Some Issues in Flat Direction Baryogenesis

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Abstract

Motivated by recent developments, we explore some issues in Affleck-Dine baryogenesis. We consider in greater detail the role of thermal effects in the production of baryon number. We find that these effects are important even for rather flat potentials, and obtain somewhat different estimates of the baryon asymmetry than those in the literature. We also consider the decay of the condensate, and possible implications of these observations for the formation of Q-balls.
1 Introduction

Through the years, there have been a number of proposals to understand the origin of the asymmetry between matter and antimatter. If nature is supersymmetric, baryogenesis through the semiclassical evolution of a scalar condensate provides a mechanism which is particularly attractive, both for simplicity and efficiency. In the original analysis, it was supposed that in a theory like the MSSM there would be some exact flat directions, in the limit of unbroken supersymmetry, and that it would be natural for fields in these directions to start off with very large expectation values. Several sources of baryon number and CP violation could be imagined, which could easily generate an enormous baryon asymmetry\[1\].

However, the assumptions of the original work were rather naive, and in considering Affleck-Dine baryogenesis, there are a number of issues one must address\[2\].

- Just how flat are the flat directions? Invariably, higher dimension terms in the superpotential lift the flat directions, unless they are protected by discrete R symmetries (or perhaps by stringy effects). For most D-flat directions, there are candidate terms in the superpotential quartic in the fields (or even low order).

- In considering the evolution of the flat directions, one must take into account the supersymmetry breaking effects of the early universe. Not only are these effects dominant in the period when $H > m_{3/2}$, they can provide an explanation why the fields start at large values. In general, the effective masses of scalars are of order $H$. If these masses are negative, they force the fields to large values, providing the initial conditions for baryogenesis. In the case of flat directions lifted by quartic terms in the superpotential, $\delta W = \frac{\phi^4}{M}$ one has $\phi^2 \sim HM$. More generally, one has, for a superpotential

$$W_n = \frac{\phi^{n+3}}{M^n}, \quad \phi^{2n+2} \sim H^2 M^{2n}. \quad (1)$$

The evolution of the system, as well as the ultimate amount of baryon production, are crucially dependent on $n$.

- In considering how baryon number is produced, one must determine the sources of baryon number violation and CP violation. In most cases the most important effect is supersymmetry violating $A$ terms. In addition to terms proportional to $m_{3/2}$, there are also terms scaled by $H$, and it is usually the mismatch in the phases of the various terms which accounts for baryon production. For example, if one has a superpotential term proportional

\[2\]
to $\phi^{n+3}$, as above, one expects an $A$ term proportional to $H(\phi^{n+3} + \text{c.c.})$. This term typically violates both baryon (and/or lepton) number and $C$ $P$. In generating a baryon number, it is crucial that the phase of this term is different from the phase that exists in flat space.

- One must carefully consider the mechanisms by which the condensate may decay. In [2], it was pointed out that the condensate generally decays as a result of scattering with the thermalized decay products of the inflaton. These have a temperature which behaves roughly as

$$T \sim (T_R^2 H M_p)^{1/4},$$

and this is substantially higher than the reheat temperature for some period. The fields which couple to $\phi$, the AD scalar, gain mass. $\phi$, however, damps as $1/t$, so eventually some of the fields have mass of order $T$ and come to equilibrium. The scattering of these particles off the condensate causes the condensate to quickly evaporate. Generally, these interactions preserve $B$ and $L$, so the previously produced baryon number is unchanged.

- Concerning the last point, it was argued in [2] that if the condensate is not very large, thermal effects can even lead to evaporation of the condensate before the baryon number is produced. If $y$ is the smallest Yukawa coupling in a particular flat direction, there are fields with mass of order $y\phi$. If $y$ is of order $10^{-2} - 10^{-4}$, and if $\phi$ is not too large, then it is possible for some of these fields to be in thermal equilibrium. Using eqn. (3) if $T_R = 10^8$ GeV, for example, then at $H \approx m_{3/2}$, $T \approx 10^9$ GeV; if the reheating temperature is $10^{10}$, then the temperature is of order $10^{10}$ at this time. On the other hand, in the case $n = 1$, the initial amplitude of the $\phi$ oscillations (when $H \sim m_{3/2}$) are of order $\phi \sim \sqrt{m_{3/2} M_p} \sim 10^{11}$ GeV, so if some Yukawa couplings are smaller than $10^{-3}$ or so, then even if the reheating temperature is of order $10^8$ GeV, such states will be in thermal equilibrium. Even though the Yukawa coupling is small, the rate for scattering of particles off the condensate is enormous compared to the age of the universe in these circumstances. Assuming the mass of the condensate particles is of order $m_{3/2}$, a typical cross section for scattering off the condensate will be of order

$$\sigma \sim \frac{y^2 \alpha}{m_{3/2} T},$$

Multiplying by $T^3$, yields a reaction rate compared to the Hubble density of order

$$\frac{\Gamma}{H} \sim y^2 \alpha \frac{M_p}{m_{3/2}}.$$
In other words, the condensate disappears almost immediately.

As a result of these latter considerations, it was argued in [2] that for \( n = 1 \), in order to produce a baryon asymmetry, one must either consider flat directions with only large Yukawa couplings, or one must assume a low reheat temperature, in order to have baryogenesis.

Recently, Campbell et al [3] have made an important observation concerning this picture. They note that if the scalar fields are not too large, so that some of the fields coupled to the condensate are in thermal equilibrium, not only is there rapid evaporation, but there are also large, positive, thermal masses for the scalars, as a result of which they oscillate earlier than otherwise expected. Moreover, the evaporation of the condensate is slow enough that the system undergoes several oscillations before it disappears. Finally, they argue that there are similar thermal \( A \) terms, as a result of which one can produce a significant asymmetry before the condensate evaporates.

We will explore these issues more thoroughly here. These thermal effects are particularly important in the case \( n = 1 \), and can be important for \( n = 2 \). We will see that generically the thermal \( A \) terms exploited by [3] to generate the required CP violation vanish. We study other possible sources of CP violation, and find that in some circumstances these can be effective.

We will see (as observed in [3]), if thermal masses are important, the estimate above of the rate of evaporation is far too large. Clearly, in the cases where particles coupled to the condensate are in thermal equilibrium, the appropriate mass for the condensate particles is not the zero temperature mass but the effective thermal mass (this, after all, determines the oscillation rate). This means that the evaporation cross section is far smaller than in the earlier estimates, and in fact the condensate evaporates well after the formation of the asymmetry. We do not agree in detail with the estimate of the cross section in [3], but a reliable calculation requires the use of real time finite temperature methods, and we will only make crude estimates here. We will assume that the cross section is dominated by processes such as scattering of thermalized fermions off the condensate. This cross section is proportional to two powers of the Yukawa coupling, \( y^2 \) and of the gauge coupling, \( g^2 \), divided by the center of mass energy. This energy is of order \( \sqrt{yT^2} \), the factor \( yT \) corresponding to the thermal mass of the condensate particles, and the factor of \( T \) the typical energy of the fermions. In other words, we will use as our working formula for the reaction rate:

\[
\Gamma = y\alpha T. \tag{5}
\]
Once one has considered thermal masses, one realizes that even in the case where the fields coupled to the condensate are not in thermal equilibrium there are other thermal effects which are important at high temperatures. Couplings to light fields through higher dimension operators leads to contributions to the condensate potential which can dominate for a significant period, and which are potentially crucial to determining the baryon number, even for rather large values of \( n \) \( (n = 2 - 4, \) and possibly larger depending on parameters).

2 Thermal Effects

To get some feeling for the issues involved, suppose, for definiteness, that the inflaton, \( I \), has a mass of order \( 10^{13} \) GeV, while the inflaton amplitude immediately after inflation is of order \( M_p \), and its associated auxiliary field, \( F_I \sim 10^{13} M_p \). Then the inflaton width is of order \( 10^3 \) GeV or so, while the Hubble constant during inflation is of order \( 10^{13} \). After inflation, the temperature quickly rises to order \( 10^{13} \) GeV, and then falls roughly as \( t^{-1/4} \). The inflaton decays when \( H \sim 10^3 \) GeV, with a reheat temperature of order \( 10^{10} \) GeV. This reheat temperature is perhaps somewhat high from the point of view of gravitino production, but we view it as a conservative choice from the perspective of the issues we address in this paper.

If the flat directions are lifted by a term with \( n = 1 \) (i.e. a \( \phi^4 \) term in the superpotential), then when inflation ends, \( \phi \approx \sqrt{H M} \sim 10^{16} \). If there are Yukawa couplings (generically denoted \( y \) in the flat direction (as there typically are) less than about \( 10^{-3} \), then the corresponding fields are in thermal equilibrium. For larger Yukawa couplings, the system still may come to thermal equilibrium well before the inflaton decays. As a result, the scalar fields have thermal masses of order \( y T \). Assuming a negative contribution to the masses of order \( H^2 \), these lead to oscillations once \( y T \approx H \). This is long before \( H \sim m_{3/2} \). Thermal effects also lead to evaporation. Typically, as pointed out in [3], however, the evaporation timescale is somewhat suppressed relative to the oscillation timescale, and one might hope to produce baryons. Indeed, examining our formula for the scattering rate, we see that this differs from the oscillation time by a factor of \( \alpha \).

On the other hand, in the picture developed in [2], the crucial element in the generation of the baryon asymmetry is a misalignment between the phases of the \( A \) terms due to the oscillating inflaton and those which exist at zero curvature. When \( \phi \) begins to oscillate at times of order \( m_{3/2} \), the difference in these phases leads to a net torque. If the fields start to oscillate much earlier, however, the effect of the zero curvature terms is suppressed, and one may have
difficulty generating an asymmetry. (One can see that misalignment is necessary by noting that otherwise one can eliminate any phase by a suitable field redefinition.)

The authors of [3] argued that there are additional $A$ terms, proportional to $T$, and that these could be responsible for the asymmetry. It is easy to see, however, that this is not the case in a generic situation. The issue is one of symmetries. The usual picture for the formation of the $A$ terms is to suppose that there is an inflaton coupling of the form

$$\int d^4\theta \frac{I^*}{M_p} f(\phi) + c.c.$$  

which gives an $A$ term scaled by $H$. It is important here that the coupling to the dilaton breaks the $R$ symmetry of the renormalizable (susy-preserving) terms of the MSSM. The authors of [3] consider terms in the superpotential of the form (taking the case $n = 1$ for simplicity)

$$W = \frac{\phi^4}{M} + h\phi\chi\chi$$  

where $\chi$ represents a field coupled to $\phi$ with a sufficiently small Yukawa coupling, $h$, that it is in thermal equilibrium. Then the potential includes terms such as

$$\delta V = \frac{\phi^3}{M}\chi^*\chi^*.$$  

They then assumed that in the thermal bath, $<\chi\chi>\sim T^2$, so that one has, effectively, a quite large $A$ term. However, in general, symmetries suppress this correlation function, and the result is quite a bit smaller. In particular, the superpotential of eqn. 8 respects an $R$ symmetry, under which the $\chi$ fields transform, so we must ask what terms violate this symmetry. The $A$ terms, such as $H\phi\chi\chi$ are examples of such terms, as are the gaugino mass terms. Using this coupling, one has:

$$\langle\chi\chi\rangle = hHT\phi\int \frac{d^3k}{(2\pi)^3} \frac{1}{(k^2 + m^2)^2} \approx hHT\phi \frac{2\pi m}{2\pi m}$$  

where $m$, by assumption, is of order $T$ (in general, (e.g. if $\chi$ couples to gauge bosons of an unbroken symmetry, then the mass is of order $g^2T^2$). Even if one supposes that $m$ is smaller, e.g. $m\sim H$, the resulting $A$ term is of order

$$V_A \approx h^2 \frac{T}{M_p} \phi^4$$  

which, for $h \sim 10^{-2}$, is smaller than the non-thermal term. It is possible that one could find larger sources of symmetry violation in particular models, but we believe that this is the generic behavior; there is no temperature enhancement of the $A$ term.
There are other possible sources of $A$ terms which can be relevant. The largest contribution is likely to come from terms which behave like $H \frac{I}{M_p}$, where $I$ is the inflaton term. These terms are not drastically suppressed at the time when oscillations begin, and they have a different time dependence than the leading $A$ term, which is simply proportional to $H$. We will see in the next section that these terms, in the case $n = 1$, may barely yield an adequate asymmetry. In the case $n = 2$, things are better.

These additional $A$ terms arise in a simple way. In addition to the term of eqn. 6, there may be a coupling of the form:

$$
\delta W = \frac{1}{M_p} (aI + b \frac{I^2}{M_p}) \phi^n,
$$

(11)

where $a$ and $b$ are complex constants. $I$ decreases as $t^{-1}$, so the second term is suppressed by $\frac{I}{M_p t}$, and this need not be a very large suppression. These two terms need not have the same phase, so there is the possibility of generating a reasonable baryon number. We will explore the effects of these terms shortly.

Another possibility, which we will not explore in great detail here, arises in the higher $n$ cases. If the suppression of, say, the $n = 1$ terms arises because of discrete symmetries, it is possible that there is less suppression of the $A$ terms with smaller $n$. To see this, let us consider the structure of eqn. 6 in more detail. As an example, take the flat direction labeled by $H_uL$. In order that this direction be lifted by terms of order, say $(H_uL)^3/M_p^2$ (corresponding to $n = 2$), one might hypothesize a suitable discrete symmetry. Now $A$ terms are generated by terms involving the inflaton field. By simply postulating a suitable $R$ transformation for $I$, one might allow the coupling $I(H_uL)^2$. In this case, the effective $A$ term is much larger than naively expected; it is not of order $H$, but of order $H(M^2/(H_uL))$. Indeed, these terms can be so large as to require modification of the whole picture. One might try to do the same thing in the case $n = 2$, but the presence of a large mixing term, $m_{3/2} \mu H_uL$ might lead to phenomenological difficulties.

All of this is relevant only to directions which are not too flat. Otherwise, the AD field is very large during the relevant time period, and the fields to which it couples are very massive, with mass much larger than the temperature. This is the case for $n \geq 2$ or so (the precise value depending on the value of the Yukawa coupling). In these other cases, however, there are other thermal effects which much be considered. In particular, integrating out massive fields can generate couplings of the AD field to remaining light fields, such as

$$
\frac{A}{16\pi^2} \ln(|\phi|^2) F^2_{\mu\nu},
$$

(12)
To understand the effect of these terms at finite temperature, note that this is a correction to the associated gauge coupling. So we can compute the finite temperature $\phi$ potential by simply calculating the free energy of the gauge theory as a function of the gauge coupling.

As an example, consider the flat direction $H_U L$. In this flat direction, the unbroken gauge group is $SU(3) \times U(1)$. The $u$ quarks gain mass in this direction, as do the right handed leptons and $H_D$. The coefficient, $A$, is obtained by integrating out these fields: $A = 3$. To understand the effect of this coupling, note that for slowly varying $\phi$, this is just a modification of the $SU(3)$ coupling. So the leading effect of this term can be determined by considering the free energy as a function of the coupling constant. The leading contribution of gluons, gluinos and quarks to the free energy can be readily obtained from calculations in the literature:

$$\delta \Omega = \frac{Ng}{144}(5N_f/4 + 7N/2)g^2T^4$$  \hfill (13)

The contribution of the scalars requires an additional computation which we have not found in the literature, but at the level of accuracy of our calculations below, this contribution will not be significant (the calculation is currently in progress by one of us). The effective potential for $\phi$ is then

$$V_{\text{eff}}(\phi) = a\alpha_s(T)^2T^4\ln(\phi^2)$$  \hfill (14)

which is obtained from the formula for the free energy as a function of temperature where

$$a = \frac{3Ng}{288}(5N_f/4 + 7N/2)$$  \hfill (15)

is a bit larger than one.

3 Estimating the Baryon Number

We will now study the effects of these thermal terms for various values of $n$. The full parameter space we might explore is very large. Possible parameters include: the coefficients of the $\phi^n$ terms in the superpotential, the coefficients of the $A$ terms proportional to $H$, the coefficient of the scalar mass terms proportional to $H^2$, the size of CP violating couplings, the coefficients of the curvature independent mass and $A$ terms, the value of $m_{3/2}$, the reheat temperature, $T_R$, the value of the Hubble constant during inflation, $H_I$, and so on. In addition, there are many discrete choices we might examine, including the particular flat direction. We will leave a more careful survey of the parameter space to subsequent work[4], and here instead will choose some particular points in the parameter space in order to illustrate the possibilities. We will see that
for \( n = 1 \) directions, with small Yukawa couplings (less that about \( 10^{-2} \)) that it may just barely be possible to produce an adequate asymmetry. For \( n = 2 \), the results are quite sensitive to different parameters. In particular, depending on parameters, different types of thermal effects are important. For \( n = 3 \) and \( n = 4 \), it is not difficult to produce an appreciable asymmetry, but thermal effects do alter the asymmetry relative to the expectations in [3], in some cases by several orders of magnitude.

Some words about notation are in order. We will, in any given case, refer to \( H_o \) as the value of the Hubble constant when the system begins to oscillate. \( H_I \) will be the value of the Hubble constant during inflation, which we will generally take to be \( 10^{13} \) GeV. We will usually take the reheating temperature to be \( T_R = 10^{10} \) GeV. All of our estimates are easily modified for other choices of these parameters.

Let us consider, first, the case where some of the fields which couple to the condensate are in thermal equilibrium. As a model, we take:

\[
V = \left( -H^2 + m_{3/2}^2 + y^2 T^2 \right) |\phi|^2 + \left| \frac{\partial W}{\partial \phi} \right|^2 + aHW + b \left( \frac{H^2}{M} \right)^2 W + A m_{3/2} W \quad W = \frac{\phi^{n+3}}{M^n}. \tag{16}
\]

We can at first ignore the \( m_{3/2} \) terms. Then we can ask: under what circumstances are the fields in thermal equilibrium. We have seen that in the case \( n = 1 \), the fields can easily be in equilibrium immediately after inflation. Consider the case \( n = 2 \). Again taking units with \( 10^{13} \) GeV = 1 for the Hubble constant during inflation and for the value of the initial temperature we have \( \phi^6 \sim H^2 M^4 \). Requiring \( y\phi \sim T \), with \( T \sim H^{1/4} \), gives

\[
H \sim y^{-12} M^{-8}. \tag{17}
\]

Obviously the result is quite sensitive to \( y \) and to constants of order one (e.g. the Planck mass vs. the reduced Planck mass in these formulas); for \( y = 10^{-3} \) and \( M = 10^5 \), one obtains \( H_o = 10^{-4} = 10^9 \) GeV. In this case, \( y\phi = 10^{-1} \approx T \) at this time, so thermal effects are important. If one used the Planck mass this would become \( H = 10^{-12} = 10 \) GeV, so the \( m_{3/2} \) term is already dominant. (When oscillations begin, \( y\phi \approx 1 \) while \( T \sim 10^{-2.5} \), so thermal effects are irrelevant).

Now we estimate the baryon number. In the case \( n = 1, 2 \), we expect that the \( A \) term suppressed by \( I/M \) discussed in equation [11] to be the principle source of baryon number. So we expect the rate of change of the baryon number per unit time is given roughly by:

\[
\frac{dn_B}{dt} = H \frac{\phi^4 b I}{M M} \sin(\delta) \tag{18}
\]
where \( \sin(\delta) \) represents an appropriate combination of \( CP \)-violating phases. If \( H_o \) and \( \phi_o \) are the values of \( \phi \) and \( H \) when oscillations begin, then the baryon number is of order:

\[
n_B = \frac{\phi_o^4 b H_o}{M} M b H_o \sin(\delta).
\]

To estimate the final baryon/entropy ratio we can multiply by \( t^2 = H_o^{-2} \), and divide by \( T_R^4 t_D^2 \), where \( T_R \) is the reheating temperature and \( t_D \sim \frac{m_i^2}{M^2} \) is the decay time. Putting this together, for \( T_R = (HM_I^2)^{1/4} \), we obtain the estimate, for our canonical inflaton model:

\[
\frac{n_B}{T^3} = b \sin(\delta) \frac{\phi_o^4}{MH_o T_R^4 t_D^2}.
\]

In the case \( n = 1 \), \( \phi_o^2 \approx H_o M \), and \( H_o \approx y^{4/3} M_I \), so plugging in the formulas above, gives the crude estimates:

\[
n = 1 : n_B = b \left( \frac{y}{10^{-2}} \right)^{4/3} \sin(\delta) 10^{-14}.
\]

This is four order of magnitudes less then desired ratio. Actual numerical study of the differential equation for the evolution of the scalar field gives results, for a range of parameters, within an order of magnitude of this. However, if the reheating temperature is somewhat lower one might be able to produce larger asymmetry.

The case \( n = 2 \) (with the choice \( M = 10^{18} \) discussed above) is more promising. Now \( \phi^6 = M^4 H^2 \). Oscillations begin much later, as indicated above. Taking the case where oscillations start at \( H_o = 10^7 \text{ GeV} \), we have for the baryon density, proceeding as before,

\[
\frac{n_B H_o^2}{T^3} = \frac{M^{1/3} H_o^{2/3}}{T_R^4 t_D^2} b \sin(\delta) \approx \left( 10^{-8} - 10^{-10} \right) b \sin(\delta)
\]

As before, if the reheat temperature is lower one can produce an even larger asymmetry.

For \( n = 3 \), the field typically comes to equilibrium only after the inflaton has decayed.

In the case \( n \geq 3 \) (or \( n = 2 \) or \( n = 1 \) if the Yukawa coupling is not too small), we should ask about the effects of the thermal potential. In such cases the potential we wish to study has the form:

\[
V = (-H^2 + m_{3/2}^2)|\phi|^2 + a T^4 \ln(|\phi|^2) + |\frac{\partial W}{\partial \phi}|^2 + a HW + b \frac{H^2}{M} W + Am_{3/2} W \quad W = \frac{\phi^{n+3}}{M^n}.
\]

We first should ask: when is the \( T^4 \) term relevant. Comparing the derivatives of the \( H^2 \phi^2 \) and the \( T^4 \) terms gives:

\[
n = 2 : H_o = 10^{-2} M^{-4/5} \approx 10^{-6} \quad n = 3 : H_o = 10^{-2} M^{-1} = 10^{-7} \quad n = 4 : H_o = 10^{-9}.
\]
This is before the decay of the inflaton in each case. At this time, for \( n = 3 \), \( \phi \sim 2 \times 10^{15} \) GeV, while \( T \sim 10^{11} \), so thermal effects should be unimportant, except for extremely small Yukawa couplings. Evaporation typically occurs well after the decay of the inflaton. To estimate the decay time, as in [2], one notes that the leading couplings of thermal particles to the condensate are through dimension 5 operators, so that the annihilation rate behaves as:

\[
\Gamma_{\text{ann}} \approx \left( \frac{\alpha}{4\pi} \right)^2 \frac{T^3}{\phi^2}
\]

where \( \alpha \) is an appropriate gauge coupling (two factors of \( \alpha \) arise from loops). This is typically less than \( H \) until the inflaton decays. After this time, as in [2], the rate slows more rapidly than \( H \), and evaporation eventually occurs.

Again, we can make a crude estimate of the baryon number. For the case \( n = 2 \), \( \phi^6 = M^4 H^2 \); for \( n = 3 \), \( \phi^8 = M^6 H^2 \). We expect that the baryon number is suppressed by \( \frac{m_{3/2}}{H_o} \), the ratio of the curvature independent to the curvature-dependent \( A \) terms, so we estimate:

\[
n = 2 : n_B t^2 \approx \frac{\phi_0^5}{M^2 H^2} \frac{m_{3/2}}{H_o} \sin(\delta) \approx 10^{18} \sin(\delta) \text{GeV}
\]

\[
n = 3 : n_B t^2 \approx \frac{\phi_0^6}{M^3 H^2} \frac{m_{3/2}}{H_o} \sin(\delta) \approx 10^{23} \sin(\delta) \text{GeV}
\]

where, as usual, \( \sin(\delta) \) represents some combination of CP-violating phases. Thus the baryon to photon ratio is of order \( 10^{-10} \sin(\delta) \) in the \( n = 2 \) case, which is clearly in an interesting range. For the case \( n = 3 \), the ratio is roughly \( 10^{-4} \sin(\delta) \), which can be substantial even for small values of the CP violating phases.

It is interesting to compare this result to what is obtained without the inclusion of thermal effects, when oscillation starts at \( H \sim m_{3/2} \), i.e. about four (\( n=2 \)) or two (\( n=3 \)) orders of magnitude later. The present baryon to photon ratio is about six orders of magnitude smaller in the first case, three orders of magnitude smaller in the second. This difference traces to two factors: first, there is the suppression by \( \frac{m_{3/2}}{H_o} \); second, the baryon number violating term times \( t^2 \) scales as \( H_o^{-1/2} \), accounting for another order of magnitude.

For the case \( n = 4 \) the results are similar. One still produces a substantial baryon number, now suppressed by about two orders of magnitude relative to the non-thermal analysis.

Numerical study, however, indicates that these estimates are not always reliable. The baryon number is often significantly larger. A systematic study of the parameter space will appear elsewhere [4]. For the moment, our main point is that the inclusion of thermal effects, even for rather flat potentials, can significantly alter the prediction of the asymmetry.
4 Q Balls

In the last few years, several authors have observed that supersymmetric theories often contain $Q$-balls in their spectra, and that these might be produced in the evolution of the Affleck-Dine condensate. The usual picture of $Q$-ball formation is to note that under certain circumstances, the evolution of the homogeneous condensate is unstable for small momenta, and to argue that this instability is likely to lead to $Q$-ball formation. At this point, there is support for such a picture from numerical simulations in some cases.

Before considering thermal effects, we would comment that it is possible for a condensate to exhibit instability even when it carries no baryon number, so it is by no means clear that any instability one finds is a signal of $Q$-ball formation. For example, the potentials associated with gauge mediated models have such instabilities, whether or not the scalar fields carry non-trivial phases. More detailed study is then necessary to determine whether $Q$-balls form.

More relevant to our present discussion, though, is the fact that the evolution of the condensate, in light of the various thermal effects described here, is rather different than usually expected. This point has already been noted in ref. The issue, in general, is whether $Q$-balls can form before the evaporation of the condensate, and whether they survive the evaporation process.

As we have seen, for a range of $n$, thermal effects control the evolution of the condensate. In the case where the $\chi$ field is in equilibrium, the potential includes not only quadratic terms, but a negative, cubic term,

$$V(T) = a g^2 T^2 |\phi|^2 - b g^3 T |\phi|^3$$  \hspace{1cm} (28)

It is easy to check that this equation satisfies the conditions of for growing instabilities. On the other hand, it is also true that any $Q$-ball which forms will evaporate in much the same way as the condensate. We will present a more detailed analysis of this problem in a subsequent publication (including issues such as the evolution of the instabilities in the thermal potential).

In cases of larger $n$, where the $\ln(\phi^2)$ terms in the potential are important, evaporation eventually destroys the condensate but not necessarily the $Q$-balls. Here the required estimates are similar to those performed in . There are now particles in thermal equilibrium which couple to the condensate through higher dimension operators. If the operators are dimension
five, the interaction rate is of order

\[ \Gamma = \frac{\alpha^3}{16\pi^2\phi^2} T^3 \]  

(29)

In the case \( n = 2 \) (assuming that there are no fields in equilibrium with the condensate), for example, one finds that the condensate eventually evaporates through interactions with the thermal bath. Consider, first, the homogeneous condensate. If one substitutes the expressions for \( T \) and \( \phi \) as a function of \( H \) in the expression for the reaction rate, one finds that immediately after inflation,

\[ \frac{\Gamma}{H} \approx 10^{-8} \]  

(30)

and that it grows as \( H^{-5/6} \). So interactions with the condensate are expected to destroy the condensate shortly after crossover. As for Q-balls, on the one hand, the mean free paths of the individual \( \chi \) particles is large compared to the size of the \( Q \)-balls. On the other, the scattering off the particles in any would-be Q-balls is very rapid.

In the case \( n \geq 3 \), the condensate evaporates much later, and Q-balls are likely to survive. More detailed investigation and further simulations would be worthwhile in these cases.

5 Conclusions

Thermal effects significantly alter the predictions for baryogenesis due to the coherent evolution of scalar fields. In agreement with [3], in the case that the potential is not terribly flat (e.g. \( n = 1 \)) and that the Yukawa couplings are small enough, we have seen that the thermal potential plays an important role in the evolution of the condensate. Oscillations start earlier than in the picture of [2], and the baryon number is produced earlier. This leads to a more promising result than predicted by [2], who argued that the rapid evaporation of the condensate in these cases could prevent any asymmetry from developing. On the other hand, our estimates are less optimistic than those of [3] in these cases, since, as we have argued, \( CP \) violation is suppressed.

We have seen that thermal effects are potentially important even in cases in which the potential is relatively flat \( (n = 2, 3, 4) \). After integrating out heavy particles, interactions with the remaining light particles, while suppressed, can be significant. This can alter by several orders of magnitude the predictions for the asymmetry. We have also seen that in some cases, the evaporation of the condensate tends to eliminate also any would-be \( Q \)-balls. The point is that the light particles in the plasma have a long mean free path that the Q-balls are transparent, but also have large enough interactions with the condensate to destroy it.
For large enough \( n \), thermal considerations are irrelevant. It is certainly of interest to further explore the possible roles of \( Q \)-balls in these cases. More refined numerical studies, both of the condensate evolution described here, and of the non-linear evolution of fluctuations, will be necessary to develop a comprehensive picture.

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