THE ANALYTICITY AND EXPONENTIAL DECAY OF A STOKES-WAVE COUPLING SYSTEM WITH VISCOELASTIC DAMPING IN THE VARIATIONAL FRAMEWORK

JING ZHANG*
Department of Mathematics and Economics
Virginia State University
Petersburg, VA 23806, USA
(Communicated by Irena Lasiecka)

ABSTRACT. In this paper, we study a fluid-structure interaction model of Stokes-wave equation coupling system with Kelvin-Voigt type of damping. We show that this damped coupling system generates an analyticity semigroup and thus the semigroup solution, which also satisfies variational framework of weak solution, decays to zero at exponential rate.

1. Introduction. Fluid-structure interaction model has been intensively studied for the last decade because of its wide applications to engineering, physics, biology and biomedicine sciences. The model in this paper describes the motion of a solid inside a fluid. The mathematical challenge of this model stems from the coupling of the hyperbolic component (elastic system) and the parabolic component (fluid). The mismatch of the parabolic-hyperbolic regularity at the interface raises difficulty in the establishment of well-posedness of the solutions in suitable functional spaces [14, 8, 9, 24, 25]. Furthermore, the parabolic-hyperbolic coupling is also the main obstacle in the study of the stability and stabilization of the fluid-structure system. As indicated in [27], [26], the dissipative propagation from the parabolic component is of weak type, thus can not overcome the conservative hyperbolic component to yield uniform stability for the overall evolution if the system is undamped. Therefore, a damping has to be added if one expects to get uniform stabilization for the system. The damping can be added on fluid-structure interface as a boundary control for the stabilization of the solid. Past literature on this topic includes [3, 4, 28] and the references therein. It can also be included directly as the structural damping in the hyperbolic component, the recent papers [30, 40, 5] work in this direction.

This paper is a subsequent work of [30, 40]. The distinct feature of [30, 40] is that the model couples a heat equation with a viscoelastic damped wave equation, in which the damping is of Kelvin-Voigt type. The physical importance of this model is that the solid (inside the fluid) described in this system exhibits viscoelastic property, that is, gradual deformation and recovery when it is subject to loading and unloading [36]. In fact, such viscoelastic property presents in a large group of materials - polymer plastics, almost all biological materials, and etc. Thus, our model

2010 Mathematics Subject Classification. Primary: 35M10, 35B35; Secondary: 35A01.

Key words and phrases. Fluid-structure interaction, stokes equation, wave equation, Kelvin-Voigt damping, analyticity, uniform stabilization.
encompasses practical applications in engineering and biomedical science. From the mathematical point of view, compared to [41], where an undamped heat-wave coupling cannot achieve uniform exponential decay under any geometric condition, [30] shows that the coupling of heat equation and Kelvin-Voigts type damped wave equation with suitable trace formulation on the interface yields very strong dissipative property for the overall system: the semigroup generated by the system is analytic. As Stokes equation and heat equation share many similarities, specially, the semigroup generated by Stokes operator also exhibits analyticity in certain function spaces, [16, 19]. It is natural to raise the question: Does the Stokes-wave coupling with Kelvin-Voigt damping still generate an analytic semigroup? In this paper, we address this problem under variational framework of the system.

[11] has shown that the semigroup generated from the viscoelastic damped wave equation is indeed analytic. Thus the two components in our model are both analytic. However, the coupling formulation within the system makes the proof of overall analyticity challenging. It is not with the range of relatively bounded perturbation, thus we can not apply any perturbation theories for analytic semigroup in hope of obtaining the analyticity of the overall coupling system. The estimates need to be developed from scratch. The second difficulty comes from the presence of the pressure in the Stokes equation. The standard method of treating pressure term is Leray’s projection in variational framework. But in our Stokes-wave coupling, pressure also involves in the boundary condition of the interface of the coupling system. Thus it persists in the structure and may potentially contribute to the singularities of the coupling system. Handling this needs introduction of new multipliers and new weights in the energy calculations. To the best of our knowledge, the present paper is the first one that considers the presence of pressure within the framework of analytic semigroup in the context of fluid-structure coupling system.

In the related literature, the Westervelt type wave equation introduced in [13, 22, 23] also includes a Kelvin-Voigt-damping-like term in the hyperbolic equation. In [33], a detailed theoretical approach using the tool of maximal \( L_p \) regularity [15], [21] shows that the global solution of this Westervelt equation exponentially decays to zero. In addition, [34, 35, 10] explore the full Navier-Stokes fluid coupled with the viscoelastic damped structures using numerical methods: the ALE(Arbitrary Lagrangian-Eulerian) mapping and Lie splitting scheme. The existence of weak solution is obtained, the stability of the weak solution is also studied.

![Fig. 1: The Fluid–Structure Interaction](image)

Let \( \Omega \) be a bounded domain in \( \mathbb{R}^n \), \( n = 2, 3 \), \( \Omega \) is partitioned into two subsets \( \Omega_f \) and \( \Omega_s \), with \( \Omega_s \) being surrounded by \( \Omega_f \), as shown in figure 1. The figure describes the scenario of a structure \( \Omega_s \) fully immersed in a fluid \( \Omega_f \). The boundary
of $\Omega_f$ is $\Gamma_f \cup \Gamma_s$, and the boundary of $\Omega_s$ is $\Gamma_s$. Thus $\Gamma_s$ is the interface where the interactions between the fluid and the structure take place.

The dynamics of this fluid-structure system is described by the following equation system:

\[
\begin{aligned}
    u_t - \Delta u + \nabla p &= 0 \quad \text{in } \Omega_f \\
    \text{div } u &= 0 \quad \text{in } \Omega_f \\
    w_{tt} - \Delta w - \Delta w_t + bw &= 0 \quad \text{in } \Omega_s \\
    u &= w_t \quad \text{in } \Gamma_s \\
    \frac{\partial (w + w_t)}{\partial \nu} &= \frac{\partial u}{\partial \nu} - p \cdot \nu \quad \text{in } \Gamma_s \\
    u &= 0 \quad \text{in } \Gamma_f \\
    (u(0), w(0), w_t(0)) &= (u_0, w_0, w_1)
\end{aligned}
\]  

(1.1)\)

In (1.1), $u$ is a vector function representing the velocity field of the fluid. $p$ is a scalar function representing the fluid pressure. $w$ and $w_t$ are both vectors and represents the displacement and velocity of the structure respectively. $\nu$ is the unit outward normal vector on $\Gamma_s$ with respect to the region $\Omega_s$. For simplicity, the constant $b$ equals either 0 or 1.

In addition, we assume the interface $\Gamma_s$ is static. This assumption is appropriate if we further assume that the structure component presents the property of small and rapid oscillations [18].

Throughout the paper we will consider the dynamic system in the following energy space

$$\mathcal{H} \equiv H \times (H^1(\Omega_s))^n \times (L_2(\Omega_s))^n$$

where $H = \{ u \in (L_2(\Omega_f))^n : \text{div } u = 0 \text{ in } \Omega_f, u \cdot \nu = 0 \text{ on } \Gamma_f \}$. The inner product in $\mathcal{H}$ is defined as

$$\left( \begin{array}{c}
  u_1 \\
  w_1 \\
  z_1
\end{array} \right), \left( \begin{array}{c}
  u_2 \\
  w_2 \\
  z_2
\end{array} \right) \in H$$

$$= (u_1, u_2)_{\Omega_f} + (\nabla w_1, \nabla w_2)_{\Omega_s} + b(w_1, w_2)_{\Omega_s} + (z_1, z_2)_{\Omega_s}$$

In addition, we define the following space:

$$V = \{ u \in (H^1(\Omega_f))^n : \text{div } u = 0 \text{ in } \Omega_f, u = 0 \text{ on } \Gamma_f \}$$

It should be noted that in the rest part of this paper, we will omit the exponent $n$ in $(H^1(\Omega_f))^n$, $(L_2(\Omega_f))^n$, $(H^1(\Omega_s))^n$ and $(L_2(\Omega_s))^n$, and denote these spaces as $H^1(\Omega_f)$, $L_2(\Omega_f)$, $H^1(\Omega_s)$ and $L_2(\Omega_s)$ for simplicity. The energy for the system is defined as follows:

$$E_u(t) = \int_{\Omega_f} |u(t)|^2 \, d\Omega_f; \quad E_w(t) = \int_{\Omega_s} b|w(t)|^2 + |\nabla w(t)|^2 + |w_t(t)|^2 \, d\Omega_s$$ \hspace{1cm} (1.2)

and

$$E(t) = E_u(t) + E_w(t) = |u(t)|^2_{\Omega_f} + b|w(t)|^2_{\Omega_s} + |\nabla w(t)|^2_{\Omega_s} + |w_t(t)|^2_{\Omega_s}$$ \hspace{1cm} (1.3)

2. Main Theorem. We first define the weak solution for (1.1) as follows:

**Definition 2.1.** Let $(u_0, w_0, w_1) \in H$ and $T > 0$. A triple $(u, w, w_t) \in L_\infty(0, T; \mathcal{H})$ is a weak solution for system (1.1) if

- $(u(0), w(0), w_t(0)) = (u_0, w_0, w_1)$
\[ u|_{\Gamma_s} = w_t|_{\Gamma_s} \in L_2(0, T; H^{1/2}(\Gamma_s)) \]

\[ \frac{\partial(w + w_t)}{\partial \nu} \in L_2(0, T; H^{-1/2}(\Gamma_s)) \]

\((u, w, w_t)\) satisfies the variational form for a.e. \(t \in (0, T)\)

\[ (u_t, \phi)_{\Omega_f} + (\nabla u, \nabla \phi)_{\Omega_f} + \left( \frac{\partial(w + w_t)}{\partial \nu}, \phi \right)_{\Gamma_s} = 0 \quad (2.1) \]

\[ (w_{tt}, \psi)_{\Omega_s} + (\nabla w, \nabla \psi)_{\Omega_s} + (\nabla w_t, \nabla \psi)_{\Omega_s} + (bw, \psi)_{\Omega_s} - \left( \frac{\partial(w + w_t)}{\partial \nu}, \psi \right)_{\Gamma_s} = 0 \quad (2.2) \]

for every \(\phi \in V\) and \(\psi \in H^1(\Omega_s)\).

We now define the notation \(\rho(w, z) = \nabla w + \nabla z\), and the operator \(A : V \times H^1(\Omega_s) \times H^1(\Omega_s) \to V'\) as:

\[ (A(u, w, z), \phi) = (\nabla u, \nabla \phi)_{\Omega_f} + (\rho(w, z) \cdot \nu, \phi)_{\Gamma_s} \]

where \(\phi \in V\).

Under these definitions, the weak solution in Definition 2.1 can be rewritten as

\[ (u_t + A(u, w, w_t), \phi) = 0 \quad (2.3) \]

\[ (w_{tt} - \text{div} \rho(w, w_t) + bw, \psi) = 0 \quad (2.4) \]

\[ u|_{\Gamma_s} = w_t|_{\Gamma_s} \quad (2.5) \]

for every \(\phi \in V\) and \(\psi \in H^1(\Omega_s)\).

Define

\[ A \begin{pmatrix} u \\ w \\ z \end{pmatrix} = \begin{pmatrix} -A(u, w, z) \\ z \\ \text{div} \rho(w, z) - bw \end{pmatrix} \quad (2.6) \]

with domain of \(A\) as follows:

\[ \mathcal{D}(A) = \{ (u, w, z) \in H : u \in V, A(u, w, z) \in H, z \in H^1(\Omega_s), \text{div} \rho(w, z) \in L_2(\Omega_s), u|_{\Gamma_s} = z|_{\Gamma_s} \in H^{1/2}(\Gamma_s) \} \quad (2.7) \]

The domain is defined as such for the need in the proof of analyticity in section 4. Under the definition of \(A\) in (2.6) with its domain in (2.7), (2.3)-(2.5) can be written into an abstract evolution as

\[ \frac{d}{dt} \begin{pmatrix} u \\ w \\ w_t \end{pmatrix} = A \begin{pmatrix} u \\ w \\ w_t \end{pmatrix} \quad (2.8) \]

Remark 2.1. To this end, it should be noted that in (2.6) the term \(bw\) when \(b = 1\) can be seen as a bounded perturbation to the case \(b = 0\) and thus does not alter the well-posedness and analyticity conclusions on the abstract evolution (2.8), [17], and therefore does not change these conclusions on the original system (1.1). Hence forward, we only work on the case \(b = 0\). All the Main Theorems (theorems presented in this section) in this paper are under the assumption that \(b = 0\), but they also hold for the case \(b = 1\).

\(b = 0\), \(A\) is simplified as

\[ A = \begin{pmatrix} -A(u, w, z) \\ z \\ \text{div} \rho(w, z) \end{pmatrix} \quad (2.9) \]
with domain of $\mathcal{A}$ unchanged as (2.7). And the consequently, the energy $E(t)$ of the evolution system becomes

$$E(t) = E_u(t) + E_w(t) = |u(t)|^2_{\Omega_t} + |\nabla w(t)|^2_{\Omega_s} + |w_t(t)|^2_{\Omega_s}$$  \hspace{1cm} (2.10)

We first develop the following Theorem:

**Theorem 2.1.** The operator $\mathcal{A}$ defined in (2.9) and (2.7) generates a strongly continuous semigroup of contraction on the Hilbert space $\mathcal{H}$. Thus, the solution $(u, w, w_t)$ for the system (2.8) can be represented as

$$\begin{pmatrix} u(t) \\ w(t) \\ w_t(t) \end{pmatrix} = e^{\mathcal{A}t} \begin{pmatrix} u_0 \\ w_0 \\ w_1 \end{pmatrix} \in C([0,T], \mathcal{H})$$  \hspace{1cm} (2.11)

In addition, the energy $E(t)$ defined in (2.10) satisfies

$$E(t) = \left\| e^{\mathcal{A}t} \begin{pmatrix} u_0 \\ w_0 \\ w_1 \end{pmatrix} \right\|_{\mathcal{H}}$$  \hspace{1cm} (2.12)

The subsequent theorems are the main theorems of this paper.

**Theorem 2.2.** For $\mathcal{A}$ defined in (2.9) and (2.7), its spectrum $\sigma(\mathcal{A})$ is contained in the following region:

$$(-\infty, -2) \cup \left( \overline{S_{r=1}(x_0)} / S_{r_0} \right)$$  \hspace{1cm} (2.13)

where $\overline{S_{r=1}(x_0)}$ is the closed disk centered at $x_0 = (-1,0)$ with radius $r = 1$, $S_{r_0}$ is the open circle centered at origin with small radius $r_0 > 0$. The whole imaginary axis is in the resolvent of $\mathcal{A}$, that is $i\mathbb{R} \subset \rho(\mathcal{A})$. In addition, there is a $\tau_0 > 0$, such that for $\tau \in \mathbb{R}$ and $|\tau| > \tau_0$, the resolvent $R(i\tau, \mathcal{A})$ satisfies the estimates:

$$\|R(i\tau, \mathcal{A})\|_{\mathcal{L}(\mathcal{H})} \leq \frac{C}{|\tau|}$$  \hspace{1cm} (2.14)

Thus, the s.c. contraction semigroup generated by $\mathcal{A}$ is analytic on $\mathcal{H}$.

**Theorem 2.3.** Theorem 2.1 and Theorem 2.2 yield that the energy of the evolution system defined in (2.10) satisfies exponential decay rate: There exist $M > 0$, $\delta > 0$, such that

$$E(t) \leq Me^{-\delta t}$$  \hspace{1cm} (2.15)

Thus, the semigroup solution, which is also the weak solution under Definition 2.1, decays to zero at uniform exponential rate.

**Remark 2.2.** In our model, we describe the motion of the structure by wave equation with viscoelastic damping. This can be generalized to elastodynamic equation with Lamé coefficient with the damping term changing accordingly (Indeed, as one can see this damping is of Kelvin-Voigt type). Using similar approach presented in this paper we expect the above main theorems will still hold when wave equation is replaced by the following elastodynamic equation. That is, the following system generates an analytic semigroup and its weak solution decays to 0 at uniform
exponential rate
\[
\begin{cases}
u_t - \Delta u + \nabla p = 0 & \text{in } \Omega_f \\
\text{div } u = 0 & \text{in } \Omega_f \\
w_t\nu - \text{div } \sigma(w) - \text{div } \sigma(w_t) + bw = 0 & \text{in } \Omega_s \\
u = w_t & \text{in } \Gamma_s \\
\sigma(w + w_t) \cdot \nu = \frac{\partial u}{\partial \nu} - p \cdot \nu & \text{in } \Gamma_s \\
u = 0 & \text{in } \Gamma_f \\
(u(0), w(0), w_t(0)) = (u_0, w_0, w_1) 
\end{cases}
\] (2.16)

where \(\sigma(w)\) is the stress tensor and \(\sigma(w) = 2\mu\varepsilon(w) + \lambda tr(\varepsilon(w))I\) with \(\varepsilon(w)\) being the strain tensor and \(\lambda > 0, \mu > 0\) being the Lamé coefficient.

**Remark 2.3.** Our approach in proving the contraction and analyticity of the semigroup generated by the system (1.1) is based on the variational method. The strong convergence of the solution (uniformly decays to 0) allows to adapt this variational approach to Galerkin Approximation of the system (1.1), thus, making the construction of finite element numerical scheme of system (1.1) a feasible option. And indeed, developing a Galerkin finite element numerical scheme for system (1.1) is in the plan of our future study of this fluid-structure interaction model.

3. **Proof of Theorem 2.1.** We now verify the assumptions of Lumer-Phillips Theorem, which yields exactly Theorem 2.1. That means we are going to prove (i) \(\mathcal{A}\) is dissipative; (ii) \(\lambda I - \mathcal{A}\) is maximal for some \(\lambda > 0\). In the following proof, we set \(\lambda = 1\).

3.1. **Dissipativity of \(\mathcal{A}\).**

**Proposition 3.1.** The operator \(\mathcal{A}\) defined in (2.9) and (2.7) satisfies
\[
\text{Re} \left( \mathcal{A} \left( \begin{array}{c} u \\ w \\ z \end{array} \right), \left( \begin{array}{c} u \\ w \\ z \end{array} \right) \right) \leq 0
\] (3.1)

**Proof.** Let \((u, w, z) \in \mathcal{D}(\mathcal{A})\), then \(u \in V, w \in H^1(\Omega_s)\) and \(z \in H^1(\Omega_s)\), thus \(w|_{\Gamma_s} \in H^{1/2}(\Gamma_s)\) and \(z|_{\Gamma_s} \in H^{1/2}(\Gamma_s) \subset H^{-1/2}(\Gamma_s)\). Therefore, \((\mathcal{A}(u, w, z), u) = (\nabla u, \nabla u)_{\Omega_f} + (\rho(w, z) \cdot \nu, u)_{\Gamma_s}\) is well defined.

\[
\mathcal{A} \left( \begin{array}{c} u \\ w \\ z \end{array} \right) = -\left( \begin{array}{c} u \\ w \\ z \end{array} \right)_{\Omega_f} + (\nabla z, \nabla w)_{\Omega_s} + (\text{div } w, z)_{\Omega_s}
\]

\[
= -(\nabla u, \nabla u)_{\Omega_f} - (\rho(w, z) \cdot \nu, u)_{\Gamma_s} + (\nabla z, \nabla w)_{\Omega_s} + (\Delta w + \Delta z, z)_{\Omega_s}
\] (3.2)

Because \(w \in H^1(\Omega_s)\) and \(z \in H^1(\Omega_s)\), the last term
\[
(\Delta w + \Delta z, z)_{\Omega_s} = (\rho(w, z) \cdot \nu, z)_{\Gamma_s} - (\nabla w, \nabla z)_{\Omega_s} - (\nabla z, \nabla z)_{\Omega_s}
\] (3.3)

Plug in this result, keep in mind \(w|_{\Gamma_s} = z|_{\Gamma_s}\), we get
\[
\mathcal{A} \left( \begin{array}{c} u \\ w \\ z \end{array} \right) = -(\nabla u, \nabla u)_{\Omega_f} + (\nabla z, \nabla w)_{\Omega_s} - (\nabla w, \nabla z)_{\Omega_s} - (\nabla z, \nabla z)_{\Omega_s}
\] (3.4)
Therefore,
\[
\text{Re} \left( \mathcal{A} \begin{pmatrix} u \\ w \\ z \end{pmatrix}, \begin{pmatrix} u \\ w \\ z \end{pmatrix} \right)_{\mathcal{H}} = -(\nabla u, \nabla u)_{\Omega_f} - (\nabla z, \nabla z)_{\Omega_s} \leq 0 \tag{3.5}
\]
The dissipativity of operator \(\mathcal{A}\) is proved.

3.2. Maximaliy of \(I - \mathcal{A}\). We want to show that \(I - \mathcal{A}\) is surjective: \(\mathcal{D}(\mathcal{A}) \rightarrow \mathcal{H}\). For every \([f, g, h] \in \mathcal{H}\), that is, \([f, g, h] \in H \times H^1(\Omega_s) \times L_2(\Omega_s)\), we need to find \((u, w, z) \in \mathcal{D}(\mathcal{A})\) satisfying
\[
(I - \mathcal{A}) \begin{pmatrix} u \\ w \\ z \end{pmatrix} = \begin{pmatrix} f \\ g \\ h \end{pmatrix} \tag{3.6}
\]
which is
\[
\begin{align*}
&\begin{cases}
u|\Gamma_s = z|\Gamma_s \\
u + A(u, w, z) = f \\
w - z = g \\
z - \text{div} \rho(w, z) = h
\end{cases} \tag{3.7}
\end{align*}
\]
Here and throughout this paper equation (3.8) is in variational sense, that is, for any \(\phi \in V\),
\[
(u + A(u, w, z), \phi) = (f, \phi) \tag{3.12}
\]
We write in the form of (3.8) for simplicity, when detailed calculations are involved, we use the variational form (3.12).

Eliminate \(z\), we get
\[
\begin{align*}
u + A(u, w, w - g) &= f \tag{3.13} \\
w - 2\Delta w &= h + g - \Delta g \tag{3.14} \\
w|\Gamma_s &= (u + g)|\Gamma_s \tag{3.15}
\end{align*}
\]
From elliptic theory, we know for \(u \in V, g \in H^1(\Omega_s), h \in L_2(\Omega_s)\), there exists \(w \in H^1(\Omega_s)\) satisfies the system (3.14) and (3.15):
\[
\begin{align*}
w - 2\Delta w &= h + g - \Delta g \in H^{-1}(\Omega_s) \tag{3.16} \\
w|\Gamma_s &= (u + g)|\Gamma_s \in H^{1/2}(\Gamma_s) \tag{3.17}
\end{align*}
\]
The solution \(w\) is continuously dependent on \(u, h, g\). Thus we write it as \(w(u, h, g)\), and it satisfies the following inequality.
\[
\begin{align*}
||w||_{1, \Omega_s} + \left|\right|\frac{\partial w}{\partial \nu} \left|\right|_{-1/2, \Gamma_s} &\leq C \left[||g||_{H^1(\Omega_s)} + ||\Delta g||_{H^1(\Omega_s)} + ||h||_{H^1(\Omega_s)} + ||g||_{1/2, \Gamma_s} + ||u||_{1/2, \Gamma_s}\right] \tag{3.18}
\end{align*}
\]
Indeed, multiply both sides of (3.16) and integrate over the domain \(\Omega_s\), we have
\[
\begin{align*}
(w - 2\Delta w, w)_{\Omega_s} &= (w, w)_{\Omega_s} - 2w(\frac{\partial w}{\partial \nu}, w)_{\Gamma_s} + 2(\nabla w, \nabla w)_{\Omega_s} = (h + g - \Delta g, w)_{\Omega_s} \tag{3.19}
\end{align*}
\]
Recall the boundary condition (3.17), the above equation becomes

\[
(w, w)_{\Omega_s} - 2\left(\frac{\partial w}{\partial \nu}, u + g\right)_{\Gamma_s} + 2(\nabla w, \nabla w)_{\Omega_s} = (h + g - \Delta g, w)_{\Omega_s}
\]

(3.20)

This yields,

\[
\|w\|^2_{H^1(\Omega_s)} + \left\|\frac{\partial w}{\partial \nu}\right\|_{H^{-1/2}(\Gamma_s)} \leq C \|h + g - \Delta g\|_{H^1(\Omega_s)} \|w\|_{H^1(\Omega_s)}
\]

(3.21)

And (3.21) implies (3.18).

Linear theory shows that \(w(u, h, g)\) can be decomposed as \(w(u, h, g) = w(0, h, g) + w(u, 0, 0)\). \(w(0, h, g)\) is independent from \(u\), and elliptic theory guarantees that the solution \(w(0, h, g)\) exists. Thus, (3.13) can be written as

\[
u + A(u, w_u, w_u) = -A(0, w_{hg}, w_{hg} - g) + f
\]

(3.22)

where \(w_u = w(u, 0, 0)\), \(w_{hg} = w(0, h, g)\). For \(g \in H^1(\Omega_s)\), \(h \in L^2(\Omega_s)\), \(\frac{\partial (2w_{hg} - g)}{\partial \nu} \in H^{-1/2}(\Gamma_s)\). Also reminding that \(f \in H\), we have the right side of (3.22),

\(-A(0, w_{hg}, w_{hg} - g) + f \in V'\). By now, to prove the maximality of \(I + A\), we only need to prove that \(I + A : V \rightarrow V'\) defined by the left side of (3.22) is surjective from \(V\) to \(V'\). By [7], this is to show \(I + A\) is continuous, coercive and monotone in \(u\).

**Continuity of \(I + A\) on \(L(V, V')\):** Compare with (3.18), we know that \(w_u\), the solution to (3.13)-(3.15) with \(g = 0\) and \(h = 0\), satisfies

\[
\|w_u\|^2_{1, \Omega_s} + \left\|\frac{\partial w_u}{\partial \nu}\right\|^2_{-1/2, \Gamma_s} \leq C \|u\|^2_{1, \Gamma_s}
\]

(3.23)

Thus, together with Poincare’s inequality,

\[
(u + A(u, w_u, w_u), \phi) = (u, \phi)_{\Omega_s} + (\nabla u, \nabla \phi)_{\Omega_s} + (2\frac{\partial w_u}{\partial \nu}, \phi)_{\Gamma_s}
\]

\[
\leq C \|u\|_{1, \Omega_s} \cdot \|\phi\|_{1, \Omega_s} + 2\|\phi\|_{1, \Gamma_s} \cdot \left\|\frac{\partial w_u}{\partial \nu}\right\|^2_{-1/2, \Gamma_s}
\]

\[
\leq C \left[ \|u\|^2_{1, \Omega_s} \cdot \|\phi\|^2_{1, \Omega_s} + 2\|\phi\|^2_{1/2, \Gamma_s} \cdot ||u||^2_{1/2, \Gamma_s} \right]
\]

(3.24)

So \(I + A\) defined in (3.22) is bounded from \(V\) to \(V'\), together with the fact that it is also linear on \(V\), we obtain its continuity from \(V\) to \(V'\).

**Coercivity of \(I + A\) on \(V\):** Let \(u \in V\),

\[
(u + A(u, w_u, w_u), u) = (u, u)_{\Omega_s} + (\nabla u, \nabla u)_{\Omega_s} + (\rho(w_u, w_u) \cdot \nu, u)_{\Gamma_s}
\]

recall that \((u, w_u)\) is the solution to the following system

\[
u + A(u, w, z) = f
\]

(3.25)

\[
w - z = 0
\]

(3.26)

\[
z - \text{div} \rho(w, z) = 0
\]

(3.27)

\[
z|_{\Gamma_s} = u|_{\Gamma_s}
\]

(3.28)
Thus,
\[ (\rho(w_u, w_u) \cdot \nu, u)_{\Gamma_s} = (\rho(w_u, z) \cdot \nu, z)_{\Gamma_s} \]
\[ = (z, z)_{\Omega_s} + 2(\nabla w, \nabla z)_{\Omega_s} = (z, z)_{\Omega_s} + 2(\nabla z, \nabla z)_{\Omega_s} > 0 \quad (3.29) \]
Thus,
\[ (u + A(u, w_u, w_u), u) \]
\[ = (u, u)_{\Omega_f} + (\nabla u, \nabla u)_{\Omega_f} + (\rho(w_u, w_u) \cdot \nu, u)_{\Gamma_s} \geq (1 - \epsilon)\|u\|^2_{L^2(\Omega_f)} \geq C\|u\|^2_{V'} \]

Coercivity of \( I + A \) on \( V \) is proved.

**Monotonicity of \( I + A \) on \( D(A) \):** The monotonicity of \( I + A \) defined in (3.22) is obtained from the following more general result:

**Proposition 3.2.** The operator \( A \) defined in (2.9) and (2.7) satisfies that \( I - A \) is strictly accretive on \( D(A) \).

**Proof.** For \((u_1, w_1, z_1) \in D(A)\) and \((u_2, w_2, z_2) \in D(A)\), we have
\[
(A(u_1, w_1, z_1) - A(u_2, w_2, z_2), u_1 - u_2)
\]
\[= -(\nabla u_1, \nabla (u_1 - u_2))_{\Omega_f} + (\rho(u_1, z_1) \cdot \nu, u_1 - u_2)_{\Gamma_s} \]
\[- (\nabla u_2, \nabla (u_1 - u_2))_{\Omega_f} - (\rho(u_2, z_2) \cdot \nu, u_1 - u_2)_{\Gamma_s} \]
\[= (\nabla (u_1 - u_2), \nabla (u_1 - u_2))_{\Omega_f} + ((\rho(u_1, z_1) - \rho(u_2, z_2)) \cdot \nu, u_1 - u_2)_{\Gamma_s} \quad (3.31) \]

And
\[
(\nabla (u_1 - u_2), \nabla (z_1 - z_2))_{\Omega_s} - (\nabla (z_1 - z_2), \nabla (z_1 - z_2))_{\Omega_s} \quad (3.33) \]

(3.32) and (3.33) combine with the facts \( u_1 = z_1 \) and \( u_2 = z_2 \) simplify (3.31) as
\[
(A(u_1, w_1, z_1) - A(u_2, w_2, z_2), u_1 - u_2)
\]
\[= -(\nabla (u_1 - u_2), \nabla (u_1 - u_2))_{\Omega_f} - (\nabla (z_1 - z_2), \nabla (z_1 - z_2))_{\Omega_s} \]
\[- (\nabla (u_1 - u_2), \nabla (z_1 - z_2))_{\Omega_s} - (\nabla (z_1 - z_2), \nabla (u_1 - u_2))_{\Omega_s} \quad (3.34) \]

Thus,
\[
\text{Re} \left( (I - A) \begin{pmatrix} u_1 \\ w_1 \\ z_1 \end{pmatrix} - (I - A) \begin{pmatrix} u_2 \\ w_2 \\ z_2 \end{pmatrix} \right)
\]
\[= \|u_1 - u_2\|^2_{\Omega_f} + \|\nabla w_1 - \nabla w_2\|^2_{\Omega_s} + \|z_1 - z_2\|^2_{\Omega_s} + \|\nabla (u_1 - u_2)\|^2_{\Omega_f} \quad (3.35) \]
\[+ \|\nabla (z_1 - z_2)\|^2_{\Omega_s} > 0 \]

Strict accretivity of \( I - A \) is proved. \( \square \)
So far, we have proved the continuity, coercivity and monotonicity of the operator $I + A$ defined in (3.22). Thus the surjectivity of $I + A$ is established, and therefore the maximality of the operator $I - A$ is established. Combined with the dissipativity of $A$ in Proposition 3.1. We have proved that $A$ defined in (2.9) and (2.7) indeed generates a strongly continuous semigroup of contraction. Theorem 2.1 is thus proved. And it is natural to obtain the following corollary:

**Corollary 3.3.** Let the initial condition $(u_0, w_0, w_1) \in D(A)$, the semigroup solution $y(t) = (u(t), w(t), w_1(t))$ obtained in Theorem 2.1 satisfies the following regularity:

1. $y(t) \in C([0, T], H)$ and $\frac{d}{dt} y(t) \in L_\infty([0, T], H)$ for any $T > 0$.
2. $A(u, w, w_t) \in H$, $\text{div} \, \rho(w, w_t) \in L_2(\Omega_s)$
3. $u = w_t \in C([0, T], H^{1/2}(\Gamma_s))$ on $\Gamma_s$, $\rho(w, w_t) \cdot \nu \in C([0, T], H^{-1/2}(\Gamma_s))$ on $\Gamma_s$

And $(u(t), w(t), w_1(t))$ also satisfies the variational formulation (2.1) and (2.2) in Definition 2.1. Thus, $(u(t), w(t), w_1(t))$ is also a weak solution defined by Definition 2.1.

4. **Analyticity of $A$.** Proof of Theorem 2.2. In this section, we want to further study the analyticity of $A$. We first introduce a new multiplier and establish an identity, which will be used later in the proof the analyticity of the semigroup generated by $A$. Because this new multiplier involves the pressure $p$ in the system (1.1). We need the following lemma. Similar results can be found in [8] and [1]

**Lemma 4.1.** Let $y = (u, w, w_t)$ be the semigroup solution to (1.1) with $(u_0, w_0, w_1) \in D(A)$. Then for all $T > 0$, there exists pressure $p \in L_2((0, T); L_2(\Omega_f))$, such that

$$
\Delta u - \nabla p \in L_2((0, T); L_2(\Omega_f)), \quad \frac{\partial u}{\partial \nu} - p \cdot \nu \in H^{-1/2}(\Gamma_s)
$$

(4.1)

And $(u, w, w_t, p)$ satisfies (1.1).

**Proof.** By Corollary 3.3, we know that $(u, w, w_t)$ is also the weak solution to (1.1), thus, for $\phi \in V$, and $\psi \in H^1(\Omega_s)$, we have

$$
\langle \nabla u, \nabla \phi \rangle_{\Omega_f} + (\rho(w, w_t) \cdot \nu, \phi)_{\Gamma_s} = -(u_t, \phi)_{\Omega_f}
$$

(4.2)

$$
\langle \nabla w, \nabla \psi \rangle_{\Omega_s} + (\nabla w_t, \nabla \psi)_{\Omega_s} - (\rho(w, w_t) \cdot \nu, \psi)_{\Gamma_s} = -(w_t, \psi)_{\Omega_s}
$$

(4.3)

$$
u|_{\Gamma_s} = w_t|_{\Gamma_s}, \xi \in L_2((0, T); H^{1/2}(\Gamma_s))
$$

(4.4)

Take $\phi \in V \cap H^1_0(\Omega_f)$ and $\psi \in H^1_0(\Omega_s)$, such that $\phi = \psi \equiv 0$ on $\Gamma_s$. By Corollary 3.3, $u_t \in L_2((0, T); H)$, $w_t \in L_2((0, T); L_2(\Omega_s))$, we have

$$
-\Delta u(t) + u_t(t), \phi)_{\Omega_f} = 0 \quad \text{a.e.} \ t \in (0, T)
$$

(4.5)

$$w_t - \Delta w - \Delta w_t = 0 \quad \text{in} \ H^{-1}(\Omega_s), \text{a.e.} \ t \in (0, T)
$$

(4.6)

Because $u_t \in V'$, a.e. $t \in (0, T)$, DeRham Theorem yields,

$$-\Delta u + \nabla p + u_t = 0
$$

(4.7)

in the sense of distribution for some $p \in L_2((0, T); L_2(\Omega_f))$.

Because by Corollary 3.3, $u_t \in L_2((0, T); L_2(\Omega_f))$, $w_t \in L_2((0, T); L_2(\Omega_s))$, we have

$$-\Delta u + \nabla p + u_t = 0 \quad \text{in} \ L_2((0, T); L_2(\Omega_f))
$$

(4.8)

$$w_t - \Delta (w + w_t) = 0 \quad \text{in} \ L_2((0, T); L_2(\Omega_s))
$$

(4.9)
This yields $\Delta u - \nabla p \in L_2((0, T); L_2(\Omega_f))$ and $\Delta(w + w_t) \in L_2((0, T); L_2(\Omega_s))$.

By Corollary 3.3, we have $\rho(w, w_t) \cdot \nu \in C([0, T], H^{1/2}(\Gamma_s))$, (4.7) and (4.2) yields

$$
\left( \rho(w, w_t) \cdot \nu + \left( \frac{\partial u}{\partial \nu} - p \cdot \nu \right), \phi \right)_{\Gamma_s} = 0 \quad \text{for any } \phi \in V \quad (4.10)
$$

Thus,

$$
\frac{\partial u}{\partial \nu} - p \cdot \nu + \rho(w, w_t) \cdot \nu = 0 \quad \text{a.e. on } \Gamma_s \quad (4.11)
$$

That is $\frac{\partial u}{\partial \nu} - p \cdot \nu \in H^{-1/2}(\Gamma_s)$.

Furthermore, 4.7 implies $\text{div } (u_t - \Delta u + \nabla p) = 0$ in $\Omega_f$. Recall that $\text{div } u = 0$, we have $\text{div } u_t = 0$, also $\text{div } (\Delta u) = \Delta(\text{div } u) = 0$. Thus $\Delta \nabla p = \nabla p = 0$. \hfill \Box

Next, we are going to introduce the new multiplier and the identity that is crucial in the proof of analyticity of the semigroup generated by $A$

**Lemma 4.2.** Let $(u_0, v_0, w_1) \in D(A)$, and $(u, w, w_t, p)$ be the quadruple obtained by Lemma 4.1. Then for any $(\pi, \phi, \psi) \in H$, the operator $A : D(A) \to H$ defined in (2.9) and (2.7) satisfies the following identity:

$$
\left( A \begin{pmatrix} u \\ w \\ z \\ \pi \\
\phi \\
\psi 
\end{pmatrix}, \begin{pmatrix} \pi \\
\phi \\
\psi 
\end{pmatrix} \right)_H = \left( -A(u, w, z), \begin{pmatrix} \pi \\
\phi \\
\psi 
\end{pmatrix} \right)_H + \left( \text{div } \rho(w, z), \begin{pmatrix} \pi \\
\phi \\
\psi 
\end{pmatrix} \right)_H \quad (4.12)
$$

**Proof.** We only need to prove the second equation. The left of the second equation satisfies:

$$
\left( -A(u, w, z), \begin{pmatrix} \pi \\
\phi \\
\psi 
\end{pmatrix} \right)_H = (-A(u, w, z), \pi) + (\nabla z, \nabla \phi) + (\text{div } \rho(w, z), \psi)
$$

$$
= - (\nabla u, \nabla \pi)_{\Omega_f} - (\rho(w, z) \cdot \nu, \pi)_{\Gamma_s} + (\nabla z, \nabla \phi)_{\Omega_s} + (\rho(w, z) \cdot \nu, \psi)_{\Gamma_s} - (\nabla w + \nabla z, \nabla \psi)_{\Omega_s} \quad (4.13)
$$

The right of the second equation satisfies

$$
\left( \text{div } \rho(w, z), \begin{pmatrix} \pi \\
\phi \\
\psi 
\end{pmatrix} \right)_H = (\Delta u - \nabla p, \pi) + (\nabla z, \nabla \phi) + (\Delta(w + z), \psi)
$$

$$
= - (\nabla u, \nabla \pi)_{\Omega_f} + \left( \frac{\partial u}{\partial \nu} - p \cdot \nu, \pi \right)_{\Gamma_s} + (\nabla z, \nabla \phi)_{\Omega_s} + (\rho(w, z) \cdot \nu, \psi)_{\Gamma_s} - (\nabla w + \nabla z, \nabla \psi)_{\Omega_s}
$$

$$
= - (\nabla u, \nabla \pi)_{\Omega_f} - (\rho(w, z) \cdot \nu, \pi)_{\Gamma_s} + (\nabla z, \nabla \phi)_{\Omega_s} + (\rho(w, z) \cdot \nu, \psi)_{\Gamma_s} - (\nabla w + \nabla z, \nabla \psi)_{\Omega_s} \quad (4.14)
$$

Thus, the left and the right of the second equation equals. Lemma 4.2 is proved. \hfill \Box

We want to prove the analyticity of $A$ using the tool introduced in [29] and [32], which require to show that the imaginary axis is in the resolvent set. Specially, we need to show $0$ is in the resolvent set.
Theorem 4.3. \( \theta \) is in the resolvent of operator \( \mathcal{A} : \mathcal{D}(\mathcal{A}) \rightarrow \mathcal{H} \) defined in (2.9) and (2.7).

Proof. For \((u, w, z) \in \mathcal{D}(\mathcal{A})\), let

\[
-\mathcal{A} \begin{pmatrix} u \\ w \\ z \end{pmatrix} = \begin{pmatrix} u^* \\ w^* \\ z^* \end{pmatrix}
\]

then we can write, \( R(0, \mathcal{A}) \begin{pmatrix} u^* \\ w^* \\ z^* \end{pmatrix} = \begin{pmatrix} u \\ w \\ z \end{pmatrix} \) (4.15)

We want to show the resolvent operator \( R(0, \mathcal{A}) \) is bounded, that is \( \|(-\mathcal{A})^{-1}\|_{\mathcal{L}(\mathcal{H})} \leq C \), which is to show

\[
\left\| \begin{pmatrix} u \\ w \\ z \end{pmatrix} \right\|_\mathcal{H} \leq C \left\| \begin{pmatrix} u^* \\ w^* \\ z^* \end{pmatrix} \right\|_\mathcal{H}
\]

or

\[
\|u\|^2_{\Omega_j} + \|\nabla w\|^2_{\Omega_j} + \|z\|^2_{\Omega_j} \leq C \left[ \|u^*\|^2_{\Omega_j} + \|\nabla w^*\|^2_{\Omega_j} + \|z^*\|^2_{\Omega_j} \right]
\]

(4.16)

Expand the first equation in (4.15), we have

\[
A(u, w, z) = u^*
\]

(4.18)

\[
- z = w^*
\]

(4.19)

\[
- \Delta(w + z) = z^*
\]

(4.20)

(4.18) multiply by \( u \) and integral over space \( \Omega_j \) yields

\[
(\nabla u, \nabla u) + (\rho(w, z) \cdot \nu, u) = (u^*, u)
\]

(4.21)

(4.20) multiply by \( z \) and integral over space \( \Omega_s \) yields

\[
(\nabla w, \nabla z) + (\nabla z, \nabla z) - (\rho(w, z) \cdot \nu, z) = (z^*, z)
\]

(4.22)

Also keep in mind that \( \nabla z = -\nabla w^* \), and \( u|_{\Gamma_s} = z|_{\Gamma_s} \), adding (4.21) and (4.22), we have

\[
(\nabla u, \nabla u) + (\nabla z, \nabla z) = (u^*, u) + (\nabla w, \nabla w^*) + (z^*, z)
\]

\[
\leq C_{\epsilon_0}(\|u^*\|^2 + \|\nabla w^*\|^2 + \|z^*\|^2) + \epsilon_0(\|u\|^2 + \|z\|^2) + \epsilon_0^2\|\nabla w\|^2
\]

(4.23)

and Poincare inequality together imply

\[
(1 - \epsilon_0)(\|\nabla u\|^2 + \|\nabla z\|^2) \leq C_{\epsilon_0}(\|u^*\|^2 + \|\nabla w^*\|^2 + \|z^*\|^2) + \epsilon_0^2\|\nabla w\|^2
\]

(4.24)

Moreover, since \((u, w, z) \in \mathcal{D}(\mathcal{A})\), we have in (4.21), \( u|_{\Gamma_s} \in H^{1/2}(\Gamma_s) \), and by trace theorem, \( \rho(w, z) \cdot \nu|_{\Gamma_s} \in H^{-1/2}(\Gamma_s) \), sort (4.21) to obtain

\[
\|\rho(w, z) \cdot \nu|^{-1/2, \Gamma_s} u\|_{0, \Omega_j} + \|\nabla u\|^2 \leq C_{\epsilon_0}(\|u^*\|^2 + \|z^*\|^2) + \epsilon_0\|\nabla w\|^2
\]

(4.25)

By Trace Theorem, we have \( c_1 \|u\|_{1, \Omega_j} \leq \|u\|_{1/2, \Gamma_s} \leq c_2 \|u\|_{1, \Omega_s} \), for some \( 0 < c_1 \leq c_2 \), combined Poincare inequality, we have the following estimates

\[
\|\rho(w, z)\|_{-1/2, \Gamma_s} \leq C_1\|u^*\|^2 + C_2\|\nabla u\|
\]

(4.26)

Also notice that by adding (4.21) and (4.22),

\[
(\nabla w, \nabla z) + (\nabla u, \nabla u) + (\nabla z, \nabla z)
\]

\[
\leq (u^*, u) + (z^*, z) \leq C_{\epsilon_0}(\|u^*\|^2 + \|z^*\|^2) + \epsilon_0(\|u\|^2 + \|z\|^2)
\]

By Poincare inequality, we have the following estimates

\[
(\nabla w, \nabla z) \leq C_3(\|u^*\|^2 + \|z^*\|^2 + \|\nabla u\|^2 + \|\nabla z\|^2)
\]

(4.27)
Now (4.20) multiply by \( w \) and integral over space \( \Omega_x \),
\[
(\nabla w, \nabla w) = (z^*, w) - (\nabla z, \nabla w) + (\rho(w, z) \cdot \nu, w)_{\Gamma_x}
\]
Plug in estimates (4.26) and (4.28) to (4.29), and recall the trace theorem, we have
\[
(\nabla w, \nabla w) \leq C_1 ||z^*||^2 + \epsilon_1 ||w||^2 + C_3(||u^*||^2 + ||z^*||^2 + ||\nabla u||^2 + ||\nabla z||^2) + \epsilon \rho(w, z) \|w\|_{1/2, \Gamma_x}
\]
\[
\leq C_1 ||z^*||^2 + C_3(||u^*||^2 + ||z^*||^2 + ||\nabla u||^2 + ||\nabla z||^2) + 4(2C_1^2 ||u^*||^2 + 2C_2^2 ||\nabla u||^2)
\]
\[
+ \frac{1}{4} ||w||_{1, \Omega_x}^2 + \epsilon \rho(w, z) \|w\|_{1/2, \Gamma_x}^2
\]
Let \( C^* = C_1 + C_3 + 8C_1^2 \), \( C = C_3 + 8C_2^2 \). Then (4.30) becomes
\[
\frac{3}{4} - \epsilon_0 - \epsilon_1 \|\nabla w\|^2 \leq C^* \|w^*\|^2 + ||z^*||^2 + C^* \|\nabla w\|^2 + ||z^*||^2 \leq C^* \|w^*\|^2 + ||z^*||^2 + C \|\nabla w\|^2 + ||z^*||^2
\]
\[
\leq C^* + (2C^* + C) \|w^*\|^2 + ||z^*||^2 + 2C \|\nabla w\|^2 + ||z^*||^2
\]
Now, we set \( \epsilon_0 < \frac{1}{2C^*} \), set \( C^* > (C^* + 2C \cdot C_0) \). Thus, the following estimation hold
\[
\frac{3}{4} - \epsilon_0 - \epsilon_1 \|\nabla w\|^2 \leq C^* \|w^*\|^2 + ||z^*||^2
\]
Adding (4.24) and (4.32), we obtain
\[
\frac{3}{4} - \epsilon_0 - \epsilon_1 \|\nabla w\|^2 + ||\nabla w||^2 + ||z||^2 \leq 2C_\epsilon \|w^*\|^2 + ||\nabla w^*||^2 + ||z^*||^2
\]
By Poincare inequality again, we get for some constant \( C > 0 \)
\[
\|u\|^2 + ||\nabla w||^2 + ||z||^2 \leq C \|u^*\|^2 + ||\nabla w^*||^2 + ||z^*||^2
\]
Thus, 0 is in the resolvent of \( \mathcal{D}(\mathcal{H}) \).

Following Theorem 4.3, because a resolvent set is an open set, we have the following corollary:

**Corollary 4.4.** there exists a small \( r_0 > 0 \), such that on the complex plane, the open disk \( \mathcal{S}_{r_0} \) centered at the origin with radius \( r_0 \) is in the resolvent set of \( \mathcal{A} \).

To prove that the operator \( \mathcal{A} \) generates an analytic semigroup on \( \mathcal{H} \). We will first establish the following theorem

**Theorem 4.5.** For \( p \in L_2((0, T); L_2(\Omega_x)) \) satisfying system (1.1) obtained in Lemma 4.1, the resolvent operator \( R(\lambda, \mathcal{A}) = (\lambda - \mathcal{A})^{-1} \) for the generator \( \mathcal{A} \) defined in (2.9) and (2.7) satisfies the following estimates:
\[
||R(\lambda, \mathcal{A})||_{L(\mathcal{H})} \leq \frac{C}{||\lambda||} \text{ for all } \lambda \in \mathbb{C} \setminus K
\]
Set \( K = (-\infty, -2) \cup \mathcal{S}_{r=1}(x_0) \), where \( \mathcal{S}_{r=1}(x_0) \) is the closed disk centered at \( x_0 = (-1, 0) \) with radius \( r = 1 \).

**Proof.** Let \((u, w, z) \in \mathcal{D}(\mathcal{A}) \), \( \lambda = \sigma + \tau i \), then
\[
(\lambda - \mathcal{A}) \begin{pmatrix} u \\ w \\ z \end{pmatrix} = ((\sigma + \tau i) - \mathcal{A}) \begin{pmatrix} u \\ w \\ z \end{pmatrix} = \begin{pmatrix} u^* \\ w^* \\ z^* \end{pmatrix} \in \mathcal{H}
\]
Then
\[ R(\lambda, A) \begin{pmatrix} u^* \\ w^* \\ z^* \end{pmatrix} = \begin{pmatrix} u \\ w \\ z \end{pmatrix} \] (4.37)

and
\[ AR(\lambda, A) \begin{pmatrix} u^* \\ w^* \\ z^* \end{pmatrix} = A \begin{pmatrix} u \\ w \\ z \end{pmatrix} = \begin{pmatrix} -A(u, w, z) \\ z \\ \text{div} \rho(w, z) \end{pmatrix} \in \mathcal{H} \] (4.38)

since \( AR(\lambda, A) = \lambda R(\lambda, A) - I \), if we prove that \( AR(\lambda, A) \) is bounded, then \( \lambda R(\lambda, A) \) is consequently bounded. Thus (4.35) is proved.

To prove \( AR(\lambda, A) \) is bounded, we just need to show
\[ \left\| \left( \begin{array}{c} u^* \\ w^* \\ z^* \end{array} \right) \right\|_{\mathcal{H}} \leq C \left\| \left( \begin{array}{c} u^* \\ w^* \\ z^* \end{array} \right) \right\|_{\mathcal{H}} \] (4.39)

which is,
\[ \left\| \left( \begin{array}{c} -A(u, w, z) \\ \text{div} \rho(w, z) \end{array} \right) \right\|_{\mathcal{H}} \leq C \left\| \left( \begin{array}{c} u^* \\ w^* \\ z^* \end{array} \right) \right\|_{\mathcal{H}} \] (4.40)

By Lemma 4.2, we need to prove
\[ \left\| \left( \begin{array}{c} \Delta u - \nabla p \\ z \\ \Delta (w + z) \end{array} \right) \right\|_{\mathcal{H}} \leq C \left\| \left( \begin{array}{c} u^* \\ w^* \\ z^* \end{array} \right) \right\|_{\mathcal{H}} \] (4.41)

That means to show
\[ \left\| \left( \begin{array}{c} \Delta u - \nabla p \\ z \\ \Delta (w + z) \end{array} \right) \right\|_{\mathcal{H}} \leq \left\| \left( \begin{array}{c} u^* \\ w^* \\ z^* \end{array} \right) \right\|_{\mathcal{H}} \] (4.42)

or
\[ \|\Delta u - \nabla p\|^2_{\Omega_f} + \|\nabla z\|^2_{\Omega_f} + \|\Delta (w + z)\|^2_{\Omega_f} \leq C(\|u^*\|^2_{\Omega_f} + \|\nabla w^*\|^2_{\Omega_f} + \|z^*\|^2_{\Omega_f}) \] (4.43)

Expand (4.36) as follows:
\[ (\sigma + \tau i)u + A(u, w, z) = u^* \] (4.44)
\[ (\sigma + \tau i)w - z = w^* \] (4.45)
\[ (\sigma + \tau i)z - \text{div} \rho(w, z) = z^* \] (4.46)

Let \( p \in L_2((0, T); L_2(\Omega_f)) \) be the pressure solves (1.1) as indicated in Lemma 4.1, multiply (4.44) by \( \Delta u - \nabla p \), integrate over the space \( L_2(\Omega_f) \) and apply Green’s Theorem. Recall that div \((\Delta u - \nabla p) = 0 \) we get
\[ \text{LHS} = (\sigma + \tau i)u + A(u, w, z, \Delta u - \nabla p) = (\sigma + \tau i)(u, \Delta u - \nabla p) + (A(u, w, z), \Delta u - \nabla p) \]
\[ = -(\sigma + \tau i) \left[ (\nabla u, \nabla u) + (u, \rho(w, z) \cdot \nu) \right] - (\Delta u - \nabla p, \Delta u - \nabla p) \] (4.47)
\[
\text{RHS} = \langle u^*, \Delta u - \nabla p \rangle = -\langle u^*, \partial_u \nabla \nu - p \cdot \nu \rangle - \langle \nabla u^*, \nabla u \rangle \\
= -\langle u^*, \rho(w, z) \cdot \nu \rangle - \langle \nabla u^*, \nabla u \rangle
\] (4.48)
multiply (4.46) by \(\Delta(w + z)\), integrate over the space \(L_2(\Omega_s)\) and apply Green’s Theorem,

\[
\text{LHS} = \langle (\sigma + \tau i)z - \text{div} \rho(w, z), \Delta(w + z) \rangle \\
= (\sigma + \tau i)\langle [z, \rho(w, z) \cdot \nu] - (\nabla z, \nabla(w + z)) \rangle - \langle \Delta(w + z), \Delta(w + z) \rangle
\] (4.49)

\[
\text{RHS} = \langle z^*, \Delta(w + z) \rangle = \langle z^*, \rho(w + z) \cdot \nu \rangle - \langle \nabla z^*, \nabla(w + z) \rangle
\] (4.50)

Adding (4.47) and (4.49) yields

\[
- (\sigma + \tau i)\langle [\nabla u, \nabla u] + \langle u, \rho(w, z) \cdot \nu \rangle \rangle - \|\Delta u - \nabla p\|^2 \\
+ (\sigma + \tau i)\langle [z, \rho(w, z) \cdot \nu] - (\nabla z, \nabla(w + z)) \rangle - \|\Delta(w + z)\|^2 \\
= \langle u^*, \Delta u - \nabla p \rangle + \langle z^*, \Delta(w + z) \rangle
\] (4.51)

Simplify (4.51), consider that \(u|_{\Gamma_\circ} = z|_{\Gamma_\circ}\), we have

\[
- (\sigma + \tau i)\langle \|\nabla u\|^2 + \|\nabla z\|^2 \rangle \\
= \|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2 + (\sigma + \tau i)\langle \nabla z, \nabla w \rangle + \langle u^*, \Delta u - \nabla p \rangle \\
+ \langle z^*, \Delta(w + z) \rangle
\] (4.52)

Multiplying (4.45) by \(\sigma - \tau i\) we get \(w = \frac{\sigma - \tau i}{\sigma^2 + \tau^2}[z + w^*]\), thus

\[
(\nabla z, \nabla w) = (\nabla z, \frac{\sigma - \tau i}{\sigma^2 + \tau^2}(\nabla z + \nabla w^*)) = \frac{\sigma + \tau i}{\sigma^2 + \tau^2}\langle \nabla z, \nabla z + \nabla w^* \rangle
\] (4.53)

Plug (4.53) into (4.52), we get

\[
- (\sigma + \tau i)\langle \|\nabla u\|^2 + \|\nabla z\|^2 \rangle - \frac{\sigma^2 - \tau^2 + 2\sigma \tau i}{\sigma^2 + \tau^2}\|\nabla z\|^2 \\
= \|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2 + \frac{\sigma^2 - \tau^2 + 2\sigma \tau i}{\sigma^2 + \tau^2}\langle \nabla z, \nabla w^* \rangle \\
+ \langle u^*, \Delta u - \nabla p \rangle + \langle z^*, \Delta(w + z) \rangle
\] (4.54)

Take the real part of the (4.55), we get

\[
\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2 \\
= -\sigma\langle \|\nabla u\|^2 + \|\nabla z\|^2 \rangle - \frac{\sigma^2 - \tau^2 + 2\sigma \tau i}{\sigma^2 + \tau^2}\|\nabla z\|^2 - \Re\left[\frac{\sigma^2 - \tau^2 + 2\sigma \tau i}{\sigma^2 + \tau^2}\langle \nabla z, \nabla w^* \rangle\right] \\
- \Re\langle u^*, \Delta u - \nabla p \rangle - \Re\langle z^*, \Delta(w + z) \rangle
\] (4.55)

Consider that \(\left|\frac{\sigma^2 - \tau^2}{\sigma^2 + \tau^2}\right| \leq 1\), \(\|\sigma^2 - \tau^2 + 2\sigma \tau i\|_{\sigma^2 + \tau^2} = 1\), we have the following estimate:

\[
\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2 \\
\leq |\sigma|\langle \|\nabla u\|^2 + \|\nabla z\|^2 \rangle + \epsilon (\langle \|\nabla z\|^2 + \|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2 \rangle + C_\epsilon (\langle \|u^*\|^2 + \|\nabla w^*\|^2 + \|z^*\|^2 \rangle)
\] (4.56)
or

\[(1 - \epsilon)\|\Delta u - \nabla p\|^2 + (1 - \epsilon)\|\Delta(w + z)\|^2\]
\[\leq (|\sigma| + 1 + 2\epsilon)(\|\nabla u\|^2 + \|\nabla z\|^2) + C_\epsilon (\|u^*\|^2 + \|\nabla w^*\|^2 + \|z^*\|^2) \quad (4.57)\]

Take the imaginary part of (4.55), we get

\[\tau \|\nabla u\|^2 + \tau (1 + \frac{2\sigma}{\sigma^2 + \tau^2}) \|\nabla z\|^2\]
\[= -\Im\left[\frac{\sigma^2 - \tau^2 + 2\sigma \tau i}{\sigma^2 + \tau^2} (\nabla z, \nabla w^*)\right] - \Im(u^*, \Delta u - \nabla p) - \Im(z^*, \Delta(w + z))\]
\[= 1, \text{ then}\]
\[\frac{\epsilon^2}{2} \|\nabla z\|^2 + C_\epsilon (\|\nabla w^*\|^2 + \epsilon^2 \|\Delta u - \nabla p\|^2 + C_\epsilon \|u^2\| + \epsilon^3 \|\Delta(w + z)\| + C_\epsilon \|z^*\| \quad (4.60)\]

Let \(\epsilon < \min(1, |\tau|)\), then

\[\frac{\epsilon^2}{2} (\|\nabla u\|^2 + \|\nabla z\|^2) \leq |\tau| \|\nabla u\|^2 + (\epsilon |\tau| - \frac{\epsilon^2}{2}) \|\nabla z\|^2\]
\[\leq \epsilon^2 (\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\| + C_\epsilon (\|\nabla w^*\|^2 + \|u^*\|^2 + \|z^*\|^2) \quad (4.61)\]

and (4.57) indicates for sufficient small \(\epsilon > 0\)

\[(1 - \epsilon - 2\epsilon (|\sigma| + 1 + 2\epsilon)) (\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|)\]
\[\leq C_\epsilon (\|\nabla w^*\|^2 + \|u^*\|^2 + \|z^*\|^2)\]
\[(4.62)\]

Thus,

\[\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\| \leq C_\epsilon (\|\nabla w^*\|^2 + \|u^*\|^2 + \|z^*\|^2) \quad (4.63)\]

plug in (4.61), we get

\[\|\nabla u\|^2 + \|\nabla z\|^2 \leq C_\epsilon (\|\nabla w^*\|^2 + \|u^*\|^2 + \|z^*\|^2) \quad (4.64)\]

Adding (4.63) and (4.64), we have

\[\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2 + \|\nabla u\|^2 + \|\nabla z\|^2 \leq C_\epsilon (\|\nabla w^*\|^2 + \|u^*\|^2 + \|z^*\|^2) \quad (4.65)\]

From here we get

\[\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2 + \|\nabla z\|^2 \leq C_\epsilon (\|\nabla w^*\|^2 + \|u^*\|^2 + \|z^*\|^2) \quad (4.66)\]

Which is exactly (4.43). The above proof shows that for \(\lambda\) in the set \(\{\lambda = \sigma + i\tau : (\sigma + 1)^2 + \tau^2 > 1\}\), (4.43) holds. That means for \(\lambda\) outside the closed region \(S_{\tau=1}(x_0)\).
with the coordinate of \( x_0 = (-1, 0) \), \( AR(\lambda, A) \) is bounded, and thus \( \lambda R(\lambda, A) \) is bounded, which means
\[
\|R(\lambda, A)\|_{L(H)} \leq \frac{C}{\|\lambda\|}
\]
This proves Theorem 4.5.

Theorem (4.5) and Proposition (4.4) combined yields the following result.

**Proposition 4.6.** The spectrum \( \sigma(A) \) of operator \( A \) defined in (2.9) and (2.7), is contained in the following region:
\[
K/S_\rho = (-\infty, -2) \cup (S_{r_0+1}(x_0)/S_{r_0})
\]
with \( x_0 = (-1, 0) \) and \( r_0 \) small. Thus, the whole imaginary axis are in the resolvent, i.e.
\[
\rho(A) \ni i\tau \equiv \{i\tau, \tau \in \mathbb{R}\}
\]
And now we are in the position to

**Proof of Theorem 2.2** Theorem 2.2 is to establish the estimates of resolvent operator \( \lambda - A \) for the special case \( \lambda = i\tau, \tau \neq 0 \) in Theorem 4.5. Thus, in (4.55), set \( \sigma = 0 \), we get
\[
\tau i(\|\nabla u\|^2 + \|\nabla z\|^2)^2 + \|\nabla z\|^2
= \|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2 - (\nabla z, \nabla w^*) + (u^*, \Delta u - \nabla p) + (z^*, \Delta(w + z))
\]  
(4.69)

Take the real part of identity (4.69), we have
\[
\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2
= \|\nabla z\|^2 + \text{Re}(\nabla z, \nabla w^*) - \text{Re}(u^*, \Delta u - \nabla p) - \text{Re}(z^*, \Delta(w + z))
\]  
(4.70)

Thus, for \( \epsilon_1 > 0 \) sufficiently small, we have
\[
(1 - \epsilon_1)(\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2) \leq (1 + \epsilon_1)\|\nabla z\|^2 + C_{\epsilon_1}(\|u^*\| + \|\nabla w^*\| + \|z^*\|)
\]  
(4.71)

Take the image part of identity (4.69), we
\[
\tau(\|\nabla u\|^2 + \|\nabla z\|^2) = -\text{Im}(\nabla z, \nabla w^*) + \text{Im}(u^*, \Delta u - \nabla p) + \text{Im}(z^*, \Delta(w + z))
\]  
(4.72)

For \( \epsilon_2 > 0 \) sufficiently small, we have
\[
\|\nabla u\|^2 + \|\nabla z\|^2
\leq \frac{\epsilon_2}{|\tau| - \epsilon_0}(\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2) + \frac{C_{\epsilon_2}}{|\tau| - \epsilon_0}(\|u^*\| + \|\nabla w^*\| + \|z^*\|)
\]  
(4.73) plus (4.71), set \( \epsilon_0 = \max(\epsilon_1, \epsilon_2) \), and sorting out, we get
\[
(1 - \epsilon_0 - \frac{2\epsilon_0}{|\tau| - \epsilon_0})(\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2) + (1 - \epsilon_0)\|\nabla z\|^2 + 2\|\nabla u\|^2
\leq (C_{\epsilon_1} + \frac{2C_{\epsilon_2}}{|\tau| - \epsilon_0})(\|u^*\| + \|\nabla w^*\| + \|z^*\|)
\]  
(4.74)

For \( |\tau| \geq 5\epsilon_0 \), we have for \( C = \max(C_{\epsilon_1}, C_{\epsilon_2}) \)
\[
\frac{1}{2} - \epsilon_0)(\|\Delta u - \nabla p\|^2 + \|\Delta(w + z)\|^2 + \|\nabla z\|^2) \leq C(1 + \frac{2}{|\tau| - \epsilon_0})(\|u^*\| + \|\nabla w^*\| + \|z^*\|)
\]  
(4.75)
which is equivalent to (4.43)
\[ \|\Delta u - \nabla p\|_2^2 + \|\Delta (w + z)\|_2^2 + \|\nabla z\|_2^2 \leq C(\|u^*\| + \|\nabla w^*\| + \|z^*\|) \] (4.76)
Thus, Let \(\tau_0 = 5\epsilon_0\), for \(\epsilon_0\) sufficiently small, if \(|\tau| \geq \tau_0\),
\[ \|i\tau R(i\tau, \mathcal{A})\|_{L(H)} \leq C \] (4.77)
that is,
\[ \|R(i\tau, \mathcal{A})\|_{L(H)} \leq \frac{C}{|\tau|} \leq \frac{C}{\tau_0} \text{ uniformly for } \tau \geq \tau_0 > 0 \] (4.78)
Thus by Theorem 3E.3 in [29] and also [32], (4.78) together with Proposition 4.6
and the \(\mathcal{A}\) being the generator of s.c. semigroup of contraction on \(\mathcal{H}\), yields that \(\mathcal{A}\) also generates an analytic semigroup of contraction on \(\mathcal{H}\). Theorem 2.2 is proved.

5. Proof of Theorem 2.3. From Theorem 2.2, we know that
\[ \|R(i\tau, \mathcal{A})\|_{L(H)} \leq \frac{C}{|\tau|} \leq \frac{C}{\tau_0} \text{ uniformly for } \tau > \tau_0 \] (5.1)
That is \(R(i\tau, \mathcal{A})\) is uniformly bounded by a constant on \(\mathcal{H}\). By [38], the analytic semigroup \(e^{\mathcal{A}t}\) is uniformly exponentially bounded, that is, there exists a \(M \geq 1\) and \(\delta > 0\), such that
\[ \|e^{\mathcal{A}t}\|_{L(H)} \leq Me^{-\delta t} \] (5.2)
Combining with (2.12), we get
\[ E(t) \leq \|e^{\mathcal{A}t}\|_{L(H)} \left\| \begin{pmatrix} u_0 \\ w_0 \\ w_1 \end{pmatrix} \right\|_{\mathcal{H}} \leq Me^{-\delta t} \] (5.3)
for some \(M > 0\) and \(\delta > 0\). Thus, Theorem 2.3 is proved.

Remark 5.1. Compare the above results with [41], we know the damping term \(\Delta w_t\) steers the system into exponential decay. An interesting question is what will happen if we replace \(\Delta w_t\) by small structural damping such as \(\alpha \Delta w_t\), or \(\Delta (\alpha w_t)\), with \(0 < \alpha < 1\). In fact, for any fixed \(\alpha > 0\), the proofs of contraction (Theorem 2.1) and analyticity (Theorem 2.2) of the generated semigroup still hold, thus, the new system also keep the uniform stability property. However, if we let \(\alpha \to 0\), the semigroup is still contractive, but analyticity is lost (The proof of Theorem 4.3 runs into trouble), and uniform stability is lost too. However, in this scenario, from [27], we do have the solution \((u, w, w_t)\) decays to zero in the energy space \(\mathcal{H}\).

Remark 5.2. Another interesting extension to our current problem is what happens if the structural damping \(-\Delta w_t\) is replaced by fractional structural damping \((-\Delta)^{\alpha} w_t\) where \(0 < \alpha < 1\). Following [11, 31], when \(1/2 \leq \alpha < 1\), the structure component of the coupling model is analytic. Thus, the coupling is still analytic-analytic type. Under this scenario, we hope to see the overall evolution system remains analytic. This is still an open question that needs further investigation.
For the situation \(0 < \alpha < 1/2\), [12] indicates that the structure component generates a semigroup of Gevrey class \(\delta\), with \(\delta > \frac{1}{2\alpha}\). One can expect the Gevrey regularity to propagate through the interface. The regularity of the overall coupling under such situation \((0 < \alpha < 1/2)\) is also an interesting problem for further study.
Acknowledgments. The author is grateful to Prof. Irena Lasiecka and Roberto Triggiani for bringing this problem to the attention, and their valuable suggestions that have greatly helped the author in solving this problem. The author also thanks to the anonymous referee for the insightful comments that greatly improve this manuscript.

REFERENCES

[1] G. Avalos and R. Triggiani, The coupled PDE system arising in fluid-structure interaction. Part I: Explicit semigroup generator and its spectral properties, AMS Contemporary Mathematics, Fluids and Waves, 440 (2007), 15–54.

[2] G. Avalos, I. Lasiecka and R. Triggiani, Higher regularity of a coupled parabolic-hyperbolic fluid-structure interactive system, Georgian Math. J., Special issue dedicated to the memory of J. L. Lions, 15 (2008), 403–437.

[3] G. Avalos and R. Triggiani, Uniform stabilization of a coupled PDE system arising in fluid-structure interaction with boundary dissipation at the interface, Discr. Cont. Dynam. Sys., 22 (2008), 817–833.

[4] G. Avalos and R. Triggiani, Boundary feedback stabilization of a coupled parabolic-hyperbolic Stokes-Lamé PDE system, J. Evol. Equ., 9 (2009), 341–370.

[5] G. Avalos and R. Triggiani, Fluid-structure interaction with and without internal dissipation of the structure: A contrast study in stability, Evolution Equations and Control Theory, 2 (2013), 563–598.

[6] W. Arendt and C. J. K. Batty, Tauberian theorems and stability of one-parameter semigroups, Transactions of the American Mathematical Society, 306 (1988), 837–852.

[7] V. Barbu, Nonlinear Semigroup and Differential Equations in Banach Spaces, Springer, 1976.

[8] V. Barbu, Z. Grujic, I. Lasiecka and A. Tuffaha, Smoothness of weak solutions to a nonlinear fluid-structure interaction model, Indiana Univ. Math. J., 57 (2008), 1773–1207.

[9] V. Barbu, Z. Grujic, I. Lasiecka and A. Tuffaha, Existence of the energy-level weak solutions for a nonlinear fluid-structure interaction model, Contemporary Mathematics, 440 (2007), 55–82.

[10] S. Canic, B. Muha and M. Bukac, Stability of the Kinematically Coupled β-Scheme for fluid-structure interaction problems in hemodynamics, International Journal for Numerical Analysis and Modeling, 12 (2015), 54–80.

[11] S. Chen and R. Triggiani, Proof of the extensions of two conjectures on structural damping for elastic system, Pacific Journal of Mathematics, vol. 136 (1989), 15–55.

[12] S. Chen and R. Triggiani, Gevrey class semigroups arising from elastic systems with gentle dissipation: the case $0 < \alpha < \frac{1}{2}$, Proc. Amer. Math. Soc., 110 (1990), 401–415.

[13] C. Clason, B. Kaltenbacher and S. Veljović, Boundary optimal control of the Westervelt and the Kuznetsov equation, J. Math. Anal. Appl., 356 (2009), 738–751.

[14] D. Coutand and S. Shkoller, Motion of an elastic inside an incompressible viscous fluid, Arch. Rational Mech. Anal., 176 (2005), 25–102.

[15] R. Denk, M. Hieber and J. Prüss, R-boundedness, Fourier multipliers and problems of elliptic and parabolic type, Memoirs Amer. Math. Soc., 166 (2003), viii+114 pp.

[16] W. Desch, M. Hieber and J. Pruss, $L_p$ theory of the Stokes equation in a half space, J. Evolution Equ., 1 (2001), 115–142.

[17] W. Desch and W. Schappacher, Some perturbation results for analytic semigroups, Mathematische Annalen, 281 (1988), 157–162.

[18] Q. Du, M. D. Gunzburger, L. S. Hou and J. Lee, Analysis of a linear-fluid structure interaction model, Discr. Dynam. Sys., 9 (2003), 633–650.

[19] Y. Giga, Analyticity of the semigroup generated by the Stokes operator in $L_r$ space, Mathematische Annalen, 178 (1981), 297–329.

[20] Y. Giga, Weak and strong solutions of the Navier-Stokes initial value problem, Publ. RIMS, Tokyo Univ., 19 (1983), 887–910.

[21] M. Hieber and J. Prüss, Heat kernels and maximal $L^p - L^q$ estimates for parabolic evolution equations, Comm. Partial Differential Equations, 22 (1997), 1647–1669.

[22] B. Kaltenbacher and I. Lasiecka, Global existence and exponential decay rates for the Westervelt equation, Discr. Cont. Dynam. Sys., Series S, 2 (2009), 503–523.
[23] B. Kaltenbacher, Boundary observability and stabilization for Westervelt type wave equations, *Appl. Math. & Optim.*, 62 (2010), 381–410.
[24] I. Kukavica, A. Tuffaha and M. Ziane, Strong solutions to a nonlinear fluid structure interaction system, *J. Diff. Eq.*, 247 (2009), 1452–1478.
[25] I. Kukavica, A. Tuffaha and M. Ziane, Strong solutions to a nonlinear fluid structure interaction system, *Adv. Diff. Eq.*, 15 (2010), 231–254.
[26] I. Lasiecka, *Mathematical Control Theory of Coupled PDEs*, SIAM, 2002.
[27] I. Lasiecka and Y. Lu, Asymptotic stability of finite energy in Navier Stokes-elastic wave interaction, *Semigroup Forum*, 82 (2011), 61–82.
[28] I. Lasiecka and Y. Lu, Interface feedback control stabilization to a nonlinear fluid-structure interaction model, *Nonlinear Anal.*, 75 (2012), 1449–1460.
[29] I. Lasiecka and R. Triggiani, *Control Theory for Partial Differential Equations: Continuous and Approximation Theories, I: Abstract Parabolic Systems*, Encyclopedia of Mathematics and its Applications, 74 Cambridge University Press, 2000.
[30] I. Lasiecka and R. Triggiani, Heat-structure interaction with viscoelastic damping: Analyticity with sharp analytic sector, exponential decay, fractional powers, *Communications on Pure and Applied Analysis*, 15 (2016), 1515–1543.
[31] K. Liu and Z. Liu, Analyticity and differentiability of semigroups associated with elastic systems with damping and gyroscopitc forces, *J. Diff. Eq.*, 141 (1997), 340–355.
[32] Z. Liu and S. Zheng, *Semigroups Associated with Dissipative Systems*, Chapman & Hall/CRC Research Notes in Mathematics, 1999.
[33] S. Meyer and M. Wilke, Optimal regularity and long-time behavior of solutions for the Westervelt equation, *Appl. Math. and Optim.*, 64 (2011), 257–271.
[34] B. Muha and S. Canic, Existence of a weak solution to a nonlinear fluid-structure interaction problem modeling the flow of an incompressible, viscous fluid in a cylinder with deformable walls, *Archives for Rational Mechanics and Analysis*, 207 (2013), 919–968.
[35] B. Muha and S. Canic, Existence of a solution to a fluid-multi-layered-structure interaction problem, *Journal of Differential Equations*, 256 (2014), 658–706.
[36] N. Özkaya, M. Nordin, D. Goldsheyder and D. Leger, *Fundamentals of Biomechanics-Equilibrium, Motion, and Deformation*, Springer-Verlag, New York, 2012.
[37] A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer Verlag, 1983.
[38] J. Pruss, On the spectrum of $C_0$ semigroup, *Transactions of American Mathematics Society*, 284 (1984), 847–857.
[39] G. Simonett and M. Wilke, Well-posedness and long-time behaviour for the Westervelt equation with absorbing boundary conditions of order zero, To appear in in J. of Evol. Eqns.
[40] R. Triggiani, A heat-viscoelastic structure interaction model with Neumann or Dirichlet boundary control at the interface: Optimal regularity, control theoretic implications, *Applied Mathematics and Optimization*, special issue in memory of A. V. Balakrishnan, 73 (2016), 571–594.
[41] X. Zhang and E. Zuazua, Long-time behavior of a coupled heat-wave system in fluid-structure interaction, *Arch. Rat. Mech. Anal.*, 184 (2007), 49–120.

Received May 2016; Revised August 2016.

E-mail address: jizhang@vsu.edu