Galaxy rotation curves: the effect of the Ampere force

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ABSTRACT

Using an example of the galaxy, an attempt is made to explain the flat rotational curves of gas and plasma in galaxies by means of the Ampere force. Using acceptable models for the galactic magnetic field and plausible physical parameters, we find that the flat rotational curves for gas and plasma can be obtained based purely on the observed baryonic (visible) matter distribution and the ampere force term in the static MHD equation of motion. We also study effects of strength of the magnetic field, its pitch angle and length scale on the rotational curves.

Subject headings: Galaxy: kinematics and dynamics – Galaxy: fundamental parameters – galaxies: magnetic fields – (cosmology:) dark matter.

1. Introduction

Observed flat rotational curves of many galaxies have been subject of long-term controversy. The observational fact that the azimuthal velocity of gas and stars in the galactic plane is constant over a large range of the distances from the centre of a galaxy has yielded two main explanations. In an attempt to save the assertion that the Newtonian gravitational theory holds over the cosmological distances, one such theory assumes the presence of non-baryonic massive dark halo surrounding a spiral disk. In this scenario, gravitational acceleration $GM(<r)/r^2$ which balances the centrifugal acceleration $V^2(r)/r$, is assumed to vary as $1/r$. This means that the mass enclosed within a certain radius $r$, $M(<r)$, scales as $\propto r$. However, this is not what is observed at large radii of the galaxy. The second possible explanation of the flat rotational curves is that the Newtonian gravity does not apply on cosmological scales and further modifications are due (Milgrom 1983). Historically the latter explanation was not favoured due to absence of the general relativistic extension of the theory. However, this drawback was alleviated by the formulation of the generalisation of Einstein’s general relativity based on a pseudo-Riemannian metric tensor and a skew-symmetric rank three tensor field, called metric-skew-tensor gravity (MSTG). The latter leads to a modified acceleration law that can explain the flat rotation curves of galaxies and cluster lensing without postulating exotic dark matter (Moffat 2005). Recently, Brownstein & Moffat (2006) have shown that MSTG can provide a good explanation to the flat rotational curves of a large sample of low and high surface brightness galaxies and an elliptical galaxy. Their MSTG fits were compared to those obtained
using Milgrom’s phenomenological MOND model and to the predictions of the Newtonian / Kepler acceleration law.

In this work, we revisit yet another, the third possibility: we show that the flat rotational curves of galaxies can be obtained based purely on the observed baryonic (visible) matter distribution and the ampere force term in the static MHD equation of motion. It should be mentioned that the present work was complete when author became aware of a similar earlier work by Nelson (1988). The latter studied the dynamical effect of magnetic stress on the tenuous outer gaseous discs of galaxies. Nelson (1988) used an earlier, less observationally constrained model (Sofue et al. 1986) for the magnetic field. No fit to an observational rotational galactic curve was presented. Here we advance the earlier hypothesis by choosing a more realistic model for the galactic magnetic field, as well as perform fit to the Milky Way rotational curve. Other significant previous developments include: Battaner et al. (1992) who argue that an azimuthal magnetic field can carry slightly ionized gas with the general galactic rotation, rendering dark matter unnecessary. It was shown for the illustrative case of M31, that a magnetic field of 6 µG is required, and the synchrotron emission of relativistic electrons in this field is compatible with the observations. However this was not without a subsequent debate (Katz 1994). More recent arguments in favour of magnetic fields, in this context, were also presented in (Battaner & Florido 2000; Battaner et al. 2002; Battaner & Florido 2007). However, based on virial constraints Sanchez-Salcedo & Reyes-Ruiz (2004), on contrary, show that azimuthal magnetic fields hardly speed up HI disks of galaxies as a whole. This demonstrates that the role of magnetic fields in the rotational curves of galaxies is an active area of research and there is no general agreement to date.

2. The model

Let us consider balance of forces acting on a small volume element of a galaxy. This is given by MHD equation of motion, which in its static (\( \partial / \partial t = 0 \)), form reads as (e.g. Gosling & Pizzo (1999)):

\[
(V \nabla)V + 2 \vec{\omega} \times V + \vec{\omega} \times (\vec{\omega} \times \vec{r}) = -\frac{GM(<r)}{r^2} \hat{r} + \frac{\vec{j} \times \vec{B}}{\rho(r)}. \tag{1}
\]

The latter is written in the frame rotating with the galaxy, which does so with uniform angular momentum \( \vec{\omega} \parallel z \). \( \hat{r} \) is a unit vector along radial coordinate, \( r \). We use cylindrical coordinate system \( (r, \phi, z) \). As it is common in the MHD we ignore the displacement current and assume \( \vec{j} = \nabla \times \vec{B} / \mu_0 \). Note that we ignore gas pressure in Eq.(1) which generally is a good approximation for galactic media.

In the simplest possible case, whilst retaining essential physics, we assume that the radial velocity of the galactic matter is zero, \( V_r = 0 \), and the only component of the velocity is azimuthal.
Thus, we end up with the only $r$-component of the MHD equation of motion Eq.(1):

$$-\frac{V_\phi^2(r)}{r} - 2\omega V_\phi(r) - \omega^2 r = -\frac{GM_<(r)}{r^2} + \frac{(\nabla \times \vec{B} \times \vec{B})_r}{\mu_0 \rho(r)}.$$  

(2)

The latter quadratic equation can be solved to yield

$$V_\phi(r) = -\omega r \pm \sqrt{\frac{GM_<(r)}{r} - \frac{r(\nabla \times \vec{B} \times \vec{B})_r}{\mu_0 \rho(r)}}.$$  

(3)

Here $V_\phi(r)$ is the azimuthal velocity in the rotating frame. It is related to the rotational (azimuthal) velocity in the laboratory frame, $\tilde{V}_\phi(r)$, via

$$\tilde{V}_\phi(r) = V_\phi(r) + \omega r = \pm \sqrt{\frac{GM_<(r)}{r} - \frac{r(\nabla \times \vec{B} \times \vec{B})_r}{\mu_0 \rho(r)}}.$$  

(4)

As expected, we could have arrived at the same result by writing the MHD equation of motion in the non-inertial frame rotating with the galaxy as MOND and MSTG models do. Note that $\pm$ signs refer to the galactic rotational curves for the either side of the galactic centre on a given azimuth direction.

Naturally, one could perform a study that would include many galaxies. However, our aim here is to demonstrate the principle. Thus, our case study will be the galaxy and in what follows we fix $|\vec{\omega}| = \omega$ at $\omega = 2\pi/(250 \times 10^6 \times 365.25 \times 24 \times 60 \times 60)$ rad s$^{-1}$, as we know that the galaxy rotates once in 250 million years.

As far as the distribution of ordinary, baryonic matter is concerned, we use observationally constrained model presented in (Brownstein & Moffat 2006). In particular, they used a simple model for $M(r)$

$$M(r) = M \left(\frac{r}{r_c + r}\right)^3,$$  

(5)

with the best fit parameters for the galaxy being $M = 9.12 \times 10^{10}$ $M_\odot$ and $r_c = 1.04$ kpc. The density prescribed by the same model (Brownstein & Moffat 2006), is

$$\rho(r) = \frac{3}{4\pi r^3} M(r) \left(\frac{r_c}{r_c + r}\right).$$  

(6)

The key ingredient of our model is the magnetic field, which manifests itself in the rotational curve via the Ampere force $\vec{j} \times \vec{B}/\rho(r) = \nabla \times \vec{B} \times \vec{B}/(\rho(r)\mu_0)$ in Eq.(4). There is a body of work that is dedicated to the determination of galactic magnetic fields (see for a review Vallée (2004)) including the galaxy (Han & Qiao 1994) or to modelling these by the numerical simulations (Dobbs & Price 2008). The galactic magnetic field models are commonly used in the study of cosmic rays (Stanev 1997; Alvarez-Muniz et al. 2002), due to important ability of magnetic component of the Lorentz force to alter or trap charged cosmic ray particle paths. We, on contrary, use the galactic
magnetic field here in the context of the galactic rotational curves. There are several methods that enable to infer galactic magnetic fields. These include: studies of starlight polarisation, background radio emission, Zeeman splitting, the rotational and dispersion measures of pulsars and extragalactic radio sources (Han & Qiao 1994). The main outcome of these studies is that the magnetic field of the galaxy has Bi-Symmetric Spiral (BSS) configuration, which is given by

\[
B_\phi = B_0(r) \cos(\phi - \beta \ln(r/r_0)) \cos p, \tag{7}
\]

\[
B_r = B_0(r) \cos(\phi - \beta \ln(r/r_0)) \sin p, \tag{8}
\]

\[
B_z = 0. \tag{9}
\]

Here \( \beta = 1/\tan(p) \). Han & Qiao (1994) used Eqs.(7-9) to fit them to the rotation measures of pulsars and extragalactic courses. They used \( B_0(r) = \text{const} \) approximation for their fit, and found the following best fit parameters: \( B_0(r) = (1.8 \pm 0.3) \mu \text{G} \), the pitch angle \( p = -8.2^\circ \pm 0.5^\circ \), and \( r_0 = (11.9 \pm 0.15) \text{kpc} \).

Note that taking into account Eqs.(7-9), Eq.(4) can be rewritten as

\[
\tilde{V}_\phi(r) = \pm \sqrt{\frac{GM_\odot(r)}{r} + \left( \frac{B_\phi}{r} + \frac{\partial B_\phi}{\partial r} - \frac{1}{r} \frac{\partial B_r}{\partial \phi} \right) \frac{rB_\phi}{\mu_0 \rho(r)}}. \tag{10}
\]

3. results

We performed fit of our analytical model (Eq.(10)) to the observational rotational curve of the galaxy (see Fig.(3) from (Brownstein & Moffat 2006)) using non-linear least squares Marquardt-Levenberg algorithm. A set of vastly different ”initial guess” parameters used, and when converged, the algorithm always yielded values similar to e.g. \( B_0(r) = 12 \mu \text{G} \) (this parameter was fixed, while \( p \) and \( r_0 \) were variable), \( p = -38.683\% \pm 2.26 \% \), \( r_0 = 15.3714 \text{kpc} \pm 0.71 \% \). Thus, we regard these values as the best fit parameters of our model. The rotational curve with the best fit parameters is shown with a thin solid line in Figs.(1)-(3). Note that solar system position is at \( \phi = 0 \) (fixed value used in this work).

We gather from Fig.1 that our model, that is the Newtonian gravity plus Ampere force (with no non-baryonic dark matter or MOND or MSTG) provides good fit to the observation rotational curve of the galaxy. We also show how model prediction varies with the strength of the magnetic field (dotted and dash-dotted curves in Fig.1). As expected, increase (by 20 \%) in the magnetic field yields commensurate increase in the rotational velocity, while decrease (by 20 \%) in the magnetic field yields decrease in the rotational velocity. The case of no magnetic field \( B_0 = 0 \) recovers the Newtonian rotational curve (thick solid line) which is well below of the observational data points.

In Fig.2 we study dependence of the rotational velocity on variation in the pitch angle. We gather that an increase (by 20\%) in the magnetic field pitch angle \( p \) results in an increase of the rotational velocity for large \( r \) and a flatter shape. While decrease in \( p \) produces a lower velocity beyond \( r = 16 \text{kpc} \). This conclusion is similar to that of Nelson (1988) (also see discussion below).
Fig. 1.— High resolution rotation curve of the Milky Way (from (Brownstein & Moffat 2006)) and model fit. The dashed line with open symbols with error bars are the observational data. Thick solid line is the Newtonian galaxy rotation curve that is essentially Eq.(10) with $B_\phi = B_r = 0$. Thin solid line is our model best fit that is Eq.(10) with the magnetic fields specified by Eqs.(7)-(9) and $B_0 = \text{const} = 12 \, \mu\text{G}, \, p = -38.6837^\circ, \, r_0 = 15.3714 \, \text{kpc}$. In order to show variation with the magnetic field strength the dotted line shows the model with $B_0 = 12 \times 1.2 \, \mu\text{G}$, and dash dotted line shows the model with $B_0 = 12 \times 0.8 \, \mu\text{G}$ (while keeping the same best fit $p = -38.6837^\circ$ and $r_0 = 15.3714 \, \text{kpc}$)
Fig. 2.— The same as in Fig.(1), but here in order to show variation with the pitch angle, \( p \), the dotted line shows the model with \( p = -38.6837 \times 1.2^\circ \), and dash dotted line shows the model with \( p = -38.6837 \times 0.8^\circ \) (while keeping the same best fit \( B_0 = 12 \mu \text{G} \) and \( r_0 = 15.3714 \text{ kpc} \))
Fig. 3.— The same as in Fig. (1), but here in order to show variation with the length scale, $r_0$, the dotted line shows the model with $r_0 = 15.3714 \times 1.1$ kpc, and dash dotted line shows the model with $r_0 = 15.3714 \times 0.9$ kpc (while keeping the same best fit $B_0 = 12 \mu$G and $p = -38.6837^\circ$).
In Fig. 3 we investigate how variation in the length scale of the magnetic field \( r_0 \) affects the rotational curve. Note that we choose a smaller variation margin (10 %) because the dependence on \( r_0 \) turns out to be strong. We gather from this graph that an increase in \( r_0 \) yields in the increase (overshoot over data points) in the rotational velocity, while decrease in \( r_0 \) produces small velocities. Again, this result seems reasonable because \( r_0 \) essentially quantifies the extent of the magnetic field. Hence, as \( r_0 \) decreases, the effect of Ampere force is weakened and rotation velocity sharply falls down.

4. Conclusions

Earlier work of Nelson (1988) conjectured about importance of the dynamical effect of magnetic stress on the tenuous outer gaseous discs of galaxies. The main useful conclusion of Nelson (1988) was that increase in the pitch angle of the magnetic field yields higher rotational velocities (this result is also corroborated in our model – see our Fig. 2). The model presented here seems superior to Nelson (1988) in that we used a more advanced magnetic field model of Han & Qiao (1994), and we perform an actual fit to the Milky Way rotational curve. The latter is possible because our model is simpler and presents an analytical expression for the rotational velocity Eq. (10) as opposed to the need for solving an ordinary differential equation (Eq. (7) from Nelson (1988)). Other significant previous developments include (Katz 1994; Battaner et al. 1992; Battaner & Florido 2000; Battaner et al. 2002; Battaner & Florido 2007; Sanchez-Salcedo & Reyes-Ruiz 2004).

In conclusion, it seems that the need for dark matter decreases over the last few decades. Initially, cold dark matter models would typically assume 90% of non-baryonic content of a galaxy (Blumenthal et al. 1984). After observations of Perlmutter et al. (1999) it is now believed that the fraction of the total energy density of the universe contained in dark matter (non-baryonic content) is 30%. In the present study via inclusion of the Ampere force, \( \vec{j} \times \vec{B} \), we show that as far as rotational curve of gas and plasma of the Milky Way is concerned, the need for non-baryonic matter is negligible, as well as, there is no need to modify usual laws of gravity. Further study is needed whether the model formulated in this work can be used to fit rotational curves of other known galaxies. The key ingredient of our model is the magnetic field, thus it is knowledge is necessary to apply the model.

Other weaknesses of this model could be e.g. how well galactic plasma couples to the magnetic field (for the Ampere force, \( \vec{j} \times \vec{B} \), to be effective). Naturally this coupling is prescribed by the degree of ionisation of the medium, which in turn, is prescribed by the Saha equation and is sensitive to the temperature. In general, initial temperatures of galaxies are expected to be high because so called virial temperature (page 557 from (Gilmore et al. 1989)) \( T_{\text{virial}} \approx \frac{GMm_p}{(kR)} \), where symbols have usual meaning, for a typical size galaxy is of the order of \( 10^6 \) K. However, after cooling phase galactic discs are much cooler at about \( \approx 10^4 \) K. Quireza et al. (2006) quote electron temperatures in the disc of galaxy of the order of \( 10^4 \) K which means that degree of ionisation of the galactic disc is sufficient to couple plasma to the magnetic field and the Ampere force. After
all, solar photosphere which is at temperature of only 6000K is commonly described by MHD approximation, despite low degree of ionisation and the presence of large concentration of neutrals. Also, in addition to thermal collisions some significant ionisation may be provided by the cosmic rays (mostly protons) that are accelerated at the bow and termination shocks. A substantial flux of cosmic rays is produced in a shock at Galactic north, a direction toward which our Galaxy has long been known to be moving in the Local Supercluster with the velocity of 200 km s$^{-1}$ (Medvedev & Melott 2007).

The origin of the magnetic field in the galaxy itself is deeply coupled with the Galaxy’s dynamics and MHD via the dynamo mechanism. The field strength and morphology are dependent on the dynamics of the plasma, which is a function of density, temperature, turbulent velocity, and galactic rotation. Therefore, the centrifugal force due to galactic rotation acts both on the plasma and magnetic field, and not on the plasma alone. Such advanced topics are naturally beyond the scope of the simple model presented here.

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