On the Thermal History of Calculable Gauge Mediation

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Abstract

Many messenger models with realistic gaugino masses are based on meta-stable vacua. In this work we study the thermal history of some of these models. Analyzing R-symmetric models, we point out that while some of the known messenger models clearly prefer the supersymmetric vacuum, there is a vast class of models where the answer depends on the initial conditions. Along with the vacuum at the origin, the high temperature thermal potential also possesses a local minimum far away from the origin. This vacuum has no analog at zero temperature. The first order phase transition from this vacuum into the supersymmetric vacuum is parametrically suppressed, and the theory, starting from that vacuum, is likely to evolve to the desired gauge-mediation vacuum. We also comment on the thermal evolution of models without R-symmetry.

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I. INTRODUCTION

It is well-known that low scale supersymmetry (SUSY) breaking mediated to the Standard Model through gauge interactions [1, 2, 3, 4, 5] can successfully solve the supersymmetric flavor puzzle. Moreover, models of gauge mediation are often calculable. Thus gauge-mediation models should be considered as compelling candidates for the UV completion of the Minimal Supersymmetric Standard Model (MSSM).

It has been suggested a long time ago (see e.g. [6]) that the SUSY-breaking minimum can be metastable. It was further argued in [7] that the gauge-mediated vacuum is likely to be metastable because of the messenger structure. For additional early examples of meta-stable supersymmetric models see e.g. [8, 9, 10]. Later it was shown in ISS [11] that metastable SUSY-breaking minima are indeed generic and can be found, for example, in massive SQCD in the free magnetic phase.

Some recent developments even more strongly advocate the idea of meta-stability of the gauge-mediated vacuum. It was proven in [12] that if gauginos acquire realistic soft masses within a calculable model, the gauge-mediation vacuum must be meta-stable within IR effective theory. This feature leads us to the question of which vacuum of the theory is cosmologically preferred.

The issue of the thermal evolution of supersymmetric models with both supersymmetric and non-supersymmetric vacua was intensively studied in [13, 14, 15, 16] in the context of ISS. All of these studies agreed that the ISS minimum is indeed cosmologically preferable. Moreover, as it was emphasized in [13] the theory is likely to end up in the non-supersymmetric vacuum even if it begins its thermal evolution from the supersymmetric vacuum.

Nevertheless ISS-like meta-stability is not the type of meta-stability which is required in order to get a viable phenomenology. The ISS vacuum is stable within IR renormalizable effective theory, and the supersymmetric vacuum becomes manifest only if non-renormalizable corrections are considered. Hence one cannot get leading order gaugino masses in ISS even if R-symmetry is maximally broken by some deformation. The absence of leading order gaugino masses was explicitly shown in the ISS-based models of direct gauge mediation [17, 18, 19, 20]. In order not to exacerbate the little hierarchy problem we will further consider in our analysis only those theories which have leading order gaugino masses.
Thus we are forced to consider models of a rather different structure than ISS; dynamical examples of models of that type have been introduced e.g. in [21] and more recently in [22]. These examples, as well as other models one can potentially consider, are particular dynamical realizations of a much broader class of models of gauge mediation, so called extraordinary gauge mediation (EOGM) [23]. EOGM supplies the most generic ansatz for any calculable R-symmetric model with F-term SUSY breaking one can consider.\footnote{As has been recently discussed in [24], D-terms are only generated from F-terms in dynamical models of SUSY breaking. Thus, usually D-terms are parametrically suppressed and we will not consider this possibility in our current study.} The thermal history of the EOGM models is the focus of our current work.

R-symmetry is an important guideline in analyzing the models of EOGM. Spontaneously broken R-symmetry, even if it is an approximate accidental symmetry of the IR effective theory, is well motivated by the Nelson-Seiberg argument [25] and because it ensures that leading order soft terms are CP-conserving. R-symmetry enables clear classification of the EOGM models, and endows these models with some useful generic properties, elaborated in [23].

In this work we systematically study the thermal history of EOGM models. We first concentrate on a special subset of EOGM, assuming the “minimal” UV completion. Namely we assume that the dynamics of the gauge mediation messengers is also responsible for SUSY breaking and R-symmetry breaking [26]. The results which we obtain do not apply to the most general EOGM models. Nonetheless, as we show later, they turn out to be helpful in analyzing models, which are more complicated than just the “minimal completion”.

Analyzing various EOGM models we stick to the definitions of [23], distinguishing three different types of these models. Here we briefly describe these three types, while more precise definitions will be given in section II.

- Type I models. These are models with the full-rank messenger mass matrix. Those models are the most straightforward generalization of O’Raifeartaigh models. Type I models necessarily have vanishing leading order gaugino masses.

- Type II models. These are models with full-rank coupling matrix between the messengers and the pseudomodulus. The simplest possible example of such models is minimal gauge mediation.
• Type III models. Neither mass nor coupling matrices in these models are full-rank matrices. Those models share some similarities both with type I and type II models.

We argue that type II models are cosmologically disfavored if the Universe is reheated above the SUSY-breaking scale.\(^2\) Hence, in order to render these models viable one can obtain a new stringent bound on the reheat temperature. Interestingly, this bound goes precisely in the same direction as bounds on the reheat temperature from gravitino cosmology, if the gravitino is more massive than a keV \(^{28, 29}\).

Studying the type I models we point out that at high temperatures they might possess two different vacua. One of them is a global vacuum at the origin of the field space and the second vacuum is local and can potentially emerge very far away from the global vacuum. While the messengers in the local vacuum are at the origin of the field space, the pseudomodulus is stabilized far away from its origin. This second vacuum has no analog in the zero temperature field theory, and it disappears when the temperature drops below the SUSY-breaking scale. In spite of the fact that the free energy of this vacuum is larger than the free energy at the origin, the first order phase transition from this vacuum into the vacuum at the origin is strongly parametrically suppressed.\(^3\) We also point out a necessary condition for this vacuum to exist. We show that this condition holds in type I models, that it does not hold in type II models and that it might hold in type III models.

Turning to the type III models, we demonstrate that if the vacuum far away from the origin exists, the thermal history of the Universe strongly depends on the initial conditions. If it starts the evolution from the origin, it will inevitably slide into phenomenologically undesirable minimum (or runaway). On the contrary, if the Universe is trapped after inflation in the vacuum far away from the origin, its transition rate into the vacuum at the origin is highly suppressed. When this minimum disappears the system slides to the gauge-mediation minimum.

Our analysis assumes that the reheating temperature is higher than the messenger scale and that the Universe cools down adiabatically. We restrict ourselves in our analysis to

\(^2\) Regarding some similar models, this observation has recently appeared in \(^{27}\). Here we are studying these models in more generality.

\(^3\) Some examples of the type I models, e.g. the minimal O’Raifeartaigh model, have been analyzed in \(^{14}\). While we agree with their results regarding the minimum near the origin, we point out that one more thermal minimum might exist, significantly changing the analysis of the thermal history of such models.
calculable and renormalizable models. We assume that the Kähler potential is canonical at tree level. For the purposes of the current analysis we also assume that the Standard Model (SM) interactions are much weaker than the messenger model interactions; the SM interactions are completely neglected in our current study. It would be interesting to study the more general setup in the future.

Our paper is organized as follows. In the second section we survey the thermal behavior of different types of “minimally completed” EOGM while also reviewing some facts about thermal field theory which are necessary for our analysis. In that section we show that a new vacuum far away from the origin may exist in type I models and explain intuitively why one should expect these minima to show up. In the third section we use these results in order to explain the behavior of some dynamical UV completions of EOGM found in the literature. We also comment on the behavior of some models which are not R-symmetric. Finally, in the fourth section we conclude. Some technical details are relegated to the appendix.

II. SURVEYING REHEATED EOGM MODELS

In this section we will survey all three types of R-symmetric EOGM models and analyze the thermal behavior of each. We will closely follow the notation of \[23\]. Consider the following superpotential

\[ W = (\lambda_{ij} X + m_{ij}) \varphi_i \bar{\varphi}_j - f X \equiv M_{ij} \varphi_i \bar{\varphi}_j - f X . \tag{1} \]

\( X \) may acquire both supersymmetric and SUSY-breaking VEVs \( X = \langle X \rangle + \theta^2 F \). Note that the scalar component of \( X \) is a pseudomodulus, which is stabilized at loop level.

Hereafter we will assume that the SUSY-breaking scale \( f \) is much lower than the messenger scale \( m \). This assumption will further allow us to consider the couplings in the hidden sector to be of order \( \mathcal{O}(1) \) without sacrificing the zero-temperature meta-stability, and justify the formal assumption which we made \( \lambda \gg g_{SM} \).

At finite temperature \( T \) the thermal potential of the model is given by \[30\]:

\[ V(T) = V(T = 0) \pm \frac{T^4}{2\pi^2} \int_0^\infty dx x^2 \ln \left( 1 \mp e^{-\frac{x^2 + \frac{m^2(\varphi)}{4T^2}}{T^2}} \right) + \ldots \tag{2} \]

\[ \equiv V(T = 0) + V_{1\text{-loop}}(T = 0) + V_{\text{th}} . \]

Plus and minus signs stand for bosons and fermions respectively, and summation over all
degrees of freedom is implied. The ellipses stand for higher loop terms. Note that $V(T = 0)$
denotes the tree-level zero-temperature potential.

Since we are interested in understanding orders of magnitude of the phase transition
temperatures, we will use the expansion of the expression (2) at temperatures much higher
than the masses of the particles:

$$V_{th} = -\frac{\pi^2}{90} T^4 \left( N_b + \frac{7}{8} N_f \right) + \frac{T^2}{24} \left( \sum_{\text{bosons}} m^2(\phi) + \sum_{\text{fermions}} \frac{m^2(\phi)}{2} \right) + \ldots$$

(3)
The contribution of the particles much heavier than the temperature is Boltzmann suppressed
and can be safely neglected.

A. Type II models

We first consider the models with $\det m = 0$ and $\det \lambda \neq 0$ (the matrices $m$ and $\lambda$
are defined in (1)). These models are clearly phenomenologically interesting since unlike type I
models they provide viable gaugino masses. The reason for this is that leading-order gaugino
masses are proportional to

$$m_{1/2} \propto \partial_X \ln \det \mathcal{M}.$$  (4)

On the other hand it was proven in [23] that if $m$ is a full rank matrix, then R-symmetry
necessarily renders the matrix $\mathcal{M}$ to be $X$-independent. Thus the models with $\det m = 0$
are of special phenomenological interest.

Before discussing the thermal history of these models let us first briefly discuss their
vacuum structure at zero temperature. Since gaugino masses are non-vanishing at leading
order, the SUSY-breaking vacuum is necessary metastable. This feature is manifest: since
$\det m = 0$, we necessarily have messengers whose masses are given solely by $\lambda X$. For
sufficiently small values of $X$ these messengers become tachyonic. Along the moduli space
of $X$ we have a stable messenger spectrum only for $X > X_{\text{min}}$ for some specific value of
$X_{\text{min}}$, which is of order of magnitude $\frac{T}{X}$. Following the tachyonic direction the theory will
either be stabilized at some supersymmetric minimum or will slide down to a runaway.

Now let us turn to the thermal history of such models. To simplify the analysis we bring
the matrix $\lambda$ to the diagonal form such that the messenger mass matrix is

$$\mathcal{M}_{ij} = \lambda_i \delta_{ij} X + m_{ij}.$$  (5)
It is straightforward now to calculate the thermal potential of this theory at temperatures $T \gg m$, where by $m$ we mean a rough messenger scale. The resulting expression is

$$V_{th} = \text{const} + \frac{T^2}{4} \left( \sum_{i=1}^{N} |\lambda_i|^2 |\phi|^2 + |\lambda_i|^2 |\tilde{\phi}|^2 \right) + \frac{T^2}{4} \text{Tr}(\lambda^\dagger \lambda |X|^2)$$  \hspace{1cm} (6)

It is also clear in light of the results of appendix A that this thermal potential must be independent of $m_{ij}$ and $f$. The global minimum of the finite temperature potential is at the origin of the field space. One can also verify that no other minimum emerges far away from the origin, and at temperatures $T \gg m$ the minimum at the origin of the field space is unique.

Since the origin is not a stable point of the zero temperature effective potential, we should estimate the temperature at which a phase transition occurs. The messengers acquire tachyonic masses of order $\sqrt{f}$, so one expects that the second order phase transition will occur at a comparable temperature. More precisely one can estimate for $\lambda \ll 1$ using expression (6)

$$T_{cr} \approx 2 \sqrt{\frac{T}{\lambda}}. \hspace{1cm} (7)$$

Now we analyze the behavior of the minimum at $\varphi_i = \tilde{\varphi}_i = 0$ with $X$ getting a VEV at the messenger scale. Once the system is stabilized at this vacuum, SUSY is spontaneously broken and the MSSM fields get their soft masses. We will further call this minimum the “EOGM vacuum”. Let $X_{EOGM}$ denote the value of $X$ at that minimum at zero temperature. When the temperature is well above the messenger scale, the effective potential for $X$ scales as $V \sim T^2 X^2$ and the EOGM vacuum does not exist. Once the temperature drops below $\lambda X_{EOGM}$ all the messengers near the point $X = X_{EOGM}$ become heavy (since $\lambda$ is a full rank matrix) and the EOGM minimum emerges. Nonetheless the number of thermalized degrees of freedom in the EOGM minimum is smaller than at the origin. In general, one can notice that the free energy of the EOGM minimum is bigger than the free energy of the origin (or, alternatively, the supersymmetric vacuum after the second order phase transition occurs) at any temperature.

This claim can be explained as follow. It is easy to see from (3) that a vacuum with a larger number of light degrees of freedom has lower free energy. When the EOGM vacuum is formed, the only light supermultiplet in this vacuum is $X$. It becomes heavy at a temperature of order $\sqrt{\frac{\alpha \lambda}{4\pi f}}$. On the other hand, the vacuum at the origin has more light
degrees of freedom: both $X$ and some messenger pairs are light (such messengers exist since rank $\lambda > \text{rank} \ m$). Once we get to the temperature of order $f$, the free energy of the EOGM vacuum is already governed by the temperature independent term, $f^2$, which arises since EOGM vacuum breaks SUSY. Hence the thermal potential at the EOGM vacuum below the temperature (7) scales as

$$V_{EOGM}(T^2 < f) = f^2 - \frac{\pi^2}{24} T^4 + \ldots \approx f^2 . \quad (8)$$

Clearly this value is bigger than the thermal potential along the path from the origin to the supersymmetric vacuum. The key point is that along that path the SUSY-breaking terms $f^2$ is absent and the value of the thermal potential is governed by the number of the massless degrees of freedom, which is necessarily smaller than (8). Thus, type II minimally-completed EOGM models are always cosmologically disfavored unless the reheat temperature is significantly smaller than the messenger scale. Therefore in this context we find a new bound on reheat temperature.

**B. Type I models**

In this subsection we discuss the models with $\det m \neq 0$ and $\det \lambda = 0$. Since these models cannot produce reasonable phenomenology, we will analyze this scenario as a toy model. The results will be useful in the analysis of type III models.

Before analyzing genuine type I models, consider as an example the minimal O’Raifeartaigh model (let the fields $\varphi$, $\tilde{\varphi}$ be gauge singlets in this example):

$$W = \lambda X (\tilde{\varphi}^2 - f) + m \varphi \tilde{\varphi} . \quad (9)$$

It is straightforward to calculate the thermal potential for small $X$ in this model and verify that at sufficiently high temperatures all the fields can be stabilized at the origin.

Let us now analyze this model at very large values of $X$, namely in the range

$$X \gg T, \quad X \gg m . \quad (10)$$

The key point for understanding this regime is noting that at zero temperature one finds very light particles in the O’Raifeartaigh models for $X \gg m$. Integrating out $\tilde{\varphi}$ in (9) for large $X$ one finds the effective superpotential

$$W = -\lambda X f - \frac{m^2}{4\lambda X} \varphi^2 . \quad (11)$$
Notice that the mass of \( \varphi \) is suppressed, rather than enhanced, by \( X \). Since the zero-temperature potential for \( X \) vanishes at tree level, the non-thermal mass of the pseudomodulus is also strongly suppressed, such that both \( \varphi \) and \( X \) are thermalized in the regime (10). The mass of \( \varphi \) becomes exactly zero when \( X \) gets to infinity, so we should expect in this range thermal runaway for \( X \). In order that this regime be reliable we need both inequalities in (10) to hold. If the temperature exceeds \( X \), the field \( \tilde{\varphi} \) is also thermalized and the runaway behavior is lost.

Now let us ask whether this runaway is stabilized. The thermal potential for \( X \) is

\[
V_{th} \sim \frac{m^4 T^2}{|\lambda X|^2} . \tag{12}
\]

But the zero temperature potential is a monotonically increasing function. The leading order term of the one-loop effective potential is

\[
V_{1-loop} \sim \frac{\alpha \lambda}{4\pi} f^2 \ln |X|^2 . \tag{13}
\]

which will inevitably stabilize the runaway. The mass matrix for the field \( \varphi \) is positive definite in the regime (10) if \( m^2 > f \). Thus this simple O’Raifeartaigh model, besides the vacuum at the origin, gets a local minimum of the thermal potential far away from the origin, \( X \gg T \). More precisely the potential is balanced at

\[
X_* \sim \left( \frac{\alpha \lambda}{4\pi} \right)^{-1/2} \frac{m^2 T}{\lambda f} . \tag{14}
\]

The free energy of this local vacuum is bigger than the free energy of the vacuum at the origin, so we are supposed to check if the thermal transition rate from this vacuum to the origin is suppressed. It turns out that the transition rate for the first order phase transition to the vacuum at the origin is parametrically suppressed since \( X_* \gg T \). We will justify this statement in the subsection II C after extending our analysis to broader class of models.

Now we are ready to generalize this idea to the type I EOGM models. Consider for example a model with \( N \) messenger pairs and with the following superpotential:

\[
W = m_{(i)} \varphi_i \tilde{\varphi}_i + \lambda_{(i)} X \varphi_i \tilde{\varphi}_{i+1} - f X \tag{15}
\]

with \( i \) running over the values 1 . . . \( N \). At large values of \( X \) all of the messengers excluding \( \varphi_N \) and \( \tilde{\varphi}_1 \) are supposed to be integrated out. After integrating out the heavy messengers one gets the following superpotential:

\[
W = -f X + (-)^{(N-1)} \frac{\det m}{\prod_{i} \lambda_{(i)} X^N \varphi_N \tilde{\varphi}_1} . \tag{16}
\]
Note that the power of $X$ in this expression can be determined, without any calculation, from R-symmetry considerations.

Now we check if the messenger mass-squared matrix is positive definite at zero temperature. The mass-squared matrix for the scalars is given by two $2 \times 2$ diagonal blocks of the form

$$M^2 = \begin{pmatrix} \left( \frac{\det m}{\prod \lambda_i} \right)^2 \frac{1}{X^{2N-2}} & \pm (N-1) \frac{\det m}{\prod \lambda_i} \frac{f}{X^N} \\ \pm (N-1) \frac{\det m}{\prod \lambda_i} \frac{f}{X^N} & \left( \frac{\det m}{\prod \lambda_i} \right)^2 \frac{1}{X^{2N-2}} \end{pmatrix}. \quad (17)$$

Since we are discussing the case where $X$ is large, this matrix is not necessarily positive definite. This is of course nothing but a manifestation of the claim of [23] that the moduli space of the type I EOGM might become unstable when $X$ exceeds some $X_{\text{max}}$. Nonetheless, for $N = 2$ (or, alternatively, multiple decoupled species of $N = 2$) the matrix (17) is always positive definite.

Consider the thermal potential. Up to $O(1)$ numbers the leading terms of the thermal potential are

$$V_{\text{th}} \sim \left( \frac{\det m}{\prod \lambda_i} \right)^2 \frac{T^2}{|X|^{2N-2}} + \left( \frac{\det m}{\prod \lambda_i} \right)^2 \frac{T^2}{|X|^{2N}} \left( |\phi_1|^2 + |\tilde{\phi}_N|^2 \right). \quad (18)$$

As expected, we get runaway behavior for $X$, and thermal masses for the light messengers which are highly suppressed compared to the naive expectation. The minimum always exists in models with two messenger pairs; in models with more messenger pairs it might exist for some definite range of temperatures. In these models one should check if the matrix (17) is positive definite for a given choice of parameters.

Now we are ready to estimate the position of the vacuum. We will continue doing analysis for generic $N$, but one should keep in mind that only for $N = 2$ (or the set of decoupled models of this type) the existence of this vacuum is guaranteed. The one-loop effective potential far away from the origin is given by (13). The minimum of the thermal potential scales as

$$X_* \sim \left( \left( \frac{\alpha_\lambda}{4 \pi} \right)^{-1/2} \frac{\det m}{\prod \lambda_i} \frac{T}{f} \right)^{\frac{1}{N-1}}. \quad (19)$$

Consider now that we start the thermal evolution in the vacuum far away from the origin and gradually lower the temperature. The vacuum will disappear either if $X_* \sim \sqrt{\det m}$ and

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4 For detailed discussion of this instability see [31].

5 Now this expression may be imprecise since we have different coefficients $\lambda_i$. Of course the dominant contribution here will come from the largest value of $\lambda$.  

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integrating some of the messengers becomes unjustified, or if the messengers at that vacuum become heavier than the temperature, whatever happens earlier. The second condition is stronger; the vacuum disappears at temperature:

\[ T \sim \left( \frac{\alpha \lambda}{4\pi f} \right)^{1/2}, \]

namely right below SUSY-breaking scale.

Note also that only for \( N = 2 \) this vacuum survives up arbitrarily high temperatures. If the number of messenger pairs is larger than 2, we get an upper bound on the temperature demanding that matrix (17) is positive definite. This upper bound reads

\[ T < \sqrt{\frac{\alpha \lambda}{4\pi} \left( \frac{\det m}{\prod \lambda_i f} \right)^{1/(N-2)}}. \]

Evidently for sufficient separation between the messenger scale and the SUSY-breaking scale, the window defined by (20) and (21) can be very large.

One can generalize these considerations to other messenger models, not necessarily of type I. Assume that we have several sets of messengers such that the matrix \( \mathcal{M} \) is block diagonal. We note that a necessary (but not sufficient!) condition in order to get this behavior is to have some subset of messengers which fulfill the condition

\[ \text{rank } m > \text{rank } \lambda . \]

If all of our messengers fulfill this condition, we get an EOGM model of type I, but in general type III models can also exhibit this behavior.

C. Type III models

Now we turn to the most phenomenologically interesting part of our analysis. These models have both \( \det \lambda = 0 \) and \( \det m = 0 \) and share the features of both type I and type II models. In particular these models are well motivated since they lead to non-vanishing gaugino masses at leading order. The vacuum structure of these models at zero temperature is also non-trivial: they might develop instability both for very small values of the pseudomodulus (as in the case of type II models) and for very big values (as in the case of type I).
The thermal histories of these models also share similar features with both type I and type II theories. It is clear that the thermal potential possesses a global minimum at the origin. If the Universe starts its evolution from this minimum, it will undergo a second order phase transition at the temperature (7), as was described in subsection II A, and slide into the supersymmetric vacuum or runaway.

Nonetheless, sliding into the supersymmetric direction is not the only possibility. Since these models might have some set of messengers fulfilling the condition (22), we can expect that certain models will also possess a local thermal vacuum far away from the origin. Here we show an explicit example of a model which indeed exhibits this behavior. Moreover, we show that starting the evolution from this minimum one ends up in the EOGM minimum, rather than in the supersymmetric one.

Consider a model with following messenger mass matrix:

\[
\mathcal{M} = \begin{pmatrix}
\lambda X & 0 & 0 & 0 \\
0 & m & \lambda X & 0 \\
0 & 0 & M & \lambda X \\
0 & 0 & 0 & m
\end{pmatrix}.
\] (23)

This model was analyzed in [23] as an example of the “minimal completion” of type III EOGM. It was shown that for sufficient separation of scales \(m\) and \(M\) the pseudomodulus can be stabilized between these two scales.\(^6\)

Consider now this model thermalized when \(X\) is sufficiently far away from the origin. The first pair of messenger decouples, three other pairs maintain the condition (22). We can expect that the vacuum far away from the origin will show up. In order to know the exact scaling of this vacuum we just substitute \(N = 3\) in the formulas which we derived in the previous subsection. We find from (19)

\[
X_* \sim \left( \frac{\alpha \lambda}{4\pi} \right)^{-1/2} \frac{m^2 M T}{\lambda^2 f} \right)^{1/2}.
\] (24)

As explained in the previous subsection, the vacuum far away from the origin for \(N > 2\) exists only at certain range of temperatures. In our case this range is given by

\[
\left( \sqrt{\frac{\alpha \lambda}{4\pi} f} \right)^{1/2} < T < \sqrt{\frac{\alpha \lambda}{4\pi} \frac{m^2 M}{\lambda^2 f}}.
\] (25)

\(^6\) As explained in [22], the separation of masses is a necessary condition for the radiative stabilization of moduli in this kind of model.
Once the vacuum dissipates, the modulus $X$ is at the messenger scale. Substituting the lower bound on the temperature from (25) into (24) we explicitly verify that when the vacuum far away from the origin dissipates, the value of $X_*$ still respects an inequality $X_* \geq X_{EOGM}$.

In order to understand what subsequently happens to the system which evolves through the vacuum far away from the origin, we analyze the thermal potential at the SUSY-breaking temperature. All the messengers are stable at the messenger scale, they have masses of order $M$ and the potential rises in these directions, such that the pseudomodulus direction $X$ is the only relevant direction for this discussion. Since at the temperature $T \sim \sqrt{f}$ none of the particles besides the multiplet $X$ are thermalized in the vicinity of the EOGM vacuum, the shape of the thermal potential is the same as it is at zero temperature. Namely the potential monotonically rises at $X > X_{EOGM}$ approximately as $V \sim \log X + \ldots$, and decreases for smaller $X$ towards the local SUSY-breaking EOGM minimum. Therefore we conclude that when the vacuum far away from the origin dissipates, the system undergoes second order phase transition to the EOGM minimum. On the other hand, the supersymmetric minimum is located near the origin. Since the shape of the potential is similar to the shape of the zero temperature potential, the supersymmetric vacuum is separated from the EOGM minimum by a barrier with height of order $\sqrt{f}$, and the distance between two these minima is of order $M$. Consequently, a second order phase transition from the vacuum far away from the origin into the supersymmetric vacuum is impossible.

Now let us briefly discuss the hypothetical possibility of a first order phase transition from the vacuum far from the origin. The rate for this phase transition is given by

$$\Gamma \sim T^4 e^{-S_3 \over T}.$$  

In our case the potential rises from the origin as a quadratic function of $X$, up to the values $X \sim T$. At this value the potential for $X$ starts falling as a negative power. The three dimensional bounce action can be approximated through the triangular approximation [32]:

$$S_3 \over T \sim 4\pi (\delta X)^3 \sqrt{\delta V T} \sim 4\pi X_*^3 \over T^3.$$  

(27)

Since we obtained $X_* \gg T$, this rate can be made arbitrarily small by appropriate choice of parameters for any temperature when the vacuum far away from the origin exists. Note that by the same reason the first order phase transition rate from the EOGM minimum at the temperature $\sqrt{f}$ (or lower) to the supersymmetric vacuum is also parametrically suppressed. In this last case one should of course substitute $M$ instead of $X_*$ into (27).
III. COMMENTS ON DYNAMICAL MODELS

A. R-symmetric Dynamical and Retrofitted Models

In the previous section we studied models of messengers with the “minimal” completion. It is plausible that at low energies this is a correct description, while all the small mass scales are just retrofitted as in [33]. In this case the description of the thermal behavior of these models is exhaustively explained in the previous section. Nonetheless, we will also be interested in applying our results to models with extra matter, which reproduce the EOGM ansatz dynamically.

Consider first the type III models. Very few dynamical examples of this kind of model are known, one of them presented, e.g. in the model of “uplifted vacua in SQCD” [22]. One can wonder if the mechanism of “rescuing” the meta-stable vacuum through evolution far away from the origin is still available in this case. The answer is negative. The crucial difference between the model presented in [22] and the “retrofitted” type III model is that in the retrofitted models the masses are “replaced” by the VEVs of some gaugino condensates. These condensates are usually formed at very high temperatures, and at low temperatures the masses behave as fundamental parameters. On the other hand, in dynamical models, these masses do not emerge before the thermal vacuum at the origin is destabilized and the relevant fields acquire VEVs. This usually happens at relatively low temperatures. Above that temperature the system evolves as if it did not have any masses at all, in other words similarly to the type II models. For example in the “uplifted vacua model” the masses do not emerge before the Universe is cooled down below the messenger scale, and hence the mechanism described in subsection II C for rescuing the EOGM vacuum is inapplicable. Therefore, a stringent bound on the reheat temperature of these models applies.

The next question we address is the behavior of (R-symmetric) models where messengers are added “by hand” and they are not part of the SUSY breaking sector. To illustrate this kind of model, consider an R-symmetric model of messengers and SM singlets which couple to the pseudomodulus. If the origin of some field is destabilized at sufficiently high temperatures, they must have vanishing R-charge. Hence, at sufficiently high temperatures $X$ has

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7 An earlier example was given in [21]. The thermal behavior of these two models does not differ.
8 We prove this statement in the appendix.
a minimum at the origin independent of the details of the SM singlets to which it couples. If the theory starts from this vacuum, either by choice of initial conditions or because it is the only vacuum of the theory, it will inevitably cool down to the supersymmetric vacuum. The possibility of utilizing the rescuing mechanism through the vacuum far away from the origin strongly depends on the details of the theory. If the rank of the matrix $m$ exceeds the rank of the matrix $\lambda$ for any subset of messengers or SM singlets, the vacuum far away from origin might emerge, as discussed in section [11]. As previously noted, the masses should emerge at sufficiently high temperatures.

Applying these arguments e.g. to the model [34], which is R-symmetric and includes both “hidden sector” fields and messenger fields (which behave as in type II), we conclude that the rescuing mechanism through the vacuum far away from the origin does not work. Even though the hidden sector fields perfectly fulfill the condition (22), the masses form only at the SUSY breaking scale, which is not sufficient to ensure the vacuum far away from the origin. We note, however, that the interactions between the messengers and the pseudomodulus in [34] can we extremely suppressed. While the model with the low scale $M_2$ will favor the supersymmetric minimum, unless the reheat temperature is lower than the SUSY-breaking scale, the thermal history of the model with high $M_2$ will be largely governed by the SM interactions, which are not taken into account in our current study. If the SM D-terms indeed prevail, the final answer can change since the temperature for the second order phase transition into the EOGM vacuum can exceed the phase transition temperature in the supersymmetric direction.\(^9\)

**B. Comments on models without R-symmetry**

In this subsection we briefly comment on messenger models which lack R-symmetry. As an example we will analyze here models presented in [35, 36, 37].\(^{10}\) In these models the messenger sector was coupled to the ISS model, while the messenger masses were added “by hand”.\(^{11}\) These models exhibit a peculiar behavior: unlike R-symmetric models they might

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\(^9\) I am grateful to S. Abel, J. Jaeckel and V. Khoze for pointing me out the importance of the SM interactions.

\(^{10}\) A similar model has been recently introduced in [38]. Since in [38] one should take $\lambda \ll 1$, the SM effects may become important.

\(^{11}\) These masses were further “retrofitted” in [37] but it does not affect the thermal behavior of the model.
be stabilized at high temperatures at a point which is not a zero of the $\det(\lambda X + m)$.\textsuperscript{12}

Consider for simplicity the following superpotential:

$$W = \lambda X \varphi_i \bar{\varphi}_i + \lambda' X \phi_j \bar{\phi}_j - m \varphi_i \bar{\varphi}_i - f X.$$  \hspace{1cm} (28)

The messengers $\varphi, \bar{\varphi}$ are charged under the SM model while $\phi, \bar{\phi}$ are uncharged.\textsuperscript{13} It is easy to see that the thermal potential of the model (28) up to an overall numerical factor is

$$V_{th} \propto T^2 \left( N_\varphi |\lambda X|^2 + N_\phi |\lambda X - m|^2 \right).$$  \hspace{1cm} (29)

At sufficiently high temperature the pseudomodulus is stabilized at

$$X = \frac{\lambda N_\varphi m}{\lambda N_\varphi + \lambda N_\phi}. \tag{30}$$

Both $X = 0$ and $X = m$ are zeros of $\det(\lambda X - m)$, but the stabilization is at some intermediate value between them.\textsuperscript{14}

Now we trace the evolution of this vacuum when the temperature drops down. Of course, we assume that the scale $m$ is much larger than the SUSY breaking scale (otherwise even the zero-temperature meta-stability may be spoiled). At the scale $m$ the vacuum at (30) is destabilized and the second order phase transition occurs. The zero temperature masses of the fields $\varphi, \bar{\varphi}$ and $\phi, \bar{\phi}$ for $X = X_*$ are $(\lambda X_* - m)$ and $\lambda X_*$ respectively. Those particles which are more massive will stop affecting the thermal potential at higher temperatures since their contributions will be Boltzmann suppressed, and the theory will slide into the direction where the lightest particles attract it. We can draw the conclusion that the models of ISS with messengers are not automatically safe; their thermal history strongly depends on the ratio $\frac{\lambda N_\varphi}{\lambda N_\phi}$. If $\lambda N_\varphi$ “wins”, the theory will be attracted to the direction of the supersymmetric minimum, but since these models have no restrictions on the hidden sector, this potential problem can circumvented by choosing a big enough magnetic group in the ISS sector.

Now let us consider a more generic picture and try to draw some conclusions. A thermal potential of an arbitrary messenger model without R-symmetry is a sum of quadratic functions with the origins sitting at the zeros of $\det(\lambda X + m)$. Thus the minimum of the potential

\textsuperscript{12} Note, that in the ISS model $\langle X \rangle = 0$.

\textsuperscript{13} Those fields may be multiplets of some other symmetry group, but it is irrelevant for our discussion.

\textsuperscript{14} As a consequence of appendix A, an R-symmetric O’Raifeartaigh model cannot exhibit such behavior unless the R-charge of $X$ vanishes.
is likely to be somewhere between these zeros. At zero temperature each of these zeros, if it is simple, is necessarily an attractive point of the potential. I.e. there is some vicinity of each of these points where the potential for $X$ attracts it to the zero, independently of whether this point has tachyonic messengers $^{39}$. Namely each of these zeros can govern the thermal evolution, and an exact answer is highly model-dependent, as illustrated in the example of (28).

IV. CONCLUSIONS AND OUTLOOK

In this paper we studied the thermal behavior of various gauge mediated models, mostly with, but also without R-symmetries. We showed that at high temperatures all R-symmetric messenger models possess a minimum at the origin of field space. Starting from this minimum the models generally evolve to the supersymmetric vacuum. In order to avoid this pitfall one should demand that the reheat temperature is lower than the messenger scale and that the Universe is trapped in the false vacuum after inflation. Demanding reheat temperature lower than the messenger scale can be quite a severe constraint on low-scale gauge-mediated models.

We also showed that this stringent demand of the low reheat temperature can be circumvented in certain models, where some particles, coupling to the pseudomodulus maintain the condition $^{22}$. In this case an additional minimum can emerge far away from the origin which can drive the thermal evolution of the Universe. In this case the Universe may end up in the SUSY-breaking minimum even for relatively high reheat temperature. Clearly the thermal evolution of models of this class strongly depend on the initial conditions after inflation. The subject of the initial conditions was not studied here and it would be interesting to study this subject in future works.

We have presented convincing evidence that the thermal history is more special if the model is R-symmetric. It would be interesting to study further the connection between R-symmetry and the structure of the thermal potential.

We also notice that we did not study the effects of the SM gauge interactions, concentrating solely on the messenger sector analysis. While the messenger sector effects are extremely

$^{15}$ In a generic model without R-symmetry we expect all zeros to be simple.
important, and the thermal history of lots of models can be determined only by analyzing
the messenger dynamics, it would be interesting to accomplish this analysis including the
SM effects. Clearly the approximation where the SM effects are completely neglected cannot
be valid in all models and in all parts of the parameter space, so it would be important to
understand in the future, where precisely these effects are important and whether they can
relax some of the constrains on the gauge-mediated models.

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APPENDIX A: PROPERTIES OF THE THERMAL POTENTIAL IN R-
SYMMETRIC MODELS

In this appendix we consider a thermalized R-symmetric O’Raifeartaigh model. We prove
that at sufficiently high temperatures the origin of the field is destabilized only if its R-charge
vanishes. We also show that the field can possess vanishing thermal mass at the origin only
if there is more than one field in the theory with precisely the same R-charge.

Consider the most generic renormalizable R-symmetric Wess-Zumino model:

\[ W = f_i \phi_i + \frac{m_{ij}}{2} \phi_i \phi_j + \frac{\lambda_{ijk}}{6} \phi_i \phi_j \phi_k . \]  

\[ \text{(A1)} \]

The couplings \( m_{ij} \) and \( \lambda_{ijk} \) are symmetric in all indices.

Now consider temperatures much higher than the masses in (A1) and derive the lead-
ing order field dependent thermal potential. Analyzing the approximation [3] we notice
an important feature which significantly simplifies our analysis: the leading order thermal
potential is merely sensitive to the trace of the mass squared matrix. Namely, we will not be
be interested either in the SUSY-breaking terms in the zero temperature potential or in any
other off-diagonal terms in the mass squared matrix.\(^{16}\) In order to obtain a correct leading

\(^{16}\) To use this property we need an additional assumption \( \sqrt{T} \ll m \). Fortunately it holds in the vast majority
of realistic gauge-mediated models.
term thermal potential it is sufficient to take a sum over the terms in the zero-temperature tree-level potential of the form

$$ V \supset m_i^2(\phi_i \phi_i^*) . \quad (A2) $$

The quadratic terms in the thermal potential are always positive since they are formed from the interaction in $(A1)$. So the only terms which can cause destabilization in the thermal potential are either tadpoles or non-diagonal quadratic terms.

In order to get a tadpole in the thermal potential we should have trilinear term in the zero temperature potential. The only way to get these interactions is

$$ V \supset m_{kl}^* \phi_l^* \lambda_{ijk} \phi_i \phi_j + c.c. \quad (A3) $$

Since terms which may give contributions to the thermal potential should be of the form $(A2)$, either index $i$ or index $j$ should be equal to $l$. Namely the field $\phi_j$ will have a tadpole in the thermal potential only if the terms $\phi_k \phi_l$ and $\phi_k \phi_l \phi_j$ are both present in the superpotential. This can happen only if the R-charge of $\phi_j$ is zero.

Another source of thermal instability can potentially come from non-diagonal masses in the thermal potential. Such masses emerge from the quartic terms in the zero-temperature potential:

$$ V \supset \lambda_{ijk}^* \lambda_{ijl} |\phi_j|^2 \phi_l^* \phi_l + c.c. \quad (A4) $$

But if $k \neq l$ this necessarily means that the fields $\phi_k$ and $\phi_l$ possess the same R-charge. Working out the eigenvalues of the thermal mass matrix for $\phi_k$ and $\phi_l$ we conclude that one of the modes has positive thermal mass while the second is exactly massless. If this happens the fate of the massless mode depends on its zero-temperature mass.

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