Complete energy conversion between light beams carrying orbital angular momentum using coherent population trapping for a coherently driven double-$\Lambda$ atom-light coupling scheme

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We propose a procedure to achieve a complete energy conversion between laser pulses carrying orbital angular momentum (OAM) in a cloud of cold atoms characterized by a double-$\Lambda$ atom-light coupling scheme. A pair of resonant spatially dependent control fields prepare atoms in a position-dependent coherent population trapping state, while a pair of much weaker vortex probe beams propagate in the coherently driven atomic medium. Using the adiabatic approximation we derive the propagation equations for the probe beams. We consider a situation where the second control field is absent at the entrance to the atomic cloud and the first control field goes to zero at the end of the atomic medium. In that case the incident vortex probe beam can transfer its OAM to a generated probe beam. We show that the efficiency of such an energy conversion approaches the unity under the adiabatic condition. On the other hand, by using spatially independent profiles of the control fields, the maximum conversion efficiency is only $1/2$.

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I. INTRODUCTION

The interaction of coherent light with atomic systems allow observation of several important and interesting quantum interference effects such as coherent population trapping (CPT) [1–5], electromagnetically induced transparency (EIT) [6–9] and stimulated Raman adiabatic passage (STIRAP) [10–12]. These phenomena are based on the coherent preparation of atoms in a so-called dark state which is immune against the loss of population through the spontaneous emission. Besides their fundamental interest, these coherent optical effects have a number of useful applications in various areas, such as enhanced nonlinear optics [13–16], slow light [6–8, 17, 18], optical switching [19] and storage of quantum information [20].

A light beam can carry orbital angular momentum (OAM) due to its helical wave front. Such a light beam with a spiral phase $e^{il\phi}$ has an optical OAM of $\hbar l$ [21–23], where $\phi$ denotes the azimuthal angle with respect to the beam axis and $l$ is the winding number representing a number of times the beams makes the azimuthal $2\pi$ phase shifts. The phase singularity at the beam core of the twisted beam renders its donut-shaped intensity profile. A number of interesting effects appear when this type of optical beams interact with the atomic systems [21–39]. Among them optical vorticities of slow light [32, 36, 37, 39, 40] have caused a considerable interest, as the OAM brings a new degree of freedom in manipulation of the optical information during the storage and retrieval of the slow light [41, 42].

The previous studies on the interplay of optical vortices and atomic structures deal with the EIT situation where the atoms are initially in their ground states. A four-level atom-light coupling of the tripod-type was suggested to transfer optical vortices between different frequencies during the storage and retrieval of the probe field [35, 39]. It has been demonstrated that without switching off and on of the control fields, transfer of optical vortices take place by applying a pair of weaker probe fields in the closed loop double-$\Lambda$ [32] or double-tripod [36] schemes. The exchange of optical vortices in non-closed loop structures has been recently shown to be possible under the condition of weak atom-light interaction in coherently prepared atomic media [38]. Such a medium is known as the phaseonium [13, 49].

The present paper extends the previous studies to a complete vortex conversion based on CPT by employing a double-$\Lambda$ coherently driven system. The double-$\Lambda$ configuration has been widely employed due to its important applications in coherent control of pulse propagation characteristics [50, 63]. Here we consider the propagation of laser pulses carrying optical vortex in the double-$\Lambda$ system prepared in a superposition state (dark state) of the lower $\Lambda$ subsystem induced by a pair of spatially dependent control fields. It is shown that in an adiabatic basis a single vortex beam initially acting on one transition of the upper $\Lambda$ system generates an extra laser beam with the same

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vorticity as that of the incident vortex beam with a unit conversion efficiency. On the other hand, if the control fields have spatially independent profiles, maximum vortex conversion efficiency is restricted by the coherence between lower levels and can reach only 1/2. The proposed method to exchange optical vortices and the complete vortex conversion is similar to the STIRAP, but the process is reversed. In the usual STIRAP the atomic states are transferred by properly choosing the time dependence of the light fields, whereas here the energy is transferred between light beams by properly choosing the position dependence of the atomic states, created by the position dependence of the control light fields.

II. FORMULATION

Let us consider the double-Λ scheme, depicted in Fig. 1. We are interested in the propagation of two laser pulses with the Rabi frequencies $\Omega_{p_1}$ and $\Omega_{p_2}$ in a quantum medium consisting of atoms characterized by the double-Λ configuration of the atom-light coupling. The two atomic lower states $|g_1\rangle$ and $|g_2\rangle$ are coupled to two excited states $|e_1\rangle$ and $|e_2\rangle$ via four laser fields characterized by the slowly varying amplitudes $\Omega_{p_1}$, $\Omega_{p_2}$, $\Omega_{c_1}$, and $\Omega_{c_2}$.

Applying the rotating-wave approximation, the Hamiltonian for the double-Λ atomic system reads

$$H = -\Omega_{p_1}|e_2\rangle\langle g_1| - \Omega_{p_2}|e_2\rangle\langle g_2| - \Omega_{e_1}|e_1\rangle\langle g_1| - \Omega_{e_2}|e_1\rangle\langle g_2| + H.c..$$

(1)

The Maxwell–Bloch equations (MBE) governing the dynamics of the probe fields $\Omega_{p_1}$ and $\Omega_{p_2}$ and two atomic coherences $\rho_{e_2g_1}$ and $\rho_{e_2g_2}$ are given by

$$\frac{\partial}{\partial t} \begin{bmatrix} \rho_{e_2g_1} \\ \rho_{e_2g_2} \end{bmatrix} = -i(\delta + \Gamma) \begin{bmatrix} \rho_{e_2g_1} \\ \rho_{e_2g_2} \end{bmatrix} + i \begin{bmatrix} \rho_{g_1e_1} - \rho_{e_2g_2} \\ \rho_{g_2e_2} - \rho_{e_2g_2} \end{bmatrix} \begin{bmatrix} \Omega_{p_1} \\ \Omega_{p_2} \end{bmatrix} - i\rho_{e_2e_1} \begin{bmatrix} \Omega_{e_1} \\ \Omega_{e_2} \end{bmatrix},$$

(2)

and

$$\frac{\partial}{\partial z} \begin{bmatrix} \Omega_{p_1} \\ \Omega_{p_2} \end{bmatrix} + c^{-1} \frac{\partial}{\partial t} \begin{bmatrix} \Omega_{p_1} \\ \Omega_{p_2} \end{bmatrix} = i\frac{\alpha \Gamma}{2L} \begin{bmatrix} \rho_{e_2g_1} \\ \rho_{e_2g_2} \end{bmatrix}.$$

(3)

Equation (2) for the atomic coherences is modeled by means of the Liouville equation $\frac{\partial}{\partial t} \rho = -i[H, \rho]/\hbar + L_\rho$ with $L_\rho$ describing the decay of the system. On the other hand, Eq. (3) for the evolution of the probe fields is written under the slowly varying envelope approximation. In the latter equation $\alpha$ determines the optical density of the medium with a length $L$, and $\Gamma$ is the rate of spontaneous decay rate of the excited states. We have assumed the same single photon detuning for both probe fields in above equations, $\delta_1 = \delta_2 = \delta$, where $\delta_1 = \omega_{p_1} - \omega_{e_2g_1}$, and $\delta_2 = \omega_{p_2} - \omega_{e_2g_2}$, with $\omega_{p_i}$ being the central frequency of the corresponding probe fields. The control fields are taken to be in exact resonance with the corresponding atomic transitions.

The diffraction terms containing the transverse derivatives $(2k_{p_1})^{-1}\nabla^2_{\bot} \Omega_{p_1}$ and $(2k_{p_2})^{-1}\nabla^2_{\bot} \Omega_{p_2}$ have been disregarded in the Maxwell equation (3), where $k_{p_1} = \omega_{p_1}/c$ and $k_{p_2} = \omega_{p_2}/c$ represent the central wave vectors of the first and second probe beams. These terms can be evaluated as $\nabla^2_{\bot} \Omega_{p_i(2)} \sim w^{-2} \Omega_{p_i(2)}$, where $w$ is a characteristic transverse dimension of the laser beams. It can be a width of the vortex core if the beam carries an optical vortex or
a characteristic width of the beam when there is no vortex. The change of the phase of the probe beams due to the diffraction term after passing the medium is then estimated to be $L/2kw^2$, where $L$ is the length of the atomic cloud, with $k \approx k_{\mu(2)}$. One can neglect the phase change $L/2kw^2$ when the sample length $L$ is not too large, $L\lambda/w^2 \ll \pi$, where $\lambda = 2\pi/k$ is an optical wavelength. For example, by taking the length of the atomic cloud to be $L = 100 \mu m$, the characteristic transverse dimension of the beams $w = 20 \mu m$ and the wavelength $\lambda = 1 \mu m$, one obtains $L\lambda/w^2 = 0.25$. Under these conditions the diffraction terms do not play an important role and one can safely drop it out in Eq. (3).

Let us assume a situation where a pair of resonant strong control fields acting on the lower legs of the double-Λ scheme.

Consequently, a weak probe pulse pair propagates in a coherently prepared medium interacting with the upper legs

$$\rho = \begin{bmatrix} \rho_{g2g1} \\ \rho_{g1g2} \end{bmatrix} = \begin{bmatrix} -\sin \theta_c \cos \theta_c \\ \cos^2 \theta_c + \sin^2 \theta_c \end{bmatrix},$$

where a mixing angle $\theta_c$ is defined by the equation

$$\cos \theta_c = \frac{1}{\sqrt{\Omega_{c1}^2 + \Omega_{c2}^2}} \left[ \begin{array}{c} \Omega_{c2} \\ \Omega_{c1} \end{array} \right].$$

Then, to the first order, the following steady state solutions for the coherences $\rho_{g1g2}$ as well as the populations $\rho_{g11}$ and $\rho_{g2g2}$ change little during the propagation of the probe fields, giving

$$\begin{bmatrix} \rho_{g2g1} \\ \rho_{g1g2} \end{bmatrix} = \begin{bmatrix} -\sin \theta_c \cos \theta_c \\ \cos^2 \theta_c + \sin^2 \theta_c \end{bmatrix}.$$  

In addition, when $\Omega_{p1} \ll \Omega_{p2}$, the excited states are weakly populated, $\rho_{g1g2}, \rho_{g2g2} \ll 1$, and Eq. (2) changes to

$$\frac{\partial}{\partial t} \begin{bmatrix} \rho_{g2g1} \\ \rho_{g2g2} \end{bmatrix} = -(i\delta + \Gamma) \begin{bmatrix} \rho_{g2g1} \\ \rho_{g2g2} \end{bmatrix} + i \begin{bmatrix} \cos^2 \theta_c - \sin \theta_c \cos \theta_c \\ -\sin \theta_c \cos \theta_c \end{bmatrix} \begin{bmatrix} \Omega_{p1} \\ \Omega_{p2} \end{bmatrix}.$$  

Then, to the first order, the following steady state solutions for the coherences $\rho_{g2g1}, \rho_{g2g2}$ are obtained

$$\begin{bmatrix} \delta - i\Gamma \end{bmatrix} \begin{bmatrix} \rho_{g2g1} \\ \rho_{g2g2} \end{bmatrix} = \begin{bmatrix} \cos^2 \theta_c - \sin \theta_c \cos \theta_c \\ -\sin \theta_c \cos \theta_c \end{bmatrix} \begin{bmatrix} \Omega_{p1} \\ \Omega_{p2} \end{bmatrix}.$$  

Substituting Eq. (8) into the Maxwell equations (3), one arrives at the following coupled equations for the propagation of the pulse pair

$$\frac{\partial}{\partial z} \begin{bmatrix} \Omega_{p1} \\ \Omega_{p2} \end{bmatrix} = -iK \begin{bmatrix} \Omega_{p1} \\ \Omega_{p2} \end{bmatrix},$$

with

$$K = \beta \begin{bmatrix} \cos^2 \theta_c - \sin \theta_c \cos \theta_c \\ -\sin \theta_c \cos \theta_c \end{bmatrix},$$

and

$$\beta = \frac{\alpha \Gamma}{2L(i\Gamma - \delta)}.$$
FIG. 2. Dependence of the dimensionless intensities of the light fields $|\Omega_{p_1}(z)|^2/|\Omega_{p_1}(0)|^2$ and $|\Omega_{p_2}(z)|^2/|\Omega_{p_1}(0)|^2$ given by Eq. (13) on the dimensionless distance $z/L_{\text{abs}}$ for $\alpha = \Gamma$, $\delta = 0$ and $\alpha = 40$.

where $l$ is the orbital angular momenta along the propagation axis $z$, and $\phi$ is the azimuthal angle, $r$ describes a cylindrical radius, $w$ is a beam waist, and $\Omega_{p_{10}}$ represents the strength of the vortex beam. The solutions to the Eq. (9) read

$$
\left[ \begin{array}{c} \Omega_{p_1}(z) \\ \Omega_{p_2}(z) \end{array} \right] = \Omega(r) \left[ \begin{array}{c} \cos^2 \theta_c e^{-i\beta z} + \sin^2 \theta_c \\ -\sin \theta_c \cos \theta_c (e^{-i\beta z} - 1) \end{array} \right].
$$

(13)

Evidently, the laser beam $\Omega_{p_1}$ transfers its vortex to the generated beam $\Omega_{p_2}$. The intensity of both new probe field vortices can be manipulated by the Rabi-frequencies of control fields (see Eq. (6)).

At the beginning of the atomic cloud where the first probe vortex beam $\Omega_{p_1}(0)$ has just entered, the second probe beam is still absent ($\Omega_{p_2}(0) = 0$), and it is generated going deeper into the atomic medium. As can be seen from Eq. (13), both vortex beams are not affected considerably by the losses during their propagation as long as the optical density length is large enough. In fact, the losses take place mostly at the entrance of the medium. If optical density of the resonant medium is sufficiently large $\alpha \gg 1$, the absorption length $L_{\text{abs}} = L/\alpha$ constitutes a fraction of the whole medium $L_{\text{abs}} \ll L$. For larger propagating distances $z \gg L_{\text{abs}}$, the losses disappear

$$
\left[ \begin{array}{c} \Omega_{p_1}(z \gg L_{\text{abs}}) \\ \Omega_{p_2}(z \gg L_{\text{abs}}) \end{array} \right] = \Omega(r) \left[ \begin{array}{c} \sin^2 \theta_c \\ \sin \theta_c \cos \theta_c \end{array} \right],
$$

(14)

and the system goes to the dark state given by Eq. (4) (see also Fig. 2).

Let us consider the efficiency of frequency conversion from initial vortex beam $\Omega_{p_1}$ to the generated vortex beam $\Omega_{p_2}$ for $z \gg L_{\text{abs}}$ described by Eq. (14). The efficiency of vortex conversion between different frequencies is

$$
\eta = \frac{|\Omega_{p_2}(z \gg L_{\text{abs}})/\Omega_{p_1}(0)|}{|\rho_{p_{1g_2}}|} = \sin \theta_c \cos \theta_c.
$$

(15)

Clearly, the maximum frequency conversion between vortex beams achieved in this way is only $1/2$ (when the coherence $\rho_{p_{1g_2}}$ is maximum). In the next section, we present another scenario for transfer of optical vortices in such a medium with the possibility to achieve the unit efficiency of the vortex conversion.

III. SPATIALLY DEPENDENT CONTROL FIELDS

Let us consider a situation where two strong resonant control laser fields $\Omega_{c_1}(z)$ and $\Omega_{c_2}(z)$ are spatially dependent. This is the case if the control beams propagate perpendicular to the probe beams, the latter beams propagating along the $z$ axis. In that case $\Omega_{c_1}(z)$ and $\Omega_{c_2}(z)$ represent the transverse profiles of the control beams. Application of such control fields prepare the system in the spatially dependent dark state $|D(z)\rangle$ immune to the spontaneously decay...
with

$$
\begin{bmatrix}
\cos \theta(z) \\
\sin \theta(z)
\end{bmatrix}
= \frac{1}{\sqrt{\Omega_{c_1}(z)^2 + \Omega_{c_2}(z)^2}}
\begin{bmatrix}
\Omega_{c_2}(z) \\
\Omega_{c_1}(z)
\end{bmatrix},
$$

(17)

where $\theta(z)$ is the position-dependent mixing angle which differs from the constant mixing angle $\theta_c$ used in previous section. Again the interaction of probe fields with the medium is considered to be sufficiently weak, so it does not affect significantly the evolution of the density matrix, which is determined mostly by the coupling fields entering the density matrix equations. This means that one can use the results from a strongly driven $\Lambda$ system for the lower atomic states when the probe fields only act as a weak perturbation that does not change the density matrix elements

$$
\begin{bmatrix}
\rho_{g_2 g_1}(z) \\
\rho_{g_1 g_1}(z) \\
\rho_{g_2 g_2}(z)
\end{bmatrix}
= \begin{bmatrix}
-\sin \theta(z) \cos \theta(z) \\
\cos^2 \theta(z) \\
\sin^2 \theta(z)
\end{bmatrix},
$$

(18)

The approximate solutions to the density matrix equations read

$$
(\delta - i\Gamma)\begin{bmatrix}
\rho_{c_2 g_1}(z) \\
\rho_{c_2 g_2}(z)
\end{bmatrix}
= \begin{bmatrix}
\cos^2 \theta(z) & -\sin \theta(z) \cos \theta(z) \\
-\sin \theta(z) \cos \theta(z) & \sin^2 \theta(z)
\end{bmatrix}
\begin{bmatrix}
\Omega_{p_1}(z) \\
\Omega_{p_2}(z)
\end{bmatrix},
$$

(19)

and the propagation equations become

$$
\frac{\partial}{\partial z} \begin{bmatrix}
\Omega_{p_1}(z) \\
\Omega_{p_2}(z)
\end{bmatrix}
= -i K(z) \begin{bmatrix}
\Omega_{p_1}(z) \\
\Omega_{p_2}(z)
\end{bmatrix},
$$

(20)

with

$$
K(z) = \beta \begin{bmatrix}
\cos^2 \theta(z) & -\sin \theta(z) \cos \theta(z) \\
-\sin \theta(z) \cos \theta(z) & \sin^2 \theta(z)
\end{bmatrix}.
$$

(21)

There is no general analytical solution of the propagation equation (20) as the coefficients of the propagation matrix Eq. (21) are spatially dependent. Equation (20) is similar to a time-dependent Schrödinger equation with the time-dependence replaced with spatial dependence ($t \rightarrow z$). In this case, the propagation matrix $K(z)$ featured in Eq. (21) resembles a time-dependent Hamiltonian. When the Hamiltonian is time-dependent, general solutions are available if the dynamics satisfies adiabaticity [11, 48]. Therefore, we follow a standard adiabatic approximation method to study the evolution of the system. Yet the adiabatic evolution here refers to the spatial evolution. The eigenstates of the propagation matrix $K(z)$ in Eq. (21) can be written as

$$
a^0 = \begin{bmatrix}
\sin \theta(z) \\
\cos \theta(z)
\end{bmatrix},
$$

(22)

$$
a^\beta = \begin{bmatrix}
\cos \theta(z) \\
-\sin \theta(z)
\end{bmatrix},
$$

(23)

where the two eigenstates $a^0$ and $a^\beta$ are orthogonal. It is straightforward to verify that the corresponding eigenenergies of the propagation matrix (21) are

$$
\lambda^0 = 0,
$$

(24)

$$
\lambda^\beta = \beta.
$$

(25)

Let us form a unitary matrix $U$ as
\[
U(z) = \begin{bmatrix}
\sin(\theta(z)) & \cos(\theta(z)) \\
\cos(\theta(z)) & -\sin(\theta(z))
\end{bmatrix}.
\] (26)

The matrix \(U\) then transforms Eq. (20) to the adiabatic basis given by Eqs. (22)-(25)

\[
\frac{\partial}{\partial z} \begin{bmatrix}
\tilde{\Omega}_{p1}(z) \\
\tilde{\Omega}_{p2}(z)
\end{bmatrix} = -i\tilde{K}(z) \begin{bmatrix}
\tilde{\Omega}_{p1}(z) \\
\tilde{\Omega}_{p2}(z)
\end{bmatrix},
\] (27)

where

\[
\begin{bmatrix}
\tilde{\Omega}_{p1}(z) \\
\tilde{\Omega}_{p2}(z)
\end{bmatrix} = U(z)^{-1} \begin{bmatrix}
\Omega_{p1}(z) \\
\Omega_{p2}(z)
\end{bmatrix},
\] (28)

with the transformed propagation matrix \(\tilde{K}(z)\) given by

\[
\tilde{K}(z) = -iU(z)^{-1} \frac{\partial}{\partial z} U(z) + U(z)^{-1} K(z) U(z).
\] (29)

In the matrix form it reads

\[
\tilde{K}(z) = i \begin{bmatrix}
0 & \cos(\theta(z)) \frac{\partial}{\partial z} \sin(\theta(z)) - \sin(\theta(z)) \frac{\partial}{\partial z} \cos(\theta(z)) \\
-\cos(\theta(z)) \frac{\partial}{\partial z} \sin(\theta(z)) - \sin(\theta(z)) \frac{\partial}{\partial z} \cos(\theta(z)) & \beta
\end{bmatrix}.
\] (30)

As can be seen, the eigenvalue \(\beta\) appears in the diagonal of \(\tilde{K}(z)\) which generally describes attenuation. However, for sufficiently long propagation distances, the field component along the eigenstate \(a^\beta\) vanishes. The adiabaticity limit constrains that the off-diagonal elements of \(\tilde{K}(z)\) are negligible compared to the difference of the diagonal ones

\[
|\cos(\theta(z)) \frac{\partial}{\partial z} \sin(\theta(z)) - \sin(\theta(z)) \frac{\partial}{\partial z} \cos(\theta(z))| \ll |\beta|.
\] (31)

When the adiabaticity condition (31) is fulfilled, the solution of the propagation equation (20) reads

\[
\begin{bmatrix}
\Omega_{p1}(z) \\
\Omega_{p2}(z)
\end{bmatrix} = \left[U(z)WU(z)^{-1}\right]^{\Omega_{p1}(z)} \begin{bmatrix}
\Omega_{p1}(z_i) \\
\Omega_{p2}(z_i)
\end{bmatrix},
\] (32)

where

\[
W = \begin{bmatrix}
e^{-i\int_{z_i}^z K_{11}(z) dz} & 0 \\
0 & e^{-i\int_{z_i}^z K_{22}(z) dz}
\end{bmatrix}.
\] (33)

For a long propagation distance \((z \gg L_{abs})\), the exponential term \(e^{-i\int_{z_i}^z K_{22}(z) dz}\) vanishes. In this limit the solutions to the Eq. (32) read

\[
\begin{bmatrix}
\Omega_{p1}(z) \\
\Omega_{p2}(z)
\end{bmatrix} = \Omega(r) \begin{bmatrix}
\sin(\theta(z_i)) \sin(\theta(z)) \\
\sin(\theta(z_i)) \cos(\theta(z))
\end{bmatrix},
\] (34)

where we have assumed that at the entry of the medium \(\Omega_{p1}(z_i)\) is a vortex \((\Omega_{p1}(z_i) = \Omega(r))\) and \(\Omega_{p2}(z_i) = 0\). As one can see from Eq. (34), in this case the same vorticity of the first vortex beam \(\Omega_{p1}(z_i)\) is transferred to the second generated field \(\Omega_{p2}(z)\).

Let us consider a situation where at the beginning of the medium the first control field is present \((|\Omega_{c1}(z_i)| = 1)\) while the second control field is zero \((|\Omega_{c2}(z_i)| = 0)\), giving...
FIG. 3. Dependence of the (a) dimensionless spatially dependent control fields given in Eq. (38) as well as the dimensionless intensities of the probe fields $|\Omega_{p_1}(z)|^2/|\Omega_{p_1}(z_0)|^2$ and $|\Omega_{p_2}(z)|^2/|\Omega_{p_1}(z_0)|^2$ given in Eq. (34) on the dimensionless distance $z/L_{abs}$. Here, $\tilde{z} = 2L_{abs}$, $z_0 = 20L_{abs}$, $\alpha = 40$ and $\delta = 0$.

\[
\begin{bmatrix}
|\sin \theta(z_i)| \\
|\cos \theta(z_i)|
\end{bmatrix} = \begin{bmatrix}
1 \\
0
\end{bmatrix}.
\]  

Moreover, we assume that the first control field is zero at the end of the medium $|\Omega_{c_1}(z_f)| = 0$ while we have $|\Omega_{c_2}(z_f)| = 1$, resulting

\[
\begin{bmatrix}
|\sin \theta(z_f)| \\
|\cos \theta(z_f)|
\end{bmatrix} = \begin{bmatrix}
0 \\
1
\end{bmatrix}.
\]  

In that case the efficiency of vortex conversion between different frequencies is obtained to be the unity using Eqs. (34)-(36):

\[
\eta = |\Omega_{p_2}(z_f)/\Omega_{p_1}(z_i)| = 1,
\]

This is illustrated in Fig. 3 showing the dependence of the intensities $|\Omega_{p_1}(z)|^2/|\Omega_{p_1}(0)|^2$ and $|\Omega_{p_2}(z)|^2/|\Omega_{p_1}(0)|^2$ given by Eq. (34) on the dimensionless distance $z/L_{abs}$ for the resonance case $\delta = 0$ and for the optical depth $\alpha = 40$.

The amplitudes of the spatially dependent control fields satisfying Eqs. (35)-(36) can be chosen as

\[
\begin{bmatrix}
\Omega_{c_1}(z) \\
\Omega_{c_2}(z)
\end{bmatrix} = \Omega_c \begin{bmatrix}
\sqrt{\frac{1}{1+e^{-(z-z_0)/\tilde{z}}}} \\
\sqrt{\frac{1}{1+e^{-(z-z_0)/\tilde{z}}}}
\end{bmatrix},
\]

where $\Omega_{c_2}(z) + \Omega_{c_1}(z) = \Omega_c^2$ (see Fig. 3(a)). Figure 3(b) illustrates that although initially at the beginning of the atomic medium the first probe field $\Omega_{p_1}(z)$ exists, going deeper through the medium it disappears while the second probe field $\Omega_{p_2}(z)$ is generated.

Note that although Eq. (34) does not show to have any limitations, we have used the adiabatic approximation to obtain it. This approximation is not valid when the light fields change too fast in space (or in time). Hence, one needs to find an optimal condition, when the adiabatic approximation is valid. Inserting the spatially dependent control fields given by Eq. (38) into the adiabatic condition (31) one gets for the resonant case $\delta = 0$:

\[
1 \sqrt{\frac{1}{2+2 \cosh ((z-z_0)/\tilde{z})}} \ll \frac{1}{L_{abs}}.
\]

At the position where the left hand side of Eq. (39) is the largest ($z = z_0$), the condition (39) requires that
To satisfy the adiabaticity requirement given by Eq. (40), we can take, for example \( \xi = 2L_{\text{abs}} \). The position \( z_0 \) should be in the middle of the atomic medium \( z_0 = \alpha L_{\text{abs}}/2 \). Furthermore to satisfy Eqs. (35)–(36), the length of the medium should be much larger than \( \xi \). Thus we can take \( z_0 = 20L_{\text{abs}} \) and thus the total length \( 40L_{\text{abs}} \).

IV. CONCLUDING REMARKS

In summary, we have considered the propagation of the optical vortices in a cloud of cold atoms characterized by the double-\( \Lambda \) configuration of the atom-light coupling and derived an approximate adiabatic equation describing such a propagation. The equation shows that when the adiabaticity condition is satisfied, the OAM can be exchanged between probe pulse pair with a maximum efficiency of 100%. It has been recently shown that a double-\( \Lambda \) scheme can be employed for the exchange of optical vortices based on EIT. Moreover, the transfer of optical vortices in coherently prepared \( n + 1 \)-level structures has been also shown. However, a complete energy conversion between optical vortices is not possible in both previous studies.

The proposed approach for the exchange and complete conversion of optical vortices resembles the STIRAP. Yet the process is reversed. It is the atomic states in STIRAP which are transferred by properly choosing the time dependence of light fields. In contrast here we transfer energy between the light beams by properly choosing the position dependence of atomic states. The proposed double-\( \Lambda \) setup can be implemented experimentally for example using the \(^{87}\)Rb atoms. The lower levels \(|g_1\rangle\) and \(|g_2\rangle\) can then correspond to the \(|5S_{1/2}, F = 1, m_F = 0\rangle, |5S_{1/2}, F = 2, m_F = 0\rangle \) hyperfine states. For the two excited states \(|e_1\rangle\) and \(|e_2\rangle\) one can use \(|e_1\rangle = |5P_{1/2}, F = 2, m_F = 1\rangle\) and \(|e_2\rangle = |5P_{3/2}, F = 2, m_F = 1\rangle\), respectively.

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