Unidirectional excitation of polaritons is of immense importance in nano-photonics as this allows for efficient channeling of energy between a source and a detector. In a reciprocal material, polaritons excited by a light source typically propagate in all directions, hence, a significant portion of the energy carried away from the detector is lost. There are two broad approaches that can be used to mitigate the problem. The first one involves non-reciprocal materials that support uni-directional polaritonic modes that can only propagate along a given set of directions. This includes magnetoplasmons in systems subjected to a strong static magnetic field, chiral plasmons in systems with non-zero Berry curvature and topologically protected modes in photonic crystals with topologically non-trivial band structure. Such an approach provides a number of advantages, including independence of the plasmon propagation direction on the light source and reduced backscattering with defects. However, to implement these non-reciprocal systems proves to be quite cumbersome.

An alternative approach relies on exploiting spin-orbit interactions of light for unidirectional excitation of polaritonic modes in otherwise reciprocal materials. Particularly, control over the direction of propagation of guided modes in plasmonic systems and dielectric waveguides has been achieved by launching the guided modes from nano-antennas illuminated by circular polarized light fields. Alternatively, plasmonic modes can be directionally launched from metallic slit edges. Moreover, the near-field of circular polarized electric and magnetic dipoles has been extensively used for unidirectional excitation of guided modes in different reciprocal systems. All these realizations mentioned above were implemented in isotropic plasmonic systems. Recently, there are several studies which use 3D uniaxial hyperbolic materials for realizing similar spin-orbit coupling of light into plasmons. Marrying the highly anisotropic optical density-of-states in a hyperbolic medium with the directional coupling via spin-orbit interaction would then enable highly efficient directional launching of surface plasmon modes.

Here, we present the theory for optimal coupling of light into polaritonic modes in anisotropic and hyperbolic 2D materials via general elliptical dipoles, and address the possibility of unidirectional excitation. A hyperbolic material is a highly anisotropic material that has conductive-type response along one of the material’s optical axes and dielectric-type response along the others. Such a material supports polaritons propagating in the form of narrow sub-diffractional rays, hence, it is particularly promising for waveguiding applications and for enhancement of dipole-dipole interactions between emitters placed in the vicinity of the material. In this paper, we focus on hyperbolic 2D materials which are conducting sheets of zero thickness supporting surface hyperbolic waves, with a conductivity tensor given by \[ \sigma = \text{diag}\{\sigma_x, \sigma_y\} = \text{diag}\{\sigma'_x + i\sigma''_y, \sigma''_x + i\sigma'_y\}, \] where \(\sigma'_x\) and \(\sigma''_x\) designate real and imaginary parts. In hyperbolic 2D materials, surface polaritons channel energy along four narrow rays as is shown in Fig. 1(a) when excited by a linear polarized electric dipole, \(p = -ie_z\ A\ m\). The circular polarized electric dipole does not allow for efficient one way excitation either, as can be seen from Fig. 1(b). Surprisingly, only an elliptically polarized dipole, \(p = p_x e_x - ie_z A m\), allows for efficient suppression of two out of four hyperbolic rays (Fig. 1(c)). Here, we found \(p_x = \sqrt{1 + |\sigma''_y/\sigma'_z|}\), which depends only on the 2D material conductivity. The simulations in Figs. 1(a-d) were performed using the Maxwell’s equation solver COMSOL Multiphysics RF Module.
the dispersion relation for the surface plasmons in a hyperbolic material \( k(q_x, q_y) = \sqrt{q_x^2 + q_y^2 - k_0^2} \), where \( q_x = \epsilon_0 \mu_0 \sigma_x \sigma_y / 4 \) is the vacuum wavenumber, and \( \epsilon_0, \mu_0 \) are vacuum permittivity and permeability, respectively. The \( k \) surface (i.e., \( \omega(q_x, q_y) = \text{const} \)) of a hyperbolic material is presented in Fig. 1(e). We see that the \( k \) surface takes a hyperbolic shape, with the hyperbola asymptotes making angles \( \pm \phi, \pi \pm \phi \) with the \( x \)-axis, where \( \tan \phi = \sqrt{\sigma_x'' / \sigma_y''} \) and \( \phi = 60^\circ \).

In a hyperbolic material, plasmons carry energy in the form of four sub-diffractional rays (one in each of the four quadrants) in the directions making angles \( \pm \theta, \pi \pm \theta \) with the \( x \)-axis, where \( \tan \theta = \sqrt{\sigma_x'' / \sigma_y''} = 1 / \tan \phi \). This explains excitation of four well-pronounced hyperbolic rays by an electric dipole placed in the center of a disk of an hyperbolic material, as can be seen in Figs. 1(a-d). We want to point out that points where the hyperbolic rays hit the disk edges serve as sources for secondary hyperbolic rays.

The amount of energy deposited by the dipole into each of the rays can be controlled by choosing the ellipticity of the dipole. Particularly, the electric dipole with the momentum \( \mathbf{p} = 2e_x - ie_z \) Am, excites hyperbolic rays propagating in the second and third quadrants while suppressing the rays propagating in the first and second quadrants (see Fig. 1(c)). We estimate the efficiency of the ray suppression by calculating the ratio, \( I_1 / I_3 \), between intensities carried by the hyperbolic rays in the first and third quadrants (see Fig. 1(f)), where

\[
I_i = \int_{S_i} d\mathbf{r} |\mathbf{E}(\mathbf{r})|^2.
\]

Here \( S_i \) is the cross-section of the detector placed at a distance 300 nm away from the point source (see white dashes on Figs. 1(a-d)) and oriented orthogonally to the direction of the ray in each of the four quadrants (i.e., a normal to \( S_i \) always points along the direction of the ray), \( i = 1, 2, 3, 4 \).

In Fig. 1(f), we study the efficiency of hyperbolic ray suppression in four hyperbolic materials distinguished by an angle, \( \theta \), between the rays and the \( x \)-axis, i.e., \( \theta = 15^\circ \) (\( \sigma_x'' = -0.19 \) mS), \( \theta = 20^\circ \) (\( \sigma_x'' = -0.36 \) mS), \( \theta = 30^\circ \) (\( \sigma_x'' = -0.95 \) mS), and \( \theta = 45^\circ \) (\( \sigma_x'' = -2.85 \) mS). We considered an elliptically polarized electric dipole, \( \mathbf{p} = px e_x - ie_z \) Am, and assumed that \( \sigma_x'' = 2.85 \) mS was the same for all the materials. We observed efficient suppression of two out of four hyperbolic rays for an optimum value of the dipole momentum, with the intensity of the suppressed rays more than an order of magnitude weaker than that of the excited rays. We want to emphasize that the circular polarized dipole, \( \mathbf{p} = e_x - ie_z \), does not provide efficient one-way guiding of the hyperbolic rays. Instead, the optimum value of \( px \) depends on the material conductivity and changes between 1.44 \( (\theta = 45^\circ) \) and 4 \( (\theta = 15^\circ) \). The precise relation between \( px \) and optical constants will be derived below.

In order to explain the uni-directional excitation of surface plasmons in hyperbolic materials, we study the problem analytically in the quasi-static approximation. The electrostatic potential of an electric dipole, \( \mathbf{p} = (px, py, pz) \), placed at a height \( z_0 \) above the 2D material, can be written as (see SI for details)

\[
\Phi(\mathbf{r}, \mathbf{r}_0) = \Phi_{ext}(\mathbf{r}, \mathbf{r}_0) + \Phi_{ind}(\mathbf{r}, \mathbf{r}_0),
\]
The electric field, $|E| = |-\nabla \Phi|$, of the electrostatic plasmon induced in a hyperbolic material ($\theta = 30^\circ$) by an electric dipole $p = 2e_x - i e_y$ A.m. $\Phi$ is defined by Eq. (2). The ratio of intensities, $I_1/I_3$, carried by electrostatic plasmons in the first and third quadrants through detectors (white lines in panel (a)), is shown in Fig. 2a. The ratio of intensities, $|\Phi_1|/|\Phi_3|$ (Eq. (3)), at the surface of a hyperbolic material due to a dipole $p = 2e_x - i e_y$ A.m. (d) $\Phi_2(x,-x\tan\theta)$ calculated using exact Eq. (2) (integration over the first quadrant in q-space) and approximate Eq. (4). $p = e_z$ A.m.

As a next step we obtained analytical approximation of integral (2) by taking into account that the dominant contribution to the integral comes from the poles in $t_q$ (see SI). The approximate electrostatic potential induced by an electric dipole, $p = (p_x, p_y, p_z)$, at the surface of hyperbolic material can be written as the sum of the contributions from four quadrants in q-space

$$\Phi_{appr}(x,y,z = 0) = \sum_{i=1}^{4} \Phi_i(x,y) (e_i \cdot p)$$ (5)

where $\Phi_1(x,y) = \Phi_2(x,y)$, $\Phi_2(x,y) = \Phi_3(-x,y)$, $\Phi_3(x,y) = \Phi_2(-x,-y)$, $\Phi_4(x,y) = \Phi_2(x,-y)$, $e_1 = (i \cos \phi, i \sin \phi, 1)$, $e_2 = (-i \cos \phi, i \sin \phi, 1)$, $e_3 = (-i \cos \phi, -i \sin \phi, 1)$, and $e_4 = (i \cos \phi, -i \sin \phi, 1)$, and

$$\Phi_2(x,y) = -\theta(r_-) \frac{q_0 e^{i q_0 r_- r_+/2}}{4 e_0 \pi \sin 2\phi} e^{-q_0 \gamma_0 |r_-|/2}$$

where $\theta$ is the Heaviside function, $r_- = x/(\sqrt{2} \sin \phi) \pm y/(\sqrt{2} \cos \phi)$, $q_0 = \sqrt{2q_0/\omega'/\pi} \sin 2\phi |\sigma|/2$, $z_0 = z_0/(\sqrt{2} \sin 2\phi |\sigma|)$, $\gamma_0 = (1/8) (\sigma'_{zz}/|\sigma| \sin^2 \phi + \sigma'_{yy}/(\sigma \cos^2 \phi))$. It should be pointed out that Eq. (4) for $\Phi_2$ was obtained by evaluating the integral in Eqs. (3) over the first quadrant in q-space for the electric dipole $p = e_z$ A.m. The distribution of the plasmon electrostatic potential was calculated using approximate Eq. (5) (see Fig. 2c) in good agreement with the results obtained by direct numerical integration of Eq. (2) (see Fig. 2a). This can also be seen from Fig. 2d where the distribution of the electrostatic potential was calculated along the line $y = -x \tan \theta$ in the fourth quadrant, which coincides with the direction of one of the hyperbolic rays.

Let us take into account that the coupling between the dipole and the hyperbolic rays is defined by $e_i \cdot p$, while the spatial distribution of the rays intensity is defined by $\Phi_i(x,y)$. We consider $\Phi_1$, which is maximum when real part of the denominator is zero, i.e. $r_+ = x/(\sqrt{2} \sin \phi) + y/(\sqrt{2} \cos \phi) = 0$. This defines a line, $y = -x \tan \phi = -x \tan \theta$, along the direction of hyperbolic rays in the second and fourth quadrants. However, $\Phi_1$ is non-zero only when $r_- = x/(\sqrt{2} \sin \phi) - y/(\sqrt{2} \cos \phi) > 0$, or $y < x \tan \theta$ (due to $\theta(r_-)$ factor in Eq. (4)). This inequality is satisfied for the fourth quadrant and thus the term $\Phi_1$ describes the hyperbolic ray carrying energy in the fourth quadrant only. Similarly, it is straightforward to demonstrate that $\Phi_2$ describes the hyperbolic ray in the 3rd quadrant, $\Phi_3$ — in the 2nd quadrant, and $\Phi_4$ — in the first quadrant. Note, that the degree of collimation depends on the dipole distance $z_0$ in a manner consistent with general arguments from the incertitude principle. Namely, the shorter the distance $z_0$, the larger the range of contributing wavevectors $q_0$ and, in turn, the shorter the variable $\tilde{x} + \tilde{y}$, i.e., sharper collimation.

Let us consider the dipole with momentum $p = p_x e_x - i e_z$ A.m. In this case, $e_1 \cdot p = e_4 \cdot p = i(p_x \cos \phi - 1) = 0$.

FIG. 2. Quasi-static model. (a) Electric field, $|E| = |-\nabla \Phi|$, of the electrostatic plasmon induced in a hyperbolic material ($\theta = 30^\circ$) by an electric dipole $p = 2e_x - i e_y$ A.m. $\Phi$ is defined by Eq. (2). (b) Ratio of intensities, $I_1/I_3$, carried by electrostatic plasmons in the first and third quadrants through detectors (white lines in panel (a)). (c) Electrostatic potential, $|\Phi|$ (Eq. (3)), at the surface of a hyperbolic material due to a dipole $p = 2e_x - i e_y$ A.m. (d) $\Phi_2(x,-x\tan\theta)$ calculated using exact Eq. (2) (integration over the first quadrant in q-space) and approximate Eq. (4). $p = e_z$ A.m.
of magnitude lower than the intensity of the excited hyperbolic ray (see Fig. 3(d)). Also, hypothetically, one could switch off the third beam by using magnetic dipole moment (albeit gigantic) \cite{22}, in a combination with an electric dipole moment.

Alternatively, the efficient launching of the unidirectional tightly confined plasmonic rays is possible in a highly anisotropic material that is not hyperbolic, i.e. \( \sigma''_x \sigma''_y > 0 \), as can be seen in Fig. 3(b). In the anisotropic material, the \( k \) surface for the plasmons resembles a highly elongated ellipse with the group velocities predominantly pointing along the optical axis of the material (see Fig. 3(c)). This leads to the plasmons carrying their energy in the form of the narrow rays along the optical axis of the material (Fig. 3(b)). By using the circular polarized dipole, \( \mathbf{p} = p_x e_x - i e_z \) A-m, we can achieve the efficient uni-directional launching of a single plasmonic ray in such a material (Figs. 3(b,d)). However, the ray can only propagate along the direction of the optical axis of the anisotropic material.

Concluding, we studied unidirectional excitation of the surface plasmons in a hyperbolic 2D material. We demonstrated that efficient unidirectional launching of hyperbolic rays requires an elliptically polarized electric dipole rather than a circular polarized one. Moreover, the dipole ellipticity depends on the direction of the hyperbolic rays propagation, i.e. on the material conductivity. In general, we can only suppress two out of four hyperbolic rays by using an electric dipole. However, we can excite a single hyperbolic ray by launching plasmons at the edge of the hyperbolic material. Finally, we can experimentally implement the unidirectional launching of the hyperbolic rays by placing a metallic sphere on the 2D material surface and illuminating it with an elliptically polarized plane wave, as can be seen from Fig. 4 where two out of four hyperbolic rays were suppressed in the case of elliptical polarization \( \mathbf{e}_p = 2e_x - i e_z \) (see Fig. 4(b)). Recent development of resonant metal antenna for 2D plasmonics suggests such experimental setup is feasible \cite{11}.

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