Features of Single Lepton Production at Hadron Collider

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March 26, 2022

Abstract

In the Born approximation, as in the next-to-leading order approximation, Single lepton production $pp \rightarrow l^\pm \nu X$ at Fermilab Tevatron is discussed. The effects of non standard model physics ($W'$ - production) are also considered. It was shown that $2 \rightarrow 3$ subprocesses play important role when a high transverse momentum cut on the lepton is imposed. On the other hand the NLO corrections do not cause qualitative changes of Born approximation results. The contributions of $W'$ bosons for this signature become more important for large values of $P_T^{cut}$.

Nearly all experimental data, which are available at present, support the Standard Model as being the correct description of observables at current energies. The signature of $W$ boson production at hadron colliders is characterized by observing isolated leptons at a large transverse momentum accompanied by a significant missing transverse momentum due to the neutrino. The cross section for these processes is roughly a nanobarn. As in the near future at the Fermilab Tevatron it will be possible to measure these cross sections of the order 0.1pb, there is a chance to find the effects beyond the Standard Model. To be sure that these effect are the results of the new physics, one needs more precise calculations in the frame of the Standard Model (like higher order corrections and etc).

One of the possibilities of the new physics is the existence of new neutral or charged vector bosons, what is a common feature of many extensions of the Standard Model. Different models give different predictions for the production of new heavy vector bosons [1]-[10]. Below we consider a simple model obtained by taking for the heavy $W'^\pm$ and $Z'$

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gauge bosons the same coupling to fermions as ordinary $W^\pm$ and $Z$. In this model there exist also the trilinear couplings $W'WZ$ and $Z'WW$, which give the important contributions to the decay widths of $W'$ and $Z'$ bosons.

We shall consider events with isolated leptons in the final state with a large transverse momentum - $p\bar{p} \rightarrow l^\pm \nu X$. A possible contribution to this signature from the new physics can be for example $W'$ boson production. It is important to know whether the cross section is fully determined by the Standard Model or not.

In the lowest order the cross section of the reaction $p\bar{p} \rightarrow l^\pm \nu X$ is determined by the subprocess

$$q\bar{q}' \rightarrow W^\pm \rightarrow l^\pm \nu$$ \hspace{1cm} (1)

On the other hand it is also necessary to consider the subprocesses:

$$q\bar{q}' \rightarrow W^\pm g \rightarrow l^\pm \nu g, \quad qg \rightarrow W^\pm q \rightarrow l^\pm \nu q$$ \hspace{1cm} (2)

If the kinematical parameters (the transverse momentum $p_T$ or the rapidity $|\eta|$) of a gluon or a quark in the final state do not satisfy the triggers requirements they can be not detected. So there are kinematical regions, where (1) and (2) subprocesses give the similar signatures. Therefore it is important to calculate the contribution of subprocesses (2) to the cross section of $p\bar{p} \rightarrow l^\pm \nu X$.

Taking into account subprocesses (1), (2), below we consider the $p\bar{p} \rightarrow l^\pm \nu X$ cross section. For calculation of this cross section we use the well known in QCD expression of $\sigma(AB \rightarrow CX)$, which after making the following change of variables

$$w = x_a x_b, \quad z = x_a / x_b$$

we can rewrite in more convenient for us form:

$$\sigma(AB \rightarrow C_1 X) = \frac{1}{2} \sum_{a,b} \int_{w_0}^{1} dw \int_{z_-}^{z_+} \frac{d z}{z} f_{a/A}(\sqrt{wz}, Q^2) f_{b/B}(\sqrt{wz}, Q^2) \hat{\sigma}(ab \rightarrow c_1 + \cdots)$$ \hspace{1cm} (3)

where the sum is taken over all the subprocesses which lead to the production of particles $C_1, \ldots$ in the final state. $\hat{\sigma}$ is the total cross section for subprocess $a + b \rightarrow c_1 + \cdots$ and $f_{h/H}$ is a distribution function of parton $h$ in a hadron $H$. $z_- = w, z_+ = 1/w$ and $w_0 = \hat{s}_{\text{min}}/S$ ($\hat{s}_{\text{min}}$ is the threshold value of partonic squared center of mass energy $\hat{s} = wS$ for the reaction of interest).

The total cross section $\hat{\sigma}$ of $2 \rightarrow 3$ subprocess can be written in a standard form

$$\hat{\sigma}(p_a + p_b \rightarrow p_1 + p_2 + p_3) \propto \frac{1}{2(2\pi)^9 \hat{s}} \int_{l_1^-}^{l_1^+} dl_1 \int_{s_2^+}^{s_2^-} ds_2 \int_{l_2^-}^{l_2^+} dl_2 \int_{s_3^+}^{s_3^-} ds_3 \frac{|\hat{M}|^2}{\sqrt{-\Delta_4}} \hspace{1cm} (4)$$

where $|\hat{M}|^2$ is a squared matrix element and $\Delta_4$ is Gram determinant of the fourth order.
The selection procedure of events used in experiments at hadron colliders requires certain experimental cut-offs of kinematical parameters. As a rule the finite resolution of a detector requires the cut-off of a transverse momentum of jets (leptons)

\[ |\vec{p}_T| \geq |\vec{p}^{\min}_T| \equiv q_T \]

and their polar angles (pseudorapidity)

\[ \pi - \theta_0 \geq \theta \geq \theta_0 \]

\( p_T \) and \( \theta \) are assumed to measure in the c.m.s. of the colliding hadrons.

If we consider the case when one of the jets’ (leptons’) transverse momentum \( p_T \) and polar angle \( \theta \) satisfy the above requirements and ignore all masses of particles (partons) in the initial and final states, then \( s_{\min} = 4q_T^2 \) and consequently \( w_0 = 4q_T^2/S \). From the relations \( (\sin \theta)_{\text{min}} = 2q_T/\sqrt{s} = 2q_T/\sqrt{wS} \), \( (\sin \theta)_{\text{min}} = 2q_T/\sqrt{S} \) (\( \hat{\theta} \) is a polar angle in the c.m.s. of partons) we can determine the values of \( z_+, z_- \) in the case of kinematical cut-offs \( |\vec{p}_T| \geq |\vec{p}^{\min}_T| \equiv q_T, \pi - \theta_0 \geq \theta \geq \theta_0 \):

\[ z_+ = \min \left[ \frac{1}{\omega}, \frac{(1 + a)(1 + A)}{(1 - a)(1 - A)} \right], \quad z_- = \max \left[ \omega, \frac{(1 - a)(1 - A)}{(1 + a)(1 + A)} \right] \]  

(5)

where \( a \equiv \sqrt{1 - 4q_T^2/wS}, A \equiv \min(\sqrt{1 - 4q_T^2/S}, \cos \theta_0) \). When \( \theta_0 \to 0 \) \( z_+, z_- \) transfer into the standard expressions \( z_- = w, z_+ = 1/w \) for the three-body phase space.

Very often in experiments we deal with the case when the transverse momentum and polar angle of one jet (lepton) are above \( q_T \) and \( \theta_0 \), but the transverse momentum of another jet (lepton) is below \( k_T - |\vec{p}_T| \leq k_T \). It should be noted that the expressions (5) are valid for this case too.

The kinematical cut-offs of \( p_{1T} \) and \( \theta \) (for the particle in the final state with the momentum \( p_1 \)) define the bounds of integrals in (4). By using the expressions for \( p_{1T} \) and \( \tan(\theta/2) \),

\[ p_{1T} = \sqrt{-\hat{t}_1(\hat{t}_1 + \hat{s} - \hat{s}_2)}; \quad \tan(\theta/2) = \sqrt{-\frac{\hat{t}_1}{z(\hat{t}_1 + \hat{s} - \hat{s}_2)}} \]  

(6)

and after taking into account the kinematical cut-offs \( \hat{p}_{1T} \geq q_T, \theta_0 \leq \theta_1 \leq \pi - \theta_0 \) we get:

\[ \hat{s}_1 = \max[0, \hat{t}_2 - \hat{t}_1], \quad \hat{s}_1^+ = \min[\hat{s} - \hat{s}_2, \hat{s} + \hat{t}_2] \]  

(7)

\[ \hat{t}_2^- = \hat{t}_1 - \hat{s}_2, \quad \hat{t}_2^+ = 0 \]  

(8)

\[ \hat{s}_2^- = \max \left[ 0, \hat{t}_1 + \hat{s} + \frac{\hat{t}_1}{z \tan^2(\theta_0/2)} \right]; \quad \hat{s}_2^+ = \min \left[ \hat{t}_1 + \hat{s} + \frac{q_T^2 \hat{s}}{\hat{t}_1}, \hat{t}_1 + \hat{s} + \frac{\hat{t}_1 \tan^2(\theta_0/2)}{z} \right] \]
\[
\hat{t}_1^+ = \min \left[ -\frac{2q_T\sqrt{s}}{1 + \frac{4q_T^2}{s}}; -\frac{\hat{s}}{2} + \frac{\hat{s}}{2}\sqrt{1 - \frac{4q_T^2}{\hat{s}}} \right]
\]
\[
\hat{t}_1^- = \max \left[ -\frac{\hat{s}}{1 + \frac{4q_T^2}{s}}; -\frac{\hat{s}}{2} - \frac{\hat{s}}{2}\sqrt{1 - \frac{4q_T^2}{\hat{s}}} \right]
\]

As it was mentioned above, because of a finite resolution of a detector used in experiments at hadron colliders, only particles with \( p_T \geq q_T \) and \( \theta \geq \theta_0 \) can be detected. So it can happen that from three partons’ in a final state, produced via \( 2 \rightarrow 3 \) subprocess, only two (for which \( p_T \geq q_T, \theta \geq \theta_0 \)) are detected. To distinguish this signature from the other one, produced via ordinary \( 2 \rightarrow 2 \) subprocess, we need additional cut-off of a transverse momentum of the third particle (jet or lepton) - \( |p_{TJ}| \leq k_T \). In this case according to the expression

\[
p_{TJ} = \sqrt{-\hat{t}_2(\hat{t}_2 + \hat{s} - \hat{s}_1)}
\]

the values of bounds \( \hat{s}_1^-, \hat{s}_1^+ \) must be changed:

\[
\hat{s}_1^- = \max \left[ 0, \hat{t}_2 - \hat{t}_1, \hat{s} + \hat{t}_2 + \frac{\hat{s}k_T^2}{t_2} \right]; \quad \hat{s}_1^+ = \min \left[ \hat{s} - \hat{s}_2, \hat{s} + \hat{t}_2 \right]
\]

The corresponding squared amplitudes of subprocesses (1) and (2), summed over final state spins and colors and averaged over initial spins and colors have the following form:

\[
\frac{d\hat{\sigma}}{dt}(qq' \rightarrow l^+\nu) = \frac{\pi\alpha^2|V_{qq'}|^2}{12\hat{s}^2 \sin^4 \theta_W} \frac{\hat{t}^2}{(\hat{s} - \hat{t}_1)(\hat{s} - \hat{t}_1 + \hat{t}_2)\left[ (\hat{s} - \hat{t}_1 - \hat{t}_2)^2 + m_W^2 \Gamma_W^2 \right]}
\]

\[
|M(qq' \rightarrow l^+\nu)|^2 = \frac{2(4\pi)^3\alpha_S\alpha^2|V_{qq'}|^2}{9 \sin^4 \theta_W} \frac{\hat{t}^2}{\hat{s}_1\left[ \hat{t}_1^2 + (\hat{s}_2 + \hat{t}_2 - \hat{t}_1)^2 \right]} \quad (11)
\]

\[
|M(qg \rightarrow l^+\nu)|^2 = \frac{(4\pi)^3\alpha_S\alpha^2|V_{qq'}|^2}{12 \sin^4 \theta_W} \frac{\hat{s}_1(\hat{t}_1^2 + \hat{s}_2^2)}{\hat{s}(\hat{t}_2)(\hat{s}_1 - m_W^2)^2 + m_W^2 \Gamma_W^2} \quad (12)
\]

where \( \alpha_S \) is the strong coupling constant, \( \alpha \) - the fine structure constant, \( V_{qq'} \) - the Cabibbo-Kobayashi-Maskava mixing matrix, \( m_W, \Gamma_W \) - the mass and the total width of \( W \) boson. Our results for \( 2 \rightarrow 3 \) squared amplitudes coincide with the corresponding expressions of [13], but are written in a more convenient form.

In our computations we choose \( Q^2 = \hat{s} \) and we take the structure functions from [16]. For the events which could represent the signals of the new physics - \( p_T^J \geq m_W \), the area of relatively large \( x_1(x_1 \geq 0.03) \) is the most important. Therefore different choices for the structure functions do not make a significant difference for this area. As at CDF additional cuts are used to reject hadronic clusters with transverse energy \( E_T \geq 7 GeV \),
we take $k_T = 7 GeV/c$. Therefore it is required that the transverse momenta of partons $g$ and $q$ in the final state of subprocesses (2) are less than $7 GeV/c$ ($|p_{3T}| \leq k_T = 7 GeV/c$).

In Fig.1 the total cross section of $p \bar{p} \to l^+ \nu X$ reaction ($\sqrt{S} = 1.8 TeV$) is presented. The dotted line corresponds to the production of a single species of lepton with the transverse momentum $p_T \geq p_T^{cut}$ and $|\eta^l| \leq 2$, generated by $2 \to 2$ subprocess (1), while the dashed line corresponds to the contribution generated by $2 \to 3$ subprocesses (2). For this last case it is assumed that there are no hadronic clusters (generated by outgoing partons) with $E_T \geq 7 GeV$. The solid line represents their sum - the total cross section. It is evident that for the values of the transverse momentum $p_T^{cut} \geq 50 GeV/c$ the cross section with $2 \to 3$ subprocesses will prevail over the ordinary cross section generated by $2 \to 2$ subprocess. The ratio $\sigma(2 \to 3)/\sigma(2 \to 2)$ for large values of $p_T^{cut}$ reaches to 3 and therefore the role of higher order contributions to the cross section become important while making a decision about the role of new physics in this area.

In the case of existence of $W'$ boson there are analogous to (1), (2) contributions to the cross section of the reaction $p \bar{p} \to l^+ \nu X$. In fig.2 the total cross sections of this reaction generated by $W'$ bosons of mass $m_{W'} = 200 GeV/c$ are presented. The dotted line corresponds to $2 \to 2$ subprocess (1), while the dashed line to the sum of $2 \to 3$ subprocesses (2). The solid line represents their sum. All parameters are the same as in fig.1: $|\eta^l| \leq 2$ and $k_T = 7 GeV/c$.

In fig.3 we plot the values of the total cross section $p \bar{p} \to l^+ \nu X$ in the case - of the Standard Model and of the existence of $W'$ bosons [3]. Here the sum over the $2 \to 2$ and $2 \to 3$ subprocesses is assumed and $|\eta^l| \leq 2$, $k_T = 7 GeV/c$. It is clear that according to the model [3] the contributions of $W'$ bosons for this signature become important for large values of $p_T^{cut}$. It proves once again the necessity of more precise estimation of this cross section in the Standard Model.

For more precision, when considering $O(\alpha_s)$ corrections to the lowest-order diagrams, besides the real emission subprocesses (2), one has to include the interference between the loop corrections to $q \bar{q}' \to g \bar{l} \nu$ and the Born graphs. There are several computations on higher order corrections to $W$ boson production including also the decays [12],[13],[14]. For calculation of soft collinear and virtual singularities we use the method of [15], dividing the phase space into singular and nonsingular regions. According to [15] we introduce two cutoffs $\delta_s$ and $\delta_c$. When

$$E_g < \delta_s \sqrt{s}/2$$

the soft-gluon approximation is used to evaluate $q \bar{q}' \to g \bar{l} \nu$ subprocess and when

$$|\hat{t}| < \delta_c \hat{s}$$

the diagrams are evaluated in the leading-pole approximation.

Again the cross section of the reaction $p \bar{p} \to l^\pm \nu X$ is determined by two contributions

$$\sigma(p \bar{p} \to l^\pm \nu X) = \sigma_{2 \to 2}^{NLO} + \sigma_{2 \to 3}$$
where $\sigma_{2\to2}^{NLO}$ according to [15] has the following form (we assume that $\lambda_{FC} = 1$ and $\hat{s} = Q^2 \equiv M^2$):

$$\sigma_{2\to2}^{NLO} = \sigma^{HC} + \sum_{q_1,q'_2} \int dx_a dx_b f_{q_1/A}(x_a,Q^2) f_{q'_2/B}(x_b,Q^2) \hat{\sigma}^{NLO}$$

(16)

Here $\sigma^{HC}$ is the contribution from the hard collinear remnants, and

$$\hat{\sigma}^{NLO} = \sigma^{Born}[1 + \frac{\alpha_s}{2\pi} \left[1 + \frac{5\pi^2}{3} + 2\ln(\delta_s)^2 + 3\ln(\delta_s)\right]]$$

(17)

There are two contributions from the remnants of the hard collinear singularities $\sigma^{HC} = \sigma_{qq}^{HC} + \sigma_{qg}^{HC}$

$$\sigma_{qq}^{HC} = \sum_{q_1,q'_2} \int dx_a dx_b \frac{\alpha_s}{2\pi} \int_{x_b}^{1-\delta_s} \frac{dz}{z} f_{q_1/A}(x_a,Q^2) f_{q'_2/B}(\frac{x_b}{z},Q^2) \sigma^{Born} * \frac{4}{3} \left[1 + \frac{z^2}{1-z^2} \ln(\delta_c) + \frac{3}{2} \frac{1}{1-z} - 2 - 3z\right] + (x_a \leftrightarrow x_b)$$

(18)

and

$$\sigma_{qg}^{HC} = \sum_{q_1,q'_2} \int dx_a dx_b \frac{\alpha_s}{2\pi} \int_{x_b}^{1} \frac{dz}{z} f_{q_1/A}(x_a,Q^2) f_{g/B}(\frac{x_b}{z},Q^2) \sigma^{Born} * \frac{1}{2} \left[(z^2 + (1-z)^2) \ln(\delta_c) + 1 - 6z(1-z)\right] + (x_a \leftrightarrow x_b)$$

(19)

In the case of the next-to-leading order calculations, in principle, we should use the NLO distribution functions, however below we use the LL distributions [16].

For calculation of the $\sigma_{2\to3}$ cross section we have to consider that

$$E_g > \delta_s \sqrt{\hat{s}}/2$$

$$|\hat{t}_2| > \delta_c \hat{s}$$

(20)

(21)

which leads to the following changes in equations (8) and (10)

$$t_2^+ = -\delta_c \hat{s}$$

$$\hat{s}_1^+ = \min[\hat{s} - \hat{s}_2, \hat{s} + \hat{t}_2, \hat{s}(1-\delta_s)]$$

(22)

(23)

$\delta_s$ and $\delta_c$ are not the physical parameters. So the inclusive cross section must not depend on them, or at the worst, must depend on them weakly. The analytical and numerical analyses of phase space show that for experimental cuts used at CDF (considered above) the total cross section $\sigma_{2\to3}$ does not depend on $\delta_s$ in the case when $\delta_s < 0.05$. (large values of $\delta_s$ and $\delta_c$ are not desirable as we use the soft gluon and the leading-pole approximations.
while evaluating the diagrams). So, the dependence on $\delta_s$ of inclusive cross section is due to $\hat{\sigma}^{NLO}$ and $\sigma^{HC}_{qq'}$ terms. As the dependence of the cross section on $\delta_s$ must be weak, we use the equation

$$\frac{\partial \hat{\sigma}^{NLO}_{2\rightarrow2}}{\partial \delta_s} = 0$$

to determine the corresponding values of $\delta_s$. We get

$$\delta_c \approx \exp\left(\frac{3}{4}\right)\delta_s^2$$  \hspace{1cm} (24)

This relation practically does not depend on the shape of the distribution functions.

On the other hand, according to the numerical calculations of $p\bar{p} \rightarrow l^{\pm}\nu X$, the values of $\delta_c$, determined as the solutions of the equation

$$\frac{\partial \sigma(p\bar{p} \rightarrow l^{\pm}\nu X)}{\partial \delta_c} = 0$$

are not constant and depend weakly on the values of $P_T^{cut} \equiv q_T$.

In the case of experimental cuts and distribution functions considered above, the values of $\delta_c$ vary in the interval 0.002 - 0.004 for $20\,GeV < P_T^{cut} < 160\,GeV$. If we consider that approximately $\delta_c = 0.003$, than according to (24) $\delta_s \approx 0.04$. We use these values to calculate the total cross section $\sigma(p\bar{p} \rightarrow l^{\pm}\nu X)$ in the NLO approximation. We present our result in fig.4. The solid line represents the NLO approximation, while the dashed line - the Born contributions ($2 \rightarrow 2$ and $2 \rightarrow 3$ subprocesses). The calculation shows that the ratio $\sigma^{NLO}/\sigma^{Born}$ for the given interval of $P_T^{cut}$ varies in the range 0.7 - 1.7. So these corrections do not cause qualitative changes of $\sigma^{Born}$ results, what was mentioned also in [15].

Therefore we can conclude that $2 \rightarrow 3$ subprocesses play important role when a high transverse momentum cut on the lepton is imposed. On the other hand the NLO corrections do not cause qualitative changes of Born approximation results. The contributions of $W'$ bosons for this signature become more important for large values of $P_T^{cut}$.

We are grateful to V.Kartvelishvili and M.Margvelashvili for fruitful discussions.

References

[1] J.L.Hewett, J.Rizzo, Phys.Rep. 183, (1989) 193

[2] F.Feruglio, L.Maiani, A.Masiero, Phys.Lett. B233, (1989) 512

[3] G.Altarelli, B.Mele, M.Ruiz-Altaba, Z.Phys. C45, (1989) 109
[4] R. Casalbuoni, P. Chiappetta, D. Dominici, F. Feruglio, G. Gatto, Nucl. Phys. B310, (1989) 181
[5] M. Kuroda, D. Schildknecht, K. H. Schwarzer, Nucl. Phys. B261, (1985) 432
[6] V. A. Bednjakov, Journal of Nuclear Physics, v.53, (1991) 777
[7] A. A. Pankov, Journal of Nuclear Physics, v.52, (1990) 1069
[8] T. M. Aliev, A. A. Bairamov, Journal of Nuclear Physics, v.52, (1990) 1082
[9] F. del Aguila, M. Quiros, F. Zwirzer, Nucl. Phys. B287, (1987) 419
[10] D. London, J. L. Rosner, Phys. Rev. D34, (1986) 1530
[11] E. Byckling, K. Kajantie, Particle Kinematics, 1973
[12] G. Altarelli, R. K. Ellis, G. Martinelli, Nucl. Phys. B157, (1979) 461
[13] P. Aurenche, J. Lindfors, Nucl. Phys. B185, (1981) 274
[14] P. Arnold, M. H. Reno, Nucl. Phys. B319, (1989) 37; B330, (1990) 284E
[15] H. Baer, M. H. Reno, Phys. Rev. D43, (1991) 2892
[16] D. W. Duke, J. F. Owens, Phys. Rev. D30, (1984) 49
Figure Captions

**Fig. 1** The total cross section for events containing a single species of lepton with transverse momentum $p_T \geq P_{T_{cut}}$ and $|\eta| \leq 2$.

**Fig. 2** The total cross section for events containing a single species of lepton generated by $W'$ bosons

**Fig. 3** The total cross section for events containing a single species of lepton generated by the Standard Model and $W'$ bosons

**Fig. 4** The total cross section for events containing a single species of lepton in the Born and next-to-leading order approximations
This figure "fig1-1.png" is available in "png" format from:

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