Coherent Thomson backscattering from laser-driven relativistic ultra-thin electron layers

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Received: date / Revised version:

Abstract. The generation of laser-driven dense relativistic electron layers from ultra-thin foils and their use for coherent Thomson backscattering is discussed, applying analytic theory and one-dimensional particle-in-cell simulation. The blow-out regime is explored in which all foil electrons are separated from ions by direct laser action. The electrons follow the light wave close to its leading front. Single electron solutions are applied to initial acceleration, phase switching, and second-stage boosting. Coherently reflected light shows Doppler-shifted spectra, chirped over several octaves. The Doppler shift is found $\propto \gamma^2 = 1/(1 - \beta^2)$, where $\beta$ is the electron velocity component in normal direction of the electron layer which is also the direction of the driving laser pulse. Due to transverse electron momentum $p_n$, the Doppler shift by $4\gamma^2 = 4\gamma^2/(1 + (p_n/m_e)^2) \approx 2\gamma$ is significantly smaller than full shift of $4\gamma^2$. Methods to turn $p_n \rightarrow 0$ and to recover the full Doppler shift are proposed and verified by 1D-PIC simulation. These methods open new ways to design intense single attosecond pulses.

PACS. 41.75.Jv Laser-driven acceleration, – 52.38.-f Intense particle beams and radiation sources in physics of plasmas, – 52.59.Ye Plasma devices for generation of coherent radiation

1 Introduction

There is a quest for high-quality x-ray sources in many fields of science [1]. Coherent VUV- and X-ray sources now become available at XFEL facilities [2]. Also the generation of high harmonics from optical laser pulses has provided us with very useful VUV-sources recently [3].

In this paper, we deal with Thomson scattering from relativistic electrons. Incoherent Thomson scattering converts photons of incident frequency $\omega_L$ to frequencies $\omega = \gamma^2(1 - \beta \cos \theta)^2 \omega_L$ due to relativistic Doppler shift [4]; here $\beta = v/c$ is electron velocity normalized to light velocity $c$, $\theta$ is the scattering angle, and $\gamma = 1/\sqrt{1 - \beta^2}$ the usual relativistic factor. Thomson scattering has been measured from high-$\gamma$ electrons at conventional accelerator facilities [5] and recently also from laser generated electron beams [6]. In these cases, light was backscattered incoherently.

Coherently backscattered photons can be obtained from electron layers provided they are dense enough so that several backscattering electrons reside in the volume $\lambda^2_R$, where $\lambda_R$ is the wavelength of the light in the rest frame of the electron layer. When moving close to velocity of light, we refer to these layers as relativistic mirrors [7]. Such mirrors have been created recently in form of nonlinear plasma waves near the point of wave-breaking when they develop diverging density spikes; values of $\gamma_m = 4 - 5$ could be deduced from the backscattered spectrum [8].

Relativistic mirrors are also formed when irradiating solid surfaces with laser amplitudes $a_0 = e A_{L0}/m_e c^2 > 1$ [9,10,11]; here $e$ and $m$ are electron charge and mass, respectively, and $A_{L0}$ is the amplitude of the vector potential of the light wave. For $a_0 > 1$, the solid surface turns into dense plasma, and electrons move close to velocity of light. For linear polarized light, the ponderomotive force has a component oscillating at twice the laser frequency $\omega_L$. This leads to an oscillating mirror front and to high harmonics in the reflected light. These surface harmonics have been observed recently [12]. The results show harmonic orders up to 3200 corresponding to 3.7 keV photons. They are in agreement with theory [13].

In the present paper, we consider laser irradiation of ultra-thin (nm) foils [14,15,16,17] at intensities high enough to separate all electrons from ions. The electrons then move with the front of the laser pulse, forming a dense relativistic electron layer. This regime has been investigated recently by Kulagin et al. [18]. Here it is studied for Thomson backscattering of a counter-propagating probe pulse.

For separating electrons from ions, the laser field $a_0 = E_{L0}/k_L$ has to be larger than the total electric field $\varepsilon_0$ that builds up due to charge separation. Here both the laser field $E_{L0} = k_L A_{L0}$ with $k_L = \omega_L/c$ and the longitudinal field $\varepsilon_0$ are taken in units of $E_0 = m_e c^2/\omega_L$. For a planar foil with electron density $n_e$ and thickness $d$, we find $\varepsilon_0 = N k_L d$. Here $N = n_e/n_{crit} = \omega_0^2/\omega_L^2$ is the density normalized to the critical density $n_{crit}$ and can be...
conveniently expressed by the ratio of plasma frequency $\omega_p = (4\pi e^2 n_e/m)^{1/2}$ and laser frequency $\omega_L$. The condition for complete electron blow-out is

$$a_0 \gg N k_L d. \quad (1)$$

As a reference foil material, we consider in the following graphite which consists of a mono-atomic layer of carbon atoms arranged in regular hexagons with a side length of $s = 0.141 \text{ nm}$. Having an area of $A = (3\sqrt{3}/2)s^2 = 5.2 \times 10^{-16} \text{ cm}^2$ and a total of 12 electrons attached to each hexagon, the electron areal density is $n_e d = 12/A = 2.3 \times 10^{-16} \text{ cm}^{-2}$, and one finds $\varepsilon_0 = N k_L d \approx 1$. Graphene layers have been successfully produced in pieces up to $100 \times 100 \text{ mm}^2 \ [19]$. This is sufficient in principle for experiments with existing laser pulses in the range of $a_0 \approx 3 - 10$, corresponding to intensities of $10^{19} - 10^{20} \text{ W/cm}^2$. It is interesting to notice that an area of $10 \times 10 \text{ mm}^2$ contains a total charge of about $1 \text{ nC}$, comparable and exceeding values in other recent experiments using laser wakefield acceleration and gas targets. Alternatively, multi-layer carbon foils with thicknesses of a few nm are available for experiments, but require higher laser intensities for complete electron blow-out.

For illustration, a two-dimensional (2D) particle-in-cell (PIC) simulation of a graphene layer irradiated by a laser pulse is shown in Fig. 1. Electron density is plotted for three different times, and also the driving laser pulse is shown. We have here chosen a two-cycle pulse $E(\tau)/E_0 = a_0 \sin \tau$ with $0 \leq \tau \leq 4\pi$ and $a_0 = 5$. At the corresponding intensity of $3 \times 10^{19} \text{ W/cm}^2$, all electrons within the focal spot are taken as free initially, i.e. ionization is not treated explicitly. It is seen that the electrons of the irradiated spot have separated from the ions and are moving with the laser pulse, surfing as a dense layer near the pulse front. One should notice that these ultra-thin foils are transparent to the light, even though they have densities far above the critical density. This is because they are much thinner than the skin depth $\lambda_{\text{skin}} \approx c/\omega_p \approx \lambda_L/10$. The light is therefore penetrating through the foil, and each electron is accelerated according to the local laser field. This means that we can use the analytic solution for relativistic electron motion in a plane laser field as an approximation, supplemented by the space charge electric field $E_x$. It is constant in time for each sub-layer of the foil, as long as these sub-layers keep their relative order and do not overtake each other. This option for analytical treatment is an important feature of ultra-thin foil acceleration. It is quite different from usual relativistic laser plasma interactions, e.g. encountered in laser wakefield acceleration, where often complicated plasma fields dominate.

Here it is direct interaction with the external laser fields that accelerates the electrons. As we shall discuss in section 2, electrons gain momentum up to a maximum of $p_{\text{max}} = (p_x, p_y) = (2a_0^2 \gamma mc, 2a_0 \gamma mc)$ and energy up to $W_{\text{max}} = (\gamma - 1)mc^2 = 2a_0^2 \gamma mc^2$ in the first half-wave of the light pulse, at least those at the layer front where $E_x = 0$. For the laser pulse $a_0 = 5$ used in Fig.1, the electron energy rises up to $W_{\text{max}} \approx 25 \text{ MeV}$ according to these estimates, but then drops again to zero in the second half of the laser wave. This behavior reflects the Lawson-Woodward theorem [20], stating that there is no net energy transfer to charged particles by light waves in vacuum. Having in mind generation of X-rays by Thomson scattering, this is acceptable, because there is plenty of time for back-scattering of a probe pulse by the dense electron sheath while surfing on the wave. Actually, the time available is approximately $t = (3a_0^2/8) \tau_L$, which is about 10 laser periods $\tau_L$ for $a_0 = 5$. For $\gamma \approx 2a_0^2 = 50$, one might be tempted to expect coherent photons Doppler-shifted by a factor $4\gamma^2 = 10^4$. Unfortunately, this is not the case because the effective $\gamma$-factor at which the layer
is moving in normal $x$-direction is
\[ \gamma_x = 1/\sqrt{1 - \beta_x^2} = \gamma / \sqrt{1 + (p_y/mc)^2} \approx \sqrt{\gamma/2}, \]  
and the frequency is shifted by a factor \( 4\gamma_x^2 \approx 2\gamma \) rather than \( 4\gamma^2 \). The reason for this is inherent to direct laser acceleration which necessarily involves large transverse momenta \( p_y \propto a_0 \) for large \( \gamma \propto p_x \propto a_0^2 \). Electrons move under an angle \( \theta \approx p_y/p_x \propto 1/a_0 \) relative to the laser direction. For incoherent Thomson scattering, the reflected signal is emitted close to the direction of the individual high-$\gamma$ electrons and show the angle $\theta$. In the case of coherent scattering from a planar layer, however, this main component is suppressed by destructive interference and only the component scattered into the normal direction of the sheet will survive. Since incoherent scattering is proportional to the number $N_x$ of scatterers, while coherent scattering is $\propto N_x^2$, the latter is bound to win as the layer becomes dense enough. In practice, one may encounter cases intermediate between incoherent and coherent scattering and observe the change in direction, frequency, and intensity.

The present paper is intended to explore basic features of electron sheet acceleration and back-scattering in 1D geometry. The analysis is restricted to schematic laser pulses of single-cycle and half-cycle form. In section 2, we review the analytic solution of single-electron motion in a planar light wave and discuss three relevant situations: (1) acceleration of an electron initially at rest, (2) collision of a relativistic electron with a counter-propagating wave, and (3) boost acceleration of an electron entering the light wave already at relativistic energy. Results of cases (1) and (2) are then compared with 1D-PIC simulations in section 3, including back-scattering of a counter-propagating probe pulse. The coherent reflectivity of an ultra-thin relativistic layer is derived analytically in a companion paper, referred to as paper II \cite{22}. The full analytical description of the driven electron layer, including self-radiation and space charge fields, was developed by Kulagin \cite{23}; it will be applied to the present reference case in another companion paper \cite{24}.

2 Single electron in plane laser wave

2.1 Equations of motion

Here we review the analytical treatment of a relativistic electron interacting with a plane light wave in vacuum, to the extent it is needed in the following discussion \cite{25}. The linear polarized light wave propagates in $x$-direction and has electric and magnetic field components $E_y$ and $B_z$, respectively. In addition, we allow for a longitudinal $E_z$ field. The equation of motion then read

\[ \frac{dp_x}{dt} = -E_x - \beta_y B_z, \]
\[ \frac{dp_y}{dt} = -E_y + \beta_x B_z, \]
\[ \gamma^2 = 1 + p_x^2 + p_y^2, \]
\[ \frac{d\gamma}{dt} = -\beta_x E_x - \beta_y E_y, \]
\[ \frac{dx}{dt} = \beta_x p_x / \gamma, \]
\[ \frac{dy}{dt} = \beta_y p_y / \gamma. \]

Here we use dimensionless variables corresponding to dimensional ones according to

\[ (t, x, y) \equiv (\omega L t, k_L x, k_L y), \]
\[ (p_x, p_y) \equiv (p_x/mc, p_y/mc), \]
\[ (E_x, E_y, B_z) \equiv (E_x, E_y, B_z) e/mc \omega_L, \]

where $\omega_L$ and $k_L$ are circular frequency and wave-number of the light wave, $e$ and $m$ are charge and mass of the electron, and $c$ is the velocity of light. For a plane wave moving in vacuum in $x$-direction with dispersion relation $\omega_L = k_L c$, the laser fields satisfy $E_y(\tau) = B_z(\tau)$ with propagation coordinate $\tau = t - x$. For an electron moving along $x(t)$, we have $d\tau/dt = 1 - \beta_x$. With these relations, the equations of motion can be written with $\tau$ as independent coordinate in the form

\[ \frac{d}{d\tau} (\gamma - p_x) = E_x, \]
\[ \frac{d}{d\tau} p_y = -E_y. \]

Integration gives the functions $\kappa(\tau) = \gamma - p_x$ and $p_y(\tau)$, and, recalling $\gamma^2 = 1 + p_x^2 + p_y^2$, we obtain

\[ \gamma(\tau) = \frac{1 + p_x^2}{2\kappa}, \]
\[ \frac{1 + p_y^2}{2\kappa} - \frac{1}{2}, \]
\[ p_x(\tau) = \frac{1 + p_y^2}{2\kappa} - \frac{1}{2}. \]

Making use of $\kappa = \gamma - p_x = (1 - \beta_x)$, the electron trajectories $x(\tau)$, $y(\tau)$ follow from

\[ \frac{dx}{d\tau} = p_x / \kappa, \]
\[ \frac{dy}{d\tau} = p_y / \kappa, \]

in parametric form with time $t(\tau) = \tau + x(\tau)$.

Let us consider solutions for a single cycle light wave of the form $E_y = a_0 \sin \tau$, limited to the interval $0 \leq \tau \leq 2\pi$ and with no longitudinal field, $E_z = 0$. In this case, the solutions of Eqs. (11) and (12) and are

\[ p_y(\tau) = p_{y0} - a_0 (1 - \cos \tau), \]
\[ \kappa(\tau) = \gamma(\tau) - p_x(\tau) = \kappa_0, \]

where $\kappa_0$ and $p_{y0}$ are integration constants. We now discuss three important cases.

2.2 Acceleration of electron initially at rest: $\kappa_0 = 1$

An electron initially at rest with $p_{x0} = p_{y0} = 0$ and $\gamma_0 = 1$ corresponds to $\kappa_0 = 1$. In this case the explicit solution is

\[ p_y(\tau) = -a_0 (1 - \cos \tau), \]
The second half, one has to shift the transverse momentum and the light wave, unless additional fields interfere. This corresponds to the Lawson-Woodward theorem \[20\]. For a clear interpretation of this theorem, one should refer to Esarey et al. [21]. In order to continue acceleration also in the middle, and \( E_x = 1 \) to the left-hand surface facing the ions of the graphene layer. For units see text.

For \( \tau = \pi \), the electron reaches the maximum energy of \( \gamma_1 = 1 + 2a_0^2 \) at time \( t_1 = \pi + x(\pi) = \pi + (1 + 3a_0^2/4) \), and momenta and positions then are \( p_{x1} = 2a_0 \), \( p_{y1} = -2a_0 \), \( x_1 = (3\pi/4)a_0^2 \), and \( y_1 = -\pi a_0 \). Momenta \( p_x \) and \( p_y \) are plotted in Fig. 2 as function of time. It is seen that they grow while interacting with the first half of the driving wave (\( 0 \leq \tau \leq \pi \)); afterwards, in the second half, the particle decelerates again and comes to rest at \( \tau = 2\pi \). There is no net energy transfer between a particle and the light wave, unless additional fields interfere. This corresponds to the Lawson-Woodward theorem [20]. For a clear interpretation of this theorem, one should refer to Esarey et al. [21]. In order to continue acceleration also in the second half, one has to shift the transverse momentum \( p_y \) at \( \tau = \pi \) from negative to positive values. In principle, this is possible by interaction with an additional counter-propagating laser pulse and will be discussed next as a second interesting solution of the basic equations.

\[
\begin{align*}
p_x(\tau) &= \frac{a_0^2}{2}(1 - \cos \tau)^2, \\
\gamma(\tau) &= 1 + \frac{a_0^2}{2}(1 - \cos \tau)^2, \\
y(\tau) &= a_0 \int_0^\tau (\cos \tau' - 1) d\tau' = -a_0(\tau - \sin \tau), \\
x(\tau) &= \frac{a_0^2}{2} \int_0^\tau (\cos \tau' - 1)^2 d\tau' \\
&= \frac{a_0^2}{2}(3\tau - 2 \sin \tau + \frac{1}{4} \sin 2\tau), \\
t(\tau) &= \tau + x(\tau).
\end{align*}
\]

For \( \tau = \pi \), the electron reaches the maximum energy of \( \gamma_1 = 1 + 2a_0^2 \) at time \( t_1 = \pi + x(\pi) = \pi + (1 + 3a_0^2/4) \), and momenta and positions then are \( p_{x1} = 2a_0 \), \( p_{y1} = -2a_0 \), \( x_1 = (3\pi/4)a_0^2 \), and \( y_1 = -\pi a_0 \). Momenta \( p_x \) and \( p_y \) are plotted in Fig. 2 as function of time. It is seen that they grow while interacting with the first half of the driving wave (\( 0 \leq \tau \leq \pi \)); afterwards, in the second half, the particle decelerates again and comes to rest at \( \tau = 2\pi \). There is no net energy transfer between a particle and the light wave, unless additional fields interfere. This corresponds to the Lawson-Woodward theorem [20]. For a clear interpretation of this theorem, one should refer to Esarey et al. [21]. In order to continue acceleration also in the second half, one has to shift the transverse momentum \( p_y \) at \( \tau = \pi \) from negative to positive values. In principle, this is possible by interaction with an additional counter-propagating laser pulse and will be discussed next as a second interesting solution of the basic equations.
as \( \Delta \gamma = \Delta p_x = 2a_0^2/(1 + 4a_0^2) \approx 1/2 \). This takes place in the short time and space interval \( \Delta x \approx \Delta t \approx \pi/2 \) corresponding to a quarter cycle. A half-cycle laser pulse of same strength as the drive pulse, but running in opposite direction, is therefore sufficient to switch the phase of the electron from deceleration back to acceleration. In case the switch pulse extends over a full cycle, the situation of both half- and full-cycle pulses will be compared with 1D-PIC simulation in section 3.

2.4 Boosting energy of a relativistic electron: \(|\kappa_0| \ll 1\)

A third, very instructive solution of Eqs. (11) to (13) is obtained for \(|\kappa_0| \ll 1\). Here we start with an electron that has already been accelerated by the first half-wave of case 1 and has interacted with the half-wave switch pulse of case 2. Initial momenta and energy now are

\[
\begin{align*}
p_{y2} &= 0, \\
p_{x2} &= -\frac{1}{2(1 + 4a_0^2)} + \frac{1 + 4a_0^2}{2}, \\
\gamma_2 &= \frac{1}{2(1 + 4a_0^2)} + \frac{(1 + 4a_0^2)}{2}, \\
\kappa_2 &= \gamma_2 - p_{x2} = \frac{1}{1 + 4a_0^2}.
\end{align*}
\]

Entering with these values into the negative half-wave of \( E_y = a_0 \sin \tau \) at time \( \tau = \pi \), the electron moves in the time interval \( \pi \leq \tau \leq 2\pi \) according to

\[
\begin{align*}
p_y(\tau) &= a_0(1 + \cos \tau), \\
p_x(\tau) &= \frac{(1 + 4a_0^2)(1 + a_0^2(1 + \cos \tau))^2}{2} - \frac{1}{2(1 + 4a_0^2)}, \\
\gamma(\tau) &= \frac{(1 + 4a_0^2)(1 + a_0^2(1 + \cos \tau))^2}{2} + \frac{1}{2(1 + 4a_0^2)}. \\
\end{align*}
\]

For \( \tau = 2\pi \), we find \( \gamma_2 \approx p_{x2} \approx 8a_0^4 \), and the leading orders of the shifts in x and y direction are \( \Delta x \approx 12a_0^6 \pi \) and \( \Delta y \approx 4a_0^3 \pi \), always supposed that \( a_0 \gg 1 \). Choosing a laser pulse with \( a_0 = 5 \) and wavelength \( \lambda_L = 0.8 \, \mu m \), the single-cycle sin-pulse accelerates the electron to an energy of \( W = \gamma mc^2 = 2.5 \, GeV \). For this it needs a distance of \( \Delta x \approx 6a_0^3 \lambda_L \approx 7.5 \, cm \) and a time interval of \( \Delta t \approx \Delta x/c \approx 250 \, ps \).

We show these results here as an interesting application of the one-dimensional relativistic equations of motion. Of course, the example is difficult to realize experimentally for several reasons. It is not clear how to produce and transport the half-cycle switch pulse. It has a spectrum with a non-zero dc component and cannot be transported over a longer distance due to strong diffraction. An alternative method to switch the transverse momentum \( p_y \to 0 \) would be a local static \( B_z \) field. This would require \( |p_y| = a_z B_L D = 2a_0 \), where \( D \) is the thickness of the magnetic layer and \( a_z = B_z/B_0 \) with \( B_0 = mcw_L/e \approx 10^4 \, T \cdot \mu m/\lambda_L \). Since the switch layer should not be thicker than a few \( \lambda_L \), we would need a magnetic field in the order of \( B_z \approx B_0a_0/10 \approx a_0 \, kT \), which is quite demanding, but may be not impossible.

2.5 Charge separation and longitudinal fields \( E_x \)

So far, we have considered solutions for single electrons only. In the case of a solid-density layer and the expulsion of all electrons, large longitudinal fields \( E_x \) will emerge due to space charge and will modify the solutions given above. For a plane foil of thickness \( d \) and electron density \( n_e \), the electric field between the layers of the expelled electrons and the ions left behind is given by

\[
\varepsilon_0 = Nk_Ld, \tag{10}
\]

where \( \varepsilon_0 \) is the electric field in units of \( E_0 = mc\omega_L/e \) and

\[
N = n_e/n_{crit} = \omega_p^2/\omega_L^2 \tag{11}
\]

is the electron density, normalized to the critical density \( n_{crit} = \pi mc^2/(e\lambda_L)^2 \approx 1.1 \times 10^{24} \, cm^{-3}/\lambda_L^2 [\mu m] \). Electrons, initially located at \( x = x_0 \) inside the electron layer, experience the electric field

\[
E_x = \varepsilon_0(1 - x_0/d), \tag{12}
\]

which is largest for \( x_0 = 0 \) on the laser-irradiated front side of the foil and vanishes at \( x_0 = d \) on the rear side. Neglecting the initial time period of electron expulsion from the foil, which is short for ultra-thin foils, the field \( E_x \) is constant in time. This holds as long as the total charges to the left and to the right of \( x_0 \) stay the same, i.e. sub-layers do not change their relative order. We can then integrate \( dk/d\tau = E_x \) and find

\[
k(\tau) = 1 + \varepsilon_0(1 - x_0/d)\tau. \tag{13}
\]

The full solution is obtained by integrating Eqs. (11) - (13). In Fig. 2 we have also plotted the curves \( p_z(t) \) including the effect of \( E_x \) for the surface on the laser side (\( x_0/d = 0 \)) and the middle interface (\( x_0/d = 0.5 \)) of the electron layer. For graphene the fields are \( E_x = 1 \) and \( E_x = 0.5 \), respectively. It is seen that the back of the ions is very strong when compared to the leading front of the layer at \( x_0/d = 1 \). Only at early times the layer moves as a whole, but then electrons facing the ions lag behind and even turn around and fall back to the ion layer.

It should be noticed that the evolution of the electron layers is still more complex. Almost quantitative agreement with 1D-PIC simulations is obtained when applying the full analytical model of Kulagin [23].

3 Simulation

3.1 Acceleration of ultra-thin electron layer

We have performed one-dimensional particle-in-cell (1D-PIC) simulations to investigate thin layer acceleration and
Fig. 3. Results of 1D-PIC simulation: (a) Snapshot of laser field $E_y$ and electron density $n_e$ after one laser cycle when the single-cycle pulse has just passed the graphene layer located at $x = 0$. Electrons are seen as a thin layer surfing on the first half-cycle and marked by sharp density spikes at the surface facing the drive laser. (b) Values of $\gamma$ and $\gamma_x$ of electrons, located at the right surface of the layer, are plotted as function of $t$.

Fig. 4. Thomson backscattering of a semi-infinite probe pulse by the electron layer shown in Fig. 3. The weak probe pulse with $a_p = 0.001$ is polarized perpendicular to the drive pulse, and its sharp front reaches the foil at $t = 0$. (a) Signal versus time as seen by an observer at $7 \lambda_L$ behind foil, (b) corresponding spectrum, which shows a sharp cutoff at $\omega/\omega_L \approx 4 \gamma_x^2 \approx 36$, well corresponding to the maximum value of $\gamma_x \approx 3$ in Fig. 3b.

Thomson back-scattering. For illustration, we choose as before a single-cycle laser pulse $E_y = a_0 \sin \tau$ with $0 \leq \tau \leq 2\pi$, $a_0 = 5$, and a layer with $\varepsilon_0 = 1$, corresponding to graphene. A snapshot of electron sheet acceleration is shown in Fig. 3a and the temporal evolution of $\gamma(t)$ and $\gamma_x(t) = \gamma/(1 + p_y^2)^{1/2}$ in Fig. 3b. All electrons are blown out of the foil. The electron density distribution is plotted here three laser cycles after the laser front has first interacted with the foil. The thickness of the electron layer is set to $d/\lambda_L = 0.001$ initially and expands to $\Delta x/\lambda_L \approx 0.06$. The density profile shows a pronounced peak at the surface facing the drive laser. It surfs on the laser wave and has substantially depleted the front part of the wave due to energy transfer to electrons. The electrons move close to velocity of light, but lag somewhat behind the light front at $x/\lambda_L = 3$. Electron $\gamma$-values corresponding to the density spike are seen to rise to a maximum of $\gamma \approx 19$ and then fall back to unity. Though this behavior is qualitatively similar to that of the analytical solution displayed in Fig. 2, the peak value is far less than the analytical maximum of $\gamma = 2a_o^2 = 50$. The deviation is attributed to modifications in the radiation field due to
energy depletion and self-radiation of the layer. Both effects are not taken into account in the present analytical treatment.

3.2 Thomson backscattering

We now add a weak probe pulse to the simulation above. It propagates antiparallel to the drive pulse and is polarized orthogonal to it so that the fields of probe and drive pulse can be clearly separated in the simulation. The probe pulse is sinusoidal and semi-infinite with a sharp front, which first hits the foil at $t = 0$, simultaneously with the drive pulse. The back-scattered signal is shown in Fig. 4a, as detected by an observer located at $x/\lambda_L = 7$. The scattered pulse is strongly chirped due to the acceleration of the electron layer. Notice that it is also strongly compressed. It vanishes after $t/\tau_L \approx 8$ due to density decay of the layer. The corresponding spectrum in Fig.4b shows a broad plateau with a sharp cut-off at $\omega/\omega_L \approx 40$. This corresponds nicely to $\gamma_{x,\text{max}} \approx 3.2$ (see Fig. 3b) and the relativistic Doppler shift $\omega/\omega_L = \gamma_{x,\text{max}}^2 (1 + \beta_{x,\text{max}})^2 \approx 39$.

Apparently, only the velocity component $\beta_x$ in the normal direction of the electron layer and the corresponding
\[ \gamma_x = 1/\sqrt{1 - \beta_x^2} = \gamma/\sqrt{1 + \beta_y^2} \] is relevant for the Doppler shift. The full shift of \( \omega/\omega_L = 4\gamma_{\text{max}}^2 \approx 1400 \), expected for \( \gamma_{\text{max}} \approx 19 \) (see Fig. 3b), is suppressed by the transverse momentum \( p_y \) of the electrons in the layer. Of course, in the present 1D-PIC simulation accounting for three momentum coordinates \( (p_x, p_y, p_z) \), but only one spatial coordinate \( x \), scattered radiation is obtained only in \( x \)-direction. But also in 2D/3D PIC simulation, coherently scattered radiation will not be emitted into the oblique direction of electron motion. This is because of the plane symmetry of the coherently accelerated electrons and destructive interference of radiation scattered into directions outside a very narrow cone in normal direction. Incoherently back-scattered radiation may be observed in other directions, in particular, when the density of the layer has decreased strongly, but this incoherent radiation scales with the number scatterers \( N_{\text{sc}} \) rather than \( N_{\text{sc}}^2 \) for coherent scattering and is therefore much weaker.

### 3.3 Reflected attosecond pulses after single-cycle switch pulse

In order to see the full \( 4\gamma^2 \) shift, one has to turn the transverse momentum \( p_y \to 0 \) by some additional interaction. Such interaction can be provided by a short counter-propagating switch pulse. As we have discussed in section 2.3, this pulse has to have similar strength as the drive pulse and same polarization. Here we present a simulation in which a single-cycle switch pulse, having \( a_0 = 2.5 \) and propagating in \((x, -z)\)-direction, hits the electron layer at time instant \( t/\tau_L = 1 \), when the \( \gamma \)-factor of the electrons is \( \approx 11.5 \) as seen in Fig. 5b. We choose this early interaction time, because it is found that the layer rapidly expands in this case and that the coherently scattered signal would be significantly weaker, had we waited until \( t/\tau_L = 5 \), when \( \gamma_{\text{max}} \approx 19 \) is reached.

In Figs. 5a and 5b, the effect of the switch pulse is demonstrated for electrons located at the right surface of the electron layer (see Fig. 3a). The transverse momentum makes a sudden excursion to \( p_y \approx 0 \), while the longitudinal momentum \( p_x \) is hardly affected. This correlates to a sudden jump in \( \gamma_x \) to almost \( \gamma_x \approx \gamma \approx 11.5 \). We expect that this interaction causes a burst of short wavelength radiation from the coherently scattered probe pulse. Indeed, we observe two spikes of attosecond duration in the reflected wave (see Fig. 6a and zoom in Fig. 6b), a smaller one at \( t/\tau_L = 7.285 \) and a prominent one at \( t/\tau_L = 7.345 \). Correspondingly, the scattered spectrum (Fig. 6c) exhibits strongly enhanced emission in the frequency range \( \omega/\omega_L \approx 40 - 200 \). Apparently, the two spikes, separated by a time interval of \( \Delta t/\tau_L \approx 0.06 \), originate from scattering at front and rear side of the electron layer (see Fig. 3a). They are separated by a distance \( \Delta x/\lambda_x \approx 0.06 \). Actually, the two signals interfere and produce the conspicuous oscillations seen in the high-frequency part of the spectrum with \( \Delta \omega/\omega_L \approx 1/0.06 \approx 17 \). This interference documents the coherence of two back-scattered signals, emitted from electron interfaces at different depth in the layer.

Similar to the harmonics spectra arising from reflection from plasma surfaces [13], the high-frequency spectrum falls off according to a power law up to \( \omega/\omega_L \approx 4\gamma^2 \approx 450 \), consistent with \( \gamma_x \approx \gamma \approx 11 \) at switch time. Also the spectral part beyond \( \omega/\omega_L \approx 450 \) is characteristic for attosecond bursts, showing no clear cut-off. The power law exponent is approximately \(-2\), i.e. the fall-off is smaller than for the value \(-8/3\) observed for surface harmonics [12, 13]. We attribute the smaller exponent to the fact that \( p_y(t) \) is just touching the \( p_y = 0 \) axis in Fig. 5a rather than intersecting it, as it does in the case of surface harmonics (compare Pukhov et al. [26]).

### 3.4 Relativistic mirror after half-wave switch pulse

Let us finally discuss how a half-cycle switch pulse would change the results. Again we apply an additional pulse \( E = E_x \sin(\tau - \tau_s) \) with \( 0 \leq \tau = t + x \leq \pi \), the phase \( \tau_s \) chosen such that it hits the electron layer at \( t/\tau_L \approx 1 \), having same polarization as drive pulse and an amplitude \( E_x \) such that \( p_x \to 0 \) is obtained approximately. The results depicted in Fig. 7a and 7b show that we indeed find \( p_y \approx 0 \) and \( \gamma_x \to \gamma \approx 11 \) for \( t/\tau_L > 1 \) when selecting the appropriate parameters. Probing this relativistic mirror as before by a perpendicularly polarized probe pulse, we find the back-scattered signal of Fig. 8a with the spectrum of Fig. 8c. On top of the reflected signal without switch pulse (compare Fig. 4a), we observe a high-frequency feature (zoomed in Fig. 8b), now showing many fast oscillations after the switch pulse has turned the electron momentum into normal direction. The zoom in Fig. 8b exhibits two regions, low-amplitude fast oscillations for \( 13.285 \leq t/\tau_L \leq 13.345 \) corresponding to the front interface of the electron sheet and high-amplitude chirped oscillations corresponding to the prominent density spike at the rear side of the layer, which is already decelerating. Since the probe pulse now sees the full \( \gamma \approx 11 \) over an extended time, the reflected spectrum exhibits a clear peak at \( \omega/\omega_L \approx 4\gamma^2 \approx 480 \).

### 4 Conclusions

In this paper, we have explored coherent Thomson backscattering from relativistic electron layers, blown out from ultra-thin solid foils by high-contrast few-cycle laser pulses. For this to happen, the laser amplitude \( a_0 \) has to be larger than the electrostatic field \( \varepsilon_0 \) building up when separating all electrons from ions in the foil. It requires ultra-thin foils: already a mono-atomic carbon layer (graphene, used here as reference material) produces \( \varepsilon_0 \approx 1 \). A second requirement is ultra-high contrast of the laser pulses to avoid premature foil destruction. Though demanding, these conditions may be met in near-future experiments.

Here we have investigated the generation of relativistic electron layers by schematic single-cycle laser pulses with \( a_0 = 5 \), making use of well known analytical theory and 1D/2D PIC simulation. The ultra-thin foils are transparent to light, and electron acceleration occurs by direct
Fig. 7. Same as Fig. 5, but for a half-cycle switch pulse. In this case, the switch pulse turns (a) the transverse momentum $p_y \to 0$ and (b) $\gamma_x \to \gamma \approx 11$ for a longer period of time.

Fig. 8. (a,b) Thomson signal and (c) spectrum of probe pulse corresponding to the case of Fig. 7, but with observer position moved to $x = 13\lambda_L$. The backscattered signal now exhibits fast oscillations in the time interval $13.25 \leq t/\tau_L \leq 13.40$, shown as zoom in (b). The small amplitude oscillations for $13.275 \leq t/\tau_L \leq 13.335$ correspond to the right edge of the electron layer (see Fig. 3a). It acts as a relativistic mirrors moving with $\gamma_x \approx \gamma \approx 11$ and generates the sharp peak at $\omega/\omega_L \approx 4\gamma_x^2 \approx 484$ in the spectrum. The larger amplitude oscillations, seen in Fig. 8b for $13.335 \leq t/\tau_L \leq 13.375$, correspond to the density spike at the left layer surface. This peak is already decelerating, and this causes the negative chirp.

action of the laser fields, admitting an approximate analytical description in terms of single-electrons in a plane light wave. Equivalent electrons then move in planes, and coherent backscattering of probe radiation occurs according to normal mirror rules due to phase selection. Conditions for coherent backscattering are discussed in a companion paper (paper II), and the scaling of reflectivity with foil density, thickness, and $\gamma$-factor are derived there [22].

An important point is that, due to the laws of direct laser acceleration, high-$\gamma$ electrons necessarily have high transverse momentum $p_y$ and move under an angle relative to the direction of laser propagation, which is also the normal direction of the electron layer. The relativistic Doppler shift of reflected light is therefore reduced to $\omega/\omega_L \approx 4\gamma_x^2 = 4\gamma^2/(1 + p_y^2)$, since only $\gamma_x = 1/\sqrt{1 - \beta_x^2}$ related to normal $x$-direction is relevant. Methods to switch $p_y \to 0$ by an additional counter-propagating laser pulse or by a static magnetic field have been discussed. Various cases are demonstrated by 1D-PIC simulation.

Probe radiation backscattered from a driven electron layer typically shows a reflected signal with a broad chirped
spectrum and a cut-off at $\omega/\omega_L \approx 4\gamma^2$. The chirp is due to layer acceleration and may well extend over several octaves. Disrupting the smooth layer motion by a counter-propagating single-cycle switch pulse changes the spectrum of the back-scattered probe radiation significantly. Having an appropriate amplitude, the switch pulse turns $p_y \rightarrow 0$ for a short moment, which is much shorter than a laser cycle. The probe radiation then experiences the full $4\gamma^2$ Doppler shift for this short moment, and a single attosecond burst of high-frequency radiation is backscattered. A broad spectrum in the region of $\omega/\omega_L \approx 4\gamma^2$ is observed, which looks quite similar to the surface harmonics, recently observed experimentally [12]. We have also investigated the more speculative case of a unipolar half-wave switch pulse which may turn $p_y \rightarrow 0$ for a longer time interval. The PIC simulation then shows an extended period of full $4\gamma^2$ Doppler shift and, correspondingly, a strong peak at $\omega/\omega_L \approx 4\gamma^2$ in the spectrum.

Though the long term goal is to use the scattered probe radiation for flexible generation of controlled attosecond VUV- and X-ray pulses, the immediate application will be to analyze the electron layer and its angular and temporal evolution in all detail. The present study is only a first step. Many questions arise concerning what happens with more realistic few-cycle pulses than those used here. Also mono-atomic foils may not be available immediately, and acceleration of foils a few nanometer thick needs to be studied. This is planned for subsequent investigations.

5 Acknowledgements

The authors would like to thank D. Habs, F. Krausz, A. Pukhov, S. Rykovanov, and L. Veisz for helpful discussions concerning this paper. H.C. Wu thanks the Alexander von Humboldt Foundation for a scholarship. This work was supported in part by the DFG project Transregio TR18, by the Munich Centre for Advanced Photonics (MAP), and by the Association EURATOM - Max-Planck-Institute for Plasma Physics.

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