A Model-Independent Photometric Redshift Estimator

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Abstract.
We derive a simple empirical photometric redshift estimator using a training set of galaxies with multiband photometry and measured redshifts in the Hubble Deep Field (HDF). This estimator is model-independent; it does not use spectral templates. The dispersion between the estimated redshifts and the spectroscopically measured ones is small; the dispersions range from $\sigma_z \simeq 0.03$ to 0.1 for $z < 2$ galaxies, and from $\sigma_z \simeq 0.14$ to 0.25 for $z > 2$ galaxies. The predictions provided by our empirical redshift estimator agree well with recently measured galaxy redshifts. We illustrate how our empirical redshift estimator can be modified to include flat spectrum galaxies with $1.4 < z < 2$.

1. Introduction

Our aim is to derive a simple but accurate empirical photometric redshift estimator which is model-independent. We do not use spectral templates; we only use a training set of galaxies with multiband photometry and measured redshifts. We have applied our method to the Hubble Deep Field (HDF) (Williams et al. 1996) and derived a catalog of estimated redshifts for 848 HDF galaxies with $I < 27$ and with measured fluxes in $U$, $B$, $V$, and $I$ (Wang, Bahcall, & Turner 1998).

To derive formulae for the estimated redshifts, we first divide the galaxy redshift sample (the training set) into regions of high and low redshifts ($z > 2$ and $z < 2$) based on empirical color cuts. We then divide both regions into color ranges to minimize dispersion. The physical motivation for this latter step is to reflect color shifts with $z$ for different type galaxies. We then find the best linear fit between redshift and colors for each color range,

$$z_a = c_1 + c_2(U - B) + c_3(B - V) + c_4(V - I),$$

where $c_i$ ($i = 1, 2, 3, 4$) are constants. Here we have assumed that photometry has been done in four bands, $U$, $B$, $V$, and $I$. The dispersion ($\sigma_z$) between our estimated redshift, $z_a$, and the measured spectroscopic redshift, $z$, is calculated using the jack knife method. For a sample with $N$ data points, we make $N$ subsamples, each omitting one data point. We then carry out linear fits $N$ times on the $N-1$ data point subsamples. We use each fit to make a prediction for the omitted data point; the rms of the true values of the omitted points about the fit predictions, scaled by $((N-1)/N)^{1/2}$, is the rms dispersion of our fit. The estimated dispersions of our formulae are therefore quite robust.
The error in \( z_a \) due to photometric errors is

\[
\Delta z_a = \left[ (c_2 \Delta U)^2 + |(c_3 - c_2) \Delta B|^2 + |(c_4 - c_3) \Delta V|^2 + (c_4 \Delta I)^2 \right]^{1/2}.
\] (2)

2. Application to the HDF

We used 82 galaxies with measured spectroscopic redshifts \( z \simeq 0.1 - 3.5 \) in the HDF as our training set (Cohen et al. 1996; Hogg et al. 1998; Lowenthal et al. 1997; Phillips et al. 1997; Steidel et al. 1996). We did not use all 90 (excluding three \( z > 2 \) galaxies with erroneous/uncertain \( z \)'s) HDF galaxies with measured \( z \)'s. The eight outlying galaxies have \( \sigma_z = 0.45 \), mostly near the boundaries of the color ranges; these galaxies were not used in deriving our redshift estimator.

We found that for \( z > 2 \), the galaxies satisfy at least one of the following three color selection criteria:

\[
U \geq 25.66, \quad U - B \geq 0.91, \quad B - V \leq 1.37, \quad V - I \leq 0.5; \quad (3)
\]

\[
I > 23.5, \quad U - B > 2.2; \quad (4)
\]

\[
I > 23.5, \quad B - V > 2.2, \quad U - B > -0.5. \quad (5)
\]

The galaxies with \( z < 2 \) in the training set do not satisfy any of the above relations, suggesting that \( z < 2 \) galaxies generally fall outside these color-magnitude regions.

We divide \( z < 2 \) galaxies into three color ranges (cr), and determine the best-fit redshift estimate formula and dispersion for each color range:

1. cr=1: \( (U - B) < (B - V) - 0.1 \) (28 galaxies)

\[
z_a = 0.4111 - 0.1852(U - B) - 0.3062(B - V) + 0.7301(V - I), \quad \sigma_z = 0.034; \quad (6)
\]

2. cr=2: \( (U - B) \geq (B - V) - 0.1 > (V - I) \) (21 galaxies)

\[
z_a = 0.163 - 0.171(U - B) + 0.340(B - V) + 0.194(V - I), \quad \sigma_z = 0.095; \quad (7)
\]

3. cr=3: \( (U - B) \geq (B - V) - 0.1 \leq (V - I) \) (19 galaxies)

\[
z_a = 1.126 + 0.480(U - B) - 0.513(B - V) - 0.250(V - I), \quad \sigma_z = 0.097. \quad (8)
\]

We divide \( z \geq 2 \) galaxies into two color ranges, and determine their redshift estimate formulas and dispersions:

1. cr=4: \( (B - V) - 0.5 > (V - I) \) (8 galaxies; \( z \geq 3 \))

\[
z_a = 2.37 + 0.02(U - B) + 1.61(B - V) - 2.47(V - I), \quad \sigma_z = 0.14; \quad (9)
\]

2. cr=5: \( (B - V) - 0.5 \leq (V - I) \) (6 galaxies; \( z \leq 3 \))

\[
z_a = 2.18 + 0.10(U - B) + 0.20(B - V) + 0.75(V - I), \quad \sigma_z = 0.25 \quad (10)
\]

Fig.1 shows our estimated redshift \( z_a \) (given by Eqs.(6)-(10)) versus the spectroscopic redshift \( z \) for 90 HDF galaxies.
Figure 1. Our empirically estimated redshift $z_a$ (given by Eqs. (8)-(10)) versus the spectroscopic redshift $z$ for 90 HDF galaxies. The galaxies with known measurement errors in $UBVI$ are plotted with error bars in $z_a$. Error bars without points represent the eight galaxies not used in fitting.
Figure 2.  (a) Our empirically estimated redshift $z_a$ (given by Eqs.(6)-(10)) versus the Sawicki et. al. (1997) template-fitting photometric redshift $z_{temp}$, for 848 galaxies in the HDF with $I < 27$ and measured $UBVI$. The symbols are the same as in Fig.1. The solid diagonal line indicates $z_a = z_{temp}$; the dotted lines mark the region $|z_a - z_{temp}| \leq 0.5$. (b) Same as Fig.2(a), but with the addition of Eq.(11) to select flat spectrum galaxies with estimated redshifts given by Eq.(12).

Our empirical photometric redshift estimator is consistent with template-based photometric redshift estimators where calibrating redshift samples are available. Fig.2(a) compares our empirically estimated redshift $z_a$ (given by Eqs.(6)-(10)) with the template-fitting photometric redshift $z_{temp}$ of Sawicki et. al. (1997), for 848 galaxies in the HDF with $I < 27$ and measured $UBVI$. The symbols are the same as in Fig.1. The solid diagonal line indicates $z_a = z_{temp}$; the dotted lines mark the region $|z_a - z_{temp}| \leq 0.5$. About 90% of the galaxies fall within the dotted lines.

There are 32 new spectroscopic redshifts of $z < 4$ HDF galaxies (Dickinson 1998, Phillips et al. 1998, Steidel et al. 1999) listed in the recent paper by Fernández-soto et al. (1999) which were not available to us when we derived our photometric estimator (Wang et al. 1998). Fig.3(a) shows the difference between our estimated redshift $z_a$ and the spectroscopic redshift $z$, scaled by $(1 + z)$, as a function of $z$. As can be seen, our empirical redshift estimator yields excellent predictions for $z < 2$, with $\sigma_z = 0.13$ for all 18 $z < 2$ galaxies, and with $\sigma_z = 0.08$ if we omit the most discrepant object. For $z > 2$ galaxies, we find $\sigma_z = 0.34$ for all 14 galaxies, and $\sigma_z = 0.31$ if we omit the galaxy with $z = 2.002$. The larger dispersion between $z_a$ and $z$ for $z > 2$ is largely due to the small number of spectroscopic redshifts which were used in deriving the redshift estimator.
3. Flat spectrum galaxies

Galaxies in the redshift range $1.4 \leq z \leq 2$ have flat spectra, which makes the measurement of their spectroscopic redshifts difficult. The absence of calibrators in the range $1.4 \leq z \leq 2.2$ (see Fig.1) leads to an artificial gap in Fig.2(a) for $1.5 \leq z_a \leq 2$ in our estimated redshift $z_a$.

To illustrate how the gap in Fig.2(a) can be filled, we select galaxies which satisfy

$$23.4 \leq I < 26.84, \quad U \geq 26,$$

$$|V - I| \leq 0.6, \quad |B - V| \leq 0.8, \quad 0.6 \leq U - B \leq 2.$$  \hspace{1cm} (11)

We assume that these galaxies satisfy the same color redshift relation as the color range $cr= 3$ galaxies (which are closest to the intermediate redshift galaxies in color and redshift), but with a different constant offset (see Eq.(8)), i.e.,

$$z_a = 1.5 + 0.480(U - B) - 0.513(B - V) - 0.250(V - I).$$  \hspace{1cm} (12)

The constant offset value has been obtained by calibrating with a galaxy which satisfies Eq.(11) and has spectroscopic redshift $z = 2.002$. Fig.2(b) presents our estimated redshift $z_a$ versus the Sawicki et al. template based photometric redshift $z_{\text{temp}}$; it is the same as Fig.2(a), but with the addition of Eq.(11) to select flat spectrum galaxies (represented by star symbols) with estimated redshifts given by Eq.(12). This addition improves on the previous comparison by moving a number of galaxies with $z_{\text{temp}} \sim 2$ and $z_a \gtrsim 2.5$ or $z_a \lesssim 0.5$ in Fig.2(a) into the redshift gap of Fig.2(b).

We can modify our empirical photometric redshift estimator by adding Eq.(11) to select flat spectrum galaxies with estimated redshifts given by Eq.(12),
before applying Eqs. (4)-(10). Fig. 3(b) shows the difference between our modified estimated redshift \( z_a \) and the spectroscopic redshift \( z \), scaled by \((1 + z)\), as a function of \( z \), for 32 newly measured galaxy redshifts. The modified redshift estimator makes the estimated redshifts for \( z > 2 \) galaxies less biased, with \( \sigma_z = 0.30 \) for 13 \( z > 2 \) galaxies (excluding the calibrator galaxy with \( z = 2.002 \)).

4. Discussion

The advantage of our method is that it is model-independent and simple. The disadvantage of the method is that it requires a galaxy redshift sample for training; this is clearly seen in Fig. 2(a), where an artificial gap for \( 1.5 \leq z_a \leq 2 \) exists as a result of the absence of calibrators with measured redshifts in the range \( 1.4 \leq z \leq 2.2 \) (see Fig. 1). In this paper, we have illustrated how this gap in estimated redshifts can be filled by modifying the empirical photometric redshift estimator to include flat spectrum galaxies (see §3). It is important to fill the gap in measured spectroscopic redshifts for \( 1.4 \leq z \leq 2 \), as it would enable better calibration of photometric redshift estimators, and better understanding of the nature of intermediate redshift galaxies.

Since our method is purely empirical, it is sensitive to the data properties of the training set. In order to find and remove possible systematic effects in the photometric redshift estimator due to selection effects in the redshift measurements, it is important to have spectroscopic redshift surveys which are complete to a given magnitude limit.

The photometric redshift estimator described here is simple and accurate. It can be further improved by additional data.

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