Simulation Analysis of Emulsion Droplet Excited by Chaotic Frequency Pulse Electric Field

Zhixiang Liao¹, Haifeng Gong², and Ye Peng¹

¹Chongqing Key Laboratory of Manufacturing Equipment Mechanism Design and Control, Chongqing Technology and Business University, Chongqing 400067, China
²Engineering Research Centre for Waste Oil Recovery Technology and Equipment of Ministry of Education, Chongqing Technology and Business University, Chongqing 400067, China

Abstract. The frequency of the chaotic pulse electric field is random in a limited range, which can be adjusted and controlled, thus can effectively cover the harmonic resonance frequency of the droplets in the oil, and realize the efficient coalescence and demulsification of the droplets. In this study, the chaotic frequency pulse electric field is constructed by embedding the chaotic sequence into the pulse interval with the Chaotic-Pulse-Position Modulation method, and the vibration dynamics simulation model of the emulsion droplet in the chaotic frequency electric field is established. The vibration dynamics response of the emulsion droplet under the action of the chaotic frequency pulse electric field is obtained by numerical calculation, and the dynamic analysis, phase diagram analysis and maximum Lyapunov exponent calculation of the response output are carried out. It is found that under the action of chaotic frequency pulse electric field, the emulsion droplet can obtain large tensile deformation at high frequency, but relatively small at low frequency, and the fluctuation is stable, which can qualitatively and quantitatively identify the chaotic vibration characteristics of the droplet.

1 Introduction

In recent years, the newly developed pulse electric field demulsification method has been widely used in the field of oil-water separation because of its simple structure, fast demulsification speed and low energy consumption [1]. Eow et al. [2-3] think that using appropriate frequency of pulse electric field can improve the efficiency of oil demulsification and dewatering. The research[4] shows that the main reason for the high efficiency demulsification of pulsed electric field is that the droplets in the oil will undergo harmonic resonance with sharp expansion and deformation in a specific pulse frequency range, which will increase the collision probability of adjacent droplets and weaken the mechanical intensity of the oil-water interfacial film, resulting in effective coalescence and demulsification.

However, in the emulsification system, the droplet size distribution span is large, there are micro droplet and millimeter droplet, which leads to the difference and uncertainty of the droplet harmonic resonance frequency. Thus the constant frequency or compound double frequency pulse electric field cannot cover the resonance frequency of all droplets in
the emulsified oil, which weakens the demulsification effect. Peng et al. [5] tried to manually adjust the frequency of pulse electric field to cope with the uncertain change of droplets size. However, manually adjusting the frequency of electric field can only cover the resonance frequency of some droplets in the oil, and the implementation is inconvenient. Jing et al. [6] pointed out that the demulsification of chaotic electric field is the development direction of electric dewatering technology in the future, and involved chaotic signals into the research of electric dewatering of crude oil for the first time, and provided various methods to obtain pulse chaotic signals. Based on the linear vibration theory, the dynamic model of droplet deformation was established, the working states of the droplets under the conditions of pulse interval chaos, pulse width chaos and pulse amplitude chaos are obtained by simulation. Which provides inspiration and guidance for the study of the dewatering of chaotic pulsed electric field. However, this study does not fully consider the nonlinearity of droplet vibration and its chaotic vibration response characteristics, which has some limitations. Through literature [7] it is found that the high-voltage electric field generated by the frequency chaotic sequence pulse has a large frequency span and can completely cover the resonance frequency of the droplets in the oil. Therefore, the high-voltage pulse electric field with constant amplitude, equal pulse width and frequency chaos is proposed to realize the efficient coalescence of the droplets in the oil. In this paper, from the perspective of nonlinear dynamics, the dynamic behavior of droplets in oil under the excitation of chaotic frequency electric field is investigated by numerical simulation, and the deformation law and response characteristics are explored, so as to lay a foundation for the establishment of a new efficient and practical demulsification method by pulse electric field.

2 The vibration dynamic model

By using the Chaotic-Pulse-Position-Modulation method [8-9], the chaotic sequence is introduced into the pulse interval of electric field to make its duty cycle chaotic, thus realizing frequency chaos. Firstly, the one-dimensional chaotic sequence is constructed by using full logistic mapping [10], and then mapped to the low-level period of the pulse electric field. The chaotic signal after mapping is simulated by DSP controller programming, and the switching of the pulse switch Q is controlled to realize the occurrence of the chaotic frequency pulse electric field. The equivalent circuit model and signal waveform of the chaotic frequency pulse electric field are shown in Figure 1.

![Figure 1. Electric field generating circuit and signal waveform of chaotic frequency pulse.](image)

According to the waveform of the pulse signal shown in Figure 1, all pulses have a width of \( \tau \) and an amplitude of \( E \), making the pulse interval \( t_n \) chaotic within \([T_l, T_u]\), where \( T_l \) is the lower bound and \( T_u \) is the upper bound. Then the pulse interval after modulation mapping is
\[ t_n = \frac{a_n + 1}{2} (T_n - T_i) + T_i, n = 1, 2, \ldots \] (1)

where \( a_n \) is the chaotic value generated by full logistic map, \( a_{n+1} = 1 - 2a_n^2, n = 1, 2, \ldots \)

The chaotic frequency pulse electric field can be expressed as

\[
E(t) = \begin{cases} 
E, & 0 \leq t \leq \tau, \sum_{i=1}^{n} \frac{2\pi}{w_i} \leq t \leq \tau + \sum_{i=1}^{n} \frac{2\pi}{w_i}; \\
0, & \text{other.} \quad n = 1, 2, \ldots
\end{cases}
\] (2)

where \( w_i = \frac{2\pi}{\tau + t_i} \) is the electric field angle frequency, \( E \) is the electric field intensity amplitude.

The deformation vibration of emulsion droplet under chaotic frequency pulse electric field will be affected by four forces. They are as follows: the inertia force of droplet \( F_i \), the resistance of viscous oil \( F_r \), the interfacial tension of oil-water \( F_h \) and the electric field excitation force \( F_e \). Which force balance equation[11] can be described as

\[ F_i + F_r + F_h = F_e \] (3)

It is assumed that a droplet in the oil is in the shape of a sphere with a radius of \( R \) in the static state. Under the action of the chaotic frequency pulse electric field, the stretching vibration occurs. The droplet is always in the shape of a long sphere, and its volume and geometric center remain unchanged. Taking the right hemisphere of the droplet as the research object, at a certain instant of vibration, its long half axis is \( a \), and its short half axis is \( b \), and its expansion deformation velocity is \( \dot{a}, \dot{b} \) respectively. The coordinate origin of Cartesian coordinate system is established on the geometric center of the droplet, and its forced deformation is shown in Figure 2.

![Figure 2](image-url) The forced deformation diagram of right hemisphere of droplets in oil.

At this instantaneous condition, the elongation of the half axis of the droplet is \( \delta x = a - R \), let \( \chi = \frac{\delta x}{R} \), putting the chaotic electric field expression (2) substitute into Dynamic model of droplet vibration[12,13,14], the dynamic model expression of emulsion droplets in chaotic frequency pulse electric field is obtained, which is

\[
\frac{d^2 \chi}{dt^2} + A\varphi(\chi)\frac{d\chi}{dt} + Bf(\chi) = Gc(t)e(\chi)
\] (8)
where \( A = \frac{4\mu}{R^2\rho} \), \( B = \frac{8\gamma}{R^3\rho} \), \( G = \frac{4\varepsilon\varepsilon_0 E^2}{R^2\rho} \) are the constant of each forced terms; \( \varphi(\chi) \), \( f(\chi) \) and \( e(\chi) \) are nonlinear function of each forced terms; \( c(t) \) is chaotic signal generating function, \( c(t) = \begin{cases} 1, & 0 \leq t \leq \tau, \sum_{i=1}^{n} \frac{2\pi}{w_i} \leq t \leq \sum_{i=1}^{n} \frac{2\pi}{w_i} \\ 0, & \text{other.} \end{cases} \quad n = 1, 2, \ldots \)

### 3 Simulation modeling based on Simulink

According to the mathematical description of the system, the simulation model of the chaotic electric field vibration system of emulsion droplet is built by selecting the module Simulink, as shown in Figure 3.

In this simulation model, subsystem is used to construct the complex nonlinear function \( \varphi(\chi) \) and \( e(\chi) \), user-defined function is used to construct nonlinear function items \( f(\chi) \) and various complex expressions, and the interpreted MATLAB function is used to construct the signal function \( c(t) \) of chaotic frequency pulse electric field.

![Figure 3. Simulink model of chaotic electric field vibration of emulsion droplets.](image)

It is assumed that the initial velocity and the initial \( \chi \) value of the droplet vibration in the chaotic pulsed electric field are 0 and 0.0001 respectively. Set the initial condition of integrator 1 to 0.0001, integrator 2 to 0, the simulation start time to 0, end time to 1 s, the maximum solution step to 0.001, and use the variable step solver ode45 to solve the model.

### 3 Numerical experiment and analysis

#### 3.1 Experimental condition

There is a droplet with a radius of \( R = 0.6 \times 10^{-3} \text{ m} \) in the emulsified oil, its density is \( \rho = 10^3 \text{ kg \cdot m}^{-3} \), the viscosity of oil is \( \mu = 47.2 \times 10^{-3} \text{ Pa \cdot s} \), the relative dielectric constant is \( \varepsilon_2 = 5 \), the interfacial tension of oil-water is \( \gamma = 5 \times 10^{-3} \text{ N \cdot m}^{-1} \), and the applied chaotic electric field intensity is \( E = 3 \times 10^5 \text{ V \cdot m}^{-1} \). It is assumed that the initial chaos value \( a_1 = 0.2 \), \( \tau = 0.01 \text{ s} \), \( T_u = 0.03 \text{ s} \), \( T_l = 0.001 \text{ s} \), the pulse electric field angular frequency \( \omega \) is controlled between 150 rad\text{\cdot{s}^{-1}} and 600 rad\text{\cdot{s}^{-1}}, and the change of pulse electric field and frequency is shown in Figure 4.
In this experimental condition, the vibration response of emulsion droplets in the pulse electric field with chaotic frequency within one second is calculated.

3.1 kinetic analysis

The vibration amplitude, velocity and acceleration of the droplet calculated by simulation are as shown in the figure 5, and the various forced situation of the droplet are as shown in the figure 6.

It can be seen from Fig. 5 that the stretching vibration deformation of emulsion droplet is unsteady state in the chaotic electric field. The vibration frequency of emulsion droplet is equal to the frequency of pulse electric field. In the high frequency region, the vibration amplitude of the emulsion drops is high, while in the middle and low frequency region, the vibration amplitude is relatively low, and the fluctuation is not large. There is no obvious resonance phenomenon near the resonance frequency of the droplet. In Fig. 6, we can see
that in the period of droplet stretching, the electric field excitation force $F_e$ and oil resistance $F_r$ decrease gradually, the interfacial tension $F_h$ increases gradually, and the inertia force of the droplet suddenly decreases to 0. In the period of droplet contraction, $F_e$ suddenly changes to 0, $F_h$ gradually decreases, $F_r$ direction suddenly, and then decreases rapidly. The direction of droplet inertia force first changes, then decreases rapidly, then increases to a smaller value in the reverse direction, and then tends to 0.

This is due to the fact that the electric field force overcomes the oil resistance and makes

![Figure 6. The various forced situation of droplet vibration response.](image)

The decreasing functions of $\chi$, so they decrease with the increase of $\chi$. In the low electric level, the electric field force is lost, and the interfacial tension as the driving force overcomes the oil resistance to make the drop contract, so the $F_e$ direction changes abruptly, $F_h$ is the increasing function of $\chi$, which changes equally with the change of $\chi$, $F_i$ is the result of the comprehensive effect of other forces, which is proportional to the acceleration of the droplet vibration. In addition, because the vibration of emulsion droplets in the electric field is relatively high resistance of the oil, and the free vibration is not obvious, the emulsion droplet is mainly driven by the electric field force to generate stretching vibration, and its response frequency is consistent with the electric field frequency, the frequency of electric field is chaotic and never repeated, and the vibration of droplets is unsteady, no obvious resonance phenomenon has been found near the resonance frequency.

### 3.3 Chaotic vibration

This study has made chaos recognition[15] for the vibration characteristics of emulsion droplets in the chaotic frequency pulse electric field by using the qualitative and quantitative methods.
Firstly, the phase diagram method[16] is used to identify the chaotic vibration of droplets. The $\chi$ value and its velocity $v$ obtained from the model can be used to draw the phase plan, which can directly analyze the vibration characteristics of droplets. The phase trajectory of droplet vibration is shown in Figure 7.

![Figure 7. Droplet vibration phase trajectory.](image)

In Figure 8, the phase trajectory is an unclosed curve, which is infinitely wound, folded and never repeated, and moves back and forth in a bounded area, which conforms to the characteristics of chaotic vibration. It can be concluded that the vibration of emulsion droplets in chaotic electric field is chaotic vibration.

Furthermore, the largest Lyapunov exponent is used to quantitative identify the chaotic vibration of droplets. The first step is to use C-C method[17] to determine the best embedding dimension $m$ and the best time delay $\sigma$ of the simulation data, and then reconstruct the phase space. Finally, the maximum Lyapunov exponent of droplet vibration response is calculated by Wolf method[18].

The C-C method calculates $\Delta S(\sigma)$ and $s_{cor}(\sigma)$ of data series, selects the first local minimum value of $\Delta S(\sigma)$ as the best $\sigma$ value, and the global smallest value of $s_{cor}(\sigma)$ as the embedded window $\sigma_w$, then obtains the best $m$ value though $\sigma_w = (m-1)\sigma$. The calculated $\Delta S(\sigma)$ and $s_{cor}(\sigma)$ variation curves are shown in Figure 8.

![Figure 8. The $\Delta S(\sigma)$ and $s_{cor}(\sigma)$ variation diagram of droplet vibration time series.](image)

It can be obtained from Figure 8 that the first local minimum value of $\Delta S(\sigma)$ is 29, the global smallest value of $s_{cor}(\sigma)$ is 133. Therefore, the best time delay $\sigma$ is 29, the embedding window $\sigma_w$ is 148, obtained the best embedding dimension $m$ is 6.
The time series obtained by simulation are \( \chi(t_1), \chi(t_2), \ldots, \chi(t_N) \), the phase space reconstruction \( Y(t_i) \) is

\[
Y(t_i) = (\chi(t_i), \chi(t_i + \sigma), \ldots, \chi(t_i + (m-1)\sigma)) \quad (i = 1, 2, \ldots, M)
\]

(9)

Finally, the evolution of phase trajectory of \( Y(t_i) \) is calculated by Wolf method to get the maximum Lyapunov exponent \( \Gamma \).

\[
\Gamma = \frac{1}{t_M - t_0} \sum_{i=0}^{M} \ln \frac{d'_i}{d_i}
\]

(10)

where \( t_0 \) is the beginning of evolution, \( t_M \) is the end of evolution, \( M \) is the total number of iterations in the evolution process, \( M = N - (m - 1)\sigma \), \( N \) is the length of time series, \( d_i \) is the distance when the two-phase rail is less than the specified value at \( t_i \), \( d'_i \) is the distance when the two-phase rail is greater than the specified value at \( t_i \).

Though calculation, the maximum Lyapunov exponent is obtained, \( \Gamma = 0.0024 > 0 \). It can be concluded that the vibration of emulsion droplet in chaotic electric field is chaotic vibration.

4 Conclusion

In this work, based on the nonlinear vibration model of the emulsion droplet under the action of the pulse electric field, the vibration dynamic model of emulsion droplet in the pulse electric field with chaotic frequency is established by introducing chaotic signal through using the Chaotic-Pulse-Position Modulation method, which can accurately describe the vibration response of the droplet in the chaotic frequency pulse electric field, and the Simulink model can effectively simulate and analyze the vibration system of emulsion droplets. The simulation analysis of the vibration system of emulsion droplet in the chaotic electric field shows that the vibration response of droplets is more intense in the high frequency region, and relatively stable in the middle and low frequency region, without obvious resonance, and the vibration response frequency of the emulsion droplet is equal to that of the chaotic electric field, the vibration phase trajectory is bounded, infinite winding and never repeated, and the maximum Lyapunov exponent is greater than 0. It is proved that the vibration of the emulsion droplet in the chaotic frequency pulse electric field is chaotic vibration, which further shows that the chaotic output of the droplet vibration system can be generated by introducing the chaotic signal.

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