Interplay between single-particle and two-particle tunneling in normal metal-$d$-wave superconductor junctions probed by shot noise

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We discuss how life-time broadening of quasiparticle states influences single- and two-particle current transport through zero-energy states at normal metal/$d$-wave superconductor junctions. We distinguish between intrinsic broadening (imaginary part $\eta$ of the energy), which couples the bound states with the superconducting reservoir, and broadening due to leakage through the junction barrier, which couples the bound states with the normal metal reservoir. We show that shot noise is highly sensitive to the mechanism of broadening, while the conductance is not. In the limit of small but finite intrinsic broadening, compared to the junction transparency $D$, $\eta/\Delta_0 \ll D$, the low-voltage shot noise at zero frequency and zero temperature becomes proportional to the magnitude $\eta$ of intrinsic broadening ($\Delta_0$ is the maximum $d$-wave gap).

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1. Introduction

Tunneling experiments have for a long time been an important probe of superconducting properties\cite{1}. According to early theoretical analyses in terms of the tunneling Hamiltonian model\cite{2}, the conductance of a normal metal-$s$-wave superconductor tunnel junction is proportional to the superconductor density of states. Within this model only single-particle tunneling is taken into account. For more transparent interfaces, Blonder, Tinkham and Klapwijk\cite{3} (BTK) presented a way to calculate the conductance within Bogoliubov-de Gennes quantum mechanics. Their approach is valid for arbitrary transparency $D$ of the interface, but assumes that the junction system has no impurities and that inelastic scattering is negligible. BTK showed that for intermediate-to-high transparency, so-called Andreev reflection\cite{4} plays an important role in determining the conductance. Andreev reflection is the process where an electron with an energy near the Fermi surface, incident on a superconductor from a normal metal, is converted into a hole with essentially unchanged momentum, but with opposite group velocity. The reversed process is also possible. In the Andreev reflection process a Cooper pair is formed on the superconductor side and a charge $2e$ is transferred over the NS interface, which corresponds to a two-particle tunneling event\cite{5}. In the low-transparency limit, Andreev reflection is suppressed since it is of second order in the transparency $D$, while ordinary single particle tunneling is of first order in $D$. Therefore, in the tunnel limit, $D \ll 1$, the BTK result reduces to the tunnel model result.

In contrast to the clear situation for $s$-wave junctions described above, the interpretation of the conductance of $d$-wave NIS junctions\cite{6–8} where interface resonant states are present is more complicated. Surface/interface states with zero energy are necessarily formed as a consequence of the $d$-wave symmetry of the order parameter, if the interface is oriented so that the order parameter changes its sign along quasiparticle trajectories involving normal reflection at the interface\cite{9}. For a recent review of the implications of these surface/interface zero-energy states (ZES), see Ref. \cite{10}. It is important that two physically distinct situations can be realized depending on the relation between (i) intrinsic broadening due to e.g. surface roughness, impurities, or phonons, which connects the inter-
face states with the bulk superconductor, and (ii) broadening due to leakage to the normal reservoir through the barrier. When intrinsic broadening is negligible, i.e. when the interface states are decoupled from the superconducting reservoir, the interface states can participate in current transport only via the Andreev reflection process. In fact, the Andreev current is resonantly enhanced by the ZES, and is of first order in the transparency $D$. On the other hand, when the intrinsic broadening is dominating, the interface states will instead assist single particle tunneling. Thus, for a $d$-wave superconductor with ZES, a zero-bias conductance peak (ZBCP) is present in both limits, but the type of tunneling responsible for it is very different. Even if both types of life times are large, it is the quotient $q = \frac{\tau_r}{\tau_I}$ which will determine what type of tunneling is dominating: single particle tunneling if $q \gg 1$ and two-particle tunneling if $q \ll 1$ [10,11]. The intrinsic relaxation time $\tau_r \propto \hbar/\eta$ is set by the damping introduced by an imaginary part $\eta$ of the quasiparticle energy, while the life time $\tau_\eta \propto \hbar/D\Delta_0$ due to leakage through the barrier is determined by the transparency $D$ of the barrier ($\Delta_0$ is the maximum $d$-wave gap).

In the present paper we will discuss the interplay between single-particle and two-particle tunneling through the ZES, with emphasis on the limit of small but finite damping. We will show that it is possible to discriminate between the two types of tunneling in a measurement of low-voltage shot noise at zero temperature and zero frequency. Shot noise was recently calculated for $d$-wave NIS junctions in Refs. [12,13], under the implicit assumption of negligible intrinsic broadening ($q = 0$). It was found that the zero-voltage differential shot noise level is zero, $\langle \delta S/\delta V \rangle(V = 0) = 0$, because the effective Andreev reflection probability is enhanced to unity near zero energy by the ZES resonance. Below we will study the effect of intrinsic broadening, and show that the noiseless character of the ZES is quickly lost when $\eta$ increases from zero. This happens because the effective Andreev reflection probability is reduced from unity with increasing broadening: single-particle tunneling via the ZES then becomes possible on the expense of two-particle tunneling, and fluctuations between the two channels is introduced. When broadening is small, the low-voltage shot noise will be proportional to $\eta$, which makes it possible to probe the magnitude of intrinsic broadening by a measurement of noise.

2. Calculation of current and shot noise

In our calculation we use the coherent BTK scattering approach for a specular $d$-wave NIS junction, modified to include intrinsic broadening on a phenomenological level by an imaginary part of the energy. We assume that the normal metal is at $x < 0$, while the superconductor is at $x > 0$. We let a $d$-wave gap node point towards the interface, so that the spectral weight of the ZES is large. It is known that for this orientation the gap is substantially suppressed near the interface [14]. However, in order to make clear our points on the presence of an interplay between single-particle and two-particle tunneling as a function of $q$, and that shot noise will be sensitive to the nature of tunneling, it is sufficient to assume a step function form of the gap function $\Delta(\theta, x) = \Theta(x)\Delta_0 \sin 2\theta$, where $\theta$ is the angle of quasiparticle propagation relative to the interface normal. The exact form of the specular barrier is not important and we assume a $\delta$-function potential characterized by a transmission probability $D(\theta) = \cos^2 \theta/(\cos^2 \theta + Z^2)$, where the dimensionless quantity $Z$ is a measure of the strength of the potential.

In the original BTK theory, effective normal reflection $R_N$ and Andreev reflection $R_A$ probabilities are calculated and then the charge current spectral density is expressed through them. We will follow a slightly different route [15], and express the charge current density in terms of probability current densities flowing along the energy axis. This viewpoint makes it possible to rigorously divide the total charge current $I(V)$ into single-particle $I_1(V)$ and two-particle $I_2(V)$ currents. When an electron at energy $E$ is injected from the normal reservoir into the junction, it
is accelerated by the voltage drop $eV$ over the barrier. At the superconductor it is Andreev reflected into a hole having the opposite charge, and is also accelerated by $eV$ when it tunnels over the barrier back to the normal metal. Thus, the Andreev reflected hole is at the energy $E + 2eV$. Each particle carries a certain probability current, proportional to the absolute value squared of the wavefunction. It is natural to change focus from the charge current flow through the junction (along the $x$-axis), to the probability current flow along the energy axis ($E$-axis), see Figs. 1, 2. In the figures, we have introduced a small auxiliary normal region ($\tilde{N}$) in between the barrier (the insulator, I) and the superconductor (S). The final results will not be affected by the presence of $\tilde{N}$ and in the end we let the thickness of this region go to zero. On each of the electron and hole legs in $\tilde{N}$ (see Fig. 2), we can define the total probability current densities $j^p_e$ and $j^p_h$, respectively, including the effects of normal backscattering at the insulator. For energies outside the gap, the Andreev reflection probability is less than unity, which leads to a leakage $j^p_e - j^p_h$ of probability current during Andreev reflection. The total leakage is the single-particle current. The rest of the probability current (the part surviving Andreev reflection) continues out into the normal metal as a hole current, and contributes to the two-particle current. For subgap energies, the Andreev reflection probability is unity; probability current is then conserved during Andreev reflection, the leakage vanishes, and single particle tunneling is quenched.

In the zero-temperature limit, the current then takes the form:

$$ eR_n I(V) = eR_n \left[ I_1(V) + I_2(V) \right] = \frac{1}{2 \langle D \rangle} \int_{-\pi/2}^{\pi/2} d\theta \cos \theta \left[ I_1(V, \theta) + I_2(V, \theta) \right], $$

$$ I_1(V, \theta) = \int_{-eV}^{0} dE \frac{D(1 - |a|^2)(1 + R|\bar{a}|^2)}{|1 - Ra\bar{a}|^2}, $$

$$ I_2(V, \theta) = 2 \int_{-eV}^{0} dE \frac{D^2|a|^2}{|1 - Ra\bar{a}|^2}, $$(2)

where $R_n = \pi\hbar/e^2k_FL_y \langle D \rangle$ is the normal state junction resistance, $L_y$ is the junction width, $k_F$ is the Fermi wave vector, $\langle D \rangle = \int d\theta \cos \theta D(\theta)/2$, and $a = a(\theta, E)$ is the Andreev reflection amplitude. The amplitude $\bar{a}$ is calculated at the angle $\pi - \theta$, the angle of propagation after normal reflection at the barrier. The differential conductance $G(V)$ is obtained by differentiation with respect to $V$. 

Figure 1. Quasiclassical paths illustrating the structure of the scattering state in real space (the $xy$-plane) due to an incoming electron from the normal metal side (N). In between the barrier (I) and the superconductor (S), we introduce a small normal region ($\tilde{N}$). Normal scattering takes place at the insulator and Andreev reflection takes place at the NS interface.

Figure 2. Structure of the scattering state in energy space (the $xE$-plane).
Figure 3. (a) The total current, (b) the differential conductance, (c) the single particle current, and (d) the two-particle current, for a $d$-wave NIS junction oriented so that a gap node is pointing towards the interface. The solid lines are calculated for $\eta = 0$, while the dashed lines are calculated for $\eta = 0.001\Delta_0$. The zero-energy states resonantly enhance the two-particle current, which is of first order in the transparency. Increasing broadening enhances the single-particle current on expense of the two-particle current, but leaves the total current unaffected. The transparency of the junction is $\langle D \rangle = 0.026$, and the temperature is zero.

For the calculation of shot noise we follow the technique in Ref. [16], see also [12]. The formula for the noise, here limited to zero temperature and zero frequency, is

$$R_n S(V) = \frac{1}{\langle D \rangle} \int_{-\pi/2}^{\pi/2} d\theta \cos \theta \int_{-eV}^0 dE S(\theta, E),$$

$$S(\theta, E) = R_N(1 - R_N) + R_A(1 - R_A) + 2R_NR_A,$$

where $R_N = R|1 - a\bar{a}|^2/(1 - R\bar{a}^2)$, and $R_A = D^2|a|^2/(1 - R\bar{a}^2)$. In the subgap region, for zero temperature and negligible broadening, the only surviving source of noise is a term involving scattering states separated in energy by $2eV$: the first scattering state consists of an electron incoming from the normal metal reservoir at the energy $E$, Andreev reflected as a hole which is ejected back into the normal metal at the energy $E + 2eV$; the second scattering state consists of a hole incoming at $E + 2eV$, Andreev reflected as an electron emerging at $E$.

We plot the total current $I(V)$, the conductance $G(V)$, the single-particle current $I_1(V)$, and two-particle current $I_2(V)$ in Fig. 3(a)-(d). From Fig. 3 it is clear that in the absence of broadening, the solid lines ($q = 0$), the ZBCP is due to the two-particle current only. One can show that the two-particle current is of first order in the transparency, $I_2 \propto D$, because of the ZES resonance. The width of the ZES resonance is $\Gamma = D|\Delta(\theta)|/\left[2\sqrt{R(\theta)}\right]$ for each quasiparticle trajectory angle $\theta$, see also e.g. Ref. [12].

To study the effects of intrinsic broadening, the quasiclassical Green’s function technique [16] can be used. Recently, a highly useful parameterization was introduced by Nagato, Nagai, and Hara [19] for the equilibrium case, and by Eschrig [20] for the non-equilibrium case, which can be used to separate electron and hole parts of the Green’s functions. Probability currents flowing along the energy axis are then easily found, and one can divide the total charge current into single- and two-particle currents, in a one-to-one mapping to the approach outlined above. Eventually, the result is the same expression as in Eq. (3), but with the Andreev reflection amplitudes substituted by generalized Andreev reflection amplitudes containing the effects of damping in the form of an imaginary part of the energy [3][4]. When broadening increases, the two-particle current is suppressed and the single-particle current is enhanced. However, the total current and the conductance are not particularly affected, as shown in Fig. 3(a)-(d) by the dashed lines. In fact, in the limit of large broadening, $q \gg 1$, the ZBCP is solely due to single-particle tunneling. In this case, it is convenient to instead of the above approach apply the tunnel formula for the conductance and calculate the local density of states at the surface, as in e.g. Ref. [21]. In this limit the width of the ZBCP is directly pro-
Figure 4. Differential shot noise $\partial S/\partial V$ at zero temperature and zero frequency, calculated for the same junction as in Fig. 3. In (a) for $\eta = 0$, and in (b) for $\eta = 0.001 \Delta_0$. The noiseless character of the zero-energy states is quickly lost when broadening is introduced.

In Fig. 4 we plot the differential shot noise, $\partial S/\partial V$. The ZES resonance does not produce any noise $(\partial S/\partial V)(V = 0) = 0$, see Fig. 3(a), as found in Refs. [12,13]. However, when a small amount of intrinsic broadening is introduced, the noiseless character of the ZES is lost, see Fig. 3(b). This behavior can be understood in the following qualitative way. When there is no broadening, the ZES resonantly enhances the Andreev current. The effective Andreev reflection probability is unity, despite the fact that $D \ll 1$, c.f. Eq. 3 for $E = 0$ and Fig. 3(d). The result of zero noise follows directly from Eq. 3, since the relation $R_N + R_A = 1$ holds in the subgap region (probability current conservation). When broadening increases from zero, the effective Andreev reflection probability is reduced from unity, and the noise becomes finite. In the limit of small intrinsic broadening, $q \ll 1$, the noise at zero-voltage is proportional to the magnitude of broadening, $(\partial S/\partial V)(\theta, V = 0) \propto \eta/\Gamma(\theta)$, where $\Gamma(\theta)$ is the width of the ZES in the absence of damping.

3. Summary

We have discussed the interplay between single-particle and two-particle tunneling through zero-energy states (ZES) in $d$-wave NIS junctions. For small intrinsic broadening of the ZES, compared to the broadening due to leakage over the barrier to the normal metal reservoir, $q \ll 1$, current is transported via two-particle tunneling. On the other hand, for large damping, $q \gg 1$, only single particle tunneling is present. We have shown that shot noise is highly sensitive to the type of tunneling, although the conductance is not. The noiseless character of the ZES found in Ref. [12,13] is quickly lost when damping is introduced. For small but finite intrinsic broadening, $q \ll 1$, the low-voltage shot noise is directly proportional to the magnitude of broadening.

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