Investigation of the Behaviour of Cohesive Powder in the Biaxial Tester

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Abstract

The results from experiments with cohesive limestone powder in a Biaxial Tester are presented. The biaxial tester allows a plane homogeneous deformation and measurement of the complete stress-strain states. It was found that proportional strain paths lead to associated proportional stress paths. Strain paths with a sharp bend lead to asymptotic stress paths. Under pure shear deformation, the limestone exhibits steady-state flow in the critical state. The critical state stress was found to be a function of the volumetric strain, i.e. the porosity, but not of the consolidation procedure.

The Cam-Clay model of Roscoe has been extended to include the cohesion exhibited by many fine powders. The cohesive strength is represented by an isotropic tensile stress measure, used to shift the yield curves into the tensile regime. Significant cohesive strength can only be achieved in fairly dense packings.

1. Introduction

Advances in bulk solids handling and silo technology have always been based on an increased understanding of the bulk solids mechanical behaviour. Janssen [1] introduced the notions of wall friction and lateral stress ratio in 1895 to calculate the stresses in the bin and the loads on the bin walls. In 1964, Jenike [2] made use of the concepts of yield locus, effective yield locus and internal friction to perform his investigations on the stresses in a hopper during discharge. More recently, several workers [3, 4, 5] applied the finite element method and constitutive models from soil mechanics to silo problems.

One major obstacle to the use of advanced constitutive models has been the lack of relevant experimental data for the materials and loading situations encountered in silo technology. Nearly all investigations have been done on soils such as sand or clay and stress levels of about 1000 kPa. Experiments at lower stress levels have been performed, e.g. by Maltby [6], Haaker [7], and Luong [8].

In this paper, experiments with a fine, cohesive limestone powder ($x_{50} \approx 5 \mu m$) in a true biaxial tester specifically designed for small stresses will be reported. The results gathered from the biaxial tests have been utilized to extend the Cam-Clay model to cohesive bulk solids.

2. The Biaxial Tester

The Biaxial Tester is one of several devices available to investigate the mechanical behaviour of the bulk solids. In contrast to the shear testers more common in silo technology, the Biaxial Tester allows measurement of the complete stress-strain state in the sample; a fact which predestines the results from the biaxial tester for use with advanced constitutive models. Figure 1 shows the biaxial tester of the
Institut für Mechanische Verfahrenstechnik of the Technische Universität Braunschweig. The material sample in the centre of the device is brick shaped. Its vertical (z) faces are covered by a top and a bottom plate.

In the horizontal directions (x and y), the sample is constrained by four steel platens (1 to 4). These platens are supported in such a fashion that each pair (1-3 or 2-4) can be moved without changing the distance of the other pair. The application of silica grease to the loading platens keeps the transmitted shear stresses to a minimum (μ ≤ 0.02) such that the measured normal stresses can be considered principal stresses. In all experiments the deformation of the sample has been prescribed and the resulting stresses have been measured.

The way of conducting experiments in the biaxial tester ensures that no shear strains or shear stresses can develop on the x, y and z planes. All measured normal stresses and strains must therefore be principal stresses or principal strains. This means that the experimental results can be fully described in the principal space. The line linking all measured strain states in the principal strain space is referred to as “strain path”. Similarly, a “stress path” is defined in the principal stress space. As the biaxial tester allows deformations in the x-y plane only, the strain path must lie completely in the $\varepsilon_x - \varepsilon_y$ plane. The stress path, however, is a general three-dimensional curve in the $\sigma_x - \sigma_y - \sigma_z$ plane. For convenience, the diagrams will nevertheless be limited to two dimensions.

3. Experiments with Fine Limestone Powder

3.1 Proportional Strain Paths

Experiments with a linear strain path belong to the simplest deformations possible in the biaxial tester. The experiment starts with $\varepsilon = 0$ and progresses with a constant deformation rate in the x and y directions. As the ratio of $\dot{\varepsilon}_y / \dot{\varepsilon}_x$ remains constant, these strain paths are referred to as proportional strain paths. For proportional strain paths, the ratio $\dot{\varepsilon}_y / \dot{\varepsilon}_x$ equals the ratio $\varepsilon_y / \varepsilon_x$, therefore in Figure 2 the $\varepsilon_y / \varepsilon_x$ ratio is used to characterize the depicted strain paths [9].

The resulting stress path is also linear and proportional. For each strain path, the measured stress paths from two independent experiments are shown. One can see clearly that to each direction of the strain path, a distinct direction of the stress path can be correlated. These paths are therefore called “associated stress and strain path”, their directions “associated directions”.

3.2 Asymptotic Stress Paths

In Figure 3, results from experiments are shown where the strain path shows a sharp bend. In the first part of the experiment the sample was consolidated with $\varepsilon_y / \varepsilon_x$. After 10 to 15% of volume change, the $y$ platens were stopped and deformation commenced in the x direction (curve $C_0...C_2$). The measured stress paths are made up of two parts as well. Initially, the stress paths follow the associated direction already known from Figure 2. After the change in the respective strain path, however, $\sigma_y$ decreases rapidly and the stress paths finally evolve parallel to the stress path $A_2$, which is associated to the strain path with $\varepsilon_y / \varepsilon_x = 0$. The ordering of the stress paths $C_0...C_2$ with respect to $A_2$ follows the amount of volume change during the first part of the strain path. This behaviour is called “asymptotic behaviour” and was first investigated with sand, where the influence of the initial straining proved to be negligible.

3.3 Pure Shearing

The investigation of the material behaviour in pure shearing is especially important since the interpretation of the standard Jenike and ring shear tests depends on the concepts of pure shearing and steadystate flow. All deformations without volume change will be considered “pure shear” deformations. In the biaxial tester, this requires $\dot{\varepsilon}_y / \dot{\varepsilon}_x = -1$ and means that the sample always has to be consolidated before it can be subjected to a pure shear loading.

Figure 4 shows the stress and strain paths of six experiments with pure shear loading. Starting from the same initial density, the samples were subjected to consolidation up to two stress levels under three initial path directions. One can see that the stress paths initially follow the directions known from Figure 2. As soon as the shear part of the strain path begins, the stresses decrease down to some stationary level. Subsequent shearing results in scatter around this point but no significant change in stress. This behaviour – deformation without change in volume and stress – is considered “Steady-State Flow” in silo technology. From Figure 4, one can also see the influence of the loading history. While six different consolidation paths have been used, only two levels of steady-state flow arise. Stress paths which belong to strain paths with the same level of volumetric strain end at the same steady-state stress level.

Further experiments showed that all steady-state stresses lie on a line through the origin, the “critical state line” [10].
Fig. 2 Consolidation under proportional deformation

Fig. 3 Asymptotic behaviour

Fig. 4 Material response under shear loading
4. Cam-Clay for Cohesive Bulk Solids

The constitutive model of Roscoe [11], commonly referred to as Cam-Clay Model, is based on the observation of steady-state flow and the concept of the critical state line.

The Cam-Clay Model was initially derived from Triaxial tests of cohesionless soils only. It involves a yield curve in \( s - llm \) space, where \( s \) is the deviatoric stress (a general shear stress measure) and \( llm \) is the hydrostatic pressure. For each bulk density (or porosity) a separate yield curve exists. Figure 5 shows a typical Cam-Clay yield curve. The curve shows the maximum deviatoric stress \( s \) the material can sustain under a given hydrostatic pressure \( llm \). This means that only stress states are permissible which lie either on or under the yield curve. One can see that the maximum sustainable deviatoric stress is associated with the intersection of the critical state line and the yield curve. It cannot sustain any deviatoric stress if the hydrostatic pressure becomes negative; i.e., the model is not defined for tensile stress states.

The experiments with the biaxial tester at the TU Braunschweig were performed with a very fine limestone powder which exhibits cohesion. Thus the original version of the Cam-Clay model is not applicable here. Nowak [10] has therefore extended the Cam-Clay model to include cohesion. Following the ideas of Höhl [5], it was assumed that any compressive deformation induces a small cohesive strength into the bulk solid. This cohesive strength originates from development of a small tensile strength, measurable by the isotropic tensile stress \( llt \) sustainable by the material. \( llt \) will be referred to as the isotropic tensile strength of the material. A saturation function ensures that the cohesive strength of the material cannot grow beyond all bounds.

The introduction of the cohesive strength into the Cam-Clay model leads to a shift of the left-hand part of the yield curve while the overall shape remains, see Figure 6. The point of no shear strength, which in Figure 4 was associated with \( \sigma_m = 0 \), has now moved into the tensile regime to the point \( \sigma_m = \sigma_t \).

In the Cam-Clay model, each yield curve belongs to a specific porosity \( n \), which in turn leads to a specific stress state on the critical state line. Using these two pieces of information, one can calculate the isotropic tensile strength \( \sigma_t \) belonging to a certain porosity. Figure 7 shows the isotropic strength \( \sigma_t \) plotted against the porosity \( n \) for the limestone powder used. It can be seen that the cohesive strength represented by \( \sigma_t \) remains fairly low for a porosity greater than 0.6 and that it changes rapidly for porosities between 0.5 and 0.6.

5. Symbols

- \( n \) [1] porosity
- \( s \) [kPa] deviatoric stress
- \( \sigma \) [kPa] stress
- \( x_{50} \) [\( \mu \)m] mean particle diameter
- \( \xi \) [1] strain tensor
- \( \varepsilon^{(0)} \) normal strains
- \( \dot{\varepsilon}^{(0)} \) strain rates
$\mu$ friction coefficient ($\mu = \tau / \sigma$)

$\sigma$ normal stress

$\sigma_1, \sigma_2, \sigma_3$ principal stresses

$\sigma_m$ hydrostatic pressure

$\sigma_i$ isotropic tensile strength

$\tau$ shear stress

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Study of mechanical and chemical engineering at the Technical Universities of Karlsruhe and Munich

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Dr.-Ing. 1971 at Karlsruhe (Prof. Rumpf) with a thesis on the shear properties of slightly compressed cohesive granular materials.

1971 – 1976 with Bayer AG at Leverkusen as head of a research group working in the field of mechanical process engineering.

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1982 – 1984: Vicepresident of the Technical University of Braunschweig
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