Virtual photon scattering in $e^+e^-$ collisions can result in events with the electron-positron pair at large rapidity separation with hadronic activity in between. The BFKL equation resums large logarithms that dominate the cross section for this process. We report here on a Monte Carlo method for solving the BFKL equation that allows kinematic constraints to be taken into account. The application to $e^+e^-$ collisions is in progress.

1 Introduction

High energy $e^+e^-$ collisions can lead to the scattering of virtual photons emitted by the initial electron and positron. When the virtuality $Q^2$ of these photons is small compared to the center-of-mass energy $\hat{s}$ of the $\gamma^*\gamma^*$ system, the scattering cross section is dominated by contributions in which the photons split into quark-antiquark pairs, with t-channel gluon exchange. The emission of additional soft gluons from the t-channel gluon gives rise to large logarithms that lead to corrections in powers of

$$\alpha_s(Q^2) \ln(\hat{s}/Q^2),$$

which is of order one in this kinematic regime. These logarithms must therefore be resummed in the calculation of the cross section. The events that result from this process are characterized by electron-positron pairs with a large rapidity separation, and hadronic activity in between.

The large-logarithm resummation is performed by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation, where its analytic solution gives a rise in the cross section $\hat{\sigma} \sim (\hat{s})^\lambda$, with $\lambda = 4C_A \ln 2 \alpha_s/\pi \approx 0.5$. The BFKL equation applies not only to virtual photon scattering as described above, but also to dijet production at large rapidity difference in hadron-hadron collisions and to forward jet production in lepton-hadron collisions.

The BFKL equation can be solved analytically, but to do so requires giving up energy-momentum conservation, because it involves integration over arbitrarily large momenta of emitted gluons. Furthermore, because the sum

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over gluons is implicit, only leading-order kinematics can be included. This leads to predictions that do not correspond to any real experimental situation. In principle the corrections due to kinematic effects are in subleading, but in practice, as we will see below, they can be quite important.

2 A Monte Carlo for BFKL Physics

The solution to the problem of lack of kinematic constraints in analytic BFKL predictions is to unfold the implicit gluon sum to make it explicit, and to implement the result in a Monte Carlo event generator. This is achieved as follows. The BFKL equation contains separate integrals over real and virtual emitted gluons. We combine the ‘unresolved’ real emissions — those with transverse momenta below some minimum value (small compared to the momentum threshold for measured jets) — with the virtual emissions. Schematically, we have

\[
\int_{\text{virtual}} + \int_{\text{real}} = \int_{\text{virtual}+\text{real,unres.}} + \int_{\text{real, res.}}
\]  

We perform the integration over virtual and unresolved real emissions analytically.

We then solve the BFKL equation by iteration, and we obtain a differential cross section that contains an explicit sum over emitted gluons along with the appropriate phase space factors. In addition, we obtain an overall form factor due to virtual and unresolved emissions. The subprocess cross section is

\[
d\hat{\sigma} = d\hat{\sigma}_0 \times \sum_{n \geq 0} f_n
\]

where \( f_n \) is the iterated solution for \( n \) real gluons emitted and contains the overall form factor. It is then straightforward to implement the result in a Monte Carlo event generator. Emitted real (resolved) gluons appear explicitly, so that conservation of momentum and energy is based on exact kinematics for each event. In addition, we include the running of the strong coupling constant. See [2] for further details.

3 Results and Prospects

We have used this BFKL Monte Carlo approach to study dijet production at hadron colliders in detail. The most important conclusion is that the effects of kinematic constraints can be very large, because they suppress radiation of the gluons that give rise to what are considered to be characteristic
BFKL effects. As a result, the predictions can change substantially. This is illustrated in Fig. which shows the dijet cross section at the Tevatron for two different center-of-mass energies as a function of the dijet rapidity difference. The naive analytic BFKL prediction lies above the leading QCD curve, as expected. But when kinematic constraints are included, the BFKL prediction gets pushed below that of leading-order QCD. Clearly it is important to incorporate kinematic constraints in our BFKL predictions.

We are currently completing the application of our BFKL Monte Carlo to virtual photon scattering in $e^+e^-$ collisions and in forward jet production at HERA. In both cases we expect kinematic constraints to be large and to lead to some suppression of BFKL effects.

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Figure 1: The dependence on the dijet rapidity separation of the BFKL and asymptotic QCD leading-order dijet cross sections at the Tevatron. The three curves at each collider energy use: (i) 'improved' BFKL MC (solid lines), (ii) 'naive' BFKL (dashed lines), and (iii) the asymptotic ($\Delta \gg 1$) form of QCD leading order (dotted lines).