Weakness Ranking Method for Subsystems of Heavy-Duty Machine Tools Based on FMECA Information

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Abstract

Heavy-duty machine tools are composed of many subsystems with different functions, and their reliability is governed by the reliabilities of these subsystems. It is important to rank the weaknesses of subsystems and identify the weakest subsystem to optimize products and improve their reliabilities. However, traditional ranking methods based on failure mode effect and critical analysis (FMECA) does not consider the complex maintenance of products. Herein, a weakness ranking method for the subsystems of heavy-duty machine tools is proposed based on generalized FMECA information. In this method, eight reliability indexes, including maintainability and maintenance cost, are considered in the generalized FMECA information. Subsequently, the cognition best worst method is used to calculate the weight of each screened index, and the weaknesses of the subsystems are ranked using a technique for order preference by similarity to an ideal solution. Finally, based on the failure data collected from certain domestic heavy-duty horizontal lathes, the weakness ranking result of the subsystems is obtained to verify the effectiveness of the proposed method. An improved weakness ranking method that can comprehensively analyze and identify weak subsystems is proposed herein for designing and improving the reliability of complex electromechanical products.

Keywords: Complex electromechanical products, Weakness ranking method, Failure mode effect and critical analysis (FMECA), Cognition best worst method (CBWM), Technique for order preference by similarity to an ideal solution (TOPSIS)

1 Introduction

As technologically advanced machines, computer numerical control (CNC) machine tools are the foundation of the equipment manufacturing industry [1, 2]. Among them, heavy-duty machine tools with multisystem construction and multitechnology integration are important guarantees for the quality of national defense machine tools. However, the domestic heavy-duty machine tool poses serious reliability problems, which seriously affect its market share [3, 4], as well as hidden dangers to national strategies. Therefore, the reliability of heavy-duty machine tools must be evaluated urgently to design and improve its reliability.

A few studies have focused on the reliability evaluation of heavy-duty machine tools. To solve the problem of insufficient failure data for heavy-duty machine tools, Zhang et al. [5] proposed an evaluation method based on the Bayes method for a small sample. Huang et al. [6] focused on the reliability modeling and analysis of heavy-duty machine tool spindles under hybrid uncertainty. Wang et al. [7] established a reliability model for an operating table and accomplished the reliability prediction of the operating table for heavy-duty machine tools. However, the heavy-duty machine tool was composed of many subsystems with different functions; as such, its reliability primarily depended on the reliabilities of their subsystems. Therefore, it is important to rank the weaknesses of
subsystems and identify the weakest subsystem to design and improve the reliability of heavy-duty machine tools.

Typical failure analysis methods include failure mode effect and critical analysis (FMECA), failure tree analysis, reliability block diagram, and potential path analysis. Currently, FMECA is the most influential and mature analysis method; it is a systematic approach for identifying all possible causes of failures and their effects on the design, manufacturing, and assembly process of a product [8]. FMECA is composed of two separate analyses: the failure mode and effects analysis (FMEA) and the criticality analysis (CA). In FMEA, different failure modes and their effects on a system are analyzed, whereas CA prioritizes the level of importance based on the failure rate and severity of the effect of failure [9]. Subsequently, a team that is familiar with the system conducts the ranking of subsystems. FMECA is widely used in the military industry, aviation [10], automobiles [11], energy industry [12], ships [13], gas turbines [14], distribution network lines [15], and other fields [16, 17].

In previous studies involving FMECA, the failure analysis of a product is generally performed based on a risk priority number (RPN), which is evaluated through interpreted linguistic expressions: (1) severity, which indicates the gravity of the effect of a failure mode; (2) occurrence, which indicates the probability of a failure occurring; and (3) detection, which measures the visibility of a failure and is the attitude of a failure mode to be identified by controls or inspections. For example, Piumatti et al. [18] identified the critical faults of a cyber–physical system used for driving a three-phase motor for industrial compressors via FMECA, where a functional simulation was performed for each fault considered to calculate its criticality based on the RPN. Thoppil et al. [19] calculated the RPNs of the failure modes for each component of a CNC lathe based on FMECA, and the spindle unit was identified as the most critical subsystem of the CNC lathe. Jomde et al. [20] investigated the reliability of the components of a linear compressor based on FMECA, by which the failure criticalities of seven failure modes were calculated based on the RPN. Finally, the flexure bearing was determined as the component with the highest failure level. Goo et al. [21] proposed an efficient systematic design methodology that combined the strengths of axiomatic design and FMECA, where the RPN was used as the reliability index. Ghali et al. [22] proposed a computer-aided design model considering functional and manufacturing requirements in the early phase of digital mock-up via FMECA, where the RPN was used as the reliability index. More examples are available in [13, 23, 24].

However, the reliability index used in traditional FMECA methods is limited and incomplete. For heavy-duty machine tools, complex maintenance is required when failure occurs; therefore, the maintenance time and maintenance cost incurred during usage are non-negligible in FMECA [25]. Otherwise, the weakest subsystem of the heavy-duty machine tool determined by the traditional FMECA method will be inaccurate.

Herein, a weakness ranking method based on generalized FMECA information is proposed for a heavy-duty machine tool. In this method, eight reliability indexes, including failure rate, failure impact, detection difficulty, RPN, matrix analysis (MA), analytical formula method (AFM), maintainability and maintenance cost were considered. Subsequently, the cognition best worst method (CBWM) was used to calculate the weight of each screened index, and the weaknesses of subsystems were using the technique for order preference by similarity to an ideal solution (TOPSIS).

The main contributions of this paper are as follows:

i. Considering that a heavy-duty machine tool is composed of many subsystems with different functions and its reliability depends primarily on the reliability of the weakest subsystem, a weakness ranking method based on the generalized FMECA information is proposed to determine the weakest subsystem of the heavy-duty machine tool.

ii. Considering the complex maintenance of the heavy-duty machine tool, the maintainability and maintenance cost are considered in the generalized FMECA information. Subsequently, eight reliability indexes are considered as the FMECA information to comprehensively analyze the failure of the subsystems.

iii. To reduce the effects of subjective cognitive differences on the analysis result, the CBWM is applied to calculate the weight of each screened index, and the TOPSIS is used to rank the weaknesses of all subsystems accurately and rapidly.

The remainder of the paper is organized as follows: Section 2 presents the generalized FMECA information and the preprocessing of the failure data. The reliability indexes that affect the weakness ranking of subsystems are analyzed in Section 3. The weight of each screened index is calculated based on the CBWM in Section 4. The weaknesses of subsystems are ranked using the TOPSIS, and the weakest subsystem is identified in Section 5. In Section 6, based on the failure data collected from certain domestic heavy-duty horizontal lathes, the weakness ranking of subsystems and the weakest subsystem are identified to verify the effectiveness of the proposed method.
2 Generalized FMECA Information and Data Preprocessing

2.1 Generalized FMECA Information

FMECA is an inductive analysis method that analyzes all possible failure modes of a product [26]. The traditional FMECA information that can affect the identification of the weakest subsystem includes the failure rate, failure impact, and damage degree [27]. To complete the information, the generalized FMECA information is proposed herein, as will be described in the following.

2.1.1 Failure Rate

The failure rate refers to the probability of failure in the unit time of a product that has not yet failed at a certain time. The failure rate of a complex electromechanical product is the sum of these subsystems, as shown in Eq. (1), and the observed average failure rates of the subsystems can be calculated using Eq. (2):

\[
\lambda(t) = \sum_{i=1}^{n} \lambda_i(t),
\]

\[
\tilde{\lambda}_i(t) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \lambda_i(t) dt,
\]

where \(\lambda(t)\) is the failure rate of the machine tool; \(\lambda_i(t)\) is the failure rate of the subsystem; \(\tilde{\lambda}_i(t)\) is the average failure rate of the subsystem during time \((t_1, t_2)\), and \(n\) is the number of subsystems.

2.1.2 Failure Impact

The failure impact is the failure effect of each failure mode of a subsystem, including the effect of the failure mode on the subsystem, system, personnel, environment, etc. To express the fuzzy information and fuzzy preference of the evaluators, the failure impact is expressed by the effect severity ranking (ESR), as shown in Table 1. For each trapezoidal fuzzy number \(\tilde{a} = (d_1, d_2, a_1, a_2)\), the membership function is obtained using Eq. (3). In Table 1, the membership functions \(\tilde{R}_{11}, \tilde{R}_{12}, \tilde{R}_{13}, \tilde{R}_{14}\) are obtained using Eq. (4), and the precision numbers \(a_{R_{11}}, a_{R_{12}}, a_{R_{13}}, a_{R_{14}}\) are obtained using the center average method expressed as Eq. (3) to characterize the corresponding severity classes.

\[
a_a = \int_{a_{R_{11}}}^{a_{R_{14}}} f(x) dx, \quad \tilde{u}_{R_{1}}(x) = \begin{cases} \frac{x - a_{R_{11}}}{a_{R_{12}} - a_{R_{11}}}, & x \in [a_{R_{11}}, a_{R_{12}}] \\ 1, & x \in [a_{R_{12}}, a_{R_{13}}] \\ \frac{x - a_{R_{13}}}{a_{R_{14}} - a_{R_{13}}}, & x \in [a_{R_{13}}, a_{R_{14}}] \\ 0, & x \notin [a_{R_{11}}, a_{R_{14}}] \end{cases}
\]

where \(f(x) : [a_{R_{11}}, a_{R_{14}}] \rightarrow [0, 1]\), \(f_{10}(x) : [a_{R_{12}}, a_{R_{13}}] \rightarrow [0, 1]\), and \(\tilde{u}_{R_{1}}\) is the centrobaric abscissa of the trapezoidal fuzzy number \(\tilde{a}\), i.e., the precision number.

When the failure impact of the \(j\)th failure mode for the \(i\)th subsystem is analyzed, the ESR score is provided by the trapezoidal fuzzy number \(\tilde{e}_{ij} = (e_{ij}, e_{ij}, e_{ij}, e_{ij})\). The precision number is obtained using Eq. (4) and substituted into the membership function of ESR classes \(u_{R_{11}}, u_{R_{12}}, u_{R_{13}}, u_{R_{14}}\) to obtain the membership degree of the score for each ESR class. The final score of the \(j\)th failure mode is weighted using Eq. (5), and the failure impact of the \(i\)th subsystem can be calculated using Eq. (6).

\[
E_{ij} = a_{R_{11}} u_{R_{11}}(a_{e_{ij}}) + a_{R_{12}} u_{R_{12}}(a_{e_{ij}}) + a_{R_{13}} u_{R_{13}}(a_{e_{ij}}) + a_{R_{14}} u_{R_{14}}(a_{e_{ij}}),
\]

\[
E_i = \sum_{j=1}^{m_j} E_{ij},
\]

where \(a_{e_{ij}}\) is the precision number of the ESR score for the \(j\)th failure mode of the \(i\)th subsystem; \(E_{ij}\) is the score of the failure impact for the \(j\)th failure mode of the \(i\)th subsystem; \(E_i\) is the score of the failure impact for the \(i\)th subsystem.

| Category         | Description                                                                 | ESR Score | Trapezoidal fuzzy number ESR Score |
|------------------|-------------------------------------------------------------------------------|-----------|-----------------------------------|
| Class I (Disastrous) | Major failure occurs; loses specified function; causes major safety accidents, casualties, and significant damage | 10, 9     | \(\tilde{R} = (8, 9, 10, 10)\)    |
| Class II (Deadly)  | Severe damage; loses specified function; no casualties occurs                |           | \(\tilde{R}_2 = (6, 7, 8, 9)\)     |
| Class III (Crisis) | Specified function degrades; loses partial performance                        | 8, 7      | \(\tilde{R}_3 = (3, 4, 6, 7)\)     |
| Class IV (Mild)   | Minor failure occurs; specified function degrades; acceptable performance     | 3, 2, 1   | \(\tilde{R}_4 = (1, 1, 3, 4)\)     |
subsystem, and \( m_i \) is the total number of failure modes for the \( i \)th subsystem.

### 2.1.3 Detection Difficulty

The detection difficulty is the possibility of determining the various causes of a failure mode through scheduled inspection procedures. The detection difficulty is indicated by the detection difficulty rank (DDR) shown in Table 2, where DDR is categorized into five classes by the trapezoidal fuzzy number.

When the detection difficulty of the \( j \)th failure mode for the \( i \)th subsystem is analyzed, the DDR score is provided by the trapezoidal fuzzy number

\[
\tilde{d}_{ij} = (d_{l_{ij}}, d_{m_{ij}}, d_{u_{ij}}),
\]

The precision number is obtained using Eq. (4) and then substituted into the membership function of DDR classes \( \tilde{u}_{r_1}, \tilde{u}_{r_2}, \tilde{u}_{r_3}, \tilde{u}_{r_4} \) to obtain the membership degree of the score for each DDR class. The final score of the \( j \)th failure mode is weighted using Eq. (7), and the detection difficulty of the \( i \)th subsystem can be calculated using Eq. (8):

\[
D_{ij} = a_{\tilde{d}_{ij}} u_{\tilde{r}_1} (a_{\tilde{d}_{ij}}) + a_{\tilde{d}_{ij}} u_{\tilde{r}_2} (a_{\tilde{d}_{ij}}) + a_{\tilde{d}_{ij}} u_{\tilde{r}_3} (a_{\tilde{d}_{ij}}) + a_{\tilde{d}_{ij}} u_{\tilde{r}_4} (a_{\tilde{d}_{ij}}),
\]

\[
D_i = \sum_{j=1}^{m_i} D_{ij},
\]

where \( a_{\tilde{d}_{ij}} \) is the precision number of the DDR score for the \( j \)th failure mode of the \( i \)th subsystem; \( D_{ij} \) is the score of the detection difficulty for the \( j \)th failure mode of the \( i \)th subsystem, and \( D_i \) is the score of the detection difficulty for the \( i \)th subsystem.

### 2.1.4 Criticality Degree

The criticality degree combines the probability of the failure mode and the severity of each failure mode to comprehensively assess the effects of various possible failures on the system. CA can be conducted using the RPN, MA, and AFM.

#### 1. RPN

When CA is performed, the severity degree, occurrence degree, and the detection difficulty of each failure mode are scored. Subsequently, the RPN of the \( j \)th failure mode for the \( i \)th subsystem is calculated using Eq. (9), and the RPN of the \( i \)th subsystem is calculated using Eq. (10):

\[
RPN_{ij} = E_{ij} \times OPR_{ij} \times D_{ij},
\]

\[
RPN_i = \sum_{j=1}^{m_i} RPN_{ij}
\]

where \( RPN_{ij} \) is the criticality degree of the \( j \)th failure mode for the \( i \)th subsystem; \( OPR_{ij} \) is the occurrence degree of the \( j \)th failure mode for the \( i \)th subsystem, which is scored based on Table 3.

#### 2. MA

MA is a method to evaluate the criticality degree of failure modes by figures. The severity degree is regarded as the abscissa with the occurrence degree as the ordinate, and the distance from the coordinate point to the original point is used to represent the criticality degree. Therefore, the criticality degree \( C_{ij} \)

### Table 2 Classification of DDR

| Category            | Description                          | DDR Score | Trapezoidal fuzzy number DDR Score |
|---------------------|--------------------------------------|-----------|-----------------------------------|
| Class I (Cannot detect) | Almost impossible to be detected      | 10        | \( \tilde{n}_1 = (9, 10, 10, 10) \) |
| Class II (Very difficult to detect) | Slight possibility of being detected  | 9, 8, 7   | \( \tilde{n}_2 = (6, 7, 9, 10) \) |
| Class III (Difficult to detect) | Can be detected on the spot or during disassembly | 6, 5, 4 | \( \tilde{n}_3 = (3, 4, 6, 7) \) |
| Class IV (Able to detect) | Self-warning                         | 3, 2      | \( \tilde{n}_4 = (1, 2, 3, 4) \) |
| Class V (Easy to detect)  | Visual detection                     | 1         | \( \tilde{n}_5 = (1, 1, 1, 2) \) |

### Table 3 Classification of occurrence degree

| Category      | Description            | Reference value of failure frequency | OPR score |
|---------------|------------------------|--------------------------------------|-----------|
| Class I       | Occur frequently       | > 20%                                | 10        |
| Class II      | Occur sometimes        | 10%–20%                              | 9, 8, 7   |
| Class III     | Occur occasionally     | 1%–10%                               | 6, 5, 4   |
| Class IV      | Occur less             | 0.1%–1%                              | 3, 2      |
| Class V       | Occur rarely           | < 0.1%                               | 1         |
of the \( j \)th failure mode for the \( i \)th subsystem is calculated using Eq. (11), and the criticality degree \( C_i \) of the \( i \)th subsystem is calculated using Eq. (12):

\[
C_{ij} = \sqrt{E_{ij}^2 + OPR_{ij}^2},
\]

\[
C_i = \sum_{j=1}^{m_i} C_{ij},
\]

(11)

(12)

3. AFM
The analytical solution of the criticality degree \( CR_{ij} \) for the \( j \)th failure mode of the \( i \)th subsystem is calculated using Eq. (13), and the criticality degree of the \( i \)th subsystem is calculated using Eq. (14):

\[
CR_{ij} = \alpha_{ij} \beta_{ij} \sum_{t} t,
\]

(13)

\[
CR_i = \sum_{j=1}^{m_i} CR_{ij},
\]

(14)

where \( \alpha_{ij} \) is the frequency ratio of the failure for the \( j \)th failure mode of the \( i \)th subsystem during the cumulative operating time and is calculated using Eq. (15); \( \beta_{ij} \) is the impact level of the failure for the \( j \)th failure mode of the \( i \)th subsystem, as described in Table 4.

\[
\alpha_{ij} = \frac{n_{ij}}{n_i},
\]

(15)

\[\text{Table 4. Failure impact level}\]

| Classes of occurrence degree | Description |
|------------------------------|-------------|
| Destroys the product or nullifies functions | 1.00 |
| Renders the product inoperable or degrades the function | 0.1–1.00 |
| Reduces the product function or causes defects | 0–0.1 |
| No appreciable impact | 0 |
| Destroys the product or nullifies the functions | 1.00 |

2.1.5 Maintainability
The maintainability of a product can be reflected by time factors such as failure detection, isolation, and maintenance time. In this study, the mean time to repair \( T_i \) of the \( i \)th subsystem was regarded as the maintainability index, i.e.,

\[
T_i = \sum_{j=1}^{m_i} T_{ij},
\]

(17)

where \( T_{ij} \) is the maintenance time of the \( j \)th failure mode for the \( i \)th subsystem at the \( k \)th time, and \( k_{ij} \) is the total number of failure modes for the \( i \)th subsystem.

2.1.6 Maintenance Cost
The maintenance cost of complex electromechanical products includes the profit without failure, labor and machine loss, labor cost, and maintenance material cost, expressed as follows:

\[
F_j = \left(F_{1j} + F_{2j} + F_{3j}\right) t_{1j}^{1k} + F_{4j} + F_{5j} t_{2j}^{2k},
\]

(18)

\[
F_{ij} = \sum_{k=1}^{k_{ij}} F_{ij}^{kj},
\]

(19)

\[
F_i = \sum_{j=1}^{m_i} F_{ij},
\]

(20)

where for the \( j \)th failure mode of the \( i \)th subsystem, \( F_{ij}^{kj} \) is the total maintenance cost of the \( k \)th failure; \( F_{ij}^{1k} \) is the profit in the unit time; \( F_{ij}^{2k} \) is the charges of operators in the unit time; \( F_{ij}^{3k} \) is the equipment cost in the unit time; \( F_{ij}^{4k} \) is the material cost; \( F_{ij}^{5k} \) is the charges of maintenance workers; \( t_{ij}^{2k} \) is the required maintenance time; \( F_{ij} \) is the total maintenance cost of all failures under the \( j \)th failure mode, and \( F_i \) is the total maintenance cost of the \( i \)th subsystem.

2.2 Ranking Indexes
When the generalized FMECA information is used to rank the subsystem weaknesses, the indicators representing the degree of weakness should be a function of the generalized FMECA information shown in Eq. (21):

\[
G_i = g(\bar{Z}_i, E_i, D_i, RPN_i, C_i, CR_i, T_i, F_i),
\]

(21)

where \( G_i \) is the ranking index of the subsystem weakness for the \( i \)th subsystem, and \( g(\cdot) \) is the ranking function of the subsystem weakness.

2.3 Preprocessing
For the convenience of subsequent processing, all indexes must be preprocessed, including normalization and assimilation processing.
1. Normalization processing

When judging the weakest subsystem, the size of the same index for different subsystems is relative; therefore, all indexes must be normalized using Eq. (22):

$$t_i^k = \frac{V_i^k}{\sum_{i=1}^{n} V_i^k},$$  \hspace{1cm} (22)

where $t_i^k$ is the dimensionless value of the $j$th failure mode for the $i$th subsystem; $V_i^k$ is the original value of the $j$th failure mode for the $i$th subsystem, $i = 1, 2, ...$, $n$, and $k = 1, 2, ... , 8$.

2. Assimilation processing

All indexes are assimilated using Eq. (23), i.e.,

$$h_i^k = \begin{cases} \frac{t_i^k}{}, & \text{High-quality index,} \\ 1/t_i^k, & \text{Low-quality index,} \\ 1/(1 + |t_i^k - 1|), & \text{Medium-quality index,} \end{cases}$$  \hspace{1cm} (23)

where $i = 1, 2, ...$, $n$, $k = 1, 2, ... , 8$, and $h_i^k$ is the assimilation conversion value of the dimensionless value for the $j$th failure mode of the $i$th subsystem.

### 3 Screening and Determination of Indexes

Because the generalized FMECA information contains many repeated indexes, similar information should be screened artificially and useful information determined based on the screening result.

#### 3.1 Index Screening

Among the three indexes representing the criticality degree, one index should be selected to ensure the most significant effect when judging the weakness. Therefore, the multistrategic weighting method was applied to complete the index screening based on the combined weight shown in Eq. (24) [28]:

$$w_X = w_X^\delta + w_X^\rho + w_X^\sigma,$$  \hspace{1cm} (24)

where $w_X$ is the combined weight of the $x$th index; $w_X^\delta$ is the subjective weight of the $x$th index; $w_X^\rho$ is the relevance weight of the $x$th index, and $w_X^\sigma$ is the information weight of the $x$th index.

1. Subjective weighting

The analytic hierarchy process (AHP) was applied to calculate the subjective weight. Using 1–9 and their backward count as the scale, the relative importance degree between indexes was subjectively provided by experienced professionals, and a judgment matrix was established using Eq. (25). Subsequently, the product-sum-gravity method was used to calculate the weight of each index using Eqs. (26)–(28). Finally, the consistency was verified [29].

$$\Delta = (\Delta_{XY})_{l_x \times l_y},$$  \hspace{1cm} (25)

$$\delta_{XY} = \frac{\Delta_{XY}}{\sum_{X=1}^{l_x} \Delta_{XY}},$$  \hspace{1cm} (26)

$$\delta_X = \sum_{Y=1}^{l_y} \delta_{XY},$$  \hspace{1cm} (27)

$$w_X^\delta = \frac{\delta_X}{\sum_{X=1}^{l_x} \delta_X},$$  \hspace{1cm} (28)

where $X = 1, 2, ..., l_x$, $Y = 1, 2, ..., l_y$, $l_x$ is the number of indexes in the same category; $\Delta$ is the judgment matrix; $\delta_{XY}$ is the element of the judgment matrix after normalizing processing by column; $\delta_X$ is the value of $\delta_{XY}$ after summing by rows, and $\Delta_{XY}$ is the relative importance degree between the $x$th and $y$th indexes.

2. Relevance weighting

The relevance between two indexes was calculated using the cosine similarity expressed in Eq. (29), and the correlation index between the indexes was calculated using Eq. (30). The relevance weight was obtained after normalizing the correlation weight using Eq. (31).

$$\rho_{XY} = \frac{\sum_{i=1}^{n} X_i Y_i}{\sqrt{\sum_{i=1}^{n} (X_i)^2} \sqrt{\sum_{i=1}^{n} (Y_i)^2}},$$  \hspace{1cm} (29)

$$\bar{\rho}_X = \frac{1}{l_x - 1} \left( \sum_{Y=1}^{l_y} |\rho_{XY} - 1| \right),$$  \hspace{1cm} (30)

$$w_X^\rho = \frac{\bar{\rho}_X}{\sum_{X=1}^{l_x} \bar{\rho}_X},$$  \hspace{1cm} (31)

where $X = 1, 2, ..., l_x$; $\rho_{XY}$ is the cosine similarity between the $x$th and $y$th indexes; $X_i$ and $Y_i$ are the $i$th components of the $x$th and $y$th indexes, respectively; $\bar{\rho}_X$ is the correlation index between the $x$th index and the other index.
3. Information weighting

The more information a certain indicator contains, the higher is the screening ability. As the simplest method to reflect the amount of information, the variance method is widely used. The larger the variance, the more information is present, as depicted in the following:

$$\sigma^2_X = \frac{\sum_{i=1}^{n} (X_i - \frac{\sum_{i=1}^{n} X_i}{n})^2}{n}, \quad (32)$$

where $\sigma^2_X$ is the variance of the $X$th index, and $X = 1, 2, ..., I$.

3.2 Index Determination

After screening the similar indexes, the remaining indexes were determined based on the existing results. The final screening results are shown in Table 5.

After the screening and determination of indexes, a linear space vector was formed (as shown in Eq. (34)) using the indexes that affected the $i$th subsystem, which is defined as the impact factor vector of the subsystems.

$$L_i = (L^1_i, L^2_i, ..., L^{v_s}_i), \quad (34)$$

where $i = 1, 2, ..., n$, and $v_s$ is the number of indexes after screening.

4 Determination of Information Weight Based on CBWM

The weakness ranking of a subsystem is affected by many factors, but the degree of each factor differs. Therefore, it is necessary to weight each factor. The linear space vector is formed by the weights of all factors $1 - v_s$ and is defined as the weight vector of the impact factors, as expressed in Eq. (35):

$$W = (W^1, W^2, ..., W^{v_s}). \quad (35)$$

To reduce the effect of subjective cognitive differences on the analysis result, the CBWM was applied to obtain the accurate weight of each component in the impact factor vector [30]. The process to determine the information weight based on the CBWM is as follows.

Step 1: Determine the set of criteria. The set of criteria in this study is the effects of weak links after screening, defined as $G = \{g^1, g^2, ..., g^{v_s}\}$.

Step 2: Determine the best criteria $g_B$ and the worst criteria $g_w$.

Step 3: Compare $g_B$ with other criteria to establish a comparison vector $A_B = (a_{1B}, a_{2B}, ..., a_{v_sB})$ based on the scale shown in Table 6.

Step 4: Compare $g_W$ with other criteria to establish the comparison vector $A_W = (a_{1W}, a_{2W}, ..., a_{v_sW})$, according to the same scale shown in Table 6.

Step 5: Judge the consistency of $A_B$ and $A_W$ based on the consistency index shown in Eq. (36):

$$C_{CBWM} = \frac{1}{v_s} \sum_{v=1}^{v_s} \left( \frac{a_{BV} + a_{vW} - a_{BW}}{\kappa} \right)^2, \quad (36)$$

where $a_{BW}$ is the difference between the most different criteria, and $\kappa$ is the maximum scale.

Step 6: Calculate the weight. When $A_B$ and $A_W$ are exactly the same, the weight is calculated using Eqs. (37)–(38). Otherwise, calculate the weight by minimizing the maximum deviation method based on Eqs. (39)–(40).

Table 5 Final screening result

| Maximum-criticality index | $\bar{X}_i$ | $E_i$ | $D_i$ | $T_i$ | $F_i$ |
|---------------------------|-------------|-------|-------|-------|-------|
| $RPN_i$                   | $\sqrt{\cdot}$ |       | $\sqrt{\cdot}$ | $\sqrt{\cdot}$ | $\sqrt{\cdot}$ |
| $C_i$                     | $\sqrt{\cdot}$ | $\sqrt{\cdot}$ | $\sqrt{\cdot}$ | $\sqrt{\cdot}$ |
| $CR_i$                    | $\sqrt{\cdot}$ | $\sqrt{\cdot}$ | $\sqrt{\cdot}$ | $\sqrt{\cdot}$ |

Table 6 Difference scale

| Scale                   | Difference |
|-------------------------|------------|
| Equally important       | 0          |
| Slightly important      | 1          |
| More important          | 2          |
| Moderately important    | 3          |
| Obviously important     | 4          |
| Very important          | 5          |
| Especially important    | 6          |
| Greatly important       | 7          |
| Extremely important     | 8          |
where \( \tau_v \) is the intermediate variable; \( \tau_B \) and \( \tau_W \) are the corresponding values of the optimal and worst criteria, respectively.

5 Weakness Ranking of Subsystems Based on TOPSIS

The TOPSIS is applied to the weakness ranking of subsystems [31], and the detailed process is described as follows:

Step 1: Based on the impact factor vector of each subsystem in Eq. (34), the optimal and worst vectors are constructed using Eqs. (41), (42), which are conditions of the weakest and least weak subsystems, respectively.

\[
\tau_v = \frac{1}{v_a} \sum_{v_i=1}^{v_a} a_{Bv} + \kappa - a_{Bv}, \tag{37}
\]

\[
\tau_v = \kappa - \frac{1}{v_a} \sum_{v_i=1}^{v_a} a_{Wv} + a_{Wv} \tag{38}
\]

\[
W^v = \frac{\tau_v}{v_a \kappa} (v = 1, 2, \cdots, v_a), \tag{39}
\]

\[
\min \max_{v} |\tau_B - \tau_v - a_{Bv}|, |\tau_v - \tau_W - a_{Wv}|, \tag{40}
\]

where \( \tau_v \) is the intermediate variable; \( \tau_B \) and \( \tau_W \) are the corresponding values of the optimal and worst criteria, respectively.

Step 2: Substitute the weights and influence factors into Eqs. (43)–(44) and calculate the distance between all indexes and the distance between \( L^+ \) and \( L^- \).

\[
d^+ = \frac{1}{v_a} \sum_{v_i=1}^{v_a} W^v (L^+ - L^+_i)^2, \tag{43}
\]

\[
d^- = \frac{1}{v_a} \sum_{v_i=1}^{v_a} W^v (L^- - L^-_i)^2, \tag{44}
\]

where \( L^+_v \) and \( L^-_v \) are the \( v \)th screening index components of \( L^+ \) and \( L^- \), respectively.

Step 3: Calculate the closeness between the weakest conditions of each subsystem using Eq. (45):

\[
G_i = \frac{d^-}{d^+_i + d^-_i}. \tag{45}
\]

Step 4: The subsystems are ranked based on the value of \( G_i \). The closer the value is to 1, the closer is the subsystem to the weakest condition. The closer the value is to 0, the closer is the subsystem to the non-weak condition. In other words, the subsystem with the largest value is the weakest subsystem.

6 Numerical Example

A heavy-duty horizontal lathe is a heavy-duty machine tool with a large transverse dimension that is widely used in aerospace, thermal power, and other industries [32]. Herein, a certain heavy-duty horizontal lathe is presented as an example to rank the weakness of subsystems based on the proposed method. The failure data were collected from field tests in cooperative enterprises by researchers.

The failure rate is presented as an example to analyze the FMECA of each subsystem, and the drilling device is presented as an example to analyze other indexes. The 565 failure data of the heavy-duty horizontal lathe collected were sorted into 17 subsystems, and the result is shown in Table 7.

The failure data of the drilling device are listed in Table 8. The calculation results of the generalized

| Name                  | Code | Frequency | Frequency |
|-----------------------|------|-----------|-----------|
| Basic component       | BC   | 0         | 0         |
| Headstock             | BB   | 17        | 0.030     |
| Feed system           | FS   | 47        | 0.083     |
| Tool holder           | TU   | 20        | 0.035     |
| CNC system            | NC   | 28        | 0.050     |
| Electrical system     | ES   | 60        | 0.106     |
| Chip removal system   | CC   | 33        | 0.058     |
| Hydraulic system      | HS   | 143       | 0.253     |
| Center rest           | CF   | 48        | 0.085     |
| Spider device         | CD   | 13        | 0.023     |
| Tailstock             | TS   | 14        | 0.025     |
| Grinding device       | GD   | 7         | 0.012     |
| Drilling device       | DD   | 2         | 0.004     |
| Protective device     | PD   | 41        | 0.073     |
| Cooling system        | CS   | 62        | 0.110     |

Table 7 Failure frequency of subsystems in heavy-duty horizontal lathe

Table 8 Failure data of the drilling device
FMECA information are shown in Table 8. After normalization and assimilation, the final results obtained are as shown in Table 10. The indexes of other subsystems were calculated in the same manner.

Three indices of the criticality degree were screened, and the judgment matrix is shown in Table 11. The weights of each index are shown in Table 12.

Based on the weights, \( CR_i \) was selected to represent the critically degree, and other indexes selected were denoted by "★" in Table 10. The weights of the five selected indexes were determined based on the CBWM using the parameters shown in Table 13.

The weakness of the subsystems was ranked using the TOPSIS, and the result is shown in Table 14.

As shown in Table 14, the hydraulic system was the weakest subsystem of the heavy-duty horizontal lathe. This is because the heavy-duty machine tool must support a heavy load in many locations, such as the hydrostatic bearing of the spindle, the hydrostatic guide of the tool holder, and the drive lifting of the center rest. Because the hydraulic system is the key subsystem of the

| Table 8 Failure information of drilling device |
|-----------------------------------------------|
| Item                                      | No. 1                             | No. 2 |
| Failure phenomenon                        | Scorpion cannot move              | Squat motor was not functioning |
| Failure type                              | Technology type                    | Loosening type                    |
| Failure mode                              | Positioning accuracy exceeds the standard | Poor contact                     |
| Failure reason                            | Squat center is not on the established axis | Poor line contact                 |
| Causality classification                  | Poor assembly                      | Loose                             |
| Failure treatment                         | Adjust the center of the sley to the specified axis | Rewiring                          |

| Table 9 FMECA information indexes of drilling device |
|-------------------------------------------------------|
| Index                          | Unknown assignment | Result |
| \( \lambda_{13} \) Failure frequency of subsystems        | 0.004               |
| \( E_{13} \) \[3, 3, 4, 5, 3\] \[3, 4, 4, 5\] | 9.393               |
| \( D_{13} \) \[2, 3, 4, 4\] \[2, 3, 4\] | 5.556               |
| \( RPN_{13} \) \( OPR_{13,1} = 2, OPR_{13,2} = 1 \) | 38.423              |
| \( C_{13} \) \[3, 3, 3, 3\] | 9.891               |
| \( CR_{13} \) \[1, 2, 3, 4\] | 0.6                 |
| \( T_{13} \) \[2, 2, 2, 1\] | 3                  |
| \( F_{13} \) \[300, 300, 300, 300, 300\] \[500, 500, 500, 500, 500\] | 2970               |

FMECA information are shown in Table 9. After normalization and assimilation, the final results obtained are as shown in Table 10. The indexes of other subsystems were calculated in the same manner.

| Table 10 Preprocessing values of FMECA information for drilling device |
|-------------------------------------------------------------|
| Index                                      | \( T_j \) | \( *E_j \) | \( *D_j \) | \( RPN_j \) | \( C_j \) | \( *CR_j \) | \( *T_j \) | \( *F_j \) |
| Basic component                              | 0         | 0          | 0         | 0          | 0         | 0          | 0         | 0          |
| Headstock                                    | 0.030     | 0.056      | 0.037     | 0.017      | 0.041     | 0.047      | 0.135     | 0.128      |
| Feed system                                  | 0.083     | 0.092      | 0.047     | 0.052      | 0.080     | 0.127      | 0.191     | 0.187      |
| Tool holder                                  | 0.035     | 0.072      | 0.051     | 0.032      | 0.052     | 0.058      | 0.215     | 0.207      |
| CNC system                                   | 0.050     | 0.069      | 0.080     | 0.173      | 0.055     | 0.087      | 0.022     | 0.021      |
| Electrical system                            | 0.106     | 0.117      | 0.086     | 0.336      | 0.164     | 0.069      | 0.018     | 0.018      |
| Chip removal system                          | 0.058     | 0.038      | 0.027     | 0.027      | 0.030     | 0.017      | 0.014     | 0.012      |
| Hydraulic system                             | 0.253     | 0.140      | 0.189     | 0.082      | 0.229     | 0.344      | 0.111     | 0.116      |
| Center rest                                  | 0.085     | 0.073      | 0.117     | 0.060      | 0.073     | 0.039      | 0.028     | 0.036      |
| Spider device                                | 0.023     | 0.015      | 0.024     | 0.006      | 0.011     | 0.027      | 0.004     | 0.004      |
| Tailstock                                    | 0.025     | 0.034      | 0.068     | 0.019      | 0.026     | 0.027      | 0.076     | 0.079      |
| Grinding device                              | 0.012     | 0.038      | 0.040     | 0.011      | 0.024     | 0.011      | 0.021     | 0.021      |
| Drilling device                              | 0.004     | 0.011      | 0.013     | 0.003      | 0.008     | 0.004      | 0.001     | 0.001      |
| Protective device                            | 0.073     | 0.127      | 0.038     | 0.051      | 0.089     | 0.099      | 0.090     | 0.099      |
| Cooling system                               | 0.110     | 0.034      | 0.079     | 0.084      | 0.056     | 0.028      | 0.033     | 0.032      |
| Lubrication system                           | 0.030     | 0.045      | 0.067     | 0.042      | 0.037     | 0.012      | 0.035     | 0.032      |
| Other                                        | 0.023     | 0.039      | 0.037     | 0.005      | 0.025     | 0.006      | 0.006     | 0.007      |
heavy-duty machine tool, it imposes a significant negative effect on the entire machine tool when the hydraulic system malfunctions. In addition, most failures of the hydraulic system are caused by solid particles in the oil, as indicated by field statistic data; as such, significant costs will be incurred to clean the oil and resume production. It is clear that the ranking result obtained using the proposed method is practicable for engineering practice.

To verify the effectiveness of the proposed method, the analysis was performed using the traditional FMECA method, of which the result is shown in Table 15.

As shown in Table 15, the electrical system was the weakest subsystem of the heavy-duty horizontal lathe, followed by the CNC system, cooling system, hydraulic system, and center rest. According to the comparison of weakness ranking of subsystems between the traditional FMECA and the proposed method in Figure 1,
the ranking result obtained using the traditional FMECA method differed significantly from that obtained using the proposed method. The main failure modes of an electrical system are short circuit and loose connection; however, they are easy to rectify and incur low maintenance costs. Therefore, the failure of an electrical system imposes minimal effect on the personnel, machine, and environment. In addition, in the actual condition, the typical failure modes of the center rest and cooling system are motor failure and insufficient coolant, respectively, which are easy to detect and maintain preventatively. Therefore, they are unlikely to cause severe faults in the machine tool and should be assigned with lower critical degrees. In summary, the weakness ranking result for the subsystems of a heavy-duty machine tool deviated from the actual situation when using the traditional FMECA method. Therefore, it can be concluded that the proposed weakness ranking method of subsystems for heavy-duty machine tools is more effective than the traditional method. Additionally, the weakness of the subsystems should be ranked while considering the maintainability and maintenance cost of the heavy-duty machine tool.

In summary, the proposed method is effective and will contribute positively to the design and reliability improvement of complex electromechanical products, including heavy-duty machine tools.

Acknowledgements
Not applicable.

Authors' Contributions
ZY and HT were in charge of the whole trial; JG and CC wrote the manuscript; YZ and JL assisted with sampling and data analyses. All authors read and approved the final manuscript.

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Funding
Supported by National Natural Science Foundation of China (Grant Nos. 51675227, 51975249), Jilin Province Science and Technology Development Funds (Grant Nos. 201802010070G, 2019C0362017G), Technology Development and Research of Jilin Province (Grant No. 2019JC037-01), Changchun Science and Technology Planning Project (Grant No. 1955011), and National Science and technology Major Project (Grant No. 2014ZX04015031).

Competing Interests
The authors declare no competing financial interests.

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[2] Y Li, Y Wang, Y He, et al. Modeling method for flexible energy behaviors in CNC machining systems. Chinese Journal of Mechanical Engineering, 2018, 31: 6.

[3] Y Xiong, Y Cheng, M Xu, et al. Reliability assessment of heavy-duty computer numerical control machine tools based on multi-performance multi-sequence hidden Markov model. Quality Engineering, 2020, 32(3): 409-420.

[4] T Jin, Z Yang, D Wang, et al. Reliability modeling for hydraulic components of heavy duty machine tools in distribution of degradation amount for oil contamination profile. China Mechanical Engineering, 2020, 31(13): 1613-1620, 1628. (In Chinese)

[5] F Zhang, S Han, J Liu, et al. Research of heavy NC machine reliability evaluation method based on Bayes theory. Modern Manufacturing Engineering, 2015, 8: 122-1251.

[6] H Huang, Z Liu, J Mi, et al. Reliability modeling and analysis of heavy-duty CNC machine tool spindle under hybrid uncertainty. Scientia Sinica, 2018, 1: 42-53.

[7] J Wang, Z Wang, Q Liu, et al. Reliability prediction of rotary working table for CXK5463 turning and milling machining center based on analogous argumentation method of similar products. Machine Tool and Hydraulics, 2016, 44(21): 164-167.

[8] M Catelani, L Ciani, M Venzì. Failure modes, mechanisms and effect analysis on temperature redundant sensor stage. Reliability Engineering and System Safety, 2018, 160: 425-433.

[9] K.W Yun. Failure modes and risk assessment of rotary compressor under extraordinary operating conditions. 6th International Compressor Engineering Conference, West Lafayette, IN, July 9–12, 2000: 1382.

[10] O Latachi, T Rachidi, M Karim, et al. Reusable and reliable flight-control software for a fail-safe and cost-efficient cubesat mission. Design and Implementation, 2020, 7(46): 146.

[11] I B Brahim, S A Addouche, A E Mhamedi, et al. Build a Bayesian network for FMECA in the production of automotive parts: diagnosis and prediction. IFAC-PapersOnLine, 2019, 52(13): 2572-2577.

[12] A K Mohanty, P R Dash, P K Pradhan. FMECA analysis and condition monitoring of critical equipment in super thermal power plant. International Journal of System Assurance Engineering and Management, 2020, 11: 583-599.

[13] A Certa, F Hopp, R Inghilleri, et al. A Dempster-Shafer theory-based approach to the failure mode, effects and criticality analysis (FMECA) under epistemic uncertainty: Application to the propulsion system of a fishing vessel. Reliability Engineering and System Safety, 2017(159): 69-79.

[14] A E Brom, I N Omelchenko, O V Belova. Lifecycle costs for energy equipment FMECA for gas turbine. Procedia Engineering Oil and Gas Engineering. Procedia Engineering, 2016, 152: 177-181.

[15] Y Wu, H Cao, C Gao, et al. Calculation of optimal segment number of N-segment n-connection mode for overhead lines. Smart Power, 2019, 47(12): 98-102.

[16] M Mazzuqi, S Carpitella, A Certa, et al. Assessing supply chain risks in the automotive industry through a modified MCDM-based FMECA. Proceses, 2020, 8(5): 579.

[17] B Suo, L Zhao, Y Yan. A novel Dempster-Shafer theory-based approach with weighted average for failure mode and effects analysis under uncertainty. Journal of Loss Prevention in the Process Industries, 2020, 65: 104145.

[18] D Piulatti, J Sini, S Borlo, et al. Multilevel simulation methodology for FMECA study applied to a complex cyber-physical system. Electronics, 2020, 9(10): 1736.

[19] N M Thoppil, V Vasu, C S P Rao. Failure mode identification and prioritization using FMECA: A study on computer numerical control lathe for predictive maintenance. Journal of Failure Analysis and Prevention, 2019, 19(4): 1153-1157.

[20] A Jomde, V Bhojwani, S Kedia, et al. Failure modes effects and criticality analysis of the linear compressor. Materials Today: Proceedings, 2017, 4(9): 10184-10188.

[21] B Goo, J Lee, S Seo, et al. Design of reliability critical system using axiomatic design with FMECA. International Journal of Naval Architecture and Ocean Engineering, 2017, 11(1): 11-21.

[22] M Ghali, M Tlilja, N Aflaoui, et al. A CAD method for tolerance allocation considering manufacturing difficulty based on FMECA tool. International Journal of Advanced Manufacturing Technology, 2017, 91: 2435-2446.

[23] R Deolath, J Jinghoanie, C Riverol. Direct methanol fuel cell system reliability analysis. International Journal of Hydrogen Energy, 2017, 42(16): 12032-12045.

[24] U Okoro, A Koliis, L Cui. Multi-criteria risk assessment approach for components risk ranking - The case study of an offshore wave energy converter. International Journal of Marine Energy, 2016, 17: 21-39.

[25] A Zhang, L Cui, P Zhang. Research on the maintenance strategy of CNC machine tool. Proceedings of 20th International Conference on Industrial Engineering and Engineering Management, Baochu, China, 2013: 583-589.

[26] D J Lawson. Failure mode, effect and criticality analysis. Electronic Systems Effectiveness and Life Cycle Costing, 1983, 3: 55-74.

[27] H Wang, L Sun, L Shi. Failure mode, effect and criticality analysis of CNC grinder. Applied Mechanics and Materials, 2012, 141: 284-288.

[28] Z Huang. Evaluating intelligent residential communities using multi-strategic weighting method in china. Energy and Buildings, 2014, 69(1): 144-153.

[29] M Ohki, S Hayashi, M Ohkita. Fast computational algorithm of a fuzzy reasoning using the product-sum-gravity method. Systems and Computers in Japan, 2000, 31(3): 40-48.

[30] J Rezaei. Best-worst multi-criteria decision-making method. Omega, 2015, 53: 49-57.

[31] J D Martinez-Morales, E R Palacios-Hernández, G A Velázquez-Carrillo. Artificial neural network based on genetic algorithm for emissions prediction of a SI gasoline engine. Journal of Mechanical Science and Technology, 2014, 28(6): 2417-2427.

[32] U Goark, J C Francisco, J Z Juan, et al. Preventing chatter vibrations in heavy-duty turning operations in large horizontal lathes. Journal of Sound and Vibration, 2015, 340: 317-330.