The Crucial Role of Inert Source in the Magnetic Aharonov-Bohm Effect

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PACS No: 03.65.-w, 03.50.De

Abstract:

The role of the inert magnetic source used in the Tonomura experiment that has confirmed the magnetic Aharonov-Bohm effect is discussed. For this purpose, an analysis of a thought experiment is carried out. Here the permanent magnet is replaced by a classical source which is made of an ideal coil. A detailed calculation of this noninert source proves that in this case the effect disappears. This outcome provides another support for the crucial role of an inert source in the Aharonov-Bohm effect. A new aspect of quantum nonlocality is pointed out.
1. Introduction

As is well known, the Lorentz force (see [1], p. 51) is the classical equation of motion of a charged particle. This equation is written in terms of electromagnetic fields. Here the potentials are auxiliary mathematical quantities that may (or may not) be used for solving problems. On the other hand, fundamental quantum mechanical equations (like the Schroedinger and the Dirac equations) depend explicitly on electromagnetic potentials. Differences between classical and quantum mechanical equations may arise from this structure of the theories. However, the consistency of these two kinds of theories is proved by Ehrenfest theorem (see [2], pp. 25-27, 137, 138) which shows that the classical limit of quantum mechanics is consistent with classical physics. This theorem settles the main problem and shows that classical physics and quantum mechanics can live side by side. Thus, classical physics holds only for experiments that belong to a the classical limit of quantum mechanics whereas quantum mechanics holds for a much larger set of experiments. The main advantage of classical physics is that its equations are much simpler than the corresponding equations of quantum mechanics.

In their work [3], Aharonov and Bohm (AB) examine phase properties of a quantum mechanical charged particle that moves in field free region where the external potentials do not vanish. Evidently, vanishing fields guarantee that a classical experiment carried out under these conditions should yield null results. On the other hand, AB argue that the dependence of the phase on the potentials should yield a phase shift in cases where a quantum mechanical charged particle moves in a multi-
ply connected field free region. This phase shift affects the interference pattern of a quantum mechanical charge.

It is well known that microscopic phenomena generally cannot be explained by classical physics. Some macroscopic phenomena, like superconductivity, superfluidity and EPR related experiments are also outside the scope of classical physics. Similarly, an AB experiment that measures phase difference of a massive particle, has no classical analog. In this sense, it provides another kind of a macroscopic quantum mechanical effect because the interference pattern depends on fields that are quite far from all possible trajectories of the electron.

The AB ideas about the magnetic AB effect have been confirmed by the work of Tonomura et al. who have constructed an appropriate experiment [4]. (Below, the experiment described in [4] is called the Tonomura experiment.) This experiment uses a ring of a single domain of a ferromagnetic material. Evidently, the magnetic ring behaves like an inert object throughout the experiment. A general remark on the importance of this property of the magnetic component of Tonomura’s experiment has been pointed out in the literature [5,6]. The main purpose of the present work is to analyze an experiment where the Tonomura quantized magnet is replace by an analogous classical coil and thereby, to show that the existence of an inert magnetic source is crucial for a nonvanishing AB phase shift. The result also shows a new aspect of quantum mechanical nonlocality.

Expressions are written in units where $\hbar = c = 1$. The second section describes the Tonomura experiment. Calculations of a classical analog of the Tonomura experiment are presented in the third section. Concluding remark on consequences of the analysis are included in the
2. The Tonomura Experiment

The Tonomura device aims to test the phase shift predicted by the magnetic AB effect, where the electrons move in an external nonsimply connected field free region. A phase dependent interference pattern of an electronic beam is obtained by means of electron holography. The beam passes near a circular ring made of a ferromagnetic material (see fig. 1). The ring is a (nearly) perfect magnetic single domain. It is covered by a superconducting material and by copper. This arrangement prevents the beam’s electrons from entering the region where the magnetic field does not vanish. The usefulness of dividing the beam into two subbeams is explained later. One subbeam $G_1$ passes through the ring’s inner region and the second subbeam $G_2$ passes at the ring’s outer region (see fig. 1). Both subbeams move in a magnetic field free region.

For phase-difference calculation, one has to examine the develop-

![Figure 1: The main elements of the Tonomura experiment (see text).](image)

last section.
ment of the action along possible trajectories of the subbeams. Below, the electron whose interference is analyzed is called the traveling electron. Quantities pertaining to the traveling electron are denoted by the subscript \((e)\). Other quantities pertain to the magnet.

Let us calculate the rate of phase accumulated. Thus, the action is the time integral of the Lagrangian of this system

\[
\mathcal{L}_{\text{total}} = \mathcal{L} + \mathcal{L}_{(e)} - e\mathbf{v}(e) \cdot \mathbf{A}. \quad (1)
\]

The terms on the right hand side represent the Lagrangian of the magnet, of the traveling electron and of their interaction, respectively. (Here \(-e\) denotes the electronic charge.) Obviously, the state of the ferromagnetic ring is independent of the traveling electron. Therefore, the first term of \((1)\) makes the same contribution to all possible trajectories pertaining to the electronic beam. The same is true for the second term of \((1)\), since, due to the field free region where the electronic beam moves, the self (kinetic) energy of the electron is constant.

In order to calculate the required interference pattern, one must have an expression for phase difference accumulated on any pair of possible trajectories of the electronic beam. Now, due to the constant value of the first and the second terms of \((1)\), these terms make no contribution to the phase difference. Let \(l_1\) and \(l_2\) denote two trajectories that begin at the beam’s origin and meet at a point on the screen where the interference is measured. Integrating \((1)\) on time, writing \(\mathbf{v} \, dt = d\mathbf{x}\) and using vector analysis, one takes the last term of \((1)\) and obtains the required phase difference

\[
\Delta \Phi = -\int_{l_1} e\mathbf{A} \cdot d\mathbf{x} + \int_{l_2} e\mathbf{A} \cdot d\mathbf{x} = \oint_l e\mathbf{A} \cdot d\mathbf{x}
\]
\[
\int_s e(\nabla \times \mathbf{A}) \cdot ds = \int_s e\mathbf{B} \cdot ds
\]

This result means that the phase shift of two possible trajectories of the traveling electron depends on the magnetic flux passing through an area whose boundary is determined by the closed line defined by \(l_1\) and \(l_2\). Here a usage of the two sets of beams \(G_1\) and \(G_2\) yields straightforwardly the required result. Thus, if both \(l_1\) and \(l_2\) belong to the same set then no magnetic flux is found and a null phase shift is obtained. On the other hand, the same nonvanishing phase shift is obtained for two trajectories that belong to different sets.

The Tonomura experiment [4] has confirmed the AB’s prediction [3] which is described in the previous lines.

3. A Classical Analog of the Tonomura Experiment

The following discussion proves the crucial role of an inert source in a test of the magnetic AB effect [3]. Let us consider a thought experiment where the Tonomura’s magnetic ring is replaced by a coil having these properties. The coil is a closed pipe which is made of an insulating material (see fig. 2). The pipe contains a uniformly charged incompressible liquid that flows frictionlessly along the pipe. The pipe itself is covered uniformly with an appropriate density of electric charge of the opposite sign. Thus, outside the pipe there is no electric field and a ring of a magnetic flux exists at the coil’s inner part. This is a "classical analog" of Tonomura magnet.
Let us compare the interference pattern of Tonomura experiment with that which is expected to be found in an experiment with the classical source described herein. Like in the standard presentations of the AB effect [3], the following calculations are carried out within the nonrelativistic limit. The calculations are analogous to those of [5,6] and the result provides a further justification for the indispensable role of an inert source in the AB effect.

Let $a$ denote the inner radius of the pipe and $R$ the radius of the inner part of the coil where the magnetic field does not vanish. The relation $a \ll R$ simplifies the calculations presented below. The symbols $\rho$ and $v$ denote the linear charge density and the velocity of the charged liquid flowing along the pipe, respectively.

Let us examine the Lagrangian (1) for the case where the coil replaces the permanent magnet. The calculations take a simpler form if the coil is regarded as a dense assembly of identical loops, each of which contains the same uniformly charged liquid that flows at the same velocity $v$. Thus, the problem of the traveling electron and one loop is analyzed (see fig. 3). The result for the entire coil will be derived from this analysis.
The loop’s vector potential is obtained from an integration on the charged liquid flowing along the loop

\[ A = \oint \frac{\rho v}{r} dl, \]  

(3)

where \( r \) denotes the distance from the line element \( dl \) to the field point where \( A \) is calculated. Thus, the interaction term of (1) is cast into the following form

\[ -e v_{(e)} \cdot A = -e \rho v \oint \frac{v_{(e)} \cdot dl}{r} \]  

(4)

Now, due to the insulating material of the pipe, the charge that covers it is static throughout the experiment. Therefore, its self energy is a constant of the motion and it also does not screen the fields of the traveling electron. The last point means that the kinetic energy \( T \) of the rotating liquid as well as the associated Lagrangian may change during the process [7]. Furthermore, for the rotating liquid the ratio of the charge density to the mass density is very very small (relative to the corresponding ratio of an electron). Hence, the calculation is simplified if the negligible change in the liquid’s velocity is ignored. For a time instant \( t \), one uses vector analysis and Maxwell equations and finds the change of the kinetic energy of the rotating charged liquid.
\[ \Delta T = \int_{-\infty}^{t} \rho v [\oint E_{(e)} \cdot dl] dt = \int_{-\infty}^{t} \rho v \left[ \int_S (\nabla \times E_{(e)}) \cdot ds \right] dt = -\int_{-\infty}^{t} \rho v \left[ \int_S \frac{\partial B_{(e)}}{\partial t} \cdot ds \right] dt = -\rho v \int_S B_{(e)} \cdot ds = -\rho v \int_S \nabla \times A_{(e)} \cdot ds = -\rho v \oint A_{(e)} \cdot dl = e \rho v \oint \nabla \times A_{(e)} \cdot dl \]

This calculation proves that (4) and (5) cancel each other. Thus, no phase shift is found for one loop of current. It follows that the combined interaction of the traveling electron with the coil and its field makes no contribution to the phase shift.

The present experiment has the same magnetic flux and the same multiply connected field-free region as that of the Tonomura experiment. However, as stated above, an examination of (4) and (5) proves that their contribution to the rate of phase accumulated cancel each other. It follows that, unlike the inert single domain used in the Tonomura experiment, a classical magnetic source does not alter the interference pattern.

4. Concluding Remarks

Several effects proving the macroscopic scale of quantum mechanics and of its nonlocality are mentioned in the introduction. The mag-
netic AB effect, whose existence is demonstrated by the Tonomura experiment certainly belongs to this category. Indeed, scaling length by atomic size, one finds that the distance between the electron’s path and the magnetic field is very large. In spite of this fact, the electronic state is affected by the relatively remote magnetic field and ”remembers” it even for the macroscopic distance between the interference region and the magnetic source.

The analysis presented above shows a new aspect of quantum mechanical nonlocality. Thus, the electronic interference depends not only on the magnetic field as is, but also on the specific device that produces this magnetic field. In the case of the ferromagnetic single domain used in the Tonomura experiment, the magnetic AB effect exists. On the other hand, if the same magnetic field is produced by the ideal coil described above, then the phase shift disappears and the (macroscopically far) interference pattern changes. Now, the traveling electron touches neither the magnetic field nor the device that produces this field. However, the interference pattern proves that the electronic beam interacts not only with the magnetic field which it does not touch but also with the device that produces this field.

The inherent nonlocality of the AB effect is summarized in the following statements. The interference pattern is an assembly of dots, each of which is created by the collision of one electron with the screen. The structure of the interference pattern reflects the probability of finding the traveling electron at any small area on the screen. This probability is the absolute value of the square of the overall amplitude and this amplitude is obtained by taking the appropriate sum of the contribution of all relevant trajectories of the traveling electron. This sum is
very sensitive to the phase factor and it determines the location of constructive and destructive interference regions. The phase accumulated on any possible trajectory is the action ($\hbar = 1$) obtained as the time integral of the Lagrangian (1). Now, the Lagrangian (1) depends on all coordinates of the system. Thus, the source contributes to the phase accumulated on every trajectory and the traveling electron ”remembers” it. Now, a dot on the screen is certainly a local property created by the collision of the traveling electron with the screen. However, this local property is affected by the source and its magnetic field even if the traveling electron has never made any contact with them.

Other conclusions can also be inferred from the discussion presented above:

1. The existence of a phase shift crucially depends on a source that behaves as an inert object throughout the experiment.

2. The multiply-connectedness topology is not sufficient for having an AB effect. This requirement must be augmented by demanding the usage of an inert source of the magnetic field.

References:

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[1] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Elsevier, Amsterdam, 2005).

[2] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1955).
[3] Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959).

[4] A. Tonomura et al., Phys. Rev. Lett. 56, 792 (1986).

[5] E. Comay, Phys. Lett. A250, 12 (1998).

[6] E. Comay, Phys. Rev. A62, 042102 (2000).

[7] The usefulness of the specific structure of the coil is now clear. Indeed, here one only needs to carry out the straightforward calculation (5). On the other hand, in the case of a metallic coil, the traveling electron induces changes of the coil’s charge density and current. Thus, for a metallic coil, one must cope with the tedious task of calculating the time dependence of these quantities and of the associated self-energy of the coil.