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Asymmetries in the spectral density of an interaction-quenched Luttinger liquid

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Abstract. The spectral density of an interaction-quenched one-dimensional system is investigated. Both direct and inverse quench protocols are considered and it is found that the former leads to stronger effects on the spectral density with respect to the latter. Such asymmetry is directly reflected on transport properties of the system, namely the charge and energy current flowing to the system from a tunnel coupled biased probe. In particular, the injection of particles from the probe to the right-moving channel of the system is considered. The resulting fractionalization phenomena are strongly affected by the quench protocol and display asymmetries in the case of direct and inverse quench. Transport properties therefore emerge as natural probes for the observation of this quench-induced behavior.

1. Introduction
Isolated one-dimensional (1D) systems are an ideal playground where both non-equilibrium phenomena and interaction effects can be explored [1, 2, 3, 4]. Many of them fall in the class of integrable systems, in which the presence of an infinite number of local conserved quantities lead to peculiar relaxation properties after they have been driven far from equilibrium. Indeed, it has been shown that integrable systems reach a steady state that is locally described by a non-thermal density matrix, which can be obtained within the framework of the generalized Gibbs ensemble (GGE), retaining strong memory of initial conditions [5]. To drive the system out-of-equilibrium and study its dynamics, it is possible to perform a quantum quench [6]. Here, the system is initially prepared in the ground state of a certain Hamiltonian and then some of its parameters are changed in time. Such a procedure can be implemented i.e. in state-of-the-art setups of cold atoms, where the system parameters can be varied with high precision and in a time-dependent way [7, 8, 9].

It is well established that the equilibrium low energy sector of interacting 1D systems is described by the Luttinger Liquid (LL) model [10]. Already in equilibrium, LLs exhibit peculiar spectral properties, with characteristic power-law behavior, stemming from their non-Fermi liquid nature [11, 12, 13, 14, 15, 16, 17, 18]. In addition, interacting 1D systems exhibit charge and energy fractionalization [16, 19, 20, 21, 22], i.e. the split-up of a particle injected into an interacting 1D system in two counter-propagating excitations which carry a fraction...
of the original charge and energy of the particle. In the last decade, the LL model has also been successfully employed to study the dynamics of 1D system after a quench of the inter-particle interaction [23, 24]. In particular, many efforts have been devoted to investigate how the unusual LL features are modified in the presence of such an out-of-equilibrium situation [25, 26, 27, 28, 29, 30, 31, 32], usually focusing on the case of a “direct” quench. This consists in a quench protocol which brings the system towards a state with stronger interactions.

In this paper we instead compare the direct quench scenario with the case of an “inverse” quench, which reduces the interaction strength. In particular, we analyze the spectral density [31] of a quenched 1D system and show that, given two interaction strengths, the direct quench always leads to a larger spectral density with respect to the inverse protocol. We argue that this strong asymmetry between the two cases is directly reflected also on transport properties. In particular, we consider a non-interacting biased 1D probe, whose left-moving particles are coupled to the right moving channel of the system through a non-local tunneling region, and we focus on the charge and energy differential conductance.

The paper is organized as follows: in Sec. 2 the model of the system, the quenched spectral density and the charge and energy differential conductances are introduced. In Sec. 3 the effect of a direct and inverse quench on the spectral density of the system is explored and its influence on the transport properties is outlined. Section 4 contains our conclusions.

2. Model and methods
We consider a 1D system of interacting fermions with two counter-propagating channels, labeled by the index \( r = R, L \). Its Hamiltonian is \( H_s(t) = H_s^{(0)} + H_s^{(1)}(t) \), where \( \hbar = 1 \)

\[
H_s^{(0)} = v_F \sum_{r=R,L} \theta_r \int_{-\infty}^{\infty} dx \, \psi_r^\dagger(x)(-i\partial_x)\psi_r(x),
\]

is the free kinetic part, with \( v_F \) the Fermi velocity, \( \psi_r(x) \) the Fermi field operator, and \( \theta_{R/L} = \pm 1 \). The interaction term is

\[
H_s^{(1)}(t) = \frac{g_{24}(t)}{2} \sum_{r=R,L} \int_{-\infty}^{\infty} dx \left[ n_r(x) \right]^2 + g_{24}(t) \int_{-\infty}^{\infty} dx \, n_R(x)n_L(x),
\]

with \( n_r(x) = :\psi_r^\dagger(x)\psi_r(x) : \) the electron density of the \( r \)-th channel and \( g_{24}(t) \) the intra/inter-channel interaction parameters [10]. Due to a sudden quench of the interactions [24, 29, 31], the latter parameters rapidly switch in time according to

\[
g_{24}(t) = g_{24}^{(i)}(t)\vartheta(-t) + g_{24}^{(f)}(t)\vartheta(t),
\]

with \( \vartheta(x) \) the step function and the indices \( i/f \) referring to situation before and after the quench. Standard bosonization procedure is applied [10], with the Fermi operator of the system expressed in terms of free bosonic fields \( \phi_r(x) \) as

\[
\psi_r^\dagger(x) = \frac{1}{\sqrt{2\pi a}} e^{i\sqrt{2\pi} \phi_r(x)} e^{-i\theta_r k_F x},
\]

with \( a \propto k_F^{-1} \) a short length cutoff. The bosonized Hamiltonian density of the system reads

\[
\mathcal{H}_s^{(\nu)}(x, t) = \frac{u_{\nu}}{2} \sum_{\eta=\pm} \left[ \partial_x \phi_{\nu,\eta}(x - \eta u_{\nu} t) \right]^2
\]
with \( \nu = i \) \((f)\) for \( t < 0 \) \((t > 0)\). Note that the fields \( \phi_{\nu,\eta}(x) \) are chiral, with \( \eta \) denoting their propagation direction and \( u_\nu = (2\pi)^{-1} \sqrt{2\pi v_F + g_4^{(\nu)}} \) \((g_2^{(\nu)})\) their propagation velocity. For \( t > 0 \) one has the relation

\[
\phi_t(x) = \sum_{\eta=\pm} A_{\eta,\nu} \phi_{\nu,\eta}(x) \quad A_{\pm} = \frac{1}{2} \left( \frac{1}{\sqrt{K_i}} \pm \sqrt{K_f} \right),
\]

with \( K_f = \sqrt{\frac{2\pi v_F + g_4^{(\nu)}}{2\pi v_F + g_4^{(\nu)} + g_2^{(\nu)}}} \). The boson operators before quench \( \phi_{i,\eta}(x) \) are different, due to the different interaction strength \( K_i \), and connected to the post-quench operators via \( \phi_{f,\eta}(x) = \sum_{\xi=\pm} \alpha_{\eta,\xi} \phi_{i,\xi}(x) \), with

\[
\alpha_{\pm} = \frac{1}{2} \left( \frac{K_i}{K_f} \pm \sqrt{\frac{K_i}{K_f}} \right).
\]

We focus on the stationary regime reached long after the quench, whose properties are captured by the non-equilibrium quenched spectral function \([27, 31]\) of the \( R \) channel

\[
A(k, \epsilon) = \int_{-\infty}^{\infty} dx d\tau \ e^{-i(kx - \epsilon \tau)} \lim_{t\to\infty} A(x, \tau, t),
\]

with

\[
A(x, \tau, t) = \{ \{ \psi_R(x, t + \tau), \psi_R^\dagger(0, t) \} \}.
\]

Here, \( \langle \ldots \rangle \) denotes the average over the pre-quench ground state and the time evolution is performed in the Heisenberg picture. The spectral function evaluates to (for details, see \([31]\))

\[
A(k, \epsilon) = \pi \left( \frac{a}{u_f} \right)^{(A_2^2 + A_2^2)(\alpha_+^2 + \alpha_-^2) - 1} \sum_{j=\pm} \prod_{\xi=\pm} \frac{I(A_2^2 \alpha_2^2, A_2^2 \alpha_2^2; j \Delta_\xi)}{I(A_2^2 \alpha_2^2; j \Delta_\xi) \Gamma(A_2^2 \alpha_2^2)},
\]

where \( 2\Delta_\pm = \epsilon \pm u_f k \) and

\[
I(\mu_1, \mu_2, \Delta) = \int_0^\infty dE \ E^{\mu_1 - 1}(E - \Delta)^{\mu_2 - 1} \vartheta(E - \Delta) e^{a(\Delta - 2E)/\epsilon u_f}.
\]

To address transport properties we now consider a setup obtained by tunnel-coupling the above system to a 1D reservoir of non-interacting fermions (henceforth, the probe). The probe is modeled as a pair counter-propagating channels, labeled themselves by the index \( r = R, L \), and is described by the Hamiltonian

\[
H_p^{(0)} = v_F \sum_{r=R,L} \theta_r \int_{-\infty}^{\infty} dx \ \chi_r^\dagger(x)(-i\partial_x)\chi_r(x),
\]

with \( \chi_r(x) \) the Fermi operator in the probe. The latter is subject to a bias voltage \( V \) with respect to the system. Non-local tunneling between the system and the probe is switched on right after the quench. An interesting scenario is the one which couples only the \( L \) \((or \ R)\) movers of the probe to the \( R \) \((or \ L)\) channel of the system, allowing to study the charge and energy fractionalization induced by interactions. In solid state devices, this can be achieved via momentum-resolved tunneling, employing a magnetic field \( B \) orthogonal to the plane of the two channels \([33, 34]\). The magnetic field also controls the momentum \( k_0 \propto B \) of the tunneling
Figure 1. Spectral density $A(k_0, \epsilon)$ (units $au_f^{-1}$) as a function of $\epsilon$ (units $u_f k_0$) for the case of (a) direct quench with $K_i = 0.9$ and $K_f = 0.7$ (blue), $K_f = 0.5$ (red), $K_f = 0.3$ (green); (b) inverse quench with $K_f = 0.9$ and $K_i = 0.7$ (blue), $K_i = 0.5$ (red), $K_i = 0.3$ (green). In the inset, the spectral density for the non-quenched case ($K_i = K_f$) with $K_i = 0.9$ (blue), $K_i = 0.7$ (red), $K_i = 0.5$ (green). In all panels, $k_0 = 0.3a^{-1}$.

Electrons (measured with respect to the Fermi momentum $k_F$). Considering a non-local tunneling along the channels and assuming the coupling of the $L$ probe states to the $R$ system channels only, the tunneling Hamiltonian is [19, 31, 35, 36]

$$H_t = \lambda \theta(t) \int_T dx \left[ \psi_R^\dagger(x) \chi_L(x) + \chi_L^\dagger(x) \psi_R(x) \right],$$

with $\lambda$ the tunneling amplitude and $T$ the extended tunneling region with length $L_T$, which is greater than any other length scale of the system. In the stationary state, long after the quench and the switching on of the tunnel-coupling, from the knowledge of the spectral density of the system of Eq. (10) one can obtain the chiral charge [37] $G^Q_\eta(V) = \partial I^Q_\eta / \partial V$ and energy [19] $G^E_\eta(V) = \partial I^E_\eta / \partial V$ differential conductances,

$$G^Q_\eta(V) = \frac{e^2 |\lambda|^2 L_T \sqrt{K_f A_\eta}}{4 \pi^2 v_F} A(k_0, V), \quad G^E_\eta(V) = \frac{e |\lambda|^2 L_T}{4 \pi^2 v_F} \left( \frac{eV + \eta u_f k_0}{2} \right) A(k_0, V), \quad (14)$$

where $I^Q/E_\eta(V)$ is the chiral charge/energy current [31].

3. Results and discussion

We start the discussion showing results for the quenched spectral density in the two cases of direct and inverse quench protocol. We focus on the stationary regime ($t \rightarrow \infty$) obtained long after the sudden quench. Figure 1 shows $A(k_0, \epsilon)$ in the case of a “direct” quench – i.e. for $K_f < K_i$ (panel (a)) – and for an “inverse” quench with $K_f > K_i$ (panel (b)). In both cases, the main difference with respect to the non-quenched case (shown in the inset of panel (a)) is that the spectral density is non-vanishing in the energy window $|\epsilon| < u_f k_0$. Indeed, a quench of the interactions effectively injects energy into the system, which is then no longer in the ground state.
Figure 2. Chiral energy conductance $G_{\eta}^{E}(V)$ (units $e|\lambda|^{2}L_{T}/(4\pi^{2}v_{F})$) as a function of $eV$ (units $u_{f}k_{0}$) for the case of (a) direct quench with $K_{i} = 0.9$ and $K_{f} = 0.5$ and (b) inverse quench with $K_{i} = 0.5$ and $K_{f} = 0.9$. In all panels, blue is for $\eta = +$, red for $\eta = -$ and $k_{0} = 0.3a^{-1}$.

of the final Hamiltonian. The larger the difference between $K_{i}$ and $K_{f}$, the larger the injected energy [29], which allows for a larger spectral weight at low energy. The spectral density is clearly modified also in the “allowed” energy range $|\epsilon| > u_{f}k_{0}$, where crossings between the curves corresponding to different quench protocols can even be observed, in sharp contrast to the non-quenched case.

Interestingly, we observe the strong asymmetry between the direct and the inverse quench protocols. Indeed, as can be seen in panel (a) and (b), in the former case $A(k_{0}, \epsilon)$ is strongly enhanced with respect to the inverse case, resulting in stronger amplitude. This fact can be explained as follows: after an interaction quench, the steady state of the system can be characterized by a generalized Gibbs ensemble [5] which, roughly speaking, behaves as an excited Luttinger liquid with an effective interaction parameter $K_{\text{eff}}$ intermediate between $K_{i}$ and $K_{f}$ [29]. In particular, in the case of a direct quench the system settles onto a state which is effectively more interacting and thus with a larger spectral function (compare the inset of Figure 1(a)). On the other hand, for an inverse quench the effective interactions in the steady state of the system are smaller, resulting into a smaller spectral function. We also note that in the direct case the behavior of the spectral density for $|\epsilon| > u_{f}k_{0}$ can be better resolved in comparison to the inverse one.

The strong asymmetry between the direct and inverse case discussed above is also reflected in the behavior of transport properties of the system. The charge differential conductance needs no further discussion, being simply proportional to $A(k_{0}, eV)$ through a constant which only depends on $K_{i}$ and $K_{f}$ - see Eq.(14). The energy differential conductance shows even more marked differences between the quench protocols, as one can see in panels (a) and (b) of Fig. 2, which report $G_{\eta}^{E}(V)$ for the case of a direct and of an inverse quench, respectively. It is also worthwhile to notice that for small bias $|eV| < u_{f}k_{0}$, the sign of the two chiral energy conductances is always opposite regardless the quench protocol, in contrast to the large bias regime. This implies that the energy flow in the two counter-propagating channels of the system is always opposite in this regime.
4. Conclusions
In this paper we have analyzed the steady state of an interaction-quenched two channels Luttinger liquid, coupled to a non-interacting 1D probe via a extended tunnel region. Exploiting momentum-resolved tunneling, we have considered the injection of a left-mover from the probe into the right-moving channel of the system. We have studied the quenched spectral density of the system, showing that it exhibits different behavior depending on the quench protocols. In particular, fixed two interaction strengths, a direct quench (increasing interactions) results in a larger amplitude of the spectral density with respect to the opposite process. These features are directly observed in related transport properties such as charge and energy differential conductances. As a consequence, these properties are natural candidates to experimentally probe this asymmetry.

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