A Look at Motion Planning for Autonomous Vehicles at an Intersection

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Abstract— Autonomous vehicles are currently being tested in a variety of scenarios. As we move towards autonomous vehicles, How do Intersections need to look? To answer that question, We break down an intersection management into the different conundrums and scenarios involved in the trajectory planning and current approaches to solve them. Then a brief analysis of current works in autonomous intersection is performed. With a critical eye, we try to delve into the current solutions’ discrepancies while providing some critical and important factors that have been addressed and look at open issues that have to be addressed. We also try to answer the question of how to benchmark intersection management algorithms by providing some factors that have an impact on the system.

I. INTRODUCTION

Autonomous Driving is a developing field with multiple significant developments in the recent past. Any vehicle traverses through a variety of different scenarios ranging from platooning, Urban environments, intersections, highway driving, parking. Each of these scenarios from an autonomous perspective require a different set of parameters and priorities and different challenges ranging from perception, decision making, tracking, path planning, motion planning and communication to name a few. Despite recent developments in Autonomous driving, the navigation of a fleet of autonomous connected cars at an intersection is still an arduous task. Motion planning techniques for Autonomous Vehicle at an intersection have evolved with time and the recent past has seen many interesting approaches proposed to solve the same. This work is an attempt at covering those proposed approaches for trajectory planning of Autonomous Vehicles and how they have been adapted for Autonomous intersections whilst also covering a variety of other approaches that attempt to achieve a vehicle’s autonomous navigation of an intersection. Motion planning techniques for Autonomous Vehicle have to generate a trajectory for autonomous vehicles whilst accounting for the static and dynamic obstacles in the environment, trajectory limits, actuator limits, pedestrians, interactions between vehicles, other moving obstacles and also spend as little time as possible at an intersection. Thus, it is a challenging problem that will have to account for a multitude of constraints and also a multi agent system in which the agents may have to cooperate or be non cooperative depending on their direction of travel and current states.

With the recent advancements in Vehicle to Vehicle (V2V), Vehicle to everything(V2X), Vehicle-to-Pedestrian (V2P), Vehicle-to-device(V2D) and Vehicle-to-grid (V2G) communication between vehicles have been utilized by the Intersection coordinators with many of the Intersection algorithms utilizing a Vehicle to Intersection (V2I) communication for data transmission. This reduces the communication problem

In this work, we attempt to analyze the current algorithms that have been presented for autonomous vehicle’s navigation at an intersection. We look at the limitations of current works and then provide a look at the open problems in the field of autonomous vehicles at intersection.

The rest of the paper is organized as, in section II we look at the problem of vehicles at an intersection and then in section III the trajectory generation of autonomous vehicles for mobile robots is looked at. Section IV Trajectory planning of autonomous vehicle in an obstacle filled environment is looked at. Section V Provides a look at Trajectory generation for mobile robots in dynamic environment and section VI looks at Multi Robot trajectory generation. Section VII Looks at autonomous vehicles at intersection and their approaches and in section VIII we provide the drawbacks and Section IX looks at factors that need to be utilized for intersection algorithms and Section X concludes the paper by answering the question of How do Intersections need to look.

II. Problem

In this work we try to analyze a problem of N number of vehicles that are at an intersection and would like to traverse across it. The Intersection is defined here as a M different roads with an arbitrary J different lanes connecting across each other. We also assume that at the intersection there may or may not be any obstacles that try to affect the vehicle.

The aim of the vehicle in the environment to traverse across the intersection in minimum time whilst also ensuring that none of the vehicles don’t collide with other vehicles and/or obstacles that are available in the environment, while also ensuring that the passengers in all of the vehicles are comfortable.

III. Trajectory generation for Autonomous Vehicles

Autonomous Vehicle can fundamentally be seen as a subset of ground based mobile robots that attempt to traverse an ever changing and dynamic environment autonomously at higher velocities than mobile robots whilst also being larger in size. Trajectory generation for mobile robots is generally done by utilizing means like RRT, Polynomials,
Trajectory planning for autonomous vehicles is complicated by the bigger size which thereby means that dynamics are considerably much more complicated and are utilized in a structured or semi structured environment. One of the first works that attempted to achieve the same is [5] wherein they utilised a A* for discrete trajectory planning, following which they utilised a Non Linear Program for smoothing the trajectory with the optimisation problem’s objective consisting of distance between obstacle and vehicle, change in velocity and change in tangential angle at the vertex. This optimised solution is locally optimal and is hence utilized to interpolate a cubic trajectory that is tracked. This work was one of the foremost of works that utilized Optimisation techniques to solve for trajectories.

High speed Trajectory planning for Autonomous Vehicle utilizing the dynamic constraints of the vehicle was taken into account was done by [7] to plan trajectories with reference points for the vehicle in a Model Predictive Control formulation. This trajectory is at the near speed limits of the vehicle.

IV. TRAJECTORY PLANNING IN OBSTACLE FILLED ENVIRONMENT

Trajectory planning in obstacle filled environment is a harder problem to solve than just a trajectory generation. Trajectory planning and tracking for autonomous mobile robots in an unknown environment is still an unsolved and an interesting problem.

In [6] utilised an covariant Hamiltonian based trajectory optimisation that was one of the first works to utilise for the obstacle avoidance an artificial potential that consisted of the line integral parametrised arc length of the path that the both passed through the obstacle which is integrated as a product of velocity to be invariant to the velocity along the path with the function being the Euclidean Distance transform. This work used a Hamiltonian based monte Carlo sampling that is utilized for sampling from the probability distribution. Although this work was tested of quadrupeds, it is an effective method that can be utilized for a robot trajectory planning effectively.

[13] proposed an online trajectory re planning and obstacle avoidance wherein they solved an online linear program for generating an obstacle free lateral trajectory and then tracked the generated trajectory using an online linear Model Predictive Control. This work while considering vehicle dynamics, assumed that longitudinal velocity of the vehicle to be constant and non-varying. In [9] a linear programming based approach to plan time optimal trajectories for autonomous vehicles in tube like roads were undertaken incorporating the system dynamics as a spatial model. They also incorporate the spatial dynamics of the vehicle and also corridor constraints of the vehicle. This method also accounts for collision avoidance constraints with the road but utilises a kinematic based model of the vehicle.

In [10] a collision avoidance based trajectory generation for autonomous vehicle consisting of geometric constraints, actuator limits and kinematic dynamics. They transform the vehicle dynamics spatially and then utilize that to generate a reference trajectory for the vehicle along the road coordinate frame and then linearise the obstacles. This optimization problem is then solved to generate a trajectory by using a Sequential Linear Program. This proposed methods then provides the vehicle with linear velocity and steering commands to drive the vehicle. The system was tested in driving in a tight maneuvering scenario thereby showing a good performance.

In [11] a spatial based model predictive control is proposed for performing obstacle avoidance in an adaptive cruise control environment. They utilized a nonlinear bicycle model with the inputs being the throttle, braking, steering and gears. Then they use a geometric corridor that is formulated by sampling at a discrete points the velocity and trajectory based obstacle filled environment for the vehicle. Then the vehicle’s trajectory is planned constraining it to be within the geometric corridor. For this work to function in a dynamic environment continuous re-planning has to be done.

Trajectory planning for autonomous vehicles under obstacles generally utilize a kinematic model of the vehicle to spatially transform the trajectory for optimizing time. The above mentioned works are all tested in a simulated environment and the feasibility of the mentioned works in a real time environment is still unknown.

V. TRAJECTORY PLANNING IN DYNAMIC ENVIRONMENTS

Trajectory planning in a dynamic environment is complicated by the dynamic nature of the obstacles. Thus this requires a dynamic model being utilized in these environments and the quality of the generated trajectory depends on the accuracy of the prediction model. Thus, to overcome this a
continuous re-planning of the trajectory has to be done to account for the inaccuracies in the predicted trajectory of the dynamic obstacles. The dynamic nature of the obstacles also results in a possibly non convex optimization problem.

One of the first works for trajectory planning in a dynamic environment is [12] which formulated a closed form solution based on the kinematic model of the vehicle as dependent on the vehicle’s collision probability. This solution is utilized for the formulation of a collision avoidance constraint based on which a trajectory is generated, differentiated twice and updated every time the environment changes.

In [8], a model predictive control based trajectory generation algorithm was presented for autonomous vehicles. They utilized a trajectory planned by minimizing the distance to the desired point, jerk, velocity while also utilizing artificial potential field for collision avoidance between the autonomous vehicles. The also formulated the vehicle as a box and added lane constraints to the vehicle. They tested the algorithm in simulated scenarios using other vehicles. The other vehicle’s path were simulated as constant acceleration based model.

In [14] a time scaled collision cone is utilized for trajectory planning in dynamic environments. This is done by using a band of predicted trajectories and utilize that predicted trajectory to formulate the collision avoidance. To find the nonlinear time scale, which becomes the optimal trajectory of the vehicle; they utilized from the sampled solutions the cost that is minimum of the time difference and collision.

In [15] a kinematic model is utilized for optimal path planning for 2D environments. The model is transformed to a set of finite differential equations which incorporate junctions of the robots and free space. This separation and junctions are utilized to solve the problem as a stochastic differential equation based optimization. The method also solves for time and minimizes the distance traveled.

All these above mentioned work utilizes a trajectory optimization based methods for generating the trajectory. In [16] sampling based approaches for motion planning are presented with two types of algorithms. The first one assumes a non cooperative motion planning between the vehicles and second a cooperative motion planning which are feasible in an environment like platooning or overtaking. The works were all tested on a real life platform. Furthermore, the thesis also experimented upon utilizing receding horizon principles for trajectory planning thereby allowing fixed time to look forward. The thesis also proposed a bounding volume based hierarchy for collision that checks for dynamic obstacle configurations if a collision looks probable.

A Survey of Motion Planning techniques for self driving cars is presented in [22],[21] puts forth a survey of the different motion prediction models are conducted.

VI. MULTI AGENT TRAJECTORY PLANNING

Until now in this paper we were looking at works proceeded under the assumption of a single robot in an obstacle filled environment. An intersection is a multi agent system with dynamic and static obstacles hence it is pertinent to look at methods for planning trajectories for multi agent systems.

In [17] a Decentralised planning for multi Agent systems collaboration was proposed using polygonal based representations for non convex obstacles. They utilized a hybrid method to detect collisions and utilized a switching method of the systems to achieve the same.

In [18] an online distributed system is presented for Multi robot holonomic vehicles using Alternating Direction Method of Multipliers. The Vehicle is formulated by a kinematic model and the collision avoidance constraints are enforced using separating hyperplanes. Then a slack of the trajectory of other vehicles is available for the Ego vehicle to generate feasible collision free trajectories.

[19] proposed a bi directional vehicle coordination scheme utilizing prioritized decoupling of path planning and an aim to reach platooning of vehicles as quickly as possible. This path planning is done by a centralized system and the reference trajectories are published back to the respective vehicles.

In [20], a centralized multi robot trajectory planner for obstacle free environments was proposed utilizing tools from non linear optimization and calculus of variation. Furthermore, the approach utilized a two step process for planning trajectories wherein a piecewise linear trajectory is generated based upon geometric constraints in the first step and in the subsequent step a higher order polynomial parametrized trajectory is generated with the multi robot trajectory planning performed based upon a quadratic programming approach. They tested the method using quadrotors in an obstacle free environment, but it is tractable for mobile robots.

VII. INTERSECTION MANAGEMENT

While all the above works presented the approaches to trajectory planning for autonomous vehicles in different scenarios, Trajectory generation for intersection is going to be a dynamically evolving in terms of vehicle quantity position and obstacles this sections attempts to review some of the works with respect to autonomous intersection management system. In the current scenario, an automated intersection manager will have to take into consideration the semi autonomous, manually driven vehicles also.

From a high level perspective autonomous intersection management systems can be classified based on the methods utilized as mentioned below:

- **Multi Agent Systems**: These management systems consider autonomous intersections as a series of multi agent navigation problems with each agent having a set of goals to reach with a trajectory being generated for each agent, [33], [35], [40]

- **Slot Based Systems**: The following works consider autonomous intersections as slots provided to vehicles from different sides as utilized for air traffic management systems with the vehicles requesting for reservation to utilize the intersection area [26] [23], [24], [25]

- **Detection Based**: These algorithms are not designed for autonomous intersection management systems but rather
A. Slot Based systems

Slot based systems are something that is being looked at currently as it allows vehicle coming from a direction slots to traverse in [23] proposed a multi agent approach to autonomous vehicles that utilized a scheduling based system. A vehicle upon arrival at the intersection send a request and the intersection manager provides a response based on analyzing the algorithm provided a velocity, trajectory and outbound lane of the vehicle. This method utilizes a multiagent system with the intersection split into grids and the collision of vehicle checked along the grid points, resulting in a discrete time approach. An improvement over this was proposed in [32], where a reservation based approach for reserving space time for a vehicle is utilized that bypasses the requirements to communicate and request for space for the vehicles. They introduced a mixed integer linear program to optimize for reservation and then using temporal rolling horizon they move the horizon forward at each instance. They further improved upon the discrete time based proposed in [23] by utilizing continues time trajectory collision avoidance checking by using conflict points and checking for collision only at the collision points.

In [28] a set of rules for coordination at an intersection based on Chinese traffic rules(Right hand Driving) was proposed that employed whether another vehicle has to yield or move in the right of way for the system. In [30] a distributed algorithm for intersection crossing that transmits state estimation, uncertainty and desired lanes based on which the priority based vehicle traversal occurs. The messages passed are acceleration, velocity, longitudinal and lateral position. In [25], an all direction turn lanes are proposed which employs the fact that Autonomous Vehicle needn’t have to utilize directional turning lanes and can function optimally in scenarios that require the vehicles to turn out in different directions from different lanes due to their high rate of control. Based upon this paradigm a collision free regions are provided for the vehicles depending on its position and other available vehicles.

[31] proposed a method to optimize reservation algorithms that uses a tile based/grid based trajectory discretization. They addressed the problem of resolving non-cooperating(conflicting) requests that have to optimize. An Integer program was put to use to solve for the optimal reservation strategies.

[29] utilized a mixed integer linear program to solve for optimal scheduling of the vehicles using the bi directional communication of the vehicles by trying to minimize the time of access for vehicles at the intersection and constraining the maximum access times for the vehicle.

In [26] They propose a framework for comparing different intersection algorithms by using queuing theory and then A slot based system is proposed. The proposed two methods are based upon queuing theory and were formulated to balance the paradigms of vehicle flows or capacity with each one emphasizing on either one of the paradigms. Based upon these paradigms and flows of the vehicles time scale of intersection access is defined.

B. Multi Agent approaches

The above discussed methods utilized communication based strategies for collision avoidance, which do not account for how the agent’s motion changes accurately or provide an optimal path to vehicles. This is a drawback as for optimal traversal vehicles a decentralized or a mixed system being utilized will provide a better efficiency and we look at trying to employ some methods to find out optimal paths to achieve the same.

In [33] to compensate for such drawbacks, a multi objective evolutionary algorithm was developed that was utilized to develop a fuzzy rule base for controlling the speed of the vehicles entering the intersection. The objective for optimization was taken to consist of the number of vehicles, the vehicles position and velocity and assume that the vehicles don’t collaborate with each other and then give trajectory for vehicle speeds for it to traverse along the intersection. In [34] a game theoretic approach to optimize traffic flow along multiple intersections. They attempt to develop an optimized intersection of the vehicles along multiple intersections utilizing the availability of connections amongst the intersection controllers. They formulate the traffic intersection as a non-cooperative game amongst different players and then based upon requests from different players (vehicles) the signal lights are changed. While the above proposed method function is utilized only for traffic light control. The approach though can be utilized by both Autonomous Vehicle and non automated vehicles alike.

In [35] the autonomous vehicle space-time trajectory is transformed to a spatial dynamic model thereby reducing the problem’s dimensional space from a higher dimension. Based upon this formulation a centralized optimization problem for all vehicles available along the intersection’s control radius is solved easily as it is a convex region for collision. The objective is also reformulated to spatial reformulation appropriately and penalised as a quadratic function, thereby solving a Quadratic Program. Based upon this, the authors of [36] proposed a centralized Model Predictive Control for autonomous intersections that utilized the spatial reformulation of dynamics and solved a Quadratic Program. They added slack variables to compensate spatial discretization of dynamics and the collision avoidance constraint that constraints only one vehicle at a point in the critical region of an intersection. This results in an easy collision avoidance but isn’t scalable for heavy volumes of traffic flow and increases time spent at an intersection in case of many vehicles. The mentioned works also require that the vehicles have a reference trajectory throughout the intersection which may not be available at all times.

Furthermore, [37] proposed an online optimization algorithm for controlling the vehicle speeds utilizing Minkowski addition and formulated unsignalised intersection problem with speed ratios for collision avoidance while also using
Minkowski addition to transform vehicle to points and solved a Quadratic Program for control actions. The above work was simulated for scalability but in real life assumes control of the vehicle i.e is a centralized system hence requires a huge computation as the vehicle number scales.

To overcome the problems with centralized approaches to autonomous intersection, [38] proposed a distributed Model Predictive Control for autonomous intersections utilizing parallel optimization. The distribution is done by giving every individual vehicle a local objective and overall constraints thereby resolving the centralized optimization problem into a quadratically constrained quadratic program. Non convex collision constraints are prioritized and converted to a semi definite programming that is solved online. In [39] proposed a combinatorial optimization problem that is decomposed into two problems, a first problem of finding the optimal coordination of the vehicles centrally and then subsequently in each vehicle a local optimal control problem is solved depending on the systems constraints and timing slots specified to each vehicle.

[40] Proposed a reservation based optimization wherein the problem is broken down into two parts of scheduling vehicle’s traversal times and then in the second step formulating a continuous time trajectory for the same. They propose a heuristic method to find locally optimal solutions for continuous time trajectory.

The above mentioned works all required that the vehicles have a complete set of data of the vehicle’s current states and desired states. This raises privacy constraints and/ or lack of data from the vehicle’s transmission sides. In the recent past, some works have attempted to overcome that problem.

1) Under data unavailability/ uncertainty: A Mixed Observability Markov Decision Process was utilized in [42] as a method of intersection merging, wherein they utilize a probabilistic model consisting of the two states (move and stop) and utilize the velocity profile of the other vehicles that are in the environment. They tested the method for the presence of human drivers and their intent. Then in [41], a partially observable Markov decision process was utilized with the intention of other vehicles as hidden variables. They tried to formulate an optimization problem for finding the optimal acceleration of the vehicle along pre planned paths. They then utilized a decomposition to formulate a low dimensional continuous time path planning for vehicles that considered future layout and future uncertainties. These works due to an abstraction of desired goals currently can at most perform worse than the methods that function without utilizing privacies.

VIII. DRAWBACKS OF THE CURRENT STATE OF THE ARTS

Reservation based intersection management systems provide vehicles with a time slot for their traversal and a speed at which they have to travel at an intersection while reducing the burden on the system also generally results in a vehicle having to slow down at the intersection or have to comparatively slow down at the intersection. These algorithms also sample the trajectories at grids and tiles and the collision avoidance’s accuracy is dependent on the discretization. Moreover the first come first serve treatment of these algorithms result in sub optimal trajectories. These algorithms albeit are much better for heterogeneous vehicles

On the other hand, the optimal control/MPC based approaches couple problems of trajectory optimization and tracking and spatially reform the trajectories. This reformulation requires that the vehicles utilize time as a function of position. These approaches work primarily as a centralized approach and the resultant problem’s complexity scales with number of agents and/or vehicles with different directions of flow. These complexities result from a higher dimension space and higher interactions to be accounted for thereby reducing the feasible space. Moreover, with the exception of some, many of these algorithms were tested for only one scenario and/or with limited number of vehicles.

Therefore as a whole, drawbacks of current intersection management systems are:

1) Reinitialization of algorithms are difficult: This is an important problem as in the optimal control based approaches, reinitialising the solver in the next iteration appropriately with respect to either the trajectory of the previously available vehicles or utilize previously generated trajectories for the vehicles and employ them as a starting point for next iteration when new vehicles might be added and some vehicles might leave the intersection. Hence, this makes it important that we utilize appropriate solutions to reinitialize the numerical optimal control problems.

2) Scalability of the algorithms: Majority of the optimal control based formulations, that use MPC based approaches, have been tested for their performance with a meager amount of vehicles. This performance may not be tractable when the number of vehicles is in tens and hundreds as this adds huge computational regions.

3) Conservative/non realistic collision approximations: The approximation of the vehicle sizes are either modeled as a circle or polygon. Formulation of vehicles as polygons is more appropriate formulation but many prevalent algorithms utilize vehicle shape as circular, which is a conservative approximation of the vehicle’s region of interest. On the other side of the spectrum, approaches at times formulate an intersection region that allows only one vehicle into it. This region is a huge region and thereby wastes a lot of the available space, thereby making it sub optimal. A step towards latter has been attempted by [25].

4) Discrete Intersection regions: The reservation and slot based approaches simulate discrete space grids and then simulates the vehicle’s collision at those grids or tiles. Despite recent advances that evaluate the intersections only at important points the usage of discrete approaches always have a probability of excluding critical points of collisions.

5) Systems developed for specific types of intersections: The developed algorithms are implemented and tested for specific type of intersections like a four way, ‘T’
Some factors which we think are important to analyze to compare and contrast intersection managers is important. A. Factors to consider for Intersection has to be done by the vehicle. 

developed intersection managers, a local trajectory planning section. This means that for utilizing the actions from the count lane marking constraints, static obstacles, pedestrians, 

1) Scalability: An intersection manager should be able to handle a high number of vehicles efficiently while also showcasing an ability to showcase similar performance when the number of vehicles at the intersection are less

2) Time Spent at intersection: It is important that all vehicles spend as little time as possible at intersections. An analysis of the time spent at intersection will provide an understanding of the performance of the intersections as it reduces overall commute times and reduces emissions, energy expenditures

3) Smoothness of trajectories: The trajectories that vehicles follow when traversing across intersections need to be at least continuous until jerk of the vehicles as autonomous vehicles have third order dynamics but a higher order of smoothness offer a higher comfort for passengers and increase actuator lifetimes.

IX. ANALYSIS AND OPEN PROBLEMS

A. Factors to consider for Intersection

With the recent developments in intersection a method to compare and contrast intersection managers is important. Some factors which we think are important to analyze intersection management systems are:

1) Scalability: An intersection manager should be able to handle a high number of vehicles efficiently while also showcasing an ability to showcase similar performance when the number of vehicles at the intersection are less

2) Time Spent at intersection: It is important that all vehicles spend as little time as possible at intersections. An analysis of the time spent at intersection will provide an understanding of the performance of the intersections as it reduces overall commute times and reduces emissions, energy expenditures

3) Smoothness of trajectories: The trajectories that vehicles follow when traversing across intersections need to be at least continuous until jerk of the vehicles as autonomous vehicles have third order dynamics but a higher order of smoothness offer a higher comfort for passengers and increase actuator lifetimes.

4) Usage of intersection space: The intersection space if necessary can be completely filled and an analysis of how the intersection space is utilized will provide intersection managers to be utilized much better. Furthermore, the intersection spaces are generally non convex space but it will still be useful to analyze algorithms by the amount of empty space left during peak handling capacities and/or the restrictions they apply for occupancies.

5) Heterogeneity: The ability of an intersection to handle vehicles of different sizes, dynamics, limits and constraints is an important point to consider to analyze and compare intersection management systems. A greater ability to handle heterogeneity is an important consideration for intersection managers

6) Robustness: The usage of data uncertainty in the measured data by intersection managers is an important consideration for analyzing intersection algorithms. This is an important factor for consideration as sensory data have uncertainties and utilizing these uncertainties will increase the possibility of collision free trajectories for autonomous vehicles during higher volumes of vehicles at the intersection. Moreover due to forward simulation nature of autonomous intersection the uncertainty is bound to increase and accounting for that might result in conservativeness with respect to time spent at intersections hence, an appropriate tradeoff between the two will have to be considered and that is another method that can be utilized for analyzing different algorithms.

7) Versatility: With many of the current algorithms being tested only for certain configurations and developed keeping in mind certain configurations, it is important to analyze the performance of intersections algorithms when layouts of the intersections change.

While the previous factors looked through at possible factors to analyze algorithms from trajectories and controlling them, Intersection management algorithms generally require data and data sharing. It is also important to consider the working of the system from a data usage perspective.

1) Data required to manage trajectories: Some intersection managers require that the vehicle share their overall control and state trajectories to the intersection managers system and this data sharing results in a large amount of data that is being shared. The sharing of such large data results increases the probability of data losses and increases the latency of communicated data. The amount of data required by an intersection manager per vehicle is also an important consideration for benchmarking intersection management algorithms.

2) Vehicle Privacy: With recent concerns with respect to privacy and data theft. It is impertinent to analyze whether the intersection managers ensures privacy with respect to storage and utilization of data and what data the vehicles are required to transmit amongst themselves. The reduction in the amount of data shared
also increases the probability of predicted trajectories of the vehicles deviating heavily from the actual trajectory that the vehicle will execute. This makes this another essential aspect to consider for intersection management algorithms.

B. Open Problems

Autonomous Intersection is still a developing field with multitude of algorithms developed for navigating through an intersection, some problems that still remain are:

1) **Decentralized Algorithms for navigation:** While current algorithms for intersection managements are progressing towards distributed optimization, A completely decentralized approach will have to have decentralized communication amongst vehicles and thereby result in the planning of trajectories taking into consideration vehicles within the vicinity. These methods will share the computational burden amongst vehicles and also allow for individual decisions amongst the vehicles, in such a way that it not only prevents collision but also prioritizes its requirements.

2) **Privacy constraints not affecting solution:** Recently, intersection management algorithms have been proposed that perform without knowing the intentions of the other vehicles in the environment. Such systems are still in nascent stages and perform less favorably than systems that know drivers intentions. Employing techniques for vehicle prediction and intention is an avenue for further research.

3) **Learning for different subsystems:** Learning based algorithms will provide a method to utilize experience from previous observations to aid in current maneuvers. Learning can be added in prediction of trajectories, optimization of trajectories or in assigning priorities to vehicles and their movements. This will allow a system to gain from how other vehicles react and thereby provide an optimum level to compensate for the uncertainties associated with the systems and provide safety to the systems.

4) **Obstacles in the environment:** Despite the dynamic nature of intersection and the presence of multiple different obstacles ranging from pedestrians, shops, cyclists; intersection algorithms tend to ignore them or compensate for them by requiring the vehicles provide a collision free reference. The consideration of such obstacles will also account for the most important aspect of a transportation management system, that is the safety of passengers and humans that utilize the roads for different purposes.

5) **Hybrid models for prediction:** Intersections require a method to predict the trajectories of the other vehicles and/or obstacles that are contained in the environment. These predictions account for either the dynamic nature, interaction between different vehicles and or maneuver based predictions. Maneuver based predictions offer a good understanding of the vehicle’s direction but doesn’t account for the interactions possible, which can be compensated for by utilizing interactions. A combination of these different methods for prediction will allow for a much more versatile, accurate prediction thereby resulting in lesser non smooth evasion or better collision avoidance by autonomous vehicles.

6) **Data uncertainty:** Data uncertainty is another important field that when added into autonomous intersections will aid in robust and versatile collision avoidance amongst the vehicles. Adding data uncertainty into the system will also allow for intersection algorithms to incorporate real life uncertainties that are going to exist due to inaccuracies and model mismatch. Moreover, data uncertainty will allow accounting for the stochasticity with the vehicle maneuvers that are going to be difficult to predict.

Majority of the above proposed problems tend to overlap with solving another problem. All of these problems are an important consideration for getting zero collision and minimum time as the vehicle traverses the intersection. An optimal trade off between many of the factors will have to be achieved.

X. Conclusion

The current state of the art intersection algorithms tries to solve the problem as a reservation for the vehicles. These reservation based algorithms discretize the trajectory into tiles, and simulate for collisions at the tiles. These methods are not feasible in real life scenarios and may not be scalable. The Second set of algorithms try to formulate a centralized problem that aims at formulating optimal trajectories for vehicles at an intersection utilizing techniques from optimal control, convex optimization and fuzzy systems. These systems have to solve an online optimization problem, everytime a new set of data is received by the intersection manager. Finding out solutions as the problem’s dimensions increase (number of vehicles) results in an increasingly harder optimization problem to solve. Hence, approximation of the solutions need to be utilized.

Revisiting the question of, How do Autonomous Intersections need to look, we believe that intersections being a dynamic environment should have continuous trajectory replanning and utilize the full space for the intersections. The utilization of full space of the intersection necessitates the removal of lanes. Furthermore, safety of humans is of at most importance in intersections and hence the consideration of pedestrians and cyclists and other human related obstacles are of prime importance in intersection algorithms. The recent data issues have also highlighted that privacy is of utmost concern. Thus, moving forward utilizing decentralized algorithms for trajectory replanning and communication is of utmost importance as it reduces computational burden but also adhere to privacy, prioritizing and cooperative driving amongst the vehicles.

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