Supersymmetric $SU(3)^3$ Unification with Anomalous $U(1)_A$ Gauge Symmetry

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We consider supersymmetric unification based on the gauge symmetry $SU(3)_C \times SU(3)_L \times SU(3)_R$ supplemented by an anomalous $U(1)_A$ gauge symmetry. Fermion masses and mixings consistent with experiments are realized, including large mixings in the neutrino sector. We also consider the supersymmetric flavor problem, gauge coupling unification and proton decay. The dominant proton decay mode is predicted to be $p \to e^+ \pi^0$, and the lifetime is estimated to be $\sim 10^{34} - 10^{35}$ years.

§1. Introduction

Supersymmetric grand unified theories offer a particularly elegant scenario for unifying the strong and electroweak interactions. Recently, realistic models based on $SO(10)$ and $E_6$ have been proposed in which an anomalous $U(1)_A$ gauge symmetry plays a critical role. The anomaly is cancelled via the Green Schwarz mechanism, and the resulting phenomenology has several attractive features. In particular, all interactions allowed by the symmetries are included in the analysis. (There can be undetermined order unity coefficients in the interactions.) These models naturally resolve the doublet-triplet problem using the mechanism of Ref. 10) (see also Refs. 11)–13)). Realistic structures of quark and lepton mass matrices, including large neutrino mixings, are realized using the Froggatt-Nielsen (FN) mechanism. The anomalous $U(1)_A$ also helps explain the hierarchical symmetry-breaking scales and the masses acquired by the superheavy particles, and the models incorporating it reduce precisely to the minimal supersymmetric standard model (MSSM) at low energies. Even though the gauge couplings are unified in these schemes slightly below the usual GUT scale, dimension-five proton decay is sufficiently suppressed, and the decay $p \to e + \pi$ via gauge mediated dimension-six operators may be seen in the near future. Finally, in these models, the cutoff scale is lower than the Planck scale, $M_P$, and the $\mu$ problem is also resolved.

However, the above-mentioned models require two adjoint Higgs fields to realize DT splitting, which is not so easily realized in the framework of superstring models. In this paper, we examine the application of the above approach to grand unified theories with semi-simple unification, whose symmetry breaking to MSSM (the minimal supersymmetric standard model) does not require an adjoint Higgs field. A particularly attractive example is provided by the gauge symmetry $SU(3)^3 \equiv SU(3)^3 \times SU(3)^3 \times SU(3)^3$.
SU(3)_C × SU(3)_L × SU(3)_R, which is a maximal subgroup of E_6, and which arises as an effective four-dimensional symmetry from the compactification of the E_8 × E_8 heterotic superstring theory on a Calabi-Yau manifold. Phenomenology based on SU(3)^3 has been extensively studied. Our goal here is to apply the techniques of Refs. 1)–5) to the gauge symmetry SU(3)^3 and elucidate the most important consequences. In particular, we show how fermion masses and mixings consistent with experiments are obtained, including large mixings in the neutrino sector (with the exception of U_e3). We also consider SUSY breaking and flavor changing neutral currents, gauge coupling unification and proton decay. While dimension-five proton decay is found to be strongly suppressed, dimension-six operators yield a proton lifetime of \(\sim 10^{34} - 10^{35}\) years for the decay channel \(p \to e^+\pi^0\).

§2. Matter sector

The matter sector for this model has essentially the same structure as that of the E_6 model, with 27 of E_6 given in terms of SU(3)_C × SU(3)_L × SU(3)_R as

\[27 \to (3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3}).\] (2.1)

Three 27-plets, \(\Psi_i\) (i = 1, 2, 3), are introduced, and the Yukawa interactions contain appropriate powers of the VEV of the FN field \((\Theta) = \lambda \Lambda\), which has an anomalous \(U(1)_A\) charge, \(\theta = -1\),

\[\lambda^{\psi_i + \psi_j + \phi} \Psi_i \Psi_j \Phi,\] (2.2)

where \(\Phi\) is a Higgs field. Throughout this paper we use units in which the cutoff is \(\Lambda = 1\). We denote all superfields by uppercase letters and their anomalous \(U(1)_A\) charges by the corresponding lowercase letters. Here, for simplicity, we have assumed an E_6 like charge assignment in the matter sector, but in principle, we can assign these charges without respecting E_6 symmetry. Using the definitions of the fields \(Q(3, \bar{2}, \bar{1})_2, U(3, \bar{1}, \bar{1})_(-1), D^c(\bar{3}, 1, \bar{1})_2, L(1, 2, \bar{1})_2, E(1, 1, 1)_2, N^c(1, 1, 0)_0, L'(1, 2, \bar{1})_2, L'(1, 2, \bar{1})_2, D'(\bar{3}, 1, \bar{1})_2, S(1, 1)\) and their conjugate fields under the standard model (SM) gauge symmetry, the fields \((3, \bar{3}, 1), (\bar{3}, 1, 3),\) and \((1, 3, \bar{3})\) under SU(3)_C × SU(3)_L × SU(3)_R are

\[(3, \bar{3}, 1) \to Q + D',\] (2.3)

\[(\bar{3}, 1, 3) \to U^c + D^c + D'^c,\] (2.4)

\[(1, 3, \bar{3}) \to L + E^c + N^c + L' + S.\] (2.5)

For future reference, under the breaking \(E_6 \to SO(10)\), we have

\[27 \to \left[Q + U^c + E^c + D^c + L + N^c\right]_{16} + \left[L'^c + L' + D' + D'^c\right]_{10} + S_{1}.\] (2.6)

Because \(D'(L'^c)\) can acquire a superheavy mass by combining it with a linear combination of \(D^c\) and \(D'^c\) (\(L\) and \(L'\)) after the breaking SU(3)_C × SU(3)_L × SU(3)_R to the SM gauge group, the remaining massless fields form the three generation matter content of MSSM. Because the Yukawa couplings are determined mainly by the
anomalous $U(1)_A$ charges of the massless fields, we would like to know which of the fields among $D^c_i$ and $D^c_i$ ($L_i$ and $L'_i$) ($i = 1, 2, 3$) are massless.

To this end, we discuss how $SU(3)_C \times SU(3)_L \times SU(3)_R$ breaks to MSSM. We introduce the Higgs fields with non-vanishing VEVs $\Phi(1,3,3)$, $\Phi(1,3,3)$, $\Phi(1,3,3)$ and $\Phi(1,3,3)$. The VEVs $|\langle \Phi \rangle| = |\langle \Phi \rangle| \sim \lambda^{-(\phi+\bar{\phi})/2}$ break $SU(3)_C \times SU(3)_L \times SU(3)_R$ to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, while the VEVs $|\langle \Phi \rangle| = |\langle \Phi \rangle| \sim \lambda^{-(c+\bar{c})/2}$ break $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ to the SM gauge group. Here the VEVs are determined by the anomalous $U(1)_A$ charges. This point can be roughly understood as follows (while determination of the VEVs themselves is explained later):

1. Because the interactions are determined by the anomalous $U(1)_A$ charges, the VEV of the gauge invariant operator $O$ with negative charge $o$ is determined as $\langle O \rangle \sim \lambda^{-o}$.

2. For $O = \Phi \Phi$, the VEV becomes $\langle \Phi \Phi \rangle \sim \lambda^{-(\phi+\bar{\phi})}$.

3. Because the $D$-flatness condition requires $|\langle \Phi \rangle| = |\langle \Phi \rangle|$, we obtain $|\langle \Phi \rangle| = |\langle \Phi \rangle| \sim \lambda^{-\frac{1}{2}(\phi+\bar{\phi})}$.

The massless modes can be determined from the superpotential

$$W_1 = \lambda \psi_1 \psi_1 + \phi \psi_1 \psi_2 \psi \psi_3 \Phi + \psi_1 \psi_2 + c \psi_1 \psi_2 \psi_3 C$$

and the VEVs $\langle \Phi \rangle \sim \lambda^{-\frac{1}{2}(\phi+\bar{\phi})}$ and $\langle C \rangle \sim \lambda^{-\frac{1}{2}(c+\bar{c})}$. The mass matrices of $D^{c(t)}$ and $D^{c(t)} (L^{c(t)}$ and $L^{c(t)}$) are obtained from

$$M_I = \begin{pmatrix}
I_1 & I_2 & I_3 \\
I_1' & I_2' & I_3' \\
I_1'' & I_2'' & I_3''
\end{pmatrix}
\begin{pmatrix}
\lambda^2 \psi_1 + r & \lambda^2 \psi_2 + r & \lambda^2 \psi_3 + r \\
\lambda^2 \psi_2 + r & \lambda^2 \psi_3 + r & \lambda^2 \psi_1 + r \\
\lambda^2 \psi_3 + r & \lambda^2 \psi_1 + r & \lambda^2 \psi_2 + r
\end{pmatrix}
\left(\begin{pmatrix}
\lambda \psi_1 + \psi_2 + \psi_3 \\
\lambda \psi_2 + \psi_3 + \psi_1 \\
\lambda \psi_3 + \psi_1 + \psi_2
\end{pmatrix} + \frac{1}{2}(\phi+\bar{\phi})
\right),$$

where $I = L, D^{c(t)}$ and we have defined the parameter $r$ as

$$r \equiv \frac{1}{2}(c - \bar{c} - \phi + \bar{\phi}),$$

which we use frequently in the following discussion. Note that the mass matrices are determined by the anomalous $U(1)_A$ charges. Therefore, the massless modes are also determined by the charges. Since, without loss of generality, we can take $\psi_1 \geq \psi_2 \geq \psi_3$ (as discussed in Ref. 4), as long as we ignore the cases with vanishing coefficients arising from some SUSY based mechanism, the main components of the massless modes can be obtained as follows:

1. $\psi_1 - \psi_3 < r : (I_1, I_2, I_3)$.

2. $0 < r < \psi_1 - \psi_3 : (I_1, I_1', I_2)$.

3. $\psi_3 - \psi_1 < r < 0 : (I_1, I_1', I_2')$.

4. $r < \psi_3 - \psi_1 : (I_1', I_1', I_3')$.

The case $(I_1, I_1', I_2)$ is interesting, because bi-large neutrino mixing angles can be realized without tan $\beta$ being too small if we take account of the mixing of subcomponents. Indeed, the massless modes $(I_{10}, I_{10}', I_{10}')$ are given by

$$I_{10} = I_1 + \lambda \psi_1 - \psi_3 I_3 + \lambda \psi_1 - \psi_2 + r I_2' + \lambda \psi_1 - \psi_3 + r I_3',$$

(2.10)
where the first terms on the right-hand sides are the main components of these massless modes, and the other terms represent mixing with the other states, \( I_3, I_2^0 \) and \( I_3^0 \). Here we adopt a basis such that the main components are not mixed.

The mass matrices for quarks and leptons are obtained from the superpotential

\[
W_2 = \lambda \psi_1 + \psi_2 + h \Psi_3 \eta H,
\]

where \( H(1, 3, 3) \) contains the MSSM Higgs doublets. Here we introduced an additional field as \( H \), but in §4, we see that \( \Phi \) can play the same role as \( H \). If we adopt the charges \( \psi_1 = 3 + n, \psi_2 = 2 + n, \psi_3 = n \) and \( h = -2n \), the mass matrices are given by

\[
M_U = \begin{pmatrix}
Q_1 & Q_2 & Q_3 \\
U_1^c & U_2^c & U_3^c \\
\end{pmatrix}
\begin{pmatrix}
\lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^5 & \lambda^4 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1 \\
\end{pmatrix}
\langle L'(H) \rangle,
\]

\[
M_D(M_E^T \eta^{-1}) = \begin{pmatrix}
Q_1(E_1^c) & Q_2(E_2^c) & Q_3(E_3^c) \\
D_1^0(L_1^0) & D_2^0(L_2^0) & D_3^0(L_3^0) \\
\end{pmatrix}
\begin{pmatrix}
\lambda^6 & \lambda^6 - r & \lambda^5 \\
\lambda^5 & \lambda^5 - r & \lambda^4 \\
\lambda^3 & \lambda^3 - r & \lambda^2 \\
\end{pmatrix}
\langle L'(H) \rangle.
\]

Here \( \eta \sim 2 - 3 \) is the renormalization group factor. Then, we can obtain the CKM matrix as

\[
U_{\text{CKM}} = \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1 \\
\end{pmatrix},
\]

which reproduces the experimental results if we take \( \lambda \sim 0.2 \). Because the ratio of the Yukawa couplings of the top and bottom quarks is \( \lambda^2 \), a small value of \( \tan \beta \sim m_t/m_b \cdot \lambda^2 \) is predicted by these mass matrices.

The Dirac neutrino mass matrix is given by the \( 3 \times 6 \) matrix

\[
L_1^0 \begin{pmatrix}
S_1 & S_2 & S_3 & N_1^c & N_2^c & N_3^c \\
\lambda^r + 6 & \lambda^r + 5 & \lambda^r + 3 & \lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^6 & \lambda^5 & \lambda^3 & \lambda^6 - r & \lambda^5 - r & \lambda^3 - r \\
\lambda^r + 5 & \lambda^r + 4 & \lambda^r + 2 & \lambda^5 & \lambda^4 & \lambda^2 \\
\end{pmatrix}
\langle L'(H) \rangle \eta,
\]

which we simply express as

\[
M_N = \left( \begin{array}{cc}
\lambda^r + 2 & \lambda^2 \\
\end{array} \right) \otimes \left( \begin{array}{ccc}
\lambda^4 & \lambda^3 & \lambda \\
\lambda^4 - r & \lambda^3 - r & \lambda^{1 - r} \\
\lambda^3 & \lambda^2 & 1 \\
\end{array} \right) \langle L'(H) \rangle \eta.
\]
The $6 \times 6$ matrix for right-handed neutrinos ($S_i, i = 1, 2, 3$, and $N^c_k, k = 1, 2, 3$) is obtained as

$$M_R = \lambda^{2n} \begin{pmatrix}
\lambda^{\bar{\phi} - \phi} & \lambda^{(\bar{\phi} - \phi + \bar{c} - c)/2} \\
\lambda^{(\bar{\phi} - \phi + \bar{c} - c)/2} & \lambda^{\bar{\bar{c}} - \bar{c}}
\end{pmatrix} \otimes \begin{pmatrix}
\lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^5 & \lambda^4 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix} \quad (2.19)
$$

from the interactions

$$\lambda^{\psi_i + \psi_j + 2\bar{\phi}} \Psi_j \bar{\Phi} \bar{\Phi} + \lambda^{\psi_i + \psi_j + \bar{c} + \bar{\phi}} \Psi_j \bar{\Phi} \bar{C} + \lambda^{\psi_i + \psi_j + 2\bar{c}} \Psi_j \bar{C} \bar{C}, \quad (2.20)
$$

by developing the VEVs $\langle \bar{\Phi} \rangle \sim \lambda^{-\frac{1}{2}(\bar{\phi} + \bar{\phi})}$ and $\langle \bar{C} \rangle \sim \lambda^{-\frac{1}{2}(c + \bar{c})}$. Using the seesaw mechanism,\(^{20}\) we obtain the neutrino mass matrix

$$M_\nu = M_N M_R^{-1} M_N^T = \lambda^{4 - 2n + c - \bar{c}} \begin{pmatrix}
\lambda^2 & \lambda^{2-r} & \lambda \\
\lambda^{2-r} & \lambda^{2-2r} & \lambda^{1-r} \\
\lambda & \lambda^{1-r} & 1
\end{pmatrix} \langle L'c(H) \rangle^2 \eta^2. \quad (2.21)
$$

Then, we finally obtain the Maki-Nakagawa-Sakata matrix as

$$U_{MNS} = \begin{pmatrix}
1 & \lambda^r & \lambda \\
\lambda^r & 1 & \lambda^{1-r} \\
\lambda & \lambda^{1-r} & 1
\end{pmatrix}. \quad (2.22)
$$

If we adopt $r = 1/2$, which implies

$$c - \bar{c} = \phi - \bar{\phi} + 1, \quad (2.23)
$$

bi-large neutrino mixing angles are realized, because of $\lambda^{1/2} \sim 0.5$, as in the $SO(10)$ model investigated in Ref. 1) and the $E_6$ model investigated in Ref. 4). Moreover, this results in the prediction $U_{e3} \sim \lambda$. Future experiments should be able to determine whether or not there is such a large $U_{e3}$ just below the CHOOZ upper limit, $U_{e3} \leq 0.15$.\(^{21}\) For the neutrino masses, the present model predicts $m_{\nu_3}/m_{\nu_e} \sim \lambda$, which is consistent with the most probable LMA MSW solution for the solar neutrino puzzle.\(^{14}\)

If we define

$$l \equiv \bar{\phi} - \phi + 2n - 10, \quad (2.24)
$$

the neutrino mass matrix is given by

$$M_\nu = \lambda^{-(5+l)} \begin{pmatrix}
\lambda^2 & \lambda^{1.5} & \lambda \\
\lambda^{1.5} & \lambda & \lambda^{0.5} \\
\lambda & \lambda^{0.5} & 1
\end{pmatrix} \langle L'c(H) \rangle^2 \eta^2, \quad (2.25)
$$

where we have used the relation (2.23). The parameter $l$ can be determined by

$$\lambda^l = \lambda^{-5} \frac{\langle L'c(H) \rangle^2 \eta^2}{m_{\nu_3} A}. \quad (2.26)$$
We are supposing that the cutoff scale \( \Lambda \) is in the range \( 5 \times 10^{15} \text{GeV} < \Lambda < 10^{20} \text{GeV} \), which allows \( -3 \leq l \leq 2 \). If we choose \( l = -2 \), the neutrino masses are given by \( m_{\nu_e} \sim \lambda^{-3} \langle L' (H) \rangle^2 \eta^2 / \Lambda \sim m_{\nu_\mu} / \lambda \sim m_{\nu_e} / \lambda^2 \). If we take \( \eta \langle L' (H) \rangle = 100 \text{ GeV} \), \( \Lambda \sim 2 \times 10^{16} \text{ GeV} \) and \( \lambda = 0.2 \), then we get \( m_{\nu_e} \sim 6 \times 10^{-2} \text{ eV} \), \( m_{\nu_\mu} \sim 1 \times 10^{-2} \text{ eV} \) and \( m_{\nu_\tau} \sim 2 \times 10^{-3} \text{ eV} \). These values are consistent with the experimental data for atmospheric neutrinos and with the large mixing angle (LMA) MSW solution for the solar neutrino problem. \(^{22}\)

\[\text{§3. SUSY breaking and FCNC}\]

Let us now discuss SUSY breaking. In this paper, we do not address how to realize degenerate sfermion masses. Here, we concentrate on the characteristic SUSY breaking effect of our scenario. Because the anomalous \( U(1)_A \) charges depend on flavor to produce the hierarchy of Yukawa couplings, generically non-degenerate scalar fermion masses are induced through the anomalous \( U(1)_A \) \( D \)-term. \(^{23}\) Under the \( E_6 \) like charge assignment of the matter sector, the SUSY contribution to \( K^0 - \bar{K}^0 \) mixing is naturally suppressed, as in the \( E_6 \) GUT scenario. The essential point is that the anomalous \( U(1)_A \) charge of \( D_1^0 \) becomes the same as that of \( D_2^0 \), because the fields \( D_1^0 \sim D_2^0 \) and \( D_3^0 \sim D_2^0 \) arise from a single field \( \Psi_1 \) of \( E_6 \). Because the constraints from \( K^0 - \bar{K}^0 \) mixing on the ratio \( \delta \equiv \Delta / \bar{m}^2 \) (where \( \Delta \) is the mixing of sfermion mass matrices and \( \bar{m} \) is the average of sfermion mass) are

\[
\sqrt{|\text{Re} (\delta_{12}^D)_{LL} (\delta_{12}^D)_{RR}|} \leq 2 \times 10^{-3} \left( \frac{\bar{m}_q (\text{GeV})}{500} \right),
\]

\[
|\text{Re} (\delta_{12}^D)_{LL}|, |\text{Re} (\delta_{12}^D)_{RR}| \leq 4 \times 10^{-2} \left( \frac{\bar{m}_q (\text{GeV})}{500} \right),
\]

and the former constraint is much stronger than the latter, suppression of \( (\delta_{12}^D)_{RR} \) makes the constraint on the SUSY breaking sector much weaker. Here we use the notation \( (\delta_{ij}^F)_{XY} \), where \( F = U, D, N, E \), the chirality index is \( X, Y = L, R \), and the generation index is \( i, j = 1, 2, 3 \), as defined in Ref. 26). As in the usual anomalous \( U(1)_A \) scenario, \( \Delta \) can be estimated as

\[
(\Delta_{ij}^F)_{XX} \sim \lambda |f_i - f_j| |(f_i - f_j)| \langle D_A \rangle. \tag{3.3}
\]

Therefore, the coincidence of the anomalous \( U(1)_A \) charges of \( D_1^0 \) and \( D_2^0 \) leads to the suppression of \( (\Delta_{12}^F)_{RR} \). This weakens the constraints from \( K^0 - \bar{K}^0 \) mixing.

To see how weak the constraints become in our scenario, we fix the SUSY breaking sector as follows. At the cutoff scale, we adopt the common gaugino mass \( M_{1/2} \) and the \( D \)-term of anomalous \( U(1)_A \) gauge symmetry \( D_A \) as non-vanishing SUSY breaking parameters. Then, the scalar fermion mass squared at low energy scales is estimated as

\[
m_{F_i}^2 \sim f_i R M_{1/2}^2 + \eta_F M_{1/2}^2, \tag{3.4}
\]

where \( \eta_F \) is a renormalization group factor and

\[
R \equiv \frac{\langle D_A \rangle}{M_{1/2}^2}. \tag{3.5}
\]
Table I. Lower bound on gaugino mass $M_{1/2}$ at the GUT scale (in GeV).

| $R$   | 0.1 | 0.3 | 0.5 | 1   | 2   |
|-------|-----|-----|-----|-----|-----|
| $(\delta_{12}^D)_{LL}$ | 17  | 43  | 61  | 87  | 105 |
| $\sqrt{(\delta_{12}^D)_{LL}(\delta_{12}^E)_{RR}}$ | 78  | 191 | 268 | 373 | 437 |
| $| (\delta_{12}^E)_{RR} |$ | 431 | 304 | 221 | 161 | 116 |

The constraint (3.2) for $(\delta_{12}^D)_{LL}$ is rewritten

$$M_{1/2} \geq 1.25 \times 10^4 \frac{\lambda^2(\psi_1 - \psi_2)R}{(\eta_D L + \psi_1 + \psi_2 R)^{3/2}} \text{(GeV)}.$$  

(3.6)

Though the main contribution to $(\delta_{12}^D)_{RR}$ vanishes, through the mixing in Eqs. (2.10) and (2.11), $(\delta_{12}^D)_{RR}$ is estimated as

$$(\delta_{12}^D)_{RR} \sim \frac{\lambda^2}{\eta_D \Delta + \psi_1 R}.$$  

(3.7)

From Eq. (3.1) for $\sqrt{(\delta_{12}^D)_{LL}(\delta_{12}^E)_{RR}}$, the constraint on the gaugino mass $M_{1/2}$ is given by

$$M_{1/2} \geq 1.8 \times 10^5 \frac{\lambda^{1.75} R(\psi_1 - \psi_2)}{(\eta_D + \psi_1 R)^{1.5}}.$$  

(3.8)

As another condition, the $\mu \rightarrow e\gamma$ process gives

$$| (\delta_{12}^E)_{LL} |, | (\delta_{12}^E)_{RR} | \leq 3.8 \times 10^{-3} \left( \frac{\tilde{m}_l \text{(GeV)}}{100} \right)^2,$$  

(3.9)

where $\tilde{m}_l$ is the average mass of the scalar leptons. This constraint can be rewritten

$$M_{1/2} \geq 1.6 \times 10^3 \frac{\lambda(\psi_1 - \psi_2)R^{1/2}}{\eta_{E_R} + \psi_1 + \psi_2 R} \text{(GeV)}.$$  

(3.10)

Taking the values $\psi_1 = 9/2$, $\psi_2 = 7/2$, $\eta_D L \sim \eta_D R \sim 6$ and $\eta_{E_R} \sim 0.15$, we obtain the rough lower limits on the gaugino mass given in Table I.

Note that in a certain range of values of $R$, the $\mu \rightarrow e\gamma$ process gives the severest constraint among the FCNC processes.\(^{27}\) This gives reason to believe that the lepton flavor violating processes\(^{27,28}\) might be seen in the near future.

§4. Higgs sector

In addition to the Higgs with non-vanishing VEVs $\Phi, \bar{\Phi}, C, \bar{C}$, we introduce $C'(1, \bar{3}, 3)$ and $\bar{C}'(1, \bar{3}, 3)$ with vanishing VEVs and several singlets, $S$ and $Z$, in order to give superheavy masses to these Higgs fields. The Higgs content is presented in Table II.

It is interesting that in this model, the Higgs field $H$ is contained in $\Phi$, as in the $E_6$ case. Actually the VEVs $\langle \Phi \rangle = \langle \bar{\Phi} \rangle$ break $SU(3)^3$ into $SU(3) \times SU(2)_L \times SU(2)_R$. 

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SU(2)_{R} \times U(1)_{B–L}, under which $\Phi(1, 3, \bar{3})$ is divided as $\Phi(1, 2, 2) + \Phi(1, 2, 1) + \Phi(1, 1, 2) + \Phi(1, 1, 1)$, and $\Phi(1, 2, 1) + \Phi(1, 1, 2) + \Phi(1, 1, 1)$ is absorbed by the Higgs mechanism, and remaining field, $\Phi(1, 2, 2)$, can be only a pair of doublet Higgs fields in MSSM. The Higgs fields $Q_L, \bar{Q}_L, Q_R$ and $\bar{Q}_R$ are introduced only to realize the same Kac-Moody levels of the three $SU(3)$ gauge groups, and they play no other role in the following argument.

In this model, the singlet composite operator $\Phi\Phi$ plays the same role as the FN field $\Theta$. The $D$-flatness condition for the anomalous $U(1)_A$ gauge symmetry is

$$D_A = g_A \left( \xi^2 + |\phi|^2 + |\bar{\phi}|^2 \right) = 0, \quad (4.1)$$

where $\xi^2$ is the parameter of the Fayet-Illiopoulos $D$-term. Because the $D$-flatness conditions of $SU(3)_L$ and $SU(3)_R$ require $|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle|$, the $D$-flatness condition for the anomalous $U(1)_A$ gauge symmetry can be rewritten

$$D_A = g_A \left( \xi^2 + (\phi + \bar{\phi}) |\phi|^2 \right) = 0. \quad (4.2)$$

Thus we obtain $\xi^2 + (\phi + \bar{\phi}) |\phi|^2 = 0$, namely, $|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| = \xi$. In this case, because $\Phi\Phi$ plays the same role as $\Theta$, the unit of the hierarchy becomes $\langle \Phi\Phi \rangle = \lambda \sim \xi^2$, which is different from that in the usual case, in which the FN field is just a singlet field $\Theta$ and $\langle \Theta \rangle = \xi$. This means that even if $\xi$ has a milder hierarchy, the unit of the hierarchy becomes larger. Using gauge rotation and the $D$-flatness condition for $SU(3)_L \times SU(3)_R$ gauge symmetry, the VEV can be taken as

$$|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda^{-\frac{1}{2}}(\phi + \bar{\phi}) \end{pmatrix}, \quad (4.3)$$

which breaks $SU(3)_C \times SU(3)_L \times SU(3)_R$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B–L}$. In order to determine the VEVs of the other Higgs fields, we examine the superpotential

$$W = W_{C'} + W_{\bar{C}'} + W_{NV}, \quad (4.4)$$

where $W_X$ denotes the terms linear in the field $X$, which has vanishing VEV, and $W_{NV}$ includes only the fields with non-vanishing VEVs. From the superpotential

$$W_{NV} = \bar{\Phi}^3 + \bar{\Phi}^2 \bar{C}, \quad (4.5)$$
The mass spectrum of the remaining fields consists of \( \langle S(C) \rangle \neq 0, \langle N^c(C) \rangle \neq 0 \) if \( \langle CC \rangle \neq 0 \). We are interested in the former vacuum.

The superpotentials \( W_{C'} \) and \( W_{C'} \) are given by

\[
W_{C'} = \lambda^{c+\phi}C'\bar{\Phi}(1 + \bar{\phi}^{c+\phi}C + \lambda^{z+\bar{Z}_i}Z_i + \lambda^{\bar{Z}_i}Z_i)
\]

\[
+ \lambda^{c+\phi}C'\bar{C}(1 + \lambda^{\bar{Z}_i}Z_i), \quad (4.6)
\]

\[
W_{C'} = \lambda^{c+\phi}C'\Phi(1 + \lambda^{\bar{Z}_i}Z_i) + \lambda^{c+\phi}C'\bar{C}(1 + \lambda^{\bar{Z}_i}Z_i).
\]

Here we ignore \((\Phi\Phi)^2\) for simplicity, but its effect is critical. After developing VEVs, the above interactions do not respect \( SU(3)_L \times SU(3)_R \) gauge symmetry. For example, the coefficient of \( N^c(C')\bar{N}^c(C) \) is different from that of \( L(C')\bar{L}(C) \). This is important to align the VEVs and to give superheavy masses to these fields. The F-flatness conditions \( F_{S(C')} = F_{S(C')} = F_{N^c(C')} = F_{N^c(C')} = 0 \) determine four VEVs, \( \langle C' \rangle \sim \lambda^{-(c+\phi)} \) and \( \langle Z_i \rangle \sim \lambda^{-z_i}(i = 1, 2, 3) \). Then, all the VEVs are determined by the anomalous \( U(1)_A \) charges.

We now examine the mass spectrum of the Higgs sector. The mass matrix \( M_L \) for \( L \) and \( \bar{L} \) is obtained from the interactions below

\[
\begin{pmatrix}
    L'_{\Phi} & L'_{C} & \bar{L}'_{\Phi} & L_{C} & L'_{C'} & L_{C'} & L_{\Phi} & L_{C} & L_{C'} \\
    L'_{C} & 0 & 0 & 0 & 0 & 0 & C'\Phi & 0 & 0 \\
    \bar{L}'_{\Phi} & 0 & 0 & \bar{\Phi}^3 & \bar{\Phi}^2\bar{C} & \bar{\Phi}C' & \bar{C}'\bar{\Phi}^2 & \bar{\Phi}\bar{C} & 0 & C'\bar{C}\bar{\Phi} \\
    \bar{L}'_{C} & 0 & 0 & \bar{\Phi}^2\bar{C} & \bar{\Phi}C^2 & \bar{C}'\bar{\Phi} & \bar{C}'C'\bar{\Phi} & \bar{C}'C & 0 & C'\bar{C}'C'\bar{\Phi} \\
    L_{C'} & 0 & 0 & C'\Phi & C'\bar{C} & C'\bar{\Phi}C' & C'\bar{C}' & 0 & C'\bar{C}'C' \bar{\Phi} & C'\bar{C}'\bar{C}'C' \bar{\Phi} \\
    L_{\Phi} & 0 & 0 & 0 & 0 & 0 & C'\Phi\bar{C} & 0 & 0 & C'\bar{C} \\
    L_{C} & 0 & 0 & 0 & 0 & 0 & C'\Phi\bar{C} & 0 & 0 & C'\bar{C} \\
    L_{C'} & 0 & 0 & \bar{\Phi}^2C'\bar{C} & \bar{\Phi}C'\bar{\Phi}C' & 0 & C'\bar{C}'\bar{C}' \bar{\Phi} & C'\bar{C}' & C'\bar{C}' & C'\bar{C}' \bar{\Phi} \\
\end{pmatrix}
\]

(4.8)

It is obvious that a linear combination of \( L'_{\Phi} \) and \( \bar{L}'_{\Phi} \) and that of \( L'_{\Phi} \) and \( L_{\Phi} \) are massless and they form the doublet Higgs fields of MSSM. \( L_{\Phi} \) and \( \bar{L}_{\Phi} \) are absorbed by the Higgs mechanism in breaking \( SU(3)_L \times SU(3)_R \) into \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \). The mass spectrum of the remaining fields consists of \( \lambda^{c+\bar{\phi}}, \lambda^{c+\bar{\phi}}, \lambda^{c+\bar{\phi}}, \lambda^{c+\bar{\phi}}, \lambda^{c+\bar{\phi}} \).

The mass matrix \( M_E \) for \( E^c \) and \( \bar{E}^c \) is obtained from the interactions

\[
\begin{pmatrix}
    E^c_{\Phi} & E^c_{C} & E^c_{C'} \\
    E^c_{\Phi} & 0 & 0 & C'\bar{\Phi} \\
    E^c_{C} & 0 & 0 & 0 \\
    E^c_{C'} & C'\bar{\Phi} & 0 & C'\bar{C}' \\
\end{pmatrix}
\]

(4.9)

The fields \( E^c_{C} \) and \( \bar{E}^c_{C} \) are eaten by the Higgs mechanism in breaking \( SU(2)_R \times U(1)_{B-L} \) into \( U(1)_Y \). The mass spectrum of the remaining fields is \( \lambda^{c+\bar{\phi}} \) and \( \lambda^{c+\bar{\phi}} \).

The mass matrix \( M_D^c \) for the fields \( D^c, D'^c, \bar{D}^c, \bar{D}'^c \) is obtained from the follow-
The mass of the fields $U$ only by the anomalous effective anomalous.

The determinants of the reduced mass matrices are estimated as simple sums of the effective charges. We will use this result in calculating the running gauge couplings.

Then all the elements of the mass matrices are estimated as simple sums of the effective anomalous $U(1)_A$ charges.

\[
\begin{align*}
D_{Q_R}^c & = \begin{pmatrix} D_{Q_R}^c & D_{Q_R}^\prime \\ \bar{Q}_R Q_R & \bar{Q}_R \bar{Q}_R C \\ 0 & \bar{Q}_R Q_R & 0 \end{pmatrix}. \\
D_{Q_L}^c & = \begin{pmatrix} \bar{Q}_L \bar{Q}_R \bar{C} & Q_L \bar{Q}_R \bar{F} & \bar{Q}_L Q_L \end{pmatrix}.
\end{align*}
\]

(4-10)

The mass spectrum consists of $\lambda^{q_1+q_r}$, $\lambda^{q_1'+q_r'}$ and $\lambda^{q_4+q_l}$.

By the above argument, the mass spectrum of superheavy particles is determined only by the anomalous $U(1)_A$ charges, and therefore we can examine whether or not coupling unification is realized. Before beginning the analysis in the next subsection, we define the reduced mass matrices $\bar{M}_I$ by getting rid of the massless modes from the original mass matrices $M_I$. The ranks of the reduced matrices in our semi-simple model are $\bar{r}_Q = \bar{r}_U = 1$, $\bar{r}_E = 2$, $\bar{r}_L = 7$ and $\bar{r}_D = 3$. It is useful to define the effective anomalous $U(1)_A$ charges,

\[
\begin{align*}
x_I & \equiv i + \frac{1}{2} \Delta \phi, \quad \bar{x}_I \equiv - \frac{1}{2} \Delta \phi, (x = \phi, l', c', d', d') \\
x_I & \equiv i + \Delta c - \frac{1}{2} \Delta \phi, \quad \bar{x}_I \equiv - \Delta c + \frac{1}{2} \Delta \phi, (x = l, d') \\
x_I & \equiv i, \quad \bar{x}_I \equiv - (x = q, u', e')
\end{align*}
\]

(4-11)

(4-12)

(4-13)

where $I = \Phi, C, C', Q_L, Q_R$ ($i = \phi, c, c', q_1, q_r$), $\Delta \phi \equiv \frac{1}{2} (\phi - \bar{\phi})$, and $\Delta c \equiv \frac{1}{2} (c - \bar{c})$.

The determinants of the reduced mass matrices are estimated as simple sums of the effective anomalous $U(1)_A$ charges of massive modes:

\[
\begin{align*}
\det \bar{M}_Q & = \lambda^{q_1+q_r} \\
\det \bar{M}_U & = \lambda^{q_1'+q_r'} \\
\det \bar{M}_E & = \lambda^{e_{\Phi} + e_{\bar{\Phi}} + e_{C} + e_{\bar{C}}} \\
\det \bar{M}_D & = \lambda^{d_{\phi} + d_{\bar{\phi}} + d_{c} + d_{\bar{c}} + d_{\bar{q}_1} + d_{\bar{q}_r} + d_{q_1} + d_{q_r}} = \lambda^{q_1 + q_2 + q_c + q_d} \\
\det \bar{M}_L & = \lambda^{b_{\phi} + b_{\bar{\phi}} + b_{c} + b_{\bar{c}} + b_{\bar{q}_1} + b_{\bar{q}_r} + b_{q_1} + b_{q_r}}
\end{align*}
\]

(4-14)

(4-15)

(4-16)

(4-17)

(4-18)

Then all the elements of the mass matrices are estimated as simple sums of the effective charges of superheavy particles if they are not vanishing, and the determinants of mass matrices are also determined by simple sums of the effective charges. We will use this result in calculating the running gauge couplings.

§5. Coupling unification

In this section, we apply the general discussion on the gauge coupling unification given in Ref. 3) to our scenario. The pattern of the breaking of the gauge symmetry in our model is as follows. At the scale $\Lambda_\Phi \sim \lambda^{-(\phi + \bar{\phi})/2}$, $SU(3)^3$ is broken into

\[
\begin{align*}
\end{align*}
\]
Supersymmetric $SU(3)^3$ Unification

$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Also, $SU(2)_R \times U(1)_{B-L}$ is broken into $U(1)_Y$ at the scale $A_C \sim \lambda^{-\frac{c+c'}{2}}$. We base our analysis on one-loop renormalization group equations. The conditions expressing the gauge coupling unification are given by

$$\alpha_3(A) = \alpha_2(A) = \frac{5}{3} \alpha_Y(A) \equiv \alpha_1(A),$$

(5.1)

where $\alpha_1^{-1}(A_\Phi > \mu > A_C) \equiv \frac{3}{5} \alpha_1^{-1}(A_\Phi > \mu > A_C) + \frac{2}{5} \alpha_{B-L}^{-1}(A_\Phi > \mu > A_C)$, $\alpha_2^{-1}(\mu > A_\Phi) \equiv \frac{4}{5} \alpha_{2R}(\mu > A_\Phi)$ and $\alpha_3^{-1}(\mu > A_\Phi)$ and $\alpha_2^{-1}(\mu > A_\Phi) \equiv \alpha_{3L}(\mu > A_\Phi) + \frac{2}{5} \alpha_{B-L}(\mu > A_C)$. Here, $\alpha_X = \frac{\alpha_X}{4\pi}$, and the parameters $g_X (X = 3, 3L, 3R, 2, R, B - L, Y)$ are the gauge couplings of $SU(3)_C$, $SU(3)_L$, $SU(3)_R$, $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$ and $U(1)_Y$, respectively.

Using the fact that the three gauge couplings of the minimal SUSY standard model (MSSM) meet at the scale $A_C \sim 2 \times 10^{16}$ GeV, the above conditions expressing gauge coupling unification can be rewritten

$$b_1 \ln \left( \frac{A}{A_C} \right) + \Sigma_I \Delta b_{1I} \ln \left( \frac{A_{\text{I}}^\text{I}}{\det M_I} \right) - \frac{12}{5} \ln \left( \frac{A}{A_C} \right) - 2 \ln \left( \frac{A}{A_\Phi} \right),$$

(5.2)

$$= b_2 \ln \left( \frac{A}{A_C} \right) + \Sigma_I \Delta b_{2I} \ln \left( \frac{A_{\text{I}}^\text{I}}{\det M_I} \right) - 2 \ln \left( \frac{A}{A_\Phi} \right),$$

(5.3)

$$= b_3 \ln \left( \frac{A}{A_C} \right) + \Sigma_I \Delta b_{3I} \ln \left( \frac{A_{\text{I}}^\text{I}}{\det M_I} \right),$$

(5.4)

where $(b_1, b_2, b_3) = (33/5, 1, -3)$ are the renormalization group coefficients for MSSM and $\Delta b_{aI}$ ($a = 1, 2, 3$) are the corrections to the coefficients of the massive fields $I = Q + \bar{Q}, U^c + \bar{U}^c, E^c + \bar{E}^c, D^c + \bar{D}^c,$ and $L + \bar{L}$. The third term in Eq. (5.2) is from the breaking $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ by the VEV $(C)$, and the last terms in Eqs. (5.2) and (5.3) are from the breaking $SU(3)_L \times SU(3)_R \rightarrow SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ by the VEV $(\Phi)$. Because all the mass matrices and the symmetry breaking scales appearing in the above conditions are determined by the anomalous $U(1)_A$ charges, these conditions can be translated to the constraints on the effective charge and the cutoff scale:

$$\alpha_1(A) = \alpha_2(A) \rightarrow A \sim A_C \lambda^{-\frac{1}{36} (5\bar{\phi} - \phi + 6(c + \bar{c}))},$$

(5.5)

$$\alpha_2(A) = \alpha_3(A) \rightarrow A \sim A_C \lambda^{-\frac{1}{36} (7\bar{\phi} + \phi + 6(c + \bar{c} + c'))},$$

(5.6)

$$\alpha_1(A) = \alpha_3(A) \rightarrow A \sim A_C \lambda^{-\frac{1}{36} (25\bar{\phi} + 19\phi + 36(c + \bar{c}) + 18(c' + \bar{c}'))},$$

(5.7)

A naive calculation leads to the relation between the charges

$$59\bar{\phi} + 41\phi + 42(c + \bar{c}) + 54(c' + \bar{c}') \sim 0,$$

(5.8)

which is difficult to satisfy in our scenario. However, careful calculation shows that gauge coupling unification is possible, though somewhat larger ambiguities of order 1 coefficients are required than in a simple group unification. Actually, with the typical charge assignment in Table I, coupling unification is realized as in Fig. 1, using the ambiguities of order 1 coefficients represented by the relation $\lambda \leq y \leq \lambda^{-1}$. 

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Fig. 1. Here, we adopt $\lambda = 0.22$, $\alpha_1^{-1}(M_Z) = 59.47$, $\alpha_2^{-1}(M_Z) = 29.81$, $\alpha_3^{-1}(M_Z) = 8.40$, the SUSY breaking scale $m_{SB} \sim 1$ TeV, and the following anomalous $U(1)_A$ charges: $(\psi_1, \psi_2, \psi_3) = (9/2, 7/2, 3/2)$, $\phi = -3$, $\bar{\phi} = 2$, $c = -3$, $\bar{c} = 1$, $z = -1$, $c' = 0$ and $\bar{c}' = 4$. Using the ambiguities of coefficients expressed by $\lambda \leq y \leq \lambda^{-1}$, the three gauge couplings meet at around $10^{16}$ GeV.

Note that the unified gauge coupling can remain in the perturbative region in the $SU(3)^3$ model, which differs from the situation for the $E_6$ model. This is because the Higgs sector in $SU(3)^3$ unification is much simpler than that in $E_6$ unification, since we do not have to introduce adjoint Higgs fields to realize the doublet-triplet splitting.

In the present model, the cutoff scale tends to be lower than the Planck scale. Indeed, the cutoff is taken as $10^{16}$ GeV in Fig. 1. Since the cutoff scale is so low, we have to treat proton decay via dimension-five operators,\(^{(31)}\) which are obtained from

$$\lambda \bar{\phi} \psi_i \bar{\psi}_j \bar{\psi}_k \bar{\psi}_l \bar{\Phi}$$

by developing the VEV $\langle \bar{\Phi} \rangle \sim \lambda^{-\frac{1}{2}}(\phi + \bar{\phi})$. The coefficients are suppressed not only by the usual small Yukawa factor but also by the suppression factor $\lambda^{4n + \frac{1}{2}(\bar{\phi} - \phi)} = \lambda^{8.5}$. Even if we take the cutoff as $\Lambda \sim 10^{16}$ GeV, the ‘effective’ colored Higgs mass is around $\lambda^{-8.5} \Lambda \sim 10^{22}$ GeV, which is much larger than the experimental bound of $10^{18}$ GeV. Thus, proton decay via dimension-five operators is adequately suppressed.

On the other hand, our model suggests that proton decay $p \rightarrow e^+\pi^0$ via dimension six operators from the Kähler potential

$$K = \frac{1}{\Lambda^2} \bar{\psi}_1 \bar{\psi}_1 \bar{\psi}_1,$$

which are allowed by the symmetry in our scenario by taking the unification scale $\Lambda_U$ as the cutoff $\Lambda$, may be seen in future experiments. If we roughly estimate the
lifetime of the proton using the formula in Ref. 33) and the recent result of the lattice calculation for the hadron matrix element parameter $\alpha$34 we find

$$\tau_p(p \to e^+\pi^0) \sim 4.5 \times 10^{34} \left( \frac{A}{10^{16}\text{GeV}} \right)^4 \left( \frac{0.015(\text{GeV})^3}{\alpha} \right)^2 \text{years.} \quad (5.11)$$

This estimate, albeit a rough one, provides a strong motivation for continuing the search for proton decay.

§6. Discussion and summary

In addition to $SU(3)^3$, $E_6$ has the other maximal semi-simple subgroups $SU(6) \times SU(2)_L$ and $SU(6) \times SU(2)_R$.36 The matter sector can be applied to these subgroups in a straightforward way. However, in the Higgs sector, it is difficult to realize the situation in which only one pair of doublet Higgs is massless. In particular, it is difficult to make the partner of the doublet Higgs massive, while keeping the latter massless. Contrastingly, in $SU(3)^3$ gauge symmetry, because the partners of the doublet Higgs $L$, $E^c$ and $N^c$ are absorbed by the Higgs mechanism, it is possible for only one pair of doublet Higgs to be massless.

In the typical charge assignment, the charges of the matter sector respect the $E_6$ symmetry, while those of the Higgs sector do not. It is difficult to respect $E_6$ symmetry in the Higgs sector without additional massless fields other than the fields in the MSSM.

By introducing singlet fields, we can build models with integer Kac-Moody level. For example, in addition to the fields in Table I, we introduce a singlet field with charge 10, one with charge $-8$, 43 singlet fields with charge $3/2$, and 62 singlet fields with charge $1/2$. Then using the relation

$$\frac{C_a}{k_a} = \frac{1}{3k_A} \text{tr} Q_A^3 = \frac{1}{24} \text{tr} Q_A, \quad (6.1)$$

where $k_A$ and $k_a$ are the Kac-Moody levels of $U(1)_A$ and $SU(3)_a (a = C, L, R)$, these Kac-Moody levels can be calculated as

$$k_A = 4, \quad k_C = k_L = k_R = 2. \quad (6.2)$$

Note that introducing the singlets with charges 10 and $-8$, the $\mu$ problem is solved by the mechanism proposed in Ref. 2).

In our model, the difference between the mass matrices of down-type quarks and charged leptons is realized because the matrices are from different Yukawa interactions. However, if this model is regarded as the low energy theory of $E_8 \times E_8$ heterotic superstring theory, we have to break the gauge symmetry $E_6$ into $SU(3)^3$. Because the matter sector respects $E_6$ symmetry, it is natural to conjecture that the Yukawa interactions also respect it. In order to realize different Yukawa interactions, we have to implement the breaking. In the brane world scenario, there is an

---

$^*) C_a \equiv \text{Tr}_{G_a} T(R) Q_A$. Here, $T(R)$ is the Dynkin index of the representation $R$, and we use the convention in which $T(\text{fundamental rep.}) = 1/2$. 

---
interesting mechanism to break the gauge symmetry.\textsuperscript{9)} However, it seems difficult to realize this breaking in the Yukawa coupling of matter that resides on the brane. To enforce $E_6$ breaking, some of the matter must be in the bulk, where the $E_6$ gauge symmetry is not respected.

In this paper, we have proposed a realistic semi-simple unified theory with the $SU(3)^3$ gauge group. Since generic interactions have been introduced, we can define the model by the anomalous $U(1)_A$ charges. Large neutrino mixing angles can be realized in the model. Moreover, the FCNC process is automatically suppressed. The half-integer charges of the matter sector automatically play the role of $R$-parity.

The model has the same matter structure as the $E_6$ model,\textsuperscript{4)} but different and simpler Higgs structure. Actually, we do not need the adjoint Higgs fields to realize doublet-triplet splitting, and the gauge couplings at the cutoff scale can be in the perturbative region. Note that in the $SU(3)^3$ model, in contrast to $SU(5)$, $SO(10)$ and $E_6$, the lightest magnetic monopole carries three (instead of one) quanta of Dirac magnetic charge.\textsuperscript{37)} This is readily seen by noting that it is allowed, in principle, to include non-bifundamental vectorlike representations, such as $(1, 3, 1) + (1, \bar{3}, 1)$, that, despite their color singlet nature, carry fractional ($e/3$) electric charge. The Dirac quantization then requires that the corresponding magnetic charges have three units. The number density of primordial $SU(3)^3$ monopoles depends, of course, on the underlying cosmological scenario, and should not exceed the nominal Parker bound of about $10^{-16} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$. The discovery of magnetic monopoles would be a truly remarkable event, and measurement of their magnetic charge would allow us to distinguish between a variety of unified gauge theories.

It would be interesting to extend the approach presented here to other semi-simple unification schemes. For instance, the gauge symmetry $SU(3)^3$ with three $27$s of $E_6$ can be embedded, in principle, in $SU(4) \times SU(3) \times SU(3)$.\textsuperscript{38)} This is worth pursuing.

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References

1) N. Maekawa, Prog. Theor. Phys. 106 (2001), 401, hep-ph/0104200; hep-ph/0110276.
2) N. Maekawa, Phys. Lett. B 521 (2001), 42, hep-ph/0107313.
3) N. Maekawa, Prog. Theor. Phys. 107 (2002), 597, hep-ph/0111205.
4) M. Bando and M. Maekawa, Prog. Theor. Phys. 106 (2001), 1255, hep-ph/0109018.
5) N. Maekawa and T. Yamashita, Prog. Theor. Phys. 107 (2002), 1201, hep-ph/0202050.
6) E. Witten, Phys. Lett. B 149 (1984), 351.
    M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B 289 (1987), 589.
    J. J. Atick, L. J. Dixon and A. Sen, Nucl. Phys. B 292 (1987), 109.
    M. Dine, I. Ichinose and N. Seiberg, Nucl. Phys. B 293 (1987), 253.
7) M. Green and J. Schwarz, Phys. Lett. B 149 (1984), 117.
8) E. Witten, Phys. Lett. B 105 (1981), 267.
    A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. 115 (1982), 380.
B. Grinstein, Nucl. Phys. B 206 (1982), 387.
K. Inoue, A. Kakuto and T. Takano, Prog. Theor. Phys. 75 (1986), 664.
E. Witten, Nucl. Phys. B 258 (1985), 75.
T. Yanagida, Phys. Lett. B 344 (1995), 211.
9) Y. Kawamura, Prog. Theor. Phys. 105 (2001), 691; ibid. 105 (2001), 999.
L. Hall and Y. Nomura, Phys. Rev. D 64 (2001), 055003.
10) S. Dimopoulos and F. Wilczek, NSF-ITP-82-07.
M. Srednicki, Nucl. Phys. B 202 (1982), 327.
11) S. M. Barr and S. Raby, Phys. Rev. Lett. 79 (1997), 4748.
12) Z. Chacko and R. N. Mohapatra, Phys. Rev. D 59 (1999), 011702; Phys. Rev. Lett. 82 (1999), 2836.
13) K. S. Babu and S. M. Barr, Phys. Rev. D 48 (1993), 5354; ibid. 50 (1994), 3529.
14) Y. Fukuda et al. (The Super-Kamiokande Collaboration), Phys. Lett. B 436 (1998), 33; Phys. Rev. Lett. 81 (1998), 1562; ibid. 86 (2001), 5656.
15) C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B 147 (1979), 277.
16) See, for instance, M. Green, J. Schwartz and E. Witten, Superstring Theory (Cambridge University Press, 1987), and references therein.
17) G. Lazarides, C. Panagiotakopoulos and Q. Shafi, Phys. Lett. B 315 (1996), 325.
18) G. Dvali and Q. Shafi, Phys. Lett. B 326 (1994), 258; ibid. 339 (1994), 241.
19) G. Dvali and Q. Shafi, Phys. Lett. B 403 (1997), 65.
20) T. Yanagida, in Proceedings of the Workshop on the Unified Theory and Baryon Number in the Universe, ed. O. Sawada and A. Sugamoto (KEK report 79-18, 1979).
M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, ed. P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam, 1979).
21) The CHOOZ Collaboration, Phys. Lett. B 420 (1998), 397.
22) L. Wolfenstein, Phys. Rev. D 17 (1978), 2369.
23) S. P. Mikheev and A. Smirnov, Yadern. Fiz. 42 (1985), 1441; Nouvo Cim. C 9 (1986), 17.
24) S. Dimopoulos and G. F. Giudice, Phys. Lett. B 357 (1995), 573.
25) A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. B 388 (1996), 588.
26) F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B 477 (1996), 321.
27) K. Kurosawa and N. Maekawa, Prog. Theor. Phys. 102 (1999), 121.
28) F. Borzumati and A. Masiero, Phys. Rev. Lett. 57 (1986), 961.
29) J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B 357 (1995), 579.
J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D 53 (1996), 2442.
J. Sato and K. Tobe, Phys. Rev. D 63 (2001), 116010.
30) M. Drees, Phys. Rev. B 181 (1986), 279.
Y. Kawamura, H. Murayama and M. Yamaguchi, Phys. Lett. B 324 (1994), 52; Phys. Rev. D 51 (1995), 1337.
31) M. Bando and T. Kugo, Prog. Theor. Phys. 101 (1999), 1313.
M. Bando, T. Kugo and K. Yoshioka, Prog. Theor. Phys. 104 (2000), 211.
32) N. Sakai and T. Yanagida, Nucl. Phys. B 197 (1982), 533.
33) T. Goto and T. Nihei, Phys. Rev. D 59 (1999), 115009.
34) J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B 402 (1993), 46.
35) JLQCD Collaboration, S. Aoki et al., Phys. Rev. D 62 (2000), 014506.
36) Super-Kamiokande Collaboration, Phys. Rev. Lett. 81 (1998), 3319; ibid. 83 (1999), 1529.
37) Q. Shafi, Proc. of NATO ASI “Monopole ’83”, Vol. 111, p. 47.
38) T. W. Kephart and Q. Shafi, Phys. Lett. B 520 (2001), 313, hep-ph/0105237.