Focusing Highly Squinted SAR Data of a Short-Range Wide-Swath Region Using Azimuth-Dependent High-Order RCMC and Frequency Extended Nonlinear Chirp Scaling Based on Equidistant Sphere Model

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This work was supported in part by the Science Foundation of ATR Key Laboratory under Grant JKWATR210202, in part by the National Natural Science Foundation of China under Grant 61301248, in part by the Zhejiang Province Science and Technology Plan Project under Grant LGG18F010009, in part by the China Postdoctoral Science Foundation under Grant 2018M630589, and in part by the Science and Technology on Sonar Laboratory under Grant 6142109KF201807.

ABSTRACT Focusing highly squinted synthetic aperture radar (SAR) data of a short-range wide-swath (SRWS) region is a challenging task because the reference range linear range cell migration correction (LRCMC) and the inherent range-dependent squint angle produce two-dimensional (2-D) spatial-variant RCM and azimuth-dependent Doppler parameters. In this paper, a two-step imaging algorithm (strategy) based on a novel equidistant sphere model (ESM) is proposed to accommodate these issues. The ESM, which is used for depicting the spatial-variant property of the echo data, is established by an investigation into the property of the origin range after the keystone transform (KT) operation and the reference range RCMC. Based on the ESM, an azimuth-dependent high-order RCMC (ADH-RCMC) method is adopted in the first step to implement the residual azimuth-dependent RCMC. In the second step, a frequency extended nonlinear chirp scaling (FENLCS) algorithm is introduced to achieve the highly varying residual Doppler centroid correction and the azimuth-dependent high-order Doppler parameter equalization. Simulation results in the case of SRWS SAR demonstrate the superior performance of the proposed algorithm.

INDEX TERMS Synthetic aperture radar (SAR), short-range wide-swath (SRWS), equidistant sphere model (ESM), azimuth-dependent, frequency extended nonlinear chirp scaling (FENLCS).

I. INTRODUCTION Synthetic aperture radar (SAR) [1], [2] has been widely adopted on many types of platforms because it can produce high-resolution images of desired scenes at any time of day and night regardless of weather conditions [3], [4]. As one of the most important imaging modes, squinted SAR provides more flexibility for observation in the region of interest [5], [6]. However, focusing highly squinted SAR data of a short-range wide-swath (SRWS) region is a challenging task due to the spatial-variant properties of the echo data, which must be taken into consideration in designing an image formation approach in order to produce fine-focused images.

When the highly squinted SAR is illuminating a SRWS region, the squint angle exhibits an inherent range-dependent property, which brings two issues into the imagery. First,
due to the varying squint angle along the range direction, all
the range cell migration (RCM) is totally range-dependent,
which is unable to be corrected with an unified RCM cor-
rection (RCMC) function and further causes echo energy
spread across several range cells. Another issue faced by
the SAR of the SRWS region is the linear RCM (LRCM),
which dominates all the RCM and accounts for the Bulk
of the range-azimuth coupling. Generally, a reference range
LRCM correction (LRMC) function is adopted to mitigate
the coupling, and the residual LRCM is often overlooked in
the conventional squinted SAR data focusing for a relatively
long-range region where the squint angle and LRCM are
spatial-invariant [7]. However, when it comes to the SRWS
scene, the residual LRCM induces the azimuth-variance of
the RCM and the Doppler parameters. In conclusion, the
two-dimensional (2-D) spatial-variant RCM and azimuth-
dependent Doppler parameters including residual Doppler
centroid and high-order Doppler parameters, make the image
formation for SRWS region a challenging task. Hence,
exploring the echo property of the SRWS scene to meet the
demand for precise imaging is a worthwhile study.

Up to now, many types of approaches have been proposed
to improve the focusing quality of highly squinted SAR
images, for example, the back projection (BP) algorithm [8],
[9], the Omega-K algorithm [10], [11], the nonlinear chirp
scaling (NLCS) algorithm [6], [7], [12], [13] and its extended
version [14], [15], [16], [17], [18], [19], [20], [21]. Although
all of the aforementioned algorithms greatly improve the
quality of squinted SAR images, they still have some limita-
tions in dealing with the two issues in the field of SRWS SAR.
Theoretically, although the BP algorithm can be applied to
focus the highly squinted SAR data of the SRWS region with
the 2-D spatial-variant correlator, its application is limited
by the high complexity and low efficiency. The Omega-K
algorithm processes the SAR data in the 2-D wavenumber
domain without any approximation. However, it can hardly
handle the spatially variant squint angle, and it is inefficient
due to the Stolt interpolation operation [8], [9]. In [6] and [7],
the NLCS algorithm is proposed by Wong and Yeo in the case
of the stripmap mode for monostatic or bistatic SAR, and
an improved NLCS with a modified chirp scaling in range
was proposed to correct the 2-D spatial-variant RCM and
azimuth-dependent Doppler parameters caused by the range-
dependent squint angle in [20]. However, this publication
utilizes a linear fitting method to acquire the expression of
range-dependent squint angle, which could lead to a large
approximation error when it is directly applied in the case
of imaging for an SRWS scene. Reference [21] elaborates
the removal of the highly varying residual Doppler centroid
by proposing a frequency extended nonlinear chirp scaling
(FENLCS) while there is no description of how the 2-D
spatial-variant RCM is addressed.

In this paper, a two-step imaging algorithm (strategy) based
on a novel equidistant sphere model (ESM) is proposed to
focus the highly squinted SAR data of the SRWS region.
The ESM, which is used for depicting the spatial-variant

property of the echo data, is established by an investiga-
tion into the property of the origin range after the key-
stone transform (KT) operation and the reference range
RCMC (i.e., reference LRCMC and Bulk RCMC). Based on
the ESM, an azimuth-dependent high-order RCMC (ADH-
RCMC) method is adopted in the first step to implement
the residual azimuth-dependent RCMC. In the second step,
a FENLCS algorithm is introduced to achieve the highly
varying residual Doppler centroid correction and the azimuth-
dependent high-order Doppler parameter equal-izat-
the elevation angle of \( P_0 \), respectively. \( r_{c0} = QP_0 \) represents the reference slant range at the beam center, and \( r_{g0} = OP_0 \) denotes the reference ground-range which is the projection of \( QP_0 \). When the platform reaches point \( S \) at azimuth time \( t_a = t_c \), \( P(x, y, 0) \) is illuminated by the beam center with an inherent range-dependent squint angle \( \theta \). Similarly to \( P_0 \), the geometry parameters of \( P \) are \( \alpha \), \( r_c \) and \( r_g \), respectively. The yaw angle \( \beta \) is fixed during the flight, which means the projections of slant ranges \( QP_0 \) and \( SP \) on the ground are parallel with each other.

It is obvious that the inherent range-dependent squint angle \( \theta \) can be expressed as

\[
\sin \theta(r_c) = \frac{r_g'(r_c) \cos \beta}{r_c} = \frac{\sqrt{r_c^2 - (r_{c0} \sin \alpha_0)^2} \cos \beta}{r_c} \tag{1}
\]

With reference to Fig. 1, the instantaneous slant range from the SAR platform to the target \( P \) is

\[
R(t; r_c, t_c, \theta(r_c)) = \sqrt{r_c^2 + v^2(t - t_c)^2 - 2r_c v(t - t_c) \sin \theta(r_c)} \tag{2}
\]

where \( t \) is the azimuth slow time, and \( t_c \) is the beam center crossing time.

Expanding instantaneous slant range into its biquadratic Taylor series at \( t = t_c \) yields

\[
R(t; r_c, t_c) = r_c + A(t-t_c) + B(t-t_c)^2 + C(t-t_c)^3 + D(t-t_c)^4 \ldots \tag{3}
\]

where \( A, B, C, \) and \( D \) are the range expanding coefficients,

\[
\begin{align*}
A &= -v \sin \theta, \\
B &= \frac{v^2 \cos^2 \theta}{2r_c}, \\
C &= \frac{2v^3 \sin \theta \cos^2 \theta}{r_c^2}, \\
D &= -\frac{v^4 \cos^2 \theta (-3 + 5 \cos 2\theta)}{16r_c^3}
\end{align*} \tag{4}
\]

In (3), term \( A(t-t_c) \) is the LRCM, term \( B(t-t_c)^2 \) is the quadratic RCM (QRCM), and the rests are higher-order \((\geq 3)\) RCM. Generally, LRCM dominates all of the RCM. Note that all the RCM depends on the beam center slant range \( r_c \) and the range-dependent squint angle \( \theta \), which means all the RCM is range-dependent.

Here, an evaluation of how the spatial squint angle \( \theta \) varies with the slant range \( r_c \) under different reference angles \( \theta_0 \) at \( r_{c0} = 10 \text{ km} \) is conducted on the basis of (1), as shown in Fig. 2(a). It is observed that the squint angle tends to be constant when the illuminated scene is located in the long-range region \((\geq 20 \text{ km})\), where the conventional image formation algorithms work well. In contrast, the squint angle varies rapidly in the short-range area \((\leq 15 \text{ km})\), which leads to a severe deterioration of imaging performance if the impact of this spatial-variance is not taken into account. Under this evaluation parameter configuration, the area where the slant range is less than 15 km and the spatial squint angle which varies obviously with the increase of the swath width can be called the SRWS region.

Here we define a slant range error (SRE) function to evaluate the impact,

\[
\text{SRE} = \int_{t_c-T_a/2}^{t_c+T_a/2} (R(t; r_c, t_c, \theta(r_c)) - R(t; r_c, t_c, \theta_0(r_{c0}))) \, dt \tag{5}
\]

where \( T_a \) is synthetic aperture time, \( R(t; r_c, t_c, \theta(r_c)) \) is the actual instantaneous slant range defined by (2), and \( R(t; r_c, t_c, \theta_0) \) is the conventional instantaneous slant range acquired at reference range in which the squint angle is \( \theta_0 \). As shown in Fig. 2(b), SRE is dramatically large in the short-range region, which will result in an enormous RCMC error if it is simply neglected.

Considering that the range dependence of cubic and biquadratic RCM is negligible, we pay close attention to the LRCM and QRCM errors originating from the range-dependent squint angle, which are defined by

\[
\begin{align*}
\Delta R_{\text{LRCM}}(r_c) &= v (\sin \theta(r_c) - \sin \theta_0(r_{c0})) T_a \\
\Delta R_{\text{QRCM}}(r_c) &= \left( \frac{v^2 \cos^2 \theta(r_c)}{2r_c} - \frac{v^2 \cos^2 \theta_0(r_{c0})}{2r_c} \right) \left( \frac{T_a}{2} \right)^2
\end{align*} \tag{6}
\]

In Fig. 3, the error curves of LRCM and QRCM along the range direction are provided with the parameters listed in Table 1. One can see that the LRCM error exceeds half of a range resolution cell when the range position is greater than 170 m off the reference point, which means that the reference LRCM only works in an extremely small region along the range direction if the range dependence of the squint angle

![Figure 2](image2.png)

**FIGURE 2.** (a) Spatial squint angle curves vary with different reference squint angle. (b) Slant range error curve and its zoomed-in at the range position of 10 km.

![Figure 3](image3.png)

**FIGURE 3.** RCM errors vary with range position. (a) LRCM error. (b) QRCM error. It can be seen that the range dependence of QRCM is negligible while the one of LRCM is severe.
TABLE 1. Simulation parameters.

| Simulation parameters      | Values |
|---------------------------|--------|
| Carrier frequency         | 10 GHz |
| Platform velocity         | 150 m/s|
| Range bandwidth           | 150 MHz|
| Reference squint angle    | 50°    |
| Doppler bandwidth         | 150 Hz |
| Synthetic aperture time   | 2.57 s |
| Sampling frequency        | 378 MHz|
| Pulse repetition frequency| 598 Hz |
| Platform height           | 3 km   |
| Reference beam center slant range | 10 km |

is not considered. In other words, the residual LRCM is too considerable to be ignored. Conversely, Fig. 3(b) indicates that the QRCM error is much smaller than half of a range resolution cell within a relatively wide extension along the range direction, which implies that the range dependence of QRCM can be neglected without sacrificing the image quality. Therefore, in the subsequent RCMC procedure, we neglect the range dependence of QRCM.

**B. LRCM ANALYSIS**

Obviously, from the expression (3) we know that there is a severe range-azimuth coupling which makes the squinted SAR data focusing difficult. A LRCM method at the reference range is generally utilized to mitigate the coupling. However, the LRCM for the SRWS region echo induces non-negligible negative effects, which deserve further investigation. The details are as follows.

Assume a linear frequency modulation (LFM) signal is transmitted, and the echo demodulated to baseband can be expressed as

\[
s(t, f; r_c, t_c) = w_t \left( t - \frac{2R(t; r_c, t_c)}{c} \right) w_a \left( \frac{t - t_c}{T_a} \right) \cdot \exp \left\{ -j4\pi f_c \frac{t}{c} R(t; r_c, t_c) \right\} \cdot \exp \left\{ j\pi K_t \left( \tau - \frac{2R(t; r_c, t_c)}{c} \right)^2 \right\}
\]  
(7)

where \( w_t(\cdot) \) and \( w_a(\cdot) \) are the range and azimuth envelopes respectively. \( t \) is the range fast time, \( \tau \) is the azimuth slow time, \( f_c \) is the carrier frequency, \( K_t \) is the range chirp rate, and \( c \) is the speed of light.

Using the principle of stationary phase (PSP) to transform the signal (7) into the range frequency domain, we have

\[
S(f, f_c; r_c, t_c) = \exp \left\{ -j\pi f^2 \frac{f_c}{K_t} \right\} \cdot \exp \left\{ -j4\pi f_c \frac{f_c + f_t}{c} \right\} \left[ (c(t - t_c)^3 + D(t - t_c)^2) \right]\}
\]  
(8)

where \( f_t \) is the range frequency. For highly squinted SAR data, the range-azimuth coupling is generally mitigated by a reference range LRCM, expressed as

\[
H_{LRCM}(f_t, t) = \exp \left\{ j4\pi f_c \frac{f_t + f_c}{c} A_0 t \right\}
\]  
(9)

where \( A_0 = -\sin(\theta_0) \) denotes the reference range LRCM slope.

Multiplying (9) with (8), the echo becomes

\[
S(f, f_c; r_c, t_c) = \exp \left\{ -j\pi f^2 \frac{f_c}{K_t} \right\} \cdot \exp \left\{ -j4\pi f_c \frac{f_c + f_t}{c} \right\} \left[ (r_c - A_0 t_c + A - A_0) (t - t_c) \right] \cdot \exp \left\{ -j4\pi f_c \frac{f_c + f_t}{c} \right\} \left[ B(t - t_c)^2 + C(t - t_c)^3 + D(t - t_c)^2 \right]\}
\]  
(10)

where the term \((r_c - A_0 t_c)\) denotes the new range position of the target after the reference range LRCM. The term \((A - A_0) (t - t_c)\) represents the residual range-dependent LRCM, which ought to be treated differently for the data from the long-range region and the SRWS region. In the rest of this section, using figure illustrations elaborates how the reference range LRCM works for two cases.

In the long-range case, the squint angle \( \theta \) and coefficient \( A \) in (3) are considered spatial-invariant, so the residual LRCM component approaches zero, which means that the LRCM of an arbitrary target \( P \) can be fully corrected by the reference range LRCM. In order to show how LRCM works more clearly, the RCM trajectories of a five-point target array are shown in Fig. 4, where target \( P_0 \) is the reference point; targets \( P_1 \) and \( P_2 \) are in the same range position as \( P_0 \) after LRCM, and targets \( P_3 \) and \( P_4 \) are in the same azimuth position as \( P_0 \). Note that for the RCM trajectory of each target, the black dotted line represents the LRCM, and the red curve represents the nonlinear RCM. After the reference range LRCM, it can be seen that the LRCMs of all targets are well-corrected, the subsequent RCMC and azimuth focusing procedure can be easily accomplished. Fig. 5 shows a well-performed

**FIGURE 4.** (a) RCM trajectories of targets in long-range region before the reference range LRCM. (b) After the reference range LRCM.

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**FIGURE 5.** (a) RCM trajectories of targets in long-range region before the reference range LRCM. (b) After the reference range LRCM.
LRCMC result for a long-range case, where the reference beam center slant range is 30 km.

When it comes to the SRWS case, the LRCMC at the reference range is no longer applicable for all targets in the illuminated area due to the range-dependent squint angle. As displayed in Fig. 6, once the reference range LRCMC is applied, the LRCM of the reference point $P_0$ is well corrected, while the RCM trajectories of the targets $P_1$ and $P_2$ are in the shape of a sloping curve, and so are the RCM trajectories of $P_3$ and $P_4$, which means that the RCM is 2-D spatial-variant. This conclusion is further confirmed by the experimental results obtained at a short reference range (10 km), shown in Fig. 7.

Figure 6. (a) RCM trajectories of targets in SRWS region before the reference range LRCMC. (b) After the reference range LRCMC.

Besides, another issue induced by the LRCMC is the azimuth-dependent Doppler parameters including residual Doppler centroid and high-order Doppler parameters, which will be discussed in Section III-E. To produce high quality images, both issues have to be taken into account by the imaging algorithm.

III. PROPOSED IMAGING ALGORITHM

The procedure of the proposed algorithm in this section is summarized as follows. First, a reference range LRCMC is adopted to remove most of the LRCM component and to mitigate the range-azimuth coupling. Then, the residual LRCM is corrected and the range dependence of RCM is eliminated through KT operation. Next, a novel ESM is proposed according to an investigation into the property of origin range. Based on the ESM, an ADH-RCMC is implemented to correct the azimuth-dependent high-order RCM, and a FENLCS is derived to remove the highly varying residual Doppler centroid and to equalize the azimuth-variant Doppler parameters. The flowchart of the proposed algorithm is summarized in Fig. 8, in which the major parts of improvement are selected by red-lined boxes, and more detailed explanations of each step are provided in Section III-A.

Figure 8. Flowchart of the proposed algorithm.

A. DESCRIPTION OF THE PROPOSED ALGORITHM

Prior to developing the imaging algorithm, a graph representation of the processing scheme is provided in Fig. 9 for better understanding each step of the algorithm. An array of five targets is distributed in a SRWS region as previously described, shown in Fig. 9 (a), and the distribution of RCMs trajectories, which consists of LRCM and nonlinear components, is displayed in Fig. 9(b). The reference range LRCMC is first applied to mitigate the range-azimuth coupling, which makes the RCM trajectory 2-D spatial-variant, demonstrated in Fig. 9(c). Then, a KT operation is adopted to remove the residual LRCM and so the range dependence of RCM is eliminated which is mainly caused by the LRCM component, demonstrated in Fig. 9(d). Following which, a Bulk RCMC at the reference range can be performed to correct the unified high-order RCM, shown in Fig. 9(e). To reveal the azimuth correlation of the residual high-order RCM and the Doppler parameters, a novel ESM is established based on the conception of an equidistant origin range that the targets sharing the same origin range will be shifted to the same range.
accounts for the residual LRCM and the Doppler centroid, this generally non-negligible residual component, which approaches zero in the conventional constant RCM, as follows.

The graph illustration in Fig. 9 gives us a simple understanding of the proposed algorithm and the details are developed in (10), the term $(A-A_0)(t-t_c)$ is left by the reference range LRCM, which approaches zero in the conventional squint configuration. However, for the SRWS region focusing, this generally non-negligible residual component, which accounts for the residual LRCM and the Doppler centroid shift, will seriously impact the focused image quality, if it is ignored simply.

To accommodate this issue, a KT operation, which is generally used to correct the range walk or range curvature of moving targets without knowing their velocities [22], is adopted to eliminate the residual LRCM. It is performed by substituting $t = f_c t_m / (f_c + f_r)$ into (10), and the result is expanded into a cubic series in terms of $f_r$,

$$S_2(f_r, t_m; r_c, t_c) = \exp \left\{ j \pi \left[ \varphi_0(t_m; r_c, t_c) + 2 \varphi_1(t_m; r_c, t_c) f_r \right] \right\} \cdot \exp \left\{ j \pi \left[ \varphi_2(t_m; r_c, t_c) f_r^2 + \varphi_3(t_m; r_c, t_c) f_r^3 \right] \right\}$$

The expression of $\varphi_i(t_m; r_c, t_c)$ can be found in (12), as shown at the bottom of the page, where $\varphi_0$ represents the azimuth modulation term, $\varphi_1$ involves the RCM term, $\varphi_2$ and $\varphi_3$ are high-order coupling terms. Here $\varphi_i$ is $\varphi_i(t_m; r_c, t_c)$ for short.

It is found that the linear coupling of $t_m$ and $f_r$, named LRCM, has been fully corrected. Then, the unified high-order

$$\begin{align*}
\varphi_0(t_m; r_c, t_c) &= -\frac{4f_c}{c} \left[ (r_c - A_0 t_c) + (A - A_0) (t_m - t_c) + B(t_m - t_c)^2 + C(t_m - t_c)^3 + D(t_m - t_c)^4 \right] \\
\varphi_1(t_m; r_c, t_c) &= \frac{2}{c} \left[ (At_c - B t_c^2 + C t_c^3 - D t_c^4 - r_c) + \left( B - 3C t_c + 6D t_c^2 \right) j t_m^2 + (2C - 8D t_c) t_m^3 + 3D t_m^4 \right] \\
\varphi_2(t_m; r_c, t_c) &= -\frac{1}{K t_m} - \frac{4B - 12C t_c + 24D t_c^2 j t_m^2}{c f_c} - \frac{(12C - 48D t_c) t_m^3 + 24D t_m^4}{c f_c} \left[ c f_c^2 \right] \\
\varphi_3(t_m; r_c, t_c) &= \frac{4B - 3C t_c + 6D t_c^2 j t_m^2}{c f_c^2} + \frac{4(4C - 16D t_c) t_m^3 + 40D t_m^4}{c f_c^2} \left[ c f_c^2 \right]
\end{align*}$$

\[12\]
RCM can be compensated by a Bulk RCMC,

$$H_{BRCMC}(f_t, t_m; r_c0) = \exp\left\{-j\frac{4\pi f_t}{c} \left[ B_0 t_m^2 + 2C_0 t_m^3 + 3D_0 t_m^4 \right]\right\}$$

(13)

where $B_0$, $C_0$, and $D_0$ are the expanding coefficients at the reference slant range,

$$\begin{align*}
B_0 &= \frac{v^2 \cos^2 \theta_0}{2r_c0} \\
C_0 &= \frac{v^3 \cos^2 \theta_0 \sin \theta_0}{2r_c^2} \\
D_0 &= -\frac{v^4 \cos^2 \theta_0 (-3 + 5 \cos 2\theta_0)}{16r_c^3}
\end{align*}$$

(14)

To validate the effectiveness of the aforementioned range preprocessing method, the measurement on the RCM trajectories of the echoes after the KT and Bulk RCMC are conducted based on the parameters in Table 1.

Assume five targets, $P_0$-$P_4$, are distributed in a SRWS region, where $P_0$ is the reference point. $P_1$ and $P_2$ are set in the same range cell as $P_0$ after the KT and Bulk RCMC, and the azimuth interval between them is 750 m. Targets $P_0$, $P_3$, and $P_4$ are placed at the azimuth center, whose beam center slant ranges are 10 km, 9 km, and 11 km, respectively. The measured RCM trajectories of the five targets are shown in Fig. 10. It is clear that before KT, the differences among the RCM curves are large, and the migration errors of the azimuth margin targets $P_1$ and $P_2$ are over 2.5 m, as shown in Fig. 10(a). In addition, it is easy to find that the residual LRCM is 2-D spatial-variant, shown in Fig. 10(b). In this case, large migration errors will occur if the Bulk RCMC is performed directly. After KT, the RCM curves of the azimuth central targets are almost identical. Meanwhile, the RCM trajectories of $P_1$ and $P_2$ are close to the extension of the reference range trajectory, as shown in Fig. 10(c), which implies that the residual 2-D variant LRCM is well corrected and the range dependence of RCM is also reduced. Apparently it is suitable to perform the Bulk RCMC, and the result is presented in Fig. 10(d). One can see that the migration errors of all azimuth central targets approach zero, while those of the azimuth margin targets are still greater than 1m. Therefore, we can undoubtedly say that the residual RCM after Bulk RCMC is high-order and azimuth-dependent, and the correction of it will be carried out in Section III-D.

Multiply (11) with (13), after the Bulk RCMC, the range delay signal becomes

$$s_{\text{delay}}(t_m; r_c, t_c) = \exp\left\{-j\frac{4\pi f_t}{c} [\mu_0 + \Delta \mu (t_m; r_c, t_c)] \right\}$$

(15)

where

$$\begin{align*}
\mu_0 &= r_c - At_c^2 + Br_c^2 - Ct_c^3 + Dt_c^4 = R (0; r_c, t_c) \\
\Delta \mu (t_m; r_c, t_c) &= -(B - B_0 - 3Ct_c + 6Dt_c^2)t_m^2 \\
&- (2C - 2C_0 - 8Dt_c)t_m^3 - 3(D - D_0)t_m^4 \\
\end{align*}$$

(16)

In (16) $\mu_0$ represents the new range position of the target after the Bulk RCMC, and $\Delta \mu (t_m; r_c, t_c)$ represents the aforementioned residual azimuth-dependent high-order RCM which is overlooked in published literature [15]. The correction of the residual RCM plays an important role in producing well-focused images, especially for the SRWS cases. As previously discussed, the coefficients in $\Delta \mu (t_m; r_c, t_c)$ depend on the slant range $r_c$ and the reference slant range $r_{c0}$. Therefore, the relationship between $r_c$ and $r_{c0}$ is vital to correcting the residual azimuth-dependent high-order RCM.

According to (15), if $\Delta \mu (t_m; r_c, t_c)$ could be corrected, then the range position of the echo will be shifted from the initial beam center slant range $r_c$ to the $\mu_0$. Note that $\mu_0 = R (0; r_c, t_c)$ represents the range from a target to the radar platform when the azimuth slow time equals to zero, which is so-called the origin range. It means that the RCM trajectories of those targets sharing the same origin range will be moved into the same range position after the whole RCMC. This is the reason why we propose the ESM as the points on the sphere surface share the same origin range, if the initial position of the radar platform is considered as the center of the sphere.

C. NEW EQUIDISTANT SPHERE MODEL

In order to obtain the relationship between $r_c$ and $r_{c0}$, the ESM is established according to the conception of equidistant origin range, as shown in Fig. 11(a).

At azimuth time $t = 0$, the radar platform is located at point $O$, which is chosen as the origin point and the spherical center. Let $P_0$ and $P$ denote two targets sharing the same origin range $R(0;r_c, t_c)$ in the observed region, where $P_0(x_0, y_0, 0)$ locates at the scene center and $P(x, y, 0)$ is an arbitrary one. $r_{c0}$ and $r_c$ are the beam center slant range of $P_0$ and $P$, and $r_{g0}$, $r_g$ are the ground-range of $P_0$ and $P$, respectively. Due to sharing the same origin range, targets $P_0$ and $P$ are located on the
FIGURE 11. Proposed ESM. (a) The geometric configuration of the sphere, where O is chosen as the spherical center. P0 and P in the ground share the same origin range while they have the different beam center slant range. (b) The circle intersected by the sphere surface and the ground plane, viewed from the top.

circle line intersected by the sphere surface and the ground plane, and the radius of the circle equals to \( r_{g0} \) as shown in Fig. 11(b), viewed from the top of the sphere.

According to Fig. 11(b), the ground coordinates of \( P_0 \) and \( P \) can be expressed as

\[
\begin{align*}
  x_0 &= r_{g0} \cos \beta \\
  y_0 &= r_{g0} \sin \beta \\
  x &= r_g \cos \beta + vt_c \\
  y &= r_g \sin \beta
\end{align*}
\]

and the equation of the circle is

\[
x_0^2 + y_0^2 = x^2 + y^2 = r_{g0}^2
\]

In Fig 11(a), we have

\[
r_c = \frac{r_g \cos \beta}{\sin \theta}, \quad r_{c0} = \frac{r_{g0} \cos \beta}{\sin \theta_0}
\]

The relationship between \( r_c \) and \( r_{c0} \) can be obtained by solving (17), (18), and (19) simultaneously, and the result expanded into a quadratic series in terms of \( t_c \) is given by

\[
r_c = r_{c0} - v \sin(\theta_0) t_c + \frac{v^2 [\cos(2 \theta) - \sin^2(\theta_0)]}{2r_{c0}} t_c^2
\]

\[
= r_{c0} - v \sin(\theta_0) t_c + \frac{v^2 [\sin^2(\theta_0) - \cos^2(\theta_0) - \sin^2(\theta_0)]}{2r_{c0}} t_c^2
\]

(20)

where \( \theta_0 = \pi/2 - \beta \) denotes the projection of the reference squint angle \( \theta_0 \). Note that when \( \theta_0 = \theta_0 \), point \( O \) and \( O' \) in Fig. 11(a) are superpositioned with each other and (20) will degrade to the equidistant circle model in [23], which validates the accuracy of the established ESM in 2D geometry configuration.

Particularly, the beam center slant range \( r_c \) is modeled as a polynomial of the beam center crossing time \( t_c \), which can be used to correct the residual azimuth-dependent high-order RCM in Section III-D and to equalize the azimuth-dependent Doppler parameters in Section III-E. To facilitate the derivation procedures in the mentioned sections, the azimuth-variant polynomials of the instantaneous slant range expanding coefficients \( A, B, C, \) and \( D \) are deduced in advance.

Substituting (1) and (20) into (4), after a series of derivations, we have

\[
\begin{align*}
  A(r_{c0}, t_c) &\approx A_0 + A_1 t_c \\
  B(r_{c0}, t_c) &\approx B_0 + B_1 t_c + B_2 t_c^2 \\
  C(r_{c0}, t_c) &\approx C_0 + C_1 t_c \\
  D(r_{c0}) &\approx D_0
\end{align*}
\]

(21)

where

\[
\begin{align*}
  A_1(r_{c0}) &= v^2 \cos^2 \beta \sin^2 \alpha_0 \\
  B_1(r_{c0}) &= \frac{v^3 \sin \theta_0 (\cos^2 \theta_0 + 2 \cos^2 \beta \sin^2 \alpha_0)}{2r_{c0}} \\
  B_2(r_{c0}) &= \frac{3v^4 \sin^2 \theta_0 \cos \theta_0}{4r_{c0}} \\
  &+ \frac{v^4 \sin^2 \theta_0 \cos^2 \beta \sin^2 \alpha_0}{2r_{c0}^3} \\
  &+ \frac{v^4 \sin \theta_0 \cos \beta \sin \theta_0 \sin^2 \beta}{r_{c0}^3} \\
  &- \frac{2 \cos \theta_0 r_{c0}^3}{r_{c0}^3} \\
  &+ \frac{v^4 \left( \cos 2\beta \cos^2 \theta_0 + 2 \cos^4 \beta \sin^4 \alpha_0 \right)}{4r_{c0}^4} \\
  C_1(r_{c0}) &= v^4 \left( \cos^2 \beta \left( 1 - 3 \cos 2\theta_0 \sin^2 \alpha_0 + \sin^2 2\theta_0 \right) \right)
\end{align*}
\]

(22)

D. AZIMUTH-DEPENDENT HIGH-ORDER RCM

In this section, we would like to introduce an ESM-based ADH-RCMC to correct the residual azimuth-dependent high-order RCM.

Substituting (21) into (15), the range delay signal can be rewritten as

\[
S_{\text{delay-1}}(f_r, t_m, t_c) = \exp \left\{ -\frac{4\pi f_r}{c} \left[ \mu_0 - (B_1 t_c + B_2 t_c^2 - 3(C_0 + C_1 t_c)) t_c + 6D_0 t_c^2 t_m^2 + (-C_1 t_c + 8D_0 t_c) t_m^3 \right] \right\}
\]

(23)

where the sum of the second and the third terms approximately equals to \( \Delta \mu(t_m, t_c) \), i.e., the residual azimuth-dependent high-order RCM. Note that the range expansion coefficient \( D \), whose azimuth-variant property has negligible influence on the RCM, can be replaced by \( D_0 \).

To correct the residual RCM, the ADH-RCMC has been adopted now, whose idea works as follows. For the targets \( P_0 \),
where $p_3$ and $p_4$ are the coefficients to be determined.

Multiplying (24) with (23), and expanding the result into a Taylor series in terms of $(t_m - t_c)$, we have

$$S_{\text{delay}}(f_r, t_m; r_c, t_c) = \exp \left\{ \frac{4\pi f_r}{c} (p_3 t_m^3 + p_4 t_m^4) \right\}$$

where $R_{\text{delay}}(t_m; r_c, t_c)$

$= (B_1 t_c^3 - 3C_0 t_c^3 + p_3 t_c^3 + B_2 t_c^4 - C_1 t_c^4 - 2D_0 t_c^4$
$+ p_4 t_c^4 + \mu_0) + (2B_1 t_c^2 - 6C_0 t_c^2 + 3p_3 t_c^2 + 2B_2 t_c^3$
$- 12D_0 t_c^2 + 4p_4 t_c^2)(t_m - t_c) + (B_1 t_c^3 - 3C_0 t_c^3 + p_3 t_c^3$
$+ B_2 t_c^4 + 3C_1 t_c^4 - 18D_0 t_c^4 + 6p_4 t_c^2)(t_m - t_c)^2$
$+ (p_3 + 2C_0 t_c - 8D_0 t_c + 4p_4 t_c)(t_m - t_c)^3 + O(t_m - t_c)^4$  

To better understand the idea of ADH-RCMC, Fig. 12 provides the measured RCM trajectories of the targets $P_0$, $P_1$, and $P_2$ in the same range cell throughout the ADH-RCMC. One can see that before ADH-RCMC, the migration errors of azimuth margin targets $P_1$ and $P_2$ are greater than one range cell, as shown in Fig. 12(a). More specifically, components of the RCM of target $P_1$ are provided in Fig. 12(b), where the solid and dashed lines represent the linear and nonlinear components, respectively. It is obvious that the linear component dominates the residual RCM, and the nonlinear one is much smaller than half of a range cell. Therefore, the residual RCM can be minimized by setting the coefficient of the linear term in (26) to zero, and then the undetermined coefficients $p_3$ and $p_4$ are obtained,

$$
\begin{align*}
p_3(r_c) &= -\frac{2(B_1 - 3C_0)}{3} \\
p_4(r_c) &= -\frac{(B_2 - 6D_0)}{2}
\end{align*}
$$

Substituting (27) into (26), we have

$$R_{\text{delay}}(t_m; r_c, t_c)$$

$= \mu_0 + (B_1 t_c^3 - 3C_0 t_c^3 + \frac{B_2 t_c^4 - C_1 t_c^4 + D_0 t_c^4}{2})$
$+ (-B_1 t_c + 3C_0 t_c - 2B_2 t_c^2 + 3C_1 t_c^2)(t_m - t_c)^2$
$+ \left(\frac{-2B_1}{3} + 2C_0 - 2B_2 t_c + 2C_1 t_c + 4D_0 t_c\right)(t_m - t_c)^3$
$+ O(t_m - t_c)^4$

Inspecting (28), the second term denotes the range offset, which is too small to have a noticeable influence on the azimuth focusing. The third and fourth terms represent the migration error after ADH-RCMC. The measured RCM trajectories of $P_0$, $P_1$, and $P_2$ along the azimuth direction are shown in Fig. 12(c). It is observed that the migration error is much smaller than half of a range cell, which fully meets the requirement of focusing the data of a SRWS region.

Till now, the whole RCMC has been accomplished. In order to evaluate the effectiveness of the aforementioned RCMC for the data acquired at different reference squint angles, we define an RCMC error function, which is the sum of the range offset and residual RCM after ADH-RCMC, given by

$$E_{\text{RCMC}}(\theta) = \max\{|R_{\text{delay}}(t_m; r_c, t_c) - \mu_0|\}$$

The measured $E_{\text{RCMC}}$ of margin target $P_1$ at different reference squint angles is shown in Fig 13. It is observed that when the reference squint angle is around $33^\circ$, there is the maximum $E_{\text{RCMC}}$. However, after the ADH-RCMC, the
maximum error is still smaller than half of a range cell, which validates that the proposed method is effective for correcting the residual azimuth-dependent high-order RCM over a wide range of reference squint angle.

The final step of range processing is accomplished by the phase compensation called Bulk Second Range Compression (BSRC) in the range frequency domain via setting the reference range squint angle. The residual azimuth-dependent high-order RCM over a wide range of reference squint angle.

The azimuth modulation signal in (11) is rewritten as

\[ S_{az}(t_m; r_c, t_c) = \exp \left\{ j \cdot \left[ f_{az}(t_m - t_c^0) + f_{ad}(t_m - t_c) \right] \right\} \]

where the constant term is omitted as it contributes little to the azimuth processing. \( f_{az} \) is the Doppler centroid, \( f_{az} = -2A_0/\lambda \) denotes the Doppler centroid at the reference range, \( f_{ad} \) represents the Doppler FM rate, \( f_{ad} \) and \( f_{ad} \) are the cubic and biquadratic Doppler parameters, respectively. \( f_{ad}(t) \) for short, is azimuth-dependent. Note that there is still a linear residual component in (32) that is negligible in the most approaches [16], [17], but in fact, it has a serious impact on the azimuth focusing and will be processed in the subsequent.

Transforming (32) into the azimuth frequency domain using the method of series revision (MSR) [24], the result is simplified as

\[
S_{az}(f; t, r_c, t_c) = \exp \left\{ -\frac{2\pi}{c} (\xi_2 f_2 + \xi_3 f^3) \right\}
\]

where

\[
\begin{align*}
\xi_2 &= \frac{1}{K_c} + \frac{4B_0 t_m^2}{c f_c} + \frac{12C_0 t_m^3}{c f_c^2} + \frac{24D_0 t_m^4}{c f_c^3} \\
\xi_3 &= -\frac{4B_0 t_m^2}{c f_c^2} - \frac{16C_0 t_m^3}{c f_c^3} + \frac{40D_0 t_m^4}{c f_c^4}
\end{align*}
\]

\( E. \) AZIMUTH COMPRESSION VIA FENLCS

As previously explained, after the RCMC, targets located in the same range bin may have different closest approaches, i.e., they have different Doppler parameters, which leads to the failure of direct application of a single matched filter to compress the whole azimuth array accurately. Therefore, the spatial variance of Doppler parameters should be removed before the azimuth focusing operation. With respect to the foregoing, an improved FENLCS with the residual Doppler Centroid correction is introduced in this section to equalize the azimuth-dependent Doppler parameters.

The azimuth modulation signal in (11) is rewritten as

\[ S_{az}(t_m; r_c, t_c) = \exp \left\{ j \cdot \left[ f_{az}(t_m - t_c^0) + f_{ad}(t_m - t_c) \right] \right\} \]

where the constant term is omitted as it contributes little to the azimuth processing. \( f_{az} \) is the Doppler centroid, \( f_{az} = -2A_0/\lambda \) denotes the Doppler centroid at the reference range, \( f_{ad} \) represents the Doppler FM rate, \( f_{ad} \) and \( f_{ad} \) are the cubic and biquadratic Doppler parameters, respectively. \( f_{ad}(t) \) for short, is azimuth-dependent. Note that there is still a linear residual component in (32) that is negligible in the most approaches [16], [17], but in fact, it has a serious impact on the azimuth focusing and will be processed in the subsequent.

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\]

where

\[
\begin{align*}
\xi_2 &= \frac{1}{K_c} + \frac{4B_0 t_m^2}{c f_c} + \frac{12C_0 t_m^3}{c f_c^2} + \frac{24D_0 t_m^4}{c f_c^3} \\
\xi_3 &= -\frac{4B_0 t_m^2}{c f_c^2} - \frac{16C_0 t_m^3}{c f_c^3} + \frac{40D_0 t_m^4}{c f_c^4}
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\]

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where the constant term is omitted as it contributes little to the azimuth processing. \( f_{az} \) is the Doppler centroid, \( f_{az} = -2A_0/\lambda \) denotes the Doppler centroid at the reference range, \( f_{ad} \) represents the Doppler FM rate, \( f_{ad} \) and \( f_{ad} \) are the cubic and biquadratic Doppler parameters, respectively. \( f_{ad}(t) \) for short, is azimuth-dependent. Note that there is still a linear residual component in (32) that is negligible in the most approaches [16], [17], but in fact, it has a serious impact on the azimuth focusing and will be processed in the subsequent.

Transforming (32) into the azimuth frequency domain using the method of series revision (MSR) [24], the result is simplified as

\[
S_{az}(f; t, r_c, t_c) = \exp \left\{ -\frac{2\pi}{c} (\xi_2 f_2 + \xi_3 f^3) \right\}
\]

where

\[
\begin{align*}
\xi_2 &= \frac{1}{K_c} + \frac{4B_0 t_m^2}{c f_c} + \frac{12C_0 t_m^3}{c f_c^2} + \frac{24D_0 t_m^4}{c f_c^3} \\
\xi_3 &= -\frac{4B_0 t_m^2}{c f_c^2} - \frac{16C_0 t_m^3}{c f_c^3} + \frac{40D_0 t_m^4}{c f_c^4}
\end{align*}
\]
By observing (32), we know that the azimuth dependence of the Doppler parameters exactly agrees with the ones of the derived range expanding coefficients in (21),

\[
\begin{align*}
 f_{d2} &= -2A/\lambda, \\
 f_{d3} &= -4B/\lambda, \\
 f_{d4} &= -4C/\lambda
\end{align*}
\]

\[
\begin{align*}
 \hat{f}_{d2} &\approx f_{d20} + f_{d21}t_c \\
 \hat{f}_{d3} &\approx f_{d30} + f_{d31}t_c \\
 \hat{f}_{d4} &\approx f_{d40}
\end{align*}
\]

(36)

In particular, the Doppler FM rate \( f_{d2} \) inseparably connected with azimuth-focusing is kept up to the second order term of \( t_c \), and the first-order dependences of cubic Doppler parameters \( f_{d3} \) are also considered. But in the literature [20], only the linear approximation of \( f_{d2} \) is adopted, and the azimuth dependence of \( f_{d3} \) is neglected, which is not sufficient for SRWS cases.

With the newly derived Doppler parameters, the azimuth equalization can be readily completed by applying FENLCS [21], following which a well-focused image can be obtained with the azimuth compression.

**F. IMAGING SWATH WIDTH ANALYSIS**

The size of the imaging swath is limited by a number of factors, but the dominant factors often are the residual RCM error (RCME) and the quadratic phase error (QPE), and the discussion that follows is limited to these two factors only.

In range dimension processing, after utilizing the ADH-RCMC to remove most of the 2D-variant RCM, the residual RCME can be express as

\[
\text{RCME} = e_0 + e_1 + e_2
\]

where

\[
\begin{align*}
 e_0 &= \frac{B_1}{3}t_c^3 - C_0t_c^2 + \frac{B_2}{2}t_c^4 - C_1t_c^4 + D_0t_c^4 \\
 e_1 &= -(B_1t_c + 3C_0t_c^2 - 2B_2t_c^3 + 3C_1t_c^4) \cdot (T_a/2)^2 \\
 e_2 &= -(\frac{2B_1}{3} + 2C_0 - 2B_2t_c^2 + 2C_1t_c + 4D_0t_c) \cdot (T_a/2)^3
\end{align*}
\]

where the calculations of \( e_0, e_1, \) and \( e_2 \) can be derived by (28).

Similarly, in azimuth dimension processing, after equalizing the azimuth spatial-variant Doppler centroid and FM rate, the QPE can be expressed as

\[
\text{QPE} = \frac{4\pi}{\lambda} (f_{d2} - \hat{f}_{d2}) \cdot (T_a/2)^2
\]

(38)

where the \( f_{d2} \) and \( \hat{f}_{d2} \) are defined by (36).

Fig. 14(a) presents the residual RCME curves of before and after ADH-RCMC. It can be observed that the RCME curve before ADH-RCMC increases drastically with the target position along the range direction, and the residual RCM can be well compensated by the proposed method. In this case, the proposed method expands the imaging scene along the range minus direction from 630 m to about 1710 m, which is a considerable improvement. The relationship between the QPE and the azimuth position is shown in Fig. 14(b). With the model [20], the error exceeds the threshold value \( \pm\pi/4 \) at only 750 m due to the lack of modeling precision, while the QPE induced by the proposed model stays within the threshold line along a wider azimuth extension, from -1300 m to 1250 m.

**G. COMPUTATIONAL COMPLEXITY ANALYSIS**

We analyze the computational complexity of the proposed algorithm as follows.

The proposed algorithm in this paper needs FFT operations, complex multiplications, and a KT. Generally speaking, it needs \( 5N\log_2(N) \) floating-point operations (FLOPs) to compute \( N \)-point FFT or inverse FFT (IFFT), and \( 6N \) FLOPs for one-time complex multiplication. The KT can be efficiently implemented by Chirp-Z transform and its computation complexity is \( 6N_1(4N_4 + 3N_3\log_2(N_2) + (N_1\log_2(N_2))/2) \), where \( N_1 \) and \( N_2 \) represent the range sample number and azimuth sample number, respectively. Thus, the total computation complexity of the proposed algorithm is

\[
C_{\text{proposed}} = 10N_1N_2\log_2(N_1 + 36N_3\log_2(N_2) + 78N_1N_2)
\]

(39)

Assume that the number of range sample is the same as that of azimuth sample, \( N_a = N_r = N \), then the total computation complexity of the proposed algorithm is on the order of \( O(N^2\log_2(N)) \), which is in the same level as the ENLCS algorithms in [17].

**IV. SIMULATION RESULTS**

In this section, simulation results are conducted to validate the effectiveness of the proposed algorithm for focusing highly squinted SAR data of the SRWS region based on the parameters in Table 1. The purpose of doing so is to show the superiority of the proposed algorithm compared to the two reference algorithms [17], [20].

The simulation involves an array of twenty five targets located in the SRWS region, marked \( P_0-P_{24} \). The target \( P_0 \) is chosen as the reference point. The \( 5 \times 5 \) point target array are evenly distributed in the imaging scene after RCMC, as illustrated in Fig. 15. The intervals both in range and in azimuth are 500 m. In other words, the width of the observed region is 2.0 km \( \times \) 2.0 km in range and in azimuth. For fair comparison, no windowing function or side-lobe control approach is used. The results of the ENLCS algorithm in [17] and the ENLCS-MCS algorithm in [20] are also provided to compare the performance.
Fig. 16 provides the RCM trajectory and imaging results obtained by the conventional ENLCS algorithm. Fig. 16(a) presents the RCM trajectory after RCMC and Bulk SRC. It can be seen that the energy of margin targets $P_3$ and $P_{24}$ crosses several bins in both range and azimuth on account of the ignoring of 2-D spatial-variant RCM. Besides, the results obtained by the conventional ENLCS algorithm are shown in Fig. 16(b). It is observed that not only the envelopes of margin targets $P_3$ and $P_{24}$ spread across several range cells, but also defocus in azimuth direction due to lack compensation for the influence of the highly varying Doppler centroid.

The imaging results obtained by the ENLCS-MCS algorithm are presented in Fig. 17. Since the 2-D spatial-variant RCM is preliminarily tackled through ENLCS and MCS, one can see that the energy distributions of margin targets $P_3$ and $P_{24}$ are more concentrated, as shown in Fig. 17(a). However, it is observed that in Fig. 17(b) imaging performance of targets $P_3$ and $P_{24}$ suffers some deterioration due to the unconsidered azimuth dependence of cubic phase and an inaccurately matched Doppler FM rate.

In the proposed algorithm, the issues of 2-D spatial-variant RCM and highly varying Doppler centroid are addressed based on the ESM. Therefore, it is observed that the energy of margin targets $P_3$ and $P_{24}$ is gathered into one range bin in both range and azimuth directions as shown in Fig. 18(a). Furthermore, compared to the two reference algorithms, the proposed method obtains fine-focused imaging results in both range and azimuth direction, as seen in Fig. 18(b).

To observe the compressed target variation properties in both range direction and azimuth direction more clearly, the subimages of targets $P_0$, $P_{17}$, $P_{22}$, $P_{13}$, $P_{14}$, which are extracted from entire focused SAR images, are analyzed in the following.

The focused results of the targets with the same azimuth position and different range positions, $P_0$, $P_{17}$, $P_{22}$, by different algorithms are shown in Fig. 9. The subimages listed at the top row are processed by the ENLCS algorithm which ignores the 2-D spatial-variant RCM and the Doppler centroid compensation. Therefore, one can see that the image quality of $P_{17}$ is worse than that of $P_0$, and the image quality of $P_{22}$ is much worse than that of the other two. The second row in Fig. 19 shows the results obtained by the ENLCS-MCS algorithm. Similarly, as the distance between the target and the reference position increases, the quality of the compressed target deteriorates due to the mismatched Doppler parameters, especially the azimuth FM rate. The imaging results obtained by the proposed algorithm are shown in the
The measured parameters of the selected targets are provided in Table 2. It is found that the PSLR and ISLR obtained by the proposed method agree well with the theoretical value, which reveals that the proposed algorithm corrects the 2-D spatial-variant RCM and equalizes the Doppler parameters well. On the other hand, the measured parameters of the targets in the focused images obtained by the two reference methods are relatively worse, especially in the range direction.

V. CONCLUSION

This paper proposed an imaging algorithm for focusing high-squint SAR data of the SRWS region. In the algorithm, a reference range LRCMC operation is first adopted to mitigate the range-azimuth coupling, following which KT operation and Bulk RCMC are performed to correct the residual LRCM and unified high-order RCM. According to an analysis of the origin range, a novel ESM is proposed to reveal the spatial-variant properties of echo. On the basis of the ESM, an ADH-RCMC method and a newly derived FENLCS are developed to perform the residual azimuth-dependent high-order RCM correction, and azimuth equalization, respectively, which show better performance compared with existing imaging algorithms. The simulation results are provided to confirm the effectiveness of the proposed algorithm.

TABLE 2. Measured parameters of the selected targets.

| Imaging Method | Target | Range | Azimuth |
|----------------|--------|-------|---------|
|                |        | PSLR/dB | ISLR/dB | PSLR/dB | ISLR/dB |
| ENLCS algorithm | 13.21  | -9.99  | -13.17  | -9.98  |
| 17             | -11.99 | -8.41  | -10.79  | -7.93  |
| 22             | -10.13 | -7.01  | -8.12  | -5.67  |
| 13             | -12.58 | -8.55  | -11.21  | -8.69  |
| 14             | -10.94 | -7.30  | -9.01  | -6.89  |
| 6              | -13.19 | -9.89  | -13.20  | -9.91  |
| ENLCS-MCS algorithm | -12.69 | -8.94  | -11.12  | -7.93  |
| 22             | -11.66 | -7.52  | -10.01  | -6.37  |
| 13             | -13.15 | -9.63  | -13.11  | -9.88  |
| 14             | -13.01 | -9.58  | -12.87  | -9.15  |
| 3              | -13.21 | -9.99  | -13.22  | -9.97  |
| 17             | -13.19 | -9.89  | -13.17  | -9.81  |
| Proposed algorithm | -13.21 | -9.90  | -13.19  | -9.86  |
| 13             | -13.20 | -9.90  | -13.21  | -9.88  |
| 14             | -13.19 | -9.87  | -13.20  | -9.89  |

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