SEMI-HARD PARTON RESCATTERINGS
IN NUCLEAR COLLISIONS AT VERY HIGH ENERGIES

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ABSTRACT

Representing the semi-hard partonic interactions by the exchange of Lipatov’s perturbative Pomeron, we express the semi-hard nuclear cross section as a self shadowing cross section. With the help of a generating functional technique, we obtain average numbers of multiple semi-hard partonic collisions without any need of using explicit expressions for the multi-parton distributions. The average number of semi-hard interactions of a given projectile parton against a target nucleus is estimated quantitatively and it is shown to grow very rapidly above one with increasing the c.m. energy.
1. Introduction

One of the main problems in the physics of semi-hard interactions is to take into account unitarity corrections. The semi-hard regime, in fact, although perturbative in the elementary interaction, gives rise to cross sections that may violate the unitarity limit. Directly related features are the large size of the QCD-parton model cross section to produce large $p_t$ partons, which is obtained when the cut-off $p_t^{\text{min}}$, defining the perturbative regime, is lowered down to the mini-jet region[1], and the large value of the mini-jet cross section, which has been observed by UA1[2]. The standard way to unitarize the semi-hard cross section is by eikonalizing the elementary semi-hard partonic interaction[3]: The semi-hard component of the hadronic interaction is included in the eikonal phase as an additive term, which represents the inclusive cross section to produce large $p_t$ partons, integrated down to the lower limit $p_t^{\text{min}}$ and depending explicitly on the impact parameter of the hadronic collision. The cross section, including all inelastic events with at least one semi-hard partonic interaction, may be easily derived and expressed as a series of multiple semi-hard partonic interactions. The resulting distribution in multiple semi-hard partonic interactions is a Poissonian, with average number depending on the impact parameter and representing different pairs of partons interacting independently at different points of the transverse plane. The underlying physical assumption is a Poisson distribution for the input multi-parton distributions to the multi-parton interaction and, in addition, the absence of any semi-hard rescattering for each parton which has interacted already once with semi-hard momentum transfer exchange[4].

To include semi-hard interactions in high energy nuclear collisions, the eikonalized expression of the nucleon-nucleon interaction can be used as a input to construct the nuclear interaction[5] according with Glauber and Gribov[6]. With respect to the input parton distributions to the semi-hard component of the nuclear interaction, both possibilities of input nuclear parton distributions, with and without shadowing corrections, have been considered[5]. Alternatively[7], the whole semi-hard nuclear interaction has been expressed by a Poisson distribution of multiple semi-hard parton collisions, as in the hadron-hadron case. The only difference, with respect to the case of semi-hard hadron-hadron interaction, is in the function of the impact parameter, which describes the overlap of the matter distribution in the transverse plane, and in the nuclear parton distributions, which,
for nucleus-nucleus, include shadowing corrections[8]. In both of these approaches a parton is allowed to interact, with momentum transfer larger than $p_{t}^{\text{min}}$, only once. Semi-hard parton rescatterings, namely multiple collisions of a given parton, each with momentum transfer larger than $p_{t}^{\text{min}}$, are typically estimated to be a negligible effect up to energies of order of $10 TeV$ in the nucleon-nucleon c.m. system[8]. In this respect the very different conclusion, that semi-hard parton rescatterings are, on the contrary, one of the major features in nuclear collisions already at energies of the order of $1 TeV$ in the nucleon-nucleon c.m. system, has been drawn[9]. This different conclusions is the origin of the rather peculiar physical picture of high energy nuclear interactions where most of the transverse energy produced is the result of multiple semi-hard parton collisions[10].

The purpose of the present note is to gain a better insight into this last point; namely the amount of semi-hard parton rescatterings in very high energy nuclear collisions, which appears as a critical element in the physics of nuclear interactions at very high energy. We will in fact review the arguments of ref.[9, 10] making clear which are the underlying assumptions and, by means of an approach which is much more general with respect to the approach used there, we will produce estimates of the average number of semi-hard rescatterings which are of much wider validity. In fact our result will not depend on any specific form for the input multi-parton distributions.

The paper is organized in two parts: In the first we comment on the general framework to discuss the semi-hard cross section and we derive the quantities of interest for the actual discussion. In the second part we present a few numerical results and our conclusions.

2. General framework and average number of rescatterings

The inelastic channels are obtained from the nuclear cross section, expressed according with Glauber and Gribov, by the use of the cutting rules by Abramovskii, Gribov and Kancheli[11]. The rules connect different cut diagrams, which are shown to be proportional to one another, in such a way that the evaluation of the contribution of the various inelastic channels to the inelastic cross section simplifies enormously. Actually the result, which is obtained after summing all different cut diagrams, is that the cross section acquires the simple probabilistic meaning
of sum of multiple inelastic interaction probabilities[12]. Remarkably the probabilistic picture is the quantum mechanical result of the sum of a very complicated set of cut amplitudes. The systematic use of the cutting rules allows also to recognize the validity of general properties of the interaction, such as the possibility to single out the self shadowing cross sections[13]. For these quantities the unitarity corrections have the peculiar propriety that all contributions from all processes, different from the one which is considered, cancel. A stronger statement is that also the functional form of the unitarity corrections for a self shadowing cross section is given, once the forward nuclear amplitude is expressed in terms of "elementary" nucleon-nucleon amplitudes[14]. The question of interest for the present discussion is whether semi-hard interactions are quantities of this kind. Two are the requirements which are to be satisfied. The first is that the soft corrections to the semi-hard process have to cancel, when all contributions, real and virtual, are taken into account. The proof of factorization at the level of power corrections[15] shows that this is the case when inclusive processes are considered. The second requirement, which allows the probabilistic picture of the interaction, is that the cutting rules, relating different cut diagrams, are to be valid also for semi-hard interactions. While this proof is missing in the general case, it has nevertheless been obtained for one of the components of the semi-hard cross section which is leading in the high energy limit[16]. One may therefore argue that it is a good working hypothesis to assume that, in high energy nuclear collisions, semi-hard interactions self shadow, in such a way that the semi-hard nuclear interaction can be expressed in a probabilistic way as a function of "elementary" semi-hard parton-parton interaction probabilities. The physical picture of the semi-hard nuclear interaction discussed in ref.[9] and in ref.[10] and the estimates of the rates of semi-hard parton rescattering, which we are going to present here, are done under this basic hypothesis.

The self-shadowing hypothesis and the probabilistic picture which follows are not sufficient to write the semi-hard nuclear cross section. In fact, differently with respect to soft interactions, where the number of "elementary" interacting objects, which in that case are the nucleons, is fixed, when considering semi-hard interactions and partons as elementary interacting objects, the number is, on the contrary, varying. Actually the configurations with many interacting partons involve the multi-parton distributions[17] that are non perturbative quantities, independent of the single-parton distributions of large $p_t$ physics. The complete description of
the nuclear semi-hard interaction needs therefore, as a input, the infinite set of non perturbative quantities, represented by the multi-parton distributions. Nevertheless, for the more limited program to evaluate a few average quantities, much less information is needed as a input. In this case it is very convenient to approach the problem by means of a generating functional formalism which allows to avoid explicit expressions for the multiparton distributions[18].

To construct the generating functional for the multi-parton distributions one may start with the exclusive multi-parton distributions, namely, at a given scale provided by the cut off $p_{t}^{min}$, the probabilities for each of the configurations with a given number of partons. The $W^{(n)}(u_{1} \ldots u_{n})$ are then the exclusive $n$-body parton distributions, where $u_{i} \equiv (b_{i}, x_{i})$ represents the transverse partonic coordinate ($b_{i}$) and longitudinal fractional momentum ($x_{i}$). The generating functional $Z[J]$ is defined as:

$$Z[J] = \sum_{n} \frac{1}{n!} \int J(u_{1}) \ldots J(u_{n}) W^{(n)}(u_{1} \ldots u_{n}) du_{1} \ldots du_{n}, \quad (1)$$

with the normalization condition

$$Z[1] = 1. \quad (2)$$

While the exclusive distributions are obtained from the expansion of the generating functional around $J = 0$, the inclusive distributions are obtained by expanding $Z[J]$ around $J = 1$. Explicitly the one-body and two-body inclusive distributions $D^{(1)}$ and $D^{(2)}$ are given by:

$$D^{(1)}(u) = W^{(1)}(u) + \int W^{(2)}(u, u') du' + \frac{1}{2} \int W^{(3)}(u, u', u'') du' du'' + \ldots$$

$$= \frac{\delta Z}{\delta J(u)} \bigg|_{J=1},$$

$$D^{(2)}(u_{1}, u_{2}) = W^{(2)}(u_{1}, u_{2}) + \int W^{(3)}(u_{1}, u_{2}, u') du'$$

$$+ \frac{1}{2} \int W^{(4)}(u_{1}, u_{2}, u', u'') du' du'' + \ldots$$

$$= \frac{\delta^{2} Z}{\delta J(u_{1}) \delta J(u_{2})} \bigg|_{J=1} \equiv D^{(1)}(u_{1}) D^{(1)}(u_{2}) + \frac{1}{2} C^{(2)}(u_{1}, u_{2}) \quad (3)$$
where the two body correlation $C^{(2)}(u_1, u_2)$, which measures the deviation from
the Poisson distribution[18], has been introduced.

The semi-hard nucleus-nucleus cross section $\sigma_H$ is constructed by multiplying
the exclusive parton distributions of the two nuclei, $A$ and $B$, by the probability
of interaction, which, after the assumption that semi-hard interactions self-
shadow, can be constructed from the elementary parton-parton interaction probability $\hat{\sigma}(u_i, u'_j)$, which represents the probability for the parton $i$ of the $A$-nucleus
to have an hard interaction with the parton $j$ of the $B$-nucleus:

$$\sigma_H = \int d\beta \sigma_H(\beta)$$

$$\sigma_H(\beta) = \int \sum_n \frac{1}{n!} \frac{\delta}{\delta J(u_1)} \cdots \frac{\delta}{\delta J(u_n)} Z_A[J]$$
$$\times \sum_m \frac{1}{m!} \frac{\delta}{\delta J'(u'_1 - \beta)} \cdots \frac{\delta}{\delta J'(u'_m - \beta)} Z_B[J']$$
$$\times \left\{ 1 - \prod_{i=1}^n \prod_{j=1}^m [1 - \hat{\sigma}(u_i, u'_j)] \right\} \prod du du' \bigg|_{J=J'=0}$$

Here $\beta$ is the impact parameter between the two interacting nuclei and the semi-
hard cross section is constructed by summing over all possible partonic configurations
of the two interacting nuclei (the sums over $n$ and $m$) and, for each configuration
with $n$ partons from $A$ and $m$ partons from $B$, summing over all possible multiple partonic interactions. This last sum is constructed by asking for the probability
of no interaction between the two configurations (actually $\prod_{i=1}^n \prod_{j=1}^m [1 - \hat{\sigma}_{i,j}]$).
The difference from one of the probability of no interaction gives the sum over all semi-hard interactions. To obtain the average number of semi-hard partonic collisions one expands the interaction probability as a sum of multiple interactions.
Since the interaction probability is multiplied by a symmetric expression, one can make the replacement:

$$1 - \prod_{i=1}^n \prod_{j=1}^m [1 - \hat{\sigma}_{i,j}] \rightarrow S[1 - \prod_{N=1}^Q \hat{\sigma}_N]$$
$$= S \sum_{N=1}^Q \binom{Q}{N} \hat{\sigma}_1 \cdots \hat{\sigma}_N (1 - \hat{\sigma}_{N+1}) \cdots (1 - \hat{\sigma}_Q)$$

5
where the index \( N \) counts the possible interactions, in such a way that \( Q = nm \), \( S \) is a symmetrizing operator\(^9\), and, in the second line in Eq.(5), the interaction probability has been expressed as a sum of multiple interactions. The average number of partonic collisions at a given value of \( Q \), \( \langle N \rangle_Q \), is given by:

\[
\langle N \rangle_Q = S \sum_{N=1}^{Q} N \binom{Q}{N} \hat{\sigma}_1 \cdots \hat{\sigma}_N (1 - \hat{\sigma}_{N+1}) \cdots (1 - \hat{\sigma}_Q) \tag{6}
\]

When in the expression for \( \sigma_H(\beta) \), as given in Eq.(4), the interaction probability is replaced with \( \langle N \rangle_Q \), the overall average number of parton interactions \( \langle N(\beta) \rangle \) is obtained. Since \( S \hat{\sigma}_1 = mn \hat{\sigma}_{1,1} \), as a result of the the sums on \( m \) and \( n \) both arguments of \( Z_A \) and of \( Z_B \) are shifted by one unit. One can then write:

\[
\langle N(\beta) \rangle = \int \frac{\delta}{\delta J(u_1)} Z_A[J+1] \bigg|_{J=0} \frac{\delta}{\delta J'(u'_1 - \beta)} Z_B[J' + 1] \bigg|_{J'=0} \hat{\sigma}(u_1, u'_1) du_1 du'_1 \tag{7}
\]

The overall average number of partonic collisions is therefore expressed, on quite general grounds, by the single scattering term, where the parton structure of each interacting nucleus enters only with the inclusive one-body parton distribution:

\[
\langle N(\beta) \rangle = \int \sum_{n} \frac{1}{n!} \frac{\delta}{\delta J(u_1)} \cdots \frac{\delta}{\delta J(u_n)} Z_A[J] \times \left\{ 1 - \prod_{i=1}^{n} Z_B \left[ 1 - \hat{\sigma}(u_i, \cdot) \right] \right\} \prod_{i=1}^{n} du_i \bigg|_{J=0} \tag{8}
\]

An analogously general result can be obtained for the average number of collisions of each parton. The sum over \( m \) in Eq.(4) can be performed explicitly in such a way that \( \sigma_H(\beta) \) is expressed as:

\[
\sigma_H(\beta) = \int \sum_{n} \frac{1}{n!} \frac{\delta}{\delta J(u_1)} \cdots \frac{\delta}{\delta J(u_n)} Z_A[J] \times \prod_{i=1}^{n} \left\{ 1 - Z_B \left[ 1 - \hat{\sigma}(u_i, \cdot) \right] \right\} \prod_{i=1}^{n} du_i \bigg|_{J=0} \tag{9}
\]

In Eq.(9) every configuration with a given number of partons from the \( A \) nucleus is required to interact at least once with nucleus \( B \). The probability for one of the partons form \( A \), which we label with \( i \), not to interact with the whole nucleus \( B \) is \( Z_B \left[ 1 - \hat{\sigma}(u_i, \cdot) \right] \). One minus this probability gives the probability for that parton
to interact at least once. Let us expand this interaction probability as a sum on successive interactions:

\[ 1 - Z_B[1 - \hat{\sigma}(u_i, \cdot)] = \int \frac{1}{\text{d} \delta J'_{u'_i - \beta}} \cdots \frac{\delta}{\delta J'(u'_m - \beta)} Z_B[J'] \bigg|_{J' = 0} \]

\[ \times \sum_{k=1}^{m} \binom{m}{k} \hat{\sigma}_{i,1} \cdots \hat{\sigma}_{i,k} (1 - \hat{\sigma}_{i,k+1}) \cdots (1 - \hat{\sigma}_{i,m}) \prod du' \] \hspace{1cm} (10)

the average number of interactions of the parton under consideration is obtained by multiplying by \( k \) each term in the sum on \( k \). Both sums on \( k \) and on \( m \) can be done explicitly in a way similar to the case already considered. The resulting average number is expressed as:

\[ \langle k(u_i) \rangle = \int D_B^{(1)}(u'_1 - \beta) \hat{\sigma}(u_i, u'_1) du'_1 = \hat{\sigma}_{i,1} \otimes D_B^{(1)}(u'_1 - \beta) \] \hspace{1cm} (11)

where the structure of the target nucleus enters only with \( D_B^{(1)} \).

One can notice that, while the average number of interactions of the parton with the target does not feel more than the inclusive one-parton distribution of the target, if one looks instead to the average number of rescatterings of the same parton, one is sensible to the multi-parton correlations. Let us in fact evaluate the average number of rescatterings \( \langle r \rangle \), namely the average number of interactions when the number of interactions is at least two. For a fixed configuration with \( m \) target partons one needs to compute:

\[ \langle r(u_i) \rangle_m = \sum_{k=2}^{m} k \binom{m}{k} \hat{\sigma}_{i,1} \cdots \hat{\sigma}_{i,k} (1 - \hat{\sigma}_{i,k+1}) \cdots (1 - \hat{\sigma}_{i,m}) \]

\[ = m^3 \hat{\sigma}_{i,1} - m^2 \hat{\sigma}_{i,1} (1 - \hat{\sigma}_{i,2}) \cdots (1 - \hat{\sigma}_{i,m}) \] \hspace{1cm} (12)

The average number of rescatterings \( \langle r(u_i) \rangle \), for an incoming parton with a given kinematical configuration represented by \( u_i \), is obtained by multiplying the expression in Eq.(12) by the sum on \( m \), giving all the different multi-parton configurations, and by performing the sum. The actual result is:

\[ \langle r(u_i) \rangle = \hat{\sigma}_{i,1} \frac{\delta}{\delta J'} [Z_B[J' + 1] - Z_B[J' + 1 - \hat{\sigma}_{i,1}]] \bigg|_{J' = 0} \] \hspace{1cm} (13)

By expanding \( Z_B[J' + 1 - \hat{\sigma}] \) for \( \hat{\sigma} \) small and keeping only the first term different from zero one obtains:
\[ \langle r(u_i) \rangle = \hat{\sigma}_{i,1} \otimes \hat{\sigma}_{i,2} \otimes \left[ D_B^{(1)}(u'_1 - \beta) D_B^{(1)}(u'_2 - \beta) + \frac{1}{2} C_B^{(2)}(u'_1 - \beta, u'_2 - \beta) \right] \]  

(14)

in such a way that combining \( \langle r(u_i) \rangle \) with \( \langle k(u_i) \rangle \) the correlation \( C_B^{(2)} \) is obtained.

A similar observation is valid for \( \langle N(\beta) \rangle \): By starting the sum in Eq.(6) from \( N = 2 \), one identifies the average number \( \langle \nu(\beta) \rangle \), which counts those semi-hard collisions with at least two semi-hard parton interactions. At the lowest order in \( \hat{\sigma}, \langle \nu(\beta) \rangle \) is expressed as:

\[ \langle \nu(\beta) \rangle = D_A^{(2)} \otimes \hat{\sigma}_{1,1} \otimes D_B^{(2)} \]  

(15)

In Eq.(15) the dominant term is obtained by neglecting the correlations in the expression \( D^{(2)}(u_1, u_2) = D^{(1)}(u_1) D^{(1)}(u_2) + 1/2 C^{(2)}(u_1, u_2) \) and keeping only the term with the larger number of \( D^{(1)} \)'s:

\[ \langle \nu(\beta) \rangle \approx D_A^{(1)} \otimes \hat{\sigma}_{1,1} \otimes D_B^{(1)} \]  

(16)

which represents the independent semi-hard scattering of two uncorrelated parton pairs localized at two different points in the transverse plane, as it can be seen by looking at the dependence on the transverse coordinates in Eq.(16). Actually this is the expression that has been used first to make predictions on the rate for double parton collisions, in high energy hadronic interactions[19], and later to analyze the actual experimental signal[20]. In Eq.(15) the most important term containing correlation is:

\[ D_A^{(1)} \otimes D_A^{(1)} \otimes \hat{\sigma}_{1,1} \otimes \hat{\sigma}_{2,2} \otimes \frac{1}{2} C_B^{(2)} + A \leftrightarrow B \]

One may notice that \( \langle r(u_i) \rangle \) and \( \langle \nu(\beta) \rangle \) give a different information about the correlation. In the first case the transverse coordinate of the two correlated partons differ only by an amount whose scale is given by the range of \( \hat{\sigma} \), as it is seen by looking at the convolutions in Eq.(14). In the second case the correlation is averaged over the whole transverse plane.
3. Numerical estimates

The analysis of the previous paragraph shows that, although multiple parton collisions involve a whole infinite set of unknown non-perturbative inputs, namely the multi-parton distributions, to construct average quantities, only a limited amount of input information is needed. Actually the average number of partonic interactions in an inelastic event with given impact parameter $\beta$, $\langle N(\beta) \rangle$, and the average number of semi-hard interactions of a projectile parton, with given transverse coordinate and fractional momentum $u_i$, against a target nucleus, $\langle k(u_i) \rangle$, are both constructed from the one-body inclusive parton distribution $D^{(1)}$.

In order to perform any estimate we need, as a input, $D^{(1)}$. When the "elementary" interaction probability is small, semi-hard rescatterings can be neglected. In this case the integral on $\beta$ of $\langle N(\beta) \rangle$ is equal to the semi-hard cross section multiplied by the multiplicity of partons which have suffered a semi-hard interaction[4], and, therefore, it is equal to the inclusive cross section. As a consequence, in this case one is allowed to identify $D^{(1)}$ with the nuclear structure function which is used to describe large $p_t$ physics. Notice that this conclusion holds at any order in the parton correlations of the multi-parton inclusive distributions. It is therefore valid for the particular case of the Poissonian distribution in multiplicity of the semi-hard parton collisions, which corresponds to the limit of neglecting all correlations[18]. The identification of $D^{(1)}$ is much less clear when semi-hard parton rescatterings are taken into account. If looking to deep inelastic scattering on a nucleus, in the kinematical regime of shadowing corrections, the fluctuation of the virtual photon in a $q\bar{q}$ pair lasts long enough to cross the whole nucleus[21]. In this case, the interaction of the photon is hadronic, or, more precisely, it can be described as the sum of $q\bar{q}$ states, with frozen transverse distance interacting according to the Glauber expression for hadron-nucleus amplitude in the forward direction[21]. On the other hand, in the Glauber expression, shadowing is the result of all multiple rescatterings of the incoming state, both soft and semi-hard. When the amount of semi-hard rescatterings is sizeable, the problem which rises is that the quantity entering in the total cross section is the probability to interact at least once, while the quantity of interest for the present purpose is rather the average number of interactions. Nuclear structure functions, with shadowing corrections, are therefore an underestimate for $D^{(1)}$. On the other hand nuclear
structure functions without shadowing corrections are an overestimate, since also soft interactions alone produce shadowing at low $x$.

To have a quantitative feeling of the importance of semi-hard rescatterings, we compute $\langle k(u_i) \rangle$, as given in Eq.(11), using as a input for $D^{(1)}$ both nuclear structure functions with and without shadowing corrections. The two choices represent a lower and an upper bound for $\langle k(u_i) \rangle$. To perform the actual calculation we use the set E and set B structure functions by HMRS[22] and, as scale factor, we consider two possible choices, $p_{t \min}^2/2$ and $p_{t \min}^2$. Shadowing corrections have been introduced by parametrizing the ratio of the nuclear-nucleon structure functions as in ref.8.

As a further input, we need to specify the elementary interaction. The regime of interest is the one of mini-jet production. In the typical configuration two mini-jets, namely jets with transverse momentum of a few Gev, have a large separation in rapidity. As one expects from asymptotic estimates[23], the rapidity interval is filled with radiated gluons, in such a way that higher order corrections in $\alpha_S$ play an important role. Asymptotically one obtains that, in the inclusive cross section, the first correction to the lowest order result is of order $(\alpha_S y)^2$[24], with $y$ the rapidity interval between the two interacting partons. To keep into account the main features of higher order corrections, we parametrize the ”elementary” partonic interaction by the exchange of the Lipatov’s perturbative Pomeron[23]. More explicitly we use the expression of the inclusive cross section to produce mini-jets derived by Mueller and Navelet[24].

The inclusive cross section to produce mini-jets with transverse momentum larger than 5 GeV, in $p\bar{p}$ collisions with c.m. energy range $200 GeV \leq \sqrt{s} \leq 900 GeV$, is compared with the cross section measured by UA1[2] in fig.1. Using as a input HMRS(B) structure functions and $p_{t \min}^2/2$ as scale factor (continuous curve in the figure) the gross figures of the data are reproduced. The other choices, which we have considered, underestimate the cross section by roughly a factor two. The average number of interactions, that a given parton undergoes with the target nucleus, $\langle k(b,x) \rangle$, as expressed by Eq.(11), is shown in fig. 2 as a function of the fractional momentum of the projectile parton $x$ and at zero impact parameter $b$. The two continuous curves refer to the two different choices of the cut off, $p_{t \min}^2 = 5 GeV$ and $p_{t \min}^2 = 6 GeV$, together with HMRS(B) structure functions and $p_{t \min}^2/2$ as scale factor, which, according with the UA1 data, is the favorite choice for the input parameters. The dependence on the scale factor is shown by
the dashed line in the same figure, where the input is $p_t^{\text{min}} = 5\text{GeV}$, HMRS(B) structure functions and $p_t^{\text{min}}$ as scale factor. With the same input, the effect of using structure functions, with shadowing corrections included, is shown by the dotted line. The dependence on the c.m. energy $\sqrt{s}$ is shown in fig.3, where the value of the cut-off has been fixed to $p_t^{\text{min}} = 5\text{GeV}$. Solid, dashed, dotted lines refer to the three values for the fractional momentum of the projectile parton $x = 1$, $x = 0.5$ and $x = 0.25$ respectively. In fig.4 we show the dependence on the impact parameter at fixed $\sqrt{s}$ and $p_t^{\text{min}}$. In all cases the atomic mass number is $A = 208$ and the nucleus is represented as an uniform sphere of radius $R$ with sharp boundaries.

Our conclusion is that the average number of semi-hard collisions, that an incoming parton undergoes with a target nucleus, is sizeable larger than one, also keeping the lower cut-off $p_t^{\text{min}}$ as high as $5\text{GeV}$. We mainly limit our analysis to the case of interactions between nuclei with sub-energies of the order of $1\text{TeV}$ in the nucleon-nucleon c.m. system. The input to the nuclear case is therefore constrained by the comparison with the available experimental result for the inclusive cross section to produce mini-jets, as measured by UA1[2] in $p\bar{p}$ collisions at similar energies. A reason for the big effect, in the number of semi-hard parton rescatterings, is in the large size of the mini-jet cross section which has been observed by UA1: almost $20\text{mb}$ at $900\text{GeV}$ c.m. energy in $p\bar{p}$ collisions, with the cut-off $p_t^{\text{min}} = 5\text{GeV}$. Nevertheless an important role is played by the actual representation of the "elementary" partonic interaction. When the "elementary" partonic interaction is represented by the exchange of Lipatov’s perturbative Pomeron, the large value of the cross section observed by UA1 is the result of a relatively large "elementary" cross section rather than of a large flux of incoming partons. In a nucleus the probability of semi-hard parton rescattering is therefore enhanced.

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Figure captions

Fig. 1 Inclusive cross section to produce mini-jets with transverse momentum larger than 5\( GeV \), in \( p \bar{p} \) collisions as a function of the c.m. energy. Continuous line: HMRS(B) structure functions, scale factor \( p_t^{\text{min}}/2 \). Dashed line: HMRS(B), scale factor \( p_t^{\text{min}} \). Dotted line: HMRS(E), scale factor \( p_t^{\text{min}}/2 \). Experimental data from UA1 (ref. [2]).

Fig. 2 Average number of interactions, \( \langle k(b = 0, x) \rangle \), Eq.(11) in the text, as a function of \( x \) in nuclear collisions with sub-energy, in the nucleon-nucleon c.m. system, \( \sqrt{s} = 1\, TeV \). Continuous lines: cut off \( p_t^{\text{min}} = 5\, GeV \) and \( p_t^{\text{min}} = 6\, GeV \), HMRS(B) structure functions, scale factor \( p_t^{\text{min}}/2 \). Dashed line: cut-off \( p_t^{\text{min}} = 5\, GeV \), HMRS(B) structure functions, scale factor \( p_t^{\text{min}} \). Dotted line: HMRS(B) structure functions, with shadowing corrections included, cut-off \( p_t^{\text{min}} = 5\, GeV \), scale factor \( p_t^{\text{min}}/2 \). Atomic mass number of the target \( A = 208 \).

Fig. 3 Dependence of \( \langle k(b = 0, x) \rangle \) on the nucleon-nucleon c.m. energy \( \sqrt{s} \). Cut-off \( p_t^{\text{min}} = 5\, GeV \), HMRS(B) structure functions, scale factor \( p_t^{\text{min}}/2 \). Solid line \( x = 1 \), dashed line \( x = 0.5 \), dotted line \( x = 0.25 \). Atomic mass \( A = 208 \).

Fig. 4 Dependence of \( \langle k(b, x = 1) \rangle \) on the impact parameter. Nucleon-nucleon c.m. energy \( \sqrt{s} = 1\, TeV \), cut-off \( p_t^{\text{min}} = 5\, GeV \), HMRS(B) structure functions, scale factor \( p_t^{\text{min}}/2 \), atomic mass \( A = 208 \).
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