Power Control in Cognitive Radio Networks:
How to Cross a Multi-Lane Highway

Wei Ren, Qing Zhao*, Ananthram Swami

Abstract

We consider power control in cognitive radio networks where secondary users identify and exploit
instantaneous and local spectrum opportunities without causing unacceptable interference to primary
users. We qualitatively characterize the impact of the transmission power of secondary users on the
occurrence of spectrum opportunities and the reliability of opportunity detection. Based on a Poisson
model of the primary network, we quantify these impacts by showing that (i) the probability of spectrum
opportunity decreases exponentially with respect to the transmission power of secondary users, where
the exponential decay constant is given by the traffic load of primary users; (ii) reliable opportunity
detection is achieved in the two extreme regimes in terms of the ratio between the transmission
power of secondary users and that of primary users. Such analytical characterizations allow us to
study power control for optimal transport throughput under constraints on the interference to primary
users. Furthermore, we reveal the difference between detecting primary signals and detecting spectrum
opportunities, and demonstrate the complex relationship between physical layer spectrum sensing and
MAC layer throughput. The dependency of this PHY-MAC interaction on the application type and the
use of handshake signaling such as RTS/CTS is illustrated.

Index Terms

Power Control, Cognitive Radio, Opportunistic Spectrum Access, Transport Throughput, Poisson
Random Network.

This work was supported in part by the Army Research Laboratory CTA on Communication and Networks under Grant
DAAD19-01-2-0011 and by the National Science Foundation under Grants ECS-0622200 and CCF-0830685. Part of this work
was presented at ITA, January 2008, and ICASSP, April 2008.

W. Ren and Q. Zhao are with the Department of Electrical and Computer Engineering, University of California, Davis, CA
95616. A. Swami is with the Army Research Laboratory, Adelphi, MD 20783.

* Corresponding author. Phone: 1-530-752-7390. Fax: 1-530-752-8428. Email: qzhao@ece.ucdavis.edu
I. INTRODUCTION

Cognitive radio (CR) for opportunistic spectrum access (OSA) addresses critical challenges in spectrum efficiency, interference management, and coexistence of heterogeneous networks in future generations of wireless systems [1]. The basic idea of OSA is to achieve spectrum efficiency and interoperability through a hierarchical access structure with primary and secondary users. Secondary users, equipped with cognitive radios capable of sensing and learning the communication environment, can identify and exploit instantaneous and local spectrum opportunities without causing unacceptable interference to primary users [2].

While conceptually simple, CR for OSA presents new challenges in every aspect of the system design. In this paper, we focus on transmission power control. We show that unique features of CR systems give a fresh twist to this classic problem and call for a new set of fundamental theories and practical insights for optimal design.

A. Power Control in Cognitive Radio Networks

In wireless networks, transmission power defines network topology and determines network capacity. The tradeoff between long-distance direct transmission and multi-hop relaying, both in terms of energy efficiency and network capacity, is now well understood in conventional wireless networks [3, 4].

This tradeoff in CR systems is, however, much more complex. The intricacies of power control in CR systems may be illustrated with an analogy of crossing a multi-lane highway, each lane having different traffic load. The objective is to cross the highway as fast as possible subject to a risk constraint. Should we wait until all lanes are clear and dash through, or cross one lane at a time whenever an opportunity arises? What if our ability to detect traffic in multiple lanes varies with the number of lanes in question?

We show in this paper that similar questions arise in power control for secondary users. The transmission power of a secondary user not only determines its communication range but also affects how often it sees spectrum opportunities. If a secondary user is to use a high power to reach its intended receiver directly, it must wait for the opportunity that no primary receiver is active within its relatively large interference region, which happens less often. If, on the other hand, it uses low power, it must rely on multi-hop relaying, and each hop must wait for its own opportunities to emerge.
A less obvious implication of the transmission power in CR networks is its impact on the reliability of opportunity detection. As shown in this paper, the transmission power of a secondary user affects the performance of its opportunity detector in terms of missed spectrum opportunities and collisions with primary users. Optimal power control in CR systems thus requires a careful analysis of the impact of the transmission power on both the occurrence of opportunities and the reliability of opportunity detection.

B. Contributions

The key contribution of this paper lies in the characterization of the impact of secondary users’ transmission power on the occurrence of spectrum opportunities and the reliability of opportunity detection. These impacts of secondary users’ transmission power lead to unique design tradeoffs in CR systems that are nonexistent in conventional wireless networks and have not been recognized in the literature of cognitive radio. The recognition and characterization of these tradeoffs contribute to the fundamental understanding of CR systems and clarify two major misconceptions in the CR literature, namely, that the presence/absence of spectrum opportunities is solely determined by primary transmitters and that detecting primary signals is equivalent to detecting spectrum opportunities. We show in this paper the crucial role of primary receivers in the definition of spectrum opportunity, which results in the dependency of the occurrence of spectrum opportunities on the transmission power of secondary users, in addition to the well understood dependency on the transmission power of primary users. Furthermore, we show that spectrum opportunity detection is subject to error even when primary signals can be perfectly detected. Such an non-equivalence between detecting primary signals and detecting spectrum opportunities is the root for the connection between the reliability of opportunity detection and the transmission power of secondary users, a connection that has eluded the CR research community thus far.

The above qualitative and conceptual findings are generally applicable to various primary and secondary network architectures, different traffic, signal propagation, and interference models. To quantify the impact of transmission power on the occurrence of opportunities and the reliability of opportunity detection, we adopt a Poisson model of the primary network and a disk model for signal propagation and interference. Closed-form expressions for the probability of opportunity and the performance of opportunity detection (measured by the probabilities of false alarm and miss detection) are obtained. These closed-form expressions allow us to establish the
exponential decay of the probability of opportunity with respect to the transmission power and
the asymptotical behavior of the performance of opportunity detection. Specifically, we show that
the probability of opportunity decreases exponentially with $p_t^{2/\alpha}$, where $p_{tx}$ is the transmission
power of secondary users and $\alpha$ is the path-loss exponent. In terms of the impact of $p_{tx}$ on
spectrum sensing, we show that reliable opportunity detection is achieved in the two extreme
regimes of the ratio between the transmission power $p_{tx}$ of secondary users and the transmission
power $P_{tx}$ of primary users: $\frac{p_{tx}}{P_{tx}} \to 0$ and $\frac{p_{tx}}{P_{tx}} \to \infty$. These quantitative characterizations lead to
a systematic study of optimal power control in CR systems. Adopting the performance measure
of transport throughput, we examine how a secondary user should choose its transmission power
according to the interference constraint, the traffic load and transmission power of primary users,
and its own application type (guaranteed delivery vs. best-effort delivery).

While the disk propagation and interference model is simplistic, it leads to clean tractable
solutions that highlight the main message regarding the dependencies of the definition, the
occurrence, and the detection of spectrum opportunities on the transmission power of secondary
users. It is our hope that this paper provides insights for characterizing such dependencies under
more complex and more realistic network and interference models.

Other interesting findings include the difference between detecting primary signals and detect-
ing spectrum opportunities and how it affects the performance of spectrum sensing. Furthermore,
we demonstrate the complex relationship between physical layer spectrum sensing and MAC
layer throughput. The dependence of this PHY-MAC interaction on the application type and the
use of handshake signaling such as RTS/CTS are illustrated.

C. Related Work

The main objectives of power control in conventional wireless networks are to improve the
energy efficiency by appropriately reducing the transmission power without degrading the link
throughput and/or to increase the total throughput of the network by enhancing the spatial reuse
of the channel [5, 6]. Under these objectives, the tradeoff in power control between fewer hops
and spatial reuse is demonstrated in [3, 4], and the impact of transmission power on network
performance, such as delay, connectivity, network throughput, is summarized in [5, 7].

Power control in CR systems has been studied under different network setups and various
performance measures. The design tradeoffs in power control in conventional wireless networks
can still be found in CR systems. The unique design tradeoffs in power control in CR systems,
i.e., the impact of transmission power on the occurrence of opportunities and the reliability of opportunity detection, however, has not been recognized or analytically characterized in the literature.

In [8, 9], power control for one pair of secondary users coexisting with one pair of primary users is considered. The use of soft sensing information for optimal power control is explored in [8] to maximize the capacity/SNR of the secondary user under a peak power constraint at the secondary transmitter and an average interference constraint at the primary receiver. In [9], the secondary transmitter adjusts its transmission power to maximize its data rate without increasing the outage probability at the primary receiver. It is assumed in [9] that the channel gain between the primary transmitter and its receiver is known to the secondary user. In [10], a power control strategy based on dynamic programming is developed for one pair of secondary users under a Markov model of primary users’ spectrum usage.

Power control for OSA in TV bands is investigated in [11, 12], where the primary users (TV broadcast) transmit all the time and spatial (rather than temporal) spectrum opportunities are exploited by secondary users.

D. Organization and Notation

The rest of this paper is organized as follows. Sec. II provides a qualitative characterization of the impact of transmission power in CR systems, and lays out the conceptual foundation for subsequent sections. In Sec. III closed-form expressions and properties of the probability of spectrum opportunity and the performance of opportunity detection are obtained as functions of the transmission power $p_{tx}$ based on a Poisson primary network model. Based on these analytical results, we study power control for optimal transport throughput in Sec. IV. The impact of RTS/CTS handshake signaling on the performance of opportunity detection and the optimal transmission power is examined in Sec. V. Sec. VI concludes the paper, which also includes some simulation results on the energy detector and some comments about the effect of multiple secondary users.

Throughout the paper, we use capital letters for parameters of primary users and lowercase letters for secondary users.

II. Impact of Transmission Power: Qualitative Characterization

This section lays out the conceptual foundation for subsequent sections. The impact of transmission power on the occurrence of opportunities is revealed through a careful examination of
the definitions of spectrum opportunity and interference constraint. The impact of transmission power on the reliability of opportunity detection is demonstrated by illuminating the difference between detecting primary signals and detecting spectrum opportunities.

A. Impact on the Occurrence of Spectrum Opportunity

A formal investigation of CR systems must start from a clear definition of spectrum opportunity and interference constraint. To protect primary users, an interference constraint should specify at least two parameters \( \{ \eta, \zeta \} \). The first parameter \( \eta \) is the maximum allowable interference power perceived by an active primary receiver; it specifies the noise floor and is inherent to the definition of spectrum opportunity as shown below. The second parameter \( \zeta \) is the maximum outage probability that the interference at an active primary receiver exceeds the noise floor. Allowing a positive outage probability \( \zeta \) is necessary due to opportunity detection errors. This parameter is crucial to secondary users in making transmission decisions based on imperfect spectrum sensing as shown in [13].

Spectrum opportunity is a local concept defined with respect to a particular secondary transmitter and its receiver. Intuitively, a channel is an opportunity to a pair of secondary users if they can communicate successfully without violating the interference constraint. In other words, the existence of a spectrum opportunity is determined by two logical conditions: the reception at the secondary receiver being successful and the transmission from the secondary transmitter being “harmless”. Deceptively simple, this definition has significant implications for CR systems where primary and secondary users are geographically distributed and wireless transmissions are subject to path loss and fading.

For a simple illustration, consider a pair of secondary users (A and B) seeking to communicate in the presence of primary users as shown in Fig. 1. A channel is an opportunity to A and B if the transmission from A does not interfere with nearby primary receivers in the solid circle, and the reception at B is not affected by nearby primary transmitters in the dashed circle. The radius \( r_I \) of the solid circle at A, referred to as the interference range of the secondary user, depends on the transmission power of A and the parameter \( \eta \) of the interference constraint,

\[ 1 \text{Here we use channel in a general sense: it represents a signal dimension (time, frequency, code, etc.) that can be allocated to a particular user.} \]
whereas the radius $R_I$ of the dashed circle (the interference range of primary users) depends on the transmission power of primary users and the interference tolerance of $B$.

The use of a circle to illustrate the interference region is immaterial. This definition applies to a general signal propagation and interference model by replacing the solid and dashed circles with interference footprints specifying, respectively, the subset of primary receivers who are potential victims of $A$’s transmission and the subset of primary transmitters who can interfere with reception at $B$. The key message is that spectrum opportunities depend on both transmitting and receiving activities of primary users. Spectrum opportunity is thus a function of (i) the transmission powers of both primary and secondary users, (ii) the geographical locations of these users, and (iii) the interference constraint. Notice also that spectrum opportunities are asymmetric. A channel that is an opportunity when $A$ is the transmitter and $B$ the receiver may not be an opportunity when $B$ is the transmitter and $A$ the receiver. This asymmetry leads to a complex dependency of the optimal transmission power on the application type and the use of MAC handshake signaling such as RTS/CTS as shown in Sec. IV and Sec. V.

It is clear from the definition of spectrum opportunity that a higher transmission power (larger $r_I$ in Fig. 1) of the secondary user requires the absence of active primary receivers over a larger area, which occurs less often. The impact of transmission power on the occurrence of opportunity thus follows directly.

### B. Impact on the Performance of Opportunity Detection

Spectrum opportunity detection can be considered as a binary hypothesis test. We adopt here the disk signal propagation and interference model as illustrated in Fig. 1. The basic concepts presented here, however, apply to a general model.

Let $\mathbb{I}(A, d, \text{rx})$ denote the logical condition that there exist primary receivers within distance $d$ to the secondary user $A$. Let $\overline{\mathbb{I}(A, d, \text{rx})}$ denote the complement of $\mathbb{I}(A, d, \text{rx})$. The two hypotheses for opportunity detection are then given by

$$
\mathcal{H}_0 : \text{opportunity exists, i.e., } \mathbb{I}(A, r_I, \text{rx}) \cap \mathbb{I}(B, R_I, \text{tx}),
$$

$$
\mathcal{H}_1 : \text{no opportunity, i.e., } \mathbb{I}(A, r_I, \text{rx}) \cup \mathbb{I}(B, R_I, \text{tx}),
$$

where $\mathbb{I}(B, R_I, \text{tx})$ and $\overline{\mathbb{I}(B, R_I, \text{tx})}$ are similarly defined, and $R_I$ and $r_I$ are, respectively, the interference range of primary and secondary users under the disk model. Notice that $\mathbb{I}(A, r_I, \text{rx})$ corresponds to the logical condition on the transmission from $A$ being “harmless”, and $\overline{\mathbb{I}(B, R_I, \text{tx})}$ the logical condition on the reception at $B$ being successful.
Detection performance is measured by the probabilities of false alarm $P_F$ and miss detection $P_{MD}$: $P_F = \Pr\{\text{decides } \mathcal{H}_1 \mid \mathcal{H}_0\}$, $P_{MD} = \Pr\{\text{decides } \mathcal{H}_0 \mid \mathcal{H}_1\}$. The tradeoff between false alarm and miss detection is captured by the receiver operating characteristic (ROC), which gives $P_D = 1 - P_{MD}$ (probability of detection or detection power) as a function of $P_F$. In general, reducing $P_F$ comes at the price of increasing $P_{MD}$ and vice versa. Since false alarms lead to overlooked spectrum opportunities and miss detections are likely to result in collisions with primary users, the tradeoff between false alarm and miss detection is crucial in the design of CR systems [13].

Without assuming cooperation from primary users, the observations available to the secondary user for opportunity detection are the signals emitted from primary transmitters. This basic approach to opportunity detection is commonly referred to as “listen-before-talk” (LBT). As shown in Fig. 2, $A$ infers the existence of spectrum opportunity from the absence of primary transmitters within its detection range $r_D$, where $r_D$ can be adjusted by changing, for example, the threshold of an energy detector. The probabilities of false alarm $P_F$ and miss detection $P_{MD}$ for LBT are thus given by

$$P_F = \Pr\{\overline{I}(A, r_D, \text{tx}) \mid \mathcal{H}_0\}, \quad P_{MD} = \Pr\{\overline{I}(A, r_D, \text{tx}) \mid \mathcal{H}_1\}. \quad (1)$$

Uncertainties, however, are inherent to such a scheme even if $A$ listens to primary signals with perfect ears (i.e., perfect detection of primary transmitters within its detection range $r_D$). Even in the absence of noise and fading, the geographic distribution and traffic pattern of primary users have significant impact on the performance of LBT. Specifically, there are three possible sources of detection errors: hidden transmitters, hidden receivers and exposed transmitters. A hidden transmitter is a primary transmitter that is located within distance $R_I$ to $B$ but outside the detection range of $A$ (node $X$ in Fig. 2). A hidden receiver is a primary receiver that is located within the interference range $r_I$ of $A$ but its corresponding primary transmitter is outside the detection range of $A$ (node $Y$ in Fig. 2). An exposed transmitter is a primary transmitter that is located within the detection range of $A$ but transmits to a primary receiver outside the interference range of $A$ (node $Z$ in Fig. 2). For the scenarios shown in Fig. 2, even if $A$ can perfectly detect the presence of signals from any primary transmitters located within its detection range $r_D$, the transmission from the exposed transmitter $Z$ is a source of false alarms whereas the transmission from the hidden transmitter $X$ and the reception at the hidden receiver $Y$ are sources of miss detections. As illustrated in Fig. 3, adjusting the detection range $r_D$ leads to
different points on the ROC. It is obvious from (1) that $P_F$ increases but $P_{MD}$ decreases as $r_D$ increases.

From the definition of spectrum opportunity, when $\frac{p_{tx}}{P_{tx}}$ is small (small $\frac{r_I}{R_I}$ in Fig. [1]), the occurrence of spectrum opportunity is mainly determined by the logical condition on the reception at $B$ being successful. In this case, the error in detecting opportunity is mainly caused by hidden transmitters. Since the distance between $A$ and $B$ is relatively small due to the small transmission power, $A$ can accurately infer the presence of primary transmitters in the neighborhood of $B$, leading to reliable opportunity detection. On the other hand, when $\frac{p_{tx}}{P_{tx}}$ is large (large $\frac{r_I}{R_I}$ in Fig. [1]), the occurrence of spectrum opportunity is mainly determined by the logical condition on the transmission from $A$ being “harmless”. Due to the relatively small transmission power of the primary users, primary receivers are close to their corresponding transmitters. Node $A$ can thus accurately infer the presence of primary receivers from the presence of primary transmitters and achieve reliable opportunity detection. To summarize, in the two extreme regimes in terms of the ratio between the transmission power of secondary users and that of primary users, the two logical conditions for spectrum opportunity reduce to one. As a consequence, reducing $P_F$ does not necessarily increase $P_{MD}$, and perfect spectrum opportunity detection is achieved. A detailed proof of this statement is given in Sec. III-C. Note that we focus on detection errors caused by the inherent uncertainties associated with detecting spectrum opportunities by detecting primary transmitters. Such uncertainties vary with the transmission power of the primary and secondary users. We ignore noise and fading that may cause errors in detecting primary transmitters, since they are not pertinent to the issue of power control for secondary users.

III. IMPACT OF TRANSMISSION POWER: QUANTITATIVE CHARACTERIZATION

In this section, we quantitatively characterize the impact of the transmission power $p_{tx}$ of secondary users by deriving closed-form expressions for the probability of opportunity and the performance of opportunity detection as functions of $p_{tx}$ in a Poisson primary network. The exponential decay of the probability of opportunity with respect to $p_{tx}^{2/\alpha}$ and the asymptotic behavior of ROC are established based on these closed-form expressions.

A. A Poisson Random Network Model

Consider a decentralized primary network, where potential primary transmitters are distributed according to a two-dimensional homogeneous Poisson process with density $\lambda$. At the beginning of each slot, each potential primary transmitter has a probability $p$ to transmit, and receivers are
in turn located uniformly within the transmission range $R_p$ of each transmitter. In the following analysis, we will frequently use the following two classic results on Poisson processes.

**Fact 1: Independent Coloring (Thinning) Theorem** [14, Chapter 5]
Let $\Pi$ be a potentially inhomogeneous Poisson process on $\mathbb{R}^d$ with density function $\lambda(x)$, where $x = (x_1, x_2, ..., x_d) \in \mathbb{R}^d$. Suppose that we obtain $\Pi'$ by independently coloring points $x \in \Pi$ according to probabilities $p(x)$. Then $\Pi'$ and $\Pi - \Pi'$ are two independent Poisson processes with density functions $p(x)\lambda(x)$ and $(1 - p(x))\lambda(x)$, respectively.

**Fact 2: Displacement Theorem** [14, Chapter 5]
Let $\Pi$ be a Poisson process on $\mathbb{R}^d$ with density function $\lambda(x)$. Suppose that the points of $\Pi$ are displaced randomly and independently. Let $\rho(x, y)$ denote the probability density of the displaced position $y$ of a point $x$ in $\Pi$. Then the displaced points form a Poisson process $\Pi'$ with density function $\lambda'$ given by $\lambda'(y) = \int_{\mathbb{R}^d} \lambda(x)\rho(x, y) \, dx$. In particular, if $\lambda(x)$ is a constant $\lambda$ and $\rho(x, y)$ is a function of $y - x$, then $\lambda'(y) = \lambda$ for all $y \in \mathbb{R}^d$.

Note that in the independent coloring theorem, the original Poisson process $\Pi$ does not have to be homogeneous and the coloring probability $p(x)$ can depend on the location $x$. This theorem is more general than the commonly known thinning theorem for homogeneous Poisson processes. In our subsequent analysis, we rely on this general version of the thinning theorem to handle location-dependent coloring/thinning.

Based on Fact 1 and Fact 2, we arrive at the following property.

**Property 1: Distributions of Primary Transmitters and Primary Receivers**
Both primary transmitters and receivers form a homogeneous Poisson process with density $p\lambda$. Note that although the two Poisson processes have the same density, they are not independent.

**B. Impact of Transmission Power on Probability of Opportunity**
Let $d$ be the distance between $A$ and $B$. Let $S_I(d, r_1, r_2)$ denote the common area of two circles centered at $A$ and $B$ with radii $r_1$ and $r_2$, respectively, and $S_c(d, r_1, r_2)$ denote the area within a circle with radius $r_1$ centered at $A$ but outside the circle with radius $r_2$ centered at $B$ (see Fig. 4). Then we have the following proposition.

**Proposition 1: Closed-form Expression for Probability of Opportunity**
Under the disk signal propagation and interference model characterized by $\{r_I, R_I\}$, the probability of opportunity for a pair of secondary users $A$ and $B$ in a Poisson primary network with
density \( \lambda \) and traffic load \( p \) is given by

\[
\Pr[H_0] = \exp \left[ -p\lambda \left( \iint_{S_c(d,r_I+R_p,R_I)} \frac{S_i(r,R_p,r_I)}{\pi R_p^2} r dr d\theta + \pi R_I^2 \right) \right],
\]

where the secondary transmitter \( A \) is chosen as the origin of the polar coordinate system for the double integral, and \( d \) the distance between \( A \) and \( B \).

**Proof:** Based on the definition of spectrum opportunity, we have

\[
\Pr[H_0] = \Pr\{\bar{\Pi}(A,r_I,rx) \cap \bar{\Pi}(B,R_I,tx)\},
\]

\[
= \Pr\{\bar{\Pi}(A,r_I,rx) | \bar{\Pi}(B,R_I,tx)\} \Pr\{\bar{\Pi}(B,R_I,tx)\}.
\]

Based on Property 1, the second term of (3) is given by

\[
\Pr\{\bar{\Pi}(B,R_I,tx)\} = \exp(-p\lambda R_I^2).
\]

Next we obtain the first term \( \Pr\{\bar{\Pi}(A,r_I,rx) | \bar{\Pi}(B,R_I,tx)\} \) of (3) based on Fact 1 with location-dependent coloring.

Let \( \Pi_{tx} \) denote the Poisson process formed by primary transmitters. If we color those primary transmitters in \( \Pi_{tx} \) whose receivers are within distance \( r_I \) to \( A \), then from Fact 1 we obtain another Poisson process \( \Pi_{tx}' \) formed by all the colored primary transmitters with density \( p\lambda \frac{S_i(r,R_p,r_I)}{\pi R_p^2} \), where \( \frac{S_i(r,R_p,r_I)}{\pi R_p^2} \) is the coloring probability for a primary transmitter at distance \( r \) to \( A \).

Given \( \bar{\Pi}(B,R_I,tx) \), i.e., there are no primary transmitters within distance \( R_I \) to \( B \), those primary receivers within distance \( r_I \) to \( A \) can only communicate with those primary transmitters inside \( S_c(d,r_I+R_p,R_I) \). Thus, the event \( \bar{\Pi}(A,r_I,rx) \) (conditioned on \( \bar{\Pi}(B,R_I,tx) \)) occurs if and only if \( \Pi_{tx} \) does not have any points inside \( S_c(d,r_I+R_p,R_I) \), i.e.,

\[
\Pr\{\bar{\Pi}(A,r_I,rx) | \bar{\Pi}(B,R_I,tx)\} = \exp \left( - \iint_{S_c(d,r_I+R_p,R_I)} p\lambda \frac{S_i(r,R_p,r_I)}{\pi R_p^2} r dr d\theta \right).
\]

Then by substituting (4, 5) into (3), we arrive at (2).

While the closed-form expression for \( \Pr[H_0] \) given in (2) appears to be complex with a double integral, it has a simple structure that allows us to establish the monotonicity and the exponential decay rate of \( \Pr[H_0] \) with respect to \( r_I^2 \) as given in Theorem 1. Furthermore, as shown in Appendix A, by integrating with respect to \( \theta \) first, we can reduce the double integral in (2) to a single integral

\[
\int_0^{r_I+R_p} \frac{S_i(r,R_p,r_I)}{\pi R_p^2} r \theta(r) dr,
\]

where \( \theta(r) \) is a function of the radial coordinate \( r \) and is determined by the shape of \( S_c(d,r_I+R_p,R_I) \). The basic idea is that the integrand in (2) is not a function of the angular coordinate \( \theta \) and the range of \( \theta \) as a function of the radial coordinate.
coordinate \( r \) can be obtained in an explicit form. In the integrand of the obtained single integral, \( S_I(r, R_p, r_I) \) that depends on \( r \) is also in an explicit form as obtained in [15] and provided in Appendix A. As a consequence, the resulting single integral is easy to compute.

From (2), we obtain the following theorem that characterizes the impact of transmission power \( p_{tx} \) on the probability of opportunity.

**Theorem 1: Impact of Transmission Power on Opportunity Occurrence**

T1.1. \( \Pr[\mathcal{H}_0] \) is a strictly decreasing function of \( p_{tx} \propto r_I^2 \).

T1.2. \( \Pr[\mathcal{H}_0] \) decreases exponentially \(^2\) with \( p_{tx}^{2/\alpha} \propto r_I^2 \), where the decay constant is proportional to \( p\lambda \), i.e., \( \exp(-p\lambda \pi r_I^2) < \frac{\Pr[\mathcal{H}_0]}{\exp(-p\lambda \pi r_I^2)} \leq 1 \), with equality when \( r_I \geq d + R_I + R_p \).

T1.3. \( \Pr[\mathcal{H}_0] \) decreases exponentially with \( p\lambda \), where the decay constant is \( \pi r_I^2 \propto p_{tx}^{2/\alpha} \).

**Proof:** Theorem 1 is obtained by examining the closed-form expression for \( \Pr[\mathcal{H}_0] \) given in (2). Details are given in Appendix B.

From T1.2, we can see that when the transmission power of secondary users is high \((r_I \geq d + R_I + R_p)\), the probability of opportunity \( \Pr[\mathcal{H}_0] \) has a simple expression: \( \Pr[\mathcal{H}_0] = \exp(-p\lambda \pi r_I^2) \).

When \( r_I \geq d + R_I + R_p \), the absence of primary receivers within distance \( r_I \) to \( A \) automatically implies the absence of primary transmitters within distance \( R_I \) to \( B \). Thus, the opportunity occurs if and only if there is no primary receiver within distance \( r_I \) to \( A \), which leads to the simple expression for \( \Pr[\mathcal{H}_0] \). Moreover, from T1.2 we can see that the traffic load \( p\lambda \) of primary users determines the exponential decay rate of \( \Pr[\mathcal{H}_0] \) with respect to \( p_{tx}^{2/\alpha} \). Similarly, T1.3 shows that the area \( \pi r_I^2 \) “consumed” by the secondary transmitter, a concept introduced in [4], is the decay constant of \( \Pr[\mathcal{H}_0] \) with respect to \( p\lambda \).

A numerical example is given in Fig. 5(a), where \( \Pr[\mathcal{H}_0] \) and its lower and upper bounds \( \exp[-p\lambda \pi (r_I^2 + R_I^2)] \) and \( \exp(-p\lambda \pi r_I^2) \), respectively) are plotted as a function of \( r_I \). The exponential decay rate of \( \Pr[\mathcal{H}_0] \) can be easily observed by noticing the log scale. Fig. 5(b) demonstrates \( \Pr[\mathcal{H}_0] \) as a function of \( p\lambda \) for different \( r_I \). It shows that the exponential decay constant of \( \Pr[\mathcal{H}_0] \) with respect to \( p\lambda \) increases as \( r_I \) increases.

\(^2\)A quantity \( N \) is said to decrease exponentially with respect to \( t \) if its decay rate is proportional to its value. Symbolically, this can be expressed as the following differential equation: \( \frac{dN}{dt} = -\lambda N \), where \( \lambda > 0 \) is called the decay constant.
C. Impact of Transmission Power on Detection Performance

In the following, we focus on the performance of the spectrum opportunity detector for one pair of secondary users $A$ and $B$, where there are no other secondary users in the network.

For LBT, false alarms occur if and only if there exist primary transmitters within the detection range $r_D$ of $A$ under $\mathcal{H}_0$, and miss detections occur if and only if there is no primary transmitter within the detection range $r_D$ of $A$ under $\mathcal{H}_1$. We thus have the following proposition.

**Proposition 2:** Closed-form Expressions for $P_F$ and $P_{MD}$

Under the disk model characterized by $\{r_I, R_I\}$, let $r_D$ be the detection range. The probabilities of false alarm $P_F$ and miss detection $P_{MD}$ for a pair of secondary users $A$ and $B$ in a Poisson primary network with density $\lambda$ and traffic load $p$ are given by

$$P_F = 1 - \exp \left[ -p\lambda \left( \pi r_D^2 - S_I(d, r_D, R_I) - \int_{S_o} \frac{S_I(r, R_p, r_I)}{\pi R_p^2} rdrd\theta \right) \right],$$

(6)

$$P_{MD} = \frac{\exp(-p\lambda\pi r_D^2) - \exp \left[ -p\lambda \left( \pi (r_D^2 + R_p^2) - S_I(d, r_D, R_I) + \int_{S_o(d, r_I + R_p, R_I) - S_o} \frac{S_I(r, R_p, r_I)}{\pi R_p^2} rdrd\theta \right) \right]}{1 - \exp \left[ -p\lambda \left( \int_{S_o(d, r_I + R_p, R_I)} \frac{S_I(r, R_p, r_I)}{\pi R_p^2} rdrd\theta + \pi R_p^2 \right) \right]},$$

(7)

where the secondary transmitter $A$ is chosen as the origin of the polar coordinate system for the double integral, $d$ the distance between $A$ and $B$, and $S_o = S_c(d, r_D, R_I) \cap S_c(d, r_I + R_p, R_I)$.

**Proof:** Similar to Proposition 1, the proof uses Fact 1 with location-dependent coloring. For details, see Appendix C.

Similarly to (2), the double integral in (6, 7) can be simplified to a single integral due to the independence of the integrand with respect to the angular coordinate $\theta$ and the special shape of $S_o$. Due to the page limit, the details are left in [16].

From Proposition 2, we can show the following theorem that characterizes the impact of the transmission power of secondary users (represented by $r_I$) on the asymptotic behavior of the ROC for spectrum opportunity detection.

**Theorem 2:** Impact of Transmission Power on Detection Performance

There exist two points on ROC that asymptotically approach $(0, 1)$ as $\frac{R_p}{R_I} \rightarrow 0$ and $\infty$, respectively. Specifically,

\[3\] Since the minimum transmission power for successful reception is in general higher than the maximum allowable interference power, it follows that the transmission range $R_p$ of primary users is smaller than $R_I$. Furthermore, under the disk signal propagation and interference model, we have $R_p = \beta R_I \, (0 < \beta < 1)$. A similar relationship holds for $d$ and $r_I$. \\

\[
\lim_{r_I \to 0} (P_F(r_D = R_I), P_D(r_D = R_I)) = (0, 1),
\]
\[
\lim_{r_I \to \infty} (P_F(r_D = r_I - R_I), P_D(r_D = r_I - R_I)) = (0, 1).
\]

Proof: The intuitive reasons for choosing \( r_D = R_I \) and \( r_D = r_I - R_I \) in the two extreme regimes are discussed in Sec. II-B. For details of the proof, see Appendix D.

Since \((0, 1)\) is the perfect operating point on a ROC, we can asymptotically approach perfect detection performance by choosing \( r_D = R_I \) when \( \frac{r_I}{R_I} \to 0 \) or \( r_D = r_I - R_I \) when \( \frac{r_I}{R_I} \to \infty \). A numerical example is shown in Fig. 6 where ROC approaches the corner \((0, 1)\) as \( \frac{r_I}{R_I} \) increases or decreases.

IV. POWER CONTROL FOR OPTIMAL TRANSPORT THROUGHPUT

In this section, the impact of the transmission power on the occurrence of opportunities and the impact on opportunity detection are integrated together for optimal power control. Under the performance measure of transport throughput subject to an interference constraint, we examine how a secondary user should choose its transmission power according to the interference constraint, the traffic load and transmission power of primary users, and its own application type (guaranteed delivery vs. best-effort delivery).

A. Transport Throughput

From Sec. II-B and Sec. II-C, it seems that the transmission power \( p_{tx} \) should be chosen as small as possible to maximize the probability of opportunity and improve detection quality. Such a choice of the transmission power, however, does not lead to an efficient communication system due to the small distance covered by the transmission. We adopt here transport throughput as the performance measure, which is defined as

\[
C(r_I, r_D) = d(r_I) \cdot P_S(r_D, d(r_I)),
\]

where \( d(\infty r_I) \) is the transmission range of the secondary user, and \( P_S \) is the probability of successful data transmission which depends on both the occurrence of opportunities and the reliability of opportunity detection. Then power control for optimal transport throughput can be formulated as a constrained optimization problem:

\[
r_I^* = \arg \max_{r_I} \{ C \} = \arg \max_{r_I} \{ d(r_I) \cdot P_S(r_D, d(r_I)) \} \text{ s.t. } P_C(r_D, r_I) \leq \zeta,
\]

where \( \zeta \) is maximum allowable collision probability, and \( P_C \) the probability of colliding with primary users which depends on the reliability of opportunity detection. Note that the detection range \( r_D \) is not an independent parameter; it is determined by maximizing \( P_S(r_D, d(r_I)) \) subject to \( P_C(r_D, r_I) \leq \zeta \) for every given interference range \( r_I \).
In order to solve the above constrained optimization problem, we need expressions for \( P_C \) and \( P_S \) which collectively measure the MAC layer performance.

\[ \text{B. MAC Performance of LBT} \]

We first consider \( P_S \), which is application dependent. For applications requiring guaranteed delivery, an acknowledgement (ACK) signal from the secondary receiver \( B \) to the secondary transmitter \( A \) is required to complete a data transmission. Specifically, in a successful data transmission, the following three events should occur in sequence: \( A \) detects the opportunity \( \mathbb{I}(A, r_D, \text{tx}) \) and transmits data to \( B \); \( B \) receives data successfully \( \mathbb{I}(B, R_I, \text{tx}) \) and replies to \( A \) with an ACK; \( A \) receives the ACK \( \mathbb{I}(A, R_I, \text{tx}) \) which completes the transmission. We thus have

\[
P_S = \Pr\{\mathbb{I}(A, r_D, \text{tx}) \cap \mathbb{I}(B, R_I, \text{tx}) \cap \mathbb{I}(A, R_I, \text{tx})\},
\]

where \( r_E = \max\{r_D, R_I\} \).

For best-effort delivery applications \[17\], acknowledgements are not required to confirm the completion of data transmissions. In this case, we have

\[
P_S = \Pr\{\mathbb{I}(A, r_D, \text{tx}) \cap \mathbb{I}(B, R_I, \text{tx})\}. \tag{11}
\]

The probability of collision is defined as \[4\]

\[
P_C = \Pr\{A \text{ transmits data } | \mathbb{I}(A, r_I, \text{rx})\}. \tag{12}
\]

Note that \( P_C \) is conditioned on \( \mathbb{I}(A, r_I, \text{rx}) \) instead of \( H_1 \). Clearly, \( \Pr[\mathbb{I}(A, r_I, \text{rx})] \leq \Pr[H_1] \).

Since the secondary transmitter \( A \) transmits data if and only if \( A \) detects no nearby primary transmitters, we thus have

\[
P_C = \Pr\{\mathbb{I}(A, r_D, \text{tx}) | \mathbb{I}(A, r_I, \text{rx})\}. \tag{13}
\]

By considering the Poisson primary network and the disk model, we obtain the closed-form expressions for \( P_C \) and \( P_S \) given in the following proposition.

\[ \text{Proposition 3: Closed-form Expressions for } P_C \text{ and } P_S \]

Under the disk model characterized by \( \{r_I, R_I\} \), let \( r_D \) be the detection range. The probabilities

\[ \text{In obtaining the definition (12) of } P_C, \text{ we have assumed that the interference caused by the ACK signal is negligible due to its short duration.} \]
of collision $P_C$ and successful transmission $P_S$ for a pair of secondary users $A$ and $B$ in a Poisson primary network with density $\lambda$ and traffic load $p$ are given by

$$P_C = \frac{\exp(-p\lambda \pi r_D^2)[1 - \exp(-p\lambda \pi (r_I^2 - I(r_D, r_I, R_p)))]}{1 - \exp(-p\lambda \pi r_I^2)},$$

(14)

$$P_S = \begin{cases} 
\exp[-p\lambda(\pi(r_D^2 + R_I^2) - S_I(d, r_E, R_I))], & \text{for guaranteed delivery,} \\
\exp[-p\lambda(\pi(r_D^2 + R_I^2) - S_I(d, r_D, R_I))], & \text{for best-effort delivery,}
\end{cases}$$

(15)

where $I(r_D, r_I, R_p) = \int_0^{r_D} 2r \frac{S_I(r, r_I, R_p)}{\pi R_p^2} dr$.

Proof: Similar to Proposition 2, the derivation of $P_C$ uses Fact 1 with location-dependent coloring. For details, see Appendix E. The expressions for $P_S$ follow immediately from (10, 11) and Property 1.

Based on the expression for $S_I(r, r_I, R_p)$, we can obtain $I(r_D, r_I, R_p)$ in an explicit form without integral. Details are left in [16] due to the page limit. Notice that the above expressions for $P_C$ and $P_S$ are in explicit form without integrals. With the explicit expressions for $P_C$ (14) and $P_S$ (15), the constrained optimization given in (9) can be solved numerically.

C. Numerical Examples

Shown in Fig. 7 is a numerical example where we plot transport throughput $C$ as a function of $r_I$. Notice that $r_I^*$ is the interference range for optimal transport throughput. We can see that $r_I^*$ for best-effort applications is different from that for guaranteed delivery, and neither of them is not in the two extreme regimes. We can also see that the optimal transport throughput for best-effort delivery is larger than that for guaranteed delivery. This example thus demonstrates that OSA based on cognitive radio is more suitable for best-effort applications as compared to guaranteed delivery due to the asymmetry of spectrum opportunities.

Fig. 8 shows how the optimal transmission power of the secondary user changes with the traffic load and transmission power of the primary users, as well as the application type of the secondary user. Specifically, the optimal interference range $r_I^*$ decreases as the traffic load increases. This agrees with our intuition from the analogy of crossing a multi-lane highway. Furthermore, the optimal transmission power of the secondary user is related to that of the primary user. We can see from Fig 8 that when the traffic load is low, $r_I^*$ is close to the interference range $R_I$ of primary transmitters. When the traffic load is high, $r_I^*$ is much smaller than $R_I$.

When the traffic load is low, $r_I^*$ for both application types are the same. This is because for both application types, the optimal detection range $r_D$ corresponding to each $r_I$ in the neighborhood
of \( r^*_I \) is larger than \( R_I \), which leads to the same probability of successful transmission \( P_S \) (see (15)) and thus the same maximum point \( r^*_I \) for transport throughput \( C \). When the traffic load is high, \( r^*_I \) for guaranteed delivery is smaller than that for best-effort delivery, which is consistent with the case shown in Fig. 7. This is because for guaranteed delivery, the optimal detection range \( r_D \) corresponding to each \( r_I \) in the neighborhood of \( r^*_I \) is smaller than \( R_I \), which leads to a smaller \( P_S \) than that for best-effort delivery (see (15)) and thus a smaller maximum point \( r^*_I \).

V. RTS/CTS-ENHANCED LBT

The sources of the detection error floor of LBT in the absence of noise and fading resemble the hidden and exposed terminal problem in the conventional ad hoc networks of peer users. It is thus natural to consider the use of RTS/CTS handshake signaling to enhance the detection performance of LBT. For RTS/CTS-enhanced LBT (ELBT), spectrum opportunity detection is performed jointly by the secondary transmitter \( A \) and the secondary receiver \( B \) through the exchange of RTS/CTS signals. The detailed steps are given below.

- \( A \) detects primary transmitters within distance \( r_D \). If it detects none, \( A \) sends \( B \) a Ready-to-Send (RTS) signal.
- If \( B \) receives the RTS signal from \( A \) successfully, then \( B \) replies with a Clear-to-Send (CTS) signal.
- Upon receiving the CTS signal, \( A \) transmits data to \( B \).

Since for ELBT, the observation space comprises the RTS and CTS signals, we have the following for the probabilities of false alarm \( P_F \) and miss detection \( P_{MD} \).

\[
P_F = \Pr\{\text{failed RTS/CTS exchange} \mid H_0\},
\]

\[
= \Pr\{\mathbb{I}(A, r_D, \text{tx}) \cup \mathbb{I}(B, R_I, \text{tx}) \cup \mathbb{I}(A, R_I, \text{tx}) \mid H_0\},
\]

\[
= \Pr\{\mathbb{I}(A, r_E, \text{tx}) \mid H_0\},
\]

where the last step follows from \( \Pr\{\mathbb{I}(B, R_I, \text{tx}) \cap H_0\} = 0 \).

\[
P_{MD} = \Pr\{\text{successful RTS/CTS exchange} \mid H_1\},
\]

\[
= \Pr\{\mathbb{I}(A, r_D, \text{tx}) \cap \mathbb{I}(B, R_I, \text{tx}) \cap \mathbb{I}(A, R_I, \text{tx}) \mid H_1\},
\]

\[
= \Pr\{\mathbb{I}(A, r_E, \text{tx}) \cap \mathbb{I}(B, R_I, \text{tx}) \mid H_1\}. \tag{17}
\]

Since \( A \) transmits data if and only if an successful RTS/CTS exchange occurs, it follows that

\[
P_C = \Pr\{\mathbb{I}(A, r_E, \text{tx}) \cap \mathbb{I}(B, R_I, \text{tx}) \mid \mathbb{I}(A, r_I, \text{rx})\}. \tag{18}
\]

5 In obtaining the definition (18) of \( P_C \), we have assumed that the interference caused by the RTS, CTS and ACK signals is negligible due to their short durations.
Unlike LBT, miss detections always lead to successful data transmissions for ELBT. This is because miss detections can only occur after a successful RTS-CTS exchange. Then \( B \) can receive data successfully as it can receive RTS. We thus have

\[
P_S = (1 - P_F) \cdot \Pr[H_0] + P_{MD} \cdot \Pr[H_1],
\]

\[
P_S = \Pr\{A(r_E, \text{tx}) \cap B(R_i, \text{tx})\}. 
\]  (19)

Notice that \( P_S \) of ELBT is identical to that of LBT for guaranteed delivery in (10). Due to the requirement on the successful reception of CTS in opportunity detection, \( P_S \) for ELBT is independent of the application, \( i.e., \) whether or not ACK is required.

Based on (16-19), we obtain the following proposition for the Poisson primary network model.

**Proposition 4:** Closed-form Expressions for \( P_F, P_{MD}, P_C, \) and \( P_S \)

Under the disk model characterized by \( \{r_I, R_I\} \), let \( r_D \) be the detection range. The probabilities of false alarm \( P_F \), miss detection \( P_{MD} \), collision \( P_C \) and successful transmission \( P_S \) for a pair of secondary users \( A \) and \( B \) in a Poisson primary network with density \( \lambda \) and traffic load \( p \) are given by

\[
P_F = 1 - \exp \left[ p\lambda \left( \pi r_D^2 - S_{A}(d, r_E, R_I) \right) - \int_{S_o} \frac{S_I(r, R_p, R_I)}{\pi R_p^2} r dr d\theta \right],
\]  (20)

where \( S_o = S_c(d, r_E, R_I) \cap S_c(d, r_I + R_p, R_I) \).

\[
P_{MD} = \frac{\exp(-p\lambda(\pi r_D^2 + R_I^2) - S_I(d, r_E, R_I)) \left[ 1 - \exp \left( -p\lambda \int_{S_o} \frac{S_I(r, R_p, R_I)}{\pi R_p^2} r dr d\theta \right) \right]}{1 - \exp \left( -p\lambda \int_{S_o} \frac{S_I(r, R_p, R_I)}{\pi R_p^2} r dr d\theta + \pi R_I^2 \right)}.
\]  (21)

\[
P_C = \frac{\exp(-p\lambda(\pi r_D^2 + R_I^2) - S_I(d, r_E, R_I)) \left[ 1 - \exp \left( -p\lambda \int_{S_o} \frac{S_I(r, R_p, R_I)}{\pi R_p^2} r dr d\theta \right) \right]}{1 - \exp(p\lambda r_D^2)}.
\]  (22)

\[
P_S = \exp(-p\lambda(\pi r_D^2 + R_I^2) - S_I(d, r_E, R_I)).
\]  (23)

Furthermore, Theorem 2 still holds for ELBT, \( i.e., \) perfect detection performance can be achieved at \( r_D = R_I \) when \( \frac{r_D}{R_I} \rightarrow 0 \) and at \( r_D = r_I - R_I \) when \( \frac{r_D}{R_I} \rightarrow \infty \).

**Proof:** The derivations of the above expressions and the proof of Theorem 2 are very similar to those for LBT, and they can be found in [16].

Similarly, based on (22, 23), we can obtain numerical solutions to the constrained optimization problem (9) for ELBT. Fig. 9 shows the maximal transport throughput as a function of the traffic load \( p \) obtained by optimizing \( r_I \). We observe from Fig. 9 that RTS/CTS handshake signaling
improves the performance of LBT when the traffic load is low, but it degrades the performance of LBT with best-effort delivery when the traffic load is high. This suggests that even when the overhead associated with RTS/CTS is ignored, RTS/CTS may lead to performance degradation due to the asymmetry of spectrum opportunities. When the traffic load is high, LBT with best-effort delivery gives the best transport throughput. This is consistent with our previous observation obtained from Fig. 7 that best-effort is a more suitable application to be considered for overlaying with a primary network with relatively high traffic load.

It can also be observed from Fig. 9 that the maximal transport throughput of the “genie aided” system (in which all opportunities are correctly detected) provides an upper bound for LBT and ELBT, which matches our intuition very well. Since LBT allows the secondary user to access the channel within the interference constraint even when it is not an opportunity, compared with the “genie aided” system, the increase of transport throughput of LBT brought by accessing the busy channel can partially compensate the loss of transport throughput due to missed detection of opportunities. When the traffic load of the primary network is high, spectrum opportunities occur infrequently, and the compensated transport throughput of LBT is comparable to the transport throughput of the “genie aided” system. Thus, we see in Fig. 9 that the maximal transport throughput of LBT for best-effort delivery approaches that of the “genie aided” system in this case.

VI. CONCLUSION AND DISCUSSION

We have studied transmission power control of secondary users in CR networks. By carefully examining the concepts of spectrum opportunity and interference constraint, we have revealed and analytically characterized the impact of transmission power on the occurrence of spectrum opportunities and the reliability of opportunity detection. Based on a Poisson model of the primary network, we have quantified these impacts by showing the exponential decay rate of the probability of opportunity with respect to the transmission power and the asymptotic behavior of the ROC for opportunity detection. In the analysis, the independent coloring theorem and displacement theorem have played a significant role, especially the former one. Such analytical characterizations allow us to design the transmission power for optimal transport throughput under constraints on the interference to primary users.

Furthermore, the non-equivalence between detecting primary signals and detecting spectrum opportunities has been illuminated. It has been specified that besides noise and fading, the
geographical distribution and traffic pattern of primary users have significant impact on the performance of physical layer spectrum sensing. The complex dependency of the relationship between PHY and MAC on the application types and the use of MAC handshake signaling such as RTS/CTS is also illustrated.

In the above analysis, the interference region of primary users is represented by a circle with radius $R_I$. It is possible that the interference contributions from multiple interferers outside of this circle cause an outage at the secondary receiver $B$, but by choosing a conservative interference range $R_I$, this possibility is negligible \[18\]. To verify the validity of the interference range $R_I$, we take into account the accumulated interference power from all primary transmitters in the simulation, which directly determines the Signal-to-interference ratio (SIR) at the secondary receiver $B$. In this case, a channel is an opportunity for one pair of secondary users if there is no primary receiver within the interference range $r_I$ of the secondary transmitter $A$ and the total power of the interference $I_B$ from all primary transmitters to the secondary receiver $B$ is below some prescribed level $\tau_B$, i.e., $\{A, r_I, rx\} \cap (I_B < \tau_B)$. To detect spectrum opportunity, the secondary transmitter $A$ uses an energy detector, which is given by

$$I_A \overset{\mathcal{H}_0}{\preceq} \tau_A,$$

where $I_A$ is the total received power at the secondary transmitter $A$ and $\tau_A$ is the threshold of the energy detector. Let $\Pi_{tx}$ denote the Poisson point process of primary transmitters, $r_i^A$ and $r_i^B$ the distance from the $i^{th}$ primary transmitter to $A$ and $B$ respectively, and $\alpha$ the path-loss exponent, then we have that

$$I_A = \sum_{i \in \Pi_{tx}} P_{tx} \cdot (r_i^A)^{-\alpha}, \quad I_B = \sum_{i \in \Pi_{tx}} P_{tx} \cdot (r_i^B)^{-\alpha}.$$

Fig. 10 shows the simulated ROC of the energy detector and the analytical ROC of LBT. We can see that reliable opportunity detection can be achieved when $\frac{Ptx}{P_{rx}} \to 0$ and $\frac{Ptx}{P_{tx}} \to \infty$. In other words, the asymptotic property of ROC (Theorem 2) still holds in this case.

As an initial analysis, we focus on one pair of secondary users in this paper. In the scenario of multiple secondary users, our definition of spectrum opportunity can still be applied, although determining the interference range $r_I$ of secondary users needs careful consideration due to the accumulation of the interference powers from multiple secondary transmitters. Moreover, since the reception at one secondary user can be affected by other secondary transmissions,
the detection of opportunities should probably be performed cooperatively to avoid the possibly violent contention of accessing the idle channel among secondary users. Our results can easily be extended to incorporate the trade-offs between sampling vs. opportunity to transmit by incorporation in a spatial Poisson model. To specify the effect of multiple secondary users on power control is an interesting future direction, and some preliminary results about the impact of transmission power of secondary users on the connectivity of the secondary network can be found in [19].

APPENDIX A: SIMPLIFICATION OF DOUBLE INTEGRAL FOR $P_T[H_0]$

By considering the shape of $S_c(d, r_1 + R_p, R_I)$ (see [13]), we can use the independence of the integrand on the angular coordinate $\theta$ to reduce the double integral to a single integral with respect to the radial coordinate $r$. Here we choose the secondary transmitter $A$ as the origin of the polar coordinate system and the line from the secondary transmitter $A$ to the secondary receiver $B$ as the polar axis. Due to the symmetry of $S_c(d, r_1 + R_p, R_I)$ with respect to the polar axis, there is a coefficient 2 before each integral below. Let $Q = \int_{S_c(d)} \frac{S_t(r, R_p, r_I)}{\pi R_p^2} r dr d\theta$,

$I(r, R_p, r_I) = 2\pi \int_0^r d\theta_0(r) = \arccos \left( \frac{d^2 + r^2 - R_p^2}{2dr} \right)$, then we have two cases:

- **Case 1**: $R_I \geq d$.
  - If $r_1 + R_p \leq R_I - d$, then $Q = 0$.
  - If $R_I - d < r_1 + R_p < R_I + d$, then $Q = \pi r_1^2 - I(R_I - d, R_p, r_I) - 2 \int_{R_I - d}^{r_1 + R_p} S_t(r, R_p, r_I) \pi R_p^2 r \theta_0(r) dr$.
  - If $r_1 + R_p > R_I + d$, then $Q = \pi r_1^2 - I(R_I - d, R_p, r_I) - 2 \int_{R_I - d}^{r_1 + R_p} S_t(r, R_p, r_I) \pi R_p^2 r \theta_0(r) dr$.

- **Case 2**: $R_I < d$.
  - If $r_1 + R_p \leq d - R_I$, then $Q = \pi r_1^2$.
  - If $d - R_I < r_1 + R_p < d + R_I$, then $Q = \pi r_1^2 - 2 \int_{d - R_I}^{r_1 + R_p} S_t(r, R_p, r_I) \pi R_p^2 r \theta_0(r) dr$.
  - If $r_1 + R_p > d + R_I$, then $Q = \pi r_1^2 - 2 \int_{d - R_I}^{d + R_I} S_t(r, R_p, r_I) \pi R_p^2 r \theta_0(r) dr$.

The expression for $I(r, R_p, r_I)$ is obtained in an explicit form as listed below.

- **Case 1**: for $r \leq |R_p - r_I|$, $I(r, R_p, r_I) = \pi r^2 \min \{1, \frac{r_1^2}{R_p^2}\}$.
- **Case 2**: for $|R_p - r_I| < r < R_p + r_I$, then $I(r, R_p, r_I) = \frac{1}{2} \pi r_1^2 + \pi^2 \left( \frac{R_p^2 + r^2 - r_I^2}{2r_1 R_p} \right) + \pi^2 \left( \frac{r_1^2 + r^2 - R_p^2}{2r_1 R_p} \right) - \frac{r_1^2 \arcsin \left( \frac{r_1^2 + R_p^2 - r^2}{2r_1 R_p} \right) - r^2 \arcsin \left( \frac{R_p^2 + r^2 - r_I^2}{4r_1 R_p} \right) - \frac{r_1^2 + R_p^2 - r^2}{2r_1 R_p}}{\sqrt{(r_1 + R_p + r)(r_1 + R_p - r)(r_1 - R_p - r)(R_p - r_1 + r)}}$.
- **Case 3**: for $r \geq R_p + r_I$, then $I(r, R_p, r_I) = \pi r_1^2$.

To compute the remaining integral $\int \frac{S_t(r, R_p, r_I)}{\pi R_p^2} \theta_0(r) dr$ numerically, we need an explicit-form expression for $S_t(r, R_p, r_I)$. Let $r_1 = \min \{R_p, r_I\}$ and $r_2 = \max \{R_p, r_I\}$, then the expression for $S_t(r, R_p, r_I)$ is given by

- **Case 1**: for $0 \leq r \leq r_2 - r_1$, $S_t(r, R_p, r_I) = \pi r_1^2$.
- **Case 2** [13]: for $r_2 - r_1 < r < r_1 + r_2$,

$$S_t(r, R_p, r_I) = r_2^2 \cos^{-1} \left( \frac{r_2^2 + r^2 - r_1^2}{2r_2 r} \right) + r_1^2 \cos^{-1} \left( \frac{r_1^2 + r^2 - r_2^2}{2r_1 r} \right) - r \sqrt{r_2^2 - \left( \frac{r_1^2 + r^2 - r_2^2}{2r_2} \right)^2}$$.

- **Case 3** for $r \geq r_1 + r_2$, $S_t(r, R_p, r_I) = 0$. 


APPENDIX B: PROOF OF THEOREM 1

Since the integrand \( \frac{S_l(r, R_p, R_I)}{\pi R_p^2} \) and the region of the double integral \( S_e(d, r_I + R_p, R_I) \) are both increasing functions of \( r_I \) \([16]\), T1.1 follows from the monotonicity of the exponential function. Obviously, T1.3 also follows from the monotonicity of the exponential function.

We now prove T1.2. Recall the definition of spectrum opportunity,

\[
Pr[H_0] = Pr\{I(A, r_I, rx) \cap I(B, R_I, tx) \} = Pr\{I(A, r_I, rx) | I(B, R_I, tx)\} \cdot Pr\{I(B, R_I, tx)\}. \tag{B1}
\]

Based on Property 1, we have that for all \( r_I > 0 \),

\[
Pr\{I(B, R_I, tx)\} = \exp(-p\lambda\pi R_I^2), \tag{B2}
\]

\[
Pr\{I(A, r_I, rx) | I(B, R_I, tx)\} > Pr\{I(A, r_I, rx)\} = \exp(-p\lambda\pi r_I^2). \tag{B3}
\]

In the last inequality, we have used the fact that the logical condition \( I(B, R_I, tx) \) reduces the number of primary transmitters that can communicate with primary users within distance \( r_I \) to \( A \), which results in a more probable occurrence of \( I(A, r_I, rx) \). Then by substituting (B2, B3) into (B1), we have \( Pr[H_0] > \exp[-p\lambda\pi(r_I^2 + R_I^2)] \) for all \( r_I \).

Obviously, \( Pr[H_0] \leq \exp(-p\lambda\pi r_I^2) \). Moreover, when \( r_I \geq d + R_I + R_p \), we have (see \([16]\))

\[
\int_{S_e(d, r_I + R_p, R_I)} \frac{S_l(r, R_p, R_I)}{\pi R_p^2} r drd\theta = \pi(r_I^2 - R_I^2).
\]

So \( Pr[H_0] = \exp(-p\lambda\pi r_I^2) \) when \( r_I \geq d + R_I + R_p \).

APPENDIX C: PROOF OF PROPOSITION 2

From (1), we have

\[
P_F = Pr\{I(A, r_D, tx) | H_0\} = 1 - Pr\{I(A, r_D, tx) | H_0\}, \tag{C1}
\]

\[
P_{MD} = Pr\{I(A, r_D, tx) | H_1\} = Pr\{I(A, r_D, tx) | I(A, r_I, rx) \cup I(B, R_I, tx)\}. \tag{C2}
\]

Based on the definition of spectrum opportunity, we have

\[
P_F = 1 - \frac{Pr\{I(A, r_D, tx) \cap I(A, r_I, rx) \cap I(B, R_I, tx)\}}{Pr[H_0]}, \tag{C3}
\]

\[
P_{MD} = \frac{Pr\{I(A, r_D, tx)\} - Pr\{I(A, r_D, tx) \cap I(A, r_I, rx) \cap I(B, R_I, tx)\}}{1 - Pr[H_0]}. \tag{C4}
\]

Since \( Pr[H_0] \) is known, we only need to calculate the two probabilities in the numerators.

Based on Property 1, we have

\[
Pr\{I(A, r_D, tx)\} = \exp(-p\lambda\pi r_D^2), \tag{C5}
\]

\[
Pr\{I(A, r_D, tx) \cap I(B, R_I, tx)\} = \exp[-p\lambda(\pi r_D^2 + r_I^2) - S_l(d, r_D, R_I)]]. \tag{C6}
\]

By using techniques similar to those used in obtaining \( Pr\{I(A, r_I, rx) | I(B, R_I, tx)\} \) (see the proof of Proposition 1 in Sec. [III-B], we have
\[ \Pr\{ \prod(A, r_I, rx) \mid \prod(A, r_D, tx) \cap \prod(B, R_I, tx) \} = \exp \left( - \int \int_{S_c(d,r_I+R_p,R_I) - \Delta A} \frac{p\lambda S_I(r, R_p, r_I)}{\pi R_p^2} r dr d\theta \right) . \]  

(C7)

Since \( \Pr\{ \prod(A, r_D, tx) \cap \prod(A, r_I, rx) \cap \prod(B, R_I, tx) \} = \Pr\{ \prod(A, r_I, rx) \mid \prod(A, r_D, tx) \cap \prod(B, R_I, tx) \} \cdot \Pr\{ \prod(A, r_D, tx) \cap \prod(B, R_I, tx) \} \), by substituting (2, C5-C7) into (C3, C4), we obtain (6), (7).

**APPENDIX D: PROOF OF THEOREM 2**

Consider first \( \frac{d}{R_I} \rightarrow 0 \). As discussed in Sec. II-B we choose \( r_D = R_I \) in this case. Recall that \( S_o = S_c(d, r_D, R_I) \cap S_c(d, r_I + R_p, R_I) \) as given in Proposition 2, and then we have

\[ \lim_{n \rightarrow 0} |S_c(d, r_I + R_p, R_I)| = 0, \quad \lim_{n \rightarrow 0} |S_o| = 0, \quad 0 \leq \lim_{n \rightarrow 0} \int \int_{S_o} \frac{S_I(r, R_p, r_I)}{\pi R_p^2} r dr d\theta \leq \lim_{n \rightarrow 0} \int \int_{S_o} \frac{r^2}{R_p^2} r dr d\theta = 0. \]

So by substituting the above limits into (6, 7) and applying the continuity of functions in (6, 7), we conclude that \( P_F(r_D = R_I) \rightarrow 0, \quad P_{MD}(r_D = R_I) \rightarrow 0 \) as \( r_D / R_I \rightarrow 0 \).

Consider next \( \frac{r_I}{R_I} \rightarrow \infty \). As discussed in Sec. II-B we choose \( r_D = r_I - R_I \) in this case. Then we have

\[ \lim_{n \rightarrow \infty} \int \int_{S_c(d, r_I + R_p, R_I)} \frac{S_I(r, R_p, r_I)}{\pi R_p^2} r dr d\theta = \pi (r_I^2 - R_I^2), \quad \lim_{n \rightarrow \infty} \int \int_{S_o} \frac{S_I(r, R_p, r_I)}{\pi R_p^2} r dr d\theta = \pi \left[ (r_I - R_I)^2 - R_I^2 \right]. \]

Similarly, we can show that \( P_F(r_D = r_I - R_I) \rightarrow 0, \quad P_{MD}(r_D = r_I - R_I) \rightarrow 0 \) as \( r_D / R_I \rightarrow \infty \).

**APPENDIX E: DERIVATION OF COLLISION PROBABILITY \( P_C \) IN PROPOSITION 3**

Recall (13) and use the total probability theorem to obtain,

\[ P_C = \frac{\Pr\{ \prod(A, r_D, tx) \cap \prod(A, r_I, rx) \}}{\Pr\{ \prod(A, r_I, rx) \}} = \frac{\Pr\{ \prod(A, r_D, tx) \} - \Pr\{ \prod(A, r_I, rx) \mid \prod(A, r_D, tx) \} \cdot \Pr\{ \prod(A, r_D, tx) \}}{\Pr\{ \prod(A, r_I, rx) \}}. \]

It follows from Property 1 that \( \Pr\{ \prod(A, r_I, rx) \} = 1 - \exp(-p\lambda \pi r_I^2), \quad \Pr\{ \prod(A, r_D, tx) \} = \exp(-p\lambda \pi r_D^2) \). Then by using arguments similar to those used in obtaining \( \Pr\{ \prod(A, r_I, rx) \mid \prod(B, R_I, tx) \} \) (see the proof of Proposition 1 in Sec. III-B), we obtain the expression for \( \Pr\{ \prod(A, r_I, rx) \mid \prod(A, r_D, tx) \} \), and (14) follows immediately. Notice that from (14) we can show that \( P_C \) decreases as \( r_D \) and \( p\lambda \) increases [16]. It follows that for fixed \( r_I, R_D \) decreases as \( p\lambda \) increases in order to satisfy the collision constraint.

**REFERENCES**

[1] J. Mitola, III and G. Maguire, Jr., “Cognitive radio: making software radios more personal,” *IEEE Personal Communications*, vol. 6, pp. 13–18, Aug. 1999.

[2] Q. Zhao and B. M. Sadler, “A Survey of Dynamic Spectrum Access,” *IEEE Signal Processing Magazine*, vol. 24, no. 3, pp. 79-89, May, 2007.

[3] M. Haenggi and D. Puccinelli, “Routing in Ad Hoc Networks: A Case for Long Hops,” *IEEE Communications Magazine*, Oct. 2005.

[4] P. Gupta and P. R. Kumar, “The capacity of wireless networks,” *IEEE Trans. Inform. Theory*, vol. 46, March 2000.
[5] M. Krunz, A. Muqattash, and S. J. Lee, “Transmission Power Control in Wireless Ad Hoc Networks: Challenges, Solution, and Open Issues,” *IEEE Network*, vol. 48, issue 5, Sep.-Oct., 2004.

[6] J. P. Monks, V. Bharghavan, W. M. Huw, “Transmission Power Control for Multiple Access Wireless Packet Networks,” *25th Annual IEEE Conference on Local Computer Networks*, Nov. 2000.

[7] N. A. Pantazis, D. D. Vergados, “A survey on power control issues in wireless sensor networks,” *IEEE Communications Surveys & Tutorials*, vol. 9, issue 4, fourth quarter 2007.

[8] S. Srinivasa and S. A. Jafar, “Soft Sensing and Optimal Power Control for Cognitive Radio,” *IEEE 2007 Global Telecommunications Conference*, Nov. 2007.

[9] Y. Chen, G. Yu, Z. Zhang, H. Chen, and P. Qiu, “On Cognitive Radio Networks with Opportunisitic Power Control Strategies in Fading Channels,” *IEEE Transactions on Wireless Communication*, vol. 7, no. 7, July 2008.

[10] L. Gao, P. Wu, and S. Cui, “Power and Rate Control with Dynamic Programming for Cognitive Radios,” *IEEE 2007 Global Telecommunications Conference*, Nov. 2007.

[11] L. Qian, X. Li, J. Attia, and Z. Gajic, “Power Control for Cognitive Radio Ad Hoc Networks,” *15th IEEE Workshop on Local & Metropolitan Area Networks*, June 2007.

[12] M. H. Islam, Y. Liang, and A. T. Hoang, “Joint Power Control and Beamforming for Cognitive Radio Networks,” *IEEE Transactions on Wireless Communication*, vol. 7, no. 7, July 2008.

[13] Y. Chen, Q. Zhao, and A. Swami, “Joint design and separation principle for opportunistic spectrum access in the presence of sensing errors,” *IEEE Transactions on Information Theory*, vol. 54, no. 5, pp. 2053-2071, May, 2008 (also see Proc. of IEEE Asilomar Conference on Signals, Systems, and Computers, Nov. 2006).

[14] J.F.C. Kingman, “Poisson Processes,” Clarendon Press, Oxford, 1993.

[15] A. Martin, “To find the Area Common to Two Intersecting Circles,” *the Analyst*, vol. 1, No. 2, pp. 33-34, Feb. 1874.

[16] W. Ren, Q. Zhao, ”Opportunity Detection at the Secondary Transmitter and Receiver,” Technical Report (TR-07-05), Aug. 2007, [http://www.ece.ucdavis.edu/~qzhao/TR-07-05.pdf](http://www.ece.ucdavis.edu/~qzhao/TR-07-05.pdf).

[17] T. Sheldon, ”Encyclopedia of Networking & Telecommunications,” Osborne/McGraw-Hill, 2001.

[18] N. Jindal, J. G. Andrews, S. Weber, “Bandwidth Partitioning in Decentralized Wireless Networks,” *IEEE Transactions on Wireless Communications*, vol. 7, issue 12, part 2, Dec. 2008.

[19] W. Ren, Q. Zhao, D. Cheowtirakul, A. Swami, “Connectivity of Cognitive Radio Networks: Proximity vs. Opportunity,” submitted to *the 10th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, May 2009.
Fig. 1. Illustration of spectrum opportunity (secondary user $A$ wishes to transmit to secondary user $B$, where $A$ should watch for nearby primary receivers and $B$ nearby primary transmitters).

Fig. 2. Spectrum opportunity detection: A common approach that detects spectrum opportunities by observing primary signals (the exposed transmitter $Z$ is a source of false alarms whereas the hidden transmitter $X$ and the hidden receiver $Y$ are sources of miss detections).
Fig. 3. ROC of LBT with perfect ears (the ROC is obtained by varying the detection range $r_D$).

$$P_D = 1 - P_{MD}$$

Fig. 4. Illustration of $S_I(d, r_1, r_2)$ (the common area of two circles with radius $r_1$ and $r_2$ and centered $d$ apart) and $S_c(d, r_1, r_2)$ (the area within a circle with radius $r_1$ centered at $A$ but outside the circle with radius $r_2$ centered at $B$ which is distance $d$ away from $A$).
Fig. 5. (a) $\Pr[H_0]$ vs $r_I$ ($p = 0.01$, $\lambda = 10/200^2$, $d = 50$, $R_p = 200$, $R_I = 250$); (b) $\Pr[H_0]$ vs $p$ ($\lambda = 10/200^2$, $d = 50$, $R_p = 200$, $R_I = 250$). Note that both y-axes use log-scale.

Fig. 6. ROC for LBT ($p = 0.01$, $\lambda = 10/200^2$, $R_p = 200$, $R_I = R_p/0.8 = 250$, $d = 0.9r_I$)
Fig. 7. Transport Throughput vs $r_I$ ($p = 0.1, \lambda = 10/200^2, R_p = 200, R_I = R_p/0.8 = 250, d = 0.9r_I, \zeta = 0.05$)

Fig. 8. Ratio between optimal interference range $r_I^*$ for transport throughput and $R_I$ vs traffic load $p$ of primary users ($\lambda = 10/200^2, R_p = 200, R_I = R_p/0.8 = 250, d = 0.9r_I, \zeta = 0.05$)
Fig. 9. Transport throughput vs traffic load $p$ of primary users ($\lambda = 10/200^2$, $R_p = 200$, $R_I = R_p/0.8 = 250$, $d = 0.9r_I$, $\zeta = 0.05$)

Fig. 10. Simulated ROC of energy detector vs. analytical ROC of LBT ($p = 0.01$, $\lambda = 10/200^2$, $P_{tx} = 10$, $R_p = 200$, $R_I = R_p/0.8 = 250$, $\tau_B = P_{tx} \cdot R_I^{-\alpha}$, $\alpha = 3$, $r_I = (P_{tx}/\tau_B)^{1/\alpha}$, $d = 0.9 \cdot r_I$)