Analysis of terrain navigability in underwater terrain aided navigation

Miao ZHAO, Zhenyang XU, and Weiqiang MA
Naval University of Engineering, Wuhan, 430033, China

Zhaomiao0911@163.com, 1451731382@qq.com, 877765907@qq.com

Abstract. Aiming at the problem that the single terrain characteristic parameter cannot comprehensively evaluate the terrain navigability of the matching areas, the weighted vector correlation degree is introduced, and the terrain characteristic parameters such as the standard deviation of terrain elevation, roughness, correlation coefficient, terrain information entropy and terrain slope are comprehensively considered. A multi-index comprehensive decision-making evaluation method based on grey fuzzy theory is proposed. In addition, aiming at the problem that different matching routes have different navigability for the specific terrain, in order to select matching routes with better navigability, a navigation coefficient based on terrain statistical parameters is proposed to analyze the navigability of different routes. And the point mass filter (PMF) algorithm is used to simulate the navigability of the matching areas with different terrain features and matching routes. The simulation results show that the proposed evaluation method and navigation coefficient are feasible in the evaluation of terrain navigability. The stronger the navigability is, the smaller the matching error is and the better the navigation performance is. This research results can provide theoretical basis for analysis of terrain navigability, selection of matching areas and matching routes planning in terrain aided navigation.

1. Introduction
At present, with the diversification of underwater missions, submarine, unmanned underwater vehicles, torpedoes, self-propelled mines and other underwater vehicles put forward higher requirements for navigation accuracy. Terrain aided navigation (TAN) technology, as a positioning method with non-cumulative error, can assist in correcting the inertial navigation error in real time, especially be suitable for long-range offensive weapons’ carrying out tasks [1-2]. It mainly utilizes the rich terrain information of the seabed, uses the measuring equipment to correlate the measured depth data in real time with the elevation data in the baseline digital map, and estimate the position of the current underwater vehicle to correct the cumulative error caused by the inertial navigation system [3-6], so the navigation positioning accuracy depends largely on the terrain features [7-11].
The richness of terrain features and the rationality of selecting the matching areas directly affect the terrain matching accuracy and reliability of underwater vehicles [12-14]. In areas with rich terrain features, terrain aided navigation technology can obtain stable and reliable positioning accuracy, but areas with flat terrain usually cannot provide enough terrain features, which leads to mismatching and reduction of navigation accuracy. Therefore, according to the known terrain information, determining a matching route with strong navigability in the working area of underwater vehicles is an effective means to reduce the mismatching rate and improve the positioning accuracy and reliability. And the evaluation and selection of matching areas is the basis for matching routes planning. The richer the terrain features are, the stronger the recognition ability of the matching algorithm is and the more reliable the result is.

Terrain navigability usually needs to be described by different parameters from different angles. It is difficult to obtain accurate and comprehensive evaluation results with a single parameter, and it is easy to select the matching areas with poor navigability. In [15], a fuzzy decision-making method is proposed, which can take into account multiple terrain characteristic parameters. However, when the number of candidate areas increases, the evaluation results tend to be evaluated by a single terrain index, so it can only be used for navigability evaluation of two to three matching areas [16]. In [16], a grey decision-making method is proposed, which is not limited by the number of candidate areas. However, when the amount of terrain elevation data is small, the lack of terrain information will lead to evaluation error, which cannot truly reflect the terrain navigability.

Combining the underwater terrain elevation information with the terrain matching algorithm and selecting the underwater matching areas have always been the research hot spot of underwater terrain aided navigation technology [17-18]. The application of terrain aided navigation technology began in the 1960s, but it has not been possible to define an effective method for evaluating terrain navigability. A multi-index comprehensive decision-making evaluation method based on grey fuzzy theory is proposed by analyzing terrain information and comprehensively considering the standard deviation of terrain elevation, roughness, correlation coefficient, terrain information entropy and terrain slope. Moreover, aiming at the problem that different matching routes have different navigability in certain areas, the navigation coefficient based on terrain statistical parameters is proposed to evaluate the navigability of different matching routes in a certain terrain. And the PMF algorithm is used to simulate the navigability of the matching routes with different terrain features and different navigation coefficients. The research results can provide reference for the analysis of terrain navigability, the selection of matching areas and matching routes planning.

2. Evaluation methods of navigability

2.1. Selection of terrain characteristic parameters

In terrain aided navigation, the digital elevation model (DEM) is the basis of terrain information analysis. Common terrain characteristic parameters include the standard deviation of terrain elevation, roughness, correlation coefficient, terrain information entropy and terrain slope, etc. Different characteristic parameters reflect different attributes of the terrain[19-20]. In terrain aided navigation, digital maps typically use the digital elevation model to store elevation data and divide the terrain area into $M \times N$ grids according to a certain grid size, and the elevation of the grid point $(i, j)$ is $h(i, j)$. In order to analyze the statistical characteristics of the local terrain, a local moving calculation window with size $m \times n$ is defined to calculate various statistical parameters of the local terrain. When the center of the calculation window is moved over all the grid points of the entire area, the statistical parameters of the respective parts of the entire terrain area can be obtained. Common terrain characteristic parameters used to study the relationship between terrain features and positioning accuracy of matching areas are as follows:

(1) Standard deviation of terrain elevation
The standard deviation of terrain elevation $\sigma$ is a macroscopic parameter that describes the overall fluctuation of the terrain and reflects the degree of a terrain elevation sample set’s generally deviating from its average elevation $\bar{h}$. The calculation formulas are as follows:

$$\begin{align*}
\sigma &= \sqrt{\frac{1}{m(n-1)} \sum_{i=1}^{m} \sum_{j=1}^{n-1} [h(i, j) - \bar{h}]^2} \quad (1) \\
\bar{h} &= \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} h(i, j) \quad (2)
\end{align*}$$

(2) Terrain roughness

The terrain roughness $r$ is a characteristic parameter that describes the degree of local fluctuation of the terrain and reflects the complexity of the terrain fluctuation within a certain range. The greater the roughness is, the larger the surface area is and the rougher the local terrain is; the smaller the terrain roughness is, the smaller the surface area is and the smoother the terrain is. According to its direction, it can be divided into absolute roughness in the longitude direction $r_x$ and absolute roughness in the latitude direction $r_y$. The calculation formulas are as follows:

$$r = \frac{(r_x + r_y)}{2} \quad (3)$$

$$r_x = \frac{1}{(m-1)n} \sum_{i=1}^{m-1} \sum_{j=1}^{n} [h(i, j) - h(i + 1, j)] \quad (4)$$

$$r_y = \frac{1}{m(n-1)} \sum_{i=1}^{m} \sum_{j=1}^{n-1} [h(i, j) - h(i, j + 1)] \quad (5)$$

(3) Terrain correlation coefficient

The terrain correlation coefficient $R$ is a measure of the degree of terrain correlation of the terrain profile in its corresponding direction. According to its direction, it can be divided into the longitude direction correlation coefficient $R_\perp$ and the latitude correlation coefficient $R_\parallel$. The calculation formulas are as follows:

$$R = \frac{(R_\perp + R_\parallel)}{2} \quad (6)$$

$$R_\perp = \frac{1}{(m-1)n} \sum_{i=1}^{m-1} \sum_{j=1}^{n} [h(i, j) - \bar{h}] [h(i + 1, j) - \bar{h}] \quad (7)$$

$$R_\parallel = \frac{1}{m(n-1)} \sum_{i=1}^{m} \sum_{j=1}^{n-1} [h(i, j) - \bar{h}] [h(i, j + 1) - \bar{h}] \quad (8)$$

(4) Terrain slope

The terrain slope $S$ is a characteristic parameter that describes the degree of inclination of the terrain surface and reflects the variation of terrain, which is determined by the rate of change of the terrain in the latitude direction $S_\parallel$ and the rate of change in the longitude direction $S_\perp$. The calculation formulas are as follows:
Terrain information entropy

The terrain information entropy $H$ is a characteristic parameter that measures the richness of terrain information. The richer the terrain information is, the smaller the entropy value is and the poorer the terrain information is, the larger the entropy value is. The calculation formulas are as follows:

$$S = \frac{1}{C_\lambda} \left[ h(i+1, j+1) + h(i+1, j) + h(i+1, j-1) - h(i-1, j+1) - h(i-1, j) - h(i-1, j-1) \right]$$

$$S_v = \frac{1}{C_\phi} \left[ h(i+1, j) + h(i, j+1) + h(i+1, j-1) - h(i-1, j) - h(i-1, j+1) - h(i-1, j-1) \right]$$

$$S = \arctan \left( \sqrt{\frac{S^2}{\Sigma \phi^2} + \frac{S^2}{\Sigma \lambda^2}} \right)$$

Evaluation of terrain navigability

2.2.1 Comparison of commonly used analysis methods of terrain navigability

Terrain navigability refers to an attribute of terrain. For a limited terrain area, this attribute reflects the ability of the terrain profile in a certain direction to provide information on the plane location. In order to meet the navigation positioning accuracy requirements, the selection of appropriate matching areas needs to be taken into account when using the terrain aided navigation technology. Generally, the process of selecting appropriate matching areas is called analysis of terrain navigability. At present, analysis of terrain navigability mainly focuses on the terrain characteristic parameters. The selection methods of matching areas can be roughly divided into three categories, which is multi-parameter comprehensive analysis method of terrain navigability, entropy analysis method and comprehensive analysis method based on fuzzy theory.

1. Multi-parameter comprehensive analysis method of terrain navigability

Because different terrain characteristic parameters describe terrain features from different angles, there is a problem that the evaluation results are not comprehensive when using a single parameter to analyze the terrain navigability. Therefore, the multi-parameter comprehensive analysis method is adopted, which is a comprehensive analysis of the influence of multiple parameters on performance of terrain navigation, supplemented by a large number of simulation calculations to determine the threshold value of each characteristic parameter, so as to formulate the criterion of the selection of matching areas.

The comprehensive analysis method of the standard deviation of terrain elevation, roughness, standard deviation of total noise of the system and the comprehensive analysis method of standard deviation of terrain elevation, the correlation coefficient are two typical multi-parameter comprehensive analysis methods. The first comprehensive analysis method is based on a large number of simulation results, and the threshold values of standard deviation of terrain elevation, roughness and standard deviation of total noise of the system are obtained and the logic function is built to serve as a criterion function of selecting matching areas. The second comprehensive analysis method analyzes the relationship between correlation coefficient, roughness, standard deviation, terrain slope and matching performance from two directions of latitude and longitude, and determines the threshold values of the standard deviation of terrain elevation and the correlation coefficient as the criterion of selecting matching areas.
(2) Entropy analysis method of terrain navigability

The terrain information entropy is based on the physical quantity of entropy in thermodynamics and measures the terrain changes with average information. Terrain information entropy is a statistic to measure the degree of the terrain’s containing the average amount of information, so it can be used as a parameter to describe the terrain features. The severer the terrain fluctuation is, the more obvious and unique the terrain features are, the richer the terrain information is, the smaller the entropy value calculated is and the better the matching performance is. And the flat terrain has less terrain information, so the terrain entropy value is large, which means that the terrain is not conducive to matching operation. Therefore, terrain information entropy can be used as an evaluation index for analysis of terrain navigability.

(3) Comprehensive analysis method of terrain navigability based on fuzzy theory

Analysis the impact of multiple parameters usually requires determining the importance of factors and making a comprehensive evaluation of multiple parameters. Comprehensive analysis method based on fuzzy theory is an effective multi-parameter decision-making method. In analysis of the terrain navigability, the fuzzy comprehensive evaluation theory can be referred to, the relationship between the various parameters and the terrain matching performance is analyzed, the weights of the terrain characteristic parameters are determined, and the terrain navigability of candidate matching areas is analyzed with calculation and simulation verification. Finally, the ideal matching area that meets the requirements is selected.

Although the evaluation result of the multi-parameter comprehensive analysis method is more comprehensive, the simulation time is quite long, and the feasibility of analysis of large-scale maps are not reliable. On the other hand, for some special cases, such as not all the alternative terrain characteristic parameters can meet the criterion of selecting matching areas, it can take a lot of time, and cannot get deterministic evaluation results. In addition, the threshold of the terrain characteristic parameters determined by the method is not universal, and the thresholds of the terrain parameters determined of different matching areas are different, so the selection of matching areas cannot be determined by a unified standard.

The entropy analysis method is simple, but there are also problems that the evaluation results are not comprehensive. Especially for the terrain matching areas with similar terrain entropy values, directly relying on the terrain information entropy to evaluate the pros and cons of the navigability sometimes does not match the results of the simulation. Therefore, the method needs to be further improved.

The comprehensive analysis method based on fuzzy theory is essentially a multi-parameter comprehensive analysis method, but it does not require a large number of calculations and simulations to determine the thresholds of terrain characteristic parameters, and there is no need to consider the problem that different matching areas have different thresholds.

Therefore, the comprehensive analysis method of terrain navigability based on fuzzy theory is more effective and feasible than other methods. In this paper, a multi-index comprehensive decision-making evaluation method is proposed by combining fuzzy theory and grey theory to evaluate the terrain navigability more comprehensively.

2.2.2 Comprehensive decision-making evaluation method based on grey and fuzzy theory

Because the description of terrain features by single terrain characteristic parameter has limitations, it cannot fully and accurately reflect the terrain navigability of matching areas. Therefore, based on the idea of multi-feature fusion, the standard deviation of terrain elevation, terrain roughness, terrain correlation coefficient, terrain information entropy and terrain slope are merged to form a terrain features set, and the features set is analyzed by grey and fuzzy comprehensive theory. Thus, the comprehensive evaluation results of the
Navigability of matching areas are obtained, which provides a theoretical basis for the selection of the matching areas in the engineering application. The specific calculation steps are as follows:

1) Construction of the evaluation index matrix

It is assumed that the number of matching areas that need to be evaluated for navigability is \( n \), and the number of evaluation index (terrain characteristic parameters) is \( m \), then the \( m \times n \) evaluation index matrix can be constructed as:

\[
X = \begin{pmatrix}
X_{11} & X_{12} & \cdots & X_{1n} \\
X_{21} & X_{22} & \cdots & X_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
X_{m1} & X_{m2} & \cdots & X_{mn}
\end{pmatrix}
\] (14)

Among them, \( x_{ij} \) represents the \( i \)-th terrain characteristic parameter of the \( j \)-th matching areas, the row vector of the matrix \( X \) represents a set of the \( i \)-th terrain characteristic parameters of all the matching areas to be evaluated, and the column vector of the matrix \( X \) represents the set of all the terrain characteristic parameters of the \( j \)-th matching area.

2) Classification and normalization of the evaluation index

According to the relationship between the evaluation index and the matching effects, the index set composed of all terrain characteristic parameters is divided into two categories: the positive index of “the bigger the better” (income type) and the negative index of “the smaller the better” (cost type). In addition, because the dimensions of different evaluation index are different, the parameters need to be normalized to obtain a dimensionless matrix, that is \( R = (r_{ij})_{m \times n} \), \( r_i \in [0,1] \). The calculation methods of positive and negative index are as follows:

When the \( i \)-th line is a positive index:

\[
\rho^+_{ij} = \frac{X_{ij} - \min_{1 \leq j \leq n} \{X_{ij}\}}{\max_{1 \leq j \leq n} \{X_{ij}\} - \min_{1 \leq j \leq n} \{X_{ij}\}}
\] (15)

When the \( i \)-th line is a negative index:

\[
\rho^-_{ij} = \frac{\max_{1 \leq j \leq n} \{X_{ij}\} - X_{ij}}{\max_{1 \leq j \leq n} \{X_{ij}\} - \min_{1 \leq j \leq n} \{X_{ij}\}}
\] (16)

3) Calculation of the multi-target grey correlation evaluation matrix

According to the correlation theory of the grey system, the optimal decision of the system needs to consider the correlation degree between each scheme and the relatively optimal scheme. \( P = (p_{ij})_{m \times n} \) is usually measured by the following formula:

\[
p_{ij} = \frac{\mu \max_j \max_i |\rho^-_{ij} - 1|}{|\rho^+_{ij} - 1| + \mu \max_j \max_i |\rho^-_{ij} - 1|}
\] (17)

Among them, \( \mu \) is the distinguishing coefficient. By reasonably selecting the value of \( \mu \), the integrity and anti-interference of the correlation degree can be improved, and the value of \( \mu \) usually ranges from 0.5 to 1.

4) Calculation of the weight of the evaluation index:
\[ W = \begin{bmatrix} W_1, & W_2, & \cdots, & W_n \end{bmatrix} = \left[ \frac{d_1}{\sum_{j=1}^{n} d_j}, \frac{d_2}{\sum_{j=1}^{n} d_j}, \ldots, \frac{d_n}{\sum_{j=1}^{n} d_j} \right] \quad (18) \]

Among them, \( d_i = \exp[\lambda \sum_{j=1}^{n} (p_j - r_s)(1 - \rho)] \), \( \lambda \) is the balance coefficient between the two attributes, and the value ranges from 0 to 1. For the convenience of calculation, this value of \( \lambda \) is set to 0.5.

(5) Calculation of the weighted vector correlation degree:

\[ E = (E_1, E_2, \ldots, E_n) = W \cdot P \quad (19) \]

Among them, \( E_j = \sum_{i=1}^{n} w_{ij} \), \( j = 1, 2, \ldots, n \). Finally, the navigability of matching areas whose number is \( n \) can be evaluated according to the value of each element in the weighted vector correlation degree set \( E \) and the principle of “the bigger the better”.

2.2. Proposal of the navigation coefficient of matching routes

For the matching area selected by a specific evaluation method, it is impossible to know the navigability of the matching routes. In order to make full use of the terrain information, it is necessary to plan the matching route of the specific terrain, which requires the analysis of navigability of the matching routes. The standard deviation of terrain elevation and terrain information entropy can be used as a quantitative index to measure the local terrain features. And the absolute roughness, correlation coefficient and terrain slope in the two directions of longitude and latitude can be used to express the terrain changes in the longitude direction and the latitude direction. Therefore, this paper uses the combination of the standard deviation of terrain elevation, terrain information entropy and absolute roughness, correlation coefficient and terrain slope in the longitude and latitude direction to define the navigation coefficient \( \rho \) of a certain matching route. The navigation coefficient \( \rho \) can be defined as:

\[ \rho = \sigma^* + (r^* \cos \theta + r^* \sin \theta) + (s^* \cos \theta + s^* \sin \theta) - H^* - (r^* \cos \theta + r^* \sin \theta) \quad (20) \]

Among them, \( \theta \) is the heading of the underwater vehicles, and “*” represents the normalization of the characteristic parameters, that is \( \sigma = \frac{\sigma^*}{\sum \sigma_i} \) and so on.

3. Results and discussion

3.1 Analysis of terrain information

The real depth data of a certain sea area are selected, and the terrain characteristic parameters are calculated by adopting a moving calculation window of size \( m \times n \) on the topographic map, and the value of \( m \) and \( n \) are set to 6 in this paper. In order to facilitate the comparative analysis, the calculation results are displayed by the contour map, and the analysis results of the selected terrain are shown in Figure 1. It can be seen from Figure 1 that the terrain information of the selected terrain is rich, and the parameters of the longitude direction and the latitude direction are different, which means that the matching effect in the corresponding direction is also different. The larger the standard deviation of terrain elevation, the terrain roughness and the terrain slope are, the smaller the correlation coefficient and the terrain information entropy are, the more obvious the terrain features are and the richer the navigation information is. The single parameter can
represent the terrain features to different extents, which can be used as the evaluation index of analysis of terrain navigability. However, using multiple terrain characteristic parameters can more fully describe the terrain features, making the subsequent analysis of navigability more accurate and reliable.
3.2 Evaluation of terrain navigability with theoretical calculation and simulation

3.2.1 Theoretical calculation of terrain navigability. The underwater terrain of the selected sea area is shown in Figure 2. The selected matching area is divided into six sub-areas with the size of 4000×3000 m², as shown in Figure 2(a), the numbers are respectively I–VI, corresponding to the underwater terrain as shown in Figure 3. The characteristic parameters of subareas and the grey fuzzy comprehensive evaluation values are calculated, and the navigability of each subarea is ranked according to the evaluation results. The values of terrain characteristic parameters of each subarea are shown in Table 1. It is not hard to see from Table 1 that the ranking based on the single parameter is inconsistent, so only relying on a single parameter index is not sufficient to accurately evaluate the terrain navigability.

Table 1 Values of terrain characteristic parameters of subareas

| Matching areas | \( \sigma/\text{m} \) | \( r/\text{m} \) | \( R \) | \( H/\text{bit} \) | \( S/\degree \) |
|---------------|------------------|----------------|------|----------------|---------|
| I             | 57.9852          | 24.9414        | 0.7630 | 1.5561         | 1.4861  |
| II            | 57.1480          | 24.0430        | 0.7698 | 1.5553         | 1.5070  |
| III           | 51.7387          | 23.0069        | 0.7674 | 1.5560         | 1.4876  |
| IV            | 47.7749          | 21.0662        | 0.7647 | 1.5562         | 1.4780  |
| V             | 53.4245          | 23.7632        | 0.7696 | 1.5557         | 1.5099  |
| VI            | 34.4479          | 15.0475        | 0.7667 | 1.5562         | 1.4457  |

The numerical calculation steps are as follows:
1) Constructing the evaluation index matrix according to formula (14):
2) Normalizing the index matrix \( X \) according to formula (15) and (16):

\[
R = \begin{bmatrix}
1 & 0.94431 & 0.73413 & 0.80727 & 0.80727 \\
1 & 0.90919 & 0.80447 & 0.80832 & 0.80832 \\
1 & 0 & 0.0032941 & 0.48900 & 0.48900 \\
0.111111 & 1 & 0.22222 & 0 & 0.0032941 \\
0.0029238 & 0.964829 & 0.74272 & 0.003111 & 0.003111
\end{bmatrix}
\]

3) Constructing the multi-target grey correlation evaluation matrix according to formula (17):

\[
P = \begin{bmatrix}
1 & 0.933387 & 0.8062 & 0.72070 & 0.333333 \\
1 & 0.843074 & 0.7188 & 0.80727 & 0.333333 \\
1 & 0.333333 & 0.43897 & 0.48900 & 0.48900 \\
0.3 & 0.917143 & 0.90074 & 0.000734 & 0.000734 \\
0.74239 & 0.907143 & 0.90074 & 0.333333 & 0.333333
\end{bmatrix}
\]

4) Calculating the index weight according to formula (18):

\[
W = \begin{bmatrix}
0.34572 & 0.555555 & 0.011373 & 0.0000734 & 0.081477
\end{bmatrix}
\]

5) Calculating the weighted vector correlation degree according to formula (19):

\[
E = \begin{bmatrix}
0.964829 & 0.8750 & 0.5815 & 0.5489 & 0.7853 & 0.3355
\end{bmatrix}
\]

Finally, according to the value of each element in the weighted vector correlation degree set, the terrain navigability of each subarea is ranked as I>II>V>III>IV>VII.

**Figure 3** Topography of different matching areas

(a) Topography of matching area I  (b) Topography of matching area II  (c) Topography of matching area III

(d) Topography of matching area IV  (e) Topography of matching area V  (f) Topography of matching area VI
3.3.2 Simulation verification of terrain navigability. In order to verify the reliability of the above theoretical calculation results, matching and positioning simulations are carried out in each subarea, and 100 routes in each subarea are randomly matched and positioned, and 100-time Monte Carlo simulations are carried out for each route, and the average value is taken as the match result. The matching results are shown in Table 2.

| Matching areas | RMSE/m | Ranking of navigability |
|----------------|--------|-------------------------|
| I              | 61.4398| 1                       |
| II             | 64.9882| 2                       |
| III            | 71.7476| 4                       |
| IV             | 80.9741| 5                       |
| V              | 68.7898| 3                       |
| VI             | 96.8965| 6                       |

It can be seen from Table 2 that the area with stronger navigability ranking has smaller matching error, and the simulation results are consistent with the theoretical calculation results. This demonstrates that the selected terrain characteristic parameters can effectively reflect the terrain navigability, and the feasibility and effectiveness of the grey fuzzy comprehensive decision-making evaluation method for evaluating terrain navigability are verified.

3.3 Evaluation of routes navigability with theoretical calculation and simulation

3.3.1 Theoretical calculation of routes navigability. Four matching routes are selected on the topographic map shown in Figure 2. The headings θ are 20°, 30°, 40°, and 50°, and the numbers are i~iv respectively. The terrain characteristic parameters of the matching routes are calculated, and the navigation information of each matching route is obtained as shown in Table 3. The normalized values of the characteristic parameters in Table 3 are shown in Table 4.

According to formula (20), the navigation coefficients of each matching route are calculated, and the navigability of the matching routes is evaluated and ranked according to the calculation results, and the corresponding results are shown in Table 5.

| Matching routes | α/s | ρ/s | r/s | R/s | H/| S°/S° |
|-----------------|-----|-----|-----|-----|---|-------|
| i               | 67.0942| 16.2399| 32.1722| 0.8340| 0.6939| 1.5562| 1.3656| 1.3909 |
| ii              | 71.1079| 16.3276| 34.8554| 0.8455| 0.6830| 1.5561| 1.3705| 1.4123 |
| iii             | 74.5309| 16.0816| 37.0382| 0.8498| 0.6808| 1.5559| 1.3737| 1.4223 |
| iv              | 79.5742| 16.2751| 40.1562| 0.8607| 0.6710| 1.5555| 1.3743| 1.4385 |

| Matching routes | α*/s | ρ*/s | r*/s | R*/s | H*/bit | S°*/S°* |
|-----------------|------|------|------|------|--------|--------|
| i               | 0.192082| 0.198061| 0.189960| 0.248881| 0.201358| 0.200028| 0.199754| 0.163553 |
| ii              | 0.203573| 0.199130| 0.205803| 0.252313| 0.198195| 0.200471| 0.19954 | 0.166069 |
| iii             | 0.213372| 0.196130| 0.218692| 0.253596| 0.197557| 0.199990| 0.200939| 0.167245 |
| iv              | 0.227811| 0.198490| 0.237102| 0.256849| 0.194713| 0.199938| 0.201027| 0.169150 |
### Table 5 Navigation coefficients and evaluation results of matching route

| Matching routes | $\rho$  | Ranking of navigability |
|----------------|---------|-------------------------|
| i              | 0.14321 | 4                       |
| ii             | 0.184046| 3                       |
| iii            | 0.217952| 2                       |
| iv             | 0.244377| 1                       |

#### 3.3.2 Simulation verification of routes navigability

In order to verify the feasibility of using the navigation coefficient to evaluate the navigability of matching routes, the PMF algorithm is used to simulate the four matching routes in the large fluctuation area shown in Figure 2. The matching track is shown in Figure 4. The value of matching error of each matching route is obtained by simulation, and the navigability is ranked according to the matching results. The corresponding results are shown in Table 6.

### Table 6 Matching error and evaluation results of matching routes

| Matching routes | RMSE/m | Ranking of navigability |
|-----------------|--------|-------------------------|
| i               | 71.2903| 4                       |
| ii              | 60.6656| 3                       |
| iii             | 57.2316| 2                       |
| iv              | 44.9875| 1                       |

With the comparative analysis of Table 5 and Table 6, it can be seen that the simulation evaluation results are consistent with the theoretical calculation results and the feasibility of the navigation coefficient for evaluating the navigability of the matching routes is proved. In addition to the simulation in the large fluctuation area, in order to further verify the feasibility of the navigation coefficient for evaluating the navigability of the matching route, the simulation analysis is carried out in the small fluctuation area. The selected truly measured terrain of the Mulan Lake is shown in Figure 5.

The real depth data of the Mulan Lake in Wuhan are analyzed and two routes with headings of 40° and 60° are selected as matching routes. The corresponding navigation coefficients and matching error are shown in Table 7 and the corresponding matching tracks and real-time error are shown in Figure 6 and Figure 7.
Table 7 Navigation coefficients and matching error of matching routes

| Matching routes | $\rho$   | RMSE/m |
|-----------------|----------|--------|
| 1 (40°)         | 0.614682 | 135.4441 |
| 2 (60°)         | 0.844953 | 122.3699 |

Figure.4 Track of matching routes

Figure.5 Underwater topography of the Mulan Lake
Figure 6 Matching results of route 1

(a) Matching route 1(θ=40°)  (b) Matching error of route 1

(c) Matching route 2(θ=60°)  (d) Matching error of route 2

Figure 7 Matching results of route 2

* Corresponding author: author@e-mail.org
4. Conclusion

Terrain aided navigation is an effective method using terrain features as an auxiliary information source and utilizing the diversity, complexity and difference of terrain to correct the inertial navigation system to accurate navigation and positioning error. The system has better navigation performance on rugged and complex terrain, and the navigation performance is obviously reduced on flat or similar terrain, and even serious navigation deviation may occur.

In this paper, a multi-index grey fuzzy comprehensive decision-making evaluation method is proposed to analyze the terrain navigability, the index weights of multi-feature parameters of the matching areas are determined, and the fusion of multi-feature parameters is realized, which overcomes the problem of inaccurate and incomplete evaluation based on a single evaluation index and can provide correct evaluation results when the amount of terrain elevation data is small and the number of candidate matching areas is large. The research results provide a theoretical basis for the analysis of terrain navigability of candidate matching areas, enriches the method of selecting underwater matching areas in navigation aided navigation and has high engineering application value.

In addition, the multiple statistical parameters of the measured terrain are analyzed and calculated under the local moving calculation window. The navigation coefficient $\rho$ with the fusion of multi-feature parameters is proposed as the criterion of selecting matching routes. Simulation analysis of matching routes with different navigation coefficients based on PMF algorithm on measured terrain is carried out and the matching error is compared and analyzed. The results show that the satisfactory matching performance can be obtained by selecting the appropriate matching route with strong navigability, and the feasibility of the navigation coefficient for analysis of routes navigability is proved. The research results can be used as the reference of the matching route planning in terrain aided navigation.

5. Acknowledgments

This work thanks to the funding of Defense Advance Research Program of Science and Technology(No.3020603030404), thanks to the technical guidance of the professors in the teaching and research section and the members of the research group for their great help in the terrain measurement of the Mulan Lake.

6. References

[1] Paull L, Saeedi S, Seto M, et al. AUV navigation and localization: A review[J]. IEEE Journal of Oceanic Engineering, 2014, 39(1): 131-149.

[2] Xu Xiao-su, Tang Jun-jun, Zhang Tao, et al. Underwater navigation method based on terrain and environmental features [J]. Journal of Chinese Inertial Technology, 2015, 23(5): 590-596.

[3] LI Lin. Current Status and Development of Seabed Terrain Matching Aided Navigation Technology[J]. Ship Electronic Engineering, 2008, 28(2): 17-19.

[4] GAO Yong-qi, LIU Hong, ZHANG Yi. The Study on Measurement Errors about Underwater Terrain Matching Performance[J]. Journal of Projectiles, Rockets, Missiles and Guidance, 2014, 34(1): 180-183.

[5] ZOU Wei, SUN Yu-chen. Summary of Underwater Terrain Matching Aided Navigation Technology[J]. Ship Electronic Engineering, 2017, 37(8): 5-10.

[6] CLAUS B, BACHMAYER R. Terrain-aided Navigation for an Underwater Glider[J]. Journal of Field Robotics, 2015, 32(7): 935-951.

[7] MA Yan. Underwater terrain navigability analysis based on multi-beam data[D]. Harbin: Harbin Engineering University, 2010.

[8] LI K H, XIONG L, CHENG L, et al. The research of matching area selection criterion for gravity gradient aided navigation[J]. Communication in Computer and Information Science, 2014, 483: 21-30.

[9] ZHANG X C, HE Z W, LIANG Y H, et al. Selection method for scene matching area based on information entropy[C]// Computational Intelligence and Design(ISCI), Fifth International Symposium, 2012: 364-368.

[10] SUN Y, ZHANG J S, QIAO Y K, et al. Matching area intelligent selection method in geomagnetic navigation[C]//Intelligent Computing and Intelligent Systems(ICIS), 2010: 860-864.
[11] WANG P, HU X P, WU M P, et al. Study on the geomagnetic features for matching suitability analysis[C]// Fifth International Conference on Intelligent Human-Machine Systems and Cybernetics, 2013: 70-73.

[12] LI Xiong-wei, LIU Jian-ye, KANG Guo hua. Analysis of terrain information using elevation matching based on entropy[J]. Journal of Applied Sciences, 2006, 24(6): 608-612.

[13] CHENG Li, ZHANG Ya-jie, CAI Ti-jing. Selection criterion for matching area in gravity aided navigation[J]. Journal of Chinese Inertial Technology, 2007, 15(5): 559-563.

[14] CHENG Hua, MA Jie, GONG Jun-bin, et al. Selection of suitable 3D terrain matching field based on least squares support vector machines[J]. Journal of Huazhong University of Science and Technology: Nature Science Edition, 2008, 36(1): 34-37.

[15] ZHANG T, XU X S, LI P J. Selection criteria for matching area in terrain aided navigation based on fuzzy decision[J]. Journal of Dalian Maritime University, 2009, 35(1): 5-8.

[16] SHEN J, ZHANG J Y, LI H. Selection criteria for matching area of terrain aided navigation based on grey decision-making[J]. Journal of Naval University of Engineering, 2012, 24(5): 48-53.

[17] LIU C, WANG H L, YAO P. On Terrain-aided Navigation for Unmanned Aerial Vehicle Using B-spline Neural Network and Extended Kalman Filter[C]// 2014 IEEE Chinese Guidance, Navigation and Control Conference. Piscataway: IEEE, 2015: 2258-2263.

[18] LUDVIGSEN M, JOHNSEN G, SORENSEN A J, et al. Scientific Operations Combining ROV and AUV in the Trondheim Fjord[J]. Marine Technology Society Journal, 2014, 48(2): 59-71.

[19] WANG Lihui, GAO Xianzhi, LIANG Bingbing, et al. Optimized Method of Building Underwear Terrain Navigation Database Based on Triangular Irregular Network[J]. Journal of Chinese Inertial Technology, 2015, 23(3): 15-18.

[20] ELMAZI D, KULLA E, ODA T, et al. A Comparison Study of Two Fuzzy-based Systems for Selection of Actor Node in Wireless Sensor Actor Networks[J]. Journal of Ambient Intelligence and Humanized Computing, 2015, 6(5): 635-645.