Study of some $B_s \rightarrow f_0(980)$ decays in the fourth generation model

Rukmani Mohanta

School of Physics, University of Hyderabad, Hyderabad - 500 046, India

Abstract

We study some non leptonic and semileptonic decays of $B_s$ meson into a final scalar meson $f_0(980)$ in the fourth quark generation model. Since the $f_0(980)$ meson is dominantly composed of $(s\bar{s})$ pair, the mixing induced CP asymmetry in the decay mode $B_s \rightarrow J/\psi f_0(980)$ would a priori give $\sin 2\beta_s$, where $\beta_s$ is the $B_s - \bar{B}_s$ mixing phase. In the standard model this asymmetry is expected to be vanishingly small. We find that in the fourth generation model a large mixing induced CP asymmetry could be possible for this process. Similarly the branching ratios of the rare semileptonic decays $B_s \rightarrow f_0(980)l^+l^-$ and $B_s \rightarrow f_0(980)\nu\bar{\nu}$ are found to be enhanced significantly from their corresponding standard model values.

PACS numbers: 13.25.Hw, 13.20.He, 12.60.-i, 11.30.Er
I. INTRODUCTION

Although the standard model (SM) of electroweak interaction has been very successful in explaining the observed experimental data so far, but still it is believed that it is a low energy manifestation of some more fundamental theory, whose true nature is not yet known. Therefore, intensive search for physics beyond the SM is now being performed in various areas of particle physics. In this context, the rare $B$ decays mediated through flavor changing neutral current (FCNC) transitions provide an excellent testing ground to look for new physics. In the SM, these transitions occur at the one-loop level and are highly suppressed. Hence, they are very sensitive to any new physics contributions.

The spectacular performance of the two asymmetric $B$ factories Belle and Babar provided us an unique opportunity to understand the origin of CP violation in a very precise way. Although, the results from the $B$ factories do not provide us any clear evidence of new physics, but there are few cases observed in the last few years, which have 2-3 $\sigma$ deviations from their corresponding SM expectations [1]. For example, the difference between the direct CP asymmetry parameters between $B^- \to \pi^0 K^-$ and $\bar{B}^0 \to \pi^+ K^-$, which is expected to be negligibly small in the SM, but found to be nearly 15%. The measurement of mixing-induced CP asymmetry in several $b \to s$ penguin decays is not found to be same as that of $B_d \to J/\psi K_s$. Recently, a very largish CP asymmetry has been observed by the CDF and D0 collaborations [2, 3] in the tagged analysis of $B_s \to J/\psi \phi$ with value $S_{\psi\phi} \in [0.24, 1.36]$. Within the SM this asymmetry is expected to be vanishingly small, which basically comes from $B_s - \bar{B}_s$ mixing phase. A further effect has recently been observed in the exclusive decay $B_d \to K^{*0} \mu^+ \mu^-$ [4, 5], the forward-backward asymmetry is found to deviate somewhat from the predictions of the SM. Although this disagreement is not statistically significant, the Belle experiment [6] claims this result as a clear indication of new physics. The upcoming Super-B factories and the LHCb experiments are expected to make many important measurements in $b$ quark decays. These measurements may in turn reveal the presence of new physics in the $b$-sector.

In this paper, we intend to study some decays of $B_s$ meson involving a scalar meson $f_0(980)$ in the final state, such as $B_s \to J/\psi f_0(980)$, $B_s \to f_0(980) l^+ l^-$ and $B_s \to f_0(980) \nu \bar{\nu}$. These modes are particularly interesting because of several reasons. First, as particle physics is entering the era of LHC, $B_s$ physics has attracted significant attention in recent times and
hence it could play a dominant role to corroborate the results of the $B_{u,d}$ mesons and also to look for new physics signature. Secondly, the structure of the scalar meson $f_0(980)$ is not yet well understood. Therefore, the experimental observations of these modes would provide us a better understanding of the nature of the scalar mesons. We intend to analyze these decay channels both in the SM and in the fourth quark generation model, usually known as SM4. SM4 is a simple extension of the standard model with three generations (SM3) with the additional up-type ($t'$) and down-type ($d'$) quarks, which basically retains all the properties of the SM3. The fourth generation model has received a renewed interest in the recent years and it has been shown in Refs. [8–10], that the addition of a fourth family of quarks with $m_{t'}$ in the range (400-600) GeV provides a simple explanation for the several deviations that have been observed involving CP asymmetries in the $B$, $B_s$ decays. Furthermore, the fourth generation could also help to explain the observed baryon asymmetry of the Universe [11].

The paper is organized as follows. In section II we discuss the nonleptonic decay process $B_s \to J/\psi f_0(980)$. The semileptonic decays $B_s \to f_0(980)\ell^+\ell^-$ and $B_s \to f_0(980)\nu\bar{\nu}$ are discussed in section III and the results are summarized in section IV.

II. $B_s \to J/\psi f_0(980)$ PROCESS

In this section we will discuss the nonleptonic decay mode $B_s \to J/\psi f_0(980)$. Before proceeding for the analysis, first we would like to briefly discuss about the structure of the scalar meson $f_0(980)$. The light scalar mesons with masses below 1 GeV is considered as a controversial issue for a long time. Even today, there exists no consensus on the nature of the $f_0(980)$ and $a_0(980)$ mesons. While the low-energy hadron phenomenology has been successfully understood in terms of the constituent quark model, the scalar mesons are still puzzling and the quark composition of the light scalar mesons are not understood with certainty. The structure of the scalar meson $f_0(980)$ has been discussed for decades but still it is not clear. There were attempts to interpret it as $K\bar{K}$ molecular states [12], four quark states [13] and normal $q\bar{q}$ states [14]. However, recent studies of $\phi \to \gamma f_0$ ($f_0 \to \gamma\gamma$) [15, 16] and $D_s^+ \to f_0\pi^+$ decays [17] favor the $q\bar{q}$ model. Since $f_0(980)$ is produced copiously in $D_s$ decays, this supports the picture of large $s\bar{s}$ component in its wave function, as the dominant mechanism in the $D_s$ decay is $c \to s$ transition. The prominent $s\bar{s}$ nature of $f_0(980)$ has been supported by the radiative decay $\phi \to f_0(980)\gamma$ [18]. However, there are some experimental
evidences indicating that $f_0(980)$ is not a pure $s\bar{s}$ state. For example, the same order of measured branching ratios of the processes $J/\psi \rightarrow f_0(980)\phi$ and $J/\psi \rightarrow f_0(980)\omega$ clearly indicate that $f_0(980)$ contains both strange and non-strange quark content \[19\]. Thus, the structure of $f_0(980)$ is usually viewed as a mixture of $s\bar{s}$ and $n\bar{n}$ ($\equiv (u\bar{u} + d\bar{d})/\sqrt{2}$) components, i.e.,

$$|f_0(980)| = |s\bar{s}| \cos \theta + |n\bar{n}| \sin \theta,$$

(1)

where $\theta$ is the $f_0 - \sigma$ mixing angle, whose value is not yet precisely known. As discussed in Ref. \[19\], its value can be extracted from the decay rates $J/\psi \rightarrow f_0(980)\phi$ and $J/\psi \rightarrow f_0(980)\omega$ as

$$\frac{\text{Br}(J/\psi \rightarrow f_0(980)\phi)}{\text{Br}(J/\psi \rightarrow f_0(980)\omega)} = \frac{1}{\lambda} \tan^2 \theta.$$

(2)

From the measured branching ratios of these decay modes, it is found that

$$\theta = (34 \pm 6)^\circ, \quad \text{or} \quad \theta = (146 \pm 6)^\circ.$$  

(3)

However, it should be noted that only $s\bar{s}$ component of $f_0(980)$ will give nonzero contribution to the $B_s \rightarrow J/\psi f_0$ process as the spectator quark in the tree and penguin topologies of $B_s$ decays is a strange quark. Thus, the decay channel $B_s \rightarrow J/\psi f_0(980)$ involves the quark level transition $b \rightarrow c\bar{c}s$, as in the case of $B_s \rightarrow J/\psi \phi$ and hence, the CP violating phase $\beta_s$ can also be extracted from this channel.

In the $B_s$ sector, $B_s \rightarrow J/\psi \phi$ is considered as the golden mode to investigate CP violation. The CDF and D0 collaborations \[2, 3\] have obtained the value of $B_s$ mixing parameter $\phi_s = -2\beta_s$ much larger than expected in the SM, modulo a large experimental uncertainty. Hence, it is of prime importance to consider other processes to measure $\beta_s$ and in this context $B_s \rightarrow J/\psi f_0(980)$ decay mode could provide an alternate option to confirm the presence of new physics in the $B_s - \bar{B}_s$ mixing phenomenon. Furthermore, the advantage of the mode $B_s \rightarrow J/\psi f_0$ over $B_s \rightarrow J/\psi \phi$ mode is that since the final state is a CP eigenstate, no angular analysis is required to disentangle the various CP components as needed for $B_s \rightarrow J/\psi \phi$. The reconstruction of $f_0$ seems to be feasible, since $f_0$ essentially decays into $2\pi$ systems. A first qualitative attempt to predict the ratio,

$$R_{f_0/\phi} = \frac{\Gamma(B_s^0 \rightarrow J/\psi f_0(980), f_0(980) \rightarrow \pi^+\pi^-)}{\Gamma(B_s^0 \rightarrow J/\psi \phi, \phi \rightarrow K^+K^-)},$$

(4)

was made by Stone and Jhang \[20\] and was found to be of the order of $(20 \sim 30)\%$. Recently, this ratio has been measured by the LHCb collaboration \[21\]. Using a fit to the $\pi^+\pi^-$ mass
spectrum they obtained

\[ R_{f_0/\phi} = \frac{\Gamma(B_s^0 \rightarrow J/\psi f_0, f_0 \rightarrow \pi^+ \pi^-)}{\Gamma(B_s^0 \rightarrow J/\psi \phi, \phi \rightarrow K^+ K^-)} = 0.252^{+0.046 +0.027}_{-0.032 -0.033}. \]  

Furthermore, the Belle Collaboration [22] has also reported the observation of \( B_s \rightarrow J/\psi f_0(980) \) with the branching ratio

\[ \text{Br}(B_s \rightarrow J/\psi f_0(980); f_0(980) \rightarrow \pi^+ \pi^-) = (1.16^{+0.31}_{-0.19} \text{ (stat)}^{+0.15}_{-0.17} \text{ (syst)}^{+0.26}_{-0.18} \text{ (N}_{B_s^0B_s^0}) \times 10^{-4}, \]  

with a significance of 8.4\( \sigma \). Using the branching ratio \( \text{Br}(f_0(980) \rightarrow \pi^+ \pi^-) = 0.45^{+0.19}_{-0.18} \), one can obtain

\[ \text{Br}(B_s \rightarrow J/\psi f_0(980)) = (2.58 \pm 0.82) \times 10^{-4}. \]  

The effective Hamiltonian describing the transition \( b \rightarrow c\bar{c}s \) is given as [23]

\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{cb} V_{ts}^* \sum_{i=1,2} C_i(\mu) O_i - V_{tb} V_{ts}^* \sum_{i=3}^{10} C_i(\mu) O_i \right], \]  

where \( C_i(\mu) \)'s are the Wilson coefficients evaluated at the renormalization scale \( \mu \), \( O_{1,2} \) are the tree level current-current operators, \( O_{3-6} \) are the QCD and \( O_{7-10} \) are electroweak penguin operators.

Here we will use the QCD factorization approach to evaluate the hadronic matrix elements as discussed in [24]. The matrix elements describing \( \bar{B}_s \rightarrow f_0 \) transitions can be parameterized in terms of the form factors \( F_0(q^2) \) and \( F_1(q^2) \) [25] as

\[ \langle f_0(p')|\bar{s}\gamma^\mu \gamma_5 b|\bar{B}_s(p)\rangle = -i \left\{ F_1(q^2) \left[ (p + p')^\mu - \frac{m_{B_s}^2 - m_{f_0}^2}{q^2} q^\mu \right] \right. \]

\[ + \left. F_0(q^2) \frac{m_{B_s}^2 - m_{f_0}^2}{q^2} q^\mu \right\} \]

\[ \langle f_0(p')|\bar{s}\sigma^{\mu\nu} \gamma_5 q^\nu b|\bar{B}_s(p)\rangle = - \frac{F_T(q^2)}{m_{B_s} + m_{f_0}} \left[ q^2(p + p')^\mu - (m_{B_s}^2 - m_{f_0}^2) q^\mu \right], \]

where \( q = p - p' \). Using the decay constant of \( J/\psi \) meson as

\[ \langle J/\psi(q, \epsilon)|\bar{c}\gamma^\mu c|0\rangle = f_\psi m_\psi \epsilon^\mu, \]  

one can obtain the transition amplitude for the process

\[ \text{Amp}(\bar{B}_s \rightarrow J/\psi f_0) = i \frac{G_F \cos \theta f_\psi m_\psi F_1(m_{\psi}^2)}{\sqrt{2}} (\epsilon \cdot p) \left[ \lambda c a_2 - \lambda t (a_3 + a_5 + a_7 + a_9) \right] \] (11)
where $\lambda_q = V_{qb}V_{qs}$. The parameters $a_i$’s are related to the Wilson coefficients $C_i$’s and the corresponding expressions can be found in Ref. [24]. Since $\lambda_u$ is negligibly small one can replace $\lambda_t$ by $-\lambda_c$ using unitarity relation $\lambda_u + \lambda_c + \lambda_t = 0$. Thus we obtain the decay width as

$$\Gamma = \frac{|p_{cm}|^3}{4\pi} G_F^2 \cos^2 \theta f_\psi^2 F_1^2(q^2) |\lambda_c(a_2 + a_3 + a_5 + a_7 + a_9)|^2.$$  
(12)

For numerical analysis, we use the particle masses, lifetimes and the values of the CKM matrix elements from [26]. The decay constants used are (in GeV) $f_{B_s} = (0.259 \pm 0.032)$ and $f_\psi = (0.416 \pm 0.006)$ [24]. The values of the Wilson coefficients are taken from [24]. We use the values of the form factors evaluated in the LCSR approach [25] as

$$F_i(q^2) = \frac{F_i(0)}{1 - a_i(q^2/m_{B_s}^2) + b_i(q^2/m_{B_s}^2)^2},$$ 
(13)

with $(i = (1, 0, T))$. The parameters $F_i(0)$’s, $a_i$’s and $b_i$’s are given in Table-1.

It should be noted that the hard scattering contributions depend on the $f_0$ meson decay constant. However, it is well known that the decay constant of $f_0$ (which is a neutral scalar meson), $f_{f_0}$ defined as $\langle 0 | \bar{q}_2 \gamma^\mu q_1 | f_0(p) \rangle = f_{f_0} p^\mu$ vanishes due to charge conjugation invariance. Therefore, the distribution amplitude for the $f_0$ meson is normalized to the scalar decay constant $\bar{f}_{f_0}$ [24, 27] defined as

$$m_{f_0} \bar{f}_{f_0} = \langle 0 | \bar{q}_2 q_1 | f_0 \rangle.$$ 
(14)

Using the equation of motion, one can obtain a relation between the scalar and vector decay constants i.e., between $\bar{f}_{f_0}$ and $f_{f_0}$ as

$$\bar{f}_{f_0} = \frac{m_{f_0}}{m_1(\mu) - m_2(\mu)} f_{f_0}.$$ 
(15)

Since $\bar{f}_{f_0}$ is nonzero, $m_{f_0}/(m_1(\mu) - m_2(\mu))$ is finite in the limit $m_1(\mu) \to m_2(\mu)$. In our analysis we use the value of the scalar decay constant of $f_0$ meson as $\bar{f}_{f_0}(1 \text{ GeV}) = (0.37 \pm 0.02)$ GeV [24], as only the $s\bar{s}$ component of $f_0(980)$ will give nonzero contribution to the decay process.

In the QCD factorization approach there are large theoretical uncertainties associated with the weak annihilation and the chirally enhanced power corrections to the hard scattering contributions due to the end point divergences. The hard scattering contributions are parameterized as

$$X_H = \left( 1 + \rho_H e^{i\phi_H} \right) \ln \frac{m_{B_s}}{\Lambda_h}.$$ 
(16)
We use $\Lambda_h = 0.5\,\text{GeV}$ and vary the hard scattering parameters within their allowed ranges i.e., $\rho_H = 1.85 \pm 0.07$ and $\phi_H = 255.9^\circ \pm 24.6^\circ$ \cite{24}. Thus, with these values we obtain the branching ratio for the process to be

$$\text{Br}(B_s \to J/\psi f_0) = (1.97 \pm 0.62) \times 10^{-4},$$

where the uncertainties are due to the form factors, decay constants and the CKM matrix elements and the hard spectator scattering contributions. Our predicted branching ratio is slightly lower than the present experimental value with a deviation of nearly 1-$\sigma$.

Next we proceed to evaluate the mixing-induced CP asymmetry for the process, which is defined as

$$S_{\psi f_0} = \eta_{\psi f_0} \frac{2 Im \lambda}{1 + |\lambda|^2},$$

where

$$\lambda = \frac{q}{p} \frac{A(B_s \to J/\psi f_0)}{A(\bar{B}_s \to J/\psi f_0)},$$

and $\eta_{\psi f_0}$ is the CP parity of the final state $\psi\phi$, which is $-1$. $q/p$ is the $B_s - \bar{B}_s$ mixing parameter and its value in the SM is given as $q/p = \exp(-2i\beta_s)$. Since the amplitude for $B_s \to J/\psi f_0$ is real in the SM, therefore the mixing induced CP asymmetry for this process in the SM is expected to be

$$S_{\psi f_0} = \sin 2\beta_s,$$

same as (modulo a sign) $S_{\psi\phi}$.

Now we will analyze this process in the fourth generation model. In the presence of a sequential fourth generation there will be additional contributions due to the $t'$ quark in the penguin and box diagrams. Furthermore, due to the additional fourth generation there will be mixing between the $b'$ quark the three down-type quarks of the standard model and the resulting mixing matrix will become a $4 \times 4$ matrix ($V_{CKM4}$) and the unitarity condition becomes $\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0$, where $\lambda_q = V_{qb}V_{qs}^*$. The parametrization of this unitary matrix

| $F_i(q^2 = 0)$ | $a_i$       | $b_i$       |
|---------------|------------|------------|
| $F_1$         | 0.185 ± 0.029 | 1.44$^{+0.13}_{-0.09}$ | 0.59$^{+0.07}_{-0.05}$ |
| $F_0$         | 0.185 ± 0.029 | 0.47$^{+0.12}_{-0.09}$ | 0.01$^{+0.08}_{-0.09}$ |
| $F_T$         | 0.228 ± 0.036 | 1.42$^{+0.13}_{-0.10}$ | 0.60$^{+0.06}_{-0.05}$ |

TABLE I: Numerical values of the form factors $F_i(0)$ and the parameters $a_i$’s and $b_i$’s.
requires six mixing angles and three phases. The existence of the two extra phases provides the possibility of extra source of CP violation. It is also found that SM4 also contributes significantly to $\Lambda_b$ decays \[28\].

In the presence of fourth generation there will be additional contribution both to the $B_s \rightarrow J/\psi f_0$ decay amplitude as well as to the $B_s - \bar{B}_s$ mixing phenomenon. Since in the SM, $B_s \rightarrow J/\psi f_0$ decay amplitude receives dominant contribution from color suppressed tree diagram, new physics contribution to its amplitude is negligible as it is induced at the one-loop level. Therefore, there will be no significant change in its branching ratio in SM4. However, for completeness we would like to present the result here.

Thus, including the fourth generation and replacing $\lambda_t \approx \lambda_c + \lambda_{t'}$, the modified Hamiltonian becomes

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \lambda_c (C_1 O_1 + C_2 O_2) - \lambda_t \sum_{i=3}^{10} C_i O_i - \lambda_{t'} \sum_{i=3}^{10} C_{i'} O_i \right]$$

$$= \frac{G_F}{\sqrt{2}} \left[ \lambda_c \left( C_1 O_1 + C_2 O_2 + \sum_{i=3}^{10} C_i O_i \right) - \lambda_{t'} \sum_{i=3}^{10} \Delta C_i O_i \right]$$

(21)

where $\Delta C_i$’s are the effective ($t$ subtracted) $t'$ contribution.

To find the new contribution due to the fourth generation effect, first we have to evaluate the new Wilson coefficients $C_{i'}$. The values of these coefficients at the $M_W$ scale can be obtained from the corresponding contribution from $t$ quark by replacing the mass of $t$ quark by $t'$ mass in the Inami Lim functions \[29\]. These values can then be evolved to the $m_b$ scale using the renormalization group equation \[30\]. Thus, the obtained values of $\Delta C_{i=1-10}(m_b)$ for a representative $m_{t'} = 400$ GeV are as presented in Table-II.

Thus one can obtain the transition amplitude in SM4, using the QCD factorization approach as in \[24\]

$$Amp(B_s \rightarrow J/\psi f_0) = \frac{iG_F}{\sqrt{2}} \cos \theta_f m_{\psi} F_1(m_{\psi}^2) 2(\epsilon \cdot p) \left[ \lambda_c (a_2 + a_3 + a_5 + a_7 + a_9) - \lambda_{t'} (a'_3 + a'_5 + a'_7 + a'_9) \right] ,$$

(22)

\[\begin{array}{cccccccccc}
\Delta C_1 & \Delta C_2 & \Delta C_3 & \Delta C_4 & \Delta C_5 & \Delta C_6 & \Delta C_7 & \Delta C_8 & \Delta C_9 & \Delta C_{10} \\
0 & 0 & 0.628 & -0.274 & 0.042 & -0.206 & 0.443 & 0.168 & -1.926 & 0.443 \\
\end{array}\]

TABLE II: Values of the Wilson coefficients $\Delta C_i$’s (in units of $10^{-2}$) at $m_b$ scale for $m_{t'} = 400$ GeV.
where $a'_i$ are related to $\Delta C_i$’s analogous to $a_i$’s are to $C_i$’s.

The above amplitude can be symbolically written as

$$Amp = \lambda_c A_c - \lambda_{t'} A_{t'},$$

(23)

where $\lambda_i$’s contain the weak phase information and $A_i$’s are associated with strong phases. One can explicitly separate the strong and weak phases and write the amplitude as

$$Amp = \lambda_c A_c \left[1 - r a e^{i(\delta + \phi_s)}\right]$$

(24)

where $a = |\lambda_{t'}/\lambda_c|$, $\phi_s$ is the weak phase of $\lambda_{t'}$, $r = |A_{t'}/A_c|$ and $\delta$ is the relative strong phase between $A_{t'}$ and $A_c$. Thus, the CP averaged branching ratio is found to be

$$Br(B_s \to J/\psi f_0(980)) = Br^{SM}(1 + r^2 a^2 - 2 r a \cos \delta \cos \phi_s).$$

(25)

For numerical evaluation using the values of the new Wilson coefficients as presented in Table-II, we obtain $r \approx 2.4 \times 10^{-2}$ and $\delta \approx -61.5^\circ$. For the new CKM elements $\lambda_{t'}$, we use the allowed range of $|\lambda_{t'}| = (0.08 - 1.4) \times 10^{-2}$ and $\phi_s = (0 \to 80)^\circ$ for a representative $m_{t'} = 400$ GeV, extracted using the available observables mediated through $b \to s$ transitions \[8\]. We find that in the presence of a fourth generation, the branching ratio becomes

$$Br(B_s \to J/\psi f_0(980)) = (1.4 - 2.6) \times 10^{-4}.$$  

(26)

Thus, one can see that the new physics contribution to the decay amplitude is almost negligible.

Now we consider the new physics contribution to the $B_s - \bar{B}_s$ mixing amplitude following \[31\]. In order to estimate the NP contribution to the $B_s - \bar{B}_s$ mixing, we parameterize the dispersive part of $B_s - \bar{B}_s$ mixing amplitude as

$$M_{12} = |M_{12}| e^{i\Phi_{B_s}} = M_{12}^{SM} + M_{12}^{NP} = M_{12}^{SM} C_{B_s} e^{i\Delta \theta_s}.$$  

(27)

In the SM, $M_{12}$ receives dominant contribution due to the top quark exchange in the box diagram and is given as

$$M_{12}^{SM} = \frac{G_F^2 M_W^2}{12 \pi^2} M_{B_s} B_{B_s} f_{B_s}^2 \lambda_t^2 \eta \ S_0(x_t),$$  

(28)

where $x_t = m_t^2 / M_W^2$ and

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1 - x)^2} - \frac{3}{2} \frac{x^3 \ln x}{(1 - x)^3}.$$  

(29)
In the presence of fourth generation, there will be additional contributions due to $t'$ exchange in the loop and the mixing amplitude is given as \[32\]

$$M_{12} = \frac{G_F^2 M_W^2}{12\pi^2} M_{B_s} B_{B_s} f_{B_s}^2 \left[ \lambda_l^2 \eta_t S_0(x_t) + \lambda_{t'}^2 \eta_{t'} S_0(x_{t'}) + 2 \eta_{t'} \lambda_t \lambda_{t'} S_0(x_t, x_{t'}) \right],$$

(30)

where

$$S_0(x, y) = xy \left\{ \frac{1}{4} + \frac{3}{2} \left( \frac{1}{1-y} - \frac{3}{4} \right) - \frac{3}{4(1-y)^2} \right\} \ln \frac{y-x}{y}$$

$$+ \left[ \frac{1}{4} + \frac{3}{2(1-x)} - \frac{3}{4(1-x)^2} \right] \ln \frac{x-y}{(x-y) - \frac{3}{4(1-x)(1-y)}} \right\}$$

(31)

and $\eta_{t'} = \frac{\alpha_s(m_{t'})}{\alpha_s(m_t)} \frac{M_{t'}^2}{M_t^2} \frac{\alpha_s(m_{t'})}{\alpha_s(m_t)} \approx \eta_{t'}$. Now parameterizing the new physics contribution to the $B_s - \bar{B}_s$ mixing amplitude as

$$M_{12} = M_{12}^{SM}(x_t) + M_{12}(x_{t'}) + M_{12}(x_t, x_{t'}) = M_{12}^{SM} C_{B_s} e^{2\theta_s},$$

(32)

one can obtain the $B_s - \bar{B}_s$ mixing phase from \[27\] as

$$\phi_{B_s} = 2\beta_s + 2\theta_s.$$

(33)

where the new contribution due to SM4 is given as

$$2\theta_s = \arctan \left( \frac{-b \ p \sin(\phi_s - \beta_s) + b^2 \ q \sin(2\phi_s - 2\beta_s)}{1 - b \ p \cos(\phi_s - \beta_s) + b^2 \ q \cos(2\phi_s - 2\beta_s)} \right),$$

(34)

with $b = |\lambda_{t'}/\lambda_t|$ and

$$p = \frac{2\eta_{t'} S_0(x_t, x_{t'})}{\eta_t S_0(x_t)}, \quad q = \frac{\eta_{t'} S_0(x_{t'})}{\eta_t S_0(x_t)}.$$

(35)

Thus, we obtain the mixing induced CP asymmetry in the presence of fourth generation as

$$S_{t'\psi_0} = \frac{\sin(2\theta_s + 2\beta_s) + 2ar \cos \delta \sin(\phi_s - 2\theta_s - 2\beta_s) - (ar)^2 \sin(2\phi_s - 2\theta_s - 2\beta_s)}{1 + (ar)^2 - 2ar \cos \delta \cos \phi_s}.$$

(36)

Now varying $\chi_t'$ between $(0.08 - 1.4) \times 10^{-2}$ and $\phi_s$ between $(0 - 80)^\circ$, we show the mixing induced CP asymmetry parameter $S_{\psi_0}$ in Figure-1. From the figure it can be seen that large CP violation could be possible for this decay mode in the fourth generation model.

### III. $B_s \to f_0(980)l^+l^-$ AND $B_s \to f_0\nu\bar{\nu}$

Now we will discuss the semileptonic decay processes $B_s \to f_0(980)l^+l^-$ and $B_s \to f_0(980)\nu\bar{\nu}$. These processes are studied in Ref. \[25\] in the SM and the branching ratios are found to $\mathcal{O}(10^{-8})$ and $\mathcal{O}(10^{-7})$ respectively.
FIG. 1: The mixing-induced CP asymmetry in $B_d \to J/\psi f_0(980)$ process ($S_{\psi f_0}$) versus $|\lambda_t|$. 

The decay process $B_s \to f_0(980) l^+ l^-$ is described by the quark level transition $b \to s l^+ l^-$. The effective Hamiltonian describing these processes can be given as [30]

$$H_{\text{eff}} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left[ C_9^{\text{eff}} (\bar{s} \gamma_{\mu} L b)(\bar{\ell} \gamma^\mu \ell) + C_{10}^{\text{eff}} (\bar{s} \gamma_{\mu} L b)(\bar{\ell} \gamma^\mu \gamma_5 \ell) - 2C_7^{\text{eff}} m_b (\bar{s} i \sigma_{\mu\nu} \frac{q^{\mu}}{q^2} R b)(\bar{\ell} \gamma^\nu \ell) \right], \quad (37)$$

where $q$ is the momentum transferred to the lepton pair, given as $q = p_- + p_+$, with $p_-$ and $p_+$ are the momenta of the leptons $l^-$ and $l^+$ respectively. $L, R = (1 \pm \gamma_5)/2$ and $C_i$'s are the Wilson coefficients evaluated at the $b$ quark mass scale. The values of these coefficients in NLL order are $C_7^{\text{eff}} = -0.31$, $C_9 = 4.154$, $C_{10} = -4.261$ [33].

The coefficient $C_9^{\text{eff}}$ has a perturbative part and a resonance part which comes from the long distance effects due to the conversion of the real $c\bar{c}$ into the lepton pair $l^+ l^-$. Therefore, one can write it as

$$C_9^{\text{eff}} = C_9 + Y(s) + C_9^{\text{res}}, \quad (38)$$

where $s = q^2$ and the function $Y(s)$ denotes the perturbative part coming from one loop matrix elements of the four quark operators and is given by [30]

$$Y(s) = g(m_c, s)(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \frac{1}{2} g(0, s)(C_3 + 3C_4) - \frac{1}{2} g(m_b, s)(4C_3 + 4C_4 + 3C_5 + C_6) + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6), \quad (39)$$
where

\[ g(m_i, s) = - \frac{8}{9} \ln(m_i/m_i^{\text{pole}}) + \frac{8}{27} + \frac{4}{9} y_i - \frac{2}{9} (2 + y_i) \sqrt{1 - y_i} \]

\[ \times \left\{ \Theta(1 - y_i) \left[ \ln \left( \frac{1 + \sqrt{1 - y_i}}{1 - \sqrt{1 - y_i}} \right) - i \pi \right] + \Theta(y_i - 1) 2 \arctan \frac{1}{\sqrt{y_i - 1}} \right\}, \quad (40) \]

with \( y_i = 4m_i^2/s \). The values of the coefficients \( C_i \)'s in NLL order are taken from [33].

The long distance resonance effect is given as [34]

\[ C_9^{\text{res}} = \frac{3\pi}{\alpha^2} (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \sum_{V_i = \psi(1S), \ldots, \psi(6S)} \kappa_{V_i} \frac{m_{V_i} \Gamma(V_i \rightarrow l^+l^-)}{m_{V_i}^2 - s - i m_{V_i} \Gamma_{V_i}}. \quad (41) \]

The phenomenological parameter \( \kappa \) is taken to be 2.3, so as to reproduce the correct branching ratio of \( \text{Br}(B \rightarrow J/\psi K^* l^+l^-) = \text{Br}(B \rightarrow J/\psi K^*) \text{Br}(J/\psi \rightarrow l^+l^-) \).

The matrix elements of the various hadronic currents in (37) between initial \( B_s \) and the final \( f_0(980) \) meson, which are parameterized in terms of various form factors as defined in Eq. (43). Thus, one can obtain the decay rate for \( B_s \rightarrow f_0 l^+l^- \) as [25]

\[ \frac{d\Gamma(B_s \rightarrow f_0 l^+l^-)}{ds} = \frac{G_F^2 \alpha^2 \cos^2 \theta |\lambda_i|^2 v_1 \sqrt{\lambda}}{512 m_B^3 \pi^5} \left\{ |C_{10}|^2 \left[ 6 m_1^2 (m_{B_s}^2 - m_{f_0}^2)^2 F_0^2(q^2) + \lambda(s - 4m_1^2) F_1^2(q^2) \right] + \lambda(s + 2m_1^2) \left| C_9 F_1(q^2) + \frac{2 C_{eff}^b (m_b - m_s) F_T(q^2)}{m_{B_s} + m_{f_0}} \right|^2 \right\} \quad (42) \]

where \( \lambda \equiv \lambda(m_{B_s}^2, m_{f_0}^2, s) = (m_{B_s}^2 - s - m_{f_0}^2)^2 - 4s m_{f_0}^2, v_1 = \sqrt{1 - 4m_1^2/s}. \) Using the particle masses and CKM elements from [26], the form factors from Eq. (43), \( \alpha = 1/129, \) we show the variation of the differential decay distribution in the SM with respect to the dilepton mass for \( B_s \rightarrow f_0(980)\mu^+\mu^- \) in Figure-2.

Integrating the differential branching ratio between \( 4m_1^2 \leq s \leq (m_{B_s} - m_{f_0})^2 \), the total branching ratios for \( B_s \rightarrow f_0 l^+l^- \) in the SM are found to be (where we have not taken into account the contributions coming from charmonium-like resonances)

\[ \text{Br}(B_s \rightarrow f_0(980)\mu^+\mu^-) = (8.8 \pm 1.97) \times 10^{-8}, \]

\[ \text{Br}(B_s \rightarrow f_0(980)\tau^+\tau^-) = (8.9 \pm 2.0) \times 10^{-9}. \quad (43) \]

These results are in agreement with predictions of Ref. [25]. Since these values are within the reach of LHCb experiment, there is a possibility that these decay modes could be observed soon.
In the presence of fourth generation, the Wilson coefficients $C_{7,9,10}$ will be modified due to the new contributions arising from the virtual $t'$ quark in the loop. Thus, these modified coefficients can be represented as

\[
C_{7}^{\text{tot}}(\mu) = C_{7}(\mu) + \frac{\lambda_{t}'}{\lambda_{t}} C_{7}'(\mu),
\]

\[
C_{9}^{\text{tot}}(\mu) = C_{9}(\mu) + \frac{\lambda_{t}'}{\lambda_{t}} C_{9}'(\mu),
\]

\[
C_{10}^{\text{tot}}(\mu) = C_{10}(\mu) + \frac{\lambda_{t}'}{\lambda_{t}} C_{10}'(\mu).
\]

The new coefficients $C_{7,9,10}'$ can be calculated at the $M_W$ scale by replacing the $t$-quark mass by $m_t'$ in the loop functions. These coefficients then to be evolved to the $b$ scale using the renormalization group equation as discussed in \[30\]. The values of the new Wilson coefficients at the $m_b$ scale for $m_{t'} = 400$ GeV is given by $C_{7}'(m_b) = -0.355$, $C_{9}'(m_b) = 5.831$ and $C_{10}' = -17.358$.

Thus, one can obtain the differential branching ratio in SM4 by replacing $C_{7,9,10}$ in Eqs (42) by $C_{7,9,10}^{\text{tot}}$. Varying the values of the $|\lambda_{t}'|$ and $\phi_s$ for $m_{t'} = 400$ GeV in their corresponding allowed ranges, the differential branching ratio for $B_s \rightarrow f_0(980)\mu^+\mu^-$ is presented in Figure-3, where we have not considered the contributions from intermediate charmonium resonances. From the figure it can be seen that the differential branching ratio of this mode is significantly enhanced from its corresponding SM value. Similarly for the process $B_s \rightarrow f_0(980)\tau^+\tau^-$ as seen from Figure-4, the branching ratio significantly enhanced from
its SM value.

FIG. 3: The differential branching ratio versus $|\chi^\prime_t|$ for the process $B_s \rightarrow f_0(980)\mu^+\mu^-$ (red region) whereas the corresponding SM value is shown by the blue region.

FIG. 4: Same as figure-3 for the process $B_s \rightarrow f_0(980)\tau^+\tau^-$. 

Next, we will discuss the decay mode $B_s \rightarrow f_0(980)\nu\bar{\nu}$. Rare $K$ and $B$ decays involving a $\nu\bar{\nu}$ pair in the final state belong to the theoretically cleanest decays in the field of flavor changing neutral current processes. Over the last twenty years, extensive analyses of the decays $K \rightarrow \pi\nu\bar{\nu}$ have been performed in the literature and several events have already been observed [35]. However, neither the inclusive nor the exclusive $b \rightarrow s\nu\bar{\nu}$ decay modes have been observed in experiments so far. With the advent of super $B$ facilities, the prospect of measuring these branching ratios seems to be not fully unrealistic and it seems appropriate
to have a closer look at these decays.

The effective Hamiltonian for $b \rightarrow s \nu \bar{\nu}$ transition is generally given as
\[
H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \frac{\alpha V_{tb} V_{ts}^*}{2 \pi \sin^2 \theta_W} \eta_X X(x_t) O_L , \tag{45}
\]
with the operator $O_L$ is given as
\[
O_L = (\bar{s} \gamma_\mu (1 - \gamma_5) b)(\bar{\nu} \gamma^\mu (1 - \gamma_5) \nu) , \tag{46}
\]
and
\[
X(x) = \frac{x}{8} \left[ 2 + \frac{x}{x - 1} + \frac{3x - 6}{(x - 1)^2} \ln x \right] , \tag{47}
\]
while $\eta_X \approx 1$.

Using the form factors as defined in Eq. (9) one can obtain the differential decay width to be
\[
\frac{d\Gamma(B_s \rightarrow f_0 \nu \bar{\nu})}{ds} = |C_L|^2 \frac{\lambda^{3/2}(m_{B_s}^2, m_{f_0}^2, s)}{32 m_{B_s}^3 \pi^3} \cos^2 \theta |F_1(q^2)|^2 , \tag{48}
\]
where
\[
C_L = \frac{G_F}{\sqrt{2}} \frac{\alpha V_{tb} V_{ts}^*}{2 \pi \sin^2 \theta_W} \eta_X X(x_t) . \tag{49}
\]
Using the values of form factors as given in Eq. (13), $m_t = 170$ GeV, $m_W = 80.4$ GeV, the total branching ratio in the SM is found to be
\[
\text{Br}(B_s \rightarrow f_0 \nu \bar{\nu}) = (3.81 \pm 0.85) \times 10^{-7} , \tag{50}
\]
which is slightly lower than the prediction of Ref. [25].

In the SM4 model, the decay width can be obtained from Eq. (48) by replacing $C_L$ with $\tilde{C}_L$ which is given as
\[
\tilde{C}_L = C_L \left( 1 + \frac{\lambda_t}{\lambda_t} \frac{X_0(x_t')}{X_0(x_t)} \right) . \tag{51}
\]
Now varying $\lambda_t$ between $0.0008 \leq |\lambda_t| \leq 0.0014$ and $\phi_s$ between $(0 - 80)^\circ$ we have shown in Figure -5 the differential branching ratio for $B_s \rightarrow f_0(980)\nu \bar{\nu}$. Form the figure it can be seen that the branching ratio is significantly enhanced from its standard model value.

**IV. CONCLUSION**

In this paper we have studied some decays of the $B_s$ meson involving the scalar meson $f_0(980)$ in the final state in the fourth quark generation model. This model is a very simple
extension of the SM with three generations and it can easily accommodate the observed anomalies in the $B$ and $B_s$ CP violation parameters for $m_\nu$ in the range of (400-600) GeV. We assumed the $f_0$ structure to be dominated by $(ss\bar{s})$ quark composition. We found that in the fourth generation model the branching ratio for the nonleptic decay $B_s \rightarrow J/\psi f_0(980)$ remains unaffected whereas the mixing-induced CP asymmetry of this mode could be significantly enhanced from its SM value. For the semileptonic decays $B_s \rightarrow f_0(980)l^+l^-$ and $B_s \rightarrow f_0(980)\nu\bar{\nu}$, the branching ratios could also be increased significantly from their standard model predictions. These branching ratios are within the reach of LHCb experiments. Hence, the observation of these modes will provide us an indirect evidence for new physics, such as the presence of an extra generation of quarks or else will support the $s\bar{s}$ composition of $f_0(980)$ scalar meson.

**Acknowledgments**

The author would like to thank Department of Science and Technology, Government of India, for financial support through Grant No. SR/S2/RFPS-03/2006.

[1] Heavy Flavor Averaging Group, [http://www.slac.stanford.edu/xorg/hfag](http://www.slac.stanford.edu/xorg/hfag).

[2] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 100, 161802 (2008), arXiv: 0712.2397 [hep-ex].

[3] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 101, 241801 (2008), [arXiv:0802.2255](https://arxiv.org/abs/0802.2255)
[hep-ex], V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 102, 032001 (2009), arXiv:0812.0037 [hep-ex].

[4] J. T. Wei et al, [Belle Collaboration], Phys. Rev. Lett. 103, 171801 (2009), arXiv:0904.0770 [hep-ex].

[5] B. Aubert et al, [Babar Collaboration], Phys. Rev. D 79, 031102 (2009), arXiv: 0804.4412 [hep-ex].

[6] Belle finds a hint of new physics in extremely rare B decays, reported in August 2009 (as Press Release): [http://www.kek.jp/itra-e/press/2009/BNLLPress14e.html](http://www.kek.jp/itra-e/press/2009/BNLLPress14e.html)

[7] W. -S. Hou, A. Soni and H. Steger, Phys. Lett. B 192, 441 (1987); W. S. Hou, R. S. Willey and A. Soni, Phys. Rev. Lett. 58, 1608 (1987).

[8] A. Soni, A. Alok, A. Giri, R. Mohanta and S.Nandi, Phys. Lett. B 683, 302 (2010), arXiv:0807.1971 [hep-ph]; Phys. Rev. D. 82, 033009 (2010), [arXiv:1002.0595 [hep-ph]].

[9] A. J. Buras et al, JHEP 1009, 106 (2010), arXiv:1002.2126 [hep-ph].

[10] W. S. Hou and C. Y. Ma, Phys. Rev. D 82, 036002 (2010), arXiv: 1004.2186 [hep-ph].

[11] M. E. Shaposhnikov, Nucl. Phys. B 287, 757 (1987); W. S. Hou, Chin. J. Phys. 47, 134 (2009), arXiv:0803.1234 [hep-ph].

[12] J. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659 (1982); Phys. Rev. D 27, 583 (1983); Phys. Rev. D 41, 2236 (1990); M. P. Locher et. al., Euro. Phys. J. C 4, 317 (1998).

[13] R. L. Jaffe, Phys. Rev. D 15, 267 (1997); M. Alford and R. L. Jaffe, Nucl. Phys. B 578, 367 (2000).

[14] N. A. Tornqvist, Phys. Rev. Lett. 49, 624 (1982); N. A. Tornqvist and M. Roos Phys. Rev. Lett. 76, 1575 (1996).

[15] A. V. Anisovich, V. V. Anisovich and V. A. Nikonov, hep-ph/0011191.

[16] F. D. Fazio and M. R. Pennington, Phys. Lett. B 521, 15 (2001), R. Delborgo, D. Liu, M. D. Scadron, Phys. Lett. B 446, 332 (1992); T. M. Aliev et al., Phys. Lett. B 527, 193 (2002).

[17] F. Kleefeld et al. Phys. Rev. D 66, 034007 (2002); E. van Beveren G. Rupp and M. D. Scadron, Phys. Lett. B 495, 300 (2000); A. Deandrea et al., Phys. Lett. B 502, 79 (2001).

[18] M.N. Achasov, SND Collaboration, Phys. Lett. B 440, 442 (1998).

[19] H. Y. Cheng, Phys. Rev. D 67, 034024 (2003).

[20] S. Stone and L. Zhang, Phys. Rev. D 79, 074024 (2009); arXiv:0909.5442 [hep-ex].

[21] R. Aaij et al., [LHCb Collaboration], Phys. Lett. B 698, 115 (2011), arXiv:1102.0206 [hep-ex].
[22] J. Li et al. [Belle Collaboration], Phys. Rev. Lett. 106, 121802 (2011), arXiv:1102.2759 [hep-ex].

[23] Y. H. Chen, H. Y. Cheng, B. Tseng and K. C. Yang, Phys. Rev. D 60, 094014 (1999).

[24] O. Leitner, J.-P. Dedonder, B. Loiseau and B. El-Bennich, Phys. Rev. D 82, 076006 (2010), arXiv: 1003.5980 [hep-ph].

[25] P. Colangelo, F. De Fazio and Wei Wang, Phys. Rev. D 81, 074001 (2010); F. De Fazio, Nucl. Phys. B (Proc. Suppl.) 207-208, 261 (2010).

[26] K. Nakamura et al, Particle Data Group, J. Physics G 37, 075021 (2010).

[27] H. Y. Cheng, C. K. Chua and K. C. Yang, Phys. Rev D 73, 014017 (2006).

[28] R. Mohanta and A. K. Giri, Phys. Rev. D 82, 094002 (2010); V. Bashiry and K. Azizi, JHEP 07, 064 (2007).

[29] T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981); ibid 65, 1772 (1981).

[30] G. Buchalla, A.J. Buras, M. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).

[31] R. Mohanta and A. K. Giri, Phys. Rev. D 78, 116002 (2008); R. Mohanta and A. K. Giri, Phys. Rev. D 76, 075015 (2007).

[32] A. Arhrib and W. S. Hou, Euro Phys. J C 27, 555 (2003).

[33] M. Beneke, Th. Fledmann and D. Seidel, Nucl. Phys. B 612, 25 (2001).

[34] C. S. Lim, T. Morozumi and A. I. Sanda, Phys. Lett. B. 218, 343 (1989); N. G. Deshpande, J. Trampetic and K. Ponose, Phys. Rev. D 39, 1461 (1989); P. J. O’Donnell and H. K.K. Tung, Phys. Rev. D 43, R2067 (1991); P. J. O’Donnell, M. Sutherland and H. K.K. Tung, Phys. Rev. D 46, 4091 (1992); F. Krüger and L. M. Sehgal, Phys. Lett. B 380, 199 (1996).

[35] A. V. Artamonov et al., [E949 Collaboration], Phys. Rev. Lett. 101, 191802 (2008).