Prompt and accurate sky localization of gravitational-wave sources

Soichiro Morisaki
Research Center for the Early Universe (RESCEU), Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan
Department of Physics, Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan

Vivien Raymond
Gravity Exploration Institute, School of Physics and Astronomy, Cardiff University, The Parade, Cardiff CF24 3AA, UK

Abstract. Accurate, precise and prompt sky localization of gravitational-wave sources is essential to the success of multi-messenger astronomy. One of the most accurate sky localizations we can obtain is from full signal parameter estimation obtained with the LIGO-Virgo LALInference software, done after a quick initial sky localization with the Bayestar software. While more accurate, the improved analysis can take on the order of months to complete for a binary neutron star event. To solve this issue, we develop a new technique to speed up the parameter estimation. Our technique speeds up the parameter estimation by a factor of $O(10^4)$ and enables updating the sky localization within 10 minutes after the detection.

1. Reduced Order Quadrature

Parameter estimation of gravitational-wave signal is computationally costly, especially for the signal from a binary neutron star (BNS) merger. It is because this process needs millions of likelihood calculations to properly sample a high-dimensional parameter space, and each likelihood calculation is costly. The likelihood is given by

$$p(d|\vec{\theta}) = A \exp \left[ -\frac{1}{2} (d - h(\vec{\theta}), d - h(\vec{\theta})) \right],$$

where $A$ is a normalization constant, $\vec{\theta}$ is a vector of source parameters, such as masses, spins, and locations, and $d$ and $h$ are data and signal observed by a detector respectively. Here we consider only one detector for the simplicity. The inner product, $(x, y)$, is defined by

$$(x, y) \equiv 4\Re \left[ \Delta f \sum_{k=1}^{L} \frac{\tilde{x}^*(f_k) \tilde{y}(f_k)}{S_n(f_k)} \right],$$

where $S_n(f)$ is the (one-sided) power spectral density of the detector and $L$ is the number of frequency samples. To represent the signal, $L$ needs to be at least $\sim f_{\text{high}} \tau$, where $f_{\text{high}}$ and $\tau$...
are the highest frequency and the duration of the signal respectively. For the signal from a BNS observed by ground-based detectors, $L$ is $\mathcal{O}(10^5)$. Therefore, the likelihood calculation needs $\mathcal{O}(10^5)$ floating-point operations, and it is computationally costly.

One of the techniques to reduce the operations is known as reduced order quadrature (ROQ) [1, 2], which is being used by the LIGO-Virgo collaboration. The basic idea is to approximate a waveform, $\tilde{h}(f_l)$, by the linear combination of basis waveforms much fewer than $\mathcal{O}(10^5)$,

$$
\tilde{h}(f_l) \simeq \sum_{k=1}^{K} c_k \tilde{e}_k(f_l).
$$

Then the inner product can be reduced to

$$
(h, d) \simeq \sum_{k=1}^{K} c_k^* (e_k, d).
$$

Since the inner product, $(e_k, d)$, can be pre-computed before the parameter estimation, the likelihood calculation can be reduced to $K$ floating-point operations, and the parameter estimation is sped up by a factor of $\sim L/K$.

The previous works [1, 2] show $K \sim \mathcal{O}(10^3)$ and the parameter estimation can be sped up by a factor of $\mathcal{O}(100)$ for BNSs. While this is significant improvement, the parameter estimation still takes from 6 hours to a day. Given that the optical radiation continues for a few days after the merger, it is still too slow. To solve this problem, we develop a technique to speed up the parameter estimation further.

The idea of our technique is to tune the parameter-estimation follow-up with the trigger values reported by detection pipelines. The detection pipeline filters data with millions of template waveforms corresponding to various values of masses and spins. Then, it reports a set of source parameter values maximizing the significance of the signal. With these trigger values, we can roughly guess the true values of the source parameters. Especially, we can restrict the source parameter space explored in the parameter estimation. This restriction significantly reduces the basis size, $K$, and speeds up the parameter estimation.

2. Restriction of parameter space based on trigger values

Our strategy is to rely on some combinations of source parameters and restrict their ranges before parameter estimation. Such combinations should be the best measurable combinations otherwise we can not restrict the parameter space and speed up the parameter estimation significantly.

2.1. The best measurable combinations

The previous work [3] studies the best measurable combinations. We apply the same method to the waveform incorporating up to the 1.5 Post-Newtonian (PN) phase contribution with assuming the design sensitivity of the LIGO detector and find the following combinations,

$$
\mu^1 = 0.980\psi^0 + 0.185\psi^2 + 0.0702\psi^3,
$$

$$
\mu^2 = -0.195\psi^0 + 0.837\psi^2 + 0.512\psi^3,
$$

where $\psi^0$, $\psi^2$ and $\psi^3$ are the 0PN, 1PN and 1.5PN phase contributions at the reference frequency of 200 Hz.
2.2. The ranges of $\mu^1$ and $\mu^2$

We need an algorithm to decide the ranges of $\mu^1$ and $\mu^2$ given the trigger values. Their ranges should be broad enough to accommodate the following two uncertainties: systematic errors due to the mismatch between the trigger values and the true values, and statistical errors due to instrumental noise. In this study, we consider up to the 2PN phase contributions, $\psi^\alpha (\alpha = 0, 2, 3, 4)$.

The systematic errors arise because the trigger values are slightly off from the true values in general due to the discreteness of the template bank. We consider the limit of high signal-to-noise ratio and show that the true value of $\psi^\alpha$, $\dot{\psi}^\alpha$, is within the ellipsoid,

$$\tilde{\Gamma}_{\alpha\beta}(\dot{\psi}^\alpha - \dot{\psi}^\alpha)(\dot{\psi}^\beta - \dot{\psi}^\beta) < 2,$$

where $\dot{\psi}^\alpha$ is the trigger value of $\psi^\alpha$. $\tilde{\Gamma}$ is the matrix obtained by projecting the constant time and phase out of the Fisher matrix (See the discussions around Eq. (17) of [3]).

The statistical error broadens the posterior distribution, and the prior range needs to include a finite range around the position of the true values. We show that in the limit of high signal-to-noise ratio the region which needs to be included by the prior range is

$$\tilde{\Gamma}_{\alpha\beta}(\psi^\alpha - \dot{\psi}^\alpha)(\psi^\beta - \dot{\psi}^\beta) < \left( \frac{N}{\rho_{\text{net}}} \right)^2,$$

where $N = \mathcal{O}(1)$ constant and $\rho_{\text{net}}$ is the network signal-to-noise ratio. This condition is derived based on the assumption that the likelihood can be approximated by the Gaussian distribution, and provides its N-sigma error region. Since we typically calculate the 90% credible regions of the parameters, $N = 3$ provides a broad enough region and we apply this value. On the other hand, $\rho_{\text{net}}$ should be the lowest value. Since the detection usually requires the signal-to-noise ratio of $\gtrsim 8$, we apply conservative threshold, $\rho_{\text{net}} = 5$.

We also note that the prior constraints on the masses and spins significantly affect the possible ranges of $\mu^1$ and $\mu^2$. In this work, we consider the following two prior configurations.

astrophysical-spin prior: $0 \ M_\odot < m_1, m_2 < 3 \ M_\odot$, $-0.05 < \chi_1, \chi_2 < 0.05$. (9)

high-spin prior: $0 \ M_\odot < m_1, m_2 < 3 \ M_\odot$, $-0.7 < \chi_1, \chi_2 < 0.7$. (10)

The former one is motivated by the fact that even PSR J0737-3039A [4], which are one of the observed binary neutron star system that will merge within a Hubble time and contains the most extremely spining pulsar among them, will have $|\chi| \lesssim 0.04$ at the merger. The latter one is motivated by the fact that the maximum spin parameter of a uniformly rotating star is $\sim 0.7$ for various realistic nuclear equations of state [5].

The ranges of $\mu^1$ and $\mu^2$ are their possible ranges given the constraints (7), (8) and (9) or (10).

3. Performance

Finally, we implement $\mu^1$ and $\mu^2$ as sampling parameters to LALInference [6], and test our technique with test parameter estimation runs on a injected signal from a 1.4$M_\odot$-1.4$M_\odot$ non-spinning binary neutron star. First, we calculate the prior ranges of $\mu^1$ and $\mu^2$ assuming the trigger values of $m_1 = m_2 = 1.4 \ M_\odot$, $\chi_1 = \chi_2 = 0$, and construct the ROQ basis within this range. The resultant basis sizes are 20 for the astrophysical-spin prior and 48 for the high-spin prior. This is significant improvement given that $K = \mathcal{O}(1000)$ in the conventional ROQ technique. With this basis, the parameter estimation can be sped up by $\sim L/K = \mathcal{O}(10^4)$. The run times are reduced to $\sim 10$ minutes for the astrophysical-spin prior and $\sim 20$ minutes for the high-spin prior, which enables updating the sky localization of a BNS event within $\mathcal{O}(10)$ minutes after the detection.
4. Conclusion

The full signal parameter estimation can provide the most accurate and precise sky localization of gravitational-wave sources. On the other hand, it takes months to complete and is too slow for the follow-up observations. To solve this issue, we develop a technique to speed up the parameter estimation. The idea of our technique is to utilize the trigger values from the detection pipelines and restrict and optimally parametrize the parameter space explored by the parameter estimation. As a result, we speed up the parameter estimation by a factor of $O(10^4)$, which enables providing the accurate and precise sky localization of a BNS event within $O(10)$ minutes after its detection, which will allow for a higher chance to identify electromagnetic counterpart signals.

References

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