Neutrino propagation in matter with general interactions

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Abstract

We present a general analysis of the effective potential for neutrino propagation in matter, assuming a generic set of Lorentz invariant non-derivative interactions. We find that in addition to the known vector and axial vector terms, in a polarized medium also tensor interactions can play an important role. We compute the effective potential arising from a tensor interaction. We show that the components of the tensor potential transverse to the direction of the neutrino propagation can induce a neutrino spin-flip, similar to the one induced by a transverse magnetic field.
I. INTRODUCTION

Neutrino physics currently provides the strongest experimental evidence for physics beyond the Standard Model (SM). The atmospheric neutrino anomaly [1] and the solar neutrino [2] problem are best explained by neutrino oscillations.

Neutrino oscillations occur when the produced neutrinos are not eigenstates of the Hamiltonian that describes their propagation. In vacuum, this is the case if the flavor eigenstates are non-trivial linear combinations of different mass eigenstates. This requires massive neutrinos that mix. It is well known that the neutrino propagation in matter can be very different from that in vacuum. The crucial fact is that coherent interactions with the background give to the neutrino an “index of refraction” which depends on its flavor. This is because normal matter, which contains only first generation fermions, is flavor asymmetric. For example, for standard weak interactions, only electron neutrinos can have charged current interactions with the background electrons. Thus, in matter the effective electron neutrino mass depends on the electron density and is enhanced with respect to the other flavors. This allows for the possibility of level crossing between different neutrino eigenstates in matter. If the electron neutrinos are produced with an effective mass above the level crossing (the “resonance”) an adiabatic transition through the resonance can induce a significant amplification of neutrino oscillations. This is known as the Mikheyev-Smirnov-Wolfenstein (MSW) effect [3]. If light sterile neutrinos exist, then also neutral current interactions are important since only the active neutrinos are subject to it [4]. In a polarized medium the neutrino effective mass also depends on the average polarization of the background, and on the angle between the neutrino momentum and the polarization vector [5,6].

Many extensions of the SM imply massive neutrinos. It is important to stress that these new physics models often predict also new neutrino interactions. In this case the SM picture can be significantly changed [7–9], since the neutrino effective mass will depend on both the SM and the new interactions. For example, non-universal interactions may give rise to matter effects that distinguish between muon and tau neutrinos. Lepton flavor violating interactions can induce an effective mixing in matter, allowing for a resonant conversion even in the absence of vacuum mixing. The two effects combined together could induce neutrino flavor transitions even for massless neutrinos.

Most of the discussions of neutrino oscillations in matter are based on the effective Hamiltonian

\[
\mathcal{H}_{(V,A)} = \frac{G_F}{\sqrt{2}} \sum_{a=V,A} (\bar{\nu} \Gamma_a \nu) \left[ \bar{\psi}_f \Gamma^a \left( g_a + g_a' \gamma_5 \right) \psi_f \right] + \text{h.c.,} \tag{1.1}
\]

where \( \Gamma^V = \gamma^\mu \), \( \Gamma^A = \gamma^\mu \gamma^5 \), \( \psi_f \) are the field operators for the background fermions \( f = e, p, n, \nu \) and \( g_a, g_a' \) are suitable coupling constants parametrizing the strength of the interactions.
Clearly, the standard neutral current and the Fierz rearranged charged current 
\((V - A) (V - A)\) structures are included in (1.1). However, \(\mathcal{H}_{(V,A)}\) describes in fact a larger 
set of interactions. For example, several models where neutrinos couple to new heavy scalars 
(like supersymmetric models without \(R\)-parity and left-right symmetric models) imply low 
energy effective interactions of the form \(\bar{\nu}(S \pm P)\psi_f \bar{\psi}_f (S \mp P) \nu\) that, after Fierz rearrangement, 
are also accounted for by (1.1). The interactions in (1.1) only induce transitions 
between neutrinos of the same chirality. Therefore the couplings between different helicity 
states that would flip the neutrino spin are suppressed by the ratio between the neutrino 
mass and its energy, \(m/E\), and can be safely neglected. Thus the matter effects induced 
by (1.1) only allow for flavor transitions that conserve the neutrino spin. (Note that tran-
sitions into sterile neutrinos [4] are no exception. The sterile neutrino is a SM singlet, but 
the state that is produced via oscillations has negative helicity.)

In contrast, neutrino transitions induced by a magnetic field result in a spin-flip [10]: a 
left-handed neutrino is rotated into a right-handed one. The rate of this transition depends 
on the neutrino magnetic moment and on the strength of the component of the magnetic 
field orthogonal to the direction of the neutrino propagation. If the SM is extended just by 
introducing right-handed neutrinos, the resulting neutrino magnetic moment is vanishingly 
small, and spin-flipping transitions are negligible even for the largest conceivable magnetic 
fIELDS. Therefore, spin-flipping transitions can be relevant for solar or supernova neutrinos 
only in the presence of new physics that induce a very large neutrino magnetic moment.

While the couplings in (1.1) account for the SM weak interactions as well as for some 
new physics interactions they are clearly not the most general ones. In this paper we sys-
tematically study the effects of all Lorentz invariant non-derivative interactions of neutrinos 
with the background fermions. Namely, we add scalar (\(S\)), pseudoscalar (\(P\)) and tensor (\(T\)) 
interactions, to the vector (\(V\)) and axial-vector (\(A\)) interactions in (1.1). In our analysis 
we reproduce the known results for \(V\) and \(A\) interactions [5]. The \(S\) and \(P\) interactions 
that couple states with opposite chirality but the same helicity are suppressed by \(m/E\) and 
therefore are negligible. Our main result is that transverse tensor interactions induce effects 
which are not helicity suppressed, because they couple states of both opposite chirality and 
opposite helicity. We find that in a polarized medium these interactions can flip the neu-
trino spin coherently. The overall effect depends on the strength of the interaction, on the 
density of the background and on the average polarization of the medium. The physics is 
similar to the electromagnetic spin-flip, however in this case spin-flipping transitions can be 
effective even for a vanishing neutrino magnetic moment. We note that an effective tensor 
potential does not need to arise from a fundamental tensor interaction. It can also result 
after Fierz reordering from some specific scalar and pseudoscalar couplings of the neutrinos 
to the background fermions.
II. NEUTRINO PROPAGATION IN MATTER WITH GENERAL INTERACTIONS

In this section we derive the neutrino propagation equation in matter in the presence of the most general pointlike and Lorentz invariant four-fermion interaction with the background fermions \((f = e, p, n, \nu)\). That is, we generalize (1.1) to

\[
\mathcal{H}_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_a (\bar{\nu} \Gamma^a \nu) \left[ \bar{\psi}_f \Gamma_a \left( g_a + g'_a \gamma^5 \right) \psi_f \right] + \text{h.c.},
\]

(2.1)

where \(\Gamma^a = \{I, \gamma^5, \gamma^\mu, \gamma^\mu \gamma^5, \sigma^{\mu\nu}\}, \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\) and \(a = \{S, P, V, A, T\}\). Here the neutrino \(\nu\) is assumed to be of the Dirac type (we will comment on the Majorana case later). In general, \(\nu\) is a vector of the different neutrino types, and \(g_a, g'_a\) are 10 matrices in the space of neutrino flavors that describe the coupling strengths. In (2.1) the Fermi constant \(G_F\) has been factored out so that all the couplings are dimensionless. From the hermiticity of \(\mathcal{H}_{\text{int}}\) in (2.1) it follows that all \(g_a\) as well as \(g'_V, g'_A\) are hermitian while \(g'_S, g'_P\) and \(g'_T\) are antihermitian. In particular, the diagonal elements in \(g_a\) and \(g'_V, g'_A\) are real while those of \(g'_S, g'_P\) and \(g'_T\) are imaginary. We stress that new interactions in general include both flavor diagonal and off-diagonal couplings. The SM charged current interactions of a \(\nu_e\) with background electrons correspond to \(g_V = -g'_V = g_A = -g'_A = 1\) for the \(\nu_e - \nu_e\) entries, while all the other couplings vanish.

The derivation of the equation of motion describing the neutrino propagation in a medium proceeds as follows. First we average the effective interactions over the background fermions. We are not interested in incoherent effects that become negligible after averaging. Therefore, while we do allow for neutrino spin-flipping interactions, we require that the background fermions do not undergo spin-flip. That is, we select coherent transitions that leave the many-fermion background system in the same state. Next we add the effective neutrino interaction to the free Lagrangian, and we derive the equation of motion for neutrino propagation in matter. Finally, we study the neutrino dynamics described by the equation of motion, under the assumption that the masses and potential terms are much smaller than the neutrino energy.

A. Computing the effective neutrino potential

The effect of the medium on the neutrino propagation in the presence of the general interactions (2.1) can be described by the Lagrangian

\[
- \mathcal{L}_{\text{int}} = \sum_{a,f} (\bar{\nu} \Gamma^a \nu) V'_a f,
\]

(2.2)

where
\[ V^f_a = \frac{G_F}{\sqrt{2}} \sum_\lambda \int \frac{d^3 p}{(2\pi)^3} \rho_f(p, \lambda) \mathcal{M}^f_a, \]  

(2.3)

is given by the expectation value of the background fermion current \( \mathcal{M}^f_a \), averaged over the fermion distribution \( \rho_f(p, \lambda) \). Here \( p \) and \( \lambda \) denote, respectively, the momentum and polarization vectors of the background fermion \( f \). According to the requirement of leaving the many-fermion background system unmodified, the matrix element

\[ \mathcal{M}^f_a \equiv \langle f, p, \lambda | \bar{\psi}_f \Gamma_a (g_a + g'_a \gamma^5) \psi_f | f, p, \lambda \rangle \]  

(2.4)

is taken between initial and final states with the same quantum numbers. The computation of the various \( \mathcal{M}^f_a \) is straightforward and is given in the Appendix. We find

\[ V^S = \frac{G_F}{\sqrt{2}} n_f g_S \left\langle \frac{m_f}{E_f} \right\rangle, \]  

(2.5)

\[ V^P = \frac{G_F}{\sqrt{2}} n_f g'_P \left\langle \frac{m_f}{E_f} \right\rangle, \]  

(2.6)

\[ V^V_\mu = \frac{G_F}{\sqrt{2}} n_f \left[ g_V \left\langle \frac{p_\mu}{E_f} \right\rangle + g'_V m_f \left\langle \frac{s_\mu}{E_f} \right\rangle \right], \]  

(2.7)

\[ V^A_\mu = \frac{G_F}{\sqrt{2}} n_f \left[ g_A \left\langle \frac{p_\mu}{E_f} \right\rangle + g_A m_f \left\langle \frac{s_\mu}{E_f} \right\rangle \right], \]  

(2.8)

\[ V^{T}_{\mu\nu} = \frac{G_F}{\sqrt{2}} n_f \left[ -g_T \epsilon_{\mu\nu\rho} \left\langle \frac{p^\rho s^\sigma}{E_f} \right\rangle + ig'_T \left\langle \frac{p_{\mu} s_{\nu} - p_{\nu} s_{\mu}}{E_f} \right\rangle \right], \]  

(2.9)

where the spin-vector \( s \), which satisfies \( s^2 = -1 \) and \( s_\mu p^\mu = 0 \), is given explicitly in \( (A3) \), and

\[ n_f = \sum_\lambda \int \frac{d^3 p}{(2\pi)^3} \rho_f(p, \lambda), \quad \langle x \rangle = \frac{1}{n_f} \sum_\lambda \int \frac{d^3 p}{(2\pi)^3} \rho_f(p, \lambda) x(p, \lambda) \]  

(2.10)

denote, respectively, the number density of the fermion \( f \) and the average of some function \( x(p, \lambda) \) over the fermion distribution.

We can now perform the contractions \( \Gamma^a V^f_a \) in \( (2.4) \), which yield

\[ \Sigma^{SP} \equiv \Sigma^0 \left[ V^S + V^P \gamma^5 \right] = \frac{G_F}{\sqrt{2}} n_f \left\langle \frac{m_f}{E_f} \right\rangle \left( g_S + g'_P \gamma^5 \right) \]  

(2.11)

\[ \Sigma^{VA} \equiv \gamma^\mu \left[ V^V_\mu + V^A_\mu \gamma^5 \right] = \frac{G_F}{\sqrt{2}} n_f \left[ \left\langle \frac{p_\mu}{E_f} \right\rangle \left( g_V + g'_A \gamma^5 \right) + m_f \left\langle \frac{s_\mu}{E_f} \right\rangle \left( g'_V + g_A \gamma^5 \right) \right] \]  

(2.12)

\[ \Sigma^{T} \equiv \Sigma^i \left[ V^B_i + i V^E_i \gamma^5 \right] = \frac{G_F}{\sqrt{2}} n_f \left[ \frac{i \gamma^i p}{E_f} \right] \left( g'_T + g_T \gamma^5 \right) \]  

(2.13)

where \( \Sigma^\mu \equiv \text{diag} (\sigma^\mu, \sigma^\mu) \) with \( \sigma^\mu = (\sigma^0, \sigma^i) \) and \( \sigma^0 = I \), and we have used \( \sigma^{ij} = \epsilon^{ijk} \Sigma^k \) and \( \sigma^{0i} = i \Sigma^i \gamma^5 \). In \( (2.13) \) we have decomposed the tensor term \( V^{T}_{\mu\nu} \), in analogy to the electro-magnetic field tensor \( F_{\mu\nu} \), as \( V^B_i = \epsilon_{ijk} V^T_{jk} \) and \( V^E_i = 2 V^T_{0i} \). Note that the second equality in \( (2.13) \) makes apparent that the tensor interaction can contribute only in the presence of a polarized background.
B. Equations of Motion

We turn now to study the effects of the potential on the neutrino propagation. The equation of motion can be deduced from the neutrino Lagrangian

\[ \mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} = \bar{\nu}(i\partial - m - \Sigma)\nu \]  

(2.14)

where the matrix of the potentials

\[ \Sigma \equiv \Sigma^{SP} + \Sigma^{VA} + \Sigma^{T} \]  

(2.15)

depends on the background density and polarization, and in general will vary along the neutrino propagation path. In the general case both \( \Sigma \) and \( m \) are matrices in the space of neutrino types. It is instructive to write the interaction part in (2.14) explicitly in the chiral basis, see \((A12)\)

\[ - \mathcal{L}_{\text{int}} = \bar{\nu} \Sigma \nu = \begin{pmatrix} \nu_L^1 \cr \nu_i^1 \cr \nu_R^1 \end{pmatrix}^T \begin{pmatrix} V_{\mu}^{LL} & V_{\mu}^{LR} \sigma^\mu & V_{\mu}^{LL} \sigma^\mu \\ V_{\mu}^{RL} \sigma^\mu & V_{\mu}^{RR} \sigma^\mu & V_{\mu}^{RL} \sigma^\mu \end{pmatrix} \begin{pmatrix} \nu_L \cr \nu_i \cr \nu_R \end{pmatrix}, \]  

(2.16)

where \( \bar{\sigma}^\mu = (\sigma^0, -\sigma^i) \) and

\[ V_{\mu}^{LL} \equiv V_{\mu}^V - V_{\mu}^A, \quad V_{\mu}^{RR} \equiv V_{\mu}^V + V_{\mu}^A, \]  

(2.17)

\[ V_{0}^{RL} \equiv V^S - V^P, \quad V_{0}^{LR} \equiv V^S + V^P, \]  

(2.18)

\[ V_{i}^{RL} \equiv V_i^B - i V_i^E, \quad V_{i}^{LR} \equiv V_i^B + i V_i^E. \]  

(2.19)

The explicit form (2.16) makes apparent that the (axial)vector potentials (contained in \( V_{\mu}^{LL} \) and \( V_{\mu}^{RR} \)) couple neutrinos of the same chirality, while the (pseudo)scalar and tensor potentials (in \( V_{0}^{RL} \) and \( V_{0}^{LR} \)) couple neutrinos of opposite chirality.

From (2.14) it follows that the equations of motion for neutrinos and antineutrinos are, respectively,

\[ \gamma_0 (\not{k} - m - \Sigma) u = 0, \quad \gamma_0 (\not{k} + m + \Sigma) v = 0. \]  

(2.20)

We note that the signs of \( m \) and \( \Sigma \) are opposite for the antineutrinos. The dispersion relations for the neutrino propagation are given by the solutions of

\[ \det [\mathcal{O}] = \det [\gamma_0 (\not{k} - m - \Sigma)] = 0. \]  

(2.21)

Solving (2.21) is simplified by working in the following approximation. Let us chose the neutrino momentum along the \( z \)-axis \( (\mathbf{k} = k\mathbf{\hat{z}}) \). Then \( \sigma_{0,3} \) couple between states of the same helicity while \( \sigma_{1,2} \) couple neutrinos of opposite helicity. Hence, for ultra-relativistic neutrinos, \( V_{1,2}^{LL} \) and \( V_{1,2}^{RR} \) in the chirality conserving diagonal blocks in (2.16) and \( V_{0,3}^{LR} \) and \( V_{0,3}^{RL} \) in the chirality flipping off-diagonal blocks are suppressed as \( m/E \ll 1 \), and can be
neglected. Thus, the relevant potential terms in (2.16) are $V_{0,3}^{LL}, V_{0,3}^{RR}$ and the tensor potential components $V_{1,2}^{LR}$ and $V_{1,2}^{RL}$ that are transverse with respect to the neutrino propagation direction. In this approximation we get

$$
\mathcal{O} = \begin{pmatrix}
E + k - V_{0+3}^{LL} & 0 & -m & -V_{-3}^{LR} \\
0 & E - k - V_{0-3}^{LL} & -V_{+3}^{LR} & -m \\
-m & -V_{-3}^{RL} & E - k - V_{0-3}^{RR} & 0 \\
-V_{+3}^{RL} & -m & 0 & E + k - V_{0+3}^{RR}
\end{pmatrix},
$$

(2.22)

where $V_{0,3} \equiv V_{0,3}^{LL}$ and $V_{\pm} \equiv V_{1,2}^{LR} \pm i V_{1,2}^{RL}$. Note that since $V_{V,A,T}^{V,A,T}$ are hermitian, $(V_{\pm}^{LR})^\dagger = V_{\pm}^{LR}$ and the matrix (2.22) is manifestly hermitian. Solving the determinant equation for (2.22) under the assumption that $V_{V,A,T}^{V,A,T}, m \ll E$ yields the neutrino energies:

$$
E_{\pm} = k + m^2/2k + \frac{1}{2} \left[ V_{0-3}^{LL} + V_{0-3}^{RR} \pm \sqrt{(V_{0-3}^{LL} - V_{0-3}^{RR})^2 + 4 V_{+3}^{LR} V_{-3}^{RL}} \right],
$$

(2.23)

where the plus (minus) sign refers to neutrinos that are mainly left(right)-handed states. Eliminating the two helicity suppressed states from the equations of motion we obtain a Schrödinger-like equation that governs the neutrino propagation:

$$
i \frac{d}{dt} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = \mathcal{H}_\nu \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \quad \text{with} \quad \mathcal{H}_\nu = k + m^2/2k + \begin{pmatrix} V_{0-3}^{LL} & V_{0-3}^{LR} \\ V_{0-3}^{RL} & V_{0-3}^{RR} \end{pmatrix}.
$$

(2.24)

The two energy eigenvalues of the effective Hamiltonian $\mathcal{H}_\nu$ are the solutions (2.23) of the determinant equation (2.21). The equation for the antineutrinos, and the corresponding eigenvalues, can be obtained from (2.24) and (2.23) by changing the sign of the potentials ($V \to -V$). Note that the contribution to the energy levels from the tensor term, which is quadratic in $V^T$, does not change sign. In the case of more than one neutrino flavor (2.23) is a matrix equation in the space of the neutrino types. It is interesting to note that in general we should not expect that the various interactions in (2.23) will be diagonal in the same basis. In this case even in the massless limit (or for degenerate neutrinos) flavor oscillations can occur in matter. In the one flavor case, the energy gap between the two states is

$$
\Delta E_\nu = \sqrt{(V_{0-3}^{LL} - V_{0-3}^{RR})^2 + 4 V_{+3}^{LR} V_{-3}^{RL}}.
$$

(2.25)

In the limit of vanishing tensor interaction ($V_T = 0$) $\nu_L$ decouples from $\nu_R$, and we obtain

$$
E_L = k + m^2/2k + V_{0-3}^{LL}, \quad E_R = k + m^2/2k + V_{0-3}^{RR}.
$$

(2.26)

Clearly in this case we can have oscillations only between different neutrino flavors. Moreover, if there is a basis where the full $V^{LL}$ (or $V^{RR}$, for the SM sterile states) is flavor diagonal, then oscillations can occur only in the presence of non-trivial mixings in the mass matrix. Setting $V^{RR} = 0$ and $V^{LL}$ equal to the SM charged current and neutral current interactions, we recover the SM case, with non-interacting right-handed states.
So far we only discussed the case of neutrinos propagating in a background of particles. If also antiparticles (e.g. positrons) are present in the background, one has to take into account the corresponding interactions. Assuming CP conservation we find that neutrino scattering off antifermions leads to the Hamiltonian in (2.24), but with opposite sign for the potential matrix.

III. IMPLICATIONS AND DISCUSSION

The general interactions that we have studied in the previous section can give rise to several effects for neutrino oscillations in matter. It is well-known that the vector and axial-vector interactions can be very important for neutrino propagation in dense matter. These interactions do not change the neutrino spin, but they can enhance flavor transitions when the neutrino moves through a resonance [3].

To recover the SM result for the potential felt by an electron neutrino propagating in an electron background, we set \( g_V = -g'_A = g_A = -g'_V = 1 \) in (2.12) and obtain

\[
\Sigma^{SM} = \sqrt{2} G_F n_e \left( \left\langle \frac{\hat{p}}{E_e} \right\rangle - m_e \left\langle \frac{\hat{k}}{E_e} \right\rangle \right) P_L, \tag{3.1}
\]

with \( P_L = \frac{1}{2}(1 - \gamma_5) \). Defining \( \hat{k} = k/|k| \) as a unit vector in the direction of the neutrino momentum \( k \) and using the explicit expression (A5) for the spin vector \( s \) we obtain

\[
V_{\nu,\bar{\nu}}^{SM} = \pm \sqrt{2} G_F n_e \left[ 1 - \left\langle \frac{\hat{k} \cdot \hat{p}}{E_e} \right\rangle - \left\langle \frac{\hat{p} \cdot \lambda}{E_e} \right\rangle + m_e \left\langle \frac{\hat{k} \cdot \lambda}{E_e} \right\rangle + \left( \frac{\hat{k} \cdot \hat{p}}{E_e(m_e + E_e)} \right) \right], \tag{3.2}
\]

which is valid for an arbitrary neutrino direction. The plus-sign in (3.2) refers to neutrinos and the minus-sign to antineutrinos. We note that (3.2) is in agreement with the results given in [3].

Our main result is, however, that in the presence of a neutrino tensor interaction with the background fermions, the neutrino can undergo spin-flip. This effect is similar to the spin-precession induced by a transverse magnetic field \( B_{\perp} \), that couples to the neutrino magnetic dipole moment \( \mu_\nu \). In fact, if we substitute in (2.24) the off-diagonal term \( V_{LR}^{\pm} \) by \( \mu_\nu B_{\perp} \) we obtain the equation of motion for a neutrino that propagates in a magnetic field [10,11]. Thus, while these two scenarios originate from different physics, formally they can be treated in the same way.

To illustrate the effects of neutrino oscillations due to the presence of a non-zero transverse tensor potential, we consider the simplest case of one neutrino generation. A left-handed neutrino that was produced at \( t = 0 \) and propagates for a time \( t \) in a constant medium will be converted into a right-handed neutrino with a probability

\[
P_{\nu}^{LR}(t) = \sin^2 2\theta \sin^2 \left( \frac{\Delta E_\nu t}{2} \right). \tag{3.3}
\]
The effective mixing angle $\theta$ is given by

$$\sin^2 2\theta = \frac{|2V_{LR}^L|^2}{(\Delta E_\nu)^2}, \quad (3.4)$$

where the energy splitting $\Delta E_\nu$ is defined in (2.25). (Note that for one neutrino flavor we have $V_{LR}^U V_{RL}^L = |V_{LR}^U|^2$.) In the case of more than one neutrino flavor, propagation in a medium with changing density can lead to resonance effects in complete analogy to the magnetic field induced resonant spin-flip. We will not discuss the details of the resonant case here (which can be found in the existing literature [11]), but we want to discuss shortly the results for different types of background matter.

First consider a medium where the average momentum of the background fermions vanishes $(\langle p \rangle = 0)$. This is in particular the case for an isotropic momentum distribution. Then, the relevant (transverse) component of the tensor potential which determines the effective mixing in (3.4) is given by

$$|V_{LR}^L| = \sqrt{2} G_F n_f \sqrt{|g_T|^2 + |g_T'|^2} \left\langle \lambda_\perp \left( \sin^2 \vartheta + \frac{m_f}{E_f} \cos^2 \vartheta \right) \right\rangle, \quad (3.5)$$

where $\vartheta$ is the angle between the momentum and the transverse polarization of the background fermion and $\lambda_\perp = \sqrt{\lambda_1^2 + \lambda_2^2}$. Note that $|V_{LR}^L|$ vanishes if the neutrino propagates along the direction of the average background polarization $(\lambda_\perp = 0)$. For a non-relativistic background (where $E_f \simeq m_f \gg p_i$) we obtain from (3.5) that the effective mixing angle is determined by

$$|V_{LR}^L| = \sqrt{2} G_F n_f \sqrt{|g_T|^2 + |g_T'|^2} \langle \lambda_\perp \rangle. \quad (3.6)$$

In the ultra-relativistic limit the effective mixing depends on $\langle \lambda_\perp \sin^2 \vartheta \rangle$ which is equal to $\langle \lambda_\perp/2 \rangle$ if $\lambda_\perp$ is uncorrelated to the momentum of the background fermion. Finally, for a degenerate background in the presence of a magnetic field, only the fermions in the lowest Landau level contribute to the polarization, with the spin oriented antiparallel to the momentum. In this case the background is not isotropic, and eq. (3.5) is not applicable. One obtains for this case

$$|V_{LR}^L| = \sqrt{2} G_F n_f \sqrt{|g_T|^2 + |g_T'|^2} \left\langle \lambda_\perp \frac{m_f}{E_f} \right\rangle, \quad (3.7)$$

which vanishes in the ultra-relativistic limit.

Let us now comment on the possible source of the tensor interaction. Of course, one cannot rule out elementary tensor interactions. However, it is interesting to note that also certain neutrino scalar interactions can generate, after Fierz rearrangement, effective tensor couplings. For example, consider the tree level Lagrangian

$$- \mathcal{L}_{\text{tree}} = \lambda_\phi \phi (T_L e_R) + \lambda_\phi' \tilde{\phi} (T_L \nu_R) + \text{h.c.}, \quad (3.8)$$
where \( L_L \) is the left-handed lepton \( SU(2)_L \) doublet, \( e_R \) (\( \nu_R \)) is the right-handed electron (neutrino) singlet, \( \phi \) is a doublet scalar field, of mass \( m_\phi \) and \( \tilde{\phi} = i\sigma_2 \phi^* \) and \( \lambda_\phi, \lambda'_\phi \) are real elementary couplings. At low energy \( E \ll m_\phi \), the interaction in (3.8) induces a set of four-fermion effective interactions, which also contains the following coupling

\[
H^\phi_{\text{int}} = \frac{\lambda'_\phi \lambda_\phi}{m_\phi^2} \left( \overline{\nu}_R \nu_L \right) \left( \overline{e}_R e_L \right) = -\frac{\lambda'_\phi \lambda_\phi}{m_\phi^2} \left[ \frac{1}{2} \left( \overline{\nu}_R \nu_L \right) \left( \overline{e}_R e_L \right) + \frac{1}{8} \left( \overline{\nu}_R \sigma_{\mu\nu} \nu_L \right) \left( \overline{e}_R \sigma^{\mu\nu} e_L \right) \right].
\]  

(3.9)

From (3.9) it follows that \( g_T \sim \lambda'_\phi \lambda_\phi / m_\phi^2 \). Finally, we mention that the above four-fermion operator can also be generated when different scalar fields mix. This possibility exists, for example, in supersymmetric models without \( R \)-parity.

Throughout this paper we assumed the neutrinos to be of the Dirac type. For the case of Majorana neutrinos there are additional constraints on some of the couplings. Namely, one can show \(^{[12]}\) that the flavor diagonal elements of the vector couplings \( g_V, g_V' \) as well as the tensor couplings \( g_T, g_T' \) vanish identically, while the axial-vector couplings are twice the value corresponding to the Dirac case. As a consequence the standard MSW effect does not distinguish between Dirac and Majorana neutrinos, but a tensor-induced spin-flip requires at least two neutrino flavors in the Majorana case.

Let us now address shortly the issue whether the new tensor term could be relevant for real physical systems, like the Sun or a supernova. The crucial point is that the effective tensor potential is proportional to the tensor couplings and to the average background polarization. From eqs. (3.3)–(3.7) it follows that in general it is suppressed by a factor

\[
\epsilon \equiv \left| \frac{V^{LR}_{+}}{V^{LL}_{0}} \right| \lesssim \sqrt{|g_T|^2 + |g_T'|^2} \langle \lambda_\perp \rangle
\]  

(3.10)

with respect to the SM vector potential. New physics effects can be relevant to neutrino oscillations only if they are large enough to affect sizably the standard results obtained with the usual SM interactions. The problem of estimating the minimum size required to render these effects observable was addressed in \(^{[8,9]}\). These analyses imply that the tensor interaction could be relevant respectively for solar and supernova neutrino oscillations, if \( \epsilon \) satisfies the following lower limits:

\[
\epsilon_{\text{sun}} \gtrsim 10^{-2} \quad \text{and} \quad \epsilon_{\text{SN}} \gtrsim 10^{-4}.
\]  

(3.11)

According to (3.10) the effect is maximal for the maximum allowed values of \( g_T, g_T' \) and \( \langle \lambda_\perp \rangle \). Clearly, the excellent agreement between the SM predictions for processes involving neutrinos and the corresponding experimental results, suggests that the tensor couplings are small. We expect that, besides the direct limits from decays and from neutrino scattering data, in some cases one can also derive severe constraints from \( SU(2)_L \) related interactions \(^{[13]}\) as well as from the bounds on neutrino masses. While a detailed phenomenological analysis is needed to give definite upper bounds on the tensor couplings \(^{[14]}\), we believe that they will not exceed the few percent level.
However, the suppression of the tensor potential due to the average polarization is by far the most important factor. In the solar interior, the magnetic field can be at most of the order of several kG. This can result in a tiny polarization of the (non-relativistic) electrons, 

\[ \langle \lambda_e \rangle \sim \frac{eB}{m_e T} \approx 10^{-8} \left[ \frac{B}{1 \text{ kG}} \right] \left[ \frac{1 \text{ keV}}{T} \right], \quad (3.12) \]

where \( B \) and \( T \) denote, respectively, the magnetic field and the temperature in the relevant region of propagation. We conclude that quite likely neutrino propagation in the Sun cannot be affected by the new tensor interactions.

In a proto-neutron star during the early cooling phase, a few seconds after the supernova explosion, the magnetic field strength can reach extremely large values. However, the temperature is also large, thus suppressing the induced polarization. Since the electron number density is only about 10% of the nucleon density, and due to the fact that the electrons are relativistic and degenerate, the effect of neutron polarization can be comparable, and even dominant, with respect to the effect of electron polarization. We estimate

\[ \langle \lambda_{p,n} \rangle \approx 10^{-5} \left[ \frac{B}{10^{13} \text{ G}} \right] \left[ \frac{10 \text{ MeV}}{T} \right] \quad \text{and} \quad \langle \lambda_e \rangle \approx 10^{-4} \left[ \frac{B}{10^{13} \text{ G}} \right] \left[ \frac{20 \text{ MeV}}{k_F} \right]^2, \quad (3.13) \]

where \( k_F \) is the Fermi momentum of the degenerate electrons. The above suggests that for conservative values of the magnetic field, of the order of \( B \lesssim 10^{13} \text{ G} \), it is unlikely that the tensor interaction could affect the propagation of supernova neutrinos. However, one cannot rule out completely the possibility of large enhancements of the effects of the tensor interaction. First, inside the supernova core the neutrinos are not freely streaming and suffer collisions. In general, the effect of collisions is to increase the production of the right-handed states. Second, the magnetic field inside the core is poorly known. It has been proposed that at early times inside the proto-neutron star the magnetic field could be as strong as \( 10^{16} \text{ G} \) \[13\]. This would imply an enhancement of the polarization of about three orders of magnitude, opening the possibility of observing these effects. Finally, it has also been speculated that very dense and neutron rich matter could have a ferromagnetic phase \[16\] (even if this is unlikely to occur at the time of neutrino emission, when the temperature is very high, there could still be some large enhancement of \( \langle \lambda_{p,n} \rangle \)). Also in this case neutrino tensor interactions with the highly polarized background could be effective for inducing transitions into right-handed states at a sizable rate. Of course, since the presence of right-handed neutrinos implies in general a non-vanishing magnetic moment, the effect of the tensor interaction will be accompanied by a similar effect of the neutrino magnetic moment coupled to the strong magnetic field. In this case, both effects have to be taken into account simultaneously. This and related issues will be discussed elsewhere \[14\].

To conclude, in this paper we have studied the effects on neutrino propagation in matter due to the most general Lorentz-invariant interactions with the background fermions. Scalar, pseudo-scalar and longitudinal tensor interactions couple states of opposite chirality but do
not flip the helicity, and hence are suppressed by the ratio between the neutrino mass and its energy. Our crucial observation is that transverse tensor interactions are not suppressed by this ratio, since they couple states of both opposite chirality and opposite helicity and they can be coherently enhanced in the presence of a non-vanishing background polarization. As a result, such interactions can induce a neutrino spin-flip during propagation, much alike the magnetic moment spin-precession \[10,11\].

Applying our scenario to astrophysical neutrino sources, we find that the suppression from the average background polarization and the tensor couplings, implies that this effect is probably irrelevant for solar neutrinos. For supernova neutrinos the effect could become observable only in the presence of extremely large magnetic fields or, more speculatively if some new mechanism can enhance by a few orders of magnitude the conversion rate. Clearly, a definite conclusion about the relevance of this effect for different physical systems requires further investigation \[14\].

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APPENDIX A:

We present here the details of the computation of the matrix elements $\mathcal{M}_a$ [c.f. (2.4)] that determine the potentials $V_a$ [c.f. (2.3)]. We have

$$\mathcal{M}_a^f \equiv \langle f, p, \lambda | \psi_f \Gamma_a (g_a + g'_a \gamma^5) \psi_f | f, p, \lambda \rangle \quad (A1)$$

$$= \frac{1}{2E_f} \bar{u}_f (p, \lambda) \Gamma^a (g_a + g'_a \gamma^5) u_f (p, \lambda) \quad (A2)$$

$$= \frac{1}{4E_f} \text{Tr} \left[ \Gamma^a (g_a + g'_a \gamma^5) (\not p + m_f) (1 + \gamma^5 \not s) \right], \quad (A3)$$

where $E_f$ and $m_f$ denote respectively the energy and the mass of the background fermion $f$. In (A2) we have assumed the background fermions to be free, so that a plane wave expansion for the field operators can be used. In obtaining (A3) we have used the identity

$$u_f (p, \lambda) \bar{u}_f (p, \lambda) = \frac{1}{2} \left( \not p + m_f \right) (1 + \gamma^5 \not s), \quad (A4)$$

where the spin vector $s$ is defined as

$$s \equiv \left( \frac{p \cdot \lambda}{m_f}, \lambda + \frac{p (p \cdot \lambda)}{m_f (m_f + E_f)} \right), \quad (A5)$$

and satisfies $s^2 = -1$ and $s^\mu p^\mu = 0$. Using $\gamma^5 \sigma^{\mu\nu} = \frac{i}{2} \epsilon^{\mu\nu\rho\sigma} \sigma^{\rho\sigma}$ and the elementary traces

$$\frac{1}{4} \text{Tr} \left[ \Gamma^S, P; V, A, T \left( \not p + m_f \right) (1 + \gamma^5 \not s) \right] = m_f, 0, p^\mu, s^\mu, -\epsilon^{\nu\rho\sigma\tau} p_\nu s_\rho$$

we obtain

$$\mathcal{M}^S = g_S \frac{m_f}{E_f}, \quad (A6)$$

$$\mathcal{M}^P = g'_S \frac{m_f}{E_f}, \quad (A7)$$

$$\mathcal{M}^V = g_V \frac{p^\mu}{E_f} + g'_V \frac{m_f}{E_f} s^\mu, \quad (A8)$$

$$\mathcal{M}^A = g_A \frac{p^\mu}{E_f} + g'_A \frac{m_f}{E_f} s^\mu, \quad (A9)$$

$$\mathcal{M}^T = -g_T \epsilon^{\mu\nu\rho\sigma} \frac{p_\rho s_\sigma - p_\sigma s_\rho}{E_f} + ig'_T \frac{p^\mu s^\nu - p^\nu s^\mu}{E_f}. \quad (A10)$$

While the identity (A4) provides a simple way to calculate $\mathcal{M}_a$ by means of standard “trace technology”, we find it useful to present also an alternative calculation which is based on the spinorial expression (A2) for $\mathcal{M}_a$. In this derivation the details of the fermion polarization

$$\lambda = \xi_f^\dagger \sigma \xi_f \quad (\xi_f^\dagger \xi_f = 1) \quad (A11)$$

are more transparent ($\xi_f$ denotes the two-component spinor of the fermion $f$).

To compute $\bar{u}(p, \lambda) \Gamma_a (g_a + g'_a \gamma^5) u(p, \lambda)$ we choose the chiral representation for $\Gamma_a$, where
\[ 1 = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}, \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad (A12) \]

with \( \sigma^\mu = (I, \sigma^i) \) and \( \bar{\sigma}^\mu = (I, -\sigma^i) \). Since

\[ u(p, \lambda) \equiv \left( \begin{array}{c} u_L(p, \lambda) \\ u_R(p, \lambda) \end{array} \right) = \left( \begin{array}{c} \sqrt{p \bar{\sigma}} \xi_f \\ \sqrt{p \bar{\sigma}} \xi_f \end{array} \right) = \sqrt{\frac{E_f + m_f}{2} \left( \frac{I - \frac{p \sigma}{E_f + m_f}}{I + \frac{p \sigma}{E_f + m_f}} \xi_f \right)}, \quad (A13) \]

it is sufficient to calculate \( u^*_C(p, \lambda) \sigma^\mu u_C(p, \lambda) \) for \( C, C' \in \{ L, R \} \). Using the identities

\[ (p \cdot \sigma) \sigma_i + \sigma_i (p \cdot \sigma) = 2p_i \quad (A14) \]
\[ (p \cdot \sigma) \sigma_i - \sigma_i (p \cdot \sigma) = 2i\epsilon_{kji} \sigma_{kpj} \quad (A15) \]
\[ (p \cdot \sigma) \sigma_i (p \cdot \sigma) = 2p_i (p \cdot \sigma) - |p|^2 \sigma_i \quad (A16) \]

we obtain

\[ u_{L,R}^\dagger(p, \lambda) I u_{L,R}(p, \lambda) = (E_f \mp p \cdot \sigma) \quad (A17) \]
\[ u_{L,R}^\dagger(p, \lambda) I u_{R,L}(p, \lambda) = m_f \quad (A18) \]
\[ u_{L,R}^\dagger(p, \lambda) \sigma u_{L,R}(p, \lambda) = m_f \lambda + \left( \frac{p(p \cdot \sigma)}{E_f + m_f} \mp p \right) \quad (A19) \]
\[ u_{L,R}^\dagger(p, \lambda) \sigma u_{R,L}(p, \lambda) = E_f \lambda \pm i(p \times \lambda) - \frac{p(p \cdot \sigma)}{E_f + m_f} \quad (A20) \]

This allows us to compute

\[ J_a \equiv \Pi(p, \lambda) \Gamma_a u(p, \lambda) \quad (A21) \]

for \( a = S, P, V, A \):

\[ J_S = u_L^\dagger u_R + u_R^\dagger u_L = 2m_f \quad (A22) \]
\[ J_P = u_L^\dagger u_R - u_R^\dagger u_L = 0 \quad (A23) \]
\[ J_V = u_R^\sigma u_R + u_L^\sigma u_L = 2p_\mu \quad (A24) \]
\[ J_A = u_L^\sigma u_R - u_R^\sigma u_L = 2m_f s_\mu \quad (A25) \]

where \( s_\mu \) is defined in \( (A3) \). From the above one immediately obtains \( \mathcal{M}_a \) for \( a = S, P, V, A \) as in \( (A6-A9) \). To compute the tensor terms we define

\[ \Sigma_i \equiv \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}, \quad \Sigma_i' \equiv \Sigma_i \gamma^5 = \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}. \quad (A26) \]

The respective currents are

\[ J_{\Sigma} = u_L^\dagger \sigma u_R + u_R^\dagger \sigma u_L = 2E_f \lambda - \frac{2p(p \cdot \lambda)}{E_f + m_f} = 2E_f s - 2ps_0 \quad (A27) \]
\[ J_{\Sigma'} = u_L^\dagger \sigma u_R - u_R^\dagger \sigma u_L = 2i(p \times \lambda) = 2i(p \times s). \quad (A28) \]

Noting that \( \sigma_{ij} = \epsilon_{ijk} \Sigma_k, \sigma_{ij} \gamma_5 = \epsilon_{ijk} \Sigma'_k, \sigma_{0i} = i \Sigma_i' \) and \( \sigma_{0i} \gamma_5 = i \Sigma_i \) one can easily verify the expression for \( \mathcal{M}_T \) in eq. \( (A10) \).
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