Testing dark energy beyond the cosmological constant barrier

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Although well motivated from theoretical arguments, the cosmological constant barrier, i.e., the imposition that the equation-of-state parameter of dark energy \( \omega_x \equiv p_x/\rho_x \) is \( \geq -1 \), seems to introduce bias in the parameter determination from statistical analyses of observational data. In this regard, phantom dark energy or superquintessence has been proposed in which the usual imposition \( \omega \geq -1 \) is relaxed. Here, we study possible observational limits to the phantom behavior of the dark energy from recent distance estimates of galaxy clusters obtained from interferometric measurements of the Sunyaev-Zel’dovich effect/X-ray observations and Type Ia supernova data. We find that there is much observationally acceptable parameter space beyond the \( \Lambda \) barrier, thus opening the possibility of existence of more exotic forms of energy in the Universe.

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Dark energy or quintessence is the invisible fuel that seems to drive the current acceleration of the Universe. Phenomenologically, this energy component is usually described by an equation-of-state parameter \( \omega_x \) which represents the ratio of the dark energy pressure to its energy density, \( \omega_x \equiv p_x/\rho_x \). In order to achieve cosmic acceleration, Einstein Field Equations (EFE) require \( \omega_x \) to be \( < -1/3 \) for a universe described by a single component whereas for a dark matter/energy dominated universe the required value, \( \omega_x < - (\Omega_m/3\Omega_x + 1/3) \), depends on the ratio between the baryonic/dark matter (\( \Omega_m \)) and dark energy density parameters (\( \Omega_x \)). In other words, what EFE mean with these upper limits is that any physical field with positive energy density and negative pressure, which violates the strong energy condition \( (\rho + 3p) > 0 \), may cause antigravity regimes (see [1] for a review on classical energy conditions).

Since cosmic acceleration from EFE provides only an upper limit to \( \omega_x \), a point of fundamental importance associated with this parametrization for the dark energy equation of state (ES) is related to the physical and/or observational lower limits that may be imposed on the parameter \( \omega_x \). Physically, if one wants dark energy to be stable, then it must obey the null energy condition which, in the Friedmann-Robertson-Walker metric, is equivalent to \( \rho + p > 0 \). This energy condition implies \( \omega_x \geq -1 \) when applied to a dark energy component described by \( \omega_x \equiv p_x/\rho_x \) or, equivalently, that the vacuum energy density or the cosmological constant (\( \Lambda \)), which is characterized by \( \omega_x = -1 \), would constitute the natural lower limiting case. Following this reasoning, firstly explicited in [2], a number of theoretical and observational analyses in which the restriction \( -1 \leq \omega_x < 0 \) is imposed have appeared in the recent literature [3]. However, by focussing our attention only on the observational side, what would current observations have to tell us about that? As well observed by Caldwell [4], it is curious that most of the observational constraints on \( \omega_x \) are consistent with models that go right up to the \( \omega_x = -1 \) border. Thus, paraphrasing him, one might ask what lies on the other side of the cosmological constant barrier.

The answer to this question has been given by several authors who also have pointed out some strange properties of phantom dark energy \( (\omega_x < -1) \) as, for instance, the fact that its energy density increases with the expansion of the Universe in contrast with usual quintessence \( (\omega_x \geq -1) \); the possibility of rip-off of the large and small scale structure of matter; a possible occurrence of future curvature singularity, etc. [1]. Although having these unusual characteristics, a phantom behavior is predicted by several scenarios, e.g., kinetically driven models [4] and some versions of brane world cosmologies [6] (see also [1] and references therein). Moreover, from the observational point of view, phantom energy is found to be compatible with most of the classical cosmological tests and seems to provide a better fit to type Ia supernovae (SNe Ia) observations than do \( \Lambda \)CDM or generic quintessence scenarios \((\omega_x \geq -1)\) [5]. Therefore, given our state of complete ignorance about the nature of dark energy, it is worth asking whether current observations are able to shed some light on the other side of the \( \Lambda \) barrier.

Our aim, in this Letter, is to seek possible observational limits to the phantom behavior of the dark energy ES, as well as to detect the bias in the ES parameter determination due to the imposition \( \omega_x \geq -1 \), from recent distance estimates of galaxy clusters obtained from interferometric measurements of the Sunyaev-Zel’dovich effect (SZE) and X-ray observations. We use, for that, the largest homogeneously analyzed sample of the SZE/X-ray clusters with angular diameter distance (ADD) determinations thus far, as provided by Reese et al. [6]. In order to constrain more precisely regions of the parameter space, we also combine SZE/X-ray ADD data with the newest SNe Ia sample of the Supernova Cosmology Project [7], recent determinations of the matter density parameter and the latest measurements of the Hubble parameter as given by the \( HST \) key project [8]. In agreement with other independent analyses, it is shown that with or without

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Angular diameter distance (Mpc)

10000

1000

100

1

thus far. From these intermediary and high-redshift ranges, it constitutes the largest homogeneously analyzed sample of clusters with redshifts ranging from 0.14 to 0.78, which were estimated (see [13] for recent summaries). In this analysis, the authors estimated the Hubble parameter $H(z)$ for three different cosmologies, with the uncertainties due to inverse Compton scattering of CMBR photons passing through an intracluster medium is of particular importance to estimating distances to galaxy clusters. This is so because for a given temperature such effect, known as Sunyaev-Zel’dovich effect [12], is proportional to the line integral of the electron number density through the cluster, $\Delta T \propto \int n_e T_e d\ell$, while the X-ray bremsstrahlung surface brightness scales as $S_X \propto \int n_e^2 d\ell$. Thus, by using X-ray spectroscopy to find the temperature of the gas and by making some assumptions on the cluster geometry, the distance to the cluster may be estimated (see [13] for recent summaries).

By applying this technique, suggested long ago [14], Reese et al. [4] determined the distance to 18 galaxy clusters with redshifts ranging from 0.14 to 0.78, which constitutes the largest homogeneously analyzed sample of the SZE/X-ray clusters with distance determinations thus far. From these intermediary and high-$z$ measurements, the authors estimated the Hubble parameter for three different cosmologies, with the uncertainties agreeing with the HST key project results [11], which probes the expansion rate in the nearby universe. Since the redshift range of the galaxy cluster sample is comparable to the intermediary and high-$z$ SNe Ia data compiled by the Supernova Cosmology Project [10, 15] and the High-$z$ Supernova Team [16], we understand that it may also provide an independent crosscheck of the cosmic acceleration mechanism. Thus, in what follows, we use these data as well as a combination of them with SNe Ia measurements to place observational limits on the ES parameter of the phantom dark energy.

**Analysis.** With the usual assumption that the effective equation of state, $\omega \sim \int \omega_x(z)\Omega_x(z)dz/\int \Omega_x(z)dz$, is a good approximation for a wide class of dark energy scenarios [17], the angular diameter distance as a function of the redshift can be written as

$$D_A(z; \Omega_m, \omega) = \frac{3000h^{-1}}{(1 + z)} \int_0^z \frac{dz'}{E(z'; \Omega_m, \omega)} \text{ Mpc}, \quad (1)$$

where the dimensionless function $E(z'; \Omega_m, \omega)$ is given by

$$E = \left[ \Omega_m (1 + z')^3 + (1 - \Omega_m)(1 + z')^{3(1+\omega)} \right]^{1/2}. \quad (2)$$

Figure 1a shows the SZE/X-ray determined distances for 18 clusters as a function of redshift for a fixed value of $\Omega_m = 0.3$ and selected values of the ES parameter. Note that Abell 370 cluster (the open circle) is clearly an
outlier in the sample so that, following [3, 12], we exclude it from the statistical analyses that follow. In Fig. 1b we show the confidence regions (68%, 95% and 99%) in the plane $\Omega_m - \omega$ from SZE/X-ray ADD data. Since we have nowadays good estimates of the dark matter density [19], we have assumed a Gaussian prior on the matter density parameter, i.e., $\Omega_m = 0.35 \pm 0.07$. Such a value, which is in good agreement with dynamical estimates on scales up to about $2h^{-1}$ Mpc [13], is derived by combining the ratio of baryons to the total mass in clusters determined from SZE/X-ray measurements with the latest estimates of the baryon density $\Omega_b = (0.020 \pm 0.002)h^{-2}$ [24] and the final value of the Hubble parameter obtained by the HST key Project $H_o = 72 \pm 8$ km s$^{-1}$ Mpc$^{-1}$ [11]. As the Figure shows, given the $\Lambda$ barrier, the best-fit converges to $\omega = -1$ (and $\Omega_m = 0.32$), with a 68% confidence bound of $\omega \leq -0.84$.

A generalization of this analysis to a parameter space that extends $\omega$ to values smaller than $-1$ is presented in Figure 1c (in all extended analyses, we have used a $\chi^2$ minimization for the range of $\Omega_m$ and $\omega$ spanning the interval [0,1] and [-15,0], respectively). There, it is shown that there is much observationally acceptable parameter space beyond the $\Lambda$ barrier, in full agreement with other similar analyses [21, 22, 23]. In actual fact, the best-fit model for these data sets occurs for $\Omega_m = 0.36$ and $\omega = -3.5$ ($\chi^2_{min} = 10.8$) with a 68% confidence bound of $-5.5 \leq \omega \leq -2.2$ (0.30 $\leq \Omega_m \leq 0.41$). In particular, this best-fit model corresponds to a very accelerating universe with deceleration parameter $q_0 \simeq -2.8$ and total expanding age of $9.7h^{-1}$ Gyr. It is also worth noticing that extreme values of $\omega$ are allowed because for intermediary and high redshifts, angular diameter distances become quite insensitive to large variations of the ES parameter (see Fig. 1a). For example, at $z = 0.78$ (the redshift of MS1137, the farthest galaxy cluster), the angular diameter distance for $\Omega_m = 0.3$ and $\omega = -3$ ($D_A \simeq 1840$ Mpc) is only ~ 10% smaller than in a model with the same amount of dark matter and $\omega = -10$ ($D_A \simeq 2045$ Mpc). This particular behavior is quite similar to what happens in analyses involving age estimates. There, like here, the function of the cosmological parameters ($\Omega_m$ and $\omega$) quickly asymptotes for large values of $\omega$ [24].

In our search for possible lower limits to the ES of the phantom component, we now perform a joint analysis of SZE/X-ray ADD and SNe Ia data. For that, we use the newest SNe Ia sample of the Supernova Cosmology Project [16] (with stretch and extinction correction applied) and follow the analytical marginalization method for the “zero point magnitude” $M$ as given in Ref. [28]. The results of the present analysis are shown in Panels 2a and 2b. In Panel 2a we show the 68%, 95% and 99% c.l. in the $\Omega_m - \omega$ plane by imposing the $\Lambda$ barrier ($\omega \geq -1$). From this combination of observational data sets we find that the best-fit model occurs exactly on the $\omega = -1$ border with $\Omega_m = 0.29$ and $\chi^2_{min}/\nu \simeq 1.26$. At 95% c.l., we obtain $\omega \leq -0.83$ and $0.19 \leq \Omega_m \leq 0.37$. Figure 2b generalizes the former analysis to include more neg-
ative values of the dark energy ES. Again, we find that there is much acceptable parameter space beyond the line \( \omega = -1 \) and that the confidence regions are modified by its presence, what clearly indicates the existence of bias in the parameter determination due to the \( \Lambda \) barrier. This particular analysis provides a 68\% confidence bound of \(-1.98 \leq \omega \leq -1.42 \) and \( 0.30 \leq \Omega_m \leq 0.45 \), with the best-fit model happening at \( \omega = -1.7 \) and \( \Omega_m = 0.38 \) \( (\chi^2_{\text{min}}/\nu \simeq 1.2) \), which corresponds to a accelerating universe with deceleration parameter \( q_o \simeq -1.0 \) and total expanding age of 9.3\( h^{-1} \) Gyr. Therefore, if one combines this 68\% confidence bound on \( \omega \) with the upper limit from EFE, one would have \(-1.9 \leq \omega < -1/3 \) instead of the usual \(-1 \leq \omega < -1/3 \).

At this point we compare our results with other recent limits on the ES parameter of the phantom energy derived from independent methods. For example, in Ref. [21] data from CMBR, large scale structure (LSS) and SNe Ia were combined to find a 95\% confidence bound of \(-2.68 < \omega < -0.78 \). Such results agree with the constraints obtained from a combination of Chandra observations of the X-ray luminosity of galaxy clusters with independent measurements of the baryonic matter density and the latest measurements of the Hubble parameter. From this latter analysis, it was found \(-2.0 \leq \omega \leq -0.6 \) at 68\% c.l. [24] while a combination of these X-ray data with measurements of the angular size of milliarcsecond radio sources provide \(-2.22 \leq \omega \leq -0.62 \) at 95\% c.l. [27]. Recently, constraints from several CMBR experiments (including the latest WMAP results [28]) along with LSS data, Hubble parameter measurements from the \( HST \) key project and SNe Ia data were obtained, with the ES parameter ranging from \(-1.38 \) to \(-0.82 \) at 95\% c.l. [21]. More recently, the authors of Ref. [22] used a sample of 57 SNe Ia to find a 95\% confidence bound of \(-2.4 < \omega < -1 \) whereas estimates of the age of the Universe as given by WMAP \((t_o = 13.7 \pm 0.2 \) Gyr) provide \(-1.18 < \omega < -0.93 \), which corresponds to an accelerating scenario with the deceleration parameter \( q_o \) lying in the range \(-0.8 < q_o < -0.52 \) [22]. All these results agree at some level with the ones found in this work.

**Conclusion.** This paper, as many of its predecessors, is mainly motivated by our present state of ignorance concerning the nature of the so-called dark energy (or dark pressure). In a first moment, vacuum energy density or cosmological constant was thought of (also motivated by the *old* age of the Universe problem) as the most viable explanation for the evidence of cosmic acceleration as given by SNe Ia observations. Observationally, \( \Lambda \) remains as a good candidate for dark energy although, from a theoretical viewpoint, one has to face a fine-tuning of 120 orders of magnitude in order to make its observed value compatible with quantum field theory expectations [30]. Later on, a first generalization of this former description, in which a “X-matter” component with ES parameter ranging from a cosmological constant \((\omega = -1)\) to pressureless matter \((\omega = 0)\), was proposed as a possible description for current observations [2]. More recently, a new generalization, the so-called *phantom* energy, in which the \( \Lambda \) barrier \((\omega = -1)\) is removed, has received increasing attention among theorists. Naturally, all these theoretical attempts to describe dark energy would not be valid without observational support. But that is not the case once several observational analyses support these parametrizations for dark energy. Here, we have explored the prospects for constraining the *phantom* behavior of the dark energy from SZE/X-ray distance estimates of galaxy clusters and SNe Ia data. We have shown that these data allow much acceptable parameter space beyond the line \( \omega = -1 \), what indicates not only the possibility of bias in the parameter determination when the \( \Lambda \) barrier is imposed but also the possibility of existence of more exotic forms of energy in the Universe. Naturally, we do not expect such results to be completely free of observational and/or theoretical uncertainties, mainly because there still exist considerable systematics uncertainties associated with SZE/X-ray distance determinations [4]. What we do expect is that in the near future, new sets of observations along with more theoretical effort will be able to decide on which side of the \( \Lambda \) barrier lies the so far mysterious dark energy.

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[1] S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D68, 023509 (2003)
[2] P. M. Garnavich et al., Astrophys. J.509, 74 (1998)
[3] M. S. Turner and M. White, Phys. Rev. D56, 4439 (1997); T. Chiba, N. Sugiyama and T. Nakamura, Mon. Not. Roy. Astron. Soc., 289, L5 (1997); Z.-H. Zhu, Mod. Phys. Lett. A 15, 1023 (2000); J. S. Alcaniz and J. A. S. Lima, Astrophys. J., 550, L133 (2001) (astro-ph/0109047); Z.-H. Zhu, M.-K. Fujimoto, and D. Tatsumi, Astron. Astrophys. 372, 377 (2001); J. Kujat, A. M. Linn, R. J. Scherrer and D. H. Weinberg, Astrophys. J., 572, 1 (2002); D. Jain et al., Int. J. Mod. Phys. D 12, 953 (2003)
[4] R. R. Caldwell, Phys. Lett. B545, 23 (2002)
[5] B. McInnes, [astro-ph/0210321], V. Faraoni, Int. J. Mod. Phys. D11, 471 (2002); R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett., 91, 071301 (2003); P. F. Gonzalez-Diaz, Phys. Rev. D68,
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I. INTRODUCTION

Dark energy or \textit{quintessence} is the invisible fuel that seems to drive the current acceleration of the Universe. Phenomenologically, this energy component is usually described by an equation-of-state parameter \(\omega_x\) which represents the ratio of the dark energy pressure to its energy density, \(\omega_x \equiv p_x/\rho_x\). In order to achieve cosmic acceleration, Einstein Field Equations (EFE) require \(\omega_x\) to be \(< -1/3\) for a universe described by a single component whereas for a dark matter/energy dominated universe the required value, \(\omega_x < -1/3 (\Omega_m/3\Omega_x + 1/3)\), depends on the ratio between the baryonic/dark matter (\(\Omega_m\)) and dark energy density parameters (\(\Omega_x\)). In other words, what EFE mean with these upper limits is that any physical field with positive energy density and negative pressure, which violates the strong energy condition \((p + 3\rho > 0)\), may cause antigravity regimes (see [1] for a review on classical energy conditions).

Since cosmic acceleration from EFE provides only an upper limit to \(\omega_x\), a point of fundamental importance associated with this parametrization for the dark energy equation of state (ES) is related to the physical and/or observational \textit{lower} limits that may be imposed on the parameter \(\omega_x\). Physically, if one wants dark energy to be stable, then it must obey the null energy condition which, in the Friedmann-Robertson-Walker metric, is equivalent to \(\rho + p > 0\). This energy condition implies \(\omega_x \geq -1\) when applied to a dark energy component described by \(\omega_x \equiv p_x/\rho_x\) or, equivalently, that the vacuum energy density or the cosmological constant (\(\Lambda\)), which is characterized by \(\omega_x = -1\), would constitute the natural lower limiting case. Following this reasoning, firstly explicitied in [2], a number of theoretical and observational analyses in which the restriction \(-1 \leq \omega_x < 0\) is imposed have appeared in the recent literature [3]. However, by focussing our attention only on the observational side, what would current observations have to tell us about that? As well observed by Caldwell [4], it is curious that most of the observational constraints on \(\omega_x\) are consistent with models that go right up to the \(\omega_x = -1\) border. Thus, paraphrasing him, one might ask what lies on the other side of the cosmological constant \textit{barrier}.

The answer to this question has been given by several authors who also have pointed out some strange properties of \textit{phantom} dark energy \((\omega_x < -1)\) as, for instance, the fact that its energy density increases with the expansion of the Universe in contrast with usual \textit{quintessence} \((\omega_x \geq -1)\); the possibility of rip-off of the large and small scale structure of matter; a possible occurence of future curvature singularity, etc. [3]. Although having these unusual characteristics, a \textit{phantom} behavior is predicted by several scenarios, e.g., kinetically driven models and some versions of brane world cosmologies (see also [1] and references therein). Moreover, from the observational point of view, \textit{phantom} energy is found to be compatible with most of the classical cosmological tests and seems to provide a better fit to type Ia supernovae (SNe Ia) observations than do \(\Lambda\)CDM or generic \textit{quintessence} scenarios \((\omega_x \geq -1)\). Therefore, given our state of complete ignorance about the nature of dark energy, it is worth asking whether current observations are able to shed some light on the other side of the \(\Lambda\) \textit{barrier}.

Our aim, in this \textit{Letter}, is to seek possible observational limits to the \textit{phantom} behavior of the dark energy ES, as well as to detect the bias in the ES parameter determination due to the imposition \(\omega_x \geq -1\), from recent distance estimates of galaxy clusters obtained from interferometric measurements of the Sunyaev-Zel’dovich effect (SZ) and X-ray observations. We use, for that, the largest homogeneous sample of the SZE/X-ray clusters with angular diameter distance (ADD) determinations thus far, as provided by Reese et al. [5]. In order to constrain more precisely regions of the parameter space, we also combine SZE/X-ray ADD data with the newest SNe Ia sample of the \textit{Supernova Cosmology Project} [6], recent determinations of the matter density parameter, WMAP distance estimates and the latest measure-
II. SZE, X-RAY EMISSION AND DISTANCE ESTIMATES

Among the sources of temperature fluctuations in the Cosmic Microwave Background Radiation (CMBR), a small distortion due to inverse Compton scattering of CMBR photons passing through an intracluster medium is of particular importance to estimating distances to galaxy clusters. This is so because for a given temperature such effect, known as Sunyaev-Zel’dovich effect, is proportional to the line integral of the electron number density through the cluster, $\Delta T \propto \int n_e T_e d\ell$, while the X-ray bremsstrahlung surface brightness scales as $S_X \propto \int n_e^2 d\ell$. Thus, by using X-ray spectroscopy to find the temperature of the gas and by making some assumptions on the cluster geometry, the distance to the cluster may be estimated (see [14] for recent summaries).

By applying this technique, suggested long ago [15], Reese et al. [9] determined the distance to 18 galaxy clusters with redshifts ranging from 0.14 to 0.78, which constitutes the largest homogeneously analyzed sample of the SZE/X-ray clusters with distance determinations thus far. From these intermediary and high-z measurements, the authors estimated the Hubble parameter for three different cosmologies, with the uncertainties agreeing with the HST key project results [12], which probes the expansion rate in the nearby universe. Since the redshift range of the galaxy cluster sample is comparable to the intermediary and high-z SNe Ia data compiled by the Supernova Cosmology Project [10, 16] and the High-z Supernova Team [17], we understand that it may also provide an independent crosscheck of the cosmic acceleration mechanism. Thus, in what follows, we use these data as well as a combination of them with SNe Ia measurements to place observational limits on the ES parameter of the phantom dark energy.

III. ANALYSIS

With the usual assumption that the effective equation of state, $\omega \sim \int \omega_x(z) \Omega_x(z)dz / \int \Omega_x(z)dz$, is a good approximation for a wide class of dark energy scenarios [18], the angular diameter distance as a function of the redshift can be written as

$$D_A(z; \Omega_m, \omega) = \frac{3000h^{-1}}{(1+z)} \int_o^z \frac{dz'}{\varepsilon(z'; \Omega_m, \omega)} \text{ Mpc}, \quad (1)$$
where the dimensionless function \( \mathcal{E}(z'; \Omega_m, \omega) \) is given by

\[
\mathcal{E} = \left[ \Omega_m (1 + z')^3 + (1 - \Omega_m) (1 + z')^{3(1+\omega)} \right]^{1/2}.
\]

Figure 1a shows the SZE/X-ray determined distances for 18 clusters as a function of redshift for a fixed value of \( \Omega_m = 0.3 \) and selected values of the ES parameter. Note that Abell 370 cluster (the open circle) is clearly an outlier in the sample so that, following [9, 19], we exclude it from the statistical analyses that follow. In Fig. 1b we show the confidence regions (68%, 95% and 99%) in the plane \( \Omega_m - \omega \) from SZE/X-ray ADD data. Since we have nowadays good estimates of the dark matter density [20], we have assumed a Gaussian prior on the matter density parameter, i.e., \( \Omega_m = 0.35 \pm 0.07 \). Such a value, which is in good agreement with dynamical estimates on scales up to about 2h\(^{-1}\) Mpc [20], is derived by combining the ratio of baryons to the total mass in clusters determined from SZE/X-ray measurements with the latest estimates of the baryon density \( \Omega_b = 0.020 \pm 0.004 \)h\(^{-2}\) [21] and the final value of the Hubble parameter obtained by the HST key Project \( H_0 = 72 \pm 8 \) km s\(^{-1}\) Mpc\(^{-1}\) [22]. As the Figure shows, given the \( \Lambda \) barrier, the best-fit converges to \( \omega = -1 \) (and \( \Omega_m = 0.32 \)), with a 68% confidence bound of \( \omega \leq -0.84 \).

A generalization of this analysis to a parameter space that extends \( \omega \) to values smaller than \(-1\) is presented in Figure 1c (in all extended analyses, we have used a \( \chi^2 \) minimization for the range of \( \Omega_m \) and \( \omega \) spanning the interval [0,1] and [-15,0], respectively). There, it is shown that there is much observationally acceptable parameter space beyond the \( \Lambda \) barrier, in fully agreement with other similar analyses [22, 23, 24]. In actual fact, the best-fit model for these data sets occurs for \( \Omega_m = 0.36 \) and \( \omega = -3.5 \left( \chi^2_{min} = 10.8 \right) \) with a 68% confidence bound of \( -5.5 \leq \omega \leq -2.2 \) (0.30 \( \leq \Omega_m \leq 0.41 \)). In particular, this best-fit model corresponds to a very accelerating universe with deceleration parameter \( q_0 \approx -2.8 \) and total expansion age of 9.7h\(^{-1}\) Gyr. It is also worth noticing that extreme values of \( \omega \) are allowed because for intermediary and high redshifts, angular diameter distances become quite insensitive to large variations of the ES parameter (see Fig. 1a). For example, at \( z = 0.78 \) (the redshift of MS1137, the farthest galaxy cluster), the angular diameter distance for \( \Omega_m = 0.3 \) and \( \omega = -3 \) (\( D_A \approx 1840 \) Mpc) is only \( \sim 10\% \) smaller than in a model with the same amount of dark matter and \( \omega = -10 \) (\( D_A \approx 2045 \) Mpc). This particular behavior is quite similar to what happens in analyses involving age estimates. There, like here, the function of the cosmological parameters (\( \Omega_m \) and \( \omega \)) quickly asymptotes for large values of \( \omega \) [25].

In our search for possible lower limits to the ES of the phantom component, we now perform a joint analysis of SZE/X-ray ADD and SNe Ia data. For that, we use...
the newest SNe Ia sample of the Supernova Cosmology Project (with stretch and extinction marginalization applied) and follow the analytical marginalization method for the “zero point magnitude” $M$ as given in Ref. 28. The results of the present analysis are shown in Panels 2a and 2b. In Panel 2a we show the 68%, 95% and 99% c.l. in the $\Omega_m - \omega$ plane by imposing the $\Lambda$ barrier ($\omega > -1$). From this combination of observational data sets we find that the best-fit model occurs exactly on the $\omega = -1$ border with $\Omega_m = 0.29$ and $\chi^2_{\text{min}}/\nu \simeq 1.26$. At 95% c.l., we obtain $\omega \leq -0.83$ and $0.19 \leq \Omega_m \leq 0.37$. Figure 2b generalizes the previous analysis to include more negative values of the dark energy $\omega$. Again, we find that there is much acceptable parameter space beyond the line $\omega = -1$ and that the confidence regions are modified by its presence, what clearly indicates the existence of bias in the parameter determination due to the $\Lambda$ barrier. This particular analysis provides a 68% confidence bound of $-1.98 \leq \omega \leq -1.42$ and $0.30 \leq \Omega_m \leq 0.45$, with the best-fit model happening at $\omega = -1.7$ and $\Omega_m = 0.38$ ($\chi^2_{\text{min}}/\nu \simeq 1.2$), which corresponds to an accelerating universe with deceleration parameter $q_0 \approx -1.0$ and total expanding age of $9.3h^{-1}$ Gyr. If one combines this 68% confidence bound on $\omega$ with the upper limit from EFE, one would have $-1.9 \leq \omega < -1/3$ instead of the usual $-1 \leq \omega < -1/3$.

At this point, it is important to observe that the very low-$\omega$ region of the above analyses can be considerably reduced by combining them with high-$z$ data as, for instance, the current CMB measurements (see, e.g., 27). To better visualize that, Fig. 2c shows the results of a combined test involving SZE/X-ray ADD + SNe Ia data and the “WMAPext” constraint (which includes other CMB experiments in addition to WMAP) on the angular size distance to the decoupling surface at $z = 1089$, i.e., $d = 14.0^{+0.2}_{-0.3}$ Gpc 11. This analysis shows that the best-fit model moves up to converge at $\omega = -1.2$ (and $\Omega_m = 0.27$), with a 68% confidence bound of $-1.38 \leq \omega \leq -1.09$. These results, along with the gradual decrease of the low-$\omega$ region seen from Figs. 1a to 2c, clearly show that SNe Ia and CMB measurements dominate the analyses over SZE/X-ray ADD data, which can be directly associated with the current systematics uncertainties on these latter measurements. As commented in Ref. 9, such systematics are observationally approachable and will be addressed in the coming years through the current generation of X-ray satellites (Chandra & XMM-Newton) and radio observatories (OVRO, BIMA & VLA). Surely, these improvements will be very welcome once SZE/X-ray determined distances are measurements independent of the extragalactic distance ladder that may provide distance to high-$z$ galaxy clusters. With such a future sample of high-$z$ objects, it is expected that SZE/X-ray ADD data will be able to provide a valuable independent check of SNe Ia and primary CMB power spectrum results.

We now compare our results with other recent limits on the ES parameter of the phantom energy derived from independent methods. For example, in Ref. 22 data from CMBR, large scale structure (LSS) and SNe Ia were combined to find a 95% confidence bound of $-2.68 < \omega < -0.78$. Such results agree with the constraints obtained from a combination of Chandra observations of the X-ray luminosity of galaxy clusters with independent measurements of the baryonic matter density and the latest measurements of the Hubble parameter. From this latter analysis, it was found $-2.0 \leq \omega \leq -0.6$ at 68% c.l. 23 while a combination of these X-ray data with measurements of the angular size of milliarcsecond radio sources provide $-2.22 \leq \omega \leq -0.62$ at 95% c.l. 23. Recently, constraints from several CMBR experiments (including the latest WMAP results) along with LSS data, Hubble parameter measurements from the HST key project and SNe Ia data were obtained, with the ES parameter ranging from $-1.38$ to $-0.82$ at 95% c.l. 27. More recently, the authors of Ref. 28 used a sample of 57 SNe Ia to find a 95% confidence bound of $-2.4 < \omega < -1$ whereas estimates of the age of the Universe as given by WMAP ($t_0 = 13.7 \pm 0.2$ Gyr) provide $-1.18 < \omega < -0.93$, which corresponds to an accelerating scenario with the deceleration parameter $q_0 = -0.52$ 19. All these results agree at some level with the ones found in this work.

IV. CONCLUSION

This paper, as many of its predecessors, is mainly motivated by our present state of ignorance concerning the nature of the so-called dark energy (or dark pressure). In a first moment, vacuum energy density or cosmological constant was thought of (also motivated by the old age of the Universe problem) as the most viable explanation for the evidence of cosmic acceleration as given by SNe Ia observations. Observationally, $\Lambda$ remains as a good candidate for dark energy although, from a theoretical viewpoint, one has to face a fine-tuning of 120 orders of magnitude in order to make its observed value compatible with quantum field theory expectations 30. Later on, a first generalization of this former description, in which a “X-matter” component with ES parameter ranging from a cosmological constant ($\omega = -1$) to presssureless matter ($\omega = 0$), was proposed as a possible description for current observations 3. More recently, a new generalization, the so-called phantom energy, in which the $\Lambda$ barrier ($\omega = -1$) is removed, has received increasing attention among theorists. Naturally, all these theoretical attempts to describe dark energy would not be valid without observational support. But that is not the case once several observational analyses support these parametrizations for dark energy. Here, we have explored the prospects for constraining the phantom behavior of the dark energy from SZE/X-ray distance estimates of galaxy clusters, SNe Ia data and CMB-based distance estimates. We have shown that these data allow much acceptable parameter space beyond the line $\omega = -1$, what
indicates not only the possibility of bias in the parameter determination when the $\Lambda$ barrier is imposed but also the possibility of existence of more exotic forms of energy in the Universe. Naturally, we do not expect such results to be completely free of observational and/or theoretical uncertainties, mainly because there still exist considerable systematics uncertainties associated with SZE/X-ray distance determinations. What we do expect is that in the near future new sets of observations along with more theoretical effort will be able to decide on which side of the $\Lambda$ barrier lies the so far mysterious dark energy.

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[1] S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D68, 023509 (2003)
[2] P. M. Garnavich et al., Astrophys. J. 509, 74 (1998)
[3] M. S. Turner and M. White, Phys. Rev. D56, R4439 (1997); T. Chiba, N. Sugiyama and T. Nakamura, Mon. Not. Roy. Astron. Soc., 289, L5 (1997); Z.-H. Zhu, Mod. Phys. Lett. A 15, 1023 (2000); J. S. Alcaniz and J. A. S. Lima, Astrophys. J., 550, L133 (2001) [astro-ph/0109047]; Z.-H. Zhu, M.-K. Fujimoto, and D. Tsutsumi, Astron. Astrophys. 372, 377 (2001); J. Kujat, A. M. Linn, R. J. Scherrer and D. H. Weinberg, Astrophys. J., 572, 1 (2002); D. Jain et al., Int. J. Mod. Phys. D 12, 953 (2003)
[4] R. R. Caldwell, Phys. Lett. B545, 23 (2002)
[5] B. McInnes, astro-ph/0210321; V. Faraoni, Int. J. Mod. Phys. D11, 471 (2002); R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett., 91, 071301 (2003); P. F. Gonzalez-Diaz, Phys. Rev. D68, 021303 (2003); S. Nojiri and S. D. Odintsov, Phys. Lett. B562, 147 (2003), hep-th/0303117; Phys. Lett. B565, 1 (2003); Y.-S. Piao and E. Zhou, Phys. Rev. D68, 083515 (2003); G. W. Gibbons, hep-th/0302199; P. H. Frampton, hep-th/0302007; M. Sami and A. Toporensky, gr-qc/0312099; J. M. Cline, S. Jeon, G. D. Moore, hep-ph/0311312
[6] T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D62, 023511 (2000).
[7] V. Sahni and Y. Shtanov, gr-qc/0205111 Int. J. Mod. Phys. 11, 1 (2002); JCAP 311, T4 (2003); astro-ph/0209346
[8] U. Alam, V. Sahni, T. D. Saini and A. A. Starobinsky, astro-ph/0311364; T. R. Choudhury and T. Padmanabhan, astro-ph/0311622
[9] E. D. Reese, J. E. Carlstrom, M. Joy, J. J. Mohr, L. Grego and W. L. Holzapfel, Astrophys. J. 581, 53 (2002)
[10] R. A. Knop et al., astro-ph/0309368
[11] D. N. Spergel et al., Astrophys. J.Suppl. 148, 175 (2003)
[12] W. L. Freedman et al., Astrophys. J. 553, 47 (2001)
[13] R. A. Sunyaev and Ya. B. Zel’dovich, Comm. Astrophys. Space Phys., 4, 173 1972
[14] M. Birkinshaw, Phys. Rep., 310, 97 (1999); J. E. Carlstrom, G. P. Holder and E. D. Reese, ARA&A, 40, 643 (2002)
[15] A. Cavaliere, L. Danese and G. de Zotti, Astrophys. J., 217, 6 (1977); J. Silk and S. D. M. White, Astrophys. J., 226, L3 (1978); M. Birkinshaw, Mon. Not. Roy. Astron. Soc. 187, 847 (1979)
[16] S. Perlmutter et al., Astrophys. J., 517, 565 (1999)
[17] A. Riess et al., Astron. J. 116, 1009 (1998)
[18] L. Wang, R. R. Caldwell, J. P. Ostriker, P. J. Steinhardt, Astrophys. J., 530, 17, (2000)
[19] Z.-H. Zhu and M.-K. Fujimoto, Astrophys. J., (in press)
[20] R. G. Calberg et al., Astrophys. J., 462, 32 (1996); A. Dekel, D. Burstein and S. White S., In Critical Dialogues in Cosmology, edited by N. Turok World Scientific, Singapore (1997)
[21] S. Burles, K. M. Nollett and M. S. Turner, Astrophys. J., 552, L1 (2001)
[22] S. Hannestad and E. Mortsell, Phys. Rev. D66, 063508 (2002)
[23] P. Singh, M. Sami and N. Dadhich, Phys. Rev. D68, 023509 (2003), hep-th/0305110
[24] V. B. Johri, astro-ph/0311293
[25] J. A. S. Lima and J. S. Alcaniz, Mon. Not. Roy. Astron. Soc. 317, 893 (2000), astro-ph/0005441; L. M. Krauss, astro-ph/0212369
[26] M. Goliath, R. Amanullah, P. Astier, A. Goobar and R. Pain, Astron. Astrop., 380, 6 (2001)
[27] A. Melchiorri, L. Mersini, C. J. Odman and M. Trodden, Phys. Rev. D68, 043509 (2003)
[28] J. A. S. Lima, J. V. Cunha and J. S. Alcaniz, Phys. Rev. D68, 023510 (2003); astro-ph/0303388
[29] Z.-H. Zhu, M.-K. Fujimoto and X.-T. He, Submited to Astrophys. J., 638, 043509 (2003); astro-ph/0303388
[30] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989); V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D9, 373, (2000); P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003); T. Padmanabhan, Phys. Rept. 380, 235 (2003)