Dynamic Behavior of Variable Cross-Section Offshore Wind Turbine with Flexible Foundation Using Finite Element Method

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Abstract. A Finite Element model for variable cross section offshore wind turbine with flexible foundation is developed to accurately determine the fundamental frequency of the tower-nacelle system. The tower is modeled using two-noded Euler beam element with two degrees of freedom, namely transverse displacement and rotation, and Hermite polynomial shape function. The governing equations are obtained using Euler-Lagrangian energy-based approach, including the effect of variable tower cross section and the nacelle mass. The soil-tower interaction is modeled by rotational and lateral springs at the lower end of the tower. Four offshore wind turbines were selected as case studies in terms of validation and comparison. The results of the current model, compared to those of an experimental, clearly show that it better simulates the dynamic behavior of the tower-nacelle system compared to those of the constant cross-section approximated models.

1. Introduction
The fast growing demand of energy, accompanied with the depletion of the traditional oil resources, and the increasing restrictions on the environmental echo of the proposed technologies, made the green energy resources as a worldwide strategic goal. The most influential limitation to invest in such clean energy resources, is its relatively high operational cost. Wind energy can be considered as one of the most appealing energy resources due to its availability and its promising cost efficiency. Offshore wind turbines, in particular, is gaining more interest due to its high-energy density, lower turbulence, lower wind shear, and fewer civil complaints. Despite of all such attractive aspects, compared to onshore wind turbine, its maintainability still present a significant constraint which calls for more research on the factors that affect its dynamic behavior and hence its service life.

The stability, performance, and lifetime of wind turbine systems can be fundamentally traced back to their dynamic behavior, and most importantly its natural frequencies. Consequently, much attention has been paid to the dynamic analysis and simulation of such turbines which include experimental modal and theoretical modal analyses. The main challenge in simulating the dynamic behavior of wind turbines,
using theoretical models, is the proper consideration and estimation of all the parameters which will have a significant effect on the accuracy of the proposed dynamic model.

An extensive research work has been found in the literature in topics related to wind turbine dynamics; either in terms of the dynamics of its main components, their dynamic interaction, and the main parameters presumed significant; or in terms of different proposed mathematical and experimental techniques which strived toward better accuracy along with being computationally and experimentally economic. Thresher [1] reviewed the various analysis methods of the dynamic simulation of horizontal axis wind turbines, and compared it with experimental data. Halfpenny [2] used the finite element method to develop a new frequency domain model, able to analyze fully flexible on and offshore wind turbines. Ahlstrom [3] proposed a finite element model to simulate the aeroelastic dynamic response of horizontal axis wind turbines. Kessentini et al. [4] used the differential quadrature method (DQM) to develop a mathematical model of a horizontal axis turbine with flexible tower and blades. The model incorporates the nacelle pitch angle and structural damping. Since this paper will focus, particularly, on the dynamic behavior of wind turbine tower – nacelle interaction of offshore wind turbines with flexible foundation, a literature review on the different aspects of such problem will be presented hereinafter.

Abrate [5] presented simple formulas that can be used to predict the fundamental frequency of non-uniform beams with various boundary conditions. Oz [6] provided a finite element model for the dynamics of an Euler-Bernoulli beam with a concentrated mass located at different locations. Li et al. [7] investigated the free flexural vibration of a non-uniform bar, under the effect of various cases of axial loads, by reducing its differential equation to Bessel’s equations. It was found that the fundamental frequency of cantilevered tall structures, which was simulated by their model, proved to have values closer to the measured field data than that of the computed without considering the existing axial forces. Mehmet et al. [8] presented an analytical solution for the modal analysis of variable cross-section isotropic beams with different boundary conditions. They came to the conclusion that the non-uniformity of the cross-section would have an influence on the natural frequencies and mode shapes. Coşkun et al. [9] presented a transverse vibration analysis of uniform and non-uniform Euler-Bernoulli beams by applying analytical approximate techniques. Zamorska [10] provided a series solution for the fourth order differential equation with various coefficients occurring in the vibration problem of Euler-Bernoulli beam, which can be extended to model the dynamic behavior variable cross wind turbine towers. Kumar et al. [11] presented a finite element model to accurately predict the higher frequencies of uniform cantilever beams with tip concentrated mass.

Molenaar [12] provided a comprehensive review on the theoretical basics and the design options of wind turbine dynamics, including offshore machines under wave action. Oh et al. [13] provided a comprehensive literature review on the different types of wind turbine foundation, in addition to the different modelling techniques of the structure-soil interaction. Xu et al. [14] utilized the finite element analysis to study the static and dynamic behavior of a tower structure supporting a 600 KW wind turbine, while including the effect of elastic subgrades. Bhattacharya et al. [15] presented a summary the results from a series of 1:100 scale tests of a V120 Vestas turbine supported while being supported by two types of foundation; monopiles and tetrapod suction caissons. The results were found useful in terms of providing an insight into the long term performance and in identifying some issues related to the soil-structure interaction. Adhikari et al. [16] presented a closed form approximate expression that predicts the fundamental frequency of wind turbine towers with flexible foundation. Their analytical model is based on Euler-Bernoulli beam-column theory with elastic end supports. Bhattacharaya et al. [17] introduced novel experimental techniques to obtain the parameters needed for the dynamic model of offshore wind turbines. Experimental results showed that the natural frequencies and the damping factors of the wind turbine tower vary, significantly, based on the model of the soil - foundation interaction. Adhikari et al. [18] characterized the dynamic behavior of offshore wind turbines using a closed form solution based on Euler-Bernoulli beam-column with elastic end supports. Arany et al. [19] provided an analytical model of offshore wind turbine with flexible foundation in order to have a fast and reasonably accurate estimate for its fundamental frequency. Their model was found suitable for preliminary design or to verify the finite element results. Alamo et al. [20] studied the dynamic effect of foundation parameters of offshore wind turbine systems. Their results simulated the effect of considering the structure – soil interaction on the accuracy of the estimated fundamental frequency and equivalent
Kumar and Nasar [21] modeled the dynamic behavior of wind turbines subject to environmental and rotor vibration forces. The tower was modeled as Euler-Bernoulli beam-column with elastic end supports. Wang et al. [22] investigated the wind turbine dynamic behavior by employing an improved model based on vibration signal analysis. This model is formulated using the Euler–Lagrangian approach in which the nacelle–tower and tower–foundation dynamic interactions were considered. Bouzid et al. [23] developed a nonlinear finite element model to simulate the lateral behaviors of monopiles which support offshore wind turbines, taking into account the flexibility of the foundation. So far, and to the authors’ knowledge, the tower–nacelle dynamic interaction, under the combined effect of the flexible foundation and variable tower cross-section, has not been investigated yet neither by analytical nor numerical simulation.

In this paper, a finite element model for variable cross-section offshore wind turbine with flexible foundation is developed to accurately determine the fundamental frequency of the tower-nacelle system. The tower is modeled using two-nodded Euler beam elements with two degrees of freedom, namely transverse displacement and rotation, and Hermite polynomial shape function. The governing equations are obtained using Euler-Lagrangian energy-based approach, including the effect of variable tower cross-section and the nacelle mass. The soil-tower interaction is modeled by rotational and lateral springs at the lower end of the tower.

2. Modeling

Physical model

A typical wind turbine tower–nacelle system, with variable cross-section and flexible foundation, will be studied in the current work. The tower is modeled as an Euler beam, while the nacelle is considered as concentrated mass $M_t$ at the tip of rotary inertia $J$, as shown in Fig. 1. The bending stiffness of the beam is $EI(x)$, where $E$ is the young’s modulus, $I$ is the second moment of area, and $x$ is a spatial coordinate which starts from the bottom of the tower and directed up. The interaction of the structure with the foundation is modeled using two springs; rotational spring with spring stiffness $k_r$ and lateral spring with spring stiffness $k_l$. The compressive effect of the top mass, due to the weight of the nacelle, will be included as a geometric stiffness term along with the elastic stiffness and is denoted as $P$.

![Figure 1. Idealization of a tower–nacelle system with flexible foundation](image-url)
Mathematical models

The tower is discretized using two-nodded beam element of length ‘L’ with two degrees of freedom, namely, transverse displacement and rotation at each node as shown in Fig. 2. To describe the displacement at in between nodal points, Hermite polynomial shape functions are used. Therefore, the transverse displacement \( w(x,t) \) can be written as

\[
 w(x,t) = [N][q] = [N_1 N_2 N_3 N_4] \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}
\]  

(1)

where \( N \) is the shape function and \( q \) is the nodal displacement.

\[
 N_1 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \quad N_2 = x - \frac{2x^2}{L} + \frac{x^3}{L^2} \quad N_3 = \frac{3x^2}{L^2} - \frac{2x^3}{L^3} \quad N_4 = -\frac{x^2}{L} + \frac{x^3}{L^2}
\]  

(2)

Figure. 2 Two nodded Beam element

For more general structural elements, the model is formulated using the Euler–Lagrangian energy-based approach as follows:

\[
 \frac{d}{dt} \frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = F
\]  

(3)

where \( T \) is the total kinetic energy of the system and \( U \) is the total potential energy of the system; \( q \) is the nodal displacement; \( \dot{q} \) is the nodal velocity; and \( F \) is the nodal force.

Elemental potential energy \( U^e \) of the beam is given by

\[
 U^e = \int_0^L \frac{1}{2} EI(x) \left[ \frac{\partial^2 w(x,t)}{\partial x^2} \right]^2 dx - \int_0^L \frac{1}{2} M_t g \frac{\partial w(x,t)}{\partial x} dx
\]  

(4)

where the first term in eqn. (4) is the effect of the elastic stiffness and the second one is the effect of the top mass (geometrical stiffness).

Elemental kinetic energy \( T^e \) of the beam element is given by

\[
 T^e = \int_0^L \frac{1}{2} (\dot{w}^2) dx = \int_0^L \frac{1}{2} \rho A(x) (\dot{w}^2) dx
\]  

(5)

where \( \rho \) is the mass density per volume of the tower and \( A(x) \) is cross sectional area of the tower.
By differentiate we lead to

\[
[m] \{ \dot{q} \} + [k] \{ q \} = 0
\]

, where \( m \) is elemental mass matrix for the beam and \( k \) is the elemental stiffness matrix for the beam.

\[
m = \int_0^L \rho A(x) [N]^T [N] \, dx
\]

\[
[k] = k_e + k_g
\]

\[
k_e = \int_0^L E I(x) [B]^T [B] \, dx \quad \text{where} \quad [B] = \frac{\partial^2 [N]}{\partial x^2}
\]

\[
k_g = -\int_0^L M_t g \frac{\partial N}{\partial x} \left( \frac{\partial ^2 N}{\partial x^2} \right) \, dx
\]

, where \( k_e \) is the elemental elastic stiffness matrix and \( k_g \) is the elemental geometrical stiffness matrix, noting that the negative sign is due to the compressive effect of the top mass.

From eqn. (10), a generalized eigenvalue problem can be stated as follows:

\[
| - \omega^2 [M] + [K] | = 0
\]

, the eigenvalues are the natural circular frequencies of oscillation of the structural system, and the eigenvectors are the amplitude vectors (mode shapes) corresponding to the natural frequencies.
Boundary Conditions

Another important step, in the current modelling process, which is the inclusion of 3 boundary conditions; the lateral stiffness $k_l$, the rotational stiffness $k_r$ at the bottom of the tower and the Top mass $M_t$ at the top of the tower.

The mentioned boundary conditions are applied as follows:

$$
K = \begin{bmatrix}
  k_{11} + k_l & k_{12} & \cdots & \cdots \\
  k_{21} & k_{22} + k_r & \cdots & \cdots \\
  \vdots & \vdots & \ddots & \vdots \\
  k_{n1} & \cdots & \cdots & k_{nn}
\end{bmatrix}
$$

(16)

$$
M = \begin{bmatrix}
  m_{11} & \cdots & \cdots & m_{1n} \\
  \vdots & \ddots & \vdots & \vdots \\
  \vdots & \vdots & \ddots & \vdots \\
  m_{n1} & \cdots & \cdots & m_{nn}
\end{bmatrix}
$$

where $n$ is the number of degrees of freedom.

3. Results

Comparative Study

To validate the current proposed finite element model, modal analysis will be conducted using the real wind turbine data presented in Table 1 and the results of the current model are compared with the analytical results presented by Adhikari et al. [18]. Noting that, and for the sake of validation, we followed Adhikari et al. [18] in their assumption of constant cross-section. Three non-dimensional parameters is defined here to utilized here and after; which are non-dimensional axial load $\nu = \frac{\nu H^2}{EI}$, non-dimensional rotational foundation stiffness $\eta_r = \frac{K_r H^3}{EI}$, and non-dimensional lateral stiffness ($\eta_l = \frac{K_l H^3}{EI}$).

| Table 1. Material and geometric properties of the turbine tower (Tempel and Molenaar [12]) |
|---------------------------------------------------------------|------------------|
| Turbine Structure Properties | Numerical Values |
| Length, $H$ | 81 m |
| Average Diameter | 3.5 m |
| Thickness | 0.075 m |
| Mass Density | 7800 Kg/m$^3$ |
| Young's Modulus | 2.1E+11 Pa |
| Top Mass $M_t$ | 130000 Kg |

Four study cases will be presented in order to assess the validity of the current proposed model. The first case is to study the variation of the fundamental frequency with the non-dimensional axial load $\nu$, while keeping the non-dimensional rotational foundation stiffness and non-dimensional lateral stiffness constant to be $\eta_r = 2$ and $\eta_l = 1$, respectively. Figure 3.a shows the results of the first case and it is found that the results of the current formulation deviate from to those of Adhikari et al. [18] by a maximum of 0.26%. The second case is similar to the first one but the values of the non-dimensional rotational foundation stiffness and non-dimensional lateral stiffness constant are changed to $\eta_r = 500$ and $\eta_l = 100$, respectively. Figure 3.b shows the results of the second case and the maximum deviation found to be 0.16 %. The third case represents the variation of the fundamental frequency with the non-dimensional rotational foundation stiffness $\eta_r$, while assuming $\nu = 0.001$ and $\eta_l = 100$. Figure 3.c shows the results of the second case and the maximum deviation found to be 0.04 %. The fourth case represents the variation of the fundamental frequency with the non-dimensional lateral foundation stiffness $\eta_l$, while assuming $\nu = 1$ and $\eta_r = 5$. Figure 3.d shows the results of the fourth case and the maximum deviation found to be 0.19 %. Therefore, the results of the current formulation almost coincide with those of Adhikari et al. [18].
It is worth to highlight that the fundamental frequency of the system increases with the increasing value of the stiffness parameters $\eta_r$ and $\eta_l$, due to the fact that the increase in the rotational and lateral stiffness properties stabilizes the system. In addition, it is noticed that after reaching certain values of $\eta_r$ and $\eta_l$ (typically above 100), any further increase in their values do come up with any change in the frequency, and consequently the foundation - structure interaction can be essentially considered as fixed support. In terms of the nacelle mass effect, the natural frequency of the system is found to decrease, as expected, with the increasing value of $\nu$, and this attributed to the reduced stiffness of the system due to the compressive effect of the nacelle. Therefore, all these parameters should be carefully estimated because the natural frequency of the system should not interfere with the blade passing frequency.

**Variable Cross-Section Towers**

The main aim of the current formulation is to study the quantitative effect, and hence the worthiness, of including cross-sectional variation of wind turbine towers on the simulated values of the fundamental frequency. Toward this end, the results of the current formulation, which considers the variation of tower cross-section, will be compared with other simulations which assumed constant cross-section tower. The data of three real wind turbines, presented in Table 2 and provided by Arany et al. [19], will be used as study cases while their experimental values, provided by [19], will be used as a reference for comparison.
Table 2. Technical Data of Three Real Wind Turbines [19]

| Wind Turbine                  | Lely A2: NM41 2-bladed | Irene Vorrink 600 kW | Walney 1 S 3.6 MW |
|-------------------------------|------------------------|----------------------|------------------|
| Equivalent bending stiffness  | – EI [GNm]             | 22                   | 21.5             | 274              |
| Young’s modulus of the tower  | – E [GPa]              | 210                  | 210              | 210              |
| Shear modulus of the tower    | – G [GPa]              | 97.3                 | 97.3             | 97.3             |
| Tower height – L [m]          |                       | 41.5                 | 51               | 83.5             |
| Bottom diameter – D_b [m]     |                       | 3.2                  | 3.5              | 5                |
| Top diameter – D_t [m]        |                       | 1.9                  | 1.7              | 3                |
| Tower wall thickness range – t |                       | 12                   | 8-14             | 20-80            |
| Lateral foundation stiffness  | – kL [GN/m]            | 0.83                 | 0.76             | 3.65             |
| Rotational foundation stiffness | – kR [GN/m/rad]       | 20.6                 | 15.5             | 254.3            |
| Cross-coupling foundation stiffness | – kLR [GN]        | -2.22                | -2.35            | -20.1            |
| Top mass (rotor-nacelle assembly) – m | | 32,000               | 35,700            | 234,500          |
| Tower mass – m [kg]           |                       | 31,440               | 31,200           | 260,000          |
| Average wall thickness – t_b [mm] |                       | 12                   | 11               | 40               |
| Shear coefficient – k [-]     |                       | 0.5328               | 0.5326           | 0.5327           |

Table 3. Convergence Test of the FE Model

| No. of Elements | Lely A2: NM41 2-bladed | Irene Vorrink 600 kW | Walney 1 S 3.6 MW |
|-----------------|------------------------|----------------------|------------------|
| f (Hz)          | R.E. (%)               | f (Hz)               | R.E. (%)         |
| 10              | 0.753873192            | 0.5354378            | 0.339998327      |
| 20              | 0.753872349            | 0.5354363            | 0.339997991      | 9.8925E-05      |
| 30              | 0.753872253            | 0.5354362            | 0.339997976      | 4.3914E-06      |
| 40              | 0.753872381            | 0.5354362            | 0.339997963      | 3.6978E-06      |
| 50              | 0.753872284            | 0.5354361            | 0.339997925      | 1.1185E-05      |
| 60              | 0.753872468            | 0.5354362            | 0.339997969      | 1.291E-05       |
| 70              | 0.753872087            | 0.5354358            | 0.339997977      | 2.1906E-06      |
| 80              | 0.753871529            | 0.5354373            | 0.339998112      | 3.9937E-05      |

Table 4 presents a comparison between the results of the current formulation and the measured fundamental frequency of the three real wind turbines presented in Table 2. It was found that the current model is in good agreement with the measured values of Irene Vorrink 600 kW and Walney 1 S 3.6 MW, showing a maximum error of 2.86%. Therefore, the current formulation result proved to be closer to the experimental data compared to those presented in [18, 19] which assumes constant cross-section tower. While the result of Lely A2: NM41 2-bladed did not show the same but, surprisingly, it is case with all the other models found in the literature. When this case was further studied, it was found that, all of the simulated frequencies are closer to the frequency of the infinitely stiff foundation, which might question the accuracy of the measured value in this particular case.
Table 4. Results of the current formulation compared with measured values

| Type                     | Lely A2: NM41 2-bladed | Irene Vorrink 600 kW | Walney 1 S 3.6 MW |
|--------------------------|------------------------|----------------------|------------------|
| Measured                 | 0.634                  | 0.546                | 0.35             |
| Current Formulation (Error %) | 0.753 (18.9%)          | 0.535 (1.9%)         | 0.339 (2.85%)    |
| Analytical solution [18] | 0.74 (16.7%)           | 0.457 (16.3%)        | -                |
| Euler–Bernoulli [19]     | 0.735 (15.9%)          | 0.456 (16.5%)        | 0.331 (5.9%)     |
| Timoshenko [19]          | 0.734 (15.8%)          | 0.456 (16.5%)        | 0.331 (5.9%)     |
| Fixed Base frequency     |                        |                      |                  |
| (infinitely stiff foundation) | 0.765                  | 0.475                | 0.345            |

In order to demonstrate the advantage, in terms of accuracy, of modeling the tower as variable cross section, the “Walney 1 S 3.6 MW” wind turbine as study case for comparison. Table 5 presents the results of the current modelling of this wind turbine, once modeled as of constant cross section tower with average diameter 4m, then with a variable cross-section and both results are compared with the measured frequency. This comparison shows the worthiness of simulating the tower as of variable cross-section rather than being of constant cross-section, which is reflected by decreasing the error from being 20 %, in the case of constant cross-section, to be 2.86 % in the variable cross-section one.

Table 5. Worthiness of Variable Cross-Section Formulation for the “Walney 1 S 3.6 MW” Wind Turbine Case

| Current Formulation | Constant Cross-Section | Variable Cross-Section | Measured Frequency | % Error for Variable Cross-Section | % Error for Constant Cross-Section |
|---------------------|------------------------|------------------------|-------------------|-----------------------------------|------------------------------------|
|                     |                        | 0.28                   | 0.34              | 0.35                              | 2.86                               | 20                                 |

4. Conclusion

A finite element model for variable cross section offshore wind turbine with flexible foundation is developed to accurately determine the fundamental frequency of the tower-nacelle system. The tower is modeled using two-noded Euler beam elements with two degrees of freedom, namely transverse displacement and rotation, and Hermite polynomial shape function. The governing equations are obtained using Euler- Lagrangian energy-based approach, including the effect of variable tower cross section and the nacelle mass. The soil-tower interaction is modeled by rotational and lateral springs at the lower end of the tower. The main aim of the current formulation is to study to the quantitative effect, and hence the worthiness, of including cross-sectional variation of wind turbine towers on the simulated values of the fundamental frequency.

The results of the current formulation, was compared with other simulations which assumed constant cross-section tower using the data of three real wind turbines. The results showed that the effect of simulating the tower as of variable cross-section rather than being of constant cross-section, can be significant as the error, of a certain wind turbine case, dropped from being 20 % when using constant cross-section assumption, to be 2.86 % in the variable cross-section one.

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