On prospects for dark matter indirect detection in the Constrained MSSM

Leszek Roszkowski\textsuperscript{a,}\textsuperscript{*}, Roberto Ruiz de Austri\textsuperscript{b}, Joe Silk\textsuperscript{c}, Roberto Trotta\textsuperscript{c}

\textsuperscript{a} Department of Physics and Astronomy, University of Sheffield, Sheffield S3 7RH, UK
\textsuperscript{b} Departamento de Física Teórica C-XI and Instituto de Física Teórica C-XVI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
\textsuperscript{c} Oxford University, Department of Astrophysics, Denys Wilkinson Building, Keble Road, Oxford, OX1 3RH, UK

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\textbf{A B S T R A C T}

We apply a rigorous statistical analysis to the Constrained MSSM to derive the most probable ranges of the diffuse gamma radiation flux from the direction of the Galactic center and of the positron flux from the Galactic halo due to neutralino dark matter annihilation, for several different choices of the halo model and propagation model parameters. We find that, for a specified halo profile, and assuming flat priors, the 68% probability range of the integrated $\gamma$-ray flux spans about one order of magnitude, while the 95% probability range can be much larger and extend over four orders of magnitude (even exceeding five for a tiny region at small neutralino mass). The detectability of the signal by GLAST depending primarily on the cuspiness of the halo profile. The positron flux, on the other hand, appears to be too small to be detectable by PAMELA, unless the boost factor is at least of order ten and/or the halo profile is extremely cuspy. We also briefly discuss the sensitivity of our results to the choice of priors.

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1. Introduction

There is currently much evidence for the existence of large amounts of dark matter (DM) in the Universe. While its nature remains unknown, DM is likely to be made up of an exotic species of weakly interacting massive particles (WIMPs). A particularly popular WIMP candidate is the lightest neutralino $\chi$ of effective low-energy supersymmetry (SUSY), which is stable due to R-parity\textsuperscript{[1,2]}. In addition to collider searches for SUSY and direct detection (DD) searches for a cosmic WIMP, a promising strategy is that of indirect detection (ID), i.e., a search for traces of WIMP pair-annihilation in the Milky Way. Since the annihilation rate is proportional to the square of the WIMP number density, of particular interest are the Galactic center (GC) and nearby clumps in the halo where the density of DM is believed to be enhanced. The aim of this Letter is to provide, for the first time, a statistical measure for the prediction of $\gamma$-ray and positron signatures in low-energy SUSY, thus allowing one to assess high-probability regions for DM-annihilation signatures that could be observed by the GLAST (in orbit since June 2008) and PAMELA (launched 2006) satellites. Existing data from EGRET suggest a spectrally distinct excess of $\gamma$-rays up to $\sim 10$ GeV and the HEAT data indicate a possible excess in positron flux between 5 to $\sim 30$ GeV. GLAST and PAMELA will provide an order of magnitude more sensitivity.

In assessing detection prospects of WIMPs there are two main sources of uncertainties. One comes from the underlying particle physics model where WIMP mass and annihilation cross section can vary over a few orders of magnitude. The other is astrophysical in nature and stems from substantial uncertainties in the DM distribution, both locally (local DM density and the existence of clumps) and towards the GC. Since the general Minimal Supersymmetric Standard Model (MSSM) suffers from a lack of predictability due to a large number of free parameters, it is interesting and worthwhile to assess WIMP detection prospects in more constrained and more well-motivated low-energy SUSY models, among which particularly popular is the Constrained MSSM (CMSSM)\textsuperscript{[3]}, which includes the minimal supergravity model\textsuperscript{[4]}. By applying a statistical approach, we derive in the CMSSM most probable ranges of fluxes, thus bringing under control all the uncertainties of the particle physics side of WIMP detection. This is a major improvement over existing methods which are usually limited to the consideration of a few representative choices of points or slices in the parameter space. Detection prospects then become a function of specific astrophysical uncertainties only.

In this Letter we employ a Bayesian Markov Chain Monte Carlo (MCMC) technique to efficiently explore the multi-dimensional parameter space of the CMSSM, and to include all relevant sources of uncertainty on the particle physics side\textsuperscript{[5,6]} (for a similar study, see\textsuperscript{[7]}). Our Bayesian approach allows us to produce probability maps for all relevant observable quantities, thus establishing a complete set of predictions of the CMSSM.
2. Bayesian analysis of the CMSSM

The CMSSM is described in terms of four free parameters: a ratio of Higgs vacuum expectation values \( \tan \beta \), and common soft SUSY-breaking mass parameters of gauginos, \( m_{1/2} \), Scalars, \( m_0 \), and tri-linear couplings, \( A_0 \). The parameters \( m_{1/2}, m_0 \) and \( A_0 \) are specified at the GUT scale, \( M_{\text{GUT}} \simeq 2 \times 10^{16} \text{GeV} \), which serves as a starting point for evolving the MSSM renormalization group equations for couplings and masses down to a low energy scale \( M_{\text{SUSY}} \equiv m_{\mu} m_{\tau} / m_t^2 \) (where \( m_{\mu}, m_{\tau}, m_t \) denote the masses of the scalar partners of the top quark), chosen so as to minimize higher order loop corrections. At \( M_{\text{SUSY}} \) the (1-loop corrected) conditions of electroweak symmetry breaking (EWSB) are imposed. The sign of the Higgs/higgsino mass parameter \( \mu \), however, remains undetermined. Here we set \( \mu > 0 \).

In deriving predictions for the observable quantities, one also needs to take into account the uncertainty coming from our imperfect knowledge of the values of some relevant Standard Model (SM) parameters, namely the pole top quark mass, \( M_t \), the bottom quark mass at \( m_b \), and the electromagnetic and the strong coupling constants at the Z pole mass \( M_Z \), \( \alpha_{\text{em}}(M_Z) / \sqrt{\alpha_s(M_Z)} \), respectively (the last three quantities are all computed in the \( \overline{\text{MS}} \) scheme). These four “nuisance parameters” are the most relevant ones for accurately predicting the SUSY spectrum and its observable signature. In our analysis we thus consider an 8-dimensional parameter space spanned by the above four SM and the four CMSSM parameters.

In general, the results of a Bayesian analysis are expressed in terms of a posterior probability distribution (or more briefly, “a posterior”). By virtue of Bayes’ theorem, the posterior is the state of knowledge about the parameters before seeing the data, conditioned on the prior knowledge of the values of some relevant Standard Model (SM) parameters, namely the pole top quark mass, \( M_t \), the bottom quark mass at \( m_b \), and the electromagnetic and the strong coupling constants at the Z pole mass \( M_Z \), \( \alpha_{\text{em}}(M_Z) / \sqrt{\alpha_s(M_Z)} \), respectively (the last three quantities are all computed in the \( \overline{\text{MS}} \) scheme). These four “nuisance parameters” are the most relevant ones for accurately predicting the SUSY spectrum and its observable signature. In our analysis we thus consider an 8-dimensional parameter space spanned by the above four SM and the four CMSSM parameters.

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mass spectra of the neutralino, the other superpartners and the Higgs bosons, the log prior allows one to examine the low mass region in more detail, in particular by "expanding" the volume of the region $100 \text{ GeV} \lesssim m_{1/2}, m_0 \lesssim 1 \text{ TeV}$. As we discuss below, the flat prior appears to produce an optimistic scenario as far as indirect detection signatures are concerned, while the log prior can give lower values of the fluxes and hence it leads to more pessimistic prospects for indirect detection. Ways of mediating between the two scenarios and to assess their relative plausibility will be explored in future work.

Since the log prior gives more "weight" to lower values of both $m_{1/2}$ and $m_0$, not surprisingly, we have found that it leads to a large widening of mostly the lower boundary of the 68% probability range at low $m_X$, while not affecting the flux ranges at larger values of the neutralino mass. For example, the 68% probability range widens to nearly three decades and, in the case of the Moore profile with adiabatic compression, can be as low as $1.2 \times 10^{-10} \text{ cm}^{-2} \text{s}^{-1}$ at $m_1 \sim 100 \text{ GeV}$, but then it quickly raises and for $m_X \gtrsim 200 \text{ GeV}$ is not very different from the case of the flat prior. A more detailed discussion of the implications for CMSSM parameters of employing a log prior is given in Ref. [10].

For a given prior, choosing a different halo profile merely amounts to shifting the total flux by the ratio of the values of $\tilde{J}$ given in Table 1. As expected, more cuspy profiles lead to higher predicted fluxes. We find that, in the CMSSM in the case of the Moore profile (with and without adiabatic compression) and the NFW profile with adiabatic compression, the continuum flux signal will be within the reach of GLAST, while for profiles with $\tilde{J}(10^{-5} \text{ sr}) \lesssim 10^3$ it will not be detectable by GLAST. (We have checked that the case of $\mu < 0$ and flat priors gives qualitatively similar results.)

The differential $\gamma$-ray flux from DM annihilations is expected to exhibit a sharp drop-off in the energy spectrum as $E_{\gamma}$ approaches $m_X$. In Fig. 2 we plot 68% and 95% probability regions for the $\gamma$-ray differential flux for the NFW profile, averaged over a solid angle $\Delta \Omega = 10^{-3} \text{ sr}$ (to allow a comparison with EGRET data), for the flat prior choice. Clearly, the current uncertainty on CMSSM parameters and hence on $m_X$ introduces a considerable spread in the predicted spectral shape of the signal. Additional uncertainty comes from the dependence on the priors. For example, for the log prior given above the 68% probability range of the differential photon range extends between $2.4 \times 10^{-11} \text{ GeV cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$ and $6.7 \times 10^{-7} \text{ GeV cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$. Thus, even if a positive signal were detected by GLAST, it would be difficult to infer from it the mass of the WIMP, especially at its lower values below some 200 GeV, with any reasonable accuracy.

4. Positron flux from the Galactic halo

Positrons can be produced either in direct DM annihilation, or from decays and hadronization of other products (gauge and Higgs bosons, etc.), with the continuum spectrum from the latter usually dominating. Once produced, they propagate through the Galactic medium and their spectrum is distorted due to synchrotron radiation and inverse Compton scattering at large energies, bremsstrahlung and ionization at lower energies. The effects of positron propagation are computed following a standard procedure described in [24,25], by solving numerically the diffusion-loss equation for the number density of positrons per unit energy $dn_{e^+}/dE$. The diffusion coefficient is parameterized as $K(\epsilon) = K_0(3\alpha + \epsilon^4)$, with $K_0 = 5.8 \times 10^{27} \text{ cm}^2 \text{s}^{-1}$, $\alpha = 0.6$ and $\epsilon = E_{e^+}/(1 \text{ GeV})$, mimicking re-acceleration effects. The energy loss rate is given by $b(\epsilon) = \tau_\epsilon \epsilon^2$, with $\tau_\epsilon = 10^{-16} \text{ s}^{-1}$, and we describe the diffusion zone (i.e., the Galaxy) as an infinite slab of height $L = 4 \text{ kpc}$, with free escape boundary conditions. Changes in the above positron propagation model, especially $K(\epsilon)$ (see e.g.

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\[^2\] Resolution and sensitivity in the range $30 \text{ MeV} \lesssim E_{\gamma} \lesssim 10 \text{ GeV}$ are energy-dependent and would require a more careful analysis.
In the framework of the Constrained MSSM, we have performed a Bayesian analysis of prospects for indirect dark matter detection via a diffuse $\gamma$-ray signal or a positron flux from the Galactic center. This has allowed us to provide a statistically rigorous assessment of the uncertainty from the particle physics side of the problem.

We found that the prospects for GLAST to detect a diffuse $\gamma$-ray signal from the Galactic center depend primarily on the cuspiness of the DM profile at small radii. For the choice of flat priors in the CMSSM parameters, the NFW model appears to be a borderline
case, while a more cuspy halo would guarantee a signal for a 68% range of the CMSSM parameter space, except near the bottom end of the neutralino mass around 100 GeV, below the 68% probability range of the CMSSM parameter space, except near the bottom end of the 68% probability range of the $\gamma$-ray flux towards lower values at low $m_\chi < 200$ GeV, but at larger $m_\chi$ gives similar results as with flat priors.

On the other hand, a positron flux is unlikely to be detectable by PAMELA for both choices of priors, unless it is strongly enhanced by a nearby clump with a boost factor of at least of order ten. The latter conclusion is valid for a specific (although well motivated) choice of propagation model parameters. Assumptions regarding propagation parameters could however be easily relaxed in our framework. It would be straightforward to extend our treatment to include propagation model parameters as nuisance parameters and marginalize over them, as well. It is expected that such a procedure would increase the present, very substantial uncertainty as to the spectral shape, which we have shown is a consequence of the current lack of knowledge as to the preferred regions of the CMSSM parameters. Finally, it would also be interesting to repeat this analysis in a more general phenomenological SUSY model than the Constrained MSSM. While a richer phenomenology might help in explaining future signals should they be detected, it is also clear that a larger number of free parameters on the particle physics side will add to the difficulty of reliably predicting the shape and strength of both the $\gamma$-ray and the positron spectra.

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