CONVECTIVE INSTABILITY OF A RELATIVISTIC EJECTA DECELERATED BY A SURROUNDING MEDIUM: AN ORIGIN OF MAGNETIC FIELDS IN GAMMA-RAY BURSTS?

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ABSTRACT

Global linear stability analysis of a self-similar solution describing a relativistic shell decelerated by an ambient medium is performed. The system is shown to be subject to the convective Rayleigh–Taylor instability, with a rapid growth of eigenmodes having angular scale much smaller than the causality scale. The growth rate appears to be largest at the interface separating the shocked ejecta and shocked ambient gas. The disturbances produced at the contact interface propagate in the shocked media and cause nonlinear oscillations of the forward and reverse shock fronts. It is speculated that such oscillations may affect the emission from the shocked ejecta in the early afterglow phase of gamma-ray bursts, and may be the origin of the magnetic field in the shocked circum-burst medium.

Key words: gamma rays: bursts – hydrodynamics – instabilities – magnetic fields – relativity – turbulence

1. INTRODUCTION

Following a gamma-ray burst explosion and a rapid acceleration phase, the relativistic ejecta accumulated in the process starts decelerating, owing to its interaction with the surrounding medium. At early times a double shock structure forms, consisting of a forward shock that propagates in the ambient medium, a reverse shock crossing the ejecta, and a contact interface separating the shocked ejecta and the shocked ambient medium. In the fireball scenario commonly adopted, the naive expectation has been that the crossing of the reverse shock should produce an observable optical flash. Despite considerable observational efforts, such flashes seem to be rare. The afterglow emission, which is produced behind the forward shock, seems to indicate strong amplification of magnetic fields in the post shock region, by some yet unknown mechanism. Moreover, the light curve of the afterglow emission deviates at early time from that predicted by a simple blast wave model.

A question of considerable interest is the stability of the double shock system. Hydrodynamic instabilities may lead to strong distortions of the system that may generate turbulence, amplify magnetic fields, and affect the emission processes in the afterglow phase. Such effects have been studied in the non-relativistic case in connection with young supernova remnants (SNRs). In fact, the idea that the Rayleigh–Taylor (R–T) instability may play an important role in the deceleration of non-relativistic ejecta dates back to Gull (1973), who performed one-dimensional simulations of young SNRs that incorporate a simple model of convection. Chevalier et al. (1992) performed a global linear stability analysis of a self-similar solution describing the interaction of a nonrelativistic ejecta with an ambient medium and found that it is subject to a convective instability. They analyzed self-similar perturbations and showed that the flow is unstable for modes having angular scale smaller than some critical value. The convective growth rate was found to be largest at the contact discontinuity surface and to increase with increasing number of the eigenmodes. They also performed two-dimensional hydrodynamical simulations that verified the linear results and enabled them to study the nonlinear evolution of the instability. The simulation exhibits rapid growth of fingers from the contact interface that saturates, in the nonlinear state, by the Kelvin–Helmholtz (K–H) instability. Strong distortion of the contact and the reverse shock was observed with little effect on the forward shock. Jun & Norman (1996) performed two- and three-dimensional MHD simulations of the instability to study the evolution of magnetic fields in the convection zone. They confirmed the rapid growth of small scale structure reported by Chevalier et al. (1992), and in addition found strong amplification of ambient magnetic fields in the turbulent flow around R–T fingers. On average, the magnetic field energy density reaches about 0.5% of the energy density of the turbulence, but it could well be that the magnetic field amplification was limited by numerical resolution in their simulations. The simulations of Chevalier et al. (1992) and Jun & Norman (1996) support earlier ideas that the clumpy shell structure observed in young (pre-Sedov stage) SNRs such as Tycho, Kepler, and Cas A is due to the R–T and K–H instability.

In this Letter we extend the linear stability analysis of Chevalier et al. (1992) into the relativistic regime. We find that denser ejecta sweeping a lighter ambient gas are subject to the R–T instability also in the relativistic case. The stability of a double-shock system has been investigated by Wang et al. (2002) using the thin shell approximation. However, this study is limited to large-scale modes and neglects pressure gradients and, therefore, excludes the convective instability. The role of the R–T instability in magnetized photon-rich shells has been considered by Thompson (2006). Gruzinov (2000) performed a linear stability analysis of a Blandford–McKee (BMK) blast wave solution, and found that the BMK solution is stable but non-universal, in the sense that some modes decay very slowly as the system evolves. Furthermore, the onset of oscillations of an eigenmode of order $l$ has been seen in the simulation once the Lorentz factor evolved to $\Gamma < l$. The conclusion drawn based on Gruzinov’s findings is that distortion of the shock front at early times may cause significant oscillations during a large portion of its evolution. If the amplitude of these oscillations is sufficiently large and if the same behavior holds in the nonlinear regime, then this can lead to generation of vorticity in the post shock region (Milosavljevic et al. 2007; Goodman & MacFadyen 2008), and the consequent amplification of magnetic fields, as demonstrated recently by Zhang et al. (2009).

2. ANALYSIS

We consider the interaction of a cold unmagnetized shell with a cold ambient medium having a density profile $\rho_1 = br^{-k}$. 

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The structure formed at early stages consists of a forward shock propagating in the ambient medium, a reverse shock propagating in the ejecta and a contact discontinuity separating the shocked ambient medium and the shocked ejecta. The equations governing the dynamics of the flow on each side of the contact interface can be written in the form

$$
\rho h \gamma^2 \frac{d \ln \gamma}{dt} + \gamma \frac{d P}{dt} = \frac{\partial P}{\partial t},
$$

(1)

$$
\frac{d}{dt} \ln \left( P / \rho \gamma \right) = 0,
$$

(2)

$$
\rho \gamma \frac{d}{dt} (h \gamma v_T) + \nabla_T P = 0,
$$

(3)

where $\gamma = u^0$ is the Lorentz factor of the fluid, $v_T$ is the tangential component of the 3-velocity (which we express as $v = v_r \hat{r} + v_T \hat{T}$), $\rho$, $P$, and $h$ are the proper density, pressure, and specific enthalpy, respectively, $\gamma$ is the adiabatic index, and $\dot{\gamma}$ is the self-similarity parameter, and $\nabla_T$ is the adiabatic index.

Equations (1)–(3) admit self-similar solutions (Nakamura & Shigeyama 2006) in cases of a freely expanding ejecta characterized by a velocity $v_r = r/t$ at time $t$ after the explosion, and a proper density profile

$$
\rho_e = \frac{a}{t^3 \gamma_e^m},
$$

(5)

where $\gamma_e = 1/\sqrt{1 - v_T^2}$ is the corresponding Lorentz factor of the unshocked ejecta. For this unperturbed solution $v_T = \nabla_T = 0$ and Equation (3) is redundant. The Lorentz factors of the forward shock, reverse shock, and the contact discontinuity surface, denoted by $\Gamma_f(t)$, $\Gamma_r(t)$, and $\Gamma_c(t)$, respectively, evolve with time as $\Gamma_f^2 = A \Gamma^m$, $\Gamma_r^2 = B \Gamma^m$, $\Gamma_c^2 = C \Gamma^m$, with the constants $A$, $B$, $C$, and $m$ determined by matching the solutions in the shocked ejecta and in the shocked ambient medium at the contact discontinuity. In particular, $m = (6 - 2k)/(m + 2)$ (Nakamura & Shigeyama 2006). The velocity of the ejecta just upstream the reverse shock is $v_r = r_0(t) = \rho v(r) = r_0(t)/t = 1 - 1/[2(m + 1) \Gamma_c^2]$, where $r_0(t) = \int (1 - 1/2 \Gamma_c^2) dt$ is the trajectory of the reverse shock, implying $\gamma_e^2 = (m + 1) \Gamma_c^2$. Thus, the self-similar solution is applicable only to situations where the ejecta is sufficiently dense, such that the reverse shock is non or at most mildly relativistic.

We have carried out a global linear stability analysis of the self-similar solution outlined above. The technical details are presented elsewhere (Levinson 2009). A brief account of the method and results is given below. This Letter focuses on the astrophysical implications of the R–T instability.

There are total of eight independent variables, four on each side of the contact: $P$, $\rho$, $\gamma$, $v_T$. The perturbations of these variables were expanded in spherical harmonics. To be more concrete, a perturbed quantity $Q$ ($Q = P$, $\rho$, etc.) is expressed as $Q(t, \chi, \theta, \phi) = Q_0(t, \chi) [1 + \xi_Q(\chi, t) Y_{lm}(\theta, \phi)]$, where $\chi = [1 + 2(m + 1) \Gamma_c^2] (1 - r/t)$ is the self-similarity parameter, and $Q_0$ denotes the unperturbed value. The linearized equations on each side of the contact discontinuity were then obtained upon substitution of the perturbed quantities into Equations (1)–(3).

Perturbations of the shock fronts and the contact discontinuity of the form

$$
\delta r_j(t, \theta, \phi) = \frac{t \delta f_j(t)}{\Gamma_j} Y_{lm}(\theta, \phi),
$$

(6)

where $j = 1, 2, c$ refers to the forward shock, reverse shock, and the contact discontinuity, respectively, were assumed. The perturbed shock normals are then $\xi_{k\mu} = \xi_0^{k\mu} + \delta \xi_{k\mu}$ ($k = 1, 2$), with $\xi_0^{k\mu} = (-\Gamma_k s, \Gamma_k, 0)$ being the unperturbed normal and

$$
\delta n_{k\mu} = (-\Gamma_k^2 s V_k, \Gamma_k s V_k, -\Gamma_k s \hat{n}_k).
$$

(7)

Here $V_k$ denotes the 3-velocity of the unperturbed shock front and $\delta V_k = d\delta s_k/dt$. The linearized jump conditions at the forward and reverse shocks are given in terms of the 4-velocity $u^\mu$ and the energy-momentum tensor $T^{\mu\nu}$ as

$$
\left[ \rho u_\mu \delta n_{k\mu} + \Delta_k (\rho u^0 n_{k\nu}^0) \right] = 0,
$$

(8)

$$
\left[ T_{\mu\nu}^0 \delta n_{k\mu} + \Delta_k T_{\mu\nu} n_{k\nu}^0 \right] = 0.
$$

(9)

where $\Delta_k = \delta Q = (\delta Q/\delta \xi_k) \delta \xi_k$ denotes the Lagrange perturbation of the quantity $Q$ at the perturbed surface $k$. The relations (8) and (9) provide six boundary conditions for the perturbation equations, 3 on each side of the contact. Two additional boundary conditions are obtained from pressure balance and the no flow condition, viz., $v = dr_j/dt = 0$, at the contact discontinuity.

Unlike in the non-relativistic case, the boundary conditions at the shock fronts break self-similarity of the perturbations. The main reason is the inherent coupling, via the Lorentz factor, between the radial and tangential velocity perturbations. Specifically, it can be shown that at the forward shock surface the perturbation of the tangential velocity is related to the perturbations of the radial velocity and pressure through $\xi_T = 3(\xi_T - \xi_R)/(2(m + 2) \Gamma_c^2)$ for $l \neq 0$, with a similar relation, though somewhat more involved, at the reverse shock front. Hence, numerical simulations of the perturbation equations are needed. The only exception is the spherical mode ($l = 0$) for which a self-similar solution was obtained analytically. For this solution $\delta Q \propto t^s$ with $s < 0$. It can be shown that one eigenmode of order $l = 0$ is associated with linear time translation of the self-similar solution. For this mode $s = -(m + 1)$. We found another eigenmode of order $l = 0$ that decay somewhat slower. The analytic solution for the $l = 0$ mode has been used both to test the code and as initial condition for the evolution of the higher order eigenmodes. Numerical integration of the perturbation equations was performed after transforming to the so-called Riemann invariants. In the region containing the shocked ambient medium they are related to the dimensionless perturbations of radial velocity, $\xi_k$, pressure, $\xi_k$, density $\xi_k$, and tangential velocity, $\xi_T$, through $\xi_k = \xi_k / \sqrt{3} / \xi_T^2$, $\xi_3 = \xi_3 - \xi_R / 2 - \xi_T / 4$, $\xi_4 = \xi_4$. Similar relations are obtained in the shocked ejecta, though somewhat more involved (Levinson 2009). We identified three variables ($\xi_1$, $\xi_2$, $\xi_3$) that propagate from the forward shock to the contact, one ($\xi_4$) that propagates from the contact to the forward shock, three that propagate from the reverse shock to the contact, and one that propagates from the contact to the reverse shock, and have chosen the boundary conditions for the Riemann invariants accordingly. It is generally found that eigenmodes having an angular scale larger than the horizon scale, specifically $l(l + 1) < \Gamma^2$, are stable.
order modes are found to be unstable with a growth rate that increases with increasing \( l \). An example is exhibited in Figure 1. The onset of oscillations, resulting from sound waves crossing, followed by a rapid growth of the initial perturbations is clearly seen. The difference between the oscillation frequencies on each side of the contact is due to the vastly different sound speeds of the shocked ambient medium and shocked ejecta. The distortion of the contact discontinuity surface becomes nonlinear very early on. The growth is algebraic in time \( t \) with a growth rate of about 10 in the example shown in Figure 1; that is, \( \delta Q \propto t^{10} \) for \( Q = P, \rho, v \). As seen, the reverse shock responds quickly to the distortion of the contact. The forward shock, on the other hand, responds much later, at time when the instability near the contact already reached a nonlinear state. From our numerical simulations we find the scaling \( s = A\sqrt{l}/\Gamma_{10} \), with the constant \( A \) depending predominantly on \( m \). For \( m = 0.645 \), corresponding to the choice of parameters in Figure 1, we find \( A \approx 1 \), and for \( m = 2 \) we find \( A = 2.45 \). This scaling is in accord with the growth rate derived heuristically in Thompson (2006; see detailed discussion in Levinson 2009).

3. DISCUSSION

The stability analysis described above seems to indicate strong convective instability at early stages of the evolution of a dense ejecta as it sweeps a lighter ambient gas. The growth rate appears to be largest at the contact discontinuity and for higher order modes. Disturbances at the interface separating the shocked ejecta and the shocked ambient medium propagate away from the contact discontinuity and cause nonlinear distortions of the shock fronts. The reverse shock responds quickly to the distortion of the contact. Propagation of the signal to the forward shock is much slower. At any rate, the instability near the contact becomes nonlinear well before the signal arrives at the forward shock, so full MHD simulations are needed to resolve the effect of the instability on the forward shock. It is naively expected that the instability will be strongly suppressed in cases where the ejecta is highly magnetized (see Thompson 2006) and/or if the reverse shock is highly relativistic. On the other hand, if the magnetic field strength in the unshocked ejecta is smaller than that required to suppress the instability but still much larger than that of the ambient medium, then at early stages mixing of the magnetized ejecta with the shocked ambient gas via growth of R–T fingers can give rise to a strong amplification of the magnetic fields behind the forward shock. Full simulations are required to quantify the conditions under which the instability is effective.

The nonlinear distortions of the contact and the shock fronts should generate turbulence in the shocked fluids on both sides of the contact discontinuity. At early stages this may strongly affect particle acceleration and the emission processes. It is tempting to speculate that the lack of observed optical flashes, that are anticipated in the “standard” model, and the fact that the early afterglow emission observed in many sources is inconsistent with the prediction of the blast wave model may be attributed to the instability discussed here. In any case, it is clear that a careful analysis that takes account of this process is required to better understand the observational characteristics of the emission during the early afterglow phase.

The stability analysis of the BMK solution performed by Gruzinov (2000) suggests that it may be a very slow attractor.

Figure 1. Time evolution of the perturbations for \( n = 1.1, k = 2 \) and \( l(l+1)/\Gamma_{10}^2(t_0) = 10^4 \), here \( \Gamma_{10}(t_0) \) is the initial Lorentz factor of the forward shock. Upper panels: perturbation of the contact discontinuity surface (left panel) and relative pressure perturbation, \( \zeta_{pc} = \delta P_c(t)/\delta P_c(t_0) \), of the shocked ambient medium at the contact (right panel). Bottom panels: perturbations of the forward (solid line) and reverse (dashed line) shock surfaces (left), and relative pressure perturbation at the forward \( (\zeta_{pf}) \) and reverse \( (\zeta_{pr}) \) shock fronts (right). The initial perturbations of the contact discontinuity and the shock surfaces in this example are \( \delta r_c/r_c = 6 \times 10^{-3} \) and \( \delta r_{f01}/r_{f01} = 10^{-3} \), respectively.
Linear perturbations of the forward shock in the BMK phase decay very slowly. Whether this behavior continues also in the nonlinear regime is unclear yet. If it does then it is anticipated that the growth of R–T fingers and, perhaps, nonlinear oscillations of the forward shock itself that are induced by the convective instability may be a source of vorticity during a long portion of the evolution of the blast wave. As demonstrated recently by Zhang et al. (2009), the induced turbulence can amplify weak magnetic fields. Their simulation seems to converge at a saturation level of $\epsilon_B \sim 5 \times 10^{-3}$, weakly dependent on the initial magnetic field strength. This process may provide an explanation for the origin of the strong magnetic fields inferred behind the collisionless shock in the afterglow phase.

Unfortunately, the linear analysis outlined above is restricted to a limited set of conditions under which the unperturbed self-similar solution of Nakamura & Shigeyama (2006) is applicable. Full three-dimensional MHD simulations are required to study this process in other situations, and to follow the evolution of the convective instability in the nonlinear state. As illustrated above, high-resolution simulations that can resolve angular scales $\Delta \theta \ll 1/\Gamma$ are required, posing a great numerical challenge. We believe that our findings strongly motivate such efforts.

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