A quantum pseudo-random number generation scheme

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Abstract. Quantum random key distribution based on physical properties of quantum mechanics has high security and true randomness, but it has the disadvantages of low key generation efficiency and high cost. The classic pseudo-random number generator has the advantages of simple algorithm and high key generation efficiency, but it has extremely strict requirements on the security and randomness of the "seed". Based on the analysis of two unrelated technologies, this paper proposes a pseudo-random number generation scheme based on quantum key distribution. This scheme makes full use of quantum key distribution to share a short quantum random number-seed between the two communicating parties, then inputs the seed into a pseudo-random number generator. Finally, the two communicating parties share a long random number to encrypt the communication content. Research shows that compared with classic quantum key distribution and classic pseudo-random number generator, this scheme has stronger security, better randomness, and higher key generation efficiency.

1. Introduction

Quantum secure communication is a new discipline combining quantum mechanics, information theory and cryptography. It is a new communication method that uses the quantum characteristics of microscopic particles as an information carrier to convey secret information. Compared with classical communication methods, quantum secure communication has huge advantages in various aspects such as security, transmission efficiency, and channel capacity. This has made quantum secure communication as a new hot spot and direction in scientific research and technological development in the field of communication and information. Among the numerous applications of quantum communication, Quantum Key Distribution (QKD) which shares a random quantum key between two parties in communication, not only is the most in-depth research, it is also a significant direction which is closest to practicality. Scholars studying QKD have proposed a crowd of important quantum cryptographic protocols. In 1984, Bennett and Brassard jointly developed first QKD (BB84 protocol) in the world which depends on single-photon polarization states[1]; in 1992, Bennett Proposed QKD depends on non-orthogonal single photon bits (B92 protocol)[2]; in 1991, Ekert of the University of Oxford in UK first proposed QKD depends on Bell state entanglement[3]; Bennett, Brassard and Mermim had ameliorated the protocol of Ekert to make it more concise which implement QKD without utilizing Bell state[4]. In the current, Quantum Identity Authentication (QIA)[5-7], Quantum Secret Sharing (QSS)[8-13] and Quantum Private Comparison (QPC)[14-16] are rapidly developing towards practical aspects.

In 1917, G.S.Vernam proposed a one-time pad (OTP) encryption method[17], and G. Shannon proved the unconditional security of this encryption method in 1949[18]. OTP refers to a symmetric
cryptographic algorithm in which the communicating parties use a random key equal to the plaintext to encrypt and decrypt the plaintext, and the key is discarded after being used once. Although OTP is extremely secure, it is difficult to share a sufficiently long random key between the two parties in the communication. Quantum key distribution can achieve high-strength security key distribution between the two communicating parties, but it faces an adverse situation in lower generation efficiency of key and higher cost. If the amount of data between the two parties in the communication is copious, this problem of QKD will be more critical.

Pseudo Random Number Generator (PRNG) is an algorithm that inputs "seed" into a preset mathematical algorithm and outputs a stable pseudo-random sequence at an extremely fast rate, and the statistical characteristics of sequence are guaranteed by the algorithm. The method of PRNG is characterized by simple devices, mature algorithms, and high generation efficiency. Therefore, it is widely used in gaming activities, statistical sampling, and numerical simulation. Due to the algorithm of the classic PRNG is public, the security of the PRNG entirely depends on the confidentiality of the "seed", and the randomness of sequence is extremely correlating with the randomness of the "seed".

In order to realize the sharing of high-efficiency random numbers between two communication parties, this paper proposes a quantum pseudo-random number generation scheme, to alleviate the problem of low key generation rate and high cost in QKD technology, and the problem that the PRNG requires a high level of confidentiality and randomness of the "seed". This scheme uses a method which QKD will share a "seed" key with the same binary bits between the two parties in communication, and the "seed" will be inputted into a pseudo-random number generator of the same algorithm. In the result, both parties share a same Binary bit of pseudo-random number used as a one-time pad secret key.

Compared with the classic QKD protocol, this scheme makes full use of the security and randomness of QKD, and alleviates the problems of slow generation rate and high cost in the QKD. Compared with the classic PRNG scheme, in this scheme, the "seeds" with unconditional security and true randomness ensure the security and randomness of pseudo-random numbers. Therefore, the random number shared by this scheme makes grate contributions to strong security, good randomness and high rate of key generation.

2. Basic knowledge of relevant issues

Pseudo-random number generation algorithms generally include square method, Fibonacci method, shift method, linear congruence method, nonlinear and inverse congruence method, and taking the minimum number method.

The advantages of the square method are simple calculation, easy implementation on a computer, and low memory consumption. While there are some existing disadvantages such as the phenomenon of a bias against a small number, the poor nature of uniformity, the difficulty in uncertain length and periodicity, the serious dependence on initial data and a problem that the randomness of the sequence of numbers is prone to degradation. The advantages of the Fibonacci method are simple calculations, fast speeds of calculation, and long periods. The disadvantages are that the numbers in the random sequence are easy to repeat, the independence is poor, there is a phenomenon of non-uniform and they have significant sequence correlation. The advantage of the shift method is the fast operation speed; the disadvantage is that it depends heavily on the initial value. If the initial value is too small, the length of the pseudo-random number sequence is short and the independence is poor. And the period of the random sequence is related to the computer word length. The calculation speed of the linear congruence method is acceptable, but the disadvantage is that there are sparse grids at high latitudes and long-period correlation. The non-linear and inverse congruence methods are complicated to calculate and have a large amount of calculation. So, its disadvantages are that the random sequence has low generation efficiency, there is a long period phenomenon, and the period depends on the computer word length. The advantage of taking the minimum number method is that it is simple to calculate and easy to implement; the disadvantage is that the "seed" is a pure decimal number with a long digit.
Analysis of six PRNG algorithms in five dimensions: "computation complexity", "Degree of dependence on seed", "random sequence uniformity", "random sequence periodicity" and "random sequence generation efficiency". The results are shown in Table I.

**Table 1.** Table of analysis results of six pseudo-random generation algorithms.

| Method                                      | Computation complexity | Degree of dependence on seed | Random sequence uniformity | Random sequence periodicity | Random sequence generation efficiency |
|---------------------------------------------|------------------------|------------------------------|----------------------------|-----------------------------|---------------------------------------|
| Square method                               | simple                 | high                         | poor                       | short                       | high                                  |
| Fibonacci method                            | simple                 | general                      | poor                       | long                        | high                                  |
| Linear congruence method                    | general                | low                          | good                       | long                        | general                               |
| Nonlinear and inverse congruence method     | complex                | low                          | good                       | long                        | low                                   |
| Taking the minimum number method            | simple                 | high                         | poor                       | short                       | high                                  |

The analysis shows that the linear congruence method has low dependence on "seeds", good random sequence quality, and acceptable computational complexity and generation efficiency. Therefore, it is a suitable quantum pseudo-random number generation scheme combining the linear congruence method with QKD. The linear congruence method is a pseudo-random number generation algorithm which is widely used at present. The basic idea is to perform a linear operation on the previous number and take a modulus to obtain the next number. The recursive formula is:

\[ x_{n+1} = (ax_n + c) \mod m \]

In the formula: \( a \) is a multiplier, \( 0 < a < m \); \( c \) is an increment, \( 0 \leq c < m \); \( m \) is a modulus, \( m > 0 \); \( x_0 \) is an initial value, \( 0 \leq x_0 < M \); \( x_{n+1} \) is a random number, \( 0 \leq x_{n+1} < M \).

The quantum key distribution used in this scheme involves two kinds of measurement bases, X-base and Z-base, \( \{ + \}, \{ - \} \) is a set of standard orthogonal bases, X-basis; \( \{ + \}, \{ - \} \) is another set of standard orthogonal bases, Z-basis. X-basis and Z-basis are non-orthogonal basis, and they satisfy the following relationship:

\[
|+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) , \quad |-\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) , \\
|0\rangle = \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle) , \quad |1\rangle = \frac{1}{\sqrt{2}} (|+\rangle - |-\rangle) .
\]

The results of the four different states measured by the two measurement bases are shown in Table II.

**Table 2.** Measurement results of different states.

|       | \( |0\rangle \) | \( |1\rangle \) | \( |+\rangle \) | \( |-\rangle \) |
|-------|----------------|----------------|----------------|----------------|
| Z-basis | \( |0\rangle \) | \( |1\rangle \) | 50%\( |0\rangle \) or 50%\( |1\rangle \) | 50%\( |0\rangle \) or 50%\( |1\rangle \) |
### 3. Quantum pseudo-random number generation scheme

| X-basis                             | 50%|+⟩ or 50%|−⟩ | 50%|+⟩ or 50%|−⟩ | +⟩ | −⟩ |

**Step 1:** Alice and Bob share a linear congruent random number generation algorithm in the open channel.

**Step 2:** Alice randomly prepares a quantum sequence $S_1$ containing $4n$ single photons whose polarization state is one of \( |0⟩, |1⟩, |+⟩, |−⟩ \). Alice sends $S_1$ to Bob via a quantum channel.

**Step 3:** After Bob receives $S_1$, he uses the X-basis or Z-basis to measure $S_1$ randomly, and publishes the sequence of the bases which is used through the common channel.

**Step 4:** Alice compares the base sequence used by Bob with the base sequence of $S_1$ prepared by himself. In order to form the quantum sequence $S_2$, the single photon state corresponding to the same base should be retained, while the single photon state corresponding to the different base should be discarded.

**Step 5:** Alice randomly selects a part of the single photon in $S_2$ and publicly announces its preparation status. Bob compares the preparation status and measurement status of this part of the single photon for eavesdropping inspection.

**Step 6:** If Bob finds that the bit error rate of the temptation particle exceeds the threshold, there is an eavesdropping in the communication and the protocol is terminated; otherwise, there is no eavesdropping in the communication and the protocol continues.

**Step 7:** Alice and Bob round off the photons for eavesdropping inspection at the same time, and the remaining particles form a quantum sequence $S_3$. Since \( |0⟩, |−⟩ \) is encoded as 0 and \( |1⟩, |+⟩ \) is encoded as 1 in $S_3$, Alice and Bob obtain the binary bit sequence $k_1$ according to $S_3$.

**Step 8:** Alice and Bob use $k_1$ as a "seed" to input a linear congruence algorithm and they obtain a pseudo-random sequence binary bit sequence $k_2$.

The scheme flow is shown in Figure 1.
4. Experimental results
Through the quantum key distribution equipment, as shown in Figure 2, a “seed” key—$k_1$ is shared between users Alice and Bob. The quantum key distribution device obtains $k_1$ by steps such as stabilizing the transmission voltage, finding synchronization light and automatically synchronizing the delay, as shown in Figure 3.

**Figure 1.** Flow chart of quantum pseudo-random number generation scheme.
Figure 2. Quantum key distribution equipment.

Figure 3. Generation of quantum random keys.

The 20-bit binary bit key $k_1$ generated by the quantum key distribution equipment is used as a "seed" in the linear congruence algorithm to obtain a 200-bit binary bit pseudo-random number $k_2$. The code and running result of the linear congruence algorithm are shown in Figure 4.
5. Security analysis of the scheme

5.1. Intercept/resend attack
If there is an eavesdropper Eve over the course of the communication between Alice and Bob, most of the information of this scheme is disclosed through the public channel, so the eavesdropping behavior can only occur on the quantum channel. If Eve intercepts and eavesdrops on the qubits in $S_1$, then sends the quantum sequence to Bob. Because the quantum states used in this scheme are non-orthogonal to each other, Eve does not know the specific quantum states and cannot obtain information. According to the quantum unclonable theorem (it is impossible to clone the quantum without destroying the unknown quantum), it can be known that Eve’s eavesdropping behavior will inevitably bring disturbance to $S_1$. Furthermore, this eavesdropping behavior will be discovered by Alice and Bob through the bit error rate, so the intercept/resend attack cannot be successful.

5.2. Entanglement attack
If the eavesdropper Eve intercepts the qubit sequence $S_1$, and performs a normal operation on the particles in $S_1$ to make it form a larger Hilbert space.

$$E \otimes |0e\rangle = a|0e_{00}\rangle + b|1e_{01}\rangle$$

$$E \otimes |1e\rangle = b'|0e_{10}\rangle + a'|1e_{11}\rangle$$

$$E \otimes |+e\rangle = \frac{1}{\sqrt{2}} \left( a|0e_{00}\rangle + b|1e_{01}\rangle + b'|0e_{10}\rangle + a'|1e_{11}\rangle \right)$$

$$= \frac{1}{2} \left[ |+\rangle \left( a|0e_{00}\rangle + b|1e_{01}\rangle + b'|0e_{10}\rangle + a'|1e_{11}\rangle \right) + |-\rangle \left( a|0e_{00}\rangle - b|1e_{01}\rangle + b'|0e_{10}\rangle - a'|1e_{11}\rangle \right) \right]$$

Figure 4. The code and running result.
\[ E \otimes |e\rangle = \frac{1}{\sqrt{2}} \left( a|0e_{00}\rangle + b|1e_{01}\rangle - b'|0e_{01}\rangle - a'|0e_{11}\rangle \right) \]

\[ = \frac{1}{2} \left[ + \left( a|0e_{00}\rangle + b|1e_{01}\rangle - b'|0e_{01}\rangle - a'|0e_{11}\rangle \right) + - \left( a|0e_{00}\rangle + b|1e_{01}\rangle - b'|0e_{01}\rangle + a'|0e_{11}\rangle \right) \right] \]

(3)

The four pure states \( \{e_0, e_1, e_0', e_1'\} \) determined by the operator E satisfy the normalization conditions:

\[ \sum_{a,b,a',b'} \langle e_{a,b} | e_{a',b'} \rangle = 1 \]

(4)

The matrix of Eve's unitary operation E is expressed as

\[ E = \begin{pmatrix} a & b' \\ b & a' \end{pmatrix} \]

(5)

Since \( EE^* = 1 \), a, b, a', b' satisfy the following relationship

\[ |a|^2 + |b|^2 = 1 \]
\[ |a'|^2 + |b'|^2 = 1 \]
\[ ab^* = (a')^* b' \]

(6)

Then, we get the result

\[ |a|^2 = |a'|^2, |b|^2 = |b'|^2 \]

(7)

If Eve attacks particles in an entangled state, the eavesdropper’s interference will inevitably introduce errors, so that the presence of the eavesdropper can be detected with a probability of P:

\[ P = |\beta|^2 = 1 - |\alpha|^2 = 1 - |\alpha'|^2 \]

(8)

When Eve does not want to introduce error, the total particles must be related to Eve’s direct state of the auxiliary quantum states. However, the stochastic state indicates that there is no correlation between the auxiliary particle and the particle of \( S_1 \), so the eavesdropper does not get any useful information, thus proving that the entanglement attack will not be successful.

6. Random analysis of the scheme

The quantum random number obtained by experiments is \( k_1 \) which is used as the "seed" to input the linear congruence algorithm to obtain the pseudo-random sequence decimal bit sequence \( k_2 \). The value of \( k_1 \) and \( k_2 \) are shown in Table III.

| \textbf{Table 3. The value of } k_1 \textbf{ and } k_2. | \hline
| \textbf{The value} | \hline
| \textbf{k1} | 00010011101100011100 | \hline
| \textbf{k2} | 28348386471240871623729309818022537734895730878752287647166228920310180 8468184600825770658753038122432174921422916505856310581267548082226439 88021630928033033058950898988439020249812478106312562966528 | \hline

Use the ENT (Pseudo-random Number Sequence Test Program) test program to test the random characteristics of \( k_2 \). The test results are as follows:

The arithmetic mean of \( k_2 \) is \( \alpha, \alpha = 4.41 \);

The variance of \( k_2 \) is \( \beta, \beta = 8.6619 \);

The first-order autocorrelation function of \( k_2 \) is \( \gamma, \gamma = 0.07 \).
Therefore, the pseudo-random number \( k_2 \) satisfies a random number characteristic.

7. Effectiveness analysis of the scheme

The generation time of \( k_1 \) is \( t_1 \), and \( t_1 = 0.00256s \). The generation time of \( k_2 \) is \( t_2 \), and \( t_2 = 0.00047s \).

The quantum random key generation efficiency is \( \eta_1 \), \( \eta_1 = 7799.54b / s \), and the pseudo-random number generation efficiency obtained by the linear congruence algorithm is \( \eta_2 \).

\[
\eta_2 = \frac{600 \text{bit}}{(t_1 + t_2)} = \frac{600 \text{bit}/(0.00256s + 0.00047s)}{198216.06b / s}
\]

In the experiment, the ratio of quantum pseudo-random number generation efficiency to quantum random key efficiency is \( p \),

\[
p = \frac{\eta_2}{\eta_1} = 198216.06/7799.54 = 25.42
\]

As the length of the random number key further increases, the generation efficiency of the quantum pseudo-random number will further improve, and the ratio to the quantum random key will be higher than 25.42. Therefore, quantum pseudo-random numbers are more efficient in key generation than quantum random keys.

8. Conclusion

The algorithm used in this solution is simple and easy to implement. The key generation speed is fast. The security of the key is based on the Heisenberg uncertainty principle in quantum mechanics, the quantum non-clonable theorem, the relevance and non-locality of entangled particles. Therefore, random numbers have unconditional security and true randomness. By combining quantum key distribution with classical pseudo-random number generation algorithms, the quantum pseudo-random number generation scheme combines the advantages of both and overcomes the shortcomings of the two technologies. This scheme improves the security, randomness, and key generation efficiency of shared random numbers, and reduces the cost of key generation, thereby making the one-time pad encryption method easier to implement in real life.

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