The recent observation of the pulsar PSR J1614-2230 with a mass of $1.97 \pm 0.04 \, M_\odot$ gives a strong constraint on the quark and nuclear matter equations of state (EoS). We explore the parameter ranges for a parameterized EoS for quark stars. We find that strange stars, made of absolutely stable strange quark matter, comply with the new constraint only if effects from the strong coupling constant and color-superconductivity are taken into account. Hybrid stars, compact stars with a quark matter core and a hadronic outer layer, can be as massive as $2 \, M_\odot$, but only for a significantly limited range of parameters. We demonstrate that the appearance of quark matter in massive stars crucially depends on the stiffness of the nuclear matter EoS. We show that the masses of hybrid stars stay below the ones of hadronic and pure quark stars, due to the softening of the EoS at the quark–hadron phase transition.

**Key words:** equation of state – stars: neutron

**Online-only material:** color figures

1. INTRODUCTION

The densities in the interior of neutron stars by far exceed the ground state density of atomic nuclei, $n_0 \sim 0.16 \, \text{fm}^{-3}$. This naturally raises the idea that compact stars might contain a deconfined and chirally restored quark phase. Recently, Demorest et al. (2010) found a new robust mass limit for compact stars by determining the mass of the millisecond pulsar PSR J1614-2230 to be $M = 1.97 \pm 0.04 \, M_\odot$. This value, together with the mass of pulsar J1903+0327 of $M = 1.667 \pm 0.021 \, M_\odot$ (Freire et al. 2010) is much larger than the Hulse–Taylor limit of $M \sim 1.44 \, M_\odot$ (Thorsett & Chakrabarty 1999), which for a long time has been the highest precisely measured pulsar mass. In this Letter, we explore the implications of this new measurement on the possible presence of quark matter in compact stars. Moreover, our aim is to map out the parameter range for the widely used quark bag model with respect to its ability to reproduce high-mass compact stars such as PSR J1614-2230.

There are two classes of compact stars that contain quark matter. The first class is the so-called hybrid stars, with quarks only in their interior either in the form of a pure quark matter core or a quark–hadron mixed phase. The size of the core depends hereby on the critical density for the quark–hadron phase transition $n_{\text{crit}}$ under neutron star conditions. The second class of so-called (strange) quark stars is realized for the special scenario of absolutely stable strange quark matter (see, e.g., Itoh 1970; Bodmer 1971; Witten 1984). It is based on the idea that the presence of strange quarks can lower the energy per baryon of the mixture of up, down, and strange quarks in weak equilibrium below the one of $^{56}\text{Fe}$ ($\sim$930 MeV). As a consequence, this strange quark matter forms the true ground state of nuclear matter and occupies the entire compact star (Alcock et al. 1986; Haensel et al. 1986).

The mass measurement for PSR J1614-2230 sets for the first time very strong limits for the parameters of any zero temperature equations of state (EoSs), and thereby also for that of quark matter. Usually, the appearance of strangeness in quark and hadronic matter provides an additional degree of freedom and thereby softens the nuclear EoS, that is, decreases the pressure for a given energy density. As a result, quark and hybrid stars cannot reach high masses. However, many studies found that effects from the strong interaction, such as one-gluon exchange or color-superconductivity can stiffen the quark matter EoS and increase the maximum mass of quark and hybrid stars (Rüster & Rischke 2004; Horvath & Lugones 2004; Alford et al. 2007; Fischer et al. 2010; Kurkela et al. 2010a, 2010b; Özel et al. 2010) and Lattimer & Prakash (2010) presented the first studies on the implications of the new mass limits from PSR J1614-2230 for quark and hybrid stars in the quark bag model. However, as we will show below, a systematic analysis of the whole allowed parameter range is still missing.

Lattimer & Prakash (2010) include strange quark matter in the form of a bag model EoS for quark stars as well as hybrid stars. The authors do not study strong effects from color-superconductivity and impose the additional constraint of $n_{\text{crit}} \gtrsim n_0$. This is a reasonable (Lattimer & Prakash 2010), but not necessary condition (Witten 1984). Moreover, they exclude a priori the existence of a quark–hadron mixed phase and come to the conclusion that the existence of a $2.5 \, M_\odot$ star would exclude the quark–hadron phase transition in compact star interiors. As we will show in the next sections our results cannot confirm this statement; furthermore we find that a quark–hadron mixed phase in fact plays a major role in supporting high-mass hybrid stars.

In a different analysis by Özel et al. (2010), the authors studied the implications of the new measurement on hybrid stars with a parameterized quark bag model including effects from color-superconductivity and QCD corrections. They find that both effects are required to support the mass of PSR J1614-2230. However, Özel et al. (2010) adjust the bag constant to obtain a fixed density of $n_{\text{crit}} = 1.5n_0$ for the phase transition to quark matter from the relatively soft APR nucleonic EoS (Akmal et al. 1998). We find that the stiffness of the hadronic EoS is important for large hybrid star masses and also that the maximum mass of hybrid star configurations experiences a minimum at around $n_{\text{crit}} \approx 0.1–0.2 \, \text{fm}^{-3}$—values close to the critical density which Özel et al. (2010) choose for their calculations.

Therefore, the aim of this Letter is to fully and systematically exploit the constraints on the quark bag EoS provided by the
new mass limit of Demorest et al. (2010). Quark matter is described by a bag model EoS with first-order corrections from the strong interaction coupling constant and effects from finite strange quark mass and color-superconductivity. For the hybrid star calculations, we use the two different relativistic mean-field (RMF) parameter sets TM1 (Sugahara & Toki 1994) and NL3 (Lalazissis et al. 1997), to explore the influence of the hadronic part of the EoS. In our calculations, we do not include hyperons which can alter the quark–hadron phase transition (Bhattacharyya et al. 2010). However, their exact role is currently an open question (see, e.g., Yasutake et al. 2010). Therefore, in this work, we will focus on non-strange hadronic matter. We consider the two possible extreme cases for the phase coexistence between quark and hadronic matter: the Maxwell transition, corresponding to a very large surface tension of quark matter (Heiselberg et al. 1993), and the Gibbs construction (Glendenning 1992) which completely neglects Coulomb and surface energies.

In the following we will describe our results and compare them with the aforementioned studies. Sections 2.1 and 2.2 are devoted to quark stars with unpaired and color-superconducting quark matter in the color–flavor–locked (CFL) phase, respectively. Hybrid stars are discussed in Section 3.

2. QUARK STARS

2.1. Unpaired Quark Matter

For the strange quark matter, we take the modified bag model:

$$\Omega_{QM} = \sum_{i=u,d,s,e} \Omega_i + \frac{3\mu^4}{4\pi^2}(1 - a_4) + B_{\text{eff}},$$

(1)

where $\Omega_i$ are the Grand potentials for the up, down, and strange quarks and electrons describing these as non-interacting fermions. We choose the strange quark mass to be $m_s = 100$ MeV (Amsler et al. 2008) while the masses of the up and down quarks and electrons are set to zero. In the sense of the generic quark matter EoS from Alford et al. (2005), we have added the $a_4$ term with the baryon chemical potential $\mu$ of the quarks in order to account for corrections from strong interaction. The usual approach in quark bag models is to unite all non-perturbative effects of the strong interactions into a bag constant $B$. The EoS can then be extended by including first-order corrections in the strong coupling constant (see, e.g., Fraga et al. 2001). The quark bag model in Equation (1) is motivated by this approach. However, since quark star matter is not in the perturbative regime, we consider $a_4$ and the bag constant as effective parameters, denoting the latter by $B_{\text{eff}}$, and explore their whole parameter range. Therefore, we vary $a_4$ from $a_4 = 1$, which corresponds to no QCD corrections, to small values when the corrections are strong. Equation (1) enables us to compute the pressure, energy density, and baryon number density assuming charge neutrality and $\beta$-equilibrium. By solving the Tolman–Oppenheimer–Volkoff equations we obtain the maximum quark star masses.

Following Farhi & Jaffe (1984), we require non-strange quark matter in bulk to have a binding energy per baryon higher than that of the most stable atomic nucleus, $^{56}$Fe, which is $930$ MeV, plus a $4$ MeV correction coming from surface effects. By imposing that $E/A \geq 934$ MeV for two-flavor quark matter at ground state, we ensure that atomic nuclei do not dissolve into their constituent quarks. Thereby we obtain an upper limit on the maximum mass of strange quark stars denoted as “two-flavor line” in Figure 1. Another constraint is given by the implementation of the strange matter hypothesis (Bodmer 1971; Witten 1984) as described in the introduction, with $E/A \leq 930$ MeV for strange quark matter at ground state (Farhi & Jaffe 1984). This condition results in the “three-flavor line” in Figure 1 and gives a lower limit on the maximum masses. Figure 1 also shows lines of constant maximum mass. The three dotted lines enclosing the red shaded area represent the mass of PSR J1614-2230 with its 1σ error (Demorest et al. 2010). The two-flavor and the three-flavor lines cross on the left outside the plot range at $a_4 = 0.247, B_{\text{eff}}^{1/4} = 102.24$ MeV which correspond to a maximum mass star with $M = 3.36 M_\odot$ and a radius of $19$ km. The Kepler line at low $B_{\text{eff}}$ represents a limit for quark stars which can rotate with a Keplerian frequency of at least $716$ Hz (Hessels et al. 2006). Therefore, the green shaded area is the allowed quark star parameter region with a minimum mass of $2.54 M_\odot$ at $a_4 \approx 0.53$ and $B_{\text{eff}}^{1/4} \approx 123.7$ MeV. However, the Kepler line is obtained from a parameterization of Lattimer & Prakash (2007) and gives a rough estimate when applied to strange stars. For a more reliable Kepler limit, the presented quark EoS should be applied in general relativistic calculations of rotating quark stars similar to the studies of Haensel et al. (2009) or Lo & Lin (2011). From Figure 1, we see that for $a_4 = 1$ the two-flavor line requires $M_{\text{max}} \lesssim 1.92 M_\odot$ which is ruled out by the new mass limit, at least within its 1σ error. Thus we find that $a_4 < 1$, i.e., QCD corrections must be included to ensure the compatibility of the model with observational data.

2.2. Color-superconducting Strange Matter Stars

At large densities, such as in compact star interiors, up, down, and strange quarks are assumed to undergo pairing and form the so-called CFL phase. We adopt the EoS from Alford et al. (2001) which introduces the pairing energy $\Delta$ as a new free parameter:

$$\begin{align*}
\Omega_{\text{CFL}} &= \frac{6}{\pi^2} \int_0^\infty dp \ p^3(p - \mu) + \frac{3}{\pi^2} \int_0^\infty dp \ p^3(\sqrt{p^2 + m_i^2} - \mu) \\
&\quad + (1 - a_4) \frac{3\mu^4}{4\pi^2} - \frac{3\Delta^2 \mu^2}{2\pi^2} + B_{\text{eff}}.
\end{align*}
$$

(2)
where \( v = 2\mu - \sqrt{\mu^2 - m_i^2/3} \). We added again the \( a_4 \) term to account for QCD corrections. The results for \( a_4 = 1.0 \) and \( m_i = 100 \) MeV are shown in Figure 2 where we have imposed the same constraints as in Figure 1. The "three-flavor" and "two-flavor" lines give again a lower and upper limit on the maximum mass. The green shaded area is the maximally allowed parameter region. Its upper edge is solely given by the constraint from the mass measurement of PSR J1614-2230. Note that for small values of \( \Delta \) quark stars are not allowed. For large gaps starting at \( \Delta \gtrsim 20 \) MeV the allowed area opens up and one can obtain high maximum masses. The largest mass allowed within the plot range is 2.34 \( M_\odot \) at \( a_4 = 1.0 \), \( B_{\text{eff}}^{1/4} = 145 \) MeV, and \( \Delta = 100 \) MeV. If we assume the same constraint from the Keplerian frequency as before, we find, that it has no influence on our results within the plot range of Figure 2. More exotic parameter combinations, as e.g., \( a_4 = 0.66, B_{\text{eff}}^{1/4} = 130.5 \) MeV, \( \Delta = 50 \) MeV and \( a_4 = 0.75, B_{\text{eff}}^{1/4} = 134.9 \) MeV, \( \Delta = 100 \) MeV with maximum masses of 2.5 \( M_\odot \) and 2.8 \( M_\odot \), respectively, are also allowed. Taking into account QCD corrections, i.e., lowering \( a_4 \), gives maximum mass lines shifted to only slightly lower values of \( B_{\text{eff}} \). Together with the lowered two-flavor and three-flavor lines this means that at some point the three-flavor constraint will become important. Still we can obtain quite large maximum masses at a sufficiently high gap value. Varying the strange quark mass basically results in shifting the maximum mass lines along the gap axis as the crucial contribution to the EoS comes from a term \( m_i^2 - 4\Delta^2 \) (Aalford et al. 2005). Thus, for fixed \( a_4 \) and a higher strange mass, the allowed area opens up at a larger value of \( \Delta \).

3. HYBRID STARS

For the hybrid star calculations we use again the bag model EoS of Equation (1) and the same quark masses as in the previous sections. As hadronic EoSs we choose the TM1 and NL3 RMF parameter sets, with maximum neutron masses of 2.2 \( M_\odot \) and 2.78 \( M_\odot \), respectively (Sugahara & Toki 1994; Lalazissis et al. 1997). The critical density \( n_{\text{crit}} \) for the quark–hadron phase transition is mainly given by the effective bag constant \( B_{\text{eff}} \) while corrections from the strong interaction \( a_4 \) affect the stiffness of the quark EoS. Combinations of small values for \( a_4 \) and \( B_{\text{eff}} \) lead therefore to stiff hybrid EoSs with a low critical density (Fischer et al. 2010). As a consequence, the corresponding hybrid stars can be very massive and have a large quark matter core. Similar to Özel et al. (2010) we omit all parameter combinations which lead to several transitions between quark and hadronic matter, which happens for small \( B_{\text{eff}} \) and \( a_4 \).

The phase transitions are modeled by the Gibbs and Maxwell constructions (see, e.g., Glendenning 1992) corresponding to a small (Gibbs) and to a large value of the surface tension (Maxwell; see discussion in Page & Reddy 2006). The former leads to the presence of an extended quark–hadron mixed phase region in the compact star interior where the pressure rises smoothly with density. With the Maxwell approach, matter experiences a direct transition from hadronic to quark matter in cold compact stars, accompanied by a density jump from lower (hadronic phase) to higher (quark phase) densities. Applying both models for the phase transition we calculate the hybrid star maximum masses and plot them as a function of the bag constant \( B_{\text{eff}} \) for fixed values of \( a_4 \). Figure 3 shows the hybrid star maximum mass curves for the TM1 EoS for the hadronic phase, while in Figure 4 we show the results for the NL3 model. The lines which extend from low values to high values of \( B_{\text{eff}} \)
correspond to calculations using the Gibbs phase transition. Stars on the solid lines have a pure quark matter core while the dashed lines represent stars where only a mixed phase is present. The maximum masses for the Maxwell transition are represented by the gray shaded area. Due to the absence of a mixed phase, hybrid stars in the Maxwell approach can only contain a pure quark matter core. A too large density jump from hadronic to quark matter in their interior leads to a gravitational instability against radial oscillations. As a consequence, we find from Figures 3 and 4 that the parameter range for hybrid stars in the Maxwell approach is significantly reduced in comparison to the “Gibbs hybrid stars.” Furthermore, in most of the cases we find that for the same combinations of $B_{\text{eff}}$ and $a_4$ the Maxwell phase transition leads to lower hybrid star maximum masses than the ones with a Gibbs construction. Only for low values of $a_4$ when quark matter becomes very stiff, the maximum masses of the “Maxwell hybrid stars” can become significantly larger than their Gibbs counterparts.

For stable hybrid stars with the Maxwell and Gibbs transition, the pure quark and the mixed phase extend over almost the whole star when $B_{\text{eff}}$, i.e., the critical density, $n_{\text{crit}}$, is low. However, as can be seen from Figures 3 and 4, a pure quark matter core in hybrid stars occurs only for small values of $B_{\text{eff}}$. For the TM1 hadronic EoS, sufficiently massive Gibbs hybrid stars contain only a mixed phase in the core. On the other hand, for the NL3 hadronic EoS with low values of $B_{\text{eff}}$ and $n_{\text{crit}}$, large cores of pure quark matter exist down to $a_4 \sim 0.5$. This is a consequence of the much stiffer NL3 hadronic EoS in comparison to the TM1 parameter set. Nevertheless, hybrid stars with a pure quark core and a mass $\geq 1.93 M_\odot$ are only obtained for $a_4 \lesssim 0.6$.

A common feature which is seen for hybrid stars in the Gibbs approach is that for a fixed value of $a_4$ the maximum masses decrease with lower critical densities and experience a minimum around $n_{\text{crit}} \sim 0.2$ fm$^{-3}$. For smaller $n_{\text{crit}}$, the quark EoS starts to dominate and the maximum masses increase again as they approach the limit of absolutely stable strange quark matter. We plot the mass of PSR J1614-2230 with its 1σ error as a gray band as well as lines at $M = 1.44 M_\odot$ and $M = 1.67 M_\odot$ to indicate the masses of the Hulse–Taylor pulsar and of J1903+0327, respectively (Thorsett & Chakrabarty 1999; Freire 2009). Especially in Figure 3 it can be seen that the new mass limit significantly tightens the constraints on the model parameters, as the whole area below the gray band is now excluded while formerly this was only the case below the 1.67 $M_\odot$ line. In this work, we did not consider effects of a finite surface tension on the quark–hadron mixed phase (Heiselberg et al. 1993) which has been found to be an intermediate of the Gibbs and the Maxwell constructions (Maruyama et al. 2007; Endo et al. 2006). However, the $B_{\text{eff}} - a_4$ parameter space for stable hybrid stars with finite surface tension can be expected to be between the Gibbs and the Maxwell constructions. Therefore our result, that hybrid stars can be massive, remains valid.

4. CONCLUSION

We present for the first time a comprehensive and systematic study on the constraints of the new compact star mass limit from the millisecond pulsar PSR J1614-2230 on the properties of quark and hybrid stars modeled within an extended quark bag model. The parameters of the bag model are an effective bag constant, corrections from the strong interaction coupling constant and color-superconductivity. We find that the new mass limit does not rule out the possibility of having quark matter in compact stars but provides tight bounds on its properties. High compact star masses where quark matter is the dominant component, require strong QCD corrections and/or a large contribution from color-superconductivity. In this case strange stars can reach masses far beyond $2 M_\odot$. For hybrid stars with a sizeable quark matter phase, our investigation shows that pure quark matter cores are obtained only for a small parameter range when the hadronic EoS is stiff and the critical density for the quark–hadron phase transition is around saturation density. Our results agree with Özel et al. (2010) concerning the importance of effective QCD corrections to reach high compact star masses. Contrarily to one of the statements of Lattimer & Prakash (2010), we find that pairing helps to increase the maximum mass and that corrections from the strong interaction have a significant effect. For hybrid stars we demonstrate that the allowed parameter region hinges crucially on the stiffness of the hadronic EoS and can therefore be much larger than in the case of Özel et al. (2010). Recently, van Kerkwijk et al. (2011) reported the possible existence of a $2.4 M_\odot$ star. If such a measurement is confirmed, even larger corrections from the strong coupling constant and larger values of the CFL gap will be required for strange stars. Hybrid stars could exist only for a stiff hadronic EoS and would contain only a core with a quark–hadron mixed phase in our approach.

An investigation similar to the one we presented here would also be desirable for other effective models of QCD such as the Nambu–Jona–Lasinio (NJL) and the PNJL models (Klähn et al. 2007; Pagliara & Schaffner-Bielich 2008; Blaschke et al. 2010; Ippolito et al. 2008), as well as Schwinger–Dyson approaches, as recently proposed by Li et al. (2011). Although mass measurements are very useful for constraining the nuclear matter EoS, additional observational information is required to probe the existence of quark matter in compact stars. Cooling, $r$-modes calculations, and gravitational wave signals of mergers (Bauswein et al. 2009) are promising tools. Also heavy-ion collisions experiments provide crucial information on the nuclear matter EoS: an extended analysis on the quark models parameters which includes both astrophysical constraints, as the mass of PSR J1614-2230, and terrestrial laboratories constraints would be extremely interesting.

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