Experimental study on fractal characteristics of coarse-grained soil filler for high-speed railway subgrade

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Abstract. This paper takes coarse-grained soil filler of high-speed railway subgrade as object of study, completes a series of coarse-grained soil compaction tests under different moisture content with electric heavy compaction apparatus type YX-30, and obtains the granularity fractal distribution curve with coarse-grained granularity fractal model. The result shows that coarse-grained soil filler has obvious fractal characteristics after compaction, and particle breakage fractal dimension between 2.61-2.82. With the increase of fractal dimension, the maximum dry density increases first and then decreases, and the maximum fractal dimension is under the optimum water content. The coarse-grained soil with high clay content has lower particle size and smaller fractal dimension before and after compaction. Water content and clay content are two important factors that affect the crushing of coarse-grained soil. The effect of clay content on particle size is more significant than that of clay content.

1. Introduction
Coarse-grained soil is a kind of rock-soil aggregate mixture whose particle size is 0.075 ~ 60 mm, and its mass fraction is more than 50%. It has good engineering properties such as fine degree, high permeability and high shear strength. Therefore, it is widely used in the construction of high-speed railway roadbed. Due to the point contact of coarse-grained soil particles, the particles are easily broken under the action of load, and the particles are distributed uniformly, which leads to the change of soil particles, and then affects the physical and mechanical properties of the coarse-grained soil particles [1]. Therefore, the research on the variation law of coarse-grained soil particle breakage in high-speed railway subgrade has guiding significance for engineering practice.

In the past, the researchers conducted a lot of experimental studies and analysis on the particle breakage of coarse-grained soil. The research contents mainly include the influence of soil particle breakage on engineering properties and the factors affecting soil particle breakage[2-4], the measurement of soil particle breakage and the describe of its essence[5-7], and consider the soil constitutive model affected by particle breakage[8-9]. Due to the uncertainty and randomness of coarse-grained soil fillers, it is difficult for the traditional theoretical model to describe and analyze this complex and regular system. Previous studies have shown that fractal theory can easily deal with the problems of fragmentation and extremely irregular materials in nature. The fractal theory can be used to deeply understand the particle size distribution characteristics of coarse grained soil and better analyze the relationship between the particle size distribution and the engineering performance of...
fillers. At present, the fractal characteristics of coarse-grained soil in high-speed railway roadbed are rarely analyzed and studied, even if it is limited to sand or dam material. Based on the previous studies, according to the requirements of high-speed railway coarse-grained soil filling grading, 10 sets of indoor compaction tests with 2 grades and 5 water contents were designed, and the fractal distribution characteristics of coarse-grained soil particles with different grades were studied, the variation law of particle breakage fractal and water content is obtained. The research conclusion can provide technical reference for the selection and filling of coarse-grained soil filler for high-speed railway subgrade.

2. Test materials and equipment

2.1. Test materials
In order to obtain qualified subgrade fillers, two kinds of graded coarse soil were obtained by mixing gravel and clay. Cumulative grading curve of sample size is shown in Figure 1. The content of different particle size components is shown in Table 1. The curvature of the mixed filler is 2.034 of the mixed filler. According to the regulations, the filler is fine breccia soil and the gradation good, and the filler belongs to group A. Therefore, it can be used as high-speed railway subgrade filler. The samples were air-dried before the test. And the water content of the test design samples was 2%, 4%, 6%, 8% and 10%. Samples were prepared and placed for 24 hours. Then the compaction test was carried out. The samples were compacted in 5 layers, 88 strike for each layer.

![Figure 1. Grain cumulative grading curves of samples.](image)

Table 1. Each size fraction content in different gradations.

| Name    | Content of each particle group with different particle diameter (mm)% | 25-40 | 20-25 | 10-20 | 5-10 | 2.5-5 | 0.63-2.5 | 0.75-0.63 | <0.075 |
|---------|---------------------------------------------------------------------|-------|-------|-------|------|-------|----------|-----------|--------|
| Sample 1 |                                                                     | 9.33  | 6.76  | 14.26 | 13.91| 17.99 | 17.07    | 11.95     | 8.72   |
| Sample 2 |                                                                     | 8.43  | 7.69  | 21.46 | 9.54 | 5.02  | 5.61     | 10.52     | 31.73  |

2.2. Test equipment
The test equipment used in the test is YX-30 heavy electric compactor for coarse grained soil. The diameter is 30cm, and the highly is 28.8cm of the compacting cylinder. The weight of the hammer is equal, the height of the fall is quite high, and the diameter of the hammer is quite large.

3. Fractal characteristics of coarse-grained soil

3.1. Coarse grain size fractal model.
Fractal geometry is used to describe irregular phenomena and behaviors in nature. At present, more applications are linear similarity, that is, self-similarity. France has put forward the fractal theory, and has established the two-dimensional granularity fractal dimension model.
In 1992, Tyler et al.[10] proposed a standardized equation for the relationship between mass and pore size, and established a weight distribution model of the fractal dimension of soil particle size distribution by assuming that different soil fractions have the same density.

According to the fractal theory, a sieve with a pore diameter \( r \) can be used to sieve the sample, and the total number of materials under the sieve is calculated as \( M_1(r) \), and the total number of materials on the sieve is calculated as \( M_2(r) \), and the total mass of the sample is \( M_t \), that is:

\[
M_t = M_1(r) + M_2(r)
\]  

(1)

Define the correlation function \( C(r) \) as:

\[
C(r) = \frac{M_1(r)}{M_t} = 1 - \frac{M_2(r)}{M_t}
\]  

(2)

Coarse-grained soil is an open self-organizing system. The particle size distribution describes the spatial structure of the material composition of the system. In three dimensions, the volume of coarse-grained soil particles larger than a certain size \( d_i \) is \( V \), there is:

\[
V = C_v \left[1 - \left( \frac{d_i}{\lambda_v} \right)^{3-D} \right]
\]  

(3)

Where: \( D \) is the fractal dimension; \( C_v \) and \( \lambda_v \) are constants, which are related to the size and shape of the coarse-grained soil particles.

According to Taylor’s and sub-fineness hypothesis, the particle size of coarse-grained soil has the same density, and the particle mass is larger than a certain particle size.

\[
M_2(d_i) = \rho C_v \left[1 - \left( \frac{d_i}{\lambda_v} \right)^{3-D} \right]
\]  

(4)

Let \( d_i = 0 \), then \( M_2(d_i) \) in formula (4) is the total mass of all particles \( M_t \):

\[
M_t = \rho C_v
\]  

(5)

Assuming that the maximum particle size in the coarse-grained soil sample is \( d_{max} \), then \( M_2(d_{max}) = 0 \), substituting into equation (4):

\[
\lambda_v = d_{max}
\]  

(6)

The fractal relationship between the mass of coarse-grained soil particles and the particle size can be obtained from formulas (2) to (6):

\[
\frac{M_1(d_i)}{M_t} = 1 - \frac{M_2(d_i)}{M_t} = \left( \frac{d_i}{d_{max}} \right)^{3-D}
\]  

(7)

The logarithm of the two sides is:

\[
\log \left[ \frac{M_1(d_i)}{M_t} \right] = (3-D) \log \left( \frac{d_i}{d_{max}} \right)
\]  

(8)

Where: \( M_t(d)/M_t \) can be obtained by particle sieving, and the slope \( k \) of the straight line part is obtained according to linear regression, which is the formula (8) \( (3-D) \) value, and then the fractal dimension is obtained:

\[
D = 3 - k
\]  

(9)
By calculation, the fractal dimension $D$ values of sample 1 and sample 2 before compaction were 2.792 and 2.615, respectively.

3.2. Analysis of particle breakage characteristics

The compaction test curve of coarse grained soil with different grades is shown in Figure 2.

![Compaction curves of coarse-grained soils with different gradations.](image)

Figure 2. Compaction curves of coarse-grained soils with different gradations.

The results show that there is a certain relationship between water content and dry density when the gradation composition of coarse soil is constant. When the water content is low, the dry density increases with the increase of water content, and when the water content increases to a certain value, the dry density increases with the increase of water content. The dry density decreases with the increase of water content. When the clay content in the sample is low, the optimum water content is lower and the maximum dry density is larger. This phenomenon shows that when the coarse particles play the role of skeleton in the sample, the fine particles can fully fill the pores, and the two particles can be filled closely with each other, so that the density reaches the maximum value.

At present, the determination of particle fragmentation degree is mostly based on the quantitative statistics of different particle size composition before and after the whole sample test. In this paper, the crushing rate $B_g$ defined by Marsal[4] is used as the measurement of particle fragmentation degree in this test, and the content difference value $W$ of each particle group before and after the test is calculated, and the sum of all positive values of $W$ is taken, that:

$$B_g = \sum |W_{ki} - W_{kf}|$$

(10)

Where: $W_{ki}$ is the mass fraction of a certain grain group on the pre-gradating blending line of coarse soil test; $W_{kf}$ is the mass fraction of the same grain group on the blending line of coarse graded soil after the test.

The breaking rate of the coarse soil particles was calculated and the relation with the water content was established, as shown in Figure 3 and Table 1.

![Relationships between broken rates and moisture contents.](image)

Figure 3. Relationships between broken rates and moisture contents.
The results show that under the same conditions, the filling rate of coarse soil increases first and then decreases with the increase of water content, and the water content corresponding to the maximum fragmentation rate is less than the best water content. The coarse grain content of soil increased with the increase of coarse grain content, and the grain bulk density rate of sample 2 was significantly higher than that of sample 1.

3.3. Fractal analysis of particle reakage

According to the fractal model of particle size, take \( \log(d_i/d_{\text{max}}) \) as the abscissa and \( \log(M_1(d_i)/M_t) \) as the ordinate, the fractal distribution curve of particle size after compaction under different gradation conditions is shown in Figure 4.

![Fractal distribution curve of particle size after compaction](image)

(a) Sample 1  
(b) Sample 2

Figure 4. Fractal distribution curve of particle size after compaction.

The linear function is used to fit the data in Figure 4, the correlation coefficient ranges from 0.92 to 0.98. The results show that \( \log(d_i/d_{\text{max}}) \) has a good linear relationship with \( \log(M_1(d_i)/M_t) \), and the particle size distribution of coarse-grained soil after crushing has fractal characteristics, the \( D \) value of fractal dimension of particle breakage is shown in Table 2 and Figure 5.

| Moisture content | Sample 1 | Sample 2 |
|------------------|----------|----------|
| 2%               | 1.85     | 2.798    |
| 4%               | 1.91     | 2.804    |
| 6%               | 1.99     | 2.809    |
| 8%               | 2.01     | 2.798    |
| 10%              | 1.96     | 2.803    |
| 12%              | 1.96     | 2.803    |
| 14%              | 2.01     | 2.798    |

| Dry density      | Sample 1 | Sample 2 |
|------------------|----------|----------|
| 1.85             | 2.798    | 2.615    |
| 1.91             | 2.804    | 2.668    |
| 1.99             | 2.809    | 2.676    |
| 2.01             | 2.798    | 2.679    |

| Fractal dimension| Sample 1 | Sample 2 |
|------------------|----------|----------|
| 2.798            | 2.803    | 2.615    |
| 2.804            | 2.809    | 2.668    |
| 2.809            | 2.803    | 2.676    |
| 2.803            | 2.809    | 2.679    |

| Difference of fractal dimension before and after compaction | Sample 1 | Sample 2 |
|-------------------------------------------------------------|----------|----------|
| 0.006                                                       | 0.007    | 0.011    |
| 0.012                                                       | 0.017    | 0.011    |
| 0.017                                                       | 0.006    | 0.053    |
| 0.011                                                       | 0.011    | 0.061    |
| 0.006                                                       | 0.007    | 0.061    |
| 0.011                                                       | 0.007    | 0.064    |
| 0.017                                                       | 0.011    | 0.057    |
| 0.011                                                       | 0.007    | 0.051    |


This can be seen in Table 2 and Figure 5, with the increase of water content, the fractal dimension of samples with different particle gradations increases first and then decreases as a whole, and the fractal dimension of optimal water content is the largest, but the overall value of fractal dimension does not change much with the change of water content. The reason is that as the moisture content of coarse soil increases gradually, the interparticle lubrication and friction resistance decrease. Under the action of compaction work, the effective contact pressure between particles increases, and the effective contact pressure increases significantly. Under the optimum water content, the fractal dimension of particles increases gradually and reaches the maximum value. With the increasing of the water content of coarse-grained soil, the water content between the particles increases and the void is filled, the particles are gradually in the overhead state, and the effective contact stress between the particles decreases gradually. Under the action of compaction load, the particle size decreases gradually, and the fractal dimension decreases gradually. The local fluctuation of the relationship between fractal dimension and water content may be caused by the inversion of coarse and fine particles.

It can also be seen from Table 2 that the difference of fractal dimension of sample 1 with relatively high clay content is relatively small before and after compaction, indicating that the degree of particle breakage of coarse soil with relatively high clay content is relatively low after compaction, and the difference of fractal dimension before and after compaction can be used to characterize the degree of particle breakage.

By comparing the fragmentation rate of coarse-grained soil and the relationship between the fractal dimension of particle size and the optimum water content, it is found that the correlation between the fractal dimension of particle fragmentation and the optimal moisture content is better than that of the crushing rate. Furthermore, it is shown that fractal dimension can be used to describe the degree and process of particle fragmentation in coarse-grained soil.

4. Conclusion

In this paper, two groups of graded coarse grained soil fillers for high-speed railway subgrade were prepared, and compaction tests of five moisture content states were conducted. Fractal geometry was used to analyze the fractal characteristics of coarse grained soil before and after compaction, and the results were as follows:

1) The coarse filler of high-speed railway subgrade has obvious fractal characteristics after compaction and crushing, and the fractal dimension of particle fragmentation is between 2.61~2.82.

2) With the increase of water content, the fractal dimension of samples at different particle levels increases first and then decreases, and the fractal dimension of the optimum water content is the largest.

3) The particle fragmentation degree of coarse soil with large clay content before and after compaction is lower, and the fractal dimension is relatively small.
(4) Water content and clay content are two important factors affecting the crushing of coarse-grained soil. In contrast, clay content has a more significant effect on the fragmentation of coarse-grained soil.

Acknowledgments
This study was partially supported by the National Natural Science Foundation of China under the Grants No.51508097, and also supported by the Provincial Undergraduate Training Program for Innovation and Entrepreneurship under the Grants No.201710388043 and No.201810388087, and also supported by Fujian Province outstanding scientific research personnel training project under the Grants No.GY-Z160147.

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