Stress-strain state simulation of large-sized cable-stayed shell structures

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Abstract. This paper studies the opportunities for applying framed cable-stayed shell structures to generate innovative structures in civil engineering. Numerical solution methods for stress-strain state problems of these kinds of geometrically nonlinear structures were developed. Developed methods efficiency is presented by a range of large-dimensional space antenna reflectors.

1. Introduction

At the moment, civil engineering is influenced by distinct tendencies in generating ultra-lightweight transformable structures of large dimensions, having the potential to sustain heavy loads. The application “tensegrity” is evidence for popularization of framed cable shelled structures (FCSS).

Tensegrity is a structural principle based on the use of prestressed and compressed elements. The term was coined by scientist and architect Richard Buckminster Fuller [1]. Structures of this kind became widely used in architecture, art, urban construction, the aerospace industry, architectural hardware, and biomechanics [2]. In some cases, these structures are of hundreds of meters in characteristic dimension. The experimental development of large-sized structures requires expenditures in material and time. For this reason, computer-generated simulation takes center stage in the design and development of these structures. It allows tracing the process of structure generation, its settings, etc. and predicting its behavior under different conditions.

In order to provide the most efficient performance of every acting element of the structure, it is necessary to carry out a careful review of the stress-strain state of the whole structure. This creates the need to generate the proper structural model, not merely carry out a limited examination of representative elements. At another point, the analysis of structures of large dimensions composed of cable rods, and thin-walled shells is obstructed by the geometrical nonlinearity of these kinds of systems, which creates the need to develop an approach permitting the numerical analysis of the above described structures.

2. Application of cable-stayed shell structures in the building industry

2.1. Examples of tensegrity structures

Figures 1 - 4 show tensegrity structures used to create space antennas and civil engineering facilities [3, 4]. The advantage of the structures shown is their small volume in folded position, relatively small
design and material costs, and eco-friendliness. The advantage of space antennas design is the ability to withstand heavy loads at launch.

![Deployable space antenna AstroMesh](image)

**Figure 1.** Deployable space antenna AstroMesh.

![The Millennium dome in London](image)

**Figure 2.** The Millenium dome in London.

![Roof of the stadium "La Plata Stadium" in Argentina](image)

**Figure 3.** Roof of the stadium "La Plata Stadium" in Argentina.

![Frame cable-stayed bridge in Australia](image)

**Figure 4.** Frame cable-stayed bridge in Australia.

2.2. **Shell structures**

Shell structures represent a surface stretched by elastic cords or special power frame made of rigid bars and beams (Figures 1, 2, 3).

The level of prestressing in the surface of the shell should be sufficient within the operational lifetime to maintain the level of tension and the shape of the object at one hand, and to allow the shell material to remain in the elastic deformation zone on the other. Under the influence of loads such as wind or snow, the tension on the surface may be increased by 6-8, even 10 times. For these reasons, the shell material must have a prestress equal to 1/20 of breaking strength.

The strength of the material may change due to changes in temperature, humidity, and creep. Therefore, the production requirements of the materials’ mechanical properties are constantly verified using special tests. In general, research is necessary to find material which is not expensive, fire-resistant, and easy to use and transport [5].

When designing space antennas, apart from the effect of prestressing and the temperature on the reflecting surface, it is very important to control such a geometric characteristic as a reflecting surface root-mean-square deviation (RMSD). RMSD should be within 2 - 3% of the operating wavelength. Such restrictions impose more stringent requirements on antenna design and material properties of the reflecting surface than on some civil engineering projects.

Materials for membranes can be made of canvas, fiber glass, and organic synthetic fibers such as polyester. At the moment there is a large selection of materials on the market, but the most commonly used are polyvinyl chloride (PVC), polytetrafluoroethylene (PTFE), and glass silicone coating. For example, the Millennium Dome, located on the Greenwich Peninsula in south-east London (Figure 2), was originally planned to be made of PVC. However, to extend its operational life, it was made of PTFE [5].

For the manufacturing of the reflective surface of space antenna AstroMesh, a thin conductive mesh made of molybdenum wire with diameter 0.03mm, coated with gold is used. [6]
3. Physical-mathematical model of stress-strain state of cable-stayed shell structures

There is considerable number of research on physical and mathematical models of FCSS. For example, Correa [7] deals with the formulation and solution of the problem of finding the FCSS state of tensegrity structures affected by external forces, using the virtual work. Murakami [8, 9] describes static and dynamic analyses of these kinds of structures, as well as the quasi-static approach to their analysis.

The authors consider modeling designs from a perspective of nonlinear elasticity theory. Shell elements are modeled by a thin membranous screen. FCSS extended structural elements, such as ropes, bars and other elements, were modeled by the core elements with effective characteristics, which give these model elements the same stiffness properties of the model as in real structural elements.

Stress-strain state of FCSS can be described by a stationary nonlinear system of equations of elasticity theory

\[
\frac{\partial}{\partial x_k} \left( \sigma_{ij} \left( \delta_{ij} + \frac{\partial u_i}{\partial x_j} \right) \right) = 0
\]

\[
\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

\[
\sigma_{ij} = \frac{E_m}{2(1+\nu_m)} \left( \varepsilon_{ij} + \frac{\nu_m}{1-2\nu_m} \delta_{ij} \varepsilon_{kk} \right) + \sigma_{ij}^0,
\]

where \( \delta_{ij} \) – Kronecker index; \( u_i, \sigma_{ij}, \sigma_{ij}^0, \varepsilon_{ij} \) – second Piola-Kirchhoff stress, prestressing tensor, strain tensor; \( E_m, \nu_m \) – modulus of elasticity and Poisson ratio of m-material.

Boundary conditions are as follows:

\[
u_i(\tilde{x}) = u_{i0}, \quad \tilde{x} \in S_1
\]

\[
n_i \sigma_{ij} \left( \delta_{ij} + \frac{\partial u_i}{\partial x_j} \right) = p_i^a, \quad \tilde{x} \in S_2,
\]

where \( S = S_1 + S_2 \) – domain boundary \( \Omega \) FCSS; \( p_i^a \) – boundary stress \( S_2 \), characterized by the normal vector \( n \), \( u_{i0} \) – boundary displacement \( S_1 \) [10].

4. Numerical solution to the problem of the stress-strain state of FCSS

Problem (1) - (5) was solved with the finite element method using the software package ANSYS [10]. In the numerical solution of equations (1) - (5), a major role is played by the choice of the initial approximation. Since the convergence domain is small, it is difficult to determine a good initial approximation which makes it possible to obtain a solution to the stationary problem. Therefore, we make a sequence of solutions where each new solution uses the previous one as the initial approximation (Figure 5).
Moreover, the solution sought would be the last in this sequence. Solutions correspond to different boundary conditions. For the initial solution, an additional boundary condition is applied:

\[ u_i(x) = 0, \quad x \in \Omega \quad (6) \]

Condition (6) corresponds to the full consolidation of FCSS. The process of solving will continue until the solution with the boundary conditions (4) and (5) is worked out. This procedure was developed by S. V. Ponomarev and V. A. Solonenko and used for numerical simulation of a single umbrella reflector of a large dimension. The procedure for finding the initial state does not have a clear algorithm yet, so changing the design of FCSS can lead to a substantial change in the sequence of removal of restraints [11].

5. Numerical models of cable-stayed shell structures on the example of large-sized space reflectors

When modeling large-sized space reflector the assumptions for application of finite element method are introduced:

- the dimension of the problem decreases; for example, for a reflective surface with a thickness on the order of a millimeter we can assume that the variables of the problem are constant throughout the thickness, and thus solve the two-dimensional problem;
- it is presumed that cable-stayed antenna design elements make no resistance to compressive forces, which introduces significant nonlinearity to the behavior of the structure [12].

Figures 6-13 show the finite-element model of large-sized space reflectors and their corresponding numerical model as developed in the software package ANSYS. Figure 7 shows a front-tension network reflector due to its tension cords. Figure 9 presents the strain of the reflector as a result of pressure on its surface. Figure 11 shows the displacement of the reflector assemblies reflecting the surface tension.

The calculation of the deviation of the reflecting surface of a paraboloid, given by the equation

\[ z = \frac{2(y^2 + y^2)}{4F}, \quad \text{where} \quad F - \text{focal length}, \]  
was carried out for the reflector in Figure 12. More information about these models can be found in works [13, 14].
Figure 6. Umbrella type reflector with aperture of 50 m.

Figure 7. Strain of front reflector network, Pa.

Figure 8. Inflatable reflector with aperture of 50 m.

Figure 9. Reflector strain.

Figure 10. Reflector with flexible spokes with aperture of 50 m.

Figure 11. Displacement of reflector assemblies, m.

Figure 12. Reflector with tensegritied rim with aperture of 48 m.

Figure 13. Distribution of deviations from the reflecting surface of paraboloid where RMSD is $4.05 \times 10^{-3}$ m.
All the numerical models are developed by using the approach proposed in Section 4. For all prescribed designs and initial solutions were predetermined zero boundary conditions on the displacement of assemblies. Further, to obtain the intermediate solutions, these boundary conditions for part of the assemblies are not predetermined, and the process continued until the solution sought was found. The main reason for its use is the large geometric nonlinearity arising in cable-stayed shell structures.

6. Conclusion
The problem of cable-stayed shell structures applicability was considered to generate innovative designs in building industry. The technique of numerical solution of the problem of stress-strain state of structures with high geometric nonlinearity was developed. The effectiveness of the developed technique is shown in a number of designs of large-sized space antenna reflectors.

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Corrigendum: Stress-strain state simulation of large-sized cable-stayed shell structures

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