Study of Fungal Decomposition Rate

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Abstract. Fungi are critical to the carbon cycle. The growth rate model of fungi was constructed from three aspects of temperature, humidity and pH, and the moisture tolerance model of fungi was constructed from two aspects of competitive ranking and water ecotone width. Combined with related references, a fungal decomposition rate model was constructed based on the growth rate and moisture tolerance models. Under the given environmental conditions, the decomposer mass change curve was obtained by solving with MATLAB software, and the effect of environmental changes on the curve trend was investigated by the control variable method. The curve of decomposer mass change was obtained by calculating the competitive rank of the population and combining it with the fungal decomposition rate model.

Keywords: Growth rate, moisture tolerance, decomposition rate, $E$ index.

1. Introduction
In May 2019, the Mauna Loa Observatory in Hawaii observed atmospheric CO$_2$ concentrations as high as 415.26×10$^{-8}$, setting a human record ever [1]. In recent years, the increasing concentration of atmospheric CO$_2$ has led to a significant increase in global temperature. As an important process affecting the atmospheric CO$_2$ concentration, the carbon cycle has become a frontier and hot topic in the field of global change and earth science research.

An important realization of the carbon cycle is the decomposition of compounds, and a key component of this process is the decomposition of plants and woody fibers. As the main decomposer of waste and wood, fungi play a key role in the global carbon cycle. It has been found that the growth rate and moisture tolerance of fungi are positively correlated with the rate of decomposition [2].

Fungi are largely hidden players, and we are well aware that they are critical to the carbon cycle [3]. Determining the effects of different fungal interactions in different environments on decomposition rates and the impact of environmental changes on the long-term dynamics of fungal decomposition and competition is extremely relevant to better predict the carbon cycle across the globe under current and future climates.
2. Fungal Decomposition Model

2.1. Model Overview
The decomposition rate of wood fiber by fungi is mainly influenced by two characteristics: the growth rate of fungi and the moisture tolerance of fungi. Based on this, the growth rate of fungi was modeled from three aspects of environmental temperature, humidity and pH in this paper. The fungal moisture tolerance model was constructed from the relationship between the competitive ranking of fungi and the width of water ecotone. The growth rate model of fungi and moisture tolerance model were combined into a multivariate nonlinear model of fungal decomposition rate. And by controlling the environmental variables, the relationship between the mass and time of decomposing wood of Monascus and Aspergillus niger under different control variables was studied.

2.2. Growth Rate Model of Fungi
The growth of fungi requires nutrients from the external environment, so changes in external environmental conditions will affect the growth and reproduction of fungi. In this paper, a mathematical model of fungal growth rate will be developed from three aspects of the environment: temperature, humidity, and pH, and a model for predicting fungal growth in a given environment will be obtained.

**Defining fungal growth rate**

The maximum colony growth rate $\mu_{\text{max}}$ was defined to consider the growth rate of the fungus. Considering the relative effects of pH and temperature on the growth rate, the estimation of the maximum colony growth rate $\mu_{\text{max}}$ was obtained using a Barani model [5] based on the gamma concept and the Rosso equation, in combination with the gamma factor of humidity. The equation is as follows.

$$\mu_{\text{max}} = \mu_{\text{opt}} \cdot \gamma(a_w) \cdot \tau(T) \cdot \rho(pH)$$

(1)

Where $\gamma(a_w)$, $\tau(T)$, $\rho(pH)$ are the relative effects of ambient humidity, temperature, and pH on the maximum colony growth rate, respectively; $\mu_{\text{opt}}$ is the value of maximum colony growth rate at optimum pH and temperature per day.

**Defining the Gamma Factor of Humidity**

Where $a_w$ is the water activity; $a_{\text{wmin}}$ is the minimum value of water activity.

$$\gamma(a_w) = \frac{a_w - a_{\text{wmin}}}{1 - a_{\text{wmin}}}$$

(2)

Water activity $a_w$ is the ratio of the equilibrium vapor pressure of water in a sample in a closed container to the vapor pressure of pure water under the same conditions. It is a dimensionless quantity, taking values between 0 and 1. Relative humidity $H_r$ is the ratio of the water vapor pressure in the air to the saturation water vapor pressure at the same temperature. It is also a dimensionless quantity and takes values between 0% and 100%. When the environment is relatively stable, the $a_w$ in the sample equilibrates with the $H_r$ in the container to obtain $a_w = H_r$, [6], and thus the relative humidity of the environment can be described by the water activity.
Defining the relative effect of ambient temperature on fungal growth rates

\[ \tau(T) = \frac{(T - T_{\text{min}})^2 (T - T_{\text{max}})}{(T_{\text{opt}} - T_{\text{min}})(T_{\text{opt}} - T_{\text{max}}) - (T_{\text{opt}} - T_{\text{max}})(T_{\text{opt}} + T_{\text{min}} - 2T)} \] (3)

Where \( T_{\text{opt}} \) is the ambient temperature at the maximum growth rate of the fungus; \( T_{\text{max}} \) is the highest ambient temperature at which the fungus can grow; \( T_{\text{min}} \) is the lowest ambient temperature at which the fungus can grow.

Defining the relative effect of environmental pH on fungal growth rates

\[ \rho(pH) = \frac{(pH - pH_{\text{min}})^2 (pH - pH_{\text{max}})}{(pH_{\text{opt}} - pH_{\text{min}})C} \]

\[ C = [(pH_{\text{opt}} - pH_{\text{min}})(pH - pH_{\text{opt}}) - (pH_{\text{opt}} - pH_{\text{max}})(pH_{\text{opt}} + pH_{\text{min}} - 2pH)] \] (4)

Where \( pH_{\text{opt}} \) is the environmental pH at the maximum growth rate of the fungus; \( pH_{\text{max}} \) is the maximum pH at which the fungus can grow; \( pH_{\text{min}} \) is the minimum pH at which the fungus can grow.

2.3. Humidity Tolerance Model of Fungi

Defining fungal moisture tolerance

According to the description of the topic, the moisture tolerance of fungi is the difference between the competitive ranking of each fungus and the moisture niche width, so the moisture tolerance of fungi can be expressed as:

\[ M = r - w_s \] (5)

Where \( M \) is the moisture tolerance of the fungus; \( r \) is the competitive ranking of the fungus; \( w_s \) is the moisture niche width of the fungus after standardized treatment.

Defining fungal competitive ranking

The competitive hierarchy of a fungus can be understood as a measure of the ability of a fungus to outcompete other fungi in a series of paired tests under similar conditions. In this paper, the growth rate of a fungus is used to determine the position of the fungus in the overall competitive hierarchy.

\[ r = \frac{\mu_i}{\sum_{i=1}^{N} \mu_i} \] (6)

Where \( \mu_i \) is the growth rate of the \( i \)th fungus, assuming that \( N \) fungi are present in the current environmental species.

When only one fungus is present in the environment, the competitive ranking is 1.

Defining the moisture niche width

The moisture niche width is the range of moisture that supports at least half of the fungal community that can sustain its fastest growth rate. It can be expressed as the difference between the maximum moisture content and the minimum moisture content.

\[ w = w_{\text{max}} - w_{\text{min}} \] (7)
Where \( w_{\text{max}} \) is the moisture niche maximum; \( w_{\text{min}} \) is the moisture niche minimum. Their units are MPa.

### 2.4. Fungal Decomposition Rate Model

#### Decomposition rate model

From the reference paper [7] given in this question, it can be seen that the average decomposition rate of fungi on wood fibers shows a linear relationship with the Spillman correlation coefficient of mycelial extension rate and fungal moisture tolerance, respectively. And Spearman’s correlation coefficient is also used to describe data with a nonlinear relationship. Therefore, when only two traits, fungal growth rate and moisture tolerance, are considered on the rate of fungal decomposition of wood fiber, the fungal decomposition rate can be expressed as a linear model of fungal growth rate and moisture tolerance.

\[
\ln(D) = a \ln(\mu_{\text{max}}) + bM + d
\]  

(8)

Where \( \mu_{\text{max}} \) is the growth rate of the fungus; \( M \) is the moisture tolerance of the fungus; \( D \) is the rate of decomposition of wood fibers by fungi; \( a, b, d \) are the coefficients of the multivariate linear model respectively.

#### Wood quality change model

Let the mass of the wood be \( m \) and the initial mass be \( m_0 \). The equation for the change in mass of the wood is as follows:

\[
\begin{align*}
\dot{m}(t) &= -Dm \\
m(0) &= m_0
\end{align*}
\]  

(9)

### 2.5. Model Solving

**Step 1. Selecting Monascus and Aspergillus niger for the study**

**Step 2. Standardization of the moisture niche width**

In the fungal moisture tolerance calculation formula (5), the competition level of the fungus is a value between 0 and 1, and there is no unit. The unit of the result obtained from the equation (7) for the width of the moisture ecotone is MPa. Thus, it needs to be standardized to eliminate the influence of the magnitude on the calculation results.

The standardized formula is as follows:

\[
w_s = 1 - \frac{w - w_{\text{min}}}{w_{\text{max}}}
\]  

(10)

Where \( w_s \) is the moisture niche width after standardization.

**Step 3. Coefficients of the decomposition rate model**

**Step 4. Solving algorithms**

**Algorithm 1: Calculation procedure of fungal decomposition rate**

**Input:** \( \mu_{\text{opt}} \text{ (days}^{-1}) \), \( a_{\mu, \text{min}} \), \( pH_{\text{min}} \), \( pH_{\text{max}} \), \( pH_{\text{opt}} \), \( T_{\text{max}} \text{ (°C)} \), \( T_{\text{min}} \text{ (°C)} \), \( T_{\text{opt}} \text{ (°C)} \)

**Output:** \( D \)

The relative effects of ambient temperature, humidity and pH on the growth rate of this fungus were calculated according to equations (2) to (4), respectively.
The growth rate $\mu_{\text{max}}$ of the fungus was calculated by equation (1).

The competitive ranking $r$ of the fungus was calculated according to equation (6).

The moisture niche width $w$ of the fungus was first calculated using equation (7), and then normalized using equation (10) to treat $w$ to obtain $w'$. 

Substituting $r$ and $w'$ into equation (5), we obtained the moisture tolerance $M$ of the fungus.

Substitute $\mu_{\text{max}}$ and $M$ into equation (8) and combine equation (9) when $d$ is taken between 0 and 0.5 to obtain the image of the mass change of wood versus time.

The ambient temperature and humidity are controlled separately and the image of the mass change of wood versus time is obtained.

**Step5. Analysis of results**

According to the slope change of the image, it can be found that the wood mass decreases significantly in the initial period, which indicates that the fungal decomposition rate is faster. After that, the wood mass decreases slowly, which indicates that the fungal decomposition rate decreases.

**AB:** Comparing the curves of Monascus and Aspergillus niger, comparing the time for a 10% decrease in wood quality when each of the two fungi is in its optimal growth environment, it was found that Monascus decomposes wood quality faster than Aspergillus niger.

**CD:** Keeping other variables constant and taking values of ambient temperature separately, C, D can both show that too high or too low ambient temperature will reduce the decomposition rate of the fungus on wood. And comparing the two images, both fungi showed the best decomposition rate when the ambient temperature was at 35°C. At this time, the decomposition of wood by Monascus is faster than that by Aspergillus niger, which further confirms the conclusion of AB.

**EF:** Keeping other variables constant, the values are taken separately for the ambient humidity. Observing the images of E and F shows that the higher the ambient humidity, the faster the rate of fungal decomposition.

![Figure 1](image-url)

**Figure 1** Change in mass of decomposer with time
3. Modeling of Decomposition Rates During Fungal Interactions

3.1. Population Competition Model
In nature, microorganisms such as fungi rarely exist alone, and always multiple populations are clustered together. When multiple populations occur in a space with limited resources, they interact with each other and present complex relationships. In general, the relationships between fungal populations can be classified into the following eight categories: neutral symbiosis, parphylytic symbiosis, reciprocal symbiosis, symbiosis, predation, parasitism, antagonism, and competition [4].

Fungi decompose wood fibers to convert lignin into the nutrients they need. Thus, when multiple fungi coexist, competition for survival is inevitable among similar populations for limited living space and nutrient sources. Therefore, this paper only considers the competitive relationship between populations as the interaction between multiple fungi.

Assuming that there are \(N\) populations in a given environment and \(x_1, x_2, \ldots, x_N\) denotes the number of each of these \(N\) populations, the model of competition between populations of organisms can be expressed as:

\[
\begin{align*}
\dot{X}(t) &= RX\varphi(x_1, x_2, \ldots, x_N) \\
R &= \begin{bmatrix} r_1 & \cdots & r_N \end{bmatrix}, X &= \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \\
\varphi(x_1, x_2, \ldots, x_N) &= \begin{pmatrix}
1 - \frac{x_1}{q_1} - s_2\frac{x_2}{q_2} - \cdots - \frac{x_N}{q_N} \\
1 - s_1\frac{x_1}{q_1} - \frac{x_2}{q_2} - \cdots - \frac{x_N}{q_N} \\
\vdots \\
1 - s_1\frac{x_1}{q_1} - s_2\frac{x_2}{q_2} - \cdots - \frac{x_N}{q_N}
\end{pmatrix}
\end{align*}
\]

(11)

Where \(r_1, r_2, \ldots, r_N\) denotes the intrinsic growth rate of the population, respectively; \(q_1, q_2, \ldots, q_N\) denotes the maximum capacity of the current environment for this population, respectively; \(s_1, s_2, \ldots, s_N\) indicates the proportion of blocking effects between species; \(G\) is the initial value of the population size.

3.2. A Decomposition Rate Model for Multiple Fungal Interactions

Relationship between population size and fungal diameter

According to model assumption 3, let \(d_{\text{single}}\) be the diameter of a single fungus and \(x(t)\) be the change in population size with time, the diameter of fungal colony coverage \(d_{\text{sum}}\) can be expressed as the following equation.

\[
d_{\text{sum}}(t) = 2\sqrt{x(t)} \cdot \pi \left(\frac{d_{\text{single}}}{2}\right)^2 / \pi
\]

(12)
Using the above equation, the change in population size was converted to the diameter of the fungus as a function of time.

**Fungal growth rate when multiple fungi coexist**

Let the fungal growth rate be \( v \). Since the mycelial extension rate is essentially the growth rate of the fungus, the growth rate of the fungus can be expressed as

\[
v(t) = \frac{d_{\text{sum}}(t + \Delta t) - d_{\text{sum}}(t)}{\Delta t}
\]

(13)

**Humidity tolerance of fungi in the coexistence of multiple fungi**

According to section 4.3, the growth rate of a fungus is known and the competition level of the fungus in the current environment can be obtained by equation (6).

\[
\ln(D(t)) = a \ln(v(t)) + bM(t) + d
\]

(14)

The moisture niche width of the fungi was obtained from Eqs. (7) and (10), and finally the moisture tolerance of each fungus was obtained from Eq. (5) when multiple fungi coexisted under current environmental conditions.

\[
M(t) = r(t) - w_s
\]

(15)

**Decomposition rate of wood fiber by fungi in coexistence of multiple fungi**

According to equation (8) in section 4.4, we can obtain

\[
\ln(D(t)) = a \ln(v(t)) + bM(t) + d
\]

(16)

The equation for the change in wood mass when multiple fungi coexist is obtained from the decomposition rate \( D(t) \) as follows.

\[
\begin{align*}
    m(t + \Delta t) &= m(t) - \sum_{i=1}^{N} D_i(t) \Delta t \\
    m(0) &= m_0
\end{align*}
\]

(17)

### 3.3. Calculation of population competition model parameters

**Population intrinsic growth rate**

The intrinsic growth rate of a population is the growth rate of that species when only that species is present in the current environment, without the influence of other population interactions, i.e., the growth rate model in section 4.2 above.

Equation (1) describes the growth rate of a fungus in terms of the length of mycelium growth per day. The intrinsic growth rate of a population should be the amount of growth of that population per day. In this paper, the relationship between the intrinsic growth rate \( r_g \) of the population and the mycelial expansion rate \( d_{\text{var}} \) will be described by the following equation.
\[ r_g = \frac{\pi [(d_{\text{vary}} + d_{\text{single}})^2 - d_{\text{single}}^2] d_{\text{vary}}}{8d_{\text{single}}} \]  

(18)

Where \( d_{\text{single}} \) is the initial fungal diameter; \( d_{\text{vary}} \) is the length of mycelial expansion per day, which is the fungal growth rate derived from equation (1).

**Maximum capacity of the population**

In the presence of a single strain, let the duration of decomposition be \( T \), the daily expansion rate of mycelium be \( d_{\text{vary}}(t) \), and the final length of mycelium expansion be

\[ d_{\text{length}} = \sum_{t=1}^{T} d_{\text{vary}}(t) \]  

(19)

The maximum holding capacity of the population can be expressed as

\[ q = \frac{\pi (d_{\text{length}} + d_{\text{single}})^2 d_{\text{length}}}{8d_{\text{single}}} \]  

(20)

**Hysteresis coefficient of the population**

The coefficient of the blocking effect between populations can be calculated by the following equation.

\[ s_i = \frac{q_i}{\sum_{i=1}^{N} q_i} \]  

(21)

### 3.4. Solution algorithm

Algorithm 2: A fungal decomposition rate model with multiple fungal interactions

**Input:** \( d_{\text{single}} \)

**Output:** \( D \)

The growth rate of the fungus \( d_{\text{vary}} \) was calculated by equation (1), and the intrinsic growth rate of the population \( r_g \) was obtained using equation (18).

The maximum capacity \( q \) of the population is calculated by equation (19)(20). Then, the hysteresis coefficient \( s \) of the population was obtained by using equation (21). Substitute \( r_g \), \( q \), and \( s \) into Eq. (11) to solve for the change in population size over time.

The change in population size was converted to the value of the change in fungal diameter over time according to equation (12).

The variation of fungal growth rate was obtained according to equation (13).

The change in moisture tolerance of the fungus was obtained by (14)(15).

The final graph of wood mass with time is obtained by (16) (17).
4. Conclusions

(a) Changes in fungal populations 
(b) Attrition from multiple fungal interactions

Figure 2 The number of red Monascus

The graph (a) illustrates that the number of red Monascus is higher than that of Aspergillus niger when they show a competitive relationship. For an initial period of time, the environment is well supplied with nutrients, oxygen, etc., and the two fungi grow at different degrees of rapidity. Monascus has a higher demand for nutrients than Aspergillus niger. When nutrients and oxygen in the environment are insufficient, Monascus first reaches its peak number, and then the number drops. At this time, the nutrients in the environment can still meet the demand of Aspergillus niger, and the number of Aspergillus niger continues to grow, and then stabilizes.

The graph (b) illustrates that the decomposition rate of wood is significantly faster when multiple fungi coexist. The decomposition of Monascus is higher than that of Aspergillus niger at the same time, which also confirms the conclusion that the growth rate of Monascus is higher than that of Aspergillus niger as shown in (a).

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