Flood characteristic description using the singularity method

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Abstract. An advanced understanding of the flood mechanism will provide scientific evidence for decision makers for assessing water resources and development planning. Mathematical models can be used to develop new indices for flood characteristics description and forecasting. In this study, a singularity index (fractal model) was developed by using peak flow and flow discharge data with time to describe flooding events in the Oak Ridges Moraine (ORM) area, especially for characterizing flow change properties. It was found that the singularity index can be used to describe flood characteristics, which describes the changing behaviour around the peak of the flood flow. The higher the singularity index, the more quickly the flood process changes, and vice versa. It was also observed that the singularity index is higher when more watersheds have urban land cover, and it is lower when fewer watersheds have urban land cover and more have agricultural land cover.

1. Introduction
Floods are a common phenomenon, which are a result of an increase in streamflow beyond the normal stream channel that contains water [1]. The Oak Ridges Moraine (ORM) area in southern Ontario is a major aquifer complex and hydrogeologically unique. The ORM area is affected by extreme hydrological events, such as floods. An advanced understanding of the mechanism of flood forecasting will provide scientific evidence that will help decision makers in assessing water resources and in development planning.

Mathematical models can be used to develop new indices for flood characteristics description and forecasting. Caldarola [2] and Bonora [3] reviewed the mathematical modeling for urban water networks and developed new mathematical framework for urban water distribution system. Di Nardo [4] used the fractal theory for the assessment of water distribution network. Natural patterns, such as floods, normally appear very complex but they may display an underlying simplicity through scale-invariance, which means the pattern appears unchanged under contraction. The key idea in fractal geometry is self-similarity, which means an object can be decomposed into small copies of itself. Fractals are a natural consequence of self-similarity resulting from scale-independent processes [5] and fractal models can be used to characterize complex physical processes and dynamic systems [6] [7]. The Power-law distribution is the only type of distribution that shows scale invariance and the self-similarity property and the Power law relation is a common tool used in fractal analysis, which plot the log(y) versus the log(x) and check whether there exists a linear relation between log(y) and log(x).
The concept of singularity is used for characterizing the anomalous behaviors of singular physical processes that result in anomalous amounts of energy release within a narrow spatial-temporal interval and these nonlinear processes can be modeled as fractals or multifractals [8]. The singularity characterizes how the statistical behavior varies as measuring scale changes. For example, those locations can be called singular location that the mean value proportionally depend on the size of the vicinity [9]. The local singularity index proposed by Cheng [10] can be used to characterize flood events because they release anomalous amounts of water within a relatively short time. The local singularity method has been widely used for geochemical anomaly identification [11] and mineral resource assessment [12]. Cheng et al. [13] indicated that the singularity model is superior to the power-law flow model, which is commonly used to characterize peak flow events. Further study indicates that singularity can be used to characterize the property of the flow change.

This study applies the singularity fractal model to describe flood characteristics for characterizing flow change properties. The spatial distribution pattern of the singularity index in the ORM area is obtained by mapping this index.

2. Methods

2.1. Datasets
The datasets used in this study were National HYDAT CD produced by Environment Canada, which provides access to the National Water Data Archive, containing daily, monthly, and instantaneous streamflow and water level data for over 2500 active and 5500 discontinued hydrometric stations in Canada. In the ORM area, 25 gauging stations were selected for the study, because at least 8 years of continued stream flow data were recorded for these stations and they are still operating. The selected stream gauging stations and corresponding watersheds were mapped using ArcGIS (Figure 1). The coordinate information of the stream gauging stations is obtained from the HYDAT CD. Each stream gauging station within the study area is used as a seed point, meaning that all the precipitation collected within each watershed boundary eventually flows through the stream gauging station.

Figure 1. Selected watersheds and stream gauging stations in the ORM area.
The Canadian Disaster Database (CDD) contains detailed disaster information that affected Canadians over the past century. Flood disasters that occurred in Ontario from 1900 to 2000 were used for this study.

**Figure 2.** Relation between peak flow data and time duration (Straight lines are fitted using the least squares method) R2 is linear correlation coefficient.

2.2. Singularity for flood analysis
The flood events at the 02HB001 station were selected for the singularity analysis. Assume $Q_1, Q_2, \ldots,$ and $Q_k$ are the daily flow following the peak flow, usually in the descending order. The average flow value $Q^*(k)$ within $k$ days from the flow peak may follow the power-law relation with the measuring unit $k$. 
where the exponent $\Delta \alpha = 1-\alpha$ and the constant $c$ are two indexes that are independent of the averaging unit $k$. The constant value $c$ indicates the height of the curve, whereas the exponent $\Delta \alpha$ characterizes the shape of the curve [7].

Therefore, we plot the value of $\mu(\epsilon)$ against $\epsilon$ on a log-log plot. If these values of $\mu(\epsilon)$ show a linear trend with $\epsilon$, then the average flow value and the measuring unit $k$ are in a power-law relationship. The slope estimated from this linear relationship ($\Delta \alpha$) can be considered as a singularity index.

The singularity index $\alpha$ has the following properties [10]:

1. $\Delta \alpha = 0$, if $\mu(\epsilon)$ = constant, which is independent of the vicinity size of $\epsilon$.
2. $\Delta \alpha<0$, iff $\mu(\epsilon)$ is a decreasing function of $\epsilon$, which normally implies relatively low river flow at time $t$.
3. $\Delta \alpha>0$, iff $\mu(\epsilon)$ is an increasing function of $\epsilon$, which normally implies a relatively high flow at the time $t$.

3. Results

3.1. Flood characteristic description using the singularity index

Figure 2 shows the plots peak flow and time duration for selected flood events at the O2HB001 gauge station. $Q^*$ is calculated using model (1), and the straight lines are fitted by the least squares method.

The slope estimated from the above linear relationship can be considered as an estimate of the singularity index $\Delta \alpha$. Table 1 lists the singularity index and $R^2$ for different flood events at the 02HB001 gauging station.

| Year | Start Date | End Date | Peak Flow (m$^3$/s) | Singularity Index | $R^2$ |
|------|------------|----------|---------------------|-------------------|-------|
| 1916 | April 26   | May 1    | 8.21                | 0.35              | 0.97  |
| 1920 | March 17   | March 22 | 35.1                | 0.41              | 0.92  |
| 1947 | April 7    | April 10 | 17.3                | 0.26              | 1.00  |
| 1948 | March 21   | March 26 | 33.7                | 0.39              | 0.94  |
| 1950 | April 4    | April 15 | 56.6                | 0.64              | 0.98  |
| 1954 | October 17 | October 29| 23.3                | 0.60              | 1.00  |
| 1956 | August 30  | September 10 | 12.3                | 0.49              | 0.95  |
| 1974 | May 17     | May 28   | 15.3                | 0.52              | 0.99  |
| 1977 | March 13   | March 26 | 17.7                | 0.52              | 0.96  |
| 1985 | February 21| March 4  | 11.7                | 0.42              | 0.94  |
| 1992 | August 29  | September 6 | 5.2                | 0.39              | 1.00  |
| 1997 | February 22| February 26 | 25.1                | 0.20              | 0.98  |
| 1998 | March 27   | April 14 | 19.7                | 0.53              | 0.98  |
| 2002 | May 16     | May 29   | 6.21                | 0.33              | 1.00  |

The singularity can be used to characterize the property of the flow change. In Table 1, the singularity index for flood events ranges from 0.2 to 0.64 for the 02HB001 station, the April 1950 flood has the highest singularity index value (0.64), and the February 1997 flood has the lowest value (0.2).

For the April 1950 flood, the peak flow was 56.6 m$^3$/s on April 4, 1950, and the flow rapidly dropped to 32.6 m$^3$/s on April 5, 1950. However, for the February 1997 flood, the peak flow was 23.3 m$^3$/s on Feb. 22, 1997, and the flow gradually dropped to 6.17 m$^3$/s on Feb. 25, 1997. The more
quickly the flow process changes, the higher the singularity index. Based on the above analysis, it can be concluded that the singularity index can be used for description of flood characteristics; the higher the singularity index, the more quickly the flood process changes. On the other hand, the lower the singularity index, the more slowly the flood process changes.

3.2. Spatial distribution of the singularity index

Furthermore, the August 1992 flood events are used to study the spatial distribution of singularity index. Table 2 shows the singularity indexes for August 1992 flood events at each gauging station. Table 3 shows the flow discharge of the August 1992 flood event at 02ED003 and 02HC030 gauging stations. Figure 3 show the singularity index curves for 02HC030 station and Figure 4 show the singularity index curves for 02ED003 station.

| Gauge Station | Singularity index |
|---------------|-------------------|
| 02EC008       | 0.45              |
| 02EC009       | 0.74              |
| 02EC010       | -                 |
| 02ED003       | 0.33              |
| 02ED100       | -                 |
| 02HB001       | 0.39              |
| 02HB018       | 0.36              |
| 02HB025       | 0.42              |
| 02HC003       | 0.61              |
| 02HC009       | 0.64              |
| 02HC018       | 0.54              |
| 02HC019       | 0.65              |
| 02HC022       | -                 |
| 02HC024       | 0.72              |
| 02HC027       | 0.87              |
| 02HC028       | 0.67              |
| 02HC030       | 0.82              |
| 02HC031       | 0.64              |
| 02HC032       | 0.53              |
| 02HC033       | 0.9               |
| 02HD003       | 0.48              |
| 02HD008       | 0.41              |
| 02HD009       | 0.46              |
| 02HD013       | 0.78              |
| 02HG002       | -                 |
Table 3. Flow discharge of August 1992 flood event at 02HC033 and 02ED003 gauge stations.

| Year | Start Date | End Date | Peak Flow (m$^3$/s) | Singularity Index |
|------|------------|----------|---------------------|-------------------|
| 1992 | 8          | 29       | 18.100              | 13.900            |
| 1992 | 8          | 30       | 2.130               | 10.200            |
| 1992 | 8          | 31       | 0.620               | 7.170             |
| 1992 | 9          | 1        | 0.421               | 5.770             |
| 1992 | 9          | 2        | 0.313               | 4.800             |
| 1992 | 9          | 3        | 0.251               | 4.670             |

Figure 3. Singularity index at 02HC033 Station (August 1992 flood).

Figure 4. Singularity index at 02ED003 Station (August 1992 flood).

The 02HC033 gauge station has the highest singularity index (0.9), and the 02ED003 station has the lowest (0.33). From the 02HC033 station, peak flow (18.1 m$^3$/s) was observed on August 29, and it rapidly dropped to 2.13 m$^3$/s on August 30; thus, the flow change was rapid, and the singularity index was very high. In contrast, at the 02ED003 station, peak flow (13.9 m$^3$/s) was observed on August 29.
It gradually dropped to 10.2 m$^3$/s on August 30, and then to 7.17 m$^3$/s on August 31, until it reached 5.77 m$^3$/s on September 01. The flow change was slow, so the singularity index was the lowest.

The spatial distribution of the singularity index was mapped to find the spatial distribution pattern of the singularity index in the ORM area (Figure 5). The south part watersheds have the most urban land cover type and a higher singularity index. Moving from the south to central part and north part, the singularity index reduces further because of less urban land cover and more agricultural land cover.

**Figure 5. Singularity index spatial distribution of August 1992 flood in the ORM area.**

4. Conclusions
Because flood events usually release large amounts of water in a short time, a singularity index (fractal model) was created by using peak flow and flow discharge data with time to describe flooding events, especially for characterizing flow change properties in this study. Based on the analysis conducted, it can be concluded that the singularity index can be used to describe flood characteristics, which describes the changing behavior around the peak of the flood flow. The higher the singularity index, the more quickly the flood process changes, and vice versa. It was also observed that the singularity index is higher when more watersheds have urban land cover, and it is lower when fewer watersheds have urban land cover and more have agricultural land cover.

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