Urban traffic congestion is on the increase worldwide; therefore, it is critical to maximize the capacity and throughput of the existing road infrastructure with optimized traffic signal control. For that purpose, this paper builds on the body of work in mixed integer linear programming (MILP) approaches that attempt to optimize traffic signal control jointly over an entire traffic network and specifically on improving the scalability of these methods for large numbers of intersections. The primary insight in this work stems from the fact that MILP-based approaches to traffic control used in a receding horizon control manner (that replan at fixed time intervals) need to compute high-fidelity control policies only for the early stages of the signal plan. Therefore, coarser time steps can be used to see over a long horizon to adapt preemptively to distant platoons and other predicted long-term changes in traffic flows. To that end, this paper contributes the queue transmission model (QTM), which blends elements of cell-based and link-based modeling approaches to enable a nonhomogeneous time MILP formulation of traffic signal control. Experimentation is then carried out with this novel QTM-based MILP control in a range of traffic networks, and it is demonstrated that the nonhomogeneous MILP formulation achieves (a) substantially lower delay solutions, (b) improved per vehicle delay distributions, and (c) more optimal travel times over a longer horizon in comparison with the homogeneous MILP formulation with the same number of binary and continuous variables.

Urban traffic congestion is on the increase worldwide, with estimated productivity losses in the hundreds of billions of dollars in the United States alone and immeasurable environmental impact; therefore, it is critical to maximize the capacity and throughput of the existing road infrastructure through optimized traffic signal control (1). Unfortunately, many large cities still use some degree of fixed-time control (2) even if they also use actuated or adaptive control methods, such as the Sydney Coordinated Adaptive Traffic System (SCATS) (3) or the split-cycle offset optimization technique (SCOOT) (4). However, there is further opportunity to improve traffic signal control even beyond adaptive methods through the use of optimized controllers (that incorporate elements of adaptive and actuated control) as evidenced in a variety of approaches, from mixed integer (linear) programming (5–10), heuristic search (11, 12), queuing delay with pressure control (13), and linear program (LP) control (14) to scheduling-driven control (15, 16) and reinforcement learning (2). Such optimized controllers hold the promise of maximizing the existing infrastructure capacity by finding more complex (and potentially closer to optimal) jointly coordinated intersection policies in comparison with heuristically adaptive policies such as SCATS and SCOOT. However, optimized methods are computationally demanding and often do not guarantee jointly optimal solutions over a large intersection network because they either (a) consider only coordination of neighboring intersections or arterial routes or (b) fail to scale to large intersection networks simply for computational reasons. The latter scalability issue is endemic to many mixed integer programming approaches to optimized signal control.

This work builds on the body of work in mixed integer linear programming (MILP) approaches that attempt to jointly optimize traffic signal control over an entire traffic network (rather than focus on arterial routes) and specifically on improving the scalability of these methods for large urban traffic networks. In the investigation of existing approaches in this vein, namely, exemplar methods in the spirit of Lo (7) and Lin and Wang (9), who use a (modified) cell transmission model (CTM) (17, 18) for their underlying prediction of traffic flows, a major drawback is the CTM-imposed requirement to choose a predetermined homogeneous (and often necessarily small) time step for reasonable modeling fidelity. This need to model a large number of CTM cells with a small time step leads to MILPs that are exceedingly large and often intractable to solve.

The primary insight in this work stems from the fact that MILP-based approaches to traffic control used in a receding horizon control manner (that replan at fixed time intervals) need to compute high-fidelity control policies only for the early stages of the signal plan. Therefore, coarser time steps can be used to see over a long horizon to preemptively adapt to distant platoons and other predicted long-term changes in traffic flows. This need for nonhomogeneous control in turn spawns the need for an additional innovation: a traffic flow model that permits nonhomogeneous time steps and properly models the travel time delay between lights is required. For that purpose, one might consider CTM extensions, such as the variable cell length CTM (19), stochastic CTM (20, 21), CTM extensions for better modeling of freeway–urban interactions (22) including CTM hybrids with link-based models (23), asymmetric CTMs for better handling of flow imbalances in merging roads (24), the situational CTM for better modeling of boundary conditions (25), and the lagged CTM...
for improved modeling of the flow–density relationship (26). However, despite the widespread varieties of the CTM and use for a range of applications (27), there seems to be no extension that permits nonhomogeneous time steps as proposed in this paper’s novel MILP-based control approach.

For that reason, as a major contribution of this work to enable the nonhomogeneous time MILP-based model of joint intersection control, the queue transmission model (QTM) that blends elements of cell-based and link-based modeling approaches is contributed, as illustrated and summarized in Figure 1. For each \( n \), the following parameters are shown: elapsed time \( t \), nonhomogeneous time step length \( \Delta t \), cumulative duration of two light phases for \( l_n \), phase \( p \) of light \( l_n \), and traffic volume of different queues \( q \) linearly interpolated between time points. There is technically a binary \( p \) for each phase, but just the current active phase is shown: NS for north–south green between time points. There is technically a binary \( p \) for each phase, but just the current active phase is shown: NS for north–south green and EW for east–west green, assuming that the top of the map is north. In Figure 1, traffic progresses from \( q_1 \) to \( q_2 \) to \( q_3 \) according to light phases and traffic propagation delay with nonhomogeneous time steps only at required change points.

The QTM technique offers the following key benefits:

- Unlike previous CTM-based joint intersection signal optimization, the QTM technique is intended for nonhomogeneous time steps that can be used for control over large horizons (7, 9).
- Any length of roadway without merges or diverges can be modeled as a single queue, leading to compact QTM MILP encodings of large traffic networks (i.e., large numbers of cells and their associated MILP variables are not required between intersections). Further, the free-flow travel time of a link can be modeled exactly, independent of the discretization time step, while the CTM requires a further increased discretization to approach the same resolution.
- The QTM technique accurately models fixed travel time delays critical to green wave coordination, as in Gartner et al. (5), Gartner and Stamatiadis (6), and He et al. (8), through the use of a non–first order Markovian update model, and further combines this model with fully joint intersection signal optimization in the spirit of Lo (7), Lin and Wang (9), and Han et al. (10).

In the rest of this paper, the novel QTM model of traffic flow with nonhomogeneous time steps is first formalized and then the way to encode it as an LP for computing traffic flows is shown. The paper proceeds to allow the traffic signals to become discrete phase variables that are optimized subject to a delay-minimizing objective and standard minimum and maximum time constraints for cycles and phases. This approach results in the final MILP formulation of traffic signal control. Experimentation is then carried out with this novel QTM-based MILP control in a range of traffic networks, and it is demonstrated that the nonhomogeneous MILP formulation achieves (a) substantially lower delay solutions, (b) improved per vehicle delay distributions, and (c) more optimal travel times over a longer horizon in comparison with the homogeneous MILP formulation with the same number of binary and continuous variables.

**QUEUE TRANSMISSION MODEL**

A QTM is the tuple \((Q, L, \Delta t, I)\), where \( Q \) and \( L \) are, respectively, the set of queues and lights; \( \Delta t \) is a vector of size \( N \) representing the homogeneous, or nonhomogeneous, discretization of the problem horizon \([0, T]\), and the duration in seconds of the \( n \)th interval is denoted as \( \Delta t_n \); and \( I \) is a matrix \([|Q| \times T]\) in which \( I_n \) represents the flow of vehicles requesting to enter queue \( i \) from outside the network at time \( n \).

A traffic light \( \ell \in L \) is defined as the tuple

\[
\left( \psi^\text{min}_\ell, \psi^\text{max}_\ell, \varphi_\ell, \Phi^\text{min}_\ell, \Phi^\text{max}_\ell \right)
\]

where

- \( \varphi_\ell = \text{set of phases of } \ell \),
- \( \psi^\text{min}_\ell(\psi^\text{max}_\ell) = \text{minimum (maximum) allowed time for } \ell \),
- \( \Phi^\text{min}_\ell(\Phi^\text{max}_\ell) = \text{minimum (maximum) allowed time for phase } k \in \varphi_\ell \).

A queue \( i \in Q \) represents a segment of road that vehicles traverse at free-flow speed; once they traverse it, the vehicles are vertically stacked in a stop line queue. Formally, a queue \( i \) is defined by the tuple

\[
\left( Q, T^\text{max}_i, F^\text{max}_i, F^\text{min}_i, \Phi^\text{max}_i, \Phi^\text{min}_i, Q^\text{max}_i \right)
\]

![FIGURE 1](image_url)  
*Example of real traffic network modeled with QTM: (a) correspondence between QTM links and nodes and streets and intersections and (b) preview of QTM model parameters as function of nonhomogeneous discretized time intervals indexed by \( n \).*
where
\[ Q_i = \text{maximum capacity of } i; \]
\[ T_i^{\text{prop}} = \text{time required to traverse } i \text{ and reach stop line}; \]
\[ F_i = \text{maximum traffic flow from } i \text{ to outside of modeled network}; \]
\[ F_i \] and \( \tilde{F}_i \) are vectors of size \([Q]\) and their \( j \)th entry (i.e., \( F_{ij} \) and \( \tilde{F}_{ij} \)) representing the maximum flow from queue \( i \) to \( j \) and the turn probability from \( i \) to \( j \) (where \( \sum_j \tilde{F}_{ij} = 1 \), respectively; and
\[ Q_i^0 = \text{set of traffic light phases controlling the outflow of queue } i, \text{ where the pair } (\ell, k) \in Q_i^0 \text{ denotes phase } k \text{ of light } \ell. \]

In contrast to the CTM formulation, the QTM formulation does not assume that \( \Delta_t = T_i^{\text{prop}} \) for all \( n \), that is, the QTM can represent nonhomogeneous time intervals (Figure 1b) (9, 17). The only requirement over \( \Delta_t \) is that no traffic light maximum phase time be smaller than any \( \Delta_t \) since phase changes occur only between time intervals; formally, \( \Delta_t \leq \min_{\ell, k, \text{known}} \Phi_{\ell k}^{\text{max}} \) for all \( n \in \{1, \ldots, N\} \).

**COMPUTING TRAFFIC FLOWS WITH QTM**

This section presents the way traffic flows are computed with QTM and nonhomogeneous time intervals \( \Delta_t \). For the rest of this section, it is assumed that a valid control plan for all traffic lights is fixed and given as a parameter; formally, for all \( \ell \in \mathcal{L}, k \in \mathcal{P}_\ell \) and interval \( n \in \{1, \ldots, N\} \), the binary variable \( p_{\ell k n} \) is known a priori and indicates whether phase \( k \) of light \( \ell \) is active (i.e., \( p_{\ell k n} = 1 \)) on interval \( n \). Each phase \( k \in \mathcal{P}_\ell \) can control the flow from more than one queue, allowing arbitrary intersection topologies to be modeled, including “all-red” phases as a switching penalty and modeling lost time from amber lights.

The problem of finding the maximal flow between capacity-constrained queues is represented as an LP over the following variables defined for all intervals \( n \in \{1, \ldots, N\} \) and queues \( i \) and \( j \):

- \( q_{in} \in [0, Q_i] = \text{traffic volume waiting in stop line of queue } i \text{ at beginning of interval } n \),
- \( f_{in}^{\text{in}} \in [0, I_i] = \text{inflow to network via queue } i \text{ during interval } n \),
- \( f_{in}^{\text{out}} \in [0, F_i] = \text{outflow from network via queue } i \text{ during interval } n \), and
- \( f_{ijn} \in [0, F_i] = \text{flow from queue } i \text{ into queue } j \text{ during interval } n \).

The maximum traffic flow from queue \( i \) to queue \( j \) is enforced by Constraints C1 and C2. Constraint C1 ensures that only the fraction \( \text{Pr}_{ij} \) of the total internal outflow of \( i \) goes to \( j \), and since each \( f_{ijn} \) appears on both sides of Constraint C1, the upstream queue \( i \) will block if any downstream queue \( j \) is full. Constraint C2 forces the flow from \( i \) to \( j \) to be zero if all phases controlling \( i \) are inactive [i.e., \( p_{\ell k n} = 0 \) for all \( (\ell, k) \in Q_i^0 \)]. If more than one phase \( p_{\ell k n} \) is active, then Constraint C2 is subsumed by the domain upper bound of \( f_{ijn} \):

\[
f_{ijn} \leq \text{Pr}_{ij} \sum_{k \in Q_i^0} f_{ijn} \tag{C1}
\]
\[
f_{ijn} \leq F_{ij} \sum_{(k,\ell) \in Q_i^0} p_{\ell k n} \tag{C2}
\]

To simplify the presentation of the rest of the LP, the helper variables \( q_{in}^0 \) (Constraint C3), \( q_{in}^{\text{out}} \) (Constraint C4), and \( t_i \) (Constraint C5) are defined to represent the volume of traffic to enter and leave queue \( i \) during interval \( n \) and the time elapsed since the beginning of the problem until the end of interval \( \Delta_t \), respectively.

\[
q_{in}^0 = \Delta_t \left( f_{in}^{\text{in}} + \sum_{j=1}^n \text{Pr}_{ij} f_{ijn} \right) \tag{C3}
\]
\[
q_{in}^{\text{out}} = \Delta_t \left( f_{in}^{\text{out}} + \sum_{j=1}^n f_{ijn} \right) \tag{C4}
\]
\[
t_i = \sum_{n=1}^N \Delta_t \tag{C5}
\]

To account for the misalignment of the different \( \Delta_t \) and \( T_i^{\text{prop}} \), one needs to find the volume of traffic that entered queue \( i \) between two arbitrary points in time \( x \) and \( y \) (\( x \in [0, T], y \in [0, T] \), and \( x < y \)), that is, \( x \) and \( y \) might not coincide with any \( t_i \) for \( n \in \{1, \ldots, N\} \). This volume of traffic, denoted as \( V_i(x, y) \), is obtained by integrating \( q_{in}^0 \) over \([x, y]\) and is defined in Equation 1, where \( m \) and \( w \) are the index of the time intervals subject to \( t_m \leq x < t_{m+1} \) and \( t_w \leq y < t_{w+1} \). Because the QTM dynamics are piecewise linear, \( q_{in}^0 \) is a step function with regard to time, and this integral reduces to the sum of \( q_{in}^0 \) over the intervals contained in \([x, y]\) and the appropriate fraction of \( q_{in}^0 \) representing the misaligned beginning and end of \([x, y]\).

\[
V_i(x, y) = \left( t_{m+1} - x \right) \frac{q_{in}^{\text{in}}}{\Delta_t} + \left( \sum_{k=m+1}^w q_{in}^{\text{out}} \right) + \left( y - t_w \right) \frac{q_{in}^{\text{out}}}{\Delta_t} \tag{1}
\]

Using these helper variables, Constraint C6 represents the flow conservation principle for queue \( i \) where \( V(t_{w+1} - T_i^{\text{prop}}, t_w - T_i^{\text{prop}}) \) is the volume of vehicles that reached the stop line during \( \Delta_t \). Since \( \Delta_t \) and \( T_i^{\text{prop}} \) for all queues are known a priori, the indexes \( m \) and \( w \) used by \( V_i \) can be precomputed to encode Equation 1; moreover, Constraint C6 represents a non–first order Markovian update because the update considers the previous \( w - m \) time steps. To ensure that the total volume of traffic traversing \( i \) [i.e., \( V(t_{w+1} - T_i^{\text{prop}}, t_w) \)] and waiting at the stop line does not exceed the capacity of the queue, Constraint C7 is applied. When queue \( i \) is full, \( q_{in}^{\text{out}} = 0 \) by Constraint C7, which forces \( f_{ijn} \) to 0 in Constraints C3 and C4. This in turn allows the queue in \( i \) to spill back into the upstream queue \( j \).

\[
q_{in} = q_{in}^{\text{out}} + V_i(t_{w+1} - T_i^{\text{prop}}, t_w - T_i^{\text{prop}}) \tag{C6}
\]
\[
V_i(t_{w+1} - T_i^{\text{prop}}, t_w) + q_{in} \leq Q_i \tag{C7}
\]

As with MILP formulations of CTM [e.g., Lin and Wang (9)], QTM is also susceptible to withholding traffic, that is, the optimizer might prevent vehicles from moving from \( i \) to \( j \) even though the associated traffic phase is active and \( j \) is not full; for example, this situation may reserve space for traffic from an alternate approach that allows the MILP to minimize delay in the long term even though it leads to unintuitive traffic flow behavior. This well-known issue is addressed through the objective function (O1) by maximizing the total outflow \( q_{in}^{\text{out}} \) (i.e., internal and external outflow) of \( i \) plus the inflow \( f_{in}^{\text{in}} \) from outside the network to \( i \). This quantity is weighted
by the remaining time until the end of the problem horizon $T$ to force the optimizer to allow as much traffic volume as possible into the network and move traffic to the outside of the network as soon as possible.

$$\max \sum_{n=1}^{N} \sum_{i=1}^{P} \Delta t_i (T - t_i + 1) (f_{i,n}^m + f_{i,n}^m)$$  \hspace{1cm} (O1)$$

The objective O1 corresponds to minimizing delay in CTM models; for example, O1 is equivalent to the objective function O3 in Lin and Wang for their parameters $\alpha = 1$, $\beta = 1$ for the origin cells and $\beta = 0$ for all other cells (9). Figure 2 depicts this equivalence by using the cumulative number of vehicles entering and leaving a network as a function of time. The delay experienced by the vehicles traveling through this network (red curve in Figure 2) equals the horizontal difference at each point between the cumulative departure and arrival curves (less the free-flow travel time through the network). Maximizing $f_{i,n}^m$ weighted by $(T - t_i + 1)$ in objective O1 is the same as forcing the departure curve to be as close as possible to the arrival curve as early as possible; therefore, the area between arrival and departure is minimized, which in turn minimizes the delay.

To illustrate the representation trade-off offered by nonhomogeneous time intervals, flows and queue volumes were computed for a fixed signal control plan derived for homogeneous $\Delta t_i = 1$ s (ground truth) with different discretizations. Figure 3a shows the approximation of the ground truth with homogeneous $\Delta t = 2.5$ and $\Delta t = 5.0$, and Figure 3b shows the approximation with nonhomogeneous time intervals that linearly increase from 1 to 2.5 s, that is, $\Delta t_i = 0.9956n + 0.9044$ for $n \in \{1, \ldots, 17\}$. As Figure 3a shows, large time steps can be rough approximations of the ground truth. Nonhomogeneous discretization (Figure 3b) exploits this fact to provide a good approximation in the initial time steps and progressively decreases precision for points far in the future. Figure 3b shows accuracy in the near term that somewhat degrades over the long term, for which accuracy will be less critical for receding horizon control.

**TRAFFIC CONTROL WITH QTm ENCODED AS AN MILP**

This section removes the assumption that a valid control plan for all traffic lights is given and extends the LP (Objective O1, Constraints C1–C7) to a mixed-integer LP (MILP) that also computes the optimal control plan. Formally, for all $\ell \in \mathcal{L}$, $k \in \mathcal{P}_\ell$, and interval $n \in \{1, \ldots, N\}$, the phase activation parameter $p_{\ell,k,n} \in \{0,1\}$ becomes a free variable to be optimized. To obtain a valid control plan, one enforces that one phase of traffic light $\ell$ is always active at any interval $n$ (Constraint C8) and ensures cyclic phase policies in which phase changes follow a fixed ordered sequence (Constraint C9), that is, if phase $k$ was active during interval $n-1$ and has become inactive in interval $n$, then phase $k+1$ must be active in interval $n$. Constraint C9 assumes that $k+1$ equals 1 if $k=|\mathcal{P}_\ell|$.

$$\sum_{k=1}^{\mathcal{P}_{\ell}} p_{\ell,k,n} = 1$$  \hspace{1cm} (C8)$$

$$p_{\ell,k-1,n} \leq p_{\ell,k,n} + p_{\ell,k+1,n}$$  \hspace{1cm} (C9)$$

Next, the minimum and maximum phase durations (i.e., $\Phi_{\ell,k}^{\min}$ and $\Phi_{\ell,k}^{\max}$) are enforced for each phase $k \in \mathcal{P}_\ell$ of traffic light $\ell$. To encode these constraints, one uses the helper variable $d_{\ell,k,n} \in [0, \Phi_{\ell,k}^{\max}]$ (defined by Constraints C10 through C14) that (a) holds the elapsed time since the start of phase $k$ when $p_{\ell,k,n}$ was active (C10, C11), (b) is constant and holds the duration of the last phase until the next activation when $p_{\ell,k,n}$ is inactive (C12, C13), and (c) is restarted when phase $k$ changes from inactive to active (C14). Constraints C10 through C14 use the big-M method to turn the cases that should not

![Figure 2](image-url)  
**FIGURE 2** Cumulative arrival (blue), departure (green), and delay (red) curves resulting from horizontal difference between arrival and departure curves less free-flow travel time. Arrival curve is fixed by demand profile, and departure curve is maximized by objective function, which has same effect as minimizing area under delay curve.
be active into subsumed constraints according to the value of $p_{i,k,n}$. $\Phi_{\ell}^{\text{max}}$ is used as the large constant since $d_{i,k,n} \leq \Phi_{\ell}^{\text{max}}$ and $\Delta_{\ell} \leq \Phi_{\ell}^{\text{max}}$. Similarly, Constraint 15 ensures the minimum phase time of $k$ and is not enforced while $k$ is still active. Figure 4, a through c, presents an example of how Constraints C10 through C15 work together as a function of the time $n$ for $d_{i,k,n}$; the domain constraint $0 \leq d_{i,k,n} \leq \Phi_{\ell}^{\text{max}}$ for all $n \in \{1, \ldots, N\}$ is omitted for clarity.

\[ d_{i,k,n} \leq d_{i,k,n-1} + \Delta_{n-1} p_{i,k,n-1} + \Phi_{\ell}^{\text{max}} (1 - p_{i,k,n-1}) \quad (C10) \]

\[ d_{i,k,n} \geq d_{i,k,n-1} + \Delta_{n-1} p_{i,k,n-1} - \Phi_{\ell}^{\text{max}} (1 - p_{i,k,n-1}) \quad (C11) \]

\[ d_{i,k,n} \geq d_{i,k,n-1} + \Phi_{\ell}^{\text{max}} p_{i,k,n} \quad (C12) \]

\[ d_{i,k,n} \geq d_{i,k,n-1} - \Phi_{\ell}^{\text{max}} p_{i,k,n} \quad (C13) \]

\[ d_{i,k,n} \leq \Phi_{\ell}^{\text{max}} (1 - p_{i,k,n} + p_{i,k,n-1}) \quad (C14) \]

\[ d_{i,k,n} \geq \Phi_{\ell}^{\text{max}} (1 - p_{i,k,n}) \quad (C15) \]

Last, the sum of all phase durations for light $\ell$ is constrained to be within the cycle time limits $\Psi_{\ell}^{\text{min}}$ (Constraint C16) and $\Psi_{\ell}^{\text{max}}$ (Constraint C17). In Constraints C16 and C17, the duration of phase 1 of $\ell$ from the previous interval $n - 1$ is used instead of the current interval $n$ because Constraint C14 forces $d_{i,\ell,n}$ to be 0 at the beginning of each cycle; however, from the previous end of Phase 1 until $n - 1$, $d_{i,\ell,n-1}$ holds the correct elapse time of Phase 1. In addition, Constraint C16 is enforced right after the end of each cycle, that is, when its first phase is changed from inactive to active. The value of Constraints C16 and C17 over time for a traffic light $\ell$ is illustrated in Figure 4d.

\[ d_{i,\ell,n} + \sum_{k=2}^{K} d_{i,k,n} \geq \Psi_{\ell}^{\text{min}} (p_{i,k,n} - p_{i,k,n-1}) \quad (C16) \]

\[ d_{i,\ell,n} + \sum_{k=2}^{K} d_{i,k,n} \leq \Psi_{\ell}^{\text{max}} \quad (C17) \]

The MILP that encodes the problem of finding the optimal traffic control plan in a QTM network is defined by (Objective O1, Constraints C1–C17).
This section compares the solutions for traffic networks modeled as a QTM with homogeneous and nonhomogeneous time intervals with regard to two evaluation criteria: the quality of the solution and convergence to the optimal solution versus the number of time steps. Specifically, the quality of solutions based on the total travel time is compared, and the third quartile and maximum of the observed delay distribution are also considered. The hypotheses to be evaluated in this paper are (a) the quality of the nonhomogeneous solutions is at least as good as that of the homogeneous solutions when the number of time intervals $N$ is fixed and (b) the nonhomogeneous approach requires fewer time intervals (i.e., smaller $N$) than the homogeneous approach to converge to the optimal solution. In the rest of this section, the traffic networks considered in the experiments, the method, and the results are presented.

**Networks**

Three networks of increasing complexity are considered (Figure 5): an avenue crossed by three side streets, a 2-by-3 grid, and a 3-by-3 grid with a diagonal avenue. The queues receiving vehicles from outside the network are marked in Figure 5 and referred to as input queues. The maximum queue capacity ($Q_i$) is 60 vehicles for non-input queues and infinity for input queues to prevent interruption of the input demand because of spill-back from the stop line. The traversal time of each queue $i$ ($T_{i}^{\text{prop}}$) is set at 9 s (a distance of 125 m with a free-flow speed of 50 km/h). For each street, flows are defined from the head of each queue $i$ into the tail of the next queue $j$; there is no turning traffic ($P_{i,j} = 1$), and the maximum flow rate between queues, $F_{i,j}$, is set at five vehicles/s. All traffic lights have two phases, north–south and east–west, and Lights 2, 4, and 6 of Network 3 have the additional northeast–southwest phase to control the diagonal avenue. For Networks 1 and 2, $\Phi_{\ell,k}^{\text{min}} = 1$; $\Phi_{\ell,k}^{\text{max}} = 3$; $\Psi_{\ell,k}^{\text{min}} = 2$; and $\Psi_{\ell,k}^{\text{max}} = 6$ for all traffic light $\ell$ and phase $k$. For Network 3, $\Phi_{\ell,k}^{\text{min}} = 1$ s and $\Phi_{\ell,k}^{\text{max}} = 6$ s for all $\ell$ and $k$; and $\Psi_{\ell,k}^{\text{min}} = 2$ s and $\Psi_{\ell,k}^{\text{max}} = 12$ s for all lights except Lights 2, 4, and 6 (i.e., lights also used by the diagonal avenue), for which $\Psi_{\ell,k}^{\text{min}} = 3$ s and $\Psi_{\ell,k}^{\text{max}} = 18$ s.

**Experimental Method**

For each network, constant background-level traffic is injected into the network in the first 55 s to allow the solver to settle on a stable policy. Then a spike in demand is introduced in the queues marked in Figure 5 and referred to as input queues. The maximum queue capacity ($Q_i$) is 60 vehicles for non-input queues and infinity for input queues to prevent interruption of the input demand because of spill-back from the stop line. The traversal time of each queue $i$ ($T_{i}^{\text{prop}}$) is set at 9 s (a distance of 125 m with a free-flow speed of 50 km/h). For each street, flows are defined from the head of each queue $i$ into the tail of the next queue $j$; there is no turning traffic ($P_{i,j} = 1$), and the maximum flow rate between queues, $F_{i,j}$, is set at five vehicles/s. All traffic lights have two phases, north–south and east–west, and Lights 2, 4, and 6 of Network 3 have the additional northeast–southwest phase to control the diagonal avenue. For Networks 1 and 2, $\Phi_{\ell,k}^{\text{min}} = 1$; $\Phi_{\ell,k}^{\text{max}} = 3$; $\Psi_{\ell,k}^{\text{min}} = 2$; and $\Psi_{\ell,k}^{\text{max}} = 6$ for all traffic light $\ell$ and phase $k$. For Network 3, $\Phi_{\ell,k}^{\text{min}} = 1$ s and $\Phi_{\ell,k}^{\text{max}} = 6$ s for all $\ell$ and $k$; and $\Psi_{\ell,k}^{\text{min}} = 2$ s and $\Psi_{\ell,k}^{\text{max}} = 12$ s for all lights except Lights 2, 4, and 6 (i.e., lights also used by the diagonal avenue), for which $\Psi_{\ell,k}^{\text{min}} = 3$ s and $\Psi_{\ell,k}^{\text{max}} = 18$ s.
and then reduced to zero for all input queues. The problem horizon \( T \) is extended until all vehicles have left the network. By clearing the network, one can easily measure the total travel time for all of the traffic as the area between the cumulative arrival and departure curves measured at the boundaries of the network. The background travel time intervals (i.e., \( 0, \Delta t \)) are used in a receding horizon manner: a control plan and the flow through the network is computed with the QTM LP formulation with a fixed (homogeneous) \( \Delta t \) of 0.25 s. Both receding horizon approaches are also compared against the optimal solution obtained by computing a single control plan for the entire control horizon \([0, T]\) with a fixed \( \Delta t \) of 0.25 s.

For all of the experiments, Gurobi was used as the MILP solver with 12 threads on a 3.1 GHz AMD Opteron 4334 processor with 12 cores. The gap accuracy was limited to 0.1% MILP and the time cutoff for solving a major frame was limited to 3,000 s for the receding horizon approaches and unbounded to determine the optimal minimum travel time solution to which all other solutions were compared. All results were averaged over five runs to account for Gurobi's stochastic strategies.

RESULTS

Figure 7, a, c, and e, shows, for each network, the increase in the total travel time with regard to the optimal solution as a function of \( N \). As hypothesized, the nonhomogeneous discretizations require fewer time intervals (i.e., smaller \( N \)) to obtain a solution with the same total travel time, and \( \Delta t_2 \) converges before \( \Delta t_1 \). That finding is important because the size of the MILP, including the number of binary variables,
FIGURE 6  Receding horizon control: (a) problem horizon $T = 40$ s, with major frames for MILP optimization area discretized in 12 time intervals ($N = 12$) that span 15 s and 30 s for homogeneous and nonhomogeneous discretizations, respectively (minor frames represent prefix of major frame MILP optimization that is executed; horizon recedes by minor frame duration after each execution); and (b) two nonhomogeneous discretizations used in experiments, shown with major frame duration of 40 s (from end of minor frame time, $\Delta t$ is linearly interpolated over 10 s, from 0.25 to 0.5 for nonhomogeneous $\Delta t_1$, and 0.25 to 1.0 for nonhomogeneous $\Delta t_2$; $\Delta t$ is then held constant to end of major time frame).

scales linearly with $N$; therefore, the nonhomogeneous approach can scale up better than the homogeneous approach (e.g., Figure 7e). Also, for homogeneous and nonhomogeneous discretizations, finding the optimal solution of major frames with large $N$ might require more time than the imposed 3,000-s time cutoff, and in this case, Gurobi returns a feasible control plan that is far from optimal. The effect on the total travel time of these poor solutions can be seen in Figure 7e for $N > 120$.

For each row in Figure 7, the Roman numeral on top of the box plots corresponds to points on the travel time plot marked with the same numeral. The mean of total delay is presented as a red square in box plots. Plots in the $i$th row correspond to results for the $i$th network in Figure 5. Figure 7 shows that nonhomogeneous (NH) achieves much better solutions at smaller $N$ than does homogeneous (H). The distribution of the total delay observed by each vehicle while traversing the network is shown in Figure 7, b, d, and f. Each group of box plots represents a different value of $N$: when the nonhomogeneous $\Delta t_1$ first converges, when the homogeneous $\Delta t$ first converges, and the final solution itself. In all networks, the quality of the solutions obtained with $\Delta t_1$ and $\Delta t_2$ is better than or equal to the quality obtained with homogeneous $\Delta t$ for fixed $N$ in both total travel time and fairness, that is, a smaller third quartile and maximum delay.

To illustrate further the differences between homogeneous and nonhomogeneous discretizations, Figure 8 shows the cumulative arrival and departure curves and how the delay evolves over time for $q_1$ of Network 2 (Figure 5b). In Figure 8a, the comparison is done when nonhomogeneous $\Delta t_2$ first converges (i.e., point I in Figure 7c), and for this value of $N$, the major frame size in seconds of the nonhomogeneous approach is 19.125 s longer than the homogeneous approach. This fact allows the MILP solver to see 19 s farther in the future when nonhomogeneous discretization is used and to find a coordinated signal policy along the avenue to dissipate the extra traffic that arrives at time 55 s. The shorter major frame of the homogeneous discretization does not allow the solver to adapt this far in advance, and its delay observed after 55 s is much larger than the nonhomogeneous one. Once the homogeneous $\Delta t$ has converged (Figure 8b), it is also able to anticipate the increased demand and adapt well in advance, and both approaches generate solutions close to optimum (Figure 8c).

CONCLUSION

This paper shows how to formulate a novel queue transmission model (QTM) of traffic flow with nonhomogeneous time steps as a linear program. The traffic signals were allowed to become discrete variables subject to a delay-minimizing optimization objective and standard traffic signal constraints leading to a final MILP formulation of traffic signal control with nonhomogeneous time steps. Experimentation was then carried out with this novel QTM-based MILP control in
FIGURE 7  Results showing total travel time convergence and delay distribution of each network: (a, c, and e) show increase in total travel time with regard to optimal solution as function of $N$ and (b, d, and f) represent distribution of total delay of each car for several values of $N$ (NH = nonhomogeneous; H = homogeneous).
Figure 8 shows the cumulative arrival and departure curves and delay for Queue 1 in a 2-by-3 network (see Figure 5): (a) comparison when NH $\Delta t_2$ first converges, (b) comparison once H $\Delta t$ converges, and (c) the same curves presented for optimal solution (labels on top of each plot match the labels shown in Figure 7c). NH $\Delta t_2$ provides near-optimal signal plans over a longer time horizon than does H when $N$ is small.

...a range of traffic networks, and it was demonstrated that the non-homogeneous MILP formulation achieved (a) substantially lower delay solutions, (b) improved per-vehicle delay distributions, and (c) more optimal travel times over a longer horizon in comparison with the homogeneous MILP formulation with the same number of binary and continuous variables. Altogether, this work represents a major step forward in the scalability of MILP-based jointly optimized traffic signal control via the use of a non-homogeneous time traffic model and thus helps pave the way for fully optimized joint urban traffic signal controllers as an improved successor technology to existing signal control methods.

Future work will include learning the QTM parameters (e.g., turn probabilities $p_{rt}$ and expected incoming flows $I_n$) from loop detector data and evaluating the impact in scalability of different non-homogeneous discretizations and the size of the computer cluster used for computing the control plans.

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