Bose-Einstein correlations from "within"

O.V. Utyuzh*, G. Wilk* and Z.Włodarczyk†

*The Andrzej Sołtan Institute for Nuclear Studies; Hoża 69; 00-681 Warsaw, Poland
†Institute of Physics, Świętokrzyska Academy, Świętokrzyska 15; 25-406 Kielce, Poland

Abstract. We describe an attempt to model numerically Bose-Einstein correlations (BEC) from "within", i.e., by using them as the most fundamental ingredient of some Monte Carlo event generator (MC) rather than considering them as a kind of (more or less important, depending on the actual situation) "afterburner", which inevitably changes original physical content of the MC code used to model multiparticle production process.

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Introduction. The problem of BEC is so well known that we shall skip introductory remarks (referring in that matter to other presentations at this conference and to [1] for more information) and instead we shall start right away with our subject concerning the proper numerical modelling of BEC, which we call BEC from within. Although phenomenon of BEC is with us from the very beginning of the systematic investigation of multiparticle production processes, its modelling is still virtually nonexisting. With the exception of attempts presented in [2, 3] all other approaches are tacitly assuming that on the whole BEC constitute only a small effect and it is therefore justify to add it in some way to the already known outputs of the MC event generators widely used to model results of high energy collisions (in the form of the so called afterburner) 1. There are two types of such afterburners:

(a) those modifying accordingly energy-momenta of identical secondaries (and correcting afterwards the whole sample for energy-momentum conservation) - they apply to each event separately;

(b) those selecting events which already have (due to some fluctuation present in any MC code) right energy-momenta of identical secondaries and counting them (by introducing some weights) many times (see [1] for references) - in this case energy-momentum balance is left intact but instead the particle spectra provided by MC code are distorted (albeit in most cases only slightly); they apply only to all events.

Example of modifications introduced by afterburner. It must be stressed that modifications that such afterburners introduce to physical background behind a given MC code were never investigated. Tacit assumption made is that they are small and therefore irrel-

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1 Methods presented in [2] were never used in practice, only [3] applied their approach to \(e^+e^-\) annihilation data.
The problem is that with the MC codes in use at present it is practically impossible to estimate the nature and strength of such changes. In can be done only by using some simple scheme of cascading, for example simple cascade model for hadronization developed by us some time ago \cite{4}. This model per se leads to no BEC (cf. left panel of Fig. 1). To obtain effect of BEC one has to add to it some simple afterburner, for example the one proposed by us in \cite{5}. In it one preserves the whole space-time and energy-momentum structure of each event but changes the charge assignment to secondaries in such way that clusters of identically charged particles occur (the original number of $(+)$, $(-)$ and neutrals remains the same). This automatically leads to BEC \cite{3}. The right panel of Fig. 1 shows what are the changes introduced by this afterburner in the original cascade: the multicharged vertices occur now (with total charge in the whole event remaining conserved). It means that, in principle, one could obtain BEC directly from MC (i.e., without any afterburner) allowing in it for the appearance of such vertices (according to some prescribed scheme - for example as some multicharged clusters, such possibility has been already mentioned in \cite{6}). However, it would be then extremely difficult to run such cascade till the very end without producing spurious multicharged particles not observed in nature. Our afterburner can be then considered as a kind of shortcut realization of such possibility with well defined physical consequences. We argue therefore that each afterburner changes original MC code it is attached to in a similar fashion but this statement cannot be at the moment substantiated\cite{4}.

**BEC from "within".** The above observation was one of our motivations to look for the MC scheme, which would be build around BEC rather than starting from some single-particle observables. This idea has been for the first time used in \cite{3} where particles were selected from a grand-canonical-like distribution with temperature $T$ and chemical potential $\mu$ chosen in such way as to describe rapidity and multiparticle distributions\cite{5} and particles were then distributed into some rapidity cells of given width, each cell containing only particles of the same charge; the method was then (quite successfully) used to describe $e^+e^-$ annihilation data. Introduction of these cells was the real origin of the BEC observed. Actually the idea that BEC demands that particles of the same charge are emitted from some cells (named elementary emitting cells - EEC) was reasonably

\[ T \] and $\mu$ are, in fact, two lagrange multipliers obtained from energy conservation and charge conservation constraints.
FIGURE 1. Example of charge flows in MC code using simple cascade model for hadronization [4]: left panel - no effect of BEC observed; right panel - after applying afterburner described in [5] (based on new assignment of charges to the produced particles) one has BEC present at the cost of appearance of multicharged vertices.

proposed earlier in [8] and consist also cornerstone of our present proposal, which can be regarded as generalization of that presented in [3]. This is nothing else but attempt to numerically realize the bunching of particles as quantum statistical effect used already in connection with BEC long time ago [7] and is done by dividing all available energy among particles (taken here as being pions) in such way that a number \( n_{\text{cell}} \) of EEC’s, each containing particles of the same charge, is formed, with multiplicity of particles in each EEC following Bose-Einstein (or geometrical) distribution\(^6\). When energy distribution from which particles are selected is thermal-like then \( P(n_{\text{cell}}) \) is Poissonian and the total multiplicity distribution is of Pólya-Aeppli type [9], closely resembling Negative Binomial distribution obtained in the so called clan model [10] (which differs only by the fact that particles in clans are distributed according to logarithmic distribution, not geometrical one [1]). Particles in a given EEC can have energies spread around the energy \( E_1 \) of the first particle defining this EEC with some width \( \sigma \). With such energy spreading allowed one gets quite reasonable results for \( C_2(Q_{\text{inv}}) \) distributions (see [1] for details)\(^7\).

Here we would like to present extension to this algorithm, which in addition to bunching accounts also for the symmetrization of the two-particle wave function (not used before) and allows to obtain in addition to \( C_2(Q_{\text{inv}}) \) also \( C_2(Q_{x,y,z}) \), i.e., in a sense it is 3-dimensional extension of our algorithm. This extension is based on the observation that symmetrization correlates the energy-momenta of particles with their space-time locations. The bunching of particles considered before was done only in the energy-momentum space and left us with a number of EEC’s, each with a number of particles with well defined energies, \( E_i \) (and momenta \( p_i = (E_i^2 - m^2)^{1/2} \)). Each EEC is build

\(^6\) To this end to the first particle selected in a given EEC one adds (up to first failure after which new EEC is selected) another particles with probability \( P = P_0 \cdot e^{-E/T}, \) where \( P_0 \in (0, 1) \) is constant, \( E \) is energy of the first particle and \( T \) parameter (corresponding to temperature in thermal models). Such form of \( P \) ensures the characteristic Bose-Einstein form of energy distribution.

\(^7\) Actually, as was shown in [11] (see also [12]), this spreading is crucial to obtain the proper shape of \( C_2 \) function.
around some "seed" particle, which is taken as particle \( i = 1 \). This was enough to get \( C_2(Q_{lm}) \) (see [1]), but not \( C_2(Q_{x,y,z}) \) involving components of \( p_i, p_i(x,y,z) \). To assign them one has first choose some space-time positions for particles in a given EEC taking them from some distribution function \( \rho(r,t) \). Actually in what follows we shall use only static source approximation, i.e., hadronization is instantaneously and therefore \( \rho(r,t) = \rho(r) \). Now \( p_i(x,y,z) \) have to be correlated with the corresponding space positions, \( r_i = (x_i, y_i, z_i) \), in the way emerging from the symmetrization of the wave functions resulting (for the plane wave approximation) in the famous \( 1 + \cos(\delta p \cdot \delta r) \) expression. Technically this is achieved by accepting only such momenta \( p_i(x,y,z) \), which for given \( r_i \) lead to \( \cos(\delta p \cdot \delta r) \leq 2 \cdot \text{Rand} - 1 \) where \( \text{Rand} \) is random number uniformly chosen from interval \((0,1)\).

In Fig. 2 we present examples of our new results (the elder results can be found in [1]) obtained with full, 3-dimensional version of our model for \( \rho(r) \) being sphere of radius \( R = 1 \, \text{fm} \) and assuming that all \( p_i(x,y,z) \) are spherically symmetric. As one can see now in addition to \( C_2(Q_{lm}) \) one has also corresponding to it \( p_i(x,y,z) C_2(Q_{x,y,z}) \).

So far we are assuming direct pion production. However, the inclusion of Coulomb
and other final state interactions in our approach is straightforward - one must simply change \( \cos(\cdots) \) term arising from plane wave approximation used to form obtained by some distorted wave function. One can also easily include resonances and allow for finite life time of the emitting source. Finally, so far, for simplicity reason, only two particle symmetrization effects have been accounted for. Namely, in a given EEC all particles are symmetrized with the particle number 1 being its seed, they are not symmetrized between themselves. This seems to be justified because majority of our EEC’s contain only 1 – 2 or 3 particles. But to fully account for multiparticle effects one should simply add other terms in addition to the \( \cos(\cdots) \) used above. This, however, would result in dramatic increase of the calculational time\(^8\). Nevertheless the effect of including at least terms when symmetrization between, say, particles 2 and 3 are added to the already present symmetrization between 1 and 2 and 1 and 3, must be carefully investigated before any final conclusion is to be reached.

**Summary.** To summarize, we are proposing numerical scheme of modelling quantum statistical phenomenon represented by BEC occurring in all hadronization processes. Distinctive features of our scheme not present in other propositions are:

- identical particles are emitted from EEC’s and only these particles are subjected to BEC;
- inside each EEC particles are distributed according to the geometrical (or Bose-Einstein) distribution;
- altogether they show characteristic Bose-Einstein form of distribution of energy.

As result we obtain a kind of *quantum clan model* with Negative-Binomial-like multiplicity distributions and characteristic shape of \( C_2(Q_{\text{inv}}) \) function [1] (notice that we automatically include in this way BEC to all orders given by the maximal number of particles in a given EEC). To get also \( C_2(Q_{x,y,z}) \) one has to use some additional space-time information and the characteristic \( 1 + \cos(\delta r \cdot \delta p) \) correlations between space-time and energy-momenta induced by the symmetrization of the respective wave functions. So far this is only a case study, we cannot yet offer any attempt to compare it with experimental data. On the other hand our approach offers new understanding of the way in which BEC are entering hadronization process.

We shall close with remark that there are attempts in the literature to model numerically BE condensation [13] (or to use notion of BE condensation in other branches of science as well [14]) using ideas of bunching of some quantities in the respective phase spaces.

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\(^8\) In fact, this converges to the proposition presented long time ago in [2], the only hope is that in our case symmetrization is performed within given EEC and therefore the number of terms of \( \cos(\cdots) \) type involved is rather limited, whereas in [2] the whole source had to be symmetrized at once resulting with number of terms growing like \( n! \) where \( n \) is observed multiplicity.
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