Robust Adaptive Learning-Based Path Tracking Control of Autonomous Vehicles Under Uncertain Driving Environments

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Abstract—This paper investigates the path tracking control problem of autonomous vehicles subject to modeling uncertainties and external disturbances. The problem is approached by employing a 2-degree of freedom vehicle model, which is formulated into a newly defined parametric form with the system uncertainties being lumped into an unknown parametric vector. On top of the parametric system representation, a novel robust adaptive learning control (RALC) approach is then developed, which estimates the system uncertainties through iterative learning while treating the external disturbances by adopting a robust term. It is shown that the proposed approach is able to improve the lateral tracking performance gradually through learning from previous control experiences, despite only partial knowledge of the vehicle dynamics being available. It is noteworthy that a novel technique targeting at the non-square input distribution matrix is employed so as to deal with the under-actuation property of the vehicle dynamics, which extends the adaptive learning control theory from square systems to non-square systems. Moreover, the convergence properties of the RALC algorithm are analysed under the framework of Lyapunov-like theory by virtue of the composite energy function and the $\lambda$-norm. The effectiveness of the proposed control scheme is verified by representative simulation examples and comparisons with existing methods.

Index Terms—Adaptive learning control, autonomous vehicles, trajectory tracking, convergence analysis.

ABBREVIATIONS

AV Autonomous Vehicle.
CEF Composite Energy Function.
DOF Degree of Freedom.
ILC Iterative Learning Control.
MPC Model Predictive Control.
MIMO Multiple Inputs Multiple Outputs.
RALC Robust Adaptive Learning Control.

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WITH the rapid advancement of communication and control technologies, artificial intelligence and the digitalisation of the automotive industry, the study on autonomous driving has become even more popular in recent years [1]–[5]. It is acknowledged that autonomous vehicles (AVs) can benefit from the transportation system from various aspects, including 1) increasing ride comfort and driving safety; 2) reducing traffic congestions and emissions; 3) improving overall energy efficiency [6]. Nevertheless, the development in the vehicle automation technology also poses new research challenges.

One of the most fundamental and important control problems of driverless vehicles is path tracking control, which concerns the design of a steering control strategy for following a trajectory generated by the path planning module. In the past years, to improve the path tracking control performance of AVs, many significant results have been reported in this area with the applications of advanced linear and nonlinear control techniques [7]–[15]. In addition to the tracking accuracy, the robustness also plays a key role in AV trajectory control due to the nonlinearities, modelling uncertainties and measurement disturbances involved in the vehicle dynamics [12]. A number of control techniques have been proposed in the literature, including PID and fuzzy control [10], [11], [13]–[16]. However, both PID and fuzzy control usually require a labor-intensive tuning process of the controller parameters. An active disturbance rejection control approach is proposed in [17] to perform the steering control for AVs experiencing uncertainties and external disturbances. The method has several design parameters to tune to ensure a good control performance. Furthermore, in [9], [18], [19] sliding mode control (SMC) approaches are proposed. Despite the robustness of SMC mechanism, the chattering phenomenon in control signal incurred by the fast switching logic could impair the practicality of the SMC-based methods [20]. Additionally, model predictive control (MPC) represents another attractive solution method for the tracking control for its constraints handling ability and optimality [7], [21]–[23]. However,
MPC-based approaches are known to be computationally expensive in the real-time implementation. It is noteworthy that practical AVs involve a large amount of repetitive maneuvers in certain circumstances, such as racing or parking. Therefore, it is possible to take the advantages of system repetition and in turn, improve the tracking performance based on the historical driving data/experience [24], which motivates the learning-based control techniques. A model-reference reinforcement learning control approach is proposed for the driving stabilization and uncertainty compensation of an autonomous surface vehicle [25]. The work presented in [26] introduces a parameterized batch reinforcement learning algorithm for near-optimal longitudinal control of autonomous land vehicles. In [27], a reinforcement-learning-based adaptive path tracking control approach is developed for simultaneous optimisation of the tracking accuracy and riding comfort. Furthermore, a vision-based lateral control combining with deep learning and reinforcement learning methods is proposed in [28] for lateral and steering control. The main drawback of the aforementioned machine learning based method is that there is a lack of rigorous stability and robustness analysis, instead a large amount of valid data and time consuming training processes are usually required. Furthermore, these methods are developed based on stochastic information and models, which may lead to non-smooth control signals with high frequency oscillations that hinder their practical implementation.

As an intelligent control approach, it is well known that iterative learning control (ILC) is effective in terms of tracking for uncertain dynamic systems [29]–[35]. By taking the advantages of the system repetition and utilizing the past control experience, ILC is able to improve the control performance gradually from trial to trial. In the past years, due to its appealing features and ease of implementation, ILC has been applied in many areas including autonomous driving. For instance, [36] develops a spatial ILC method for semi-autonomous driving, which enables iterative learning of human driving behaviours to achieve satisfactory tracking performance. A learning control algorithm is proposed for path following task of an AV in [37] to deal with predefined periodic trajectories with unknown periods. Moreover, [38] proposes a PD-type ILC and a quadratically optimal ILC for multiple lap path tracking of an autonomous race vehicle. While it is noteworthy that most of these works focus on linearized vehicle models, which may result in great deviations when being applied to practical driving and eventually lead to unsatisfactory control performance, especially under a uncertain condition.

In the present work, we propose a learning-based path tracking controller for AVs whose dynamics are governed by an under-actuated nonlinear system with various uncertainties. In order to properly deal with the system uncertainties and facilitate the controller design, the vehicle model is firstly reformulated into a parametric form, where uncertainties are lumped together. A robust adaptive learning control (RALC) algorithm with non-square input distribution matrix is then developed to realize the path tracking task. To perform a rigorous convergence and robustness analysis, techniques of composite energy function (CEF), as well as the $\lambda$-norm are adopted simultaneously. In contrast to existing works, the main contributions of this work are summarized as follows:

1) In contrast to the non-learning control techniques [7], [13], we propose a novel RALC approach for AVs to achieve enhanced tracking performance driving under uncertain circumstances by taking the advantages of system repetitions and historical control experience. With the proposed RALC algorithm, the point-wisely high-precision tracking instead of asymptotic convergence property along time axis can be proved;

2) Different from the reinforcement learning and deep learning techniques [25], [27], [28], the proposed RALC methodology is developed based on Lyapunov-like theory where the convergence of the proposed controller is analyzed rigorously. Moreover, the RALC algorithm requires only the most recent system input and output data instead of a vast number of training data as required by many data-driven approaches;

3) The under-actuation property of the vehicle dynamics (that induces a non-square input distribution matrix) is addressed by an ingenious technique, which can yield a pseudo-inverse-like matrix for the feedback control design. From the theoretical point of view, the proposed approach is novel, which extends the traditional ALC from square systems to non-square systems for the first time. This technique makes it possible to tackle nonlinear systems with the non-square input distribution matrix, which has important practical implications.

4) The proposed learning-based controller is shown to be effective in path tracking control of AVs, which does not only ensure a high-accuracy tracking performance but also is capable of nullifying the effect of various system uncertainties and iteration-varying external disturbances. The effectiveness, robustness, adaptiveness and learning ability of the proposed learning-based path tracking controller have been verified numerically.

This paper is organized as follows. The vehicle dynamics and path tracking control problem formulation are presented in Section II, and Section III shows some preliminary results on system reformulation and convergence properties. The learning controller design and convergence analysis are investigated in Section IV. Furthermore, an illustrative example is demonstrated in Section V. Section VI draws a conclusion. Throughout the present work, let’s denote $\mathbb{N}$ the set of natural numbers. For a given vector $x = [x_1, x_2, \cdots, x_n]^T$, $\|x\|$ denotes the Euclidean norm. For a vector $x(t) \in \mathbb{R}$, $t \in [0, T]$, $\|x(t)\|_\lambda \overset{\Delta}{=} \max_{t\in[0,T]} e^{-\lambda t} \|x(t)\|$ represents the $\lambda$-norm. For any matrix $A \in \mathbb{R}^{m \times n}$, $\|A\|$ is the induced matrix norm. Denote $|\cdot|$ the absolute value.

II. SYSTEM MODELLING AND PROBLEM FORMULATION

In this section, the dynamic and kinematic models of the vehicle will be described, which are followed by the path tracking control problem formulation.
respectively, For the angles vehicle longitudinal velocity for the front and rear wheels, \( \delta \) and \( \theta \) are the nominal tyre lateral forces of the front and rear wheels are the front and real tyre cornering stiffness, and \( \mu \) is the yaw moment of inertia,

\[ \gamma = \frac{F_f l_f - F_r l_r}{J_z} \]

where \( \gamma = \frac{v_y}{v_x} \) represents the vehicle sideslip angle with \( v_x \) and \( v_y \) being the vehicle longitudinal and lateral velocity respectively, \( \gamma \) is the yaw rate, \( m \) is the mass of the vehicle, \( l_f \) and \( l_r \) are the distances from the center of gravity to the front and rear axles respectively, \( F_f \) and \( F_r \) are the lateral forces of the front and rear axles respectively. In this work, similar as [5], [7], [19], [40], we will mainly focus on the lateral vehicle motion while ignoring the longitudinal dynamics. It is assumed that the vehicle is always moving forward with a fixed speed, namely, \( v_x > 0 \) is a constant.

Considering the effect of unknown external disturbances in uncertain driving environments, the lateral forces of the front and rear axles can be described as

\[ F_f = F_{fn} + d_f(\beta, \gamma), \]
\[ F_r = F_{rn} + d_r(\beta, \gamma), \]

where

\[ F_{fn} = 2 \mu C_f (\delta - \theta_f), \]
\[ F_{rn} = 2 \mu C_r (-\theta_r), \]

are the nominal tyre lateral forces of the front and rear wheels with \( \delta \) being the steering angle, \( d_f(\beta, \gamma) \) and \( d_r(\beta, \gamma) \) are the lateral external disturbances acting on the front and rear wheels, \( \mu \) is the tyre-road adhesion coefficient, \( C_f \) and \( C_r \) are the front and rear tyre cornering stiffness, and \( \theta_f \) and \( \theta_r \) represent the angles between the vehicle velocity and the vehicle longitudinal velocity for the front and rear wheels, respectively. For the angles \( \theta_f \) and \( \theta_r \), they can be calculated by the following relationships

\[ \tan(\theta_f) = \frac{v_y + l_f \gamma}{v_x}, \]
\[ \tan(\theta_r) = \frac{v_y - l_r \gamma}{v_x}. \]

Moreover, to focus on the path tracking ability, the kinematic model of the vehicle also needs to be taken into account, which is governed by the following differential equations [5]

\[ \dot{y}_L = v_x \beta + T_p \dot{v}_y \gamma + v_x \phi_L, \]
\[ \dot{\phi}_L = \gamma - v_x \rho, \]

where \( y_L \) represents the lateral deviation, \( \phi_L \) is the heading error, \( T_p \) is the driver’s preview time and \( \rho \) denotes the road curvature. The schematic of the kinematics is shown in Fig. 2.

By combining (1)-(9), the whole vehicle dynamics can be described as follows

\[ \dot{\beta} = \frac{2 \mu C_f}{m v_x} \delta - \gamma - \frac{2 \mu C_f}{m v_x} \arctan \left( \frac{\beta + l_f \gamma}{v_x} \right) - \frac{2 \mu C_r}{m v_x} \arctan \left( \frac{\beta - l_r \gamma}{v_x} \right) + d_1(\beta, \gamma), \]
\[ \dot{\gamma} = \frac{2 \mu C_f l_f}{J_z} \delta - \frac{2 \mu C_r l_r}{J_z} \arctan \left( \frac{\beta + l_f \gamma}{v_x} \right) + \frac{2 \mu C_r l_r}{J_z} \arctan \left( \frac{\beta - l_r \gamma}{v_x} \right) + d_2(\beta, \gamma), \]
\[ \dot{y}_L = v_x \beta + T_p \dot{v}_y \gamma + v_x \phi_L, \]
\[ \dot{\phi}_L = \gamma - v_x \rho, \]

where

\[ d_1(\beta, \gamma) \triangleq \frac{d_f + d_r}{m v_x}, \]
\[ d_2(\beta, \gamma) \triangleq \frac{d_f l_f - d_r l_r}{J_z}. \]

Furthermore, in order to fully consider the uncertain cornering characteristics of tyre, the cornering stiffness are modelled by

\[ C_f = C_{f0} + \Delta_f \]
\[ C_r = C_{r0} + \Delta_r, \]

where \( C_{f0} \) and \( C_{r0} \) are the nominal cornering stiffness values, and \( \Delta_f \) and \( \Delta_r \) are the uncertainties due to the unknown driving environments. In this paper, the following assumption is imposed to the uncertainties to facilitate the controller design.

Assumption 1: (1) The lateral external disturbances \( d_f(\beta, \gamma) \) and \( d_r(\beta, \gamma) \) are bounded. (2) The cornering
stiffness uncertainties $\Delta_f, \Delta_r$ are assumed to be bounded and the following inequality holds for $\Delta_f$
\[
\left| \frac{\Delta_f}{C_f0} \right| < \xi < 1 \tag{14}
\]
with $\xi > 0$.

**Remark 1:** In practice, it is reasonable to assume that the external disturbances $d_f, d_r$ and the cornering stiffness uncertainties $\Delta_f, \Delta_r$ are bounded in a normal operation status [12], [40], [41]. Moreover, it also makes sense to assume that the magnitude of the cornering stiffness uncertainty $\Delta_f$ is smaller than the nominal cornering stiffness $C_f0$. Otherwise, the vehicle may experience an abnormal driving environment.

### B. Control Problem Formulation

Let $\beta_r, \gamma_r, y_{L,r}, \varphi_{L,r}$ be the reference trajectories to be followed and $\delta_k$ be the reference input. Given the vehicle dynamics (10)-(13) which operates in a repetitive manner, our control objective is to find a sequence of control inputs $\delta_k$ such that the system states can track the references as $k$ increases, namely,
\[
\lim_{k \to \infty} [\beta_k, \gamma_k, y_{L,k}, \varphi_{L,k}] = [\beta_r, \gamma_r, y_{L,r}, \varphi_{L,r}].
\]
where $k \in \mathbb{N}$ represents the trial or iteration index.

### III. Preliminary Results

From the last section, it is obvious that the vehicle dynamics (10)-(13) are under-actuated. For path tracking control of such system, the main difficulty lies in that to achieve a high-accuracy tracking performance in presence of system uncertainties and external disturbances by using the only control variable $\delta_k$. To facilitate the controller design and convergence analysis, we will firstly reformulate (10)-(13) with new state variables into a parametric form and then provide some preliminary results for the upcoming convergence analysis.

#### A. Reformulation of the Vehicle System Model

Define two new state variables $x_1$ and $x_2$ as
\[
\begin{align*}
x_1 &= \beta + \frac{1_f}{v_x} \gamma, \\
x_2 &= \beta - \frac{1_r}{v_y} \gamma.
\end{align*}
\tag{15}
\]
The system (10)-(13) can be rewritten as follows
\[
\begin{align*}
\dot{z} &= Az + Bx + B_p\rho \\
\dot{x} &= f(x)\Theta + \bar{B}(C_f0 + \Delta_f)u + \omega
\end{align*}
\tag{16}
\]
where $x = [x_1, x_2]^T$, $z = [y_L, \varphi_L]^T$, $\omega = [d_1, d_2]^T$.

\[
A = \begin{bmatrix} 0 & v_x \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} v_x l_f + T_p \omega_y & v_x l_f - T_p \omega_y \\ \frac{l_f + l_r}{v_x} & \frac{l_f + l_r}{v_y} \end{bmatrix}, \quad B_p = \begin{bmatrix} 0 \\ -\frac{1}{v_x} \end{bmatrix},
\]

\[
u = \delta - \arctan(x_1), \quad \bar{B} = \begin{bmatrix} 2 & 2 \frac{l_f}{l_f + l_r} \\ \frac{2}{l_f + l_r} \end{bmatrix}, \quad \Theta = \begin{bmatrix} 1 + \frac{\Delta_f}{C_f0} \end{bmatrix}, \quad \bar{f}(x) = \begin{bmatrix} \bar{T}_{11}(x) \bar{T}_{12}(x) \\ \bar{T}_{21}(x) \bar{T}_{22}(x) \end{bmatrix}
\]
with
\[
\begin{align*}
\bar{T}_{11}(x) &= -\frac{v_x}{1_f + l_f} (x_1 - x_2), \\
\bar{T}_{12}(x) &= -\left(2 \frac{\mu C_f0}{mv_x} - \frac{2 \mu l_f C_f0}{J_f v_x} \right) \arctan(x_2), \\
\bar{T}_{21}(x) &= -\frac{v_x}{l_f + l_r} (x_1 - x_2), \\
\bar{T}_{22}(x) &= -\left(2 \frac{\mu_l^2 C_f0}{J_r v_x} + \frac{2 \mu C_f0}{mv_x} \right) \arctan(x_2),
\end{align*}
\]
and $\omega = [\omega_1, \omega_2]^T$ with
\[
\omega_1 \equiv d_1(\beta, \gamma) + \frac{l_f}{v_x} d_2(\beta, \gamma), \quad \omega_2 \equiv d_1(\beta, \gamma) - \frac{l_r}{v_y} d_2(\beta, \gamma).
\]
Since the disturbances $d_1$ and $d_2$ are bounded, there exists positive constant $\kappa$ such that
\[
\|\omega\| \leq \kappa. \tag{18}
\]

With the newly introduced state variables $x_1$ and $x_2$, it is obvious that the original vehicle system (10)-(13) has been reformulated into a cascade form with a linear part and a nonlinear part, where the control input only acts on the nonlinear part. Moreover, the nonlinear part (17) has been reformulated into a parametric form with (1) the function $f(x)$ being known and global Lipschitz continuous with respect to $x_1$ and $x_2$, (2) the rear cornering stiffness being inserted into parametric uncertainties and (3) the front cornering stiffness being treated as the input distribution uncertainties. Finally, to indicate the system repetition, the iteration index $k \in \mathbb{N}$ is added as subscripts and the vehicle dynamics becomes
\[
\begin{align*}
\dot{z}_k &= Az_k + Bx_k + B_p\rho, \\
\dot{x}_k &= \bar{f}(x_k)\Theta + \bar{B}(C_f0 + \Delta_f)u_k + \omega_k, \\
\delta_k &= u_k + \arctan(x_{1,k}),
\end{align*}
\tag{19}
\]
where $x_k = [x_{1,k}, x_{2,k}]^T$, $z_k = [y_{L,k}, \varphi_{L,k}]^T$. Furthermore, to simplify the convergence analysis, it is assumed that the reference trajectories are generated by the following dynamics
\[
\begin{align*}
\dot{z}_r &= Az_r + Bx_r + B_p\rho, \\
\dot{x}_r &= \bar{f}(x_r)\Theta + \bar{B}(C_f0 + \Delta_f)u_r, \\
\delta_r &= u_r + \arctan(x_{1,r}),
\end{align*}
\tag{20}
\]
with $z_r = [y_{L,r}, \varphi_{L,r}]^T$ and $x_r = [x_{1,r}, x_{2,r}]^T$. Additionally, to ensure the perfect tracking performance, the following identical initial condition assumption is imposed.

**Assumption 2:** At each iteration, the initial states of the system (19)-(20) can be reset to the same position with the reference trajectories, namely, it is assumed that $x_k(0) = x_r(0)$ and $z_k(0) = z_r(0)$.

**Remark 2:** The identical initialization condition (i.i.c.) is a common assumption in ILC area, which is imposed to


B. Preliminary Result On System Convergence

For the system (19)-(21), the control objective is to find a sequence of the control input \( \delta_k \) such that the system states \( x_0 \) and \( z_k \) are able to track the predefined reference \( x_r \) and \( z_r \) as the iteration number \( k \) approaches to infinity. Thanks to the reformulation of the vehicle dynamical model from (10)-(13) into (19)-(21), we can establish the following preliminary result which would simplify the controller design and convergence analysis of the whole system.

**Proposition 1:** Consider the system (19) under the Assumption 2, the convergence of \( x_k(t) \rightarrow x_r(t) \), \( t \in [0, T] \) implies the convergence of \( z_k(t) \rightarrow z_r(t) \), \( t \in [0, T] \) in the iteration domain.

**Proof of Proposition 1.** Denote \( e_{x,k} \triangleq x_k - x_r \) and \( e_{z,k} \triangleq z_k - z_r \). From the systems (19) and (22), we can obtain that

\[
\dot{e}_{z,k} = Ae_{x,k} + Be_{z,k}
\]  

(25)

whose solution can be represented as

\[
e_{z,k}(t) = e^{\lambda t} e_{z,k}(0) + \int_0^t e^{\lambda(t-\tau)} Be_{z,k}(\tau) d\tau
\]

(26)

where \( e_{z,k}(0) = 0 \) is utilized. By taking the norm on both sides of (26), it gives

\[
\|e_{z,k}(t)\| \leq \int_0^t \|e^{\lambda(t-\tau)}\|B\|e_{z,k}(\tau)\|d\tau,
\]

(27)

where \( \lambda_{\text{max}} > 0 \) can be any positive constant. Further taking the \( \lambda \)-norm, we have

\[
\|e_{z,k}(t)\| \leq \max_{t \in [0,T]} e^{-\lambda t} \int_0^t e^{\lambda(t-\tau)} \|B\|\|e_{z,k}(\tau)\|d\tau
\]

\[
\leq \max_{t \in [0,T]} e^{(\lambda_{\text{max}} - \lambda)} \|B\| \int_0^t e^{(\lambda_{\text{max}} - \lambda)\tau} d\tau \|e_{z,k}\|_{\lambda}
\]

\[
= \|B\| \left( 1 - e^{(\lambda_{\text{max}} - \lambda)T} \right) \|e_{z,k}\|_{\lambda}
\]

\[
\triangleq \zeta_0 \|e_{z,k}\|_{\lambda},
\]

(28)

where \( \lambda > \lambda_{\text{max}} \). From (28) and the norm equivalence theorem, it is obvious that the convergence of \( e_{x,k}(t) \) can be ensured as long as \( e_{x,k} \) converges in the iteration domain. That is, the convergence of \( z_k \) can be implied directly from the convergence of \( x_k \). Therefore, when designing the tracking controller, only the convergence of the system state \( x_k \) needs to be taken into account, which thus will simplify the controller design and convergence analysis for the whole system. ■

IV. LEARNING-BASED LATERAL CONTROLLER DESIGN

Based on Proposition 1, this section will focus on the subsystem dynamics (20) and design the trajectory tracking controller for the AV.

In view of the subsystem (20), to facilitate and simplify the controller design process, let’s reformulate it as follows by adding and subtracting the derivative of the reference \( x_r \)

\[
\dot{x}_k = f(x_k)\Theta + \dot{x}_r + b(1 + \Pi_f)u_k + o_k
\]

(29)

where \( b \triangleq \bar{B}C_{f0}, \Pi_f \triangleq \Delta_f/C_f \), and

\[
f(x_k) \triangleq \left( \frac{T_{11}(x_k) - \dot{x}_1}{T_{21}(x_k) - \dot{x}_2}, \frac{T_{12}(x_k)}{T_{22}(x_k)} \right).
\]

From (29), the error dynamics can be easily derived, which is described as

\[
\dot{e}_{x,k} = \dot{x}_k - x_r = f(x_k)\Theta + b(1 + \Pi_f)u_k + o_k.
\]

(30)

In order to realize the path tracking control, the following robust adaptive learning-based control scheme is proposed for the subsystem (29)

\[
\begin{align*}
\dot{u}_k & = \begin{cases} 
\frac{b^T}{\|bb^T\|}[-K e_{x,k} - f(x_k)\dot{\Theta}_k] \\
- \frac{\|e_{x,k}\|}{\|e_{z,k}\|} K e_{x,k} \\
+ f(x_k)\Theta_k \|e_{x,k}\| \\
- \frac{\|e_{x,k}\|}{\|e_{z,k}\|} K e_{x,k} \zeta(e_{x,k}), \quad e_{x,k}^T b \neq 0 \\
\frac{b^T}{\|bb^T\|}[-K e_{x,k} - f(x_k)\dot{\Theta}_k], \quad e_{x,k}^T b = 0
\end{cases} \\
\dot{\Theta}_k & = \dot{\Theta}_{k-1} + \Gamma\left( h(x_k) - e_{x,k} \right) e_{x,k}^T e_{x,k}
\end{align*}
\]

where \( K \in \mathbb{R}^{2 \times 2} \) is the control gain matrix to be designed that is assumed to be positive definite, and \( \dot{\Theta}_k \in \mathbb{R}^2 \) is the estimate of the parametric uncertainty \( \Theta \). The updating law for \( \dot{\Theta}_k \) is developed as follows

\[
\dot{\Theta}_k(t) = \dot{\Theta}_{k-1}(t) + \Gamma^T(x_k)e_{x,k},
\]

\[
\Theta_0(t) = 0, \quad t \in [0, T]
\]

(32)

where \( \Gamma \) is the learning gain matrix to be determined that is diagonal with positive diagonal elements.

**Remark 3:** Note that the robustness of the proposed path tracking controller to the system uncertainties and external disturbances are mainly determined by the second and third terms on the right hand side of (31) when \( e_{x,k}^T b \neq 0 \). The second term is designed to compensate the uncertainties \( \Delta_f \) in the front cornering stiffness, while the third term is incorporated to reject the external disturbances \( \bar{d}_f \) and \( \bar{d}_s \).

In terms of the rear cornering stiffness uncertainty \( \Delta_r \), it is adaptively estimated by the parametric updating law (32).
Moreover, when implementing the proposed path tracking controller, the initial input $u_0$ may be set to zero directly or generated by any prespecified feedback controller that leads to bounded tracking error $e_{x,0}$.

**Remark 4:** From (31) and the relationship (21), it is obvious that the steering angle $\delta_t$ is at least second order differentiable with respect to the time $t$ for both the scenarios of $e_{x,k}^T b \neq 0$ and $e_{x,k}^T b = 0$ because of the smoothness of the function $\tanh()$ and Euclidean norm. Furthermore, in order to avoid numerical singularities that may occur when $\|e_{x,k}^T b\|^2$ is very close to zero, the controller (31) can be adjusted in implementation as follows

$$ u_k = \frac{b^T}{\|b\|^2} (-K e_{x,k} - f(x_k) \hat{\Theta}_k) $$

where the i.i.c. in the Assumption 2 is utilized. By substituting the controller (31) into the error dynamics (30), it gives

$$ \dot{e}_{x,k} = f(x_k) + \frac{b b^T (1 + \Gamma_f)}{\|b\|^2} [-K e_{x,k} - f(x_k) \hat{\Theta}_k] $$

and

$$ \begin{align*}
\dot{e}_{x,k} &= \frac{-b(1 + \Gamma_f) b^T e_{x,k}}{(1 - \zeta) \| e_{x,k} b \|^2} (2 + \zeta) \| K e_{x,k} + f(x_k) \hat{\Theta}_k \| \| e_{x,k} b \| \\
&\quad - b(1 + \Gamma_f) b^T e_{x,k} \kappa e_{x,k} \xi (e_{x,k}) + \alpha_k
\end{align*} $$

Considering Assumption 1, there has $\xi + \Gamma_f > 0$. Hence, by using the equation $1 + \Gamma_f = (1 - \zeta) + (\xi + \Gamma_f)$, we can obtain the following inequality

$$ -e_{x,k}^T b (1 + \Gamma_f) b^T e_{x,k} \leq -\| e_{x,k} b \|^2 - \frac{\|b\|^2}{\|b\|^2} (\xi + \Gamma_f) e_{x,k}^T b e_{x,k} $$

which gives

$$ -e_{x,k}^T b (1 + \Gamma_f) b^T e_{x,k} \leq -\| e_{x,k} b \|^2 $$

Moreover, the following inequality holds

$$ e_{x,k}^T b (1 + \Gamma_f) b^T e_{x,k} \leq \| e_{x,k} b \|^2 - \frac{\|b\|^2}{\|b\|^2} (\xi + \Gamma_f) e_{x,k}^T b e_{x,k} $$

Consequently, by premultiplying $e_{x,k}^T$ on both sides of (36) and substituting (38) and (39) into it, we can obtain that

$$ e_{x,k}^T \dot{e}_{x,k} \leq -e_{x,k}^T K e_{x,k} - e_{x,k}^T f(x_k) \Phi_k - \kappa e_{x,k} \xi (e_{x,k}) + e_{x,k}^T \alpha_k $$

Claim [47]: The following inequality holds for any given $\epsilon > 0$ and for any $u \in \mathbb{R}$

$$ 0 \leq |u| - u \tanh\left(\frac{u}{\epsilon}\right) \leq \sigma \epsilon $$

where $\sigma$ is a constant that satisfies $\sigma = e^{-(\sigma + 1)}$, i.e., $\sigma = 0.2785$. Now, by applying the above claim, we have

$$ -\kappa e_{x,k} \xi (e_{x,k}) + e_{x,k}^T \alpha_k \leq \kappa (|e_{x1,k}| + |e_{x2,k}|) - \kappa e_{x,k} \xi (e_{x,k}) \leq 2\kappa \epsilon $$

By combining (41) with (40), it yields

$$ e_{x,k}^T \dot{e}_{x,k} \leq -e_{x,k}^T K e_{x,k} - e_{x,k}^T f(x_k) \Phi_k + 2\kappa \epsilon $$

This completes the proof of Theorem 1.
According to (35) and (42), we have
\[
\frac{1}{2}e_{x,k}^TKe_{x,k} + \int_0^T [e_{x,k}^TKe_{x,k} + 2\kappa\sigma e^T]dt + 2\kappa\sigma eT. 
\]
(43)

In terms of the last term of (34), it holds that
\[
\Phi_k^T\Gamma^{-1}\Phi_k - \Phi_{k-1}^T\Gamma^{-1}\Phi_{k-1} \\
= (\hat{\Theta}_{k-1} - \hat{\Theta}_k)^T\Gamma^{-1} [(\hat{\Theta}_k - \hat{\Theta}_{k-1}) - 2\Phi_k] \\
= -(\hat{\Theta}_k - \hat{\Theta}_{k-1})^T\Gamma^{-1}(\hat{\Theta}_k - \hat{\Theta}_{k-1}) \\
-2(\hat{\Theta}_k - \hat{\Theta}_{k-1})^T\Gamma^{-1}\Phi_k \\
= -e_{x,k}^Tf(x_k)\Gamma f(x_k)e_{x,k} + 2e_{x,k}^Tf(x_k)\Phi_k \\
\leq 2e_{x,k}^Tf(x_k)\Phi_k 
\]
(44)
where the updating law (32) is applied. Therefore, the last term on the right hand side of (34) becomes
\[
\frac{1}{2}\int_0^T (\Phi_k^T\Gamma^{-1}\Phi_k - \Phi_{k-1}^T\Gamma^{-1}\Phi_{k-1})dt \leq \int_0^T e_{x,k}^Tf(x_k)\Phi_kdt. 
\]
(45)

Finally, combining (43), (45) with (34), we have
\[
\Delta E_k \leq -\int_0^T e_{x,k}^TKe_{x,k}dt - \frac{1}{2}e_{x,k-1}^T + 2\kappa\delta eT \\
\leq -\frac{1}{2}\|e_{x,k-1}\|^2 + 2\kappa\sigma eT 
\]
(46)
as \( K \) is positive definite. Due to the existence of the constant term \( 2\kappa\sigma eT \), the negative definite property of \( \Delta E_k \) cannot be ensured for all iterations. That is, the perfect tracking performance would not be achieved when the external disturbances exist. Instead, we will show that the tracking error \( e_{x,k} \) will enter a neighborhood of zero within finite iterations.

**Part II. Convergence property of the tracking error:** From (46), it is not difficult to derive that
\[
\sum_{j=1}^k \Delta E_j(t) \leq -\frac{1}{2}\sum_{j=1}^k (\|e_{x,j-1}\|^2 - 4\kappa\sigma eT) 
\]
(47)
which gives
\[
E_k(t) \leq E_0(t) - \frac{1}{2}\sum_{j=1}^k (\|e_{x,j-1}\|^2 - 4\kappa\sigma eT). 
\]
(48)

Note that \( E_k(t) \) is positive, and \( E_0(t) \) is bounded as both \( e_{x,0} \) and \( \hat{\Theta}_0 \) are finite. Therefore, we can obtain from (48) that \( \|e_{x,k}(t)\|, t \in [0, T], \) is bounded for any finite \( k \). Furthermore, for any given constant \( \tau > 0 \), there exists a finite integer \( k_0 \in \mathbb{Z}^+ \) such that
\[
\|e_{x,k}(t)\|^2 \leq 4\kappa\sigma eT + \tau 
\]
(49)
for any \( k > k_0 \). The argument can be obtained by proof by contradiction. If the condition (49) does not hold, we have, \( \|e_{x,k}(t)\|^2 \geq 4\kappa\sigma eT + \tau \) holds for \( k \rightarrow \infty \). In such case, we have that the RHS of (48) will go to minus infinity, which contradicts the positive definiteness of \( E_k(t) \). Finally, the tracking error \( \|e_{x,k}(t)\| \) will enter into the specified neighborhood of zero with the radius of \( 2\sqrt{\kappa\sigma eT + \tau} \) within finite iterations.

**Remark 5:** From the expression of \( 2\sqrt{\kappa\sigma eT + \tau} \), it is obvious that the convergence accuracy of the tracking error is related to the bound of the external disturbances \( \omega_e \) and any prespecified non-zero bound of tracking error can be obtained by tuning the design parameters \( \epsilon \) and \( \tau \) appropriately. It is worthwhile to mention that in the controller (31), to ensure the smoothness/continuity of the steering angle \( \delta_k \), a hyperbolic tangent function \( \varsigma \) is employed, which leads to a non-zero convergence result of the tracking errors. In fact, this can be avoided by replacing the hyperbolic tangent function with a sign function, which may guarantee the zero convergence of the tracking errors by sacrificing the smoothness of the control input. In real-time applications, one can select/tune the parameter \( \epsilon \) in the function \( \varsigma \) according to practical demand to ensure a continuous control input signal or to achieve a high-accuracy track performance.

**Remark 6:** From the convergence analysis of the Theorem 1, it is obvious that the positive definiteness of the feedback gain \( K \) and the learning gain \( \Gamma \) is the only condition to guarantee the convergence for the proposed control algorithm. Moreover, from the inequalities presented in (44) and (46), we can find that the difference of the CEF \( \Delta E_k \) depends on the magnitudes of the minimal eigenvalues of the feedback gain \( K \) and the learning gain matrix \( \Gamma \). That is, the convergence and learning rates of the proposed control algorithm may be improved by increasing the minimal eigenvalues of \( K \) and \( \Gamma \). However, high feedback and learning gains usually lead to increased control effort, which may eventually be saturated due to the actuator limits. Meanwhile, high-frequency components (noise) can be excited. Therefore, there exists a trade-off between the convergence rate and the control efforts as well as the steady-state accuracy. This implies that both the constraints of the actuator and system noise should be considered when designing \( K \) and \( \Gamma \).

**Remark 7:** According to the proposed RALC scheme (31)-(32), we can see that the control inputs to the vehicle system are updated along the iteration axis, and the input signal \( u_k \) at the \( k \)th iteration only requires the system information at the \( k \)th and \( k-1 \)th iterations, i.e., \( e_{x,k}, f(x_k), \hat{\Theta}_k \). The historical system data before the \( k-1 \)th iteration is not adopted in the proposed learning algorithm. Moreover, different from other data-driven methods, such as deep learning and neural networks, the time-consuming training process is not needed for the proposed RALC.

**Corollary 1:** For system (19) and (20), the proposed learning control law (31) and (32) ensure the convergence of the tracking errors \( e_{x,k}(t) \) and \( e_{x,k}(t) \), \( t \in [0, T] \) simultaneously as the iteration number increases.

Finally, according to the results presented in Theorem 1 and Corollary 1, the path tracking results of the AV can be summarized as follows.

**Theorem 2:** Consider the vehicle system (10)-(13) operating in a repetitive manner, the learning controller (31) and
the parametric updating law (32) ensure that the system states $\beta_k, \gamma_k, y_{L,k}, \phi_{L,k}$ are able to approach to the references $\beta_r, \gamma_r, y_{L,r}, \phi_{L,r}$ as close as possible when the iteration number increases.

Remark 8: In contrast to the traditional ILC that is developed based on the contraction-mapping method, where system nonlinearities has to be globally Lipschitz [32], the proposed RALC applies a Lyapunov-like method to design the learning rules, which is effective to handle more generic nonlinear systems. Furthermore, according to the controller design and convergence analysis presented in Theorem 1, the proposed RALC is proven to be able to deal with the iteration-time-varying system uncertainties, input distributed uncertainties and external disturbances, which might be extremely hard to be compensated with the traditional ILC. Therefore, the proposed approach is more effective in dealing with various system uncertainties. In addition, for the concerned control problem in this work, the vehicle dynamics have only an input and four outputs, and thus the Markov parameter matrix $CB$ is not invertible, which implies that the necessary condition for the traditional ILC to converge does not hold. That is, for the problem considered in the present work, the traditional ILC is not applicable.

V. CASE STUDIES AND DISCUSSIONS

In order to verify the effectiveness of the proposed control approach, this section presents two case studies respectively for vehicle kinematics with zero and nonzero road curvatures with consideration of different longitudinal speeds and adhesion coefficients. Also, in the nonzero road curvature scenario, the proposed method is benchmarked against three conventional control methods: the traditional PI control [11], the backstepping variable structure control (BVSC) [40] and the PD-type ILC [38]. The vehicle parameters adopted in simulations are presented in Table I. The operation time interval is assumed to be $[0, 20]$ s. To show the robustness of the proposed control strategy to disturbances, the system uncertainties are assumed to be $d_f = 10 \cos(3t) + \sin(t)$, $d_r = 10 \sin(3t) + \sin(t)$. $\Delta f = 5 \sin(2t) + \cos(t)$, $\Delta r = 5 \cos(2t) + \cos(t)$. All the simulations are performed in MATLAB2019b on a 32GB RAM desktop PC with i7-9700 CPU.

Case 1: With zero road curvature.

In this case study, simulations based on double lane-changing trajectories are conducted with the road curvature $\rho = 0$. The longitudinal speed is firstly set to be $v_1 = 10$ m/s, and the feedback gain $K$ and the learning gain $\Gamma$ are selected as follows

$$K = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 1000 & 0 \\ 0 & 1000 \end{bmatrix}. \quad (50)$$

The threshold is set to be $\eta = 1.5 \times 10^{-4}$. When $\|e_{x,k}^T b\|^2 > \eta$, we apply the controller presented in (31) together with the parametric updating law (32), and $\xi, \kappa$ are selected as $0.1, 6 \times 10^{-6}$, respectively. When $\|e_{x,k}^T b\|^2 \leq \eta$, the controller is adjusted as follows

$$u_k = \frac{b^T e_{x,k}}{\|b^T\|} \left[-K e_{x,k} - f(x_k) \hat{\theta}_k \right]$$

The simulation results are presented in Figs. 3-4, where the reference trajectories are presented in red lines. As shown in Fig. 3, thanks to the learning ability of the proposed controller, all the states of the vehicle are able to gradually converge to the predefined reference trajectories as the iteration number increases, despite the presence of system uncertainties and external disturbances. From Fig. 4, we can see that the

| Parameter | Value |
|-----------|-------|
| $I_e$ | 0.2280 kg m^2 |
| $C_f$ | 0.67810 N rad |
| $I_f$ | 0.1192 m |
| $l_e$ | 1.598 m |
Fig. 4. The maximal tracking error profile along the iteration axis for case 1 with $v_x = 10$.

almost perfect tracking performance can be achieved within 250 iterations with the maximal tracking error $\|e_k\|_{\text{sup}}$ drops by around 90%, where

$$\|e_k\|_{\text{sup}} \triangleq \max_{t \in [0, T]} \sqrt{e_{\beta,k}^2 + e_{\gamma,k}^2 + e_{\phi,k}^2 + e_{yL,k}^2}$$

with $e_{\beta,k}$, $e_{\gamma,k}$, $e_{\phi,k}$, $e_{yL,k}$ being the tracking errors for $\beta_k$, $\gamma_k$, $\phi_k$, $yL_k$, respectively. Moreover, according to Fig. 3, it is worthwhile to note that in contrast to traditional feedback controllers that focus on the asymptotic convergence performance along the time axis, the proposed RALC method is able to improve the tracking performance at each time instant gradually. The computation time for each iteration of learning is 0.46s, which justify the practical application value of the proposed method.

Furthermore, to show the robustness of the proposed control scheme against longitudinal speed variations, simulations with $v_x = 15\text{m/s}$, $v_x = 20\text{m/s}$ and $v_x = 30\text{m/s}$ are also conducted. The results are presented in Fig. 5, including the convergence of the maximal tracking error along iteration axis as well as the tracking error profiles for the lateral deviation and heading error profiles at the 250th iteration. From Fig. 5, it can be seen that with the increasing of the longitudinal speed, the convergence of the proposed RALC can still be guaranteed. Although the tracking errors are slightly increased when $v_x$ increases, they are reasonable and acceptable from practical point of view, as the real-world driving speed is constrained (e.g., legal speed limit).

Additionally, we would like to mention that the entire computation time for 250 iterations of learning is 115.75 seconds, in which the computation for the reference trajectories takes 0.23 seconds and the computation time for each iteration of learning is just 0.46 seconds, which justify the practical application value of the proposed method.

Case 2: With nonzero road curvature.

To further validate the proposed control approach, we first assume the curvature of the reference path to be $\rho = 0.02e^{-0.02t} \sin(0.5t)$. The longitudinal speed is set to $v_x = 10$ for illustration purpose. The simulation results are presented in Figs. 6-8. In Fig. 6, the convergence of the maximal tracking error is shown, from which we can see that the convergence of the proposed RALC can still be guaranteed when the reference path is winding. The tracking performance for lateral deviation $yL_k$ and the heading error $\phi_k$ at the 150th iteration is presented in Figs. 7 and 8. It is shown that after 150 iterations of learning, the vehicle can precisely track the reference trajectories at each time instant. Furthermore, the comparisons with the traditional PI control, the BVSC and the PD-type ILC are also shown in Figs. 7 and 8. We can see that owing to the adaptive learning ability along the iteration axis, the proposed RALC displays more impressive tracking performance than the PI, BVSC and PD-type ILC approaches. In particular, as the PD-type ILC is developed based on the linearized vehicle model, its performance is degraded in presence of the system nonlinearities and uncertainties. In addition, due to the utilization of the derivative of tracking errors, oscillations can be introduced at the beginning of the operations. For the PI control and BVSC, although
they also show great ability to handle the system uncertainties, it is not easy to achieve accurate tracking performance within a finite time interval. Furthermore, their control performances remain the same as the vehicle performs repetitive tasks.

Moreover, owing to the robustness, adaptiveness and learning ability, the proposed learning-based path tracking controller can deal with other potential limiting conditions, such as severe road surface conditions, shaped curve road conditions, etc. As a representative example, the results of the driving along an 8-shaped track after 100 iterations of learning are depicted in Fig. 9 with different adhesion coefficients, namely, $\mu = 0.5$, $\mu = 0.75$, $\mu = 1$. It can be observed that although a decrease in the friction condition leads to a slight increase in the steady-state error, the proposed controller ensures a satisfactory tracking accuracy under limiting conditions.

VI. CONCLUSION

The present work concerns the robust path tracking control of autonomous driving with the consideration of various system uncertainties and disturbances. Inspired by the newly introduced parametric representation of the vehicle dynamics, a novel robust adaptive learning control (RALC) algorithm is developed by taking advantages of the system repetition. Moreover, a rigorous convergence analysis for the proposed control strategy is conducted under the framework of the composite energy function methodology with aid of the $\lambda$-norm. The effectiveness, robustness, adaptiveness and learning ability of the RALC have been verified via illustrative simulations, where it has shown that the tracking error can be reduced gradually to an acceptable small value from iteration to iteration. Furthermore, it is noteworthy that a specialised technique is developed to handle the non-square input distribution matrix caused by the under-actuated vehicle dynamics. Such a novel methodology in the field of adaptive iterative learning control (AILC) has the potential to extend the application scope of AILC. Future research efforts will be devoted to the development of a hardware-in-loop testbed so as to enable real-time validation of the proposed RALC.

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