Testing Multiple Gluon Dynamics at the Tevatron

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Abstract

We propose the measurement of the ratio $R_{CSS}(Q_T^{\text{min}}) \equiv \frac{\sigma(Q_T>Q_T^{\text{min}})}{\sigma_{\text{Total}}}$ to study the effects of the multiple soft gluon radiation, predicted by QCD, on the transverse momentum ($Q_T$) distribution of the weak gauge bosons $W^\pm$ and $Z^0$ produced at the Tevatron. We compare the prediction of the extended Collins-Soper-Sterman resummation formalism with the next-to-leading and next-to-next-to-leading order calculations. We show that both the rich dynamics of the QCD multiple soft gluon radiation and the non-perturbative sector of QCD can be tested by measuring $R_{CSS}$.

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I. INTRODUCTION

With 100 pb$^{-1}$ luminosity at the Tevatron, about $2 \times 10^6 W^\pm$ and $6 \times 10^5 Z^0$ bosons are produced, and the data sample will increase by a factor of 20 in the Run 2 era. In view of this large event rate, a careful study of the transverse momentum distributions of vector bosons can provide a stringent test of the rich dynamics of the multiple soft gluon emission predicted by Quantum Chromodynamics (QCD). The increasing precision of the experimental data demands a high precision theoretical calculation of the distributions of the $W^\pm$ and $Z^0$ bosons, which takes the effects of multiple gluon radiation into account. In this work, within the extended Collins-Soper-Sterman formalism [1–3], we illustrate the effect of the multiple gluon radiation on the vector boson transverse momentum distribution.

To test the dynamics of the multiple soft gluon radiation, in this work we propose the measurement of the ratio $R_{CSS}(Q_T^{\text{min}}) \equiv \frac{\sigma(Q_T>Q_T^{\text{min}})}{\sigma_{\text{Total}}}$ for the $W^\pm$ and $Z^0$ bosons produced at the Tevatron. We show that for vector boson transverse momenta less than about 30 GeV, the difference between the resummed and the fixed order predictions (either at the $\alpha_S$ or $\alpha_S^2$ order) can be observed by measurement. This suggests that in this kinematic region, the effects of the multiple soft gluon radiation are important, and hence, this case provides an ideal opportunity to test this aspect of the QCD dynamics. For $Q_T$ less than about 10 GeV, the $Q_T$ distribution is largely determined by the non-perturbative part of QCD. At the Tevatron, for $W^\pm$ and $Z^0$ production, this non-perturbative physics, when parametrized by Eq. (4), is dominated by the parameter $g_2$, which was shown to be related to properties of the QCD vacuum [4]. Therefore, precisely measuring the $Q_T$ distributions in the low $Q_T$ region, e.g. from $Z^0$ events can advance our knowledge of the non-perturbative QCD physics.

The next section summarizes the relevant formulae of our extension of the Collins-Soper-Sterman resummation formalism, which describes the production and decay of vector bosons at hadron colliders. In Section III, we present the differences in the next-to-leading order (NLO, e.g. $O(\alpha_S)$), next-to-next-to-leading order (NNLO, e.g. $O(\alpha_S^3)$), and the resummed predictions for $R_{CSS}$, and show that the differences due to the soft gluon effects are measurable. We illustrate that, using the experimental data, the improvement of the non-perturbative sector of the resummation formalism is also possible. Finally, we draw our conclusions, based upon these theoretical results, about the importance of the multiple soft gluon radiation in $W^\pm$ and $Z^0$ production at the Tevatron.
II. SUMMARY OF THE RESUMMATION FORMALISM

The fully differential cross section of the hadronic production and decay of a vector boson, within the extended Collins-Soper-Sterman resummation formalism, is characterized as follows. The kinematics of the vector boson $V$ (real or virtual) can be expressed in the terms of its mass $Q$, rapidity $y$, transverse momentum $Q_T$, and azimuthal angle $\phi_V$, measured in the laboratory frame (the center-of-mass frame of hadrons $h_1$ and $h_2$). The kinematics of the lepton $\ell_1$ is described by $\theta$ and $\phi$, the polar and the azimuthal angles defined in the Collins-Soper frame [5], which is a special rest frame of the $V$-boson [6]. (A more detailed discussion of the kinematics can be found in Ref. [3].) The resummed cross section is given by the following formula in Ref. [2]:

\[
\frac{d\sigma(h_1 h_2 \to V(\to \ell_1 \ell_2) X)}{dQ^2 dy dQ_T^2 d\phi_V d\cos \theta d\phi} = \frac{Q^2}{96\pi^2 S} \left( \frac{Q^2 - M_V^2}{Q^2} \right)^2 + \frac{Q^4 \Gamma_V^2 / M_V^2}{Q^2}
\]

\[
\times \left\{ \frac{1}{(2\pi)^2} \int d^2b e^{i\hat{Q}_T \cdot \hat{b}} \sum_{j,k} \tilde{W}_{jk}(b_s, Q, x_1, x_2, \theta, \phi, C_1, C_2, C_3) \tilde{W}_{jk}^{NP}(b, Q, x_1, x_2)
\right. \\
\left. \\
+ Y(Q_T, Q, x_1, x_2, \theta, \phi, C_4) \right\}. 
\]  

(1)

In the above equation the parton momentum fractions are defined as $x_1 = e^y Q / \sqrt{S}$ and $x_2 = e^{-y} Q / \sqrt{S}$, where $\sqrt{S}$ is the center-of-mass (CM) energy of the hadrons $h_1$ and $h_2$. For $V = W^\pm$ or $Z^0$, we adopt the LEP line-shape prescription of the resonance behavior. The renormalization group invariant quantity $\tilde{W}_{jk}(b)$, which sums to all orders in $\alpha_S$ all the singular terms that behave as $\alpha_S^m Q_T^{-2} \ln^{2m-1} (Q_T^2 / Q^2) (1 \leq m \leq n)$ for $Q_T \to 0$, is

\[
\tilde{W}_{jk}(b, Q, x_1, x_2, \theta, \phi, C_1, C_2, C_3) = \exp \{- S(b, Q, C_1, C_2)\} | V_{jk} |^2
\]

\[
\times \left[ \left( C_{ja} \otimes f_{a/h_1} \right)(x_1) \left( C_{kb} \otimes f_{b/h_2} \right)(x_2) + \left( C_{ka} \otimes f_{a/h_1} \right)(x_1) \left( C_{jb} \otimes f_{b/h_2} \right)(x_2) \right]
\]

\[
\times (g_L^2 + g_R^2)(f_L^2 + f_R^2)(1 + \cos^2 \theta)
\]

\[
+ \left[ \left( C_{ja} \otimes f_{a/h_1} \right)(x_1) \left( C_{kb} \otimes f_{b/h_2} \right)(x_2) - \left( C_{ka} \otimes f_{a/h_1} \right)(x_1) \left( C_{jb} \otimes f_{b/h_2} \right)(x_2) \right]
\]

\[
\times (g_L^2 - g_R^2)(f_L^2 - f_R^2)(2 \cos \theta) \right). 
\]  

(2)

where $\otimes$ denotes the convolution

\[
\left( C_{ja} \otimes f_{a/h_1} \right)(x_1) = \int_{x_1}^1 \frac{d\xi}{\xi} C_{ja} \left( \frac{x_1}{\xi} ; g_3, b, \mu = \frac{C_3}{b}, C_1, C_2 \right) f_{a/h_1} \left( \xi, \mu = \frac{C_3}{b} \right),
\]

and the $V_{jk}$ coefficients are given by

\[
V_{jk} = \begin{cases} 
\delta_{jk} & \text{for } V = W^\pm \\
\text{Cabibbo – Kobayashi – Maskawa matrix elements} & \text{for } V = Z^0.
\end{cases}
\]

\[
3
\]
The $q\bar{q}V$ and $\ell_1\bar{\ell}_2V$ vertices are defined as $i\gamma_\mu [g_L(1 - \gamma_5) + g_R(1 + \gamma_5)]$ and $i\gamma_\mu [f_L(1 - \gamma_5) + f_R(1 + \gamma_5)]$, respectively. For example, for $V = W^+$, $q = u$, $\bar{q}' = d$, $\ell_1 = \nu_e$, and $\bar{\ell}_2 = e^+$, the couplings are $g_L^2 = f_L^2 = G_F M_W^2/\sqrt{2}$ and $g_R^2 = f_R^2 = 0$, where $G_F$ is the Fermi constant. The Sudakov exponent $S(b, Q, C_1, C_2)$ in Eq. (2) is defined as

$$S(b, Q, C_1, C_2) = \int_{c_1/Q^2}^{c_2/Q^2} \frac{d\mu^2}{\mu^2} \left[ A(\alpha_S(\mu), C_1) \ln \left( \frac{C_2 Q^2}{\mu^2} \right) + B(\alpha_S(\mu), C_1, C_2) \right].$$

The explicit forms of the $A$, $B$ and $C$ functions and the renormalization constants $C_i$ ($i=1,2,3$) are summarized in Refs. [1,2].

In Eq. (1) the magnitude of the impact parameter $b$ is integrated from 0 to $\infty$. However, in the region where $b > 1/\Lambda_{QCD}$, the Sudakov exponent $S(b, Q, C_1, C_2)$ diverges as the result of the Landau pole of the QCD coupling $\alpha_S(\mu)$ at $\mu = \Lambda_{QCD}$, and the perturbative calculation is no longer reliable. In this region of the impact parameter space (i.e. large $b$), a prescription for parametrizing the non-perturbative physics in the low $Q_T$ region is necessary. Following the idea of Collins and Soper [7], the renormalization group invariant quantity $\tilde{W}_{jk}(b)$ is written as

$$\tilde{W}_{jk}(b) = \tilde{W}_{jk}(b_s)\tilde{W}_{jk}^{NP}(b).$$

Here $\tilde{W}_{jk}(b_s)$ is the perturbative part of $\tilde{W}_{jk}(b)$ and can be reliably calculated by perturbative expansions, while $\tilde{W}_{jk}^{NP}(b)$ is the non-perturbative part of $\tilde{W}_{jk}(b)$ that cannot be calculated by perturbative methods and has to be determined from experimental data. To test this assumption, one should verify that there exists a universal functional form for this non-perturbative function $\tilde{W}_{jk}^{NP}(b)$. This is similar to the general expectation that there exists a universal set of parton distribution functions (PDF’s) that can be used in any perturbative QCD calculation to compare it with experimental data. In the perturbative part of $\tilde{W}_{jk}(b)$,

$$b_s = \frac{b}{\sqrt{1 + (b/b_{max})^2}},$$

and the non-perturbative function was parametrized by (cf. Ref. [1])

$$\tilde{W}_{jk}^{NP}(b, Q, Q_0, x_1, x_2) = \exp \left[ -F_1(b) \ln \left( \frac{Q^2}{Q_0^2} \right) - F_{j/h_1} (x_1, b) - F_{k/h_2} (x_2, b) \right], \quad (3)$$

where $F_1$, $F_{j/h_1}$ and $h_{k/h_2}$ have to be first determined using some sets of data, and later can be used to predict the other sets of data to test the dynamics of multiple gluon radiation predicted by this model of the QCD theory calculation. As noted in Ref. [1], $F_1$ does not depend on the momentum fraction variables $x_1$ or $x_2$, while $F_{j/h_1}$ and $F_{k/h_2}$ in general depend
on those kinematic variables. The \(Q^2/Q_0^2\) dependence associated with the \(F_1\) function was predicted by the renormalization group analysis \(\text{[1]}\). Furthermore, \(F_1\) was shown to be universal, and its leading behavior (\(~b^2\)) can be described by renormalon physics \(\text{[4]}\). Various sets of fits to these non-perturbative functions can be found in Refs. \(\text{[8]}\) and \(\text{[9]}\).

In our numerical calculations, we use the Ladinsky-Yuan parametrization of the non-perturbative function (cf. Ref. \(\text{[9]}\)):

\[
\widetilde{W}_{jk}^{NP}(b, Q, Q_0, x_1, x_2) = \exp \left[ -g_1 b^2 - g_2 b^2 \ln \left( \frac{Q}{2Q_0} \right) - g_1 g_3 b \ln (100x_1 x_2) \right],
\]

where \(g_1 = 0.11^{+0.04}_{-0.03}\) GeV\(^2\), \(g_2 = 0.58^{+0.1}_{-0.2}\) GeV\(^2\), \(g_3 = -1.5^{+0.1}_{-0.1}\) GeV\(^{-1}\), and \(Q_0 = 1.6\) GeV. (The value \(b_{\text{max}} = 0.5\) GeV\(^{-1}\) was used in determining the above \(g_i\)’s.) These values were fit for CTEQ2M PDF with the canonical choice of the renormalization constants, i.e. \(C_1 = C_3 = 2e^{-\gamma_E}\) (\(\gamma_E\) is the Euler constant) and \(C_2 = 1\). In principle, for a calculation using a different set of PDF, these non-perturbative parameters should be refit using a data set that should include the recent high statistics \(Z^0\) data from the Tevatron.

In Eq. \(\text{[1]}\), \(\widetilde{W}_{jk}\) sums over the soft gluon contributions that grow as \(\alpha_S^n Q_T^{-2} \ln^{2m-1}(Q_T^2/Q^2)\) \((1 \leq m \leq n)\) to all orders in \(\alpha_S\). Contributions less singular than those included in \(\widetilde{W}_{jk}\) should be calculated order-by-order in \(\alpha_S\) and included in the \(Y\) term, introduced in Eq. \(\text{[1]}\). This would in principle extend the applicability of the CSS resummation formalism to all values of \(Q_T\). The \(Y\) term, which is defined as the difference between the fixed order perturbative contribution and those obtained by expanding the perturbative part of \(\widetilde{W}_{jk}\) to the same order, is given by

\[
Y(Q_T, Q, x_1, x_2, \theta, \phi, C_4) = \int_{x_1}^{1} \frac{d\xi_1}{\xi_1} \int_{x_2}^{1} \frac{d\xi_2}{\xi_2} \sum_{n=1}^{\infty} \left[ \frac{\alpha_S(C_4 Q)}{\pi} \right]^n \times f_{a/h_1}(\xi_1, C_4 Q) R_{ab}^{(n)}(Q_T, Q, \frac{x_1}{\xi_1}, \frac{x_2}{\xi_2}, \theta, \phi) f_{b/h_2}(\xi_2, C_4 Q),
\]

where the functions \(R_{ab}^{(n)}\) contain contributions less singular than \(\alpha_S^n Q_T^{-2} \ln^{2m-1}(Q_T^2/Q^2)\) \((1 \leq m \leq n)\) as \(Q_T \to 0\). Their explicit expressions are summarized in Refs. \(\text{[2],[3]}\).

\(^1\)Here, and and throughout this work, the flavor dependence of the non-perturbative functions is ignored, as it is postulated in Ref. \(\text{[1]}\).

\(^2\)It is shown in Ref. \(\text{[3]}\) that since the \(A, B, C,\) and \(Y\) functions are only calculated to some finite order in \(\alpha_S\), the CSS resummed formula as described above will cease to be adequate when the value of \(Q_T\) is in the vicinity of \(Q\). Hence, in practice, one has to switch from the resummed prediction to the fixed order perturbative calculation as \(Q_T \geq Q\).
III. THE RATIO $R_{CSS}$

In this work we propose to measure the ratio $R_{CSS}(Q_T^{\text{min}}) = \frac{\sigma(Q_T > Q_T^{\text{min}})}{\sigma_{\text{Total}}}$ to distinguish the predictions of the resummed, NLO and NNLO calculations. In Fig. 1 we show the distributions of $R_{CSS}$, which is defined by

$$R_{CSS}(Q_T^{\text{min}}) \equiv \frac{\sigma(Q_T > Q_T^{\text{min}})}{\sigma_{\text{Total}}} = \frac{1}{\sigma_{\text{Total}}} \int_{Q_T^{\text{min}}}^{Q_T^{\text{max}}} dQ_T \frac{d\sigma(h_1 h_2 \rightarrow V)}{dQ_T},$$

where $Q_T^{\text{max}}$ is the largest $Q_T$ allowed by the phase space. In the NLO calculation, $\sigma(Q_T > Q_T^{\text{min}})$ grows without bound near $Q_T^{\text{min}} = 0$, as the result of the singular behavior $1/Q_T^2$ in the matrix element. The NLO curve runs well under the resummed one in the 2 GeV < $Q_T^{\text{min}}$ < 30 GeV region, and the $Q_T$ distributions from the NLO and the resummed calculations have different shapes even in the region where $Q_T$ is of the order of 15 GeV.

With a large number of fully reconstructed $Z^0$ events at the Tevatron, one should be able to use the data to clearly discriminate these two theoretical calculations. The experimental uncertainty in the total cross sections of the $W^\pm$ and $Z^0$ productions, based on 19.7 pb$^{-1}$ CDF data, is in the ballpark of 5% [10]. Fig. 1 shows that, in the 10 GeV < $Q_T^{\text{min}}$ < 30 GeV region, even with this experimental precision we should see deviations between the experiment and the NLO predictions, in which the effects of the multiple gluon radiation are not included. In view of this result it is not surprising that the D0 analysis of the $\alpha_S$ measurement [11] based on the measurement of $\sigma(W + 1 \text{jet})/\sigma(W + 0 \text{jet})$ does not support the NLO calculation. We expect that if this measurement were performed demanding the transverse momentum of the jet to be larger than about 50 GeV, at which scale the resummed and NLO distributions cross (cf. Ref. [3]), the NLO calculation would adequately describe the data.

To show that for $Q_T$ below 30 GeV the QCD multiple soft gluon radiation is important to explain the D0 data [11], we also include in Fig. 1 the prediction for the $R_{CSS}$ distribution at the order of $\alpha_S^2$. As shown in the figure, the $\alpha_S^2$ curve is closer to the resummed curve, which proves that for this range of $Q_T$ the soft gluon effect included in the $\alpha_S^2$ calculation is important for predicting the vector boson $Q_T$ distribution. In other words, in this range of $Q_T$, it is more likely that soft gluons accompany the $W^\pm$ boson than just a single hard jet associated with the vector boson production. For large $Q_T$, it becomes more likely to have hard jet(s) produced with the vector boson.

The measurement of $R_{CSS}$ can also provide information about the non-perturbative physics associated with the initial state hadrons. As shown in Ref. [9], the effect of the non-perturbative physics on the $Q_T$ distributions of the $W^\pm$ and $Z^0$ bosons produced at the
FIG. 1. The ratio $R_{CSS}$ as a function of $Q_T^{\text{min}}$ for $W^\pm$ bosons. The fixed order [$\mathcal{O}(\alpha_S)$ short dashed, $\mathcal{O}(\alpha_S^2)$ dashed] curves are ill-defined in the low $Q_T$ region. The corresponding distributions for the $Z^0$ boson are indistinguishable from those for the $W^\pm$ in this plot.

Tevatron is important for $Q_T$ less than about 10 GeV. This is evident if one observes that different parametrizations of the non-perturbative functions do not change the $Q_T$ distribution for $Q_T > 10$ GeV, but can dramatically change its shape for $Q_T < 10$ GeV. Since for $W^\pm$ and $Z^0$ production, the $\ln(Q^2/Q_0^2)$ term is large, the non-perturbative function, as defined in Eq. (3), is dominated by the $F_1(b)$ term which is expected to be universal for all Drell-Yan type processes and which is related to the physics of the renormalon [4]. Hence, the measurement of $R_{CSS}$ can be used to probe this part of non-perturbative physics for $Q_T < 10$ GeV, in addition to probing the dynamics of multiple soft gluon radiation for 10 GeV $< Q_T < 40$ GeV. It is therefore important to measure $R_{CSS}$ at the Tevatron. With a large sample of $Z^0$ data at the Run 2, it will be possible to determine the dominant non-perturbative function which can then be used to determine the $W^\pm$ boson $Q_T$ distribution to improve the accuracy of the $M_W$ and the charged lepton rapidity asymmetry measurements.

IV. CONCLUSIONS

In conclusion, the measurement of $R_{CSS}$ provides an accurate test of the dynamics of the multiple soft gluon radiation predicted by QCD. With high enough luminosity, the NLO, NNLO, and resummed theoretical predictions can be distinguished by the $W^\pm$ and $Z^0$ data at the Tevatron. The comparison of the NNLO and resummed predictions shows that the soft gluon effect is important in the $Q_T < 30$ GeV region. Additionally, in the $Q_T < 10$ GeV region, the $R_{CSS}$ measurement, using the recent $Z^0$ data at the Tevatron, can
provide valuable information on the non-perturbative sector of the resummation formalism. Therefore, a careful measurement of the $Q_T$ distribution of the vector bosons $W^\pm$ and $Z^0$ in the $Q_T < Q/2$ region at the Tevatron can further our knowledge of the perturbative dynamics and the non-perturbative domain of QCD.

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