COULOMB EFFECTS AND ELECTRON TRANSPORT THROUGH A COHERENT CONDUCTOR

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We analyze electron transport through relatively short coherent conductors in the presence of Coulomb interaction. We evaluate the current-voltage characteristics of such conductors taking into account the effect of an external environment. Within our model, at large conductances and low $T$ the conductance is suppressed by a universal factor which depends only on the type of the conductor. We also argue that at $T = 0$ the system "scatterer+shunt" can be either an insulator or a metal depending on whether its total resistance is larger or smaller than $R_Q = h/e^2 \approx 25.8$ kΩ. In a metallic phase the Coulomb gap is fully suppressed by quantum fluctuations.

1 Introduction

Discrete nature of the electron charge plays a crucial role in various phenomena in mesoscopic physics, causing, for instance, Coulomb blockade of electron tunneling in metallic junctions and shot noise in mesoscopic conductors. It is of particular importance to understand how an interplay of charge discreteness, Coulomb interaction and coherent scattering may affect electron transport in disordered conductors at sufficiently low temperatures.

Recently we argued that the interaction term in the current-voltage characteristics of a (relatively short) coherent conductor is controlled by the parameter

$$\beta = \frac{\sum_n T_n (1 - T_n)}{\sum_n T_n},$$

which is already well known in the theory of shot noise. Here $T_n$ are the transmissions of conducting modes. A similar conclusion was also reached in Ref. 6 in the limit of a single conducting mode, in which case the parameter reduces to $\beta = 1 - T_1$.

In this paper we extend our previous results by considering the effect of an external environment on quantum transport through a coherent scatterer in the presence of Coulomb interaction.

2 Quasiclassical Langevin equation

As in Ref. 5 we will consider electron transport through an arbitrary coherent scatterer between two big reservoirs. The scatterer conductance without interactions is defined by the standard Landauer formula $1/R = (2e^2/h) \sum_n T_n$. Phase and energy relaxation are only allowed in the reservoirs and not during scattering, i.e. the scatterer is shorter than both dephasing and inelastic relaxation lengths $L_\varphi$ and $L_{in}$. Coulomb effects in the scatterer region are described by an effective capacitance $C$. We also assume that the scatterer is attached to an external voltage source $V_x$ via a linear impedance $Z_S(\omega)$.

In the limit of sufficiently large energies and/or at large scatterer conductances $g = R_Q/R \gg 1$ it is convenient to describe the system by means of the quasiclassical Langevin equation approach. This equation can be derived from the real time path integral technique. For a coherent conductor to be considered here this derivation was performed in Ref. 5. The effect of
a linear external impedance can be incorporated in the same way as it was done for the case of tunnel junctions.\[1.2.3\] Combining the results\[1.2.3\] one arrives at the following Langevin equation

$$\frac{C}{e} \dot{\varphi} + \frac{1}{eR} \varphi + \int Z_S^{-1}(t-t') \dot{\varphi}(t') \, dt' - \frac{V_x}{Z_S(0)} = \xi_1 \cos \varphi + \xi_2 \sin \varphi + \xi_3 + \xi_s.$$  \tag{2}$$

Here $\dot{\varphi}(t)/e = V(t)$ is the fluctuating voltage across the conductor. Eq. \tag{2} is sufficient provided $eV$ and $T$ are smaller than the typical inverse traversal time $1/\tau_{trav}$ (e.g. the Thouless energy in the case of diffusive conductors). Below we will mainly address the limit $RC > \tau_{trav}$, however some of our results should remain valid in the opposite case as well. The terms in the right-hand side of \tag{2} account for the current noise. They are defined by the correlators

$$\langle |\xi_1|^2 \rangle = \langle |\xi_2|^2 \rangle = \frac{\beta}{R} \omega \coth \frac{\omega}{2T}, \quad \langle |\xi_3|^2 \rangle = \frac{1-\beta}{R} \omega \coth \frac{\omega}{2T},$$  \tag{3}$$

$$\langle |\xi_s|^2 \rangle = \text{Re} \left( \frac{\omega}{Z_S(\omega)} \right) \coth \frac{\omega}{2T}.$$  \tag{4}$$

If we set $\varphi(t) = eV_x t$ and define the total fluctuating current $\delta I(t) = \xi_1 \cos eV_x t + \xi_2 \sin eV_x t + \xi_3$, we will immediately reproduce the standard result\[4.5.6.7.8.9.10\].

$$\langle |\delta I_s|^2 \rangle = \frac{1}{RQ} \left\{ 2\omega \coth \frac{\omega}{2T} \sum_n T_n^2 + \left[ \sum_{\pm} (\omega \pm eV_x) \coth \frac{\omega \pm eV_x}{2T} \right] \sum_n (1 - T_n) \right\}.$$  \tag{5}$$

The $I-V$ curve for a conductor can be obtained by averaging eq. \tag{2} over noise. We find

$$IR = V - \langle |\xi_1 \cos \varphi + \xi_2 \sin \varphi| \rangle.$$  \tag{6}$$

The last term in eq. \tag{2} describes the effect of Coulomb interaction. We note that this term depends only on the two stochastic variables $\xi_1$ and $\xi_2$. Since the correlation functions for both these variables\[3\] are proportional to the parameter $\beta$, the magnitude of the whole interaction term in \tag{2} should scale with the same parameter. Thus, the result\[3\] takes the form

$$\frac{dI}{dV} = 1 - \beta f(V,T),$$  \tag{7}$$

where $f(V,T)$ is the universal function which depends on $R$ and $Z_S(\omega)$. This function was already evaluated in the case of tunnel junctions.\[1.4.5.6.7.8.9.10\] Defining $1/Z(\omega) = 1/R - i\omega C + 1/Z_S(\omega)$ and proceeding perturbatively in $\text{Re} Z$, at $T \to 0$ one finds (cf., e.g., Refs. 9 and 6)

$$\frac{d^2 I}{dV^2} = \frac{e^2 \beta}{\pi RV} \text{Re}[Z(eV)].$$  \tag{8}$$

In a special case of a linear Ohmic environment $Z_S(\omega) \simeq R_S$ we find

$$f(V,T) = \frac{e}{\pi} \int_0^\infty dt \frac{(\pi T)^2}{\sinh^2(\pi Tt)} e^{-F(t)} \left( 1 - e^{-|\delta t|} \right) \sin[eVt],$$  \tag{9}$$

$$F(t) = -\frac{1}{g_0} \int_{-\infty}^{+\infty} dt' \frac{(\pi T)^2}{\sinh^2(\pi Tt')} \left( \beta g \cos[eVt'] + (1-\beta)g + gs \right) \times \left[ |t' - t| - |t'| + R_0 C \left( e^{-|t' - t|/R_0 C} - e^{-|t'|/R_0 C} \right) \right].$$  \tag{10}$$

where $g_0 \equiv R_Q/R_0 = g + gs$ and $gs = R_Q/R_S$. Eqs. \tag{3}, \tag{4} work well provided either $g_0 \gg 1$ or $\max(T,eV) \gg E_C$. In the limit $g_0 \gg 1$ and $\max(eV,T) \gg g_0 E_C \exp(-g_0/2)$ one can set $\exp(-F(t)) \simeq 1$. Then eqs. \tag{7}, \tag{8} yield the result

$$I = \frac{V}{R} - e\beta T \text{Im} \left[ w \Psi \left( 1 + \frac{w}{2} \right) - i w \Psi \left( 1 + \frac{i w}{2} \right) \right].$$  \tag{11}$$
where $\Psi(x)$ is the digamma function, $w = u + iv$, $u = g_0 E_C/\pi^2 T$ and $v = eV/\pi T$. At $T \to 0$ from (11) we obtain

$$R \frac{dI}{dV} = 1 - \frac{\beta}{g_0} \ln \left(1 + \frac{1}{(eV R_0 C)^2}\right),$$

(12)

while in the limit $eV/E_C \gg \max(1, g)$ we find $RI = V - \beta e/2C$. At $V \to 0$ from (11) we get

$$f(0, T) = \frac{2}{g_0} \left[\gamma + 1 + \ln \left(\frac{g_0 E_C}{2\pi^2 T}\right)\right], \quad \gamma \simeq 0.577.$$

(13)

The above logarithmic dependencies on $eV$ and $T$ should also hold for $R_0 C < \tau_{\text{trav}}$, in which case in eqs. (12) and (13) with the logarithmic accuracy one can set $g_0 E_C \to 1/\tau_{\text{trav}}$.

At very low temperatures and voltages $\max(T, eV) < g_0 E_C \exp(-g_0/2)$ the interaction correction to the conductance of a coherent scatterer saturates at a universal value which does not depend on the interaction but only on the transmission distribution. This result follows immediately from Eqs. (11), (13). Evaluating (11) at long times we find

$$F(t) \simeq (2/g_0)(\ln(t/R_0 C) + \gamma)$$

and performing the integral in (11), in the leading order in $\beta/g_0 \ll 1$ we obtain

$$G = \frac{1 - \beta}{R} = \frac{2e^2}{h} \sum_n T_n^2,$$

(14)

This formula successfully reproduces a complete Coulomb blockade of tunneling $G \to 0$ in the limit $\beta \to 1$ (tunnel junctions), demonstrates the absence of it for ballistic scatterers ($\beta \to 0$), and yields suppression of the Landauer conductance by the factor 2/3 for diffusive conductors.

3 Comparison with experiments

In order to compare our theoretical results with recent experiments we will use the data reported recently by two groups.

Weber et al. [1] experimentally investigated the $I-V$ curve of a short ($L \sim 90 \text{ nm} \ll L_{\varphi}, L_{\text{in}}$) diffusive conductor fabricated as a bridge between two big metallic reservoirs. Even though the conductance of this bridge was large $g \approx 2000$ the Coulomb blockade effect on the $I-V$ curve was clearly visible becoming more pronounced with decreasing temperature, see Fig. 3 of Ref. 11. It was observed that the experimental data [1] were well described by the formula

$$G(V, T) = G(0, T_0) + A \ln(T/T_0) + A \Phi(eV/T),$$

(15)

where $A$ was determined to be $A \approx (0.4 \div 0.7) \times R_Q^{-1}$ depending on the sample and $\Phi(eV/T)$ was found to be a universal function of $eV/T$ which tends to zero at $V \to 0$. The logarithmic dependence of $G(0, T)$ on temperature [13] agrees with our eqs. (11), (13). Both the function $\Phi(eV/T)$ and the value $A$ can easily be evaluated within our theory. Making use of eq. (11) in the experimentally relevant interval of temperatures and voltages we obtain $A = 2\beta/R_Q$ and

$$\Phi(eV/T) = \Re \left[\Psi \left(1 + i \frac{eV}{2\pi T}\right) + \gamma + i \frac{eV}{2\pi T} \Psi' \left(1 + i \frac{eV}{2\pi T}\right)\right].$$

(16)

The function (16) is plotted in Fig. 1 together with the experimental data [1] for the sample No. 1. One observes a very good agreement between theory and experiment except at high voltages where heating effects become dominant. From the same comparison for the sample No. 1 one finds $\beta \approx 0.25$ slightly smaller than the value $\beta = 1/3$ expected in a diffusive limit.

In another experiment Krupenin et al. [1] studied the $I-V$ curves of $Cr$ resistors fabricated in the form of 2d strips and also clearly observed Coulomb blockade effects. The $\sim 1\mu m$ long samples [12] were not diffusive (their resistances were too large to be described by the Drude
Figure 1: The function (16) plotted together with the experimental data for the sample No 1. Different experimental curves correspond to different temperatures.

Figure 2: Eq. (12) plotted together with the experimental data for the sample 1 with $R \approx 2.5$ kΩ per square.

formula) and most likely had a granular structure. For our comparison we will adopt the model of a granular array with grains connected via short coherent scatterers with some average value of the parameter $\beta$. For the sample with the resistance 2.5 kΩ per square one can estimate the
average conductance of one scatterer as \( g \approx 10 \). Again, a very good agreement between theory and experiment is observed, see Fig. 2. From the best fit one can extract the value \( \beta \approx 0.35 \).

A relatively low value of the scatterer conductance \( g \approx 10 \) reached in the experiments also allows to enter the regime \( \max(T, eV) < gE_C \exp(-g/2) \), where the conductance was predicted to saturate at the level \( \left( \frac{1}{2}\right)^{L_\varphi} \). According to this prediction the ratio between the values of \( \beta \) measured in the limits of low and high voltages should approach \( 1 - \beta \). From the data we extract \( \beta \approx 0.31 \). This value is close to one found from the fit in Fig. 2. Thus the data clearly support both our theoretical predictions \( (12) \) and \( (14) \). It is also interesting that both estimates give the values of \( \beta \) very close to \( \beta = 1/3 \) which one would expect for diffusive scatterers.

Finally, we notice that the results of recent conductance measurements on multi-wall carbon nanotubes, with typical lengths of order \( L_\varphi \) or shorter, are also consistent with the logarithmic dependence predicted in eq. (2). Further experimental work on the subject would be highly desirable.

4 Instantons and metal-insulator phase transition

Under which conditions does the result \( (14) \) remain valid? One of them was already formulated above: \( g_0 \gg 1 \), i.e. either \( g \) or \( g_S \) should be much larger than one. In this case the conductance \( G \) saturates at the value \( \left( \frac{1}{2}\right)^{L_\varphi} \) for \( \max(eV, T) < g_0E_C \exp(-g_0/2) \). In order to establish another important limitation for the result \( (14) \) let us recall that, even though our quasiclassical Langevin equation approach does account for nonlinear in \( \varphi \) effects, this approach may nevertheless become insufficient at very low energies even for \( g_0 \gg 1 \) because it does not include instantons \( (16) \), i.e. nontrivial saddle points for the exact effective action.

An important part of the instanton analysis was carried out by Nazarov who derived the renormalized Coulomb energy \( \tilde{E}_C \) for a general coherent conductor within the exponential accuracy. The result \( (16) \) can be written in the form \( \tilde{E}_C \propto E_C \exp(-ag) \). One can also go beyond the exponential accuracy and estimate the pre-exponent in the expression for \( \tilde{E}_C \). This was done in Ref. 16. Without going into details here, we only quote the result

\[
\tilde{E}_C/E_C \sim \left[ \prod_n R_n \right] \ln \left[ \prod_n R_n^{-1} \right] \sim ag \exp(-ag).
\]

This formula is valid for \( ag \gg 1 \), i.e. either at large conductances \( g \gg 1 \) or, if \( g \sim 1 \), for very small values \( R_n \) implying \( a \gg 1 \).

The quantity \( \tilde{E}_C \) plays the role of an effective Coulomb gap in our problem. At \( T < \tilde{E}_C \) our Langevin equation analysis is insufficient and, hence, eq. (14) becomes inaccurate. In this regime the conductance decreases with \( T \) and the system is an insulator at \( T = 0 \).

Let us now take into account the effect of an external linear Ohmic environment \( Z_S \approx R_S \). In this case quantum fluctuations of the charge in the shunt resistor will affect the Coulomb gap \( \tilde{E}_C \) and – as we shall see – may even lead to its total suppression provided \( R_S \) is sufficiently low.

In order to proceed we will follow the analysis \( (16) \). Treating the charge \( q \) as a quantum variable and integrating out the phase \( \varphi \) one can map our problem onto that of a linearly damped quantum particle \( q \) in a periodic potential. This is a well-known problem \( (16) \) which can be treated, e.g., by means of the renormalization group (RG) technique. Successively reducing the high frequency cutoff \( \omega_c \) and integrating out charges with higher frequencies one arrives at the standard RG equations \( (16) \). After trivial manipulations these equations can be rewritten directly in terms of the combination \( ag = \sum_n \ln R_n^{-1} \). One obtains

\[
d(\Sigma)/d(\ln \omega_c) = (1 - g_\Sigma)(1 + 1/\Sigma), \quad dg_\Sigma/d(\ln \omega_c) = 0,
\]

where \( g_\Sigma = g g_S / g_0 \) is the total dimensionless conductance. Eqs. (18) are valid as long as \( ag \gg 1 \).

One observes that for \( g_\Sigma < 1 \) the quantity \( ag \) decreases in the course of renormalization. Hence,
in that case the Coulomb gap remains nonzero, the charge $q$ is localized and the system is an insulator at $T = 0$. The effective Coulomb gap $\tilde{E}_C$ can be defined as the energy scale at which the renormalized value $ag$ becomes of order one. Then from (18) one finds

$$\tilde{E}_C \sim E_C[ag \exp(-ag)]^{1-g\Sigma}. \quad (19)$$

On the other hand, for $g\Sigma > 1$ the combination $ag$ always scales to larger values. In this case the Coulomb gap is fully suppressed by quantum fluctuations, $\tilde{E}_C = 0$, the charge $q$ is delocalized and the conductance remains nonzero [14] even at $T = 0$. This is a metallic phase. A quantum phase transition between the insulating and metallic phases occurs at $g\Sigma = 1$.

Finally, let us briefly discuss possible implications of our results for recent experiments which strongly indicate the presence of a metal-insulator phase transition in various 2d disordered systems. One can consider a (sufficiently small) coherent scatterer with the dimensionless conductance $g$ viewing all other scatterers in the system as an effective environment with the conductance $g_S$. Assuming this environment to be Ohmic at sufficiently low frequencies, one immediately arrives at the conclusion about the presence of a quantum metal-insulator phase transition at $g\Sigma = 1$. In 2d systems one has $g \sim g_S \sim g_0 \sim g\Sigma$. Therefore in such systems this phase transition should be expected at conductances $\sim 1/R_Q$, exactly as it was observed in many experiments [3]. Local properties of the insulating and metallic phases are expected to be very different. In the insulating phase charges should be localized around inhomogeneities (puddles) due to Coulomb blockade, while in the metallic phase the Coulomb gap is suppressed and the charge distribution should be much more uniform. These expectations are fully consistent with experimental observations [4]. Thus, there might be a direct relation between the experimental results [3,4] and the old problem of a dissipative quantum phase transition [7].

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