Development of A four-Degrees-of-Freedom Diffraction Sensor

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Abstract. The development of a diffraction-type four-degrees-of-freedom sensor based on the three-dimensional diffraction method and collimation method is described in this paper. This sensor is designed to be integrated with a linear laser encoder to allow five-degrees-of-freedom measurement. It is composed of a miniature collimation-type sensor, a reflective diffraction grating and a quadrant detector to simultaneously measure a straightness error and three rotational angles. Based on the diffraction method, the reflective diffraction grating reflects the incident laser from the collimation-type sensor to several diffractive laser rays and only the 0 and +1-order diffractive laser rays are detected by the collimation-type sensor and a quadrant detector respectively. According to the changed spot positions of the diffraction laser lights on the collimation-type sensor and the quadrant detector, a straightness error and the three rotational errors can be solved simultaneously.

1. Introduction

A high precision multi-degrees-of-freedom measuring system is important for measuring the pose of an object on a motion stage with high accuracy. Current trends require a multi-degrees-of-freedom measuring system that can simultaneously monitor on-line the linear and rotational displacement of an object along three orthogonal axes. The laser interferometer calibration system is the most appropriate instrument for the measurement of displacement, straightness, pitch and yaw angular errors. The most representative instruments are the HP/Aglient five-degrees-of-freedom laser interferometer system using two plane mirrors and six fiber-couple heterodyne laser interferometers for the measurement of X and Y-axes displacements, pitch, roll and yaw angular errors. The SIOS Messtechnik GmbH

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integrated three miniature fiber-couple circularly polarized laser interferometers, three plane mirrors and two angular sensors with the positive sensitive detectors for the measurement of six-degrees-of-freedom. Some new methods and measuring systems have also been developed to measure multiple geometric errors simultaneously to speed up the calibration. Nakamura et al. [1] used four laser interferometers and a corner cube to measure the spatial coordinates of a microscopic scanning stage. Liu et al. [2] also developed a measurement system using a miniature fiber-couple laser interferometer, three quadrant detectors and multiple laser paths setup to have six-degrees-of-freedom measurement. Kim et al. [3] successfully developed a six degrees-of-freedom measuring system using a circle grating target and three quadrant detectors. The small measuring range of the six-degrees-of-freedom errors is limited within the diverged laser spot size covered on the circle grating.

In this paper, a diffraction-type four-degrees-of-freedom sensor which can be integrated with the type of laser linear encoder that is widely used on a motion stage to measure linear displacement is developed. The linear laser encoder uses the diffraction grating as the target and detects the interference fringe of the diffraction rays based on the Doppler principle but it can measure only the linear displacement. Thus, this paper proposed a diffraction-type sensor which can be integrated with the laser linear encoder to measure an additional three angular errors and one straightness error of a high precision motion stage simultaneously.

2. The design of the diffraction-type four-degrees of freedom sensor

The schematic graph of the diffraction-type four-degrees-of-freedom sensor is shown in Fig. 1. The system is composed of a miniature collimation-type sensor (including a laser diode, quadrant detector (Q1), collimated lens and beam splitter), a quadrant detector (Q2) and a reflective diffraction grating. The collimated laser from the collimation-type sensor is projected on a diffraction grating and diffracted into several diffraction rays. The 0-order diffraction ray, which has a larger intensity, is reflected back to the sensor and focused on Q1. The +1 order diffraction ray is projected on Q2. The collimation-type sensor is designed to measure the rotational angles, $\theta_1$ and $\theta_2$, defined in Fig. 1 of the diffraction grating. By using the measuring results of the collimation-type sensor and the quadrant detector (Q2), the displacement (Z-axis) and three rotational angles ($\alpha$, $\beta$ and $\gamma$) of the diffraction grating can be solved by optical ray analysis.

3. Measurement easurement principles

The principle of the measurement utilizes the directional change of the 0 and 1-order diffraction rays and the laser spots on two quadrant detectors (Q1 and Q2) to obtain the displacement and three angular motion errors. When a laser beam projects on a diffraction grating, it is diffracted into several directions. The directions of the diffraction rays are a function of the wavelength of the incident ray, the grating pitch and the angle between the incident ray and the normal direction of the grating surface. As also shown in Figure 1, a grating coordinate {O} is fixed to the diffraction grating. The X-axis is perpendicular to the grating pitch direction. {O1} and {O2} are the detector coordinates attached to the quadrant detectors.

![Figure 1](attachment:Figure_1.png)

**Figure 1.** Schematic graph of the diffraction-type four-degrees-of-freedom sensor.

![Figure 2](attachment:Figure_2.png)

**Figure 2.** Photograph of the experimental setup.
3.1. Measurement principle of collimated-type sensor

The measurement principle of the $\gamma$ and $\beta$ angles is based on the autocollimation method. As shown in Figure 1, the beam reflected by the diffraction grating and focused on to the Q1 is used to measure two angles. Then, the $\gamma$ and $\beta$ angles from the rotation of grating can be calculated by:

$$\begin{align*}
\beta &= k_1 X_{Q1} = k_1 \left[ (V_1 + V_4) - (V_2 + V_3) \right] / \left[ (V_1 + V_2 + V_3 + V_4) \right] \\
\gamma &= -k_2 Y_{Q1} = -k_2 \left[ (V_1 + V_2) - (V_3 + V_4) \right] / \left[ (V_1 + V_2 + V_3 + V_4) \right]
\end{align*}$$

(1) (2)

Where V1, V2, V3 and V4 are the transform voltages outputted from the four quadrants of Q1. $k_1$ and $k_2$ represent the angular proportional gains.

3.2. Measurement principle of diffraction method

Basing on the three-dimensional diffraction analysis, the other two measurement elements, the $\alpha$ angle and the displacement along the z axis, can be obtained. Let the unit vector of the incident laser from the collimation type sensor be L in the grating coordinate system $\{O\}$. Here

$$L = \begin{bmatrix} 0 & 0 & -1 & 1 \end{bmatrix}^T$$

(3)

The rotational transformation matrix, R, describes a new coordinate system due to the rotation of the grating relative to the original grating coordinate system $\{O\}$, and can be represented as

$$R = \begin{bmatrix}
\cos \alpha \cos \beta & \cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma & \sin \alpha \sin \gamma + \sin \alpha \sin \beta \cos \gamma & 0 \\
\sin \alpha \cos \beta & \cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma & \sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma & 0 \\
-\sin \beta & \cos \beta \sin \gamma & \cos \beta \cos \gamma & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

(4)

Three rotational parameters ($\alpha$, $\beta$, $\gamma$) are represented as the z-y-x angular motion errors. Thus, a new coordinate, $L'$, of the incident beam L related to the rotated grating coordinate system can be transformed by means of eq. (5).

$$L' = R^{-1} L = \begin{bmatrix} t_x' & t_y' & t_z' & 1 \end{bmatrix}^T$$

(5)

From the three-dimensional grating equation, the vectors of the diffraction laser lights related to the rotated grating coordinate system can be represented as $L_m$. Here

$$L_m = \begin{bmatrix} \bar{l}_{mx} & \bar{l}_{my} & \bar{l}_{mz} & 1 \end{bmatrix}^T = \begin{bmatrix} t_x' + m(x/d) \sqrt{1 - t_x'^2 - t_y'^2} & t_y' \end{bmatrix}^T$$

(6)

Where $\bar{l}_{mi}$ ($i=x, y, z$) are the components of the mth-order diffraction vector. $\lambda$ is the wavelength of the incident ray and d is the pitch of the diffraction grating. Let $\{O2\}$ be the +1-order detector coordinate attached on the center of the quadrant detector. Thus, the coordinate transformation matrix between the $\{O2\}$ coordinate system and $\{O\}$ grating coordinate system can be

$$T = \begin{bmatrix}
\cos \theta & 0 & \sin \theta & t_x \\
0 & 1 & 0 & 0 \\
-\sin \theta & 0 & \cos \theta & t_y \\
0 & 0 & 0 & 1
\end{bmatrix}$$

(7)

The $\theta$ is the diffraction angle of the 1-order diffracted laser light. The $t_x$ and $t_y$ are the related translation displacements between the $\{O2\}$ coordinate system and the $\{O\}$ coordinate system. As we known that a diffraction grating is incapable of changing the direction of diffracted laser light against in-plane motion and the spot position on a quadrant detector (Q2), the directional change is only affected by the error along the z axis and three rotational angles ($\alpha$, $\beta$ and $\gamma$) of the diffraction grating. Let the error along the z axis of the diffraction grating be the ez. Thus, the vector of the error is represented as

$$E = \begin{bmatrix} 0 & 0 & e_z & 1 \end{bmatrix}^T$$

(8)
The vector $E$ can be transformed into the $\{O_2\}$ coordinate system and described:

$$E_{o_2} = T^{-1}E = \begin{bmatrix} e_{+1x} & e_{+1y} & e_{+1z} \end{bmatrix}^T,$$

Also, the vector of the +1-order diffracted laser light related to the rotated grating coordinate system can be transformed into the $\{O_2\}$ coordinate system by means of the following equations.

$$\mathbf{L} = \mathbf{R} \mathbf{L} = \begin{bmatrix} \hat{r}_{+1x} & \hat{r}_{+1y} & \hat{r}_{+1z} \end{bmatrix}^T.$$

Here $\mathbf{R}$ represents the rotational transformation matrix of the rotated grating coordinate system related to the $\{O_2\}$ coordinate system. Combining the line equations of the +1-order diffracted laser light after diffraction and the X-Y plane equation of the quadrant detector, the spot coordinate on the quadrant detector of the $\{O_2\}$ coordinate system is determined as

$$f(e_x, \alpha, \beta, \gamma) = x_{+1} = e_{+1x} - (\hat{r}_{+1x}/\hat{r}_{+1z}) e_{+1z},$$

$$g(e_x, \alpha, \beta, \gamma) = y_{+1} = e_{+1y} - (\hat{r}_{+1y}/\hat{r}_{+1z}) e_{+1z}.$$

Eqns. (11) and (12) are functions with four variables. From the discussion above, the variables the $\beta$ and $\gamma$ measured from the collimation-type sensor are taken into eqns. (11) and (12). Thus, the $e_x$ and $\alpha$ can be obtained by solving simple algebraic functions.

4. Calibration test of the individual element sensors
The resolution and calibration tests of the individual element sensors, collimation-type sensor, and the quadrant detector are first completed and then integrated as the new sensor to measure the four-degrees-of-freedom signals.

4.1. Resolution and calibration test of the collimation-type sensor
The experimental setup is shown in Figure 2. The rotational PZT stage was mounted on a moving stage. A miniature dual-beams laser interferometer was used as reference to check the resolution and verify the collimation-type sensor. The interferometer can simultaneously measure the displacement and $\gamma$ angle of the rotational PZT stage. The displacement resolution of the interferometer was 1.24 nm and the angular resolution is 0.01 arcsec. A diffraction grating with a closely spaced 1200 lines/mm was mounted on the PZT rotational stage. A plane mirror, which was used to reflect the dual laser beams of the interferometer, was mounted at the rear of the grating. A small square voltage signal was applied to the PZT driver of the PZT stage to rotate the grating and plane mirror simultaneously. The output of the interferometer showed the amplitude of 0.3 arcsec well and the collimation-type sensor also responded to the 0.3 arcsec quite well. The angular resolution of the collimation type sensor can be estimated better than 0.3 arcsec. The calibration test of the collimation-type sensor was performed three times, using increasing step voltages to drive the PZT. The input voltages were from -50 V to +50 V with a step 10 V. Within the calibration range of +30 arcsec, the maximum error difference was about 0.8 arcsec. The standard deviation was less than 0.25 arcsec which shows good repeatability for the calibration test results.

4.2. Calibration test of the quadrant detector (Q2)
The calibration tests for two positioning errors of the quadrant detector (Q2) were also carried out with the interferometer. The accuracy calibration of X-and Y-axis positioning errors was made in direct comparison with the interferometer. The residual error of the quadrant detector was found to be within +0.4 $\mu$m for a measuring range of +30 $\mu$m. The standard deviation was less than 0.5 $\mu$m. The straightness error will be less than 2 $\mu$m within a long traveling range. Thus, the calibrated result is sufficient for the straightness error measurement.

5. Experimental results for the diffraction-type four-degrees-of-freedom sensor
For verification of the diffraction-type four-degrees-of-freedom sensor, an experiment was set up, as also shown in Figure 2. In this experiment, the moving stage was moved point-by-point along the Z-axis. In the initial setup, the diffraction type four-degrees-of-freedom sensor was carefully adjusted to
make the diffraction rays light on the centers of the collimation-type sensor and the quadrant detector (Q2). The sensor measured and calculates the displacement, and three rotation errors via the outputs of the collimation-type sensor and the quadrant detector (Q2), and the laser interferometer measured the displacement and γ angular error simultaneously. In this experiment, measurements were taken by moving the stage for a fixed distance of +30µm with an increment of 10 µm, for 3 runs. The measurement errors and the comparison results between the sensor and the laser interferometer were calculated and are shown in Figures. 3–8. The maximum position error was less than 0.4µm. The maximum residual error of the γ angular was less than 0.9 arcsec and the standard deviation was less than 0.4 arcsec. The maximum α angular error and β angular error were about 10arcsec and 1.5arcsec, respectively. These measurement results show the feasibility of the proposed diffraction-type four-degrees-of-freedom sensor.

Figure 3. The comparison measuring result of $e_z$ in the Z-axis.

Figure 4. The average measuring error between the proposed sensor and laser interferometer along the Z-axis.

Figure 5. The measuring errors and comparison results of γ angular error between the proposed sensor and the dual beam laser interferometer.

Figure 6. The error difference and standard deviation of γ angular error measurement.

Figure 7. The measuring result of α angular error of the moving stage.

Figure 8. The measuring result of β angular error manual moving stage.

6. Conclusion

A new sensor for increasing the linear laser encoder measuring ability to that of have five-degrees-of-freedom measurement has been proposed in this paper. This improvement allows the five motion errors (linear displacement, straightness, pitch, roll and yaw) of a high precision motion stage to be measured and further compensated via the control method. This study shows that the five-degrees-of-freedom laser linear encoder can be implemented. Further study will be carried out in order to miniaturize the sensor and integrate it with a high resolution laser linear encoder of a high precision motion stage for precision motion error compensation.
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