Cosmological evolution for magnetic universe based in a simple nonlinear electrodynamics

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Abstract. We investigate the cosmic dynamics of Friedmann-Robertson-Walker universes – flat spatial sections – which are driven by nonlinear electrodynamics (NLED) Lagrangian. We pay special attention to the check of the sign of the square sound speed since, whenever the latter quantity is negative, the corresponding cosmological model is classically unstable against small perturbations of the background energy density. Besides, based on causality arguments, one has to require that the mentioned small perturbations of the background should propagate at most at the local speed of light. We also look for the dynamics by mean of dynamical system tools. Our results indicate that in the case of the power law NLED Lagrangian, it is not possible to explain the current accelerated expansion stage of the universe because the model is not stable for the possible values of the free parameter that allows such expansion. Finally, in the second case, whose Lagrangian has a power law NLED term plus a Maxwell term, the model is stable and causal for a certain range of values of the free parameter.

1. Introduction
Many observations lead to the conclusion that the present universe is accelerating [1]-[3]. The reason for this acceleration was attributed to the dominant presence of a new cosmic component called dark energy. The ΛCDM model came out as the most successful one for explaining this late time acceleration of the universe with $\omega = -1$, where $\omega$ is the barotropic parameter of the equation of state (EoS) for the material content of the universe. In this model the cosmological constant is being considered as dark energy but the model is plagued with severe drawbacks [4]. For single scalar field models, it has been shown that the equation of state cannot cross the phantom divide line ($\omega = -1$). So, models with a combination of phantom and quintessence [5], and scalar field models with scalar-dependent coupling in front of kinetic term [6] as well as fluid models [7] have also been constructed to realize the crossing of the phantom divide line, which still seems to be allowed by recent observations [8].

Recently, the idea of non-linear electrodynamics (NLED) has been proposed as a solution to source the Universe acceleration [9]-[11]. In the early Universe the effect of the NLED may have been very strong and, in principle, this may also explain the inflation. In late time epochs, the reason to use NLED may be different than the early universe: it can be implemented as a phenomenological approach, in which the cosmic substratum is modeled as a material media with electric permeability and magnetic susceptibility that depend in nonlinear way on the fields [12]. Another argument is based on the view that General Relativity is a low energy quantum
The four-dimensional (4D) Einstein-Hilbert action of gravity coupled to NLED is given by

\[ S = \int d^4x \sqrt{-\tilde{g}} [R + L_m + L(F, G)], \]

(1)

where \( R \) is the curvature scalar, \( L_m \) – the background perfect fluid’s Lagrangian density associated with ordinary matter that contain the universe, and \( L(F, G) \) is the gauge-invariant electromagnetic (EM) Lagrangian density which is a function of the electromagnetic invariants: \( F = 2(B^2 - E^2) \) and \( G = -4E \cdot B \).

Standard (linear) Maxwell electrodynamics is given by the Lagrangian \( L = -F/4 \). The corresponding field equations can be derived from the action (1) by performing variations with respect to the space-time metric \( g_{\mu\nu} \), to obtain \( G_{\mu\nu} = T_{\mu\nu}^m + T_{\mu\nu}^{EM} \) where \( T_{\mu\nu}^{m} = (\rho_m + p_m) u_\mu u_\nu - p_m g_{\mu\nu} \), and \( T_{\mu\nu}^{EM} = g_{\mu\nu} [L(F) - GL_G] - 4F_{\mu\nu\alpha} F^{\alpha}_{\nu\sigma} L_F \) with \( \rho_m = \rho_m(t) \), \( p_m = p_m(t) \), where \( u_\mu \) is the normalized \((u_\mu u^\mu = 1)\) velocity of the reference frame where the fields are measured, while \( L_F \equiv dL/dF \), \( L_{FF} \equiv d^2L/dF^2 \), etc. Variation with respect to the components of the electromagnetic potential \( A_\mu \) yields to the electromagnetic field equations

\[ \left( F^{\mu\nu} L_F + \frac{1}{2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} L_G \right)_{,\mu} = 0. \]

(2)

Since the observations have shown that the current universe is very close to a spatially flat geometry [15], a result which is quite natural within primordial inflation scenarios [16], in this paper we shall consider a homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe with flat spatial sections, which is described by the metric \( ds^2 = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j \), where \( a(t) \) is the cosmological scale factor, Latin indexes run over three-space. Since the spatial sections of the FRW spacetime are isotropic, the electromagnetic (EM) fields can be compatible with such a universe only if an averaging procedure is performed. (In particular, the energy density and the pressure of the NLED field should be evaluated by averaging over volume.) Following the standard approach [17] (for details see also [18, 19, 20] and references therein) we define the volumetric spatial average of a quantity \( X \) at the time \( t \) by

\[ \bar{X} = \lim_{V \to V_0} \frac{1}{V} \int d^3x \sqrt{-\tilde{g}} \bar{X} \text{ where } V = \int d^3x \sqrt{-\tilde{g}} \text{ and } V_0 \text{ is a sufficiently large time-dependent} \]

...
three-volume. Besides, for the EM field to act as a source for the FRW model we need to impose that (The averaging procedure is independent of the equations of the EM field so it can be safely applied in the NLED case [18]): $E_i = 0, B_i = 0, E_iB_j = 0, E_iE_j = -\frac{1}{2}E^2g_{ij}, B_iB_j = -\frac{1}{3}B^2g_{ij}$.

Additionally it has to be assumed that the electric and magnetic fields, being random fields, have coherent lengths that are much shorter than the cosmological horizon scales. Under these assumptions the energy-momentum tensor of the EM field – associated with the Lagrangian density $L = L(F,G)$ – can be written in the form of the energy-momentum tensor for a perfect fluid $T_{\mu\nu}^{EM} = (\rho_{EM} + p_{EM}) u_\mu u_\nu - p_{EM}g_{\mu\nu}$, where $\rho_{EM} = -L + GLG - 4LF^2$, and $p_{EM} = L - GLG - \frac{4}{3}(2B^2 - E^2)L_F$ where $E^2$ and $B^2$ being the averaged electric and magnetic fields squared, respectively. In this paper we shall restrict to the case of nonlinear theories defined by $L = L(F)$ [20], so that $\rho_{EM} = -L - 4LF^2$ and $p_{EM} = L - \frac{4}{3}(2B^2 - E^2)L_F$.

In what follows, to simplify the analysis, we shall consider a flat FRW universe which is filled in the three-volume. Besides, for the EM field to act as a source for the FRW model we need to impose that $E_i = 0, B_i = 0, E_iB_j = 0, E_iE_j = -\frac{1}{2}E^2g_{ij}, B_iB_j = -\frac{1}{3}B^2g_{ij}$.

As mentioned before, one issue of interest when one explores cosmological models of dark energy is the possibility of crossing the so called "phantom divide" barrier $\omega = -1$. In consequence, it is very important to check the sign of the square sound speed for the different cosmological models.

If the matter considering in the model is a perfect fluid, then the barotropic parameter is $\omega = \frac{p}{\rho}$ where $\rho$ is the energy density and $p$ is the pressure of the perfect fluid. In this case, we can define the barotropic parameter for the magnetic universes in the NLED as

$$\omega_B = \frac{p_B}{\rho_B} = -1 + \frac{4}{3} \frac{F L_F}{L} = -1 + \frac{4}{3} \frac{F}{L_F} d \ln (L),$$

where we have used the expressions for $\rho_B$ and $p_B$.

The condition here for there to be a crossing phantom line is that $F \frac{d}{dF} \ln (L)$ must have a sign change at some stage in the evolution of the electromagnetic field.

Another quantity of cosmological importance is the adiabatic square sound speed which, for the cases of interest in this paper, can be written as

$$c_s^2 = \frac{dp_B}{d\rho_B} = \frac{dp_B}{d\rho_B} \frac{dF}{dL_F} = \frac{1}{3} + \frac{4}{3} \frac{F L_{FF}}{L_F} = \frac{1}{3} + \frac{4}{3} \frac{F d}{dF} \ln (L_F).$$

In the case when $c_s^2 < 0$, the energy density perturbations uncontrollably grow resulting in a classical instability of the cosmological model. The increment of instability is inversely proportional to the wavelength of the perturbations, and the models where $c_s^2 < 0$, are violently unstable, so that these should be rejected [22].
Even if $c_s^2$ is a positive quantity, a causality issue may arise whenever the square sound speed is greater than the local speed of light (for a critical review on this issue see [23]). As a matter of fact, it is usually assumed that $c_s \leq 1$, while the complementary bound $c_s > 1$ is used as a criterion for rejecting theories [24, 25]. In particular, low-energy effective field theories – even when these are based in Lorentz-invariant Lagrangians – have been rejected if they admit superluminal fluctuations [26].

3. NLED with a $\gamma F^\alpha$ term
Recently in [13] showed, from the point of view of the observations, that a non-linear electrodynamics whose Lagrangian is given by $L = \gamma F^\alpha$ con $\gamma < 0$ reproduces an accelerated universe as dark energy when $\alpha = -1/4$ and $\alpha = -1/8$.

Considering the expressions for barotropic parameter and squared speed sound for the magnetic universes in the NLED we have: $\omega_B = -1 + \frac{1}{3} \alpha = c_s^2$. So for there to be an accelerated expansion behavior, it is required that $\alpha < \frac{1}{7}$. Whereas when there is a crossing of phantom line $\omega_B = -1$, necessarily the model will be unstable because $c_s^2 = -1$.

According to the results shown in [13] when $\alpha = -1/4$ y $\alpha = -1/8$, then $\omega_B = c_s^2 = -4/3$ and $\omega_B = c_s^2 = -7/6$, respectively. These results show us that the model is unstable. And, if $\alpha = -1$, then $\omega_B = c_s^2 = -7/3$ and, also the model is unstable.

4. NLED with a $-\frac{1}{4}F - \gamma F^\alpha$ term
In this section will be consider la NLED Lagrangian given by $L = -\frac{1}{4}F - \gamma F^\alpha$, which describe a universe with Maxwell’s radiation and a nonlinear term of the form studied in [13]. The barotropic parameter and squared speed sound for the magnetic universes for this NLED Lagrangian are given by:

$$\omega_B = \frac{(16 \alpha - 12) \gamma F^\alpha + F}{3F + 12 \gamma F^\alpha}, \quad c_s^2 = \frac{16 \gamma \alpha F^\alpha (\alpha - 3/4) + F}{12 \gamma \alpha F^\alpha + 3F}.$$

We are going to consider the two cases studied previously, when $\alpha = 2$ we obtain the squared speed sound showed in section V of [14], so the results obtained in that work are confirmed, that is, for $\gamma > 0$ is unstable whereas $a(t) < a_*$, value from which it becomes stable until the end of its evolution where $c_s^2 \rightarrow 1$ when $a \rightarrow \infty$. While when $\gamma < 0$, the model is unstable from the beginning of its evolution to a value $a(t) = a_*$ where the model stabilizes and continues up to an evolution of Maxwell’s radiation. Now, and given that the EoS parameter was not studied in [14], we have that:

$$\omega_B = \frac{1 + 20 \gamma F}{3 + 12 \gamma F} = \frac{a^4 + 20 \gamma F_0}{3a^4 + 12 \gamma F_0},$$

(5)

where we can observe that, when $\gamma > 0$, there is no line crossing ($\omega_B = \frac{5}{2}$ when $a \rightarrow 0$ and evolves to $\omega_B = \frac{1}{7}$ when $a \rightarrow \infty$). Finally when $\gamma < 0$, the EoS parameter $\omega_B$ has a line crossing but at some point of its evolution is indeterminate.

On the other hand, if we substitute $\alpha = -1$ we recover the squared speed sound given in section VI in [14], so if $\gamma < 0$ then the model is unstable at all times of its evolution. While if $\gamma > 0$ the model is stable very early in its evolution and subsequently it is completely unstable. The EoS barotropic parameter is given by:

$$\omega_B = \frac{F^2 - 28 \gamma}{3F^2 + 12 \gamma} = \frac{F_0^2 - 28 \gamma a^8}{3F_0^2 + 12 \gamma a^8}.$$
Figure 1. The behavior of the squared speed sound ($c^2_s$) and of the EoS parameter $\omega$ is shown when $\gamma = 0.1$ and $F_0 = 1$. In the left panel the behavior of $c^2_s$ is shown, which is unstable when $\alpha < 0$ except at an early stage of its evolution. When $\alpha > 0$ we observe a range of $\alpha$ for which the model is stable, outside that it presents a discontinuity that makes the model not physical. The range of values of $\alpha$ in which $c^2_s$ is positive less than unity is shown in the middle panel. And in the right panel the behavior of the EoS parameter is shown, where we can see that there is no crossing phantom line $\omega_B = -1$ for values $\alpha \geq 0$. However, if there is such a cross for values $\alpha < 0$, at a point of evolution after $a(t = 0) = 0$.

When we have take $\gamma > 0$ we have two cases: if $a \to 0$ then $\omega_B \to 1/3$ and if we have $a \to \infty$ then $\omega_B \to -\frac{\gamma}{3}$, so there is a crossing phantom line at $a = 2^{-1/4}\gamma^{-1/8}$ and the rest of its evolution remains as a phantom field. The model is decelerated from the beginning of its evolution and when $a = 12^{-1/8}\gamma^{-1/8}$ begins to accelerate and remains so until the end of its evolution. Finally if $\gamma < 0$, the model does not have a crossing phantom line.

In figure 4 the squared speed sound and the EoS parameter are shown as a function of $\alpha$. When $\alpha < 0$, $c^2_s$ presents a singularity at some point in its evolution. When $\alpha < 0$, $c^2_s$ presents a singularity at some point in its evolution. There is a range of values of $\alpha$ where $c^2_s$ is stable and causal as described below. When $\alpha \to 0$, later in the evolution the stability of the model is achieved.

On the other hand, if $\alpha > 0$, the model presents a singularity in $c^2_s$ at some point in its evolution, except in a certain range of values of $\alpha$, $3/4 \leq \alpha \leq 3/2$ as shown in the central panel of the figure 4, regardless of the value of $\gamma$.

In the right panel of the figure, the graph of the parameter $\omega_B$ is shown as a function of the scale factor $a(t)$ (substituting $F = 1/a^3$) and $\alpha$. Here we can see that for values $\alpha \geq 0$ there is no crossing phantom line, although this is the range of values of $\alpha$ where the model is stable. While for negative values of $\alpha$ there is such a crossing phantom line from some value $a(t) > 0$ after the Big Bang, although this is not the region of values of the parameter $\alpha$ that interests us.

When $\alpha = -1$ the crossing phantom line occurs at a very early time in the evolution of the model, this result was not shown in [14].

5. Conclusions
In this work we have studied a magnetic FRW universe with a nonlinear electrodynamics previously studied but without conclusively testing if the model works to describe the accelerated expansion of the current universe.

In view of the previous results we can conclude that it is not possible for a Lagrangian electromagnetic form $L = \gamma F^\alpha$ to describe a stable universe for the values reported in [13].
However, if $\alpha \geq 3/4$ the model will be stable, although it will not be able to model a universe that accelerates since for this to happen it is required that $\alpha < 1/2$.

From the observational results given in [13], from the theoretical study of the squared speed sound made in [14] and the analysis made in this work about the EoS parameter $\omega_B$, we can conclude that a universe modeled by a nonlinear electrodynamics given by Lagrangian $L = -\frac{1}{4} F \gamma F \alpha$ is stable and causal for a certain range of values of the free parameter. Therefore we can say that is not a completely good model to describe the evolution of the universe.

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