Damping of a Yukawa Fermion at Finite Temperature

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The damping of a massless fermion coupled to a massless scalar particle at finite temperature is considered using the Braaten-Pisarski resummation technique. First the hard thermal loop diagrams of this theory are extracted and effective Green’s functions are constructed. Using these effective Green’s functions the damping rate of a soft Yukawa fermion is calculated. This rate provides the most simple example for the damping of a soft particle. To leading order it is proportional to \( g^2 T \), whereas the one of a hard fermion is of higher order.

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I. INTRODUCTION

A consistent calculation, i.e. leading to gauge independent results that are complete in the order of the coupling constant, of damping rates in quantum field theories at high temperature requires the use of resumed propagators and vertices according to the effective perturbation theory developed by Braaten and Pisarski \[1\]. Using this method the damping rates of quarks and gluons in a quark-gluon plasma have been discussed in great detail \[2–19\]. For example, the damping rates of soft partons, i.e. of partons with momenta of the order of \( g T \) or smaller, were found to be finite and proportional to \( g^2 T \) \[16–19\]. In particular, the long standing puzzle of the gauge dependence of the gluon damping rate at rest was solved in this way \[16\]. However, the computation of soft rates is rather cumbersome since effective propagators as well as vertices have to be taken into account.

In the case of the damping rates of hard partons with momenta of the order of \( T \) or larger, on the other hand, it is sufficient to include only an effective gluon propagator. These rates represent the most simple application of the Braaten-Pisarski method in QCD. Therefore they have been considered in a number of papers for studying the resummation technique, e.g. the gauge independence of its results \[3,18,20,21\]. Furthermore these rates are closely related to interesting properties of the quark-gluon plasma, such as thermalization time, viscosity, and mean free path and energy loss of partons \[3,14,22–25\]. However, owing to the absence of static magnetic screening in the resumed gluon propagator the damping rate of hard partons turns out to be logarithmic infrared divergent even using the resummation technique \[3,14,22\].

Here we will investigate the case of a massless fermion coupled to a massless scalar particle. The damping rate of this “Yukawa” fermion is of interest because it provides a simple example for the application of the Braaten-Pisarski method. Furthermore it shows some interesting features which are not observed in gauge theories.

II. HARD THERMAL LOOPS AND EFFECTIVE GREEN FUNCTIONS

We start from the following Lagrangian

\[
\mathcal{L} = i \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + g \bar{\psi} \psi \phi ,
\]

(1)

describing the coupling of a massless fermion \( \psi \) to a massless scalar \( \phi \) with a coupling constant \( g \).

According to the Braaten-Pisarski technique we first have to extract the hard thermal loop diagrams of this theory from which we construct the effective Green’s functions by resummation. Starting with the hard thermal loop self energy of the scalar particle we consider the diagram shown in Fig.1. Standard Feynman rules give \( (K^2 = k_0^2 - k^2) \)

\[
\Pi(P) = -ig^2 \int \frac{d^4K}{(2\pi)^4} \, tr \left[ S(K - P)S(K) \right] .
\]

(2)

Evaluating the trace over the gamma matrices and adopting the imaginary time formalism at finite temperature, we find
\[ \Pi(P) = -4 g^2 T \sum_{k_0} \int \frac{d^3k}{(2\pi)^3} (k_0 q_0 - k \cdot q) \tilde{\Delta}(K) \tilde{\Delta}(Q), \]  

where \( Q = P - K \), \( \tilde{\Delta}(K) = 1/K^2 \) and the sum extends over discrete fermionic energies, \( k_0 = (2n + 1)i\pi T \). The sum over \( k_0 \) can easily be performed by introducing the Saclay representation of the propagators [26] 

\[ \tilde{\Delta}(K) = - \int_0^{1/T} d\tau e^{k_0\tau} \frac{1}{2k} \left \{ [1 - n_F(k)] e^{-k\tau} - n_F(k) e^{k\tau} \right \}, \]

where \( n_F(k) = 1/[\exp(k/T) + 1] \) is the Fermi distribution.

The calculation of the scalar self energy in the hard thermal loop approximation, \( K \gg P \), can be done analogously to the gluon polarization tensor [1]. The only difference between (3) and the corresponding expression for the longitudinal part of the gluon self energy by quark polarization is, apart from an overall factor, a different sign between the \( k_0 q_0 \)- and \( k \cdot q \)-term in (3) coming from additional gamma matrices due to the quark-gluon vertices in the trace of the gluon polarization tensor. This sign, however, is essential since the minus sign in (3) leads to a cancellation of the momentum dependent terms, from which the simple result

\[ \Pi(P) = \frac{g^2 T^2}{6}. \]  

follows, whereas in the gluon case a complicated momentum dependent term survives. Furthermore the gluon hard thermal loop polarization tensor shows an imaginary part corresponding to Landau damping below the light cone.

The effective scalar propagator is given by resuming the hard thermal loop self energy (3) within a Dyson-Schwinger equation resulting in

\[ \Delta^*(K) = \frac{1}{K^2 - m_S^2}, \]

where \( m_S^2 = g^2T^2/6 \) describes an effective thermal mass of the scalar particle generated by the interaction with the fermions of the heat bath.

The hard thermal loop fermion self energy caused by the diagram of Fig.2 differs from the quark self energy [27] only by a factor 8/3. Thus we simply may replace the effective quark mass \( m_q^2 = g^2T^2/6 \) by the effective Yukawa fermion mass \( m_Y = g^2T^2/16 \). The effective fermion propagator in the helicity eigenstate representation is given by [18,28,29]

\[ S^*(P) = \frac{1}{2D_+(P)} (\gamma_0 - \hat{p} \cdot \gamma) + \frac{1}{2D_-(P)} (\gamma_0 + \hat{p} \cdot \gamma), \]

where

\[ D_\pm(P) = -p_0 \pm \frac{\hat{p}}{p} \left ( \pm 1 - \frac{\pm p_0 - p}{2p} \ln \frac{p_0 + p}{p_0 - p} \right ). \]

In contrast to the effective scalar propagator (3) the effective fermion propagator (6) shows an imaginary part giving rise to damping effects, as we will see below.

The Yukawa theory [1] contains no effective vertices on the hard thermal loop level. Consider for instance the correction to the three-point vertex shown in Fig.3. According to the rules of power counting [1] this correction is suppressed compared to the bare vertex by a factor of \( g \) even if all external legs are soft. The reason for this is the fact that the scalar propagator in Fig.3 is cancelled by a factor \( K^2 \) coming from the fermion propagators in the hard thermal loop limit. In QCD there is an additional gamma matrix in the vertex correction, \( K \gamma_\mu K = 2K_\mu K - K^2 \gamma_\mu \). Owing to the first term the cancellation of the gluon propagator is upset [1]. Similar considerations apply to higher n-point functions. The absence of effective vertices is not surprising as there are no Ward identities in the Yukawa theory, which relate effective propagators to effective vertices.

III. DAMPING OF A SOFT YUKAWA FERMION

Now we turn to the damping of a Yukawa fermion. First we consider the damping of a hard fermion with momentum of the order of \( T \) or larger. Comparing with the quark damping rate the lowest order contribution proportional to
$g^2 T [2,13]$ should come from the imaginary part of the fermion self energy containing an effective scalar propagator (Fig.4). However, since the scalar propagator has no imaginary part (virtual Landau damping) the diagram of Fig.4 is real. The fermion self energy diagram containing an effective fermion propagator, but a bare scalar propagator due to energy momentum conservation at the vertex, contributes to higher order $g^4$, as can be seen from the similar case of photon damping [21]. Thus there is no damping of a hard fermion to order $g^2$ in the Yukawa theory.

The damping rate of a soft Yukawa fermion follows from the diagram of Fig.5, where we have to use an effective scalar as well as fermion propagator, since the external momentum is soft, $p_0, p \sim gT$. The corresponding self energy diagrams for soft quark or gluon damping are much more involved [19,29] due to the presence of effective momentum dependent vertices and the momentum dependence of the effective gluon propagator. The damping mechanism corresponding to the fermion self energy of Fig.5 can be seen by cutting the diagram [31]. Since the fermion propagator has an imaginary part there is a cut-pole contribution describing "Compton scattering".

The damping rates of a soft fermion and a plasmino, corresponding to a negative helicity eigenstate, at rest, $p = 0$, are equal and given by [18]

$$\gamma_{\pm}(0) = -\frac{1}{8} tr \left[ \gamma_0 \, Im \Sigma^*(m_Y + i\epsilon, 0) \right] ,$$

where $\Sigma^*$ is given by Fig.5. Substituting the effective scalar and fermion propagators (3) and (6) into $\Sigma^*$ and evaluating the trace over the gamma matrices yields

$$tr \left[ \gamma_0 \, \Sigma^*(P) \right] = -2 \, g^2 \, T \sum_{k_0} \int \frac{d^3k}{(2\pi)^3} \, \Delta^*(K) \left[ \frac{1}{D_+(Q)} + \frac{1}{D_-(Q)} \right] ,$$

where $Q = P - K$. The sum over the Matsubara frequencies can be performed easily by using the Saclay method again. In the case of the effective scalar propagator the Saclay representation reads

$$\Delta^*(K) = -\int_{0}^{1/T} d\tau \, e^{i\omega \tau} \frac{1}{2\omega} \left\{ [1 + n_B(\omega_k)] e^{-\omega \tau} + n_B(\omega_k) e^{\omega \tau} \right\} ,$$

where $n_B(\omega_k) = 1/[e^{\omega_k/kT} - 1]$ denotes the Bose distribution and $\omega_k^2 = k^2 + m^2$. In the case of the effective fermion propagator it is convenient to introduce the spectral representation [32]

$$\frac{1}{D_{\pm}(Q)} = -\int_{0}^{1/T} d\tau' \, e^{i\omega \tau'} \int_{-\infty}^{\infty} d\omega \, \rho_{\pm}^{\text{disc}}(\omega, q) [1 - n_F(\omega)] e^{-\omega \tau'} ,$$

where the spectral functions $\rho_{\pm}$ are given in Ref. [29]. Since the damping comes from the cut term of the fermion propagator we only need the discontinuous part of the spectral function defined by

$$\rho_{\pm}^{\text{disc}}(x, y) = \frac{1}{\pi} \, Im \, \frac{1}{D_{\pm}(x, y)} \, \theta(y^2 - x^2) .$$

Carrying out the sum over $k_0$ and the integrations over $\tau$ and $\tau'$ [40] reduces to

$$tr \left[ \gamma_0 \, \Sigma^*(P) \right] = g^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k} \int_{-\infty}^{\infty} d\omega \, \left\{ \rho_{\pm}^{\text{disc}}(\omega, q) + \rho_{\pm}^{\text{disc}}(\omega, -q) \right\} [1 - n_F(\omega) + n_B(\omega_k)] \frac{1}{p_0 - \omega_k - \omega} + [n_F(\omega) + n_B(\omega_k)] \frac{1}{p_0 + \omega_k - \omega} .$$

Putting the self energy $\Sigma^*$ on the mass shell, $p_0 = m_Y + i\epsilon$, $p = 0$, extracting the imaginary part via

$$Im \, \frac{1}{m_Y + \omega_k - \omega + i\epsilon} = -\pi \, \delta(m_Y \pm \omega_k - \omega) ,$$

and expanding the distribution functions for soft energies, $n_F(\omega) \simeq 1/2$ and $n_B(\omega_k) \simeq T/\omega_k$, we obtain after integrating over the angles and $\omega$ the leading term of the damping rate

$$\gamma_{\pm}(0) = \frac{g^2 T}{16\pi} \int_{0}^{\infty} dk \frac{k^2}{\omega_k} \left[ \rho_{\pm}^{\text{disc}}(\omega_+, k) + \rho_{\pm}^{\text{disc}}(\omega_-, k) + \rho_{\pm}^{\text{disc}}(\omega_+, k) + \rho_{\pm}^{\text{disc}}(\omega_-, k) \right] ,$$

where $\omega_{\pm} = m_Y \pm \omega_k$. 

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Inserting (13) into this expression, the step function of the discontinuous part of the spectral function cannot be satisfied for $\omega_+$ which is always larger than $k$. In the case of $\omega_-$ the step function restricts $k$ to

$$k \geq \sqrt{\left(\frac{m_S^2 + m_Y^2}{2m_Y}\right)^2 - m_S^2} = \frac{5}{24} g T.$$  \hspace{1cm} (17)

Carrying out the remaining integration over $k$ numerically, we end up with the final result

$$\gamma_{\pm}(0) = 0.24 \frac{g^2 T}{16\pi}. \hspace{1cm} (18)$$

**IV. CONCLUSIONS**

We have considered the Yukawa theory at finite temperature. First we have extracted the hard thermal loops and constructed effective Green’s functions from them. In contrast to gauge theories the boson propagator shows no momentum dependence and is real for all values of the momentum and the energy. Also there are no effective vertices in this theory.

Next we have calculated the damping rate of a Yukawa fermion. Owing to the absence of an imaginary part of the effective scalar propagator the damping rate of a hard fermion is at least of the order $g^4$. The damping rate of a soft fermion, on the other hand, is of the order $g^2$ due to the simultaneous use of an effective scalar and fermion propagator. As a new feature, not observed in the case of quarks or gluons, the damping rate of a Yukawa fermion at rest contains a kinematic restriction depending on the effective thermal masses of the scalar field and the Yukawa fermion.

The computation of the Yukawa fermion damping rate at rest provides the most simple example for a soft rate using the Braaten-Pisarski method since no effective vertices are involved and the effective scalar propagator is momentum independent.

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FIG. 1. Hard thermal loop contribution to the scalar self energy.

FIG. 2. Hard thermal loop contribution to the fermion self energy.

FIG. 3. Correction to the three point vertex.

FIG. 4. Fermion self energy containing an effective scalar propagator denoted by a blob.

FIG. 5. Fermion self energy containing an effective scalar and fermion propagator.
This figure "fig1-1.png" is available in "png" format from:

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