Gauged $L_\mu - L_\tau$ with Large Muon Anomalous Magnetic Moment and the Bimaximal Mixing of Neutrinos

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Abstract

We consider the gauging of $L_\mu - L_\tau$ as an explanation of a possibly large muon anomalous magnetic moment. We then show how neutrino masses with bimaximal mixing may be obtained in this framework. We study the novel phenomenology of the associated gauge boson in the context of present and future high-energy collider experiments.
In the minimal standard model of quarks and leptons with no right-handed neutrino singlet, one of the three lepton number differences ($L_e - L_\mu$, $L_e - L_\tau$, $L_\mu - L_\tau$) is anomaly-free and may be gauged [1]. If one right-handed neutrino singlet $N_R$ is added, then one of the three combinations ($B - 3L_e$, $B - 3L_\mu$, $B - 3L_\tau$) is also anomaly-free and may be gauged $[2, 3]$. For example, we could have both $L_e - L_\mu$ and $B - 3L_\tau$. On the other hand, even with just one $N_R$, we may choose to consider $L_\mu - L_\tau$ as the only additional gauge symmetry.

Specifically, under this extra gauge symmetry $U(1)_X$, $(\nu_\mu, \nu_\tau, \mu, \tau)_L$, $(\nu_\mu, \mu)_R$ have charge +1; $(\nu_\tau, \tau)_L$, $\tau_R$ have charge −1; all other fields including $N_R$ have charge 0. It has already been noted [4] that the extra gauge boson $X$ of this model contributes to the muon anomalous magnetic moment as shown in Fig. 1. Its contribution [5] is easily calculated to be

$$\Delta a_\mu = \frac{g_X^2 m_\mu^2}{12\pi^2 M_X^2}. \quad (1)$$

To complete the model, we add two extra Higgs doublets: $(\eta^+_1, \eta^0_1)$ have charge +1 and $(\eta^+_2, \eta^0_2)$ have charge −1. This differs from Model C of Ref. [4] in their $U(1)_X$ charge assignments. Thus our model has no flavor-changing couplings in the charged-lepton sector, but because we also add the one $N_R$, realistic neutrino oscillations are allowed, as shown below.

The mass matrix spanning $X$ and the standard $Z$ boson is given by

$$M_{XZ}^2 = \begin{bmatrix}
2g_X^2 (v_1^2 + v_2^2) & g_X g_Z (v_1^2 - v_2^2) \\
g_X g_Z (v_1^2 - v_2^2) & (g_Z^2/2)(v_0^2 + v_1^2 + v_2^2)
\end{bmatrix}, \quad (2)$$

where $v_{0,1,2}$ are the vacuum expectation values of the standard-model $\phi^0$ and $\eta^0_{1,2}$ respectively, with $v_0^2 + v_1^2 + v_2^2 = (2\sqrt{2}G_F)^{-1}$. If we assume $v_1 = v_2$, then there is no $X - Z$ mixing and $M_X = 2g_X v_1$. This implies

$$\Delta a_\mu = \frac{m_\mu^2}{48\pi^2 v_1^2} > \frac{G_F m_\mu^2}{6\pi^2\sqrt{2}} = 1.555 \times 10^{-9}. \quad (3)$$

In other words, such a model actually predicts a lower bound on $\Delta a_\mu$. 

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Experimentally, the muon magnetic moment has been measured precisely \[^6\] and a large positive discrepancy \[^7\] of \((4.26 \pm 1.65) \times 10^{-9}\) from the prediction of the standard model is possible, although there is no universal consensus regarding the uncertainties of the hadronic contributions \[^8\]. Note that \(M_X/M_Z = 2\sqrt{2}(g_X/g_Z)(v_1/\sqrt{v_0^2 + 2v_1^2})\), which means that \(M_X\) is allowed to be much heavier than \(M_Z\) even though Eq. (3) is independent of it. For example, if \(v_0 = v_1 = v_2\), then \(\Delta a_\mu = 2.33 \times 10^{-9}\), and \(M_X/g_X \approx 200\) GeV.

To obtain a desirable pattern of neutrino masses to explain the atmospheric \[^9\] and solar \[^10\] neutrino data, we add a singlet charged scalar \(\zeta^+\) which also has a \(U(1)_X\) charge of +1 (but since it is a scalar, it does not contribute to the axial vector anomaly), and supplement our model with a discrete \(Z_2\) symmetry, under which \(\eta_{1,2}\) and \(N_R\) are odd but all other fields are even. The relevant Yukawa interaction terms are then given by

\[ L_Y = f_1 \bar{N}_R(\nu_\mu \eta_2^0 - \mu_L \eta_2^+ ) + f_2 \bar{N}_R(\nu_\tau \eta_0^1 - \tau_L \eta_1^+) + h \zeta^+ (\nu_e \tau_L - e_L \nu_\tau) + H.c. \] (4)

Since \(N_R\) is allowed a large Majorana mass \(M_N\), the canonical seesaw mechanism \[^11\] generates one small neutrino mass

\[ m_3 = \frac{f_1^2 v_2^2 + f_2^2 v_1^2}{M_N} = \frac{2f_1^2 v_1^2}{M_N} \] (5)

corresponding to the eigenstate

\[ \nu_3 = \frac{f_1 \nu_\mu + f_2 \nu_\tau}{\sqrt{f_1^2 + f_2^2}}. \] (6)

We now allow the \(Z_2\) discrete symmetry to be broken softly, i.e. by terms of dimension 2 or 3 in the Lagrangian. However, given the gauge symmetry and particle content of our model, the only possible such term is the trilinear scalar interaction

\[ L_S = \lambda \, \zeta^- (\eta_1^+ \phi^0 - \eta_0^0 \phi^+) + H.c. \] (7)

This generates a radiative \(\nu_e \nu_\tau\) mass as shown in Fig. 2. As a result, the \(3 \times 3\) neutrino mass
matrix in the \((\nu_e, \nu_\mu, \nu_\tau)\) basis is given by

\[
\mathcal{M}_\nu = \begin{bmatrix}
0 & 0 & m' \\
0 & m_3 c^2 & m_3 s c \\
m' & m_3 s c & m_3 s^2
\end{bmatrix}, \tag{8}
\]

where \(s \equiv \sin \theta\) and \(c \equiv \cos \theta\) with \(s/c = f_2/f_1\).

Assuming \(m'\) to be much smaller than \(m_3\), the eigenvalues are easily determined to be

\[
\pm cm' - \frac{s^2 m'^2}{2m_3}, \quad m_3 + \frac{s^2 m'^2}{m_3}, \tag{9}
\]

corresponding to the eigenstates

\[
\nu_1 = \frac{1}{\sqrt{2}} \left[ 1 - \frac{s^2 m'}{4cm_3} \right] \nu_e - \frac{s}{\sqrt{2}} \left[ 1 + \frac{(4 - 3s^2)m'}{4cm_3} \right] \nu_\mu + \frac{c}{\sqrt{2}} \left[ 1 - \frac{3s^2 m'}{4cm_3} \right] \nu_\tau, \tag{10}
\]

\[
\nu_2 = \frac{1}{\sqrt{2}} \left[ 1 + \frac{s^2 m'}{4cm_3} \right] \nu_e + \frac{s}{\sqrt{2}} \left[ 1 - \frac{(4 - 3s^2)m'}{4cm_3} \right] \nu_\mu - \frac{c}{\sqrt{2}} \left[ 1 + \frac{3s^2 m'}{4cm_3} \right] \nu_\tau, \tag{11}
\]

\[
\nu_3 = \frac{sm'}{m_3} \nu_e + c \nu_\mu + s \nu_\tau. \tag{12}
\]

If \(f_1 = f_2\) so that \(s = c = 1/\sqrt{2}\), we then obtain nearly bimaximal mixing of neutrinos for understanding the atmospheric and solar data as neutrino oscillations. In addition,

\[
\Delta m_{23}^2 \simeq \Delta m_{13}^2 \simeq m_3^2 + (2s^2 - c^2)m'^2 = m_3^2 + \frac{1}{2}m'^2, \tag{13}
\]

\[
\Delta m_{12}^2 \simeq \frac{2s^2 m'^3}{m_3} = \frac{m_3^3}{\sqrt{2}m_3}. \tag{14}
\]

Using \(m_3 = 0.05\, \text{eV}, m' = 0.016\, \text{eV}\), we find \(\Delta m_{23}^2 \simeq 2.6 \times 10^{-3}\, \text{eV}^2\), and \(\Delta m_{12}^2 \simeq 5.8 \times 10^{-5}\, \text{eV}^2\), in good agreement with data \[12\]. Note also that \(|U_{e3}| \simeq 0.22\) (close to the maximum value allowed) in this model, due to the form \[13\] of Eq. (8).

Referring back to Fig. 2, we calculate the \(\nu_e\nu_\tau\) mass term to be

\[
m' \simeq \frac{\hbar \lambda m^2 v_1}{16\pi^2 v_0 m_3^2}. \tag{15}
\]

Let \(v_0 = v_1, m_\xi = 1\, \text{TeV}\), then \(m' = 0.016\, \text{eV}\) implies \(\hbar \lambda = 0.8\, \text{MeV}\). This is consistent with our assumption that the term in Eq. (7) breaks the assumed \(Z_2\) discrete symmetry softly, so
that $\lambda$ may be naturally small \[14\]. Note also that $m_\zeta$ is assumed to be heavy in order that it does not contribute significantly to $\tau \rightarrow e\nu_\tau \bar{\nu}_e$.

To obtain $v_1 = v_2$, we assume that the Higgs potential containing $\Phi$, $\eta_{1,2}$ and $\zeta$ is invariant under the interchange of $\eta_1$ and $\eta_2$. In that case, the components of $(\eta_1 - \eta_2)/\sqrt{2}$ are mass eigenstates. If they are the lightest scalars, they would be stable because neither $\eta_1$ nor $\eta_2$ could decay into light fermions [see Eq. (4)]. However, the $\eta_1 - \eta_2$ interchange symmetry cannot be exact because of Eq. (4) and other terms of the Standard Model; hence we expect some small mixing between $(\eta_1 - \eta_2)/\sqrt{2}$ and $\Phi$, which will allow it to decay, but with an enhanced lifetime. Note also that $X \rightarrow -X$ under the interchange of $\eta_1$ and $\eta_2$; hence even (odd) states under this symmetry may decay into lighter odd (even) states + $X$ (either real or virtual) in this model.

Assuming the typical range of $M_X/g_X \sim 200$ GeV for explaining the muon anomalous magnetic moment, one expects interesting phenomenological signatures of the $X$ boson at present and future high-energy collider experiments. Let us discuss them one by one.

Firstly one can search for the $Z \rightarrow f\bar{f}X$ decay in the LEP-I data, where $f = \mu, \tau$ or $\nu_\mu, \nu_\tau$. The squared decay amplitude averaged over the $Z$ polarizations is

$$|M|^2 = 16E_1E_2g^2_Zg^2_X\left[(1 + \cos \theta)\left\{\frac{1}{(M_Z - 2E_1)^2} + \frac{1}{(M_Z - 2E_2)^2}\right\}\right. \right.
\left. + \frac{4(1 - \cos \theta)}{(M_Z - 2E_1)(M_Z - 2E_2)}\left\{1 - \frac{E_1 + E_2}{M_Z} + \frac{E_1E_2(1 - \cos \theta)}{M^2_Z}\right\}\right] (16)$$

where $g^2_Z = (2e^2/\sin^2 2\theta_W)(I^2_{3f} + 2\sin^4 \theta_W Q^2_{3f} - 2\sin^2 \theta_W I_{3f}Q_{3f})$ and $E_{1,2}$ are the energies of $\bar{f}, f$ and $\theta$ the angle between them in the $Z$ rest frame. In particular the decay $Z \rightarrow \mu^+\mu^-X$, follows by $X \rightarrow \mu^+\mu^- (BR = 1/3)$, leads to a clean 4-muon final state. We have computed this signal cross-section incorporating a $p_T > 3$ GeV cut on each muon, as required for muon identification at LEP, and made a comparison with the ALEPH data [15]. This corresponds to 1.6 million hadronic $Z$ events and shows 20 4-muon events against the SM prediction of
20.0 ± 0.6. Moreover the smaller $\mu^+\mu^-$ invariant mass for all these events as well as the SM prediction is < 20 GeV. Thus the 95% CL upper bound on the number of signal events for $M_X > 20$ GeV is 3, corresponding to the 0 observed events. Fig. 3 shows the resulting lower limit on $M_X$ as a function of $g_X$, i.e. $M_X > 50(70)$ GeV for $g_X \gtrsim 0.1(1)$.

Secondly the model predicts a small deviation from the universality of $Z$ boson coupling to $e^+e^-$, $\mu^+\mu^-$ and $\tau^+\tau^-$ channels, since the latter ones have an extra one-loop radiative correction from $X$. The resulting contribution to the $Z$ width is given by

$$\frac{\Delta \Gamma}{\Gamma} = -\frac{g_X^2}{4\pi^2} \left[ \frac{7}{4} + \delta + \left( \delta + \frac{3}{2} \right) \ell n\delta + (1 + \delta^2) \left\{ Li_2 \left( \frac{\delta}{1 + \delta} \right) + \frac{1}{2} \ell n^2 \left( \frac{\delta}{1 + \delta} \right) - \frac{\pi^2}{6} \right\} \right],$$

where $\delta \equiv M_X^2/M_Z^2$, and $Li_2(x) = -\int_0^x (dt/t)\ell n(1-t)$ is the Spence function. The measured $Z$ partial widths at LEP-I

$$\Gamma_e = 84.02 \pm 0.14 \text{ MeV}, \quad \Gamma_\mu = 84.00 \pm 0.21 \text{ MeV},$$

(18)
correspond to a 95% CL limit of $\Delta \Gamma/\Gamma < 0.006$ on adding the two errors in quadrature. The resulting upper limit on $g_X$ is shown in Fig. 3 as a function of $M_X$. It does not give any serious constraint on the mass or coupling of the $X$ boson.

We have estimated the signal cross-section for $X$ boson production at LEP 200 and LC energies via $e^+e^- \rightarrow \mu^+\mu^-X$, followed by the $X \rightarrow \mu^+\mu^-$ decay. The squared Feynman amplitude for $e^+e^- \rightarrow \mu^+\mu^-X$ was evaluated using the FORM program [18]. The resulting 4-muon signal cross-sections are shown in Fig. 4 for $g_X = 1$, where we have again imposed a $p_T^\mu > 3$ GeV cut as required for muon identification. The signal can be easily distinguished from the SM background of Drell-Yan pairs via the clustering of a $\mu^+\mu^-$ invariant mass at $M_X$. Thus a signal size of $\sim 10$ events should be adequate for discovery of the $X$ boson. With the integrated luminosity of $\sim 0.7 \text{ fb}^{-1}$ at LEP 200, this corresponds to a signal cross-section of $\sim 10 \text{ fb}$. Thus we see from Fig. 4 that the LEP 200 limit on $X$ mass is $M_X > 60$
GeV for $g_X = 1$, which is no better than the LEP-I limit. With the projected luminosity of $\sim 100 \, \text{fb}^{-1}$ at LC, a signal cross-section of 0.1 fb should be viable. This corresponds to a discovery limit of $M_X = 300(500) \, \text{GeV}$ at LC 500 (LC 1000) for $g_X = 1$. Note that the signal cross-section scales like $g_X^2$. Thus the LC discovery limit goes down to 200 (250) GeV for $g_X = 0.3$ and to 100 GeV for $g_X = 0.1$.

We have also estimated the $X$ signal cross-section for TEV 2 and LHC energies of 2 and 14 TeV respectively. In each case we have computed the 3-muon and 4-muon signals from $u\bar{d} \rightarrow \mu \nu X$ and $u\bar{u}(d\bar{d}) \rightarrow \mu^+\mu^- X$ respectively, followed by $X \rightarrow \mu^+\mu^-$. We have imposed a $p_T^\mu > 10 \, \text{GeV}$ and $|\eta_\mu| < 2.5$ cut on each muon as required for muon identification at these colliders. The resulting signal cross-sections are shown in Fig. 5. Even in this case we expect that the clean 3-muon and 4-muon signal events can be distinguished from the SM background via the clustering of a $\mu^+\mu^-$ invariant mass at $M_X$. Thus we again consider a signal size of $\sim 10$ events as adequate for the discovery of $X$ boson. With the expected luminosity of $\sim 2 \, \text{fb}^{-1}$ in Run II of the Tevatron, this corresponds to a signal cross-section of $\sim 10 \, \text{fb}$. This means a discovery limit of $M_X = 70 \, \text{GeV}$ for $g_X = 1$, i.e. similar to LEP 200. The projected luminosity of $100 \, \text{fb}^{-1}$ at LHC implies a viable signal cross-section of 0.1 fb. This corresponds to a discovery limit of 400 GeV for $g_X = 1$, going down to 200 (100) GeV for $g_X = 0.3(0.1)$. These are very similar to the corresponding discovery limits of LC. While they do not exhaust the full range of $M_X/g_X$, they do cover the interesting range of $M_X/g_X \sim 200 \, \text{GeV}$. Finally one expects copious production of the $X$ boson at muon colliders right upto $M_X = \sqrt{s}$, because of its gauge coupling to the $\mu^+\mu^-$ pair.

In conclusion, we have proposed in the above a verifiable explanation of the possible discrepancy of the newly measured muon anomalous magnetic moment as coming from the realization of the gauged $L_\mu - L_\tau$ symmetry at the electroweak energy scale. Our specific model has the added advantage of allowing a simple neutrino mass matrix which can
explain the present data on atmospheric and solar neutrino oscillations. We discuss the phenomenology of the associated gauge boson $X$ and show that it can indeed be relatively light, i.e. $M_X/g_X \sim 200$ GeV, and be observed through its distinctive decay into $\mu^+\mu^-$ at future high-energy colliders.

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Figure 1: Contribution of $X$ to muon magnetic moment.

Figure 2: Radiative contribution to the $\nu_e\nu_\tau$ mass.
Figure 3: The LEP-I constraints on the mass and coupling of the $X$ boson from $Z \rightarrow X\mu\mu$ decay and the universality of $Z$ coupling to the $e^+e^-$ and $\mu^+\mu^-$ channels. The region above the curves is excluded at 95% CL.
Figure 4: The $X$ boson signal cross-section in the 4-muon channel at LEP and LC energies of 200, 500 and 1000 GeV for $g_X = 1$. 
Figure 5: The $X$ boson signal cross-section in the 3-muon and 4-muons channels at the Tevatron and LHC energies for $g_X = 1$. 