DO WE OBSERVE QUANTUM GRAVITY EFFECTS AT GALACTIC SCALES? *

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Abstract. The nonperturbative renormalization group flow of Quantum Einstein Gravity (QEG) is reviewed. It is argued that there could be strong renormalization effects at large distances, in particular a scale dependent Newton constant, which mimic the presence of dark matter at galactic and cosmological scales.

1 Introduction

By now it appears increasingly likely that Quantum Einstein Gravity (QEG), the quantum field theory of gravity whose underlying degrees of freedom are those of the spacetime metric, can be defined nonperturbatively as a fundamental, “asymptotically safe” theory (Lauscher 2002). By definition, its bare action is given by a non–Gaussian renormalization group (RG) fixed point. In the framework of the “effective average action” a suitable fixed point is known to exist within certain approximations. They suggest that the fixed point should also exist in the exact theory, implying its nonperturbative renormalizability.

The general picture regarding the RG behavior of QEG as it has emerged so far points towards a certain analogy between QEG and non–Abelian Yang–Mills theories, Quantum Chromo–Dynamics (QCD) say. For example, like the Yang–Mills coupling constant, the running Newton constant \( G = G(k) \) is an asymptotically free coupling, it vanishes in the ultraviolet (UV), i.e. when the typical momentum scale \( k \) becomes large. In QCD the realm of asymptotic freedom is realized for momenta \( k \) larger than the mass scale \( \Lambda_{\text{QCD}} \) which is induced dynamically. In QEG the analogous role is played by the Planck mass \( m_{\text{Pl}} \). It delimits the asymptotic scaling region towards the infrared (IR). For \( k \gg m_{\text{Pl}} \) the RG flow is well described by its linearization about the non–Gaussian fixed point. Both in QCD and QEG simple local approximations (truncations) of the running Wilsonian action (effective average action) are sufficient above \( \Lambda_{\text{QCD}} \) and \( m_{\text{Pl}} \), respectively. However, as the scale \( k \) approaches \( \Lambda_{\text{QCD}} \) or \( m_{\text{Pl}} \) from above, many complicated,

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typically nonlocal terms are generated in the effective action. In fact, in the IR, strong renormalization effects are to be expected because gauge (diffeomorphism) invariance leads to a massless excitation, the gluon (graviton), implying potential IR divergences which the RG flow must cure in a dynamical way. Because of the complexity of the corresponding flow equations it is extremely difficult to explore the RG flow of QCD or QEG in the IR, far below the UV scaling regime, by analytical methods. In QCD, lattice results and phenomenology suggest that the nonperturbative IR effects modify the classical Coulomb term by adding a confinement potential to it which increases (linearly) with distance: $V(r) = -a/r + \kappa r$.

The problem of the missing mass or “dark matter” is one of the most puzzling mysteries of modern astrophysics. It is an intriguing idea that the apparent mass discrepancy is not due to an unknown form of matter but rather indicates that we are using the wrong theory of gravity, Newton’s law in the non-relativistic and General Relativity in the relativistic case. If one tries to explain the observed non–Keplerian rotation curves of galaxies or clusters in terms of a modified Newton law, a nonclassical term needs to be added to the $1/r$-potential whose relative importance grows with distance. In “MOND”, for instance, a point mass $M$ produces the potential $\phi(r) = -GM/r + \sqrt{a_0 GM} \ln(r)$ and it is tempting to compare the $\ln(r)$-term to the qualitative similar confinement potential in (quenched) QCD. It seems not unreasonable to speculate that the “confinement” potential in gravity is a quantum effect which results from the antiscreening character of quantum gravity (Lauscher 2002) in very much the same way as this happens in Yang–Mills theory. If so, the missing mass problem could get resolved in a very elegant manner without the need of introducing dark matter on an ad hoc basis. In (Reuter 2004a,b) this idea has been explored within a semi–phenomenological analysis of the effective average action of quantum gravity.

2 RG running of the gravitational parameters

The effective average action $\Gamma_k[g_{\mu\nu}]$ is a “coarse grained” Wilsonian action functional which defines an effective field theory of gravity at the variable mass scale $k$. Roughly speaking, the solution to the effective Einstein equations $\delta\Gamma_k/\delta g_{\mu\nu} = 0$ yields the metric averaged over a spacetime volume of linear extension $k^{-1}$. (From the technical point of view $k$ is a IR cutoff introduced into the functional integral over the microscopic metric in such a way that only quantum fluctuations of wavelengths smaller than $k^{-1}$ are integrated out.) In a physical situation with a typical scale $k$, the effective field equation $\delta\Gamma_k/\delta g_{\mu\nu} = 0$ “knows” about all quantum effects relevant at this particular scale. For $k$ fixed, the functional $\Gamma_k$ should be visualized as a point in the space of all action functionals. When the RG effects are “switched on”, one obtains a curve in this space, the RG trajectory, which starts at the bare action $S \equiv \Gamma_{k \to \infty}$ and ends at the ordinary effective action $\Gamma \equiv \Gamma_{k \to 0}$. At the exact level, $\Gamma_k$ contains all the infinitely many invariants one can construct from $g_{\mu\nu}$, their $k$-dependent prefactors having the interpretation of scale dependent gravitational coupling constants. To become technically feasible most of the investigations using the effective average action formalism employ the
so-called Einstein-Hilbert approximation which retains only Newton’s constant $G(k)$ and the cosmological constant $\Lambda(k)$ as running parameters. If one introduces the dimensionless couplings $g(k) \equiv k^2 G(k)$ and $\lambda(k) \equiv \Lambda(k)/k^2$ the RG equations governing their scale dependence read $k \partial_k g = \beta_g(g, \lambda)$, $k \partial_k \lambda = \beta_\lambda(g, \lambda)$ with known beta–functions $\beta_g$ and $\beta_\lambda$. The RG flow on the $g$-$\lambda$–plane displays two fixed points: a Gaussian fixed point (GFP) at the origin, and the non-Gaussian fixed point (NGFP) at $g_\ast > 0$, $\lambda_\ast > 0$ which is necessary for asymptotic safety. The RG trajectories are classified as of Type Ia, IIa (separatrix), and IIIa depending on whether, when $k$ is lowered, they run towards negative, vanishing, and positive values of the cosmological constant, respectively. In (Reuter 2004b) the very special trajectory which seems realized in Nature has been identified and its parameters were determined. This trajectory is of Type IIIa; see fig. 1.

![Fig. 1. Nature’s Type IIIa trajectory and the separatrix. The dashed line is a classical RG trajectory along which $G(k), \Lambda(k) = \text{const}$. (From (Reuter 2004b).)](image)

For $k \to \infty$ it starts infinitesimally close to the NGFP. Then, lowering $k$, the trajectory spirals about the NGFP and approaches the “separatrix”, the distinguished trajectory which ends at the GFP. It runs almost parallel to the separatrix for a very long “RG time”; only in the “very last moment” before reaching the GFP, at the turning point $T$, it gets driven away towards larger values of $\lambda$. In fig. 1 the points $P_1$ and $P_2$ symbolize the beginning and the end of the regime in which classical general relativity is valid (“GR regime”). The classical regime starts soon after the turning point $T$ which is passed at the scale $k_T \approx 10^{-30} m_{\text{Pl}}$.

In (Reuter 2004b) we argued that to the right of the point $P_2$ there starts a regime of strong IR renormalization effects which might become visible at astrophysical and cosmological length scales. In fact, within the Einstein-Hilbert approximation, trajectories of Type IIIa cannot be continued to the extreme IR ($k \to 0$). They terminate at a non-zero value of $k$ as soon as the trajectory reaches $\lambda = 1/2$. (Close to the question mark in fig. 1.) Before it starts becoming invalid and has to be replaced by a more precise treatment, the Einstein-Hilbert approximation suggests that $G$ will increase, while $\Lambda$ decreases, as $\lambda \nearrow 1/2$. 

The Type IIIa trajectory of QEG which Nature has selected is highly special in the following sense. It is fine-tuned in such a way that it gets extremely close to the GFP before “turning left”. The coordinates $g_T$ and $\lambda_T$ of the turning point are both very small: $g_T = \lambda_T \approx 10^{-60}$. The coupling $g$ decreases from $g(k) = 10^{-70}$ at a typical terrestrial length scale of $k^{-1} = 1$ m to $g(k) = 10^{-92}$ at the solar system scale of $k^{-1} = 1$ AU, and finally reaches $g(k) = 10^{-120}$ when $k$ equals the present Hubble constant $H_0$.

In fact, the Hubble parameter $k = H_0$ is approximately the scale where the Einstein-Hilbert trajectory becomes unreliable. The observations indicate that today the cosmological constant is of the order $H_0^2$. Interpreting this value as the running $\Lambda(k)$ at the scale $k = H_0$, the dimensionless $\lambda(k)$, at this scale, is of the order unity: $\lambda(H_0) \equiv \Lambda(H_0)/H_0^2 = O(1)$. So it is roughly near the present Hubble scale where the IR effects should have grown large.

In principle it should be possible to work out the predictions of the theory for cosmological scales by an ab initio calculation within QEG. Unfortunately, because of the enormous technical complexity of the RG equations, this has not been possible in practice yet. In this situation one can adopt a phenomenological strategy, however. One makes an ansatz for the RG trajectory which has the general features discussed above, derives its consequences, and confronts them with the observations. In this manner the observational data can be used in order to learn something about the RG trajectory in the nonperturbative regime which is inaccessible to an analytic treatment for the time being. Using this strategy, the cosmological consequences of a very simple scenario for the $k \to 0$ behavior has been worked out; the assumption proposed in (Bonanno 2002) is that the IR effects lead to the formation of a second NGFP into which the RG trajectory gets attracted for $k \to 0$. This hypothesis leads to a phenomenologically viable late-time cosmology with a variety of rather attractive features. It predicts an accelerated expansion of the universe and explains, without any fine tuning, why the corresponding matter and vacuum energy densities are approximately equal.

3 Galaxy rotation curves

Given the encouraging results indicating that the IR effects are “at work” in cosmology, by continuity, it seems plausible to suspect that somewhere between solar system and cosmological scales they should first become visible. In (Reuter 2004a,b) we therefore investigated the idea that they are responsible for the observed non-Keplerian galaxy rotation curves. The calculational scheme used there was a kind of “RG improvement”, the basic idea being that upon identifying the scale $k$ with an appropriate geometric quantity comparatively simple (local) truncations effectively mimic much more complicated (nonlocal) terms in the effective action. Considering spherically symmetric, static model galaxies only, the scale $k$ was taken to be the inverse of the radial proper distance which boils down to $1/r$ in leading order. Since the regime of galactic scales turned out to lie outside the domain of validity of the Einstein–Hilbert approximation the only practical option was to make an ansatz for the RG trajectory $\{G(k), \Lambda(k), \cdots\}$ and to explore its
observable consequences. In particular a relationship between the $k$-dependence of $G$ and the rotation curve $v(r)$ of the model galaxy has been derived.

The idea was to start from the classical Einstein–Hilbert action and to promote $G$ and $\Lambda$ to scalar fields: $S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \{ R/G(x) - 2 \Lambda(x)/G(x) \}$. Upon adding a matter contribution this action implies the modified Einstein equation $\mathcal{G}_{\mu\nu} = -\Lambda(x) g_{\mu\nu} + 8\pi G(x) (T_{\mu\nu} + \Delta T_{\mu\nu})$ with $\Delta T_{\mu\nu} \equiv \frac{1}{8\pi} (D_\mu D_\nu - g_{\mu\nu} D^2) G^{-1}$. In (Reuter 2004a) we analyzed the weak field, slow–motion approximation of this theory for a time–independent Newton constant $G = G(x)$ and $\Lambda \equiv 0$. In this (modified) Newtonian limit the equation of motion for massive test particles has the usual form, $\ddot{x}(t) = -\nabla \phi$, but the potential $\phi$ obeys a modified Poisson equation:

$$\nabla^2 \phi = 4\pi \frac{G}{\rho_{\text{eff}}} \rho_{\text{eff}} \quad \text{where} \quad \rho_{\text{eff}} \equiv \rho + (8\pi G)^{-1} \nabla^2 \mathcal{N} \quad (3.1)$$

Here it is assumed that $T_{\mu\nu}$ describes pressureless dust of density $\rho$ and that $G(x)$ does not differ much from the constant $\mathcal{G}$. Setting $G(x) \equiv \mathcal{G} [1 + \mathcal{N}(x)]$ we assumed that $\mathcal{N}(x) \ll 1$. Apart from the rest energy density $\rho$ of the ordinary ("baryonic") matter, the effective energy density $\rho_{\text{eff}}$ contains an additional contribution $(8\pi G)^{-1} \nabla^2 \mathcal{N}(x) = (8\pi G)^{-1} \nabla^2 G(x)$ due to the position dependence of Newton’s constant. Since it acts as a source for $\phi$ on exactly the same footing as $\rho$ it mimics the presence of “dark matter”.

Up to this point the discussion applies to an arbitrary prescribed position dependence of Newton’s constant, not necessarily related to a RG trajectory. In the case of spherical symmetry the natural choice of the geometric cutoff is $k = \xi/r$ with $\xi$ a constant of order unity. Hence we obtain the position dependent Newton constant $G(x) \equiv G(r)$ as $G(r) \equiv G(k = \xi/r)$. Writing again $G \equiv \mathcal{G} [1 + \mathcal{N}]$, $G(k)$ should be such that $\mathcal{N} \ll 1$.

Let us make a simple model of a spherically symmetric “galaxy”. For an arbitrary density profile $\rho = \rho(r)$ the solution of the modified Poisson equation reads

$$\phi(r) = \int_0^r \frac{d\rho'}{r'^2} \frac{\mathcal{M}(r')}{r^2} + \frac{1}{2} \mathcal{N}(r) \quad (3.2)$$

where $\mathcal{M}(r) \equiv 4\pi \int_0^r \rho(r') r'^2 \, d\rho'$ is the mass of the ordinary matter contained in a ball of radius $r$. On circular orbits test particles in the potential (3.2) have the velocity $v^2(r) = r \phi'(r)$ so that we obtain the rotation curve

$$v^2(r) = \frac{\mathcal{G} \mathcal{M}(r)}{r^2} + \frac{1}{2} r \frac{d}{dr} \mathcal{N}(r) \quad (3.3)$$

We identify $\rho$ with the density of the ordinary luminous matter and model the luminous core of the galaxy by a ball of radius $r_0$. The mass of the ordinary matter contained in the core is $\mathcal{M}(r_0) \equiv \mathcal{M}_0$, the “bare” total mass of the galaxy. Since, by assumption, $\rho = 0$ and hence $\mathcal{M}(r) = \mathcal{M}_0$ for $r > r_0$, the potential outside the core, in the halo, is $\phi(r) = -\frac{\mathcal{G} \mathcal{M}_0}{r} + \mathcal{N}(r)/2$. 
As an example, let us adopt the power law $G(k) \propto k^{-q}$ with $q > 0$ which was motivated in (Reuter 2004a,b). We assume that this $k$–dependence starts inside the core of the galaxy so that $G(r) \propto r^q$ everywhere in the halo. For the modified Newtonian limit to be realized, the position dependence of $G$ must be weak. Therefore we shall tentatively assume that the exponent $q$ is very small. Expanding to first order in $q$ we obtain $N(r) = q \ln(r)$. In the halo, this leads to a logarithmic modification of Newton’s potential: $\phi(r) = -G M_0 / r + q^2 \ln(r)$. The corresponding rotation curve is $v^2(r) = G M_0 / r + q / 2$. At large distances the velocity approaches a constant $v_\infty = \sqrt{q / 2}$. Obviously the rotation curve implied by the $k^{-q}$–trajectory does indeed become flat at large distances – very much like those we observe in Nature. Typical measured values of $v_\infty$ range from 100 to 300 km/sec, implying $q \approx 10^{-6}$ which is indeed very small. Including the core region, the complete rotation curve reads $v^2(r) = G M(r) / r + q / 2$. For a realistic $M(r)$ its $r$–dependence is in rough qualitative agreement with the observations.

Our $v^2(r)$ is identical to the one obtained from standard Newtonian gravity by postulating dark matter with a density $\rho_{DM} \propto 1/r^2$. We see that if $G(k) \propto k^{-q}$ with $q \approx 10^{-6}$ no dark matter is needed. The resulting position dependence of $G$ leads to an effective density $\rho_{\text{eff}} = \rho + q / (8\pi G r^2)$ where the $1/r^2$–term, well known to be the source of a logarithmic potential, is present as an automatic consequence of the RG improved gravitational dynamics.

Thus it seems that if the observed non–Keplerian rotation curves are due to a renormalization effect, the scale dependence of Newton’s constant should be roughly similar to $G(k) \propto k^{-q}$. Knowing this, it will be the main challenge for future work to see whether a corresponding RG trajectory is actually predicted by the flow equations of QEG. For the time being an ab initio calculation of this kind, while well–defined conceptually, is still considerably beyond the state of the art as far as the technology of practical RG calculations is concerned. In performing such calculations it might help to rewrite the nonlocal terms generated during the flow in terms of local field monomials by introducing extra fields besides the metric. This is a standard procedure in the Wilsonian approach which often allows for a simple local description of the effective IR dynamics. It is tempting to speculate that the resulting local effective field theory might be related to the generalized gravity theory in (Papers I) which includes a Kalb–Ramond field; it is fully relativistic and explains the galaxy and cluster data with remarkable precision.

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