The Application of Harmonic Analysis Methods to Modeling and Simulating the Deformation of the Ring Shell of a Rotating Furnace under High Temperature Influence

Pankratov V.M., Golikov A.V., Barulina M.A., Pankratova E.V., Markelova O.V.

Institute of Precision Mechanics and Control, Russian Academy of Sciences, Saratov, Russian Federation, 410028, Saratov, 24 Rabochaya str., Russia

marina@barulina.ru

Abstract. In the paper, the problem of calculating the deformed state of the inner ring shell of a rotating furnace under high temperature is solved by using harmonic analysis methods. The constructed mathematical model allows us to obtain parameters of furnace’s shell deformation based on an array of displacement, which is measured at the reference points. The reference points define the shape of the shell in the polar coordinate system in a discrete way. The results of the work allow automated processing of the metrological data in order to develop practical recommendations of the constructive and methodological way for the complementation of structural elements and equipment units or thermal control systems, if necessary.

1. Introduction

The leading technical equipment for the production of the various kind of friable materials is a rotating furnace [1-3], which used in the manufacture of mortar, materials and some consumable raw materials that required drying. That type of equipment may have a different design, size, and unique characteristics. The primary purpose of rotating furnace – is to produce cement clinker by burning dry and wet raw mixture(slimes). The rotating furnaces are quite complex industrial equipment, which is both physical and chemical reactor and furnace chamber. During the work process of such a furnace, the deformation of the ring shell occurs regularly. For example, it occurs due to high-temperature influences, which lead to bending of the geometrical axis of the shell and, consequently, to the irregularity of distribution of applied loads. The non-regular distribution of the applied loads is the main source of some dangerous accidents [4-8]. Therefore, the necessity of control of temperature and mechanical processes inside and outside of the furnace is the actual problem. Various methods exist for the process of monitoring of thermal deformation [6,7]. One of these methods is to obtain a discrete array of deformations from laser sensors, measuring the deviation of some points of the thin shell from its nominal value.

In this work, the problem of obtaining the parameters of the deformed state of the inner ring shell of the furnace is solved based on empirical input data and in consideration of the high-temperature effects. For solving the formulated problem, the mathematical model of furnace’s deformation was created based on methods of the harmonic analysis.

2. Mathematical model
The input data for solving this problem are:
- nominal radius $r_0$ of the thin ring shell,
- the inner radius of shell ring $r_1$,
- the array of fiducial (reference) points $r_i(\phi_i) = r_0 + \Delta r_i(\phi_i), \quad (i = 1,N, \quad N \geq 36)$ describes the shell’s shape in the polar coordinate system (fig.1).

Herewith, deviations $\Delta r_i(\phi_i)$ from the nominal radius are small (for example, up to 10%), thus $\Delta r_i/r_0 << 1$.

An array of reference points is formed from data received from laser sensors, which make measurements of the changeable value $r_i$ during the furnace rotation. The frequency of the measurements is specified.

The output data is:
- coefficients $a_n, c_n, \quad n = 1,M$ of Fourier series;
- value of angles $\phi_i$ and appropriate arrays $\Delta R_i(\phi_i), \Delta R_r(\phi_i)$ of small radial deviations with their signs in dimensional and dimensionless types and an array $h_i(\phi_i)$ of intervals from ring shell to shell surface, in case of $i = 1,L, \quad L = 10N$ or $L = 100N$;
- radius $R_T$ of the new circle, appropriate for the deformed shell;
- coordinates $x_C, y_C$ of the geometrical center of the new circle with radius $R_T$.

The main methodology for solving the problem with the initial data defined above is as follows.

The input array $r_i(\phi_i) = r_0 + \Delta r_i(\phi_i)$ reduced to a dimensionless form:
$$r_i(\phi_i) = \frac{r_i(\phi_i)}{r_0} = 1 + \frac{\Delta r_i(\phi_i)}{r_0}.$$ (1)

Let us present an array (1) as a Fourier series [6,7].

The Fourier series of $M$ terms for the deformed shell with variable radius $R(\phi)$, has the form:
- in dimensional form
$$R(\phi) = R_T + \delta \sum_{n=1}^{M} a_n \cos n\phi + c_n \sin n\phi ;$$ (2)
- in dimensionless form
$$R_*(\phi) = \frac{R(\phi)}{R_T} = 1 + \frac{\delta}{R_T} \sum_{n=1}^{M} a_n \cos n\phi + c_n \sin n\phi .$$ (3)
The value of the new radius \( R_T \), which is the average value of the numbers of the array (1), and the values of Fourier coefficients \( a_n, c_n \), reflecting the shape of the deformed shell, are calculated using the appropriated definite integrals by the method of Fourier harmonic analysis. The number \( M \) terms of Fourier series (2) or (3) is selected from the condition of accomplishment quite matching up of harmonic function (2) or (3) with the discrete response (1) on expectation function and mean square deviation.

In (2), (3) \( \delta \) – small parameter, which is necessary for analytical research.

To control the calculation of radius \( R_T \) of the new circle corresponding to the new deformed shell (1), using the least square adjustment method (LSAM). Find the circle with radius \( R_T \), which is the best approximation of the discrete function defined by (1) or the Cartesian coordinate system as:

\[
x_i = r_i \cos \varphi_i, \quad y_i = r_i \sin \varphi_i, \quad 0 \leq \varphi_i \leq 2\pi.
\]

According to the least square adjustment method [9,10], the residual values are calculated:

\[
x_i^2 + y_i^2 - R_i^2 = e_i.
\]

The function \( U \) is introduced and minimized:

\[
U = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} \left( x_i^2 + y_i^2 - R_i^2 \right)^2 \rightarrow \min.
\]

Calculating \( \frac{\partial U}{\partial R_T} = 0 \Rightarrow NR_T^2 - \sum_{i=1}^{N} \left( x_i^2 + y_i^2 \right) = 0 \Rightarrow NR_T^2 - \sum_{i=1}^{N} r_i^2 = 0 \).

The final formula for the new radius \( R_T \), according to the deformed shell, using the least square adjustment method is:

\[
R_T = \sqrt{\frac{\sum_{i=1}^{N} r_i^2}{N}}.
\]

The following formulas perform the calculation of the array of the small radial deviations with their signs and intervals \( h_i \) from the ring shell to the shell surface using the obtained based on (2) or (3):

- in dimensional form

\[
\Delta R_i(\varphi_i) = R(\varphi_i) - R_T = \delta \sum_{n=1}^{M} a_n \cos n\varphi_i + c_n \sin n\varphi_i ;
\]

- in dimensionless form

\[
\Delta R_i(\varphi_i) = R(\varphi_i) - R_T = \delta \sum_{n=1}^{M} a_n \cos n\varphi_i + c_n \sin n\varphi_i .
\]

- in dimensional form

\[
h_i(\varphi_i) = r_i - (r_0 \pm \Delta R_i) .
\]

Here \( i = 1, L \), \( r_0 \) is \( L > N \) in specified number of times, for example, \( L = 10N \) or \( L = 100N \).

Such detailed division (up to \( L = 100N \)) by the angle \( \varphi \) (4)-(6) allows, if necessary, to take into account the tolerances for installing the locking devices at the specified point of the shell.

We perform analytical and numerical findings of the coordinates \( x_C, y_C \) of the geometric center of the new circle with radius \( R_T \).

According to the known provisions [10], the determination of the geometric center \( x_C, y_C \) of the new circle with radius \( R_T \) is realized through the calculation of double integrals in the polar coordinate system by closed area \( G \) by the following formulas:

\[
x_C = \iint_G r^2 \cos \varphi rdrd\varphi \quad \int_G rdrd\varphi , \quad y_C = \iint_G r^2 \sin \varphi rdrd\varphi \quad \int_G rdrd\varphi ;
\]
In correlations (8), describing a closed area $G$ with the representation (2), and using the representation of $R(\varphi)$ in the form:

$$R(\varphi) = R_r + \delta \sum_{n=1}^{M} a_n \cos n\varphi + c_n \sin n\varphi = R_r + \delta f(\varphi).$$  \hspace{1cm} (9)

Calculating integrals in (7) taking into account (8), (9) and neglecting small terms of the second and higher order, we get:

$$x_c = \frac{2\pi}{2\pi} \cos \varphi \int_{0}^{R_r} \int_{0}^{r} dr \int_{0}^{\varphi} d\varphi \int_{0}^{R(\varphi)} rdr = \delta a_1, \hspace{1cm} (10)$$

$$y_c = \frac{2\pi}{2\pi} \sin \varphi \int_{0}^{R_r} \int_{0}^{r} dr \int_{0}^{\varphi} d\varphi \int_{0}^{R(\varphi)} rdr = \delta c_1. \hspace{1cm} (11)$$

As we can see, the displacement $x_c$ (on the horizontal axis $x$) of the geometrical center defined by the Fourier coefficient $a_1$ at $\cos \varphi$.

The offset $y_c$ (on the vertical axis $y$) of the geometrical center defined by the Fourier coefficient $c_1$ at $\sin \varphi$.

Other Fourier coefficients in (9) are not affected by the geometrical center displacement, which is quite consistent with the general provision of theoretical mechanics and mathematics.

On the fig.2 shows an example of calculating the radial diagram of the ring shell deformation based on the input data.

The main calculation results in graphical form are the value of the radial deviation (shell deformation) and displacement of the rotation axis. The numerical data in the table in the figure shows the offset of the geometric center of rotation and the eccentricity.

3. Conclusion.

A mathematical model of the deformed state of the inner ring shell of a rotating furnace is constructed using harmonic analysis methods. The possibility of analytical and numerical findings of the coordinates $x_c$, $y_c$ of the geometrical center of the new circle with radius $R_T$ was shown. The
constructed mathematical model is rather simple and can be implemented both in universal environments for mathematical modeling and in specialized software.

4. Bibliography

[1] Khodorov E I 1968 *The cement industry furnaces* (Leningrad: Stroiisdat) p. 456
[2] Rotary kilns for cement plants Available from: https://flsmidth-prod-cdn.azureedge.net/-/media/brochures/brochures-products/pyro/2000-2017/rotary-kilns-for-cement-plants.pdf [Accessed 1th Marth 2020]
[3] Vaccaro M 2006 Burning alternative fuels in rotary cement kilns *Cement Industry Technical Conference (IEEE Conference Record)* 10
[4] Tuishev A I and Tokarev D G 2015 Methods for determining the deformation state of rotating furnace *Vestnik NSIEI* 12 (55) 82-84
[5] Sholomitskii A A, Kovalev P S, Medvedskaya T M and Martinov A V 2017 The effect of heating the furnace on the straightness of its axis of rotation *Vestnik SSGA* 22(4) 18-24
[6] Petrov V V, Turin S V and Kopitov A N 2010 Control of the geometrical characteristics of the rotation furnaces *Cement and its usage* 2 78–82.
[7] Zhang Y and Zheng Weifei 2011 Dynamic detecting method of the axial distortion and shell distortion of rotary kiln *Cement Engineering* 02 20-26
[8] Boateng A A 2011 *Rotary Kilns: Transport Phenomena and Transport Processes* (Butterworth-Heinemann)
[9] Bronstein I N, Semendyayev K A, Musiol G and Mühlig H 2007 *Handbook of Mathematics* (Springer Berlin Heidelberg)
[10] Adler Y P, Markova E V and Granovsky Y V 1976 *Experiment planning when searching for optimal conditions* (Moscow: Nauka)

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