Ambient-Data-Driven Modal-Identification-Based Approach to Estimate the Inertia of an Interconnected Power System

DEYOU YANG\textsuperscript{1}, BO WANG\textsuperscript{1}, JIN MA\textsuperscript{2}, (Member, IEEE), ZHE CHEN\textsuperscript{3}, (Fellow, IEEE), GUOWEI CAI\textsuperscript{1}, ZHENGLONG SUN\textsuperscript{1}, (Member, IEEE), AND LIXIN WANG\textsuperscript{1}

\textsuperscript{1}Electrical Engineering Department, Northeast Electric Power University, Jilin 132013, China
\textsuperscript{2}School of Electrical and Information Engineering, The University of Sydney, Sydney, NSW 2006, Australia
\textsuperscript{3}Department of Energy Technology, Aalborg University, 9220 Aalborg, Denmark

Corresponding author: Bo Wang (eebowang@hotmail.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 51977031 and Grant 51877032, and in part by the Science and Technology Project of State Grid of China under Grant 52230019000D.

\textbf{ABSTRACT} A novel approach for estimating the inertia of an interconnected power system is presented using the identification of interarea oscillation modes (frequency, damping and mode shape) extracted from ambient data. The proposed method concentrates on estimating the values of the effective inertia of each area rather than the equivalent inertia of the entire system. Based on an equivalent two-machine system (ETmS) obtained by combining small signal stability analysis (SSSA) with the structure of the power grid, we derive a mathematical relationship between the effective inertia of each area and the interarea oscillation modes. Furthermore, the interarea oscillation modes can be extracted from ambient data, and the developed scheme enables an online estimation of the inertia only by using the outputs measured by PMUs. The performance of the proposed methodology is tested via numerical simulation cases and real data.

\textbf{INDEX TERMS} Inertia constant, small signal stability analysis, modal identification, ambient data, interconnected power system.

\textbf{ABBREVIATIONS AND NOMENCLATURE}

\textbf{ABBREVIATIONS}
SG Synchronous generator
ASG Aggregated synchronous generator
ETmS Equivalent two-machine system
PMU Phase measurement unit

\textbf{NOMENCLATURE}

\begin{itemize}
\item $V, \theta$ Bus voltage magnitude and phase
\item $I, \alpha$ Bus current magnitude and phase
\item $x_l$ Transmission line reactance
\item $E$ Internal voltage magnitude
\item $\delta$ Power angle
\item $x$ Inner reactance
\item $f_d$ Oscillation frequency of a mode
\item $\rho$ Decaying coefficient of a mode
\item $\lambda$ Eigenvalue of a mode
\item $\varphi$ Eigenvector of a mode
\item $K_s$ Synchronous power coefficient
\item $0$ Normal value
\item $e$ Equivalent value
\end{itemize}

\section{I. INTRODUCTION}

Inertia directly affects the response of a power system’s frequency, power angle and other electrical quantities on the electromechanical time scale; it is also one of the important parameters that is used to measure the immunity of the system [1], [2]. Due to the high penetration of large-scale renewable sources connected to the system through power electronics, the inertia decreases [3], [4], and its distribution also changes [5], [6]. Therefore, monitoring and determining the inertia level and distribution are very important to ensure the security of a power system for a transmission system operator (TSO).
In modern power systems, elements such as the frequency dependence of the load and the power electronic interfaces for generation, storage and the load contribute to the effective inertial response of the system, besides conventional synchronous generators (SGs) [7], [8], and this characteristic will become even more prevalent in the future. The inertia is complex and strongly time-varying. Estimating the effective inertia using PMU is considered a viable technology. This study presents an approach for estimating the effective inertia of different areas of an interconnected power system from ambient PMU measurements.

Several techniques have been proposed for the estimation of inertia using PMU measurements. In the early stage, the estimation of the inertia of entire systems was developed and tested using the PMU measurements of frequency events [9]–[15]. Such methods are mainly based on the relationship between the unbalanced active power and the ratio of change of frequency (RoCoF) and depend on data recorded in a large disturbance. Typical research cases including the inertia of the Western Electricity Coordination Council (WECC) system [10] and the Great Britain (GB) power system [15] were estimated using recorded disturbances, and an improved online simulation was subsequently realized. In [16], the relationship between the equivalent inertia and the root of the swing equation is derived, so that the inertia can be estimated by the oscillation signal rather than the frequency deviation caused by sudden power mismatch. In [17], the inertia is estimated by the eigenvalues and eigenvectors extracted from the PMU-measured oscillation power and frequency signals.

However, the inertia estimated based on recorded data for a large disturbance only reflects the inertial response of the system when the disturbance occurs, which makes it difficult to achieve continuous online inertia estimation.

In an actual system, the PMU records a large amount of ambient data in real time. Moreover, previous studies have indicated that the ambient data contain rich electromechanical response information [18], [19]. Some scholars have carried out studies on ambient data-based inertia estimation methods. Based on the Laplace transform of the swing equation, the close-loop frame [20] and ARMAX model [21] were used to estimate the inertia based on ambient data of the active power and frequency deviation. In [22], the relationship between inertia and frequency was established based on a training algorithm, which examined the inertia estimation problem from a statistical point of view. Based on an aggregated power system model, the inertia estimation problem was built and described as a regression equation that can be solved by dynamic regressor extension and mixing [23]. In [24], the variant feature of the system inertia is expressed as a stochastic model according to the random behaviour of the power system, so that the inertia can be estimated by a likelihood function.

At present, most data-driven inertia estimation approaches have been developed on the basis of the relationship between the active power and frequency changes. As one of the most important electromechanical parameters of a power system, the inertia not only affects the frequency response but also influences other electromechanical behaviors of the system, such as electromechanical oscillation. The technical solutions for estimating the inertia based on the oscillation modes (frequency, damping and mode shape) extracted from the PMU have been proven to be feasible. However, the existing techniques using modal information are limited to post-event analysis using an equivalent system model and are unrealistic in terms of obtaining the effective inertia of different areas.

To address the above concerns, this paper develops a methodology for online estimation of the effective inertia of different areas of an interconnected power system by employing the values of interarea oscillation modes, i.e., modal frequencies, damping and mode shape, calculated from online ambient data using PMU. The main contribution of this paper and the presented method are as follows:

1) According to the results of small signal stability analysis (SSSA) and the cut set, an equivalent two-machine system (ETmS) is constructed; then, on the basis of existing research [25], we develop an expression for the ratio of the effective inertia of the different areas using the interarea modal eigenvalue and mode shape;

2) This paper builds an effective inertia estimation strategy that relies solely on ambient data for the interconnected power system;

3) Both simulations and real measured case studies confirm the effectiveness of the proposed effective inertia estimation scheme.

The remainder of this paper is organized as follows. Section 2 discusses the details of the proposed approach. Section 3 reviews recursive stochastic subspace identification (RSSI) and discusses an online application scheme for the proposed effective inertia estimation approach. Sections 4 and 5 present two cases to evaluate the performance of the proposed method. Section 6 presents the conclusions.

II. PROPOSED METHODOLOGY

A. PROBLEM STATEMENT

For the interconnected power grid in Figure 1 (a), the system can be reduced to an ETmS, which is based on the definition of the generator group and the critical cut set (comprising multiple tie-lines), as shown in Figure 1 (b), where ASG1 and ASG2 are the aggregated synchronous generators (ASGs) of generator group 1 and generator group 2, respectively. Then, the dynamics of the ETmS can be described by the swing equation [26], expressed as

\[ \dot{\delta}_e = (\omega_e - 1) \omega_0 \]  
\[ 2H_e \omega_e = P_{m,e} - P_e - D_e \Delta \omega_e \] (1) (2)

where \( \delta_e = \delta_{ASG1} - \delta_{ASG2}, \omega = \omega_{ASG1} - \omega_{ASG2} \) and \( H_e = H_{ASG1}H_{ASG2}/(H_{ASG1} + H_{ASG2}) \).

Linearizing (1) and (2) at the operation point and assuming \( P_{m,e} = 0 \), the homogeneous differential equation with a state
variable is formed and can be expressed as

\[ 2H_e \Delta \dot{\delta}_e + D_e \Delta \ddot{\delta}_e + \omega_n P_e \cot(\delta_e) \Delta \delta_e = 0 \quad (3) \]

Subsequently, the model of the interconnected system, which is hiding in (3), can be obtained by the solution of the differential equation. According to modal analysis theory, the real and imaginary parts of the solution correspond to the oscillation frequency and decaying coefficient of the electromechanical modal information, respectively. As a result, the relationship between the system parameters and the modal information of the electromechanical oscillation can be expressed as

\[ j2\pi f_d = \frac{D^2_e - 8\omega_n H_e \cot(\delta_e)}{AH_e} \quad (4) \]

\[ \rho = -D_e/4H_e \quad (5) \]

Although the interarea exchange power \( P_e \) can be measured by PMU equipped at a tie-line, directly determining the equivalent power angle \( \delta_e \) is difficult. Because the determination of the equivalent power angle \( \delta_e \) based on network reduction, which is used in [27], has low adaptability for large systems, the determination of the equivalent power angle \( \delta_e \) based on the PMU measurements is developed in this paper.

Figure 1 (c) is an equivalent circuit of the ETmS shown in Figure 1 (b), where \( \dot{V}_i = V_i \dot{\theta}_i, i = 1, 2 \) is the voltage vector at buses 1 and 2; \( \dot{I}_i = I_i \dot{\alpha}_i, i = 1, 2 \) is the equivalent injection current vector of buses 1 and 2; \( x_1 \) is the equivalent reactance between bus 1 and bus 2; and \( \dot{E}_i = E_i \delta_i, i = 1, 2 \) is the internal voltage vector of ASG1 and ASG2.

For the area where ASG \( i \) is located, it is assumed that there are \( M \) boundary buses connected to the critical cut set, and there are \( N \) lines connected to the boundary buses inside the area, as shown in Figure 2 (a). Thus, the equivalent injection current of bus \( i \) can be expressed as

\[ \dot{I}_i = \sum_{j=1}^{N} I_{ij} \angle \alpha_{ij}, i = 1, 2 \]

\[ \dot{V}_i = \sum_{j=1}^{M} V_{ij} \angle \theta_{ij}/M, \]

\[ i = 1, 2. \]

For ASG \( i \), the phase relationship between the internal voltage \( \dot{E}_i \), terminal voltage \( \dot{V}_i \), and injection current \( \dot{I}_i \) is shown in Figure 2 (b) [28]. Let \( E_i = 1.0 \) p.u.; the following equation can be written using the sine rule for the triangle shown in Figure 2 (b).

\[ \delta_{ASGi} = \cos^{-1}(V_i \cos \alpha_i) - \alpha_i \quad (6) \]

The estimate of \( \delta_{ASGi} \) is generated by solving (1) at every time step. Considering that the ambient data used in this paper have a certain randomness, to reduce the estimation error, this paper uses the statistical mean in a fixed time period as the basis for the subsequent calculation.

Based on the determined power \( P_e \) and angle \( \delta_e \), the equivalent inertia \( H_e \) can be estimated by extracting the electromechanical oscillation modal frequency and damping from PMU measurements. This idea is basically the same as that proposed in [27]; however, the method proposed in [27] mainly depends on the trajectory after a large disturbance to estimate the equivalent inertia and the effective inertia of the entire system.

\[ \delta_{ASGi} = \cos^{-1}(V_i \cos \alpha_i) - \alpha_i \]

**B. QUANTIFICATION OF THE EFFECTIVE INERTIA OF DIFFERENT AREAS**

It should be noted that the estimated inertia \( H_e = H_{ASGi}H_{ASG2}/(H_{ASGi} + H_{ASG2}) \) using (4) and (5) is the equivalent inertia of the ETmS rather than the effective inertia of the area. \( H_{ASGi} \) and \( H_{ASG2} \) can be determined based on the quantitative relationship between them.

In [25], it was proven that the inertia can be estimated using the modal eigenvalue and its corresponding mode shape as

\[ H_{ASGi} = -\Re \left\{ \frac{\lambda D_{ASGi} \phi_{i\lambda}}{\lambda^2 \phi_{i\lambda}} + \sum_{j} K_{ij} \phi_{i\lambda} \right\} \quad (7) \]

In theory, the inertia can be estimated using (7) once the modal eigenvalues and mode shape are identified. However, it should be emphasized that in (7), the mode shape must be the element of the right eigenvector corresponding to the power angle. In practice, measuring the power angle is a difficult task, especially for the ETmS and ambient data used in this paper. Therefore, equation (7) is mainly employed to determine the quantitative relationship between \( H_{ASGi} \).
and $H_{\text{ASG2}}$. Ignoring the equivalent damping of the ASG and assuming that the interarea oscillation modal eigenvalue and corresponding eigenvectors extracted by the average frequency of the two areas are $\lambda$ and $\varphi$, respectively, it is known that $K_{e,12} = K_{e,21}$ for the ETmS according to the definition of the synchronous power coefficient. Thus, we can obtain the following:

$$
\frac{H_{\text{ASG1}}}{H_{\text{ASG2}}} = \text{Re} \left\{ \frac{\varphi_{\text{ASG2}}}{\lambda^2 \varphi_{\text{ASG1}}} \right\} \text{Re} \left\{ \frac{\varphi_{\text{ASG1}}}{\lambda^2 \varphi_{\text{ASG2}}} \right\}
$$

(8)

At this point, the effective inertia of ASG1 and ASG2 can be simultaneously estimated while extracting the modal information and initial power angle from the ambient data.

III. AMBIENT-DATA-DRIVEN INERTIA ESTIMATION SCHEME

The scheme is the basis for realizing an effective inertia online estimation to accurately extract the interarea electromechanical oscillation modes (frequency, damping and mode shape). In this section, the RSSI algorithm is briefly reviewed first, and then the ambient data-driven effective inertia estimation scheme is presented.

A. EXTRACTION OF OSCILLATION MODES BASED ON RSSI

As a mature system identification method, the stochastic subspace identification (SSI) algorithm has been successfully applied to ambient-data-driven electromechanical oscillation mode extraction. The covariance-driven stochastic subspace identification (SSI-COV) algorithm constructs the covariance matrix based primarily on observation data and then estimates the state matrix $A$ and observation matrix $C$ in the state space model shown in (9) on the basis of a singular value decomposition of the covariance matrix.

$$
\begin{cases}
x_{k+1} = Ax_k + w_k \\
y_k = Cx_k + v_k
\end{cases}
$$

(9)

where $x_k$ is the system state variable, $y_k$ is the measurement, $w_k$ is process noise and $v_k$ is measurement noise. The oscillation frequency, damping and mode shape can be obtained by analyzing the eigenvalue of the following continuous time matrix [29]:

$$
\begin{cases}
A_c = \frac{1}{\Delta t} \log A \\
C_c = C
\end{cases}
$$

(10)

where $\Delta t$ is the sampling period of the measurement data.

The traditional SSI algorithm is time consuming in single calculations, which is not suitable for an online application. Therefore, Prof. Vaithianathan Venkatasubramanian proposed the recursive adaptive subspace identification (RASSI) algorithm [30], which increases computational efficiency and realizes online tracking of the oscillation modes.

B. EFFECTIVE INERTIA ONLINE ESTIMATION SCHEME

Currently, PMUs have been installed in large-capacity power plants, large substations and interarea tie-lines. An increasing number of PMUs will be installed in future power grids. In the presented scheme, ambient data of the voltage, current and frequency measured by the PMUs are used to obtain the effective inertia.

Before performing the scheme for the effective inertia estimation based on ambient data, it is necessary to construct the ETmS according to the results of an SSSA of the operational structure and power flow and determine the required measurements and the area where the effective inertia can be estimated. Then, the relative effective inertia of different areas can be estimated using the following steps.

1) Use the measured ambient data of the voltage and current of the boundary buses connected to the critical cut set to estimate the equivalent power angle $\delta_{\text{ASGi}}$ and extract the modes from the measured ambient data of the frequency of each area. This provides the key intermediate parameters for the inertia estimation scheme.

2) Determine the quantitative relationship between $H_{\text{ASG1}}$ and $H_{\text{ASG2}}$ based on the extracted modal information.

3) Estimate the equivalent inertia $H_e$ using the equivalent power angle and modal frequency and damping extracted in step 1.

4) Calculate the area inertias $H_{\text{ASG1}}$ and $H_{\text{ASG2}}$ using the results of steps 2 and 3 combined with the equation

$$
H_e = \frac{H_{\text{ASG1}} - H_{\text{ASG2}}}{H_{\text{ASG1}} + H_{\text{ASG2}}}.
$$

This concept is illustrated in Figure 3 for the estimation of the effective inertia of the relative area for any ETmS.

IV. NUMERICAL SIMULATIONS

In this section, the IEEE 16-generator system is considered to demonstrate the accuracy of the proposed scheme. The IEEE 16-generator system is a typical interconnected power system, and the system consists of five areas, as shown in Figure 4. Among these areas, area 3, area 4 and area 5...
are equivalent systems. Area 1 (New England system) and area 2 (New York system) are interconnected via three lines (line 60-61, line 27-53, and line 54-53), and area 3, area 4 and area 5 deliver power to area 2 via line 41-40, line 18-49 and line 18-50. To simulate the ambient response, the Power System Toolbox (PST) is used to model the test system. In the established system, the generator is adopted the complex model, and the two typical exciters, i.e., IEEE ST1A and IEEE DC1, and power system stabilizers (PSS) are also considered. More detailed parameters and information on the configuration of the test system can be found in [31].

**A. OSCILLATION MODE EXTRACTION AND INTERAREA EQUIVALENTS**

The modal analysis based on the SSSA for the system with basic operating conditions (Opc) is carried out. Four electromechanical oscillation modes with different oscillation frequencies and damping ratios are obtained, as shown in Table 1. Correspondingly, the mode shapes calculated by SSSA are shown in Figure 5 (a). As seen from the mode shapes, for mode 1, the generators in area 1 and area 2 oscillate against the generators in area 3, area 4 and area 5, and for mode 2, the generators in area 1 mainly oscillate against the generators in area 2.

The model analysis for the IEEE 16-generator system network revealed that two critical cut sets are formed by the tie-line in the system. The first set is Cut set No. 1, consisting of line 14-40, line 18-49 and line 18-50, and the second is Cut set No. 2, consisting of line 60-61, line 27-53 and line 54-53.

According to the mode shapes of mode 1 and mode 2, two ETmSs can be built based on the two critical cut sets, i.e., ETmS No. 1 and ETmS No. 2, which are marked by the red and blue separated lines, respectively, in Figure 4. In ETmS No. 2, ASG1 and ASG2 are the equivalent generators for area 1 and area 2, respectively. Based on ETmS No. 2, the effective inertias of area 1 and area 2 can be estimated using the extracted modal information of mode 2. Similarly, the overall effective inertia $H_{345}$ of area 3, area 4 and area 5 could be estimated based on ETmS No. 1 and the extracted modal information of mode 1.

In the case of basic Opc, we use PST to obtain a continuous 3 min set of ambient data under the condition that the load in the system randomly fluctuates by 5% of the base value. The average frequency of the relative area is calculated by employing the weighted average method proposed in [21].

Using the per-unit values of the average frequency of the relative area as input signals, RASSI is applied to extract the interarea oscillation modes. The mode extraction results are shown in Table 1. It can be observed that the number of electromechanical modes extracted by RASSI from the ambient data is consistent with that of the model analysis. Moreover, the statistical index, i.e., the mean value and standard deviation, shows that the extracted modes concentrate around the theoretical value within a small range. Correspondingly,

---

**TABLE 1.** Model parameters calculated by different methods.

| Mode | Method | Frequency (Hz) | Damping Ratio (%) |
|------|--------|----------------|-------------------|
|      |        | Mean  | Std. | Mean  | Std. |
| 1    | SSSA   | 0.4018|      | 6.17  |      |
|      | RASSI  | 0.4147| 0.0272| 6.4   | 0.13 |
| 2    | SSSA   | 0.5317|      | 2.38  |      |
|      | RASSI  | 0.5371| 0.0272| 2.58  | 0.26 |
| 3    | SSSA   | 0.6525|      | 4.5   |      |
|      | RASSI  | 0.6454| 0.0162| 4.78  | 0.27 |
| 4    | SSSA   | 0.7792|      | 3.74  |      |
|      | RASSI  | 0.7723| 0.0206| 4.05  | 0.46 |

---

**FIGURE 4.** Single-line diagram and cut set of the IEEE 16-generator, 5-area power system.
the interarea oscillation mode shape extracted by the RASSI algorithm is shown in Figure 5(b), from which it can be seen that the oscillation between the areas in each mode is consistent with the theoretical mode shape.

B. EFFECTIVE INERTIA ESTIMATION

On the basis of accurately extracting the electromechanical oscillation modes, the effective inertia estimation method presented in Section III. B is used to estimate $H_1$, $H_2$, and $H_{345}$. To demonstrate the accuracy of the estimation results, the sum of the inertia of each component is calculated, which is regarded as the actual value [22]. The estimation results are shown in Figure 6. In this paper, when using RASSI to extract the electromechanical oscillation modes, continuous data with a length of 30 s are used as the input window length, so the inertia cannot be estimated in the first 0.5 min. It can be observed from Figure 6 that the estimation results show random fluctuation characteristics. However, the fluctuation is around the actual value, marked as the dotted line, within a small range.

Base on the estimation results, the statistical analysis is carried out, as shown in Table 2. The statistical analysis shows that the mean values of the effective inertias $H_1$ and $H_2$ of area 1 and area 2 are 32.31 and 63.92, respectively, which are close to the theoretical values (30.67 and 65.48), and the standard deviations are small. The mean value of the estimated result of $H_{345}$ deviates from the theoretical value (108.95) by 15.57, which is larger than the deviation between the actual value and the estimated value of area 1 and area 2. The main reason is that area 3, area 4 and area 5 are equivalent systems themselves, and the electrical distance between the three equivalent areas (area 3, area 4 and area 5) is large; it is difficult to aggregate the three equivalent areas (area 3, area 4 and area 5) into an equivalent generator.

To verify the adaptability and robustness of the proposed approach to the operation conditions and electromechanical oscillation modes, two operation conditions (Opc 2 and Opc 3) are constructed by changing the exchange power between the areas. Furthermore, we obtain two sets of new modes. The electromechanical oscillation modal frequency and damping corresponding to the three operation conditions are shown in Figure 7. From the spider plot shown in Figure 7, it can be clearly seen that the electromechanical oscillation modes of the system are obviously different under the three operation conditions considered in this paper.

Table 2 also lists the mean and standard deviation of the effective inertia estimation results corresponding to Opc2 and Opc3. Similar to the estimation results of the basic Opc, the estimation results of area 1 and area 2 under Opc2 and
Opc3 are close to the actual value. It can be observed from Table 2 that for area 1 and area 2, the deviation between the mean and the theoretical values is within 8% of the theoretical value. The results show that the proposed method has a small deviation in the effective inertia estimation of the area in which the electrical connection is tight, and the proposed method has a strong robustness to the operating condition and electromechanical oscillation modes.

Limited by the randomness of the ambient data, there are a small number of values in the effective inertia estimation results that greatly deviate from the theoretical value in the continuous time period. However, this part of the value does not represent the actual estimation results. In practical applications, the statistical mean of the estimation results in a continuous time period can be taken as the expected effective inertia.

### V. CASE STUDIES BASED ON REAL MEASUREMENTS

This section considers real measurements of a power grid in North China as an example to verify the validity and adaptability of the proposed method.

The simplified schematic of the system is shown in Figure 8. The DB power grid is relatively large and consists of three closely connected small grids. The NM power grid is an energy base that consists of four large thermal power plants and delivers power to the DB power grid through four 500 kV transmission lines. The transmission distance of 4 lines exceeds 800 km.

The SSSA results show that there is an interarea oscillation mode with a frequency of 0.45 Hz. This mode is mainly expressed in that all generators of the NM power grid oscillate against most generators of the DB power grid. Therefore, the system can be equivalent to an ETmS by using the above four 500 kV tie-lines as a critical cut set.

Based on the ambient data recorded using PMUs during a period of time, the effective inertia of the NM power grid and the DB power grid are estimated. During this time period, two generators with a rated capacity of 667 MV A, three generators with a rated capacity of 733 MV A and one generator with a rated capacity of 556 MV A are connected to the NM power grid. The frequency and active power measured by the PMU are shown in Figure 9. During normal operation, the power system continues to suffer from ambient excitation caused by the load behavior or other stochastic processes.

Thus, the measured frequency and power randomly fluctuate around the operation point.

Next, the effective inertia of the two power grids is estimated by the method proposed in this paper. The statistical

---

### TABLE 2. Inertia estimation result in different case.

| Item | Actual value | Estimated Value | Basic Opc | Opc 2 | Opc 3 |
|------|--------------|----------------|----------|-------|-------|
|      | Mean         | Std.            | Mean     | Std.  | Mean  |
| H₁   | 30.67        | 32.31           | 33.15    | 3.07  | 28.31 |
| H₂   | 65.48        | 63.92           | 62.59    | 2.96  | 66.09 |
| H₃   | 108.95       | 93.38           | 92.48    | 14.15 | 95.74 |

---

FIGURE 8. Simplified schematic and topology of the North China power grid.

FIGURE 9. PMU real measurement data of (a) frequency signal and (b) power signal of the actual power system.
results of the estimated values for 5 min are shown in Table 3. The statistical results show that the mean of the estimated effective inertia of the NM power grid is close to the theoretical inertia based on the synchronous generator in the area, while the deviation of the DB power grid is larger. The main reasons are as follows: 1) some generators in the DB power grid do not directly participate in the electromechanical oscillation mode, which is used to build the ETmS, and all the generators in the NM power grid are involved; 2) the motor-based industrial load in the NM power grid accounts for a very low proportion of the overall load. However, the DB power grid is the load center containing a large number of motors, which can also contribute to the effective inertia of the power grid.

The results of the estimation of the effective inertia of the NM power grid verify the effectiveness and feasibility of the proposed algorithm for practical systems and real measurements.

VI. CONCLUSIONS

Based on an in-depth study of the coupling relationship between inertia and electromechanical oscillation modes, an effective inertia estimation method based on ambient data is proposed in this paper. The scheme proposed in this paper relies only on the measured output of PMUs.

Test cases of the IEEE 16-generator system show that the proposed effective inertia estimation method can provide reliable estimation results. In contrast to most existing algorithms, the RoCoF of the center of inertia and the total active power deficit, which are very difficult to obtain in an actual system, are not required in the proposed scheme. An archived case with real measurement data shows that the proposed method can feasibly estimate the effective inertia of each area from ambient data. The focus of future research may investigate the effective inertias of systems with invasive power electronic devices and propose a targeted estimation method.

REFERENCES

[1] P. Tielens and D. Van Hertem, “The relevance of inertia in power systems,” Renew. Sustain. Energy Rev., vol. 55, pp. 999–1009, Mar. 2016.
[2] A. Ulbig, T. S. Borsche, and G. Andersson, “Impact of low rotational inertia on power system stability and operation,” in Proc. Conf. IFAC, Cape Town, South Africa, 2014, pp. 7290–7297.
[3] E. Ørutm, M. Kuivanieniemi, M. Laasonen, A. I. Bruseth, E. A. Jansson, A. Danell, K. Elkingston, and N. Modig, “Future system inertia,” ENTSOE, Brussels, Belgium, Tech. Rep., 2015.
[4] D. Gautam, V. Vittal, and T. Harbour, “Impact of increased penetration of DFIG-based wind turbine generators on transient and small signal stability of power systems,” IEEE Trans. Power Syst., vol. 24, no. 3, pp. 1426–1434, Aug. 2009.
[5] E. Spahic, D. Varma, G. Beck, G. Kahn, and V. Hild, “Impact of reduced system inertia on stable power system operation and an overview of possible solutions,” in Proc. IEEE Power Energy Soc. Gen. Meeting (PESGM), Boston, MA, USA, Jul. 2016, pp. 1–5.
[6] H. Pulgar-Painelam, Y. Wang, and H. Silva-Saravia, “On inertia distribution, inter-area oscillations and location of electronically-interfaced resources,” IEEE Trans. Power Syst., vol. 33, no. 1, pp. 995–1003, Jan. 2018.
[7] Y. Bian, H. Wyman-Pain, F. Li, R. Bhakar, S. Mishra, and N. P. Padhy, “Demand side contributions for system inertia in the GB power system,” IEEE Trans. Power Syst., vol. 33, no. 4, pp. 3521–3530, Jul. 2018.
[8] J. M. Mauricio, A. Marano, A. Gomez-Exposito, and J. L. Martinez Ramos, “Frequency regulation contribution through variable-speed wind energy conversion systems,” IEEE Trans. Power Syst., vol. 24, no. 1, pp. 173–180, Feb. 2009.
[9] T. Inoue, H. Taniguchi, Y. Ikeguchi, and K. Yoshida, “Estimation of system inertia constant and capacity of spinning-reserve support generators using measured frequency transients,” IEEE Trans. Power Syst., vol. 12, no. 1, pp. 136–143, Feb. 1997.
[10] D. P. Chassin, Z. Huang, M. K. Donnelly, C. Hassler, E. Ramirez, and C. Ray, “Estimation of WECC system inertia using observed frequency transients,” IEEE Trans. Power Syst., vol. 20, no. 2, pp. 1190–1192, May 2005.
[11] S. Sharma, S.-H. Huang, and N. D. R. Sarma, “System inertia frequency response estimation and impact of renewable resources in ERCOT interconnection,” in Proc. IEEE Power Energy Soc. Gen. Meeting, Jul. 2011, pp. 1–6.
[12] P. M. Ashton, G. A. Taylor, A. M. Carter, M. E. Bradley, and W. Hung, “Application of phasor measurement units to estimate power system inertial frequency response,” in Proc. IEEE Power Energy Soc. Gen. Meeting, Jul. 2013, pp. 1–5.
[13] P. Wall and V. Terzija, “Simultaneous estimation of the time of disturbance and inertia in power systems,” IEEE Trans. Power Del., vol. 29, no. 4, pp. 2018–2031, Aug. 2014.
[14] M. Shamirzaee, H. Ayoubzadeh, D. Farokhzad, F. Aminifar, and H. Haeri, “An improved method for estimation of inertia constant of power system based on polynomial approximation,” in Proc. Smart Grid Conf. (SGC), Dec. 2014, pp. 1–7.
[15] P. M. Ashton, C. S. Saunders, G. A. Taylor, A. M. Carter, and M. E. Bradley, “Inertia estimation of the GB power system using synchrophasor measurements,” IEEE Trans. Power Syst., vol. 30, no. 2, pp. 701–709, Mar. 2015.
[16] R. K. Panda, A. Mohapatra, and S. C. Srivastava, “Online estimation of system inertia in a power network utilizing synchrophasor measurements,” IEEE Trans. Power Syst., vol. 35, no. 4, pp. 3122–3132, Jul. 2020, doi: 10.1109/TPWRD.2019.2958603.
[17] D. Yang, B. Wang, G. Cai, Z. Chen, J. Ma, Z. Sun, and L. Wang, “Data-driven estimation of inertia for multi-area interconnected power systems using dynamic mode decomposition,” IEEE Trans. Ind. Informat., early access, May 27, 2020, doi: 10.1109/TII.2020.2998074.
[18] J. W. Pierre, D. J. Trudnowski, and M. K. Donnelly, “Initial results in electromechanical mode identification from ambient data,” IEEE Trans. Power Syst., vol. 12, no. 3, pp. 1245–1251, Aug. 1997.
[19] N. Zhou, J. W. Pierre, D. J. Trudnowski, and R. T. Gutormson, “Robust RLS methods for online estimation of power system electromechanical modes,” IEEE Trans. Power Syst., vol. 22, no. 3, pp. 1240–1249, Aug. 2007.
[20] J. Zhang and H. Xu, “Online identification of power system equivalent inertia constant,” IEEE Trans. Ind. Electron., vol. 64, no. 10, pp. 8098–8107, Oct. 2017.
[21] K. Tuttleberg, J. Kilter, D. Wilson, and K. Uhlen, “Estimation of power system inertia from ambient wide area measurements,” IEEE Trans. Power Syst., vol. 33, no. 6, pp. 7249–7257, Nov. 2018.
[22] X. Cao, B. Stephen, I. F. Abdulhadi, C. D. Booth, and G. M. Burt, “Switching Markov Gaussian models for dynamic power system inertia estimation,” IEEE Trans. Power Syst., vol. 31, no. 5, pp. 3394–3403, Sep. 2016.
[23] J. Schiffer, P. Aristidou, and R. Ortega, “Online estimation of power system inertia using dynamic regressor extension and mixing,” IEEE Trans. Power Syst., vol. 34, no. 6, pp. 4993–5001, Nov. 2019.

TABLE 3. Inertia estimation results for the actual system.

| Item         | Theoretical value based on SG | Estimated value |
|--------------|--------------------------------|-----------------|
| NM power grid | 16.14                          | 18.43           |
| DB power grid | 307.85                         | 375.17          |

D. Yang et al.: Ambient-Data-Driven Modal-Identification-Based Approach to Estimate the Inertia
[24] F. Allella, E. Chiado, G. M. Giannuzzi, D. Lauria, and F. Mottola, “On-line estimation assessment of power systems inertia with high penetration of renewable generation,” IEEE Access, vol. 8, pp. 62689–62697, Mar. 2020.

[25] A. Gorbunov, A. Dymarsky, and J. Bialek, “Estimation of parameters of a dynamic generator model from modal PMU measurements,” IEEE Trans. Power Syst., vol. 33, no. 1, pp. 53–62, Jan. 2020.

[26] A. Chakrabortty, J. H. Chow, and A. Salazar, “A measurement-based framework for dynamic equivalencing of large power systems using wide-area phasor measurements,” IEEE Trans. Smart Grid, vol. 2, no. 1, pp. 68–81, Mar. 2011.

[27] G. Cai, B. Wang, D. Yang, Z. Sun, and L. Wang, “Inertia estimation based on observed electromechanical oscillation response for power systems,” IEEE Trans. Power Syst., vol. 34, no. 6, pp. 4291–4299, Nov. 2019.

[28] G. Chavan, M. Weiss, A. Chakrabortty, S. Bhattacharya, A. Salazar, and F.-H. Ashrafi, “Identification and predictive analysis of a multi-area WECC power system model using synchrophasors,” IEEE Trans. Smart Grid, vol. 8, no. 4, pp. 1977–1986, Jul. 2017.

[29] L. S. Shieh, H. Wang, and R. E. Yates, “Discrete-continuous model conversion,” Appl. Math. Model., vol. 4, no. 6, pp. 449–455, Dec. 1980.

[30] S. A. Nezam Sarmadi and V. Venkatasubramanian, “Electromechanical mode estimation using recursive adaptive stochastic subspace identification,” IEEE Trans. Power Syst., vol. 29, no. 1, pp. 349–358, Jan. 2014.

[31] J. H. Chow and K. W. Cheung, “A toolbox for power system dynamics and control engineering education and research,” IEEE Trans. Power Syst., vol. 7, no. 4, pp. 1559–1564, Nov. 1992.

DEYOU YANG received the B.S. and M.S. degrees in electrical engineering from Northeast Electric Power University, Jilin, China, in 2005 and 2009, respectively, and the Ph.D. degree in electrical engineering from North China Electric Power University, Beijing, China, in 2014. From 2009 to 2010, he was a Research Assistant (RA) with The Hong Kong Polytechnic University. He is currently an Associate Professor of electrical engineering with Northeast Electric Power University. His research interests include power system stability analysis and control.

BO WANG received the B.S. and M.S. degrees in electrical engineering from Northeast Electric Power University, Jilin, China, in 2014 and 2017, respectively, where he is currently pursuing the Ph.D. degree in electrical engineering. His research interest includes stability and dynamic analysis of the renewable power systems.

JIN MA (Member, IEEE) received the B.S. and M.S. degrees in electrical engineering from Zhejiang University, Hangzhou, China, in 1997, and the Ph.D. degree in electrical engineering from Tsinghua University, Beijing, China, in 2000 and 2004, respectively. He was a Faculty Member with North China Electric Power University, China, from 2004 to 2013. He is currently an Associate Professor with the School of Electrical and Information Engineering, The University of Sydney, Australia. His major research interests include power system modeling, dynamic power systems, power system economics, energy informatics, and data analytics on smart grid operation.

ZHE CHEN (Fellow, IEEE) received the B.Eng. and M.Sc. degrees in electrical engineering from the Northeast China Institute of Electric Power Engineering, Jilin, China, in 1990 and 1993, respectively, and the Ph.D. degree in electrical engineering from the Harbin Institute of Technology, Harbin, China, in 1999. He is currently a Professor of electrical engineering with Northeast Electric Power University. His research interests include power system stability analysis and control, and smart grid with renewable power generation.

GUOWEI CAI received the B.S. and M.S. degrees in electrical engineering from Northeast Electric Power University, China, in 1990 and 1993, respectively, and the Ph.D. degree in electrical engineering from the Harbin Institute of Technology, Harbin, China, in 1999. He is currently a Professor of electrical engineering with Northeast Electric Power University. His research interests include power system stability analysis and control.

ZHENGLONG SUN (Member, IEEE) received the B.S., M.S., and Ph.D. degrees in electrical engineering from Northeast Electric Power University, Jilin, China, in 2011, 2014, and 2018, respectively. His research interest includes power system dynamic analysis.

LIXIN WANG received the B.S. and M.S. degrees in electrical engineering from Northeast Electric Power University, Jilin, China, in 2014 and 2017, respectively, where she is currently pursuing the Ph.D. degree in electrical engineering.