A two-state multiserver retrial queueing model with balking

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Abstract. This paper discusses a two-state multiserver retrial queueing system with balking. Entering customer is admitted to join any idle server and receive his service immediately. On the other hand, if all the servers are busy, then the entering customer joins the orbit or balks from the system. Arrival of primary calls and repetition of repeating calls both follow Poisson distribution. Service times of each server follow exponential distribution. By solving the difference-differential equations recursively, we obtained transient probabilities of exact number of arrivals and exact number of departures at when all or none servers are busy. Results for particular cases are also discussed.

1. Introduction

Retrial queues are characterized by the phenomenon in which on finding all servers busy upon arrival, the arriving customer must leave the service area and join a group of unserved customers called ‘orbit’. If upon re-request for service from the orbit, a customer finds all the ‘c’ servers busy, then he/she may again rejoin the orbit and this manner continues until he/she gets service. Such queueing systems play an important role in the analysis of telephone switching systems, telecommunication networks and computer systems. For example, a subscriber who obtains a busy signal will repeat his request until his connection is made. Reviews of retrial queue can be found in Yang and Templeton [1], Falin and Templeton [2], Artalejo [3,4] and Gomez-Corral [5]. Most recently Kim and Kim [6,7] provided a detailed survey on retrial queueing systems.

If an arriving customer finds that the expected waiting time in the system seem to be more than his available time then he refuses to join the system. In this situation, the customer is said to be balked. The concept of balking was first studied by Haight [8]. A very few works have been done in this area. Retrial queueing systems with balking are mainly applicable in web access, call centers and computer networks.

Multiserver queueing systems find a range of applications in telecommunication and computer systems. Customers arrive from a population or source into the system to receive their service from one or more servers. Single server queues have been investigated extensively. Multiserver queues are less investigated due to the more complicated structure of the stochastic process which describes their behavior. Falin [9] discussed a model with non-persistent primary customers which can be considered as a retrial queue with
constant balking probabilities. Khalil, Falin and Yang [10] have considered a multiserver markovian queueing system with balking. Falin and Artalejo [11] investigated a different multiserver retrial queue in which customers join a classical waiting line or the orbit depending on the number of customers in the queue.

In the present paper, we follow the concept of ‘two-state’ queueing models which is firstly analyzed by Pegden and Rosenshine [12]. In the paper, they described the state of the system which is given by $i$ and $j$, where ‘$i$’ is the number of arrivals in the system and ‘$j$’ is the number of departures from the system until time $t$. They also provided the probabilities of exact number of units arrived in the system and exact number of units departed from the system by time $t$. Indra and Renu [13] presented the time dependent probabilities of exact arrivals and departures by time $t$ for M/M/1 queueing model with Bernoulli schedule and multiple working vacations. Garg and Kumar [14] obtained explicit time dependent probabilities of exact number of arrivals and departures from the orbit by time $t$ of a single server retrial queue with impatient customers.

Considering all the above ideas in this paper, we are able to obtain the time dependent probabilities for the exact number of arrivals in the system and exact number of departures from the system by a given time when all, some or none servers are busy for a multiserver retrial queueing system with balking.

The paper is organized as follows: The brief description of the queueing model is given in Section 2. The two-dimensional state model and the difference-differential equations of the model are derived in Section 3. Time dependent solution for when all, some or none servers are busy for the model is obtained in Sections 4. Some interesting measures of performance and verification of results are provided in Section 5. The paper ends with a suitable conclusion.

2. Model Description

Consider a multiserver retrial queueing system with balking in which customers arrive according to a Poisson process with rate $\lambda$. Service of an arriving customer gets started immediately if any free service is available. On the other hand, if an arriving primary customer finds all the servers busy, the customer either balks the system with probability $(1 - \beta)$ or joins the virtual queue (orbit) with probability $\beta$. For getting service from the servers, the customers can retry from the orbit with rate $\theta$. Service time of each of the customers follow a common exponential distribution with parameter $\mu$. The Stochastic processes involved viz. arrivals of units, departures of units and retrials are statistically independent.

Laplace transformation $\hat{f}(s)$ of $f(t)$ is given by

$$\hat{f}(s) = \int_0^\infty e^{-st} f(t) \, dt, \quad \text{Re}(s) > 0$$

The Laplace inverse of

$$\frac{\tilde{Q}(p)}{P(p)} = \sum_{k=1}^n \sum_{l=1}^{m_k} \frac{t^{m_k-l} \lambda^l \theta^k}{(m_k-l)!} \frac{d^{l-1}}{dp^{l-1}} \left( \frac{\tilde{Q}(p)}{P(p)} \right) (p - a_k)^{m_k}$$

\[ \forall p = a_k, \, a_i \neq a_k \text{ for } i \neq k. \]

where,

$$P(p) = (p - a_1)^{m_1} (p - a_2)^{m_2} \ldots \ldots (p - a_n)^{m_n}$$
Q(p) is a polynomial of degree < $m_1 + m_2 + m_3 + \ldots + m_n - 1$.

The Laplace inverse of $\mathcal{N}_{a,b,c}^{(n_1,n_2,n_3)}(s) = \frac{1}{(s+a)^{n_1}(s+b)^{n_2}(s+c)^{n_3}}$ is

$$\mathcal{N}_{a,b,c}^{(n_1,n_2,n_3)}(t) = \sum_{i=1}^{n_1} \sum_{m=1}^{i} \frac{e^{-at} t^{n_2-i} (-1)^{m+1} \Gamma(l-m-1)(m-i)!}{(n_1-i)!(m-1)!} (b-a)^{n_2+m-1} (c-a)^{n_1+i-m}$$

$$+ \sum_{i=1}^{n_2} \sum_{m=1}^{i} \frac{e^{-bt} t^{n_2-i} (-1)^{m+1} \Gamma(l-m-1)(m-i)!}{(n_2-i)!(m-1)!} (a-b)^{n_2+m-1} (c-a)^{n_1+i-m}$$

$$+ \sum_{i=1}^{n_3} \sum_{m=1}^{i} \frac{e^{-ct} t^{n_2-i} (-1)^{m+1} \Gamma(l-m-1)(m-i)!}{(n_3-i)!(m-1)!} (a-c)^{n_3+m-1} (b-c)^{n_2+i-m}$$

If $L^{-1}\{p(s)\} = P(t)$ and $L^{-1}\{q(s)\} = Q(t)$, then

$$L^{-1}\{p(s)q(s)\} = \int_0^t P(u)Q(t-u)\,du = P \ast Q, P \ast Q$$

is convolution of $P$ and $Q$.

3. The Two-Dimensional State Model

3.1 Definitions

$P_{i,j,0}(t) =$ Probability that there are exactly $i$ arrivals in the system and $j$ departures from the system by time $t$ when server is idle.

$P_{i,j,m}(t) =$ Probability that there are exactly $i$ arrivals in the system and $j$ departures from the system by time $t$ when $m$ servers are busy. $1 \leq m \leq c - 1$.

$P_{i,j,c}(t) =$ Probability that there are exactly $i$ arrivals in the system and $j$ departures from the system by time $t$ when all the $c$ servers are busy.

$P_{i,j}(t) =$ Probability that there are exactly $i$ arrivals in the system and $j$ departures from the system by time $t$.

$$P_{i,j}(t) = P_{i,j,0}(t) + \sum_{m=1}^{c-1} P_{i,j,m}(t) + P_{i,j,c}(t) \quad \forall \ i, j \quad i \geq j$$

also

$P_{i,j,c}(t) = 0$ & $P_{i,j,m}(t) = 0$ for $i \leq j, 1 \leq m \leq c - 1$; $P_{i,j,0}(t) = 0, i < j$.

Initially

$P_{0,0,0}(0) = 1; \ P_{i,j,0}(0) = 0, P_{i,j,c}(0) = 0 \ & P_{i,j,m}(t) = 0, \forall \ i, j \neq 0 \ & 1 \leq m \leq c - 1$.

3.2 The difference – differential equations governing the system model are

$$\frac{d}{dt} P_{i,j,0}(t) = -\lambda (i - j + 1) P_{i,j,0}(t) + \mu P_{i-j,1,1}(t) \quad i \geq j \geq 0$$  (1)
\[
\frac{d}{dt} P_{i,j,m}(t) = - (\lambda + m\mu + (i - j - m)\theta) P_{i,j,m}(t) + \lambda P_{i-1,j,m-1}(t) + (i - j - (m - 1))\theta P_{i,j,m-1}(t) + (m + 1)\mu P_{i,j+1,m-1}(t) \\
i > j \geq 0, 1 \leq m < c
\]

\[
\frac{d}{dt} P_{i,j,c}(t) = -(\lambda\beta + c\mu) P_{i,j,c}(t) + \lambda P_{i-1,j,c-1}(t) + \lambda\beta (1 - \delta_{i-c,j}) P_{i-1,j-1,c}(t) + (i - j - (c - 1))\theta P_{i,j,c-1}(t) \\
i > 1, i > j \geq 0
\]

where \(\delta_{i-c,j} = \begin{cases} 1, \text{ when } i - c = j \\ 0, \text{ otherwise} \end{cases}\)

Using the Laplace transformation \(\bar{f}(s)\) of \(f(t)\) given by

\[
\bar{f}(s) = \int_0^\infty e^{-st} f(t) \, dt, \quad \text{Re}(s) > 0
\]

in the equations (1) - (3) along with the initial conditions, we have

\[
(s + \lambda + (i - j)\theta)\bar{P}_{i,j,0}(s) = \mu \bar{P}_{i,j+1,1}(s) \quad i \geq j \geq 0
\]

\[
(s + \lambda + m\mu + (i - j - m)\theta) \bar{P}_{i,j,m}(s) = \lambda \bar{P}_{i-1,j,m-1}(s) + (i - j - (m - 1))\theta \bar{P}_{i,j,m-1}(s) + (m + 1)\mu \bar{P}_{i,j+1,m-1}(s) \\
i > j \geq 0, 1 \leq m < c
\]

\[
(s + \lambda\beta + c\mu) \bar{P}_{i,j,c}(s) = \lambda \bar{P}_{i-1,j,c-1}(s) + \lambda\beta (1 - \delta_{i-c,j}) \bar{P}_{i-1,j-1,c}(s) + (i - j - (c - 1))\theta \bar{P}_{i,j,c-1}(s) \\
i > j \geq 0
\]

where \(\delta_{i-c,j} = \begin{cases} 1, \text{ when } i - c = j \\ 0, \text{ otherwise} \end{cases}\]

4. Solution of the Problem

Solving equations (4) to (6) recursively, we have

\[
\bar{P}_{0,0,0}(s) = \frac{1}{s + \lambda}
\]

\[
\bar{P}_{i,1,0}(s) = \frac{\mu}{(s + \lambda)} \bar{P}_{i-1,1,1}(s) \quad \text{for } i \geq 1
\]

\[
\bar{P}_{m,0,m}(s) = \frac{\lambda}{s + \lambda + m\mu} \bar{P}_{m-1,0,m-1}(s) \quad \text{for } 1 \leq m \leq c - 1
\]
\[
\begin{align*}
\tilde{P}_{l,i-m,m}(s) &= \frac{\lambda}{s + \lambda + m\mu} \tilde{P}_{l-1,i-m,m-1}(s) + \frac{(m+1)\mu}{s + \lambda + m\mu} \tilde{P}_{l,i-m-1,m+1}(s) \\
& \quad \text{for } m = 1 \text{ to } c - 2, i = m + 1 \text{ to } c - 1 \\
\tilde{P}_{c1,c-1}(s) &= \frac{\lambda}{s + \lambda + (c-1)\mu} \tilde{P}_{c-1,1,c-2}(s) + \frac{cu}{s + \lambda + (c-1)\mu} \tilde{P}_{c,0,c}(s) \\
\tilde{P}_{l1,c-1}(s) &= \frac{c\mu}{s + \lambda + (c-1)\mu + (i-j-(c-1))\theta} \left( \frac{\lambda}{s + \lambda + \beta + c\mu} \right)^{(i-c-1)} \beta^{i-c} \tilde{P}_{c-1,0,c-1}(s) \\
& \quad \text{for } i > c \\
\tilde{P}_{l0,c}(s) &= \left( \frac{\lambda}{s + \lambda + \beta + c\mu} \right)^{(i-c-1)} \beta^{i-c} \tilde{P}_{c-1,0,c-1}(s) \quad \text{for } i \geq c \\
\tilde{P}_{l,j,c}(s) &= \left[ \sum_{k=1}^{i-j-(c-2)} \left( \frac{\lambda}{s + \lambda + \beta + c\mu} \right)^{(i-j-(c-2)-k)} \beta^{i-j-(c-2)-(k+1)} \eta_k(s) \right] \tilde{P}_{j+k+(c-2),j,c-1}(s) \\
& \quad \text{for } i \geq j + c, j \geq 1 \\
\text{where } \eta_k(s) &= \begin{cases} 
1 & \text{for } k = 1 \\
\frac{1}{(k-1)\theta} & \text{for } k = 2 \text{ to } i - j - (c - 1) \\
\frac{(k-1)\theta}{(s + \lambda + \beta + c\mu)} & \text{for } k = i - j - (c - 2) 
\end{cases}
\end{align*}
\]
\[
\begin{align*}
\tilde{P}_{l,j,c-1}(s) &= \frac{\lambda}{s + \lambda + (c-1)\mu + (i-j-(c-1))\theta} \tilde{P}_{l-1,j,c-2}(s) + \frac{(i-j-(c-2))\theta}{s + \lambda + (c-1)\mu + (i-j-(c-1))\theta} \tilde{P}_{l,j,c-2}(s) \\
& \quad + \frac{c\mu}{s + \lambda + (c-1)\mu + (i-j-(c-1))\theta} \left[ \sum_{k=1}^{i-j-(c-3)} \left( \frac{\lambda}{s + \lambda + \beta + c\mu} \right)^{(i-j-(c-3)-k)} \beta^{i-j-(c-3)-(k+1)} \eta_k(s) \tilde{P}_{j+k+1+j-1,c-1}(s) \right] \\
& \quad \text{for } i \geq (c - 1) + j, j > 1 \\
\text{where } \eta_k(s) &= \begin{cases} 
1 & \text{for } k = 1 \\
\frac{1}{(k-1)\theta} & \text{for } k = 2 \text{ to } i - j - (c - 2) \\
\frac{(k-1)\theta}{(s + \lambda + \beta + c\mu)} & \text{for } k = i - j - (c - 3) 
\end{cases}
\end{align*}
\]
\[
\frac{(m+1)\mu}{(s+\lambda+(m-1)\mu+(i-j-m)\theta)} \left\{ \frac{\lambda}{(s+\lambda+(m+1)\mu+(i-j-m)\theta)} \tilde{P}_{l-1,j-1,m}(s) + \frac{(i-j-(m-1))\theta}{(s+\lambda+(m+1)\mu+(i-j-m)\theta)} \tilde{P}_{l,j-1,m}(s) + \frac{(m+2)\mu}{(s+\lambda+(m+1)\mu+(i-j-m)\theta)} \tilde{P}_{l,j-2,m+2}(s) \right\} \\
\text{for } 1 \leq m \leq c-2, \ i \geq j + m, j > c - m
\]

(16)

\[
\tilde{p}_{l,j,0}(s) = \frac{(m+1)\mu}{s+\lambda+(i-j-1)\theta} \left\{ \frac{\lambda}{s+\lambda+2\mu+(i-j)\theta} \tilde{P}_{l-1,j-1,0}(s) + \frac{(i-j+1)\theta}{s+\lambda+2\mu+(i-j)\theta} \tilde{P}_{l-1,j-2,m+1}(s) + \frac{(m+2)\mu}{s+\lambda+2\mu+(i-j-1)\theta} \tilde{P}_{l,j-3,m+3}(s) \right\} \\
\text{for } i > j \geq c
\]

(17)

Taking the Inverse Laplace transform of equations (7) to (17), we have

\[
P_{0,0,0}(t) = e^{-\lambda t}
\]

(18)

\[
P_{l,0,0}(t) = \mu e^{-\lambda t} * P_{l-1,1}(t) \quad \text{for } i \geq 1
\]

(19)

\[
P_{m,0,m}(t) = \lambda e^{-(\lambda+\mu)t} * P_{m-1,0,0}(t) \quad \text{for } 1 \leq m \leq c - 1
\]

(20)

\[
P_{l,1,m}(t) = \lambda e^{-(\lambda+\mu)t} * P_{l-1,1,m-1}(t) + (m+1)\mu e^{-(\lambda+\mu)t} * P_{l,1,m-1}(t) \\
\text{for } m = 1 \text{ to } c-2, i = m + 1 \text{ to } c - 1
\]

(21)

\[
P_{c,1,0}(t) = \lambda e^{-(\lambda+(c-1)\mu)t} * P_{c-1,1,0}(t) + c\mu e^{-(\lambda+(c-1)\mu)t} * P_{c,0,0}(t)
\]

(22)

\[
P_{1,1,0}(t) = c\mu \lambda^{(c-1)} \beta^{(c-1)} e^{-(\lambda+(c-1)\mu+(i-j-(c-1))\theta) t} \\
\left\{ \frac{1}{(\gamma)^{(c-1)-1}} - e^{-\lambda t} \sum_{r=0}^{c-1} \frac{t^r}{r!} \left( \frac{\lambda}{(i-j-(c-1))\theta} \right)^{(c-1)-r} \right\} \ast P_{c-1,0,c-1}(t) \\
\text{for } i > c
\]

(23)

\[
P_{0,0,c}(t) = \lambda^{(c-1)} \beta^{(c-1)} e^{-(\lambda+\mu)(c-1) t} \ast P_{c-1,0,c-1}(t) \\
\text{for } i \geq c
\]

(24)
\[ P_{ij,c-1}(t) = \lambda e^{-(\lambda(1+c)-(1))}\theta(t) + \left(i - j - (c - 1)\right) \theta e^{-\left(\frac{c}{\beta}t\right)} \sum_{r=0}^{i-j-1} \frac{r!}{\lambda^r (c-j-1)^{r+1}} e^{-\left(\frac{c}{\beta}t\right)} P_{i-1,j,c-2}(t) \]

for \( i \ge j + c, j \ge 1 \) (25)

\[ P_{ij,c-1}(t) = \lambda e^{-(\lambda(1+c)-(1))}\theta(t) + \left(i - j - (c - 1)\right) \theta e^{-\left(\frac{c}{\beta}t\right)} \sum_{r=0}^{i-j-1} \frac{r!}{\lambda^r (c-j-1)^{r+1}} e^{-\left(\frac{c}{\beta}t\right)} P_{i-1,j,c-2}(t) \]

for \( i \ge j + c, j \ge 1 \) (26)

\[ P_{ij,m}(t) = \lambda e^{-(\lambda(1+m)-(1))}\theta(t) + \left(i - j - (m - 1)\right) \theta e^{-(\lambda(1+m)-(1))}\theta(t) + \]

\[ e^{-(\lambda(1+m)-(1))}\theta(t) + \left(i - j - (m - 1)\right) \theta e^{-(\lambda(1+m)-(1))}\theta(t) + \]

\[ (m + 2)\mu^2 e^{-(\lambda(1+m)-(1))}\theta(t) + \left(i - j - (m - 1)\right) \theta e^{-(\lambda(1+m)-(1))}\theta(t) + \]

\[ P_{i-1,j,m-1}(t) \]

for \( 1 \le m \le c - 2, i \ge j + m, j \ge (c - m) \) (27)
5. Some Important Performance Measures of the Model and Verification of Results

5.1 The Laplace transform of $P_l(t)$ that exactly $i$ units arrive by time $t$ is:

$$\bar{P}_l(s) = \sum_{j=0}^{i} \bar{P}_{l,j}(s) = \frac{\lambda^i}{(s+\lambda)^{i+1}} ; \quad i > 0 \tag{29}$$

And its Inverse Laplace transform is

$$P_l(t) = \frac{e^{-\lambda t} (\lambda t)^i}{i!} \tag{30}$$

The assumption that primary arrivals form a Poisson process is a basic assumption and analysis of the above abstract solution also verifies the same.

5.2. Taking summation over $i$ and $j$ on equations (7) - (17) and adding, we verified that the sum of all possible probabilities is one.

$$\sum_{i=0}^{\infty} \sum_{j=0}^{i} \{P_{l,j,0}(s) + \bar{P}_{l,j,m}(s) + \bar{P}_{l,j,c}(s)\} = \frac{1}{s}.$$

After taking the inverse Laplace transformation, we get

$$\sum_{i=0}^{\infty} \sum_{j=0}^{i} \{P_{l,j,0}(t) + P_{l,j,m}(t) + P_{l,j,c}(t)\} = 1.$$

which is a verification of our results.

5.3 Define $Q_{n,m}(t)$ as the probability that there are $n$ customers in the system at time $t$ and $m$ ($m = 0, 1, 2, \ldots, c$) servers are busy.

$$Q_{n,m}(t) = \sum_{j=0}^{\infty} P_{l+n+m,j,m}(t) \quad (m = 0, 1, 2, \ldots, c)$$
The number of customers i.e. 'n' in the orbit is obtained by using the relation:

\[ n = \text{(number of arrivals – number of departures – m)} \]

Using above relation from the equations (1) to (3) the set of equations in statistical equilibrium are:

\[ (\lambda + m + n\theta)Q_{n,m} = \lambda Q_{n,m-1} + (n + 1)\theta Q_{n+1,m-1} + (m + 1) Q_{n,m+1} \]
\[ 0 \leq m \leq c - 1, n \geq 0 \]

\[ (\lambda\beta + c\mu)Q_{n,c} = \lambda Q_{n,c-1} + (n + 1)\theta Q_{n+1,c-1} + \lambda\beta Q_{n-1,c} (1 - \delta_n, 0) \]
\[ (\text{case } m = c), n \geq 0 \]

where \( \delta_{n,0} = \begin{cases} 1, & \text{when } n = 0 \\ 0, & \text{when } n \geq 1 \end{cases} \)

### 5.4 Special Cases

(a) Assuming \( \beta = 1 \) in equations (31) and (32), we get

\[ (\lambda + m + n\theta)Q_{n,m} = \lambda Q_{n,m-1} + (n + 1)\theta Q_{n+1,m-1} + (m + 1) Q_{n,m+1} \]
\[ 0 \leq m \leq c - 1, n \geq 0 \]

\[ (\lambda + c\mu)Q_{n,c} = \lambda Q_{n,c-1} + (n + 1)\theta Q_{n+1,c-1} + \lambda Q_{n-1,c} \]
\[ (\text{case } m = c), n \geq 0 \]

The above equations coincide with the equations of (2.17) and (2.18) of Falin & Templeton [2].

(b) Putting \( \beta = 1 \) and \( c = 1 \) in equations (18) to (28), we get the following equations:

\[ P_{0,0,0}(t) = e^{-\lambda t} \]
\[ P_{l,1,0}(t) = \mu e^{-(\lambda + (l-1)\theta) t} * P_{l,0,1}(t) \quad \text{for } i \geq 1 \]
\[ P_{l,0,0}(t) = \left[ (\lambda\mu) e^{-(\lambda + \theta) t} \left( \frac{1}{\mu} - \frac{e^{-\mu t}}{\mu} \right) * P_{l-1,l-1,0}(t) + (\mu\theta) e^{-\lambda t} \left( \frac{1}{\mu} - \frac{e^{-\mu t}}{\mu} \right) * P_{l,l-1,0}(t) \right] \]
\[ \quad \text{for } i > 1 \]
\[ P_{l,0,1}(t) = \lambda^i e^{-\lambda t} \left( \frac{1}{(\mu t)^i} - \frac{e^{-\mu t}}{\mu^i} \right) \sum_{r=0}^{l-1} (t)^r \frac{1}{r!} \]
\[ \quad \text{for } i \geq 1 \]
\[ P_{l,l-1,1}(t) = 2^l e^{-(\lambda + \mu) t} * P_{l-1,l-1,0}(t) + \theta e^{-(\lambda + \mu) t} * P_{l,l-1,0}(t) \]
\[ \quad \text{for } i > 1 \]
\[ P_{i,j,0}(t) = \mu \lambda^{j-1} e^{-(\lambda+\mu)t} (\frac{1}{\mu})^{\mu} \sum_{r=0}^{j-1} \frac{(t)^r}{r!} \cdot P_{j-1,0}(t) + e^{-(\lambda+\mu)t} \sum_{r=0}^{j-1} \frac{(t)^r}{r!} \cdot P_{j-1,1}(t) \quad \text{for } j > 1 \]  

\[ P_{i,j,1}(t) = \lambda^{j-1} e^{-(\lambda+\mu)t} \frac{(t)^{i-j-2}}{(i-j-2)!} \cdot P_{j+1,0}(t) + e^{-(\lambda+\mu)t} \sum_{k=2}^{j-1} \lambda^{j-k} \frac{(t)^{i-j-k-1}}{(i-j-k-1)!} \cdot P_{k,j-1,0}(t) + e^{-(\lambda+\mu)t} \sum_{k=2}^{j-1} k\theta \lambda^{j-k} \frac{(t)^{i-j-k}}{(i-j-k)!} \cdot P_{j+1,k,j,0}(t) + (i-j)\theta e^{-(\lambda+\mu)t} \cdot P_{j,0}(t) + \lambda^{j-1} e^{-(\lambda+\mu)t} \frac{(t)^{i-j-2}}{(i-j-2)!} \cdot P_{j+1,j,1}(t) \quad \text{for } i > j > 1 \]

The above results coincide with those of Singla and Kalra [15].

Conclusion

In this paper, we have analyzed an M/M/c queueing system with balking and retrials. We have formulated the difference differential equations based on our queueing model and obtained transient state probabilities by solving the difference differential equations recursively. Some interesting special cases of the queueing model have been obtained.

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