Detuned Electromagnetically Induced Transparency in \( N \)-type Atom System

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Abstract

The electromagnetically induced transparency (EIT) in an \( N \) configuration is studied under both resonant and off-resonant conditions. In a certain off-resonant condition the dark state of the four level system, which is almost the same as the resonant dark state in \( \Lambda \) configuration, is rebuilt. The actual system with damping is examined using optical Bloch equation, both numerically and analytically. Based on this detuned dark state, some new applications with frequency shifts can be realized.

Key words: EIT, \( N \)-type atom, dark state

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Recently, manipulations of the refraction and absorption properties by means of quantum coherence and interference (QCI) has been studied extensively for resonant interaction between laser and three-level atom ensembles with \( \Lambda \)-, \( V \)- and \( \Xi \)-type configurations and show the effects of electromagnetically induced transparency (EIT) \([1]\), electromagnetically induced absorption (EIA) \([2,3]\), and etc. These effects lead to possibilities of the manipulation of light pulse group velocity, and the subluminal and superluminal propagation of light pulse have been demonstrated experimentally \([4]\).

It is straightforward to consider the possibility of adjusting an external driving field into this three-level system to modulate the absorption and dispersion properties. The interaction between light pulse and four level system is also studied for \( N \) configurations \([5]\), with the observations of the nonlinearity at low light level \([6]\), as well as the interchange between subluminal and superluminal propagation \([7]\).

Other important applications of EIT, especially in the \( \Lambda \) configuration, are light storage and quantum memory. The possibility of light storage based on EIT is proposed first by Fleischhauer and Lukin \([8,9]\) with the dark state polarization model and experimentally realized in ultracold sodium atoms \([10]\), in hot rubidium vapor \([11]\), and in solids \([12]\). For quantum memory, single photon storage has been realized both in room-temperature atomic gas \([13]\) and in cold atom cloud \([14]\).

As is well known that, in a strict sense, the \( \Xi \)- and \( V \)-type three-level schemes do not show EIT owing to the absence of a meta-stable dark state \([15]\). Dark state is very important in applications of EIT, especially in light storage. However, in those studies about \( N \) configuration, no exact dark state is achieved in resonant condition. At the same time, in a \( \Lambda \) system where a dark state condition is satisfied, the external control is impossible and its single transparency window brings some limitations in applications, especially in real communications.

Here, an \( N \) configuration atomic system is studied in...
both resonant and off-resonant conditions. In some off-resonant conditions, dark states can be reached, with more than one transparency windows. Hence, some new applications under those off-resonant conditions are proposed.

1. Detuned Dark States

Let us consider an \( N \)-type atom system shown in Fig. 1, the Hamiltonian of which (under the rotating wave frame) can be written as

\[
\hat{H} = \hbar \Delta_a |1\rangle \langle 1| + \hbar \Delta_b |3\rangle \langle 3| + \hbar (\Delta_b - \Delta_c) |4\rangle \langle 4| \\
- \frac{\hbar}{2} (\hat{\Omega}_a^+ |1\rangle \langle 2| + \hat{\Omega}_a^- |2\rangle \langle 1|) + \hat{\Omega}_c^+ |3\rangle \langle 4| + \text{H.c.},
\]

where \( \hat{\Omega}_i \) (\( i = a, b, c \)) is the unit vector, \( \hat{\mathcal{E}}_i \) is the envelop of the incident light and \( \omega_i \) (\( i = a, b, c \)) is the central frequency.

A general expression of the eigenstates of the Hamiltonian is very complicated. For simplicity, the coupling and driving field are set to be resonant, while the situation of \( \Delta_b = 0, \pm \omega_c/2 \) will be examined.

If the driving light \( \Omega_c \) is not introduced, Fig. 1 shows exactly a \( \Lambda \) configuration with dark state

\[
|\Psi_i\rangle = \cos \theta |1\rangle - \sin \theta |3\rangle,
\]

where

\[
\theta = \tan^{-1} \left( \frac{\Omega_a}{\Omega_b} \right),
\]

with eigenvalue 0 [16].

However, when applying the driving field \( \Omega_c \), the dark state in resonant condition vanishes. Eigenstates of the Hamiltonian then become

\[
|\Psi_i\rangle = \frac{1}{\sqrt{2}} (\cos \theta_1 |1\rangle - \sin \theta_1 |3\rangle + \cos \theta_2 |2\rangle - \sin \theta_2 |4\rangle),
\]

where

\[
\theta_1 = \tan^{-1} \left( \frac{\Omega_a \Omega_b}{\Omega_b^2 + \Omega_c^2 - 4 \Omega_c^2} \right), \quad \theta_2 = \tan^{-1} \left( \frac{\Omega_c \Omega_b}{\Omega_c^2 - 4 \Omega_c^2} \right),
\]

and \( \hbar \lambda_i \) (\( i = 1, 2, 3, 4 \)) are the eigenvalues of the Hamiltonian, satisfying the equation:

\[
(2 \lambda - \Omega_a) (2 \lambda + \Omega_a) (4 \lambda^2 - \Omega_b^2 - \Omega_c^2) = \Omega_a^2 \Omega_b^2.
\]

Compared with Eq. (2), state shown in Eq. (3) is the superposition of all four levels and hence is not a dark state at all. The driving field introduces state shifts and destroys the resonant dark state. However, Eq. (3) shows some new properties.

It is apparent that there is no strict dark state in this resonant condition, although in Eq. (3) some amplitude of the states can be very small. \( \theta_1 \) and \( \theta_2 \) show that the population are transferring between two lower levels and two upper levels independently, each shares a half. So, more than EIT, some applications related to population transfer is available here.

However, when

\[
\Delta_b = \pm \Omega_c/2,
\]

one of the eigenstates of the Hamiltonian becomes

\[
|\Psi_{\pm}\rangle = -\frac{\Omega_b}{\Omega_a} |1\rangle + |3\rangle \pm |4\rangle,
\]

with eigenenergy 0, while the other three eigenstates are

\[
|\Psi_{\pm}\rangle = -2 \Omega_a \lambda |1\rangle + 4 \lambda |3\rangle - \Omega_b (2 \lambda |3\rangle + \Omega_b |4\rangle),
\]

with \( \lambda_{\pm} \) (\( i = 1, 2, 3 \)) satisfying

\[
8 \lambda_{\pm}^3 \lambda_{\pm}^2 - (2 \Omega_a^2 + 2 \Omega_c^2) \lambda \pm (2 \Omega_a^2 + \Omega_c^2) \Omega_c = 0.
\]

The state shown in Eq. (4) is the same as in \( \Lambda \) configuration except \( |4\rangle \) term. It has no contribution from \( |2\rangle \) and therefore is a dark state, which implies that if the atom is prepared in this state, the probability of the transition to \( |2\rangle \) is zero and subsequently, there is no absorption. Thus, the medium is transparent to the probing light with detuning \( \Delta_b = \pm \Omega_c/2 \).

2. Modulations by the driving light

The driving field is introduced in order to modulate the \( \Lambda \)-type EIT properties. With both the coupling and driving fields detunings being zero, the detuned dark states [Eq. (4)] have been derived. Next, we will study the influence on this modulation of both the detuning and intensity, which is represented by Rabi frequency of the driving field here.

The coupling light is assumed to be resonant for the maximum \( \Lambda \)-type EIT signal. In this case, it is evident that the eigenstate with zero eigenvalue of the Hamiltonian [Eq. (1)] is dark for the probing light, since there is no energy exchange between the probing light and the atom. The Hamiltonian reaches its zero eigenvalue when

\[
\Delta_b^\pm = \frac{1}{2} (\Delta_c \pm \sqrt{\Delta_c^2 + \Omega_c^2}).
\]

And the case \( \Delta_c = 0, \Delta_b = \pm \Omega_c/2 \) is exactly the same as derived in previous section, and correspondingly, the detuned dark states are

\[
|D_{\pm}\rangle = \Omega_b |1\rangle - \Omega_a |3\rangle + \frac{\Omega_c}{\Delta_b \pm \sqrt{\Delta_b^2 + \Omega_c^2}} |4\rangle,
\]

According to this result, the influence of the driving field can be understood as following, while \( \Delta_c \) is assumed to be positive for convenience: (i) If the driving field is resonant, the detuned effects are the same on both sides of the detunings so that a symmetrical detuned EIT can be drawn.

(ii) When the driving field is not resonant, or if it is far detuned with \( \Delta_c \gg \Omega_c \), then the dark state can be achieved at \( \Delta_b \approx 0 \) and \( \Delta_b \approx \Delta_c \). However, it is no longer symmetric because when \( \Delta_b \approx 0 \), the state remains \( \Omega_b |1\rangle \in \Omega_a |3\rangle \), which is exactly the dark state in three-level \( \Lambda \)-type EIT profile; but nearly all the population will be pumped to the level \( |4\rangle \) if \( \Delta_b \approx \Delta_c \) and merely a EIT signal will be available.

3. The dressed state picture

The detuned dark state [Eq. (6)] can be understood more clearly in a dressed state picture. Since the influence of the
driving field is interesting here, the states dressed only by the driving field is sufficiently evident for analysis.

\[
\begin{align*}
&|2\rangle \rightarrow |3\rangle, \\
&|4\rangle \rightarrow |1\rangle,
\end{align*}
\]

According to the dressed state theory, the driving field, with no influence on \(|1\rangle\) and \(|2\rangle\), dressed the states \(|3\rangle\) and \(|4\rangle\) into \(|3'\rangle\) and \(|4'\rangle\), respectively,

\[
|3'\rangle = \cos \vartheta |3\rangle - \sin \vartheta |4\rangle,
|4'\rangle = \sin \vartheta |3\rangle + \cos \vartheta |4\rangle,
\]

where \(\tan 2\theta = -\Omega_c/\Delta_c\), and the energy shifted is exactly the same as that in Eq. (5), as illustrated in Fig. 2. Thus, the detuned EIT effect can, in this case, be viewed as an Autler-Townes splitting in an EIT medium.

From the above analysis, the detuning of the driving field is set to be zero for symmetric detuned dark states.

4. The density matrix approach with spontaneous dampings included and double transparency windows

In the above analysis, no dampings are included. However, in realistic situations, dampings should be considered, especially for the case of the detuned dark state [Eqs. (4) or (6)] derived previously, in which an excited state \(|4\rangle\) is contained. In this case, the spontaneous emission of \(|4\rangle\) will surely affect the transparency features.

For simplicity, we assume that there exists only radiative damping, for the collision-induced dampings can be well controlled in cold atoms, so the coherence dephasing rates are

\[
\Gamma_{12} = \frac{1}{2}(\gamma_a + \gamma_b),
\Gamma_{13} = 0,
\Gamma_{14} = \frac{1}{2}\gamma_c,
\Gamma_{23} = \frac{1}{2}(\gamma_a + \gamma_b),
\Gamma_{24} = \frac{1}{2}(\gamma_a + \gamma_b + \gamma_c),
\Gamma_{34} = \frac{1}{2}\gamma_c.
\]

To see the absorption and dispersion characteristics of the probing light in such a system, we give numerically the steady state solution of \(\rho_{23}\), as illustrated in Fig. 3. It is shown that the pumping light generates one (when \(\Omega_c = 0\)) or more (when \(\Omega_c \neq 0\)) transparency windows, which is very evident when \(\Omega_c\) is large.

Next, an analytical solution is given. For simplicity, all the damping rates are assumed to be equal (\(\gamma_a,b,c = \gamma\)), and \(\Delta_a\) and \(\Delta_c\) are set to be zero for resonant pumping lights. If the requirement for laser energy in \(\Lambda\) system is satisfied [17], i.e. \(\Omega_a \gg \Omega_b\), one has

\[
\rho_{23} = \frac{g_b}{2g_b(1 + 2g_b^2)}\left[(A(\delta_+) + B(\delta_-))E(\delta_-) + [A(\delta_-) - B(\delta_-)]E(\delta_+) - [E(\delta_+) + E(\delta_-)]\right]
/\left(E(\delta_+)E(\delta_-) + \frac{16}{g_c^2}[E(\delta_+) - E(\delta_-)]^2\right),
\]

where

\[
A(x) = -g_c(1 + g_c^2)(4x + i),
B(x) = (1 - 2ix)(1 - g_c^2),
E(x) = 1 - 2g_c^2 + 2(i + 2x)(2i + 2x).
\]

Here, \(\delta_{\pm} = \delta_b \pm \frac{\Delta_b}{\gamma}\) denotes the positive and negative frequency shifts, relative to two-photon resonance. \(g_b = \Omega_b/\gamma\), \((i = a,b,c)\) and \(\delta_b = \Delta_b/\gamma\) represent the normalized Rabi frequencies and probing light detunings, respectively.

The interest here is focused on the order of the magnitude of \(g_c\), which is proportional to the square root of the intensity of the coupling light. If \(g_c \gg 1\), \(B\) can be much smaller than \(A\) and \(\frac{16}{g_c^2}[E(\delta_+) - E(\delta_-)]^2\) can be ignored.

Then the result becomes

\[
\rho_{23} = \frac{1}{2}[f(\delta_+) + f(\delta_-)],
\]

where \(f(x) = -g_b \frac{i + 4\delta_b}{2} E(x)\), as illustrated in the inset of Fig. 3. The numerical results are also shown in the inset of Fig. 3 for comparison.

From Fig. 3, one finds that there are three transparent windows at \(\pm \Omega_c/2\) detunings and resonance. As mentioned above, the dark states generate the detuned transparency and the resonant transparency can be achieved directly from Eq. (3). Since \(\Omega_b\) is relatively small, we have \(4x^2 \approx w^2\) and \(\theta_2 \approx \pi/2\), which makes [2] in Eq. (3) vanish and leads to a dark-like state and therefore the transparency.

The numerical solution shows that if the shift is large enough, say, about \(10\gamma\), all the transparency windows are separated evidently. Experimental conditions often guarantee the assumption we have made to be valid.

The EIT element \(f\) is important since, from which, one finds that the three lights play different roles: \(g_b\) determines the transparency shift, \(g_a\) determines the line shape and width, and \(g_b\) is a separate term which only affects the amplitude. This property makes it easy for manipulation of the state and the coherence.
For further consideration, the dark state [shown in Eq. (4)] is of great interest since it totally agrees with the dark state in Λ configuration, which is of great importance in many respects. It should be emphasized that the driving Rabi frequency $\Omega$ has the impact only on the frequency shift of probing light but has nothing to do with the dark state itself. Also, since the amplitude is the same between $|3\rangle$ and $|4\rangle$, there is always enough population, more than half, to interact with the light. So, the detuned EIT looks like a perfect shift of the Λ-type resonant EIT scheme since even the dark states have been shifted to both sides, and the techniques, such as the adiabatic evolution, the dark state polariton and those based on the dark state, can be used here in a symmetric frequency condition.

5. Further Analysis for $\gamma_c$

However, from the above analysis, one may find out that the absorption at $\pm \Omega_c/2$ is not zero. This is because only $|2\rangle$ is dark, while $|4\rangle$, which keeps damping out, is still populated. If the dampings are included, the states expressed in Eq. (4) are not the ideal dark-states and it has an absorption determined by $\gamma_c$.

The influence of $\gamma_c$ is illustrated in Fig. 4 and 5, which show that the absorption is proportional to $\gamma_c$ when $\gamma_c$ is small.

From the above analysis, it is clear that the damping rate from $|4\rangle$, i.e., $\gamma_c$, plays an important role. If $\gamma_c$ is approximately equal to $\gamma$, the expressions have been given already. If $\gamma_c$ is small compared to $\gamma$, as required experimentally, the imaginary part of $\rho_{23}$ can be derived as

$$\text{Im}[\rho_{23}(\delta_b = 0)] = \frac{\zeta \kappa^2 g_a}{4 \kappa^2 + (\kappa^2 - 1)^2 g_a^2},$$

where $\kappa = g_c/g_a$, $\zeta = g_b/g_a$. This approximate expression is derived under the condition $\kappa \geq 1$, $g_a \geq 1$, $\zeta \ll 1$ and $\gamma_c \leq \zeta \gamma$, which requires a strong coupling light and an even stronger driving light, compared with a weak probing light and a small damping rate. If all the conditions are satisfied, the absorption $|\delta_b = (g_c \pm g_a)/2|$ and transparency $(\delta_b = \pm g_c/2)$ can be derived in the same way, which are

$$\text{Im}[\rho_{23}]^{\text{trans}} = \frac{\Gamma \zeta \left[12 \kappa^2 + (1 - 3 \kappa^2 + 4 \kappa^4) g_a^2\right]}{32 \kappa^2 g_a + 2 (1 - 4 \kappa^2)^2 g_a^3},$$
$$\text{Im}[\rho_{23}]^{\text{absorp}} = \frac{\Gamma \zeta \left[2 (1 + 2 \kappa)^2 + 4 (\kappa + \kappa^2)^2 g_a^2\right]}{8 (\kappa + \kappa^2)^2 g_a (4 \Gamma + \zeta^2 g_a^2)},$$

where $\Gamma = \gamma_c/\gamma$. It is apparent that when the probing light is resonant, the absorption is independent of $\gamma_c$. But when the probing light is detuned to be around $\pm \Omega_c/2$, the absorption characteristics strongly depends on $\gamma_c$.

However, it should be noted that the level $|4\rangle$ should not be too stable, as illustrated in Fig. 6. In the limit $\gamma_c \to 0$, the relationship between the absorption and $\Delta_b$ would undergo great changes (Fig. 6): The absorption at all frequencies has been reduced to less than two percent of that at $\gamma_c = 0.1$. That is to say, when one probing light injecting into this system, the system is almost transparent, so the difference between transparency window and other frequencies can be neglected. In this sense, when level $|4\rangle$ is too stable, the windows we create would collapse. Moreover, the overall property of absorption curve (inset of Fig. 6) is also significantly changed.

6. Discussion and Conclusion

In conclusion, we apply a driving optical field to the Λ configuration and destroy the dark state in resonant condition. However, at $\pm \Omega_c/2$ detunings of the probing field, the dark states are rebuilt with almost complete fidelity. The influences of the three lights and destructive effects by dampings are discussed.

For experimental realization, a closed four-level N-type atom system is required. One of this kind of atom configurations is available in the D2 line of $^{133}$Cs. The four levels can be chosen, according to respective dipole moments, as: $|6^2S_{1/2}, F = 3\rangle \to |1\rangle$, $|6^2P_{3/2}, F = 4\rangle \to |2\rangle$, $|6^2S_{1/2}, F = 4\rangle \to |3\rangle$ and $|6^2P_{3/2}, F = 5\rangle \to |4\rangle$. In this configuration, the four levels compose a closed system and all the damp-
Fig. 6. (color online) Illustration of the collapse of transparency windows. Here, $\Omega_a/\gamma = 2, \Omega_b/\gamma = 0.2, \Omega_c/\gamma = 20$. Both the coupling and driving lights are resonant. (red: $\gamma_c = 0.1$ case; blue: $\gamma_c = 10^{-7}$ case; inset: enlarged figure of $\gamma_c = 10^{-7}$ case.) We find that when $\gamma_c$ is reduced to $10^{-7}$, the transparency window has collapsed.

Rates are equal ($\gamma_a = \gamma_b = \gamma_c = 2\pi \times 5.22\text{MHz}$). Of course, $\gamma_c$ is not chosen to be sufficiently small here to restrain the absorption, as required by the above theoretical analysis. However, compared with the experimental results in V-type and cascade configuration EIT experiments [4], this problem is not crucial for a light pulse storage experiment. But one has to notice that for single photon storage, it might bring much more disadvantages.

This phenomenon can, hopefully, be widely used in many areas of quantum optics and quantum information, such as slow light, light storage and quantum memory, based on the current well-founded quantum manipulation techniques. In concrete, it may give a possibility of storing two light pulse with two different frequencies with two different techniques based on dark state polariton [9], which gives potential possibility for wavelength division multiplexing [18].

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