Brueckner-AMD study of light nuclei

Kiyoshi Katō¹, Yuhei Yamamoto² and Tomoaki Togashi³

¹ Department of Physics, Faculty of Science, Hokkaido University, Sapporo 060-0810, Japan
² Division of Physics, Graduate School of Science, Hokkaido University, Sapporo 060-0810, Japan
³ Graduate School of Advanced Life Science, Hokkaido University, Sapporo, 060-0812, Japan

E-mail: kato@nucl.sci.hokudai.ac.jp

Abstract. In many states of light nuclei, the cluster structure is observed in addition to the shell structure. To understand the mechanism of clustering, we apply the Brueckner theory to the Antisymmetrized Molecular Dynamics (AMD) based on realistic nuclear interactions. The Bethe-Goldstone equation in the Brueckner theory is solved for every nucleon pair described by wave packets of AMD, and the $G$-matrix is calculated with single-particle orbits in AMD self-consistently. We show applicability of this method not only to self-conjugate nuclei but also to $N \neq Z$ nuclei with $A \leq 12$. It is confirmed that these results present reasonable description of cluster structures and energy-level schemes in comparison with the experimental ones in light nuclei. For $^8$Be having a typical $\alpha + \alpha$ cluster structure, the structure dependence of the $G$-matrix is investigated and the role of tensor force is shown to be important in understanding the clustering mechanism.

1. Introduction

Cluster structure of nuclei has been extensively studied. In light nuclei, many cluster states are observed in low excitation energies. These observations mean that much energy is not needed to divide a nucleus to clusters of smaller nuclear systems. Therefore the clustering mechanism has been studied based on the overall saturation property of the binding energy.

About three decades ago, to study the mechanism of $\alpha$ clustering in $^8$Be and $^{12}$C, Bandō et al. [1] applied the Brueckner theory to the 2$\alpha$ and 3$\alpha$ models, respectively, using the molecular orbits as single-particle states and discussed the $G$-matrix based on the realistic nuclear forces in the cluster model. Their pioneer works to study the cluster structures based on the realistic nuclear interactions have also provided us with the promising method of studying the clustering mechanism. In their study, they adopted the Brink wave function [2] to $^8$Be and $^{12}$C where the $\alpha$-cluster of four nucleons forming the $(0s)^4$ configuration was assumed at a localized center. They also discussed the important contributions of the tensor force through analyses of the triplet-even term of the $G$-matrices in the Brueckner theory.

In the last decade, the antisymmetrized molecular dynamics (AMD) [3, 4] calculations have resulted in understanding and predicting the properties of both stable and unstable nuclei while the phenomenological effective interactions have been used. A great advantage of AMD is that AMD can be used to microscopically describe both shell and cluster structures without any assumption of configurations as results of the energy variation. Recently, we have developed AMD into a framework starting from realistic nuclear forces combining the Brueckner theory.
with the AMD+Hartree-Fock method \[5, 6\] where the single-particle orbits in AMD can be defined. We call this framework "Brueckner-AMD".[7, 8, 9, 10, 11, 12]

In section 2 of this presentation, we briefly explain the framework of the Brueckner-AMD, and show its applicability to nuclear structure of several light nuclei based on realistic nuclear interactions in section 3. Using the method, we elucidate the effects of the tensor force on the \(\alpha-\alpha\) clustering of \(^8\)Be in section 4.

2. Framework of Brueckner-AMD

2.1. Antisymmetrized Molecular Dynamics (AMD)

For a mass number \(A\) nucleons, the Antisymmetrized Molecular Dynamics (AMD) wave function is expressed as follows \[3, 4\]:

\[
|\Phi\rangle = \mathcal{A} \left( \prod_j^A |\phi_j\rangle \right) = \mathcal{A} \left( \prod_j^A |Z_j\rangle \cdot |\chi_j\rangle \right),
\]

\[
\langle \mathbf{r}|Z_j\rangle = \left( \frac{2\nu}{\pi} \right)^{3/4} \exp \left[ -\nu \left( \mathbf{r} - \frac{Z_j}{\sqrt{\nu}} \right)^2 + \frac{Z_j^2}{2} \right].
\]

Here \(\mathcal{A}\) is the antisymmetrization operator, and \(|\phi_j\rangle = |Z_j\rangle \cdot |\chi_j\rangle\) is the single-particle wave function consisting of a spatial part \(|Z_j\rangle\) and a spin-isospin part \(|\chi_j\rangle\). The spatial part \(|Z_j\rangle\) is defined by a Gaussian form as shown in Eq. (2).

In the Brueckner theory \[13\], the single-particle orbits and energies are needed to determine the \(G\)-matrix. Using the AMD+Hartree-Fock method \[5, 6\], we obtain the single-particle orbits \(\bar{\mathbf{f}}_p = \sum_j g_{jp}|\phi_j\rangle\) and energies \(\epsilon_p\).

2.2. The Single-Particle Energies in the Brueckner-AMD

We cannot calculate straightforward the expectation value of the realistic interaction with the AMD wave functions because of its strong repulsive core at a short-range distance. We construct the \(G\)-matrix originated from the realistic nuclear interaction by applying the Brueckner theory. We perform this procedure using the method proposed by Bandô \textit{et al.} [1]. In Brueckner-AMD, we construct the \(G\)-matrix, in the usual sense, solving the Bethe-Goldstone equation \[14, 15\]:

\[
\hat{G} = \hat{v}_2 + \hat{v}_2 \frac{Q}{\epsilon_1 + \epsilon_2 - (\hat{t}_1 + \hat{t}_2)} \hat{G}.
\]

Details of the solving method of Eq.(3) were discussed in our papers \[7, 8, 9, 10, 11\]. Using the \(G\)-matrix, the single-particle energy \(\epsilon_p\) is given as

\[
\epsilon_p = \langle \bar{\mathbf{f}}_p | \hat{G} | \bar{\mathbf{f}}_p \rangle + \sum_q \langle \bar{\mathbf{f}}_p | \hat{G} | \bar{\mathbf{f}}_q \rangle \mathcal{A} \{ \bar{\mathbf{f}}_p | \bar{\mathbf{f}}_q \}.
\]

We calculate the \(G\)-matrix using this single-particle energy \(\epsilon_p\) and orbits \(|\hat{\mathbf{f}}_p\rangle\), and again solve the single-particle orbits and energies applying this \(G\)-matrix to the AMD+Hartree-Fock equation as an effective interaction. This procedure is repeated until the convergence is obtained.

The \(G\)-matrix can be represented with the correlation function \(F_{ij}\)[8] as

\[
\psi_{ij} = \phi_{ij} + \frac{Q}{\epsilon_i + \epsilon_j - (\hat{t}_i + \hat{t}_j)} \hat{G} \cdot \phi_{ij} \equiv \tilde{F}_{ij} \cdot \phi_{ij},
\]

and then,

\[
\langle \phi_{ij} | \hat{G} | \phi_{ij} \rangle = \langle \phi_{ij} | \hat{v}_2 | \psi_{ij} \rangle = \langle \phi_{ij} | \hat{v}_2 \cdot \tilde{F}_{ij} | \phi_{ij} \rangle,
\]
where \( \psi_{ij} \) is the solution of the Bethe-Goldstone equation and \( \phi_{ij} \) is the relative part between two wave packets \( |\phi_i\rangle \) and \( |\phi_j\rangle \) in AMD.

The single-particle potential energy \( \varepsilon_p^{\text{pot}} \) is presented as

\[
\varepsilon_p^{\text{pot}} = \sum_q \langle \tilde{\hat{f}}_p f_q | \hat{G} | A \{ \tilde{\hat{f}}_p f_q \} \rangle \\
= \sum_q \sum_{ijkl} \tilde{g}_{ip} \tilde{g}_{jq} \tilde{g}_{kp} \tilde{g}_{lq} \langle \phi_i \phi_j | \tilde{\hat{v}}_2 \cdot \hat{F}_{kl}^{pq} | A \{ \phi_k \phi_l \} \rangle .
\]  
(7)

When the realistic nuclear interaction \( \tilde{\hat{v}}_2 \) has the central \( \tilde{\hat{v}}_2^C \), the tensor \( \tilde{\hat{v}}_2^T \), and the spin-orbital \( \tilde{\hat{v}}_2^{LS} \) forces, we decompose the single-particle potential energy into the contributions \( \varepsilon_p^C, \varepsilon_p^T, \) and \( \varepsilon_p^{LS} \) from the central, the tensor, and the spin-orbital forces, respectively:

\[
\varepsilon_p^{\text{pot}} = \varepsilon_p^C + \varepsilon_p^T + \varepsilon_p^{LS} ,
\]  
(8)

where each term is expressed as follows:

\[
\varepsilon_p^X = \sum_q \sum_{ijkl} \tilde{g}_{ip} \tilde{g}_{jq} \tilde{g}_{kp} \tilde{g}_{lq} \langle \phi_i \phi_j | \tilde{\hat{v}}_2^X \cdot \hat{F}_{kl}^{pq} | A \{ \phi_k \phi_l \} \rangle \quad (X = C, T, LS).
\]  
(9)

### 2.3. The Cooling Method with a Constraint

We determine a configuration of a nucleus solving the following equation of the constraint cooling method [16] which is an energy variation method:

\[
\frac{dZ_i}{dt} = -\frac{\partial H}{\partial Z_i^*} - \frac{\partial C}{\partial Z_i^*}, \quad C = C_0(\langle W \rangle - W)^2 ,
\]  
(10)

where \( H = \langle \Phi | \hat{H} | \Phi \rangle / \langle \Phi | \Phi \rangle \) is an expectation value of the Hamiltonian \( \hat{H} = \sum_i t_i + \sum_{i>j} \tilde{\hat{v}}_2 \cdot \hat{F}_{ij} \), and \( C_0 \) is the positive constant and the term \( W \) is a value of the constraint quantity \( W \). In the case of \(^8\text{Be}\), we adopt the distance between quasi-clusters as the constraint quantity. It is defined as

\[
\langle d^2 \rangle = |R_n - R_m|^2 ,
\]  
(11)

where the \( R_n(R_m) = \frac{1}{A_n(A_m)} \sum_{i \in C_n(A_n)} \frac{1}{\sqrt{\nu}} \), represent the center of mass for the quasi-cluster \( C_n(A_n) \) consisting of \( A_n(A_m) \) nucleons.

In Eq. (10), the derivative \( \partial H / \partial Z_i \) cannot be calculated analytically, and then we calculate this derivative as the difference in \( Z_i \). We use a common Gaussian width value for every wave packet; \( \nu = 0.195 \text{ fm}^{-2} \) in Eq.(2). We perform the parity projection alone before variation, that is, \( H \) in Eq. (10) is replaced by \( H = \langle \hat{P}^\pm \Phi | \hat{H} | \hat{P}^\pm \Phi \rangle / \langle \hat{P}^\pm \Phi | \hat{P}^\pm \Phi \rangle \), where \( \hat{P}^\pm \) is a parity projection operator. When we calculate a fixed spin state, the spin projection is further performed in addition to the parity projection.

### 3. Binding Energies and Low-Lying Levels of Light Nuclei

We apply the Brueckner-AMD method to various light nuclei with \( A \leq 12 \). In this report, we present the results of \(^4\text{He}, \, ^7\text{Li}, \, ^8\text{Be}, \, ^9\text{Be}, \, ^{11}\text{B} \), and \(^{12}\text{C}\). In these calculations, we use the Argonne \( v'8' \) (AV8') [17, 18] as the realistic nuclear force and switch off Coulomb interactions.

The binding energies 24.6 MeV and 44.0 MeV are calculated for \(^4\text{He}\) and \(^8\text{Be}\). This result indicates that the experimental values 28.3 MeV and 56.5 MeV for \(^4\text{He}\) and \(^8\text{Be}\), respectively, are still large and suggests the necessity of a three-nucleon force. On the other hand, the \(^8\text{Be}\) binding energy from two \( \alpha \) threshold is -5.2 MeV. A preliminary calculation for \(^8\text{Be}\) suggests
that about 5 MeV binding energy is added by solving the zero point oscillation of the relative motion between two $\alpha$-clusters. Thus the present calculation reproduces the reasonable $^8\text{Be}$ binding energy measured from the two-$\alpha$ threshold.

The ground and excited energy levels in $p$-shell nuclei for each $J^\pm$ state are shown in Figs. 1 − 2, where the energies of the lowest-energy states corresponding to the experimental ground states are normalized to zero energy.

From Figs. 1 and 2 of the energy-level schemes, we see that the results of the Brueckner-AMD with the spin and parity projection (B-AMD) well agree with the experimental data (EXP) on the whole. Here, note that the excitation energies of the excited parity states in these calculations are consistent with the experimental data, while their excitation energies in the shell model calculations [19, 20] generally become much higher than the experimental ones. In our results, the $3/2^-$ and $1/2^-$ states in $^9\text{Be}$ and $^{11}\text{Be}$, and the $5/2^+$ and $3/2^+$ states in $^9\text{Be}$ are almost degenerated. This means that the effect of the spin-orbit interaction is not taken into account appropriately in the present calculation.

4. Tensor Force Effects on $\alpha-\alpha$ Clustering of $^8\text{Be}$

The calculated $^8\text{Be}$ binding energy of the parity-projected intrinsic state is shown in the left panel of Fig. 3 as a function of the distance between two quasi-$\alpha$ clusters. The energy minimum appears at the large distance $\sim 3.0$ fm, which indicates the realization of the $\alpha-\alpha$ structure.
Figure 3. Left panel: The energy surface as a function of the distance of quasi-clusters. Inside figure presents the density distribution at $d = 3$ fm. Right panel: Single-particle energies of the eight-nucleon orbits in $^8$Be. Inside figures present single particle densities of $\epsilon_1$ and $\epsilon_2$, respectively, at $d = 3$ fm.

The single-particle energies degenerate into almost two sets of energies $\epsilon_{1,2}$ as seen in the right panel of Figure 3. These $\epsilon_{1,2}$ are understood to correspond to 0s- and 0p-orbits of a one-center potential at a small distance but to 0s-orbits of a two-center potential at a large distance.

We investigate the mechanism why the energy minimum appears at the large distance $d \sim 3.0$ fm from the view of the nuclear force.[12] For this purpose, we decompose the single-particle potential energies $\epsilon^\text{pot}_{1,2}$ in Eq.(7) into the channel components for such as $^1E$, $^3E$, $^1O$ and $^3O$ channels. Although this $d$ dependence is caused by wave function and $G$-matrix, our interest exists in the $d$ dependence of the effective interaction $G$. To see the $d$ dependence of the $G$-matrix on the single-particle energies, we compare the $d$ dependence of the single-particle energies, $\epsilon^{3E,T}_{1,2}[G(d)]$, with $\epsilon^{3E,T}_{1,2}[G(d = 4.5)]$ obtained using the no state-dependent $G$-matrix fixed at $d = 4.5$ fm. In the left panel of Figure 4, we show the single-particle energies of the $^3E$ tensor channel which has the largest $d$ dependence among all channel components included in

Figure 4. Left panel: single-particle energy components $\epsilon^{3E,T}_{1,2}(G(d))$ of the tensor force in the $^3E$ channel is compared with $\epsilon^{3E,T}_{1,2}(G(d = 4.5))$ which are calculated using the fixed $G$-matrix for the $^3E$ channel at $d = 4.5$ fm. Right panel: The solid line, $G$, is the total energy surfaces same as Figure 3, and $G^{1E}[d = 4.5]$ and $G^{3E}[d = 4.5]$ are the total energy surfaces calculated using the fixed $G$-matrices at $d = 4.5$ fm for $^1E$ and $^3E$ channel, respectively.
the AV8' force. In the right panel, the total energy surface is also shown in comparison with calculations using the fixed $G$-matrices at $d = 4.5$ fm for $1E$ or $3E$ channel.

Comparing $\varepsilon_{1,2}^{3E,T}$ and $\varepsilon_{1,2}^{3E,T}[G(d = 4.5)]$ in the left panel of Fig. 4, $\varepsilon_2$ shows a small difference but $\varepsilon_1$ indicates a large difference at a small distance $d$. This result means that the tensor force causes the strong state-dependence in the $3E$ channel, and makes the binding of the $\varepsilon_1$ orbit weak. As a result of the weak binding of this single-particle orbit, the total energy surface becomes shallow at the small distance (d) region in comparison with the fixed-$G$-matrix energy surface described by $G^{3E}(d = 4.5)$ in the right panel in Figure 4. This is understood as the result of the state-dependent $G$-matrix dominantly coming from the weakened tensor-force contribution. We call this effect of the state-dependence in the $G$-matrix as a "tensor suppression”.

5. Summary

We showed the framework of the Brueckner-AMD and the procedure of the spin-parity projection. The $G$-matrix elements in a single Slater determinant are calculated for single-particle orbits in AMD. In the projection procedure, the $G$-matrix between the different Slater determinants is defined by the two-body correlation functions derived from the solutions of the Bethe-Goldstone equation. We applied this framework to some light nuclei and confirmed that it can reproduce low excited levels based on the realistic nuclear force AV8'. Furthermore, we investigated the reason why an $\alpha$-$\alpha$ cluster structure is formed in $^8$Be. It is shown that the single-particle energies $\varepsilon_{1}^{3E,T}$ of the $3E$ channel become shallow at the small distance of quasi-clusters. This feature of $3E$ is caused by the tensor force, and such a property of the tensor force is understood as the suppression of the tensor force role.[1]

The Brueckner-AMD can be extended easily to the multi-configuration calculation calculating the $G$-matrix with the correlation functions. We attempt to describe, for example, the second or higher $0^+$ states in $^{12}$C.

References

[1] Bandō H, Nagata S and Yamamoto Y 1970 Prog. Theor. Phys. 44 646; 1971 Prog. Theor. Phys. 45 1515
[2] Brink D, 1965 Proc. Int. School of Physics "Enrico Fermi" Course 36 ed. C. Bloch (Academic Press, New York and London, 1966), p. 247
[3] Kanada-En’yo Y, Horiuchi H and Ono A 1995 Phys. Rev. C 52 628
[4] Kanada-En’yo Y and Horiuchi H 2001 Prog. Theor. Phys. Suppl. No. 142 205
[5] Doté A, Kanada-En’yo Y and Horiuchi H 1997 Phys. Rev. C 56 1844
[6] Doté A and Horiuchi H 2000 Prog. Theor. Phys. 103 91
[7] Togashi T and Katō K 2007 Prog. Theor. Phys. 117 189
[8] Togashi T, Murakami T and Katō K 2009 Prog. Theor. Phys. 121 299
[9] Togashi T, Murakami T and Katō K 2008 Int. J. Mod. Phys. E 17 2081; 2008 J. Phys. Conference Series 111 012026
[10] Togashi T, Katō K and Murakami T 2009 Int. J. Mod. Phys. A 24 2142
[11] Murakami T, Togashi T and Katō K 2009 Mod. Phys. Lett. A 24 1013
[12] Yamamoto Y, Togashi T and Katō K 2010 Prog. Theor. Phys. 124 315
[13] K. A. Brueckner K A, Levinson C A and Mahmoud H M 1954 Phys. Rev. 95 217
[14] Bethe H A 1956 Phy. Rev. 103 1353
[15] Goldstone J 1957 Proc. R. Soc. A 239 267
[16] Taniguchi Y, Kimura M and Horiuchi H 2004 Prog. Theor. Phys. 112 475
[17] Pudliner B S, Pandharipande V R, Carlson J, Pieper S C and Wiringa R B 1997 Phys. Rev. C 56 1720
[18] Wiringa R B and Pieper S C 2002 Phys. Rev. Lett. 89 192501
[19] Navrátíl P, Vary J F and Barrett B R 2000 Phys. Rev. Lett. 84 5728
[20] Forssén C, Navrátíl P, Ormand W E and Caufier E 2005 Phys. Rev. C 71 044312