Combustion Models in Finance

C. Tannous and A. Fessant

Laboratoire de Magnétisme de Bretagne, UMR-CNRS 6135,
Université de Bretagne Occidentale, BP 809 Brest CEDEX, 29285 FRANCE

(Dated: March 31, 2022)

Combustion reaction kinetics models are used for the description of a special class of bursty Financial Time Series. The small number of parameters they depend upon enable financial analysts to predict the time as well as the magnitude of the jump of the value of the portfolio. Several Financial Time Series are analysed within this framework and applications are given.

PACS numbers: 01.75.+m, 82.40.P, 89.90.+n

INTRODUCTION

Stock exchange behavior is traditionally analysed with Statistical tools (Descriptive statistics, Time-series...) and more recently with models derived from High-Energy physics or Statistical Physics. The importance of financial stakes suffices to justify interest in these new methods.

Moreover, any possible analogy that might be drawn with Cooperative physical phenomena involving a large number of degrees of freedom is favourably welcome in the burgeoning field of Financial Physics. The unambiguous identification of the precursory patterns or aftershock signatures of the market are some of the very important questions to deal with. It is noted, however, that the US market is the favourite candidate analysed in detail so far in the literature and a deeper study of the European market scene is lacking, in particular, the analysis of individual stock behavior.

In some cases, the stock trend is easy to guess, however the largest value its rate might attain is very difficult to predict, because of the interplay of many parameters drawn from political or economical interests.

Obviously, the estimation of this figure is absolutely essential for investors and any prediction tool in this field provides a leading edge to business people who are constantly tapping the market searching for opportunities of growth and profit.

We treat this problem by exploiting an analogy with combustion models occurring in Physics and Engineering. Surprisingly, the application of combustion theory to stocks picked from various economical and manufacturing sectors leads to very realistic estimations close to current trading values.

In order to build a framework for our work, we describe a special class of Financial Time Series (FTS) that display bursty behaviour at some instant of time with a large jump with respect to a prior stable level of activity lasting for a comparatively long time before the burst.

The interest in this behaviour for Market Analysts and Financial Companies is to be able to predict both the time at which the value jumps as well as the amplitude of the jump in order to assess the magnitude of benefit or loss in spite of a stable level of activity for some time. The ability to detect such behaviour provides a new speculation tool for gauging the potential of some companies and possibly predict the maximum amplitude of their growth in the near or far future.

Models borrowed from Combustion theory display spectacular behaviour of that sort with a very low pre-ignition state for a long time followed by a surprising explosion. The resulting behaviour looks dramatically like the FTS we are interested in. These Deterministic models are based on the assumption that some chemical concentration behaves as a disturbance of the combustion kinetics and is responsible for the sudden explosion with a long somehow latent pre-ignition state.

This modeling can cope with upward or downward bursts with the proviso of a prior stable activity for a while before the burst. For instance, our approach cannot cope with situations like the recent crisis of the NASDAQ [1]. Market shares and startup companies belonging to the "New Economy" belong to a different class of FTS and their analysis is suggested by Time series analysis along the lines of the work of Johansen and Sornette [1] by Stochastic Ito Calculus [2] or with Statistical/Field Theory techniques. Nevertheless, our Deterministic approach embodies patterns of behaviour that are akin to what is observed in Cooperative phenomena based modelling.

This paper is organised according to the following: In the next section, we describe a set of Combustion models that display bursty behaviour with a previously long pre-ignition state. We extend these models to downward explosive models and discuss the features of these models. In section 3, we discuss the optimisation procedure and the objective function that will allow us to predict the
burst time and magnitude of the explosion of the FTS. Section 4 reports on the application of these models to actual financial series and section 5 contains a discussion of the results with our conclusion.

**COMBUSTION MODELS**

We consider a simple combustion model described by the concentration of a chemical \( y(t) \) that obeys the non-linear evolution equation:

\[
\frac{dy}{dt} = y^2(1 - y)
\]

(1)

The initial concentration \( y(0) = \epsilon \).

This differential equation can be analytically integrated as:

\[
ln \left( \frac{y}{1 - y} \right) - \frac{1}{y} = t + C
\]

(2)

where \( C = ln \left( \frac{1}{1 - \epsilon} \right) - \frac{1}{\epsilon} \) is a constant defined from the initial condition.

The time \( t^* \) at which the value jumps is defined as the time the curvature of \( y(t) \) changes sign, i.e:

\[
t^* = ln(2) - 3/2 - ln \left( \frac{\epsilon}{1 - \epsilon} \right) + \frac{1}{\epsilon}
\]

(3)

This shows the jump time is on the order of \( \frac{1}{\epsilon} \) whereas the width of the transition region around the jump is defined by:

\[
\Delta = ln \left( \frac{y_2(1 - y_1)}{y_1(1 - y_2)} \right) + \left( \frac{1}{y_1} - \frac{1}{y_2} \right)
\]

(4)

with \( y_1 \) and \( y_2 \) are the values defining the transition region around \( t^* \). We relate these coordinates to \( \epsilon \) through: \( y_1 = \phi_1 \epsilon \) and \( y_2 = 1 - \phi_2 \epsilon \) with the constraint: \( \phi_1 + \phi_2 \leq \frac{1}{\epsilon} \).

These expressions show that for an initial value \( \epsilon \) and in the simple symmetric case \( \phi_1 = \phi_2 = \phi \), the width of the transition region is given by:

\[
\Delta = 2ln \left( \frac{1 - \phi \epsilon}{\phi \epsilon} \right) + \frac{1}{\phi \epsilon} - \frac{1}{1 - \phi \epsilon}
\]

(5)

This model is dubbed a Singular perturbation problem since decreasing \( \epsilon \) induces a divergence of both \( t^* \) and \( \Delta \). An example of the behaviour of \( y(t) \) is depicted in fig. [1] where \( \epsilon = 10^{-3} \).

The down burst model is described by the differential equation:

\[
\frac{dy}{dt} = -y(1 - y)^2
\]

(6)

with the initial condition \( y(0) = 1 - \epsilon \).

This differential equation can be analytically integrated with the result:

\[
ln \left( \frac{1 - y}{y} \right) - \frac{1}{1 - y} = t + C
\]

(7)

where the constant \( C \), the burst time \( t^* \) and the transition width are exactly the same as the previous case if we redefine the values of \( y_1 \) and \( y_2 \) in a symmetric fashion as: \( y_1 = 1 - \phi \epsilon \) and \( y_2 = \phi \epsilon \). The obtained time series for \( \epsilon = 10^{-2} \) is depicted in fig. [2].

**OPTIMISATION PROCEDURE**

The parameter space consists of three variables: \( \epsilon, \phi \) and \( \delta t \) the adaptive unit of time. The reason for the existence of the additional parameter \( \delta t \) is that we have to find simultaneously the best transition time and ratio \( \Delta^t \) for the FTS.

The optimization procedure consists of defining an objective function and finding its minimum in the four-dimensional parameter space \( \epsilon, \delta t, \phi_1, \phi_2 \). The objective function is based on a least mean squares approximation of the difference between the FTS and the Combustion model defined as a functional \( F[\epsilon, \delta t, \phi_1, \phi_2] \).

The optimization program itself is based on a globally convergent method for solving non-linear system of
equations: the multidimensional secant method developed by Broyden. It is based on a fast and accurate method for the iterative evaluation of the Jacobian of the objective function needed during the minimisation procedure. It is a Quasi-Newton method that consists of approximating the Jacobian and updating it with an iterative procedure. It converges superlinearly to the solution like all secant methods.

In order to run the Optimisation, we tried several strategies based on the following observations:

1. The differential equation can be integrated by starting for several trial values of epsilon and the results for \( t^* \) and the ratio \( \Delta t^* \) stored and interpolated in order to speed up the Optimisation procedure.

2. The analytical solution can be used in order to build the objective function explicitly, however this is valid only in certain cases and with the inversion of the roles of y and t enforced by the implicit relation between them (see e.g. equation 7).

3. The parameter space being four dimensional was reduced in some cases to two dimensions by fixing the values of the parameters \( \phi_1 \) and \( \phi_2 \) on the basis of statistical grounds.

All the above operations should yield roughly the same values for the parameters before running the final check in order to test the accuracy of the fitting to the FTS at hand. Once the fitting is validated a prediction for the largest magnitude of the plateau value can be made and compared whenever possible to the available data.

RESULTS

We have studied the PPS evolution of several companies over a period ranging from one to five years. These were chosen from different industrial and economic sectors with a variety of total market capitalizations. During the same period, the Paris CAC 40 index grew regularly and steadily except for the July-November 1998 period.

These trading shares were chosen because of their peculiar behaviour of underperforming the CAC 40 index over a period of one to five years with a growth rate that is weaker than other shares pertaining to the same sector. Some of the shares are Alcatel, GFI, STMicroelectronics belonging to the TMT sector (Technology, Media and Telecommunications) whereas DMC, Rochette and Suez belong to traditional economy sectors. In the Suez case we extended the study to seven years.

For illustration, we review the cases of these companies one by one highlighting the validity of the associated combustion model while giving some historical perspective in order to provide a background interpretation of the model parameters.

Starting with Alcatel, its price per share (PPS) started rising, after a stable period of five years despite a strong dip in November 1997 following the withdrawal of the American Pension Funds. The beginning of the growth period spans a period of 2 years with a PPS start value trading around 20 Euros and a upper value of 90 Euros representing 450 % growth. The combustion model predicts an upper value of the PPS trading around 80 Euros which is an estimation below the actual rate by about 12% (see fig. 3).

Next is DMC, a textile company that went through difficult times with a serious drop of its PPS due to a overwhelmingly hostile economic situation. It managed to stabilise its PPS to trade it at 4 to 5 Euros at the end of 1998. Ultimately it underwent restructuring and recentered its activities around Sportswear and Creative Leisure cutting down on less profitable activities. That action sent a strong signal to stock brokers, investors and financiers. Its PPS grew 450 % in less than 2 months to reach a trading value of about 20 Euros. In this case, the combustion model overestimates the actual value by 15 % as displayed in 4.

GFI Informatique is a computer services company belonging to the TMT sector with obviously a total market capitalisation volume much smaller than Alcatel’s. After making its entry on the TMT market in 1998, its PPS started growing steadily slowly at first then faster around August 1999 due to the interest at that time in
FIG. 3: Alcatel time series: The fitting parameters are $\epsilon = 0.0796$, $\delta t = 1.4324$, and $\phi_1 = \phi_2 = 2$.

FIG. 4: Dolfuss-Mieg time series: The fitting parameters are $\epsilon = 0.0118$, $\delta t = 17.922$ and $\phi_1 = \phi_2 = 2$.

FIG. 5: GFI time series: The fitting parameters are $\epsilon = 0.0082$, $\delta t = 17.576$ and $\phi_1 = \phi_2 = 2$.

FIG. 6: Rochette time series: The fitting parameters are $\epsilon = 0.00535$, $\delta t = 47.557$ and $\phi_1 = \phi_2 = 2$.

TMT companies and excellent prospective growth potential of GFI. The PPS traded 60 Euros at the beginning of the year 2000 reaching a growth rate of 600%. The value predicted by the Combustion model is 60 Euros in perfect agreement with the actual value (see fig. 3).

Rochette is a traditional Pulp and Paper company with a PPS that suffered more than 25% slip during the general slowdown period of the year 1998 second semester. Its PPS stayed constant for nearly a year trading around 2.5 Euros. At the end of 1999, it grew to 7.7 Euros in good agreement with the suggested Combustion model (see fig. 3).

The PPS of STMicroelectronics, a company dealing with the design and testing of Semiconductor Components had a stable history until the end of 1998. It exploited fully the development of the High Technology sector and it rose to 70 Euros and that is equivalent to a 700% progress as predicted by the Combustion model (see fig. 3).

The final study case is about Suez. Initially, Suez is a company geared toward Energy and Water resources distribution; it extended its activity to Telecommunications around 1996 merging with Lyonnaise des Eaux in June 1997. That event triggered a 230% growth rate in its PPS rising from 75 Euros to 175 Euros. Again, this is in with perfect agreement with the suggested combustion model (see fig. 3). The long term stability of the PPS pushed us in this case to extend the study period to over seven years.
FIG. 7: Stmicro time series: The fitting parameters are $\epsilon = 0.00317$, $\delta t = 41.268$ and $\phi_1 = \phi_2 = 2$.

FIG. 8: Suez time series: The fitting parameters are $\epsilon = 0.00571$, $\delta t = 42.713$ and $\phi_1 = \phi_2 = 2$.

DISCUSSION AND CONCLUSIONS

The special class of FTS we consider, suffer some lag spanning periods that range any time duration from several months to several years with respect to the progress of other FTS belonging to the same business sector or to the stock market index.

The behaviour we describe seems compatible with the overall picture that any lag in the progress of the PPS of some company in comparison to other similar companies makes it attractive to the investors. That interest even increases as the lag gets more pronounced. In some cases, this induces a recovery phase in which the PPS readjusts with a rise that is larger the more important the respective PPS difference is.

The length of the recovery phase depends intimately on the overall Economical situation that might interfere with the growth, nevertheless it is reasonable to expect a readjustment of the price share in such a way it conforms to the other shares belonging to the same category.

The region of validity of the results we obtain spans the stable as well as the period after the jump of the PPS. We observed that the long term evolution of the PPS after the jump is highly variable depending on the market reaction. In the framework of our model, this means that after the jump, we are in a stage where the strict conditions for the validity of the combustion model are no longer obeyed.

The simple combustion model is a straightforward translation of the above facts with the proviso of a sound interpretation of its basic parameters that ought to be evaluated from the portfolio of the company of interest and the background Economical context. Identifying the parameters of the combustion model from Economical data and assessing them is presently work in progress.

* Electronic address: tannous@univ-brest.fr
[1] A. Johansen and D. Sornette: Euro. Phys. J. B 17, 317 (2000).
[2] R. O’Malley Singular perturbation methods for ordinary differential equations (Springer-Verlag, New York, 1991).
[3] W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery Numerical Recipes (Cambridge University Press, Second Edition, New-York, 1992).
[4] C. G. Broyden: Mathematics of Computation, 19, 557 (1965).
[5] C. W. Gardiner Handbook of Stochastic Methods: For Physics, Chemistry and the Natural Sciences (Springer Series in Synergetics, Vol 13, 2nd Edition, 1996).