Heat transfer theoretical study of a single borehole system used to remove heat from the ground

B Horbaniuc, A Dumencu, Gh Dumitrașcu

Automotive and Mechanical Engineering Department, “Gheorghe Asachi” Technical University of Iasi, Iasi, Romania

E-mail: bogdan_horbaniuc@yahoo.com

Abstract. Borehole heat exchangers are used to extract heat from the ground in ground-source heat pump applications. Thus, atmospheric pollution with greenhouse gases is reduced by reducing the fuel consumption for residential or social buildings heating. Heat removal by means of the borehole results in a radial propagation of the perturbation (ground cooling). The magnitude of this perturbation and the removed heat are important parameters of the process that must be evaluated. The theoretical model presented in the paper uses the decomposition finite difference technique to determine the temperature distribution in the ground at different moments (1, 3, and 6 months). Although the model is two-dimensional, at this stage a one-dimension heat diffusion is considered in order to determine the penetration depth of the perturbation during the operation period which has been set to 6 months per year.

1. Introduction

Nowadays increasing amounts of fossil fuels are being burnt in order to satisfy the growing energy demand with the direct consequence of affecting the environment which is about to reach the threshold of radical and irreversible degradation. The greenhouse effect already has put its fingerprint on the global climate, triggering the melting of the polar caps and of the mountain glaciers and altering the weather patterns, thus making weather increasingly unpredictable. These upsetting phenomena have generated the interest towards renewable energies as an alternate for the fossil fuel-based energies.

One solution to this issue is to use the heat contained in the environment and to increase its temperature via heat pumps. The ground represents a heat source of immense quasi-isothermal capacity all year long and therefore it can be used as such in order to operate heat pumps in heating applications. Such devices are called Ground-Source Heat Pumps (GSHPs) and can be utilized to heat individual houses.

European countries with rich geothermal potential (such as Island and in a much lesser proportion France, and Hungary [1]) can harness geothermal heat at much higher temperatures, which make GSHPs useless. Such privileged regions are very few; almost all of the European countries are constrained at using the low-temperature heat removed from the ground.

The installed capacity worldwide is about 12,000 MWt and the global energy use is around 72,000 TJ (20,000 GWh) per year [2].

The technology dates from the 1940’s [8], but the interest in GSHP applications grew during the first oil crisis of the 1970’s, especially because of the very low surface area required for the boreholes, which made them suitable for individual houses [3].
Presently, 80,000 units are installed annually in the USA. The most illustrative example is the Galt House Hotel (Louisville, Kentucky), which uses a 19.6 MW heating/15.8 MW cooling unit for 100 apartments and 89,000 m² of office surface area (161,650 m² in total) and saves 47 percent of the necessary energy, which represents $25,000 per month [2].

In Europe, one must mention the campus of the National Technical University of Athens, where the heating and cooling of the Mining Engineering Building are provided via a GSHP, as a result from a pilot project from 1993, accomplished with Swiss support [1].

There are three configurations for the underground heat exchangers [3], [4]: horizontal loop, spiral loop, and vertical loop (borehole). When the available surface area is small, the third solution is recommended. A specific advantage of this configuration is the fact that below a depth of 15-20 meters, the ground temperature remains practically constant all year long, which provides a quasi-isothermal heat reservoir. The depth of the borehole is usually in the range 23 … 100 m [4].

The paper deals with the investigation of the conduction heat transfer in the ground during the heat extraction by a single borehole. A six months operation is supposed, roughly representing the cold season when there is a heat demand in order to operate a heat pump.

Because analytical methods are very difficult to apply (and only in very simple situations) [5], [6], the approach uses finite differences.

In this line of approach one can mention Bandos et al., [7] and Lamarche and Beauchamp [8], who use the finite-source model to determine the effect of vertical temperature variations, and Philippe et al., [9], who compare the results provided by three analytical methods: the infinite line source, the infinite cylindrical source and the finite line source respectively.

Actual situations involve two- or three-dimension transient conduction heat transfer with Neumann boundary conditions, making the analytical approach almost impossible, due to the complications associated with finding solutions for the differential equations that describe the phenomenon. A different, more pragmatic, approach involves the use of numerical techniques, among which one of the most popular is the finite difference method [6], [10], [11].

Two variations of the finite difference technique can be used: the explicit finite difference method (EFD), which allows the calculation of the present time temperature in the current node, respectively the implicit scheme (IFD), where the three nodal temperatures are simultaneously considered at the present time step. The EFD determines the unknown temperature in a sequential manner, but the magnitude of the time step is drastically limited, whereas in the EFD such a constraint does not operate, but the method involves the necessity to solve a set of linear algebraic equations corresponding to the total number of nodes [11], [12].

The paper deals with the use of the decomposition technique (DT) of the IFD scheme in two dimensions, for the heat removal from the ground by a single borehole, in cylindrical coordinates.

2. The physical model
The configuration of the borehole-ground system consists of a long hollow cylinder of length $L$, inner radius $R_0$, and outer radius $R_d$. The initial temperature of the ground is $T_\infty$.

A fluid of temperature $T_L$, lower than $T_\infty$, flows along the central channel (tube) that represents the borehole, cooling the cylindrical domain (the ground). Heat is removed by convection with the heat transfer coefficient $k_r$, via a cooling fluid. The other surfaces (frontal at $z = 0$ and in depth at $z = L$), respectively at $r = R_d$ (the outer limit of the ground domain) are adiabatic - see figure 1.

The essence of the decomposition approach is described in [4]. During a time-step of the procedure, the two-dimension conduction heat transfer is “decomposed” into two one-dimension heat transfer processes that occur sequentially: first in radial direction, and then axially. The start temperatures (known) of a new time step $\tau$ of magnitude $\Delta \tau$ are the final temperatures at the end of the previous time step (superscript $\tau - 1$). The intermediate temperatures (superscript $i$) resulting from the application of the decomposition procedure in its first (intermediate) step are the input temperatures for the second (final) one the output of which is represented by the final temperature (superscript $p$).
From the standpoint of the finite difference approach, this “trick” leads to a significant simplification, because one only must deal with very simple finite difference equations, which are very easy to manipulate.

The heat conduction equation in cylindrical coordinates that describes the radial heat transfer is [13]:

$$\frac{\partial \theta}{\partial \tau} = \lambda \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right)$$  \hspace{1cm} (1)

The heat transfer equation in the axial direction corresponds to the one-direction heat transfer in Cartesian coordinates:

$$\frac{\partial \theta}{\partial \tau} = \frac{\lambda}{\rho c} \frac{\partial^2 \theta}{\partial z^2}$$  \hspace{1cm} (2)

In the above equations, \( \lambda \) accounts for the heat conduction coefficient, \( \rho \) for the density, \( \tau \) for time, \( c \) for the heat capacity, and \( \theta \) for the dimensionless temperature defined as:

$$\theta = \frac{T - T_{\infty}}{T_H - T_{\infty}}$$  \hspace{1cm} (3)

where \( T_H \) is the temperature of the hot fluid that flows along the central tube during the charging phase. This manner of defining the dimensionless temperature was necessary to obtain a unique criterion when considering both heat removal from the ground (discharging) and heat injection (charging).

The boundary conditions are:

− at \( r = R_0 \):

$$k \frac{\partial \theta}{\partial r} \bigg|_{r=R_0} = -\lambda \left( \frac{\partial \theta}{\partial r} \right) \bigg|_{r=R_0}$$  \hspace{1cm} (4)

− at \( z = 0 \):

$$\left( \frac{\partial \theta}{\partial z} \right) \bigg|_{z=0} = 0$$  \hspace{1cm} (5)
\[-\text{at } r = R_d:\]
\[
\left( \frac{\partial \theta}{\partial r} \right)_{r=R_d} = 0 \quad (6)
\]
\[-\text{at } z = L:\]
\[
\left( \frac{\partial \theta}{\partial z} \right)_{z=L} = 0 \quad (7)
\]

where \( \theta_0 \) represents the dimensionless temperature of the cylinder’s inner surface.

3. Finite difference approach
A mesh has been attached to the cylindrical domain, consisting of \( N_r \) nodes of step \( h_r \) in the radial direction and \( N_z \) nodes of step \( h_z \) in the axial direction respectively (see figure 2).

The heat transfer equation (1) can be rewritten in finite difference form for a time step \( \Delta \tau \), by using the finite difference operator \( D_r^2 \) as follows [10], [11]:
\[
\frac{\theta_{m,n}^p - \theta_{m,n}^{p-1}}{\Delta \tau} = a D_r^2 \theta_{m,n}^p \quad (8)
\]
where \( a = \frac{\lambda}{\rho c} \) stands for the thermal diffusivity of the cylinder material, and \( p \) for the current time step number.

The operator \( D_r^2 \) can be expressed as:
\[
D_r^2 \theta_{m,n}^p = \frac{1}{h_r^2} \left( \frac{c_m-1}{c_m} \theta_{m+1,n}^p - 2\theta_{m,n}^p + \frac{c_m+1}{c_m} \theta_{m-1,n}^p \right) \quad (9)
\]

where:
\[
c_m = 2 \left( N_z \frac{R_d}{R_d - R_0} - m \right) \quad (10)
\]

Figure 2. The mesh attached to the domain.
Equation (2) can be written in finite difference form as:

\[ \frac{\theta^\rho_{m,n} - \theta^\rho_{m,n}}{\Delta \tau} = a D^2 \theta^\rho_{m,n} \]  

(11)

where

\[ D^2 \theta^\rho_{m,n} = \frac{1}{h_i^2} \left( \theta^\rho_{m,n-1} - 2\theta^\rho_{m,n} + \theta^\rho_{m,n+1} \right) \]  

(12)

The convergence and stability criteria of the finite difference scheme are:

\[ \alpha_r = \frac{a \Delta \tau}{h_i^2} \text{ (radial direction)} \]  

(13)

\[ \alpha_z = \frac{a \Delta \tau}{h_i^2} \text{ (axial direction)} \]  

(14)

After a series of manipulations, equation (8) becomes:

\[ -\theta^\rho_{i-1,n} + \sigma_r \theta^\rho_{i,n} - \gamma_r \theta^\rho_{i+1,n} = \beta_r \]  

(15)

where:

\[ \sigma_r = \frac{c_m}{c_m-1} \left( 2 + \frac{1}{\alpha_r} \right) \]  

(16)

\[ \gamma_r = \frac{c_m+1}{c_m-1} \]  

(17)

\[ \beta_r = \frac{c_m}{c_m-1} \frac{1}{\alpha_r} \theta^\rho_{i,n} \]  

(18)

\[ c_m = 2 \left( N_r R_0 \frac{R_0}{R_d - R_0} + m \right) \]  

(19)

In a similar manner, one obtains the finite difference form of equation (12):

\[ -\theta^\rho_{m,n-1} + \sigma_z \theta^\rho_{m,n} - \theta^\rho_{m,n+1} = \beta_z \]  

(20)

where:

\[ \sigma_z = 2 + \frac{1}{\alpha_z} \]  

(21)

\[ \beta_z = \frac{1}{\alpha_z} \theta^\rho_{m,n} \]  

(22)

The temperatures on the boundaries of the domain result from the boundary conditions written in the finite difference form.

4. Results and discussion

The finite difference method described above has been applied to a single borehole consisting of a 20 m long pipe 0.25 m across. The radius of the storage system boundary was \( R_d = 5.125 \) m. The temperatures were: \( T_H = 100^\circ \text{C}, T_m = 10^\circ \text{C}, T_L = 5^\circ \text{C} \). The thermo-physical properties of the ground are: \( \lambda = 0.25 \text{ Wm}^{-1}\text{K}^{-1}, a = 0.18 \times 10^{-6} \text{ m}^2\text{s}^{-1} \). The convection heat transfer coefficient from the channel to the cold fluid was \( k_c = 1000 \text{ Wm}^{-2}\text{K}^{-1} \). The grid attached to the cylindrical domain had \( N_r = 50 \) and \( N_z = 50 \) nodes respectively.

A computer code was written for the algorithm of the decomposition technique and was run in order to obtain the nodal temperatures at 30, 120 and 180 days respectively. Due to the imposed boundary conditions, although the finite difference scheme models a two-dimension unsteady heat conduction, the heat propagation is actually only radial (one-dimension), and the results provided by
this model will be compared with those obtained by applying a one-dimension propagation model presented in a previous paper [5] to check whether they describe accurately this situation.

Figure 3 shows the 3-D plot of the temperature field after 30 days. One can see that the heat diffusion is very slow, only about 10 nodes being affected by the perturbation, which represent roughly 1 meter of penetration depth in the radial direction. Beyond radial node #10, practically there is no noticeable temperature variation of the ground.

Figure 4. 3-D plot of the temperature distribution after 120 operation days.
The same trend can be noticed by analyzing figures 4 and 5 respectively that illustrate the temperature radial distribution across the ground in terms of 3-D representation, for 120 days and 180 days.

As time progresses, so does the temperature perturbation, reaching the 30th node (3 meters) after 120 days and the 40th node (4 meters) after 180 days respectively. The result might be slightly different if one considers the actual situation when the cylindrical domain is infinite (boundless) radially. Yet, since the finite difference approach cannot operate with infinite values, one is constraint to consider a boundary of finite radius at which an adiabatic boundary condition must be imposed. The longer this radius, the closer the considered domain is to the actually infinite radius domain. The results described by figures 3 through 5 show that the chosen value for the outer radius (5 m) was a correct supposition since within the 180 days interval, the perturbation only affects 80% of the entire radius.

![3-D plot of the temperature distribution after 180 operation days.](image)

Figure 5. 3-D plot of the temperature distribution after 180 operation days.

The validity of the 2-D decomposition finite difference approach has been verified by comparing its numerical results with those supplied by the one-dimension finite difference scheme. Table 1 displays the dimensionless temperatures and the relative errors in percents that results by using the two approaches. One can see that that the results are practically identical, the highest error being 0.004 percent.

Figure 6 joins the three temperature plots allowing to better interpret the results.

The slow propagation can be additionally explained by the low temperature difference between the ground and the cold fluid, which is only 5 degrees. Thus, the temperature gradients are feeble, and consequently the heat diffusion is slow.

The same reason (low temperature difference) explains the relatively low values of the heat removed from the ground: 2627 kJ after 30 days, 7792 kJ after 120 days, and 10830 kJ after 180 days respectively.

The above finds suggest that by merely removing heat from the ground is not a very smart option, especially when the ground temperature is not re-established during the warm season by heating it via the heat pump that operates in the air conditioning (cooling) mode.
### Table 1. Comparison between the results provided by the two finite difference approaches: one-dimension (1-D) and two-dimension (2-D) respectively.

| Node | Dimensionless temperatures according to the two models | Relative error (percent) |
|------|--------------------------------------------------------|-------------------------|
|      | 30 days 120 days 180 days 30 days 120 days 180 days |                         |
| 5    | -1.83E-02 -0.01826 -0.025827 -0.02583 -2.76E-02 -0.0275 | 0.002 0.004 -0.004     |
| 10   | -0.00685 -0.00685 -0.01527 -0.01527 -0.0175 -0.0175   | 0 0 0                   |
| 15   | -2.27E-03 -0.00227 -9.28E-03 -0.00928 -1.16E-02 -0.01157 | -0.004 0.001 -0.004    |
| 20   | -6.32E-04 -6.32E-04 -0.00557 -0.00557 -0.00769 -0.00769 | 0 0 0                   |
| 25   | -1.43E-04 -1.43E-04 -0.00324 -0.00324 -0.00505 -0.00505 | 0 0 0                   |
| 30   | -2.63E-05 -2.63E-05 -0.00182 -0.00182 -0.00328 -0.00328 | 0 0 0                   |
| 35   | -3.89E-06 -3.89E-06 -9.82E-04 -9.82E-04 -0.00212 -0.00212 | 0 0 0                   |
| 40   | -4.61E-07 -4.61E-07 -5.22E-04 -5.22E-04 -0.0014 -0.0014   | 0 0 0                   |
| 45   | -4.41E-08 -4.41E-08 -2.99E-04 -2.99E-04 -0.00102 -0.00102 | 0 0 0                   |
| 50   | -6.58E-09 -6.58E-09 -2.34E-04 -2.34E-04 -8.98E-04 -8.98E-04 | 0 0 0                   |

**Figure 6.** Temperature distributions in the ground after 30, 120, and 180 days.

The alternate (and much better) option is to store heat in the ground during the warm season and to remove it during the cold season, thus improving the effectiveness of the entire system (heat pump coupled with the heat reservoir/sink represented by the ground).

### 5. Conclusions

Borehole heat exchangers represent a good solution to harness heat contained in the ground via heat pumps in systems dubbed ground-source heat pumps (GSHP).

A decomposition finite difference technique has been used to study the radial heat diffusion in the ground in order to validate the correctness of the model and to determine the temperature distribution after 30, 120, and 180 operation days.

The model resulted in a pure one-dimension conduction heat transfer, the isothermal surfaces being cylinders co-axial with the central tube’s axis.

The numerical results show that the heat propagation process is a slow one and that the system’s performance is poor, due to the low temperature difference between the ground and the cold fluid.
This suggests to adopt the solution of storing heat in the ground and to remove it when necessary, thus significantly improving the performance.

Further work will focus on considering the convection heat transfer through the ground surface to the air, which will lead to a two-dimension heat transfer (radial and axial). This improved model will be further be used to study the removal of the heat that has previously stored in the ground in a ground heat storage unit.

6. References

[1] Sanner B, Karytsas C, Mendrinos D, and Rybach L 2003 Current status of ground source heat pumps and underground thermal energy storage in Europe *Geothermics* 32 pp 579-588

[2] Lund J, Sanner B, Rybach L, Curtis R, and Hellström G 2004 Geothermal (Ground-Source) Heat Pumps a World Overview *GHC Bulletin* September pp 1-10

[3] Yang H, Cui P, Fang Z 2003 Vertical-borehole ground-coupled heat pumps: A review of models and systems *Applied Energy* 87 pp 16–27

[4] Omer AM 2008 Ground-source heat pumps systems and applications, *Renewable and Sustainable Energy Review* 12 pp 344–371

[5] Causon D M, and Mingham C G 2010 *Introductory Finite Difference Methods for PDEs,* (Frederikberg: Ventus Publishing Aps)

[6] Croft D R, and Lilley D G 1977 *Heat Transfer Calculations Using Finite Difference Equations,* (London: Applied Science Publishers)

[7] Bandos T, Monter A, Fernandez E, Santander JLG, Isidro JM, Perez J, Fernandez de Cordoba PJ, and Urchueguia JF 2009 Finite line-source model for borehole heat exchangers: effect of vertical temperature variations *Geothermics* 38 pp 263-270

[8] Lamarche L, and Beauchamp B 2007 A new contribution to the finite line-source model for geothermal boreholes *Energy and Buildings* 39 pp 188-198

[9] Philippe M, Bernier M, and Marchio D 2009 Validity ranges of three analytical solutions to heat transfer in the vicinity of single boreholes *Geothermics* 38 pp 407–413

[10] LeVeque R 2007 *Finite Difference Methods for Ordinary and Partial Differential Equations* (Philadelphia: Society for Industrial and Applied Mathematics)

[11] Recktenwald G W 2004 Finite difference approximations to the heat equation *Mechanical Engineering* 10 pp 1-27

[12] Godunov S K, and Reabenki V S 1977 *Finite Difference Computing Schemes –* Romanian translation – (Bucharest: Editura Tehnică)

[13] Horbaniuc B, Dumitrașcu Gh, and Dumencu A 2014 Étude du stockage thermique dans le sol en utilisant un schéma à différences finies unidimensionnel *Actes du Colloque Francophone en Energie, Environnement, Economie et Thermodynamique COFRET ’14 Septième edition*