A TOPS Improved Algorithm Based on Eigenvalue Discrimination

Zhang Jinfeng, Li Lichun
University of Information Engineering, Henan zhengzhou, 450001

Abstract: Under low SNR, if the signal subspace estimation of TOPS (Test of orthogonality of projected subspaces, TOPS) algorithm reference frequency point is inaccurate, the error will spread to the whole frequency point, thus causing the problem of algorithm performance degradation. In this paper, a TOPS improved algorithm based on eigenvalue discrimination is proposed. This method selects the optimal reference frequency point by judging the distinguishing degree of characteristic value between signal subspace and noise subspace at each frequency point, and then uses the transfer matrix to project the signal subspace at the optimal reference point to each frequency point, reducing the error of the whole frequency band caused by the inaccurate selection of reference frequency point. This algorithm has higher resolution accuracy under low SNR, and the threshold of 100% resolution probability SNR is about 4dB lower than the original algorithm. Simulation results verify the effectiveness of the proposed algorithm.

1. INTRODUCTION

Wideband signals are widely used in civil and military high-tech fields, such as radar, sonar, medical imaging, etc, due to their many advantages such as carrying a large amount of information, being conducive to target detection and information extraction [1]. Wideband signal DOA estimation as an important content in the field of array signal processing, it is developed on the basis of narrow-band direction-finding algorithm, but also different from narrowband DOA estimation algorithm, because the array of broadband signal flow pattern is the joint function of frequency and orientation, to make the different frequency of the signal subspace, so the classic Capon algorithm [2] and subspace algorithm [3][4][5] and other narrowband kind of direction finding algorithms cannot be directly applied to the wideband signal direction finding.

At present, wideband direction-finding algorithms can be roughly divided into two categories. One is methods based on statistical inference, such as maximum likelihood method [6] and Bayesian algorithm [7]. This kind of algorithm has better estimation performance, but contains multi-dimensional optimization process and high computational complexity, which is difficult to be applied in practical projects. The other is algorithm based on signal subspace, Incoherent signal-subspace Method (ISBM) [8] and Coherent signal subspace Method (CSSM) [9]. The ISSM algorithm is simple, but it has poor performance under low SNR, and cannot be applied to Coherent signal sources. CSSM algorithm can distinguish coherent sources and has better estimation performance under low SNR. However, this algorithm requires Angle estimation and pre-selected reference frequency points, and DOA estimation accuracy is easily affected by prior information. In recent years, scholars have applied the idea of sparse reconstruction to DOA estimation and proposed many DOA estimation methods based on sparse representation and reconstruction [10][11][12], which have good estimation accuracy and spatial resolution. However, the key of such methods lies in how to establish the broadband signal sparse
model and select the appropriate sparse reconstruction algorithm, and the performance of the algorithm depends on the model parameters[1].

Yoon et al proposed TOPS algorithm in 2003 [13]. In this algorithm, a diagonal matrix related to frequency and Angle is used to project the signal subspace on the reference frequency point to each frequency point, and then a new matrix is constructed with the noise subspace. DOA estimation is obtained by judging the lack of rank of the matrix. This algorithm does not need Angle estimation, but if the signal subspace estimation at the reference frequency point is not accurate, false peak will appear in the spectral peak search process, and the estimation performance is poor under low SNR. To solve these problems, scholars have proposed Modified TOPS algorithm [14], Square TOPS algorithm [15], Weighted TOPS algorithm [16] and the Test of Orthogonality of Frequency Subspaces (TOFS) [17]. These methods improve the performance of the algorithm to some extent, but the selection of the reference frequency points has not been solved.

This paper mainly discusses and studies the optimal reference frequency point selection based on TOPS algorithm, proposes the signal subspace and noise subspace eigenvalue discrimination method, and selects the optimal reference frequency point by judging the maximum difference between the eigenvalues of each frequency point. This method can be applied to some improved algorithms such as TOPS algorithm, square TOPS algorithm and weighted TOPS algorithm. The improved algorithm has higher direction-finding accuracy than the original algorithm, lower Rmse, the SNR threshold of 100% resolution probability is reduced by about 4dB compared with the original algorithm.

2. Wideband signal model

Suppose there are P Wideband signals in the far field space, and the bandwidth is $B = [f_L, f_H]$. The Angle of incidence to a uniform linear array composed of M elements is $\Omega = \{\theta_1, \theta_2, \ldots, \theta_P\}$. Element spacing is the half a wavelength of the highest frequency. The signal received by the Mth element is:

$$x_m(t) = \sum_{p=1}^{P} s_p(t - \tau_m) + n_m(t)$$

In the formula: $s_p(t)$ is the Pth wideband signal, $\tau_m = (m - 1)d \sin \theta_p/c$. $c$ is the speed of light, $n_m(t)$ is a complex Gaussian white noise with zero mean and $\sigma_n^2$ variance.

The observation time $T_0$ is divided into $K$ segments, The DFT transform at point J is performed on the receiving data $x_m(t)$ of the element, and the received data of the entire element is expressed as:

$$\mathbf{X}_k(f_j) = A(f_j, \theta) \mathbf{S}_k(f_j) + \mathbf{N}_k(f_j)$$

In the formula, $\mathbf{X}_k(f_j), \mathbf{S}_k(f_j), \mathbf{N}_k(f_j)$ are the frequency domain expressions of the received data, signal and noise of the array at $f_j, A(f_j, \theta)$ represents the array manifold of $f_j$. Its expression as:

$$A(f_j, \theta) = [a(f_j, \theta_1), a(f_j, \theta_2), \ldots, a(f_j, \theta_P)]^T$$

The covariance matrix of $\mathbf{X}(f_j)$ is:

$$R(f_j) = E[\mathbf{X}(f_j)\mathbf{X}^H(f_j)] = A(f_j, \theta) \mathbf{R}_s(f_j) A^H(f_j, \theta) + \sigma_n^2 I_M$$

In the formula, $R_s(f_j) = E[\mathbf{S}(f_j)\mathbf{S}^H(f_j)]$, $I_M$ is the $M \times M$ dimensional unit matrix.

If the signal sources is unrelated, By eigenvalue decomposition of the covariance matrix $R_s(f_j)$, the signal subspace $\mathbf{U}_j$ and the noise subspace $\mathbf{G}_j$ can be obtained at the point of $f_j$:

$$\mathbf{U}_j = [v_{j,1} \ v_{j,2} \ \cdots \ v_{j,P}]$$

$$\mathbf{G}_j = [v_{j,p+1} \ v_{j,p+2} \ \cdots \ G_{j,p+M}]$$
\( \psi_{k\lambda} \) is the eigenvector corresponding to the eigenvalues, which are arranged from the largest to the smallest.

3. TOPS ALGORITHM

The basic idea of TOPS algorithm is to use diagonal transformation matrix to judge whether signal subspace and noise subspace are orthogonal with each possible direction within the incidence Angle range. If orthogonality exists, the direction is the true signal incidence direction.

The transition matrix of TOPS algorithm is a matrix related to frequency and angle. The Kth element can be expressed as:

\[
[\Psi(f_j, \theta_j)]_{k\lambda} = \exp\{-j2\pi f(k-1)d \sin \theta_j/c\}
\]

(6)

The direction vector of the array is \( a(f_0, \theta_0) \), after the transformation matrix projection, the new array direction vector is

\[
a(f_k, \theta_k) = \Psi(f_j, \theta_j)a(f_0, \theta_0)
\]

(7)

Where, frequency and signal incidence angle satisfy the following formula:

\[
f_k = f_l + f_j
\]

\[
\sin \theta_k = \frac{f_j}{f_k} \sin \theta_l + \frac{f_l}{f_k} \sin \theta_j
\]

(8)

The above formula shows that any array flow pattern matrix can be transformed to the specified frequency by transformation matrix. If \( \theta_l = \theta_j \), then \( \theta_k = \theta_l = \theta_j \), so, the guide vector can be transformed from frequency point \( f_l \) to frequency point \( f_k \) without changing its DOA information.

When \( \Delta f = f_j - f_l \), literature\(^{(13)}\) proves that the following two range spaces are the same:

\[
\text{span}\{\Phi(\Delta f, \phi)F_j\} = \text{span}\{A(f_j, \theta)\}
\]

(9)

where \( \phi \) is the assumed azimuth angle, the value of \( \hat{\theta} \) angle is related to \( \phi \) and satisfies the following relation:

\[
[\hat{\theta}] = \arcsin \left\{ \frac{f_j}{f_k} \sin \theta_l + \frac{2f_l}{f_j} \sin \phi \right\}
\]

(10)

The above formula shows that a matrix can be constructed, and when the assumed azimuth is equal to a real incidence Angle, the rank of the constructed matrix will be missing.

According to formula (7) and (10), it can be obtained that when \( M \geq 2P, K \geq P + 1 \), the signal subspace at the frequency point \( f_j \) is \( U_j(\phi) \), which can be expressed as:

\[
U_j(\phi) = \Phi(\Delta f_j, \phi)F_0, \quad j = 1, \ldots, l
\]

(11)

where, \( \Delta f_j = f_j - f_l \); \( \phi \) is the assumed incidence direction of the source, and \( F_0 \) is the signal subspace at the reference frequency point.

Therefore, the signal subspace and noise subspace at each frequency point can be obtained, and a judgment matrix of \( P \times j(M-P) \) can be constructed:

\[
D(\phi) = [U^H_1 G_1 U^H_2 G_2 \cdots U^H_l G_l]
\]

(12)

From the previous analysis, we can obtain:

1) When the assumed signal incidence angle \( \phi \) is equal to the true incidence angle \( \theta \), \( D(\phi) \) will lack rank.

2) When the assumed signal incidence angle \( \phi \) is not equal to the true incidence angle \( \theta \), \( D(\phi) \) will be full rank.

In practical engineering applications, due to the presence of noise, the estimation of signal subspace will have certain errors, so \( D(\phi) \) generally will not have rank absence. SVD (singular value decomposition) of \( D(\phi) \) is usually adopted. DOA estimation can be obtained from the angle.
corresponding to the minimum singular value $\sigma(\theta)_{\text{min}}$:

$$\hat{\theta} = \arg\max_{\phi} \frac{1}{\sigma(\phi)_{\text{min}}}$$  \hspace{1cm} (13)

DOA estimation can be obtained by using equation (13) to search spectral peaks.

4. SELECTION OF REFERENCE FREQUENCY POINTS

TOPS algorithm avoids the estimate of the impact of the initial angle, focusing matrix is omitted, and reduces the computational complexity, but the algorithm under low SNR estimation performance is weak, susceptible to pseudo peak, analysis the reason, it is mainly due to the core of the algorithm is to reference frequency point of the signal subspace by transfer matrix projection to other frequency points, transfer matrix is a diagonal matrix related to the array of flow patterns, the antenna array is determined, array flow model can accurately said, so the reference frequency point signal subspace to build the decline in performance is likely to be the main factor, this is because under the low SNR, the signal subspace will leak to the noise subspace to different degrees. If the reference frequency point is not selected properly, the estimation error of the signal subspace will be extended to the entire frequency band, resulting in the decline of algorithm performance. To solve this problem, this paper proposes to discriminate the maximization of different eigenvalues and select the optimal reference frequency point, because at each frequency point, the signal subspace of leakage has different degrees of noise subspace. After the eigenvalue decomposition of the covariance matrix $R(f_j)$, the degree of leakage at each frequency point can be judged according to the eigenvalue differentiation degree at each frequency point. the greater the eigenvalue differentiation, the orthogonality of signal subspace and noise subspace will be better.

Assume that the eigenvalues corresponding to the signal subspace and the noise subspace at frequency point $f_j$ are $\lambda_{j,s}$ and $\lambda_{j,n}$, where $\hat{\lambda}_{j,s} = \{\hat{\lambda}_{j,1} \cdots \hat{\lambda}_{j,p}\}$, $\hat{\lambda}_{j,n} = \{\hat{\lambda}_{j,1} \cdots \hat{\lambda}_{j,\hat{p}}\}$. The distinguishing degree of eigenvalues is defined as follows:

$$\eta_j = \overline{\lambda_{j,s}} - \overline{\lambda_{j,n}}$$ \hspace{1cm} (14)

where $\overline{\lambda_{j,s}} = \frac{1}{p} \sum_{i=1}^{p} \lambda_{j,i}$, $i = 1, \cdots, p; \overline{\lambda_{j,n}} = \frac{1}{M-p} \sum_{i=p+1}^{M} \lambda_{j,i}$, $i = p+1, \cdots, M$ the reference frequency point can be obtained from equation (15).

$$k_0 = \arg\max_{\phi} \{\eta_j\}$$ \hspace{1cm} (15)

Then the signal subspace at the optimal reference frequency point is projected to each frequency point by the transfer matrix.

Usually, the null space projection matrix of the signal subspace is used to eliminate the estimation error of the signal subspace. Its projection matrix $P_j(\phi)$ is defined as follows:

$$P_j(\phi) = I - \left( a_j^H(\phi)a_j(\phi) \right)^{-1} a_j(\phi)a_j^H(\phi)$$ \hspace{1cm} (16)

The projected matrix can be expressed as:

$$U'_j(\phi) = P_j(\phi)R(f_j)P_0$$ \hspace{1cm} (17)

Finally, the modified matrix is formed from equation (12), and $U'_j(\phi)$ is replaced by $U_j(\phi)$.

The algorithm steps of this paper are as follows:

1) Segmenting the received data in the time domain and performing DFT for each segment to obtain the covariance matrix $R(f_j)$ of each frequency point;

2) Eigenvalue decomposition of $R(f_j)$ is performed to obtain signal subspace $U_j$ and noise subspace $G_j$ at each frequency point;

3) The reference frequency point $k_0$ and signal subspace $U_{k_0}$ are obtained by using Equations (14) and (15).
4) Using the transfer matrix \( \Phi(f_j, \theta_j) \), the signal subspace \( U_{k_5} \) is transferred to each frequency point, and the null space is projected.

5) Equation (12) is used to generate the judgment ma-trix \( D(\phi) \) for each possible incidence Angle;

6) The singular value decomposition of matrix \( D(\phi) \) is carried out, and the local P maximum points are obtained through one-dimensional Angle search, and the signal direction is obtained.

5. SIMULATION
In order to verify the effectiveness of the proposed algorithm, the proposed method is compared with the classical TOPS method. The simulation conditions are as follows: the number of uniform linear array is \( M=8 \), the array element spacing is the half wavelength corresponding to the highest broadband frequency, the farfield independent broadband signal, the lowest frequency is 100HZ, the highest frequency is 6100HZ, and the array noise is complex Gaussian white noise.512 fast beats, 512 DFT points, and 100 Monte Carlo experiments.

5.1 Spatial spectrum with different SNR
The direction of the two independent far field wideband signals is -10º and 30º respectively. Figure 1 shows the spatial spectrum estimated by DOA when the SNR is -4, 0 and 10dB respectively.

From the simulation results in Figure 1, the proposed algorithm has a sharper spectral peak, lower sidelobe, higher estimation accuracy, and better overall performance than the original algorithm. Especially under low SNR, the proposed algorithm can accurately estimate the source direction, while the original algorithm's estimation error is between 1º and 4º.
5.2 DOA estimation accuracy and effective estimation probability under different SNR

Simulation conditions remain unchanged, SNR changes from -6dB to 15dB, and the root-mean-square error of DOA estimation is defined as \( \text{RMSE} = \frac{1}{p} \sum_{p=1}^{P} \frac{1}{N} \sum_{n=1}^{N} (\hat{\theta}_{n,p} - \theta_p)^2 \). \( \hat{\theta}_{n,p} \) is the experimental result of the nth signal, \( \theta_p \) is the true incidence Angle of the P signal, when \( |\hat{\theta}_{n,p} - \theta_p| \leq 1^\circ \), it is an effective estimate. Figure 2 shows the RMSE of DOA estimation under different SNR, Figure 3 shows DOA effective es-timation probability under different SNR.

![FIG. 2 RMSE under different SNR](image1)

![FIG. 3 Effective estimation probability under different SNR](image2)

From Figure 2 and Figure 3, it shows that under the condition of low SNR, the angle estimation variance of the proposed algorithm is slightly lower than the original algorithm. With the improvement of SNR, the performance of this algorithm is improved obviously, and the SNR threshold of 100% effective resolution probability decreases by about 4dB. This is because the reference frequency points selected by the algorithm in this paper can better ensure the orthogonality of signal subspace and noise subspace, and effectively avoid the error spread to the whole frequency band due to the inaccurate selection of reference frequency points.

5.3 DOA effective estimation probability of different reference frequency points

Under the same conditions, 9 reference frequency points (700, 1450, 2200, 2950, 3250, 3700, 4450, 5200, 5950) were randomly selected to calculate the DOA effective estimation probability, the SNR was 0dB. Other simulation conditions remain the same.

![Effective estimation probability of different frequency](image3)

Figure 4: Effective estimation probability at different reference frequency points

As shown in Figure 4, the effective estimation probability of DOA of this algorithm is above 0.9, and the effective estimation probability of other reference frequency points is between 0.5 and 0.9. It can be seen that the reference frequency points selected by this algorithm are better.
6. CONCLUSION
For traditional TOPS algorithm under low SNR, estimated that the problem of poor performance, this article obtains from the select reference frequency point, reduce the reference frequency point signal subspace to spread to the whole band of error, and effectively improve the DOA estimation performance of the original algorithm, especially under low SNR, the algorithm in this paper estimate error is reduced about 17% compared to the original algorithm, effectively estimate probability increased by about 15%, 100% effective estimate probability SNR threshold decreased 4 db, the simulation results verify the effectiveness of the proposed method. Meanwhile, this algorithm is also applicable to modified TOPS, square TOPS, weighted TOPS and other improved algorithms.

REFERENCES
[1] Chen Mingjian, Hu Zhenbiao et al. New Direction-of-Arrival Estimation Method for Wideband Sources Using Weighted TOPS [J]. Journal of Data Acquisi-tion and Processing, 2019, 34(3): 453-461.
[2] Rubseman M, Pesavento M. Maximally robust capon beamformer[J]. IEEE Trans on Signal Processing,2013,61(8):2030-2041
[3] Li Shuai, Chen Hui.DOA Estimation of Coherent Sources in the Presence of Impulsive Noise [J].Radar Science and Technology, 2017, 15(2): 178-184.
[4] Qian Cheng, Huang Lei, SO H C. Improved unitary root-music for DOA estimation based on pseudo-noise resampling[J].IEEE Signal Processing Letters,2014,21(2):140-144
[5] Ye Zhongfu, Luo Dawei et al. Review for Coherent DOA Estimation Technique[J]. Journal of Data Acquisi-tion and Processing, 2017, 32(2): 258-265.
[6] Chen J C, Hudson R E, Yao K. Maximum-like source localization and unknown sensor location estimation for wideband signals in the near field[J]. Signal Processing, IEEE Transactions on,2002,50(8): 1843-1854
[7] Feng Mingyue, He Minghao et al. DOA Estimation for Coprime Array Based on Fast Sparse Bayesian Learning Using Bessel Priors [J]. Journal of Electonices & Information Technology, 2018, 40(7): 1604-1644.
[8] Han K, Nehorai A. Wideband gaussian source processing using a linear nested array[J]. IEEE Signal Processing Letters,2013,20(11):1110-1113.
[9] Kumar D S, Hinduja I S, Mani V V.DOA estimation of IR-UWB signals using coherent signal processing[C]// IEEE 10th International Colloquium on Signal Processing and Its Application, Kuala Lumpur, Malaysia: IEEE,2014:288-291.
[10] Shen Q,Liu W, Cui W, et al.. Focused compressive sensing for underdetermined wideband DOA estimation exploiting high-order difference coarrays [J]. IEEE Signal Processing Letters,2017(99): 86-90
[11] Sun Bing,Ruan Hualin et al. Direction of Arrival Estimation with Coprime Array Based on Toeplitz Covariance Matrix Reconstruction [J]. Journal of Electonices & Information Technology, 2019, 41(8):1924-1930
[12] Wang Y, Chen B, Zheng Y, et al. Joint power distribution and direction of arrival estimation for wideband signals using sparse Bayesian learning [J]. IET Radar Sonar & Navigation,2017,11(1):52-59
[13] Yoon Y S, Kaplan L M, Mcclellan J H. TOPS: New DOA estimation for wideband signals [J]. IEEE Transactions on Signal Processing, 2006,54(6): 1977-1989.
[14] Shaw A K. Improved wideband DOA estimation using modified TOPS (mTOPS) algorithm [J]. IEEE Signal Processing Letters,2016,23(12):1697-1701.
[15] Okane K,Ohnishi T. Resolution improvement of wideband direction of arrival estimation “squared TOPS”[C]//2010 IEEE International Conference on Communications Foundations of Computational Mathematics.CapeTownSouthAfrica:IEEE,2010:1550-3607.
[16] Hayashi H, Ohtsuki T. DOA estimation for wideband signals based on weighted squared TOPS [J]. Eurasip Journal on Wireless Communications & Networking ,2016(1):243-254.
[17] Yu H, Liu J, Huang Z, et al. A new method for wideband DOA estimation [C]// International Conference on Wireless Communication, Networking and Mobile Computing. Shanghai, China: IEEE, 2007: 598-601.