Experimental determination of the quasi-particle decay length $\xi_{\text{Sm}}$ in a superconducting quantum well.\footnote{Submitted to Phys. Rev. B, Rap. Comm.}

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(July 11, 2021)

Abstract

We have investigated experimentally the electronic transport properties of a two-dimensional electron gas (2DEG) present in an AlSb/InAs/AlSb quantum well, where part of the toplayer has been replaced by a superconducting Nb strip, with an energy gap $\Delta_0$. By measuring the lateral electronic transport underneath the superconductor, and comparing the experimental results with a model based on the Bogoliubov-de Gennes equation and the Landauer-Büttiker formalism, we obtain a decay length $\xi_{\text{Sm}} \approx 100 \text{ nm}$ for electrons. This decay length corresponds to an interface transparency $T_{\text{SIN}} = 0.7$ between the Nb and InAs. Using this value, we infer an energy gap in the excitation
spectrum of the SQW of $\Delta_{\text{eff}} = 0.97\Delta_0 = 0.83$ meV.
A superconducting quantum well (SQW) can be defined as a system in which one of the barriers of a quantum well, in our case InAs in between AlSb barriers, is replaced by a superconductor, here Nb. In a quantum well, particles are confined by normal reflections at the boundaries. In a SQW, also Andreev reflection at the superconducting barrier can occur, changing the confinement. Due to this superconducting barrier, an energy gap $\Delta_{\text{eff}}$ appears in the excitation spectrum of the two dimensional electron gas (2DEG) present in the SQW. The magnitude of this gap depends on the interface transparency $T_{\text{SIN}}$ between the superconductor and the InAs, and the superconducting energy gap $\Delta_0$. In the limit $T_{\text{SIN}} \ll 1$, Volkov et al. have shown that the SQW can be described as a 2 dimensional superconductor with an effective order parameter $\Delta_{\text{eff}} e^{i\phi} (\Delta_{\text{eff}} \ll \Delta_0)$, where $\phi$ is the macroscopic phase of the superconductor on top.

The physics of the SQW is of importance to understand transport in co-planar super-normal-superconductor (SNS) junctions. From a technological point of view there are two kinds of SNS junctions, sandwich- (inline planar) and co-planar junctions. In sandwich-type junctions, the junction length $L$ is well defined. In co-planar structures however, electrons can travel a certain distance underneath the superconductor before being Andreev reflected, thus effectively enlarging the junction length. The distance electrons penetrate underneath the superconductor can be associated with a decay length $\xi_{\text{Sm}}$, similar to the superconducting coherence length $\xi_0 = \hbar v_F/\Delta$.

It is important to have a good estimate of the actual junction length, $L_{\text{eff}} = L + 2\xi_{\text{Sm}}$, because it is a relevant parameter in calculations for the critical current $I_c$ in SNS junctions. This was also appreciated recently by Nguyen et al. who measured Nb-InAs-Nb junctions with varying length, using a transmission line model (TLM). When plotting the resistance versus the junction length, they obtained a straight line, intersecting the length axis at a negative value. They interpreted this length to be the average distance $x_A = 1.5 \mu m$ an electron needs to travel underneath the superconductor, before it is Andreev reflected.

The system under study is a 15 nm InAs layer sandwiched between a 2 $\mu$m AlSb layer and a superconductor, Nb, see Fig 2b. The 2DEG present in the InAs has a high electron
mobility, resulting in a long elastic mean free path $\ell$. The ballistic regime is therefore easily accessible. Furthermore the absence of a Schottky barrier in metal-InAs contacts enables one to make highly transparent interfaces. At the InAs-AlSb interface, the barrier for electrons is assumed to be infinite, at the Nb-InAs interface a $\delta$-function potential barrier is present, characterized by a dimensionless parameter $Z = \frac{2p_F H}{\mu} = \frac{H}{\hbar v_F}$ as introduced by Blonder et al. Due to the high interface transparency in our system the limit $\Delta_{\text{eff}} \ll \Delta_0$, used by Volkov et al., no longer applies, therefore also the quasi particle decay length is expected to be different from $\hbar v_F/\Delta_{\text{eff}}$. The classical description used by Nguyen et al. takes into account multiple Andreev reflections, but ignores phase coherence between these multiple reflections. This approach will in general not lead to an exponential decay of the laterally transmitted wave functions in the SQW. Here we will calculate both the energy gap and the decay length of quasi particles at the Fermi-level in the SQW, using a quantum mechanical description.

We will assume that only the lowest 2D subband in the QW is filled, which is the case in our samples. In order to calculate the wave functions we have to solve the Bogoliubov-de Gennes equation,

\[
\begin{bmatrix}
\mathcal{H} & \Delta(r) \\
\Delta^*(r) & -\mathcal{H}
\end{bmatrix}
\begin{bmatrix}
u(r) \\
u^*(r)
\end{bmatrix}
= E
\begin{bmatrix}
u(r) \\
v(r)
\end{bmatrix},
\]

where the Hamiltonian $\mathcal{H}$ is defined as

\[
\mathcal{H} = -\frac{\hbar^2}{2m^*} \nabla^2 + U(r) - \mu.
\]

For the effective mass we assume $m^* = m_0$, the free electron mass, in the Nb, and $m^* = 0.023 m_0$ in the InAs. For the potential $U(r)$ we take

\[
U(r) = U(z) = H \delta(z) - E_{F, \text{Nb}} \theta(-z)
- E_{F, \text{InAs}} \theta(z) \theta(L - z) + V_0 \theta(z - L),
\]

where the Fermi energies are $E_{F, \text{Nb}} = 5.3$ eV and $E_{F, \text{InAs}} = 0.11$ eV. The potential barrier at the InAs-AlSb interface, $V_0$, is assumed to go to infinity. In Eq.(1) the pair potential $\Delta(r)$ is assumed to be $\Delta_0$ in the Nb ($z < 0$), and zero elsewhere.
The solutions to the Bogoliubov-de Gennes equation, Eq.(1), are given by electron- and hole wave functions in the InAs-quantum well \((0 < z < L)\),

\[
\Psi(r) = \begin{bmatrix} u(r) \\ v(r) \end{bmatrix} = \begin{Bmatrix} u(z) \\ v(z) \end{Bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{i(k_x z + k_y y)},
\]

and by mixed quasi-particle wave functions in the Nb superconductor \((z < 0)\), with \(u^2 = 1 - v^2 = \frac{1}{2}(1 + \Omega/E)\). \(\Omega^2 = E^2 - \Delta^2\), \(E\) being the energy with respect to the Fermi-level.

These wave functions and their derivatives have to be matched at the boundaries \(z = 0\) and \(z = L\), see Ref. 2 for a detailed analysis. Numerical solutions of Eq.(1) are given in Fig. 1.

For \(k_z\) real, where \(k_z\) is the wave vector in the \(z\)-direction, there are no solutions of \(E(k)\) with \(|E|\) smaller than an effective energy gap \(|\Delta_{\text{eff}}|\). This \(|\Delta_{\text{eff}}|\) is calculated as function of the transparency \(T_{\text{SIN}} = 1/(1 + Z^2)\) of the Nb-InAs interface, by finding the minimum of \(E(k)\), Fig. 1a. At low transparency it can be shown that \(\Delta_{\text{eff}}\) depends linearly on \(T_{\text{SIN}}\):

\[\Delta_{\text{eff}} \approx \frac{1}{4} T_{\text{SIN}} E_0,\]

where \(E_0 = \frac{\hbar^2}{2m^*} \left(\frac{\pi}{L}\right)^2\) is the confinement energy in the QW. At \(E = 0\) there are only solutions for Eq.(1) for complex \(k_z\). The total energy \(\frac{\hbar^2}{2m^*}(k_z^2 + k_{\|}^2)\) must be real, hence \(k_{\|} = \sqrt{k_x^2 + k_y^2}\) has an imaginary part. By taking the \(y\)-direction along the boundary of the SQW, wave function matching requires \(\text{Im}(k_y) = 0\). For the decay length we can thus write \(\text{Im}(k_x)^{-1} = -\text{Re}(k_x)/\text{Re}(k_z) \text{Im}(k_z) \approx -k_F \cos(\alpha)/\text{Re}(k_z) \text{Im}(k_z) = \xi_{\text{SIN}} \cos(\alpha)\), where \(\alpha\) is the angle of incidence. The decay length \(\xi_{\text{SIN}}\) is plotted in Fig. 1b. The decay length \(\xi = \hbar k_F/m^* \Delta_{\text{eff}}\), analogous to the expression used for the decay length of quasi-particles in a superconductor, is shown for comparison. As can be seen, at high transparency, there is a substantial difference between both decay lengths. We will show that the former one is the relevant one in the SQW.

Samples are based on a 15 nm InAs quantum well with AlSb barriers. Prior to any processing the top AlSb layer is removed. The parameters of the quantum well with an exposed InAs surface are (measured at 4.2 K): \(n_S = 1.1 \times 10^{16} \text{ m}^{-2}\), \(\mu_e = 2.2 \text{ m}^2/\text{Vs}\) and \(\ell = 380 \text{ nm}\). We want to measure the laterally transmitted signal through a SQW, depending on the width \(d\). For this purpose, the Nb pattern is defined, using electron beam
lithography (EBL), as a narrow strip, either \( d = 100 \) or \( 200 \) nm, with probes at either side at distances of 200 and 300 nm, see Fig. 2a. Inspection with the electron microscope shows \( d = 100 \) and 236 nm. Prior to the Nb deposition, the InAs surface is cleaned using low energy Ar-sputtering. This can reduce the thickness of the quantum well by a maximum of 2 nm, and might alter the carrier density \( n_s \), and mobility \( \mu_e \). To define the width of the InAs channel, a mesa-etch is performed in the 200 nm strip sample, \( W = 0.9 \) \( \mu \)m. This was not done for the sample with a 100 nm strip, which means that in this sample the junctions to the SQW have a width equal to the length of the strip, 3.5 \( \mu \)m. Evidently some parallel conductance will be present.

Prior to the measurements we checked the continuity of the strips, together with the critical temperature \( T_c \), by measuring the resistance through contacts 3 and 4 (Fig. 2a). Measurements are done using a standard 4-point lock-in technique, at 1.3 K. By applying an ac modulation current on top of a dc-bias, we can measure the energy dependence of transport in the SQW. Transport through the SQW can be modelled in the spirit of the Landauer-Büttiker formalism, using normal- and Andreev reflection probabilities, \( R_{e\rightarrow e} \) and \( R_{e\rightarrow h} \), and transmission probabilities \( T_{e\rightarrow e} \) and \( T_{e\rightarrow h} \), see Fig 2b. Conservation of particles requires \( 1 = R_{e\rightarrow e} + R_{e\rightarrow h} + T_{e\rightarrow e} + T_{e\rightarrow h} \). By expressing the current in terms of these reflection and transmission probabilities, we can translate the region underneath the strip into a schematic resistor network, shown in Fig. 2c, where

\[
R = \frac{1}{G_S 2(R_{e\rightarrow h} + T_{e\rightarrow e})}, \quad (4a)
\]

\[
R_C = \frac{1}{G_S} \frac{(T_{e\rightarrow e} - T_{e\rightarrow h})}{4(R_{e\rightarrow h} + T_{e\rightarrow h})(R_{e\rightarrow h} + T_{e\rightarrow e})}. \quad (4b)
\]

Here \( G_S = \frac{2e^2}{h} \frac{W}{2\lambda_F} \) is the Sharvin conductance. Although these resistors do not have physical relevance, they are useful for calculating the the various measurable quantities:

\[
\frac{\partial V_1}{\partial I_1} \approx \frac{\partial V_2}{\partial I_2} \approx \frac{R_{||}}{R_{||} + R + R_C} (R + R_C) = \frac{R_{||}}{R_{||} + R + R_C} \times \frac{1}{G_S} \frac{2(R_{e\rightarrow h} + T_{e\rightarrow h}) + (T_{e\rightarrow e} - T_{e\rightarrow h})}{4(R_{e\rightarrow h} + T_{e\rightarrow h})(R_{e\rightarrow h} + T_{e\rightarrow e})}, \quad (5a)
\]
\[
\frac{\partial V_2}{\partial I_1} \approx \frac{R_\parallel}{R_\parallel + R + R_C} - \frac{R_C}{R_\parallel + R + R_C}
\]
\[
= \frac{R_\parallel}{R_\parallel + R + R_C}
\times \frac{(T_{e\rightarrow e} - T_{e\rightarrow h})}{2(R_{e\rightarrow h} + T_{e\rightarrow h}) + (T_{e\rightarrow e} - T_{e\rightarrow h})},
\] (5b)

where \(R_\parallel\) accounts for any parallel conductance that might be present. For small \((T_{e\rightarrow e} - T_{e\rightarrow h})\), the angular distribution of incoming electrons is taken into account by calculating the following integral:

\[
T_{e\rightarrow e} - T_{e\rightarrow h} = \frac{|\Psi(x = d)|^2}{|\Psi(x = 0)|^2}
\]
\[
= \frac{1}{2} \int_{-\pi/2}^{\pi/2} \cos(\alpha) \exp \left( -2d \xi_{Sm} \cos(\alpha) \right) d\alpha.
\] (6)

In Fig. 3 the experimental data are shown. For the wide strip sample, \(d = 236\) nm, we measure junction resistances of \(\partial V_1/\partial I_1|_{V_1=0} = 425\) \(\Omega\) and \(\partial V_2/\partial I_2|_{V_2=0} = 625\) \(\Omega\), whereas from the Sharvin conductance, with \(W = 0.9\) \(\mu m\), we would expect \(1/G_S \approx 180\) \(\Omega\), which is approximately 3 smaller. This can be explained by elastic scattering, present in the samples. In the limit \(T_{e\rightarrow e}, T_{e\rightarrow h} \ll 1\), which we assume to be the case, we can write \(\partial V_1/\partial I_1 \approx \frac{1}{G_S} \frac{1}{2R_{e\rightarrow h}}\), Eq. (5a). Using \(\partial V_1(2)/\partial I_1(2) \approx 525\) \(\Omega\) we get \(R_{e\rightarrow h} \approx 0.17\), or \(R_{e\rightarrow e} \approx (1 - R_{e\rightarrow h}) \approx 0.83\). This value for \(R_{e\rightarrow h}\) is used in further analysis. For the narrow strip sample, \(d = 100\) nm, the junction width is \(W = 3.5\) \(\mu m\). By scaling \(\partial V_1(2)/\partial I_1(2)\) from the wide strip sample, we expect \(\partial V_1(2)/\partial I_1(2) \approx 135\) \(\Omega\). The measured values are \(\partial V_1/\partial I_1|_{V_1=0} = 75\) \(\Omega\) and \(\partial V_2/\partial I_2|_{V_2=0} = 68\) \(\Omega\). These lower values are explained by a parallel resistance of \(R_\parallel \approx 150\) \(\Omega\), which is in agreement with expectation on geometrical grounds. We will first focus on the zero voltage bias transfer signal \(\partial V_2/\partial V_1\). From the wide strip, \(d = 236\) nm, we obtain \(\partial V_2/\partial V_1|_{V_1=0} = \partial V_1/\partial V_2|_{V_2=0} \approx 0.01\) which, with the aid of Eq. (5b) leads to \((T_{e\rightarrow e} - T_{e\rightarrow h}) \approx 0.0035\). From Eq. (6) we can then calculate \(\xi_{Sm} \approx 100\) nm. Using again Eq. (6) and (5b) for the narrow strip, \(d = 100\) nm, we expect \(\partial V_2/\partial V_1 \approx 0.097\), which is in reasonable agreement with the observed values \(\partial V_2/\partial V_1|_{V_1=0} \approx 0.04\) and \(\partial V_1/\partial V_2|_{V_2=0} \approx 0.05\). The discrepancy is mainly due to the fact that we assume \(R_{e\rightarrow h}\) to be equal for both samples, whereas in general it will depend on \(d\). Using \(\xi_{Sm} \approx 100\) nm we can estimate from Fig. 3b that the transparency \(T_{\text{Sin}} = 0.7\) for the Nb-InAs interface.
At finite energy, $\xi_{Sm} \propto \text{Im}(k_x)^{-1}$ will increase with $E$, until $E \geq \Delta_{\text{eff}}$, where the decay length will diverge, for $E > \Delta_{\text{eff}}$ there will be propagating states. Therefore the transfer signal $\partial V_2/\partial V_1(V_1)$ is expected to increase with increasing bias voltage $V_1$. In both samples we observe a dip at low voltage bias, with a width of approximately 0.8 mV. From the estimated interface transparency $T_{\text{SIN}} = 0.7$ we infer an induced energy gap $\Delta_{\text{eff}} = 0.97\Delta_0 = 0.83$ meV, where $\Delta_0 = 0.86$ meV is the superconducting energy gap of the strip, which is very close to width of the observed dip. This dip is however only observed when the current is injected from one probe, when the current is injected from the opposite probe, there is no dip present in the transfer signal, see Fig. 3. This indicates that we can not understand the voltage dependence within the presented model. When the elastic scattering length is small, $\ell \leq \xi_{Sm}$, we do not measure $\xi_{Sm}$, but $\xi_{\text{eff}} = \sqrt{\xi_{Sm}\ell}$. This will lead to a somewhat larger value for $\xi_{Sm}$. In the InAs used $\ell$ is reduced due to the Ar-sputtering, and we expect that this will influence especially the voltage dependent signal. Furthermore we see a small dip at $V = 2$ mV $\approx (\Delta_{0,\text{probe}} + \Delta_{0,\text{strip}})/e$. This is not expected from our model, but is probably related to the fact that we do not inject electrons from a normal reservoir, but use a superconducting injector instead.

Nguyen et al. obtained an Andreev transfer length $x_A = 1.5 \mu m$, which is equivalent to a decay length for the wave functions of 3 $\mu m$. Comparing this value to the decay length obtained from our experiment, $\xi_{Sm} \approx 100 \text{ nm}$, we see a huge difference. Nguyen et al. obtain their $x_A$ by extrapolating resistances of long Nb-InAs-Nb junctions (20 to 200 $\mu m$) to lower junction lengths. This is however not allowed, since a junction of zero length still has a finite resistance, of the order of the Sharvin resistance.

In conclusion, we have investigated lateral transport in a Nb-InAs-AlSb superconducting quantum well. At zero voltage bias we can describe the transport properties in terms of transmission and reflection probabilities. For quasi-particles in the SQW, a decay length of $\xi_{Sm} \approx 100 \text{ nm}$ is inferred from the experiments. This $\xi_{Sm}$ corresponds to an interface transparency between the Nb and InAs of $T_{\text{SIN}} = 0.7$, which, according to calculations, results in an induced energy gap in the excitation spectrum of the Nb-InAs SQW of $\Delta_{\text{eff}} = 0.97\Delta_0$. 
From the experiment there are some indications for the presence of this gap, although the exact voltage dependence of the transfer signal can not be understood within the presented model. We believe that the transfer signal at finite bias is greatly influenced by elastic scattering.

This work was supported by the Dutch Science Foundation NWO/FOM. B. J. van Wees acknowledges support from the Royal Dutch academy of Sciences (KNAW).
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11 Due to the highly non-uniform density of states in a superconductor, the energy distribution of electrons injected from a superconductor will in general be non-linear in voltage bias.

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FIGURES

FIG. 1. The normalized induced energy gap $\Delta_{\text{eff}}/\Delta_0$ (Eq.(1), $\min E(k)$) as function of the barrier transmittance $T_{\text{SIN}} = 1/(1 + Z^2)$ of the Nb-InAs interface (a), the dashed line shows the linear dependence for $T_{\text{SIN}} \ll 1$. Figure (b) shows the calculated decay length, $\xi_{\text{Sm}} = |k_F/(\text{Re}(k_z)\text{Im}(k_z))|$ (Eq.(1), $E = 0$). The quasi-particle decay length $\xi = \hbar k_F/m^*\Delta_{\text{eff}}$ analogous to that of a superconductor is given for comparison (dashed curve). For these curves we used $k_F = 0.26$ nm$^{-1}$, and $\Delta_0 = 1.0$ meV ($T_c \sim 6$ K).

FIG. 2. Schematic picture of the sample under study, a narrow strip, width $d$, with two probes on either side (a). The distances of the probes to the strip are 200 (1) and 300 nm (2). The width of the InAs channel is defined by a mesa-etch. In the absence of a mesa-etch, the length of the strip determines the junction width. For electrons underneath the narrow strip we can define normal- and Andreev-like reflection and transmission coefficients $R_{e\rightarrow e,(e\rightarrow h)}$ and $T_{e\rightarrow e,(e\rightarrow h)}$ (b). This region can be represented by a simple network of resistors $R$ and $R_C$ (c). In the absence of a mesa-etch, some parallel conductance, characterized by $R_\parallel$, is present.

FIG. 3. Differential resistance, $\partial V_1/\partial I_1$ and $\partial V_2/\partial I_2$, of the junctions formed by probe 1 and the strip, and probe 2 and the strip, as function of the bias, and the transmitted signals $\partial V_2/\partial I_1$, and $\partial V_1/\partial I_2$, for a strip with $d = 100$ nm and a strip with $d = 236$ nm. All curves are measured at $T = 1.3$ K.
InAs

AlSb

| R | R |
|---|---|
| 1 | 2 |

\( R_{e-e} \) \( R_{e-h} \)

\( R_{e-h} \) \( T_{e-h} \)

\( R_{c} \)

\( R_{\parallel} \)

\( R_{\parallel} \)
