Just-So Oscillation: as Just as MSW?

Zurab G. Berezhiani

Istituto Nazionale di Fisica Nucleare, sezione di Ferrara, I-44100 Ferrara, Italy
and
Institute of Physics, Georgian Academy of Sciences, Tbilisi 380077, Georgia
and
Anna Rossi

Instituto de Fisica Corpuscular - C.S.I.C., Universitat de Valencia
46100 Burjassot, Valencia, Spain

Abstract

The neutrino long wavelength (just-so) oscillation is reconsidered as a solution to the solar neutrino problem. In the light of the presently updated results of the four solar neutrino experiments, the data fit in the just-so scenario substantially improves and becomes almost as good as in the MSW scenario. Surprising result of our analysis is that best fit is achieved when the oscillation occurs only between two neutrino states: switching on the oscillation into third neutrino increases the $\chi^2$ value. Namely, we consider the vacuum oscillation scenario in the three-neutrino system (4 parameters) and find out that the $\chi^2$ minimum is always achieved in the two parameter subspace in which actually only two neutrino states oscillate. This holds in the framework of any solar model with relaxed prediction of the various neutrino fluxes. The possible theoretical implications of this observation are also discussed.
There are strong arguments to believe that the Solar Neutrino Problem (SNP), i.e.
the deficit of the solar neutrino fluxes indicated by four operating experiments [1, 2, 3, 4] as compared to the predictions of the Standard Solar Models (SSM) [5, 6], cannot be
explained by the nuclear/astrophysical reasons. Namely, the results of different detectors
cannot be reconciled among each other by varying the SSM parameters as are the central solar
temperature, nuclear cross sections etc. [7]. The problem is essentially related to the peculiar
energy dependence needed for the suppression of different solar neutrino components, so that
the intermediate energy $^7$Be neutrinos are to be killed more than the lower energy ($pp$) or
higher energy ($^8$B) neutrinos. This points that the SNP is rather due to the “non-standard”
neutrino properties. The most natural and plausible possibility is the oscillation of the solar $\nu_e$ into another type of neutrino $\nu_x$.

The neutrino oscillation picture can provide the necessary energy dependence in two
regimes, which are known as the Mikheyev-Smirnov-Wolfenstein (MSW) [8] and the long
wavelength vacuum oscillation (so called just-so) [9] scenarios. The MSW resonant conversion
in matter offers a natural possibility of selective strong reduction of the $^7$Be neutrinos and
thus appears to be the most attractive and elegant solution to the SNP. It provides a very
good fit of the experimental data, implying the mass range $\delta m^2 \sim 10^{-5}$ eV$^2$ and small mixing angle, $\sin^2 2\theta \sim 10^{-2}$ [12]. The just-so scenario, with the oscillation wavelength comparable
to the sun-earth distance, needs $\delta m^2$ of about $10^{-10}$ eV$^2$ and large mixing angle, $\sin^2 2\theta \sim 1$ [13, 14], which parameter range can be naturally generated by non-perturbative quantum
gravitational effects [10, 11].

Both the MSW and just-so mechanisms are theoretically motivated and so far the
experimental data do not allow to discriminate them. Hence, both these scenarios should
be considered as realistic candidates to the SNP solution. However, the peculiar "just-so"
predictions as are the seasonal time variation of the neutrino signals and the specific deforma-
tion of the original solar neutrino spectra [13, 14], will allow to test the long wavelength oscillation scenario at the future real time detectors like Superkamiokande, SNO, Borexino,
etc., and to discriminate it from the MSW picture.

In the present paper we present an updated analysis of the just-so scenario. The earlier
analysis [13, 14] has shown that the experimental data fit in this scheme is acceptable, but
somewhat worse than that of MSW picture. However, after publication of these papers the
experimental data have been changed. In particular, the result of the Homestake experiment
has been recalibrated [1] and the new data of GALLEX experiment became available [3].
On the other hand, the SSM predictions have been reconsidered by taking into account
the helium and heavy element diffusion [6]. The changes are small and rather confirm the
stability of both the experimental and theoretical results. Nevertheless, as we show below,
these small changes make the "just-so" data fit substantially better so that it becomes almost
as good as in the MSW picture.

Another issue which we address in this paper is the following. Up to now, in the literature
all the analysis of the vacuum oscillation scenario was performed within the simplified case
of two neutrino ($2\nu$) system, which case employs only two parameters: the $\nu_e - \nu_x$ mixing

\[1\]In the language of the Standard Model (SM) the neutrino masses emerge through the higher order
operators of the type $\frac{1}{M}(l_iCl_j)HH$, where $l_i$ ($i = 1, 2, 3$) and $H$ are respectively the lepton and Higgs
doublets and $M$ is some regulator scale. (For example, the famous “seesaw” scenario effectively reduces to
these operators with $M$ of the order of the right-handed neutrino mass). Then the neutrino mass range
needed for the just-so scenario corresponds to the Planck scale, $M \sim 10^{19}$ GeV, whereas the MSW scenario
requires $M$ to be of the order of the supersymmetric grand unification scale, $M \sim 10^{16}$ GeV.
angle \( \theta \) and the mass square difference \( \delta m^2 \) between two neutrino eigenstates. However, the nature knows that there exist three neutrino species and if their masses are originated by the same mechanism, then the relevant case should be the oscillation picture in the \( 3\nu \) system, in which case the solar neutrino data are determined by four parameters. Here explore the full parameter space for the 3 neutrino just-so oscillations. As we will see, surprisingly the introduction of additional parameters does not improve the data fit and the best description of the existing data is essentially provided by the \( 2\nu \) oscillation, i.e. by the case in which the oscillation into the third neutrino is decoupled. We briefly discuss the implications of this observation for the structure of the neutrino mass matrix.

We perform our analysis in the spirit adopted earlier for the MSW picture in refs. \[1,2\] and for the just-so scenario in our previous paper \[14\]. Namely, instead of studying the just-so scenario in the context of one particular SSM with including in the \( \chi^2 \) analysis the theoretical uncertainties, we prefer to scan various realistic SSMs. We take as a reference model the recent Bahcall and Pinsonneault (BP) SSM \[6\] without the underlying theoretical uncertainties, while the variations of the SSM parameters are taken into account by parametrizing various neutrino fluxes as \( \phi^k = f_k \phi_0^k \) (\( k = ^8B, \ 7Be, \ 13N, \ 15O, \ pep \ and \ pp \)), where the subscript ‘0’ denotes the values predicted by the BP SSM \[6\] and the factors \( f_k \) account for theoretical uncertainties. Thus, the cases of other SSMs \[5\] will be effectively recovered by relaxing \( \phi^k \). In particular, for a SSM characterized by a given set of \( f_k \), the signals of the radiochemical detectors are expected to be

\[
\begin{align*}
R_{Cl}^{SSM} &= 7.36f_B + 1.24f_{Be} + 0.22f_{pep} + 0.11f_N + 0.37f_O \text{ (SNU)} \quad (1) \\
R_{Ga}^{SSM} &= 69.7f_{pp} + 16.1f_B + 37.7f_{Be} + 3.0f_{pep} + 3.8f_N + 6.3f_O \text{ (SNU)} \quad (2)
\end{align*}
\]

as compared to the experimental rates \( R_{Cl}^{exp} = 2.55 \pm 0.25 \text{ SNU} \[1\] \) and \( R_{Ga}^{exp} = 74 \pm 8 \text{ SNU} \) (for the gallium data we use the weighted average of the GALLEX result \( R_{GALLEX} = 77 \pm 8 \pm 6 \text{ SNU} \[3\] \) and the SAGE one \( R_{SAGE} = 69 \pm 11 \pm 6 \text{ SNU} \[4\] \)). Thus, the observed signal in each detector can be given as a ratio of the measured rate to that is expected in a given SSM,

\[ Z_a = \frac{R_a^{exp}}{R_a^{SSM}} \quad (a = Cl, Ga) \]

and it is a function of the factors \( f_k \). On the other hand, the Kamiokande result \[2\] implies that the ratio of the experimental rate to the prediction of SSM with given \( f_B \) is

\[ Z_K = \frac{R_K^{exp}}{R_K^{SSM}} = \frac{1}{f_B} \quad (0.45 \pm 0.08) \quad (3) \]

where obviously the case \( f_B = 1 \) corresponds to the BP SSM \[3\].

Several numerical studies have shown that in practice the effects of independent variations of the metal fraction, the opacities, the astrophysical factor \( S_{pp} \) and the age of the sun are well reproduced by variations of the central solar temperature \( T_c \) according to the power laws \[13\]:

\[ \phi^k = \phi_0^k \left( \frac{T_c}{T_c^0} \right)^{\beta_k} \]

Thus, we consider \( f_B = (T_c/T_c^0)^{\beta_B} \) as a free parameter while for other coefficients we correspondingly obtain \( f_k = f_k^{\beta_k} \), where \( \xi_k = \beta_k/\beta_B \). Following ref. \[13\], we take \( \xi_{pp} = -0.034 \pm 0.006, \xi_{Be} = 0.47 \pm 0.08, \xi_N = 1.0 \pm 0.4, \xi_O = 1.2 \pm 0.4 \) and \( \xi_{pep} = 0 \pm 0.1 \) In this

\footnote{In the following computations we take the central values neglecting the theoretical uncertainties. These uncertainties are indeed small for the \( pp \) and \( Be \) neutrinos. For the CNO components these are bigger but the contribution of the latter to the signals \[3\] are not substantial.}
way, the case of other SSM’s will be effectively recovered by varying $f_B$. For example, by taking $f_B \simeq 0.7$ the Turk-Chieze and Lopez SSM \cite{5} is reproduced. The factor $f_B$ is varied in the range $0.4 - 1.6$ (the lower limit $f_B \simeq 0.4$ is in fact set by the Kamiokande result), which in fact corresponds to the variation of the central solar temperature within $(-4 \div +2)$ %.

Let us first consider the vacuum oscillation solution in the case of two neutrino flavours: $\nu_e \rightarrow \nu_x$ where $\nu_x$ is $\nu_\mu$ or $\nu_\tau$ (alternatively, it can be also a sterile state $\nu_s$). In this case the survival probability for solar $\nu_e$’s with energy $E$ depends on two parameters, the mixing angle $\theta$ and the mass square difference $\delta m^2$:

$$P(E) = 1 - \sin^2 2\theta \sin^2 \left( \frac{\delta m^2 L_t}{4E} \right)$$

(4)

The sun-earth distance depends on time as $L_t = L[1 - \varepsilon \cos(2\pi t/T)]$, where $L = 1.5 \cdot 10^{11}$ m, $T = 1$ yr, and $\varepsilon = 0.0167$ is the ellipticity of the orbit. Then the time averaged detection rates expected in the radiochemical experiments read as

$$R = \int dE \sigma(E) \sum_k \langle P(E) \phi^k \rangle_T \lambda_k(E)$$

(5)

where $\sigma(E)$ is the detection cross section, $\phi^k = f_k \phi^0_k$ are the fluxes of the relevant components of the solar neutrinos ($k = B, Be, \text{etc.}$), $\lambda_k(E)$ are their energy spectra normalized to 1, and $\langle \ldots \rangle_T$ stands for the average over the whole time period $T$. For the Kamiokande detector we have

$$R_K = \int_{E_{th}} dE \lambda_B(E) \left[ \langle P(E) \phi^B \rangle_T \sigma_{\nu_e}(E) + \left( \langle \phi^B \rangle_T - \langle P(E) \phi^B \rangle_T \right) \sigma_{\nu_x}(E) \right]$$

(6)

where $\sigma_{\nu_y}$ ($y = e, x$) is the $\nu_y e^-$ scattering cross section (in the case of conversion into sterile neutrino $\sigma_{\nu_s} = 0$) and $E_{th} = \frac{1}{2}(T_e + \sqrt{T_e(T_e + 2m_e)})$, where $T_e = 7.5$ MeV is the recoil electron kinetic energy threshold.

We accept the hypothesis that the solar neutrino luminosities are constant in time, and use the averaged data of the chlorine, gallium and Kamiokande experiments to perform the standard $\chi^2$ analysis for various cases. We find it instructive to separate the experimental and theoretical (SSM) uncertainties, and do not include the latter in $\chi^2$ analysis once these are simulated by varying the factors $f_k$. We define

$$\chi^2 = \sum_a \left( \frac{R_{a,exp} - R_a}{\sigma_a} \right)^2$$

(7)

where the index $a = Cl, Ga, K$ labels three different data and $\sigma_a$ are the corresponding experimental errors. Once we describe the three experimental results by means of two parameters ($\delta m^2$ and $\sin^2 2\theta$), the number of the degrees of freedom is 1.

We have performed the $\chi^2$ analysis for various values of $f_B$, using for the numerical minimization procedure the MINUIT package provided by the CERN program library. The corresponding best fit points of $\chi^2_{min}$ and ‘1σ’ areas containing the true parameter values with 68 % probability, once the solution is assumed, are shown in Fig. 1. We see that compared to the previous analysis \cite{14}, the fits have become much better. E.g. in the case
\( f_B = 1 \) we have \( \chi_{\text{min}}^2 = 1.6 \) (versus \( \chi_{\text{min}}^2 = 4.4 \) obtained in our previous paper 4), so that the just-so oscillation is allowed as a SNP solution at the 20 \% confidence level (C.L.).

By varying \( f_B \), the relevant range of \( \delta m^2 \) remains rather stable, while \( \sin^2 2\theta \) tends to be smaller with decreasing \( f_B \). The lowering (increasing) of \( f_B \) implies the weakening (strengthening) of the oscillation. However, as a general tendency, by decreasing \( f_B \) the fit becomes worse: e.g. for \( f_B = 0.4 \) we have \( \chi_{\text{min}}^2 = 3.2 \), while it improves for \( f_B > 1 \): e.g. for \( f_B = 1.6 \) we have \( \chi_{\text{min}}^2 = 0.9 \), which is acceptable at 34 \% C.L.

Thus our conclusion is that for \( f_B \geq 0.7 \) the relevant parameter range is limited by the values \( \delta m^2 = (0.5 - 1) \times 10^{-10} \text{eV}^2 \) and \( \sin^2 2\theta = 0.7 - 1 \). For \( f_B \leq 0.7 \) the parameter range is somewhat wider (see Fig. 1) but as we have seen in this case the quality of the fit is less good). The dependence on \( \sin^2 2\theta \) is simpler – the just so scenario prefers the large (close to the maximal) mixing 5 whereas the dependence on \( \delta m^2 \) is more sophisticated, due to the different oscillation length of solar neutrinos with different energies. To give some more numerical insight on this dependence, in the Figs. 2a,b we show \( \chi^2 \) as a function of \( \delta m^2 \) for the cases \( \sin^2 2\theta = 1 \) and 8/9, which correspond to the cases of the maximally symmetric (democratic) mixing in the 2 and 3 neutrino system (see below).

Let us consider now the case of three neutrino mixing, when the neutrino mass eigenstates \( \nu_i \) with masses \( m_i \) \((i = 1, 2, 3)\) are related to the weak eigenstates \( \nu_{\alpha} \) \((\alpha = e, \mu, \tau)\) as \( \nu_{\alpha} = U_{\alpha i} \nu_i \), where \( U \) is the 3x3 unitary matrix of the neutrino mixing. The general expression for the \( \nu_{\alpha} \to \nu_{\beta} \) transition probability is:

\[
P_{\alpha\beta}(E) = \left| \sum_i U_{\beta i} U_{\alpha i}^* \exp(-iE_i t) \right|^2
\]

(8)

The radiochemical detectors are sensitive only to \( \nu_e \) state. On the other hand, the Kamiokande experiment does not distinguish between the contributions of \( \nu_\mu \) and \( \nu_\tau \), provided that the latter have no non-standard interactions with electrons (\( \sigma_{\nu_\mu} = \sigma_{\nu_\tau} \) in eq. (3)). Hence, the analysis of the solar neutrino data in the 3\( \nu \) case needs only the expression of the \( \nu_e \to \nu_e \) survival probability:

\[
P(E) = 1 - 4 |U_{e1} U_{e2}|^2 \sin^2\left(\frac{\delta_{21} L_t}{4E}\right) - 4 |U_{e1} U_{e3}|^2 \sin^2\left(\frac{\delta_{31} L_t}{4E}\right) - 4 |U_{e2} U_{e3}|^2 \sin^2\left(\frac{\delta_{32} L_t}{4E}\right)
\]

(9)

where \( \delta_{ij} = m_i^2 - m_j^2 \). This expression depends on four independent variables which we choose as \( U_{e2}, U_{e3} \) and \( \delta_{21}, \delta_{31}, \) whereas \( \delta_{32} = \delta_{31} - \delta_{21} \) and the unitarity constraints fix \( |U_{e1}|^2 = 1 - |U_{e2}|^2 - |U_{e3}|^2 \). One can use e.g. Maiani-like parametrization by identifying \( U_{e1} = \cos \theta \cos \phi \), \( U_{e2} = \sin \theta \cos \phi \) and \( U_{e3} = \sin \phi \).

The 2\( \nu \) oscillation is a limiting case which is achieved if the third neutrino is decoupled. E.g. in the limit \( U_{e3} = 0 \) (sin \( \phi = 0 \)), in which case the dependence \( \delta_{31} \) is also lost, we have \( 4 |U_{e1} U_{e2}|^2 = \sin^2 2\theta \) and thus eq. (9) reduces to the familiar expression (8). On the other hand, when e.g. \( \delta_{32} = 0 \), we can identify \( \delta_{31} = \delta_{21} = \delta m^2 \) and still \( 4 |U_{e1}|^2 (1 - |U_{e1}|^2) = \sin^2 2\theta \).

3 Since the theoretical uncertainties are not included, here and in the following the quantitative statements on the C.L. are somewhat informal, and reflect only the ideal situation with no uncertainties in SSM predicted values of the neutrino fluxes, exactly fixed by the choice of the factors \( f_k \). For the case of \( \nu_e \) being a sterile neutrino we obtain \( \chi_{\text{min}}^2 = 5.5 \).

4 According to ref. [17], such a large mixing puts the just-so oscillation in contradiction with the observed pattern of the neutrino signal from SN 1987A. See, however, the recent paper [18].
In principle, the presence of four parameters versus only three experimental data makes the system overdetermined and thereby the standard meaning of $\chi^2$ analysis is not directly applicable. However, even in this case the canonical $\chi^2$ function provides a numerical insight on the deviation of the theoretical estimates from the experimental data, within their errors. Its minimization with respect to all the four parameters can allow to know whether the $3\nu$ just-so scenario shows a better “fit” (i.e. smaller $\chi^2$) in some relevant parameter region as compared to the $2\nu$ case. As smaller the value of $\chi^2$ is, as closer is the theoretical ”just-so” prediction with experimental data.

The result of our analysis is rather surprising. Performing numerically the minimization procedure in the four parameter space, we always find that the best fit point (for which $\chi^2$ achieves the minimum) is always located in the two parameter subspace which effectively corresponds to the $2\nu$ case, i.e. when e.g. $U_{e3} = 0$ or $\delta_{32} = 0$, for any feasible value of $f_B$. The values of the minimal $\chi^2_{\text{min}}$ coincides with that is obtained above in the $2\nu$ scheme. For example, in the case $f_B = 1$ we obtain $\chi^2_{\text{min}} = 1.6$, which occurs for the parameter values $|U_{e3}|^2 = 0, \ |U_{e2}|^2 = 0.40$ (i.e. $\sin^2 2\theta = 0.96$) and $\delta_{21} = 0.6 \cdot 10^{-10} \text{eV}^2$. Of course, there are also trivially equivalent minima obtained by permutations $U_{ei} \rightarrow U_{ej}$, or by identifying $\delta_{ij} = \delta_{ik}$.

The feature seems rather obvious in the limit when say $\delta_{31}$ is taken to be large, i.e. $m_3 \gg 10^{-5} \text{eV}$. In this case the two last terms in (3) are in the regime of the averaged vacuum oscillation, and the $\nu_e$ survival probability reads

$$P(E) = 1 - \frac{1}{2} \sin^2 2\phi - \cos^4 \phi \sin^2 2\theta \sin^2 \left( \frac{\delta_{21}L}{4E} \right)$$

Then for non-zero $\phi$ the data fit has to become worse, since the contribution of the energy dependent term becomes smaller which makes it more difficult to reconcile the Homestake and Kamiokande data. This feature, however, is not so obvious when $\delta_{31}$ is also in the just-so ($\sim 10^{-10} \text{eV}^2$) range.

As we have seen, the best just-so description of solar neutrino data is achieved when in fact only 2 neutrino states participate the oscillation. Let us discuss briefly how this situation could be obtained in the context of various neutrino mass generation schemes.

The neutrino mass range needed for the just-so oscillation can naturally emerge from the Planck scale effects \[10, 11\]. In the minimal standard model (or SU(5) GUT) the neutrinos are massless as far as renormalizable interactions are concerned. In other words, in the absence of the right-handed (RH) neutrino states the lepton number conservation arises as an accidental global symmetry of the theory due to the joint requirement of gauge invariance and renormalizability. However, the global symmetries need not be respected by nonperturbative quantum gravity effects. Thus, if the SM (or SU(5)) is a true theory up to the Planck scale, the neutrino masses can be induced only by the non-renormalizable operators cutoff by the Planck scale $M_{Pl}$:

$$\frac{\alpha_{ij}}{M_{Pl}} (l_i H) C(l_j H)$$

(11)

where $l_i = (\nu_i, e_i)^T$ ($i = 1, 2, 3$) are the left-handed (LH) lepton doublets, $H$ is the standard Higgs doublet, and $\alpha_{ij}$ are the order 1 constants. (In fact, this is equivalent to the seesaw mechanism with RH neutrino states having $\sim M_{Pl}$ Majorana masses and $\sim 1$ Yukawa constants; in particular, such a situation is automatically obtained in a supersymmetric grand unified model $SU(6)$ \[19\]). As a result, the neutrinos get Majorana masses of the
order of $\delta m^2 \sim \hat{\delta} m^2$ thus naturally emerges in the range needed for the just-so oscillation.

In general case, this operator induces mass terms for all three neutrinos $\nu_e, \nu_\mu, \nu_\tau$. If the matrix $\alpha_{ij}$ of the effective coupling constants is a general one, then we have the $3 - \nu$ oscillation case. However, it could occur that $\alpha_{ij}$ is a rank-1 matrix,

$$\alpha = U^T P U, \quad P = diag(0, 0, 1)$$

where $U$ is some unitary matrix. Then only one neutrino eigenstate gets mass of the order of $\hat{\delta} m$, while others stay massless. In this case the neutrino oscillation picture effectively reduces to the $2\nu$ case. For example, it was argued in [11] that once the operator (11) is induced by nonperturbative gravitational effects (virtual black holes or wormholes), the flavour blindness of gravity could imply the following pattern for the neutrino mass matrix

$$\hat{m}_\nu^0 = \alpha \hat{\delta} m \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = U^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3\alpha \hat{\delta} m \end{pmatrix} U$$

in which case we effectively have $2\nu$ oscillation with $\delta m^2 = 9\alpha^2 \hat{\delta} m^2$ and $\sin^2 2\theta = 8/9$, in good agreement with the data fit in the just-so picture (see Fig. 2b).

It is however also possible that one of the neutrino eigenstates acquires a mass larger than $\hat{\delta} m$ from sources different from the Planck scale induced operators (11). In particular, the neutrino mass matrix may have a structure

$$\hat{m}_\nu = m \tilde{T} + \hat{m}_\nu^0, \quad m \gg \hat{\delta} m$$

where $T$ is some rank-1 matrix. If $T = P$, where $P$ is defined in eq. (12), then the $\nu_\tau$ state acquires mass $m$ and for $m \sim 5 - 7$ eV it can play a role of the cosmological hot dark matter component [21]. In this case for the SNP solution remains the just-so oscillation $\nu_e \to \nu_\mu$ with maximal mixing $\sin^2 2\theta = 1$, which still provides a good fit to the solar neutrino data (see Fig. 2a).

Alternatively, one can imagine that the matrix $T$ has a democratic structure in the space of the $\nu_\mu$ and $\nu_\tau$ weak eigenstates:

$$T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

In this case the $\nu_\mu \to \nu_\tau$ oscillation emerges with $\sin^2 2\theta_{\mu\tau} = 1$ and $\delta m^2_{\mu\tau} = 4m^2$, which for $m \simeq 5 \cdot 10^{-3}$ eV could explain the the atmospheric neutrino deficit [20], while the just-so oscillation is still effective in explaining the SNP.

Concluding, we have seen that the just-so oscillation scenario provides a very good fit to the recently updated experimental data on the solar neutrinos, and its present status is as reasonable as that of the widely popular MSW scenario. We have shown that best data fit is achieved within the two neutrino system, and with incorporating the third neutrino the fit can only get worse. This points that the third neutrino state can be involved into the games of solving other present neutrino puzzles as are the atmospheric neutrino problem or the problem of the cosmological hot dark matter.
Acknowledgements.

We are grateful to Gianni Fiorentini for illuminating conversations. Useful discussions with Giovanni Di Domenico, Barbara Ricci, Alexei Smirnov and Jose Valle are also gratefully acknowledged.

References

[1] B.T. Cleveland et al., Nucl. Phys. B (Proc. Suppl.) 38 (1995) 47.
[2] Kamiokande Collaboration, Nucl. Phys. B (Proc. Suppl.) 38 (1995) 55.
[3] GALLEX Collaboration, P. Anselmann et al., preprint LNGS 95/37.
[4] SAGE Collaboration, J.S. Nico et al., Proc. 27th Int. Conf. on High Energy Physics, Glasgow, UK (July 1994).
[5] J. N. Bahcall and R. K. Ulrich, Rev. Mod. Phys. 60 (1989) 297; J.N. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. 64 (1992) 885; S. Turk-Chi`eze and I. Lopez, Ap. J. 408 (1993) 347; S. Turk-Chi`eze et al., Phys. Rep. 230 (1993) 57; V. Castellani, S. Degl’Innocenti and G. Fiorentini, Astron. Astrophys. 271 (1993) 601.
[6] J.N. Bahcall and M.H. Pinsonneault, preprint IASSNS-AST 95/24.
[7] V. Castellani, S. Degl’Innocenti and G. Fiorentini, Phys. Lett. B 303 (1993) 68; V. Castellani et al., Phys. Lett. B 324 (1994) 425; J.N. Bahcall and H.A. Bethe, Phys. Rev. Lett. 65 (1993) 2233; J.N. Bahcall et al., preprint IASSNS-AST 94/13 (1994); J.N. Bahcall, preprints IASSNS-AST 94/14, IASSNS-AST 94/37 (1994); S. Bludman et al., Phys. Rev. D 47 (1993) 2220; Phys. Rev. D 49 (1994) 3622; V.S. Berezinsky, Comments Nucl. Part. Phys. 21 (1994) 249; A.Yu. Smirnov, preprint DOE/ER/40561-136-INT94-13-01 (1994).
[8] S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. 42 (1985) 1441; Nuovo Cimento 9C (1986) 17; L. Wolfenstein, Phys. Rev. D 17 (1978) 2369; D 20 (1979) 2634.
[9] V. Gribov and B. Pontecorvo, Phys. Lett. 28 (1967) 493; J.N. Bahcall and S. Frautschi, Phys. Lett. 29B (1969) 623; V. Barger, R.J.N. Phillips and K. Whisnant, Phys. Rev. D 24 (1981) 538; S.L. Glashow and L.M. Krauss, Phys. Lett. B 190 (1987) 199.
[10] R. Barbieri, J. Ellis and M.K. Gaillard, Phys. Lett. B 90 (1980) 249.
[11] E. Akhmedov, Z. Berezhiani and G. Senjanovi´c, Phys. Rev. Lett. 69 (1992) 3013.
[12] P.I. Krastev and A.Yu. Smirnov, Phys. Lett. B 338 (1994) 282; V. Berezinsky, G. Fiorentini and M. Lissia, Phys. Lett. B 341 (1994) 38; N. Hata and P. Langacker, Phys. Rev. D 50 (1994) 632.
[13] P. Krastev and S. Petcov, Phys. Rev. Lett. 72 (1994) 1960; preprint SISSA 41/94/EP (1994).
[14] Z.G. Berezhiani and A. Rossi, Phys. Rev. D 51 (1995) 5229.
[15] V. Castellani et al., Phys. Rev. D 50 (1994) 4749.
[16] A. Acker, A.B. Balantekin and F. Loreti, Phys. Rev. D 49 (1994) 328.
[17] J.N. Bahcall, A.Yu. Smirnov and D.N. Spergel, Phys. Rev. D 49 (1994) 1389.
[18] P.J. Kernan and L.M. Krauss, Nucl. Phys. B 437 (1995) 243.
[19] Z.G. Berezhiani, hep-ph/9503306, Phys. Lett. B (in press).
[20] Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. B 335 (1994) 237.
[21] R. Shaefer and Q. Shafi, Nature 359 (1992) 199.
Figure Captions

Fig. 1. The long-dashed, dott-dashed, solid, dashed and dotted contours mark the 68 % C.L. regions respectively for $f_B=0.4$, 0.7, 1, 1.3, 1.6. The corresponding best fit points are shown by diamonds, with $\chi^2_{\text{min}}$ being respectively 3.2, 2.5, 1.6, 1.5, 0.9.

Fig. 2. $\chi^2$ as a function of $\delta m^2$ for a fixed mixing angle: $\sin^2 2\theta = 1$ (a) and $\sin^2 2\theta = 8/9$ (b). The dot-dashed, solid and dashed curves correspond to the cases $f_B = 0.7$, 1 and 1.3, respectively.

Fig. 3. $\chi^2$ minimized with respect to parameters $\sin^2 2\theta$ and $\delta_{21}$, as a function of the ‘13’ mixing $\sin^2 2\phi$, for the case of large $\delta_{31}$ ($m_3 \gg \tilde{m}$). The dot-dashed, solid and dashed curves correspond to the cases $f_B = 0.7$, 1 and 1.3, respectively.