Joint Optimization of a Bilevel Logistics Network and an Arc Interdiction Location-routing Problem

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**Abstract**

There is a high interest in optimization of transportation and logistics networks due to its high impact on the economic performance of supply chain networks. This paper presents a bilevel mixed-integer programming model as well as a solution method to manage distribution process in a logistic network, where two decision makers, called distributor and interdictor, make efforts to achieve their contradictory targets. This problem, known as arc interdiction location-routing problem (AI-LRP), is in fact a new, extended version of the classical LRP. The distributor strives to deliver goods to customers with minimal risk and cost, while the interdictor, by contrast, endeavors to disrupt products flow through a few critical arcs. AI-LRP has wide applications in reality, including distribution of particular goods like money, precious metals, hazardous materials, and prisoners that may need security measures. The interplay between two decision makers is formulated as a bilevel model. To solve the model, a novel genetic algorithm (NGA) is devised in which the density ordered heuristic of the knapsack problem is applied to generate an initial population of solutions. Computational results illustrate that NGA outperforms a commercial solver in terms of computational time and quality of solutions.

**Keywords**

Logistics, Network Interdiction, Location-routing Problem, Stackelberg Game

**1. Introduction**

Recent researches on the interdiction problem show that terrorists mainly tend to bring chaos by interrupting service delivery or exacerbating emergency situations rather than murdering. Examples can be seen in Afghanistan, where a terrorist group called Taliban attacked to telecommunication masts, and in Northern Ireland where an assault happened on ambulance stations or a threat to attack to electrical grids [1]. Therefore, any attempts to combat such attacks would improve the stability and reliability in supply chain networks. This is the primary reason why there is a growing interest among not only scholars but also business owners in taking defense-attack strategies to deal with probable intelligent threats. The combination of reliability analysis with game theory is a common approach that many researchers have adopted to formulate the interplay between distributor and interdictor. Other initiatives, moreover, have been taken to provide insights into interdiction location-routing networks, including adversarial risk analysis, system survivability, and interdiction networks [2]. Network interdiction addresses distributor-interdictor models, known as Stackelberg game, in which decisions made by two intelligent players, called distributor and interdictor, affect network performance [3]. The distributor tries to optimize his objective function, for example, detecting the shortest path [4], maximizing shipping flow [5, 6], and minimizing the maximum profit [7]. On the other hand, the interdictor take steps to hinder the distributor’s optimal plan by attacking some network arcs so as to reduce the profit, to increase the traversing time, or to increase transportation cost. However, there is a restricted budget available to the interdictor that limits the amount of potential damage to the network [8]. With regards to the bilevel programming, any decision taken by either player would

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significantly affect circumstances and objective function of the other player.

Location-routing problem (LRP) has been identified as one of the most integrated logistics problems. Simultaneous attention to these two concepts seems necessary [9]. It consists of two primary problems, each of which is known as a hard combinatorial optimization problem; facility location problem (FLP) and vehicle routing problem (VRP). Simultaneous consideration of these two concepts will complicate the problem [10-12]. Although these two problems have traditionally been taken into account at different levels of planning, it has been shown that the integrated approach of LRP can reduce the overall cost of the long-term planning horizon significantly. This problem is still of extreme scientific interest because of the increasing structural complexity of the models due to the constraints [13, 14]. In LRP, a set of capacitated candidate depots, a set of customers with known demands and a fleet of vehicles with predefined capacity are involved. The fixed costs of establishing the depots and the use of vehicles are considered as well. The total demands of customers allocated to each depot must not exceed its capacity. Furthermore, each vehicle has to begin and end its route in a single depot. The problem is to decide what depot to open and what vehicle routes to choose while total cost, including opening the facilities, employing vehicles and traveling, is minimized. Distribution networks, emergency and reverse logistics are some examples of practical LRP applications [15-16].

In the field of transportation, although random disruptions may adversely affect delivery of goods, interdiction plan meticulously designed by smugglers or attackers appear to be by far more devastating. In logistics systems with security considerations, such as transportation of fuel, hazardous material, money or prisoners, classical LRP are incapable of handling interdiction-related assumptions. On the other hand, the traditional LRP perspective by a decision maker alone does not allow us to simultaneously look at the strategies associated with a committed decision maker with conflicting objectives [7]. In reality, considering the development of LRP strategies, a single-decision maker cannot respond to the various needs associated with the possibility of having multiple conflicting decision makers [17-18]. As far as we know, no research on this version of LRP has been published, and this paper is the first attempt to address these essential needs [19].

1. Applications and Modeling Approaches
In recent years, there has been a growing interest in interdiction-based problems amongst not only researchers but also policy-makers. Greater efforts, however, have been made to deal with two main problems: Facility interdiction problem and network interdiction problem. In general, facility interdiction problem has a focus on attacking the most critical facilities, so that this field has drawn more attention after a terrorist attack to the World Trade Center in 2001 [20]. As pioneers, Church et al. devised two mathematical formulations for R-Interdiction Median (RIM) and R-Interdiction Covering (RIC) problems [21].

Nevertheless, the emergence of interdiction models dates back to 1964 when Wollmer [6] investigated the problem of removing some predetermined arcs in a network to minimize the maximum flow between origin and destination nodes. The concept of network interdiction and related issues have been developed in several fields such as drug trafficking, military planning, protection of power transmission networks against terrorist attacks, checking contagious diseases and trafficking of nuclear materials. Table 1, that is an extension of what Bidgoli and Kheirkhah [7] has already presented, summarizes the application areas of network interdiction problems.

A variety of network interdiction problems has been studied in the literature, including the maximum flow, shortest path and distribution, as well as routing. One well-studied problem in network interdiction is the “maximum flow network interdiction problem”, which has been considered by many researchers [5, 6, 22-25]. This problem minimizes the maximum source to sink flow through a network by attacking a few arcs with limited interdiction resources. Another network interdiction problem is the “shortest path interdiction problem”. In this problem, some arcs are interdicted by interdictor to maximize the shortest path length, and to impose an additional cost on the network while the distributor delivers goods to customers. This problem was first presented by Fulkerson and Harding [4] and followed later by other researchers [8, 19, 26].

Recently, another network interdiction was carried out in the field of routing problem. In work of Bidgoli and Kheirkhah [7], vehicle routing problem (VRP) was investigated under network interdiction assumption, in which a distributor attempts to maximize his profits through a network restricted by an interdictor, whose resources are constrained. Information asymmetry was considered for this problem that is similar to the assumption made by Bayrak and Bailey [19]. They formulated the problem as a bilevel model, and developed two meta-heuristic algorithms to solve the model.

To the best of our knowledge, fairly intense investigations have been conducted into facility interdiction problems, yet routing problems in network interdiction have received scant attention. Moreover, since decisions on locating new facilities as well as on routing can significantly affect network interdiction, this study takes these issues into account as well. To do so,
network interdiction is embedded in location-routing problem (LRP), which results in a new problem named Arc Interdiction Location-Routing Problem (AI-LRP). As Salhi and Rand [13] have proven, integrating location and routing problem can make substantial cost savings compared to a situation in which these two problems are solved separately. This study formulates the AI-LRP as a bilevel programming model.

1. 2. Solution Approaches for Bilevel Models

Generally, the bi-level models are classically known as NP-hard due to complexity of a sequence optimal decisions for two levels [54-56]. In this regard, heuristics and metaheuristics are popular in the literature [57-58]. In this regard, Table 2 illustrates some solution-based studies on the bilevel model in chronological order, including the author name, year of publication, kind of problem and the method used to solve the problem.

| Author(s)                     | Application area                        |
|-------------------------------|-----------------------------------------|
| Yaghlaene et al. [2]          | Network survivability                   |
| Bidgoli and Kheirkhah [7]     | Vehicle routing problem                 |
| Sadeghi et al. [8], Zhang et al. [27] | Transportation network                |
| McMasters and Mustin [22], Ghare et al. [23] | Military operations     |
| Akgun et al. [24], Afshari Rad and Kahki [28], Zenklusen [29], Altner et al. [30] | Network flow             |
| Wood [25], Washburn and Wood [31] | Drug interdiction               |
| Banusiewicz [32], Garcia [33], Brown et al. [34] | Security-related areas |
| Apostolakis and Lemon [35], Aksen and Aras [36] | Terrorism                           |
| Morton et al. [37], Pan et al. [38], Witt [39] | Nuclear smuggling interdiction |
| Pan [40]                      | Border control                         |
| Assimakopoulos [41]          | Infection control in hospitals          |
| Lim and Smith [42]           | Supply chain networks                   |
| Garg and Smith [43], Luss and Wong [44], Clarke and Anandalingam [45] | Network survivability analysis |
| Salmeron et al. [46]         | Protecting electric power grids against terrorist attacks |
| Anandalingam and Apprey [47] | Conflict resolution                     |
| Jiang and Liu [48]           | Water supply networks                   |
| Granata et al. [49]          | Critical Disruption Path               |
| Rocco et al. [50]            | Shortest-path network                   |
| Fathollahi-Fard et al. [51]  | Closed-loop supply chain                |
| Hajighaee-Keshhteli and Fathollahi-Fard [52] | Distribution network |
| Jabbarzare et al. [53]       | Illicit Supply Chains                   |

2. PROBLEM STATEMENT

The concept of network interdiction was first introduced by Bidgoli and Kheirkhah [7] in the area of VRP. As already mentioned, this type of problem is formulated by a Stackelberg game between an intelligence interdictor and a distributor. The interdictor’s target is to maximize its dominance over a network of system services in accordance with available interdiction budget. On the other hand, taking initiatives to fulfill customers’ demands, the distributor aims to minimize transportation cost and risk [20].
TABLE 2. Some various solution approaches for solving bilevel models

| Author(s)          | Year | Type of problem | Solution approach                        |
|--------------------|------|----------------|------------------------------------------|
| Bard and Falk [59] | 1982 | general max-min problem | Branch and bound algorithm               |
| Bialas and Karwan [60] | 1984 | Two-level linear programming | Extreme point algorithm                  |
| Bard [61]          | 1988 | Quadratic bilevel programming | Branch and bound algorithm               |
| White and Anandalingam [62] | 1993 | Two-level linear programming | Penalty function method                  |
| Vicente et al. [63]  | 1994 | Quadratic bilevel programming | Descent method                           |
| Israeli and Wood [64] | 2002 | Shortest path | Decomposition method                      |
| Muu and Van Quy [65] | 2003 | Quadratic bilevel programming | Branch and bound algorithm               |
| Arroyo and Galiana [66] | 2005 | Terrorist threat problem | Reformulation                            |
| Sun et al. [57]    | 2008 | Location of logistics distribution centers | Heuristic algorithm                      |
| Bayrak and Bailey [19] | 2008 | Shortest path | Reformulation                            |
| Ramirez-Marquez [26] | 2010 | Shortest path | Evolutionary algorithm based on PSDA      |
| Kuo and Han [67]   | 2011 | Supply chain management | Hybrid of GA and PSO                      |
| Lan et al. [17]    | 2011 | inventory control of deteriorating items | Genetic algorithm                        |
| Aksen and Aras [31] | 2012 | Facility location with imminent attack | Tabu search algorithm                     |
| Wei et al. [68]    | 2013 | Regional Bus Scheduling Problems | Genetic algorithm                        |
| Angulo et al. [69] | 2014 | expansion of highway networks | Particle swarm optimization algorithm      |
| Szeto and Jiang [70] | 2014 | transit route design and frequency setting | Hybrid artificial bee colony              |
| Camacho-Vallejo et al. [71] | 2015 | humanitarian logistics | Reformulation and reduction method        |
| Aliakbarian et al. [1]  | 2015 | hierarchical facilities | SA, VDNS, hybrid SA- VDNS                |
| Nandi et al. [72]   | 2016 | Interdicting attack graphs | Exact (branch and bound) and heuristic algorithm |
| Saranwong and Likasiri [73] | 2017 | distribution center problem | four heuristic algorithms                  |

In AI-LRP, there are two decision makers (distributor and interdictor) with conflicting objective functions. The distributor seeks to minimize total risk and transportation cost through the network, while the interdictor tries to disrupt the network by interdicting some selected arcs [78]. In reality, at the lower-level problem, the distributor with incomplete information about the interdictor’s efforts seeks to supply its customers economically, with the lowest transportation cost and fine paid. On the other hand, the interdictor at the upper-level problem, with no information about routing attempts to maximize his/her penalties by forcing the distributor to take out the low-risk arcs by imposing a fine on some specific arcs. Fortunately, the interplay between the interdictor and the distributor in AI-LRP with asymmetric information, known as Stackelberg game, can be formulated as a bilevel model.

The AI-LRP can be defined with the aid of a graph $G=(V,A)$, where $V = \{ i,j \}$ is a set of vertices in which $I$ and $J$ represent the candidate depots and customers nodes, respectively, $A = \{(i, j): i, j \in V \}$ is the set of arcs. Each arc $(i, j)$ has a non-negative cost (distance) $TC_{ij}$ that is measured using Euclidian distance, and triangular inequality holds (i.e., $TC_{ij} + TC_{jk} \geq TC_{ik}$). Each customer $j \in J$ has a demand of $D_j$, Capacity $P$, and fixed cost of opening depot $OC_i$ are taken into account for depot $i \in I$. Moreover, a capacity $Q$ and fixed operating cost $FC$ are related to a limited fleet of homogeneous vehicles. The following notations are applied to represent a bilevel model for AI-LRP.

**Sets:**

$I$: Set of candidate depots indexed by $i$, where $I=\{1,2,\ldots,M\}$ and $M$ is the number of depots

$J$: Set of customer nodes indexed by $j$, where $J=\{1,2,\ldots,N\}$ and $N$ is the number of customers

$V$: Set of all points: $V = I \cup J$ and $V=\{1,2,\ldots,M,M+1,M+2,\ldots,M+N\}$

$A$: Set of arcs $(i, j)$ connecting every pair of nodes $i, j \in V$ with $

$K$: Set of vehicles indexed by $k$ and $K=\{1,2,\ldots,K\}$ in which $K$ is the number of vehicles

**Parameters:**

$H$: Risk factor conversion to cost
B: Total interdiction budget
Q: Capacity of a vehicle. It is assumed that all vehicles are homogeneous
P_i: Capacity of candidate depot i
D_i: Demand of customer at node j
OC_i: Fixed cost of opening a depot at candidate site i
FC: Fixed cost of employing a vehicle
TC_i,j: Traveling cost or distance for each arc (i, j) ∈ A
F_i,j: The fine imposed on traversing through arc (i, j) ∈ A
\( \overline{F}_i,j \): The estimation of fine on traversing through arc (i, j)
\( R_{i,j} \): The risk associated with traversing a vehicle through arc (i, j) ∈ A
\( B_{i,j} \): The budget required for interdiction arc (i, j) ∈ A

**Decision variables:**

\[ z_i = \begin{cases} 1 & \text{if a depot at candidate site } i \text{ is opened} \\ 0 & \text{otherwise} \end{cases} \]

\[ y_{i,j} = \begin{cases} 1 & \text{if customer node } j \text{ is assigned to the depot at candidate site } i \\ 0 & \text{otherwise} \end{cases} \]

\[ w_{i,j} = \begin{cases} 1 & \text{if arc } (i, j) \text{ is interdicted by the interdictor} \\ 0 & \text{otherwise} \end{cases} \]

\[ x_{i,j,k} = \begin{cases} 1 & \text{if arc } (i, j) \text{ is traveled by vehicle } k \\ 0 & \text{otherwise} \end{cases} \]

\[ u_{i,j,k} = \text{Auxiliary variables for sub-route elimination constraints in route } k. \]

The corresponding bilevel model for the AI-LRP is demonstrated in Relaions (1)-(16):

\[ \text{[Upper_level]}: \max_{x, z, w} \quad \text{OF}_i = \sum_{i,j,k} (F_{i,j} + HR_{i,j} x_{i,j,k} w_{i,j}) \quad (1) \]

Subject to

\[ \sum_{i,j} B_i w_{i,j} \leq B \quad (2) \]

\[ w_{i,j} \leq \sum_{k} x_{i,j,k} \quad \forall \ i, j \in V \quad (3) \]

Where

\[ \text{[Lower_level]}: \quad \arg\{ \min_{z, y, w} \quad \text{OF}_L = \sum_{i,j} OC_{i,j} + \sum_{i,j,k} FC_{i,j,k} + \sum_{i,j,k} gC \overline{F}_{i,j,k} \} \quad (4) \]

Subject to

\[ \sum_{i,j} D_i x_{i,j,k} \leq Q \quad \forall \ k \in K \quad (5) \]

\[ \sum_{j,k} D_{j,k} y_{i,j,k} \leq P_{i,j} \quad \forall \ i \in I \quad (6) \]

\[ \sum_{i,j,k} x_{i,j,k} = 1 \quad \forall \ j \in J \quad (7) \]

Objective Functions (1) and (4) are related to the interdictor and distributor, respectively. By this we mean that \( \text{OF}_i \) indicates the objective function of Upper-level’s problem, and \( \text{OF}_L \) computes the objective function of Lower-level’s problem. In Equation (1), the interdictor attempts to disrupt some arcs in the network so that the risk and fine imposed on the distributor for traversing through the network is maximized. On the other hand, in the objective Function (4), the distributor takes evasive actions by choosing optimal routes in a way that total cost of the interdicted network, including the sum of the fixed depot costs, the fixed costs of employing vehicles, the transportation costs, and the fines paid are minimized. In these equations, \( w \) is the decision vector of the upper-level problem, and \( x \) is the decision vector of the lower-level one. The relationship between \( w \) and \( x \) can be represented by a so-called response function, which is denoted by \( x = x(w) \). The contributory factor in dealing with the bilevel model is to obtain the response function through solving the lower-level problem and substitute \( x(w) \), that is a function in terms of \( w \), for \( x \) in the upper-level problem. Needless to say, the response function connects the upper and lower-level decision variables, which makes the two programming models dependent on one another. By this we mean that optimizing only one level cannot optimize the overall system performance, which indicates the nature of the response function.

Constraints (2) represent the total resources needed to disrupt the arcs that must not exceed the available budget. Constraints (3) make sure that the interdictor...
can only attack those arcs through which a route is passed, meaning that if there is no planned route which runs across the arc \((i, j)\), the relevant interdiction variable \((w'_{ij})\) will inevitably take zero. Constraints (5-16) are related to the formulation of the constrained location-routing problem (CLRP). Indeed, Constraints (5) and (6) guarantee that all customer nodes are served within vehicle capacity and are assigned within depot capacity, respectively. Constraints (7) assure that each customer should be visited by exactly one route and that each customer is reachable through one preceding node from the route. The constraints of the sub-route elimination are expressed in (8). Continuity of routes and return to the starting depot are ensured through Restrictions (9). Constraints (10) state that each vehicle must be employed once. Constraints (11) warrant that a customer has to be assigned to a depot only if there is a route between them. Finally, Constraints (12)-(16) specify the variables used in the formulation.

A straightforward solution scheme for the proposed model is to hierarchically solve the upper and lower-level problems. For example, the lower-level problem (Formulation (4)-(16)) is first solved optimally after substituting all \(w'_{ij}\) with 1, and then the obtained solution is checked in the constraints relevant to the upper-level problem (i.e., Constraints (2)-(3)). If the solution meets these constraints, then the lower-level objective function (i.e., Relation (4)) can be swapped with Constraint (17).

Otherwise, we need to solve the lower-level problem and check the results in upper-level constraints, but this time with substituting zero for one \(w'_{ij}\) based on heuristic information. This process continues until a feasible solution is achieved to the upper-level problem. In Relation (17), \(OF^*_L\) is the optimum value of the lower-level problem. In fact, in this way, the bilevel programming problem simplified as a single-level programming problem, and consequently the upper-level objective function can be solved by considering objective Function (1) and Constraints (2)-(3) and (5)-(17).

\[
\sum_{i\in I} OC_i z_i + \sum_{i\in I} \sum_{j\in J} FC_{ij} + \sum_{i\in I} \sum_{k\in K} (JC_{ij} + w'_{ij} F_{ij}) x_{ij} \leq OF^*_L 
\]  

(17)

The steps of simplifying the bilevel model into a single-level one can be summarized as follows:

Step 0: Set \(w'_{ij}=1\) for all \(i, j \in V\). Determine the set of \(A_{w,ij}\) for each arc \((i,j)\) and sort it in descending order.

Step 1: Solve the lower-level problem (formulation 4-16) and determine \(OF_L\) and vector \(x\).

Step 2: Check the vector \(x\) in constraints (2-3). If the solution meets these constraints, then go to step 3; otherwise, go to step 4.

Step 3: Set \(OF^*_L\) with \(OF_L\). Replace the relation (4) with relation (17), then stop.

Step 4: Set \(w'_{ij}=0\) for the arc \((i,j)\) that has the highest value in set \(A_{w,ij}\). Then withdraw arc \((i,j)\) from the set of \(A_{w,ij}\), and go to step 1.

3. SOLUTION APPROACH

In this section, a Novel Genetic Algorithm (NGA), firstly introduced by Holland [78], is presented to solve the AI-LRP. To develop the NGA, at first, an initial population of solutions based on density ordered heuristic of the knapsack problem is generated through a few steps. Next, the representation scheme of the solutions, the selection operation of an individual, the crossover and mutation operators are shown by some figures. Then, the population of solutions is evaluated by a proper fitness function. After that, the best solutions are applied to form the next generation. This improvement procedure will be continued until the termination condition (max generation) is matched. When a better solution is obtained, the past best-known solution will be supplanted by the new best solution.

The NGA begins with generating an initial population of solutions. This step has been inspired by the density ordered heuristic of the knapsack problem. Successful applications of this algorithm have been reported in many studies [79]. The procedure is based on the “Route first-cluster second” strategy, which consists of constructing a few giant routes followed by splitting each into some feasible sub-routes. Firstly, a giant route must be constructed for each opened depot in such a way that begins from the depot and serves a set of customers, whose total demands are commensurate with the depot’s capacity. In this route, each customer is visited once and the route finishes at the starting depot, like in VRP, considering depot’s capacity constraint. Then, this route is divided into a number of sub-routes where each one represents a feasible route.

In more details, let an AI-LRP test instance, with \(M\) candidate depots and \(N\) customer nodes. The method begins with calculating the density vector between each candidate depot and all customer nodes, named by \(DV_{M,N}\). The density vector of \(DV_{M,N}\) is calculated by the following heuristic formula:

\[
DV_{M,N}(i) = \sum_{j=1}^{N} \frac{(P_i - D_j)^{\alpha} F_{ij}^{\beta} HR_{ij}^{\gamma}}{B_{ij}^{\theta}} \quad \forall i \in I
\]  

(18)

where parameters \(P_i, D_j, F_{ij}, H_i\) and \(R_{ij}\) have previously been introduced. Parameters \(\alpha, \beta, \gamma\) and \(\theta\) are considered as heuristic information that significantly affects the solution’s quality and their optimal levels.
should be calculated. In section 5, a Multiple Linear Regression analysis is carried out to identify their best values.

To create the first giant route, the depot \( i_1 \) with the greatest value of \( DV_{MN} \) is selected to be opened (i.e., \( i_1 = \max \{ DV_{MN}(i) \} \)). The opened depot \( i_1 \) is considered as a starting point for the giant route. In the next step, customer node \( j_1 \) should be included in the giant route based on the highest density value with the opened depot (i.e., \( j_1 |DM_a(j_1) = \max \{ F_{j_1}^D \frac{HR_{j_1}^D}{D_{j_1}^D B_{j_1}^D} \} \forall j \in J \)). Then, the next customer, whose density value with the previous included customer \( j_1 \) in the density matrix is the highest, is selected to visit. Density value, indicated by \( DM_a \), is computed as follows:

\[
DM_a(j_{next}) = \max_j \left\{ F_j^D \frac{HR_j^D}{D_j^D B_j^D} \right\} \quad \forall j \in J \setminus \{j_1, \ldots, j_1\}
\]  

(19)

Equation (19) indicates that the higher fine and risk, and the lower demand a customer has, the more likely to be selected as the next customer. The process continues until either all customers are inserted into the giant route(s) or the depot’s capacity exceeds. If the depot’s capacity is outstripped, another giant route should be constructed. Similarly, the new giant route begins from a new, unopened depot which has the greatest density value with non-assigned customers. The process finishes when all customers are visited by a giant route. Needless to say, this procedure yields only one solution. Other initial solutions, however, can be produced by selecting a random first customer \( j_1 \) instead of applying the relation \( j_1 |DM_a(j_1) = \max_j \left\{ F_j^D \frac{HR_j^D}{D_j^D B_j^D} \right\} \forall j \in J \).

Consequently, the final step of “Route first-cluster second” strategy is applied to build some feasible sub-routes based on the ordering customer nodes in each giant route. To do this, the splitting process keeps on adding the customers one by one to the first sub-route until the vehicle capacity constraint exceeds. Then, a second sub-route starts to cover those customers who have not visited yet in the giant route, and continues adding customers. Eventually, the process ends when all customers in the giant route are visited by a sub-route. The splitting process, finally, breaks each giant route into some feasible sub-routes, considering the vehicle’s capacity. Each sub-route serves as many customers as possible, and they are visited in the same order as in the giant route. The output of the first phase in the NGA includes pop_size initial solutions. Each solution contains a few opened depots, a set of feasible sub-routes along with a visiting plan for each sub-route. The generation of the initial population through the first step in the NGA can be illustrated by Algorithm 1:

**Algorithm 1: Route first-cluster second strategy to create the initial population.**

**Input:** Test instance of AI-LRP

1. Calculate the density vector between unopened depots and all non-assigned customers (\( DV_{MN}(i) \)).
2. Calculate the density matrix for non-assigned customers (\( DM_a \)).
3. Sort unopened depots based on \( DV_{MN} \) in descending order, and select the top-ranked one to be opened.
4. Select the first customer who has the highest \( DM_a \) value with current depot and join it with the giant route, and select the first customer randomly for the other initial solutions.
5. Include next customer based on \( DM_a(j_{next}) \), and repeat it until the depot is filled to capacity.
6. Repeat step 1 to 5 until all customers are assigned to a depot.
7. Create some feasible sub-routes for each giant route base on the splitting process.

**Output:** Opened depots as well as feasible sub-routes for each initial solution.

Figure 1 illustrates an example of creating an initial solution for the AI-LRP by the proposed approach. This example consists of 16 customers (customer 5 to 20) along with 4 candidate depots (depot 1 to 4), all distributed in random coordinates. Figure 1(a) shows the geographic distribution of customers and candidate depots. In Figure 1(b), two depots are opened, and for each a giant route is constructed, according to the steps presented in Algorithm 1. The first giant route starts from the depot 3 which has the highest density vector \( DV_{MN} \) calculated by relation (18). Then, the giant route visits customer 17 as the first customer, whose density with depot 3 (i.e., \( DV_{MN} \)) is the highest. The next customers are included in the giant route based on the value of \( DM_a(j_{next}) \). In Figure 1(c), two giant routes are divided into four feasible sub-routes without exceeding vehicle’s capacity constraint.

After producing offspring, their feasibility is checked. The generated offspring is infeasible if the capacity of a depot, a vehicle or both are not met. For example, after the crossover operator, if the total demand of customers assigned to a depot is more than its capacity, the offspring is infeasible. When this is the case, the value of fit(S) will be decreased by subtracting a penalty. After considering the feasibility and the penalization of infeasible offspring, the new generation is selected from the old generation as well as new generated offspring. The general structure of the NGA is given by Algorithm 2.

**Algorithm 2: Novel Genetic Algorithm overview**

**Step 1: Initiation**
- Set \( k = 0 \) and generate pop_size chromosomes by applying Algorithm 1:

**Step 2: Evaluation**
- Calculate the fitness value for each chromosome by objective function (1);
- Put elite_size=\( p \times pop_size \) chromosomes with the best fitness values into the next population;
Step 3: Selection
Calculate the selection probability for each chromosome by equation (20);
Select \((1-p_e) \times \text{pop}_\text{size}\) chromosomes by roulette wheel method and send them to next population;
\[ k \leftarrow k + 1 \]

Step 4: Crossover
Select 100\(p_c\) percent of chromosomes by roulette wheel method as parents;
Apply single-point crossover operator on selected parents to produce offsprings;
Add offsprings to current population;

Step 5: Mutation
Select 100\(p_m\) percent of chromosomes from the current population by roulette wheel method;
Apply mutation operator on selected chromosomes to produce offsprings;
Add new offsprings to current population;

Step 6: Swap operator
Select 100\(p_s\) percent of chromosomes from current population by roulette wheel method;
Apply swap operator on selected chromosomes to produce offsprings;
Add new offsprings to current population;

Step 7: Reproduction
Check the feasibility of all offsprings in current population;
If the offspring \(S\) is feasible  Then
Go to step 8;
else
Subtract a penalty from the fitness value \(f(S)\);
endIf

Step 8: Termination
If \(k < \max \text{gen}\) Then
Go to step 2;
else
Stop and return the best solution \(I_{\text{best}}\).
endIf.

4. COMPUTATIONAL RESULTS

To solve the problem, we have used the benchmarks of Bidgoli and Kheirkhah [7] for arc interdiction vehicle routing problem (AI-VRP). The algorithm is tuned by response surface method and now, we check its validation by using a commercial exact software. The NGA is encoded in MATLAB R2008a on a computer, holding Intel® Core™ i7-6700HQ CPU @ 2.60 GHz and 16 GB of RAM. For computational experiment presented in Table 3, each test instance is run 10 times, and the average solution of the upper level \((OF_U)\) and its execution time are displayed in the second and third column of the table, respectively. To evaluate the validity of the NGA solutions, the results are compared with a commercial solver. To do this, each test instance was solved by using an ILP solver in GAMS 24.8.5 with the simplified formulation given at the end of section 3 and the related results are provided in three last columns of Table 3. It is necessary to note that, the GAMS solutions (optimum or lower and upper bounds) were reported after 8 hours of running time with the BARON solver.

As shown in Table 3, NGA and GAMS are both capable of reaching optimal solutions to four test instances, marked in bold. In these instances, the upper and lower bounds, found by BARON solver, are equal. In five test instances, NGA produced solutions that are equal to the lower bounds offered by GAMS, which are in italics. Moreover, GAMS couldn’t find even feasible solutions for three instances after 8 hours of running time. In summary, computational experiments carried out by these two solution methods reach a conclusion that NGA outperforms BARON solver in GAMS. In addition to the quality of solutions, NGA is generally able to solve the problem in a shorter computational time which is the second advantage of this algorithm compared to GAMS.
TABLE 3. Computational results of the NGA on test instances with asymmetric information (ε = 0.6)

| Test instance | NGA | GAMS 24.8.5 (BARON solver) |
|---------------|-----|----------------------------|
|               | OFU | CPU time (sec)  | Lower bound OFU | Upper bound OFU | CPU time (sec) |
| 2 × 10        | 67403 | 1 | 67403 | 67403 | 3 |
| 2 × 12        | 79823 | 2 | 79823 | 79823 | 3 |
| 2 × 15        | 101725 | 5 | 101725 | 101725 | 36 |
| 2 × 20        | 144089 | 15 | 144089 | 144089 | 322 |
| 2 × 25        | 164644 | 18 | 158600 | 165209 | 28800 |
| 2 × 30        | 213924 | 26 | 213924 | 227088 | 28800 |
| 2 × 35        | 241288 | 26 | 241288 | 248001 | 28800 |
| 3 × 40        | 278491 | 30 | 253785 | 280893 | 28800 |
| 3 × 45        | 355942 | 29 | 349011 | 361021 | 28800 |
| 4 × 50        | 355307 | 33 | 355307 | 376407 | 28800 |
| 4 × 55        | 386869 | 33 | 385200 | 394021 | 28800 |
| 4 × 65        | 473857 | 42 | No solution found | No solution found | 28800 |
| 4 × 70        | 485527 | 48 | 485527 | 501413 | 28800 |
| 4 × 75        | 475630 | 50 | 472691 | 477120 | 28800 |
| 4 × 80        | 474008 | 47 | 474008 | 496211 | 28800 |
| 4 × 85        | 480408 | 51 | 477125 | 494905 | 28800 |
| 4 × 90        | 487787 | 51 | No solution found | No solution found | 28800 |
| 4 × 100       | 511185 | 64 | No solution found | No solution found | 28800 |

Average 31.2 19254.2

1): Bold numbers represent optimal solutions produced by either NGA or GAMS.

5. CONCLUSION AND FUTURE DIRECTIONS

This study presented a bilevel programming model as well as a genetic algorithm for a new version of LRP called Arc Interdiction Location-Routing Problem (AI-LRP) in a network where security measures should be taken to decrease vulnerability and to ensure the reliability of the delivery process. Since making decisions on locating and routing can significantly be affected by the potential risk entailed in natural disasters like earthquake and flooding, or other incidents like random failures and terrorist attacks, this paper took risk management into consideration. In the AI-LRP, there are two decision makers, a distributor and an interdictor, with conflicting targets. While the distributor makes efforts to deliver goods at the lowest possible risk and transportation cost, the interdictor aimed to take devastating action to frustrate the distributor’s plan by interrupting product flow through the network.

This research can be extended in a few directions. Our study assumes that only a sub-set of arcs are at risk of disrupting in the network. However, facilities may be disrupted because of several reasons, such as terrorist attacks, floods, labor strikes and power failures. It is also assumed that the interdictor knows the distributor’s estimation of some parameters like fine, while in reality the interdictor parameters may be stochastic. If this is the case, the AI-LRP can be formulated as a bi-level stochastic programming model. Moreover, the network can be investigated with fuzzy parameters, like fuzzy demands or fuzzy transportation costs. Finally, this study can be extended by deploying other novel metaheuristics like Red Deer Algorithm [80] and Social Engineering Optimizer [81] that have shown a promising performance in dealing with optimization problems.

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Persian Abstract

چکیده

علاوه زیادی به بهینه‌سازی شبکه‌های لجستیک و حمل و نقل با توجه به تاثیر زیاد آن بر عملکرد اقتصادی شبکه‌های زنجیره تأمین وجود دارد. در این مقاله، یک مدل برنامه‌ریزی دو سطحی به صورت عدد صحیح مخلوط و همچنین روش حل آن برای یک شبکه لجستیک آرازه‌ای ارائه شده است. مدل شامل دو سطح تصمیم‌گیری وجود دارد که یکی توزیع کننده و دیگری بازاریاب است. این مساله به نام مساله کمان ممانعت پذیر مکانیابی و مسیریابی شناخته شده است که در واقع توزیع آیی بر مساله مکانیابی و مسیریابی کلاسیک است. توزیع کننده نالش می‌کند که بازویی را ایجاد می‌کند و بازاریاب یا برای ایجاد که توزیع بازاریابی می‌کند. این مساله کاربرد‌های زیادی دارد، از جمله توزیع راه‌آهن، فلزات، مواد شیمیایی و زندانیانی که نیاز به اقدامات امنیتی داشته باشند. تکنیک‌ها و روش‌های توزیع منابع به علت کارایی شبکه لجستیکی و سطحی فرآیند شده است. برای حل مسئله، الگوریتم ژنتیک جدیدی معرفی شده است که به بهبود کارایی و کیفیت جواب‌ها کمک می‌کند.