Free Convection of Water near its Density Maximum in a Heat Generating Porous Cavity with Sinusoidal Heating

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Abstract. The aim of the present numerical investigation is to understand the unsteady free convective flow and heat transfer of cold water near its density maximum temperature in a two-dimensional square porous cavity with internal heat generation. The present study uses the Brinkmann–Forchheimer extended Darcy model for porous medium to study the effects of density inversion parameter, heat generation parameter, Darcy number and porosity. The finite volume method is used to solve the governing equations. The nonlinear behavior of local heat transfer is observed due to sinusoidal heating. The heat transfer enhances for heat absorption case than that of heat generation case in the presence of density maximum.

Keywords: Heat transfer; Porous medium; Sinusoidal temperature; Density inversion.

1. Introduction

Free convection in porous media plays a vital role in many engineering applications such as grain storage installations, solar energy system, oil recovery, underground water flow, heat exchangers, planar reactors and industrial furnaces. In general, density of the fluid decreases when temperature increases. This behavior is reversed in pure water when it attains its maximum density of 999.972[kgm⁻³] around 4°/g at a pressure of one atmosphere. The effect of density maximum on free convection has been studied by many authors on the basis of different phenomena. Sivasankaran and Ho [1, 2] numerically investigated the effect of density maximum and observed that the temperature of maximum density leaves strong effects on fluid flow and heat transfer. Varol et al. [3] numerically analyzed the free convection of cold water near 4°C in a thick bottom walled porous cavity.

The numerical study on convective flow in cavities with sinusoidal temperature distribution is made by several authors. Sivasankaran et al. [4] numerically investigated the effect of non-uniform heating on both side walls on mixed convection in a lid-driven cavity. Bhuvaneswari et al. [5] numerically examined the effect of magneto-convective flow with non-uniform heating on side walls. It is observed that the heat transfer rate increases in the case of non-uniform heating of side walls. Sivasankaran et al. [6] numerically investigated the effects of sinusoidal boundary condition on mixed convection in a square cavity in the presence of magnetic field. Sivasankaran and Bhuvaneswari [7] numerically deliberate the convection in a porous cavity with sinusoidal heating on both sidewalls.

Since no work is reported on convection of cold water in enclosure in the presence of density maximum and heat generation, the present study involves on the solution of this problem numerically.
2. Mathematical formulation

In the present study, cold water saturated square porous cavity of size $L$ with internal heat generation is considered as shown in Figure 1. The right side wall is sustained with fixed temperature $\theta_e$ and left side wall of the cavity is maintained with sinusoidal temperature. The horizontal walls act in adiabatic condition. The gravity functions in downward direction. The velocity components $u$ and $v$ are considered in $x$ and $y$ directions respectively. A homogeneous, isotropic porous medium is considered in this study. The porous medium is modeled with Brinkman-Forchheimer extended Darcy model and it is assumed thermodynamic equilibrium with the water. Except density, all the thermal properties of the water are considered as constant and the density of the water follows the relation described by Sivasankaran and Ho [2]. The Boussinesq approximation is valid and it is further assumed that the viscous dissipation is negligible.

The equations governed the system are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$1 \frac{\partial u}{\epsilon \partial t} + \frac{1}{\epsilon} \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\nu}{\epsilon} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \frac{\nu}{K} u \frac{F_c}{\sqrt{K}} \sqrt{u^2 + v^2} \quad (2)$$

$$1 \frac{\partial v}{\epsilon \partial t} + \frac{1}{\epsilon} \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\nu}{\epsilon} \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - \frac{\nu}{K} v \frac{F_c}{\sqrt{K}} \sqrt{u^2 + v^2} + g \beta (\theta - \theta_a) \quad (3)$$

$$\alpha \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{q}{\rho C_p} \quad (4)$$

The following are the initial and boundary conditions for the problem:

$$t = 0, \quad u = v = 0, \quad \theta = \theta_e, \quad 0 \leq (x, y) \leq L$$

$$t > 0, \quad u = v = 0, \quad \frac{\partial \theta}{\partial y} = 0, \quad y = 0 \& L \quad (5)$$

$$u = v = 0, \quad \theta = \theta(y) = \sin \pi y, \quad x = 0$$

$$u = v = 0, \quad \theta = \theta_e, \quad x = L$$

The non-dimensional equations are obtained using the following dimensionless variables: $X = \frac{x}{L}$, $Y = \frac{y}{L}$, $U = \frac{uL}{v}$, $V = \frac{vL}{v}$, $T = \frac{\theta - \theta_e}{\theta_h - \theta_e}$, $\tau = \frac{tL^2}{v}$, $\xi = \frac{\omega}{vL^2}$ and $\Psi = \frac{\psi}{v}$. Then we get,

$$1 \frac{\partial \xi}{\epsilon \partial t} + \frac{1}{\epsilon^2} \left[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right] = \frac{1}{\epsilon} \nabla \xi - \frac{1}{Da} \nabla \xi - \frac{F_c}{\sqrt{Da}} \nabla \xi^1 + \frac{Ra}{Pr} \frac{\partial \xi}{\partial X} \xi^1 \quad (6)$$

$$\frac{\partial^2 \xi}{\partial X^2} - \frac{\partial \xi}{\partial Y^2} = -\xi \quad (7)$$

$$\frac{\partial T}{\partial \tau} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \nabla^2 T + H_g \quad (8)$$

$$U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X} \quad \text{and} \quad \zeta = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y}, \quad F_c = \frac{1.75}{\sqrt{150 v^{1/2}}}, \quad \text{and} \quad |\xi| = \sqrt{U^2 + V^2}.$$
The present problem is governed by the following non-dimensional parameters: Prandtl number, $Pr = \frac{V}{\alpha}$; Darcy number, $Da = \frac{K}{L^2}$; Rayleigh number, $Ra = \frac{gLHg}{\nu^3}$; density inversion parameter, $T_a = \frac{\rho_a - \rho}{\rho_a - \rho_c}$ and $Hg = \frac{qL^2}{\rho C_p V (\theta_a - \theta_c)}$, the heat generation parameter.

The dimensionless form for the initial and boundary conditions are as follows:

$\tau = 0,$ \quad $U = V = \psi = 0, \quad \zeta = T = 0, \quad 0 \leq X \leq 1, \quad 0 \leq Y \leq 1$

$\tau > 0,$ \quad $U = V = \psi = 0, \quad \zeta = \frac{\partial \psi}{\partial Y^2} \frac{\partial T}{\partial Y} = 0, \quad Y = 0 \& 1$

$\tau > 0,$ \quad $U = V = \psi = 0, \quad \zeta = \frac{\partial \psi}{\partial X^2} \cdot T = \sin \pi Y, \quad X = 0$

$\tau > 0,$ \quad $U = V = \psi = 0, \quad \zeta = \frac{\partial \psi}{\partial X^2} \cdot T = 0, \quad X = 1$

The local heat transfer across the hot wall is calculated by the local Nusselt number $Nu = \left(\frac{ET}{\partial Y}\right)_{\tau=0}$ and the average Nusselt number is given by $\overline{Nu} = \int_0^1 Nu dX$. The non-dimensional governing equations are solved using finite volume method. The detailed method of solution and code validation can be found in Janagi et al. [8].

3. Results and discussion

![Streamlines for various density inversion parameter and heat generation parameter with Ra=10^6, Da=10^{-3}, and \epsilon=0.6.](image)

Figure 2. Streamlines for various density inversion parameter and heat generation parameter with $Ra=10^6$, $Da=10^{-3}$, and $\epsilon=0.6$.

In the present investigation, the controlling parameters are the heat generation parameter ($-3 \leq Hg \leq 3$), Rayleigh number ($10^3 \leq Ra \leq 10^6$), Darcy number ($10^{-1} \leq Da \leq 10^{-5}$), porosity ($0.4 \leq \epsilon \leq 0.8$) and density inversion parameter ($0 \leq T_a \leq 1$). Fig. 2(a)-(e) presents the flow for different values of density inversion parameter and heat generation parameter with $Ra = 10^6$, and...
This is because the density maximum plane exists along the cold wall and hence the water rises along the hot wall and falls along the cold wall. As of the cavity, which shows the strong convective motion inside the cavity. When $H_g = -3$, there appears a bicellular pattern along with a small eddy near the bottom left corner of the cavity. Though there is no density inversion inside the cavity, bicellular pattern exists due to heat absorption. As $T_m$ raises, the cell near the hot wall strengthens and cell near the cold wall is pushed towards the cold wall. Also, it is observed that the small eddy disappears as shown in Fig. 2(a). There appears a tricellular pattern for $T_m = 0$ on increasing $H_g$, see Fig. 2(b). As $T_m$ raises, the cell near the hot wall strengthens and suppressing the secondary cell as described in the previous case. A single largest cell is formed when $T_m = 0$ in the absence of heat generation parameter. This is because the density maximum plane exists along the cold wall and hence the water rises along the hot wall and falls along the cold wall. As $T_m$ increases, a hot cell is suppressed by the formation of secondary cell due to density inversion and the secondary cell enlarges in size to occupy the entire cavity. With increase in heat generation parameter and when $T_m = 0$, two small secondary cells formed near the hot wall. The secondary cells grow in size as $T_m$ raises. When $H_g = 3$, $T_m = 0$, the small eddy grows in size and rotates with more dense near the top-left corner of the cavity. As $T_m$ raises, the bicellular pattern is formed. In addition, the multicellular pattern is formed when $T_m = 1$ and $H_g = 3$. When $H_g = 3$, the counterclockwise rotating primary cell pushes the secondary cell towards the right wall as $T_m$ raises. Further increase in $T_m$, the tri-cellular pattern is formed.

The isotherms are depicted for different density inversion parameter and heat generation parameter with $Ra = 10^6$, $Da = 10^{-3}$ and $\varepsilon = 0.6$ in the fig. 3(a) – 3(e). When there is no heat generation or absorption ($H_g = 0$), $T_m = 0$, thermal boundary layer forms near the left-bottom wall and right-top wall of the cavity, which shows the strong convective motion inside the cavity. When $H_g = -3$, there is no considerable change in the convective heat transfer as $T_m$ raises. The strong thermal boundary layer exists along left wall for all values of $T_m$ in the case of heat absorption. But, we found the reverse phenomena in the case of heat generation, that is, the strong boundary layers are formed along right
wall in the case of heat generation, which is evidently seen in Fig. 3(a,b,d,e). The vertical temperature stratification occurs in the upper portion of the cavity in the case of strong heat absorption and heat generation. Fig. 4 shows the local Nusselt number for different values of heat generation parameter. The local Nusselt number profiles clearly indicate that the local heat transfer is directly affected by sinusoidal heating. It is observed from Fig. 4(a)-4(c) that the local heat transfer is very high in the upper portion of the cavity. Also local heat transfer rate behaves nonlinearly with the height of the wall ($Y$).

![Graph](image)

**Figure 4** Local Nusselt number for different heat generation parameters.

Fig. 5(a) presents average Nusselt number for different values of the density inversion parameter and heat generation parameter. The average heat transfer rate raises on increasing density inversion parameter except $Hg = 1$. The heat transfer rate is higher for heat absorption case than that of heat generation case. This is because the heat energy is transformed into the cavity rapidly due to heat absorption. Fig. 5(b) exemplifies the average Nusselt number against the Darcy number for different values of the density inversion parameter and heat generation parameter. It is observed that higher heat transfer rate is found at $Hg = -1$ than $Hg = 1$ for all values of $T_m$ and $Da$. In the presence of heat absorption, the average heat transfer rate decreases on increasing the values of the Darcy numbers for all values of $T_m$. In the case of heat generation, average Nusselt number decreases slightly and then it raises on increasing the values of the Darcy numbers. The average Nusselt number raises
on increasing the values of $T_m$ in the case of heat absorption. Also the average heat transfer rate decreases on increasing the value of $T_m$ for low values of $Da$ (10^{-1},10^{-2})$ in the case of heat generation. The opposite behavior is observed for higher values of $Da$ (10^{-3},10^{-4})$ in the case of heat generation. It is interesting to note that two different behaviors on heat transfer rate are observed in the presence of heat generation and heat absorption, respectively.

![Graphs](image)

**Figure 5.** Average Nusselt number for (a) different density inversion parameter, (b) Darcy number, (c) Rayleigh number and (d) porosity.

Fig. 5(c) indicates the average Nusselt number for different values of heat generation parameter and Rayleigh number. It is found that the average heat transfer rate decreases on increasing the value of $Ra$ for $Hg = 1$ in the presence of density maximum inside the enclosure. The average heat transfer rate decreases first ($Ra \leq 10^4$) and then raises in the absence of heat generation/absorption. Here we observed a contradict result from the universal fact that average Nusselt number raises with $Ra$. This is because of the density maximum plane exists inside the enclosure and non-uniform heating. Also the average heat transfer rate raises with $Ra$ in the presence of heat absorption. Further scrutinizing the curves, it is observed that heat transfer rate is high in the presence of heat absorption/generation with comparing to the non-heat generating case. Fig. 5(d) shows the effect of porosity on average Nusselt number for different values of the heat generating parameter. It is clearly observed that heat absorption/generation enhances the heat transfer rate comparing the non-heat generating ($Hg = 0$) case. There is no significant change on average heat transfer rate when increasing the porosity for given values of $Hg$. Comparing heat absorption and heat
generation cases, the enhancement of heat transfer is observed for heat absorption case. This is because of the fluid inside the cavity absorbs the heat energy, so the heat is transferred quickly from the vertical walls.

4. Conclusions

The effects of heat generation/absorption and sinusoidal heating on free convection in a square porous cavity are studied numerically in the presence of density maximum of water. It is detected that the strong thermal boundary layers are formed along the left wall in the case of heat absorption and along the right wall in the case of heat generation. The multi-cellular flow patterns are formed inside the enclosure due to sinusoidal heating and density inversion effect. The local heat transfer behaves nonlinearly and it is directly affected by the sinusoidal heating. The average heat transfer rate decreases first and then enhances on increasing the values of the Rayleigh number in the absence of heat generation/absorption, which contradicts the universal fact that average Nusselt number enhances with Ra. There is no significant effect on average Nusselt number when the porosity raises in the presence of density maximum for different values of heat generation parameter. The average heat transfer rate diminishes on rising $Da$ in the presence of heat absorption. The average heat transfer rate decreases first and increases on increasing $Da$ in the presence of heat generation. The heat transfer enhances for heat absorption case than that of heat generation case for all values of $Ra$, $Da$, $\varepsilon$ in the presence of density maximum.

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