1. Introduction

The proton and neutron are the lightest members of the lowest flavour SU(3) baryon octet, and neutron β− decay

\[ n \rightarrow p + e^- + \nu_e \]  

is a strangeness conserving \((\Delta S = 0)\) semi-leptonic decay, whose rate is governed by the weak vector and axial vector coupling constants \(G_V\) and \(G_A\). The anomalously long lifetime of the neutron \([1]\), \(\tau_n = (885.7 \pm 0.8)\) s, is purely a consequence of the extremely low energy release which is less than 0.1 % of the nucleon mass. The decay is interpreted according to the universal V-A theory of weak interactions \([2]\) with a conserved vector current derived from an approximate global symmetry of the QCD Lagrangian \([3]\). Thus \(G_V\) is expressible in terms of the Fermi coupling constant according to the relation \([4]\)

\[ G_V (\Delta S = 0) = G_F (1 + \Delta_b - \Delta_m) V_{ud}, \]  

where \(\Delta_b\) and \(\Delta_m\) are inner radiative corrections associated with beta and muon decay respectively, and \(V_{ud}\) is the leading element of the Cabibbo-Kobayashi-Maskawa mixing matrix \([3]\). Since the axial current is not conserved, \(G_A\) is renormalised by the strong interactions and possibly by non-Standard Model contributions to the weak interactions \([2]\). It must therefore be...
determined by experiment. It follows that neutron decay can be characterised by \( G_\lambda \), together with the ratio [1]

\[
\lambda = G_\lambda / G_\nu = -1.2695 \pm 0.0029 .
\]  

(3)

Although \( G_\lambda \) is itself of great theoretical interest, it is the determination of \( G_\nu \) which attracts the greatest attention because it offers the only method for determining \( V_{ud} \) and verifying the unitarity of the CKM matrix. \( G_\nu \) may be determined from the measured ft-values of the pure Fermi 0’ – 0’ superallowed \( \beta \)-transitions within isospin triplets provided the relevant nuclear physics corrections may be applied with confidence [5]. An alternative approach to the determination of \( G_\nu \) which is independent of nuclear structure effects is offered by neutron decay alone for which the factor \( G_\nu^2[1 + 3 \lambda^2] \) may be derived from the neutron lifetime as before, and the value of \( \lambda \) from observations on the parity-violating neutron-spin electron-momentum correlation coefficient \( a \) in polarized neutron decay [6].

2. The Electron-Antineutrino Angular Correlation Coefficient \( a \)

There is, however, another route to the determination of \(|\lambda|\) which has not been fully explored, namely the measurement of the electron-antineutrino angular correlation coefficient \( a \). This is given in lowest order by the expression [6]:

\[
a_a = \left[ (1 - \lambda^2) / (1 + 3 \lambda^2) \right] .
\]  

(4)

Since this is a parity conserving correlation which does not contain interference terms proportional to \( \lambda \), its observation does not require that the neutrons be polarized. It possesses the further advantage, which it shares with the electron asymmetry coefficient \( A \), that it is proportional to the anomaly \(|\lambda|-1\) rather than to \(|\lambda|\) itself.

To date the most successful method for determining \( a_a \) relies on a measurement [7] of the proton kinetic energy spectrum \( g(E) \) which has the form [8]

\[
g(E) = g_i(E) + a_a g_2(E), \quad 0 \leq E \leq E_m = 0.75 \text{ keV} ,
\]  

(5)

where the function \( g_i(E) \geq 0 \) reaches a maximum near the middle of the spectrum at about the same point where the function \( g_2(E) \) changes sign from negative to positive. The behaviour of \( g_2(E) \) is a reflection of the fact that, for a Fermi (Gamow-Teller) transition, the momenta \( p_e \) and \( p_\nu \) are predominantly parallel (anti-parallel). It should be remarked that \( a_a \), as it appears in Eq. (5) is correctly given by Eq. (4).

3. Measurement of the Integral Proton Spectrum using an Ion Trap

The experimental technique is based on a modification of the apparatus used to measure the neutron lifetime where protons from neutron decay, of energy \(<0.75 \text{ keV} \), are stored in a quasi-Penning trap aligned along the neutron beam [9,10]. This is formed by the superposition of an axially symmetric \( =5 \text{ T} \) magnetic field on a coaxial system of electrodes with \( =1 \text{ kV} \) electrostatic barriers at the trap ends. The end electrode facing the detector is designated as the “gate” while the far end electrode is designated as the “mirror.” In the measurement of \( a_a \), the neutron beam was collimated by a 16 mm diameter aperture fixed at the entrance to the cryostat in combination with a 20 mm aperture located 5 m upstream. The possible means by which this apparatus could be employed to determine a value for \( a_a \) have been analysed in detail [11].

In the simplest method the potential on the gate electrode is kept constant at about 0.85 kV while the mirror electrode may be set at different potentials \( V_0 \) in order to measure the number \( N_s(V_0) \) of protons trapped behind a barrier of variable height \( V_0 \). This is what is meant by the “one-dimensional integrated spectrum,” since protons are trapped if the energy \( m_0 \text{ in longitudinal degree of freedom} \) is less than \( e V_0 \). However Monte Carlo simulations based on the known spatial variation of the magnetic and electric fields [11] and the theoretical proton spectrum [8] indicated that this spectrum was rather insensitive to the value of \( a_a \). It was therefore found advantageous to exploit the action of a nonuniform magnetic field by establishing the mirror electrode in a region where the magnetic field has fallen to a very low value. The modified apparatus and magnetic field map are shown in Figs. 1 and 2.

In this variant, which is the inverse of the normal magnetic mirror effect whereby energy is transferred from the longitudinal to the transverse mode when a charged particle is transported into a region of high magnetic field, nearly all the transverse energy is transferred into the longitudinal mode, an effect described as adiabatic focusing or collimation. Thus the measured integrated spectrum coincides very closely with the full three-dimensional spectrum. It is, however, essential that the magnetic field be uniform at the position of the mirror electrode, and to this end a permanent magnet in the form of an annulus has been built into the mirror.
electrode. Under these conditions, and assuming exact adiabatic invariance, approximately 90% of the total proton energy should appear in the longitudinal degree of freedom [11]. In order to avoid the necessity for computing the detection efficiency for protons created in the region of inhomogeneous magnetic field, the experiment is carried out with two different trap lengths; a “long” trap and a “short” trap, with the difference in counts giving the number $N_3(V_0)$ of protons created in the region of high uniform magnetic field and trapped in the region of low uniform magnetic field [10]. At the end of each trapping cycle the mirror potential is lowered to zero to permit any protons or electrons which remained trapped to escape on to the mirror.

According to the field plot shown in Fig. 2, the spatial variation of the magnetic field in the middle range where $B \approx 2$ T is such that the field changes by about 7% in one period of cyclotron oscillation for a proton of average energy moving with a velocity $\approx 250$ km s$^{-1}$. This change might reasonably be described as adiabatic. However at $B=1$ T the field change per cycle increases to about 30% per period and the assumption of exact adiabaticity fails. Thus transverse and longitudinal motions are coupled in this region and an oscillating proton visits every state open to it by conservation of energy and angular momentum. The result is that “unbound” protons with energy above the barrier set by the mirror, but with initial longitudinal energy below the barrier, will ultimately escape. However, the time scale must be determined from the experimental data.

4. Experimental Results

The first experiments were performed during two cycles of beam-time on the high-flux cold neutron beam PF1 at the Institut Laue-Langevin in Grenoble,
France in 1997. The maximum count rate for trapped protons was about $4\,\text{s}^{-1}$ with a background/signal ratio less than 1%. The integral spectra observed in both long and short 10 ms traps were recorded in 50 V steps of $V_0$ up to a maximum of 900 V with the same statistical error at each point corresponding to about 1% at the highest counting rate. Approximately $10^6$ events were recorded in total [12].

The final experiments were performed during two cycles in 1998 and the results including many of the technical details have been published [13]. For this work the counting rates were raised from about $4\,\text{s}^{-1}$ to about $20\,\text{s}^{-1}$. Thus, to keep the deadtime correction [13] associated with the simultaneous release of two or more protons from the trap at an appropriate level, the trapping time was reduced from 10 ms to 1 ms or 2 ms with a corresponding increase in background which was determined separately. In these runs the mirror setting was altered in 10 V steps with a precision of $\pm 10\,\text{mV}$, from 0 V up to 850 V with the gate set a fixed value of 850 V. At the end of each data-taking cycle the gate potential was lowered to zero to allow a background count. The results are shown in Figs. 3 and 4.

**Fig. 3.** Comparison of experimental data with theory for summed 1 ms runs. The vertical axis shows the integrated counts in arbitrary units and the horizontal axis shows the mirror potential in volts.

**Fig. 4.** Comparison of experimental data with theory for summed 2 ms runs. The vertical axis shows the integrated counts in arbitrary units and the horizontal axis shows the mirror potential in volts.
The results of an analysis based on the *assumption* that all particles which are not energy bound escape over a time scale which is negligible compared with the trapping time are displayed in Table 1.

Since the difference between the two mean values of \( a_0 \) is non-zero at a level of significance < 0.1 \%, the hypothesis of an infinitesimal lifetime for unbound particles in the trap must be rejected. Indeed, since unbound particles are released at a constant rate into the trap, whereas the loss rate is proportional to the number present at any one time, it follows that the number of unbound particles which are trapped must eventually reach an equilibrium value. The difference in the measured values of \( a_0 \) may then be used to determine this lifetime, under the weaker hypothesis that the number of unbound trapped particles reaches equilibrium in a time < 1 ms. The results of this re-analysis are shown in Table 2.

Table 1. Results for six runs each at 1 ms and 2 ms trapping times. It is tentatively assumed that “unbound” protons escape in times << 1 ms

| Run No. | \( a \) (1 ms) | std. dev. | \( \chi^2/\text{dof} \) | \( a \) (2 ms) | std. dev. | \( \chi^2/\text{dof} \) |
|---------|----------------|-----------|------------------|----------------|-----------|------------------|
| 34      | –0.241         | 0.031     | 1.58             | –0.178         | 0.021     | 1.05             |
| 35      | –0.266         | 0.024     | 1.78             | –0.189         | 0.02      | 1.18             |
| 36      | –0.212         | 0.026     | 1.16             | –0.186         | 0.023     | 1.23             |
| 37      | –0.234         | 0.03      | 1.65             | –0.15          | 0.024     | 1.6              |
| 38      | –0.259         | 0.028     | 1.38             | –0.172         | 0.027     | 1.66             |
| 40      | –0.262         | 0.03      | 1.31             | –0.196         | 0.025     | 1.44             |

\[<a (1 \text{ ms})>= \text{–0.246} \pm 0.009 \]

\[<a (2 \text{ ms})>= \text{–0.179} \pm 0.007 \]

Table 2. Values of \( a_0 \) and standard errors for six runs each at 1 ms and 2 ms trapping times and a best fit value for the mean lifetime of unbound particles in the trap of \((0.303 \pm 0.019) \text{ ms}\)

| Run No. | \( a \) (1 ms) | std. dev. | \( \chi^2/\text{dof} \) | \( a \) (2 ms) | std. dev. | \( \chi^2/\text{dof} \) |
|---------|----------------|-----------|------------------|----------------|-----------|------------------|
| 34      | –0.1016        | 0.027     | 1.31             | –0.1052        | 0.021     | 1.02             |
| 35      | –0.123         | 0.021     | 2.02             | –0.1158        | 0.019     | 1.04             |
| 36      | –0.0706        | 0.024     | 1.28             | –0.1117        | 0.0222    | 1.09             |
| 37      | –0.0919        | 0.029     | 1.82             | –0.0768        | 0.022     | 1.45             |
| 38      | –0.1206        | 0.028     | 1.26             | –0.0991        | 0.025     | 1.62             |
| 40      | –0.1252        | 0.031     | 1.11             | –0.1237        | 0.027     | 1.4              |

\[<a (1 \text{ ms})>= \text{–0.10548} \pm 0.0054 \]

\[<a (2 \text{ ms})>= \text{–0.10538} \pm 0.00668 \]

\[<a>= \text{–0.1054} \pm 0.0055 \chi^2/\text{dof}=0.63 \]

The mean value of \( a_0 \) derived from all the measurements listed is

\[ a_0 = \text{–0.1054} \pm 0.0055 \]

where the error quoted is the standard error on the mean for 10 degrees of freedom. The final value for \( |\lambda| \) is obtained by application of Eq. (4) with the result

\[ |\lambda| = \text{1.271} \pm 0.018 \]

5. References

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