Decoupling Spurious Baryon States in the $1/N_c$ Expansion of QCD

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We identify “spurious” states in meson-baryon scattering, those that appear in QCD for $N_c > 3$ but decouple for $N_c = 3$. The key observation is that the relevant flavor SU(3) Clebsch-Gordan coefficients contain factors of $1 - 3/N_c$. We show that this method works even if SU(3) is badly broken. We also observe that resonant scattering poles lying outside naive quark model multiplets are not necessarily large $N_c$ artifacts, and can survive via configuration mixing at $N_c = 3$.

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I. INTRODUCTION

A large body of literature convincingly demonstrates that the $1/N_c$ expansion about the limit $N_c \to \infty$ provides a fertile harvest of qualitative and semi-quantitative information about QCD. However, the study of baryons in large $N_c$ QCD presents a formidable complication absent from the meson sector, namely, the emergence of spurious states, which appear in $N_c > 3$ worlds but have no analog in the physical $N_c = 3$ world. Clearly, the measurable properties of states that exist at $N_c = 3$ must decouple from the spurious states as $N_c \to 3$ in a systematic $1/N_c$ expansion. In this context, decoupling refers both to static and dynamical properties: The matrix elements of neither type of observable for physical states at $N_c = 3$ may be permitted to depend on couplings to states that do not exist in the $N_c = 3$ world. To keep the discussion concise, we denote “physical” states as ones whose analogs survive at $N_c = 3$ and “spurious” as ones that do not. One can imagine starting with a world of large (but finite) $N_c$; then the spurious states are allowed and couple to physical states, in the sense that they can be reached from the physical states via the emission or absorption of some number of light mesons. As $N_c$ is reduced towards 3, such couplings must diminish; and eventually as one reaches the world of $N_c = 3$, all couplings between the physical and spurious states must vanish. This paper explores how this decoupling comes about.

In fact, this is a multifaceted and complicated problem. Before proceeding any further we must clarify the precise attributes of spurious states. We mean here states that, by virtue of their quantum numbers, cannot exist in $N_c = 3$ QCD. We distinguish such states from those that merely fail to exist in the most naive quark model (such as pentaquarks). To make this distinction clear, let us begin by considering a world with two light flavors $(c=3)$ and large $(but\ finite)\ N_c$.

As is well known these states have $I = J$, and their low-lying excitation energies [including a common mass $M_0 = O(N_c^1)$] are given by

$$M_J = M_0 + \frac{J(J+1)}{2I} + O(1/N_c^2), \quad (1)$$

where $I = O(N_c^1)$ is a moment of inertia parameter. As $N_c \to \infty$ the system is represented by a contracted SU(4) symmetry [1], and one finds an arbitrarily large number of states degenerate in mass to within $O(1/N_c)$. Note that for any nonzero $m_\pi = O(N_c^0)$, states with $I = J = O(N_c^0)$ are stable since there is no phase space for decay. Of course, in the real world approximate chiral symmetry ensures an anomalously small $m_\pi$. In the exact chiral limit of $m_\pi = 0$ the phase space for the decay of such states is not zero. However, even in this limit, the states with $I = J = O(N_c^0)$ remain narrow, with $O(1/N_c)$ widths.

For finite $N_c$ the preceding picture for ground-state baryons clearly breaks down, and Eq. (1) breaks down with it. In the context of a naive quark model with all of the quarks in s waves, $J_{\text{max}} = \frac{N_c}{2}$. Moreover, states with $I > \frac{N_c}{2}$ are impossible in the context of a naive quark model allowing only minimal quark content. Thus, states with $I = J > \frac{N_c}{2}$ might then be labeled spurious. However, this argument is based upon a naive quark model description and not upon full QCD. So, what happens in QCD for large but finite $N_c$? The answer is that no one knows in detail; one would have to be able to solve QCD completely to find out. But there does exist a qualitative idea that is phenomenology consistent. One notes that the level spacings from Eq. (1) scale as

$$M_{J+1} - M_J = \frac{J+1}{I} = O(J/N_c), \quad (2)$$

Thus, for $J < O(N_c^1)$ the level spacing is smaller than $O(N_c^0)$. The phase space for the decay of the $J+1$ state into the $J$ state plus a $\pi$ is small [either is below $m_\pi$ and hence zero, or if one works in the chiral limit it is driven by the $O(1/N_c)$ mass splitting], and hence the particle masses are well defined. Now suppose one took this result seriously for larger values of $J$; it becomes
clear that something must break down. For example, as \( J \) increases towards \( O(N_c^3) \) the width of the state becomes \( O(N_c^0) \), and one can no longer speak of narrow states given by Eq. \( \text{(1)} \). In one qualitative sense this is similar to the fate of the naive quark model at large but finite \( N_c \). In both cases Eq. \( \text{(1)} \) breaks down for \( I = J = O(N_c^1) \).

However, one also finds an important distinction. In the naive quark model with minimal quark content, the states with \( I = J > \frac{1}{2} \) are spurious. In a more general picture of large \( N_c \) QCD, the breakdown of Eq. \( \text{(1)} \) is gradual as \( J \) increases, and no point occurs at which one can identify the states as spurious. QCD with a large but finite value of \( N_c \) allows baryon number unity states with \( I = J \) quantum numbers for all half-integral values of \( I = J \). These states may not all be narrow or indeed may not even be resonant, but they are not forbidden. The possibility of states with \( I = J > \frac{1}{2} \) existing in the context of large \( N_c \) models such as the Skyrme model has long been known \[2\]. Thus, in the language used here these states are not spurious. We denote such states as “exotic”, since they are states beyond those accessible in the naive quark model with minimal quark content.

The example considered above illustrates an important issue, that the worrisome states—regardless of whether labeled “spurious” or “exotic”—are precisely those with large \( I \) and \( J \). Note that these states only connect to the nucleon via the emission of a large number of mesons, which is necessary to support a large isospin value. Thus, if one restricts to baryon resonances obtainable in meson-nucleon scattering, states with large \( I \) or \( J \) are not a concern. Scattering states are useful to examine not only because a large fraction of the known resonances are in fact observed in these channels, but also because there has been considerable theoretical progress in understanding these processes at large \( N_c \) \[2, 4, 5, 6, 7, 8, 9, 10, 11\].

Although the situation is quite similar for a 2-flavor world, the generalization to 3 flavors is drastically different. The allowable 2-flavor representations in an \( N_c = 3 \) world are equivalent to those found in a large \( N_c \) world. However, none of the 3-flavor multiplets seen at \( N_c = 3 \) exist in a large \( N_c \) world, and conversely, all of the 3-flavor multiplets for large \( N_c \) contains states that are spurious at \( N_c = 3 \). The reason for this disparity is clear: In the exact SU(3) limit \( I \)-spin, \( U \)-spin, and \( V \)-spin all exist on equal footing. In the SU(2) flavor space one may consider only representations where the isospin is small, i.e., \( I = O(N_c^0) \) as opposed to \( I = O(N_c^1) \). However, in SU(3) flavor if one chooses representations with \( I = O(N_c^0) \), then necessarily \( U = O(N_c^1) \) and \( V = O(N_c^1) \). Thus, for example, the spin \( \frac{1}{2} \) baryons of the ground-state band form the nucleon isodoublet for 2 flavors at any \( N_c \), but for 3 flavors occupy a \( \{p,q\} = [1,\frac{1}{2}(N_c-1)] \equiv \text{“8” representation} \). For \( N_c = 3 \) this representation is the familiar octet, but as \( N_c \to \infty \) it grows arbitrarily large. When \( N_c \) is large but finite most of the states in this representation are spurious, having no \( N_c = 3 \) analog. Even the ground-state baryon representations include spurious states.

As noted above, we wish to access excited baryons via meson-baryon scattering on a target baryon in its ground state. Inasmuch as one can study this scattering using an arbitrary value of \( N_c \), it must be possible to use the continuous set of results connecting the limit \( 1/N_c = 0 \) to the physical point \( 1/N_c = 1/3 \). In the latter case, all spurious states must therefore decouple; the theory must forbid access to these states at \( N_c = 3 \). Moreover, this decoupling must be smooth; such couplings must diminish gradually and vanish as \( N_c \) reaches 3.

The goal of the present paper is to understand how this decoupling occurs. It is a critical task since \( 1/N_c \) is not very small for \( N_c = 3 \); in order to obtain more than semi-quantitative predictions it is essential to be able to handle the \( 1/N_c \) corrections. The coupling to spurious states clearly represents a key class of \( 1/N_c \) corrections. Moreover, the nature of these corrections is rather special: Unlike typical power series corrections, which become progressively less significant at higher orders, the couplings to spurious states impose selection rules and completely forbid access to certain states.

II. TYPES OF 1/Nc CORRECTIONS IN MESON-BARYON SCATTERING

Since large \( N_c \) QCD has a smooth \( 1/N_c \to 0 \) limit that supports a large number of spurious states, it must be true that at \( N_c = 3 \) the sum of the various \( 1/N_c \) corrections conspire to cancel exactly the leading-order result for all couplings between physical and spurious states, while retaining all physical states. At first sight it may seem daunting to show how such a conspiracy can come about. Clearly, it is necessary to understand first the possible sources of \( 1/N_c \) corrections to the theory.

It is well known that the scattering amplitudes are connected via linear relations at leading order in \( 1/N_c \). These relations follow from a master equation in which the physical amplitudes are linear superpositions of various “reduced” amplitudes \[12\]. In light of these relations, it is easy to see that \( 1/N_c \) corrections to the scattering amplitudes may grouped into three classes:

i) There are \( O(1/N_c) \) corrections to the leading-order \( [O(N_c^0)] \) reduced amplitudes \( \tau \); one can show \[11\] that all such amplitudes multiply structures obeying the simple \( t \)-channel rules \( I_t = J_t \) and \( Y_t = 0 \). The master expression obtained in Ref. \[11\] is:

\[
S_{LL'S_{B}} S_{B'} J_t J_{t_2} R_t \gamma_{1,t} \gamma_{1,t_2} Y_{t_1} Y_{t_2} = \delta J_t J_{t_2} \delta J_{t_1} J_{t_2} \delta R_t R_{t_2} \delta I_t I_{t_2} \delta Y_{t_1} Y_{t_2} \delta Y_{t_1} Y_{t_2} \]
where we have imposed $Y_i = 0$ but left coefficients originating as $I_t$ or $J_t$ distinct in this expression.

[The notation used here and in Eq. (4) is fully explained, among other places, in Sec. II of Ref. [11]. For our purposes, the most important details are: Unprimed (primed) variables represent initial (final) quantities, $\phi$ ($\tilde{B}$) denote mesons (baryons); square brackets denote representation multiplicities [$R$ for SU(3)], and the factors with double vertical bars are SU(3) isoscalar factors.]

ii) There are also manifestly subleading terms, whose independent reduced amplitudes multiply structures with $Y_i \neq 0$ and/or $I_t \neq J_t$; as shown in Ref. [11], the contributions from such amplitudes scale as $N_c^{-|Y_i|/2}$ and $N_c^{-|I_t-J_t|}$, respectively. One may easily transcribe an expression such as Eq. (4) to allow for terms with $Y_i \neq 0$ or $I_t \neq J_t$ by inserting the appropriate $1/N_c$ power in front of the new reduced amplitudes; this procedure has been carried out in the 2-flavor case [3, 8].

 Corrections of types i) and ii) are dynamical in origin. We know of no way to obtain them without solving QCD completely. Even if such corrections were possible, we would not be able to decouple the spurious states unless the subleading amplitudes turned out to provide precisely the factors of $-3$ necessary to cancel the leading-order ones. A priori it would require a truly remarkable coincidence for this to occur. Moreover, one would expect the values of the subleading coefficients (being of dynamical origin) to vary with quark masses (for illustration purposes here, taken equal and nonzero), making decoupling for every value of quark mass even more remarkable. Adding to the implausibility, the reduced amplitudes carry information not only about spurious but ordinary states as well. Thus, if such a remarkable cancellation did occur, one would expect it also to decouple states that must remain in the $N_c = 3$ theory. We therefore argue that such $1/N_c$ corrections are almost certainly not responsible for the exact decoupling of spurious $N_c > 3$ states.

iii) Unlike the 2-flavor case, the Clebsch-Gordan coefficients (CGC) of the 3-flavor case (or more precisely, the SU(3) isoscalar factors employed above) are not $O(N_c^0)$ numbers, as is the case for 2-flavor SU(2) CGC [7]. This difference is a consequence of the phenomena described above, that even 3-flavor representations containing physical states contain in addition numerous spurious states, the subset surviving at $N_c = 3$ occupying only small corners of the weight diagrams of these multiplets. For transitions between spurious and non-spurious states, the SU(3) CGC turn out to be inhomogeneous functions of $N_c$, and couplings that are forbidden at $N_c = 3$ are all explicitly proportional to factors of $(1 - 3/N_c)$. In a strict large $N_c$ counting, factors such as these are set to unity, but in considering whether states decouple completely at $N_c = 3$ in the $1/N_c$ expansion, it is appropriate to set such factors to zero [12]. Consider for example $K\Sigma$ scattering. The product $8 \otimes 8$ contains $1^+$ as the singlet state, but states with $\Xi$ quantum numbers as well. The SU(3) CGC for such a process, which is finite as $N_c \to \infty$ but vanishes for $N_c = 3$, is

$$\left( 1^+, \frac{N_c}{2} - 1, \frac{1}{2}, -1 \right) \left( 1^+, \frac{N_c}{2}, -2 \right) = \frac{1}{2} \sqrt{3(N_c - 3) \over N_c + 5}. \quad (4)$$

This is the primary method of decoupling spurious states in meson-baryon scattering. Note the smoothness of decoupling as $N_c \to 3$: the nature of SU(3) CGC provides a continuous path to decoupling. In the notation of Ref. [7], the surviving intermediate states are $N^*$ (nonexotic) and $E_0^*$ [an exotic state, such as a pentaquark, not allowed for $q\bar{q}q$ but still in an allowed $N_c = 3$ SU(3) representation], but not $E_+^*$ (spurious: only allowed for $N_c > 3$).

Additional examples serve to illustrate this effect, and we refer to Ref. [7] for the necessary background. In the product $8 \otimes 8$ for $N_c > 3$ one finds the SU(3) representation $"S" \equiv [2, \frac{1}{2}, (N_c - 5)]$ (so named because its row of highest hypercharge carries $\Sigma$ quantum numbers), and in $10 \otimes 8$ ("10" $\equiv [3, \frac{1}{2}, (N_c - 3)]$) for $N_c > 3$ one finds an additional "10" representation, denoted "101" (the "102" defined as the one that survives at $N_c = 3$). The tables of Ref. [7] provide the means to obtain a number of their CGC via unitarity when only one product representation has been omitted for given values of $Y$ and $I$; the normalization sign is not determined using this method, but it can be fixed using the methods outlined in Ref. [7] and is in any case irrelevant for our current purposes.

\[ \times (-1)^S_{\phi} - S_\phi + J_\phi - J_t + (J_\phi' - S_\phi') - \sqrt{1/N_c} \left( [S_B][I_B'] [J_B'][|J_\phi'|/|I_\phi'|][I_\phi'][J_t] \right)^{1/2} \]

\[ \times \sum_{l \in R_{\phi}, l' \in R_{\phi}', Y \in R_{\phi} \cup R_{\phi}'}\left( R_{\phi} R_{\phi}' \gamma_{l l'} I_{\phi} Y_{\phi} J_t 0 \right) \left( R_{\phi} R_{\phi}' \gamma_{l l'} I_{\phi} Y_{\phi} J_t 0 \right) \]

\[ \times \left( R_B S_B, + \frac{N_c}{2} J_t 0 I_{\phi} Y_{\phi} J_t 0 \right) \left( R_B S_B, + \frac{N_c}{2} I_{\phi} Y_{\phi} J_t 0 \right) \]

\[ \times \sum_{K \bar{K} K', L, L'}(-1)^{K - \frac{1}{2}} \left[ K \right]\left[ (\bar{K}' [\bar{K}])^{1/2} \right] \left\{ J_\phi J_\phi' I' J_t 0 \right\} \left\{ J_\phi J_\phi' I' J_t 0 \right\} \left\{ J_\phi J_\phi' I' J_t 0 \right\} \]

\[ \times \left\{ I' I' K \right\} \left\{ I' I' \bar{K} \right\} \left\{ I' I' \bar{K} \right\} \left\{ I' I' \bar{K} \right\} \left\{ I' I' \bar{K} \right\} \left\{ I' I' \bar{K} \right\} \]
the case of $KN \rightarrow \Sigma$, a relevant CGC that survives the $N_c \rightarrow \infty$ limit but vanishes for $N_c=3$ is

$$
\begin{pmatrix}
\text{"8"} & 8 & \text{"S"} \\
\frac{1}{2}, \frac{N_c}{3} & \frac{1}{2}, -1 & 1, \frac{N_c}{3}, -1
\end{pmatrix} = \pm \sqrt{\frac{(N_c+3)(N_c-3)}{(N_c+5)(N_c+1)}}
$$

while for the $\pi\Delta \rightarrow \Delta$ channel one encounters the CGC

$$
\begin{pmatrix}
\text{"10"} & 8 & \text{"10"} \\
\frac{3}{2}, \frac{N_c}{3} & 1, 0 & \frac{3}{2}, \frac{N_c}{3}
\end{pmatrix} = \pm \sqrt{\frac{5(N_c+5)(N_c-3)}{2(3N_c^2+14N_c-9)}}.
$$

Note that this behavior is generic. Had we lived in an $N_c=5$ world, some of the states that are spurious at $N_c=3$ would be physical. However, spurious states would still occur. The coupling between spurious states and physical states for $N_c=5$ all have CGC proportional to $(1-5/N_c)$. Analogous results hold for any $N_c$.

There is one other set of constraints, which originates from recognizing that ordinary baryon resonances are obtained by scattering mesons from baryons stable against strong decays. In the $N_c=3$ world, these are only the $J^P = \frac{1}{2}^+$ SU(3) 8 baryons (less the $\Sigma^0$) and the $\Omega^-$ in the $J^P = \frac{3}{2}^+$ 10. The other 10 resonances are unstable because they decay primarily via a single $\pi$ to a state in the 8. In a true large $N_c$ world, all these states [those described in Eq. (1)] belong to the generalization of the spin-flavor 56, and we have noted that the ones with $J = O(N_c^0)$ are stable against strong decay. The full “56” for $N_c$ large consists of SU(3) multiplets with $J^P = \frac{1}{2}^+, \frac{3}{2}^+, \ldots \frac{N_c}{2}^+$. Therefore, all resonances only reachable via scattering with the putatively stable but exotic $\frac{5}{2}^+, \frac{7}{2}^+, \ldots$ baryons are spurious. In the notation of Ref. [3], the initial states are labeled $\mathcal{E}$ (as opposed to nonexotic $\mathcal{N}$). Thus, only $\mathcal{NN}^*$ and $\mathcal{N}^*\mathcal{E}_0^*$ amplitudes survive our selection rules.

III. COMPLETELY BROKEN SU(3) FLAVOR

While we have made use of the full 3-flavor expression Eq. (3), one should not construe that using SU(3) CGC to identify spurious states depends on SU(3) symmetry being exact. Indeed, as we show in this section, it is meaningful even in the case that flavor SU(3) is completely broken to SU(2)$\times$U(1). One begins by noting that the 3-flavor $s$-channel expression (3) is found to hold separately for each SU(3) representation $R_s$:

$$
S_{sll's'j,r,s'\gamma'ly} = (-1)^{sa-sb'}([R_B][R_B'][S][S'])^{1/2}/[R_s] \sum_{l \in R_s, l' \in R_{b'}, I'' \in R_s, Y \in R_{b'} R_{b'}} (-1)^{l+l'+Y} [I'']
$$

$$
\times \left( \begin{array}{c c}
R_B & R_{b'} \\
S_B & I_Y \\
\end{array} \right) I'' Y + \frac{N_c}{3} \left( \begin{array}{c c}
R_{b'} & R_{s'} \\
S_{b'} & I'Y \\
\end{array} \right) I'' Y + \frac{N_c}{3}
$$

$$
\times \sum_{K,K',K''} [K][\tilde{K}][\tilde{K}'])^{1/2} \left\{ \begin{array}{c c c}
L & I & \tilde{K} \\
S & S_B & S_{b'} \\
J_s & I'' & \tilde{K}' \\
\end{array} \right\} [I'' Y] \left\{ \begin{array}{c c c}
L' & I' & \tilde{K}' \\
S' & S_{b'} & S_{b'} \\
J_s & I'' & K \\
\end{array} \right\} \tau_{K,K',K''}.
$$

Even if the poles in the scattering amplitudes do not fall into SU(3) representations $R_s$ (completely broken SU(3)), as a matter of pure mathematics it remains possible to separate $s$-channel contributions according to the flavor symmetry properties of the amplitude. The full physical amplitude, still containing SU(3) CGC, assumes the form

$$
M_{ll's's'j, r, s'\gamma'ly} = \sum_{R_s, s'} f(R_s; m_s) S_{sll's's'j, r, s'\gamma'ly} ,
$$

where the coefficient functions $f(R_s; m_s)$ equal unity in the SU(3) limit $m_s \rightarrow m_{u,d}$ (and do not depend upon $\gamma_s$) because no physical measurement can distinguish values...
of this degeneracy quantum number).

We note that the proof presented in Ref. [10], that the 3-flavor expression reduces to the well-known 2-flavor expression [12], uses an SU(3) CGC completeness relation (Eq. (13) in [14]); in the current language, that proof implicitly uses $f = 1$. However, we have found the proof to work equally well in the large $N_c$ limit regardless of the value of $f$. In fact, all the steps required are precisely those used to prove the $I = J$ rule in Ref. [11]. Starting with the expression relevant to nonstrange initial states (and similarly for the final states):

$$
\sum_{R_s,\gamma_s,Y} f(R_s; m_s) \left( \frac{R_B}{S_B, N \frac{\gamma_s}{\phi}} Y \left( \frac{R_s}{I''}, Y + \frac{N}{\gamma_s} \right) \left( \frac{R_B}{S_B, N \frac{\gamma_s}{\phi}} Y \left( \frac{R_s}{I''}, 0 \right) \right) \right), 
$$

(9)

we note that the SU(3) completeness relation can reduce the CGC to $\delta_{IL, \delta_{I'P, I}}$ (which is precisely what is needed to prove the result) if one can show that, at leading order in $N_c$, only one $R_s$ and one $Y$ is allowed in the given CGC, so that $f$ can effectively be pulled out of the sum, making its precise value is irrelevant. But this is precisely what is shown for an equivalent case, Eq. (3) of [11]. The 2-flavor result therefore follows from the 3-flavor expression, even for $f \neq 1$, i.e., for an arbitrary value of $m_s$.

### IV. ABOUT $\kappa$ POLES AND DECOUPLING

We have noted since our earliest papers on the subject [3] that the poles appearing in a given meson-baryon scattering amplitude often exceed the number found in a naive quark potential model. For example, we found that three poles, corresponding to $K = 0, 1,$ and $2$, explain the nonstrange members of the SU(6)$\times$O(3) (“70”, 1”). For $N_c \geq 5$, this multiplet boasts two $\Delta_\pm$ states, which is precisely the number of reduced amplitudes found in the $I = \frac{3}{2}, J = \frac{1}{2}$ channels ($K = 1$ and 2). The matching of such SU(6)$\times$O(3) multiplets to collections of resonances arising from poles with a given $K$ was denoted “compatibility” [4], and was suggested as a possible explanation for the similarity of physically observed to quark-model predicted baryon resonance multiplets.

In the case of $N_c = 3$, however, only one state with $\Delta_\pm$ quantum numbers survives in the (“70”, 1”). Does this mean that, as one dials $N_c$ from large values down to 3, either the $K = 1$ or $K = 2$ pole decouples from the theory? Clearly, such poles are not excluded on the basis of group theory alone, because the presence or absence of poles is a dynamical effect, as discussed in Sec. [4]. But then the concept of compatibility appears to falter somewhat for small finite $N_c$, since SU(6)$\times$O(3) multiplets no longer match resonance multiplets labeled by $K$.

Indeed, this should not trouble us, for SU(6)$\times$O(3) is not a true symmetry of baryon resonances, owing to the presence of large $[O(N_c^6)]$ configuration mixing [5]. In the case of the $\Delta_\pm$, the (“70”, 1”) state mixes with, e.g., a (“70”, 3”) state. In the limit of heavy quarks (and large $N_c$) these states become narrow and split in mass, but for physical values all possible mixed states should appear. The upshot is that in a given process poles labeled by $K$ might be dynamically suppressed or shifted in mass, but in principle all of them appear for finite $N_c$.

This result begs the question of how to use $N_c = 3$ phenomenology to identify which $K$ pole appears in a given process when some might be dynamically suppressed. As it turns out, the mixed partial-wave amplitudes ($L \neq L'$) for $\pi N \to \pi \Delta$ prove very convenient for this purpose: As may be determined from Eq. (3), for a given value of parity at most one value of $K$ appears in each such amplitude: $K = \frac{1}{2}(L + L') = J$, and $P = (-1)^K$. As an illustration, we note that the light negative-parity $N_\pm$ states $N(1535)$ and $N(1650)$, which we previously found from their $\pi$ and $\eta$ couplings to be mostly $K = 0$ and 1, respectively [3], have very different $\pi N \to \pi \Delta$ branching fractions through the $SD_{11}$ mixed partial wave ($K = 1$) [14]. For $N(1535)$ it is < 1%, and for $N(1650)$ it is 1–7%, confirming the previous assignment.

### V. CONCLUSIONS

We have shown that the properties of the SU(3) Clebsch-Gordan coefficients ensure that spuriously baryon states arising in large $N_c$ QCD decouple from the physical ones surviving at $N_c = 3$. Moreover, this decoupling is smooth; all couplings between physical and spurions states have factors proportional to $1/N_c$. This critical result allows the coupling to spurions states to be treated in a self-consistent way with other $1/N_c$ corrections. We have furthermore seen that employing the SU(3) factors does not require full SU(3) symmetry, and that resonant poles representing states lying outside the naive quark model can easily occur in our $N_c = 3$ world.

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