The Effects of Spring Stiffness on Vortex-Induced Vibration for Energy Generation

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Abstract. Vortex-induced vibration (VIV) is the turbulent motion induced on bluff body that generates alternating lift forces and results in irregular movement of the body. VIV-powered system seems a good idea in greening the energy sector and most importantly is its ability to take advantages of low current speed of water to generate electricity. This paper aims to investigate the effects of spring stiffness on the characteristic of VIV. The study is important in order to maximize these potentially destructive vibrations into a valuable resource of energy. Five cylinders with the range of 0.25 to 2.00 inch diameter are tested to study the behavior of VIV. Results from this experiment indicates that, the 2.0 inch cylinder gave the lowest error in frequency ratio which is 1.1% and have a high potential of lock-in condition to occur. In term of maximum amplitude, this cylinder gave the highest amplitude of oscillation motion that is equal to 0.0065 m.

1. Introduction
Renewable energy is an energy that originates from resources which is naturally replenished and continuously exists in the environment. Unlike the fossil fuels, renewable energy resources generate energy with no pollution and global warming emissions. It is a key in greening the energy sector by shifting the usage of fossil fuels to renewable energy in order to minimize the carbon-dioxide emissions and effects of global warming from burning of fossils fuels. However, alternative energy sources technologies exist currently such as solar, wind and hydro energy are unable to meet the demand of perpetually and ecological sources of energy. For instance, solar and wind energy are strongly reliant on the weather condition and work only where there is good exposure to sunlight [1]. Similarly, hydro energy can be generated only when there is abundant amount of flowing water. Besides, energy based on wind turbines does have an environmental cost as their effect on migratory paths of birds which could be killed by the rotating blades of wind turbines [2].

Ocean which is known as the world’s largest storage of renewable energy seems an ideal alternative source of energy as it can offer an unlimited amount of clean energy [3]. However, the limitation of this source of energy is it low ocean current speed to generate energy. This is because ocean technologies based on tides and currents turbines can only work for current flow stronger than 2

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m/s [4]. Thus, newer technology needs to be developed to efficiently convert this slow moving water into valuable energy.

One possible solution that meets these criteria is a hydroelectric power extraction system based on Vortex Induced Vibration (VIV) which can generate energy in low speed. VIV is a turbulence motions induced on bluff body that generates periodic irregularities lift forces on the body and pushing it up and down perpendicular to fluid flow [5]. Usually, engineers have been trying to suppress the presence of VIV because of its disruptive power on vibrating structures. A dramatic example of their disruptive power occurred on the Tacoma bridge that collapsed in 1940 due to the present of VIV. In this research, instead of suppressing the effect of VIV, these potentially destructive vibrations will be maximized and exploited in order to transform these vibrations into a valuable resource of energy [6].

Harnessing the phenomenon of vortex-induced vibrations (VIV) is originally designed by Professor Michael Bernitsas from the University of Michigan in 2005. The concept has been named Vortex-Induced Vibrations for Aquatic Clean Energy (VIVACE), and it works by mounting a cylinder horizontally in a fixed frame to allow an alternating movement of oscillation body in order to turn these potentially destructive vibrations into renewable and clean energy [6]. Based on previous study, VIV-powered system has the ability to generate power in the range of current speed 0.26 to 2.6 m/s [6]. Thus, this power generation system is suitable in generating alternative energy on slowing moving water as the typical current ocean are within the range of 0.41 to 1.18 m/s [7].

To the authors’ best knowledge, the study still lacks fundamental reasoning as it focuses more on the application side. This paper aims to investigate the effects of spring stiffness on the behavior of VIV for further enhancement of energy generation system. When the vortex shedding frequency approaches the natural frequency, amplitudes of bodies’ oscillation grow and start forming a strong interaction with the shedding mechanism in the flow [8]. This phenomenon is known as “lock-in” condition which is similar to linear resonance. Therefore, the stiffness of a spring is a significant parameter in this research as it affects the natural frequency for controlling structural resonance. Thus, proper understanding on the effects of spring stiffness is studied.

2. Literature review

2.1. Reynolds number

The range of Reynolds number, $Re$ targeted in this research are in the range of $300 < Re < 3 \times 10^5$ which is known as “Fully Turbulent Vortex Street” regime for the natural low-flow [6]. $Re$ can be determined using the equation (1):

$$Re = \frac{UD}{v}$$ (1)

2.2. Strouhal number

Another vital parameter that describes the vortex shedding frequency to the oscillating flow mechanism in this research is known as Strouhal number, $St$ as shown in equation (2):

$$St = \frac{f_s D}{U}$$ (2)
Figure 1. Strouhal number vs. Reynolds number [9].

Figure 1 shows that $St$ is almost 0.2 for smooth surfaces within the range of $300 < Re < 10^5$, which matches well to the fully developed turbulent vortex street. Therefore $St = 0.2$ will be used as a constant value in any calculations for this research [9].

2.3. Lock-in phenomenon

In order to provide strong amplitude of the body oscillation, the natural frequency of the system must be synchronized with the shedding frequency. This phenomenon is known as lock-in condition. It is important to find the range of shedding frequency that matches with natural frequency, $f_n$ to design energy harnessing device. The natural frequency of the body can be calculated using equation (3), where $k$ is spring stiffness and $m_{app}$ is mass applied to the cylinder:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m_{app}}} \tag{3}$$

Mass applied, $M_{app}$, for testing cylinder properties is determined using equation (4), (5) and (6):

$$Volume = \frac{\pi}{4} \times D^2 \times L = 8.596 \times 10^{-4} \text{ m}^3 \tag{4}$$

$$M_{\text{dis}} = \rho_{\text{water}} \times Volume = 0.857 \text{ kg} \tag{5}$$

$$M_{\text{app}} = M_{\text{pipe}} + M_{\text{dis}} + M_{\text{add}} = 1.145 \text{ kg} \tag{6}$$

Then, the value of natural frequency obtained, the frequency ratio, $f^*$ can be determined using equation (7):

$$f^* = \frac{f_s}{f_n} \tag{7}$$

According to [10], lock-in condition can occur when the value of frequency ratio is close to 1 and within the ± 30 % lock-in range. Another parameter that determined the range of lock-in condition is known as reduced velocity as shown in equation (8):
Shedding frequency can lock in and shift to match the natural frequency within the range of \(3 < U^* < 8\) \([11]\).

3. Theoretical calculation

This section will express the result of spring stiffness for one testing cylinder based on theoretical calculation. The cylinder and flow parameters examined are listed in table 1. Example calculation is shown using cylinder E.

**Table 1.** Cylinder and water parameters

| Cylinder | Label | A     | B     | C     | D     | E     |
|----------|-------|-------|-------|-------|-------|-------|
| Diameter [m] |       | 0.0213 | 0.02683 | 0.0339 | 0.0486 | 0.0604 |
| Mass\(_{pipe}\) [kg] |       | 0.0817 | 0.10357 | 0.1205 | 0.1944 | 0.2778 |
| Mass\(_{add}\) [kg] |       |       |       | 0.0105 [Board apparatus] |       |       |
| Length [m] |       |       |       |       | 0.30 m |       |

**Water**

| Velocity [m/s] | 0.453 m/s (using data from water tunnel flow speed experiment) \([12]\) |
| Kinematic viscosity | 9.0x 10\(^{-7}\) m\(^2\)/s |

The \(Re\) for cylinder E is given by equation (1):

\[
Re = \frac{UD}{V} = 3.04 \times 10^5
\]

For lock in phenomenon to occur it is an essential to match the vortex shedding frequency with natural frequency to enlarge the amplitudes of bodies’ oscillation. The vortex shedding frequency, \(f_s\), will be determined by the flow condition and cylinder size as shown in equation (2):

\[
f_s = \frac{StU}{D} = \frac{1.501}{s}
\]

In order calculate the value of natural frequency, \(f_n\), of the oscillation body, the value of the spring stiffness which is equal to 100 N/m will be used to match the natural frequency to the shedding frequency. Mass applied, \(M_{app}\), for testing cylinder properties is determined using equation (4), (5) and (6):

\[
Volume = \frac{\pi}{4} \times D^2 \times L = 8.596 \times 10^{-4} \text{ m}^3
\]

\[
M_{dis} = \rho_{water} \times Volume = 0.857 \text{ kg}
\]

\[
M_{app} = M_{pipe} + M_{dis} + M_{add} = 1.145 \text{ kg}
\]
Then, the value of natural frequency can be determined using equation (3):

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m_{\text{app}}}} = \frac{1.487}{s} \]

Based on the value of natural frequency obtained, the frequency ratio, \( f^* \) can be determined using equation (7):

\[ f^* = \frac{f_o}{f_n} = \frac{1.501}{1.487} = 1.01 \]

The frequency ratio obtained is equal to 1.01, which is well within the ± 30% lock-in range [10].

In order to validate the range of lock-in condition on the testing cylinder, the value of reduced velocity is calculated using equation (8):

\[ U^* = \frac{U}{Df_n} = 4.997 \]

It showed that, shedding frequency for cylinder E can achieve the lock-in and shift to match the natural frequency as the value of reduced velocity falls within the range of \( 3 < U^* < 8 \).

4. Experimental set-up
The goal of this paper is to study the effects of spring stiffness on vortex-induced vibration. The amplitude of testing cylinder is studied in order to validate the theoretical result obtained. In order to vary the mass applied on the cylinder, different diameters of cylinder are used in this experiment. All cylinders are made of PVC pipes with diameter of 0.25, 0.5, 1.0, 1.5 and 2.0 inch are labelled as A, B, C, D and E respectively. A constant value of spring stiffness which is 100 N/m will be used in this experiment in order to investigate the relationship between amplitude of cylinder and frequency ratio. The experiment is conducted in the low-turbulence free surface in the water tunnel constructed. In order to measure the amplitude of the oscillating cylinder, the Vernier displacement sensor is used to measure the displacement of the cylinder during experiment. Diameter of cylinder is measured using caliper and mass is measured using an electronic scale.

5. Results and discussion

5.1. Theoretical results
Based on calculation obtained, the results of theoretical values are tabulated in table 2.
Table 2. Theoretical results calculation

| Cylinder | A    | B    | C    | D    | E    |
|----------|------|------|------|------|------|
| Reynolds Number, Re | 10710 | 13511 | 17068 | 24504 | 30401 |
| Shedding frequency, f_s | 4.265 | 3.381 | 2.677 | 1.864 | 1.501 |
| Required spring stiffness, k [N/m] | 142 | 128 | 113 | 104 | 102 |
| Used spring stiffness, k [N/m] | | | | 100 | |
| Natural frequency, f_n | 3.609 | 3.022 | 2.540 | 1.843 | 1.487 |
| Frequency Ratio, f* | 1.182 | 1.119 | 1.054 | 1.012 | 1.009 |
| Reduced Velocity, U* | 5.908 | 5.594 | 5.269 | 5.058 | 4.997 |

Based on the results calculated in table 2, it showed that, natural frequency of the cylinder generally decreases with an increasing of mass applied on the cylinder [13]. A large mass causes the natural frequency to be small because of the larger inertia were produced on the system. This will increase the time of cylinder oscillation to return to the midpoint from a displaced position, so the period is increased and the frequency is reduced.

From the result calculated, all cylinders gave the values of frequency ratio within the lock in phenomenon range. Cylinder E showed the lowest error in frequency ratio which is 1.1 %. While, cylinder A showed the highest error in frequency ratio which is 15.37 %. The highest result on cylinder A is due to the high different of spring stiffness used which is 100 N/m compared to the required spring stiffness which is 142 N/m.

However, lock–in is most likely to happen for all of these cylinders as the frequency error is less than 30%. Besides, the entire cylinder tested were achieved at $U^*$ between 3 and 8 which fulfill the lock-in theory. The relationship between frequency ratio and reduced velocity for each cylinder is shown (figure 2).

Based on the graph plotted, it clearly showed that as the frequency ratio closes to unity, the reduced velocity will be collected at the center of the lock-in range. As the frequency ratio is increased, the values of reduced velocity started to move away from the center of lock-in range. In this research, cylinder E most likely will create the large amplitude of oscillation due to the closeness of its natural frequency with shedding frequency which results in frequency ratio close to 1. Thus, it showed that, the potential of lock-in to occur is increased with the decrease of frequency error of the cylinder.
5.2. Experimental results

In order to validate the results obtained in table 2, the corresponding cylinder amplitude is measured using motion sensor to give an overview of characteristic of VIV through stiffness of spring. The result of the cylinder amplitude was shown in (figure 3) [12].

Based on the figure 3, the maximum amplitude obtained by cylinder E is equal to 0.0065 m while Cylinder A gave the lowest maximum amplitude that is equal to 0.0035 m. The highest value of amplitude obtained by Cylinder E is due to the closeness of its natural frequency with shedding frequency which results in frequency ratio close to 1 and increase the possibility of lock-in condition to occur. Thus, it showed that the results obtained on the theoretical calculation were proven.
Figure 3. Amplitude profile of different sizes of cylinder.
6. Conclusion
The VIV application in generating alternative energy is a viable solution of the current energy extraction problem for daily application. In this paper, the effects of spring stiffness were investigated theoretically and experimentally with the range Reynolds numbers (300 < Re < 300,000). Based on this research, it can be concluded that, the stiffness of spring is a vital parameter that affects the natural frequency of the oscillation body for the lock-in condition to occur. Results were shown on the Cylinder E which gave the highest maximum amplitude when it frequency ratio is close to 1. As the frequency ratio is close to 1, the possibility of lock-in condition to happen will increase. Then, amplitudes of bodies’ oscillation grow and start forming a strong interaction with the shedding mechanism in the flow.

Nomenclature

\begin{align*}
Re & \quad \text{Reynolds Number} \\
St & \quad \text{Strouhal Number} \\
U & \quad \text{Current Speed, m/s} \\
D & \quad \text{Diameter of Cylinder, m} \\
v & \quad \text{Dynamic Viscosity, Pa.s} \\
f_s & \quad \text{Shedding Frequency, Hz} \\
f_n & \quad \text{Natural Frequency, Hz} \\
u^* & \quad \text{Reduced Velocity} \\
k & \quad \text{Stiffness of Spring, N/m} \\
\rho_{\text{fluid}} & \quad \text{Density of Fluid, kg/ m}^3 \\
\rho_{\text{cylinder}} & \quad \text{Density of Cylinder, kg/ m}^3 \\
M_{\text{app}} & \quad \text{Mass of Application, kg} \\
M_{\text{cyl}} & \quad \text{Mass of Cylinder, kg} \\
M_{\text{dis}} & \quad \text{Mass of Fluid Displaced, kg} \\
M_{\text{add}} & \quad \text{Additional Mass, kg} \\
f^* & \quad \text{Frequency Ratio} \\
g & \quad \text{Gravitational Force, m/ s}^2
\end{align*}
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