Searching for orbits around the triple system 45 Eugenia

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Abstract. Asteroids are small bodies that raise high interest, because they have unknown characteristics. The present research aims to study orbits for a spacecraft around the triple asteroid 45 Eugenia. The quality of the observations made by the spacecraft depends on the distance the spacecraft remains from the bodies of the system. It is used a semi-analytical model that is simple but able to represent the main characteristics of that system. This model is called "Precessing Inclined Bi-Elliptical Problem" (PIBEP). A reference system centered on the main body (Eugenia) and with the reference plane assumed to be in the orbital plane of the second more massive body, here called Petit-Prince, is used. The secondary bodies are assumed to be in elliptical orbits. In addition, it is assumed that the orbits of the smaller bodies are precessing due to the presence of the flattening of the main body (J₂). This work analyzes orbits for the spacecraft with passages near Petit-Prince and Princesses, which are the two smaller bodies of the triple system.

1. Introduction

According to [1], asteroids are bodies that orbit the Sun, but are too small to be considered planets. They may receive different classifications according to their orbital, physical, chemical and mineralogical characteristics. Most of the asteroids in the Solar System are located between the orbits of the planets Mars and Jupiter. This region is designated as the main asteroid belt.

Several missions have had or have an asteroid as the main target. Near-Shoemaker is designed to study the 433 Eros asteroid. In 1996, it collected images from 253 Mathilde. In 2000, after the approach, it landed on the asteroid 433 Eros [2]. In 2005, Hayabusa collected images and landed on the asteroid 24143 Itokawa [3]. Dawn visited, between 2011 and 2012, the asteroid 4 Vesta. In 2015, it reached 1 Ceres [4]. The Osiris-Rex spacecraft was launched in September 2016 and is expected to reach the asteroid Bennu (formerly called the 1999 RQ36) in 2018. The objective of the Osiris-Rex spacecraft is to collect materials from this asteroid and return them to Earth in 2023 [5].

2. The triple system 45 Eugenia

The asteroid 45 Eugenia was discovered at the Paris Observatory on June 27, 1857, by the astronomer Hermann Mayer Salomon Goldschmidt. The system consists of a main body (Eugenia) with 217 km in diameter and two smaller bodies (Petit-Prince and Princessse) with diameters of 5 and 7 km, respectively [6]. In Table 1, the characteristics of the triple asteroid are shown.
Table 1. Physical and orbital characteristics of the Eugenia system [6].

| Asteroid   | Main body | \( a \) (AU) | \( e \) | \( i \) \(^{\circ} \) | Period (years) | Radius (km) | Mass (kg) |
|------------|-----------|---------------|--------|----------------|----------------|--------------|-----------|
| Eugenia    | Sun       | 2.721         | 0.083  | 6.61          | 4.49           | 108.5       | 5.63 x 10\(^{18} \) |
| Petit-Prince| Eugenia  | 1164.5        | 0.006  | 9             | 4.7            | 3.5         | 2.5 x 10\(^{14} \) |
| Princesse  | Eugenia  | 610           | 0.069  | 18            | 1.8            | 2.5         | 2.5 x 10\(^{14} \) |

3. Description

The objective of this work is to search for orbits around the main body of the system, with the goal that they can be used to observe all the bodies of the system. The first step is to choose the initial conditions for the orbits of the spacecraft that allow close passages by the asteroids. It is used the same technique developed in [7, 8]. After finding the initial conditions, the orbits are numerically integrated using the mathematical model called PIBEP [7, 8]. This model uses a reference system with origin on the main body (Eugenia) and have a reference plane in the orbital plane of Petit-Prince. The secondary bodies are assumed to be in elliptical orbits precessing due to the flattening of the main body (\( J_2 \)). The spacecraft moves under the gravity forces of the three bodies involved in the system and the \( J_2 \) term of the gravity field of the main body. The evolution of the distances between the spacecraft and all the bodies of the system is monitored, and the time the vehicle remains close to those bodies is computed and shown in the results.

In a first analysis, a study of the eccentricities of the orbits is performed for all orbits obtained. Those with eccentricity greater than 1 are discarded, since they are hyperbolic orbits, therefore, not allowing repeated passages between the spacecraft and the secondary bodies. A second analysis is made to verify the distances between the spacecraft and Eugenia (R1), between the spacecraft and Petit-Prince (R2), and between the spacecraft and Princesse (R3), all of them as a function of time. After choosing initial orbits around the central body, it is also eliminated orbits ending in collisions with one of the bodies. After that, the remaining trajectories are numerically integrated using the model PIBEP. This process also takes into account other aspects of the four-body dynamics, such as close approaches [9, 10] and gravitational captures [11, 12].

4. Mathematical model

The model used in the present research is the one shown in [6, 7]. It was noted that, in this system, the effects of the solar radiation pressure was not so important, as in the 2001SN\(_{26} \) [13] studied in [7], due to the larger masses of the bodies, which generates higher gravitational forces. The distances between the spacecraft and Eugenia (R1), spacecraft and Petit-Prince (R2) and spacecraft and Princesse (R3) are given by Eqs. 1-3, where \((x_1, y_1, z_1)\) are the coordinates of Eugenia, \((x_2, y_2, z_2)\) are the coordinates of Petit-Prince and \((x_3, y_3, z_3)\) are the coordinates of Princesse.

\[
R_1 = \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} \tag{1}
\]
\[
R_2 = \sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2} \tag{2}
\]
\[
R_3 = \sqrt{(x-x_3)^2 + (y-y_3)^2 + (z-z_3)^2} \tag{3}
\]

The equations of motion for the spacecraft in the inertial system, considering the above assumptions, are given by Eqs. 4-6, shown below:
where $J_2 = 0.06$ [6] is the flatness of the central body and $r = 108.5$ km is the value of the radius of Eugenia. The gravitational parameters of Eugenia, Petit-Prince and Princesse are given by $\mu_1$, $\mu_2$ and $\mu_3$, respectively.

5. Results

The evolution of the distances between the spacecraft and the bodies of the system was analyzed in two intervals: up to 50 km and 50-100 km. The simulations considered only the situation where the asteroids start their motion at the periapsis of their orbits. The next figures illustrate the evolution of the distances between the spacecraft and all the bodies of the system. The term resonance is used in the sense of the instantaneous initial orbit only. It means that the trajectory obtained from a given resonance would have several encounters with the resonant body, if the dynamics was the "two-body problem". But, since the dynamics is much more complex, the trajectory is modified and real resonances may not occur. In that sense, it is just a technique used to find initial conditions, which proofed to work very well. See [7, 8] for more explanations. A large number of resonances was simulated, and the best trajectories are shown next.

5.1. Internal orbit to Petit-Prince, initially in resonance 1:2, with the spacecraft starting at the apoapsis of its orbit

In this orbit, the spacecraft does not observe the main body Eugenia from the distances limit defined (up to 50 km and 50-100 km). In the interval between 50-100 km it remains 4.02 days observing Petit-Prince. The time is not long, but there are many passages, which can be useful for the observations. Regarding Princesse, it is noted that the spacecraft remains less than 1 day close to the body, but also making several passages. Figure 1 shows the evolution of the distances, as a function of time, between the spacecraft and the bodies. Figures 2 and 3 show a zoom of the passages of the spacecraft in the intervals simulated.

![Fig. 1. Distances of the spacecraft as a function of time until Eugenia (blue), Petit-Prince (red) and Princesse (green).](image-url)
Fig. 2. Distances of the spacecraft as a function of time until Eugenia (blue), Petit-Prince (red) and Princesse (green) in the interval less than 50 km.

Fig. 3. Distances of the spacecraft as a function of time until Eugenia (blue), Petit-Prince (red) and Princesse (green) in the interval 50-100 km.

5.2. *Internal orbit to Princesse initially in resonance 5:9, when the spacecraft starts its motion at the periapsis of its orbit*

The spacecraft does not observe the main body Eugenia and Petit-Prince. When the spacecraft is in its periapsis, Princesse is observed for 0.40 days at a distance of up to 50 km and in the range 50-100 km for 3.64 days, with several passages. Figure 4 shows the evolution of the distances, as a function of time, between the spacecraft and the three bodies. Figures 5 and 6 show a zoom of the passages of the spacecraft in the simulated intervals.
Fig. 4. Distances of the spacecraft according to the time until Eugenia (blue), Petit-Prince (red) and Princesse (green).

Fig. 5. Distances of the spacecraft according to the time until Eugenia (blue), Petit-Prince (red) and Princesse (green) in the interval less than 50 km.

Fig. 6. Distances of the spacecraft according to the time until Eugenia (blue), Petit-Prince (red) and Princesse (green) in the interval 50-100 km.
6. Conclusions

Based in the results shown in the present paper, it is noted that, using initially internal orbits in resonance with Petit Prince, the spacecraft had no close encounter with the central body, while using orbits which are initially in internal resonance with Princesse, the spacecraft did not observe the main body and Petit-Princes.

It means that the strategy to search for orbits developed in [7] does not give orbits with large observational times, as found for the triple system 2001SN263. The reason is that the smaller bodies of the 45 Eugenia system are very far from the central body, generating close encounters between the spacecraft and the bodies of the system, but with shorter durations. Those passages are important, even if the time is not too long. It also means that it is necessary to use more orbital maneuvers to paste together the different orbits found with the present technique, since no adequate orbits were found to observe all the bodies in a single trajectory. On the other side, the distances between the bodies are larger, and this implies in a system much less disturbed, so the orbits are more stable in general, requiring less fuel consumption on station-keeping maneuvers required to compensate effects of the perturbations.

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