THE GRAVITATIONAL WAVE SIGNAL FROM CORE-Collapse SUPERNOVAE

VIKTORIY A MOROZOVA1, DA VID RADICE1,2, AD AM BURROWS1, AND DA VID VARTANYAN1

Accepted for publication in The Astrophysical Journal

ABSTRACT

We study gravitational waves (GWs) from a set of two-dimensional multi-group neutrino radiation hydrodynamic simulations of core-collapse supernovae (CCSNe). Our goal is to systematize the current knowledge about the post-bounce CCSN GW signal and recognize the templatable features that could be used by the ground-based laser interferometers. We demonstrate that starting from \( \sim 400 \) ms after core bounce the dominant GW signal represents the fundamental quadrupole \((l = 2)\) oscillation mode (f-mode) of the proto-neutron star (PNS), which can be accurately reproduced by a linear perturbation analysis of the angle-averaged PNS profile. Before that, in the time interval between \( \sim 200 \) and \( \sim 400 \) ms after bounce, the dominant mode has two radial nodes and represents a g-mode. We associate the high-frequency noise in the GW spectrograms above the main signal with p-modes, while below the dominant frequency there is a region with very little power. The collection of models presented here summarizes the dependence of the CCSN GW signal on the progenitor mass, equation of state, many-body corrections to the neutrino opacity, and rotation. Weak dependence of the dominant GW frequency on the progenitor mass motivates us to provide a simple fit for it as a function of time, which can be used as a prior when looking for CCSN candidates in the LIGO data.

Keywords: gravitational waves — hydrodynamics — equation of state — supernovae: general

1. INTRODUCTION

After decades of development, the ground-based laser interferometers LIGO and Virgo have detected the gravitational-wave (GW) signal of merging binary systems of black holes (Abbott et al. 2016, 2017a) and neutron stars (Abbott et al. 2017b). The latter event was especially interesting, because it was subsequently observed in the X-ray, UV, optical, infrared, and radio bands (Abbott et al. 2017c). Detection of the GWs from a galactic core-collapse supernova (CCSN), potentially accompanying by detection of neutrinos and electromagnetic observations in all available bands, could be the next major breakthrough. Given that the estimate for the galactic CCSNe rate is \( 3.2^{+7.3}_{-2.6} \) per century (Li et al. 2011; Adams et al. 2013), and the youngest known galactic CCSN remnant is \( \sim 100 \) years old (Reynolds et al. 2008; Borkowski et al. 2017), chances are high that we will not have long to wait.

Our ability to recognize the CCSN GW signal and extract it from the nonstationary and non-Gaussian background noise of the detectors largely depends on our knowledge of the signal’s time-frequency structure (see, for example, Hayama et al. 2015; Gossan et al. 2016, and references therein). Attempts to characterize the GW emission from CCSNe started in 1960s with the analytical estimates of Wheeler (1966) and evolved into the fully relativistic multidimensional numerical simulations of the early 2000s (see reviews of Ott 2009 and Kotake 2013). Indeed, since a spherically-symmetric star does not emit GWs, and the CCSN mechanism relies on a complex hydrodynamical evolution of the stellar core, including neutrino interactions, fluid instabilities, and shocks, a two-dimensional (2D) radiation-hydrodynamics code is the minimum capability required to model the CCSN GW signal. Previous studies of the GW signal from 2D CCSN models may be found, for example, in Marek et al. (2009); Murphy et al. (2009); Kotake et al. (2009); Müller et al. (2013) and Cerdá-Durán et al. (2013). Once computationally unaffordable, general-relativistic 3D simulations of CCSNe with state-of-the-art neutrino physics have been recently performed by a number of groups (Melson et al. 2015b,a; Lentz et al. 2015; Müller et al. 2017; Ott et al. 2017; see also Takizaki et al. 2012; Ott et al. 2013; Takizaki et al. 2014; Müller 2015; Roberts et al. 2016; Pan et al. 2017). Yakunin et al. (2017); Andresen et al. (2017); Kuroda et al. (2016) and Kuroda et al. (2017) have provided the GW signal from their most recent 3D simulations (for earlier work, see Fryer et al. 2004; Scheidegger et al. 2008, 2010; Müller et al. 2012; Ott et al. 2012; Kuroda et al. 2014). Here, we show the results of our current 2D study of the GW signals from the recent Fornax models, building upon the earlier efforts of our group to simulate CCSN explosions (Dolence et al. 2015; Skinner et al. 2016; Burrows et al. 2017; Radice et al. 2017).

One of the main difficulties in the study of the GW signal of CCSNe is related to the stochasticity of the processes responsible for its generation. For example, the early \( \sim 100 \) Hz signal arising in the first tens of milliseconds after core bounce is commonly associated with the shock oscillations driven by prompt convection (Marek et al. 2009; Murphy et al. 2009; Müller et al. 2013; Yakunin et al. 2010, 2015), which makes its parameterization very difficult. In the next few hundred milliseconds, a stronger signal follows with the frequency gradually increasing in the range \( 300 - 1000 \) Hz. This signal is usually associated with the surface g-modes of the newly formed proto-neutron star (PNS) excited by the downstreams from the postshock convection region or from convection inside the PNS itself (Murphy et al. 2009; Marek et al. 2009; Müller et al. 2013). Some features of the GW signal are known to be associated with the standing accretion shock instability (SASI) (Cerdá-Durán et al. 2013; Kuroda et al. 2016; Andresen et al. 2017; Pan et al. 2017).

A number of attempts has been made to systematize and identify the features of the CCSNe GW signal by means of asteroseismology, specifically, applying linear perturba-

1 Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544, USA, vsg@astro.princeton.edu
2 Schmidt Fellow, Institute for Advanced Study, 1 Einstein Drive, Princeton, NJ 08540
tion analysis to the PNS and its surrounding region (Fuller et al. 2015; Sotani & Takiwaki 2016; Torres-Forné et al. 2017; Camelo et al. 2017). For example, Fuller et al. (2015) showed that the fundamental quadrupolar oscillation mode of the PNS may be responsible for the early post-bounce signal of the rapidly rotating core. Recently, Torres-Forné et al. (2017) presented a relativistic formalism to identify the eigenfrequencies of the PNS and its surrounding postshock region, which they used to analyze the rotating 2D CCSN model from Cerdà-Durán et al. (2013). In the current study, we go one step further and relax the Cowling approximation used in Torres-Forné et al. (2017). After doing so, we can firmly relate the dominant component of the GW signal from our models with the fundamental f-mode quadrupole oscillation of the PNS. This association holds for different progenitor masses, equations of state (EOS), and numerical prescriptions for gravity and neutrino interactions used in our study.

We find that the dominant GW frequency depends weakly on the progenitor zero-age main-sequence (ZAMS) mass, without any clear systematic trend. Instead, it is sensitive to the EOS and the details of neutrino microphysics. Motivated by its simple time evolution, we fit the dominant GW frequency as a function of time with a quadratic polynomial, which can be used as a prior in the GW data analysis, when looking for the CCSN candidates. We identify a new feature in the form of a power ‘gap’ across the GW spectrogram, which, if proven physical, may provide some information about the structure of the inner PNS core. We study the influence of rotation on the GW signal and find that, while increasing the power of the core bounce signal, it may weaken the GW emission in the post-bounce phase.

The paper is organized in the following way. Section 2 outlines our numerical setup and summarizes the CCSN models used in our study. In Section 3.1, we present an example of the GW spectrogram obtained and describe its key features common between all our models. In Section 3.2, we explain the physical origin of some of these features by means of the linear perturbation analysis. Section 3.3 is devoted to the comparison of the spectrograms from different simulations, which shows the key dependences of the GW signal on the parameters of the models and the details of the numerical setup. Discussion and conclusions are given in Section 4. For simplicity, in the sections describing the linear analysis (Section 3.2 and Appendix B), we use the geometrized system of units $G = c = 1$, where $c$ is the speed of light and $G$ is Newton’s gravitational constant. In other sections, $G$ and $c$ are shown explicitly in the equations.

**2. NUMERICAL SETUP**

The 2D core-collapse supernova simulations analyzed in our study were performed with the neutrino-radiation-hydrodynamics code Fornax (Skinner et al. 2016; Burrows et al. 2017; Vartanyan et al. 2018, Skinner et al. 2018, in prep.). Fornax solves the hydrodynamic equations using a directionally-unsplit Godunov-type finite-volume scheme in spherical coordinates, with the HLLC approximate Riemann solver (Toro et al. 1994). The majority of simulations presented here use a monopole approximation for the approximate general-relativistic (GR) gravitational potential, following Case A of Marek et al. (2006). Some simulations were performed with a multipole gravity solver (Müller & Steinmetz 1995), where we set the maximum spherical harmonic order equal to twelve. Fornax offers a possibility to include rotation in 2D, which is used in one of our simulations.

In Fornax, we distinguish three species of neutrino, i.e., electron neutrinos $\nu_e$, anti-electron neutrinos $\bar{\nu}_e$, and heavy lepton neutrinos $\nu_L$, with the latter including $\nu_\mu$, $\nu_\tau$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$ taken together (Burrows et al. 2017). The transport of neutrinos is followed using an explicit Godunov characteristic method, with the HLLE approximate Riemann solver (Einfeldt 1988), modified as in Audit et al. (2002) and O’Connor (2015) to reduce the numerical dissipation in the diffusive limit. We use an M1 tensor closure for the $2^{nd}$ and $3^{rd}$ moments of the radiation fields (Vayet et al. 2011; Shibata et al. 2011; Murchikova et al. 2017). The neutrino energy is discretized in twenty groups, varying logarithmically in the range $1 - 300$ MeV for the electron neutrinos and $1 - 100$ MeV for the other neutrino species.

We follow the prescription for the neutrino-matter interactions outlined in Burrows et al. (2006). For more details about the neutrino microphysics implemented in Fornax see Burrows et al. (2017) and Radice et al. (2017) and references therein. We include the effects of many-body corrections to the axial-vector term in the neutrino-nucleon scattering rate, as described in Horowitz et al. (2017). One of our models was simulated without the many-body correction in order to distinguish its influence on the GW signal.

In our models, we use three different equations of state (EOS), namely, the SFHo EOS (Steiner et al. 2013), the Lattimer-Swesty EOS with nuclear incompressibility parameter $220$ MeV (Lattimer & Swesty 1991), and the DD2 EOS (Fischer et al. 2014; Banik et al. 2014).

In our simulations, we use the progenitor models obtained with the stellar evolution code Kepler by Sukhbold et al. (2016) with the ZAMS masses of $10$, $13$, and $19$ $M_\odot$. Our radial grid consists of $678$ points for the $10 M_\odot$ model and $608$ points for the $13$ and $19 M_\odot$ models, spaced evenly with $\Delta r = 0.5$ km for $r \lesssim 10$ km and logarithmically for $r \gtrsim 100$ km, smoothly transitioning in between. The outer boundary is placed at $20,000$ km. The angular resolution smoothly varies between $\approx 0.95^\circ$ at the poles and $\approx 0.65^\circ$ at the equator in $256$ zones. To avoid the overly restrictive Courant conditions close to the coordinate center, the angular resolution decreases in the innermost radial zones, representing a so-called dendritic grid (Skinner et al. 2018, in prep.).

To extract the GW signal measured by a distant observer we employ the standard formula for the trace-free quadrupole moment of the source in the slow-motion approximation (Finn & Evans 1990; Murphy et al. 2009):

$$F_{jk} = \int \rho \left( x^j x^k - \frac{1}{3} \delta^{jk} x^i x^i \right) d^3 x .$$

(1)

For axisymmetric sources, this has only one independent component along the symmetry axis, $F_{zz}$. In spherical coordinates, the time derivative of this component can be rewritten in terms of the fluid velocity $v_i$ as (Equation (38) of Finn & Evans 1990, corrected in Murphy et al. 2009):

$$\frac{d}{dt} F_{zz} = \frac{8\pi}{3} \int_{-1}^{1} d \cos \theta \int_{r_1}^{r_2} d r r^3 \rho \times$$

$$\left[ P_2(\cos \theta) v_r + \frac{1}{2} \frac{\partial}{\partial \theta} P_2(\cos \theta) v_\theta \right] ,$$

(2)

where $P_2(\cos \theta)$ is the second Legendre polynomial. After that, the axisymmetric GW strain can be computed as

$$h_+ = \frac{3}{2} \frac{G}{D c^3} \sin^2 \theta \frac{d^2}{dt^2} F_{zz} ,$$

(3)
where $D$ is the distance to the source and $\theta'$ is the angle between the symmetry axis and the line of sight of the observer (henceforth, we assume $\sin^2 \theta' = 1$). Following Murphy et al. (2009), we compute the total energy emitted in GWs as

$$E_{GW} = \frac{3}{10} G \frac{c^3}{r} \int_0^t \left( \frac{d^3 f_{zz}}{dt^3} \right)^2 dt,$$

and compute the spectrogram of this energy by means of the short-time Fourier transform (STFT)

$$\frac{dE_{GW}}{df}(f, \tau) = \frac{3}{5} G \frac{c^3}{r^2} (2\pi f)^2 \left| \tilde{S}(f, \tau) \right|^2,$$

where

$$\tilde{S}(f, \tau) = \int_{-\infty}^{\infty} A(t) H(t - \tau) e^{-2\pi i ft} dt,$$

$A \equiv \frac{\partial}{\partial t} f_{zz}$, and $H(t - \tau)$ is the Hann window function with the time offset $\tau$. The sampling frequency of the GW strain output in our simulations is $16,384$ Hz, and we use the window size of $40$ ms, when performing the STFT. The high sampling frequency is necessary to avoid the aliasing in GW spectrograms, seen in some of the early studies.

In addition to the matter motion, we compute the GW signal associated with the neutrino emission, first recognized by Epstein (1978) (see more in Thorne 1992; Burrows & Hayes 1996; Mueller & Janka 1997). We use Eq. (24) from Mueller & Janka (1997) for the transverse-traceless part of the gravitationnal strain from neutrinos, $h_{ij}^{TT}$, which we provide here for completeness (see also Yakunin et al. 2015):
GW signal due to neutrinos, and the spectrogram in the top panel of Figure 1 takes into account only the matter contribution.

As in previous studies (see, for example, Marek et al. 2009; Murphy et al. 2009; Müller et al. 2013; Yakunin et al. 2010, 2017), we see the early signal associated with the prompt PNS convection in the first ∼50 ms after bounce. The duration and strength of this signal depend upon the progenitor mass and EOS, but this component is generally weak compared to the other, more dominant features in the spectrogram. The only exception is the rotating 13 M⊙ model, which manifests a very energetic prompt convection signal and will be shown later in Section 3.3.3. This is expected based on previous work devoted to the GWs from rotating core collapse (Dimmelmeier et al. 2008; Abdikamalov et al. 2014; Richers et al. 2017; Torres-Forné et al. 2017). The prompt convection signal is followed by a short, ∼50 ms, quiescent phase, in agreement with previous results (Marek et al. 2009; Murphy et al. 2009; Müller et al. 2013; Yakunin et al. 2017).

The dominant part of the signal lasts from ∼150 ms after core bounce until the end of the simulation, with the frequency growing from ∼300 to ∼2000 Hz. Despite the high-frequency noise, most of the energy is concentrated along a relatively thin stripe, as can be seen from the linear 3D visualization of the spectrogram in Figure 2. Some of the earlier work predicted the abrupt reduction in the high frequency signal at the onset of explosion due to the cessation of down-flowing plume excitation of the inner core (Murphy et al. 2009; Yakunin et al. 2015). However, as was shown in Müller et al. (2013), the high frequency signal may persist for a certain time before this happens, and we see the same in our model. As in Müller et al. (2013), the post-explosion signal from our model M10_SFHo consists of distinct ‘bursts’ of emission, presumably caused by the continuing accretion episodes. For another exploding model in our study (19 M⊙), the post-explosion signal stays strong until the end of the simulation at ∼1.5 s after bounce, without decaying in energy (see more in Section 3.3). The explosion is marked by the offset of h+D from zero, which indicates that the shock is not spherical (the prolate explosion shifts the strain up, while the oblate explosion shifts it down; see Murphy et al. 2009; Müller et al. 2013; Yakunin et al. 2015).

A number of recent works (Cerdá-Durán et al. 2013; Kuroda et al. 2016, 2017; Pan et al. 2017; Andresen et al. 2017) pointed to a separate GW feature associated with the SASI (see more about the SASI phenomenon, for example, in Blondin et al. 2003; Foglizzo et al. 2007). This signal is expected to reside at lower frequency, typically 100 ∼ 200 Hz, and coincides in time with the periods of enhanced shock oscillations. To test this regime in model M10_SFHo, we plot its entropy along the polar axis in the top panel of Figure 3. The plot shows that the shock oscillates mildly in the period 100 ∼ 400 ms after bounce (these oscillations, though, are not as vigorous as typically seen when the SASI is identified) and before the explosion sets in. The early part of the GW spectrogram, plotted in the bottom panel of Figure 3, indeed shows some power excess at low frequencies in this period, and we associate it with the oscillations of the shock, but this signal is very weak compared to the higher frequency signal from the same model. In the rest of this paper, we concentrate on the dominant part of the GW signal at higher frequencies.

One curious feature seen in all our models is a ‘gap’ crossing the GW spectrogram at ∼1300 Hz. We checked the dependence of this feature, which at first glance looked like a numerical artifact, on simulation parameters, such as timestep, resolution and output frequency. For example, the dependence on the GW signal on the grid resolution for the model M10_LS220_no_manybody is given in Appendix A. The ‘gap’ persisted at exactly the same location for all combinations of numerical parameters we considered for a given model, varying only slightly between the different models. We do not exclude the possibility that the ‘gap’ is physical
and attempt to explain it in part with the trapped g-mode of the PNS inner core in the next subsection.

3.2. Analytical explanation of the key features of GW signal from the M10 SFHo model

In this subsection we focus on explaining the dominant features of the GW spectrogram, using the model M10 SFHo as an example (shown in Figure 1), by means of a linear perturbation analysis.

The system of equations we solve combines the linearized equations of general-relativistic hydrodynamics in a spherically-symmetric conformally-flat background metric (Banyuls et al. 1997; Torres-Forné et al. 2017) together with the Poisson equation. It can be summarized in the form

\[
\partial_r \eta_r + \left[ \frac{2}{r} + \frac{1}{\Gamma_1} \frac{\partial_r P}{P} + \frac{6}{\psi} \frac{\partial_r \psi}{\psi} \right] \eta_r + \frac{\psi^2}{\alpha^2 c_s^2} \left( \sigma^2 - \mathcal{L}^2 \right) \eta_\perp - \frac{1}{\alpha c_s^2} \delta \hat{\alpha} = 0 ,
\]

\[
\partial_r \eta_\perp - \left( 1 - \frac{N^2}{\sigma^2} \right) \eta_r + \left[ \partial_r \ln q - \hat{G} \left( 1 + \frac{1}{c_s^2} \right) \right] \eta_\perp - \frac{1}{\alpha \hat{G}} \mathcal{L}^2 \delta \hat{\alpha} = 0 ,
\]

\[
\partial_r f_\alpha + \frac{2}{r} f_\alpha + 4 \pi \left[ \partial_r \rho - \frac{\rho}{P T_1} \partial_r P \right] \eta_r - \frac{4 \pi \rho}{P T_1} g \sigma^2 \eta_\perp + \frac{4 \pi \rho^2 h}{P T_1 \alpha} \left( \frac{1}{r} \frac{l(l+1)}{r^2} \right) \delta \hat{\alpha} = 0 ,
\]

and

\[
\partial_r \delta \hat{\alpha} = f_\alpha \alpha - \hat{G} \delta \hat{\alpha} .
\]

Here, scalar functions \( \eta_r = \eta_r(r) \) and \( \eta_\perp = \eta_\perp(r) \) represent the amplitudes of the decomposition of radial (\( \xi^r \)) and polar (\( \xi^\theta \)) Lagrangian displacements of a fluid element with respect to its equilibrium position in terms of spherical harmonics:

\[
\xi^r = \eta_r Y_{lm} e^{-i \sigma t} ,
\]

\[
\xi^\theta = \eta_\theta \left( \frac{1}{r} \frac{\partial \theta}{\partial \theta} \right) Y_{lm} e^{-i \sigma t} ,
\]

where \( \sigma \) is the mode frequency. The scalar function \( \delta \hat{\alpha} = \delta \hat{\alpha}(r) \) is the amplitude of the lapse function perturbation

\[
\delta \alpha = \delta \hat{\alpha} Y_{lm} e^{-i \sigma t} ,
\]

and we define \( f_\alpha = \partial_r (\delta \hat{\alpha}/\alpha) \). The conformal factor of the metric, \( \psi \), is equal to 1 in our numerical setup. The details of the derivation of Eqs. (8)-(11) are given in Appendix B. In the limit \( \delta \hat{\alpha} = 0 \) (the Cowling approximation), Eqs. (8)-(9) coincide with Eqs. (31)-(32) of Torres-Forné et al. (2017).

In Eqs. (8)-(11), \( P \) is the pressure, \( \rho \) is the rest-mass density of the matter, \( h \) is the specific enthalpy, \( c_s \) is the relativistic speed of sound, \( \Gamma_1 \) is the adiabatic index, \( \hat{G} \equiv -\partial_r \ln \alpha \) is the radial component of the gravitational acceleration, \( q \equiv \rho \alpha - 2 \psi^2 \), \( N \) is the relativistic Brunt-Väisälä frequency, which in our case is equal to (see also Müller et al. 2013)

\[
N^2 = \frac{\alpha \partial_r \alpha}{\psi^4} \left( \frac{1}{\Gamma_1} \frac{\partial_r P}{P} - \frac{\partial_r \psi}{\rho h} \right) ,
\]

and \( \mathcal{L} \) is the relativistic Lamb frequency

\[
\mathcal{L}^2 = \frac{\alpha^2 \psi^4 c_s^2 l(l+1)}{P T_1} .
\]

These quantities describe the spherically symmetric equilibrium background configuration, which we find by averaging the hydrodynamical output of our 2D simulations over polar angle.

Figure 4 shows the Brunt-Väisälä frequency for the averaged profile of the M10 SFHo model (middle panel). Black lines show the radial coordinates where the density is equal to \( 5.0 \times 10^9 \) g cm\(^{-3}\), \( 10^9 \) g cm\(^{-3}\), and \( 10^{11} \) g cm\(^{-3}\), with the latter density surface commonly used as a definition of the PNS boundary. Colored gray are the regions where the Brunt-Väisälä frequency is imaginary (\( N^2 < 0 \)), which means they are convectively unstable. To further emphasize the convection, in the bottom panel of Figure 4 we plot the anisotropic velocity defined as (Takahashi et al. 2012; Pan et al. 2017)

\[
\nu_{\text{aniso}} = \sqrt{\frac{\rho}{4 \pi} \left( \left( v_r - \langle v_r \rangle_{4\pi} \right)^2 + v_\theta^2 \right)} \langle \rho \rangle_{4\pi} ,
\]

where \( \langle \rangle_{4\pi} \) denotes spherical averaging. From this plot, one can see a convective layer inside the PNS, between \( \sim 10 \) and \( \sim 20 \) km, but the convective velocities there are much smaller than the ones above the PNS surface. The outer convective zone between the PNS boundary radius and the shock radius (shown in the top panel of Figure 4) is recognized as the main driving region for the GW signal (Murphy et al. 2009). The middle panel of Figure 4 shows the imaginary Brunt-Väisälä

3 To compute the lapse function of the equilibrium background configuration from the simulation output, we use the formula \( \alpha = \exp \left( \Phi_{\text{eff}}/c_s^2 \right) \), where \( \Phi_{\text{eff}} \) is the approximate relativistic gravitational potential (Marek et al. 2006; Case A).
To solve Eqs. (8)-(11), we place the outer boundary condition at the radial coordinate where $\rho = 10^{10} \text{ g cm}^{-3}$ (solid black line in Figure 4). There, we impose the condition $\Delta P = 0$ on the Lagrangian perturbation of the pressure, which physically corresponds to a free surface of the PNS (see, for example, Reisenegger & Goldreich 1992). Mathematically, this boundary condition can be written as

$$\alpha q \sigma^2 \eta_{\perp} - \frac{\rho h}{\alpha} \delta \alpha + \partial_r P \eta_r = 0 .$$

Our treatment of the outer boundary condition is, therefore, different from the one in Torres-Forné et al. (2017), where $\eta_r = 0$ at the shock position is imposed instead. At the innermost point, we impose a small radial displacement, use the regularity condition (Reisenegger & Goldreich 1992)

$$\eta_r |_{r = 0} = l \eta_{\perp} |_{r = 0} ,$$

and assume $\delta \alpha |_{r = 0} = f \rho |_{r = 0} = 0$. As in Torres-Forné et al. (2017), we apply a trapezoidal rule to discretize the radial derivatives in Eqs. (8)-(11). Starting from the innermost point, we integrate the equations outwards, inverting a $4 \times 4$ matrix of coefficients at every step. We use the bisection method to find the solutions satisfying Eq. (17) at the outer boundary. The frequencies $\sigma / 2\pi$ corresponding to these solutions are the eigenfrequencies of our model.

Figure 5 shows the eigenfrequencies of $l = 2$ (quadrupolar) modes overplotted on the GW spectrogram for the model M10_SFHo. Each eigenfrequency is represented by a number of nodes in the corresponding mode, i.e., the number of times the radial displacement function $\eta_r$ changes its sign along the radial coordinate. To avoid crowding the numbers, we show only the modes with the number of nodes $< 7$ above the frequency 700 Hz and $< 4$ below that frequency. Since the GW signal itself was obtained from the numerical simulations using a quadrupole formula (Finn & Evans 1990), we primarily focus on $l = 2$ modes in this study. At the same time, we cannot exclude the case of non-linear coupling between the $l = 2$ modes and the modes of different $l$, which can explain certain features of the GW signal (see, for example, Torres-Forné et al. 2017). For the interested reader, the $l = 3$ and $l = 4$ modes are shown in Appendix C.

The left panel of Figure 5 shows the results for the modes obtained under the Cowling approximation ($\delta \alpha = 0$ and $f = 0$ in Eqs. (8)-(11)). Starting from $\sim 0.4 \text{ s}$ after bounce, a fundamental mode (the $f$-mode, with zero radial nodes) can be clearly identified. Above this mode, one can see $p$-modes (acoustic), for which the frequencies increase with the number of nodes, while below it there are $g$-modes, for which the frequencies decrease with the number of nodes. Before $\sim 0.4 \text{ s}$ after bounce, as in Torres-Forné et al. (2017), we see the mixing and crossing between the different modes, during which they change the number of nodes.

The right panel of Figure 5 shows the full solution of Eqs. (8)-(11), for $\delta \alpha \neq 0$. As in the left panel, the fundamental ($f$) mode clearly stands out after $\sim 0.4 \text{ s}$ post-bounce time, but in this case it agrees very well with the strongest component of the GW radiation. This result is expected from physical grounds, and it shows that the Cowling approximation can indeed affect the analysis and should be used with caution when interpreting the GW signal from numerical simulations of CCSNe. The $p$- and $g$-modes can be also identified in the left panel of Figure 5. Interestingly, the GW spectrogram from our simulations shows almost no power below the $f$-mode, suggesting that the higher order $g$-modes of the PNS are not excited. Aside from the possible SASI and neutrino signal, which are expected to operate at the frequencies $\lesssim 100 \text{ Hz}$ (see Kuroda et al. 2016; Andresen et al. 2017), there is no other apparent mechanism that could fill this ‘excluded region’ of the spectrogram.

Figure 6 shows the dependence of the obtained results on the position of the outer boundary, placed at the radial coordinate where the density reaches a given value. We remind the reader that the three choices of boundary density correspond to the three black lines in Figure 4 with the middle value, $\rho = 10^{10} \text{ g cm}^{-3}$, being our default choice. From Figure 6, it is seen that our approach does not let us capture the outer $p$-mode frequencies very accurately, because the result is very sensitive to the position of the outer boundary. This is probably related to the fact that the $p$-modes represent the sound waves propagating between the PNS surface and the shock position, a region which is not taken into account in our analysis. This, however, does not affect the qualitative conclusion that the high frequency noise on the GW spectrogram above the dominant feature is at least partially associated with these modes. Another possible source of this noise is the turbulent convection between the PNS and the shock front, which is
Figure 5. Eigenfrequencies $\nu/2\pi$ of the $l = 2$ modes compared to the GW spectrum from model X1.0.5FHz. Each digit represents the number of nodes in the corresponding mode. The left panel shows the results obtained using the Cowling approximation, while the right panel shows the solution of the full system of Eqs. (8)-(11). In the right panel, the dominant feature of the spectrogram is well described by the fundamental (0 radial nodes) mode starting from $\sim$400 ms after bounce.

Figure 6. Dependence of the derived eigenfrequencies on the position of the outer boundary in our analysis. This plot demonstrates that the frequencies of p-modes are only approximately captured by our calculations. At the same time, the frequencies of the f-mode and the low order g-modes are almost insensitive to the position of the outer boundary, which demonstrates the robustness of our main result, i.e., the association between the dominant GW feature and the fundamental ($l = 2$) PNS mode.

The chaotic and does not necessarily represent any simple eigenmode of the system.

At the same time, the frequency of the fundamental mode in Figure 6 is almost insensitive to the position of the outer boundary, and the low-order g-modes depend weakly on it. Importantly, this shows that the dominant GW frequency is not just proportional to the Brunt-Väisälä frequency at the surface of the PNS, as was suggested in earlier work. Indeed, Figure 4 shows that the three black lines corresponding to the different outer boundary locations pass through very different values of the Brunt-Väisälä frequency. The fact that the fundamental quadrupolar eigenfrequency in Figure 6 is nearly independent on the position of the outer boundary tells us that the dominant frequency of the GW signal is defined by the entire structure of the PNS, rather than by its surface characteristics alone.

The left panel of Figure 7 illustrates the time evolution of the radial eigenfunction $\eta_r$ for the $l = 2$ modes associated with the dominant frequency of the GW signal. The eigenfunctions are normalized to 1 and plotted as a function of radial coordinate from the innermost grid point up to the location of the outer boundary. In Figure 7, they are shifted along the $y$-axis according to the time after bounce at which they are calculated (the time is indicated on the left side of the panel and directed downwards). As we already mentioned, starting from $\sim$400 ms after bounce and until the end of the simulation the main signal is represented by the f-mode, which has the largest amplitude at the PNS boundary surface and gradually decreases towards the center. Before that, in the time interval between $\sim$200 and $\sim$400 ms, this mode is smoothly connected to a g-mode having two radial nodes (see also the left panel of Figure 5). The right panel of Figure 7 shows the energy density $\mathcal{E}$ defined as (Torres-Forné et al. 2017)

$$\mathcal{E} = \frac{\sigma^2}{8\pi} \rho \left[ \eta_r^2 + l(l+1) \frac{\eta_t^2}{\eta_r^2} \right]$$

(19)

for the corresponding eigenfunctions of the left panel. The figure shows that the shape of the fundamental eigenfunction is very similar in the case of the Cowling approximation (black lines) and in the case when $\delta\alpha \neq 0$ (red lines). The
energy density of the modes shows less agreement. Note that the definition of $\mathcal{E}$ contains the mass density, which is larger in the inner region than at the the surface of the PNS. Therefore, even a barely visible disagreement between the eigenfunctions in the inner region may lead to a large disagreement between the energy density distributions (see, for example, the 0.48 s snapshot in Figure 7).

Finally, in Figure 8 we attempt to address the nature of the ‘gap’ seen in our spectrograms by performing the linear perturbation analysis of the PNS inner core only. For that, we place the outer boundary at the inner maximum of the Brunt-Väisälä frequency, which roughly corresponds to the radial coordinate of 10 km (see Figure 4), and solve the system of Eqs. (8)-(11) in that inner region, using the boundary condition (17). This approach is not strictly accurate, but it gives us an idea about the eigenfrequencies of the inner core. In Figure 8, the digits show the number of radial nodes in the corresponding modes. All modes with the number of nodes less than 7 are shown, without selection. The resolution of the PNS core in our simulations does not exceed a few tens of grid points, which leads to the spurious nodes and the step-like behavior of the eigenfrequencies is a result of the insufficient resolution of the PNS core in the simulations. Nevertheless, the core eigenfrequencies lie close to the position of the ‘gap’ in the GW spectrogram and roughly resemble its morphology. We speculate that the ‘gap’ may appear as a result of interaction between the high-order $p$-modes and the trapped mode of the PNS inner core, e.g., by means of an avoided crossing.

We speculate that the ‘gap’ may be the result of interaction between the trapped PNS core mode and the other ($p$- or $f$-) modes of the system, probably by means of an avoided crossing (Christensen-Dalsgaard 1981; Yoshida & Erguchi 2001; Stergioulas 2003). One of the simplest examples of the avoided crossing phenomenon is the case of two coupled classical oscillators, where the eigenfrequencies demonstrate characteristic splitting in the strong coupling regime (Novotny 2010). Analogously, one may view the PNS as a coupled system of the inner core and the outer convectively stable shell, mediated by the inner PNS convection region (Dessart et al. 2006). In this picture, the modes of the inner core may repel the modes of the shell, leading to an empty region in the frequency space with the width related to the strength of the coupling between the two. At the same time, the inner mode itself is most likely not excited, because it is shielded from the down-falling plumes of the postshock convection region by the PNS surface (though, the inner PNS convection itself may be a source of mode excitation; see, for example, Müller et al. 2013). To clarify the nature of the ‘gap’, higher resolution simulations are necessary. If this spectrogram feature is real, it could serve as an interesting analysis tool to probe the structure of the inner PNS.

3.3. Dependence of the GW signal on parameters

In this section, we outline the key dependences of the GW signal on the progenitor mass and rotational angular velocity, on the EOS, and on the details of the microphysics, such as the inclusion of the many-body corrections to the neutrino-nucleon scattering rate and the implementation of the gravity
3.3.1. Dependence of the GW signal on the progenitor mass

Figure 9 shows the GW spectrograms and waveforms for the models M10\textsubscript{SFHo}, M13\textsubscript{SFHo}, and M19\textsubscript{SFHo}, which are simulated with the identical numerical setup and differ only in the progenitor mass. Two of the models, M10\textsubscript{SFHo} and M19\textsubscript{SFHo}, explode at \(\sim400\) and \(\sim350\) ms after bounce, respectively, and have a characteristic explosion ‘tail’ in their waveforms (Murphy et al. 2009; Yakunin et al. 2010; Müller et al. 2013). The model M13\textsubscript{SFHo} does not explode. White markers on the spectrograms indicate the eigenfrequencies of the fundamental \(l = 2\) modes, found as described in Section 3.2 for each model. In general, we see good agreement between the analytical eigenfrequencies and the dominant GW signal, with the largest deviation seen in the post-explosion phase of the 19\(M_{\odot}\) model.

For comparison, the red lines in the top panel of Figure 9 show the peak GW frequency \(f_{\text{peak}}\) computed as suggested in Murphy et al. (2009) and Müller et al. (2013), where it is associated with the surface value of the Brunt-Väisälä frequency divided by \(2\pi\). In Figure 9, we use Equation (17) of Müller et al. (2013), omitting the factor \(\left(1 - \frac{GM_{\text{PNS}}}{R_{\text{PNS}}^2}\right)^2\), where \(M_{\text{PNS}}\) and \(R_{\text{PNS}}\) are the mass and radius of the PNS, respectively. As in Pan et al. (2017), we find that removing this factor results in better agreement between \(f_{\text{peak}}\) and the dominant feature of the GW spectrogram\(^4\). The three lines correspond to the three different density isosurfaces of \(\rho = 5.0 \times 10^{10}\, \text{g cm}^{-3}, 10^{11}\, \text{g cm}^{-3},\) and \(10^{11}\, \text{g cm}^{-3}\), which can represent the PNS surface. The plot shows that using the conventional definition for the PNS surface, \(\rho = 10^{11}\, \text{g cm}^{-3}\), the analytical formula for \(f_{\text{peak}}\) may provide a good fit to the dominant GW signal. At the same time, the value of \(f_{\text{peak}}\) is sensitive to the location of the PNS surface, which currently lacks strict physical definition.

The agreement between the analytic eigenfrequencies and the GW spectrograms allows us to compare the spectrograms by comparing the frequencies. Figure 10 shows the full results of the linear perturbation analysis for the three considered models, performed as in Section 3.2. In the bottom panel of Figure 10, the filled symbols show the modes with larger than zero number of nodes, while the empty symbols show the \(f\)-mode frequencies. This plot demonstrates that the PNS eigenfrequencies in general, and the frequencies of the fundamental quadrupolar mode in particular, are strikingly similar between the models, despite the large difference in their progenitor masses and even in the waveforms themselves.

This may reflect the fact that the evolution of the PNS radius is very similar between the models with different progenitor masses, which is shown in the top panel of Figure 10, and was already noticed in the literature for a wide range of progenitors differing not only by the ZAMS mass, but also in the metallicity (see, for example, Figure 7 of Bruenn et al. 2016, Figure 10 of Summa et al. 2016, Figure 15 of Radice et al.\footnote{This may be related to the fact that we use the approximate relativistic gravitational potential in our simulations.}).

\(4\) This may be related to the fact that we use the approximate relativistic gravitational potential in our simulations.
Figure 10. Top panel: The radius at which the angle-averaged density of the models M10_SFHo, M13_SFHo, and M19_SFHo is equal to $10^{10}$ g cm$^{-3}$. This represents the outer boundary in the linear perturbation analysis of Section 3.2, and it can be used as a proxy for the PNS radius (although $\rho = 10^{11}$ g cm$^{-3}$ is more commonly used in the literature for that). Bottom panel: 1−2 eigenfrequencies of these models, calculated using linear perturbation analysis, as described in Section 3.2. Large empty symbols represent the fundamental (0-nodes) mode, which is also shown in Figure 9. This plot demonstrates that the dominant frequency of the GW signal depends weakly on the progenitor ZAMS mass.

Indeed, if the GW signal from CCSNe is so tightly related to the PNS eigenmodes, the structure of the PNS should be the main factor defining the time-frequency structure of this signal.

The same argument cannot be applied to the amplitude of the GW signal, which, instead, must depend on the mechanism of excitation of the PNS modes. It was shown in many previous studies that the GW signal from CCSNe experiences sudden increases in amplitude at the moments when the PNS surface is hit by the downflowing accretion ‘plumes’ (Murphy et al. 2009; Müller et al. 2013; Yakunin et al. 2015). It is, therefore, natural to expect that the GW power will depend on the details of the postshock accretion, which takes place above the PNS surface and is largely determined by the core structure of the progenitor. Figure 11 shows the energy emitted in GWs due to the matter motions alone as a function of time for the models M10_SFHo, M13_SFHo and M19_SFHo. In these models, we don’t see a monotonic dependence of the GW power on the progenitor mass, with the model M13_SFHo producing the weakest signal among the three. In fact, it is hard to expect such a monotonic dependence, because the dependence of the progenitor core structure itself on the progenitor ZAMS mass is not monotonic (Sukhbold et al. 2016) and, moreover, may be intrinsically chaotic (Sukhbold et al. 2017). For this reason, we advise using caution when deducing the dominant signal frequency based on the total GW energy spectrum, especially if it is done for the purpose of comparing models with different progenitor masses. Accretion downflows hitting the PNS surface at random moments of time may give more weight to the system eigenfrequencies in those moments, complicating the overall picture. Instead, the comparison of the time-frequency spectrograms serves this purpose best.

The strongest signal among all our models is produced by the model M19_SFHo. In Figure 12, we present the linear 3D representation of the GW spectrogram from this model. This figure emphasizes the point made in Section 3.1, that the GW signal may stay strong for a long time after the explosion (more than a second in the case of M19_SFHo). The large offset from zero seen in the GW strain of this model at late times (the bottom panel of Figure 9) suggests a very asymmetric character for its explosion. This is indeed the case, as demonstrated in Figure 6 of Vartanyan et al. (2018), which shows snapshots of the electron fraction and entropy of this model at different moments of time. For the analogous snapshots of the 10 $M_\odot$ model, we refer the reader to Radice et al. (2017).

The dependence of the CCSN GW signal on the progenitor mass was previously studied in a number of works (Murphy et al. 2009; Müller et al. 2013; Yakunin et al. 2015). For example, Müller et al. (2013) report ~30% differences in the typical emission frequencies between their 11.2 and 25 $M_\odot$ models, which they admit to be small for this large a mass difference. Our comparison, however, shows even smaller scatter, no more than ~5−10% in frequency across the considered mass range, without any systematic trend. On one hand, we cannot exclude that at least part the difference between the 11.2 and 25 $M_\odot$ models of Müller et al. (2013) may come from the fact that they were simulated with a slightly different EOS (we discuss the dependence of the signal on EOS in the next subsection). On the other hand, the models of Müller et al. (2013) treat general relativity more accurately by solving the relativistic equations of hydrodynamics in the conformally-flat approximation, while our work uses the effective potential approach, which may also affect the dominant frequency of the signal (see Müller et al. 2013).

$^5$ Note that in the top panel of Figure 10 we show the radii where the angle-averaged density is equal to $10^{10}$ g cm$^{-3}$, which also serves as the outer boundary in our analysis. It is more common in the literature to use $\rho = 10^{11}$ g cm$^{-3}$ as the definition of the PNS radius. In our models, the radii at the density $10^{11}$ g cm$^{-3}$ are nearly the same as the radii at the density $10^{10}$ g cm$^{-3}$.
3.3.2. Dependence of the GW signal on the equation of state

Figure 13 shows the GW spectrograms and waveforms for the models M10\_LS220, M10\_SFHo, and M10\_DD2, which were simulated with three different EOSes. All other numerical parameters and the details of microphysics are the same between these models. We find that the EOS has a large impact on the amplitude of the GW signal, its dominant frequency, total emitted energy $E_{GW}$ (see Table 1), and even the qualitative outcome of the simulation (the model M10\_SFHo explodes, unlike the other two). Similar EOS sensitivity of the simulation outcome was recently reported by Pan et al. (2017) in the context of the GW signal from black hole formation in failed SNe.

In order to emphasize the dependence of the dominant GW frequency on the EOS, we overplot the GW spectrograms of these models in the bottom panel of Figure 14. Empty markers represent the f-mode eigenfrequencies found from the linear perturbation analysis of the models, as described in Section 3.2. The top panel of Figure 14 shows the evolution of the PNS radii taken at the value of density $10^{10}$ g cm$^{-3}$. Compared to the top panel of Figure 10, the difference between the PNS radii in Figure 14 is slightly larger and more systematic, which translates into the systematic $\sim 10$–15% difference in the dominant frequencies of the GW signal, which, in turn, is well captured by our analysis (the largest disagreement is seen in the M10\_DD2 model). Interestingly, among the three EOSes used in our study, SFHo is the ‘softest’ one, while DD2 is the ‘hardest’. Nevertheless, the smallest PNS radius and the largest GW frequency are produced by the LS220 EOS. This suggests that the EOS dependence of the GW signal, as well as the overall core evolution, may not necessarily be described in terms of a single stiffness parameter defined at zero temperature.

We quantify the dependence of the f-mode eigenfrequency on time for models M10\_LS220, M10\_SFHo, and M10\_DD2 by fitting it with a polynomial. We find that a simple quadratic function in the form $f = At^2 + Bt + C$, where $f$ is frequency in Hz, $t$ is time in seconds, and $A$, $B$, and $C$ are coefficients, adequately describes the dependence over the first $\gtrsim 1.5$ seconds after bounce, while the core keeps shrinking. Eventually, the PNS will cool down and deleptonize, which could lead to

Figure 12. Linear 3D representation of the GW spectrogram from the model M10\_SFHo. This model starts exploding at $\sim 350$ ms after the core bounce, but the dominant component of the GW signal does not decay and stays strong until the end of the simulation, for more than a second after the explosion.

Figure 13. GW spectrograms and waveforms from the models M10\_LS220, M10\_SFHo, and M10\_DD2, differing only in the EOS. Gray hatched regions simply fill the blank space left after aligning the simulations in time.
the flattening of the frequency-time curve. The quadratic fits are shown with the lines of corresponding color in Figure 14 and explicitly written down in the right bottom corner of the figure. These fits can be used as priors when looking for the CCSN GW signal in the data from ground-based laser interferometers, such as LIGO, Virgo, or KAGRA. At the same time, we emphasize that the accuracy of these fits may be affected by the details of the physics and microphysics used in our (and other) codes. For example, to demonstrate the sensitivity of the GW signal to the details of the neutrino opacity, we compare the spectrograms from models M10_LS220 and M10_LS220_no_manybody in Figure 15. The many-body corrections to the neutrino-nucleon scattering cross section decrease the neutrino opacity, which leads to the faster contraction of the PNS, as shown in the top panel of Figure 15. The bottom panel of Figure 15 shows that neglecting these corrections results in a \( \sim 10\% \) shift in the dominant GW frequency. Another factor influencing the GW frequency is the description of the gravitational field (see, e.g. Müller et al. 2013). Taking all these factors into account, we expect the accuracy of the fits from Figure 14 to be not worse than \( \sim 30\% \).

On the other hand, the power of the GW signal does demonstrate monotonic dependence on the stiffness of the EOS, with the hardest EOS (DD2 in our case) producing the weakest signal. As we already mentioned in the previous subsection, the amplitude of the GW signal is largely determined by accretion and post-shock convection, which act as driving forces for the excitation of the PNS oscillations. It was found in previous work (Marek et al. 2009; Kuroda et al. 2016, 2017) that softer EOSes result in more vigorous SASI activity. While we do not clearly identify SASI in any of the three models, we also find that the shock oscillations are strongest in the M10_SFHo model and weakest in the M10_DD2 model. This leads to stronger excitation of the PNS modes and more powerful GW signals in case of the softest EOS.

### 3.3.3. Dependence of the GW signal on rotation

Simulations of rotating core collapse were the first to predict and study the GW emission from CCSNe (Ott et al. 2007, 2012; Dimmelmeier et al. 2007, 2008; Abdikamalov et al. 2010). Because of the symmetry breaking introduced by rotation, these models produce strong GW signals already at the early stages of collapse and bounce, which makes even short (few tens of milliseconds) simulations very informative. Not very demanding in terms of the neutrino physics, these simulations progressed enough to establish the connection between the properties of the GW signal and the progenitor core parameters (Summerscales et al. 2008; Logue et al. 2012; Röver et al. 2009; Abdikamalov et al. 2014; Engels et al. 2014; Fuller et al. 2015; Powell et al. 2016; Richers et al. 2017). The main limitation of these papers is that fast rotating cores are not very common among CCSN progenitors (Heger et al. 2005; Woosley & Heger 2006). Here, we focus on the GW signal from a moderately \( (\Omega_0 = 0.2 \text{ rad s}^{-1}) \) rotating progenitor, and follow it for a full second after bounce, which, to the best of our knowledge, is currently the longest simulation of its kind, for which the GW signal has been extracted.

In the rotating model, we use the multipole gravity solver of Müller & Steinmetz (1995). For all other models shown before, we used a monopole approximation for the grava-
The main findings of our study can be briefly summarized as follows:

- We reproduce the dominant, long-lasting GW signal from CCSNe by means of linear perturbation analysis and associate it with a g-mode having 2 radial nodes at the early stage (~200–400 ms after bounce) and with the f-mode later on (from ~400 ms until more than a second after bounce). This finding presages future opportunities for the analytical study of the CCSN GW signal.

- We demonstrate a weak dependence of the dominant GW frequency on the progenitor ZAMS mass and provide a simple quadratic fit for it as a function of time for three different EOSes. This may help identify possible

Figure 16. GW spectrograms and waveforms of the models M13_SFHo and M13_SFHo\_multipole, differing only in the gravity implementation (see Section 2).

Figure 17. Comparison of the GW spectrograms and waveforms from the models M10_SFHo\_multipole and M10_SFHo\_rotating, differing only in the angular velocity. Gray hatched regions simply fill the blank space left after aligning the simulations in time.

persists in the GW spectrograms of both models.

4. CONCLUSIONS AND DISCUSSION

The main findings of our study can be briefly summarized as follows:

- We reproduce the dominant, long-lasting GW signal from CCSNe by means of linear perturbation analysis and associate it with a g-mode having 2 radial nodes at the early stage (~200–400 ms after bounce) and with the f-mode later on (from ~400 ms until more than a second after bounce). This finding presages future opportunities for the analytical study of the CCSN GW signal.

- We demonstrate a weak dependence of the dominant GW frequency on the progenitor ZAMS mass and provide a simple quadratic fit for it as a function of time for three different EOSes. This may help identify possible
CCSN candidates in the GW data from ground-based laser interferometers.

- We identify a new feature in the GW spectrogram, which looks like a ‘gap’ across the noisy GW emission in the first $\sim 200-700$ ms after bounce. Our attempts to explain it as a numerical artifact failed. We explain the ‘gap’ as the interaction between the outer p-modes and g-modes of the PNS inner ($\sim 10$ km) core, probably as a result of avoided crossing.

- We show the effect of moderate ($0.2 \, \text{rad} \, \text{s}^{-1}$) initial progenitor rotation on the GW signal. The rotation strengthens the bounce signal, but weakens the dominant part of the post-bounce GW emission.

All simulations analyzed in our study are 2D, which raises a question about how our conclusions will change in the full 3D case. It is known from previous studies that the success of an explosion in the CCSN simulations largely depends on the hydrodynamical instabilities and the associated turbulent pressure behind the stalled shock (Burrows et al. 1995; Murphy et al. 2013; Couch & Ott 2015; Müller & Janka 2015; Abdikamalov et al. 2016; Takahashi et al. 2016; Müller et al. 2017), which also increases the exposure of matter to neutrino heating (Buras et al. 2006; Murphy & Burrows 2008). However, it is known that turbulence has different properties in 2D and 3D (Kraichnan 1967), and it has been shown that this difference artificially facilitates explosion (Hanke et al. 2012; Dolence et al. 2013; Takiwaki et al. 2014; Couch & O’Connor 2014; Couch & Ott 2015; Abdikamalov et al. 2015). Therefore, if the properties of turbulence in the gain region were directly reflected in the GW spectrogram, we would expect it to differ in 3D. Instead, our analysis suggests that the strongest component of the GW signal is associated with the fundamental mode of the PNS itself, which is expected to be nearly spherical even in the 3D case. Turbulence in this case acts only as a driving force exciting the mode oscillations. This makes us believe that the time-frequency structure of the GW signal shown here and its linear analysis will still be applicable for 3D models, while the amplitude may change (become smaller). The same was recently suggested in Yakunin et al. (2017), where the authors obtained similar behavior of the GW signal for a 2D and a 3D model. In their 3D case, convection was characterized by a larger number of relatively small scale structures, as opposed to the few massive accretion funnels in 2D. This led in 3D to smoother GW energy emission, but caused only moderate changes in its spectral distribution, vis-à-vis their 2D results, during the first 450 ms of the signal. More about the comparison between the 2D and 3D GW signals from CCSNe may be found in Andrensen et al. (2017).

Interestingly, our linear analysis presents an opportunity to predict the dominant frequency of the GWs from CCSNe based on 1D simulations. However, this approach should be applied with great caution, because, for example, the evolution of the PNS radius differs between the 1D and 2D simulations for the same models (Radice et al. 2017). At the same time, such an analysis allows one to quickly cover large regions of parameter space related to the EOS and microphysics, in order to investigate which of the parameters has the strongest influence on the GW signal.

Pan et al. (2017) suggested that increasing the sensitivity of the next generation GW detectors in the $\sim 1000$ Hz window is very important for the study of the BH formation in failed SNe (see also Kuroda et al. 2018). We add to this statement that high frequency sensitivity is crucial for the detection of the GW signal from the successful SN explosions as well. Increasing the sensitivity of aLIGO and KAGRA in this band would help us to fully exploit the luck of the next nearby SN discovery and trace the high-frequency GW signal of a newborn NS.

We thank Aaron Skinner and James Stone for helpful discussions and feedback. We thank Pablo Cedrén-Durán and José Antonio Font for finding an error in the original calculations, and for other helpful suggestions. The authors would like to acknowledge support of the U.S. NSF under award AST-1714267, the Max-Planck/Princeton Center (MPPC) for Plasma Physics (under award NSF PHY-1144374), and the DOE SciDAC Grant DE-SC0018297 (under subaward 00009650). The authors employed computational resources provided by the TIGRESS high performance computer center at Princeton University, which is jointly supported by the Princeton Institute for Computational Science and Engineering (PICSciE) and the Princeton University Office of Information Technology. They also acknowledge a supercomputer allocation by the National Energy Research Scientific Computing Center (NERSC), which is supported by the Office of Science of the US Department of Energy (DOE) under contract DE-AC03-76SF00098. DR acknowledges support from a Frank and Peggy Taplin Membership at the Institute for Advanced Study and the Max-Planck/Princeton Center (MPPC) for Plasma Physics (NSF PHY-1523261).

REFERENCES

Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2016, ApJ, 818, L22
—. 2017a, Physical Review Letters, 119, 141101
—. 2017b, Physical Review Letters, 119, 161101
—. 2017c, ApJ, 848, L12
Abdikamalov, E., Gossan, S., DeMaio, A. M., & Ott, C. D. 2014, Phys. Rev. D, 90, 044001
Abdikamalov, E., Zhaksylykov, A., Radice, D., & Berdíbek, S. 2016, MNRAS, 461, 3864
Abdikamalov, E., Ott, C. D., Radice, D., et al. 2015, ApJ, 808, 70
Abdikamalov, E. B., Ott, C. D., Rezzolla, L., et al. 2010, Phys. Rev. D, 81, 044012
Adams, S. M., Kochanek, C. S., Beacom, J. F., Vagins, M. R., & Stanek, K. Z. 2013, ApJ, 778, 164
Andrensen, H., Müller, B., Müller, E., & Janka, H.-T. 2017, MNRAS, 468, 2032
Audit, E., Charrier, P., Chièze, J., & Dubroca, B. 2002, ArXiv Astrophysics e-prints
Baník, S., Hempel, M., & Bandyopadhyay, D. 2014, ApJS, 214, 22
Banyuls, F., Font, J. A., Ibáñez, J. M., Martí, J. M., & Miralles, J. A. 1997, ApJ, 476, 221
Blondin, J. M., Mezzacappa, A., & DeMarino, C. 2003, ApJ, 584, 971
Borkowski, K. J., Gwynne, P., Reynolds, S. P., et al. 2017, ApJ, 837, L7
Bruenn, S. W., Lentz, E. J., Hix, W. R., et al. 2016, ApJ, 818, 123
Buras, R., Janka, H.-T., Rampp, M., & Kifonidis, K. 2006, A&A, 457, 281
Burrows, A., & Hayes, J. 1996, Physical Review Letters, 76, 352
Burrows, A., Hayes, J., & Fryxell, B. A. 1995, ApJ, 450, 830
Burrows, A., Reddy, S., & Thompson, T. A. 2006, Nuclear Physics A, 777, 356
Borkowski, K. J., Gwynne, P., Reynolds, S. P., et al. 2017, ApJ, 837, L7
Burs, A., Janka, H.-T., Marek, A., & Müller, E. 2007, Physical Review Letters, 98, 251101
Burs, A., Varatyan, D., Dolence, J. C., Skinner, M. A., & Radice, D. 2017, Accepted to Space Science Reviews
Camelio, G., Lovato, A., Guaglieri, L., et al. 2017, Phys. Rev. D, 96, 043015
Cerdá-Durán, P., DeBrye, N., Aloy, M. A., Font, J. A., & Obergaulinger, M. 2013, ApJ, 779, L18
Christensen-Dalsgaard, J. 1981, MNRAS, 194, 229
Couch, S. M., & O’Connor, E. P. 2014, ApJ, 785, 123
Couch, S. M., & Ott, C. D. 2015, ApJ, 799, 5
Cessi, S., Burrows, A., Livine, E., & Ott, C. D. 2006, ApJ, 645, 534
Dimmelmeier, H., Ott, C. D., Janka, H.-T., Marek, A., & Müller, E. 2007, Physical Review Letters, 98, 251101
Dimmelmeier, H., Ott, C. D., Marek, A., & Janka, H.-T. 2008, Phys. Rev. D, 78, 064056
Dolence, J. C., Burrows, A., Murphy, J. W., & Nordhaus, J. 2013, ApJ, 765, 110
A. RESOLUTION DEPENDENCE OF THE GW SIGNAL FOR THE MODEL M10_LS220_NO_MANYBODY.

Figure 18 shows the GW spectrograms and waveforms of the model M10_LS220_no_manybody for three different grid resolutions, of which the lowest (‘standard’) is used in all other models of this study. It is clear from the figure that the overall structure of the GW signal and its spectrogram depend weakly on resolution, demonstrating the robustness of our results.

B. DERIVATION OF THE LINEAR PERTURBATION EQUATIONS INCLUDING LAPSE VARIATION

Here, we derive the set of equations describing linear perturbations of a spherically-symmetric background, including the perturbation of the lapse function. Generally, our calculations follow the same scheme as described in Torres-Forné et al. (2017), and for simplicity of notation we use the same notation where possible. All equations are given in geometrized units. We start with a static spherically-symmetric conformally flat space-time metric in isotropic coordinates ($t, x$):

\[ ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2dt^2 + \psi^4 f_{ij}dx^i dx^j, \]

where $\alpha$ and $\psi$ are the lapse and shift functions, respectively, $f_{ij}$ are the perturbation components, and $g_{\mu\nu}$ is the background space-time metric.

APPENDIX
where \( \alpha \) is the lapse function, \( \psi \) is the conformal factor, and \( f_{ij} \) is the flat spatial 3-metric. In this metric, the equations of general-relativistic hydrodynamics for a perfect fluid can be rendered in the form (Torres-Forné et al. 2017; Banyuls et al. 1997):

\[
\frac{1}{\sqrt{\gamma}} \partial_t [\sqrt{\gamma} D] + \frac{1}{\sqrt{\gamma}} \partial_i [\sqrt{\gamma} \nu^i] = 0 ,
\]

\[
\frac{1}{\sqrt{\gamma}} \partial_t [\sqrt{\gamma} S_j] + \frac{1}{\sqrt{\gamma}} \partial_i [\sqrt{\gamma} S_j \nu^i] + \alpha \partial_i P = \frac{\alpha \beta h}{2} u^\mu u^\nu \partial_\nu g_{\mu \nu} ,
\]

\[
\frac{1}{\sqrt{\gamma}} \partial_t [\sqrt{\gamma} E] + \frac{1}{\sqrt{\gamma}} \nabla_i [\sqrt{\gamma} (E \nu^i + \alpha P \nu^i)] = \alpha^2 (T^\mu_\nu \partial_\mu \ln \alpha - T^\mu_\nu T^\nu_\mu) .
\]

Here, \( T^\mu_\nu = \rho w^\mu w^\nu + P g^\mu_\nu \) is the energy-momentum tensor of a perfect fluid, where \( \rho \) is its rest-mass density, \( P \) is the pressure, \( w^\mu \) is the 4-velocity, \( h \equiv 1 + \epsilon + P/\rho \) is the specific enthalpy, and \( \epsilon \) is the specific internal energy. \( \Gamma^\mu_{\nu \lambda} \) denotes the Christoffel symbols, and \( \gamma = \psi^{12} r^4 \sin^2 \theta \) is the determinant of the three-metric, \( \gamma_{ij} = \psi^4 f_{ij} \). The conserved rest-mass density \( D \), momentum density in the \( j \)-direction \( S_j \), and the total energy density \( E \) are defined as

\[
D = \rho W , \quad S_j = \rho h W^2 \nu_j , \quad E = \rho h W^2 - P ,
\]

where \( W = 1/\sqrt{1 - \nu_\mu \nu^\mu} \) is the Lorentz factor, \( \nu^i \) and \( \nu^{*i} \) represent the Eulerian and “advective” velocities, in the spherically-symmetric case equal to \( u^i/W \) and \( \alpha u^i/W \), respectively.

As in Torres-Forné et al. (2017), we consider the linear perturbations of the system with respect to the equilibrium static background, for which the only non-zero radial component of Eq. B3 is

\[
\frac{1}{\rho h} \partial_r P = -\partial_r \ln \alpha \equiv G_r ,
\]

where \( G_r = \tilde{G} \) is the radial component of the gravitational acceleration. At the same time, in addition to the perturbation of density, pressure, and velocity, we introduce the non-zero perturbation of the lapse function, \( \alpha \). This addition does not fully relax the Cowling approximation, but it closely mimics the conditions of our numerical simulations, where the shift vector \( \beta^i = 0 \) and the conformal factor is fixed \( \psi = 1 \). Following Torres-Forné et al. (2017), we denote the Eulerian perturbations of the quantities by \( \delta \) and the Lagrangian perturbations by \( \Delta \), where the relation between the two for any quantity, e.g. \( \rho \), is

\[
\Delta \rho = \delta \rho + \xi^i \partial_i \rho .
\]
Here, $\xi^i$ is the Lagrangian displacement of a fluid element, related to the advective velocity as

$$\partial_t \xi^i = \delta v^i.$$  \hspace{1cm} (B8)

After perturbing the quantities by substituting, e.g., $\rho \rightarrow \rho + \delta \rho$, and leaving only terms of linear order, Eqs. B2 and B3 can be rewritten as

$$\frac{\Delta \rho}{\rho} = - \left( \partial_t \xi^i + \xi^i \partial_i \ln \sqrt{\gamma} \right),$$  \hspace{1cm} (B9)

$$\rho h \partial_t \delta \nu_j + \alpha \partial_i \delta P + \delta \alpha \partial_i \rho = - \delta (\rho h) \partial_j \alpha - \rho h \partial_j \delta \rho,$$  \hspace{1cm} (B10)

with the three components of Eq. B10 taking the form

$$\rho h \psi^4 \alpha^{-2} \frac{\partial^2 \xi^r}{\partial t^2} + \partial_r \delta P + \frac{\delta \alpha}{\alpha} \partial_r \rho = \delta (\rho h) \tilde{G} - \frac{\rho h}{\alpha} \partial_r \delta \rho,$$  \hspace{1cm} (B11)

$$\rho h \psi^4 \alpha^{-2} r^2 \frac{\partial^2 \xi^\theta}{\partial t^2} + \partial_\theta \delta P = - \frac{\rho h}{\alpha} \partial_\theta \delta \rho,$$  \hspace{1cm} (B12)

$$\rho h \psi^4 \alpha^{-2} r^2 \sin^2 \theta \frac{\partial^2 \xi^\phi}{\partial t^2} + \partial_\phi \delta P = - \frac{\rho h}{\alpha} \partial_\phi \delta \rho.$$  \hspace{1cm} (B13)

The condition of adiabaticity of the perturbations

$$\frac{\Delta P}{\Delta \rho} = \frac{\hbar c_s^2}{\rho} \frac{P}{\Gamma_1},$$  \hspace{1cm} (B14)

where $c_s$ is the relativistic speed of sound and $\Gamma_1$ is the adiabatic index, allows one to write (Torres-Forné et al. 2017):

$$\delta (\rho h) = \left( 1 + \frac{1}{c_s^2} \right) \delta P - \rho h \xi^i B_i,$$  \hspace{1cm} (B15)

where

$$B_i = \frac{\partial v^j}{\rho h} - \frac{1}{\Gamma_1} \frac{\partial P}{P},$$  \hspace{1cm} (B16)

is the relativistic version of the Schwarzschild discriminant and $e = \rho (1 + \epsilon)$. For a spherically symmetric background, the only non-zero component of $B_i$ is $B_r = B$. Due to the adiabatic nature of perturbations, Eq. B4 does not add any information.

To close the system of Eqs. B2-B3, we use the Poisson equation

$$\nabla^2 \delta \Phi = 4 \pi \delta \rho,$$  \hspace{1cm} (B17)

where $\Phi$ is the gravitational potential. Using the relation $\alpha = e^\phi$ we rewrite it as

$$\nabla^2 \left( \frac{\delta \alpha}{\alpha} \right) = 4 \pi \delta \rho.$$  \hspace{1cm} (B18)

Following Torres-Forné et al. (2017), we consider only polar perturbations and expand them in terms of spherical harmonics as

$$\delta P = \delta \tilde{P} Y_{lm} e^{-i \sigma t}, \quad \delta \alpha = \delta \tilde{\alpha} Y_{lm} e^{-i \sigma t},$$

$$\xi^r = \eta_r Y_{lm} e^{-i \sigma t}, \quad \xi^\theta = \eta_\perp \frac{1}{r^2} \partial_\theta Y_{lm} e^{-i \sigma t},$$

where $\delta \tilde{P}$, $\delta \tilde{\alpha}$, $\eta_r$, and $\eta_\perp$ are scalar functions depending only on radial coordinate. With this ansatz, and using the adiabaticity condition B14, Eq. B18 may be brought to the form

$$\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} \left( \frac{\delta \tilde{\alpha}}{\alpha} \right) - \frac{1}{\alpha} \frac{l(l+1)}{r^2} \delta \tilde{\alpha} = 4 \pi \left[ \frac{\rho}{\Gamma T_1} \left( \delta \tilde{P} + \eta_r \partial_r P \right) - \eta_r \partial_r \rho \right].$$  \hspace{1cm} (B20)

To conveniently find the numerical solution, we introduce the function $f_\alpha = \partial_r (\delta \tilde{\alpha}/\alpha)$ and break this second-order equation into two first-order equations:

$$\frac{2}{r} f_\alpha + \partial_r f_\alpha - \frac{1}{\alpha} \frac{l(l+1)}{r^2} \delta \tilde{\alpha} = 4 \pi \left[ \frac{\rho}{\Gamma T_1} \left( \delta \tilde{P} + \eta_r \partial_r P \right) - \eta_r \partial_r \rho \right],$$  \hspace{1cm} (B21)

$$- \frac{\partial_r \alpha}{\alpha^2} \delta \tilde{\alpha} + \frac{1}{\alpha} \partial_r \alpha \delta \tilde{\alpha} = f_\alpha.$$  \hspace{1cm} (B22)

Eq. B12 results in

$$\delta \tilde{P} = q \sigma^2 \eta_\perp - \frac{\rho h}{\alpha} \delta \tilde{\alpha},$$  \hspace{1cm} (B23)
where, after Torres-Forné et al. (2017), we have defined $q \equiv \rho \alpha^{-2} \psi^4$. Using Eqs. B14, B15, and B23 in Eqs. B9 and B11, we get

$$\partial_r \eta_r + \left[ \frac{2}{r} + \frac{1}{\Gamma_1} \frac{\partial_r P}{P} + 6 \frac{\partial_r \psi}{\psi} \right] \eta_r + \frac{\psi^4}{\alpha^2 c_s^4} \left( \sigma^2 - \mathcal{L}^2 \right) \eta_\perp - \frac{1}{\alpha c_s^2} \delta \hat{\alpha} = 0 ,$$  

(B24)

$$\partial_r \eta_\perp - \left( 1 - \frac{N^2}{\sigma^2} \right) \eta_r + \left[ \partial_r \ln q - \tilde{G} \left( 1 + \frac{1}{c_s^2} \right) \right] \eta_\perp - \frac{1}{\alpha G} \frac{N^2}{\sigma^2} \delta \hat{\alpha} = 0 ,$$  

(B25)

where $N$ is the relativistic Brunt-Väisälä frequency defined as

$$N^2 \equiv \frac{\alpha^2}{\psi^4} G^2 B_i = \frac{\alpha^2}{\psi^4} \tilde{G} B ,$$  

(B26)

and $\mathcal{L}$ is the relativistic Lamb frequency

$$\mathcal{L}^2 \equiv \frac{\alpha^2}{\psi^4} c_s^2 \frac{l(l+1)}{r^2} .$$  

(B27)

Finally, using Eq. B23, we bring Eqs. B21 and B22 to the form

$$\partial_r f_\alpha + \frac{2}{r} f_\alpha + 4\pi \left[ \partial_r \rho - \frac{\rho}{\Gamma_1} \partial_r P \right] \eta_r - \frac{4\pi \rho^2 h}{\Gamma_1 \alpha} q \sigma^2 \eta_\perp + \left[ \frac{4\pi \rho^2 h}{\Gamma_1 \alpha} - \frac{1}{\alpha^2} \frac{l(l+1)}{r^2} \right] \delta \hat{\alpha} = 0 ,$$  

(B28)

$$\partial_r \delta \hat{\alpha} = f_\alpha \alpha + \frac{\partial_r \alpha}{\alpha} \delta \hat{\alpha} .$$  

(B29)

To find the eigenfrequencies of the linear perturbation modes, $f = \sigma/(2\pi)$, we numerically solve the system of first-order differential equations B24, B25, B28, and B29.
C. RESULTS FOR $L = 3$ AND $L = 4$ MODES

Figure 19. Eigenfrequencies $\alpha/2\pi$ of the $l = 3$ (top two panels) and $l = 4$ (bottom two panels) modes compared to the GW spectrogram for the model M10_SFHl. Each digit represents the number of nodes in the corresponding mode. The left panels show the results obtained using the Cowling approximation, while the right panels show the solution of the full system of Eqs. (8)-(11). The fundamental $l = 3$ mode in the top left panel seems to coincide with the dominant GW frequency, but it shifts upwards once we relax the Cowling approximation in the top right panel.