On the Quantum Chromodynamics of a Massive Vector Field in the Adjoint Representation

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Abstract
In this paper, we explore the possibility of constructing the quantum chromodynamics of a massive color-octet vector field without introducing higher structures like extended gauge symmetries, extra dimensions or scalar fields. We show that gauge invariance is not enough to constraint the couplings. Nevertheless the requirement of unitarity fixes the values of the coupling constants, which otherwise would be arbitrary. Additionally, it opens a new discrete symmetry which makes the coloron stable and avoid its resonant production at a collider. On the other hand, a judicious definition of the gauge fixing terms modifies the propagator of the massive field making it well-behaved in the ultra-violet limit. The relation between our model and the more general approach based on extended gauge symmetries is also discussed.

1 Introduction
Many extensions of the Standard Model, such as non-minimal Technicolor [1, 2, 4, 5, 3], Extra-dimensions [6], Top-color [7, 8] and Chiral-color [9, 10, 11, 12], predict the existence of massive color-octet spin-1 particles which we will collectively call “colorons”. In principle, it is expected that, if a coloron exists in the appropriated mass range, it should be copiously produced at hadron colliders such as the Tevatron or the LHC [5, 13]. Indeed, some renewed interest on this kind of particles has arisen [14, 15] because some sort of color-octet spin-1 resonance may be the origin of the large $t\bar{t}$ forward-backward asymmetry measured by CDF [16, 17] and D0 [18].

From the phenomenological point of view, it is convenient, given the large variety of models predicting colorons, to find an effective model-independent description which can grasp their essential features. This is the origin, for example, of the deconstruction idea [19, 20]: the initial intension was to describe

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the Kaluza-Klein excitations of the gluon independently of the details of the underlying extra-dimensional theory. A similar problem, but in the context of non-minimal Technicolor, motivated another effective description [3, 5] based on the observed low-energy symmetries. This phenomenological analysis produced two important results: it was shown that the s-channel production of a single coloron is plagued by theoretical uncertainties [5] and it was argued that the coloron pair production would be almost model-independent [5, 13], being determined exclusively by QCD gauge invariance.

Interestingly, when one tries to make the quantum chromodynamics of a coloron, one finds that there are at least two different, although gauge equivalent, formulations [3, 21]. Both of them lead to theories with a bad ultraviolet behavior due to the presence of the massive spin-1 field. This is a quite frustrating situation because we know how to construct consistent and renormalizable quantum field theories with scalars or fermions (massive or not) as matter fields but things seem to be very different when a massive spin-1 field is considered.

In this work, we revisit the construction of a gauge theory for the coloron and we examine the possibility that such a theory be consistent with renormalizability and unitarity without introducing neither scalar fields nor higher structures such as extra-dimensions or extended gauge symmetries. It is important to emphasize that our aim is not to present an alternative to the well established method of introducing colorons through the breaking down of an enlarged gauge sector, but rather to investigate what are the minimal requirements for a consistent coloron model only in the framework of the observed QCD gauge symmetry. For this purpose, we organized this paper in the following way. In section 2, we describe the construction of a general classical gauge theory with a massive spin-one field in the adjoint representation. In section 3, we move to the quantum version of the theory, paying special attention to the gauge fixing and the ghost terms, and implementing the BRST symmetry. Section 4 is devoted to study the constraints imposed by requiring that perturbative unitarity of the S-matrix holds at tree-level. Finally, we summarize our conclusions in section 5.

2 A Gauge Theory for a Massive Vector Field

2.1 Global Symmetry

Usually, the starting point for studying the physical properties of a massive spin-one field is the Proca Lagrangian. So, let’s consider a generalization of the Proca theory for a non-Abelian global continuous symmetry:

\[ \mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} M^2 V^a_{\mu} V^{a\mu} - V^a_{\mu} J^{a\mu}(+\mathcal{L}_{\text{int}}) \]  (1)

where \( F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu \) and \( V^a_\mu \) transforms homogeneously under the global symmetry \( V^a_\mu \to U^{\dagger} V^a_\mu U \). We have included an external source \( (J^{a\mu}) \) which is supposed to be a conserved current. Additionally, all the other invariant terms
that can be constructed with $V_\mu$ and $\partial_\mu V_\nu$ can be eventually included in $\mathcal{L}_{\text{int}}$, but they will not be relevant for the present analysis and, for simplicity, will not be taken explicitly into account. As it is well known, the field equation obtained from Lagrangian (1) can be written as:

$$\partial_\mu F^{\mu\nu} + M^2 V^\nu = J^\nu$$

(2)

and the anti-symmetry of $F_{\mu\nu}$ automatically implies the Lorenz condition:

$$\partial_\mu V^{\mu\nu} = 0$$

(3)

Let us recall, for completeness, that equation (3) eliminates one degree of freedom from $V_\mu$. Naturally, the remaining degrees of freedom correspond to the three polarization states of a massive spin-one particle.

Unfortunately, when a quantum theory is constructed from Lagrangian (1), it leads to the following propagator:

$$\Delta_{\mu\nu} = -\frac{i}{q^2 - M^2} \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2} \right)$$

(4)

which spoils the renormalizability of the theory due to the bad ultraviolet behavior of the last term. Interestingly, the form of this propagator, and its ultraviolet divergence, can be traced to the anti-symmetric structure of $F_{\mu\nu}$. So, we can raise the reasonable question of whether an anti-symmetric $F_{\mu\nu}$ is essential to our theory, or not. In other words, is there a fundamental principle that compels us to include $\partial_\mu V_\nu \partial^\mu V^\nu$ and $\partial_\mu V_\nu \partial^\mu V^\nu$ in the Lagrangian with the same weight? Certainly, the answer is negative. In gauge theory, the anti-symmetry of $F_{\mu\nu}$ is dictated by the gauge principle since its necessary to cancel the inhomogeneous part of the transformation of the gauge field, but this is not the case here because in our construction $V_\mu$ transforms homogeneously. Consequently, it is possible to write down a more general Lagrangian:

$$\mathcal{L} = -\frac{1}{2} \partial_\mu V_\nu \partial^\mu V^{\nu\nu} + \frac{(1 + a)}{2} \partial_\mu V_\nu \partial^\mu V^{\nu\mu} + \frac{1}{2} M^2 V_\mu V^\mu - V_\mu J^\mu (+\mathcal{L}_{\text{int}})$$

(5)

Of course, the Proca Lagrangian is recovered for $a = 0$. This time, the field equation is:

$$\partial^2 V_\mu - a\partial_\mu \partial^2 V_\nu + M^2 V_\mu = J_\mu$$

(6)

where we have dropped the group index. Differentiating (6), it follows that a generalized Lorenz condition is satisfied:

$$\partial_\mu V^\mu = f(x)$$

(7)

where $f(x)$ is a solution of the following equation:

$$[a\partial^2 - M^2] f(x) = 0$$

(8)
Of course, \( f(x) = 0 \) is a solution of (8) and the usual Lorenz condition can be used.

A more important consequence, however, is the fact that the propagator obtained from (5) can be written in following way:

\[
\Delta_{\mu\nu} = \frac{-i}{q^2 - M^2} \left( g_{\mu\nu} - \frac{(1 + a) q_\mu q_\nu}{aq^2 - M^2} \right)
\]

Notice that this new propagator has the same form of the propagator of a massive gauge boson in the context of spontaneously broken gauge symmetries. This modified propagator behaves adequately in the ultraviolet limit.

In the Abelian case, the theoretical construction presented so far is similar to the one resulting from the Stueckelberg theory when the compensating scalar field is gauged away. Indeed, equations (6) and (8) are formally equal to those obtained from the Stueckelberg Lagrangian [22]. It is worth to recall that the Stueckelberg formalism makes the theory of a massive photon renormalizable. For this reason, Lagrangian (5) seems to be a good starting point in the attempt of making a consistent theory for the coloron.

### 2.2 Local Symmetry

Evidently, the most direct way to turn the previous construction into a local gauge theory is to take Lagrangian (5), replace partial derivatives by covariant ones, include a Yang-Mills term for the gluon, include in \( \mathcal{L}_{\text{int}} \) all the gauge invariant and renormalizable terms we can form with \( V_\mu, D_\mu V_\nu \) and \( G_{\mu\nu} \) (field-strength of gluons) with arbitrary coefficients. In principle, this Lagrangian should contain a kinetic mixing term, nevertheless, it can be removed by a simple redefinition of the fields (see, for instance, [21]). This standard procedure leads us to the following Lagrangian:

\[
\mathcal{L} = -\frac{1}{2} Tr \{ G_{\mu\nu} G^{\mu\nu} \} - Tr \{ D_\mu V_\nu D^\mu V^\nu \} + (1 + a) Tr \{ D_\mu V_\nu D^\mu V^\nu \} + a_{11} Tr \{ D_\mu V_\nu V^\mu V^\nu \} + a_{12} Tr \{ D_\mu V_\nu V^\mu V^\nu \} + a_{21} Tr \{ V_\mu V_\nu V^\mu V^\nu \} + a_{22} Tr \{ V_\mu V_\nu V^\mu V^\nu \} + a_{3} Tr \{ G_{\mu\nu} [V^\mu, V^\nu] \} + M^2 Tr \{ V_\nu V^\nu \}
\]

We recall that this Lagrangian is invariant under the local gauge transformations:

\[
G_\mu \rightarrow U G_\mu U^{-1} - \frac{1}{g} (\partial_\mu U) U^{-1}
\]

\[
V_\mu \rightarrow U V_\mu U^{-1}
\]

Notice that the third term in (10) includes the implementation of the Stueckelberg trick and, in the quantum version of the theory developed so far, the
propagator of the coloron would be the one shown in (9). Nevertheless, the
same term modifies the $GVV$ and the $GGV$ vertex (the last one is also mod-
ified by the term proportional to $a_3$). This fact is important because it means
that it is not guaranteed that, in the general case, a massive color-octet spin-one
particle interacts with gluons with a typical QCD strength as it is commonly
believed.

At this point it is important to notice that this Lagrangian is written in a very
specific basis, which we call the physical basis, where the mass matrix is diagonal
and the massive field transforms homogeneously under gauge transformations.
Nevertheless, it will be convenient for our purposes to write Lagrangian (10) in
terms of a general basis. For this reason, we define new fields $A^1_{1\mu}$ and $A^2_{2\mu}$, by
rotating $G$ and $V$, in such a way that

$$G_\mu = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} A^1_{1\mu} + \frac{g_1}{\sqrt{g_1^2 + g_2^2}} A^2_{2\mu} \quad (11)$$

$$V_\mu = \frac{g_1}{\sqrt{g_1^2 + g_2^2}} A^1_{1\mu} - \frac{g_2}{\sqrt{g_1^2 + g_2^2}} A^2_{2\mu} \quad (12)$$

In these expressions $g_1$ and $g_2$ are constant that satisfy the constrain

$$g \equiv \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \quad (13)$$

where $g$ is the usual QCD coupling constant.

In terms of the new basis, the Lagrangian (10) can be re-organized as:

$$\mathcal{L} = -\frac{1}{2} Tr [F_{1\mu\nu} F_{1\mu\nu}^{\mu\nu}] - \frac{1}{2} Tr [F_{2\mu\nu} F_{2\mu\nu}^{\mu\nu}]$$

$$+ \frac{M^2}{g_1^2 + g_2^2} Tr [(g_1 A_{1\mu} - g_2 A_{2\mu})^2]$$

$$+ \frac{a}{g_1^2 + g_2^2} Tr [(g_1 D_\mu A_1 - g_2 D_\mu A_2) (g_1 D_\nu A_1 - g_2 D_\nu A_2)]$$

$$+ \mathcal{L}_{NM} \quad (14)$$

where

$$F_{j\mu\nu} = \partial_\mu A_{j\nu} - \partial_\nu A_{j\mu} - ig [A_{j\mu}, A_{j\nu}]$$

and

$$D_\mu = \partial_\mu - ig^2 \frac{a}{g_1} [A_{1\mu}, -] - ig^2 \frac{a}{g_2} [A_{2\mu}, -]$$

$\mathcal{L}_{NM}$ represents all the interaction terms depending on arbitrary constants.

The gauge transformation of the new fields can be easily obtained form
the transformation laws of the gluon and the massive field $V_\mu$. The resulting
transformation law is:
\[ A_{i\mu} \to UA_{i\mu}U^{-1} - \frac{1}{g_i} (\partial_{\mu} U) U^{-1} \quad (i = 1, 2) \quad (15) \]

This means that the fields \( A_{1\mu} \) and \( A_{2\mu} \) transform like connections. Of course, Lagrangian (14) is invariant by construction under this gauge transformation. The important point is that now the coloron Lagrangian is written in such a way that it formally seems a gauge theory with two connections. In the next section we will use this fact to write the quantum version of the theory and implement the BRST symmetry by applying twice the Fadeev-Popov prescription.

3 Quantum Theory: BRST Symmetry

Strictly speaking, what we have done so far is to develop a classical theory. If we want to quantize the theory using the path integral method, it is necessary to add gauge fixing terms and ghost fields as dictated by the Fadeev-Popov procedure. Fortunately, this is an easy task in the version of the model developed in the previous section. A good starting point is Lagrangian (14). Because we have two gauge-like fields, all we need to do is to duplicate the standard Fadeev-Popov prescription and add to (14) the following Lagrangian:

\[
\mathcal{L}_{GF} = \frac{1}{2} \xi_1 B_1^a B_1^a - B_1^a \partial^\mu A_{1\mu}^a + \bar{c}_1^a \partial^\mu D_{1\mu}^{ab} c_b^b \\
+ \frac{1}{2} \xi_2 B_2^a B_2^a - B_2^a \partial^\mu A_{2\mu}^a + \bar{c}_2^a \partial^\mu D_{2\mu}^{ab} c_b^b \quad (16)
\]

where, as usual, \( B_1^a \) and \( B_2^a \) are auxiliary fields, \( c, \bar{c}_1 \) and \( \bar{c}_2 \) are ghost and anti-ghost fields and \( \xi_1 \) and \( \xi_2 \) are gauge parameters. Additionally, we have used the notation \( D_{i\mu} = \partial_{\mu} - ig_i [A_{i\mu}, \cdot] \). In order to avoid the introduction of kinetic mixing terms in the physical basis (because we expect that in the basis formed by \( G_{\mu} \) and \( V_{\mu} \), everything is diagonal), we chose \( \xi_1 = \xi_2 = \xi \). Of course, by construction, the whole Lagrangian (that is, (14) + (16)) is invariant under the BRST transformations:

\[
\delta_B A_{1\mu}^a = \frac{1}{g_1} D_{1\mu}^{ab} c_b^b \\
\delta_B c^a = -\frac{1}{2} f^{abc} c_b^b c_c^c \\
\delta_B \bar{c}_1^a = B_1^a \\
\delta_B B_1^a = 0
\quad (17) \quad (18) \quad (19) \quad (20)
\]

In the physical basis, Lagrangian (16) takes the form:

\[
\mathcal{L}_{GF} = \left( \partial^\mu G_{\mu}^a \right)^2 - \frac{1}{2\xi} \left( \partial^\mu V_{\mu}^a \right)^2 + \bar{c}^a \partial^\mu D_{\mu}^{ab} c_b^b \\
+ \alpha f^{abc} \partial^\mu c^a V_{\mu}^c b^b + \beta f^{abc} \partial^\mu \eta^a V_{\mu}^c b^b
\quad (21)
\]
where we have already eliminated the auxiliary fields. Here $\alpha$ and $\beta$ are some combinations of the original coupling constants $g_1$ and $g_2$ but their exact expressions are not important for our purposes. On the other hand, $\bar{c}$ and $\bar{\eta}$ are related to the previous anti-ghosts by the following definitions:

$$\bar{c} \equiv \bar{c}_1 + \bar{c}_2$$

$$\bar{\eta} \equiv \bar{c}_2 - \bar{c}_1$$

An important characteristic of Lagrangian (21) is that $\bar{\eta}$ doesn't have a kinetic term and hence its equation of motion is only a constraint:

$$f^{abc} \partial^\mu (V_c^\mu c^b) = 0$$

Interestingly, this is exactly the kind of constraint needed to implement the Lorenz condition for a massive field transforming homogeneously under the symmetry group. Putting (24) back in the Lagrangian, we find:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi} \left( \partial^\mu G_a^\mu \right)^2 - \frac{1}{2\xi} \left( \partial^\mu V^a_\mu \right)^2 + \bar{c}^a \partial^\mu D^{ab}_\mu c^b$$

So finally, we have the correct gauge fixing and ghost terms for our model. The most important consequence of this procedure is that the second term of (25) contributes to the propagator of $V_\mu$. Indeed, we can write now the correct propagator for the gluon and the coloron in the complete theory:

$$\Delta_G = -\frac{i\delta^{ab}}{q^2} \left( g^{\mu\nu} + (\xi - 1) \frac{q^\mu q^\nu}{q^2} \right)$$

$$\Delta_V = -\frac{i\delta^{ab}}{q^2 - M^2} \left( g^{\mu\nu} + (\xi + \xi a - 1) \frac{q^\mu q^\nu}{(1 - \xi a)q^2 - \xi M^2} \right)$$

4 Unitarity

4.1 Unitarity Constraints

Hitherto, we have constructed a general quantum gauge theory of the coloron with operators of dimension four or less. It can be seen as a good starting point for an effective theory and its main consequence is that the coupling of the coloron to gluon may sensibly deviate from the general expectation. However, we would like to recall that the aim of this work is to explore the construction of a coloron theory which can be well behaved in the ultraviolet limit and, eventually, renormalizable. In this sense, it is necessary to compel the theory to preserve the perturbative unitarity of the S-matrix. For this purpose, we compute the amplitudes for the processes $V_L V_L \rightarrow V_L V_L$ and $G G \rightarrow V_L V_L$ (where $G$ is the gluon and $V_L$ is the longitudinally polarized coloron) at tree-level and we impose the condition that the terms which are divergent in the
ultraviolet limit ($\epsilon/M^2 \to \infty$) vanish. In these calculations, we use a simplified version of (10):

$$\mathcal{L} = -\frac{1}{2} Tr \{ G_{\mu\nu}G^{\mu\nu} \} - Tr \{ D_{\mu}V_{\nu}D^{\mu}V^{\nu} \} + (1 + a) Tr \{ D_{\mu}V_{\nu}D^{\mu}V^{\nu} \}$$
$$+ a_1 Tr \{ (D_{\mu}V_{\nu} - D_{\nu}V_{\mu}) [V^{\mu}, V^{\nu}] \}$$
$$+ a_2 Tr \{ [V_{\mu}, [V^{\mu}, V^{\nu}] \} + M^2 Tr \{ V_{\mu}V^{\nu} \}$$

$$+ \frac{1}{2\xi} (\partial\mu G^{\mu})^2 \left( \frac{1}{2\xi} (\partial\mu G^{\mu})^2 + \epsilon^a \partial\mu D^{\mu} c^b \right)$$

This simplification is well motivated, however, because the terms in (10) containing three and four $V$ fields would give origin in the amplitude to terms proportional to the $d$ terms of (10):

$$M = \frac{1}{3} Tr \{ G_{\mu\nu}G^{\mu\nu} \}$$
$$L = \frac{1}{3} Tr \{ V_{\mu}V^{\nu} \}$$

$$\frac{1}{2} \left( \partial\mu G^{\mu} \right)^2 \left( \frac{1}{2\xi} \left( \partial\mu G^{\mu} \right)^2 + \epsilon^a \partial\mu D^{\mu} c^b \right)$$

This vanishing in these calculations, we use a simplified version of (10):

This vanishing in these calculations, we use a simplified version of (10):

$$\mathcal{M} = \frac{(a_2 + g^2 + a_1^2)}{4s (M^2 - s) (M^2 - t) (M^2 - u) M^4} f^{abc} f^{cde}$$
$$+ \frac{(a_2 + g^2 + a_1^2)}{4s (M^2 - s) (M^2 - t) (M^2 - u) M^4} f^{abc} f^{bde}$$
$$- \frac{(a_2 + g^2)}{4s (M^2 - s) (M^2 - t) (M^2 - u) M^2} f^{abc} f^{cde}$$
$$- \frac{(t^4 + 14t^3u - 20t^2u^2 - 28tu^3 - 2u^4)}{4s (M^2 - s) (M^2 - t) (M^2 - u) M^2} a_1^2 (t + u) \epsilon^{abc} f^{cde}$$
$$- \frac{(t^4 + 14t^3u + 40t^2u^2 + 14tu^3 + u^4)}{4s (M^2 - s) (M^2 - t) (M^2 - u) M^2} a_2 (t + u) \epsilon^{abc} f^{bde}$$
$$- \frac{(t^4 + 17t^3u + 46t^2u^2 + 17tu^3 + u^4)}{4s (M^2 - s) (M^2 - t) (M^2 - u) M^2} \epsilon^{abc} f^{bde}$$

$$+ \mathcal{O} \left( \frac{M^2}{s} \right)$$

It can be easily seen that, in order to cancel the problematic terms, it is enough to satisfy the following conditions:

$$a = 0 \quad (30)$$
$$a_1 = 0 \quad (31)$$
$$a_2 = -g^2 \quad (32)$$
Indeed, with this election of parameters, the amplitude for the $V_L V_L \rightarrow V_L V_L$ scattering completely vanishes. Surprisingly, the previous conditions avoid the presence of the Stueckerberg term and forbid the coloron triple vertex.

In a similar way, imposing unitarity to the $GG \rightarrow V_L V_L$ scattering amplitude, we get an additional condition:

$$a_3 = -g$$  \hspace{1cm} (33)

Consequently, taken into account the restrictions due to unitarity, the Lagrangian takes the simple form:

$$\mathcal{L} = -\frac{1}{2} \text{Tr} \{ G_{\mu\nu} G^{\mu\nu} \} - \text{Tr} \{ D_\mu V_\nu D^\mu V^\nu \} + \text{Tr} \{ D_\mu V_\nu D^\nu V^\mu \} - g^2 \text{Tr} \{ [V_\mu, V_\nu] [V^\mu, V^\nu] \} - g \text{Tr} \{ G_{\mu\nu} [V^\mu, V^\nu] \} + M^2 \text{Tr} \{ V_\mu V^\mu \} - \frac{1}{2\xi} (\partial^\mu G_\mu)^2 - \frac{1}{2\xi} (\partial^\mu V_\mu)^2 + \bar{c}^a \partial^\mu D_\mu c^b$$

and the propagators are:

$$\Delta_G = -\frac{i\delta^{ab}}{q^2} \left( g^{\mu\nu} + (\xi - 1) \frac{q^\mu q^\nu}{q^2} \right)$$  \hspace{1cm} (35)

$$\Delta_V = -\frac{i\delta^{ab}}{q^2 - M^2} \left( g^{\mu\nu} + (\xi - 1) \frac{q^\mu q^\nu}{q^2 - \xi M^2} \right)$$  \hspace{1cm} (36)

Notice that the coloron propagator is the same one we would have obtained for a massive spin-one field in a gauge theory with spontaneous symmetry breaking.

### 4.2 Consequences of Unitarity

Interestingly, Lagrangian (34) posses a new $Z_2$ symmetry under which $V$ is odd and $G$ is even. This discrete symmetry makes the coloron to be stable. Hence, an unexpected consequence of unitarity is that the coloron, conveniently dressed by gluons, will form a new kind of stable hadron that can be a cold dark matter candidate. Another implication of the $Z_2$ symmetry is that this kind of coloron cannot be resonantly produced at a collider. The easiest way to create it, is pair production. Naturally, the produced colorons will hadronize producing two jets. Because of the huge background for two jets at a hadron collider and the absence of any distinctive kinematic structure, we expect that the observation of this kind of colorons at the LHC would be very challenging.

In the “Two-Connections” picture, on the other hand, the $Z_2$ symmetry translates as a symmetry of the Lagrangian under the interchange of the two connections ($A_1 \leftrightarrow A_2$). Imposing this symmetry to Lagrangian (14), we obtain that the coupling constants must satisfy the condition $g_1 = g_2 = \sqrt{2} g$ and the
Lagrangian (including the gauge fixing and ghost sectors) takes the simple form:

\[
\mathcal{L} = -\frac{1}{2} \text{Tr} [F_{1\mu\nu} F_{1}^{\mu\nu}] - \frac{1}{2} \text{Tr} [F_{2\mu\nu} F_{2}^{\mu\nu}] + \frac{M^2}{2} \text{Tr} [(A_{1\mu} - A_{2\mu})^2] \\
- \frac{1}{2\xi} (\partial^\mu A^{a}_{1\mu})^2 - \frac{1}{2\xi} (\partial^\mu A^{a}_{2\mu})^2 + \bar{c}^{a} \frac{\partial^\mu D^{ab}_{1\mu}}{2} c^{b} + \bar{c}^{a} \frac{\partial^\mu D^{ab}_{2\mu}}{2} c^{b} \tag{37}
\]

Obviously, Lagrangian (34) is automatically obtained from (37) after the diagonalization of the mass matrix.

5 Summary and Conclusions

Finally, we have arrived to our goal and now it is time to recapitulate our main results. First, we studied the construction of a general local gauge theory for the coloron (with operators of dimension up to 4). We saw that it is plagued of undetermined coupling constants, but, nevertheless, this general theory can be a good starting point for effective models. A direct consequence of this degree of arbitrariness is that the expectation of the coloron interacting with gluon with “typical QCD intensity” is not guaranteed. In part, this theoretical uncertainty is due to the presence of the Stueckelberg term which, as fas as we know, has not been considered before in the context of coloron phenomenology.

In a second step, we were able to construct a particular gauge theory for the coloron which is BRST invariant, consistent with perturbative unitarity. Surprisingly, this model produces propagators with acceptable ultraviolet behavior. Additionally, the conditions imposed by unitarity are protected by the emergence of a discrete symmetry. For all these reasons and from the point of view of power counting, we can expect that the theory should be renormalizable. However, a formal proof must still be provided.

Of course, the method described here is not the only possible construction of a coloron model which is consistent at the quantum level. In [23], for example such a model is constructed by extending the QCD gauge group to \(SU(3) \times SU(3)\). In principle, one could expect the obvious difference in the chosen gauge groups should imply different structures at the quantum level. For example, in [23], it is necessary to introduce two ghost fields, one of which becomes massive due to the symmetry breaking process, in order to fix all the gauges present in the model. In our case, on the other hand, only one ghost field remains because the physical coloron is not treated as a true gauge field. Beside that, in [23], it is necessary to introduce scalar fields (the would-be Goldstone bosons) in the symmetry breaking process. Of course, in our approach, they are completely absent. Nevertheless, those differences are only apparent since our model can be obtained from the \(SU(3) \times SU(3)\) one if an interchange symmetry is imposed between the two groups and if the broken symmetry is non-linearly realized in the unitary gauge (see equation (37)). Since the usual methodology based on two groups is more general, the construction presented in this paper should be viewed as a bottom-up approach which explore the minimal conditions needed for obtaining a consistent coloron theory.
The model developed here may be phenomenologically challenging because the observation of and stable coloron at the LHC seems to be difficult.

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