Searching for possible $\Omega_c$–like molecular states from meson-baryon interaction

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Stimulated by the observations of recent observation of $\Omega_c$ as well as former one of $P_c(4380)$ and $P_c(4450)$, we perform a coupled channel analysis of $\Xi_c^*\bar{K}/\Omega_c\eta/\Xi_c^*\bar{K}/\Xi_c^*\bar{K}/\Omega_c\omega$ systems to search for possible $\Omega_c$–like molecular states by using a one-boson-exchange potential. Our results suggest there exists a loosely bound molecular state, a $\Xi_c^*\bar{K}/\Omega_c\eta/\Xi_c^*\bar{K}/\Xi_c^*\bar{K}/\Omega_c\omega$ with $I(J^P)=0(3/2^-)$, it is mainly composed of the $\Xi_c^*\bar{K}$ system. Two-body strong decay width is also studied, where we find that $\Xi_c^*\bar{K}$ is the dominant decay channel.

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I. INTRODUCTION

In the past 14 years, a series of novel phenomena relevant to the XYZ states have been reported with the accumulation of experimental data. These observations provide us good chance to identify the exotic configurations (multiquark states, glueball, hybrid) of hadron (see review papers [1, 2] for more details). In fact, experimental and theoretical studies on exotic states are a good approach to deepen our understanding of nonperturbative behavior of quantum chromodynamics (QCD).

In 2015, $P_c(4380)$ and $P_c(4450)$ have been reported by the LHCb Collaboration in the $\Lambda_{b}^0 \to J/\psi pK^-$ decay [3], which stimulated theorists’s great enthusiasm of investigating the heavy pentaquarks. As shown in figure 1, the masses of $P_c(4380)$ and $P_c(4450)$ are close to the thresholds of a pair of charmed baryon and anti-charmed meson, theorists have tried to describe their inner structures by hadronic molecules [4–16]. In particular, $P_c(4380)$ was interpreted as a loose molecular pentaquark made up by $\Sigma_c^*D$ with several MeV binding energy. Analogously, with a simple replacement of quarks, it is natural to conjecture whether there exist possible $\Omega_c$–like molecules composed of a charmed-strange baryon and an anti-strange meson. In a sense, this study can enhance our understanding of these $P_c$ states.

![Diagram](image)

FIG. 1: A comparison of the masses of the heavy pentaquarks and the mass thresholds of a pair anti-meson and baryon systems.

In fact, the studies of the interaction between one hadron and one strange meson have carried out for several decades. In these works, observations near a pair of hadron thresholds are often explained as possible molecular states as showed in table I.

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Therefore, searching for exotic states from hadron and strange meson interaction is very important, and a further exploration on a charmed-strange baryon and an anti-strange meson interaction is needed.

### TABLE I: A summary of the newly observations and their corresponding molecular explanations.

| Observations | I(JP) | Explanations | Observations | I(JP) | Explanations |
|--------------|-------|--------------|--------------|-------|--------------|
| X(5568)     | 1(0⁺) | BR [17–23]   | D(2317)      | 0(0⁺) | DK [24–29]   |
| D₁(2460)    | 0(1⁺) | D⁺K [29–32]  | f₀(980)      | 0(0⁺) | KK [33–35]   |
| f₁(1285)    | 0(1⁺) | K⁺K [36–39]  |             |       | N⁰K [40–42]  |

The LHCb Collaboration announced that five new Ωc states were observed in Ξ⁺cK⁻ channel [43]. We should emphasize that no resonant-like signals were found in the Ξ⁺cK⁻ channel, which is a natural case if the observed states are standard three quark states. As their masses are very close to the thresholds of a pair of charmed-strange baryon and anti-strange meson, molecular pentaquarks have been proposed in Ref. [44–48]. In this paper, a comprehensive investigation of Ξ⁺cK⁻/ΩcK⁻/Ωcη/Ξ⁺cK⁺/Ξ⁺cK⁻/ΩcK⁻ interaction will be adopted, which is also helpful to understand the natures of these new Ωc states.

For obtaining the effective potential describing the interaction between the charmed-strange baryon and anti-strange meson, we adopt one-boson-exchange (OBE) model, including π, σ, ρ, ω and φ, which is often used to identify and predict exotic states, such as X/Y/Z/P states. Both the S-D mixing effect and the coupled channel effect will be considered here. This study will provide valuable information to search for Ωc-like molecular states composed of charmed-strange baryon and anti-strange meson. What is more important is that it will also give a test of molecular state picture for P₄ (4380) and P₅ (4450).

This paper is organized as follows. After introduction, in Sec. II, the interaction of charmed-strange baryon and anti-strange meson is studied under considered the OBE model and the coupled channel effect. The decay of the possible molecular candidates is further discussed in Sec. III. The paper ends with a discussion and conclusion in Sec. V.

### II. THE Ξ⁺c⁻K⁺⁻ INTERACTIONS

#### A. Lagrangians

Let us start with the effective Lagrangian approach in the hadronic level. If considering the heavy quark limit and chiral symmetry [49], the effective Lagrangians relevant to the charmed-strange baryon are

\[
L_{B₁} = \frac{i g}{2 f_π} \bar{B}_3 \sigma^{\mu\nu} \left( \pi_\mu - \rho_\mu \right) B_3, \quad L_{B₅} = \frac{i g_π}{2 f_π} \bar{S}_5 \sigma^{\mu\nu} \left( \pi_\mu - \rho_\mu \right) S_5 + \frac{3}{2} \frac{g_π}{f_π} \epsilon^{\mu\nu\lambda\kappa} v_\lambda \left( \bar{S}_5 \sigma^{\mu\nu} \right) + \frac{g_π}{f_π} \bar{S}_5 F^{\mu\nu} (\rho) S_5, \quad L_{B₅B₅} = i g_3 \bar{S}_5 \sigma^{\mu\nu} \pi_\mu v_\nu + h.c.
\]

In the above formula, \(A_μ\) and \(V_μ\) are the axial current and vector current, respectively

\[
A_μ = \frac{1}{2} (\xi squared) \frac{\partial_μ \xi}{ξ \partial_μ \xi²} = \frac{i}{2 f_π} P, \quad V_μ = \frac{1}{2} (\xi squared) \frac{\partial_μ \xi}{ξ \partial_μ \xi²} = \frac{i}{2 f_π} [P, \partial_μ P] + \ldots
\]

with \(ξ = \exp (ip/f_π)\) and the pion decay constant \(f_π = 132\ MeV\), \(ρ_μ = ig_π V_μ / \sqrt{2}\), and \(F^{μν}(ρ) = \partial^ρ ρ^ν - \partial^ν ρ^μ + [ρ^μ, ρ^ν]\). \(S\) is defined as a superfield, which includes \(B_3\) with \(J^P = 1/2⁺\) and \(B₅\) with \(J^P = 3/2⁺\) in the 6F flavor representation, i.e.,

\[
S = \sqrt{2} (γ_μ + ν_μ) γ⁵ B_3 + B₅ γ⁵. \text{ Matrices } B₃, B₅(ρ), P and V are expressed as
\]

\[
B₃ = \begin{pmatrix} \Lambda⁺ & Ξ⁺ & \Xi⁺ \\ -Ω⁺ & 0 & Ξ⁰ \\ -Ξ⁺ & -Ξ⁰ & 0 \end{pmatrix}, \quad B₅(ρ) = \begin{pmatrix} Σ⁺ & Σ⁰ & Ξ⁺ \\ Σ⁺ & Σ⁰ & Ξ⁺ \\ Ξ⁺ & Ξ⁺ & Ω⁺ \end{pmatrix} \quad P = \begin{pmatrix} Σ⁺ & Σ⁺ & Σ⁺ \\ Ξ⁺ & Ξ⁺ & Ξ⁺ \\ Ω⁺ & Ω⁺ & Ω⁺ \end{pmatrix} \quad V = \begin{pmatrix} ρ⁺ & K⁺ & K⁺ \\ K⁺ & K⁺ & K⁺ \\ K⁺ & K⁺ & K⁺ \end{pmatrix}
\]

For the strange meson part, the effective Lagrangians are constructed as [50, 51]

\[
\mathcal{L} = \mathcal{L}_{PPV} + \mathcal{L}_{VVV} + \mathcal{L}_{VVV} = \frac{ig}{\sqrt{2}} (\partial^μ P (PV_μ - V_μ P) + \frac{g_{VVV}}{\sqrt{2}} (\partial_μ P, V_λ V_ν P) + \frac{ig}{2 \sqrt{2}} (\partial^μ V_ν (V_μ V_ν - V_ν V_μ)).
\]
By expanding Eqs. (1)-(4), one can further obtain

\[ L_{B_0 B_\sigma} = l_B \langle \bar{B}_3 \sigma B_3 \rangle, \]

\[ L_{\bar{g}g}^{\sigma_{\mu\nu}} = -l_S \langle \bar{B}_0 \sigma B_0 \rangle + l_S \langle \bar{B}_{3\mu} \sigma B_0^\nu \rangle - \frac{l_S}{\sqrt{3}} \langle \bar{B}_{3\mu} \sigma (\gamma^\mu + \gamma^\nu) \gamma^5 B_0 \rangle + h.c., \]

\[ L_{B_0 B_\sigma V} = \frac{1}{\sqrt{2}} \beta g \bar{V} \langle \bar{B}_3 \gamma^5 \cdot V B_3 \rangle, \]

\[ L_{\bar{g}g}^{\sigma_{\mu\nu}} = \frac{g_\rho}{2 f_\pi} \rho^{\mu\nu} \bar{V}_3 \langle \bar{B}_3 \gamma^5 (\partial^\mu V^\nu - \partial^\nu V^\mu) B_3 \rangle + h.c., \]

\[ L_{\bar{g}g}^{\sigma_{\mu\nu}} = \frac{-g_\rho}{2 f_\pi} \rho^{\mu\nu} \bar{V}_3 \langle \bar{B}_3 \gamma^5 (\partial^\mu V^\nu - \partial^\nu V^\mu) B_3 \rangle + h.c., \]

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Here, \( \tau \) is Pauli matrix, \( \pi \) and \( \rho \) denote the pion and rho meson isospin triplets, respectively. For the \( \sigma \) exchange vertex, we use

\[ L_{\sigma K^0 K^*} = g_\sigma \bar{K} K \sigma + g_\sigma \bar{K} K^* \sigma. \]

Coupling constants in above Lagrangians are collected in table II. The corresponding phase factors between these coupling constants are determined by quark model.

| \( l_8 \) | \( \beta g \bar{V} \) | \( l_5 \) | \( g_1 \) | \( \beta g \bar{V} \) | \( \lambda g \bar{V}(\text{GeV}^{-1}) \) | \( g_4 \) | \( \lambda g \bar{V}(\text{GeV}^{-1}) \) | \( g_\sigma \) | \( g \) |
|---|---|---|---|---|---|---|---|---|---|
| -3.65 | -6.0 | 7.3 | 1.0 | 12.0 | 19.2 | 1.06 | -6.8 | 3.65 | 12.0 |

By adopting the Breit approximation, the effective potentials can be related to the scattering amplitudes by

\[ \chi_{h_1 h_2 \to h_3 h_4}(q) = \frac{M(h_1 h_2 \to h_3 h_4)}{\sqrt{\left|M_f \right| \left|M_f \right|}}. \]

For this scattering process \( h_1 h_2 \to h_3 h_4 \), \( M_f \) and \( M_i \) are the masses of the initial states (\( h_1, h_2 \)) and final states (\( h_3, h_4 \)), respectively. \( M(h_1 h_2 \to h_3 h_4) \) denotes the scattering amplitude for the \( h_1 h_2 \to h_3 h_4 \) process by exchanging one boson (\( \sigma, \pi, \rho, \omega, \phi \)) in the channel. The effective potential in the coordinate space \( V(r) \) can be obtained through performing Fourier transformation, i.e.,

\[ V_{h_1 h_2 \to h_3 h_4}(r) = \int \frac{d^3 q}{(2\pi)^3} e^{iq \cdot r} \chi_{h_1 h_2 \to h_3 h_4}(q) F^2(q^2, m_E^2). \]

In the above equation, a monopole form factor \( F(q^2, m_E^2) = (\Lambda^2 - m_E^2)/(\Lambda^2 - q^2) \) is introduced at every interactive vertex to express the off-shell effect of the exchanged boson, where \( \Lambda, m_E \) and \( q \) are the cutoff, mass and four-momentum of the exchanged meson, respectively.
B. A single channel analysis

In this subsection, we discuss the single $\Xi^cK$ system before performing our full coupled channel study. This is useful to indicate the importance of the coupled channels of $\Xi^cK/\Xi^c\bar{K}/\Xi^cK^*$. The OBE effective potential for the S-wave $\Xi^cK$ system is given as

$$V_{1}^{\Xi^cK}(r) = -\frac{1}{2}k_g g_{xy} Y(\Lambda, m_\sigma, r) - \frac{1}{16} \beta g_g g \left[ G(I) Y(\Lambda, m_\rho, r) + Y(\Lambda, m_\omega, r) - 2Y(\Lambda, m_\phi, r) \right],$$

(19)

where $G(I)$ is the isospin factor; 3 for the isoscalar system, and -1 for the isovector system. The corresponding flavor wave functions $|l, l_0\rangle$ for $\Xi^cK$ system are $|0, 0\rangle = (|\Xi^cK^+\rangle + |\Xi^cK^-\rangle)/\sqrt{2}$, $|1, 1\rangle = |\Xi^cK^0\rangle$, $|1, -1\rangle = |\Xi^cK^-\rangle$ and $|1, 0\rangle = (|\Xi^cK^+\rangle - |\Xi^cK^-\rangle)/\sqrt{2}$, respectively. In addition, function $Y(\Lambda, m, r)$ denotes

$$Y(\Lambda, m, r) = \frac{1}{4\pi r} (e^{-mr} - e^{-\Lambda r}) - \frac{\Lambda^2 - m^2}{8\pi\Lambda} e^{-\Lambda r}.$$

(20)

Here the cutoff $\Lambda$ is a parameter of the OBE potential. As is known well, its reasonable values are around $\Lambda \sim 1$ GeV \cite{53, 54}, as corresponding to the typical hadronic scale or to the intrinsic size of hadrons. As we will see in the following sections, in the present paper, we attempt to find bound state solutions by varying the cutoff parameter to obtain binding energy around a few to ten MeV for loosely bound molecular hadrons. The physical relevance of the results will be discussed in terms of the cutoff parameters.

In figure 2, we present the $r$ dependence of the effective potentials for $\Xi^cK$ system with $I = 0, 1$, and its cutoff $\Lambda$ is taken as 1.25 GeV.

According to the effective potential in figure 2, we can summarize that:

- There does not exist $\pi$–exchange contribution as the spin-parity conservation forbids the vertex $\bar{K} - \bar{K} - \pi$.
- The interactions from $\sigma$–exchange is attractive, and it is dominant, which are consistent with the conclusions in Ref. \cite{52}.
- As discussed in Ref. \cite{52}, the interaction of $\omega$–exchange is different for a hadron-hadron system with light quark-quark or quark-antiquark combinations. For the $\Xi^cK$ system, its quark configuration is $(c s \bar{q}) - (\bar{s} q)$. The $\omega$–exchange from the combination of $q - \bar{q}$ provides attractive interaction.
- In comparison with the $\omega$–exchange, $\phi$ meson couples to the strange quark(s) and/or anti-strange quark(s). Here, the $s - \bar{s}$ combination from the $\Xi^cK$ system implies that the interaction from the $\phi$–exchange is repulsive.
- The $\rho$ meson couples to the isospin charge, which explains that in the isoscalar $\Xi^cK$ system, the interaction from $\rho$–exchange is attractive and almost three times stronger than that in the isovector $\Xi^cK$ system.

By adopting the effective potential in equation (19), we numerically solve the Schrödinger equation, with the cutoff $\Lambda$ being scanned from 1 to 5 GeV. Finally, only the $\Xi^cK$ state with $|I(J^P) = 0(3/2^-)|$ can generate bound solutions when $\Lambda$ is taken larger than 1.60 GeV. As the cutoff $\Lambda$ is increased, it binds deeper and deeper. In particular, when $\Lambda$ is fixed as 2.30 GeV, its binding energy reaches as -21.15 MeV. Then the mass of the $\Xi^cK$ state with $I(J^P) = 0(3/2^-)$ appears close to the observed mass of the newly reported $\Omega_c(3119)$ in \cite{43}. However, if we take the cutoff value $\Lambda \sim 1$ GeV \cite{53, 54} as a typical hadron scale and is more reasonable than $\Lambda \sim 2$ GeV, our results indicate there don’t exist any bound solutions for the S-wave $\Xi^cK$ system, although the total OBE effective potential is attractive. For higher partial waves, due to the repulsive centrifugal potential, it is not likely for bound states exist.

To summarize, for the single $\Xi^cK$ system, the intermediate range and short range forces from the OBE model are not strong enough to generate bound molecular states. Therefore, the newly $\Omega_c(3119)$ cannot be a pure $\Xi^cK$ molecular state. In Ref. \cite{48},
the authors proposed that the newly $\Omega_c(3119)$ can be associated with a meson-baryon state strongly coupled to $\Xi^*K$ and $\Omega^*_c\eta$ with $J^P = 3/2^-$ by using the vector meson exchange interaction in the local hidden gauge method. We can find that the vector exchange model provides an attractive interaction, and the coupled channel effect is very important, which is also consistent with our present results: only by considering the single $\Xi^*K$ system with vector exchange model cannot reproduce the $\Omega_c(3119)$. Thus, in the next section, we will adopt the coupled channel approach to do further study. In our present study, we take into account the long range force from pion exchange, tensor force and S-D wave mixing, which we believe important and was not considered in Ref. [48].

C. $\Xi^*/\Omega_c/\eta/\Omega^*_c/\Xi^*/\Xi^*/\Omega^*_c/\omega$ coupled channel system

In this section, we consider coupled channel studies with the one-pion exchange interaction included, as expected from the nucleon-nucleon interaction. We include the three channels $\Xi^*/\Omega_c/\eta/\Omega^*_c/\Xi^*/\Xi^*/\Omega^*_c/\omega$, and possible quantum numbers are summarized in table III.

| $J^P$ | Channels |
| --- | --- |
| $1/2^-$ | $\eta/\Omega_c: \left| \frac{3}{2} \right|, \Omega^*_c: \left| \frac{3}{2} \right|, \Xi^*: \left| \frac{3}{2} \right|, \Xi^*: \left| \frac{3}{2} \right|, \omega/\Omega_c: \left| \frac{3}{2} \right|$ |
| $3/2^-$ | $\eta/\Omega_c: \left| \frac{3}{2} \right|, \Omega^*_c: \left| \frac{3}{2} \right|, \Xi^*: \left| \frac{3}{2} \right|, \Xi^*: \left| \frac{3}{2} \right|, \omega/\Omega_c: \left| \frac{3}{2} \right|$ |
| $5/2^-$ | $\eta/\Omega_c: \left| \frac{3}{2} \right|, \Omega^*_c: \left| \frac{3}{2} \right|, \Xi^*: \left| \frac{3}{2} \right|, \Xi^*: \left| \frac{3}{2} \right|, \omega/\Omega_c: \left| \frac{3}{2} \right|$ |

The general expressions of spin-orbit wave functions $|\frac{3}{2} \pm 1, L_J\rangle$ for the investigated systems are constructed as

$$\Omega_c/\eta : |\frac{3}{2} \pm 1, L_J\rangle = \sum_{m, m_J} |C_I^{LM}_{m, m_j} \chi_{4, m} Y_{L, m_J}\rangle,$$

$$\Xi^*/\Omega_c/\eta : |\frac{3}{2} \pm 1, L_J\rangle = \sum_{m, m_J} |C_I^{LM}_{m, m_j} \Phi_{m, m_j} Y_{L, m_J}\rangle,$$

$$\Xi^*/\Xi^*/\Omega_c/\omega : |\frac{3}{2} \pm 1, L_J\rangle = \sum_{m, m_J} |C_I^{LM}_{m, m_j} C_{\mu_{m, m_j}} \chi_{4, m} \epsilon^m m | Y_{L, m_J}\rangle.$$

Here, $C_I^{LM}_{m, m_j}$, $C_{\mu_{m, m_j}}$, and $C_{\mu_{m, m_j}}$ are the Clebsch-Gordan coefficients, $\chi_{4, m}$ and $Y_{L, m_J}$ stand for the spin wave function and the spherical harmonics function, respectively. The polarization vector $\epsilon$ for the vector meson is defined as $\epsilon^m = \frac{1}{\sqrt{2}} (\epsilon^m_0 + i \epsilon^m_1)$ and $\epsilon^m_0 = \epsilon^m_1$, which satisfy $\epsilon^m_{\pm1} = \frac{1}{\sqrt{2}} (0, \pm 1, 0)$ and $\epsilon_0 = (0, 0, 0, -1)$. The polarization tensor $\Phi_{m, m_j}$ for baryon $\Xi^*$ with spin-3/2 is constructed as $\Phi_{m, m_j} = \sum_{m, m_J} |\frac{3}{2} \pm 1, L_J\rangle Y_{L, m_J}$.

The total effective potential for the $\Xi^*/\Omega_c/\eta/\Omega^*_c/\Xi^*/\Xi^*/\Omega^*_c/\omega$ system is given in a matrix form,
where subscripts $I$ and $J^P$ are the isospin and spin-parity for the $\Xi_c^* \bar{K}/\Omega_c \eta/\Omega_c^* \eta/\Xi_c^* \bar{K}^*/\Xi_c^* \bar{K}^*/\Omega_c \omega$ system, respectively. The superscript stands for the corresponding scattering process. The concrete expressions for each component shown in equation (21) are presented in section appendix A.

With the preparation of the effective potentials, in the following, we will solve the coupled Schrödinger equation to search for bound states, where cutoff $\Lambda$ is scanned from 1 GeV to 5 GeV. Before we show our numerical results, several remarks are in order for the relation between a loosely bound molecular scenario and the cutoff parameter $\Lambda$ introduced in the form factor.

1. For a molecular state composed of hadrons $A$ and $B$, its mass is determined as $M = M_A + M_B + E$, here, $E$ is the binding energy. For large $r$ where the interaction is sufficiently suppressed, the asymptotic form of wave function for the $S$–wave $AB$ molecular state has the form $\psi(r) \sim e^{-\sqrt{2M}r}/r$, where $\mu = M_AM_B/(M_A + M_B)$ is the reduced mass. By using the approximate wave function, we find the relation between the size and the binding energy of the molecular state, $R \sim 1/\sqrt{2\mu E}$. For a loosely bound state, the size of the system should be much larger than the size of all component hadrons, say a few fm or larger. Therefore a good candidate of molecular states should have a binding energy around several MeV or less.

2. In the coupled channel approach, the binding energy $E$ is measured from the the lowest mass threshold, $M_{th}$, among various channels. Here we introduce another energy $E'$ measured form the mass threshold of the most dominant channel $M_{dom}$, $E' = E + (M_{th} - M_{dom})$. For example, if there is a bound state for the $\Xi_c^* \bar{K}/\Xi_c^* \bar{K}^*/\Xi_c^* \bar{K}^*/\Omega_c \bar{K}^*/$ system, and $\Xi_c^* \bar{K}^*$ is the dominant channel, $E' = E + (M_{\Xi_c^* \bar{K}^*} - M_{\Xi_c^* \bar{K}^*})$, which may amount to be several hundreds MeV. If $E' \sim 200$ MeV, by using the relation $R \sim 1/\sqrt{2\mu E'}$, we find that the size $R$ turn out to be about 0.5 fm or even less, which can not be identified with a molecular state.

3. Because the hadrons are not point-like particles, we introduce a form factor in a monopole manner in every interaction vertex. As a typical hadronic scale, in Ref. [53, 54], a reasonable value of cutoff $\Lambda$ should be around 1 GeV.

According to the above discussions, in table IV, results for the binding energies, root-mean-square radii $r_{RMS}$, probabilities of various components of wave functions are shown for all possible spin-parity configurations. There, the binding energies are measured from the lowest threshold, for which we show the results around a few MeV by tuning the cutoff $\Lambda$ to respect the criterion for the molecular state. For each case, we show the results for three different $\Lambda$ values to show possible $\Lambda$ dependence. From the the numerical results presented in table IV, we see that:

- For $I(J^P) = 0(3/2^-)$, when the cutoff $\Lambda$ is taken around 1.30 GeV, there exists one bound state solution with several MeV binding energy and its root-mean-square (RMS) radius larger than 1 fm. Compared to the results for the deuteron [53, 54], the $\Xi_c^* \bar{K}/\Xi_c^* \bar{K}^*/\Xi_c^* \bar{K}^*/\Omega_c \bar{K}^*$ state with $I(J^P) = 0(3/2^-)$ is a candidate of a molecular state. In this state, $\Xi_c^* \bar{K}^*|S_\perp\rangle$ channel has dominant contribution, which is more than fifty percent for the $\Xi_c^* \bar{K}/\Omega_c \eta/\Omega_c^* \eta/\Xi_c^* \bar{K}^*/\Xi_c^* \bar{K}^*/\Omega_c \omega$ molecular state. Since this state satisfies the requirements for a loosely bound system, with the size a few fm, and the cutoff around 1 GeV, it is a good candidate of a hadronic molecule with the quantum numbers of $\Omega_c$.

- For $I(J^P) = 0(1/2^-)$, it is possible to find a bound state with a binding energy around a few MeV with the cutoff $\Lambda$ around 1 GeV. However, the resulting bound states have a small size as their corresponding RMS radii are around 0.5 fm. We shall come back to this point shortly.

- For $I(J^P) = 0(3/2^+)$, when cutoff $\Lambda$ tuning around 2 GeV, a loosely bound solution is obtained, and its RMS radius is around 1 fm with several MeV binding energy. The $\Xi_c^* \bar{K}$ and $\Xi_c^* \bar{K}^*$ systems are the dominant channels. If the value of cutoff around 2 GeV is a reasonable input, we may conclude that there exist a loosely bound state.

- For $I(J^P) = 0(1/2^+)$ and $0(5/2^+)$, bound state solutions can be also obtained, by choosing the cutoff $\Lambda$ around 2 GeV or even larger. Moreover, the resulting sizes are too small around 0.5 fm or less.

- For $I(J^P) = 0(5/2^-)$, as its cutoff $\Lambda$ is far away from 1 GeV, it cannot be a reasonable molecular candidate.

A possible reason for too small sizes for $I(J^P) = 0(1/2^+, 5/2^+) \Xi_c^* \bar{K}/\Omega_c \eta/\Omega_c^* \eta/\Xi_c^* \bar{K}^*/\Xi_c^* \bar{K}^*/\Omega_c \omega$ states is that they are dominated by higher mass channels such as $\Xi_c^* K^*$ and/or $\Xi_c^* K^*$. Measuring the binding energies of these states from the higher mass thresholds, they turn to be around a few hundred MeV, which explains the small size of the bound state of the higher channels.

According to the above analysis, we have seen that there is one candidate of a molecular like state in the present approach, $\Xi_c^* \bar{K}/\Omega_c \eta/\Omega_c^* \eta/\Xi_c^* \bar{K}^*/\Xi_c^* \bar{K}^*/\Omega_c \omega$ with $I(J^P) = 0(3/2^-)$, which is dominated by $\Xi_c^* \bar{K}$. In comparison with the analysis of a single $\Xi_c^* \bar{K}$ channel which does not accommodate a bound state solution with a cutoff $\Lambda \sim 1$ GeV, it is implied that the coupled channel effect is very important in the $\Xi_c^* \bar{K}/\Omega_c \eta/\Omega_c^* \eta/\Xi_c^* \bar{K}^*/\Xi_c^* \bar{K}^*/\Omega_c \omega$ interaction.

Besides the loose molecular state composed by the lowest channel, $\Xi_c^* \bar{K}$, we also obtain several bound solutions, where the dominant channels are $\Xi_c^* \bar{K}^*$ and/or $\Xi_c^* \bar{K}^*$ channels. Their binding energies measured from these higher mass threshold $\bar{E}$ reach several hundreds MeV, implying that the relevant OBE interaction is strongly attractive. In many of these cases, however, the
TABLE IV: Bound state properties (binding energy $E$ and root-mean-square radius $r_{RMS}$) for all the investigated systems after the coupled channel effects are considered. Probability for the different channels are also given. $E$, $r_{RMS}$, and $\Lambda$ are in units of MeV, fm, and GeV, respectively. To emphasize the dominant channel, we label the probability for the corresponding channel in a bold manner.

| $I(J^P)$ | $\Lambda$ | $E$ | $r_{RMS}$ | $\Xi^*\bar{K}$(%) | $\Omega_\eta$(%) | $\Omega_\eta^*$$(\%) | $\Xi_\epsilon\bar{K}^*$(%) | $\Xi_\epsilon\bar{K}^*$(%) | $\Omega_\epsilon\omega$(%) |
|----------|----------|-----|---------|----------------|---------------|----------------|-----------------|----------------|----------------|
| 0(1/2−) | 1.182 | -0.53 | 0.74 | 15.97 | 0.63 | 0.75 | 70.21 | 11.98 | 6.03 |
|         | 1.186 | -3.28 | 0.71 | 15.63 | 0.65 | 0.75 | 70.46 | 12.06 | 6.06 |
|         | 1.190 | -6.06 | 0.69 | 15.34 | 0.66 | 0.76 | 70.67 | 12.12 | 6.10 |
| 0(1/2+) | 1.709 | -0.26 | 0.59 | 1.85 | 0.28 | 0.19 | 52.85 | 18.66 | 26.22 |
|         | 1.712 | -4.18 | 0.50 | 1.16 | 0.28 | 0.16 | 55.00 | 18.94 | 26.15 |
|         | 1.715 | -8.15 | 0.48 | 0.84 | 0.28 | 0.14 | 53.03 | 19.11 | 26.59 |
| 0(3/2−) | 1.270 | -0.58 | 5.17 | 98.22 | ~0 | 0.21 | 1.50 | ~0 | 0.07 |
|         | 1.320 | -6.00 | 2.17 | 92.04 | ~0 | 0.99 | 6.68 | 0.03 | 0.24 |
|         | 1.370 | -20.55 | 1.19 | 79.27 | ~0 | 2.48 | 27.44 | 0.21 | 0.50 |
| 0(3/2+) | 1.945 | -0.16 | 1.44 | 38.93 | 0.01 | 6.57 | 36.92 | 14.63 | 2.84 |
|         | 1.949 | -3.23 | 0.95 | 36.12 | 0.01 | 6.90 | 38.59 | 15.39 | 2.99 |
|         | 1.953 | -6.44 | 0.81 | 34.58 | 0.01 | 7.09 | 39.39 | 15.85 | 3.07 |
| 0(5/2−) | 4.480 | -1.98 | 0.26 | 6.86 | 0.09 | 4.84 | 31.52 | 29.67 | 27.01 |
|         | 4.482 | -6.07 | 0.26 | 6.88 | 0.09 | 4.86 | 31.54 | 29.68 | 26.94 |
|         | 4.484 | -10.19 | 0.26 | 6.90 | 0.09 | 4.88 | 36.79 | 29.69 | 26.87 |
| 0(5/2+) | 2.096 | -0.63 | 0.73 | 17.94 | … | 0.48 | 45.32 | 26.75 | 9.50 |
|         | 2.100 | -3.47 | 0.58 | 17.31 | … | 0.49 | 45.80 | 26.82 | 9.58 |
|         | 2.104 | -6.35 | 0.53 | 16.99 | … | 0.50 | 46.10 | 26.78 | 9.63 |

cutoff $\Lambda$ is much larger than 1 GeV. By decreasing $\Lambda$ the interaction can be weakened, and these tightly bound states of the higher mass channels may become looser. As a result, they may appear as Feschbach type resonances. This is an interesting issue to be studied, which we would like to leave as a future study.

III. DECAY BEHAVIOR FOR THE PREDICTED $\Omega_\epsilon$ STATE

In this section, we will discuss the two-body strong decay behaviors for the $\Xi^*_\epsilon\bar{K}/\Omega_\epsilon\eta/\Omega_\epsilon^*\bar{K}^*/\Xi^*_\epsilon\bar{K}^*/\Omega_\epsilon\omega$ state with $I(J^P) = 0(3/2^-)$. A relevant diagram is presented in the left of the figure 3.

![Diagram](image)

FIG. 3: Two body strong decay diagram for a molecular state $MS$ into a baryon $C$ and a meson $D$ (left). On the right hand, it is shown that $MS$ is composed by two colorless hadrons, a baryon $A$ and a meson $B$. $Q$ corresponds to the exchanged hadron.
In the rest frame of the molecular state $MS$ with mass $m_{MS}$, its two-body decay width for $MS \rightarrow C + D$ is

$$
d\Gamma = \frac{1}{2J + 1} \frac{m_c |p_c|}{8 \pi^2 m_{MS}} |M_f(\Sigma^+ \rightarrow C + D)|^2 d\Omega, \tag{22}
$$

$$
|p_c| = |p_D| = \frac{1}{2m_{MS}} \left( (m_c^2 + m_D^2)^2 \right)^{1/2}, \tag{23}
$$

where particles $C$ and $D$ are the final fermion and boson, respectively.

In figure 3, the scattering amplitude for the process $\Sigma^+ \rightarrow C + D$ is related to that from process $A + B \rightarrow C + D$,

$$
M_f(\Sigma^+ \rightarrow C + D) = \int d^3 r \frac{d^3 k}{(2\pi)^3} e^{-i k \cdot r} \psi(r) \frac{M_f(A + B \rightarrow C + D)}{\sqrt{64 \pi m_A m_B m_{MS}}}, \tag{24}
$$

where $I$ stands for the isospin for $MS$ state, $\psi(r)$ is the wave function for the obtained $\Omega_c$ state, which has been obtained in the last section.

The allowed two-body decay channels for the $\Xi_c \bar{K}/\Omega_c \eta/\Omega_c^{*} \eta/\Xi_c^{*} \bar{K}/\Omega_c^{*} \omega$ state with $I(J^P) = 0(3/2^-)$ include $\Xi_c \bar{K}$ and $\Xi_c^{*} \bar{K}$. The relevant scattering amplitude are written as

$$
M_f^{(\Xi_c \bar{K})}(\Omega_c \eta \rightarrow \Xi_c \bar{K}) = \frac{i \lambda g_{\bar{K}V}}{\sqrt{6}} (\epsilon^{\mu \nu \rho \lambda} - \epsilon^{\mu \nu \rho \lambda}) \bar{u}_c \gamma^5 \gamma_\rho q_{\lambda} A_{\lambda} \gamma^{\mu} g_{\beta \beta} q_{\beta} m_{c}^2 \frac{4}{q^2 - m_{c}^2} (P_{B} + P_{D}), \tag{25}
$$

$$
M_f^{(\Xi_c \bar{K})}(\Omega_c \eta \rightarrow \Xi_c^{*} \bar{K}) = \frac{i \lambda g_{\bar{K}V}}{\sqrt{6}} (\epsilon^{\mu \nu \rho \lambda} - \epsilon^{\mu \nu \rho \lambda}) \bar{u}_c \gamma^5 \gamma_\rho q_{\lambda} A_{\lambda} \gamma^{\mu} g_{\beta \beta} q_{\beta} m_{c}^2 \frac{4}{q^2 - m_{c}^2} (P_{B} + P_{D}), \tag{26}
$$

$$
M_f^{(\Xi_c \bar{K})}(\Xi_c^{*} \bar{K}/\Omega_c \eta \rightarrow \Xi_c \bar{K}) = \frac{\lambda g_{\bar{K}V}}{\sqrt{6}} (\epsilon^{\mu \nu \rho \lambda} - \epsilon^{\mu \nu \rho \lambda}) \bar{u}_c \gamma^5 \gamma_\rho q_{\lambda} A_{\lambda} \gamma^{\mu} q_{\lambda} (P_{B} + P_{D}), \tag{27}
$$

$$
M_f^{(\Xi_c \bar{K})}(\Xi_c^{*} \bar{K}/\Omega_c \eta \rightarrow \Xi_c^{*} \bar{K}) = \frac{\lambda g_{\bar{K}V}}{\sqrt{6}} (\epsilon^{\mu \nu \rho \lambda} - \epsilon^{\mu \nu \rho \lambda}) \bar{u}_c \gamma^5 \gamma_\rho q_{\lambda} A_{\lambda} \gamma^{\mu} q_{\lambda} (P_{B} + P_{D}), \tag{28}
$$

$$
M_f^{(\Xi_c \bar{K})}(\Xi_c \bar{K}/\Omega_c ^{*} \eta \rightarrow \Xi_c \bar{K}) = \frac{\lambda g_{\bar{K}V}}{\sqrt{6}} (\epsilon^{\mu \nu \rho \lambda} - \epsilon^{\mu \nu \rho \lambda}) \bar{u}_c \gamma^5 \gamma_\rho q_{\lambda} A_{\lambda} \gamma^{\mu} q_{\lambda} (P_{B} + P_{D}), \tag{29}
$$

$$
M_f^{(\Xi_c \bar{K})}(\Xi_c \bar{K}/\Omega_c ^{*} \eta \rightarrow \Xi_c^{*} \bar{K}) = \frac{\lambda g_{\bar{K}V}}{\sqrt{6}} (\epsilon^{\mu \nu \rho \lambda} - \epsilon^{\mu \nu \rho \lambda}) \bar{u}_c \gamma^5 \gamma_\rho q_{\lambda} A_{\lambda} \gamma^{\mu} q_{\lambda} (P_{B} + P_{D}), \tag{30}
$$

$$
M_f^{(\Xi_c \bar{K})}(\Xi_c \bar{K}/\Omega_c ^{*} \eta \rightarrow \Xi_c \bar{K}) = \frac{\lambda g_{\bar{K}V}}{\sqrt{6}} (\epsilon^{\mu \nu \rho \lambda} - \epsilon^{\mu \nu \rho \lambda}) \bar{u}_c \gamma^5 \gamma_\rho q_{\lambda} A_{\lambda} \gamma^{\mu} q_{\lambda} (P_{B} + P_{D}), \tag{31}
$$

$$
M_f^{(\Xi_c \bar{K})}(\Xi_c \bar{K}/\Omega_c ^{*} \eta \rightarrow \Xi_c \bar{K}) = \frac{\lambda g_{\bar{K}V}}{\sqrt{6}} (\epsilon^{\mu \nu \rho \lambda} - \epsilon^{\mu \nu \rho \lambda}) \bar{u}_c \gamma^5 \gamma_\rho q_{\lambda} A_{\lambda} \gamma^{\mu} q_{\lambda} (P_{B} + P_{D}), \tag{32}
$$

Using the wave function of the $\Xi_c \bar{K}/\Omega_c \eta/\Xi_c^{*} \bar{K}/\Xi_c^{*} \bar{K}/\Omega_c ^{*} \eta$ state with $I(J^P) = 0(3/2^-)$, the decay widths for all the allowed decay channels are collected in three groups, which are based on the choices of cutoff $\Lambda$ in table IV.

In table V, the total decay width for the $\Xi_c \bar{K}/\Omega_c \eta/\Xi_c^{*} \bar{K}/\Xi_c^{*} \bar{K}/\Omega_c ^{*} \eta$ state with $I(J^P) = 0(3/2^-)$ is around several MeV, and $\Xi_c \bar{K}$ is the dominant decay channel. With the increasing of cutoff, the total decay width becomes larger and larger. For the $\Xi_c \bar{K}$ channel, the decay width is less than 1 MeV. A measurement in the $\Xi_c \bar{K}$ channel is helpful to further understand the structure of $\Omega_c$ states.
### TABLE VI: Bound state properties (binding energy $E$ and root-mean-square radius $r_{\text{RMS}}$) for all the investigated systems after the coupled channel effects are considered. Probability for the different channels are also given. $E$, $r_{\text{RMS}}$, and $\Lambda$ are in units of MeV, fm, and GeV, respectively. To emphasize the dominant channel, we label the probability for the corresponding channel in a bold manner.

| $I(J^P)$ | $\Lambda$ | $E$ (MeV) | $r_{\text{RMS}}$ (fm) | $\Xi_c^+ \bar{K}$ (%) | $\Xi_c^+ \bar{K}^*$ (%) | $\Xi_c^+ \bar{K}^{**}$ (%) | $\Omega_c^*\rho$ (%) |
|----------|-----------|-----------|------------------------|------------------------|------------------------|------------------------|---------------------|
| 1(1/2⁻) | 1.656     | -0.96     | 0.36                   | 0.61                   | 46.95                  | 50.48                  | 1.85                |
| 1(1/2⁺)  | 1.660     | -5.06     | 0.35                   | 0.60                   | 46.82                  | 50.72                  | 1.98                |
|          | 1.664     | -9.21     | 0.35                   | 0.60                   | 46.45                  | 50.94                  | 2.00                |
| 1(3/2⁻)  | 1.460     | -0.67     | 4.85                   | 95.89                  | 3.60                   | 0.24                   | 0.27                |
|          | 1.510     | -4.32     | 2.37                   | 90.00                  | 8.71                   | 0.53                   | 0.76                |
|          | 1.560     | -11.35    | 1.47                   | 81.86                  | 13.93                  | 0.79                   | 1.42                |
| 1(3/2⁺)  | 2.230     | -1.90     | 0.48                   | 4.58                   | 23.55                  | 48.32                  | 23.53               |
|          | 2.232     | -4.26     | 0.45                   | 4.43                   | 23.51                  | 48.44                  | 23.61               |
|          | 2.234     | -6.63     | 0.44                   | 4.32                   | 23.45                  | 48.53                  | 23.68               |
| 1(5/2⁻)  | 2.812     | -0.90     | 0.40                   | 0.61                   | 20.58                  | 56.55                  | 22.26               |
|          | 2.816     | -5.34     | 0.40                   | 0.61                   | 20.48                  | 56.58                  | 22.31               |
|          | 2.816     | -5.34     | 0.40                   | 0.61                   | 20.40                  | 55.60                  | 22.38               |
| 1(5/2⁺)  | 3.590     | -1.27     | 0.43                   | 0.92                   | 37.30                  | 26.38                  | 35.39               |
|          | 3.600     | -4.76     | 0.42                   | 0.88                   | 37.25                  | 26.39                  | 35.48               |
|          | 3.610     | -8.28     | 0.42                   | 0.85                   | 37.19                  | 26.39                  | 35.56               |

### IV. OTHER PARTNERS

After we discuss the isoscalar $\Xi^- \bar{K}/\Omega_\eta/\Omega_\eta’/\Xi^- \bar{K}^*/\Omega_\omega\omega$ systems, in the following, we study their isovector partners by using the same model. The $\Xi_c^+ \bar{K}$, $\Xi_c^+ \bar{K}^*$, and $\Omega_c^*\rho$ channels are considered. The corresponding numerical results are presented in table VI.

Finally, we find that the $\Xi_c^+ \bar{K}$ channel is also dominant. For the $I(J^P) = 1(3/2^−)$, $1(3/2^+)$, and $1(5/2^−)$, because the value of the cutoff $\Lambda$ is away from 1 GeV and/or their RMS radii are very small, they are not possible loose molecular candidates.

As a byproduct, we further extend our study on the $\Xi^- \bar{K}$ system. According to the G-parity rule [55], the properties of the $\omega$ and $\phi$ exchanges are exactly opposite to those in the $\Xi^- \bar{K}$ system. After solving the Schrödinger equation, our results indicate that

- For the isoscalar $\Xi_c^+ \bar{K}$ system, bound solutions can be obtained when cutoff is taken larger than 1.7 GeV. Thus, the interaction from considered one single channel is not strong enough to generate a loosely bound state.
• For the isovector $\Xi^*_c K$ system, there doesn’t exist any binding solutions by tuning cutoff $\Lambda$ from 1 to 5 GeV.

When the $\Xi^*_c K/\Sigma^*_c \rho/\Sigma^*_c \omega/\Lambda^*_c K^*/\Xi^*_c K^*$ coupled channel system is further considered, finally, we can predict two possible loosely bound molecular states, the isoscalar $\Xi^*_c K/\Sigma^*_c \rho/\Sigma^*_c \omega/\Xi^*_c K^*$ with $J^P = 3/2^-$ and the isovector $\Xi^*_c K/\Sigma^*_c \rho/\Sigma^*_c \omega/\Lambda^*_c K^*/\Xi^*_c K^*$ state with $J^P = 3/2^-$. For the isoscalar state, the $\Xi^*_c K$ channel is dominant with more than 80 percent probability. For the isovector state, the remaining channels $\Sigma^*_c \rho/\Sigma^*_c \omega/\Lambda^*_c K^*$ with $J^P = 1(3/2^-)$ play much more important roles, as their larger probabilities.

V. SUMMARY

Since the observation of the hidden-charm pentaquarks $P_c(4380)$ and $P_c(4450)$, searching for heavy pentaquarks becomes a very hot topic of hadron physics. In this work, by adopting the OBE model and coupled channel approach, we carry out a systematic investigation of possible $\Omega_c$–like molecular state from the $\Xi^*_c K/\Omega_c \eta/\Omega_c \eta$/$\Xi^*_c K^*$/$\Xi^*_c K^*$/$\Omega_c \omega$ interaction.

In order to justify the existence of a molecular state, we borrow the experience in the deuteron. Finally, we find a reasonable loose $\Omega_c$–like molecular state, the $\Xi^*_c K/\Omega_c \eta/\Omega_c \eta/\Xi^*_c K^*$/$\Xi^*_c K^*$/$\Omega_c \omega$ state with $I(J^P) = 0(3/2^-)$. Compared to the probabilities for all the discussed channels, it is mainly made up by the $S$-wave $\Xi^*_c K$ component. And then, we further study the two-body strong decay behavior of the $\Xi^*_c K/\Omega_c \eta/\Omega_c \eta/\Xi^*_c K^*$/$\Xi^*_c K^*$/$\Omega_c \omega$ state with $I(J^P) = 0(3/2^-)$. Our results indicate that its two-body strong decay width is around several MeV, and the $\Xi^*_c$ decay channel is the dominant. In comparison with the observation of the LHCb [43], the mass of this $\Xi^*_c K/\Omega_c \eta/\Omega_c \eta/\Xi^*_c K^*$/$\Xi^*_c K^*$/$\Omega_c \omega$ state with $I(J^P) = 0(3/2^-)$ is very close to their threshold, especially the $\Omega_c(3090)$ and $\Omega_c(3119)$. If the spin-parities of the five $\Omega_c$ states is further determined in the further, the $\Omega_c$ with $I(J^P) = 0(3/2^-)$ can be easily related to the $\Xi^*_c K/\Omega_c \eta/\Omega_c \eta/\Xi^*_c K^*$/$\Xi^*_c K^*$/$\Omega_c \omega$ state with $I(J^P) = 0(3/2^-)$. Meanwhile, we may predict an isovector molecular partner, the $\Xi^*_c K/\Xi^*_c K^*$/$\Xi^*_c K^*$/$\Omega_c \rho$ state with $I(J^P) = 1(3/2^-)$.

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Appendix A: Relevant subpotentials

The concrete subpotentials in equation (21) are expressed as

\[
\chi_{L,J^P}^{\Xi_l^* K - \Xi_l^* K}(r) = -\frac{1}{16} \beta_{SE} g_{\sigma \omega} D_1 (J^P) Y(\Lambda, m_{\sigma}, r) - \frac{1}{16} \beta_{SE} g_{\sigma \omega} D_1 (J^P) \left[ \mathcal{G}(I) Y(\Lambda, m_{\omega}, r) + Y(\Lambda, m_{\sigma}, r) - 2 Y(\Lambda, m_{\phi}, r) \right],
\]

\[
\chi_{L,J^P}^{\Xi_l^* K - \Xi_l^* K}(r) = \frac{1}{8} \frac{g_{SE}}{f_\pi} \sqrt{\frac{1}{M_{\Xi^*_c} M_{K}}}, \left[ \frac{1}{3} E_1 (J^P) \nabla \nabla + \frac{1}{3} F_1 (J^P) r \frac{\partial}{\partial r} \frac{\partial}{\partial r} \right] \mathcal{G}(I) U(\Lambda, m_{\sigma}, r) - \frac{1}{8} \frac{g_{SE}}{f_\pi} \sqrt{\frac{1}{M_{\Xi^*_c} M_{K}}}, \left[ \frac{1}{3} E_1 (J^P) \nabla \nabla - \frac{1}{3} F_1 (J^P) r \frac{\partial}{\partial r} \frac{\partial}{\partial r} \right] Y(\Lambda, m_{\sigma}, r),
\]

\[
\chi_{L,J^P}^{\Xi_l^* K - \Xi_l^* K}(r) = \left[ \frac{\lambda_{SE} g_{\sigma \omega} D_2 (J^P) Y(\Lambda, m_{\omega}, r) + \frac{1}{8} \beta_{SE} g_{\sigma \omega} D_2 (J^P) \left[ \mathcal{G}(I) Y(\Lambda, m_{\omega}, r) + Y(\Lambda, m_{\sigma}, r) - 2 Y(\Lambda, m_{\phi}, r) \right] \right].
\]
\[
\mathcal{V}_{l,jr}^{\Xi_{l}^{K} \rightarrow \Xi_{l}^{K}}(r) = \frac{1}{4} \frac{g_{4g_{V}V_{P}}}{\sqrt{3}} \left[ \frac{1}{3} E_{2}(J_{P}) \nabla^{2} + \frac{1}{3} H_{2}(J_{P}) \frac{\partial}{\partial r} \frac{1}{3} \frac{\partial}{\partial r} \right] Y(\Lambda, m_{\omega}, r) \\
- \frac{1}{4} \frac{g_{4g_{V}V_{P}}}{\sqrt{3}} \left[ \frac{1}{3} E_{2}(J_{P}) \nabla^{2} + \frac{1}{3} H_{2}(J_{P}) \frac{\partial}{\partial r} \frac{1}{3} \frac{\partial}{\partial r} \right] Y(\Lambda, m_{\omega}, r) \\
+ \frac{1}{4} \frac{\lambda_{l}g_{V}V_{P}}{M_{K}} H_{1}(J_{P}) \frac{1}{r} \frac{\partial}{\partial r} \left[ g(\lambda)(\Lambda, m_{\omega}, r) + Y(\Lambda, m_{\omega}, r) - 2Y(\Lambda, m_{\omega}, r) \right] \\
+ \frac{\lambda_{l}g_{V}V_{P}}{M_{K}} \left[ 2 \frac{1}{3} E_{2}(J_{P}) \nabla^{2} - \frac{1}{3} H_{2}(J_{P}) \frac{1}{r} \frac{\partial}{\partial r} \frac{1}{3} \frac{\partial}{\partial r} \right] Y(\lambda, m_{\omega}, r), (A5)
\]

\[
\mathcal{V}_{l,jr}^{\Xi_{l}^{K} \rightarrow \Xi_{l}^{K}}(r) = \frac{1}{4} \frac{g_{4g_{V}V_{P}}}{\sqrt{3}} \left[ \frac{1}{3} E_{2}(J_{P}) \nabla^{2} + \frac{1}{3} H_{2}(J_{P}) \frac{\partial}{\partial r} \frac{1}{3} \frac{\partial}{\partial r} \right] Y(\lambda, m_{\omega}, r) \\
+ \frac{\sqrt{3} g_{4g_{V}V_{P}}}{12} \left[ \frac{1}{3} E_{3}(J_{P}) \nabla^{2} + \frac{1}{3} H_{3}(J_{P}) \frac{\partial}{\partial r} \frac{1}{3} \frac{\partial}{\partial r} \right] Y(\lambda, m_{K}, r) \\
+ \frac{1}{6} \frac{g_{1}g_{V}}{M_{\Xi}} \left[ \frac{1}{3} E_{3}(J_{P}) \nabla^{2} + \frac{1}{3} H_{3}(J_{P}) \frac{\partial}{\partial r} \frac{1}{3} \frac{\partial}{\partial r} \right] Y(\lambda, m_{K}, r), (A5)
\]
In equation (A16), the function \( U(\Lambda_i, m_{\pi i}, r) \) is defined as
\[
U(\Lambda_i, m_{\pi i}, r) = \frac{1}{4\pi r}(\cos(m_{\pi i}r) - e^{-\Lambda_i r}) - \frac{\Lambda_i^2 + m_{\pi i}^2}{8\pi \Lambda_i} e^{-\Lambda_i r}, \quad i = 0, 1, 2.
\]
The variables in Eqs. (A1)-(A20) are
\[
q_0 = \frac{M_{\pi 0}^2 + M_{\pi 0}^2 - M_{\pi 0}^2}{2(M_{\pi 0}^2 + M_{\pi 0}^2)}, \quad q_1 = \frac{M_{\pi 1}^2 - M_{\pi 2}^2 - M_{\pi 3}^2}{2(M_{\pi 1}^2 + M_{\pi 2}^2)}, \quad q_2 = \frac{M_{\pi 0}^2 - M_{\pi 3}^2}{2(M_{\pi 0}^2 + M_{\pi 3}^2)}, \quad q_3 = \frac{M_{\pi 0}^2 - M_{\pi 0}^2 + M_{\pi 3}^2}{2(M_{\pi 0}^2 + M_{\pi 3}^2)},
\]
\[
q_4 = \frac{M_{\pi 1}^2 + M_{\pi 2}^2 - M_{\pi 3}^2}{2(M_{\pi 1}^2 + M_{\pi 2}^2)}, \quad q_5 = \frac{M_{\pi 0}^2 - M_{\pi 2}^2 + M_{\pi 3}^2}{2(M_{\pi 0}^2 + M_{\pi 2}^2)}, \quad q_6 = \frac{M_{\pi 0}^2 + M_{\pi 3}^2}{2(M_{\pi 0}^2 + M_{\pi 3}^2)}, \quad q_7 = \frac{M_{\pi 0}^2 + M_{\pi 0}^2 + M_{\pi 3}^2}{2(M_{\pi 0}^2 + M_{\pi 3}^2)},
\]
\[
q_8 = \frac{M_{\pi 1}^2 + M_{\pi 2}^2 - M_{\pi 3}^2}{2(M_{\pi 1}^2 + M_{\pi 2}^2)}, \quad q_9 = \frac{M_{\pi 0}^2 - M_{\pi 2}^2 - M_{\pi 3}^2}{2(M_{\pi 0}^2 + M_{\pi 2}^2)}, \quad q_{10} = \frac{M_{\pi 0}^2 - M_{\pi 2}^2 - M_{\pi 3}^2}{2(M_{\pi 0}^2 + M_{\pi 2}^2)}, \quad q_{11} = \frac{M_{\pi 0}^2 + M_{\pi 0}^2 - M_{\pi 3}^2}{2(M_{\pi 0}^2 + M_{\pi 3}^2)},
\]
\[
q_{12} = \frac{M_{\pi 1}^2 + M_{\pi 2}^2 + M_{\pi 3}^2}{2(M_{\pi 1}^2 + M_{\pi 2}^2)}, \quad q_{13} = \frac{M_{\pi 0}^2 + M_{\pi 3}^2 + M_{\pi 3}^2}{2(M_{\pi 0}^2 + M_{\pi 3}^2)}, \quad \Lambda_i^2 = \Lambda^2 - q_i^2, \quad mE_i^2 = |mE^2 - q_i^2|.
\]
In addition, \( D_1(J^p), E_i(J^p), F_i(J^p) \) and \( H_i(J^p) \) in Eqs. (A1)-(A20), which are the spin-spin, tensor, and spin-orbit operators, respectively. For example,
\[
D_1 = \sum_{a,b} C_{1/2,1/1,1/1}^{a/b} \frac{1}{1/2,1/1,1/1} x_1^a e_{i1}^a \cdot e_{i1}^b, \quad D_2 = \chi^3 \sigma \cdot e_2^1 \cdot e_4^1,
\]
\[
D_3 = \sum_{a,b} C_{1/2,1/1,1/1}^{a/b} (\sigma \cdot e_1^b) \chi^3, \quad E_1 = \sum_{a,b} C_{1/2,1/1,1/1}^{a/b} e_{i2}^a \cdot (i e_3^a \times \sigma) \chi_1, \quad E_2 = \chi^3 (\sigma \cdot e_2) \chi_1,
\]
\[
F_1 = \sum_{a,b} C_{1/2,1/1,1/1}^{a/b} (\sigma \cdot e_1^b) \cdot S(\hat{r}, e_2, i e_3^b \times \sigma) \chi_1, \quad F_2 = \chi^3 (\sigma \cdot e_2) \chi_1, \quad H_1 = \chi^3 (\sigma \cdot e_2) \chi_1,
\]
\[
S(\hat{r}, a, b) = 3(\hat{r} \cdot a)(\hat{r} \cdot b) - a \cdot b.
\]
The subscripts (1, 2, 3, 4) labeled in the polarization vector $\epsilon$ and spin wave function $\chi$ correspond to the hadrons in the process $B(1)M(2) \rightarrow B(3)M(4)$, where $B$ and $M$ stand for the baryon and meson, respectively. When performing the numerical calculations, these corresponding operators in equation (A22) will be replaced by numerical matrixes, which are summarized in table VII.

**TABLE VII: Non-zero matrix elements ($\langle f | \mathcal{A} | i \rangle$) in various channels for the operators $\mathcal{A}$ in equation (A22).**

| $\mathcal{A}$ | 1/2$^-$ | 1/2$^+$ | 3/2$^-$ | 3/2$^+$ | 5/2$^-$ | 5/2$^+$ |
|---------------|---------|---------|---------|---------|---------|---------|
| $\mathcal{D}_1$ | (1)     | (1)     | (1,0)   | (1)     | (1)     | (1)     |
| $\mathcal{D}_2$ | (1,0)   | (1,0)   | (1,0,0) | (1,0)   | (1,0)   | (1)     |
| $\mathcal{D}_3$ | (0)     | (0)     | (0,1/$\sqrt{3}$) | (-1/$\sqrt{3}$) | (-2/$\sqrt{3}$) | (0)     |
| $\mathcal{E}_1$ | (0,1)   | (0,1)   | (1,0,0) | (0,1)   | (0,1)   | (1)     |
| $\mathcal{E}_2$ | (-2,0)  | (-2,0)  | (1,0,0) | (-2,0)  | (-2,0)  | (1)     |
| $\mathcal{E}_3$ | ($\sqrt{3}$,0) | ($\sqrt{3}$,0) | (0,$\sqrt{3}$,0) | ($\sqrt{3}$,0) | ($\sqrt{3}$,0) |                  |
| $\mathcal{F}_1$ | (-$\sqrt{2}$,1) | (-$\sqrt{2}$,1) | (0,1,-1) | (-$\sqrt{2}$,1) | ($\sqrt{2}$,-$\sqrt{2}$) | ($\sqrt{2}$,-$\sqrt{2}$) |
| $\mathcal{F}_2$ | ($0$,-$\sqrt{2}$) | ($0$,-$\sqrt{2}$) | (1,0,-1) | ($\sqrt{2}$,1) | ($\sqrt{2}$,-$\sqrt{2}$) | ($\sqrt{2}$,-$\sqrt{2}$) |
| $\mathcal{F}_3$ | ($0$,-$\sqrt{3}$) | ($0$,-$\sqrt{3}$) | ($\sqrt{3}$,0,-$\sqrt{3}$) | ($\sqrt{2}$,0) | ($\sqrt{2}$,0) | ($\sqrt{2}$,0) |
| $\mathcal{H}_1$ | ($\frac{2}{3}$,-$\frac{2}{3}$) | ($\frac{2}{3}$,-$\frac{2}{3}$) | (0,0,0) | (-$\frac{2}{3}$,-$\frac{2}{3}$) | (-$\frac{2}{3}$,-$\frac{2}{3}$) | (-$\frac{2}{3}$,-$\frac{2}{3}$) |
| $\mathcal{H}_2$ | (0)     | (0)     | ($\sqrt{3}$) | ($\sqrt{3}$) | ($\sqrt{3}$) | ($\sqrt{3}$) |
| $\mathcal{H}_3$ | (0,0)   | (-$\frac{2}{3}$,-$\frac{2}{3}$) | ($0$,-$\sqrt{3}$,-$\sqrt{3}$) | ($\frac{1}{3}$,-$\frac{2}{3}$) | ($\frac{1}{3}$,-$\frac{2}{3}$) | ($\frac{1}{3}$,-$\frac{2}{3}$) |
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