Negative group delay of reflected Weyl quasiparticles

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Abstract
When an electron is incident from a Weyl material to an insulator and totally reflected, it suffers a reflection group delay and a reflection shift (Goos–Hänchen and/or Imbert–Fedorov shifts). We found the group delay is negative for half of the incident states. The negative group delay does not mean the electron is bounced back before its injection, but is an effective acceleration of the electron near the interface induced by self-interference. The reflection shift orients circulating the points at which the surface-bulk state transition occurs. The reflection shift and the group delay cause velocity correction of the bound states in the Weyl material sandwiched by two insulators. The velocity correction features induced by the negative group delay were verified by a tight-binding calculation, in which the concept of group delay is not used.

1. Introduction
Electrons in Dirac materials behave like photons in medias. The electronic counterparts of many optical effects can be found in condensed matter physics. The 2D electron optics was firstly noticed in graphene systems. A pn graphene junction serves as a negative refraction interface for electrons [1], and its Veselago focus effect was verified by transport experiment [2]. Due to the variety of degrees of freedom and the complexity of electronic dispersion, lots of unusual refraction and diffraction effects, were subsequently discussed [3–10]. The Weyl semimetal, that accommodates pairs of Dirac points named by Weyl nodes, can be regarded as the 3D analogue of graphene [11–13]. The Weyl nodes of one pair possess opposite chiralities, which results in chiral anomaly [14, 15] and negative longitudinal magnetoresistence [16–19]. Electron optics in 3D version, such as Klein tunneling, negative refraction, Veselago focusing and deflected diffraction [20–24], in Weyl materials were proposed too. From the view of wave packet dynamics, if an electron is injected onto a junction interface, the spot of reflection on the interface does not exactly overlap with that of the incident one. The longitudinal and transverse components of the relative shift are called Goos–Hänchen shift and Imbert–Fedorov shift. The reflection shift can happen both in graphene [25–27] and Weyl materials [28–32]. The reflection shift along the interface consumes a period of time, that was not recognized in previous literatures [27, 30], and the shift-induced in-plane velocity has not been well addressed up to now.

In this paper, we calculate the reflection phase when the electron is injected from a Weyl material to an insulator and investigated the reflection shift as well as the group delay. The reflection shift orients as a semi-vortex circulating the point at which the surface-bulk state transition occurs. The group delay is negative for some incident parameters, that is an effect of self-interference. When the Weyl material is sandwiched by two insulators, bound states are formed by multiple reflections at the beside interfaces and the in-plane velocity of the bound states is modified by the reflection shift. The paper is organized as follows. In section 2, we establish the minimal model of the Weyl-insulator interface to solve the reflection problem and figure out the features of the reflection shift and group delay. In section 3, we use the wave packet dynamics to show physical origination of the group delay and demonstrate that the positive or negative group delay can be viewed as the deceleration or acceleration effect near the interface. In section 4, the velocity correction in the sandwiched structure is derived analytically by means of the reflection phase

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and is calculated numerically using the tight-binding model, and the consistency between the two methods implies the negative group delay results in correct physics. In section 5, we discuss the relationship between the negative group delay with the Hartman effect and summarize our main conclusions. Additionally, a few appendix sections are presented to explain some calculation details.

2. Reflection shift and group delay of Weyl electrons

The minimal bipartite Hamiltonian, that can describe Weyl materials as well as insulators, can be written as \( H = \mathbf{q} \cdot \mathbf{\sigma} \), where \( \mathbf{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \) is the vector of Pauli matrices in spin (pseudo-spin) space, and \( \mathbf{q} \) has different meanings when applying the equation on different types of materials. If the material is a Weyl semimetal (material 1), \( \mathbf{q} = \mathbf{k} - \mathbf{k}_W \) with \( \mathbf{k}_W = (k_0, 0, 0) \) being the position of positive chirality Weyl node, and if it is an insulator (material 2), \( \mathbf{q} = (k_x^2 + \Delta, k_y, k_z) \) with \( \Delta > 0 \) being the parameter to produce an energy gap \( 2\Delta \). The Eigen pair of the Hamiltonian are

\[
E = \pm q, \quad |\psi\rangle = \sqrt{\frac{1}{2E}} \left( e^{i\phi/2} \sqrt{E + q_z} \pm e^{-i\phi/2} \sqrt{E - q_z} \right),
\]

where \( \phi \) is the azimuthal angle of \( \mathbf{q} \). In the following, the quantities for the two materials will be distinguished by labeling them with subscripts 1 and 2. In the above description, the reduced Planck constant \( \hbar \) and the Fermi velocity of the Weyl material \( v_F \), which should appear in the Weyl Hamiltonian as \( H = \hbar v_F \mathbf{k} \cdot \mathbf{\sigma} \), are set to be unit. To compare the calculations of analytical and tight-binding methods, the lattice constant \( a_0 \) is adopted as the length unit, and correspondingly the units of wave vector, time and energy are \( 1/a_0, a_0/v_F \) and \( \hbar v_F / a_0 \) respectively.

When a plane wave of the conduction band \( (E > 0) \) from the Weyl semimetal is incident to the insulator, the incident-reflected and the transmitted waves are connected at the interface locating at, say, \( z = 0 \). We apply the continuity equation \( |\psi_i\rangle + r |\psi_r\rangle = t |\psi_t\rangle \) at the interface, where \( r \) and \( t \) are the reflection and transmission coefficients, and \( |\psi_i\rangle, |\psi_r\rangle \) and \( |\psi_t\rangle \) are the incident, reflected and transmitted states, respectively. The reflection coefficient is worked out as

\[
r = -\frac{e^{i\Delta\phi/2} \cos \frac{q_z}{2} \sin \frac{\theta}{2} - e^{-i\Delta\phi/2} \sin \frac{q_z}{2} \cos \frac{\theta}{2} e^{-E + q_z}}{e^{i\Delta\phi/2} \sin \frac{q_z}{2} \sin \frac{\theta}{2} - e^{-i\Delta\phi/2} \cos \frac{q_z}{2} \cos \frac{\theta}{2} e^{-E + q_z}},
\]

where \( \theta \) is the polar angle of \( \mathbf{q} \) and \( \Delta\phi = \phi_2 - \phi_1 \) is the azimuthal angle difference. We consider that the incident energy lies in the energy gap of the insulator \( (E < \Delta) \). In the case, \( q_z \) is pure imaginary so that the transmitted wave is evanescent and the reflection probability is unit, i.e. \( r = e^{i\beta} \), where \( \beta \) is the complex angle of \( r \) and reveals the additional phase acquired by the electron when reflected.

The reflection phase plays important role in wave packet dynamics. If a wave packet hits on the interface, the reflection outgoing point is not overlap with the hitting point and a reflection shift occurs. The reflection does not happen instantly but takes a while called group delay or phase time. The reflection shift and group delay are related with the reflection phase by

\[
s = -\nabla_E \beta - \nabla_E \phi_1 \cos \theta_1, \tag{3}
\]

\[
\tau = \partial_E \beta + \partial_E \phi_1 \cos \theta_1, \tag{4}
\]

where \( \nabla_E \) is the gradient operator in \( q_{1x}, q_{1y} \) plane for fixed \( z \). Equation (3) has ever appeared in reference [32]. By means of 3D wave packet dynamics presented in appendix A, both equations (3) and (4) can be proven, as shown in appendix B.

Figure 1 shows the reflection shift and the group delay as functions of \( q_{1x} \) and \( q_{1y} \). The incidence can only be defined in the circle defined by \( q_{1x}^2 + q_{1y}^2 = E^2 \), and from here on, the circle will be referred as the incident circle. Because of the alike calculation between equations (3) and (4), the reflection shift and the group delay share some similar features: they both vary slowly near the circle center, and become very large near the circle edge. For the reflection shift, there is a semi-vortex circulating around the point \( (0, -E) \), at which the surface band joints with the bulk states, as reported in references [31, 32]. Interestingly, the reflection group delay is negative in half region of the incident circle. The negative group delay effect for total reflection is seldom noticed in condensed matter physics. The negative group delay effect could also be found in other types of interfaces, as proven in appendix C.
3. Reflection of wave packets

The reflection group delay is often used to describe the duration of reflection retarded of injection. Because the electron cannot be reflected before it is injected, the physical meaning of negative group delay needs to be clarified. According to reference [33], the group delay consists of two components: one is the dwell time, which describes the mean time of the electron stored behind the interface as an evanescent wave before it is reflected back, the other is called the self-interference time, which is caused by the interference between the injected and the reflected waves. We notice that the former is always positive but the latter is not so. To find out what happens when the group delay becomes negative, we construct a 1D incident wave packet \( \Psi_1 = \int f(q_{1z})e^{iq_{1z}z} dz dq_{1z} \), where \( f(q_{1z}) = e^{-i(q_{1z}-q_{1z0})/\Gamma^2} \) is the Gaussian distribution function, \( q_{1z0} \) is the Gaussian peak center, and \( \Gamma \) reflects the wave packet width. The corresponding reflection wave function is \( \Psi_r = \int f(q_{1z})e^{i(q_{1z}z)} dz dq_{1z} \). The total wave function in the region \( z < 0 \) is thus \( \Psi = \Psi_1 + \Psi_r \). Behind the interface, the transmitted wave function decays off, but it is not our concern.

Figure 2 shows time evolution of wave packets with negative and positive delays. We set \( \Delta \) to be quite large so as to the dwell time is negligible and the self-interference time in the group delay is dominant. The two wave packets are initiated far away from the interface and propagates in \( z \)-direction. When they move close to the interface, each wave packet hits the interface and is rebounded. The reflected front tail overlaps and interferes with the remained part and the self-interference makes the wave packets for negative and positive delays deformed in different ways. When a small portion of the peaks is reflected, the difference of the two peak profiles only appears in their front parts, and when the most portion of the peaks is reflected, the separation between the two peaks is formed. Due to the self-interface, the wave packet of the negative (positive) group delay looks like being accelerated (decelerated), and the acceleration (deceleration) only takes place near the interface. Though the two peaks begin at the same position, they are separated by a distance \( \Delta z \) after the reflection is completed. According to the above analysis, the negative group delay does not mean the reflection happens before the injection, but is the time advance of the electron return its initial position relative to case without picking up the reflection phase shift.

4. Velocity correction in sandwiched structures

The reflection shift and group delay induce a velocity correction of electron in-plane motion. However, if one detects the velocity near the interface, the reflection is not completed, while if does far away the interface, the portion of reflection time in the total time is too small to show its impact. We overcome this problem by considering a double interface system in which the Weyl material is sandwiched by two insulators. Because the bound states in the Weyl material are formed by electrons being reflected forth and back between the interfaces, the reflection phase effect is accumulated and the velocity of each bound state is corrected.

Apart from the interface at \( z = 0 \), we set the other interface locating at \( z = -L \). When the electron is reflected on the lower interface, another reflection phase is gained. A similar continuity equation can be established and a new reflection coefficient \( r' (\phi) = r(-\phi) \) can be obtained. Correspondingly, the reflection phase is \( \beta' = \beta(-\phi) \) and the reflection shift and group delay are \( \beta' = s(-\phi) \) and \( r' = r(-\phi) \) respectively. The bound states have to meet the constraint of Sommerfeld quantization condition

\[
\oint q_{1z} dz + \beta + \beta' = 2n\pi
\]  

(5)
with \( n \) being integer. Because the phase shift accumulated by the free propagation in a round of forth and back is \( 2q_{1z}L \), we have

\[
q_{1z}L = n\pi - \beta_A,
\]

where \( \beta_A = (\beta + \beta')/2 \) is the average phase shift per reflection.

The energy levels (energy bands) are determined by the energy relation \( E^2 = q_{1x}^2 + q_{1y}^2 + q_{1z}^2 \) with \( q_{1z} \) being quantized by equation (6). Taking the gradient operation in \( q_{1x} - q_{1y} \), space upon the energy relation by two sides, it yields

\[
2E\nabla E = \nabla(q_{1x}^2 + q_{1y}^2) + 2q_{1x} \nabla q_{1z}.
\]

Here \( q_{1z} \) is not a free parameter but a function of \( q_{1x}, q_{1y}, \) and \( E \), saying, \( q_{1z} = q_{1z}(q_{1x}, q_{1y}, E) \), so we have \( \nabla q_{1z} = \nabla_E q_{1z} + (\partial_E q_{1z}) \nabla E \). Because of \( q_{1z} = (n\pi - \beta_A)/L \), we have the equations \( \nabla_E q_{1z} = -\nabla_E \beta_A/L \) and \( \partial_E q_{1z} = -\partial_E \beta_A/L \). Applying these substitutions, equation (7) is changed to

\[
\nabla E = \frac{\nabla(q_{1x}^2 + q_{1y}^2)}{2E} - \frac{q_{1z}}{EL} (\nabla_E \beta_A + \partial_E \beta_A \nabla E).
\]

In the equation, \( \nabla E \equiv u \) is the velocity of bound state, \( \nabla(q_{1x}^2 + q_{1y}^2)/2E \equiv u_p \) and \( q_{1z}/E \) are respectively the in-plane projection and the \( z \)-projection of 3D velocity in the sandwiched Weyl material, \( EL/q_{1z} \equiv t_p \) is the time for the electron traveling from side to side. With the help of equations (3) and (4), one can find that \(-\nabla_E \beta_A = (s + s')/2 \) and \( \partial_E \beta_A = (\tau + \tau')/2 \equiv t_s \) are respectively the shift and group delay per reflection (see appendix B). By defining the shift velocity \( u_s \equiv -\nabla_E \beta_A/\partial_E \beta_A \), the above equation is transformed as

\[
u = \frac{t_p}{t_p + t_s} u_p + \frac{t_s}{t_p + t_s} u_s.
\]

The physical meaning of the equation is quite clear. The bound state can be regarded as a weighted composite motion. One motion is the free in-plane motion with the velocity \( u_p \) and the other is the anomaly drift with the velocity \( u_s \) caused by the reflection shift. The velocity correction is defined by \( \Delta u = u - u_s \), and it reads

\[
\Delta u = \frac{t_s}{t_p + t_s} (u_s - u_p).
\]

Figure 2. Time-evolution of two wave packets being reflected with positive (red curves) and negative group delay (blue curves). The blue wave packet is invisible in (a) because it is overlapped by the red one. The parameters are \( k_\pi = \pi/2, \Delta = 10, E = 1, \) and \( \Gamma = 1/3 \). The filled triangle marks the position of the wave packet peak at \( t = -50 \).
Figure 3. (a) The $x$-component of velocity correction as a function of $k_x - k_W$ and $k_y$. (b) The energy contours (bold black curves), the velocity correction (blue vectors), and its amplitude (red 3D curves) obtained from the tight-binding calculation. The parameters are $k_W = \pi/2$, $\Delta = k_W$, $L = 100$, and $E = k_W/10$.

For our minimal model, the $y$-components of $u_p$ and $u_u$ cancel with each other and only $\Delta u_x$ needs to be calculated, but for more complex models, $\Delta u_y$ does not vanish exactly.

Figure 3(a) shows the calculated velocity correction as function of $q_1^x$ and $q_1^y$ (also $k_x - k_W$ and $k_y$). The sign of $\Delta u_x$ keeps unchanged in the whole incident circle, though the orientation of reflection shift varies in the incident circle (see figure 1). The velocity correction is relatively large near the line $q_1^x = 0$ and decreases to zero at the circle edge. At points $(0, \pm E)$, there are two promontories like a pair of horse ears. The two points are just those at which the surface bands belong to the upper and lower interfaces connected with the bulk states in the incident circle.

If $\Delta$ and $L$ are very large, the calculation can be simplified by taking $\phi_2 = 0$. On the line of $q_1^x = 0$, the velocity correction can be reduced to $\Delta u_x = 1/EL$, which is constant and independent of $\theta_1$ (see the dashed line in figure 3(a)). On the line of $q_1^y = 0$, we have $\Delta u_x = \cos^2 \theta_1/EL$, which depends on $\theta_1$ explicitly (see the solid curve in figure 3(a)). The velocity correction is most notable when the electron is injected vertically.

The results of our simple model can be compared with those of tight-binding calculations. We construct the tight-binding model by discretizing the Hamiltonian

$$H_{tb} = \sigma_x (2 - \cos k_x - \cos k_y - \cos k_z + \cos k_W)$$

$$+ \sigma_y \sin k_y + \sigma_z \sin k_z$$

in $z$-direction. The model describes a multilayer Weyl material laying along $x$–$y$ plane sandwiched by vacuum. The vacuum plays the role of insulators with infinite energy gap. By solving the tight-binding Hamiltonian, we have all the conduction and valence bands $E_n(k_x, k_y)$. The lowest conduction band consists of surface states and bulk states, and the others are bulk bands. At a given positive energy, every conduction band is reduced to a contour if the energy is greater than the band minimum.

Figure 3(b) shows the energy contours and the velocity correction on them. All the contours lie in the incident circle except for the outmost one that crosses it. The outmost contour corresponds to the lowest conduction band, the states on which out of the incident circle are surface ones. The velocity correction of all the states is approximately but not strictly along the direction of Weyl point connection, because the Hamiltonian in equation (11) is different from that in the minimal model. The velocity correction leads to the contours drift oppositely (in the figure, the velocity correction points to $k_x$ direction, the contour drift is along $-k_x$ direction) and the contour centers are deviated from the Weyl point. If there is no contour drift,
the outmost contour will be slightly smaller than the incident circle and be just accommodated in it. Now we turn on the contour drift, almost half of the contour then runs out of the incident circle. However, no bulk state is allowed outside, so the contour fragment remained in the incident circle has to be connected with the surface energy bands to be continuous and enclosed. The 3D curve of $\Delta u$ for the outmost contour is truncated at two points, of which the projections locate on the incident circle. At the truncating points, $\Delta u$ is much larger than otherwhere, that reflects the promontories mentioned previously. The velocity correction in the figure is straight obtained from the tight-binding calculation without calling for the concept of reflection phase. Figures 3(a) and (b) are consistent with each other, which implies our analytical calculation of the group delay is reliable.

5. Discussion and summary

The group delay is an old problem that have confused researchers for many years [34]. When a particle tunnels through a barrier, the group delay will saturate at the limit of large barrier length, which is named by the Hartman effect [35] and its analogs in optics, electromagnetics and acoustics were also discovered [34]. In early ages, the group delay is interpreted as the traversal time, which results in a dangerous conclusion: particle’s velocity has no upper bound in barrier media and the so-called superluminece happens [36]. A few years ago, H G Winful proposed that the group delay is not the traversal time, but a life time of energy stored in the traversal region [34], so the superliminence does not really exist.

For most cases, the group delay referred to is the one of transmission. However, as argued by Winful [34], the transmission wave packet, which is typically very weak, is not the smaller copy of the incident one, but can be regarded as the deformation of any part of the incident wave packet (for example, the wave packet tail far from the incident peak center). Therefore, the group delay does not mean the traversal time and the transmission velocity is not the traversal velocity. We avoid this ambiguous problem by studying the total reflection at the Weyl-insulator junction interface when the electron is injected from the Weyl side. We found the group delay is negative for half of the possible incident states. The negative group delay for the total reflection can be view as a new version of superluminece. When the electron is reflected, it has to travel two times (forth and back) through the self-interference region with the thickness of the order of the wave packet width. In our version, the traversal time is abnormally negative for the reflected electron traveling through the self-interference region, and in the usual version, the traversal time is abnormally short for the transmitted electron traveling through the barrier region. Unlike the transmission cases, the out-going wave packet for the total reflection is exactly the incident one, and the group delay is undoubtedly a real ‘delay’. By using the wave packet dynamics, we demonstrated that, the group delay, either positive or negative one, is induced by self-interference effect close to the interface, and the negative group delay does not break the causality law. The reflection group delay, combining with the reflection shift, causes the velocity correction of the bound states in the Weyl material sandwiched by two insulators. The analytically calculated velocity correction is well consistent with the tight-binding model in which the concept of group delay is used. The group delay and the velocity correction of Weyl quasiparticles can be detected in photonic crystal systems, in which the group delay [37], reflection shift [38] and velocity [39] have ever been measured successfully, and different types of Weyl photonic crystals were experimentally fabricated in recent years [40, 41]. Our research not only gives the correct description of the in-plane velocity in Weyl sandwiched structures, but also sheds new light on the old topic of group delay.

Acknowledgments

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix A. 3D wave packet dynamics of hitting interface

A wave packet in real space can be constructed by summing up infinite plane waves weighted by profile function $f(k)$ (the weight of $k$ component is $|f(k)|^2$). A typical choice of the profile function is the Gaussian distribution $f \propto \exp[(k - \bar{k})^2/\Gamma^2]$ with $\bar{k}$ being the wave packet center and $\Gamma$ describing the wave packet...
width. The real space wave packet without normalization of certain spin component is
\[
\Psi_{wp} = \int e^{-(k-k')/2} e^{i\alpha} e^{ikr} dk,
\]  
(A1)
where \(\alpha = \alpha(k)\) is a spin dependent phase factor associated with injection or reflection.

If the wave packet width \(\Gamma\) is small enough to ensure \(\alpha\) and \(E\) can be viewed as slow varying functions, we can expanding \(\alpha\) and \(E\) near the wave packet center \(k\) up to the first order,
\[
e^{i(\alpha-E)t} = e^{i(\alpha-E)k} e^{\nabla_3(\alpha-E)k} (k-k'),
\]  
(A2)
where \(\nabla_3\) stands for the 3D gradient operator in \(k\)-space (in the main text, \(\nabla\) denotes the gradient operator in 2D). Inserting equation (A2) into equation (A1), the wave packet turns to be
\[
\Psi_{wp} = \int e^{-(k-k')/2} e^{i(k-k')[r + \nabla_3(\alpha-E)k]} e^{i(k r + \alpha - Et)} dk
\]  
(A3)
\[
\Psi_{wp} = \Gamma \theta_{1/2} e^{\Gamma [\alpha + (\alpha - Et)]^2} e^{i\alpha} e^{i(k r - Et)},
\]
where \(\Gamma\) is the 2D gradient operator in \(k\)-space (in the main text, \(\nabla\) denotes the gradient operator in 2D). Inserting equation (A2) into equation (A1), the wave packet turns to be
\[
\Psi_{wp} = \int e^{-(k-k')/2} e^{i(k-k') [r + \nabla_3(\alpha - Et)k]} e^{i(k r + \alpha - Et)} dk
\]  
(A3)
\[
\Psi_{wp} = \Gamma \theta_{1/2} e^{\Gamma [\alpha + (\alpha - Et)]^2} e^{i\alpha} e^{i(k r - Et)},
\]
where \(\nabla_3\) stands for the 3D gradient operator in \(k\)-space (in the main text, \(\nabla\) denotes the gradient operator in 2D). Inserting equation (A2) into equation (A1), the wave packet turns to be
\[
\Psi_{wp} = \int e^{-(k-k')/2} e^{i(k-k') [r + \nabla_3(\alpha - Et)k]} e^{i(k r + \alpha - Et)} dk
\]  
(A3)
\[
\Psi_{wp} = \Gamma \theta_{1/2} e^{\Gamma [\alpha + (\alpha - Et)]^2} e^{i\alpha} e^{i(k r - Et)},
\]
The wave packet in real-space is of Gaussian type too. It describes a wave packet that is centered at \(\mathbf{r} = \nabla_3(-\alpha + Et)\mathbf{k}\) and travels with the velocity \(\mathbf{v}(\mathbf{k}) = (\nabla_3 E)\mathbf{k}\). From now on, we neglect the subscript for brevity. The time when the wave packet center hits the interface locating at \(z = 0\) is
\[
t = \frac{\partial \alpha}{\partial k_x} \left( \frac{\partial E}{\partial k} \right)^{-1} = \frac{\partial \alpha}{\partial E}.
\]  
(A4)
By setting the time in the position of wave pocket center as the above hit time, we have the position of hit spot on the interface,
\[
\mathbf{R} = -\nabla_\parallel \alpha + \nabla_\perp E \frac{\partial \alpha}{\partial E},
\]  
(A5)
where \(\nabla_\parallel\) is the 2D gradient operator in \(k_x - k_y\) plane. The variable \(\alpha(k)\) can also be regarded as a function of \(k_x\) and \(E\), so we have \(\nabla_\parallel \alpha = \nabla_\parallel E(k_x, E) + \partial_{k_x} \alpha \nabla_\parallel E\), where \(\nabla_\parallel\) is the \(k_x - k_y\) gradient operator acting on variables as explicit functions of \(E\), same meaning as in the main text, and the hit location is reduced to be
\[
\mathbf{R} = -\nabla_\perp E \alpha.
\]  
(A6)
It is not surprising that the hit spot location and moment are calculated similarly as above because \(k \cdot r\) and \(-Et\) appears in the plane wave on an equal footing.

### Appendix B. Reflection shift and group delay

When the electron is incident from the Weyl material to the insulator, the incident and reflected states are
\[
|\psi_i\rangle = \begin{pmatrix} e^{-i\phi_1/2} \cos(\theta_1/2) \\ e^{i\phi_1/2} \sin(\theta_1/2) \end{pmatrix}, \quad |\psi_r\rangle = \begin{pmatrix} e^{-i\phi_1/2} \sin(\theta_1/2) \\ e^{i\phi_1/2} \cos(\theta_1/2) \end{pmatrix},
\]  
(B1)
where \(\phi_1\) and \(\theta_1\) are the azimuthal and polar angles of the injection, same as in the main text. For the spin-up (or spin-down) component of the incident wave packet, we have \(\alpha = -\phi_1/2\) and it weighted by \(\cos^2(\theta_1/2)\) (or \(\alpha = \phi_1/2\), weighted by \(\sin^2(\theta_1/2)\)), the average hit position and the average hit moment are
\[
R_i = \frac{1}{2} \nabla_\parallel E \phi_1 \cos \theta_1, \quad t_i = -\frac{1}{2} \frac{\partial E}{\partial \phi_1} \cos \theta_1.
\]  
(B2)
For the spin-up (or spin-down) component of the reflection wave packet, taking into account the reflection coefficient \(r = e^{i\beta}\), we have \(\alpha = \beta - \phi_1/2\) and it weighted by \(\sin^2(\theta_1/2)\) (or \(\alpha = \beta + \phi_1/2\), weighted by \(\cos^2(\theta_1/2)\)), the average position and the average moment for the reflection are
\[
R_r = -\nabla_\parallel E \beta - \frac{1}{2} \nabla_\parallel E \phi_1 \cos \theta_1, \quad t_r = \frac{\partial E}{\partial \phi_1} + \frac{1}{2} \frac{\partial E}{\partial \phi_1} \cos \theta_1.
\]  
(B3)
One can find that the reflection spot does not overlap with the incident spot and there is reflection shift \(s = R_r - R_i\) between the reflection and injection. The longitudinal and transverse components of the shift are the Goos–Hänchen shift and Imbert–Fedorov shift, respectively. The outgoing moment is different from
the hit moment by the value \( \tau = t_r - t_i \), which is the group delay time. Explicitly, the reflection shift and group delay are

\[
s = -\nabla_E \beta - \nabla_E \phi_1 \cos \theta_1, \\
\tau = \partial_E \beta + \partial_E \phi_1 \cos \theta_1.
\]

So we have equations (3) and (4) in the main text.

The reflection shift and delay result in a shift velocity along the interface. When the wave packet propagates in a sandwiched Weyl material of width \( L \), the reflection coefficient on the interface locating at \( z = -L \) is denoted as \( s' = e^{i\beta'} \). Its reflection shift and group delay are just those in the above equation pair by taking the replacement \( \phi \rightarrow -\phi \). The replacement leads to \( \beta \rightarrow \beta' \) and \( \phi_1 \rightarrow -\phi_1 \), so we have

\[
s' = -\nabla_E \beta' + \nabla_E \phi_1 \cos \theta_1, \\
\tau' = \partial_E \beta' - \partial_E \phi_1 \cos \theta_1.
\]

The shift velocity along the interfaces, \( u_s = (s + s')/(\tau + \tau') \), is calculated as

\[
u_s = -\frac{\nabla_E \beta_A}{\partial_E \beta_A},
\]

where \( \beta_A = (\beta + \beta')/2 \) means the average of reflection phase of the two interfaces. We reach the same shift velocity by means of wave packet dynamics, instead of the Sommerfeld quantization condition in the main text.

**Appendix C. Sign of group delay**

When the electron reflected on the upper interface, the transmitted wave decays away, and the \( z \)-component of the transmitted wave vector is imaginary, i.e. \( q_{xz} = i\kappa \) with \( 1/\kappa \) characterizing the decay length in the insulator. The derivatives of the reflection phase for variable \( \mu \) (here \( \mu \) can be \( k_x, k_y \), or \( E \)) is

\[
\partial_\mu \beta = \frac{\partial_\mu (\Delta \phi - \xi) \cos \theta_1 + \sin(\Delta \phi - \xi) \partial_\mu \theta_1}{1 - \sin \theta_1 \cos(\Delta \phi - \xi)},
\]

where \( \xi = \text{arg}(E + i\kappa) \). The reflection shift and group delay can both be calculated by means of the above equation.

If the energy gap of the insulator is very large, saying, \( \kappa/E \gg 1 \), we have \( \xi = \pi/2 \). Specifying \( \mu \) in equation (C1) as \( E \), recalling \( \partial_E \phi_1 = \partial_E \phi_2 = 0 \), and applying \( \partial_\mu \theta_1 = -\sin \theta_1/q_{1z} \), the group delay on the upper interface is reduced to

\[
\tau = \partial_E \beta = \frac{1}{q_{1z}} \frac{\cos(\Delta \phi) \sin \theta_1}{1 - \sin \theta_1 \sin(\Delta \phi)}.
\]

According to the equation, the sign of \( \tau \) depends on that of \( \cos(\phi_2 - \phi_1) \). If \( \phi_2 \) cannot synchronously follow the change of \( \phi_1 \), the negative group delay could take place. In our model, when \( \phi_1 \) varies from 0 through \( 2\pi \), while \( \phi_2 \) is almost zero, so \( \tau \) is negative in the left half and positive in the right half of the incident circle, as displayed in figure 1(b).

The non-following change between \( \phi_1 \) and \( \phi_2 \) always happens if the energy valley of one side material of the junction is mismatched with that of other side. The Weyl-insulator junction, of which the energy valleys of besides materials locate at \( (k_{\Gamma W}, 0, 0) \) and \( \Gamma \)-point, respectively, is not the only system to produce negative group delay, other valley-mismatch junctions could have similar effect.

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