Deep Inelastic Scattering from Holographic Spin-One Hadrons

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### Searching for a 'Holographic QCD'

#### AdS/CFT [Maldacena/Gubser-Klebanov-Polyakov/Witten]
- IIB SUGRA
- Dual to an $\mathcal{N} = 4$ SYM with gauge group $SU(N)$ in $D = 4$ (Too many SUSY’s)
- No matter in the fundamental representation of the gauge group

#### AdS/non-CFT [Polchinski-Strassler/Maldacena-Núñez/Klebanov-Strassler]
- Broken conformal symmetry
- Less SUSY’s ($\mathcal{N} = 1$)
- Still no matter in the fundamental representation of the gauge group

#### AdS/non-CFT with flavours [Karch-Katz]
- Matter in the fundamental representation
- Missed spontaneous $\chi_{SB}$
### Example 1: D3-D7 [Kruczenski-Mateos-Myers-Winters]
- Several QCD phenomena obtained
- Missed spontaneous $\chi_{SB}$

### Example 2: D4 − D8 − $\overline{D8}$ [Sakai-Sugimoto]
- IIA SUGRA
- Realizes spontaneous $\chi_{SB}$
- Same Universality Class as QCD (*false for high energies).
- Extra $SO(5)$ global symmetry

### Aim of this Work
- Study a QCD process (DIS) from both of these models
- We develop a method based on that by Polchinski and Strassler
- We obtain analytic expressions all the *structure functions* for vector and scalar mesons in both models
Introduction

Holographic Mesons
- Models with matter fields in fundamental representation
- Modeled as fluctuations of flavour branes (probe approximation)
- We are interested in the structure of holographic mesons in DIS

Previous Works
- Glueballs and spin-1/2 hadrons [Polchinski-Strassler hep-th/0209211]. Confining gauge theories conformal for $P \gg \Lambda$. No flavours.
- Scalar mesons and unpolarized vector mesons [Ballon Bayona-Boschi-Filho-Braga]
- Nucleons [Gao-Xiao; Gao-Mou]

Our Approach
- Derive the holographic mesons directly from $\mathcal{L}_{DBI}$ up to 2nd order (no $\mathcal{L}_{eff}$)
- Find a Noether current in the bulk

Our Results
- Obtain the full set of analytical structure functions
- Discuss their relation to OPE results
It is a high energy process which allows to investigate the hadronic structure.

It is the scattering of a lepton from a hadron in the high energy regime.

It can be described as the electromagnetic scattering of a lepton by a quark or a parton inside the hadron.

Deep Probes at a small scale into the hadron: \( \lambda_e \ll \lambda_H \)

The hadronic target will fragment in new hadrons

\[ x = -\frac{q^2}{2(p \cdot q)} = 1 + \frac{M_X^2 - M^2}{2(p \cdot q)}, \quad 0 \leq x \leq 1. \]

We take \( q^2 \to \infty \) keeping \( x \) fixed
Cross Section

\[
\frac{d^2 \sigma}{dE'd\Omega} = \frac{e^4}{16\pi^2 q^4} \frac{E'}{ME} \Pi^{\mu\nu} W_{\mu\nu}(P, q)_{h' h} 
\]

Leptonic Tensor

\[
\Pi^{\mu\nu} = \sum_{\text{final spin}} \langle k' | J_\nu^\mu(0) | k, s_l \rangle \langle k, s_l | J_\mu^\nu(0) | k' \rangle
\]

- Computed perturbatively within QED

Hadronic Tensor

\[
W_{\mu\nu}(P, q)_{h' h} = \frac{1}{4\pi} \int d^4x e^{i q \cdot x} \langle P, h' | [J_\mu(x), J_\nu(0)] | P, h \rangle,
\]

- Cannot be calculated within perturbative QCD
The resulting form for the $W_{\mu\nu}$ for spin-0 targets is

$$W_{\mu\nu} = F_1 \eta_{\mu\nu} - \frac{F_2}{P \cdot q} P_\mu P_\nu$$
The Hadronic Tensor $W_{\mu\nu}$

The resulting form for the $W_{\mu\nu}$ for spin-1 targets is \textit{Hoodbhoy–Jaffe–Manohar1988}

\[
W_{\mu\nu} = F_1 \eta_{\mu\nu} - \frac{F_2}{P.q} P_\mu P_\nu
+ b_1 r_{\mu\nu} - \frac{b_2}{6} (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) - \frac{b_3}{2} (s_{\mu\nu} - u_{\mu\nu}) - \frac{b_4}{2} (s_{\mu\nu} - t_{\mu\nu})
- \frac{ig_1}{P.q} \epsilon^{\mu\nu\lambda\sigma} q^\lambda s^\sigma - \frac{ig_2}{(P.q)^2} \epsilon^{\mu\nu\lambda\sigma} q^\lambda (P.q s^\sigma - s.q P^\sigma).
\]

\[
\begin{align*}
    r_{\mu\nu} &\equiv \frac{1}{(P.q)^2} \left( q.\zeta^* q.\zeta - \frac{1}{3} (P.q)^2 \kappa \right) \eta_{\mu\nu}, \\
    s_{\mu\nu} &\equiv \frac{2}{(P.q)^3} \left( q.\zeta^* q.\zeta - \frac{1}{3} (P.q)^2 \kappa \right) P_\mu P_\nu, \\
    t_{\mu\nu} &\equiv \frac{1}{2(P.q)^2} \left( q.\zeta^* P_\mu \zeta_\nu + q.\zeta^* P_\nu \zeta_\mu + q.\zeta P_\mu \zeta^*_\nu + q.\zeta P_\nu \zeta^*_\mu - \frac{4}{3} (P.q) P_\mu P_\nu \right), \\
    u_{\mu\nu} &\equiv \frac{1}{P.q} \left( \zeta^*_\mu \zeta_\nu + \zeta^*_\nu \zeta_\mu + \frac{2}{3} M^2 \eta_{\mu\nu} - \frac{2}{3} P_\mu P_\nu \right), \\
    s^\sigma &\equiv \frac{-i}{M^2} \epsilon^{\sigma\alpha\beta\rho} \zeta^*_\alpha \zeta_\beta P_\rho, \\
    \kappa &\equiv 1 - 4x^2 t \\
\end{align*}
\]
Calculating $W_{\mu\nu}$

**Structure functions**
- $F_i; b_i; g_i$ depend on the adimensional variables

\[
x = -q^2 / 2(P \cdot q) \quad 0 < x \leq 1 \text{ (Bjorken parameter)}
\]
\[
t = P^2 / q^2 \quad t < 0
\]

**Forward Compton scattering scattering amplitude** $T^{\mu\nu}$

\[
\text{Im } T^{\mu\nu} = i \langle P, Q | T(\tilde{J}^\mu(q)J^\nu(0)) | P, Q \rangle
\]

**The optical theorem implies**

\[
\text{Im } T^{\mu\nu} = 2\pi W_{\mu\nu}
\]
## Choosing the QFT

- We want a QFT with a gravity dual, as similar as possible to QCD.
  - Using the correspondence $\rightarrow$ Conformal at some regime.
  - QCD-like $\rightarrow$ Confining
- They choose $\mathcal{N} = 1\star$ SYM. This is $\mathcal{N} = 4$ SYM (conformal) with a conformal symmetry breaking at an energy $m$.

## Geometry

\[
\begin{align*}
    ds^2 &= \frac{r^2}{R^2} \eta_{\mu\nu} dy^\mu dy^\nu + \frac{R^2}{r^2} dr^2 + R^2 \hat{ds}_W^2 ; \\
    R &= (4\pi gN)^{1/4} \alpha'/2
\end{align*}
\]

- Introduce a cutoff $r_0 \equiv \Lambda R^2$
- Regime of interest $q \gg \Lambda \Rightarrow r \gg r_0 \equiv \Lambda R^2$ (Asymptotic $AdS_5 \times S^5$)
- $(g_s N)^{-1/2} \ll x \lesssim 1 \Rightarrow \alpha' s \ll 1$ (Supergravity regime)
A Holographic Dual Description (Polchinski-Strassler)

Incoming hadron: 10D free dilaton. Massless. Spin-0. Represents glueball.

\[ \Phi = e^{iP \cdot y} \psi(r, \Omega). \]

Current insertion

Perturbation in the B.C. at \( r = \infty \). It excites non-normalizable modes in the \( AdS \) interior. Isometry of \( W \) with Killing vector \( \nu_j \)

\[ \delta g_{mj} = A_m(y, r)\nu_j(\Omega). \]
The models

The D3-D7-brane model [Kruczenski-Mateos-Myers-Winters]
- SUGRA IIB
- $N_f = 1$ hypermultiplets
- $\mathcal{N} = 2$ in $D = 4$
- Has dynamical quarks
- Conformal for $r \rightarrow \infty$ (asymptotically $AdS_5 \times S^3$)

The D4-D8-\overline{D8}-brane model [Sakai-Sugimoto]
- SUGRA IIA with 1 compact direction
- $N_f = 1$ hypermultiplets
- $\mathcal{N} = 0$ in $D = 4$ (SUSY completely broken)
- Chiral symmetry breaking: $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$
- Not conformal for $r \rightarrow \infty$
Holographic Mesons

- They are open string excitations of the $D7(D8)$ probe brane.
- Studied as fluctuations of the $S_{DBI}$.

**DBI action for the D7(8)-brane**

\[
S_{D7(8)} = -\mu_{7(8)} \int d^{8(9)} \xi \sqrt{-\det\{P[G]_{ab} + 2\pi\alpha' F_{ab}\}} \\
\quad + \frac{(2\pi\alpha')^2}{2} \mu_7 \int P[C^{(4(5))}] \wedge F \wedge F.
\]
Scalar mesons

Scalar mesons emerge from transversal fluctuations
D7(8)-brane

\[ L_{\text{sc}a} = -\mu_7(8)(\pi \alpha')^2 \sqrt{|\text{det}g|} \left( \frac{R}{r} \right)^{\frac{4}{2}} \left( \frac{3}{2} \right) g^{ab} \left[ \partial_a \Phi \partial_b \Phi^* - \partial_a \Phi^* \partial_b \Phi \right] \]

\[ \Phi = e^{i P \cdot y} \phi(r) Y^\ell(S^3(4)) . \]
Vector mesons emerge from fluctuations parallel to the D7(8)-brane

\[ \mathcal{L}_{\text{vec}} = -\mu_{7(8)}(\pi\alpha')^2 \sqrt{|\det g|} F^{ab} F^{*}_{ab} \]

\[ B_\mu = \zeta_\mu e^{iP \cdot y} \phi(r) Y^\ell(S^3(4)) . \]

\[ B_r = B_i = 0 \ ; \quad P \cdot \zeta = 0 \]
Insertion of a $U(1)$ current in the $AdS_5$ boundary induces a perturbation on the BC and excites a non-normalizable mode $A_m$ in the $AdS_5$ interior. This is the extension of $A_\mu|_{4D}$ into the bulk.

The perturbation propagates into the $AdS_5$ inducing a metric fluctuation

$$
\delta g_{mj} = A_m(y, r)\nu_j(\Omega).
$$

$$
\nu^j \partial_j Y^\ell = iQ_\ell Y^\ell
$$

$$
D_m F^{mn} = 0 \Rightarrow
$$

$$
A_\mu = n_\mu e^{i q \cdot y} a(r) ; \quad A_r = e^{i q \cdot y} b(r)
$$
Interaction and Holographic Identification

**Interaction**

- The gauge field propagating into the boundary couples to the Noether current of the quadratic Lagrangian for the scalar (vector) mesons

\[ \mathcal{L} = Q\sqrt{-g}A^m j_m \]

**Holographic Identification**

\[ S_{int} = (2\pi)^4 \delta^4 (P_X - P - q)n_\mu \langle P + q, X | J^\mu (0) | P, Q \rangle \]

**Calculation of \( T^{\mu\nu} \)**

\[ \text{Im } T^{\mu\nu} = \sum_X \delta (M_X^2 + [P + q]^2) \langle P, Q | J^\nu (0) | P + q, X \rangle \]
\[ \times \langle P + q, X | J^\mu (0) | P, Q \rangle \]
Scalar Mesons

\[ F_1 = 0 \; ; \; F_2 = A'_0 Q^2 \left( \frac{\mu_t^2 \alpha'^4}{\Lambda^8} \right) \left( \frac{\Lambda^2}{q^2} \right)^{\nu+1} x^{\nu+3} (1 - x)^\nu. \]

Vector Mesons

\[ F_1 = A(x) \frac{1}{12x^3} (1 - x - 2xt - 4x^2 t + 4x^3 t + 8x^3 t^2), \]
\[ F_2 = A(x) \frac{1}{6x^3} (1 - x + 12xt - 14x^2 t - 12x^2 t^2), \]
\[ b_1 = A(x) \frac{1}{4x^3} (1 - x - xt), \]
\[ b_2 = A(x) \frac{1}{2x^3} (1 - x - x^2 t), \]
\[ b_3 = A(x) \frac{1}{24x^3} (1 - 4x + 8x^2 t), \]
\[ b_4 = A(x) \frac{1}{12x^3} (-1 + 4x - 2x^2 t), \]
\[ g_1 = A(x) \frac{1}{8x^4} (-3 + 3x + 4xt + 5x^2 t - 6x^3 t - 8x^3 t^2), \]
\[ g_2 = A(x) \frac{1}{16x^4} (3 - 3x - 4xt + 2x^2 t). \]

\[ A(x) = A_0 Q^2 \left( \frac{\mu_t^2 \alpha'^4}{\Lambda^8} \right) \left( \frac{\Lambda^2}{q^2} \right)^{\nu+5} (1 - x)^{\nu-1}, \]
Scalar Mesons

\[ F_1 = 0 \quad ; \quad F_2 = E_0' Q^2 \left( \frac{\mu_R^2 \alpha'^4}{\Lambda^{10}} \right) \left( \frac{\Lambda^2}{q^2} \right)^{\nu+1} x^{\nu+7/2} (1 - x)^\nu. \]

Vector Mesons

\[ F_1 = E(x) \frac{1}{12x^3} (1 - x - 2xt - 4x^2t + 4x^3t + 8x^3t^2), \]
\[ F_2 = E(x) \frac{1}{6x^3} (1 - x + 12xt - 14x^2t - 12x^2t^2), \]
\[ b_1 = E(x) \frac{1}{4x^3} (1 - x - xt), \]
\[ b_2 = E(x) \frac{1}{2x^3} (1 - x - x^2t), \]
\[ b_3 = E(x) \frac{1}{24x^3} (1 - 4x + 8x^2t), \]
\[ b_4 = E(x) \frac{1}{12x^3} (-1 + 4x - 2x^2t), \]
\[ g_1 = E(x) \frac{1}{8x^4} (-3 + 3x + 4xt + 5x^2t - 6x^3t - 8x^3t^2), \]
\[ g_2 = E(x) \frac{1}{16x^4} (3 - 3x - 4xt + 2x^2t). \]

\[ E(x) = E_0 Q^2 \left( \frac{\mu_R^2 \alpha'^4}{\Lambda^{10}} \right) \left( \frac{\Lambda^2}{q^2} \right)^{\nu} x^{\nu+11/2} (1 - x)^{\nu-1}, \]
Conclusions

- We studied the most general case for a spin-one hadron under parity-invariant interactions, in the regime where the entire hadron is scattered.
- We obtained analytical expressions for DIS structure functions for the scalar and vector case.

Scalar mesons

- $F_1 \propto \text{Casimir of scattered hadron under Lorentz Group:}$
  
  \[ F_1 = 0 \]

- $F_2 \propto q^{-2(\tau - 1)}$ in OPE $\Rightarrow \tau = \nu + 2$
Conclusions

Vector mesons - Good Stuff

- Exact modified Callan-Gross relation (for \( t \approx 0 \)):
  \[ 2F_1(x) = F_2(x) \text{ and } 2b_1(x) = b_2(x) \]
- \( b_1 \propto O(F_1) \) for relativistic spin-1/2 constituents
- \( F_2 \propto q^{-2(\tau-1)} \) in OPE \( \Rightarrow \tau = \nu + 2 \)
- Peak at \( W_i \approx \frac{1}{2} = \frac{1}{n} \)
- Same behaviour in both models
- Structure functions do not diverge and right signature.
- \( 2b_3 = -b_4 \); \( 2g_2 = -g_1 \); \( 3F_2 = b_2 \); \( 3F_1 = b_1 \)

Vector mesons - Bad Stuff

- Not valid for \( x \ll 1 \)
- \( 2F_1 = F_2 \Rightarrow \) Full hadron
- \( 3F_1 = b_1 \Rightarrow \) Relativistic spin-1/2 constituents
- All \( W_i \propto q^{-2\nu} = q^{-2(\tau-1)} \Rightarrow \) All same twist. \( \neq \) OPE
THANK YOU!
Sakai and Sugimoto proposed the D4-D8-\(\overline{D8}\) model, which is dual to a theory in the same universality class as QCD in 4D for large \(N\).

Brane configuration:
- \(N\) D4-branes compactified on a \(S^1\).
- \(N_f\) pairs of D8 and \(\overline{D8}\)-branes, transversal to the \(S^1\).

\[
\begin{align*}
\text{D4} &: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad \cdots \\
\text{D8} - \overline{\text{D8}} &: \quad 0 \quad 1 \quad 2 \quad 3 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9
\end{align*}
\]

Chiral symmetry \(U(N_f)_L \times U(N_f)_R\) from QCD is realized as the gauge theory in the world-volume of the \(N_f\) pairs of D8-\(\overline{D8}\)-branes.
Antiperiodic boundary conditions are imposed for the fermions on the circle $S^1$ on the D4-branes $\Rightarrow$ SUSY breaks completely and the spurious fields become massive.

D8 and $\overline{D8}$-branes merge. On the resulting D8-brane there is only one $U(N_f)$ factor left as a gauge symmetry. This is interpreted as a holographic realization of spontaneous chiral symmetry breaking from $U(N_f)_L \times U(N_f)_R$.

In the region of interest, $U \gg U_0 \equiv \Lambda^2 R^3$, the induced metric on the $D8$-brane can be approximated by

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \eta_{\mu\nu} dy^\mu dy^\nu + \left(\frac{R}{U}\right)^{3/2} dU^2 + R^{3/2} U^{1/2} d\Omega_4^2.$$
Gauge fields $A_m$

$$
A_\mu = \frac{2}{\Gamma(5/4)} n_\mu e^{iqy} q^{5/4} \left( \frac{R^3}{U} \right)^{5/8} K_{5/4} \left( \frac{2qR^{3/2}}{U^{1/2}} \right),
$$

$$
A_U = iq.n RE^{i\Phi y} \left( \frac{R}{U} \right)^{17/8} K_{13/4} \left( \frac{2qR^{3/2}}{U^{1/2}} \right).
$$

The fields associated with scalar mesons emerge as fluctuations on the D8-brane.

The obtained solutions are:

$$
\Phi_{IN/OUT} = c_i(\Lambda U)^{-9/8-\nu/2} e^{iP\cdot y} Y(\Omega),
$$

$$
\Phi_X = c_X \Lambda^{-13/8} s^{1/4} U^{-9/8} J_\nu \left( \frac{2s^{1/2} R^{3/2}}{U^{1/2}} \right) e^{iP_X \cdot y} Y(\Omega),
$$

with $\nu = (1/4) \sqrt{81 + 64 \ell (\ell + 3)}$.

The structure functions are

$$
F_1 = 0, \quad F_2 = E_0' Q^2 \left( \frac{\mu_8^2 Q^2}{\Lambda^{10}} \right) \left( \frac{\Lambda}{q^2} \right)^{\nu + 1} x^{\nu + 7/2} (1 - x)^\nu,
$$
Vector mesons in the D4 – D8 – D8 model

Vector mesons emerge from fluctuations of the DBI fields in the directions parallel to the D8-brane.

The solution is

\[ B_{\mu \text{IN/OUT}} = \zeta_\mu c_i \Lambda^{-1} (\Lambda U)^{-\nu/2 - 9/8} e^{iP \cdot y} Y^\ell (S^4), \]

\[ B_{X\mu} = \zeta_{X\mu} c_X \Lambda^{-13/8} s^{-1/4} U^{-9/8} J_\nu \left( \frac{2s^{1/2} R^{3/2}}{U^{1/2}} \right) e^{iP_x \cdot y} Y^\ell (S^4), \]

with \( \nu = (1/4) \sqrt{81 + 64\ell (\ell + 3)} \).
Results and Discussion
$F_1 = 0$ for scalar mesons in both models was expected, since this function is proportional to the Casimir of the scattered hadron under the Lorentz group.

$F_2(x)$ is similar in both models. This is because of the model-independent factor. $F_2(q) \sim q^{-2(\tau_p-1)}$ for scalar mesons corresponds to the twist $\tau_p = \nu + 2$. 
Vector Mesons

$F_1(x)$ - Both models

$F_2(x)$ - Both models

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DIS from Holographic Spin-One Hadrons
Vector Mesons

\[ b_1(x) - \text{Both models} \]

\[ b_2(x) - \text{Both models} \]
Vector Mesons

\[ b_3(x) - \text{Both models} \]

\[ b_4(x) - \text{Both models} \]
Vector Mesons

$g_1(x)$ - Both models

$g_2(x)$ - Both models
The dependences on $x$ are similar in both models, as well as in the scalar mesons case.

We obtained the modified Callan-Gross relation as previously obtained by Polchinski and Strassler for spin-$\frac{1}{2}$ hadrons in the supergravity regime: $F_2(x) = 2F_1(x)$ (entire hadron scattered).

Other Callan-Gross-like relation found: $b_2(x) = 2b_1(x)$

Other relations found: $b_4(x) = -2b_3(x)$, $g_1(x) = -2g_2(x)$, $b_2(x) = 3F_2(x)$ and $b_1(x) = 3F_1(x)$, all of them valid for both models in the $t \to 0$ limit.

The relation $b_1 \sim \mathcal{O}(F_1)$ is consistent with the case of the $\rho$-meson, consisting of relativistic half-spin constituents (Conflict with modified Callan-Gross relations?).

It is expected for $b_3$, $b_4$ and $g_2$ to be non-leading twist, being the rest of the structure functions of leading twist. In our model these are of the same order: $W_j(q) \sim q^{-(\tau_p-1)}$, being the twist $\tau_p = \nu + 1$. 

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DIS from Holographic Spin-One Hadrons
Conclusions

- Deep Inelastic Scattering from scalar and vector mesons was studied by using the holographic models D3-D7 and D4-D8-D8.
- We developed a systematic method based on calculating the Noether current in the DBI Lagrangian in order to obtain $\mathcal{L}_{int}$ in the dual supergravity theory, guaranteeing the existence of a conserved current.
- Analytical expressions were obtained for the wavefunctions of the scalar and vector mesons in the 5D effective space for the D4-D8-D8 model.
- Analytical expressions for the structure functions were found, for the full spectra of scalar and vector mesons in both models.
- We obtained the full tensor structure for the hadronic tensor corresponding to the DIS with polarized spin-1 targets. All the structure functions were calculated, namely $F_1$, $F_2$, $b_1$, $b_2$, $b_3$, $b_4$, $g_1$ and $g_2$.
- These structure functions satisfy the modified Callan-Gross relation $2F_1(x) = F_2(x)$ for $t = 0$.
Outlook

- Extensions to other models like D4 – D6 – \( \overline{D6} \) by KMMW can be done.
- Extension to \( N_f > 1 \) can be studied with the same method.
- Apply this approach to flavoured versions of the Klebanov-Strassler solutions and to the Maldacena-Núñez solution with the addition of matter fields in the fundamental representations.
THANK YOU!
Stuff
Las variables cinemáticas son:

- **M** La masa del blanco hadrónico.
- **E** La energía del leptón incidente.
- **k** El momento del leptón incidente. \( k = (E, 0, 0, E) \) si la masa del leptón se desprecia.
- **Ω** El ángulo sólido que determina la dirección en la cual el leptón es dispersado (el eje \( \hat{z} \) corresponde a la dirección del leptón incidente).
- **E’** La energía del leptón dispersado.
- **k’** El momento del leptón dispersado. \( k’ = (E’, E’\sin\theta\cos\phi, E’\sin\theta\sin\phi, E’\cos\phi) \). 
- **p** El momento del blanco, \( p = (M, 0, 0, 0) \), para una experiencia con el blanco fijo.
- **q** = \( k – k’ \), el momento transferido durante la dispersión (el momento del fotón virtual).
- **ν** = \( E’ – E = p.q/M \), la variación de energía del leptón.
- **y** = \( \nu/E = p.q/p.k \), la fracción de la variación de energía del leptón.
- **q^2** = \( 2EE’(1 – \cos\theta) = 4EE’\sin^2\theta/2 \)
- **x** = \( –q^2/2M\nu = –q^2/2p.q = –q^2/2MEy \).
- **ω** = \( 1/x \).
El tensor $T^{\mu\nu}$ caracteriza la dispersión de Compton y tiene las mismas propiedades de simetría que $W^{\mu\nu}$.

\[
\text{Im } T^{\mu\nu} = 4\pi^2 \sum_{X} \delta(M_X^2 + [P + q]^2) \langle P, Q | J^\nu(0) | P + q, X \rangle \\
\times \langle P + q, X | J^\mu(0) | P, Q \rangle .
\]

El teorema óptico implica

\[
2 \text{Im } \tilde{F}_j = F_j ,
\]

donde $\tilde{F}_j$ es la función de estructura $j$ de $T_{\mu\nu}$ y $F_j$ la de $W_{\mu\nu}$. 
Cross Section

**DIS Amplitude**

\[ i\mathcal{M} = (-ie)^2 \left( \frac{-i\eta_{\mu\nu}}{q^2} \right) \langle k'|J^\mu_i(0)|k, s_i\rangle \langle X|J^\nu_h(0)|P, h\rangle \]

**Cross Section**

\[ \frac{d^2\sigma}{dE'd\Omega} = \frac{e^4}{16\pi^2q^4} \frac{E'}{ME} l_{\mu\nu} W_{\mu\nu}(P, q)_{h' h} \]

- \( E \) Energy of the incoming lepton
- \( E' \) Energy of the scattered lepton
- \( M \) Mass of the target hadron
- \( l_{\mu\nu} \) Leptonic Tensor
- \( W_{\mu\nu} \) Hadronic Tensor
Calculating $I^{\mu\nu}$

The leptonic tensor is defined by

$$I^{\mu\nu} = \sum_{\text{final spin}} \langle k' | J_i^\nu(0) | k, s_l \rangle \langle k, s_l | J_i^\mu(0) | k' \rangle$$

- It can be computed pertubatively within quantum electrodynamics.

For a spin-$\frac{1}{2}$ lepton it is given by

$$I^{\mu\nu} = 2[k^{\mu}k'^{\nu} + k'^{\nu}k^{\mu} - \eta^{\mu\nu}(k \cdot k' - m_l^2) - i\epsilon^{\mu\nu\alpha\beta} q_\alpha s_l \beta]$$

where $m_l$ is the lepton mass and

$$2s_l^{\mu} = \bar{u}(k, s_l) \gamma^{\mu} \gamma_5 u(k, s_l).$$
Calculating $W_{\mu\nu}$

The hadronic tensor is defined as

$$W_{\mu\nu}(P, q)_{h'h} = \frac{1}{4\pi} \int d^4x \ e^{iq\cdot x} \langle P, h'|[J_\mu(x), J_\nu(0)]|P, h\rangle,$$

- This tensor completely characterizes the relevant hadronic structure for DIS.
- It cannot be calculated within perturbative QCD.
- It can be decomposed into structure functions, related to parton distribution functions.

The structure functions are adimensional functions of $P^2$, $P \cdot q$, $q^2$.

We define the new adimensional variables:

$$x = -\frac{q^2}{2(P \cdot q)} \quad 0 < x \leq 1 \ (Bjorken \ parameter)$$

$$t = \frac{P^2}{q^2} \quad t < 0$$
Calculating $W_{\mu\nu}$

### Physical requirements for $W_{\mu\nu}$

- **Current conservation** $\partial_\mu J^\mu(x) = 0$, since we are dealing with parity-preserving interactions. This implies
  
  $$q^\mu W_{\mu\nu}(P, q, h) = q^\nu W_{\mu\nu}(P, q, h) = 0$$

- **Parity invariance:** $W_{\mu\nu}(P, q, h) = W_{\mu_p\nu_p}(P_p, q_p, h_p)$

- **Time-reversal symmetry:** $W_{\mu\nu}(P, q, h) = W^*_{\nu_T\mu_T}(P_T, q_T, h_T)$

- **Translation invariance:** $W_{\mu\nu}(P, q, h) = -W_{\nu\mu}(P, -q, h)$
Calculating $W_{\mu\nu}$

The resulting form for the $W_{\mu\nu}$ for spin-0 targets is

$$W_{\mu\nu} = F_1 \eta_{\mu\nu} - \frac{F_2}{P.q} P_\mu P_\nu$$
Calculating $W_{\mu\nu}$

The resulting form for the $W_{\mu\nu}$ for spin-1 targets is $\text{Hoodbhoy} - \text{Jaffe} - \text{Manohar}1988$

\[
W_{\mu\nu} = F_1 \eta_{\mu\nu} - \frac{F_2}{P.q} P_\mu P_\nu \\
+ b_1 r_{\mu\nu} - \frac{b_2}{6} (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) - \frac{b_3}{2} (s_{\mu\nu} - u_{\mu\nu}) - \frac{b_4}{2} (s_{\mu\nu} - t_{\mu\nu}) \\
- \frac{ig_1}{P.q} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma - \frac{ig_2}{(P.q)^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (P.q s^\sigma - s.q P^\sigma).
\]

\[
r_{\mu\nu} \equiv \frac{1}{(P.q)^2} \left( q.\zeta^* q.\zeta - \frac{1}{3} (P.q)^2 \kappa \right) \eta_{\mu\nu},
\]

\[
s_{\mu\nu} \equiv \frac{2}{(P.q)^3} \left( q.\zeta^* q.\zeta - \frac{1}{3} (P.q)^2 \kappa \right) P_\mu P_\nu,
\]

\[
t_{\mu\nu} \equiv \frac{1}{2(P.q)^2} \left( q.\zeta^* P_\mu \zeta_\nu + q.\zeta^* P_\nu \zeta_\mu + q.\zeta P_\mu \zeta^*_\nu + q.\zeta P_\nu \zeta^*_\mu - \frac{4}{3} (P.q) P_\mu P_\nu \right),
\]

\[
u_{\mu\nu} \equiv \frac{1}{P.q} \left( \zeta^*_\mu \zeta_\nu + \zeta^*_\nu \zeta_\mu + \frac{2}{3} M^2 \eta_{\mu\nu} - \frac{2}{3} P_\mu P_\nu \right),
\]

\[
s^\sigma \equiv \frac{-i}{M^2} \epsilon^{\sigma\beta\rho} \zeta^*_\alpha \zeta_{\beta} P_\rho,
\]

\[
\kappa \equiv 1 - 4x^2 t
\]
The structure functions are non-perturbative in QCD.

How to study them?

- OPE
- Phenomenological models (e.g., parton model)
- Lattice QCD
- AdS/CFT
We study Deep Inelastic Scattering in a regime with:

- $N \to \infty$.
- 't Hooft parameter $\lambda \equiv g_{YM}^2 N \equiv g_s N$ is $\lambda \gg 1$, but finite.
- Bjorken parameter is $x \sim 1$ (with $x < 1$).
The tensor $T_{\mu \nu}$ characterizes the Compton scattering and has the same symmetry properties as $W_{\mu \nu}$.

\[
\text{Im } T_{\mu \nu} = 4\pi^2 \sum_X \delta(M_X^2 + [P + q]^2) \langle P, Q| J^{\nu}(0)|P + q, X\rangle \\
\times \langle P + q, X| J^{\mu}(0)|P, Q\rangle.
\]

The optical theorem implies

\[
2 \text{Im } T_j = W_j,
\]

being $T_j$ the j-th structure function for $T_{\mu \nu}$ and $W_j$ that of $W_{\mu \nu}$.
A Holographic Dual Description (Polchinski-Strassler)
Selecting the QFT

- We want a QFT with a gravity dual, as similar as possible to QCD.
  - Using the correspondence $\rightarrow$ Conformal at some regime.
  - QCD-like $\rightarrow$ Asymptotically free

- $SU(N)$ theories with $\mathcal{N} = 0$ and $\mathcal{N} = 1$ realize this, being “almost-conformal” for $P \gg \Lambda$, as QCD is.

- We choose $\mathcal{N} = 1^* \text{SYM}$. This is $\mathcal{N} = 4 \text{SYM}$ (conformal) with a conformal symmetry breaking at an energy $m$. It turns out a theory with broken SUSY to $\mathcal{N} = 1$, keeping the same non-massive fields and adding scalars and fermions in the adjoint representation at $E = m$.

- $m \gg \Lambda \Rightarrow$ Perturbative QFT (small $g$) but strongly-coupled gravity theory

- $m \sim \Lambda \Rightarrow$ Non-perturbative QFT (large $g$) and weakly-coupled gravity theory

$$R \sim (gN)^{1/4}$$
AdS$_5 \times W$ space:

\[ ds^2 = \frac{r^2}{R^2} \eta_{\mu \nu} dy^\mu dy^\nu + \frac{R^2}{r^2} dr^2 + R^2 \tilde{ds}_W^2, \]
\[ R = (4\pi gN)^{1/4} \alpha'^{1/2} \]

- Gravitational red-shift: \( \tilde{p}_\mu = \frac{R}{r} p_\mu \). \( \Rightarrow E^{(4)} \sim \frac{r}{R^2} \).
- Asymptotic AdS$_5 \times S^5$ for \( r \gg r_0 \equiv \Lambda R^2 \) (i.e.: \( q \gg \Lambda \)).
- 10D scale: \( \tilde{s} = \frac{(1/x-1)}{\alpha'(4\pi g_s N)^{1/2}} \) then
  \[ (g_s N)^{-1/2} \ll x \lesssim 1 \Rightarrow \alpha' \tilde{s} \ll 1 \] (Supergravity regime)
String Theory Approach to DIS

Incoming hadron: 10D free dilaton. Massless. Spin-0.

$$\Phi = e^{iP \cdot y} \psi(r, \Omega).$$

Current insertion

Perturbation in the B.C. at $r = \infty$. It excites non-normalizable modes in the $AdS$ interior. Isometry of $W$ with Killing vector $\nu_j$

$$\delta g_{mj} = A_m(y, r)\nu_j(\Omega).$$
String Theory Approach to DIS

Gauge Fields

\[ A_\mu = n_\mu e^{iqy} \frac{qR^2}{r} K_1[qR^2/r], \]
\[ A_r = -i(q \cdot n)e^{iqy} \frac{R^4}{r^3} K_0[qR^2/r], \]

Holographic Hadron

\[ \Phi_i = e^{iP.y} \frac{c_i}{\Lambda R^4} \left[ \frac{r}{r_0} \right]^{-\Delta} Y(\Omega), \]
\[ \Phi_X = e^{i(P+q).y} \frac{c_X}{r^2} J_{\Delta-2} \left[ \frac{s^{1/2} R^2}{r} \right] Y(\Omega). \]
String Theory Approach to DIS

Interaction

\[
S_{int} = \int d^{10}x \sqrt{-g} A^m_{\nu j} \partial_m \Phi \partial_j \Phi^* . \\
= iQ \int d^{10}x \sqrt{-g} A^m (\Phi \partial_m \Phi^*_X - \Phi^*_X \partial_m \Phi) \\
\nu^j \partial_j Y^\ell (\Omega) = iQ_{\ell} Y^\ell (\Omega)
\]

Ansatz

\[
S_{int} = (2\pi)^4 \delta^4 (P_X - P - q) n_{\mu} \langle P + q, X | J^\mu (0) | P, Q \rangle .
\]
String Theory Approach to DIS

We calculate the tensor for Compton scattering

\[
\text{Im } T^{\mu\nu} = 4\pi^2 \sum_X \delta(M_X^2 + [P + q]^2) \langle P, Q | J^{\nu}(0) | P + q, X \rangle \\
\times \langle P + q, X | J^{\mu}(0) | P, Q \rangle
\]

Sum over radial excitations

\[
\sum_n \delta(M_n^2 - s) \sim \left( \frac{\partial M_n^2}{\partial n} \right)^{-1} \sim (2\pi s^{1/2}\Lambda)^{-1}
\]

It turns out

\[
F_1 = 0, \quad F_2 = A_0 Q^2 \left( \frac{\Lambda^2}{q^2} \right)^{\Delta-1} x^{\Delta+1} (1 - x)^{\Delta-2}, \\
A_0 = 2^{2\Delta} \pi |c_i|^2 |c_X|^2 \Gamma(\Delta)^2
\]
DIS from mesons in the D3-D7-brane model
The D3 − D7 model

- We include $N_f$ matter hypermultiplets in the fundamental representation by introducing $N_f$ D7-branes in the $AdS_5 \times S^5$ background \cite{Karch-Katz-2002}.

**Brane configuration**

| D3 | 0 1 2 3 − − − − − − |
| D7 | 0 1 2 3 4 5 6 7 − − |

- We take $N_f = 1 \Rightarrow$ flavour symmetry $U(1)$. Theory $\mathcal{N} = 2 \ SU(N)$ SYM.
- We approximate the metric by $AdS_5 \times S^3$ in the interaction region.

**DBI action for the D7-brane**

\[
S_{D7} = -\mu_7 \int d^8 \xi \sqrt{-\text{det}\{P[G]_{ab} + 2\pi\alpha' F_{ab}\}} \\
+ \frac{(2\pi\alpha')^2}{2} \mu_7 \int P[C^{(4)}] \wedge F \wedge F.
\]
Scalar mesons emerge as transversal fluctuations of the D7-brane. The solution for these mesons is

\[
\Phi_{\text{IN/OUT}} = c_i (\Lambda \rho)^{-\nu - 1} e^{iP \cdot y} Y(\Omega),
\]

\[
\Phi_\chi = c_\chi \Lambda^{-3/2} s^{1/4} \rho^{-1} J_\nu \left( \frac{s^{1/2} R^2}{\rho} \right) e^{iP_\chi \cdot y} Y(\Omega),
\]

with \( \nu = \sqrt{m_\ell^2 + 1} = \ell + 1 \) and \( s = -(P + q)^2 = M_\chi^2 \).

The interaction Lagrangian \( \mathcal{L}_{\text{int}} \) calculated from the metric fluctuation \( \delta g^{mj} = A^m(y, \rho) \nu^j(\Omega) \) is

\[
\mathcal{L}_{\text{int}} = i Q \mu_7 (\pi \alpha')^2 \sqrt{|\det g|} R^2 \rho^{-2} A^m (\Phi \partial_m \Phi^*_\chi - \Phi^*_\chi \partial_m \Phi),
\]

\[
\equiv Q \sqrt{|\det g|} A^m j_m.
\]

Structure functions are

\[
F_1 = 0,
\]

\[
F_2 = A'_0 Q^2 \left( \frac{\mu_7^2 \alpha'^4}{\Lambda^8} \right) \left( \frac{\Lambda^2}{q^2} \right)^{\nu + 1} x^{\nu + 3} (1 - x)^\nu.
\]
Vector mesons in the D3 − D7 model

- Vector mesons emerge from fluctuations of the DBI fields in directions parallel to the D7-brane.

Solution

\[ B_{\mu \text{IN/OUT}} = \zeta_{\mu} c_{i} \Lambda^{-1}(\Lambda \rho)^{-\nu-1} e^{i P \cdot y} Y^{\ell}(S^3), \]
\[ B_{\mu \chi} = \zeta_{\chi \mu} c_{\chi} s^{-1/4} \Lambda^{-3/2} \rho J_{\nu} \left( \frac{s^{1/2} R^{2}}{\rho} \right) e^{i P_{\chi} \cdot y} Y^{\ell}(S^3), \]

with \( \nu = \ell + 1. \)

- The interaction Lagrangian \( \mathcal{L}_{\text{int}} \) calculated from the metric fluctuation \( \delta g^{mj} = A^{m}(y, \rho)\nu^{j}(\Omega) \) is

\[
\mathcal{L} = 2i Q_{\mu 7}(\pi \alpha')^{2} \sqrt{\det g} A_{m}[B_{xn}^{*} F^{nm} - B_{n}(F_{X}^{nm})^{*}] \\
\equiv Q \sqrt{\det g} A^{m} j_{m}.
\]
Structure Functions

\[ F_1 = A(x) \frac{1}{12x^3} (1 - x - 2xt - 4x^2t + 4x^3t + 8x^3t^2), \]

\[ F_2 = A(x) \frac{1}{6x^3} (1 - x + 12xt - 14x^2t - 12x^2t^2), \]

\[ b_1 = A(x) \frac{1}{4x^3} (1 - x - xt), \]

\[ b_2 = A(x) \frac{1}{2x^3} (1 - x - x^2t), \]

\[ b_3 = A(x) \frac{1}{24x^3} (1 - 4x + 8x^2t), \]

\[ b_4 = A(x) \frac{1}{12x^3} (-1 + 4x - 2x^2t), \]

\[ g_1 = A(x) \frac{1}{8x^4} (-3 + 3x + 4xt + 5x^2t - 6x^3t - 8x^3t^2), \]

\[ g_2 = A(x) \frac{1}{16x^4} (3 - 3x - 4xt + 2x^2t). \]

where \( A(x) \) is given by

\[ A(x) = A_0 Q^2 \left( \frac{\mu_{\perp}^2 (\alpha')^4}{\Lambda^8} \right) \left( \frac{\Lambda^2}{q^2} \right)^\nu x^{\nu+5} (1 - x)^{\nu-1}, \]

and \( A_0 = |c_i|^{2} |c_X|^{2} 2^{4+2\nu} [\Gamma(2 + \nu)]^{2} \pi^{5} \) is a dimensionless normalization constant.
DIS from mesons in the D4-D8-D8-brane model
Sakai and Sugimoto proposed the D4-D8-D8 model, which is dual to a theory in the same universality class as QCD in 4D for large $N$.

**Brane configuration:**
- $N$ D4-branes compactified on a $S^1$.
- $N_f$ pairs of D8 and $\overline{D8}$-branes, transversal to the $S^1$.

\[
\begin{align*}
\text{D4} & : 0 \ 1 \ 2 \ 3 \ 4 \ -\ -\ -\ -\ - \\
\text{D8} - \overline{\text{D8}} & : 0 \ 1 \ 2 \ 3 \ -\ 5 \ 6 \ 7 \ 8 \ 9
\end{align*}
\]

Chiral symmetry $U(N_f)_L \times U(N_f)_R$ from QCD is realized as the gauge theory in the world-volume of the $N_f$ pairs of D8-$\overline{\text{D8}}$-branes.
Antiperiodic boundary conditions are imposed for the fermions on the circle $S^1$ on the D4-branes $\Rightarrow$ SUSY breaks completely and the spurious fields become massive.

D8 and $\overline{\text{D}8}$-branes merge. On the resulting D8-brane there is only one $U(N_f)$ factor left as a gauge symmetry. This is interpreted as a holographic realization of spontaneous chiral symmetry breaking from $U(N_f)_L \times U(N_f)_R$.

In the region of interest, $U \gg U_0 \equiv \Lambda^2 R^3$, the induced metric on the D8-brane can be approximated by

$$ds^2 = \left( \frac{U}{R} \right)^{3/2} \eta_{\mu\nu} dy^\mu dy^\nu + \left( \frac{R}{U} \right)^{3/2} dU^2 + R^{3/2} U^{1/2} d\Omega_4^2.$$
Gauge fields $A_m$

\[
A_\mu = \frac{2}{\Gamma(5/4)} n_\mu e^{iqy} q^{5/4} \left( \frac{R}{U} \right)^{5/8} K_{5/4} \left( \frac{2qR^3/2}{U^{1/2}} \right),
\]
\[
A_U = iq n R e^{iqy} \left( \frac{R}{U} \right)^{17/8} K_{13/4} \left( \frac{2qR^3/2}{U^{1/2}} \right).
\]

The fields associated with scalar mesons emerge as fluctuations on the $D8$-brane.

The obtained solutions are:

\[
\Phi_{IN/OUT} = c_i (\Lambda U)^{-9/8 - \nu/2} e^{i \mathbf{P} \cdot \mathbf{y}} Y(\Omega),
\]
\[
\Phi_X = c_X \Lambda^{-13/8} s^{1/4} U^{-9/8} J_\nu \left( \frac{2s^{1/2} R^{3/2}}{U^{1/2}} \right) e^{i \mathbf{P} \cdot \mathbf{x}} y Y(\Omega),
\]

with $\nu = (1/4) \sqrt{81 + 64 \ell (\ell + 3)}$.

The structure functions are

\[
F_1 = 0, \quad F_2 = E'_0 Q^2 \left( \frac{\mu_8^2 \alpha'^4}{\Lambda^{10}} \right) \left( \frac{\Lambda^2}{q^2} \right)^{\nu + 1} x^{\nu + 7/2} (1 - x)^\nu,
\]
Vector mesons emerge from fluctuations of the DBI fields in the directions parallel to the $D8$-brane.

The solution is

$$
B_{\mu \text{IN/OUT}} = \zeta_{\mu} c_i \Lambda^{-1} (\Lambda U)^{-\nu/2-9/8} e^{iP \cdot y} Y^\ell (S^4),
$$

$$
B_{X\mu} = \zeta_{X\mu} c_X \Lambda^{-13/8} s^{-1/4} U^{-9/8} J_\nu \left( \frac{2s^{1/2} R^{3/2}}{U^{1/2}} \right) e^{iP_{X \cdot y}} Y^\ell (S^4),
$$

with $\nu = (1/4) \sqrt{81 + 64\ell (\ell + 3)}$. 

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DIS from Holographic Spin-One Hadrons
Results and Discussion
$F_1 = 0$ for scalar mesons in both models was expected, since this function is proportional to the Casimir of the scattered hadron under the Lorentz group.

$F_2(x)$ is similar in both models. This is because of the model-independent factor. $F_2(q) \sim q^{-2(\tau_p - 1)}$ for scalar mesons corresponds to the twist $\tau_p = \nu + 2$. 
Vector Mesons

$F_1(x)$ - Both models

$F_2(x)$ - Both models
Vector Mesons

\[ b_1(x) - \text{Both models} \]

\[ b_2(x) - \text{Both models} \]
Vector Mesons

$b_3(x) -$ Both models

$b_4(x) -$ Both models
Vector Mesons

$g_1(x)$ - Both models

$g_2(x)$ - Both models
The dependences on $x$ are similar in both models, as well as in the scalar mesons case. We obtained the modified Callan-Gross relation as previously obtained by Polchinski and Strassler for spin-$\frac{1}{2}$ hadrons in the supergravity regime: $F_2(x) = 2F_1(x)$ (entire hadron scattered). Other Callan-Gross-like relation found: $b_2(x) = 2b_1(x)$.

Other relations found: $b_4(x) = -2b_3(x)$, $g_1(x) = -2g_2(x)$, $b_2(x) = 3F_2(x)$ and $b_1(x) = 3F_1(x)$, all of them valid for both models in the $t \rightarrow 0$ limit.

The relation $b_1 \sim O(F_1)$ is consistent with the case of the $\rho$-meson, consisting on relativistic half-spin constituents (Conflict with modified Callan-Gross relations?).

It is expected for $b_3$, $b_4$ and $g_2$ to be non-leading twist, being the rest of the structure functions of leading twist. In our model these are of the same order: $W_j(q) \sim q^{-2(\tau_p-1)}$, being the twist $\tau_p = \nu + 1$. 

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DIS from Holographic Spin-One Hadrons
Conclusions and Outlook
Conclusions

- Deep Inelastic Scattering from scalar and vector mesons was studied by using the holographic models D3-D7 and D4-D8-D8.

- We developed a systematic method based on calculating the Noether current in the DBI Lagrangian in order to obtain $\mathcal{L}_{int}$ in the dual supergravity theory, guaranteeing the existence of a conserved current.

- Analytical expressions were obtained for the wavefunctions of the scalar and vector mesons in the 5D effective space for the D4-D8-D8 model.

- Analytical expressions for the structure functions were found, for the full spectra of scalar and vector mesons in both models.

- We obtained the full tensor structure for the hadronic tensor corresponding to the DIS with polarized spin-1 targets. All the structure functions were calculated, namely $F_1$, $F_2$, $b_1$, $b_2$, $b_3$, $b_4$, $g_1$ and $g_2$.

- These structure functions satisfy the modified Callan-Gross relation $2F_1(x) = F_2(x)$ for $t = 0$. 
Outlook

- Extensions to other models like D4 – D6 – \( \overline{D6} \) by KMMW can be done.
- Extension to \( N_f > 1 \) can be studied with the same method.
- Apply this approach to flavoured versions of the Klebanov-Strassler solutions and to the Maldacena-Núñez solution with the addition of matter fields in the fundamental representations.
THANK YOU!
Stuff
Las variables cinemáticas son:

- \( M \): La masa del blanco hadrónico.
- \( E \): La energía del leptón incidente.
- \( k \): El momento del leptón incidente. \( k = (E, 0, 0, E) \) si la masa del leptón se desprecia.
- \( \Omega \): El ángulo sólido que determina la dirección en la cual el leptón es dispersado (el eje \( \hat{z} \) corresponde a la dirección del leptón incidente).
- \( E' \): La energía del leptón dispersado.
- \( k' \): El momento del leptón dispersado.
  \[ k' = (E', E'\sin\theta\cos\phi, E'\sin\theta\sin\phi, E'\cos\phi) \]
- \( p \): El momento del blanco, \( p = (M, 0, 0, 0) \), para una experiencia con el blanco fijo.
- \( q = k - k' \), el momento transferido durante la dispersión (el momento del fotón virtual).
- \( \nu = E' - E = p.q/M \), la variación de energía del leptón.
- \( y = \nu/E = p.q/p.k \), la fracción de la variación de energía del leptón.
- \( q^2 = 2EE'(1 - \cos\theta) = 4EE'\sin^2\theta/2 \)
- \( x = -q^2/2M\nu = -q^2/2p.q = -q^2/2MEy \)
- \( \omega = 1/x \)
El tensor $T^{\mu\nu}$ caracteriza la dispersión de Compton y tiene las mismas propiedades de simetría que $W^{\mu\nu}$.

\[
\text{Im } T^{\mu\nu} = 4\pi^2 \sum_X \delta(M_X^2 + [P + q]^2) \langle P, Q | J^{\nu}(0) | P + q, X \rangle \\
\times \langle P + q, X | J^{\mu}(0) | P, Q \rangle.
\]

El teorema óptico implica

\[
2 \text{Im } \tilde{F}_j = F_j,
\]

donde $\tilde{F}_j$ es la función de estructura $j$ de $T_{\mu\nu}$ y $F_j$ la de $W_{\mu\nu}$.