Optimization of the Study Materials Design in the Conditions of the COVID-19 Pandemic

D Knežo
Department of Natural Sciences and Humanities, Faculty of Manufacturing Technologies of the Technical University of Kosice with a seat in Prešov, Bayerova 1, 080 01 Prešov, Slovak Republic

Abstract. Numerical and statistical methods are frequently used as a research tool in technical fields and therefore are an important part of studies within technical study programmes. The COVID-19 pandemic has affected university education resulting in the distance form of education. Experience has shown that to train the application of numerical and statistical methods as well as to create homework, which is a condition for students to complete their courses, it is important to provide a large number of exercises and tasks. The work describes a model of automated study materials design using computer technology with emphasis on optimizing the design in terms of three optimization conditions: any number of exercises and tasks, minimizing the design time required and a sufficient variety of exercises and tasks. Free Pascal programming language and the LaTeX typographic system were used as the basis for the model.

1. Introduction
The articles [2], [3] and [4] deal with the implementation of information technology in education and research. This article builds on the mentioned publications and their results that enabled the automated design of study materials based on information technology.

The COVID-19 pandemic has also had a severe impact on educational process at universities. The most significant change in the educational process, which was the reaction of universities to the pandemic, was the transition to a distance form of education. One of the conditions for achieving maximum efficiency and quality of the educational process was to provide a sufficient amount of study materials in electronic form so that the requirements for student training, students' homework and the process of evaluating students' work and results are met. This work describes a model of automated design of the study materials in the field of numerical and statistical methods based on the programming language Free Pascal and the typographic system LaTeX. The model enables to create any number of examples and tasks from the selected numerical and statistical methods.

2. Professional orientation of the study materials design
Automated creation of the study materials is focused on the two areas: numerical methods and statistical methods. Examples and tasks of the following types can be created from the numerical methods:

- type 1 - numerical solution of the equation (bisection or regula falsi method),
- type 2 - interpolation (Lagrange or Newton interpolation polynomial),
- type 3 - least squares method (polynomial regression function - linear or quadratic function),
• type 4 - numerical calculation of the definite integral (Richardson extrapolation - trapezoidal or Simpson's method)
• type 5 - numerical solution of the Cauchy problem (Euler's method or Runge-Kutta method).

Examples and tasks of the following types can be created from the statistical methods:
• type 6 - descriptive statistics,
• type 7 - distributions of continuous random variables (uniform, exponential and normal distribution),
• type 8 - interval estimates,
• type 9 - testing statistical hypotheses.

From all these areas, the three types of output are created and then stored in the three PDF files:
• file 1 - examples with the result and a brief solution procedure designed to explain the use of the method,
• file 2 - tasks with the result, but without the solution procedure intended for method training,
• file 3 - tasks without stating the result and the solution procedure intended for testing students.

3. Model of automated study materials design
The model of automated study materials design is shown in fig. 1. The Free Pascal program produces source code for the LaTeX typographic system, which processes the source code and produces study materials in PDF format.

![Figure 1. Study materials design model.](image)

The block diagram of the Free Pascal program is shown in fig. 2. The program requires only two inputs: the type of task and the number of tasks. Having entered the input data, the program performs the following operations in a following cycle:
• generates random data needed for task creation,
• creates a solution to the task,
- creates and writes the appropriate LaTeX source code to three .tex files.

![Block diagram of the program in the Free Pascal language](image)

**Figure 2.** Block diagram of the program in the Free Pascal language.

4. Generation of random data
Depending on the type of tasks, the program generates random data, based on which the source code of the text of the task, a brief procedure of its solution and the result are created.
4.1. Type 1 – numerical solution of the equation
Within this type of a task, it is necessary to find all the roots of the equation of the following form

\[ a_n x^n + \cdots + a_3 x + a_0 = a(x - x_1)(x - x_2) \cdots (x - x_n) = 0, \]

where \( n = 3 \) or \( n = 4 \), with accuracy \( \varepsilon \) by the prescribed method. The program

- randomly selects one of the two methods of solution (bisection or regula falsi method),
- generates \( a \) coefficient \( a \) (rational number),
- generates roots \( x_1, \ldots, x_n \) (rational numbers), where \( x_i \neq x_j \) for \( i \neq j \),
- randomly selects the accuracy \( \varepsilon \) from a set of predefined values.

4.2. Type 2 – interpolation polynomial
Within this type of a task, it is necessary to find the Lagrange or Newton interpolation polynomial for the data \((x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)\). The program

- randomly selects Lagrange or Newton interpolation polynomial,
- generates \( n \), where \( 2 \leq n \leq 5 \),
- generates data \((x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)\), where \( x_i \neq x_j \) for \( i \neq j \).

4.3. Type 3 – least squares method
Within this type of a task, it is necessary to find a regression function for data \((x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)\). The program

- randomly selects a linear or quadratic function,
- generates \( n \), where \( 4 \leq n \leq 15 \),
- generates data \((x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)\), where \( x_i \neq x_j \) for \( i \neq j \), and at least three (in the case of a linear function) or four (in the case of a quadratic function) values of \( x \) are different from each other.

4.4. Type 4 - numerical calculation of the definite integral
Within this type of a task, it is necessary to calculate the value of the definite integral

\[ \int_{a}^{b} f(x) \, dx, \]

with an accuracy \( \varepsilon \), while the type of the function \( f \) is one of the predetermined types (the set of types can be supplemented) and the function also depends on the constants \( k_1, k_2, \ldots, k_m \). The program

- randomly selects the trapezoidal or Simpson method,
- randomly selects the type of function,
- generates constants \( k_1, k_2, \ldots, k_m \),
- generates the values \( a \) and \( b \) so that the function \( f \) on the interval \([a, b]\) is continuous,
- randomly selects the accuracy \( \varepsilon \) from a set of predefined values.

4.5. Type 5 – numerical solution of the Cauchy problem
Within this type of a task, it is necessary to solve the Cauchy problem

\[ y' = f(x, y), \quad y(x_0) = y_0, \]

numerically with the step \( h \) on the interval \([x_0, x_0 + n \cdot h]\) by the Euler method or the Runge-Kutta method. The type of the function \( f \) is one of the predefined types (the set of types can be supplemented) and the function also depends on the constants \( k_1, k_2, \ldots, k_m \). The program

- randomly selects the Euler method or the Runge-Kutta method,
randomly selects the type of function,
• generates constants $k_1, k_2, \ldots, k_m$,
• generates the values $x_0, y_0, n$ and $h$ so that the function $f$ is continuous in sufficient neighbourhood of the point $(x_0, y_0)$.

4.6. Type 6 – descriptive statistics
Within this type of a task, it is necessary to create a probability table for the data set $x_1, x_2, \ldots, x_m$ and calculate the corresponding numerical. The program
• generates the number of data in the file (natural number) $m$, where $10 \leq m \leq 30$,
• generates the number of mutually different data in the file (natural number) $k$, where $5 \leq k \leq m/2$,
• generates mutually different data in the file (real numbers) $z_1, z_2, \ldots, z_k$, where $z_1 < z_2 < \cdots < z_k$,
• generates absolute frequencies (natural numbers) $n_1, n_2, \ldots, n_k$ of data $z_1, z_2, \ldots, z_k$, where $n_1 + n_2 + \cdots + n_k = m$.

4.7. Type 7 – distributions of continuous random variables
Within this type of a task, it is necessary to calculate the probabilities $P(k_1 \leq X \leq k_2)$, $P(k_3 \leq X)$, $P(X \leq k_4)$ for a randomly selected distribution of the random variable $X$. The program
• randomly selects distributions (uniform, exponential, or normal),
• in the case of the uniform distribution $Un(a, b)$ it generates $a, b, k_1, k_2, k_3, k_4$, where $a < b$, $k_1 < k_2$ and $k_1, k_2, k_3, k_4 \in [a, b]$,
• in the case of the exponential distribution $Ex(\alpha, \lambda)$ it generates $\alpha, \lambda, k_1, k_2, k_3, k_4$, where $\alpha > 0$, $k_1 < k_2$ and $k_1, k_2, k_3, k_4 \in [0, \infty)$,
• in the case of the normal distribution $N(m, \sigma^2)$ it generates $m, \sigma, k_1, k_2, k_3, k_4$, where $\sigma > 0$ and $k_1 < k_2$.

4.8. Type 8 – interval estimates
Within this type of a task, it is necessary to determine the two-sided, left-hand and right-hand confidence intervals for the mean value $m$ if the variance $\sigma^2$ is known or unknown for the random selection $x_1, x_2, \ldots, x_n$ from the normal distribution at the significance level $\alpha$. The program
• randomly selects the significance level $\alpha$ from a set of predefined values,
• randomly decides whether the variance $\sigma^2$ is known or unknown,
• in the case of known variance it generates $m$,
• in the case of unknown variance it generates $m$ and $\sigma$, where $\sigma > 0$,
• generates the values $x_1, x_2, \ldots, x_n$ from the normal distribution $N(m, \sigma^2)$.

4.9. Type 9 – testing statistical hypotheses
Within this type of a task, for a random sample $x_1, x_2, \ldots, x_n$ from the normal distribution at the significance level $\alpha$ it is necessary to perform a test of the null hypothesis that the mean value $m$ of the population is equal to the given value $m_0$, i.e. $H_0: m = m_0$ compared to the alternative hypothesis $H_1: m \neq m_0$ or $H_1: m > m_0$ or $H_1: m < m_0$. The program
• randomly selects the significance level $\alpha$ from a set of predefined values,
• randomly decides whether the variance $\sigma^2$ is known or unknown,
• generates $m$ from the given neighborhood of $m_0$,
• in the case of unknown variance it generates $\sigma$, where $\sigma > 0$,
• generates the values $x_1, x_2, \ldots, x_n$ from the normal distribution $N(m, \sigma^2)$. 

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5. Source code for LaTeX
After generating the necessary data and calculating the results, the program creates the source code for the LaTeX typographic system, while the resulting code includes

- code for plain text,
- code for mathematical text, including numbered and unnumbered relations,
- code for tables.

The code is gradually written into the three files as described in section 2. After the end of the program, the files are ready for processing by the LaTeX system.

6. Processing of source files by LaTeX system
The files containing the source code are processed by the LaTeX typographic system resulting in the three files in PDF format. Figure 3 illustrates one task from a PDF file 3.

Calculate the integral
\[ \int_{0}^{2} \ln \left( x^2 + 2 \right) \, dx \]
using the trapezoidal rule with an accuracy \( \varepsilon = 0.005 \) using Richardson extrapolation.

Trapezoidal rule:
\[ \int_{a}^{b} f(x) \, dx \approx I_n = \frac{h}{2} \left[ f(x_0) + f(x_n) + 2 \left( f(x_1) + f(x_2) + \ldots + f(x_{n-1}) \right) \right]. \]

Calculation completion test for Richardson extrapolation:
\[ \left| \frac{I_{2n} - I_n}{3} \right| < \varepsilon \]

Richardson extrapolation:

| \( n \) | \( I_n \) | \( (I_n - I_{n/2})/3 \) |
|------|------|----------------|
| 2    | 2.3411 |                |
| 4    | 2.2995 | -0.0139           |
| 8    | 2.2890 | -0.0035           |

Table of function values for \( n = 8 \):

| \( i \) | \( x_i \) | \( f(x_i) \) |
|------|-----|---------|
| 0    | 0.0000 | 0.6931 |
| 1    | 0.2500 | 0.7239 |
| 2    | 0.5000 | 0.8109 |
| 3    | 0.7500 | 0.9410 |
| 4    | 1.0000 | 1.0986 |
| 5    | 1.2500 | 1.4469 |
| 6    | 1.5000 | 1.6219 |
| 7    | 1.7500 | 1.7918 |
| 8    | 2.0000 |         |

Figure 3. Task from PDF file 3.

7. Conclusion
The core of the model is a program written in the Free Pascal language that enables both to design the source text for the LaTeX typographic system and consider all the conditions under which the designed study material is correct in terms of statistical and numerical methods. The described automated system largely relieves pedagogical staff of the demanding process of study materials design including tests compilation and allows them to focus on other important
aspects of the educational process that the COVID-19 pandemic has brought. From the students' point of view, the automated system creates an almost unlimited space to train statistical and numerical methods application and if necessary, it also provides brief usage instructions. Finally, the system can also be used to design printed texts and textbooks.

The automated system for educational materials design was created for educational needs in the academic year 2019/2020. It has also been verified in practice. After the modification, it was used in the academic year 2020/2021 and in this year more than two thousand tasks were created and used in pedagogical practice. Experience has confirmed the usefulness of the automated program to meet the needs of the educational process.

Looking forward, the author would like to focus on WEB applications design for e-learning and the presented automated system could provide a good starting point for further research in the area.

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