Understanding the redshift evolution of the luminosity functions of Lyman $\alpha$ emitters

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Accepted 2009 June 12. Received 2009 June 12; in original form 2009 February 5

ABSTRACT

We present a semi-analytical model of star formation which explains simultaneously the observed ultraviolet (UV) luminosity function (LF) of high-redshift Lyman break galaxies (LBGs) and LFs of Lyman $\alpha$ emitters. We consider both models that use the Press–Schechter (PS) and Sheth–Tormen (ST) halo mass functions to calculate the abundances of dark matter haloes. The Lyman $\alpha$ LFs at $z \lesssim 4$ are well reproduced with only $\lesssim 10$ per cent of the LBGs emitting Lyman $\alpha$ lines with rest equivalent width greater than the limiting equivalent width of the narrow band surveys. However, the observed LF at $z > 5$ can be reproduced only when we assume that nearly all LBGs are Lyman $\alpha$ emitters. Thus, it appears that $4 < z < 5$ marks the epoch when a clear change occurs in the physical properties of the high-redshift galaxies. As Lyman $\alpha$ escape depends on dust and gas kinematics of the interstellar medium (ISM), this could mean that on an average the ISM at $z > 5$ could be less dusty, more clumpy and having more complex velocity field. All of these will enable easier escape of the Lyman $\alpha$ photons. At $z > 5$, the observed Lyman $\alpha$ LF are well reproduced with the evolution in the halo mass function along with very minor evolution in the physical properties of high-redshift galaxies. In particular, up to $z = 6.5$, we do not see the effect of evolving intergalactic medium opacity on the Lyman $\alpha$ escape from these galaxies.

Key words: galaxies: formation – galaxies: high-redshift – galaxies: luminosity function, mass function – cosmology: theory – early Universe.

1 INTRODUCTION

Determining the star formation history of the high-redshift universe is one of the major goals of ongoing observations. Available observational data mainly consist of ultraviolet (UV) luminosity functions (LFs) of high-redshift Lyman break galaxies (LBGs) which can in turn give the star formation rate density of the universe. The galaxies have been identified even up to redshift $z \sim 10$ using so called photometric ‘drop-out’ technique (Bouwens et al. 2004; Hopkins & Beacom 2006; Richard et al. 2006). However, very good constraints are available only up to $z \sim 7$ (Bouwens et al. 2008).

In addition to the ‘drop-out’ techniques, narrow band searches for high-redshift galaxies emitting a strong Lyman $\alpha$ line are successful in detecting galaxies at $3 \lesssim z \lesssim 6$ (Cowie & Hu 1998; Hu, Cowie & McMahon 1998; Rhoads et al. 2000; Taniguchi et al. 2005; Kashikawa et al. 2006; Shimasaku et al. 2006; Dawson et al. 2007; Gronwall et al. 2007; Murayama et al. 2007; Ota et al. 2008; Ouchi et al. 2008). Unlike the drop-out technique used in detecting the LBGs, the searches for Lyman $\alpha$ emitters (LAEs) are not biased by UV luminosity. However, the detectability depends on the Lyman $\alpha$ emissivity and radiative transport. Thus, these two techniques pick up galaxies with different types of selection biases. Availability of the UV LFs of Lyman $\alpha$ selected galaxies allows us to understand these biases and provides joint constraints on models of galaxy formation at $z > 3$.

The star formation rate is a key quantity for both UV as well as Lyman $\alpha$ emission from a galaxy. Hence, it is interesting to obtain a semi-analytical model of star formation for these high-redshift galaxies that can explain both these sets of observations. In our previous work by Samui, Srianand & Subramanian (2007, hereafter Paper I), we have built a semi-analytic model of star formation taking account of several feedback processes in order to explain the observed UV LFs of LBGs at $3 \leq z \leq 10$. By fitting the observed data, we put constraints on the nature of the star formation in this redshift range. In Samui, Subramanian & Srianand (2009, hereafter Paper II), we studied the effect of assumed form of the halo mass function on the results of semi-analytical galaxy formation models, in detail. As a continuation of these works, here we compute the LF of LAEs using the same star formation model and compare it with the three sets of available observations, which are the...
Table 1. Flux at various wavelengths as predicted by ‘STARBURST99’ for a continuous mode of star formation at a rate of 1 $M_{\odot}$ yr$^{-1}$. The quoted values are at the time $t = 1.4 \times 10^8$ yr after the star formation began. We also show the equivalent width calculated at $t = 10^7$ yr.

| Model | IMF $M_{low}$ | IMF $M_{up}$ | $f_{905}^{a,b}$ | $f_{1505}^{a,b}$ | Ratio $f_{905}/f_{1505}$ | Hi-UV$^c$ | $f_{1215}^{d}$ | EW$^f$ (Å) |
|-------|--------------|-------------|----------------|----------------|------------------|--------|------------|----------|
| model1 | 1            | 100         | 0.050          | 40.12          | 40.37            | 0.57   | 53.13      | 40.44    | 49.4   |
| model2 | 1            | 100         | 0.040          | 40.10          | 40.32            | 0.60   | 53.23      | 40.36    | 74.3   |
| model3 | 1            | 100         | 0.020          | 40.15          | 40.42            | 0.53   | 53.33      | 40.45    | 75.8   |
| model4 | 1            | 100         | 0.008          | 40.16          | 40.46            | 0.50   | 53.45      | 40.45    | 100.3  |
| model5 | 1            | 100         | 0.004          | 40.18          | 40.48            | 0.51   | 53.50      | 40.46    | 110.6  |
| model6 | 1            | 100         | 0.001          | 40.21          | 40.48            | 0.53   | 53.57      | 40.44    | 135.3  |
| model7 | 1            | 100         | 0.0004         | 40.24          | 40.50            | 0.54   | 53.61      | 40.48    | 133.4  |
| model8 | 0.1          | 100         | 0.0004         | 39.83          | 40.10            | 0.54   | 53.20      | 40.07    | 139.4  |
| model9 | 5            | 100         | 0.0004         | 40.58          | 40.75            | 0.66   | 53.94      | 40.80    | 135.5  |
| model10 | 10           | 100         | 0.0004         | 40.75          | 40.79            | 0.91   | 54.11      | 40.92    | 157.6  |
| model11 | 20           | 100         | 0.0004         | 40.89          | 40.78            | 1.29   | 54.29      | 40.95    | 216.7  |
| model12 | 40           | 100         | 0.0004         | 40.95          | 40.77            | 1.54   | 54.39      | 40.93    | 288.8  |

$^{a}$log of flux (erg s$^{-1}$ Å$^{-1}$) at 905 Å.
$^{b}$Note that ‘STARBURST99’ gives flux at 905 and 1505 Å.
$^{c}$log of flux (erg s$^{-1}$ Å$^{-1}$) at 1505 Å.
$^{d}$log of no. of H$^+$ ionizing photons per sec.
$^{e}$log of flux (erg s$^{-1}$ Å$^{-1}$) at 1215 Å.
$^{f}$Equivalent widths are calculated taking $f_{esc}^{Ly}$ = 1 and $f_{esc} = 0.1$ and $\eta = 1$.

2 SEMI-ANALYTICAL MODELS

In order to compute the LF of high redshift galaxies, one needs to model both the star formation in an individual galaxy and the abundance of dark matter haloes in which the galaxies form. We compute the abundance and formation rate of dark matter haloes as a function of redshift in the framework of Lambda cold dark matter cosmology. For this purpose, we consider two halo mass functions, the analytically motivated Press–Schecter (PS) halo mass function (Press & Schechter 1974) and the Sheth–Tormen (ST) halo mass function (Sheth & Tormen 1999), which gives a better fit to numerical galaxy formation simulations. For the PS halo mass function, we use the formalism of Sasaki (1994), to calculate the net formation rate of haloes. However, the Sasaki formalism is not easily generalizable to the other form of mass functions. Hence, for the ST halo mass function, we simply take recourse to its derivative to calculate the net formation rate of dark matter haloes (also see Paper II).

The star formation rate of an individual galaxy of dark matter mass $M$ is assumed to be (Chiu & Ostriker 2000),

$$M_{sf}(M, z, z_e) = f_s \left( \frac{\Omega_b}{\Omega_m} \right) \frac{\kappa}{\kappa_{bins}(z_e)} \left( \frac{t(z) - t(z_e)}{t_{bins}(z_e)} \right),$$

where, the amount and duration of the star formation is determined by the values of $f_s$ and $\kappa$, respectively. We can fix these two parameters by fitting the observed UV LFs of high-redshift LBGs (see Paper I and II for details). Further, $t(z)$ is the age of the universe; thus $t(z) - t(z_e)$ gives the age of the galaxy at $z$ that has formed at an earlier epoch $z_e$, and $\kappa_{bins}$ is the dynamical time at that epoch. The star formation rate is converted to luminosity at 1500 Å assuming an initial mass function (IMF) of the stars formed (see equation 6–8 of Paper I). The observed luminosity is less by a factor, $\eta$, than the actual luminosity because of the dust reddening inside the galaxy. In principle, the value of $\eta$ depends on the wavelength and the functional form is governed by the nature of the dust grains.

As in Paper I, we calculate the reionization history of the universe and the radiative feedback of the meta-galactic background UV radiation on the star formation in a self-consistent manner for each model (also see Thoul & Weinberg 1996; Benson et al. 2002; Bromm & Loeb 2002; Dijkstra et al. 2004). We assume a steep cut-off of star formation in haloes with mass $M \geq 10^{12} M_{\odot}$ which is attributed to active galactic nuclei feedback (Bower et al. 2006; Best et al. 2006).

We compute the Lyman $\alpha$ luminosity of a star-forming galaxy assuming case-B recombination. In this case, two Lyman $\alpha$ photons are produced out of three hydrogen ionizing photons (Osterbrock 1989) that are confined within the interstellar medium (ISM) of the galaxy. Hence, the Lyman $\alpha$ luminosity produced in any star-forming region is related to its star formation rate by

$$L_{Ly\alpha} = 0.68 h v_{Ly\alpha}(1 - f_{esc}) N_{\gamma} M_{sf}.$$

Here, $h v_{Ly\alpha}$ is $10.2$ eV and $f_{esc} = 0.1$ is the energy of a Lyman $\alpha$ photon and the escape fraction of UV ionizing photons, respectively. Further, $N_{\gamma}$ is the rate of ionizing photon production per unit solar mass of star formation. This mainly depends on the initial mass function of the stars and also on the metallicity. Values of number of ionizing photons per baryon of star formation for different IMFs and different metallicities can be found in table 1 of Paper I.

The observed Lyman $\alpha$ luminosity is given by

$$L_{Ly\alpha}^{obs} = f_{esc}^{Ly\alpha} L_{Ly\alpha}.$$
Here, $f_{\text{Ly}^\alpha}^{\text{esc}}$ is the escape probability of the Lyman $\alpha$ photons. This is decided by the dust optical depth, velocity field of the ISM in the galaxies and the Lyman $\alpha$ optical depth due to ambient intergalactic medium (IGM) around the galaxies.

As Lyman $\alpha$ is a resonant transition, we expect the effective dust optical depth for Lyman $\alpha$ in the ISM to be much larger than that for the UV continuum photons (i.e. $f_{\text{Ly}^\alpha}^{\text{esc}} < 1/\eta$). However, if Lyman $\alpha$ emission comes from some outflows in the star-forming region (Malhotra & Rhoads 2002; Dijkstra, Wyithe & Haiman 2007b; Verhamme et al. 2008) or through inhomogeneous ISM (Neufeld 1991; Hansen & Oh 2006; Finkelstein et al. 2008, 2009) then there may not be any correlation between $\eta$ and $f_{\text{Ly}^\alpha}^{\text{esc}}$.

The escape fraction of Lyman $\alpha$ also depends on the optical depth of the IGM in the immediate neighbourhood of the galaxy, in particular the proximate region that is affected by excess ionization by the galaxy itself. Thus, the redshift evolution of $f_{\text{Ly}^\alpha}^{\text{esc}}$ can be a useful probe of the reionization history of the universe (Malhotra & Rhoads 2004; Haiman & Cen 2005; Stern et al. 2005; Dijkstra 2007b; Dayal, Ferrara & Galli 2008) and/or the redshift evolution of dust abundance (Mao et al. 2007), velocity field and gas clumping factor in galaxies.

It may be possible that all the LBGs do not have a detectable Lyman $\alpha$ emission. The spectroscopic observations of LBGs by Shapley et al. (2003) show that only 25 per cent of the LBGs at $z \sim 3$ have Lyman $\alpha$ emission with rest equivalent width $W_0 \geq 20$ $\text{Å}$ (also see Steidel et al. 2000). Also the observations of UV LF of LAEs show similar results. Hence, we consider that only a fraction $G_f$ of the entire galaxy population will be detected as LAEs in surveys as they are usually sensitive to galaxies having Lyman $\alpha$ equivalent widths above certain limiting value.

The rest-frame equivalent width of the Lyman $\alpha$ emission is given by

$$ W_0 = L_{\text{Ly}^\alpha}/(L_{\text{cont}}/\eta) = f_{\text{Ly}^\alpha}^{\text{esc}} L_{\text{Ly}^\alpha}/(L_{\text{cont}}/\eta), \tag{4} $$

where $L_{\text{cont}}$ is the continuum luminosity per unit wavelength near 1215 $\text{Å}$. We obtained this from the stellar synthesis code 'STARBURST99'\(^1\) (Leitherer et al. 1999). For our continuous mode of star formation, we use the same prescription as in Paper I for the UV continuum flux, to calculate the 1215 $\text{Å}$ continuum flux. We tabulate the continuum luminosities at different wavelengths and rest-frame equivalent width of Lyman $\alpha$ emission in Table 1 for various physical parameter related to the nature of the star formation for a continuous constant star formation model as obtained from 'STARBURST99' at the time $t = 1.4 \times 10^8$ yr after the star formation began. We also quote the equivalent width calculated at $t = 10^7$ yr. Note that for a constant continuous star formation model, the number of high-mass star remains constant after typical lifetime of OB stars that dominate in $L_{\text{Ly}^\alpha}$. Hence, the equivalent width decreases with time as contribution from the low-mass stars to $L_{\text{cont}}$ continuously adds up.

In our model, the Lyman $\alpha$ equivalent width of a galaxy is independent of its mass as both Lyman continuum as well as line flux would scale with mass. It depends on the value of $k_{\text{Ly}^\alpha}$ and most importantly the values of $\eta$ and $f_{\text{Ly}^\alpha}^{\text{esc}}$. It also depends on the value of $f_{\text{esc}}$, the escape fraction of the ionizing photons. In Fig. 1, we show the time evolution of the intrinsic rest-frame equivalent width of a galaxy as predicted by our models. The observed equivalent width would be scaled by a factor of $\eta f_{\text{Ly}^\alpha}^{\text{esc}}$. We also assume a Salpeter IMF in the mass range 1–100 $M_\odot$ (solid line) and 10–100 $M_\odot$ (dashed line). As can be seen from Table 1 as well as from Fig. 1, the intrinsic equivalent width depends on the assumed IMF. Note that throughout this work, we will use $\kappa = 1$ and Salpeter IMF with the mass range 1–100 $M_\odot$ with metallicity 0.0004 (i.e. model7 of Table 1).

Note that we have mainly three sets of observations that can be used to constrain our model parameters. These are (i) UV LF of LBGs, (ii) Lyman $\alpha$ LF of LAEs and (iii) UV LF of LAEs. Along with these we have the information about the equivalent width distribution of the LAEs. The first set of observations can be used to constrain $f_{\text{esc}}/\eta$ combination. The second set can be used to constrain $f_{\text{esc}} f_{\text{Ly}^\alpha}^{\text{esc}}$ and the last one can be used to obtain $G_f$. Then, we will be able to calculate the mean $W_0$. The spread in the equivalent width of the detected galaxies will come in two ways: (i) distribution in $\eta$ and $f_{\text{Ly}^\alpha}^{\text{esc}}$ and (ii) the spread in their ages. Since in our model we assume only the average value for both $\eta$ and $f_{\text{Ly}^\alpha}^{\text{esc}}$, we will have distribution in $W_0$ only coming from the spread in the ages of detected galaxies. We show this distribution in the following section while discussing our results.

### 3 LUMINOSITY FUNCTIONS

In Fig. 2, we compare our model predictions for both UV and Lyman $\alpha$ LFs with the observed data points. For each redshift bin, we have used the most recent measurement of the UV LF of LBGs that covers a wide range in luminosity. Below we provide details of observational data used in each redshift bin. LFs of LAEs are taken from Ouchi et al. (2008). The solid and dashed lines are our model predictions using ST and PS halo mass functions, respectively.

At a particular redshift, we first fit the observed UV LFs of LBGs by adjusting $f_{\alpha}/\eta$. For this, we use $\chi^2$ minimization technique (see Paper II for details). Then, we fit the observed UV LF of LAEs by changing $G_f$ and keeping same $f_{\alpha}/\eta$ obtained for the nearest

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\(^1\) http://www.stsci.edu/science/starburst99

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**Figure 1.** The time evolution of intrinsic Lyman $\alpha$ equivalent width of a galaxy as predicted by our models. Solid and dashed lines are for the Salpeter IMF in the mass ranges 1–100 and 10–100 $M_\odot$, respectively (i.e. model7 and model10 of Table 1). We have assumed $\kappa = 1$. The dynamical time ($t_{\text{dyn}}$) depends on redshift of collapse ($z_c$). For example, at $z_c = 10$, 5 and 3, $t_{\text{dyn}} = 8.8 \times 10^7$, $2.2 \times 10^7$ and $4 \times 10^6$ yr, respectively. We also show the actual time that corresponds to $z_c = 10$ on the top axis.

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**Figure 2.** Comparison of UV LF of LBGs with the observed data points for different redshift bins.
available redshift. Note that, we did not try to get $G_f$ through $\chi^2$ minimization as there are only few data points in the observed LF (also there are issues related to the completeness of the samples). Finally, we match our model predictions with observed Lyman $\alpha$ LF at the same redshift by adjusting $f_\star/\eta$ and keeping $G_f$ fixed. This is also done using $\chi^2$ minimization. In Table 2, we summarize the best-fitting parameters along with the $\chi^2$ values at different redshifts for models with both PS and ST mass functions. Below we describe these results for specific redshifts.

3.1 Luminosity functions at $z \sim 3$

At $z \sim 3$, we have all the three observed LFs and the data are quite well established, i.e. different groups have confirmed the data by different methods. For $z \sim 3$, we use the observed UV LF of LBGs given by Reddy & Steidel (2009) which covers the low luminosity end. We show in the left most panels of Fig. 2 (panel (a) and (b)), both UV and Lyman $\alpha$ LFs at $z \sim 3$ as predicted by our model along with the observed data points. The best-fitting model parameters are given in Table 2. In panel (a) of Fig. 2, the set of thin curves in the top is the predictions of UV LFs of LBGs. A good agreement with the observed UV LF of LBGs is obtained for $f_\star/\eta = 0.044 \pm 0.001$ and $0.055 \pm 0.001$ for models with the PS and ST mass functions, respectively. The corresponding reduced $\chi^2$ for these fits is 3.9 and 0.97. Thus, the shape of the observed UV LF of the LBGs is better reproduced by the model with the ST halo mass function. We fit the UV LF of the LAEs by multiplying the UV LF of LBGs with a fraction $G_f$. As mentioned earlier, we did not try to get $G_f$ through $\chi^2$ minimization. The observed data points are well reproduced for $G_f = 0.07$ with same $f_\star/\eta$. These curves are also shown in the figure by the thick blue lines [the bottom set of curves in panel (a)] for both PS and ST mass functions. The declining trend in the UV LF of LAEs seen in the low luminosity end (open triangles) is mainly due to incompleteness. Apart from these points, other data points do not require luminosity dependent $G_f$. This is

\begin{table}
\centering
\begin{tabular}{llllll}
\hline
$z$ & $f_\star/\eta$ & $G_f$ & $f_\star f_{\text{esc}}^\lambda$ & EW$^\star$ (Å) & $f_\star/\eta$ & $G_f$ & $f_\star f_{\text{esc}}^\lambda$ & EW$^\star$ (Å) \\
\hline
3.1 & 0.044(3.90) & 0.07 & 0.059 $\pm$ 0.011 (0.95) & 179 & 0.055(0.97) & 0.07 & 0.076 $\pm$ 0.011 (2.49) & 183 \\
3.7 & 0.046(2.32) & 0.10 & 0.051 $\pm$ 0.014 (0.68) & 148 & 0.042(1.09) & 0.10 & 0.050 $\pm$ 0.015 (1.08) & 159 \\
5.7 & 0.081(1.19) & 1.00 & 0.044 $\pm$ 0.017 (0.91) & 72 & 0.050(0.63) & 1.00 & 0.028 $\pm$ 0.021 (0.42) & 75 \\
6.5 & – & 1.00 & 0.054 $\pm$ 0.012 (2.30) & – & – & 1.00 & 0.031 $\pm$ 0.015 (2.32) & – \\
\hline
\end{tabular}
\caption{Comparison of model predictions between the PS and ST mass functions.}
\end{table}

\footnotesize
1 Obtained using $\chi^2$ minimization and also corrected for dust opacity at $\lambda = 1500$ Å; the $\chi^2$ per degree of freedom is given in bracket (see Paper II for details).
2 Values indicated inside the bracket are best-fitting $\chi^2$ per degree of freedom.
3 The average equivalent width is calculated at $t = 10^8$ yr.
consistent with our implicit assumption that $G_f$ is independent of halo mass (or galaxy luminosity).

The value of $G_f$ is in agreement with the measurements of Shapley et al. (2003) where they found that the fraction of LBGs having Lyman $\alpha$ emission with equivalent width $W_0 \geq 60$ Å is $\sim 8$ per cent (see their fig. 8). Note that the sample of LAEs of Ouchi et al. (2008) at $z = 3.1$ has $W_0 \geq 60$ Å. Hence our results are consistent with both these observations. From the fact that $G_f$ matches with the prediction from fig. 8 of Shapley et al. (2003), we can conclude that both the techniques of detecting $\sim 3$ galaxies appear to pick a subset of the same parent population of galaxies.

We now turn to Lyman $\alpha$ LF of LAEs. We show this in panel (b) of Fig. 2 along with the observational data taken from Ouchi et al. (2008). To fit the Lyman $\alpha$ LF, only free parameter is $f_{\alpha} f_{esc}^{Ly}$ as $G_f$ has already been fixed by fitting the UV LF of the LAEs. The best fit with the observational data is obtained with $f_{\alpha} f_{esc}^{Ly} = 0.059 \pm 0.011$ and 0.076 $\pm 0.011$ for the PS and ST mass functions, respectively. The corresponding best-fitting $\chi^2$ is 0.95 and 2.49. If we consider $\eta = 4.5$ as obtained by Reddy et al. (2006) then we have $f_{esc}^{Ly} = 0.29$ and 0.30, respectively, for the models with the PS and ST mass functions. Taking $\eta = 4.5$ also implies $f_{\alpha} = 0.20$ and $f_{\alpha} = 0.25$ for the models with the PS and ST mass functions, respectively.

We now calculate the average rest-frame equivalent width of the Lyman $\alpha$ emission of the star-forming galaxies that are contributing to the LF. Note that for given values of $f_{\alpha}/\eta$ and $f_{\alpha} f_{esc}^{Ly}$, the equivalent width is solely determined from the IMF we assume. This can be easily understood if we rewrite equation (4) as

$$W_0 = \frac{L_{Ly}(f_{\alpha} f_{esc}^{Ly})}{L_{esc}(f_{\alpha}/\eta)}.$$  

(5)

The ratio $L_{Ly}/L_{esc}$ depends on the IMF and the metallicity of the gas (see Table 1) and $f_{\alpha}/\eta$ and $f_{\alpha} f_{esc}^{Ly}$ come from the fit. Therefore, fitting simultaneously the UV and Lyman $\alpha$ LFs uniquely specify the average equivalent width of the Lyman $\alpha$ emission line. For the fit presented in panels (a) and (b) of Fig. 2, the average equivalent widths are 179 and 183 Å for the models with the PS and ST mass functions, respectively. Note that $\eta$ reflects extinction at $\lambda \sim 1500$ Å; the relative extinction at $\lambda = 1215$ Å will be higher than that at $\lambda = 1500$ Å. Therefore, the actual equivalent width will be higher depending upon the adopted extinction correction. The Lyman $\alpha$ rest equivalent width distribution predicted for the best-fitting model parameters is shown in Fig. 3. Note that the spread in $\eta$ and $f_{esc}^{Ly}$ around their best-fitting values will make this distribution spread over wider equivalent width range. Hence, one should not directly compare this histogram with observations although the mean value itself is relevant. In all results presented here, we use a lower mass cut-off of 1 $M_\odot$ in the assumed IMF. Increasing this to $\geq 10 M_\odot$ to mimic a top-heavy IMF would increase the predicted equivalent width by a factor of $\sim 1.4$ (see Fig. 1).

### 3.2 Luminosity functions at $z \sim 4$

We show our model prediction as well as the observed data points at $z \sim 4$ in panels (c) and (d) of Fig. 2. The observed UV LF of LBGs at $z = 4$ is taken from Bouwens et al. (2007). The LF of LAEs is at $z = 3.7$ and we compare this with UV LF of LBGs at $z = 4$. We see from our model predictions that there is no significant change in the properties of the galaxies from $z \sim 3$ to $\sim 4$. The UV LF of LBGs can be well fitted with $f_{\alpha}/\eta = 0.046 \pm 0.001$ and 0.042 $\pm 0.001$ with all other parameters being same as at $z = 3$ for the models using the PS and ST mass functions, respectively (see Table 2). The corresponding reduced $\chi^2$ is 2.32 and 1.09. Thus, even for this redshift bin, the model with the ST mass function provides a better fit to the observed data. If we assume $\eta = 4.5$, we get $f_{\alpha} = 0.21$ and 0.19 for the models with PS and ST mass functions, respectively.

In order to fit the observed UV LF of LAEs of Ouchi et al. (2008) at $z = 3.7$, we need $G_f = 0.1$. Comparing the values of $G_f$, we conclude that there is no strong evolution in the percentage of LBGs showing up as LAEs from $z = 3$ to $4$. Assuming no redshift evolution in the equivalent width distribution of Lyman $\alpha$ and taking the limiting rest equivalent width of 45 Å (as in Ouchi et al. 2008), we estimate $G_f \sim 0.1$ from the fig. 8 of Shapley et al. (2003). However, Reddy et al. (2008) report an evolution in the Lyman $\alpha$ equivalent-width distribution of LBGs between $1.9 \leq z \leq 3.4$. Continuation of this trend to higher redshifts will mean $G_f$ more than 10 per cent. Our model predictions match reasonably well with these observational predictions given the error in measurements.

The good agreement with the data of Lyman $\alpha$ LF at $z = 3.7$ (taken from Ouchi et al. 2008) is obtained for $f_{\alpha} f_{esc}^{Ly} = 0.051 \pm 0.014$ for the model with the PS mass function with best-fitting $\chi^2$/d.o.f. = 0.68. The mean Lyman $\alpha$ equivalent-width distribution of the LBGs as predicted from this model is 148 Å. For the model with the ST mass function, one needs $f_{\alpha} f_{esc}^{Ly} = 0.050 \pm 0.015$ (with best-fitting $\chi^2$/d.o.f. = 1.08) and the average equivalent width predicted by this model is 183 Å. The Lyman $\alpha$ rest equivalent width distribution predicted for the best-fitting model parameters is shown in Fig. 3. For $\eta = 4.5$, we get $f_{esc}^{Ly} = 0.25$ and 0.26 for PS and ST mass functions, respectively. These values are consistent with that we derived for $z \sim 3$. Therefore, with no or minor evolutions in the physical conditions in the LBGs, our models reproduce the observed LF for $3 \leq z \leq 4$. However, from Fig. 3, it is clear that our models predict a mild decrease in the rest equivalent width of Lyman $\alpha$ with increasing redshift.

### 3.3 Luminosity functions at $z \sim 6$

In the panels (e) and (f) of Fig. 2, we show our model prediction of LFs at $z \sim 6$. The observed UV LFs of LBGs at $z = 6$ are taken from Bouwens et al. (2007). First, the required values of $f_{\alpha}/\eta$ are...
0.081 ± 0.001 and 0.050 ± 0.001 for the model with the PS and ST mass functions, respectively. The corresponding reduced \( \chi^2 \) is 1.19 and 0.63, suggesting both PS and ST mass functions produce good fit to the data. We need \( G_f = 1.0 \) to reproduce the UV LF of LAEs at this redshift. Hence, 100 per cent of the LBGs are detected as LAEs at \( z \sim 6 \). This is considerably different from \( z = 3 \) or 4 where only \( \lesssim 10 \) per cent of LBGs are detectable as LAEs. Assuming no redshift evolution in the equivalent width distribution of Lyman \( \alpha \) and taking the limiting rest equivalent width of 25 Å (as in Ouchi et al. 2008), we estimate \( G_f \sim 0.25 \) from the fig. 8 of Shapley et al. (2003). This means that the physical properties related to the Lyman \( \alpha \) emission have changed considerably from \( z = 3.7 \) to 5.7. This conclusion depends very much on the accuracy of the observed LFs. While data of Shimasaku et al. (2006) are consistent with that of Ouchi et al. (2008), there are some discrepancies in the fraction of Lyman break selected galaxies that are also LAEs (see Ajiki et al. 2003; Rhoads et al. 2003; Hu et al. 2004; Dow-Hygelund et al. 2007). We will come back to this issue in the discussion section. As we have been using Ouchi et al.'s data in all redshift bins, we base our conclusions on their data.

We show our model predictions for the Lyman \( \alpha \) LF of LAEs at \( z = 5.7 \) in panel (f) of Fig. 2. The observed data points are taken from Ouchi et al. (2008). To fit the observed Lyman \( \alpha \) LF, one needs \( f_{\alpha}f_{esc}^{\alpha} = 0.044 ± 0.017 \) for the model using the PS mass function. The best-fitting \( \chi^2 \) per degree of freedom is 0.91. Therefore, even though the fraction of LAEs has increased considerably from \( z = 3 \) to 6, the value of \( f_{\alpha}f_{esc}^{\alpha} \) in the galaxies identified as LAEs which characterizes the Lyman \( \alpha \) escape (for fixed \( f_{\alpha} \)) has changed negligibly (within the uncertainty of the best-fitting values). However, this is true only for the model with the PS mass function. The model that uses the ST mass function predicts a change in the escape of the Lyman \( \alpha \) photons at \( \sim 3\sigma \) level. For this model, the best fit is obtained with \( f_{\alpha}f_{esc}^{\alpha} = 0.028 ± 0.021 \) (with best-fitting \( \chi^2/d.o.f. = 0.42 \)). The calculated mean equivalent widths are 72 and 75 Å for the model with the PS and ST mass functions, respectively. The predicted rest equivalent width distribution is shown in Fig. 3. As noted above, we see a decrease in the average equivalent width with increasing redshift.

### 3.4 Cumulative luminosity function at \( z = 6.5 \)

Fan et al. (2006) have shown, based on the spectra of QSOs, that there is a significant increase in the IGM neutral fraction at \( z \gtrsim 6 \). As Lyman \( \alpha \) escape also depends on the IGM opacity one expects a significant change in the Lyman \( \alpha \) LF at \( z \gtrsim 6 \). Kashikawa et al. (2006) have given the integrated Lyman \( \alpha \) LF and the UV LF of LAEs at \( z = 6.5 \). The observed data and our model predictions are compared in Fig. 4. We use \( f_{\alpha}/\eta = 0.081 \) and 0.050, respectively, for the models with the PS and ST mass functions. These are the best-fitting values for \( z \sim 6 \) UV LF of LBGs. They provide a good fit to the observed UV LF of LAEs at \( z \sim 6 \) for \( G_f = 1 \). Our model predictions of the Lyman \( \alpha \) LF match reasonably well with the observed data (bottom panel in Fig. 4). The good agreement with the data is obtained for \( f_{\alpha}f_{esc}^{\alpha} = 0.054 ± 0.012 \) and 0.031 ± 0.015 for the PS and ST mass function, respectively. The corresponding best-fitting \( \chi^2 \) per degree of freedom is 2.30 and 2.32. These two values are similar to those at \( z = 5.7 \). Hence, we conclude that the evolution in the dark matter halo mass function is sufficient to explain the observed evolution in the Lyman \( \alpha \) LF from \( z = 5.7 \) to 6.5 without any major changes in other physical properties related to the star formation in the high-redshift galaxies.

![Figure 4](https://academic.oup.com/mnras/article-abstract/398/4/2061/983383)

**Figure 4.** Upper panel: the UV LFs of LAEs at \( z = 6.5 \). The observed data are from Kashikawa et al. (2006). The predicted UV LFs of LBGs at \( z = 6.5 \) for the ST and PS halo mass functions are shown by solid and dashed lines, respectively. We take the values of \( f_{\alpha}/\eta \) that fits the UV LFs at \( z = 6 \). Lower panel: the cumulative Lyman \( \alpha \) LF at \( z = 6.5 \). The solid line is for the ST mass function and dashed is for PS mass function. The spectroscopic (filled triangles) and photometric (filled circles) data are taken from Kashikawa et al. (2006). The model parameters are adjusted to fit the LF obtained from the photometric data.

### 4 Conclusion and Discussion

We have built a semi-analytical model of star formation for high-redshift galaxies which simultaneously reproduces the observed UV LFs of LBGs and LAEs and the Lyman \( \alpha \) LF of LAEs in the redshift range \( 3 \leq z \leq 6.5 \). We fit the UV LFs of LBGs by changing \( f_{\alpha}/\eta \) while we adjust \( G_f \), the fraction of LBGs detected as LAEs, to match the UV LFs of LAEs. Finally, to fit the Lyman \( \alpha \) LFs of LAEs, we vary \( f_{\alpha}f_{esc}^{\alpha} \). The best-fitting values of our model parameter at different redshifts allow us to probe the redshift evolution of properties of galaxies. In our models, we make an implicit assumption that the LAEs are a subset of a parent population of normal galaxies detected through Lyman break technique.

Within the observational uncertainties, we are able to reproduce the observed UV LFs of LAEs by simply scaling the best-fitting UV LFs of LBGs by a constant factor \( G_f \). This basically means that at a given \( z \) the fraction of LBGs that are seen as LAEs is independent of the UV luminosity of galaxies and mass of the dark
matter haloes. Improving the errors in the UV LFs of LAEs will allow us to investigate the possible dependence of \( G_f \) on the mass of the galaxies.

The most interesting results from our study is the redshift evolution of \( G_f \). We showed that for \( z \sim 3.1 \) the well-measured fraction of LAEs among the LBGs is consistent with the \( G_f \) we require to fit the three LF at this redshift. Our model fits to the observations clearly show a strong evolution in \( G_f \) between \( z < 4 \) and \( > 5 \). Physically \( G_f \) at any given redshift will be given by the distribution in the Lyman \( \alpha \) escape among the population of LBGs. This will be governed by \( E(B - V) \), line of sight \( H_\alpha \) column density, velocity field in the Lyman \( \alpha \) emitting region and/or the duty cycle of the burst of star formation. It is interesting to note that even if there is absolutely no change in the distribution of Lyman \( \alpha \) equivalent width (absorption as well as emission) as a function of redshift one expects \( G_f \) to increase with \( z \) mainly because of the decrease in the liming rest equivalent width of Lyman \( \alpha \) emission in Ouchi et al.’s (2008) survey. For example, based on fig. 8 of Shapely et al. we expect \( G_f \) to be 0.25 at \( z \sim 6 \). From Fan et al. (2006), we note that the IGM transmission decreases by at least a factor of 3 between \( z = 3.1 \) and 5.7 due to Gunn–Peterson optical depth. The actual change in the IGM optical depth in the proximity of the LAE is difficult to quantify as it depends on the ionization efficiency of the galaxy. Therefore, we expect \( G_f \lesssim 0.25 \) if the properties of LBGs do not change between \( z \sim 3.1 \) and \( > 5.7 \). Thus, our results giving \( G_f \) = 1.0 at \( z \sim 6 \) strongly support an evolution in the physical properties of these galaxies with redshift.

Ouchi et al. (2008) provides LFs only at \( z = 3.1, 3.7 \) and 5.7. From our analysis, we see a sudden jump in \( G_f \) between \( z = 3.7 \) and 5.7. In order to explore whether this change is gradual or not, we consider few other observations in the intermediate redshift. At \( z = 4.5 \), Dawson et al. (2007) have measured Lyman \( \alpha \) LF of LAEs. In absence of UV LF of their sample, we are unable to follow the same procedure as earlier. However, we note that values of \( G_f \) and \( f_{\text{esc}}^{\text{Ly}\alpha} \) that fit the Lyman \( \alpha \) LF of Ouchi’s sample at \( z = 3.7 \) produce a good fit to the Dawson et al. data where as using the best-fitting parameters at \( z = 5.7 \) over produces the abundance of \( z = 4.5 \) LAEs. There are two independent measurements of LFs of LAEs available at \( z = 4.86 \): one by Ouchi et al. (2003) and other by Shioya et al. (2009). Ouchi et al. (2003) cover the low luminosity end of the LF \( \left[ 5 \times 10^{41} < L_{\text{Ly}\alpha} \left( \text{erg s}^{-1} \right) < 2 \times 10^{44} \right] \) where as Shioya et al. (2009) cover the high end \( \left[ 8 \times 10^{42} < L_{\text{Ly}\alpha} \left( \text{erg s}^{-1} \right) < 4 \times 10^{43} \right] \) with slight overlap between them. The Ouchi et al. (2003) measurements are consistent with \( G_f = 0.1 \). However, if we also consider Shioya et al. (2009) data, \( G_f \) could be as large as 0.3. Note that the completeness of the sample is always an issue in this case. Hence more observations are needed in this redshift range in order to probe in detail how \( G_f \) increases to unity by \( z = 5.7 \).

Unlike at \( z \sim 3.1 \), the LF of LAEs obtained by different groups for \( z \sim 5.7 \) disagree up to a factor of 5 (see Ajiki et al. 2003; Rhoads et al. 2003; Hu et al. 2004; Shimasaku et al. 2006; Murayama et al. 2007; Ouchi et al. 2008). The difference could be due to differences in the colour selection criteria used in the narrow band survey and the depth of the broad-band photometry. Dow-Hygelund et al. (2007) have found that only 30 per cent of the LBGs at \( z \sim 6 \) selected through i-dropout selection show Lyman \( \alpha \) emission with rest equivalent width \( \geq 20 \text{ Å} \). It is also important to remember that while the narrow band imaging picks object within very narrow redshift range the broad-band colour techniques pick objects over a much wider redshift range. Incompleteness levels in these two types of surveys are also very different. Dow-Hygelund et al. (2007) have shown that the i-band selection misses considerable number of LAEs at \( z < 5.8 \). On the other hand, the narrow band technique of Ouchi et al. (2008) picks object at \( z = 5.70 \pm 0.01 \). After taking into account this effect, Dow-Hygelund et al. (2007) conclude that up to 40 per cent of the i-dropout galaxies could be LAEs. From our models, we find the redshift evolution between the mean \( z \) of LBGs and LAEs will account for an additional 10 per cent increase in \( G_f \). Even after taking into account all these effects, one needs \( G_f \) to be a factor of 2 higher to explain the available observed LFs.

Thus, we can conclude that there is an increase in \( G_f \) as a function of \( z \) but to get the actual amount we need lot more observations at \( z > 5 \). Recent results from the narrow band survey of LAEs at \( z \sim 4.7 \) by Shioya et al. (2009) are also consistent with increasing value of \( G_f \) with increasing \( z \). As Lyman \( \alpha \) escape depends on the amount of dust and gas kinematics, the higher value of \( G_f \) implies that on an average the ISM of \( z > 5 \) galaxies are less dusty, more clumpy and having complex velocity field making the escape of Lyman \( \alpha \) photons easier.

Further, the evolution in the observed Lyman \( \alpha \) LF at \( z \geq 5.7 \) can be understood as evolution in the number density of the dark matter haloes arising from the structure formation model with modest change in the physical properties of these galaxies. This is independent of the horizont mass function we assume. Dijkstra et al. (2007a) have arrived at same conclusion in the evolution of LFs for \( z \geq 5.7 \) while considering no evolution in the IGM transitivity in this redshift range.

Our best-fitting models at different redshifts show that average Lyman \( \alpha \) equivalent width decreases with increasing redshift. This is contrary to some preliminary observational results that suggest an increase of equivalent width with increasing redshift (Nilsson et al. 2009). This result needs to be confirmed with larger number of spectroscopic data. In our model, it is possible to get such a trend by allowing the initial stellar mass function to vary with redshift (see Fig. 1). Also, relaxing our assumption that the equivalent width is independent of galaxy mass will have some effect on the equivalent width distribution. Indeed such mass dependence of equivalent width distribution is indicated by observations of Ando et al. (2006).

There are a number of other attempts to fit the UV and Lyman \( \alpha \) LFs using galaxy formation models. Kobayashi et al. (2007) using their hierarchical galaxy formation models fitted the LF of LAEs by varying the escape fraction. However, according to their models all LBGs would be detected as LAEs. Mao et al. (2007) fitted LFs using semi-analytic models that compute \( E(B - V) \) and relate it to the escape fraction of Lyman \( \alpha \) photons. In this model, preferably low metallicity dust free galaxies will be seen as LAEs. However, recent observations suggest that the LAEs need not be confined to primordial low dust populations (Pentericci et al. 2009, also see Fynbo et al. 2003; Scannapieco, Schneider & Ferrara 2003; Dawson et al. 2007). Nagamine et al. (2008) used the hierarchical structure formation models to fit the LAEs assuming a normal galaxy is a LAE for a brief period of time (duty cycle argument). They find the duty cycle increases with increasing redshift as we find for \( G_f \).

It is important to realize that high redshift LFs are based on deep field observations covering small volumes. The effect of cosmic variance may be large. The UV LFs used here for LBGs are mainly based on photometric data with large redshift uncertainty. Therefore, more observations are needed to get a clearer picture on the evolution of physical properties of the galaxies. In the case of modelling, one requires a clear physical model for \( G_f \). It is possible that simple ideas of duty cycle based on dust properties may not be sufficient since the velocity field in the Lyman \( \alpha \) emitting regions may play an important role. Indeed, all the high-z LBGs show signatures of outflows that can enable easy transport of Lyman \( \alpha \) photons.
Thus, physical understanding of $G_f$ based on a dynamical model (e.g. Verhamme et al. 2008) that will also fit the LFs is the next step in this subject. Such models may also explain the observed wide spread in the rest equivalent width distribution.

ACKNOWLEDGMENTS

We thank an anonymous referee for useful comments that has helped in improving our paper. We thank Masami Ouchi for providing data on Lyman α LFs at $z = 3.1, 3.7$ and $5.7$ as well as some useful discussion. We also thank Nobumari Kashikawa for providing the observational data at $z = 6.5$. SS thanks CSIR, India for the grant award No. 9/545(23)/2003-EMR-I.

REFERENCES

Ajiki M. et al., 2003, AJ, 126, 2091
Ando M., Ohta K., Iwata I., Akiyama M., Aoki K., Tamura N., 2006, ApJ, 645, 9
Benson A. J., Lacey C. G., Baugh C., Cole S., Frenk C. S., 2002, MNRAS, 333, 156
Best P. N., Kaiser C. R., Heckman T. M., Kauffmann G., 2006, MNRAS, 368, L67
Bouwens R. J. et al., 2004, ApJ, 616, L79
Bouwens R. J., Illingworth G. D., Franx M., Ford H., 2007, ApJ, 670, 928
Bouwens R. J., Illingworth G. D., Franx M., Ford H., 2008, ApJ, 686, 230
Bower R. G., Benson A. J., Malbon R., Healy J. C., Frenk C. S., Baugh C. M., Cole S., Lacey C. G., 2006, MNRAS, 370, 645
Bromm V., Loeb A., 2002, ApJ, 575, 111
Cowie L. L., Hu E. M., 1998, AJ, 115, 1319
Chiu W. A., Ostriker J. P., 2000, ApJ, 534, 507
Dawson S., Rhoads J. E., Malhotra S., Stern D., Wang J., Dey A., Spinrad H., Jannuzi B. T., 2007, ApJ, 671, 1227
Dayal P., Ferrara A., Galleroni S., 2008, MNRAS, 389, 1683
Dijkstra M., Haiman Z., 2005, ApJ, 623, 627
Dijkstra M., Lazio A., Wyithe J. S. B., 2007a, MNRAS, 377, 1175
Dijkstra M., Wyithe J. S. B., Haiman Z., 2007b, MNRAS, 379, 253
Dow-Hygelund C. C. et al., 2007, ApJ, 660, 47
Dunkley J. et al., 2009, ApJS, 180, 306
Fan X. et al., 2006, AJ, 131, 1203
Finkelstein S. L., Rhoads J. E., Malhotra S., Grogin N., Wang J., 2008, ApJ, 678, 655
Finkelstein S. L., Rhoads J. E., Malhotra S., Grogin N., 2009, ApJ, 691, 465
Fynbo J. P. U. et al., 2003, A&A, 406, 36
Gronwall C. et al., 2007, ApJ, 667, 79
Haiman Z., Spaans M., 1999, ApJ, 518, 138
Haiman Z., Cen R., 2005, ApJ, 623, 627
Hansen M., Oh S. P., 2006, MNRAS, 367, 979
Hopkins A. M., Beacom J. F., 2006, ApJ, 651, 142
Hu E. M., Cowie L. L., McMahon R. G., 1998, ApJ, 502, L99
Hu E. M., Cowie L. L., Capak P., McMahon R. G., Hayashino T., Komiyama Y., 2004, AJ, 127, 563
Kashikawa N. et al., 2006, ApJ, 637, 631
Kobayashi M. A. R., Totani T., Nagashima M., 2007, ApJ, 670, 919
Le Delliou M., Lacey C., Baugh C. M., Guiderdoni B., Bacon R., Courtois H., Soubse T., Morris S. L., 2005, MNRAS, 357, L11
Le Delliou M., Lacey C. G., Baugh C. M., Morris S. L., 2006, MNRAS, 365, 712
Leitherer C. et al., 1999, ApJS, 123, 3
Malhotra S., Rhoads J. E., 2002, ApJ, 565, 71
Malhotra S., Rhoads J. E., 2004, ApJ, 617, 5
Mao J., Lapi A., Granato G. L., de Zotti G., Danese L., 2007, ApJ, 667, 655
Murayama T. et al., 2007, ApJS, 172, 523
Nagamine K., Ouchi M., Springel V., Hernquist L., 2008, preprint (arXiv:0802.0228)
Neufeild D. A., 1991, ApJ, 370, 85
Nilsson K. K., Tapken C., Möller P., Freudling W., Fynbo J. P. U., Meisenheimer K., Laursen P., Oetlin G., 2009, A&A, 498, 13
Osterbrock D. E., 1989, Astrophysics of Gaseous Nebulae and Active Galactic Nuclei. University Science Books, Mill Valley, CA
Ota K. et al., 2008, ApJ, 677, 12
Ouchi M. et al., 2003, ApJ, 582, 60
Ouchi M. et al., 2008, ApJS, 176, 301
Pentericci L., Grazian A., Fontana A., Castellano M., Giallongo E., Salimbeni S., Santini P., 2009, A&A, 494, 553
Press W. H., Schechter P., 1974, ApJ, 187, 425
Reddy N. A., Steidel C. C., 2009, ApJ, 692, 778
Reddy N. A., Steidel C. C., Pettini M., Adelberger K. L., Shapley A. E., Erb D. K., Dickinson M., 2008, ApJ, 175, 48
Reddy N. A., Steidel C. C., Fadda D., Yan L., Pettini M., Shapley A. E., Erb D. K., Adelberger K. L., 2006, ApJ, 644, 792
Rhoads J. E., Malhotra S., Dey A., Stern D., Spinrad H., Jannuzi B. T., 2000, ApJ, 545, L85
Rhoads J. E. et al., 2003, ApJ, 125, 1006
Richard R., Pello R., Schaerer D., Le Borgne J. F., Kneib J. P., 2006, A&A, 456, 861
Samui S., Srianand R., Subramanian K., 2007, MNRAS, 377, 285 (Paper I)
Samui S., Subramanian K., Srianand R., 2009, New Astron., 14, 591 (Paper II)
Sasaki S., 1994, PASJ, 46, 427
Scannapieco E., Schneider R., Ferrara A., 2003, ApJ, 589, 35
Shapley A. E., Steidel C. C., Pettini M., Adelberger K. L., 2003, ApJ, 588, 65
Sheth R. K., Tormen G., 1999, MNRAS, 308, 119 (ST)
Shimasaku K. et al., 2006, PASJ, 58, 313
Shioya Y. et al., 2009, ApJ, 696, 546
Stark D. P., Loeb A., Ellis R. S., 2007, ApJ, 668, 627
Steidel C. C., Adelberger K. L., Shapley A. E., Pettini M., Dickinson M., Giavalisco M., 2000, ApJ, 532, 170
Stern D., Yost S. A., Eckart M. E., Harrison F. A., Helfand D. J., Djorgovski S. G., Malhotra S., Rhoads J. E., 2005, ApJ, 619, 12
Taniguchi Y. et al., 2005, PASJ, 57, 165
Thommes E., Meisenheimer K., 2005, A&A, 430, 877
Thoul A. A., Weinberg D. H., 1996, ApJ, 465, 608
Verhamme A., Schaerer D., Atek H., Tapken C., 2008, A&A, 491, 89

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