Regression Models and Response Surfaces to Assess the External Stability of Soil Nailing Walls

Modelos de regresión y superficies de respuesta para evaluar la estabilidad externa de muros de soil Nailing

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Abstract

Due to its fast and economical execution, the geotechnical solution known as soil nailing walls is widely used for the stabilization of slopes and ground excavations. At the design stage, verification of the external stability of the soil nailing wall is one of the most important acceptability criteria. The main objective of this work is to evaluate the external stability of soil nailing walls, considering the variability influence of their height and the geomechanical parameters of the soil in-situ. The probabilistic 2k factorial design methodology has been applied to generate 32 experiments. A vertical soil nailing wall, with variable height, under pseudo-static load conditions, and executed in residual soil from granitic rock has been used as a prototype model. Based on the analysis of the observations of the 32 experiments, three regression models have been developed, which can be used to predict the value of the factors of safety with arbitrary realizations. Furthermore, the observations show that the factors that most influence the external stability of soil nailing walls are the height of the wall, the cohesion and the friction angle of the soil in-situ.

Keywords: Soil nailing wall, external stability, factor of safety, experimental design, response surface

Resumen

Debido a su rápida y económica ejecución, la solución geotécnica conocida como muros de soil nailing se utiliza ampliamente para la estabilización de taludes y excavaciones del terreno. En la etapa de diseño, la verificación de la estabilidad externa del muro de soil nailing es uno de los criterios de aceptabilidad más importantes. El principal objetivo de este trabajo es evaluar la estabilidad externa de muros de soil nailing, considerando la influencia de la variabilidad de su altura y los parámetros geomecánicos del suelo in-situ. La metodología probabilística de diseño factorial 2k ha sido aplicada para generar 32 experimentos. Un muro de soil nailing vertical, con altura variable, bajo condiciones de carga pseudo-estática, y ejecutado en suelo residual de roca granítica ha sido utilizado como modelo prototipo. Basándose en el análisis de las observaciones de los 32 experimentos, se han desarrollado tres modelos de regresión, los que pueden ser usados para predecir el valor de los factores de seguridad con realizaciones arbitrarias. Además, las observaciones muestran que los factores que más influencia tienen sobre la estabilidad externa de muros de soil nailing son la altura del muro, la cohesión y el ángulo de fricción del suelo in-situ.

Palabras clave: Muro de soil nailing, estabilidad externa, factor de seguridad, diseño experimental, superficie de respuesta

1. Introduction

Soil nailing is an onsite stabilization technique, whose main principle is the soil reinforcement through passive elements resisting tensile loads. Soil nailing allows forming an equivalent gravity-wall structure, with higher shear strength when compared to the original in-situ soil (Juran et al., 1190). In civil engineering, soil nail walls have been used for over five decades given their fast execution and low costs (Lazarte et al., 2015). In general, they are used for stabilizing natural slopes, urban cut excavations, roadway cuts, foundations, bridge piers and underground excavations (Bruce and Jewell, 1986); (Juran, 1987); (Briaud and Lim, 1990); (Alston, 1991); (Gu et al., 2014); (Lazarte et al., 2015).

In terms of ultimate limit states, the geotechnical design of soil nail walls (SNW) should meet three main requirements: external, internal and facing stability. As for the external stability, the analysis consider the following failure modes: overall failure, sliding failure (i.e. base length cut) and bearing capacity failure (i.e. basal elevation in soils with low shear strength, particularly soft soils) (Zevgolis and Daffas, 2018).

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There are usually uncertainties regarding the design parameters in soil nail walls to assess their external stability (Gässler and Gudehus, 1983); (Lazarte et al., 2003b); (Lazarte et al., 2011); (Babu and Singh, 2009); (Lin and Liu, 2017), (Lin et al., 2017)). The external failure may occur even if the calculated factor of safety is higher than the minimum required by the design codes of different countries. Therefore, it is important that the external stability analyses of soil nail walls consider the variability of design parameters. In this sense, and because the behavior of soil nail walls is significantly affected by the complex nature of the interaction between their main components (loading conditions and reinforcement elements), as well as the uncertainty induced by the variability of geomechanical parameters of the in-situ soil, it is necessary to apply probabilistic methods to assess the soil nail wall stability.

This work has developed an experimental design using limit equilibrium methods combined with $2^k$ factorial design. Subsequently, and based on results, three regression models are proposed to predict the values of (i) the overall factor of safety, (ii) the sliding factor of safety, and (iii) the bearing capacity factor of safety. Finally, it defines the design parameters with the biggest impact on the external stability of soil nail walls.

2. Stability of Soil Nail Walls

(Figure 1) shows the different failure modes of soil nail walls under instability conditions. The geotechnical design of SNW must guarantee the safety and reliability in the face of different failure modes. In general, terms, the failure modes of soil nail walls can be classified into three groups, external failure mode, internal failure mode and facing failure mode (Lazarte et al., 2003a), (Lazarte et al., 2011), (Lazarte et al., 2015). This study offers an analysis of the external failure modes.

![Figure 1. Failure modes in soil nail walls adapted from (Lazarte et al., 2015)](image-url)
The external failure modes deal with the failures related to the overall stability, sliding and bearing capacity of the soil nail wall. The overall stability refers to the loss of the general stability of the reinforced soil mass, which may occur when the total loads exceed the strengths provided by the soil along the critical failure surface and the nails throughout it. The sliding stability refers to the horizontal movement of the entire reinforce soil mass along its base. The bearing capacity stability refers to the failure that may occur when soil nail walls are built on soils with low shear strength. (Figure 2) shows the potential external failure modes of soil nail walls under instability conditions.

Currently, the geotechnical design of SNW is based on the Allowable Stress Design (ASD) (Lazarte et al., 2003a), (Lazarte et al., 2011), (Lazarte et al., 2015), where factors of safety are assigned to control the safety levels of a structure against different potential failure modes. The factor of safety is defined as the relationship between a system’s general strength and the total loads that the system has to bear.

The limit equilibrium methods, such as the approximation through a bi-linear sliding surface or sliding blocks (Gässler and Gudehus, 1981), can model quite well the potential failure mechanism generated in soil nail walls. However, the use of this type of models to study the external stability aspects of SNW should consider the great influence of the variability of geomechanical parameters of the in-situ soil, such as unit weight, cohesion, internal friction angle and pullout strength, as well as the height of the SNW. In order to quantify the influence of these factors on the external stability of SNW, an experimental design was developed, which uses limit equilibrium methods combined with $2^k$ factorial design, which is discussed in detail in the following sections.

3. Methodology

3.1. $2^k$ Factorial Design

Factorial design has been widely used in experiments involving several factors, thus it is necessary to the study the factors’ joint effects on a response variable. The term joint effects of the factors generally refers to the main effects and interactions between the different factors that intervene in the response. A special case of factorial design is that where each factor $k$ selected has only two levels. Given the fact that each replication of this design has exactly two experimental designs, it is usually called $2^k$ factorial design. A complete replication of this kind of experimental design requires $2 \times 2 \times \ldots \times 2 = 2^k$ observations. (Montgomery, 2001), (Myers and Montgomery, 2002), (Myers et al., 2016) discuss and analyze in detail this type of experimental design.

The $2^k$ factorial design provides a small number of experiments, which allow studying the influence of $k$ factors on a response. As mentioned earlier, the present research considers five factors for the experimental design, that is, $k = 5$. Therefore, the analysis is limited to the $2^5$ factorial design.
3.2. Limit Equilibrium Models

A vertical soil nail wall was used to demonstrate how the proposed methodology was applied. (Figure 3) shows the complete geometry of the soil nail wall. Since this work is focused on studying the influence of the variability of geomechanical parameters of in-situ residual soil (as a result of weathering and subsequent decomposed granite) and the SNW height, all the other parameters involved in the external stability analysis have been considered in a deterministic way, and are indicated in (Table 1). Thus, the SNW height, unit weight, cohesion, internal friction angle, and pullout strength of the in-situ soil have been considered in a probabilistic way.

![Figure 3. Overall geometry of the soil nail wall and the failure mechanism](image)

| Parameter                        | Value |
|----------------------------------|-------|
| Horizontal acceleration coefficient, $k_h$ [-] | 0.20  |
| Overloading, $q$ [kPa]           | 12    |
| Nail length, $L$ [m]             | 0.70H |
| Nail diameter, $D$ [mm]          | 90    |
| Nail inclination, $\alpha$ [°]   | -15   |
| Nail strength, $T_{adm}$ [kN]    | 170   |
| Spacing between nails, $S$ [m]   | 1.50  |
| Inclination, $\beta$ [°]         | 90    |
| Facing thickness, $e$ [cm]       | 20    |
| Facing concrete strength, $f'_c$ [MPa] | 25    |
| Horizontal acceleration coefficient, $k_h$ [-] | 500   |

The proposed methodology can be applied within a range of limit equilibrium methods. A simple geometry (vertical SNW), subjected to pseudo-static loading conditions, modeled with the GGU-Stability software (GGU, 2016), was used to demonstrate its application. This commercial software can model the SNW through the sliding blocks method, whose main characteristic is that the failure mechanism is generated by the interaction of two blocks, a passive and an active one. Villalobos et al. (2013, 2018) have studied the suitability of using this method to evaluate the stability of soil nail walls under pseudo-static loading conditions in granite residual soils.
(Table 1) summarizes the deterministic parameters used in all limit equilibrium models, including the coefficient of peak horizontal ground acceleration, crest overloading, SNW inclination, diameter, length, strength, inclination and spacing ($S_h = S_v$, i.e. horizontal spacing equal to vertical spacing) of the nails, and the type of facing. The GGU-Stability software automatically defines the number of potential sliding surfaces and as a result produces the values of the minimum external safety factors (i.e. failures: sliding and bearing capacity).

### 3.3. 2$^5$ Factorial Design

(Table 2) summarizes the distribution assigned to each probabilistic parameter and their statistical moments, where $H$ represents the SNW height, $G$, $C$, $F$ and $R$, the unit weight, the cohesion, the internal friction angle and the pullout strength of the in-situ residual soil, respectively. It should be mentioned that, in general, parameters $H$ and $G$ represent part of the loads acting on the soil nail wall; while $C$, $F$ and $R$ are part of the SNW strength. Additionally, the relative, maximum and minimum values for each selected factor or parameter have been calculated with the following expression (Equation 1):

$$x_{i\text{ max/min}} = \mu_{xi} \pm 1.65\sigma_{xi}$$

Where, $\mu_i$ and $\sigma_i$ are the median and standard deviation of the parameter of interest $i$. The choice of the characteristic values of the geomechanical parameters ($H$, $C$, $F$, and $R$) is based on an interval of confidence of 95% (Orr, 2000). These values assume that the geomechanical parameters of the in-situ residual soil follow a normal distribution; and that the relative upper limit (maximum) and lower limit (minimum) values have 5% and 95% probabilities of being exceeded, respectively. Furthermore, the values of $\sigma_{xi}$ considered for the geomechanical parameters have been corroborated based on COV values reported in the technical literature (Phoon and Kulhawy, 1999); (Duncan, 2000).

| Parameter                  | Designation | Distribution | $m$  | $s$  | $x_{\text{min}}$ | $x_{\text{max}}$ |
|----------------------------|-------------|--------------|------|------|------------------|------------------|
| Height, $H$ [m]            | $H$         | -            | -    | -    | 3.0              | 12.0             |
| Unit weight, $\delta$ [kN/m$^3$] | $G$         | Normal       | 18.5 | 1.1  | 16.7             | 20.3             |
| Cohesion, $c$ [kPa]        | $C$         | Normal       | 10   | 3.2  | 4.2              | 15.8             |
| Internal friction angle, $\phi$ [$^\circ$] | $F$         | Normal       | 32   | 2    | 28.7             | 35.3             |
| Pullout strength, $r_s$ [kPa] | $R$         | Normal       | 150  | 45   | 75.8             | 224.3            |

The experimental design considers the effect of five factors in a process called 2$^5$ factorial design, which generates 32 different combinations of soil nail wall. Table 3 summarizes the combinations defined for each SNW execution, based on the relative maximum and minimum values of each selected factor. Execution #1 represents the combination of the relative minimum value calculated for each factor, while execution #32 represents the combination of the relative maximum value calculated for each factor. The other 30 executions are generated based on the combinations of relative maximum and minimum values of each factor. The generation of these 32 executions allows identifying the most important factors of interest within the design process.
Table 3. Codified $2^5$ factorial design and calculated factors of safety

| Realization | H | G | C | F | R | $F_{SC}$ | $F_{SD}$ | $F_{CS}$ |
|-------------|---|---|---|---|---|----------|----------|----------|
| 1           | - | - | - | - | - | 0.80     | 2.35     | 2.05     |
| 2           | + | - | - | - | - | 0.64     | 2.06     | 1.31     |
| 3           | - | + | - | - | - | 0.72     | 2.23     | 1.80     |
| 4           | + | + | - | - | - | 0.57     | 2.01     | 1.24     |
| 5           | - | - | + | - | - | 1.51     | 3.33     | 4.91     |
| 6           | + | - | + | - | - | 0.85     | 3.04     | 2.80     |
| 7           | - | - | - | + | - | 0.96     | 3.19     | 5.40     |
| 8           | + | - | - | + | - | 0.81     | 3.41     | 5.17     |
| 9           | - | - | - | - | + | 1.24     | 2.35     | 2.05     |
| 10          | + | - | - | - | + | 0.94     | 2.06     | 1.31     |
| 11          | - | + | + | - | - | 1.36     | 3.22     | 4.39     |
| 12          | + | + | + | - | - | 0.75     | 2.87     | 2.52     |
| 13          | - | + | - | + | - | 0.87     | 3.22     | 5.27     |
| 14          | + | + | - | + | - | 0.72     | 3.34     | 5.00     |
| 15          | - | + | - | - | + | 1.09     | 2.23     | 1.80     |
| 16          | + | + | - | - | + | 0.84     | 2.02     | 1.24     |
| 17          | - | - | + | + | - | 1.70     | 4.03     | 10.14    |
| 18          | + | - | + | + | - | 1.02     | 3.89     | 6.97     |
| 19          | - | - | + | - | + | 1.81     | 3.33     | 4.91     |
| 20          | + | - | + | - | + | 1.15     | 3.04     | 2.80     |
| 21          | - | - | - | + | + | 1.52     | 3.19     | 5.40     |
| 22          | + | - | - | + | + | 1.19     | 3.41     | 5.17     |
| 23          | - | + | + | + | - | 1.51     | 3.94     | 9.28     |
| 24          | + | + | + | + | - | 0.90     | 3.84     | 6.70     |
| 25          | - | - | + | + | + | 2.10     | 4.03     | 10.14    |
| 26          | + | + | + | - | + | 1.01     | 2.87     | 2.52     |
| 27          | - | + | + | - | + | 1.71     | 3.22     | 4.39     |
| 28          | + | + | - | + | + | 1.06     | 3.34     | 5.00     |
| 29          | - | + | - | + | + | 1.34     | 3.22     | 5.27     |
| 30          | + | - | + | + | + | 1.40     | 3.89     | 6.97     |
| 31          | - | - | + | - | + | 1.99     | 3.94     | 9.28     |
| 32          | + | + | + | + | + | 1.24     | 3.84     | 6.70     |

4. Results

As mentioned earlier, the factors of safety related to overall failure ($F_{SC}$), sliding ($F_{SD}$) and bearing capacity ($F_{CS}$) are the acceptance criteria for assessing the external stability of soil nail walls. Given the importance of these criteria, the main results of this work are analyzed below.

4.1 Factors of Safety

The last three columns in (Table 3) indicate the values for the factors of safety obtained from each SNW execution using the sliding blocks method. The combination #4, where all factors of interest, except H and G, have the relative minimum value, obtained the lowest value of all safety factors, while combination #25, where all the factors of interest, except H and G, have the relative maximum value, obtained the highest value of all safety factors.

The influence of the height on the SNW stability is observed as a significant factor in the analysis. Figures 4-5 show the failure mechanisms obtained for soil nail walls, considering experiments of minimum and maximum height, respectively. Figure 4a shows the failure mechanism for combination #25, which is precisely where the highest safety factor values were obtained, with relative maximum values of C, F and R, except H and G, that is,
with the highest strength values and lowest loading values. While Figure 5a shows the failure mechanism for combination #4, where the lowest safety factor values were obtained, with relative minimum values of C, F and R, except H and G, that is, with the lowest strength values and highest loading values. Figure 4b shows the failure mechanism for combination #1, which considered the relative minimum values of all factors of interest, that is, for strength and loading. While Figure 5b shows the failure mechanism for combination #32, which considered the relative maximum values of all factors of interest, that is, for strength and loading.
4.2 Sensitivity Analysis and ANOVA

In order to identify which factors of interest have the greatest influence on the external stability of soil nail walls, a sensitivity analysis and ANOVA were carried out, thereby calculating the sum of squares, the estimated effect, the sum of least squares, and the values $F_0$ and $p$, for all five factors evaluated and their double interactions (Table 4) (Table 5) (Table 6). This study has considered a significance level $\alpha = 0.05$, which has been recommended by (Montgomery 2001), (Myers and Montgomery, 2002) and (Myers et al., 2016).

For each variability source or factor of interest, the value of $F_0$ (statistical parameter of Fisher) was used to assess the significance level of each factor within the process. The value of $F_0$ was calculated by dividing the sum of squares of each factor by the error of the sum of squares. Then, the value of $F_0$ is compared to the $F$ value (obtained from the $F$-test) of each factor. If $F_0$ is higher than $F$, the null hypothesis of the design is rejected, and the corresponding factor is identified as an important source of variability. Moreover, if the corresponding $p$ value of
each factor is lower than the considered significance level ($\alpha = 0.05$), the null hypothesis is rejected and the factor of interest is identified as having high significance.

**Table 4. Sensitivity analysis and ANOVA for $F_{Sc}$**

| Factor | Estimated Effect | Regression Coefficient | Sum of Squares | Degrees of freedom | Least Squares | $F_0$ | $p$ |
|--------|------------------|------------------------|----------------|-------------------|--------------|-------|-----|
| $H$    | -0.44625         | 0.00553                | 1.59311        | 1                 | 1.59311      | 2641.43005 | 0.00000 |
| $G$    | -0.12250         | 0.00393                | 0.12005        | 1                 | 0.12005      | 199.04663  | 0.00000 |
| $C$    | 0.41875          | 0.07770                | 1.40281        | 1                 | 1.40281      | 2325.90674 | 0.00000 |
| $F$    | 0.20875          | 0.03582                | 0.34861        | 1                 | 0.34861      | 578.01036  | 0.00000 |
| $R$    | 0.37125          | 0.00097                | 1.10261        | 1                 | 1.10261      | 1828.16580 | 0.00000 |
| $HG$   | 0.00875          | 0.00504                | 0.00061        | 1                 | 0.00061      | 1.01554    | 0.32858 |
| $HC$   | -0.22500         | -0.00431               | 0.40500        | 1                 | 0.40500      | 671.50259  | 0.00000 |

**Table 5. Sensitivity analysis and ANOVA for $F_{So}$**

| Factor | Estimated Effect | Regression Coefficient | Sum of Squares | Degrees of freedom | Least Squares | $F_0$ | $p$ |
|--------|------------------|------------------------|----------------|-------------------|--------------|-------|-----|
| $H$    | -0.13125         | -0.15879               | 0.13781        | 1                 | 0.13781      | 62.88770  | 0.00000 |
| $G$    | -0.07875         | -0.09732               | 0.04961        | 1                 | 0.04961      | 22.63957  | 0.00021 |
| $C$    | 0.79375          | 0.23714                | 5.04031        | 1                 | 5.04031      | 2300.03565 | 0.00000 |
| $F$    | 0.96875          | 0.09624                | 7.50781        | 1                 | 7.50781      | 3426.02496 | 0.00000 |
| $R$    | 0.00000          | 0.00000                | 0.00000        | 1                 | 0.00000      | 0.00000   | 1.00000 |
| $HG$   | -0.00625         | -0.00039               | 0.00031        | 1                 | 0.00031      | 0.14260   | 0.71067 |
| $HC$   | -0.08875         | -0.00170               | 0.06301        | 1                 | 0.06301      | 28.75437  | 0.00006 |
| $HF$   | 0.15625          | 0.00526                | 0.19531        | 1                 | 0.19531      | 89.12656  | 0.00000 |
| $HR$   | 0.00000          | 0.00000                | 0.00000        | 1                 | 0.00000      | 0.00000   | 1.00000 |
| $GC$   | -0.02625         | -0.00126               | 0.00551        | 1                 | 0.00551      | 2.51551   | 0.13229 |
| $GF$   | 0.03375          | 0.00284                | 0.00911        | 1                 | 0.00911      | 4.15829   | 0.05830 |
| $GR$   | 0.00000          | 0.00000                | 0.00000        | 1                 | 0.00000      | 0.00000   | 1.00000 |
| $CF$   | -0.15875         | -0.00415               | 0.20161        | 1                 | 0.20161      | 92.00143  | 0.00000 |
| $CR$   | 0.00000          | 0.00000                | 0.00000        | 1                 | 0.00000      | 0.00000   | 1.00000 |
| $FR$   | 0.00000          | 0.00000                | 0.00000        | 1                 | 0.00000      | 0.00000   | 1.00000 |
| Error  | -                | -                      | 0.009650       | 16                | -            | -       | -    |
| Total  | -                | -                      | 5.027750       | 31                | -            | -       | -    |
4.3 Regression Models and Response Surfaces

(Equation 2) (Equation 3) (Equation 4) use the regression coefficients of the linear interaction models with two factors (third column of (Table 4) (Table 5) (Table 6)), which establish a simple mathematical relationship for each prediction model of the factors of safety. These functions were used to build the response surfaces, which serve to predict the values of the safety factors for different random executions, based on the variability of the geomechanical parameters of the in-situ residual soil and the height of the soil nail wall. These functions are expressed as follows:

\[
\begin{align*}
FS_G &= -0.299 + 0.00553 \cdot H + 0.00393 \cdot G + 0.0777 \cdot C + 0.0358 \cdot F + 0.000970 \cdot R \\
& \quad - 0.00431 \cdot H \cdot C - 0.0000748 \cdot H \cdot R + 0.0000969 \cdot F \cdot R \\
FS_D &= -0.159 \cdot H + 0.237 \cdot C + 0.0962 \cdot F - 0.00415 \cdot CF \\
FS_{CS} &= -9.164 + 0.0219 \cdot H + 0.0870 \cdot C + 0.5827 \cdot F - 0.911 \cdot H \cdot C
\end{align*}
\]

The response surfaces can be represented as 3D charts showing the value of the factors of safety versus the variation of two other input factors, thereby keeping an average value of all the other factors constant. The response surfaces indicate the range of safety factors in the region of interest. Furthermore, once the response surface is available, the combination of the input factors reaching the maximum and minimum values, that is, the best and worst scenarios, respectively, can be rapidly identified, even if they have not been executed in a limit equilibrium model.
The charts in Figures 6-8 show the response surfaces obtained for the linear prediction models, with interaction between two selected factors. Regarding $FS_{G}$, the five factors of interest are individually relevant, whereas only the double interaction between height and cohesion, height and pullout strength, cohesion and pullout strength, and friction angle and pullout strength, are relevant. With regard to the $FS_{D}$, all factors, except the pullout strength, are individually relevant, while only the double interaction between height and cohesion, height and friction angle, and cohesion and friction angle, are relevant. Finally, in relation to $FS_{CS}$, as in $FS_{D}$, all factors, except the pullout strength, are individually relevant, whereas only the double interaction between height and cohesion, and cohesion and friction angle, are relevant.

Figure 6. Response surfaces for $FS_{G}$ interaction: (a) $H - C$, (b) $H - R$, (c) $C - R$, (d) $F - R$
Figure 7. Response surfaces for \( FS_{D} \), interaction: (a) \( H \sim C \), (b) \( H \sim F \), (c) \( C \sim F \).

Figure 8. Response surfaces for \( FS_{CS} \), interaction: (a) \( H \sim C \), (b) \( C \sim F \).
4.4 Height Effect

The decreased safety factor values are reflected by the increase of the height of the soil nail wall, and vice versa. This can be observed in Table 3, where the highest safety factor values are obtained with H (-), while the lowest safety factor values are obtained with H (+). Moreover, the estimated height factor value (Table 4) (Table 5) (Table 6) is negative in all cases, that is, the external instability of the soil nail wall increases. In relation to the FS_D, it has the lowest estimated effect value (EE_D = -0.13125), while the FS_CS has the highest estimated effect (EE_CS = -1.44125), which evidences that the FS_CS is the most affected by the height increase of the soil nail wall.

(Figure 6a) (Figure 6b), (Figure 7a) (Figure 7b) and (Figure 8a) show the response surfaces where safety factor values is based on the SNW height. All cases of double interaction, except the H-F interaction of FSD, evidence the negative effect of the height on the safety factor values.

4.5 Effect of the Geomechanical Parameters

The increased safety factor values are reflected by the increase of the cohesion, the internal friction angle and the pullout strength of the in-situ residual soil, and vice versa. This can be observed in Table 3, where the highest safety factor values are obtained with C (+), F (+) and R (+), while the lowest safety factor values are obtained with C (+), F (-) and R (-). Furthermore, the estimated effect value of C, F and R (Tables 4-6) is positive in all cases, that is, the external stability of the SNW increases. The results of the sensitivity analysis and ANOVA (Tables 4-6) show that the interaction between unit weight of the in-situ residual soil (G) and the other factors of interest is not highly relevant.

(Figure 6a), (Figure 6c), (Figure 7a), (Figure 7c) and (Figure 8a) (Figure 8b) show the response surfaces, where the safety factor values are based on the cohesion of the in-situ residual soil. All cases of double interaction evidence the positive effect of cohesion on the safety factor values. (Figure 6d), (Figure 7b) (Figure 7c), and 8b show the response surfaces where the safety factor values are based on the friction angle of the in-situ residual soil. All cases of double interaction evidence the positive effect of the friction angle on the safety factor values. Figure 6b-d show the response surfaces, where the value of FS_C is based on the pullout strength of the in-situ residual soil. All cases of double interaction show the positive effect of the pullout strength on the FS_C value. Regarding FS_D and FS_CS, there is a null estimated effect of the pullout strength of the in-situ residual soil.

4.6 Estimated Factors of Safety

(Figure 9) shows the charts of the factors of safety observed versus the factors of safety estimated by the statistical regression models given by (Equation 2) (Equation 3) (Equation 4), for the 32 executions of the limit equilibrium models. These charts allow observing that, in all cases, the regression models can properly predict the values of the factors of safety, with an adequate goodness of fit in all cases (R^2 > 0.99).
5. Case studies

The models developed in this document were used to calculate the values of the factors of safety from other references concerning soil nail walls previously built, which had enough data to allow making the calculations. Four previous researches were selected as case studies, which had the data corresponding to the height of a vertical soil nail wall and geomechanical parameters of the in-situ soil. (Table 7) shows the details of the four case studies. In all four studied cases, the factors of safety were calculated with the GGU Stability software and were compared to the estimated ones through the proposed regression models.

Figure 9. Factors of safety observed (limit equilibrium models) versus estimated (statistical regression models) for 32 executions: (a) overall, (b) sliding and (c) bearing capacity
In general, and specifically in (Figure 10), it can be observed that, in all four case studies, the proposed models estimates safety factor values in agreement with those calculated by the GGU Stability software. Complementing the analyses with (Table 7), and concerning all the results, the obtained absolute error range was 0.25 to 15.33% (for $FS_G$ and $FS_D$, respectively). In 50% of the estimated factors of safety, the proposed model estimated a value that was lower than the calculated one. Considering the number of variables involved and the small variation of the results, it can be concluded that the proposed model satisfactorily predicts the value of the factors of safety that verify the external stability of soil nail walls.

![Figure 10. Comparison of the calculated and estimated factors of safety through the proposed regression models](image-url)
6. Conclusions

This paper has assessed the external stability of soil nail walls in granite residual soils under pseudo-static loading conditions. It is known that there is a source of uncertainty due to the variability of the factors involved in the analysis, such as the height of the soil nail wall and the geomechanical parameters of the in-situ soil. Therefore, the assessment of the external stability of SNW through deterministic method is not clear enough to solve the problem. Consequently, this research has evaluated the external stability of SNW using a probabilistic method, which considers the variability of five factors involved in the analysis.

A $2^5$ factorial design was developed to assess the influence of the SNW height and the geomechanical parameters of the in-situ soil (σ, c, φ and $r_s$) on the three external failure modes (overall, sliding and bearing capacity), commonly considered as acceptance criteria in this kind of geotechnical projects.

The linear model with double interaction provides an adequate arrangement for the obtained data. The generation of 32 executions allowed identifying the factors of interest having the greatest impact on the stability of soil nail walls. The generation of response surfaces or predictive models describing the output response (i.e. factors of safety) has been especially interesting for any random combination with different input parameters (height of the soil nail wall and geomechanical parameters of the in-situ soil).

The factors of interest having the greatest influence on the external stability of soil nail walls are the height, the cohesion and the friction angle of the in-situ soil. Only in the case of $F_{S_C}$, the pullout strength of the in-situ soil is a factor having a high impact on the response. The interactions between the height, cohesion, friction angle, and pullout strength of the in-situ soil significantly control the external stability of soil nail walls.

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