Spectral running and non-Gaussianity from slow-roll inflation in generalized two-field models

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Abstract. Theories beyond the standard model such as string theory motivate low energy effective field theories with several scalar fields which are not only coupled through a potential but also through their kinetic terms. For such theories we derive the general formulae for the running of the spectral indices for the adiabatic, isocurvature and correlation spectra in the case of two-field inflation. We also compute the expected non-Gaussianity in such models for specific forms of the potentials. We find that the coupling has little impact on the level of non-Gaussianity during inflation.

Keywords: cosmological perturbation theory, inflation, physics of the early universe
1. Introduction

String theory and theories with supersymmetry usually contain many scalar fields which can play an important role in the early Universe. For example, in string theory they often describe the dynamics of extra spatial dimensions and other degrees of freedom living in higher dimensions. These scalar degrees of freedom couple to matter fields propagating in the three large dimensions we perceive, see for example [1]–[3]. If these extra dimensions exist, they will have had an influence on the evolution of the universe at some point. It is usually thought that in particular the dynamics of the very early universe is affected by the existence of extra dimensions. As such, they will alter the predictions of inflationary cosmology. If apart from the inflaton field(s) other scalar fields are present, they will generally alter the evolution of the field(s) driving inflation and affect the production of cosmological perturbations.

Generally speaking, the existence of multiple fields during the inflationary epoch modifies some of the single-field predictions. For example, apart from the usual adiabatic perturbations produced during inflation, there could be isocurvature (entropy) perturbations produced, whose existence is constrained by observations (see, for example [4]–[13] and references therein).
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Related to this, in single-field inflation, the curvature perturbation on constant energy density hypersurfaces, \( \zeta \), is constant on super-horizon scales and can be evaluated at the horizon crossing. However, in the presence of multiple fields during the inflationary epoch, \( \zeta \) does not remain constant and varies since the non-vanishing isocurvature perturbations act as a source term for the change of \( \zeta \) [5, 6].

In order to distinguish between inflationary models, cosmologists need to extract as much information as possible from the data. Usually the spectral index and the scalar–tensor ratio are used to distinguish between inflationary models. With the advent of precision data (in particular the data obtained by WMAP [14]), the running of the spectral index can be added as an additional quantity in order to distinguish between the models. One can expect considerable improvement with future data coming from the PLANCK satellite and the mapping of the large scale structures in the universe, as well as a better understanding of small scale clustering as obtained from the Ly\( \alpha \) forest.

Another potential observable which could distinguish between inflationary models is the amount of non-Gaussianity generated during inflation (see [15] and references therein). Although it is small for many inflationary models (e.g. [16]–[18]), there are examples in which perturbations show a considerable amount of non-Gaussianity [19]–[26].

In this paper we study the slow-roll regime of generalized two-field inflation, where the scalar fields are also coupled through kinetic terms [5, 6, 9]. In general, a multi-field action can be given by

\[
S = \int d^4x \sqrt{-g} \left( M_P^2 R - \frac{1}{2} G_{I,J} \partial_\mu \phi^I \partial^\mu \phi^J - W(\phi) \right). \tag{1.1}
\]

For simplicity we consider a system of two scalar fields, whose dynamics are governed by the following action:

\[
S = \int d^4x \sqrt{-g} \left( M_P^2 R - \frac{1}{2} g^{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} e^{2b(\varphi)} g^{\mu \nu} \partial_\mu \chi \partial_\nu \chi - W(\varphi, \chi) \right). \tag{1.2}
\]

In this expression \( M_P = 1/\sqrt{8\pi G} \) is the reduced Planck mass. Note that, in this case, \( G_{I,J} \) is symmetric.

We obtain the general formulae for the running of the spectral indices (adiabatic, non-adiabatic and correlated), following [27] and [28]. The running is affected by the presence of the coupling and therefore future experiments constraining the running of the spectral index will give vital information about the existence of fields which couple non-trivially to the field(s) driving inflation. Furthermore, we derive the general formulae for non-Gaussianity and apply them to some specific models.

The paper is organized as follows. In the next section we summarize the equations governing the background and perturbation evolution. In section 3 we derive the general expressions for the running of the spectral indices (for the adiabatic and isocurvature power spectra as well as the correlation spectrum). In section 4 we derive the expected non-Gaussianity for two cases. In sections 5 and 6 we then apply our results to specific examples. Numerical results are presented in section 7. Our conclusions can be found in section 8. Useful formulae are collected in the appendices.
2. Background and perturbation equations

In this section we set our notation and briefly review some results found in previous work. It will closely follow [27] and [28]. The equations of motion for the two scalar fields in a Friedmann–Robertson–Walker spacetime follow from the action (1.2) and are given by

\[
\ddot{\varphi} + 3H \dot{\varphi} + W_\varphi = b_\varphi e^{2b} \dot{\chi}^2,
\]

\[
\ddot{\chi} + (3H + 2b_\varphi \dot{\varphi}) \dot{\chi} + e^{-2b} W_\chi = 0,
\]

(2.1)

where \( b_\varphi = \partial b(\varphi)/\partial \varphi, \ W_\varphi = \partial W(\varphi, \chi)/\partial \varphi, \) etc. Einstein’s equations lead to

\[
H^2 = \frac{1}{3M_p^2} \left[ \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} e^{2b} \dot{\chi}^2 + W \right],
\]

\[
\dot{H} = \frac{1}{2M_p^2} \left[ \dot{\varphi}^2 + e^{2b} \dot{\chi}^2 \right] = -\frac{\dot{\sigma}^2}{2M_p^2}.
\]

(2.2)

We define e-folding number, \( N \), as

\[
N(t_e, t_s) \equiv \int_{t_s}^{t_e} H \, dt.
\]

(2.3)

It is useful to separate the perturbations into components of adiabatic and isocurvature modes. Following [27, 29], we define the average (adiabatic) and orthogonal (entropy) fields \( \sigma \) and \( s \) as

\[
ds = \sigma \cos \theta \, d\varphi + \sin \theta e^b \, d\chi,
\]

\[
ds = e^b \cos \theta \, d\chi - \sin \theta \, d\varphi,
\]

(2.4)

with

\[
\cos \theta = \frac{\dot{\varphi}}{\sqrt{\dot{\varphi}^2 + e^{2b} \dot{\chi}^2}}, \quad \sin \theta = \frac{e^b \dot{\chi}}{\sqrt{\dot{\varphi}^2 + e^{2b} \dot{\chi}^2}}.
\]

(2.5)

These fields satisfy the equations of motion

\[
\ddot{\sigma} + 3H \dot{\sigma} + W_\sigma = 0,
\]

\[
\dot{\theta} = -\frac{W_s}{\sigma} - b_\varphi \dot{\sigma} \sin \theta,
\]

(2.6)

where

\[
W_\sigma = W_\varphi \cos \theta + e^{-b} W_\chi \sin \theta,
\]

\[
W_s = -W_\varphi \sin \theta + e^{-b} W_\chi \cos \theta.
\]

(2.7)

We now turn to cosmological perturbations. We work in the longitudinal gauge, in which the perturbed metric has the form

\[
ds^2 = -(1 + 2\Phi) \, dt^2 + a^2(1 - 2\Phi) \, dx^2,
\]

(2.8)

where \( a = a(t) \) is the scale factor and \( \Phi = \Phi(t, x) \) is the metric perturbation. We now calculate the perturbations in the adiabatic and entropy fields. Instead of working with the field perturbation \( \delta \sigma \), it is more convenient to work with the Sasaki–Mukhanov variable...
\[ Q_\sigma = \delta\sigma + (\dot{\sigma}/H)\Phi. \] The equation of motion for \( Q_\sigma \) is given by
\[
\ddot{Q}_\sigma + 3H\dot{Q}_\sigma + \left[ \frac{k^2}{a^2} + W_{\sigma\sigma} + \dot{\theta} \frac{W_s}{\dot{\sigma}} - \frac{1}{M_p^2a^3} \left( \frac{a^3\dot{\sigma}^2}{H} \right) - b_{\varphi}\dot{\varphi}\frac{W_\chi e^{-b}}{\dot{\sigma}} \sin \theta \right] Q_\sigma \\
= -2 \left( \frac{W_s}{\dot{\sigma}} \delta s \right) + 2 \left( \frac{W_s}{\dot{H}} + \frac{\dot{H}}{H} \right) \frac{W_s}{\dot{\sigma}} \delta s,
\]
whereas for \( \delta s \) it is given by
\[
\ddot{\delta s} + 3H\dot{\delta s} + \left[ \frac{k^2}{a^2} + W_{ss} + 3\dot{\theta}^2 - b_{\varphi}\dot{\varphi}\dot{\sigma}^2 + b_\varphi^2g(t) + b_{\varphi}f(t) \right] \delta s = -\frac{k^2}{a^2} \frac{\Phi}{2\pi G} \frac{W_s}{\dot{\sigma}^2},
\]
with
\[
g(t) = -\dot{\sigma}^2(1 + 3\sin^2 \theta), \\
f(t) = W_\varphi(1 + \sin^2 \theta) - 4W_s \sin \theta. \tag{2.11}
\]
We have taken the notation of [28] and use
\[
W_{\sigma\sigma} = W_{\varphi\varphi} \cos^2 \theta + W_{\chi\chi} \sin 2\theta e^{-b} + V_{\chi\chi} \sin^2 \theta e^{-2b}, \tag{2.12}
\]
\[
W_{ss} = W_{\varphi\varphi} \sin^2 \theta - W_{\chi\chi} \sin 2\theta e^{-b} + V_{\chi\chi} \cos^2 \theta e^{-2b}. \tag{2.13}
\]
We define the curvature and isocurvature fluctuations as
\[
\zeta = \frac{H}{\dot{\sigma}} Q_\sigma, \quad S = \frac{H}{\dot{\sigma}} \delta s, \tag{2.14}
\]
then the time derivative of \( \zeta \) is related to \( S \) by
\[
\dot{\zeta} = \frac{H}{\dot{\sigma}} k^2 \frac{\Phi}{a^2} - 2 \frac{W_s}{\dot{\sigma}} S. \tag{2.15}
\]
During inflation, the fields are assumed to follow the slow-roll limit,
\[
\dot{\varphi} = \dot{\sigma} \cos \theta \simeq -\frac{W_\varphi}{3H}, \quad \dot{\chi} = \dot{\sigma} \sin \theta e^{-b} \simeq -\frac{W_\chi}{3H} e^{-2b}, \tag{2.16}
\]
\[
H^2(\varphi, \chi) \simeq \frac{1}{3M_p^2} W(\varphi, \chi). \tag{2.17}
\]
The slow-roll parameters are defined in appendix A for convenience.

In the slow-roll limit and on large scales, we find the evolutions of curvature and isocurvature perturbation can be written in terms of slow-roll parameters,
\[
\dot{\zeta} \simeq BHS, \quad \dot{S} \simeq -\gamma HS, \tag{2.18}
\]
where
\[
B = -2\eta_{ss} - \text{sign}(b_{\varphi}) \text{sign}(\frac{W_\chi}{W}) \sqrt{\epsilon_b\epsilon_\chi \sin^2 \theta},
\]
\[
\gamma = -\eta_{ss} + 2\epsilon + \eta_{ss} + \frac{1}{2} \text{sign}(b_{\varphi}) \text{sign}(\frac{W_\chi}{W}) \sqrt{\epsilon_b\epsilon_\chi} \sin \theta \cos \theta \\
+ \frac{1}{2} \text{sign}(b_{\varphi}) \text{sign}(\frac{W_\varphi}{W}) \sqrt{\epsilon_b\epsilon_\varphi}(1 + \sin^2 \theta). \tag{2.19}
\]
Generally speaking on large scales the evolutions of adiabatic and isocurvature fluctuations follow the following set of equations
\[
\dot{\zeta} = \alpha(t)H(t)S, \\
\dot{S} = \delta(t)H(t)S. 
\]  
We use the formalism of transfer matrix [4],
\[
\begin{pmatrix} \zeta(t) \\ S(t) \end{pmatrix} = \begin{pmatrix} 1 & T_{\zeta S} \\ 0 & T_{SS} \end{pmatrix} \begin{pmatrix} \zeta(t_*) \\ S(t_*) \end{pmatrix},
\]
where
\[
T_{SS}(t_*, t) = \exp \left( \int_{t_*}^{t} \delta(t')H(t')dt' \right),
\]
\[
T_{\zeta S}(t_*, t) = \int_{t_*}^{t} \alpha(t')H(t')T_{SS}(t_*, t')dt'.
\]
Then the power spectra are
\[
P_\zeta = (1 + T_{\zeta S}^2)P_\zeta|_\ast = P_\zeta|_\ast(1 + \cot^2 \Delta),
\]
\[
P_S = T_{SS}^2P_\zeta|_\ast,
\]
\[
P_C = T_{\zeta S}T_{SS}P_\zeta|_\ast,
\]
where the cross-correlation angle \(\Delta\) is
\[
\cos \Delta = \frac{P_C}{\sqrt{P_\zeta P_S}}.
\]
We note that \(T_{\zeta S} = \cot \Delta\) and \(\Delta\) has range \(0 \leq \Delta \leq \pi\) giving positive and negative correlation depending on the sign of \(\cos \Delta\). The spectral indices are defined as
\[
n_X - 1 \equiv \frac{d\ln P_X}{d\ln k}, \quad X = \zeta, S, C.
\]
The spectral indices are best written in terms of slow-roll parameters. They have been calculated in [28] and are given by
\[
n_\zeta - 1 = -6\epsilon + 4\epsilon(\cos \Delta)^2 + 2\eta_{\sigma\sigma}(\sin \Delta)^2 + 4\eta_{ss} \sin \Delta \cos \Delta + 2\eta_{ss}(\cos \Delta)^2
\]
\[+ 2 \text{sign}(b_\varphi) \text{sign} \left( \frac{W_\chi}{W} \right) \sqrt{\epsilon_b \epsilon_\chi} (\sin \theta)^2 \sin \Delta \cos \Delta
\]
\[+ \text{sign}(b_\varphi) \text{sign} \left( \frac{W_\varphi}{W} \right) \sqrt{\epsilon_b \epsilon_\varphi} (1 + \sin^2 \theta) \cos^2 \Delta
\]
\[- \text{sign}(b_\varphi) \text{sign} \left( \frac{W_\chi}{W} \right) \sqrt{\epsilon_b \epsilon_\chi} \sin \theta \cos \theta \sin^2 \Delta,
\]
\[
n_S - 1 = -2\epsilon + 2\eta_{ss} + \text{sign}(b_\varphi) \text{sign} \left( \frac{W_\varphi}{W} \right) \sqrt{\epsilon_b \epsilon_\varphi}(1 + \sin^2 \theta)
\]
\[
\text{and}
\]
\[
n_C - 1 = n_S - 1 - B \tan \Delta.
\]
In these expressions, all the slow-roll parameters are evaluated at horizon crossing.
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Table 1. Full set of model parameters to second order, listed by physical fields \((\phi, \chi)\) or component fields \((\sigma, s)\). \(H, T_{SS}\) and \(\xi_{sss}^2\) are not required for the spectral running and can be disregarded.

| \(\phi, \chi\) | \(\sigma, s\) | \(H\) | \(T_{SS}\) | \(\Delta\) | \(\epsilon_\phi\) | \(\epsilon_\chi\) | \(\eta_{\phi\phi}\) | \(\eta_{\phi\chi}\) | \(\eta_{XX}\) | \(\epsilon_b\) | \(\xi_{\phi\phi}\) | \(\xi_{\chi\chi}\) | \(\xi_{\phi\chi}^2\) | \(\xi_{SS}^2\) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

3. Running of the spectral indices

To calculate the running of the spectral indices we need to know the second-order slow-roll parameters and the derivative of first-order slow-roll parameters and the transfer functions. For convenience, we list them extensively in appendix A.

When \(b_\phi \neq 0\) we have 14 parameters at second order in general, as listed in table 1. However, considering the component fields \((\sigma, s)\), \(H, T_{SS}\) and \(\xi_{sss}^2\) do not appear in the running of spectral indices, which reduces the parameters to 11. Therefore the running spectral indices are given by: \(\theta, \Delta\) and 9 slow-roll parameters. We define the running spectral indices \(\alpha_X\) as

\[
\alpha_X \equiv \frac{dn_X}{d\ln k}, \quad X = \zeta, S, C. \quad (3.1)
\]

To leading order in slow roll, the runnings of the spectral indices after inflation are:

\[
\alpha_\zeta \equiv \left( \frac{dn_\zeta}{d\ln k} \right)_{b=0} + \frac{1}{16} \left[ 5 - 3 \cos 2\theta + 8(1 - \cos 2\theta)(\sin 2\theta \sin 2\Delta + \cos 2\theta \cos 2\Delta) \\
- 2 \sin 2\theta(2 - 3 \cos 2\theta + \cos^2 2\theta) \sin 4\Delta \\
- \{3 + \cos 2\theta(3 - 6 \cos 2\theta + 2 \cos^2 2\theta)\} \cos 4\Delta \epsilon_{\phi}^\epsilon \epsilon \\
- \frac{1}{4} [1 + \cos 2\theta + 2 \cos 2\Delta + (1 - \cos 2\theta)(\sin 2\theta \sin 2\Delta + \cos 2\theta \cos 2\Delta)] \xi_{\phi} \epsilon \\
+ [2 \cos^3 \theta + \cos \theta(3 - 2 \cos 2\theta) \cos 2\Delta + \cos \theta(-2 + 2 \cos 2\theta) \cos 4\Delta \\
+ 2 \sin^3 \theta(2 \sin 2\Delta - \sin 4\Delta)] \sign(b_\phi) \sign \left( \frac{V_\sigma}{V} \right) \sqrt{\epsilon_{\phi}^\epsilon \epsilon} \\
+ \left\{ - \cos \theta + \sin \theta(1 - \cos 2\theta)(-\sin 2\Delta + \frac{1}{2} \sin 4\Delta) \right\} \eta_{\sigma\sigma} \\
+ \frac{1}{2} \cos \theta(-3 + 2 \cos 2\theta) \cos 2\Delta + \frac{1}{2} \cos \theta(2 - \cos 2\theta) \cos 4\Delta \eta_{\sigma s} \\
+ \left\{ \frac{3}{2} \sin \theta + \frac{1}{2} \cos \theta(3 + \cos 2\theta) \sin 2\Delta - \cos \theta(2 - \cos 2\theta) \sin 4\Delta \\
- \frac{1}{2} \sin \theta(6 + \cos 2\theta) \cos 2\Delta + \sin \theta(1 - \cos 2\theta) \cos 4\Delta \right\} \eta_{s s} \\
+ \left\{ \frac{3}{2} \cos \theta + \frac{1}{2} \sin \theta(3 + \cos 2\theta) \sin 2\Delta - \sin^3 \theta \sin 4\Delta + \cos^3 \theta \cos 2\Delta \\
+ \frac{1}{2} \cos \theta(-2 + \cos 2\theta) \cos 4\Delta \right\} \eta_{s s} \right\} \sign(b_\phi) \sign \left( \frac{V_\sigma}{V} \right) \sqrt{\epsilon_{\phi}^\epsilon \epsilon}.
\]

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\[ \begin{align*}
\alpha_S \equiv \frac{d\alpha_S}{d\ln k} &= \left( \frac{d\alpha_S}{d\ln k} \right)_{b=0} + \sin^2 \theta \cos^4 \theta \epsilon_b - \frac{1}{2} (1 + \sin^2 \theta) \cos^2 \theta \xi_b \\
&\quad + \left[ 2 \cos \theta - (1 + \sin^2 \theta) \cos \theta \eta_{ss} + \sin \theta (-1 + 3 \sin^2 \theta) \eta_{\alpha} + 2 \cos^3 \theta \eta_{ss} \right] \\
&\quad \times \text{sign}(\epsilon_b) \text{sign} \left( \frac{V_{\sigma}}{V} \right) \sqrt{\epsilon_b},
\end{align*} \]

\[ \alpha_C \equiv \frac{dn_C}{d\ln k} = \frac{dn_C}{d\ln k} + \tan \Delta \left[ 4 \eta_{\alpha} (\eta_{\sigma} - \eta_{ss}) - 2 \xi_{\sigma \sigma} + \frac{1}{2} \sin^2 \theta \sin 2 \theta \cos 2 \theta \epsilon_b \right] \\
&\quad - \frac{1}{2} \sin^3 \theta \cos \theta \xi_b + \{ \sin \theta (3 \cos^2 \theta - 1) \eta_{ss} - 5 \sin^2 \theta \cos \theta \eta_{\alpha} \} \\
&\quad \times \text{sign}(\epsilon_b) \text{sign} \left( \frac{V_{\sigma}}{V} \right) \sqrt{\epsilon_b} + \tan^2 \Delta [-4 \eta_{\alpha}^2 - 4 \eta_{\sigma} \sqrt{\epsilon_b} \sin^3 \theta - \sin^6 \theta \epsilon_b],
\]

where
\[ \left( \frac{dn_C}{d\ln k} \right)_{b=0} = 2 (-7 + 4 \cos 2 \Delta - \cos 4 \Delta) \epsilon^2 + 2 (4 - 3 \cos 2 \Delta + \cos 4 \Delta) \epsilon \eta_{\sigma} \]
\[ + (2 \sin 2 \Delta - \sin 4 \Delta) \epsilon \eta_{\alpha} + 2 (2 + \cos 2 \Delta - \cos 4 \Delta) \eta_{ss} + \sin^2 \Delta (\eta_{\sigma} \eta_{\alpha} - \eta_{\sigma} \eta_{ss}) + 2 \cos^2 \Delta \eta_{\sigma} \eta_{ss} - (1 - \cos 4 \Delta) \eta_{\sigma} \eta_{ss} \\
+ 2 \sin 4 \Delta (\eta_{\sigma} \eta_{\alpha} - \eta_{\sigma} \eta_{ss}) - (1 - \cos 2 \Delta) \xi_{\sigma \sigma} \\
- 2 \sin 2 \Delta \xi_{\sigma \sigma} - (1 + \cos 2 \Delta) \xi_{\sigma \sigma}^2.
\]

\[ \left( \frac{dn_C}{d\ln k} \right)_{b=0} = -8 \epsilon^2 + 4 \epsilon (\eta_{\alpha} + \eta_{ss}) + 4 \eta_{\alpha}^2 - 2 \xi_{\sigma \sigma}^2.
\]

All slow-roll parameters are evaluated at horizon crossing. We note that the running spectral index of isocurvature perturbation, \( \alpha_S \), is independent of \( \Delta \), which means that the \( \alpha_S \) is determined at horizon crossing and does not change thereafter.

4. **Non-Gaussianity during inflation**

Another potential discriminator between inflationary models is the level of non-Gaussianity produced during inflation. Current observations limit the non-linearity parameter, \( |f_{NL}| < 100 \) [30]. A perfect CMB experiment cannot hope to detect \( |f_{NL}| < 3 \) [31]. Using the \( \delta N \) formalism [32]–[36], the non-linearity parameter is given by [37]
\[ -6 \frac{f_{NL}}{5} = \frac{A_{\delta N}^I K N_{\delta N} N_{\delta N}}{(N_{\delta N} G^{I})^2} + 4 \frac{G^{I M} G^{K N} N_{\delta N} N_{\delta N}}{(N_{\delta N} G^{I})^2}, \]

which is valid in the slow-roll limit. The first term, which we call \( -6 \frac{f_{NL}^{(3)}}{5} \), can be written as [38],
\[ -6 \frac{f_{NL}^{(3)}}{5} = \frac{r}{16} (1 + f), \]

where \( r \equiv \mathcal{P}_T / \mathcal{P}_s \) is tensor-to-scalar ratio and \( f \) is a function of the momentum triangle with the range of values \( 0 \leq f \leq \frac{4}{3} \) [16]. In appendix C, we have shown equation (4.2) is valid for non-canonical kinetic terms. From [28], the tensor-to-scalar ratio is calculated
to be $r \lesssim 16\epsilon$ and therefore we can approximate
\begin{equation}
\left| \frac{a}{3} f^{(3)}_{NL} \right| \lesssim \epsilon
\end{equation}
which is certainly too small to be observable in CMB experiments. Also see [39]. Due to this, we will concentrate on the second term, $f^{(4)}_{NL}$, only in the next sections and consider two cases: separable potentials of product and sum.

4.1. Product potential $W(\varphi, \chi) = U(\varphi)V(\chi)$

We first consider the case of a separable potential by product, for which we derive the analytic formula for the non-linear parameter $f_{NL}$. To use equation (4.1), we need to know the dependence of the number of e-foldings on the fields $\varphi$ and $\chi$. Following the calculations in [6], we can obtain the derivatives of $N$ by $\varphi_*$ and $\chi_*$.

First we find the number of e-foldings,
\begin{equation}
N(\varphi_*, \chi_*) = -\frac{1}{M_p^2} \int_{t_*}^\epsilon \frac{U}{U_\varphi} \, d\varphi = -\frac{1}{M_p^2} \int_{\chi_*}^\epsilon \frac{V}{V_\chi} e^{2b(\epsilon)} \, d\chi,
\end{equation}
where superscript (or subscript) $^e$ and $^*$ denotes the values evaluated at the end of inflation and horizon crossing respectively. From the equations of motion, in the slow-roll regime, it is possible to find a constant of motion along the trajectory, as in [6]:
\begin{equation}
C_1 = -\frac{1}{M_p^2} \int e^{-2b(\epsilon)} \frac{U}{U_\varphi} \, d\varphi + \frac{1}{M_p^2} \int V \, d\chi.
\end{equation}
Using this constant of motion, in the slow-roll regime we find the first derivatives of $N(t_*, t_*)$ with respect to the fields,
\begin{equation}
\frac{\partial N}{\partial \varphi_*} = \frac{\text{sign}(U/U_\varphi)}{M_p \sqrt{2\epsilon_\varphi}} \left[ 1 - \frac{\epsilon_\varphi}{\epsilon} e^{2b_\varphi - 2b_*} \right],
\end{equation}
\begin{equation}
\frac{\partial N}{\partial \chi_*} = \frac{\text{sign}(V/V_\chi)}{M_p \sqrt{2\epsilon_\chi}} \left[ \frac{\epsilon_\chi}{\epsilon} e^{2b_\chi - b_*} \right].
\end{equation}
From the first derivatives we can find the second derivatives,
\begin{equation}
M_p^2 \frac{\partial^2 N}{\partial \varphi_*^2} = 1 - \frac{\eta_{\varphi\varphi}}{2\epsilon_\varphi} + \frac{1}{2} \text{sign} \left( b_\varphi \right) \text{sign} \left( \frac{U_\varphi}{U} \right) \sqrt{\frac{\epsilon_\varphi^e}{\epsilon_\varphi^e} e^{2b_\varphi - 2b_*} - \frac{1}{\epsilon_\varphi} e^{4b_\varphi - 4b_*} A_P},
\end{equation}
\begin{equation}
M_p^2 \frac{\partial^2 N}{\partial \chi_*^2} = e^{2b_*} \left[ 1 - \frac{\eta_{\chi\chi}}{2\epsilon_\chi} \frac{\epsilon_\chi^e}{\epsilon_\chi^e} e^{2b_\chi - 2b_*} - \frac{1}{\epsilon_\chi} e^{4b_\chi - 4b_*} A_P \right],
\end{equation}
\begin{equation}
M_p^2 \frac{\partial^2 N}{\partial \varphi_* \partial \chi_*} = \text{sign} \left( \frac{U_\varphi}{U} \right) \text{sign} \left( \frac{V_\chi}{V} \right) \frac{1}{\sqrt{\epsilon_\varphi \epsilon_\chi}} e^{4b_* - 3\epsilon_*} A_P,
\end{equation}
where
\begin{equation}
\eta_{ss} = \frac{\epsilon_\varphi \epsilon_\chi}{\epsilon}
\end{equation}
\begin{equation}
A_P = -\frac{\epsilon_\varphi \epsilon_\chi}{\epsilon^2} \left[ \eta_{ss} - \frac{1}{2} \text{sign} \left( b_\varphi \right) \text{sign} \left( \frac{U_\varphi}{U} \right) \frac{(\epsilon_\chi^e)^2}{\epsilon_\chi^e} \sqrt{\frac{\epsilon_\varphi^e}{\epsilon_\varphi^e} - 4 \frac{\epsilon_\varphi \epsilon_\chi}{\epsilon}} \right].
\end{equation}
We have followed the notation of [38] as much as possible, for reasons that become apparent in the next section.
From equations (4.6) and (4.7) we find the second term of equation (4.1),

\[-\frac{6}{5}f_{\text{NL}}^{(4)} = \frac{2e^{-2b_e + 2b_\chi}}{((u^2\alpha^2/e^\phi_\chi)+(v^2/e_{\chi}^*)^2)^2}\left[\frac{u^3\alpha^3}{e_{\phi}^*}\left(1 - \frac{\eta_{\phi\phi}^*}{2e_{\phi}^*}\right) + \frac{v^3}{e_{\chi}^*}\left(1 - \frac{\eta_{\chi\chi}^*}{2e_{\chi}^*}\right)\right] + \frac{1}{2}\text{sign}(b_\phi)\text{sign}\left(\frac{U_\phi}{U}\right)\frac{vu^2\alpha^2}{(e_{\phi}^*)^4}\sqrt{e_{\phi}^*e_{\chi}^*} - \left(\frac{u\alpha}{e_{\phi}^*} - \frac{v}{e_{\chi}^*}\right)^2e^{2b_e - 2b_\chi}\cdot A_F\right], \tag{4.10}
\]

with the definitions

\[u \equiv \frac{e_{\phi}^*}{e_{\phi}^*}, \quad v \equiv \frac{e_{\chi}^*}{e_{\chi}^*}, \quad \alpha \equiv e^{-2b_e + 2b_\chi}\left[1 + \frac{e_{\phi}^*}{e_{\phi}^*}(1 - e^{2b_e - 2b_\chi})\right]. \tag{4.11}\]

When \(b(\phi) = 0\), then \(\alpha = 1\) and \(e_\phi = 0\), which means that the symmetry between \(\phi\) and \(\chi\) in equation (4.10) is restored.

4.2. Sum potential \(W(\phi, \chi) = U(\phi) + V(\chi)\)

The second case we consider consists of separable potential models by sum. Similar cases with canonical kinetic terms were previously studied in [38], where it was shown that they do not generate significant non-Gaussianity. We will now derive the general formula for \(f_{\text{NL}}\) in the presence of non-canonical kinetic terms, closely following the derivation found in the above-mentioned paper.

The total number of e-foldings along a trajectory in slow-roll regime is given by

\[N(\varphi, \chi) = -\frac{1}{M_P^2}\int_{\varphi_*}^{\varphi} \frac{U}{U_\varphi} d\varphi - \frac{1}{M_P^2}\int_{\chi_*}^{\chi} e^{2b(\phi)} \frac{V}{V_\chi} d\chi. \tag{4.12}\]

We note that in principle \(\varphi\) in the integration of \(\chi\) can be rewritten in terms of \(\varphi_*, \chi_*\) and \(\chi\) along the trajectory using equations of motion. For sum potentials, \(\varphi\) and \(\chi\) have the relation along the trajectory through

\[\int_{\varphi_*}^{\varphi} e^{-2b(\phi)} \frac{1}{U_\varphi} d\varphi = \int_{\chi_*}^{\chi} \frac{1}{V_\chi} d\chi. \tag{4.13}\]

As before, the integral of motion along the trajectory leads to a constant of motion

\[C_2 \equiv -M_P^2\int e^{-2b(\phi)} \frac{d\varphi}{U_\varphi} + M_P^2\int \frac{d\chi}{V_\chi}. \tag{4.14}\]

This enables us to derive the first derivatives of \(N\)

\[M_P\frac{\partial N}{\partial \varphi_*} = \text{sign}\left(\frac{U_\varphi}{W_\varphi}\right)\frac{1}{\sqrt{2e_{\phi}^*W_*}}(U^* + Z^*) - G, \]

\[M_P\frac{\partial N}{\partial \chi_*} = \text{sign}\left(\frac{V_\chi}{W_\chi}\right)\frac{e^{b_\chi}}{\sqrt{2e_{\chi}^*W_*}}(V^* - Z^*) - H. \tag{4.15}\]
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using the definitions,

\[ Z^e = \frac{(V^e e^e - U^e e^e)}{e^e} e^{2b_v - 2b_s}, \tag{4.16} \]

\[ G(\varphi_s, \chi_s) \equiv \frac{1}{M_p} \int_s^e 2b_v e^{2b(\varphi)} V \frac{\partial \varphi}{\partial \varphi_s} d\chi; \tag{4.17} \]

\[ H(\varphi_s, \chi_s) \equiv \frac{1}{M_p} \int_s^e 2b_v e^{2b(\varphi)} V \frac{\partial \varphi}{\partial \chi_s} d\chi. \tag{4.18} \]

In the same way the second derivatives can be derived:

\[ M_p^2 \frac{\partial^2 N}{\partial \varphi_s^2} = 1 - \frac{\eta_{\varphi}^e U_s^e + Z^e}{2 \epsilon_{\chi}^e} + \text{sign} \left( \frac{U_{\varphi}}{W_s} \right) \frac{M_p^e \frac{\partial Z^e}{\partial \varphi_s}}{W_s \sqrt{2 \epsilon_{\chi}^e} \frac{\partial \varphi_s}{\partial \varphi_s}} - M_p \frac{\partial G}{\partial \varphi_s}; \]

\[ M_p^2 \frac{\partial^2 N}{\partial \chi_s^2} = e^{2b_s} \left[ 1 - \frac{\eta_{\chi}^e V_s^e - Z^e}{2 \epsilon_{\varepsilon}^e} - \text{sign} \left( \frac{V_{\chi}}{W_s} \right) \frac{M_pe^{-b_s} \frac{\partial Z^e}{\partial \chi_s}}{W_s \sqrt{2 \epsilon_{\varepsilon}^e} \frac{\partial \chi_s}{\partial \chi_s}} \right] - M_p \frac{\partial H}{\partial \chi_s}; \tag{4.19} \]

\[ M_p^2 \frac{\partial^2 N}{\partial \varphi_s \partial \chi_s} = e^{b_s} \left[ \frac{1}{2} \text{sign}(b_{\varphi}) \text{sign} \left( \frac{V_{\chi}}{W_s} \right) \sqrt{\frac{\epsilon_{\chi}^e V_s^e - Z^e}{W_s}} - \text{sign} \left( \frac{V_{\chi}}{W_s} \right) \frac{M_pe^{-b_s} \frac{\partial Z^e}{\partial \chi_s}}{W_s \sqrt{2 \epsilon_{\varepsilon}^e} \frac{\partial \varphi_s}{\partial \varphi_s}} \right] \]

\[ - M_p \frac{\partial H}{\partial \varphi_s}. \]

The order of the second derivatives, \( \partial^2 N/\partial \varphi_s \partial \chi_s = \partial^2 N/\partial \chi_s \partial \varphi_s \), and the last two equations emphasize that \( G \) and \( H \) are not independent quantities. In fact, they are related by

\[ M_p \frac{\partial G}{\partial \chi_s} = M_p \frac{\partial H}{\partial \varphi_s} - \text{sign}(b_{\varphi}) \text{sign} \left( \frac{V_{\chi}}{W_s} \right) \frac{e^{b_s} V_s^e}{2 W_s} \frac{\epsilon_{\varepsilon}^e}{\epsilon_{\varepsilon}^e}. \tag{4.20} \]

A similar expression relating \( \partial G/\partial \varphi_s \) to \( \partial H/\partial \varphi_s \) and \( \partial H/\partial \chi_s \) can be obtained, but it is irrelevant for our purposes and is therefore not given. From the definition of \( Z^e \) in equation (4.16), we can calculate

\[ \text{sign} \left( \frac{U_{\varphi}}{W_s} \right) \sqrt{\frac{\epsilon_{\varphi}^e \frac{\partial Z^e}{\partial \varphi_s}}{\partial \varphi_s}} = \frac{\sqrt{2}}{M_p} W_s \left[ A_S + B_S + C_S \right], \tag{4.21} \]

\[ \text{sign} \left( \frac{V_{\chi}}{W_s} \right) \sqrt{\frac{\epsilon_{\chi}^e \frac{\partial Z^e}{\partial \chi_s}}{\partial \chi_s} e^{-b_s}} = - \frac{\sqrt{2}}{M_p} W_s \left[ A_S + B_S \right], \tag{4.22} \]
where we define
\[
\mathcal{A}_S \equiv -\frac{W^2 e^\epsilon}{W^*_e} \left(1 - \frac{\eta_{ss}}{e^\epsilon} - \frac{1}{2} \text{sign}(b_\varphi) \text{sign} \left( \frac{U_\varphi}{W} \right) \frac{e^\epsilon}{(e^\epsilon)^2 \sqrt{e^\epsilon_b} e^\epsilon} \right) e^{4b_\varphi + 4b_\chi},
\]
\[
\mathcal{B}_S \equiv \frac{1}{2} \text{sign}(b_\varphi) \text{sign} \left( \frac{U_\varphi}{W} \right) \frac{e^\epsilon}{e^\epsilon} \sqrt{e^\epsilon_b e^\epsilon} Z e^{2b_\varphi - 2b_\chi},
\]
\[
\mathcal{C}_S \equiv -\frac{1}{2} \text{sign}(b_\varphi) \text{sign} \left( \frac{U_\varphi}{W} \right) \frac{Z e^\epsilon}{W^*_e} \sqrt{e^\epsilon_b e^\epsilon}.
\]

From the results of the first and second derivatives, it is straightforward to calculate
\[
-\frac{6}{5} f_{\text{NL}}^{(4)} = \frac{2}{((u^2 \alpha_u^2 / \epsilon_\varphi^*) + (v^2 \alpha_v^2 / \epsilon_\chi^*))^2} \left[ u^2 \alpha_u^2 \left( 1 - \frac{\eta_{ss}}{2 \epsilon_\varphi^*} u - M_P \frac{\partial \mathcal{G}}{\partial \varphi^*} + \mathcal{C}_S \right) \right.
\]
\[
- 2 M_P \text{sign} \left( \frac{U_\varphi}{W} \right) \text{sign} \left( \frac{V_\chi}{W} \right) \frac{u \alpha_u v \alpha_v e^{-2b_\chi}}{\sqrt{e_\varphi^*} \sqrt{e_\chi^*} \partial \chi^*} \partial \mathcal{G} \biggr],
\]
\[
+ \frac{v^2 \alpha_v^2}{\epsilon_\chi^*} \left( 1 - \frac{\eta_{ss}}{2 \epsilon_\chi^*} v - M_P \frac{\partial H}{\partial \chi^*} e^{-2b_\chi} \right) + \left( \frac{u \alpha_u v \alpha_v}{\epsilon_\varphi^*} \frac{v \alpha_v}{\epsilon_\chi^*} \right) \left( \mathcal{A}_S + \mathcal{B}_S \right) \biggr],
\]

where we have defined
\[
u \equiv \frac{U^* + Z^e}{W^*_e}, \quad v \equiv \frac{V^* - Z^e}{W^*_e},
\]
\[
\alpha_u \equiv 1 + \text{sign} \left( \frac{U_\varphi}{W} \right) \sqrt{2 \epsilon_\varphi^*} u \mathcal{G}, \quad \alpha_v \equiv 1 + \text{sign} \left( \frac{U_\chi}{W} \right) \frac{\sqrt{2 \epsilon_\chi^*} e^{-b_\chi}}{v} \mathcal{H}.
\]

When \( b(\varphi) = 0 \), this calculation of \( f_{\text{NL}}^{(4)} \) simplifies, since
\[
\mathcal{B}_S = \mathcal{C}_S = \mathcal{G} = \mathcal{H} = 0, \quad \alpha_u = \alpha_v = 1,
\]

and the result is identical to that found in [38].

5. Examples of scalar–tensor theories with product potentials

At first, we consider a separable potential by product with exponents in the \( \varphi \) field, as in the case of a massless dilaton, \( \varphi \),
\[
W(\varphi, \chi) = U(\varphi)V(\chi) = e^{4\epsilon(\varphi)}V(\chi).
\]

When considering the physical fields \((\varphi, \chi)\), only \(H\) and \(T_{SS}\) can be discarded. Therefore, the set of parameters consists of \( \Delta \) and 11 slow-roll parameters as shown in table 1. However, with the potential chosen above, the set reduces to only five independent slow-
roll parameters, which are useful for the running spectral indices:

$$\epsilon_\varphi = 8M_p^2\epsilon_{\varphi}^2, \quad \epsilon_\chi = \frac{M_p^2}{2}\left(\frac{V_\chi}{V}\right)^2 e^{-2b},$$

$$\eta_{\chi\chi} = M_p^2\left(\frac{V_{\chi\chi}}{V}\right)e^{-2b}, \quad \xi_{\chi\chi\chi}^2 = M_p^4\left(\frac{V_{\chi\chi\chi\chi}}{V^2}\right)e^{-4b},$$

$$\xi_c = 8M_p^2\epsilon_{\varphi},$$

since the remaining eight are not independent:

$$\epsilon_c = \epsilon_\varphi,$$

$$\eta_{\varphi\varphi} = \frac{1}{2}\xi_c + 2\epsilon_\varphi, \quad \eta_{\varphi\chi} = 2\text{sign}(\epsilon_\varphi)\text{sign}\left(\frac{V_\chi}{V}\right)\sqrt{\epsilon_\varphi\epsilon_\chi} = \epsilon\sin2\theta,$$

$$\xi_{\varphi\varphi\varphi}^2 = 4\epsilon_\varphi^2 + 3\xi_c\epsilon_\varphi + \xi_{\varphi\varphi}\sqrt{\epsilon_\varphi}, \quad \xi_{\varphi\chi\chi}^2 = 4\epsilon_\varphi\epsilon_\chi + \epsilon_\chi\xi_c,$$

$$\xi_{\varphi\chi\chi\chi} = 2\text{sign}(\epsilon_\varphi)\text{sign}\left(\frac{V_\chi}{V}\right)\sqrt{\epsilon_\varphi\epsilon_\chi\eta_{\chi\chi}}.$$  

5.1. Jordan–Brans–Dicke theory

First we apply our results to the Jordan–Brans–Dicke (JBD) theory with quadratic and quartic potentials. In this theory,

$$U(\varphi) = e^{4b(\varphi)}; \quad b(\varphi) = -\beta \frac{\varphi}{M_p},$$

where we assume $\beta$ is a positive constant. For this potential choice, $\epsilon_\varphi = \epsilon_b = \eta_{\varphi\varphi}/2 = 8\beta^2$ and the slow-roll ends when $\epsilon^e = \epsilon_\chi^e + \epsilon_\varphi^e = 1$.

Two potentials are chosen for $\chi$:

- Model 1a: $V(\chi) = (\lambda/4)\chi^4$
- Model 1b: $V(\chi) = \frac{1}{2}m_\chi^2\chi^2$

where $\lambda$ is a dimensionless parameter and $m_\chi$ is the mass of the $\chi$ field. The slow-roll parameters for both quartic and quadratic potentials for the $\chi$ field are shown in the first two columns of table 2. As seen in the table, the choice of potentials reduces the five independent slow-roll parameters to only two: $\epsilon_\varphi$ and $\epsilon_\chi$. Equivalently, since $\epsilon_\varphi = \epsilon\cos^2\theta$ and $\epsilon_\chi = \epsilon\sin^2\theta$, we can use the parameters $\epsilon$ and $\theta$. Hence, all the first- and second-order slow-roll parameters are proportional to $\epsilon$ and $\epsilon^2$ respectively which make the primordial spectral indices proportional to $\epsilon^2$ in the lowest order.

One final parameter is required, $\tan\Delta$, to describe the evolution after inflation. Then $(1/\epsilon^2)(dn_{\zeta,C,S}/d\ln k)$ are just a function of $\theta$ and $\Delta$. We note that $\theta$ is calculated at horizon crossing and depends on $\beta$ and the initial values of $\varphi$ and $\chi$, whereas $\Delta$ represents the evolution after horizon crossing and depends on the late time evolution of the Universe.

In figure 1 we show the running spectral indices $\alpha_\zeta$ and $\alpha_C$ for model 1a. The result for model 1b is almost identical and is not shown. As said before, $\alpha_S$ is independent of $\Delta$, thus we plot the $\alpha_S$ dependence on $\theta$ separately in figure 3. We plot in the range of
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Figure 1. The spectral index \((n_\zeta - 1)/\epsilon\) and running of \(n_\zeta\) and \(n_C\) in terms of \(\epsilon\) for model 1a.

Table 2. Four Jordan–Brans–Dicke models considered in the text and their slow-roll parameters for a product potential, \(W = e^{b(\phi)}V(\chi)\). In these specific models, five parameters are reduced further to only two and three independent parameters for models 1 and 2 respectively. \(\epsilon_\phi\) and \(\epsilon_\chi\) are directly related to \(\epsilon\) through \(\theta\). The three remaining parameters \((\eta_{XXX}, \xi_{XXX}, \xi_b)\) are shown to be also functions of \(\epsilon\) through \(\epsilon_\phi\) and \(\epsilon_\chi\).

| Model | 1a | 1b | 2a | 2b |
|-------|----|----|----|----|
| \(b(\phi)\) | \(-\beta \phi M_P\) | \(-\beta \phi M_P\) | \(-\beta \phi^2 M_P\) | \(-\beta \phi^2 M_P\) |
| \(V(\chi)\) | \(\frac{1}{4} \chi^4\) | \(\frac{1}{2} m_\chi^2 \chi^2\) | \(\frac{1}{4} \chi^4\) | \(\frac{1}{2} m_\chi^2 \chi^2\) |
| \(\eta_{XX}\) | \(\frac{3}{2} \epsilon_\chi\) | \(\epsilon_\chi\) | \(\frac{3}{2} \epsilon_\chi\) | \(\epsilon_\chi\) |
| \(\xi_{XXX}\) | \(\frac{3}{2} \epsilon_\chi^2\) | 0 | \(\frac{3}{2} \epsilon_\chi^2\) | 0 |
| \(\xi_b\) | 0 | 0 | \(-16\beta\) | \(-16\beta\) |

\(0 < \theta < \pi\) since the potential has the reflection symmetry \(\chi \rightarrow -\chi\) which corresponds to the change of \(\theta\) to \(-\theta\).

For the correlated spectral running, \(\alpha_C\), there is an divergence at \(\Delta \rightarrow \pi/2\). This divergence is not observable. From equation (2.23), it can be seen that, for \(\Delta = \pi/2\) \((T_{\zeta S} = 0)\), there is no evolution in \(\mathcal{P}_\zeta\) and the amplitude of the correlation spectrum is zero. It is therefore impossible to define a spectral index or running at this point. As \(\Delta \rightarrow \pi/2\), \(T_{\zeta S}\) is non-zero, but very small. In this limit, the correlation amplitude would be also small and therefore unobservable.

As an aside, it can be noted that, for the specific case of model 1a, \(B\) (defined in equation (2.19)) is identically zero, due to a cancelling of the slow-roll parameters. This means that from equation (2.18) the curvature perturbation remains constant after horizon crossing \((\zeta = 0)\) for this JBD model with quartic potential.

It is possible to consider the case \(V(\chi) = \lambda \chi^n\). In this JBD theory, and using the fact that at the end of inflation \(\epsilon^e = \epsilon^e_\phi + \epsilon^e_\chi = 1\), we can analytically solve \(N\) in the slow-roll
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limit,
\[
N = \frac{1}{8\beta^2} \ln \left[ 1 + \frac{4\beta^2}{nM_P^2} e^{-2\beta \varphi_*/M_P} \right] + \frac{1}{8\beta^2} \ln \left[ 1 + \frac{1 - 8\beta^2}{1 + (2n - 8)\beta^2} \right].
\]  
(5.10)

We find that $\beta < 0.05$ is required to obtain enough e-foldings for $\varphi^*/\chi^* < 50M_P$. From this fact, $8\beta^2N \lesssim 1$ and the limit of equation (5.10) gives $e^e \simeq n/4N < 0.004n$, where we have taken $N \approx 60$. In this limit of $\beta$, we can approximate equation (5.10) further:
\[
N \approx \frac{\lambda^2}{2n} - \frac{n}{4} - \beta \varphi^* \chi^2.
\]  
(5.11)

For models 1a and 1b, we see from figure 1 that
\[-10(e^e)^2 \lesssim \alpha \lesssim 0.\]  
(5.12)

Hence $|\alpha| \lesssim 1.6n^2 \times 10^{-4}$ which is quite compatible with observations [14].

From the slow-roll equations of motion, equation (2.16), we find the solution of $\varphi$
\[
\varphi = \varphi_* + M_P \text{sign} \left( \frac{U_\varphi}{U} \right) \sqrt{2e\varphi}N,
\]  
(5.13)

which gives $b_e - b_\varphi = -4\beta^2N$. Using this we can express equation (4.10) at the end of inflation (we take $\epsilon^e = \epsilon^\chi + \epsilon^\varphi = 1$) in terms of slow-roll parameters at horizon crossing. In the limit $8\beta^2N \ll 1$, we find $\alpha \approx 1 + N$, $u \approx \epsilon^\varphi$ and $v \approx 1 - \epsilon^\varphi$. This leads to the simplified form of $f^{(4)}_{NL}$,
\[
-\frac{6}{5} f^{(4)}_{NL} \approx 2\epsilon^\varphi - \eta^\varphi + 2\epsilon^\chi \epsilon^\varphi (N + 1) - \epsilon^\varphi.
\]  
(5.14)

It can be noted that in this small $\beta$ limit, $\epsilon^\varphi < \epsilon^\chi$ and the last two terms in this equation are negligible compared to the first two terms.

Using equation (5.11), $f^{(4)}_{NL}$ can be written in terms of the required e-folding number in the small $\beta$ limit (required for enough inflation)
\[
-\frac{6}{5} f^{(4)}_{NL} \approx -\frac{1}{2N} + \frac{\beta \varphi_*}{N M_P}.
\]  
(5.15)

It is clear that, in order to achieve enough inflation, the parameters are required to be small and hence $-\frac{6}{5} f^{(4)}_{NL} \approx 1/2N$, which is unobservable.

5.2. Brans–Dicke type models with quadratic exponent

In order to observe the effect of higher powers in $b(\varphi)$, we consider a quadratic function, such that $\xi_b \neq 0$. To this end, we consider a Brans–Dicke type model with
\[
U(\varphi) = e^{4b(\varphi)}, \quad b(\varphi) = -\beta \varphi^2/M_P^2.
\]  
(5.16)

With this choice, $\epsilon_\varphi = 32\beta^2\varphi^2/M_P^2$ and $\eta_{\varphi\varphi} = -8\beta + 64\beta^2\varphi^2/M_P^2$.

Once again, two example potentials are chosen for $\chi$:
- Model 2a: $V(\chi) = (\lambda/4)\chi^4$
- Model 2b: $V(\chi) = \frac{1}{2}m_\chi^2\chi^2$

where $\lambda$ and $m_\chi$ have the same definitions as before.
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Figure 2. Spectral index \((n_\zeta - 1) / \epsilon\) and running of \(n_\zeta\) and \(n_C\) for model 2a. We used \(\beta = 0.1\epsilon\) here.

Figure 3. Running of \(n_S\) for models 1 and 2. \(\alpha_S\) is independent of \(\Delta\).

The slow-roll parameters for these potentials are shown in the last two columns of table 2. Once again, most of the slow-roll parameters can be written in terms of the single parameter, \(\epsilon^2\). The exception is \(\xi_b\), which is related to the coefficient of the coupling term, \(\beta\). In order to analyse the expected levels of running from these cases, it is necessary to relate \(\beta\) to \(\epsilon\) and we take the assumption that \(\beta = 0.1\epsilon\). For model 2a, the numeric calculations of the spectra for this special case (in relation to \(\epsilon, \theta\) and \(\Delta\)) are shown in figures 2 and 3. Model 2b is almost identical to model 2a and the numerical results are not shown.

Models 2a and 2b have a reflection symmetry about \(\varphi\) \((\theta \rightarrow -\theta)\) and/or \(\chi\) \((\theta \rightarrow \pi - \theta)\). However since the sign of \(\tan \Delta\) is determined by the sign of \(\alpha\) in equation (2.20) and \(\alpha\) is proportional to \(\sin 2\theta\) for the potential in equation (5.1), The reflection of \(\varphi\) or \(\chi\) changes also the sign of \(\tan \Delta\) before the end of inflation. For this reason, we see the symmetry in figure 2 around the point \((\theta = \pi/2, \Delta = \pi/2)\). However after inflation, \(\Delta\) is no longer dependent on the fields \(\varphi\) and \(\chi\), thus this symmetry is not necessarily valid any more.

In this model, the solutions for \(\varphi\) and \(\epsilon^e_\varphi\) are given by

\[
\varphi_e = \varphi_* e^{8\beta N}, \quad \epsilon^e_\varphi = 32\beta^2 \varphi_*^2 / M_P^2 = \epsilon^*_{\varphi} e^{16\beta N}.
\]
Due to the quadratic coupling, the potential is steeper than that of traditional Brans–Dicke models, so that a much smaller $\beta$ is required to give enough e-folding number. In this case, if we assume $16\beta N \ll 1$, then we obtain

$$N \approx \frac{1}{16\beta} \ln \left[ 1 + \frac{8\beta}{n} e^{-2\beta \phi^2} \chi^2 \right] - \frac{1}{16\beta} \ln \left[ 1 + \frac{4\beta n^2}{1 - 32\beta^2 \phi^2 \chi^2} \right]. \quad (5.18)$$

In order to obtain enough inflation, for $\phi_*, \chi_* < 50M_P$, then we require $\beta < 0.0005$. In this very small limit, we can approximate equation (5.18):

$$N \approx \frac{\chi^2}{2n} - \frac{n^2}{4} + \beta \left( \frac{n^4}{2} - \frac{\phi^2 \chi^2}{n} - \frac{2\chi^4}{n^2} \right). \quad (5.19)$$

As in model 1, it is possible to estimate the non-Gaussianity for small $\beta$. Again, we take $\epsilon^e = 1$, so that $\alpha, u$ and $v$ are given as before. In the limit $16\beta N \ll 1$, we find

$$-\frac{6}{5} f_{\text{NL}}^{(4)} \approx 2\epsilon^e - \eta^e + 2\epsilon^e \epsilon^e (N + 1) - \epsilon^e - \frac{1}{2} (\epsilon^e)^2 \epsilon^e \epsilon^e (N + 1)^3. \quad (5.20)$$

The final term is directly due to $b_{\varphi \varphi} \neq 0$. Using the approximation in equation (5.19), we find

$$-\frac{6}{5} f_{\text{NL}}^{(4)} \approx \frac{1}{2N} + \frac{\beta}{N M_P^2}. \quad (5.21)$$

As in model 1, in order to achieve enough inflation, the parameters are required to be small and hence $-\frac{6}{5} f_{\text{NL}}^{(4)} \approx 1/2N$.

### 6. Examples of scalar–tensor theories with sum potentials

The second models we consider are those with sum potentials. If we define the potential

$$W(\varphi, \chi) = U(\varphi) + V(\chi) = \frac{1}{2} m_\varphi^2 \varphi^2 + \frac{1}{2} m_\chi^2 \chi^2, \quad (6.1)$$

then the slow-roll parameters simplify greatly and are given by

$$\epsilon_\varphi = \frac{M_P^2}{2} \left( \frac{m_\varphi^2}{W} \right)^2, \quad \epsilon_\chi = \frac{M_P^2}{2} \left( \frac{m_\chi^2}{W} \right)^2 e^{-2b}, \quad (6.2)$$

$$\eta_{\varphi \varphi} = M_P^2 \frac{m_\varphi^2}{W}, \quad \eta_{\chi \chi} = M_P^2 \frac{m_\chi^2}{W} e^{-2b}, \quad \eta_{\varphi \chi} = 0, \quad (6.3)$$

$$\xi_{\varphi \varphi \varphi} = \xi_{\varphi \chi \chi} = \xi_{\chi \chi \chi} = 0. \quad (6.4)$$

It is no longer possible to directly relate the parameters to $\epsilon$ and $\theta$. However, if we represent the ratio of masses by

$$r = \frac{m_\chi^2}{m_\varphi^2}, \quad (6.5)$$

then the slow-roll parameters can be rewritten in terms of $\epsilon, \beta$ and $re^{-2b}$ only:

$$\eta_{\varphi \varphi} = \epsilon \left( \cos^2(\theta) + \frac{\sin^2(\theta)}{re^{-2b}} \right), \quad \eta_{\chi \chi} = \epsilon \left( \sin^2(\theta) + \cos^2(\theta) re^{-2b} \right), \quad (6.6)$$

$$\epsilon_b = 64 \frac{\beta^2}{\epsilon} \frac{\cos^2(\theta)}{\left( \cos^2(\theta) + \sin^2(\theta) / re^{-2b} \right)}. \quad (6.7)$$

For this sum potential, the remaining choice is in the function of $b(\varphi)$ and we consider the two obvious cases:
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Figure 4. Spectral index $(n_\zeta - 1)$ and running of $n_\zeta$ and $n_C$ in terms of $\epsilon$ for model 3. Values of $\beta = 0.1\sqrt{\epsilon}$ and $re^{-2b} = 2$ have been assumed.

Figure 5. Running of $n_S$ for models 3 and 4. $\alpha_S$ is independent of $\Delta$.

- Model 3: $b(\phi) = -\beta \phi/M_P$
- Model 4: $b(\phi) = -\beta \phi^2/M_P^2$.

We can fix $\beta$ and the mass ratio, in order to estimate the spectral index and running. We take $re^{-2b} = 2$ and, for models 3 and 4 respectively, we assume $\beta = 0.1\sqrt{\epsilon}$ and $\beta = 0.1\epsilon$. The results of the spectrum for model 3 are shown in figures 4 and 5. The results for model 4 are very similar to those of model 3 and are therefore not shown.

7. Numerical analysis of non-Gaussianity

In order to verify our analytical results of the previous section, we numerically solved the background equations of motion, equation (2.1) and the Friedmann equation, equation (2.2), not assuming slow-roll. Multiple trajectories were considered, with initial values, $\phi_i$ and $\chi_i$, given by a $\phi$-$\chi$ grid and $\dot{\phi}_i = \dot{\chi}_i = 0$. The end of inflation is taken to be the time at which the slow-roll parameter $\epsilon = \epsilon_\phi + \epsilon_\chi$ becomes unity and the total number of e-foldings until this point is given by $N_f$.

At horizon crossing, the fields have reached values of $\phi_*$ and $\chi_*$ and the remaining number of e-folds is denoted by $N$. The calculation of non-Gaussianity requires the
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Figure 6. Level of non-Gaussianity for models 3 (left) and 4 (right) calculated from the second term of equation \((4.1)\). We have assumed \(m_\varphi = m_\chi = 5 \times 10^{-3}M_P\). \(\beta = 0.1\).

gradient of the number of e-foldings between horizon crossing and the end of inflation, \(N\) with respect to each of the fields at crossing, \(\varphi_*\) and \(\chi_*\).

It is useful to notice, quite trivially, that if for a given trajectory, \(\varphi_i = \varphi_*\) and \(\chi_i = \chi_*\), then \(N_f = N\). It is therefore possible to calculate \(N_{,i}\), \(N_{,ij}\) etc by differentiating the grid of \(N_f\) with respect to \(\varphi_i\) and \(\chi_i\).

Each of the cases considered above (models 1–4) were modelled numerically. The analytical slow-roll approximations given in equations (5.14) and (5.20) were seen to agree within 5%. For models 3 and 4, no analytical approximation was found. For these sum potentials, the numerical level of non-Gaussianity was calculated from the second term in equation \((4.1)\) and the results are shown in figure 6.

In previous work, [38], for theories with canonical kinetic terms and a sum potential, \(f_{\text{NL}} \approx 1/N\). We find numerically that the \(f_{\text{NL}}\) is of that order of magnitude, even with non-canonical couplings. Therefore we conclude that \(\beta\) has little impact on the level of non-linearity.

8. Conclusion

The focus of this paper was slow-roll inflation in theories with two scalar fields, coupled through a potential as well as their kinetic terms. We have derived the general formulae for the running of the spectral indices for adiabatic and entropic perturbations as well as for the cross-correlation spectrum. In addition, using the \(\delta N\)-formalism and specializing to the product potential \((W(\varphi, \chi) = U(\varphi)V(\chi))\) and the sum potential \((W(\varphi, \chi) = U(\varphi) + V(\chi))\) we have derived the general expressions for the non-linearity parameter \(f_{\text{NL}}\) during inflation.

We have calculated the spectral indices and runnings for several example models. The results are within present observational ranges.

One of the specific examples we have considered is the Jordan–Brans–Dicke theory with quadratic and quartic potentials. In this case, the slow-roll parameter for the field \(\varphi\) is given by the (constant) coupling parameter \(\beta\), defined in equation \((5.9)\). The slow-roll condition for the field \(\varphi\) implies that \(\beta\) itself has to be small. In this case, an analytical formula for \(f_{\text{NL}}\) during inflation is given by equation \((5.14)\) and is of order \(1/N\), where \(N\) is the total number of e-foldings. Assuming \(\beta\) to be positive, the effect of the coupling is
to enhance \( f_{\text{NL}} \) by an additional term \( \beta \varphi / N M_P \). When the coupling is quadratic in the \( \varphi \) field (as in equation (5.16)), the non-linearity is enhanced by a term \( \beta \varphi^2 / N M_P^2 \). Neither of these terms are significant and in the slow-roll limit, \( f_{\text{NL}} \) is not much enhanced by the coupling.

We also considered models with a sum of two quadratic potentials. Due to the complexity of the analytic formula for \( f_{\text{NL}} \), it is difficult to find an approximate equation. We therefore provide numerical results for simple models and showed that \( f_{\text{NL}} \ll 1 \) and is unobservable for the slow-roll limit.

To conclude, in order to distinguish between inflationary models, future observations will search for deviations from standard one-field inflation. In order to do so one has to go beyond the spectral index as an observable. Future experiments, such as PLANCK, will tighten the constraints on the spectral running and non-linearity. The results of this paper can be used to calculate these parameters in other multi-field models with non-canonical kinetic terms, in order to constrain the parameter ranges.

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**Appendix A. Slow-roll parameters**

The slow-roll parameters we shall use are first order:

\[
\epsilon = -\frac{\dot{H}}{H^2} = -\frac{1}{2M_P^2} \frac{\dot{\sigma}}{H^2} = \frac{M_P^2}{2} \left( \frac{V_{\varphi}}{V} \right)^2 = \epsilon_\varphi + \epsilon_\chi, \tag{A.1}
\]

\[
\epsilon_\varphi = \frac{M_P^2}{2} \left( \frac{V_{\varphi}}{V} \right)^2 = \epsilon \cos^2 \theta, \quad \epsilon_\chi = \frac{M_P^2}{2} \left( \frac{V_\chi}{V} \right)^2 e^{-2b} = \epsilon \sin^2 \theta, \tag{A.2}
\]

\[
\epsilon_b = 8M_P^2 b_\varphi, \tag{A.3}
\]

\[
\eta_{\sigma\sigma} \equiv \eta_{\varphi\varphi} \cos^2 \theta + \eta_{\chi\chi} \sin 2\theta + \eta_{\varphi\chi} \sin^2 \theta,
\]

\[
\eta_{\sigma\chi} \equiv (\eta_{\chi\chi} - \eta_{\varphi\varphi}) \sin \theta \cos \theta + \eta_{\varphi\chi} (\cos^2 \theta - \sin^2 \theta),
\]

\[
\eta_{ss} \equiv \eta_{\varphi\varphi} \sin^2 \theta - \eta_{\varphi\chi} \sin 2\theta + \eta_{\chi\chi} \cos^2 \theta, \tag{A.4}
\]

where

\[
\eta_{\varphi\varphi} = M_P^2 \frac{V_{\varphi\varphi}}{V}, \quad \eta_{\varphi\chi} = M_P^2 \frac{V_{\varphi\chi}}{V} e^{-b}, \quad \eta_{\chi\chi} = M_P^2 \frac{V_{\chi\chi}}{V} e^{-2b}, \tag{A.5}
\]

and second order:

\[
\xi_{\sigma\sigma\sigma} \equiv \xi_{\varphi\varphi\varphi} \cos^2 \theta + \xi_{\varphi\varphi\chi} 3 \cos^2 \theta + \xi_{\varphi\chi\chi} 3 \sin^2 \theta + \xi_{\chi\chi\chi} \sin^2 \theta,
\]

\[
\xi_{\sigma\sigma\chi} \equiv (\xi_{\varphi\varphi\varphi} - \xi_{\varphi\varphi\chi}) \sin \theta \cos \theta + \xi_{\varphi\chi\chi} (2 \sin^2 \theta - \cos^2 \theta) + \xi_{\varphi\varphi\chi} (\cos^2 \theta - 2 \cos^2 \theta),
\]

\[
\xi_{ssss} \equiv \xi_{\varphi\varphi\varphi} \sin^2 \theta + \xi_{\varphi\varphi\chi} (2 \cos^2 \theta - \sin^2 \theta) + \xi_{\varphi\chi\chi} (\cos^2 \theta + 2 \sin^2 \theta) + \xi_{\chi\chi\chi} \cos^2 \theta, \tag{A.6}
\]
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\[ \xi^2_{\phi\phi\phi} = M_p^4 \frac{V_{\phi\phi\phi} V_\phi}{V^2}, \quad \xi^2_{\chi\chi\chi} = M_p^4 \frac{V_{\chi\chi\chi} V_\chi e^{-2b}}{V^2}, \]
\[ \xi^2_{\phi\chi\chi} = M_p^4 \frac{V_{\phi\chi\chi} V_\phi}{V^2}, \quad \xi^2_{\phi\chi\chi} = M_p^4 \frac{V_{\phi\chi\chi} V_\phi e^{-b}}{V^2} = \xi^2_{\phi\chi\chi} \cot \theta, \]  
\[ \xi^2_{\phi\chi\chi} = M_p^4 \frac{V_{\phi\chi\chi} V_\chi e^{-3b}}{V^2}, \quad \xi^2_{\phi\chi\chi} = M_p^4 \frac{V_{\phi\chi\chi} V_\phi e^{-2b}}{V^2} = \xi^2_{\phi\chi\chi} \cot \theta. \]

Here we note that \( \xi^2_{\phi\phi\phi} \) does not appear in the running of spectral indexes.

\[ \xi_b = 8 M_p^2 b_{\phi\phi}. \]  

Appendix B. Time derivative of first-order slow-roll parameters

\[ \frac{1}{H} \dot{\epsilon} = 4 \epsilon^2 - 2 \epsilon \eta_{\sigma\sigma} + \frac{1}{2} \text{sign}(b_\phi) \text{sign} \left( \frac{V_\sigma}{V} \right) \sqrt{\epsilon_b \epsilon} \sin^2 \theta \cos \theta \epsilon \]  
\[ \frac{1}{H} \dot{\epsilon}_b = - \text{sign}(b_\phi) \text{sign} \left( \frac{V_\sigma}{V} \right) \sqrt{\epsilon_b \epsilon} \cos \theta \xi_b \]  
\[ - \frac{\dot{H}}{H^3} = -2 \epsilon^2 + \frac{1}{H} \dot{\epsilon} \]  
\[ \frac{1}{H} \dot{\eta}_{\sigma\sigma} = 2 \epsilon \eta_{\sigma\sigma} - 2 \eta_{\sigma\sigma} - \xi^2_{\sigma\sigma\sigma} \]
\[ + (\eta_{\sigma\sigma} \sin \theta + 2 \eta_{\sigma\sigma} \cos \theta) \sin \theta \cos \theta \text{sign}(b_\chi) \text{sign} \left( \frac{V_\sigma}{V} \right) \sqrt{\epsilon_b \epsilon} \]  
\[ \frac{1}{H} \dot{\eta}_{\sigma\sigma} = 2 \epsilon \eta_{\sigma\sigma} + \eta_{\sigma\sigma}(\eta_{\sigma\sigma} - \eta_{ss}) - \xi^2_{\sigma\sigma\sigma} \]
\[ + \left( \frac{1}{2} \eta_{\sigma\sigma} \cos \theta + \eta_{ss} \cos^2 \theta \sin \theta \right) \text{sign}(b_\phi) \text{sign} \left( \frac{V_\sigma}{V} \right) \sqrt{\epsilon_b \epsilon} \]  
\[ \frac{1}{H} \dot{\eta}_{ss} = 2 \epsilon \eta_{ss} + 2 \eta_{ss}^2 - \xi^2_{\sigma\sigma ss} + \eta_{ss} \cos^3 \theta \text{sign}(b_\sigma) \text{sign} \left( \frac{V_\sigma}{V} \right) \sqrt{\epsilon_b \epsilon} \]

\[ \frac{\partial}{H_s \partial t_s} \ln(1 + T_{\xi S}^2) = 2 \sin \Delta \cos \Delta (-\alpha_s - \delta_s T_{\xi S}) \]
\[ \frac{\partial}{H_s \partial t_s} \ln T_{SS} = -\delta_s, \]  
\[ \frac{\partial}{H_s \partial t_s} \ln T_{S} = -\frac{\alpha_s - \delta_s T_{\xi S}}{T_{\xi S}}. \]
Appendix C. Calculation of $-\frac{6}{5}f_{NL}^{(3)}$ for non-canonical terms

The first term of equation (4.1), $f_{NL}^{(3)}$, can be written by

$$-\frac{6}{5}f_{NL}^{(3)} = \frac{A^IJK N_{I}N_{J}N_{K}}{(N_{I}N_{J})G^{IJ})^2 \sum i k^3}$$  \hspace{1cm} (C.1)

where summations over $I, J, K$ are implied and $A^IJK$ is given by [37]

$$A^IJK(k_1,k_2,k_3) = \frac{1}{M^2} \sum \text{perms} \hat{\varphi}_I^4 H G_{JK} M_{123},$$  \hspace{1cm} (C.2)

with momentum dependent $M_{123} = M(k_1,k_2,k_3)$. We can perform the sum and permutations in the numerator of equation (C.1):

$$\sum_{IJK} A^IJK N_{I}N_{J}N_{K} = \sum_{IJK} N_{I}N_{J}N_{K} \frac{1}{M^2} \sum \text{perms} \hat{\varphi}_I^4 H G_{JK} M_{123}.$$  \hspace{1cm} (C.3)

Following from this, the numerator in equation (C.1) can be reduced to

$$-\frac{1}{4M^2} \left( \sum_{IJ} N_{I}N_{J} G_{IJ} \right) \sum \text{perms} M_{123},$$  \hspace{1cm} (C.4)

where we have used $\sum_{I} N_{I} \hat{\varphi}_I = -H$. We define

$$f = f(k_1,k_2,k_3) \equiv -1 - \frac{\sum \text{perms} M_{123}}{2 \sum_{I} k^3}.$$  \hspace{1cm} (C.5)

Finally, if we consider the power spectra of curvature perturbations and gravitational waves,

$$P_\zeta = P_* \sum_{IJ} N_{I}N_{J} G_{IJ}, \quad P_* \equiv \frac{H^2}{4\pi^2}, \quad P_g = \frac{8P_*}{M^2},$$  \hspace{1cm} (C.6)

we obtain

$$-\frac{6}{5}f_{NL}^{(3)} = \frac{r}{16}(1 + f),$$  \hspace{1cm} (C.7)

where $r$ is scalar-to-tensor ratio, $r \equiv P_g/P_\zeta$.

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