THE HUNGARIAN METHOD FOR THE ASSIGNMENT PROBLEM, WITH GENERALIZED INTERVAL ARITHMETIC AND ITS APPLICATIONS

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Abstract: In this paper, we focus on the solution procedure for fully interval assignment problem (FIAP), Hungarian method is considered into account. In consideration method the given FIAP is decayed into Interval Assignment Problem, solving it with existing method and by using its optimal solutions, an optimal interval assignment solution to the given FIAP is obtained. Although the assignment problem can be solved as an ordinary transportation problem or as Linear programming problem, its unique structure can be exploited, resulting in special purpose algorithm, is called Hungarian method. A numerical example is provided to illustrate the solution procedure developed in this paper.

Keywords: Interval numbers, Assignment problems, Hungarian assignment problem.

1. INTRODUCTION

Assignment problem describes that one individual be able to perform one work at a time, to get the optimal solution by maximize the total profit (or) minimize the total cost. Assignment problem can be declared in the form of m x n matrix (cij) also known as Cost matrix, where cij is the cost of assigning ith device to jth job as ‘n’ works to be performed on ‘m’ machine (one work to one machine). This is also known as Fully interval Assignment problem, it can be solved by using a known process called Hungarian method. Jayalakshmi [6] introduced computation of intervals without using arithmetic operations, as decomposed FIAP into crisp AP using midpoint technique. Sarangam Majumdar [11] proposed interval Assignment method to solve real world linear problems whose cost matrix are in interval form. Ramesh Kumar A et.al [8][9] proposed the application of assignment problem and converting crisp AP into interval forms. For both maximization and minimization of Hungarian method using its extension of intervals was solved by Amutha et.al[1], Deepa et.al[3]introduced a new method known as Best assignment then solving it by intervals, Humayra Dil Afroz et.al[6] proposed a new assignment method comparative with existing method. Whereas Sundaresan et.al and Ramon.E.Moore[12][10] proposes a basic definitions of OR and existing intervals. In real world consumption, there are many values i.e for eg time, ability, etc are in crisp form which does not give a perfect value, In this paper using Interval arithmetic by Hungarian method, the problem can be solved more to get precisely with the basic assumption of one job to one person for minimizes total cost (or) maximizes the profit. The rest of the paper includes review on interval arithmetic are highlighted.

2. Preliminaries

This section includes some notations, notions which results in our further consideration. Let \( \hat{a} = [a_1,a_2] = \{x : a_1 \leq x \leq a_2, x \in \mathbb{R} \} \). If \( \hat{a} = a_1 = a_2 = a \), then \( \hat{a} = [a,a] = a \) is a real number (or a degenerate interval). Let \( IR = \{ \hat{a} = [a_1,a_2] : a_1 \leq a_2 \text{ and } a_1, a_2 \in \mathbb{R} \} \) be the set of all proper intervals and \( IR = \{ \hat{a} = [a_1,a_2] : a_1 > a_2 \text{and } a_1, a_2 \in \mathbb{R} \} \) be the set of all improper intervals on the real line \( \mathbb{R} \). We shall use the terms interval and interval number interchangeably. The Midpoint and Width of an interval number \( \hat{a} = [a_1,a_2] \) are...
defined as \( m(\tilde{a}) = \left( \frac{a_1 + a_2}{2} \right) \) and \( w(\tilde{a}) = \left( \frac{a_2 - a_1}{2} \right) \). The interval number \( \tilde{a} \) can also be expressed in terms of its midpoint and width as: \( \tilde{a} = [a_1, a_2] = \langle m(\tilde{a}), w(\tilde{a}) \rangle \).

### 2.1. A new interval arithmetic

Ming Ma et al. [7] have proposed a new fuzzy arithmetic based upon both fuzziness index and location index function. The fuzziness index functions are in use to follow the lattice rules then they are the least upper bound and greatest lower bound in the lattice \( L \). That is for \( a, b \in L \) we define \( a \vee b = \max\{a, b\} \) and \( a \wedge b = \min\{a, b\} \), whereas the location index number is considered as the ordinary arithmetic which includes basic concepts.

For two intervals \( \tilde{a} = [a_1, a_2], \tilde{b} = [b_1, b_2] \in IR \) and \( +, -, \times, \div \), the arithmetic operations on \( \tilde{a} \) and \( \tilde{b} \) are defined as:

\[
\tilde{a} \times \tilde{b} = [a_1, a_2] \times [b_1, b_2] = \langle m(\tilde{a}), w(\tilde{a}) \rangle \times \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) \times m(\tilde{b}), \max \{w(\tilde{a}), w(\tilde{b})\} \rangle.
\]

In Particular

(i). Addition: \( \tilde{a} + \tilde{b} = \langle m(\tilde{a}), w(\tilde{a}) \rangle + \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) + m(\tilde{b}), \max \{w(\tilde{a}), w(\tilde{b})\} \rangle \).

(ii). Subtraction: \( \tilde{a} - \tilde{b} = \langle m(\tilde{a}), w(\tilde{a}) \rangle - \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) - m(\tilde{b}), \max \{w(\tilde{a}), w(\tilde{b})\} \rangle \).

(iii). Multiplication: \( \tilde{a} \times \tilde{b} = \langle m(\tilde{a}), w(\tilde{a}) \rangle \times \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) \times m(\tilde{b}), \max \{w(\tilde{a}), w(\tilde{b})\} \rangle \).

(iv). Division: \( \tilde{a} \div \tilde{b} = \langle m(\tilde{a}), w(\tilde{a}) \rangle \div \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) \div m(\tilde{b}), \max \{w(\tilde{a}), w(\tilde{b})\} \rangle \), provided \( m(b) \neq 0 \).

### 2.2. Ranking of interval numbers

Sengupta and Pal [2] proposed a simple and efficient index for comparing any two intervals on \( IR \) through decision maker’s satisfaction.

**Definition 2.2.1:** Let \( \preceq \) be an extended order relation between the interval numbers \( \tilde{a} = [a_1, a_2], \tilde{b} = [b_1, b_2] \in IR \), then for \( m(\tilde{a}) < m(\tilde{b}) \), we construct a premise \( (\tilde{a} \preceq \tilde{b}) \) which implies that \( \tilde{a} \) is inferior to \( \tilde{b} \) (or \( \tilde{b} \) is superior to \( \tilde{a} \)).

An acceptability function is defined as: \( A_{\preceq} : IR \times IR \rightarrow [0, \infty) \)

\[
A_{\preceq}(\tilde{a}, \tilde{b}) = A(\tilde{a} \preceq \tilde{b}) = \frac{m(\tilde{b}) - m(\tilde{a})}{w(\tilde{b}) + w(\tilde{a})}, \text{ where } w(\tilde{b}) + w(\tilde{a}) \neq 0.
\]

may be interpreted as the grade of acceptability of the the first interval number to be inferior to the second interval number. For any two interval numbers \( \tilde{a} \) and \( \tilde{b} \) in \( IR \) either \( A(\tilde{a} \preceq \tilde{b}) \geq 0 \) (or ) \( A(\tilde{b} \succeq \tilde{a}) \) (or) \( A(\tilde{a} \preceq \tilde{b}) = 0 \) (or) \( A(\tilde{a} \preceq \tilde{b}) + A(\tilde{b} \succeq \tilde{a}) = 0 \). If \( A(\tilde{a} \preceq \tilde{b})=0 \) and \( A(\tilde{b} \succeq \tilde{a})=0 \), then
we say that the interval numbers $\tilde{a}$ and $\tilde{b}$ are equivalent (non-inferior to each other) and we denote it by $\tilde{a} \sim \tilde{b}$. Also if $A(\tilde{a} \leq \tilde{b}) \geq 0$, then $\tilde{a} \leq \tilde{b}$ and if $A(\tilde{b} \leq \tilde{a}) \leq 0$, then $\tilde{b} \leq \tilde{a}$.

3. MAIN RESULTS

3.1. General Interval Assignment Problem

Let there are $m$ work and $n$ people available with dissimilar skills. If the cost of doing $j^{th}$ work by $i^{th}$ people is $c_{ij}$. Now the problem is which the work is to be assigned to whom so that the cost of completion of work will be minimum. Mathematically, we express the problem as follows:

Minimize $\hat{Z}(\text{cost}) = \sum \sum \tilde{c}_{ij} \tilde{x}_{ij}; i=1,2,\ldots,n; j=1,2,\ldots,n$

Where $\tilde{x}_{ij} = \begin{cases} 1; & \text{if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\ 0; & \text{if } i^{th} \text{ person is not assigned the } j^{th} \text{ work} \end{cases}$

with the restrictions

$\sum \tilde{x}_{ij} = 1; j=1,2,\ldots,n$

i.e., $i^{th}$ person will do only one work

$\sum \tilde{x}_{ij} = 1; i=1,2,\ldots,n$

i.e., $j^{th}$ work will be done only by one person

3.2. Interval Hungarian method

In this section an algorithm to solve assignment problem with generalized interval arithmetic using Hungarian method:

Step 1: Find out the mid values of each interval in the cost matrix.

Step 2: Subtract the interval which have smallest mid value in each row from all the entries of its row.

Step 3: Subtract the interval which have smallest mid value from those columns which have no intervals contain zero from all the entries of its column.

Step 4: Draw lines through appropriate rows and columns so that all the intervals contain zero of the cost matrix are covered and the minimum number of such lines is used.

Step 5: Test for optimality (i) If the minimum number of covering lines is equal to the order of the cost matrix, then optimality is reached. (ii) If the minimum number of covering lines is less than the order of the matrix, then go to step 6.

Step 6: Determine the smallest mid values of the intervals which are not covered by any lines. Subtract this entry from all uncrossed elements and add it to the crossing having an interval contain zero. Then go to step 4.
3.3. *Tabular form of the problem*

The cost matrix of the interval assignment problem is known in the chart

| People | 1    | 2    | 3    | \ldots j | \ldots n |
|--------|------|------|------|----------|----------|
| Jobs   | \tilde{C}_{11} | \tilde{C}_{12} | \tilde{C}_{13} | \tilde{C}_{1j} | \tilde{C}_{1n} |
| 1      | \tilde{C}_{11} | \tilde{C}_{12} | \tilde{C}_{13} | \tilde{C}_{1j} | \tilde{C}_{1n} |
| 2      | \tilde{C}_{21} | \tilde{C}_{22} | \tilde{C}_{23} | \tilde{C}_{2j} | \tilde{C}_{2n} |
| \ldots | \ldots | \ldots | \ldots | \ldots | \ldots |
| i      | \tilde{C}_{i1} | \tilde{C}_{i2} | \tilde{C}_{i3} | \tilde{C}_{ij} | \tilde{C}_{in} |
| \ldots | \ldots | \ldots | \ldots | \ldots | \ldots |
| n      | \tilde{C}_{n1} | \tilde{C}_{n2} | \tilde{C}_{n3} | \tilde{C}_{nj} | \tilde{C}_{nn} |

4. **NUMERICAL EXAMPLE**

**Example 1:**

Let us consider an assignment method discussed by Ramesh kumar et al.[8]. The problem is solved by interval Hungarian method as follows: A corporation has four salesmen and four open territories available for assignment, that they are not equally rich in their sales potential and also it is estimated that the usual salesman operation in each territory would bring in the following annual Sales:

| Territory | I   | II  | III | IV  |
|-----------|-----|-----|-----|-----|
| Annual sales (RS): | 60,000 | 50,000 | 40,000 | 30,000 |

It is considered that the four salesmen will differ in ability, that working under the same conditions their yearly is estimated would be proportionately as follows:
Salesman: A  B  C  D  
Proportion:  7  5  5  4

If the criterion is maximum expected total values, the intuitive answer is to assign the best salesman to the richest territory; the next best salesmen to the second richest territory and so on verify this answer by the Hungarian method.

Solution

Step 1: By Constructing the effectiveness of the matrix. By taking 10,000 as one unit as the sales proportion and the maximum sales matrix is obtained as follows:

Table (a) Sales in 10,000 rupees

| Sales proportion | 6 | 5 | 4 | 3 |
|------------------|---|---|---|---|
|                  | I | II | III | IV |
| 7 A              | 42 | 35 | 28 | 21 |
| 5 B              | 30 | 25 | 20 | 15 |
| 5 C              | 30 | 25 | 20 | 15 |
| 4 D              | 24 | 20 | 16 | 12 |

The Value of C_{11} = Sales Proportion × Sales Territory

= 7×6 = 42

By applying traditional assignment method Ramesh kumar et al. got an optimal as I,II,III,IV are assign to A,B,C,D with an maximum cost is [95,103]. They converted this assignment problem into interval Hungarian method. Now the cost matrix of the interval Hungarian problem is

Step 2: Crisp entries of Cost matrix

|     | I     | II    | III   | IV    |
|-----|-------|-------|-------|-------|
| A   | [41,43]| [34,36]| [27,29]| [20,22]|
| B   | [29,31]| [24,26]| [19,21]| [14,16]|
| C   | [29,31]| [24,26]| [19,21]| [14,16]|
| D   | [23,25]| [19,21]| [15,17]| [11,13]|
Applying their Assignment method, Ramesh kumar et al.[8] obtained the optimal assignment as A, B, C,D machines are assign I, II, III, IV operators respectively and the optimum assignment cost as [95,103]. After stating that the above assignment is optimal, they claim that the solution is not unique and another optimal solution can be obtained as A,B,C,D are assign to IV, II, III, I respectively and the minimum cost is [89,91]. Hence the result and their conclusion violate the concept of optimality.

Now we shall solve the same interval assignment problem given in the step 2 by applying the method in this paper. Let us express any two interval parameters as

\[ \tilde{a} = [a_1, a_2] \]

in terms of midpoint and width as \( \tilde{a} = [m(\tilde{a}), w(\tilde{a})] \).

Now all the given interval assignment problem becomes

**Step 3**: Interval entries of Cost matrix

|     | I   | II  | III | IV  |
|-----|-----|-----|-----|-----|
| A   | <42,1> | <35,1> | <28,1> | <21,1> |
| B   | <30,1> | <25,1> | <20,1> | <15,1> |
| C   | <30,1> | <25,1> | <20,1> | <15,1> |
| D   | <24,1> | <20,1> | <16,1> | <12,1> |

**Step 4**: Subtract the smallest interval mid value in each row and column from all the entries of its row and column

|     | I   | II  | III | IV  |
|-----|-----|-----|-----|-----|
| A   | <9,1> | <6,1> | <3,1> | <0,1> |
| B   | <3,1> | <2,1> | <1,1> | <0,1> |
| C   | <3,1> | <2,1> | <1,1> | <0,1> |
| D   | <0,1> | <0,1> | <0,1> | <0,1> |

**Step 5**: Interval entries of Cost matrix

|     | I   | II  | III | IV  |
|-----|-----|-----|-----|-----|
| A   | <8,1> | <5,1> | <2,1> | <0,1> |
| B   | <2,1> | <1,1> | <0,1> | <0,1> |
| C   | <2,1> | <1,1> | <0,1> | <0,1> |
| D   | <0,1> | <0,1> | <0,1> | <1,1> |
Step 6: Interval entries of Cost matrix

|     | I    | II   | III  | IV   |
|-----|------|------|------|------|
| A   | <7,1> | <4,1> | <2,1> | <0,1> |
| B   | <1,1> | <0,1> | <0,1> | <0,1> |
| C   | <1,1> | <0,1> | <0,1> | <0,1> |
| D   | <0,1> | <0,1> | <1,1> | <2,1> |

The optimal assignment schedule is given by

A → IV, B → II, C → III, D → I

The optimal assignment cost = <21,1> + <25,1> + <20,1> + <24,1>

= <90, 1>

= [89,91]

It is noted that our solution [89,91] is very much sharper than the solution [95,103] obtained by Ramesh kumar et al[8]

Example 2:

Beta Corporation has four plants each of which can manufacture any one of the four goods. Production costs differ from one plant to another as do sales revenue. Given the revenue and cost data below, attain which product each plant should manufacture to maximize profit.

Sales revenue (Rs. 000s product)

| Plant | 1 | 2 | 3 | 4 |
|-------|---|---|---|---|
| A     | 50| 68| 49| 62|
| B     | 60| 70| 51| 74|
| C     | 55| 67| 53| 70|
| D     | 58| 65| 54| 69|

Production cost (Rs. 000s product) plant
Solution

Step 1: The profit matrix by using production cost sales revenue.

\[
\text{Profit} = \text{sales} - \text{cost}
\]

Profit matrix

|   | 1  | 2  | 3  | 4  |
|---|----|----|----|----|
| A | 49 | 60 | 45 | 61 |
| B | 55 | 63 | 45 | 69 |
| C | 52 | 62 | 49 | 68 |
| D | 55 | 64 | 48 | 66 |

Step 2: Cost matrix with crisp entries

|   | 1    | 2    | 3    | 4    |
|---|------|------|------|------|
| A | [0,2] | [7,9] | [3,5] | [0,2] |
| B | [4,6] | [6,8] | [5,7] | [4,6] |
| C | [2,4] | [4,6] | [3,5] | [1,3] |
| D | [2,4] | [0,2] | [5,7] | [2,4] |

Applying the interval assignment method, Ramesh kumar et al. obtained the optimal solution as A,B,C, D as II, IV, I, III operators respectively and the optimum assignment cost is [18,26]. After stating the above assignment is optimal, they claim that the solution is not unique and another optimal solution can be obtained as I, III, IV, II are assign to A,B,C,D respectively and the minimum cost is [9,11]. Hence their result and their conclusion violate the concept of optimality.

Now we shall solve the same interval assignment problem given in the step 2 by applying the method in this paper. Let us state all the interval parameter
\( \tilde{a} = [a_1, a_2] \) in terms of midpoint and width as \( \tilde{a} = [m(\tilde{a}), w(\tilde{a})]. \)

Now all the given interval assignment problem becomes

**Step 3:** Cost matrix with interval entries

|       | 1    | 2    | 3    | 4    |
|-------|------|------|------|------|
| A     | <1,1>| <8,1>| <4,1>| <1,1>|
| B     | <5,1>| <7,1>| <6,1>| <5,1>|
| C     | <3,1>| <5,1>| <4,1>| <2,1>|
| D     | <3,1>| <1,1>| <6,1>| <3,1>|

**Step 4:** Subtract the least interval mid value in both row and column from all the entries of its row and column

|       | 1    | 2    | 3    | 4    |
|-------|------|------|------|------|
| A     | <0,1>| <7,1>| <2,1>| <0,1>|
| B     | <0,1>| <2,1>| <0,1>| <0,1>|
| C     | <1,1>| <3,1>| <1,1>| <0,1>|
| D     | <2,1>| <0,1>| <4,1>| <2,1>|

The optimal assignment schedule is given by

A\( \rightarrow \)I, B\( \rightarrow \)III, C\( \rightarrow \)IV, D\( \rightarrow \)II

The optimal assignment cost

\[
= <1,1> + <6,1> + <2,1> + <1,1> \\
= <10, 1>
\]

\( = [9, 11] \)

It is noted that our solution \([9, 11]\) is very much sharper than the solution \([18, 26]\) obtained by Ramesh kumar et al.[8]

**Example 3**

An air line operates seven days a week has time table shown below. Crews must have a least layover (rest) time of 5 hrs, between flights. Obtain the pair of flights that minimizes layover time away from quarters. For any given the crews will e based at the city that effect is the least layover
| Flight No | Depart | Arrive | Flight No | Depart | Arrive |
|-----------|--------|--------|-----------|--------|--------|
| 1         | 7.00AM | 8.00AM | 101       | 8.00AM | 9.15AM |
| 2         | 8.00AM | 9.00AM | 102       | 8.30AM | 9.45AM |
| 3         | 1.30PM | 2.30PM | 103       | 12.00Noon | 1.15PM |
| 4         | 6.30PM | 7.30PM | 104       | 5.30PM | 6.45PM |

For each pair, state the town where the crews should be based

**Solution**

**Step1:** Constructing the table for layover times between flights that the crew is based at Delhi,

Consider 15 mins = 1 unit

Flight arrives and Depart from Jaipur, The minimum layover is 5 hrs among flights.

| Flight no 1 | Timing | Flight no 2 | Timing |
|-------------|--------|-------------|--------|
| Arrives    | Depart |
| 101         | 24 hrs = 96 units | 8.00AM | 8.00AM |
| 102         | 24hrs+30 min =98 units | 8.00AM | 8.30AM |
| 103         | 24hrs+4 min =112 units | 8.00AM | 12.00Noon |
| 104         | 9 hrs+30 min =38 units | 8.00AM | 5.30PM |
|             |        | 101         | 23 hrs = 92 units | 9.00AM | 8.00AM |
|             |        | 102         | 23hrs+30 min =94 units | 9.00AM | 8.30AM |
|             |        | 103         | 24hrs+3 min =108 units | 9.00AM | 12.00Noon |
|             |        | 104         | 8 hrs+30 min =34 units | 9.00AM | 5.30PM |
| Flight no 3 | Timing          |
|------------|----------------|
|            | Arrives        | Depart       |
| 101        | 17 hrs +30 min =70 units | 2.30PM       | 8.00AM       |
| 102        | 18 hrs =72 units    | 2.30PM       | 8.30AM       |
| 103        | 21hrs+30 min =86 units | 2.30PM       | 12.00Noon    |
| 104        | 24 hrs+3 hrs =108 units | 2.30PM       | 5.30PM       |

| Flight no 4 | Timing          |
|------------|----------------|
|            | Arrives        | Depart       |
| 101        | 12 hrs + 30 min = 50 units | 7.30PM       | 8.00AM       |
| 102        | 13hrs =52 units    | 7.30PM       | 8.30AM       |
| 103        | 16hrs+30 min =66 units | 7.30PM       | 12.00Noon    |
| 104        | 22 hrs =88 units    | 7.30PM       | 5.30PM       |

Layover times when crew is based on Delhi (Table 1)

| Flight No | 101 | 102 | 103 | 104 |
|-----------|-----|-----|-----|-----|
| 1         | 96  | 98  | 112 | 38  |
| 2         | 92  | 94  | 108 | 34  |
| 3         | 70  | 72  | 86  | 108 |
| 4         | 50  | 52  | 66  | 88  |

Flight arrives and Depart from Delhi, The minimum place over is 5 hrs between flights.

|            | Flight no 101 | Timing          |
|------------|--------------|----------------|
|            | Arrives      | Depart (ND)     |
| 1          | 21 hrs + 45 min = 87 units | 9.15AM       | 7.00AM       |
| 2          | 22hrs+45 min =91 units    | 9.15AM       | 8.00AM       |
| 3          | 28hrs+15 min =113 units   | 9.15AM       | 1.30PM       |
| 4          | 9 hrs+ 15 min =37 units   | 9.15AM       | 6.30PM       |
### Flight no 102

|   | Timing |   |
|---|--------|---|
|   | Arrives | Depart (ND) |
| 1 | 21 hrs + 15 min = 85 units | 9.45AM | 7.00AM |
| 2 | 22 hrs + 15 min = 89 units | 9.45AM | 8.00AM |
| 3 | 27 hrs + 45 min = 111 units | 9.45AM | 1.30PM |
| 4 | 8 hrs + 45 min = 35 units | 9.45AM | 6.30PM |

### Flight no 103

|   | Timing |   |
|---|--------|---|
|   | Arrives | Depart (ND) |
| 1 | 17 hrs + 45 min = 71 units | 1.15PM | 7.00AM |
| 2 | 18 hrs + 45 min = 75 units | 1.15PM | 8.00AM |
| 3 | 24 hrs + 15 min = 97 units | 1.15PM | 1.30PM |
| 4 | 5 hrs + 15 min = 21 units | 1.15PM | 6.30PM |

### Flight no 104

|   | Timings |   |
|---|---------|---|
|   | Arrives | Depart (ND) |
| 1 | 12 hrs + 15 min = 49 units | 6.45PM | 7.00AM |
| 2 | 13 hrs + 15 min = 53 units | 6.45PM | 8.00AM |
| 3 | 18 hrs + 45 min = 75 units | 6.45PM | 1.30PM |
| 4 | 23 hrs + 45 min = 95 units | 6.45PM | 6.30PM |

### Layover times when crew is based on Jaipur (Table 2)

| Flight No | 101 | 102 | 103 | 104 |
|-----------|-----|-----|-----|-----|
| 1         | 87  | 85  | 71  | 49  |
| 2         | 91  | 89  | 75  | 53  |
| 3         | 113 | 111 | 97  | 75  |
| 4         | 37  | 35  | 21  | 95  |
**Step 2:** Construct the table for minimum layover times among flights with the help of Table 1 and 2

| Flight No | 101 | 102 | 103 | 104 |
|-----------|-----|-----|-----|-----|
| 1         | 87  | 85  | 71  | 38  |
| 2         | 91  | 89  | 75  | 34  |
| 3         | 70  | 72  | 86  | 75  |
| 4         | 37  | 35  | 21  | 88  |

**Step 3:** Cost matrix with interval entries

| Flight No | 101   | 102   | 103   | 104   |
|-----------|-------|-------|-------|-------|
| 1         | [86,88]| [84,86]| [70,72]| [37,39]|  
| 2         | [90,92]| [88,90]| [74,76]| [33,35]|  
| 3         | [69,71]| [71,73]| [85,87]| [74,76]|  
| 4         | [36,38]| [34,36]| [20,22]| [87,89]|  

**Step 4:** Cost matrix with interval entries

| Flight No | 101   | 102   | 103   | 104   |
|-----------|-------|-------|-------|-------|
| 1         | <87,1> | <85,1> | <71,1> | <38,1> |  
| 2         | <91,1> | <89,1> | <75,1> | <34,1> |  
| 3         | <70,1> | <72,1> | <86,1> | <75,1> |  
| 4         | <37,1> | <35,1> | <21,1> | <88,1> |  

**Step 5:** Subtract the smallest entries in each row and column, we get

| Flight No | 101   | 102   | 103   | 104   |
|-----------|-------|-------|-------|-------|
| 1         | <49,1> | <45,1> | <33,1> | <0,1>  |  
| 2         | <57,1> | <53,1> | <41,1> | <0,1>  |  
| 3         | <0,1>  | <0,1>  | <16,1> | <5,1>  |  
| 4         | <16,1> | <12,1> | <0,1>  | <67,1> |  

Step 6:

| Flight No | 101   | 102   | 103   | 104   |
|-----------|-------|-------|-------|-------|
| 1         | <4,1> | <0,1> | <0,1> | <0,1> |
| 2         | <12,1> | <8,1> | <8,1> | <0,1> |
| 3         | <0,1> | <0,1> | <28,1> | <50,1> |
| 4         | <4,1> | <0,1> | <0,1> | <100,1> |

Flight 1→102
Flight 2→104
Flight 3→101
Flight 4→103

5. CONCLUSION:

In this paper, we have proposed interval versions of Hungarian method for solving the interval assignment problems without converting them to classical assignment problems. The main objective is to form the solution more precisely. Numerical examples solved and the results obtained are discussed. It is to be noted that the optimal solution obtained by our method is sharper than the solution obtained by others.

6. REFERENCES

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