A lattice determination of $g_A$ and $\langle x \rangle$ from overlap fermions

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We present results for the nucleon’s axial charge $g_A$ and the first moment $\langle x \rangle$ of the unpolarized parton distribution function from a simulation of quenched overlap fermions.

1. INTRODUCTION

The axial charge $g_A$ of the nucleon describes the beta decay of the neutron. It can be defined by

$$2\gamma_{\mu}g_A = \langle p(\vec{p}), \vec{s} | (A^{(u)}_\mu - A^{(d)}_\mu)|p(\vec{p}), \vec{s} \rangle \tag{1}$$

where $\vec{s}$ is the spin vector of the proton and $A^{(q)}_\mu = \bar{q}\gamma_\mu\gamma_5q$. Thus, $g_A$ can be computed on the lattice from a flavour non-singlet proton matrix element. There have been lattice determinations of $g_A$ in the past; most of them used Wilson-like fermions, which makes it hard to go to small quark masses. For recent results from Wilson fermions see e. g. [1].

For the first moment of the unpolarized parton distribution $\langle x \rangle$, which measures the fractional momentum carried by the quarks, the comparison of lattice and phenomenological data has been problematic [2]. To settle the problem, one needs to do simulations at smaller quark masses.

Here, we present results for these two quantities obtained with quenched overlap fermions.

2. SIMULATION DETAILS

We use the overlap operator given by

$$D = \rho(1 + \frac{m_q}{2\rho} + (1 - \frac{m_q}{2\rho})\gamma_5 \text{sgn}(H_W(-\rho))) \tag{2}$$

where $H_W(-\rho) = \gamma_5(D_W - \rho)$, $D_W$ being the Wilson Dirac operator. We approximate the sign function appearing in [2] by minmax polynomials [3]. For the gauge part we chose the Lüscher-Weisz action [4]

$$S[U] = \frac{6}{g^2} \left\{ c_0 \sum_{\text{plaquette}} \frac{1}{3} \text{Re Tr} [1 - U_{\text{plaquette}}] \right\}$$
Figure 1. Comparison between $Z_V$ obtained from the nucleon matrix element (eq. (5)) and $Z_A$ obtained from the Ward identity (eq. (6)).

$$+ c_1 \sum_{\text{rectangle}} \frac{1}{3} \text{Re Tr} [1 - U_{\text{rectangle}}]$$

$$+ c_2 \sum_{\text{parallelogram}} \frac{1}{3} \text{Re Tr} [1 - U_{\text{parallelogram}}]$$

with coefficients $c_1$, $c_2$ ($c_0 + 8c_1 + 8c_2 = 1$) taken from tadpole improved perturbation theory [5]. We ran our computations at $\beta = 8.45$, corresponding to $a = 0.095\text{fm}$, at two volumes, $V_1 = 16^332$, $V_2 = 24^348$, the physical volumes being $(1.5\text{fm})^3$ and $(2.3\text{fm})^3$, respectively. The parameter $\rho$ was set to 1.4. Our quark masses are $m_q = 0.028, 0.056, 0.084$ and 0.14, which corresponds to pion masses ranging from $440 - 950\text{MeV}$. On the $24^348$ lattice we have simulated two more masses down to $m_\pi \approx 300\text{MeV}$, but with our current statistics the errors on these data points are too large to draw any conclusions from them.

In order to remove $O(a)$ errors from the three-point functions, we employ the method of [6], which amounts to replacing propagators $D^{-1}\Psi$ by $\frac{1}{1 + 2c_0} D^{-1}\Psi - \frac{1}{2c_0(1+2c_0)} U_{\text{rectangle}} \Psi$. Jacobi smeared point sources [7] with parameters $N_s = 50$ and $\kappa_s = 0.21$ have been used in order to obtain a good overlap with the ground state.

The computation of matrix elements follows the procedure outlined in [8]: We form the ratio

$$R = \frac{\langle N(t_{\text{sink}})O(t)\bar{N}(t_{\text{source}}) \rangle}{\langle N(t_{\text{sink}})\bar{N}(t_{\text{source}}) \rangle}$$

from which the matrix element can be extracted in the region $t_{\text{source}} < \tau < t_{\text{sink}}$. We always set $t_{\text{source}} = 0$ and $t_{\text{sink}} = 13$ (in lattice units), which corresponds to a distance between source and sink of 1.2fm. So far we have analysed 250 (40) configurations on the $16^332$ ($24^348$) lattice.

3. RENORMALIZATION

The operators appearing inside the three-point functions have to be renormalized. For $g_A$, the operator to be used is the axial current $A_\mu$, the renormalization of which is particularly simple because it can be obtained from a Ward identity [8] as

$$Z_A = \lim_{t \rightarrow \infty} \frac{2m_q}{m_\pi} \frac{\langle P(t)P(0) \rangle}{\langle A_\mu(t)P(0) \rangle}$$

For overlap fermions, $Z_A$ should agree with $Z_V$ in the chiral limit. We can compute $Z_V$, making use of current conservation, from the nucleon matrix element:

$$Z_V \langle N|V_\mu|N \rangle = 1$$

The comparison of $Z_A$ obtained from eq. (5) to $Z_V$ obtained from eq. (6) is shown in fig. 1; extrapolating linearly to the chiral limit, we obtain $Z_A = 1.416(20)$, $Z_V = 1.426(7)$. 
The nucleon axial charge $g_A$ as a function of the quark mass.

The renormalization of the operators used for the first moment of the structure function is more difficult; at present, we do not have a nonperturbative calculation, so we use the value obtained in tadpole-improved perturbation theory $Z \nu_2 = 1.4566$.

4. RESULTS

In fig. 2 we show the ratio of three-point function to two-point function for the case of $g_A$. One can observe a reasonable plateau in the region $t_{\text{source}} < \tau < t_{\text{sink}}$.

We show our results for $g_A$ in fig. 3. A linear extrapolation to the chiral limit yields $g_A = 1.05(5)$ for $V_1 = (1.5 \text{fm})^3$ and $g_A = 1.13(5)$ for $V_2 = (2.3 \text{fm})^3$, which is somewhat lower than the experimental value $g_A = 1.27$.

The result for $\langle x \rangle^{u-d}$ is plotted in fig. 4. There appears to be almost no dependence on the quark mass or on the lattice volume. In the chiral limit, we obtain $\langle x \rangle^{\overline{MS}}(2 \text{GeV}) = 0.20(2)$, which is closer to the phenomenological value than previous results obtained with Wilson-like fermions. One must bear in mind however that we are still lacking a nonperturbative value for the renormalization constant.

5. CONCLUSIONS

We have determined $g_A$ and $\langle x \rangle^{u-d}$ from quenched overlap fermions. For $g_A$, our results are in agreement with most previous determinations: there is some dependence on the lattice volume; on our larger volume ($V = (2.3 \text{fm})^2$ we find $g_A = 1.13(5)$.

For $\langle x \rangle^{u-d}$ we see only a weak dependence on volume and quark mass; the result is considerably closer to the phenomenological value than most previous lattice results; however, a large systematic uncertainty remains as long as we do not have a nonperturbative value for the renormalization constant.

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