Analytical approach to subhalo population in dark matter haloes

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ABSTRACT

In the standard model of cosmic structure formation, dark matter haloes form by gravitational instability. The process is hierarchical: smaller systems collapse earlier, and later merge to form larger haloes. The galaxy clusters, hosted by the largest dark matter haloes, are at the top of this hierarchy and representing the largest as well as the last structures formed in the Universe, while the smaller and first haloes are those Earth-sized dark subhaloes that have been both predicted by theoretical considerations and found in numerical simulations, though there do not exist any observational hints of their existence. The probability that a halo of mass \( m \) at redshift \( z \) will be part of a larger halo of mass \( M \) at the present time can be described in the frame of the extended Press & Schechter theory making use of the progenitor (conditional) mass function. Using the progenitor mass function, we calculate analytically, at redshift zero, the distribution of subhaloes in mass, formation epoch and rarity of the peak of the density field at the formation epoch. That is done for a Milky Way size system, assuming both a spherical and an ellipsoidal collapse model. Our calculation assumes that small progenitors do not lose mass due to dynamical processes after entering the parent halo, and that they do not interact with other subhaloes. For a \( \Lambda \) cold dark matter power spectrum, we obtain a subhalo mass function \( dn/dm \) proportional to \( m^{-\alpha} \) with a model-independent \( \alpha \sim 2 \). Assuming that the dark matter is a weakly interacting massive particle, the inferred distributions are used to test the feasibility of an indirect detection in the \( \gamma \)-ray energy band of such a population of subhaloes with a \textit{Gamma-ray Large Area Space Telescope} like satellite.

Key words: methods: analytical – galaxies: haloes – cosmology: theory – dark matter.

1 INTRODUCTION

The present-day description of the universe includes the presence of a large amount of cold dark matter (CDM) whose nature and distribution is unknown. This DM provides about 26 per cent of the energy budget of the universe.

The amount and properties of CDM are well constrained by astrophysical observations such as the anisotropies in the cosmic microwave background, large-scale structure and distant Type Ia supernovae (Spergel et al. 2003; Astier et al. 2006; Tegmark et al. 2006). On the other hand, two main open questions arise. The first concerns the particle physics nature of the CDM. Weakly interacting massive particles are attractive candidates since their relic abundance can fit the observed one (Dimopoulos 1990). Stable neutralinos in supersymmetric extensions of the standard model (Jungman, Kamionkowski & Griest 1996; Bertone, Hoooper & Silk 2005) or Kaluza–Klein particles in theories with a TeV−1 size universal extra dimension (Appelquist, Cheng & Dobrescu 2001; Servant & Tait 2003) are the most commonly studied particles. Since these particles have never been observed, there is a large uncertainty on the prediction of their effects, which has to be taken into account. The other open question regards the distribution of DM inside the haloes. Numerical \( N \)-body simulations (Navarro et al. 1997, hereafter NFW; Navarro et al. 2003; Diemand et al. 2004b), whose scale resolution is about \( \sim \)1 kpc, allow solely an extrapolation of the very inner slope of the DM profile and do not take into account interactions with the baryons which fall in the DM potential well or the controversial effect of the presence of a black hole (BH) at the centre of the halo (Ullio, Zhao & Kamionkowski 2001; Merritt et al. 2002; Bertone & Merritt 2005a, b). Indeed, if the BH has accreted adiabatically, it can create a pronounced cusp in an influence region ranging from a fraction of parsec to a few parsec depending on the velocity dispersion and the BH mass; on the other hand, in the case of BH merging events a core of 10–100 pc could be formed. Experimental data on DM distribution in the haloes of galaxies and clusters are not conclusive too (see e.g. the discussion in Fornengo, Pieri &
Scopel 2004). In the hierarchical formation scheme of the CDM scenario, large systems are the result of the merging and accretion of smaller haloes (subhaloes), whose dense central cores would survive the merging event and continue to orbit within the parent halo, as shown by high-resolution N-body simulations (Moore et al. 1999; Blaši & Sheth 2000; Ghigna et al. 2000; Giocoli, Tormen & van den Bosch 2008). CDM models are characterized by an excess of power on small scales. The arising divergence of the linear density contrast at large wavenumbers has been proved to be damped by collisional processes and free streaming, respectively, before and after kinetic decoupling, leading to exponential damping of the linear CDM density contrast and to the existence of a typical scale (Jeans scale) for the first haloes corresponding to a Jeans mass of about $10^{-6} M_{\odot}$ (Hofmann, Schwarz & Stöcker 2001; Green, Hofmann & Schwarz 2004, 2005). Numerical simulations have indeed reproduced hierarchical clustering in CDM cosmologies with a mass resolution sufficient to resolve the Jeans mass (Diemand, Moore & Stadel 2005b) with particle mass $m_p = 1.2 \times 10^{-10} M_{\odot}$ and force resolution of $\epsilon = 0.01$ pc; however, such a high-resolution run could be evolved only to $z = 26$, in a very small spatial patch, and producing haloes of mass $[10^{-6}, 10^{-2}] M_{\odot}$.

Among the simulations evolved on larger scales and to redshift $z = 0$, present milestones are the Millennium Simulation (Springel et al. 2005) and the Via Lactea Simulation (Diemand, Kuhlen & Madau 2007a). The first is a cosmological N-Body run with over 10 billion particles in a cubic region 500 Mpc $h^{-1}$ on a side (particle mass $m_p = 1.23 \times 10^9 M_{\odot}$; force resolution $\epsilon = 7$ kpc); the second was done to obtain a simulated Milky Way (MW) with the highest possible mass resolution (particle mass $m_p = 2.09 \times 10^6 M_{\odot}$; force resolution $\epsilon = 90$ pc). However, a simulation with the mass and force resolution similar to that of Diemand et al. (2005b), evolved to redshift zero over a region containing a mass comparable to that of our Galaxy, would require about $10^{12}$ particles and a time-resolution of a few hundred years. Such requirements are way beyond the computational capabilities of present-day supercomputers: applying Moore's law and starting from present-day state-of-the-art, a run like this could be performed in roughly 50 yr from now.

A reasonable alternative is to study the clustering properties of MW-like systems through an analytical approach. We use the fact that the probability that a halo of mass $m$ at redshift $z$ will be part of a larger halo of mass $M$ at the present time is described by the progenitor conditional mass function $f(m, z|M, z_0 = 0)$, according to the so-called extended Press & Schechter theory. Using the progenitor mass function, we can calculate analytically, at redshift zero, the distribution of subhaloes in mass formation epoch and rarity of the peak of density field at the formation epoch. That is done for a MW-size system, assuming both a spherical and an ellipsoidal collapse model.

Numerical simulations described in Diemand, Madau & Moore (2005a) show that the distribution of material originating from the earliest branches of the merger tree within the present-day haloes depends on the $\sigma$-peaks of the primordial density fluctuation field it belonged to. We extend their numerical results by performing an analytical estimate of the density peak distribution as a function of the halo mass traced back to the smallest-scale haloes, thus avoiding the limitation imposed by numerical simulations. In this way, we obtain a realistic estimate of the distribution and mass function of the whole population of subhaloes.

Such an analytical estimate can provide a powerful tool to take into account the effect of early high-density peaks in present-day haloes.

This is particularly important in the framework of DM indirect detection, since a high $\sigma$-peak halo translates into a higher concentration and thus a higher value for the density squared which has to be integrated along the line-of-sight (LOS) to obtain a prediction for particle fluxes coming from DM annihilation.

Given some model for the hierarchical formation of our Galaxy and for the internal structure of subhaloes, DM may be in fact indirectly detected using annihilation rates predicted from particle physics (Bergström 2000; Bertone et al. 2005) through the observation of high-density point source or extended regions inside our Galaxy. If we restrict ourselves to $\gamma$-ray observations, these can be obtained using either atmospheric Cerenkov telescopes (Aharonian et al. 1997; Weekes et al. 1997; Baixeras 2003) or satellite-borne detectors like $\text{Gamma-ray Large Area Space Telescope (GLAST)}$ (Michelson 2001). The detectability of DM substructures with GLAST has been widely discussed in the literature (see e.g. Pieri, Branchini & Hofmann 2005; Pieri, Bertone & Branchini 2008 and references therein). The small mass haloes have been found to give the main contribution to an unresolved $\gamma$-ray foreground arising from DM annihilation, while their detection as resolved objects has been proved to be very unlikely. Indeed, the unresolved subhalo foreground is prominent above the MW smooth foreground far from the Galactic Centre (GC), where the overall flux is still too low to be detected.

In this paper, we apply the analytical derivation of the subhalo population properties, such as the $\sigma$-peak distribution, on the indirect detection of $\gamma$-rays. We thus study the possibility that high $\sigma$-peak material could bring the foreground level above the detectability threshold of a GLAST-like large-field-of-view satellite.

As in Pieri et al. (2008), we use different models for the virial concentration of subhaloes.

This paper is organized as follows. In Section 2, we review the spherical and ellipsoidal collapse models and their properties. In Section 3, we describe the original analytical derivation of the density peak distribution as a function of the halo mass, and the subhaloes mass function for a present-day halo with mass $M = 10^{12} M_{\odot} h^{-1}$. In Section 4, we estimate the upper bound for the contribution to the $\gamma$-ray flux due to the presence of a population of subhaloes inside the MW. In Section 5, we study the prospects for detection of substructures with a GLAST-like experiment in our best-case scenario. A discussion of our results can be found in Section 6.

2 EXTENDED PRESS & SCHECHTER THEORY: FROM PROGENITORS TO SUBHALOES

In the hierarchical picture of galaxy formation, structures up to protogalactic scale grow as a consequence of repeated merging events. Smaller systems collapse at high redshifts, when the universe is denser, and subsequently assemble to form bigger and bigger haloes (Lacey & Cole 1993). This merging history is often represented by the so-called ‘merger-trees’.

Smaller systems accreted on to a larger halo along its merging history tree and still surviving at a later time are called ‘substructures’ or ‘subhaloes’ (Ghigna et al. 1998; De Lucia et al. 2004; Gao et al. 2004; Tormen, Moscardini & Yoshida 2004; van den Bosch, Tormen & Giocoli 2005). In what follows, we will discuss an analytical approach to derive the mass function of subhaloes. We will use the simplifying assumption that no tidal stripping or merging events among substructures happen. In this approach, the mass of each subhalo remains constant in time, and equals the original virial mass (Eke, Cole & Frenk 1996) of the progenitor halo at the
considered redshift. A similar study was carried out by Sheth (2003), who calculated the subhalo mass function using the creation rate of the progenitors of a present-day DM halo. Our approach is different: we derive the subhalo mass function from the entire population of progenitors (as shown by equation 4), in order to allow a direct comparison with the N-Body results of Diemand et al. (2005a).

2.1 Conditional mass function

Let us consider a halo with virial mass $M$ at some final redshift $z_0$. According to the hierarchical picture of galaxy formation, going backward in time the halo will be split into smaller and smaller systems, called ‘progenitors’. Mass conservation tells us that the sum of all masses of progenitor haloes at any given redshift equals the mass of the halo at $z_0$. Let us define the conditional mass function $f(m, z | M, z_0) \, dm$ as the fraction of mass belonging to haloes with mass between $m$ and $m + dm$ at redshift $z$, which are progenitors of a halo of mass $M$ (an $M$-halo) at a later redshift $z_0$.

Assuming the spherical collapse model (Press & Schechter 1974), we can express $m$ and $z$ as a function of the new variables $s$ and $\delta_{sc}$. The conditional mass function is independent of the power spectrum of density fluctuations and it is described as (Lacey & Cole 1993):

$$ f(s, \delta_{sc} | S, \delta_0) \, ds = \frac{\delta_{sc} - \delta_0}{\sqrt{2\pi(s - 3)}} \exp \left[ -\frac{(\delta_{sc} - \delta_0)^2}{2(s - S)} \right] \frac{ds}{s - S}, \quad (1) $$

where $s = \sigma^2(m)$ is the square of the mass variance of an $m$-halo, and $\delta_{sc}$ is the spherical collapse overdensity at redshift $z$. $S$ and $\delta_0$ are the mass variance of $M$-halo and the spherical collapse overdensity at the present time, respectively. To compute the mass variance, we have chosen a power spectrum with primordial spectral index $n = 1$, and a transfer function obtained from cmbfast (Seljak & Zaldarriaga 1996) for a concordance $\Lambda$ CDM universe ($\Omega_m, \Omega_\Lambda, h = 0.3, 0.7, 0.7$) with $\sigma_8 = 0.772$, extended down to a mass $M = 10^6 M_\odot/ h^{-1}$.

We have integrated this power spectrum using a top-hat filter in the real space. To obtain the mass variance until the typical Jeans neutrino mass, we linearly extrapolate the power spectrum down to a scale corresponding to $M = 10^6 M_\odot$. At small scales, this power spectrum has a slightly steeper slope than the one proposed by Green et al. (2004), obtained for a bino-like neutrino.

Over the last 10 yr, N-Body simulations have shown that the collapse of DM haloes is actually not well described by an isolated spherical model; the influence of surrounding protohaloes can be reproduced using an ellipsoidal model (Sheth, Mo & Tormen 2001; Sheth & Tormen 2002).

In the excursion set approach, the progenitor mass function of a halo is described by the conditional probability of first upcrossing distribution. Such a probability is well fitted by a random walk in the plane $(s, \delta)$, starting from $(S, \delta_0)$ (Bond et al. 1991). In the spherical collapse model, this barrier has a constant height, defined by the collapse redshift: $B_{sc}(s, \delta_{sc}) = \delta_{sc}$. For the ellipsoidal collapse case, the barrier height is not constant but depends on $s$ and on $\delta_{sc}$ as described by the following equation:

$$ B_{sc}(s, \delta_{sc}) = \sqrt{q} \delta_{sc} \left[ 1 + \beta \left( \frac{s}{q \delta_{sc}} \right)^\gamma \right]. \quad (2) $$

Sheth et al. (2001) found $q = 0.707, \beta = 0.5$ and $\gamma = 0.6$; the value of the last two parameters is motivated by an analysis of the collapse of homogeneous ellipsoids, whereas the value of $q$ comes from requiring that the predicted halo abundances match what is found in the simulations.

Considering the barrier described in equation (2), Sheth & Tormen (2002) found an approximate solution for the diffusion equation, expressed as follows:

$$ f(s, \delta_{sc} | S, \delta_0) \, ds = \frac{|T(s, \delta_{sc} | S, \delta_0)|}{\sqrt{2\pi(s - S)}} \exp \left[ -\frac{[B(s, \delta_{sc}) - B(S, \delta_0)]^2}{2(s - S)} \right] \frac{ds}{s - S}, \quad (3) $$

with $T(s | S)$:

$$ T(s, \delta_{sc} | S, \delta_0) = \sum_{n=0}^5 \frac{(S - s)^n}{n!} \frac{\partial^n [B(s, \delta_{sc}) - B(S, \delta_0)]}{\partial s^n}. $$

In Fig. 1, we show the conditional mass function at five different redshifts for a halo with present-day mass $M = 10^{12} M_\odot/ h^{-1}$, both for the spherical (dotted curves) and for the ellipsoidal (solid curves) collapse predictions. It can be observed that the halo is split into smaller and smaller progenitors at higher redshifts; discrepancies between the two models depend both on mass and on redshift.

Comparing the two predictions at fixed redshift, one can note that the spherical model predicts more progenitors at intermediate mass, and fewer at both very small and very large masses, compared to the ellipsoidal model (Sheth & Tormen 2002). In other words, the two predictions cross each other in two points, although these crossings do not necessarily fall in the range of masses plotted in the figure.

A direct consequence of this is that massive progenitors exist at higher redshifts in the ellipsoidal collapse, and the distribution of formation redshifts (defined as the earliest epoch when a halo assembles half of its final mass in one system) is consequently shifted to earlier epochs (Giocoli et al. 2007).

From $f(s, \delta_{sc} | S, \delta_0) \, ds$, we can write the total number of progenitors at any given redshift as

$$ N(m, \delta_{sc} | M, \delta_0) \, dm = \frac{M(S)}{m(s)} \int f(s, \delta_{sc} | S, \delta_0) \, ds. \quad (4) $$

Considering a scale-free power spectrum $P(k) \propto k^n$, the mass variance scales as $s(m) \propto m^{-(n+3)/3}$, and the number of progenitors can
be explicitly written in terms of s:

\[ N(m, \delta_{sc} | M, \delta_0)dm = \left( \frac{s}{S} \right)^{(\alpha+3)/3} f(s, \delta_{sc} | S, \delta_0)ds. \]  

(5)

2.2 Number of progenitors

Iterating equation (4) over mass, we obtain the total number of progenitors in the given mass interval, as a function of redshifts:

\[ dn(z, \Delta m) = \int_{m_s}^{m_f} N(m, \delta_{sc} | M, \delta_0)dm = N(z)_{m_f}^{m_s}, \]  

(6)

where \( m_s \) and \( m_f \) represent the bounds of the interval. For a white-noise power spectrum (scale free with \( n = 0 \)) and a spherical collapse mass function, a primitive of this integral can be written as

\[ N(z) = \frac{1}{S\sqrt{2\pi}} \left[ e^{-\frac{(\delta_{sc}-\delta_0)^2}{2\sigma^2}} \right] \]  

\[ \times \left[ 2\sqrt{\frac{2}{\pi}}(\delta_{sc} - \delta_0) - e^{-\frac{(\delta_{sc}-\delta_0)^2}{2\sigma^2}} \right] \]  

\[ \times \sqrt{2\pi}(S - (\delta_{sc} - \delta_0)^2) \text{erf} \left( \frac{\delta_{sc} - \delta_0}{\sqrt{2}(S - \delta_0)} \right) \].  

(7)

In Fig. 2, we show the total number of progenitors multiplied by the mass, in five different mass decades, for a halo with mass \( M = 10^{12} M_\odot\ h^{-1} \) at \( z_0 \), as a function of redshifts. We have assumed a concordance ΛCDM power spectrum and have integrated equation (4) numerically. The solid lines represent the prediction for the ellipsoidal collapse model, while the dotted lines refer to the spherical collapse one. From the top to bottom panel, the curves represent the following mass bins: \([h^{-6}, 10^{-5}], [10^{-5}, 1], [10^{-4}, 10^{-3}], [10^{-3}, 10^{-2}]\) and \([10^{-2}, 10^{-1}]\), all but the first expressed in term of \( M_\odot\ h^{-1}\). The dashed lines show, for comparison, the prediction for the ellipsoidal collapse model using the Green et al. (2004) power spectrum linear trend down to small masses. As expected, the mass function differs from the concordance ΛCDM at small masses and high redshift.

It can be observed that the spherical collapse, for a fixed mass bin, underpredicts the number of haloes at high redshifts compared to the ellipsoidal model. We will see in the following sections that if we consider the variable \( v(z, m) = \delta_{sc}(z)/\sigma(m) \), for any given mass this will result in the inequality \( v_{sc}(m) > v_{s}(m) \).

3 UNEVOLVED SUBHALO MASS FUNCTION FROM THE MERGER TREE OF A PARENT M-HALO

The progenitor mass function, integrated over \( \delta_{sc} \), gives the total number of progenitors of mass between \( m \) and \( m + dm \) that a halo of final mass \( M \) has had at all times:

\[ \frac{dn(m)}{dm} = \int_{\delta_0}^{\infty} \frac{M}{m} f(s, \delta_{sc} | S, \delta_0)d\delta_{sc}. \]  

(8)

In the case of the spherical collapse, this integral results in

\[ \frac{dn(m)}{dm} = \frac{M}{\sqrt{2\pi} \sigma S} \propto m^{-\xi}, \]  

(9)

with \( \xi \approx 1 \) for a ΛCDM power spectrum. Since the same system may be a progenitor of the same final halo at more than one redshift, integrating the progenitor mass function overcounts the total number of progenitors. The result of this integration must then be properly re-normalized by imposing the constraint coming from Diemand et al. (2005b) that roughly 10 per cent of the total MW mass (\( M = 5 \times 10^{11} M_\odot\ h^{-1} \)) is in systems with mass ranging from \( 10^7 \) to \( 10^{10} M_\odot\ h^{-1} \):

\[ \int_{10^{-7}}^{10^{10}} \frac{m}{M}dn = 0.1. \]  

(10)

In Fig. 3, we plot the differential mass distribution of subhaloes in a \( 10^{12} M_\odot\ h^{-1} \) (MW-like) DM halo. The distribution has a power-law behaviour approximately described by the relation

\[ \frac{dn(m)}{dm} = Am^{-\alpha}, \]  

(11)

with \( \alpha \approx 2 \) for both the spherical and the ellipsoidal collapse model, respectively. Once the normalization factor is fixed, we find that the differential distribution of the subhaloes is independent of the mass of the progenitor halo, \( M \), considering all the progenitors with mass from \( 10^{-7} \) to \( m/M = 0.01 \).

3.1 Progenitors \( \sigma \)-peak in the host halo

Using high-resolution N-Body simulations, Diemand et al. (2005a) studied the spatial distribution at \( z = 0 \) of matter belonging to high-redshift progenitors of a given system. They found that this distribution mainly depends on the rareness of the density peak corresponding to the progenitor, expressed in terms of \( \delta_{sc}/\sigma(M, z) \), and is largely independent of the particular value of \( z \) and \( M \); matter from high-\( \nu \) progenitors ends up at smaller distances from the centre of the final system.

\footnote{A least squares fitted on the points gives \( \alpha_{sc} = -1.9494 \pm 0.0005 \) and \( \alpha_{sc} = -1.9561 \pm 0.0008 \).}
Analytical subhalo population

Figure 3. Differential distribution of subhaloes in a $10^{12} \, M_\odot \, h^{-1}$ DM halo. The distribution has a slope approximatively equal to 1 and has been normalized considering that 10 per cent of the total mass is in subhaloes with mass from $10^7$ to $10^{10} \, M_\odot \, h^{-1}$.

We can understand this in terms of the revised secondary infall (Quinn & Zurek 1988; Zaroubi, Naim & Hoffman 1996): the formation of haloes in N-Body simulations preserves ranking of particle binding energy, that is, particles in the cores of progenitor haloes will end up in the core of the final system. Equally, particles from progenitors accreted at earlier times, hence possessing more negative initial binding energies, will likely have a more negative final energy, and so be more centrally concentrated than average matter.

At fixed redshift (hence at fixed $\delta_{sc}$), higher mass progenitors have a larger $\nu$, are more self-bound than smaller mass ones, and thus end up closer to the centre of the final system. Analogously, for a fixed progenitor mass, higher redshift progenitors have a larger $\delta_{sc}$, hence a larger $\nu$, since at higher redshift the universe is denser. They are also more self-bound than lower redshift siblings, and so end up closer to the centre of final system.

In Fig. 4, we plot the subhalo mass function in terms of $\nu$. To compute the factor $\nu$ for each progenitor, we integrated the total number of progenitors in a given mass bin equation (6), at all redshifts. In the top panel, we consider all the progenitors at all redshifts with mass in the full range $h \, 10^6$–$10^{10} \, M_\odot \, h^{-1}$. In the bottom panel, we show the similar distribution only for the smallest and larger decade of the progenitors mass.

4 $\gamma$-RAY FLUX FROM GALACTIC SUBSTRUCTURES

4.1 Modelling Galactic halo and substructures

We model the distribution of DM in our Galaxy after Diemand et al. (2005a). For the smooth component of the MW, we use the best fitting to the high-resolution numerical experiments of Diemand et al. (2005b):

$$\rho_s(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)^\gamma \left[1 + \left(\frac{r}{r_s}\right)^\alpha\right]^{(\beta - \gamma)/\alpha}},$$

with $(\gamma, \beta, \alpha) = (1.2, 3, 1)$. The scale radius $r_s$ and density $\rho_s$ are constrained by the virial properties of the halo. Following Diemand et al. (2005a), we adopt $r_s = 26$ kpc, while $\rho_s$ has to be normalized to the virial mass of the smooth DM halo. We include a physical cut-off $r_{cut} = 10^{-8}$ kpc, which represents the distance at which the self-annihilation rate equals the dynamical time of spike formation.

We shape the spatial distribution of subhaloes according to the fact that it traces the mass distribution of the parent halo from $r_{vir}$ down to a minimum radius $r_{min}(M)$ where tidal effects become important. We use equation (12) together with the fact that the dependence on the initial conditions when the haloes accreted on to the present-day MW halo is set through the parameter $\nu(M)$. We then use the parametrization obtained in Diemand et al. (2005a):

$$r_s \rightarrow r_v = f_v r_s,$$

$$\beta \rightarrow \beta_v = 3 + 0.26^{\nu/1.6}.$$  \hspace{1cm} (13)

This parametrization reflects the fact that material accreted in areas with high-density fluctuations is more concentrated towards the...
centre of the galaxy, and has a steeper outer slope. We also use the mass function, derived in Section 2, to model the number density of subhaloes per unit mass at a distance \( r \) from the GC, for a given \( v \) (\( M \)):

\[
\rho_{\text{halo}}(M, r, v) = \frac{AM^2}{[r/r_{\text{min}}(M)]} \left( 1 + [r/r_{\text{min}}(M)] \right)^{\beta - 1} \frac{\sigma^*}{2\pi^2 r} \tag{14}
\]

in units of \( M^{-1}\Omega \text{kpc}^{-3} \). The mass dependence on \( r \), reflects the mass dependence of the virial parameter \( r_v = r_{\text{vir}}/r_{\text{c}} \). The effect of tidal disruption is taken into account through the step function \( \theta[r - r_{\text{min}}(M)] \), where \( r_{\text{min}}(M) \) is estimated following the Roche criterion. A is a normalization factor obtained by imposing that 10 per cent of the MW mass is distributed in subhaloes with masses in the range \( 10^{-10} \text{M}_\odot \) (Diemand et al. 2005b) as in Section 2.

As a result about 50 per cent of the MW mass is contained within \( \sim 2 \times 10^{10} \) subhaloes in the mass range \( [10^{-6}, 10^{10}] \text{M}_\odot \). The solar neighbourhood density is \( \sim 280 \text{ pc}^{-3} \), mainly constituted by haloes with mass of \( 10^{-6} \text{M}_\odot \). The halo closest to the Earth is expected to be located \( \sim 9.5 \times 10^{-2} \text{ pc} \) away.

The remaining 50 per cent of the MW mass is assumed to be smoothly distributed, and we use this half mass value to normalize \( \rho_0 \) in equation (12).

Few constraints exist on the density profile of each subhalo. Numerical simulations (Diemand, Moore & Stadel 2005b; Diemand, Kuhlen & Madau 2006, 2007b) suggest that they were formed with a NFW profile, which is described by equation (12) with \( (\gamma, \beta, \alpha) = (1, 3, 2) \). Even if subhaloes probably underwent tidal stripping and consequent mass loss after merging, their higher central density should prevent the inner regions from being affected. Pieri et al. (2008) explored different possibilities for the concentration parameter \( c_{\text{vir}} = r_{\text{vir}}/r_{\text{c}} \), where \( r_{\text{vir}} \) is defined as the radius at which the mean halo density is 200 times the critical density. Following their guidelines, we use two models for the concentration \( c_{\text{vir}} \); we assume that the inner structure of subhaloes is either fixed at the time they merge on to the parent halo (\( z \)-labelled model) or that it evolves with redshift until the present time (0 model). In model \( B_{\text{ref}} \), the NFW concentration is computed at \( z = 0 \) according to Bullock et al. (2001) (hence the prefix \( B \)), and extrapolated to low masses. In model \( B_{\text{ref},0} \), the values of \( c_{\text{vir}}(M, z) \) are obtained from those at \( z = 0 \) using the evolutionary relation \( c_{\text{vir}}(M, z) = c_{\text{vir}}(M, z = 0)/(1 + z) \), where the merging redshift \( z \) is determined by the knowledge of the value of \( v \) assigned to each progenitor. It is then assumed that the halo parameters such as scale radius and density freeze at the merging epoch without being affected by the evolution of the environment at later times. Therefore, subhaloes are much denser in model \( B_{\text{ref},0} \) than in model \( B_{\text{ref}} \).

The values \( c_{\text{vir}} \) thus found to refer to progenitors formed from average-density fluctuations (\( v = 1\sigma \) peaks of the fluctuation density field). However, haloes with equal mass at redshift \( z_1 \) may have assembled at different previous epochs; specifically, if we call \( z < z_1 \), the redshift of mass assembly for progenitors observed at redshift \( z_1 \), the amplitude of the initial density fluctuations producing the progenitors is an increasing function of \( z \). Therefore, their concentration \( c(M, z) \) is also an increasing function of the peak amplitude \( v \). To account for this effect, we use the relation \( c_{\text{vir}}(M, v) = v(M) c_{\text{vir}}(M, v = 1) \).

### 4.2 Modelling the \( \gamma \)-ray flux from DM annihilation

We model the photon flux from neutralino annihilation in the population of Galactic subhaloes following Pieri et al. (2008). Given a direction of observation defined by the angle-of-view \( \psi \) from the GC, and a detector with angular resolution \( \theta \), the \( \gamma \)-ray flux can be parametrized as

\[
\frac{d\Phi_{\gamma}}{dE_{\gamma}}(E_{\gamma}, \psi, \theta) = \frac{d\Phi_{\gamma}}{dE_{\gamma}}(E_{\gamma}) \Phi_{\text{cosmo}}(\psi, \theta). \tag{15}
\]

The particle physics dependence in equation (15) is given by the annihilation spectrum and DM properties and is embedded in the term

\[
\frac{d\Phi_{\gamma}}{dE_{\gamma}}(E_{\gamma}) = \frac{1}{4\pi} \frac{\sigma_{\text{ann}} v}{2m_{\chi}^2} \int \frac{dN_f}{dE_f} B_f,
\]

where \( m_{\chi} \) is the DM particle mass, \( \sigma_{\text{ann}} v \) is the self-annihilation cross-section times the relative velocity of the two annihilating particles, and \( dN_f/dE_f \) is the differential photon spectrum for a given final state \( f \) with branching ratio \( B_f \), which we take from Fornengo et al. (2004).

The LOS integral defined as

\[
\Phi_{\text{cosmo}}(\psi, \Delta\Omega) = \int dM \int d\nu \int_{\Delta\Omega} d\nu d\Omega_{\text{LOS}} d\phi \int dc x J(x, y, z | \lambda, \theta, \phi)
\]

(17)

accounts for the influence of cosmology in the flux computation. \( \Delta\Omega \) is the solid angle defined by the angular resolution of the instrument, \( J(x, y, z | \lambda, \Delta\Omega) \) is the Jacobian determinant, \( R = \sqrt{\lambda^2 + R^2 - 2\lambda R \cos \psi} \) is the galactocentric distance and \( r \) is the radial distance inside the single subhalo, \( R_C \) is the distance of the Sun from the GC and \( C = \cos \theta \cos (\psi - \cos \psi) \sin \theta \sin \psi \). \( P[1(v|M)] \) is the probability distribution function for the peak ratio \( v(M) \) calculated using the extended Press–Schechter formalism. \( P[c(M)] \) is the lognormal probability distribution for \( c \) centred on \( c_{\text{vir}}(M) \) as it is computed in our models, while \( P[1(v|M)] \) is determined by the merging history of each subhalo and \( P[c(M)] \) describes the scatter in concentration for haloes of equal mass (Bullock et al. 2001; Neto et al. 2007). Therefore, the two probabilities may be assumed independent. The single halo contribution to the total flux is given by

\[
\Phi_{\text{halo}}(M, r, v, c) = \int d\nu \int_{\Delta\Omega} d\nu d\Omega_{\text{LOS}} d\phi \int dc x J(x, y, z | \lambda, \theta, \phi)
\]

(18)

This equation is also used to derive the contribution of the smooth component of the MW itself.

Equation (17) gives the average subhalo contribution to the Galactic annihilation flux within \( \Delta\Omega \) along the direction \( \psi \).

This contribution is shown in Fig. 5 together with the MW smooth halo component obtained with equation (18), for the two models considered in this analysis, for \( \Delta\Omega = 10^{-5} \text{ sr} \), corresponding to an experimental angular resolution of 0.1. The sum of the MW smooth and clumpy diffuse contributions is shown as well. We define this sum as our ‘annihilation signal’, which will be multiplied by equation (16) to obtain the predicted \( \gamma \)-ray diffuse flux from neutralino annihilation in our Galaxy. In the small box, we show a zoom-in at small angles of the annihilation signal and we superimpose the signal obtained in Pieri et al. (2008) for two similar models (we refer to their paper for the detailed explanation of models). Our models give a higher flux at the GC, where the signal is dominated by the
MW smooth contribution. This is due to the different MW profile adopted. Yet, we find one order of magnitude enhancement at the GC in our approach, the enhancement is greater close to the GC: indeed, at the anticentre it goes down to a factor of 2.

We have used the $P[v(M)]$ for the ellipsoidal collapse in equation (17). We have checked that using the corresponding probability function for the spherical collapse does not change the result on $\Phi^{\text{new}}$. This is due to the fact that the main difference between the two models resides at small values of $v$. A small $v$ gives low-concentration parameter and its contribution to equation (18) is then depressed with respect to that of a haloes with a higher $v$.

4.3 Normalization to EGRET data

In order to make predictions on detectability, we impose the best value of $\Phi^{\text{pp}}$ compatible with the available experimental limits. As in Pieri et al. (2008), we first assume the optimistic model where $m_f = 40 \text{GeV}$, $\sigma_{\text{ann}} = 3 \times 10^{-26} \text{cm}^2$ and the branching ratio is 100 per cent in quarks $b\bar{b}$. We then integrate equation (16) above $3 \text{GeV}$. This choice of parameters gives a value of $\Phi^{\text{pp}} = 2.6 \times 10^{-9} \text{cm}^2 \text{kpc}^{-1} \text{GeV}^{-2} \text{s}^{-1} \text{sr}^{-1}$.

We then compute the expected number of photons above $3 \text{GeV}$ in 1 yr for a solid angle of $10^{-5} \text{sr}$ corresponding to the angular resolution of a GLAST-like satellite. The result for the $B_{\text{ref,0}}$ (dashed curve) and $B_{\text{ref,d}}$ (dotted curve) models is shown in Fig. 6.

We compare the obtained number of events with the EGRET data for the diffuse Galactic component parametrized according to Bergström, Ulio & Buckley (1998):

$$\frac{\text{d}N}{\text{d}E} = N_0(l, b) \times 10^{-6} E^{-2.7} \frac{\gamma}{\text{cm}^2 \text{s sr GeV}}.$$ (19)

and with the diffuse extragalactic $\gamma$ emission, as extrapolated from EGRET data at lower energies (Sreekumar et al. 1998):

$$\frac{\text{d}N}{\text{d}E} = 1.38 \times 10^{-3} E^{-2.1} \frac{\gamma}{\text{cm}^2 \text{s sr GeV}}.$$ (20)

The normalization factor $N_0$ in equation (19) depends only on the interstellar matter distribution. The resulting number of photons above $3 \text{GeV}$ in 1 yr for $\Delta \Omega = 10^{-3} \text{sr}$, computed along $l = 0$ where its value is minimum, is shown in Fig. 6 (solid curve).

We find an excess of annihilation signal photons towards the GC in both models. Yet, the angular resolution of EGRET corresponding to $\Delta \Omega = 10^{-3} \text{sr}$ does not allow to reconstruct a spiky source such as ours. We have checked that if we compute the number of annihilation signal photons towards $\psi = 0$ smeared in a cone of view of $1^\circ$, it is below the number of EGRET-detected photons for the same angular resolution.

Yet, the $B_{\text{ref,0}}$ model exceeds the extragalactic diffuse measured background too, which is dominant above $\psi = 40^\circ$. Since the extragalactic background is not due to any point source, we safely expect that it will scale with the solid angle. The number of annihilation signal photons produced in the $B_{\text{ref,0}}$ model should then be less or at most comparable with the number of measured background photons. We make the optimistic assumption that the two numbers are comparable at $\psi = 40^\circ$ where the discrepancy is larger and we thus fix $\Phi^{\text{pp}}_{\text{ref,0}} = 2.0 \times 10^{-9} \text{cm}^4 \text{kpc}^{-1} \text{GeV}^{-2} \text{s}^{-1} \text{sr}^{-1}$ for the $B_{\text{ref,0}}$ model, correctly normalized to EGRET data, while we keep $\Phi^{\text{pp}}_{\text{ref,d}} = 2.6 \times 10^{-9} \text{cm}^4 \text{kpc}^{-1} \text{GeV}^{-2} \text{s}^{-1} \text{sr}^{-1}$ for the $B_{\text{ref,d}}$ model. Assuming that the Green et al. (2004) power spectrum would result in a difference of less than a factor of 2 in the number of predicted photons, we therefore do not further consider this case in our analysis.
5 PROSPECTS FOR DETECTION

In this section, we study the sensitivity of a GLAST-like apparatus for 1 yr of effective data taking.

We define the experimental sensitivity $\sigma$ as the ratio of the number $n_\gamma$ of annihilation signal photons and the fluctuation of background events $n_{bg}$:

$$\sigma \equiv \frac{n_\gamma}{\sqrt{n_{bg}}} = \frac{\sqrt{T_\xi \epsilon_{\Delta \Omega}}}{\sqrt{\int \frac{\Delta_1}{\Omega_1} d\phi_{bg} \frac{dE d\Omega}{dE d\Omega}}} \int \frac{A_{si}(E, \theta)}{\phi_{bg}(E, \phi)} d\phi_{bg} dE d\Omega,$$

(21)

where $T_\xi = 1$ yr is the effective observation time and $\phi_{bg}$ is the background flux given by equations (19) and (20), computed along $l = 0$, that we assume to be composed of astrophysical photons only. The quantity $\epsilon_{\Delta \Omega}$ is the fraction of signal events within the optimal solid angle $\Delta \Omega$ corresponding to the angular resolution of the instrument and it is optimistically set to 1. $A_{si}$ is the effective detection area defined as the detection efficiency times the geometrical detection area. We use $A_{si} = 10^7$ cm$^2$, independent of the energy $E$ and the incidence angle $\theta_i$. Finally, we assume an angular resolution of 0.1 and an energy threshold of 3 GeV.

The resulting sensitivity curves as a function of the angle of view $\psi$ are shown in Fig. 7 for the $B_{\text{ref},0}$ (solid curve) and $B_{\text{ref},z}$ (dotted curve) annihilation signal models. In the small box, a zoom-in at GC is shown. An almost 1.5 $\sigma$ around 10$^{-4}$ is found for the $B_{\text{ref},z}$ model. The same model would be detected at about 30 $\sigma$ at the GC. As far as the $B_{\text{ref},0}$ model is concerned, it would show up with $\sim 40\sigma$ effect towards the GC that would rapidly fall down 1$\sigma$ after 0.5. A 5 $\sigma$ detection at the GC would be possible for both models with a value of $\phi_{\gamma}$ even six times lower. In the case of a striking excess detection along the GC, a milder excess at larger angles could be a hint for the discrimination about the models, though no discovery could be claimed.

Pieri et al. (2008) studied the detectability of resolved haloes which would shine above the Galactic foreground, finding in their best case scenario that only a tenth of large mass haloes would be detected, with a mass slope of $-2$ for the halo mass function.

Repeating their analysis is beyond the goal of this paper. Yet, we note that the effect of including the $P[v(M)]$ factor in equation (17) with respect to the concentration models in Pieri et al. (2008) leads to an enhancement of the Galactic foreground. We thus expect that including $P[v(M)]$ will be compensated by the increased foreground and we do not expect a dramatic change in the number of detectable haloes.

As a further test, we have computed the sensitivity of a GLAST-like experiment for a $B_{\text{ref},z}$, halo once $\phi_{\gamma}$ has been normalized to the EGRET data. We chose the closer $M = 10^{-6}$ M$_\odot$ halo, located at 9.5 $\times$ 10$^{-2}$ pc from the Sun. We chose $v = 2.4$ given from the probability of finding one halo with such a value in a 1 pc$^3$ sphere around the Sun. We conservatively considered only the astrophysical background in equation (22), while the annihilation signal foreground should be considered too. Even in these very optimistic hypotheses, the source would produce only a 0.1 $\sigma$ detection signal in 1 yr. The concentration parameter should be further multiplied by a factor of about 6 in order to obtain a 5 $\sigma$ effect. This could be achieved using the lognormal probability $P[c(M)]$ but with a ridiculously small probability.

We conclude that the effect of introducing the $P[v(M)]$ can only be observed in a global enhancement of the diffuse Galactic annihilation foreground.

6 CONCLUSIONS

In this paper, we have, for the first time, derived an analytical description of the mass function and distribution of rareness of density peaks in the subhalo population of our Galaxy, applying the extended Press & Schecter formalism. To make the calculation possible, tidal interactions and close encounters between subhaloes have been neglected. Very small (micro solar mass) subhaloes are extremely concentrated; therefore, at least for them, our approximation is a reasonable one.

The obtained results are valid over the whole range of subhalo masses [$10^{-6}$, $10^{3}] M_\odot$ and thus confirm and extend the results of the N-body simulations, whose resolution is still far too low in order to simulate coherently this mass range.

Making use of the results of Diemand et al. (2005a) on the distribution of different $\sigma$-peak material inside our Galaxy, we have been able to shape and model the total expected annihilation $\gamma$-ray foreground, statistically taking into account the merging history of each progenitor.

We have used the best-case particle physics scenario to derive predictions for the detectability of such a signal with a GLAST-like experiment. We have shown how both the merging history and the intrinsic properties of the halo formation can contribute to an enhancement of the expected flux by arising the inner concentration of subhaloes. Yet, the real concentration of the single subhalo today remains an open question. We use two models, which result in very different inner densities inside the haloes. In the first model, we assume that the inner shells of the subhaloes remain frozen at the moment they enter the parent halo and thus compute the concentration parameter at the merging epoch, as it is derived in our calculations. Alternatively, we assume that the subhaloes continue to evolve with redshift, and thus compute the halo properties today. We use the Bullock et al. (2001) model for the concentration

![Figure 7. Sensitivity curves for a GLAST-like experiment for the $B_{\text{ref},0}$ (solid curve) and the $B_{\text{ref},z}$ (dotted curve) models described in the text. A zoom-in at small angles is provided in the superimposed frame.](https://academic.oup.com/mnras/article-abstract/387/2/689/1022961/689)
parameter at $z = 0$, extrapolated at low masses. We refer to Pieri et al. (2008) for the effect of using different models.

Our results on detectability show that the detection would be possible and impressivte towards the GC for both models. This detection would be mainly due to the spike in the MW halo at the GC. Unfortunately, a reliable modelling of the astrophysical background coming from the GC and of the effect of the central supermassive BH on the inner DM density profile is still poorly known.

A 1.5$\sigma$ effect would show up as well, around $\sim 10^{-9}$ from the GC, only for the $B_{\text{gal}}$ model. Though no discovery could be claimed for, this could be a significant hint for the existence of such a population of subhaloes, and it would be propulsive for successive studies with upcoming experimental technologies.

A final note on the methodology. In this work, we derived the final subhalo mass function starting from all progenitor haloes at any redshift. We did so in order to directly compare our analytical results to the results obtained by Diemand et al. (2005a) using $N$-Body simulations. However, the subhalo population should indeed be derived starting from the population of ‘satellite haloes’ directly accreted by the protohalo (also called main progenitor) at all previous times (Tormen 1997), since only a fraction of progenitors at redshift $z$ merge directly with the main halo progenitor. Unfortunately, the mass function of satellite haloes cannot be obtained analytically: it requires Monte Carlo simulations of the merging history tree of halo formation (Somerville & Kolatt 1999; van den Bosch 2002; Zentner et al. 2005).

We are currently working on this issue (Giocoli et al., in preparation), and it will be interesting to compare the results obtained using the two methods.

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