The kinematics and strength study of indexing bevel gear mechanism in rotating device driver

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Abstract. Indexing spatial gear mechanisms are widely used in automatic machines. The movement in gear mechanisms with stops is defined by complex design shapes that can lose their stiffness at critical loads. To measure the ultimate bearing stress of these mechanisms is possible by both the physical and numerical models tests results. The paper considers the numerical model for the spatial gear mechanism with driven disk indexing at a constant angular velocity of a driving disk. The presented mechanism kinematics and geometry parameters and finite element model were analyzed in the SolidWorks design environment. The structure and kinematics analysis revealed the possible reasons of mechanism failures. The numerical calculations results showing the design performance under the active area contact stresses and teeth bending stresses are represented. The maximal bend of the shaft was determined to avoid engagement failure.

Key words: rotary mechanism, gear mechanism, intermittent motion, orthogonal mechanism

1. Introduction
The consideration is made of bevel gear based spatial mechanism with output member indexing at constant angular speed of the driving member. Similar constructions are used, for example, in advertising installations [1]. A large number of inventions [2, 3] highlights such constructions relevance. Study subject (Fig. 1) includes driving and driven disks and represents a transmission gear with possible stops in three positions. Bevel gear based mechanism is a transmission with a complex spatial interaction of driving and output members [4]. Stopping in one of the three positions is performed by flat surface sliding, present on mechanism members. Therefore, the numerical simulation in the Solidworks environment was chosen to study kinematics and strength. Examining similar models in the computer environment provides comprehensive information on the study subject [3]. Providing kinematic motion by the given law gives no definite information on the design normal operation. For this cause it is necessary to perform power analysis and field tests [6]. The research objective is to study the mechanism kinematics and to detect the design defects, to assess the strength indices at effective loads and to develop means to eliminate harmful factors arising during operation.
2. Problem statement

The aims of the study are as follows:

- the design analysis by the performance at high loads, and the mechanism seizure and jamming inability;
- the calculation of loads acting on the mechanism computer model elements;
- the calculation of the design elements strength by contact and bending stresses.

The analysis of the full-scale specimen enabled us to conclude that the design normal operation declines when the first engaging tooth slips at the driving disk. Therefore, it is relevant to develop means of eliminating such a drawback. The slipping can occur: 1) when a tooth breaks; consequently, one should check a tooth by contact stresses; 2) when a tooth changes its shape or bends [7]; therefore, it calls for bending resistance calculation; 3) when the axes of driving and driven disks displace; thus, the threshold displacements are to be determined.

To calculate the loads, we applied the computer model that allowed us to obtain the system behaviour approximate evaluation by performing computational experiments [8]. Moreover, it is also necessary to define missing initial data used in calculations and required for the future model research.

Table 1. Initial data for calculation

| Properties                                             | Value      |
|--------------------------------------------------------|------------|
| Torque on the shaft with installed driving disk $M_1$, N·m | 10         |
| Maximal overload torque at starting $M_{max}$, N·m      | 25.33      |
| Rotation frequency of driving disk and driven disk $\omega$, rad/s | 1.256     |
| Operation time, sec                                    | 5          |
| Out-of-service time, sec                               | 15         |
| Lifespan in the stresses variation cycles for a period of one year, $N_{k1}$ | 525600 |
| Lifespan in the stresses variation cycles for a period of three years, $N_{k3}$ | 1576800 |

Based on the given initial data in Table 1, it is obvious that at 5 sec operation and 15 sec standstill each tooth of the driven disk experiences the stress once a minute. Loading cyclogram for operation in two modes looks as shown in Fig. 2. Despite the transmitted torque and idling torque, there is also a peak torque arising at the starting moment (driving and output
members engagement). In the following calculations, rating torque $M_1$ is taken as equal to peak one $M_{\text{max}}$ and effective for whole life cycle.

![Figure 2. Loading cyclogram](image)

3. Theory
Transmission gear design analysis. The transmission gear design analysis is aimed at identifying the factors influencing the parts operation.

The main working surfaces that are supposed to experience sliding friction are (Fig. 3): ring surface 1 (no load operation provides kinematics), flat surface 2 and 6, spherical surface 3. Working surfaces that are supposed to have rolling friction, with a share of sliding, are teeth surfaces 4.

Flat surface 2 operates under gravity, but it is partly compensated by force $F_a = 200.5 \text{ N}$, occurring in tooth engagement 4; spherical surface 3 works under axial force from transmission $F_a = 200.5 \text{ N}$. Friction may arise on surface 6 as a result of the driven disk axis bend, but only at significant wear of surface 2.

The circumferential speed is defined by the following equation:

$$V = \frac{\pi \cdot 0.857 \cdot d_{1e} \cdot n_e}{60000} = \frac{\pi \cdot 0.857 \cdot 68 \cdot 12}{6000} = 0.038 \text{ m/s},$$

where $d_{1e}$ is reference diameter of the driven disk. Consequently, the transmission is low-speed one: lubrication conditions are without lubricant or with grease.

Powers in engagement at peak load $M_{\text{max}}$ are:

Circumferential $F_{c} = \frac{2000 \cdot M_{\text{max}}}{d_{1e}} = \frac{2000 \cdot 25.33}{65} = 779.3 \text{ N};$

Radial $F_{r1} = F_{r2} = F_{c} \cdot \tan 20^\circ \cdot \cos 45^\circ = 200.5 \text{ N};$

Axial $F_{a1} = F_{a2} = F_{c} \cdot \tan 20^\circ \cdot \sin 45^\circ = 200.5 \text{ N}.$

![Figure 3. The diagram of forces and friction surface: 1 – ring surface; 2, 6 – flat surface; 3 – spherical surface; 4 – teeth surfaces; 5 – gutter sharp edges](image)
4. Experimental results

1. Kinematic design analysis
The model kinematics study consists of determining the relevant position of the members during operation and detecting unacceptable operation conditions. Fig. 4 (on the left) shows the moment of normal gear segment engagement, and Fig. 4 (on the right) demonstrates the first tooth sliding operation moment. The consequence is the mechanism jamming, as shown in Fig. 5. Fig. 6 shows the motion path in terms of backlashes arising under the load.

![Figure 4. Normal gear segment engagement (left) and driving disk tooth slipping (right)](image)

![Figure 5. The consequence of jamming at the first engaging tooth slipping](image)

![Figure 6. The motion path of the chosen point for one complete revolution of the driving disk](image)

2. Design power analysis
Power analysis is aimed at determining the stresses, displacements and lifespan of the driving and driven disks (Fig. 7, 8, 9).
The force is to be determined in modeling, choosing by the shear absence condition. Maximal power causing no shear for a driving disk is $F = 1200 \text{ N}$ at $73.3 \text{ MPa}$; and for a driven disk it is $F = 1600 \text{ N}$ at $77.3 \text{ MPa}$.

![Stress profiles for driving disk (left) and driven disk (right) by maximum distortion energy theory (von Mises theory)](image)

**Figure 7.** Stress profiles for driving disk (left) and driven disk (right) by maximum distortion energy theory (von Mises theory)

![Displacement profiles for driving disk (left) and driven disk (right)](image)

**Figure 8.** Displacement profiles for driving disk (left) and driven disk (right)

![Service lifetime and the place (in black colour) of the most probable failure of the driving disk (left) and the driven disk (right)](image)

**Figure 9.** Service lifetime and the place (in black colour) of the most probable failure of the driving disk (left) and the driven disk (right)

![Normal force on the contact surfaces](image)

**Figure 10.** Normal force on the contact surfaces

5. **Results discussion**
One of the main drawbacks of the gear transmissions is the teeth edge contact at disengagement. This edge creates point load adversely affecting the gear materials wear. The edge contact at the tooth root can be compensated by dulling the contacting edges at the tooth top (Fig. 11).
Driving disk material fatigue for the effective loads sets in at 333783 stresses variation cycles (Fig. 9), signifying the lifespan of about 7.5 months. For the driving disk, fatigue fractions happen after 686223 cycles (Fig. 9), corresponding to the lifespan of 16 months. The tooth failure point, under the effect of the contact stresses, coincides with the fatigue fraction area for driving and driven disks. The buckling failure analysis is not necessary for bulk constructions, since the failure occurs later due to higher stresses.

Since the tooth operates in bending, the stresses are at the tooth root (Fig. 8). Such model behaviour is in agreement with the gear transmission operation practice. The tooth core performs almost no work, which signals about the presence of the surface stresses and irrational material utilization. Threshold displacements are distributed along the tooth unevenly. They are concentrated on the inner back cone due to less structural stiffness in this area. The maximum deformation value for the driven disk is 0.137 mm (Fig. 8) and for the driving one it is 0.209 mm (Fig. 8). The maximum shaft bending according to the point tracing path on the driving disk at the teeth engagement in the time moment of 0.8 seconds (Fig. 6) is as follows $\Delta x = 10.19 - 7.26 = 2.93$ mm. Analytical calculations of the powers in the engagements correspond to the data obtained in computer modelling (Fig. 25). The value of the rating torque $M_1$ acting upon construction is, however, 30% larger. If circumferential force $F_t = 500$ N, then $M_1 = \frac{500 \cdot 65}{2000} = 16.25$ N·m. Therefore, we get the improved loading cyclogram (Fig. 12) based on the numerical modelling finding (Fig. 10).

6. Conclusions
As a result of the investigation, the mechanism kinematics was studied and the design defects were found. The results of the computer modelling by the contact stresses are obtained, and the test of bending stiffness and components lifespan is performed. As a result of the undertaken investigation, the model drawbacks were detected, namely, large axial forces arising at engagement. These forces affect not only the teeth engagement but shafts and bearings as well. The maximum shaft bending is limited by 3 mm, to avoid tooth slipping.
The mechanism jamming may occur when the first tooth coming into engagement slips. This is unacceptable and consequently, the driving disk tooth bending stiffness should be improved to lower the bending effect.

The mechanism seizure is possible due to the sharp edges contact; these edges are to be altered in the defined area (Fig. 11) to avoid the mechanism jamming.

The comparison of the components lifespan demonstrates that the driven disk works 48.6% less. Therefore, the model design should be modified to increase the lifespan.

The obtained modelling results show the model application for the given initial data to be acceptable. The marginal stresses for the driving disk are, however, 30% larger than those of the driven disk. It is necessary to create a balanced-life design to distribute whole load along the whole component body and to move the stresses concentration from the most loaded areas. The design balanced life can be achieved: a) by creating stiffening ribs; b) by creating hollows inside the material bulk and by filling it with the stiffening mesh having a shape performing well in bending.

Stresses acting along the line from the tooth to the locking hub centre of the driving and driven disks are distributed unevenly. The strength is excessive in some areas, so balanced-life design is to be found for the model driving and driven disks. It will allow the weight to be lowered and the design balanced life to be increased.

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