Complementary Filter for Attitude Estimation Based on MARG and Optical Flow Sensors

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Abstract. The combination of tri-axial magnetometer, accelerometer, and gyroscope has been widely used for three-dimensional attitude measurement, and this combination is also called MARG sensor. However, the measurement of accelerometer contains both the gravitational and motional acceleration, and only the former is useful for attitude estimation. As a result, MARG-based attitude estimation is easily disturbed by motion acceleration. In this paper, we introduce a complementary filter that estimates gravity and geomagnetic vectors in parallel, and utilize an optical flow sensor to detect and compensate motion acceleration. Experiment results show that the proposed algorithm has better performance than the existing ones when experiencing linear acceleration.

1. Introduction
Attitude and heading reference systems (AHRS) are widely used for three-dimensional attitude determination, especially in the flight control of unmanned aerial vehicles (UAV). AHRS uses three different sensors, including a tri-axial magnetometer to measure the geomagnetic field, a tri-axial accelerometer for the measurement of gravity vector, as well as a tri-axial gyroscope to gauge the angular motion. The latest micro-electro-mechanical system (MEMS) technology can integrate the above three types of sensors in a single package, which is usually referred to as MARG sensor with nine degrees of freedom (9-DOF) [1].

Data fusion is one of the key issues for MARG-based AHRS design, since it is critical to extract 3D attitude information from the measurements of MARG sensor. Extended Kalman filter (EKF) and complementary filter (CF) are the most common solutions in practice, mainly because their low computational complexity.

However, MARG-based AHRS can be influenced by various error sources and external disturbances. Time-invariant errors, including the scale factors, misalignments, constant bias, etc., can be compensated through adequate calibration procedure. But the changing disturbances, especially the motion acceleration, can significantly impact the performance of AHRS.

AHRS calculate the tilt angles (including pitch and roll) according to the gravity vector measured by accelerometer. However, the accelerometer is sensitive to both the gravitational and motional acceleration, i.e. its measurement is actually the sum of these two measurands. Therefore, motion acceleration will cause inevitable errors of the calculated 3D attitude.
The existing algorithms to eliminate motion acceleration in MARG-based AHRS can be categorized into the threshold-based [2-4] and model-based [5-9] ones. Although the effectiveness of these algorithms are proven, they can only rely on the gyroscope when dealing with motion acceleration. Hence they are essentially no more than trade-offs between the accumulative attitude error caused by gyro drift and the dynamic attitude error caused by motion acceleration.

In this paper, we proposed a dual-vector complementary filter for MARG-based AHRS, which makes parallel estimations of the geomagnetic and gravity vectors, so as to reduce the impact of motion acceleration on AHRS. Moreover, we make use of optical flow sensor to acquire auxiliary information of linear motion, in order to further enhance the dynamic performance of AHRS.

2. Previous Research

2.1. Complementary filter
A classic complementary filter consists of a high-pass sub-filter and a low-pass sub-filter. The transfer functions of these two sub-filters are complementary, i.e. the sum of these two transfer functions form an all-pass filter with the gain of 1.

In the case of AHRS, 3D attitude is usually represented by Euler angles (including heading, pitch, and roll), quaternion, or direction cosine matrix (DCM), and it can be calculated through two different ways. The first way to determine 3D attitude is to use the gravity and geomagnetic vectors (e.g. the TRIAD algorithm [10]), and the corresponding attitude estimation is denoted as \( \mathbf{\theta}_A \). On the other hand, 3D attitude can also be calculated by integration of angular velocity, and the result is denoted as \( \mathbf{\theta}_B \). It is noteworthy that the integration of angular velocity will lead to accumulative error, and thus the attitude estimation \( \mathbf{\theta}_B \) should be put through the high-pass sub-filter. Meanwhile, the attitude estimation \( \mathbf{\theta}_A \) should pass through the low-pass sub-filter, so that the noise of accelerometer and magnetometer can be suppressed.

R. Mahony proposed a modified architecture of complementary filter for attitude estimation, namely the generalized complementary filter (GCF) [11], which is shown in Fig. 1. In recent years, GCF has been frequently adopted in UAV flight control [12-14]. It has been proven that GCF can be viewed as a fixed gain version of EKF but with much lower computational burden [14], and thus it is easier to be deployed on microcontrollers with limited processing power.

![Figure 1. Schematic diagram of generalized complementary filter.](image)

2.2. Motion acceleration elimination
The threshold-based algorithms [2-4] utilize one or more criteria to detect motion acceleration. For instance, the norm of the measured gravity vector is the most commonly used criterion. Once the norm exceeds the pre-set threshold, it indicates the presence of motion acceleration. In such case, the measured gravity vector becomes unreliable, and thus it should have less weight in attitude estimation, or even be totally discarded. Such algorithms are also called switching filters, since they change their own parameters (e.g. the noise covariance or cut-off frequency of accelerometer) whenever motion acceleration arises.

The model-based algorithms [5-9] eliminate the motion acceleration by properly modelling and estimating it. This type of algorithms seem to preserve more information from accelerometer than those threshold-based ones. However, unless auxiliary sensor is available (e.g. GPS/GNSS module or airspeed meter), the only way to estimate motion acceleration is to subtract the gravity vector (which is calculated by integration of angular velocity) from the measurement of accelerometer. Consequently, both the
model-based and threshold-based algorithms are faced with the dilemma: the accumulative error resulting from integration of angular velocity, versus the dynamic error caused by motion acceleration.

2.3. Optical flow sensor
Optical flow refers to the motion of pixels on image plane. In recent years, optical flow sensor has been frequently used for flight control of UAV [15-16]. When an optical flow sensor is equipped on a UAV and pointed to the ground, as shown in Fig. 3, its measurement reflects both the linear and angular motion of the UAV. The corresponding measurement model can be described by (1), in which $\delta$ denotes the optical flow, $f$ is the scale factor, $d$ is the distance from the camera to the ground, $n$ is the unit vector along the camera's optical axis, $v_\perp$ is the perpendicular component of the UAV's linear velocity $v$ with respect to $n$ (i.e. $v_\perp = v - v \cdot n$), and $\omega$ is the UAV's angular velocity.

\[
\delta = -f \left( \frac{v_\perp}{d} + n \times \omega \right) \tag{1}
\]

According to (1), the optical flow measurement $\delta$ contains the translational movement, and thus we will use optical flow sensor to detect and compensate the motion acceleration.

3. Complementary attitude filter design

3.1. Dual-vector parallel complementary filter
Fig. 3 shows a dual-vector based complementary filter, which estimates the gravity vector $g$ and geomagnetic vector $h$ in parallel. This architecture can help to suppress the impact of motion acceleration on the estimated geomagnetic vector $\hat{h}$. Meanwhile, the two sub-filters for $g$ and $h$ can choose different parameters that better suited to the corresponding sensors. The Euler angles, including heading, pitch, and roll, are calculated according to the two estimated vectors $\hat{g}$ and $\hat{h}$ through TRIAD algorithm [10]. In Fig. 3, $g^*$, $h^*$, and $\omega^*$ denote the measurement of $g$, $h$, and $\omega$, respectively.

The complementary filter in Fig.3 can be implemented using (2)-(4).

\[
\begin{align*}
\omega_{\text{err}, g} & = g^* \times \hat{g} \\
\omega_{\text{err}, h} & = h^* \times \hat{h} \\
\end{align*}
\tag{2}
\]

\[
\begin{align*}
b_{\omega, g} & = K_{i,g} \omega_{\text{err}, g} \\
b_{\omega, h} & = K_{i,h} \omega_{\text{err}, h} \\
\end{align*}
\tag{3}
\]

\[
\begin{align*}
\hat{g} & = \hat{g} \times (\omega^* - b_{\omega, g} - K_{p,g} \omega_{\text{err}, g}) \\
\hat{h} & = \hat{h} \times (\omega^* - b_{\omega, h} - K_{p,h} \omega_{\text{err}, h}) \\
\end{align*}
\tag{4}
\]
3.2. Using optical flow sensor to compensate motion acceleration

According to the measurement model in (1), we can get the velocity $\mathbf{v}_k$ at the $k$th time step (denoted as $\mathbf{v}_{k,k}$), as described by (5). Note that each symbol with the subscript $k$ indicates the corresponding value at the $k$th time step, and $T$ is the sampling period. Then the motion acceleration at the $k$th time step (denoted as $\mathbf{a}_{k,k}$) can be approximately calculated using (6).

$$\mathbf{v}_{k,k} = -\left(\frac{\delta_k}{T} + \mathbf{n} \times \mathbf{w}_k\right) d_k$$  (5)

$$\mathbf{a}_{k,k} \approx \frac{\mathbf{v}_{k,k} - \mathbf{v}_{k,k-1}}{T}$$  (6)

It is worth mentioning that only the acceleration that perpendicular to the optical axis of camera can be measured by optical flow sensor. But in the case of UAV, since MARG-based AHRS is mainly affected by horizontal acceleration most of the time, a single optical flow sensor pointed to the ground (as shown in Fig. 2) can effectively detect and compensate the motion acceleration.

It should also be noticed that the distance $d_k$ in (5) should be known. Thus a distance-measuring sensor (e.g. ultrasonic sensor or laser rangefinder) should be used along with the optical flow sensor.

4. Experiments

To evaluate the proposed algorithm, we use a MARG-based AHRS accompanied with an optical flow module. The AHRS contains a single-chip 9-DOF MARG sensor MPU9250, while the optical flow module consists of an optical flow sensor PMW3901 and a laser ranging sensor VL53L1X.

The AHRS module moves along two horizontal guide rails, as shown in Fig. 4, and thus it experiences linear acceleration on x-axis. Meanwhile, the optical flow sensor is vertically pointed to the ground, with a distance of approximately 1m. Fig. 5 and 6 shows the raw data from accelerometer and optical flow sensor, respectively, both of which reflect the impact of motion acceleration.
We compare the performance of following four algorithms:

- Complementary filter with no compensation of motion acceleration (CF-UC): It is implemented according to the architecture introduced in Section 3.1, with $K_p,g = K_p,h = 0.3$ and $K_l,g = K_l,h = 0.1$.

- Complementary filter with threshold-based compensation of motion acceleration (CF-SW): It turns CF-UC into a switching filter. When $|g^*|/g_0 < 1.1$ ($g_0 = 9.8 \text{m/s}^2$ is the theoretical norm of gravity vector), the filter uses the above settings; otherwise $K_p,g = K_p,h = 0.03$ and $K_l,g = K_l,h = 0.01$.

- Complementary filter with model-based compensation of motion acceleration (CF-EC): It uses the algorithm in [9] to estimate and compensate motion acceleration.

- Complementary filter with optical flow sensor (CF-OF): Based on CF-SW, it uses optical flow data to further compensate motion acceleration, as introduced in Section 3.2.

The pitch angle estimated by the above four algorithms are plotted in Fig. 7 to Fig. 10. It can be seen in Fig. 7 that the motion acceleration causes large deviation of pitch angle estimated by CF-UC. In Fig. 8 and Fig. 9, the disturbance caused by motion acceleration can be partially suppressed by both CF-SW and CF-EC. With the aid of optical flow sensor, CF-OF can further eliminate the impact of motion acceleration and greatly reduce the dynamic attitude error, as can be seen in Fig. 10.

Table 1 lists the root-mean-square (RMS) of pitch angle that estimated by each algorithm, in which we can further evaluate the performance of the above four algorithms. It is clear that the proposed algorithm (CF-OF) has much better performance than the other three, since it can effectively utilize the optical flow measurement to compensate the motion acceleration.
Table 1. Comparison between different algorithms.

| Algorithm | RMS of pitch |
|-----------|--------------|
| CF-UC     | 15.31°       |
| CF-SW     | 8.29°        |
| CF-EC     | 8.55°        |
| CF-OF     | 2.36°        |

5. Conclusions
In this paper, we present a dual-vector parallel complementary filter for 3D attitude estimation, which makes use of optical flow data to compensate motion acceleration. Experiment results prove that the proposed algorithm can effectively enhance the dynamic performance of AHRS against motion acceleration, and it can achieve higher accuracy compared with existing methods.

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