Fluid added polar inertia and damping for the torsional vibration of a Kaplan turbine model runner considering multiple perturbations

A Soltani Dehkhargani, J Aidanpää, F Engström, M J Cervantes

Department of Engineering Science and Mathematics, Luleå University of Technology, Luleå, Sweden

E-mail address: arasol@ltu.se

Abstract. A water turbine runner is exposed to several perturbation sources with different frequencies, phases, and amplitudes both at the design and off-design operations. Rotor-stator interaction, cavitation, rotating vortex rope, and blade trailing edge vortices are examples of such perturbations which can disturb the runner. The rotor dynamic coefficients require being determined to perform a reliable dynamic analysis. Fluid added inertia, damping, and stiffness have previously been investigated for individual perturbation frequencies for the torsional vibration of a Kaplan turbine model runner. However, a number of perturbation sources mostly take place simultaneously and alter the dynamics of the runner. Soltani et al. [1] have evaluated the torsional added parameters for a Kaplan turbine runner using numerical simulations considering single perturbation frequency. In the present work, the fluid added parameters are assessed in the presence of multiple perturbation sources. A similar methodology is used. A single-degree-of-freedom (SDOF) model for the dynamic model and unsteady Reynolds-averaged Navier–Stokes approach for the flow simulations are assumed. Perturbations with different frequencies are applied to the rotational speed of the runner to determine the fluid added parameters for the torsional vibration. A number of previously investigated frequencies are chosen and their combinations are investigated. In addition, two different phase shifts are considered between the applied perturbations to study the effect of phase. Two more test cases with higher perturbation amplitude are also conducted to investigate its influence on the fluid added inertia and damping. The results are compared with the previous study and the interaction of multiple perturbations on the added parameters is investigated.

1. Introduction
Among all renewable resources, hydropower is the largest and provides about 17% of the global electricity capacity [2]. The extent of intermittent resources such as solar and wind increases the off-design operations in hydro-turbines. Hence undesired loads and flow pulsations with a wide frequency spectrum are induced on the runner. The presence of pressure pulsations with various frequencies and amplitudes on hydro turbine runners has been shown and investigated at steady and transient operating conditions [3-5]. Not only an individual perturbation but also the superposition of multiple perturbations may alter the dynamic response of the turbine by modifying the rotor dynamic parameters.
A limited number of studies has been conducted on the influence of fluid added properties in hydro-turbines. In 2009, Karlsson et al. [6] have studied the Hölleforsen Kaplan runner. The added polar inertia increased with perturbation frequency at best efficiency point (BEP) but decreased at off-design operating conditions. However, the added damping showed an almost linear trend in the studied frequency range, 10-40 times the runner operating frequency. An increase of 30-80% in damping and reduction of 5-65% in eigenfrequency were obtained in the investigated frequency range.

In 2013, Puolakka et al. [7] have investigated three individual Kaplan runners. The axial, polar-axial, and polar added mass increased and added damping declined with runner reduced frequency, $\kappa = \frac{\omega c}{\U}$. $\omega$, $c$, and $\U$ are imposed angular frequency, half-chord blade section, and upstream velocity, respectively. A slight variation of the added mass and damping for the reduced frequency range larger than one was obtained for all three investigated runners.

Most recently, the added damping was estimated for a propeller turbine runner at high load for the various amplitude of prescribed modal motion [8]. The results showed that the added damping is almost independent of the vibration amplitude. In addition, any significant change was observed in the flow field by the structural deflection. Hence, one-way fluid-structure interaction analysis would be sufficient. The fluid added mass was found 3.59 times of the runner structural mass and the added modal stiffness was found to be about 2% of the structural modal stiffness.

A number of numerical and experimental investigations showed that the natural frequencies of model and prototype Francis turbines are reduced due to the added mass effect. Modal analysis was performed to obtain the natural frequencies of Francis runners in the air and immersed in water [9-11]. Another study showed that the operating condition of a Francis turbine affects the fluid-structure interaction [12]. One-way fluid-structure interaction simulations were performed at different operating conditions. The largest reduction of the natural frequency was found at part load.

Soltani et al. [1] have investigated the added polar inertia, damping, and stiffness of the U9 Kaplan turbine model runner. Unsteady Reynolds-Averaged Navier-Stokes (URANS) simulations combined with SDOF vibration model were used to obtain the added properties. A method was proposed to obtain the added stiffness. Both added polar inertia and damping increased with perturbation frequency. The effect of stiffness was found negligible. The results were in good agreement with the previous study performed by Puolakka et al. [7].

A good turbine design should take into account the fluid influence on the structure. The current study presents the effect of multiple perturbations on the fluid added parameters for the torsional vibration. The simulations and modeling of the system are similar to previous research [1]. Three test cases with different perturbation frequencies, two with different phases, and two with higher perturbation amplitudes are considered; the frequencies of 4, 7, and 10 times the runner operating frequency, the phase shifts of 30 and 60, and the amplitudes of 0.5% and 1% of the rotational speed. These perturbations are applied to the rotational speed of the runner. The least square fitting method is used to fit the runner moment. Thereafter, the fluid added polar inertia and damping are obtained. The fluid added stiffness is disregarded in this study as it was previously found negligible [1].

2. Methodology

2.1. Numerical simulation

The Projus U9 model located in Porjus, Sweden has been investigated in the present study. This Kaplan turbine model is a 1:3.1 scale of the 10 MW prototype turbine, which has a runner diameter of 1.55 m. The turbine is composed of six runner blades, twenty guide vanes, and eighteen unequally distributed stay vanes. The model runner has a diameter of 500 mm, and its hub-to-tip ratio is 0.52. The runner shroud and tip solidity are 0.87 and 1.2, respectively.

The computational domain is composed of the guide vanes, runner and draft tube, see Figure 1. The complete geometry of the guide vanes and the runner have been used in the domain. The mesh used in the simulations has been selected based on the mesh study of Mulu et al.[13]. Hexahedral-type elements have been used for all parts, including the guide vanes, runner and draft tube. The number of cells in the
hub and tip clearance in the radial direction is 5-10. The total number of elements in this study is approximately 17 million.

The fluid part has been solved using the commercial software ANSYS-CFX 16.2. The unsteady Reynolds-averaged Navier-Stokes approach has been applied to perform the simulations. The high-resolution discretization scheme was applied for the continuity and momentum equation advection term. The first-order upwind scheme has been used for the advection terms in the turbulence equations. The simulations required a sufficiently small time-step to resolve various phenomena in the flow. The time step is $3.6 \times 10^{-4}$ second which corresponds to approximately $1.5^\circ$ of the runner revolution. The simulations have been run until a periodic flow was achieved at specific monitor points. After reaching convergence, the data have been recorded for further analysis in the present investigation.

![Figure 1. Computational domain of the Kaplan turbine model.](image)

The standard k-$\varepsilon$ model with a scalable wall function has been used to model the turbulent flow [14]. The turbulence model has been selected based on a research performed by Mulu et al.[13] which investigated many turbulence models (k – $\varepsilon$, RNG k – $\varepsilon$, SST, and SAS-SST) to simulate the Porjus U9 model at the BEP. A transient rotor-stator interface has been selected as the domain interface between the rotating and stationary domains and a sliding interface has been used between the domains to resolve the transient interactions. The simulations have been performed at the BEP for the U9 model. The operating parameters are presented in Table 1.

| Parameter                        | Value  |
|----------------------------------|--------|
| Turbine head (m)                 | 7.5    |
| Flow rate ($m^3$/s)              | 0.71   |
| Runner angular Velocity (rad/s)  | 72.92  |
| Guide vane angle (°)             | 26     |
| Blade shroud stagger angle (°)   | 42     |
| Blade tip stagger angle (°)      | 21     |
2.2. Modeling of the system

The runner of the Kaplan turbine has been assumed to be rigid and the equation of angular motion is derived considering a SDOF model.

\[ J_f \ddot{\phi}(t) + C_f (\dot{\phi}(t) - \omega_0) + K_f (\phi(t) - \omega_0 t) = M(t) \]  

where \( \phi(t) \), \( \omega_0 \) and \( M(t) \) are the angular displacement, runner angular velocity and torsional moment of the turbine, respectively. \( t \) is time and the “dot” denotes time derivative. The added polar inertia \( J_f \), damping \( C_f \) and stiffness \( K_f \) are parameters given by the interaction of the runner with the surrounding water.

To investigate the added properties, a prescribed harmonic perturbation \( \ddot{\phi} \) is applied to the runner angular velocity \( \omega_0 \). This component includes two harmonics which have two different frequencies with an individual amplitude. A phase shift is also considered between the harmonics. The total angular velocity \( \dot{\phi}(t) \) of the turbine can be expressed as

\[ \dot{\phi}(t) = \omega_0 + \ddot{\phi}(t) = \omega_0 (1 + A_1 \sin(k_1 \cdot \omega_0 \cdot t) + A_2 \cos(k_2 \cdot \omega_0 \cdot t + \phi)) \]  

where \( A_1, k, \) and \( \phi \) are the perturbation amplitude, the perturbation frequency factor and the phase shift between the perturbations, respectively. The subscripts 1 and 2 refer to the first and second applied perturbations, respectively.

Seven test cases have been studied with different frequency combination, phase shift, and perturbation amplitude. Table 2 presents the detail of the cases.

| Test case | \( k_1 \) and \( k_2 \) | \( \phi \) | \( A \) |
|-----------|----------------|-------|-----|
| 1         | 4 and 7        | 0°    | 5-10-3 |
| 2         | 4 and 10       | 0°    | 5-10-3 |
| 3         | 7 and 10       | 0°    | 5-10-3 |
| 4         | 4 and 7        | 30°   | 5-10-3 |
| 5         | 7 and 10       | 60°   | 5-10-3 |
| 6         | 4 and 7        | 0°    | 10-10-3 |
| 7         | 7 and 10       | 0°    | 10-10-3 |

An additional moment, \( M(t) \), affects the turbine operation due to these perturbations. The governing equation is written as

\[ J_f \ddot{\phi}(t) + C_f \dot{\phi}(t) + K_f \phi(t) = \dot{M}(t) \]  

The additional moment can be decomposed into four components and expressed as

\[ \dot{M}(t) = M_1 \sin(k_1 \cdot \omega_0 \cdot t) + M_2 \cos(k_1 \cdot \omega_0 \cdot t) + M_3 \sin(k_2 \cdot \omega_0 \cdot t) + M_4 \cos(k_2 \cdot \omega_0 \cdot t) \]  

Assuming a negligible fluid added stiffness, the fluid added polar inertia and damping can be found from the following equations for each individual applied frequency.
\[ J_f_1 = \frac{M_1}{A_1 k_1 \omega_0}, J_f_2 = \frac{M_2}{A_2 k_2 \omega_0} \]  

\[ C_{f_1} = \frac{M_3}{A}, C_{f_2} = \frac{M_4}{A} \]  

3. Results

Figure 2 illustrates the velocity streamlines initiated at the inlet of the guide vanes at the BEP operating condition. The streamlines pattern indicates a stable flow at the BEP. Within the runner, the flow velocity increases and then decreases continuously towards the draft tube bend. As the kinematic energy leaving the runner is recovered into pressure, the flow velocity reduces.

![Figure 2](image_url)

**Figure 2.** Flow streamlines initiated at the inlet of the guide vanes at the BEP operating condition where the velocity distribution shown in the stationary frame of reference.

As a preliminary, Figure 3 shows the numerical result of the additional moment of the turbine due to the applied perturbations for the perturbation factors of 4 and 7. The fitted curve is obtained using the last three periods of the results.

![Figure 3](image_url)

**Figure 3.** Additional moment and fitted curve for case 1.
The added polar inertia is presented for different simulations along with the reference values obtained for single perturbation frequencies in Figure 4. The results show that the deviation from the reference values for all the cases is small for the perturbation factors of 4 and 10. Although more discrepancy is seen for the perturbation factor of 7 for some cases, the maximum difference with the reference value is approximately 10%, see Table 4. This indicates that the added polar inertia can be considered independent of the presence of multiple perturbations with different frequencies, phase shifts, and amplitudes.

Figure 4. Fluid added polar inertia as a function of perturbation frequency.

Figure 5 presents the fluid added damping for different cases. The results illustrate a different trend compared to the added polar inertia. More added damping appears in cases with multiple perturbations, especially at high perturbation frequencies. As shown in Figure 5, the case 1-7 gain approximately the same added damping. It means that added damping only depends on the perturbation frequency and the phase shift between the harmonics and higher amplitude does not have any significant effect.

Figure 5. Fluid added damping as a function of perturbation frequency.
Table 4 presents the comparison between the added properties with the reference values given in Table 3. The added polar inertias are seen approximately unchanged. However, the added damping increases at high frequencies due to the interaction of the frequencies particularly.

Table 3. Reference added properties for single perturbation frequencies [1].

|   | Jf  | Cf  |
|---|-----|-----|
| 4 | 0.25| 30.97 |
| 7 | 0.26| 54.16 |
| 10| 0.27| 93.30 |

Table 4. The difference of added polar inertia and damping for cases with multiple perturbations compared to reference values (%).

| Test case | Jf  | Cf  |
|-----------|-----|-----|
|           | k = 4 | k = 7 | k = 10 | k = 4 | k = 7 | k = 10 |
| 1         | 2.76 | 7.40 | -     | 16.32 | 41.00 | -     |
| 2         | 1.69 | -    | 1.88  | 14.88 | -     | 64.82 |
| 3         | -    | 2.66 | 1.16  | -     | 32.14 | 63.31 |
| 4         | 2.63 | 10.94 | -     | 16.13 | 40.85 | -     |
| 5         | -    | 3.28 | 0.56  | -     | 28.35 | 64.13 |
| 6         | 2.05 | 4.97 | -     | 14.60 | 36.33 | 64.13 |
| 7         | -    | 1.53 | -1.94 | -     | 30.05 | 52.20 |

4. Discussion and conclusion
The fluid added properties of a Kaplan turbine model have been obtained considering multiple perturbations. The perturbations have been applied to the rotational speed of the turbine runner. Seven different cases with different frequency combination, amplitude and phase shift have been examined. The additional moment due to these perturbations has been analyzed and the added polar inertia and damping calculated. The results illustrate that the added polar inertia can be assumed independent of the presence of multiple perturbations as a small deviation from the reference values has been observed. On the other hand, the interaction of the harmonics has modified the added damping value, particularly at high perturbation frequencies. The added damping has increased by 14-65%. As results illustrated, any effect of phase shift and amplitude has not been observed in the added properties. Therefore, the increase of added damping can be attributed to the presence of multiple perturbations, not a different phase shift or amplitude.

5. Acknowledgments
The study was performed as a part of the "Swedish Hydropower Centre - SVC". SVC was established by the Swedish Energy Agency, Elforsk and Svenska Kraftnät with Luleå University of Technology, The Royal Institute of Technology, Chalmers University of Technology and Uppsala University (www.svc.nu).
6. References

[1] Soltani Dehkharqani A, Cervantes M J and Aidanpää J 2017 Numerical analysis of fluid-added parameters for the torsional vibration of a Kaplan turbine model runner Adv. Mech. Eng. 9 1-10

[2] Sawin J L, Seyboth K and Sverrisson F 2017 Renewables 2017 Global status report

[3] Trivedi C, Gandhi B and Michel C J 2013 Effect of transients on Francis turbine runner life: a review J. Hydraul. Res. 51 121-132

[4] Amiri K, Mulu B, Raisee M and Cervantes M J 2015 Unsteady pressure measurements on the runner of a Kaplan turbine during load acceptance and load rejection J. Hydraul. Res. 54 56-73

[5] Soltani Dehkharqani A, Amiri K and Cervantes M 2015 Steady and transient pressure measurements on the runner blades of a Kaplan turbine model IAHR Int. Meeting of the Workgroup on Cavitation and Dynamic Problems in Hydraulic Machinery and Systems (Ljubljana, Slovenia, September 9-11)

[6] Karlsson M, Nilsson H and Aidanpää J 2009 Numerical estimation of torsional dynamic coefficients of a hydraulic turbine Int. J. Rotating Mach. 2009 1-7

[7] Puolakka O, Keto-Tokoi J and Matusiak J 2013 Unsteady load on an oscillating Kaplan turbine runner J. Fluids Struct. 37 22-33

[8] Gauthier J P, Giroux A M, Etienne S and Gosselin F P 2017 A numerical method for the determination of flow-induced damping in hydroelectric turbines J. Fluids Struct. 69 341-354

[9] Rodriguez C G, Egusquiza E, Escaler X, Liang Q W and Avellan F 2006 Experimental investigation of added mass effects on a Francis turbine runner in still water J. Fluids Struct. 22 699-712

[10] Liang Q W, Rodríguez C G, Egusquiza E, Escaler X, Farhat M and Avellan F 2007 Numerical simulation of fluid added mass effect on a francis turbine runner Comput. Fluids. 36 1106-1118

[11] Lais S, Liang Q, Henggeler U, Weiss T, Escaler X and Egusquiza E 2009 Dynamic analysis of Francis runners-experiment and numerical simulation IJFMS. 2 303-314

[12] Liu D, Liu S, Wu Y and Liu X 2008 Numerical simulation of hydraulic turbine based on fluid-structure coupling The 4th Int. Symp. on Fluid Machinery and Fluid Engineering (Beijing, China, November 24-27)

[13] Mulu B, Cervantes M, Devals C, Vu T and Guibault F 2015 Simulation-based investigation of unsteady flow in near-hub region of a Kaplan Turbine with experimental comparison Eng. Appl. Comput. Fluid Mech. 9 139-156

[14] Ansys C 2013 Release 15.0