QCD Factorization For B Decays To Two Light Pseudoscalars
Including Chirally Enhanced Corrections *

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Abstract

Since b quark mass is not asymptotically large, chirally enhanced corrections which arise from twist-3 wave functions may be important in B decays. We thus evaluate the hadronic matrix elements with the emitted meson described by leading twist and twist-3 distribution amplitudes $\Phi_p(x)$. After summing over the four "vertex correction" diagrams, we obtain the results with infrared finiteness which shows that chirally enhanced corrections arise from $\Phi_p(x)$ can be consistently included in QCD factorization. We also briefly discuss the contributions from "hard spectator" diagrams.

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It is well known that two-body, non-leptonic charmless B decays are crucial for extracting CKM matrix elements. However, due to our ignorance on how to calculate hadronic matrix elements, we conventionally resort to Bjorken’s color transparency argument [1] which lead to “naive factorization assumption”,

\[
\langle M_1 M_2 | Q | B \rangle = \langle M_2 | J_1 | 0 \rangle \langle M_1 | J_2 | B \rangle, \tag{1}
\]

This assumption makes the hadronic matrix elements scale-independent. Noting that Wilson Coefficients are scheme- and scale-dependent, the theoretical calculations on the branching ratios would then be scheme- and scale-dependent which is unacceptable. To save factorization hypothesis, a phenomenological parameter \(N_{\text{eff}}\) is introduced [4] which is commonly called generalized factorization. However this approach is not satisfactory yet because in principle \(N_{\text{eff}}\) is process dependent.

Recently, Beneke, Buchalla, Neubert and Sachrajda [2,3] proposed a promising method: in the heavy quark limit, they show that the emitted light meson can be described with leading twist-2 distribution amplitude, which means that we should show the infrared finiteness using twist-2 distribution amplitudes, which means that we should show the infrared finiteness using

\[
\frac{2m_b^2}{m_b(m_u+m_d)} \approx 1.18 \text{ which makes the power suppression completely fail. This parameter is multiplied by } a_6 \text{ and } a_8, \text{ where } a_6 \text{ is very important numerically in penguin-dominated B decays. So an evaluation of the hadronic matrix elements including chirally enhanced corrections may be phenomenologically or numerically important. In this letter, we shall examine this problem in some detail.}
\]

Chirally enhanced corrections arise from twist-3 light-cone distribution amplitudes, generally called \(\Phi_p(x)\) and \(\Phi_\sigma(x)\). For light pseudoscalar mesons, they are defined as [3]

\[
\langle P(p')|\bar{q}(y)i\gamma_5 q(x)|0\rangle = f_p\mu_p \int_0^1 du \ e^{i(\bar{u}p'\cdot y + \bar{y}p'\cdot x)} \phi_p(u), \tag{3}
\]

\[
\langle P(p')|\bar{q}(y)\sigma_{\mu\nu}\gamma_5 q(x)|0\rangle = i f_p\mu_p \left( \frac{p'_{\mu}z_{\nu} - p'_{\nu}z_{\mu}}{2} \right) \int_0^1 du \ e^{i(\bar{u}p'\cdot y + \bar{y}p'\cdot x)} \phi_\sigma(u), \tag{4}
\]

where \(\mu_p = \frac{M_b^2}{m_u+m_d}\), \(z = y - x\). We notice that in Ref. [2] color transparency is demonstrated in one-loop level in the heavy quark limit. If we want to include chirally enhanced corrections consistently, we should describe the emitted light meson with leading twist-2 and twist-3 distribution amplitudes, which means that we should show the infrared finiteness using
twist-3 distribution amplitudes after summing over the ”vertex correction” diagrams. In this paper, we shall restrict ourselves to \( \Phi_p(x) \) while postpone the discussion of \( \Phi_\sigma(x) \) to ref [7] because of the complicated derivation in proving the infrared finiteness of the ”vertex correction” diagrams using \( \Phi_\sigma(x) \).

We notice that in Ref [8], the authors have used twist-3 distribution amplitude \( \Phi_p(x) \) to calculate the strong penguin corrections (Fig.(e)-(f)). The difference of our work from ref [8] is that we calculate ”vertex correction” diagrams and show the infrared finiteness of \( a_6 \) and \( a_8 \) at the order of \( \alpha_s \).

In the following, we take leading twist and twist-3 wave functions \( \Phi_p(x) \) to describe the emitted light meson, and we will show the infrared finiteness under this approach.

The \( |\Delta B| = 1 \) effective Hamiltonian is given by [3]

\[
\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[ \sum_{q=u,c} v_q \left( C_1(\mu)Q_1^q(\mu) + C_2(\mu)Q_2^q(\mu) + \sum_{k=3}^{10} C_k(\mu)Q_k(\mu) \right) - v_\ell (C_{7\gamma}Q_{7\gamma} + C_{SG}Q_{SG}) \right] + h.c.,
\]

(5)

where \( v_q = V_{qb}V_{qb}^{\ast} \) (for \( b \to d \) transition) or \( v_q = V_{qb}V_{qs}^{\ast} \) (for \( b \to s \) transition) and \( C_i(\mu) \) are Wilson coefficients which have been evaluated to next-to-leading order approximation. The four-quark operators \( Q_i \) are

\[
\begin{align*}
Q_1 &= (\bar{u}_a b_\alpha)_{V-A}(\bar{q}_\beta u_\beta)_{V-A} \\
Q_2 &= (\bar{u}_a u_\alpha)_{V-A}(\bar{q}_\beta b_\beta)_{V-A} \\
Q_3 &= (\bar{q}_\alpha b_\alpha)_{V-A}(\sum_{q'} \bar{q}_\beta q'_\beta)_{V-A} \\
Q_4 &= (\sum_{q'} (\bar{q}_\beta q'_\beta)_{V-A}(\bar{q}_\alpha b_\alpha)_{V-A} \\
Q_5 &= (\bar{q}_a b_\alpha)_{V-A} \sum_{q'} \bar{q}_\beta (\bar{q}_\beta q'_\beta)_{V-A} \\
Q_6 &= -2 \sum_{q'} (\bar{q}_\beta q'_\beta)_{S-P} (\bar{q}_\alpha b_\alpha)_{S-P} \\
Q_7 &= \frac{3}{2} (\bar{q}_a b_\alpha)_{V-A} \sum_{q'} e_q (\bar{q}_\beta q'_\beta)_{V-A} \\
Q_8 &= -3 \sum_{q'} e_q (\bar{q}_\beta q'_\beta)_{S-P} (\bar{q}_\alpha b_\alpha)_{S-P} \\
Q_9 &= \frac{3}{2} (\bar{q}_a b_\alpha)_{V-A} \sum_{q'} e_q (\bar{q}_\beta q'_\beta)_{V-A} \\
Q_{10} &= \frac{3}{2} \sum_{q'} e_q (\bar{q}_\beta q'_\beta)_{V-A}(\bar{q}_\alpha b_\alpha)_{V-A}
\end{align*}
\]  

(6)

and

\[
Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{q}_a \sigma^{\mu\nu}(1 + \gamma_5)b_\alpha F_{\mu\nu}, \quad Q_{SG} = \frac{g}{8\pi^2} m_b \bar{q}_a \sigma^{\mu\nu}t_{\alpha\beta}b_\beta G_{\mu\nu}, \quad (q = d \text{ or } s).
\]

(7)

The amplitude of the decays of B to two light pseudoscalar mesons in QCD factorization can be written as:

\[
A(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \left[ \sum_{p=u,c} \sum_{i=1,10} v_p a_i^p \langle M_1 M_2 | Q_i | B \rangle_F, \right.
\]

(8)

where \( v_p \) is CKM factor and \( \langle M_1 M_2 | Q_i | B \rangle_F \) is the factorized matrix elements.

We calculate QCD coefficients \( a_i^p \) with the emitted mesons described by light-cone distribution amplitudes. For instance, let we consider the ”vertex correction” diagrams. Twist-3 distribution amplitude \( \Phi_p(x) \) makes no contribution when considering \( (V - A) \otimes (V - A) \) and \( (S + P) \otimes (S - P) \) currents because of their Lorentz structure. As to \( (V - A) \otimes (V + A) \) current, there is some subtlety in regularizing the infrared divergences. If we use dimension regularization, the infrared finiteness would not hold for the case of \( \Phi_p(x) \) after summing
over those four ”vertex correction” diagrams. That is because wave functions are defined in 4-dimensions, it may be inconsistent to extend its usage directly to D-dimensions. Thus we assign a virtual mass to the gluon propagator and regularize the infrared integrals in four dimensions, it may be inconsistent to extend its usage directly to D-dimensions. Thus we assign a virtual mass to the gluon propagator and regularize the infrared integrals in four dimensions. Then the ”vertex correction” contributions of \((V - A) \otimes (V + A)\) current to \((S + P) \otimes (S - P)\) operator is infrared finite:

\[
V = \frac{\alpha_s}{4\pi N} \int_0^1 dx \Phi_p(x) \{ i\pi \log \frac{x}{\bar{x}} - \frac{1 + x}{x} \log \bar{x} + \frac{1 + \bar{x}}{\bar{x}} \log x - Li_2(-\frac{x}{\bar{x}}) + Li_2(-\frac{\bar{x}}{x}) \},
\]

where \(\bar{x} = 1 - x\) and \(Li_2(x)\) is dilogarithm function. This means that we can consistently include twist-3 distribution amplitude \(\Phi_p(x)\) in the framework of QCD factorization. The explicit expressions of \(a_i^\nu\) for \(i = 1\) to \(10\) (for symmetric light-cone distribution amplitudes of light pseudoscalar mesons) are obtained as:

\[
a_1^u = C_1 + \frac{C_2}{N} + \frac{\alpha_s C_F}{4\pi N} C_2 F,
\]

\[
a_2^u = C_2 + \frac{C_1}{N} + \frac{\alpha_s C_F}{4\pi N} C_1 F,
\]

\[
a_3 = C_3 + \frac{C_4}{N} + \frac{\alpha_s C_F}{4\pi N} C_4 F,
\]

\[
a_4^p = C_4 + \frac{C_3}{N} - \frac{\alpha_s C_F}{4\pi N} \left\{ \left( \frac{4}{3} C_1 + \frac{44}{3} C_3 + \frac{4}{3} (C_4 + C_6) \right) \ln \frac{\mu}{m_b} \right. \\
+ \left( G_{M_2}(s_p) - \frac{2}{3} \right) C_1 + (G_{M_2}(0) + G_{M_2}(1) - f_{M_2} + f'_{M_2} + \frac{50}{3} \bar{C}_3) \\
- \frac{2f}{3} C_4 + (3G_{M_2}(0) + G_{M_2}(s_c) + G_{M_2}(1))(C_4 + C_6) + G_{M_2,s} C_6 \right\},
\]

\[
a_5 = C_5 + \frac{C_6}{N} + \frac{\alpha_s C_F}{4\pi N} C_6 (-F - 12),
\]

\[
a_6^p = C_6 + \frac{C_5}{N} - \frac{\alpha_s C_F}{4\pi N} \left\{ \left( C_1 + 2C_3 + f(C_4 + C_6) \right) \ln \frac{\mu}{m_b} \right. \\
+ \left( G_{M_2}'(s_p) - \frac{7}{12} \right) C_1 + (G_{M_2}'(0) + G_{M_2}'(1) - \frac{7}{6} \bar{C}_3 - \frac{7}{12} f C_4) \\
+ (3G_{M_2}'(0) + G_{M_2}'(s_c) + G_{M_2}'(1))(C_4 + C_6) - \frac{f}{12} C_6 + \frac{3}{2} C_8 \right\},
\]

\[
a_7 = C_7 + \frac{C_8}{N} + \frac{\alpha_s C_F}{4\pi N} C_8 (-F - 12),
\]

\[
a_8 = C_8 + \frac{C_7}{N} - \frac{\alpha_s C_F}{4\pi N} \left\{ \frac{9\pi}{6C_7} \right. \\
+ \frac{\alpha_{em}}{9\pi} \left\{ -((\frac{C_1}{N} + C_2) + \frac{1}{2} (C_4 + C_3) + \frac{1}{2} (C_6 + C_5)) \ln \frac{\mu}{m_b} \right. \\
+ \left( \frac{7}{12} - G_{M_2}'(s_p) \right) C_1 + (\frac{7}{24} - G_{M_2}'(s_c) + \frac{1}{2} G_{M_2}'(1))(C_4 + C_6 + C_3) \\
+ \left( \frac{3}{24} - G_{M_2}'(s_c) + \frac{1}{2} G_{M_2}'(1)(\frac{C_6}{N} + C_5) - \frac{3}{4} C_7 \right) \left. \right\},
\]

\[
a_9 = C_9 + \frac{C_{10}}{N} + \frac{\alpha_s C_F}{4\pi N} C_{10} F,
\]
\[ a_{10}^{p} = C_{10} + \frac{C_{9}}{N} + \frac{\alpha_{s} C_{F}}{4\pi} C_{9} F + \frac{\alpha_{em}}{9\pi} \left\{ \left( -\frac{2}{3} (2C_{2} + C_{1}) + (C_{3} + \frac{C_{4}}{N}) + (C_{5} + \frac{C_{6}}{N}) \right) \ln \frac{\mu}{m_{b}} + \left( \frac{2}{3} - G_{M_{2}}(s_{p}) \right) (C_{2} + \frac{C_{1}}{N}) + \left( \frac{1}{3} - G_{M_{2}}(s_{c}) + \frac{G_{M_{2}}(1)}{2} \right) (C_{3} + \frac{C_{4}}{N}) + \left( -G_{M_{2}}(s_{c}) + \frac{G_{M_{2}}(1)}{2} \right) (C_{5} + \frac{C_{6}}{N}) - \frac{1}{2} C_{7} \gamma_{M_{2},8} \right\}. \]

Here \( N = 3 \) (\( f = 5 \)) is the number of color (flavor), \( C_{F} = \frac{N^{2} - 1}{2N} \) is the factor of color, \( s_{p} = m_{p}^{2}/m_{b}^{2} \) for \( p = u, c \) and we define the symbols in the above expressions as: (most of them are as the same as Beneke's except for \( G_{s, M_{2}}(s) \) and \( G_{s, M_{2}}'(s, x) \))

\[ F = -12 \ln \frac{\mu}{m_{b}} - 18 + f_{M_{2}}^{I} + f_{M_{2}}^{II}, \]

\[ f_{M_{2}}^{I} = \int_{0}^{1} dx \, g(x) \Phi_{M_{2}}(x), \quad G_{M_{2},8} = \int_{0}^{1} dx \, G_{8}(x) \Phi_{M_{2}}(x), \]

\[ G_{M_{2}}(s) = \int_{0}^{1} dx \, G(s, x) \Phi_{M_{2}}(x), \]

\[ G_{M_{2}}'(s) = \int_{0}^{1} dx \, G'(s, x) \Phi_{M_{2}}^p(x), \]

here \( \Phi_{M_{2}}(x) (\Phi_{M_{2}}^p(x)) \) is leading twist (twist-3) wave function of the emitted meson \( M_{2} \), and the hard-scattering functions are

\[ g(x) = 3 \frac{1 - 2x}{1 - x} \ln x - 3i\pi, \quad G_{8}(x) = \frac{2}{1 - x}, \]

\[ G(s, x) = -4 \int_{0}^{1} du \, u(1 - u) \ln(s - u(1 - u)(1 - x) - i\epsilon), \]

\[ G'(s, x) = \frac{3}{4} G(s, x). \]

As to \( f_{M_{2}}^{II} \) which labels the contributions from the hard spectator scattering diagrams (Fig. (g)-(h)), We take the wave function of B meson as \( \gamma_{5}(\Gamma_{B} - M_{B})\Phi_{B}(\xi) \) and find that, when considering twist-3 \( \Phi_{p}(x) \) distribution contributions, the hard spectator scattering contributions are proportional to:

(i) for the case of \( (V - A) \otimes (V - A) \) and \( (S + P) \otimes (S - P) \) currents,

\[ \int_{0}^{1} d\xi \, \frac{\Phi_{B}(\xi)}{\xi} \int_{0}^{1} dy \, \frac{\Phi_{M_{1}}(y)}{y} \int_{0}^{1} dx \left\{ \frac{\Phi_{M_{2}}(x)}{x} + \frac{\Phi_{M_{2}}^{p}(x) \mu_{M_{2}}}{M_{B}}(x - (1 - x)) \right\}; \]

(ii) for the case of \( (V - A) \otimes (V + A) \) currents,
\[ \int_0^1 d\xi \frac{\Phi_B(\xi)}{\xi} \int_0^1 dy \frac{\mu_{M_1}}{M_B} \Phi_{M_1}^p(y) \int_0^1 dx \frac{\Phi_{M_2}^p(x)}{x} \frac{\mu_{M_2}}{M_B} \frac{m_B}{x(1-x)}. \] (28)

Under the symmetric distributions of final state light mesons, the logarithmically divergent integrals are canceled after summing over two hard spectator scattering diagrams. As a result there is no hard spectator scattering contributions to \( a_6 \) and \( a_8 \), the contribution of hard spectator scattering to other \( a_i \) is as the same as ref \[2,10\]:

\[ f_{M_2}^{II} = \frac{4\pi^2}{N C_B} \frac{f_{M_1} f_B}{F_{B \rightarrow M_1(0)} m_B^2} \int_0^1 d\xi \frac{\Phi_B(\xi)}{\xi} \int_0^1 dx \frac{\Phi_{M_1}(x)}{x} \int_0^1 dy \frac{\Phi_{M_2}(y)}{y}. \] (29)

In ref \[5\], the authors have discussed the contributions of asymmetric distributions and find that numerically this effect is very small. In our case, when considering the asymmetric distributions, there would appear divergent integrals, but in this case the asymmetric distribution corrections would also be small if we parametrized the divergent integrals as an unknown parameter (as what have done in ref \[5\]) and could be safely neglected.

We notice that the above approach of evaluating "hard spectator" contribution is naive. For instance, the scale of "hard spectator" contribution should be different from the "vertex correction" contribution. While it seems reasonable, for the "vertex correction" diagrams, to take the scale \( \mu \sim \mathcal{O}(m_b) \) to avoid large logarithm \( \alpha_s \log \frac{\mu}{m_b} \), a natural choice of the scale of "hard spectator" contribution may be around \( \mathcal{O}(1 \text{ GeV}) \) because the average momentum square of the exchanged gluon is about \( 1 \text{ GeV}^2 \). Another disturbing feature of "hard spectator" contribution is that, as have been pointed out in ref \[5\], when including the contribution of \( \Phi_{\sigma} \), there would appear divergent integral \( \int_0^1 dx \frac{k_T^2}{x} \) even if the symmetric distribution amplitude is applied. This divergent integral implies that the dominant contribution comes from the end-point region, or in another word, it is dominated by soft gluon exchange. However the transverse momentum may not be omitted in the end-point region, if so, the corresponding divergent integral would then changed to:

\[ \int dx d^2 k_T \frac{\phi(x,k_T)}{x m_b^2 + k_T^2}. \] (30)

As an illustration, we do not consider the \( k_T \) dependence of wave functions (though it is certainly not a good approximation), then the above integral proportions to:

\[ \int \frac{dx dk_T^2}{x m_b^2 + k_T^2} \propto \int \frac{dx dy}{x + y}. \] (31)

The above integration is convergence now, furthermore it is not dominated by end-point contribution. This illustrates that the treatment of "hard spectator" diagrams may need further discussions.

In summary, we consider some chirally enhanced corrections arise from twist-3 distribution amplitudes \( \Phi_p(x) \). To include chirally enhanced corrections consistently in QCD factorization, we describe the emitted light meson with leading twist and twist-3 \( \Phi_p(x) \) distribution amplitudes and show the infrared finiteness of "vertex correction" diagrams (fig.(a)-(d)). We also briefly discuss the disturbing "hard spectator" contributions.
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REFERENCES

[1] J.D. Bjorken, Nucl. Phys. (Proc. Suppl.) **B11**, 325 (1989).
[2] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999).
[3] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Nucl. Phys. **B591**, 313 (2000).
[4] H.Y. Cheng, Phys.Lett.B, **335**, 428 (1994); Phys.Lett.B, **395**, 345 (1997).
A. Ali, G. Kramer and C.D. Lu, Phys.Rev. D **59**, 014005 (1999) and Phys.Rev. D **58**, 094009 (1998).
[5] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, to appear in the proceedings of ICHEP2000, [hep-ph/0007256](http://arxiv.org/abs/hep-ph/0007256).
[6] V.M. Braun and I.E. Filyanov, Z.Phys. **C48**, 239 (1990);
M. Beneke and T. Feldmann, Nucl. Phys. **B592**, 3 (2000).
[7] D. Du, D. Yang and G. Zhu, in preparation.
[8] T. Muta, A. Sugamoto, M. Yang and Y. Yang, [hep-ph/0006022](http://arxiv.org/abs/hep-ph/0006022).
[9] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[10] D. Du, D. Yang and G. Zhu, Phys. Lett. **B488**, 46 (2000).
| QCD Coefficients | $\mu = 5.0 \, \text{GeV}$ | $\mu = 2.5 \, \text{GeV}$ |
|------------------|-----------------------------|-----------------------------|
| $a_u^1$          | $1.043 + 0.012i$            | 1.019                       | 1.067 + 0.025i | 1.039 |
| $a_u^2$          | $0.034 - 0.075i$            | 0.184                       | $-0.011 - 0.104i$ | 0.104 |
| $a_3$            | $0.007 + 0.002i$            | 0.003                       | $0.010 + 0.004i$ | 0.005 |
| $a_4^u$          | $-0.030 - 0.014i$           | $-0.029$                    | $-0.033 - 0.018i$ | $-0.041$ |
| $a_4^c$          | $-0.035 - 0.006i$           | $-0.029$                    | $-0.040 - 0.006i$ | $-0.041$ |
| $a_5$            | $-0.007 - 0.003i$           | $-0.005$                    | $-0.009 - 0.006i$ | $-0.010$ |
| $r_\chi a_6^u$  | $-0.047 - 0.003i$           | $-0.044$                    | $-0.050 - 0.003i$ | $-0.052$ |
| $r_\chi a_6^c$  | $-0.049 - 0.006i$           | $-0.044$                    | $-0.053 - 0.006i$ | $-0.052$ |
| $a_7 \times 10^5$ | $11.1 + 2.7i$              | 9.1                         | $2.8 + 5.5i$ | 3.6 |
| $r_\chi a_8^u \times 10^5$ | $49.0 - 3.0i$   | 45.3                       | $570 - 1.2i$ | 49.2 |
| $r_\chi a_8^c \times 10^5$ | $47.3 - 4.6i$   | 45.3                       | $56.3 - 1.9i$ | 49.2 |
| $a_9 \times 10^5$ | $-939.8 - 13.3i$          | $-913.3$                    | $-969.6 - 25.4i$ | $-941.4$ |
| $a_{10} \times 10^5$ | $22.4 + 57.8i$       | $-135.7$                    | $68.1 + 88.9i$ | $-68.0$ |
| $a_{10}^c \times 10^5$ | $19.1 + 63.0i$       | $-135.7$                    | $66.2 + 91.9i$ | $-68.0$ |

**TABLE I.** The QCD coefficients $a_\ell^\pi(\pi\pi)$ at NLO and LO for the renormalization scales at $\mu = 5 \, \text{GeV}$ and $\mu = 2.5 \, \text{GeV}$. 
FIG. 1. Order of $\alpha_s$ corrections to hard-scattering kernels. The upward quark lines represent the ejected quark pairs from $b$ quark weak decays.