The recently announced Super-Kamiokande data on atmospheric neutrino oscillation seems to require a maximal mixing between the $\nu_\mu$ and $\nu_\tau$ within the conventional three neutrino picture. It is then tempting to suggest as has been done in literature, that the solar neutrino deficit be also understood as resulting from a maximal mixing between $\nu_e$ and $\nu_\mu$. In this letter, we propose a left-right symmetric extension of the standard model where permutation symmetry leads to one of the maximal mixing patterns in a technically natural manner. The double seesaw mechanism gives small Majorana masses for neutrinos needed to understand the atmospheric as well as the solar neutrino puzzles.

I. INTRODUCTION

The announcement by the Super-Kamiokande collaboration \[1\] of evidence for neutrino oscillation (and hence nonzero neutrino mass) in their atmospheric neutrino data is a major milestone in the search for new physics beyond the standard model. An outstanding feature of these oscillations is the maximal mixing between the $\nu_\mu$ and $\nu_\tau$ ($\sin^2 2\theta_{\mu-\tau} \approx 0.7 - 1$) in sharp contrast with the mixing pattern in the quark sector. Also the inferred $\Delta m^2_{\mu-\tau} \approx 5 \times 10^{-4} - 6 \times 10^{-3}$ eV$^2$ is lower than most “see-saw motivated” extrapolations from $\Delta m^2_{e-\mu}$ values in the small or large angle MSW solutions to the solar neutrino problem \[2\]: $\Delta m^2_{e-\mu} \approx 3 \times 10^{-6} - 7 \times 10^{-6}$ eV$^2$ with $\sin^2 2\theta \approx 3 - 5 \times 10^{-3}$ and $\Delta m^2 \approx 10^{-5} - 10^{-4}$ eV$^2$ with $\sin^2 2\theta \approx 0.8 - 1$.

The solar neutrino problem provided the first indication for neutrino oscillation and this evidence keeps building up. It can also be resolved by the maximal $\nu_e-\nu_\mu$ vacuum oscillation with fine tuned small mass difference $\Delta m^2_{e-\mu} \approx 10^{-10}$ eV$^2$. Maximal mixing with larger $\Delta m^2$ values yield an energy independent suppression of all solar neutrinos \[1\] (except when it is in the large angle MSW range mentioned above). While this does not resolve the solar neutrino problem at present, it does considerably ameliorate it.

All the above suggests considering maximal ($\nu_e-\nu_\mu$) mixing alongside maximal ($\nu_\mu-\nu_\tau$) mixing \[5\]. Three specific "bimaximal mixing" patterns \[6–8\] having particularly simple forms are:

**Case (A) \[6\]:**

$$ U_\nu = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{pmatrix} $$  \hspace{1cm} (1)

where $\omega = e^{2\pi i \frac{2}{3}}$; we will call this the symmetric mixing pattern.

**Case (B) \[5\]:**

$$ U_\nu = \begin{pmatrix} 1 & -1 & 0 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} $$  \hspace{1cm} (2)

This has been called in the literature as bimaximal mixing \[5\].

**Case (C) \[7\]:**

$$ U_\nu = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{pmatrix} $$  \hspace{1cm} (3)

\[1\] More precisely, the suppression factor in the radio chemical experiments is by 50% whereas in the Super-Kamiokande it is 57%. 

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We call this democratic mixing.

In the above equations, we have defined $U_\nu$ as follows:

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U_\nu
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
$$

with $\nu_{e,\mu,\tau}$ being the weak eigenstates and $\nu_{1,2,3}$, the mass eigenstates.

Our goal is to explore possible extensions of the standard model that may provide a theoretical understanding of the maximal mixing patterns. Attempts to understand the pattern (A), made in our previous paper [13] were largely unsuccessful. Also the CHOOZ upper bound [10] on $\nu_e - \nu_\mu$ oscillation with $\Delta m^2_{13} \geq 10^{-3}$ eV$^2$, tends together with Super-Kamiokande data, to disfavor this possibility. There has been several recent attempts to derive the second pattern (B) [10]. Here we will show by using an extension of the standard model, that it is possible to generate the pattern (C) in a consistent and natural way. Our motivation is quite clear: if nature presents us with such a neutrino mixing pattern, we must seek an extension of the standard model that can naturally lead to it. Hopefully a theory that naturally provides this pattern will have other testable predictions.

In Ref. [6], permutation symmetry was imposed on the charged lepton mass matrix and not on the neutrino mass matrices in order to motivate the pattern (C). No underlying theoretical justification was discussed for such a hypothesis. In the framework of gauge theories, such an assumption is hard to understand a priori since the charged leptons and the neutrinos are members of the same isodoublet of the standard model gauge group $SU(2)_L$ and therefore the permutation symmetry could lead to a similar mass matrix for both the charged lepton sector as well as the neutrino sector. If that happens, the neutrino mixing matrix which is given by $U^T_{\text{dir}} U_\nu$ could substantially differ from (C). It is therefore important to investigate whether the above mixing pattern arises in a complete theory. Also the putative mass pattern $\Delta m^2_{32} \gg \Delta m^2_{21}$ should be provided by the model rather than arbitrarily fixed. It is considerations such as these which motivate us to add this brief note to the exploding literature on neutrino models.

We find that by combining the permutation symmetry $S_3$ with a $Z_4 \times Z_3 \times Z_2$ symmetry in the left-right symmetric extension of the standard model, we can obtain the maximal mixing pattern (C) in a technically natural manner (i.e. without setting any parameters to zero by hand). In the limit of exact permutation symmetry, all the neutrinos are degenerate as are the electron and the muon. As a result, the mixing angles can be rotated away. However, once one admits permutation breaking terms to accomodate the electron muon mass difference, the neutrino degeneracy is removed and the democratic form (pattern C) of the maximal mixing pattern remains. In fact, the masses of $\nu_e$ and $\nu_\mu$ get related to the electron and muon masses arising completely from radiative corrections. To avoid arbitrary deviations from the maximal pattern, we assume that the permutation symmetry (but not the $Z_4 \times Z_3 \times Z_2$) is softly broken. This adds only small, finite, corrections to the mixing pattern and one obtains a complete and realistic gauge theoretic derivation of the maximal mixing pattern C.

$$\text{II. PERMUTATION SYMMETRY AND A GAUGE THEORY OF MAXIMAL MIXING}$$

We consider a left-right symmetric extension of the standard model with the usual fermionic field content $\bar{10}$. We omit the discussion of the quark sector for now. Denoting the leptons by $\psi_a \equiv (\nu_a, e_a)$, the $\psi_{L,R}$ transform as the $SU(2)_{L,R}$ doublets respectively under the left-right gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The subscript $a$ stands for the generations. We choose the following set of Higgs bosons to achieve the symmetry breaking: three sets of left and right doublets denoted by $\chi_{a,L,R}$ ($a = 1,2,3$): the $\chi_{a,R}$ vev will break the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry down to the standard model $U(1)_Y$ group. We choose three bidoublets $\phi_{0,1,2}$ to break the electroweak $SU(2)_L \times U(1)_Y$ symmetry and give mass to the quarks and charged leptons as well as the Dirac mass for the neutrinos. In order to implement the double seesaw $\bar{10}$ mechanism for neutrino masses, we introduce three gauge singlet fermion fields, $\sigma_a$.

In order to get the desired pattern for lepton masses, we demand the theory to respect the symmetry $S_3 \times Z_4 \times Z_3 \times Z_2$ for all dimension four terms. We assume that all interactions of dimension four are invariant under permutation of the three indices $a = 1,2,3$ for the fields that carry the subscript $a$ [14]. This symmetry will be softly broken by terms of dimension $\leq 3$. We assume that under left-right symmetry $\phi_0 \leftrightarrow \phi_1^0$ and $\phi_1 \leftrightarrow \phi_2^0$. The transformation of the various fields under symmetry $Z_4 \times Z_3 \times Z_2$ is given in Table I. The quark fields are assumed to be singlets under the above groups.

The Yukawa couplings invariant under the above symmetries are:

$$
\mathcal{L}_Y = h_0 \sum_a \bar{\psi}_a L \phi_0 \psi_{aR} + h_1 (\bar{\psi}_{1L} \phi_1 \psi_{2R} + \bar{\psi}_{2L} \phi_1 \psi_{3R} + \bar{\psi}_{3L} \phi_1 \psi_{1R})
+ h_1 (\bar{\psi}_{1R} \phi_0^1 \psi_{2L} + \bar{\psi}_{2R} \phi_0^1 \psi_{3L} + \bar{\psi}_{3R} \phi_0^1 \psi_{1L}) h.c.
$$

(5)
It is then clear that after the $\phi_{0,1,2}$ acquire vev’s, they will give Dirac mass to the charged leptons and the neutrinos. To get the desired pattern of charged lepton masses and the Dirac mass for the neutrinos, we choose the vev pattern for the $\phi$’s as follows: $<\phi_0> = \left( \begin{array}{c} \kappa_0 \\ 0 \\ \kappa_0' \end{array} \right)$. On the other hand, for the fields $\phi_{1,2}$, we choose $<\phi_{1,2}> = \left( \begin{array}{c} 0 \\ 0 \\ \kappa_{1,2}' \end{array} \right)$. Due to left-right symmetry, one can assure that $\kappa_1' = \kappa_2'$. It is crucial that the the vev pattern for $\phi_{1,2}$ is stable since this is what distinguishes the neutrino sector from the charged lepton sector and leads to the maximal mixing pattern $(C)$ of democratic type for the neutrinos. It is important for this that there be no tadpole terms involving the $\sigma$ fields. This is verified by making the observation that all the $\phi_\alpha$ have same $Z_4$ quantum number; as a result terms like $Tr(\phi_\alpha\phi_\alpha)$ which could generate the tadpoles are not present in the potential.

The above vev pattern has the consequence that all elements of the charged lepton mass matrix are nonzero whereas the Dirac mass matrix for the neutrinos is diagonal. To see the resulting mixing matrix, let us write the charged lepton mass matrix:

$$M_L = m_0 \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$$

(6)

where $m_0 = h_1\kappa_1'$ and $m_0a = h_0\kappa_0'$. Three eigen vectors of this mass matrix can be written as:

$$\begin{pmatrix} c \\ \mu \\ \tau \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ 2 \end{pmatrix} \text{ with } \begin{pmatrix} e^0 \\ \mu^0 \\ \tau^0 \end{pmatrix}$$

(7)

where the particles in the above equation with superscript zero denote that they are the weak eigenstates prior to the diagonalization of the mass matrix. Note that this matrix is precisely the matrix in Eq. (2). The corresponding eigenvalues are:

$$m_e = m_0(a - 1)$$

$$m_\mu = m_0(a - 1)$$

$$m_\tau = m_0(a + 2)$$

(8)

It is then easy to see that if the Majorana mass matrix for the neutrinos is diagonal, the consequent neutrino mixing matrix determined by the diagonalization of the charged lepton matrix above and is precisely of type $(C)$ described above. Note however that the muon and the electron masses are equal. In order for them to be much less than the $\tau$ mass as observed, we must have $a \simeq 1$. It is interesting to note that if instead of $S_4$ symmetry, one assumes $S_{3L} \times S_{3R}$ then indeed one ends up with $a = 1$ as has been noted already [7]. As alluded to before, we must have permutation symmetry breaking terms to split their masses. Before proceeding to that discussion, let us turn to the neutrino sector to make sure that no mixing angles emerge in the neutrino sector that could vitiate the maximal pattern. Equation (3) and the $\phi_\alpha$ vev pattern imply that the Dirac mass matrix for the three neutrinos is diagonal with all $m_{\nu_\alpha}$ given by $m_{\nu_\alpha}(\kappa_0'/?\kappa_0)$. In order to understand the small neutrino masses we must implement a seesaw mechanism. It turns out that in this case the appropriate one for us is the double seesaw mechanism discussed in [3]. From the terms involving the gauge singlet fermions $\sigma_\alpha$’s in the Lagrangian:

$$\mathcal{L}_\sigma = \Sigma_\alpha f(\bar{\psi}_a R \chi_\alpha R \sigma_a + R \rightarrow L + m_{\nu_\alpha} \sigma_a^2) + h.c.$$ 

(9)

the $(\nu_L, \nu_R, \sigma)$ mass matrix comes out to be

$$M_\nu = \begin{pmatrix} 0 & m_{\nu e} & 0 \\ m_{\nu e} & 0 & f v_R \\ 0 & f v_R & M_\sigma \end{pmatrix}$$

(10)

with $<\chi_\alpha R> = v_R$ providing the largest mass scale in the problem. Each of the entries in Eq. (7) except the $M_\sigma$ is a $3 \times 3$ unit matrix. In the limit of exact permutation symmetry, $M_\sigma$ would also be a unit matrix. The light neutrino eigenvalues are given by:

$$m_{\nu_e} \simeq \frac{m_{\nu e}^2 m_{\sigma_\alpha}}{f^2 v_R^2}$$

(11)
It is important to emphasize that there is no mixing in the purely neutrino sector so that in the basis where the charged leptons are diagonal, we have the desired maximal mixing pattern. This in our opinion is the big model building challenge that we have solved in this article. Clearly, if permutation symmetry had not been broken by the different $\sigma_a$ masses, the mixing matrix would have been arbitrary.

To get a feeling for the scale of new physics $v_R$, we note that $m_{\mu,0} \simeq (\kappa_0/\kappa_0)(m_\tau/3)$ GeV. Therefore, assuming $\kappa_0/\kappa_0 \sim 0.1$, we get $m_{\mu,0} \simeq 0.06$ GeV; and for $v_R = 10^5$ GeV and $f = 2$, we get $m_{\nu_a} \simeq 0.9 \times 10^{-4}(m_{\sigma_a}/\text{GeV})$ eV. If we choose $m_{\sigma_3} \simeq 500$ GeV and $m_{\sigma_{1,2}} \ll m_{\sigma_3}$, we get $m_{\nu_\tau} \simeq 4.5 \times 10^{-2}$ eV, which is in the range required to solve the atmospheric neutrino puzzle.

At this stage it might appear that the muon- and electron-neutrino masses can be chosen at will by adjusting the $m_{\sigma_{1,2}}$. But this is not so since the muon and electron masses which are tiny at the tree level (if we choose $a = 1$) must also arise out of the mass splitting among the $\sigma_a$’s at the one loop level. The radiative contributions to the muon and electron masses arise from the diagram of type shown in Fig.1 and we can estimate this contribution to be:

$$m^{(1)}_{\ell_a} \simeq \frac{f^2 m_{\sigma_a}^2 \mu^3 \beta v_R}{16\pi^2 \lambda (\beta v_R)^5} \tag{12}$$

where $\beta v_R$ is the typical heavy Higgs boson mass that appears in the loop. We have also used the fact the vevs of $\chi_{aL,R}$ satisfy the relation $v_{aL}v_{aR} \simeq \kappa_0 \mu_\lambda$.

![Feynman diagram](image)

**FIG. 1.** The Feynman diagram responsible for one loop radiative corrections to the muon and the electron masses. The dashed lines are the scalar bosons with appropriate quantum numbers.

Choosing $\beta \simeq 0.14$ and $\mu \simeq v_R$, $\lambda \simeq 1$ and $f \simeq 2$, we estimate

$$m^{(1)}_{\ell_a} \simeq 10^{-5} m_{\sigma_a}^2 \tag{13}$$

Note that since we need to get the entire masses for the muon and the electron from the one loop correction, we must choose $m_{\sigma_2} \simeq 100$ GeV and $m_{\sigma_1} \simeq 7$ GeV. This then implies that $m_{\nu_\tau} \simeq 9 \times 10^{-3}$ eV and $m_{\nu_e} \simeq 6 \times 10^{-4}$ eV. We thus see that $\Delta m^2_{12}$ relevant for solving the solar neutrino problem is $\simeq 8.1 \times 10^{-5}$ eV$^2$. This is comfortably in the right range for solving the solar neutrino problem using the large angle MSW solution.

**III. HIGGS POTENTIAL AND SYMMETRY BREAKINGS**

Let us now discuss the vev pattern assumed in the preceding analysis. Two points need to be discussed are (i) the specific vev pattern for the field $\phi_{1,2}$ that differentiates the neutrino Dirac mass from the charged lepton mass
matrix and (ii) the induced $\chi_{aL}$ vev. Note that due to the nontrivial transformation of the $\phi_1$ field under the $Z_3$
symmetry, the only allowed terms involving it in the potential are $Tr(\phi_1^2 \phi_1)$, $Tr(\phi_1 \phi_1 \phi_1)$, $Tr(\phi_1 \phi_0 \phi_0 \phi_0 \phi_0)$. Similar thing happens for $\phi_2$. Note further that the $Z_4$ symmetry forbids terms like $Tr\phi_1^2 \phi_2$. The absence of terms of the form $Tr\phi_1^2 \phi_2$ guarantees that once we choose the vev of the form $Diag < \phi_{1,2} > = (0, \kappa_{1,2})$, there are no tadpole like terms that can destabilize that vacuum. Finally the fact that under left-right symmetry $\phi_1 \leftrightarrow \phi_2$ guarantees there is a discrete symmetry between these two fields leading to a stable minimum with $\kappa_1' = \kappa_2'$.

Turning now to the second point, note that the potential involving the $\chi_{aL}, R$ fields has the form

$$V(\chi_{aL}, \chi_{aR}) = -M_0^2(\chi_{aL}^\dagger \chi_{aL} + \chi_{aR}^\dagger \chi_{aR}) + \lambda_+ (\chi_{aL}^\dagger \chi_{aL} + \chi_{aR}^\dagger \chi_{aR})^2 + \lambda_-(\chi_{aL}^\dagger \chi_{aL} - \chi_{aR}^\dagger \chi_{aR})^2 + \mu aL \phi_0 \chi_{aL} + h.c.$$  \hspace{1cm} (14)

where sum over $a$ has been omitted for simplicity. Minimizing this we get that $v_{aL} v_{aR} \simeq (\mu \kappa_0)/\lambda$.

A few comments about the model are in order.

(i) It is worth pointing out that in presence of the permutation symmetry breaking terms in the singlet fermion sector, there will be small deviations from the equality of the $\mu_a$'s and consequently of the scalar doublet masses. But these effects are small and they do not alter any of our conclusions.

(ii) The quark fields are assumed to be singlets under $S_3 \times Z_3 \times Z_2$. Therefore, their masses arise from the $\phi_0$ couplings only and thus are not constrained by the patterns in the lepton sector.

(iii) The lightest of the singlet fermions $\sigma_1$ which couples to electrons can be produced at LEP energies but has a cross section of order $\sigma_{ee} \sim f^4 E^2/\nu_R^2$ which at the highest LEP energies is about $\sim 10^{-44} f^4$ cm$^2$ and is thus practically invisible.

IV. CONCLUSION AND OUTLOOK

In conclusion, we have succeeded in constructing a natural gauge model for the democratic maximal mixing for neutrinos suggested by the present neutrino data if LSND results are not included. The models also predicts a small mass difference between the $\nu_e$ and $\nu_\mu$ as needed for the large angle MSW solution to the solar neutrino problem. To the best of our knowledge, this is the first time that a gauge model for understanding the democratic lepton mass matrix in an extension of the standard model has been constructed. The model is essentially an electroweak scale model with low scale for the right handed W’s and uses the double seesaw mechanism to generate small neutrino masses. Some of the new fermions of the model are light in the sense of collider physics. But their couplings to known particles are weak and thus there is no conflict with existing data.

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| Fields | $Z_4$ | $Z_3$ | $Z_2$ |
|--------|-------|-------|-------|
| $\psi_{aL}$ | 1 | $\omega^a$ | 1 |
| $\psi_{aR}$ | $-i$ | $\omega^a$ | 1 |
| $\phi_0$ | $i$ | 1 | 1 |
| $\phi_1$ | $i$ | $\omega^{-1}$ | 1 |
| $\phi_2$ | $i$ | $\omega^{-2}$ | 1 |
| $\chi_{aR}$ | $-i$ | $\omega^a$ | $(-1)^a$ |
| $\chi_{aL}$ | 1 | $\omega^a$ | $(-1)^a$ |
| $\sigma_0$ | 1 | 1 | $(-1)^a$ |

TABLE I. Transformation properties of the various fields under $Z_4 \times Z_3 \times Z_2$