Active mode-locking of mid-infrared quantum cascade lasers with short gain recovery time

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Abstract: We investigate the dynamics of actively modulated mid-infrared quantum cascade lasers (QCLs) using space- and time-domain simulations of coupled density matrix and Maxwell equations with resonant tunneling current taken into account. We show that it is possible to achieve active mode locking and stable generation of picosecond pulses in high performance QCLs with a vertical laser transition and a short gain recovery time by bias modulation of a short section of a monolithic Fabry-Perot cavity. In fact, active mode locking in QCLs with a short gain recovery time turns out to be more robust to the variation of parameters as compared to previously studied lasers with a long gain recovery time. We investigate the effects of spatial hole burning and phase locking on the laser output.

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1. Introduction

Generation of ultrashort pulses in mid-infrared (mid-IR) quantum cascade lasers (QCLs) is one of the last remaining fundamental challenges in QCL physics and laser physics in general. The gain recovery time of a typical QCL is of the order of 1 ps, which is much shorter than the cavity round-trip time of order 50 ps. This effectively prohibits passive mode locking with a saturable absorber of any kind. Indeed, any initial intensity fluctuation in a cavity will quickly get damped if the gain recovers to its small-signal value immediately after the passage of the pulse, leading to preferential amplification of the tails of the pulse.

The situation with active mode locking (AML) is not so obvious, although there is a widespread belief that a long gain recovery time is essential in this case as well. AML has been observed in terahertz QCLs where the gain recovery time is believed to be significantly longer than in the mid-IR QCLs; see e.g. [1, 2]. AML was also observed in two-section mid-IR QCLs of a ”super-diagonal” design in which the laser transition is diagonal in real space so that the upper-state lifetime can be as long as 40-50 ps [3, 4]. The latter papers also pointed out that active mode locking in monolithic Fabry-Perot lasers appears to be limited by the spatial hole burning (SHB), i.e. a transient population grating created by a standing wave pattern inside the cavity. Scattering of the laser field off the population grating leads to the proliferation of modes with uncorrelated phases, which destroys single-pulse operation and leads to a chaotic output [3–5]. To avoid the detrimental effects of the SHB, [6] proposed active modulation of an external ring cavity QCL where the field propagates in only one direction and the population grating is not formed. The active device modeled in [6] was assumed to have a vertical laser transition and a short gain recovery time. Simulations in [6] showed a stable and robust generation of mode-locked pulses, but the pulse duration was very long for modulation at the round-trip period. Reaching the output pulse duration of ~ 5 ps required the gain modulation with ultrashort current pulses of 20-30 ps, which would be difficult to synthesize.

The studies in [6] revealed that short gain recovery does not prevent mode locking as long as the net gain window is opened for a short time as compared to the cavity round-trip time. To achieve this, the modulation of only a small part of the laser cavity seems to be crucial. Indeed if the gain responds to modulation instantaneously, modulation of the whole cavity will destroy an isolated pulse as it will experience both gain and loss during each roundtrip. According to our simulations below, in this case the steady-state output is either intensity modulated or simply dies out.

Opening a short gain window is straightforward in an external cavity. However, for many applications a compact monolithic design is preferable. Here we show that generation of mode-locked pulses of a few ps duration is feasible in a monolithic Fabry-Perot cavity even for QCLs with a short gain recovery time, as long as only a short section of the cavity is modulated at the cavity round-trip time. The cavity design is similar to the one employed in [3, 4]. However in contrast to those works we focus on a high-performance active region design with a vertical laser transition and a short gain recovery time. We compare the performance of lasers with long and short gain recovery times and find that standard QCLs with gain relaxation times of the order of 1 ps are in fact preferable for the AML. Indeed, in super-diagonal lasers electrons accumulate in the upper laser state and the modulation of injection is rather weak. The gain modulation could possibly be achieved by varying bias, which leads to variation of the oscillator strength. However, the DC bias modulation in the active region of a super-diagonal laser is reduced by a strong space charge effect and is accompanied by the modulation of the upper state lifetime and the shift of the intersubband transition energy. These effects can reduce the resulting gain modulation and are not easy to control and utilize. In vertical transition lasers the density of upper-state electrons is much smaller than the electron density in the injector. The dominant effect of the bias modulation is the modulation of injection current and gain.
We use a more realistic model of the QCL active region and transport which includes resonant tunneling injection, electron distribution over in-plane k-vectors, and space charge. We also perform time-domain and space-domain simulations of the propagating field, in which we can directly follow the pulse formation and the pulse instabilities in time. Note that all previous studies cited above used the frequency-domain modal approach and a two- or three-level laser model without current injection. We find the single-pulse operation to be quite robust to the variation of parameters such as the DC bias, bias modulation, and the length of the modulated section. At the same time the phase coherence and pulse duration are sensitive to even slight variations the modulation period: at the level of 0.3%. Moreover the optimal modulation period values for the shortest pulse and longest phase coherence time do not coincide.

2. The model of the active region and main equations

We use a four-subband model to describe the QCL active region. It includes the injector/extractor states (hereafter the ground state), upper laser state, and lower laser state, denoted as $g$, $u$ and $l$, respectively. A schematic diagram of the model and the two-section cavity are shown in Fig. 1.

![Schematic diagram of the active region model and the two-section cavity](image)

The system dynamics is described by coupled density matrix and Maxwell equations:

\[
\begin{align*}
\partial_t n_g &= \frac{n_u}{T_{ug}} + \frac{n_l}{T_{lg}} - J + D \frac{\partial^2 n_g}{\partial z^2}, \\
\partial_t n_u &= J - \frac{n_u}{T_{ul}} - \frac{n_l}{T_{lg}} - i \frac{dE}{\hbar} (\rho_{ul} - \rho_{ul}^*) + D \frac{\partial^2 n_u}{\partial z^2}, \\
\partial_t n_l &= \frac{n_u}{T_{ul}} - \frac{n_l}{T_{lg}} + i \frac{dE}{\hbar} (\rho_{ul} - \rho_{ul}^*) + D \frac{\partial^2 n_l}{\partial z^2}, \\
\partial_t \rho_{ul} &= -\left( i \omega + \frac{1}{T_2} \right) \rho_{ul} - i \frac{dE}{\hbar} (n_u - n_l), \\
\frac{\partial^2 E}{c^2} - \frac{n^2}{\epsilon_0 c^2 L_p} \frac{\partial^2}{\partial t^2} (\rho_{ul} + \rho_{ul}^*) = \frac{\Gamma d}{\epsilon_0 c^2 L_p} \frac{\partial_t}{\partial t} \left( \rho_{ul} + \rho_{ul}^* \right),
\end{align*}
\]
where \( n_g, n_u \) and \( n_l \) are the sheet densities of the corresponding states, \( \rho_{ul} \) is the off-diagonal density matrix element for a laser transition, \( d \) is the dipole moment, \( E \) is the electric field of the laser mode, \( n \) is the refractive index, \( L_\rho \) is the thickness of one period, \( \Gamma \) is the overlap factor, \( h\omega \) is the resonance energy of optical transition, and \( D \) is the diffusion coefficient, which we will take to be zero in simulations in order to consider the most unfavorable for AML case of a strong SHB. The injection current \( J \) is given by [7]

\[
J = \frac{e\Omega^2\gamma}{\hbar(\Delta^2 + \gamma^2)} \left\{ \theta(\Delta) \left( n_g - n_u e^{-|\Delta|/k_BT} \right) + \theta(-\Delta) \left( n_g e^{-|\Delta|/k_BT} - n_u \right) \right\} .
\]  

(2)

Here \( \Omega \) and \( \gamma \) are the coupling energy and the broadening of the states across the injection barrier, \( \theta(x) \) is the Heaviside function, and \( \Delta \) is the energy detuning from alignment. If we define the bias at alignment to be 0, then \( \Delta = eE_{\text{ext}}\delta \), where \( E_{\text{ext}} \) is the bias electric field, and \( \delta \) is the separation between the centroids of states \( g \) and \( u \).

Next, we generalize the ansatzes made in [5]:

\[
E(z,t) = \frac{1}{2} \left[ E_+(z,t)e^{-i(\omega_0 t - kz)} + E_+^*(z,t)e^{i(\omega_0 t - kz)} \right] + \frac{1}{2} \left[ E_-(z,t)e^{-i(\omega_0 t + kz)} + E_-^*(z,t)e^{i(\omega_0 t + kz)} \right],
\]

\[
\rho_{ul}(z,t) = \eta_+ e^{-i(\omega_0 t - kz)} + \eta_- e^{-i(\omega_0 t + kz)},
\]

\[
n_g(z,t) = n_{g0} + n_{g2} e^{2ikz} + n_{g2}^* e^{-2ikz},
\]

\[
n_u(z,t) = n_{u0} + n_{u2} e^{2ikz} + n_{u2}^* e^{-2ikz},
\]

\[
n_l(z,t) = n_{l0} + n_{l2} e^{2ikz} + n_{l2}^* e^{-2ikz},
\]

(3)

where \( k = n\omega/c \). \( E_+ \) and \( E_- \) are amplitudes of the fields traveling in right and left directions respectively. All populations contain the slowly varying average part and the grating part, denoted by subscripts 0 and 2. The injection current \( J \) is a linear combination of \( n_g \) and \( n_u \) when the space charge effect is not included. So, \( J(z,t) \) should have the same form as electron populations:

\[
J(z,t) = J_0 + J_2 e^{2ikz} + J_2^* e^{-2ikz}.
\]

(4)

Although we include the space charge in our work, if we ignore the effect of the population grating on energy detuning \( \Delta \) as the first order approximation, the conclusion above still holds.

Substituting these expressions into the density matrix equations, and making the slowly varying envelope approximation, we get the following equations for the envelope functions,

\[
\partial_t n_{g0} = \frac{n_{d0}}{T_{ag}} + \frac{n_{l0}}{T_{lg}} - J_0 ,
\]

(5)

\[
\partial_t n_{g2} = \frac{n_{d2}}{T_{ag}} + \frac{n_{l2}}{T_{lg}} - J_2 - 4k^2 D n_{g2} ,
\]

(6)

\[
\partial_t n_{u0} = J_0 - \frac{n_{d0}}{T_{ad}} - \frac{n_{u0}}{T_{ag}} + i \frac{d}{2\hbar} \left[ E_+ \eta_+^* + E_- \eta_-^* - \text{c.c.} \right],
\]

(7)

\[
\partial_t n_{u2} = J_2 - \frac{n_{d2}}{T_{ad}} - \frac{n_{u2}}{T_{ag}} - 4k^2 D n_{u2} + i \frac{d}{2\hbar} \left[ E_+ \eta_+^* - E_- \eta_-^* \right],
\]

(8)

\[
\partial_t n_{l0} = \frac{n_{d0}}{T_{ad}} - \frac{n_{l0}}{T_{lg}} - i \frac{d}{2\hbar} \left[ E_+ \eta_+^* + E_- \eta_-^* - \text{c.c.} \right],
\]

(9)

\[
\partial_t n_{l2} = \frac{n_{d2}}{T_{ad}} - \frac{n_{l2}}{T_{lg}} - 4k^2 D n_{l2} - i \frac{d}{2\hbar} \left[ E_+ \eta_+^* - E_- \eta_-^* \right],
\]

(10)
\[ \partial_t \eta_+ = -i \frac{d}{2\hbar} \left[ (n_{a0} - n_{i0}) E_+ + (n_{a2} - n_{i2}) E_- \right] - \frac{\eta_+}{T_2}, \tag{11} \]
\[ \partial_t \eta_- = -i \frac{d}{2\hbar} \left[ (n_{a0} - n_{i0}) E_- + (n_{a2} - n_{i2}) E_+ \right] - \frac{\eta_-}{T_2}, \tag{12} \]
\[ \left( \frac{n_c}{\epsilon} \partial_t + \partial_z \right) E_+ = i \frac{\Gamma \omega}{n_\text{egc} L_p} \eta_+ - i u E_+, \tag{13} \]
\[ \left( \frac{n_c}{\epsilon} \partial_t - \partial_z \right) E_- = i \frac{\Gamma \omega}{n_\text{egc} L_p} \eta_- - i u E_. \tag{14} \]

Here the waveguide loss \( l_w \) is added in the equations for the fields. These are the partial differential equations we solve numerically. The derivatives are approximated by the 2nd order expression, e.g. a quantity \( f(t) \) is updated at each step as

\[ f((n+1)\Delta t) = f(n\Delta t) + \Delta t f'(n\Delta t) + \frac{1}{2} (\Delta t)^2 f''(n\Delta t). \tag{15} \]

In experiment the laser oscillations in a cavity grow starting from the spontaneous noise, i.e. quantum fluctuations of the field and polarization. To include them in our modeling in a rigorous way we would have to perform second quantization of the field and electrons which would make the problem intractable. To adequately simulate the spontaneous noise we add a complex random source term to the right-hand side of the polarization equations Eqs. (11) and (12) which has zero average and is delta-correlated in space and time within the accuracy of the space-time numerical grid. The maximum magnitude of the random source is \( \beta n_{a0}/T_{\text{rad}} \) where \( T_{\text{rad}} \approx 10^{-7} \) s is the spontaneous radiative lifetime of the laser transition and \( \beta \) is the small geometric factor measuring the fraction of spontaneous emission coupled to the transverse laser mode. We also added a random delta-correlated electric field along the cavity at the initial moment of time to model initial fluctuations of the field. We performed numerous simulations with different values of the \( \beta \) parameter and initial noise. Figure 5 below shows one example of simulations for \( \beta = 10^{-4} \) and the initial random field of magnitude \( 10^{-3} \) V/cm. We found that the magnitude of fluctuations only affects the field dynamics during the initial transient stage of the laser field formation. The resulting pulses are not sensitive to the spontaneous noise level. We start observing noticeable changes to the shape of the pulses only when \( \beta \) approaches 1.

3. Base set of parameters

For QCLs with a short gain recovery time, the parameters used in our simulation are: (1) Resonant tunneling: \( \Omega = 1.5 \) meV, \( \gamma = 6.58 \) meV, \( \delta = 10 \) nm; (2) Lifetimes: \( T_{\text{ul}} = 1 \) ps, \( T_{\text{ag}} = 3 \) ps, \( T_{\text{ig}} = 0.1 \) ps, \( T_2 = 0.05 \) ps, \( D = 0 \); (3) Optical parameters: \( n = 3.3 \), \( l_w = 10 \) cm\(^{-1} \), \( \Gamma = 1 \), \( d = e \times 2 \) nm, \( \alpha_0 = 6.2 \) \( \mu \)m; (4) QCL structure parameters: \( L_p = 580 \) Å, \( n_{\text{doping}} = 8.0 \times 10^{10} \) cm\(^{-2} \), \( T_c = 300 \) K, cavity length \( L_{\text{act}} = 3 \) mm, facet reflection coefficients \( R_l = 0.5 \) and \( R_r = 0.5 \). Those parameters are typical for mid-IR QCLs. The threshold bias field obtained is 8.31 kV/cm below the alignment bias. In this paper, we use the bias at alignment as a reference, defined to be 0, then the threshold bias field is \( V_{th} = -8.31 \) kV/cm. For a laser with a long gain recovery time \( T_{\text{ul}} = 50 \) ps, three parameters in the active region are different from the short gain recovery time design, namely \( \Omega = 0.15 \times 1.5 \) meV, \( T_{\text{ul}} = 50 \) ps, and \( T_{\text{ag}} = 100 \) ps. The corresponding threshold bias is \( V_{th} = -9.31 \) kV/cm, which is close to the threshold in the \( T_{\text{ul}} = 1 \) ps case.

We choose the modulated section near the left facet, so that the modulation needs to be applied at the cavity round-trip time. The modulated section length is denoted by \( L_{\text{mod}} \), in units of \( L_{\text{act}} \). The DC bias \( V_{\text{DC}} \) is applied to the remaining part of the cavity. The bias on the modulated section is written as

\[ V_{\text{Mod}} = V_{\text{Mod,DC}} + V_{\text{Mod,Amp}} \sin \left( \frac{2\pi t}{T_{\text{mod}}} \right). \tag{16} \]
All biases are normalized to the threshold bias $V_{th}$. The modulation period $T_{mod}$ and all times on the plots are normalized to the phase roundtrip time $T_{\text{round}} = 2n_rL_{\text{act}}/c = 66$ ps. In order to study the dependence of the QCL performance on modulation parameters, we choose the following base set of parameters: $I_{\text{mod}} = 0.1$, $V_{\text{DC}} = 1.0$, $V_{\text{Mod,DC}} = 1.0$, $V_{\text{Mod,Amp}} = 0.5$, and $T_{\text{mod}} = 1.0$. We will change only one parameter at a time. Note that the threshold bias $V_{th}$ is negative, so $V > 1$ ($V < 1$) means below (above) threshold.

4. The effects of injection pumping and spatial hole burning

First, we analyze the effect of resonant tunneling. In previous works [3, 4], the current or gain were assumed to be uniform in each section of the cavity and sinusoidally modulated. This assumption looks reasonable at small modulation amplitudes but becomes problematic with deeper modulation. In particular, a large modulation amplitude as compared to the DC pumping level in [4] would create large negative swings in the current and gain, which is unrealistic for QCLs. Here we adopt a more realistic point of view that the injection current to the upper laser state results from electron transport under the applied bias and cannot be directly controlled. We assume that the input parameter that we can control and modulate is an applied bias whereas the current is calculated from Eqs. (2) and (5)-(14) assuming the Fermi-Dirac distribution of electrons in the injector with a given temperature $T_e$. In this case, a large modulation amplitude would move the injector out of resonance with the upper laser state, thus decreasing the gain but not flipping its sign and not reversing the current.

Figure 2 shows the spectrum of the bias, injection current, and gain (c) near the left facet assuming a sinusoidal modulation of the bias at a relatively large amplitude $V_{\text{Mod,Amp}} = 0.9$. The zero frequency part has been subtracted, since it only contains information about the average of the corresponding quantities. From this picture we can see that the response of the current to the sinusoidal modulation of the bias is noticeably nonlinear as it contains a number of higher harmonics. For a smaller modulation amplitude $V_{\text{Mod,Amp}} = 0.5$ only the second harmonic remains in the spectrum at a magnitude smaller than -10 dB; therefore, a sinusoidal modulation of the current would be a reasonable approximation.

![Figure 2](image-url)
Fig. 3. The output field amplitude over one roundtrip time taken close to the end of the simulation time for different values of the DC bias $V_{DC}$ in units of threshold bias for lasers with (a) $T_{ul} = 1$ ps and (b) $T_{ul} = 50$ ps. The DC biases in the modulated section and DC section are set to be equal.

Fig. 4. Same as Fig. 3, but without population grating.

The effect of the spatial hole burning (SHB) described by the population and injection current gratings at the half-wavelength period in Eqs. (3) and (4) is significant, although in contrast to statements in previous studies, we do not find SHB in a monolithic Fabry-Perot cavity to be the main obstacle for the AML. The effect of SHB is more nuanced as illustrated in Figs. 3 and 4 which show the dependence of the pulse shape on the DC bias for lasers with short and long relaxation time. Here we keep the DC part of the bias in the modulation section $V_{mod,DC}$ equal to the bias in the DC section $V_{DC}$, to minimize the number of free parameters. For a laser with $T_{ul} = 1$ ps, both the intensity and pulse duration increase when the DC bias is increased. Also, rapid oscillations within the pulse envelope are developed at high enough bias. Comparison with
simulations carried out in the absence of the populations grating, \( n_{(\mu,\lambda)}=0 \) and \( J_2=0 \) (Fig. 4) show that the substructure is due to the SHB instability because it is not present in the absence of SHB. The same conclusion is true for the case of \( T_{\text{ul}} = 50 \text{ ps} \), where the multiple pulsations disappear when \( n_{(\mu,\lambda)}=0 \). At the same time, an overall increase in the pulse duration is due to an increase in the net gain window and resulting amplification of the tails of the pulse. We again notice that in a laser with a short gain recovery time the single pulse regime is more robust.

5. The effect of the modulation period on the pulse duration and phase coherence

An apparent optimal choice of the modulation period is the phase round-trip time, i.e. normalized \( T_{\text{mod}} = 1 \) in Eq. (16). However, this turns out to be not the best choice for the long-time stability of the output. In Fig. 5 we plot the output field at the left facet for the whole simulation range of 10,000 roundtrips. As one can see, it takes a long time before the output intensity eventually stabilizes, although there is still a slight long-term variation in the magnitude of the pulses. Furthermore, although the amplitude of the output field is nearly periodic, the real (Fig. 5(b)) and imaginary (not shown) parts of the field are not. This means that the spectrum of the output field is not phase locked. The autocorrelation function for the real part of the field, defined as

\[
R_E(\tau) = \int_{t_1}^{t_2} \text{Re}[E(t)]\text{Re}[E(t+\tau)]dt \Bigg/ \int_{t_1}^{t_2} \text{Re}[E(t)]^2 dt ,
\]

Fig. 5. (a) The output field amplitude and (b) real part of the field at the left facet for the whole simulation range of 10,000 roundtrips and the base set of parameters. After the output is stable, there are still some small oscillations in the pulse amplitude. The real part of the field is not periodic in the modulation period, indicating that the phase is not locked.
experiences oscillations between 1 and 0 with a period of about 200 round-trips.

Fig. 6. (a) The amplitude of the output laser field on the left facet. (b) The injection current at the point adjacent to the left facet. (c) The difference between the gain and waveguide loss $g - l_w$ at the point adjacent to the left facet. The gain follows the injection current almost instantaneously, due to the short gain recovery time. The peak of the pulse has a delay with respect to the maximum of the gain. (d, e) A frame in the movie (Media 1, size: 3.50 MB) of the evolution of field intensity (d) and $g - l_w$ (e) in the same laser.

To understand the reason for these long-time variations, in Fig. 6 we show the output laser field from the left facet for the base set of parameters over the time interval of 5 periods, taken close to the end of the simulation time. Along with the laser field we show the injection current and net waveguide gain $g - l_w$ (excluding mirror losses) near the left facet. The duration of intensity pulses is about 10% of the round-trip time, which is around 7 ps. Although the injection current and gain are shown for a point near the left facet, this point can represent the whole modulated section since the gain saturation effect is small at these intensities. From Fig. 6(a) we can see that the gain follows the injection current, while the peak of the pulse is delayed with respect to the gain maximum, namely the pulse is experiencing the tailing edge of the gain. This can be explained by the fact that the group velocity of the pulse is smaller than the phase velocity and therefore the gain modulation is slightly out of phase with the circulating pulse. This can be visualized in a movie (Media 1, Figs. 6(d) and 6(e)) which shows the dynamics of the field intensity and gain over the two modulation periods. Therefore, the gain modulation is slightly out of resonance with the circulating pulse which causes long-period oscillations in the output.

To eliminate the phase mismatch we slightly increase the modulation period to $T_{mod} = 1.003$ to compensate for the group delay of the pulses. As shown in Fig. 7, this leads to the stable output after the laser field build-up time of about 200 round-trips. Moreover, the real and imaginary parts of the field are also exactly periodic as can be verified by the autocorrelation function which has a constant value equal to 1. At the same time, the pulse duration is longer in this case as illustrated in Fig. 8 which shows the pulse shape as a function of the modulation period $T_{mod}$. When $T_{mod} = 1$, the pulses generated by a laser with a short gain recovery time have the shortest duration. When $T_{mod} = 1.003$, the intensity of the pulse increases significantly as it has the best overlap with the maximum of the gain in a modulated section. At the same time, the time window of the net gain is increased in this case, leading to a longer pulse duration.
Fig. 7. (a) The output field amplitude and (b) real part of the field at the left facet over 1,000 roundtrips in a laser with a short gain recovery time for the base set of parameters but a slightly longer modulation period $T_{\text{mod}} = 1.003$, which matches the group roundtrip time of the pulse. Both the field amplitude and the real part of the field become strictly periodic after about 200 roundtrips.

6. The output dependence on the modulation amplitude and length of the modulated section

Two other parameters that strongly affect the output pulses are the modulation amplitude and length of the modulated section. Figure 9 shows the effect of varying modulation amplitude. As expected, for both $T_{\text{ul}} = 1$ ps and $T_{\text{ul}} = 50$ ps cases, the peak intensity of the pulse increases with the modulation amplitude. At the same time, the pulse becomes more asymmetric in the $T_{\text{ul}} = 1$ ps case, which is the result of the phase mismatch between the modulation period and group round-trip time. For a laser with a long gain recovery time, the $T_{\text{mod}} = 1$ modulation period is closer to resonance with the group round trip time and the pulses are more symmetric. Also the gain saturation effect is stronger, so the tailing edge of pulse is not amplified.

With increasing length of the modulated section, the pulse experiences a stronger round-trip gain but at the same time the window of the net gain becomes wider which leads to the pulse broadening. Fig. 10 shows the dependence of the output on the length of a modulated section. In the $T_{\text{ul}} = 1$ ps case, when $l_{\text{mod}}$ increases from 0.1 to 0.5, the intensity and duration of the pulse increase at the same time. For longer $l_{\text{mod}}$ the pulse eventually broadens to the whole cavity and its peak intensity drops. However, the pulsation still exists even when the modulated section is 90% of the whole chip. In the $T_{\text{ul}} = 50$ ps case, one only gets a good pulse around $l_{\text{mod}} = 0.3$, and this pulse is shorter as compared to the $T_{\text{ul}} = 1$ ps design. However, with increasing $l_{\text{mod}}$ the
Fig. 8. The output field amplitude over one roundtrip time taken close to the end of the simulation time for different values of the modulation period $T_{\text{mod}}$ measured in units of phase roundtrip time for lasers with (a) $T_{ul} = 1$ ps and (b) $T_{ul} = 50$ ps and the base set of parameters.

Fig. 9. The output field amplitude over one roundtrip time taken close to the end of the simulation time for different values of the modulation amplitude $V_{\text{Mod,Amp}}$ in Eq. (16), measured in units of threshold bias for lasers with (a) $T_{ul} = 1$ ps and (b) $T_{ul} = 50$ ps and the base set of parameters. No significant output is generated when $V_{\text{Mod,Amp}} \leq 0.1$. 
output quickly turns into multiple chaotic pulsations. It is indeed expected that a laser with a long population relaxation time comparable to the round-trip time is more prone to instabilities than the laser with a short recovery time in which the pulsations of the population inversion are strongly damped.

When the whole QCL cavity is modulated, no isolated pulses are obtained in both cases, and only the intensity is modulated. We checked that this conclusion remains true for the parameters typical for THz QCLs. The observation of isolated pulses in [1, 2] could be the result of the nonuniform injection of modulated pumping along the cavity. Indeed the THz waveguide forms a microstrip transmission line for the microwave modulation which forms a standing wave along the cavity.

Fig. 10. The output field amplitude over one roundtrip time taken close to the end of the simulation time for different lengths of the modulated section \( l_{\text{mod}} \) measured in units of the total cavity length \( L_{\text{act}} \) for lasers with (a) \( T_{\text{ul}} = 1 \) ps and (b) \( T_{\text{ul}} = 50 \) ps and the base set of parameters.

7. Conclusion

In conclusion, we showed that standard high-performance mid-IR QCLs with a short gain recovery time can be actively modulated to generate mode-locked pulses of a few ps duration when the modulation is applied only to a short section of a monolithic Fabry-Perot cavity to create a net gain window for a circulating pulse. The performance of QCL with a short gain recovery time is more robust to varying parameters as compared to QCLs with a long gain recovery time. By fine tuning the modulation period to the group round-trip time of the pulse, one can phase lock the electric field of the pulse and use its spectrum as a frequency comb.

The peak power of the generated pulses grows with increasing modulation amplitude and DC bias, but is limited by the concomitant pulse broadening due to widening gain window, as is seen in Figs. 9 and 3. For a base set of parameters the peak power of the pulses is about 16 times higher than the CW output power for the DC bias equal to the time-averaged modulated bias of Eq. (16).
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