On the Density of Cold Dark Matter.

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I. INTRODUCTION

The new results on Cosmic Microwave Background (CMB) anisotropy from TOCO ([3], BOOMERanG ([4], [5]), MAXIMA ([6]), and DASI ([7]) experiments represent an extraordinary success for the standard cosmological model of structure formation based on Cold Dark Matter (CDM) and adiabatic primordial perturbations (see e.g. [10]). Furthermore, early data releases from the 2dFGRS and SDSS galaxy redshift surveys ([11], [12]) are living up to expectations and combined analysis of all these datasets are placing strong constraints on most cosmological parameters ([4], [28]).

However, even if theory and observations are in spectacular agreement on large (\(\gtrsim 1\)Mpc) scales, various discrepancies seem to present on smaller (sub-galactic) scales. In particular, numerical simulations of CDM models yield an excess of small-scale power, producing, for example, more satellite galaxies than observed ([20]) and cuspy galactic halos with excessive dark mass concentrations ([8], [9]), while on the smallest scales probed, gravitational lensing ([1], [2]) prefers CDM substructure. Many authors have addressed these issues and many solutions have been proposed, ranging from modifications of the properties of the CDM particles to the introduction of new physics and to a refinement of the astrophysical processes involved in the simulations.

One way to address these issues is by detection of dark matter. The key quantity of relevance to dark matter searches is the CDM density, which specifies the dark matter annihilation cross-section. In this paper, we use the latest CMB and deep redshift surveys data in order to simultaneously constrain the most important cosmological observables related to CDM: the CDM density \(\Omega_{cdm}h^2\), the spectral tilt \(n_S\) of the primordial power spectrum and the rms mass fluctuations by a pre-factor \(C_{01}\) in units of \(C_{10}^{COBE}\) with \(0.5 < C_{10} < 1.40\). In order to test the stability of our results under variation of some of the theoretical assumptions, we also consider variations in the dark energy equation of state \(\omega_Q = -1\) (quintessence) or an extra background of relativistic particles \(\Delta N_e = 1\). Further- more, under the assumption of CDM, we extrapolate the cosmological constraints into a measurement of the linear rms density fluctuations on sub-galactic mass scales.

II. COSMOLOGICAL CONSTRAINTS ON CDM

A. Method

We compare the recent CMB observations with a set of models with 6 parameters sampled as follows: \(\Omega_{cdm}h^2 \equiv \omega_{cdm} = 0.01, \ldots, 0.40\), in steps of 0.01; \(\Omega_b h^2 \equiv \omega_b = 0.01, \ldots, 0.040\), in steps of 0.001; \(\Omega_{\Lambda} = 0.0, \ldots, 1.0\), in steps of 0.05 and \(\Omega_m = 0.1, \ldots, 1.0\), in steps of 0.05. The value of the Hubble constant is not an independent parameter, since:

\[
h = \sqrt{\frac{\omega_{cdm} + \omega_b}{1 - \Omega_k - \Omega_{\Lambda}}}. \tag{1}
\]

We vary the spectral index of the primordial density perturbations within the range \(n_s = 0.60, \ldots, 1.40\) (in steps of 0.02) and we rescale the amplitude of the fluctuations by a pre-factor \(C_{10}\), in units of \(C_{10}^{COBE}\) with \(0.5 < C_{10} < 1.40\). In order to test the stability of our results under variation of some of the theoretical assumptions, we also consider variations in the dark energy equation of state parameter \(\omega_Q = -1.0, \ldots, -0.5\) as expected in quintessence models (see e.g. [1], [4], [8], [9]) and of an extra background of relativistic particles with \(\Delta N_e = 0.3, \ldots, 9\) in steps of 0.3 (see [1] and references therein).
We also include the COBE data using Lloyd Knox’s analytical marginalization method presented in [22].

For a given cosmological model is then defined by

\[
\frac{\sigma}{C} \sim \frac{1}{\sqrt{\ell}}
\]

where the integral of the likelihood is 0

The theoretical models are computed using a modified version of the publicly available CMBFAST code [36] and are compared with the recent BOOMERanG-98, DASI and MAXIMA-1 results. The power spectra from these experiments were estimated in 19, 9 and 13 bins respectively, spanning the range 25 \( \ell \) ≤ 1150. For the DASI and MAXIMA-I experiments we use the publicly available correlation matrices and window functions. For the BOOMERanG experiment we assign a flat interpolation for the spectrum in each bin \( \ell(\ell+1)C_{\ell}/2\pi = C_{\ell} \), we approximate the signal \( C_{\ell} \) inside the bin to be a Gaussian variable and we consider \( \sim 10\% \) correlations between the various bins. The likelihood for a given cosmological model is then defined by

\[
-2\ln L = (C_{\ell} - C_{\ell}^0)^2/M_{BB}^{-1}(C_{\ell} - C_{\ell}^0)
\]

where \( M_{BB}^{-1} \) is the Gaussian curvature of the likelihood matrix at the peak. We consider 10\%, 4\% and 5\% Gaussian distributed calibration errors for the BOOMERanG-98 [?], DASI [7] and MAXIMA-I [?] experiments respectively and we included the beam uncertainties by the analytical marginalization method presented in [22].

We also include the COBE data using Lloyd Knox’s RADPack packages.

In addition to the CMB data we incorporate the real-space power spectrum of galaxies in the 2dF 100k galaxy redshift survey using the data and window functions of the analysis of Tegmark et al. [37].

To compute \( \mathcal{L}_{2dF} \), we evaluate \( p_i = P(k_i) \), where \( P(k) \) is the theoretical matter power spectrum and \( k_i \) are the 49 k-values of the measurements in [37]. Therefore we have

\[
-2\ln \mathcal{L}_{2dF} = \sum_i (P_i - (Wp)_i)^2/dP_i^2,
\]

where \( P_i \) and \( dP_i \) are the measurements and corresponding error bars and \( W \) is the reported 27 \( \times \) 49 window matrix. We restrict the analysis to a range of scales where the fluctuations are assumed to be in the linear regime (\( k < 0.2h^{-1}\text{Mpc} \)). When combining with the CMB data, we marginalize over a bias \( b \) considered to be an additional free parameter.

We attribute a likelihood to each value of \( \omega_{cdm}, n_S, \sigma_8 \) by marginalizing over the nuisance parameters. We then define our 68\% (95\%), confidence levels to be where the integral of the likelihood is 0.16 (0.025) and 0.84 (0.975) of the total value.

B. CMB results and test for theoretical assumptions.

The results of our analysis are shown in Table 1. In the first row, we restrict our analysis to CMB data with a combination of “weak priors”: \( h = 0.65 \pm 0.2 \), age \( t_0 > 10\text{Gyr} \) and \( \Omega_m > 0.1 \). Since the value of \( \sigma_8 \) is degenerate with \( \Omega_m \), in the table we report \( \sigma_8^* \) defined as the value of \( \sigma_8 \) with \( \Omega_m = 0.30 \pm 0.05 \).

We see that, under the class of models considered, the CMB data suggest a \( \sim 2\sigma \) detection of CDM, a scalar spectral index \( n_S \sim 0.92 \) and a value of \( \sigma_8 \sim 0.66 \) at \( \Omega_m \sim 0.3 \). The reason why the present data

FIG. 1. Top panel: CMB anisotropies and CDM. Bottom panel: Allowed region for the matter power spectrum from CMB and from other cosmological observables obtained under the assumption of adiabatic CDM primordial fluctuations. The data from the 2dF redshift survey (Tegmark and Hamilton, 2002) is also plotted in the figure.

FIG. 2. Constraints in the \( (\Omega_m)^0.6\sigma_8 - \Omega_{cdm}h^2 \) plane. The results of the 3 combined analysis CMB+HST, CMB+SN-Ia and CMB+2dFGRS are shown together with the 68\% c.l. cluster constraints.
favour CDM is evident in Fig.1, top panel, where we plot the recent CMB data with the best fit purely baryonic (BDM) and CDM models. BDM models fail to reproduce the observed power at $\ell \geq 700$. The new CMB data provide independent support for the presence of non-baryonic dark matter in the universe.

Before including information from complementary cosmological datasets, it is important to test the stability of our CMB results by removing some of the theoretical assumptions used in the analysis. We restrict our analysis to $\Lambda$CDM models. However another candidate that could possibly explain the observations of an accelerating universe is a dynamical scalar “quintessence” field. The common characteristic of quintessence models is that their equation of state $w_Q = p/\rho$, varies with time and can be greater than $w_Q = -1$, the value corresponding to the cosmological constant. Adopting ‘quintessence’ instead of a cosmological constant does not change the results of our analysis, but increases the error bars.

Another possibility is to consider an extra background of relativistic particles (see e.g. [1]), parametrized by a larger number of effective massless neutrinos $N_{\nu}^{eff}$. Since increasing $N_{\nu}^{eff}$ changes the epoch of equality, which is well determined by CMB observations (2), larger values of $\Omega_{cdm} h^2$ are needed in order to compensate for this variation. The constraints on $\sigma_8^*$ are not greatly affected since the matter power spectrum is mainly sensitive to changes in the redshift of equality, which is kept constant via the CMB data.

In the entire analysis, we assume negligible reionization and an optical depth $\tau_e \sim 0$. This is in agreement with recent estimates of the redshift of reionization $z_{re} \sim 6 \pm 1$ (see e.g. [1]). We have also removed this assumption. Due to the well known degeneracy with the scalar spectral index $n_s$, the effect of including variations in $\tau_e$ leaves the CMB data in better agreement with higher values of $n_s$ and of $\sigma_8$.

A background of gravity waves and/or of isocurvature modes can modify the theoretical CMB spectrum on large angular scales. In order to test for these hypotheses, we repeat the analysis without the COBE data. This has the effect of relaxing our constraints.

The high-$\ell$ part of the observed spectrum can be contaminated by different systematics (see e.g. [2]): beam reconstruction, detector noise, foregrounds. All 3 different experiments can be affected by different systematics and the fact that the 3 datasets are in reasonable agreement suggests that the systematics are under control. However, we repeated the analysis removing the data points at $\ell > 650$. As we can see, again, apart from an increase in the error bars, there is no significant difference.

C. Comparison with complementary datasets.

Since the CMB results are stable under variations of different theoretical assumptions, we can now assume that the A-CDM models are valid and investigate the effects of applying various prior probabilities and of incorporating complementary cosmological datasets. Including the gaussian prior $0.8 \Omega_m - 0.6 \Omega_\Lambda = -0.2 \pm 0.1$ from type Ia supernova luminosity distances [10] or $h = 0.71 \pm 0.07$ from measurements with the Hubble Space Telescope [23] does not change our conclusions and improves the constraints on the 3 parameters.

In Figure 1, bottom panel, we check for consistency of the 2dF data with the set of CDM models used in the CMB analysis by plotting a convolution of all the matter power spectra from the theoretical models in agreement with the CMB data, together with the
recent 2dF analysis of [33]. As one can see, the region consistent with CMB alone is quite broad (due to the weak CMB constraint on \( \Omega_\Lambda \)) and contains the shape of the 2dF spectrum. Including other cosmological constraints from SN-Ia and HST shrinks the CMB constraint into a region consistent with the shape inferred from 2dF. This 'consistency' is reflected in the results of Table 1: including information on the shape of the 2dF matter power spectrum improves our constraints in a similar direction to the SN-Ia and HST priors.

On similar scales, recent analyses of the local cluster number counts can be summarised as giving different results for \( \sigma_8 \) mainly due to systematics in the calibration between cluster virial mass and temperature: a high value \( \sim \Omega_{m,0}^{0.6} \sigma_8 = 0.55 \pm 0.05 \) in agreement with the results of (24, 26) and a lower one, \( \sim \Omega_{m,0}^{0.6} \sigma_8 = 0.40 \pm 0.05 \) following the analyses of [29] and [44]. It is interesting to plot these constraints in the \( \Omega_{m,0}^{0.6} \sigma_8 - \Omega_{cdm} h^2 \) plane. We do this in Figure 2, where we plot the 95% confidence level contour of the combined CMB+HST, CMB+SN-Ia and CMB+2dFGRS analyses (obtained again with the assumption of CDM) together with the high and low constraints on \( \Omega_{m,0}^{0.6} \sigma_8 \) at 68% c.l.

As we can see, a correlation appears from the present CMB data between these 2 quantities: namely, increasing \( \Omega_{cdm} h^2 \) enhances the rms mass fluctuations. The independent constraint on \( \Omega_{m,0}^{0.6} \sigma_8 \) from clusters can be used to break the degeneracy. Using the high constraint, we obtain a higher value for the density in CDM (see Table 1) with 0.17 > \( \Omega_{cdm} h^2 > 0.11 \) at 95% c.l.. Using the low value, the constraint becomes 0.15 > \( \Omega_{cdm} h^2 > 0.09 \), again at 95% c.l.. The 2 results are consistent and favour CDM; however, when additional information such as SN-Ia and 2dF are included, the CMB tends to prefer the lower value.

We further analyze this possible discrepancy in Figure 3 where we plot constraints in the \( \Omega_{m} - \sigma_8 \) plane together with the 68% c.l. results of [23] and of [30]. For values of \( \Omega_{m} < 0.35 \), the high \( \sigma_8 \) constraint is in slight disagreement with the CMB+2dF result at more than 1 \( \sigma \).

On smaller (sub-galactic) scales, recent and strong constraints on the linear rms mass fluctuations \( \sigma(M) \) have been obtained through lensing measurements (4). In Figure 3, we plot the predictions of the CDM models that are within the 95% CMB constraints in the \( \sigma(R) - R \) plane together with a region indicative of the constraint obtained from lensing. A more careful comparison and a better study of all the assumptions and possible systematics should be done. However, here we wish to point out that, even in our simple analysis, the two contours overlap and the CDM paradigm seems consistent over a range of scales from \( 10^4 \text{Mpc} h^{-1} \) to \( 10^{-1} \text{Mpc} h^{-1} \).

### III. DISCUSSION

In this paper we have provided strong constraints on the amount of non-baryonic dark matter in the universe by comparing adiabatic inflationary models with a set of cosmological observations. Combining CMB anisotropy measurements, high redshift supernovae observations, constraints on the Hubble parameter from the HST key project, and the matter power spectrum data from the 2dFGRS, we find \( \Omega_{cdm} h^2 = 0.11 \pm 0.04 \), \( n_S = 0.93 \pm 0.08 \), \( \sigma_8 = 0.66 \pm 0.10 \) at 95% confidence level. These results provide strong evidence for the presence of non-baryonic dark matter in a way independent of the local cluster abundance data.

If the dark matter is the neutralino, a knowledge of \( \Omega_{cdm} h^2 \) can set strong bounds on the parameter space of the simplest and most direct implementations of supersymmetry (see e.g. [4, 1, 13]).

The constraint on \( \Omega_{cdm} h^2 \) obtained here can be considered stable under the removal of some of the theoretical assumptions made in the analysis. Including quintessence, reionization and a large-scale CMB component such as expected from gravity waves (see e.g. [43, 44]) or isocurvature CDM perturbations (see e.g. [45]) can relax the constraints but does not change the conclusions. Models with \( \Omega_{cdm} h^2 \) as large as 0.2 are disfavoured in all cases except under the exotic hypothesis of an extra background of relativistic particles.

Depending on the external data we incorporate, the CMB constraints in the \( \sigma_8 \Omega_M h^2 - \Omega_{cdm} h^2 \) plane can be 1–2 \( \sigma \) lower than most of the determinations inferred from the local cluster X-ray temperature function (see e.g. [24, 27, 28] and cosmic shear data (27, 28, 14, 15), while in better agreement with the 'new'

![FIG. 4. The predictions of the CDM models that are within the 95% CMB constraints in the \( \sigma(R) - R \) plane together with a region indicative of the constraint obtained from lensing.](image-url)
analyses of $\sigma_8$ and $n_S$. However, we showed that use of the high or low priors on $\sigma_8$ has only a marginal effect on the constraints on $\Omega_{cdm}h^2$.

With reference to previous analyses, we are in agreement with the recent paper by Lahav et al. [38], although our value of $\sigma_8^*$ appears to be slightly lower. However we allow for variations in the spectral index $n_S$ while in [38] this parameter has been set to $n_S = 1$. We also included the COBE data using the lognormal approximation as in [40] and the 2dF data is taken from the independent analysis of [37].

The low values of $\sigma_8$ and $n_S$ can possibly alleviate some of the problems of CDM on subgalactic scales. We will investigate this in a forthcoming paper [39]. However, we also find that the models compatible with the CMB can satisfy the recent lensing constraints. This result suggests that a solution of the subgalactic CDM problem can most likely be obtained by a refinement of the astrophysical processes involved in the numerical simulations, rather than by an ad hoc modification of the dark matter properties. However, if future cluster temperature or cosmic shear analyses were to converge towards a higher $\sigma_8$ value, then this could lead to a possible discrepancy with the CMB+2dF result. It will be the task of future experiments and analysis to verify this interesting result.

Acknowledgments

The authors would like to thank Celine Boehm and Marco Peloso for useful discussions. AM acknowledges support from PPARC.

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