Kuchowicz gravastars in the braneworld formalism

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(Dated: March 16, 2022)

In the current letter, we study isotropic static spherically symmetric gravastars without charge under the framework of braneworld gravity (dimensionally reduced RS-II braneworld with positive brane tension) using the metric potential of Kuchowicz type (which is physically acceptable, non-singular and stable) in the Mazur-Mottola conjuncture. We derived the Kuchowicz free parameters from the junction conditions assuming Schwarzschild vacuum without cosmological constant. As well, we assume that the interior is filled with Dark Energy (DE-like) fluid, shell with ultrarelativistic stiff fluid. Adopting this conditions, we probed several physical aspects of the gravastars, such as proper length, shell energy and entropy, surface redshift and adiabatic index, interior region mass.

I. INTRODUCTION

Over the past decade, interest in gravastars has increased greatly. There was written a large number of papers, that investigate the research of charged and non-charged gravastars (for example, see [1–8]). Gravastars (gravitational condensate stars) was first ever proposed in works of Mazur and Mottola [9, 10]. This stars are probable alternative to the black holes. Gravastars are usually have the following structure:

- Interior region $D_1$ (from $r = 0$ to $R_1$): de Sitter fluid with Equation of State (further - EoS) $p = -\rho$.
- Intermediate (shell) region $D_2$ (from $R_1$ to $R_2$): stiff Zeldovich fluid with EoS $p = \rho$.
- Exterior region $D_3$ (from $R_2$ to $r = \infty$): empty spacetime with Schwarzschild, Schwarzschild-de Sitter or Reissner-Nordstrom geometry and EoS $p = \rho = 0$.

Exterior spacetime could be considered as the Schwarzschild Black Hole (BH) metric.

On the Figure (1) we illustrated the geometry of the gravastar spacetime on the conformal diagram. We expanded static gravastar spacetime as the Reissner-Nordstrom spacetime. Metric potentials of the gravastar interior spacetime are non-singular, and therefore, at $r = 0$ we have space without any singularities. On the conformal diagram, triangles mean de Sitter (further - dS) spacetime (interior gravastar region with EoS parameter $\omega = -1$). As well, $i_0$ stands for the infinitely distant spacelike point and $\mathcal{I}^+/-$ stands for the null-like hypersurface.

Mainly, there was the research in gravastars field done only in the Einstein General Theory of Relativity. But, despite the fact that Einstein’s relativism still describes the universe quite well, we cannot quantize relativistic systems, and recent cosmological observations and theoretical works require modifying the classical Einstein-Hilbert action. There was many attempts done to properly modify GR gravity, and one of the most viable theories of modified gravity is $f(R)$ theory, that replaces Ricci scalar in classical EH action by arbitrary function of Ricci scalar. This theory was originally proposed in [11]. One of the interesting features of this kind of modified gravity is that this MOG could describe cosmological inflation [12–14] as well as the late time acceleration, solve the dark energy problem [15, 16]. There was done some research on the various topics in other kinds of MOG’s. For example, Das et al [17] have derived exact solutions of gravastars in the $f(R, T)$ gravity. In this model, they defined pressure as the negative energy density, shell region was filled with the ultrarelativistic fluid and exterior region was assumed to be the vacuum in the non-rotating Schwarzschild-de Sitter spacetime (with the present $\Lambda$ term). Gravastar solutions in this gravity were non-singular and exact. In the $f(G, T)$ gravity, gravastar model was first ever constructed by Shamir et al. [6]. In turn, there was also probed the electromagnetic nature of the gravastars by [18] in $f(T)$ gravity with: $T = 0$ (traceless) or $f_{TT} = 0$. It was

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shown that with $T = 0$ there is no physically acceptable solutions, but with $f_{TT} = 0$ authors constructed non-singular and exact solutions for three gravastar regions.

As we know, gravity is much weaker than other three natural forces - strong and weak nuclear force and electromagnetic force. In the particle physics, this problem is known as the hierarchy problem. In attempt to solve this problem, Randall and Sundrum proposed RS-I model [19, 20]. This model includes two $(3 + 1)$ dimensional branes with the positive and negative tension in the $5D$ bulk (usually Anti-de Sitter). In the RS braneworlds, only gravity could freely propagate through bulk, and other forces located on the branes. We will investigate the second model, namely RS-II, in which the second brane with negative tension is sent to the infinity, and thus we have only one positive tension brane. In this model at the lower energies, we could also recover Newtonian gravity. During the last few decades, many papers were devoted to the investigation of braneworlds, and some of them were particularly aimed on the gravastars (for example see [23–25] and references therein. In our paper we will investigate the non-charged Kuchowicz gravastars in the framework of braneworld gravity, which is exactly the dimensionally reduced RS-II model with five dimensional bulk.

We have found some analytical and numerical solutions for different regions of gravastars structure. Also, we investigated the physical aspects of the gravastars, such as proper length, energy, entropy and interior region mass.

Our letter is organised as follows: in the Section (I) we provide the brief introduction into the topic of charged and non-charged gravastars, modified theories of gravity. In the Section (II), we describe the formalism of braneworld gravity, derive energy density and pressure for the spherically symmetric interior spacetime from modified Einstein Field Equations. In the Section (III), we provide effective Equation of State for different gravastar regions and describe Kuchowicz-like metric potential that we will use across our paper. In the Section (IV) we probe the physical aspects of the gravastars in the framework of modified gravity. Finally, we summed up everything in the last Section (V).

II. BRANEWORLD FORMALISM

In braneworld theory of gravity the Einstein-Hilbert (EH) action integral is modified as follows [26]:

$$S_{BWG} = \frac{1}{2\kappa^2_{4+d}} \int d^{4+d}x \sqrt{-g} \left( (4+d)R - 2\Lambda_{4+d} + \frac{1}{2} \nabla^2 \right) - \frac{1}{2} \kappa^2_{4+d} \left( \nabla^2 + \mathcal{L}_m \right) \right.$$

where $\mathcal{L}_m$ is the Lagrangian of the matter fields. Then, by varying the EH action we could obtain (modified) EFE [22]:

$$G_{AB} - \frac{1}{2} R g_{AB} = \nabla^2 g_{AB} + \kappa^2_{4+d} T_{AB}$$

FIG. 1. Conformal diagram of static gravastar
where four dimensional terms above could be defined through the five dimensional ones:

\[
E_{\mu \nu} = (5)C_{ACBD} n^A n^D g_\mu^\alpha g_\nu^\beta,
\]

\[
\Lambda = \frac{1}{2} (\Lambda_5 + \kappa_4^2 \sigma) \xrightarrow{\Lambda_5 \to 0} -\kappa_4^2 \sigma
\]

\[
\kappa_4^2 = \frac{1}{6} \lambda \kappa_5^4
\]

Finally, we as well could derive curvature from the Israel–Darmois junction conditions:

\[
K_{\mu \nu} = -\frac{1}{2} k_5^2 \left[ T_{\mu \nu} + \frac{1}{3} (\sigma - T) g_{\mu \nu} \right]
\]

After the use of some tedious algebra and Israel’s junction conditions, we could finally come up with the most simplified form of the on brane field equations [27]:

\[
G_{\mu \nu} = T_{\mu \nu} + \frac{6}{\sigma} S_{\mu \nu} + E_{\mu \nu}
\]

where 4 + d dimensional energy-momentum tensor is related to the 4-dimensional on brane one by Dirac delta function:

\[
(4+d) T_{AB} = -\Lambda_{4+d} 4^{+d} g + (-\sigma g_{\mu \nu} + T_{\mu \nu}) \delta(y - y_0)
\]

In the equation above \( y_0 \) means the location of the brane in the additional fifth bulk coordinate \( y \). As well, aforementioned EFE’s for 5D Randall-Sundrum II braneworld configuration could be rewritten in the more simplified form [26]

\[
G_{\mu \nu} = -\frac{1}{2} \Lambda_5 g_{\mu \nu} + \frac{2}{3} k_5^2 \left[ (5) T_{AB} g_{\mu}^A g_{\nu}^B + \left( (5) T_{AB} h^A h^B - \frac{1}{4} (5) T \right) g_{\mu \nu} \right] + KK_{\mu \nu} - K_{\mu}^\alpha K_{\alpha \nu} + \frac{1}{2} \left[ K^{\alpha \beta} K_{\alpha \beta} - K^2 \right] g_{\mu \nu} - \mathcal{E}_{\mu \nu}
\]

where the expressions for unknown tensors are

\[
S_{\mu \nu} = \frac{TT_{\mu \nu}}{12} - \frac{T_{\mu \alpha} T_{\nu}^\alpha}{4} + \frac{g_{\mu \nu}}{24} (3T_{\alpha \beta} T^{\alpha \beta} - T^2)
\]

\[
E_{\mu \nu} = -\frac{6}{\sigma} \left[ Uu_\mu u_\nu + P \chi_\nu \chi_\nu + h_{\mu \nu} \left( \frac{U - P}{3} \right) \right]
\]

In the equations above, \( \sigma \) is the (3 + 1) dimensional brane tension, \( G_{\mu \nu} \) is the Einstein tensor, \( T_{\mu \nu} \) is the brane stress-energy tensor \( (T = g_{\mu \nu} T_{\mu \nu} \text{ is it’s trace}) \), \( U \) and \( P \) are bulk energy density and isotropic pressure, finally \( u_\mu \) is the four-velocity and \( U = 1/\sqrt{g_{\mu \nu} e^\nu} \) is radial spacelike unitary vector, \( h_{\mu \nu} = g_{\mu \nu} + u_\mu u_\nu \). For simplicity, we will use bulk Equation of State (EoS) \( P = \omega U \) with \( U = Ap + B \) where \( p \) is the brane energy density. We will study the (3 + 1) dimensional gravastar geometry with the following interior spherically symmetric spacetime (metric signature is \( -, +, +, +, + \)):

\[
ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2
\]

Using the line element above and EFE’s from Equation (2), we could derive energy density and isotropic pressure [23]:

\[
e^{-\lambda} \left( \frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = \left[ \frac{\rho (r)}{\sigma} \left( 1 + \frac{\rho (r)}{2 \sigma} \right) + \frac{6U}{\sigma} \right]
\]

\[
e^{-\lambda} \left( \frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = \left[ \frac{\rho (r)}{\sigma} \left( 1 + \frac{\rho (r)}{2 \sigma} \right) + \frac{2U}{\sigma} + \frac{4P}{\sigma} \right]
\]

\[
e^{-\lambda} \left( \frac{\nu''}{2} - \frac{\lambda \nu'}{4} + \frac{\nu^2}{4} + \frac{\nu' - \lambda'}{2r} \right) = \left[ \frac{\rho (r)}{\sigma} \left( 1 + \frac{\rho (r)}{2 \sigma} \right) + \frac{2U}{\sigma} - \frac{2P}{\sigma} \right]
\]
where we have used stress-energy tensor of form [6]:

\[ T_{\mu\nu} = (\rho + p_t)u_\mu u_\nu - p_t g_{\mu\nu} + (p_r - p_t)\chi_\mu \chi_\nu \] (16)

Here, we define \( p_r \) and \( p_t \) as radial and tangential pressures respectively, \( u_\mu \) is timelike four-velocity and \( \chi_\mu \) is radial four-vector. We consider isotropic case for simplicity and therefore \( p_r = p_t = p \). It is also necessary to define 'effective' energy density and isotropic pressure [24]

\[ \rho^{\text{eff}} = \rho + \frac{p}{2\sigma} + \frac{6U}{\sigma} \] (17)

\[ p^{\text{eff}} = p + \frac{1}{2\sigma} \left( \rho(p + 2p) + 4U \right) \] (18)

Finally, in braneworld gravity for a given line element, energy conservation (well known Tolman-Oppenheimer-Volkov) equation reads [28–31]:

\[ \frac{dp}{dr} + \frac{v'(r)}{2}(\rho + p) + F_{\text{ex}} = 0 \] (19)

where \( F_{\text{ex}} \) is the external force, that is present because of stress-energy tensor non-continuity in braneworld MOG. To properly analyze such compact astrophysical object as gravastar, one could assume the physically viable metric potential, namely Kuchowicz-like metric potential of form [32]:

\[ e^{\nu(r)} = e^{CR^2 + 2\ln D} \] (20)

where \( C \) and \( D \) are arbitrary constants. Interior spacetime with the given forms of metric potentials is often called as Kuchowicz spacetime.

A. Junction conditions

Gravastars need to satisfy continuity equations below (equations obtained at the surface of gravastar of radius \( R \), therefore \( r = R \) [33]):

Continuity of \( g_{tt} \):

\[ 1 - \frac{2M}{R} = e^{CR^2} D^2 \] (21)

Continuity of \( \frac{dg_{tt}}{dr} \):

\[ \frac{2M}{R^2} = 2 C R e^{CR^2} D^2 \] (22)

So the solutions of equations above for Kuchowicz constants are

\[ C = - \frac{M}{R^2(2M - R)} \] (23)

\[ D = \frac{\sqrt{Me^{-CR^2}}}{\sqrt{C} R^{3/2}} \] (24)

Therefore, we could proceed to the gravastars in the next section.

III. GRAVASTARS IN THE BRANEWORLD GRAVITY

A. Interior region

Spacetime of gravastars is separated on three different regions. First region - interior region. Fluid in this region has the following effective Equation of State (further - EoS) [9, 10]:

\[ p = -\rho \] (25)

Also, for the interior region one equation is true [7, 34]:

\[ p = -\rho = -\rho_c \] (26)

where \( \rho_c \) is constant energy density. Then, by adopting the effective EoS \( p = -\rho_c \), we could derive metric tensor component (metric potential). For five dimensional RS-II braneworld, gravastar with K metric potential has the energy density of form

\[ e^{-\lambda} \left( \frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = \left[ \rho_c \left( 1 + \frac{\rho_c}{2\sigma} \right) + \frac{6}{\sigma} (A\rho_c + B) \right] \] (27)

Therefore, one left metric potential has the following solution:

\[ e^{-\lambda} = -6A\rho_c r^3 + 6Br^3 + \rho_c r^3(\rho_c + \sigma) - 3r\sigma + 3c_1 \]

\[ \frac{3\sigma}{3r\sigma} \] (28)

To obtain regular solution at the origin we impose \( c_1 = 0 \) so that

\[ e^{-\lambda} = 1 - \frac{r^2(\rho_c(6A + \rho_c + \sigma) + 6B)}{3\sigma} \] (30)
Now we could also derive the brane tension from continuity condition for $g_{rr}$:

$$
\text{Continuity of } g_{rr} : \left( \frac{1 - 2M}{R} \right)^{-1} = e^{\lambda(R)} \quad (31)
$$

Using equation above, brane tension is

$$
\sigma = \frac{R^3(\rho_c(12A + \rho_c) + 12B)}{12M - 2\rho_c R^3} \quad (32)
$$

For the above expression, brane tension is positive (as expected), if $A > 0$ for relatively small constant density $\rho_c \ll 1$ and if $A < 0$ for bigger values of $\rho_c$. Using that assumption for brane tension we finally obtain simplified form of metric potential

$$
e^{-\lambda} = 1 - \frac{2Mr^2}{R^3} \quad (33)
$$

Also, we could see that both energy density and isotropic pressure do not suffer from the central singularity, which is common for gravastars models with non-singular metric potentials. Consequently, while we already defined stress-energy tensor components, we finally could also calculate the total interior region mass from the formula [35]:

$$
\mathcal{M} = \int_{R_1}^{R} 4\pi r^2 \rho \, dr = \frac{4\pi R^3}{3} \rho_c \quad (34)
$$

As we see, interior region mass is invariant under the change of gravitation framework, since energy density is constant. But we also could define the active gravitational mass of interior region in terms of effective energy density as follows:

$$
\tilde{M} = \int_{0}^{R} 4\pi r^2 \rho_{\text{eff}} \, dr = \frac{4\pi R^3}{3} \left( \frac{6A \rho_c}{\sigma} + \rho_c \left( \frac{\rho_c}{2\sigma} + 1 \right) \right) \quad (35)
$$

We plot the gravitational mass for both regular and effective energy densities on the Figure (1). As we see, quantity of the active gravitational mass grows exponentially as we get nearer to the envelope (shell), which is expected (for example, the same results were obtained for gravastars, admitting conformal motion in $f(R, T^2)$ gravity [36]).

### B. Intermediate region: shell

The shell of the gravastar is usually very thin, but finite. It separate interior and exterior regions of gravastar and contain all of the collapsing star mass. We will assume, that the matter in the shell obey EoS equation $\rho = p$ (with $\omega = 1$). Also, from the thin-shell approximation, $0 < e^{-\lambda(r)} < 1$ [7]. Thus, with the given EoS, we could say the fluid in the shell is stiff fluid (found by [38]). For stiff fluid-like equation of state general form of braneworld EFE’s are rewritten as follows [23]

$$
e^{-\frac{\lambda}{r}} + \frac{1}{r^2} = \left[ \rho \left( 1 + \frac{6A}{\sigma} \right) \rho_c \left( \frac{\rho_c}{2\sigma} + 1 \right) \right], \quad (36)
$$

$$
- \frac{1}{r^2} = \left[ \rho \left( 1 + \left( \frac{1 + 2\omega}{\sigma} \right) 2A \right) \right] + \frac{3\rho^2}{2\sigma} + \left( \frac{1 + 2\omega}{\sigma} \right) 2B, \quad (37)
$$

$$
- \frac{\lambda^'}{r} = \frac{3A \rho_c e^{-Gr^2}}{CD^2\sigma} - \frac{3B\rho^2}{\sigma} \rho_c e^{-2Gr^2} + \frac{9\rho^2}{8CD^4\sigma} + \frac{\rho_c}{2CD^2} + \log(r) - c_1 \quad (38)
$$

Solving EFE’s and using $\rho = \rho_c e^{-\nu(r)}$ with Kuchowicz-like $\nu(r)$ we could obtain analytically second unknown metric potential

$$
e^{-\lambda} = \frac{3A \rho_c e^{-Gr^2}}{CD^2\sigma} - \frac{3B\rho^2}{\sigma} \rho_c e^{-2Gr^2} + \frac{9\rho^2}{8CD^4\sigma} + \frac{\rho_c}{2CD^2} + \log(r) - c_1 \quad (39)$$
Without the loss of generality it is convenient to assume that \( c_1 = 0 \).

\[
ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2
\]  

(40)

where \( M \) indicates total gravastar mass. For exterior spacetime, since EoS is vacuum one, EFE’s has the very simplified form below

\[
e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} = \frac{6B}{\sigma}
\]

(41)

Solution is therefore

\[
e^{-\lambda} = 1 - \frac{2M}{r} - \frac{2B}{\sigma} r^2
\]

(42)

To get rid of effective brane cosmological constant \( \Lambda = 6B/\sigma \), we will assume that \( B = 0 \) and therefore solution above mimics regular component of Schwarzschild spacetime line element (constant of integration for sake of regularity at the origin is already assumed to vanish).

IV. PHYSICAL ASPECTS OF GRAVASTARS IN MODIFIED GRAVITY

A. Proper length

Gravastar shell proper length:

\[
\ell = \int_{R}^{R+\epsilon} \frac{dr}{\sqrt{e^{-\lambda(r)}}}
\]

(43)

For the Starobinsky, gamma and exponential gravity we have the following proper length of the gravastar shell (using Tolman-Kuchowicz metric potentials):

On the Figure (3) we as usual numerically solved equation (43) with \( A = 1 \) and \( B = 0 \) (vanishing effective brane cosmological constant), \( R = 9.69 \) and varying shell thickness. As one may notice, values of the shell proper length \( \ell \) grows with \( \epsilon \rightarrow \infty \).

B. Energy

Energy of the gravastar shell is defined as follows:

\[
\mathcal{E} = \int_{R}^{R+\epsilon} 4\pi r^2 \rho_{\text{eff}} \, dr
\]

(44)

On the Figure (4) we numerically solved Equation (44) for braneworld gravity with regular energy density \( \rho \) and effective energy density \( \rho_{\text{eff}} \). Remarkably, as we see if \( \epsilon \) grows, \( \mathcal{E} \rightarrow \infty \), which is expected behavior of shell energy. Also, shell energy for effective energy density is bigger that for regular \( \rho \).
C. Entropy

Mazur and Mottola \cite{9, 10} stated that interior region of gravastar must have zero entropy density, which is stable for the single condensate area. But, entropy on the shell is generally non-zero. The entropy of the relativistic star system (static) gravastar could be easily determined by the formula below:

\[ S = \int_{R}^{R+\epsilon} 4\pi r^2 \frac{s(r)}{\sqrt{e^{-\lambda(r)}}} dr \]  

(45)

where

\[ s(r) = \xi \frac{k_B}{\hbar} \sqrt{\frac{p}{2\pi}} \]  

(46)

We assumed that \( k_B = \hbar \). On the Figure (5) variation of entropy within the shell is shown for RS-II braneworld gravastar. During the numerical analysis of the gravastar shell entropy, we noticed that shell entropy grows as shell thickness becomes bigger. Also, for effective pressure (plugged in the definition of \( s(r) \) function), entropy is slightly bigger that for regular isotropic pressure \( p \).

![Shell entropy for PSRJ1416 – 2230](image)

**FIG. 5.** Shell entropy w.r.t shell thickness \( \epsilon \). For model we use PSR J1416-2230 compact star with mass \( M = 1.97M_\odot \) and radius \( R = 9.69\text{km} \) \cite{37}. As well, we assume that \( \rho_c = 0.01 \), \( \xi = 0.235 \) and \( \sigma = 20.0762 \), \( A = 1 \).

D. Surface redshift

Gravastar surface redshift is defined in the following way:

\[ Z_s = \left| g_{tt} \right|^{1/2} - 1 = \frac{e^{-\frac{1}{2}R \left( C^2 \right)}}{D} - 1 \]  

(47)

Surface redshift for the isotropic compact star fluid must not exceed 2 (for the spacetimes with present cosmological constant surface redshift must not exceed 5). We plot the surface redshift on the Figure (6) for different compact objects with stellar nature. As we noticed from numerical investigation, for each compact star \( Z_s \) at the whole interior domain does not exceed 2, which is necessary condition.

![Surface redshift for different compact objects](image)

**FIG. 6.** Surface redshift w.r.t radial coordinate \( r \).

E. Adiabatic index

We could check the dynamical stability of the relativistic stellar against infinitesimal adiabatic perturbations by following the pioneering work of Chandrasekhar \cite{39}. Chandrasekhar predicted that for the relativistic system to be stable the adiabatic index should exceed \( 4/3 \). This adiabatic index is defined as \cite{40}:

\[ \Gamma = \frac{p + \rho \frac{dp}{d\rho}}{p} \]  

(48)

Then:

- For the interior region with the EoS \( p = -\rho \), \( \Gamma = 0 \).
- For the intermediate shell region with the EoS \( p = \rho \), \( \Gamma = 2 \).

Therefore, we could conclude that for gravastars with braneworld gravity formalism, from the adiabatic index interior region is unstable and shell region is stable.

V. CONCLUSIONS

In the present letter we have studied the static and spherically symmetric non-charged gravastars under the framework of braneworld gravity model (we assume that bulk is five dimensional and brane configuration is of RS-II type) with the Kuchowicz metric potential. In this section we want to summarize all of the key results, that was obtained in the paper.

As well, we have derived several physical parameters, such as: interior region mass, proper length, shell
energy and entropy, surface redshift and adiabatic index. We have discussed the nature of this parameters both analytically and graphically. From the numerical solutions we have noted that:

- Interior region mass: we investigate the interior region gravitational mass on the Figure (1) for both regular and effective energy densities. As one could notice, gravitational mass grows exponentially with radius $R$, which represents the common behavior of interior region DE-like matter.

- Proper length: The proper length of the gravastar shell $\ell$ is plotted w.r.t. shell thickness. As we noticed, shell proper length is increasing with growing shell thickness. We plotted the results on the Figure (3).

- Energy: The energy of the shell $E$ was probed and illustrated on the Figure (4). As well, the shell energy behaves as expected.

- Entropy: For the interior region, entropy density $S$ is zero, but for the shell region generally not. We plotted the entropy for regular braneworld gravity on Figure (5). As we see, on-shell entropy monotonously grow with shell thickness.

- Surface redshift: from the values of the surface redshift $Z_s$ we could judge whether the compact object is stable or not. For the isotropic fluid (with $p_r = p_t$), surface redshift must not exceed the value of 2, which is obeyed for some compact stars and results are plotted on the Figure (6).

As we already said, gravastars are usually separated on three different regions: interior, shell and exterior. With Kuchowicz metric potential, from Equation of State for each region we have derived analytically second unknown metric potential for $g_{rr}$ component.

In the end, we came to the conclusion that we have derived new, non-singular and horizonless gravastar model for braneworld gravity with the impact of Kuchowicz metric potential. Generally, with special metric potential it is more challenging to obtain physically acceptable solutions, but, as one could notice, interest to the Tolman-Kuchowicz metric potentials for the compact objects (exotic stars for example) have grown in this decade (see [41–46]), and thus it is important to test the Kuchowicz spacetime on static gravastars.

**ACKNOWLEDGMENT**

PKS acknowledges National Board for Higher Mathematics (NBHM), No.: 02011/3/2022 NBHM(R.P.)/R&D II/2152 Dt.14.02.2022, Govt. of India under Department of Atomic Energy (DAE). We are thankful to the honorable anonymous referee for helpful comments, which have significantly improved our work in terms of research quality and presentation.

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