MERGING OF GLOBULAR CLUSTERS IN INNER GALACTIC REGIONS. II. NUCLEAR STAR CLUSTER FORMATION

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ABSTRACT

In this series of papers, we present the results of detailed N-body simulations of the interaction of a sample of four massive globular clusters in the inner region of a triaxial galaxy for two different sets of initial conditions that correspond to different initial density concentrations. A full merging of the clusters takes place, leading to a slowly evolving cluster that is quite similar to observed nuclear clusters. Actually, both the density and the velocity dispersion profiles match qualitatively, and also quantitatively after scaling to a larger number of merger globulars, with the observed features of many nucleated galaxies. In the case of dense initial clusters, the merger remnant shows a density profile more concentrated than that of the progenitors, with a central density higher than the sum of the progenitors’ central densities and an effective radius compatible with observed nuclear values. These findings support the idea that a massive nuclear cluster may have formed in the early phases of the mother galaxy’s evolution and led to the formation of a nucleus, which in many galaxies has a luminosity profile similar to that of an extended King model. A correlation with galactic nuclear activity is suggested.

Subject headings: galaxies: nuclei — galaxies: star clusters — globular clusters: general — methods: n-body simulations

1. INTRODUCTION

In the first paper of this series devoted to the study of the interaction of globular clusters (GCs) in triaxial galaxies, we analyzed the face-on collision between two GCs moving on quasi-radial orbits in the central galactic region, with the aim of understanding the effectiveness of tidal distortion. Our first finding was that the face-on collision is practically ineffective with respect to the role of the external tidal field. Actually, the tidal erosion has been shown to destroy loose GCs (those with an initial King concentration parameter $c < 1$) in a few passages through the galactic center, while tight clusters ($c \geq 1.6$) keep a substantial amount of their mass bound until complete orbital decay. Regarding this last point, another important finding is that the orbital energy dissipation due to tidal interaction is of the same order as that caused by dynamical friction. In light of these results, and given that dynamical friction has been shown to be important in segregating massive GCs in triaxial potentials (Pesce et al. 1992; Capuzzo-Dolcetta 1993; Capuzzo-Dolcetta & Vicari 2005), a natural further step in the present program of investigation is to study the possible merging of a set of GCs that have orbitally decayed in the central galactic region, to see whether a sort of super star cluster (SSC) results from the merging, and what its morphological and dynamical characteristics could be.

The existence of very bright ($10^7 - 10^8$ $L_\odot$) clusters of stars in the central region of galaxies across the Hubble sequence has been definitively ascertained thanks to Very Large Telescope (VLT) and Hubble Space Telescope (HST) observations (Walcher et al. 2005; van der Marel et al. 2007; Wehner & Harris 2006). On the basis of integrated colors and of the estimated $M/L$ ratios, these clusters are usually thought to be young, or at least to contain a significant population of young stars. In any case, the question of age is a controversial one, because, for instance, the nuclear star clusters (NCs) in M82 show evidence of mass segregation despite the fact that their spectra are well fitted by stellar population synthesis models with ages 10–50 Myr (McCready 2004). Böker et al. (2004) fitted analytical models to HST images of 39 NCs in order to determine their effective radii after correction for the instrumental point-spread function. They compared the luminosities and sizes of NCs to those of other ellipsoidal stellar systems, in particular the Milky Way globular clusters, finding a narrow size distribution for NCs statistically indistinguishable from that of Galactic GCs, even though the NCs are, on average, 4 mag brighter than the old GCs. They discuss some possible interpretations of the similarity between NCs and Galactic GCs, and from a comparison of NC luminosities with various properties of their host galaxies, find that more luminous galaxies harbor more luminous NCs. It remains unclear whether this correlation reflects the influence of galaxy size, mass, and/or star formation rate.

However, Rossa et al. (2006), by means of spectroscopic data from the HST Space Telescope Imaging Spectrograph (STIS), derived NC ages for 19 galaxies and found that they range from 10 Myr to 10 Gyr, with a non-negligible presence of old clusters. A comparable result has also been deduced for 9 NCs of very late type bulgeless spirals by Walcher et al. (2006) by means of a high-resolution spectroscopic survey with the VLT Ultraviolet and Visual Echelle Spectrograph (UVES).

Genzel et al. (2003), in their study of the stellar cuspy distribution around the supermassive black hole (SBH) in the Galactic center, found that the $K$-band luminosity function of the local NC (within 9$''$ of Sgr A*) resembles that of the large-scale Galactic bulge, but shows an excess of stars at $K_s \leq 14$. This result fits with population synthesis models of an old, metal-rich stellar population with contributions from young, early-, and late-type stars at the bright end. In the central arcsecond, Genzel et al. (2003) argued that a stellar-merger model is the most appealing explanation. These stars may thus be “super blue stragglers,” formed and “rejuvenated” through mergers of lower mass stars in the very dense ($\geq 10^8 M_\odot$ pc$^{-3}$) environment of the cusp.

Another intriguing piece of the puzzle of the structure of SSCs is given by Baumgardt et al. (2003). Through a comparison
between the observational data on the kinematical structure of the very bright cluster G1 in M31 (obtained with the HST WFP Camera 2 and STIS instruments) and their results of dynamical simulations carried out using the special purpose computer GRAPE-6, they obtained very good fits when starting simulations with initial conditions extracted from the end product of a previous simulation of a merging between two pre-existent star clusters. The merging explains observed features without the need to invoke the presence of an intermediate-mass black hole in the center of G1.

Not many simulations have been presented in the literature that study possible formation scenarios for SSCs. Of those that there are, note those by Fellhauer & Kroupa (2002, 2005), which find that star cluster aggregates, such as the ones found in the Antennae or Stephan Quintet, are very likely the merger progenitors of SSCs. Interestingly enough, these authors also find that the resulting SSC is a stable and bound object, whose density profile is well fitted by a King profile, even if the mass loss of the merger product occurs through every perigalacticon passage.

2. GLOBULAR CLUSTER MERGING AND GALACTIC NUCLEI

The increasing amount of data for massive clusters in galaxies, together with that for GC systems in galaxies, especially of the early type but also of spirals (Harris 1986, 1991, 2001; Ashman & Zepf 1998, etc.), makes interesting the investigation started in the first paper of this series (Miocchi et al. 2006, hereafter Paper I) of the fate of massive GCs moving in the parent galaxy field that are subjected to dynamical friction braking and tidal interaction. In Paper I, we studied the dynamical evolution of two GCs with masses $\approx 10^7 M_\odot$ moving in quasi-radial orbits in a triaxial galactic potential for eight passages across the galactic center. The scope of that paper was to investigate the GCs’ chances of surviving possible tidal disruption induced both by the external field and by mutual cluster-cluster interaction; with this aim, we maximized these effects by giving the clusters initial conditions corresponding to quasi-radial orbits.

In this paper we have another aim, namely to study whether and how the merging of various massive GCs decayed by dynamical friction in the inner galactic region may occur. The main questions to answer are: (1) Given some (realistic) initial conditions for a set of GCs that have experienced a significant orbital decay, are they undergoing a full merge? (2) If so, what is the time needed? (3) What is the final structure of the merged NC? (4) Does it attain a quasi–steady state?

The answers to these questions are of overwhelming importance to give substance to the interpretation of the formation of early-type galaxy nuclei via merging of decayed GCs, a hypothesis first raised by Tremaine et al. (1975) and subsequently extensively studied by Capuzzo-Dolcetta (1993), Capuzzo-Dolcetta & Tesseri (1997, 1999), and Capuzzo-Dolcetta & Vicari (2005).

This scenario of galactic nuclei formation has recently been proposed again due to the independent and almost simultaneous findings that NC masses obey scaling relationships with host galaxy properties similar to those of SBHs (see Rossa et al. 2006 for spirals, Wehner & Harris 2006 for dwarf elliptical [dE] galaxies, and Côté et al. 2006 for elliptical galaxies). In particular, Côté et al. (2006) give evidence of NC luminosity distributions in early-type galaxies in the Virgo Cluster that are much better fitted by an extended profile (King’s) than by a point source (see Böker et al. 2002 for a similar finding in late-type spirals and Graham & Guzmán 2003 for dE galaxies in the Coma Cluster). The half-mass radii of nuclei ($r_h$) are in the range 2 pc $< r_h < 62$ pc, with $\langle r_h \rangle = 4.2$ pc, and correlate with the nucleus luminosity $r_h \propto L_g^5 \pm 0.01$. The mean of the frequency function for the nucleus-to-galaxy luminosity ratio in nucleated galaxies, log $\eta = -2.49 \pm 0.09$, is indistinguishable from that of the SBH-to-bulge mass ratio, log ($M_{\rm BH}/M_{\rm gal}$) = $-2.61 \pm 0.07$, calculated in 23 early-type galaxies. On these bases, Côté et al. (2006) argue that resolved stellar nuclei are the low-mass counterparts of nuclei-hosting SBHs detected in the bright galaxies. If this view is correct, then one should think in terms of central massive objects, either SBHs or SSCs, that accompany the formation and/or early evolution of almost all early-type galaxies.

All these characteristics of galactic nuclei are fully compatible with a formation through multiple GC merging in the inner galactic regions (which we call the “dissipationless” scheme) and represent an alternative to the “dissipational” scenario, which is based mainly on qualitative hypotheses (see, e.g., van den Bergh 1986; Silk et al. 1987; Babul & Rees 1992) supported by some quantitative results (Mihos & Hernquist 1994). The first step towards proof of the validity of the dissipationless hypothesis requires a detailed $N$-body simulation of the interaction of stellar clusters in the inner region of their parent galaxy, taking into account both the cluster-cluster and the cluster-galaxy dynamical interaction. This latter interaction includes tidal distortion, which acts on the cluster’s internal motion, and dynamical friction, which acts on the cluster’s orbital motion.

In this context, we cite the encouraging results of the numerical simulations made by Oh & Lin (2000), who revisited the hypothesis of dE nuclei formation through the orbital decay of GCs and suggested that this occurs mainly in galaxies with relatively weak extragalactic tidal perturbation, which leads to the formation of compact nuclei within a Hubble timescale. They also showed that the central galactic field does not destroy the integrity of the clusters and facilitates the merging. Moreover, they found that the observed central structures of some nucleated Virgo Cluster dEs are well reproduced by superimposing a small number of globular clusters on the galactic stellar distribution. However, Oh & Lin (2000) used a very small number of particles (500 in each cluster) to simulate the final merger stage, during which they neglected the role of dynamical friction as well. Thus, their results could be affected by spurious collisional effects, even though a rather large smoothing radius was adopted in the interparticle interactions.

More recently, Bekki et al. (2004) showed how the formation of NCs via multiple merging of GCs leads to systems with global scaling relations that are compatible with those observed for galactic nuclei. Unfortunately, as mentioned before, these results cannot be considered conclusive, because in these authors’ simulations the important role of the external field was neglected.

3. THE SIMULATIONS

We consider GCs as $N$-body systems moving within a triaxial galaxy represented by an analytical potential and subjected to deceleration due to dynamical friction (DF). To maintain a high level of resolution in the simulations (i.e., having a large number of particles to represent each GC without exceeding computational capabilities), we decided to limit our study to the interaction of a limited number (four) of GCs.

We studied the merging process of these GCs in two separate simulations (cases 1 and 2) as if they had already decayed to the inner region of the galaxy. A quick orbital decay induced by DF is due to large initial values for the total mass of the clusters. These large masses are compatible with those of many young GCs actually observed in various galaxies (e.g., Kissler-Patig et al.
TABLE 1  
INITIAL CLUSTER PARAMETERS  

| GC Model | $M$       | $r_i$   | $c$      | $r_e$   | $r_h$  | $\rho_0$ | $t_{bc}$ | $\sigma_k$ | $\rho_0/r^3$ |
|----------|----------|---------|----------|---------|--------|-----------|----------|------------|-------------|
| a1       | $1.5 \times 10^{-2}$ | 0.16    | 1.2      | $1.1 \times 10^{-2}$ | $2.3 \times 10^{-2}$ | 770   | $2.6 \times 10^{-2}$ | 0.36 | $3.2 \times 10^0$ |
| b1       | $1.7 \times 10^{-2}$ | 0.16    | 1.0      | $1.5 \times 10^{-2}$ | $2.4 \times 10^{-2}$ | 410   | $2.8 \times 10^{-2}$ | 0.36 | $1.7 \times 10^0$ |
| c1       | $1.8 \times 10^{-2}$ | 0.14    | 0.99     | $1.4 \times 10^{-2}$ | $2.2 \times 10^{-2}$ | 610   | $2.3 \times 10^{-2}$ | 0.41 | $1.7 \times 10^0$ |
| d1       | $1.4 \times 10^{-2}$ | 0.14    | 0.89     | $1.8 \times 10^{-2}$ | $2.4 \times 10^{-2}$ | 250   | $3.3 \times 10^{-2}$ | 0.34 | $1.2 \times 10^0$ |
| a2       | $1.5 \times 10^{-2}$ | 0.77    | 1.2      | $5.3 \times 10^{-2}$ | 0.11   | 6.1  | 0.30 | 0.16 | 290 |
| b2       | $1.7 \times 10^{-2}$ | 0.78    | 1.0      | $7.3 \times 10^{-2}$ | 0.12   | 3.3  | 0.31 | 0.16 | 150 |
| c2       | $1.8 \times 10^{-2}$ | 0.68    | 0.99     | $6.9 \times 10^{-2}$ | 0.11   | 4.9  | 0.26 | 0.18 | 160 |
| d2       | $1.4 \times 10^{-2}$ | 0.72    | 0.89     | $9.3 \times 10^{-2}$ | 0.12   | 2.0  | 0.37 | 0.15 | 110 |

Note.—Parameters for the initial cluster models, expressed in galactic units. Reported are the GC mass ($M$), the limiting radius ($r_i$), the half-mass radius ($r_h$), the central density ($\rho_0$), the half-mass crossing time ($t_{bc}$), the King velocity parameter ($\sigma_k$), and the central phase-space-density estimate (where $\sigma = \sqrt{3}\sigma_k$).

Only the spherically symmetric part, $\rho_0(r)$, of the total density contributes to the mass in the generic sphere of radius $r$, giving

$$M(r) = 4\pi M_b \left\{ \ln \left[ \frac{r}{r_{bc}} + \left( \frac{r}{r_{bc}} \right)^2 \right] + \frac{1}{2} \right\},$$

which gives an infinite total mass. However, there is no need for a cutoff in the model, because only the gravitational potential (which is not divergent) is important for the purpose of our simulations.

As in Paper I, the origin of the reference frame lies at the galactic center, and the $x$ and $z$ axes lie along the maximum and minimum axes of the triaxial ellipsoid, respectively. The centers of mass of the clusters were initially located well within the galactic core (see Table 2).

With regard to DF, we used the generalization to the triaxial case (see Pece et al. 1992) of the Chandrasekhar formula (Chandrasekhar 1943), with a self-consistent evaluation of the velocity dispersion tensor, also taking into account that the GCs are extended objects. See Paper I for more details.

3.2. The Cluster Model

Our numerical study involved two sets of four clusters evolved in two separate simulations. The GCs’ initial internal distribution was sampled from a stellar King (1966) isotropic model with total mass $M$, velocity parameter $\sigma_k$, limiting radius $r_i$, and King radius $r_c$; $c = \log (r_i/r_c)$ is the concentration parameter, $t_{bc} = [r_h^3/(GM_b)^{1/2}]$ is the half-mass crossing time, and $\rho_0$ is the central density.

The parameter is such that at the center $\sigma = r^2 \simeq 3\sigma_k^2$, the approximation being as much better as the model concentration is higher (Binney & Tremaine 1987, § 4.4c).

TABLE 2  
ORBITAL INITIAL CONDITIONS FOR THE CLUSTERS  

| Cluster Model | $x_0$ | $y_0$ | $z_0$ |
|---------------|-------|-------|-------|
| a1, a2        | −0.4  | 0.25  | −0.35 |
| b1, b2        | 0.3   | −0.35 | 0.31  |
| c1, c2        | 0.375 | 0.35  | −0.325|
| d1, d2        | −0.26 | −0.225| 0.425 |

Note.—Initial conditions of the orbits of the four clusters. All started with zero initial velocity.
density. The limiting radius is the radius at which the King distribution function drops to zero in order to reproduce the presence of the external field (King 1966). The limiting radii are slightly larger than the actual local tidal radii; this choice was also made to model the expected presence of some tidal debris around the clusters as a result of the previous orbital evolution and decay (see discussion in Paper I, § 2.2). The initial values of these parameters are listed in Table 1.

Each cluster was represented by $N = 2.5 \times 10^5$ "particles," whose individual masses were assigned according to a Salpeter mass distribution ($dN \propto m^{-2.35} dm$), cut off at $m_{\text{min}} = 100 m_{\text{min}}$ with average $\langle m \rangle = 3.1 m_{\text{min}}$, where $m_{\text{min}}$ is adjusted to give the desired total cluster mass. When simulating massive GCs, the assumption $N = 2.5 \times 10^5$ gives a high value of $m_{\text{min}}$, which shifts the chosen mass distribution toward large masses, thus causing it to lose its ability to represent a real GC stellar mass spectrum. However, as discussed in Paper I, we use unequal mass particles to verify that no collisional relaxation takes place during the simulation, as demanded by the two-body relaxation time of the clusters, which in any case is longer than the merging time.

Actually, even for the most compact case considered here (model a1; see Table 1), the half-mass relaxation time of the simulated cluster is longer than the merging time $t_m \sim 18$ (see § 4), after which the clusters lose their individuality and evolve as a system with a higher $N$ (see discussion in § 6). The relaxation time of the real cluster (which has a larger number of stars) is obviously longer than that of the simulated (sampled) cluster.

Moreover, if the GCs are massive enough to decay rapidly into the galactic core, it can be safely assumed (see also Paper I, § 2) that their age at the beginning of the simulation is less than their internal two-body relaxation time, so that mass segregation has not occurred and thus is not included in the initial stellar distribution. In the following section, we often refer to the system’s center of density (CD), i.e., the average particle position weighted with the local density instead of the mass, as defined in Casertano & Hut (1985). As in Paper I, in most cases we took the CD as the origin of the best suitable reference frame for the study of the internal system properties.

### 4. RESULTS

Although the truly conserved quantity, in the absence of any external dissipation, is the center-of-mass orbital energy, it is preferable to refer to the CD orbital energy instead, because the CD identifies the current location of the GC better than the center of mass when the outer part of the cluster is highly distorted and dispersed. Defining $E_{\text{orb}}$ as the orbital energy (per unit mass) of the cluster CD, its dissipation due to both the DF braking effect and to tidal interaction with the environment can be quantified by the parameter (Paper I)

$$\xi_{\text{orb}}(t) = \frac{E_{\text{orb}}(t) - \Psi_0}{E_{\text{orb}}(0) - \Psi_0},$$

where $\Psi_0 = -4\pi G M_b/r_{bc}$ is the central galactic potential well. In Figure 1, $\xi_{\text{orb}}(t)$ is plotted for all the clusters in the two cases studied.

The merging occurs rather quickly, as indicated by the snapshots of the system as projected onto one of the coordinate planes (see Figs. 2 and 3 for case 1) and as clearly seen from the time behavior of the Lagrangian radii of the whole system (Fig. 4). Lagrangian radii attain steady values at a "merging" time $t_m \sim 18$, when the innermost radius shown also stabilizes; note that the smaller the Lagrangian radius, the more time is necessary for a stable state to be achieved. A similar merging time can also be deduced by analyzing the time behavior of the distances of the clusters’ CDs to the galactic center (Fig. 5).

Of course, the energy dissipation rate is closely related to the prescription adopted for the evaluation and treatment of DF. For instance, our evaluation (Paper I, Appendix A) could lead, in principle, to an overestimate of the effect, because we consider the cluster as a constant-mass point. Obviously, DF could be accounted for accurately by adopting a full $N$-body representation of the galactic region in which GCs move; for this reason, we are running a self-consistent simulation that we will present in a forthcoming paper (Capuzzo-Dolcetta & Micocchi 2008). However, preliminary results show a clearly shorter orbital decaying time for the same clusters as those in case 1 when moving in a self-consistent representation of the same galaxy sampled by $5 \times 10^5$ "particles." This is shown by the time behavior of the distance of the clusters’ CDs from the galactic center, as depicted in Figure 6, from which a merging time $t_m \sim 13$ is deduced, i.e., a

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2 To partially overcome this problem, we conservatively used a halved cluster mass as the mass parameter in the DF formula.
Fig. 2.—Snapshots of the simulation in case 1 (projection on the x-y plane) from $t = 0$ to $t = 20.3$. 
Fig. 3.—Continuation of Fig. 2 from $t = 21.6$ to $t = 42$. 
Fig. 4.—Evolution of the Lagrangian radii evaluated with respect to the global CD. The lines refer to 10%, 30%, 50%, and 90% of the total mass of the system. *Left*: case 1; *right*: case 2.

Fig. 5.—Time behavior of the distance of the clusters’ CDs to the galactic center. *Left*: NC₁ progenitors; *right*: NC₂ progenitors. Symbols are as in Fig. 1.
factor 1.4 smaller than in the simulations with the analytic DF form. This finding agrees with some recent results that show how stars that have been stripped from the cluster by the field, but that are still close enough to the system, continue to contribute to the mass of the decelerating system (Fellhauer & Lin 2007). Moreover, recent, fully self-consistent $N$-body simulations confirm that the real DF effect can be even stronger than that estimated by the usual Chandrasekhar formula (probably due to the greater friction caused by tidal effects; see Fujii et al. 2007).

4.1. The NC Morphology

To study NC morphology, we evaluated the eigenvalues \( \{I_1, I_2, I_3\} \) and the eigenvectors \( \{e_1, e_2, e_3\} \) of the inertia tensor,

\[
I_\ell = \frac{1}{N} \sum_{k=1}^{N} (x_k^\ell - x_k^0) (x_k^\ell - x_k^0) - \frac{1}{N^2} \sum_{k=1}^{N} (x_k^0 - x_k^0) \sum_{k=1}^{N} (x_k^0 - x_k^0),
\]

where \((x_k^\ell, x_k^0, x_k^0)\) is the position vector of the \(k\)th particle, and \((x_k^0)\) is the CD position. In Figure 7, the square root of the eigenvalues are plotted as a function of time. They are evaluated using equation (4) considering only particles closer than \(r_h\) to the system CD, where \(r_h\) is the half-mass radius of the NC at the end of the simulations. Because \(I_1^{1/2}\) is equal to the length of the \(\ell\)th axes of the ellipsoid fitted to the matter distribution, the ratios of the axes give a direct measure of the triaxiality of the system; e.g., if \(I_1 = I_2 = I_3\), then the system is spherical.

It can be seen that after the initial merging phase, the NC as a whole attains a stable configuration. Only in case 2 (Fig. 7, top) is the system shape moderately axisymmetric (oblate) and nonrotating. Examining the orientation of the eigenvectors shows it to have its minor axis nearly aligned to the z-direction and axial ratios \(I_1^{1/2} : I_2^{1/2} : I_3^{1/2} \approx 1.4 : 1.4 : 1\). This alignment suggests that the final morphology is influenced, to some extent, by the galactic morphology. Thus, it is not surprising that in the NC2 outskirts, the stars tend, eventually, to move on orbits compatible with the underlying galactic triaxial configuration, as we will see from the velocity anisotropy (§ 4.1.2). Nevertheless, the self-gravity of the simulated system is important enough to make the shape of the NC2 different from that of the galaxy. Accordingly, the configuration of the NC1 is almost spherical, due to its stronger self-gravity (Fig. 7, bottom).

4.1.1. Density Profile

The radial density profiles of the NCs, which we have seen to be quasi-spherical at the end of the merging, are plotted in Figure 8, where, for comparison, the profile corresponding to the spatial superposition of the four progenitor clusters in their initial configurations is also shown. This gives an immediate visual indication of the efficiency of the merging process; the merging remnant density profile may be even more concentrated than that given by the mere sum of the initial profiles of the four progenitors. Actually, the NC1 merger profile has a smaller core (showing a central density 1.25 times higher) and a more extended envelope. In fact, it is remarkably well fitted by a high-concentration (c = 2.2) King profile (see Fig. 8). This result, which agrees with what has been found in the different context of merging galaxies since the pioneering White (1978) simulations, suggests that a sort of violent relaxation took place during the merging. This relaxation is likely due to the rapidly varying potential acting on each star in the clusters.

In the initial cluster models, a stellar mass spectrum was included without mass segregation in order to investigate possible relaxation phenomena due to either spurious collisional effects or induced by the external tidal field (Gnedin et al. 1999). The analysis of the behavior of the average mass as a function of the distance from the center in the final NC configuration indicates no evidence of mass segregation, thus enforcing the above-mentioned hypothesis that violent relaxation is a cause of the increased concentration of the merging product.
4.1.2. Velocity Distribution

To compare our simulation results with observational data giving the radial profile of the line-of-sight velocity dispersion, we must evaluate the total velocity dispersion, $\sigma_{\text{tot}}$, by properly accounting for the contribution of the underlying galaxy, which is given directly by the self-consistent model (according to the fitting formulae [A3] and [A4] in Paper I). Figure 9 clearly shows a tendency for $\sigma_{\text{tot}}$ to decrease toward the galactic center, where the surface density of the NC dominates, and thus $\sigma_{\text{tot}}$ drops to the velocity dispersion of the NC, which is “colder” than the host galactic core. The behavior shown in Figure 9 may seem peculiar for a self-gravitating system, which normally keeps its equilibrium through a negative (or null, if isothermal) gradient of $\sigma^2$. This apparent peculiarity is due to the fact that the central observed velocity dispersion is dominated by the NC, which is not a self-gravitating system. This feature was found by a Keck II spectroscopic analysis (Geha et al. 2002; see their Fig. 5) for most of the nucleated Virgo Cluster dEs in the sample. The same $\sigma_{\text{tot}}$ radial behavior is also shown by the solution of the Jeans equations for a sample of NCs observed in late-type spirals (Walcher et al. 2005) and is also deduced on the basis of direct N-body simulations by Oh & Lin (2000), which are concerned with GCs merging at the centers of dEs. These simulations of Oh & Lin (2000) confirm the occurrence of a merging event within the central potential well of the host galaxy, even though the validity of their results is not completely guaranteed because the low number of particles forces them to adopt a rather large gravitational smoothing radius in order to suppress fictitious collisional effects.

As regards the velocity anisotropy, we studied the anisotropy parameters $\beta_{\phi,\theta} = 1 - \sigma_{\phi}^2/\sigma_{r}^2$ of the stellar velocity distribution in the NCs as a function of the distance from the system CD, where $\phi$ and $\theta$ denote the azimuthal and polar angles, respectively. These parameters are compared with those of the host galaxy.

We have found that NC1 has a nearly isotropic velocity distribution ($|\beta_{\phi,\theta}| < 0.2$), while the influence of the external galactic potential induces $\beta_{\phi} < \beta_{\theta} < 0$ (i.e., $\sigma_{\phi}^2 > \sigma_{\theta}^2$) in the outer region ($r \gtrsim 0.5r_{\text{nc}}$) of the less compact NC2, which means that the velocity distribution in the outskirts is preferentially tangential and parallel to the $x$-$y$ plane, as in the galactic model. Noticeably, Geha et al. (2002) found that the $\sigma_{\text{tot}}$ decrease in the...
about 5 times lower than the usually adopted values based on the
gential velocity distribution.

cases, by a model with a nearly isotropic or moderately tan-
center of their observed nucleated de can be reproduced, in most
density (solid line) where \(\rho_0^\mathrm{NC} \) is obtained with a virial parameter

\( K \), the central velocity dispersion of the progenitors) such that it
decreases the phase-space density by a factor of 0.4.

Another relevant parameter is the central value of \( \rho/\sigma^3 \), which
is proportional to the density in the phase space. Its value (see
Table 3) in the merged systems is smaller than that of the four
individual progenitor clusters; this corresponds to the expected
rarefaction in the phase space of the inner part of the system after
the mutual interaction of the merging clusters, and especially the
interaction with the external field. This time-dependent inter-
action leads to a certain diffusion in phase space, even if the four
GCs constitute a collisionless system. Note how the rise of the
central space density with respect to the linear summation of in-
dividual GC densities in the case of NC 1 is strongly balanced by
expansion in velocity space (\( \sigma^3 \) of NC 1 is \( \sim 3 \) times higher than
the average velocity dispersion of the progenitors) such that it
decreases the phase-space density by a factor of 0.4.

As regards the virial equilibrium of NCs, it is worth noting that
the King model fitting of NC 1 (Fig. 8) gives a velocity parameter
\( \sigma_K = (4\pi G \rho_0)^{1/2} r_c / 3 \simeq 0.36 \), which is significantly lower than
the true central value of the line-of-sight velocity dispersion \( \sim 0.52 \).
This discrepancy is due to the non-negligible influence of the ex-
ternal galactic potential, which kinematically “heats up” the NC.
This has an important implication for the reliability of the NC
mass determination by means of the virial relation \( M = \eta \sigma_p^2 r_b / G \),
where \( \sigma_p \) is the projected velocity dispersion. The true mass
value for our NCs is obtained with a virial parameter \( \eta \simeq 2 \) that is
about 5 times lower than the usually adopted values based on the
unrealistic assumption of isolated King models. This finding is in
remarkable agreement with the results of Walcher et al. (2005) on
the dynamical mass estimate of NCs observed in bulgeless spiral
galaxies.

5. ARE RESOLVED GALACTIC NUCLEI COMPATIBLE
WITH A MERGING ORIGIN?

An important result of this work is that a quasi-stationary NC
may form as a merger product of orbitally decayed massive clusters.
The projected density profiles in the two cases studied here are
given in Figure 10. Their similarity to the resolved nuclear
profiles of many of the galaxies observed by, e.g., Böker et al.
(2002), Geha et al. (2002), Walcher et al. (2005), and Côté et al.
(2006) is clear; of course, the ratio between the central surface
total density and that of the central galaxy plateau depends on the
number of merged GCs. This ratio is \( \approx 18 \) for the four merging
GCs of case 1 and \( \approx 1.5 \) for case 2. An analysis of Figure 4 in
Côté et al. (2006) indicates ratios of the central value of the sur-
facial luminosity of the nucleus to the inward extrapolated galactic
profile in the interval \( 1 \leq L_0 / L_{gal} \leq 100 \). Under the assumption
that the density contrast scales linearly with the number of merged
GCs (\( N_m \)), the range of the observed ratio corresponds to the in-
terval \( 0 \leq N_m \leq 22 \) for case 1 and \( 0 \leq N_m \leq 267 \) for case 2. Of
course, the linearity is not guaranteed; actually, the modes of the
merging of a more abundant sample of GCs deserve careful fu-
ture study. Moreover, a fully self-consistent study of the feed-
back among the merging GCs and the surrounding stellar field
is needed to confirm the modes and time of orbital decay and

![Fig. 10](image-url)

**Fig. 10.**—Projected surface density profile for the last NC configuration, summed to the galaxy background profile (long-dashed line) so as to give the total surface density (solid line). Left: case 1; right: case 2.
merging. This study will constitute a check of the reliability of the dynamical friction treatment done here under the assumptions of the Pesce et al. (1992) scheme, which, if proven, will result in a huge possible savings in CPU time in future simulations.

6. THE MERGER REMNANT STATIONARITY

In both cases studied here, merging occurs at $\sim 20 t_h$, and the characteristics of the resulting NC remain almost unchanged over the remaining simulation time (a further $25 t_h$). This does not necessarily imply the secular stationarity of the merged system, which should be investigated by sufficiently extending the N-body simulations in time. However, simple considerations of the relaxation times convince us that the lifetime of the merged system should be quite long. The velocity dispersion of the stars of the NC is larger than required by its virial equilibrium as an isolated system (as seen in § 4.1.2); hence, the half-mass relaxation time as evaluated by the classical Spitzer (1987) formula,

$$ t_{rh} = \frac{N}{7 \ln N} \left( \frac{M_h}{M} \right)^{1/2} \left( \frac{r_h}{r_{hc}} \right)^{3/2} t_h, $$

represents a lower bound on the relaxation time of the NC embedded in the galactic stellar field. Equation (5) gives $t_{rh} = 1.1 \times 10^4 t_h$ for the values $N = 10^6$, $r_h/r_{hc} = 0.12$, and $M/M_h = 6.4 \times 10^{-2}$, corresponding to the four merged clusters of case 1, and $t_{rh} = 2.6 \times 10^3 t_h$ for the four merged clusters of case 2, where only the ratio $r_h/r_{hc}$ has a different value (0.21). In physical units, this means that $t_{rh} = 5.8$ Gyr and $t_{rh} = 13.5$ Gyr in cases 1 and 2, respectively, assuming that $r_{hc} = 200$ pc and $M_h = 10^9 M_\odot$. Consequently, the merged system should remain almost stationary for times on the order of the age of galaxies, especially when one considers that the “true” number of stars in the system is from $64$ to $6.4 \times 10^3$ times the one used in the simulations for $M_0$ in the range $10^8$–$10^{11} M_\odot$, implying that one must scale all the relaxation times estimated above by a factor in the range 50–3900. If other clusters merge to the center, $N$ increases, as $r_h$ likely does, further increasing $t_{rh}$. In conclusion, a collisional evolution for the nuclear cluster seems unlikely, but larger scale instabilities cannot be ruled out. Therefore, the dynamical evolution of the merger remnant deserves a simulation that is more extended in time.

With the likely hypothesis that the NC keeps its stability during subsequent merging events, the growth of the central density $\rho_0$ shown by Figure 8 can be reasonably scaled with a number $N_m$ of merging clusters, giving $\rho_0 \simeq 575 N_m$ (case 1) and $\rho_0 \simeq 3.7 N_m$ (case 2) in units of the central galactic density, $\rho_0$. With $M_h = 3 \times 10^8 M_\odot$ and $r_{hc} = 200$ pc as typical values of normal elliptical galaxies, one has $\rho_0 = 375 M_\odot$ pc$^{-3}$; consequently, for the NC to reach a high density on the order of $10^7 M_\odot$ pc$^{-3}$, which is the typical environmental star density needed by a massive object to accrete at the order of $1 M_\odot$ yr$^{-1}$ (Capuzzo-Dolcetta 2002), $N_m \simeq 46$ (case 1) and $N_m \simeq 7150$ (case 2) are needed. Even with all the necessary cautions required by the various assumptions, these numbers (particularly for case 1) indicate that a NC formed by merging of individual substructures may actually have reached central densities large enough, and in a time short enough, to provide the mass necessary to grow a massive central galactic object, as suggested in various previous studies (e.g., Capuzzo-Dolcetta 1993; Capuzzo-Dolcetta & Tesseri 1999; Capuzzo-Dolcetta & Vicari 2005). In light of this, the importance of further, more detailed, and complete study of the modes of formation and evolution of a NC via the merging process (Capuzzo-Dolcetta & Miocchi 2008) is clear.

7. CONCLUSIONS

In this paper, we have studied the modes of interaction of a sample of a few (four) globular clusters, whose orbital motion is limited to the core of the galactic triaxial field because they have experienced significant and rapid dissipation of their orbital energy by dynamical friction acting on their large initial masses. We studied two sets of four GCs with different initial conditions, characterized by a higher (case 1) and a lower (case 2) central density. Dynamical friction was properly taken into account by the triaxial generalization of the classical Chandrasekhar formula (Pesce et al. 1992), and it is the main cause of orbital energy dissipation up to the beginning of the merging process, when the individual cluster sizes are comparable with the orbital size. From this point forward, tidal torque is the main cause of the loss of residual orbital energy. The merger is completed in $\sim 18$ galactic core-crossing times, i.e., in much less than a Hubble time and, in any case, less than the total orbital decaying time.

The merged system keeps some of the characteristics of the pre-existing objects, attaining a relaxed structure which has, in case 2, a mildly triaxial shape (inherited by the environment) and a halo that is diffusing in the external field. Case 1 results in a merger configuration that better conserves the spherical symmetry of the “progenitors” and whose radial density profile is more concentrated toward the center than what would be expected on the basis of the mere space superposition of the four progenitors. This is likely the consequence of some violent relaxation of the merged cluster to the state of a quasi-stationary super star cluster, whose stellar density distribution maintains a King shape and is very similar to nuclear clusters observed at the centers of many galaxies. Even within the limits of our simulations (both on the number of merging objects and on the time extension of the integration), we may infer from a comparison of our results with the characteristics (surface brightness profile, integrated light and velocity dispersion; see also Table 4, which contains some examples of physical values of relevant NC parameters) of the nuclei of many galaxies (Böker et al. 2002; Geha et al. 2002; Walcher et al. 2005; Côté et al. 2006) that such nuclei may have actually formed by the merging of a few tens (when

### Table 4: Examples of Physical Values for the NC Parameters

| Parameter                              | NC1             | NC2             |
|----------------------------------------|-----------------|-----------------|
| Effective radius (pc)                  | 8.4–25          | 17–51           |
| Total mass ($M_\odot$)                 | $2.8 \times 10^7$–$2.8 \times 10^8$ | $2.8 \times 10^7$–$2.8 \times 10^8$ |
| Central density ($M_\odot$ pc$^{-3}$)  | $4 \times 10^4$–$1.1 \times 10^7$ | 250–6.6 $\times 10^4$ |
| Central line-of-sight velocity dispersion (km s$^{-1}$) | 36–280 | 14–110 |

Notes.—Intervals of values of some relevant parameters characterizing the NCs resulting from the four merged GCs, expressed in physical units for choices of length and mass in the ranges 100–300 pc and $10^9$–$10^{11} M_\odot$, respectively. The effective radius is defined as that containing half of the cluster mass in projection.
merging progenitors are quite compact) up to a few hundreds (when progenitors are looser) of massive GCs that orbitally decayed into the inner galactic regions.

In a remarkable result, the radial profile of the line-of-sight velocity dispersion of the merger remnant shows the same minimum at the galactic center as actually found by observations of a sample of nucleated Virgo Cluster dEs (Geha et al. 2002) and of late-type spirals (Walcher et al. 2005). Finally, an important theoretical result is that the high stellar densities ($\lesssim 10^7 M_\odot$ pc$^{-3}$; see Capuzzo-Dolcetta 2002) that typically needed to allow significant accretion ($\sim 1 M_\odot$ yr$^{-1}$) onto a massive object may be reached by means of a merging of fewer than 50 clusters of the type considered in case 1. This may have important implications for the role of NCs in contributing to the innermost galactic activity.

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