Horizontal Symmetries for the Supersymmetric Flavor Problem

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Abstract

The heaviess of the third family fermions and the experimental absence of large flavor-violating processes suggest, in supersymmetric theories, that the three families belong to a $2 + 1$ representation of a horizontal symmetry $G_H$. In this framework, we discuss a class of models, based on the group $U(2)$, that describe the flavor structure of the fermion masses and are compatible with an underlying GUT. We study the phenomenology of these models and focus on two interesting scenarios: In the first one, the first and second family scalars are assumed to be heavier than the weak scale, allowing for complex soft supersymmetry-breaking terms. In the second one, all the CP-violating phases are assumed to be small. Both scenarios present a rich phenomenology in agreement with constraints from flavor-violating processes and electric dipole moments.
1 Introduction

The flavor structure of the Standard Model (SM) is an open problem, but also a hint in the search for a more fundamental theory. From the experimental data we have learned that the third-family fermions are very special. They are very heavy and almost decoupled from the fermions of the other two families. In the supersymmetric version of the SM (MSSM), data from flavor-violating processes provide us with more hints about the flavor structure. The experimental indication of small flavor changing neutral currents (FCNC) implies the need of a super-GIM mechanism in the scalar-quark sector, i.e. the first- and second-family squarks have to be highly degenerate \(^1\). To satisfy such a requirement, the scalar masses were assumed to be universal (universality condition) \(^1\). Nevertheless, if no symmetry protects such universality, it will be spoiled by higher-order effects \(^2\). Then, even if universality holds at the scale where supersymmetry is broken, it will not hold at a lower scale. This departure from universality induced by radiative corrections can be particularly significant in grand unified theories (GUTs) where large representations and large Yukawa couplings are present \(^3\).

The above considerations suggest that any low-energy effective supersymmetric theory must have an approximate \(U(2)_L\times U(2)_R^d\times U(2)_R^u\) symmetry of flavor under which the 1\(^{st}\) and 2\(^{nd}\) families transform as a doublet and the 3\(^{rd}\) as a singlet. Such flavor symmetry could just be accidental. In this paper, however, we will consider that a horizontal symmetry \(G_H\) with the three families belonging to the \(2 + 1\) representation is actually realized at some high-energy scale. Furthermore, inspired by GUTs, we will consider that \(G_H\) is a subgroup of only one \(U(2)\) factor instead of allowing for independent left and right rotations\(^1\). The \(G_H\) symmetry could be gauged, global or discrete. If the horizontal group is gauged, however, gauge-breaking effects can arise in the squark sector (D-term contributions), which, as we will see, are too large. This suggests that \(G_H\) could be a discrete subgroup of \(U(2)\). (Moreover, in the latter case one does not need to worry about the cancellation of the \(U(1)\) anomaly).

Of course, \(G_H\) has to be broken to give mass to the lightest families. Motivated by the pattern of fermion masses and mixing angles, we will consider that the horizontal symmetry breaking is realized in two steps. In the first step, the second-family fermions get masses, while it is only after the second step that the first family becomes massive.

We will not make any specific assumption about the soft supersymmetry breaking (SSB) terms. Based on the above symmetry-breaking pattern, we will be able to infer the flavor structure of the scalars mass matrices and analyse the phenomenological implications of the model. The horizontal symmetry will force an approximate \(1^{st} - 2^{nd}\) family squark degeneracy, and FCNC processes will be suppressed (by the super-GIM mechanism). We will also study the possibility that the first two families of scalars are heavier than the other particles and allow for complex SSB terms. These two ingredients will lead to a phenomenology very different from that in the MSSM with universal soft masses.

Horizontal symmetries under which the quarks transform as \(2 + 1\) have been proposed previously in the literature to solve the supersymmetric flavor problem \(^4\). \(^5\) (see also ref. \(^6\)). Our approach, however, will be more general and address new aspects, such as

\(^1\) Our results can be easily generalized to the case \(U(2)_L\times U(2)_R^d\times U(2)_R^u\).
the $G_H$-breaking effects in the SSB terms, the problems with gauging $G_H$, CP violation, third-family contribution to FCNC, and electric dipole moment (EDM) contributions.

In section 2, we discuss the problem of the quark masses and mixing angles from a phenomenological point of view, and look for the possible approximate horizontal symmetries $G_H$ of the SM that allow us to reproduce the observations. Considering in particular the non-Abelian case with the fermions in the $2 + 1$ representation, we present two toy models, one leading to a texture form for the quark mass matrices, the other allowing for more general structures compatible with the symmetry-breaking pattern. In section 3, we present the flavor structure of the scalar mass matrices imposed by the horizontal symmetry breaking. We also discuss the possibility that the first- and second-family squarks are heavier than the other scalars. In section 4, we analyse the phenomenology of the model. We study the contribution to FCNC, EDMs, lepton-number-violating processes, proton decay and the effects of the CP-violating phases in the particle mass spectrum. Section 5 is devoted to conclusions. In the appendix, we show how the SSB terms can be modified when the horizontal symmetry is broken.

## 2 Fermion masses and horizontal symmetries

### 2.1 General considerations

In a purely phenomenological approach, the observed quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix $V_{CKM}$ can be written in the Wolfenstein parametrization as

$$M_d^{\text{diag}} = m_b \begin{pmatrix} d \lambda^4 & 0 & 0 \\ 0 & s \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M_u^{\text{diag}} = m_t \begin{pmatrix} u \lambda_u^4 & 0 & 0 \\ 0 & c \lambda_u^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A \lambda^3(\rho + i \eta) \\ -\lambda & 1 - \lambda^2/2 & A \lambda^2 \\ A \lambda^3(1 - \rho + i \eta) & -A \lambda^2 & 1 \end{pmatrix}, \quad (2)$$

where $d, s, c \approx 1$, $u \lesssim 1$, $\lambda \approx 0.2$ is essentially the Cabibbo angle, $\lambda_u \approx 0.06$, and all the coefficients in eq. (2) are of order 1.

In general, the above structure is not sufficient to determine the form of the down- and up-quark mass matrices $M_d$ and $M_u$ in the basis of the gauge eigenstates. However, the Cabibbo angle turns out to be given with a good approximation by the same parameter $\lambda \approx \sqrt{m_d/m_s}$ that determines the matrix $M_d^{\text{diag}}$, and from $\lambda > \lambda_u$ we can expect that the full CKM matrix is given in a first approximation by the (left-handed) down-quark rotation $U^d_L$, which relates the gauge to the mass eigenstates. This consideration is reflected in eqs. (1) and (2), where the same parameter $\lambda$ has been used to describe the down-quark masses and the CKM mixing matrix.
Neglecting the up-quark rotation, and assuming for the moment that $M_d$ is a symmetric matrix (so that $U_R^d = U_L^d$), we can guess the form of the down-quark mass matrix in the gauge eigenstates basis

$$M_d = U_R^d M_d \text{ diag } U_L^d \simeq V_{\text{CKM}}^* M_d \text{ diag } V_{\text{CKM}}^\dagger \simeq \begin{pmatrix} (d + s)\lambda^4 & s\lambda^3 & A(\rho - in)\lambda^3 \\ s\lambda^3 & s\lambda^2 & A\lambda^2 \\ A(\rho - in)\lambda^3 & A\lambda^2 & 1 \end{pmatrix} m_b, \quad (3)$$

at the lowest order in the expansion parameter $\lambda$. Then, from these semi-phenomenological considerations, we find the following ‘onion’ structure for the mass matrix $M_d$: i) the entry 33 is of order $\lambda^0 = 1$; ii) the 22, 23 and 32 entries appear at order $\lambda^2$; iii) the off-diagonal elements of the first row (and column) are of order $\lambda^3$; iv) the 11 entry arises only at $O(\lambda^4)$. This structure is expected even if $M_d$ is not symmetric, provided that $(U_R^d)_{ij} \sim (U_L^d)_{ij}$. In this case, eq. (3) should be understood as an order-of-magnitude estimate for the entries of $M_d$, with undetermined coefficients of order 1 for each $\lambda^n$ term.

Equation (3) suggests that a spontaneously broken horizontal symmetry, acting on the flavor indices, could be responsible for the generation of the quark masses. Assuming that the parameter $\lambda$ measures the amount of the breaking of the symmetry, we can guess that the subsequent ‘onion’ layers correspond to different breaking steps. We anticipate that symmetry arguments alone cannot explain the orders of magnitude of the ratios of the different layers of eq. (3), and in particular they do not predict the suppression in the 11 entry of the mass matrix $M_d$ since, according to eq. (3), it should be a factor $\lambda$ smaller than the other elements in the first row (and column). These problems can be solved in the class of models that we will discuss, where the different powers in $\lambda$ appearing in eq. (3) will be related to the order of the non-renormalizable operator contributing to the given mass entry. Since we do not necessarily require $M_u$ and $M_d$ to have texture zeros [4], we will generally content ourselves to predict just the dependence of the entries in eq. (3) on $\lambda$. In other words, we will just try to explain the orders of magnitude for the mixing angles, not the precise values. Nevertheless, we will also provide an example where a texture structure is obtained, so that accurate predictions for the CKM mixing matrix can be made, following e.g. ref. [9].

Once the CKM matrix is assumed to be essentially related to the down-quark rotation, the flavor structure of the matrix $M_u$ remains undetermined. However, if the horizontal symmetry acts in the same way on the up- and down-quark sectors, then $M_u$ and $M_d$ can be expected to have similar structures. In GUTs such as SO(10), the matrices $M_u$ and $M_d$ (more precisely, the Yukawa couplings at the GUT scale) are related by Clebsch-Gordan coefficients [10], which should take into account the fact that the scaling factor for the up-quarks $\lambda_u$ is smaller than the factor $\lambda$ determining the down-quark mass ratios. We will not try to explain the large ratio $m_t/m_b$; it could arise from a large $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$ or from a small coupling of the down sector with the light Higgs.

Motivated by the previous phenomenological considerations, our aim here is to classify the approximate horizontal groups $G_H$ leading to eq. (3). In the limit of vanishing Yukawa couplings, the SM has a global flavor symmetry $U(3)$ for each field in the basis of the left-handed fields $Q = (u \ d) \, ^T$, $w^\dagger$, $d^c$, $L = (\nu \ e) \, ^T$ and $e^c$ (with a flavor index understood, and $\psi_{\nu \ e} \equiv \psi_{\nu \ e} \equiv C(\overline{\psi}_{\nu \ e})^T$). This would be the maximal horizontal symmetry allowed for any effective theory, containing no new fields in addition to the known SM (or MSSM) particles. In the following we will assume that all the SM fermions in the basis given above
transform in the same way under \( G_H \). This is a necessary condition if \( G_H \) is assumed to commute with some GUT group such as SO(10). In other words, we will not consider here the alternative that the horizontal symmetry is unified in a non-trivial way within a larger GUT.

We assume that \( G_H \) allows (just) the third family masses, \textit{i.e.} mass matrices of the form

\[
M^0_d = M_d(\lambda = 0) = \text{diag}(0, 0, 1)m_b.
\]

(4)

Since we have assumed that the quark fields \( Q \) and \( d \) transform in the same way, then \( G_H \) is the group of (unitary) matrices \( U \) satisfying the condition

\[
U^T M^0_d U = M^0_d,
\]

(5)

where we have also assumed that the Higgs field responsible for the breaking of the SM gauge symmetry is invariant under \( G_H \). This latter simplifying assumption characterizes our approach, which can be easily generalized to the case that the light Higgs field transforms as a singlet with a non-trivial phase under \( G_H \). The most general solution \( U \in U(3) \) of eq. (5) is a block matrix \( U = \left( \begin{array}{ccc} U_{2 \times 2} & 0_{2 \times 1} \\ 0_{1 \times 2} & \pm 1 \end{array} \right) \), that is

\[
G_H \subseteq U(2) \times Z_2 = U(1) \times SU(2) \times Z_2,
\]

(6)

where the \( U(1) \) and SU(2) factors correspond to global phase and special unitary transformations for the first two families, and the \( Z_2 \) describes a sign ambiguity in the definition of the third family, which is singlet under \( G_H \) (the factor \( Z_2 \) in eq. (6) would be a U(1) if we allowed the light Higgs field to transform with a non-trivial phase under \( G_H \)). We are then left with two possibilities for the first two families: they can belong either to A) two singlets under \( G_H \), so that the three-flavor components quark field belongs to the representation \( 1 + 1 + 1 \), or to B) a doublet, so that the quarks belong to \( 2 + 1 \).

The first case (A) results, for instance, if \( G_H = U(1) \) in eq. (6). It is interesting to notice that this minimal continuous group, or even its \( Z_3 \) subgroup (consisting of discrete phase transformations given by the third roots of the unity), is sufficient to force the form eq. (6) of the mass matrix. In fact, it can be shown that a theory based on \( G_H = U(1)' \times Z_2 \), where the \( U(1)' \) is generated by a suitable combination of the \( U(1) \) and SU(2) generators in eq. (6), can provide mass matrices of the form \( 3 \). A similar result can be obtained using the \( Z_8 \) discrete subgroup of \( U(1)' \). However, this case (A) is disfavored in the supersymmetric version of the theory, since it cannot provide a super-GIM mechanism for the scalar sector.

### 2.2 \( G_H \) with the quarks in the \( 2 + 1 \) representation

In the case (B) mentioned in the previous paragraph, \( G_H \) should be non-Abelian in order to have a two-dimensional irreducible representation. From eq. (6), it should then contain either the SU(2) factor, or one of its (discrete) non-Abelian subgroups having the same doublet representation, such as the double dihedral groups \( Q_{2N} \equiv D^D_{N} \,[12] \), that have \( 4N \)

\footnote{Models based on a U(1) horizontal symmetry can also reproduce texture structures for the mass matrices [11].}
elements. For definiteness, we will consider the cases $Q_{2N}$ with either $N = 3$ or $N = 4$, since they reproduce all the properties of the continuous case for what concerns the fermion masses in the cases we will consider. We remark again that in the supersymmetric version of the theory the case of the discrete symmetry will be selected as phenomenologically preferred, since a gauge horizontal symmetry leads, in general, to unacceptably large FCNC.

It is interesting to notice that the choice $G_H = SU(2)$ (or $G_H = Q_{2N}$) is also sufficient by itself to ensure the form eq. (4) of $M_0^d$, if the quark Yukawa couplings to the SM Higgs field are symmetric in the flavor indices. This is the case in SO(10) models where the light Higgs field belongs to the (symmetric) 10 representation. However, an additional U(1) factor is helpful to generate acceptable mass matrices after the spontaneous horizontal symmetry breaking. The corresponding factor in the discrete group case is $Z_{2N}$, which has $2N$ elements, as the $\simeq Z_{2N}$ Abelian subgroup of $Q_{2N}$, which consists of the diagonal matrices

$$
\begin{pmatrix}
e^{i\pi k/N} & 0 \\
0 & e^{-i\pi k/N}
\end{pmatrix}, \quad (k = 0, \ldots, 2N - 1),
$$

(7)

Let us study first what can be deduced on the quark masses from symmetry arguments alone in models based on the full group of eq. (6), or on its discrete subgroup $Z_{2N} \times Q_{2N}$. In the continuous case, we will consider the symmetry-breaking chain

$$
U(1) \times SU(2) \rightarrow U(1)_1 \rightarrow 1,
$$

(8)

where the $U(1)_1$ acts only on the first flavor. The corresponding symmetry-breaking pattern in the discrete case is

$$
Z_{2N} \times Q_{2N} \rightarrow Z'_{2N} \rightarrow 1,
$$

(9)

where $Z'_{2N}$ corresponds to the discrete phases for the first family (i.e. it is the discrete $Z_{2N}$ subgroup of $U(1)_1$), and can be thought to result from the combinations of the 2N elements of $Z_{2N}$ with the 2N diagonal matrices of $Q_{2N}$, eq. (7). Actually the $Z'_{2N}$ acts as a $Z'_N$ on the doublet quark field, since it is represented by the matrices

$$
\begin{pmatrix}
e^{2i\pi k/3N} & 0 \\
0 & 1
\end{pmatrix}.
$$

The first symmetry-breaking step in eq. (8) [or in eq. (9)] generates non-zero 22, 23 and 32 entries in the mass matrix $M_0^d$, giving rise to the second family masses and the CKM mixing angle $V_{ts}$ (and $V_{cb}$). If the 22, 23 and 32 entries are of the same order of magnitude, then $V_{ts} \sim m_s/m_b$, which is in the experimentally correct range. Since the $U(1)_1$ (or $Z'_{2N}$) symmetry protects the first row and column of the mass matrix, the first family mass and its mixing angles can only be generated after it is broken. In general, the 11 entry is expected to be of the same order as the 12, 13, 21 and 31 ones, in contradiction with the ‘onion’ pattern discussed in the previous section. Nevertheless, if the SU(2) (or $Q_{2N}$) symmetry is broken only by the VEVs of $G_H$-doublet scalar fields, the 11 entry turns out

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3 These discrete groups were also used in ref. [13] to predict texture forms for the quark masses in the framework of non-unified theories.

4 Taking into account the possible additional $Z_2$ factor, the symmetry breaking chain is $U(1) \times SU(2) \times Z_2 \rightarrow U(1)_1 \times Z'_2 \rightarrow Z''_2$, where $Z'_2$ describes a simultaneous change of the second and third families indices, and $Z''_2$ is the global sign ambiguity.
to be proportional to the square of the $U(1)_1$ (or $Z'_{2N}$) breaking parameter and can be conveniently suppressed, as we will see explicitly below.

Let $G_H = U(1) \times SU(2) \times Z_2$ (or $G_H = Z_{2N} \times Q_{2N} \times Z_2$), allowing the Yukawa Lagrangian

$$\mathcal{L}_Y^0 = h_d d_3^c H_d Q_3 + h_u u_3^c H_u Q_3 + h.c.,$$

(10)

where for each term we have understood a Charge Conjugation matrix, and $H_d$ ($H_u$) is the MSSM Higgs field giving mass to the down- (up-) quarks. The quark-mass terms involving the first two families will arise from non-renormalizable operators involving the scalar fields whose VEVs break $G_H$. These contributions are suppressed by inverse powers of the energy scale $\Lambda$, the cutoff of our effective theory, which can be close to the GUT scale $M_G$. We then assume that the first breaking step in eq. (8) [eq. (9)] occurs at some scale below $\Lambda$, due to an SU(2)-doublet scalar field $\Phi$, which is singlet under the SM group (see table 1). Without losing generality, it is always possible to rotate to a basis in which the first component of $\langle \Phi \rangle$ vanishes. Then we can write

$$\langle \Phi \rangle = \Lambda \left( \begin{array}{c} 0 \\ \epsilon \end{array} \right),$$

(11)

where $\epsilon$ is a small adimensional parameter (we are assuming that $i \sigma_2 \Phi$ transforms as the quark $G_H$-doublet); $\langle \Phi \rangle$ preserves the residual $U(1)_1$ (or $Z'_{2N}$) symmetry of eq. (8) [or (1)], acting on the first family, which is generated in this basis by the charge $Q_1 = T_3 + Q/2$ (where $T_3$ and $Q$ are the SU(2) and U(1) generators). Then non-renormalizable terms involving $(\langle \Phi \rangle / \Lambda)^2$ will be responsible for the generation of the 22 entry of the mass matrix at the order $\epsilon^2$, while linear terms in $\langle \Phi \rangle / \Lambda$ will contribute to the 23 and 32 entries. According to the ‘onion’ structure of eq. (3), all these entries should be of the same order of magnitude, so that the 23 and 32 elements should also be suppressed by an extra power in $\epsilon$. This can be guaranteed by the symmetry $Z_2$ if a $U(1) \times SU(2)$ (or $Z_{2N} \times Q_{2N}$) invariant field $\chi$ is introduced, and the transformation rules of table 1 are assumed. The field $\chi$ is supposed to get a $Z_2$-breaking VEV, which can be written in terms of an adimensional parameter $\epsilon_\chi$ as

$$\langle \chi \rangle = \Lambda \epsilon_\chi.$$  

(12)

The $G_H$-symmetric Yukawa Lagrangian, including the lowest-dimension non-renormalizable interactions, can be written as

$$\mathcal{L}_Y = h_d \left( d^c T \right) \left( d_3^c \right)^T \left( \frac{h_4}{\Lambda} \Phi \Phi^T \frac{h_4}{\Lambda} \Phi \chi \right) \left( \begin{array}{c} Q \\ Q_3 \end{array} \right) H_d \{ d \rightarrow u \} + h.c.,$$

(13)

5If the underlying GUT is SO(10), these fields should actually belong to non-trivial representations such as the 45, in order to generate different $M_u$ and $M_d$ matrices.

6The rotation leading to eq. (11) does not belong in general to the discrete group $Q_{2N}$, and we should assume that the $Z'_{2N}$ symmetry is left invariant by $\langle \Phi \rangle$. Actually, in the models discussed below the hierarchy amongst the entries in $M_d$ can be due to an increasing negative power in $\Lambda$ rather than to a hierarchy between the $Q_{2N}$ and the $Z'_{2N}$ symmetry-breaking scales.

7The discrete symmetry $Z_{2N} \times Q_{2N}$ allows for few more invariant terms, such as $d^c T \sigma_1 \sigma_2 \Phi \Phi^T \sigma_2 \sigma_1 Q$ and $d^T \sigma_2 \Phi \Phi^T \sigma_2 Q - d^T \sigma_3 \sigma_2 \Phi \Phi^T \sigma_2 \sigma_3 Q$. It can be seen that the form of the mass matrices below is not changed by such additional invariants, which affect only the actual coefficients of the different entries.
where $h_{\Phi\Phi}^d$, $h_{\Phi\chi}^d$ and $h_{\chi\Phi}^d$ are Yukawa couplings normalized to $h_d$, which can be expected to be of $O(1)$. As mentioned above, in GUTs such as SO(10) it has to be assumed that the difference between the up-quark and down-quark Yukawa couplings is due to different Clebsch-Gordan factors that depend on the GUT. When the electroweak symmetry is broken by $\langle H_d \rangle$, the mass matrix

$$M_d = \begin{pmatrix}
0 & 0 & 0 \\
0 & h_{\Phi\Phi}^d \epsilon^2 & 0 \\
h_{\Phi\chi}^d \epsilon \epsilon_{\chi} & 0 & 1
\end{pmatrix} h_d \langle H_d \rangle$$

is generated, and we will assume that $\epsilon_{\chi} \sim \epsilon$, which is natural if the horizontal SU(2) and $Z_2$ symmetries are broken at a common scale. Now the identification $\epsilon \sim \lambda$ will provide the correct orders of magnitude for the 22, 23 and 32 entries of $M_d$, as in eq. (3).

For the first row and column of the mass matrix to be generated, the U(1)$_1$ (or $Z_{2N}'$) symmetry has to be broken. As shown in table 1, this can be due to either a second SU(2)-doublet $\Phi'$ (case I), or II) an SU(2)-singlet $\chi_1$ (case II).

**Model I:** The addition of the second SU(2)-doublet $\Phi'$, getting the general VEV

$$\langle \Phi' \rangle = \Lambda \begin{pmatrix}
\delta' \\
\epsilon'
\end{pmatrix},$$

Table 1: Horizontal symmetry assignments for the fields in a model based either on the group $U(1) \times SU(2) \times Z_2$, or on the discrete symmetry $Z_{2N} \times Q_{2N} \times Z_2$. The U(1)-generator $Q$ can also be used to define the representation of $Z_{2N}$, except in the case of the field $\chi_1$, which should transform under $Z_8$ as the quark field squared. The label $q_i$ refers to any of the quark fields, $q_i = Q_i$, $d_i$, $c_i$, $u_i$.

| Field | U(1) (or $Z_{2N}$) | SU(2) (or $Q_{2N}$) | $Z_2$ |
|-------|-------------------|-------------------|-----|
| $q \equiv (q_1 \  q_2 \  q_3)$ | 1 | 2 | +1 |
| $\Phi \equiv (\Phi^1 \ \Phi^2)$ | 0 | 1 | -1 |
| I) $\Phi' \equiv (\Phi'_1 \ \Phi'_2)$ | -1 | 2 | -z |
| II) $\chi_1$ | -2/3 | 1 | +1 |
allows us to generate the first family mass and its mixing angles. Omitting coefficients of order 1, the full mass matrix now reads

\[ M_d \simeq \begin{pmatrix} \delta'^2 & \epsilon' \delta' + \epsilon \delta' & \delta' \epsilon'_x \\ \epsilon' \delta' + \epsilon \delta' & \epsilon^2 + \epsilon' \epsilon' & \epsilon \epsilon' \epsilon'_x + \epsilon' \epsilon_x \\ \delta' \epsilon_x & \epsilon \epsilon_x + \epsilon' \epsilon_x & 1 \end{pmatrix} h_d \langle H_d \rangle. \]  

(16)

If the U(1)_1-breaking parameter \( \delta' \) is of the order of \( \epsilon^2 \), eq. (16) reproduces the ‘onion’ pattern of eq. (3). This condition is satisfied naturally if \( \Phi' \) is an effective field given by the product of two fields (in this case we also expect \( \epsilon' \sim \epsilon^2 \)). For instance, we can consider a model containing a doublet \( \Phi'' \) having \( Q = -2 \) (and \( Z_2 \)-parity = \(-z\)), and an SU(2)-singlet \( \chi'' \) (\( Z_2 \)-even) carrying \( Q = 1 \). Then we can define \( \Phi' \equiv \Phi'' \chi'' \), and we have

\[ \delta', \epsilon' \sim \frac{\langle \Phi'' \rangle \langle \chi'' \rangle}{\Lambda^2} \sim \epsilon^2, \]  

(17)

assuming that all the \( G_H \)-breaking VEVs are generated at the same energy scale. Then eq. (16) reproduces the structure of eq. (3) if the common order of magnitude for the adimensional parameters \( \epsilon \) and \( \epsilon' \chi \) is \( \lambda \).

We remark that in this example the mass matrix eq. (16) is not necessarily symmetric in the flavor indices, so that the comparison with eq. (3) cannot be strict. In fact, in SO(10)×\( G_H \) models the fields \( \Phi \) and \( \chi \) can belong to the 45 representation of SO(10). As a consequence, the mass matrix is in general not symmetric, since the product 45 × 10 contains the 120 antisymmetric representation. However, we expect from eq. (16) that in orders of magnitude \( (M_d)_{ij} \sim (M_d)_{ji} \), so that \( U_L^d \sim U_R^d \). The precise coefficients in eq. (16) are related to the ratios of different Yukawa couplings, which are arbitrary numbers expected to be of order 1. For this reason, in model I there is no hope to predict more than the orders of magnitude for the CKM mixing angles as functions of the quark masses.

**Model II:** In this case, the U(1)_1 (or \( Z_{2N}^2 \)) symmetry is broken by the VEV of an SU(2)-singlet field \( \chi_1 \), as given in table 1, which can be written in terms of an adimensional parameter \( \epsilon_1 \) as

\[ \langle \chi_1 \rangle = \Lambda \epsilon_1. \]  

(18)

Then the non-renormalizable Yukawa Lagrangian eq. (13) gets an additional operator involving the antisymmetric SU(2) invariant \( d^T i \sigma_2 Q = d_1^T Q_2 - d_2^T Q_1 \):

\[ \frac{1}{\Lambda^3} d^T i \sigma_2 Q \chi_1^2 H_d, \]  

(19)

[the discrete symmetry that should be considered in this case is \( Z_8 \times Q_8 \) (i.e. \( N = 4 \)) (see table 1)], and the mass matrix eq. (14) becomes

\[ M_d \simeq \begin{pmatrix} 0 & \epsilon_3^3 & 0 \\ -\epsilon_1^3 & \epsilon_2^2 & \epsilon \epsilon'_x \\ 0 & \epsilon \epsilon'_x & 1 \end{pmatrix} h_d \langle H_d \rangle, \]  

(20)

where we understand that the coefficients are of order 1. The texture structure of eq. (20) is similar to those considered in the literature [3]. We will assume that the CKM mixing
matrix and the quark masses can be recovered also in this case, although a complete analysis including also the up-quark masses would be needed. Even without performing the detailed analysis, we see that the correct orders of magnitude can be obtained if \( \epsilon_1 \sim \epsilon \simeq \lambda \), which is the case if the \( \text{U}(1)_1 \) and \( \text{SU}(2) \) symmetries are broken at the same scale. Again, we find from eq. (21) that the left-handed and the right-handed rotations are expected to be of the same order of magnitude.

As we have discussed, the quark mass matrix can be understood as arising from horizontal symmetries in several ways. From a phenomenological point of view, there is no way to distinguish amongst the different models that lead to a satisfactory description of the quark masses. As we will see, this arbitrariness is reduced since the absence of large FCNC, which can be induced by the scalar partners of the quarks, favors a discrete horizontal symmetry with the quark superfields in the \(2+1\) representation.

3 The squark mass matrices

We will consider that the SSB terms are the most general ones \[14\] allowed by the symmetry of the theory and that they are generated at a high scale \( \approx M_P \). If the first and second family superfields, \( Q, D^c \) and \( U^c \), transform as a doublet under a horizontal symmetry \( G_H \), the corresponding squarks are degenerate. Nevertheless, the horizontal symmetry has to be broken to generate the masses of the quarks \( u, c \) and \( d, s \). The \( G_H \)-breaking can affect the SSB parameters and spoil the degeneracy of the squarks of the first two families \[15\]. In the ignorance of the underlying theory above the scale \( \Lambda \), one can proceed as in the fermion sector and write all possible non-renormalizable operators allowed by the symmetry of the theory that can contribute (when \( G_H \) is broken) to the SSB terms. This is explained in detail in the appendix. These operators determine the effective theory below \( \Lambda \). Of course, if we knew the underlying theory above \( \Lambda \), we could know which operators are in fact generated and which are not. For example, if in the theory above \( \Lambda \) the quark superfields do not mix with other superfields, then no operator contributing to the squark soft masses (and involving superfields that break \( G_H \)) can be generated \[15\].

In these theories the super-GIM cancellation is guaranteed and only charged currents can generate flavor-violating processes. The non-renormalizable operators contributing to the fermion masses could be assumed to arise from Higgs mixings.

Here we will adopt a more conservative approach. Based on symmetry grounds, we will parametrize the scalar masses in a model-independent way. We will assume the ‘onion’ flavor symmetry breaking suggested in the previous section for the fermions. This breaking pattern allows us to write the down-scalar masses using an expansion in \( \lambda \) (we neglect the \( \text{SU}(2)_L \times \text{U}(1)_Y \) gauge contributions arising from the D-terms):

\[
M^2_d = \begin{pmatrix}
\text{m}_Q^2 + M_d^\dagger M_d & m_{LR}^2 \\
m_{LR}^2 & m_D^2 + M_d M_d^\dagger
\end{pmatrix},
\]  

(21)
where

\[
\begin{align*}
\mathbf{m}_{Q,D}^2 &\simeq m_{Q,D}^2 \begin{pmatrix} 1 & \lambda^3 & \lambda^3 \\ \lambda^3 & (1 + \lambda^2) & \lambda^2 \\ \lambda^3 & \lambda^2 & m_{Q_3,D_3}^2/m_{Q,D}^2 \end{pmatrix}, \\
\mathbf{m}_{LR}^2 &\simeq A_0 m_b \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 & A_b/A_0 \end{pmatrix} + \mu M_d \tan \beta,
\end{align*}
\] (22, 23)

where \(\mu\) is the Higgs mass in the superpotential. We are only showing the order of magnitude of the entries of the squark masses based on symmetry considerations. In a given model, however, the off-diagonal entries could be smaller than those in eqs. (22) and (23). For example, in our model I (see appendix), the squark mass matrix is of the form (21) but in our model II the 12 and 13 entries of \(\mathbf{m}_{Q,D}^2\) are of order \(\lambda^5\) [see eq. (57)]. As we will see later, however, only the diagonal entries are relevant for phenomenological purposes. We will neglect the scale evolution of the different entries in eq. (21), since this effect is smaller than the uncertainty in the parameters of the model. The soft mass matrix of the \(U^c\)-squark, \(\mathbf{m}_{U}^2\), is arbitrary if the CKM matrix is assumed to come from the down sector. Nevertheless, the off-diagonal entries of \(\mathbf{m}_{D}^2\) cannot be larger than those in eq. (22).

Eq. (22) can be diagonalized by a matrix \(\tilde{V}_{Q,D}\) of order

\[
\tilde{V}_{Q,D} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},
\] (24)

that is of the order of \(V_{\text{CKM}} \sim U_{L,R}^d\). We obtain the \(Q\)-squark soft masses:

\[
\mathbf{m}_Q^2 \simeq \text{diag}(m_{Q_1}^2, m_{Q_2}^2(1 + \lambda^2), m_{Q_3}^2),
\] (25)

and similarly for \(D^c\). The horizontal symmetry only guarantees a degeneracy between the first and second families up to \(O(\lambda^2) \sim 4 \times 10^{-2}\). As we will see in the next section, this is not enough to suppress the supersymmetric contribution to the \(\varepsilon_K\) parameter if the CP-violating phases are of \(O(1)\). One possibility to further suppress this contribution will be to assume that

\[
m_{Q_1}^2 \gg m_{Q_2}^2 \sim m_H^2 \sim m_Z^2,
\] (26)

and equivalently for the \(U^c\) and \(D^c\). Notice that such a splitting is not possible in a scenario with universal soft masses but it is possible here since we are considering general SSB terms.

Eq. (26) arises several questions. First, one can wonder how to generate such a splitting. Second, it is not clear that eq. (26) is stable under radiative corrections. It is known
that supersymmetry stabilizes the scale hierarchies if the supersymmetry-breaking scale $m_S$ is close to the weak scale $m_Z$. If the supersymmetry-breaking masses of the first families are much larger than $m_Z$, one could be worried about whether such a splitting destabilizes the weak scale.

In principle, eq. (24) could be a consequence of the supersymmetry-breaking pattern at $M_P$. A large hierarchy in SSB parameters is possible in string theories [16]. A second possibility is that eq. (26) is generated after integrating out some heavy states. It is shown in refs. [7, 8, 9] that, after integrating out the heavy states, new contributions to the soft masses of the light sparticles can be induced. Some of these contributions (see fig. 4d of ref. [18]) depend on ratios of VEVs and masses of the heavy states, and could be larger than $m_S^2$. It is crucial, in order to preserve the splitting (26) under radiative corrections, to generate it at a scale below $\Lambda$. At this scale the 1st and 2nd families are almost decoupled from the 3rd family and the Higgs, and the small soft masses of the latter can be maintained naturally. Nevertheless, the splitting (26) cannot be very large since the first and second families couple to the third one and to the Higgs through gauge interactions. At the one-loop level, there are D-term contributions to the renormalization group equations (RGEs) of the Higgs $H_{u,d}$ and third family scalars given by

$$\frac{dm_i^2}{dt} = \frac{3Y_i\alpha_1}{10\pi} \left( m_Q^2 - 2m_U^2 + m_D^2 - m_L^2 + m_E^2 \right),$$

where $Y_i$ is the hypercharge of the particle $i$, $Y_i = (1, -1, 1/3, 2/3, -4/3, -1, 2)$ for $i = (H_u, H_d, Q_3, D_3^c, U_3^c, L_3, E_3^c)$ where $L$ and $E^c$ denote the leptons and $t = \ln Q$. However, this contribution cancels if the soft masses respect an SU(5) symmetry, i.e. $m_Q^2 = m_U^2 = m_D^2$ and $m_L^2 = m_E^2$. At the two-loop level, the dominant contributions to the RGEs are given by [20]

$$\frac{dm_i^2}{dt} = \beta_m^{(1)} + \beta_m^{(2)} + \beta_m^{(3)},$$

where

$$\beta_m^{(3)} = \frac{2\alpha_3^2}{3\pi^2} \left( 2m_Q^2 + m_U^2 + m_D^2 \right),$$

for $i = \text{color triplet}$,

$$\beta_m^{(2)} = \frac{3\alpha_2^2}{8\pi^2} \left( 3m_Q^2 + m_L^2 \right),$$

for $i = \text{weak doublet}$, and

$$\beta_m^{(1)} = \frac{3Y_i^2\alpha_1^2}{20\pi^2} \left( m_Q^2 + 3m_L^2 + 8m_U^2 + 2m_D^2 + 6m_E^2 \right)$$

\[+ \frac{3Y_i\alpha_1}{20\pi^2} \left[ \frac{8\alpha_3}{3} \left( m_Q^2 - 2m_U^2 + m_D^2 \right) + \frac{3\alpha_2}{2} \left( m_Q^2 - m_D^2 \right) \right] \]

\[+ \frac{\alpha_1}{30} \left( m_Q^2 - 32m_U^2 + 4m_D^2 - 9m_L^2 + 36m_E^2 \right), \tag{31}\]

for $Y_i \neq 0$. Eq. (29) can induce a large contribution to the third-family-squark mass when the evolution from $M_G$ to $m_Q$ is taken into account:

$$\Delta m_{Q_3,D_3,U_3}^2 \simeq \frac{4}{9\pi} \left[ \alpha_3(M_G) - \alpha_3(m_Q) \right] \left( 2m_Q^2 + m_U^2 + m_D^2 \right). \tag{32}$$
For equal 1st and 2nd family soft masses, eq. (32) only allows for a splitting $m_Q^2/m_Q^2 \simeq 30$. If only $m_D^2$ dominates (32), the splitting can be $m_D^2/m_Q^3 \simeq 120$. Of course, there are other contributions to the RGEs of the third-family squarks coming from the top Yukawa coupling and gluino that will change the estimate eq. (32) —recent analysis can be found also in ref. [21]. Also the splitting (26) could be generated at a scale below $M_G$. As a conservative value, we will allow for a factor 10 splitting between the soft masses of the 1st - 2nd families and the 3rd one.

4 Phenomenological implications

4.1 FCNC processes and EDMs

We will work in the basis in which the squark mass matrices $m_Q^2$, $m_D^2$ and $m_U^2$ are diagonal. We can go to this basis through a superfield rotation $V_{Q,D,U}$ that will redefine the quark mass matrices as

$$
M_d \rightarrow \tilde{V}_D^T M_d \tilde{V}_Q = \tilde{V}_D^T U_R M_d^{\text{diag}} U_L^T \tilde{V}_Q,
$$

$$
M_u \rightarrow \tilde{V}_U^T M_u \tilde{V}_Q = \tilde{V}_U^T U_R M_u^{\text{diag}} U_L^T \tilde{V}_Q,
$$

and similarly for the left–right mixing mass matrices $m_{LR}^2$. Hence all the flavor mixings will arise from the four matrices

$$
V^Q \equiv U_L^d \tilde{V}_Q, \quad \nabla^Q \equiv U_L^u \tilde{V}_Q,
$$

$$
V^D \equiv U_R^d \tilde{V}_D^*, \quad V^U \equiv U_R^u \tilde{V}_U^*,
$$

and $m_{LR}^2$ that is not diagonal in the above basis. The $\lambda$-dependence of the matrices (34) is easily derived from (24) and the assumptions $U_{d,L,R} \sim V_{\text{CKM}}$ and $U_{R} \sim V_{\text{CKM}}(\lambda \rightarrow \lambda_u)$:

$$
V^{Q,D} \sim \nabla^Q \simeq \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix},
$$

and similarly for $V^U$ with $\lambda \rightarrow \lambda_u$. Had we taken the off-diagonal entries of $m_{Q,D}^2$ smaller than those in (32), as in our model II, the matrices $V^{Q,D}$ would have been of the same order of (35), since they would still have been originated from $U_L^d$.

Let us start considering the contributions to FCNC processes and EDMs induced by the 1st and 2nd family squarks. The most stringent constraints on the model come from gluino-mediated contributions to CP-violating observables. The contribution to the CP-violating $\varepsilon_K$ parameter of the $K^{-}\bar{K}$ system is dominated by diagrams involving $Q$ and $D^c$ squarks in the same loop [22, 23]. If $m_Q^2 \simeq m_D^2 \gg m_{\tilde{g}}$, where $m_{\tilde{g}}$ is the gluino mass, we have [23]:

$$
\varepsilon_K \simeq \frac{\alpha_3^2}{54\sqrt{2}} \frac{f_K^2 m_K}{m_Q^2 \Delta m_K} \left[ 1 + \frac{2}{3} \left( \frac{m_K}{m_s + m_d} \right)^2 \right] \text{Im} \left\{ \frac{V_{11}^Q \delta m_Q^2 V_{21}^Q V_{11}^D \delta m_D^2 V_{21}^D}{m_Q^2 m_D^2} \right\},
$$

(36)
where $\delta m_{Q,D}^2$ is the mass-squared difference between the first and second families of squarks. Note that we have CP violation even if only two families are considered \[24\]. Using the experimental value $\varepsilon_K \simeq 2.3 \times 10^{-3}$, we get the constraint

$$\left( \frac{1 \text{ TeV}}{m_Q} \right)^2 \left| V_{12}^Q \frac{\delta m_{Q}^2}{m_Q^2} V_{12}^D \frac{\delta m_{D}^2}{m_D^2} \right| \sin \varphi < 1.2 \times 10^{-6}, \quad (37)$$

where $\varphi = \text{Arg}(V_{12}^Q V_{12}^Q V_{12}^D V_{12}^D)$. From eqs. (25) and (35), we obtain

$$\left| V_{12}^Q \delta m_{Q,D}^2 V_{12}^D \right| \simeq \lambda^3 \sim 8 \times 10^{-3} \Rightarrow m_{Q,D} > 5 \text{ TeV} \sqrt{\frac{\sin \varphi}{1/2}}. \quad (38)$$

We then see that an approximate horizontal symmetry under which the first two families transform as a doublet cannot guarantee a small contribution to $\varepsilon_K$. One is forced to have large squark masses or small CP-violating phases. If the horizontal symmetry is gauged, extra gauge contributions to the squark masses arise when the symmetry is broken:

$$\Delta m_{Q,D}^2 \simeq T_A m_A^2, \quad (39)$$

where $T_A$ are the generators of $G_H$ and $m_A^2$ are SSB parameters of the order of the soft masses of the scalars that break $T_A$. Such contributions induce a $\delta m_{Q,D}^2 \sim m_A^2$ that for $m_A^2 \sim m_{Q,D}^2$ spoil the super-GIM cancellation. Hence a gauged $G_H$ will only be allowed if $m_{Q,D}^2$ are much larger than the other soft masses.

Contributions to the neutron EDM can also be important, in particular due to one-loop gluino diagrams. The induced down-EDM for $m_Q^2 \sim m_D^2 \gg m_{\tilde{g}}^2$ is given by \[25\]

$$d_d = \frac{\alpha_3}{9 \pi m_Q^4} \text{Im}\{m_{\tilde{g}}^*(V^D m_{\text{LR}}^2 V^Q)_{11}\}. \quad (40)$$

If the phase in $m_{\tilde{g}}$ or in the entry $(V^D m_{\text{LR}}^2 V^Q)_{11}$ is of order 1, eq. (10) gives a too large contribution to the neutron EDM $|d_n| \simeq (4d_d - d_u)/3$ and $|d_u|_{\text{exp}} < 10^{-25} \text{e cm}$, unless the squark masses are above the TeV-scale. Notice that the phase in $(V^D m_{\text{LR}}^2 V^Q)_{11}$ is related to those in $V^{Q,D}$ and then has the same origin that the CKM phase, which is known to be of $O(1)$.

The above constraints suggest the following possible scenarios:\[9\]

(a) The squark masses of the first and second families are larger than the other soft masses\[10\]: $m_i^2 \sim 10^2 m_s^2$. As we already mentioned, this does not lead to a naturalness problem, since these two families are almost decoupled from the Higgs. This scenario allows for phases in the SSB terms of $O(1)$ and for gauged horizontal symmetries.

(b) All the CP-violating phases are small, $\varphi \sim 10^{-2}$ (we now denote by $\varphi$ a generic CP-violating phase). This is technically natural, since the phases are

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\[9\]Scenarios similar to (a) and (b) were also suggested in ref. [3].

\[10\] Actually, eq. (17) and the down-EDM eq. (40) can be suppressed by only increasing $m_Q^2$ or $m_D^2$. This suggests an alternative scenario where only $m_D^2$ is large (in an SU(5) GUT also $m_L^2$ would be large).
protected by an extra symmetry, \textit{i.e.} the CP-symmetry. This possibility can arise if CP is broken spontaneously. In this case, small complex parameters could be generated in the same way that one generates the small Yukawa couplings (see section 2).

(c) The phases in \((V_{11}^Q V_{21}^{Q*L})\) and \(m_3^2 V_L^D m_L^{2*} V_R^{Q*}\) can be rotated away (\textit{i.e.} they are not physical). As we will see, the first condition is satisfied in models with special texture zeros in the fermion mass matrices such as our model II. The second condition is fulfilled if supersymmetry is assumed to be broken by a hidden sector such that the SSB terms are real.

In scenario (a), contributions to FCNC processes and EDMs (of the first two families) can arise from one-loop diagrams involving the third-family squarks \([2, 27]\). Even that they can be close to the experimental values. Stop-loops can generate a large up-EDM:

\[
d_u = \frac{2 e \alpha_3}{9 \pi m_{\tilde{g}}} |V_{13}^{Q*} V_{13}^{Q}| \sin \varphi \sin 2 \theta_t [x_1 I(x_1) - x_2 I(x_2)],
\]

where \(x_i = m_3^2/m_{\tilde{t}_i}^2\) and \(I(x) = [(1+x)/2 + x \ln x/(1-x)]/(1-x)^2\); the angle \(\sin \theta_t\) defines the left–right stop mixing, \textit{i.e.} \(\tilde{t}_1 = - \sin \theta_t \tilde{t}_R + \cos \theta_t \tilde{t}_L\). Taking \(V_{13}^{U} \sim \lambda_u^3 \sim 2 \times 10^{-4}\), and \((\sin \varphi \sin 2 \theta_t) \sim 0.25\), the contribution \([23]\) to the neutron EDM is close to the experimental value for \(m_{\tilde{t}_2} \gg m_{\tilde{g}} \sim 100 \text{ GeV} \gg m_{\tilde{t}_1}\).

Larger effects arise from the bottom sector. Constraints on the sbottom masses can be obtained from \(\varepsilon_K\). Taking \(m_{Q,3}^2 \simeq m_{D,3}^2\), we obtain \([23]\):

\[
\left(\frac{1 \text{ TeV}}{m_{Q,3}}\right)^2 |V_{13}^{Q} V_{23}^{Q} V_{13}^{D} V_{23}^{D}| \sin \varphi < 10^{-7} \times \begin{cases} 4 & \text{for } m_{Q,3}^2 \gg m_{\tilde{g}}^2 \\ 1.6 & \text{for } m_{Q,3}^2 \simeq m_{\tilde{g}}^2 \end{cases},
\]

which implies (since \(V_{13}^{Q*} V_{23}^{Q*} \sim \lambda^5\))

\[
m_{Q,3, D,3} > \sqrt{\frac{\sin \varphi}{1/2}} \times \begin{cases} 350 \text{ GeV} & \text{for } m_{Q,3}^2 \gg m_{\tilde{g}}^2 \\ 550 \text{ GeV} & \text{for } m_{Q,3}^2 \simeq m_{\tilde{g}}^2 \end{cases}.
\]

The contribution to the down-EDM is given by (for \(m_{Q,3}^2 \simeq m_{D,3}^2\))

\[
d_d = \frac{e \alpha_3 m_b}{9 \pi m_{Q,3}^4} |V_{13}^{Q*} V_{13}^{Q}| \sin \varphi m_{\tilde{g}} (A_b + \mu \tan \beta) \times \begin{cases} 1 & \text{for } m_{Q,3}^2 \gg m_{\tilde{g}}^2 \\ 1/6 & \text{for } m_{Q,3}^2 \simeq m_{\tilde{g}}^2 \end{cases},
\]

which can put stronger constraints if \(\tan \beta\) is large. Notice that the constraints eqs. \([12]\) and \([14]\) can be relaxed if either \(m_{Q,3}\) or \(m_{D,3}\) are increased. Since \(D_3^2\) is decoupled from \(H_u\) at tree level, \(m_{D,3}\) could be larger than \(m_{H_u} \sim m_Z\) without an extreme fine-tuning. Constraints on only \(m_{Q,3}\) can also be obtained from \(\varepsilon_K\) (or \(\Delta m_K\)) but they are about an order of magnitude smaller than eq. \([14]\) \([23]\).

Contributions to \(\Delta m_{B_d}\) and \(b \rightarrow s \gamma\) can also be important. They are, however, always suppressed by a mixing factor of order \(\lambda^3 \sim 8 \times 10^{-3}\) and \(\lambda^2 \sim 4 \times 10^{-2}\) respectively. Several diagrams contribute to these processes (involving chargino–stop, gluino–sbottom,
Higgs–top and W–top loops). They give similar contributions with an arbitrary relative sign \[20\], rendering it difficult to get predictions for these processes.

In scenario (b), the 3\(^{rd}\) family contributions to \(\varepsilon_k\) and EDMs are very small since they are suppressed by the small CP-violating phases. In fact, also the usual SM contribution to \(\varepsilon_k\) arising from a box diagram involving a \(W\) is small; \(\varepsilon_k\) is dominated by diagrams involving first- and second-family squarks and gluinos [eq. (30)], and we have to rely on these contributions to explain the experimental value of \(\varepsilon_k\). Even if \(\varphi\) is small, constraints on the squark masses could arise from \(\Delta m_K\), but they are around an order of magnitude smaller than those from \(\varepsilon_k\) \[23\].

Let us finally consider scenario (c). The phases of the \(ij\) entries \((i, j = 1, 2)\) of \(V^Q\) and \(V^D\) can be rotated away in models where (in the basis where \(m^2_{Q,D}\) are diagonal) the 11 entry of \(M_d\) is zero. This is because in a two-family model one has the freedom to redefine the phases in the down quarks such that only the 11 entry of \(M_d\) is complex. An example where \((M_d)_{11} = 0\) is our model II [see eq. (21)]. The condition \(\text{Im}\{m^*_{\tilde{g}}(V^D m^2_{\tilde{L}R} V^Q)_{11}\} = 0\) requires, first, that supersymmetry is broken by a hidden sector. In this case (see appendix), \(m^2_{\tilde{L}R} \simeq (A_0 + \mu \tan \beta) M_d\), which implies \(\text{Im}\{m^*_{\tilde{g}}(V^D m^2_{\tilde{L}R} V^Q)_{11}\} \simeq m_d \text{Im}\{m^*_{\tilde{g}}(A_0 + \mu \tan \beta)\}\). If the SSB parameters are real (as suggested in certain theories \[23\]), the contributions to the EDMs vanish. Of course, this does not mean that all the physical CP-violating phases are zero. When the third family is considered, there are phases that cannot be rotated away such as the CKM phase.

### 4.2 \(\mu \to e\gamma\) and proton decay in GUTs

In GUTs such as SU(5) or SO(10), the lepton Yukawa matrix is related to the quark Yukawa matrix. If our model is embedded in such GUTs, the lepton and sleptons mass matrices will be of the form of eqs. (9) and (27), respectively, but with different Clebsch factors \[11\] acting on the different entries. Such Clebsch factors depend on the GUT and they are usually a number between 0.1–10. The lepton number is violated and flavor-violating processes such as \(\mu \to e\gamma\) can be induced. In scenario (a), only the contributions from the stau are relevant \[27\]. From the experimental value, \(\text{BR}(\mu \to e\gamma)|_{\text{exp}} < 4.9 \times 10^{-11}\), one has, taking \(m^2_{L_3} \simeq m^2_{E_3}\),

\[
\left(\frac{100 \text{ GeV}}{m_{L_3}}\right)^2 \left[\frac{m_B m_\tau (A_\tau + \mu \tan \beta)}{m^2_{L_3} m_\mu}\right] V^E_{13} V^E_{23} < 10^{-3} \times \begin{cases} 1/2 & \text{for } m^2_{L_3} \gg m^2_B, \\ 3 & \text{for } m^2_{L_3} \simeq m^2_B, \end{cases}
\]

where \(V^L, V^E\) define the rotations that diagonalize the lepton mass matrix \(M_l\) (in the basis where \(m^2_{E,\mu}\) are diagonal) and \(m_B\) is the mass of the bino that is assumed to be a mass-eigenstate. Taking \(V^L, V^E \sim V^Q\), eq. (13) can be satisfied for \(m_{L_3} \sim m_B \sim (A_\tau + \mu \tan \beta) \sim 150\) GeV. The above constraint comes from the one-loop diagram involving a left–right stau mixing and again can be loosened if either \(m_{L_3}\) or \(m_{E_3}\) is large. Bounds on only \(m_{L_3}\) or \(m_{E_3}\) are about an order of magnitude weaker. In scenarios (b) and (c) there are also contributions to \(\mu \to e\gamma\) from diagrams involving the first- and second-family squarks. Particularly dangerous contributions are those involving a left–right mixing. They lead to the constraint (for \(m^2_{L} \simeq m^2_{E}\))

\[
\left(\frac{100 \text{ GeV}}{m_L}\right)^2 \left[\frac{m_B \left(V^E m^2_{L,R} V^{L\dagger}\right)_{12}}{m^2_{L} m_\mu}\right] < 10^{-3} \times \begin{cases} 1/2 & \text{for } m^2_{L} \gg m^2_B, \\ 3 & \text{for } m^2_{L} \simeq m^2_B, \end{cases}
\]

15
where $m^2_{LR}$ now denotes the left–right mixing in the selectron and smuon sector. For general SSB terms (see appendix), $m^2_{LR}$ is not proportional to $M_l$ and then $V_E^* m^2_{LR} V^L_\dagger$ is non-diagonal. The 12 entry of the latter is expected to be $\sim A_0 \lambda^3 m_\tau$, leading to the bound

$$m_L > 700 \text{ GeV for } m_L \simeq A_0 \simeq m_\tilde{B}. \quad (47)$$

If supersymmetry is assumed to be broken by a hidden sector, one has for a two-family model (see appendix) that $V_E^* m^2_{LR} V^L_\dagger \simeq M^{\text{diag}}_l (A_0 + \mu \tan \beta)$ and the 12 entry is zero up to $O(\lambda^5)$. In this case, the constraint (46) can be easily accommodated.

It should be kept in mind, however, that the above bounds present a large uncertainty due to the unknown Clebsch factors that relate $V_{L,E}$ with $V_{Q,D}$.

Another feature in SUSY GUTs is the proton decay that arises from dimension-five operators induced by colored Higgsinos. The proton decay rate depends not only on the coupling of the colored Higgsinos to the quarks and leptons (which depends on Clebsch factors) but on the squark and slepton masses as well [29]. For universal soft masses, the dominant proton decay modes are $p \rightarrow K^+ \nu_e \mu$ and $p \rightarrow \pi^+ \nu_e \mu$; they arise from loop diagrams involving the scalars of first two families and the $\tilde{W}$. In scenario (a) such diagrams are suppressed since $m^2_i \gg m^2_{\tilde{W}}$ for $i = 1^{\text{st}} - 2^{\text{nd}}$ family scalars. Nevertheless, diagrams involving only the third-family sparticles can also contribute to proton decay. The dominant mode is now $p \rightarrow K^+ \nu_\tau$. It is important to notice that, unlike the case with universal soft masses, in scenario (a) either the gluino or the wino exchange contribute to this decay mode.

Gluino exchange diagrams can also contribute to $p \rightarrow K^0 \mu$ if there is a large splitting between the squark masses. This decay could then be used to probe scenario (a). Note, however, that $p \rightarrow K^0 \mu$ needs an extra mixing factor between second and third family that is not necessary in the decay $p \rightarrow K^+ \nu_\tau$.

4.3 CP-violating phases in the SSB terms

In scenario (a), the SSB parameters (and the Yukawas) are allowed to have phases of $O(1)$ giving rise to a phenomenology richer than that in the MSSM with real SSB parameters. Charginos, neutralinos and stops can have complex mass matrices and their interactions will violate CP. It is interesting to analyse the effects on the upper bound on the lightest-Higgs mass. If phases in the SSB terms are allowed, the parameters of the Higgs potential can be complex. It is possible, however, to rotate the phases away from the tree-level potential. CP-violating phases will be present in the stop left–right mixing

$$m^2_{LR} \equiv m_t \left( A_t e^{i\phi_t} + \mu e^{i\phi_\mu} \cot \beta \right), \quad (48)$$

that, at the one-loop level, will affect the Higgs mass. It can be shown [12] that the upper bound on the mass of the lightest Higgs, written as a function of $|m^2_{LR}|$, is not modified by the phases in eq. (48). Only the relative phase between $\mu$ and $A_t$ can affect $|m^2_{LR}|$, and consequently modify the Higgs mass bound.

11 Contributions to $p \rightarrow K^+ \nu_\tau(\nu_\mu)$ are extra suppressed by the small mixing of the first (second) with the third family.

12 We thank H. Haber for his collaboration on this point.
There are also important effects of the CP-violating phases on the dark-matter density of the lightest supersymmetric particle (LSP). It has been shown in ref. [31] that when phases are included in the sparticle mass matrices, the limits on the LSP mass can be relaxed by a factor 2–3.

In general, CP-violating phases change the relation between masses and interactions of the sparticles and then modify the experimental bounds from direct detection.

In scenario (b) all the CP-violating phases are small, \( \varphi \sim \mathcal{O}(10^{-2}) \), and the above effects are not present. The best place to probe this scenario will be in future B-factories. Since the CP-asymmetries in B decays are proportional to the CP-violating phases in the CKM matrix, one expects much smaller asymmetries in these models than in models with \( \varphi \sim \mathcal{O}(1) \).

5 Conclusions

We have discussed how the observed quark masses and CKM mixings point to a definite flavor structure for the down-quark mass matrix in the gauge eigenstates, that can be described in terms of an approximate horizontal symmetry \( G_H \) for the SM. Assuming that \( G_H \) is realized at some superheavy scale and commutes with some GUT group such as SO(10), and that it leaves invariant the light Higgs field, we have shown that it should be a subgroup (either continuous or discrete) of \( U(2) \times Z_2 \), characterized by the symmetry-breaking pattern \( U(2) \rightarrow U(1)_1 \rightarrow 1 \). This class of non-Abelian groups has a two-dimensional representation, so that the quark families can belong to the \( 2 + 1 \) representation; this choice is favored in the supersymmetric version of the theory since it allows for a super-GIM mechanism. We have provided explicitly two models based on \( U(2) \times Z_2 \) (or on one of its discrete subgroups). Model I predicts the correct orders of magnitude for the entries of the mass matrix, leading to the most general structure consistent with the observations; in model II, on the contrary, texture zeros are generated so that a more predictive form of the mass matrix is generated.

In the scalar sector, the horizontal symmetry forces the squarks of the first two families to be degenerate. Nevertheless, the breaking of \( G_H \) spoils such degeneracy unless the quark superfields do not mix with other heavy superfields [15]. The breaking of the degeneracy in the down sector is expected to be

\[
\frac{\delta m_Q^2}{m_Q^2} \sim \frac{m_s}{m_b},
\]

which is sufficient to suppress the squark contributions to FCNC processes except for those to \( \varepsilon_K \). We have proposed three possibilities that lead to a \( \varepsilon_K \) in agreement with the data: (a) the first- and second-family scalars are heavier (\( \sim \) few TeV) than the other scalars (\( \sim 100 \) GeV); (b) all CP-violating phases are small, \( \varphi \sim \mathcal{O}(10^{-2}) \). (Both possibilities provide also a solution to the EDM problem [23] in the MSSM.) A third possibility (c) is to have a down-quark mass matrix with specific texture zeros such that the dominant contribution to \( \varepsilon_K \) vanishes. We have shown an example where this scenario is realized.

The phenomenology of these scenarios is very different from that in the MSSM with universal soft masses. In scenario (a), we have shown that the third-family squarks can give important contributions to FCNC processes and EDMs. For sbottom masses around
300 GeV, these processes are predicted to be around the experimental values. Scenario (a) also leads to a very different phenomenology in the context of GUTs. The proton decay and $\mu \rightarrow e\gamma$ are induced by the third-family scalars and then they depend on the mixing angles of the first and second family with the third one. The proton decay mode $p \rightarrow K^0\mu$ is enhanced (compared to the case with universal soft masses) and could test this scenario.

Scenario (a) allows for phases of order 1 in the soft terms. These phases can have implications in the Higgs mass, dark-matter density and $B$ decays.

The phenomenology of scenarios (b) and (c) is more similar to that in the MSSM with universal soft masses. These scenarios could be distinguished, however, in the future $B$-factories where the CP-violating phases in the $B$-decays will be measured.

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Appendix. \( G_H \)-breaking effects in the SSB terms

To analyse how the \( G_H \)-breaking affects the SSB terms, it is convenient to work within the superfield formalism. Softly broken supersymmetric theories can be formulated in the superfield formalism by using a spurion external field, \( \eta \). Supersymmetry is broken by giving to this superfield a \( \theta \)-dependent value, \( \eta \equiv m_S \theta^2 \). We will follow ref. [18] but extending that analysis to include non-renormalizable operators. The SSB terms for the quark superfield \( Q = (Q, Q_3) \) can be written by

\[
L_{\text{soft}} = \int d^4 \theta Q^\dagger \left[ \bar{\eta} \Gamma_Q(\phi) + \eta \Gamma_Q(\phi) - \bar{\eta} \eta \{ Z_Q(\phi) - \Gamma_Q(\phi) \} \right] Q - \left( \int d^2 \theta \eta W'(Q, \phi) + \text{h.c.} \right),
\]

and similarly for the other superfields; \( \Gamma_Q(\phi) \) and \( Z_Q(\phi) \) are matrix operators made of the superfields \( \phi \) that break \( G_H \). When the \( \phi \) get VEVs, such matrices contribute to the SSB parameters of \( Q \). Explicitly, the contribution to the trilinears and soft masses can be obtained by writing eq. (50) in component fields [and after replacing \( \eta(\bar{\eta}) \) by \( m_S \theta^2(\bar{\theta}^2) \)]:

\[
A_{D_i Q_j H_d} = m_S \left[ Y'_{ij H_d} + Y_{ij H_d} \langle \Gamma_D \rangle_{ii} + Y_{ij H_d} \langle \Gamma_Q \rangle_{ij} + Y_{ij H_d} \langle \Gamma_{H_d} \rangle \right],
\]

\[
m_Q^2 = m_S^2 \langle Z_Q \rangle,
\]

where

\[
Y'_{ij H_d} \equiv \frac{\partial W_{\text{eff}}}{\partial D_i^c \partial Q_j \partial H_d},
\]

\( W_{\text{eff}} \) being the low-energy effective superpotential; similarly for \( Y'_{ij H_d} \), but replacing \( W_{\text{eff}} \) by \( W'(Q, \langle \phi \rangle) \). The \( W' \) is a general holomorphic function of the superfields that is in principle different from the superpotential \( W \). However, in theories where supersymmetry is broken by a hidden sector that does not couple to the observable sector in the superpotential, one has that (\( W \)-proportionality condition [18])

\[
W' = aW,
\]

where \( a \) is a constant. Eq. (53) is not modified either by higher-order corrections (by the non-renormalization theorem) or by integrating out heavy superfields [18].

To analyse the contributions to the soft masses, we write all non-renormalizable operators \( Q^\dagger Z(\phi) \eta \bar{\eta} Q \) allowed by \( G_H \) and the MSSM group in the effective theory below \( \Lambda \). We are assuming that \( G_H \) is not gauged —otherwise there are additional contributions, eq. (39). The \( G_H \)-invariant renormalizable operator is given by (hereafter we will omit the \( \eta \) superfield)

\[
Q^\dagger \left( \begin{array}{cc} 1 & 1 \\ 1 & b \end{array} \right) Q,
\]

which leads to equal soft masses for the first two families. Additional non-renormalizable operators involving \( \Phi \) and \( \chi \) (see table 1) are

\[
Q^\dagger \left( \begin{array}{ccc} \Phi^* \Phi^T + i \sigma_2 \Phi \Phi^T i \sigma_2 & \Phi^* \chi \\ \chi^\dagger \Phi^T & 0 \end{array} \right) Q,
\]

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where for each term we understand a coefficient divided by the scale $\Lambda^2$, and we omit terms proportional to the operators of eq. (54). The operators in eqs. (54) and (55) are common to the both models I and II. The specific operators to each model are given below.

**Model I**: The addition of the field $\Phi'$ gives rise to operators similar to those in eq. (55) since $\Phi$ and $\Phi'$ belong to the same representation of $G_H$. When the fields $\Phi$, $\chi$ and $\Phi'$ get VEVs [eqs. (11), (12) and (15)], soft mass matrices of the form of eq. (22) are generated if we identify $\epsilon \sim \lambda$.

**Model II**: The field $\chi_1$ gives rise to the following invariants

$$Q^i \left( i\sigma_2 \Phi \Phi^T (\chi^3)^\dagger - \chi^3 \chi^i \Phi \right) \sigma_2 \right) Q.$$  

Then, from eqs. (11), (12) and (18), and the identification $\epsilon \sim \lambda$, we find

$$m^2_Q \simeq m^2_Q \begin{pmatrix} 1 & \lambda^5 & \lambda^5 \\ \lambda^5 & (1 + \lambda^2) & \lambda^2 \\ \lambda^5 & \lambda^2 & m^2_{Q3} / m^2_{Q1} \end{pmatrix},$$

where we have absorbed in the parameter $m^2_Q$ a $O(\lambda^2)$-contribution to the 11 entry.

The contributions to the trilinear SSB terms are more subtle. If the $W$-proportionality condition (53) does not hold, the effective terms induced in $W$ will also be induced in $W'$, but with different coefficients, i.e. $Y_{ijH_d} \sim Y'_{ijH_d}$. Then the $\lambda$-dependence of $(m^2_{LR})_{ij} = A_{D_iQ_jH_d} \langle H_d \rangle + \mu (M_d)_{ij} \tan \beta$ will be the same as that of $(M_d)_{ij} = Y_{ijH_d} \langle H_d \rangle$. Nevertheless, since they are not exactly equal, they cannot be diagonalized and made real by the same rotation, and then flavor-violating processes and EDMs can be induced.

If the condition (53) holds, one has from eq. (51)

$$m^2_{LR} = m_S [a M_d + (\Gamma^T_D) M_d + M_d \langle \Gamma_Q \rangle + M_d \langle \Gamma_{H_d} \rangle] + \mu M_d \tan \beta,$$

and the breaking of the proportionality $m^2_{LR} \propto M_d$ only arises from the operators $Q^i \eta \Gamma_Q Q$ and $D^c \eta \Gamma_D D^c$ that are of the same type as those in eqs. (54)–(56). Hence, if only two families are considered, one has in model I that $\langle \Gamma_{Q,D} \rangle = (\Gamma^\text{diag}_{Q,D}) + O(\lambda^2)$ and

$$m^2_{LR} \simeq (A_0 + \mu \tan \beta) M_d + m_S m_b \begin{pmatrix} \lambda^6 & \lambda^5 \\ \lambda^5 & \lambda^4 \end{pmatrix},$$

where $A_0 = m_S [a + \langle H_d \rangle + \langle \Gamma^\text{diag}_{Q} \rangle + \langle \Gamma^\text{diag}_{D} \rangle]$. 

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