Problem On The Acoustic Cloak By 0 to $R_1$ Transformation

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In this paper, we prove that for incident acoustic wave $p_i(\vec{r})$ in outside of whole sphere $r_s \geq R_2$, in the annular layer $R_1 \leq R \leq R_2$, with anisotropic acoustic media induced by 0 to $R_1$ spherical radial transformation, pressure acoustic wave $P(\vec{R})$, satisfies the anisotropic acoustic equation and no scattering interface condition on outer interface boundary $\vec{R} = \vec{R}_2$, then on the inner interface boundary $\vec{R} = \vec{R}_1$, $P(\vec{R}_1)$ is nonzero constant, $P(\vec{R}_1^+) \neq 0$, and the inner sphere $R \leq R_1$ can not be cloaked, otherwise, $P(\vec{R}_1) = 0$, in $R \leq R_1$ that does cause contradiction between $P(\vec{R}_1^+) \neq 0$ and $P(\vec{R}_1^-) = 0$, $P(\vec{R}_1^+) \neq P(\vec{R}_1^-)$, the contradiction shows that the nonzero acoustic wave is propagation to penetrate the inner sphere, the inner sphere $R \leq R_1$, can not be cloaked. Inversely, suppose that the inner sphere $R \leq R_1$ is cloaked, the $P(\vec{R}) = 0$ is the zero solution of acoustic equation in inner sphere, if the pressure acoustic wave $P(\vec{R})$, satisfies the anisotropic acoustic equation in annular layer $R_1 \leq R \leq R_2$, and satisfies zero interface continuous conditions on the inner interface boundary $\vec{R} = \vec{R}_1$, then on the outer interface boundary $\vec{R} = \vec{R}_2$, $P(\vec{R}_2) \neq p_i(\vec{R}_2)$,the pressure wave is not equal to incident wave, which is contradiction with no scattering interface continuous condition on outer interface boundary $\vec{R} = \vec{R}_2$, then there exist scattering wave to disturb the incident wave in outside of the whole sphere, $R \leq R_2$, the whole sphere will be detected and exposed. The above two basic contradictions are caused by the inconsistent between induced anisotropic acoustic media in annular layer $R_1 \leq R \leq R_2$ by 0 to $R_1$ spherical radial transformation and background media in inner sphere. The inconsistent anisotropic acoustic media causes there exist no acoustic wave solution to satisfy the acoustic wave equations and interface continuous condition global acoustic equations system. If the pressure wave $P(\vec{R})$ is the solution of the acoustic wave equation (14) and satisfy the necessary no scattering interface conditions (26) and (27) on the outer interface boundary $\vec{R} = \vec{R}_2$, moreover, for infinity countable angular frequencies $\omega_m$ that make $j_1(k_{b,m}R_1) = 0$, then the acoustic wave is propagation to penetrate into the inner sphere, $P(\vec{R}) = \frac{k^2_{b,m}R_2^2 - k_{b,m}r_s^2}{r_s^2}j_1(k_{b,m}R_1)j_0(k_{b,m}R)$,and the wave does satisfy interface continuous condition on the inner interface boundary $\vec{R} = \vec{R}_1$, the inner sphere can not be cloaked. In particular, in this paper, we propose a novel anisotropic media in the sphere $R \leq R_1$ that is the induced anisotropic acoustic media (15) to (17) or (20) to (22), by the 0 to $R_1$ sphere radial transformation (7) or (18),respectively. The anisotropic acoustic media formulas in the sphere $R \leq R_1$ is same as that in the annular layer $R_1 \leq R \leq R_2$. We prove if the anisotropic acoustic media are installed in the sphere $R \leq R_1$, then the acoustic wave solution of the acoustic wave equation (14) is continuous propagation to penetrate into the inner sphere $R \leq R_1$, the inner sphere $R \leq R_1$, can not be cloaked. Therefore, 0 to $R_1$ spherical radial transformation can not be used to induce acoustic no scattering cloak. We propose our Global and Local field method and novel approach to prove the cloak in paper [4] is not "No scattering acoustic cloak". First, we define "the global acoustic equation system", Second, we define "acoustic no scattering cloak" as follows that suppose that the anisotropic acoustic media is created in the annular layer that make there exist acoustic wave solution to satisfy the global acoustic equation system, if in the outside of whole sphere the acoustic wave solution equal to incident wave, $p(\vec{r}) = p_i(\vec{r})$, i.e. there exist no scattering wave to disturb the incident wave, and the acoustic wave solution is zero in the inner sphere, i.e. the inner sphere is cloaked, then the annular layer, $R_1 \leq R \leq R_2$ and inner sphere $R \leq R_1$ is called the "acoustic no scattering cloak. In Statement 7 in this paper, we prove if the induced anisotropic acoustic media (73) - (75) (i.e. (25),(24), (26) in paper [4]) by 0 to $R_1$ linear transformation "0R1SRLT" in (71)-(22) is installed in the annular layer $R_1 \leq R \leq R_2$, and $j_1(k_bR_1) \neq 0$, then there exist no acoustic wave solution to satisfy the "global acoustic wave equation system" in (61) to (68). That prove that the cloak in paper [4] is not "acoustic No scattering Cloak". In the statement 8 in this paper, we prove if the induced anisotropic acoustic media (73) - (75) (i.e. (25),(24), (26) in paper [4]) by 0 to $R_1$ linear transformation "0R1SRLT" in (71)-(72) is installed in the annular layer $R_1 \leq R \leq R_2$, and $j_1(k_bR_1) = 0$, then there exist acoustic wave solution to satisfy the "global acoustic wave equation system" in (61) to (68), the nonzero and continuous bounded acoustic wave solution is propagation to penetrate into the inner sphere, the inner sphere $R \leq R_1$ is not cloaked. That prove that the cloak in paper [4] is not "acoustic No scattering Cloak".
novel statement 9, we prove if the induced anisotropic acoustic media (73) - (75) by 0 to $R_1$ linear transformation "$0R1SRLT" in (71)-(72) is installed in the annular layer $R_1 \leq R \leq R_2$ and the inner sphere $R \leq R_1$, then there exist acoustic wave solution to satisfy the "global acoustic wave equation system" in (61) to (65) and (91) to (93), the nonzero and continuous bounded physical acoustic wave solution is propagation to penetrate into the inner sphere, $R \leq R_1$, the inner sphere $R \leq R_1$ is not cloaked. That prove that the cloak in paper [4] is not "acoustic No scattering Cloak". Moreover, that prove that the "Physically ....." explanation in paper [4] is not correct and is a mistake in "No Scattering Cloak Super Physical Sciences". We proved that 0 to $R_1$ spherical radial transformation can not be used to induce acoustic no scattering cloak. The 0 to $R_1$ spherical radial transformation can not be used to induce static electric conductivity no scattering cloak.

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I. INTRODUCTION

There are two methods to make electromagnetic invisible cloak, one is our GILD and GL no scattering modeling and inversion method, another one is transformation method. Using GL no scattering modeling and inversion and without transformation, we proposed electromagnetic (EM) practicable GLLH [1] double layer invisible cloak with relative refractive index not less than 1 and EM GLHUA[2] double layer invisible cloak with relative parameters not less than 1, both without exceeding light speed propagation. In paper [2], we discover a novel electromagnetic invisible cloak media with relative parameter not less than 1 in the sphere layer $R_1 \leq R \leq R_2$. In particular, we find exact analytic electromagnetic wave propagation in the novel practicable double layer electromagnetic invisible cloak in the paper [2]. Using 0 to $R_1$ sphere radial linear transformation, Pendry et al proposed EM invisible cloak [3]. It is proved that by [5][6][7][8] the Pendry EM cloak [3] is invisible cloak with infinite speed and exceeding light phase velocity fundamental difficulties. In this paper, we define "the acoustic no scattering cloak" as follows, given $R_2 > R_1 > 0$, a whole sphere $R \leq R_2$ is located in the background acoustic space, to divide the sphere $R \leq R_2$ into an annular layer $R_1 \leq R \leq R_2$ and an inner sphere $R \leq R_1$, the 3D full Space is split into the three domains and two interfaces $\vec{R} = \vec{R}_2$ and $\vec{R} = \vec{R}_1$, the outside of the whole sphere $R \geq R_2$ with background isotropic acoustic media, the annular layer $R_1 \leq R \leq R_2$ with anisotropic acoustic media, the inner sphere $R \leq R_1$ with background isotropic acoustic media, the 3 acoustic equations in the three domains respectively and the 4 interface continuous conditions equations of wave and its derivative on the interfaces $\vec{R} = \vec{R}_2$ and $\vec{R} = \vec{R}_1$ that compose global acoustic equation system. Suppose that the anisotropic acoustic media is created in the annular layer that make there exist acoustic wave solution to satisfy the global acoustic equation system with Sommerfeld far field radiation condition, if in the outside of whole sphere the acoustic wave solution equal to incident wave, i.e. there exist no scattering wave to disturb the incident wave, and the acoustic wave solution is zero in the inner sphere, i.e. the inner sphere is cloaked, then the annular layer and inner sphere is called the acoustic no scattering cloak, the anisotropic acoustic media is called no scattering materials, the annular layer is called no scattering cloaking layer, the inner sphere is called no scattering cloaked concealment. We prove that the acoustic cloak in paper [4] is not "no scattering acoustic cloak". We proved that the 0 to $R_1$ spherical radial transformation method can not be used to induce acoustic no scattering cloak, and prove that the induce anisotropic acoustic medium in annular layer $R_1 \leq R \leq R_2$ by linear 0 to $R_1$ sphere radial transformation in [4] is inconsistent with background isotropic media in inner sphere that causes there exist no acoustic wave solution to satisfy the above global acoustic equation system. Because in the 0 to $R_1$ radial spherical coordinate transformation, the value range of the acoustic field is invariant, Let $p_i(\vec{r})$ be incident pressure acoustic wave in background space before transformation, $\vec{r}$ be radial variable in background space before transformation, $\vec{R}$ be radial variable after transformation. Let the 0 to $R_1$ sphere radial transformation to be $\vec{R}(r) = R_1 + Q(r)$, and inverse transformation to be $r = Q^{-1}(R - R_1)$, the $P_i(\vec{R})$ be pressure acoustic wave in the sphere annular layer $R_1 \leq R \leq R_2$ after transformation, $P_i(\vec{R}) = p_i(r(R)) = p_i(Q^{-1}(R - R_1))$, $p_i(\vec{r})$ be pressure acoustic wave in background space outside sphere $r \geq R_2$, if $p_i(\vec{r}) = p_i(\vec{r})$ then there exist no scattering wave to disturb the incident wave, the $P_2(\vec{R})$ be pressure acoustic wave in the inner sphere $\vec{R} \leq R_1$. Suppose that on the outer interface boundary $\vec{R} = \vec{R}_2$, acoustic wave is continuous and its radial derivative with parameter is also continuous between acoustic wave $P_1(\vec{R})$ and incident acoustic wave $p_i(\vec{r})$ , $P_1(\vec{R}^-_2) = p_i(\vec{R}^+_2)$ and $\frac{1}{\rho_i} \frac{\partial^2}{\partial r^2} P(\vec{R}^-_2) = R_2^2 \frac{\partial}{\partial r} p_i(\vec{R}^+_2)$, then there exist no scattering acoustic wave to disturb the incident wave $p_i(\vec{r})$ in outside sphere, both interface

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continuous conditions are necessary condition for "there exist no scattering acoustic wave from whole sphere". We call the above acoustic wave \( P_1(\vec{R}) \) to be "NO Scattering Acoustic Wave" that does satisfy "no scattering acoustic wave condition" (26) and (27) on the outer interface boundary \( \vec{R} = \vec{R}_2 \). If the acoustic wave satisfies the no scattering interface conditions (26) and (27) on the outer interface boundary \( \vec{R} = \vec{R}_2 \), then on the inner interface boundary \( \vec{R} = \vec{R}_1 \), the acoustic wave \( P_1(\vec{R}) \) is nonzero constant \( P_1(R_1^+) \neq 0 \), and then the inner sphere \( R \leq R_1 \) can not be cloaked, otherwise, the \( P_2(\vec{R}) = 0 \) in \( R \leq R_1 \) that causes contradiction between \( P_2(R_1^-) = 0 \) and \( P_1(R_1^+) \neq 0 \), \( P_2(R_1^-) \neq P_1(R_1^+) \). The basic interface continuous conditions are destroyed, Inversely, if the inner sphere is cloaked that will cause the pressure wave does not satisfy no scattering interface condition on the outer interface boundary \( \vec{R} = \vec{R}_2 \), \( P(R_1^+) \neq P_1(R_1^+) \). In particular, in this paper, we propose a novel anisotropic media in the sphere \( R \leq R_1 \) that is the induced anisotropic acoustic media (15) to (17) or (20) to (22), by the 0 to \( R_1 \) sphere radial transformation (7) or (18), respectively. The anisotropic acoustic media formulas in the sphere \( R \leq R_1 \) is same as that in the annular layer \( R_1 \leq R \leq R_2 \). We prove if the anisotropic acoustic media are installed in the sphere \( R \leq R_1 \), then the acoustic wave solution of the acoustic wave equation (14) is continuous propagation to penetrate into the inner sphere \( R \leq R_1 \), the inner sphere \( R \leq R_1 \), can not be cloaked. Therefore, 0 to \( R_1 \) spherical radial transformation can not be used to induce acoustic no scattering cloak. In some published papers on acoustic cloak, the 0 to \( R_1 \) spherical radial transformation was wrong used to induce their acoustic cloak, we prove in paper [4] that is not "no scattering acoustic cloak". We propose our Global and Local field method and novel approach to prove the cloak in paper [4] is not "No scattering acoustic cloak". First, we define "the global acoustic equation system". Second, we define "acoustic no scattering cloak" as follows that suppose that the anisotropic acoustic media is created in the annular layer that make there exist acoustic wave solution to satisfy the global acoustic equation system, if in the outside of whole sphere the acoustic wave solution equal to incident wave, \( p(\vec{r}) = p_i(\vec{r}) \), i.e. there exist no scattering wave to disturb the incident wave, and the acoustic wave solution is zero in the inner sphere, i.e. the inner sphere is cloaked, then the annular layer, \( R_1 \leq R \leq R_2 \) and inner sphere \( R \leq R_1 \) is called the "acoustic no scattering cloak. In Statement 7 in this paper, we prove if the induced anisotropic acoustic media (73) - (75) (i.e. (25), (24), (26) in paper [4]) by 0 to \( R_1 \) linear transformation "0R1SRLT" in (71) - (72) is installed in the annular layer \( R_1 \leq R \leq R_2 \), and \( j_1(k_0 R_1) \neq 0 \), then there exist no acoustic wave solution to satisfy the "global acoustic wave equation system" in (61) to (68), the nonzero and continuous bounded acoustic wave solution is propagation to penetrate into the inner sphere, \( R \leq R_1 \), the inner sphere \( R \leq R_1 \) is not cloaked. That prove that the cloak in paper [4] is not "No scattering Cloak". Moreover, that prove that the "Physically ....." explanation in paper [4] is not correct and is a mistake in "No Scattering Cloak Super Physical Sciences". We proved that 0 to \( R_1 \) spherical radial transformation can not be used to induce acoustic no scattering cloak. The 0 to \( R_1 \) spherical radial transformation can not be used to induce static electric conductivity no scattering cloak. The contents of this paper are as follows. The isotropic pressure acoustic wave equation in the 3D spherical coordinate space is described in section 2. In section 3, we describe that 0 to \( R_1 \) spherical radial coordinate transformation does induce relative acoustic media for pressure acoustic wave propagation. In section 4, we prove that if acoustic wave solution does satisfy necessary no scattering conditions on the outer interface boundary \( \vec{R} = \vec{R}_2 \), then the pressure wave is nonzero constant on the inner interface boundary. In section 5, we proved that cloaked inner sphere that causes the wave solution of the acoustic equation (14) is different from the incident wave on outer interface boundary, \( \vec{R} = \vec{R}_2 \). In section 6, we prove that the 0 to \( R_1 \) spherical radial transformation method can not be used to induce acoustic no scattering cloak, in this section we proposal a novel anisotropic acoustic media in the inner sphere that making the acoustic wave solution of the acoustic equation (14) can be continuous propagation to penetrate into the inner sphere \( R \leq R_1 \), the inner sphere can be not cloaked. The discussions is described in section 7. We write physical letter to summary this paper in section 8, in this section, we propose our Global and Local field method and novel approach to prove the cloak in paper [4] is not "No scattering acoustic cloak". Conclusion is presented in section 9.
II. THE ISOTROPIC ACOUSTIC WAVE EQUATION IN THE 3D SPHERICAL COORDINATE SPACE

A. The isotropic pressure acoustic wave equation in the 3D spherical coordinate space with background acoustic speed and incident pressure acoustic wave

In the 3D spherical coordinate in space, the isotropic pressure acoustic wave equation is Helmholtz equation as follows:

\[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial p(r)}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial p(r)}{\partial \theta} + k_b^2 r^2 p(r) = S(r, r_s), \]  

(1)

where \( p(r) \) is the pressure wave field in the background space. \( c_b \) is the background constant acoustic speed, for example, the pressure wave in background deep ocean, \( \omega \) is the angular frequency, let \( k_b = \frac{\omega}{c_b} \). \( S(r, r_s) \) is the acoustic source located in \( r_s \) in some device in ocean. For example,

\[ S(r, r_s) = \frac{1}{\sin \theta} \delta(r - r_s) \delta(\theta - \theta_s) \delta(\phi - \phi_s), \]  

(2)

The incident pressure wave field \( p_i(r) \) does satisfy the acoustic equation (1) in infinite space which is excited by source (2).

\[ p_i(r) = -\frac{1}{4\pi} e^{ik_b|\vec{r} - \vec{r}_s|}, \]  

(3)

B. "Background no scattering sphere" \( r \leq R_2 \) and homogeneous acoustic wave equation

Suppose that the source is located outside the whole sphere with radius \( R_2 > 0 \). \( r_s > R_2 \), in the sphere \( r \leq R_2 \), with background acoustic medium, the pressure acoustic wave equation (1) become to the homogeneous acoustic wave equation,

\[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial p(\vec{r})}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial p(\vec{r})}{\partial \theta} + k_b^2 r^2 p(\vec{r}) = 0, \]  

(4)

on the interface boundary \( \vec{r} = \vec{R}_2 \), the pressure acoustic wave field \( p(\vec{r}) \) and its radial derivative satisfy the continuous interface conditions

\[ p(\vec{R}_2^+) = p(\vec{R}_2^-) = p_i(\vec{R}_2^+) \]  

(5)

\[ \frac{\partial}{\partial r} p(\vec{R}_2^-) = \frac{\partial}{\partial r} p(\vec{R}_2^+) = \frac{\partial}{\partial r} p_i(\vec{R}_2^+) \]  

(6)

where \( p_i(\vec{r}) \) is incident acoustic wave in (3), \( \vec{R}_2^+ = (R_2^+, \theta, \phi) \). \( \vec{R}_2^- = (R_2^-, \theta, \phi) \). The \( p(\vec{r}) \) in (3) in sphere \( r \leq R_2 \) is the solution of equation (4) with continuous interface conditions (5) and (6). The sphere \( r \leq R_2 \) with background acoustic medium is called the background no scattering sphere, the interface continuous conditions (5) and (6) on the interface boundary \( r = R_2 \) is sufficient and necessary conditions for no scattering from the whole sphere \( r \leq R_2 \) to disturb the incident wave in outside of whole sphere in \( r \geq R_2 \).
III. 0 TO R1 SPHERICAL RADIAL COORDINATE TRANSFORMATION DOES INDUCE RELATIVE ACOUSTIC MEDIA FOR PRESSURE ACOUSTIC WAVE PROPAGATION

A. 0 to R1 sphere radial coordinate transformation

For \( R_1 > 0, R_2 > R_1 \), inside of the sphere \( r \leq R_2 \), the 0 to \( R_1 \) sphere radial continuous coordinate transformation is

\[
R(r) = R_1 + Q(r), 0 \leq r \leq R_2, \quad (7)
\]

\[
R(0) = R_1, Q(0) = 0, \quad (8)
\]

\[
R(R_2) = R_2, Q(R_2) = R_2 - R_1, \quad (9)
\]

\[
\frac{\partial}{\partial r} R(r) \geq 0, \quad (10)
\]

where \( R = R(r) \) is radial coordinate in the physical spherical annular layer \( R_1 \leq R \leq R_2 \). We use \( r, \vec{r}, \rho(\vec{r}) \) to denote sphere radial variable, sphere radial vector, and pressure acoustic wave propagation in background space, and use \( R, \vec{R}, P(\vec{R}) \) to denote sphere radial variable, sphere radial vector, and pressure acoustic wave propagation in sphere annular layer space domain, \( R_1 \leq R \leq R_2 \), respectively.

By 0 to \( R_1 \) sphere radial transformation (7)-(10), the "background no scattering background sphere" is compressed into the physical sphere annular layer domain, \( R_1 \leq R \leq R_2 \) and a new absolute empty topology 3D inner sphere \( R \leq R_1 \) is created. We will install some acoustic media in the inner sphere, for example, the background medium is in the inner sphere, \( R \leq R_1 \). We prove that the background medium in the inner sphere is inconsistent with the induced anisotropic acoustic media by transformation in the annular layer.

The inverse transformation of (7) is

\[
r = r(R) = Q^{-1}(R - R_1), R_1 \leq R \leq R_2, \quad (11)
\]

B. The induced acoustic equation with relative anisotropic acoustic media

To substitute 0 to \( R_1 \) radial coordinate transformation (7)-(10) into the pressure acoustic equation (4) in the background sphere \( r \leq R_2 \), and by the following transformations, Let \( \vec{R} = (R, \theta, \phi) = (R(r), \theta, \phi) \)

\[
\begin{align*}
\frac{\partial \vec{R}}{\partial \vec{r}} &= \left( \begin{array}{c} \frac{\partial R}{\partial r} \frac{\partial R}{\partial \theta} \frac{\partial R}{\partial \phi} \\ \frac{1}{\sin \theta} \frac{\partial p(\vec{r})}{\partial \theta} + k_0^2 \frac{1}{\rho \sin \theta} R^2 \frac{\partial p(\vec{r})}{\partial \phi} = 0, \\
\frac{1}{\sin \theta} \frac{\partial p(\vec{r})}{\partial \phi} + k_0^2 \frac{1}{\rho \sin \theta} R^2 \frac{\partial p(\vec{r})}{\partial \phi} = 0,
\end{array} \right)
\end{align*}
\]

by substitution \( P_1(R) = p_i(r(R)) = p_i(Q^{-1}(R - R_1)) \), the equation (4) is translated to the following anisotropic homogeneous acoustic equation in physical sphere annular layer domain, \( R_1 \leq R \leq R_2 \)

\[
\frac{\partial}{\partial R} \frac{1}{\rho \gamma} R^2 \frac{\partial P_1(\vec{R})}{\partial R} + \frac{1}{\rho \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial P_1(\vec{R})}{\partial \theta} + \frac{1}{\rho \sin \theta} \frac{\partial^2 P_1(\vec{R})}{\partial \phi^2} + \frac{1}{R^2} k_0^2 R^2 P_1(\vec{R}) = 0, \quad (14)
\]

by the 0 to \( R_1 \) sphere radial transformation (7-10), the induced relative anisotropic parameters in physical sphere annular layer domain \( R_1 \leq R \leq R_2 \) are

\[
\rho_r = \frac{R^2}{R^2} \frac{dr}{dR}, \quad (15)
\]

\[
\rho_\theta = \rho_\phi = \frac{dR}{dr}, \quad (16)
\]

\[
K_t = \frac{R^2}{R^2} \frac{dR}{dR}, \quad (17)
\]

Use \( P(\vec{R}) \) to replace \( P_1(\vec{R}) \) in the equation (14), the equation (14) is the general anisotropic acoustic equation. In particular, for 0 to \( R_1 \) sphere radial linear transformation in the paper [4],

\[
R = R_1 + Q(r) = R_1 + \frac{R_2 - R_1}{R_2} r, \quad (18)
\]

the linear transformation (18) does satisfy transformation conditions (7)-(10), and its inverse transformation

\[
r = Q^{-1}(R - R_1) = \frac{R_2}{R_2 - R_1}(R - R_1), \quad (19)
\]

The anisotropic relative parameters induced by linear transformation (18) are

\[
\rho_r = \frac{R^2}{R^2} \frac{dr}{dR} = \frac{(R_2 - R_1)}{R_2} \frac{R^2}{(R - R_1)^2}, \quad (20)
\]

\[
\rho_\theta = \rho_\phi \frac{dR}{dr} = \frac{R_2 - R_1}{R_2}, \quad (21)
\]

\[
K_t = \frac{R^2}{R^2} \frac{dR}{dR} = \frac{(R_2 - R_1)^3}{R_2^3} \frac{R^2}{(R - R_1)^2}, \quad (22)
\]

The inner radius is \( a \) and outer radius is \( b \) the paper [4], corresponding, inner radius is \( R_1 \) outer radius is \( R_2 \) in this paper, \( a \) is \( R_1 \), \( b \) is \( R_2 \), respectively, by 0 to \( R_1 \) sphere radial linear transformation (18), relative anisotropic acoustic parameters in (20) to (22) in this paper is same as the relative acoustic parameters [(24)-(26)] in paper [4] induced by same linear transformation in (18).
Substitute the background acoustic medium,
\[ p_r = p_\theta = p_\phi = K_t = 1, \]  
(23)
into the equation (14), the anisotropic pressure acoustic equation (14) becomes to isotropic homogeneous pressure acoustic equation (4) without source. On the interface boundary \( \vec{r} = \vec{R} = \vec{R}_2, \) the general pressure wave field solution of equation (14), \( P(\vec{R}), \) and pressure acoustic wave solution of equation (1), \( p(\vec{r}), \) satisfy the following continuous interface conditions
\[ P(\vec{R}_2^-) = p(\vec{R}_2^+), \]
(24)
and their derivatives satisfy the following continuous interface conditions
\[ \frac{1}{p_r} R_2^2 \frac{\partial}{\partial R} P(\vec{R}_2^-) = R_2^2 \frac{\partial}{\partial r} p(\vec{R}_2^+), \]
(25)
where \( \vec{R}_2^- = (R_2^-, \theta, \phi), \) \( \vec{R}_2^+ = (R_2^+, \theta, \phi). \) If substitute the pressure acoustic wave \( P_1(\vec{R}) \) by transformation of \( p_i(\vec{r}) \) and the incident wave and its derivative into right hand of (24) and (25), the interface continuous conditions (24) and (25) become no scattering interface continuous conditions (26) and (27),
\[ P_1(\vec{R}_2^-) = p_i(\vec{R}_2^+), \]
(26)
\[ \frac{1}{p_r} R_2^2 \frac{\partial}{\partial R} P_1(\vec{R}_2^-) = R_2^2 \frac{\partial}{\partial r} p_i(\vec{R}_2^+), \]
(27)
equation (26) and (27) are the sufficient and necessary condition of that no scattering acoustic wave from the whole sphere \( R \leq R_2 \) to disturb incident wave in outside of the whole sphere. The pressure acoustic wave \( P_1(\vec{R}) \) is no scattering acoustic wave by transformation of the incident wave \( p_i(\vec{r}). \)

IV. OUTER INTERFACE NO SCATTERING CONTINUOUS CONDITIONS GOVERN THAT THE SOLUTION OF ACOUSTIC WAVE EQUATION IS NONZERO CONSTANT ON THE INNER INTERFACE BOUNDARY

Outer interface continuous no scattering conditions (26) and (27) govern that the pressure acoustic wave solution of acoustic equation (14) is nonzero constant on the inner interface boundary \( \vec{r} = \vec{R}_1, P_1(\vec{R}_1) = -\frac{1}{\rho_r} \frac{\partial p_i(\vec{r})}{\partial r} \neq 0. \)

Statement 1 : Suppose that the point delta source (2) is in outside of whole sphere, \( r_s > R_2, \) the pressure wave \( P_1(\vec{R}) \) does satisfy the acoustic equation (14) with relative anisotropic parameters (15)-(17) or (20)-(22) by transformation (7) or (18), respectively, and does satisfy the wave and derivative no scattering continuous interface continuous conditions (26) and (27), for the transformation (7) and induced anisotropic parameters (15)-(17), then \( P_1(\vec{R}) \) has the analytic express
\[ P_1(\vec{R}) = p_i(r(R), \theta, \phi) = -\frac{1}{4\pi} \frac{e^{ikb|Q^{-1}(R-R_1), \theta, \phi)-r_s|}}{|Q^{-1}(R-R_1), \theta, \phi)-r_s|}, \]
(28)
Where \( r = Q^{-1}(R - R_1). \) For \( P_1(\vec{R}) \) is solution of he acoustic equation (14) with relative anisotropic parameters (20)-(22) by linear transformation (18)
\[ P_1(\vec{R}) = p_i(r(R), \theta, \phi) = \frac{1}{4\pi} \frac{e^{ikb|Q^{-1}(R-R_1), \theta, \phi)-r_s|}}{|Q^{-1}(R-R_1), \theta, \phi)-r_s|}, \]
(29)
Proof : By inverse transformation in (11) or (19), \( P_1(\vec{R}) \) is put back to \( p_i(\vec{r})\)
\[ \begin{align*}
& \text{left hand side of (14)} \\
& = \frac{\partial}{\partial r} \left( r^2 \frac{\partial p_i(\vec{r})}{\partial r} \right) - \frac{1}{\rho_r} \frac{\partial}{\partial r} \left( \frac{1}{\rho_r} \frac{\partial p_i(\vec{r})}{\partial r} \right) + \frac{1}{\rho_r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial p_i(\vec{r})}{\partial \theta} \right) \\
& + \frac{1}{\rho_r \sin \theta} \frac{\partial}{\partial \phi} \left( \sin \theta \frac{\partial p_i(\vec{r})}{\partial \phi} \right) + k_b^2 r^2 p_i(\vec{r}),
\end{align*} \]
(30)
Substitute \( \rho_r = \frac{R_2^2}{\vec{R} \cdot \vec{r}} \) in (15), \( \rho_\theta = \rho_\phi = \frac{4\pi}{k_b} \) in (16) and \( K_t = \frac{R_2^2}{\vec{R} \cdot \vec{r}} \) in (17) into the equation (30), we have
\[ \begin{align*}
& \text{left hand side of (14)} \\
& = \frac{\partial}{\partial r} \left( r^2 \frac{\partial p_i(\vec{r})}{\partial r} \right) + \frac{1}{\rho_r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial p_i(\vec{r})}{\partial \theta} \right) \\
& + \frac{1}{\rho_r \sin \theta} \frac{\partial}{\partial \phi} \left( \sin \theta \frac{\partial p_i(\vec{r})}{\partial \phi} \right) + k_b^2 r^2 p_i(\vec{r}) = 0.
\end{align*} \]
(31)
the right hand side of (31) is the equation (4), because \( p_i(\vec{r}) \) satisfies equation (4), therefore, \( P_1(\vec{R}) \) in (28) or (29) does satisfy the acoustic equation (14). Because the interface boundary \( \vec{R} = \vec{R}_2 \) is invariant under transformation condition (7) or (18), and by interface condition (5), we have
\[ P_1(\vec{R}_2^-) = p_i(\vec{R}_2^+), \]
(32)
\( P_1(\vec{R}) \) in (28) does satisfy the no scattering interface condition (27).
\[ \begin{align*}
& \text{left hand side of (27)} \\
& = \frac{1}{\rho_r} R_2^2 \frac{\partial}{\partial r} P_1(\vec{R}_2^-) \\
& = \frac{1}{\rho_r} R_2^2 \frac{\partial}{\partial r} \frac{\partial}{\partial r} p_i(\vec{R}_2^+),
\end{align*} \]
(33)
Substitute \( \rho_r = \frac{R_2^2}{\vec{R} \cdot \vec{r}} \) in (15) into (33), by transformation condition (9) and derivative interface continuous continuous condition (6), we have
\[ \begin{align*}
& \text{left hand side of (27)} \\
& = \frac{1}{\rho_r} R_2^2 \frac{\partial}{\partial r} P_1(\vec{R}_2^-) \\
& = \frac{1}{\rho_r} R_2^2 \frac{\partial}{\partial r} \frac{\partial}{\partial r} p_i(\vec{R}_2^+),
\end{align*} \]
(34)
the derivative interface continuous condition (27) is satisfied. Substitute \( r = Q^{-1}(R - R_1) = \frac{R_2^2}{R_2^2 - R_1} (R - R_1) \)
in (19) into (28), we obtain (29). The Statement 1 is proved.

**Statement 2**: Suppose that the point delta source (2) is in outside of whole sphere, \( r_s > R_2 \), the pressure acoustic wave \( P_1(\vec{R}) \) is solution of the pressure acoustic equation (14) in the sphere annular layer domain \( R_1 \leq R \leq R_2 \) with anisotropic relative acoustic parameters in (15)-(17) or (20)-(22), also, the pressure acoustic wave \( P_1(\vec{R}) \) satisfies no scattering interface continuous conditions (26) and (27) on the interface boundary \( \vec{R} = \vec{R}_2 \), then on the inner interface boundary \( \vec{R} = \vec{R}_1 \), \( P_1(\vec{R}_1) \) is nonzero constant,

\[
P_1(\vec{R}_1) = \frac{1}{4\pi} \frac{e^{ik_s |\vec{r}_s|}}{|\vec{r}_s|} \neq 0.
\]

Proof: In this Statement, outer interface boundary is \( \vec{R} = \vec{R}_2 = (R_2, \theta, \phi) \), inner interface boundary is \( \vec{R} = \vec{R}_1 = (R_1, \theta, \phi) \). Because pressure acoustic wave \( P_1(\vec{R}) \) is solution of the pressure acoustic equation (14) in the sphere annular layer domain \( R_1 \leq r \leq R_2 \) with anisotropic relative acoustic parameters in (15)-(17) by transformation (7), and also \( P_1(\vec{R}) \) does satisfy the no scattering interface conditions (26)-(27), by (28) in Statement 1,

\[
P_1(\vec{R}) = -\frac{1}{4\pi} \frac{e^{ik_s |Q^{-1}(R-R_1),\theta,\phi) - \vec{r}_s|}}{|(Q^{-1}(R-R_1),\theta,\phi) - \vec{r}_s|},
\]

then on the inner interface boundary \( \vec{R} = \vec{R}_1 \), by transform property (8), we have

\[
P_1(\vec{R}_1) = -\frac{1}{4\pi} \frac{e^{ik_s |Q^{-1}(R-R_1),\theta,\phi) - \vec{r}_s|}}{|(Q^{-1}(R-R_1),\theta,\phi) - \vec{r}_s|} \neq 0.
\]

For \( P_1(\vec{R}) \) is solution of the pressure acoustic equation (14) in the sphere annular layer domain \( R_1 \leq r \leq R_2 \) with anisotropic relative acoustic parameters in (20)-(22) by linear transformation (18), Because pressure acoustic wave \( P_1(\vec{R}) \) also satisfies the no scattering interface continuous conditions (26)-(27), by (29) in Statement 1,

\[
P_1(\vec{R}_1) = -\frac{1}{4\pi} \frac{e^{ik_s |(0,\theta,\phi) - \vec{r}_s|}}{|(0,\theta,\phi) - \vec{r}_s|} \neq 0.
\]

The Statement 2 is proved.

V. CLOAKED INNER SPHERE THAT CAUSES THE WAVE SOLUTION OF THE ACOUSTIC EQUATION (14) IS DIFFERENT FROM THE INCIDENT WAVE ON OUTER INTERFACE BOUNDARY, \( \vec{R} = \vec{R}_2 \).

In this section, we propose if inner sphere is cloaked, then the wave solution of the anisotropic acoustic equation (14) is different from the incident wave on outer interface boundary, \( \vec{R} = \vec{R}_2 \). By 0 to \( R_1 \) sphere radial transformation (7)-(11) or (18)-(19), the background sphere \( R \leq R_1 \) is compressed into the physical sphere annular layer domain, \( R_1 \leq R \leq R_2 \). A new sphere \( R \leq R_1 \) is inflated from zero. Originally, the new sphere \( R \leq R_1 \) is absolute empty topological space without physical acoustic geometry medium. We should make acoustic media in the new inner sphere \( R \leq R_1 \). We prove the background acoustic speed medium \( c_0 \) in the inner sphere \( R \leq R_1 \) that is inconsistent with the induced anisotropic acoustic media in \( R_1 \leq R \leq R_2 \) by 0 to \( R_1 \) sphere radial transformation.

Suppose that the inner sphere \( R \leq R_1 \) is cloaked, pressure acoustic wave \( P_2(\vec{R}) = 0 \) in inner sphere, it obvious \( P_2(\vec{R}) = 0 \) is solution of acoustic equation (4) in inner sphere \( R \leq R_1 \) with background medium. Let \( P(\vec{R}) \) is wave solution of the anisotropic acoustic equation (14) in the annular layer, \( R_1 \leq R \leq R_2 \). The \( P_2(\vec{R}) = 0 \) in \( R \leq R_1 \) induces

\[
P(\vec{R}_1) = P_2(\vec{R}_1) = 0,
\]

Next, we find wave solution of the anisotropic acoustic equation (14) in the annular layer, \( R_1 \leq R \leq R_2 \), with interface continuous conditions (39) and (40)

**Statement 3**: Suppose that the point delta source (2) is in outside of whole sphere, \( r_s > R_2 \), the pressure acoustic wave \( P(\vec{R}) \) is solution of the pressure acoustic equation (14) in the sphere annular layer domain \( R_1 \leq R \leq R_2 \) with anisotropic relative acoustic parameters in (15)-(17) or (20)-(22), also, satisfies the interface continuous condition (39) and (40) on the inner interface boundary \( \vec{R} = \vec{R}_1 \), then

\[
P(\vec{R}) = -\frac{1}{4\pi} \frac{e^{ik_s |Q^{-1}(R-R_1),\theta,\phi) - \vec{r}_s|}}{|(Q^{-1}(R-R_1),\theta,\phi) - \vec{r}_s|} + \frac{1}{4\pi} \frac{e^{ik_{0s} |(k_0Q^{-1}(R-R_1))h_0^{(1)}(k_0r_s)|}}{|(k_0Q^{-1}(R-R_1))h_0^{(1)}(k_0r_s)|}, R_1 \leq R \leq R_2,
\]

where \( j_0 \) is 0 order sphere Bessel function, \( h_0^{(1)} \) zero order first Hankel function. Moreover, on the outer interface boundary, acoustic wave \( P(\vec{R}_2) \) is different from incident wave \( P_1(\vec{R}_2) \),

\[
P(\vec{R}_2) = -\frac{1}{4\pi} \frac{e^{ik_s |(R,R_2,\theta,\phi) - \vec{r}_s|}}{|(R,R_2,\theta,\phi) - \vec{r}_s|} + \frac{1}{4\pi} \frac{e^{ik_{0s} |(k_0R_2h_0^{(1)}(k_0r_s)|}}{|(k_0R_2h_0^{(1)}(k_0r_s)|}, R_1 \leq R \leq R_2,
\]

Proof: substitute anisotropic relative acoustic parameters in (15)-(17) into pressure acoustic equation (14),

\[
\frac{\partial P}{\partial R} \frac{\partial}{\partial R} (Q^{-1}(R-R_1))^2 \frac{\partial P}{\partial \theta} \frac{\partial}{\partial \theta} \frac{\partial P}{\partial \phi} \frac{\partial}{\partial \phi} + \frac{\partial^2 P}{\partial \theta^2} + \frac{1}{\sin \theta} \frac{\partial^2 P}{\partial \phi^2} + \frac{\partial^2 P}{\partial R^2} (Q^{-1}(R-R_1))^2 k_0^2 P = 0,
\]
Substitute inverse transformation (11) into (43), equation (43) is translated to
\[
\frac{\partial}{\partial r} r^2 \frac{\partial P}{\partial r} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial P}{\partial \theta} + \frac{1}{\sin \theta} \frac{\partial^2 P}{\partial \phi^2} + r^2 k_0^2 P = 0. \tag{44}
\]

Substitute inverse transformation (11) into (41), the pressure acoustic wave \( P(\vec{R}) \) to be
\[
P(\vec{R}) = -\frac{1}{4\pi} e^{ik_b s} \left| \vec{R} - \vec{r}_s \right|
+ \frac{1}{4\pi} ik_b j_0 (k_b R) h_0^{(1)}(k_b r_s)
= -\frac{1}{4\pi} e^{ik_b s} \left| \vec{R} - \vec{r}_s \right|
+ \frac{1}{4\pi} ik_b j_0 (k_b R) h_0^{(1)}(k_b r_s),
\]
where the anisotropic acoustic equation (14) is different from the interface continuous condition (39), \( P(\vec{R}_+^1) = 0 \), is satisfied.

From equation (50) and transformation (7) and (11),
\[
\frac{\partial}{\partial \vec{r}} P(\vec{R}) = -\sum_{l=1}^\infty \frac{ik_b}{\pi} \left[ \frac{dP}{dr} \right]_{r=\vec{R}_+^1} q_l(-ik_b Q^{-1}(R - R_1))
- k_b \frac{d}{dr} h_0^{(1)}(k_b r_s)
\sum_{m=-l}^l Y_l^{(m)}(\theta, \phi) Y_l^{(m)*}(\theta_s, \phi_s), Q^{-1}(r - r_s) \leq r_s.
\]
From (40),
\[
(Q^{-1}(R - R_1))^2 \frac{dP}{dr} P(\vec{R}) \big|_{\vec{R}=\vec{R}_+^1} =
-\sum_{l=1}^\infty i k_b \left( (Q^{-1}(R - R_1)) j_l(k_b Q^{-1}(R - R_1))
- k_b \left( Q^{-1}(R - R_1) \right)^2 j_{l+1}(k_b Q^{-1}(R - R_1)) h_0^{(1)}(k_b r_s)
\sum_{m=-l}^l Y_l^{(m)}(\theta, \phi) Y_l^{(m)*}(\theta_s, \phi_s), R = R_1,
= -\sum_{l=1}^\infty i k_b \left( (Q^{-1}(R - R_1))^2 j_l(k_b Q^{-1}(R - R_1))
- k_b \left( (Q^{-1}(R - R_1))^2 j_{l+1}(k_b Q^{-1}(R - R_1)) h_0^{(1)}(k_b r_s)
\sum_{m=-l}^l Y_l^{(m)}(\theta, \phi) Y_l^{(m)*}(\theta_s, \phi_s), R = R_1,
\]
therefore,
\[
\frac{1}{R^2} R^2 \frac{\partial}{\partial R} P(\vec{R}_+^1) = (Q^{-1}(R - R_1))^2 \frac{dP}{dr} P(\vec{R}) \big|_{\vec{R}=\vec{R}_+^1} = 0,
\]
we have proved that the wave \( P(\vec{R}) \) in (41) does satisfy the anisotropic acoustic equation (14) and interface zero wave value condition (39) and zero derivative condition (40) on the interface boundary \( \vec{R} = \vec{R}_1 \). The acoustic wave \( P(\vec{R}) \) in (41) is caused by the condition that the inner sphere is cloaked”. Moreover, on outer interface boundary \( \vec{R} = \vec{R}_2 \), the value of the acoustic wave in (41) is,
\[
P(\vec{R}_2) = -\frac{1}{4\pi} e^{ik_b s} \left| \vec{R}_2 - \vec{r}_s \right|
+ \frac{1}{4\pi} ik_b j_0 (k_b R_2) h_0^{(1)}(k_b r_s)
= p_i(\vec{R}_2) + P_s(\vec{R}_2),
\]
We have (42)
\[
P(\vec{R}_2) = -\frac{1}{4\pi} e^{ik_b s} \left| \vec{R}_2 - \vec{r}_s \right|
+ \frac{1}{4\pi} ik_b j_0 (k_b R_2) h_0^{(1)}(k_b r_s)
= p_i(\vec{R}_2) + P_s(\vec{R}_2)(k_b r),
\]
Cloaked inner sphere causes the wave solution (41) of the anisotropic acoustic equation (14) is different from
the incident wave on outer interface boundary $\vec{R} = \vec{R}_2$, the necessary no scattering condition on the outer interface boundary $\vec{R} = \vec{R}_2$ is destroyed. Statement 3 is proved.

VI. 0 TO $R_1$: SPHERICAL RADIAL TRANSFORMATION CAN NOT BE USED TO INDUCE ACOUSTIC NO SCATTERING CLOAK

A. 0 to $R_1$ spherical radial transformation can not be used to induce acoustic no scattering cloak

Statement 4: For source incident acoustic wave or plane wave $p_i(\vec{r})$ is in outside of whole sphere $r_s \geq R_2$, in annular layer domain, $R_1 \leq R \leq R_2$, the anisotropic relative acoustic media (15)-(17) or (20)-(22) is induced by 0 to $R_1$ spherical radial transformation (7) or (18), the background medium is in the inner sphere, if pressure acoustic wave $P_1(\vec{R})$ is solution of acoustic equation (14) in annular layer domain, $R_1 \leq R \leq R_2$, and on the interface boundary $\vec{R} = \vec{R}_2$, $P_1(\vec{R})$ satisfies no scattering interface condition (26) and (27), then the insizers sphere $R \leq R_1$, can not be cloaked. Inversely, if the insizers sphere $R \leq R_1$, is cloaked, then on the interface boundary $\vec{R} = \vec{R}_2$, the no scattering interface continuous condition (26) and (27) of the acoustic wave $P(\vec{R})$, can not be satisfied, the whole sphere can cause scattering wave to disturb the incident wave and will be detected. Therefore, 0 to $R_1$ spherical radial transformation can not be used to induce acoustic no scattering cloak.

Proof: For source incident acoustic wave $p_i(\vec{r})$ is in outside of whole sphere, $r_s \geq R_2$, and the anisotropic acoustic media (15)-(17) in annular layer $R_1 \leq R \leq R_2$ is induced by the $0$ to $R_1$ spherical radial transformation (7) or (20)-(22) by linear 0 to $R_1$ spherical radial transformation (18), and background medium is in inner sphere. If on the outer interface boundary $\vec{R} = \vec{R}_2$, no scattering interface continuous conditions (26) and (27) is satisfied by the wave solution $P_1(\vec{R})$ of the acoustic wave equation (14) in layer $R_1 \leq R \leq R_2$, based on Statement 2, the acoustic wave $P_1(\vec{R})$ is nonzero constant on the inner interface boundary $\vec{R} = \vec{R}_1$, $P_1(\vec{R}_1) = -\frac{1}{4\pi} \frac{e^{ik_b r_s}}{r_s} \neq 0$ in (37), then inner sphere $R \leq R_1$, can not be cloaked, otherwise, if inner sphere, $R \leq R_1$, is cloaked, $P_2(\vec{R}) = 0$, it does cause $P_1(\vec{R}_1) = 0$, that is contradiction with $P_1(\vec{R}_1) = -\frac{1}{4\pi} \frac{e^{ik_b r_s}}{r_s} \neq 0$, that induces the acoustic wave equation in the inner sphere with nonzero Dirichlet boundary value on the boundary $\vec{R} = \vec{R}_1$, solve the equation, the nonzero scattering acoustic wave propagation in the inner sphere $R \leq R_1$, if $j_0(k_b R_1) \neq 0$, $P_2(\vec{R}) = -\frac{1}{4\pi} \frac{e^{ik_b r_s}}{r_s} j_0(k_b R_1)$.

The inner sphere $R \leq R_1$ is not cloaked. Inversely, if inner sphere $R \leq R_1$ is cloaked, $P_2(\vec{R}) = 0$, in $R \leq R_1$, it is obvious that wave $P_2(\vec{R}) = 0$, satisfies background homogeneous acoustic equation (4) in inner sphere $R \leq R_1$, moreover, the $P_2(\vec{R}) = 0$, in $R \leq R_1$, that causes $P(\vec{R}_1) = 0$ in (39) and its zero derivative condition in (40). Under the zero interface conditions (39) and (40), based on Statement 3, on the outer interface boundary $\vec{R} = \vec{R}_2$, the wave solution of the acoustic equation (14) is different from the incident wave $p_i(\vec{R})$,

$$P(\vec{R}_2) = -\frac{1}{4\pi} e^{ik_b \vec{r} \cdot \vec{r}_s} j_0(k_b R_2) h_1^{(1)}(k_b r_s) \frac{1}{r_s} \left(1 - \frac{1}{4\pi} \frac{e^{ik_b r_s}}{r_s} j_0(k_b R_1) h_1^{(1)}(k_b r_s) \right)$$

(42)

that is basic contradiction with the necessary no scattering interface continuous condition on the outer interface boundary $\vec{R} = \vec{R}_2$ and (26)/(27), and for $l = 0$, the value of $P(\vec{R}_2)$ in (42) that induces the acoustic wave equation (1) (the zero order sphere Bessel equation) in outside of whole sphere, $R \geq R_2$, with zero Dirichlet boundary value on the boundary $\vec{R} = \vec{R}_2$, solve the equation, the nonzero scattering acoustic wave propagation in the outside of the whole sphere, $R \geq R_2$,

$$p_s(\vec{r}) = \frac{ik_b}{4\pi} j_0(k_b R_2)$$

(56)

the incident wave in outside sphere can be disturbed, whole sphere can be detected, therefore, 0 to $R_1$ spherical radial transformation can not be used to induce acoustic no scattering cloak. The statement 4 is proved.

B. Pressure acoustic wave does satisfy the "no scattering interface continuous condition" (26) and (27) on the outer interface boundary $\vec{R} = \vec{R}_2$, the wave is propagation to penetrate into the inner sphere with background acoustic medium, if $j_1(k_b R_1) = 0$

Statement 5: Suppose that the pressure acoustic wave $P_2(\vec{R})$ is solution of the pressure acoustic equation (4) in the inner sphere $R \leq R_1$, with background acoustic speed medium , moreover, the pressure acoustic wave $P_2(\vec{R})$ does satisfy interface continuous conditions on the interface boundary $\vec{R} = \vec{R}_1$.

$$P_2(\vec{R}_1) = -\frac{1}{4\pi} \frac{e^{ik_b r_s}}{r_s}$$

(55)

$$\frac{\partial P_2(\vec{R})}{\partial R} |_{R=R_1} = \frac{R_2 - R_1}{R_2 - R_1} (R - R_1)^2 \frac{\partial P_2(\vec{R})}{\partial R} |_{R=R_1} = 0,$$

(56)

If $j_1(k_b R_1) = 0$, then bounded pressure acoustic wave

$$P_2(\vec{R}) = \frac{1}{4\pi} \frac{e^{ik_b r_s}}{r_s} j_0(k_b R_1) j_0(k_b R)$$

(57)
is propagation in inner sphere \( R \leq R_1 \).

Proof: Because the interface continuous condition (55) and (56) are spherical symmetry, the pressure wave \( P_2(\vec{R}) \) in (57) is sphere symmetry function only depend on the radial variable \( R \), \( P_2(\vec{R}) \) in (57) does satisfy the following 0 order sphere Bessel equation
\[
\frac{\partial}{\partial R} R^2 \frac{\partial P_2}{\partial R} + k_0^2 R^2 P_2 = 0, \tag{58}
\]
\[
P_2(\vec{R}) = \frac{k_0^2 R^2}{4\pi} e^{ikb\vec{r}_s} \int n_1(k_0R_1)j_0(k_0R_1)\, (k_0R_1^2 - k_0R^2) \, \sin(k_0R_1) \, k_0 R_1 - \frac{1}{k_0 R_1} \sin(k_0R_1) \sin(k_0R) \Bigg|_{R=R_1}.
\]

The inner sphere can not be cloaked.

In the Statement 4, for \( j_1(k_0R_1) \neq 0 \), we prove that the isotropic background medium in the inner sphere \( R \leq R_1 \) and induced anisotropic acoustic media (15)-(17) or (20)-(22) in the annular layer \( R_1 \leq R \leq R_2 \) is inconsistent that does cause the anisotropic acoustic equation (14), the interface continuous conditions on the outer interface boundary \( \vec{R} = \vec{R}_2 \) and on the inner interface boundary \( \vec{R} = \vec{R}_1 \) are contradiction equation system and there exist no physical wave solution to satisfy the above global equations system if \( j_1(k_0R_1) \neq 0 \). In statement 5, we prove that if \( j_1(k_0R_1) = 0 \), then there exist acoustic wave solution to satisfy the above global acoustic equation system.

The pressure acoustic wave \( P_1(\vec{R}) \) does satisfy the anisotropic acoustic equation (14) and does satisfy the no scattering interface continuous conditions (26) and (27) on the outer interface boundary \( \vec{R} = \vec{R}_2 \), then on the inner interface boundary \( \vec{R} = \vec{R}_1 \), the acoustic wave \( P_1(\vec{R}) \) is nonzero constant and is propagation to penetrate into the inner sphere and does satisfy acoustic equation and interface continuous conditions on the interface boundary \( \vec{R} = \vec{R}_1 \), the inner sphere can not be cloaked.

**C. Pressure acoustic wave does satisfy the "no scattering interface continuous condition" (26) and (27) on the outer interface boundary \( \vec{R} = \vec{R}_2 \), the wave is propagation to penetrate into the inner sphere with novel acoustic anisotropic media**

In this section, we propose a novel anisotropic medium in the sphere \( R \leq R_1 \) that is the induced anisotropic acoustic media (15) to (17) or (20) to (22) by 0 to \( R_1 \) sphere radial linear transformation (7) or (18), respectively. The anisotropic acoustic medium formulas in the sphere \( R \leq R_1 \) is same as that in the annular layer \( R_1 \leq R \leq R_2 \). We prove if the anisotropic acoustic media are installed in the sphere \( R \leq R_1 \), and the acoustic wave solution of the acoustic wave equation (14) and does satisfy no scattering interface continuous condition (26) and (27) on the outer interface boundary \( \vec{R} = vec\vec{R}_2 \), then the acoustic wave is continuous propagation to penetrate into the inner sphere \( R \leq R_1 \), the inner sphere \( R \leq R_1 \), can not be cloaked. The anisotropic acoustic wave equation (14) with anisotropic media (20) to (22) is,
\[
\frac{\partial}{\partial R} R^2 \frac{\partial P(\vec{R})}{\partial R} + \frac{1}{R^2} \frac{\partial}{\partial R} R \frac{\partial P(\vec{R})}{\partial R} + \frac{1}{R^2} \frac{\partial^2 P(\vec{R})}{\partial \theta^2} + \left( \frac{R}{R_2 - R_1} \right)^3 (R - R_1)^2 k_0^2 P(\vec{R}) = 0, \tag{59}
\]

**Statement 6:** Suppose that anisotropic acoustic media (20)-(22) is installed in sphere annular layer \( R_1 \leq R \leq R_2 \) which is induced by linear transformation (18), the anisotropic acoustic media by same formulas (20)-(22) is installed in the sphere \( R \leq R_0 \). Then the acoustic
wave $P_1(\vec{R})$ is solution of the acoustic equation (59) or (14) in the annular layer $R_1 \leq R \leq R_2$, the wave $P_1(\vec{R})$ and incident wave $p_i(\vec{R})$ satisfy "no scattering interface continuous conditions" (26) and (27) on the outer interface boundary $\vec{R} = \vec{R}_2$, the acoustic wave $P_2(\vec{R})$ is solution of the acoustic equation (59) or (14) in the inner sphere $R \leq R_1$, the wave $P_2(\vec{R})$ and $P_1(\vec{R})$ satisfy the interface continuous conditions on the outer interface boundary $\vec{R} = \vec{R}_1$, the acoustic wave $P_1(\vec{R})$ by (29) in $R_1 \leq R \leq R_2$ annular layer is proved in statement 1,

$$P_1(\vec{R}) = p(r(R), \theta, \phi)$$

and (62) on the interface boundary $\vec{R} = \vec{R}_1$, the wave $P_2(\vec{R})$ and its radial derivative are only constant and it is not depend on angular variable, therefore the anisotropic acoustic equation (59) or (14) in inner sphere $R \leq R_1$ becomes the one dimensional sphere Bessel equation

$$\frac{\partial}{\partial R} \frac{R_2}{R_2 - R_1} (R-R_1)^2 \frac{\partial P_2}{\partial R} + k_0^2 \left( \frac{R_2}{R_2 - R_1} \right)^3 (R-R_1)^2 P_2 = 0,$$

(64)

Substitute $P_2(\vec{R}) = -\frac{1}{4\pi} e^{ik_b r_s} j_0 \left( k_b \frac{R_2}{R_2 - R_1} (R-R_1) \right)$ in (60) into the above one dimensional sphere Bessel equation (64), Let $r = \frac{R_2}{R_2 - R_1} (R-R_1)$, the equation (64) becomes

$$\frac{\partial}{\partial r} \frac{R_2}{R_2 - R_1} \frac{\partial P_2}{\partial r} + k_0^2 \left( \frac{R_2}{R_2 - R_1} \right)^3 (R-R_1)^2 P_2 = 0,$$

(65)

$$R_2 \frac{\partial}{\partial r} \frac{R_2}{R_2 - R_1} \left( \frac{R_2}{R_2 - R_1} (R-R_1) \right)^2 \frac{\partial P_2}{\partial r} + k_0^2 \left( \frac{R_2}{R_2 - R_1} \right)^3 (R-R_1)^2 P_2 = 0,$$

(66)

$$\frac{\partial}{\partial R} \frac{R_2}{R_2 - R_1} (R-R_1)^2 \frac{\partial P_2}{\partial R} + k_0^2 \left( \frac{R_2}{R_2 - R_1} \right)^3 (R-R_1)^2 P_2 = 0,$$

(67)

Let $r = \frac{R_2}{R_2 - R_1} (R-R_1)$, it is obvious that $P_2(\vec{R})$ in (60) can be translated to

$$P_2(\vec{R}) = -\frac{1}{4\pi} e^{ik_b r_s} r_s \frac{R_2}{R_2 - R_1} (R-R_1) j_0 \left( k_b \frac{R_2}{R_2 - R_1} (R-R_1) \right)$$

(68)

so, the $P_2(\vec{R})$ in (60) or (69) is solution of the equation (68), therefore, $P_2(\vec{R}) = -\frac{1}{4\pi} e^{ik_b r_s} r_s \frac{R_2}{R_2 - R_1} (R-R_1) j_0 \left( k_b \frac{R_2}{R_2 - R_1} (R-R_1) \right)$ in (60) is solution of the acoustic equation (64),(59) or (14). Also

$$P_2(\vec{R})|_{\vec{R} = \vec{R}_1} = -\frac{1}{4\pi} e^{ik_b r_s} r_s \frac{R_2}{R_2 - R_1} (R-R_1)$$

(69)

$$= P_1(\vec{R})|_{\vec{R} = \vec{R}_1},$$

$$= \frac{R_2}{R_2 - R_1} (R-R_1)^2 \frac{\partial P_2}{\partial R} |_{R=R_1} = \frac{R_2}{R_2 - R_1} (R-R_1)^2 \frac{\partial P_2}{\partial R} |_{R=R_1} = 0,$$

(70)

Therefore pressure wave solution of (14) or (59), $P_1(\vec{R})$ and $P_2(\vec{R})$ satisfy the interface continuous conditions
Using 0 to $R_1$ sphere radial linear transformation, Pendry et al proposed EM invisible cloak [3]. It is proved that by [5][6] Pendry EM cloak [3] is invisible cloak with infinite speed and exceeding light speed fundamental difficulties. However, in the paper [4], authors did use 0 to $R_1$ spherical radial linear coordinate transformation to induce acoustic cloak, but we proved that is not “no scattering acoustic cloak”. In the page 024103-4 of paper [4], from interface boundary derivative continuous condition (28) in [4], $C_n = 0$, for all $n = 0, 1, 2 \cdots$ was obtained. However, for $n = 0$, left hand side of (27) in [4], $K_0j_0(k_0r-a) |_{r=a} = K_0 \neq 0$, the right hand side of (27).$C_n j_0(k_0r)|_{r=a} = 0$. So the basic interface continuous condition equation (27) in [4] is not satisfied. Authors of [4] used interface boundary derivative continuous conditions (28) to derive zero scattering in the inner sphere $R \leq R_1$ (in $r \leq a$ in [4]), which inversely destroyed wave field interface continuous conditions (27) that caused contradiction equation (27). Authors in paper [4] used $| \vec{R}_1|$ Physically, this is because the radial mass density tends to infinity at the inner edge of the shell which reduces all radial particle motion to zero | to explain equation (28) in [4], but that can not explain the basic contradiction equation (27) in [4]. The contradiction equation (27) in the paper [4] is caused by inconsistent between the relative induced anisotropic acoustic media (20)-(22) in layer $R_1 \leq R \leq R_2$ by 0 to $R_1$ (or 0 to a in [4]) linear transformation (18) and background medium in inner sphere $R \leq R_1$ and $j_1(k_0 R_1) \neq 0$, | (or $j_1(k_0 a) \neq 0$ in [4]). The inconsistent causes there exist no solution of the anisotropic acoustic equations and interface continuous equations system. In statement 5, we proved to chose infinity countable suitable angular frequency $\omega$, inner sphere radius $R_1$, and background acoustic speed $c_b$, that making $j_1(k_0 R_1) = j_1(k_\omega R_1) = 0$, and the acoustic wave solution of the acoustic wave equation (14) and does satisfy "no scattering interface continuous condition" (26) and (27) on the outer interface boundary $\vec{R} = vecR_2$, then the acoustic wave $P_1(\vec{R})$ is propagation to penetrate into the inner sphere $R \leq R_1$ and becomes to $P_2(\vec{R}) = -\frac{1}{4\pi} e^{ik_0 r} j_0(k_0 r)$ in (57), and the $P_1(\vec{R})$ and $P_2(\vec{R})$ satisfy interface continuous conditions without contradiction equation in [4], the inner sphere can not be cloaked. note that the anisotropic media by (20) to (22) in the annular layer $R_1 \leq R \leq R_2$ that is same as anisotropic media in [4], we only add the condition $j_1(k_0 R_1) = j_1(k_\omega R_1) = 0$, and the acoustic wave solution of the acoustic wave equation (14) and does satisfy "no scattering interface continuous condition" (26) and (27) on the outer interface boundary $\vec{R} = vecR_2$, the acoustic wave is propagation and continuous to penetrate into the sphere $R \leq R_1$ and without the contradiction equation [for example, (27) in [4]]. Therefore, explain of authors in paper [4] [Physically, this is because the radial mass density tends to infinity at the inner edge of the shell which reduces all radial particle motion to zero | that can not be used to explain the basic contradiction equation (27) in [4]. The acoustic cloak in [4] is not acoustic no scattering cloak, the inner sphere can not be cloaked. 0 to $R_1$ sphere radial linear transformation can not be used to induce acoustic no scattering cloak. By 0 to $R_1$ sphere radial transformation (7)-(10), the background sphere $r \leq R_2$ is compressed into the sphere annular layer domain, $R_1 \leq R \leq R_2$. A new sphere $r \leq R_1$ is expanded by inflated of zero. Originally, A new sphere $r \leq R_1$ is absolute empty space without physical acoustic media. We should install acoustic medium in the new inner sphere $r \leq R_1$. We proved that the induced anisotropic media (15) to (17) or (20) to (22) in the annular layer $R_1 \leq r \leq R_2$ is inconsistent with the background acoustic speed medium in the inner sphere $r \leq R_1$. The inconsistent shows that the induced anisotropic media (15) to (17) or (20) to (22) in the annular layer $R_1 \leq r \leq R_2$ is not "No scattering acoustic cloak" media. we prove that the acoustic cloak[4] is not "no scattering acoustic cloak", and we prove that the 0 to $R_1$ spherical radial transformation method can not be used to induce acoustic no scattering cloak.

In statement 6, we propose a novel anisotropic media in the sphere $R \leq R_1$ that is the induced anisotropic acoustic media (15) to (17) or (20) to (22) by 0 to $R_1$ sphere radial linear transformation (7) or(18), respectively. The anisotropic acoustic medium formulas in the sphere $R \leq R_1$ is same as that in the annular layer $R_1 \leq R \leq R_2$. We prove if the anisotropic acoustic media are installed in the sphere $R \leq R_1$, and the acoustic wave solution of the acoustic wave equation (14) and does satisfy "no scattering interface continuous condition" (26) and (27) on the outer interface boundary $\vec{R} = vecR_2$, then the acoustic wave solution of the acoustic wave equation (14) is continuous propagation to penetrate into the inner sphere $R \leq R_1$, the inner sphere $R \leq R_1$, can not be cloaked. In our statement 6, we install the anisotropic acoustic media (20) – (22) in the annular layer $R_1 \leq R \leq R_2$ and inner sphere $R \leq R_1$ that are same as [(6) – (8) in page 024301-2 of paper [4], the induced density are going to infinity on the both sides of the interface boundary edge $\vec{R} = R_1$, if by explain of authors in paper [4], the
pressure $P(\vec{R})$ should be zero in the inner sphere $R \leq R_1$, however, in statement 6, we proved that the nonzero and bounded acoustic wave is continuous propagation to penetrate into the inner sphere $R \leq R_1$, and in inner sphere $R \leq R_1$, the nonzero acoustic wave $P_2(\vec{R})$ is bounded and continuous solution of the acoustic equation (59) or (14) with the induced anisotropic acoustic media (20) – (22),

$$P_2(\vec{R}) = \frac{1}{4\pi} \frac{e^{ik_r r_s}}{r_s} j_0 \left( k_r \frac{R_2}{R_2 - R_1} (R_1 - R) \right),$$

therefore, the explain of the authors in [4] [ Physically, this is because the radial mass density tends to infinity at the inner edge of the shell which reduces all radial particle motion to zero ] that can not be used to explain the basic contradiction equation (27) in paper [4]. The physical explain in paper [4] is not physical base of the "no scattering acoustic cloak" in [4]. The cloak in paper [4] is not "acoustic No scattering cloak", the inner sphere $R \leq R_1 [(r \leq a) in [4]] can not be cloaked.

In statement 4, we proved that if the inner sphere is cloaked that causes the interface continuous condition (26) and (27) can not be satisfied that is contradiction with the necessary "No Scattering interface continuous condition" (26) and (27) on the outer interface boundary $\vec{R} = \vec{R}_2$. Because radial mass density does no tend to infinity at the outer edge of the shell, $\vec{R} = \vec{R}_2$, the physical explain in [4] is fail to explain the contradiction in outer interface boundary, $\vec{R} = \vec{R}_2$. These undisputed evidences and theoretical proofs prove that the physical explain in paper [4] is not physical base of the "no scattering acoustic cloak" in [4]. The cloak in paper [4] is not "acoustic No scattering cloak", the inner sphere $R \leq R_1 [(r \leq a) in [4]] can not be cloaked.

0 to $R_1$ spherical radial transformation can not be used to induce acoustic no scattering cloak. We have proposed the Dirichlet to Neumann boundary value nonlinear operator method and one dimensional well posed acoustic wave coefficient inversion in 1986 [10] and three dimensional acoustic wave coefficient inversion in 1988 [11]. Based on our acoustic wave velocity inversion, without transform, we proposed GILD and GL no scattering method [12],[13] and propose GLLH and GLHUA double layer electromagnetic invisible cloak without exceeding light speed violation propagation [1][2][5][7], which method can be used to make acoustic no scattering cloak in next paper will be submitted.

Fianally, we discuss the case $k_b = 0$ that can be caused by the zero frequency or infinity background acoustic speed $c_0$. When $k_b = 0$, the acoustic equation becomes to the variable coefficient homogeneous Laplace-Betrami equation In [14], author consider static electric conductivity equation in absence source, that is variable coefficient homogeneous Laplace-Betrami equation. 1 to 0 (D-to-N) is boundary map of any variable coefficient Laplace-Betrami equation in absence source. The no uniqueness of this inverse scattering problem is obvious, Counterexamples in [14] is no necessary and trivial for variable coefficient Laplace-Betrami equation. Transformation hole in [14] is not "acoustic no scattering cloak", because for any space domain with background material or any continuation material hole, unity field 1 is always obvious solution of variable coefficient Laplace-Betrami equation in absence source. The field 1 is always to penetrate into the hole if boundary D to N map is 1 to 0. Inversely, to cloak hole that causes the boundary map to become 0 to 0, causes scattering field to disturb incident map 1 to 0 to map 0 to 0, the whole domain is detected and exposed, Counterexamples in paper [14] is not acoustic no scattering cloak, Counterexamples in paper [14] is not relative to the electromagnetics invisible cloak in [1][2][3][5][7].

VIII. PHYSICAL LETTER

We propose our Global and Local field method and novel approach to prove the cloak in paper [4] is not "No scattering acoustic cloak". First, we define "the global acoustic equation system", Second, we define "acoustic no scattering cloak" as follows that suppose that the anisotropic acoustic media is created in the annular layer that make there exist acoustic wave solution to satisfy the global acoustic equation system, if in the outside of whole sphere the acoustic wave solution equal to incident wave, $p(\vec{r}) = p_i(\vec{r})$, i.e. there exist no scattering wave to disturb the incident wave, and the acoustic wave solution is zero in the inner sphere, i.e. the inner sphere is cloaked, then the annular layer, $R_1 \leq R \leq R_2$ and inner sphere $R \leq R_1$ is called the "acoustic no scattering cloak. In Statement 7 in this paper, we prove if the induced anisotropic acoustic media (73) - (75) (i.e. (25),(24),(26) in paper [4]) by 0 to $R_1$ linear transformation "0R1SRLT" in (71)-(22) is installed in the annular layer $R_1 \leq R \leq R_2$, and $j_1(k_b R_1) \neq 0$, then there exist no acoustic wave solution to satisfy the "global acoustic equation system" in (61) to (68). That prove that the cloak in paper [4] is not "acoustic No scattering Cloak". In the statement 8 in this paper, we prove if the induced anisotropic acoustic media (73) - (75) (i.e. (25),(24),(26) in paper [4]) by 0 to $R_1$ linear transformation "0R1SRLT" in (71)-(72) is installed in the annular layer $R_1 \leq R \leq R_2$, and $j_1(k_b R_1) = 0$, then there exist acoustic wave solution to satisfy the "global acoustic wave equation system" in (61) to (68), the nonzero and continuous bounded acoustic wave solution is propagation to penetrate into the inner sphere, the inner sphere $R \leq R_1$ is not cloaked. That prove that the cloak in paper [4] is not "acoustic No scattering Cloak". In the novel statement 9, we prove if the induced anisotropic acoustic media (73) - (75) by 0 to $R_1$ linear transformation "0R1SRLT" in (71)-(72) is installed in the annular layer $R_1 \leq R \leq R_2$ and the inner sphere $R \leq R_1$, then there exist acoustic wave solution to satisfy the "global acoustic wave equation system" in (61) to (65) and (91) to (93), the nonzero and continuous bounded physical acoustic wave solution is propagation...
to penetrate into the inner sphere, \( R \leq R_1 \), the inner sphere \( R \leq R_1 \) is not cloaked. That prove that the cloak in paper [4] is not "acoustic No scattering Cloak". That prove that the "Physically ...." explanation in paper [4] is not correct and is a mistake in "acoustic No Scattering Cloak Super Physical Sciences". We proved that 0 to \( R_1 \) spherical radial transformation can not be used to induce acoustic no scattering cloak. The 0 to \( R_1 \) spherical radial transformation can not be used to induce static electric conductivity no scattering cloak. We define "the global acoustic equation system" as follows, chose \( R_2 > R_1 > 0 \), a whole sphere \( R \leq R_2 \) is located in the 3D full space, the 3D full Space is split into the outside of the whole sphere \( R \geq R_2 \) with background isotropic acoustic media, the annular layer \( R_1 \leq R \leq R_2 \) with anisotropic acoustic media, the inner sphere \( R \leq R_1 \) with background isotropic acoustic media, and two interfaces \( R = R_2 \) and \( R = R_1 \). The isotropic acoustic equation in the outside of the whole sphere, \( R \geq R_2 \)

\[
\begin{align*}
\frac{\partial}{\partial r} \left( r^2 \frac{\partial p(\vec{r})}{\partial r} \right) + \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \sin \vartheta \frac{\partial p(\vec{r})}{\partial \vartheta} \\
+ \frac{1}{\sin^2 \vartheta} \frac{\partial^2 p(\vec{r})}{\partial \varphi^2} + k_0^2 r^2 p(\vec{r}) = S(\vec{r}, \vec{r}_s).
\end{align*}
\]  

(71)

Sommerfeld radiation condition equation

\[
\lim_{r \to \infty} r \left( \frac{\partial p}{\partial r} - ipk p(\vec{r}) \right) = 0,
\]

(72)

where \( p(\vec{r}) \) is the pressure wave field in the background space. \( \omega_0 \) is the background constant acoustic speed, \( \omega_0 = \frac{k_0}{\rho_0} \), \( S(\vec{r}, \vec{r}_s) \) is the acoustic source located in \( \vec{r}_s \), \( r_s > R_2 \). The anisotropic acoustic equation is in the annular layer \( R_1 \leq R \leq R_2 \),

\[
\begin{align*}
\frac{\partial}{\partial r} \left( R^2 \frac{\partial p(\vec{r})}{\partial r} \right) + \frac{1}{\rho_s \sin \vartheta} \frac{\partial}{\partial \vartheta} \sin \vartheta \frac{\partial p(\vec{r})}{\partial \vartheta} \\
+ \frac{1}{\rho_s \sin^2 \vartheta} \frac{\partial^2 p(\vec{r})}{\partial \varphi^2} + \frac{k^2}{R^2} P_1(\vec{R}) = 0,
\end{align*}
\]  

(73)

Pressure interface continuous equation on \( R = R_2 \),

\[
P_1(\vec{R}_2) = p(\vec{R}_2^+),
\]

(74)

velocity interface continuous equation on \( R = R_2 \),

\[
\begin{align*}
\frac{1}{\rho_r} \frac{\partial}{\partial R} P_1(\vec{R}_2) &= \frac{\partial}{\partial r} p(\vec{R}_2^+),
\end{align*}
\]  

(75)

The acoustic wave equation in the inner sphere \( R \leq R_1 \)

\[
\begin{align*}
\frac{\partial}{\partial r} \left( R^2 \frac{\partial p(\vec{r})}{\partial r} \right) + \frac{1}{\rho_s \sin \vartheta} \frac{\partial}{\partial \vartheta} \sin \vartheta \frac{\partial p(\vec{r})}{\partial \vartheta} \\
+ \frac{1}{\rho_s \sin^2 \vartheta} \frac{\partial^2 p(\vec{r})}{\partial \varphi^2} + \frac{k^2}{R^2} P_2(\vec{R}) = 0,
\end{align*}
\]  

(76)

Pressure interface continuous equation on \( R = R_1 \),

\[
P_2(\vec{R}_1^+) = P_1(\vec{R}_1^-),
\]

(77)

velocity interface continuous equation on \( R = R_1 \),

\[
\frac{\partial}{\partial R} P_2(\vec{R}_1^-) = \frac{1}{\rho_r} \frac{\partial}{\partial r} P_1(\vec{R}_1^+).
\]

(78)

The equations (71) to (78) compose to the "global acoustic wave equation system". The source in (71) is

\[
S(\vec{r}, \vec{r}_s) = \delta(r - r_s) \delta(\vartheta - \theta_s) \delta(\varphi - \phi_s) / \sin \theta.
\]

(79)

In the 3D background media full space, the solution of the equation (71) with source (79) is called the incident acoustic wave \( p_i(\vec{r}) \),

\[
p_i(\vec{r}) = -0.25 e^{ik_0 |\vec{r} - \vec{r}_s|} / |\vec{r} - \vec{r}_s| / \pi.
\]

(80)

We define "acoustic no scattering cloak" as follows: Suppose that the anisotropic acoustic media is created in the annular layer that make there exist acoustic wave solution to satisfy the global acoustic equation system, if in the outside of whole sphere the acoustic wave solution equal to incident wave, \( p(\vec{r}) = p_i(\vec{r}) \), i.e. there exist no scattering wave to disturb the incident wave, and the acoustic wave solution is zero in the inner sphere, i.e. the inner sphere is cloaked, then the annular layer, \( R_1 \leq R \leq R_2 \) and inner sphere \( R \leq R_1 \) is called the "acoustic no scattering cloak", the anisotropic acoustic media is called no scattering materials, the annular layer is called no scattering cloaking layer, the inner sphere is called no scattering cloaked concealment. 0 to \( R_1 \) sphere radial coordinate linear transformation "0R1SRLT" is

\[
R = R_1 + (R_2 - R_1) r / R_2,
\]

(81)

and its inverse transformation

\[
r = R_2 (R - R_1) / (R_2 - R_1),
\]

(82)

The induced anisotropic acoustic media by the above "0R1SRLT" in (81)-(82) is

\[
\rho_r = (R_2 - R_1) R_2 / (R_2 (R - R_1)^2),
\]

(83)

\[
\rho_0 = \rho_0 = (R_2 - R_1) / R_2,
\]

(84)

\[
\lambda = (R_2 - R_1) R_2 / (R_2 (R - R_1)^2),
\]

(85)

Statement 1: If the induced anisotropic acoustic media (83) - (85) by 0 to \( R_1 \) linear transformation "0R1SRLT" in (81)-(82) is installed in the annular layer \( R_1 \leq R \leq R_2 \), and \( j_1(k_0 R_1) \neq 0 \), then there exist no acoustic wave solution to satisfy the "global acoustic wave equation system" in (71) to (78).

Proof: To solve (71) to (75) in the global acoustic equation system, the acoustic wave solution equal to incident wave, \( p(\vec{r}) = p_i(\vec{R}) \) in outside of whole sphere, \( R \geq R_2 \), by the statement 1 in our paper [15] in arXiv:1706.05375v7, the acoustic wave solution \( P_1(\vec{R}) \) in the annular layer \( R_1 \leq R \leq R_2 \), is

\[
P_1(\vec{R}) = p_i(r(\vec{R}), \vartheta, \phi) = - \frac{1}{4 \pi i \rho_s} \frac{\Delta \vartheta \varphi}{\rho_0 (\vec{R} - (R_2 - R_1), \vartheta, \phi - \vec{r}_s)}.
\]

(86)
on the inner boundary $\vec{R} = \vec{R}_1$, the pressure acoustic wave $P_1(\vec{R}^+_1)$ is nonzero constant,

$$P_1(\vec{R}^+_1) = -\frac{1}{4\pi} \frac{e^{ik_0|r_s|}}{|r_s|} \neq 0. \quad (87)$$

and velocity is zero. Because on the inner boundary $\vec{R} = \vec{R}_1$, the pressure $P_1(\vec{R}_1)$ is localized to nonzero constant in (87), and the velocity is localized to zero, which are independent with the angular $\theta$ and $\phi$, the pressure interface continuous equation on the interface $\vec{R} = \vec{R}_1$. (77) becomes

$$P_2(\vec{R}^+_1) = P_2(\vec{R}^-_1) = P_1(\vec{R}^+_1) = -\frac{1}{4\pi} \frac{e^{ik_0|r_s|}}{|r_s|}, \quad (88)$$

the fluid velocity interface continuous equation on the interface $\vec{R} = \vec{R}_1$. (78) becomes

$$\frac{\partial}{\partial R} P_2(\vec{R}^+_1) = \frac{\partial}{\partial R} P_2(\vec{R}^-_1) = \frac{1}{\rho_c} \frac{\partial}{\partial r} P_1(\vec{R}^+_1) = 0. \quad (89)$$

The acoustic wave equation is the inner sphere $R \leq R_1$ (76) becomes sphere Bessel equation,

$$\frac{\partial}{\partial R} \left( R^2 \frac{\partial P_2(R)}{\partial R} \right) + k^2_0 R^2 P_2(R) = 0. \quad (90)$$

The global acoustic equation system (71) to (78) is localized to local equation system (88) to (90) in the inner sphere $R \leq R_1$, the unique solution of equation (90) with Neumann boundary condition (89),

$$P_2(\vec{R}) = 0, \quad (91)$$

does not satisfy the pressure wave continuos equation (88), the inner sphere can not be cloaked. Also, because $j_1(k_0 R_1) \neq 0$, the unique solution of equation (90) with Dirichlet boundary condition (88), if $j_0(k_0 R_1) \neq 0,

$$P_2(\vec{R}) = -\frac{1}{4\pi} \frac{e^{-ik_0 r_s}}{r_s} \frac{j_0(k_0 R)}{j_0(k_0 R_1)}. \quad (92)$$

does not satisfy the velocity interface continuous equation (89), there exist no acoustic wave solution to satisfy the local acoustic equation system (88) to (90) in the inner sphere, therefore, there exist no acoustic wave solution to satisfy the global acoustic system (71) to (78), the inner sphere $R \leq R_1$ can not be cloaked. Inversely, suppose that the inner sphere $R \leq R_1$ is cloaked, $P_2(\vec{R}) = 0$, by the statement 3 in [15], the pressure acoustic wave $P_2(\vec{R}) = 0$, in $R \leq R_1$ and $P_1(\vec{R})$ in the $R_1 \leq R \leq R_2$

$$P_1(\vec{R}) = -\frac{1}{4\pi} \frac{e^{ik_0 |((R_2/(R_2 - R_1))(R - R_1), \theta, \phi) - \vec{r}|}}{|(R_2/(R_2 - R_1))(R - R_1) - \vec{r}|} \quad + \frac{1}{4\pi} i k_0 j_0 (k_0 ((R_2/(R_2 - R_1))(R - R_1)) h^{(1)}_0(k_0 r_s), \quad (93)$$
satisfy the equations (76), (77) and (78) in the global acoustic equation system, where $j_0$ is 0 order sphere Bessel function, $h^{(1)}_0$ zero order first Hankel function. Moreover, on the outer interface boundary, acoustic wave $P_1(\vec{R}_2)$ is different from incident wave $p_0(\vec{R}_2),\quad

$$P_1(\vec{R}_2) = -\frac{1}{4\pi} \frac{e^{ik_0 |((R_2/(R_2 - R_1))(R - R_1), \theta, \phi) - \vec{r}|}}{|(R_2/(R_2 - R_1))(R - R_1) - \vec{r}|} \quad + \frac{1}{4\pi} i k_0 j_0 (k_0 ((R_2/(R_2 - R_1))(R - R_1)) h^{(1)}_0(k_0 r_s), \quad (94)$$

Substitute $P_1(\vec{R}_2)$ in (94) into the interface continuous equation (74) and (75), the equation (74) is reduced to

$$p_2(R_2) = \frac{1}{4\pi} i k_0 j_0 (k_0 R_2) h^{(1)}_0(k_0 r_s), \quad (95)$$

and the equation (75) is reduced to

$$\frac{\partial}{\partial r} p_s(\vec{R}^+_2) = -\frac{1}{4\pi} i (k_0)^2 j_1(k_0 R_2) h^{(1)}_0(k_0 r_s), \quad (96)$$

The acoustic wave equation (71) in the outside of whole sphere $R \geq R_2$ is reduced to the scattering sphere Bessel equation,

$$\frac{\partial}{\partial r} \left( r^2 \frac{\partial p_s(r)}{\partial r} \right) + k_0^2 r^2 p_s(r) = 0. \quad (97)$$

the Sommerfeld radiation equation (72) is reduced to

$$\lim_{r \rightarrow \infty} r(\partial p_s/\partial r - ik p_s(r)) = 0, \quad (98)$$

The global acoustic equation system (71) to (78) is localized to local scattering equation system (95) to (98) in the outside of whole sphere $R \geq R_2$, the unique solution of scattering equation (97) and (98) with Neumann boundary condition (96),

$$p_s(r) = \frac{i k_0}{4\pi} \frac{h^{(1)}_0(k_0 r)}{h^{(1)}_0(k_0 R_2)} j_1(k_0 R_2) h^{(1)}_0(k_0 r_s), \quad (99)$$

does not satisfy the equation (95), the unique solution of scattering equation (97) and (98) with Dirichlet boundary condition (95)

$$p_s(r) = \frac{i k_0}{4\pi} \frac{h^{(1)}_0(k_0 r)}{h^{(1)}_0(k_0 R_2)} j_1(k_0 R_2) h^{(1)}_0(k_0 r_s), \quad (100)$$

does not satisfy the equation (96). There exist no scattering wave solution to satisfy the scattering equation system (95) to (98), therefore, there exist no acoustic wave solution to satisfy the global acoustic system equations (71) to (78), the statement 1 is proved. The statement 1 shows that the induced anisotropic acoustic media (83) to (85) by linear transformation "ORISRLT" (81) and (82) in annular layer is inconsistent with background acoustic media in inner sphere. the 0 to $R_1$ sphere radial linear transformation can not be used to induce the acoustic no scattering cloak.

**Statement 2**: If the induced anisotropic acoustic media
(83) - (85) by 0 to $R_1$ linear transformation "0R1SRLT" in (81)-(82) is installed in the annular layer $R_1 \leq R \leq R_2$, and $j_1(k_bR_1) = 0$, then there exist acoustic wave solution to satisfy the "global acoustic wave equation system" in (71) to (78), the nonzero and continuous bounded acoustic wave solution is propagation to penetrate into the inner sphere, the inner sphere $R \leq R_1$ is not cloaked. **Proof:** To repeat the proof in first paragraph of the statement 1. Because on the inner bundary $\vec{R} = \vec{R}_1$, the pressure $P_1(\vec{R}_1)$ is nonzero constant in (87), and velocity is zero, which are independent with the angular $\theta$ and $\phi$, the global acoustic equation system (71) to (78) is localized to local equation system (88) to (90) in the inner sphere $R \leq R_1$, because $j_1(k_bR_1) = 0$, the unique solution of equation (90) with Dirichlet boundary condition (88), if $j_0(k_bR_1) \neq 0$,

$$P_2(\vec{R}) = -\frac{1}{4\pi} e^{i k_b r_s} j_0(k_b R) \frac{j_0(k_b R_1)}{j_0(k_b R_1)}, \quad (92)$$

does satisfy the velocity interface continuous equation (89), there exist acoustic wave solution to satisfy the local acoustic equation system (88) to (90) in the inner sphere, therefore, there exist acoustic wave solution to satisfy the global acoustic system (71) to (78), the nonzero acoustic wave $P_2(\vec{R})$ is propagation in the inner sphere, the inner sphere $R \leq R_1$ is not cloaked.

The following statement is novel that the induced anisotropic acoustic media (83) - (85) by 0 to $R_1$ linear transformation "0R1SRLT" in (81)-(82) is installed in the annular layer $R_1 \leq R \leq R_2$ and the inner sphere $R \leq R_1$, the equation (76) should be changed to the anisotropic acoustic equation in the annular layer $R \leq R_1$,

$$\frac{\partial}{\partial r} R^2 \frac{\partial P_2(\vec{R})}{\partial r} + \frac{1}{\rho_s \sin \theta} \frac{\partial}{\partial \theta} \frac{1}{\rho_s \sin \theta} \frac{\partial P_2(\vec{R})}{\partial \theta} + \frac{1}{\rho_s \sin \theta} \frac{\partial P_2(\vec{R})}{\partial \phi} + \frac{k_b^2}{2} R^2 P_2(\vec{R}) = 0, \quad (101)$$

the equation (77) is no changed, the pressure interface continuous equation on the interface $\vec{R} = \vec{R}_1$,

$$P_2(\vec{R}_1^-) = P_1(\vec{R}_1^+), \quad (102)$$

the equation (78) should be changed to fluid velocity interface continuous equation on the interface $\vec{R} = \vec{R}_1$,

$$\frac{1}{\rho_s(\vec{R}_1^-)} \frac{\partial}{\partial R} P_2(\vec{R}_1^-) = \frac{1}{\rho_s(\vec{R}_1^+)} \frac{\partial}{\partial r} P_1(\vec{R}_1^+). \quad (103)$$

**Statement 3:** If the induced anisotropic acoustic media (83) - (85) by 0 to $R_1$ linear transformation "0R1SRLT" in (81)-(82) is installed in the annular layer $R_1 \leq R \leq R_2$ and the inner sphere $R \leq R_1$, then there exist acoustic wave solution to satisfy the "global acoustic wave equation system" in (71) to (75) and (101) to (103), the nonzero and continuous bounded acoustic wave solution is propagation to penetrate into the inner sphere, $R \leq R_1$

$$P_2(R) = -\frac{1}{4\pi} e^{i k_b r_s} j_0 \left( \frac{k_b R^2}{R_2 - R_1} (R - R_1) \right), \quad (104)$$

de the inner sphere $R \leq R_1$ is not cloaked.

**Proof:** To repeat the proof in first paragraph of the statement 1. Because on the inner bundary $\vec{R} = \vec{R}_1$, the pressure $P_1(\vec{R}_1)$ is nonzero constant in (87), and velocity is zero, which are independent with the angular $\theta$ and $\phi$, the "global acoustic wave equation system" (71) to (75) and (101) to (103), is localized to the following local equation system (105) to (107) in the inner sphere $R \leq R_1$, the equation (101) is reduced to the 1-D sphere Bessel equation (105) with anistropic media,

$$\frac{\partial}{\partial R} \frac{R^2 P_2(\vec{R})}{R - R_1} - k_b^2 \left( \frac{R_2 - R_1}{R - R_1} \right)^3 (R - R_1)^2 P_2 = 0, \quad (105)$$

equation (102) is reduced to the equation (106)

$$P_2(R_1^-) = P_2(R_1^+) = P_1(R_1^+) = -\frac{1}{4\pi} e^{i k_b r_s} \frac{j_0(k_b R_1)}{j_0(k_b R_1)}, \quad (106)$$

equation (103) is reduced to the equation (107)

$$\frac{R^2 P_2(\vec{R})}{R - R_1} \frac{\partial P_2}{\partial R} \bigg|_{R=R_1} = \frac{R_2 - R_1}{R_2 - R_1} \frac{\partial P_2}{\partial R} \bigg|_{R=R_1} = 0, \quad (107)$$

Let $r = \frac{R_2 - R_1}{R - R_1}$, the equation (105) becomes

$$\frac{\partial}{\partial r} \frac{R_2}{R_2 - R_1} \frac{\partial}{\partial r} (R - R_1)^2 \frac{\partial P_2}{\partial r} \bigg|_{R=R_1} + k_b^2 \left( \frac{R_2 - R_1}{R_2 - R_1} \right)^3 (R - R_1)^2 P_2 = 0, \quad (108)$$

By coordinate transformation in statement 6 in [15], the equation (108) is reduced to the 1-D sphere Bessel equation,

$$\frac{\partial}{\partial r} r^2 \frac{\partial P_2}{\partial r} + k_b^2 r^2 P_2 = 0, \quad (109)$$

Let $r = \frac{R_2 - R_1}{R - R_1}$, it is obvious that $P_2(R)$ in (104) can be translated to $P_2(r)$

$$P_2(R) = -\frac{1}{4\pi} e^{i k_b r_s} j_0 \left( \frac{k_b R_2}{R_2 - R_1} (R - R_1) \right) = \frac{1}{4\pi} \frac{e^{i k_b r_s} j_0}{r_s} j_0(k_b r) = P_2(r), \quad (110)$$

so, the $P_2(r)$ in (110) is solution of the equation (99), therefore, $P_2(\vec{R})$ in (104)

$$P_2(\vec{R}) = -\frac{1}{4\pi} e^{i k_b r_s} j_0 \left( \frac{k_b R_2}{R_2 - R_1} (R - R_1) \right), \quad (104)$$

is solution of the acoustic equation (105), also satisfies interface boundary pressure continuous condition (106)

$$P_2(R^-) = P_2(R)|_{R=R_1} = -\frac{1}{4\pi} e^{i k_b r_s} j_0 \left( \frac{k_b R_2}{R_2 - R_1} (R_1 - R_1) \right) = -\frac{1}{4\pi} \frac{e^{i k_b r_s}}{r_s} \bigg|_{r=R_1} \quad (111)$$

and velocity continuous interface condition (107)

$$\frac{R_2 - R_1}{R_2 - R_1} (R - R_1)^2 \frac{\partial P_2}{\partial R} \bigg|_{R=R_1} = \frac{R_2 - R_1}{R_2 - R_1} (R - R_1)^2 \frac{\partial P_2}{\partial r} \bigg|_{R=R_1} = 0, \quad (112)$$
Therefore there exist acoustic wave solution to satisfy the global acoustic equation system ((71) to (75) and (101) to (103)), the acoustic wave solution in (104) is propagation to penetrate into the inner sphere \( R \leq R_1 \), the inner sphere \( R \leq R_2 \) is not cloaked. The induced anisotropic acoustic media (83), (84), (85) are same as (25), (24), (26), in the page 24301-3 in the paper [4]. In statement 1, we proved that when \( j_1(k_2 R_1) \neq 0 \), there exist no acoustic wave solution to satisfy the global acoustic equation system with anisotropic the media in paper [4], the anisotropic acoustic media (25), (24), (26), in page 24301-3 in [4] is inconsistent with the background media, the equation (27) in page 24301-3 in the paper [4] is not satisfied and is contradiction equation for \( n = 0 \). The acoustic cloak in paper [4] is not acoustic no scattering cloak. In paper [4], authors write “Physically, this is because the radial mass density tends to infinity at the inner edge of the shell which reduces all radial particle motion to zero” to explain their cloak, authors in [4] only explain the fluid velocity is zero at the inner edge of shell, but the pressure is nonzero and wave speed is nonzero in the edge of shell, moreover, we used same induced anisotropic acoustic media in paper [4], the radial mass density tends to infinity at the inner edge of the shell, we only add condition \( j_1(k_2 R_1) = 0 \), in statement 2, we proved that there exist nonzero acoustic wave propagation in (92) in the inner sphere \( R \leq R_1 \), the inner sphere \( R \leq R_1 \), is not cloaked. Novel, we install the induced anisotropic acoustic media in paper [4] in the annular layer shell \( R_1 \leq R \leq R_2 \) and inner sphere \( R \leq R_1 \), the radial mass density tends to infinity at the inner both edges of the shell, by the explain in paper [4] the inner sphere should be cloaked, however, in statement 3, we proved that there exist nonzero acoustic wave in (104) is propagation in the inner sphere \( R \leq R_1 \), the inner sphere \( R \leq R_1 \), is not cloaked. Therefore, The acoustic cloak in paper [4] is not acoustic no scattering cloak. The 0 to \( R_1 \) spherical radial transformation can not be used to induce acoustic no scattering cloak. Our proof are suitable for arbitrary 0 to \( R_1 \) sphere radial transformation and its induced anisotropic, therefore the 0 to \( R_1 \) spherical radial transformation can not be used to induce acoustic no scattering cloak [15].

IX. CONCLUSION

Summary, if \( j_1(k_2 R_1) \neq 0 \), the transformation induced anisotropic acoustic media (20) to (22) in the annular layer \( R_1 \leq R \leq R_2 \) are inconsistent with background medium in inner sphere \( R \leq R_1 \), that caused there exist no solution of the acoustic equations and the interface continuous conditions equations system. if \( j_1(k_2 R_1) = 0 \), there exist acoustic wave solution to satisfy the above anisotropic acoustic equations and interface continuous condition equations system, the acoustic wave solution is propagation to penetrate into the inner sphere, the inner sphere can not be cloaked. If we install the transformation induced anisotropic acoustic media (20) to (22) in the annular layer \( R_1 \leq R \leq R_2 \) and inner sphere \( R \leq R_1 \), the acoustic wave solution is continuous propagation to penetrate into the inner sphere, the inner sphere \( R \leq R_1 \), can not be cloaked. Therefore, 0 to \( R_1 \) spherical radial transformation can not be used to induce acoustic no scattering cloak. The 0 to \( R_1 \) spherical radial transformation can not be used to induce static electric conductivity no scattering cloak. It is totally different from transformation, the GILD and GL no scattering modeling and inversion can be used to make GLHUA and GLLH double layer acoustic no scattering cloak [1] [2] [5] [7] [8].

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[1] G. Xie, Jianhua Li, L. Xie, F. Xie, arXiv:1005.3999 (2010).
[2] Jianhua. Li, F. Xie, L. Xie, G. Xie, arXiv:1706.10147 (2016).
[3] J. B. Pendry, D. Schurig, and D. R. Smith, Science 312, 1780 (2006).
[4] Cummer, S.A., Popa, B.-I., Schurig, D., Smith, D.R., Pendry, J., Rahm, M., Starr, A.: Phys. Rev. Lett. 100, 024301(2008)
[5] Jianhua. Li, F. Xie, L. Xie, G. Xie, arXiv:1612.02857 (2016).
[6] H. Chen, B.-I. Wu, B. Zhang, and J. A. Kong, Phys. Rev. Lett. 99, 063903 (2007).
[7] Jianhua. Li, F. Xie, L. Xie, G. Xie, arXiv:1701.00534 (2017).
[8] Jianhua. Li, F. Xie, L. Xie, G. Xie, arXiv:1701.02583 (2017).
[9] Lax, P. D., Scattering Theory, Reversed Edition, Academic Press, Feb.2, 2012.
[10] Ganquan Xie, Communication on pure and applied math., vol. 39, 307-322, 1986.
[11] Ganquan Xie and J. Li, Science In China (series A) vol. 31, no.10, 1988.?
[12] G. Xie., J.H. Li, E. Major, D. Zuo, M. Geophysics, Vol. 65, No. 3, 804–822, 2000.
[13] G. Xie., F. Xie, L. Xie, and J. Li, PIER 63, 141–152, 2006.
[14] A. Greenleaf, M. Lassas, G. Uhlmann, Math. Res. Lett. 10.1 (2003).
[15] Jianhua. Li, F. Xie, L. Xie, G. Xie, arXiv:1706.05375 (2017).
(2017).