DATA AUGMENTATION CAN IMPROVE ROBUSTNESS

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ABSTRACT

Adversarial training suffers from robust overfitting, a phenomenon where the robust test accuracy starts to decrease during training. In this paper, we focus on reducing robust overfitting by using common data augmentation schemes. We demonstrate that, contrary to previous findings, when combined with model weight averaging, data augmentation can significantly boost robust accuracy. We evaluate our approach on CIFAR-10 against $\ell_\infty$ and $\ell_2$ norm-bounded perturbations of size $\epsilon = 8/255$ and $\epsilon = 128/255$, respectively. We show large absolute improvements of +2.93% and +2.16% in robust accuracy compared to previous state-of-the-art methods. In particular, against $\ell_\infty$ norm-bounded perturbations of size $\epsilon = 8/255$, our model reaches 60.07% robust accuracy without using any external data.

1 INTRODUCTION

Despite their success, neural networks are not intrinsically robust. In particular, it has been shown that the addition of imperceptible deviations to the input, called adversarial perturbations, can cause neural networks to make incorrect predictions with high confidence (Carlini & Wagner, 2017a; Carlini & Wagner, 2017b; Goodfellow et al., 2015; Kurakin et al., 2016; Szegedy et al., 2014). Starting with Szegedy et al. (2014), there has been a lot of work on understanding and generating adversarial perturbations (Carlini & Wagner, 2017b; Athalye & Sutskever, 2018), and on building defenses that are robust to such perturbations (Goodfellow et al., 2015; Papernot et al., 2016; Madry et al., 2018; Kannan et al., 2018). Among successful defenses are robust optimization techniques like the one developed by Madry et al. (2018) that learn robust models by finding worst-case adversarial perturbations at each training step. Since Madry et al. (2018), various modifications to their original implementation have been proposed (Zhang et al., 2019; Xie et al., 2019; Pang et al., 2020; Huang et al., 2020; Rice et al., 2020; Gowal et al., 2020).

Notably, Hendrycks et al. (2019); Carmon et al. (2019); Uesato et al. (2019); Zhai et al. (2019); Najaﬁ et al. (2019) showed that using additional data improves adversarial robustness, while Rice et al. (2020); Wu et al. (2020); Gowal et al. (2020) found that data augmentation techniques did not boost robustness. This dichotomy motivates this paper. In particular, we explore whether it is possible to fix the training procedure such that data augmentation becomes useful (in the setting without additional data). By making the observation that model weight averaging (WA) (Izmailov et al., 2018) helps robust generalization to a wider extent when robust overfitting is minimized, we propose to combine model weight averaging with data augmentation techniques. Overall, we make the following contributions:

- We demonstrate how, when combined with model weight averaging, data augmentation techniques such as Cutout (DeVries & Taylor, 2017), CutMix (Yun et al., 2019) and MixUp (Zhang et al., 2018) can improve robustness.
- To the contrary of Rice et al. (2020); Wu et al. (2020); Gowal et al. (2020) which all tried data augmentation techniques without success, we are able to use any of these three aforementioned techniques to obtain new state-of-the-art robust accuracies. We find CutMix to be the most effective method by reaching a robust accuracy of 60.07% on CIFAR-10 against $\ell_\infty$ perturbations of size $\epsilon = 8/255$ (an improvement of +2.93% upon the state-of-the-art).

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2 Preliminaries and Hypothesis

Adversarial training. Madry et al. (2018) formulate a saddle point problem to find model parameters $\theta$ that minimize the adversarial risk:

$$\arg \min \theta \mathbb{E}_{(x,y) \sim \mathcal{D}} \left[ \max_{\delta \in \mathcal{S}} l(f(x + \delta; \theta), y) \right]$$

where $\mathcal{D}$ is a data distribution over pairs of examples $x$ and corresponding labels $y$, $f(\cdot; \theta)$ is a model parametrized by $\theta$, $l$ is a suitable loss function (such as the $0-1$ loss in the context of classification tasks), and $\mathcal{S}$ defines the set of allowed perturbations. For $\ell_p$ norm-bounded perturbations of size $\epsilon$, the adversarial set is defined as $\mathcal{S}_p = \{ \delta \mid \|\delta\|_p \leq \epsilon \}$. In the rest of this manuscript, we will use $\epsilon_p$ to denote $\ell_p$ norm-bounded perturbations of size $\epsilon$ (e.g., $\epsilon_\infty = 8/255$) and for the inner optimization, we use the Projected Gradient Descent (PGD) with $K$ steps which we refer to as $\text{PGD}^K$.

Robust overfitting. To the contrary of standard training, which often shows no overfitting in practice (Zhang et al., 2017), adversarial training suffers from robust overfitting (Rice et al., 2020). Robust overfitting is the phenomenon by which robust accuracy on the test set quickly degrades while it continues to rise on the train set (clean accuracy on both sets continue to improve as well). Rice et al. (2020) propose to use early stopping as the main contingency against robust overfitting, and demonstrate that it also allows to train models that are more robust than those trained with other regularization techniques (such as data augmentation or increased $\ell_2$-regularization). They observed that some of these other regularization techniques could reduce the impact of overfitting at the cost of producing models that are over-regularized and lack overall robustness and accuracy. There is one notable exception which is the addition of external data (Carmon et al., 2019; Uesato et al., 2019). Fig. 1(a) shows how the robust accuracy (evaluated on the test set) evolves as training progresses on CIFAR-10 against $\epsilon_\infty = 8/255$. Without external data, robust overfitting is clearly visible and appears shortly after the learning rate is dropped (the learning rate is decayed by 10$^{-10}$ against $\epsilon_\infty$ at step $10^5$). With external data, robust overfitting completely disappears when an additional set of 500K pseudo-labeled images from 80M-T1 (Torralba et al., 2008) is introduced.

Model weight averaging. Model weight averaging (WA) (Izmailov et al., 2018) can be implemented using an exponential moving average $\theta'$ of the model parameters $\theta$ with a decay rate $\tau$ (i.e., $\theta' \leftarrow \tau \cdot \theta' + (1 - \tau) \cdot \theta$ at each training step). During evaluation, the weighted parameters $\theta'$ are used instead of the trained parameters $\theta$. Gowal et al. (2020), Chen et al. (2021) discovered that model weight averaging can significantly improve robustness on a wide range of models and datasets. Chen et al. (2021) argue (similarly to Wu et al., 2020) that WA leads to a flatter adversarial
Hypothesis. As WA results in flatter, wider solutions compared to the steep decrease in robust accuracy observed for Stochastic Gradient Descent (SGD) [Chen et al., 2021], it is natural to ask ourselves whether WA remains useful in cases that do not exhibit robust overfitting. Fig. 1(c) shows how the robust accuracy evolves as training progresses when using WA and additional external data (for which standard SGD does not show signs of overfitting). We notice that the robust performance in this setting is not only preserved but even boosted when using WA. Hence, we formulate the hypothesis that model weight averaging helps robustness to a greater effect when robust accuracy between model iterations can be maintained.

3 DATA AUGMENTATIONS

Limiting robust overfitting without external data. Rice et al. [2020] show that combining data augmentation methods such as Cutout or MixUp with early stopping does not improve robustness upon early stopping alone. While, these methods do not improve upon the “best” robust accuracy, they reduce the extent of robust overfitting, thus resulting in a slower decrease in robust accuracy compared to classical adversarial training (which uses random crops and weight decay). This can be seen in Fig. 2, where MixUp without WA exhibits no decrease in robust accuracy, whereas the robust accuracy of the standard combination of random padding-and-cropping without WA (Pad & Crop) decreases immediately after the change of learning rate.

Verifying the hypothesis. Since MixUp preserves robust accuracy, albeit at a lower level than the “best” obtained by Pad & Crop, it can be used to evaluate the hypothesis that WA is more beneficial when the performance between model iterations is maintained. Therefore, we compare in Fig. 2 the effect of WA on robustness when using MixUp. We observe that, when using WA, the performance of MixUp surpasses the performance of Pad & Crop. Indeed, the robust accuracy obtained by the averaged weights of Pad & Crop (in blue) slowly decreases after the change of learning rate, while the one obtained by MixUp (in green) increases throughout training. Ultimately, MixUp with WA obtains a higher robust accuracy despite the fact that the non-averaged model has a significantly lower “best” robust accuracy than the non-averaged Pad & Crop model. This finding is notable as it demonstrates for the first time the benefits of data augmentation schemes for adversarial training (this contradicts to some extent the findings from three recent publications: Rice et al. [2020], Wu et al. [2020], Gowal et al. [2020]).
Table 1: Robust test accuracy (against AA+MT) against $\epsilon_\infty = 8/255$ on CIFAR-10 as the model size increases. We compare Pad & Crop and CutMix.

Table 2: Clean (without adversarial attacks) accuracy and robust accuracy (against AA+MT) on CIFAR-10 as we both test against $\epsilon_\infty = 8/255$ and $\epsilon_2 = 128/255$.

Exploring data augmentations. After verifying our hypothesis for MixUp, we investigate in Sec. 4 if other augmentations can help maintain robust accuracy and also be combined with WA to improve robustness. We concentrate on image patching techniques like Cutout ( DeVries & Taylor 2017), CutMix (Yun et al. 2019) and RICAP (Takahashi et al. 2018). We also evaluate automated augmentation strategies like AutoAugment ( Cubuk et al. 2019), RandAugment (Cubuk et al. 2020).

4 EXPERIMENTAL RESULTS

Experimental setup. In all the experiments we use model weight averaging (WA) (Izmailov et al. 2018) with a decay rate $\tau = 0.999$. All the technical details, hyperparameters, architecture and evaluation procedure are described in the appendix.

Experimental results. We consider as baseline the Pad & Crop augmentation which reproduces the current state-of-the-art set by Gowal et al. (2020). In Fig. 3 we compare this baseline with various data augmentations, MixUp, Cutout, CutMix and RICAP, as well as learned augmentation policies with AutoAugment and RandAugment. Three clusters are clearly visible. The first cluster, containing AutoAugment and RandAugment, increases the clean accuracy compared to the baseline but, most notably, reduces the robust accuracy. Indeed, these automated augmentation strategies have been tuned for standard classification, and should be adapted to the robust classification setting. The second cluster, containing RICAP, Cutout and CutMix, includes the three methods that occlude local information with patching and provide a significant boost upon the baseline with +3.06% in robust accuracy for CutMix and an average improvement of +1.79% in clean accuracy. The last cluster, with MixUp, only improves the robust accuracy upon the baseline by a small margin of +0.91%. A possible explanation lies in the fact that MixUp tends to either produce images that are far from the original data distribution (when $\alpha$ is large) or too close to the original samples (when $\alpha$ is small). The appendix contains more ablation analysis on all methods.

Table 1 shows the performance of CutMix and the Pad & Crop baseline when varying the model size. CutMix consistently outperforms the baseline by at least +2.90% in robust accuracy across all the model sizes. Table 2 shows the performance of CutMix on CIFAR-10 against $\epsilon_\infty = 8/255$ and $\epsilon_2 = 128/255$. We observe that using CutMix provides a significant boost in robust accuracy for both threat models with up to +2.93% (in the $\epsilon_\infty$ setting) and +2.16% (in the $\epsilon_2$ setting) when training a WRN-70-16. Finally, we show the generality of our approach as using CutMix on CIFAR-100 significantly improves on the state-of-the-art with our best model reaching 32.43% against AUTOATTACK (in the $\epsilon_\infty$ setting). We refer to the appendix for more details on the CIFAR-100 experiments.

5 CONCLUSION

Contrary to previous works (Rice et al. 2020, Gowal et al. 2020, Wu et al. 2020), which have tried data augmentation techniques to train adversarially robust models without success, we demonstrate that combining data augmentations with model weight averaging can significantly improve robustness. Our work provides novel insights into the effect of model weight averaging on robustness, which we hope can further our understanding of robustness. All our models are available online at https://github.com/deepmind/deepmind-research/tree/master/adversarial_robustness/iclrw2021data.
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**Data Augmentation Can Improve Robustness**  
(Supplementary Material)

## A Related Work

**Adversarial training.** The adversarial training procedure (Madry et al., 2018) feeds adversarially perturbed examples back into the training data. It has been augmented in different ways – with changes in the attack procedure (e.g., by incorporating momentum; Dong et al., 2018), loss function (e.g., logit pairing; Mosbach et al., 2018) or model architecture (e.g., feature denoising; Xie et al., 2019). Another notable work by Zhang et al. (2019) proposed TRADES, which balances the trade-off between standard and robust accuracy, and achieved state-of-the-art performance against $\ell_\infty$ norm-bounded perturbations on CIFAR-10. More recently, the work from Rice et al. (2020) studied robust overfitting and demonstrated that improvements similar to TRADES could be obtained more easily using classical adversarial training with early stopping. This later study revealed that early stopping was competitive with many other regularization techniques and demonstrated that data augmentation schemes beyond the typical random padding-and-cropping were ineffective on CIFAR-10. Finally, Gowal et al. (2020) highlighted how different hyper-parameters (such as network size and model weight averaging) affect robustness. They were able to obtain models that significantly improved upon the state-of-the-art, but lacked a thorough investigation on data augmentation schemes. Similarly to Rice et al. (2020), they also make the conclusion that data augmentations beyond random padding-and-cropping do not improve robustness.

**Data augmentation.** Data augmentation has been shown to reduce the generalization error of standard (non-robust) training. For image classification tasks, random flips, rotations and crops are commonly used (He et al., 2016). More sophisticated techniques such as Cutout (DeVries & Taylor, 2017) (which produces random occlusions), CutMix (Yun et al., 2019) (which replaces parts of an image with another) and MixUp (Zhang et al., 2018) (which linearly interpolates between two images) all demonstrate extremely compelling results. As such, it is rather surprising that they remain ineffective when training adversarially robust networks.

## B Experimental Setup

**Architecture.** We use WRNs (He et al., 2016; Zagoruyko & Komodakis, 2016) as our backbone network. This is consistent with prior work (Madry et al., 2018; Rice et al., 2020; Zhang et al., 2019; Uesato et al., 2019; Gowal et al., 2020) which use diverse variants of this network family. Furthermore, we adopt the same architecture details as Gowal et al. (2020) with Swish/SiLU (Hendrycks & Gimpel, 2016) activation functions. Most of the experiments are conducted on a WRN-28-10 model which has a depth of 28, a width multiplier of 10 and contains 36M parameters. To evaluate the effect of data augmentations on wider and deeper networks, we also run several experiments using WRN-70-16, which contains 267M parameters.

**Outer minimization.** We use TRADES (Zhang et al., 2019) optimized using SGD with Nesterov momentum (Polyak, 1964; Nesterov, 1983) and a global weight decay of $5 \times 10^{-4}$. We train for 400 epochs with a batch size of 512, and the learning rate is initially set to 0.1 and decayed by a factor 10 two-thirds-of-the-way through training. We scale the learning rates using the linear scaling rule of Goyal et al. (2017) (i.e., effective LR = max(LR × batch size/256, LR)). We also use model weight averaging (WA) (Izmailov et al., 2018). The decay rate of WA is set to $\tau = 0.999$.

**Inner minimization.** Adversarial examples are obtained by maximizing the Kullback-Leibler divergence between the predictions made on clean inputs and those made on adversarial inputs (Zhang et al., 2019). This optimization procedure is done using the Adam optimizer (Kingma & Ba, 2014) for 10 PGD steps. We take an initial step-size of 0.1 which is then decreased to 0.01 after 5 steps.
Evaluation. We follow the evaluation protocol designed by Gowal et al. (2020). Specifically, we train two (and only two) models for each hyperparameter setting, perform early stopping for each model on a separate validation set of 1024 samples using PGD40, similarly to Rice et al. (2020) and pick the best model by evaluating the robust accuracy on the same validation set. Finally, we report the robust test accuracy against a mixture of AUTOATTACK (Croce & Hein, 2020b) and MULTITARGETED (Gowal et al., 2019), which is denoted by AA+MT. This mixture consists in completing the following sequence of attacks: AUTOPGD on the cross-entropy loss with 5 restarts and 100 steps, AUTOPGD on the difference of logits ratio loss with 5 restarts and 100 steps and finally MULTITARGETED on the margin loss with 10 restarts and 200 steps. The training curves, such as those visible in Fig. 1, are always computed using PGD with 40 steps and the Adam optimizer (with step-size decayed by $10 \times$ at step 20 and 30).

C ADDITIONAL EXPERIMENTS

Model weight averaging decay rate. In Fig. 4, we run an ablation study measuring the robust accuracy obtained when varying the decay rate $\tau$ of model weight averaging (WA) and using either Pad & Crop or CutMix. When using CutMix, the best robust accuracy is obtained at the highest decay rate $\tau = 0.999$. When using Pad & Crop, it is only obtained at a lower decay rate $\tau = 0.9925$. This is consistent with our observation from Sec. 3 that highlights how WA improves robustness to a greater extent when robust accuracy can be maintained throughout training. As larger decay rates average over longer time spans, they should better exploit the fact that CutMix maintains robust accuracy after the learning rate is dropped to the contrary of Pad & Crop (see Fig. 6).

Mixing rate of MixUp. For completeness, we also vary the different hyper-parameters that define the different data augmentations. In particular, for MixUp, we vary the mixing rate $\alpha$. Remember that MixUp blends images by sampling an interpolation point $\lambda \sim \beta(\alpha, \alpha)$ from a Beta distribution with both its parameters set to $\alpha$. Small values of $\alpha$ produce images near the original images, while larger values tend to blend images equally. In Fig. 5(a) we observe that smaller values of $\alpha$ are preferential (irrespective of whether we use model weight averaging). This conclusion is in line with the recommended settings from Zhang et al. (2018) for standard training, but contradicts the experiments made by Rice et al. (2020) who recommend a value of $\alpha = 1.4$ for robust training. We also note that using model weight averaging can increase robust accuracy by up to +5.79% when using MixUp.

Window length of Cutout. CutOut creates random occlusions (i.e., anywhere in the original image) of a fixed size (measured in pixels). Remember that CIFAR-10 images have a size of $32 \times 32$ pixels. The size of this occlusion is controlled by a parameter called the window length. Fig. 5(b) shows how the robust accuracy varies as a result of changing this parameter. We notice that the optimal window length is at 18 pixels whether model weight averaging (WA) is used or not. While WA is useful, it is noticeably less powerful when using CutOut (as opposed to MixUp and CutMix) bringing only an improvement of +2.05% in robust accuracy. This is clearly explained by the training curves shown in
Figure 5: Robust test accuracy against AA+MT with $\epsilon_\infty = 8/255$ on CIFAR-10 as we vary (a) the mixing rate $\alpha$ of MixUp, (b) the window length when using CutOut and (c) the window length when using CutMix. The model is a WRN-28-10 and we compare the settings without and with model weight averaging (in which case, we use $\tau = 0.999$). As a reference, the same model trained with Pad & Crop and model weight averaging reaches 54.44% robust accuracy.

Figure 6: Accuracy against $\epsilon_\infty = 8/255$ on CIFAR-10 without using model weight averaging (WA) for different data augmentation schemes. The model is a WRN-28-10 and the curves show the evolution of the robust accuracy as training progresses (against PGD$^{40}$). The jump in robust accuracy two-thirds through training is due to a drop in learning rate.

Table 3: Clean (without adversarial attacks) accuracy and robust accuracy (AA+MT) on CIFAR-100 against $\epsilon_\infty = 8/255$ obtained by different models. Robust accuracy against AUTOATTACK is also reported for select models.

**Window length of CutMix.** CutMix patches a rectangular cutout from one image onto another. In Yun et al. [2019], the area of this patch is sampled uniformly at random (this is the setting used throughout this paper). In this ablation experiment, however, we fix its size (i.e., window length) and observe its effect on robustness. In Fig. 5(c) we observe that the optimal size is not the same depending on whether model weight averaging (WA) is used. We also note that WA improves robust accuracy by $+3.14\%$. Overall, CutMix obtains the highest robust accuracy of any of the four considered augmentations (including MixUp, CutOut and Pad & Crop).

**CIFAR-100.** Finally, to evaluate the generality of our approach, we evaluate CutMix on CIFAR-100. The results are shown in Table 3. Our best model reaches 32.43% against AUTOATTACK and improves noticeably on the state-of-the-art (in the setting that does not use any external data). It is worth noting that the currently best known result on CIFAR-100 against $\epsilon_\infty = 8/255$ when using external data is 36.88% against AUTOATTACK.
Figure 7: Robust test accuracy measured by running AUTOAttack with (a) different radii $\epsilon_\infty$ and (b) different number of steps $K$. The model is a WRN-70-16 network trained with CutMix against $\epsilon_\infty = 8/255$, which obtains 60.07% robust accuracy against AA+MT at $\epsilon_\infty = 8/255$.

D ANALYSIS OF MODELS

In this section, we perform additional diagnostics that give us confidence that our models are not doing any form of gradient obfuscation or masking (Athalye et al., 2018; Uesato et al., 2018).

AUTOAttack and robustness against black-box attacks. First, we report in Table 4 the robust accuracy obtained by our strongest models against a diverse set of attacks. These attacks are run as a cascade using the AUTOAttack library available at https://github.com/fra31/auto-attack. The cascade is composed as follows:

- AUTOAttack-CE, an untargeted attack using PGD with an adaptive step on the cross-entropy loss (Croce & Hein, 2020b),
- AUTOAttack-T, a targeted attack using PGD with an adaptive step on the difference of logits ratio (Croce & Hein, 2020b),
- FAB-T, a targeted attack which minimizes the norm of adversarial perturbations (Croce & Hein, 2020a),
- SQUARE, a query-efficient black-box attack (Andriushchenko et al., 2020).

First, we observe that our combination of attacks, denoted AA+MT matches the final robust accuracy measured by AUTOAttack. Second, we also notice that the black-box attack (i.e., SQUARE) does not find any additional adversarial examples. Overall, these results indicate that our empirical measurement of robustness is meaningful and that our models do not obfuscate gradients.

| MODEL | NORM | RADIUS $\epsilon_\infty$ | AUTOAttack-CE | + AUTOAttack-T | + FAB-T | + SQUARE | CLEAN | AA+MT |
|-------|------|-----------------|---------------|----------------|--------|---------|-------|-------|
| WRN-28-10 (CutMix) | $\epsilon_\infty$ | $\epsilon = 8/255$ | 61.01% | 57.61% | 57.61% | 57.61% | 86.22% | 57.50% |
| WRN-70-16 (CutMix) | $\epsilon_\infty$ | $\epsilon = 8/255$ | 62.65% | 60.07% | 60.07% | 60.07% | 87.25% | 60.07% |

Table 4: Clean (without adversarial attacks) accuracy and robust accuracy (against the different stages of AUTOAttack) on CIFAR-10 obtained by different models. Refer to https://github.com/fra31/auto-attack for more details.

Further analysis of gradient obfuscation. In this paragraph, we consider a WRN-70-16 trained with CutMix against $\epsilon_\infty = 8/255$, which obtains 60.07% robust accuracy against AA+MT at $\epsilon_\infty = 8/255$.

In Fig. 7(a) we run AUTOAttack-CE with 100 steps and 1 restart and we vary the perturbation radius $\epsilon_\infty$ between zero and 64/255. As expected, the robust accuracy gradually drops as the radius increases indicating that PGD-based attacks can find adversarial examples and are not hindered by gradient obfuscation.
In Fig. 7(b) we run AutoPGD-CE with $\epsilon_\infty = 8/255$ and 1 restart and vary the number of steps $K$ between five and 1000. We observe that the measured robust accuracy converges after 50 steps. This is further indication that attacks converge in 100 steps.

**Loss landscapes.** Finally, we analyze the adversarial loss landscapes of the model considered in the previous paragraph. To generate a loss landscape, we vary the network input along the linear space defined by the worse perturbation found by PGD$_{40}^\infty$ ($u$ direction) and a random Rademacher direction ($v$ direction). The $u$ and $v$ axes represent the magnitude of the perturbation added in each of these directions respectively and the $z$ axis is the adversarial margin loss (Carlini & Wagner, 2017b): $z_y - \max_{i \neq y} z_i$ (i.e., a misclassification occurs when this value falls below zero).

Fig. 8 shows the loss landscapes around the first 2 images of the Cifar-10 test set. All landscapes are smooth and do not exhibit patterns of gradient obfuscation. Overall, it is difficult to interpret these figures further, but they do complement the numerical analyses done so far.

Figure 8: Loss landscapes around the first two images from the Cifar-10 test set for the WRN-70-16 network trained with CutMix. This model obtains 60.07% robust accuracy. It is generated by varying the input to the model, starting from the original input image toward either the worst attack found using PGD$_{40}^\infty$ ($u$ direction) or a random Rademacher direction ($v$ direction). The loss used for these plots is the margin loss $z_y - \max_{i \neq y} z_i$ (i.e., a misclassification occurs when this value falls below zero). The diamond-shape represents the projected $\ell_\infty$ ball of size $\epsilon = 8/255$ around the nominal image.