A Liquid Spring with High-static-low-dynamic Stiffness

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Abstract. The conventional multi-rate spring with high-static-low-dynamic (HSLD) stiffness may not apply to massive payload low frequency vibration isolation due to the poor lateral stability of the spring with pre-compression. The new type of multi-rate spring refers to as a kind of liquid spring consists of a hydraulically driven spring and several pre-compressed slave springs. The payload is supported by the piston of the liquid spring, and the excitation force from the payload is transferred to the pistons of slave springs by hydraulic pressure. The liquid spring shows an HSLD stiffness property. It is found that the proper design of pistons’ cross-section area and the number of slave springs of the liquid spring can achieve an ideal piecewise linear stiffness property. Moreover, the slave spring owns better lateral stability under the condition that both the conventional multi-rate spring and the liquid spring have the same stiffness property. The lateral stability of a pre-compressed linear spring and the stiffness model of the liquid spring have been investigated.

1. Introduction
An isolator with HSLD stiffness may overcome the dilemma that low natural frequency and small static displacement cannot be achieved at the same time for a linear spring. Many passive nonlinear isolators with HSLD stiffness have been developed recently. Carrella[1,2] applied oblique springs and attracting magnets as two types of negative stiffness elements, and parallel combined them with vertical spring to achieve two typical isolators with HSLD stiffness. Huang[3,4] utilized Euler-bulked beams as the negative stiffness element. Zhou[5,6] invented cam–roller–spring mechanisms as the negative element. Those additional mechanisms providing negative stiffness are proved benefit from the vibration isolation performance, while those complex mechanisms may need more spaces. Furthermore, the nonlinear stiffness properties of those nonlinear isolators may produce harmful nonlinear dynamic phenomena even the payload under a small excitation force.

The conventional multi-rate spring can be designed to possess a piecewise linear HSLD stiffness. As a result, the vibration isolation system applying the conventional multi-rate spring can be regarded as a linear system when the excitation force has not reached the critical value. However, the conventional multi-rate spring may not apply to massive payload low frequency vibration isolation due to the poor lateral stability of the spring with pre-compression.

The liquid spring mentioned in this paper is also a multi-rate spring characteristic of piecewise linear HSLD stiffness. And the liquid spring also utilizing pre-compressed spring, while the lateral stability of the pre-compressed linear spring can be designed to be better than the conventional counterpart. In other words, the liquid spring is potentially applying to massive payload low frequency vibration isolation. Moreover, the liquid spring can achieve rich structures by taking advantage of Pascal’s principle and the fluid connectivity.
2. Multi-rate spring with HSLD stiffness

2.1. The conventional multi-rate spring
The schematic diagram of the conventional multi-rate spring[7] with HSLD stiffness is shown in Figure 1 (a). A linear spring with high stiffness $K_0$ is series connection with a pre-compressed linear spring with low stiffness $K_1$. When the external force $F$ is smaller than the pre-compressed force $F_1$, the conventional multi-rate spring shows a high static stiffness. Otherwise, the conventional multi-rate spring presents a series stiffness of these two linear spring. Therefore, the conventional multi-rate spring shows a piecewise linear stiffness property. According to Figure 1 (a), the force-displacement relationship is given by

$$F = \begin{cases} 
K_0 \cdot x & F \leq F_1 \\
\frac{K_0 \cdot K_1}{K_0 + K_1} \cdot x & F_1 < F 
\end{cases} \quad (1)$$

Define the working stiffness as

$$K_w = \frac{K_0 \cdot K_1}{K_0 + K_1} \quad (2)$$

Then the stiffness property of the conventional multi-rate spring is shaped like Figure 1 (b).

![Conventional Multi-rate Spring](image)

**Figure 1.** Conventional multi-rate spring, (a) schematic diagram, (b) stiffness property.

2.2. The liquid spring with one slave spring
The schematic diagram of the liquid spring with one slave spring is plotted in Figure 2. The primary piston with diameter $r_1$ is subject to the external force $F$. The slave piston with diameter $r_2$ is subject to a pre-compressed force $F_2$ provided by the linear slave spring with stiffness $K_2$. The primary piston and the slave piston are communicated by the liquid. Before the slave spring operating, the stiffness of the liquid spring can be designed to be equal to $K_0$ by proper arrangement of the type of the liquid, the volume of the liquid and the size of the primary piston according to hydrostatics. While when the external force can push the slave piston moving, the movement of the primary piston not only contributed by the compressibility of the liquid but also contributed by the movement of the slave piston. Therefore, the liquid spring presents a softer spring. Theoretically, the force-displacement relationship of the liquid spring can be derived as

$$F = \begin{cases} 
K_0 \cdot x & F \leq F_2 \cdot \mu^2 \\
\frac{K_0 \cdot K_2 \cdot \mu^4}{K_0 + K_2 \cdot \mu^4} \cdot x & F_2 \cdot \mu^2 < F 
\end{cases} \quad (3)$$

where $\mu = r_1 / r_2$ is the diameter ratio between the primary piston and the slave piston.
To obtain the same stiffness property shown in Figure 1 (b) as the conventional multi-rate spring, the configuration of the liquid spring should follow the bellows relationship with the configuration of the conventional multi-rate spring according to Equation (1) and Equation (3).

\[ K_2 \cdot \mu^2 = K_1 \]  
\[ F_2 \cdot \mu^2 = F_1 \]  

The Equation (4) indicates that a linear slave spring with any stiffness can be applied to the liquid spring to achieve the same stiffness property as the conventional multi-rate spring by setting a proper size of the slave piston. Moreover, a corresponding pre-compressed force of the slave spring is acquired according to Equation (5). Then the lateral stability of each slave spring can be determined. As a result, it is significant that taking the lateral stabilities of a series of pre-compressed slave springs and the lateral stability of the pre-compressed spring to have a compare. Especially, it is noticed that when the diameter of the slave piston is equal to the diameter of the primary piston (i.e., \( \mu = 1 \)), the lateral stability of the slave spring is equal to the lateral stability of the pre-compressed linear spring in the conventional multi-rate spring.

Concerning whether the lateral stabilities of those slave springs is better than the conventional counterpart. The definition of the lateral stability of a pre-compressed linear spring is proposed in this paper. Taking no consideration of the installation of the pre-compressed linear spring, the lateral stability of a pre-compressed linear spring is only based on the stiffness and the deformation of the spring. It is well known that, as for a specific spring, the smaller deformation, the better lateral stability. Moreover, when springs under the same deformation, the spring with higher stiffness shows better lateral stability. As a result, the lateral stability of a pre-compressed linear spring is defined as

\[ S = \frac{K}{\Delta l} \]  

where \( \Delta l \) is the deformation of the spring, and \( K \) is the stiffness of the spring. It indicates the higher the value, the better the lateral stability.

Then the lateral stability of the pre-compressed spring in the conventional multi-rate spring is

\[ S_c = \frac{K_1}{F_1} \cdot \frac{K_1^2}{F_1} = \frac{K_1}{F_1} \]  

Moreover, the lateral stability of the slave spring in the liquid spring is

\[ S_L = \frac{K_2}{F_2} \cdot \frac{K_2^2}{F_2} = \frac{K_2^2}{F_2} \]  

Substituting Equation (4) and Equation (5) into Equation (8) derive

\[ S_L = \frac{1}{\mu^2} \cdot \frac{K_1^2}{F_1} \]
To design a productive liquid spring with one slave spring, the lateral stability of the slave spring in the liquid spring should be better than the lateral stability of the pre-compressed spring in the conventional multi-rate spring, i.e., \( S_c < S_L \). Then the design criteria of the liquid spring with one slave spring can be reduced as

\[
\mu < 1
\]  \hspace{1cm} (10)

Concerning to the liquid spring with one slave spring, the diameter of the slave piston should be greater than the diameter of the primary piston to ensure the slave spring has better lateral stability than the conventional counterpart. While the consequence is that the stiffness and the pre-compressed force of the slave spring are higher than the conventional counterpart according to Equation (4) and Equation (5). Theoretically, any high stiffness spring can be applied in the liquid spring to achieve an ideal low working stiffness.

2.3. The liquid spring with more than one slave spring

As for the liquid spring with more than one slave spring shown in Figure 3 (a), it can be seen that the configuration of each slave spring element may be different. The stiffness property of the liquid spring may be quite complicated on account of the various configuration of every slave spring element. While, concerning the influence of the number of slave springs on the liquid spring’s stiffness property, it assumes that all slave spring elements are with the same configuration as Figure 3 (b). Correspondingly, the force-displacement relationship of the liquid spring with more than one slave spring can be derived as

\[
F = \begin{cases} 
K_0 \cdot x & F \leq F_i \cdot \mu^2 \\
\frac{K_0 + K_2 \cdot \mu^2}{n} \cdot x & F_i \cdot \mu^2 < F 
\end{cases}
\]  \hspace{1cm} (11)

To obtain the same stiffness property as the conventional multi-rate spring, the configuration of the liquid spring with more than one slave spring should follow the bellow relationship with the configuration of the conventional multi-rate spring according to Equation (1) and Equation (11).

\[
\frac{K_2 \cdot \mu^4}{n} = K_1
\]  \hspace{1cm} (12)

\[
F_i \cdot \mu^2 = F_i
\]  \hspace{1cm} (13)

Then the lateral stability of the slave spring in the liquid spring with more than one slave spring is

\[
S_c = \frac{n^2 \cdot K_2}{\mu^4 \cdot F_i}
\]  \hspace{1cm} (14)

As a result, the design criterion of the liquid spring with more than one slave spring can be written as

\[
\mu^3 < n
\]  \hspace{1cm} (15)

where \( n \) is the number of slave springs.
The Equation (12) and the Equation (15) demonstrate that any stiffness of the slave spring, even the stiffness of the slave spring is equal to the conventional counterpart, can be applied to the liquid spring to obtain the desired stiffness property while considering the lateral stability of the slave spring. Furthermore, the diameter ratio can be greater than one due to the number of slave springs is a positive integer. In another word, the slave piston can be designed as small as possible to achieve the same stiffness property so long as the number of slave springs can obey the Equation (12).

3. Conclusion

The liquid spring presented in this paper is a new type of multi-rate spring and possesses the characteristic of piecewise linear HSLLD stiffness. And the lateral stability of the pre-compressed slave spring in the liquid spring can be designed to be better than the conventional counterpart. The stiffness property of the liquid spring is desirable by adjusting the size of pistons and the number of slave springs. A linear spring with arbitrary magnitudes stiffness is able to apply to the liquid spring to achieve the ideal piecewise linear stiffness property. The liquid spring is especially applicable for massive payload low frequency vibration isolation.

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