MeV-GeV Polarimetry with $\gamma \rightarrow e^+e^-$: Asserting the Performance of Silicon Strip Detectors-Based Telescopes

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Abstract

The polarimetry of gamma rays converting to an $e^+e^-$ pair would open a new window on the high-energy gamma-ray sky by, among other things, providing insight into the radiation mechanism in pulsars (curvature or synchrotron) or deciphering the composition of the gamma-ray emitting jets in blazars (leptonic or lepto-hadronic).

The performance of polarimeters based on homogeneous active targets (gas detectors (MeV, HARPO) or emulsions (GeV, GRAINE) has been studied both with simulation and by the analysis of data collected with telescope prototypes on linearly-polarised gamma-ray beams, and found to be excellent. The present ($\text{Fermi}$-LAT, AGILE) and future project (AMEGO, ASTROGAM) gamma-ray missions, though, are using active targets based on silicon strip detectors (SSD). No demonstration of a non-zero effective polarisation asymmetry with SSDs has been published to date, be it only with simulated data, and sensitivity estimations were obtained from an assumed value of the effective polarisation asymmetry.

I present a characterisation of the potential of SSD-based active targets for polarimetry with gamma-ray conversions to pairs and the development of various methods to improve on the sensitivity. This work could pave the way to providing the polarimetry of the brightest gamma-ray sources of the sky from the decade of data collected by the $\text{Fermi}$-LAT and by AGILE, and to guiding the design of future missions.

keywords: gamma rays, pair conversion, polarisation, polarimeter, silicon strip detector, moments method

1 High-Energy $\gamma$-Ray Polarimetry: Motivation

The gamma-ray sky is dominated by sources emitting by non thermal processes since a body hot enough to produce gamma-rays, that is, well above Eddington luminosity, would blow its emitting layers off. Several of these radiative processes such as synchrotron radiation and curvature radiation involve a magnetic field and the emission is polarised in a direction that depends on the direction of the field and/or of the charged particle flow. (Inverse) Compton scattering too can produce partially polarised photons, as the differential cross section involves a polarised component that has a specific angular distribution. In addition, Compton scattering enables an efficient transfer of the polarisation of an incident photon to the scattered one. In contrast to
the above, high-energy hadronic interactions of cosmic rays with matter produce spin-zero neutral pions that eventually decay to randomly polarised photons. The feasibility of gamma-ray polarimetry, the measurement of the polarisation fraction and of the polarisation angle of the high-energy radiation emitted by cosmic sources would be the opening of a new window on the non-thermal Universe [1].

In particular the polarimetry of $\mathcal{O}(100 \text{ MeV})$ gamma rays would sign the transition between synchrotron radiation and curvature radiation in rotation-powered pulsars [2]. “Since the pulsed radiation is emitted along the direction of relativistic particle motion which is coupled to the magnetic field direction, and the electric vector, parallel to the particle acceleration, is different for curvature and synchrotron radiation, the polarisation can be a very sensitive diagnostic” [3].

Also gamma-ray polarimetry would enable the determination of the composition of the gamma-ray-emitting high-energy charged particles in blazar jets: an electron-positron plasma is predicted to emit gamma rays with a low polarisation fraction (leptonic model) while a jet consisting of ionised matter (lepto-hadronic model) is expected to emit highly polarised gamma rays [4, 5].

Polarimetry is also a powerful tool to detect a possible Lorentz-invariance violation (LIV) in the photon sector induced by “new” physics beyond the standard model: as the induced birefringence would have the polarisation direction of the radiation emitted by a GRB rotated by a photon-energy-dependent angle during propagation, any significant broad-band observation of a non-zero linear polarisation of gamma rays from the GRB would constrain the amplitude of the violation: the larger the propagation distance and the photon energies, the stronger the constraint (a recent review is at [6]). Polarimetry is an exercise that is very demanding in terms of photon statistics though, as we shall see, and polarisation measurements of GRBs with a sufficient precision, within the $e^+e^-$-pair creation energy range, would need missions capable of sending hundreds of kilograms of sensitive material to space [7], something that might have to wait for some far-away future.

2 High-Energy $\gamma$-Ray Polarimetry: Measurement

The conversion of a photon to an $e^+e^-$ pair needs the interaction with an extra charged particle here named the target, as the effective mass of the produced pair is larger than $2mc^2$ ($m$ the electron mass), while the photon mass is zero and effective mass is bound to be conserved upon conversion. The final state then consists of three particles, the two leptons of the pair (the electron and the positron) and the recoiling target. The differential cross section is therefore five-dimensional [8]; its polarised version [9, 10, 11] includes a term that depends on the linear polarisation fraction and direction of the incoming photon and on the azimuthal angles of the final state particles.

The traditional way to perform the measurement is to use the dependence of the singly-differential cross section, function of an azimuthal angle, $\varphi$, that describes the orientation of the

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1The azimuthal angle of a particle is defined as the angle, in a plane perpendicular to the direction of the incident photon, between the direction of propagation of that particle and a fixed direction.
\[ \frac{d\sigma}{d\varphi} \propto (1 + A \times P \cos[2(\varphi - \varphi_0)]) , \]  

(1)

- \( A \), the polarisation asymmetry, depends on the energy, \( E \), of the photon and varies from \( \pi/4 \) at threshold \[12\] to 1/7 at very high-energy \[13\]. Over most of the MeV-GeV energy range which is presently accessible to experimentalists, \( A \) is close to 0.15 – 0.20 (see e.g. Fig. 3 of \[12\]).

- \( P \) is the linear polarisation fraction of the photon beam.

- \( A \times P \) is the modulation factor of the \( \varphi \) distribution.

Only the linear (not the circular) polarisation of the photon takes part in the differential cross section, at first order Born approximation \[9\].

After polarimetry with pair conversions was considered \[9, 10, 11\] it was realised that the multiple scattering of the leptons in the detector material before their azimuthal angle can be measured is a serious issue. The blurring of the azimuthal information with an angular resolution, \( \sigma_{\varphi} \), degrades the measurement of \( P \) by a dilution factor, the ratio of the polarisation asymmetry with and without experimental effects, \( D \equiv A_{\text{eff}}/A_{\text{QED}} \), of \( D = e^{-2\sigma_{\varphi}^2} \). Calculations based on the small-polar-angle approximation and on approximating the opening angle of the pair, \( \theta_{+-} \), to the most probable value of its distribution, \( \hat{\theta}_{+-} \) \[14\], indicated that the measurement of the azimuthal angle of the event must be performed in the very few \( 10^{-3} \) radiation lengths \( (X_0) \) downstream of the conversion vertex \[15, 16, 17\]. \( \sigma_{\varphi} \) increases in proportion to \( \sqrt{t} \) during propagation in a material of path length \( t \times X_0 \) \[18\], so \( D \) decreases exponentially with \( t \); for an angular resolution of several radians, there is simply no meaningful angular measurement, and the effective asymmetry \( A_{\text{eff}} \) is close to zero. The telescope then has no sizeable sensitivity to the polarisation of the incoming radiation. Since the value of the most probable opening angle varies like the inverse of the photon energy \[14\], since the value of the RMS multiple scattering angle of a relativistic charged particle varies like the inverse of the particle momentum \( p \) \[18\], and since the distribution of the momenta of the leptons scales approximately like the inverse of the photon energy \[8\], the variation of the dilution with thickness turns out to be independent of energy \[15, 16, 17\].

In the first, to my knowledge, successful experimental attempt to demonstrate the feasibility of polarimetry with pairs, the authors used the conversion of the photons of a 1.5 – 2.4 GeV beam on a 100-micron-thick carbon-foil target \((0.5 \times 10^{-3}X_0)\), with silicon detectors 2 meters downstream of the target and located on both sides of the beam, measuring the lepton track positions \[19\]. Such a set-up, which is well tuned to a high-intensity beam measurement (with an efficiency of 0.02 %), is obviously not suitable for a space mission, for which one wishes a high-effective-area high-acceptance detector.

It was then realised that, thanks to the large high-angle tail that the pair-opening-angle distribution exhibits, the most-probable approximation of \[15, 16, 17\] is actually quite pessimistic at large thicknesses (Fig. 17 of \[7\]). Also instead of using the simple expression of the scattering

\[^{2}\text{An updated version, obtained with the optimal “bisectrix” event-azimuthal angle definition from \[12\] can be found in Fig. 7 of \[20\].}\]
Table 1: Properties of some elements.

| Element | ρX₀ | X₀ | 1 / ρ  \( \frac{dE}{dx} \) | dE / dx | ρ |
|---------|-----|----|----------------|---------|---|
| C       | 42.7 | 19.3 | 1.742 | 3.85 | 2.21 |
| Al      | 24.0 | 8.90 | 1.62  | 4.36 | 2.70 |
| Si      | 21.8 | 9.37 | 1.66  | 3.88 | 2.33 |
| W       | 6.76 | 0.35 | 1.15  | 22.1 | 19.3 |

Table 2: Properties of several past and future trackers for gamma-ray space missions.

| Reading | ASTROGAM | AMEGO | LAT | AGILE |
|---------|----------|-------|-----|-------|
| Reading | DSSSD    | DSSSD | SSSSD | SSSSD |
| Wafer thickness | e | 500. | 500. | 400. | 410. |
| Layer number | N | 56 | 60 | 49 | 14 |
| Distance | \( \ell \) | 1 | 1 | 3 | 1.9 |
| Strip pitch | p | 240 | 500 | 228 | 242 |
| Critical energy | \( E_c \) | 67 | 32 | 211 | 126 |

angle after traversing a slab of matter as an estimate of the track angle RMS deflection, one can perform the reconstruction of the angle at the production vertex in an optimal way using a Kalman-filter based tracking [21], in which case the angular precision varies like \( p^{-3/4} \) in the homogeneous-detector approximation (eq. (4) of [22]). The dilution of the polarisation asymmetry then does not depend on the converter thickness only, but also on the photon energy: there is still some sizeable sensitivity in the lowest-energy part of the spectrum (Fig. 20 of [7]). This opportunity was explored by two recent projects using homogeneous detectors

- The GRAINE experiment [23] developed a high-density, (sub-micron) hyper-high-resolution emulsion tracker with which they measured the polarisation fraction of a GeV test beam [24].
- The HARPO experiment [25] developed a low-density, mild-resolution gaseous time projection chamber (TPC), with which we measured the polarisation fraction of a MeV test beam [26].

In both cases, the dilution of the polarisation asymmetry was found to be excellent.

The present-day gamma-ray missions in orbit, though, the Fermi-LAT [27] and AGILE [28] and the projects AMEGO [29] and ASTROGAM [30] use telescopes in which the active target consists of a stack of silicon strip detectors (SSDs) with thicknesses of 300 to 500 microns, that is of \((3.2 \text{ to } 5.3) \times 10^{-3} X_0\) (Tab. 1) and that therefore in the present context can be described to be thick wafers (Tab. 2), in particular when compared with the state of the art [31]. Polarimetry with such detectors has never been demonstrated, to my knowledge, be it by the analysis of the data collected from a detector prototype exposed to a polarised gamma-ray beam nor from
studies of simulated data, which means that a non-zero, strictly positive value of the effective polarisation asymmetry $A_{\text{eff}}$ has never been obtained for a SSD-based active-target polarimeter. This is most likely what has lead the authors of prospective studies to assume a value of $A_{\text{eff}}$ to issue a sensitivity $[28, 32]$. We can surmise that this inability to demonstrate a non-zero value of $A_{\text{eff}}$ may be due to a number of reasons or to a combination thereof:

- The dilution due to multiple scattering, as mentioned already. Telescopes (AMEGO, ASTROGAM) that use double-sided silicon strip detectors (DSSSDs) have a fraction of the conversion wafer, downstream of the vertex, inflicting multiple scattering on the tracks (i.e. a fraction of $\approx 4 \times 10^{-3} X_0$) even before they exit from the conversion wafer; in the case of single-sided SSDs (the LAT, AGILE), a pair of wafers is needed with orthogonal strip orientation, and the thickness is even larger.

- The impossibility to reconstruct the azimuthal angle in the second layer when the two tracks “fall” in the same strip for the two transverse directions $x$ and $y$: the first track segments of the two tracks are in this case reconstructed as a single track, and there is no azimuthal information available. Given the value of the most probable value $\hat{\theta}_{+-}$, of the pair opening angle, $\theta_{+-}$ [14]

$$\hat{\theta}_{+-} = \frac{1.6 \text{ MeV}}{E} \text{ rad},$$

and the limit pair opening angle that can be measured from the two first layers, $\theta_{c} = p/\ell$ (from pure geometry, no multiple scattering assumed) where $p$ is the strip pitch and $\ell$ the distance between layers, it is possible to define a critical photon energy value $E_c$ for which the most probable pair opening angle is equal to the limit pair opening angle by

$$E_c \equiv \frac{1.6 \text{ MeV} \ell}{p}.$$  

Since most cosmic sources have a flux that decreases strongly with photon energy, $E_c$ gives the energy scale of the photon candidates that should contribute mainly to the polarimetry measurement, would the telescope effective area already plateau at that energy. For the Fermi-LAT, $E_c = 211 \text{ MeV}$ (Tab. [2]).

- For the Fermi-LAT and for AGILE, for which a high value of the telescope effective area was obtained by interleaving the silicon detectors with high-$Z$ (tungsten) foils, the W foils must be located above the detectors (which is actually the case) and events with a conversion in the Si wafers should be identified (and those in W foils, rejected) with a high efficiency (something that was actually proven to be possible [32]).

- As a pad segmentation of the reading electrodes was out of reach due to power consumption limitations in orbit, two orthogonal series of strips are used, and in the case where the track hits produce separate clusters in both the $x$ and $y$ strips, a two-fold ambiguity arises. For homogeneous detectors, the issue is easily solved by matching the violently varying profiles of the energy deposition along the tracks (deposited-charge track matching, see Fig. 6 of [33]). For DSSSDs, deposited-charge track matching can also be considered. Even in case where no track-matching scheme is available, or if the scheme used is inefficient, the azimuthal information is still present in the data and, using the two possible configurations of the event (each weighted by a factor of 1/2), yields, at worst, an induced dilution factor of $D = 0.5$ (see Sec. [14]).
In this paper I characterise the performance of telescopes using DSSSD-based active targets for the polarimetry of linearly polarised gamma-rays.

Figure 1: Schema of the test detector model (not to scale). Adapted from [34]. The thick low-density aluminium structural material is not shown.

3 Test detector model

The test detector model used in this study is a simplified version of the Fermi-LAT active target (Fig. 1):

- The longitudinal (vertical) structure is similar to that described in [34]:
  - 18 measurement layers at a distance of 3 cm from each other, each consisting of a pair
of 400 \( \mu \text{m} \)-thick silicon wafers with a 2 mm gap between them, one measuring the \( x \), the other the \( y \), transverse (horizontal) coordinate, with a pitch of \( p = 228 \mu \text{m} \).

- Each of the top 16 layers includes a tungsten foil, the first 12 planes of 95 \( \mu \text{m} \) in thickness \( (t = 2.7\%) \), while the final four are 720 \( \mu \text{m} \) thick \( (t = 18\%) \).
- Vented aluminium honeycomb panels, with an average density of 48 kg/m\(^3\) for the heavy-converter and bottom trays and 16 kg/m\(^3\) for the others, are filling the 3 cm space between layers to give to the structure its rigidity.

- The horizontal structure, on the contrary, is different from that of the LAT: a unique tower-sized square wafer with 1536 strips in each direction, that is, 35 cm, with one of the central strips set to contain the conversion point.

The charge deposited by the passage of the leptons through the wafer is assumed to be collected in the strip right below the creation point, as charge sharing was shown to be small \cite{35}. The discrimination threshold is set at 35 keV, which corresponds to approximately 1/4 of the energy deposited by a minimum-ionising particle crossing the wafer at a right angle.

### 4 Assumptions

I simulate a point-like source and I assume that the combination of the path of the LAT on its orbit, of the variation of the orbit and of the attitude of the detector makes the exposure for that source isotropic, in LAT coordinates. I assume that the data have been analysed and that a list of photons associated with that source has been obtained and therefore that

- The direction of propagation of each incident photon is perfectly known to be coming from the source;
- An estimate of the photon energy is available.

### 5 Simulation

After an isotropic irradiation is generated, photons coming from above the tracker are selected, and a \( \cos \theta_{\text{LAT}} > 0.25 \) cut is applied, which corresponds approximately to the angle of 75° for which the LAT acceptance reaches zero \cite{36}.

The conversion is generated with the five-dimensional, polarised event generator documented in \cite{37}, after which the propagation of the leptons in the tracker is simulated with the EGS5 software, version 1.0.5 \cite{38}.

### 6 Tracking

Before I engage in the description of the details of the reconstruction, a couple of words about tracking are in order. The Kalman-filter-based technology that is commonly used for the tracking
of the trajectories of charged particles in a tracker is in principle able to make use optimally of all the information available, taking into account in an appropriate way the correlations induced by multiple scattering at each slice of the tracker \[21\]. The Fermi-LAT documentation states that their tracking is able to assign correctly hits to their original track with high certainty \[27\] but the assignment performance in the second layer, very close to the vertex and which is critical for the present study, needs to be established.

Let us define “the rest of the event” to be the hits registered in the tracker downstream with the exception of those in the first and the second layer, and let’s suppose that the hit-to-track assignment is performed perfectly in the rest of the event. Let us fit them to a track. The precision of track parameter measurements with an optimal tracking in a segmented (i.e., not homogeneous) tracker was studied in \[39, 20\]. The RMS angular resolution for a track impinging normally on the tracker, \(\sigma_\theta\), is given by

\[
\sigma_\theta = \frac{\sigma}{\ell} \sqrt{\frac{2 x^2 \left( \sqrt{4j - x^2 + \sqrt{-4j - x^2}} \right)}{(\sqrt{4j - x^2 + jx}) (\sqrt{-4j - x^2 - jx})}},
\]  

(4)

where \(x\) is the distance between wafers, \(\ell\), normalised to the detector scattering length \(\lambda\) \[40\], \(x \equiv \ell/\lambda = \sqrt{(\ell/\sigma)(p_0/p)\sqrt{\Delta/X_{0,W}}}\), \(j\) is the imaginary unit, and other variables are defined in Table 3.

| Table 3: Simple geometry for the tracking section. |
|-----------------------------------------------|
| distance between layers | \(\ell\) | 3 cm |
| single wafer precision | \(\sigma = p/\sqrt{12}\) | 66 \(\mu\)m |
| track momentum | \(p\) | |
| multiple scattering constant | \(p_0\) | 13.6 MeV/c |
| tracker scattering momentum | \(p_1\) see eq. (5) | 0.367 MeV/c |
| tracker scattering length | \(\lambda\) see [40] | |
| scatterer thickness | \(\Delta\) | 95 \(\mu\)m |
| scatterer radiation length | \(X_{0,W}\) | 0.35 cm |

The variation of \(\sigma_\theta\) with \(p\) is shown in Fig. 2 for a track traversing the full 12 layers of the “front” part of the tracker and a conversion at the very bottom of the bottom wafer of a layer. Figure 2 shows two regimes:

- at low momentum (large \(x\), coarsely-segmented detector), an optimal measurement can be obtained simply from the position measurements in the two first wafers, no Kalman filter is needed: asymptotically at very low momentum, the angular resolution is obviously that of a segment between two measurement points, \(\sigma_\theta \approx \sqrt{2\sigma/\ell}\).

- at high momentum (small \(x\)), the thin straight line is the homogeneous-detector asymptote \[7\], \(\sigma_\theta \approx (p/p_1)^{-3/4}\), where the momentum \(p_1\) that characterises the tracking-with-

\[3^4\] Note that the specific energy loss of the leptons along their path in the detector, \(dE/dx\), was neglected, so the momenta are assumed to be unchanged over the full trajectory.
Figure 2: Single-track RMS angular resolution of a segmented detector as a function of track momentum (eq. (4)). The thin high-\(p\) line is the \(\sigma_\theta \approx (p/p_1)^{-3/4}\) “homogeneous detector” asymptote \([7]\). The low-\(p\) asymptotic value is shown by the horizontal line at \(\sigma_\theta \approx \sqrt{2 \sigma / \ell}\). The inclined continuous straight lines show the RMS multiple scattering; the inclined dashed straight line show the pair opening angle for equipartition \((E = 2p)\), for comparison.
multiple-scattering properties of the homogeneous detector is

\[ p_1 = p_0 \left( \frac{2\sigma}{\ell} \right)^{1/3} \left( \frac{\Delta}{X_{0,1W}} \right)^{1/2}. \]  

(5)

The limit between the high- and low-momentum ranges can be defined as \( x = 2 \) \[39\], which corresponds to \( p = 255 \text{ MeV/c} \) for the (front part of the) \( \text{Fermi-LAT} \). The thicker curve represents the result (eq. (4)) of the exact calculation \[20\]. These expressions have been verified by comparison with the angular resolution of an actual Kalman-filter-based tracking applied on a Gaussian-generated tracker with Gaussian-distributed multiple scattering distributions (Figure 8 of \[39\]).

In addition to the angular resolution discussed above for a conversion just at the entrance of the 3 cm gap, for a conversion inside a wafer or in the tungsten foil, the multiple scattering inside that layer must be taken into account. The two parallel straight lines represent this contribution for the crossing of a full layer and for the crossing of a full single wafer, respectively.

- For a conversion in the tungsten, the multiple scattering in the first layer dominates the angular resolution below 1 GeV/c;
- For a conversion in a bottom wafer, it dominates below 0.3 GeV/c.

In addition, I have indicated with a dashed line the most probable value of the opening angle \[14\] for equipartition (\( E \approx 2p \)): wherever the conversion point (silicon, tungsten), the projection of the opening angle on the second layer, that is, the distance between the two tracks, is smaller than the resolution of the position of each of the tracks as extrapolated from the tracking over the rest of the event: we can surmise that the contribution to track matching in the second layer, from the transportation of the track matching in the rest of the event even if assumed to be perfect, is most likely to be of little help.

7 Reconstruction

The simple LAT model described above is exposed to an isotropic gamma-ray irradiation from above with \( \cos \theta_{\text{LAT}} > 0.25 \). At this point the model is a simple stack of transversely infinite planes of silicon, tungsten and low-density aluminium in vacuum. The conversion point (CP) is recorded and a single tower is built with the strips containing the CP at its centre. It is checked that the transverse leakage from that “tower” is negligible. Therefore the side effects induced by the dead zones at the wafer borders and at the tower borders of the actual \( \text{Fermi-LAT} \) are not addressed in this study. No dead channels, no wafer misalignments are simulated. I require that three successive layers see a signal above threshold, so as to emulate the main ingredient of the trigger of the \( \text{Fermi-LAT} \).

The conversion layer is defined as the first layer, starting from above, that has a non-zero signal in a silicon wafer. After discrimination over threshold has been applied, the hit pattern in the strips of the first and of the second layer is recorded. Clusters are formed from groups of consecutive hit strips. Track positions measured in that wafer are defined to be the position
The three possible configurations in the 2nd layer, with the number of reconstructed tracks mentioned. The bullets show the true, unknown to the experimentalist, position of the leptons when crossing that layer.

of the geometrical centre of clusters. The nature of the material in which the conversion took place is recorded:

- lower wafer;
- upper wafer;
- W front (a thin tungsten foil);
- W back (a thick tungsten foil).

We should keep in mind that this is a Monte Carlo information that is used in the analysis, and that is not directly available in the analysis of the data of the actual Fermi-LAT.

- Conversions in the lower wafer can be easily identified as no signal is collected in the neighbouring upper wafer (up to sub-threshold charge deposition, dead channel and border proximity analysis).
- The identification of the conversions in the upper wafer has been addressed for the actual Fermi-LAT in [32].

The cluster configurations that will be of some use in the present study are therefore:

- at least one cluster in the 1st layer;
- 0 or 1 cluster in each wafer of the 1st layer;
- 1 or 2 cluster(s) in each wafer of the 2nd layer;
- 2 clusters in at least one wafer of the 2nd layer.

I name $n_i$ the number of clusters in layer $i$ and $n_{xi}$ and $n_{yi}$ the number of clusters in wafers $x$ and $y$ of layer $i$, respectively, $i = 1, 2$. 
• Possible track crossings are built from the couple of two clusters, one \( x \) the other \( y \) in the same layer.

• Track candidates are built from pairs of track crossings, one in the first layer the other in the second.

• In the case where 2 track candidates were found, 1 photon candidate is built from that pair. In the case where 4 track candidates were found, 2 photon candidates are built from those, using track pairs that have no cluster in common.

As alluded in the previous section, the reliability of the matching of tracks that would be measured, even perfectly, in the rest of the detector (angles and momenta) with the hits in the second layer, is considered to be dubious enough, in this study, that I consider that the momenta of the track candidates are not known, and therefore I obtain the direction of a reconstructed photon as the bisectrix of the directions of its two tracks.

So there are two classes of events that are useful for polarimetry (Fig. 3):

\[(n_{x2} = 1 \cap n_{y2} = 2) \cup (n_{x2} = 2 \cap n_{y2} = 1)\]
\[n_{\text{track}} = 2 \quad n_{\gamma} = 1\]
\[n_{\text{track}} = 4 \quad n_{\gamma} = 2\]

• In the case of an \( n_{\gamma} = 1 \) event, the two tracks “fall” in the same cluster of one and only one wafer of the 2nd layer (Fig. 3 centre). The azimuthal angle cannot really be “measured” then, only a 0° or a 90° value can be assigned, in the LAT frame, depending on the orientation of the wafer that collected two clusters. Some amount of polarimetry information is still present, though, in that assignment, as we shall see.

• In the case of \( n_{\gamma} = 2 \), only one photon candidate is genuine, and the other is a combinatorial fake (Fig. 3 right). The polar angles of these two candidates, with respect to the known direction of the source, are the same, so that variable cannot be used for a discrimination. Some amount of polarimetry information is still present, though, as already alluded to in the discussion about “track matching” in Sect. 2.

As conversions in a lower wafer are identified from the absence of a signal in the corresponding upper wafer that is measuring, say, the \( x \) coordinate, the reconstruction of the event is modified:

• If \( n_{x2} = 1 \), the projection of the unit vector in the \((x, z)\) plane is taken to be that on the known direction of the incoming photon;

• If \( n_{x2} = 2 \), the directions of the tracks are taken so that the known direction of the incoming photon is their bisectrix;

The variation of the fraction of conversion events for various configurations, and in particular for the \( n_{\text{track}} = 2 \) and \( n_{\text{track}} = 4 \) classes that are of interest to this work (dashed curves), is shown in Fig. 4 as a function of the incident photon energy.
8 Definition of the event azimuthal angle

For a three-particle final state as is the case for a photon conversion to a pair, even if the recoiling nucleus cannot be detected, there was no natural, unique way to define “the” azimuthal angle of an event, that was named generically $\varphi$ in eq. (1). Since the pioneering work by Wojtsekhowski [41], a common practice has been to use the angle, $\omega$, defined as “the angle between the polarisation plane of the incident photon and the vector that connects the crossing points of the positron and the electron in the detector plane, which is perpendicular to the photon momentum” [III], see Fig. 5.

It was later discovered that another definition, the bisectrix of the azimuthal angles of the electron and of the positron, $\phi \equiv (\varphi_+ + \varphi_-)/2$, yielded a higher value of the polarisation asymmetry and, more importantly, a value that was found to be compatible with the published low-energy and high-energy asymptotic expressions for $A$ [12] (with a phase sign change with
Figure 5: Azimuthal angles definition in the momentum plane perpendicular to the direction of the incident photon (adapted from [41]). $\varphi_+$ and $\varphi_-$ are the azimuthal angles of the positron and of the electron, respectively, $\phi$ is their bisectrix, and $\omega$ is the azimuthal angle defined by [41].
Figure 6: Study of the variation of the polarisation asymmetry at generator level, for nuclear conversion of gamma rays on a naked silicon nucleus ("QED", full symbols) and on a silicon atom ("Si", open symbols). Asymmetry (left), R.M.S. width (centre), and ratio of width to asymmetry (right).

It was also found that there is still room for improvement, as an “optimal” measurement using the whole (five dimensional) differential cross section instead of the mere one-dimensional azimuthal-angle differential cross section yielded a precision of the measurement improved by a factor of two to three (Fig. 21 of [7], Fig. 3 of [12]): the value of the polarisation asymmetry is found to be similar to that of the 1D analysis, but the uncertainty is smaller.

I present results obtained with various definitions of the event azimuthal angle in the next sections.

It should be noted that following the usual notations, I refer to the azimuthal angle of “the positron” and of “the electron”, while in active targets without a magnetic field, the electric charge of the tracks cannot be measured. This misuse of language is actually innocuous, as an exchange would be equivalent to adding $\pi$ to $\phi$, that is, $2\pi$ to $2\phi$, and the cosine (and sine) would therefore be left unchanged.

9 The measurement on out-of-the-generator 4-vectors

In this work, I compute asymmetries with the moments method [7, 12], that is, as the expectation value, $E(w)$, of an optimal weight, $w$, chosen so that $E(w) = A \times P$, and computed as the average value of the distribution of $w$ for the generated sample. When an optimal weight is used, the moments method is equivalent to a likelihood fit [12]. The moments method has been routinely used in partial wave and amplitude analyses in particle physics and nuclear physics, until the development of maximisation tools such as MINUIT favoured the use of likelihood analyses ([12] and references therein).

Some differences can be noticed though, between the two methods, for example background noise contribution(s) is(are) taken into account in likelihood analyses by including their probability density functions (pdf) in the global maximised pdf, while for moments analyses only
the moments are needed. Background subtraction can then be performed by injecting events from the sidebands, or from the external regions on the region of interest (RoI), with a negative weight.

Let us first perform the measurement on simulated samples generated from the bare event generator, i.e. without any detector effects. Figure 6 shows the variation of the polarisation asymmetry $A$, as a function of incident photon energy from event samples generated with fully linearly polarised beams ($P = 1$) on silicon. We have

$$\sum_i^N w_i = N \times A \times P.$$ (6)

For the 1D differential cross section (eq. (1)), the optimal weight is $w \equiv 2 \cos 2\varphi$ [7, 12]. The centre plot shows the RMS width, $W$, of the distribution of $w$. For $A$ small (which will be the case here, as we shall see), the width is $W \approx \sqrt{2}$ (eq. (19) of [12]). The precision of the measurement of the modulation factor, $\sigma_{A \times P}$ is related to the width

$$W = \sqrt{N} \sigma_{A \times P}.$$ (7)

For a measurement on a source of unknown polarisation fraction, $P$, the effective value of $A$ must be known either from Monte Carlo studies or from prototype calibration on a beam with known polarisation fraction. The uncertainty of the measurement of $P$ is

$$\sigma_P = \frac{\sigma_{A \times P}}{A},$$ (8)

that is,

$$\sigma_P \approx \frac{W}{A \sqrt{N}}.$$ (9)

$\sigma_{A \times P}$ is found to be independent, at first order [4], of the value of $P$.

Let us examine the performance of the measurement at Monte Carlo level, that is, without taking any detector effect into account. From event samples of simulated pair conversions of fully polarised photons:

- The variation of $A$ with $E$ shows the known (eq. (16) and Fig. 3 of [12]) increase at low energy (Fig. 6 left).

- The polarisation asymmetry is found to be smaller for conversion on atoms ("Si") than on isolated nuclei ("QED") at high energy where the screening of the field of the nucleus by the electron cloud is effective. This was expected, as screening suppresses the amount of very low $q$ events who are carrying a higher polarisation content (Fig. 11 bottom left of [7]).

4See eq. (19) of [12]. A full treatment of the next-to-leading order terms in the computation of the uncertainty leads to an additional factor 3/2 in the correction term, see eq. (30) of [33]. J. Pretz, private communication, 2018.)
Figure 7: $n_{\text{track}} = 2$ events: Polarisation asymmetry as a function of incident photon energy.

- **Upper row**: For various conversion locations, from left plot to right plot: W back, W front, upper wafer, lower wafer, and various measurement methods (weights 2$\cos(2\phi)$ (open square), $-2\cos(2\omega)$ (open triangle), $\cos(2\phi) - \cos(2\omega)$ (open diamond).

- **Centre row**: For various methods, from left plot to right plot: 2$\cos(2\phi)$, $-2\cos(2\omega)$, $\cos(2\phi) - \cos(2\omega)$ and for the Monte Carlo generator 2$\cos(2\phi_{MC})$, and various conversion locations W back (bullet), W front (full square), upper wafer (upward triangle), lower wafer (downward triangle),

- **Bottom row**: The width of the distribution.

Notice the different vertical scales for the measured ($\phi$ or $\omega$) and the MC values of the azimuthal angle.
The polarisation asymmetry is found to be larger with a \( \phi \)-based weight than with a \( \omega \)-based weight as was already known \[12\]. Using the average of the cosines as the weight obviously just yields a result which is the average of the individual measurements.

Widths are energy-independent to first order (Fig. 6 centre) and are found to have the same value \( W \approx \sqrt{2} \) for both \( \phi \)- and \( \omega \)-based weights. For the combination, with weight \( \cos(2\phi) - \cos(2\omega) \), the width is \( \approx 1 \), which is a sign of uncorrelated measurements.

Finally the relative width plot shows that the combination is more precise than any of the individual measurements, by a factor of \( \approx \sqrt{2} \) (Fig. 6 right).

10 \( n_{\text{track}} = 2 \): Polarisation asymmetry from reconstructed events

We now examine the results for events reconstructed from the test detector, for events with \( n_{\text{track}} = 2 \) reconstructed tracks. The asymmetry measured using either \( \phi \)- or \( \omega \)-based weights or their combination, or the MC \( \phi \) value, for various conversion locations (lower wafer, upper wafer, front W and back W) is shown, as a function of incident photon energy in Fig. 7.

The polarisation asymmetry is found to be small, peaking at \( A \approx 0.03 \) for a conversion in the lower wafer, which corresponds to a dilution of \( D \approx 0.13 \). The peaking energy of about 200 MeV is found to be commensurate with the critical energy \( E_c \). The usable energy range ranges (sic) from 20 MeV to 2 GeV at maximum, that is, for a conversion in the lower wafer.

The polarisation asymmetry obviously decreases when the amount of multiple scattering increases, but it is interesting to note that even for conversion in a tungsten foil, some sensitivity is still present. We can surmise that the issue of deciphering conversions in an upper wafer from conversions in a “front” tungsten foil might be less critical than anticipated.

It is also interesting to note that in addition to the known energy dependence of the MC-generator asymmetry, a small additional variation can be seen, that depends also on the conversion location (Fig. 7, centre row, right plot); this is most likely due to indirect selection effects, as the value of \( A \) varies with the kinematic variables in the conversion (Fig. 11 of \[7\]).

The widths of the weight distributions show the nominal value of \( \sqrt{2} \) for a simple cosine-based weight. For the combination though, \( w = \cos(2\phi) - \cos(2\omega) \), the width is found to be larger than 1 in contrast with the results of the previous section (Fig. 6), based on the use of generator-level angles. This is the indication of a correlation between the two angles, induced by detector effects;
11 Variation of the polarisation asymmetry with kinematic observables

In this section I study the variation of the polarisation asymmetry as a function of several observables that are available for each event:

- \( \cos \theta_{\text{LAT}} \);
- \( \theta \), the polar angle between the known position of the source and the reconstructed direction of the photon candidate.

As the distributions of \( \theta \) vary violently with photon energy and with conversion location, and as they have extended tails, I use a function \( g(\theta) \) in place of \( \theta \) itself, devised so that the statistics be of the same order of magnitude for all bins. Function \( g(\theta) \) is documented in the Appendix. Figure 8 shows the polarisation asymmetry, measured using \( w = 2 \cos(2\phi) \), of reconstructed events as a function of \( g(\theta) \) and of \( \cos \theta_{\text{LAT}} \).

- The polarisation asymmetry decreases with \( g(\theta) \), either due to the known decrease at high \( q \) (Fig. 11 left bottom of [7]) or because these events have undergone a lot of multiple scattering; or both.
- The polarisation asymmetry increases with \( \cos \theta_{\text{LAT}} \), i.e. decreases with \( \theta_{\text{LAT}} \), again due to multiple scattering;

12 Better than cutting: weighting

Given the variation of the average value of the polarisation asymmetry as a function of event observables seen in the previous section, one might be tempted to select events in some range of these variables. An improved precision can be reached, though, by weighting events according to the polarisation asymmetry as averaged in Fig. 8. Later I combine events from different samples too, for example, with different conversion locations or of different energies.

Let us consider \( K \) event samples, \( k = 1 \cdots K \), each of which with number of events \( N_k \) and providing a measurement of \( P \) equal to \( P_k \) with variance \( V_k \). For separate event samples, the measurements are uncorrelated and an optimal combination is achieved by weighting each measurement by the inverse of its variance:

\[
P = \left( \sum_k \frac{P_k}{V_k} \right) \div \left( \sum_k \frac{1}{V_k} \right).
\]

From eq. (9) and the fact that the RMS widths of the \( w \) weight distributions for various samples are found to be identical, \( W_k = W \), we have:

\[
\frac{1}{V_k} = \frac{N_k A_k^2}{W^2},
\]

19
Figure 8: \( n_{\text{track}} = 2 \) events: Variation of the polarisation asymmetry as a function of event observables \( g(\theta) \) (top row) and \( \cos \theta_{\text{LAT}} \) (bottom row), for various conversion locations (from left to right: W back, W front, upper wafer and bottom wafer) and for various photon energies \( (w = 2 \cos(2\phi)) \).

Figure 9: Polarisation asymmetry as a function of incident photon energy, and various conversion locations, without and with weighting \( (n_{\text{track}} = 2 \) events, \( w = 2 \cos(2\phi) \)). Small horizontal shifts have been applied to improve on the readability.
Figure 10: $n_{\text{track}} = 2$ events: Polarisation asymmetry $A$ (left), number of events $N$ (centre) and value of $N_k A_k^2$ as a function of the conversion location (from “W back” ($k = 1$) to “down wafer” ($k = 4$)) and their sum “all”. Without and with $g(\theta)$ weighting, for each location, with global and $g(\theta)$ weighting, for all locations combined. The numbers of events are normalised to 1 incident photon on the detector.

where $A_k$ is the average effective polarisation asymmetry of sample $k$, so

$$P = \frac{\sum_k P_k N_k A_k^2}{\sum_k N_k A_k^2},$$

(12)

and given eq. (6),

$$\sum_i w_{i,k} = N_k A_k P_k,$$

(13)

we have

$$P = \frac{\sum_k A_k \sum_i N_k w_{i,k}}{\sum_k N_k A_k^2},$$

(14)

that is,

$$P = \sum_k \sum_i w'_{i,k},$$

(15)

the weighted weights (sic) for event $i$ of sample $k$, $w'_{i,k}$, being

$$w'_{i,k} = \frac{A_k w_{i,k}}{\sum_{\ell} N_{\ell} A_{\ell}^2},$$

(16)

In practice, given the fact that if a weight $w$ is an optimal weight, then $a \times w$, $a > 0$, is also an optimal weight, we can forget the denominator: an optimal combination of samples can be achieved by weighting each event by the known average value of the polarisation asymmetry of its sample, obtained from the analysis of MC samples (see, e.g., [44]).

$$w''_{i,k} = A_k w_{i,k},$$

(17)
The variance of $P$ is obtained from eq. (14)

$$\text{Var}(P) = \frac{\sum_k A_k^2 \sum_i N_{i,k} \text{Var}(w_{i,k})}{(\sum_k N_{k} A_k^2)^2} = \frac{W^2}{\sum_k N_{k} A_k^2},$$

(18)

and the precision of the measurement is then

$$\sigma_P = \frac{W}{\sqrt{\sum_k A_k^2 N_{k}}}.$$  (19)

For each conversion location, the variation of the polarisation asymmetry of $A$ with energy is presented in Fig. 9 for each weighting scheme (no weighting; weighting based on $g(\theta)$ or on $\cos \theta_{\text{LAT}}$). A sanity cut of $A_k > 0.001$ is applied in these combinations, something that induces a loss in statistics, especially at low energies.

An optimal weighting scheme would obviously be based on a two-fold segmentation of the $g(\theta)$, $\cos \theta_{\text{LAT}}$ space. This was not possible due to the limited statistics of the Monte Carlo samples. Attempts to weighting events with products of weights based on the two variables brought some minor further improvement on single-configuration event samples (plots not shown), but induced deleterious consequences when combining samples (e.g. with various conversion locations) that is presented in the next section, was attempted.

13 Combination of samples

The same weighting method that was used to take benefit of the variation of the effective polarisation asymmetry with event variables in the last section can also be used to combine events from different samples, such as that with different conversion locations. The values of $A_k$, of $N_k$ and of $N_{k} A_k^2$ are shown in Fig. 10 for $k = 1$ (W back) to $k = 4$ (down wafer), and their optimal combination (all). Results with $g(\theta)$ weighting (open symbols) are compared to that with a global weight for each conversion location.

- For $10^9$ 500 MeV incident photons, for example, $\approx 119 \times 10^6 n_{\text{track}} = 2$ photons are useable for pair polarimetry with $\sum_{k=1}^{4} N_{k} A_k^2 \approx 23000$ and $\sigma_P \approx 0.0093$.
- For $10^7$ 500 MeV incident photons, $\approx 1.19 \times 10^6 n_{\text{track}} = 2$ photons are useable for pair polarimetry with $\sum_{k=1}^{4} N_{k} A_k^2 \approx 230$ and $\sigma_P \approx 0.093$.
- The number of events (larger for W than for Si) compensates the better asymmetry for Si, to some extent (except for “back” events).

Figure 11 shows the overall sensitivity, in terms of $\sum_k N_{k} A_k^2$, as a function of energy, for various weighting schemes.
Figure 11: $n_{\text{track}} = 2$ events: Value of $\sum_k N_k A_k^2$ (i.e., all conversion locations combined) as a function of the conversion location for various weighting schemes, normalised to one incident photon on the detector.
14 Events with 4 reconstructed tracks

As mentioned already, the reconstruction of pair conversions in a two-wafer, single-sided SSDs scheme, for events with two separate clusters in each of the two directions, $x$ and $y$, in the second layer, suffers from a two-fold ambiguity (See Fig. 3 right).

14.1 Ambiguity: a simple 1D Toy model

Let us consider first a simple model with photons impinging perpendicular to the detector plane, with an azimuthal distribution in the sky frame given by (eq. (1)), and without any detector effects. The azimuthal plane is parallel to the detector wafers. The azimuthal angle of the correctly reconstructed event in the LAT frame, $\hat{\phi}$, is related to the azimuthal angle in the sky frame, $\phi$, by

$$\hat{\phi} = \phi + \delta,$$

(20)

where $\delta$ documents the azimuthal angle of the LAT wrt the sky frame. The azimuthal angle of the wrongly reconstructed candidate in the LAT frame, $\hat{\phi}'$, is obtained from the angle of the correctly reconstructed candidate, modulo $\pi$ (Fig. 12), by:

$$\hat{\phi}' = \pi - \hat{\phi};$$

(21)

hence, back to the sky frame,

$$\phi' = \hat{\phi}' - \delta = \pi - \phi - 2\delta.$$  

(22)

When no information is available to decipher which track pair is the correct one, both possible values are used with 1/2 weighting, and the original distribution (eq. (1)) becomes

$$\frac{d^2N}{d\phi d\delta} \propto \frac{1}{2} \left[(1 + A \times P \cos[2(\phi - \phi_0)]) + (1 + A \times P \cos[2(\phi + 2\delta + \phi_0)])\right].$$

(23)

Under an isotropic exposure, for each value of $\phi$, the second term of the second line cancels upon integration on $\delta$, and we are left with

$$\frac{dN}{d\phi} \propto \left(1 + \frac{A \times P}{2} \cos(2(\phi - \phi_0))\right),$$

(24)

which means that the additional dilution factor induced by the two-fold ambiguity in the 4-reconstructed-track events is of 1/2. This is illustrated with a toy-MC simulation in Fig. 13.

In case some non-zero efficiency to assign the correctly reconstructed track pair would be available, the dilution is found to vary linearly with that efficiency, from $D = 0.5$ for zero efficiency to $D = 1$ for perfect assignment [45].

Figure 14 shows the results of a toy MC exercise in which $10^4$ experiments are simulated with $10^6$ events each, sampled from eq. (1) with $A = 0.2$ and $P = 1$. We obtain
Figure 12: One-dimensional toy model: Schema of the azimuthal angle definitions for a 4-reconstructed-track event. The position of the electron and of the positron in the azimuthal plane are indicated by $e^-$ and $e^+$. The thin horizontal and vertical straight lines denote the hit strips in the second detector. The correct and wrong azimuthal angles of the event are indicated, for the two definitions of the azimuthal angle used in this study, Wojtsekhowski’s $\varphi \equiv \omega$ [41] and Gros’s $\varphi \equiv \phi$ [12]. Lines $\Delta$ and $\Delta'$ show the azimuthal directions of the bisectrix for the correctly and incorrectly reconstructed pairs, respectively.
Figure 13: $\varphi$ (left plot) and $w$ (right plot) distributions from a toy MC sample generated from the 1D differential cross section of (eq. (1)) with $A = 0.2$ and $P = 1$. Thick histogram: original distribution, i.e. with perfect track assignment available. Thin histogram: using both candidates with equal weights, as when no information on track assignment is available. Adapted from [45], see pages 39-40 of [46].
• with perfect track assignment (left plot, \( \langle A \times P \rangle = 0.2000, \sigma_{A \times P} = 0.0014 \), that is, \( \sigma_{A \times P}/\langle A \times P \rangle = 0.007 \), or

• with random \( \delta \) angle and both assignments used (right plot, \( \langle A \times P \rangle = 0.1000, \sigma_{A \times P} = 0.0010 \), that is, \( \sigma_{A \times P}/\langle A \times P \rangle = 0.010 \),

so even though the ambiguity inflicts a \( D = 0.5 \) dilution of the polarisation asymmetry, using both possible candidates in the calculation of the weight results in a reduction of \( W \) from \( W \approx \sqrt{2} \) to \( W \approx 1 \), and therefore the loss in the precision of the measurement of \( P \) due to the ambiguity is only of a factor of \( \sqrt{2} \).

The detailed simulations using the exact differential cross section and taking detector effects, that have been presented in the previous sections, confirm the results of the simple 1D Toy model, that is, \( D \approx 0.5 \) and \( W \approx 1 \) (plots not shown).

### 14.2 Track matching attempt

The transverse angular kick undergone by the pair due to the recoil momentum carried away by the nucleus has a 68% containment value of \( \theta_{68} = 1.5 \text{ rad}[E/\text{MeV}]^{-5/4} \) [22], while the most probable value of the pair opening angle is \( \hat{\theta}_{+} = 1.6 \text{ rad MeV}/E \) (eq. [2] and [14]), so the kick is smaller than the opening angle on the whole energy range (we obtain \( \theta_{68} = \hat{\theta}_{+} \) for \( E = (1.5/1.6)^{4} \approx 0.8 \text{ MeV} \)). Therefore the two tracks are produced almost back-to-back in the
Figure 15: Test of a best-candidate assignment based on acoplanarity: Ratio, $r$, of the number of correctly to the number of falsely assigned candidates, as a function of energy, for various conversion locations.
Figure 16: \( n_{\text{track}} = 4 \) events: Polarisation asymmetry as a function of incident photon energy.

- **Upper row**: For various conversion locations, from left plot to right plot: W back, W front, upper wafer, lower wafer, and various measurement methods (weights 2\cos(2\phi) (open square), \(-2\cos(2\omega) (open triangle), \cos(2\phi) - \cos(2\omega) (open diamond).\)

- **Centre row**: For various methods, from left plot to right plot: 2\cos(2\phi), \(-2\cos(2\omega), \cos(2\phi) - \cos(2\omega) and for the Monte Carlo generator 2\cos(2\phi_{MC}), and various conversion locations W back (bullet), W front (full square), upper wafer (upward triangle), lower wafer (downward triangle),

- **Bottom row**: The width of the distribution.

Notice the different vertical scales for the measured (\phi or \omega) and the MC values of the azimuthal angle.
azimuthal plane, something that could be conserved, to some extent, in correctly reconstructed photon candidates, but that would be disturbed for incorrect ones. Indeed, the spectra of the acoplanarity angle for correct and false reconstructions show a difference, something that can enable an assignment: the combination that shows the largest acoplanarity angle (closest to $\pi$) is chosen.

Figure 15 shows the ratio, $r$, of the number of correctly to the number of falsely assigned candidates, as a function of energy, for various conversion locations.

- Little improvement is seen with respect to random assignment ($r \approx 1$), except for conversions in the lower wafer and at low energies.
- For other locations, multiple scattering is the main source of acoplanarity and blurs the information initially present in the tracks.
- At higher energies, even for a conversion in the lower wafer, $r$ decreases towards unity, most likely because the opening angle at the vertex is small and multiple scattering, that is then the main contribution to an effective opening angle large enough to reach $n_{\text{track}} = 4$, ruins the information of the acoplanarity angle at vertex for both candidates.

Given these results, I will not use that best-candidate method in the following.

14.3 $n_{\text{track}} = 4$ events: measurement

The asymmetry measured using either $\phi$- or $\omega$-based weights or their combination, or the MC $\phi$ value, for various conversion locations (lower wafer, upper wafer, front W and back W) is shown as a function of incident photon energy in Fig. 16 for $n_{\text{track}} = 4$ in a way similar to Fig. 7 for $n_{\text{track}} = 2$. We see again a number of features that we observed for the $n_{\text{track}} = 2$ events:

- A polarisation asymmetry that vary with conversion location, improving from “back” to “lower wafer”;
- A poor improvement of the values of $W$, for a combination of $\omega$- and $\phi$-using measurements, with respect to measurements made using either of these azimuthal angle;

but we note also some different features:

- A polarisation asymmetry that decreases with energy,
- A value of $W$ that is close to unity, as anticipated in subsection 14.1

The variation with event variables $g(\theta)$ and $\cos \theta_{\text{LAT}}$ is presented in Fig. 17 for $n_{\text{track}} = 4$ in a way similar to Fig. 8 for $n_{\text{track}} = 2$. The variation of the polarisation asymmetry is found to be milder for $n_{\text{track}} = 4$ events than for $n_{\text{track}} = 2$. Accordingly, weighting events with an average value of the polarisation asymmetry that depends on the value of one of these variables brings little improvement, if any (Fig. 18).
Figure 17: \( n_{\text{track}} = 4 \) events: Variation of the polarisation asymmetry as a function of event observables \( g(\theta) \) (top row) and \( \cos \theta_{\text{LAT}} \) (bottom row), for various conversion locations (from left to right: W back, W front, upper wafer and bottom wafer) and for various photon energies. The measurement is performed using the moments method with weight \( 2 \cos(2\phi) \).
The values of $A_k$, of $N_k$ and of $N_k A_k^2$ are shown in Fig. 19 for $k = 1$ (W back) to $k = 4$ (down wafer), and their optimal combination (all), in a way similar to Fig. 10 for $n_{\text{track}} = 2$.

Figure 20 shows the overall sensitivity, in terms of $\sum_k N_k A_k^2$, as a function of energy, for various weighting schemes, in a way similar to Fig. 11 for $n_{\text{track}} = 2$. Most of the sensitivity is clearly at lower energies for $n_{\text{track}} = 4$ events. Not only a variable-dependent weighting does not increase the figure of merit, but at low energies is decreases it, because of the loss in statistics induced by the sanity cut.

---

Figure 18: Polarisation asymmetry as a function of incident photon energy, and various conversion locations, with and without weighting. ($n_{\text{track}} = 4$ events, $2 \cos(2\phi)$). Small horizontal shifts have been applied to improve on the readability.

Figure 19: $n_{\text{track}} = 4$ events: Polarisation asymmetry $A$ (left), number of events $N$ (centre) and value of $N_k A_k^2$ as a function of the conversion location (from “W back” $(k = 1)$ to “down wafer” $(k = 4)$) and their combination “all”. Without and with $g(\theta)$ weighting, for each location, with global and $g(\theta)$ weighting, for all locations combined. The number of events is normalised to 1 incident photon on the detector.
Figure 20: $n_{\text{track}} = 4$ events: Value of $\sum_k N_k A_k^2$ (i.e., all conversion locations combined) as a function of the conversion location with various weighting schemes, normalised to one incident photon on the detector.

Figure 21: Combination of $n_{\text{track}} = 2$ and of the $n_{\text{track}} = 4$ samples: polarisation asymmetry (left) and inverse of the precision $\sigma_P$ of the measurement of $P$, normalised to one incident photon (right), as a function of photon energy.
15 Wrap-up

The next step is the combination of the $n_{\text{track}} = 2$ and of the $n_{\text{track}} = 4$ samples. For each energy, all events are combined to a single measurement, each event affected with a weight that depends on the conversion location and on the number of reconstructed tracks. Figure 21 presents the polarisation asymmetry (left) and the inverse precision of the measurement of $P$, $1/\sigma_P$, as a function of $E$.

- The sensitivity for the $n_{\text{track}} = 2$ sample peaks at a higher energy ($\approx 500 \text{ MeV}$), than the $n_{\text{track}} = 4$ sample ($\approx 100 \text{ MeV}$).
- The precision of the combined sample peaks at ($\approx 200 \text{ MeV}$), with $1/\sigma_P \approx 0.006 \times \sqrt{N}$, where $N$ is here the total number of photons incident on the detector, that is, $\sigma_P \approx 0.005$ for the $N = 10^9$ simulated samples used in this study. The effective polarisation asymmetry of the combined sample is $A \approx 1.4\%$ at the peak.

![Figure 22: Energy distribution of the figure of merit as a function of energy for the $\epsilon \approx 2. \times 10^{11} \text{ cm}^2\text{s}$ exposure measurement of a $\Gamma = 2$, $F_0 = 1.5 \times 10^{-3}\text{ MeVcm}^{-2}\text{s}^{-1}$ bright source.](image)

16 Analysis of a full spectrum

The energy spectrum is binned, and events from different energy bins are analysed with the same combination method as presented previously.
Let us first present a simple estimate of the precision to be expected for an exposure of about
\( \epsilon \approx 2 \times 10^{11} \text{cm}^2 \text{s} \) observing a bright source with spectral index \( \Gamma = 2 \) and \( dN/dE = F_0/E^2 \), \( F_0 = 1.5 \times 10^{-3} \text{MeVcm}^{-2}\text{s}^{-1} \), with \( E > 20 \text{MeV} \). Supposing an isotropic exposure map, and taking into account the fraction of the sphere within the \( \cos \theta_{\text{LAT}} > 0.25 \) cut, we obtain the number of photon incident on the detector in each energy bin.

Taking into account the fraction of these events that convert to a pair in the active target (see Fig. 4), we obtain the number of pair-converted-and-triggered photons, of \( N_d = 3 \times 10^6 \), a number which is commensurate with the number of photons actually detected by the Fermi-LAT on the brightest sources of the \( \gamma \)-ray sky [47].

Energy bin limits are defined to be the logarithmic average of the nominal energies of successive bins, that is 20., 31.6, 70.7, 141., 316., 707., 1414., 3162., 7071. MeV.

### 16.1 Back-of-the-envelope estimation of the precision

I first estimate the precision of the measurement of \( P \) using the approximate expression of eq. (19), where the sum, \( \sum_k A_k^2 N_k \) runs on both the conversion location and the number of reconstructed tracks.

\[
\sigma_P = \frac{W}{\sqrt{\sum_k A_k^2 N_k}}. \tag{25}
\]

I obtain \( \sum_k A_k^2 N_k \approx 61 \), that is, with \( W \approx 1.2 \), \( \sigma_P = 0.15 \).

### 16.2 Full scale exercise

A sample of \( 10^9 \) events is generated between 20 MeV and 5 GeV with a \( \cos \theta_{\text{LAT}} > 0.25 \) cut, with a \( \Gamma = 2 \) spectral index and \( P = 1 \).

A measurement performed on that huge sample (\( \varphi = \phi \), all conversion locations combined, \( n_{\text{track}} = 2 \) and \( n_{\text{track}} = 4 \) combined, global weighting, with the weight applied for a given event being taken from the closest energy (in log space) of the samples described in Sections 9-15) yields \( A \times P = 0.0129 \) and \( \sigma_{A \times P} = 0.00012 \), that is \( \sigma_P = 0.0096 \), and \( W = 1.18 \).

That sample is then split into \( n = 144 \) subsamples with same sizes, for which the average number per sample of pair-converted-and-triggered events is \( \approx 3 \times 10^6 \), a number that is commensurate to the number of photons associated to a bright source in the Fermi-LAT data. The R.M.S. width of the \( n \) results is found to be \( \sigma_P \approx 0.14 \) (Fig. 23), that is, compatible with the approximate value from eq. (19), of \( \sigma_P \approx 0.15 \).
Figure 23: Distribution of the values of $A \times P$ obtained from $n = 144$ samples of photons generated between 20 MeV and 5 GeV with a $\Gamma = 2$ spectral index and $P = 1$. 
Figure 24: Study of systematics: $\langle \cos(n\phi) \rangle$ (upper row) and $\langle \sin(n\phi) \rangle$ (lower row), $n = 1 \cdots 4$, as a function of photon energy ($\varphi_0 = 0$, $\varphi \equiv \phi$, $n_{\text{track}} = 2$, global weighting). Simulations were performed with $P = 0$ (bullet) and with $P = 1$ (circle). Note the different vertical scale for $\cos(2\phi)$. An isotropic exposure map was used in this simulation and therefore no distortion from the $\cos(2\phi)$ signal was expected; actually no significant distortion is found.

17 Discussion and perspectives

I have estimated the potential of a Fermi-LAT-like active target to perform the polarimetry of a bright source of the gamma-ray sky.

From the geometry of the detector (layer spacing, strip pitch) and from the energy scaling of the distribution of the pair opening angle, the critical energy around which a significant fraction of the pair-conversion events are expected to show a large enough opening angle to yield a measurable value of the azimuthal angle is in the very lowest part of the LAT sensitivity energy range (Sect. 2).

The precision of the measurement of the direction of a track at vertex is dominated by the precision of the measurements in the two first layers on most of the energy range relevant to the present study, and, due to the multiple scattering in the next tungsten foils, it is only above 1 GeV/c that the angular precision approaches the homogeneous-detector asymptote for which the use of a Kalman filter is needed to perform an optimal fit (Sect. 6). Therefore I use only the information from the two first layers in this analysis (Sect. 7). No pattern recognition is applied and only events for which no extra clusters are present and a straightforward reconstruction can be performed are used. The main sources of event loss are then (Fig. 4)

- At low energies, the requirement that 3 layers in a row are hit (an emulation of the basic ingredient of the Fermi-LAT trigger) and the rejection of events with too many clusters in
the conversion layer ($n_{1x} > 1$ or $n_{1y} > 1$).

Also, the cross section of pair conversion decreases at low energies (less than one half that of the high-energy asymptote at 20 MeV for tungsten [48]), so a fraction of the incident photons just sail through the target without interacting.

- At high energies, the reconstruction of one single track that does not carry any usable azimuthal information, and the rejection of events with too many clusters in the next layer.

The effective polarisation asymmetry is found to be small, and, for conversions in the lower wafer of a layer,

- to peak at $A \approx 0.03$ for $E \approx 200$ MeV for $n_{\text{track}} = 2$ events (Fig. 7);
- to decrease as a function of energy for $n_{\text{track}} = 4$ events (Fig. 16).

For conversions in the upper wafer of a layer, or for conversions in a thin tungsten foil, the effective polarisation asymmetry is only worse. For conversions in a thick tungsten foil of the so-called “back” part of the active target, there is barely any significant asymmetry.

The combination of measurements of $P$ from several samples is performed in an optimal way by weighting each measurement by its inverse variance, as is usual, which turns out here to weighing each event by the polarisation asymmetry of the sample to which it belongs (Sect. 12).

I have tested a number of methods in the hope to further improve on the precision,

- weight with a weight that depends on one of the observables of the event (Sect. 11 and Figs. 9, 11, 18 and 20);
- combine measurements performed with various definitions of the azimuthal angle (Sect. 8 and Figs. 7, 16 and 21);
- in the case of $n_{\text{track}} = 4$ events, solve the photon reconstruction ambiguity by choosing the candidate with the best value of the acoplanarity angle (Subsect. 14.2).

These attempts have proven to bring a mild improvement only, and therefore I finally use a simple scheme in which

- Events are weighted with a weight that depends on the conversion location, on the number of reconstructed tracks and on the photon energy;
- The “bisectrix” azimuthal angle is used, $\varphi \equiv \phi$;
- For $n_{\text{track}} = 4$ events, both photon candidates are used with weight $1/2$.

Under the approximation of a constant value of the weight width $W$, a figure of merit is $\sum_k N_k A_k^2$, which is proportional to the combined inverse variance (eq. (18)). To that measure, conversions in the lower wafer dominate the precision of a combined measurement, as is expected,
but conversion in the upper wafer and in the thin tungsten foils contributed equally, the higher statistics for tungsten compensating for the lower effective asymmetry (Figs. 10 and 19).

For event samples simulated at various photon energies with the same number of incident events, the figure of merit then peaks at $E \approx 500 \text{ MeV}$ for $n_{\text{track}} = 2$ (Fig. 11) and at $E \approx 100 \text{ MeV}$ for $n_{\text{track}} = 4$ (Fig. 20). For realistic spectra with a $\Gamma = 2$ spectral index, peaking takes place at a much lower energy (Fig. 22).

Under that simple scheme, a measurement on a bright source of the gamma-ray sky with an exposure presently available at the Fermi-LAT would yield a precision of $\approx 15\%$.

The main avenue for improvement, besides pursuing the limited-gain methods explored above, would be the use of the pattern-recognition software of the experiment to analyse the events with a too large number of clusters.

A number of issues that are relevant to polarimetry have been left outside the scope of the present work.

- Measuring the polarisation angle together with the polarisation fraction, using the same moments’ method as was used here, can be performed based on a combination of moments of cosines and of sines ([44] and references therein). The precision of the measurement of $P$ is left unchanged (eq. (36) of [44]).

- Establishing the significance of a non-zero polarisation fraction in the presence of the nuisance parameter of a free polarisation angle can be an issue, as “Linear polarization is non-negative, i.e. one always measures something, even for an unpolarized source” (M. Weisskopf, 2014). This question has been addressed in [49]; in an experimental context, see for example [24].

- For such small effective polarisation asymmetries, and therefore such small modulation factors, systematics are obviously expected to be an issue.

  - Given the expected value of the uncertainty on the measurement of $P$, I don’t expect multiplicative systematics affecting the asymmetry, such as those originating from the imprecise knowledge of the mass model of the active target (e.g., the exact thickness of the wafers), to be a limiting issue.

  - More serious problems could originate from the variation of the effective asymmetry with $\theta_{\text{LAT}}$ (Figs. 8 and 17), and the possible dependence of the $\theta_{\text{LAT}}$ distribution on the actual exposure map (that was taken to be isotropic in the present study). Any discrepancy between the simulated and the actual exposure map, any discrepancy between the simulated and the actual dependence on $\theta_{\text{LAT}}$ of the trigger/reconstruction/selection efficiency would end up in data-to-MC differences in the higher-order spectra.

The examination of the higher-order moments, (Fig. 24) and in particular of their variation with $\theta_{\text{LAT}}$, will be critical to the control of these biases, as is usual in high-energy physics.

For polarization measurements performed using a moments method, background subtraction is usually performed by injecting events from the sidebands in the sum, with a negative weight. This should be easy in the present case of a bright source, for which the signal/noise ratio is large, but care should be taken at low energies for which the PSF gets worse [50].
18 Conclusion

The potential for $\gamma$-ray polarimetry in the pair conversion regime with a SSD-based active target is characterised in quantitative way. A non-zero, yet small, effective polarisation asymmetry is demonstrated for the first time. The main strength of this study is the use of validated and simple tools, and the generation of large MC samples, typically $10^9$ events per sample, that enabled precision studies of these small asymmetries. The main weakness is the lack of pattern recognition, so the analysis was restricted to events with the nominal numbers of clusters in the two first layers, something that induced a large loss in statistics, in particular at low energies (Fig. 4).

It is not easy to predict the performance of the AMEGO [29] and ASTROGAM [30] projects from the present study without performing a dedicated analysis. Due to the absence of high-$Z$ converter foils in the active target, and thanks to the DSSSD scheme, all pair-conversion events are of the “lower wafer” category, in my classification. The DSSSD scheme makes $(x, y)$ track matching straightforward for $n_{\text{track}} = 4$ events. Taken at face value, the geometry seems to be less favourable than for the Fermi-LAT, with a smaller distance between layers and larger strip pitches (accordingly their critical energy is lower, see Tab. 2), but these detectors are designed to enable trigger / reconstruction / selection of pair-conversion events of lower energies than the Fermi-LAT. The silicon sensitive masses are similar, 69 kg for the Fermi-LAT [34], to be compared to 40 kg for AMEGO [30].

The SSD part of the AMS-02 tracker should enable a high-performance $\gamma$-ray polarimetry, with its DSSSD scheme, thin wafers, narrow readout pitch, and large distance between layers [51]. Unfortunately the smallness of the silicon mass (4.5 kg) and the need to select photons that convert in a thin layer just above an empty space followed by a wafer, might limit the available statistics for a significant measurement.

Gamma-ray polarimetry with pair conversions with a higher value of the effective polarisation asymmetry than in the present study, may have to wait for projects using homogeneous detectors [24, 26] or thinner silicon wafers [31].
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Variables used in the text.

\( A \) (linear) polarisation asymmetry  
\( D \) dilution of the polarisation asymmetry  
\( \delta \) azimuthal angle of the LAT wrt the sky frame.  
\( \Delta \) W scatterer thickness  
\( \Delta, \Delta' \) azimuthal directions of the bisectrix for  
\( \) the correctly and incorrectly reconstructed pairs, respectively.  
\( E \) expectation value  
\( E \) photon energy  
\( E_c \) critical energy of an active target  
\( e \) wafer thickness  
\( \epsilon \) data taking exposure  
\( \varphi \) generic azimuthal angle of a conversion event  
\( \varphi_0 \) polarisation angle of the incoming radiation  
\( \phi \) one way to define the azimuthal angle \( \varphi \)  
\( \Gamma \) source spectral index  
\( j \) imaginary unit  
\( \ell \) distance between layers  
\( \lambda \) detector scattering length  
\( n_{\gamma} \) number of photon candidates in an event  
\( n \) number of subsamples  
\( n \) high-order index  
\( N \) number of layers  
\( N \) number of events in sample  
\( N \) total number of generated events  
\( N_d \) number of pair-converted-and-triggered events  
\( n_{\text{track}} \) number of track candidates in an event  

\( \text{eq. (1)} \)  
\( \text{Sect. 2} \)  
\( \text{Sect. 14} \)  
\( \text{Fig. 12} \)  
\( \text{Sect. 9} \)  
\( \text{Tab. 2} \)  
\( \text{Sect. 9} \)  
\( \text{Fig. 24} \)  
\( \text{Tab. 2} \)  
\( \text{Sect. 9} \)  
\( \text{Sect. 15} \)  
\( \text{Sect. 17} \)
$p$  silicon strip pitch
$p$  track momentum
$p_0$  characteristic momentum of multiple scattering  [18]
$p_1$  characteristic multiple-scattering momentum of an active target  [7], eq. (5)
$P$  (linear) polarisation fraction of the incoming radiation  eq. (1)
$q$  target recoil momentum upon gamma-ray conversion
$r$  ratio of the number of correctly to falsely assigned candidates  Fig. 15
$\rho$  material density
$\sigma$  RMS
$\sigma$  single-(wafer)-measurement space resolution
$\sigma_{\theta}$  (polar) angular resolution
$\sigma_{\varphi}$  RMS resolution of the measurement of the azimuthal angle
$t$  thickness through which a lepton propagate, $x$, normalised to $X_0$
$\theta$  polar angle
$\theta_{+\,-}$  pair opening angle of an event  [14], eq. (2)
$\hat{\theta}_{+\,-}$  most probable value of $\theta_{+\,-}$
$\theta_{\text{LAT}}$  angle between the photon direction and the $z$ axis of the LAT
$V$  variance
$w$  weight
$\omega$  one way to define the azimuthal angle $\varphi$  [41]
$W$  RMS width (of weight distributions)  Sect. 9
$W$  tungsten
$X_0$  material radiation length
$x$  detector longitudinal sampling normalised to scattering length  Sect. 6
$x$  axis name
$y$  axis name
$z$  axis name
In Section 11 I have used a function $g(\theta)$ in the place of variable $\theta$, so as to histogram the distribution with a decent, more or less homogenous, statistics per bin. Function $g(\theta)$ is documented in this Appendix.

$$g(\theta) = \theta / \theta_u,$$  \hspace{1cm} (26)

with

$$\theta_u = \theta_0 \left( \frac{E}{100 \text{MeV}} \right)^\alpha a^i \oplus \theta_1 b^{\text{track}}$$  \hspace{1cm} (27)

where $\theta_0$, $\alpha$, $a$, $\theta_1$, $b$ are the following parameters:

| $\theta_0$ | $\alpha$ | $a$ | $\theta_1$ | $b$ |
|-----------|---------|-----|-----------|-----|
| 0.2154    | -1.0338 | 0.5260 | -0.0014  | 2.0289 |

$i$ varying from 1 to 4 for W-back to lower-wafer.
Contents

1 High-Energy $\gamma$-Ray Polarimetry: Motivation 1

2 High-Energy $\gamma$-Ray Polarimetry: Measurement 2

3 Test detector model 6

4 Assumptions 7

5 Simulation 7

6 Tracking 7

7 Reconstruction 10

8 Definition of the event azimuthal angle 13

9 The measurement on out-of-the-generator 4-vectors 15

10 $n_{\text{track}} = 2$: Polarisation asymmetry from reconstructed events 18

11 Variation of the polarisation asymmetry with kinematic observables 19

12 Better than cutting: weighting 19

13 Combination of samples 22

14 Events with 4 reconstructed tracks 24

14.1 Ambiguity: a simple 1D Toy model . . . . . . . . . . . . . . . . . . . . . 24

14.2 Track matching attempt . . . . . . . . . . . . . . . . . . . . . . . . . . . . 27

14.3 $n_{\text{track}} = 4$ events: measurement . . . . . . . . . . . . . . . . . . 30

15 Wrap-up 34

16 Analysis of a full spectrum 34

16.1 Back-of-the-envelope estimation of the precision . . . . . . . . . . . . . 35

16.2 Full scale exercise . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 35

17 Discussion and perspectives 37
18 Conclusion

A