The transform performing algorithm for frequency domain search

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Abstract. The algorithm basic for a rotation invariant method producing frequency transform is offered. The execution is based on a blocked matrixes usage. Each group of blocks consists of four units. Transform calculation direction is different for each block in a group. The transform results have a lot of fragments for which invariance for rotation is present for angles divisible by 90 degrees. This limitation is determined by a data representation rectangular grid. The consideration is based on two-dimensional digital cosine transform, but it could be used for different transform basics and any dimensions. The Hadamard matrix can be considered as such an example. The same algorithm could be used for compressed domain image retrievals based on DCT-processing.

1. Introduction
Data compression has a significant contribution to the computer science and telecommunications. Modern image processing algorithms and feature extraction techniques are developed in a spatial (pixel) domain [1]. The JPEG compressed image retrieval methods based on DCT coefficients have been elaborated in recent years. There are various approaches and systems of image retrieval and matching investigated in the paper. A particular attention was given to studying and confrontation of experimental results for some CBIR methods and systems using DCT and its features [2].

The digital cosine transform is an embedded part of the JPEG coder and decoder. Working directly in a frequency domain is still the most promising area for compressed image processing and retrieval [3]. In addition, DCT also preserves a set of good properties such as energy compacting and image data decorrelation. The image prefiltering is usually advantageous for constructing an image retrieval system.

2. DCT encoder usage
The widely used JPEG mode is called the baseline JPEG system, which is based on a sequential mode, DCT-based coding and Huffman coding for entropy encoding.

The two dimensional Fourier transform is a DCT basis [4]. The DCT has significant advantages over the Fourier transform. This is due to the following facts. DCT has only real numbers and fewer spectral components [5].

The even DCT has the following kind for square images with \( N \times N \) pixels [6]

\[
X(u, v) = \frac{2}{N} C(u)C(v) \sum_{j=0}^{N-1} \sum_{k=0}^{N-1} x(j, k) \cdot \cos \left( \frac{\pi}{N} u \left( j + \frac{1}{2} \right) \right) \cos \left( \frac{\pi}{N} v \left( k + \frac{1}{2} \right) \right).
\]

The inverse transform expression is determined as
\[ x(j,k) = \frac{2}{N} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} C(u)C(v)X(u,v) \cos \left( \frac{\pi}{N} j \left( u + \frac{1}{2} \right) \right) \cos \left( \frac{\pi}{N} k \left( v + \frac{1}{2} \right) \right). \]

This expressions allow separable transform, i.e., a two-dimensional transformation is replaced by a sequence of one-dimensional transforms by rows and then columns [7]. But we can work with a columns and rows order. Let us consider a separable approach.

A basis functions system can be represented in a two-dimensional kind. The blocks number increase, necessary for processing, will form a blocked transformation matrix. All blocks from the matrix are oriented in the same direction. In general, the number can be arbitrary and be determined only by a lines length or a samples number. There are no symmetry elements relatively vertical or horizontal matrices axes. It could be useful in processing of coefficients placed in a new symmetry disposition.

Let us consider the orthogonality condition applied to the block matrix. If there is a basis functions system can be represented in a two-dimensional kind. The blocks number increase, necessary for processing, will form a blocked transformation matrix. All blocks from the matrix are oriented in the same direction. In general, the number can be arbitrary and be determined only by a lines length or a samples number. There are no symmetry elements relatively vertical or horizontal matrices axes. It could be useful in processing of coefficients placed in a new symmetry disposition.

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Expression (1) defines identity matrices up to products coefficient \( K \). It follows from a definition and properties of block matrices [7]. In case of product units independently (each separate block of matrix of discrete cosine transform \( A_t \)), normalizing coefficient \( K \) disappears. The identity blocks usage does not increase a matrix rank, therefore a inverse matrix, which is required for the inverse transform may be singular. The described process allows producing individual units direct transform and place a result in the desired blocked matrix location.

\[ A_b = \begin{bmatrix} A & A & \ldots & A \\ A & A & \ldots & A \\ \vdots & \vdots & \ddots & \vdots \\ A & A & \ldots & A \end{bmatrix} \times \begin{bmatrix} A^T & A^T & \ldots & A^T \\ E & E & \ldots & E \\ \vdots & \vdots & \ddots & \vdots \\ E & E & \ldots & E \end{bmatrix} = K \begin{bmatrix} E & E & \ldots & E \\ E & E & \ldots & E \end{bmatrix}, \]

where \( A^T \) - transpondet matrix \( A \).

3. DCT encoder algorithm modification
A transform with symmetry elements needs the blocks to be placed reversing the rows and columns order. There are two types [8] of placement.

The first variant of placement of the basis function corresponds to expression

\[ A_{1b}(k,n) = C(k) \cos \left( \frac{\pi(2N - (2n+1)(2N - (k+1))}{2N} \right), \]

\[ A_{2b}(k,n) = C(k) \cos \left( \frac{\pi(2n+1)(2N - (k+1))}{2N} \right), \]

\[ A_{3b}(k,n) = C(k) \cos \left( \frac{\pi(2N - (2n+1)k)}{2N} \right), \]

\[ A_{4b}(k,n) = C(k) \cos \left( \frac{\pi(2n+1)k}{2N} \right). \]

The next type is

\[ A_{1b}(k,n) = C(k) \cos \left( \frac{\pi(2n+1)k}{2N} \right), \]

\[ A_{2b}(k,n) = C(k) \cos \left( \frac{\pi(2n+1)(2N - (k+1))}{2N} \right), \]

\[ A_{3b}(k,n) = C(k) \cos \left( \frac{\pi(2N - (2n+1)k)}{2N} \right), \]

\[ A_{4b}(k,n) = C(k) \cos \left( \frac{\pi(2N - (2n+1)(2N - (k+1))}{2N} \right). \]

Each group \( A_{bm} \) of four blocks \( A_{1b} \times A_{4b} \) placement is as follows:

\[ A_{bm} = \begin{bmatrix} A_{1b} & A_{2b} \\ A_{3b} & A_{4b} \end{bmatrix} \]

Expressions (2) and (3) are transform matrix formation basis consisting of four oriented basic DCT
functions groups. The graphic views of two dimensional DCT basis functions series are shown in figure 1. All sets contain $6 \times 6$ units, but part elements directions are different. The traditional JPEG compression system has one direction. It is shown in figure 1, a.

![Figure 1. Sets of the DCT matrix blocks: a – traditional set; b – first kind modification set; c – second kind modification set.](image1)

The second and third parts of figure 1 show oriented groups (4) basic functions sets (2) and (3) expressions, accordingly. The directions for two last cases are opposite to each other. Similar matrices have vertical, horizontal and diagonal symmetry axes. In this case, the whole matrix rotation or parts of four blocks groups could be rotated at $0^\circ$, $90^\circ$, $180^\circ$, $270^\circ$. It does not change in transform matrices. This algorithm, based on (2) or (3), can be involved in the JPEG system for realising the search in the entire two dimensional data set or in its parts [9]. The search is performed in the frequency domain that will reduce the spent time.

The orthogonality condition is as follows:

$$A_b = \begin{bmatrix} A_1 & A_2 & A_1 & A_2 & \cdots & A_1 & A_2 \\ A_3 & A_4 & A_3 & A_4 & \cdots & A_3 & A_4 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ A_3 & A_4 & A_3 & A_4 & \cdots & A_3 & A_4 \\ A_1 & A_2 & A_1 & A_2 & \cdots & A_1 & A_2 \\ \end{bmatrix} = K^*, \quad (5)$$

where $E_I$ - identity matrix; $E_r$ - identity matrix with a reverse rows order.

Execution results for expressions (1) and (5) for $6 \times 6$ units are shown in figure 2.

![Figure 2. The graphic representation of single matrices sets: a – traditional type; b – proposed type.](image2)

In the first part, there is the traditional identity matrices type. The identity matrices with an auxiliary diagonal are presented in the second part.
4. **Propagation techniques on other bases**
Same algorithm can be used with other transforms, such as Walsh, Karhunen-Loeve, etc.

Let us consider the Hadamard matrix. Hadamard matrices have simple matrix structures: they are square; they have entries +1 or −1, orthogonal row vectors and orthogonal column vectors [10]. Using the same procedure (2) and (3), block structures can be created with the rotation of invariant property. Graphic views of two-dimensional Hadamard basis functions series are shown in figure 3. All sets contain 6×6 units. The transform type, where all parts have one direction, is shown in figure 1, a. Other images have blocks with different directions.

![Figure 3. Sets of the Hadamard matrix blocks: a – traditional set; b – first kind modification set; c – second kind modification set.](image)

The Hadamard transform in a modification form can be used for data search. In addition, the proposed approach can be extended in any dimension.

5. **Conclusion**
The frequency transform implementation variants are proposed in the paper. They are used for more effective search algorithms. The JPEG compression coder can be modified using one of two embodiments of the discrete cosine transform. Expressions for those realizations are described. Each of these variants is more resistant to rotation than the standard technique used in the JPEG baseline version. The questions of the modified orthogonal transformation are considered. The proposed formulas were simulated. Simulation results confirm the validity of the proposed approach. The technique can be extended to other transforms, and universality of the approach allows using any multidimensional data sets.

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