Analysis of Resistance Torque of Wheeled Steering Vehicle

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Abstract. Based on the structural characteristics of the wheeled skid vehicle, this paper combines the theory of ground vehicle mechanics and related statics theory to theoretically derive the resistance moment of the single wheel turning in place and turning around a certain point. At the same time, based on the composition and decomposition of the motion, the overall steering form and steering conditions of the wheeled skid vehicle are analyzed. Taking a specific vehicle as an example, a comparative analysis of different steering modes is provided, which provides a theoretical basis for the design of the entire vehicle’s steering system.

1. Introduction
The steering method that relies on changing the speed and steering of the wheels on both sides to steer the driving direction is slip steering, also known as differential steering. Sliding steering vehicles are classified into tracked and wheeled according to their running devices [1-2]. Wheeled vehicles are generally all-wheel drive, and each wheel is rigidly suspended [3]. Wheel steering is flexible and easy to operate and is widely used in narrow sites such as construction sites, workshops, and warehouses. For skid-steer vehicles, the analysis of steering resistance torque is the theoretical basis for steering system design [4-5]. This article analyzes the motion process of wheeled skid steer on horizontal ground, adopts the shape closer to the actual ground contact surface of the tire, obtains more accurate wheel steering resistance torque, and derives steering conditions under different steering forms.

2. Steering form of wheeled steer vehicle
As shown in Figure 1, each drive wheel number is represented by \( j \) (\( j = 1, 2, 3, 4 \)). To simplify the analysis, it is assumed that the center of gravity of the vehicle is located in the longitudinal symmetry plane, and the vehicle does not carry external loads. Speed steering travels on horizontal roads, ignoring the effects of centrifugal force. Let the center of the mass speed of the vehicle be \( v \), the wheel distance is \( B \), the axle distance is \( L \), and the linear speed of the outer drive wheels of the wheeled steering vehicle is \( v_1 \) and the linear speed of the inner drive wheels is \( v_2 \).

At this time, it can be divided into four types of steering conditions: (1) \( v_1 > v_2 > 0 \), that is, the speed direction is the same and different in size; (2) \( v_1 > v_2 = 0 \), that is, the inner brake and the outer wheel rotate; (3) \( v_1 > 0 \), \( v_2 < 0 \), \(|v_1| > |v_2|\), that is, the speed direction is different in size; (4) \( v_1 > 0 \), \( v_2 < 0 \), \(|v_1| = |v_2|\), that is, turning in place. The steering angular velocity of the vehicle under four conditions is shown in Eq. (1). The turning radius can be uniformly expressed as Eq. (2):
3. Analysis of Steering Resistance Torque of a Single Wheel

3.1 Tire deformation characteristics
The tire deforms under the load, and the contact with the ground is surface contact. The grounding shape of the tire is often measured by imprinting. Data [6-7] shows that the grounding shape of a patterned tire with a certain inflation pressure under different vertical loads is shown in Figure 2. When the load is low, the tire contacts the ground in the area near the centre of the ground, and the ground shape is approximately circular; as the load gradually increases, the ground area of the tire gradually increases, and it contacts the ground over the entire tire width, and the ground shape is approximately rectangular. Based on such experimental results, the ground contact shape of the tire under vertical load is treated as a rectangle.

3.2 Resistance torque when the tire rotates around the center of the ground print
To simplify the calculation, it is assumed that the contact surface of the tire with the ground is a rectangle of $a \times b$ ($b$ is the tire width). The width $b$ can be approximated by the value of the tire section width. The length $a$ can be calculated as shown in Figure 3.
As can be seen from the figure, \( OA = r_o \) is the free radius of the tire; \( OB = r_d \) is the dynamic radius of the tire; the length of \( AC \) is the length of the bottom surface of the tire; \( h \) is the flattening amount, which is the difference between the free radius of the tire and the power radius. According to the geometric relationship, the length of the grounding mark can be obtained as shown in Eq. (3).

\[
a = 2\sqrt{r_o^2 - (r_o - h)^2}
\]  

(3)

In the above formula, because the value of \( h \) is very small (when the tire pressure and vehicle load are in the normal range, it is only a few percent of \( m \)), then \( h^2 \) is smaller, so the term \( h^2 \) is omitted [8], as shown in Eq. (4). As shown in Figure 4, the tire rotates around the ground contact center \( O \) at an angular velocity \( \omega \). Because the tire squeezes or shears the ground, it must be subject to the steering resistance moment from the ground.

\[
a \approx 2\sqrt{2r_o h}
\]  

(4)

Figure 4. Schematic diagram of tires rotating around the centre of the ground print.

Assume that the tire ground pressure is evenly distributed on the ground mark. Let the area of a small unit on the ground surface be \( dA \). Its distance from the point \( O \) is \( l \), then the frictional moment experienced by this small unit is shown in Eq. (5).

\[
dM_j = \mu PldA = \mu F_{z,ab} ldA
\]  

(5)

\( F_z \) is the vertical load acting on the tire; \( P \) is the specific ground pressure; \( \mu \) is the coefficient of sliding friction. Integrating on both sides, total resistance moment in contact area is shown in Eq. (6).

\[
M_j = 4 \int_0^{a\tan\frac{a}{b}} \int_0^{\frac{b}{\cos\theta}} \mu F_{z,ab} r^2 drd\theta + 4 \int_0^{a\tan\frac{a}{b}} \int_0^{\frac{b}{\cos\theta}} \mu F_{z,ab} r^2 drd\theta
\]  

(6)

This result in resistance moment when single wheel rotates around ground centre is shown in Eq. (7).

\[
M_j = \frac{\mu F_{z,ab}}{6} \sqrt{a^2 + b^2} + \frac{\mu F_{z,ab} a^2}{12b} \ln \frac{\sqrt{a^2 + b^2} + b}{a} + \frac{\mu F_{z,ab} b^2}{12a} \ln \frac{\sqrt{a^2 + b^2} + a}{b}
\]  

(7)
3.3 Analysis of the tire's rotation around a point other than the ground footprint

As shown in Figure 5(a), the rotation center $O$ is not on the axis of the tire, the direction of the wheel's movement is at an angle with the tire plane, and the speed is $v$. A coordinate system is established with the rotation center $O$ as the origin, the $x$ axis parallel to the tire axis, and the $y$ axis perpendicular to the axis. The $O$ coordinate of the tire ground center is $(x_1, y_1)$, as shown in Figure 5(b).

![Figure 5. The tire rotates around any point O. (a) Speed diagram; (b) Force diagram.](image)

This movement process is divided into three movements: the rotation movement of the wheel around the ground contact center of the tire, the sliding movement in the $x$ direction, and the pure rolling movement in the $y$ direction. Let $F$ be the driving force acting on the center $O$ of the ground imprint, $F_f$ be the linear rolling resistance of the wheel, and $F_r$ be the lateral force. Then $F_f = F_{z_1}, F_r = \mu F_{z_2}$.

The total resistance moment $M_z$ when a single wheel rotates around the $O$ point is shown in Eq. (8).

$$M_z = M_f + f F_z x_1 + \mu F_r y_1$$  \hspace{1cm} (8)

4. Vehicle Steering Resistance Torque Analysis

4.1 Analysis of Steering Resistance Torque When Wheels on Both Sides Move in the Same Direction

Supposing the resistance torque of each driving wheel around the center of the imprint is $M_{q_1}$, the driving force of each wheel is $F_{z_1}$, the lateral force is $q$, the linear rolling resistance is $F_f$, the steering resistance torque of each wheel is $M_{q_2}$, and the driving torque of each wheel is $q_1$. Therefore, the total driving torque $M_{q_1}$ of the tire is shown in Eq. (9). The total resistance moment $M_{q_1}$ is shown in Eq. (10).

$$M_{q_1} = M_{q_1} + M_{q_2} + M_{q_3} + M_{q_4} = F_1(R + 0.5B) + F_2(R + 0.5B) + F_3(R - 0.5B) + F_4(R - 0.5B)$$

$$M_{q_1} = M_{q_1} + M_{q_2} + M_{q_3} + M_{q_4} = M_q + F_{f_1}(R + 0.5B) + F_{f_2}(R + 0.5B) + F_{f_3}(R - 0.5B) + F_{f_4}(R - 0.5B)$$

$$F_f = f F_z x_1 + \mu F_r y_1$$  \hspace{1cm} (9)

$$M_{q_1} = M_{q_1} + M_{q_2} + M_{q_3} + M_{q_4} = M_q + F_{f_1}(R + 0.5B) + F_{f_2}(R + 0.5B) + F_{f_3}(R - 0.5B) + F_{f_4}(R - 0.5B)$$

$$M_{q_1} = M_{q_1} + M_{q_2} + M_{q_3} + M_{q_4} = M_q + F_{f_1}(R + 0.5B) + F_{f_2}(R + 0.5B) + F_{f_3}(R - 0.5B) + F_{f_4}(R - 0.5B)$$

$$+ F_f R + F_r c + M_4 + F_r(R - 0.5B) + F_r(R - 0.5B) + F_r d + M_3$$

$$+ F_f R + F_r c + M_4 + F_r(R - 0.5B) + F_r d + M_3$$

$$+ f G R + 2 \mu G c d L$$  \hspace{1cm} (10)

4.2 Analysis of Steering Resistance Torque When One Wheel Brakes and the Other Wheel Drives

The force on one side of the wheel is braked and the force on the other side of the wheel is shown in Figure 6. At this time, the driving force of the left wheel is 0, and the steering process is completed by the driving of the right wheel. According to its force, the total driving torque of this type of steering is obtained as shown in Eq. (11).

$$M_{Q_2} = B(F_2 + F_4)$$  \hspace{1cm} (11)
The total resistance moment $M_{R2}$ is shown in Eq. (12).

$$M_{R2} = \frac{\mu G}{6} \sqrt{a^2 + b^2} + \frac{\mu G a^2}{12b} \ln \frac{\sqrt{a^2 + b^2} + b}{a} + \frac{\mu G b^2}{12a} \ln \frac{\sqrt{a^2 + b^2} + a}{b} + \frac{1}{2} fGR + 2\mu G \frac{cd}{L}$$

(12)

Figure 6. Forces when braking on one wheel and driving on the other.

4.3 Analysis of Steering Resistance Torque When Wheels on Both Sides Reverse

When the left and right wheels move in opposite directions, the turning radius of the vehicle is further reduced. The force of each wheel is shown in Figure 7. The total driving torque $Q_M$ is shown in Eq. (13).

$$Q_M = F_1(R + 0.5B) + F_2(R + 0.5B) + F_3(0.5B - R) + F_4(0.5B - R)$$

(13)

The total resistance moment $M_{R3}$ is shown in Eq. (14).

$$M_{R3} = \frac{\mu G}{6} \sqrt{a^2 + b^2} + \frac{\mu G a^2}{12b} \ln \frac{\sqrt{a^2 + b^2} + b}{a} + \frac{\mu G b^2}{12a} \ln \frac{\sqrt{a^2 + b^2} + a}{b} + \frac{1}{2} fGR + 2\mu G \frac{cd}{L}$$

(14)

If the wheels on both sides move in opposite directions and the speed is equal, the vehicle can turn in place and the turning radius reaches the minimum. Therefore, the driving torque $Q_M$ the vehicle receives when turning in place is shown in Eq. (15).

$$Q_M = 0.5B(F_1 + F_2 + F_3 + F_4)$$

(15)

The total resistance moment $M_{R4}$ is shown in Eq. (16).

$$M_{R4} = \frac{\mu G}{6} \sqrt{a^2 + b^2} + \frac{\mu G a^2}{12b} \ln \frac{\sqrt{a^2 + b^2} + b}{a} + \frac{\mu G b^2}{12a} \ln \frac{\sqrt{a^2 + b^2} + a}{b} + \frac{1}{2} fGR + 2\mu G \frac{cd}{L}$$

(16)
Figure 7. Force situation when the wheels on both sides move in the opposite direction.

5. Conclusions
According to the actual shape of the ground contact surface of the tire, a new method for calculating the steering resistance moment of the wheel is proposed in this paper, and the steering form of the slip steering of the wheeled vehicle is comprehensively analyzed. Based on this, steering conditions for three types of steering are obtained. From the above analysis, it can be seen that when the steering radius \( R \geq \frac{B}{2} \), the steering resistance of the entire vehicle is related to the size of the steering radius. When the steering radius \( \frac{B}{2} \geq R \geq 0 \), the steering resistance of the entire vehicle is related to the wheelbase and wheelbase of the vehicle itself. The research results provide a basis for the design of the wheeled vehicle steering system.

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