{b}_1(\Omega_0 + \omega) = \hat{b}_1(\Omega_0 - \omega) + \sqrt{2\omega} K \hat{c}_K(\omega)$, the conversion coefficient for the KM is obtained as

$$S_{31,K}(\omega) = \frac{\sqrt{\beta_K}}{2i} \left( \frac{\hat{b}_1(\Omega_0 - \omega)}{a_i(\omega)} + \frac{\hat{b}_1(\Omega_0 + \omega)}{a_i(\omega)} \right)$$

$$= -2\sqrt{\beta_K} \delta_K \chi_c \chi_0 \left[ \frac{g_K \chi_c \chi_0}{1 - g_K^2 \chi_c \chi_0} \left( 1 + g_K^2 \chi_c \chi_0 T_K \right) \right]$$

where $\beta_K$ is the amplification factor. Here, we point out that, if we take into account the MO conversion of only the KM ($g_M = 0$), Eq. (A17) becomes the same result as the single-mode MO conversion coefficient in Ref. [26]. In the same manner, we can induce the MO conversion coefficient for a MSM by using similar boundary conditions and amplification factor of $\beta_M$ that is given by

$$S_{31,M}(\omega) = -2\sqrt{\beta_M} \delta_M \chi_0 \left[ \frac{g_M \chi_0 \chi_0}{1 - g_M^2 \chi_0 \chi_0} \left( 1 + g_M^2 \chi_0 \chi_0 T_K \right) \right].$$

Therefore, the conversion efficiency for the two-mode MO conversion for the KM and a MSM can be obtained as

$$\eta(\omega) = \left| S_{31,K}(\omega) \right|^2 + \left| S_{31,M}(\omega) \right|^2.$$  \hspace{1cm} (A19)

Appendix B: Magnetoelastic modes in a ferromagnetic sphere

Magnons are spin excitations describing small perturbations to the magnetization of a ferromagnetic system. A small oscillating magnetic field in the plane perpendicular to the bias field can lead the alignment of spins to deviate slightly from the bias direction. The bias field exerts a torque on misaligned spins, and then the spins begin precessing around it. L.R. Walker first considered the relationship between the resonance frequency and the internal static field of a ferromagnetic sphere[25,26]. He assumed that the microwave magnetic fields in spheroids satisfy the magnetostatic approximations. The allowed resonant frequencies of MSMSs in a sphere inserted in a microwave cavity can be derived from the characteristic equation in terms of associated Legendre function

$$P_i^j(\cos \xi) \quad (i, j \geq 0),$$

$$i + 1 + \xi_0 P_i^j(\xi_0) P_i^j(\xi_0) \pm j\chi_2 = 0,$$  \hspace{1cm} (B1)

where $\xi_0^2 = 1 - \frac{1}{\chi_1}$, $\chi_1 = \frac{\gamma M_H}{\gamma H^2 - j^2}$, $\chi_2 = \frac{\gamma M_H}{\gamma H^2 - j^2}$, $H_i = H_0 - \frac{M_s}{\gamma H^2}$, and $P_i^j(\xi_0) = \frac{dP_i^j(\xi_0)}{d\xi}$. Here, $H_i$ and $H_o$ are internal and external magnetic fields, respectively. $\mu_0 M_s = 0.178 \text{ T}$ (at 298 K), is the saturation magnetization, $\mu_0$ is the vacuum permeability, $\frac{\gamma}{2\pi} = 28 \text{ GHz/T}$ is the gyromagnetic ratio, and $f$ is the frequency. $i$ and $j$ are mode indices that $i \geq 1$ is an integer and $j$ is also an integer obeying $-i \leq j \leq i$.

For a single mode solution, it is labelled with $(i, j)$. For MSMSs with $i - |j| = 0$ or 1, the relationships between the resonant frequencies and the external magnetic field can be given by

$$\omega_{ij} = \gamma \frac{H_o + \left( \frac{j}{2j + 1} - \frac{1}{3} \right) \gamma M_s}{\mu_0} \quad (i = j),$$

$$\omega_{ij} = \gamma \frac{H_o + \left( \frac{j}{2j + 3} - \frac{1}{3} \right) \gamma M_s}{\mu_0} \quad (i = j + 1).$$

Here, the $(1,1)$ FMR mode, known as the Kittel mode, is the lowest mode in which all spins precess in phase which gives a frequency given by $\omega_{11} = \mu_0 \gamma H_o$.

Appendix C: Microwave reflection spectrum

Figure 7(a) and (b) show the measured reflection spectrum $|S_{11}(\omega)|^2$ and the phase $S_{11}(\omega)$ of the hybrid system with the 0.45 mm-dia YIG as a function of the microwave frequency. From the boundary condition $\hat{a}_a(\omega) = \frac{g_i}{g_o} \hat{a}_c(\omega)$.
\[
\hat{a}_i(\omega) + \sqrt{2\kappa_e} \hat{a}(\omega) \text{ and equations } \text{(A1)} \text{ and } \text{(A4)}, \text{ we can easily obtain the reflection coefficient } S_{11}(\omega),
\]
\[
S_{11}(\omega) = 1 - \frac{i2\kappa_e}{\omega - \omega_c + i\kappa_t - \sum_m g_m^2 \chi_m}. \tag{C1}
\]
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