Interplay of soft and hard processes and hadron $p_T$ spectra in $pA$ and $AA$ collisions

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Motivated by a schematic model of multiple parton scattering within the Glauber formalism, the transverse momentum spectra in $pA$ and $AA$ collisions are analyzed in terms of a nuclear modification factor with respect to $pp$ collisions. The existing data at the CERN Super Proton Synchrotron (SPS) energies are shown to be consistent with the picture of Glauber multiple scattering in which the interplay between soft and hard processes and the effect of absorptive processes lead to nontrivial nuclear modification of the particle spectra. Relative to the additive model of incoherent hard scattering, the spectra are enhanced at large $p_T$ (hard) by multiple scattering while suppressed at low $p_T$ (soft) by absorptive correction with the transition occurring at around a scale $p_0 \sim 1 - 2$ GeV/$c$ that separates soft and hard processes. Around the same scale, the $p_T$ spectra in $pp$ collisions also change from an exponential form at low $p_T$ to a power-law behavior at high $p_T$. At very large $p_T \gg p_0$, the nuclear enhancement is shown to decrease like $1/p_T^2$. Implications of these nuclear effects on the study of jet quenching, parton thermalization and collective radial flow in high-energy $AA$ collisions are discussed.

I. INTRODUCTION

Hadron yields, spectra and correlations have been the focus of many experiments in relativistic heavy-ion collisions. They provide a snapshot of the state of matter when hadrons stopped interacting with each other, a stage often referred to as freeze-out in heavy-ion collisions. One can then infer the condition in the early stage prior to the freeze-out. Such a procedure relies crucially on our understanding of the dynamical evolution of the system. On the other hand, knowledge of the initial condition at the beginning of thermalization will also help one to unravel the history of evolution. This is especially the case when a complete thermalization cannot be achieved in certain regions of phase space (large $p_T$, for example). Therefore, it is important to study the hadron spectra in $p+p$ and $p+A$ collisions that will help us to understand the initial condition of the dense matter in high-energy heavy-ion collisions.

Hadron production at large transverse momentum in $p+p$ collisions is relatively well understood within the perturbative QCD (pQCD) parton model. Because of the large transverse momentum scale, the hard parton-parton scattering processes can be calculated in pQCD, while the non-perturbative effects can be factorized into universal parton distributions and parton fragmentation functions. These parton distributions and fragmentation functions can be independently measured in deep-inelastic electron-nucleon scattering, $e^+e^-$ annihilation and other processes. The calculated inclusive hadron spectra either in leading order [3] or next-to-leading order [2,4] agree very well with experimental data.

In $pA$ collisions, one should take into account multiple scatterings inside the nucleus. Because of the interference effect, these multiple scatterings will give rise to different $A$-scaling behavior of the spectra at different values of $p_T$. Generalizing to $AA$ collisions, one should expect similar behavior for the spectra of initially produced partons. These produced partons would undergo further interactions, possibly leading to thermalization (or partial thermalization) and expansion, which would be reflected in the final hadron spectra. Therefore, one should understand first the nuclear modification of the hadron spectra from $p+p$ to $p+A$ and to $A+A$ due to initial multiple scattering. Only then can one reliably extract information about the freeze-out conditions, e.g., temperature and flow velocity [3,4], and parton energy loss [4] from the final hadron spectra.

In this paper, we will analyze the nuclear modification of the hadron $p_T$ spectra in $p+A$ collisions motivated by a schematic model of multiple parton scattering in which coherence and absorptive corrections are included via the Glauber multiple scattering formalism. Multiple scatterings generally enhance the large $p_T$ spectra relative to the additive model of hard scattering. It can be shown [3] that absorptive corrections and the power-law behavior of perturbative parton cross section are the main reasons why the nuclear enhancement at high $p_T$ decreases like $1/p_T^2$. We will also show that the same absorptive processes suppress the spectra relative to the additive model at low $p_T$, where soft processes dominate and the $p_T$ spectra deviate from a power-law behavior. By analyzing the experimental data in terms of the proposed nuclear modification factor, we can phenomenologically determine the $p_T$ scale that separates soft and hard processes by the transition from nuclear suppression to nuclear enhancement of the $p_T$ spectra.

II. MULTIPLE PARTON SCATTERING IN $P + A$ COLLISIONS

Multiple hard parton scatterings in QCD are considered as high-twist processes and their contributions to
the cross section of hadronic collisions are generally suppressed by $1/Q^2$, where $Q$ is the momentum scale of the processes. The coefficients of these contributions are generally related to the multiple parton correlation functions inside a hadron [8]. In collisions involving a nucleus, such high-twist contributions are enhanced by $A^{1/3}$ due to the large nuclear size. For certain quantities like the transverse momentum imbalance of dijets in photon-nucleus collisions [9], parton energy loss in deep-inelastic lepton-nucleus scattering [10], broadening of the jet transverse momentum in deep-inelastic lepton-nucleus scattering [11] and Drell-Yan pairs in $p + A$ collisions [12], these high-twist terms are the leading contributions. However, such QCD treatment of multiple parton collisions is so far limited to the parton level and has not yet been extended to hadron spectra. In this paper, we will use instead a schematic Glauber model [12] of multiple parton scattering to motivate our analysis.

Let us denote $h_i^N$ as the differential cross section for a parton-nucleon scattering, $i + N \rightarrow j + X$,

$$h_i^N = \sum_{k,c} \int dx_b f_{i/N}(x_b) \frac{\hat{s}}{\pi} \frac{d \hat{\sigma}_{ib \rightarrow jc}}{dt} \delta(\hat{s} + \hat{t} + \hat{u})$$

(1)

where $f_{i/N}(x_b)$ is the parton distribution function inside a target nucleon, $\hat{s}$, $\hat{t}$ and $\hat{u}$ are Mandelstam variables and $d \hat{\sigma}_{ib \rightarrow jc}/dt$ are the differential parton-parton cross sections. In terms of $h_i^N$, we define the effective parton-nucleon total cross section as

$$\sigma_{iN}(p_i) = \frac{1}{2} \sum_j \int \frac{d^3p_j}{E_j} h_i^N(p_i, p_j),$$

(2)

and the differential nucleon-nucleon cross section for parton production as

$$E \frac{d\sigma_{NN}}{dp} = \sum_i \int \frac{dx_i}{x_i} f_{i/N}(x_i) h_i^N(p_i, p).$$

(3)

The effective parton-nucleon and nucleon-nucleon cross sections are only finite after one introduces some effective infrared cut-off in the parton-parton scattering processes. We will discuss this cut-off later. Following the approach by [12, 13], in which the Glauber approximation is used for multiple parton scattering, one can find the parton spectra in $pA$ collisions up to double scattering approximation as

$$E \frac{d\sigma_{NA}}{dp} \approx E \frac{d\sigma_{NN}}{dp} T_A(b) + \frac{1}{2} T_A^2(b) \sum_i \int \frac{dx_i}{x_i} f_{i/N}(x_i)$$

$$\times \left[ \sum_k \int \frac{d^3p_k}{E_k} h_k^N h_{kN}^N - (\sigma_{iN} + \sigma_{jN}) h_i^N \right],$$

(4)

where $T_A(b)$ is the nuclear thickness function with normalization $\int d^2b T_A(b) = A$. We take the nucleus to be a hard sphere of radius $r_A = r_0 A^{1/3}$ with $r_0 = 1.14$ fm.

One can then obtain the ratio of the differential cross sections of $N + A \rightarrow j + X$ and $N + N \rightarrow j + X$,

$$R_A = \frac{E d^3\sigma_{NA}/d^3p}{E d^3\sigma_{NN}/d^3p} = \left[ 1 + \frac{9A^{1/3}}{16\pi^2 E d\sigma^2_{NN}/d^3p} \sum_i \int \frac{dx_i}{x_i} f_{i/N}(x_i) \right] \times \left[ \sum_k \int \frac{d^3p_k}{E_k} h_k^N h_{kN}^N - (\sigma_{iN} + \sigma_{jN}) h_i^N \right].$$

(5)

Compared to the additive model of parton scattering which gives $R_A = 1$, the second term in the above equation gives us the nuclear modification of the parton spectrum due to multiple parton scattering inside a nucleus. This term contains contributions from both the double parton scattering and the absorptive correction to the single scattering processes. Notice that the absorptive contribution at this order is negative. As we will demonstrate, it is the cancellation by the absorptive correction that leads to many interesting and nontrivial features of the nuclear modification of the particle spectra.

### III. Nuclear Modification: A Schematic Study

As a demonstration of the consequences of multiple parton scattering, absorptive correction and the resultant interplay between low and high $p_T$ behavior of the particle spectra in $pA$ collisions, we illustrate the nuclear modification of parton spectra in a schematic model. Assuming parton-hadron duality, the conclusions can be applied qualitatively to hadron spectra.

For a schematic study, let us assume that all partons are identical and the differential parton-nucleon cross sections have a simple regularized power-law form in $p_T$,

$$h_i^N \equiv h(p_T) = \frac{C}{(p_T^2 + p_0^2)^n}, \quad (|y| < \Delta Y/2),$$

(6)

with the total parton-nucleon cross section,

$$\sigma_{iN} \equiv \sigma = \int dy d^2p_T \frac{C}{(p_T^2 + p_0^2)^n} = \frac{\pi \Delta Y C}{(n-1)p_0^{2n-2}}.$$  

(7)

Such a form is motivated by the fact that both the pQCD calculation and experimentally measured differential jet production cross sections show such a power-law behavior. The parameter $p_0$ is introduced as an infrared cut-off. It can be regarded phenomenologically as a scale that separates soft and hard processes, because for $p_T < p_0$ the parton spectra deviate significantly from a power-law behavior.
Using this schematic parton-nucleon cross section, the nuclear modification factor can be simplified as

$$R_A = 1 + \frac{9}{16} A^{1/3} \frac{\sigma}{\pi r_0^2} \left\{ \left( \frac{n-1}{\pi} \right) \left( 1 + \frac{p_T^2}{p_0^2} \right)^n \right\} \times \int d^2y_T \left[ 1 - \frac{p_T}{p_0} \left( \frac{p_T}{p_0} - \frac{y_T}{\gamma_T} \right)^2 \right]^{-n} - 2 \right\}. \quad (8)$$

This modification factor as a function of $p_T/p_0$ for different values of $n$ is shown in Fig. 1, where we plot $R_A - 1$ in units of the modification strength $9A^{1/3}\sigma/(16\pi r_0^2)$. One can also evaluate the nuclear modification factor analytically at two different limits of $p_T$,

$$R_A = 1 + \frac{9}{16} A^{1/3} \frac{\sigma}{\pi r_0^2} \left[ \frac{2n^2 p_0^2}{n-2} p_T^2 + O(p_T^4) \right], \quad (p_T \gg p_0);$$

$$R_A = 1 + \frac{9}{16} A^{1/3} \frac{\sigma}{\pi r_0^2} \left[ \frac{3n-1}{2n-1} + \frac{n(n-1)(2n+3)p_T^2}{2(4n^2-1) p_0^2} + O(p_T^4) \right], \quad (p_T \ll p_0). \quad (9)$$

In this regime, the absorptive parts dominate and $R_A \propto 1/A^{1/3}$. This is consistent with the wounded nucleon model for soft particle production. The coefficient $c \leq 1$ is a result of partial cancellation by contributions from multiple scattering.

It is clear that the shape of the nuclear modification of the inclusive spectra results from the interplay between multiple scattering and absorptive corrections. This is especially true at low $p_T$. Even though it gives the apparent broadening of the effective spectra in $pA$ relative to $pp$ collisions, the underlying physics is very different from the random-walk model \cite{14} for the $p_T$ broadening of soft hadrons. One can expect the same behavior in $A + A$ collisions if there is no additional nuclear effect, such as final state rescattering or jet quenching due to radiative parton energy loss. Analyzing the experimental data in this framework with respect to the above baseline behavior would allow one to identify and study new effects caused by the presence of dense matter.

Another important feature of the modification factor is its sensitivity to the form of particle spectra in $pp$ collisions. At low $p_T$, its behavior is not very sensitive to the form of differential parton-nucleon cross section. If a Gaussian form of the spectrum is used, the modification factor remains roughly the same at $p_T \sim 0$. In contrast, the high-$p_T$ behavior of the enhancement, $\sim 1/p_T^4$, is strictly the consequence of the power-law form of the parton-nucleon cross section. In fact, with a Gaussian form, the enhancement will increase with $p_T^2$ exponentially. Therefore, the observed large $p_T$ behavior of the Cronin enhancement \cite{14} is consistent with the power-law form of the jet production cross section and with the picture of multiple parton scattering in $p + A$ collisions. The nuclear modification factor at large $p_T$ also depends on the power of the $pp$ spectra. Since the power $n$ decreases with energy, one should expect $R_A$ to decrease with energy. This trend has been observed in experiments \cite{15} in the energy range $\sqrt{s} = 20$–40 GeV.

Overall, the nuclear modification factor $R_A$ has nontrivial and interesting $p_T$ dependence. It is smaller than 1 at
low $p_T$ and larger than 1 at intermediate $p_T$. The transition ($R_A = 1$) occurs at around $p_T \sim p_0/\sqrt{s} \sim \sqrt{p_T^2}$. It approaches 1 again at very large $p_T \gg p_0$. The modification strength at both low and high $p_T$ is proportional to the size of the nucleus. As we will see in the next section, this is qualitatively consistent with the experimental data of inclusive hadron spectra in $pA$ collisions. Since $p_0$ is a momentum scale that sets the onset of power-law-like spectra in $pp$ collisions, one can consider it a scale separating soft and hard processes. According to the schematic model, one should be able to determine independently this scale from the nuclear modification factor of the hadron spectra.

IV. ANALYSIS OF EXPERIMENTAL DATA

To facilitate the analysis of $pA$ and $AA$ spectra in terms of the nuclear modification factor $R_A$, one needs to know the $pp$ spectra at the same energy. Since we will have to take ratios of hadron spectra from different experiments with different $p_T$ bins, this can only be achieved by parameterizing the baseline spectra. Shown in Fig. 2 are the measured negative hadron spectra [16,17] and the parameterization in $pp$ collisions at $E_{lab} = 200$ GeV. We use a two-component parameterization,

$$f(p_T) = C_0 e^{-m_T/T_0} + C \frac{1 - 2p_T/\sqrt{s}}{(p_T^2 + p_E^2)^n}.$$  \hspace{1cm} (11)

The fit parameters are listed in Table I for different spectra at different energies. The parameters in the exponential form are mainly determined by the spectra below $p_T < 1$ GeV/$c$ while those in the power-law form are mainly determined by the large $p_T$ region. Shown as a dashed line is the contribution from the exponential component. It is clear that at $p_T > 2$ GeV/$c$, the spectrum is already dominated by the power-law behavior. The power-law form we use for the CERN-SPS energy range contains a factor $(1 - 2p_T/\sqrt{s})^n$ that is caused by the rapid decrease of quark distributions at large $x \sim 1$. This factor will become negligible at higher energies when $2p_T/\sqrt{s} \ll 1$.

In the original UA1 parameterization [18], a power-law form $A_0/(p_T + p_0)^n$ is used to fit the hadron spectra over a large range of energies. While this single power-law is sufficient to fit the spectra for $p_T > 0.2$ GeV/$c$, an exponential term is necessary to fit the spectra at low transverse momentum $p_T < 0.2$ GeV/$c$.

In order to compare to the spectra in $pA$ and $AA$ collisions for near isospin symmetrical nuclei, the data shown in Fig. 2 at large $p_T$ are for ”negative” hadrons with the $\pi^-$ contribution being replaced by an isospin averaged value, $\tilde{h}_0 = (\pi^+ + \pi^-)/2 + K^- + \bar{p}$. Since particle production at large $p_T$ is dominated by the leading hadrons from valence quark scattering and there are more up-quarks than down-quarks in a $pp$ system, one should see more $\pi^+$ than $\pi^-$ at large $p_T$. Shown in the inserted box at the lower-left corner are the ratios of $h^-/\pi^-$ and $\tilde{h}_0/\pi^\pm$ [$\pi^\pm = (\pi^+ + \pi^-)/2$]. Note that $h^-/\pi^-$ decreases with $p_T$ as expected while $\tilde{h}_0/\pi^\pm$ remains relatively constant. At

![FIG. 2. Negative hadron spectra in $pp$ collisions at $E_{lab} = 200$ GeV from Refs. [16,17] and the fit (solid line) according to Eq. (11) with fit parameters given in Table I. The dashed line is the underlying exponential component. The $\tilde{h}_0$ is defined as $\tilde{h}_0 = (\pi^+ + \pi^-)/2 + K^- + \bar{p}$. The upper inserted box shows the ratio of parameterizations (solid line) at $E_{lab} = 158$ and 200 GeV and the corresponding pQCD parton model calculation (dot-dashed line). The lower box shows the ratio of $h^-/\pi^\pm$ and $\tilde{h}_0/\pi^\pm$.](https://example.com/fig2.png)

| $h^-$ ($pp, E_{lab} = 158$ GeV) | $C_0$ (mbGeV$^{-2}$) | $T_0$ (MeV) | $C$ (mbGeV$^{-2}$) | $a$ | $p_0$ (GeV/c) | $n$ |
|---|---|---|---|---|---|---|
| 169.6 | 154 | 24565 | 9.3 | 2.43 | 6.29 |
| 174.7 | 154 | 13653 | 9.1 | 2.27 | 6.22 |
| 174.7 | 154 | 17653 | 9.8 | 2.27 | 6.29 |
| 150.3 | 154 | 13653 | 9.1 | 2.37 | 6.22 |
| 2.43 | 154 | 653 | 0 | 1.75 | 6.22 |
low $p_T$, $h^-$ and $h_0^-$ are approximately the same within a few percent accuracy. This isospin dependence can be described well by the pQCD parton model [23]. With $h_0^-$ and $h^-$, one will be able to estimate the isospin effect in $pA$ and $AA$ collisions. In the inserted box at the upper-right corner we also plot the ratio of the hadron spectra in $pp$ collisions at $\sqrt{s} = 200$ GeV. Since there are no experimental data for $pp$ collisions at $E_{\text{lab}} = 158$ GeV, we parameterize the spectrum at $E_{\text{lab}} = 158$ GeV to fit the ratio (dot-dashed line) obtained from a pQCD parton model calculation [23].

\[ R_{AB}(p_T) = \frac{d\sigma_{AB}}{dyd^2p_T} \frac{(N_{\text{binary}})}{d\sigma_{NN}}(p+p)_{\text{lab}} \]

for $AB$ collisions, where $\langle N_{\text{binary}} \rangle = \int d^2b T_{AB}(b)$ is the number of binary collisions averaged over the impact-parameter range of the corresponding centrality. Here $T_{AB}(b)$ is the nuclear overlap function for $AB$ collisions at impact parameter $b$. For minimum-biased events, $\langle N_{\text{binary}} \rangle = AB$. For the purpose of the study in this paper, we will select event centrality according to the geometrical cross sections.

Shown in Figs. 4 and 5 are the nuclear modification factors for hadron spectra in $pA$ and $AB$ collisions at $E_{\text{lab}} = 200$ GeV. The data are from Refs. [16,17,19].

At the BNL Relativistic Heavy-ion Collider (RHIC) energy, we expect the power-law contribution to become more important. Shown in Fig. 3 is the charged hadron spectrum in $p\bar{p}$ collisions at $\sqrt{s} = 200$ GeV [13] and the parameterization. The power-law is indeed more prominent than at the CERN-SPS energy. There is a general trend that both $p_0$ and $n$ of the power-law component decrease with colliding energy. However, the underlying exponential term remains the same.

With these parameterizations of hadron spectra in $pp$ collisions, we can analyze the hadron spectra in $pA$ and $AA$ collisions in terms of a nuclear modification factor which is defined in general as

\[ R_{AB}(p_T) = \frac{d\sigma_{AB}}{dyd^2p_T} \frac{(N_{\text{binary}})}{d\sigma_{NN}}(p+p)_{\text{lab}} \]

for $AB$ collisions, where $\langle N_{\text{binary}} \rangle = \int d^2b T_{AB}(b)$ is the number of binary collisions averaged over the impact-parameter range of the corresponding centrality. Here $T_{AB}(b)$ is the nuclear overlap function for $AB$ collisions at impact parameter $b$. For minimum-biased events, $\langle N_{\text{binary}} \rangle = AB$. For the purpose of the study in this paper, we will select event centrality according to the geometrical cross sections.
intermediate values of $p_T$ and decrease again to approach to 1 at very large $p_T$. However, at $p_T = 4$ GeV/$c$, one is already close to the kinetic limit of $\sqrt{s}/2$ at the CERN-SPS energy. Near the kinetic limit the spectra become sensitive to other nuclear effects such as the Fermi motion that will increase the parton distribution function at $x \sim 1$, leading to increase of $R_{AB}$ again. In $pA$ collisions at the RHIC energy, one will certainly see the decrease of $R_A$ again at large $p_T \gg 2$ GeV/$c$.

The increase of the nuclear modification factor with $p_T$ due to the onset of hard parton scatterings appears very similar to the broadening of hadron spectra due to collective radial flow in a hydrodynamic picture. If there are strong interactions among partons in the early stage and hadrons in the late stage of heavy-ion collisions, the system will be driven to local thermal equilibrium. The information of initial multiple parton scattering contained in the initial parton spectra could be partially or completely erased by the thermalization. Collective flow then will be developed and hadron spectra will become broader than in $pp$ and $pA$ collisions due to the boost by a collective flow velocity $v$ [24]. Such an effect essentially will also increase $R_{AB}$ as a function of $p_T$. The question is how one can distinguish these two apparently different dynamics that produce the same final hadron spectra. Mass dependence of the hadron spectra has been proposed as a unique measure of the collective flow effect in heavy-ion collisions [23]. Since a collective flow provides a common velocity boost for all particles, heavy particles will then acquire more transverse momentum in the nonrelativistic region ($p_T \lesssim m_h$). One then should see a linear mass dependence of the slope parameter from an exponential fit of the measured hadron spectra [23]. In the current parton model $[23]$, one cannot exclude the mass dependence of the nuclear modification factor. It is therefore important to perform the current analysis for different hadron species in $pp$ and $pA$ collisions to find out whether nuclear modification factors have a mass dependence. Only then can one find out whether there is complete or partial thermalization and to what extent the effect of initial multiple parton scatterings has survived the final state interaction and contributed to the apparent collective radial flow.

V. HIGH $p_T$ SPECTRA AND PARTON ENERGY LOSS

Since the interaction for an energetic parton inside a medium is dominated by small angle forward scatterings, thermalization will be less complete for high $p_T$ partons. Hydrodynamic description will then become less relevant. Large $p_T$ hadron spectra will be determined by how an energetic parton propagates inside the medium. Since the large $p_T$ hadron spectra are calculable in the pQCD parton model, they are good probes of the parton dynamics in dense matter. The nuclear modification factor at large $p_T$ is a convenient and efficient way to determine the effects of the final state interaction of energetic partons inside a medium.

![Graph 5](image5.png)

FIG. 5. The nuclear modification factor $R_{AB}(p_T)$ for hadrons in $AB$ collisions at $E_{lab} = 158$ GeV. The data are from Refs. [20–22]. The line is the pQCD parton model calculation.

![Graph 6](image6.png)

FIG. 6. The predicted nuclear modification factor $R_{AA}(p_T)$ for $\pi_0$ spectra in $Pb + Pb$ collisions at $E_{lab} = 158$ GeV with different centrality cuts.

To demonstrate whether the experimental data at the CERN-SPS are consistent with the picture of multiple parton scattering, we also plot in Fig. 5 the pQCD-inspired parton model calculation of the nuclear modification factor [24]. In such a model, one extends the collinear factorized parton model to include intrinsic transverse momentum and its broadening due to multiple scattering in nuclear matter. The value of the intrinsic transverse momentum and its nuclear broadening are adjusted once and the model can reproduce most of the experimental data in $pp$ and $pA$ collisions [23]. In $AA$ collisions, the effect of parton energy loss is modeled by the modification of parton fragmentation functions [24].
in which the distribution of leading hadrons from parton fragmentation is suppressed due to parton energy loss. We will not describe the model here and refer readers to Ref. [24] for detail. We note that the behavior of the nuclear modification factor at \( p_T > 2.0 \text{ GeV/c} \) in \( Pb + Pb \) collisions is well described by the model without any additional final state medium effect such as the parton energy loss or jet quenching [3]. Since the pQCD parton model cannot deal with soft processes, the calculations are only reliable for \( p_T > 2 \text{ GeV/c} \). The \( p_T \) broadening due to multiple parton scattering is essential to account for the \( p_T \) dependence of the nuclear modification factor. Different variations of the parton model studies [27] give the same conclusion. Such a good agreement is not just a coincidence. The calculated absolute differential cross section for different colliding systems \((S + S, S + Au \text{ and } Pb + Pb)\) and for different centralities also agree well with the experimental data [3]. Shown in Fig. 6a are the calculated nuclear modification factors for \( \pi^0 \) spectra in \( Pb + Pb \) collisions with different centrality cuts, assuming no parton energy loss. Though the \( p_T \) dependences of the modification factors are similar, the magnitudes increase with centrality. The impact-parameter ranges for different centralities are determined by the fractions of geometrical nuclear cross sections.

Recent theoretical studies of parton propagation in a dense medium predict a substantial parton energy loss due to induced gluon radiation as the parton interacts with the dense medium [29, 32]. If a dense medium is produced in heavy-ion collisions, large \( p_T \) jets will lose energy as they propagate through the medium. This will lead to suppression of large \( p_T \) hadrons [3]. However, as shown by Fig. 6b, the experimental data of central \( Pb + Pb \) collisions at the CERN-SPS are consistent with the picture of initial multiple parton scattering without any parton energy loss in the final state. Careful analysis of the data against the pQCD parton model can actually exclude effects of any significant amount of parton energy loss in central \( Pb + Pb \) collisions at the CERN-SPS [33]. To be consistent with the current theoretical estimates of parton energy loss in a dense medium, these data imply that the initial energy density would be small and the lifetime of the dense system is very short in \( Pb + Pb \) collisions at the CERN-SPS. On the other hand, preliminary data from RHIC experiments [24] have shown significant suppression of high \( p_T \) hadron spectra. Combined with the null results on hadron suppression in heavy-ion collisions at the CERN-SPS, RHIC data would imply the onset of parton energy loss due to the increasing initial energy density and the lifetime of the dense system created in heavy-ion collisions at RHIC energies. Study of the nuclear modification factors for hadron spectra at large \( p_T \) will help to determine the average parton energy loss in the medium produced in heavy-ion collisions [3].

The nuclear modification factor will also help us to understand the energy dependence of the parton energy loss. Earlier theoretical studies [29, 31] gave a constant or a weakly energy-dependent parton energy loss \( dE/dx \).

However, more recent studies in an opacity expansion approach [31, 32] predict a stronger energy-dependence for \( dE/dx \) when the parton energy is small \((E < 10 \mu, \mu \) being the Debye screening mass in the medium). Parameterization of the numerical results of Ref. [31] gives \( dE/dx \propto E^{1.5}/(10 + E)^{1.25} \) for \( \mu = 0.5 \text{ GeV} \). The coefficient depends linearly on the initial parton density and the system size. This form has a strong energy dependence for \( E \sim 10 \text{ GeV} \) and a weak dependence at large \( E \).

![FIG. 7. The parton model calculation of nuclear modification factor \( R_{AB}(p_T) \) for hadrons in central \( Au + Au \) collisions at \( \sqrt{s} = 130 \text{ GeV} \) with different forms of parton energy loss. \( \lambda_q \) is the value of mean free path for a propagating quarks used in the calculation.](image)

Shown in Fig. 7 are the nuclear modification factors calculated in a pQCD parton model [23] for different forms of parton energy loss. The nuclear modification factor is suppressed to become smaller than 1 for non-vanishing parton energy loss. For a constant \( dE/dx \), the modification factor increases with \( p_T \) after the initial drop. This is because that the constant energy loss becomes relatively less important for higher initial parton energy. For asymptotically large \( p_T \), the finite and constant energy loss would become negligible and the modification factor \( R_{AB} \) would approach 1. However, for an energy loss that has a strong energy dependence, the shape of the modification factor is very different. If this is the case, the energy loss for a parton in the small \( p_T \) region is still very small and only becomes sizable at large \( p_T \). Consequently, the effect of energy loss is also very small. The modification factor will then increase, following the trend of the initial parton spectra due to the transition from soft to hard scatterings. However, when larger energy loss sets in at higher \( p_T \), the hadron spectra are strongly suppressed leading to much smaller values of \( R_{AB} \). We should note that the \( dE/dx \) used in these calculations is only representative of its value averaged over the entire evolution of the dense system. Since the energy loss is directly proportional to the parton den-
sity of the medium which decreases very fast due to rapid expansion, a small averaged $dE/dx$ still corresponds to large initial energy loss and large initial parton density.

Similar to the interplay between soft and hard processes in the initial parton production, there should also be a smooth transition between effects of hydrodynamic evolution at low $p_T$ and parton energy loss at high $p_T$. The hydrodynamic picture is relevant only if there is local thermalization, which is more likely for low $p_T$ partons. It breaks down for very large $p_T$ partons because the small angle scatterings in QCD are not sufficient to bring these partons in equilibrium with the rest of the system. These high $p_T$ partons will suffer energy loss as they propagate through the medium leading to suppression of large $p_T$ hadrons. However, at intermediate $p_T$, one cannot neglect the effect of partial thermalization or the detailed balance. For example, one should consider both gluon absorption and radiation by a propagating parton. So far all theoretical studies have only considered gluon radiation. If gluon absorption is also included, the effective parton energy loss will be reduced significantly. Since the effect of gluon absorption will be small for high energy partons ($p_T \gg T$, $T$ being the temperature of the thermal medium), the absorption will further increase the energy dependence of the effective parton energy loss. The $p_T$ dependence of the nuclear modification factor $R_{AB}(p_T)$ at intermediate $p_T$ will then provide useful information about the thermalization of the dense medium.

VI. CONCLUSIONS

In this paper we have proposed to analyze the hadron transverse momentum spectra in terms of the nuclear modification factor $R_{AB}(p_T)$ which is defined in such a way that a naive additive model of incoherent hard parton scattering would give $R_{AB} = 1$. We demonstrated in a schematic model of Glauber multiple parton scattering that the modification factor $R_{AB}(p_T)$ has a nontrivial $p_T$ dependence due to the absorptive processes and the interplay between soft and hard parton scattering, excluding final state scatterings. Because of the absorptive processes, the hadron production at small $p_T \sim 0$ is coherent and the hadron spectra in $AB$ collisions are proportional to the number of participant nucleons, leading to $R_{AB} < 1$. At large $p_T$ the hard parton scatterings become incoherent. Multiple parton scatterings then enhance the hadron spectra so that $R_{AB} > 1$. The momentum scale $p_0$ at which the transition occurs [$R_{AB}(p_0) = 1$] can be identified as the scale that separates soft and hard processes underlining both $pp$ and $AB$ collisions. Analyses of the existing experimental data on $pp$, $pA$ and $AB$ collisions indicate that $p_0 \approx 1 - 2$ GeV/c.

We pointed out that such analyses of future experimental data are important to study the effect of final state interactions. At low $p_T$, collective radial flow from hydrodynamic expansion gives similar $p_T$ dependence of $R_{AB}$. Disentangling the effects of initial multiple scattering and the radial flow would require a careful study of the modification factor in $pA$ collisions, especially its dependence on the hadron mass. At large $p_T$, parton energy loss will lead to suppression of the hadron spectra. Experimental measurement of $R_{AB}$ will provide important information on the initial parton density that is produced in heavy-ion collisions. At the intermediate $p_T$, we have shown that the $p_T$ dependence of $R_{AB}$ is sensitive to the energy dependence of $dE/dx$. This in turn is related to gluon absorption by the propagating partons reflecting the detailed balance in an equilibrating system.

Although experimental data at the CERN-SPS do not indicate any sign of parton energy loss, recent preliminary experimental data from RHIC show significant suppression of large $p_T$ hadrons. Detailed studies of the nuclear modification factor $R_{AB}(p_T)$ over the whole range of $p_T$ in both $pA$ and $AA$ collisions, will help us not only to determine the parton energy loss or the initial parton density but also to find out the degree of thermalization and radial collective flow. Recent studies also pointed out that parton energy loss can also lead to azimuthal anisotropy in high $p_T$ hadron spectra which is significantly different from hydrodynamic behavior. A combination of studies on $R_{AB}$ and the azimuthal anisotropy at high $p_T$ will provide a window to the initial condition and dynamics of early evolution of the dense matter that was never possible before.

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