Estimates of the uncertainties associated with models of the nucleon structure functions in the $\Delta$ production region

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Theoretical studies of the inclusive electron-nucleus cross section at beam energies up to few GeV show that, for the region of the quasi-elastic peak is understood at quantitative level, the data in the $\Delta$ production region are sizably underestimated. We analyze the uncertainty associated with the description of the nucleon structure functions $W_1$ and $W_2$ and its impact on the nuclear cross section. The results of our study suggest that the failure to reproduce the data is to be mostly ascribed to the poor knowledge of the neutron structure functions at low $Q^2$.

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In view of the rapid development of neutrino physics, leading to significant improvements in the experimental accuracy, the treatment of nuclear effects in data analysis is now regarded as one of the main sources of systematic uncertainty [1, 2].

Much of the information needed to understand the nuclear response to neutrino interactions in the energy range $E_\nu = 0.5–3$ GeV, relevant to many neutrino experiments, can be extracted from the results of experimental and theoretical studies of electron-nucleus scattering (for a recent review see, e.g., Ref. [3]). In this kinematical regime, in which single nucleon knock out is known to be the dominant reaction mechanism, both quasi-elastic and inelastic processes, leading to the appearance of hadrons other than protons and neutrons, must be taken into account.

Non relativistic nuclear many-body theory (NMBT) provides a consistent framework, suitable to describe electron-nucleus interactions in a variety of kinematical conditions, ranging from quasi-elastic to deep inelastic scattering (see, e.g., Ref. [4]). In the impulse approximation regime, in which scattering off a nuclear target reduces to the incoherent sum of elementary processes involving individual nucleons, the basic elements of the NMBT approach are the nucleon spectral function $P(p, E)$, yielding the energy and momentum distribution of the knocked out nucleon, and the electron-nucleon cross section, that can be written in terms of the two structure functions $W_1^N$ and $W_2^N$ ($N = p, n$).

A study of inclusive electron scattering off oxygen at beam energies between 0.7 and 1.2 GeV, carried out within NMBT [4], has recently shown that, while the data in the region of the quasielastic peak are accounted for with an accuracy better than ~10%, theory fails to explain the measured cross sections at larger electron energy loss, where $\Delta$ production dominates. Based on the fact that the calculation of the cross section at the quasielastic and $\Delta$ production peak involves integrations of $P(p, E)$ extending over regions of the $(p, E)$ plane which almost exactly overlap one another, the authors of Ref. [5] argued that the disagreement between theory and data is unlikely to be imputable to deficiencies of the spectral function, and should rather be ascribed to the description of the elementary electron-nucleon cross section.

This short note is aimed at providing a quantitative estimate of the uncertainty associated with the available models of the nucleon structure functions at low $Q^2$. The impact on the nuclear cross section is also analyzed comparing theoretical results to the data of Refs. [6, 7].

The calculations of Ref. [6] were carried out using the Höhler-Brash parameterization of the nucleon form factors [8, 9], resulting from a fit which includes the recent Jefferson Lab data [10] and the Bodek and Ritchie (BR) parametrization of the proton and neutron inelastic structure functions [11], covering both the resonance and deep inelastic region.

The structure functions of Ref. [11] have been obtained from a global fit to the electron-proton and electron-deuteron cross sections measured at SLAC [12], spanning the kinematical domain $1 < Q^2 < 20$ GeV$^2$ and $0.1 \leq x \leq 0.75$, $x$ being the Bjorken scaling variable. As a consequence, as pointed out in Ref. [6], using the results of Bodek and Ritchie in the kinematical region covered by the data of Ref. [6], corresponding to $Q^2 \lesssim 0.2$ GeV$^2$ at the $\Delta$ production peak, involves an extrapolation whose validity needs to be carefully investigated.

Additional uncertainty is associated with the neutron structure functions $W_1^n$ and $W_2^n$, that have been extracted from deuteron data subtracting the proton contribution and unfolding nuclear effects. The authors of Ref. [11] followed the approach of Atwood and West [13], with the deuteron wave function obtained using the Hamada-Jonston (HJ) model of the nucleon-nucleon (NN) potential [14].

To gauge the systematic error involved in the determination of $W_1^n$ and $W_2^n$, we have compared the electron-deuteron cross sections computed using the HJ deuteron wave function to those obtained using the state-of-the-art NN potential of Ref. [15], generally referred to as Argonne v18 (A18) potential. The HJ and A18 models...
significantly differ in the description of the short range repulsive component of the NN force. The HJ exhibits a hard core of radius $r_c \sim 0.5$ fm, leading to a vanishing deuteron wave function at $r < r_c$, while the A18 potential smoothly reaches a large but finite positive value at $r = 0$. The differences in the short range behavior of the wave functions in coordinate space are reflected by sizable differences in the momentum distributions $n(p)$ at $|p| \gtrsim 1$ GeV. However, as $n(p)$ falls by a factor $\sim 10^6$ in the range $0 < |p| < 1$ GeV, these differences have negligible effect on the calculated electron-deuteron cross section. For example, at beam energy $E_e = 4$ GeV and electron scattering angle $\theta_e = 30^\circ$ the results obtained using the HJ and A18 momentum distributions differ by at most $2\%$ over the energy loss range extending from pion production threshold, corresponding to $\nu \sim 1.05$ GeV, to $\nu \sim 2.5$ GeV.

We have also investigated the effect of the ambiguity implied by the relativistic normalization of the momentum distribution, which is obtained from the nonrelativistic wave function in momentum space. A number of theoretical calculations have been carried out using the normalization of Ref. [16] instead of the one suggested by Atwood and West, which amounts to replacing

$$n(p) \to \frac{m}{\sqrt{p^2 + m^2}} n(p), \quad (1)$$

$m$ being the nucleon mass. Our results show that, as the momentum distribution is sharply peaked at small $|p|$, the presence of the additional normalization factor does not produce any appreciable effects.

Having established that the uncertainty arising from the treatment of nuclear effects is small, we can now address the question of whether the fit of Ref. [11] provides a reasonable description of the structure functions at low $Q^2$. To clarify this issue we have compared the calculated electron-deuteron cross section, obtained using the A18 momentum distribution and the BR structure functions, to the Jefferson Lab data of Refs. [17, 18] at $E_e = 2.445$ GeV and $\theta_e = 20^\circ$. This kinematical setup corresponds to the lowest $Q^2$ at the $\Delta$ production peak available in the Jefferson Lab dataset, namely $Q^2 = 0.54$ GeV$^2$.

The results, represented by the solid line in the lower panel of Fig. 1 show that in the $\Delta$ region the measured cross section is underestimated by as much as $\sim 25\%$. On the other hand, the results displayed in the upper panel show that the BR model provides a much better account of the electron-proton cross section, thus implying that the disagreement with deuteron data is to be mostly ascribed to the neutron contribution.

For comparison, in the upper panel we also include the cross sections calculated using the structure functions resulting from a recent fit including the JLab data [15, 20] and from the dynamical model of Refs. [21, 22]. It clearly appears that the different models describe the data in the region of the $\Delta$ peak with comparable accuracy. The distinctive behavior exhibited by the predictions of the model of Refs. [21, 22], at both lower and higher energy loss, is to be ascribed to the fact that it does not include the contributions of other resonances and non resonant pion production.

In Refs. [11, 12] the deuteron cross section is written in the form

$$\sigma_d = \sigma_p + \sigma_n \quad (2)$$

where the proton and neutron contributions, $\sigma_{p,n}$ can in turn be expressed in terms of the proton and neutron structure functions, smeared by nuclear binding and Fermi motion. Using the Atwood-West formalism and the measured electron-proton cross sections, $\sigma_p$, one can then compute the ratio

$$S_p = \frac{\sigma_p}{\sigma_n}, \quad (3)$$

whose value provides a measure of nuclear effects. Note that Eq. (3) is based on the premise that the ratios between the unsmeared and smeared structure functions, $W_{p,2}^p$ and $\tilde{W}_{p,2}^p$, satisfy [12]

$$S_p \approx \frac{W_{p,2}^p}{\tilde{W}_{p,2}^p} \approx \frac{W_p^p}{\tilde{W}_p^p}. \quad (4)$$
The results of numerical calculations carried out in the \( \Delta \) production region show that \( S_p \), while varying sharply as a function of the invariant mass of the hadronic final state, \( W \), exhibits a rather weak dependence on \( Q^2 \).

![Diagram](image)

**FIG. 2:** (Color online) The nucleon structure functions \( W_2^N \) (\( N = n, p \)) at \( E_e = 2.445 \text{ GeV} \) and \( \theta_e = 20^\circ \), corresponding to \( Q^2 = 0.54 \text{ GeV}^2 \) at the \( \Delta \) production peak, plotted as a function of the invariant mass of the hadronic final state. The shaded area represents the \( W_2^n \) resulting from our analysis, while the solid and dashed line correspond to \( W_2^n \) and \( W_2^p \) of Ref. [11], respectively.

Assuming that the smearing ratios for the proton and neutron cross section, \( S_p \) and \( S_n \), be close to one another, the unsmeared neutron cross section can finally be estimated from

\[
\sigma_n = S_n \bar{\sigma}_n \approx S_p \sigma_d - \sigma_p .
\]

The neutron structure function obtained applying the above procedure at beam energy \( E_e = 2.445 \text{ GeV} \) and electron scattering angle \( \theta_e = 20^\circ \), plotted as a function of \( W \), is shown by the shaded area in Fig. 2. The resulting \( W_2^n \), which accounts for the measured deuteron cross sections by construction, turns out to be significantly larger than the one obtained from the fit of Ref. [11], the difference at the \( \Delta \) production peak being \( \sim 60 \% \). For comparison, the proton structure function \( W_2^p \) is also shown.

To gauge the impact of the uncertainty in the neutron structure functions on the nuclear cross section, we have analyzed electron scattering off carbon at \( E_e = 1.3 \text{ GeV} \) and \( \theta_e = 37.5^\circ \), and compared our results to the SLAC data of Ref. [6].

The shaded area in the upper panel of Fig. 3 shows the results of our calculations, carried out within the Plane Wave Impulse Approximation (PWIA) formalism described in Ref. [2] with the spectral function obtained from the Local Density Approximation (LDA) [23]. We have used the proton structure functions of Ref. [11], while for the neutron the BR fit has been modified according to

\[
W_2^n(Q^2, W) \rightarrow S(W)W_2^n(Q^2, W) ,
\]

\( S(W) \) being the ratio between the results of the analysis described in the previous section (shaded area of Fig. 2) and the corresponding BR fit (solid line of Fig. 2).

In view of the fact that in the kinematical setup of the carbon data the value of \( Q^2 \) at the \( \Delta \) production peak is \( \sim 0.4 \text{ GeV}^2 \), using \( S(W) \) extracted at \( Q^2 \sim 0.5 \text{ GeV}^2 \) appears to be reasonable.

Comparison with the solid line, obtained using the BR fit for both the proton and the neutron, shows that the neutron structure functions that reproduce the deuteron data at \( Q^2 = 0.54 \text{ GeV}^2 \) also provide a much better description of carbon data at electron energy loss \( \nu \gtrsim 0.6 \text{ GeV} \). Discrepancies between theory and experiment still persist at lower \( \nu \), in the region of the dip between the quasi-elastic and the \( \Delta \) production peak.

The inclusive cross section in the dip region is known to be affected by processes in which the electron couples to the two-body component of the nuclear electromagnetic current arising from \( \pi \) and \( \rho \) meson exchange between nucleons. However, the amount of strength associated with meson exchange currents (MEC) is still somewhat controversial, and the possibility that the cross section in the dip region might be ascribed to an asymmetry in the shape of the quasi-elastic peak has also been suggested (see, e.g., [25] and references therein). The contribution of MEC is not taken into account in the
PWIA approach of Ref. [3], and its significance in the kinematical regime under discussion should be carefully investigated.

The lower panel of Fig. 3 shows a comparison between our theoretical results and the oxygen data discussed in Ref. [3]. It clearly appears that at the low $Q^2$ of the data of Ref. [3] the enhancement resulting from using the neutron structure functions obtained from our analysis is not sufficient to explain measured cross section.

The results discussed in this paper show that the structure functions obtained from the global fit of Refs. [11, 12], while providing a quantitative account of proton data, fail to explain the measured deuteron cross sections at low $Q^2 (< 1 \text{ GeV}^2)$.

Employing a simple and somewhat crude procedure, similar to the one followed by Bodek and Ritchie, we have extracted the neutron structure functions at $Q^2 \sim 0.5 \text{ GeV}^2$ from the Jefferson Lab electron-proton and electron-deuteron data. The resulting $W^d_2$ and $W^p_2$ turn out to be much larger than those of Ref. [11] in the region of the $\Delta$ production peak.

When used to calculate the nuclear cross section in the impulse approximation scheme, the neutron structure functions obtained from our analysis provide a satisfactory description of carbon data at $Q^2 = 0.4 \text{ GeV}^2$. On the other hand, oxygen data at lower $Q^2$ are still sizably underestimated. This feature is likely to reflect an appreciable $Q^2$-dependence of the function $S(W)$, describing the difference between our results and the BR fit.

The results of Figs. 1 and 3 have been obtained neglecting final state interactions (FSI), which are known to lead to a redistribution of the inclusive strength [3]. Within the approach of Ref. [10], FSI effects in the quasi-elastic channel are included through a folding procedure that accounts for nucleon-nucleon rescattering processes.

However, the extension to inelastic channels involves additional problems and has not been carried out yet. To estimate the relevance of FSI in the $\Delta$ production region we have folded the PWIA inelastic cross sections of Figs. 1 and 3 using the folding functions obtained from the approach of Ref. [10]. The results, showing that the main effect is a quenching of the peak of less than $\sim 2\%$ and $4\%$ in deuteron and carbon, respectively, suggest that FSI do not significantly affect our analysis.

The ability of dynamical models [21, 22, 24, 27] to explain the nuclear cross section should also be investigated. The results of a recent calculation [27] based on the model of Ref. [26], are in fairly good agreement with the data of Ref. [6] in the $\Delta$ production region at $Q^2 \sim 0.2 \text{ GeV}^2$. However, the fact that measured quasi-elastic cross section is sizably overestimated seems to point to deficiencies in the treatment of nuclear effects.

In conclusion, our study suggests that the neutron structure functions in the $\Delta$ production region at low $Q^2$, which is still poorly known, may be extracted from a fit to the upcoming deuteron data from Jefferson Lab [28]. Valuable complementary information will also come from the direct measurement of the free neutron structure functions at $1 < Q^2 < 5$, presently being carried out by the Jefferson Lab E03-12 (BoNuS) collaboration [29].

[1] Proceedings of NuInt01, Eds. J.G. Morfin, M. Sakuda and Y. Suzuki. Nucl. Phys. B (Proc. Suppl.) 112 (2002).
[2] Proceedings of NuInt04, Eds. F. Cavanna, P. Lipari, C. Keppel and M. Sakuda. Nucl. Phys. B (Proc. Suppl.) 139 (2005).
[3] O. Benhar, D. Day and I. Sick, nucl-ex/0603029 Submitted for publication in Reviews of Modern Physics.
[4] O. Benhar, V.R. Pandharipande and S.C. Pieper. Rev. Mod. Phys. 65 (1993) 817.
[5] O. Benhar, N. Farina, H. Nakamura, M. Sakuda and R. Seki. Phys. Rev. D 72 (2005) 053005.
[6] R.M. Sealock et al., Phys. Rev. Lett. 62 (1989) 1350.
[7] M. Anghinolfi et al., Nucl. Phys. A 602 (1996) 405.
[8] G. Höhler et al., Nucl. Phys. B114 (1976) 505.
[9] E.J. Brash, A. Kozlov, Sh. Li, and G.M. Huber. Phys. Rev. C 65, 051001(R) (2002) 051001(R).
[10] M.K. Jones et al., Phys. Rev. Lett. 84 (2000) 1398.
[11] A. Bodek and J.L. Ritchie, Phys. Rev. D 23 (1981) 1070.
[12] A. Bodek et al., Phys. Rev. D 20 (1979) 1471.
[13] W.B. Atwood and G.B. West, Phys. Rev. D 7 (1973) 773.
[14] T. Hamada and I.D. Johnston, Nucl. Phys. A 34 (1982) 382.
[15] R.B. Wiringa, V.G.J. Stoks and R. Schiavilla, Phys. Rev. C 51 (1995) 38.
[16] O. Benhar and V.R. Pandharipande, Phys Rev. C 47 (1993) 2218.
[17] I. Niculescu, PhD Thesis, Hampton University, 1999. Unpublished.
[18] I. Niculescu et al., Phys. Rev. Lett. 85 (2000) 1186.
[19] Y. Liang, M.E. Christy, R. Ent and C.E. Keppel (Jefferson Lab Hall C E94-110 Collaboration), nucl-ex/0410027.
[20] http://www.jlab.org/~christy/cs.fits/cs.fits.html
[21] O. Lalakulich and E.A. Paschos, Phys. Rev. D 71 (2005) 074003.
[22] O. Lalakulich, E.A. Paschos and G. Piranishvili, Nucl. Phys. B (Proc. Suppl.) 159 (2006) 133.
[23] O. Benhar, A. Fabrocini, S. Fantoni and I. Sick, Nucl. Phys. A579 (1994) 493.
[24] J. Carlson and R. Schiavilla, Rev. Mod. Phys. 70 (1998) 743.
[25] J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, A. Molinari and I. Sick, Phys. Rev. C 71 (2005) 015501.
[26] T. Sato and T.-S.H. Lee, Phys. Rev. C 54 (1996) 2660.
[27] T. Sato, B. Szczepinska, K. Kubodera and T.-S.H. Lee, Nucl. Phys. B (Proc. Suppl.) 159 (2006) 141.
[28] V. Tvaskis, J. Steinman and J. Bradford, Nucl. Phys. B (Proc. Suppl.) 159 (2006) 163.
[29] www.jlab.org/~kuhn/BoNuS Welcome.html