Brane-World Black Holes in Randall-Sundrum Models

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(November 1, 2018)

Abstract

We study brane-world black holes from Randall-Sundrum(RS) models in $(D + 1)$-dimensional anti-de Sitter spacetimes. The solutions are directly obtained by using a slightly generalized RS metric ansatz in $D+1$ dimensions. The metric of the brane world can be described by the Schwarzschild solution promoted to the black cigar solution in $D+1$ dimensions, which is compatible with the recently suggested black cigar solution in $D = 4$. Furthermore, we show that the Ricci flat condition for the brane can be easily derived from the effective gravity defined on the brane by using the RS dimensional reduction. Especially, it is shown that in two dimensions the effective gravity on the brane is described by the Polyakov action.

PACS : 04.60.-m, 04.70.Dy

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I. INTRODUCTION

Recently, there has been much interests in the idea that our universe may be embedded in some higher-dimensional spacetimes. It has been suggested that the gauge hierarchy problem can be resolved in higher dimensions \[1,2\]. This framework unifies gravitation and gauge interactions at the weak scale and explains their hierarchy in our spacetime. However, this leads to another hierarchy between weak scale and compactification one.

On the other hand, in order to solve the hierarchy problem Randall and Sundrum(RS) have proposed a two-brane model called RS1 model involving a small and curved extra dimension which is a slice of anti-de Sitter (AdS) spacetime \[3\]. The negative tension brane is regarded as our universe and the hierarchy between physical scales naturally appears in our brane. Furthermore, RS have studied a single-brane model(RS2 model) by taking \( r_c \rightarrow \infty \), where \( r_c \) is a radius of the extra dimension \[4\]. In these RS models, the nonfactorizable metric with \(-\pi r_c \leq y \leq \pi r_c\)

\[
ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2,
\]

is essential as a static vacuum, which is different from that of the conventional Kaulza-Klein(KK) style in that the extra coordinate is associated with the conformal factor. On the other hand, the cosmological aspects of the RS model are intensively studied in Ref. \[5\]. The linearized gravity on the RS brane world is also treated in Refs. \[4,6,7\] and the dilatonic brane-world model are covered in Ref. \[8\].

Apart from the RS model, the static AdS domain wall has been found as BPS domain walls of \( D = 4, N = 1 \) supergravity theoris in Ref. \[9\], which is relevant for RS model in a special case. A further study on the global spacetime and non-static solution in this subject has been shown in Ref. \[10\], and the \( D \)-dimensional approach has been studied in Ref. \[11\].

As for black holes, Chamblin, Hawking, and Reall(CHR) \[12\] have proposed that a non-rotating and uncharged black hole on the domain wall is described by a black cigar solution in five dimensions. The curvature singularity and black hole mass have been discussed in
In Ref. [12], the vacuum metric (1) is generalized by substituting $\eta_{\mu\nu}$ with $g_{\mu\nu}$ under the Ricci flat condition, and find the brane-world solution. Note that black hole solutions in intersecting domain wall backgrounds has been studied in Ref. [13].

In the four-dimensional RS model, brane-world black holes and their thermodynamics [14], and a comparison with BTZ black holes and black strings on brane-world black holes [15] have been studied. Furthermore, in Ref. [16], aspects of evaporation in brane-world black holes have been discussed.

In Sec. II, we would like to obtain the brane-world black hole solutions from $(D + 1)$-dimensional AdS gravity with $(D - 1)$-branes and show that they satisfy the Ricci flat condition in all dimensions. In Sec. III, we find the effective gravity of the brane in terms of the RS dimensional reduction. In our calculations, the Ricci flat condition naturally appears as an equation of motion even though there exists some scalar field couplings in the effective action, so that the desirable brane-world solution can be easily obtained. On the other hand, the single brane limit is well defined, and the effective gravity just becomes Einstein gravity. Furthermore, for the case of three dimensions two-dimensional effective gravity is described by the well-known Polyakov action. In Sec. IV we discuss our results comparing with the RS2 model.

II. BRANE-WORLD BLACK HOLES IN (D+1)-DIMENSIONS

Let us now consider the first RS (RS1) model in $(D + 1)$ dimensions,

$$S_{(D+1)} = \frac{1}{2\kappa_2^{(D+1)}} \int d^D x \int dy \sqrt{-g_{(D+1)}} \left[ R^{(D+1)} + D(D - 1)k^2 \right] - \int d^D x \sqrt{-\tilde{g}(+)\lambda(+) - \int d^D x \sqrt{-\tilde{g}(-)\lambda(-)}, \tag{2}$$

where $\lambda(+)\) and $\lambda(-)$ are tensions of the branes at $y = 0$ and at $y = \pi r_c$, respectively, $D(D - 1)k^2$ is a cosmological constant, and $\kappa_2^{(D+1)} = 8\pi G_N^{(D+1)}$. Note that we use $M, N, K, \cdots = 0, 1, \cdots, D$ for $(D + 1)$-dimensional spacetime indices and $\mu, \nu, \kappa, \cdots = 0, 1, \cdots, D - 1$ for branes. We assume orbifold $S^1/Z_2$ which has a periodicity in the extra coordinate $y$ and
identify $-y$ with $y$. Two singular points on the orbifold are located at $y = 0$ and $y = \pi r_c$, and two $(D-1)$-branes are placed at these points, respectively. Note that $\tilde{g}^{(+)}_{\mu\nu}$ and $\tilde{g}^{(-)}_{\mu\nu}$ are defined as

$$\tilde{g}^{(+)}_{\mu\nu} \equiv g^{(D+1)}_{\mu\nu}(x^\mu, y = 0),$$
$$\tilde{g}^{(-)}_{\mu\nu} \equiv g^{(D+1)}_{\mu\nu}(x^\mu, y = \pi r_c).$$

We assume a generalized metric as

$$ds^2_{(D+1)} = e^{\sigma(\Phi(x))} g^{(D)}_{\mu\nu}(x) dx^\mu dx^\nu + T^2(x) dy^2,$$

where the moduli field $T(x)$ is different from $\Phi(x)$ for the present. From Eq. (2), the equations of motion are given as

$$G^{(D+1)}_{MN} = T^{(D+1)}_{MN}.$$

By using the metric (4), the Einstein tensors are calculated as

$$G^{(D+1)}_{\mu y} = -\frac{1}{2} (D - 1) \left( \partial_{\mu} \Phi - \frac{\Phi}{T} \partial_{\mu} T \right) \partial_{y} \sigma,$$
$$G^{(D+1)}_{yy} = -\frac{1}{2} T^2 e^{-\sigma \Phi} \left( R^{(D)} - 4 \sigma (D - 1) \Box \Phi - \frac{1}{4} \sigma^2 (D - 1)(D - 2) (\nabla \Phi)^2 - \frac{\Phi^2}{4T^2} D(D - 1) e^{\sigma \Phi} (\partial_{y} \sigma)^2 \right),$$

and the stress-energy tensor is explicitly written as

$$T^{(D+1)}_{MN} = \frac{1}{2} g^{(D+1)}_{MN} D(D - 1) k^2 + \kappa^2_{(D+1)} \sqrt{-\tilde{g}^{(+)}} \lambda^{(y)} \delta^{(+) \mu \nu} \delta^{(+) M N}$$
$$+ \kappa^2_{(D+1)} \sqrt{-\tilde{g}^{(-)}} \lambda^{(y - \pi r_c)} \delta^{(-) \mu \nu} \delta^{(-) M N}. $$
Since the ($\mu y$)-component of Eq. (9) vanishes, from Eq. (5) we obtain the following relation,

$$G^{(D+1)}_{\mu y} = -\frac{1}{2}(D - 1) \left( \partial_\mu \Phi - \frac{\Phi}{T} \partial_\mu T \right) \partial_\gamma \sigma = 0. \tag{10}$$

It is interesting to note that the case of $\sigma(y) = \text{constant}$ corresponding to a factorizable geometry describes a conventional Kaluza-Klein (KK) theory without a KK gauge field $A_\mu(x)$. For our nonfactorizable metric (8), we take $\sigma(y) = -\frac{2}{k}|y|$ by considering RS vacuum metric (1), then Eq. (10) yields

$$\Phi(x) = T(x). \tag{11}$$

In the ($\mu \nu$)-components of Eq. (5), there exist discontinuities resulting from the delta functional source due to the presence of brane tensions at $y = 0$ and at $y = \pi r_c$. At this stage, we consider junction conditions and integrate out the Einstein equation near the branes,

$$\int_{0-}^{0+} dy G^{(D+1)}_{\mu \nu} = \int_{0-}^{0+} dy T^{(D+1)}_{\mu \nu},$$

$$\int_{\pi r_c - \epsilon}^{\pi r_c + \epsilon} dy G^{(D+1)}_{\mu \nu} = \int_{\pi r_c - \epsilon}^{\pi r_c + \epsilon} dy T^{(D+1)}_{\mu \nu}. \tag{12}$$

The jump along the extra coordinate near the $(D - 1)$-branes gives a relation,

$$\lambda_+ = -\lambda_- = \frac{2(D - 1)}{k^2_{(D+1)}} k, \tag{13}$$

where we note that the branes at $y = 0$ and at $y = \pi r_c$ have a positive tension ($\lambda_+$) and a negative one ($\lambda_-$), respectively. Using the relation (13) for this brane model, the equation of motion (5) is explicitly given as

$$R^{(D)}_{\mu \nu} - \frac{1}{2} g^{(D)}_{\mu \nu} R^{(D)} - \frac{1}{T} \left[ \nabla_\mu \nabla_\nu T - g^{(D)}_{\mu \nu} \Box T \right] + \frac{1}{2} k^2 y^2 (D - 2) \left[ 2 \nabla_\mu T \nabla_\nu T + (D - 3) g^{(D)}_{\mu \nu} (\Box T)^2 \right]$$

$$+ k |y| (D - 2) \left[ \nabla_\mu \nabla_\nu T - g^{(D)}_{\mu \nu} \Box T \right] - \frac{k |y|}{T} \left[ 2 \nabla_\mu T \nabla_\nu T + (D - 3) g^{(D)}_{\mu \nu} (\Box T)^2 \right] = 0, \quad (14)$$

$$R^{(D)} + 2 k |y|(D - 1) \Box T - k^2 y^2 (D - 1)(D - 2)(\Box T)^2 = 0. \quad (15)$$

Trace of Eq. (14) and Eq. (15) give the following simple equations,
\[ R^{(D)} + k|y|(D - 1)\Box T = 0, \]
\[ \Box T - k|y|(D - 2)(\nabla T)^2 = 0. \]  

(16)

In Eq. (16), as a simple constant solution of \( T(x) \), we set \( T(x) = 1 \), then metric solution \( g^{(D)}_{\mu\nu} \) should be determined by a condition,

\[ R^{(D)} = 0. \]  

(17)

Combining \( R^{(D)} = 0 \) and \( T(x) = 1 \) with Eq. (14), the Ricci flat condition, \( R^{(D)}_{\mu\nu} = 0 \) introduced in Ref. [12], is obtained from the equations of motion. From this condition (17), it is natural to consider the \( D \)-dimensional Schwarzschild black hole solution for uncharged nonrotating case as a brane solution,

\[ ds^2 = e^{-2k|y|} \left[ - \left( 1 - \frac{2M}{r} \right) dt^2 + \left( 1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 d\Omega_{(D-2)}^2 \right] + dy^2, \]  

(18)

where \( d\Omega_{(D-2)}^2 \) is a metric of unit \((D - 2)\)-sphere. For \( D = 4 \), it is a “black cigar” solution which describes a black hole placed on the hypersurface at the fixed extra coordinate \( y \) [12]. The black hole horizon is \( r_H = 2M \) [12], which is shown in FIG. 1.

FIG. 1. The black hole can be formed by collapsing matters on the brane and its singularities exist along the extra dimension.
On the other hand, the Arnowitt-Deser-Meisner (ADM) mass $\tilde{M}$ of the brane-world black hole measured on the brane is given as

$$\tilde{M} = Me^{-ky_0},$$

(19)

where $y_0$ is 0 and $\pi r_c$ which are $y$-coordinates of the positive and the negative tension branes, respectively.

The Ricci scalar and the square of the Ricci tensor are constant, while the square of Riemann tensor is written as

$$R_{MNKL}R^{MNKL} = 10k^4 + \frac{12M^2e^{4ky}}{r^6},$$

(20)

especially for $D = 4$. It can be shown that in Eq. (20), there exists a curvature singularity at $r = 0$ and this is extended along the $y$-coordinate such as a string in FIG. 1. The range of $y$ spans from 0 to $y_c = \pi r_c$ for the RS1 model, while for the RS2 model, $y_c$ goes to infinity. Therefore, for $y_c \to \infty$, the square of Riemann tensor diverges for finite $r$ and the ADM mass would be exponentially suppressed as $\tilde{M} = Me^{-ky_c}|_{y_c\to\infty} \to 0$ on the negative tension brane for the RS2 model.

**III. EFFECTIVE THEORY FROM THE DIMENSIONAL REDUCTION**

We now study $D$-dimensional effective theory on the brane by using the dimensional reduction with the generalized metric (11) of RS models and by including the constraint (12) to be consistent with $(D + 1)$-dimensional Einstein equation and find an explicit action describing the brane world. At first sight, a candidate for the theory seems to be Einstein gravity because of the Ricci flat condition. However, the resulting action looks like a dilaton gravity satisfying the Ricci flat condition. Using the metric (11), a Ricci scalar can be expressed as

$$R^{(D+1)} = e^{-\sigma \Phi} \left[ R^{(D)} - \sigma (D - 1) \Box \Phi - \frac{2}{T^2} \Box T - \frac{\sigma}{T}(D - 2) \nabla_\mu \Phi \nabla^\mu T 
- \frac{1}{4} \sigma^2 (D - 1)(D - 2)(\nabla \Phi)^2
- \frac{\Phi^2}{4T^2} D(D + 1)e^{\sigma \Phi}(\partial_y \sigma)^2
- \frac{\Phi}{T^2} De^{\sigma \Phi} \partial^2 \sigma \right].$$

(21)
Inserting Eq. (21) into Eq. (2) for \( \sigma(y) = -2k|y| \) and the constraint (11), we obtain

\[
S_{(D+1)} = \frac{1}{2\kappa^2_{(D+1)}} \int d^Dx \sqrt{-g(D)} T \int_{-\pi r_c}^{\pi r_c} dy e^{-(D-2)|y|/T} \left[ R(D) + 2k|y|(D-1)\Box T \right. \\
-\frac{2}{T} \Box T + \frac{2k|y|}{T} (D-2)(\nabla T)^2 - k^2 y^2 (D-1)(D-2)(\nabla T)^2 \\
- D(D+1)e^{-2k|y|/T} k^2 + \frac{4}{T} kD e^{-2k|y|/T} (\delta(y) - \delta(y - \pi r_c)) + e^{-2k|y|/T} D(D-1)k^2 \bigg] \\
- \int d^Dx \sqrt{-g(D)} e^{-Dk\pi r_c T} \lambda_+ - \int d^Dx \sqrt{-g(D)} e^{-Dk\pi r_c T} \lambda_-.
\]

(22)

After integrating out along the extra dimension \( y \) from \(-\pi r_c \) to \( \pi r_c \), the \( D \)-dimensional effective action is now obtained as

\[
S_{(D)} = \frac{1}{2\kappa^2_{(D+1)}k} \int d^Dx \sqrt{-g(D)} \left[ \frac{2}{(D-2)} \left( 1 - e^{-(D-2)\phi} \right) R(D) - 2(D-1)e^{-(D-2)\phi}(\nabla \phi)^2 \\
+ \left( 4(D-1)k^2 - 2\kappa^2_{(D+1)}k\lambda_+ \right) - e^{-D\phi} \left( 4(D-1)k^2 + 2\kappa^2_{(D+1)}k\lambda_- \right) \right],
\]

(23)

where we redefined \( \phi(x) \equiv \pi kr_c T(x) \). If we use junction condition (13) in Eq. (23), then the resulting effective action can be obtained as

\[
S_{(D)} = \frac{1}{2\kappa^2_{(D+1)}k} \int d^Dx \sqrt{-g(D)} \left[ \frac{2}{(D-2)} \left( 1 - e^{-(D-2)\phi} \right) R(D) - 2(D-1)e^{-(D-2)\phi}(\nabla \phi)^2 \right],
\]

(24)

The action (24) is reminiscent of the low-energy string theory with some modified dilaton coupling, however, it is different in that the Ricci flat condition is satisfied through the equations of motion.

From the effective action (24), equations of motion are derived as

\[
\left[ 1 - e^{-(D-2)\phi} \right] (R_{\mu\nu}^{(D)} - \frac{1}{2}g_{\mu\nu}^{(D)} R^{(D)}) + (D-2)^2 e^{-(D-2)\phi} \nabla_\mu \phi \nabla_\nu \phi \\
- (D-2)e^{-(D-2)\phi} \nabla_\mu \nabla_\nu \phi - g_{\mu\nu}^{(D)} (D-2)^2 e^{-(D-2)\phi}(\nabla \phi)^2 + g_{\mu\nu}^{(D)} (D-2)e^{-(D-2)\phi} \Box \phi \\
+ (D-1)(D-2)e^{-(D-2)\phi} \left( \frac{1}{2} g_{\mu\nu}^{(D)}(\nabla \phi)^2 - \nabla_\mu \phi \nabla_\nu \phi \right) = 0,
\]

(25)

\[
R^{(D)} - (D-1)(D-2)(\nabla \phi)^2 + 2(D-1)\Box \phi = 0,
\]

(26)

and the trace of Eq. (25) gives

\[
R^{(D)} - e^{-(D-2)\phi} \left( R^{(D)} - (D-1)(D-2)(\nabla \phi)^2 + 2(D-1)\Box \phi \right) = 0.
\]

(27)
Combining Eqs. (26) and (27), the condition $R^{(D)} = 0$ is obtained, which is consistent with the previous Eq. (17) from the $(D + 1)$-dimensional analysis. Here, if we assume a constant background $\phi$ solution and use $R^{(D)} = 0$, then the Ricci flat condition shown in the previous section can be reproduced in Eq. (25).

As for the special case of $D = 2$, we take the limit of $D \to 2$ in the action (24), then two-dimensional brane world is governed by

$$ S_{(D=2)} = \frac{1}{\kappa_3^2 k} \int d^2 x \sqrt{-g^{(2)}} \left[ \phi R^{(2)} - (\nabla \phi)^2 \right]. \quad (28) $$

Integrating out $\phi(x)$, the well-known two-dimensional Polyakov action \[17\] is obtained,

$$ S_{2D} = -\frac{1}{4\kappa_3^2 k} \int d^2 x \sqrt{-g^{(2)}} R^{(2)} \frac{1}{\Box} R^{(2)}. \quad (29) $$

This result is of interest in that the Polyakov action is derived from the classical three-dimensional brane-world model. From the beginning, if one fixes $D = 2$, then the same result comes out.

**IV. DISCUSSIONS**

Now, it seems to be appropriate to comment on a single brane model of RS2 model \[1\], the similar analysis to the RS1 model can be applied to the RS2 model, however, for simplicity, we take the limit of $r_c \to \infty$ corresponding to $\phi(x) \to \infty$ in Eq. (24). Then, the resulting action gives a desirable result as

$$ S_{(D)} = \frac{1}{(D - 2)\kappa_3^2 k} \int d^D x \sqrt{-g^{(D)}} R^{(D)}. \quad (30) $$

It is the $D$-dimensional Einstein-Hilbert action which yields a $D$-dimensional Schwarzschild black hole solution. For the present RS2 model, if $y \to \infty$, the ADM mass (19) is exponentially suppressed and the black hole horizon shrinks to zero at the AdS horizon.

Finally, the effective mass scale $M_D^2$ in $D$ dimensions from Eq. (24) can be defined as

$$ M_D^2 = \frac{2M_3^3}{(D + 1)k} \left( 1 - e^{-\frac{(D-2)\phi}{2}} \right). \quad (31) $$
where \( M_{(D+1)}^3 \) is a mass scale in \((D+1)\)-dimensional spacetimes which is defined as \(1/\kappa_{(D+1)}^2\). Note that the relation (31) is consistent with the result of the original cosmological derivation for the constant background \( \phi = \pi k r_c \) in \( D = 4 \).

In summary, we have studied the brane-world black hole solutions in the RS model. The RS dimensional reduction with the junction conditions gives the interesting brane-world gravity. The Ricci flat condition appears as one of the reduced equations of motion and the solutions are easily solved. As a comment, the effective gravity might reflect more or less holographic properties in that especially for \( D = 2 \) the Polyakov induced gravity is a conformal field theory with the central charge \( c = \frac{24\pi}{\kappa_{(3)}^2 k} \) in the spirit of AdS/CFT correspondence [RS], so that it would be interesting to study this issue in more detail.

**Acknowledgments**

We would like to thank P. P. Jung for helpful discussions. We are grateful to M. Cvetič for useful comments on domain walls, and would like to S. D. Odintsov, J. Erlich, and R. C. Myers for helpful remarks on our work. This work was supported by the Ministry of Education, Brain Korea 21 Project No. D-0055, 1999.
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