Feynman rules in $N = 2$ projective superspace II: Massive hypermultiplets

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Abstract

Manifest $N = 2$ supersymmetric hypermultiplet mass terms can be introduced in the projective $N = 2$ superspace formalism. In the case of complex hypermultiplets, where the gauge covariantized spinor derivatives have an explicit representation in terms of gauge prepotentials, it is possible to interpret such masses as vacuum expectation values of an Abelian vector multiplet. The duality transformation that relates the $N = 2$ off-shell projective description of the hypermultiplet to the on-shell description involving two $N = 1$ chiral superfields allows us to obtain the massive propagators of the $N = 1$ complex linear fields in the projective hypermultiplet. The $N = 1$ massive propagators of the component superfields in the projective hypermultiplet suggest a possible ansatz for the $N = 2$ massive propagator, which agrees with an explicit calculation in $N = 2$ superspace.
1 Introduction

A manifestly $N = 2$ supersymmetric path integral quantization of $N = 2$ massless hypermultiplets living in projective superspace [1] has recently been proposed [2] (for an alternative formulation of $N = 2$ superspace, see [3]). This off-shell representation of $N = 2$ supersymmetry contains $N = 1$ chiral, complex linear and auxiliary superfields. It can be related to the traditional on-shell hypermultiplet involving two massless $N = 1$ chiral fields via a duality transformation that acts on the complex linear field of the projective hypermultiplet.

In this paper we extend the analysis of [2] to massive hypermultiplets (for the Harmonic superspace version see [4]). As in [2] we first guess the massive propagator of the projective hypermultiplet inspired by the explicit form of the massive propagators of its $N = 1$ component superfields. The most efficient way to find these massive $N = 1$ propagators is to perform the duality transformation in the action where we have included couplings to sources.

To write the propagators in projective superspace we have to introduce a central charge in the supersymmetry algebra. It is useful to identify such a central charge with the expectation value of a background Abelian vector multiplet. We show that using the centrally extended spinor derivatives, we can find an ansatz for the $N = 2$ propagator. The ansatz takes a simple form which is the naive generalization of the massless one in [2]. When reduced to $N = 1$ components it gives the correct $N = 1$ propagators of the massive chiral-antichiral, linear-antilinear and auxiliary fields. Finally we derive the same $N = 2$ propagator directly in $N = 2$ superspace by manipulating the path integral. We end with a list of the Feynman rules used to calculate Feynman diagrams containing massive hypermultiplets in projective superspace.

There is one exception to this result; for real $O(2p)$ multiplets we cannot express the propagator in projective superspace. This is related to the fact that we cannot consistently assign a $U(1)$ charge to a real field. We may, however, find a massive $N = 2$ propagator if we use a complex $O(2p)$ multiplet. It describes two physical hypermultiplets, but otherwise it gives consistent Feynman rules. In particular, the limit $p \to \infty$ of the complex $O(2p)$ multiplet gives the (ant)arctic multiplet propagator.

2 Projective Superspace with central charges

We briefly review the basic ideas of $N = 2$ projective superspace and consider the peculiarities introduced by central charges. For a more complete review of ordinary $N = 2$ projective superspace we refer the reader to [1],[2].

The algebra of $N = 2$ supercovariant derivatives with central charges in four dimensions is

\[ \{ \mathcal{D}_a, \mathcal{D}_b \} = q C_{ab} C_{\alpha\beta} \tilde{m}, \quad \{ \tilde{\mathcal{D}}^a, \tilde{\mathcal{D}}^b \} = -q C^{ab} C^\epsilon_{\dot{\alpha}\dot{\beta}} \tilde{m}, \quad \{ \mathcal{D}_a, \tilde{\mathcal{D}}^b \} = i \delta^b_a \partial_{\alpha\beta}. \quad (1) \]

1 We will use the notation and normalization conventions of [1]; in particular we denote $D^2 = \frac{1}{2} D^\alpha D_\alpha$ and $\Box = \frac{1}{2} \partial^{\alpha\beta} \partial_{\alpha\beta}$. 

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where we have included a possible global $U(1)$ charge which will be useful later. The projective subspace of $N = 2$ superspace is parameterized by a complex coordinate $\zeta$, and it is spanned by the following projective supercovariant derivatives:

\[
\tilde{\nabla}_\alpha(\zeta) = \mathcal{D}_{1\alpha} + \zeta \mathcal{D}_{2\alpha}, \\
\tilde{\nabla}_{\dot{\alpha}}(\zeta) = \bar{\mathcal{D}}^2_{\dot{\alpha}} - \zeta \mathcal{D}^1_{\dot{\alpha}} .
\]  

(2)

The conjugate of any object is constructed in this subspace by composing the antipodal map on the Riemann sphere with hermitian conjugation. To obtain the barred supercovariant derivative we conjugate the unbarred derivative and we multiply by an additional factor $(-\zeta)$

\[
- \zeta \tilde{\nabla}_\alpha(\zeta) = \tilde{\nabla}_{\dot{\alpha}}(\zeta) .
\]  

(3)

The orthogonal combinations

\[
\tilde{\Delta}_\alpha = -\mathcal{D}_{2\alpha} + \frac{1}{\zeta} \mathcal{D}_{1\alpha} , \\
\bar{\tilde{\Delta}}_{\dot{\alpha}} = \bar{\mathcal{D}}^2_{\dot{\alpha}} + \frac{1}{\zeta} \mathcal{D}^1_{\dot{\alpha}} ,
\]  

(4)

and the projective supercovariant derivatives give the following algebra

\[
\{ \tilde{\nabla}, \tilde{\nabla} \} = \{ \tilde{\nabla}, \bar{\tilde{\nabla}} \} = \{ \tilde{\Delta}, \bar{\tilde{\Delta}} \} = \{ \tilde{\Delta}, \bar{\tilde{\Delta}} \} = 0 \\
\{ \nabla_\alpha, \tilde{\Delta}_{\beta} \} = -2qC_{\alpha\beta\bar{m}} \\
\{ \tilde{\nabla}_\alpha, \bar{\tilde{\Delta}}_{\dot{\alpha}} \} = -\{ \bar{\tilde{\nabla}}_{\dot{\alpha}}, \tilde{\Delta}_\alpha \} = 2i\partial_{\alpha\dot{\alpha}} .
\]  

(5) \hspace{1cm} (6) \hspace{1cm} (7)

For notational simplicity we denote from now on $\mathcal{D}_{1\alpha} = \mathcal{D}_\alpha ; \mathcal{D}_{2\alpha} = \mathcal{Q}_\alpha$. Superfields living in $N = 2$ projective superspace are annihilated by the projective supercovariant derivatives (2). This constraints can be rewritten as follows

\[
\tilde{\nabla}_\alpha \tilde{\Upsilon} = 0 = \bar{\tilde{\nabla}}_{\dot{\alpha}} \bar{\tilde{\Upsilon}} \implies \mathcal{D}_\alpha \tilde{\Upsilon} = -\zeta \mathcal{Q}_\alpha \tilde{\Upsilon} , \quad \bar{\mathcal{Q}}_{\dot{\alpha}} \bar{\tilde{\Upsilon}} = \zeta \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\tilde{\Upsilon}} .
\]  

(8)

Manifestly $N = 2$ supersymmetric actions have the form

\[
\frac{1}{2\pi i} \oint_C \frac{d\zeta}{\zeta} \mathcal{D}^2 \bar{\mathcal{D}}^2 f(\tilde{\Upsilon}, \bar{\tilde{\Upsilon}}, \zeta) ,
\]

(9)

where $C$ is a contour around some point of the complex plane that generically depends on the function $f(\tilde{\Upsilon}, \bar{\tilde{\Upsilon}}, \zeta)$.

The superfields obeying (3) may be classified as: i) $O(k)$ multiplets, ii) rational multiplets iii) analytic multiplets. The $O(k)$ multiplet can be expressed as a polynomial in $\zeta$ with powers ranging from 0 to $k$. If we multiply the $O(k)$ multiplet by a factor $\zeta$ it becomes a polynomial in $\zeta$ that we will denote as $O(i, i + k)$. Rational multiplets are quotients of $O(k)$ multiplets, and analytic multiplets are analytic in the coordinate $\zeta$ on some region of the Riemann sphere.

For even $k = 2p$ we can impose a reality condition on the $O(k)$ multiplet. We use the the name $\eta$ for the real $O(2p)$ superfield. The reality condition obeyed by this field can be written as
or equivalently, in terms of coefficient superfields,

\[
\tilde{\eta}_{2p-n} = (-)^{p-n}\tilde{\eta}_n .
\] (11)

The arctic multiplet, is the limit \( k \to \infty \) of the complex \( O(k) \) multiplet. It is therefore analytic on \( \zeta \) around the north pole of the Riemann sphere

\[
\tilde{\Upsilon} = \sum_{n=0}^{\infty} \tilde{\Upsilon}_n \zeta^n .
\] (12)

Its conjugate (antarctic) superfield

\[
\bar{\tilde{\Upsilon}} = \sum_{n=0}^{\infty} \bar{\tilde{\Upsilon}}_n (-\frac{1}{\zeta})^n
\] (13)

is analytic around the south pole of the Riemann sphere.

Similarly, if we consider the self-conjugate projective superfield \( \tilde{\eta}/\zeta^p \), the real tropical multiplet can be identified with the limit \( p \to \infty \) of this field

\[
V(\zeta) = \sum_{n=-\infty}^{+\infty} v_n \zeta^n ;
\] (14)

it contains a piece analytic around the north pole of the Riemann sphere (though not projective) and a piece analytic around the south pole:

\[
V(\zeta) = V_-(\zeta) + V_+(\zeta) ; \quad V_-(\zeta) = \sum_{n=-\infty}^{-1} v_n \zeta^n + \frac{1}{2} v_0 , \quad V_+(\zeta) = \frac{1}{2} v_0 + \sum_{n=1}^{+\infty} v_n \zeta^n .
\] (15)

The reality condition in terms of its coefficient superfields is the following

\[
v_{-n} = (-)^n \bar{v}_n .
\] (16)

The constraints obeyed by multiplets living in projective superspace (8) can be written in terms of their coefficients

\[
\mathcal{D}_\alpha \tilde{\Upsilon}_{n+1} = -\mathcal{Q}_\alpha \tilde{\Upsilon}_n , \quad \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\tilde{\Upsilon}}_n = \bar{\mathcal{Q}}_{\dot{\alpha}} \bar{\tilde{\Upsilon}}_{n+1} .
\] (17)

Such constraints imply that the lowest order coefficient superfield of any multiplet is antichiral in \( N = 1 \) superspace, and the next to lowest order obeys a modified antilinearity constraint. The same constraints imply that the highest order coefficient superfield is chiral in \( N = 1 \) superspace and the next to highest order obeys a modified linearity constraint

\[
\mathcal{D}_\alpha \tilde{\Upsilon}_0 = 0 , \quad \mathcal{D}^2 \tilde{\Upsilon}_1 = q m \tilde{\Upsilon}_0 , \quad \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\tilde{\Upsilon}}_0 = 0 , \quad \bar{\mathcal{D}}^2 \bar{\tilde{\Upsilon}}_{k=1} = q m \bar{\tilde{\Upsilon}}_k
\] (18)

\[
\bar{\mathcal{D}}_{\dot{\alpha}} \bar{\tilde{\Upsilon}}_0 = 0 , \quad \bar{\mathcal{D}}^2 \bar{\tilde{\Upsilon}}_1 = -q m \bar{\tilde{\Upsilon}}_0 , \quad \mathcal{D}_\alpha \tilde{\Upsilon}_k = 0 , \quad \mathcal{D}^2 \tilde{\Upsilon}_{k=1} = -q m \tilde{\Upsilon}_k .
\]
In the case of complex $O(k)$ hypermultiplets these highest and lowest order superfields are not conjugate to each other, and the complex multiplet describes twice as many physical degrees of freedom as the real one \[2\].

In the case of the real projective multiplet there is no lowest or highest order coefficient, and therefore none of the coefficient superfields is constrained in $N = 1$ superspace.

3 Massive hypermultiplet in $N = 1$ superspace

3.1 $N=1$ duality of the off-shell and on-shell hypermultiplet descriptions

To illustrate how the central charges arise in the algebra of $N = 2$ supercovariant derivatives and understand the role they play in the duality of the on-shell and off-shell realizations, we begin considering hypermultiplets that live in a projective superspace without central charges. The supercovariant derivatives in this space obey the following algebra

$$\{D_{a \alpha}, D_{b \beta}\} = 0, \quad \{D_{a \alpha}, \bar{D}_{b \dot{\beta}}\} = i \delta_a^b \partial_{\alpha \dot{\beta}}.$$  \hspace{1cm} (19)

The kinetic action of a massless (anti)arctic hypermultiplet living in this space is

$$S_0 = \int d^4x D^2 \bar{D}^2 \oint \frac{d \zeta}{2 \pi i \zeta} \bar{\Upsilon} \Upsilon.$$  \hspace{1cm} (20)

The component form of this action and the $N = 1$ duality transformation that converts it into the well known action of the hypermultiplet realizing $N = 2$ supersymmetry on shell, have been discussed in ref. \[2\]. If we include a coupling with an external Abelian vector multiplet living in $N = 2$ projective superspace \[3\] we have

$$S = \int d^4x d^4\theta \oint \frac{d \zeta}{2 \pi i \zeta} \bar{\Upsilon} e^V \Upsilon,$$  \hspace{1cm} (21)

where $V(\zeta)$ is a real tropical multiplet. This action is invariant under $N = 2$ supersymmetry transformations and also under $U(1)$ gauge transformations

$$\bar{\Upsilon}' = e^{iqA}\bar{\Upsilon}, \quad (e^V)' = e^{iA}e^V e^{-iA}, \quad \Upsilon' = e^{iqA}\Upsilon,$$  \hspace{1cm} (22)

when we assign $U(1)$ charges $q(\Upsilon) = 1, q(\bar{\Upsilon}) = -1$. The most general gauge parameter $A(\zeta)$ is a projective arctic multiplet. Splitting the tropical multiplet into the antarctic piece and the arctic piece \(13\)

$$e^{V(\zeta)} = e^{V_-(\zeta)} e^{V_+(\zeta)},$$  \hspace{1cm} (23)

we can define gauge covariantized projective spinor derivatives \[1\] :

$$\nabla_\alpha = \nabla_\alpha + q(\nabla_\alpha V_-) = \nabla_\alpha - q(\nabla_\alpha V_+) = D_\alpha + \zeta Q_\alpha$$  \hspace{1cm} (24)

and
\[ \hat{\nabla}_\alpha = \nabla_\alpha + q(\nabla_\alpha V_-) = \nabla_\alpha - q(\nabla_\alpha V_+) = Q_\alpha - \zeta \mathcal{D}_\alpha , \] (25)

which annihilate the covariantly projective (ant)arctic multiplet

\[ \tilde{\Upsilon} = e^{V^+} \Upsilon , \quad \bar{\tilde{\Upsilon}} = e^{V^-} \bar{\Upsilon} . \] (26)

This covariantly projective hypermultiplet transforms only under the residual \( U(1) \) gauge symmetry,

\[ \tilde{\Upsilon}' = e^{V^+} \Upsilon' = e^{V_i} e^{i(\Lambda_0 + \Lambda_0) - i\Lambda} e^{i\Lambda} \tilde{\Upsilon} = e^{\frac{i}{2}(\Lambda_0 + \Lambda_0)} \tilde{\Upsilon} , \] (27)

and correspondingly its conjugate transforms with the opposite \( U(1) \) charge

\[ \bar{\tilde{\Upsilon}}' = e^{-\frac{i}{2}(\Lambda_0 + \Lambda_0)} \bar{\tilde{\Upsilon}} . \] (28)

The interacting hypermultiplet action (21) can be written as the kinetic action of a covariantly projective (ant)arctic multiplet, very much like we do with a covariantly chiral \( N = 1 \) superfield

\[ S = \int d^4x d^4\theta \int \frac{d\zeta}{2\pi i\zeta} \tilde{\Upsilon} e^V \bar{\Upsilon} = \int d^4x d^4\theta \int \frac{d\zeta}{2\pi i\zeta} \tilde{\Upsilon} \bar{\Upsilon} . \] (29)

The anticommutators of gauge covariantized spinor derivatives

\[ \{ \mathcal{D}_\alpha, Q_\beta \} = qC_{\alpha\beta} D^2 v_1 , \quad \{ \bar{\mathcal{D}}_\dot{\alpha}, \bar{Q}_{\dot{\beta}} \} = qC_{\dot{\alpha}\dot{\beta}} \bar{D}^2 v_{-1} \] (30)

are proportional to the \( N = 2 \) gauge field strength \( \bar{W} = i D^2 v_1 \) and its conjugate. These scalar field strengths and the spinorial \( W_\alpha = \bar{D}^2 D_\alpha v_0 \) are the only gauge invariant fields contained in the tropical multiplet \( \bar{\Upsilon} \). The constrained \( N = 1 \) superfields in \( \Upsilon \) are a covariantly antichiral field \( \Upsilon_0 \) and a superfield obeying \( \mathcal{D}^2 \bar{\Upsilon}_1 = -iq\bar{W} \bar{\Upsilon}_0 \). A nonzero v.e.v. of the scalar field strength introduces a central charge in the algebra of \( N = 2 \) gauge covariantized spinor derivatives. Therefore, to restrict our analysis to the massive hypermultiplet we consider only the v.e.v. of the gauge multiplet

\[ V = \bar{m}(\theta_2 - \theta_1 \zeta)^2 - m(\bar{\theta}^1 + \bar{\theta}^2 \zeta)^2 \]
\[ = \left( \theta_2^2 \bar{m} - (\bar{\theta}^1)^2 m \right) \frac{1}{\zeta} - \left( \theta_2 \theta_1 m + \bar{\theta}^2 \bar{\theta}^1 \right) + \left( \theta_1^2 \bar{m} - (\bar{\theta}^1)^2 m \right) \zeta , \] (31)

and to distinguish fields that obey the constraints \( \bar{\Upsilon} \) with central charges in the algebra of spinorial derivatives from those that obey the same constraints without central charges, we refer to the former as covariantly projective and to the latter as ordinary projective.

Now that we have recovered the algebra (11) of \( N = 2 \) spinorial derivatives with central charges acting on a hypermultiplet which has \( q(\bar{\Upsilon}) = 1, q(\Upsilon) = -1 \), we want to perform the

\footnote{An interesting generalization of this constraint was proposed long time ago for \( N = 1 \) theories with complex linear multiplets in \( \bar{\Upsilon} \). In that reference it was pointed out that the mass terms of such fields mix them with chiral multiplets.}
duality transformation that in the massless case exchanges the \(N = 1\) complex linear field by a chiral one \([2]\). The action \((29)\) in components is

\[
S = \int d^4x d^4\theta \left( \tilde{\Upsilon}_0 \tilde{\Upsilon}_0 - \tilde{\Upsilon}_1 \tilde{\Upsilon}_1 + \sum_{n=2}^{+\infty}(-)^n \tilde{\Upsilon}_n \tilde{\Upsilon}_n \right). \tag{32}
\]

The path integral of the theory may be rewritten using a parent action that includes Lagrange multipliers imposing the modified \(N = 1\) linearity constraint on \(\tilde{\Upsilon}_1\) \((18)\) and the corresponding constraint on its conjugate

\[
S = \int dx d^4\theta \left[ \tilde{\Upsilon}_0 \tilde{\Upsilon}_0 - \tilde{\Upsilon}_1 \tilde{\Upsilon}_1 + \tilde{\Upsilon}_2 \tilde{\Upsilon}_2 + \ldots + Y(D^2\tilde{\Upsilon}_1 - m\tilde{\Upsilon}_0) + \bar{Y}(D^2\tilde{\Upsilon}_1 - \bar{m}\tilde{\Upsilon}_0) \right]. \tag{33}
\]

We can integrate out the unconstrained field \(\tilde{\Upsilon}_1\) to get the kinetic term of a chiral field \(\tilde{\phi} = \bar{D}^2Y\), a chiral mass term mixing \(\tilde{\Upsilon}_0\) and \(\tilde{\phi}\) (recall that \(\tilde{\Upsilon}_0\) is chiral) and its complex conjugate, plus auxiliary field terms that decouple

\[
S_{dual} = \int dx d^4\theta \left( \tilde{\Upsilon}_0 \tilde{\Upsilon}_0 + \bar{\tilde{\phi}}\tilde{\phi} + \ldots \right) - \int dx d^2\theta \tilde{\phi} m \tilde{\Upsilon}_0 - \int dx d^2\bar{\theta} \bar{\tilde{\phi}} \bar{m}\tilde{\Upsilon}_0. \tag{34}
\]

This provides the traditional description of massive hypermultiplets in terms of two \(N = 1\) chiral scalars.

We can also introduce mass terms for a complex \(O(2p)\) hypermultiplet\[^3\]. Its phase rotates under the gauge transformations \((27-28)\) with the same \(U(1)\) charge assignment as the (ant)arctic multiplet. To illustrate this we give the following explicit construction of the covariantly projective \(O(2p)\) multiplet.

As was shown in \([6]\), it is possible to make a partial \(N = 2\) supersymmetric Lindström-Roček gauge choice for the vector multiplet. The basic idea is to truncate the \(\zeta\) expansion of \(V\) by gauging away all but a finite number of its components while preserving \(N = 2\) supersymmetry. In this gauge we set \(v^L_R = 0 \forall i \neq -1,0,1\) and we gauge away all of the prepotential \(v_1\) except the pieces \(D^2v_1\) and \(\bar{Q}^2v_1\), leaving us with a \(v^L_R\) which is quadratic in Grassmann coordinates \([6]\). In this gauge it is straightforward to see that a covariantly projective hypermultiplet constructed from an ordinary projective complex \(O(2p - 2)\) field \(\rho\) is a complex \(O(2p)\) multiplet

\[
\tilde{\rho} = e^{V^L_R} \rho = e^{v^L_R (1 + v^L_R \zeta + (v^L_R)^2 \zeta^2)} \rho. \tag{35}
\]

A generic gauge transformation \((22)\) maps the gauge multiplet \(V^L_R\) into a real tropical multiplet with an infinite number of nonzero coefficients and the finite multiplet \(\rho\) transforms into an arctic multiplet\[^4\]. At the same time the \(O(2p)\) multiplet \(\tilde{\rho}\) remains a \(O(2p)\) multiplet after the gauge transformation, as follows from equation \((27)\).

\[^3\]As noted in \([3]\) the complex \(O(k)\) multiplets include chiral ghosts for odd \(k\), and although their \(N = 2\) propagator is consistent with the general form derived below, we will restrict ourselves to the case of physical fields.

\[^4\]This makes perfect sense if we regard the \(O(2p - 2)\) multiplet as a special (ant)arctic multiplet in a particular gauge, \(i.e.\) we are working with the space of \(O(\infty)\) polynomials which contains the subspace of \(O(2p - 2)\) polynomials.
Since this multiplet is a special case of the (ant)arctic multiplet, we use the same notation \( \tilde{\Upsilon} \) and \( U(1) \) charge assignment for both. The covariantly projective complex \( O(2p) \) multiplet therefore obeys the constraints (18) with \( q = 1 \) and its conjugate with \( q = -1 \). The action of the free complex \( O(2p) \) multiplet written with Lagrange multipliers is

\[
S = \int dx \, d^4 \theta \left[ \tilde{\Upsilon}_0 \tilde{\Upsilon}_0 - \tilde{\Upsilon}_1 \tilde{\Upsilon}_1 + \ldots - \tilde{\Upsilon}_{2p-1} \tilde{\Upsilon}_{2p-1} + \tilde{\Upsilon}_{2p} \tilde{\Upsilon}_{2p} + Y(D^2 \tilde{\Upsilon}_1 - m \tilde{\Upsilon}_0) + \bar{Y}(D^2 \tilde{\Upsilon}_{2p-1} - \bar{m} \tilde{\Upsilon}_{2p}) \right] .
\] (36)

Integrating the unconstrained fields as before we obtain the free action of two copies of massive chiral hypermultiplets plus auxiliary fields

\[
S_{\text{dual}} = \int dx \, d^4 \theta \left( \tilde{\Upsilon}_0 \tilde{\Upsilon}_0 + \bar{D}^2 \bar{Y} \bar{D}^2 \bar{Y} + \ldots + \bar{D}^2 X \bar{D}^2 \bar{X} \right)
- \int dx \, d^2 \theta \, m(\bar{D}^2 \bar{Y} \tilde{\Upsilon}_0 + \bar{D}^2 X \tilde{\Upsilon}_{2p}) - \int dx \, d^2 \theta \, \bar{m}(\bar{D}^2 \bar{Y} \bar{\tilde{\Upsilon}}_0 + \bar{D}^2 \bar{X} \bar{\tilde{\Upsilon}}_{2p}) .
\] (37)

### 3.2 Massive \( N = 1 \) propagators of chiral, linear and auxiliary fields

To derive the propagators of the \( N = 1 \) superfields contained in the massive (ant)arctic hypermultiplet, we introduce the following (ant)arctic source living in projective superspace

\[
S_0 + S_j = \int dx \, d^2 \theta \, \int \frac{d\zeta}{2\pi i \zeta} \tilde{\Upsilon} \bar{\Upsilon} + \frac{\bar{\zeta}}{\zeta} \bar{\Upsilon} + \zeta^2 \bar{\Upsilon} j
= \int dx \, d^2 \theta \, (\tilde{\Upsilon}_0 \tilde{\Upsilon}_0 - \tilde{\Upsilon}_1 \tilde{\Upsilon}_1 + \tilde{\Upsilon}_2 \tilde{\Upsilon}_2 + \ldots + \tilde{\Upsilon}_{2j} \tilde{\Upsilon}_0 + \bar{\tilde{\Upsilon}}_{0j2})
+ \bar{\tilde{\Upsilon}}_1 + \bar{\tilde{\Upsilon}}_{1j3} + \bar{\tilde{\Upsilon}}_{2j4} + \ldots ) .
\] (38)

After introducing Lagrange multipliers that multiply the linearity constraints modified by central charges, we integrate out the unconstrained field \( \tilde{\Upsilon}_1 \) as we did before. We obtain the free action of two massive chiral fields and auxiliary fields coupled to \( N = 1 \) unconstrained sources plus a term quadratic in sources

\[
S_0 + S_j = - \int dx \, d^2 \theta \, m \bar{\phi} \tilde{\Upsilon}_0 - \int dx \, d^2 \theta \, \bar{m} \tilde{\Upsilon}_0
+ \int dx \, d^4 \theta \, (\tilde{\Upsilon}_0 \bar{\tilde{\Upsilon}}_0 + \bar{\phi} \bar{\tilde{\Upsilon}}_0 + \tilde{\Upsilon}_2 \bar{\tilde{\Upsilon}}_2 + \ldots + \tilde{\Upsilon}_{2j} \tilde{\Upsilon}_0 + \bar{\tilde{\Upsilon}}_{0j2}
+ \bar{\phi} j_3 + \bar{\phi} j_3 + \bar{\phi} j_4 \bar{\tilde{\Upsilon}}_4 + \tilde{\Upsilon}_2 j_4 + \ldots + \bar{\phi} j_3 j_3) .
\] (39)

The path integral of this free theory can be obtained using standard \( N = 1 \) superspace technology to integrate out the chiral fields: we rewrite the chiral mass terms as nonchiral integrals and we insert chiral and antichiral projectors in the corresponding sources.
Inverting the mass-kinetic matrix we can complete squares on the chiral and antichiral superfields. The only nonvanishing propagators connecting the essential contribution to the linear-antilinear propagator. The only nonvanishing propagators connecting the $N = 1$ superfields of the massive (ant)arctic hypermultiplet are the following (we omit their complex conjugates)

\[
\langle \tilde{Y}_0(1) \tilde{Y}_0(2) \rangle = - \frac{\mathcal{D}^2 \mathcal{D}^2}{\Box - m\tilde{m}} \delta^4(\theta_{12}) \delta^4(x_{12}),
\]

\[
\langle \tilde{Y}_1(1) \tilde{Y}_1(2) \rangle = \left(1 - \frac{\mathcal{D}^2 \mathcal{D}^2}{\Box - m\tilde{m}} \right) \delta^4(\theta_{12}) \delta^4(x_{12}),
\]

\[
\langle \tilde{Y}_{n>1}(1) \tilde{Y}_{n>1}(2) \rangle = (-)^{n+1} \delta^4(\theta_{12}) \delta^4(x_{12}),
\]

\[
\langle \tilde{Y}_1(1) \tilde{Y}_0(2) \rangle = - \frac{m\tilde{D}^2}{\Box - m\tilde{m}} \delta^4(\theta_{12}) \delta^4(x_{12}),
\]

\[
\langle \tilde{Y}_0(1) \tilde{Y}_1(2) \rangle = - \frac{m\mathcal{D}^2}{\Box - m\tilde{m}} \delta^4(\theta_{12}) \delta^4(x_{12}).
\]

In the case of the complex $O(2p)$ hypermultiplet we have in addition propagators for the second copy of physical $N = 1$ superfields.
\[ \langle \tilde{\Upsilon}_{2p-1}(1) \tilde{\Upsilon}_{2p-1}(2) \rangle = \left( 1 - \frac{D^2\bar{D}^2}{\Box - m\bar{m}} \right) \delta^4(\theta_{12})\delta^4(x_{12}), \] (47)

\[ \langle \tilde{\Upsilon}_{2p}(1) \tilde{\Upsilon}_{2p-1}(2) \rangle = -\frac{m\bar{D}^2}{\Box - m\bar{m}} \delta^4(\theta_{12})\delta^4(x_{12}), \] (48)

\[ \langle \tilde{\Upsilon}_{2p-1}(1) \tilde{\Upsilon}_{2p}(2) \rangle = -\frac{mD^2}{\Box - m\bar{m}} \delta^4(\theta_{12})\delta^4(x_{12}). \] (49)

\[ \langle \tilde{\Upsilon}_{2p}(1) \tilde{\Upsilon}_{2p}(2) \rangle = -\frac{D^2\bar{D}^2}{\Box - m\bar{m}} \delta^4(\theta_{12})\delta^4(x_{12}), \] (50)

4 Massive hypermultiplet in \( N = 2 \) superspace

4.1 Ansatz for the \( N = 2 \) massive propagator

Now that we have the massive propagators of the \( N = 1 \) component fields, we may try to guess the form of the \( N = 2 \) massive propagator for the complex \( O(2p) \) multiplet. In \( N = 1 \) components it must be of the following form

\[ \langle \tilde{\Upsilon}(1) \tilde{\Upsilon}(2) \rangle \bigg|_{\theta^2_{a}=0} = \langle \tilde{\Upsilon}_{0}(1) \tilde{\Upsilon}_{0}(2) \rangle + \zeta_{1} \langle \tilde{\Upsilon}_{1}(1) \tilde{\Upsilon}_{1}(2) \rangle - \frac{1}{\zeta_{2}} \langle \tilde{\Upsilon}_{0}(1) \tilde{\Upsilon}_{1}(2) \rangle \] (51)

\[ + \left( \frac{\zeta_{1}}{\zeta_{2}} \right)^{2p-1} \langle \tilde{\Upsilon}_{2p-1}(1) \tilde{\Upsilon}_{2p-1}(2) \rangle + \frac{1}{\zeta_{2}} \left( \frac{\zeta_{1}}{\zeta_{2}} \right)^{2p} \langle \tilde{\Upsilon}_{2p-1}(1) \tilde{\Upsilon}_{2p}(2) \rangle \]

\[- \zeta_{1} \left( \frac{\zeta_{1}}{\zeta_{2}} \right)^{2p-1} \langle \tilde{\Upsilon}_{2p}(1) \tilde{\Upsilon}_{2p-1}(2) \rangle + \left( \frac{\zeta_{1}}{\zeta_{2}} \right)^{2p} \langle \tilde{\Upsilon}_{2p}(1) \tilde{\Upsilon}_{2p}(2) \rangle \]

Just as we did in the massless case, we substitute the expressions (42 - 50) and the hypermultiplet propagator is

\[ \langle \tilde{\Upsilon}(1) \tilde{\Upsilon}(2) \rangle \bigg|_{\theta^2_{a}=0} = \left( \left( \frac{\zeta_{1}}{\zeta_{2}} \right)^{2p-1} - 1 \right) \times \] (52)

\[ \left( \frac{D^2\bar{D}^2}{\Box - m\bar{m}} + \zeta_{1} \frac{\bar{m}D^2}{\Box - m\bar{m}} - \frac{mD^2}{\Box - m\bar{m}} \right) \left( \frac{\bar{D}^2D^2}{\Box - m\bar{m}} + \frac{\zeta_{1}}{\zeta_{2}} \right) \delta^4(\theta_{12})\delta^4(x_{12}) . \]

We also rewrite the identity operator in the auxiliary field propagators

\[ \frac{\bar{D}^2D^2 + D^2\bar{D}^2 - \bar{D}^2D^2 - m\bar{m}}{\Box - m\bar{m}} = 1 \] (53)

and we find
covariantized spinor derivatives (to be justified later when we derive the massive propagator in $N$)

This suggests that the correct massive $N$ triplet is the naive generalization of the massless propagator

Similarly the (ant)arctic hypermultiplet has the following reduced form in $N = 1$ superspace

When $|\zeta_1/\zeta_2| < 1$ this result is consistent with the limit $2p \to +\infty$ of (54). Not surprisingly we find the same convergence problems as in the massless case [2].

We also expect the $N = 2$ propagator to be proportional to $\nabla^4_1 \nabla^4_0$, but now we have to be careful with the central charges of the algebra. We assign a global $U(1)$ charge $q = 1$ to the covariantized spinor derivatives (to be justified later when we derive the massive propagator in $N = 2$ superspace)

Let us consider the $N = 1$ projection of the following expression

This suggests that the correct massive $N = 2$ propagator for the complex $O(2p)$ hypermultiplet is the naive generalization of the massless propagator
\[ \langle \tilde{\Upsilon}(1) \tilde{\Upsilon}(2) \rangle = -\frac{\zeta_1^{2p-1} + \zeta_2^{2p-1}}{\zeta_2^2 (\zeta_1 - \zeta_2)^3} \frac{\nabla_1^4 \nabla_2^4}{m^4} \delta^8(\theta_{12}) \delta^4(x_{12}), \]  \hspace{1cm} (58)

whereas for the (ant)arctic hypermultiplet it is

\[ \langle \tilde{\Upsilon}(1) \tilde{\Upsilon}(2) \rangle = -\frac{1}{\zeta_2^2} \sum_{n=0}^{+\infty} \left( \frac{\zeta_1}{\zeta_2} \right)^n \frac{\nabla_1^4 \nabla_2^4}{(\zeta_1 - \zeta_2)^2 (\Box - m \bar{m})} \delta^8(\theta_{12}) \delta^4(x_{12}). \]  \hspace{1cm} (59)

Note that if we take the convergent limit of the infinite sum also the last two terms in (55) are reproduced by the \( N = 1 \) projection of this expression. As in the massless case, we have to conjecture that this limit can be analytically continued to the region of no convergence.

The conjugate propagators are obtained from the projection of

\[ \langle \tilde{\Upsilon}(1) \tilde{\Upsilon}(2) \rangle = -\frac{\zeta_1^{2p-1} + \zeta_2^{2p-1}}{\zeta_1^2 (\zeta_2 - \zeta_1)^3} \frac{\nabla_1^4 \nabla_2^4}{m^4} \delta^8(\theta_{12}) \delta^4(x_{12}), \]  \hspace{1cm} (60)

and

\[ \langle \tilde{\Upsilon}(1) \tilde{\Upsilon}(2) \rangle = -\frac{1}{\zeta_1^2} \sum_{n=0}^{+\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n \frac{\tilde{\nabla}_1^4 \tilde{\nabla}_2^4}{(\zeta_1 - \zeta_2)^2 (\Box - m \bar{m})} \delta^8(\theta_{12}) \delta^4(x_{12}). \]  \hspace{1cm} (61)

where, to get the correct \( N = 1 \) projection, we must use supercovariant derivatives obeying the algebra \( \{ \} \) with \( q = -1 \).

### 4.2 Calculation of the massive hypermultiplet propagator in \( N = 2 \) superspace

To derive the massive hypermultiplet propagator in \( N = 2 \) superspace we must couple this multiplet to (ant)arctic sources. We consider the charged ordinary hypermultiplet interacting with an Abelian real tropical multiplet which will develop a v.e.v. The \( N = 2 \) and gauge invariant free action with sources that we want is the following

\[ S_0 + S_J = \int dx \, d^4\theta \, \oint \frac{d\zeta}{2\pi i \zeta} \tilde{\Upsilon} \tilde{e}^V \tilde{\Upsilon} + \int dx \, d^8\theta \, \oint \frac{d\zeta}{2\pi i \zeta} \left( \tilde{J} \tilde{e}^V \tilde{\Upsilon} + \tilde{\Upsilon} \tilde{e}^V \tilde{J} \right). \]  \hspace{1cm} (62)

The source \( J \) is an unconstrained arctic field transforming as

\[ J' = e^{i\Lambda} J \]  \hspace{1cm} (63)

with charge \( q = 1 \) under the \( U(1) \) symmetry \( \{ 22 \} \). The field \( \tilde{e}^V J \) can be regarded as a complex tropical source transforming with the same charge but with the antarctic gauge parameter \( \bar{\Lambda} \). Writing the action with \textit{redefined} (ant)arctic sources \( J \rightarrow e^{\tilde{V}} J \) and the covariantly projective (ant)arctic hypermultiplets \( \{ 26 \} \), we have

\[ S_0 + S_J = \int dx \, d^4\theta \, \oint \frac{d\zeta}{2\pi i \zeta} \tilde{\Upsilon} \tilde{e}^{V} \tilde{\Upsilon} + \int dx \, d^8\theta \, \oint \frac{d\zeta}{2\pi i \zeta} \left( \tilde{J} \tilde{e}^{V} \tilde{\Upsilon} + \tilde{\Upsilon} \tilde{e}^{V} \tilde{J} \right). \]  \hspace{1cm} (64)

If we use the covariantly projective complex \( O(2p) \) hypermultiplet \( \{ 35 \} \) we obtain an action of exactly the same form. The difference is that the \( \tilde{\Upsilon} \) fields are of finite order in \( \zeta \) and
thus the contour integration forces all but a finite number of component sources to decouple. Reducing the corresponding two actions to $N = 1$ components we recover (38) and its equivalent for the $O(2p)$ multiplet.

We want to write the kinetic action and source terms as integrals with the full $N = 2$ superspace measure, and integrate out the hypermultiplet in the free theory path integral. We analyze the complex $O(2p)$ hypermultiplet since the (ant)arctic one is simply reproduced by taking the limit $2p \to +\infty$. As in the massless case [2] we use an unconstrained $O(2p - 4)$ prepotential

$$\tilde{\Upsilon} = e^{V_+} \nabla^4 e^{-V_+} \left( e^{V_+} \psi \right) = \nabla^4 \sum_{n=0}^{2p-4} \tilde{\psi}_n \zeta^n, \quad \tilde{\Upsilon} = e^{V_-} \nabla^4 e^{-V_-} \left( e^{V_-} \tilde{\psi} \right) = \frac{\nabla^4}{\zeta^4} \sum_{m=0}^{2p-4} \frac{\tilde{\psi}_m}{(-\zeta)^m}. \quad (65)$$

The action with sources written in terms of $\tilde{\psi}$ is

$$S_0 + S_J = \int dx \ d\theta \oint d\zeta \frac{d\zeta}{2\pi i} \left( \tilde{\Upsilon} e^{V_+} \nabla^4 e^{-V_+} \tilde{\Upsilon} \phi + J e^{V_+} \nabla^4 e^{-V_+} \tilde{\psi} + \tilde{\Upsilon} e^{V_-} \nabla^4 e^{-V_-} \tilde{\psi} \phi \right). \quad (66)$$

To integrate out the prepotential we use the techniques introduced in the massless case: we insert a projector in the source term so that we can factor out the kinetic operator and complete squares by shifting the prepotential with an $O(2p - 4)$ polynomial. From the algebra (5-7) we learn that the projector operator for $q = 1$ fields living in projective superspace is given by

$$\frac{\nabla^4 \tilde{\Upsilon}^4}{16(\Box - m\bar{m})^2} \tilde{\Upsilon}^4 = \tilde{\Upsilon}^4. \quad (67)$$

We are also free to add to $\tilde{\Upsilon}^4$ any operator annihilated by the $\tilde{\Upsilon}^4$ to the left and to the right. It is particularly convenient to choose the following combinations

$$K = \frac{\left( \Delta^2 - \frac{1}{\zeta} \tilde{\Delta} \tilde{\Delta} + \frac{2}{\zeta} \bar{m} + \frac{1}{\zeta^2} \tilde{\nabla}^2 \right) \left( \tilde{\Delta}^2 - \frac{1}{\zeta} \tilde{\Delta} \tilde{\Delta} - \frac{2}{\zeta} \bar{m} + \frac{1}{\zeta^2} \tilde{\nabla}^2 \right)}{16(\Box - m\bar{m})^2} = \frac{Q^2 \bar{D}^2}{(\Box - m\bar{m})^2},$$

$$L(\zeta) = \frac{\left( \Delta^2 + \frac{1}{\zeta} \tilde{\Delta} \tilde{\Delta} - \frac{2}{\zeta} \bar{m} + \frac{1}{\zeta^2} \tilde{\nabla}^2 \right) \left( \tilde{\Delta}^2 + \frac{1}{\zeta} \tilde{\Delta} \tilde{\Delta} + \frac{2}{\zeta} \bar{m} + \frac{1}{\zeta^2} \tilde{\nabla}^2 \right)}{16(\Box - m\bar{m})^2} = \frac{D^2 \bar{Q}^2}{\zeta^4(\Box - m\bar{m})^2}, \quad (68)$$

which satisfy the following useful relation

$$\tilde{\Upsilon}^4(\zeta) L(\zeta') \tilde{\Upsilon}^4(\zeta') = \left( \frac{\zeta}{\zeta'} \right)^4 \tilde{\Upsilon}^4(\zeta) K \tilde{\Upsilon}^4(\zeta'). \quad (69)$$

Using this operators, the last source term can be rewritten

\footnote{Note that after doing the integration by parts in the second source term all covariantized projective derivatives have the same $U(1)$ charge $q = 1.$}
\[ \oint \frac{d\zeta}{2\pi i\zeta} \bar{\psi} \nabla^4 \zeta^2 K \nabla^4 J. \] (70)

The differential operator \( \bar{\psi} \nabla^4 / \zeta^2 \) is a \( O(-2p + 2, 2) \) polynomial in \( \zeta \). However the operator \( K \) annihilates the coefficients of \( \zeta \) and \( \zeta^2 \) in \( \nabla^4 / \zeta^2 \), and therefore only the \( O(-2p + 2, 0) \) piece contributes to the source coupling. This means that we can project the expression multiplying \( \bar{\psi} \nabla^4 / \zeta^2 \) onto the subspace of \( O(0, 2p - 2) \) polynomials. Using the delta functions on the Riemann sphere that we defined in \([2]\), we rewrite the source term

\[ \oint \frac{d\zeta}{2\pi i\zeta} \bar{\psi} \nabla^4 \zeta^2 J = \oint \frac{d\zeta'}{2\pi i\zeta'} \bar{\psi} \nabla^4 \zeta' J(\zeta') = \oint \frac{d\zeta}{2\pi i\zeta} \bar{\psi} \nabla^4 J(\zeta). \] (71)

where we have defined the source

\[ \mathcal{J}(\zeta) = \oint \frac{d\zeta'}{2\pi i\zeta'} \left( \delta_{(2p-4)}(\zeta, \zeta') K + \delta_{(2p-5)}(\zeta, \zeta') L(\zeta') \right) \frac{\nabla^4(\zeta')}{\zeta'^2} J(\zeta'). \] (72)

which is a \( O(0, 2p - 4) \) superfield, and due to the identity \([39]\) satisfies

\[ \nabla^4 \mathcal{J} = \oint \frac{d\zeta'}{2\pi i\zeta'} \delta_{(2p-4)}(\zeta, \zeta') K \frac{\nabla^4(\zeta')}{\zeta'^2} J(\zeta'). \] (73)

Similarly we may rewrite the conjugate source term as

\[ \oint \frac{d\zeta}{2\pi i\zeta} \bar{\psi} \nabla^4 \bar{\mathcal{J}} = \oint \frac{d\zeta'}{2\pi i\zeta'} \bar{\psi} \nabla^4 \bar{\mathcal{J}}(\zeta'), \] (74)

where we have used the fact that the operator \( L \) annihilates the coefficients of the \( \zeta^{-2} \) and \( \zeta^{-1} \) terms in the operator \( \nabla^4 / \zeta^2 \), and where we have defined

\[ \bar{\mathcal{J}} = \oint \frac{d\zeta'}{2\pi i\zeta'} \left( \delta_{(1-2p)}(\zeta, \zeta') L(\zeta') + \delta_{(1-2p)}(\zeta, \zeta') K \right) \zeta'^2 \nabla^4(\zeta') \bar{\mathcal{J}}(\zeta'). \] (75)

satisfying

\[ \nabla^4 \bar{\mathcal{J}} = \oint \frac{d\zeta'}{2\pi i\zeta'} \delta_{(2-2p)}(\zeta, \zeta') L(\zeta') \zeta'^2 \nabla^4(\zeta') \bar{\mathcal{J}}(\zeta'). \] (76)

The new sources \( \mathcal{J}, \bar{\mathcal{J}} \) have the correct order in \( \zeta \), so we may complete squares in the action

\[ S_0 + S_J = \int dx \ d^8 \theta \oint \frac{d\zeta}{2\pi i\zeta} \left( \bar{\psi} + \mathcal{J} \right) \nabla^4 \left( \bar{\psi} + \mathcal{J} \right) - \mathcal{J} \nabla^4 \mathcal{J}. \] (77)
\[
\ln Z[0, J] = -\int d^8 \theta \left\{ \frac{d \zeta}{2 \pi i} \tilde{J} \tilde{\nabla}_1^4 \tilde{\nabla}_2^4 J \right\}
\]

(78)

\[
= -\int d^8 \theta \left\{ \frac{d \zeta}{2 \pi i} \tilde{J} \tilde{\nabla}_1^4 \tilde{\nabla}_2^4 J \right\}
\]

Using the anticommutator in (81)

To obtain the form of the propagator proposed above, we must use the following identities

\[
\delta_{(0)}^{(2p-2)}(\zeta, \zeta') \tilde{\nabla}_1^4(\zeta) \tilde{\nabla}_2^4(\zeta') \frac{\tilde{\nabla}_1^4(\zeta')}{\zeta'^2} J(\zeta') .
\]

The \(N = 2\) propagator is obtained by functional differentiation with respect to the unconstrained sources that couple to the hypermultiplet

\[
\langle \tilde{\Psi}(1) \tilde{\Psi}(2) \rangle = \frac{\delta}{\delta \tilde{J}(x_1, \theta_1, \zeta_1)} \frac{\delta}{\delta \tilde{J}(x_2, \theta_2, \zeta_2)} \ln Z_0
\]

(79)

Since the operator \(K\) annihilates the two highest order coefficients in \(\tilde{\nabla}_1^4(\zeta)\) and \(\tilde{\nabla}_2^4(\zeta')\), the product

\[
\delta_{(0)}^{(2p-2)}(\zeta, \zeta') \tilde{\nabla}_1^4(\zeta) \frac{\tilde{\nabla}_1^4(\zeta')}{\zeta'^2}
\]

is \(O(0, 2p)\) in \(\zeta\) and \(O(-2p, 0)\) in \(\zeta'\). Hence, the integration of the complex coordinates gives the following propagator

\[
\langle \tilde{\Psi}(1) \tilde{\Psi}(2) \rangle = -\delta_{(0)}^{(2p-2)}(\zeta_1, \zeta_2) \frac{\tilde{\nabla}_1^4(\zeta) \tilde{\nabla}_2^4(\zeta)}{\zeta_2^2(\Box - m\tilde{m})^2} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2).
\]

To obtain the form of the propagator proposed above, we must use the following identities

\[
\tilde{\nabla}_1^2 Q^2 = \tilde{D}^2 Q^2 + \zeta_1 \tilde{m} Q^2 = \frac{\tilde{\nabla}_1^2 + \tilde{m} \tilde{\nabla}_1^2}{\zeta_2 - \zeta_1},
\]

(82)

\[
\tilde{D}^2 \tilde{\nabla}_2^2 = \tilde{D}^2 \tilde{Q}^2 + \zeta_2 m \tilde{D}^2 = \frac{\tilde{\nabla}_2^2 + m \tilde{\nabla}_2^2}{\zeta_2 - \zeta_1}.
\]

(83)

Using the anticommutator \(\{\tilde{\nabla}_1^\alpha, \tilde{\nabla}_2^\beta\} = i(\zeta_1 - \zeta_2) \partial^{\alpha\beta}\) they allow us to rewrite the numerator in (81)

\[
\tilde{\nabla}_1^2 \left( \frac{\tilde{\nabla}_1^2 \tilde{\nabla}_2^2}{(\zeta_2 - \zeta_1)^2} - \frac{\tilde{m} \tilde{\nabla}_1^2}{\zeta_2 - \zeta_1} \right) \left( \frac{\tilde{\nabla}_1^2 \tilde{\nabla}_2^2}{(\zeta_2 - \zeta_1)^2} + \frac{m \tilde{\nabla}_2^2}{\zeta_2 - \zeta_1} \right) \tilde{\nabla}_2^2 = \frac{\Box - m\tilde{m}}{(\zeta_2 - \zeta_1)^2} \tilde{\nabla}_1^4 \tilde{\nabla}_2^4.
\]

(84)
The propagator may now be written as follows

$$\langle \bar{\Upsilon}(1) \Upsilon(2) \rangle = -\delta^{(2p-2)}(\zeta_1, \zeta_2) \frac{\bar{\nabla}^4_1 \nabla^4_2}{\zeta_2^2(\zeta_2 - \zeta_1)^2(\Box - mm)} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2) , \quad (85)$$

and using the explicit form of the Riemann sphere delta function we recover the expression (58) proposed above.

To obtain the conjugate propagator we integrate by parts in (78), which has the effect of reversing the sign in the $U(1)$ charge of the covariantized derivative, and we take the corresponding functional derivatives with respect to unconstrained sources. This explains the previous $U(1)$ charge assignment in the covariant derivatives of the conjugate propagator. Using the same integration by parts we deduce that the $U(1)$ charge changes when we use transfer rules

$$\bar{\nabla}^4_2(q = 1)\delta^8(\theta_1 - \theta_2) = \delta^8(\theta_1 - \theta_2) \bar{\nabla}^4_2(q = -1) . \quad (86)$$

The (ant)arctic multiplet propagators are obtained from the propagators above by letting $p \rightarrow \infty$.

5 Feynman rules for diagram construction

The rules for constructing diagrams are very similar to those used with the massless hypermultiplet [2]. For completeness we briefly summarize them here:

- **Propagators:**
  We put the $\bar{\nabla}^4$ factors of the propagators in the vertices. With this convention the propagator of the complex $O(2p)$ multiplet becomes

$$\langle \bar{\Upsilon}(1) \Upsilon(2) \rangle = -\delta^{(2p-2)}(\zeta_1, \zeta_2) \frac{\bar{\nabla}^4_1 \nabla^4_2}{\zeta_2^2(\zeta_1 - \zeta_2)^2(\Box - mm)} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2) , \quad (87)$$

and its conjugate

$$\langle \bar{\bar{\Upsilon}}(1) \bar{\Upsilon}(2) \rangle = -\delta^{(0)}(\zeta_1, \zeta_2) \frac{\bar{\nabla}^4_1 \nabla^4_2}{\zeta_1^2(\zeta_1 - \zeta_2)^2(\Box - mm)} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2) , \quad (88)$$

The (ant)arctic multiplet propagators are obtained from the propagators above by letting $p \rightarrow \infty$.

- **Vertices:**
  These are read directly from the interaction Lagrangian. It is most convenient to put the spinor derivatives $\bar{\nabla}^4_1 , \nabla^4_2$ of a propagator in the vertices. This means that every internal line of a vertex will have a $\bar{\nabla}^4$ factor except one where the spinor derivative

---

6 This is the natural sign for the derivatives acting on $\bar{J}$ which has the opposite $U(1)$ charge.
has been used to complete the restricted superspace measure of the interaction vertex to a full $N = 2$ superspace measure.

- We may always absorb in the Grassmann measure of a projective vertex a factor of $\tilde{\nabla}^4$ from an internal line, so we will integrate over $d^8\theta$ at each vertex. There is also an integral over the $x$-space position of each vertex, or, equivalently, an integral over each loop momentum plus an overall factor $\propto \delta(\Sigma_{k_{ext}})$.

- In computing any particular diagram, we amputate the external line propagators, which means that there are no $\tilde{\nabla}^4$ factors on external lines.

- Finally, there may be symmetry factors associated with certain graphs and they are calculated in the usual fashion. The “$D$”-algebra is performed and the Grassmann coordinate dependence of the propagators is reduced until we obtain a local expression in Grassmann space.

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