Low-lying charmed and charmed-strange baryon states

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Abstract In this work, we systematically study the mass spectra and strong decays of 1P and 2S charmed and charmed-strange baryons in the framework of non-relativistic constituent quark models. With the light quark cluster–heavy quark picture, the masses are simply calculated by a potential function. The strong decays are studied by the Eichten–Hill–Quigg decay formula. Masses and decay properties of the well-established 1S and 1P states can be reproduced by our method. \(\Sigma_c(2800)^{0,+}++\) can be assigned as a \(\Sigma_{c2}(3/2^-)\) or \(\Sigma_{c2}(5/2^-)\) state. We prefer to interpret the signal \(\Sigma_c(2850)^0\) as a 2S(1/2\(^+\)) state although we cannot thoroughly exclude the possibility that this is the same state as \(\Sigma_c(2800)^0\). \(\Lambda_c(2765)^+\) or \(\Sigma_c(2765)^+\) could be explained as the \(\Lambda_c^+(2S)\) state or \(\Sigma_{c1}^+(1/2^-)\) state, respectively. We propose to measure the branching ratio of \(B(\Sigma_c(2455)^0\pi)^0\) in the future, which may disentangle the puzzle of this state. Our results support \(\Sigma_c(2980)^0\) as the first radial excited state of \(\Sigma_c(2470)^0\) with \(J^P = 1/2^+\). The assignment of \(\Sigma_c(2930)^0\) is analogous to \(\Sigma_c(2800)^{0,+,+\ldots}\), i.e., a \(\Sigma_c(3/2^-)\) or \(\Sigma_c(5/2^-)\) state. In addition, we predict some typical ratios among partial decay widths, which are valuable for experimental search for these missing charmed and charmed-strange baryons.

1 Introduction

At present, the particle data group (PDG) lists nine charmed and ten charmed-strange baryons [1]. They are \(\Lambda_c(2286)^+\), \(\Lambda_c(2595)^+\), \(\Lambda_c(2625)^+\), \(\Lambda_c(2765)^+\) (or \(\Sigma_c(2765)^+\)), \(\Lambda_c(2880)^+\), \(\Lambda_c(2940)^+\), \(\Sigma_c(2800)^{0,+,+\ldots}\), \(\Sigma_c(2470)^0\) and \(\Sigma_c(2520)^0\) with \(J^P = 1/2^+\). The assignment of \(\Sigma_c(2930)^0\) is analogous to \(\Sigma_c(2800)^{0,+,+\ldots}\), i.e., a \(\Sigma_c(3/2^-)\) or \(\Sigma_c(5/2^-)\) state. In addition, we predict some typical ratios among partial decay widths, which are valuable for experimental search for these missing charmed and charmed-strange baryons.
tion of charmed baryons should be more helpful for improving our understanding of the confinement mechanism. The spectroscopy of charmed baryons has been investigated in various models. So far, several kinds of quark potential models [6–10], the relativistic flux tube (RFT) model [11,12], the coupled channel model [13], the QCD sum rule [14–16], and the Regge phenomenology [17] have been applied to study the mass spectra of excited charmed baryons, and so did lattice QCD [18,19]. The strong decay behaviors of charmed baryons have been studied by several methods, such as the heavy hadron chiral perturbation theory (HHChPT) [20,21], the chiral quark model [22,23], the 3P0 model [24], and a non-relativistic quark model [25]. The decays of 1P Λc and Ξc baryons have also been investigated by a light front quark model [26,27], a relativistic three-quark model [28], and the QCD sum rule [29].

Although many experimental and theoretical efforts have been made for the research of charmed baryons, most of the 1P and 2S charmed baryons are not yet established. Several candidates, including Λc(2765) +, Σc(2800)0+,++ , Ξc(2930)0, and Ξc(2980)0+ are still in controversy. Λc(2765) + was first observed by the CLEO Collaboration in the decay channel of Λc(2765) + → Λc+π+π− [30], and confirmed by Belle in the Σc(2455)π channel [31,32]. Because both Λc+ and Σc+ excitations can decay through Λc+π+π− and Σc(2455)π, we even do not know whether the observed charmed baryon signal around 2765 MeV is the Λc+ or Σc+ state, or their overlapping structure [33]. In the e+e− annihilation process, an isospin triplet state, Σc(2800)0+,++ , was observed by Belle in the channel of Λc+π− , and it was tentatively identified as the Σc2 state with J P = 3/2− [34]. Interestingly, another neutral resonance was later found by BaBar in the process of B− → Σc0π− → Λc+π−ρ with the mass 2846 ± 8 ± 10 MeV and decay width 86 ± 32 MeV [35]. The higher mass and the weak evidence of J = 1/2 indicate that the signal observed by BaBar might be different from the Belle observation. In this paper, we denote the signal discovered by BaBar by Σc(2850)0, Ξc(2930)0, which was only seen by BaBar in the decay mode Λc+K− [36], still needs more confirmation. Ξc(2980)0+ was first reported by Belle in the channels Λc+K−π+ and Λc+K0S π− [37], and was later confirmed by Belle [4,38] and BaBar [39] in the channels Ξc′(2580)π, Ξc(2645)π and Ξc(2455)K, respectively. However, the decay widths reported by Refs. [4,37–39] were quite different from each other. More experimental information as regards the charmed baryons can be found in Refs. [40–43].

Obviously, a systematic study of masses and decays is required for these unestablished charmed baryons. More importantly, most of 2S and 1P charmed baryons have not yet been detected by any experiments. Such a research can also help the future experiments to find them. In the present work, we will focus on both the mass spectra and the strong decays of low-lying 1P and 2S charmed baryons. We pay attention to only the charmed baryons inside of which degrees of freedom of two light quarks are frozen. It means that two light quarks are not considered to be excited, neither radially nor orbitally. As illustrated in Ref. [44], these kinds of charmed excitations carry lower-excited energies, which means these excited charmed baryons may more likely be detected by experiments. Fortunately, our results indicate that most of the observed charmed baryons can be accommodated in this way.

The paper is arranged as follows. In Sect. 2, the masses of low-excited charmed baryons are calculated by the non-relativistic quark potential model. In Sect. 3, the Eichten–Hill–Quigg (EHQ) decay formula, which is employed to study the strong decays of excited charmed baryons, is introduced. The properties of low-lying charmed baryon states are fully discussed in Sect. 4. Finally, the paper ends with a conclusion and an outlook. Some detailed calculations and definitions are collected in the appendices.

2 The deduction of mass spectra

2.1 Treating charmed baryon system as a two-body problem

To study the baryon dynamics, one crucial question which should be answered is “What are the relevant degrees of freedom in a baryon?” [45]. In constituent quark models, a baryon system consists of three confined quarks. Thus the dynamics of a baryon resonance is surely more complex than a meson. Due to the HQS, however, the dynamics of charmed baryons could be greatly simplified. The HQS suggests that the couplings between a c quark and two light quarks are weak [46]. Therefore, two light quarks in a charmed baryon could first couple with each other to form a light quark cluster.1 Then the light quark cluster couples with a charm quark, and a charmed baryon resonance forms. With this assumption, two light quarks have the same status to a c quark, including the average distances to a c quark. In the light cluster–heavy quark picture, the dynamics of heavy baryon can be simplified. In the non-relativistic constituent quark model, the spin-independent part of the Hamiltonian is

\[
H = \sum_{i=1}^{3} \left( \frac{p_i^2}{2m_i} + m_i \right) + \sum_{i<j} \left( -\frac{2\alpha}{3r_{ij}} + \frac{b}{2r_{ij}} \right),
\]

where the Cornell potential [47] is used as a phenomenological confining term. The Jacobi coordinates are usually taken to deal with the three-body problem for baryons. ρ, λ, and R are related to the quark positions by

1 In some work, the light quark cluster may also be named a light diquark.
With the SU(3) flavor symmetry, the relation of $\tau$ is obvious.

As a two-body problem, we treat the masses of different kinds of clusters as parameters and first fix them before calculating the masses of the low-lying charmed baryons. According to the flavor and spin, the light cluster can be classified into two kinds: one is the “scalar” cluster, and another is the “vector” cluster. Constrained by the Pauli exclusion principle, the total wave function of the light quark cluster should be antisymmetric in the exchange of two quarks. Because the spatial and color parts of this light quark cluster are always symmetric and antisymmetric, respectively, the function $|\text{flavor} \times |\text{spin}|$ should be symmetric. Therefore, the scalar light quark cluster $\{qq\}$ ($S = 0$) is always flavor antisymmetric, and the axial-vector light quark cluster $\{qq\}$ ($S = 1$) is flavor symmetric. In Jaffe’s terminology [49], the “scalar” and “vector” quark clusters are named the “good” and “bad” quark clusters, respectively. The masses of the “good” light quark cluster $\{qq\}$ and $\{qs\}$ are taken from our previous work where $m_{\{qq\}}$ and $m_{\{qs\}}$ were fixed as 450 and 630 MeV by the RFT model [11], respectively. Following Jaffe’s method [49], the bad light quark cluster masses can be evaluated by the following relationships:

$$H^{\text{conf}}_{Q-\text{cl}}(\lambda) = -\frac{4\alpha_s}{3\lambda} + b\lambda^\nu - C_{Qqq}.$$
\[ 4 \times \Sigma_c(2520) + 2 \times \Sigma_c(2455) = \Lambda_c(2286) \approx 210 \text{ MeV}, \]
\[ 4 \times \Xi_c^*(2645) + 2 \times \Xi_c^*(2580) - \Xi_c(2470) \approx 150 \text{ MeV}. \]

Evidently, the masses of \([qq]\) and \([qs]\) are about 660 MeV and 780 MeV, respectively. Henceforth, we will call the \(\Lambda_c^+\) and \(\Xi_c^0\) baryons the \(G\)-type baryons, and \(\Sigma_c^{0,++}\) and \(\Xi_c^{0,+}\) the \(B\)-type baryons for convenience.

Due to an antitriplet color structure, the spin-dependent interactions between light cluster and \(c\) quark are expected to be the same as the meson systems. In a constituent quark model [53], the spin-dependent interaction is written as

\[ H_S = H_{Q-cl}^{cont} + H_{Q-cl}^{en} + H_{Q-cl}^{SO}. \tag{6} \]

The color contact interaction \(H_{Q-cl}^{cont}\) is usually given by the following form:

\[ H_{Q-cl}^{cont} = \frac{32\pi}{9} \frac{\alpha_s}{m_Q m_{cl}} \tilde{\delta}_\sigma(\lambda) \tilde{S}_Q \cdot \tilde{S}_{cl}, \tag{7} \]

where \(\tilde{S}_Q\) and \(\tilde{S}_{cl}\) refer to the heavy quark and light cluster spins. A Gaussian-smeared function \((\sigma / \sqrt{\pi})^3 e^{-\sigma^2 \lambda^2}\) is normally used for \(\delta_\sigma(\lambda)\) [54]. If the SU(3) flavor symmetry is kept for the charmed baryons, we may modify the color contact interaction as

\[ H_{Q-cl}^{cont} = \frac{32}{9\sqrt{\pi}} \frac{\alpha_s \sigma^3}{m_Q} e^{-\sigma^2 \lambda^2} \tilde{S}_Q \cdot \tilde{S}_{cl}, \tag{8} \]

where the mass of a light quark cluster is just replaced by a unit. This assumption is supported by the mass differences of the 15 \(B\)-type charmed baryons:

\[ \Sigma_c(2520)^{++} - \Sigma_c(2455)^{++} = 64.44^{+0.25}_{-0.24} \text{ MeV}, \]
\[ \Sigma_c(2645)^+ - \Sigma_c^*(2580)^+ = 70.2 \pm 3.0 \text{ MeV}, \]
\[ \Omega_c(2770)^0 - \Omega_c(2695)^0 = 70.7 \pm 2.6 \text{ MeV}. \]

The mass differences shown above are mainly due to the color contact interaction in the quark potential model. Clearly, these values are almost independent of the light quark cluster masses. The color tensor interaction in Eq. (6) is

\[ H_{Q-cl}^{en} = \frac{4}{3} \frac{\alpha_s}{m_Q m_{cl}} \frac{1}{\lambda^3} \left( \frac{3 \tilde{S}_Q \cdot \tilde{\lambda}}{\lambda^2} \left( \tilde{S}_{cl} \cdot \tilde{\lambda} \right) - \tilde{S}_Q \cdot \tilde{S}_{cl} \right). \tag{9} \]

Finally, \(H_{Q-cl}^{SO}\) denotes the spin–orbit interaction which contains two terms. One is the color magnetic interaction which arises from one-gluon exchange,

\[ H_{Q-cl}^{SO(cm)} = \frac{4}{3} \frac{\alpha_s}{\lambda^3} \left( \frac{\tilde{S} \cdot \tilde{L}}{m_Q m_{cl}} + \frac{\tilde{S}_Q \cdot \tilde{L}}{m_Q^2} + \frac{\tilde{S}_{cl} \cdot \tilde{L}}{m_{cl}^2} \right). \tag{10} \]

where \(\tilde{S}\) denotes the spin of a baryon, \(\tilde{S} = \tilde{S}_Q + \tilde{S}_{cl}\). Another spin–orbit interaction is the Thomas-precession term,

\[ H_{Q-cl}^{SO(tp)} = -\frac{1}{2\lambda} \frac{\partial H_{Q-cl}^{conf}}{\partial \lambda} \left( \frac{\tilde{S}_Q \cdot \tilde{L}}{m_Q^2} + \frac{\tilde{S}_{cl} \cdot \tilde{L}}{m_{cl}^2} \right). \tag{11} \]

To reflect the importance of the heavy quark symmetry, we rewrite the spin-dependent interactions as

\[ H_S = V_{ss} \tilde{S}_Q \cdot \tilde{S}_{cl} + V_1 \tilde{S}_Q \cdot \tilde{S}_{cl} + V_2 \tilde{S}_Q \cdot \tilde{J}_{cl} + V_3 \tilde{S}_Q \cdot \tilde{J}_{cl}. \tag{12} \]

The degrees of freedom of the light quark cluster are characterized by its total angular momentum \(\tilde{J}_{cl}\), i.e., \(\tilde{J}_{cl} = \tilde{S}_{cl} + \tilde{L}\). Obviously, the orbital angular momentum \(\tilde{L}\) of a charmed baryon in the present picture is defined by the angular momentum between light quark cluster and \(c\) quark, i.e., \(\tilde{L} = \tilde{L}_c\). The tensor operator is defined as \(\tilde{S}_{12} = \frac{3}{2} (\tilde{S}_Q \cdot \tilde{\lambda}) (\tilde{S}_{cl} \cdot \tilde{\lambda}) / \lambda^2 - \tilde{S}_Q \cdot \tilde{S}_{cl}\).

With the confining term of Eq. (5), the coefficients \(V_{ss}, V_1, V_2, V_3\) in Eq. (12) are defined by

\[ V_{ss} = \frac{1}{m_Q} \left[ \frac{32\alpha_s}{9\sqrt{\pi}} \frac{m_Q}{\lambda^2} - \frac{1}{m_Q} \left( \frac{2\alpha_s}{3\lambda^3} - \frac{b\nu}{2\lambda^{v-2}} \right) \right], \]
\[ V_1 = \frac{1}{m_{cl}} \left[ \frac{1}{m_{cl}} \left( \frac{2\alpha_s}{3\lambda^3} - \frac{b\nu}{2\lambda^{v-2}} \right) + \frac{4\alpha_s}{3\lambda^3 m_Q} \right], \]
\[ V_2 = \frac{1}{m_Q} \left[ \frac{1}{m_Q} \left( \frac{2\alpha_s}{3\lambda^3} - \frac{b\nu}{2\lambda^{v-2}} \right) + \frac{4\alpha_s}{3\lambda^3 m_{cl}} \right], \]
\[ V_3 = \frac{4\alpha_s}{3\lambda^3 m_Q m_{cl}}. \tag{13} \]

2.3 Getting masses of charmed baryons

In our calculation, the following Schrödinger equation is solved for the \(nS\) state:

\[ \left( -\frac{\nabla^2}{2m_\lambda} + H_{Q-cl}^{conf} + H_{Q-cl}^{cont} \right) \Psi = E \Psi. \tag{14} \]

The confining and contact terms have been given by Eqs. (5) and (8). For the orbital excitations, all spin-dependent interactions are treated as the leading-order perturbations. Our calculation indicates that the color contact interaction can be ignored for the orbital excitations.

Two bases are employed to extract the mass matrix elements. One is defined by the eigenstates \(|S_{cl}, L, J_{cl}, S_Q, J\rangle\) (\(jj\) coupling scheme) and the other is by \(|S_{cl}, S_Q, S, L, J\rangle\) (\(LS\) coupling scheme). The relation between these two bases is
Similarly, for two states with $J^P = 3/2^-$,

$$\langle \Phi_{3/2} | H_S | \Phi_{3/2} \rangle = \left( -V_1 + \frac{\sqrt{2}}{4} + \frac{V_5}{\sqrt{2}}, -\frac{5V_{55} + 16V_4}{4\sqrt{5}} V_1 - \frac{3V_5}{2} - \frac{3V_4}{4} + \frac{4V_3}{5} \right).$$

For the $J^P = 5/2^-$ state,

$$\langle j_1 = 2, J^P = 5/2^- | H_S | j_1 = 2, J^P = 5/2^- \rangle = V_1 + \frac{1}{2} V_{ss} - \frac{6}{5} V_1.$$
Here we have denoted the physical states by their masses (see Table 3). The mixing angles for other states in Table 3 with the same $J^P$ are listed in Table 4.

Our results of the mixing angles in Table 4 indicate that the heavier $1/2^-$ state, $\Sigma_c(2765)$, is dominated by a $|1/2^+\rangle$ component, while $\Sigma_c(2702)$ is dominated by a $|0, 1/2^-\rangle$ component. For two $3/2^-$ states, the light $\Sigma_c(2785)$ is dominated by $|2, 1/2^-\rangle$, and the heavy $\Sigma_c(2798)$ by $|1, 1/2^-\rangle$. The mixing of $2P$ states is similar to the $1P$ states. For the $1D$ states, one notices that both $3/2^+$ and $5/2^+$, with heavier masses, are dominated by smaller $f_{cl}$ components.

The uncertainty may exist in the mixing angles. Firstly, the loop corrections to the spin-dependent one-gluon-exchange potential may be important for the heavy–light hadrons. As
an example, the lower mass of $D_s(2317)^\pm$ compared with the old calculations [53] can be well explained by the corrected spin-dependent potential [59, 60]. If we use this type of potential in our calculation, of course, the mixing angle will change. Secondly, the mixing angles depend on the parameters. Thirdly, there are other mechanisms, e.g., hadron loop effects [61], which may contribute to the mixing phenomenon in hadron physics. Anyway, we expect that the mixing angles in Table 4 reflect the main features of the mixing states. Due to the uncertainties of the mixing angles, however, we ignore the mixing effects as the first step to the study of the decays of charmed excitations in the next subsection. Obviously, it is a good approximation only when the mixing effects are not large. Fortunately, this crude procedure is partially supported by the former analysis of charmed mesons [62–64]. If the decay properties obtained in this way describe the principal characteristics of the mixing states, the angles obtained by the potential model may be overestimated.

2.4 Simple harmonic oscillator (SHO) $\beta$ values

In the next section, the Okubo–Zweig–Iizuka (OZI) allowed decays will be calculated for the 1$P$, 2$P$, and 1$D$ $\Sigma_c/\Sigma_c'$ states.

Table 4: The mixing angles for the 1$P$, 2$P$, and 1$D$ $\Sigma_c/\Sigma_c'$ states

|        | 1$P$(1/2$^-$) | 1$P$(3/2$^-$) | 2$P$(1/2$^-$) | 2$P$(3/2$^-$) | 1$D$(3/2$^+$) | 1$D$(5/2$^+$) |
|--------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\Sigma_c$ | 125.4$^\circ$ | -156.8$^\circ$ | 124.8$^\circ$ | -151.4$^\circ$ | 172.2$^\circ$ | -175.6$^\circ$ |
| $\Sigma_c'$ | 125.0$^\circ$ | -153.6$^\circ$ | 124.3$^\circ$ | -145.1$^\circ$ | 168.9$^\circ$ | -173.8$^\circ$ |

contain a light flavor meson, a charmed meson, a light flavor baryon, e.g., $\pi$, $K$, $D$, $p$, and $\Lambda$. For the $\beta$ of these hadrons, the following potential will be used:

$$V(r) = F_{q_1} \cdot F_{q_2} \left( \frac{\alpha_s}{r} - \frac{3}{4} b r + \frac{3}{4} C + \frac{32\alpha_s\sigma^3 e^{-\sigma r^2}}{9\sqrt{\pi m_q m_q}} \vec{S}_{q_1} \cdot \vec{S}_{q_2} \right),$$

(18)

where

$$\langle F_{q_1} \cdot F_{q_2} \rangle = \begin{cases} -4/3 & \text{for } q_1 q_2 \quad (19a) \\ -2/3 & \text{for } q_1 q_2 \quad (19b) \end{cases}$$

Here, the parameters $\alpha_s$ and $b$ are taken as 0.45 and 0.145 GeV$^{-1}$ as in Table 1, respectively. To reproduce the masses of the light quark clusters in Table 1, the masses of $u/d$, $s$ are fixed as 0.195 and 0.380 GeV. While $\sigma$ and $C$ are treated as adjustable parameters, the masses of the $\pi/\rho$, $K/K^*$, $D/D^*$, $p/\Delta$, and $\Lambda$ families are fitted with experimental data. Meanwhile, the values of $\beta$ for the corresponding states are also obtained; they are collected in Table 6.

Before ending this section, we briefly summarize the complicated deduction presented here. Firstly, the dynamics of heavy baryon is simplified as a two-body system when the symmetric configuration is considered. Secondly, the mass matrices were calculated in the $jj$ coupling scheme. By solving the Schrödinger equation, we obtain the mass spectra and mixing angles for the relevant states. For estimating the two-body strong decays in the next section, finally, we also present the values of the SHO wave function scale for all initial and final states.

3 Strong decays

In this section, we will use the formula provided by Eichten, Hill, and Quigg (EHQ) [66] to extract the decay widths of excited charmed baryons. Since the dynamical behavior of the heavy–light hadrons is governed by the light degrees of freedom in the limit of heavy quark symmetry, a doublet formed by two states with the same $j$ but different $\lambda$ shall have similar decay properties. More specifically, the transitions between two doublets should be determined by a single amplitude which is proportional to the products of four Clebsch–Gordan coefficients [46]. Some typical ratios of excited charmed baryons with negative parity were pre-
dicted by this law [46]. Later, a more concise formula (the EHQ formula) was proposed for the widths of heavy–light mesons [66]. The EHQ formula has been applied systematically to the decays of excited open-charm mesons [62–64]. Recently, the EHQ formula has been extended to the study of the decay properties of $1^S_0$ states [11].

For the charmed baryons, the EHQ formula can be written as

$$\Gamma_{1C,\ell}^{A \rightarrow BC} = \xi \left( C_{1C,\ell,1A}^{q_j b_j b_j} \right)^2 \left| M_{1C,\ell}^{A,1B}(q) \right|^2 q^{2\ell+1} e^{-q^2/\beta^2},$$

where $\xi$ is the flavor factor given in Table 13 in Appendix B. $q = |\vec{q}|$ denotes the three-momentum of a final state in the rest frame of an initial state, $A$ and $B$ represent the initial and final heavy–light hadrons, respectively. $C$ denotes the light flavor hadron (see Fig. 3). The explicit expression of $\tilde{q}$ is given in Eq. (A.11) in Appendix A. In addition, $C_{1C,\ell,1A}^{q_j b_j b_j}$ is a normalized coefficient given by the following equation:

$$C_{1C,\ell,1A}^{q_j b_j b_j} = (-1)^{J_A+J_B+J_C+J}\sqrt{(2J_A+1)(2J_B+1)} \times \left\{ \begin{array}{c} q_j b_j b_j \\ 1C J_A J_B \end{array} \right\},$$

where $\tilde{J}_C \equiv \tilde{s}_C + \ell$. The symbols $s_C$ and $\ell$ represent the spin of the light hadron $C$ and the orbital angular momentum relative to $B$, respectively. The transition factors $M_{1C,\ell}^{A,1B}(q)$ involved in the concrete dynamics can only be calculated by various phenomenological models. For the decays of heavy–light mesons, transition factors have been calculated by the relativistic chiral quark model [67] and the $^3P_0$ model [62, 64, 68]. In our work, we will employ the $^3P_0$ model [69–71] to obtain the transition factors. More details of an estimate of the transition factors are given in Appendix A.

4 Discussion

4.1 Experimentally well-established $1^S_0$ and $1^P_0$ states

At present, all the ground states and $1^P_0 G$-type charmed states have been experimentally established [1]. These states have been observed, at least, by two different collaborations, and their properties including masses and decays have been well determined. With good precision, the strong decays of these states provide a crucial test of our method.

Among the $1^S_0$ charmed baryons, the measurements of $\Sigma_c(2455)$ and $\Sigma_c(2520)$ have been largely improved [2, 3] (see Table 7). In our calculation, the mass and decay width of $\Sigma_c(2520)^{++}$ measured by CDF will be taken as input data to fix the constant $\gamma$ peculiar to the $^3P_0$ model. With the transition factor for the process $\Sigma_c(2520) \rightarrow \Lambda_c(2286) + \pi$ [see Eq. (A.12) in Appendix A], the value of $\gamma$ is fixed as 1.296.$^2$

As shown in Tables 8 and 9, the predicted widths of other ground and $1^P_0 G$-type charmed baryons are well consistent with experiments. Our results of mass spectra and decay widths indicate that $\Lambda_c(2595)^{++}$, $\Lambda_c(2625)^{++}$, $\Sigma_c(2790)^{0,++}$, and $\Sigma_c(2815)^{0,++}$ can be accommodated with the $1^P_0$ $G$-type charmed baryons. $\Lambda_c(2595)^{++}$ and $\Sigma_c(2790)^{0,++}$ can be classified into the $1/2^-$ states while $\Lambda_c(2625)^{++}$ and $\Sigma_c(2815)^{0,++}$ can be classified into the $3/2^-$ states. The predicted mass splittings between the $1^P_0 1/2^-$ and $3/2^-$ states are 25 MeV and 27 MeV for the $\Lambda_c$ and $\Sigma_c$ baryons, respectively, which are also consistent with the experiments. The assignments of $\Lambda_c(2595)^{++}$, $\Lambda_c(2625)^{++}$, $\Sigma_c(2790)^{0,++}$, and $\Sigma_c(2815)^{0,++}$ are also supported by other work [9–12] in which the light quark cluster scenario was also employed. In addition, the mass spectra obtained by different types of the quark potential models in the three-body picture also support these assignments [7, 8, 56–58]. However, the investigations by QCD sum rules indicate that these $1^P_0$ candidates may have more complicated structures [14–16]. Especially, the work by Chen et al. suggested that $\Lambda_c(2595)^{++}$ and $\Lambda_c(2625)^{++}$ form the heavy doublet $\Lambda_{c1}(1/2^-, 3/2^-)$ (the same assignments as the case of $\Sigma_c(2790)^{0,++}$ and $\Sigma_c(2815)^{0,++}$) [16], which is

\[ \text{Table 6} \] The effective $\beta$ values in GeV for the light quark cluster and various hadrons (the second row). The values of $\sigma$ and $C$ are given in the square brackets for various hadron structures (the third row)

| $qq$ | $qq$ | $qs$ | $qs$ | $\pi$ | $\rho$ | $K$ | $K^*$ | $D$ | $D^*$ | $p$ | $\Lambda$ |
|------|------|------|------|------|------|-----|------|-----|------|-----|--------|
| 0.201 | 0.143 | 0.207 | 0.159 | 0.298 | 0.179 | 0.291 | 0.201 | 0.250 | 0.230 | 0.189 | 0.226 |
| [1.17, 0.39] | [1.57, 0.38] | [0.73, 0.63] | [0.83, 0.48] | [1.20, 0.63] | [−0.38] | [−0.26] |

\[ \text{Fig. 3} \] The two topological diagrams for an excited charmed baryon $A$ decaying into the final states $B$ and $C$. The brown line denotes a charm quark.

\[ \text{Springer} \]
different from our conclusion. Since the quantum numbers of $J^P$ have not yet been determined for these $1P$ charmed states, more experiments are required in the future.

4.2 $1P\,\Sigma_c^{0,+;++}$ states

As shown in Tables 2 and 3, the masses of $1P\,\Sigma_c$ states are predicted in the range of 2700–2800 MeV. Then $\Sigma_c(2765)^+$ and $\Sigma_c(2800)^{0,+;++}$ can be grouped into the candidates of $1P\,\Sigma_c$ family. The predicted mass of the $|1P,\,1/2^−⟩_b$ state is about 2765 MeV, which is in good agreement with the measured mass of $\Sigma_c(2765)^+$. In addition, the theoretical result for the decay width of the $\Sigma_c(1/2^-)$ state in Table 10 is about 63.52 MeV, which is also in agreement with the measurements [30–32]. Furthermore, the signal of $\Sigma_c(2765)^+$ has been observed in the $\Sigma_c(2455)\pi$ intermediate state, while there is no clear evidence for the decay of $\Sigma_c(2765)^+$ through $\Sigma_c(2520)\pi$. This is also consistent with our results of the $|1P,\,1/2^−⟩_b$ state. Based on the combined analysis of the mass spectrum and strong decays, we therefore conclude that $\Sigma_c(2765)^+$ could be regarded as a good candidate of $\Sigma_c(1/2^-)$. Considering the uncertainties of the quark potential models, the masses obtained by Refs. [9, 10, 56] are not contradictory to our assignment to $\Sigma_c(2765)^+$.

According to the predicted masses in Table 3, $\Sigma_c(2800)^{0,+;++}$ could be assigned to either the $|1P,\,3/2^-⟩_b$, or $|1P,\,3/2^-⟩_b$, or $|1P,\,5/2^-⟩_b$ states. When we consider the decay properties of these three states (see Table 10), the possibility of assignment to the $|1P,\,3/2^-⟩_b$ state can be excluded since the Belle Collaboration observed this state in the $\Lambda_c^+\pi$ mode.3 At present, the Belle Collaboration tentatively identified $\Sigma_c(2800)^{0,+;++}$ as members of the $\Sigma_c(3/2^-)$ isospin triplet, which agrees with our results of both mass spectrum and strong decays. Then the possibility of assignment to the $|1P,\,3/2^-⟩_b$ state, the predicted width is about 40.1 MeV, which is compatible with the experiment [34]. However, we notice that the quantum number $J^P$ of $\Sigma_c(2800)^{0,+;++}$ has not yet been measured. Then the possibility of this state as the $\Sigma_c(5/2^-)$ candidate cannot be excluded by our results since the decay mode of $\Lambda_c^+\pi$ is dominant for this state. In addition, the predicted mass and total width of the $\Sigma_c(5/2^-)$ state are also compatible with experimental data of the $\Sigma_c(2800)^{0,+;++}$ baryon. Therefore, we would like to point out that the signal of $\Sigma_c(2800)^{0,+;++}$ found by Belle might be their overlapping structure. We hope the future experiments measure the following branching ratios to disentangle this state.

For the $\Sigma_c(3/2^-)$ state,

$$\frac{B(\Sigma_c(3/2^-) \rightarrow \Sigma_c(2455)\pi)}{B(\Sigma_c(3/2^-) \rightarrow \Sigma_c(2520)\pi)} = 1.90;$$

(22)

$$\frac{B(\Sigma_c(3/2^-) \rightarrow \Lambda_c(2286)\pi)}{B(\Sigma_c(3/2^-) \rightarrow \Sigma_c(2455)\pi)} = 4.07.$$ (23)

For the $\Sigma_c(5/2^-)$ state,

$$\frac{B(\Sigma_c(5/2^-) \rightarrow \Sigma_c(2455)\pi)}{B(\Sigma_c(5/2^-) \rightarrow \Sigma_c(2520)\pi)} = 0.58;$$

(24)

$$\frac{B(\Sigma_c(5/2^-) \rightarrow \Lambda_c(2286)\pi)}{B(\Sigma_c(5/2^-) \rightarrow \Sigma_c(2455)\pi)} = 9.85.$$ (25)

As mentioned earlier, the signal $\Sigma_c(2850)^0$ discovered by the BaBar Collaboration may be a $J = 1/2^−$ state. If $\Sigma_c(2850)^0$ is the $1/2^+\,(2S)$ state, the corresponding ratios (see Sect. 4.5) are different from Eqs. (22)–(25). So the measurements of

---

3 Even if the possible mixing between $\Sigma_c(3/2^-)$ and $\Sigma_c(3/2^-)$ is considered, the partial width of $\Lambda_c^+\pi$ is only 3.87 MeV for the $|1P,\,3/2^-⟩_b$ state where the mixing angle obtained in Table 4 has been used.
these ratios of branching fractions can help us understand
the nature of $\Sigma_c(2800)^{0+,++}$ and $\Sigma_c(2850)^0$.

Although, at present, the $\Sigma_c(1/2^-)$ and $\Sigma_c(3/2^-)$
states are still missing in the experiments, our results
indicate that the $\Sigma_c(1/2^-)$ state may be a narrow
resonance and its predominant decay channel $\Lambda_c^+\pi$ is only
about 3.64 MeV (see Table 10). Since the decay mode of
$\Sigma_c(2520)\pi$ is the largest for $\Sigma_c(3/2^-)$ state, we sug-
gest to search this channel for this state in future experi-
ments. In the heavy quark limit, the branching ratio for the
$\Sigma_c(3/2^-)$ state,

$$B(\Sigma_c(3/2^-) \to \Sigma_c(2455)\pi)$$

is much smaller than $\Sigma_c(2800)^{0,++}$ (Eq. 22).

4.3 $1P$ $\Xi_c^{'}0,+$ states

As shown in Table 3, the predicted mass of $1P$ $\Xi_c^{'}$ is in the
range from 2840 to 2930 MeV. Then the resonance structure
observed by BaBar [36] in the decay channel $B^- \to
\Xi_c^{'}(2930)^0\Lambda_c^- \to \Lambda_c^+K^-\Lambda_c^-$ with an invariant mass of 2.93
GeV could be a good candidate of $1P$ $\Xi_c^{'}$ members. The
results of the decays in Table 11 favor $\Xi_c^{'}(2930)^0$ as the
$\Xi_c^{'}(3/2^-)$ or $\Xi_c^{'}(5/2^-)$ state. Then $\Xi_c^{'}(2930)^0$ might be
regarded as the strange partner of $\Sigma_c(2800)^{0,++}$ by our
results. Interestingly, the mass difference between $\Sigma_c(2930)^0$
and $\Sigma_c(2800)^{0,++}$ is about 130 MeV, which is comparable
with the mass differences among sextet states of the
ground charmed baryons [21]. With a chiral quark model,
Liu et al. also analyzed the $\Xi_c^{'}(2930)^0$ by the two-body
strong decays [23]. Their results support $\Xi_c^{'}(2930)^0$ as the
$\{\Sigma_c^2P_\lambda, 1/2^-\} or \{\Xi_c^2P_\lambda, 1/2^-\}$ state. Since the heavy quark
symmetry was not considered in Ref. [23], the notations of
ccharm-strange baryons in Ref [23] are different from our
$\Sigma_c^{0}(1/2^-)$ and $\Sigma_c^{1}(1/2^-)$. Although the results in Table 11
indicate that the $\Lambda_c^+K$ decay mode dominates the decay of
$\Sigma_c(1/2^-)$ state, the mass of this state is predicted to be about
2840 MeV, which is much smaller than $\Xi_c^{0}(2930)$. In addi-
tion, the $\Lambda_c^+K$ decay mode is forbidden for the $\Xi_c^{1}(1/2^-)$
state. Thus, according to our results, $\Xi_c^{1}(2930)$ is unlikely
to be a $1P$ state with $J^P = 1/2^-$. Another charm-strange baryon, $\Xi_c(2980)^{0,+}$, is slightly
higher than the predicted mass range of $1P$ $\Xi_c^{'}$ states.
This state has been observed in $\Sigma_c(2455)K$, $\Xi_c^{'}(2580)\pi$,
$\Xi_c^{'}(2645)\pi$, and nonresonant $\Lambda_c^+K\pi$ decay channels. How-
ever, it was not seen in the decay modes of $\Lambda_c^+K$ and
$\Xi_c\pi$ [37–39]. Comparing the mass and decay properties of
$\Xi_c(2980)^{0,+}$ with our results, the possibility of a $1P$ $\Xi_c^{'}$
state might be excluded. As shown in the next subsection,
we find that the $\Xi_c^{'}(1/2^-)$ and $\Xi_c^{'}(3/2^-)$ are quite narrow (see Table 11).

Table 10 The partial and total decay widths of $1P$ $\Sigma_c$ states in MeV

| Decay modes | $1/2^-$ (1$P$) | $3/2^-$ (1$P$) | $5/2^-$ (1$P$) |
|-------------|---------------|---------------|---------------|
| $\Sigma_c(2702)$ | $\Sigma_c(2765)$ | $\Sigma_c(2798)$ | $\Sigma_c(2785)$ | $\Sigma_c(2790)$ |
| $\Lambda_c\pi$ | 3.64 | x | x | 24.06 | 24.63 |
| $\Sigma_c(2455)\pi$ | x | 58.94 | 3.48 | 5.22 | 2.50 |
| $\Sigma_c(2520)\pi$ | x | 1.70 | 63.72 | 2.47 | 4.34 |
| $\Lambda_c(2595)\pi$ | 2.88 | 2.31 | 1.93 | 0.03 |
| $\Lambda_c(2625)\pi$ | 3.12 | 0.07 | 0.63 |
| Theory | 3.64 | 63.52 | 72.63 | 33.75 | 32.13 |
| Expt. [1] | $\approx$50 | 72$^{+22}_{-15}$ |

Table 11 The partial and total decay widths of $1P$ $\Xi_c^{'}$ states in MeV

| Decay modes | $1/2^-$ (1$P$) | $3/2^-$ (1$P$) | $5/2^-$ (1$P$) |
|-------------|---------------|---------------|---------------|
| $\Xi_c^{'}(2839)$ | $\Xi_c^{'}(2900)$ | $\Xi_c^{'}(2932)$ | $\Xi_c^{'}(2921)$ | $\Xi_c^{'}(2927)$ |
| $\Lambda_cK$ | 46.59 | x | x | 11.59 | 12.43 |
| $\Sigma_c\pi$ | 4.39 | x | x | 7.42 | 7.75 |
| $\Xi_c^{'}(2580)\pi$ | x | 9.44 | 0.76 | 1.20 | 0.57 |
| $\Xi_c^{'}(2645)\pi$ | x | 0.52 | 3.23 | 0.75 | 1.31 |
| $\Xi_c^{'}(2790)\pi$ | x | 0.01 | | | |
| Theory | 50.98 | 9.96 | 4.00 | 20.96 | 22.06 |
| Expt. | 36 $\pm$ 7 $\pm$ 11 [36] |
Table 12 The partial and total decay widths of $2S$ $\Lambda_c^+$ and $\Xi_c^{0,+}$ states in MeV

| $1/2^+$ ($2S$) | $1/2^+$ ($2S'$) | $3/2^+$ ($2S'$) |
|-----------------|-----------------|-----------------|
| $\Lambda_c$(2765)$^+$ | $\Xi_c$(2980) | $\Lambda_c$(2850)$^0$ | $\Xi_c'$(3000) | $\Lambda_c$(2880)$^0$ | $\Xi_c'$(3030) |
| $\Sigma_c$(2455)$\pi$ | 26.23 | $\sum_c$(2455)$K$ | 3.14 | $\Lambda_c^+\pi$ | 35.11 | $\Lambda_c^+K$ | 17.42 | $\Lambda_c^+\pi$ | 34.96 | $\Lambda_c^+K$ | 18.37 |
| $\Sigma_c$(2520)$\pi$ | 19.28 | $\sum_c'(2580)\pi$ | 11.47 | $\Lambda_c$(2520)$^+\pi$ | 17.54 | $\Xi_c$(2580)$\pi$ | 4.13 | $\Lambda_c$(2520)$^+\pi$ | 54.52 | $\Xi_c$(2645)$\pi$ | 12.92 |
| $\Sigma_c$(2625)$\pi$ | 12.83 | $\sum_c'$(2645)$\pi$ | 6.92 | $\Lambda_c$(2520)$^+\pi$ | 1.57 | $\Xi_c$(2790)$\pi$ | 5.89 | $\Lambda_c$(2520)$^+\pi$ | 1.07 | $\Xi_c$(2790)$\pi$ | 0.40 |
| $D^0n$ | 0.03 | $\sum_c$(2455)$K$ | 15.34 | $D^0n$ | 3.03 | $\sum_c$(2455)$K$ | 6.49 |

$45.51 \pm 0.03 [1]$, $28.1 \pm 2.4^{+1.0}_{-5.0} [4]$, $86^{+33}_{-22} [35]$.

4.4 $2S$ $\Lambda_c^+$ and $\Xi_c^{0,+}$ states

According to the mass spectrum (see Table 2), $\Lambda_c/\Sigma_c$(2765)$^+$ can also be regarded as the first radial ($2S$) excitation of the $\Lambda_c$(2286)$^+$ with $J^P = 1/2^+$. Interestingly, the results of strong decays in Table 12 do not contradict this assignment. Our calculation indicates that the decay channel $\Sigma_c$(2455)$\pi$ is a dominant decay channel for the $\Lambda_c^+(2S)$ state. This is in line with the observations by Belle [31,32]. At present, both $1/2^+(2S)$ $\Lambda_c^+$ and $1/2^-(1P)$ $\Sigma_c^+$ are possible for the assignment of $\Lambda_c/\Sigma_c$(2765)$^+$. However, there is the very important feature for experiments to distinguish these two assignments in future. Specifically, we suggest to search $\Lambda_c/\Sigma_c$(2765)$^+$ in the channel of $\Sigma_c$(2520)$\pi$. As shown in Table 12, the channel $\Sigma_c$(2520)$\pi$ is large enough to find the $\Lambda_c^+(2S)$ state. On the other hand, this mode seems to be too small to be detected for the $\Sigma_c$(1/2$^-$) (see Table 10). Explaining the criteria concretely, we give the following branching ratios for these two states.

For the $\Lambda_c$(2S) state,

$$B(\Lambda_c(2765) \rightarrow \Sigma_c(2520) \pi)/B(\Lambda_c(2765) \rightarrow \Sigma_c(2455) \pi) = 0.74. \quad (27)$$

For the $\Sigma_c$(1/2$^-$) state,

$$B(\Sigma_c(2765) \rightarrow \Sigma_c(2520) \pi)/B(\Sigma_c(2765) \rightarrow \Sigma_c(2455) \pi) = 0.03. \quad (28)$$

The branching ratio of $B(\Sigma_c(2520)\pi)/B(\Sigma_c(2455)\pi)$ for the $\Sigma_c$(1/2$^-$) state is roughly an order of magnitude smaller than $\Lambda_c$(2S). If $\Lambda_c$(2765) is the $2S$ excitation, $\Xi_c$(2980) could be a good candidate as its charm-strange analog [21] as seen in Table 12. The mass difference between $\Lambda_c$(2765) and $\Xi_c$(2980) is about 200 MeV, which nearly equals the mass difference between $\Lambda_c$(2287) and $\Xi_c$(2470). The predicted width of $\Xi_c$(2980) is 27.44 MeV which is in good agreement with the experimental data [4,39]. As the $2S$ excitation of $\Xi_c$(2470), the branching ratio

$$B(\Xi_c(2980) \rightarrow \Xi_c'(2580)\pi) = 0.89 \quad (29)$$

is predicted for $\Xi_c$(2980), which can be tested by future experiments. Recently, the ratio of branching fractions

$$B(\Xi_c(2980)^+ \rightarrow \Xi_c'(2580)^0\pi^+) / B(\Xi_c(2815)^+ \rightarrow \Xi_c'(2645)^0\pi^+ \rightarrow \Xi_c^{0,+}\pi^-\pi^+) \approx 75\% \quad (30)$$

has been estimated by the Belle Collaboration [4]. Combining this with the predicted partial widths of $\Xi_c$(2815) and $\Xi_c$(2645) in Tables 8 and 9, the branching fraction $B(\Xi_c(2980)^+ \rightarrow \Xi_c'(2580)^0\pi^+)$ is evaluated to be about 40%, which is consistent with our direct result of 41.8%.

4.5 $2S$ $\Sigma_c^{0,+}$ and $\Xi_c^{0,+}$ states

In Table 3, masses of the $2S$ $\Sigma_c(1/2^+, 3/2^+)$ states are predicted as 2850 and 2876 MeV, respectively. The neutral $\Sigma_c$(2850)$^0$ found by the BaBar Collaboration in the decay channel $B^-(\rightarrow \Sigma_c(2850)^0\bar{p}) \rightarrow \Lambda_c^+\pi^-\bar{p}$ [35] can be regarded as the $2S$ $\Sigma_c$ state with $J^P = 1/2^+$. The mass and width of the neutral $\Sigma_c$(2850)$^0$ and $\Sigma_c$(2850)$^+$ are collected here:

$$\Sigma_c(2800)^0: \quad m = 2806^{+5}_{-7} \text{ MeV}; \quad \Gamma = 72^{+22}_{-15} \text{ MeV}; \quad \Sigma_c(2850)^0: \quad m = 2846 \pm 8 \pm 10 \text{ MeV}; \quad \Gamma = 86^{+12}_{-12} \text{ MeV}.$$ 

For lack of experimental information, at present, PDG treated $\Sigma_c(2850)^0$ and $\Sigma_c(2800)^{0,+}$ as the same state [1]. As pointed out by the BaBar Collaboration [35], however, there are indications that these two signals detected by Belle [34] and BaBar [35] are two different $\Sigma_c^+$ states. The main reasons are listed as follows:

1. Although the widths of $\Sigma_c(2800)^{0,+}$ and $\Sigma_c(2850)^+$ are consistent with each other, their masses are $3 \sigma$ apart.
2. The Belle Collaboration tentatively identified the $\Sigma_c(2800)^{0}J^{P}=1/2^{+}$ as the $J=3/2$ isospin triple, while the BaBar Collaboration found weak evidence of $\Sigma_c(2850)^{0}$ as a $J=1/2$ state.

Our results also indicate that $\Sigma_c(2800)^{0}J^{P}=1^{++}$ and $\Sigma_c(2850)^{0}$ are the different $\Sigma_c$ excited states. One notices that the predicted mass of $1/2^{+}(2S)$ $\Sigma_c$ state in this work and in Ref. [9] are around 2876 MeV. Even the results in Refs. [10,56] are only about 50 MeV larger than the measurements. Due to the intrinsic uncertainties of the quark potential model, it is appropriate to assign $\Sigma_c(2850)$ as a $2S 1/2^{+}$ state. More importantly, the predicted decay width of $\Sigma_c(1/2^{+}, 2S)$ state is 118.36 MeV, which is comparable with the measurement by BaBar [35]. The partial width of $\Lambda_3\pi$ is 35.11 MeV, which can explain why $\Sigma_c(2850)^{0}$ was first found in this channel. We find that the decay modes of $\Sigma_c(2455)\pi$ and $\Sigma_c(2520)\pi$ are also large. Finally, we give the following branching ratios:

$$\frac{B(\Sigma_c(2850) \rightarrow \Sigma_c(2520) \pi)}{B(\Sigma_c(2850) \rightarrow \Sigma_c(2520) \pi)} = 3.26,$$

$$\frac{B(\Sigma_c(2850) \rightarrow \Sigma_c(2520) \pi)}{B(\Sigma_c(2850) \rightarrow \Sigma_c(2520) \pi)} = 0.61,$$

which can be tested by future experiments. If $\Sigma_c(2850)^{0}$ is the $1/2^{+}(2S)$ state, the mass of its doublet partner in the heavy quark effective theory is predicted as 2876 MeV (denoted $\Sigma_c(2880)$). According to the predicted decay widths in Table 12, this state might also be broad. $\Lambda_3^{++}\pi$, $\Sigma_c(2455)\pi$, and $\Sigma_c(2520)\pi$ are also dominant for the decay of $\Sigma_c(2880)$. The ratio of $\Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2850)\pi)$ for $\Sigma_c(2880)$ is different from $\Sigma_c(2850)$, whose numerical value is given by

$$\frac{B(\Sigma_c(2880) \rightarrow \Sigma_c(2520) \pi)}{B(\Sigma_c(2880) \rightarrow \Sigma_c(2520) \pi)} = 0.29.$$ (33)

Even though the strange partners of $\Sigma_c(2850)$ and $\Sigma_c(2880)$ have not been found by any experiments, their decay properties are calculated and presented in Table 12. Our results indicate that $\Lambda_3^{++}K$, $\Xi'_c(2580)\pi$, and $\Sigma_c(2455)K$ are the dominant decay modes of $\Xi'_c(3000)$ state with $J^{P}=1/2^{+}$, while $\Lambda_3^{++}K$ and $\Xi'_c(2645)\pi$ are those of the $\Xi'_c(3030)$. Besides the masses and decay widths, the following branching ratios may also be valuable for future experiments:

$$\frac{B(\Xi'_c(3030) \rightarrow \Xi'_c(2580) \pi)}{B(\Xi'_c(3030) \rightarrow \Xi'_c(2645) \pi)} = 3.04.$$

\footnote{If $\Xi'_c(2980)$ is the first radial excited state of $\Xi'_c(2765)$. Then our predicted masses for $2S$ charm-strange baryons may be about 20–30 MeV lower than the experiments. To compensate for this difference, we increase by about 25 MeV for the $2S \Xi'_c$ states in this case.}

5 Summary and outlook

In principle, both $\rho$ and $\lambda$ modes can be excited in a baryon system. For charmed baryons, the excitation energies of the $\rho$ and $\lambda$ modes are different due to the heavier mass of a charm quark. For the ordinary confining potential, such as the linear or harmonic form, the excited energy of the $\rho$ mode is larger than the $\lambda$ mode [44]. Hence the low-excited charmed baryons may be dominated by the $\lambda$ mode excitations. Recently, the investigation by Yoshida et al. confirmed this point [75]. Furthermore, they find that the $\rho$ and $\lambda$ modes are well separated for the charmed and bottom baryons, which means the component of the $\rho$ mode can be ignored for the low-excited charmed baryons. Interestingly, Refs. [9–11] have also shown that the masses of existing charmed baryons can be explained by the $\lambda$ mode. Hence, our study of strong decays of the low-excited charmed baryons is an important complement to this work [9–11,75].

Up to now, several candidates of the $1P$ and $2S$ charm and charm-strange baryons have been found by experiments, and some of them are still open to debate. To better understand these low-excited charmed baryons, in this paper, we carry out a systematical study of the mass spectra and strong decays for the $1P$ and $2S$ charmed baryon states in the framework of the non-relativistic constituent quark model. The masses have been calculated in the potential model where the charmed baryons are simply treated as a quasi-two-body system in a light quark cluster picture. The strong decays are computed by the EHQ decay formula where the transition factors are determined by the $3P_0$ model. When calculating the decays, the inner structure of a light quark cluster has also been considered. Except for the unique parameter $\gamma$ of the QPC model, the parameters in the potential model and in the EHQ decay formula have the same values.

The well-established ground and $1P$ $G$-type charmed baryons provide a good test to our method. The experimental properties including both masses and widths for these states can be well explained by our results. This success has made us more confident of our predictions for other $1P$ and $2S$ states. Our main conclusions are as follows.

The broad state $\Lambda_c(2765)^+(or \Sigma_c(2765)^+)$ which is still ambiguous could be assigned to the $1/2^+(2S)$ $\Lambda_3^+$, or the $1/2^-(1P) \Sigma_{3/2}^-$ state. The branching ratio $B(\Sigma_c(2455)\pi)/B(\Sigma_c(2520)\pi)$ is found to be different for these two assignments, which may help us understand the nature of this state. $\Sigma_c(2800)^{0,+,+++}$ observed by the Belle Collaboration in $e^+e^-$ annihilation processes [34] can be regarded as a negative-parity state with $J^{P}=3/2^-$, or $5/2^-$, or their overlapping structure. We suggest to measure the
of excited charmed baryons. As an example, the process \( \Sigma_c(2520) \to \Lambda_c(2280)\pi \) will be constructed and the transition factor for the EHQ formula will be extracted.

As pointed out in Sect. 3, there are two possible decay processes for an excited charmed baryon state (see Fig. 3). The final states of the left figure contain a charmed baryon and a light meson. The right one contains a charmed meson and a light baryon. If a baryon decays via the so-called \(^3P_0\) mechanism, a quark–antiquark pair is created from the vacuum and then regroups two outgoing hadrons by a quark rearrangement process. In the non-relativistic limit, the transition operator \( \hat{T} \) of the \(^3P_0\) model is given by

\[
\hat{T} = -3\gamma \sum_{m} \sum_{m'} \sum_{m''} \left( \langle m' | m | m'' \rangle \right) \int d^3 \vec{k}_4 d^3 \vec{k}_5 \delta^3 (\vec{k}_4 + \vec{k}_5) \times \psi_{jA}^{\gamma} (\vec{k}_4, \vec{k}_5) \psi_{jB}^{\gamma} (\vec{k}_4, \vec{k}_5)
\]

where \( \omega^{(4,5)} \) and \( \phi_0^{(4,5)} \) are the color and flavor wave functions of the \( q \bar{q} \bar{s} \) pair created from the vacuum. Thus, \( \omega^{(4,5)} = (R \bar{R} + G \bar{G} + B \bar{B})/\sqrt{3} \) and \( \phi_0^{(4,5)} = (u \bar{u} + d \bar{d} + s \bar{s})/\sqrt{3} \) are color and flavor singlets. The pair is also assumed to carry the quantum number of \( 1^{++} \), suggesting that they are the \(^3P_0\) state. The \( \xi_{3^{-1},m} \) represents the pair production in a spin triplet state. The solid harmonic polynomial \( \psi_{jA}^{\gamma} (\vec{k}_4, \vec{k}_5) \) reflects the momentum-space distribution of the \( q \bar{q} \bar{s} \) pair created from the vacuum. The value of \( \gamma \) is usually fixed by fitting the well-measured partial decay widths.

When the mock state [77] is adopted to describe the spatial wave function of a meson, the helicity amplitude \( M^{jA,jB,L \cdot C} (\vec{r}) \) can easily be constructed in the \( LS \) basis [71]. The mock state for an \( A \) meson is

\[
\begin{align*}
& | A \rangle = \left( n_A \right)^{2 \Delta_s+1} L_{A}^{jA;jB} \left( \vec{P}_A \right) \\
& \equiv \omega_{A}^{123} \phi_{A}^{123} \prod_{A} \int d^3 k_1 d^3 k_2 d^3 k_3 \delta^3 \left( \vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{P}_A \right) \\
& \times \psi_{n_A}^{L \cdot A} \left( \vec{k}_1, \vec{k}_2, \vec{k}_3 \right) | q_1 \rangle | q_2 \rangle | q_3 \rangle.
\end{align*}
\]

As for the left decay process in Fig. 3, the wave function of a \( B \) baryon can be constructed in the same way. The wave function of a \( C \) meson is

\[
\begin{align*}
& | C \rangle = \left( n_C \right)^{2 \Delta_c+1} L_{C}^{jC;jC} \left( \vec{P}_C \right) \\
& \equiv \omega_{C}^{15} \phi_{C}^{15} \prod_{C} \int d^3 \vec{k}_4 d^3 \vec{k}_5 \delta^3 \left( \vec{k}_4 + \vec{k}_5 - \vec{P}_C \right) \\
& \times \psi_{n_C}^{L \cdot C} \left( \vec{k}_4, \vec{k}_5 \right) | q_1 \rangle | \bar{q}_1 \rangle | \bar{q}_5 \rangle | \bar{q}_5 \rangle.
\end{align*}
\]

Here, the symbols of \( \prod_i (i = A, B, \text{and } C) \) represent the Clebsch–Gordan coefficients for the initial and final hadrons.
functions are employed to describe the spatial wave function of a hadron. In the momentum space, the SHO radial wave function, $\psi_L^{\mu}(q)$, is given by

$$
\psi_{Lm}^{\mu}(q) = \begin{cases}
\frac{(-1)^m}{\beta^{L+1/2}} \sqrt{\frac{2(2n-1)!}{\Gamma(n+L+1/2)}} \left(\frac{q}{\beta}\right)^L e^{-\frac{q^2}{2\beta^2}} L_{n-1}^{L+1/2} \left(\frac{q^2}{\beta^2}\right) Y_{Lm}(\Omega),
\end{cases}
$$

(A.8)

with $q = (m_1 \vec{k}_1 - m_2 \vec{k}_2)/(m_1 + m_2)$ and $Y_{Lm}(\Omega) = |q|^L Y_L(\Omega q)$. $L_{n-1}^{L+1/2}(q^2/\beta^2)$ is an associated Laguerre polynomial. The values of the SHO wave function scale parameter $\beta$ have been given in Tables 5 and 6. In the light quark cluster picture, the wave function of a charmed baryon can easily be constructed. Taking $A$ as a baryon as an example, the wave functions corresponding to the $1S, 2S,$ and $1P$ states are given as follows, respectively:

$$
\begin{align*}
\psi_{00}^0 &= \frac{3^{3/4}}{\pi^{3/2} \rho_{5dA}^2 p_A^2} e^{-\frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2 - \frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2} ; \\
\psi_{00}^0 &= \frac{3^{3/4} \sqrt{2}}{\pi^{3/2} \rho_{5dA}^2 p_A^2} e^{-\frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2 - \frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2} \times \frac{3}{2} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^3} ; \\
\psi_{1m}^0 &= \frac{3^{3/4} \sqrt{2} \sqrt{2}}{\pi^{3/2} \rho_{5dA}^2 p_A^2} e^{-\frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2 - \frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2} \times \frac{3}{2} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^3} ;
\end{align*}
$$

With the help of Eq. (A.7), the transition amplitude can be obtained. In the following, we take the process $\Sigma_c(2520) \rightarrow \Lambda_c(2280)^+ \pi$ as an example. The wave functions of initial and final states are

$$
\begin{align*}
\psi_A &= \frac{3^{3/4}}{\pi^{3/2} \rho_{5dA}^2 p_A^2} e^{-\frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2 - \frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2} ; \\
\psi_B &= \frac{3^{3/4} \sqrt{2}}{\pi^{3/2} \rho_{5dA}^2 p_A^2} e^{-\frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2 - \frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2} ; \\
\psi_C &= \frac{3^{3/4} \sqrt{2}}{\pi^{3/2} \rho_{5dA}^2 p_A^2} e^{-\frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2 - \frac{1}{2\rho_A} \left(\frac{m_1 k_1 - m_2 k_2}{m_1 + m_2 + \beta}\right)^2} ;
\end{align*}
$$

Based on Eq. (A.7), we obtain the amplitude:

$$
M_{1/2}(q) = \frac{3g}{8\pi^{5/4} f 5/2 \lambda^3 / 2 p_A^2 / 2 p_A^2 / 2 p_B^2 / 2 p_C^2 / 2 p_B^2 / 2 p_C^2 / 2 p_C^2} e^{-\mu^2/4^n - \frac{q^2}{2\lambda^2}} ,
$$

(A.9)

where

$$
\begin{align*}
f &= \frac{1}{2\beta_B^2} + \frac{1}{2\beta_B^2} + \frac{1}{2\beta_M^2} - \frac{\mu^2}{4\lambda} ; \\
g &= \frac{1}{2\beta_B^2} + \frac{\epsilon_3}{\beta_B^2} + \frac{\epsilon_4}{\beta_M^2} - \frac{\mu v}{2\chi} ;
\end{align*}
$$

where
\[ h = \frac{1}{2\beta^2} + \frac{\varepsilon}{2\beta} + \frac{\varepsilon^2}{2\beta^3} + \frac{\varepsilon^3}{2\beta^4} - \frac{v^2}{4\lambda}; \]

\[ \lambda = \frac{1}{2\beta^2} + \frac{1}{2\beta^3} + \frac{\varepsilon}{2\beta^4} + \frac{\varepsilon^2}{2\beta^5}; \]

\[ \mu = \frac{\varepsilon}{\beta^2} + \frac{\varepsilon^2}{\beta^3} + \frac{\varepsilon^3}{\beta^4} + \frac{\varepsilon^4}{\beta^5}; \]

\[ \varepsilon = \frac{1}{m_1 + m_2}; \quad \varepsilon_2 = \frac{1}{m_3 + m_4}; \]

and

\[ q = \sqrt{[M^2_A - (M_B + M_C)^2] M^2_A - (M_B - M_C)^2]} / 2M_A. \]

(A.10)

Here, \( M_A, M_B, \) and \( M_C \) are the masses of hadrons \( A, B, \) and \( C, \) respectively. Then the \( \beta \) in Eq. (20) is given by

\[ \beta = 2 \sqrt{\frac{f}{g^2 - g^2}}. \]

(A.11)

where \( f \) and \( g \) have been defined above. For the decay channel of \( \Sigma_c(2520) \to \Lambda_c(2280)\pi, \) the value of \( C^0_{i,j,k}L_k \) is \(-1). \) Therefore, we obtain

\[ \mathcal{M}^{L_0}_{ii}(q) = \frac{3g}{8\pi^{3/4}\sqrt{f}\lambda^{3/2}\beta^2_{\Lambda^A} \beta^2_{\Lambda^B} \beta^2_{\Lambda^C}}. \]

(A.12)

where a phase space factor \( (2\pi E_B E_C / M_A)^{1/2} \) is omitted. One notices that the unitary rotation between the \( LS \) coupling and \( jj \) coupling (Eq. (15)) should be performed to reduce the transition factors of \( 1P \) state with the same \( JP. \) More details for calculating the decay amplitudes of an excited baryon in the \( 3P_0 \) model can be found in Refs. [24,79].

Appendix B: Flavor factors

Based on the light SU(3) flavor symmetry, the flavor wave functions of charmed and charmed-strange baryons are given by [57]

\[ \Lambda^+ = \frac{1}{\sqrt{2}} (ud - du)c; \quad \Sigma^{++} = uuc; \]

\[ \Xi^+ = \frac{1}{\sqrt{2}} (us - su)c; \quad \Sigma^+ = \frac{1}{\sqrt{2}} (ud + du)c; \]

\[ \Xi^0 = \frac{1}{\sqrt{2}} (ds - sd)c; \quad \Sigma^0 = \frac{1}{\sqrt{2}} (dc - cd); \]

\[ \Xi^+ = \frac{1}{\sqrt{2}} (us + su)c; \quad \Xi^0 = \frac{1}{\sqrt{2}} (ds + sd)c. \]

As shown in Fig. 3, the final states of an excited charmed baryons may contain a light meson and a low energy charmed baryon or a light baryon and a charmed meson. The flavor wave functions for the final states are collected in the following:

Table 13 The flavor matrix element \( \xi \) for different decay channels of the charmed baryons

| Initial state | Final states |
|---------------|--------------|
| \( \Lambda_c^+ \) | \( \Sigma_c^{++} \) |
| \( \Xi^+ \) | \( \Lambda^+_c \) |
| \( D^+ n / D^0 p \) | \( \sqrt{1/7} \) |
| \( \sqrt{1/73} \) | \( \sqrt{1/73} \) |
| \( \Sigma^{++} \) | \( \Sigma^{++} \) |
| \( \Xi^+ \) | \( \Lambda^+_c \) |
| \( D^+ p \) | \( \sqrt{1/73} \) |
| \( \sqrt{1/75} \) | \( \sqrt{1/75} \) |
| \( \Sigma_c^{++} \) | \( \Sigma^{++} \) |
| \( \Sigma^+ \) | \( \Lambda^+_c \) |
| \( D^+ n / D^0 p \) | \( \sqrt{1/7} \) |
| \( \sqrt{1/73} \) | \( \sqrt{1/73} \) |
| \( \Sigma^0 \) | \( \Sigma^0 \) |
| \( \Xi^0 \) | \( \Lambda^+_c \) |
| \( \Xi^0 \) | \( \Lambda^+_c \) |
| \( \xi_{\pm} \) | \( \xi_{\pm} / \xi_{\mp} \) |
| \( \xi_{\pm} / \xi_{\mp} \) | \( \xi_{\pm} / \xi_{\mp} \) |
| \( 1/\sqrt{2} \) | \( 1/\sqrt{2} \) |
| \( \sqrt{1/76} \) | \( \sqrt{1/76} \) |
| \( \sqrt{1/73} \) | \( \sqrt{1/73} \) |
| \( \sqrt{1/75} \) | \( \sqrt{1/75} \) |

With the above flavor wave functions, the flavor matrix elements \( \xi \) for different decay processes are presented in Table 13.

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