The Characterizations of $\delta$ – Algebras with Their Ideals

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Abstract. We explored new algebra structures such as $\delta$ –algebra, $\delta$ –subalgebra and $\delta$ –ideal in this work. In addition, we study their fundamental characteristics and their connections with other algebras, such as $d/\rho/BCK$ – algebras, $d/\rho/BCK$ – subalgebras and $d/\rho/BCK$ – ideals. Also, some examples to illustrate our notions are given.

MSC (2010): 20G05, 20D06, 20B30, 20B35

Keywords: $d$-algebra, BCK-algebra, $\rho$ – algebra, Ideals, $d$-subalgebra.

1. Introduction

$BCK$ – Algebra [1] and $BCI$ – algebra [2] were considered by Imai and Iseki. After that, in 1983, Hu showed $BCH$ – algebras [3] and Li [4] addressed it in 1985. Next, some new algebra forms are seen ([5]-[26]). The construction of $d$ – algebra, a further extension of BCK-algebras, was demonstrated in 1999, see [27]. Moreover, Jun, Neggers and Kim [28] demonstrate the construction of the $d$ – ideal in $d$ – algebra. They also considered fuzzy algebra constructions such as fuzzy $d/B/BCCI$ – algebras, fuzzy $d/d^\rho$ – ideals and the relationships between them are seen ([29]-[31]).

The constructions of $\rho$ – algebra, $\rho/\rho$ – ideals, $\rho$ – subalgebra and permutation topological $\rho$ – algebra were first proposed by Khalil and Abdul Alradha in 2017 [5]. Next, the concepts of soft $\rho$ – algebra and soft edge $\rho$ – algebra [6] were demonstrated.
In this study, we explored new algebra structures such as $\delta$–algebra, $\delta$–subalgebra and $\delta$–ideal. We study their fundamental characteristics and their connections with other algebras, such as $d/\rho/BCK$–algebras, $d/\rho/BCK$–subalgebras and $d/\rho/BCK$–ideals. In addition, some examples to illustrate our notions are given.

2. Preliminaries

We remember the basic description and data that are required in our work in this section.

**Definition 2.1:** [27] A $d$–algebra $(\eta, \alpha, f)$ is a set $\eta \neq \emptyset$ and fulfilling the following assumptions with binary operation $\alpha$ and a fixed $f$:

i) $\alpha\alpha\eta = f$

ii) $\alpha\alpha\eta = f$

iii)$\alpha\alpha\eta = f$ and $\alpha\alpha\eta = f \rightarrow \alpha = \eta$, for all $\eta, \alpha \in \eta$.

**Remark 2.2:**[27] For any $d$–algebra $\eta$. We say $\eta$ is finite $d$–algebra if $\eta$ is a finite set.

**Definition 2.3:** [28] For any $d$–algebra $(\eta, \alpha, f)$, we say it is $BCK$–algebra, if $\eta$ such that following assumptions:

(1). $(\alpha\eta\alpha)(\alpha\eta\alpha) = f$

(2). $(\alpha\eta\alpha)(\alpha\eta\alpha) = f$, for all $\alpha, \eta \in \eta$.

**Definition 2.4:** [28] For any $\phi \neq \emptyset \subseteq \eta$. We say $\emptyset$ is a $d$–subalgebra $(d$–SA) of $\eta$ if $\alpha\eta\eta \in \emptyset$, $\forall \alpha, \eta \in \emptyset$ and $(\eta, \alpha, f)$ is a $d$–algebra.

**Definition 2.5:** [28] For any $\phi \neq \emptyset \subseteq \eta$. We say $\emptyset$ is a $d$–ideal $(d$–I) of $\eta$ if $(\eta, \alpha, f)$ is a $d$–algebra and fulfilling the following assumptions:

(1). $\alpha\eta\eta \in \emptyset$ and $\alpha \in \emptyset \rightarrow \alpha \in \emptyset$,

(2). $\alpha \in \emptyset$ and $\eta \in \emptyset \rightarrow \alpha\eta\eta \in \eta$, $\forall \alpha, \eta \in \eta$.

**Definition 2.6:** [28] For any $\phi \neq \emptyset \subseteq \eta$. We say $\emptyset$ is a $BCK$–ideal $(BCK$–I) of $\eta$ if $(\eta, \alpha, f)$ is a $BCK$–algebra and fulfilling the following assumptions:

(1). $\alpha \in \emptyset$,

(2). $\alpha\eta\eta \in \emptyset$ and $\eta \in \emptyset \rightarrow \alpha \in \emptyset$, for all $\alpha, \eta \in \eta$.

**Definition 2.7:** [5] We say $(\eta, \alpha, f)$ is a $\rho$–algebra if $\eta \in \eta$ and fulfilling the following assumptions:

i) $\alpha\eta\eta = f$

ii) $\alpha\eta\eta = f$

iii)$\alpha\eta\eta = f$ and $\alpha\eta\eta = f \rightarrow \alpha = \eta$, for all $\eta, \alpha \in \eta$.

iv) For all $\eta \neq \emptyset \in \eta$–$\{f\} \rightarrow \alpha\eta\eta = \alpha\eta\eta \neq f$.
Note: Any $\rho$-algebra is $d$-algebra. Nevertheless, the opposite is not true in general.

**Definition 2.8:** [5] For any $\phi \neq H \subseteq \Omega$. We say $H$ is a $(\rho-SA)$ of $\eta$ if $\nu \omega \in H$, $\forall \nu, \omega \in H$ and $(\Omega, o, f)$ is a $\rho$-algebra.

**Definition 2.9:** [5] For any $\phi \neq \xi \subseteq \eta$. We say $\xi$ is a $(\rho-I)$ of $\eta$ if $(\eta, o, f)$ is a $\rho$-algebra and fulfilling the following assumptions:

1. $\nu, \omega \in \xi$ imply $\nu \omega \in \xi$,
2. $\nu \omega \in \xi$ and $\omega \in \xi$ imply $\nu \in \xi$, $\forall \nu, \omega \in \eta$.

**Definition 2.10:** [5] For any $\phi \neq \xi \subseteq \eta$. We say $\xi$ is a $(\rho-I)$ of $\eta$ if $(\eta, o, f)$ is a $\rho$-algebra and fulfilling the following assumptions:

1. $f \in \xi$,
2. $\nu \in \xi$ and $\omega \in \eta \rightarrow \nu \omega \in \xi$, $\forall \nu, \omega \in \eta$.

**Definition 2.11:** [28]
Assume that $(\eta, o, e)$ is a $d$-algebra. If $\eta$ satisfies the identity $(\nu \omega \omega)\omega \nu = e$, then $\eta$ is said to be $d^*$-algebra.

**Remark 2.12:** [28]
In $d^*$-algebra any BCK-algebra is $d$-ideal and $d$-subalgebra.

3. Characterizations of $\delta$-algebra

In this section, new definitions such as $\delta$-algebra, $\delta$-ideal and fuzzy $\delta$-subalgebra in algebra will be introduced and their relationships with other algebras will be studied.

**Definition 3.1:**
We say $(\eta, o, f)$ is a $\delta$-algebra if $f \in \eta$ and fulfilling the following assumptions:

i) $\nu \omega = f$

ii) $f = f$

iii) $\nu \omega = f$ and $\omega \nu = f \rightarrow \omega = \nu$, for all $\nu, \omega \in \eta$.

iv) For all $\nu \neq \omega \in \eta - \{f\} \rightarrow \nu \omega \omega = \omega \nu \omega \neq f$.

v) For all $\nu \neq \omega \in \eta - \{f\} \rightarrow (\nu \omega (\omega \sigma))\omega (\sigma \omega) = f$
Example 3.2: Suppose that \( \eta = \{ f, \nu, \omega, \sigma \} \) and the binary operation \( \circ \) is described as a table (1)

\[
\begin{array}{cccc}
\circ & f & \nu & \omega & \sigma \\
\hline
f & f & f & f & f \\
\nu & \nu & f & \nu & \nu \\
\omega & \omega & \nu & f & \nu \\
\sigma & \sigma & \nu & \nu & f \\
\end{array}
\]

"Table (1)"

Hence \( (\eta, \circ, f) \) is a \( \delta \) – algebra

Remarks 3.3:

(1) Any \( \delta \) – algebra will be \( d \) – algebra. Nevertheless, the opposite is not true in general.
(2) Any \( \delta \) – algebra will be \( \rho \) – algebra. Nevertheless, the opposite is not true in general.

Example 3.4: Suppose that \( \eta = \{ f, \nu, \omega, \sigma \} \) and the binary operation \( \circ \) is described as a table (2)

\[
\begin{array}{cccc}
\circ & f & \nu & \omega & \sigma \\
\hline
f & f & f & f & f \\
\nu & \nu & f & \nu & \nu \\
\omega & \omega & \omega & f & \sigma \\
\sigma & \sigma & \sigma & \sigma & f \\
\end{array}
\]

"Table (2)"

Hence \( (\eta, \circ, f) \) is a \( d \) – algebra. However, it is not \( \delta \) – algebra, since \( \nu \neq \omega \in \eta \setminus \{ f \} \) and \( \nu \circ \omega \neq \omega \circ \nu \). Also, if \( \circ \) is described as a table (3)

\[
\begin{array}{cccc}
\circ & f & \nu & \omega & \sigma \\
\hline
f & f & f & f & f \\
\nu & \nu & f & \nu & \omega \\
\omega & \omega & \nu & f & \nu \\
\sigma & \sigma & \omega & \nu & f \\
\end{array}
\]

"Table (3)"

Then \( (\eta, \circ, f) \) is a \( \rho \) – algebra. However, it is not \( \delta \) – algebra, since \( \nu \neq \sigma \in \eta \setminus \{ f \} \) and \( \nu \circ (\nu \circ \sigma) = (\nu \circ \omega) \circ \nu = \nu \circ \omega \neq \nu \neq f \).

Definition 3.5: Assume that \( \phi \neq H \subseteq \Omega \), where \( (\Omega, \circ, f) \) is a \( \delta \) – algebra, we say that \( H \) is \( \delta \) – subalgebra (\( \delta \) – SA) of \( \Omega \) if \( \nu \circ \omega \in H \), whenever \( \nu \in H \) and \( \omega \in H \).

Theorem 3.6: If \( (\Omega, \circ, f) \) is a \( \delta \) – algebra, then \( H \) is \( d \) – SA of \( \Omega \), whenever \( H \) is \( \delta \) – SA of \( \Omega \).

Proof: Assume \( H \) is \( \delta \) – SA of \( \delta \) – algebra \( \Omega \). Hence we have \( \Omega \) is \( d \) – algebra and \( H \) satisfies \( \nu \circ \omega \in H \) whenever \( \nu \in H \) and \( \omega \in H \). Thus \( H \) is \( d \) – SA of \( \delta \) – algebra \( \Omega \).

Theorem 3.7: If \( (\Omega, \circ, f) \) is a \( \delta \) – algebra, then \( H \) is \( \rho \) – SA of \( \Omega \), whenever \( H \) is \( \delta \) – SA of \( \Omega \).

Proof: Assume \( H \) is \( \delta \) – SA of \( \delta \) – algebra \( \Omega \). Hence we have \( \Omega \) is \( \rho \) – algebra and \( H \) satisfies \( \nu \circ \omega \in H \) whenever \( \nu \in H \) and \( \omega \in H \). Thus \( H \) is \( \rho \) – SA of \( \rho \) – algebra \( \Omega \).
Remarks 3.8:

(1) Theorem (3.6) show that any $\delta - SA$ is $\delta - SA$. Nevertheless, the opposite is not true in general.

(2) Theorem (3.7) show that any $\delta - SA$ is $\rho - SA$. Nevertheless, the opposite is not true in general.

Example 3.9: Suppose $\Omega = \{1,2,3,4,5\}$ is a $d$-algebra, where $\omega$ is described as a table (4):

| $\omega$ | 1 | 2 | 3 | 4 | 5 |
|----------|---|---|---|---|---|
| 1        | 1 | 1 | 1 | 1 | 1 |
| 2        | 2 | 1 | 1 | 2 | 1 |
| 3        | 3 | 3 | 1 | 1 | 3 |
| 4        | 4 | 4 | 4 | 1 | 4 |
| 5        | 5 | 5 | 5 | 5 | 1 |

"Table (4)"

Then $H = \{1,4\}$ is $d - SA$ of $\Omega$. However, $\Omega$ is not $\delta$-algebra, so $H = \{1,4\}$ is not $\delta - SA$ of $\Omega$.

Example 3.10:

See table (3) in Example (3.4), we have $H = \{f, u, \omega\}$ is $\rho - SA$ of $\eta$. However, $\eta$ is not $\delta$-algebra, so $H = \{f, u, \omega\}$ is not $\delta - SA$ of $\eta$.

Proposition 3.11:

Let $\Omega = \{1,2,\ldots,n\}$, $|\Omega| = n$ and define ($\bullet$) on $\Omega$ as follows:

$$
\nu \bullet \omega = \begin{cases} 
1, & \text{if } \nu = \omega \text{ or } \nu = 1 \\
\nu, & \text{if } \nu > \omega \neq 1, \\
\nu, & \text{if } \omega < \nu 
\end{cases}
$$

for all $\nu, \omega \in \Omega$. Then $(\Omega, \bullet, 1)$ is $\delta$-algebra.

Proof:

(3) Since $\nu \bullet \omega = \begin{cases} 
1, & \text{if } \nu = \omega \text{ or } \nu = 1 \\
\nu, & \text{if } \omega < \nu 
\end{cases}$, and $\forall \nu, \omega \in \Omega$. Then for each $n > 1$, the following are hold:

i)- $\nu \bullet 1 = 1$,  
ii)- $1 \bullet \nu = 1$,  
iii)- $\nu \bullet \omega = 1$ and $\omega \bullet \nu = 1 \Rightarrow \nu = \omega \forall \nu, \omega \in \Omega$.  
iv)- For any $\nu \neq \omega \in \Omega - \{1\}$, if $\omega > \nu \rightarrow \nu \bullet \omega = \omega = \omega \bullet \nu$, and $\nu \bullet \omega = \nu \neq \omega \bullet \omega$, if $t < c$.

Hence $\nu \bullet \omega = \omega \bullet \nu \neq 1$
v)- For any \( \nu \neq \omega \in \Omega - \{1\} \), if \( \omega > \nu \) we have \((\nu \cdot (\nu \cdot \omega)) \cdot (\omega \cdot \nu) = (\nu \cdot \omega) \cdot (\nu \cdot \omega) = 1 \) and \((\nu \cdot (\nu \cdot \omega)) \cdot (\omega \cdot \nu) = (\nu \cdot \nu) \cdot (\omega \cdot \nu) = 1 \cdot (\omega \cdot \nu) = 1 \), if \( \omega < \nu \). Hence \((\nu \cdot (\nu \cdot \omega)) \cdot (\omega \cdot \nu) = 1 \). Then \((\Omega, \cdot, 1)\) is \( d^{n}-\text{algebra}. \)

Notes Referring to \( d^{n} \)-Algebra 3.12:
Let \( n = \nu > 2 \), where \( \nu \) is a prime. So \((\Omega, \cdot, 1)\) is a finite \( d^{n} \)-algebra, where \( \cdot \) is described as a table (5):

| \( \cdot \) | 1 | 2 | 3 | 4 | 5 | \( \nu \cdot 3 \) | \( \nu \cdot 2 \) | \( \nu \cdot 1 \) | \( \nu \) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | \( \nu \) |
| 2 | 2 | 1 | 3 | 4 | 5 | \( \nu \cdot 3 \) | \( \nu \cdot 2 \) | \( \nu \cdot 1 \) | \( \nu \) |
| 3 | 3 | 3 | 1 | 4 | 5 | \( \nu \cdot 3 \) | \( \nu \cdot 2 \) | \( \nu \cdot 1 \) | \( \nu \) |
| 4 | 4 | 4 | 4 | 1 | 5 | \( \nu \cdot 3 \) | \( \nu \cdot 2 \) | \( \nu \cdot 1 \) | \( \nu \) |
| 5 | 5 | 5 | 5 | 5 | 1 | \( \nu \cdot 3 \) | \( \nu \cdot 2 \) | \( \nu \cdot 1 \) | \( \nu \) |

It is obvious that for each \( 2 \leq i \leq \nu - 1 \), \((H_{i}, \cdot, 1)\) is \( d^{n} \)-SA of finite \( d^{n} \)-algebra \((\Omega, \cdot, 1)\), where \( H_{i} = \{1, 2, \ldots, i\} \). In addition, \( \forall \nu \neq \omega \in \Omega - \{1\} \) and \( \nu < \omega \), we consider that \((\nu \cdot \omega) \cdot \nu = \omega \cdot \nu = \omega \cdot 1 = 1 \). So there is no need for \( d^{n} \)-algebra to be \( d^{n} \)-algebra.

Now, if \( \Omega = \{1, 2, 3, 4\} \) let's use the table below:

| \( \omega \) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 1 | 2 | 2 |
| 3 | 3 | 2 | 1 | 2 |
| 4 | 4 | 2 | 2 | 1 |

"Table (5)"

"Table (6)"
Then \((\Omega, o, 1)\) is a \(\delta\)-algebra. Moreover, \((\Omega, o, 1)\) is not \(BCK\)-algebra, since 
\[((3o4)o(3o1))o(1o4) = (2o3)o1 = 2o1 = 2 \neq 1\). Therefore \(\delta\)-algebra need not be \(BCK\)-algebra.

**Definition 3.13:**
Assume \((\eta, o, f)\) is a \(\delta\)-algebra and \(\phi \neq \mathcal{I} \subseteq \eta\). Then \(\mathcal{I}\) is said to be \(\delta\)-ideal \((\delta-I)\) of \(\delta\)-algebra \(\eta\) if:

(1). \(\nu, \omega \in \mathcal{I} \Rightarrow \nu \omega \in \mathcal{I}\)

(2). \(\nu \omega \in \mathcal{I}\) and \(\omega \in \mathcal{I} \Rightarrow \nu \in \mathcal{I} \forall \nu, \omega \in \eta\).

**Example 3.14:**
For any \(\delta\)-algebra \(\eta\), we have \(\eta\) and \(\{f\}\) are \(\delta\)-ideals. Also, any \(\delta-I\) of \(\delta\)-algebra \(\eta\) is a \(\delta\)-SA.

**Lemma 3.15:** Every \(\delta-I\) is \((\nu/d)\)-SA.

**Proof:**
Let \(\mathcal{I}\) be \(\delta-I\) of \(\delta\)-algebra \((\eta, o, f)\), then from [Theorems (3.6) - (3.7)] we have \((\eta, o, f)\) is \((\nu/d)\)-algebra and hence from [Definition 3.13- (1)] we consider that \(\mathcal{I}\) is \((\nu/d)\)-SA.

**Theorem 3.16:** Every \(d-I\) in \(\delta\)-algebra \((\eta, o, f)\) is \(\delta-I\).

**Proof:**
Let \(\mathcal{I}\) be a \(d-I\) in \(\delta\)-algebra \((\eta, o, f)\). We want to show that:

(1). \(\nu, \omega \in \mathcal{I} \Rightarrow \nu \omega \in \mathcal{I}\)

(2). \(\nu \omega \in \mathcal{I}\) and \(\omega \in \mathcal{I} \Rightarrow \nu \in \mathcal{I} \forall \nu, \omega \in \eta\)

Since \(\mathcal{I}\) is the \(d-I\), condition (2) is held. Even \(\forall \nu, \omega \in \mathcal{I}\), we have \(\nu \in \mathcal{I}\) and \(\omega \in \eta\) (since \(\mathcal{I} \subseteq \eta\)). This implies that \(\nu \omega \in \mathcal{I}\) [By definition (2.9)-(2)]. Also, since \((\eta, o, f)\) is \(\delta\)-algebra then \(\mathcal{I}\) is \(\delta-I\) for us.

**Theorem 3.17:**
Every \(d-I\) in \(\delta\)-algebra \((\eta, o, f)\) is \(\delta-I\).

**Proof:** By [Definition (2.5)-(2)] and by the same way in proof theorem (3.16), we have every \(d-I\) in \(\delta\)-algebra \((\eta, o, f)\) is \(\delta-I\).

**Remark 3.18:** The opposite of theorem (3.17) is not true in general.
Example 3.19: Assume $\Omega = \{1,2,3,4,5\}$ is a $d$–algebra, where * is described as a table (7):

$$
\begin{array}{ccccc}
* & 1 & 2 & 3 & 4 & 5 \\
1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 1 & 2 & 1 & 2 \\
3 & 3 & 3 & 4 & 1 & 4 \\
4 & 4 & 4 & 3 & 1 & 4 \\
5 & 4 & 4 & 2 & 2 & 1 \\
\end{array}
$$

"Table (7)"

Then $\mathcal{G} = \{1,2\}$ is a $d$–1 of $\Omega$. Also, $\Omega$ is not $\delta$–algebra, thus $\mathcal{G} = \{1,2\}$ is not $\delta$–1 of $\Omega$.

Lemma 3.20: the intersection of a family of $\delta$–1 in $\delta$–algebra $\Omega$ is a $\delta$–1 in $\Omega$.

Proof:

Let $P_i$ , $i \in \mathcal{X}$ be a $\delta$–1 of $\Omega$ and let $\nu, \omega \in \bigcap_{i \in \mathcal{X}} P_i$ , then $\nu, \omega \in P_i$, so $\nu \omega \in P_i \forall i \in \mathcal{X}$, (since $P_i$ is a $\delta$–1) so $\nu \omega \omega \in \bigcap_{i \in \mathcal{X}} P_i$. Now, let $\nu \omega \omega \in \bigcap_{i \in \mathcal{X}} P_i$ and $\omega \in \bigcap_{i \in \mathcal{X}} P_i$, so $\nu \omega \omega \in P_i$. Then $\nu \omega \omega = \nu \omega \omega \in \bigcap_{i \in \mathcal{X}} P_i$.

Remark 3.21: In $\delta$–algebra $(\Omega, \omega, f)$, the $\delta$–1 of $\Omega$ is a $\delta$–SA. Nevertheless, the opposite is not true in general.

Example 3.22: Suppose $(\Omega, \omega, f)$ is $\delta$–algebra where $\Omega = \{1,2,3,4,5,6\}$ and $\omega$ is described as a table (7):

$$
\begin{array}{cccccccc}
\omega & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 1 & 3 & 4 & 5 & 6 \\
3 & 3 & 3 & 1 & 4 & 5 & 6 \\
4 & 4 & 4 & 1 & 5 & 6 \\
5 & 5 & 5 & 5 & 5 & 1 & 6 \\
6 & 6 & 6 & 6 & 6 & 6 & 1 \\
\end{array}
$$

"Table(8)"

Also, let $\phi \neq H \subseteq \Omega , \ H = \{1,2,6\} , \ H$ is a $\delta$–SA, but $H$ is not $\delta$–1 since if we take $\nu = 3$ and $\omega = 6$ then $\nu \omega \omega = 3 \times 6 = 6 \in H$ and $\omega \in H$, but $\nu \notin H$.

Theorem 3.23: Every $\delta$–1 $\mathcal{G}$ of $\delta$–algebra $\eta$ is a $BCK$–1 of $BCK$–algebra $\eta$.

Proof: Let $\mathcal{G}$ be a $\delta$–1 in $\delta$–algebra $(\eta, \omega, f)$. Then $\phi \neq \mathcal{G} \subseteq \eta$ and $(\eta, \omega, f)$ is $d$–algebra. We just need to prove that:

(1). $f \in \mathcal{G}$,

(2). $\nu \omega \omega \in \mathcal{G}$ and $\omega \in \mathcal{G}$ imply $\nu \in \mathcal{G}$, for all $\nu, \omega \in \mathcal{G}$.

Since $\mathcal{G}$ is the $\delta$–1, condition (2) is held. At least $\exists \nu \in \mathcal{G}$ (since $\phi \neq \mathcal{G}$). Therefore $\nu \omega \omega \in \mathcal{G}$[By definition(3.11)-(1)]. But $\nu \omega \omega = f$, so $f \in \mathcal{G}$. Then $\mathcal{G}$ is a $BCK$–1 of $BCK$–algebra $\eta$.

Remark 3.24: In the following diagram, we will state our results:
4. Conclusion

We introduce the notions of $\delta$– algebra, $\delta$– subalgebra and $\delta$– ideal in this review. They are shown their basic characterizations and their relationships with other algebras such as $d / \rho / BCK$ – algebras, $d / \rho / BCK$ – subalgebras and $d / \rho / BCK$ – ideals are shown. Some new forms of fuzzy algebras, such as fuzzy $\delta$– algebra, fuzzy $\delta$– subalgebra and fuzzy $\delta$– ideal will be discussed. Next, the links between them and other forms of fuzzy algebras will also be given.

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