Formation of liquid-metal jets in a vacuum arc cathode spot: Analogy with drop impact on a solid surface

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Abstract. Conditions of the liquid-metal jets formation in a cathode spot of a vacuum arc discharge are studied. Our consideration is based on the analogy between the processes, occurring in the liquid phase of the cathode spot, and the processes, accompanying a liquid drop impact on a flat solid surface. In the latter case there exists a wide variety of experimental data on the conditions under which the spreading regime of fluid motion (i.e., without formation of jets and secondary droplets) changes into the splashing one. In the present work, using the hydrodynamic similarity principle (processes in geometrically similar systems will proceed similarly when their Weber and Reynolds numbers coincide), criteria for molten metal splashing are formulated for different materials of the cathode. They are compared with the experimental data on the threshold conditions for vacuum arc burning.

1. Introduction

The main properties of a vacuum arc discharge are determined by the processes in a small near-surface region of the cathode (a cathode spot). This region includes the active (heated) part of the cathode surface and a near-cathode plasma. According to [1], the cathode spot of a vacuum arc has a cellular structure. Indeed, the cathode photographs [2] demonstrate that the vacuum arc burning leads to the formation of traces on the cathode surface that have substructures in the form of individual craters with the diameters of the order of several micrometers. The directly recorded oscillograms of the arc current have showed [2] that the time of the current flowing through a single crater is limited and is of the order of dozens of nanoseconds (for definiteness, we will take $T \approx 30$ ns). Electric current through the cell is limited from below: the discharge extinguishes at currents below some threshold, which depends on the cathode material [3].

The cathode spot is a source of liquid-metal jets and drops [4]. They are formed when a liquid metal is extruded by the pressure of explosive plasma out from craters developed on the cathode. The formation of microjets leads to the appearance of microinhomogeneities on the cathode surface that provide necessary conditions for the initiation of the explosive electron emission and, consequently, for the self-sustained operation of a vacuum arc discharge. Up to now, the formation of microjets was not observed in situ. However, as one can notice, the
patterns observed for the collision in the photographs [2] resemble the picture observed during collisions of drops with a solid surface (see, for example, [5, 6]).

In the recent papers [7–10], a new approach has been proposed for the analysis of threshold conditions of vacuum arc burning. The idea was in using the analogy between the behavior of liquid metal in the vacuum arc cathode spot and the behavior of fluid in the classical problem on a drop impact on a solid surface. Its obvious advantage is the following. On the one hand, the characteristic times for hydrodynamic processes in the cathode spot are a few or dozens of nanoseconds and the characteristic scale is a few microns, which complicates their observation in dynamics. On the other hand, the typical time and spatial scales in the problem of drop impact on a surface are as a rule milliseconds and millimeters, respectively. As a consequence, for the latter case, there are a lot of experimental data on a fluid behavior and, particularly, on the splashing conditions. The basis for matching these two phenomena is the hydrodynamic similarity principle: if the Weber (We) and Reynolds (Re) numbers for the liquid flow are equal in geometrically similar systems with different spatial and time scales, then the processes will proceed in a similar way.

In the present work, using this analogy, we will analyze the data on a vacuum arc burning at near-threshold currents for different cathode materials (W, Au, Cu, Mo). The criteria for jet formation will be compared with the results of numerous experimental researches concerning the drop impact on a flat solid surface. Note that earlier we have investigated only the separate case of copper cathode [7]. As it turns out, the main conclusions of [7] can be extended to other metals often used in laboratory experiments. The estimations for molten metal flow fall close to the boundary between splashing and spreading regimes in the parametric We–Re plane.

2. Conditions for liquid splashing
There are two main regimes of liquid motion in the problem of fluid drop impact on a solid flat surface: the splashing regime in which jets and secondary droplets are formed, and the spreading regime where the liquid flow is regular. It is convenient to analyze the transition between them in terms of the nondimensional Weber and Reynolds numbers,

$$We = \rho U^2 D / \sigma, \quad Re = UD / \nu,$$

where $U$ is the characteristic velocity of the liquid, $D$ is the characteristic size of the area occupied by the liquid, $\sigma$ is the surface tension coefficient, $\rho$ is the density, $\nu$ is the kinematic viscosity ($\nu = \eta / \rho$, where $\eta$ is the dynamic viscosity). The boundary between the regimes can be described using the results of the experiments on liquid drops falling on the solid surface.

It follows from numerous investigations (see, e.g., the review [11]) that the splashing criterion can be written in the form

$$K \geq K_c, \quad K = We^\alpha Re^\beta,$$

where $K$ is the splashing parameter, $K_c$ is its critical value, $\alpha$ and $\beta$ are some constants. If the parameter $K$ is greater than its threshold value $K_c$ then the spreading regime will be changed by the splashing one.

The experiments, which were used for determining the splashing criteria, were carried out in the following scenario: the drops fell from different heights, and the moment of collision was fixed by camera. Splashing of water drops was studied in the pioneer work [6]. Further, a number of investigated liquids was significantly extended [11]. It should be noted that the solid surface, with which a drop collides, can be in different states: dry or wet. As applied to the analysis of liquid-metal dynamics (we deal with a liquid initially present on the cathode surface), it is reasonable to use the splashing criterion for a wetted surface.

The values of the parameters $K_c$, $\alpha$, and $\beta$ were calculated in different works [6, 12–17] (table 1). It is clear that studied ranges of Weber and Reynolds numbers distinguish for
Table 1. The splashing criteria found by different researchers.

| Reference | $\alpha$ | $\beta$ | $K_c$ | $W_{\text{e min}}$ | $W_{\text{e max}}$ | $R_{\text{e min}}$ | $R_{\text{e max}}$ |
|-----------|---------|---------|------|----------------|----------------|----------------|----------------|
| [14]      | 0.817   | 0.366   | 1320 | —              | —              | —              | —              |
| [15]      | 0.8     | 0.4     | 2100 | 28             | 890            | 96             | 2600           |
| [16]      | 0.5     | 0       | 20    | 127            | 1420           | 988            | 14000          |
| [12]      | 0.8     | 0.4     | 2100  | 200            | 1600           | 100            | 18000          |
| [13]      | 0.8     | 0.4     | 2074  | —              | —              | —              | —              |
| [17]      | 1       | 0       | 410–460 | 377        | 2010           | 505            | 1695           |

Different liquids and experimental setups. As a result, the obtained approximations of the splashing criteria are valid in different ranges $W_{\text{e min}} < W_e < W_{\text{e max}}$ and $R_{\text{e min}} < R_e < R_{\text{e max}}$ (see Table 1). According to the estimates made for copper cathode [7, 8], the corresponding Weber and Reynolds numbers fall into these ranges. This gives us the opportunity to apply the dynamic similarity principle, and, hence, to compare the splashing criteria for molten metal in the cathode spot and for liquid drops that collide with the surface despite of significantly different time and spatial scales.

According to the results that are presented in Table 1, the conditions of jets formation (i.e., splashing criteria) depend substantially on the Weber number. So, according to [16, 17], there is no dependence of the splashing threshold on the Reynolds number: $\beta = 0$. This counts in favour of smallness of the viscosity influence on the splashing criteria. Note that the qualitative analysis [9] of liquid metal extrusion from the crater under the action of the pressure of the cathode plasma allowed the authors to formulate the following splashing criterion $W_e \geq 497$ that matches the criteria of [16, 17]. However, the criteria of [12–15] take into account weak dependence on the Reynolds number, i.e., on the fluid viscosity.

3. Parameters of liquid dynamics in the vacuum arc cathode spot

Preliminary estimates have been already made in [7] for the conditions of liquid metal splashing in a vacuum arc for the copper cathode. Let us show that the suggested approach is applicable for other cathode materials usually used in experiments (see [18]). From general considerations, since the molten metal parameters (their viscosity, density, and surface tension coefficient) and the vacuum arc parameters (their erosion rate, ion velocity, threshold current, and crater size) do not differ from each other significantly, then the Weber and Reynolds numbers for different cathode materials will be comparable.

Now we estimate the Weber and Reynolds numbers corresponding to the process of molten metal extrusion from the craters for W, Au, Cu, Mo cathodes. We will use the approach proposed in [8]. Suppose that liquid metal melted by Joule heating fills a semispherical cavity (microcrater) of the diameter $D_0$ on the cathode. Plasma impact on the fluid surface can be described by the momentum $p$ acquired by the liquid within a time $T$:

$$p \approx sIu_i\gamma_iT,$$

where $I$ is the current flowing through the cell, $u_i$ is the characteristic ion velocity, $\gamma_i$ is the ion erosion rate (mass removed per unit charge). Here $s$ is the coefficient that is equal to 0.5 under the assumption of isotropic plasma expansion into the half-space over the cathode and to unity for the anisotropic one-dimensional expansion in the normal direction. For the rough estimates, we will take the intermediate value of this parameter, namely, $s = 0.8$. Since we consider the threshold conditions for liquid-metal splashing, we should take the threshold values
Table 2. The parameters that determine the dynamics of molten metal in the vacuum arc cathode spot.

| Metal | \( u_i \) (m/s) | \( \gamma_i \) (g/C) | \( \rho \) (g/cm\(^3\)) | \( I_c \) (A) | \( \sigma \) (N/m) | \( \eta \) (mPa s) | \( U \) (m/s) | We | Re |
|-------|-----------------|------------------|--------------------|---------------|-----------------|----------------|--------|-----|-----|
| Cu    | 1.28            | 40               | 8                  | 1.6           | 1.37            | 4.34           | 147    | 399 | 858 |
| W     | 1.05            | 62               | 17                 | 1.6           | 2.32            | 6              | 88     | 179 | 789 |
| Au    | 0.58            | 121              | 17.4               | 1.4           | 1.13            | 5.38           | 81     | 319 | 828 |
| Mo    | 1.74            | 47               | 10.2               | 1.5           | 2.23            | 5.6            | 172    | 431 | 996 |

for the electric currents, \( I = I_c \). The characteristic velocity of the liquid can be estimated by dividing the momentum by the total fluid mass:

\[
U \approx \frac{12p}{\pi \rho D_0^3}.
\]

Notice that in [7] the Bernoulli principle was used in order to estimate the velocity. However, it is not fully correct because the flow is not stationary: the liquid accelerates only in the crater.

Since we are going to compare the liquid behavior for the drop collision with a solid surface and for the molten metal extrusion from the microcrater, it is natural to match the liquid metal filling the hemispherical crater and the drop of the diameter \( D = 2^{-1/3}D_0 \) (then the corresponding volumes coincide).

The difficulty in estimating the Weber and Reynolds numbers is in determining the parameters of the vacuum arc operation and of liquid metals. Almost all of them are easy to find in the literature: the ion velocities, cathode erosion rate, molten metal density can be taken from the review [18], and the threshold currents from [3]. The surface tension coefficients and dynamic viscosities can be found in [19–21]. As for the crater’s sizes, there is a lack of data. Daalder [22] determined the crater diameters for a copper cathode in the current range of 4–230 A. For a fixed current, the crater diameter has values according a lognormal distribution solely determined by the current. For the threshold current, the most probable diameter is equal approximately to 4 \( \mu \)m. Later, similar researches were conducted for the cathodes made of tungsten and cadmium [2, 23]. Comparing the results of these works, one can conclude that \( D \approx 4 \mu \)m for a wide spectrum of cathode materials at the threshold currents. This diameter is used in our calculations of Reynolds and Weber numbers.

Table 2 contains the parameters of different molten metals (\( \rho, \sigma, \eta \)), the parameters of the cathode spot plasma (\( u_i, \gamma_i, I_c \)), our estimations for fluid velocity (\( U \)), and, finally, the desired Reynolds and Weber numbers. Comparing the obtained values of We and Re with their ranges investigated in experimental studies on drop collision with a surface (table 1), we can conclude that, despite the difference in the nature of the processes in a cathode spot and for a drop impact, the Reynolds and Weber numbers for molten metals fall into the well-studied range (figure 1). It proves the possibility of using the dynamic similarity principle and allows us to make conclusions on the splashing conditions in the vacuum arc basing on the analogy with the drop impact process.

Our main results are illustrated in figure 1. On the parametric We-Re plane, the spreading/splashing boundaries from different researches (see table 1) and our estimates for the liquid metal (W, Cu, Au, Mo) flow parameters for the threshold vacuum arc currents are shown. One can see that calculated values of We and Re numbers occupies the compact area on the parametric plane. Moreover, they lie in the near-critical region separating the spreading regime (i.e., without jet formation) and the splashing one (i.e., where jets and droplets are formed). This fact allows us to conclude that minimal currents for vacuum arc burning (\( I_c \))
Figure 1. The curves 1–5 give the boundaries between the spreading and splashing regimes corresponding to [13–17], respectively; the points correspond to our estimations for We and Re numbers for liquid metal dynamics in the vacuum arc for different cathode materials at the threshold currents.

correspond to the threshold conditions for jet formation. For lesser We and Re numbers, the jet formation becomes impossible, and, consequently, microprotrusions, which are responsible for initiation of the explosive electron emission, cannot develop. Then spontaneous extinction of vacuum arc will occur.

4. Conclusion
There are two main parameters that determine the process of molten metal extrusion from forming craters: the crater diameter $D_0$ and the pressure of the cathode plasma $P \approx 4p/(\pi D_0^2 T)$ exerted on the free surface of a liquid. It is clear from general considerations that, in the case of relatively low pressure, the extrusion of molten metal will result in its spreading over the cathode surface. On the contrary, in case of high pressure, this process will lead to the formation of jets and droplets, i.e., to splashing. Consequently, there exist a threshold value of the pressure (depending on $D_0$ and on the liquid parameters) separating these two regimes. According to the ekton model of the cathode spot [3, 18], electric explosion of liquid-metal jets plays an important role in the self-sustained operation of a vacuum arc discharge. That is why the search of the threshold conditions of jet formation is a fundamental problem of the theory of vacuum arc. In the present paper, using empiric data on the behavior of liquid drops, the corresponding threshold conditions were constructed in terms of the Weber and Reynolds numbers (these numbers depend on $P$ and $D_0$). It was revealed that, for different cathode materials frequently used in laboratory experiments on a vacuum arc, the parameters of molten metal motion for threshold currents are close to the splashing threshold. This points to the key part of hydrodynamic processes in self-sustaining vacuum arc discharge.
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