Kaluza-Klein Picture and Nucleon-Nucleon Dynamics at Low Energies

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Abstract

In this note we present additional arguments in favour of Kaluza and Klein picture of the world. We show that geniusly simple formula provided by Kaluza-Klein approach gives an excellent description for the mass spectrum of two-nucleon system. It has also been established that the experimental data obtained at low energies where the nucleon-nucleon dynamics has been studied reveal a special sort of (super)symmetry between fermionic (dibaryon) and bosonic states predicted by Kaluza-Klein scenario.

1 Introduction

In our previous paper [1] we have presented the arguments in favour of that the Kaluza-Klein picture of the world has been been observed for a long time in the experiments at very low energies where the nucleon-nucleon dynamics has been studied. Actually, we have shown that the structure of proton-proton total cross section at very low energies has a clear signature of the existence of the extra dimensions: It was found that geniusly simple formula for KK excitations provided by Kaluza-Klein approach accurately described the experimentally observed irregularities in the spectrum of mass of the diproton system. Surely, this was a very nice fact and, certainly, it was not an accidental coincidence.

The central point of Kaluza-Klein approach is related to the existence of a new fundamental scale characterizing a size of compact internal extra space. The detailed study of the structure for the proton-proton total cross section at very low energies allowed us to calculate this scale. It turned out the fundamental scale has a clear physical meaning: This scale corresponds to the scale of distances where the strong Yukawa forces in strength come down to the electromagnetic forces. We have shown that the Kaluza-Klein tower of KK excitations built by the calculated fundamental scale was in a good correspondence with the experimentally observed picture of irregularities in the spectrum of mass of the diproton system. However, it’s clear that Kaluza-Klein scenario predict the same KK excitations in the proton-antiproton system as well, and this is a very nontrivial fact. That is why, it was intriguing for us to find an experimental confirmation of this fact. We have performed an analysis of experimental data on mass spectrum of the resonance states of proton-antiproton system above elastic threshold and compared them with Kaluza-Klein picture. The results of this analysis have been presented here.
2 Kaluza-Klein picture and KK excitations in two-nucleon system

It is well known that the basic idea of the Kaluza-Klein scenario may be applied to any model in Quantum Field Theory (see for the details e.g. the excellent review articles \[2, 3\] and many references therein). As example, let us consider the simplest case of (4+d)-dimensional model of scalar field with the action

\[ S = \int d^{4+d}z \sqrt{-G} \left[ \frac{1}{2} (\partial_M \Phi)^2 - \frac{m^2}{2} \Phi^2 + \frac{G_{(4+d)}}{4!} \Phi^4 \right], \tag{1} \]

where \( G = \det |G_{MN}|, G_{MN} \) is the metric on \( M_{(4+d)} = M_4 \times K_d \), \( M_4 \) is pseudo-Euclidean Minkowski space-time, \( K_d \) is a compact internal \( d \)-dimensional space with the characteristic size \( R \). Let \( \Delta_{K_d} \) be the Laplace operator on the internal space \( K_d \), and \( Y_n(y) \) are orthonormalized eigenfunctions of the Laplace operator

\[ \Delta_{K_d} Y_n(y) = -\frac{\lambda_n}{R^2} Y_n(y), \tag{2} \]

and \( n \) is a (multi)index labeling the eigenvalue \( \lambda_n \) of the eigenfunction \( Y_n(y) \). \( d \)-dimensional torus \( T^d \) with equal radii \( R \) is an especially simple example of the compact internal space of extra dimensions \( K_d \). The eigenfunctions and eigenvalues in this special case look like

\[ Y_n(y) = \frac{1}{\sqrt{V_d}} \exp \left( i \sum_{m=1}^{d} n_m y^m / R \right), \tag{3} \]

\[ \lambda_n = |n|^2, \quad |n|^2 = n_1^2 + n_2^2 + \ldots n_d^2, \quad n = (n_1, n_2, \ldots, n_d), \quad -\infty \leq n_m \leq \infty, \]

where \( n_m \) are integer numbers, \( V_d = (2\pi R)^d \) is the volume of the torus.

To reduce the multidimensional theory to the effective four-dimensional one we write a harmonic expansion for the multidimensional field \( \Phi(z) \)

\[ \Phi(z) = \Phi(x, y) = \sum_n \phi^{(n)}(x) Y_n(y). \tag{4} \]

The coefficients \( \phi^{(n)}(x) \) of the harmonic expansion (4) are called Kaluza-Klein (KK) excitations or KK modes, and they usually include the zero-mode \( \phi^{(0)}(x) \), corresponding to \( n = 0 \) and the eigenvalue \( \lambda_0 = 0 \). Substitution of the KK mode expansion into action (1) and integration over the internal space \( K_d \) gives

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \left( \partial_{\mu} \phi^{(0)} \right)^2 - \frac{m^2}{2} (\phi^{(0)})^2 + \frac{g}{4!} (\phi^{(0)})^4 \right\} + \sum_{n \neq 0} \left\{ \frac{1}{2} \left( \partial_{\mu} \phi^{(n)} \right)^2 - \frac{m_n^2}{2} \phi^{(n)*} \phi^{(n)} + \frac{g}{4!} (\phi^{(n)})^2 \sum_{n \neq 0} \phi^{(n)*} \phi^{(n)} \right\} + \ldots. \tag{5} \]

For the masses of the KK modes one obtains

\[ m_n^2 = m^2 + \frac{\lambda_n}{R^2}, \tag{6} \]
and the coupling constant $g$ of the four-dimensional theory is related to the coupling constant $G_{(4+d)}$ of the initial multidimensional theory by the equation

$$g = \frac{G_{(4+d)}}{V_d},$$

(7)

where $V_d$ is the volume of the compact internal space of extra dimensions $K_d$. The fundamental coupling constant $G_{(4+d)}$ has dimension $[\text{mass}]^{-d}$. So, the four-dimensional coupling constant $g$ is dimensionless one as it should be. Eqs. (6,7) represent the basic relations of Kaluza-Klein scenario. Similar relations take place for other types of multidimensional quantum field theoretical models. From four-dimensional point of view we can interpret each KK mode as a particle with the mass $m_n$ given by Eq. (6). We see that in according with Kaluza-Klein scenario any multidimensional field contains an infinite set of KK modes, i.e. an infinite set of four-dimensional particles with increasing masses, which is called the Kaluza-Klein tower. Therefore, an experimental observation of series KK excitations with a characteristic spectrum of the form (6) would be an evidence of the existence of extra dimensions.

As it was mentioned above in Introduction we have applied the main issues of Kaluza-Klein approach to our analysis of the structure of proton-proton total cross section at very low energies and calculated the fundamental scale (size) $R$ of the compact internal extra space. One obtained by this way

$$\frac{1}{R} = 41.481 \text{ MeV} \quad \text{or} \quad R = 24.1 \text{ GeV}^{-1} = 4.75 \times 10^{-13} \text{cm}.$$  

(8)

After that we have built the Kaluza-Klein tower of KK excitations by the formula

$$M_n = 2\sqrt{m_p^2 + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, \ldots)$$

(9)

and compared it with the observed irregularities in the spectrum of mass of the diproton system. The result of the comparison has been presented in Table 2 of ref. [1]. Now we would like to include in this table the experimental data on mass spectrum of the resonance states of proton-antiproton system above elastic threshold taking into account that Kaluza-Klein scenario predict $M^{pp}_n = M^{p\bar{p}}_n$. The extended version of Table 2 from ref. [1] is shown here in Table 1.

We have used Review of Particle Physics [4] and recent review article of Crystal Barrel Collaboration [5] where the experimental data on mass spectrum of the resonance states of proton-antiproton system above elastic threshold have been extracted from. As it is seen from Table 1, the nucleon-nucleon dynamics at low energies provides a quite remarkable confirmation of Kaluza-Klein picture. Moreover, Kaluza-Klein scenario predict a special sort of (super)symmetry between fermionic (dibaryon) and bosonic states, which is very nontrivial fact, and Table 1 contains an experimental confirmation of this fact as well.

Actually, we also see that there are an empty cells, especially $M_{21}$, $M_{23}$, $M_{25}$, $M_{27}$ – $M_{33}$, $M_{35}$, $M_{36}$, in the Table. In this respect we would like to request nuclear and particle physicists-experimenters to search missing two-nucleon states to fill the empty cells. We very hope that it would be possible to make it in the near future. For a convenience we also present here all possible of the calculated two-nucleon KK excitations with account of proton-neutron mass difference. There are all collected in Table 2, which may serve as a guide for the experimenters.
It should be stressed that we have not seen proton-antiproton resonances in the structure of proton-antiproton total cross section at low energies. This fact could be explained by crossing properties of the amplitudes. The crossing structure of the amplitudes is such as to result to suppression of nucleon-nucleon KK excitations in proton-antiproton channel and vice versa there is their enhancement in proton-proton channel.

### Table 1: Kaluza-Klein tower of KK excitations of \( pp(p\bar{p}) \) system and experimental data.

| n  | \( M_{pp} \) MeV | \( \Delta M_{pp} \) MeV | \( f_{\text{Rfs}} \) | \( M_{pp} \) MeV | \( \Delta M_{pp} \) MeV | \( f_{\text{Rfs}} \) |
|----|------------------|-----------------------|----------------|------------------|-----------------------|----------------|
| 1  | 1878.38          | 1877.5 ± 0.5          | [9]            | 1873 ± 2.5       | 1877.5 ± 0.5          | [9]            |
| 2  | 1883.87          | 1886 ± 1              | [6]            | 1870 ± 10        | 1870 ± 10             | [6]            |
| 3  | 1892.98          | 1898 ± 1              | [6]            | 1897 ± 1         | 1897 ± 1              | [6]            |
| 4  | 1905.66          | 1904 ± 2              | [10]           | 1910 ± 30        | 1908 ± 2              | [10]           |
| 5  | 1921.84          | 1916 ± 2              | [6]            | ~1920            | 1916 ± 2              | [6]            |
| 6  | 1941.44          | 1937 ± 2              | [6]            | 1939 ± 2         | 1939 ± 2              | [6]            |
| 7  | 1964.35          | 1965 ± 2              | [6]            | 1968             | 1968 ± 2              | [6]            |
| 8  | 1990.46          | 1980 ± 2              | [6]            | 1990 ± 15        | 1990 ± 15             | [6]            |
| 9  | 2019.63          | 2017 ± 3              | [6]            | 2020 ± 3         | 2020 ± 3              | [6]            |
| 10 | 2051.75          | 2046 ± 3              | [6]            | 2040 ± 40        | 2040 ± 40             | [6]            |
| 11 | 2086.68          | 2087 ± 3              | [6]            | 2080 ± 10        | 2080 ± 10             | [6]            |
| 12 | 2124.27          | 2120 ± 3.2            | [12]           | 2105 ± 15        | 2105 ± 15             | [12]           |
| 13 | 2164.39          | 2150 ± 12.6           | [12]           | 2165 ± 45        | 2165 ± 45             | [12]           |
| 14 | 2206.91          | 2192 ± 3              | [13]           | 2207 ± 13        | 2207 ± 13             | [13]           |

3 Conclusion

In this note we have presented additional arguments in favour of Kaluza and Klein picture of the world. We have shown that geniusly simple formula (9) provided by Kaluza-Klein approach gives an excellent description for the mass spectrum of two-nucleon system. It has also been established that the experimental data obtained at low energies where the nucleon-nucleon dynamics has been studied reveal a special sort of (super)symmetry between fermionic (dibaryon) and bosonic states predicted by Kaluza-Klein scenario. Of course it would be very desirable to state new experiments such as e.g. to fill an empty cells in Table 1 (like in Mendeleev Table!), and we hope this is a quite promising subject of investigations in particle and nuclear physics.
Table 2: Kaluza-Klein tower of KK excitations of two-nucleon system.

| $n$ | $M_{pp}^{nn}$ MeV | $M_{nn}^{pp}$ MeV | $M_{nn}^{nn}$ MeV |
|-----|-------------------|-------------------|-------------------|
| 1   | 1878.38           | 1879.67           | 1880.96           |
| 2   | 1883.87           | 1885.15           | 1886.44           |
| 3   | 1892.98           | 1894.26           | 1895.54           |
| 4   | 1905.66           | 1906.93           | 1908.21           |
| 5   | 1921.84           | 1923.11           | 1924.37           |
| 6   | 1941.44           | 1942.69           | 1943.94           |
| 7   | 1964.35           | 1965.59           | 1966.82           |
| 8   | 1990.46           | 1991.68           | 1992.89           |
| 9   | 2019.63           | 2020.84           | 2022.04           |
| 10  | 2051.75           | 2052.94           | 2054.12           |
| 11  | 2086.68           | 2087.84           | 2089.01           |
| 12  | 2124.27           | 2125.42           | 2126.56           |
| 13  | 2164.39           | 2165.52           | 2166.64           |
| 14  | 2206.91           | 2208.01           | 2209.11           |
| 15  | 2251.68           | 2252.75           | 2253.83           |
| 16  | 2298.57           | 2299.62           | 2300.68           |
| 17  | 2347.45           | 2348.49           | 2349.52           |
| 18  | 2398.21           | 2399.23           | 2400.24           |
| 19  | 2450.73           | 2451.72           | 2452.72           |
| 20  | 2504.90           | 2505.87           | 2506.84           |
| 21  | 2560.61           | 2561.56           | 2562.51           |
| 22  | 2617.76           | 2618.69           | 2619.62           |
| 23  | 2676.27           | 2677.17           | 2678.08           |
| 24  | 2736.04           | 2736.92           | 2737.81           |
| 25  | 2796.99           | 2797.86           | 2798.73           |
| 26  | 2859.05           | 2859.90           | 2860.75           |
| 27  | 2922.15           | 2922.98           | 2923.81           |
| 28  | 2986.22           | 2987.03           | 2987.85           |
| 29  | 3051.20           | 3052.00           | 3052.79           |
| 30  | 3117.04           | 3117.81           | 3118.59           |
| 31  | 3183.67           | 3184.43           | 3185.20           |
| 32  | 3251.06           | 3251.80           | 3252.55           |
| 33  | 3319.15           | 3319.88           | 3320.61           |
| 34  | 3387.90           | 3388.62           | 3389.34           |
| 35  | 3457.28           | 3457.99           | 3458.69           |
| 36  | 3527.25           | 3527.94           | 3528.63           |
| 37  | 3597.77           | 3598.44           | 3599.12           |
| 38  | 3668.81           | 3669.47           | 3670.13           |
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