Unparticle-Induced Lepton Flavor Violating Decays

\[ \tau \rightarrow \ell (V^0, P^0) \]

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Abstract

We make an evaluation of the lepton flavor violating (LFV) decays \( \tau \rightarrow \ell (V^0, P^0) \), where \( \ell = e \) or \( \mu \) and \( V^0(P^0) \) is a neutral vector (pseudo-scalar) meson, in the context of unparticle physics. The constraints are investigated systematically on the related coupling parameters from all the available experimental data, and the parameter values are specified appropriately. The results show that whereas over the whole parameter space allowed by experiments all the \( \tau \rightarrow \ell P^0 \) modes have a branching ratio too small to be measurable experimentally, in a large subspace as observed all of the \( \tau \rightarrow \ell V^0 \) modes get simultaneously a branching ratio as high as \( \mathcal{O}(10^{-10} - 10^{-8}) \), which is reachable at the LHC and super B factory. The important implications are drawn.

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1 Introduction

In the Standard Model (SM), massless neutrinos of different families are not mixed so that lepton flavors are made exactly conservative, or speaking, a lepton flavor violating (LFV) mode is forbidden absolutely in the SM. If the neutrino oscillation phenomenon takes place actually, we can affirm that the neutrinos are of a nonvanishing mass and thus lepton flavor conservation would be broken. Even so, LFV processes are still highly suppressed because of the smallness of neutrino masses. Hence, any distinct LFV signal can be deemed an indication of new physics beyond the SM. Recently, there has been an increasing interest in LFV physics. The current status of this subject is reviewed in [1].

Given the fact that the operators responsible for LFV transitions could be provided by most of the existing models beyond the SM, LFV phenomena could be explored in various theoretical frameworks. Most of efforts have been devoted to an investigation about LFV decays [2, 3, 4, 5, 6, 7, 8, 9] and lepton anomalous magnetic moments \( (g - 2) \) [9, 10, 11, 12, 13, 14]. A large branching ratio is predicted for \( \tau \rightarrow 3\ell, \ell\gamma \) and \( \ell(V^0, P^0) \) (where \( \ell = e \) or \( \mu \) and \( V^0(P^0) \) is a neutral vector (pseudo-scalar) meson) in some models such as the MSSM framework [3], SUSY seesaw mechanism [4], SUSY-GUT scenario [5] and type-III seesaw model [6]. Of all the existing discussions on LFV, those based on unparticle theory [15] are especially intriguing, because LFV processes can proceed at a tree level in this approach. The phenomenological implications of unparticle physics have been discussed intensely for the LFV transitions \( \mu \rightarrow 3e \) [7], \( \mu \rightarrow e\gamma \) [8] and \( \tau \rightarrow \mu\gamma \) [9], electron and muon \( (g - 2) \) [9, 10, 11, 12, 13, 14] and collider physics [11, 16]. Besides, the effects of unparticle have been explored on hadronic processes [12, 17, 18, 19, 20, 21]. More interestingly, the experimental constraints have been investigated on some of the unparticle coupling strengths and the important results have been obtained. For a mini-review on unparticle phenomenology one can be referred to [22]. The recent progress in unparticle physics can be found in [23]. On the other hand, a continuous experimental search has already performed for various LFV \( \tau \) decays. Very recently, an updated measurement has been reported on \( Br(\tau \rightarrow \ell V^0) \) [24] and \( Br(\tau \rightarrow \ell K^0_x) \) [25]. The estimated experimental upper limits on the branching ratios are in the range a few \( \times (10^{-8} - 10^{-7}) \) at 90% confidence level, for \( \tau \rightarrow 3\ell, \ell\gamma \) and \( \ell(V^0, P^0) \) [26].

Though no clear signal has been detected in the current extensive search for LFV decays, it is expected that the future LHC will probe \( \tau \rightarrow 3\mu \) and \( \ell(V^0, P^0) \) down to the \( 10^{-8} \) level, while a sensitivity of \( 10^{-10} - 10^{-9} \) will be reachable for a search for \( \tau \rightarrow 3\ell, \ell\gamma \) and \( \ell(V^0, P^0) \) at the super B factor [27].

Motivated by the recent progress in unparticle phenomenology and good prospect of the experiments on LFV \( \tau \) decays, in this Letter we intend to make an assessment of \( \tau \rightarrow \ell(V^0, P^0) \) in the context of unparticle physics to understand the possibility to discover them in the future experimental searches.
This Letter is organized as follows. In the following section, on a brief introduction of the basic concepts of unparticle physics, we address the effective models we use for describing unparticle interactions with the SM particles and make a simple discussion. Section 3 is devoted to a derivation of decay rates for $\tau \rightarrow \ell (V^0, P^0)$. A detailed parameter discussion and numerical evaluation is presented in section 4. The final section is reserved for summary.

2 Effective Interactions

The scale invariance in the conformal field theory, although not an exact symmetry of nature, might play an important role in exploring new physics beyond the SM. It prohibits strictly any particles with a definite nonzero mass from manifesting themselves and thus is broken in the SM. But there could be a sector, which is exactly scale invariant and interacts very weakly with SM particles at a scale much beyond the SM one. On the basis of a previous study [28], Georgi [15] suggests that there exist, in a very high energy theory, SM fields and BZ fields with a nontrivial infrared fixed point. These two sectors interact with each other by exchanging particles with a very large mass $M_U$. Below this mass scale, the heavy particles can be integrated out, resulting in the following local interactions:

$$\frac{1}{M_U^{d_{SM}+d_{BZ}-4}} O_{SM} O_{BZ},$$  \hspace{1cm} (1)

where $O_{SM}$ is a SM operator with mass dimension $d_{SM}$ and $O_{BZ}$ an operator with mass dimension $d_{BZ}$ built out of BZ fields. When the energy scale runs down to a certain scale $\Lambda_U$, at which the scale invariance in the BZ sector emerges, the renormalizable couplings of the BZ fields bring about dimensional transmutation. Then below this scale the BZ operators match onto unparticle ones and correspondingly, the interactions in (1) match onto an effective interaction of the form

$$\frac{C_U A_{d_{BZ}-d_U}}{M_U^{d_{SM}+d_{BZ}-4}} O_{SM} O_U,$$  \hspace{1cm} (2)

with $C_U$ being a coupling coefficient and $d_U$ the nonintegral number scale dimension of the unparticle operator $O_U$.

Scale invariant unparticle stuff bears the characters strikingly other than those of ordinary particles. In particular, scale invariance can be used to fix the two-point functions of unparticle operators and further their propagators. The resulting propagators read,

$$\int d^4x e^{iP \cdot x} \langle 0 | T[O_U^\mu(x)O_U^\nu(0)] | 0 \rangle = i \frac{A_{d_U}}{2 \sin(d_U \pi)} (-g^{\mu\nu} + \frac{P\mu P\nu}{P^2})(-P^2 - i\epsilon)^{d_U-2},$$  \hspace{1cm} (3)

for a transverse vector unparticle, and

$$\int d^4x e^{iP \cdot x} \langle 0 | T[O_U(x)O_U(0)] | 0 \rangle = i \frac{A_{d_U}}{2 \sin(d_U \pi)} (-P^2 - i\epsilon)^{d_U-2},$$  \hspace{1cm} (4)
for a scalar unparticle. The coefficient $A_{d_U}$ is given by

$$A_{d_U} = \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + \frac{1}{2})}{\Gamma(d_U - 1)\Gamma(2d_U)}.$$  

(5)

Since the matching procedure from the BZ operators to unparticle ones is unknown, unparticles may interact with SM particles in many possible ways. In the present case, we would like to use the effective coupling forms suggested by Georgi [15]. Then the interactions of a vector unparticle with the charged leptons can be expressed uniformly as

$$\mathcal{L}_E = 2\Lambda_{d_U}^{1-d_U}\bar{E}_L\gamma_\mu V_E E_L O_{E_U}^\mu$$

$$= 2\Lambda_{d_U}^{1-d_U}\left(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L\right)\gamma_\mu \begin{pmatrix} \lambda_{ee} & \lambda_{e\mu} & \lambda_{e\tau} \\ \lambda_{\mu e} & \lambda_{\mu\mu} & \lambda_{\mu\tau} \\ \lambda_{\tau e} & \lambda_{\tau\mu} & \lambda_{\tau\tau} \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} O_{E_U}^\mu,$$  

(6)

where a left-hand lepton vector $E_L$ is introduced, and all the related coupling constants $\lambda_{ij}$ are arranged in a $3 \times 3$ matrix $V_E$ and are treated as a real number. These coupling constants are in general viewed as a free parameter. A hierarchical relation, however we may conceive, does exist among some of them, because the LFV operators might be suppressed to a different degree by a small factor. We postulate that the following relations are respected:

$$\lambda_{\tau\tau} \geq \lambda_{\tau\mu} \geq \lambda_{\tau e} \quad \text{and} \quad \lambda_{\mu\mu} \geq \lambda_{\mu e}.$$  

In fact, such relations could be accommodated by the existing experimental data, as will be seen later.

The unparticle interactions with quarks could be discussed in parallel. Since only three light quarks are involved in the present situation, it suffices that we confine ourself to the former two generations. We have

$$\mathcal{L}_U = 2\Lambda_{d_U}^{1-d_U}\bar{U}_L\gamma_\mu V_U U_L O_{U_U}^\mu$$

$$= 2\Lambda_{d_U}^{1-d_U}\left(\bar{u}_L, \bar{c}_L\right)\gamma_\mu \begin{pmatrix} \lambda_{uu} & \lambda_{uc} \\ \lambda_{cu} & \lambda_{cc} \end{pmatrix} \begin{pmatrix} u_L \\ c_L \end{pmatrix} O_{U_U}^\mu,$$  

(7)

$$\mathcal{L}_D = 2\Lambda_{d_U}^{1-d_U}\bar{D}_L\gamma_\mu V_D D_L O_{D_U}^\mu$$

$$= 2\Lambda_{d_U}^{1-d_U}\left(\bar{d}_L, \bar{s}_L\right)\gamma_\mu \begin{pmatrix} \lambda_{dd} & \lambda_{ds} \\ \lambda_{sd} & \lambda_{ss} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix} O_{D_U}^\mu,$$  

(8)

which describe the unparticle interactions with up- and down-type quarks, respectively. For the related flavor conserving couplings $\lambda_{qq}$ ($q = u, d, s$) and flavor changing one $\lambda_{sd}$, we assume them to comply with the numerical relationship $\lambda_{uu} \sim \lambda_{dd} \sim \lambda_{ss} \geq \lambda_{sd}$. 

4
Correspondingly, the effective interactions involving scalar unparticle are of the following forms:

\[ \mathcal{L}'_E = 2\Lambda^{-d_L}_U(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L)\gamma_\mu \begin{pmatrix} \lambda'_{ee} & \lambda'_{e\mu} & \lambda'_{e\tau} \\ \lambda'_{\mu e} & \lambda'_{\mu\mu} & \lambda'_{\mu\tau} \\ \lambda'_{\tau e} & \lambda'_{\tau\mu} & \lambda'_{\tau\tau} \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \partial^\mu O_U, \]

(9)

\[ \mathcal{L}'_U = 2\Lambda^{-d_L}_U(\bar{u}_L, \bar{c}_L)\gamma_\mu \begin{pmatrix} \lambda'_{uu} & \lambda'_{uc} \\ \lambda'_{cu} & \lambda'_{cc} \end{pmatrix} \begin{pmatrix} u_L \\ c_L \end{pmatrix} \partial^\mu O_U, \]

(10)

\[ \mathcal{L}'_D = 2\Lambda^{-d_L}_U(\bar{d}_L, \bar{s}_L)\gamma_\mu \begin{pmatrix} \lambda'_{dd} & \lambda'_{ds} \\ \lambda'_{sd} & \lambda'_{ss} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix} \partial^\mu O_U, \]

(11)

where it can likewise be assumed that there are the coupling hierarchies which are of the same structures as the corresponding ones suggested in the vector unparticle cases, and it should be understood that the scale dimensions have been set identical for the two different types of unparticles.

3 Calculation of Decay Rates

Now we embark upon calculating the decay rates for \( \tau \to \ell(V^0, P^0) \) with the effective interactions (6) – (11). It is easily noticed that the scalar (vector) unparticle does not couple with a single vector (pseudo-scalar) meson. Then the decays \( \tau \to \ell V^0 \) proceed via just the vector unparticle, while the \( \tau \to \ell P^0 \) transitions do by only the scalar unparticle.

In order to discuss the vector unparticle mediated decays \( \tau \to \ell V^0 \), we could take \( \tau \to \mu \phi \) as an illustrative example. From the Feynman diagram plotted in Fig. 1, we can write down the transition amplitude as

![Feynman diagram](image-url)
\[
A(\tau \to \mu \phi) = \frac{\lambda_{\mu} \lambda_{s}}{\Lambda_{d}^{2(\Delta_{d}^{-1})}} \frac{A_{d}}{\sin\left(\Delta_{d} \pi\right)} m_{\phi} f_{\phi} \left(\frac{m_{\phi}^{2} - m_{\tau}^{2} - i\epsilon}{P_{2}}\right)^{d_{d}^{-2}} \\
\times \frac{\lambda_{s}}{\Lambda_{d}^{-1}} \langle \phi | \bar{s} \gamma_{\mu} (1 - \gamma^{5}) s | 0 \rangle, \tag{12}
\]
where \(P_{\mu}^{\tau}\) is the four momentum of the unparticle, and we have employed the ideal mixing scheme for the \(\omega - \phi\) system. Further, the above expression can be simplified as,
\[
A(\tau \to \mu \phi) = i \frac{\lambda_{\mu} \lambda_{s}}{2\Lambda_{d}^{2(\Delta_{d}^{-1})}} \frac{A_{d}}{\sin\left(\Delta_{d} \pi\right)} m_{\phi} f_{\phi} \left(\frac{m_{\phi}^{2} - m_{\tau}^{2} - i\epsilon}{P_{2}}\right)^{d_{d}^{-2}} \\
\times \sum_{\text{spin}} \langle \phi | \bar{s} \gamma_{\mu} (1 - \gamma^{5}) s | 0 \rangle, \tag{13}
\]
using the standard definition \(\langle \phi | \bar{s} \gamma_{\mu} s | 0 \rangle = m_{\phi} f_{\phi} \varepsilon_{\mu}^{*}\), with \(m_{\phi}, f_{\phi}\) and \(\varepsilon\) being the corresponding mass, decay constant and polarization vector, respectively. After summing over the spins of the final states and averaging over the spins of the initial state, the decay width is derived as
\[
\Gamma(\tau \to \mu \phi) = \frac{|\vec{p}|}{16\pi m_{\tau}^{2}} \sum_{\text{spin}} |A(\tau \to \mu \phi)|^{2}, \tag{14}
\]
where \(\vec{p}\) stands for the momentum of the outgoing particles in the \(\tau\) rest frame, and
\[
\sum_{\text{spin}} |A(\tau \to \mu \phi)|^{2} = \left[ \frac{\lambda_{\mu} \lambda_{s}}{2\Lambda_{d}^{2(\Delta_{d}^{-1})}} \frac{A_{d}}{\sin\left(\Delta_{d} \pi\right)} m_{\phi} f_{\phi} \left(\frac{m_{\phi}^{2} - m_{\tau}^{2} - i\epsilon}{P_{2}}\right)^{d_{d}^{-2}} \right]^{2} \left[ \frac{m_{\tau}^{4}}{m_{\phi}^{2}} + m_{\tau}^{2} - 2m_{\phi}^{2} \right]. \tag{15}
\]

Using (13) – (15) and making a simple algebraic manipulation, the decay rates for the other \(\tau \to \ell V^{0}\) modes are easily achieved. Here we do not give them any more.

For the scalar unparticle mediated decays \(\tau \to \ell P^{0}\), the hadronic matrix elements \(\langle P^{0}(p) | \bar{q} \gamma_{\mu} \gamma_{5} q | 0 \rangle\) enter into the expressions for the decay amplitudes. As usual, in the \(\pi^{0}\) and \(K^{0}(\bar{K}^{0})\) case these matrix elements are parameterized, for instance, as
\[
\langle K^{0}(p) | \bar{d} \gamma_{\mu} \gamma_{5} s | 0 \rangle = i f_{K} p_{\mu}, \tag{16}
\]
with \(p_{\mu}\) and \(f_{K}\), respectively, being the four momentum and decay constant of the \(K^{0}\) meson. In contrast, the \(\eta\) and \(\eta'\) situation is much more complicated because of mixing. The relevant decay constants are defined by
\[
\langle M(p) | \bar{q} \gamma_{\mu} \gamma_{5} q | 0 \rangle = \frac{i}{\sqrt{2}} f_{M}^{q} p^{\mu}, \quad \langle M(p) | \bar{s} \gamma_{\mu} \gamma_{5} s | 0 \rangle = i f_{M}^{s} p^{\mu}, \tag{17}
\]
where \(M = \eta\) or \(\eta'\) and \(q = u\) or \(d\). We would like to consider the \(\eta - \eta'\) mixing effect in the Feldmann-Kroll-Stech (FKS) scheme \cite{29}. In this scheme the physical meson states \(|\eta\rangle\) and \(|\eta'\rangle\), in term of the parton Fock states \(|\eta_{q}\rangle = |u\bar{u} + d\bar{d}|/\sqrt{2}\) and \(|\eta_{s}\rangle = |s\bar{s}\rangle\), are decomposed as
\[
\left( \begin{array}{c}
|\eta\rangle \\
|\eta'\rangle
\end{array} \right) = \left( \begin{array}{cc}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{array} \right) \left( \begin{array}{c}
|\eta_{d}\rangle \\
|\eta_{s}\rangle
\end{array} \right), \tag{18}
\]
where $\phi$ is the mixing angle. Furthermore, by defining the two basic decay constants $f_q$ and $f_s$ as

$$\langle \eta_q(p)|q\gamma^\mu\gamma_5 q|0\rangle = \frac{i}{\sqrt{2}} f_q p^\mu, \quad \langle \eta_s(p)|\bar{s}\gamma^\mu\gamma_5 s|0\rangle = i f_s p^\mu,$$  \hspace{1cm} (19)

we have the following relations:

$$f_q = f_q \cos \phi, \quad f_s = -f_s \sin \phi,$$

$$f_q' = f_q \sin \phi, \quad f_s' = f_s \cos \phi.$$  \hspace{1cm} (20)

With the aid of the data fitting results $f_q/f_\pi = 1.07$, $f_s/f_\pi = 1.34$ and $\phi = 39.3^\circ \pm 1.0^\circ$, the desired values of the decay constants $f_q$, $f_q'$, $f_s$ and $f_s'$ can be achieved \[29\].

At present, the decay rates for $\tau \rightarrow \ell P^0$ could be calculated with the known decay constants. As in the vector meson case, we illustrate our findings of $\tau \rightarrow \ell P^0$ with the resulting expression for the decay width in the $\tau \rightarrow \mu K^0$ case,

$$\Gamma(\tau \rightarrow \mu K^0) = \frac{\bar{p}}{16\pi m_t^2} \sum_{\text{spin}} |A(\tau \rightarrow \mu K^0)|^2,$$  \hspace{1cm} (21)

with

$$A(\tau \rightarrow \mu K^0) = -\frac{\lambda_{\tau\mu} \lambda_{sd}}{2 \Lambda^{2d_t}} \frac{A_{dJ}}{\sin(d_t \pi)} f_K m_K^2 (m_K^2 - i\epsilon)^{d_t-2} \times \left[m_\tau\bar{\tau}(1 + \gamma^5) + m_\mu\bar{\mu}(1 - \gamma^5)\right],$$  \hspace{1cm} (22)

and

$$\sum_{\text{spin}} |A(\tau \rightarrow \mu K^0)|^2 = \left[\frac{\lambda_{\tau\mu} \lambda_{sd}}{\Lambda^{2d_t}} \frac{A_{dJ}}{\sin(d_t \pi)} f_K m_K^2 (m_K^2 - i\epsilon)^{d_t-2}\right]^2 \times \left[(m_\tau^2 + m_\mu^2)(m_\tau^2 + m_\mu^2 - m_K^2) - m_\tau^2 m_\mu^2\right].$$  \hspace{1cm} (23)

### 4 Parameter Discussions and Numerical Evaluations

For a quantitative analysis of these LFV decays, we need to make a detailed discussion about the various parameters involved in the calculation.

The mass parameters associated with the present calculation have been well known, the decay constants of the related light mesons have been determined better experimentally too. All those are listed in Tab[1].

The non-integral scale dimension $d_t$ is calculable in principles, but difficult to estimate in practice. However, it might be limited to $1 < d_t < 2$, which is to be used here, from the unitarity [15] and convergence condition. As concerns the scale parameter $\Lambda_t$, we
could let it range from 1 TeV to a few TeV, because ones expect generally that a certain new physics, if it exists, should appear at such energy region.

Our main concern, of course, is how the underlying coupling constants take their values. For having a knowledge about the couplings $\lambda_{\tau\mu}$ and $\lambda_{\tau e}$, ones have to make a correlation discussion for the various LFV processes where these couplings are involved. Unfortunately, the currently available experimental data are not sufficient to provide them with a decisive parameter space. For the relevant unparticle-quark couplings, though we can extract them from the experimental measurements on some hadronic processes in a certain data fitting way, the uncertainties, among other things, in the long distant QCD parameters would affect greatly the accuracy of extraction.

As the case stands, it is needed to work at a level of order of magnitude, as we make a choice of parameter sets from the regions allowed experimentally. Before starting our discussion, a couple of explanations are in order: (1) Both vector and scalar unparticles could in general be responsible for a LFV transition. Including simultaneously contributions of both the unparticles can make not only the results have a large uncertainty but also the calculation extremely complicated. In the following parameter discussion we assume that they two contributes separately, as done in many studies, and consider only the vector unparticle cases. Also, we suppose that the corresponding coupling strengths are the same for the scalar and vector unparticle interactions. (2) We know that the unparticle parameters of an effective interaction contains $\Lambda_U$, $d_U$ and the coupling constant $\lambda$. The resulting transition amplitude for a process depends on the parameter function $f_{d_U}(\lambda, \Lambda_U)$. With the function values extracted from an experiment, which are generally relevant to $d_U$, the values of $\lambda$ can be determined at any $d_U$, the results being, of course, dependent on $\Lambda_U$. Accordingly, these coupling values, though changed with $\Lambda_U$, correspond to one and the same $f(\lambda, \Lambda_U)$ of a fixed value. If we want to make a theoretical prediction with these extracted coupling parameters, we could work at an arbitrarily chosen $\Lambda_U$. The final results must have nothing to do with $\Lambda_U$, for the same $f(\lambda, \Lambda_U)$ enters, which keeps its value unchanged for different $\Lambda_U$. For convenience, we will work at $\Lambda_U = 1$ TeV. (3) The unparticle couplings $\lambda_{ee}$ and $\lambda_{\mu e}$, as two important inputs in our parameter determination, have been investigated in detail in [10, 31] and [8], respectively. In the region $1.5 < d_U < 2$, the resulting bounds on $\lambda_{ee}$ and $\lambda_{\mu e}$ are available for the present case. From the findings obtained by a study on the inviable positronium decays [31], we deduce easily that $\lambda_{ee} \leq 10^{-4}$ for $d_U = 1.5$, $\lambda_{ee} \leq 10^{-3}$ for $d_U = 1.6$ and $\lambda_{ee} \leq 10^{-2}$ for $d_U \geq 1.7$. Moreover, $\lambda_{\mu e}$ has a negligibly small number, as required by the experiments on the $\mu - e$ conversion in heavy nuclei [8], so that we can set it to zero. These constraint conditions will be used below to restrict other unparticle couplings. In the region $1 < d_U < 1.5$, the study indicates that a more stringent restriction on $\lambda_{ee}$ comes from the precise measurement on long-ranged spin-spin interaction of electrons.
However, the results are not directly applicable and a revaluation is needed. In reality, if we work in the present context we have to assess not only $\lambda_{\mu\mu}$ but also the related unparticle-quark couplings in the region $1 < d_{\mu\mu} < 1.5$ in which these unparticle parameters are less known. It is possible to make such an assessment, however goes beyond the scope of this work. We will choose $1.5 \leq d_{\mu\mu} < 2$ as our work region.

We investigate, to begin with, the possible regions of $\lambda_{\tau\mu}$ and $\lambda_{\tau e}$ allowed by the existing experiments [26]. The authors of [9] manage to understand the parameter region of $\lambda_{\tau\mu}$ in a scalar unparticle model by a combined analysis of the muon $g - 2$, $\tau \rightarrow \mu \gamma$ and $\tau \rightarrow 3\mu$. They find that as $d_{\mu\mu} \geq 1.6$ the muon $g - 2$ experiment demands that $\lambda_{\mu\mu}$ have a negligibly small number, which is in agreement with what is required by the experiments of the $\mu - e$ conversion in heavy nuclei [8], and at least one of $\lambda_{\mu\tau,\mu\mu}$ be of $O(10^{-1} - 1)$. Including further the possible constraints from the experimental data $Br(\tau \rightarrow 3\mu) < 3.2 \times 10^{-8}$ and $Br(\tau \rightarrow \mu \gamma) < 6.8 \times 10^{-8}$, they conclude that one of the two couplings is of $O(10^{-1} - 1)$, while the other is at or below order $10^{-2}$. These constraints are possibly weak, because in the derivation the $\mu$ loop is assumed to dominate in the $\tau \rightarrow \mu \gamma$ transition so that the contribution of the virtual $\tau$ particle is not included. However, the same conclusion can yet be drawn in disregard of the constraint of $\tau \rightarrow \mu \gamma$. We make the same investigation within the present framework by means of the experimental observations of the muon $g - 2$, $\tau \rightarrow 3\mu$ and $\tau \rightarrow \mu^+e^-$ (with $Br(\tau \rightarrow \mu^+e^-) < 2.7 \times 10^{-8}$), and find that $\lambda_{\tau\mu}$ can range from $O(10^{-3})$ to $O(10^{-2})$ if $\lambda_{\mu\mu}$ takes a larger value of $O(10^{-2} - 1)$, and vice versa. From these possible parameter regions we can pick out our preferred parameter sets: (1) $\lambda_{\tau\mu} = 10^{-3}$ and $\lambda_{\mu\mu} = 10^{-2}$, for $d_{\mu\mu} = 1.5$. (2) $\lambda_{\tau\mu} = 10^{-3}$ and $\lambda_{\mu\mu} = 10^{-1}$, for $d_{\mu\mu} = 1.6$. (3) $\lambda_{\tau\mu} = 10^{-2}$ and $\lambda_{\mu\mu} = 1$, for $d_{\mu\mu} > 1.6$. At this point, we must emphasize the fact that the current experimental data on the tau $g - 2$ [26] do not provide more about the couplings involving $\tau$ lepton than we get above and below, because of the existing sizable uncertainty which allows us to do theoretical calculation within a considerably large space of parameter.

As far as $\lambda_{\tau e}$ is concerned, the parameter regions allowed by $\mu \rightarrow e\gamma$ have been evaluated in [8]. However, a consistent evaluation requires us to consider a combined constraint from the processes $\mu \rightarrow e\gamma$, $\tau \rightarrow 3e$ and $\tau \rightarrow e\mu^+\mu^-$ as well as electron $g - 2$. From the experimental measurements $Br(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$, $Br(\tau \rightarrow 3e) < 3.6 \times 10^{-8}$ and $Br(\tau \rightarrow e\mu^+\mu^-) < 3.7 \times 10^{-8}$, it follows that $\lambda_{\tau e}$ can be limited to the region $\lambda_{\tau e} \leq O(10^{-4})$ for $d_{\mu\mu} = 1.5$ and 1.6, while the resulting upper limits can basically remain at order of $10^{-3}$ for $d_{\mu\mu} > 1.6$. The constraint is achievable from the electron $g - 2$ experiment too, by making a replacement of the corresponding parameters in the expression for the muon $g - 2$ and then confronting the result with the numerical deviation between the SM estimate and experimental measurement $|\Delta\alpha| < 15 \times 10^{-12}$ [32]. But no new results are found. In the numerical evaluation we will use $\lambda_{\tau e} = 10^{-4}$ for $d_{\mu\mu} = 1.5$ and 1.6, and $\lambda_{\tau e} = 10^{-3}$ for
In passing, it is attractive to examine the possible region for \( \lambda_{\tau \tau} \) using the experimental bounds \( Br(\tau \to \mu\gamma) < 6.8 \times 10^{-8} \) and \( Br(\tau \to e\gamma) < 1.1 \times 10^{-7} \), along with the various constraint conditions obtained already. The results show that \( \tau \to \mu\gamma \) furnishes a stronger restriction \( \lambda_{\tau \tau} \leq O(10) \) as \( d_U \geq 1.5 \) and therefore the possibility of a sizable unparticle-tau coupling strength cannot be ruled out. Then we can conclude, according to the present study, that our hierarchy assumptions \( \lambda_{\tau \tau} \geq \lambda_{\tau \mu} \geq \lambda_{\tau e} \) and \( \lambda_{\mu\mu} \geq \lambda_{\mu e} \) are acceptable at least for the existing LFV experiments. It remains to be seen whether such relationships are true or not. We can believe that the future precision measurement on the tau \( g-2 \) would help to clarify this issue.

To turn to the discussion about the unparticle-quark couplings. The existing constraints on them come mainly from the studies on some inclusive \([20]\) and exclusive \([18, 19]\) decays of B mesons and neutral meson mixing systems \([12, 17, 18, 19]\). One expects that the inclusive process \( B \to X_s\gamma \) would provide a stringent constraint on new physics effects, as a result of the good agreement between the experimental measurement and SM prediction on \( Br(B \to X_s\gamma) \). However, it is not always this case in the face of unparticle effects \([20]\). The sensitivity of \( Br(B \to X_s\gamma) \) to the coupling parameters weakens as \( d_U > 1.5 \). As it is, the constraints would become considerably weak as \( d_U > 1.7 \) so that a sizable unparticle-quark coupling strength is allowed. To have more understanding of the unparticle parameters, in \([19]\) the impacts of unparticle are analyzed on \( B_{d,s} - \bar{B}_{d,s} \) mixing processes and exclusive channels \( B \to \pi\pi, \pi K \), and especially a detailed \( \chi^2 \) data fitting is carried out for the \( B \to \pi\pi, \pi K \) with the constraints of \( B_{d,s} - \bar{B}_{d,s} \) mixing. The fitting results with \( d_U = 1.5 \) demonstrate that there is a large coupling strength of \( O(10^n)(n = 0, 1) \) for the flavor conserving interactions, which is compatible with the findings in the \( B \to X_s\gamma \) situation. What is particularly interesting is that \( \lambda_{uu} \) and \( \lambda_{dd} \) turn out to be at the same order of magnitude and a relatively small number is implied for \( \lambda_{sd} \), as expected by us. It is claimed that with the yielded optimized values of parameters, the existing various discrepancies may be explained between the SM predictions and experimental data. We think that these constraints, though subject to an estimate of uncertainty, could serve as a valuable reference for us to select proper parameter values. We assign the following numbers to the related couplings: \( \lambda_{uu} = \lambda_{dd} = \lambda_{ss} = \lambda \sim 10^{-n} \) \((n = -1, 0, 1, 2)\) and \( \lambda_{sd} = 10^{-n}\lambda \) \((n = 1, 2)\) for \( d_U = 1.5 \). In the region \( d_U > 1.5 \), little is known about them. Nevertheless, numerous studies show that when \( d_U \) increases, the ranges allowed experimentally become large for unparticle-lepton couplings. The same should be true of the quark case, for the coupling forms are the same in the two situations. Taking this point into consideration and for simplicity, we suggest that these unparticle-quark couplings remain unchanged in the region \( d_U \geq 1.5 \), due to a certain stringent restriction condition. This is equivalent to
Table 1: Summary of the leptonic and hadronic parameters (in units of MeV).

|    | \(f_\pi\) | \(f_K\) | \(f_\rho\) | \(f_\omega\) | \(f_{K^*}\) | \(f_\phi\) | \(f_\eta\) | \(f_\eta'\) |
|----|---------|---------|---------|-----------|----------|--------|--------|--------|
|    | 130     | 160     | 209     | 195       | 217      | 231    | 139    | 174    |

|    | \(m_\pi\) | \(m_K\) | \(m_\rho\) | \(m_\omega\) | \(m_{K^*}\) | \(m_\eta\) | \(m_{\eta'}\) |
|----|---------|---------|---------|-----------|----------|--------|--------|
|    | 130     | 498     | 770     | 782       | 892      | 1020   | 547    | 958    |

\[m_\tau\quad m_\mu\]

\[1777\quad 105\]

a conservative estimate. In addition, in the case of \(\tau \to \ell (\rho^0, \pi^0)\) we need to confront a combination of two couplings \(\lambda_{uu} - \lambda_{dd}\). Since we are discussing the unparticle couplings at a level of order of magnitude, it is sound to set \(\lambda_{uu} - \lambda_{dd}\) at the same order as \(\lambda_{uu,dd}\).

Now we are a position to make a numerical evaluation. In the first place, we can notice that for both \(\tau \to \ell V^0\) and \(\tau \to \ell P^0\) an approximate order of magnitude relation exists between the branching ratios, with our selected coupling parameters. In the \(\tau \to \ell V^0\) situation, we have the following observation:

\[
\begin{align*}
Br(\tau \to \mu \rho^0) &\approx Br(\tau \to \mu \omega) \sim Br(\tau \to \mu \phi) \\
&> Br(\tau \to e \rho^0) \sim Br(\tau \to e \omega) \sim Br(\tau \to e \phi) \\
&\geq Br(\tau \to \mu K^{*0}(K^{*0})) > Br(\tau \to e K^{*0}(\bar{K}^{*0})),
\end{align*}
\]

(24)

if neglecting the mass difference between muon and electron and \(SU(3)\) breaking effects in the hadron parameters. A similar relation holds approximately for \(\tau \to \ell P^0\). However, it would suffer from a large \(SU(3)\) breaking correction. The numerical calculations denote that these order of magnitude relations are, indeed, respected better for \(\tau \to \ell V^0\) than for \(\tau \to \ell P^0\).

Let us take a closer look at the behaviors of the branching ratios in the parameter spaces we adopt. It is clearly seen that the parameter region \(\lambda \leq 1\) is allowed by the experiments, while the region \(\lambda > 1\), where the branching ratios for all the \(\tau \to \ell V^0\) go beyond their experimental upper limits, is prohibited. The allowable parameter sets are fixed as: (I) \(\lambda = 10^{-2}, 1.5 \leq d_U < 2\); (II) \(\lambda = 10^{-1}, 1.55 \leq d_U < 2\); (III) \(\lambda = 1, 1.85 < d_U < 2\). Over these parameter areas all the \(\tau \to \ell P^0\) modes show a branching ratio less than \(\mathcal{O}(10^{-20})\), which is far from the experimental reach. We will focus our discussion on the \(\tau \to \ell V^0\) case. For the set I, the branching ratios are of orders \(10^{-14} - 10^{-9}\) for \(\tau \to \mu V^0\), compared with the numerical region for \(Br(\tau \to e V^0)\) \(10^{-16} - 10^{-11}\). In the set II case, whereas the \(\tau \to \mu V^0\) modes have a branching ratio ranging from \(10^{-12}\) to \(10^{-8}\), the numerical results for \(Br(\tau \to e V^0)\) are located between \(10^{-14}\) and \(10^{-9}\). If the set III is used, the numerical
values for $Br(\tau \rightarrow \mu V^0)$ vary from $10^{-10}$ to $10^{-8}$, while those for $Br(\tau \rightarrow e V^0)$ do between $10^{-12}$ and $10^{-10}$. To illustrate the dependence of $Br(\tau \rightarrow \ell V^0)$ on $\lambda$ and $d_\ell$, we can typically consider the $\tau \rightarrow \mu \phi$ case in which the behaviors are shown of $Br(\tau \rightarrow \mu \phi)$ in some parameter regions in Fig.2. Albeit the branching ratios turn out to be sensitive to $d_\ell$ and $\lambda$, there is still a large parameter region, as will be seen, in which for any $d_\ell$ almost all the $\tau \rightarrow \mu V^0$ modes have a branching ratio as large as $\mathcal{O}(10^{-10} - 10^{-8})$, which are expected to be reachable at the LHC and super B factor.

The parameter regions experimentally favorite can be summarized as: (I) $\lambda = 10^{-2}$, $1.5 \leq d_\ell < 1.8$; (II) $\lambda = 10^{-1}$, $1.55 \leq d_\ell \leq 1.9$; (III) $\lambda = 1$, $1.85 < d_\ell < 2$. In these regions of the allowable parameter space, as a matter of fact, all the $\tau \rightarrow \mu V^0$ modes except $\tau \rightarrow \mu K^{*0}(\bar{K}^{*0})$ are accessible experimentally, and in some subregions the same observations can be obtained for $\tau \rightarrow \mu K^{*0}(\bar{K}^{*0})$ and $\tau \rightarrow e(\rho^0, \omega, \phi)$. Only $\tau \rightarrow eK^{*0}(\bar{K}^{*0})$ exhibits a branching ratio below $\mathcal{O}(10^{-10})$. The partial findings from these parameter regions, together with the current experimental upper limits on them, are collected in Tab.2.

Table 2: Some selected numerical results for $Br(\tau \rightarrow \ell V^0)$. The corresponding parameter sets ($\lambda_{\tau \mu}$, $\lambda_{\tau e}$, $\lambda_{\rho \phi}$) are $(10^{-3}, 10^{-4}, 10^{-2}, 10^{-6})$, $(10^{-3}, 10^{-4}, 10^{-1}, 10^{-2})$, $(10^{-2}, 10^{-3}, 10^{-1}, 10^{-2})$ and $(10^{-2}, 10^{-3}, 1, 10^{-1})$, respectively, for $d_\ell = 1.5$, 1.6, 1.7 and 1.8, and 1.9.

| Mode       | $d_\ell$ | 1.5  | 1.6  | 1.7  | 1.8  | 1.9  | EXP.UL |
|------------|----------|------|------|------|------|------|--------|
| $\tau \rightarrow \mu \rho^0$ | $8.0 \times 10^{-9}$ | $2.6 \times 10^{-8}$ | $6.3 \times 10^{-8}$ | $4.3 \times 10^{-9}$ | $3.6 \times 10^{-8}$ | $6.8 \times 10^{-8}$ |
| $\tau \rightarrow \mu \omega$ | $6.9 \times 10^{-9}$ | $2.2 \times 10^{-8}$ | $8.1 \times 10^{-8}$ | $3.8 \times 10^{-9}$ | $3.2 \times 10^{-8}$ | $8.9 \times 10^{-8}$ |
| $\tau \rightarrow \mu \phi$ | $8.1 \times 10^{-9}$ | $2.9 \times 10^{-8}$ | $1.2 \times 10^{-7}$ | $6.1 \times 10^{-9}$ | $5.7 \times 10^{-8}$ | $1.3 \times 10^{-7}$ |
| $\tau \rightarrow \mu K^{*0}$ | $1.2 \times 10^{-10}$ | $4.2 \times 10^{-10}$ | $1.6 \times 10^{-9}$ | $7.9 \times 10^{-11}$ | $6.9 \times 10^{-10}$ | $5.9 \times 10^{-8}$ |
| $\tau \rightarrow \mu \bar{K}^{*0}$ | $1.2 \times 10^{-10}$ | $4.2 \times 10^{-10}$ | $1.6 \times 10^{-9}$ | $7.9 \times 10^{-11}$ | $6.9 \times 10^{-10}$ | $1.0 \times 10^{-7}$ |
| $\tau \rightarrow e \rho^0$ | $8.1 \times 10^{-11}$ | $2.6 \times 10^{-10}$ | $9.4 \times 10^{-10}$ | $4.4 \times 10^{-11}$ | $3.6 \times 10^{-10}$ | $6.3 \times 10^{-8}$ |
| $\tau \rightarrow e \omega$ | $7.0 \times 10^{-11}$ | $2.2 \times 10^{-10}$ | $8.2 \times 10^{-10}$ | $3.8 \times 10^{-11}$ | $3.2 \times 10^{-10}$ | $1.1 \times 10^{-7}$ |
| $\tau \rightarrow e \phi$ | $8.3 \times 10^{-11}$ | $3.0 \times 10^{-10}$ | $1.2 \times 10^{-9}$ | $6.2 \times 10^{-11}$ | $5.8 \times 10^{-10}$ | $7.3 \times 10^{-8}$ |
| $\tau \rightarrow e K^{*0}$ | $1.2 \times 10^{-12}$ | $4.2 \times 10^{-12}$ | $1.6 \times 10^{-11}$ | $7.9 \times 10^{-13}$ | $7.0 \times 10^{-12}$ | $7.8 \times 10^{-8}$ |
| $\tau \rightarrow e \bar{K}^{*0}$ | $1.2 \times 10^{-12}$ | $4.2 \times 10^{-12}$ | $1.6 \times 10^{-11}$ | $7.9 \times 10^{-13}$ | $7.0 \times 10^{-12}$ | $7.7 \times 10^{-8}$ |

So far all the numerical calculations are performed with the fixed coupling values. However, these parameter values, as has been emphasized, should be understood as an order of magnitude and thus we have to consider the effects resulting from the variations of coupling parameters within their individual orders of magnitude. We have examined such effects. With the fixed values of $\lambda_{\tau \mu}$ and $\lambda_{\tau e}$, $\lambda$ dependence of $Br(\tau \rightarrow \mu \phi)$ is plotted in Fig.3.
Figure 2: $d_U$ dependence of $Br(\tau \rightarrow \mu \phi)$ with the different $\lambda$ values. The horizontal line denotes the present experimental upper bound on $Br(\tau \rightarrow \mu \phi)$.

Figure 3: $\lambda$ dependence of $Br(\tau \rightarrow \mu \phi)$ with the different $d_U$. The horizontal line denotes the present experimental upper bound on $Br(\tau \rightarrow \mu \phi)$. 
Obviously, the numerical results can change by up to two orders of magnitude, when $d_U$ remains fixed and $\lambda$ ranges from 0.01 to 0.1. The similar situation appears for $0.1 < \lambda \leq 1$. Exploring further the case where all the related parameters vary simultaneously, we would have a much larger numerical range. But this also indicates that there could be more theoretical results which are within the experimental reach. Those listed in Tab.2 are only some estimated lower bounds on $Br(\tau \to \ell V^0)$. In fact, when the couplings $\lambda_{\tau \mu}$ and $\lambda_{\tau e}$ change at the same time within their respective ranges, we can work within the expanded parameter regions for $\lambda$ and $d_U$: (I) $0.01 \leq \lambda < 0.04$, $d_U = 1.5$; (II) $0.01 \leq \lambda < 0.1$, $1.5 < d_U < 2$; (III) $\lambda = 0.1$, $1.55 \leq d_U < 2$; (IV) $0.1 < \lambda < 1$, $1.75 < d_U < 2$; (V) $\lambda = 1$, $1.85 < d_U < 2$. In these parameter spaces there are many favorable subspaces as observed, in which all the $\tau \to \ell V^0$ modes can get simultaneously a branching ratio of $O(10^{-10} - 10^{-8})$. By contrast, all the $\tau \to \ell P^0$ modes remain still inaccessible to experiments over these areas.

Conversely, the restrictions can be inspected on the coupling parameters from the experiment data on $Br(\tau \to \ell (V^0, P^0))$. By limiting ourself to the parameter area $0.01 \leq \lambda \leq 1$, the constraints on $\lambda_{\tau \mu}$ and $\lambda_{\tau e}$ are observed to be weaker in a large region of the parameter spaces for $\lambda$ and $d_U$, and rather loose in some subspaces like $\lambda = 0.01$ and $1.6 \leq d_U < 2$, when compared with those presented above.

Again we stress that all the presented numerical results, despite achieved at $\Lambda_{\text{U}} = 1$ TeV, maintain unchanged as $\Lambda_{\text{U}}$ increases, and besides, the scalar unparticle mediated $\tau \to \ell P^0$ are evaluated using the same scale dimensions and coupling strengths as in the vector unparticle case. Even if we regard these corresponding parameters as independent of each other, it is yet difficult to get a interesting result for $\tau \to \ell P^0$, since it is hardly conceivable that the related coupling constants have a considerably sizable number in such a case. Once the coupling parameters become better understood in the whole $d_U$ region $1 < d_U < 2$, we could make a more complete and reliable assessment of these LFV processes. But it seems likely that with the parameter sets specified adequately the hierarchical relation $Br(\tau \to \ell V^0) \gg Br(\tau \to \ell P^0)$ will be kept valid, although the branching ratios alter with change in parameter values. Of course, to do calculation with different unparticle coupling scenarios would in general lead to different results. It is desirable to enquire into these LFV processes in other unparticle coupling schemes.

Our findings for $\tau \to \ell V^0$ appear to be comparable with some of the existing estimates \cite{3, 4, 5, 6}. Nevertheless, in the $\tau \to \ell P^0$ case we have a branching ratio much less than those for $\tau \to \ell V^0$, presenting a striking contrast to the predications of the other approaches.
5 Summary

We have made a detailed analysis for the unparticle induced LFV decays $\tau \to \ell (V^0, P^0)$ in an effective model with a hierarchical relation suggested among some of the coupling constants.

To get a consistent and believable assessment, all the available experimental data have been used to constrain the unparticle couplings. From the obtained constraint conditions, the parameter values for the related couplings have been specified appropriately. As a by-product, it is found that a sizable $\lambda_{\tau\tau}$ is allowed by the current experimental data, and our hierarchy hypotheses $\lambda_{\tau\tau} \geq \lambda_{\tau\mu} \geq \lambda_{\tau e}$ and $\lambda_{\mu\mu} \geq \lambda_{\mu e}$ can be accommodated by these constraint conditions.

We have evaluated the branching ratios and examined the possibility to experimentally discover these modes in the near future. In the parameter region $\lambda > 1$, all the $\tau \to \mu V^0$ modes have a branching ratio exceeding their individual experimental upper limits. The experimentally allowed regions for $\lambda$ and $d_U$ are determined approximately as: (I) $0.01 \leq \lambda < 0.04$, $d_U = 1.5$; (II) $0.01 \leq \lambda < 0.1$, $1.5 < d_U < 2$; (III) $\lambda = 0.1$, $1.55 \leq d_U < 2$; (IV) $0.1 < \lambda < 1$, $1.75 < d_U < 2$; (V) $\lambda = 1$, $1.85 < d_U < 2$. In many regions of these parameter spaces, for all the $\tau \to \ell V^0$ modes we can have simultaneously a branching ratio of orders $10^{-10} - 10^{-8}$, which are expected to be accessible at the LHC and super B factory. Compared with the $\tau \to \ell V^0$ case, all the $\tau \to \ell P^0$ modes show a branching ratio beyond the experimental reach.

Also, we have inspected the limits imposed on the couplings $\lambda_{\tau\mu}$ and $\lambda_{\tau e}$ by the experiments on $\tau \to \ell (V^0, P^0)$, observing that there is a looser bound than those yielded by the other available LFV experiments, in a large subspace of the parameter spaces $0.01 \leq \lambda \leq 1$ and $1.5 \leq d_U < 2$.

It is explicitly too early to draw a final conclusion whether these LFV decays are observable experimentally. We have to await the improvement in experiment and progress in unparticle phenomenology. Different from the predictions of the other new physics models, however, the unparticle approach gives the numerical relation $Br(\tau \to \ell V^0) \gg Br(\tau \to \ell P^0)$, with the implication that there is a greater discovery potential of $\tau \to \ell V^0$ than that of $\tau \to \ell P^0$ in future experiments. If this gets confirmed in the future experimental searches, the present research is perhaps instructive in identifying whether or not these LFV processes are induced or dominated by unparticles.

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