MODELING SPECTRAL VARIABILITY OF PROMPT GAMMA-RAY BURST EMISSION WITHIN THE JITTER RADIATION PARADIGM

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ABSTRACT

The origin of rapid spectral variability and certain spectral correlations of prompt gamma-ray burst emission remains an intriguing question. The recently proposed theoretical model of the prompt emission is built upon unique spectral properties of jitter radiation—the radiation from small-scale magnetic fields generated at a site of strong energy release (e.g., a relativistic collisionless shock in baryonic or pair-dominated ejecta, or a reconnection site in a magnetically dominated outflow). Here we present the results of implementation of the model. We show that anisotropy of the jitter radiation pattern and relativistic shell kinematics altogether produce effects commonly observed in time-resolved spectra of the prompt emission, e.g., the softening of the spectrum below the peak energy within individual pulses in the prompt light curve, the so-called “tracking” behavior (correlation of the observed flux with other spectral parameters), the emergence of hard, synchrotron-violating spectra at the beginning of individual spikes. Several observational predictions of the model are discussed.

Key words: gamma rays: bursts – magnetic fields – radiation mechanisms: non-thermal – shock waves

1. INTRODUCTION

Gamma-ray burst (GRB) prompt emission exhibits rapid spectral variability the origin of which is not fully understood yet. While some of it—the hard-to-soft evolution at late prompt stages—can be attributed to the high-latitude emission in the standard synchrotron shock model (Kumar & Panaitescu 2000; Genet & Granot 2008), the long-acknowledged “tracking” behavior (Bhat et al. 1994; Crider et al. 1997; Frontera et al. 2000) remains unexplained within this model. The tracking behavior is quite intriguing: the low-energy spectral index \( \alpha \) follows the observed flux (Crider et al. 1997; Pothapragada 2005), with harder index corresponding to higher flux, as is illustrated in Figure 1 (we used data from Kaneko et al. 2006). To our knowledge, the synchrotron shock model (e.g., with self-absorption and comptonization) does not seem to generally predict such a correlation. The situation becomes even more dramatic when one takes into account that the hardest observed \( \alpha \)’s cannot be reconciled with the standard synchrotron theory, which constrains the lower energy photon index to be \( \alpha \leq -2/3 \) (Preece et al. 1998, 2000), and that most of the GRB spectra exhibit \( \alpha \sim -1 \), a value by no means special within the theory (Kaneko et al. 2006).3

A recent theoretical model (Medvedev 2006) of the spectral variability of prompt GRBs predicts that the above properties naturally follow from the jitter radiation theory (Medvedev 2000) of electron emission in small-scale magnetic fields generated in situ. Note that one should not use here the results by Fleishman (2006), as we discuss at the end of Section 2. Here we do not limit ourselves to a particular model of a GRB: either a kinetically driven outflow, including baryon-dominated and lepton-dominated ones (i.e., with electron–ion and electron–positron pair plasmas, respectively), or a magnetic-field-dominated outflow (e.g., driven by Poynting flux), or any combination thereof.

The prediction that relativistic shocks generate strong magnetic fields via the Weibel-like (particle streaming) instability (Medvedev & Loeb 1999) has been confirmed with a large number of numerical PIC simulations in both baryonic and pair plasmas (Silva et al. 2003; Frederiksen et al. 2004; Nishikawa et al. 2005; Spitkovsky 2008; Chang et al. 2008; Keshet et al. 2008). The generated fields are random on a very small (sub-Larmor) scale, hence the synchrotron theory is inapplicable in such environments while the jitter radiation seems to be validated in these simulations (Hededal 2005), whose immediate relevance to GRB physics has recently been strengthened (Medvedev & Spitkovsky 2009). The generation of small-scale magnetic fields has also been discovered in PIC simulations of magnetic reconnection in pair plasmas (Swisdak et al. 2008; Zenitani & Hesse 2008). We can speculate that a strong energy release, as in GRBs, can produce electron–positron plasmas in situ during a reconnection event, even if the plasma is initially lepton-poor. The strong small-scale fields are produced via the Weibel instability by the streams of accelerated particles in the reconnection exhaust funnels; hence jitter radiation is expected to be produced here.

In this paper, we study spectral variability and correlations within the jitter radiation framework using a simple numerical model of an instantaneously illuminated, relativistically expanding thin spherical shell segment within a jet (Pothapragada et al. 2007; Reynolds et al. 2008). We interpret the flux–\( \alpha \) correlation as a combined effect of temporal variation of the viewing angle, the anisotropy of the radiation field in the shell and relativistic aberration. We mostly focus on the \( F_\nu(E_{\text{peak}})-\alpha \) correlation because other parameters—the hard index, the peak energy itself \( E_{\text{peak}} \), and the bolometric flux—are more susceptible to the overall GRB energetics, details of electron acceleration, etc; so, unlike \( \alpha \), more factors can affect them and cause mutual correlations.

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3 It is worth mentioning that BATSE time-resolved spectral catalog (Kaneko et al. 2006) is the largest available to date and will remain such for at least a few years. Besides, the GBM instrument on board Fermi-GRO is very similar to BATSE in many respects, hence it is natural to compare the results with BATSE data.
2. NUMERICAL MODEL

One usually assumes that each individual “spike” or “pulse” in the prompt light curve is a single “emission episode” produced by an internal dissipation event (a shock or reconnection) that occurs within the ejecta when two thin plasma shells of different speeds and/or field orientations collide. The emission region may generally be curved, and hence we use the model of the instantaneously illuminated spherical shell. Because of the shell curvature, the photons emitted along the line of sight, $\theta' = \theta = 0$ (see Figure 2), arrive at the detector first whereas the high-latitude emission, $\theta'$ through an angle $\theta < 0$, arrive at a detector simultaneously.

Photons emitted from thin annuli or parts thereof centered on the line of sight are in the observer’s (detector) frame.) Hence, an instantaneous “flash” in the source frame translates to a light curve in the observer’s frame (we neglect the cosmological redshift effect, assuming $z = 0$). The angle-dependent spectrum (i.e., the differential solid angle of jitter radiation in the source frame, shown in Figure 3, thus translates into a time-dependent spectrum in the detector frame. Integration over the emitting volume and accounting for time dilation, relativistic Doppler effect, and relativistic angle aberration yields the energy flux $F_\nu$, seen by a distant observer.

Let us consider emission from a thin segment of a spherical shell expanding radially with the Lorentz factor $\Gamma = (1 - \beta^2)^{-1/2}$, $\beta = v/c$ and confined within a jet of opening angle $\theta_{jet}$, its axis can be misaligned with the line of sight through an angle $\theta_{abs}$, as shown in Figure 2. The observed flux from an optically thin relativistic source has been calculated by Woods & Loeb (1999), Granot et al. (1999), Ryde & Petrosian (2002), and others. Here we briefly outline the method following Woods & Loeb (1999). We use spherical coordinates with a GRB

\[
F_\nu = \int I_\nu(\alpha, \phi) \cos \alpha d\Omega \approx \int d\phi \int I_\nu(\alpha, \phi) d\alpha d\Omega, \quad (1)
\]

where $\cos \alpha$ comes from the obliqueness of the ray to the detector plane, which is negligible at small angles: $\cos \alpha \approx 1$; also $\sin \alpha \approx \alpha$. Here $I_\nu(\alpha, \phi)$ is the intensity along the ray in the direction $(\alpha, \phi)$ and $d\Omega$ is the differential solid angle of the emitting region. From simple geometry $\alpha \approx r \sin \theta/d = (r/d)(1 - \mu^2)^{1/2}$, where $\mu = \cos \theta$, so that for given $r$, $a \Delta \alpha = (-r^2/d^2)\mu d\mu$.

The intensity is calculated as follows,

\[
dI_\nu = j_\nu ds = j'_\nu D^2(\mu) dr/\mu, \quad (2)
\]

where the differential path length is $ds = (r^2d\theta^2 + dr^2)^{1/2} = dr/\mu$ and we used the Lorentz invariance of the quantity $j_\nu/v^2$, so that $j_\nu/v^2 = j'_\nu/v'^2$. The observed and emitted frequencies, $\nu$ and $\nu'$, are related as

\[
\nu = \nu'D(\mu) = \frac{\nu'}{\Gamma(1 - \beta \mu)} \quad (3)
\]

with $D = [\Gamma(1 - \beta \mu)]^{-1} = \Gamma(1 + \beta \mu)$ being the relativistic Doppler boosting factor. The emissivity in the comoving frame, $j'_\nu(\Omega'_j, r, t_e)$, measured in erg (s cm$^{-2}$ Hz sr$^{-1}$), is a function of frequency $\nu'$, position $r = (r, \theta, \phi)$, time $t_e$, and the direction toward the detector measured in the comoving frame $\Omega'_j = (\theta', \phi')$, where the angles transform due to relativistic aberration as

\[
\phi' = \phi, \quad \cos \theta' = \mu' = \frac{\mu - \beta}{1 - \beta \mu}. \quad (4)
\]

Because of the shell curvature, the photon arrival time, $t_e$, is related to the emission time, $t_e$, as $t_e = t_e - r \mu/c$, so that $t = 0$ is for a photon emitted from the origin at $t_e = 0$. Combining the
above equations together, we arrive at a general result analogous to Equation (8) of Woods & Loeb (1999),
\[ F_\nu(t) = \frac{1}{\Omega_d} \int d\Omega_d \int d\mu \int r^2 dr \, D^2 \rho \, (\gamma_d, t, r + r\mu/c), \]
where limits of integration are set by the geometry of the source, \((\phi, \mu) \in \Gamma\). In general, \(\Gamma\) is a function of position and time. Our numerical model calculates the spectral flux for an arbitrarily shaped source and \(\Gamma(\theta, \phi')\). For a jet, one gets \(\theta \in [0, \theta_0 + \theta_{obs}], \phi \in [\phi_1(\theta), \phi_2(\theta)]\) (see Figure 2). The \(\phi\)-limits are found numerically for each \(\theta\) by ensuring \(\phi\) to be within the edges of jet cone; in the absence of edges \(\phi \in [0, 2\pi]\). For simplicity, we also assume \(\Gamma = \text{const} \) hereafter.

Assuming that emission occurs at \(r = R\) and \(t_0 = t_e = R/c\) and the shell is of infinitesimal thickness, \(\Delta R \to 0\), we take the emissivity as \(j'_\nu = n_e' P(\nu', \theta')\Omega(\Delta R' / 2 - |r' - R'|)\delta(r' - t_e) \to \Sigma^1_e P(\nu', \theta')\delta(r - R)\delta(t_e - t_0)\), where \(\Theta = \delta\) and \(\delta\) are the Heaviside step-function and Dirac \(\delta\)-function. Here we used \(r' = R, t' = t/\Gamma\), and the identities \(\delta(ax) = |a|^{-1}\delta(x)\), \(\Theta(ax) = \text{sign}(a)\Theta(x)\), and also introduced the electron surface density \(\Sigma^1_e = n_e'^2\). Substituting \(j'_\nu\) into Equation (5) and integrating over \(dr\) one obtains the time-dependent \(\delta\)-function \(\propto \delta(\mu - \beta^{-1} + t_e/R)\). Upon integration over \(d\mu \) and \(d\phi\), we obtain
\[ F_\nu(t) = F_0 D^2(t) P(\nu(t'), \mu'(t)) (\phi_2(t) - \phi_1(t)), \]
where \(F_0\) is the normalization constant, \(F_1(t_e) = 1\) at the peak frequency, \(D(t) = R(\Gamma(t)\beta c t)\), \(\nu(t') = \nu(\nu' D(t)\Gamma^{-1/2} - 1)/\beta\), and \(\phi_1, \phi_2\) are found numerically, as is described above. For a conical jet, \(\phi_2 - \phi_1\) have been calculated analytically (Ioka & Nakamura 2001; Yamazaki et al. 2002).

The jet comoving spectrum \(P(\nu', \theta')\) depends on the full three-dimensional spatial spectrum of the magnetic field and the electron distribution in the emitting shell. It is calculated elsewhere in full detail; see Medvedev (2000, 2006) and Reynolds et al. (2008, 2009). In order to keep our model simple, we assume the field spatial spectra in the shell (e.g., shock plane) and normal to it (i.e., along the Weibel current filaments) to be identical and described by a broken power law peaked at \(k_B \sim \lambda_B^{-1}\), the characteristic field correlation scale. The electron energy distribution is assumed to be a power law with the low-energy cutoff at \(\gamma_c\), \(n(\gamma) \propto \gamma^{-p}\) for \(\gamma \gtrsim \gamma_c\). Hence the peak jitter frequency is \(\nu_{peak} = c\chi c/\lambda_B\). The resultant spectrum is shown in Figure 3 for several angles \(\theta'\) ranging from 0 (head-on shell/shock) to \(\pi/2\) (edge-on shell/shock in the comoving frame); at a higher angle \(\theta' > \pi/2\) the spectrum is identical to that at the angle \(\pi - \theta'\). The \(P(\nu', \theta')\) spectrum above the peak is set either by the electrons, yielding the slope \(-(p + 1)/2\), or by the field spectrum, whichever is harder. If it is set by the field (as is in this study), the variation of the high-energy index of the \(P(\nu', \theta')\) spectrum in the range \(\sim 2.5\)–\(3.5\) is real and associated with the varying viewing angle only, neither with the changes of the magnetic field spectrum, nor with the electron energy distribution. The slope of the field spectrum at \(k < k_B\) plays a minor role for the low-energy slope of the radiation spectrum. Note that the variation of the lower-energy index between 1 and 0 (or 0 and \(-1\) for the photon spectrum) is a \textit{benchmark} feature of the jitter radiation regime.

In this context, the reader shall be warned on the following. Recently an invalid statement has been made (Fleishman 2006) that the spectrum \(F_\nu \propto \nu^1\) below the peak, “valid in the presence of ordered small-scale magnetic field fluctuations, does not occur in the general case of small-scale random magnetic field fluctuations.” It has been demonstrated (Medvedev 2005, 2006) that the statement is flawed since jitter radiation from random magnetic fluctuations with a fairly general distribution function (not just “ordered small-scale” fields) does allow for \(\propto \nu^1\) spectra. Moreover, the entire range from \(\propto \nu^1\) to \(\propto \nu^0\) is allowed for monoenergetic electrons, and even softer spectra can form, depending on the electron energy distribution. Another confusing issue is related to the absorption-like \(\propto \nu^2\) spectrum (Fleishman 2006). Such a spectrum is due to plasma dispersion and shall be seen in/near the radio band, not in gamma-rays, as the reader might incorrectly infer (Medvedev 2005). We stress once again here that the spectral shape of jitter radiation depends, in general, on the viewing angle: it can vary between \(\propto \nu^1\) and \(\propto \nu^0\), and this is the effect we use in the present study.

The obtained light curve \(F_\nu(t)\) represents the full spectrum evolution in time for each individual pulse of a prompt GRB and, possibly, an X-ray flare. In order to analyze spectral correlations and to compare our model to observational results, we fit the spectrum at each time step, \(t\), with the Band function (Band et al. 1993) to obtain the peak energy \(E_{\text{peak}}\) (or, in general, the break energy), the low- and high-energy photon indices \(\alpha\) and \(\beta\), and the overall normalization \(A\). The fit is done in the linear scale of \(E\) (not log(\(E\))). This makes the fit more sensitive to the spectral shape around \(E_{\text{peak}}\) and less sensitive to the shape at energies lesser and greater than \(E_{\text{peak}}\). This is a better method, since it is known that in the region \(E < E_{\text{peak}}\), the spectral shape is not a simple power law. Many GRB spectra were analyzed similarly, hence the comparison with observations is facilitated. Since noise is absent in our spectra, in contrast to observations, the \(\chi^2\) values should not be compared. In order to mimic the spectral limitations of BATSE, we limit our six-decade spectrum to the two-decade spectral window including most of \(E_{\text{peak}}\) values, except for very soft spectra at late times. In our correlation analysis, we use the low-energy photon index \(\alpha\) and the energy flux at the peak energy, \(F_\nu(E_{\text{peak}})\), because the latter is a well-defined physical quantity characterizing an emission mechanism, in contrast to the flux within a fixed energy band often used in GRB data analysis. A detailed study of spectral correlations between other parameters goes beyond the scope of this paper and will be presented elsewhere (S. S. Pothapragada et al. 2009, in preparation).

3. RESULTS AND DISCUSSION

In our model, \(\theta_{\text{jet}}\) and \(t_e = R/(2cT^2)\) are free parameters; they were chosen to be \(t_e \sim 10s\) and \(\theta_{\text{jet}} = 5^\circ\); here we also reset \(t\) to be the “time since trigger”, i.e., the very first photons arrive at \(t = 0\). Figure 4 (left) shows the time evolution of \(\alpha\) and \(F_\nu(E_{\text{peak}})\) within a single pulse (emission episode) for three misalignment angles \(\theta_{\text{obs}} = 0, \theta_{\text{jet}}/2, \theta_{\text{jet}}\), the third case appears between the other two because we plot the normalized (not absolute) flux. The differences are minimal: the \(\alpha\)-curves overlap and the \(F_\nu(E_{\text{peak}})\)-curves start to deviate around the jet break at \(\sim 100 t_c\). The overall tracking behavior is evident: the higher flux in the beginning of the pulse correlates to the larger \(\alpha\), i.e., the steeper spectrum below \(E_{\text{peak}}\). At early times and at very late times (near the jet break) \(\alpha\) exceeds the “synchrotron line of death” \(\alpha = -2/3\). This is a prediction of our model. Figure 4 (right) represents correlation of the same spectral parameters.
The pulse starts off in the upper right corner and follows the upper part of the curve to the left—this is the early emission phase. At later times $t \gg t_\epsilon$, the path turns downward—this is the high-latitude emission phase. At very late times, one sees steepening of the spectrum. Because the magnetic field lies predominantly in the shell plane (in the internal shock scenario), we suggest that the polarization of gamma-ray emission shall increase at times when one sees the shell nearly edge-on in the comoving frame, i.e., at the time $t \sim t_\epsilon = R/(2c\Gamma^2)$. A stronger polarization is expected for a sufficiently misaligned jet, $\theta \sim 1/\Gamma < \theta_{\text{jet}}$, or a distorted internal shock front/reconnection layer. The presence of a feature in $F_\nu(E_{\text{peak}})$ at $t \sim t_\epsilon$ is another model prediction, though the exact shape can depend on the actual magnetic field distribution.

Finally, we create a synthetic GRB by overlaying 100 individual pulses (now with $t_\epsilon \sim 1$ s) at random positions and with random amplitudes drawn from a power-law distribution derived from the pulse statistics (Beloborodov et al. 2000). In order to mimic the time-resolved spectral analysis done by the BATSE team, we then bin it in time so that fluence is the same in all bins (the bin sizes, of course, vary); the binned energy flux light curve is shown in Figure 5 (left) with the red line. The bin-
averaged spectra are then fitted as usual and the dependence of the soft photon index $\alpha$ on time is overplotted in the same graph. The inset in the right panel shows that the synthetic spectra are well fitted with the Band function. Clearly, $\alpha$ tracks flux in the same way it does in Figure 1. The scatter plot, Figure 5 (right), does show a positive $F_{\nu}(E_{\text{peak}}) - \alpha$ correlation with some scatter. One can also see that there is a small population of spectra with steep (even steeper than synchrotron) $\alpha$'s and that many spectra have their soft indices clustered around $\alpha = -1$, in great agreement with observational data (Preece et al. 2000; Kaneko et al. 2006). We stress that this jitter model is the only one, to our knowledge, which can explain these facts altogether.

4. CONCLUSIONS

We developed a model of prompt GRB spectral variability built upon physical understanding of the internal structure of a dissipation region. The model presented here is rather crude, because it uses very simplified models of the magnetic field spatial spectrum, the electron distribution, and the simplified relativistic shell kinematics. Yet, the results obtained mimic the observed spectral correlations of the BATSE catalog remarkably well without any ad hoc assumptions, thus suggesting robustness of the model. We extend our analysis to other spectral parameters and use more elaborate models in forthcoming publications. The following predictions can be made. First, one shall expect steepening of the spectral index below the peak energy at very late times. Such observations are difficult because an afterglow sets in before this and because the peak shifts to low frequencies, where (self-)absorption may play a role. Second, a feature in the light curve $F_{\nu}(E_{\text{peak}}, t)$ can be expected around $t \sim R/(2c\Gamma^2)$. Third, if the jet is misaligned with $\theta_{\text{obs}} \sim 1/\Gamma$ and/or the shock front/reconnection layer is distorted, one can expect enhancement of polarization of gamma-ray emission at times $t \sim R/(2c\Gamma^2)$. In magnetic outflows, the ambient field can also contribute to polarization and (synchrotron) emission.

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REFERENCES

Band, D., et al. 1993, ApJ, 413, 281
Beloborodov, A. M., Shtern, B. E., & Svensson, R. 2000, ApJ, 353, 158
Bhat, P. N., et al. 1994, ApJ, 426, 604
Chang, P., Spitkovsky, A., & Arons, J. 2008, ApJ, 674, 378
Crider, A., et al. 1997, ApJ, 479, L39
Fleishman, G. 2006, ApJ, 638, 348
Frederiksen, J. T., Hededal, C. B., Haugbølle, T., & Nordlund, Å. 2004, ApJ, 608, L13
Frontera, F., et al. 2000, ApJS, 127, 59
Genet, F., & Granot, J. 2008, ApJ, submitted (arXiv:0812.4677)
Granot, J., Piran, T., & Sari, R. 1999, ApJ, 513, 679
Hededal, C. B. 2005, PhD thesis, Niels Bohr Univ., Copenhagen (arXiv: astro-ph/0506559)
Ioka, K., & Nakamura, T. 2001, ApJ, 561, 703
Kaneko, Y., Preece, R. D., Briggs, M. S., Paciesas, W. S., Meegan, C. A., & Band, D. L. 2006, ApJS, 166, 298
Keshet, U., Katz, B., Spitkovsky, A., & Waxman, E. 2009, ApJ, 693, L127
Kumar, P., & Panaitescu, A. 2000, ApJ, 541, L51
Medvedev, M. V. 2000, ApJ, 540, 704
Medvedev, M. V. 2005, arXiv:astro-ph/0503463
Medvedev, M. V. 2006, ApJ, 637, 869
Medvedev, M. V., & Loeb, A. 1999, ApJ, 526, 697
Medvedev, M. V., & Spitkovsky, A. 2009, ApJ, 700, 956
Nishikawa, K.-I., Harder, P., Richardson, G., Preece, R., Sol, H., & Fishman, G. J. 2005, ApJ, 632, 927
Pothapragada, S., Reynolds, S., Graham, S., & Medvedev, M. V. 2007, APS Meeting Abstracts, 8090P
Pothapragada, S. 2005, APS Meeting Abstracts, 1163P
Preece, R. D., Briggs, M. S., Mallozzi, R. S., Pendleton, G. N., Paciesas, W. S., & Band, D. L. 1998, ApJ, 506, 23
Preece, R. D., Briggs, M. S., Mallozzi, R. S., Pendleton, G. N., Paciesas, W. S., & Band, D. L. 2000, ApJS, 126, 19
Reynolds, S., Pothapragada, S., & Medvedev, M. 2009, ApJ, submitted (arXiv:0902.2369)
Reynolds, S., Pothapragada, S., & Medvedev, M. 2008, BAAS, 40, 210
Ryde, F., & Petrosian, V. 2002, ApJ, 578, 290
Silva, L. O., Fonseca, R. A., Tonge, J. W., Dawson, J. M., Mori, W. B., & Medvedev, M. V. 2003, ApJ, 596, L121
Spitkovsky, A. 2008, ApJ, 673, L39
Swisdak, M., Liu, Y.-H., & Drake, J. F. 2008, ApJ, 680, 999
Woods, E., & Loeb, A. 1999, ApJ, 523, 187
Yanazaki, R., Ioka, K., & Nakamura, T. 2002, ApJ, 571, L31
Zenitani, S., & Hesse, M. 2008, Phys. Plasmas, 15, 022101