Problems in suppressing cooling flows in clusters of galaxies by global heat conduction

Noam Soker

ABSTRACT

I use a simple analytical model to show that simple heat conduction models cannot significantly suppress cluster cooling flows. I build a static medium where heat conduction globally balances radiative cooling, and then perturb it. I show that a perturbation extending over a large fraction of the cooling flow region and with an amplitude of $\sim 10\%$, will grow to the non-linear regime within a Hubble time. Such perturbations are reasonable in clusters which frequently experience mergers and/or AGN activity. This result strengthens previous findings which show that a steady solution does not exist for a constant heat conduction coefficient.

Key words: galaxies: clusters: general — cooling flows — intergalactic medium — X-rays: galaxies: clusters

1. Introduction

Recent Chandra and XMM-Newton observations resolved the inner structure of the cooling flow region in clusters of galaxies. Traditionally, this region is defined as the region where radiative cooling time is shorter than the age of the cluster; not all clusters possess cooling flows (e.g., Fabian 1994). While hot, $T \simeq 3 - 8 \times 10^7$ K, X-ray emitting gas seems to cool at rates of $10 - 500 M_\odot$ yr$^{-1}$, there are almost no indications for gas cooling below a temperature of $\sim 10^7$ K (e.g., Kaastra et al. 2001; Peterson et al. 2001, 2002; Fabian 2002). This renewed interest in an old idea (e.g., Bregman & David 1988), that heat conduction from the intra-cluster medium (ICM) outside the cooling flow region supplies the energy carried by radiation (e.g., Narayan & Medvedev 2001; Voigt et al. 2002; Fabian, Voigt, & Morris 2002; Ruszkowski & Begelman 2002). Bregman & David (1988) show that the heat

\footnote{Department of Physics, Oranim, Tivon 36006, Israel; soker@physics.technion.ac.il.}
conduction needs a fine tuning in order to explain the temperature and density profiles in cooling flows, and that a solution with a constant heat conduction coefficient is unstable, i.e., no steady solution exist. Norman & Meiksin (1996) try to increase the heat conduction efficiency by allowing the cooling flow to stretch magnetic flux loops, which then reconnect; cooling flow is not completely suppressed, but the cooling rate is substantially reduced in their model. Other works argue, from different reasoning, for a heat conduction coefficient \( \kappa \) much bellow the Spitzer value for unmagnetized plasma, such that heat conduction does not play a global role in clusters and cannot prevent cooling flows (e.g., Pistinier & Shaviv 1996; Markevitch, Vikhlinin, & Forman 2002; Loeb 2002).

In a recent paper Zakamska & Narayen (2003; hereafter ZN03) fit five cooling flow clusters with a simple hydrostatic model where heat conduction balances radiative cooling. The thermal conductivity is \( \sim 30\% \) the Spitzer conductivity. They argue that this model is stable against local transverse perturbations. For five cooling flow clusters they could not fit such a model, as they require too high conductivity. In this paper I show that such a solution is unstable to large large-scale radial perturbation with amplitudes \( \gtrsim 10\% \) in the temperature, over the age of a cluster. Hence, either the heat conduction does not play a role, or the cooling flow is much younger than the cluster, as proposed in the moderate cooling flow model (Soker et al. 2001). In either cases, heat conduction does not help in explaining the large scale behavior in cooling flow clusters, although it may be important locally, i.e., conduction to cold clouds in the inner region. In \( \S \)2 I build a simple hydrostatic model with a power-law ICM. In \( \S \)3 I use this model to argue that the solution is not stable over a long time, hence strengthening the results of Bregman & David (1988). In light of the renewed interest in heat conduction, (sometimes ignoring previous results), I aim at reaching this conclusion via a simple model. My short summary is in \( \S \)4.

2. A Simple Steady State Hydrostatic Solution

To show that a solution where conduction balances radiative cooling is unstable, I start by building a simple spherically symmetric model with a power-law medium

\[
P = P_0 r^{-\eta}; \quad T = T_0 r^\beta; \quad n = n_0 r^{-\eta-\beta},
\]

(1)

where \( P, T, \) and \( n \) are the thermal pressure, temperature, and electron density, respectively, and \( r \) is a dimensionless radial coordinate. The profiles used in equation (1) imply that the treatment is correct for \( r > 0 \), or more practically for \( r \gtrsim 0.1 \). This does not change the conclusions of the present paper, because the instability found here and most of the radiative energy loss occur at much larger distances from the center. Like ZN03 I take the cooling
rate per unit volume and the heat flux to be \( j = K_1 n^2 T^{1/2} \), and \( F = -f K_2 T^{5/2} dT/dr \), respectively, where \( K_1 \) and \( K_2 \) are constants, and \( f \) is the ratio of the conductivity to the Spitzer value. The total radiated power, a free-free emission, inside a radius \( r \) is given by

\[
L_{\text{radS}} = \int_0^r K_1 n^2 T^{1/2} 4\pi x^2 dx = 8\pi K_1 n_0^2 T_0^{1/2} (6 - 4\eta - 3\beta)^{-1} r^{3 - 2\eta - \frac{3}{2}\beta},
\]  

(2)

where subscript ‘S’ stands for the steady state solution. Most of the energy is radiated just inward to \( r \), such that the detailed structure close to the center is not important. The rate of inward heat flow across a spherical surface at \( r \) is

\[
L_{\text{heatS}} = -4\pi r^2 F = 4\pi f K_2 \beta T_0^{7/2} r^{2\beta + 1}.
\]  

(3)

In a steady state hydrostatic model \( L_{\text{heatS}} = L_{\text{radS}} \) for each radius, which implies two equalities: the equality of the powers of \( r \) in equations (2) and (3) and the equality of the constant terms in these two equations. The first equality gives

\[
\eta = 1 - 2.5\beta,
\]  

(4)

and the second equality gives

\[
f K_2 \beta T_0^3 = 2K_1 n_0^2 (6 - 4\eta - 3\beta)^{-1}.
\]  

(5)

From the solutions of ZN03, I find \( \eta \sim 0.3 \). Equation (5) implies that when \( f \) is changed by a factor \( \alpha \), this may result in a modest change in the temperature and a change in the density by a factor of \( \sim \alpha^{1/2} \); this is compatible with the finding of ZN03.

3. Perturbing the Temperature Profile

I take a simple form for the large scale temperature radial perturbation, \( \Delta T = T_\delta(r-1) \), such that the new temperature profile is

\[
T' = T + \Delta T = T_0 r^{\beta} + T_\delta(r-1),
\]  

(6)

where \( T \) is given in equation (1), and the derivative of the temperature is (for \( r > 0 \))

\[
\frac{dT'}{dr} = \beta T_0 r^{\beta-1} + T_\delta.
\]  

(7)

For simplicity I assume that the pressure profile does not change, hence the new density profile is

\[
n' = n(T/T').
\]  

(8)
A full accurate treatment should consider the perturbation in pressure as well. The pressure perturbation is neglected here for the following reasons. (1) As stated in Section 1, the aim of the present paper is to present a simple treatment in order to clearly demonstrate the global radiative instability. (2) This assumption is accurate for a case of constant pressure. Although the pressure in clusters is not constant, it is still quite shallow. As derived below, the instability which occurs for a constant pressure is not much different from the one for a shallow pressure profile. (3) Although being simple, the result of the present treatment is in accord with the numerical results of Bregman & David (1988). This makes the treatment trustworthy.

Only the first order in $T_\delta$ is retained. For example,

$$
(T')^{5/2} = [T_0 r^\beta + T_\delta (r - 1)]^{5/2} = T_0^{5/2} r^{5\beta/2} \left[ 1 + \frac{5}{2} \frac{T_\delta}{T_0^2} r^{-\beta} (r - 1) \right] + O(T_\delta^2). \tag{9}
$$

Keeping only the first order is not justified very close to the center where the unperturbed temperature is very low. As noted earlier, this does not affect the conclusions here, because the instability occurs farther out in the cluster, and because in real clusters the very central region is expected to be dominated by the AGN activity anyhow. Substituting equations (7) and (9) in the expression for the total rate of inward heat flow across a spherical surface of radius $r$, similar to equation (3) for the steady state heat flow $L_{\text{heatS}}$, gives to first order in $T_\delta$

$$
L_{\text{heat}} = L_{\text{heatS}} \left[ 1 + \frac{T_\delta}{T_0} r^\beta \left( \frac{1}{\beta} r + \frac{5}{2} r - \frac{5}{2} \right) \right]. \tag{10}
$$

Expanding $(n')^2 (T')^{1/2}$ to first order in $T_\delta$, and integrating, similar to equation (2) for the steady state cooling rate $L_{\text{radS}}$, and using equation (4) for $\eta$, gives

$$
L_{\text{rad}} = L_{\text{radS}} \left[ 1 + \frac{3}{2} \frac{T_\delta}{T_0 r^\beta} \left( \frac{2 + 7\beta}{2 + 5\beta} - \frac{2 + 7\beta}{4 + 5\beta} r \right) \right]. \tag{11}
$$

The net cooling rate of the region inward to $r$ is given by $L_{\text{rad}} - L_{\text{heat}}$, which result in a cooling time $\tau_{\text{cool}}(r)$. Let $\tau_{\text{rad}}(r)$ be the cooling time calculated when only the radiative cooling $L_{\text{radS}}$ is considered, i.e., the “standard” cooling time at radius $r$. (The cooling time calculated at radius $r$, is not exactly the same as the average cooling time of the gas inward to $r$. However, it was noticed above that most of the radiative cooling rate inward to $r$ occurs close to $r$. I therefore neglect the difference between these two values.) The ratio between $\tau_{\text{rad}}(r)$ and $\tau_{\text{cool}}(r)$ is given by

$$
\frac{\tau_{\text{rad}}(r)}{\tau_{\text{cool}}(r)} = \frac{L_{\text{rad}} - L_{\text{heat}}}{L_{\text{radS}}}. \tag{12}
$$
Substituting for $L_{\text{heat}}$ and $L_{\text{rad}}$ from equations (10) and (11), respectively, and using $L_{\text{heat}}S = L_{\text{rad}}S$, gives to first order in $T_\delta$

$$\frac{\tau_{\text{rad}}(r)}{\tau_{\text{cool}}(r)} = \frac{T_\delta}{T(r)} \left( \frac{6 + 21\beta}{4 + 10\beta} + \frac{5}{2} - \frac{6 + 21\beta}{8 + 10\beta} r - \frac{5}{\beta} + \frac{5}{2} r \right).$$  \hspace{1cm} (13)

As mentioned above $\beta \sim 0.3$. Equation (13) gives then

$$\frac{\tau_{\text{rad}}(r)}{\tau_{\text{cool}}(r)} = \frac{T_\delta}{T(r)} (4.2 - 8.52r) = \frac{\Delta T(r)}{T(r)} \left( \frac{4.2 - 8.52r}{r - 1} \right), \hspace{1cm} \text{for} \hspace{0.5cm} \beta = 0.2, \hspace{1cm} (14)$$

and

$$\frac{\tau_{\text{rad}}(r)}{\tau_{\text{cool}}(r)} = \frac{T_\delta}{T(r)} (4.3 - 6.2r) = \frac{\Delta T(r)}{T(r)} \left( \frac{4.3 - 6.2r}{r - 1} \right), \hspace{1cm} \text{for} \hspace{0.5cm} \beta = 0.4. \hspace{1cm} (15)$$

Note that $\beta = 0.4$ is the case of constant pressure. Although this is not realistic, in the assumption made here of neglecting the pressure variation, the treatment of this case, where there is no pressure gradient, is accurate. The last two equations show that there are no fundamental differences between the instabilities in the case of constant pressure and the case of a shallow pressure gradient appropriate for clusters.

Let us examine the nature of the instability. In a case of no radiative cooling, or a slow radiative cooling, the restoring mechanism of a temperature perturbation is the heat conduction itself. Namely, when a region gets cooler the temperature gradient gets steeper, the energy flow via heat conduction from warmer to cooler regions increases, restoring the cooling region back to the equilibrium temperature. What was shown above is that under the conditions which exist in cooling flow clusters, when a large inner regions gets cooler, hence denser, the energy lose rate via radiative cooling, i.e., the radiative power, increases more than the heating power via the heat conduction does. For local perturbations, the short scale means steep temperature gradient, hence efficient heat conduction. This is why ZN03 find their perturbation to be stable. I instead consider large scale perturbations, where radiative energy lose occurs in a large region, as appropriate for the inner cooling flow region. Such large perturbations are reasonable in the violent environment near the cD galaxy in cooling flow clusters. The time evolution of the departure from the initial perturbative state occurs on the cooling time scale or longer (since some heat conduction may exist). Therefore, it is important (during the cluster age) only in the inner regions of cooling flow clusters (and only in cooling flow cluster) where cooling time is very short.

For an instability to develop, the expression inside the parenthesis in the far right-hand side of the last two equations should be negative. This occurs for $r > 1$. However, for a large scale perturbation this occurs at relatively large radii, where the standard cooling time $\tau_{\text{rad}}$ is long. There is also an instability at much smaller radii. For these, the perturbation, as
chosen here, must be a large scale perturbation: It starts at \( r = 1 \) (eq. 6), and the instability occurs only for \( r < r_i = 4.2/8.52 = 0.5 \) for \( \beta = 0.2 \), and \( r < r_i = 4.3/6.2 = 0.7 \) for \( \beta = 0.4 \). I consider the case \( \beta = 0.2 \) (the results are not sensitive to the value of \( \beta \)), at \( r = 0.25 \). For example, the perturbation starts at \( r = 1 \), which in real units I take at \( R = 60 \) kpc, and I consider the region around \( r \approx 0.25 \), or in real units \( R \approx 10 - 20 \) kpc. The “standard” cooling time in this region in cooling flow clusters is \( \tau_{\text{rad}}(R = 10 - 20 \text{ kpc}) \approx 2 - 5 \times 10^8 \text{ yr} \). Hence the cooling time, scaled with a perturbation of \( \sim 10\% \) is

\[
\tau_{\text{cool}}(15 \text{ kpc}) \approx 2 \times 10^9 \left( \tau_{\text{rad}}(15 \text{ kpc}) \right) \left( \frac{\Delta T(15 \text{ kpc})}{0.17(15 \text{ kpc})} \right) \text{ yr}, \tag{16}
\]

for a perturbation given by equation (6) with \( r = 1 \) at \( R \approx 40 - 80 \) kpc. For these parameters, the perturbation will grow to the nonlinear regime during the age of the cluster, \( \sim 10^{10} \) yr. Therefore, a model with no cooling flow requires, either that (1) the cooling flow is much younger than the cluster, e.g., there is an episodic heating of the cooling flow region on a time scale of \( \sim 10^9 \) yr (e.g., Soker 2001), or (2) there is another continuous energy source, e.g., an AGN at the center of the central cD galaxy (Ruszkowski & Begelman 2002). In either cases, heat conduction by itself cannot “kill” the cooling flow.

The following should be noted. The instability occurs for \( r < r_i \), whereas the region \( r_i < r < 1 \) is stable. However, this stable region cannot stabilize the unstable region \( r < r_i \). This is because the instability evolves on a cooling time scale, which is much shorter in the inner unstable region than in the stable region. The non-existence of a steady state solution was found by Bregman & David (1988) in their numerical calculations.

4. Summary

New results by the Chandra and XMM-Newton X-ray telescopes, which indicate a very low mass cooling rate at temperatures below \( \sim 1 \) kev (see review by Fabian 2002), renewed interest in heat conduction from the outer ICM as an energy source to compensate for radiative cooling at lower temperatures. The goal of the present paper is to use a simple analytical model to show that (simple) heat conduction models cannot significantly suppress cluster cooling flows. I used a power-law ICM and built a static medium where heat conduction globally balances radiative cooling, and then perturbed it. I showed that a large scale temperature perturbation with an amplitude of \( \sim 10\% \), will grow to the non-linear regime within a Hubble time. The perturbation should extend over a large fraction of the cooling flow region. Such perturbations are reasonable in clusters which frequently experience mergers and/or AGN activity, e.g., X-ray deficient bubbles (McNamara 2002). This result strengthens the finding of Bregman & David (1988), who use numerical calculations to show
that a steady solution does not exist for a constant heat conduction coefficient.

This instability over a time scale of $\sim 10^9$ yr, implies that another energy source should either make the cooling flow a short-lived episodic process, or that it continuously supplies energy. In either cases the heat conduction does not seem to play a significant role; the major role is played by the other energy source(s). A popular energy source is AGN activity, which was proposed to reduce the mass cooling rate in cooling flows in galaxies (e.g., Binney & Tabor 1995; Ciotti & Ostriker 1997, 2001; Jones et al. 2002), and in clusters of galaxies (e.g., Soker et al. 2001; Churazov et al. 2002)

To the instability found in the present paper, I add the failure of ZN03 to build a steady model with heat conduction to five clusters, the claim made by Sun et al. (2003) that generally heat conduction can’t compensate for radiative cooling in cluster cooling flows, the indications that heat conduction is suppressed at other regions of clusters (Loeb 2002; Markevitch et al. 2002; Nath 2003), and the required fine tuning argued by Bregman & David (1988), and conclude that heat conduction cannot explain the properties of cooling flow clusters.

I thank the referee and Larry David for useful comments. This research was supported in part by grants from the US-Israel Binational Science Foundation and the Israel Science Foundation.

REFERENCES

Binney, J., & Tabor, G. 1995, MNRAS, 276, 663
Bregman, J. N., & David, L. P. 1988, ApJ, 326, 639
Churazov, E., Sunyaev, R., Forman, W., & & Böhringer, H. 2002, MNRAS, 332, 729
Ciotti, L., & Ostriker, J. P. 1997, ApJ, 487, L105
Ciotti, L., & Ostriker, J. P. 2001, ApJ, 551, 131
Fabian, A. C. 1994, ARA&A, 32, 277
Fabian, A. C. 2002, in Galaxy Evolution: Theory and Observations, eds. V. Avila-Reese, C. Firmani, C. Frenk, & C. Allen, RevMexAA SC, in press (astro-ph/0210150)
Fabian, A. C., Voigt, L. M., & Morris, R. G. 2002, MNRAS, 335, L71
Jones, C., Forman, W., Vikhlinin, A., Markevitch, M., David, L., Warmflash, A., Murray, S., & Nulsen, P. E. J. 2002, ApJ, 567, L115
Kaastra, J. S., Ferrigno, C., Tamura, T., Paerels, F. B. S., Peterson, J. R., & Mittaz, J. P. D 2001, A&A, 365, L99
Loeb, A. 2002, NewA, 7, 279
Markevitch, M., Vikhlinin, A., Forman, W. R. 2002, in Matter and Energy in Clusters of Galaxies”, to appear in ASP Conference Series (astro-ph/0208208)
McNamara, B. R. 2002, in The High-Energy Universe at Sharp Focus: Chandra Science, Symposium at the ASP meeting, Vol. 262, Eds. Eric M. Schlegel and Saeqa Dil Vrtilek (San Francisco: ASP), 351
Narayan, R., & Medvedev, M. V. 2001, ApJ, 562, L129
Nath, B. B. 2003, MNRAS, in press (astro-ph/0302072)
Norman, C., & Meiksin, A. 1996, ApJ, 468, 97
Peterson, J. R., Ferrigno, C., Kaastra, J. S., Paerels, F. B. S., Kahn, S. M., Jernigan, J. G., Bleeker, J. A. M., & Tamura, T. 2002, in New Visions of the X-ray Universe in the XMM-Newton and Chandra Era, in press (astro-ph/0202108)
Peterson, J. R., et al. 2001, A&A, 365, L104
Pistinner, S., & Shaviv, G. 1996, ApJ, 459, 147
Ruszkowski, M., & Begelman, M. C. 2002, ApJ, 581, 223
Soker, N., White, R. E. III, David, L. P., & McNamara, B. R. 2001, ApJ, 549, 832
Sun, M., Forman, W., Jones, C. & Murray, S. S. 2003, AAS, 201.301
Voigt, L. M., Schmidt, R. W., Fabian, A. C., Allen, S. W., & Johnstone, R. M. 2022, MNRAS, 335, L7
Zakamska, N. L., & Narayan, R. 2003, ApJ, 582, 162 (ZN03)

This preprint was prepared with the AAS LATEX macros v5.0.