HEAVY-QUARK BARYONS AS SKYRMIONS

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ABSTRACT
I discuss recent development on the description of heavy-quark (such as charmed and bottom) baryons as one or more heavy mesons “wrapped” by a skyrmion. Amazingly enough, such a description naturally arises when light-quark chiral symmetry and heavy-quark spin symmetry are incorporated in an effective Lagrangian. I interpret the resulting spectrum in terms of nonabelian induced (Berry) potentials in analogy to diatomic molecular systems.

1. Introduction
There is a wide-spread feeling that the skyrmion description of the baryons is a highly predictive and coherent one, ranging from the structure of the nucleon and Δ to nuclear forces. Indeed many of the no-go arguments raised against the skyrmion picture are disappearing rapidly. As we heard in this meeting, we no longer seem to have any problem with S- and P-wave πN scatterings; yesterday’s too large energy of the ground state baryon seems no longer too large once Casimir contributions of $O(N_c^3)$ are taken into account; the missing attraction at medium range in NN potentials that binds nuclei is no longer missing; the soliton instability problem is just a red herring; whether explicit quark degrees of freedom should be present or not is becoming a non-issue. At this moment, there seem to exist no really serious arguments against the notion that the nucleon is a skyrmion in and outside of nuclear medium. Whatever deviation from nature there may be, may simply be a matter of poorly understanding the intricacy of the skyrmion, not the deficiency of the model itself. As far as I know, it is the only model available which can address simultaneously one-nucleon and many-nucleon problems with equal ease. It is of course not derived from QCD but it is consistent with it.

In this talk, I will argue that even when one has very heavy quarks in the baryons, the skyrmion picture still makes sense and in fact, it comes out strikingly close to the quark-model description which we expect gets better as the quark mass becomes heavier. This may sound amazing or, to some, unbelievable. In fact, when Riska, Scooccola and I submitted a paper a few years ago showing that the skyrmion model worked just as well in charmed and bottom baryon sectors, a referee promptly rejected it. It appeared in a different journal only after a long delay because of the referee’s repeated refusal to accept our arguments. As I shall describe below, our claim is vindicated by the recent development.
2. Diatomic Molecules

To better clarify the basic idea involved in the workings of the soliton model in complex strong interactions, let me start with a quantum mechanical problem of a diatomic molecule. This quantum mechanical problem can be put in a context that closely mimics the baryon excitation we are interested in. It shows a generic setting in which nonabelian gauge potentials are induced by interactions.

Consider the dynamics of a diatomic molecule where two atoms are separated by the internuclear separation denoted by the vector $\vec{R}$. We shall restrict ourselves to the relative motion only. Because of the symmetry of the diatom, rotational symmetry of the electrons is broken. The electronic state is characterized by the eigenvalue $\Omega$ of the operator $\hat{R} \cdot \hat{J}_{el}$. In my discussion, I will confine myself to the triplet of the states $\Omega = 0, \pm 1$. This system was recently studied by Zygelman$^6$ and reanalyzed by Lee and myself$^7$ to gain useful insight into its generic structure.

This system can be roughly categorized into two according to the size of $R$. For small $R$, the potential curve for the $\Omega = 0$ state (called $\Sigma$) which is higher lying does not overlap with that of the degenerate doublet $\Omega = \pm 1$ (called $\pi$). Thus one can focus on the doublet, ignoring the singlet. For large $R$, however, the potential curves start overlapping, the complete overlap occurring at $R = \infty$. Then the triplets become degenerate, with the restoration of the electronic rotational symmetry. We will exploit this feature later on in discussing heavy-quark symmetry.

Let me now describe how the above structure can be understood.$^7$ We will consider the molecular excitation described by the dynamical variables $\mathbf{R}(t)$ which could be vibration or rotation as slow compared with the electronic excitation which we consider to be fast. We wish to integrate out the fast degree of freedom and express the whole system on the coordinate $\mathbf{R}(t)$ of the slow degree of freedom. The resulting system can be described by the following Lagrangian

$$L = \frac{1}{2\mu} \dot{\mathbf{R}}^2 + i\theta^a(\frac{\partial}{\partial t} - i\vec{A}^\alpha T^\alpha_{ab} \cdot \dot{\mathbf{R}})\theta_b$$

where $\mu$ is the reduced mass, $\theta_a$ is a Grassmann variable labeled by $a$, $\vec{A}^\alpha$ is the Berry potential inherited from integrating out the fast degree of freedom and $T^\alpha$ is the matrix representation of the vector space in which the Berry potential lives. In our case, we have a triplet of states and $\mathbf{A} \cdot \mathbf{T} \in SU(2)$. I have kept only the essential pieces in the Lagrangian (1), leaving out some “junks”, to make the argument as simple as possible. The “junks” do not change the main thrust of my argument. Let me also mention that the Grassmannian variables for each $a$ are used as a trick to avoid writing the Lagrangian in matrix form. There is nothing deep in it at least for our purpose. Quantizing (1) in a standard way, one gets the Hamiltonian

$$H = \frac{1}{2\mu} (\mathbf{p} - \vec{A})^2.$$  

Since there is gauge invariance in the theory as one can see from (1), we are allowed
to make a gauge transformation on (2) and obtain the following Hamiltonian

$$H = -\frac{1}{2\mu R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} + \frac{1}{2\mu R^2} \left( \vec{L}_o + (1 - \kappa) \vec{I} \right)^2 - \frac{1}{2\mu R^2} (1 - \kappa)^2 (\vec{F} \cdot \hat{R})^2$$  \hspace{1cm} (3)$$

and the corresponding gauge field

$$\vec{A}' = (1 - \kappa) \frac{\hat{R} \times \vec{F}}{R^2},$$

and the magnetic field

$$\vec{B}' = -(1 - \kappa^2) \frac{\hat{R} (\hat{R} \cdot \vec{I})}{R^2}$$

where $\kappa$ indicates the extent to which electronic rotational symmetry is present, with $\kappa = 1$ indicating full symmetry and $\kappa = 0$ a complete absence of symmetry. In (3), $\vec{L}_o$ is the angular momentum lodged in the dumb-bell and $\vec{I}$ is the angular momentum contributed by the Berry potential. Neither $\vec{L}_o$ nor $\vec{I}$ separately commutes with the Hamiltonian. What commutes is the total angular momentum $\vec{L} = \vec{L}_o + \vec{I}$.

We are now ready to analyze what happens in the two extreme cases of $\kappa = 0$ which results when $R \to 0$ and $\kappa = 1$ which results when $R \to \infty$. For $\kappa = 1$, the degenerate $\Sigma$ and $\pi$ states form a representation of the rotation group and hence the Berry potential (and its field tensor) vanishes or becomes a pure gauge. The spectrum then independent of the angular momentum carried by the electronic system. *This just means that rotational symmetry is restored in the electronic sector.* For $\kappa = 0$, the $\Sigma$ and $\pi$ states are completely decoupled and only the $U(1)$ monopole field can be developed on the $\pi$ states. $\kappa$ goes to zero and the Hamiltonian can be written as

$$H = -\frac{1}{2\mu R^2} \frac{\partial}{\partial R} R^2 \frac{\partial}{\partial R} + \frac{1}{2\mu R^2} (\vec{L} \cdot \vec{L} - 1)$$  \hspace{1cm} (4)$$

which is a generic form for a system coupled to an $U(1)$ monopole field. In this case, the “magnetic charge” is quantized to $\pm 1$ or twice the basic Dirac monopole charge $\pm \frac{1}{2}$. A truly nonabelian Berry potential with non-quantized charge can be obtained only for $\kappa$ which is not equal to zero or one.

### 3. Spectrum of Heavy Baryons

I will now turn to the structure of heavy baryons described as skyrmions and summarize the recent development. What I will present below is based on work done recently in collaboration with Min and his coworkers in Seoul. Instead of making detailed analysis to compare with experiments or with quark models, let me start with a simplified Lagrangian consistent with hidden gauge symmetry (HGS).

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*We should note that the magnetic field is of the nonabelian monopole type of ’t Hooft and Polyakov with however a charge which is not quantized. This is a generic feature of induced gauge fields we encounter in various systems.*
consider two light flavors $q = u, d$ and a third flavor $Q$ which will be taken to be heavy later on. For the moment I will consider $u, d, Q$ on the same footing and write a Lagrangian built from chiral symmetry. Obviously when the mass of $Q, m_Q$, becomes large compared with the chiral scale $\Lambda_\chi \sim 1$ GeV, it would make no sense to talk about chiral symmetry associated with the quark $Q$ but let me blindly start with an $SU(3)$ chiral Lagrangian anyway and take $m_Q$ become large. I will write the Lagrangian as the sum of the $SU(2)$ Skyrme Lagrangian of the $(u, d)$ sector, $\mathcal{L}_{su(2)}$, the HGS Lagrangian without (with) the $\omega$ meson coupling, $\mathcal{L}_\Phi^\text{HGS}$ ($\mathcal{L}_\omega^\text{HGS}$), and the “anomalous parity” Lagrangian, $\mathcal{L}_{an}$:

$$\mathcal{L}^\text{HGS} = \mathcal{L}_{su(2)} + \mathcal{L}_\Phi^\text{HGS} + \mathcal{L}_\omega^\text{HGS} + \mathcal{L}_{an},$$

$$\mathcal{L}_{su(2)} = \frac{F_\pi^2}{16} \text{Tr} \left[ \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right] + \frac{1}{32\epsilon^2} \text{Tr} \left[ \Sigma^\dagger \partial_\mu \Sigma, \Sigma^\dagger \partial_\nu \Sigma \right]^2,$$  

$$\mathcal{L}_\Phi^\text{HGS} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - m_\Phi^2 \Phi^\dagger \Phi - \frac{1}{2} \Phi_\mu^\dagger \Phi^{\mu\nu} + m_\Phi^2 \left[ \Phi_\mu^\dagger + \frac{2i}{F_\pi g_{\Phi^*}} \Phi^\dagger A_\mu \right] \left[ \Phi^{\mu} + \frac{2i}{F_\pi g_{\Phi^*}} A^\mu \Phi \right]$$

$$\mathcal{L}_\omega^\text{HGS} = -\frac{iN_c}{2F_\pi^2} B_\mu \left[ \left( \Phi^\dagger D^\mu \Phi - (D^\mu \Phi)^\dagger \Phi \right) - \left( \Phi_\mu^\dagger D^\mu \Phi^{\mu\nu} - (D^\mu \Phi^{\mu^*})^\dagger \Phi^{\ast\nu} \right) \right],$$

$$\mathcal{L}_{an} = -\frac{iN_c}{2F_\pi^2} B_\mu \left( \Phi^\dagger D^\mu \Phi - (D^\mu \Phi)^\dagger \Phi \right) + \delta \mathcal{L}_{an} \tag{5}$$

where

$$D_\mu = \partial_\mu + V_\mu, \quad \Sigma = \xi \cdot \xi,$$  

$$\left( \begin{array}{c} V_\mu \\ A_\mu \end{array} \right) = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi \pm \xi \partial_\mu \xi^\dagger \right),$$

$$B_\mu = \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \text{Tr} \left\{ \Sigma^\dagger \partial^\mu \Sigma \Sigma^\dagger \partial^\nu \Sigma \Sigma^\dagger \partial^\beta \Sigma \right\},$$

$$\Phi_\mu^{\ast\nu} = \partial_\mu \Phi_\nu^{\ast} - \partial_\nu \Phi_\mu^{\ast} + V_\mu \Phi_\nu^{\ast} - V_\nu \Phi_\mu^{\ast}, \tag{6}$$

with $\epsilon_{0123} = +1$. Here, $\Sigma$ is the $SU(2)$ chiral field, $\Phi$ and $\Phi^\ast_\mu$ are, respectively, the pseudoscalar and vector meson doublets of the form $Q\bar{q}$, $F_\pi$ represents the pion decay constant and $g_{\Phi^*}$ is the $\Phi^\ast$ “gauge” coupling to matter fields. For instance, if we take the kaons to be heavy mesons, $\Phi^\dagger = (K^-, \bar{K}^0)$, $\Phi_\mu^{\ast\nu} = (K^{*-}, K^0)$. This Lagrangian is obtained from a hidden gauge symmetric Lagrangian $^{10}$ by integrating out the $\omega$ and $\rho$ meson fields and then taking the limit $m_\Phi \to \infty$, neglecting the terms that vanish as $m_{\Phi^{\ast\nu}}$ and $m_{\phi^{\ast\nu}}$ or faster. For the purpose of comparing with the heavy-quark limit $^{11}$ which we will refer to as Isgur-Wise (IW) symmetric limit, it is necessary to keep the vector mesons explicitly instead of integrating them out as done in Scoccola et al.$^{10}$ The reason for this will become clear later on.

We need to explain a bit what $\mathcal{L}_{an}$ is in the context of the heavy-meson limit that we are interested in. The first term is what one obtains from the topological Wess-Zumino term written down by Witten $^{12}$ when expanded à la Callan-Klebanov...
This is intrinsically tied to anomalies in effective theory. Later, as the heavy quark mass increases, this term will disappear. However, the second term, which is intrinsic-parity odd as the Wess-Zumino term is and involves the vectors $P^\ast$’s, need not vanish in the heavy-quark limit. We expect it to modify the constants of the main term responsible for the binding of the mesons $\Phi$ and $\Phi^\ast$ to a soliton. They are fixed at low energy by low-energy theorems and in the heavy-quark limit by heavy-quark symmetry. A priori, there is no reason why they should be related.

To see what remains of the Lagrangian (5) in the heavy-quark limit, we make the meson-field redefinition,

$$\Phi^\ast_\mu = e^{-imv_x}P^\ast_\mu/\sqrt{m},$$

$$\Phi_\mu = e^{-imv_x}P_\mu/\sqrt{m},$$

so that the fields $P^\ast_\mu$ and $P_\mu$ are independent of the meson mass and obtain

$$L_{\text{HGS}} = -iPv_\mu \stackrel{\leftrightarrow}{D} P_\mu^\dagger + iP^\ast_\mu v_\nu \stackrel{\leftrightarrow}{D} P^\ast_\mu^\dagger + i\sqrt{2} \left( PA_\mu P^\ast_\mu^\dagger + P^\ast_\mu A_\mu P^\dagger \right),$$

$$L_{\omega} = \frac{N_c}{F_\pi^2}B_\mu \left( P_\mu^\ast P^\dagger - P^\ast_\mu P_\mu^\dagger \right),$$

$$L_{\text{an}} = \delta L_{\text{an}}$$

where

$$(D_\mu P)^\dagger = (\partial_\mu + V_\mu)P^\dagger,$$

$$A \stackrel{\leftrightarrow}{D} B^\dagger = A(DB)^\dagger - (DA)B^\dagger.$$

We have not written out the term $\delta L_{\text{an}}$ since while the coefficients are known phenomenologically in the light-quark sector, they are not known in the regime we are concerned with. We expect that it will include terms of the form

$$\frac{iN_c}{F_\pi^2} \epsilon^{\mu\nu\alpha\beta} v_\mu \left( aPA_\nu A_\alpha P^\ast_\beta^\dagger - bP^\ast_\beta A_\alpha A_\nu P^\dagger \right)$$

with $a$ and $b$ unknown constants. Note that since in $^9$ we started with an apparently $SU(3)$ symmetric Lagrangian (apart from the meson mass term) with the flavor $Q$ put on the same footing as the light quarks, Eq. (5) results from the $\omega$-meson coupling terms and hence the constant $N_c/F_\pi^2$ is fixed. In the heavy-quark limit, the “primordial” Wess-Zumino term is absent. However, the rest will remain to modify effectively the coefficient of Eq.(6) which came from the $\omega$-meson coupling with the heavy mesons $P$ and $P^\ast$. That such a term must be present can be seen by bosonizing light and heavy quarks from QCD.$^{15}$

If the $P$ and $P^\ast$ are degenerate, the intrinsic-parity odd Lagrangian in which the Wess-Zumino term figures in the HGS Lagrangian$^8$ contains a term that survives in the heavy-meson mass limit

$$c_4 L_{(4)} = -c_4 2ig_\phi^2 \epsilon^{\mu\nu\alpha\beta} v_\mu P^\ast_\nu A_\alpha P^\ast_\beta^\dagger,$$
with \( c_4 \) the coefficient of \( \mathcal{L}_{(4)} \). This is effectively a four-derivative term that belongs to the same intrinsic-parity class as \( \delta \mathcal{L}_{an} \) discussed above. The coefficient \( c_4 \) is fixed in the light-quark sector to \( c_4 = iN_c/16\pi^2 \) from the decay \( \omega \to \rho\pi \). We will see what the coefficient is in the heavy-quark sector.

Our Lagrangian (8)–(13) can now be compared with the one implied by IW symmetry

\[
\mathcal{L}^\text{JMW}_\Phi = -i\text{Tr}H_a\gamma^\mu \partial_\mu H_a + i\text{Tr}H_aH_b\gamma^\mu (V_\mu)_{ba} + ig\text{Tr}H_aH_b\gamma^\mu \gamma_5 (A_\mu)_{ba} + \cdots ,
\]

with the heavy meson field \( H_a \) (where \( a \) labels the light-quark flavor) defined as

\[
H = \frac{1}{2} \left[P \gamma^\mu - P^* \gamma^\mu\right].
\]

In terms of \( P \) and \( P^* \), (14) reads

\[
\mathcal{L}^\text{JMW}_\Phi = -iP^\gamma \cdot \bar{D} P^\dagger + iP^* \gamma \cdot \bar{D} P^*\dagger \\
+2ig \left\{ P^* A^\mu P^\dagger + P A^\mu P^*\dagger \right\} + 2g\epsilon^{\lambda\mu\nu\kappa} v_\lambda P^* A_\nu P^*\kappa.
\]

We see that the HGS Lagrangian (8) with (13) is identical – except for the term (9) – to the IW symmetric Lagrangian (16) if we identify \( g = 1/\sqrt{2} \) and \( c_4 g_5^2 \). Surprisingly \( g = 1/\sqrt{2} \) is rather close to the quark-model value \( g \sim 0.75 \) and also to the recent CLEO collaboration data \( g \approx 0.6 \). Furthermore low-energy theorem \( g \approx 0.65 \approx ig \). In (16), the term of the form (9) is missing for the simple reason that \( B_\mu \) involves three derivatives, so higher order in derivative expansion. Ignoring it in pion dynamics may be justified but it is not in soliton dynamics. In fact we will see later that it is the most important contribution in our way of describing heavy baryons as it is in the Callan-Klebanov scheme. Since we do not know its coefficient in the IW limit, we will take it in the form

\[
\mathcal{L}^\text{HGS}_\omega = \alpha B_\mu j^\mu ,
\]

with \( \alpha \) an unknown constant. The Lagrangian (17) obviously satisfies both chiral symmetry and IW symmetry. Such a term arises in an approximate bosonization of QCD, through the coupling of \( H \) to the \( \omega \) meson. In (17), \( j^\mu \) is the \( U(1) \) current of the Lagrangian \( \mathcal{L}^\text{JMW}_\omega \) corresponding to the heavy-quark flavor which is conserved in our case. Although as mentioned above, the coupling constant \( \alpha \) cannot be determined by chiral and Isgur-Wise symmetries alone, we will analyze the structure of heavy baryons in units of \( -N_c/2F_\pi^2 \), i.e., the coefficient of \( \mathcal{L}^\text{HGS}_\omega \) in Eq.(17). We will normalize the meson field as

\[
\int d^3r j^0 = -2 \int d^3r \left( PP^\dagger + P^* P^*\dagger \right) = -1
\]
and work in the rest frame of the heavy meson, \( v = (1, 0, 0, 0) \). Note that \( P_0^* = 0 \) since \( v \cdot P^* = 0 \).

The Lagrangian correct to order \( O(m_\Phi^0 \cdot N_c^0) \) is given by
\[
L_B = -M_{sol} - m_\Phi + \int d^3r (\mathcal{L}_P + \mathcal{L}_W),
\]
\[
-\mathcal{L}_P = 2gi \left\{ P^{*i} A^{i} P^{\dagger} + P A^{i} P^{*i\dagger} - i \epsilon^{0ijk} P^{*i} A^{j} P^{*k\dagger} \right\},
\]
\[
-\mathcal{L}_W = 2\alpha B_0 \left( P P^{\dagger} + P_i^i P^{*i\dagger} \right).
\]
(20)

One can readily see that \( \mathcal{L}_P \) and \( \mathcal{L}_W \) are invariant with respect to the global rotation \( S \in SU(2)_V \) in the light flavor space (i.e., the isospin space) provided that the fields transform
\[
P(x) = \phi(x) S^{\dagger},
\]
\[
P_i^*(x) = \phi_i^*(x) S^{\dagger},
\]
\[
\xi(x) = S \xi_0(\vec{x}) S^{\dagger},
\]
(21)
with \( x = (t, \vec{x}) \) and \( \xi_0(\vec{x}) = \exp(i \vec{r} \cdot \vec{r} F(r)/2) \) in the hedgehog configuration. Following the standard procedure for collective quantization, we elevate \( S \) to a dynamical variable by endowing it with the time dependence \( S(t) = a_0(t) + \vec{a}(t) \cdot \vec{r} \). As defined, the fields \( \phi(x) \) and \( \phi_i^*(x) \) are fields living in the rotating frame.

The equations of motion for \( \phi(x) \) and \( \phi^*_i(x) \) gotten from the Lagrangian valid at \( O(m_\Phi^0 \cdot N_c^0) \) imply that
\[
|\phi(x)|^2 \propto \delta^3(\vec{x}),
\]
\[
|\phi_i^*(x)|^2 \propto \delta^3(\vec{x}).
\]
Given these solutions, the energy shift coming from \( - (\mathcal{L}_P + \mathcal{L}_W) \) of (21) is
\[
E_I = - \int (\mathcal{L}_P + \mathcal{L}_W) d^3r
\]
\[
= - \frac{1}{2\pi^2} \alpha \left\{ F'(0) \right\}^3
\]
(22)

with the contribution of \( \mathcal{L}_P \) vanishing identically. This differs from the result of the recent work by Manohar and his collaborators\(^{18} \) who get the entire action from \( \mathcal{L}_P \) whose contribution is non-vanishing since the meson \( H \) is not rotated in their scheme in contrast to our scheme (21). I will discuss the difference of these two approaches later.

3.1. Spectrum in IW limit

We take \( g = 1/\sqrt{2} \approx 0.7, F'(0) \approx -0.89 \) GeV from the literature and the experimental value of \( F_\pi = 186 \) MeV and \( e = 4.75 \), with which we find the \( \alpha \) value in the b-quark sector should be
\[
\alpha \approx - \frac{1}{2.8} \left( \frac{N_c}{2F_\pi^2} \right)
\]
(23)
to reproduce \( M_{\Lambda_b} - M_N = 4.65 \) GeV, the predicted value of the quark model. This corresponds to the binding energy of

\[
E_I \approx -0.55 \text{ GeV.} \tag{24}
\]

Next we consider the effects of \( O(m_\Phi^0 \cdot N_c^{-1}) \) term in the Lagrangian. For this we define

\[
S^\dagger \dot{S} = i \vec{\tau} \cdot \vec{\Omega} \tag{25}
\]

and write the corresponding Lagrangian to \( O(m_\Phi^0 \cdot N_c^{-1}) \)

\[
L_{(-1)} = \int d^3r \mathcal{L}_{(-1)} = 2I\Omega^2 - 2\vec{\Omega} \cdot \vec{Q}, \tag{26}
\]

where

\[
\vec{Q} = -\int d^3r \left( \phi \bar{n}(\vec{r}) \phi^\dagger + \phi^*_\mu \bar{n}(\vec{r}) \phi^*_\mu^\dagger \right),
\]

\[
\bar{n} = \frac{1}{2} \left( \xi^\dagger_0 \xi_0 + \xi_0 \xi^\dagger_0 \right)
= \cos F(\vec{r}) \vec{r} - (\cos F(\vec{r}) - 1)\hat{r}(\vec{r} \cdot \hat{r}), \tag{27}
\]

and \( I \) is the moment of inertia of the \( SU(2) \) soliton determined from the properties of the \( N \) and \( \Delta \). As suggested by Manohar et al., because of the \( \delta \)-function structure of the meson wavefunctions and a parity-flip at the origin, it is more convenient to transform the heavy-meson fields to

\[
\phi \rightarrow \phi' = \phi \xi_0,
\phi^*_\mu \rightarrow \phi'^*_\mu = \phi^*_\mu \xi_0,
\bar{n} \rightarrow \xi^\dagger_0 \bar{n} \xi^\dagger_0. \tag{28}
\]

The binding energy is not affected by this transformation. With the primed fields, \( \vec{Q} \) is of the form

\[
\vec{Q} = -\frac{1}{2} \int d^3r \left\{ \phi' \left( \Sigma^\dagger \vec{r} \Sigma + \vec{r} \right) \phi'^\dagger + \phi'^*_\mu \left( \Sigma^\dagger \vec{r} \Sigma + \vec{r} \right) \phi'^*_{\mu\dagger} \right\}. \tag{29}
\]

Now since in the soliton rotating frame, the “isospin” of the meson is transmuted to spin, we can identify

\[
\vec{Q} = cJ_Q, \tag{30}
\]

namely, proportional to the angular momentum lodged in the meson which is \( 1/2 \). Canonical quantization of (24) leads to an \( O(m_\Phi^0 \cdot N_c^{-1}) \) splitting in energy given by the Hamiltonian

\[
\Delta H = 2I\Omega^2 = \frac{1}{2\Omega} \left( J_I + cJ_Q \right)^2 \tag{31}
\]
with the spectrum
\[ \Delta E_{hf} = \frac{1}{2I} \left\{ cJ(J + 1) + (1 - c)J_\ell(J_\ell + 1) + c(1 - c)J_Q(J_Q + 1) \right\}, \]  
(32)
where \( \vec{J}_\ell \) is the spin lodged in the rotor. The total spin \( \vec{J} \) of the system is
\[ \vec{J} = \vec{J}_\ell + \vec{J}_Q. \]  
(33)

The Hamiltonian (31) is a heavy-baryon analog to the diatomic molecular spectrum (3) and \( c \) an analog to \( (1 - \kappa) \). One can show by an explicit calculation that with (29)
\[ c = 0. \]  
(34)
This is the analog of the vanishing \( (1 - \kappa) \) in diatomic molecules, a consequence of restored rotational symmetry. What happens here is that the first term of (29) coming from the \( P \) mesons gets cancelled exactly by the second coming from the \( P^* \) mesons. If the \( P \) and \( P^* \) were not degenerate the cancellation would not occur. This suggests the following: For not too large \( m_\Phi \), say, \( m_K \), \( c \) can be substantial, of \( O(1) \), since the \( K^* \) is rather high-lying compared with the \( K \). As \( m_\Phi \) becomes large, the \( P^* \) comes near the \( P \), thus decreasing \( c \) such that in the heavy-quark limit, we get \( c = 0 \).

Given that \( c = 0 \) in the Isgur-Wise limit, we have the splitting
\[ \Delta E_{hf} = \frac{1}{2I} J_\ell(J_\ell + 1). \]  
(35)
This \( \Delta E_{hf} \) predicts that there is an effective “fine” splitting of right sign and magnitude between \( \Lambda \) and the degenerate \( \Sigma \) and \( \Sigma^* \). The predicted mass spectrum (denoting the mass by the particle symbol) for b-quark baryons, with \( \Lambda - N = 4.65 \) GeV to fix \( \alpha \), is
\[ \Sigma_b - N = \Sigma^*_b - N = 4.84 \) GeV. \]  
(36)
These are comparable to the predictions of quark potential models
\[ (\Lambda_b - N)^{QM} = 4.65 \) GeV, \]  
\[ (\Sigma_b - N)^{QM} = 4.86 \) GeV, \]
and to those of bag models
\[ (\Lambda_b - N)^{BM} = 4.62 \) GeV, \]  
\[ (\Sigma_b - N)^{BM} = 4.80 \) GeV. \]

3.2. *Hyperfine spectrum.*
It is possible, within the scheme described so far, to discuss hyperfine splitting with a nonzero $c$. For a finite heavy-quark mass for which $m_\Phi < m_\Phi^*$, the CK model indicates that $c \sim 1/m_\Phi$. This is the hidden $m_\Phi^{-1}$ dependence buried in the hyperfine coefficient $c$ alluded above which we conjecture may have an intricate connection to a Berry potential. For a sufficiently large $m_\Phi$, we may therefore assume $c = a/m_\Phi$. Now using (32), we can write for baryons with one heavy quark $Q$

$$
\Sigma_Q - \Lambda_Q = \frac{1}{I} (1 - c_Q) \simeq 195 \text{MeV}(1 - c_Q).
$$

(37)

With the experimental value $\Sigma_c - \Lambda_c \approx 168 \text{MeV}$ for the charmed baryons, we get $c_c \approx 0.14$. This means that with $m_\Omega = 1869 \text{MeV}$, the constant $a \approx 262 \text{MeV}$. So

$$
c_\Phi \simeq 262 \text{MeV}/m_\Phi.
$$

(38)

Now for $b$-quark baryons, using $m_B = 5279 \text{MeV}$, we find $c_b \approx 0.05$ which with (37) predicts

$$
\Sigma_b - \Lambda_b \approx 185 \text{MeV}.
$$

(39)

This agrees well with the quark-model prediction. Furthermore the $\Sigma^* - \Sigma$ splitting comes out correctly also. For instance, it is predicted that

$$
\frac{\Sigma_b - \Sigma_b}{\Sigma_c - \Sigma_c} \simeq \frac{m_\Omega}{m_B} \approx 0.35
$$

(40)

to be compared with the quark-model prediction $\sim 0.32$. If one assumes that the heavy mesons $\Phi$ are weakly interacting, then we can put more than one $\Phi$’s in the soliton and obtain the spectra for $\Xi$’s and $\Omega$’s in good agreement with quark-model results.

4. Discussions

I have discussed how one can continuously “dial” from chiral symmetry to IW symmetry in the spectrum of baryons. This is a surprising outcome. In doing so, a Wess-Zumino like term plays an essential role. For $m_Q < \Lambda_\chi$, the Wess-Zumino term plays a key role in binding a pseudoscalar $\Phi$ to an $SU(2)$ soliton. For $m_Q \gg \Lambda_\chi$, the Wess-Zumino term vanishes but a term of the form $\text{Tr} \bar{H} v_\mu H B^\mu$ contributes through coupling with the light vector meson $\omega$ which given a reasonable strength again dominates the binding. The baryon structure is exactly the same as the one given by the CK model with the meson shrunk to the center of the soliton. It is not clear whether all this is just a coincidence or something profound but it is certainly intriguing.

The description of Manohar and his collaborators differs from the above scenario in that in their scheme, the heavy meson $H$ gets bound to a rotating skyrmion through

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Footnote:

It is amusing to note that this formula works satisfactorily even for the kaon for which one predicts $c_s \approx 0.53$ to be compared with the empirical value 0.62.
a residual interaction given by \( \mathcal{L}_P \) in (20) with no contribution from a Wess-Zumino like term. Since one starts here with a Lagrangian in the IW symmetry limit, the hyperfine splitting comes from an IW symmetry breaking term of the form

\[
\frac{c}{m_H} \text{Tr}(\bar{H}\sigma_{\mu\nu}H\sigma^{\mu\nu})
\]

which splits the degeneracy of \( P \) and \( P^* \). Surprisingly the results of both approaches seem rather close. The connection between the two is not yet understood.

In terms of the “Berry charge,” the limit \( c = 0 \) clearly corresponds to the restoration of the IW symmetry, namely the symmetry that makes \( P \) and \( P^* \) degenerate. It remains to be seen how this result can be obtained in the general setting of Berry potentials in the strong interactions developed recently by Lee, Nowak, Zahed and myself. More work is needed in this area.

A matter of potential interest on which I have no result to discuss is the possibility of having the chiral partner \( S \) of \( H \) which arises naturally in approximate bosonization of QCD bound to a soliton. A rich variety of spectra associated with this excitation is predicted.

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