Interplay Between Strong and Weak CP Phases

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Abstract

We discuss the subtle interplay between strong and weak CP phases that has often been ignored in the literature. We also point out the potentially important role that it plays in various models.

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Introduction

The mystery of CP violation has only grown with time. It was pointed out by Landau[1] in 1957 that the issue of CP violation is tightly coupled with that of complex phases in field theory. The latter, in turn, has proven to be one of the most subtle aspects of the theory — in part, due to its connection with anomalies. In strong interactions, it is particularly complicated because the relation between the phases of the field operators and physical states is highly non-trivial, stemming from the non-perturbative effects of confinement and chiral condensation. This is in particular true for chiral phases — in sharp contrast to vector phases, whose associated vectorial symmetries (e.g., flavor symmetries) are not dynamically broken by QCD. Chiral phase rotations can generate rather convoluted effects in general hadronic matrix elements, a fact which leads to a very subtle interplay between the strong and weak CP phases, as recently pointed out by the authors[2].

In this article, we wish to illustrate this result, using as an example a recently proposed superweak model of CP violation[3], and then discuss its significance for other models, including the standard Kobayashi-Maskawa Model[4].

The Illustrative Model

Consider the following superweak model of CP violation[3] with a $U(1)_X$ family gauge symmetry. The quark fields transform as

\begin{align*}
X &= +1 : s_R, b_R \quad X = -1 : c_R, t_R \\
X &= +2 : u_R \quad X = -2 : d_R
\end{align*}

A Higgs doublet $\phi_1$ transforming as $X = -1$ is needed to give mass to the $s$, $b$, $c$, and $t$ quarks. Another Higgs doublet $\phi_2$ with $X = 2$ is responsible for the $d$ and $u$ quark masses. We shall not worry about the lepton sector (see Ref.[3] for discussion of such issues.) The charges are arranged so that the anomaly is cancelled[3]. In particular, the $\text{Tr}(XXX)$ anomaly is cancelled between the up and down quarks. We also assume that CP symmetry is imposed on the Lagrangian so that, before breaking of the $U(1)_X$ symmetry, all couplings are real, and the strong CP-violation parameter $\theta_{QCD}$ is identically zero.
An $SU(2) \times U(1)$ singlet Higgs is introduced to break the $U(1)_X$ symmetry at a high energy scale of around 1-10 TeV. The $X$ boson generates flavor changing neutral currents and gives rise to the superweak interaction of the form

$$\mathcal{L}_{\text{sw}}^{\Delta S=2} = \frac{F_R}{M_X^2} (\bar{s}_R \gamma_\mu d_R)^2 + \text{h.c.}$$

where $F_R$ is a dimensionless coupling constant. There are also superweak interaction involving right-handed up type quarks, but which are not relevant here. Due to the rich phenomenology in the kaon system, this particular superweak interaction is the most relevant one experimentally. The $U(1)_X$ symmetry breaking may or may not break CP, and so the coupling $F_R$ may or may not be complex\[^{[3]}\]. We consider both possibilities below.

For our purposes, we can focus on the reduced effective theory, with CP-conserving Standard Model-type interactions and vanishing $\theta^{\text{QCD}}$, to which has been added the new superweak interaction $\mathcal{L}_{\text{sw}}^{\Delta S=2}$. We shall consider the scenario in which the up quark is massive, unless otherwise specified, while, for the sake of our argument, we will consider the limit in which $m_d$ is zero. Without the new interaction the parameter $\theta^{\text{QCD}}$ is then unphysical, with CP a good symmetry. By the usual argument, with a massless quark present — in this case, the $d$ quark — the right-handed component of that quark can be rotated to absorb $\theta^{\text{QCD}}$ via the axial anomaly, while otherwise leaving the lagrangian invariant. With the addition of the interaction $\mathcal{L}_{\text{sw}}^{\Delta S=2}$, however, $\theta^{\text{QCD}}$ becomes physical, as can be seen by considering the following cases:

(a) When $F_R$ is real and $\theta^{\text{QCD}} = 0$, CP is an unbroken symmetry of nature. If we rotate $d_R$ by an arbitrary phase, $F_R$ becomes complex and $\theta^{\text{QCD}}$ nonzero, but all other couplings are unchanged. Consider a “calculation” of $\epsilon$ using the usual techniques for the hadronic matrix elements, and with the lagrangian with $d_R$ rotated as described. This naive procedure will yield a non-zero $\epsilon$ due to the non-vanishing contribution of $\mathcal{L}_{\text{sw}}^{\Delta S=2}$ to $\text{Im}(M_{12})$ of the $K - \bar{K}$ mixing matrix — a result which, of course, is incorrect. CP is conserved, appearances notwithstanding. *Something* must occur, then, to correct the “usual techniques for the hadronic matrix elements”. New complex phases must enter into the calculation of the hadronic matrix elements to cancel the CP-odd contribution due to the unphysical complexity in $F_R$. The lesson here is that when one makes a chiral rotation of quarks,
not only does $\theta_{QCD}$ change, but the hadronic matrix elements must also undergo certain phase changes. If the strong force were to conserve CP, with breaking due only to small, weak (non-QCD) corrections, one could certainly choose a basis of states such that hadronic matrix elements are purely real (modulo absorptive contributions). In the case considered above, however, QCD (with $\theta_{QCD}$ non-zero) does not respect CP, via instanton contributions. The extent to which matrix elements must be adjusted in the presence of a non-zero $\theta_{QCD}$ is precisely that which cancels out the spurious non-zero $\epsilon$ in the example above. (One could alternatively reabsorb $\theta_{QCD}$ by a rotation of $u_R$, which would generate a complex up quark mass. Since we have assumed $m_d = 0$, the imaginary part of the mass matrix cannot be proportional to the identity matrix. Vacuum stability then requires reinterpretation of the usual meson states such that complex hadronic matrix elements are explicitly required.)

(b) If $F_R$ is complex and $\theta_{QCD} = 0$, CP is violated. In this case, the correct (non-zero) value for $\epsilon$ can be calculated without complication; all hadronic matrix elements (modulo absorptive contributions) can correctly be assumed to be real. It is illuminating to consider calculation of $\epsilon$ in another basis, which is obtained by a phase rotation of $d_R$, such that $F_R$ becomes real and $\theta_{QCD}$ non-zero. Since the two theories are the same, one must arrive at the same result for $\epsilon$. One can thus draw a rather surprising conclusion: $\theta_{QCD}$ can also, in certain situations, contribute to $\epsilon$.

In fact, from the way we obtain the $\theta_{QCD}$ contribution to $\epsilon$ in this example, one realizes that there is an important subtlety here. The actual contribution from $\theta_{QCD}$ to $\epsilon$ is correlated to the explicit mechanism of CP violation, which in our current example is the superweak $F_R$. A related result is that when $\theta_{QCD}$ is not zero, how each hadronic matrix element develops a phase also depends on the particular electroweak mechanism of CP violation in the theory. In the present case, the CP violating coupling also happens to be the chiral symmetry breaking phase.

Another lesson one learns is that the usual argument (see for example Ref.[7]), which concludes that the contribution of $\theta_{QCD}$ to CP-violating quantities such as the neutron electric dipole moment (edm) must be proportional to $m_u m_d$, is not strictly correct, a counterexample to which is offered by the simplified model presented above. The role of $m_d$ is replaced by the coupling $F_R$. Of course, $F_R$ breaks the chiral symmetry associated with $d$ quark, so that
the $d$ quark will certainly pick up mass at some (probably higher-loop) level, but the point is that the $F_R$ coupling plays a much more direct role in the contribution of $\theta_{QCD}$ than even the induced $m_d$!

Now we come to an apparent paradox whose resolution gives even further insight into the interplay between strong and weak CP phases.

We parenthetically noted above that if redefinition of the quark phases generates an imaginary part of the quark mass matrix *not proportional* to the identity matrix, the low-energy meson states must be suitably reinterpreted to ensure stability of the vacuum around which we carry out perturbation theory. This redefinition explicitly reintroduces the phase(s) rotated from the couplings into certain hadronic matrix elements, to ensure rephasing invariance. If both $m_d, m_u$ vanish, then arbitrary rotation of the corresponding right-handed quarks seems to have no effect on vacuum stability, since the mass matrix is left real and diagonal (only $m_s$ non-zero). Then, apparently, all phases may be arbitrarily rotated away, and with them, any possibility of CP violation. Specifically, consider the following variant of the two cases already considered:

(c) Let $F_R$ be complex, but take $\theta_{QCD}$ to be zero. If both $m_u, m_d$ are strictly zero, is CP conserved or violated? At first glance, one might claim the phase in $F_R$ to be unphysical, since a combined phase rotation of the form $u_R \rightarrow e^{-i\delta}u_R$ and $d_R \rightarrow e^{i\delta}d_R$ can make $F_R$ real and maintain $\theta_{QCD} = 0$. It is very tempting to claim that CP violation is proportional to $m_u$ for a small up quark mass and further that there is no CP violation when $m_u \rightarrow 0$ because the phase of $F_R$ then becomes.

This conclusion is *incorrect*, however, because we have ignored the vacuum degeneracy in the case of massless $u$ and $d$ quarks. Different choices of vacua would give different CP violation. It is true that there exists one very special vacuum where CP is conserved. However, a general vacuum possesses chiral condensate with a phase uncorrelated to that of $F_R$, and thus CP violation usually occurs, *even if* $F_R$ is real (since what is important is the relative phase between the vacuum and $F_R$). This idea can be demonstrated directly in the chiral effective lagrangian approach. The chiral field $\Sigma$ (3×3 unitary matrix) can be perturbed around a vacuum configuration diag$(e^{-i\phi}, e^{i\phi}, 1)$. If $F_R$ is turned off, the strong interaction is independent of $\phi$ because of the chiral symmetry. However, with $F_R$, the phase
\( \phi \) has physical meaning and has implications with respect to CP violation. Now we include effects of the real quark masses \( m_u \neq 0 \) and \( m_d \neq 0 \). Their net effect is simply to pick out a particular vacuum, with \( \Sigma = \text{diag}(1,1,1) \). In this case of vacuum alignment, a complex \( F_R \) is necessary, but also sufficient, for CP violation, since again it is the relative phase between \( F_R \) and the vacuum that is important. In some sense, the (possibly infinitesimal) up and down quark masses enforce CP violation, in the particular case that \( F_R \) is real, whereas in the massless case, CP violation is still generally expected via the vacuum phase.

**Discussions of other models**

The above argument can not only be applied to the superweak \( F_R \) interaction above, it can also be easily generalized to a wide variety of models — especially those in which \( d_R \) has additional interactions, including lepto-quark models, charged Higgs models, etc. It is the interplay between the low energy hadronic physics and CP violation that is the issue here and that is also why it is so easy to get confused.

Consider the KM model\(^\text{[4]}\). CP is explicitly broken by the dimension-four Yukawa couplings and thus the \( \theta_{QCD} \) is present and uncalculable. *A priori*, based on the previous argument, one does not know at all what kind of phases to expect for each hadronic matrix elements. The usual way to proceed is to make assumptions. After the quark mass matrices are diagonalized, and after making several chiral phase rotations to turn the charge current mixing matrix into the Kobayashi-Maskawa form, with only one complex phase (the Kobayashi-Maskawa phase) remaining, one then assumes that \( \theta_{QCD} \) is made vanishingly small order by order in this very special basis. With the lagrangian in this form, QCD conserves CP, so that a basis for strong eigenstates may be chosen in which all hadronic matrix elements can be made real. Note, however, that the change of phase convention from one convention (e.g., that with the KM phase), to another (e.g, the Chau-Keung\(^\text{[8]}\) convention) does not involve any chiral phase rotations.

Another example of the impact of our argument is to consider the Weinberg-Branco Three Doublet Model\(^\text{[9]}\) of spontaneous CP violation. This and many other examples are treated in Ref.\(^\text{[3]}\).
To summarize, in some models of CP violation, the CP violating mechanism itself also breaks chiral symmetry. In such cases, there can be very interesting and subtle interplay between the strong and weak CP violating effects that deserve careful study.

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