Current oscillations in a superlattice under non-quantizing electric and magnetic fields

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Abstract

We calculate the current density in a semiconductor superlattice with parabolic miniband under crossed non-quantizing electric and magnetic fields. The Corbino disk geometry is considered. The current-voltage curve contains oscillations with period proportional to the magnetic field. The possibility is shown of the negative absolute conductivity. The Ampere-Gauss characteristics also contain overshoots under high enough electric fields. In all cases, the peaks smear with temperature rising.

1 Introduction

In recent experiments, a number of magnetotransport phenomena were found in solids under classical high magnetic fields in presence of a dc electric field [1]. In particular, resistance oscillations have been observed in two-dimensional electron gas (2DEG) under action of both magnetic and electric fields. Such effects were observed in perfect GaAs/AlGaAs heterojunctions. A brief review of theoretical and experimental works on this subject was presented in [2]. To interpret the experimental results unambiguously, further investigations are required of the nonlinear magnetotransport including the semiconductor structures other than the heterojunctions mentioned. In this connection, attention should be paid to [3], where the layered crystal conductivity oscillations in parallel electric and magnetic fields were studied in scope of the quasi-classical approach.

In this work, the current behavior is studied under simultaneous effect of crossed non-quantizing electric and magnetic field on conduction electrons in a semiconductor superlattice (SL). It should be emphasized here, that a specific model of a SL with a parabolic miniband (see below) is used. Such a problem with reference to SLs with cosine miniband has been investigated by many authors. Some papers should be mentioned in which the electron motion in electric and magnetic fields was supposed to be non-quantized, and the electron gas was assumed to be nondegenerate. It was shown in [4], in the linear approximation on the magnetic field,
that the Hall field switching with the sign inversion was possible under high electric field. In [5] an exact solution of the electron motion equation and corresponding current density at zero temperature were found under Corbino regime. No conductivity oscillations were found there. In [6] the Boltzmann equation was solved in the \( \tau \)-approximation and was showed that the current parallel to SL axis in crossed electric and magnetic fields did not depend on the magnetic field.

The interest to the miniband models other than cosine one has been revived in connection with an idea of making a terahertz (THz) Bloch oscillator based on SL (see, e.g., [7, 8, 9]). As it was shown [8, 9], the THz field generation and amplification conditions can be realized, in particular, in SL with parabolic miniband. The latter means the electron dispersion law in form of a truncated parabola (that is the dispersion law is assumed to be parabolic up to the Brillouin zone edge). Note, that in [10] a dispersion law was considered in form of joined direct and inverted parabolas (besides the cosine-like law). The parabolic dispersion law in present work is a specific case of that model.

The magnetic field \( H \) perpendicular to the driving electric field \( E \), which is parallel to SL axis \( OX \) transforms an one-dimensional problem to a two-dimensional one, so that we have, formally, a 2DEG modulated periodically along SL axis.

## 2 The problem statement

The electron energy in the lowest parabolic miniband of SL is

\[
\epsilon(p) = \frac{p^2}{2m_L} + \epsilon(p),
\]

where \( p_\perp \) and \( m_\perp \) are the electron quasi-momentum and effective mass in the SL layer plane, respectively,

\[
\epsilon(p) = \frac{\Delta^2 p^2}{\pi^2 \hbar^2}, \quad -\frac{\pi \hbar}{d} \leq p \leq \frac{\pi \hbar}{d},
\]

\( p \) is the electron quasi-momentum along SL axis, \( \Delta/2 \) is the miniband width, \( d \) is the SL period. The longitudinal energy \( \epsilon(p) \) may be expanded into Fourier series:

\[
\epsilon(p) = \frac{1}{2} \sum_{k=1}^{\infty} \Delta_k \left( 1 - \cos \left( \frac{kpd}{\hbar} \right) \right),
\]

where \( \Delta_k = 4\Delta \frac{(-1)^{k+1}}{k^2 \pi^2} \) may be treated as a “width” of a partial cosine miniband.

We consider a quasi-classical situation: \( \Delta \gg eEd, \hbar \omega, \hbar/\tau \), where \( \omega = eH/(m_\perp c) \) is the cyclotron frequency, \( \tau \) is the mean free time, which is assumed to be constant.

It is convenient to introduce dimensionless variables by the following substitutions:

\[
\frac{d}{\pi \hbar} p \to p, \quad \frac{t}{\tau} \to t, \quad \omega \to \omega, \quad \frac{E}{E_0} \to E \quad (E_0 \equiv \frac{\pi \hbar}{ecd\tau}).
\]
To avoid cumbersome expressions, let us make an assumption that is not principal for the further conclusions, namely, we suppose the longitudinal effective mass (near the Brillouin zone edge) is equal to the transverse one, \( \frac{\pi^2 \hbar^2}{\Delta d^2} = m_\perp \). Such a condition is fulfilled, e.g., at the following parameter values: \( \Delta = 0.02 \text{ eV}, \ d = 5 \times 10^{-7} \text{ cm}, \ m_\perp = 10^{-28} \text{ g} \). The electron velocity along the SL axis \( OX \) is

\[
V_x(t) = \frac{\partial \epsilon(p)}{\partial p_x} = \frac{2\Delta d}{\pi^2 \hbar} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \sin(k\pi p_x(t))
\]

The time dependence of the quasi-momentum is found from the equation of motion

\[
\frac{dp}{dt} = E + \omega(p \times h),
\]

where \( h = H / H \) is the unit vector along the magnetic field. A situation is considered below, which is realized in the Corbino geometry,

\( E = (E, 0, 0), \ H = (0, 0, H) \).

In this case, the solution of Eq. (5) takes the form

\[
p_x(t) = p_{0x} \cos \omega t + p_{0y} \sin \omega t + \frac{E}{\omega} \sin \omega t,
p_y(t) = p_{0y} \cos \omega t - p_{0x} \sin \omega t + \frac{E}{\omega} (\cos \omega t - 1),
p_{0x, y} = p_{0x, y}(0).
\]

By Chambers method [12], the current density \( j \) along the SL axis can be found:

\[
j(E, \omega, T) = e \sum_{p_0} f_0(p_0, T) \int_0^\infty \exp(-t)V_x(t) \, dt,
\]

where the equilibrium distribution function of initial momenta of nondegenerate carriers is

\[
f_0(p_0, T) = n \left[ 2\pi T \text{erf}(1/\sqrt{2T}) \right]^{-1} \exp \left( -\frac{p_{0x}^2 + p_{0y}^2}{2T} \right).
\]

Here \( \text{erf}(z) \) is the error function, \( n \) is the carrier density, and the temperature (in energy units) is presented with the substitution \( T / \Delta \to T \). The integration over \( p_{0x} \) in Eq. (7) goes from \(-1\) to \(1\), while that over \( p_{0y} \) from \(-\infty\) to \(\infty\).

By substituting Eqs. (4) and (6) to Eq. (7), we obtain (in units of \( j_0 = en\Delta d / (\pi \hbar) \))

\[
j(E, \omega, T) = \frac{2}{\pi(1 - \exp(-2\pi / \omega))} \int_0^{2\pi / \omega} \exp(-t) \times \sum_{k=1}^{\infty} \nu_k(\omega, T) \frac{(-1)^{k+1}}{k} \sin \left( \frac{k\pi E}{\omega} \sin \omega t \right) dt,
\]

3
where
\[ \nu_k(\omega t, T) = \exp(-k^2 \pi^2 T/2) \\
\times \text{Re}(\text{erf}(\sqrt{1/2T}(1 + ik\pi \cos \omega t)))/\text{erf}(\sqrt{1/2T}). \] (10)

In absence of the magnetic field \((\omega \to 0)\), Eqs. (9) and (10) lead to the result of [9] that was found by solving the Boltzmann equation in \(\tau\)-approximation:

\[ j(E, 0, T) = E + \frac{\exp(T/2E^2)}{2\text{erf}(1/\sqrt{2T})\sinh(1/E)} \\
\times \left[ \text{erf}(\sqrt{T/2}E - \sqrt{1/2T}) - \text{erf}(\sqrt{T/2}E + \sqrt{1/2T}) \right]. \] (11)

At \(T \to 0\), the formula follows from Eq. (11) which was obtained in [8]:

\[ j(E, 0, 0) = E - \frac{1}{\sinh(1/E)}. \] (12)

3 The superlattice current-voltage characteristics in magnetic field

First, let us consider the case of extremely low temperatures \((T \to 0)\), when factor (10) is equal to 1, so that Eq. (9) takes the form

\[ j(E, \omega, T) = \frac{1}{\pi(1 - \exp(-2\pi/\omega))} \int_0^{2\pi/\omega} \exp(-t) \\
\times \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \sin \left(\frac{k\pi E}{\omega} \sin \omega t \right) dt. \] (13)

Note, that the function \(z(p) \equiv \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \sin k\pi p\) appeared here is defined on the whole number axis and represents the Fourier expansion of a periodic sawtooth (discontinuous) function, which is obtained by periodical continuation of the straight-line segment \(z = p\) from the interval \(-1 \leq p \leq 1\). It is such a circumstance that leads, ultimately, to appearance of the current oscillations. It may be mentioned that \(z(p)\) function often occurs in the theory of magnetic oscillations in metals [13].

Integrating Eq. (13) by parts and using the formula

\[ \sum_{k=1}^{\infty} (-1)^{k+1} \cos kx = \frac{1}{2} - \pi \sum_{n=-\infty}^{\infty} \delta(x - (2n + 1)\pi), \] (14)

we get

\[ j(E, \omega, 0) = \frac{E}{1 - \exp(-2\pi/\omega)} \int_0^{2\pi/\omega} \exp(-t) \cos \omega t \\
\times \left\{ 1 - 2 \sum_{n=-\infty}^{\infty} \delta \left( \frac{E}{\omega} \sin \omega t - (2n + 1) \right) \right\} dt. \] (15)
Figure 1: Current density as a function of electric field (in dimensionless units) at $T = 0$ and various values of the magnetic field: 1 — $\omega = 0$, 2 — $\omega = 0.5$, 3 — $\omega = 1$

Figure 2: Current density as a function of electric field (in dimensionless units) at $\omega = 1$ and various values of the temperature $T$ (in dimensionless units): 1 — $T = 0.1$, 2 — $T = 0.2$, 3 — $T = 0.5$
Figure 3: Ampere-Gauss characteristics at \( T = 0 \) and various values of the driving electric field \( E \) (in dimensionless units): 1 — \( E = 1 \), 2 — \( E = 2.5 \), 3 — \( E = 3.5 \).

By dividing the integration domain into four equal parts, \( \pi/2 \omega \) length each, and shifting the variable of integration, we obtain

\[
j(E, \omega, 0) = \frac{E}{1 + \omega^2} - \frac{2}{\cosh(\pi/2\omega)} \int_0^{1} \sinh \left( \frac{\pi - 2 \arcsin x}{2\omega} \right) \sinh \left( \arccos \left( \frac{\omega}{E} \right) \right) \delta(x - \omega(2n + 1)/E) \, dx \quad (\omega > 0). \tag{16}
\]

Note, that Eq. (12) follows from Eq. (16) at \( \omega \to 0 \). Thus, we get finally from Eq. (16)

\[
j(E, \omega, 0) = j_{0,1} \Theta \left( \frac{E}{\omega} \right) \left( 1 - \frac{E}{\omega} \right) + j_{1,3} \Theta \left( \frac{E}{\omega} - 1 \right) \Theta \left( 3 - \frac{E}{\omega} \right) + \sum_{s=1}^{\infty} j_{2s+1,2s+3} \Theta \left( \frac{E}{\omega} - (2s + 1) \right) \Theta \left( 2s + 3 - \frac{E}{\omega} \right), \tag{17}
\]

where

\[
j_{0,1} = \frac{E}{1 + \omega^2},
\]

\[
j_{1,3} = \frac{E}{1 + \omega^2} - \frac{2 \sinh \left( \frac{\arccos(\omega/E)}{\omega} \right)}{\cosh(\pi/2\omega)},
\]

\[
j_{2s+1,2s+3} = j_{2s-1,2s+1}
\]
Figure 4: Ampere-Gauss characteristics at $E = 2$ and various values of the temperature $T$ (in dimensionless units): 1 — $T = 0.2$, 2 — $T = 0.5$, 3 — $T = 1$

\[
\Theta(x) \text{ being the Heaviside step function.}
\]

4 Discussion

Current-voltage curve (CVC) calculated by Eqs. (17) and (18) at $T \to 0$ and various values of the magnetic field ($\omega$) is shown in Fig. 1. At $\omega \geq 0.25$, when the oscillations manifest themselves the most clearly, the CVC may be called multi-N-type characteristic. Note, that the magnetic field stimulates the current increasing in the CVC maxima (a negative magnetoresistance) and leads to appearance of regions with negative absolute conductivity.

CVC appears overshoots at $E_N = 2\omega(N + 1/2)$; $N = 0, 1, \ldots$. The oscillation period obtained from Eqs. (17) and (18) is $\Delta(E) = 2\omega$, or $\Delta(E) = \frac{2\pi}{\epsilon d} \omega$ (in dimensional units). At the parameter values mentioned and $\tau \approx 5 \times 10^{-12}$ s, the electric field unit is $E_0 \approx 750$ V/cm, while magnetic field $H = 10^4$ Oe corresponds to $\omega = 1$. Note, that the quasi-classical conditions are fulfilled at the used parameter values. At $T \neq 0$ CVC can be calculated numerically by means of Eqs. (9) and (10) (Fig. 2). In Figs. 3 and 4 the Ampere-Gauss characteristics are shown at $T = 0$ and
\( T \neq 0 \), respectively. In all the cases, the peaks smear with temperature rising. At the miniband width value used, \( T = 1 \) value corresponds to 230 K.

Thus, CVC of semiconductor superlattice with parabolic miniband in crossed classical electric and magnetic fields under Corbino geometry is a multi-N-type characteristic with oscillations, the periodic being proportional to the magnetic field. Appearance of the regions with negative absolute conductivity is possible. Ampere-Gauss characteristic contains overshoots. The peaks smear with temperature rising.

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