OBSERVATIONAL EVIDENCE FOR THE EFFECT OF AMPLIFICATION BIAS IN GRAVITATIONAL MICROLENSING EXPERIMENTS

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ABSTRACT

Recently Alard, Mao, & Guibert and Alard proposed to detect the shift of a star’s image centroid, \(\delta x\), as a method to identify the lensed source among blended stars. Goldberg & Woźniak actually applied this method to the OGLE-1 database and found that seven of 15 events showed significant centroid shifts of \(\delta x \gtrsim 0.2\). The amount of centroid shift has been estimated theoretically by Goldberg; however, he treated the problem in general and did not apply it to a particular survey or field and therefore based his estimate on simple toy model luminosity functions (i.e., power laws). In this paper, we construct the expected distribution of \(\delta x\) for Galactic bulge events based on the precise stellar luminosity function observed by Holtzman et al. using the Hubble Space Telescope. Their luminosity function is complete up to \(M_V \sim 9.0\) \((M_Y \sim 12)\), which corresponds to faint M-type stars. In our analysis we find that regular blending cannot produce a large fraction of events with measurable centroid shifts. By contrast, a significant fraction of events would have measurable centroid shifts if they are affected by amplification-bias blending. Therefore, the measurements of large centroid shifts for an important fraction of microlensing events of Goldberg & Woźniak confirm the prediction of Han & Alard that a large fraction of Galactic bulge events are affected by amplification-bias blending.

Subject headings: gravitational lensing — stars: luminosity function, mass function

1. INTRODUCTION

Experiments to detect massive astrophysical compact halo objects (MACHOs) by monitoring the light variations of stars undergoing gravitational microlensing events are being done by several groups (MACHO, Alcock et al. 1997a; EROS, Ansari et al. 1996; OGLE, Udalski et al. 1997; DUO, Alard & Guibert 1997). Since the lensing probability for a single-source star is very low, these searches are being conducted toward very dense star fields such as the Large Magellanic Cloud and the Galactic bulge. While searches toward these crowded fields result in an increased event rate, these types of searches imply that many of the observed light curves are blended by the light from unresolved stars that are not being lensed.

Depending on the source for the blended light, blending can be classified into several types. The first type, “regular blending,” occurs when a bright source star registered on the template plate is lensed and its flux is blended with the light from numerous faint unresolved stars below the detection limit imposed by crowding. Regular blending affects the results of lensing experiments in various ways. First, it makes the measured Einstein timescale shorter than the true one. Since the lens mass scales as \(M \propto t_E^2\), the lens mass determined from the measured timescale will be underestimated and the lens population will be misinterpreted. In addition, since the optical depth is directly proportional to the summation of event timescales, i.e., \(\tau \propto \sum t_{E,i}\), the Galactic MACHO fraction determined from the optical depth without a proper correction of blending is subject to great uncertainty (Di Stefano & Esin 1995).

The second type of blending, “lens blending,” occurs if the source for the blended light is the lens itself (Kamionkowski 1995; Buchalter, Kamionkowski, & Rich 1996; Buchalter & Kamionkowski 1997; Alard 1997). Beside the effects of regular blending, lens blending has several additional effects on the result of lensing experiments. First, because detecting events due to lenses close to the observer is comparatively more difficult than detecting events produced by lenses near the source, lens blending makes the optical depth depend on the geometry of the lens system. As a result, the matter distribution derived from the optical depth distribution deviates from its true distribution. Second, lens blending causes the lensing optical depth to depend on the lens mass function since more massive stars, which contribute more to the total optical depth, tend to be brighter, resulting in a larger blending effect (Nemiroff 1997; Han 1998).

Finally, “amplification bias” blending occurs when one of several unresolved faint stars in the seeing disk of a reference star is lensed, and the flux of the lensed star is associated with the flux from other stars in the integrated seeing disk (Nemiroff 1997). In current experiments, photometry is carried out by comparing template images, in which only very bright stars are resolved and registered, with a series of images taken of the same field. The result is that in amplification-biased events the brightest star appears to be the source because the lensed star is too faint to be resolved. To be detected, the amplification-biased event must be highly amplified to overcome the high-threshold flux from the brighter star. Therefore, the mean detection probability for each source star will be very low; however, if these faint stars comprise a significant fraction of the total number of stars, a considerable fraction of events might be amplification-biased (Han 1997b; Alard 1997). Moreover,
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**Fig. 1.**—Illustration of the centroid shifts for a simple case in which two stars are blended and the fraction of light from the lensed source star is $\kappa = 0.41$. In the top panel we present the PSF of each star. Since the separation between the two stars is small, the integrated PSF appears to be a single star (left-hand middle panel) with its center at the CL (cross). In the remaining panels we present the integrated PSFs and the position of CL for $A_{abs} = 3, 5, 10$ with respect to the original position of the CL before amplification. As the amplification increases, one still cannot resolve individual PSFs, but the CL shifts increasingly toward the position of the lensed star.

because of the large amount of blended light from a bright star, the effects of blending for these events would be much more significant than those caused by other blending types.

There have been several methods proposed to correct for the blending problem. The first method is to introduce an additional lensing parameter representing the residual flux from unresolved faint stars into the light-curve fitting process. This method, however, suffers from large uncertainties in the derived lensing parameters as a result of parameter degeneracies (Woźniak & Paczyński 1997). Early warning systems (for MACHO, Pratt et al. 1996; for OGLE, Udalski et al. 1994; for PLANET, Albrów et al. 1995) allow one to construct lensing light curves with high time resolution and small photometric errors, which enable one to detect small color shifts caused by blending (Buchalter et al. 1996). Because of the narrow distribution of colors for Galactic bulge stars, however, the expected color shifts are small. Han (1997b) proposed to use the Hubble Space Telescope (HST) to provide blending corrections. By using the high resolving power of HST combined with color information from ground-based observations, one can identify the lensed source star in the blended seeing disk; however, this method requires costly HST time. One can also correct the blending effect statistically if the luminosity function (LF) of stars well below the detection limit can be constructed (Alcock et al. 1997b), but in this case one loses information about individual events.

Recently Alard, Mao, & Guibert (1995) and Alard (1996) proposed to detect the shift of a star’s image centroid, $d\alpha$, as a method to identify the lensed source among blended stars. Goldberg & Woźniak (1997) actually applied this method to the OGLE-1 database and found that seven out of 15 events showed significant centroid shifts of $d\alpha \gtrsim 0.2$. The amount of centroid shift has been estimated theoretically by Goldberg (1997); he treated the problem in general, however, and did not apply it to a particular survey or field, and thus based his estimates on simple toy model luminosity functions (i.e., power laws). In this paper, we construct the expected distribution of $d\alpha$ for Galactic bulge events based on the precise stellar LF observed by Holtzman et al. (1998) using HST. Their LF is complete up to $M_V \sim 9.0$ ($M_V \sim 12$), which corresponds to faint M-type stars. In our analysis we find that regular blending cannot produce a large fraction of events with measurable centroid shifts. By contrast, a significant fraction of events would have measurable centroid shifts if they are affected by
amplification-bias blending. Therefore, the measurements of Goldberg & Wozniak of large centroid shifts for an important fraction of microlensing events confirms the predictions of Han (1997a) and Alard (1997) that a large fraction of Galactic-bulge events are affected by amplification-bias blending.

2. CENTROID SHIFT

Consider a seeing disk in which many closely spaced stars are located with positions and fluxes given by $x_i$ and $F_{0,i}$, respectively. If one of these stars is gravitationally lensed and its flux is increased by $A_{\text{abs}}$, the position of the center of light (CL) will shift toward the lensed star by an amount of

$$\delta x = \xi |\langle x \rangle - x_j|; \quad \xi = \frac{\kappa(A_{\text{abs}} - 1)}{\kappa(A_{\text{abs}} - 1) + 1} = \frac{A_{\text{obs}} - 1}{A_{\text{abs}}},$$

where $A_{\text{obs}} = 1 + \kappa(A_{\text{abs}} - 1)$ is the observed amplification, $\kappa = F_{0,j}/\sum F_{0,i}$ is the fractional light of the lensed source star before amplification, and $\langle x \rangle = (\sum_{i \neq j} F_{0,i} x_i + F_{0,j} x_j) / \sum F_{0,i}$ is the position of the CL before amplification (Goldberg 1997). Here the subscript $j$ is used to designate the lensed star.

In Figure 1, we illustrate the centroid shifts for a simple case in which two stars are blended and the light fraction of the lensed source star is $\kappa = 0.41$. In the top panel, we present the point spread function (PSF) of each star. Since the separation between the two stars is small, the integrated PSF appears to be a single star (left-hand middle panel) with its center at the CL (cross). In the remaining panels, we present the integrated PSFs and the positions of CL when $A_{\text{abs}} = 3$, 5, and 10 with respect to the original position of the CL when $A_{\text{abs}} = 1$. As the amplification increases, one still cannot resolve individual PSFs, but the CL shifts increasingly toward the position of the lensed star.

3. EXPECTED DISTRIBUTION OF CENTROID SHIFTS

In this section we actually estimate the distribution of centroid shifts, $f(\delta x)$, that is expected for the microlensing experiment toward the Galactic bulge, toward which most events are not only detected but blending is also most severe. Under current experimental strategy toward very crowded star fields, it is very likely that every event is affected by one of the three types of blending. However, the fraction of events affected by each type of blending is poorly known. Among the three types of blending, Han (1998) found that although a significant fraction of lenses might be stars, most of them are very faint, and thus lens blending has only a small effect on the result of microlensing experiments. Therefore, we begin by estimating the individual distributions of centroid shifts caused by regular and amplification-bias blending, $f_{\text{reg}}(\delta x)$ and $f_{\text{amp}}(\delta x)$, and then estimate $f(\delta x)$ for various relative combinations of both types of blending.

To estimate $f(\delta x)$ it is essential to know how densely stars are crowded and what their brightness distribution is, i.e., the precisely determined LF of Galactic bulge stars. We construct the LF by adopting the recent determination by Holtzman et al. (1998) using HST. Their LF is complete up to $M_I \sim 9.0$ ($M_V \sim 12$), corresponding to faint M-type stars. Although the expected contribution to the total event rate by stars fainter than this limit would be very small, we extend the LF using the solar neighborhood LF from Gould, Bahcall, & Flynn (1997) as derived by another HST imaging. In Figure 2 we present the model LF. We note that the faint part of both the Galactic bulge and disk LFs have similar shapes as seen by the good agreement between the two LFs in their overlapping region: $6.7 \leq M_I \leq 9.0$.

To simulate observations of microlensing events, we begin by using a Monte Carlo method to produce stars whose fluxes are distributed according to the model LF. At the same time, we assign positions to individual stars that are uniformly distributed throughout the seeing disk whose center lies at the CL. The average seeing in current microlensing experiments is $\sim 2''$. However, when a star is located at the edge of the seeing disk, one can isolate its amplified image from that of the integrated seeing disk. We therefore set the size of the “effective seeing disk,” the maximum unresolvable separation between stars, to $\theta_{\text{eff}} = 1.5''$. However, we note that, depending on the flux ratio of the component stars, some blended images closer than 1.5'' might be resolved while others could not be. Therefore, our adoption of $\theta_{\text{eff}} = 1.5''$ is the mean separation below which blended stellar images cannot be resolved. According to the model LF, there are on average $\sim 10$ stars within the effective seeing disk of the angular area of $\pi(\theta_{\text{eff}}/2)^2$. If the total flux of all stars within the effective seeing disk is greater than the detection limit imposed by crowding, we register this total as the template star brightness. Current observations reach the detection limit when the total density of stars toward Baade’s window is $\sim 10^6$ stars deg$^{-2}$. According to the model LF, this corresponds to $M_I \sim 3.1$ mag.

Next we select a lensed star. In the case of regular blending, the lensed star is the brightest one in the effective seeing disk. On the other hand, in the case of amplification-biased events, the lensed star is one of the relatively faint stars in the effective seeing disk. Once the source stars are chosen, we produce test events with lensing impact parameters, $\beta$, that are distributed randomly in the range from 0 to 1. Among these events, detectable events should satisfy the

![Figure 2](image-url)
The expected distributions of centroid shifts caused by regular \( f_{\text{reg}}(\delta x) \) and amplification-bias blending \( f_{\text{amp}}(\delta x) \) are shown in Figure 3. The upper panel displays the distribution of centroid shifts caused by regular blending \( f_{\text{reg}}(\delta x) \), while the lower panel shows the distribution of centroid shifts due to amplification-bias blending \( f_{\text{amp}}(\delta x) \).

The threshold amplification condition that the integrated flux at peak amplification is greater than the flux before amplification by a factor of at least 1.34, i.e.,

\[
1 - \int_0^{\delta x_{\text{lim}}} f(\delta x) d\delta x < 0.1,
\]

where \( \delta x_{\text{lim}} = (\beta^2 + 2)/(\beta^2 + 4)^{1/2} \). Finally, we compute the centroid shifts of individual detected events using equation (1). In actual data analysis, however, estimating the image centroid of a star by software such as DoPHOT (Schechter, Mateo, & Saha 1993) is a bit more sophisticated than finding a simple center of light. This kind of software finds the centroid by attempting to solve the least-squares problem. Therefore, we note that using the center of light to compute the centroid shift is a simplifying approximation, but it would not change our main results (C. Alard 1998, private communication).

In the upper panel of Figure 3, we present the expected distribution of centroid shifts caused by regular and amplification-bias blending. In the lower panel, we present the fraction of events with centroid shifts greater than a certain limiting value \( \delta x_{\text{lim}} \) i.e.,

\[
1 - \int_0^{\delta x_{\text{lim}}} f(\delta x) d\delta x < 0.1.
\]

The actual centroid shift distribution is obtained by \( f(\delta x) = \left( 1 - f_{\text{reg}}(\delta x) + f_{\text{amp}}(\delta x) \right) \).

In Figure 4, we show the expected centroid-shift distribution for events toward the Galactic bulge for various amplification-bias event fractions of \( f_{\text{amp}} = 20\%, 40\%, \text{and} \ 80\% \). From the known distributions of centroid shifts due to regular blending, \( f_{\text{reg}}(\delta x) \), and amplification-bias blending, \( f_{\text{amp}}(\delta x) \), the actual centroid shift distribution is obtained by \( f(\delta x) = \left( 1 - f_{\text{amp}}(\delta x) \right) \).

In Figure 5, we present the distribution of the fraction of events with centroid shifts greater than a limiting value of \( \delta x_{\text{lim}} \) for various amplification-bias event fractions of \( f_{\text{amp}} = 0\%, 20\%, 40\%, \text{and} \ 80\% \). If all events are free from amplification-bias blending \( (f_{\text{amp}} = 0) \), only \( \sim 2\% \) of events would have centroid shifts with \( \delta x \geq 0.2 \). On the other hand, to produce large centroid shifts for a significant number of events, a large fraction of Galactic bulge events should be affected by amplification-bias blending.
Once the distribution of centroid shifts caused by each type of blending is obtained, the actual centroid shift distribution is obtained by

\[ f(\delta x) = (1 - \mathcal{F}_{\text{amp}}) f_{\text{reg}}(\delta x) + \mathcal{F}_{\text{amp}} f_{\text{amp}}(\delta x) \]

with the known amplification-biased event fraction of \( \mathcal{F}_{\text{amp}} \). Han (1997a) estimated that \( \mathcal{F}_{\text{amp}} \sim 40\% \) of events detected toward the Galactic bulge are affected by amplification-bias blending; however, to see the dependency of \( f(\delta x) \) on the value of \( \mathcal{F}_{\text{amp}} \), we examine various amplification-biased event fractions of \( \mathcal{F}_{\text{amp}} = 20\%, 40\%, \) and \( 80\% \), and the resulting distributions of \( f(\delta x) \) and \( 1 - \int_{\delta x}^{\delta x_{\text{lim}}} f(\delta x) d\delta x \) are presented in Figures 4 and 5, respectively. From these figures, one first finds that the distribution \( f(\delta x) \) changes significantly for different values of \( \mathcal{F}_{\text{amp}} \). Second, in order to produce large centroid shifts for a significant number of events, a large fraction of Galactic bulge events must be affected by amplification-bias blending. Therefore, the recent finding by Goldberg & Woźniak (1997) that an important fraction of Galactic bulge events exhibit considerable centroid shifts confirms the predictions of Han (1997b) and Alard (1997) that a substantial fraction of Galactic bulge events are affected by amplification-bias blending.

4. DISCUSSION

The fact that events affected by amplification-bias blending produce large \( \delta x \) while the centroid shifts caused by regular blending are small can be understood analytically in the following way. To produce centroid shifts large enough to be measured, events should satisfy two conditions. First, the event should be highly amplified. For a very low amplification event (\( A_{\text{abs}} \sim 1 \)), the expected centroid shift will be small since \( A_{\text{abs}} \sim 1 \) and thus \( \eta \sim 0 \) in equation (1). On the other hand, for a very high amplification event (\( A_{\text{abs}} \sim \infty \)), one finds \( \delta x \sim |X - x_{\text{lim}}| \) since the factor \( \eta \) approaches unity; however, not all events that satisfy the first condition produce large centroid shifts. The second condition is that the lensed star should be one of the faint stars in the blended disk. If the lensed star is the brightest one in the effective seeing disk and its flux dominates the flux over those from other faint blended stars, i.e., \( \kappa \sim 1 \), the position of the CL before gravitational amplification will be very close to that of the lensed star, resulting in a small amount of Galactic bulge events are affected by amplification-bias blending.

![Fig. 6—Correction for amplification-bias blending by monitoring bright source stars. Upper panels: Expected distributions of the fractional light \( \kappa \) and timescale decrease factor \( \eta \) for various threshold brightness of reference image. Middle panels: Same as upper panels, but the distributions are corrected by the detection efficiency. We assume that the detection efficiency is linearly proportional to the timescale decrease factor, i.e., \( \epsilon \propto \eta \). Lower panels: Distributions of events with the fractional light \( \kappa \geq \kappa_{\text{lim}} \) (left-hand panel) and the timescale decrease factor \( \eta \geq \eta_{\text{lim}} \) (right-hand panel).](image)
of shift \(|\langle x \rangle - x_j| \sim 0\) since \(\sum t_{ij} F_{ij} x_i + F_{0,j} x_j \sim F_{0,j} x_j\) and \(\sum F_{0,i} \sim F_{0,j}\). On the other hand, if the lensed source is very faint, i.e., \(\kappa \sim 0\), the light from the source star has negligible effect on the position of \(C_L\), resulting in a high possibility of a large centroid shift. In summary, the expected centroid shifts for various extreme cases of amplification and source-star brightness are the following:

\[
\begin{align*}
\eta & \sim 0 \quad \delta x \sim 0 \quad \text{for a low-amplification event} \\
\delta x & \sim |\langle x \rangle - x_j| \sim 0 \quad \text{for a high-amplification event with a luminous source} \\
\delta x & \sim |\langle x \rangle - x_j| \quad \text{for a high-amplification event with a faint source}
\end{align*}
\]

(4)

For amplification-biased events, source stars are in general very faint (\(\kappa \sim 0\)) and mostly far below the detection limit. Despite their low luminosities, the fact that they are highly amplified for detection Although they can result in small \((A_{\text{abs}} \sim 1)\), the conditions for large centroid shifts agree well with those for amplification-biased events. On the other hand, regular blended events do not meet these conditions. First, because of a relatively small amount of blended light, regular blended events do not need to be highly amplified for detection \((A_{\text{abs}} \sim 1)\). Although they can be highly amplified \((A_{\text{abs}} \sim \infty)\), the dominance of their fluxes \((\kappa \sim 1)\) over those from other faint blended stars will result in small \(\delta x\).

As demonstrated by the large centroid shifts for a significant fraction of events, the effect of amplification-bias blending on the results of lensing experiments is important. However, the methods to correct for the blending effect mentioned in § 1 have various limitations in application. One very simple but practical method to minimize the effect of amplification-bias blending is to monitor only very bright stars. With increasing reference-image brightness, the required amplification for detection becomes higher, resulting in a lower probability of amplification-biased events. To show how the blending effect decreases with increasing reference image brightness, we simulate Galactic bulge events that are expected to be detected for various threshold reference image brightenesses. For each event, we compute the light fraction of the source star \(\kappa\) and the timescale decrease factor \(\eta\). The distributions of \(\kappa\) and \(\eta\) are shown in the upper panel of Figure 6. For a given fraction of source-star flux, the observed timescale is reduced by \(\eta = t_{\text{eff}}/t_0 = (2 - A_\text{abs}^{-1/2})^{-1/2}\). Here events are assumed to be detected as long as they can amplify the reference-image flux by more than a factor of 1.34. Highly blended events, however, will have short \(t_{\text{eff}}\), resulting in low detectability. Therefore, we correct the distributions by the detection efficiency. We assume that the detection efficiency is linearly proportional to the timescale decrease factor, i.e., \(\epsilon \propto \eta\). The efficiency-corrected distributions \(f(\kappa)\) and \(f(\eta)\) are presented in the middle panels. In the lower panels we present the distributions of the fraction of events with little blending effects \((\kappa \simeq 0.9)\) and \(\eta \gtrsim 0.9\) is \(\lesssim 10\%\). However, as the brightness of the threshold reference-image increases, this fraction gradually increases until it becomes \(\lesssim 80\%\) when only stars brighter than \(M_V \sim 0\) are monitored, which corresponds to the brightness of Galactic-bulge clump giant stars (Paczynski & Stanek 1998). The MACHO group (Alcock et al. 1997a) already applied this method and their optical depth determination is based on clump giant stars. This method to correct blending, however, requires the sacrifice of statistical precision of the results of lensing experiments owing to the lowered event rate. A more general solution for the blending correction is provided by the rapidly progressing image subtraction technique (Alard & Lupton 1997; Tomaney 1998) that is also being applied to detect microlensing events toward M31 by the Colombia-Vatt group (Crotts & Tomaney 1996; Tomaney & Crotts 1996).

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