GAUGE COUPLING UNIFICATION
IN GENERAL SUPERSYMMETRIC MODELS*

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ABSTRACT

We study the effects of additional fields on the unification of gauge couplings in supersymmetric models. We find that the effects are quite constrained by the requirement of SU(5) gauge invariance. In general, we find that any extension of the MSSM will form complete SU(5) multiplets or spoil gauge coupling unification.

1. Introduction

The standard model of elementary particle physics (SM) is in excellent agreement with present experimental results. Nonetheless, the theory suffers from a variety of theoretical shortcomings and is generally believed to be the low energy effective part of a more fundamental theory which is characterized by a higher symmetry and fewer arbitrary parameter. One very popular and promising step in this direction is to embed the SU(3)_c ⊗ SU(2)_L ⊗ U(1)_Y in a single simple gauge group. The most economical candidate for such a unified gauge group is SU(5). Here, all fermions of one generation can be embedded in a 5 dimensional and a 10 dimensional representation respectively. The quantization of the U(1) hypercharges follows automatically and the numerous constraints from the absence of triangle anomalies reduces significantly.

Maybe the most prominent feature of such a grand unified theory (GUT) is that baryon and lepton number are no longer conserved separately as a consequence of having quarks and leptons in the same multiplet. As a result, the proton is no longer a stable particle. Thus, the observed lower limit of the proton lifetime implies a severe lower bound on the scale of the GUT symmetry breaking, M_{GUT}.

Unfortunately, the SM as the complete low energy theory of an SU(5) GUT model is ruled out because (a) the gauge couplings do not meet within the theoretical and experimental errors, (b) the large hierarchy between M_{GUT} and the electro-weak scale is unstable under radiative corrections and would require excessive fine-tuning and (c) the predicted rate of proton decay is too fast. However, all these problems can

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be avoided in the minimal supersymmetric extension of the SM (MSSM). Here, the prediction of the strong coupling constant, $\alpha_s$, within the framework of the minimal supersymmetric model (MSSM) obtained by assuming gauge coupling unification without any intermediate scale is in acceptable agreement with experiment. In addition, the unification of $\tau$ and bottom Yukawa couplings is quite promising.

Despite these successes of supersymmetric grand unified theories (SUSY GUTs) based on SU(5) there remain still some problems. Maybe the most severe challenge is the so-called doublet/triplet problem of giving the colored Higgs triplet a mass of the order of $M_{\text{GUT}}$ while retaining the Higgs doublets responsible for the electro-weak symmetry breaking at the electroweak scale. There have been many attempts to try and solve these problems all of which have one thing in common: they require the introduction of new fields. Thus, an extension of the particle content of the MSSM is inevitable and one might ask whether some of these new particles can have a significant impact on the gauge coupling unification. We will consider two cases. In section 2 we will assume that all the components of an SU(5) multiplet are present in the low energy effective theory. This idea has already been explored in refs. Here any effect will come through multiplet splitting due to renormalization group (RG) evolution. In section 3 we will integrate the additional fields at $M_{\text{GUT}}$. Here the multiplet splitting occurs through a possible coupling to the vacuum expectation value (VEV) of the adjoint representation.

2. Complete SU(5) Multiplets

From the severity of the doublet/triplet problem we know how hard it is to construct a model with a large mass hierarchy between different SU(3)$\otimes$SU(2)$\otimes$U(1) members of the same SU(5) multiplet. Therefore, we will first consider extensions of the MSSM by complete SU(5) multiplets in the low energy effective theory.

The renormalization group equations (RGEs) for these models can be written at the one-loop level as

$$\frac{d\alpha_i}{dt} = \alpha_i^2 \beta_i .$$

(1)

Here, $t \equiv (2\pi)^{-1} \ln (\text{scale})$, the indices $i, j = 1, 2, 3$ refer to the U(1), SU(2) and SU(3) gauge group and summation over twice occurring indices is assumed. Furthermore, the one-loop $\beta$ functions for the gauge couplings are

$$\beta_i = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_H \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} + \left( 2N_G + \beta^X \right),$$

(2)

where the three contributions to $\beta_i$ come from the gauge sector, the Higgs doublets (in the MSSM the number of Higgs doublets, $N_H = 2$) and the contribution of complete SU(5) multiplets. It is a simple exercise to show that all such extensions will
preserve gauge coupling unification at the one-loop level. The contributions of the
gauge/gaugino sector to $\beta_i$ are non-universal since some gauge bosons and their
superpartners acquire a mass via the Higgs mechanism while others stay massless due
to gauge invariance. The contributions of the Higgs bosons are also non-universal
because the doublets are responsible for the electro-weak symmetry breaking and
should have a mass of the order of $m_z = 91.187$ GeV while the Higgs triplets have
to acquire a mass at $M_{GUT}$ in order to sufficiently suppress the rate for proton-decay.
This doublet/triplet splitting is solely motivated by experiment and has no satisfying
theoretical solution yet. The last term which also contains the contribution of $N_G = 3$
generations of quarks and leptons is universal for all three couplings. (Note, that one
family of quarks and leptons can be embedded in a $\overline{5} = d^c(3, 1, 2/3) \oplus l(1, 2, -1)$ and
a $10 = q(3, 2, 1/3) \oplus u^c(3, 1, -4/3) \oplus e^c(1, 1, -2)$; the numbers in brackets indicate
the transformation properties under the SU(3), SU(2) and U(1) gauge symmetries, respectively.)
The reason is that the inclusion of a full SU(5) multiplets with a mass, $m$, does not break the SU(5) gauge symmetry and should yield a universal contribution
to all three $\beta$ functions at any scale above $m$ at the one-loop level. The contributions
of the extensions of the MSSM can be written as
\[ \beta^X = \sum_{\Phi} T(\Phi), \]
where the sum is over all SU(5) multiplets $\Phi$. The values of $T(\Phi) \equiv d(\Phi)C_2(\Phi)/r$ are listed in Table II
for all representations of SU(5) with $d < 100$. Here, $r = 24$ is the number of generators of SU(5) and $C_2(\Phi) [d(\Phi)]$ is the quadratic Casimir operator [dimension] of the SU(5) representation $\Phi$. By imposing gauge coupling unification at $M_{GUT}$ i.e.
\[ \alpha_{GUT} \equiv \alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT}), \]
and solving eq. II to first order in perturbation theory we obtain
\[ t_0 \equiv \frac{1}{2\pi} \ln \frac{M_{GUT}}{m_z} = \frac{\alpha_1^{-1}(m_z) - \alpha_2^{-1}(m_z)}{\beta_1 - \beta_2} \simeq 5.3, \]
\[ \alpha_3^{-1}(m_z) = \alpha_2^{-1}(m_z) + t_0(\beta_3 - \beta_2) \simeq 8, \]
where we have used
\[ \alpha_1(m_z) = \frac{5}{3} \frac{\alpha_{em}}{\cos^2 \theta_{\overline{MS}}}, \]
\[ \alpha_2(m_z) = \frac{\alpha_{em}}{\sin^2 \theta_{\overline{MS}}}, \]
and $\alpha_{em}^{-1} = 127.9$ and $\sin^2 \theta_{\overline{MS}} = 0.2314$ as the low energy input values. The prediction of the strong coupling constant, $\alpha_s(m_z) \equiv \alpha_3(m_z) \simeq 0.125$ is in quite good

Table 1. The contribution of various SU(5) multiplets \( \Phi \) to \( \beta^X \).

| \( \Phi \)   | 5, 5 | 10, 10 | 15, 15 | 24 | 35, 35 | 40, 40 | 45, 45 | 50, 50 | 70, 70' | 70', 70'' | 75 |
|-------------|------|--------|--------|----|--------|--------|--------|--------|--------|----------|-----|
| \( T(\Phi) \) | 1/2  | 3/2    | 7/2    | 5  | 14     | 11     | 12     | 35/2   | 49/2   | 42       | 25 |

agreement with the world average \( \alpha_s(m_Z) = 0.117 \pm 0.005 \). In deriving eq. [5] we have assumed that there is no intermediate scale but it also holds in the case of a widely spread particle spectrum as long as the members of the different SU(5) multiplets lie close together.

Note, that the right hand side of eq. [5] is independent of \( \beta^X \). This means that any extension of the MSSM by full SU(5) multiplets will maintain the property of gauge coupling unification at one loop. The unification scale \( M_{\text{GUT}} \approx 2 \times 10^{16} \text{ GeV} \) remains also unchanged and for the unified gauge coupling we obtain

\[
\alpha^{-1} \simeq 24 - t_0 \beta^X. \tag{7}
\]

By requiring that the right hand side of eq. [7] is larger than zero we find \( \beta^X \lesssim 4.5 \) but maybe models with \( \beta^X = 5 \) are still acceptable due to higher order corrections or threshold corrections etc. and shall be included into our considerations.

In order to derive a viable model, we have to impose additional constraints. The cancellation of triangle anomalies and the fact that all additional particles have to be massive implies that complex representations only occur in pairs. Thus, there are four types of extensions satisfying the above requirements

- \( n \) additional pairs of 5 and \( \overline{5} \), where \( n = 1, 2, 3, 4, 5 \),
- one additional pair of 10 and \( \overline{10} \),
- \( n \) additional pairs of \( \overline{5} \) and 10 where \( n = 1, 2 \),
- one additional adjoint representation, \( 24 = g(8, 1, 0) \oplus w(1, 3, 0) \oplus b(1, 1, 0) \oplus x(3, 2, 5/3) \oplus \pi(3, 2, -5/3) \).

Experimental lower limits on the additional particle masses can be satisfied by adding explicit dirac or majorana mass terms to the superpotential allowed by gauge invariance in models of type 1, 2 and 4. Models of type 3 correspond to the MSSM with four or five generations. For the invisible width of the Z boson at LEP experiments we know that the number of (almost) massless neutrinos is 3 and a mechanism has to be introduced in order to give mass to the additional neutrinos larger than about \( m_Z/2 \). Also, a lower limit on the mass of an additional lepton of \( m_\tau \gtrsim m_\tau/2 \) at LEP and a lower limit on the mass of an additional down type quark of \( m_d > 85 \text{ GeV} \).
at CDF have been established\textsuperscript{10}. This implies that the Yukawa couplings for the additional fermions are bound from below since no explicit gauge invariant mass term exists. On the other hand, there is an upper limit on the masses from the infra-red fixed-point behavior of the Yukawa couplings leaving only a very constrained region in parameter space. The four generation model has been studied recently and found to be quite constrained but could still be feasible if a right-handed neutrino is introduced to raise the mass of the left-handed neutrino above the experimental bounds\textsuperscript{12}. The five generation model is even more constrained and might already be ruled out by present data. However, the model with additional 5, 5, 10, 10 with the possibility for a explicit dirac mass for all the additional particles is still allowed. It also does not require any additional fields to generate mass for the unseen neutrinos.

The assumption that all the members, $\phi$, of one SU(5) multiplet, $\Phi$, are mass degenerate is protected by gauge invariance. It acquires corrections below $M_{\text{GUT}}$ where the gauge symmetry is broken through one-loop RG evolution

\[ \frac{d\mu_{\phi}}{dt} = \mu_{\phi} \sum_{i} \alpha_i \beta_{i,\phi}, \tag{8} \]

where the $\beta_{i,\phi}$ are listed in Table\textsuperscript{2}. The splitting between the various members of an SU(5) at the electro-weak scale multiplet due to RG evolution is quite considerable and gives rise to significant threshold corrections. These corrections have to be included if we want to obtain a self-consistent renormalization group result with two-loop $\beta$ functions. If we decouple the fields $\phi$ from the RGEs at $\mu_{\phi}$ we obtain the improved one-loop formulas

\[ \alpha_i^{-1}(m_z) = \alpha_{\text{GUT}}^{-1} + t_{0i} \beta_i - \Delta_i^{MSSM} - \frac{1}{2\pi} \sum_{\phi} N_{\phi} \ln \frac{\mu_{\phi}}{m_z} \beta_{i,\phi}, \tag{9} \]

where $\beta_{i,\phi}$ are the contributions of the field, $\phi$, to $\beta_i$ given in ref.\textsuperscript{13}. Furthermore,

\[ N_{\phi} = \begin{cases} N_5 \\ N_{10} \\ N_{24} \end{cases} \quad \text{for } \phi = \begin{cases} l, d^c \\ e^c, u^c, q \\ b, w, g, x, \tau \end{cases}, \tag{10} \]

is the number of the fields $\phi$. It has been shown in ref.\textsuperscript{14} that the step approximation indeed contains the full one-loop threshold corrections due to heavy particles. The

| $\beta_{i,\phi}$ | $l$ | $d^c$ | $e^c$ | $u^c$ | $q$ | $b$ | $w$ | $g$ | $x$ | $\tau$ |
|-----------------|-----|------|------|------|----|----|----|----|----|-----|
| $U(1)$          | $\frac{5}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $0$ | $0$ | $0$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $SU(2)$         | $-\frac{3}{2}$ | $0$ | $0$ | $0$ | $\frac{3}{2}$ | $0$ | $0$ | $\frac{5}{2}$ | $\frac{5}{2}$ |
| $SU(3)$         | $0$ | $\frac{3}{2}$ | $0$ | $\frac{3}{2}$ | $0$ | $0$ | $-6$ | $\frac{3}{2}$ | $\frac{3}{2}$ |
Table 3. The one-loop (left column) and two-loop (right column) results for the ratio, \( r = M_{GUT}^X / M_{GUT} \), \( \Delta \alpha_s / \alpha_s \) [in %], \( \alpha_{GUT} \), the ratio \( m_b / m_\tau \). We have chosen \( m_0(M_{GUT}) = 200 \) GeV, \( \mu_\phi(M_{GUT}) = 1 \) TeV for all additional fields, \( \phi \), and \( \alpha_t(M_{GUT}) = \alpha_{GUT} \). Furthermore, we have fixed \( m_{1/2} \) such that \( M_3(M_3) = 200 \) GeV.

| \( N_5 \) | \( N_{10} \) | \( N_{24} \) | \( r \) | \( \Delta \alpha_s / \alpha_s \) in % | \( \alpha_{GUT} \) | \( m_b / m_\tau \) |
|---|---|---|---|---|---|---|
| MSSM | 0 | 0 | 0 | \( \text{..} \) | .72 | 1.00 | .64 | 0.0 | .040 | .042 | 1.80 | 1.87 |
| 1 | 2 | 0 | 0 | \( \text{..} \) | .75 | 1.11 | .48 | .1 | .050 | .053 | 1.88 | 1.96 |
| 2 | 4 | 0 | 0 | \( \text{..} \) | .79 | 1.29 | .27 | .1 | .066 | .073 | 1.99 | 2.09 |
| 3 | 6 | 0 | 0 | \( \text{..} \) | .85 | 1.59 | .43 | .1 | .096 | .118 | 2.15 | 2.30 |
| 4 | 8 | 0 | 0 | \( \text{..} \) | .97 | 2.16 | .63 | .1 | .175 | .333 | 2.44 | 2.65 |
| 5 | 1 | 1 | 0 | \( \text{..} \) | .71 | 1.63 | .61 | 1.3 | .070 | .081 | 2.08 | 2.31 |
| 6 | 3 | 1 | 0 | \( \text{..} \) | .76 | 2.23 | .37 | 1.5 | .104 | .149 | 2.28 | 2.64 |
| 7 | 5 | 1 | 0 | \( \text{..} \) | .83 | 4.20 | .73 | 2.1 | .203 | 1.68 | 2.64 | 3.36 |
| 8 | 0 | 2 | 0 | \( \text{..} \) | .48 | 1.57 | .93 | .2 | .088 | .118 | 2.01 | 2.28 |
| 9 | 2 | 2 | 0 | \( \text{..} \) | .46 | 2.16 | .76 | .16 | .144 | .332 | 2.18 | 2.62 |
| 10 | 0 | 0 | 1 | \( \text{..} \) | .49 | .72 | .33 | 26 | .280 | .238 | 2.32 | 3.26 |

MSSM threshold corrections, \( \Delta_{\text{MSSM}} \), studied in ref. 13 raise the predicted value of \( \alpha_s(m_Z) \) by about 10% and spoil the success of the GUT prediction to some degree.

However, additional effects on the unification of the gauge couplings arise because the universality of \( \beta^X \) in eq. 2 is violated at the two-loop level. The two-loop \( \beta \) functions of these models derived from ref 13 are listed in ref 7.

In minimal SU(5) SUSY-GUTs the down-type quark fields \( d \) and the left-handed lepton fields \( l \) are embedded in one representation and as a result the \( \tau \) and bottom Yukawa couplings are unified at \( M_{GUT} \). By inspecting the \( \beta \) functions it is easy to see that \( m_b / m_\tau \) increases with \( \alpha_{GUT} \) and hence also with \( \beta^X \). This increase can be compensated by an increase in the top Yukawa coupling, \( \alpha_t \). However, \( \alpha_t \) quickly approaches its IR fixed-point \( \alpha_t = O(\alpha_s) \). Hence, any significant effects of the top Yukawa coupling can only come from the integration close to \( M_{GUT} \) and requires unperturbatively large values of \( \alpha_t(M_{GUT}) \). Thus, in our numerical work we have used \( \alpha_t(M_{GUT}) = \alpha_{GUT} \) in order to obtain a natural prediction for \( m_b / m_\tau \) for a particular model assuming \( \tau \)-bottom Yukawa unification.

It is important to note that the additional particles discussed here have an explicit SUSY conserving mass parameter and are naturally heavier than the standard model particles and their superpartners. Thus, by decoupling they will be unconstrained by any presently available experimental data.

In Table 3 we have summarized our results for the MSSM and 10 extended models characterized by \( N_5 \), \( N_{10} \), and \( N_{24} \). We have chosen the somewhat large values of \( \mu_\phi = 1 \) TeV for all additional fields \( \phi \) in order to exploit as many models as possible. It is clear that we recover the MSSM predictions if we raise \( \mu_\phi \) to \( M_{GUT} \). The parameter independent lower limit on the gluino mass of \( M_3 > 100 \) GeV has been established.
from direct particle search at CDF\textsuperscript{13}. However, stronger limits can be derived from the chargino/neutralino search by imposing GUT constraints and we chose $M_{\tilde{g}} = 200$ GeV in order to safely avoid all the present bounds.

The first row corresponds to the MSSM where we have $\alpha_{\text{MSSM}} = 0.124$ and $M_{\text{GUT}}^{\text{MSSM}} = 2.3 \times 10^{16}$ GeV for our choice of parameters. In the different columns we present the result for the unification scale divided by the two-loop MSSM value and denoted by $r$, the relative change in the prediction of $\alpha_s$ with respect to the two-loop MSSM value, the unified gauge coupling, $\alpha_{\text{GUT}}$, and the ratio of $m_b(m_Z)$ to $m_{\tau}(m_Z)$. The two values correspond to the results obtained by using one-loop and two-loop $\beta$ functions. We see that already for the MSSM the value of $m_b/m_\tau$ for $\alpha_t(M_{\text{GUT}}) = \alpha_{\text{GUT}}$ is slightly above its experimental value of $m_b/m_\tau \simeq 1.6$ but can still be brought in agreement with experiment by a modest increase of $\alpha_t$ or by choosing $\alpha_b = O(\alpha_t)\text{\textsuperscript{3}}$. The situation becomes more problematic in all extended models. On the one hand, this ratio increases even more due to larger values of $\alpha_s$ at high scales. On the other hand, the IR fixed point of the top quark Yukawa coupling obtained from

$$\frac{d}{dt} \frac{\alpha_t}{\alpha_s} = (12 - \beta^X) \frac{\alpha_t}{\alpha_s} - 12 = 0,$$ \hspace{1cm} (11)

increases with $\beta^X$. As a result, the resent measurement of the top quark mass, $m_t = 176 \pm 8\text{(stat)} \pm 10\text{(sys)}$ GeV by the CDF collaboration\textsuperscript{20} implies that either the top Yukawa coupling is below its IR fixed point or one has to move even closer to the theoretically unfavorable value $\tan \beta = 1$ (the solution for $\tan \beta \simeq m_t/m_b$ might be ruled out entirely for large enough values of $\beta^X$).

Furthermore, we see that the models 4, 7, 9 and 10 become non-perturbative at $M_{\text{GUT}}$. This scenario of non-perturbative unification was already advocated in ref.\textsuperscript{18} in non-SUSY models and in ref.\textsuperscript{19} extended to SUSY models as being particularly attractive. The reason is that the dependence of $\alpha_t$ on $\alpha_{\text{GUT}}$ in eq.\textsuperscript{8} vanishes in the large $\alpha_{\text{GUT}}$ limit. It is interesting that the predictability of the models with non-perturbative unification is not limited by large uncertainties at $M_{\text{GUT}}$ where the couplings are large but by low energy threshold corrections. This is due to the fact that the masses of the various members of one SU(5) representation are widely scattered due to RG evolution in eq.\textsuperscript{8}.

3. SU(5) Multiplets with Mass Splitting

So far we have neglected the effects of superheavy multiplets (\textit{i.e.} multiplets that acquire mass at $M_{\text{GUT}}$). This assumption is justified in the case where all components of one multiplet are mass degenerate as they have to be if SU(5) is unbroken. However, in general the fields will not only obtain a mass from an explicit mass term but also from an interaction term $W \supset \lambda \phi 24\phi$, where the 24 dimensional representation acquires a non-zero VEV, $a$. In this case we obtain GUT threshold corrections to the
Fig. 1. Threshold corrections to $\alpha_s$ and $M_{\text{GUT}}$ due to a pair of various complex SU(5) representations $\phi$ and $\bar{\phi}$ with (a) $\phi = 5, 10, 15$ (b) $\phi = 35, 40, 45$ (c) $\phi = 50, 70, 70'$ and due to the real SU(5) multiplets, $\phi = 24, 75$. 
Table 4. The changes in $\alpha_s$ and $M_{\text{GUT}}$ for an additional pair of complex fields with dimension, $d$.

| $d$  | 5   | 10  | 15  | 35  |
|------|-----|-----|-----|-----|
| $\alpha_s/\alpha_s^0$ | -1.0 | 10. | -5. | -4.5 |
| $M_{\text{GUT}}/M_{\text{GUT}}^0$ | 0.97 | 2.2 | 2.0 | 10. |

gauge couplings from eq. (9) with the masses given by

$$\mu_{\phi,\bar{\phi}} = |\mu + Y_{\phi,\bar{\phi}} \lambda a|.$$  \hspace{1cm} (12)

In fig. 1 we present the threshold corrections to $\alpha_s$ and $M_{\text{GUT}}$ due to various SU(5) multiplets with dimension $d < 100$. The corrections shown in fig. 1(a)–(c) are due to a pair of complex representations $\phi$ and $\bar{\phi}$ ($\phi = 5, 10, 15, 35, 40, 45, 70, 70'$) and the ones in fig. 1(d) are due to one real representation $\phi = 24, 75$. They are obtained by varying $0 < a/\mu < \infty$. All curves start at $(1, 0)$ for $a = 0$ since all the threshold corrections vanish in this case. The corrections diverge logarithmically if one of the particle masses vanish. The curves terminate for $\mu = 0$. The corresponding end points are listed in table 4 for all the models where unification is possible within the $\alpha_s$–$M_{\text{GUT}}$ plane under consideration. (It looks as if these end points would lie in the middle of the contours rather than at the end. This is due to the fact that two branches of the same curves coincide.) These regions of large corrections require typically a severe fine-tuning. There are two types of exceptions. First, in the cases of real representations the neutral fields under the U(1) symmetry can be naturally massless while the charge fields can have an arbitrarily large mass. Second, the cases where the multiplet splitting is achieved via a missing-partner mechanism. In table 5 we present all cases where parts of an SU(5) multiplet can acquire a mass from a term

$$W \supset \phi_1 24 \bar{\phi}_1 \text{ or } \phi_1 75 \bar{\phi}_2,$$  \hspace{1cm} (13)

with the the dimensions of the representations smaller than 100. The fields obtaining mass from eq. (13) are the ones common to $\phi_1$ and $\bar{\phi}_2$ (third column of table 4). There are three different possibilities for the low energy particle spectrum:

- only the fields common in $\phi_1$ and $\bar{\phi}_2$ are heavy (ie. $\beta_i^X = T(\phi_1) + T(\phi_2) - 2\Delta \beta_i$)
- there is an additional explicit mass term $\mu \phi_1 \bar{\phi}_1$ (ie. $\beta_i^X = T(\phi_2) + T(\bar{\phi}_1) - 2\Delta \beta_i$)
- there is an additional explicit mass term $\mu \phi_2 \bar{\phi}_2$ (ie. $\beta_i^X = T(\phi_1) + T(\bar{\phi}_1) - 2\Delta \beta_i$)

with the $\Delta \beta_i$ listed in table 5. (Note, that $\phi_i$ and $\bar{\phi}_i$ are different fields.) It is a simple exercise to check that none of these possible extension yield a feasible model.

We see from fig. 1 in general an increase in $M_{\text{GUT}}$ favored by the lower limits on the proton decay results in an unfavorable increase in $\alpha_s$ and vice versa. The
Table 5. The various members of a SU(5) multiplet which can acquire a mass via the missing partner mechanism.

| $φ_1$ | $φ_2$ | Heavy fields          | $Δβ_1$ | $Δβ_2$ | $Δβ_3$ |
|-------|-------|-----------------------|--------|--------|--------|
| 5     | 45    | $(1, 2, 1) \oplus (3, 1, -2/3)$ | 1/2    | 1/2    | 1/2    |
| 5     | 50    | $(3, 1, -2/3)$         | 1/5    | 0      | 1/2    |
| 10    | 15    | $(3, 2, 1/3)$          | 1/10   | 3/2    | 1      |
| 10    | 40    | $(3, 1, -4/3) \oplus (3, 2, 1/3)$ | 9/10   | 3/2    | 3/2    |
| 15    | 40    | $(3, 2, 1/3)$          | 1/10   | 3/2    | 1      |
| 35    | 40    | $(3, 3, -4/3) \oplus (6, 2, 1/3)$ | 26/10  | 9      | 13/2   |
| 45    | 50    | $(3, 1, -2/3) \oplus (3, 2, -7/3) \oplus (8, 2, 1)$ | 59/10  | 11/2   | 15/2   |
| 45    | 70    | $(1, 2, 1) \oplus (3, 1, -2) \oplus (3, 3, -2/3)$ | 11/10  | 5/2    | 2      |

Exceptions are the 35 and 70' dimensional representations for some choices of $a/μ$. Here, the agreements with experiment can be mildly improved.

4. Summary

In summary, we have investigated various extensions of the MSSM with additional SU(5) multiplets. We distinguish the cases of (a) additional fields at the weak scale where the mass splitting is generated via RG evolution and (b) additional fields at $M_{GUT}$ where the mass splitting arises from a coupling to the VEV of the 24. We find that the predictions for $α_s$ and $M_{GUT}$ are quite stable in all models of type (a) with the exceptions of those models with non-perturbative unification. Here, the predictions will be softened by the low energy threshold corrections due to a widely scattered unpredictable particle spectrum. On the other hand, the models of type (b) can have a large effect due to the missing partner mechanism or fine-tuning. However, no candidate could be found that would significantly improve the predictions of the MSSM. In conclusion, we can say that additional fields in the low energy effective theory form complete SU(5) multiplets or spoil gauge coupling unification.

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6. References

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Fig. 1. Threshold corrections to $\alpha_s$ and $M_{\text{GUT}}$ due to a pair of various complex SU(5) representations $\phi$ and $\bar{\phi}$ with (a) $\phi = 5, 10, 15$ (b) $\phi = 35, 40, 45$ (c) $\phi = 50, 70, 70'$ and due to the real SU(5) multiplets, $\phi = 24, 75$. 