On the uniqueness of $D = 11$ interactions among a graviton, a massless gravitino and a three-form.

IV: Putting things together

E. M. Cioroianu, E. Diaconu, S. C. Sararu
Faculty of Physics, University of Craiova, 13 Al. I. Cuza Street Craiova, 200585, Romania

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Abstract

Under the hypotheses of smoothness of the interactions in the coupling constant, locality, Poincaré invariance, Lorentz covariance, and the preservation of the number of derivatives on each field in the Lagrangian of the interacting theory (the same number of derivatives like in the free Lagrangian), we prove that the only consistent interactions in $D = 11$ among massless gravitini, a graviton, and a 3-form are described by $N = 1$, $D = 11$ SUGRA.

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1 Introduction

In this part we use the results from Refs. [1], [2], and [3] and approach the fourth (and final) step of constructing all possible interactions in $D = 11$ among a graviton, a massless Majorana spin-$3/2$ field, and a three-form gauge field. Of course, we maintain the same working hypotheses like in the first three parts, namely smoothness of interactions in the coupling constant, locality, Poincaré invariance, Lorentz covariance, and the preservation of the number of derivatives on each field in the Lagrangian density of the interacting theory. First, we put all the fields together and investigate if there are consistent interactions vertices at order one in the coupling constant involving all of them. The answer is negative, such that the first-order deformation of the solution to the master equation is completely known from the previous steps. Second, we analyze the consistency of the first-order deformation at order two in the coupling constant. This restricts the six constants that parameterize the first-order deformation to

*e-mail address: manache@central.ucv.ro
†e-mail address: ediaconu@central.ucv.ro
‡e-mail address: scsararu@central.ucv.ro
satisfy a simple, algebraic system. There are two types of solutions, but only one is interesting from the point of view of interactions (the other allows at most the interactions between a graviton and a 3-form). Third, we analyze this solution and observe that it systematically reproduces the Lagrangian formulation of $D = 11, N = 1$ SUGRA. Therefore, we can state that all consistent interactions in $D = 11$ among a spin-2 field, a massless Majorana spin-3/2 field, and a three-form that comply with our working hypotheses are uniquely described by $D = 11, N = 1$ SUGRA.

2 No simultaneous interactions at order one in the coupling constant

Now, we put together all the three kinds of fields (graviton, massless gravitini and three-form) and start from the free action

$$S_0^b [h_{\mu\nu}, A_{\mu\nu\rho}, \psi_{\mu}] = \int d^{11} x \left( L_0^b + L_0^A + L_0^\psi \right)$$

in $D = 11$, where $L_0^b$, $L_0^A$, and $L_0^\psi$ denote the Lagrangian densities of the Pauli-Fierz model, of an Abelian three-form, and of a massless Rarita-Schwinger field respectively (see Section 2 from Ref. [1] and also Section 2 from Ref. [2]). Consequently, the BRST symmetry of the free model (1) is written as

$$s = \frac{1}{2} \left( s^h, A + s^A, \psi + s^h, \psi \right),$$

where $s^h, A$, $s^A, \psi$, and $s^h, \psi$ denote the BRST symmetries of the free models respectively approached in Refs. [1], [2], and [3]. The overall BRST differential (2) further decomposes as

$$s = \delta + \gamma,$$

where $\delta$ stands for the full Koszul-Tate differential and $\gamma$ represents the total longitudinal exterior derivative. Both operators from the right-hand side of (3) can be written in a manner similar to (2), but in terms of the corresponding operators built in Refs. [1]–[3]

$$\delta = \frac{1}{2} \left( \delta^{h, A} + \delta^{A, \psi} + \delta^{h, \psi} \right), \quad \gamma = \frac{1}{2} \left( \gamma^{h, A} + \gamma^{A, \psi} + \gamma^{h, \psi} \right).$$

The actions of $\delta$ and $\gamma$ on the generators from the BRST complex associated with theory (1) are given by

$$\delta h^{\mu\nu} = 2H^{\mu\nu}, \quad \delta A^{\mu\nu\rho} = \frac{1}{3!} \partial_{\lambda} F^{\mu\nu\rho\lambda}, \quad \delta \psi^{*\mu} = -i \partial_{\alpha} \bar{\psi} \gamma^{\alpha\beta\mu}, $$

$$\delta \eta^{*\mu} = -2 \partial_{\rho} h^{\mu\nu}, \quad \delta C^{*\mu\nu} = -3 \partial_{\rho} A^{*\mu\nu\rho}, \quad \delta \xi^{*\mu} = \partial_{\mu} \psi^{*\mu},$$

$$\delta \chi^{*\mu} = -2 \partial_{\rho} C^{*\mu\rho}, \quad \delta \chi = -\partial_{\mu} C^{*\mu}, \quad \delta \chi^{\Omega} = 0, \quad \gamma \chi^{\Omega} = 0, \quad \gamma h_{\mu\nu} = \partial_{(\mu} \eta_{\nu)}, \quad \gamma A_{\mu\nu\rho} = \partial_{\mu} C_{\nu\rho}, \quad \gamma \psi_{\mu} = \partial_{\mu} \xi, $$

$$\gamma A_{\mu\nu} = \partial_{(\mu} \eta_{\nu)}, \quad \gamma A_{\mu\nu\rho} = \partial_{\mu} C_{\nu\rho}, \quad \gamma \psi_{\mu} = \partial_{\mu} \xi,$$
\[
\gamma \eta_\mu = 0, \quad \gamma C_{\mu\nu} = \partial_{[\mu} C_{\nu]}, \quad \gamma \xi = 0, \quad \gamma C_\mu = \partial_\mu C, \quad \gamma \mathcal{C} = 0. \tag{9}
\]

In formulas (5)–(9) we denoted by \( \chi^\Omega \) the entire field/ghost spectrum and by \( \chi^\Omega_{\mathcal{C}} \) their antifields.

At this point, we will rely on the previous results exposed in Refs. [1]–[3] related to the first-order deformation of the solution to the master equation in the various sectors of theory (1) and to the associated local BRST cohomologies in order to determine the expression of the first-order deformation for the full model.

Let us denote by \( S_1 \) the first-order deformation of the solution to the master equation for theory (1), which is solution to the equation

\[
s S_1 = 0. \tag{10}
\]

The functional \( S_1 \) naturally decomposes into

\[
S_1 = S^h_1 + S^A_1 + S^\psi_1 + S^{h^-\psi}_1 + S^{A^-\psi}_1 + S^{h^-\psi}_1 + S_1^{\text{int}}. \tag{11}
\]

Some of the terms from the right-hand side of (11) have already been constructed in Refs. [1]–[3]. Their significance is as follows:

- \( S^h_1 \) means the first-order deformation in the Pauli-Fierz sector (it depends only on the BRST generators associated with the Pauli-Fierz model) and has been extensively investigated in the literature. Its nonintegrated density is given for instance in formula (47) from Ref. [1];
- \( S^A_1 \) represents the first-order deformation for the 3-form (it involves only the BRST generators corresponding to an Abelian 3-form) and was explicitly computed in Ref. [1], see formula (52).
- \( S^\psi_1 \) signifies the component of the first-order deformation in the Rarita-Schwinger sector (it comprises only the BRST generators for a massless Rarita-Schwinger vector spinor) and was deduced in Ref. [2], see formula (50);
- \( S^{h^-\psi}_1 \) denotes the first-order deformation related to the cross-couplings between a Pauli-Fierz field and a 3-form (it effectively mixes the two sorts of BRST generators) and was built in detail in Ref. [1], see formula (77);
- \( S^{A^-\psi}_1 \) is the first-order deformation describing the interactions between massless gravitini and a 3-form (again, it effectively couples the BRST generators from the vector spinor complex with those of the 3-form) and was approached in Ref. [2], see formula (110);
- \( S^{h^-\psi}_1 \) stands for the first-order deformation expressing the cross-couplings between a spin-2 field and a massless Rarita-Schwinger spinor (it effectively combines the Pauli-Fierz BRST generators with those from the vector spinor sector) and was analyzed in Ref. [3]. Its nonintegrated density is the sum between the components listed in formulas (20)–(22) from Ref. [3].
Finally, $S_{1}^{\text{int}}$ is the first-order deformation that gathers simultaneously all the three types of BRST generators and thus describes (at least cubic) interaction vertices containing the spin-2 field, the massless gravitini and the 3-form. It will be investigated in the sequel.

Since each of the first six components from the right-hand side of (11) satisfies an equation of the type (10), it follows that $S_{1}^{\text{int}}$ is subject to the equation

$$sS_{1}^{\text{int}} = 0. \quad (12)$$

In order to compute the general solution to this equation, let us denote by $a^{\text{int}}$ its nonintegrated density, such that the local form of (12) is

$$sa^{\text{int}} = \partial^{\mu}m_{\mu}^{\text{int}}, \quad (13)$$

where $m_{\mu}^{\text{int}}$ is a local current. Eq. (13) shows that $a^{\text{int}} \in H^{0}(s|d)$, where $d$ is the exterior spacetime differential in $M_{11}$. The solution to (13) is unique modulo the addition of $s$-exact terms and full divergences

$$a^{\text{int}} \rightarrow a^{\text{int}} + sc^{\text{int}} + \partial^{\mu}n_{\mu}^{\text{int}}. \quad (14)$$

If the general solution to Eq. (13) is purely trivial, $a^{\text{int}} = sc^{\text{int}} + \partial^{\mu}n_{\mu}^{\text{int}}$, then it can be taken to vanish, $a^{\text{int}} = 0$.

In order to analyze Eq. (13), we develop $a^{\text{int}}$ with respect to the antighost number

$$a^{\text{int}} = \sum_{i=0}^{I} a_{i}^{\text{int}}, \quad \text{agh} \left( a_{i}^{\text{int}} \right) = i, \quad \text{gh} \left( a_{i}^{\text{int}} \right) = 0, \quad \varepsilon \left( a_{i}^{\text{int}} \right) = 0, \quad (15)$$

and assume, without loss of generality, that decomposition (15) stops at some finite value of $I$. Replacing (15) into (13) and projecting it on the various values of the antighost number by means of (3), we obtain that (13) is equivalent to the tower of equations

$$\gamma a_{I}^{\text{int}} = \partial^{\mu}(m_{\mu}^{\text{int}}), \quad (16)$$
$$\delta a_{I}^{\text{int}} + \gamma a_{I-1}^{\text{int}} = \partial^{\mu}(m_{\mu}^{\text{int}}), \quad (17)$$
$$\delta a_{i}^{\text{int}} + \gamma a_{i-1}^{\text{int}} = \partial^{\mu}(m_{\mu}^{\text{int}}), \quad 1 \leq i \leq I - 1, \quad (18)$$

where $\left( m_{\mu}^{\text{int}} \right)_{i=0}^{I}$ are some local currents, with $\text{agh} \left( m_{\mu}^{\text{int}} \right) = i$. Eq. (16) can be replaced in strictly positive antighost numbers by

$$\gamma a_{I}^{\text{int}} = 0, \quad I > 0. \quad (19)$$

Due to the second-order nilpotency of $\gamma$ ($\gamma^{2} = 0$), the solution to (19) is unique up to $\gamma$-exact contributions

$$a_{I}^{\text{int}} \rightarrow a_{I}^{\text{int}} + \gamma c_{I}^{\text{int}}. \quad (20)$$
Meanwhile, if it turns out that $a_I^{\text{int}}$ reduces to $\gamma$-exact terms only, $a_I^{\text{int}} = \gamma c_I^{\text{int}}$, then it can be made to vanish, $a_I^{\text{int}} = 0$. In other words, the nontriviality of the first-order deformation $a_I^{\text{int}}$ is translated at its highest antighost number component into the requirement that $a_I^{\text{int}} \in H^I(\gamma)$, where $H^I(\gamma)$ denotes the cohomology of the exterior longitudinal derivative $\gamma$ in pure ghost number equal to $I$. So, in order to solve Eq. (13) (equivalent with (19) and (17)–(18)), we need to compute the cohomology of $\gamma$, $H(\gamma)$, and, as it will be made clear below, also the local cohomology of $\delta$, $H(\delta|d)$.

Using the results derived in Refs. [1]–[3] regarding the cohomology of $\gamma$, we can state that $H(\gamma)$ is generated on the one hand by $\chi^*_\Omega$, $F_{\mu\nu\rho\lambda}$, $\partial_{(\mu}\psi_{\nu)}$, and $K_{\mu\nu\alpha\beta}$, together with their spacetime derivatives and, on the other hand, by the undifferentiated ghost for ghost for ghost $C$, by the undifferentiated ghost $\xi$ as well by the ghosts $\eta_{\mu}$ and their antisymmetric first-order derivatives $\partial_{(\mu}\eta_{\nu)}$. So, the most general (and nontrivial) solution to (19) can be written, up to $\gamma$-exact contributions, as

$$a_I^{\text{int}} = \alpha_I \left( [F_{\mu\nu\rho\lambda}], [\partial_{(\mu}\psi_{\nu)}], [K_{\mu\nu\alpha\beta}], [\chi^*_\Omega] \right) \omega^I \left( C, \xi, \eta_{\mu}, \partial_{(\mu}\eta_{\nu)} \right), \quad (21)$$

where the notation $f([q])$ means that $f$ depends on $q$ and its derivatives up to a finite order, and $\omega^I$ denotes the elements of a basis in the space of polynomials with pure ghost number $I$ in the corresponding ghost for ghost for ghost, Rarita-Schwinger ghost, Pauli-Fierz ghosts and their antisymmetric first-order derivatives. The objects $\alpha_I$ (obviously nontrivial in $H^0(\gamma)$) were taken to have a finite antighost number and a bounded number of derivatives, and therefore they are polynomials in the antifields $\chi^*_\Omega$, in the linearized Riemann tensor $K_{\mu\nu\alpha\beta}$, in the antisymmetric first-order derivatives of the spin-vector, $\partial_{(\mu}\psi_{\nu)}$, and in the field-strength of the three-form $F_{\mu\nu\rho\lambda}$ as well as in their subsequent derivatives. They are required to fulfill the property $\text{agh}(\alpha_I) = I$ in order to ensure that the ghost number of $a_I^{\text{int}}$ is equal to zero. Due to their $\gamma$-closeness, $\gamma \alpha_I = 0$, and to their polynomial character, $\alpha_I$ will be called invariant polynomials.

Inserting (21) in (17), we obtain that a necessary (but not sufficient) condition for the existence of (nontrivial) solutions $a_{I-1}$ is that the invariant polynomials $\alpha_I$ are (nontrivial) objects from the local cohomology of the Koszul-Tate differential $H(\delta|d)$ in antighost number $I > 0$ and in pure ghost number zero,

$$\delta \alpha_I = \partial_{\mu} \left( (I-1)^\mu \right), \quad \text{agh} \left( \begin{array}{c} (I-1)^\mu \\ j \end{array} \right) = I - 1, \quad \text{pgh} \left( \begin{array}{c} (I-1)^\mu \\ j \end{array} \right) = 0. \quad (22)$$

We recall that the local cohomology $H(\delta|d)$ is completely trivial in both strictly positive antighost and pure ghost numbers. Using the fact that the Cauchy order of the free theory under study is equal to four, the general results from Refs. [4] and [5], according to which the local cohomology of the Koszul-Tate differential in pure ghost number zero is trivial in antighost numbers strictly greater than its Cauchy order, ensure that

$$H_J(\delta|d) = 0, \quad J > 4, \quad (23)$$
where \( H_J(\delta|d) \) denotes the local cohomology of the Koszul-Tate differential in antighost number \( J \) and in pure ghost number zero. It can be shown that any invariant polynomial that is trivial in \( H_J(\delta|d) \) with \( J \geq 4 \) can be taken to be trivial also in \( H^\text{inv}_J(\delta|d) \). \( H^\text{inv}_J(\delta|d) \) denotes the invariant characteristic cohomology in antighost number \( J \) — the local cohomology of the Koszul-Tate differential in the space of invariant polynomials.) Thus:

\[
\alpha_J = \delta b_{J+1} + \partial_{\mu}^{(J)\mu}, \quad \text{agh} (\alpha_J) = J \geq 4 \Rightarrow \alpha_J = \delta \beta_{J+1} + \partial_{\mu}^{(J)\mu}, \quad (24)
\]

with both \( \beta_{J+1} \) and \( \gamma^{(J)\mu} \) invariant polynomials. Results (23) and (24) yield the conclusion that \( H^\text{inv}_J(\delta|d) = 0 \), \( J > 4 \).

Using the results from Refs. [1]–[3], the spaces \( (H_J(\delta|d))_{J \geq 2} \) and \( (H^\text{inv}_J(\delta|d))_{J \geq 2} \) are spanned by

\[
\begin{align*}
H_4(\delta|d), H^\text{inv}_4(\delta|d) & : (C^*), \\
H_5(\delta|d), H^\text{inv}_5(\delta|d) & : (C^*\mu), \\
H_2(\delta|d), H^\text{inv}_2(\delta|d) & : (C^{\mu\nu}, \eta^{*\mu}, \xi^*).
\end{align*}
\]

(26) \( \cdots \) (28)

In contrast to the groups \( (H_J(\delta|d))_{J \geq 2} \) and \( (H^\text{inv}_J(\delta|d))_{J \geq 2} \), which are finite-dimensional, the cohomology \( H_1(\delta|d) \) in pure ghost number zero, known to be related to global symmetries and ordinary conservation laws, is infinite-dimensional since the theory is free. Fortunately, it will not be needed in the sequel.

The previous results on \( H(\delta|d) \) and \( H^\text{inv}(\delta|d) \) in strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. Based on formulas (23)–(24), one can successively eliminate all the pieces of antighost number strictly greater than four from the nonintegrated density of the first-order deformation by adding only trivial terms. Consequently, one can take (without loss of nontrivial objects) \( I \leq 4 \) into the decomposition (15). In addition, the last representative reads as in (21), where the invariant polynomial is necessarily a nontrivial object from \( (H^\text{inv}_J(\delta|d))_{2 \leq J \leq 4} \) or from \( H_1(\delta|d) \) for \( J = 1 \).

The previous discussion enforces that we can take \( I = 4 \) in (15) and work with

\[
a^\text{int} = a^\text{int}_0 + a^\text{int}_1 + a^\text{int}_2 + a^\text{int}_3 + a^\text{int}_4, \quad (29)
\]

where the components from the right-hand side of (29) satisfy Eqs. (19) and (17)–(18) for \( I = 4 \). Due to (21) and (26), we can write the nontrivial solution to (19) for \( I = 4 \) in the form

\[
a^\text{int}_4 = C^* \omega^4 \left( C, \xi, \eta_{\mu}, \partial_{[\mu} \eta_{\nu]} \right). \quad (30)
\]

Since \( a^\text{int}_4 \) already depends on \( C^* \), which is a BRST generator from the 3-form sector, we have to select from \( \omega^4 \) only those elements of pure ghost number 4.
that depend simultaneously on the ghosts from the Rarita-Schwinger and Pauli-Fierz sectors, namely involve both \( \xi \) and \( \eta \) or \( \partial_{\nu}\eta_{\rho} \). These are precisely

\[
\left\{ \bar{\xi}_{\gamma\alpha}(\xi_{\nu}\eta_{\rho}), \bar{\xi}_{\gamma\alpha\beta}(\xi_{\nu}\eta_{\rho}), \bar{\xi}_{\gamma\alpha\beta\gamma}(\xi_{\nu}\eta_{\rho}), \bar{\xi}_{\gamma\alpha}(\xi_{\nu}\eta_{\rho}), \bar{\xi}_{\gamma\alpha\beta}(\xi_{\nu}\eta_{\rho}), \bar{\xi}_{\gamma\alpha\beta\gamma}(\xi_{\nu}\eta_{\rho}) \right\},
\]

such that (31) becomes

\[
a^\text{int}_4 = C^* \left[ \frac{q_1}{2} \bar{\xi} R_{\mu\nu} \xi_{\mu}\eta_{\nu} + q_2 \sigma^{\mu\nu} \bar{\xi} R_{\mu\nu} \xi_{\mu}\eta_{\nu} \right]
\]

\[
+ \frac{q_3}{2} \sigma^{\rho\lambda} \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda},
\]

with \( q_1, q_2, \) and \( q_3 \) some real, arbitrary constants. By applying the operator \( \delta \) on (32) and further using definitions (35)–(39), we obtain that

\[
\delta a^\text{int}_4 = \partial_{\nu} \left\{ -C^* \left[ \frac{q_1}{2} \bar{\xi} R_{\mu\nu} \xi_{\mu}\eta_{\nu} + q_2 \sigma^{\mu\nu} \bar{\xi} R_{\mu\nu} \xi_{\mu}\eta_{\nu} \right]
\]

\[
+ \frac{q_3}{2} \sigma^{\rho\lambda} \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda} \right\}
\]

\[
+ \gamma \left\{ C^* \left[ \frac{q_1}{2} \bar{\xi} R_{\mu\nu} \xi_{\mu}\eta_{\nu} h_{\mu\beta} - \frac{q_2}{2} \sigma^{\mu\nu} \bar{\xi} R_{\mu\nu} \xi_{\mu}\eta_{\nu} h_{\mu\beta} \right]
\]

\[
- 2\eta_{\nu}(\partial_{\nu}\partial_{\beta}\eta_{\mu}) + q_3 \sigma^{\rho\lambda} \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\partial_{\beta}\eta_{\mu}) + q_4 \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\partial_{\beta}\eta_{\mu})
\]

\[
+ q_5 \sigma^{\rho\lambda} \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\partial_{\beta}\eta_{\mu}) \right\}
\]

\[
+ \frac{1}{2} C^* \left[ \frac{q_1}{2} \bar{\xi} R_{\mu\nu} \xi_{\mu}\eta_{\nu} - q_2 \sigma^{\mu\nu} \bar{\xi} R_{\mu\nu} \xi_{\mu}\eta_{\nu} \right] \partial_{\nu}\eta_{\rho}.\]

We observe that (33) cannot agree with (17) for \( I = 4 \) unless we set

\[
q_1 = q_2 = 0,
\]

which then replaced in (32) and (33) produces

\[
a^\text{int}_4 = \frac{q_3}{2} \sigma^{\rho\lambda} C^* \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda},
\]

\[
a^\text{int}_3 = -q_3 \sigma^{\rho\lambda} C^* \left[ \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda} + \bar{\xi} \gamma^\alpha\beta \psi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda} \right].
\]

Acting now with \( \delta \) on (36) and recalling definitions (35)–(39), we have that

\[
\delta a^\text{int}_3 = \partial_{\nu} \left\{ -2q_3 \sigma^{\rho\lambda} C^* \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda} + \bar{\xi} \gamma^\alpha\beta \psi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda} \right\}
\]

\[
+ \gamma \left\{ q_3 \sigma^{\rho\lambda} C^* \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda} + 2\psi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda} \right\}
\]

\[
- q_3 \sigma^{\rho\lambda} C^* \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda} + q_4 \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda} \right\}
\]

\[
+ q_5 \sigma^{\rho\lambda} C^* \bar{\xi} R_{\rho\lambda} \xi_{\mu}(\partial_{\nu}\eta_{\rho}) \partial_{\beta}\eta_{\lambda} \right\},
\]

such that (37) is compatible with (17) for \( I = 3 \) if

\[
q_3 = 0.
\]
If we insert $(38)$ into $(35)$, then we conclude that $a^\text{int}_4 = 0$, so we can take $I = 3$ in $(15)$.

Consequently, decomposition $(15)$ reduces to

$$a^\text{int} = a^\text{int}_0 + a^\text{int}_1 + a^\text{int}_2 + a^\text{int}_3,$$  

(39)

where the components from the right-hand side of $(39)$ satisfy Eqs. $(19)$ and $(17)$–$(18)$ for $I = 3$. Taking into account formula $(21)$ and relation $(27)$, we find that

$$a^\text{int}_3 = C^*\mu^\alpha \beta \eta \mu (C, \xi, \eta, \partial_{[\mu} \eta_{\nu]},$$  

(40)

where we have to elect again among the elements $\omega^3$ only those involving simultaneously $\xi$ and $\eta_{\mu}$ or $\partial_{[\mu} \eta_{\nu]}$, namely

$$\{ \xi \eta_{\mu}, \xi \eta_{\nu}, \xi \eta_{\alpha} \delta \xi \eta_{\mu}, \xi \eta_{\alpha} \delta \xi \partial_{[\mu} \eta_{\nu]}.$$

(41)

Only the second and fourth elements from $(41)$ allow the formation of 11-dimensional vector-like combinations, so the general form of $(40)$ reads as

$$a^\text{int}_3 = \frac{1}{2} C^* C^\nu \alpha \beta (q_4 \xi \eta_{\mu} + q_5 \xi \eta_{\mu} \partial_{[\mu} \eta_{\nu]}),$$  

(42)

with $q_4$ and $q_5$ real numbers. Next, we act like in the case $I = 4$, namely apply $\delta$ on $(42)$, and then manipulate the resulting expression with the help of definitions $(5)$–$(9)$, which further yields

$$\delta a^\text{int}_3 = \partial_{\nu} \left( \sigma^\alpha \beta C^* C^\nu \alpha \beta (q_4 \xi \eta_{\mu} + q_5 \xi \eta_{\mu} \partial_{[\mu} \eta_{\nu]} \right) + \gamma \left( \sigma^\alpha \beta C^* C^\nu \alpha \beta (2 \psi_{\nu} \eta_{\beta} - \xi h_{\beta \nu}) + q_5 \xi \eta_{\mu} \left( 2 \psi_{\nu} \partial_{[\mu} \eta_{\nu]} - \xi \partial_{\nu} h_{\rho \nu]} \right) \right) \right)$$

$$- \frac{q_4}{2} \sigma^\alpha \beta C^* C^\nu \alpha \beta (q_4 \xi \eta_{\mu} + q_5 \xi \eta_{\mu} \partial_{[\mu} \eta_{\nu]}).$$  

(43)

Formula $(43)$ does not concur with $(17)$ for $I = 3$ unless

$$q_4 = 0,$$  

(44)

which substituted in $(42)$ and $(43)$ provides

$$a^\text{int}_3 = \frac{q_5}{2} C^* C^\nu \alpha \beta \xi \eta_{\mu} \partial_{[\mu} \eta_{\nu]},$$  

(45)

$$a^\text{int}_2 = -q_5 C^* C^\nu \alpha \beta \xi \eta_{\mu} \partial_{[\mu} \eta_{\nu]}.$$  

(46)

Acting with $\delta$ on $(46)$, we can write that

$$\delta a^\text{int}_2 = \partial_{\nu} \left( 3 q_5 C^* C^\nu \alpha \beta \xi \eta_{\mu} \partial_{[\mu} \eta_{\nu]} \right) + \gamma \left( -3 q_5 C^* C^\nu \alpha \beta (2 \psi_{\nu} \partial_{[\mu} \eta_{\nu]} - \xi \partial_{\nu} h_{\rho \nu]} + \xi \eta_{\mu} \partial_{[\mu} \eta_{\nu]} \right) \right)$$

$$+ 3 q_5 C^* C^\nu \alpha \beta \xi \eta_{\mu} \partial_{[\mu} \eta_{\nu]}.$$  

(47)
so Eq. (18) for $I = 2$ cannot hold except for the case where

$$q_5 = 0.$$  (48)

Introducing (44) and (48) in (42), we deduce that we can take $a_{3 \text{int}} = 0$.

The following possibility is to stop at antighost number 2, in which situation

$$a_{\text{int}} = a_{0 \text{int}} + a_{1 \text{int}} + a_{2 \text{int}},$$  (49)

where the components of $a_{\text{int}}$ are subject to Eqs. (19) and (17–18) for $I = 2$. Due to result (28) and formula (21), the nontrivial solution to (19) for $I = 2$ takes the form

$$a_{2 \text{int}} = \xi \hat{\omega}^2 (C, \xi, \eta, \partial_{[\mu} \eta_{\nu]}).$$  (50)

The elements $\hat{\omega}^2$, $\omega^2_{\mu}$, and $\omega^2_{\mu \nu}$ have the pure ghost number equal to 2 and must at least contain ghosts belonging to the sectors respectively complementary to that including the antifield coupled to them. In other words, $\hat{\omega}^2$ compulsory contains both $C$ and $\eta$, or $\partial_{[\mu} \eta_{\nu]}$, $\omega^2_{\mu}$ must depend on both $C$ and $\xi$, and $\omega^2_{\mu \nu}$ are restricted to involve both $\xi$ and $\eta$ or $\partial_{[\mu} \eta_{\nu]}$. Since the pure ghost number of $C$ is already 3, it follows that $\hat{\omega}^2$ and $\omega^2_{\mu}$ must be discarded by putting

$$\hat{\omega}^2 (C, \xi, \eta, \partial_{[\mu} \eta_{\nu]}) = 0,$$  (51)

$$\omega^2_{\mu} (C, \xi, \eta, \partial_{[\mu} \eta_{\nu]}) = 0.$$  (52)

Regarding $\omega^2_{\mu \nu}$, we observe that the ghost $\xi$ is a spinor, so it can be mixed with $\eta$, or $\partial_{[\mu} \eta_{\nu]}$, into an antisymmetric tensor through an at least cubic combination, simultaneously involving $\bar{\xi}$, $\xi$, and $\eta$, or $\partial_{[\mu} \eta_{\nu]}$, which therefore displays a pure ghost number greater or equal to 3. For this reason we must also give up $\omega^2_{\mu \nu}$

$$\omega^2_{\mu \nu} (C, \xi, \eta, \partial_{[\mu} \eta_{\nu]}) = 0.$$  (53)

The results expressed by (51)–(53) ensure, via (50), that $a_{2 \text{int}} = 0$, so we have to consider the case $I = 1$ in (15).

Consequently, we have that

$$a_{\text{int}} = a_{0 \text{int}} + a_{1 \text{int}},$$  (54)

where $a_{0 \text{int}}$ and $a_{1 \text{int}}$ fulfill the equations

$$\gamma a_{1 \text{int}} = 0,$$  (55)

$$\delta a_{1 \text{int}} + \gamma a_{0 \text{int}} = \partial^{(0)\text{int}}_{\mu} m_{\mu}.$$  (56)

According to (21), the general, nontrivial solution to (55) is given by

$$a_{1 \text{int}} = \psi^\mu (M_\mu \xi + M_\mu^\alpha \eta_\alpha + M_\mu^{\alpha \beta} \partial_{[\alpha} \eta_{\beta]}).$$


\[ +h^{*\mu\nu}(N_{\mu\nu}\xi + N_{\mu\nu}^{\alpha}\eta_{\alpha} + N_{\mu\nu}^{\alpha\beta}\partial_{[\alpha}\eta_{\beta]}) \]
\[ +A^{*\mu\nu\rho}(P_{\mu\nu\rho}\xi + P_{\mu\nu\rho}^{\alpha}\eta_{\alpha} + P_{\mu\nu\rho}^{\alpha\beta}\partial_{[\alpha}\eta_{\beta]}) \]

where the objects generically denoted by \(M\), \(N\) or \(P\) are gauge invariant quantities. In order to provide interactions among all the three kinds of fields, each of them is required to depend at least on those gauge invariant combinations constructed out of the fields from the sector(s) that are complementary to the respectively coupled antifields/ghosts. Regarding their tensorial properties, the elements \(M^{\alpha\beta}\), \(N^{\alpha\beta}\), and \(P^{\alpha\beta}\) are antisymmetric in their upper indices \(\alpha\) and \(\beta\), all the quantities of the type \(N\) are symmetric in their lower indices \(\mu\) and \(\nu\), and all the quantities denoted by \(P\) are completely antisymmetric in their lower indices \(\mu\), \(\nu\), and \(\rho\). Moreover, each \(M_{\mu}\) is a \(2^5 \times 2^5\) matrix with bosonic, gauge invariant functions as elements. Furthermore, each of \(M^{\alpha}_{\mu}\), \(M^{\alpha\beta}_{\mu}\), \(N_{\mu\nu}\), or \(P_{\mu\nu\rho}\) is a fermionic, gauge invariant spinor tensor. Since the only fermionic, spinor fields are the gravitini and the gauge invariant quantities built out of them are their antisymmetric first-order derivatives, \(\partial_{[\gamma}\psi_{\delta]}\), it follows that \(M^{\alpha}_{\mu}\), \(M^{\alpha\beta}_{\mu}\), \(N_{\mu\nu}\), and \(P_{\mu\nu\rho}\) can be further represented in terms of some \(2^5 \times 2^5\) matrices with bosonic, gauge invariant elements, of the type:

\[
M^{\alpha}_{\mu} = \tilde{M}^{\alpha\beta\gamma\delta}_{\mu} \partial_{[\gamma}\psi_{\delta]}\quad M^{\alpha\beta}_{\mu} = \tilde{M}^{\alpha\beta\gamma\delta}_{\mu} \partial_{[\gamma}\psi_{\delta]}\quad N_{\mu\nu} = \partial_{[\gamma}\tilde{\eta}_{\delta]}\tilde{N}^{\gamma\delta}_{\mu\nu}\quad P_{\mu\nu\rho} = \partial_{[\gamma}\psi_{\delta]}\tilde{P}^{\gamma\delta}_{\mu\nu\rho}
\]

where \(\tilde{M}^{\alpha\beta\gamma\delta}_{\mu}\), \(\tilde{M}^{\alpha\beta\gamma\delta}_{\mu}\), \(\tilde{N}^{\gamma\delta}_{\mu\nu}\), or \(\tilde{P}^{\gamma\delta}_{\mu\nu\rho}\) may contain in principle additional spacetime derivatives. At this point we ask that the corresponding \(a^{\int}_{0}\) (as solution to \(\ref{56}\)) leads to interacting field equations preserving the derivative order of the free ones (derivative order assumption). This further requires that the maximum derivative order of \(a^{\int}_{0}\) is equal to two, with the precaution that each interacting field equation contains at most one spacetime derivative acting on the gravitini. In the sequel we will argue that each of the terms from the right-hand side of \(\ref{57}\), if consistent, would produce in the interacting Lagrangian terms forbidden by the derivative order assumption.

Related to the first term, \(\psi^{*\mu}M_{\mu}\xi\), since both \(\psi^{*\mu}\) and \(\xi\) belong to the Rarita-Schwinger sector, it follows that each element of the matrices \(M_{\mu}\) is constrained to be at least linear in both the linearized Riemann tensor \(K_{\mu\nu\rho}\) and the field strength \(F_{\mu\nu\rho\lambda}\) of the 3-form, so it contains at least three spacetime derivatives. If consistent, each of these terms would lead to an interacting Lagrangian density with minimum three derivatives, which is unacceptable, so we must set

\[
M_{\mu} = 0.
\]

The second element, \(\psi^{*\mu}M_{\mu}^{\alpha}\eta_{\alpha}\), already contain generators from the Rarita-Schwinger and Pauli-Fierz sectors, so \(M_{\mu}^{\alpha}\) are bound to be at least linear in \(F_{\mu\nu\rho\lambda}\). Due to the former relation from \(\ref{58}\), we conclude that \(\psi^{*\mu}M_{\mu}^{\alpha}\eta_{\alpha}\) has at least two spacetime derivatives, among which one already acts on the gravitini, so it would provide field equations with at least two derivatives acting on the gravitini. We have to forbid this by setting

\[
M_{\mu}^{\alpha} = 0.
\]
Using exactly the same arguments we eliminate the third piece, \( \psi^\mu M^\alpha_\mu \partial_{[\alpha} \eta_{\beta]} \), by putting
\[
M^\alpha_\mu = 0.
\]  
(62)

A simple analysis of the fourth component, \( h^{\mu\nu} N^{\alpha\mu}_{\mu\nu} \xi \), shows that \( N^{\mu\nu} \) is compelled to be at least linear in \( F_{\mu\nu\rho\lambda} \), and so, according to the first formula in (59), it contains at least two derivatives. Since \( \delta h^{\mu\nu} \) also contains two derivatives, this component would generate an interacting Lagrangian density with at least three derivatives. The derivative order assumption is again broken, so we must take
\[
N^{\mu\nu} = 0.
\]  
(63)

Looking at the fifth constituent, \( h^{\mu\nu} N^{\alpha}_{\mu\nu}\eta_{\alpha} \), since both \( h^{\mu\nu} \) and \( \eta_{\alpha} \) pertain to the Pauli-Fierz sector, it results that the bosonic, \( \gamma \)-invariant tensor \( N^{\alpha}_{\mu\nu} \) is simultaneously at least quadratic in the antisymmetric first-order derivatives of the Rarita-Schwinger spinors and linear in \( F_{\mu\nu\rho\lambda} \), which amounts to at least three derivatives. Thus, if consistent, this term would give rise to a Lagrangian density with at least four derivatives. The same reason can be used to eliminate \( h^{\mu\nu} N^{\alpha}_{\mu\nu}\partial_{[\alpha} \eta_{\beta]} \) from \( a^\text{int}_1 \), and hence we can write
\[
N^{\alpha}_{\mu\nu} = 0, \quad N^{\alpha\beta}_{\mu\nu} = 0.
\]  
(64)

The seventh term, \( A^{\mu\nu\rho} P_{\mu\nu\rho} \xi \), contains the fermionic, gauge invariant spinor tensor \( P_{\mu\nu\rho} \), which is required to involve the Pauli-Fierz field, so it effectively depends on \( K_{\mu\nu\rho\lambda} \). Joining this observation to the second relation from (59), we get that \( P_{\mu\nu\rho} \) includes at least three derivatives, and thus the corresponding Lagrangian density (if any) would furnish interaction vertices with at least four derivatives. This is again in contradiction with the derivative order assumption, so we must discard this term by choosing
\[
P_{\mu\nu\rho} = 0.
\]  
(65)

Finally, the last two pieces from the right-hand side of (57), \( A^{\mu\nu\rho} P^{\alpha}_{\mu\nu\rho}\eta_{\alpha} \) and \( A^{\mu\nu\rho} P^{\alpha\beta}_{\mu\nu\rho}\partial_{[\alpha} \eta_{\beta]} \), involve the bosonic, gauge invariant tensors \( P^{\alpha}_{\mu\nu\rho} \) and \( P^{\alpha\beta}_{\mu\nu\rho} \), which are required to depend on the Rarita-Schwinger spinors, and therefore they are at least quadratic in the antisymmetric first-order derivatives of gravitini. If consistent, these objects would imply interaction vertices with at least three and respectively four derivatives, and therefore must be canceled through
\[
P^{\alpha}_{\mu\nu\rho} = 0, \quad P^{\alpha\beta}_{\mu\nu\rho} = 0.
\]  
(66)

Inserting the previous results, (60)–(66), into (57), we obtain \( a^\text{int}_1 = 0 \), such that the first-order deformation of the solution to the master equation can only reduce to its antighost number zero component (we can only have \( I = 0 \) in (55)).

This final possibility is described by
\[
a^\text{int} = a^\text{int}_0, \quad a^\text{int} = a^\text{int}_0.
\]  
(67)
where $\alpha^\text{int}_0$ is subject to the equation

$$\gamma \alpha^\text{int}_0 = \partial^\mu \langle 0 \rangle^\text{int}_\mu. \tag{68}$$

In order to analyze properly the solution to (68), we split its solution as

$$\alpha^\text{int}_0 = \tilde{\alpha}^\text{int}_0 + \tilde{\alpha}'^\text{int}_0, \tag{69}$$

where

$$\gamma \tilde{\alpha}^\text{int}_0 = 0, \tag{70}$$

$$\gamma \tilde{\alpha}'^\text{int}_0 = \partial^\mu \langle 0 \rangle^\text{int}_\mu, \tag{71}$$

with $\langle 0 \rangle^\text{int}_\mu \neq 0$.

Due to (70), $\tilde{\alpha}^\text{int}_0$ is a bosonic, gauge invariant object, which is required to depend on all three kinds of fields. Consequently, $\tilde{\alpha}^\text{int}_0$ is at least quadratic in the antisymmetric first-order derivatives of the spinors, $\partial_\mu \psi$, and at least linear in both $K_{\mu\nu\rho\lambda}$ and $F_{\mu\nu\rho\lambda}$, so it contains at least five spacetime derivatives, which disagrees with the derivative order assumption. In conclusion, we eliminate it from the interacting Lagrangian density by putting

$$\tilde{\alpha}^\text{int}_0 = 0. \tag{72}$$

Now, we approach Eq. (71) in a standard manner. Namely, we decompose $\tilde{\alpha}'^\text{int}_0$ with respect to the total number of derivatives into

$$\tilde{\alpha}'^\text{int}_0 = \omega_0 + \omega_1 + \omega_2, \tag{73}$$

where $(\omega_k)_{k=0,1,2}$ comprises $k$ derivatives. By projecting (71) on the different possible values of the number of derivatives, we find that it becomes equivalent to three equations, one for each component

$$\gamma \omega_k = \partial_\mu l^\mu_k, \quad k = 0, 1, 2, \tag{74}$$

where

$$m_{(0)\mu}^{(0)\text{int}} = l_0^\mu + l_1^\mu + l_2^\mu. \tag{75}$$

In the sequel we solve (74) for each value of $k$.

We start with (74) for $k = 0$ and recall definitions (8), which produce

$$\gamma \omega_0 = \frac{\partial^R \omega_0}{\partial \psi_\mu} \partial_\mu \xi + \frac{\partial \omega_0}{\partial h_{\mu\nu}} \partial_{(\mu} \eta_{\nu)} + \frac{\partial \omega_0}{\partial A_{\mu\nu\rho}} \partial_{[\mu} C_{\nu\rho]}$$

$$= \partial_\mu \left( \frac{\partial^R \omega_0}{\partial \psi_\mu} \xi + 2 \frac{\partial \omega_0}{\partial h_{\mu\nu}} \eta_{\nu} + 3 \frac{\partial \omega_0}{\partial A_{\mu\nu\rho}} C_{\nu\rho} \right) \tag{74}$$

1One can no longer replace Eq. (68) with the homogeneous one, like in the previous cases, since now we reached the bottom value of the antighost number, namely zero.
\[-\left( \partial_\mu \frac{\partial R}{\partial \psi_\mu} \right) \xi - 2 \left( \partial_\mu \frac{\partial \omega_0}{\partial h_{\mu\nu}} \right) \eta_\nu - 3 \left( \partial_\mu \frac{\partial \omega_0}{\partial A_{\mu\nu\rho}} \right) C_{\nu\rho}\]  
(76)

From (76) we observe that (74) for \( k = 0 \) cannot hold unless
\[ \partial_\mu \frac{\partial R}{\partial \psi_\mu} = 0, \quad \partial_\mu \frac{\partial \omega_0}{\partial h_{\mu\nu}} = 0, \quad \partial_\mu \frac{\partial \omega_0}{\partial A_{\mu\nu\rho}} = 0. \]  
(77)

But \( \omega_0 \) has no derivatives acting on the fields, such that the only solution to (77) is purely trivial
\[ \omega_0 = 0. \]  
(78)

For \( k = 1 \) we have that
\[ \gamma \omega_1 = \frac{\partial^R \omega_1}{\partial \psi_\mu} \partial_\mu \xi + \frac{\partial \omega_1}{\partial h_{\mu\nu}} \partial_\mu (\eta_\nu) + \frac{\partial \omega_1}{\partial A_{\mu\nu\rho}} \partial_\mu (C_{\nu\rho}) \]
\[ + \frac{\partial^R \omega_1}{\partial \psi_\mu} \partial_\alpha \partial_\mu \xi + \frac{\partial \omega_1}{\partial (\partial_\alpha h_{\mu\nu})} \partial_\alpha \partial_\mu (\eta_\nu) + \frac{\partial \omega_1}{\partial (\partial_\alpha A_{\mu\nu\rho})} \partial_\alpha \partial_\mu (C_{\nu\rho}) \]
\[ = \partial_\mu \left[ \frac{\partial^R \omega_1}{\partial \psi_\mu} \xi + 2 \frac{\partial \omega_1}{\partial h_{\mu\nu}} \eta_\nu + 3 \frac{\partial \omega_1}{\partial A_{\mu\nu\rho}} C_{\nu\rho} + \frac{\partial^R \omega_1}{\partial \psi_\mu} \partial_\alpha \xi \right. \]
\[ + \frac{\partial \omega_1}{\partial (\partial_\alpha h_{\mu\nu})} \partial_\alpha (\eta_\nu) + \frac{\partial \omega_1}{\partial (\partial_\alpha A_{\mu\nu\rho})} \partial_\alpha (C_{\nu\rho}) \]  
\[ - 2 \left( \partial_\alpha \frac{\partial \omega_1}{\partial (\partial_\alpha h_{\mu\nu})} \right) \eta_\nu - 3 \left( \partial_\alpha \frac{\partial \omega_1}{\partial (\partial_\alpha A_{\mu\nu\rho})} \right) C_{\nu\rho} \]  
\[ - \left( \partial_\mu \frac{\delta^R \omega_1}{\delta \psi_\mu} \right) \xi - 2 \left( \partial_\mu \frac{\delta \omega_1}{\delta h_{\mu\nu}} \right) \eta_\nu - 3 \left( \partial_\mu \frac{\delta \omega_1}{\delta A_{\mu\nu\rho}} \right) C_{\nu\rho}, \]  
(79)

so (79) complies with (74) for \( k = 1 \) if
\[ \partial_\mu \frac{\delta^R \omega_1}{\delta \psi_\mu} = 0, \quad \partial_\mu \frac{\delta \omega_1}{\delta h_{\mu\nu}} = 0, \quad \partial_\mu \frac{\delta \omega_1}{\delta A_{\mu\nu\rho}} = 0. \]  
(80)

The general solutions to (80) are expressed by
\[ \frac{\delta^R \omega_1}{\delta \psi_\mu} = \partial_\nu L^{\mu\nu}, \quad \frac{\delta \omega_1}{\delta h_{\mu\nu}} = \partial_\rho L^\rho_{\mu\nu}, \quad \frac{\delta \omega_1}{\delta A_{\mu\nu\rho}} = \partial_\lambda L^{\mu\nu\rho\lambda}, \]  
(81)

where all the quantities generically denoted by \( L \) depend only on the undifferentiated fields (have no spacetime derivatives). In addition, \( L^{\mu\nu} \) is a fermionic, spinor tensor, antisymmetric in its Lorentz indices, \( L^{\mu\nu\rho\lambda} \) is a bosonic, completely antisymmetric tensor, and \( L^\rho_{\mu\nu} \) is also a bosonic tensor, antisymmetric in its first two indices
\[ L^\rho_{\mu\nu} = -L^\rho_{\nu\mu}. \]  
(82)

As \( \delta \omega_1/\delta h_{\mu\nu} \) is symmetric and \( L^{\rho\mu\nu} \) is derivative-free, it follows that this tensor must be symmetric in its last two indices
\[ L^{\rho\mu\nu} = L^{\rho\nu\mu}. \]  
(83)
Using repeatedly properties (82) and (83), it is easy to obtain $L^{\mu\nu} = 0$, and hence
\[ \frac{\delta \omega_1}{\delta h_{\mu\nu}} = 0. \] (84)
This means that $\omega_1$ may depend on the Pauli-Fierz field only through trivial combinations (full divergences), which bring no contributions to the interacting Lagrangian density, and therefore $\omega_1$ cannot assemble all the three sectors in a nontrivial way and can be taken to vanish
\[ \omega_1 = 0. \] (85)
Finally, we solve Eq. (74) for $k = 2$. If we make the notations
\[ F^\mu = \frac{\delta R \omega_2}{\delta \psi_\mu}, \quad D^{\mu\nu} = \frac{\delta \omega_2}{\delta h_{\mu\nu}}, \quad D^{\mu\nu\rho} = \frac{\delta \omega_2}{\delta A_{\mu\nu\rho}}, \] (86)
then we can write
\[ \gamma \omega_2 = - (\partial_\nu F^\mu) \xi - 2 (\partial_\mu D^{\mu\nu}) \eta_\nu - 3 (\partial_\mu D^{\mu\nu\rho}) C_{\nu\rho} + \partial_\mu u^\mu, \] (87)
with $u^\mu$ a local current. From (87) we infer that $\omega_2$ cannot be solution to (74) unless
\[ \partial_\mu F^\mu = 0, \quad \partial_\mu D^{\mu\nu} = 0, \quad \partial_\mu D^{\mu\nu\rho} = 0. \] (88)
The solutions to the last equations are known and take the general form
\[ F^\mu = \partial_\nu F^{\mu\nu}, \quad D^{\mu\nu} = \partial_\alpha \partial_\beta U^{\mu\alpha\nu\beta}, \quad D^{\mu\nu\rho\lambda} = \partial_\chi D^{\mu\nu\rho\lambda}, \] (89)
where $F^{\mu\nu}$ and $D^{\mu\nu\rho\lambda}$ are completely antisymmetric in their Lorentz indices and $U^{\mu\alpha\nu\beta}$ possesses the mixed symmetry of the Riemann tensor. In addition, $F^{\mu\nu}$ and $D^{\mu\nu\rho\lambda}$ contain precisely one spacetime derivative of the fields and $U^{\mu\alpha\nu\beta}$ depends only on the undifferentiated fields. At this stage it is useful to introduce a derivation in the algebra of the fields and of their derivatives that counts the powers of the fields and their derivatives, defined by
\[ N = \sum_{k \geq 0} \left[ \frac{\partial R}{\partial (\partial_{\mu_1} \cdots \partial_{\mu_k} \psi_\mu)} (\partial_{\mu_1} \cdots \partial_{\mu_k} \psi_\mu) + (\partial_{\mu_1} \cdots \partial_{\mu_k} h_{\mu\nu}) \frac{\partial}{\partial (\partial_{\mu_1} \cdots \partial_{\mu_k} h_{\mu\nu})} \right. \\
+ (\partial_{\mu_1} \cdots \partial_{\mu_k} A_{\mu\nu\rho}) \left. \frac{\partial}{\partial (\partial_{\mu_1} \cdots \partial_{\mu_k} A_{\mu\nu\rho})} \right]. \] (90)
Then, it is easy to see that for every nonintegrated density $\chi$, we have that
\[ N \chi = \frac{\delta R \chi}{\delta \psi_\mu} \psi_\mu + \frac{\delta \chi}{\delta h_{\mu\nu}} h_{\mu\nu} + \frac{\delta \chi}{\delta A_{\mu\nu\rho}} A_{\mu\nu\rho} + \partial_\mu s^\mu. \] (91)
If $\chi^{(l)}$ is a homogeneous polynomial of order $l > 0$ in the fields and their derivatives, then
\[ N \chi^{(l)} = l \chi^{(l)}. \] (92)
Using (86), (89), and (91), we deduce that

\[
N\omega^2 = \frac{1}{2} F^{\mu\nu} \partial_{\mu} \psi_{\nu} - \frac{1}{2} K_{\mu\alpha\nu\beta} U^{\mu\alpha\nu\beta} + \frac{1}{4} F_{\mu\nu\rho\lambda} D^{\mu\nu\rho\lambda} + \partial_{\mu} \bar{v}^{\mu}. \tag{93}
\]

We expand \(\omega^2\) as

\[
\omega^2 = \sum_{l>0} \omega^{(l)}_2, \tag{94}
\]

where \(N\omega^2 = \sum_{l>0} l\omega^{(l)}_2\), such that

\[
N\omega^2 = \sum_{l>0} l\omega^{(l)}_2. \tag{95}
\]

Comparing (93) with (95), we reach the conclusion that the decomposition (94) induces a similar decomposition with respect to \(\partial_{\mu} \psi_{\nu}\), \(U^{\mu\alpha\nu\beta}\), and \(D^{\mu\nu\rho\lambda}\), i.e.

\[
F^{\mu\nu} = \sum_{l>0} \frac{1}{l} F_{(l-1)}^{\mu\nu}, \quad U^{\mu\alpha\nu\beta} = \sum_{l>0} \frac{1}{l} U_{(l-1)}^{\mu\alpha\nu\beta}, \quad D^{\mu\nu\rho\lambda} = \sum_{l>0} \frac{1}{l} D_{(l-1)}^{\mu\nu\rho\lambda}. \tag{96}
\]

Substituting (96) in (93) and comparing the resulting expression with (95), we obtain that

\[
\omega^{(l)}_2 = \frac{1}{2l} \left( F_{(l-1)}^{\mu\nu} \partial_{\mu} \psi_{\nu} - K_{\mu\alpha\nu\beta} U_{(l-1)}^{\mu\alpha\nu\beta} + \frac{1}{2} F_{\mu\nu\rho\lambda} D_{(l-1)}^{\mu\nu\rho\lambda} \right) + \partial_{\mu} \bar{v}^{\mu}. \tag{97}
\]

Introducing (97) in (94), we arrive at

\[
\omega^2 = \frac{1}{2} \hat{F}^{\mu\nu} \partial_{\mu} \psi_{\nu} - \frac{1}{2} K_{\mu\alpha\nu\beta} \hat{U}^{\mu\alpha\nu\beta} + \frac{1}{4} F_{\mu\nu\rho\lambda} \hat{D}^{\mu\nu\rho\lambda} + \partial_{\mu} \hat{v}^{\mu}, \tag{98}
\]

where

\[
\hat{F}^{\mu\nu} = \sum_{l>0} \frac{1}{l} \hat{F}_{(l-1)}^{\mu\nu}, \quad \hat{U}^{\mu\alpha\nu\beta} = \sum_{l>0} \frac{1}{l} \hat{U}_{(l-1)}^{\mu\alpha\nu\beta}, \quad \hat{D}^{\mu\nu\rho\lambda} = \sum_{l>0} \frac{1}{l} \hat{D}_{(l-1)}^{\mu\nu\rho\lambda}. \tag{99}
\]

We will show that the second term from the right-hand side of (98) does not comply with the derivative order assumption. Indeed, the tensor \(\hat{U}^{\mu\alpha\nu\beta}\) effectively depends on both \(A_{\mu\nu}\) and \(\psi_{\mu}\) (and possibly on their derivatives) in order to describe simultaneous interactions among all the fields. Due to the presence of the linearized Riemann tensor \(K_{\mu\alpha\nu\beta}\), the term from \(\omega^2\) containing \(\hat{U}^{\mu\alpha\nu\beta}\) will contribute to the field equations for the spin-2 field with quantities involving at least two spacetime derivatives acting on \(\psi_{\mu}\), which breaks the derivative order assumption. Consequently, we must set

\[
\hat{U}^{\mu\alpha\nu\beta} = 0. \tag{100}
\]

The last result implies (via the second formula in (99)) \(U_{(l-1)}^{\mu\alpha\nu\beta} = 0\) for all \(l > 0\), which further yields (due to the second relation from (96)) that \(U^{\mu\alpha\nu\beta} = 0\),
such that the second result from (89) finally leads to $D^\mu\nu = 0$. Recalling the second notation from (86) this is the same with

$$\frac{\delta \omega_2}{\delta h_{\mu\nu}} = 0,$$

meaning that $\omega_2$ is enabled to depend on the spin-2 field only through a (trivial) full divergence, which brings no contribution to the Lagrangian action of the interacting model. We conclude that there is no nontrivial $\omega_2$ that mixes all the three field sectors, so we can take

$$\omega_2 = 0$$

(101)

without loss of generality.

Inserting (78), (85), and (101) in (73) we find that

$$\tilde{a}^{\text{int}}_0 = 0,$$

(102)

such that results (72) and (102) substituted in (69) provide

$$a^{\text{int}}_0 = 0$$

and hence the first-order deformation of the solution to the master equation that mixes all the fields cannot reduce nontrivially to its component of antighost number zero.

Since we have exhausted all the possibilities of constructing a nontrivial $a^{\text{int}}$ as in (15) ($I = 4, 3, 2, 1, 0$), we conclude that the general solution to (12) that complies with all our working hypotheses is

$$S^{\text{int}}_1 = 0.$$  (103)

In conclusion, the full expression of the first-order deformation of the solution to the master equation associated with the free theory described by (1) decomposes as

$$S_1 = S^h_1 + S^A_1 + S^\psi_1 + S^{h-A}_1 + S^{A-\psi}_1 + S^{h-\psi}_1,$$

(104)

where all the terms from the right-hand side of (104) have been reported in Refs. [1]–[3].

### 3 Second-order deformation

The scope of this section is to investigate the consistency of the first-order deformation and hence to determine the expression of the second-order deformation of the solution to the master equation. In view of this, we start from the equation

$$(S_1, S_1) + 2s S_2 = 0,$$

(105)
where \( S_1 \) reads as in (104). By direct computation we find that the antibracket \((S_1, S_1)\) naturally decomposes into
\[
(S_1, S_1) = (S_1, S_1)^h + (S_1, S_1)^A + (S_1, S_1)^\psi + (S_1, S_1)^{h-A} + (S_1, S_1)^{A-\psi} + (S_1, S_1)^{h-\psi} + (S_1, S_1)^{int},
\]
where \((S_1, S_1)^{sector(s)}\) is the projection of \((S_1, S_1)\) on the respectively mentioned sectors(s). Clearly, (106) induces a similar decomposition with respect to the second-order deformation
\[
S_2 = S_2^h + S_2^A + S_2^\psi + S_2^{h-A} + S_2^{A-\psi} + S_2^{h-\psi} + S_2^{int}.
\]
The projection of (105) on the various sectors makes (105) equivalent to the tower of equations
\[
\begin{align*}
(S_1, S_1)^h + 2s S_2^h &= 0, \\
(S_1, S_1)^A + 2s S_2^A &= 0, \\
(S_1, S_1)^\psi + 2s S_2^\psi &= 0, \\
(S_1, S_1)^{h-A} + 2s S_2^{h-A} &= 0, \\
(S_1, S_1)^{A-\psi} + 2s S_2^{A-\psi} &= 0, \\
(S_1, S_1)^{h-\psi} + 2s S_2^{h-\psi} &= 0, \\
(S_1, S_1)^{int} + 2s S_2^{int} &= 0.
\end{align*}
\]
If we denote by \( \Delta^{sector(s)} \) and \( b^{sector(s)} \) the nonintegrated densities of the functionals \((S_1, S_1)^{sector(s)}\) and \( S_2^{sector(s)} \) respectively, then Eqs. (108)–(114) take the local form
\[
\begin{align*}
\Delta^h &= -2sb^h + \partial^\mu n^h_\mu, \\
\Delta^A &= -2sb^A + \partial^\mu n^A_\mu, \\
\Delta^\psi &= -2sb^\psi + \partial^\mu n^\psi_\mu, \\
\Delta^{h-A} &= -2sb^{h-A} + \partial^\mu n^{h-A}_\mu, \\
\Delta^{A-\psi} &= -2sb^{A-\psi} + \partial^\mu n^{A-\psi}_\mu, \\
\Delta^{h-\psi} &= -2sb^{h-\psi} + \partial^\mu n^{h-\psi}_\mu, \\
\Delta^{int} &= -2sb^{int} + \partial^\mu n^{int}_\mu, 
\end{align*}
\]
with
\[
\text{gh}\left(\Delta^{sector(s)}\right) = 1, \quad \text{gh}\left(b^{sector(s)}\right) = 0, \quad \text{gh}\left(n^{sector(s)}_\mu\right) = 1,
\]
for some local currents \( n^{sector(s)}_\mu \). Recalling decomposition (104) of the first-order deformation as well as the concrete expressions of its components, we find that
\[
(S_1, S_1)^h = (S_1^h, S_1^h).
\]
By direct computation we deduce

\[ S_h^2 = \int d^{11}x \left\{ -\frac{1}{2} \left( h^2 - 2h_{\mu\nu}h^{\mu\nu} \right) - \frac{1}{4} h^{\mu\nu\rho\sigma} \left[ h_{\mu}^\rho \partial_{\nu} \left( h_{\rho\lambda\eta}^\lambda \right) + \frac{1}{2} h_{\rho\lambda} \left( \partial^\lambda h_{\mu\nu} \right) \eta^\rho + \frac{3}{2} \left( \partial_{(\mu} h_{\nu)}^\lambda - \partial_\lambda h_{\mu\nu} \right) h_{\rho}^\lambda \eta^\rho \right] + \frac{1}{8} h_{\rho\lambda} \left( h_{\mu}^\rho \partial_{(\mu} h_{\nu)}^\lambda \eta^\rho - h_{\rho\sigma} \partial_{(\mu} h_{\nu)}^\lambda \eta^\rho - \eta^\rho \partial_{\sigma} \left( h_{\rho\lambda\eta}^\lambda \right) \right) \right\}, \tag{123} \]

where \( \mathcal{L}_E^{EH} \) is the quartic vertex of the Einstein-Hilbert Lagrangian. Meanwhile, it results that

\[ (S_1, S_1)^A = 0, \]

so Eq. (109) reduces to

\[ sS_2^A = 0 \tag{124} \]

and has been solved in Ref. [1]. Namely, we have argued that the solution to (124) can be taken as trivial modulo a redefinition of the constant \( q \) that parameterizes \( S_1^A \)

\[ S_2^A = 0. \tag{125} \]

Eq. (110) has been tackled in Section 6 from Ref. [3], where we proved that the parameters \( \bar{k}, \tilde{k}, m, \) and \( \Lambda \) are restricted to satisfy the relations

\[ \bar{k}^2 + \tilde{k}^2 = 0, \quad 180m^2 - \tilde{k}\Lambda = 0, \tag{126} \]

which then grant the nonintegrated density of the second-order deformation in the Rarita-Schwinger sector to be expressed as the sum between the pieces listed in formulas (71), (72), and (74) from Ref. [3]. Eq. (111) has been worked out in detail in Ref. [1], where it was shown that the constant \( k \) (parameterizing the cross-couplings between the spin-2 field and the 3-form) is subject to the relation

\[ k(k + 1) = 0. \tag{127} \]

Taking the nontrivial solution of (127) \( k = -1 \), it follows that the second-order deformation in the mixed sector graviton-3-form is described by formula (117) din Ref. [1].

Let us investigate now Eq. (112). It is easy to see that

\[ (S_1, S_1)^{A-\psi} = \left( S_{1}^{A-\psi}, S_{1}^{A-\psi} \right) + 2 \left( S_{1}^{A}, S_{1}^{A-\psi} \right) + 2 \left( S_{1}^{A}, S_{1}^{A-\psi} \right) + 2 \left( S_{1}^{h-A}, S_{1}^{h-\psi} \right). \tag{128} \]

Recalling that \( \Delta^{A-\psi} \) denotes the nonintegrated density of \( (S_1, S_1)^{A-\psi} \) and performing the necessary computations in the right-hand side of (128), we get that \( \Delta^{A-\psi} \) decomposes into

\[ \Delta^{A-\psi} = \sum_{I=0}^{4} \Delta I^{A-\psi}, \quad \text{arg} \left( \Delta I^{A-\psi} \right) = I, \quad I = \{0, 4\}, \tag{129} \]
with
\[ \Delta^A_{4 - \psi} = \gamma \left( \frac{ikk}{8} C^* C \tilde{\xi}_\mu \xi \right) + \partial_\mu \tau^A_{4 - \psi} \mu, \]
(130)
\[ \Delta^A_{3 - \psi} = \delta \left( \frac{ikk}{8} C^* C \tilde{\xi}_\mu \xi \right) + \gamma \left[ -\frac{ikk}{8} C^* \left( C_{\mu \nu} \tilde{\xi}_\gamma \nu + C^\nu \xi (\mu \psi) \right) \right] + \partial_\mu \tau^A_{3 - \psi} \mu, \]
(131)
\[ \Delta^A_{2 - \psi} = \delta \left[ -\frac{ikk}{8} C^* \left( C_{\mu \nu} \tilde{\xi}_\nu \xi + 2 C_{\mu \nu} \xi (\nu \psi) \right) \right] + \gamma \left[ \frac{ikk}{8} C^* \left( A_{\mu \nu \rho} \tilde{\xi}^\rho \xi \right. \right.
\left. \left. - 2 C_{\mu \nu} \tilde{\xi} (\nu \psi) \right) \right] + \partial_\mu \tau^A_{2 - \psi} \mu, \]
(132)
\[ \Delta^A_{1 - \psi} = \delta \left[ \frac{ikk}{8} C^* \left( A_{\mu \nu \rho} \tilde{\xi}_\gamma \nu \xi - 2 C_{\mu \nu} \tilde{\xi}(\nu \psi) \right) \right] + \gamma \left[ \left( \frac{3ikk}{8} A^* \mu \nu \rho \tilde{\xi}_\xi \nu \lambda \right) \right.
\left. + 4i \left( \frac{k^2}{32} \right) A^* \mu \nu \rho F_{\mu \nu \rho \lambda} \tilde{\xi} \xi \lambda \right] + \partial_\mu \tau^A_{1 - \psi} \mu, \]
(133)

and
\[ \Delta^A_{0 - \psi} = \delta \left( -\frac{3ikk}{8} A^* \mu \nu \rho A_{\mu \nu \rho} \tilde{\xi}_\gamma \nu \xi \right) + \gamma \left( \frac{3ikk}{8} A^* \mu \nu \rho A_{\mu \nu \rho} \tilde{\xi}_\xi \nu \lambda \right) + \right.
\left. \frac{4i}{3} \left( \frac{k^2}{32} \right) F^* \mu \nu \rho F_{\mu \nu \rho} \beta \left( \tilde{\xi}_\mu \nu \psi \beta - \frac{1}{8} \sigma_{\alpha \beta} \tilde{\xi} \xi \psi \lambda \right) \right.
\left. - 18k \left( q + \frac{k}{3 (12)^2} \right) \varepsilon^{\mu \gamma \mu \nu} A_{\mu \nu \rho} \tilde{\xi}_\gamma \nu \psi \rho F_{\mu \nu \rho} \right. \right.
\left. \left. \varepsilon^{\mu \gamma \mu \nu} A_{\mu \nu \rho} \tilde{\xi}_\gamma \nu \psi \rho F_{\mu \nu \rho} \right) \right. \right.
\left. \left. + \text{im} k F^* \mu \nu \rho \lambda \tilde{\xi}_\gamma \mu \nu \rho \sigma \psi \sigma + \partial_\mu \tau^A_{0 - \psi} \mu. \right) \]
(134)

Because \((S_1, S_1)^{A - \psi}\) contains terms of maximum antighost number equal to four, we can assume (without loss of generality) that \(b_{A - \psi}^{0 - \psi}\) stops at antighost number five
\[ b_{A - \psi} = \sum_{I = 0}^{5} b_I^{A - \psi}, \quad \text{agh} \left( b_I^{A - \psi} \right) = I, \quad I = 0, 5, \]
(135)
\[ n_{A - \psi}^{\mu} = \sum_{I = 0}^{5} n_I^{\mu}, \quad \text{agh} \left( n_I^{A - \psi} \right) = I, \quad I = 0, 5. \]
(136)

By projecting Eq. (119) on the various (decreasing) values of the antighost number, we infer the following tower of equations
\[ \gamma b_{5}^{A - \psi} = \partial_\mu \left( \frac{1}{2} n_5^{A - \psi} \mu \right), \]
(137)
\[ \Delta^{\Lambda - \psi} = -2 \left( \delta b_i^{\Lambda - \psi} + \gamma b_4^{\Lambda - \psi} \right) + \partial_\mu n_i^{\Lambda - \psi \mu}, \quad I = 0, 4. \] (138)

Eq. (137) can always be replaced with

\[ \gamma b_5^{\Lambda - \psi} = 0. \] (139)

If we compare (130) with (138) for \( I = 4 \), then we find that \( b_5^{h - A} \) is restricted to fulfill the equation

\[ \delta b_5^{\Lambda - \psi} + \gamma b_4^{\Lambda - \psi} = \partial_\mu n_4^{\Lambda - \psi \mu}, \] (140)

where

\[ b_4^{\Lambda - \psi} = -\frac{ik\tilde{k}}{16} C^\ast C^n \tilde{\xi}^\gamma \mu \xi + \tilde{b}_4^{\Lambda - \psi}. \] (141)

The solution to (139) reads as

\[ b_5^{\Lambda - \psi} = \beta_5^{\Lambda - \psi} \left( [F_{\mu\nu\rho\lambda}] \cdot \left[ [\partial_\mu \psi_\eta] \right] \cdot \left[ \chi_\ast \right] \right) \omega_5 (C, \xi). \] (142)

Substituting the above form of \( b_5^{\Lambda - \psi} \) into (140), we infer that a necessary condition for (140) to possess solutions is that \( \beta_5^{\Lambda - \psi} \) belongs to \( H_5 \). Since for the model under consideration we know that \( H_5 = 0 \) and \( H_5^{\text{inv}} = 0 \), it follows that we can take

\[ b_5^{\Lambda - \psi} = 0, \] (143)

such that Eq. (140) reduces to \( \gamma b_4^{\Lambda - \psi} = \partial_\mu n_4^{\Lambda - \psi \mu} \), that can always be replaced (as it stands in a strictly positive value of the antighost number) with \( \gamma b_4^{\Lambda - \psi} = 0 \).

The last equation was investigated in Ref. [2] and was shown to possess only the trivial solution

\[ b_4^{\Lambda - \psi} = 0. \] (144)

Due to (141) and (144), we observe that relations (130) agree with Eq. (138) for \( I = 4, I = 3 \) and \( I = 2 \) respectively. On the contrary, \( \Delta^{\Lambda - \psi} \) given in (133) cannot be written like in (138) for \( I = 1 \) unless

\[ \chi^{\Lambda - \psi} = 4i \left( \tilde{k}^2 - \frac{k\tilde{k}}{32} \right) A^{\mu\nu\rho \lambda} F_{\mu\nu\rho\lambda} \tilde{\xi}^\gamma \lambda \xi, \] (145)

can be expressed like

\[ \chi^{\Lambda - \psi} = \delta \varphi^{\Lambda - \psi} + \gamma \omega^{\Lambda - \psi} + \partial_\mu l^{\Lambda - \psi \mu}. \] (146)

Assume that (146) holds. Then, by acting with \( \delta \) on it from the left, we infer that

\[ \delta \chi^{\Lambda - \psi} = \gamma \left( -\delta \omega^{\Lambda - \psi} \right) + \partial_\mu \left( \delta l^{\Lambda - \psi \mu} \right). \] (147)

On the other hand, using the concrete expression of \( \chi \), we have that

\[ \delta \chi^{\Lambda - \psi} = 4i \left( \tilde{k}^2 - \frac{k\tilde{k}}{32} \right) \left[ \gamma \left( -T^{\mu\nu} \tilde{\xi}^\gamma \left( \mu \psi_\nu \right) \right) + \partial_\mu \left( T^{\mu\nu} \tilde{\xi} \gamma_\nu \xi \right) \right], \] (148)
where
\[ T^{\mu\nu} = \frac{1}{3!} F^{\rho\lambda\sigma\mu} F_{\rho\lambda\sigma
u} - \frac{\sigma^{\mu\nu}}{2 \cdot 4!} F^{\rho\lambda\sigma\epsilon} F_{\rho\lambda\sigma\epsilon} \] (149)
is the stress-energy tensor of the Abelian three-form gauge field. The right-hand side of (148) can be written like in the right-hand side of (147) if the following conditions are simultaneously satisfied
\[ \delta \omega^{A-\psi} = -4i \left( \tilde{k}^2 - \frac{k \tilde{k}}{32} \right) T^{\mu\nu} \xi_{(\mu} \psi_{\nu)}, \] (150)
\[ \delta I^{A-\psi \mu} = 4i \left( \tilde{k}^2 - \frac{k \tilde{k}}{32} \right) T^{\mu\nu} \xi_{\nu} \xi. \] (151)

Since none of the quantities \( \psi_{\mu} \) or \( \xi \) are \( \delta \)-exact, we deduce that the last relations hold if stress-energy tensor of the Abelian three-form gauge field is \( \delta \)-exact
\[ T^{\mu\nu} = \delta \Omega^{\mu\nu}. \] (152)

We have shown in Section 4.3 of Ref. [1] that relation (152) is not valid, and thus neither are (150)–(151). As a consequence, \( \chi^{A-\psi} \) must vanish, which further implies
\[ \tilde{k}^2 - \frac{k \tilde{k}}{32} = 0. \] (153)

Inserting (153) in (133), we deduce that
\[ \Delta_0^{A-\psi} = \delta \left( -\frac{3i k \tilde{k}}{8} A^{*\mu\nu\rho} A_{\mu \nu} \xi_{(\rho} \psi_{\lambda)} \right) + \]
\[ -18 \tilde{k} \left( q + \frac{k}{3 \cdot (12)^2} \right) \varepsilon_{\mu_1 ... \mu_{11}} \xi_{\mu_1 \mu_2 \psi_{\mu_3}} F_{\mu_4 ... \mu_7} \xi_{\mu_8 ... \mu_{11}} \]
\[ + i m \tilde{k} F^{\mu\nu\rho\lambda} \xi_{(\mu\rho\lambda\sigma)} \psi^{\sigma} + \partial_{\mu} \tilde{\tau}_0^{A-\psi}. \] (154)

We remark that (154) satisfies Eq. (138) for \( I = 0 \) if the quantity
\[ \hat{\chi}^{A-\psi} = \text{im} \tilde{k} F^{\mu\nu\rho\lambda} \xi_{(\mu\rho\lambda\sigma)} \psi^{\sigma} \]
\[ -18 \tilde{k} \left( q + \frac{k}{3 \cdot (12)^2} \right) \varepsilon_{\mu_1 ... \mu_{11}} \xi_{\mu_1 \mu_2 \psi_{\mu_3}} F_{\mu_4 ... \mu_7} \xi_{\mu_8 ... \mu_{11}}, \] (155)
can be written as
\[ \hat{\chi}^{A-\psi} = \delta \hat{\varphi}^{A-\psi} + \gamma \hat{\omega}^{A-\psi} + \partial_{\mu} \hat{l}_{\mu}^{A-\psi}. \] (156)

Let us assume that (156) takes place. Then, we apply \( \gamma \) on (156) and find that
\[ \gamma \hat{\chi}^{A-\psi} = \delta \left( -\gamma \hat{\varphi}^{A-\psi} \right) + \partial_{\mu} \left( \gamma \hat{l}_{\mu}^{A-\psi} \right) \] (157)

Direct computation based on (155) provides
\[ \gamma \hat{\chi}^{A-\psi} = \partial_{\mu} \left[ \text{im} \tilde{k} \frac{1}{2} F^{\nu\rho\lambda\sigma} \xi_{(\nu\rho\lambda\sigma)} \xi \right] 21 \]
On the one hand, Eq. (157) requires that the current appearing in its right-hand side is trivial in $H(\gamma)$. On the other hand, the current involved in the right-hand side of (158) is clearly a nontrivial element of $H(\gamma)$. This contradiction emphasizes that relation (156) cannot be valid, and therefore we must set $\hat{\chi}^A - \psi = 0$, which leads to the conditions

$$m \tilde{k} = 0, \quad \tilde{k} \left( q + \frac{\tilde{k}}{3 \cdot (12)^3} \right) = 0.$$  \hspace{1cm} (159)

Inserting now (153) and (159) in (130)–(134), we are able to identify the components of the second-order deformation in the mixed gravitini-3-form sector as

$$b_4^{A - \psi} = -\frac{i k \tilde{k}}{16} C^{\mu} \bar{C}_\mu \bar{\xi} \bar{\xi},$$  \hspace{1.5cm} (160)

$$b_3^{A - \psi} = \frac{ik \tilde{k}}{16} C^{\mu} \left( C_{\mu \nu} \bar{\xi}^\nu \xi + C^{\nu} \gamma_{(\mu} \psi_{\nu)} \right),$$  \hspace{1.5cm} (161)

$$b_2^{A - \psi} = -\frac{ik \tilde{k}}{16} C^{\mu \nu} \left( A_{\mu \nu \rho} \bar{\xi}^\rho \xi - 2 C_{\rho}^\rho \bar{\xi} \gamma_{(\nu} \psi_{\rho)} \right),$$  \hspace{1.5cm} (162)

$$b_1^{A - \psi} = \frac{3i k \tilde{k}}{16} A^{\mu \nu \rho} A_{\mu \nu}^\lambda \bar{\xi} \gamma_{(\rho} \psi_{\lambda)},$$  \hspace{1.5cm} (163)

and

$$b_0^{A - \psi} = 0.$$  \hspace{1.5cm} (164)

Formulas (160)–(164) allow us to write

$$S_2^{A - \psi} = \int d^{11} x \left( b_4^{A - \psi} + b_3^{A - \psi} + b_2^{A - \psi} + b_1^{A - \psi} + b_0^{A - \psi} \right).$$  \hspace{1.5cm} (165)

In the following step we approach Eq. (113). From (114), we determine the first term from the left-hand side of Eq. (113) under the form

$$(S_1, S_1)^{h - \psi} = \left( S_1^{h - \psi}, S_1^{h - \psi} \right) + 2 \left( S_1^{h - \psi}, S_1^{h - \psi} \right) + 2 \left( S_1^{h - \psi}, S_1^{h - \psi} \right),$$  \hspace{1.5cm} (166)

where $\Delta^{h - \psi}$ from the left-hand side of (120) (the local form of (113) decomposes as

$$\Delta^{h - \psi} = \sum_{I=0}^{2} \Delta^{h - \psi}_I, \quad \text{agh} \left( \Delta^{h - \psi}_I \right) = I, \quad I = 0, 2,$$  \hspace{1.5cm} (167)

with

$$\Delta^{h - \psi}_2 = \gamma \left[ \frac{1}{8} \left( \bar{\xi}_\gamma^\mu \bar{\xi}_\delta^\nu \bar{\xi}_\lambda^\rho \bar{\psi}_\mu^\nu \bar{\psi}_\rho^\gamma \eta + \bar{\xi}_\gamma^\mu \bar{\xi}_\lambda^\rho \bar{\psi}_\mu^\nu \bar{\psi}_\rho^\gamma \eta \right) - \bar{\eta}^\mu \bar{\eta}^\nu \bar{\eta}^\rho \bar{\psi}_\mu^\nu \bar{\psi}_\rho^\gamma \eta \right].$$
\[
-\frac{1}{8} k (k - 1) \left[ \xi^* \gamma_{\mu \nu} \xi \left( \partial^\mu \eta^{\alpha} \right) \partial^\nu \eta^{\beta} \sigma_{\alpha \beta} + \frac{i}{2} \eta^{* \mu} \xi^\nu \partial_{[\mu} \eta_{\nu]} \right] + \partial_\mu \frac{r_k}{2} \eta_{\mu}^\nu (168)
\]

\[
\Delta_{1}^{h - \psi} = \delta \left[ -\frac{k}{8} \left( \xi^* \gamma_{\mu \nu} \xi h^\mu_{\lambda} \partial^\nu \eta^{\lambda} + \xi^* \gamma_{\mu \nu} \xi \partial_{\mu} h^\nu_{\lambda} - \eta^{* \mu} \eta^{\nu} \bar{\psi}_{[\mu} \gamma_{\nu]} \xi \right) 
\right. 
+ \bar{k} \xi^* \left( \partial_{[\mu} \bar{\psi}_{\nu]} \right) \eta^{\mu} \eta^{\nu} + 2i m \bar{k} \xi^* \gamma_{\mu} \xi \eta^{\mu} \bigg] + \gamma \left[ -\frac{k}{8} \bar{\psi}^{* \mu} \gamma_{\nu \rho} \bar{\psi}_{\mu} \left( h^\mu_{\lambda} \partial^\nu \eta^{\lambda} \right) 
\right. 
+ \partial^\mu h^\rho_{\lambda} \eta^{\lambda} \right) 
-\frac{k}{4} \bar{\psi}^{* \mu} \gamma_{\nu \rho} \bar{\psi}_{\mu} \left( \partial^\nu h^\rho_{\lambda} + \frac{1}{2} \partial_{\mu} h^\nu_{\lambda} \right) h^\lambda_{\lambda} \bar{\psi}_{\mu} \left( \gamma_{\rho} \xi h^\nu_{\nu} - 2 \gamma_{\nu \rho} \eta^{\nu} \right) 
\left. \right) 
+ \frac{k^2}{4} \bar{\psi}^{* \mu} \gamma_{\nu \rho} \bar{\psi}_{\mu} \left( \partial^\nu h^\rho_{\lambda} \right) \eta^{\lambda} - \frac{k^2}{4} \bar{\psi}^{* \mu} \left( \partial^\nu \eta^{\lambda} \right) \eta^{\lambda} - \frac{1}{2} \left( \partial^\nu \eta^{\lambda} \right) \partial_{\nu} \eta^{\lambda} \sigma_{\alpha \beta} 
\left. \right) 
-\frac{1}{8} \bar{\psi}^{* \mu \nu} \left( \xi^\gamma \xi \partial_{\mu} \eta_{\nu} - 2 \bar{\psi}_{\mu} \gamma_{\rho} \xi \partial_{\nu} \eta_{\rho} - 4 \eta^{\nu} \delta_{\gamma \nu} \partial_{\mu} \psi_{\rho} \bigg) \right] + \partial_{\mu} r_k^{h - \psi} (169)
\]

and

\[
\Delta_{0}^{h - \psi} = \delta \left[ -\frac{k}{8} \bar{\psi}^{* \mu} \gamma_{\nu \rho} \bar{\psi}_{\mu} \left( h^\mu_{\lambda} \partial^\nu \eta^{\lambda} + \partial^\nu h^\rho_{\lambda} \right) 
\right. 
-\frac{k}{4} \bar{\psi}^{* \mu} \gamma_{\nu \rho} \bar{\psi}_{\mu} \left( \partial^\nu h^\rho_{\lambda} + \frac{1}{2} \partial_{\mu} h^\nu_{\lambda} \right) h^\lambda_{\lambda} \bar{\psi}_{\mu} \left( \gamma_{\rho} \xi h^\nu_{\nu} - 2 \gamma_{\nu \rho} \eta^{\nu} \right) 
\left. \right) 
-\frac{k^2}{4} \bar{\psi}^{* \mu} \gamma_{\nu \rho} \bar{\psi}_{\mu} \left( \partial^\nu \eta^{\lambda} \right) \eta^{\lambda} \partial_{\nu} \eta^{\lambda} - \frac{k^2}{4} \bar{\psi}^{* \mu} \gamma_{\nu \rho} \bar{\psi}_{\mu} \left( \partial^\nu \eta^{\lambda} \right) \eta^{\lambda} \partial_{\nu} \eta^{\lambda} 
\left. \right) 
-\frac{1}{32} \bar{k} \left[ -2 \left( \partial^\mu \bar{\psi}_{\nu} \right) \gamma_{\mu \rho} \left( \psi_{\rho} h_{\lambda \sigma} \partial_{\sigma} \eta^{\lambda} h^{\nu \lambda} \right) - 8 \bar{\psi}_{\rho} \gamma_{\mu} \psi_{\nu} \left( h^\mu_{\lambda} \partial_{\lambda} \eta_{\rho} \right) 
\right. 
-2 k \bar{\psi}^{* \mu} \gamma_{\nu \rho} \psi_{\mu} h^\rho_{\lambda} \partial_{\nu} \eta^{\lambda} + 4 \left( \partial^\nu \bar{\psi}_{\mu} \right) \gamma_{\nu \rho} \psi_{\mu} \partial_{\rho} \eta^{\nu} \left( h^\rho_{\lambda} \partial_{\lambda} \eta^{\mu} \right) 
\left. \right) 
-\frac{1}{32} \bar{k} \left[ 8 \bar{\psi}_{\mu} \gamma_{\nu \rho} \psi_{\mu} h^\rho_{\lambda} \partial_{\nu} \eta^{\lambda} + 4 \bar{\psi}_{\mu} \gamma_{\nu \rho} \psi_{\mu} h^\rho_{\lambda} \partial_{\nu} \eta^{\lambda} 
\right. 
+ \frac{1}{32} \bar{k} \left( \partial^\mu \bar{\psi}_{\nu} \right) \gamma_{\mu \rho} \psi_{\mu} h^\rho_{\lambda} \partial_{\nu} \eta^{\nu} - 8 \bar{\psi}_{\rho} \gamma_{\mu} \psi_{\nu} \left( h^\mu_{\lambda} \partial_{\lambda} \eta^{\rho} \right) 
\left. \right) 
+ \frac{1}{32} \bar{k} \left( \partial^\mu \bar{\psi}_{\nu} \right) \gamma_{\mu \rho} \psi_{\mu} h^\rho_{\lambda} \partial_{\nu} \eta^{\nu} - 8 \bar{\psi}_{\rho} \gamma_{\mu} \psi_{\nu} \left( h^\mu_{\lambda} \partial_{\lambda} \eta^{\rho} \right) 
\left. \right) 
+ \frac{1}{32} \bar{k} \left( \partial^\mu \bar{\psi}_{\nu} \right) \gamma_{\mu \rho} \psi_{\mu} h^\rho_{\lambda} \partial_{\nu} \eta^{\nu} - 8 \bar{\psi}_{\rho} \gamma_{\mu} \psi_{\nu} \left( h^\mu_{\lambda} \partial_{\lambda} \eta^{\rho} \right) 
\left. \right) 
+ \frac{1}{32} \bar{k} \left( \partial^\mu \bar{\psi}_{\nu} \right) \gamma_{\mu \rho} \psi_{\mu} h^\rho_{\lambda} \partial_{\nu} \eta^{\nu} - 8 \bar{\psi}_{\rho} \gamma_{\mu} \psi_{\nu} \left( h^\mu_{\lambda} \partial_{\lambda} \eta^{\rho} \right) 
\right] 
\]
Introducing (171) in (168)–(170) and recalling Eq. (120), we identify the various pieces of the nonintegrated density of the second-order deformation in the mixed and respectively

\[ -4\left(\partial^{[\mu}\bar{\psi}^{\rho]}\right)\gamma_{\mu\nu}^{\rho}\left(\psi^{\lambda}\eta^{\sigma}\partial_{\lambda}\bar{h}_{\sigma}^{\lambda} - \psi^{\lambda}\eta^{\sigma}\partial_{\lambda}\bar{h}_{\sigma}^{\lambda} - \frac{1}{2}\psi^{\lambda}\eta^{\sigma}\partial_{\lambda}\eta_{\sigma} + \frac{1}{2}\xi_{\rho}\eta^{\sigma}\partial_{\lambda}\bar{h}_{\sigma}^{\lambda}\right) \]

\[ +4\left(\partial^{[\mu}\bar{\psi}^{\rho]}\right)\gamma_{\nu\rho}^{\lambda}\left(\xi_{\nu\rho}^{\lambda}\partial_{\lambda}\bar{h}_{\sigma}^{\lambda} + \psi^{\lambda}\omega^{\rho}\partial_{\lambda}\bar{h}_{\sigma}^{\lambda} + 2\psi^{\lambda}\eta^{\sigma}\partial_{\lambda}\bar{h}_{\sigma}^{\lambda}\right) \]

\[ -4\bar{\psi}_{\mu}^{\nu}\eta^{\sigma}\left(\partial_{\nu}\eta_{\sigma}\partial_{\lambda}\bar{h}_{\sigma}^{\lambda} - \frac{1}{2}\eta_{\nu}\eta_{\sigma}\partial_{\lambda}\bar{h}_{\sigma}^{\lambda}\right) + 2\eta_{\lambda}^{\eta}\left(\partial^{\rho}\partial_{\lambda}^{\rho}h_{\rho}^{\lambda} - \partial^{\rho}\partial_{\lambda}^{\rho}h_{\rho}^{\lambda}\right) \]

\[ +4\bar{\psi}_{\mu}^{\nu}\eta^{\sigma}\frac{1}{2}\psi_{\nu}^{\rho}\partial_{\lambda}\bar{h}_{\sigma}^{\lambda} + \frac{2}{2}\delta_{\alpha\beta}\partial_{\lambda}^{\alpha}\partial_{\lambda}^{\beta} + \psi_{\nu}^{\rho}\partial_{\lambda}^{\alpha}\partial_{\lambda}^{\beta} \]

\[ +2\eta_{\nu}\left(\partial^{\rho}\partial_{\nu}^{\rho}h_{\rho}^{\lambda} - \partial^{\rho}\partial_{\nu}^{\rho}h_{\rho}^{\lambda}\right) \]

\[ +4\bar{\psi}_{\mu}^{\nu}\eta^{\sigma}\left(\partial_{\nu}\partial_{\alpha}\bar{h}_{\alpha}^{\lambda} - \partial_{\nu}\partial_{\alpha}\bar{h}_{\alpha}^{\lambda}\right) = -2\left(\partial_{\nu}\partial_{\nu}\bar{h}_{\alpha}^{\lambda} - \partial_{\nu}\partial_{\nu}\bar{h}_{\alpha}^{\lambda}\right) \]

\[ +4\bar{\psi}_{\mu}^{\nu}\eta^{\sigma}\left(\partial_{\nu}\partial_{\nu}\bar{h}_{\alpha}^{\lambda} - \partial_{\nu}\partial_{\nu}\bar{h}_{\alpha}^{\lambda}\right) \]

\[ +4\bar{\psi}_{\mu}^{\nu}\eta^{\sigma}\left(\partial_{\nu}\partial_{\nu}\bar{h}_{\alpha}^{\lambda} - \partial_{\nu}\partial_{\nu}\bar{h}_{\alpha}^{\lambda}\right) \]

Pursuing a reasoning similar to the previously investigated equation we conclude that parameter \( k \) is subject to the algebraic equation

\[ \tilde{k} (\tilde{k} - 1) = 0. \] (171)

Introducing (171) in (168)–(170) and recalling Eq. (120), we identify the various pieces of the nonintegrated density of the second-order deformation in the mixed graviton-gravitin sector as

\[ b_{\mu}^{\nu\psi} = \frac{k}{16}\bar{\psi} \xi_{\rho}^{\mu} \left( h_{\lambda}^{\rho} \partial^{[\mu} h_{\nu]}^{\rho} + \eta_{\sigma}^{\lambda} \partial_{\lambda} h_{\sigma}^{\rho} \right) - \frac{i\bar{\psi} \xi_{\rho}^{\mu} \bar{\psi}_{\rho} \xi_{\sigma}^{\mu} \eta_{\sigma}^{\mu} - \frac{k}{2} \bar{\psi} \xi_{\rho}^{\mu} \bar{\psi}_{\rho} \xi_{\sigma}^{\mu} \eta_{\sigma}^{\mu}, \] (172)

\[ h_{\mu}^{\nu\psi} = \frac{k}{16} \bar{\psi} \xi_{\rho}^{\mu} \left( h_{\lambda}^{\rho} \partial^{[\mu} h_{\nu]}^{\rho} + \eta_{\sigma}^{\lambda} \partial_{\lambda} h_{\sigma}^{\rho} \right) + 2 \xi_{\rho}^{\lambda} \partial^{[\mu} h_{\nu]}^{\lambda} + \frac{1}{2} \partial_{\mu} h_{\nu}^{\rho} \right) \]

\[ -\frac{i\bar{\psi} \xi_{\rho}^{\mu} \bar{\psi}_{\rho} \xi_{\sigma}^{\mu} \eta_{\sigma}^{\mu} + \frac{2}{2} \bar{\psi} \xi_{\rho}^{\mu} \bar{\psi}_{\rho} \xi_{\sigma}^{\mu} \eta_{\sigma}^{\mu}, \] (173)

and respectively

\[ h_{0}^{\nu\psi} = \frac{9m\bar{k}}{2} \left( \bar{\psi} \xi_{\rho}^{\mu} \bar{\psi}_{\rho} \xi_{\sigma}^{\mu} \right) \]

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Finally, we solve (114) in its local form, namely (121). Taking into account
\[
\frac{ik}{64} \bar{\psi} \gamma_{\mu \nu \rho \lambda \sigma} \psi_{\nu} \partial h^\nu \partial h^\sigma + \frac{ik}{32} \partial \bar{\psi} \gamma_{\mu \nu} \partial h^\mu h^\nu + \frac{ik}{16} \frac{8}{\psi_{\mu} \gamma_{\mu \nu} \partial h^\mu h^\nu}
\]
\[
\frac{1}{2} h^e_\mu \partial h_\rho + \frac{1}{2} h^r \partial h_\rho + \frac{1}{k} \bar{\psi} g_{\mu \nu} \psi (h^e_\mu \partial h_{\lambda \nu} + h^r \partial h_{\lambda \nu})
\]
\[
+h^e_\mu \partial h_{\nu \lambda} - \frac{i k^2}{32} (\partial \bar{\psi}) \gamma_{\mu \nu \rho \lambda} \psi (\psi^\rho h + 2 \psi h^2 + 2 \psi^2 h^2 - \psi^2 h_{\lambda \sigma})
\]
\[
+ \frac{i k^2}{8} \partial (\bar{\psi} \psi) \gamma_{\nu \rho} \psi (\psi^\rho h^\mu - \psi^\mu h^\rho + \psi^\rho h^\mu) - \frac{i k^2}{16} \partial (\bar{\psi} \psi) \gamma_{\mu \nu \rho \lambda} \psi (h^\rho \partial h_{\lambda \nu} - h^\rho \partial h_{\nu \lambda} + h \partial h_{\nu \lambda})
\]
\[
- \frac{i k^2}{16} \bar{\psi} g_{\mu \nu} \psi \psi (h^e_\mu \partial h_{\nu \lambda} - 2 h_{\mu \nu} \partial h^\rho + 2 h^\rho \partial h_{\lambda \mu} - h \partial h_{\mu \nu}).
\]
(174)

Formulas (172)–(174) enable us to write
\[
S_2^{h - \psi} = \int d^{11}x \left( \bar{\psi}_2 + \bar{\psi}_1 + \bar{\psi}_0 \right). \tag{175}
\]

Finally, we solve (114) in its local form, namely (121). Taking into account one more time the concrete form of the first-order deformation, (104), we observe that
\[
(S_1, S_1)_{\text{int}} = 2 \left( S_1^{h - A}, S_1^{A - \psi} \right) + 2 \left( S_1^{h - \psi}, S_1^{A - \psi} \right), \tag{176}
\]
where $\Delta_{\text{int}}$ decomposes as
\[
\Delta_{\text{int}} = \sum_{I = 0}^{2} \Delta_{I}^{\text{int}}, \quad \text{agh} \left( \Delta_{I}^{\text{int}} \right) = I, \quad I = 0, 2, \tag{177}
\]
with
\[
\Delta_{2}^{\text{int}} = \gamma \left( 2 k \bar{\psi} C^{\mu \nu} \eta^{\rho} \xi_{\mu \nu} \psi_{\rho} + k (k + \bar{k}) C^{\mu \nu} \xi_{\mu \nu} \xi_{\rho \sigma} \sigma^{\alpha \beta} + \partial_{\mu} t_{2}^{\text{int} \mu} \right)
\]
\[
\Delta_{1}^{\text{int}} = \delta \left( 2 k \bar{\psi} C^{\mu \nu} \eta^{\rho} \xi_{\mu \nu} \psi_{\rho} - \frac{i k \bar{k}}{36} \xi (\gamma_{\mu \nu \rho \lambda} \xi \eta_{\rho \sigma} + 8 \gamma_{\mu \nu \rho} \xi \eta_{\rho} + 8 \gamma_{\mu \nu \rho} \xi \eta_{\rho}) \right)
\]
\[
+ \bar{\psi} \left( -3 k \bar{k} A^{\mu \nu \rho} (h^e_\mu \partial h_{\nu \lambda} + 2 \psi h^\rho \psi) + \frac{i k \bar{k}}{2} \psi^{\mu \nu \rho \lambda \sigma} \xi \psi \partial_{\mu \nu \rho \lambda \sigma} \right)
\]
\[
- \frac{i k \bar{k}}{9} \psi^{\mu \nu \rho \lambda \sigma} \xi \left( h^e_\mu \partial_{\sigma} A^\nu_{\rho \lambda} - 3 \frac{i k \bar{k}}{36} \psi^{\mu \nu \rho \lambda \sigma} \xi \psi \partial_{\mu \nu \rho \lambda} \right) + \frac{i k \bar{k}}{9} \psi^{\mu \nu \rho \lambda \sigma} \xi \left( h^\nu \partial^\rho A^\lambda_{\mu \rho} - 3 \frac{i k \bar{k}}{36} \psi^{\mu \nu \rho \lambda \sigma} \xi \psi \partial_{\mu \nu \rho \lambda} \right)
\]
\[
- 2 \psi^{\mu \nu \rho \lambda \sigma} \xi \left( \bar{\psi} (\partial_{\mu \nu \rho \lambda} \xi) \psi_{\sigma} + \bar{\psi} \psi_{\sigma} \xi \partial_{\mu \nu \rho \lambda} \right) + \bar{k} (k + \bar{k}) \left( 6 A^{\mu \nu \rho} \left( \eta^{\rho} (\partial_{\mu \nu \rho} \xi) \right) \psi_{\sigma} + \psi_{\sigma} \partial_{\mu \nu \rho} \xi \partial_{\mu \nu \rho} \right) \right)
\]
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+ \frac{1}{18} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \xi \left( \eta^\sigma \partial_\sigma \Phi^\rho_\lambda + 2 F_{\mu\rho\sigma} \partial_\lambda \eta_\beta \sigma^{\alpha\beta} \right) \\
+ \frac{i}{36} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \xi \left( \eta^\omega \partial_\omega \Phi^\rho_\lambda + 2 F_{\mu\rho\lambda} \partial_\lambda \eta_\omega \sigma^{\alpha\beta} \right) \right) + \partial_{\mu} \tau^\text{int}_1 \mu, \quad (179)

and

\Delta^\text{int}_0 = \delta \left[ -3 k \bar{k} A^{\mu\nu} p (\hbar^\nu \bar{\psi}_\lambda \gamma_{\mu\rho\lambda} + 2 \eta^\lambda \bar{\psi}_\mu \gamma_{\nu\rho\lambda} \psi_\lambda) + \frac{i k k}{72} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \xi \hbar^\rho \Phi^\nu_\lambda \partial_\lambda \eta_\omega \right] \\
- \frac{i k k}{9} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \xi \hbar^\rho \Phi^\lambda_\omega \partial_\omega \eta_\lambda \left( h^\rho \bar{\psi}_\lambda A^{\nu\rho\lambda} - \frac{3}{2} A_{\mu\nu\sigma} \partial_\mu \hbar^\rho \right) \\
+ \frac{i k k}{18} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \xi \hbar^\rho \Phi^\lambda_\omega \partial_\omega \eta_\lambda \left( h^\rho \bar{\psi}_\lambda A^{\nu\rho\lambda} - \frac{3}{2} A_{\mu\nu\sigma} \partial_\mu \hbar^\rho \right) \\
- 2 \eta^\sigma F_{\mu\rho\sigma} \partial_\sigma \eta_\lambda \right] + \gamma \left[ \frac{k k}{48} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \rho \lambda \omega \psi_\nu F^{\rho\lambda\omega} h - 6 A^{\rho\lambda\nu} \partial_\nu \eta_\lambda \right]

+ 4 h^\rho \partial_\rho A^{\lambda\omega} \lambda \omega \psi_\lambda F^{\rho\lambda\omega} - \frac{k k}{24} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \rho \lambda \omega \psi_\nu F^{\rho\lambda\omega} - \frac{k k}{4} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \rho \lambda \psi_\nu (h F_{\mu\rho\lambda} - 2 h^\sigma F_{\mu\rho\sigma}) \\
+ 2 h^\rho \partial_\rho A^{\lambda\omega} \lambda \omega \psi_\lambda F^{\rho\lambda\omega} - \frac{k k}{4} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \rho \lambda \psi_\nu A_{\mu\rho\sigma} \partial_\sigma \hbar^\rho \right]

+ \tilde{k} (k + \tilde{k}) \left[ \frac{1}{24} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \rho \lambda \omega \partial_\nu \eta_\rho \partial_\omega \eta_\lambda \left( h_\rho^\lambda \bar{\psi}_\nu F^{\rho\lambda\omega} - 2 \eta^\sigma F^{\rho\lambda\omega} \partial_\nu \eta_\lambda \right) \\
+ \frac{1}{12} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \rho \lambda \omega \partial_\nu \eta_\rho \partial_\omega \eta_\lambda \left( h_\rho^\lambda \bar{\psi}_\nu F^{\rho\lambda\omega} - 2 \eta^\sigma F^{\rho\lambda\omega} \partial_\nu \eta_\lambda \right) \\
- \frac{1}{2} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \rho \lambda \omega \partial_\nu \eta_\rho \partial_\omega \eta_\lambda \left( h_\rho^\lambda \bar{\psi}_\nu F^{\rho\lambda\omega} - 2 \eta^\sigma F^{\rho\lambda\omega} \partial_\nu \eta_\lambda \right) \\
- \frac{1}{2} \bar{\psi}^\mu \gamma_{\mu\rho\lambda} \rho \lambda \omega \partial_\nu \eta_\rho \partial_\omega \eta_\lambda \left( h_\rho^\lambda \bar{\psi}_\nu F^{\rho\lambda\omega} - 2 \eta^\sigma F^{\rho\lambda\omega} \partial_\nu \eta_\lambda \right) \right]. \quad (180)

Reprising the same steps like in the previous cases, we conclude that (121) cannot hold unless the parameters \( k, \tilde{k}, \) and \( \tilde{k} \) satisfy the algebraic equation

\[ \tilde{k} (k + \tilde{k}) = 0. \quad (181) \]

Assuming (181) holds, we insert (178)–(180) into (121) and identify the non-integrated density of the second-order deformation in the interacting sector (describing simultaneous interactions among graviton, gravitini, and 3-form) under the form

\[ \hat{b}^\text{int}_2 = -k \tilde{k} C^{\mu\nu\sigma} \eta^\rho \bar{\psi}_\rho \psi_\mu + \frac{i k k}{72} \tilde{\xi} (\gamma_{\mu\nu\rho\lambda} \xi \eta_\rho + 8 \gamma_{\mu\nu\rho\lambda} \eta_\rho) \Phi^{\mu\nu\lambda}, \quad (182) \]

\[ \hat{b}^\text{int}_1 = 3 k \tilde{k} A^{\nu\rho\sigma} \left( \eta^\lambda \bar{\psi}_\lambda \eta_\rho + \frac{1}{2} h^\sigma \bar{\psi}_\lambda \gamma_{\mu\nu} \xi \right) \]
\[ + \frac{i\bar{k}}{18} \psi^\mu_\sigma A_{\nu\rho\lambda} \xi \left( h^\sigma_\mu \partial_\sigma A_{\nu\rho\lambda} - \frac{3}{2} A_{\mu
u\sigma} \partial_\rho h^\sigma_\lambda \right) \]
\[ - \frac{i\bar{k}}{36} \psi^\mu_\sigma A_{\nu\rho\lambda} \xi \left( h^\nu_\sigma \partial_\sigma A_{\nu\rho\lambda} - \frac{3}{2} A^\nu_\mu\omega \partial_\lambda h^\sigma_\omega \right) \]
\[ - \frac{i\bar{k}}{144} \psi^\mu_\sigma A_{\nu\rho\lambda} \xi h^\nu_\sigma F^\rho\lambda\omega + \frac{i\bar{k}}{72} \psi^\mu_\sigma A_{\nu\rho\lambda} \psi^\omega_\sigma F^{\nu\rho\lambda\omega} \eta_\omega \]
\[ + \frac{i\bar{k}}{18} \psi^\mu_\sigma A_{\nu\rho\lambda} \xi \left( h^\sigma_\mu F_{\nu\rho\lambda\omega} - 2\psi^\sigma F_{\nu\rho\lambda\omega} \eta\sigma \right), \tag{183} \]

with the help of which we have that
\[ S_{2,\text{int}} = \int d^{11}x \left( b_{2,\text{int}}^0 + b_{1,\text{int}}^0 + b_0^0 \right). \tag{185} \]

In conclusion, we determined all the nontrivial constituents of the second-order deformation given by (107).

### 4 Redefinition of first- and second-order deformations

We showed in the previous section that the consistency of the first-order deformation at order two in the coupling constant implies a simple algebraic system for the six parameterizing constants, defined by Eqs. (126), (127), (153), (159), (171), and (181). There are two types of nontrivial solutions, namely
\[ k = -1 \text{ or } k = 0, \quad \bar{k} = \bar{k} = m = 0, \quad \Lambda, q = \text{arbitrary}, \tag{186} \]
\[ k = -\bar{k} = -1, \quad \bar{k}_{1,2} = \pm \frac{i\sqrt{2}}{8}, \quad q_{1,2} = -\frac{4\bar{k}_{1,2}}{12}, \quad m = 0 = \Lambda. \tag{187} \]

The former type is less interesting from the point of view of interactions since it maximally allows the graviton to be coupled to the 3-form (if \( k = -1 \)).

For this reason in the sequel we will extensively focus on the latter solution, (187), which forbids both the presence of the cosmological term for the spin-2 field and the appearance of gravitini 'mass' constant. In this case the first-order deformation of the solution to the master equation is expressed by relation (104), where:
(i) the density of $S^1_h$ reads as in formula (47) from Ref. [1];

(ii) $S^1_\psi = 0$ (follows from relation (50) given in Ref. [2] where we set $m = 0$);

(iii) the sum $S^A_1 + S^{h-A}_1$ is furnished by formula (118) from Ref. [1] in which we take $q \to -4\hat{k}_i/(12)^3$;

(iv) the density of $S^{h-\psi}_1$ is the sum among the right-hand sides of formulas (20), (21), and (22) in Ref. [3] where in addition we put $\bar{k} = 1$;

(v) $S^{A-\psi}_1$ is pictured by relation (110) from Ref. [2] modulo the replacement $\tilde{k} \to \tilde{k}_i$.

Consequently, the second-order deformation of the solution to the master equation is still (107), up to the following specifications:

1. $S^h_2$ follows from (123) restricted to $\Lambda = 0$;

2. the density of $S^\psi_2$ is equal to the sum among the right-hand sides of formulas (71), (72), and (74) from Ref. [3] where we put $\bar{k} = 1$;

3. $S^A_2 = 0$, in agreement with (123);

4. $S^{h-A}_2$ comes from relation (117) reported in Ref. [1] modulo the change $q \to -4\hat{k}_i/(12)^3$;

5. $S^{A-\psi}_2$ reads as in (165) with $k = -\tilde{k} = -1$;

6. $S^{h-\psi}_2$ is expressed by (175) for $k = 1$ and $m = 0$;

7. $S^{int}_2$ is pictured by (185), with $k = -\tilde{k} = -1$ and $\tilde{k} \to \tilde{k}_i$ given by (187).

In order to compare the interacting model resulting from our cohomological approach with the results known from the literature [10, 11], it is necessary to redefine the first-order deformation through a trivial, $s$-exact term, which does not modify either the cohomological class of $S_1$ or the physical contents of the coupled theory

$$S_1 \to \check{S}_1 = S_1 + sK,$$

with

$$K = -\left(\xi^*\psi^\mu \eta_\mu + \frac{1}{2} \psi^*\psi^\mu \eta_\mu \right).$$

The above redefinition brings contributions only to the mixed graviton-gravitini sector, so we can write

$$\check{S}_1 = S^h_1 + S^A_1 + S^{h-A}_1 + S^{A-\psi}_1 + \check{S}^{h-\psi}_1,$$

where

$$S^h_1 = \int d^{11}x \left[ \mathcal{L}_{EH}^{11} + h^{*\mu\rho} \left( h_{\mu\nu} \partial_\rho \eta^\nu - \eta^\nu \partial_\rho h_{\nu\rho} \right) + \frac{1}{2} \eta^{*\mu \nu} \eta^\nu \partial_\mu \eta_\nu \right],$$

and

$$S^{A-\psi}_1 = \int d^{11}x \left[ \mathcal{L}_{EH}^{11} + h^{*\mu\rho} \left( h_{\mu\nu} \partial_\rho \eta^\nu - \eta^\nu \partial_\rho h_{\nu\rho} \right) + \frac{1}{2} \eta^{*\mu \nu} \eta^\nu \partial_\mu \eta_\nu \right].$$

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\[ S_1^\lambda = -\frac{4\kappa}{(12)^4} \int d^{11}x \sum_{\mu_1\ldots\mu_{11}} A_{\mu_1\mu_2\mu_3} F_{\mu_4\mu_5\ldots\mu_{11}} F_{\mu_{12}...\mu_{11}}, \] (192)

\[ S_1^{h-A} = \int d^{11}x \left\{ \frac{1}{12} F_{\mu\nu\rho\lambda} \left[ F_{\mu\nu\rho\lambda} h_{\lambda} - 3 \partial_{(\mu} (A_{\nu\rho\lambda} h_{\lambda}) - \frac{1}{8} F_{\mu\nu\rho\lambda} \right] 
+ \frac{3}{2} A^{\mu\nu\rho} \left( \frac{2}{3} \eta^\lambda \partial_\lambda A_{\mu\nu\rho} + A_{\mu\nu} \partial_\rho \eta_\lambda + h_{\rho\lambda} \partial_\nu C_{\mu\lambda} - C_{\mu\lambda} \partial_\nu h^\lambda_\rho \right) 
+ C^{\mu\nu\rho} \left( (\partial_\mu C_{\nu\rho}) \eta^\rho + C_{\mu\rho} \partial_\nu \eta_\rho \right) + h_{\nu\rho} \partial^\rho C_{\mu\lambda} + \frac{1}{2} C_\rho \partial_\mu h_{\nu\rho} \right\}, \] (193)

\[ S_1^{h-\psi} = -\kappa \int d^{11}x \left\{ \frac{1}{4} \bar{\psi}_\mu \gamma_{\mu\nu} \psi_\lambda F^{\mu\nu\rho\lambda} + \frac{1}{2} \bar{\psi} \gamma_{\mu\nu} \psi_\lambda F^{\mu\nu\rho\lambda} \right\} \frac{1}{9} \bar{\psi}^{\mu\nu} F_{\mu\nu\rho\lambda} \gamma^{\rho\lambda} \xi 
+ \frac{1}{3} \bar{\psi}^{\mu\nu} \gamma^{\rho\lambda} \gamma^{\mu\nu} \psi_\lambda \frac{1}{2} C^{\mu\nu\rho} \xi \gamma_{\mu\nu} \psi_\rho, \] (194)

\[ \hat{S}_1^{h-\psi} = \int d^{11}x \left\{ \frac{1}{4} \bar{\psi}_\mu \gamma_{\mu\nu} \psi_\lambda F^{\mu\nu\rho\lambda} + \frac{1}{2} \bar{\psi} \gamma_{\mu\nu} \psi_\lambda F^{\mu\nu\rho\lambda} \right\} \bar{\psi}_\mu \gamma_{\mu\nu} \psi_\lambda 
- \frac{1}{2} \bar{\psi}_\mu \gamma_{\mu\nu} \psi_\lambda \partial_\nu h_{\rho\lambda} + \frac{1}{4} \left[ h^{\mu\nu} \xi \gamma_{\mu} \psi_\nu \right] 
+ \bar{\psi}_\mu \gamma_{\mu\nu} \psi_\lambda \partial_\nu \eta_{\rho\lambda} + 4 \bar{\psi}_\mu \gamma_{\mu\nu} \psi_\lambda \partial_\nu h_{\rho\lambda} + \frac{1}{4} \left[ h^{\mu\nu} \xi \gamma_{\mu} \psi_\nu \right] 
- \bar{\psi}_\mu \gamma_{\mu\nu} \psi_\lambda \partial_\nu \eta_{\rho\lambda} - \frac{1}{8} \left[ \eta^{\mu\nu} \xi \gamma_{\mu} \psi_\nu \right] \xi \gamma_{\mu\nu} \psi_\rho \right\}, \] (195)

In [191] the notation $L_1^{EH}$ means the cubic vertex of the Einstein-Hilbert Lagrangian.

Redefinition (188) induces a modification in the expression of the second-order deformation. Indeed, let us denote by $\hat{S}_2$ the second-order deformation associated with $\hat{S}_1$, namely

\[ \left( \hat{S}_1, \hat{S}_1 \right) + 2s \hat{S}_2 = 0. \] (196)

Since $S_2$ is solution to the equation

\[ (S_1, S_1) + 2s S_2 = 0, \] (197)

then (196) and (197) provide

\[ 2s \left( \hat{S}_2 - S_2 \right) + \left( \hat{S}_1, \hat{S}_1 \right) - (S_1, S_1) = 0, \]
or, equivalently (due to the bilinearity of the antibracket)

\[ 2s \left( \hat{S}_2 - S_2 \right) + \left( \hat{S}_1, \hat{S}_1 - S_1 \right) + \left( \hat{S}_1 - S_1, \hat{S}_1 \right) = 0. \]  \tag{198} 

Substituting (189) in (198), we infer the equation

\[ 2s \left( \hat{S}_2 - S_2 \right) + \left( \hat{S}_1, sK \right) + (sK, S_1) = 0. \]  \tag{199} 

Recalling the fact that the BRST differential behaves like a derivation with respect to the antibracket plus the \( s \)-closeness of both \( S_1 \) and \( \hat{S}_1 \), we find that (199) becomes

\[ s \left[ \hat{S}_2 - S_2 + \left( \frac{\hat{S}_1 + S_1}{2}, K \right) \right] = 0, \]  \tag{200} 

which further produces

\[ \hat{S}_2 = S_2 - \left( \frac{\hat{S}_1 + S_1}{2}, K \right). \]  \tag{201} 

Performing the necessary computations with the help of (189), we obtain that

\[ \hat{S}_2 = S_2^h + S_2^{h-A} + S_2^{A-\psi} + S_2^{\hat{\psi}} + \hat{S}_2^{h-\psi} + \hat{S}_2^{\text{int}}, \]  \tag{202} 

where

\[ S_2^h = \int \! d^{11} x \left\{ C^* h_{\mu\nu} \eta^\mu \partial^\nu C + C^{*\mu} \left[ \frac{3}{4} h_{\mu\nu} h^\mu C - \frac{1}{2} C^\nu \eta^\nu \partial_{[\mu} h_{\nu]} \right] \right. \]

\[ + \left. \frac{1}{4} h_{\mu\nu} \left( \partial^\rho h_{\mu\nu} \right) \eta^\rho - \frac{3}{2} \left( \partial_{[\mu} h_{\nu]\lambda} - \partial_{[\lambda} h_{\mu\nu]} \right) h^\lambda h^\rho \eta^\rho \right\}, \]  \tag{203} 

\[ S_2^{h-A} = \frac{1}{2} \int \! d^{11} x \left\{ C^* h_{\mu\nu} \eta^\mu \partial^\nu C + C^{*\mu} \left[ \frac{3}{4} h_{\mu\nu} h^\mu C - \frac{1}{2} C^\nu \eta^\nu \partial_{[\mu} h_{\nu]} \right] \right. \]

\[ + \left. \frac{1}{4} C^\nu \left( h_{\mu\nu} \eta_{[\mu} - h_{\nu\eta} \partial_{[\mu} \eta_{\nu]} \right) - h^{\nu\rho} \eta_{[\mu} \partial_{\nu] C_{\mu]} - C^{*\mu\nu} \left[ C^\rho \eta_{[\mu} \partial_{\nu]} h^\lambda \right] \right. \]

\[ + \left. \frac{1}{2} C^{\mu\nu} \left( h_{\nu\lambda} \eta_{[\mu} - h_{\mu\lambda} \partial_{[\mu} \eta_{\nu]} \right) + (\partial_{[\mu} C_{\nu]} \eta_{[\lambda} h^\rho \partial_{\nu]} \right) \eta_{\lambda} h^\rho \partial_{\nu]} \right) \]

\[ - \frac{1}{2} C_{\mu\rho} \left( h^{\rho\nu} \partial_{[\mu} \eta_{\nu]} - h^{\nu\eta} \partial_{[\mu} \eta_{\nu]} \right) + (\partial_{[\mu} C_{\nu]} \eta_{[\lambda} h^\rho \partial_{\nu]} \right) \eta_{\lambda} h^\rho \partial_{\nu]} \right) \]

\[ + \left. \frac{3}{2} A^{*\mu\nu} \left[ C_{\rho\xi} \partial_{[\mu} \left( h_{\nu\lambda} \eta_{[\xi]} + h_{\rho\xi} \partial_{[\mu} \eta_{\xi]} + 2 \sigma^{\nu\lambda} \eta_{[\xi]} \partial_{[\rho} h_{\lambda]} \right) + A_{\nu\lambda} \eta_{[\rho} h_{\xi]} \partial_{[\mu} h_{\lambda]} \right] \right] \]

\[ \right. \]
\[
\begin{align*}
-\frac{2}{3} \hbar^2 \xi \partial_\lambda A_{\mu\rho} &+ \frac{1}{8} F^{\mu\nu\rho\lambda} F_{\mu\nu} \xi_\tau \left[ \hbar^2 \hbar^2 - \frac{3}{2} \delta^2_\rho \delta^2_\lambda \left( \frac{1}{4} \hbar^2 - \hbar^\alpha_\beta \hbar^\alpha_\beta \right) \right] \\
- \frac{1}{3} \delta^\xi_\rho h^\lambda_\sigma h^\pi_\tau &+ \frac{1}{16} F^{\mu\nu\rho\lambda} \left[ A_{\xi\rho\lambda} \partial_\mu (h_{\nu\pi} h^\lambda_\pi) - h_{\mu\pi} h^\xi_\mu \partial_\nu A_{\xi\rho\lambda} \right] \\
+ \frac{4}{3} \partial_\xi A_{\mu\rho} &+ A_{\mu\nu} \left( 4 h^\pi_\rho \partial_{\pi} h^\xi_\lambda - h \partial_{\mu} h^\xi_\lambda \right) - \frac{2}{3} h^\xi_\lambda h \partial_\xi A_{\mu\rho} \\
- 2 A_{\mu\xi} \partial_\nu \left( h^\xi_\mu h^\gamma_\lambda \right) + 2 h^\xi_\mu h^\gamma_\lambda \partial_\xi A_{\mu\nu} &+ q_\xi \hat{e}^{\mu_1 \cdots \mu_{11}} \left( \frac{1}{2} h A_{\mu_1 \mu_2 \mu_3} F_{\mu_4 \cdots \mu_7} \right) \\
- 4 h^\xi_\mu A_{\mu_2 \mu_3} F_{\mu_4 \cdots \mu_7} &+ 3 h^\xi_\mu A_{\mu_2 \mu_3} F_{\mu_4 \cdots \mu_7} \right) F_{\mu_5 \cdots \mu_{11}} \\
&+ \frac{1}{16} \partial_\xi \left( h^\pi_\mu A_{\rho_\mu} \right) \left[ 9 \hat{e} \left( h^\xi_\mu A^{\mu_\nu_\tau} \right) - \frac{1}{3} \partial_\xi \left( h^\mu_\mu A^{\mu_\nu_\tau} \right) \right].
\end{align*}
\]

According to (202), only the last three components from the second-order deformation change. Thus, the Rarita-Schwinger contribution passes into

\[
\begin{align*}
\hat{S}_2^n &= \frac{1}{25} \int d^1 x \left\{ \hat{e} \left( \gamma^\mu_\nu \xi \gamma_\mu \psi_\nu + \frac{1}{16} \psi^\rho_\mu \bar{\psi}^\rho_\mu \xi \right) \\
- \frac{1}{3} \frac{24}{23} (11 \cdot 17 \psi^\rho_\mu \gamma^\mu_\nu \bar{\psi}^\rho_\nu - 29 \psi^\rho_\mu \gamma^\mu_\rho \bar{\psi}^\rho_\mu - 62 \psi^\rho_\mu \gamma^\rho_\nu \psi^\rho_\nu \bar{\psi}^\rho_\nu) \xi \gamma_\mu \psi_\nu \\
- \frac{1}{3} \frac{24}{23} (14 \cdot 37 \psi^\rho_\mu \bar{\psi}^\rho_\nu + 7 \psi^\rho_\mu \gamma^\rho_\mu \gamma^\rho_\nu \bar{\psi}^\rho_\nu - 68 \psi^\rho_\mu \gamma_\rho_\nu \psi^\rho_\nu \bar{\psi}^\rho_\nu \\
- 29 \psi^\rho_\mu \gamma_\mu_\rho \bar{\psi}^\rho_\nu + \frac{1}{3} \frac{24}{23} (\psi^\rho_\mu \gamma^\rho_\mu \gamma^\rho_\nu \bar{\psi}^\rho_\nu - 56 \psi^\rho_\mu \gamma_\nu_\rho \psi^\rho_\nu \bar{\psi}^\rho_\nu + \psi^\rho_\mu \gamma^\rho_\nu \gamma^\rho_\lambda \psi^\rho_\lambda \bar{\psi}^\rho_\lambda \psi^\rho_\nu + 14 \psi^\rho_\mu \gamma^\rho_\nu \gamma^\rho_\lambda \psi^\rho_\lambda \bar{\psi}^\rho_\nu \psi^\rho_\lambda \psi^\rho_\nu) \xi \gamma_\mu_\rho \gamma^\rho_\nu \bar{\psi}^\rho_\nu \\
- \xi \left( \psi^\rho_\mu \gamma_\nu_\rho \bar{\psi}^\rho_\nu + \frac{1}{3} \bar{\psi}^\rho_\mu \gamma^\rho_\nu \psi_\nu \right) \right] + \frac{3}{6} \left( \psi^\rho_\mu \gamma_\nu_\rho \bar{\psi}_\nu \right) \\
- \frac{1}{2} \psi^\rho_\mu \gamma_\nu_\rho \psi_\nu \left( \psi^\rho_\mu \gamma_\mu_\nu \psi^\rho_\nu + \bar{\psi}^\rho_\mu \gamma^\rho_\nu \psi^\rho_\nu \bar{\psi}^\rho_\nu \psi^\rho_\nu \bar{\psi}^\rho_\nu \psi^\rho_\nu \right) \\
- \frac{1}{4} \bar{\psi}^\rho_\mu \gamma_\nu_\rho \psi_\nu \left( \psi^\rho_\mu \gamma_\nu_\rho \psi^\rho_\nu + \bar{\psi}^\rho_\mu \gamma^\rho_\nu \psi^\rho_\nu \bar{\psi}^\rho_\nu \psi^\rho_\nu \bar{\psi}^\rho_\nu \psi^\rho_\nu \right) \\
- \frac{1}{2} \psi^\rho_\mu \gamma_\nu_\rho \psi_\nu \left( \psi^\rho_\mu \gamma_\nu_\rho \psi^\rho_\nu + \bar{\psi}^\rho_\mu \gamma^\rho_\nu \psi^\rho_\nu \bar{\psi}^\rho_\nu \psi^\rho_\nu \bar{\psi}^\rho_\nu \psi^\rho_\nu \right) \right].
\end{align*}
\]

the mixed graviton-gravitino piece takes the form

\[
\begin{align*}
\hat{S}_2^n &= \int d^1 x \left\{ \frac{1}{16} \left[ \xi^* \gamma_\mu_\nu \xi \left( \bar{\hbar}^\mu_\nu \partial_\lambda \eta^\lambda + \eta^\lambda \partial_\nu \bar{\hbar}^\mu_\nu \right) \right] \\
- \eta^\mu_\nu \bar{\xi} \gamma_\lambda \eta_\nu \right) \eta^\lambda \eta_\nu + \frac{1}{8} \xi^* \gamma_\lambda \eta_\mu \partial_\lambda \eta_\mu \right].
\end{align*}
\]
and the terms expressing the simultaneous interactions among all the three types of fields amount to

\[ \tilde{S}_{2}^{\text{int}} = \bar{\psi}_{i} \int d^{11}x \left\{ \frac{i}{12} F^{\mu\nu\rho\lambda} \left( \kappa^{\ast} \gamma_{\mu\nu} \gamma_{\lambda} \kappa + 8 \sigma_{\mu \sigma} \xi^{\ast} \gamma_{\mu\nu} \xi^{\ast} \right) \eta^{\sigma} \\
- \frac{i}{18} \bar{\psi}_{i} \gamma_{\rho \lambda} \kappa \left[ F_{\mu\rho\sigma} h_{\ast}^{\sigma} - 3 \partial_{\mu} \left( h_{\sigma} A_{\rho \lambda} \right) \right] \\
+ \frac{i}{36} \bar{\psi}_{i} \gamma_{\mu\nu} \kappa \left[ F_{\nu\rho\lambda} h_{\ast}^{\sigma} - 3 \partial_{\nu} \left( A_{\rho \lambda} h_{\sigma} \right) \right] \\
- \frac{1}{8} h F_{\mu\nu\rho\lambda} \left( \bar{\psi}_{i} \gamma^{\mu\nu} \psi \gamma_{\lambda} + \frac{1}{12} \bar{\psi}_{i} \gamma_{\alpha\beta\mu\nu\rho\lambda} \psi_{\beta} \right) \\
+ \frac{1}{12} \left( \bar{\psi}_{i} \gamma^{\mu\nu} \psi \lambda \right) + \frac{1}{2} \bar{\psi}_{i} \gamma_{\alpha\beta\mu\nu\rho\lambda} \psi_{\beta} \right\} \]  

(208)

In deriving formula (208) we used the identity

\[ \bar{\psi}_{i} \gamma_{\rho} \psi_{\beta} \bar{\psi}_{i} \gamma_{\mu\nu\rho\lambda} \psi \gamma_{i} + \frac{1}{2} \bar{\psi}_{i} \gamma_{\rho} \psi_{\lambda} \bar{\psi}_{i} \gamma_{\alpha\beta\mu\nu\rho\lambda} \psi_{\beta} \]

\[ + \frac{1}{36} \left( 4 \bar{\psi}_{i} \gamma^{\mu\nu\rho\lambda} \psi \gamma_{\rho} \gamma_{\sigma\mu\nu} \psi_{\beta} + \bar{\psi}_{i} \gamma_{\mu\nu} \gamma_{\rho\lambda} \psi \gamma_{\sigma} \gamma_{\alpha\beta\rho\lambda} \psi_{\beta} \right) + 21 \left( -4 \bar{\psi}_{i} \gamma^{\mu\nu} \psi \gamma_{\sigma} \gamma_{\rho} \psi_{\sigma} \right) \]

\[ + \bar{\psi}_{i} \gamma_{\mu\nu} \psi \gamma_{\alpha\beta} \psi_{\beta} + \bar{\psi}_{i} \gamma_{\alpha\beta} \psi \gamma_{\mu\nu} \psi \psi_{\beta} \right) = 0. \]  

(209)
5 Analysis of the deformed theory. Uniqueness of $D = 11, N = 1$ SUGRA

In Ref. [6] (Section 5) it has been shown that the local BRST cohomologies of the Pauli-Fierz model and respectively of the linearized version of vielbein formulation of spin-two field theory are isomorphic. Because the local BRST cohomology (in ghost numbers zero and one) controls the deformation procedure, it results that this isomorphism allows one to pass in a consistent manner from the Pauli-Fierz version to the linearized version of the vielbein formulation and conversely during the deformation procedure. Nevertheless, the linearized vielbein formulation possesses more fields (the antisymmetric part of the linearized vielbein) and more gauge parameters (Lorentz parameters) than the Pauli-Fierz model, such that the switch from the former version to the latter is realized via the above mentioned isomorphism by imposing some partial gauge-fixing conditions, which come from the more general ones [7]

\[\tilde{\delta}_e \sigma_{[\alpha} e_{\beta]}^\mu = 0. \quad (210)\]

In the context of this larger partial gauge-fixing, simple computations lead to the vielbein fields $e_a^\mu$, their inverses $e^a_\mu$, the inverse of their determinant $e$, and the components of the spin connection $\omega_{\mu ab}$ (up to the second order in the coupling constant) in terms of the Pauli-Fierz field as

\begin{align*}
\left(0\right)_{e_a}^{\mu} &= e_a^\mu, \\
\left(1\right)_{e_a}^{\mu} &= \lambda ^{e_a}^{(1)} e_a^{(2)} ^\mu + \cdots = \delta_a^\mu - \frac{\lambda}{2} h_a^\mu + \frac{3\lambda^2}{8} h_a^\rho h_\rho^\mu + \cdots, \\
\left(2\right)_{e_a}^{\mu} &= \delta_a^\mu + \frac{\lambda}{2} h_a^\mu - \frac{\lambda^2}{8} h_a^\rho h_\rho^\mu + \cdots, \\
\omega_{\mu ab} &= \lambda ^{(1)} \omega_{\mu ab} + \lambda ^{e_a} \omega_{(2) ab} + \cdots, \quad \text{(213)}
\end{align*}

where

\[\left(1\right)_{\omega_{\mu ab}} = - \partial_{[a} h_{b] \mu}, \quad \left(2\right)_{\omega_{\mu ab}} = - \frac{1}{4} \left[ 2 h_{c[a} \left( \partial_{b]} h_c^\mu \right) - 2 h_{[a}^\nu \partial_{b]} h_{b]}^\nu - \left( \partial_{[a} h_{b]}^\mu \right) h_{b]}^\nu \right]. \quad \text{(216)}

Based on these isomorphisms, we can further pass to the analysis of the deformed theory obtained in the previous sections.

The component of antighost number equal to zero present in $\mathcal{S}_1$ is precisely the interacting Lagrangian at order one in the coupling constant

\[\mathcal{L}_1 = \mathcal{L}^{EH}_1 + \mathcal{L}^{\text{h} - \Lambda}_1 + \left[ - \frac{i}{4} \bar{\psi}_\mu \gamma^\mu \rho \partial_\rho \psi_\mu \right] + \left[ - \frac{i}{4} \bar{\psi}_\mu \gamma^\mu \rho \partial_\rho \psi_\mu \right]
\]

\[+ \left[ - \frac{i}{4} \bar{\psi}_\mu \gamma^\mu \rho \partial_\rho \psi_\mu \right] h_{\mu \rho} + \frac{i}{4} \bar{\psi}_\mu \gamma^\mu \rho \left( \partial^\lambda \psi_\rho \right) h_{\mu \rho \lambda} + \frac{i}{4} \bar{\psi}_\mu \gamma^\mu \rho \left( \partial^\lambda \psi_\rho \right) h_{\rho \lambda}, \quad \text{(214)}\]
\[
\begin{align*}
&+ \left[ \frac{i}{8} \left( \bar{\psi}^\mu \gamma^\lambda \psi^\nu - 2 \sigma^{\nu \lambda} \bar{\psi}^\mu \gamma^\rho \psi^\rho \right) \partial_{[\mu} h_{\nu]} \right] \\
&+ \left[ -\frac{i}{8} \left( 2 \bar{\psi}^\mu \gamma^{\mu \rho} \left( \partial_\rho \psi^\lambda \right) h_{\rho \lambda} + \bar{\psi}^\rho \gamma^{\rho \mu} \psi^\lambda \partial_{[\mu} h_{\nu]} \right) \right] \\
&- \frac{\kappa_i}{48} F_{\mu \nu \rho \lambda} \left( \bar{\psi}_\alpha \gamma^{\alpha \beta \mu \nu \rho \lambda} \psi_\beta + 2 \bar{\psi}^\mu \gamma^{\mu \rho} \psi^\lambda \right) \\
\end{align*}
\]

\[
\equiv \mathcal{L}^{\text{EH}}_1 + \mathcal{L}^{\text{h-A}}_1 + \left[ -\frac{i}{2} e^{(0)} \psi^\mu \Gamma_{\mu \nu} \left( \bar{\Omega} + \bar{\hat{\Omega}} \right) \frac{(0)}{2} \psi^\nu \right] \\
\]

\[
+ \left[ -\frac{i}{2} e^{(0)} \psi^\mu \Gamma_{\mu \nu} \left( \bar{\Omega} + \bar{\hat{\Omega}} \right) \frac{(0)}{2} \psi^\nu \right] \\
+ \left[ -\frac{i}{2} e^{(0)} \psi^\mu \Gamma_{\mu \nu} \left( \bar{\Omega} + \bar{\hat{\Omega}} \right) \frac{(0)}{2} \psi^\nu \right] \\
+ \left[ -\frac{i}{2} e^{(0)} \psi^\mu \Gamma_{\mu \nu} \left( \bar{\Omega} + \bar{\hat{\Omega}} \right) \frac{(0)}{2} \psi^\nu \right] \\
+ \left[ -\frac{i}{2} e^{(0)} \psi^\mu \Gamma_{\mu \nu} \left( \bar{\Omega} + \bar{\hat{\Omega}} \right) \frac{(0)}{2} \psi^\nu \right] \\
+ \left[ -\frac{i}{2} e^{(0)} \psi^\mu \Gamma_{\mu \nu} \left( \bar{\Omega} + \bar{\hat{\Omega}} \right) \frac{(0)}{2} \psi^\nu \right] \\
+ \left[ -\frac{i}{2} e^{(0)} \psi^\mu \Gamma_{\mu \nu} \left( \bar{\Omega} + \bar{\hat{\Omega}} \right) \frac{(0)}{2} \psi^\nu \right] \\
\right],
\]

\[(217)\]

where \(\mathcal{L}^{\text{h-A}}_1\) and \(F_{\mu \nu \rho \lambda}\) are respectively listed in formulas (124) and (126) from Ref. [1] (with \(q \rightarrow q_i\) and \(q_i\) as in (187)). In the above we also made the notations

\[
(0)\ D_{(\mu} \left( \frac{\Omega + \hat{\Omega}}{2} \right) = \partial_{\mu},
\]

\[(218)\]

\[
(1)\ D_{\mu} \left( \frac{\Omega + \hat{\Omega}}{2} \right) = \frac{1}{16} \left( (\Omega)_{\mu \nu} + (\hat{\Omega})_{\mu \nu} \right) \gamma^{\alpha \beta} ,
\]

\[(219)\]

where \((\Omega)_{\mu \nu}\) and \((\hat{\Omega})_{\mu \nu}\) \((n \geq 1)\) are the net contributions of the quantities

\[
\begin{align*}
\Omega_{\mu \nu} &= \omega_{\mu \nu} + \mathcal{K}_{\mu \nu} = \omega_{\mu \nu} + \frac{i \lambda^2}{16} e^m_{\mu} \left[ \bar{\psi}^n \gamma_{mn \rho \sigma} \psi^\rho + 2 \left( \bar{\psi}_a \gamma \psi_b + \bar{\psi}_b \gamma \psi_a \right) \right] \\
\hat{\Omega}_{\mu \nu} &= \Omega_{\mu \nu} - \frac{i \lambda^2}{16} e^m_{\mu} \bar{\psi}^n \gamma_{mn \rho \sigma} \psi^\rho \equiv \omega_{\mu \nu} + \frac{i \lambda^2}{8} e^m_{\mu} \left( \bar{\psi}_a \gamma \psi_b + \bar{\psi}_b \gamma \psi_a \right)
\end{align*}
\]

\[(220)\]

to order \(n\) of perturbation theory, with \(\omega_{\mu \nu}\) given in (214). Notations \(\Gamma^{(\alpha_1 \cdots \alpha_k)}\) \((k \leq 6)\) signify the net contributions of the matrices

\[
\Gamma^{(\alpha_1 \cdots \alpha_k)} = e_{a_1}^{\alpha_1} \cdots e_{a_k}^{\alpha_k} \gamma^{(\alpha_1 \cdots \alpha_k)} ,
\]

\[(221)\]
again to order \( n \) of perturbation theory and \( (0) \psi_{\mu} = \psi_{m} \) means the zero-order approximation of the curved spin-vector

\[
(0) \psi_{\mu} = e_{\mu}^{m} \psi_{m}.
\]

Along the same line, the piece of antighost number equal to zero from the second-order deformation offers us the interacting Lagrangian at order two in the coupling constant \( L_{2} \)

\[
L_{2} = L_{2}^{EH} + L_{2}^{b-A} + T_{0} + T_{1} + T_{2} + T_{3} + T_{4} + T_{5} + T_{6},
\]

where

\[
T_{0} = \left[ \frac{1}{25} \bar{\psi}_{\mu} \gamma^{\rho} \psi_{\rho} \bar{\psi}_{\mu} \gamma^{\lambda} \psi_{\lambda} - \frac{1}{29} \left( \bar{\psi}_{\mu} \gamma_{\mu\rho\lambda\beta} \psi_{\nu} + 2 \left( \bar{\psi}_{\alpha} \gamma_{\rho} \psi_{\beta} + \bar{\psi}_{\rho} \gamma_{\alpha} \psi_{\beta} \right) \right) \right. \\
\times \left. \left( - \bar{\psi}_{\lambda} \gamma^{\rho} \lambda_{\alpha\beta} \psi_{\sigma} + 2 \left( \bar{\psi}_{\alpha} \gamma_{\rho} \psi_{\beta} + \bar{\psi}_{\rho} \gamma_{\alpha} \psi_{\beta} \right) \right) \right] \\
\equiv \left[ -\frac{1}{2} e_{\mu}^{a} e_{\mu}^{b} \left( \frac{2}{2} \bar{K}_{\mu} K_{\nu} - \bar{K}_{\mu} K_{\nu} \right) \right],
\]  

\[
T_{1} = \left[ -\frac{i}{16} \bar{\psi}_{\sigma} \gamma^{\mu\nu\rho} \left( \partial_{\nu} \psi_{\mu} \right) \left( h^{2} - 2 h_{\alpha\beta} h^{\alpha\beta} \right) \right] + \left[ -\frac{i}{8} \bar{\psi}_{\sigma} \gamma^{\mu\nu\rho} \left( \partial_{\nu} \psi_{\mu} \right) h_{\mu\sigma} \right] \\
+ \left[ \frac{i}{8} h \left( \bar{\psi}_{\sigma} \gamma^{\mu\nu\rho} \left( \partial_{\nu} \psi_{\mu} \right) h_{\mu\sigma} + \bar{\psi}_{\mu} \gamma^{\mu\nu\rho} \left( \partial_{\lambda} \psi_{\rho} \right) h_{\nu\lambda} + \bar{\psi}_{\mu} \gamma^{\mu\nu\rho} \left( \partial_{\lambda} \psi_{\rho} \right) h_{\nu\lambda} \right) \right] \\
+ \left[ \frac{1}{16} \left( \bar{\psi}_{\mu} \gamma^{\lambda} \psi_{\nu} - 2 \sigma^{\mu\nu} \bar{\psi}_{\mu} \gamma^{\lambda} \psi_{\nu} \right) h_{\mu\lambda} \right] \\
+ \left[ -\frac{i}{16} h \left( \bar{\psi}_{\sigma} \gamma^{\mu\nu\rho} \left( \partial_{\nu} \psi_{\mu} \right) h_{\mu\sigma} + \bar{\psi}_{\rho} \gamma^{\nu\mu\sigma} \psi_{\lambda} \partial_{\nu} h_{\mu\lambda} \right) \right] \\
\equiv \left[ -\frac{i}{2} e_{\psi_{\mu}} \Gamma^{(0) (0) (0) (0) \mu\nu\rho} D_{\nu} \left( \frac{\Omega + \hat{\Omega}}{2} \right) \psi_{\rho} \right] \\
+ \left[ -\frac{i}{2} e_{\psi_{\mu}} \Gamma^{(1) (0) (0) (0) \mu\nu\rho} D_{\nu} \left( \frac{\Omega + \hat{\Omega}}{2} \right) \psi_{\rho} \right] \\
+ \left[ -\frac{i}{2} e_{\psi_{\mu}} \Gamma^{(0) (1) (0) (0) \mu\nu\rho} D_{\nu} \left( \frac{\Omega + \hat{\Omega}}{2} \right) \psi_{\rho} \right] \\
+ \left[ -\frac{i}{2} e_{\psi_{\mu}} \Gamma^{(0) (0) (1) (0) \mu\nu\rho} D_{\nu} \left( \frac{\Omega + \hat{\Omega}}{2} \right) \psi_{\rho} \right] \\
+ \left[ -\frac{i}{2} e_{\psi_{\mu}} \Gamma^{(0) (0) (0) (1) \mu\nu\rho} D_{\nu} \left( \frac{\Omega + \hat{\Omega}}{2} \right) \psi_{\rho} \right],
\]  

\[
T_{2} = \left[ \frac{i}{16} \bar{\psi}_{\sigma} \gamma^{\mu\nu\rho} \left( \partial_{\nu} \psi_{\sigma} \right) h_{\sigma} h_{\mu \lambda} \right].
\]
\[ T_3 = -\frac{i}{16} \bar{\psi}_\mu \gamma^{\alpha\beta\gamma} (\partial_\nu \psi_\rho) \left( 3h^{\mu\lambda} h_{\lambda\alpha} \delta^\rho_\beta \delta^\rho_\gamma + 3\delta^\mu_\alpha h^{\nu\lambda} h_{\lambda\beta} \delta^\rho_\gamma \\
+ 3\delta^\mu_\alpha \delta^\nu_\beta h^{\rho\lambda} h_{\lambda\gamma} + 2h^{\mu\rho} h^{\nu\beta} \delta^\rho_\alpha + 2h^{\mu\rho} h^{\nu\beta} h^{\delta_\alpha} + 2\delta^\mu_\alpha \delta^\nu_\beta h^{\delta_\alpha} h^{\delta_\gamma} \right) \\
+ \frac{i}{16} \bar{\psi}_\mu \left( \delta^\lambda_\alpha \delta^\rho_\gamma + \delta^\lambda_\rho \delta^\gamma_\alpha + \delta^\lambda_\gamma \delta^\alpha_\rho \right) \psi^\rho \partial^\alpha \partial^\gamma h^\nu, \\
+ \frac{i}{16} \bar{\psi}_\rho \left( \delta^\lambda_\alpha \delta^\gamma_\rho + \delta^\lambda_\rho \delta^\gamma_\alpha + \delta^\lambda_\gamma \delta^\alpha_\rho \right) \psi^\gamma \partial^\nu h^\nu, \\
+ \frac{i}{32} \bar{\psi}_\mu \gamma^{\mu\nu\rho\alpha} \delta^\lambda_\rho \partial^\alpha \partial^\lambda h^\nu, \\
+ \left[ \frac{i}{8} \bar{\psi}_\mu \gamma^{\alpha\beta\gamma} \left( h^{\mu\rho} \delta^\gamma_\rho \delta^\rho_\beta + \delta^\mu_\alpha h^{\nu\rho} \delta^\rho_\gamma + \delta^\mu_\alpha \delta^\nu_\beta h^\rho \right) \right] \partial_\nu \left( h_{\rho\lambda} \gamma^\lambda \right) \right] \\
\equiv \left[ \frac{i}{2} \psi^\rho \Gamma D_\nu \left( \frac{\Omega + \hat{\Omega}}{2} \right) \psi^\rho \right], \quad (227) \]

\[ T_4 = \left[ \frac{i}{32} \left( \bar{\psi}^\mu \gamma^\rho \psi^\nu - 2\bar{\psi}^\mu \gamma^\sigma \psi^\sigma \psi^\rho - \frac{1}{2} \bar{\psi}^\mu \gamma^{\alpha\beta\mu\nu} \psi^\rho \right) \times \right. \\
\]

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\[
\begin{aligned}
&= \left(2h_{\lambda[\rho\sigma]} h_{\nu]}^\lambda - 2h_{\lambda[\mu} \partial^\lambda h_{\nu]}^\rho - \left(\partial_{\rho} h_{\nu]}^\lambda\right) h_{\nu]}^\lambda - \frac{i}{4} \left(\frac{1}{2} \tilde{\psi}^\lambda \gamma_{\mu\rho\lambda\sigma} \psi_{\sigma} \right)
+ 2 \left(\tilde{\psi}_{[\mu} \psi_{\nu]} + \tilde{\psi}_{[\mu} \gamma_{\rho]} \psi_{\nu]\right)\right)
\end{aligned}
\]
\[
\begin{aligned}
&= \left[- \frac{i}{16} \tilde{\psi}_{\rho} \left(\delta^\alpha_{\rho} \delta_{\beta} \gamma_{\mu} + \delta^\beta_{\rho} \delta^\gamma_{\mu} + \delta^\gamma_{\rho} \delta^\delta_{\mu} \gamma_{\nu}\right) \psi_{\lambda} h_{\rho\lambda} \partial_{[\alpha} h_{\beta]}^\nu
+ \frac{i}{16} \tilde{\psi}_{\mu} s_{\alpha [\mu} \delta_{\nu]}^\beta \psi_{\lambda} h_{\rho\lambda} \partial_{[\alpha} h_{\beta]}^\nu \right] + \left[ \frac{i}{16} \tilde{\psi}_{\mu} \gamma_{\mu\rho} \partial_{\nu} (h_{\rho\lambda} h_{\lambda}^\sigma \psi_{\sigma}) \right]
\end{aligned}
\]
\[
\begin{aligned}
&= \left[- \frac{i}{2} \tilde{\psi}_{\mu} \Gamma^\rho \psi_{\nu} \left(\Omega + \tilde{\Omega}\right) \psi_{\rho} \right]
+ \left[ \frac{i}{2} \tilde{\psi}_{\mu} \Gamma^\rho \psi_{\nu} \left(\Omega + \tilde{\Omega}\right) \psi_{\rho} \right]
+ \left[ \frac{i}{2} \tilde{\psi}_{\mu} \Gamma^\rho \psi_{\nu} \left(\Omega + \tilde{\Omega}\right) \psi_{\rho} \right].
\end{aligned}
\]
\[
T_5 = \frac{1}{3} \cdot 2^{10} \tilde{\psi}_{\rho} \left(\tilde{\psi}_{\alpha} \gamma_{\alpha\beta\mu\nu} \psi_{\beta} + 2 \tilde{\psi}_{\beta} \gamma_{\beta\rho} \psi_{\beta} \right)
= - \frac{\tilde{k}_i}{3} \cdot 2^{10} \tilde{\psi}_{\rho} \left(\tilde{\psi}_{\alpha} \gamma_{\alpha\beta\mu\nu} \psi_{\beta} + 2 \tilde{\psi}_{\beta} \gamma_{\beta\rho} \psi_{\beta} \right)
\]
\[
T_6 = \left[ - \frac{\tilde{k}_i}{3} \cdot 2^{10} \tilde{F}_{\mu\rho\lambda} \left(\tilde{\psi}_{\alpha} \gamma_{\alpha\beta\mu\nu} \psi_{\beta} + 2 \tilde{\psi}_{\beta} \gamma_{\beta\rho} \psi_{\beta} \right) \right]
+ \left[ \frac{\tilde{k}_i}{3} \cdot 2^{10} \tilde{F}_{\mu\rho\alpha} \left(\tilde{\psi}_{\alpha} \gamma_{\alpha\beta\mu\nu} \psi_{\beta} + 2 \tilde{\psi}_{\beta} \gamma_{\beta\rho} \psi_{\beta} \right) \right]
+ \left[ \frac{\tilde{k}_i}{25} \partial_{\mu} (h_{\rho\lambda} A_{\rho\lambda}) \left(\tilde{\psi}_{\alpha} \gamma_{\alpha\beta\mu\nu} \psi_{\beta} + 2 \tilde{\psi}_{\beta} \gamma_{\beta\rho} \psi_{\beta} \right) \right]
\]
\[
\begin{aligned}
&= \left[ \frac{\tilde{k}_i}{48} \left(\tilde{\psi}_{\alpha} \gamma_{\alpha\beta\mu\nu} \psi_{\beta} + 2 \tilde{\psi}_{\beta} \gamma_{\beta\rho} \psi_{\beta} \right) \right]
+ \left[ \frac{\tilde{k}_i}{48} \left(\tilde{\psi}_{\alpha} \gamma_{\alpha\beta\mu\nu} \psi_{\beta} + 2 \tilde{\psi}_{\beta} \gamma_{\beta\rho} \psi_{\beta} \right) \right]
+ \left[ \frac{\tilde{k}_i}{48} \left(\tilde{\psi}_{\alpha} \gamma_{\alpha\beta\mu\nu} \psi_{\beta} + 2 \tilde{\psi}_{\beta} \gamma_{\beta\rho} \psi_{\beta} \right) \right]
\end{aligned}
\]
\]
\]
\]
\[
37
Lagrangian (expressed in terms of the ‘curved’ spin-vector $\psi$ strength of the ‘curved’ 3-form $\bar{L}$ Lagrangian, $L$ separately coupled to a graviton or respectively to a three-form gauge field.

where it has been shown that gravitini allow no self-interactions in the gravitini vertex is permitted, by contrast to the results from Refs. [2] and [3], whereas it has been shown that gravitini allow no self-interactions in $D = 11$ if separately coupled to a graviton or respectively to a three-form gauge field.

Relying on (217) and (224), we observe that the first orders of the interacting theories give us the deformed gauge transformations for the original fields (the deformed Lagrangian reads as in relation (130) from Ref. [1] and the Levi-Civita symbol $\epsilon{}_{\alpha\beta\gamma}$ is defined via formula (132) from the same reference.

At the first sight it seems that we obtained two different interacting theories, respectively corresponding to the two different values of $\bar{k}$ and $q$ from (187). $\bar{k}_1 = \frac{i\sqrt{2}}{8}$, $q_1 = -\frac{i\sqrt{2}}{2(12)}$, and $\bar{k}_2 = -\frac{i\sqrt{2}}{8}$, $q_2 = \frac{i\sqrt{2}}{2(12)}$ respectively. Nevertheless, this is not the case since the two models are correlated through the transformation $\tilde{A}_{\mu\nu\rho} \rightarrow -\bar{A}_{\mu\nu\rho}$, so (233) is the $D = 11$, $N = 1$ SUGRA Lagrangian for both choices (see also Refs. [8] and [9]).

The pieces linear in the antifields from the deformed solution to the master equation give us the deformed gauge transformations for the original fields (the indexes $\mu$, $\nu$, $\alpha$, $\beta$, $\gamma$ are flat) as

$$\tilde{\delta} e_{\varepsilon \varepsilon} h_{\mu \nu} = \partial_{(\mu} \varepsilon_{\nu)} + \lambda \left[ \frac{1}{2} h_{(\mu} \partial_{\nu)} \varepsilon^\rho - \frac{1}{2} \varepsilon^\rho \partial_{(\mu} h_{\nu)\rho} + \varepsilon^\rho \partial_\rho h_{\mu \nu} + \frac{i}{8} \tilde{C}(\mu, \nu, \varepsilon) \right]$$

with $\mathcal{L}_2^{h - A}$ given in formula (128) from Ref. [1] (with $q \rightarrow q_i$ and $q_i$ as in (187)), and

$$\frac{\tilde{D}_\mu}{2} \left( \Omega + \hat{\Omega} \right) = \frac{1}{16} \left( \frac{\tilde{D}_\mu + \hat{\Omega}_{\mu a b}}{2} \right) \gamma^{a b}.$$  (232)

We observe from (225) that only now, in the presence of all fields, the quartic gravitini vertex is permitted, by contrast to the results from Refs. [2] and [3], whereas it has been shown that gravitini allow no self-interactions in $D = 11$ if separately coupled to a graviton or respectively to a three-form gauge field.

Relying on (217) and (224), we observe that the first orders of the interacting Lagrangian, $\mathcal{L}_0 + \lambda \mathcal{L}_1 + \lambda^2 \mathcal{L}_2 + \cdots$, come from the expansion of the following Lagrangian (expressed in terms of the ‘curved’ spin-vector $\psi_\mu$ and the field strength $\bar{F}_{\mu \nu \rho}$)

$$\mathcal{L} = \frac{2}{\lambda^2} e R(\Omega) - \frac{i e}{2} \bar{\psi}_\mu \Gamma^{\mu \nu \rho} D_\nu \left( \frac{\Omega + \hat{\Omega}}{2} \right) \psi_\rho - \frac{e}{48} \bar{F}_{\mu \nu \rho \lambda} \Gamma^{\mu \nu \rho \lambda}$$

with

$$\mathcal{L}^{(2)} = \frac{\lambda \bar{k}_i}{96} e \left( \bar{F}_{\mu \nu \rho \lambda} + \bar{F}_{\rho \mu \nu \lambda} \right) \left( \bar{\psi}_\mu \Gamma^{\alpha \beta \mu \nu \rho \lambda} \psi_\beta + 2 \bar{\psi}_\mu \Gamma^{\mu \nu \rho \lambda} \right)$$

$$- \frac{4 \lambda^2 \bar{k}_i}{(12)^4} \bar{F}_{\mu_1 \mu_2 \cdots \mu_{11}} \bar{F}_{\mu_4 \cdots \mu_7} \bar{F}_{\mu_8 \cdots \mu_{11}}.$$  (233)

The notation $D_\mu \left( \frac{\Omega + \hat{\Omega}}{2} \right) \psi_\rho$ denotes the full covariant derivatives of $\psi_\rho$

$$D_\mu \left( \frac{\Omega + \hat{\Omega}}{2} \right) \psi_\rho = \partial_\mu \psi_\rho + \frac{1}{8} \left( \Omega_{\mu a b} + \hat{\Omega}_{\mu a b} \right) \gamma^{a b} \psi_\rho.$$  (234)

and

$$\bar{F}_{\mu \nu \rho \lambda} = \bar{F}_{\mu \nu \rho \lambda} + \lambda \bar{k}_i \bar{\psi}_\mu \Gamma^{\rho \lambda} \psi_\lambda.$$  (235)
\[
\delta_{e,\varepsilon} \psi_\mu = \partial_\mu \varepsilon + \lambda \left[ -\frac{1}{2} h^{\mu}_{\varepsilon} \partial_\nu \varepsilon + (\partial_\nu \psi_\mu) \varepsilon + \frac{1}{2} \psi_\rho \partial_\mu \varepsilon \right] \\
+ \frac{1}{8} \gamma^{\alpha\beta} \varepsilon \partial_\alpha h_{\beta\mu} + \frac{i k_i}{9} \left( \gamma^{\nu\rho\lambda} \varepsilon \mathcal{F}_{\nu\rho\lambda} + \mathcal{F}_{\rho\lambda\sigma} \right) \\
- \frac{1}{8} \gamma^{\nu\rho\lambda\sigma} \varepsilon \mathcal{F}_{\nu\rho\lambda\sigma} + \lambda^2 \left[ \frac{3}{8} h^{\mu}_{\nu\rho} \partial_\rho \varepsilon - \frac{1}{32} \gamma^{\alpha\beta} \varepsilon \left( -2h^\rho_\mu \partial_\alpha h_{\beta\rho} \\
+ 2h\rho_\mu \left( \partial_\beta h^\rho_\mu - \partial_\rho h^\beta_\mu \right) - \partial_\rho h^\rho_\mu \right) \\
+ \frac{1}{16} \gamma^{\nu\rho\lambda} \varepsilon \left( h_\nu^\rho \partial_\lambda \varepsilon - \varepsilon \partial_\nu h_\lambda \varepsilon + \frac{1}{8} \psi_\rho \left( h_\nu^\rho \partial_\beta \varepsilon \right) - h_\rho^\nu \partial_\mu \varepsilon \right) \\
- \frac{1}{16} \psi_\rho \varepsilon \partial_\mu h^\rho_\nu - \frac{1}{2} \left( \partial_\mu \psi_\nu \right) \varepsilon \partial_\beta \varepsilon - \frac{1}{16} \psi_\rho \varepsilon \partial_\beta \varepsilon \\
- \frac{1}{64} \gamma^{\alpha\beta} \varepsilon \partial_\alpha h_{\beta\mu} + \frac{1}{64} \gamma^{\alpha\beta} \varepsilon \left( \varepsilon \partial_\alpha h_{\beta\mu} \right) + \psi_\mu \gamma^{\alpha\beta} \varepsilon \\
- \frac{1}{9} \cdot 2^{\delta} \gamma^{\nu\rho\lambda} \varepsilon \left( \varepsilon \partial_\nu h_{\rho\lambda} \varepsilon + \frac{i k_i}{18} \gamma^{\nu\rho\lambda} \varepsilon \left( h_\nu^\rho \mathcal{F}_{\nu\rho\lambda\sigma} + \partial_\mu \left( \partial_\rho h_{\nu\lambda} \varepsilon \right) \right) \\
+ \frac{1}{144} \gamma^{\nu\rho\lambda\sigma} \varepsilon \left( \varepsilon \partial_\nu h_{\rho\lambda} \varepsilon \right) - \frac{i k_i}{144} \gamma^{\nu\rho\lambda\sigma} \varepsilon \left( \varepsilon \partial_\nu h_{\rho\lambda} \varepsilon \right) \\
+ \cdots \right] \\
+ \left( 0 \right) \delta_{e,\varepsilon} \psi_\mu + \lambda^2 \delta_{e,\varepsilon} \psi_\mu + \cdots
\]
If we introduce the notation
\[ g_{\mu\nu} = \sigma_{\mu\nu} + \lambda h_{\mu\nu}, \]
then (236) imply some gauge transformations for the metric tensor of the form
\[ \frac{1}{\lambda} \delta_{\epsilon,\gamma} g_{\mu\nu} = \tilde{\epsilon}_{(\mu;\nu)} + \frac{i\lambda}{8} \delta_{\gamma} \Gamma_{(\mu;\nu)}, \]
where
\[ \tilde{\epsilon}_{\mu;\nu} = \partial_{\mu} \epsilon_{\nu} - \Gamma_{\mu\nu}^{\rho} \tilde{\epsilon}_{\rho}. \]
Here, \( \Gamma_{\mu\nu}^{\rho} \) are precisely the (affine) connection coefficients associated with the metric (239),

\[ \Gamma_{\mu\nu}^{\rho} = g_{\rho\lambda} \Gamma_{\lambda\mu\nu}^{\rho}, \]
where \( g_{\rho\lambda} \) are the elements of the inverse of (239), and

\[ \Gamma_{\lambda\mu\nu} = \frac{1}{2} (\partial_{\mu} g_{\nu\lambda} + \partial_{\nu} g_{\mu\lambda} - \partial_{\lambda} g_{\mu\nu}) \]
stand for the standard Christoffel symbols of the first kind. In (240) quantities \( \tilde{\epsilon}_{\mu} \) are the ‘curved’ gauge parameters of the spin-2 field

\[ \tilde{\epsilon}_{\mu} = e_a^\mu \tilde{\epsilon}_a = (0)_{\mu} + \lambda (1)_{\mu} + \lambda^2 \epsilon_{(2)\mu} + \cdots \]

Using expansions (211)–(212) and transformations (236), one can show perturbatively that the gauge transformations of the vielbein fields and of their inverses read as

\[ \frac{1}{\lambda} \delta_{\epsilon,\gamma} e^a_{\mu} = e^a_{\mu} \tilde{e}_a = (0)_{\mu} + \lambda (1)_{\mu} + \lambda^2 \tilde{e} (2)_{\mu} + \cdots \]

where

\[ \tilde{e}_{\mu} = \tilde{e}_a^\mu \equiv e_a^\mu \equiv \frac{1}{8} \epsilon_{\gamma} \Gamma^\gamma_{a\mu} \psi_a. \]

respectively. Indeed, the translation and rotation gauge parameters allow the perturbative developments

\[ \tilde{e}_\mu = (0)^\mu + \lambda (1)^\mu + \cdots = (\delta_{a}^\mu - \frac{\lambda}{2} h_a^\mu + \cdots) e^a = e_a^\mu \epsilon^a \]
and

$$\epsilon_{ab} = (0) \epsilon_{ab} + \lambda (1) \epsilon_{ab} + \cdots = \frac{1}{2} \partial_{[a} \epsilon_{b]}$$

$$= \frac{\lambda}{4} \left( \epsilon^c \partial_{[a} h_{b]c} - \frac{1}{2} h^c_{[a} \partial_{b]} \epsilon_c - \frac{1}{2} \left( \partial_c \epsilon_{[a)} h^{b]}_c + i \frac{1}{4} \bar{\epsilon} \gamma_{[a} \psi_{b]} \right) \right) + \cdots \quad (248)$$

respectively. Using now (212) combined with (236), it follows that

$$\delta_{\epsilon,\varepsilon}^{(0)} e^a_{\mu} = \delta_{\epsilon,\varepsilon}^{(0)} \delta^{(0)} = 0, \quad \quad (249)$$

$$\delta_{\epsilon,\varepsilon}^{(1)} e^a_{\mu} = \frac{1}{2} \delta_{\epsilon,\varepsilon}^{(0)} h^a_{\mu} = \frac{1}{2} \left( \partial^a e^b + \partial^b e^a \right) \sigma_{b\mu}$$

$$= 0 + \partial_\mu e^a + \frac{1}{2} \partial^{[a} e^{b]} \sigma_{b\mu}$$

$$= (0)^p e^{(0)} e^a_{\mu} + (0)^a e^a_{\mu} + (0)^{a b} e^a_{\mu} = 0, \quad \quad (250)$$

$$\delta_{\epsilon,\varepsilon}^{(2)} e^a_{\mu} = \frac{1}{2} \delta_{\epsilon,\varepsilon}^{(1)} h^a_{\mu} - \frac{1}{8} \left( \delta_{\epsilon,\varepsilon}^{(0)} \partial^a \right) h^a_{\mu} - \frac{1}{8} h^{a}_{\rho} \delta_{\epsilon,\varepsilon}^{(0)} h^a_{\mu}$$

$$= \frac{1}{4} \left[ h_{m\mu} \partial^a e^m + h^a_{m\mu} - \epsilon^m \partial^a h^a_{m\mu} - \epsilon^m \partial^a h^a_{m\mu} \right]$$

$$+ 2 \epsilon^m \partial_\mu h^a_{m\mu} + \frac{1}{4} \bar{\epsilon} \gamma^{(a} \psi^{b)} \sigma_{b\mu}$$

$$- \frac{1}{8} \left( \partial^a e^m + \partial_\mu e^a \right) h^a_{m\mu} - \frac{1}{8} h^a_{m\mu} (\partial^m e^n + \partial^n e^m) \sigma_{m\mu}$$

$$= \frac{1}{2} \partial^a e^m h^a_{m\mu} + \frac{1}{2} \left( \partial^a h^a_{m\mu} \right) + \frac{1}{2} h^a_{m\mu} \partial_\mu e^m + \frac{1}{4} \partial^{[a} e^{b]} h_{m\mu} + \left( - \frac{1}{4} \epsilon^m \partial^{a} h^b_{m\mu} \right)$$

$$+ \frac{1}{8} h^a_{m\mu} \bar{\epsilon} \gamma^{(a} \psi^{b)} \sigma_{b\mu} + \frac{1}{8} \left( \partial_\mu e^{(a} h^{b]}_{m\mu} - \frac{1}{16} \bar{\epsilon} \gamma^{(a} \psi^{b)} \right) \sigma_{b\mu} + \frac{i}{8} \bar{\epsilon} \gamma^{a} \psi^{b} \sigma_{b\mu}$$

$$= (0)^p e^{(0)} e^a_{\mu} + (0)^a e^a_{\mu} + (0)^{a b} e^a_{\mu} \epsilon$$

$$+ \left( e^a_{\mu} \right)^{(1)} e^a_{\mu} + \left( e^a_{\mu} \right)^{(2)} e^a_{\mu}$$

and thus (249)–(251) are nothing but the first three orders of the gauge transformations (246).

As we specified before, all the original fields bear flat indices, so in (236)–(238) \( A_{\alpha \beta \gamma} \) means \( A_{abc} \) and \( \psi_m \) is \( \psi \). The first three orders of the gauge transformations for the gravitini, (237), can be put under the form

$$\delta_{\epsilon,\varepsilon}^{(0)} \psi_m = (0)^{\mu} e_m D_\mu \hat{\Theta} \varepsilon, \quad \quad (252)$$
We reprise the same procedure with respect to the 3-form. The first three orders of \( \delta_{e,\varepsilon} A_{abc} \) can be organized as

\[
\delta_{e,\varepsilon} A_{abc} = \left[ \frac{(1)}{D_\mu (\omega) \varepsilon_{[ab]}} \right]^{(0)} \mu \varepsilon_{c},
\]

\[
\delta_{e,\varepsilon} A_{abc} = \left[ \frac{(1)}{D_\mu (\omega) \varepsilon_{[ab]}} \right]^{(0)} \mu \varepsilon_{c} + \left[ \frac{(0)}{D_\mu (\omega) \varepsilon_{[ab]}} \right]^{(1)} \mu \varepsilon_{c}.
\]

Consequently, we can state that formulas \((252) - (254)\) originate in the perturbative expansion of the expression

\[
\delta_{e,\varepsilon} \psi_m = e_m \mu (0) D_\mu (\hat{\Omega}) \varepsilon (0) + \left( \partial_\mu \psi_m \right) (0) \varepsilon + \left( \varphi_{\mu} \psi_m \right) (0) \varepsilon + \left( \varphi_{\mu} \psi_m \right) (0) \varepsilon + \left( \varphi_{\mu} \psi_m \right) (0) \varepsilon + \left( \varphi_{\mu} \psi_m \right) (0) \varepsilon.
\]

where

\[
D_\mu (\hat{\Omega}) \varepsilon = \partial_\mu \varepsilon + \frac{1}{8} \hat{\Omega}_{\mu ab} \gamma^{ab} \varepsilon.
\]

Taking into account \((253)\), from \((254)\) we deduce the form of the gauge transformations for ‘curved’ gravitini, \( \psi_\mu = \psi_m \varepsilon_m \), as

\[
\delta_{\varepsilon,\varepsilon} \psi_\mu = D_\mu (\hat{\Omega}) \varepsilon + \lambda \left[ \left( \partial_\mu \psi_\mu \right) \varepsilon + \varepsilon_{mn} \psi_\mu \varepsilon_{ab} + \right. \]

\[
\left. \frac{1}{4} \gamma^{ab} \psi_\mu \varepsilon_{ab} \right].
\]

We reprise the same procedure with respect to the 3-form. The first three orders of \((258)\) (with \( \alpha \beta \gamma \rightarrow abc \)) can be organized as

\[
\delta_{e,\varepsilon} A_{abc} = \left[ \frac{(0)}{D_\mu (\omega) \varepsilon_{[ab]}} \right]^{(0)} \mu \varepsilon_{c},
\]

\[
\delta_{e,\varepsilon} A_{abc} = \left[ \frac{(1)}{D_\mu (\omega) \varepsilon_{[ab]}} \right]^{(0)} \mu \varepsilon_{c} + \left[ \frac{(0)}{D_\mu (\omega) \varepsilon_{[ab]}} \right]^{(1)} \mu \varepsilon_{c}.
\]
\[ + (\partial_\mu A_{abc}) (0) \psi_{[ab} \psi_{c]}, \quad (259) \]

\[ \delta_{\epsilon, \epsilon, A_{abc}} = (D_\mu (\omega) \epsilon_{[ab} \epsilon_{c]} \mu + (\partial_\mu A_{abc}) \epsilon_{[ab} \epsilon_{c]} \mu ) + A_{m}^\mu \epsilon_{[ab} \epsilon_{c]m}, \quad (260) \]

where we denoted by \( D_\mu (\omega) \epsilon_{ab} \) the net contributions of orders zero, one, and two respectively of the covariant derivative

\[ D_\mu (\omega) \epsilon_{ab} = \partial_\mu \epsilon_{ab} + \frac{1}{2} (\epsilon_{m}^a \omega_{bm} - \epsilon_{m}^b \omega_{am}). \quad (261) \]

Therefore, relations (258)–(260) are nothing but the first three orders of the general formula

\[ \delta_{\epsilon, \epsilon, A_{abc}} \epsilon_{ab} \epsilon_{[ab} \epsilon_{c]} \mu + \lambda \left[ (\partial_\mu A_{abc}) \epsilon_{[ab} \epsilon_{c]} \mu - \tilde{k}_i \tilde{\epsilon}_{\gamma_{[ab}} \psi_{c]} - \frac{i \lambda}{8} (\tilde{\epsilon}^m \psi_{[a]} A_{bc]m}, \quad (262) \]

Due to (245) and (262), we obtain the gauge transformations of the ‘curved’ 3-form, \( \bar{A}_{\mu \nu \rho} \), are given by

\[ \delta_{\epsilon, \epsilon, A_{\mu \nu \rho}} = \partial_\mu \tilde{\epsilon}_{\nu \rho] + \lambda \left[ \tilde{\epsilon}^\lambda \partial_\lambda \bar{A}_{\mu \nu \rho} + A_{\lambda [\mu \nu} (\partial_\rho] \tilde{\epsilon}^\lambda) - \tilde{k}_i \tilde{\epsilon}_{\gamma_{[\mu \nu}} \psi_{\rho]} \right], \quad (263) \]

where \( \bar{A}_{\mu \nu \rho} = \epsilon^{a}_{\mu} \epsilon^{b}_{\nu} \epsilon^{c}_{\rho} A_{abc} \), \( \tilde{\epsilon}_{\nu \rho] = \epsilon^{a}_{\mu} \epsilon^{b}_{\nu} \psi_{ab} \).

So far, we proved that the only consistent interactions in \( D = 11 \) for a spin-2 field, a massless 3-form, and a massless (Rarita-Schwinger) spinor vector complying with our working hypotheses are nothing but the first orders of the Lagrangian formulation of \( D = 11, N = 1 \) SUGRA (action (233) and gauge transformations (245), (257), and (263)). The uniqueness of \( D = 11, N = 1 \) SUGRA to all orders in the coupling constant can be shown using exactly the same procedure like in Section 6 of Ref. [1]. Thus, it can be proved that the complete deformed solution of the master equation for a spin-2 field, a massless 3-form, and a massless Rarita-Schwinger spinor, consistent at all orders in the coupling constant

\[ \hat{S} = S_0 + \lambda \hat{S}_1 + \lambda^2 \hat{S}_2 + \cdots, \quad (264) \]

coincides at each order with the solution of the master equation for \( D = 11, N = 1 \) SUGRA modulo a redefinition of the coupling constant of the type

\[ \lambda \rightarrow \lambda (1 + k_2 \lambda^2 + k_3 \lambda^3 + \cdots), \quad (265) \]

where \( (k_m)_{m \geq 2} \) are some arbitrary, real constants.
6 Conclusion

To conclude with, in this paper we have completed the cohomological BRST approach to the consistent interactions in eleven spacetime dimensions that can be added to a free theory describing a massless spin-2 field, a massless (Rarita-Schwinger) spin-3/2 field, and an Abelian 3-form gauge field. The couplings are obtained under the hypotheses of smoothness in the coupling constant, locality, Lorentz covariance, Poincaré invariance, and the derivative order assumption (the maximum derivative order of the interacting Lagrangian density is equal to two, with the precaution that each interacting field equation contains at most one spacetime derivative acting on gravitini). Our main result is that if we decompose the metric like $g_{\mu\nu} = \sigma_{\mu\nu} + \lambda h_{\mu\nu}$, then we can couple the 3-form and the gravitini to $h_{\mu\nu}$ in the space of formal series with the maximum derivative order equal to two in $h_{\mu\nu}$ such that the resulting interactions agree with the well-known $D = 11, N = 1$ SUGRA couplings in the vielbein formulation. Only now, in the presence of all fields, the cosmological term and the gravitini ‘mass’ constant are forbidden and the quartic gravitini vertex is unfolded. Although at a first sight it seems that two different theories emerge (corresponding to the two different values of $\tilde{k}$ from (187)), in fact each of them describes $D = 11, N = 1$ SUGRA since they can be obtained one from the other by the simple 3-form redefinition $A_{abc} \to -A_{abc}$. Our approach is thus a systematic, cohomological proof of the uniqueness of $D = 11, N = 1$ SUGRA.

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