Model-Free 3D Reconstruction of Weld Joint Geometry Using Laser Scanning in Presence of Noise

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Abstract—This article presents a novel utilization of the concept of entropy in information theory to a model-free reconstruction of 3D space in presence of noise. We show that its formulation attains its global minimum at the upper edge of this space to facilitate the extraction of its volume. Furthermore, we present a Monte Carlo approach that is based on the stratification of the sensor information to compute the volume of this extracted space. Moreover, we provide a preliminary empirical analysis of the effect of variation of the noise on the extraction process of this space to realize the impact of this noise on the computation of its area and volume.

I. INTRODUCTION

This article proposes a novel approach to utilization of the sensor data to reconstruct the 3D space that is formed at the joint of two rigid bodies in a stub-to-chord structure. Figure 1 shows one such structure. They are used in oil rigs that are deployed in arctic and oceans. The primary usage of this reconstructed 3D space is to facilitate the transformation of their welding process from manual to fully-automated system.

Welding is one of the industrial applications that attracts researchers in the field of robot manipulation since early 80’s. There exists a rich body of research that addresses various aspects of this task ranging from welding process [2] and sensors [1] to welding power sources and robot programming [4], [3].

At present, the fabrication process of jack-up rig is mostly manual. In order to improve the productivity of the shipyard to reduce man-hours, the automation of this fabrication process is highly desirable. However, the preprogramed robot methodologies [5] are susceptible to failure due to the human error and misalignment of the workpiece that is involved in its preparation. Furthermore, their reliance on the models of the workpieces (e.g., CAD models) limits the scalability of these approaches in fully automated systems since workpieces come in varying sizes and shapes and designing their models is a cumbersome and time-consuming process. Moreover, a jack-up rig can have a large number of intersection joints, depending on its size and scale. These joints are formed at the intersection of two rigid cylindrical bodies (Please see Figure 1). In addition, their setups are not identical. Therefore, the conventional seam tracking technology [6], [7], [8] is not suitable for welding these intersection joints. This is mainly due to the significant amount of time that is needed to identify the start and the end points in each stub-to-chord joint [9].

In contrast, we adapt a model-free approach that utilizes the coordinates information of the point cloud that is generated by a sensor. Laser sensors produce direct distance information on the location of an object. They are deployed in a variety of applications ranging from mapping and localization in mobile robotics to terrestrial mapping in geodetic metrology. For instance, structured laser light techniques [14] are used to acquire 3D information of an object (e.g., its shape) via projection of laser stripes. However, this procedure significantly affects the accuracy of the reconstruction of the object [14]. A review of the application of the three-dimensional laser scanning is found in [13]. Laser scanners are source of variable errors at edges where the specifications that are provided by their manufacturers are incomparable [10]. Research on accuracy of laser sensors generally takes form of utilization of the synthetic models of real scanner noise to evaluate various denoising algorithms. Goesele et al. [11] compares the accuracy of two laser scanners. Their analysis is based on a slanted edge modulation.
transfer function. Genex [12] presents an accuracy analysis for their system. However, they employ planar surfaces in their assessment.

Moreover, the noise measurement is assumed to exhibit a Gaussian characteristic in a wide range of disciplines [15]. In addition, Gaussian white noise i.e., independent Gaussian noise per mesh vertex is introduced in some studies [16]. For example, an analysis of the terrestrial laser scanning systems using RiegL LMS-Z309h, suggests an upper bound error of approximately 10 mm (on its x-direction) [17].

The remainder of this article is organized as follows. Section I[1] describes the set of assumptions that are adapted in present study. Section III[2] explains the formulation of an entropy based cost function to determine the edge of the joint space. A Monte Carlo approach to calculate the volume of the extracted joint space is presented in section IV[3]. Section V[4] provides details on experimental setup and the performance of the entropy cost function to detect the edge of the joint space to extract this space. Further analysis of the effect of the noise on calculation of the volume of this space is presented in section VI[5]. Conclusion along with some insight on future direction of this study is presented in section VII[6].

II. BASIC ASSUMPTION

We assume that the sensor data and the position information of the sensor is affected by independent zero mean Gaussian noises. Furthermore, we assume that the sensor is facing the workpiece prior to the commencement of the sensing phase. It is primarily relocated along the z-axis of the frame of the world with a constant velocity during the sensing period. Additionally, it generates data in the form of stream of 3-tuples to indicate the spatial locations of a sensed positions where the x component of these tuples provide the depth information (i.e., distance of these points from the sensor). Furthermore, the x-y-z components of these tuples are affected by independent noises. We also assume that the length of the translational relocation of the sensor along the z-axis of its frame of reference is known.

III. ENTROPY COST FUNCTION

Entropy plays a central role in a variety of topics ranging from thermodynamics and data compression to cryptography and information theory. In particular, it describes the expected amount of information that is learned from a random variable, in information theory. More specifically, the entropy of a discrete random variable \( X = \{x_1, \ldots, x_n\} \) with positive probabilities \( P(X) = \{p_1, \ldots, p_n\} \) is [18]:

\[
H(X) = E[-\log_2(P(X))] = \sum_{i=1}^{n} p_i (-\log_2(p_i))
\]

Furthermore, the entropy of the random variables \( X = \{x_1, \ldots, x_n\} \) and \( Y = \{y_1, \ldots, y_m\} \) with joint probability mass function \( f(X,Y) \) is:

\[
H(X,Y) = -\sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i, y_j) \log_2(f(x_i, y_j))
\]

with \( n = \|X\| \) and \( m = \|Y\| \). It is apparent that:

\[
f(X,Y) = P(X) \times P(Y)
\]

if \( X \) and \( Y \) are independent random variables.

We utilize the depth information in every three consecutive tuples that are sampled by the sensor to form messages, of lengths three, whose expected amounts of information are computed, using (1) and (2). Let \( S \) represent the set of point cloud that is generated by the sensor during the sensing process. This implies that:

\[
S = \cup_{i=1}^{N} s_i, \quad i \neq j \Rightarrow s_i \neq s_j
\]

where \( s_i = \{x_1^{(i)}, y_1^{(i)}, z_1^{(i)}, \ldots, x_m^{(i)}, y_m^{(i)}, z_m^{(i)}\}^T \) is the \( i^{th} \) reading of the sensor in its translational relocation during the sensing period. \( N \) is the total number of readings. It is possible that \( \|s_i\| \neq \|s_j\| \) for two or more sensor readings. Let \( \tau_k^{(i)} = [x_k^{(i)}, y_k^{(i)}, z_k^{(i)}] \in s_i \) be the \( k^{th} \) tuple of the \( i^{th} \) sensor reading where \( x_k^{(i)} \) carries the depth information of the \( k^{th} \) data point in \( i^{th} \) sensor reading. We stratify these tuples, column wise, where every \( \tau_k^{(i)} \in s_i \) contributes to at most one stratum \( C^{(i)} \) and is independent of all other readings.

We use every three consecutive depths data to formulate our entropy cost function. Let \( \tau_{k-1}^{(i)}, \tau_k^{(i)} \) and \( \tau_{k+1}^{(i)} \) represent the depths of points that correspond to tuples \( \tau_{k-1}^{(i)}, \tau_k^{(i)}, \tau_{k+1}^{(i)} \in C^{(i)} \) where \( C^{(i)} \) is the \( i^{th} \) stratum of the point cloud that is generated by the sensor and \( k \) indicates the translational steps of the sensor. We define the correlation ratio between every two consecutive points as:

\[
r_{(k-1,k)}^{(i)} = \frac{\tau_{k-1}^{(i)} / \tau_k^{(i)}}{\tau_k^{(i)} / \tau_{k+1}^{(i)}}
\]

\[
r_{(k,k+1)}^{(i)} = \frac{\tau_k^{(i)} / \tau_{k+1}^{(i)}}{\tau_k^{(i)}}
\]

The probabilities that are associated with these consecutive points, given their corresponding ratios, are:

\[
P_{r_{(k-1,k)}^{(i)}} = \frac{1}{\eta} r_{(k-1,k)}^{(i)}
\]

\[
P_{r_{(k,k+1)}^{(i)}} = \frac{1}{\eta} r_{(k,k+1)}^{(i)}
\]

\[
\eta = r_{(k-1,k)}^{(i)} + r_{(k,k+1)}^{(i)}
\]

Using (2), the entropy of this message is:

\[
\hat{H}(r_{(k-1,k)}^{(i)}, r_{(k,k+1)}^{(i)}) = -\left( f(r_{(k-1,k)}^{(i)}), r_{(k,k+1)}^{(i)} \right) \times \log_2(f(r_{(k-1,k)}^{(i)}, r_{(k,k+1)}^{(i)}))
\]

and

\[
f(r_{(k-1,k)}^{(i)}, r_{(k,k+1)}^{(i)}) = p_{(k-1,k)}^{(i)} \times p_{(k,k+1)}^{(i)}
\]

as it is defined in (3). We apply (10) and (11) on every stratum \( C^{(i)} \) that is formed through the stratification of the point cloud \( S \) that is sampled by the sensor. This results in a set \( \hat{H}^{(i)} \) whose elements are the entropies of every three consecutive tuples in \( C^{(i)} \). The tuple \( \tau_j^{(i)} \in C^{(i)} \) that lies on
the upper edge of the joint space is:
\[
\tau_j^{(i)} \leftarrow \text{argmin}(\hat{H}^{(i)})
\]
\[
\hat{H}^{(i)} = \{ H(r_{(0,1)}^{(i)}, r_{(1,2)}^{(i)}), \ldots, H(r_{(n-2,n-1)}^{(i)}, r_{(n-1,n)}^{(i)})) \}
\]
\[
\forall x_{k-1}^{(i)}, x_{k}^{(i)}, x_{k+1}^{(i)} \in C^{(i)}
\]
n is the total number of tuples in stratum \(C^{(i)}\).

**Proposition 1:** Equation \((10)\) attains its global minimum at the edge.

**Proof:** Let \(H(a)\) and \(H(b)\) denote the sets of entropies that are associated with the points in the stratum \(C\) and that are above and below the upper edge of stratum \(C\), respectively. Furthermore, let the entropy value that corresponds to the point that lies on the edge in stratum \(C\) be a member of both of these sets. There are two cases to consider:

1) **Surface above the upper edge is not flat:** Equations \((5)\) through \((8)\) imply that the value of entropies \(h_{l}^{a} \in H^{(a)}, \ i = 0, \ldots \|H^{(a)}\|, \ h_{j}^{b} \in H^{(b)}, \ j = 0, \ldots \|H^{(b)}\|\) monotonically decrease as these sets move towards their respective element that is associated with the point on the edge. Without loss of generality, let \(h_{l}^{a}, h_{l+1}^{a}, h_{l+1}^{b}\) be three consecutive entropies that represent the entropy below the current value, the current value, and the element above the current value as we move towards the edge in \(H^{(a)}\), respectively. We have:
\[
h_{l+1}^{a} > h_{l}^{a} > h_{l}^{a}, \ \forall h_{l-1}^{a}, h_{l}^{a}, h_{l+1}^{a} \in H^{(a)}
\]
Similarly, for \(H^{(b)}\) we have:
\[
h_{j-1}^{a} > h_{j}^{a} > h_{j}^{b}, \ \forall h_{j-1}^{a}, h_{j}^{a}, h_{j+1}^{a} \in H^{(b)}
\]
Comparing \((14)\) and \((15)\), we get:
\[
h_{l}^{a} < h_{l+1}^{a}
\]
\[
h_{j}^{a} > h_{j+1}^{b}
\]
Furthermore, \(h_{l}^{a} = h_{l}^{b} = h\) when these sets reach their respective values at the edge. We get:
\[
h_{l+1}^{a} > h > h_{l-1}^{b}
\]
which contradicts \((15)\). Therefore, \(h_{l}^{a} = h_{l}^{b} = h\) must be the element where \(H^{(a)}\) and \(H^{(b)}\) attain their minimum.

2) **Surface above the upper edge is flat:** This implies that \((14)\) is:
\[
h_{l+1}^{a} = h_{l}^{a} = h_{l}^{a}, \ \forall h_{l-1}^{a}, h_{l}^{a}, h_{l+1}^{a} \in H^{(a)}
\]

The remainder of the proof follows as in case 1.

IV. **Calculation of the Volume of the Extracted Space**

In a welding task, it is crucial to ensure that the adapted arrangement of beads at every step of the process covers an acceptable portion of the space that is welded. More specifically, it is necessary to ensure that the geometry of beads (e.g., their widths, heights, and penetrations) at each step results in filling a satisfactory portion of the volume of the welding space. Calculation of the volume of the welding space helps analyze the outcome of the welding process as it progresses in several paths and steps. There exists a considerable amount of original research on predicting the value of bead parameters \([19, 20]\). However, the calculation of exact volume of the welding space is difficult due to the irregularity of the cut and the shape of the joint that is formed by the two rigid bodies (see Figure 1).

In this section, we propose an approach to estimate this volume, based on the stratified samples of the sensor data. The stratification of the row wise sensor readings into column strata allows the adaptation of a Monte Carlo approach to approximate the volume of the extracted space \(S'\):
\[
\int_{T \in S'} g(T) \, dA, \quad S' \subseteq S
\]
where \(x_{k} \in S'\) is the depth that is associated with the \(k\)-th tuple in the subset \(S' \subseteq S\) of the extracted space and function \(g(T)\) gives its value. Let \([y_{0}, y_{N}']\) denote the breadth of the longest sensor reading \(S'_{\text{longest}} \in S\). In addition, let \([z_{0}', z_{N}']\) indicate the range of the translation of the sensor from the first to the last sensor readings. Furthermore, let \(\rho\) be the total number of points in the longest sample \(S'_{\text{longest}} \in S\). We utilize these values to construct an imaginary plane \(\Psi\) that is always visible to the sensor. More specifically, we assume the existence of a plane that covers the breadth and the heights that are presented by these values and that there is a tuple associated with every point that lies on this plane.

It is important to note that we are not concerned with the values of these tuples. The area of \(\Psi\) and its total number of points during the sampling are:
\[
A_{\Psi} = \|y_{0} - y_{N}'\| \times \|z_{0}' - z_{N}'\|
\]
\[
\Sigma_{\Psi} = \rho \times \|z_{0}' - z_{N}'\|
\]
Given the set of all strata that represent the extracted joint space \(C \in S'\), the area of the extracted space is:
\[
A_{S'} = \frac{\Sigma_{S'}}{\Sigma_{\Psi}} \times A_{\Psi}
\]
where \(S'\) is the subset that contains all points associated with the extracted space. \(\Sigma_{S'}\) refers to the total number of these points. Therefore, the volume of the extracted space is:
\[
V_{S'} = \frac{1}{|S'|} \sum_{T \in S'} g(T) \times A_{S'}
\]
where \(g\) is defined in \((19)\).
V. Case Study

A. Simulation Setup

In order to further study the performance of our approach, we implement it in VRep simulation environment [21]. We use the simulated Hokuyo URG 04LX UG01 laser scanner that is readily available in VRep. It is mounted on a prismatic joint to enable its vertical translational relocation, with a constant velocity, within the specified range during the sensing phase. Furthermore, it generates data (at every translation step) in the form of stream of sensor readings whose every three consecutive values correspond to a 3-tuples. These tuples correspond to the coordinates information of the sensed positions of the workpiece where the x component provides the depth information (i.e., the distance of a sensed location from the sensor). Table I provides further specification details of this sensor. Moreover, we implement our controller as a Python script. This script communicates with the sensor and its prismatic joint through a remoteAPI to relocate the sensor to scan the workpiece.

B. Sampling Phase

We assume that the sensor data and its relocation information are affected by independent zero mean Gaussian noises. We minimize the effect of these noises through alternative sampling and L1 error minimization to reduce the discrepancies between consecutive sensing phases:

$$\min_{S, S'} \sum_{i=1}^{N} \| \tau_{S}^{(i)} - \tau_{S'}^{(i)} \|, \|S\| = N = \|S'\|$$

(23)

where $S$ and $S'$ refer to the same row wise sensor data at two consecutive sensing phases. The final tuples that represent the sampled data with respect to the scanned workpiece is the average of all these consecutive sensor readings once their discrepancy is below a certain threshold or if it starts diverging. Figure 2 shows the evolution of the value of this error during one simulation run.

C. Edge Detection and Volume Extraction

Figures 3 and 4 illustrate the result of the computation of the probabilities and their corresponding entropies as formulated in (7) through (11). An interesting observation that is worth noticing is that the values of the two probabilities that are associated with a depth message are inversely related. Figure 3 shows that the values of these probabilities increase and/or decrease in directions that are inversely proportional. This is in accordance with the claim of the decremental monotonicity of the entropies of the consecutive depth messages as we move towards the element of a stratum that lies on the edge of the joint space (proposition 1). Furthermore, this figure depicts the variation of these probabilities. The larger fluctuations correspond to increase in the ratio difference between the x components of a particular consecutive tuples that form a specific depth message. The flat portion of the figure (roughly 0.5) shows how the calculated probabilities (and consequently their respective entropies in Figure 4) change in response to the change in the surface of the structure. More specifically, it indicates that the change in the value of these components follows a regular pattern. These are the values that are above and farther away from the detected edge. In this figure, the largest fluctuation corresponds to the probabilities of the message whose middle depth component i.e., $x_k$ in (2) and (6) is on the edge of the joint space of its stratum. Figure 4 shows the point at which (10) attains its global minimum for a given stratum. Figures 5 and 6 present the index of the data point that lies

![Fig. 2. Minimization of the effect of the Gaussian noise on sensor reading through L1 error function.](image)

![Fig. 3. Sample probabilities calculated for $\tau_{k-1}^{(i)}, \tau_{k}^{(i)}, \tau_{k+1}^{(i)} \in C^{(i)}$ as formulated in (7) and (8) (i.e., $P_1 = p_{k-1}^{(i)}$ and $P_2 = p_{k+1}^{(i)}$, respectively) where $C^{(i)}$ is the $i^{th}$ stratum of the point cloud that is generated by the sensor.](image)
Fig. 4. Entropies as formulated in (7) through (11) for $\tau^{(i)}_{k-1}, \tau^{(i)}_k, \tau^{(i)}_{k+1} \in C^{(i)}$ where $C^{(i)}$ is the $i^{th}$ stratum of the point cloud that is generated by the sensor.

Fig. 5. Sample probabilities calculated for $\tau^{(i)}_{k-1}, \tau^{(i)}_k, \tau^{(i)}_{k+1} \in C^{(i)}$ as formulated in (7) and (8) (i.e., $P_1 = p^{(i)}_{(k-1)}$ and $P_2 = p^{(i)}_{(k+1)}$, respectively) where $C^{(i)}$ is the $i^{th}$ stratum of the point cloud that is generated by the sensor. Index of the selected point (i.e., 64) is indicated in the figure.

Fig. 6. Entropies as formulated in (7) through (11) for $\tau^{(i)}_{k-1}, \tau^{(i)}_k, \tau^{(i)}_{k+1} \in C^{(i)}$ where $C^{(i)}$ is the $i^{th}$ stratum of the point cloud that is generated by the sensor. Index of the selected point (i.e., 64) is indicated in the figure.

Fig. 7. Original point cloud from sensor readings. These points represent the final tuples that are computed based on averaging of all sensor data to minimize the effect of noise in (23).

Fig. 8. The detected edge of the joint space based on (10) through (13). The edge of this space is depicted in green.
### Table II

**Effect of the Noise (in millimeter) on the Depths of Measured Data. Column CI refers to the [2.5 − 97.5]% confidence interval of these values.**

| Noise Level | \( \sigma_{\text{noise}}^2 \) | \( \mu \) | Median | Mode | \( \sigma^2 \) | \( \sigma \) | CI          |
|-------------|-------------------------------|----------|--------|------|---------------|----------|-------------|
| Noise-Free  | 0.0002                        | 0.0161   |         |      |               |          | [0.44, 0.50]|
| 1.0 mm      | 0.459                         | 0.453    | 0.442  |      | 0.0002        | 0.0161   | [0.44, 0.50]|
| 3.0 mm      | 0.461                         | 0.455    | 0.444  |      | 0.0002        | 0.0160   | [0.45, 0.50]|
| 6.0 mm      | 0.458                         | 0.451    | 0.441  | 0.0002| 0.0162        | [0.44, 0.50]|
| 9.0 mm      | 0.455                         | 0.449    | 0.438  | 0.0002| 0.0164        | [0.44, 0.49]|            |
| 13.0 mm     | 0.463                         | 0.457    | 0.446  | 0.0002| 0.0159        |          | [0.45, 0.50]|            |

### Table III

**Effect of the Noise (in millimeter) on the Translational Motion of the Sensor. Column CI refers to the [2.5 − 97.5]% confidence interval.**

| Noise Level | \( \sigma_{\text{noise}}^2 \) | \( \mu \) | Median | Mode | \( \sigma^2 \) | \( \sigma \) | CI          |
|-------------|-------------------------------|----------|--------|------|---------------|----------|-------------|
| Noise-Free  | 0.0002                        | 0.0052   |         |      |               |          | [0.014, 0.032]|
| 1.0 mm      | 0.027                         | 0.028    | 0.031  |      | 0.00002       | 0.005    | [0.015, 0.032]|
| 3.0 mm      | 0.025                         | 0.026    | 0.031  |      | 0.00002       | 0.0052   | [0.014, 0.031]|
| 6.0 mm      | 0.028                         | 0.029    | 0.033  |      | 0.00002       | 0.0053   | [0.016, 0.034]|
| 9.0 mm      | 0.027                         | 0.028    | 0.032  |      | 0.00002       | 0.0052   | [0.016, 0.033]|
| 13.0 mm     | 0.026                         | 0.028    | 0.032  |      | 0.00002       | 0.0052   | [0.015, 0.032]|

### Table IV

**Effect of the Noise (in millimeter) on Calculation of the Volume of the Joint Space.**

| Noise Level | \( \sigma_{\text{noise}}^2 \) | Steps | Expected Depth | Hit/Miss Ratio | \( \Psi \) Area (unit²) | Joint Space Area (unit²) | Joint Space Volume (unit³) |
|-------------|-------------------------------|------|----------------|---------------|--------------------------|---------------------------|-----------------------------|
| Noise-Free  | 0.400                         | 1    | 0.0400         | 0.4139        | 7.403                    | 3.0641                    | 0.0613                      |
| 1.0 mm      | 0.411                         | 1    | 0.4138         | 0.4138        | 7.382                    | 3.0553                    | 0.0657                      |
| 3.0 mm      | 0.434                         | 3    | 0.4343         | 0.4343        | 7.355                    | 3.0441                    | 0.0663                      |
| 6.0 mm      | 0.366                         | 1    | 0.3660         | 0.4138        | 7.367                    | 3.0491                    | 0.0599                      |
| 9.0 mm      | 0.318                         | 8    | 0.42042        | 0.42042       | 7.4291                   | 3.0747                    | 0.0764                      |
| 13.0 mm     | 0.548                         | 4    | 0.5480         | 0.41386       | 7.478                    | 3.0951                    | 0.0849                      |

### VI. Effect of the Noise on Calculation of the Volume of the Joint Space

Tables II and III provide statistics on effect of the variation of the independent Gaussian noises on the sensor motion and the depth measurement of the 3-tuples that are generated by the sensor. The first entry of these tables (i.e., no noise) refers to the setting where the translational motion of the sensor and its measurements are unaffected by noise. In other words, they represent the case in which a complete and accurate information is assumed. We increase the variances of these zero mean Gaussian noises in the scale of millimeter. It is worth noting that these values are computed after averaging all sensor readings as described in subsection V-B.

The entries of these tables indicate that the process of minimization of the discrepancies among consecutive samplings in (23) is capable of reducing the effect of the change in variance of the noise to bring the sampled data close to its noise-free version. This is evident in the mean, variance, standard deviation, and the confidence interval of the depth and translational motion of the sensor. Despite small variation in the mean of these values in Tables II and III, their respective variances are in accordance with the noise-free data and stay within acceptable standard deviation.

Table IV provides statistics on the performance of our approach to calculate the volume of the extracted joint space in presence of noise in (22). The change of the variance of the Gaussian noise is in millimeter. The first entry of this table refers to the setting where the noise is absent. In other words, this entry reflects the actual value of the sensor. The first entry of these tables (i.e., no noise) refers to the setting where the translational motion of the sensor and its measurements are unaffected by noise. In other words, they represent the case in which a complete and accurate information is assumed. We increase the variances of these zero mean Gaussian noises in the scale of millimeter. It is worth noting that these values are computed after averaging all sensor readings as described in subsection V-B.

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the area and the volume of the extracted joint space. The column entry "step" refers to the number of steps that (23) takes to converge. Similarly, the "expected depth" entry is the average value of the depths of all 3-tuples that fall under the edge of the joint space as formulated by $\frac{1}{\text{area}} \sum_{\tau \in S^f} f(\tau)$ in (22). Furthermore, the entry "hit/miss ratio" corresponds to the ratio of the total number of 3-tuples that fall inside the joint space to those that are assumed to be on the imaginary plane $\Psi$, as explained in section IV and (20). Entry "area" shows the area of this imaginary plane. The last two columns of Table IV provide the calculated values of the area and the volume of the extracted joint space, respectively. The number of steps in Table IV indicates the absence of any specific pattern to converge on the minimization of the discrepancies of the consecutive sensor data. However, this approach is capable of capturing the change in the variance of the noise to reduce its effect on the ratio between the tuples on the imaginary plane and the actual joint space. This results in computation of the area of the imaginary plane that exhibits less fluctuation with the change in the variance, as it is shown in "area" entry of Table IV. As a result, the calculated area of the joint space is less affected by this change in the variance of the noise. However, this change in variance of the noise has its impact on the expected depth of the joint space. In particular, this is evidence in the last entry of Table IV. This explains the overestimations of the calculated volume of the extracted joint space in the last two entries of Table IV as compared to its original, noise-free value.

VII. CONCLUSIONS

We present an approach to a model-free 3D reconstruction of the joint space that is formed between two rigid bodies in an offshore rig structure. We show how the depth information that is generated by a sensor through scanning of this setup is used to encode messages whose entropies help identify the location of the edge of this joint space. Moreover, we demonstrate that the sampled data with minimum entropy messages correspond to the location of this edge to extract the confined under this edge to reconstruct its 3D joint space. In addition, we show how the stratification of the sensor data provides an opportunity to compute the volume of this space to facilitate the monitoring of the welding process.

Our preliminary analysis of the effect of noise on calculation of the area and the volume of this space shows that the depth information of the sample data is more affected by the introduction of the Gaussian independent noises on the translational motion of the sensor and its data sampling. This results in a situation where the area of the joint space exhibits less deviation from its value in an idealized noise-free environment. On the contrary, its volume is overestimated due to the effect of the noise on the sampled depths. The future direction of this research pertains to implementation of our approach on real sensor to realize the utility of this approach to acquire reliable information. In addition, we examine the potentials that statistical approaches (e.g., multivariate normal distribution and Gaussian Mixture Models) can offer to further minimize the effect of the noise on sensor data and its motion. Furthermore, it is crucial to integrate this setting with a real robotic system to study the effect of the various constraints that are imposed on its performance via this integration.

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