A unified statistical approach to explain the transverse momentum spectra in hadron-hadron collision

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Thermodynamical description of the system created during high energy collision requires a proper thermodynamical framework to study the distribution of particles. In this work, we have attempted to explain the transverse momentum spectra of charged hadrons formed in pp collision at different energies using the unified statistical framework. This formalism has been proved to nicely explain the spectra of particles produced in soft processes as well hard scattering processes in a consistent manner. For this analysis, we have used the highest available range of \( p_T \) published by experiments to verify the applicability of unified statistical framework at large \( p_T \).

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I. INTRODUCTION

Understanding the mechanism of particle production and dynamics of the system created during high energy hadron-hadron and heavy-ion collider experiments is the driving force behind the current understanding of particle physics. Due to the limitation in detector capabilities and the short timescale at which collision occurs, it is difficult to directly probe the initial stages of collision. Hence, we rely on the kinematic information of the final state particles to understand the thermodynamics and the evolution of extreme matter created during collision.

The transverse momentum, \( p_T \), spectra of final state particles is an essential kinematic observable used extensively to study the particle production dynamics in high energy collision. Several statistical, thermodynamical and hydrodynamical inspired models have been developed to characterise the \( p_T \)-spectra. The Tsallis statistics, developed by C. Tsallis, is a statistical thermal model used extensively to study the thermodynamics of the system created in high energy collision. It is a generalisation of standard Boltzmann-Gibbs thermodynamics with an additional parameter to take care of non-extensivity in the system. The Tsallis distribution is very effective in explaining the spectra in the low-\( p_T \) region, as shown in the works [2,4]. Some modifications in Tsallis distribution has been proposed to extend its applicability to the high-\( p_T \) part of the spectra [5,6]. A three-component Tsallis distribution [7] has also been introduced to fit the charged hadron spectra at large transverse momenta up to 200 GeV/c in 0.9 and 7 TeV pp collision data. Recently, works based on unified statistical framework [8,9], which nicely explain the contribution of soft processes and hard scattering processes in a unified manner, have been discussed for the spectra in heavy-ion collision.

In this work, we have applied the unified statistical framework to explain the spectra over a larger \( p_T \) range, upto few hundred GeV/c, with the yield spanning over several order of magnitude. We have also incorporated the three-component Tsallis formalism [7] and obtained the corresponding fit results. We have used the data of transverse momentum spectra of charged hadron produced in pp collision at four different energies (\( \sqrt{s_{NN}} = 900 \text{ GeV} \) [10], 2.76 TeV [11], 5.02 TeV [12] and 7 TeV [10]) measured by CMS experiment over wide \( p_T \) range up to 400 GeV/c. Further, we have performed an analysis on the recently released high multiplicity pp collision data at 7 TeV measured by ALICE experiment in different V0M event multiplicity classes [13] with the multiplicities corresponding to each class provided in table I. In section 2 & 3 we will discuss the Tsallis formalism followed by the unified statistical framework. We will provide corresponding fit results in section 4 and the conclusion in section 5.

II. NON-EXTENSIVE STATISTICAL APPROACH

If we consider that a purely thermal source emanates the particles in high energy collision, the most natural choice to explain the energy distribution of such particles is Boltzmann-Gibbs (BG) statistics. However, a significant deviation from data has been observed when fitting the BG-distribution with the transverse momentum spectra.

A generalisation to BG-statistics known as the non-extensive statistics has been proposed by C. Tsallis in 1988 [1]. The Tsallis statistics contains an additional parameter \( q \) which takes care of non-extensivity in the system. The Tsallis statistics replace standard exponential of BG-statistics with a \( q - \text{exponential} \) which is defined as:

\[
exp_q(x) = [1 - (q - 1)x]^{\frac{1}{q-1}} \quad (1)
\]

and the corresponding Tsallis entropy [1] will be of the
form:

\[ S_q = k \frac{1 - \sum_i p_i^q}{q - 1} \quad (2) \]

The Tsallis distribution is preferred over other formalism because it is thermodynamically consistent [2] in the sense that it satisfy the standard thermodynamic relations. The distribution function in case of Tsallis statistics is given as:

\[ E \frac{d^3N}{dp^3} = E \frac{gV}{(2\pi)^3} \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{-\frac{q}{q-1}} \quad (3) \]

We can replace \( E \) with \( m_T \cosh(y) \) where \( y \) is the rapidity of final state particles. Since we study the spectra in the midrapidity region and also chemical potential is close to zero at LHC energies, we can write the distribution function for the transverse momentum spectra as:

\[ \frac{1}{2\pi p_T} \frac{d^2N}{dp_T dy} = \frac{gV m_T}{(2\pi)^3} \left[ 1 + (q - 1) \frac{m_T}{T} \right]^{-\frac{q}{q-1}} \quad (4) \]

Here \( m_T \) is the transverse mass and is related to the transverse momentum \( (m_T = \sqrt{m^2 + p_T^2}) \) and \( g \) is the spin degeneracy factor.

The charged hadron spectra are known to primarily consist of pions, kaons and protons, hence in Ref. [7], a three-component Tsallis function has been introduced to fit the spectra and such expression can be written in the form:

\[ \frac{1}{2\pi p_T} \frac{d^2N}{dp_T dy} = 2 \frac{V}{(2\pi)^3} \sum_{i=1}^{3} g_i m_T,i \left[ 1 + (q - 1) \frac{m_T,i}{T} \right]^{-\frac{q}{q-1}} \quad (5) \]

Here the summation index \( i \) runs over \( \pi^+, K^+ \) and \( p \) with the additional factor 2 taking care of the corresponding antiparticles. The degeneracy factor for different mesons are given by \( g_{\pi^+} = g_{K^+} = 1 \) and for proton \( g_p = 2 \).

The particle production in high energy collision can be divided into two distinct class. The low-\( p_T \) regime of the spectra are dominated by particle produced in soft processes, and statistical thermal models are used to explain spectra in this region [14].

High-\( p_T \) regime contains particles produced primarily in hard processes, and we have a well established QCD inspired power-law form [13] [10] of the distribution to explain the spectra at this regime. Perturbative QCD based distribution function is given as:

\[ f(p_T) = \frac{1}{N} \frac{dN}{dp_T} = A p_T \left( 1 + \frac{p_T}{p_0} \right)^{-n} \quad (6) \]

The three-component Tsallis formalism considers the system to be populated primarily by three types particles ( \( \pi^+, K^+ \) and \( p \) ), however, in reality, the system produces many particles. Therefore, a function with more free parameters may reveal the complexity of QCD processes in a generalised way. Hence, we used the unified formalism [8] [9] to verify the spectral properties of the system.

### Table I. VZERO multiplicity classes and the corresponding multiplicity values \((dN_{ch}/d\eta)\)

| Multiplicity class | 7 TeV pp collision |
|--------------------|--------------------|
| V0M I              | 21.3 ± 0.6         |
| V0M II             | 16.5 ± 0.5         |
| V0M III            | 13.5 ± 0.4         |
| V0M IV             | 11.5 ± 0.3         |
| V0M V              | 10.1 ± 0.3         |
| V0M VI             | 8.45 ± 0.25        |
| V0M VII            | 6.72 ± 0.21        |
| V0M VIII           | 5.4 ± 0.17         |
| V0M IX             | 3.9 ± 0.14         |
| V0M X              | 2.26 ± 0.12        |

![FIG. 1. (color online) Top plot: The transverse momentum data of charged hadrons produced in pp collision at 0.9 TeV [10], 2.76 TeV [11], 5.02 TeV [12] and 7 TeV [10] measured by the CMS experiment fitted with Tsallis distribution Eq. (5). Bottom plot: Ratio of the experimental data to the corresponding value obtained from the fit function.](image)

The unified statistical framework is a generalization of Tsallis statistics, and it combines both perturbative QCD based power law and Tsallis statistics to give a unified description of the spectra. This framework consistently explains the spectra in both low- and high-\( p_T \) region and provides a tool to enhance our understanding of the \( p_T \) spectra. In the next section, we will discuss the mathematical formulation of the unified statistical framework.

### III. UNIFIED STATISTICAL FRAMEWORK

Pearson distribution is a generalised probability distribution function introduced by Karl Pearson in 1895 [17]. Different distribution function such as Gaussian, Gamma, inverse gamma, Beta, Student’s T-distribution belongs to the Pearson family and can be obtained from Pearson distribution under some limit on its parameters. This formalism has been successfully applied in diverse fields such as financial marketing, geophysics and statistics. Pearson distribution is expressed in the form of a
The transverse momentum data of charged hadrons divided into multiplicity classes produced in pp collision at 7 TeV [13] measured by the ALICE experiment fitted with Tsallis distribution Eq. (10). Bottom plot: Ratio of the experimental data to the corresponding value obtained from the fit function.

differential equation [18] as:

$$\frac{1}{p(x)} \frac{dp(x)}{dx} + \frac{a + x}{b_0 + b_1x + b_2x^2} = 0$$  \hspace{1cm} (7)$$

Here, the parameters $a$, $b_0$, $b_1$, and $b_2$ are related to first four moments $(m_1, m_2, m_3$ and $m_4$) of the distribution as:

$$a = b_1 = \frac{m_3(m_4 + 3m_2^2)}{10m_2m_4 - 18m_2^2 - 12m_3^2}$$  \hspace{1cm} (8)$$

$$b_0 = \frac{m_2(4m_2m_4 - 3m_2^2)}{10m_2m_4 - 18m_2^2 - 12m_3^2}$$  \hspace{1cm} (9)$$

$$b_2 = \frac{2m_2m_4 - 6m_2^2 - 3m_2^2}{10m_2m_4 - 18m_2^2 - 12m_3^2}$$  \hspace{1cm} (10)$$

The solution of the differential equation Eq. (7) has been modified in Ref. [8, 9] to explain the particle spectra in the heavy-ion collision. The distribution function for transverse momenta in the unified statistical framework is given as:

$$\frac{1}{2\pi p_T} \frac{d^2 N}{dp_T dy} = B' \left( 1 + \frac{p_T}{p_0} \right)^{-n} \left( 1 + (q - 1) \frac{p_T}{T} \right)^{-\frac{n}{2}}.$$  \hspace{1cm} (11)$$

Unified distribution function has also been proved to be thermodynamically consistent satisfying the standard thermodynamical relations [8]. In the case of heavy-ion collision, the unified formalism has been shown to provide better fit to the spectra than Tsallis statistics [8, 9]. These enhancements in the fit quality can be attributed to the generalisation of Tsallis statistics using the unified formalism, where both soft and hard processes for particle productions has been unified in a consistent manner.

Further, as discussed in Ref. [9], the numerical value of the fit parameters obtained by fitting the $p_T$-spectra with Eq. (11) suggest that with an increase in $p_T$, the Tsallis part of the equation decay rapidly, whereas the hard scattering part decay slowly, pointing toward the dominance of first part of Eq. (11) in the high-$p_T$ region.

### IV. RESULTS AND DISCUSSION

The presence of quenching effect beyond certain $p_T$ value in heavy-ion collision limit the application of the statistical thermal models, however, the absence of such effect in pp collision makes it an ideal choice to test whether the developed formalism covers a broader range of $p_T$. In this work, we have tested the applicability of unified statistical framework over large transverse momenta. Further, we have also verified the three-components formalism [1] and compared the results. The spectra of charged hadron produced in pp collision at different energies measured by ALICE and CMS experiment at LHC have been considered. While the pseudorapidity ranges of data at 0.9 TeV & 7 TeV is $|\eta| < 2.4$ [10], 2.76 TeV [11] & 5.02 TeV [12] is $|\eta| < 1$. At the same time, the corresponding range for the multiplicity class divided data measured by ALICE experiment at 7 TeV [13] is $|\eta| < 0.5$. Figure 1 represents the Tsallis fit to $p_T$ spectra for four different energies with $p_T$ range upto 400 GeV. The Tsallis fit for 900 GeV and 7 TeV has already been presented in Ref. [5], however, for the sake of completeness and to compare the results, these two en-

### TABLE II. Best fit value of the parameters $T$ (GeV), $q$, $p_0$ (GeV/c) & $n$ and the $\chi^2$/NDF value obtained by fitting the multiplicity class divided charged hadron spectra produced in pp collision at 7 TeV measured by the ALICE experiment [13] with the unified distribution function Eq. (11).

| Mult. class | $T$   | $q$   | $p_0$ (GeV/c) | $n$    | $\chi^2$/NDF |
|------------|-------|-------|---------------|--------|--------------|
| V0M I      | 0.221 | 1.146 | 73.878        | -7.282 | 0.367        |
|            | ±0.011| ±0.004| ±11.3         | ±0.367 | 0.996        |
| V0M II     | 0.211 | 1.145 | 86.114        | -8.973 | 0.377        |
|            | ±0.010| ±0.004| ±10.817       | ±0.377 | 0.787        |
| V0M III    | 0.202 | 1.142 | 44.988        | -5.967 | 0.274        |
|            | ±0.011| ±0.005| ±6.725        | ±0.274 | 0.639        |
| V0M IV     | 0.194 | 1.132 | 17.558        | -4.231 | 0.293        |
|            | ±0.010| ±0.005| ±1.945        | ±0.293 | 0.518        |
| V0M V      | 0.190 | 1.136 | 18.987        | -3.811 | 0.483        |
|            | ±0.017| ±0.009| ±9.757        | ±0.483 | 0.518        |
| V0M VI     | 0.182 | 1.129 | 13.101        | -3.903 | 0.522        |
|            | ±0.017| ±0.009| ±2.540        | ±0.522 | 0.321        |
| V0M VII    | 0.166 | 1.114 | 7.026         | -4.336 | 0.515        |
|            | ±0.003| ±0.001| ±1.062        | ±0.515 | 0.337        |
| V0M VIII   | 0.167 | 1.121 | 11.110        | -4.583 | 0.433        |
|            | ±0.005| ±0.002| ±3.306        | ±0.433 | 0.107        |
| V0M IX     | 0.156 | 1.135 | 13.28         | -3.342 | 0.995        |
|            | ±0.005| ±0.003| ±3.067        | ±0.995 | 0.377        |
| V0M X      | 0.126 | 1.077 | 6.417         | -9.693 | 0.557        |
|            | ±0.005| ±0.002| ±1.95         | ±0.557 | 0.726        |

FIG. 2. (color online) Top plot: The transverse momentum data of charged hadrons divided into multiplicity classes produced in pp collision at 7 TeV [13] measured by the ALICE experiment fitted with Tsallis distribution Eq. (10). Bottom plot: Ratio of the experimental data to the corresponding value obtained from the fit function.
TABLE III. Best fit value of the parameters $T$ (GeV), $q$, $p_0$ (GeV/c) & $n$ and the $\chi^2/NDF$ value obtained by fitting the charged hadron spectra produced in $pp$ collision at 0.9 TeV [10], 2.76 TeV [11], 5.02 TeV [12] and 7 TeV [13] measured by the CMS experiment with the unified distribution function Eq. (11).

| Energy | $T$     | $q$  | $p_0$ | $n$  | $\chi^2/NDF$ |
|--------|---------|------|-------|------|--------------|
| 0.9 TeV| 0.078   | 1.032| 3.603 | -25.58| 1.79         |
|        | $\pm 0.009$ | $\pm 0.003$ | $\pm 0.131$ | $\pm 3.119$ |             |
| 2.76 TeV| 0.132   | 1.07 | 4.014 | -8.926| 0.996        |
|        | $\pm 0.006$ | $\pm 0.002$ | $\pm 0.231$ | $\pm 0.363$ |             |
| 5.02 TeV| 0.146   | 1.122| 2.737 | -3.119| 3.119        |
|        | $\pm 0.007$ | $\pm 0.001$ | $\pm 0.422$ | $\pm 0.011$ |             |
| 7 TeV  | 0.125   | 1.147| 0.849 | -1.814| 4.559        |
|        | $\pm 0.001$ | $\pm 0.001$ | $\pm 0.046$ | $\pm 0.009$ |             |

FIG. 3. (color online) Top plot: The transverse momentum data of charged hadrons produced in $pp$ collision at 0.9 TeV [10], 2.76 TeV [11], 5.02 TeV [12] and 7 TeV [13] measured by the CMS experiment fitted with unified distribution Eq. (11). Bottom plot: Ratio of the experimental data to the corresponding value obtained from the fit function.

FIG. 4. (color online) Top plot: The transverse momentum data of charged hadrons divided into multiplicity classes produced in $pp$ collision at 7 TeV [12] measured by the ALICE experiment fitted with unified distribution Eq. (11). Bottom plot: Ratio of the experimental data to the corresponding value obtained from the fit function.

FIG. 5. (color online) The best fit value of $\chi^2/NDF$ obtained by fitting the $p_T$-spectra of charged hadron produced in different multiplicity classes of $pp$ collision at 7 TeV with three-component Tsallis Eq. (5) and unified distribution Eq. (11).

energies has been plotted along with other energy spectra. The corresponding fit to 7 TeV data divided into separate multiplicity classes is given in Fig. 2. Similarly, the unified function fit for different energies and multiplicity classes of 7 TeV is shown in Fig. 3 and Fig. 4 respectively.

From the plot of the ratio of experimental data to the fit function, we observe a log-periodic oscillation over a broad range of transverse momenta for both Tsallis distribution and the unified statistical framework. This form of oscillation has been discussed for Tsallis distribution in Ref. [5, 19, 20]. Further, the oscillation observed in the 7 TeV ALICE experiment data shows an interesting pattern over different multiplicity classes. Here we observe a clear reversal in the oscillation pattern as we go from the ALICE multiplicity class V0M 1 to V0M 10. This strange behaviour in data over fit needs to be further explored, and it has the potential to give interesting physics information.

In table II and III, we have provided the fitted value for different parameters that appear in the unified function Eq. (11). The $\chi^2/NDF$ values, which represent the goodness of the fit, is also presented in the table II and III. Low $\chi^2/NDF$ values in the tables suggest a good agreement between the experimental data and the unified distribution function.

Ratio plot of different energies for Tsallis fit (Fig. 1) and unified function fit (Fig. 3) show similar variation from the unity. However the ratio plots of 7 TeV at different multiplicity classes show a significant improvement with unified function fit (Fig. 1) as compared to Tsallis fit (Fig. 2), particularly, at higher $p_T$ where hard scattering processes are dominant. This improvement can also be quantified in terms of the $\chi^2/NDF$ values. From the Fig. 5 we observe lower value of $\chi^2/NDF$ for the unified function compared to the three-component Tsallis function across all multiplicity classes pointing toward a better fit to the experimental data using the unified
function.

V. CONCLUSION

In Tsallis formalism we need to assume that the charged hadron spectra are dominated by pions, kaons and protons only and hence we consider three terms in Eq. (5). However, this assumption can be relaxed in the unified formalism, where we take a single term to explain the spectra. This is due that advantageous fact that unified function naturally provides more free parameters and incorporate the physics of both soft and hard processes in a consistent manner.

In conclusion, we have presented the applicability of unified statistical framework in the small collision system and at large transverse momenta, and it has been shown to effectively fit the spectra upto very high $p_T$. Fitting at high $p_T$ is justified because unified formalism contains hard scattering in its construction.

This result further extends the applicability of unified statistical framework to $pp$ collision and to the large transverse momenta.

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