The Grover energy transfer algorithm for relativistic speeds

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Abstract
Grover’s algorithm for quantum search can also be applied to classical energy transfer. The procedure takes a system in which the total energy is equally distributed among $N$ subsystems and transfers most of it to one marked subsystem. We show that in a relativistic setting the efficiency of this procedure can be improved. We will consider the transfer of relativistic kinetic energy in a series of elastic collisions. In this case, the number of steps of the energy transfer procedure approaches 1 as the initial velocities of the objects become closer to the speed of light. This is a consequence of introducing nonlinearities in the procedure. However, the maximum attainable transfer will depend on the particular combination of speed and number of objects. In the procedure, we will use $N$ elements, as in the classical non-relativistic case, instead of the $\log_2(N)$ states of the quantum algorithm.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Physics gives new ways to analyse information transmission and computation. The traditional theories of information and communication implicitly assume an underlying classical world, with no regard to modern physical theories. However, the recent physical analysis of information has considerably enriched both fields. Taking quantum mechanics into account in communications has led to quantum cryptography [1, 2] and the applications to information processing include efficient algorithms for factoring [3] among others [4].

The description of these quantum algorithms in terms of equivalent physics problems offers a greater insight into how the algorithm works and why it can outperform classical
computer results [5, 6]. Conversely, the algorithm point of view gives a global understanding of related physical problems. In particular, Grover’s algorithm for quantum search can also be studied as an energy transfer problem.

In this paper, we take a variation of Grover’s algorithm for quantum search and translate it into a relativistic setting. This variation can be thought of as an effective procedure for energy transfer between \( N \) non-quantum objects. In the non-relativistic case, the algorithm needs a number of steps proportional to \( \sqrt{N} \) to transfer the energy to a marked object. We show that special relativity predicts a smaller number of steps, which can be reduced even to a single step when the speed of light, \( c \), is approached.

This formulation recasts previous analogues into a more general framework and shows how a Grover procedure can be improved when nonlinearities are present. On one hand, the results highlight some usually overlooked features of the original Grover’s algorithm, like the influence of the initial conditions (the number of states) in the final probability of success. On the other hand, the study opens a way to analyse communications and computation in connection to relativity.

Section 2 reviews the basics of Grover’s algorithm for quantum search and the classical Grover energy transfer. Section 3 explains the mechanical model we will use to study both non-relativistic and relativistic Grover energy transfer. Section 4 presents the results obtained from this model. Section 4.1 describes how the number of steps required for the maximum energy transfer decreases as the initial speeds approach the speed of light in vacuum, \( c \). When we are close to \( c \), a single step can give the desired energy transfer. Section 4.2 discusses the impact of the initial conditions, such as the initial velocity, in the amount of energy that can be transferred to the selected object. Finally, section 5 comments the results of the previous sections and points to future lines of research.

2. Grover’s algorithm and the Grover energy transfer

The basic form of Grover’s algorithm for quantum search gives a procedure to stand out a marked element out of \( N \) possibilities [7]. The algorithm relies on an oracle that can detect and invert the phase of a particular target state \( |t\rangle \). With a sequence of oracle calls and diffusion operators that redistribute the probability amplitudes of a state among the others, we can gradually transfer the probability amplitude from the non-marked states to the target.

The starting point is a uniform superposition \( |s\rangle \) of all the possible states. The states are originally in phase. The oracle changes the sign of the probability amplitude of the target state. During the diffusion stage, the probability amplitude of each state is evenly distributed among the rest of the states of the superposition. For a uniform superposition, all the states would gain the same probability amplitude they lose. Here, the negative target state will instead reduce the probability amplitude of the rest of the states while still receiving their probability amplitude contribution. The result is a slight reduction in the probability amplitude of the many non-marked states and an important increase in the probability amplitude of \( |t\rangle \) (see figure 1). After a number of iterations of the order of \( \sqrt{N} \), there is a high probability of finding the desired state in a measurement.

The evolution can be more easily understood if we define a state \( |t_\perp\rangle \) that collects all the non-marked states from the original superposition. This state is orthogonal to \( |t\rangle \) and close to \( |s\rangle \) in the original Hilbert space. If we observe the evolution of \( |s\rangle \) in the two-dimensional space spanned by \( |t\rangle \) and \( |t_\perp\rangle \), Grover’s algorithm reduces to a series of rotations that takes the state from \( |s\rangle \) to \( |t\rangle \) [8, 9].

A similar procedure can be defined to transfer energy between classical oscillators [6]. In the initial state, the total energy of the system is equally distributed among \( N \) oscillators.
During the Grover procedure, the energy of a marked oscillator will go from being of the order of $\frac{1}{N}$ of the total energy to being of the order of the total energy. Now, the diffusion operator redistributes energy instead of probability amplitude. The crucial difference is that in the classical case we can no longer use a quantum superposition. This results in an increase in the space complexity of the procedure. In the quantum case we could encode $N$ states in $\log_2 N$ two-level systems. Now, we need $N$ oscillators. Apart from that, the time complexity of the procedure (the number of steps) is the same in both scenarios.

The Grover energy transfer can also be better studied by defining two different modes. The first one corresponds to the marked mode to which we want to transfer the energy of the global system of $N$ oscillators. The second mode is the collective mode of all the non-marked oscillators.

3. Generalized model for the Grover energy transfer

We will discuss a simplified case based on elastic collisions [10]. The basic model will be the same for both non-relativistic and relativistic Grover energy transfer. We will consider $N$ bodies of normalized mass 1 which are moving to the right with the same initial speed $v_0$. The Grover energy transfer algorithm will transfer all the kinetic energy of the composite $N$-body system to one selected body. The transfer will occur in a series of elastic collisions.

In order to select one of the $N$ bodies, we will imagine an initial oracle that introduces a small external perturbation that slightly increases the velocity of the selected item. The new velocities will give, with time, the small spatial separation which will be enough to keep the energy transfer going. Alternatively, we can pose the problem as the detection of the single body out of $N$ which has a slightly different initial velocity. At the end of the identification procedure, that body will carry all the kinetic energy of the system.

To simplify the study, we will group the $N$ masses into two balls: a big ball of normalized mass $M = N - 1$ and a small ball of normalized mass 1. This is equivalent to the definition of the $|t\rangle$ and $|t\perp\rangle$ states.

The procedure for both the non-relativistic and the relativistic Grover energy transfer is summed up in figure 2. We begin with both balls moving to the right, with positive speeds $v_0$, for the big ball, and $v_0 + \epsilon$ for the small ball. The value of $\epsilon$ can be, in principle, as small as desired. We will imagine that it can be neglected for the calculations and that its only effect is that the smaller ball will be a little ahead of the big one.

At each iteration, the small ball will meet a rigid wall that will act as the oracle in Grover’s algorithm. Now, instead of a $\pi$ phase shift, we have a change of sign in the velocity when
The small ball rebounds. After the rebound, the balls will collide. Collisions play the role of the diffusion operator. They redistribute the energy into all the modes, in this case, the kinetic energy of the small and big balls. The selected item will slightly reduce the velocity of each of the other \( M \) bodies while receiving a small amount of energy from each of them.

The small ball will start with \( \frac{1}{N} \)th of the total kinetic energy and, at the end of the procedure, its fraction of the total kinetic energy will be close to 1. We suppose that during the whole series of collisions the big ball does not meet the wall and still moves to the right. This is not strictly necessary, but makes the analysis more direct.

In the non-relativistic case, the evolution of the system after each collision is derived from the conservation of energy and momentum. This evolution, when written in a matrix form, corresponds exactly to the evolution operator of quantum search and requires the same number of steps \([10]\). Notably, this does not depend on the initial velocity.

Here, we generalize the situation to relativistic speeds. Our analysis is based on special relativity and will pay no regard to any possible gravitational effects. All the given parameters refer to the frame in which the wall is stationary.

We consider a thought experiment in which we can have a rigid wall and the balls always meet and collide head on. We can imagine an idealized system with extensive balls. For the discussed velocities and energies this is less natural than a treatment with particle collisions, but it allows a simple analysis with no collision cross sections involved.

We assume a series of elastic collisions. The evolution after each collision can be deduced from the conservation of four-momentum. We study a simplified case in which there is only linear movement in the \( x \) direction. All the velocities given will be taken to be in the \( x \) direction and are normalized to the speed of light, using \( v_i \) as a shorthand for \( \frac{v_i}{c} \). In this case, conservation of four-momentum can be expressed with only two equations (for conservation of relativistic energy and \( x \)-momentum). Additionally, as we are dealing with elastic collisions, the rest mass of each ball will be conserved. The evolution after a collision is given by

\[
M \gamma_1 + \gamma_2 = M \gamma_1' + \gamma_2'
\]

(1)

\[
M \gamma_1 v_1 + \gamma_2 v_2 = M \gamma_1' v_1' + \gamma_2' v_2'
\]

(2)

for the conservation of energy and \( x \)-momentum, respectively. In the equations, \( \gamma_i \) represents the Lorentz factor \( \frac{1}{\sqrt{1-v_i^2}} \), corresponding to the velocity \( v_i \). The left-hand sides of the equations define the four-momentum components before the collision and the right-hand sides the situation after the collision. Variables with subindices 1 refer to parameters of the big ball and those with subindices 2 correspond to the small ball.

The resulting nonlinear system of equations can be more easily solved by adding and subtracting both equations to give the equivalent system,
\[ M\alpha_1 + \alpha_2 = M\alpha'_1 + \alpha'_2, \quad (3) \]
\[ \frac{M}{\alpha_1} + \frac{1}{\alpha_2} = \frac{M}{\alpha'_1} + \frac{1}{\alpha'_2}, \quad (4) \]

where \( \alpha_i = \sqrt{\frac{1+v_i}{1-v_i}} \). In both equations, we can group the terms with \( \alpha_1 \) and \( \alpha'_1 \) and \( \alpha_2 \) and \( \alpha'_2 \) at different sides to get

\[ M(\alpha_1 - \alpha'_1) = \alpha'_2 - \alpha_2, \quad (5) \]
\[ \frac{M(\alpha_1 - \alpha'_1)}{\alpha_1\alpha'_1} = \frac{\alpha'_2 - \alpha_2}{\alpha_2\alpha'_2}. \quad (6) \]

That way, it is easy to see two solutions. We discard the trivial solution \( \alpha'_1 = \alpha_1, \alpha'_2 = \alpha_2 \), which describes a situation with no changes, and take instead

\[ \alpha'_1 = C\alpha_2, \quad (7) \]
\[ \alpha'_2 = C\alpha_1, \quad (8) \]

with \( C = \frac{M\alpha_1 + \alpha_2}{M\alpha'_2 + \alpha'_1} \). The final velocities can then be calculated from \( v_i = \frac{\alpha^{i-1}}{\alpha^{i+1}} \).

A complete iteration consists of two steps. At the beginning of each iteration there is an oracle phase in which the small ball rebounds from the wall taking \( v_2 \) to \( v'_2 = -v_2 \) and \( \alpha_2 \) to \( \alpha''_2 = \frac{1}{\alpha''_2} \). These \( \alpha'_1 \) and \( \alpha'_2 \) become the initial conditions \( \alpha_1 \) and \( \alpha_2 \) for the ball collision of the next diffusion phase. For a complete iteration with an oracle and a diffusion phase, we can describe the situation from the iterative expressions

\[ C^i = \frac{M\alpha_1^{i-1} + \alpha_2^{i-1}}{M\alpha_2^{i+1} + \alpha'_1^{i-1}}, \quad (9) \]
\[ \alpha_1^i = \frac{C^i}{\alpha_2^{i-1}}, \quad (10) \]
\[ \alpha_2^i = C\alpha_1^{i-1}, \quad (11) \]

where \( \alpha_1^i \) and \( \alpha_2^i \) can be used to find the velocities after the iteration \( i \). The initial condition is \( \alpha_1^0 = \alpha_2^0 = \sqrt{\frac{1+v_0}{1-v_0}} = \alpha_0 \).

While we could study the energy transfer using velocity, we can understand the evolution better by looking into the relativistic kinetic energies of the balls. We have total energies \( E_1 = M\gamma_1 \) and \( E_2 = \gamma_2 \), but, as the rest masses are conserved, the only energy that can be transferred is the relativistic kinetic energy \( K_1 = M\gamma_1 - M \) and \( K_2 = \gamma_2 - 1 \). We are not interested in the particular values of these energies, but in their relationship to the total kinetic energy \( K_T \). \( K_T \) is conserved all through the energy transfer procedure and can be deduced from the initial condition

\[ K_T = K_1^{\text{ini}} + K_2^{\text{ini}} = N\gamma_0 - N, \quad (12) \]

where \( \gamma_1 = \gamma_2 = \gamma_0 \).
4. Results

4.1. Reduction in the number of steps

First, we will study the number of steps needed to achieve the maximum energy transfer. As in Grover’s algorithm, if we do not stop the procedure, the kinetic energy of the system will alternatively go from the big to the small ball. We have only considered the first maximum of kinetic energy of the small ball (the fastest energy transfer).

Figure 3 shows the results for some representative cases of the relativistic Grover energy transfer of normalized initial velocities $v_0 = 0.001$, 0.01, 0.05, 0.1, 0.3 and 0.8. The number of steps is plotted along the asymptotic limit of steps for Grover search, $\frac{\pi}{4} \sqrt{N}$ [8]. The most significant conclusion is that, for this relativistic setting, the procedure is more efficient than in the non-relativistic case.

The reduction in the number of steps is greater for higher velocities. As expected, for small velocities, the deviation from the non-relativistic case is small. However, as we come closer to the speed of light, the number of steps is dramatically reduced.

These results are valid for any total energy. Equations (1) and (2) are still valid for the general masses $m'$ for the small ball and $Mm'$ for the big ball, as we can factor out $m'$ from all of the equations. For two different masses for the small ball we have different energies, but the $K_2/K_T$ ratio and the evolution do not change.

The relevant factor is the ratio between the kinetic energy $K_2$ of the selected object (the small ball) and the total kinetic energy $K_T$. In the initial state, $\frac{K_2}{K_T}$ is exactly $\frac{1}{N}$. When we stop the procedure, $K_2$ is of the order of $K_T$. N in the x axis tells us the initial distribution of energy. For higher values of N we start with a smaller proportion of the total energy in the small ball.

It might seem that the faster energy transfer is a consequence of the higher kinetic energy of systems with a greater initial value of $v_0$. However, for any initial velocity $v_0$, we can always find a mass which will give us the same initial kinetic energy at other initial velocities. The total kinetic energy and its initial fraction in the marked object will be the same, but the number of steps until the maximum transfer will depend on the initial velocity.

Therefore, the reduction in the number of steps is a genuine effect of velocity. In the non-relativistic analogue of the quantum search of [10], the original mass and velocity are irrelevant.
and the number of steps grows always as \( \frac{\pi}{4} \sqrt{N} \). In the relativistic case, the evolution does not depend on mass either, but the initial velocity determines the total number of interactions before the maximum energy transfer. The number of steps decreases with \( v_0 \) and, for a high enough initial velocity \( v_0 \), the maximum transfer can occur even in a single collision.

We can estimate the initial velocity needed for this single step transfer from four-momentum conservation. The single step has two phases. First, we apply the oracle so that the small ball has a new velocity \(-v_0\). The big ball keeps its velocity \( v_0 \). In the collision, we will have \( \alpha_1 = \alpha_0 = \sqrt{\frac{1 + x^2}{1 - x^2}} \) and \( \alpha_2 = \frac{1}{\alpha_0} \). If all the kinetic energy of the system is transferred to the small ball after the single collision, the big ball has the velocity 0 in the final state and \( \alpha_1' = 1 \). From equation (7) we can find the initial speed for which there is a single step transfer of energy. In that case, we can see that \( \alpha_1' = C\alpha_2 = \frac{x}{\alpha_0} = 1 \), with \( C = \frac{M\alpha_0 + \frac{1}{\alpha_0}}{v_0^2 + \alpha_0} \).

The final state condition leads to the equation
\[
(\alpha_0 - 1)(\alpha_0^2 - (N - 2)\alpha_0 + 1) = 0. \tag{13}
\]
We ignore the trivial solution where the big ball is already at rest before the collision and the solution in which \( v_0 < 0 \) (\( \alpha_0 < 1 \)) and take only the initial velocity of balls moving to the right (\( \alpha_0 > 1 \)):
\[
\alpha_0 = \frac{1 + \sqrt{1 - x^2}}{x}, \tag{14}
\]
with \( x = \frac{2}{N-2} \). This corresponds to an initial velocity
\[
v_0^{ss} = 1 - \frac{x^2}{1 + \sqrt{1 - x^2}}. \tag{15}\]
This solution is only valid for \( N \geq 4 \). For \( N = 4 \) we only have the trivial solution \( \alpha_0 = 1 \) (\( v_0 = 0 \)). If \( N < 4 \), there is no initial velocity for which there is a single step transfer. For \( N \gg 1 \) we can approximate the single step velocity as \( v_0^{ss} = 1 - \frac{2}{N} \). Numerical simulations show that, given a fixed \( N \), the maximum energy transfer is achieved in a single collision for values of \( v_0 \) around \( v_0^{ss} \). Once \( v_0 \) is fixed, the number of steps of the energy transfer will still increase as \( N \) grows. However, this number of steps will be smaller than \( \frac{\pi}{4} \sqrt{N} \). The higher values of \( v_0 \) give a smaller number of steps. Furthermore, for any chosen \( N \), we can always find a velocity \( v_0 \) which gives a single step transfer. This contrasts with the non-relativistic case, where a single step transfer happens only for the \( N = 4 \) case (our limit when \( v_0 \to 0 \) [10].

Even for very small initial velocities, there will be a value of \( N \) for which relativistic effects will become relevant. This can be best noted looking at the graph of \( v_0 = 0.001 \) in figure 3. For moderate values of \( M \) the deviation from the non-relativistic scenario is negligible. However, for high values of \( M \) the curve starts to separate from the non-relativistic prediction.

We can explain the separation looking into the Taylor expansion of \( K_1 \) for small initial velocities, \( K_1 = \frac{1}{8} M v_0^4 + \frac{3}{8} M v_0^6 + O(v_0^8) \). In most problems with a small \( v_0 \), we could neglect all the terms but the non-relativistic kinetic energy. However, for high values of \( M \), the correction terms can be comparable to the other relevant energy of the problem, \( K_2 \). We have taken as a limit the \( K_1 \) of the initial state, when \( v_1 = v_0 \). During the Grover energy transfer, \( v_1 \) becomes smaller at every step as the big mass loses energy. The initial state gives us the maximum deviation of \( K_1 \) from the non-relativistic approximation.

For a given problem (a particular value of \( N \)), we can define a breakpoint initial velocity \( v_0^b \) at the value at which the first correction term of \( K_1 \) is equal to the non-relativistic approximation of \( K_2 \) so that \( \frac{1}{8} M v_0^4 = \frac{3}{8} v_0^6 \). This happens when \( v_0^b = \frac{2}{\sqrt{3} M} \). Velocities much below \( v_0^b \) will
follow the non-relativistic Grover energy transfer, while velocities well above $v_0^b$ will show a clear deviation. Similarly, for a given $v_0$, we can define a breakpoint $M_b = \frac{1}{M_b}$. This estimate corresponds well to the behaviour shown in figure 3 and is best noted for $v_0 = 0.001$ (with $N_b \approx 1.3 \times 10^6$) and $v_0 = 0.01$ (with $N_b \approx 1.3 \times 10^4$).

4.2. Transferred fraction of energy

We can also look at the fraction of the total energy that is transferred at the end of the procedure. Figure 4 shows that, while in the final step the energy of the small ball is still of the order of the total energy, there can be a significant variation for different initial conditions. This behaviour is not exclusive of the relativistic case. In the quantum and non-relativistic (classical) Grover algorithms there is also a point at which an additional collision cannot increase the energy of the marked element any more. In the Grover procedure, energy is transferred in discrete steps. The final energy of the small ball comes from adding these steps and cannot exceed the total energy. Steps with an energy transfer above the difference $K_T - K_2$ are only allowed if they take energy from the small to the big ball.

This can be compared to the end of a Game of the Goose game. If the die roll exceeds the exact value needed to reach home, the piece rebounds and moves back the excess number of positions.

In a Grover procedure, a total or almost total transfer can be achieved if the discrete transfer steps carry a low energy and we have a greater precision. When each step transfers a higher energy or probability, as it happens in both non-relativistic and quantum Grover algorithms for small values of $N$, the initial value of $N$ will determine the maximum final amplitude. The difference between the maximum transfer values vanishes as $N$ grows and the steps are more precise. For the same reason, the variation of the maximum amplitude becomes more marked at relativistic speeds. When the initial velocity is high, the energy transferred at each step is greater and we have a rougher approximation to an exact transfer of all of the kinetic energy.
Table 1. Comparison of the efficiency of equivalent quantum, non-relativistic and relativistic Grover procedures.

|                | Number of elements | Number of steps |
|----------------|-------------------|-----------------|
| Quantum        | $\log_2(N)$       | $\frac{\pi}{4} \sqrt{N}$ |
| Non-relativistic| $N$               | $\frac{\pi}{4} \sqrt{N}$ |
| Relativistic   | $N$               | $\leq \frac{\pi}{4} \sqrt{N}$ |

This means that choosing an appropriate pair of values $v_0$ and $N$ can improve the efficiency. There will be some initial values for which we can obtain a perfect fit to the maximum energy (an exact die roll).

5. Analysis

We have presented a relativistic generalization of an effective procedure for the Grover energy transfer. The results make some aspects of the original quantum Grover’s algorithm clearer and show that the effective procedure for the Grover energy transfer can be more efficient when taken from a non-relativistic to a relativistic setting.

Table 1 gives a comparison between the time and space resources required by the different Grover procedures in the quantum, non-relativistic and relativistic cases. The space resources (number of elements) involved in the non-relativistic and relativistic cases are both of the order of $N$. In the quantum Grover procedure, where we transfer probability amplitude, quantum superposition allows for a greater compactness as $\log_2 N$ two-level states are enough to encode $N$ values.

The time complexity (number of steps) grows as $\frac{\pi}{4} \sqrt{N}$ in both the quantum and the non-relativistic procedures. This bound has been shown to be optimal [11]. However, the nonlinearities that appear in the relativistic version of the procedure alter the behaviour and allow a faster transfer. For velocities close to the speed of light, the transfer can even be done in a single step. These results show that there are physical algorithmic procedures that are more efficient in situations where relativistic effects apply.

While quantum information has become an active field of research, there are few results that incorporate the principles of the theory of relativity into communications and information processing. There are already promising studies that include relativistic constraints into new communication protocols [12, 13], in particular, relativistic bit commitment protocols [14, 15]. In spite of these advances, there has been no significant work on the possibility of relativistic computing. We believe that this kind of simple physical analogues of different algorithms can help to develop a field of relativistic computation, parallel to quantum computation. In this particular case, direct application is doubtful. Our thought experiment is highly idealized and the collisions between the balls and with the wall are not realistic. For the given velocities, a detailed study could be made considering particle collisions. In this case, it would be important to consider the collision cross sections to check the possibility of a maintained series of collisions.

Nevertheless, the procedure could be implemented with other nonlinear analogues. There have already been analog simulators of interesting relativistic effects like Hawking radiation in optical fibre systems [16]. Similarly, the relativistic Grover energy transfer could be simulated with optical nonlinear systems inside optical fibres or microring resonators. This also opens...
the door to a new line of research on nonlinear Grover procedures in different systems like, for instance, a series of electronic oscillators with nonlinear coupling.

There are also many possible extensions to the presented Grover transfer procedure. The most obvious addition is checking the procedure’s efficiency when we have more than one marked object. In a quantum Grover algorithm with \( k \) targets, the number of steps can be divided by a factor of \( \sqrt{k} \) [17]. We have already performed preliminary simulations for more than one marked object and, while there is an improvement for high velocities when compared to the non-relativistic transfer, the advantage reduces as the number of marked objects increases. A detailed analysis of these data will be presented elsewhere.

Additionally, it would be desirable to have at least a rough approximation to the number of steps the procedure requires for each initial velocity. One of the greatest challenges in dealing with relativistic Grover energy transfer is finding intuitive explanations for the results. The non-relativistic analogue of Grover algorithm gives a simple physical argument of why the optimal number of steps should be of the order of \( \sqrt{N} \) [6], but, in the relativistic case, the nonlinearity of the equations obscures an easy interpretation. Our work in progress includes a search for closed expressions for the asymptotic limit of steps and the combinations of \( v_0 \) and \( N \) that optimize the transfer. Our results suggest that there could be initial velocities for which the complexity is smaller than \( \sqrt{N} \). However, without a closed bound we cannot rule out that for every \( v_0 \) there is a different constant factor replacing \( \frac{\pi}{4} \) in the \( \frac{\pi}{4} \sqrt{N} \) bound for the number of steps.

Relatedly, it is still to be proved whether the relativistic procedure is also optimal. The fact that we recover the \( \frac{\pi}{4} \sqrt{N} \) bound at the Newtonian limit \( (v_0 \to 0) \) suggests that it is, but there could be a different behaviour for higher velocities.

Such a wide array of further possibilities gives us hope that looking into different physical analogues of quantum algorithms will give their deeper understanding and could help to devise relativistic algorithms.

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References

[1] Bennett CH and Brassard G 1984 Proc. IEEE Int. Conf. on Computers, Systems and Signal Processing (Bangalore, India) pp 175–9
[2] Ekert A K 1991 Phys. Rev. Lett. 67 661–3
[3] Shor PW 1997 SIAM J. Comput. 26 1484–509
[4] Nielsen M A and Chuang I L 2000 Quantum Computation and Quantum Information (Cambridge: Cambridge University Press)
[5] Grover L K 2001 Am. J. Phys. 69 769–77
[6] Grover L K and Sengupta A M 2002 Phys. Rev. A 65 032319
[7] Grover L K 1997 Phys. Rev. Lett. 79 325–8
[8] Farhi E and Gutmann S 1998 Phys. Rev. A 57 2403–6
[9] Mermin N D 2007 Quantum Computer Science: An Introduction (Cambridge: Cambridge University Press)
[10] Zhang J and Lu Z 2003 Am. J. Phys. 71 83–6
[11] Zalka C 1999 Phys. Rev. A 60 2746–51
[12] Barrett J, Hardy L and Kent A 2005 Phys. Rev. Lett. 95 010503
[13] Bridier K, Hayden P and Panangaden P 2009 J. High Energy Phys. JHEP08(2009)074
[14] Kent A 1999 Phys. Rev. Lett. 83 1447–50
[15] Hardy L and Kent A 2004 Phys. Rev. Lett. 92 157901
[16] Philbin T, Kuklewicz C, Robertson S, Hill S, König F and Leonhardt U 2008 Science 319 1367–70
[17] Boyer M, Brassard G, Høyer P and Tapp A 1998 Fortschr. Phys. 46 493–505