Bidirectional Angle Limited Shortest Path Ray Tracing in Complex Topographical Conditions

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Abstract. Ray tracing technology in complex topographical conditions has an important influence on mountain velocity modeling and migration imaging. In order to realize the stable and high precision ray tracing in the conditions, the bidirectional angle limited shortest path ray tracing method with under the condition of irregular calculation boundary is proposed. Firstly, based on the shortest path ray-tracing method, a new algorithm of local traveltime is given under the condition of bidirectional angle limited. Secondly, combining this algorithm with narrowband technology, a ray-tracing algorithm is proposed to adapt to arbitrary complex terrain; Then, we compare the accuracy of this method and the first-order finite-difference method; Finally, the adaptability and stability of the complex surface are verified by the calculation example, and the corresponding conclusions are given.

1. Introduction
The shortest path ray-tracing method (SPR) was first derived from graphic theory and first introduced into seismic wave traveltime and ray path calculation in 1986 [1]. The conventional SPR computational accuracy is high, but it needs to set additional computing nodes on the grid line, resulting in a relatively larger computational and storage cost. To solve this problem, many scholars have improved the method in many ways over the years. In order to improve the efficiency of the calculation, bidirectional angle limited is added to the calculation of current source point near the grid point, which reduces the amount of computation and the number of replacement of traveltime value, and then improves the calculation efficiency. Furthermore, the algorithm records the ray path information in the traveltime calculation, so that the ray path from the source point to any point in the calculation area can be obtained by simple reverse (toward the source direction) search, which also improves the computational efficiency.

In addition, as a large number of seismic exploration work in China has begun to transfer to complex surface areas, seismic wave travel time and ray path computational technology in complex topographical conditions become very important, which can be used not only to analyze some propagating laws of seismic waves in complex topographical conditions but also to provide forward modeling tools for the improvement of seismic data processing technology and the development of new seismic data processing technology in complex topographical conditions. In view of this, the SPR method with the bidirectional angle limited can adapt to the complex media and the accuracy is high, which makes it able to deal with the ray path in complex topographical conditions.

Above all, in this paper, we will focus on the SPR with complex topography, by introducing bidirectional angle limited method and narrowband technology to improve the calculation accuracy of SPR, so that the new SPR method can adapt to complex topography and undulated interface problems.
2. Bidirectional angle limited SPR method

The normal SPR uses Huygens principle and Fermat principle repeatedly in traveltime calculation. The main calculation process is: first, to calculate the source near the grid node, and then these points are used as the new source (i.e., secondary source) to continue to calculate the traveltime value of the adjacent nodes under the control of the Fermat principle (minimum traveltime principle), and then repeat the traveltime value of all the grid nodes in the computational complete model space in turn, and record the location information of the upper primary source points of each grid node at the time of calculation, which is used to track the ray path. Local calculation formula used by conventional SPR (as shown in Figure 1) in the current algorithm execution is:

$$tt(j) = \min (tt(j), tt(i) + sd_{ij}), (j \in FS(i) \cap Q)$$

(1)

where i is the current sub-source point (the s point in figure 1), FS(i) represents the point set on the gridline directly adjacent to the s point, Q represents the point set before completion, s represents the slowness of the current grid cell, and dij represents the point j to point i distance. Formula (1) indicates that when the s point is a sub-source point, the traveltime value of all grid nodes that are directly adjacent to the s point and have not completed the walk-time calculation is calculated or updated.

As shown in figure 1a, the conventional SPR follows the local algorithm described in formula (1). At this time, it is necessary to calculate the traveltime value of the neighbouring 14 white mesh nodes. When the local seismic wave is propagated from the upper left corner to the lower right corner, and the medium velocity is considered to be uniformly distributed in a local small region, so the next seismic wave should not be transmitted to the region with a very large angle at the lower left and upper right of the local grid unit, which means it is not necessary to calculate its traveltime value at this position of the local grid node at this time point, because this will greatly increase the number of calculation and the replacement of the traveltime value.

In order to solve the above problem, a bidirectional angle limit algorithm is introduced, which actually includes two aspects:(1) In order to reduce the calculation amount, when determining the calculated grid nodes near the front sub-source point, we choose the grid nodes in the control range from the certain angle (the green solid line angle $\alpha$ shown in Fig.1b) of the sub-source (the green filled grid nodes shown in Fig.1b); (2) In order to improve the calculation accuracy, we choose the traveltime value of calculating a grid node (the G point shown in Fig.1b) from a certain angle of the calculated point (the red dotted lines angle shown in Fig.1b) compute the completed mesh nodes as sub-source points (grid nodes filled in red as shown in figure 1b).

Based on the above ideas, formula (1) can be modified as follows:

$$tt(j) = \min (tt(j), tt(S_i) + sd_{ij}), (j \in FS(i) \cap Q \cap \alpha_m, i \in \beta_m)$$

(2)

Formula (2) indicates that when S point is a sub-source point, only the adjacent point $j \in FS(i) \cap Q \cap \alpha_m$ is calculated (as shown in Fig.1b, $j \in FS(i) \cap Q \cap \alpha_m = G, G_1, G_2, G_3$). $\alpha_m$ denotes the
adjacent grid nodes within the limit of angle $\alpha$, and the method of determining angle $\alpha$ is as follows: firstly, the line is used to connect S point and its upper-order sub-source point S2; secondly, the two sides of the straight line SS2 and SS2 at the point of S are oriented from two points of angle S2, the rays of the direction (the green solid line shown in 2), the angle determined by the two lines is the angle $\alpha$. In addition, when calculating the travel time value of the current calculated point, we are not limited to the use of the current sub-source point, but to select a sub-source point set $i \epsilon \beta_m$, $\beta_m$ denotes all the grid nodes that have been completed in the range of traveltine adjacent to the s point (as shown in figure 2b, $i \epsilon \beta_m = S, S_1, S_2$).

3. Algorithm in complex topographical conditions
To implement the global computation of traveltime in complex topographical conditions, the narrowband technology is selected out and adopted together with bidirectional angle limited SPR method. Narrowband technology is a method to achieve narrowband expansion and evolution through the removal and incorporation of narrowband mesh nodes, and then approximates the wavefront propagation process [2]. By introducing new grid node types (including surface points, points above surface, interface points, interface lower points, etc.), we adapts narrowband technology to the surface irregular computing boundary and underground undulated interface. The implementation of the first arrival wave traveltime calculation in complex topographical conditions is as follows:

(1) Initialization: let the source point be the only accepted point, and its traveltime value is zero; the grid nodes directly adjacent to the source point calculates its traveltime and incorporates into the narrowband, and the traveltime value is calculated by analytical formula.

(2) Loop: ① select the point with the smallest traveltime in the narrow band as the extension point and change its attribute to the accepted point; ② judge the grid nodes around the extension point to keep its original attribute unchanged if it is a surface point or accepted point; update its traveltime value if it is a narrow band point; calculate its traveltime and incorporate it into the narrow band if its attribute is far point; ③ if it is an empty narrow band, the calculation is complete, otherwise jump back to (1).

The above contents are the overall implementing step of the traveltime calculation. The ray path calculation of SPR is achieved by recording the next sub-source point of each grid node in the traveltime calculation process. When the traveltime calculation is completed, the ray path is obtained from the receiving point by constantly searching the upper sub-source point until the source point. That is to say, the curve of a series of sub-source points that is searched by sequential connection is the ray path.
4. Accuracy analysis
To analyze the accuracy of the bidirectional angle limited SPR, a homogeneous media model was selected with a size of 4 km*4 km and a velocity of 1.0 km/s at (2.0 km,2.0 km). Figure 3 shows the traveltime relative error of the first-order finite difference method, the second-order finite difference method and the bidirectional angle limited SPR method at the grid spacing of 10 m respectively. The average relative errors of the three algorithms are: 0.1409%, 0.7086%, 0.1330%, and the maximum relative errors are: 0.0042, 0.0813, 0.0339. Figure 3 shows that the accuracy of the bidirectional angle limited algorithm is much higher than the first-order finite difference method and the second-order finite difference method, and the maximum error is much smaller than that of the second-order finite difference method.

![Figure 3](image)

Figure 3. Accuracy comparison of the bidirectional angle limited method with the first and second order finite difference method. (a) First-order finite difference method; (b) Second-order finite difference method; (c) Bidirectional angle limited SPR method.

5. Numerical example
To test the effectiveness of the proposed algorithm in complex topographical conditions, a model with undulated surface and undulated interface is presented. The model size is 8.5×12.0 km2 and the source point is located at (6.0 km,1.0 km). Figure 4 shows the first arrival wave traveltime and ray path in complex topographical conditions. From the calculated results showed in figure 4, it can be found that the proposed algorithm can adapt to the complex surface conditions stably, and the calculated results also show that the obvious diffracted waves are produced in the relatively small elevation valley, concave and other areas.

![Figure 4](image)

Figure 4. The first arrival wave traveltime and ray path in complex topographical conditions.
6. Conclusions and discussion

In view of the problem of seismic wave traveltime and ray path calculation in complex topographical conditions, this paper proposes a new SPR algorithm combining bidirectional angle limited and narrowband technology. The calculation and analysis show that: ① The new algorithm has higher accuracy than the first and second-order finite difference method; ② The new algorithm can adapt to complex surface condition in a stable and flexible way. For the above two advantages, this algorithm will have a broad application prospect in velocity modeling and migration imaging in complex mountain areas.

References

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