Pair production of charged Higgs bosons in the Left-Right Twin Higgs model at the ILC and LHC

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Abstract

Left-Right twin Higgs(LRTH) model predicts the existence of a pair of charged Higgs $\phi^{\pm}$. In this paper, we study the production of the charged Higgs bosons pair $\phi^{\pm}$ at the International Linear Collider(ILC) and the \textit{CERN} Large Hadron Collider(LHC). The numerical results show that the production rates are at the level of several tens fb at the ILC, the process $e^{+}e^{-} \rightarrow \phi^{+}\phi^{-}$ can produce the adequate distinct multi-jet final states. We also discuss the charged Higgs boson pair production via the process $q\bar{q} \rightarrow \phi^{+}\phi^{-}$ at the LHC and estimate there production rates. We find that, as long as the charged Higgs bosons are not too heavy, they can be abundantly produced at the LHC. The possible signatures of these new particles might be detected at the ILC and LHC experiments.

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I. Introduction

The standard model (SM) provides an excellent effective field theory description of almost all particle physics experiments. But in the SM the Higgs boson mass suffers from an instability under radiative corrections. The naturalness argument suggests that the cutoff scale of the SM is not much above the electroweak scale: New physics will appear around TeV energies. Most extensions of the SM require the introduction of extended Higgs sector to the theory. Generically, charged Higgs boson arises in the extended Higgs sector, which does not exist in the SM. It implies that the observation of a charged Higgs boson is a clear evidence for existence of the new physics beyond the SM. Many alternative new physics theories, such as supersymmetry, topcolor, and little Higgs, predict the existence of new scalar or pseudo-scalar particles. These new particles may have cross sections and branching fractions that differ from those of the SM Higgs boson. Thus, studying the production and decays of the new scalars at high energy colliders will be of special interest.

Recently, the twin Higgs mechanism has been proposed as a solution to the little hierarchy problem [1, 2, 3]. The Higgs is a pseudo-Goldstone boson of a spontaneously broken global symmetry. Gauge and Yukawa interactions that explicitly break the global symmetry give mass to the Higgs. Once a discrete symmetry is imposed, the leading quadratic divergent term respects the global symmetry, thus does not contribute to the Higgs model. The twin Higgs mechanism can be implemented in left-right models with the discrete symmetry being identified with left-right symmetry [2]. The left-right twin Higgs (LRTH) model contains $U(4)_L \times U(4)_R$ global symmetry as well as $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry. The left-right symmetry acts on only the two $SU(2)$'s gauge symmetry. The leading quadratically divergent SM gauge boson contributions to the Higgs masses are canceled by the loop involving the heavy gauge bosons. A vector top singlet pair is introduced to generate an $O(1)$ top Yukawa coupling. The quadratically divergent SM top contributions to the Higgs potential are canceled by the contributions from a heavy top partner. The LRTH model predicted the existence of the new heavy particles, such as heavy gauge boson, fermions, and scalars at or below the TeV scale, which might generate characteristic signatures at the present and future collider experiments [4, 5, 6].

The hunt for the Higgs and the elucidation of the mechanism of symmetry breaking is one
of the most important goals for present and future high energy collider experiments. Precision electroweak measurement data and direct searches suggest that the Higgs boson must be relative light and its mass should be roughly in the range of 114.4 GeV–208 GeV at 95% CL [7]. While the discovery potential of the Higgs at the LHC has been established for a wide range of Higgs masses, only rough estimates on its properties will be possible, through measurements on the couplings of the Higgs to the fermions and gauge boson for example [8]. The most precise measurements will be performed in the clean environment of the future high energy $e^+e^-$ linear collider, the International Linear Collider (ILC) with the center of mass (c.m.) energy $\sqrt{s}=300$ GeV–1.5 TeV [9] and the yearly luminosity 500 $fb^{-1}$. The running of the high energy and luminosity linear collider will open an unique window for us to understand the basic theory of particle physics. The aim of this paper is to investigate production of the charged Higgs bosons pair at the ILC and LHC and see whether the possible signatures of the LRTH model can be detected in the near future ILC and LHC experiments.

This paper is organized as follows, The relevant couplings to ordinary particles of the charged Higgs bosons $\phi^\pm$ in the LRTH model are given in section II. In section III and section IV we study the charged Higgs bosons pair productions at the ILC and LHC, respectively. The numerical results and discussions are also presented in theses sections. The conclusions are given in section V.

II. The relative couplings

The LRTH model is based on the global $U(4)_1 \times U(4)_2$ symmetry, with a locally gauged subgroup $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Two Higgs fields, $H$ and $\hat{H}$, are introduced and each transforms as $(4, 1)$ and $(1, 4)$ respectively under the global symmetry. They are written as

$$H = \begin{pmatrix} H_L \\ H_R \end{pmatrix}, \quad \hat{H} = \begin{pmatrix} \hat{H}_L \\ \hat{H}_R \end{pmatrix},$$

(1)

where $H_{L,R}$ and $\hat{H}_{L,R}$ are two component objects which are charged under the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ as

$H_L$ and $\hat{H}_L : (2, 1, 1), \quad H_R$ and $\hat{H}_R : (1, 2, 1).$ (2)

The global $U(4)_1(U(4)_2)$ symmetry is spontaneously broken down to its subgroup $U(3)_1(U(3)_2)$ with non-zero vacuum expectation values as $\langle H \rangle = (0, 0, 0, f)$ and $\langle \hat{H} \rangle = (0, 0, 0, \hat{f})$. Each
spontaneously symmetry breaking results in seven Nambu-Goldstone bosons. Three of six Goldstone bosons that are charged under $SU(2)_R$ are eaten by the heavy gauge bosons, while leaves three physical Higgs: $\phi^0$ and $\phi^\pm$. The remaining Higgses are the SM Higgs doublet $H_L$ and an extra Higgs doublet $\hat{H}_L = (\hat{H}_1^+, \hat{H}_2^0)$ that only couples to the gauge boson sector. A residue matter parity in the model renders the neutral Higgs $\hat{H}_2^0$ stable, and it could be a good dark matter candidate. A pair of vector-like quarks $q_L$ and $q_R$ are introduced in order to give the top quark a mass of the order of electroweak scale, which are singlets under $SU(2)_L \times SU(2)_R$. The masse of the light SM-like top and the heavy top are

$$m_t^2 \simeq y^2 f^2 \sin^2 x - M^2 \sin^2 x \sim (yv/\sqrt{2})^2,$$

$$m_T^2 \simeq y^2 f^2 + M^2 - m_t^2,$$

where $v = 246$ GeV is the electroweak scale and $x = v/\sqrt{2}f$, the mass parameter $M$ is essential to the mixing between the SM-like top quark and the heavy top quark. The top Yukawa coupling can then be determined by fitting the experimental value of the light top quark mass. At the leading order of $1/f$, the mixing angles for left-handed and right-handed fermions are

$$S_L \simeq \frac{M}{m_T} \sin x,$$

$$S_R \simeq \frac{M}{m_T} (1 + \sin^2 x).$$

The value of $M$ is constrained by the requirement that the branching ratio of $Z \to b\bar{b}$ remains consistent with the experiments. We define the Weinberg angle in the $LRTH$ model as

$$s_w = \sin \theta_w = \frac{g'}{\sqrt{g'^2 + 2g^2}},$$

$$c_w = \cos \theta_w = \sqrt{\frac{g^2 + g'^2}{g^2 + 2g'^2}}.$$

The unit of the electric charge is then given by

$$e = g s_w = \frac{g g'}{g^2 + 2g'^2}.$$
in the form \(i\gamma^\mu(g_V + g_A\gamma^5)\). The couplings constants of the heavy gauge bosons and the charged Higgs boson \(\phi^\pm\) to ordinary particles, which are related to our calculation, can be written as

\[
g_{\mu}^{ZH\bar{e}e} = \frac{e}{4s_Wc_W\sqrt{1 - 2s_W^2}}(-1 + 4s_W^2), \quad g_{A}^{ZH\bar{e}e} = -\frac{e}{4s_Wc_W\sqrt{1 - 2s_W^2}}, \tag{10}
\]

\[
g_{\mu}^{ZH\bar{u}u_{1,2}} = \frac{e}{4s_Wc_W\sqrt{1 - 2s_W^2}}(-1 + \frac{8}{3}s_W^2), \quad g_{A}^{ZH\bar{u}u_{1,2}} = -\frac{e}{4s_Wc_W\sqrt{1 - 2s_W^2}}, \tag{11}
\]

\[
g_{\mu}^{ZH\bar{d}d_{1,2}} = \frac{e}{4s_Wc_W\sqrt{1 - 2s_W^2}}(1 - \frac{5}{3}s_W^2), \quad g_{A}^{ZH\bar{d}d_{1,2}} = \frac{e}{4s_Wc_W\sqrt{1 - 2s_W^2}}, \tag{12}
\]

\[
g_{\mu}^{ZH\bar{b}b} = \frac{e}{4s_Wc_W\sqrt{1 - 2s_W^2}}(1 - \frac{4}{3}s_W^2), \quad g_{A}^{ZH\bar{b}b} = \frac{e}{4s_Wc_W\sqrt{1 - 2s_W^2}}, \tag{13}
\]

\[
g_{\phi^-\phi^+\gamma} = -e(p_1 - p_2)_\mu, \quad g_{\phi^-\phi^+Z_H} = \frac{es_W}{c_W}(p_1 - p_2)_\mu, \tag{14}
\]

\[
g_{\phi^-\phi^+Z} = -\frac{e(1 - 3s_W^2)}{2s_Wc_W\sqrt{1 - 2s_W^2}}(p_1 - p_2)_\mu, \quad g_{\phi^+\phi^-} = -i(S_Rm_bp_L - y_Sf_P_R)/f, \tag{15}
\]

where \(P_{L(R)} = (1 \mp \gamma_5)/2\) is the left(right)-handed projection operator. \(s_W(c_W)\) represents the sine(cosine) of the Weinberg angle, \(p_1\) and \(p_2\) refer to the incoming momentum of the first and second particle, respectively.

### III Production of the charged Higgs bosons pair at ILC

From above discussions, we can see that the charged Higgs bosons pair \(\phi^+\phi^-\) can be produced in \(e^+e^-\) annihilation via virtual photon or neutral gauge boson exchanging. The Feynman diagram of the process \(e^+e^- \rightarrow \phi^+\phi^-\) is shown in Fig.1.

At the leading order, the production amplitude of the process can be written as:

![Feynman diagram](image)

**Figure 1**: The Feynman diagrams of the process \(e^+e^- \rightarrow \phi^+\phi^-\).
\[ M_1 = M_\gamma + M_Z + M_{ZH}, \quad (16) \]

with

\[
M_\gamma = e^2 p_2 \gamma^\mu u_e(p_1) G^{\mu\nu}(p_1 + p_2, 0)(p_3 - p_4)_\nu,
\]

\[
M_Z = \frac{e s_W}{c_W} p_2 \gamma^\mu (g_{V\bar{e}e} + g_{A\bar{e}e} \gamma^5) u_e(p_1) G^{\mu\nu}(p_1 + p_2, M_Z)(p_3 - p_4)_\nu,
\]

\[
M_{ZH} = \frac{e(1 - 3 s_W^2)}{2 s_W c_W \sqrt{1 - 2 s_W^2}} p_2 \gamma^\mu (g_{V\bar{e}e} + g_{A\bar{e}e} \gamma^5) u_e(p_1) G^{\mu\nu}(p_1 + p_2, M_{ZH})(p_3 - p_4)_\nu.
\]

Here, \( G^{\mu\nu}(p, M) = -ig^{\mu\nu} p^2 - M^2 \) is the propagator of the particle. With above production amplitudes, we can obtain the production cross section directly. In the calculation of the cross section, instead of calculating the square of the amplitudes analytically, we calculate the amplitudes numerically by using the method of the references [10] which can greatly simplify our calculation.

In the numerical calculation, we take the input parameters as \( G_F = 1.166 \times 10^{-5} \text{GeV}^{-2} \), \( M_Z = 91.187 \text{ GeV} \) and \( s_W^2 = 0.2315 \) [11]. The electromagnetic fine-structure constant \( \alpha_e \) at a certain energy scale is calculated from the simple QED one-loop evolution with the boundary value \( \alpha_e = 1/137.04 \). Except for these SM input parameters, the production cross sections is dependent on the symmetry breaking \( f \), the mixing parameter \( M \) and the mass of the charged Higgs boson \( \phi^\pm \). In our analysis, we take \( M \) to be small and pick a typical value of \( M = 150 \text{ GeV} \).

The symmetry breaking scales \( f \) is allowed in the range of \( 500 \sim 1500 \text{ GeV} \) [4]. As numerical estimation, we will assume that \( M_{\phi^\pm} \) is in the range of \( 150 \sim 450 \text{ GeV} \).

In Fig.2, we plot the cross section of \( e^+e^- \rightarrow \phi^+\phi^- \) are plotted as a function of the mass parameter \( M_{\phi^\pm} \) for \( M_{ZH} = 2.5 \text{ TeV} \) and three values of the the center of mass energy. The plots show that the cross section decrease with \( M_{\phi^\pm} \) due to the phase space suppression. For \( \sqrt{s} = 500 \text{ GeV} \), the cross section falls sharply to a very small rate with \( M_{\phi^\pm} \) increasing. So, the energy 500 GeV is not suitable to search for the heavy charged Higgs bosons pair. The cross section is not sensitive to \( M_{\phi^\pm} \) when \( \sqrt{s} = 1600 \text{ GeV} \). The change of the cross section with \( \sqrt{s} \) is not monotonous because the influence of \( \sqrt{s} \) on the phase space and Z-propagator is inverse.

In the most case, the production rate is at the order of tens fb. For the light charged scalars, the production rate can be near 30fb in the case of \( \sqrt{s} = 800 \text{ GeV} \). With yearly expected luminosity about \( \mathcal{L} = 500 fb^{-1} \), then there will be \( 10^2 \sim 10^4 \) events to be generated each year.

To see the influence of the new heavy gauge boson mass \( M_{ZH} \) on the cross section, in
Figure 2: The cross section of $e^+e^- \rightarrow \phi^+\phi^-$ as a function of charged Higgs bosons mass $M_{\phi^\pm}$ for $f = 1 TeV$, $M = 150 GeV$ and various center-of-mass $\sqrt{s}$.

Figure 3: The cross section $\sigma(s)$ of $e^+e^- \rightarrow \phi^+\phi^-$ as a function of the heavy gauge boson mass $M_{ZH}$ for $\sqrt{s}=800$ GeV and different values of $M_{\phi^\pm}$.

Fig.3 we plot $\sigma(s)$ as a function of $M_{ZH}$ for $\sqrt{s}=800$ GeV and three values of $M_{\phi^\pm}=150$, 250, 350 GeV, respectively. From Fig.3, one can see that the cross section decreases slowly with
$M_{Z_H}$ increasing and is more sensitive to the charged Higgs bosons mass. This is because the production cross section are mainly aroused by the exchange of $\gamma$ and $Z$ boson. In general, the cross section is at the order of tens fb. This abundant production allows to enforce tight requirements on the event pre-selection and the mass reconstruction.

It has been shown that the charged Higgs $\phi^{\pm}$ dominantly decay into $t\bar{b}$ for larger value of the mixing parameter between the SM-like top quark and the heavy top quark [4]. In the case of $\phi^{+} \rightarrow t\bar{b}$, the signals of the charged Higgs bosons pair production is $t\bar{b}t\bar{b}$. The cross section of the irreducible $t\bar{b}t\bar{b}$ background has been estimated using the Comphep program [12] at 0.8 TeV and found to be 5.5 fb. In order to efficiently distinguish the signals from the underlying backgrounds and to measure the charged scalar mass, it is important to obtain a clean charged scalar signal in the mass distribution of the multi-jet final states. To identify the production mode $t\bar{b}t\bar{b}$, we insist on 8 jets or 1 lepton plus 6 jets(in particular, fewer than 10 visible lepton/jets so as to discriminate from the 4t final states) and possibly require that one $W$ and the associated top quark be reconstructed. In particular, since final states contain at least four b jets, in order to eliminate any residual QCD background, we need one or two b-tags without incurring significant penalty. Such b-tagging should have efficiency of 60% or better. The mistagging of b-quark and s-quark will make the $e^+e^- \rightarrow W^+W^-$ become important which significantly enhance the background. So, the efficient b tagging and mass reconstruction of the charged Higgs bosons is very necessary to reduce the background [13].

It is known that many new physics model predict similar heavy charged scalars, such as $\Pi^{\pm}$ in the topcolor-assisted technicolor model(TC2) and $H^{\pm}$ in the two-Higgs doublet model(2HDM). To distinguish the scalars in the LRTH model from the charged top-pions in the TC2 model and the Higgs in the 2HDM, we should compare the cross section of $e^+e^- \rightarrow \phi^+\phi^-$ with those of similar process $e^+e^- \rightarrow \Pi^{+}\Pi^{-}$ [14] in TC2 model and process $e^+e^- \rightarrow H^{+}H^{-}$ [15] in the 2HDM. The cross section sections values of such three process are not significantly different for the same parameters. For example, $\sigma(e^+e^- \rightarrow \Pi^{+}\Pi^{-}) = 21.51, 15.09 fb$ with $\sqrt{s}=800, 1600$ GeV and $M_{\Pi}=300$ GeV, $\sigma(e^+e^- \rightarrow H^{+}H^{-}) = 10.02, 8.1 fb$ with $\sqrt{s}=800, 1600$ GeV and $M_{H}=300$ GeV, $\sigma(e^+e^- \rightarrow \phi^+\phi^-) = 12.54, 9.26 fb$ with $\sqrt{s}=800, 1600$ GeV and $M_{\phi}=300$ GeV. So we should distinguish them depending on their different feature of decay modes and pole structure. The $t\bar{b}$ is the main decay mode for the charged scalars. However, we should probe
charged top-pions via the flavor-changing decay mode $\Pi^+ \rightarrow c\bar{b}$ to obtain the identified signals. $\tau\nu$ can also provide the identified signals of charged Higgs from the 2HDM which is not exist for the charged top-pions and $\phi^\pm$.

IV. Production of the charged Higgs bosons pair at LHC

The Large Hadron Collider (LHC) at CERN has a good potential for discovery of a charged Higgs boson. At the LHC, the charged Higgs bosons also can be produced in pair production mode. There are two important $\phi^+\phi^-$ production channels: (i) $q\bar{q} \rightarrow \phi^+\phi^-$, where $(q = u, d, c, s, b)$ (via Drell-Yan process, where a photon and a Z-boson are exchanged in the $s$-channel and the top quark in the $t$-channel.) [16] (ii) the loop-induced gluon fusion process $gg \rightarrow \phi^+\phi^-$ [17]. The Feynman diagrams of these processes are shown in Fig.4. In the case of $q = b$, there are additional Feynman diagrams involving $\phi^0$ and $H$ in the Fig.4(a), the neutral Higgs bosons $\phi^0$ and $H$ exchange in the $s$-channel can also contribute to the pair production process. However, these contributions are very smaller than those of other tree-level processes because of either the small Yukawa couplings, small patron distribution functions or both combined. On the other hand, the contributions from the Fig.4.(c-e) are also very smaller than those of the tree-level processes. This is because the Yukawa couplings depends sensitively on the parameter $M$ and $f$. For small $f$, the values of the parameter $M$ are very small [5]. Once $M$ is very small or in the limit that $M=0$, certain couplings go to zero. Although the gluon fusion get an enhancement due to larger patron distribution functions, the contributions of gluon fusion process is suppressed by the order of $(M/f)^4$. Thus, we will ignore these processes in the following estimation.

Using the relevant Feynman rules, we can write the invariant amplitude for the parton process $q(p_1)\bar{q}(p_2) \rightarrow \phi^+(p_3)\phi^-(p_4)$ as:

$$\mathcal{M}_2 = \mathcal{M}_{21} + \mathcal{M}_{22}$$ \hspace{1cm} (17)

For the process $b\bar{b} \rightarrow \phi^+\phi^-$, the invariant amplitude comes from the Fig.4(a) and (b):

$$\mathcal{M}_{21} = e\bar{v}(p_2)Q_{\gamma\nu}u(p_1)g^{\mu\nu}(p_4 - p_3)_{\mu}$$
$$+ e\frac{S_W}{c_W}\bar{v}(p_2)\gamma_{\nu}(g_{\nu}Zb\bar{b} + g_A Z_{b\bar{b}}\gamma_5) u(p_1) \frac{g^{\mu\nu}}{s - m_Z^2}(p_4 - p_3)_{\mu}$$
$$+ e\frac{1 - 3S_W^2}{2 c_W S_W \sqrt{1 - 2S_W^2}} \bar{v}(p_2)\gamma_{\nu}(g_{\nu}Z_{b\bar{b}} + g_A Z_{b\bar{b}}\gamma_5) u(p_1) \frac{g^{\mu\nu}}{s - m_{Z_H}^2}(p_4 - p_3)_{\mu}$$
Figure 4: The tree-level Feynman diagrams for the process \(q\bar{q} \rightarrow \phi^+\phi^-\) (Fig. (a-b)) and the one-loop Feynman diagrams for \(gg \rightarrow \phi^+\phi^-\) (Fig. (c-f)) in the LRTH model.

\[
+ \frac{1}{f^2} \bar{v}(p_2)(S_{Rm_b}P_L - yS_LfP_R) \frac{\hat{q} + m_t}{\hat{t} - m_t^2}(S_{Rm_b}P_R - yS_LfP_L)u(p_1) \tag{18}
\]

For \(u, c, d, c\) and \(s\) quarks, we only consider the contributions of the s-channel process to the scattering amplitude, which can be written as:

\[
\mathcal{M}_{22} = e\bar{v}(p_2)Q_\nu u(p_1)g^{\mu\nu}(p_4 - p_3)_\mu \\
+ \frac{eS_W}{c_W} \bar{v}(p_2)\gamma_\nu(g_\nu^{Zq\bar{q}} + g_A^{Zq\bar{q}}\gamma_5)u(p_1) \frac{g^{\mu\nu}}{\hat{s} - m_Z^2}(p_4 - p_3)_\mu \\
+ \frac{e(1 - 3s_W^2)}{2c_Ws_W\sqrt{1 - 2s_W^2}} \bar{v}(p_2)\gamma_\nu(g_\nu^{ZHq\bar{q}} + g_A^{ZHq\bar{q}}\gamma_5)u(p_1) \frac{g^{\mu\nu}}{\hat{s} - m_{ZH}^2}(p_4 - p_3)_\mu \tag{19}
\]

with \(Q = 2e/3\) (for \(q = u, c\)) and \(Q = -e/3\) (for \(q = d, s, b\)). Where \(\hat{s} = (p_1 + p_2)^2\) and \(\hat{t} = (p_1 - p_3)^2\) are the usual Mandelstam variables. We have neglected the light quark masses in our calculations except the bottom quark. From above equations, we can see that the production cross of the t-channel process is suppressed by the order of \((M/f)^4\). Furthermore, the cross section via the new heavy gauge boson \(Z_H\) exchange is suppressed by an order of magnitude compared to that for the s-channel \(Z\) exchange and photon exchange. Therefore, the main
Figure 5: The cross section of the process $q\bar{q} \rightarrow \phi^+\phi^-$ as a function of charged Higgs bosons mass $M_{\phi^\pm}$ for $f = 1000 GeV$ and $M = 150 GeV$.

production processes are the usual Drell-Yan processes through the $s$-channel $Z$ exchange and photon exchange.

To get the numerical results, we take the input parameters as $m_t = 172.7 GeV$ [18] and used the CTEQ6L patron distribution functions [19] and two-loop running coupling constant $\alpha_s(m_Z) = 0.118$. There are two free parameters $f$ and the value of the mixing parameter $M$. In this paper, we will take the typical values of $f = 1000$ GeV and $M = 150$ GeV. Our numerical results are shown in Fig.(5), in which we plot the production cross section $\sigma(\phi^+\phi^-)$ for the process $p\bar{p} \rightarrow \phi^+\phi^- + X$ at the LHC with $\sqrt{s} = 14 TeV$ as a function of charged Higgs bosons mass $M_{\phi^\pm}$ for $f = 1000 GeV$. To comparison, we use the solid line and dashed line to represent the contributions of the process $q\bar{q} \rightarrow \phi^+\phi^-(q = u, d, c, and s)$ and the process $b\bar{b} \rightarrow \phi^+\phi^-$, respectively. From Fig.5 one can see that the production cross section of the charged Higgs bosons $\phi^+\phi^-$ mainly comes from the usual Drell-Yan processes $q\bar{q} \rightarrow \phi^+\phi^-(q = u, d, c, and s)$ through the $s$-channel gauge bosons exchange and photon exchange. The total production cross section $\sigma(\phi^+\phi^-)$ is in the range of $134.5 fb \sim 1.7 fb$ for $150 GeV \leq M_{\phi^\pm} \leq 450 GeV$. The charged top-pions pair in the Higgsless-top-Higgs(HTH) model and charged Higgs bosons in the minimal supersymmetry standard moedl(MSSM) pair production at the LHC have been
calculated to leading and next-to-leading order \cite{20, 21}. They have shown that the total cross section for the charged top-pions and Higgs bosons pair production processes is smaller than $10 \text{fb}$ in most of the parameter spaces. Thus, we expected that the charged Higgs bosons $\phi^+\phi^-$ predicted by the LRTH model can be more easily detected at the LHC via this process than those for the charged top-pions $\pi^\pm$ in HTH model and charged Higgs bosons $H^\pm$ in the MSSM.

V. Conclusions

The SM predicts the existence of a neutral Higgs boson, while many popular models beyond the SM predict the existence of the neutral or charged scale particles. These new particles might produce the observable signatures in the current or future high energy experiments, which is different from that for the SM Higgs boson. Any visible signal from the new scalar particles will be evidence of new physics beyond the SM. Thus, studying the new scalar particle production is very interesting at the ILC and LHC.

The twin Higgs mechanism provides an alternative method to solve the little hierarchy problem. The LRTH model is a concrete realization of the twin Higgs mechanism. The cancellations of divergences occurs by alignment of vacua and existence of several new particles. The new particles in the LRTH model are heavy top quark, new gauge bosons, and new Higgs bosons, which might produce characteristic signatures at the ILC and LHC experiments.

In this paper, we discuss the pair production of the charged Higgs bosons $\phi^+\phi^-$ predicted by the LRTH model at the ILC and the LHC via suitable mechanisms. We can obtain the following conclusions: (i) For the production of $\phi^+\phi^-$ at the ILC, we found that the production rate is at the level of several tens fb in a large of the parameter space. The efficient $b$ tagging and mass reconstruction of the charged Higgs bosons is needed in order to reduce the background. We concluded that the charged scalars $\phi^\pm$ predicted by the LRTH model should be experimentally observable via the process $e^+e^- \rightarrow \phi^+\phi^-$ at the ILC. (ii) For the production of $\phi^+\phi^-$ at the LHC, we found that the total production cross section $\sigma(\phi^+\phi^-)$ is in the range of $134.5\text{fb} \sim 1.7\text{fb}$ for $150\text{GeV} \leq M_{\phi^\pm} \leq 450\text{GeV}$, which might be larger than those for the charged top-pions $\pi^\pm$ in HTH model and charged Higgs bosons $H^\pm$ in the MSSM. The main production processes are the usual Drell-Yan processes through the s-channel Z exchange and photon exchange. In conclusion, as long as the charged Higgs bosons is not too heavy, we conclude that the pair production of the charged Higgs bosons will be a good test for the LRTH
model at future ILC and LHC experiments.

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