Strangeness Saturation: Energy- and System-Size Dependence

J. Cleymans\textsuperscript{a}, B. Kämpfer\textsuperscript{b}, P. Steinberg\textsuperscript{a,}\textsuperscript{*}, S. Wheaton\textsuperscript{a}

\textsuperscript{a} Department of Physics, University of Cape Town, Rondebosch 7701, Cape Town, South Africa
\textsuperscript{b} Institut für Kern- und Hadronenphysik, Forschungszentrum Rossendorf, PF 510119, D-01314 Dresden, Germany

Relativistic heavy-ion collisions lead to a final state which has a higher degree of strangeness saturation than those of elementary collisions. A systematic analysis of this phenomenon, based on the strangeness saturation factor, $\gamma_s$, is made for C+C, Si+Si and Pb+Pb collisions at the CERN SPS collider and for Au+Au collisions at RHIC energies. Strangeness saturation is shown to increase with the number of participants within a colliding system, at both CERN SPS and RHIC energies. The saturation observed in central collisions of lighter nuclei deviates from that seen in peripheral collisions of heavier nuclei with an equivalent participant number, which could be due to the difference in nuclear density.

I. INTRODUCTION

It has been shown that statistical-thermal models are able to reproduce the multiplicities measured in relativistic heavy-ion collisions with remarkable success. This is accomplished with a very small number of parameters—the temperature, baryon-chemical potential, $\mu_B$, and a factor measuring the degree of strangeness saturation, $\gamma_s$. As is now well known, there is very little difference between the temperatures observed in $p+p$ and relativistic heavy-ion collisions. The extracted strangeness saturation factor, $\gamma_s$, is however very different in $p+p$ and heavy-ion collisions.

In this paper we focus on hadron multiplicities and extract the thermal parameters as a function of system-size and energy.

The recent study in [1] (cf. table III therein) impressively demonstrated that, with increasing system-size at SPS energies, the strangeness saturation increases. In [2] we have shown that at a beam energy of 158 AGeV, in collisions of lead-on-lead nuclei, the strangeness saturation continuously increases with centrality. However, strangeness (as measured by fully-integrated kaon and antikaon multiplicities) is clearly below saturation. A preliminary analysis [3, 4, 5] of the centrality dependence at RHIC energy of $\sqrt{s_{NN}} = 130$ GeV points to a further increase of strangeness towards saturation for central collisions of gold nuclei. An independent analysis [6, 7] confirms this finding.

This paper is divided into several sections. Firstly the system-size dependence of the thermal parameters is determined using $4\pi$-yields from central C+C and Si+Si collisions [8], and centrality-binned Pb+Pb collisions [9, 10] at 158 AGeV at the CERN SPS. For comparison, centrality-binned mid-rapidity yields from Au+Au collisions at $\sqrt{s_{NN}} = 130$ GeV [11] and Pb+Pb collisions at SPS energy [12] are analysed, despite the danger in applying the thermal model to yields in a limited rapidity window. Finally, the energy dependence of the thermal parameters is further elucidated by analysis of central Pb+Pb yields measured by NA49 at 40, 80 and 158 AGeV [12, 13, 14, 15, 16].

II. ANALYSES OF HADRON MULTIPLICITIES

In the thermal model, hadron multiplicities can be described [17, 15, 19, 20, 21] by the grand-canonical partition function $Z(V, T, \vec{\mu}) = \text{Tr} \{ e^{-\hat{H} + \vec{\mu} \cdot \vec{Q}} \}$, where $\hat{H}$ is the statistical operator of the system, $T$ denotes the temperature, and $\mu_i$ and $Q_i$ represent the chemical potentials and corresponding conserved charges respectively. In the analysis of $4\pi$-data, the net-zero strangeness and the baryon-to-electric charge ratio of the colliding nuclei constrain the components of $\vec{\mu}_i = (\mu_B, \mu_S, \mu_Q)$. The particle numbers are given, in the Boltzmann approximation, by

$$N^\text{prim}_i = V g_i |S_i| \frac{d^3p}{(2\pi)^3} dm_i e^{-\frac{E_i - \vec{\mu} \cdot \vec{Q}}{T}} BW(m_i),$$

(2.1)

* Visiting Fulbright Professor on leave of absence from the Brookhaven National Laboratory, Upton, NY, USA.
where we include phenomenologically a strangeness saturation factor, $\gamma_s$, with $|S_i|$ the number of valence strange quarks and anti-quarks in species $i$ (i.e. $\gamma_s$ for the kaons and $\gamma_s^2$ for $\phi$) to account for incomplete equilibration in this sector, $E_i = \sqrt{p_i^2 + m_i^2}$, and BW is the Breit-Wigner distribution. The particle numbers to be compared with experiment are $N_i = N_i^{\text{prim}} + \sum_j \text{Br}(j \rightarrow i) N_j^{\text{prim}}$, due to decays of unstable particles with branching ratios $\text{Br}(j \rightarrow i)$. For small participant numbers (typically $N_{\text{part}}$ below 40), one has to resort to a canonical or micro-canonical formalism.

A. System-size dependence

1. Analysis of fully-integrated yields

In order to extract the system-size dependence of the thermal parameters we analyse 4$\pi$-multiplicities of $\pi^\pm$, $K^\pm$, $\phi$ and $N_{\text{part}}$ (taken as the sum over all baryons) in 6 centrality bins in the reaction Pb+Pb [8] (at 158 AGeV) and for central Si+Si and C+C collisions at the same energy. Our previous analyses of the Pb+Pb system included $\gamma$ yields. They are excluded in this analysis in order that the Pb-, C- and Si systems be treated equivalently. No weak feed-down corrections have yet been applied to the peripheral Pb-, or C- and Si systems [25]. Due to the rather limited data set, the freeze-out temperature was fixed at 165 MeV, independent of centrality and colliding system. This is supported by a variety of fits to both heavy-ion and elementary collision systems in this energy regime [1, 2, 18, 26, 27]. Owing to the size of the C-, Si- and peripheral Pb systems, strangeness was treated canonically in all systems. As shown in [24], for systems of this size at SPS energy, it is sufficient to treat the baryon- and charge content grand-canonically. The results are displayed in Figs. 1 and 2 with the specifics of each fit explained in the captions.

The strangeness saturation factor, $\gamma_s$, shows an increasing trend with collision centrality in the Pb+Pb system, except possibly over the two most central bins (see Fig. 1). It is also clear that the C+C and Si+Si systems lie above the trend suggested by the Pb+Pb points. This suggests that peripheral Pb+Pb collisions are not equivalent, with respect to strangeness saturation, to central collisions of lighter nuclei with the same participant number. In the C+C and Si+Si systems the baryon chemical potential is also lower than in the peripheral Pb+Pb bins (refer to Fig. 2). It should be stressed that the only direct baryon information we have at our disposal in this analysis is the number of $\gamma$-mesons, in these systems scales with $f_2$, the fraction of participants which undergo multiple collisions. As shown in Fig. 3, $\gamma_s$ scales with this variable too. The strangeness saturation extracted from $p+p$ collisions at $\sqrt{s} = 19.4$ GeV (denoted in Fig. 3 by the square) suggests a strong flattening off of $\gamma_s$ for small systems. What is surprising is the approximate equivalence of $f_2$ and $\gamma_s$ for a number of points in Fig. 1 ($f_2$ in this figure is denoted by the diamonds).

2. Mid-rapidity analysis

When applying the thermal model to 4$\pi$-data, many dynamical effects cancel out in ratios of the fully-integrated hadron yields [23]. In particular, effects due to flow disappear if the freeze-out surface is characterized by a single
temperature and chemical potential. When applying the thermal model to mid-rapidity data this is true only when the Bjorken model holds. Furthermore, in a limited rapidity window there is no guarantee that the total strangeness should be zero, nor that the baryon-to-charge ratio in this kinematic region should be set by the colliding nuclei. In a grand canonical approach this affects the constraints on the chemical potentials. In the canonical formulation the ‘canonical suppression volume’ (i.e. the volume in which quantum numbers are exactly conserved and that used to calculate the densities) and the ‘normalisation volume’ (i.e. the volume required to convert densities to yields) are not necessarily the same. A further complication arises in the treatment of decays. The experimental yields include feed-down from heavier resonances into stable, final-state particles. With mid-rapidity data, this requires careful consideration of the decay kinematics, as particles in a certain rapidity range will in general decay into particles in different kinematic windows.

Despite these disclaimers we analyse the following mid-rapidity yields measured by the NA49 and PHENIX collaborations:

i) NA49 mid-rapidity yields of $\pi^\pm$, $K^\pm$, $p$ and $\bar{p}$ in the reaction Pb(158 AGeV) + Pb in 6 centrality bins [9].

ii) PHENIX mid-rapidity densities of $\pi^\pm$, $K^\pm$ and $p^\pm$ in the reaction Au + Au at $\sqrt{s} = 130$ AGeV in 5 centrality bins [11].

The PHENIX yields were not corrected for weak decays. PHENIX estimate the probability for reconstructing protons from $\Lambda$ decays as prompt protons at 32% at $p_T = 1$ GeV/c [11]. The PHENIX analysis was performed with both 0% and 50% feed-down from weak decays, while no weak feeding was included in the SPS analysis. In all cases the grand-canonical formalism was applied, with the total strangeness set to zero but $\mu_Q$ fit as a free parameter.

In Fig. 4 the system-size dependence of $\gamma_s$ at mid-rapidity is shown for SPS (left panel) and RHIC (right panel). In the SPS plot the results of our earlier analysis of fully-integrated NA49 yields [9] are included for comparison. It is seen that $\gamma_s$, as extracted from the mid-rapidity NA49 data, is well above that obtained from the analysis of the fully-integrated NA49 yields. In order to exclude the possibility that the difference in the strangeness saturation extracted from the $4\pi$- and mid-rapidity data analysed here is due solely to different strange hadrons included in the fits, the $4\pi$ NA49 analysis was repeated with the hidden-strangeness $\phi$ excluded. In this way the two NA49 analyses are equivalent with respect to strange particles. This led to a slight decrease in $\gamma_s$ in the most peripheral bins, and thus an even larger difference between mid-rapidity and fully-integrated results. Thus, certainly at SPS energies, the degree of strangeness saturation is far higher in the central rapidity region. Included in the RHIC plot of Fig. 4 is $f_2$. This fraction of multiply-struck participants parametrises the system-size dependence of the strangeness saturation factor in Au+Au collisions at RHIC energy remarkably well.

B. Energy dependence

In order to further investigate the energy dependence of the thermal parameters we analyse the fully-integrated yields of $\pi^\pm$, $K^\pm$, $\Lambda$ and $\bar{\Lambda}$ in central Pb+Pb collisions at 40, 80 and 158 AGeV, supplemented with $K^0$, $\Xi^-$, $\bar{\Xi}^+$ and $\phi$ multiplicities at 158 AGeV [12, 13, 14, 15]. In Figs. 3 and 4, $\gamma_s$ and the Wróblewski factor [31], $\lambda_s$, which measures the ratio of newly created $s\bar{s}$ pairs to newly created non-strange valence quark pairs at the primary hadron level:

$$\lambda_s = \frac{2\langle s\bar{s} \rangle}{\langle uu \rangle + \langle dd \rangle}. \quad (2.3)$$

are displayed as a function of the collision energy. Included in the figures are the results of our earlier system-size analysis of the Pb+Pb system at CERN SPS and the Au+Au system at RHIC [9]. It should be noted that the centrality cuts on the most central Pb+Pb collisions are slightly different at the various energies (7.2% at 40 and 80 AGeV, and 5% at 158 AGeV). In view of the system-size dependence extracted in the previous section, this will raise the 40 and 80 AGeV points relative to the 158 AGeV point. In Fig. 3 one observes that $\lambda_s$ for central collisions decreases with collision energy from 40 AGeV (in agreement with [32]), within a given collision system, it shows a systematic increase with participant number, while remaining above the typical value of 0.2 seen in $pp$ collisions [12]. With respect to $\gamma_s$ there is fairly good agreement between the most peripheral heavy-ion bins and the results from elementary systems of comparable energy [27].

For comparison we also show the value of $\gamma_s$ extracted from mid-rapidity Pb+Pb data [12, 13, 16] at 158 AGeV (open circle) in Fig. 3. As can be seen, the strangeness saturation in the mid-rapidity region is clearly greater than that averaged over $4\pi$. 
III. SUMMARY

In conclusion, the strangeness saturation factor, $\gamma_s$, has been shown to increase with participant number in the Pb+Pb system at the CERN SPS as well as the Au+Au system at RHIC. Central collisions of C+C and Si+Si at SPS energies deviate, with respect to strangeness saturation, from peripheral Pb+Pb collisions. However, $\gamma_s$ is seen to scale with the fraction of multiply-struck participants, $f_2$. In fact, $f_2$ remarkably tracks the $N_{\text{part}}$-dependence of $\gamma_s$ as extracted from mid-rapidity yields in Au+Au collisions at RHIC. Where both mid-rapidity and fully-integrated data was available, the degree of strangeness saturation observed at mid-rapidity was found to be consistently higher than that extracted from $4\pi$-data.

Acknowledgments

We acknowledge useful correspondence with C. Höhne and M. Gaździcki. One of us (S.W.) acknowledges the financial assistance of the National Research Foundation (NRF) of South Africa.

[1] I.G. Bearden et al. (NA44 collaboration), nucl-ex/0202019.
[2] J. Cleymans, B. Kämpfer, S. Wheaton, Phys. Rev. C 65 (2002) 027901.
[3] B. Kämpfer, J. Cleymans, K. Gallmeister, S. Wheaton, hep-ph/0202134.
[4] B. Kämpfer, J. Cleymans, K. Gallmeister, S. Wheaton, hep-ph/0204227.
[5] J. Cleymans, B. Kämpfer, S. Wheaton, hep-ph/0208247.
[6] N. Xu et al. (STAR collaboration), private communication.
[7] M. Kaneta, contribution presented at Quark Matter 2002 (QM 2002), Nantes, France, 18-24 July 2002.
[8] C. Höhne (NA49 collaboration), hep-ex/0209013.
[9] F. Sikler (NA49 collaboration), Nucl. Phys. A 661 (1999) 45c.
[10] V. Friese et al. (NA49 collaboration), Nucl. Phys. A 698 (2002) 487c.
[11] K. Adcox et al. (PHENIX collaboration), Phys. Rev. Lett. 88 (2002) 242301.
[12] S.V. Afanasiev et al. (NA49 Collaboration), Phys. Rev. C 66 (2002) 054902.
[13] A. Mischke (NA49 Collaboration), contribution presented at Quark Matter 2002 (QM 2002), Nantes, France, 18-24 July 2002.
[14] S.V. Afanasiev et al. (NA49 Collaboration), Phys. Lett. B 491 (2000) 50.
[15] S.V. Afanasiev et al. (NA49 Collaboration), Phys. Lett. B 538 (2002) 275.
[16] A. Mischke (NA49 Collaboration), nucl-ex/0209002.
[17] For a general review see e.g. K. Redlich, J. Cleymans, H. Oeschler and A. Tounsi, Acta Physica Polonica B33 (2002) 1609.
[18] F. Becattini, J. Cleymans, A. Keränen, E. Suhonen, K. Redlich, Phys. Rev. C 64 (2001) 024901.
[19] P. Braun-Munzinger, I. Hepe, J. Stachel, Phys. Lett. B 465 (1999) 15.
[20] P. Braun-Munzinger et al., Phys. Lett. B 344 (1995) 43, B 365 (1996) 1, B 465 (1999) 15, B 518 (2001) 415.
[21] J. Cleymans, K. Redlich, Phys. Rev. Lett. 81 (1998) 5284; J. Sollfrank, J. Phys. G: Nucl. Part. Phys. 23 (1997) 1903.
[22] J. Letessier, J. Rafelski and A. Tounsi, Phys. Rev. C 50 (1994) 406; C. Slotta, J. Sollfrank and U. Heinz, in Proceedings of Strangeness in Hadronic Matter, Tucson, edited by J. Rafelski, AIP Conf. Proc. No. 340 (AIP, Woodbury, 1995), p. 462.
[23] K. Redlich, Nucl. Phys. A 698 (2002) 94.
[24] A. Keränen and F. Becattini, Phys. Rev. C 65 (2002) 044901.
[25] C. Höhne, private communication.
[26] F. Becattini, Z. Phys. C 69 (1996) 485.
[27] F. Becattini and U. Heinz, Z. Phys. C 76 (1997) 269.
[28] S.A. Bass et al., Prog. Part. Nucl. Phys. 41 (1998) 225.
[29] J. Cleymans and K. Redlich, Phys. Rev. C 60 (1999) 054908.
[30] J.D. Bjorken, Phys. Rev. D 27 (1983) 140.
[31] A. Wróblewski, Acta Physica Polonica B16 (1985) 379.
[32] F. Becattini, hep-ph/0206203.
[33] P. Braun-Munzinger, J. Cleymans, H. Oeschler, K. Redlich, Nucl. Phys. A 697 (2002) 902.
FIG. 1: The system-size dependence of the strangeness saturation factor, $\gamma_s$, as extracted from centrality-binned Pb+Pb [9, 10], and central C+C and Si+Si data [8] under various fit conditions. The circles with error bars represent the results of our $\chi^2$-analysis assuming 50% feeding from weak decays, while the triangles show the results minimising the quadratic deviation (again assuming 50% feeding). For comparison, the results of our earlier Pb system analysis [5], with $\bar{p}$'s included in the fit, are included (squares). Also shown are the fraction of participants which underwent multiple collisions, $f_2$, as extracted from a Glauber calculation [8] (diamonds).
FIG. 2: The system-size dependence of the baryon chemical potential, $\mu_B$, as extracted from centrality-binned Pb+Pb [9, 10], and central C+C and Si+Si data under various fit conditions. The circles with error bars represent the results of our $\chi^2$-analysis assuming 50% feeding from weak decays, while the triangles show the results minimising the quadratic deviation (again assuming 50% feeding). For comparison, the results of our earlier Pb system analysis [5], with $\bar{p}$'s included in the fit, are included (squares).
FIG. 3: The strangeness saturation factor, $\gamma_s$, as extracted from centrality-binned Pb+Pb (triangles) and central C+C and Si+Si data (circles), as a function of $f_2$, the fraction of multiply-struck participants. The results shown are those obtained with $\mu_B$ fixed at 250 MeV and $T$ at 165 MeV, assuming 50% weak feed-down. For comparison, the strangeness saturation as extracted from p+p collisions at $\sqrt{s} = 19.4$ GeV is included (square).
FIG. 4: Left Panel: Comparison of the strangeness saturation factor, $\gamma_s$, extracted from mid-rapidity NA49 data [9] (up triangles) with the results of our earlier analysis of NA49 4$\pi$-yields (squares) [5]. Right Panel: The strangeness saturation observed in Au+Au collisions as extracted from PHENIX data [11]. The analysis was performed assuming 50% weak feed-down (down triangles) and 0% weak feed-down (up triangles). Also shown are the fraction of multiply-struck participants, $f_2$, obtained from our Glauber calculation (dashed line).
FIG. 5: The energy dependence of the strangeness saturation factor, $\gamma_s$, extracted from central Pb+Pb collisions at 40, 80 and 158 AGeV [12, 13, 14, 15] (up triangles), together with the results of our earlier analysis of centrality-binned Pb+Pb collisions at 158 AGeV (down triangles) and Au+Au collisions at RHIC (diamonds) [5]. For comparison we show the results obtained from $pp$ collisions (filled squares) and $p\bar{p}$ collisions (open squares) at various energies [27]. The open circle is extracted from mid-rapidity yields [12, 13, 16].
FIG. 6: The energy dependence of the Wróblewski factor, $\lambda_s$, extracted from central Pb+Pb collisions at 40, 80 and 158 AGeV $^{12, 13, 14, 15}$ (up triangles), together with the results of our earlier analysis of centrality-binned Pb+Pb collisions at 158 AGeV (down triangles) and Au+Au collisions at RHIC (diamonds) $^{5}$. For reference we show the typical value of 0.2 extracted from pp systems $^{32}$. 