A hierarchical search for gravitational waves from supermassive black hole binary mergers

I W Harry, S Fairhurst and B S Sathyaprakash

Cardiff School of Physics and Astronomy, Cardiff University, Queens Buildings, The Parade, Cardiff CF24 3AA, UK
E-mail: ian.harry@astro.cf.ac.uk, Stephen.Fairhurst@astro.cf.ac.uk and B.Sathyaprakash@astro.cf.ac.uk

Received 19 April 2008, in final form 11 August 2008
Published 2 September 2008
Online at stacks.iop.org/CQG/25/184027

Abstract

We present a method to search for gravitational waves from coalescing supermassive binary black holes in LISA data. The search utilizes the $F$-statistic to maximize over, and determine the values of, the extrinsic parameters of the binary system. The intrinsic parameters are searched over hierarchically using stochastically generated multi-dimensional template banks to recover the masses, sky location and coalescence time of the binary. We present the results of this method applied to the Mock LISA Data Challenge 1B data set.

PACS numbers: 04.25.Nx, 04.30.Tv, 04.80.Nn, 95.85.Sz

1. Introduction

There is growing evidence that some fraction of quasars [1], and x-ray and infrared sources [2] host supermassive binary black holes (SMBBH) that are potential sources of gravitational radiation. The late time evolution of such systems is dominated by the emission of gravitational waves, the radiation back reaction torque driving the system to coalesce. The Laser Interferometer Space Antenna (LISA) [3] targets gravitational waves from these systems in the frequency range of $[\text{few} \times 10^{-5}, 0.1] \text{ Hz}$ which corresponds to SMBBH of masses in the range $[10^4, \text{few} \times 10^7] M_\odot$. Within a redshift of $z \sim 10$, SMBBH coalescence rates could be as high as several tens per year but depending on the way galaxies and black holes at their cores formed the rates could be several hundreds per year [4, 5]. Even at redshifts $z \sim 10$ mergers can be detected by LISA, expected amplitude signal-to-noise ratio (SNR) being in excess of several thousands for sources at a redshift of $z = 1$, implying an SNR of $\sim 10$ even at redshifts of $z \simeq 10$. LISA is, therefore, an excellent probe of the seed black holes that are believed to be responsible for the formation and evolution of galaxies [4–6] and the large-scale structure in the Universe and it is important to be able to detect SMBBH mergers at as low an SNR as $\sim 10$. 


0264-9381/08/184027+11$30.00 © 2008 IOP Publishing Ltd Printed in the UK
SMBBH mergers from redshifts up to about $z \sim 3$ can be detected without any sophisticated data analysis, although accurate models of the merger dynamics would be needed for parameter extraction. Indeed, these sources will be so bright that one has to worry about systematics due to our limited theoretical understanding of their dynamics [7]. At larger redshifts, however, it would be necessary to employ data analysis techniques that are sensitive to weaker signals. This is an important goal for LISA as there is significant uncertainty in when the first seed black holes and galaxies might have formed and it would be good to be able to probe as far back as a redshift of $z \sim 10–15$. At a redshift of $z$, an SMBBH of intrinsic total mass $M$ would appear in LISA to have a red-shifted total mass of $(1 + z)M$. Thus, at $z = 10$ LISA would probe masses that are intrinsically 11 times smaller than at $z = 0$. Therefore, searching for SMBBH at higher redshifts would probe smaller masses too.

In addition to SMBBH mergers LISA will observe a host of other sources (see [8] and references therein). These include binary white dwarfs in the Milky Way (both a stochastic signal from an unresolved background population and continuous signals from resolved foreground sources), inspirals of small black holes into supermassive black holes (again a stochastic background from overlapping sources and a foreground of individual sources), etc. Analysing LISA data and resolving tens of thousands of signals belonging to different classes is unprecedented and likely to be a daunting task. Matched filtering is a very powerful approach that has been successfully used in several applications to dig weak signals from noisy backgrounds. For example, matched filtering using a bank of templates has been extensively used in searching for gravitational wave signals in ground-based detectors, see for example [9, 10]. In this paper we report the results from a hierarchical matched filtering algorithm to search for SMBBH mergers.

From a computational point of view, however, matched filtering is very expensive, computational cost increasing as a power-law of the number of search parameters. While the parameter space of stellar mass binaries consisting of non-spinning black holes is only two-dimensional, the number of parameters in the case of SMBBH, even while neglecting spins, is quite large. This is because the source’s position relative to LISA changes during the course of observation, causing a modulation in the signal’s amplitude and phase that must be taken into account in the search templates as well as the waveform’s polarization angle. Thus, the computational cost of a naive implementation of a matched filtered search would be formidable. For example if we were to conduct a one-stage match-filtered search on the MLDC 1B dataset to get parameter accuracies to similar levels to those we quote in our results section we estimate that you would require $10^{19}$ templates. Of course detecting a signal as loud as the ones used in the MLDC requires significantly less templates, and can be done by placing templates in mass space only. We therefore developed a hierarchical approach in which the goal was to zoom-in onto an interesting region of the parameter space in several steps, each of which uses a progressively greater density of templates. We tested our algorithm on the training and challenge data sets from Challenge 1B of the Mock LISA Data Challenge (MLDC)\(^1\).

In the SMBBH coalescence section of the data set the challenge was to detect and characterize one SMBBH coalescence buried in LISA instrumental noise only. Two datasets were released, one where the coalescence was in the middle of the observation period and a second where the coalescence was two months after the observation period ended. We only took part in the Challenge where the binary coalesced during the observation time.

---

\(^1\) The LISA International Science Team has put together a task force to develop a set of data analysis challenges [11] of ever increasing complexity [12] to encourage data analysts to explore and test their search algorithms on simulated data. The most recent release of challenge data sets was Challenge 1B, a rerun of the Challenge 1, consisting of the simplest possible data sets with only one inherent signal.
For our templates we used post-Newtonian waveforms at the second post-Newtonian order. We tapered the end point of our templates to prevent the bleeding of spurious power in the Fourier domain arising from the step function that is implicit if the waveform were to be terminated abruptly. The signal is characterized by nine independent parameters. We separate these into the ‘intrinsic’ parameters consisting of the two component masses, the binary’s position on the sky and its epoch of coalescence and ‘extrinsic’ parameters comprising the inclination angle, the polarization phase, the coalescence phase and the distance to the binary. We devised a search that was capable of determining all these parameters in an efficient manner, albeit not to accuracies that are theoretically possible. Let us note, however, that the goal of this exercise is not to measure the parameters accurately but to efficiently detect the signal and constrain the parameter space well enough so that other techniques, such as the Markov chain Monte Carlo, can be deployed in a follow-up study to determine the parameters more accurately.

Other groups have, of course, participated in the search for SMBBHs in the Mock LISA Data Challenges, and their methods differ from ours [11, 13–18]. The majority of these searches involve a variety of methods to detect the source and constrain somewhat its parameters followed by a Markov chain Monte Carlo follow-up to determine accurately all the parameters.

2. Search method

A search for supermassive black hole signals in the LISA data requires, in general, the determination of 17 parameters. In this paper, and in Challenge 1B of the MLDC, spins of the component black holes are ignored, restricting to non-spinning components. In addition, the orbit is assumed to be circularized sufficiently by the time it enters LISA’s sensitive band that eccentricity can be ignored. This allows us to neglect eight parameters leaving the parameters of interest to be:

- The masses of the two components of the binary, $M_1$ and $M_2$. It is often convenient to express the mass parameters in terms of the chirp mass $\mathcal{M}$ and reduced mass $\mu$, defined as
  \begin{equation}
  \mathcal{M} = (M_1 + M_2)^{2/3} \mu^{3/5} \quad \text{and} \quad \mu = \frac{M_1 M_2}{M_1 + M_2}.
  \end{equation}

- The time that the binary coalescences, $t_c$, which is assumed to be within the LISA data.
- The sky location of the binary, determined by its ecliptic latitude, $\beta$, and longitude, $\lambda$.
- The orientation of the binary system, given by the inclination angle, $\iota$, and polarization angle, $\psi$.
- The initial phase of the binary, $\phi_0$.
- The luminosity distance to the binary, $D_L$.

In this search, we make use of the ‘$\mathcal{F}$-statistic’ [19] to analytically maximize over four of the parameters introduced above (the ‘extrinsic parameters’): the distance to the binary, and its inclination, polarization and initial phase. This procedure is discussed in section 2.1. The remaining five parameters are determined by searching over stochastically generated template banks [20, 21]. Since some parameters, in particular the chirp mass and coalescence time, are more easily determined we employ a hierarchical search whereby we obtain good estimates of these parameters before refining our search to determine the full parameter set. The stochastic bank is described in section 2.2 and the hierarchical search method is discussed in 2.3. Results of this search applied to the MLDC 1B data are presented in section 3.
2.1. Matched filtering with the F-statistic

When the signal waveform is well known, the technique of matched filtering is typically used to search for the signal (see, for example, [22, 23]). The F-statistic is an elegant way to maximize over the extrinsic parameters, and thereby simplify the search. It is used in a number of other searches by other groups for the MLDC and in the pulsar search with ground-based detectors (for example [10, 24]). We quickly recap how it is used in this section. We follow closely [19] when describing this method. Given a template waveform \( h \) and the data \( s \), we calculate the likelihood function, defined as

\[
\ln \Lambda_1 = \langle s, h \rangle - \frac{1}{2} \langle h, h \rangle,
\]

where the inner product between the signal and template is given by

\[
\langle s, h \rangle = 4 \Re \left( \int_0^\infty \frac{\tilde{h}(f) \tilde{s}(f)}{S_h(f)} \, df \right),
\]

and \( S_h(f) \) is the power spectral density of the LISA detector.

For a signal present in the LISA data, it can be shown that the gravitational wave signal can be decomposed as [25]

\[
h(t) = \sum_{i=1}^{4} A_i(D_L, \varphi_o, \iota, \psi) \cdot h_i(t; t_c, M_1, M_2, \lambda, \beta).
\]

The amplitudes \( A_i \) are functions only of the extrinsic parameters: \( D_L, \varphi_o, \iota \) and \( \psi \). The \( h_i(t) \) are functions of the remaining intrinsic parameters only. The benefit of expressing the waveform in this manner is that it is straightforward to maximize the likelihood parameters over these \( A_i \) by requiring

\[
\frac{\partial \ln \Lambda_1}{\partial A_i} = 0.
\]

With a bit of algebra this can be shown to be equivalent to

\[
A_i = \sum_{j=1}^{4} M_{ij}^{-1} \langle s, h_j \rangle \quad \text{where} \quad M_{ij} = \langle h_i, h_j \rangle.
\]

Therefore, the \( A_i \) can be determined from the \( \langle h_i, h_j \rangle \) and \( \langle s, h_i \rangle \). Furthermore, for each possible set of values for \( A_i \) we obtain a unique value for the four extrinsic parameters: distance \( D_L \), initial phase \( \varphi_o \), inclination angle \( \iota \) and polarization angle \( \psi \). However, there remain implicit degeneracies in these values. Specifically, as we use only the dominant, 2\( \Phi \) harmonic in the waveform, there is a degeneracy in the initial phase corresponding to \( \varphi_o \to \varphi_o + \pi \). The same degeneracy exists for the polarization angle. Additionally, a system with polarization \( \psi \) and phase \( \varphi_o \) is indistinguishable from one with values \( \psi + \frac{\pi}{2} \) and phase \( \varphi_o + \frac{\pi}{2} \). Finally, by substituting the expression for \( A_i \) from (6), the likelihood expression becomes

\[
\ln \Lambda_1 = \frac{1}{2} \sum_{i,j=1}^{4} \langle s, h_i \rangle M_{ij}^{-1} \langle s, h_j \rangle.
\]

In the above discussion, we have used the gravitational wave strain \( h(t) \) in discussing the F-statistic. In the MLDC, the signals were released in the form of time delay interferometry (TDI) variables \( X, Y \) and \( Z \) [26, 27]. These TDI variables are used as a way of cancelling the laser phase noise in the output of LISA\(^2\). The F-statistic method is equally applicable to the laser phase noise.
TDI variables. To maximize the efficiency of our search method we simultaneously utilize two of the TDI outputs, $X$ and $Y$, to conduct our search. We do not use the $Z$ output since the gravitational wave content in it can be constructed from the other two and is therefore not independent.

It is a trivial matter to convert the one-detector search outlined above to a two-detector TDI search. We simply rewrite our likelihood function as

$$
\ln \Lambda = \langle s_X, h_X \rangle + \langle s_Y, h_Y \rangle - \frac{1}{2} \langle h_X, h_X \rangle - \frac{1}{2} \langle h_Y, h_Y \rangle, \tag{8}
$$

where the subscripts $X$ and $Y$ denote the data or template appropriate for either the $X$ or $Y$ TDI data stream$^3$. The $\mathcal{F}$-statistic maximization can similarly be extended to the two-detector search. In this case, the expressions in (6) and (7) generalize to include a summation over detector. For example,

$$
\mathcal{M}_{ij} = \langle h_{i,X}, h_{j,Y} \rangle + \langle h_{i,Y}, h_{j,Y} \rangle. \tag{9}
$$

2.2. Stochastically generated template bank

Even after maximizing over the ‘extrinsic’ variables, there are still five remaining, ‘intrinsic’ parameters that we would like to determine. We utilize a template bank to search over this five-dimensional parameter space [28, 29]. Existing templated searches for gravitational waves from binary coalescences in ground-based detectors utilize a two, or at most three, dimensional parameter space. Geometrical placement algorithms exist [30, 31] to deal with the problem of efficiently placing templates in these parameter spaces. However, when the parameter space becomes higher dimensional we have two problems with using these geometric placement methods. Firstly, there is no known optimal placement algorithm for dimensions higher than two. It has been shown [35] that placing a square lattice of templates becomes grossly inefficient in higher dimensions. Additionally, when the signal manifold becomes curved it is unclear how to construct these geometrical lattices. The signal manifold for the SMBBH search in LISA data suffers from both of these issues. Therefore, we use the method outlined in [20, 21] and create stochastically generated template banks. Other implementations of randomly generated template banks can be found in [35, 36].

The final stochastic bank is designed so that for any signal in the parameter space, at least a fraction $M$ of the potential signal power is recovered. This is most easily understood by introducing the notion of overlap between two templates as

$$
\mathcal{O}(h_1, h_2) = \frac{|h_1 \cdot h_2|}{|h_1||h_2|}, \tag{10}
$$

where the norm of the template is defined as $|h|^2 = \langle h, h \rangle$. When the two templates are identical, the overlap will be unity. Then, given any signal $s$ in the parameter space, we require that

$$
\text{Max}_I(\mathcal{O}(s, h_I)) \geq M, \tag{11}
$$

where $I$ labels the templates in the bank. The parameter $M$ is known as the minimal match.

We begin by choosing a randomly generated set of densely spaced points in the parameter space that are the candidate templates, see figure 1. There are significantly more initial points than would be needed to cover the space with an appropriate density of templates to

---

$^3$ Strictly speaking, this expression is incorrect for the $X$ and $Y$ channels as the noise in them is correlated. It is, of course, preferable to use the synthetic $A$ and $E$ variables which are generated from $X, Y, Z$ and are independent. Due to time constraints, for this challenge we did not get around to moving the code over to $A, E$ and $T$. This has since been implemented.
Figure 1. The left-hand figure shows an initial set of over-dense templates which are placed stochastically in an arbitrary flat parameter space (where the parameter space metric is the identity). These are subsequently filtered to reduce the number, demanding a maximum overlap between templates of 0.995. The right-hand figure shows the final template bank generated by the stochastic bank placement procedure. The shaded circles surrounding each template are the regions of parameter space which are covered by the template. As can be seen, the vast majority of the space is adequately covered. The axes here are arbitrary.

We subsequently go through the candidate templates and remove those which are superfluous.

More specifically, we select an initial template and remove any other templates from the bank which are redundant as they are too close to the initial template. This is simplified by calculating the metric on the parameter space,

$$g_{ij} = \left[ \frac{\partial \hat{h}}{\partial \theta_i}, \frac{\partial \hat{h}}{\partial \theta_j} \right], \quad \hat{h} = \frac{h}{|h|},$$

(12)
at the location of the initial template (whose parameters we denote by $\theta$). Then, we can approximate the overlap between this template and any other in the bank as

$$O(h(\theta), h(\theta + \delta \theta)) \approx 1 - \frac{1}{2} g_{ij}(\delta \theta)^i (\delta \theta)^j.$$ (13)

If the overlap is greater than the required minimum, the second template is discarded. Having tested every template, we then move on to one of the surviving templates and repeat the procedure, again discarding templates which are too close to the selected template. This process is repeated until all templates have been tested. The method is efficient since the costly process of computing the metric is only performed at the location of surviving templates, not at the location of all initial templates.

The final template bank will cover the majority of the space to the desired accuracy. However, since the initial process of placing points is stochastic, we cannot guarantee that the entire space will necessarily be covered appropriately. To try to be sure that the parameter space is adequately covered we choose a number of initial seed points, which we believe would be enough to cover the parameter space and after the bank has been generated we generate a further 20 000 seed points and test their overlap with the rest of the bank. The percentage of these seed points that have an overlap with the template bank of more than the required
minimum can then be used as a measure of how well the parameter space is covered. If the coverage is not sufficient we can generate more seed points and extend the template bank. An alternative method would be to, instead of using a predetermined number of initial seed points, run the template bank generation until a specific number of seed points have been rejected concurrently, we would then consider this template bank adequate [20]. Figure 1 shows an example template bank that results from this procedure. Using this method we are left with a stochastically generated template bank that is capable of covering any parameter space in any number of dimensions.

While the stochastic bank generation is generic, there are certain subtleties which arise in employing it for the $F$-statistic search for SMBBH described in section 2.1. First note that, in contrast to searches for binaries in ground-based detectors, we must include the coalescence time when generating the template bank. Binary coalescence signals in ground-based detectors last at most $\sim 1000$ s, during which the motion of the Earth, and the detector, can be neglected. Hence, the waveforms of binaries with different coalescence times differ only by a time-shift and amplitude rescaling. However, SMBBH signals spend several months in LISA’s sensitive band, during which LISA completes a significant fraction of an orbit around the Sun. Consequently, the template shape depends on the coalescence time, and this parameter must be included in the template bank.

Next, we consider the effect of maximization over the four extrinsic parameters in the $F$-statistic. This is dealt with by generating a metric on the full parameter space and projecting down to the five-dimensional subspace (see [30] for details). A complication arises in that the projected metric depends upon the value of three of the extrinsic parameters $\iota$, $\psi$ and $\phi_0$. This is a well-known issue, see for example [33]. To proceed, we simply choose a fiducial value of 0.5 radians for these angles. The value was chosen arbitrarily, ensuring that none of the four $A_i$ values was zero and they would all contain contributions from both gravitational-wave polarizations.

To generate the metric, we calculated the inner product (3) for the $X$-detector using gravitational-wave strain $h(t)$ rather than the TDI variables. This introduces two additional approximations. First, by using the strain, rather than the TDI variables, we are neglecting the directional dependence of the detector’s response function and implicitly working at the long-wavelength approximation. Second, we have performed the search using both the $X$ and $Y$ data streams while only the metric for $X$ was used to generate the template bank. The above simplifications will mean that the stated minimal match of the metric would not have been achieved. However, in performing the search, as described in section 2.3, we continually refined the template bank to determine the correct parameter values and did not rely on the minimal match to decide stopping conditions.

2.3. Hierarchical search technique

Populating even the reduced, five-dimensional parameter space with sufficient templates to determine the binary’s parameters to the required accuracy would necessitate far more templates than could feasibly be filtered (as discussed in the introduction). Thus we must employ a hierarchical method to search for the parameters.

We began by match-filtering the data against a bank comprised of templates that are sparsely spaced and placed in only the two-dimensional space of mass parameters. This bank enabled us to make an initial estimate of the binary’s masses and coalescence time with 1000 templates in the allowed range of masses, setting the sky location arbitrarily to $\lambda = 0.5$

---

4 It is immediate from the definition of the metric (12) that the distance, $D_L$, will not affect the metric at all and can safely be neglected.
and $\beta = 0.5$ for all templates and fixing the coalescence time to be the value at the beginning of the allowed range. This enabled us to estimate the chirp mass and reduced mass to within 30% accuracy and coalescence time to within 10 000 s.

We then placed a second bank of 1000 templates within a reduced range of the parameter space, using the best estimate of the coalescence time, sky locations again set to $\lambda = \beta = 0.5$ and repeat the process. By this method we could estimate the chirp mass to at least ±5%, the reduced mass to at least ±10% and the coalescence time to within 10 000 s. Using these initial estimates we were then able to place a template bank with restricted parameter ranges to determine all five of the ‘intrinsic’ parameters.

The final step in determining the parameters could be performed by two different methods. The first method involves placing a template bank over the full five-dimensional parameter space and using a hierarchical procedure to ‘zoom in’ on the true values of the binary’s parameters. While this is the preferred search method, a large number of templates are still required to fill this reduced five-dimensional template space, to do this in one step would require $10^{13}$ templates. We would thus have to use a hierarchical procedure to construct a series of five-dimensional banks, but this search can still become computationally costly. An alternative technique is to alternate between placing two-dimensional template banks in the mass space, using the best current estimates of coalescence time and sky location, and placing three-dimensional banks in sky location and coalescence time, using the best current estimates for the masses. This method is computationally quicker as we limit the template bank size to under 1000 templates for the two-dimensional case and under 10 000 templates for the three-dimensional banks. However, much more than when using five-dimensional banks, care must be taken to avoid ‘zooming in’ on secondary maxima. For example, LISA has similar sensitivity to binary systems on opposite sides of the sky, so restricting the range of sky locations used in our template bank searches is not trivial.

The figures quoted above for template bank size and parameter accuracy are those for the binary systems in the MLDC datasets where the SNR is very large (approx. 500). For SMBBH systems where the SNR is significantly lower the main issue would be whether any templates at the initial stage were similar enough to the signal to pick it up. If so, the parameter accuracies at this stage would be similar as they are limited by the template spacing. Further investigation is warranted to determine what strength of signals can be detected by this method, how many more templates are needed at initial stage to detect weaker signals and how final parameter accuracy depends on SNR.

In future searches using this method it would be desirable to automate the hierarchical technique. To do so, we would need to quantify how many iterations are needed to adequately determine the parameters and how much each iteration reduces the possible range of values for each parameter. Although this method is still under development, it is interesting to note that it uses a comparable number of templates as the MCMC search implemented in [18]. It is also worth noting that in a template-bank-based search it is straightforward to parallelize the search over numerous computers.

3. Results

The MLDC Challenge 1B data set for SMBBH consists of one year of simulated LISA data with a single supermassive binary black hole coalescence occurring during the year. In addition, a ‘training’ data set was released for which the binary’s parameters were also made public. Due to unforeseen technical issues we were unable to run as full an analysis as we would have liked on the challenge dataset, and our results reflect this. Therefore, we have also included the results from the training run, as they provide a more accurate reflection of
Table 1. Table showing the results of our analysis on a training dataset.

| Parameter                              | True value     | Our value      | Error   | Fract. error |
|----------------------------------------|----------------|----------------|---------|--------------|
| Chirp mass, $M$ ($M_\odot$)            | $1.3769 \times 10^6$ | $1.3772 \times 10^6$ | 360     | $2.6 \times 10^{-4}$ |
| Symmetric mass ratio, $\eta$           | 0.1959         | 0.1972         | 0.0013  | –            |
| Ecliptic latitude, $\beta$             | 1.028          | 1.072          | 0.044   | –            |
| Ecliptic longitude, $\lambda$          | 5.050          | 5.037          | 0.013   | –            |
| Coalescence time, $t_c$ (s)            | 17 523 096.4   | 17 523 090.6   | 6.4     | –            |
| Polarization angle, $\psi$             | 0.826          | 0.668          | 0.158   | –            |
| Inclination angle, $i$                 | 2.846          | 2.313          | 0.533   | –            |
| Initial phase, $\phi_0$                | 1.844          | 1.836          | 0.048   | –            |
| Luminosity distance, $D_L$ (Gpc)        | 36.3           | 26.6           | 9.6     | 0.27         |

Table 2. Table showing the results of our analysis on the official challenge dataset.

| Parameter                              | True value     | Our value      | Error   | Fract. error |
|----------------------------------------|----------------|----------------|---------|--------------|
| Chirp mass, $M$ ($M_\odot$)            | $2.6832 \times 10^6$ | $2.6904 \times 10^6$ | 7178.8  | $2.68 \times 10^{-3}$ |
| Symmetric mass ratio, $\eta$           | 0.2159         | 0.2316         | 0.0158  | –            |
| Ecliptic latitude, $\beta$             | 1.139          | -0.235         | 1.374   | –            |
| Ecliptic longitude, $\lambda$          | 3.931          | 3.382          | 0.549   | –            |
| Coalescence time, $t_c$ (s)            | 15 045 887.8   | 15 046 429.6   | 541.2   | –            |
| Polarization angle, $\psi$             | 6.063          | 5.941          | 0.123   | –            |
| Inclination angle, $i$                 | 1.939          | 1.252          | 0.687   | –            |
| Initial phase, $\phi_0$                | 0.213          | 1.031          | 0.818   | –            |
| Luminosity distance, $D_L$ (Gpc)        | 10.7           | 26.0           | 15.3    | 1.43         |

the sensitivity of our current search technique. The released training data parameters were not used in running the search, as it was treated as a warm up to the challenge. For both training and challenge results we have taken into account the parameter degeneracies discussed in section 2.1 by choosing the values of polarization and initial phase that are closest to the true values.

The results from the training data set are presented in table 1, while the Challenge results are shown in table 2. It is interesting to compare our results to those obtained by other groups applying different methods to search for SMBBH coalescences in the Mock LISA Data Challenge [11, 13–18]. It is clear that our Challenge results are substantially less accurate, for reasons described above. However, our results from the training data set are comparable to those obtained using other methods. In particular, it is gratifying to see that we were able to obtain the correct sky location. Furthermore, the sky location is recovered to within a few square degrees, which is the accuracy required to make an optical follow-up feasible (see, for example, [34]).

4. Summary and future plans

We have presented a hierarchical, template-based search method for SMBBH in LISA data. This method makes use of the $F$-statistic to reduce the parameter space for non-spinning black holes from nine to five dimensions, and then employs a stochastically generated template bank to search over the remaining parameter space. This method has been applied to perform a search on the data from Challenge 1B of the MLDC. We were able to successfully locate the signal and, in the case of the training data, recover its parameters with good accuracy.
In the future, we will continue our participation in the Mock LISA Data Challenges. Challenge 3 has already been started and includes an SMBBH data set where spin effects have been included in the waveform [14]. In order to participate, we must develop an analysis technique to be able to search for inspiralling supermassive black holes with spin. Initially, we want to investigate how effectively we are able to search for spinning binaries with non-spinning templates and see if this approach might enable us to get a good estimate of the masses and coalescence time of the binaries. However, to obtain good parameter estimates, we will need to incorporate the effects of spin into our signal model. Unfortunately, the $\mathcal{F}$-statistic is not directly applicable due to the added complications spinning binaries bring. We will either have to develop a new technique to analytically maximize over some of the parameters, or be forced to place templates in a much higher dimensional signal manifold.

Acknowledgments

We would like to acknowledge many useful discussions with Curt Cutler, Michele Vallisneri and Jeff Crowder. This work has been supported by an SFTC grant (IWH and BSS) and the Royal Society (SF).

References

[1] Valtonen M J et al 2008 A massive binary black-hole system in OJ 287 and a test of general relativity Nature 452 851–3
[2] Komossa S et al 2003 Discovery of a binary AGN in the ultraluminous infrared galaxy NGC 6240 using Chandra Astrophys. J. 582 L15–20
[3] Bender P L 1995 LISA: Laser Interferometer Space Antenna for the detection and observation of gravitational waves: Pre-phase: A Report (unpublished)
[4] Rees M J and Volonteri M 2006 Massive black holes: formation and evolution Proc. IAU Symp. 251–8
[5] Sesana A, Volonteri M and Haardt F 2007 The imprint of massive black hole formation models on the LISA data stream Mon. Not. R. Astron. Soc. 377 1711–6
[6] Volonteri M, Haardt F and Madau P 2003 The assembly and merging history of supermassive black holes in hierarchical models of galaxy formation Astrophys. J. 582 559–73
[7] Cutler C and Vallisneri M 2007 LISA detections of massive black hole inspirals: parameter extraction errors due to inaccurate template waveforms Phys. Rev. D 76 104018
[8] LISA Mission Science Office 2007 LISA: probing the universe with gravitational waves (www.lisascience.org/resources/talks-articles/science)
[9] Abbott B et al 2008 Search for gravitational waves from binary inspirals in S3 and S4 LIGO data Phys. Rev. D 77 062002
[10] Abbott B et al 2008 The Einstein@Home search for periodic gravitational waves in LIGO S4 data Preprint arXiv:0804.1747
[11] Arnaud K A et al 2007 Report on the first round of the Mock LISA Data Challenges Class. Quantum Grav. 24 S529–40
[12] Arnaud K A et al 2007 An overview of the second round of the Mock LISA Data Challenges Class. Quantum Grav. 24 S551–64
[13] Babak S et al 2008 Report on the Second Mock LISA Data Challenge Class. Quantum Grav. 25 114037
[14] Babak S et al 2008 The Mock LISA Data Challenges: from Challenge 1b to Challenge 3 Class Quantum Grav. 25 184026
[15] Cornish N J and Porter E K 2007 Searching for massive black hole binaries in the first Mock LISA Data Challenge Class. Quantum Grav. 24 S501–11
[16] Brown D A, Crowder J, Cutler C, Mandel I and Vallisneri M 2007 A three-stage search for supermassive black hole binaries in LISA data Class. Quantum Grav. 24 S595–606
[17] Camp J B, Cannizzo J K and Numata K 2007 Application of the Hilbert–Huang transform to the search for gravitational waves Phys. Rev. D 75 061101
[18] Cornish N J and Porter E K 2007 The search for massive black hole binaries with LISA Class. Quantum Grav. 24 5729–55
[19] Schutz B F, Jaranowski P and Królak A 1998 Data analysis of gravitational-wave signals from spinning neutron stars: The signal and its detection Phys. Rev. D 58 063001
[20] Harry I, Sathyaprakash B S and Allen B A stochastic template bank for gravitational wave data analysis, in preparation
[21] Van Den Broeck C, Brown D, Harry I, Sathyaprakash B S, Tagoshi H and Takahashi H 2008 Template banks to search for compact binaries with spinning components in gravitational wave data, in preparation
[22] Wainstein L A and Zubakov V D 1962 Extraction of Signals From Noise (Englewood Cliffs, NJ: Prentice-Hall)
[23] Finn L S and Chernoff D F 1993 Observing binary inspiral in gravitational radiation: one interferometer Phys. Rev. D 47 2196–219
[24] Whelan J T, Prix R and Khurana D 2008 Improved search for galactic white dwarf binaries in Mock LISA Data Challenge 1B using an F-statistic template bank Class. Quantum Grav. 25 184029
[25] Krolak A, Tinto M and Vallisneri M 2004 Optimal filtering of the LISA data Phys. Rev. D 70 022003
[26] Tinto M, Estabrook F B and Armstrong J W 2002 Time-delay interferometry for LISA Phys. Rev. D 65 082003
[27] Vallisneri M 2005 Geometric time delay interferometry Phys. Rev. D 72 042003
[28] Owen B 1996 Search templates for gravitational waves from inspiraling binaries: choice of template spacing Phys. Rev. D 53 6749–61
[29] Owen B and Sathyaprakash B S 1998 Matched filtering of gravitational waves from inspiraling compact binaries: computational cost and template placement Phys. Rev. D 60 022002
[30] Babak S, Balasubramanian R, Churches D, Cokelaer T and Sathyaprakash B S 2006 A template bank to search for gravitational waves from inspiralling compact binaries: I. Physical models Class. Quantum Grav. 23 S577–504
[31] Cokelaer T 2007 Gravitational waves from inspiralling compact binaries: hexagonal template placement and its efficiency in detecting physical signals Phys. Rev. D 76 102004
[32] Messenger C and Prix R A random template placement strategy in n-dimensions, in preparation
[33] Prix R 2007 Search for continuous gravitational waves: metric of the multi-detector F-statistic Phys. Rev. D 75 022004
[34] Tyson J A 2002 Large synoptic survey telescope: overview Proc. SPIE Int. Soc. Opt. Eng. 4836 10–20
[35] Prix R 2007 Template-based searches for gravitational waves: efficient lattice covering of flat parameter spaces Class. Quantum Grav. 24 S481–90
[36] Babak S 2008 Building a stochastic template bank for detecting massive black hole binaries Preprint arXiv:0801.4070