Proposed observations of gravity waves from the early Universe via “Millikan oil drops”

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Pairs of Planck-mass drops of superfluid helium coated by electrons (i.e., “Millikan oil drops”), when levitated in a superconducting magnetic trap, can be efficient quantum transducers between electromagnetic (EM) and gravitational (GR) radiation. This leads to the possibility of a Hertz-like experiment, in which EM waves are converted at the source into GR waves, and then back-converted at the receiver from GR waves back into EM waves. Detection of the gravity-wave analog of the cosmic microwave background using these drops can discriminate between various theories of the early Universe.

Keywords: Gravitational radiation; quantum mechanics; cosmic microwave background.

1. Forces of gravity and electricity between two electrons

Consider the forces exerted by an electron upon another electron at a distance $r$ away in the vacuum. Both the gravitational and the electrical force obey long-range, inverse-square laws. Newton’s law of gravitation states that

$$|F_G| = \frac{Gm_e^2}{r^2}$$  \hspace{1cm} (1)

where $G$ is Newton’s constant and $m_e$ is the mass of the electron. Coulomb’s law of electricity states that

$$|F_e| = \frac{e^2}{r^2}$$  \hspace{1cm} (2)

where $e$ is the charge of the electron. The electrical force is repulsive, and the gravitational one attractive.

Taking the ratio of these two forces, one obtains the dimensionless constant

$$\left| \frac{F_G}{F_e} \right| = \frac{Gm_e^2}{e^2} \approx 2.4 \times 10^{-43}.$$  \hspace{1cm} (3)
The gravitational force is extremely small compared to the electrical force, and is therefore usually omitted in all treatments of quantum physics. Note, however, that this ratio is not strictly zero, and therefore can in principle be amplified.

2. Gravitational and electromagnetic radiation powers emitted by two electrons

The above ratio of the coupling constants $\frac{G m^2}{\epsilon^2}$ also is the ratio of the powers of gravitational (GR) to electromagnetic (EM) radiation emitted by two electrons separated by a distance $r$ in the vacuum, when they undergo an acceleration $a$ relative to each other. Larmor’s formula for the power emitted by a single electron undergoing acceleration $a$ is

$$P_{EM} = \frac{2}{3} \frac{e^2}{c^3} a^2 .$$

(4)

For the case of two electrons undergoing an acceleration $a$ relative to each other, the radiation is quadrupolar in nature, and the modified Larmor formula is

$$P'_{EM} = \frac{\kappa}{3} \frac{e^2}{c^3} a^2 ,$$

(5)

where the prefactor $\kappa$ accounts for the quadrupolar nature of the emitted radiation. Since the electron carries mass, as well as charge, and its charge and mass co-move rigidly together, two electrons undergoing an acceleration $a$ relative to each other will also emit homologous quadrupolar gravitational radiation according to the formula

$$P'_{GR} = \frac{\kappa}{3} \frac{2 G m^2}{e^2} a^2 .$$

(6)

with the same prefactor of $\kappa$.

The Equivalence Principle demands that the lowest order of gravitational radiation be quadrupolar, and not dipolar, in nature. Hence the ratio of gravitational to electromagnetic radiation powers emitted by the two-electron system is given by the same ratio of coupling constants, viz.,

$$\frac{P'_{GR}}{P_{EM}} = \frac{G m^2}{\epsilon^2} \approx 2.4 \times 10^{-43} .$$

(7)

Thus it would seem at first sight to be hopeless to try and use any two-electron system as the means for coupling between electromagnetic and gravitational radiation.

3. The Planck mass scale

However, the ratio of the forces of gravity and electricity of two “Millikan oil drops” (to be described in more detail below; however, see Fig. 1) needs not be so hopelessly small.\(^1\)

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\(^1\)Here $\kappa = \frac{1}{3} \frac{e^2}{c^2}$, where $v$ is their relative speed and $c$ is the speed of light, if $v \ll c$. 
Suppose that each “Millikan oil drop” contains a Planck-mass amount of superfluid helium, viz.,

$$m_{\text{Planck}} = \sqrt{\frac{\hbar c}{G}} \approx 22 \text{ micrograms} \quad (8)$$

where $\hbar$ is Planck’s constant$/2\pi$, $c$ is the speed of light, and $G$ is Newton’s constant. Planck’s mass sets the characteristic scale at which quantum mechanics ($\hbar$) impacts relativistic gravity ($c, G$). Note that this mass scale is mesoscopic, and not astronomical, in size. This suggests that it may be possible to perform some novel nonastronomical, table-top-scale experiments at the interface of quantum mechanics and general relativity, which are accessible to the laboratory.

The ratio of the forces of gravity and electricity between the two “Millikan oil drops” now becomes

$$\left| \frac{F_G}{F_e} \right| = \frac{G m_{\text{Planck}}^2}{e^2} = \frac{G}{e^2} \frac{(hc/G)}{e^2} = \frac{hc}{e^2} \approx 137 \quad . \quad (9)$$

Now the force of gravity is 137 times stronger than the force of electricity, so that instead of a mutual repulsion between these two charged objects, there is now a mutual attraction between them. The sign change from mutual repulsion to mutual attraction between these two “Millikan oil drops” occurs at a critical mass $m_{\text{crit}}$ given by

$$m_{\text{crit}} = \sqrt{\frac{e^2}{hc}} m_{\text{Planck}} \approx 1.9 \text{ micrograms} \quad (10)$$
whereupon $|F_G| = |F_e|$, and the forces of gravity and electricity balance each other. This is a strong hint that mesoscopic-scale quantum effects can lead to nonnegligible couplings between gravity and electromagnetism.

The critical mass $m_{\text{crit}}$ is also the mass at which there occurs an equal amount of electromagnetic and gravitational radiation power generated upon scattering of radiation from the pair of “Millikan oil drops,” each member of the pair with a mass $m_{\text{crit}}$ and with a single electron attached to it. Now the ratio of quadrupolar gravitational to quadrupolar electromagnetic radiation power is given by

$$\frac{P'_{\text{GR}}}{P'_{\text{EM}}} = \frac{G m_{\text{crit}}^2}{e^2} = 1,$$

where the prefactors of $\kappa$ in Eqs. (5) and (6) cancel out, if the charge of the drop co-moves rigidly together with its mass. This implies that the scattered power from these two charged objects in the gravitational wave channel becomes equal to that in the electromagnetic wave channel. However, it should be emphasized that it has been assumed here that a given drop’s charge and mass move together as a single unit, in accordance with a Mössbauer-like mode (i.e., a zero-phonon mode) of response to radiation fields, which will be discussed below. This is purely quantum effect based on the quantum adiabatic theorem’s prediction that the system will remain adiabatically, and hence rigidly, in its nondegenerate ground state during perturbations arising from externally applied radiation fields.

4. “Millikan oil drops” described in more detail

Let the oil of the classic Millikan oil drops be replaced with superfluid helium ($^4\text{He}$) with a gravitational mass of around the Planck-mass scale, and let these drops be levitated in a superconducting magnetic trap with Tesla-scale magnetic fields.

The helium atom is diamagnetic, and liquid helium drops have successfully been magnetically levitated in an anti-Helmholtz magnetic trapping configuration. Due to its surface tension, the surface of a freely suspended, ultracold superfluid drop is atomically perfect. When an electron approaches a drop, the formation of an image charge inside the dielectric sphere of the drop causes the electron to be attracted by the Coulomb force to its own image. However, the Pauli exclusion principle prevents the electron from entering the drop. As a result, the electron is bound to the surface of the drop in a hydrogenic ground state. Experimentally, the binding energy of the electron to the surface of liquid helium has been measured using millimeter-wave spectroscopy to be 8 Kelvin, which is quite large compared to the milli-Kelvin temperature scales for the proposed experiment. Hence the electron is tightly bound to the surface of the drop.

Such a “Millikan oil drop” is a macroscopically phase-coherent quantum object. In its ground state, which possesses a single, coherent quantum mechanical phase throughout the interior of the superfluid, the drop possesses a zero circulation quantum number (i.e., contains no quantum vortices), with one unit (or an
integer multiple) of the charge quantum number. As a result of the drop being at ultra-low temperatures, all degrees of freedom other than the center-of-mass degrees of freedom are frozen out, so that there results a zero-phonon Mössbauer-like effect, in which the entire mass of the drop moves rigidly as a single unit in response to radiation fields. Also, since it remains adiabatically in the ground state during perturbations due to these radiation fields, the “Millikan oil drop” possesses a quantum rigidity and a quantum dissipationlessness that are the two most important quantum properties for achieving a high conversion efficiency for gravity-wave antennas.

Note that a pair of spatially separated “Millikan oil drops” have the correct quadrupolar symmetry in order to couple to gravitational radiation, as well as to quadrupolar electromagnetic radiation. When they are separated by a distance on the order of a wavelength, they should become an efficient quadrupolar antenna capable of generating, as well as detecting, gravitational radiation.

5. A pair of “Millikan oil drops” as a transducer

Now imagine placing a pair of levitated “Millikan oil drops” separated by approximately a microwave wavelength inside a black box, which represents a quantum transducer that can convert gravitational (GR) waves into electromagnetic (EM) waves. This kind of transducer action is similar to that of the tidal force of a gravity wave passing over a pair of charged, freely falling objects orbiting the Earth, which can convert a GR wave into an EM wave. Such transducers are linear, reciprocal devices.

By time-reversal symmetry, the reciprocal process, in which another pair of “Millikan oil drops,” converts an EM wave back into a GR wave, must occur with the same efficiency as the forward process, in which a GR wave is converted into an EM wave by a first pair of “Millikan oil drops.” The time-reversed process is important because it allows the generation of gravitational radiation, and can therefore become a practical source of such radiation.

This raises the possibility of performing a Hertz-like experiment, in which the time-reversed quantum transducer process becomes the source, and its reciprocal quantum transducer process becomes the receiver of GR waves in the far field of the source. Room-temperature Faraday cages can prevent the transmission of EM waves, so that only GR waves, which can easily pass through all classical matter such as the normal (i.e., dissipative) metals of which standard, room-temperature Faraday cages are composed, are transmitted between the two halves of the apparatus that serve as the source and the receiver, respectively. Such an experiment would be practical to perform using standard microwave sources and receivers, since the scattering cross-sections and the transducer conversion efficiencies of the two “Millikan oil drops” turn out not to be too small, as will be shown below. The Hertz-like experiment would allow the calibration of the “Millikan-oil-drops” receiver for detecting the gravity-wave analog of cosmic microwave background radiation from the extremely
early Big Bang.

6. Mössbauer-like response of “Millikan oil drops” in a magnetic trap to radiation fields

Let a pair of “Millikan oil drops” be levitated in a superconducting magnetic trap, where the drops are separated by a distance on the order of a microwave wavelength, which is chosen so as to satisfy the impedance-matching condition for a good quadrupolar microwave antenna. See Fig. 1.

Now let a beam of electromagnetic waves in the Hermite-Gaussian TEM$_{11}$ mode, which has a quadrupolar transverse field pattern that has a substantial overlap with that of a gravitational plane wave, impinge at a 45° angle with respect to the line joining these two charged objects. As a result of being thus irradiated, the pair of “Millikan oil drops” will be driven into motion in an anti-phased manner, so that the distance between them will oscillate sinusoidally with time, according to an observer at infinity. Thus the simple harmonic motion of the two drops relative to one another produces a time-varying mass quadrupole moment at the same frequency as that of the driving electromagnetic wave. This oscillatory motion will in turn scatter (in a linear scattering process) the incident electromagnetic wave into gravitational and electromagnetic scattering channels with comparable powers, provided that the ratio of quadrupolar Larmor radiation powers given by Eq. (11) is of the order of unity, which will be the case when the mass of both drops is on the order of the critical mass $m_{\text{crit}}$ for the case of single electrons attached to each drop. The reciprocal process should also have a power ratio of the order of unity.

The Mössbauer-like response of “Millikan oil drops” will now be discussed in more detail. Imagine what would happen if one were replace an electron in the vacuum with a single electron which is firmly attached to the surface of a drop of superfluid helium in the presence of a strong magnetic field and at ultralow temperatures, so that the system of the electron and the superfluid, considered as a single quantum entity, would form a single, macroscopic quantum ground state. Such a quantum system can possess a sizeable gravitational mass. For the case of many electrons attached to a massive drop, where a quantum Hall fluid forms on the surface of the drop in the presence of a strong magnetic field, there results a nondegenerate, Laughlin-like ground state.

In the presence of Tesla-scale magnetic fields, an electron is prevented from moving at right angles to the local magnetic field line around which it is executing tight cyclotron orbits. The result is that the surface of the drop, to which the electron is tightly bound, cannot undergo liquid-drop deformations, such as the oscillations between the prolate and oblate spheroidal configurations of the drop which would occur at low frequencies in the absence of the magnetic field. After the drop has been placed into Tesla-scale magnetic fields at milli-Kelvin operating temperatures, both the single- and many-electron drop systems will be effectively frozen into the ground state, since the characteristic energy scale for electron cyclotron motion
in Tesla-scale fields is on the order of Kelvins. Due to the tight binding of the electron(s) to the surface of the drop, this would freeze out all shape deformations of the superfluid drop.

Since all internal degrees of freedom of the drop, such as its microwave phonon excitations, will also be frozen out at sufficiently low temperatures, the charge and the entire mass of the “Millikan oil drop” should co-move rigidly together as a single unit, in a Mössbauer-like response to applied radiation fields. This is a result of the elimination of all internal degrees of freedom by the Boltzmann factor at sufficiently low temperatures, so that the system stays in its ground state, and only the external degrees of freedom of the drop, consisting only of its center-of-mass motions, remain.

The criterion for this Mössbauer-like mode of response of the electron-drop system is that the temperature of the system is sufficiently low, so that the probability for the entire system to remain in its nondegenerate ground state without even a single quantum of excitation of any of its internal degrees of freedom being excited, is very high, i.e.,

\[
\text{Prob. of zero internal excitation} \approx 1 - \exp\left(\frac{-E_{\text{gap}}}{k_B T}\right) \to 1 \quad \text{as} \quad \frac{k_B T}{E_{\text{gap}}} \to 0, \quad (12)
\]

where \(E_{\text{gap}}\) is the energy gap separating the nondegenerate ground state from the lowest permissible excited states, \(k_B\) is Boltzmann’s constant, and \(T\) is the temperature of the system. Then the quantum adiabatic theorem ensures that the system will stay adiabatically in the nondegenerate ground state of this quantum many-body system during perturbations, such as those due to weak, externally applied radiation fields, whose frequencies are below the gap frequency \(E_{\text{gap}}/\hbar\). By the principle of momentum conservation, since there are no internal excitations to take up the radiative momentum, the center of mass of the entire system must undergo recoil in the emission and absorption of radiation. Thus the mass involved in the response to radiation fields is the mass of the whole system.

For the case of a single electron (or many electrons in the case of the quantum Hall fluid) in a strong magnetic field, the typical energy gap is given by

\[
E_{\text{gap}} = \hbar \omega_{\text{cyclotron}} = \frac{\hbar e B}{mc} \gg k_B T, \quad (13)
\]

an inequality which is valid for the Tesla-scale fields and milli-Kelvin temperatures being considered here.

7. Estimate of the scattering cross-section

Let \(d\sigma_{a\to\beta}\) be the differential cross-section for the scattering of a mode \(a\) of radiation of an incident gravitational wave to a mode \(\beta\) of a scattered electromagnetic wave by a pair of “Millikan oil drops” (Latin subscripts denote GR waves, and Greek subscripts EM waves). Then, by time-reversal symmetry

\[
d\sigma_{a\to\beta} = d\sigma_{\beta\to a}. \quad (14)
\]
Since electromagnetic and weak gravitational fields both formally obey Maxwell’s equations\(^7\) (apart from a difference in the signs of the source density and the source current density), and since these fields obey the same boundary conditions, the solutions for the modes for the two kinds of scattered radiation fields must also have the same mathematical form. Let \(a\) and \(\alpha\) be a pair of corresponding solutions, and \(b\) and \(\beta\) be a different pair of corresponding solutions to Maxwell’s equations for GR and EM modes, respectively. For example, \(a\) and \(\alpha\) could represent incoming plane waves which copropagate in the same direction, and \(b\) and \(\beta\) scattered, outgoing plane waves which copropagate together in a different direction. Then for the case of a pair of critical-mass drops with single-electron attachment, there is an equal conversion into the two types of scattered radiation fields in accordance with Eq. (11), and therefore
\[
d\sigma_{a\to b} = d\sigma_{a\to \beta}, \quad (15)
\]
where \(b\) and \(\beta\) are corresponding modes of the two kinds of scattered radiations.

By the same line of reasoning, for this pair of critical-mass drops
\[
d\sigma_{b\to a} = d\sigma_{\beta\to a} = d\sigma_{\beta\to a}. \quad (16)
\]
It therefore follows from the principle of reciprocity (i.e. time-reversal symmetry) that
\[
d\sigma_{a\to b} = d\sigma_{\alpha\to \beta}. \quad (17)
\]

To estimate the size of the total cross-section, it is easier to consider first the case of electromagnetic scattering, such as the scattering of microwaves from two Planck-mass-scale drops, with radii \(R\) and a separation \(r\) on the order of a microwave wavelength (but with \(r > 2R\)). See Fig. 1. Let the electrons on the “Millikan oil drops” be in a quantum Hall plateau state, which is known to be that of a perfectly dissipationless quantum fluid, like that of a superconductor. Furthermore, it is known that the nondegenerate Laughlin ground state is that of a perfectly rigid, incompressible quantum fluid\(^8\) The two drops thus behave like perfectly conducting, shiny, mirrorlike spheres, which scatter light in a manner similar to that of perfectly elastic hard-sphere scattering in idealized billiards. The total cross section for the scattering of electromagnetic radiation from a pair of drops is therefore given approximately by the geometric cross-sectional areas of two hard spheres
\[
\sigma_{\alpha\to \text{all} \, \beta} = \int d\sigma_{\alpha\to \beta} \simeq \text{Order of } 2\pi R^2 \quad (18)
\]
where \(R\) is the hard-sphere radius of a drop.

However, if, as one might expect on the basis of classical intuitions, the total cross-section for the scattering of gravitational waves from the two-drop system is extremely small, like that of all classical matter such as the Weber bar, then by reciprocity, the total cross-section for the scattering of electromagnetic waves from the two-drop system must also be extremely small. In other words, if “Millikan oil drops” were to be essentially invisible to gravitational radiation, then they must
also be essentially invisible to electromagnetic radiation. This would lead to a contradiction with the hard-sphere cross section given by Eq. (18), or with any other reasonable estimate for the electromagnetic scattering cross-section of these drops, so these classical intuitions must be incorrect.

From the reciprocity principle and from the important properties of quantum rigidity and quantum dissipationlessness of these drops, one therefore concludes that for two critical-mass “Millikan oil drops,” it must be the case that

\[ \sigma_{a \rightarrow b} = \sigma_{\alpha \rightarrow \beta} \simeq \text{Order of } 2\pi R^2. \]

(19)

8. Cosmic Microwave Background in gravity waves

An important problem in cosmology is the detection and the measurement of the spectrum of gravitational radiation from the extremely early Universe, especially around microwave frequencies. Since gravitational radiation decouples from matter at a much earlier era of the Big Bang (i.e., the Planck era) than electromagnetic radiation, observations of these primordial gravity waves would constitute a much deeper probe of the structure of the early Universe than is the case for the usual CMB.

In particular, the string-inspired pre-Big-Bang model, the ekpyrotic model based on brane theory, and the conventional inflation model, give totally different predictions as to the gravity-wave spectrum.\(^9\) See Fig. 2. Observations in the radio- and
microwave-frequency parts of the spectrum would be decisive in determining which
model (if any) is the correct one, since the position of the maxima in the spec-
tra predicted by the pre-Big-Bang and ekpyrotic models and their strengths are
strikingly different from each other. Both models in turn yield spectra which differ
greatly from the spectrum predicted by the conventional inflation model, which is
extremely flat up to the microwave frequency range, where there is a cutoff, but
where there are no maxima at all.

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