Prompt Beta Spectroscopy as a Diagnostic for Mix in Ignited NIF Capsules

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The National Ignition Facility (NIF) technology is designed to drive deuterium-tritium (DT) internal confinement fusion (ICF) targets to ignition using indirect radiation from laser beam energy captured in a hohlraum. Hydrodynamical instabilities at interfaces in the ICF capsule leading to mix between the DT fuel and the ablator shell material are of fundamental physical interest and can affect the performance characteristics of the capsule. In this Letter we describe new radiochemical diagnostics for mix processes in ICF capsules with plastic or Be (0.9%Cu) ablator shells. Reactions of high-energy tritons with shell material produce high-energy $\beta$-emitters. We show that mix between the DT fuel and the shell material enhances high-energy prompt beta emission from these reactions by more than an order of magnitude over that expected in the absence of mix.

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I. INTRODUCTION

The projected fusion yields at NIF suggest that radiochemical analysis of neutron and charged-particle incidental reactions could provide important diagnostics for physical processes in the implosion and burn of an ICF capsule. In particular, such reactions can probe mixing at the capsule interfaces owing to hydrodynamic instabilities. Materials in current capsule designs can be used as effective radiochemical detectors for mixing processes without introducing additional dopants. For example, Stoyer et al. pointed out that the $^8\text{O}(\alpha,n)^{21}\text{Ne}$ reaction can be a sensitive probe of plastic-shell mixing.\(^1\) In the present paper we concentrate on capsule designs with either plastic or beryllium shells. The capsules involve a layer of DT ice approximately 80 $\mu$m thick with $\rho = 0.25$ g cm$^{-3}$, an interior gas fill $\rho = 0.005$ g cm$^{-3}$, and an ablator layer thickness of approximately 155 $\mu$m. With a peak hohlraum temperature of 330 eV, the capsules are expected to reach a peak $\langle \rho R \rangle_{DT} \sim 1.2$ g cm$^{-2}$ and achieve ignition with a yield up to about 18 MJ.

A microscopic understanding of mix remains one of the key issues for ICF.\(^3\) We propose to study mix of the ablator shell into the DT fuel by measuring the number of interactions of tritons with the shell material. The promising reactions emit high-energy $\beta$s, and we propose prompt $\beta$ spectroscopy as a new probe of mix. We present the physics underlying the sensitivity of these reaction rates to mix and estimates for the reaction yields under different mix scenarios.

II. DETECTION

The set of core NIF diagnostics is expected to include a Debris Collector and Rad-Chem Station.\(^4\) A suite of collectors will be placed on the target chamber wall, covering multiple lines of sight. Variations of this capability could allow for prompt diagnostics. An alternate scheme that would allow a higher collection fraction is a variation of the gas collection system designed by Lawrence Livermore National Laboratory\(^5\) and installed at OMEGA. The reactions of interest in this letter would require the gas collection system to be fast enough to isolate samples with half-lives on the order of seconds.

The triton induced reactions of most interest for our proposed studies of mix produce high-energy $\beta$ emitters. For the plastic ablator shells we consider the $^{18}\text{O}(t,n)^{20}\text{F}(\beta^-)$, and $^{13}\text{C}(t,\alpha)^{12}\text{B}(\beta^-)$ reactions, and for the Be ablator shells the $^9\text{Be}(t,\alpha)^8\text{Li}(\beta^-)$ and $^9\text{Be}(t,p)^{11}\text{Be}(\beta^-)$ reactions.

The energy of the $\beta$s is high enough for them to be clearly distinguishable from prompt laser ejected electrons. Table III summarizes the properties of all the reactions of interest. In the cases of $^{20}\text{F}$ and $^{11}\text{Be}$, the additional coincidence $\gamma$-ray aids detection and distinction from $\beta$s of similar energy. The cross sections are not known for all the reactions. The $^9\text{Be}(t,\alpha)^8\text{Li}(\beta^-)$ reaction has a cross section of about 17 mb at 0.5-MeV and about 185 mb at 1.5 MeV.\(^5\) The $^9\text{Be}(t,p)^{11}\text{Be}(\beta^-)$ reaction has a threshold at 1.55 MeV and a cross section of about 1 mb at 3 MeV.\(^5\) The $^{18}\text{O}(t,n)^{20}\text{F}$ cross section is not known but is expected to be reasonably large.

A thick aluminum window on the beta spectrometer in the Debris Collector would act as a high-energy gate to eliminate the background, and the different life-times of the $\beta$ decays of interest would be used to distinguish the
reactions. The number of decays needed to provide a half-life signature and counting statistics better than 10% is about $10^4$ atoms. The solid angle for debris collection is about $10^{-6}$, assuming a 1 cm$^2$ debris collector. The beta counter efficiency on the order of 10% includes the effect of the aluminum absorber. The minimum number of atoms needed to provide the requisite number of decays is thus $10^{10}$ atoms for a debris collection system. This limit could be lowered a few orders of magnitude by increasing the effective area for debris collection.

The $^{20}\text{F}$ gaseous products could be retrieved by a gas collection system. The efficiency for fluorine collection should be high because the NIF target chamber is constructed of aluminum. The estimated time for 99% collection of gases is 90 seconds. The reduction in the number of atoms during collection is $3.5 \times 10^{-4}$ for $^{20}\text{F}$. The efficiency for an internal (4$\pi$) gas proportional counter is near 100%. Therefore, the minimum number of atoms for a detection is about $10^8$ for $^{20}\text{F}$.

### III. MIX REACTION RATE ENHANCEMENT

#### A. Knock-on Triton Fluence

The energetic tritons in the above reactions are produced by collisions with high energy neutrons from the central DT burn. We call these knock-on tritons. The number of knock-on tritons scales with the high-energy neutron production. Some of these energetic tritons go on to fuse with deuterium, some will escape, but a significant fraction will react with detector nuclei in the shell. Stopping of these energetic tritons reduces the probability of reaction with nuclei in the shell. If the shell material is intimately mixed with the fuel, the reaction yields are enhanced by the ratio of the mixing length (the thickness of the mixed region) to the triton stopping length in the unmixed fuel. Therefore, the total number of reactions of knock-on tritons with shell material is a measure of the mixing.

In the typical implosion scenario, ignition occurs near the center of the DT volume, in the hotspot region. The DT combustion progresses outward from the hotspot, finally reaching the shell region and completing. During most of the burn, the interfacial region near the shell is inert, evolving hydrodynamically under the external driving force and the internal pressure. During the interior burn, the interfacial region is bathed in 14 MeV neutrons which produce a spectrum of energetic knock-on tritons. Because the triton stopping length in the DT plasma is relatively short, only knock-on tritons produced near the interfacial region contribute to the high-energy part of the triton spectrum as seen by the shell material. A radially localized source of high-energy tritons is created near the shell, absent of complications due to the DT-burn wave, which is still near the center of the capsule.

For the nominal design discussed in this paper, approximately $10^{19}$ high-energy (14 MeV) neutrons are produced by DT burn. The shape of the neutron spectrum does not deviate significantly from a thermally-broadened 14-MeV peak because the system is quite thin to high-energy neutrons. The differential production rate for knock-on tritons at a point in the fuel is $\phi_n n_t (d\sigma/dE)$, where $\phi_n$ is the neutron flux, $n_t$ is the triton density at the specified point, and $d\sigma/dE$ is the (normalized) distribution of tritons in the final state due to collisions with 14-MeV neutrons, a known kinematic function. The effective size (radius) of the triton source and the distortion of the triton spectrum depend on the Coulomb drag. Using an analytic approximation for path length, integrating over the position of triton production, and assuming that the triton density and the electron temperature in the fuel are approximately constant over a triton trajectory, we obtain the differential knock-on triton fluence

$$d\psi_t = \frac{\theta_e^3 \psi_n n_t}{cRn_e \sqrt{E}} \sigma_{nt}(\geq E),$$

where $\theta_e$ is the electron temperature, $\psi_n$ is the neutron fluence, $n_e$ is the electron density, $c_R \approx 1.5 \times 10^{-20}\text{keV}^2\text{cm}^2$ and $\sigma_{nt}(\geq E) = \int_{E}^{E_{\text{max}}} dE \sigma_{nt}/dE$ is the integral of the microscopic knock-on triton distribution over triton energies greater than the given energy $E$. ($E_{\text{max}} \approx 10.5$ Mev).

#### B. Scenario I: No-Mix, Smooth and Wrinkled Shells

First we consider the interaction of energetic tritons with a thick unmixed region of shell material. The tritons cross the surface of the material with a differential fluence given by Eq. (1). The rate of triton reactions with the shell nuclei is small compared to the rate of energy loss, so that the probability of reaction of a triton of energy $E$ with a shell nucleus is

$$P(E) \approx \tilde{n} \int_0^E dE' \frac{\sigma_{1,\text{shell}}(E')}{{-(dE/dx)}_{\text{shell}}}.$$
Here $\hat{n}$ is the number density of shell ions that could serve as radio-chemical detectors ($^9$Be, $^{13}$C, or $^{18}$O) and $\sigma_{l,\text{shell}}$ is the energy-dependent cross-section for the triton reaction with the shell nucleus of interest. Assuming that the shell electron density and temperature, $\hat{n}_e$ and $\theta_e$, are constant over the the distance of a triton stopping length, we then have

$$P(E) \simeq 2 \frac{\hat{n}}{\hat{n}_e c_R} \int_0^E dE' \frac{\sigma_{l,\text{shell}}(E')}{\sqrt{E'}}.$$  

If we adopt the simplest physical model for a cross section with threshold as

$$\hat{E} = \hat{E}_0 \Theta(E \geq \hat{E}_c),$$

and $\Theta$ is the Heaviside step function, the probability for a $t + \text{shell}$ reaction becomes,

$$P(E) \simeq 2 \frac{\hat{n}}{\hat{n}_e c_R} \sigma_0 \left( \sqrt{E} - \sqrt{\hat{E}_c} \right) \Theta(E \geq \hat{E}_c). \tag{2}$$

The total number of reactions of high-energy tritons with shell nuclei is obtained by integrating the product of Eq.(1) and Eq.(2), and multiplying by the shell area. We also need to introduce a geometric flux factor $g(E)$ which is dependent on the triton energy, since higher energy tritons are produced preferentially in the forward (radial) direction. For a surface described by periodic wrinkling with maximum normal deflection angle $\chi$, the geometric flux factor is given by

$$g(w,\chi) = \left\{ \begin{array}{ll} \cos \eta, & 0 \leq |\eta| \leq \pi/2 - \chi \\
\frac{1}{c_R} \frac{\sin |\eta|}{\cos \chi}, & |\eta| > \pi/2 - \chi \end{array} \right.$$  

where $w = E/E_{\text{max}}$ and $\cos \eta = \sqrt{w}$. The number of reactions with the shell material is then

$$\mathcal{R} \simeq \hat{f} \sigma_0 N_n \left( 1 - \frac{\theta_e}{c_R} \right) \int_{E_c}^{E_{\text{max}}} dE \sigma_{n,t}(E) g(E) \left( 1 - \sqrt{\frac{E_c}{E}} \right),$$

where $\hat{f} = \hat{n}/\hat{n}_e$ is the ratio of the density of detector nuclei to the density of free electrons in the shell, $f = n_t/n_e \simeq 0.5$ is the ratio of density tritons to density of free electrons in the fuel, and $N_n$ is the total number of 14 MeV neutrons produced ($\psi \times$ area). The distribution of knock-on tritons is expressed as the product of a total cross section ($\sigma_{n,t}^{\text{TOT}}$) and a triton spectral shape, which can be approximated as

$$h_{nt}(w) = \frac{\sigma_{nt}(w)}{\sigma_{nt}^{\text{TOT}}} \simeq 1 - \frac{1}{\pi} \left[ \sqrt{w(1-w)} + 2 \sin^{-1} \sqrt{w} \right].$$

The number of detector activations is then

$$\mathcal{R} \simeq 4.6 \times 10^{13} \hat{f} \sigma_0 n_{nt} \left( \frac{\sigma_0}{10^{13}} \right) \left( \frac{\sigma_{nt}^{\text{TOT}}}{1 \text{ b}} \right) \left( \frac{\theta_e}{1 \text{ keV}^2} \right)^{1/2} I(w,\chi).$$

where

$$I(w,\chi) = \int_{w_{\text{min}}}^{w_{\text{max}}} dw \ h_{nt}(w) g(w,\chi) \left( 1 - \sqrt{\frac{w}{w}} \right).$$

The sensitivity of the $t + \text{shell}$ reaction rate to the threshold energy and the shape of the fuel/shell interface is carried in the integral $I(w,\chi)$. We estimate that in the shell region the detector nucleus to electron ratio is $\hat{f} \simeq 0.25, 2 \times 10^{-3}, 5 \times 10^{-5}$ for $^9$Be, $^{13}$C and $^{18}$O, respectively. As shown in Table I, we find less than a factor of two increase in the predicted $t + \text{shell}$ reaction rates for a highly rippled interface over that for a smooth interface.

C. Scenario II: Chunk Mix

Next we consider the case of so-called chunk mix, where micron-scale pieces of shell material are injected into the fuel region by shell instabilities. This case is relatively easy to treat as a modification of the unmixed case treated above. Specifically, we assume that the chunks are large enough to stop energetic tritons within their volume and that the spacing between chunks is larger than the triton stopping length in the fuel. In this situation there is no shielding.
Non-Dimensional Integral \(I(w_*,\chi)\)

| Threshold \(w_*=E_*/E_{\text{max}}\) | \(2\chi/\pi\) | 0.0 | 0.5 | 0.9 |
|-----------------------------|-----------------|------|------|------|
| 0.75                        | 0.002           | 0.002|      |      |
| 0.50                        | 0.011           | 0.011|      |      |
| 0.30                        | 0.033           | 0.033|      |      |
| 0.25                        | 0.042           | 0.043|      |      |
| 0.20                        | 0.054           | 0.054|      |      |
| 0.15                        | 0.069           | 0.070|      |      |
| 0.10                        | 0.088           | 0.091|      |      |
| 0.05                        | 0.118           | 0.122|      |      |
| 0.01                        | 0.161           | 0.172|      |      |
| 0.0                         | 0.197           | 0.217|      |      |

**TABLE I:** Values for the non-dimensional integral \(I(w_*,\chi)\) appearing in the reaction rate for a no-mix DT fuel/shell interface. As the shape of the interface \((2\chi/\pi)\) goes from flat \((2\chi/\pi=0)\) to highly wrinkled \((2\chi/\pi =0.9)\) there is less than a factor of two increase in the reaction rate.

Effect for triton production and the total number of reactions is enhanced by a simple geometric factor \((R = E_{\text{chunk}}R)\) reflecting the increased surface area exposed to the triton flux. We find an enhancement factor,

\[E_{\text{chunk}} \simeq 1 + \frac{3}{4\pi} \frac{\ell}{R_c} \frac{\rho_c}{\rho_{\text{shell}}},\]

where \(\ell\) is a mixing length, giving the size of the region over which chunks are dispersed, \(R_c\) is the average size of chunks, and \(\rho_c/\rho_{\text{shell}}\) gives the fraction of shell material mixed into the fuel as chunks, as a fraction of the nominal shell material density. This final factor can be quite small in physically reasonable situations, although it is partially compensated by the ratio \(\ell/R_c\), which can be expected to be large.

Hydrodynamic simulations typically show interface instabilities leading to the formation of highly non-linear interface structures, not unlike our highly-wrinkled no-mix scenario. Assuming that these structures can break-off and lead to chunk mix, the chunk radii would be of the order of a micron. With an assumed mixing length \(\ell \sim 4\mu\text{m}\), this scenario would imply \(\ell/R_c \sim 1\). Thus, no enhancement over the highly-wrinkled-but-unmixed interface would be expected for large chunk-mix. On the other hand, if the chunks mixed into the gas were small (order of 0.1 \(\mu\text{m}\)) the enhancement factor \(E_{\text{chunk}}\) could be as large as a factor of about two.

### D. Scenario III: Atomic Mix

Finally we consider the situation in which an atomically mixed region exists near the shell interface, characterized by a mixing length \(\ell\). In this case the high-energy tritons interact directly with shell ions mixed into the fuel. The total number of reactions in the mix region is given by the expression,

\[R_{\text{mix}} = \int_{\text{mix region}} dr \int dE \frac{d\psi_t}{dE} \sigma_{t, \text{detector}}(E) \text{Area}(r) \hat{n}(r),\]

where \(d\psi_t/dE\) is again the differential triton fluence, given by Eq.\(^1\). For the purpose of our estimate it is reasonable to assume the basic temperature and density parameters are constant within the mix region. In this case we obtain

\[R_{\text{mix}} \simeq 2.7 \times 10^{14} \frac{N_a}{10^{19}} \frac{n_t}{10^{26} \text{ cm}^{-3}} \frac{\ell}{4 \mu\text{m}} \frac{\sigma_0}{100 \text{ mb}} \frac{\sigma_{nt}^{\text{TOT}}}{1 \text{ b}} \left( \frac{\theta_e}{1 \text{ keV}} \right)^2 I_{\text{mix}}(w_*),\]

where

\[I_{\text{mix}}(w_*) = \int_{w_*}^{-1} \frac{dw}{\sqrt{w}} h_{\text{mix}}(w).\]

In a realistic physical picture, the density of the DT gas and the shell ions are more likely to vary (as opposed to remaining constant) across the mix region. For a linear mixing profile, this variation reduces the \(t + \text{shell}\) reaction rate by a factor of 1/6, which is included in above estimate.

\(I_{\text{mix}}(w_*)\) is easily evaluated and some results are given in Table \(\text{III}\). The values for this integral are an order of magnitude larger than the values obtained for \(I(w_*,\chi)\). This enhancement reflects the immersion of the shell material in the fuel, which gives a volume enhancement proportional to the ratio of the mixing length to the triton stopping length.
TABLE II: Values for the non-dimensional integral \( I_{\text{mix}}(w_*) \) appearing in the reaction rate for atomically mixed interfaces. The reaction rates are about an order of magnitude larger than the unmixed case.

| \( w_* \) | \( E_*/E_{\text{max}} \) | \( I_{\text{mix}}(w_*) \) |
|---|---|---|
| 0.50 | 0.12 | \( 0.14 \times 10^{12} \) |
| 0.40 | 0.18 | \( 0.18 \times 10^{12} \) |
| 0.30 | 0.25 | \( 0.23 \times 10^{12} \) |
| 0.20 | 0.36 | \( 0.35 \times 10^{12} \) |
| 0.15 | 0.43 | \( 0.42 \times 10^{12} \) |
| 0.10 | 0.52 | \( 0.51 \times 10^{12} \) |
| 0.05 | 0.66 | \( 0.65 \times 10^{12} \) |
| 0.01 | 0.87 | \( 0.86 \times 10^{12} \) |
| 0.00 | 1.06 | \( 1.05 \times 10^{12} \) |

**TABLE III:** Estimated number of reactions for the different mix scenarios. For the first two reactions we took the cross sections to be 200 mb and 1 mb, respectively. Little information is available for the \( ^{18}\text{O} \) and \( ^{13}\text{C} \) reactions, and they are arbitrarily set to 100 mb. We took \( \sigma_{nt} = 1\text{b} \), \( \theta_s = 1\text{ keV} \), and \( \theta_c = 1\text{ keV} \). For the positive-Q reactions we took \( w_* = 0.05 \) to account for the Coulomb barrier. To estimate the yields for the chuck mix scenario we took \( 2\chi/\pi = 0.5 \) and \( E_{\text{chunk}} = 2.0 \).

**IV. CONCLUSIONS**

Charged particle reactions leading to prompt high-energy \( \beta^- \)-decay signals can provide an important diagnostic for mixing in ICF capsules. The predicted number of such reactions, at the yields and neutron fluences expected at the NIF facility, is sufficient for a detectable radiochemical signature of mixing processes. Our estimates for the interaction of high-energy knock-on tritons with shell material suggest that an adequate number of beta-emitters are produced to provide a unique prompt diagnostic for mix between the fuel region and the shell material. The reaction yields for the \( t + \text{shell} \) reactions of interest are enhanced by the ratio of the mixing length to the triton stopping length, and this leads to increased reaction yields of more than an order of magnitude over the no-mix situation. Table III lists and summarizes our estimated \( t +^{9}\text{Be} \), \( t +^{18}\text{O} \), and \( t +^{13}\text{C} \) reaction yields for the different mix scenarios.

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