Universal Classical Optical Computing Inspired by Quantum Information Process

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1. Introduction

The fundamental idea of optical computing, or optical information processing, is based on the benefits of encoding information by light. Using the properties of fast-propagation and coherence, the encoded information is expected to be processed by a device with low heat generation, high degree of parallelism, and fast calculation speed. During the past 60 years, a series of achievements on optical computing have been done, including joint transform correlator, vector matrix multiplier, energy-conserving gates and circuits, etc. However, several important problems are still to be solved. For example, an efficient nonlinear element for optical switching is missing.

It is the key ingredient in the optical digital computing devices which are the mimics of the current electronic computers. In recent years, the main research interests of the optical computing have turned to developing the devices for special tasks, and two outstanding works that follow the spirit have been completed. One is the analog optical computing based on metamaterial, in contrast to the digital optical computing scheme, analog optical computing employs the light amplitude distribution as the information carrier, and the engineered photonic structures with certain permittivity and permeability as the modulation approach. Using such a framework, several kinds of applications have been realized, such as the calculation of differentiation, integration, Laplace operator, solutions to linear equations, etc. The other refers to optical Ising machines. Those machines imitate the networks of interaction magnetic spins by optical setups such as correlated optical parametric oscillators; bulk optics with spatial light modulators, or etc., aiming to solve complex optimization problems. Although the full potential of the Ising machines remains to be explored, their speed-ups over conventional digital computers in several cases have already been demonstrated.

Nevertheless, a blueprint for building a universal optical computing device is absent. The investigation on this kind of blueprint is important from two aspects. First, assessing the universality of an optical computing system is a theoretical treatment for knowing the upper limit of its computing capability. For example, if an optical system could perform universal computing, it would be powerful enough to calculate all possible computable functions. Second, the discussion of the connection of the optical modules for universal computing would also specify the roadmap to large-scale devices applicable for practical issues. One good example of the universal computing model that promotes the relevant researches is quantum computing. Early exploration on universal quantum computing is started by D. Deutsch, and further explained by D. DiVincenzo and A. Barenco using simple universal gate sets. Based on that foundation, a series of quantum algorithms have been proposed, including the famous Shor’s integer factoring algorithm and Grover’s searching algorithm. Also, such foundation settles the direction of the experimental quantum
computing research which now concentrates on optimizing the quality of the gate circuits generated by the universal gate sets, and lots of exciting work have been accomplished.\cite{45-51}

Inspired by quantum computing strategy, we propose a universal optical computing scheme that can be considered as an analogy of the universal quantum computing, indicating the upper limit of the capability of optical computing to be as powerful as that of quantum computing. Specifically, the operations on light equivalent to the quantum universal gate set are given. Based on our theoretical consideration, we present an experimental verification on the analogy of the two-qubit processor. In the following, we first establish the theory of our universal optical computing.

2. Universal Computing Based on the States of Classical Beams

A physical system for computation requires two basic pillars. One is a mapping of the states of the system to the information that is to be computed. The other is the well-defined control on the system, guaranteeing that the final state after the control would give the results of certain computational tasks. For example, in the theory of quantum computing, a two-level state of a system, or a qubit, is employed to represent a bit of information and the unitary operators on them are the required controls (see S1, Supporting Information for more details). The system that performs such a computing strategy must contain the degree of freedom for encoding qubits, and the unitary controls on qubits of the system must be well-defined. One of the candidates is the optical system.

2.1. Information Encoding by the States of Classical Beams

An optical field possesses a rich degree of freedom for information encoding, some of which have a close relation with qubits.\cite{52-56} The polarization state of a light field in a 2D plane, for instance, is a well-defined two-level state, no matter whether the light is in the quantum regime or the classical regime. The two-level state based on the polarization of a classical light field can be denoted by

\[ |E\rangle = c_1 |h\rangle + c_2 |v\rangle \]  

(1)

Here, we use a slightly modified quantum bra-ket notation to denote the classical two-level state. Such a notation was first introduced by R. Speeruw for describing the optical degree of freedoms analogous to qubits.\cite{52} However, the state expressed by Equation (1) is different from that defined by Speeruw. We only adopt the spirit of the previous work for simulating a qubit by a classical optical state and also call it a cebit. |h\rangle and |v\rangle denote the basis of the state, referring to the horizontal and vertical polarization state respectively. Complex number \(c_1\) and \(c_2\) are the projection of cebit state |E\rangle on |h\rangle and |v\rangle respectively, denoted by \(c_1 = \langle h|E\rangle\) and \(c_2 = \langle v|E\rangle\) with the constrain \([c_1]^2 + [c_2]^2 = 1\). Like the bra-ket notation, the complex conjugate of |E\rangle can be denoted by \(\langle E\rangle\). Therefore, the counterpart of density operator can be denoted by \(|\langle E\rangle E\rangle\). More generally, for the distinct polarization states of different light fields, an N-cebit state |NE\rangle can be defined by

\[ |NE\rangle = \sum_{j_1,j_2,...,j_N=0}^{1} c_{j_1,j_2,...,j_N} |j_1\rangle |j_2\rangle ... |j_N\rangle \]  

(2)

where \(|j_1\rangle |j_2\rangle ... |j_N\rangle\) represents a bunch of correlated polarization states, each of which is either |h\rangle or |v\rangle, and can be measured independently. The correlation information of the state is characterized by the complex coefficient \(c_{j_1,j_2,...,j_N}\), the subscripts \(j_m\) of which is 0 or 1 for integer \(m\) ranging from 1 to \(N\). \(|j_m\rangle = |h\rangle\) when \(j_m = 0\), and \(|j_m\rangle = |v\rangle\) when \(j_m = 1\). Comparing Equations (1) and (2) with the related qubit states (Equations S1 and S2 in S1, Supporting Information), one can easily certify the correspondence between them.

There are various types of classical light fields available for encoding the cebits given by Equations (1) and (2). Here, we consider a multi-mode polarized beam for encoding a cebit. The expression of the beam field is given by

\[ E(r, t) = \sum_{p=1}^{P} F_p(r) p_c(r) \]  

(3)

where \(r\) and \(t\) are the transverse coordinate and time. \(F_p(r, t)\) is a set of normalized orthonormal modes in time domain, or spatial domain, or others, under the condition \(\int f_{p_1}(r, t) ... f_{p_k}(r, t) d\Omega = \delta_{k_1,...,k_p}\). \(\Omega\) is the parameter of the domain. \(p_c(r)\) is the polarization vector field of \(F_p\). \(P\) is the total mode number. Then, \(c_1\) in Equation (1) can be given by the projection of Equation (3) on the horizontal and vertical polarization state. Notice that the projection of Equation (3) is a summation of the polarization components of all the modes. Therefore, the analogy of a qubit we propose here is the space spanned by the measurements of the beam in the orthonormal polarization basis, not the polarization states of a single mode (such as the single photon mode). Although this is different from the traditional physical picture, we find that a well-defined information process can be given based on it. An example of the setup for obtaining projection is shown in Figure 1a. A local oscillator (LO) \(E^{10}\) is introduced whose polarization state \(e(h, v, or their superpositions) is set to be the direction of projection. Then, \(E^{10}\) and beam \(E\) of a single cebit interferes at the beam splitter (BS in Figure 1a) the outputs are collected by mode-revoluted detectors \(D_1\) and \(D_2\). The real part of \(\langle e|E\rangle\) can be obtained by measuring the difference of the signals of \(D_1\) and \(D_2\) with subtraction device \(D_3\). The imaginary part can be measured in the same way, except for adding a \(\pi/2\) phase shift to \(E^{10}\).

More generally, an N-cebit state can be encoded by \(N\) distinguishable beams of the same form with Equation (3). Then, \(c_{j_1,j_2,...,j_N}\) of Equation (2) can be given by measuring the correlation of the projections of \(N\) multi-mode polarized beams, expressed by \(c_{j_1,j_2,...,j_N} = \langle j_1|j_2| ... |j_N\rangle |NE\rangle\). (For instance, \(c_{10...0}\) is given by measuring the correlation of the projection of all the beams on horizontal polarization except for that of the first beam on the vertical, expressed by \(|\langle v|j_2| ... |j_N\rangle |NE\rangle\). Such a measurement can be realized by a two-step procedure: 1) measure the local...
The beams, the real (imaginary) part of integrating the multiplication of the local projection signals all performing the interferometry setup shown in Figure 1a. After beams (denoted by $E_1$, $E_2$, $E_3$, $E_4$) are introduced for the correlated measurement is illustrated in Figure 1b. The setup is given by the left panel, and a simple denotation of it is given by the right panel. The control cebit is encoded by $E_c$, and the target cebit is encoded by $E_t$. The output of target cebit is encoded by $E'_t$. The output control cebit has no changes. e) A simplified scheme for CCX considering a special implementation of two-cebit states, including a mode splitter (MS), a polarization beam splitter (PBS), a phase shifter (PS), and a cylinder lens (CL). $E_c$ encodes the control cebit, $E_t$ encodes the target cebit. $E_c$ and $E_t$ are the output. f) A scheme for an arbitrary 2-cebit operation (upper panel) and its corresponding quantum circuit (lower panel). $U_{1E_1}$ to $U_{4E_4}$ are single cebit operations implemented by QWPs and HWPs, corresponding to the eight single qubit gates denoted by $R_z$, $R_x$, $R_y$, $R_{z_{\pi}}$, $R_{x_{\pi}}$, $R_{y_{\pi}}$, $R_{z_{\pi/2}}$, $R_{x_{\pi/2}}$, $R_{y_{\pi/2}}$, $R_{z_{\pi/4}}$, $R_{x_{\pi/4}}$, $R_{y_{\pi/4}}$, $R_{z_{\pi/8}}$, $R_{x_{\pi/8}}$, $R_{y_{\pi/8}}$. The relations are marked by arrows.

projection of each beam, and 2) multiply the measured signal and integrate the result in the domain of $f_c$. An example of the setup for the correlated measurement is illustrated in Figure 1b. For each beam (denoted by $E_{n_{\pm}1}$, $E_{n_{\pm}2}$, $E_{n_{\pm}3}$), associated LO beams (denoted by $E_{LO,0}$, $E_{LO,1}$, $E_{LO,2}$) are introduced for performing the interferometry setup shown in Figure 1a. After integrating the multiplication of the local projection signals of all the beams, the real (imaginary) part of $E_{nj_{\pm}m_{\pm}}$ can be obtained. Such a setup corresponds to the coincidence counters applied to quantum optics experiments. A detailed theoretical analysis of the measurement schemes shown in Figure 1a,b is presented in “Theoretical Analysis of the Measurement Scheme for Cebits.”

### 2.2. The Universal Operations on Cebits

The universal computing based on cebits requires the well-defined controls on the physical systems such that all cebit states are addressable. Considering the relation between qubits and cebits, those controls are unitary. Therefore, we present the method for implementing arbitrary unitary operations on cebits encoded by multi-mode polarized beams, using the light modulations. We start from the introduction of the setup for two basic operations.

The first one is single cebit operations. Suppose that a multi-mode polarized beam $E_q$ encodes a cebit. The unitary operations on such a single cebit can be denoted by $U_{1E_q} = R_y(\xi)R_y(\eta)R_y(\zeta)$, where $R_y(\xi)$ ($R_y(\eta)$) is the Pauli-$Y$ (Pauli-$Z$) rotation by angle $\xi$ ($\eta$). The operations can be performed by letting $E_q$ pass through a quarter-wave plate (QWP), a half-wave plate (HWP) and a QWP sequentially (also called a Q-H-Q), shown in Figure 1c. The fast axes of them are at angles $\pi/4 + \xi/2$, $-\pi/4 + \eta - \zeta/4$ and $\pi/4 - \zeta/2$ respectively. The output beam is denoted by $E'_q$. Then, the transformation of the cebit encoded by $E_q$, $E'_q$ is $U_{1q}$. Notice that parameter $\xi$, $\eta$ and $\zeta$ are arbitrary real numbers. A theoretical instruction of the relation between the Q-H-Qs and the unitary operations on cebits is given in “The Unitary Operations of a Single Cebit Based on Q-H-Qs.”

The second one is called a CCX operation. For a two-cebit state $\lvert 2E \rangle = c_{00}\lvert h \rangle \lvert h \rangle + c_{01}\lvert h \rangle \lvert v \rangle + c_{10}\lvert v \rangle \lvert h \rangle + c_{11}\lvert v \rangle \lvert v \rangle$, the CCX operation from the first cebit to the second is defined by $U_{2CCX} \lvert 2E \rangle = c_{00}\lvert h \rangle \lvert h \rangle + c_{01}\lvert h \rangle \lvert v \rangle + c_{11}\lvert v \rangle \lvert h \rangle + c_{11}\lvert v \rangle \lvert v \rangle$, which is an analog of the CNOT gate in quantum circuit model (Equation S4, Supporting Information). The superscript 1 means that the first cebit is the control cebit and the second cebit is the target cebit. In the following discussion about a two-cebit process, we hold the convention and omit the superscript. For a more specific explanation of the physical picture, please see Section S6, Supporting Information. Based on Equation (3), the implementation of a two-cebit state requires two beams, denoted by $E_i = \sum_{k=1}^{P} f_i \rho_{i,k}$ and
The CCX operation can be implemented by introducing the $E_c$-dependent control on $E_t$. An example of the setup for a CCX operation is shown in the left part of Figure 1d. For the ease of applying the setup in the following subfigures, we use a blue box with dashed edges to represent the whole setup, indicated by the equals sign in Figure 1d. In such a setup, the beam $E_t$ of the control cebit is split into two parts by a BS. One part is used as the output of the beam of the control cebit. The other is measured by a mode splitter (MS)\cite{58}, and a detector $D_{M}$, which helps to record the coefficients $p_{ck}^{H} = h \cdot p_{ck}$ and $p_{ck}^{V} = v \cdot p_{ck}$. Then a programed calculator (PR) gives the matrix $M = F_1 \cdot F_2$, and

$$
F_1 = \begin{pmatrix}
    p_{c1}^{H} & O \\
    O & p_{c1}^{H} \\
    p_{c2}^{V} & O \\
    O & p_{c2}^{V}
\end{pmatrix}, \\
F_2 = \begin{pmatrix}
    p_{c}^{H} & O \\
    O & p_{c}^{H} \\
    p_{c}^{V} & O \\
    O & p_{c}^{V}
\end{pmatrix}
\tag{4}
$$

where $p_{c1}^{H} = (p_{c1}^{H1}, p_{c1}^{H2}, \ldots, p_{c1}^{Hn})$ and $p_{c1}^{V} = (p_{c1}^{V1}, p_{c1}^{V2}, \ldots, p_{c1}^{Vn})$. $O$ is a zero-vector sharing the same size with $p_{c1}^{H}$. $F_1$ is the pseudo inverse matrix\cite{59} of $F_1$. For the beam $E_c$ of the target cebit, an MS is also applied, together with a spatial light modulator (SLM) acting on each mode and a cylinder lens (CL) for bunching the modes, which implements the transformation of $E_c$ to $E_f$. The parameter of $E_f$ is given by $(p_{c}^{H1}, p_{c}^{H2}, \ldots, p_{c}^{Hn}) = T \cdot (p_{c1}^{H1}, p_{c1}^{H2}, \ldots, p_{c1}^{Hn})$, where $T$ represents the matrix transpose. Using the above definition of cebits, it can be verified that the transformation of the 2-cebit state encoded by $E_c$ and $E_f$ to that encoded by $E_c$ and $E_f$ is a CCX operation. Obviously, the setup shown in Figure 1d for the CCX operation is not optimal. The elements we consider in Figure 1d might not be the best choices to implement the transformation indicated by Equation (4), such as the SLMs whose efficiency is quite unsatisfying for information processing at present. This would cause errors in our computation scheme. However, due to the convenience of manipulating classical light, the intensities of the beam modes can be easily adjusted so that the errors can be effectively suppressed and the setup works in principle. Besides, the setup can also be simplified in special cases. For example, when $P = 2$, consider two beams $E_c = f_1 p_{c1} + f_2 p_{c2}$ and $E_f = f_1 (h + v) / \sqrt{2} + f_2 (h - v) / \sqrt{2}$, which are sufficient to implement an arbitrary 2-cebit state. The CCX operation from the cebit encoded by $E_c$ to that encoded by $E_f$ can be implemented by a much simpler setup, as shown in Figure 1e. In such a setup, the vertical component of mode $f_1$ of $E_c$ is phase-shifted by a factor of $\pi$. It can be implemented by letting $E_c$ pass an MS, picking out the vertical component of $f_1$ by a polarization beam splitter (PBS), and then shifting the phase by a phase shifter (PS). Last, a PBS and a CL collect the light modes together, giving the output $E_f$. The other beam $E_c$ is not operated. As the output of the setup shown in Figure 1e, $E_f$ and $E_c$ encode the resultant 2-cebit state of that being operated by a CCX. A theoretical background of the setup in Figures 1d,e functioning as a CCX operation is given in “The Theoretical Background of the Setup for a CCX Operation and the Simplification.”

Based on the setup of single cebit operation and CCX operation, an arbitrary unitary operation on an N-cebit state encoded by $N$ beams can be realized. We start from the two-cebit case.

Using three CCXs and eight single cebit operations, an arbitrary unitary operation on a two-cebit state can be expressed by

$$
U_{2E} = (U_{1E1} \otimes U_{1E2}) \cdot U_{CCX} \cdot (U_{1E3} \otimes U_{1E4}) \cdot U_{CCX} \cdot (U_{1E5} \otimes U_{1E6})
\tag{5}
$$

where the $U_{1E1}$, $U_{1E2}$, ..., and $U_{1E6}$ denote the arbitrary single cebit operations. Based on the previous discussion, single cebit operations can be implemented by the Q-H-Qs shown in Figure 1c, and the CCX operation can be implemented by the setup shown in Figure 1d (or Figure 1e under the constraints). Therefore, the whole setup of Equation (5) can be given by connecting the proper Q-H-Qs and the setup for CCXs, as shown in the upper panel of Figure 1f. The two beams output by the setup would encode the resultant two-cebit state of that being operated by a $U_{2E}$. In fact, the setup shown in Figure 1d (or indicated by Equation (5)) is an analogy of the universal 2-qubit processor.\cite{60} We present the circuit of the 2-qubit processor in the lower panel of Figure 1f and mark the correspondence using arrows. The detailed discussions of the optical setup for Equation (5) and the 2-qubit circuit in Figure 1f are respectively given in S2.1 and S1.1. Supporting Information.

Furthermore, an arbitrary operation on an $N$-cebit state can be constructed. First, we define a special $N$-cebit operation denoted by $M_N R_k$ in a recursive way. When $N = 2$, an $M_2 R_k$ represents a 2-cebit operation defined by $M_2 R_k (|q_1, q_2) = U_{CCX} (I_2 \otimes U_k (|q_2)) U_{CCX} (I_2 \otimes U_k (|q_1))$, where $I_2$ is a 2-by-2 identity matrix and $U_k (|q_2)$ equals to $R_k (|q_2) (R_k (|q_1)$ when $k = 1, 2$. Such an operation can be implemented by simply two CCX setups and two Q-H-Qs, as shown in Figure 2a. The angle parameters are $\xi = \eta = 0$ and $\zeta = \varphi$ for $U_k (|q_2)$, and $\xi = \zeta = 0$ and $\eta = \varphi$ for $U_k (|q_1)$. Correspondingly, an $M_1 R_k$ represents a 3-cebit operation composed of two CCX operations and two $M_2 R_k$s. Rigorously, an $M_N R_k$ is composed of two CCXs and two $N$ $-1$-cebit operation $M_{N-1} R_k$, expressed by $M_N R_k := U_{CCX} (I_2 \otimes M_{N-1} R_k) U_{CCX} (I_2 \otimes M_{N-1} R_k)$. Since one $M_N R_k$ contains two variables, an $M_N R_k$ contains four variables, and generally an $M_N R_k$ contains $2^{N-1}$ variables in total. We hide the variables here and below since they are irrelevant to the main conclusion. The optical construction of the recursive relation of $M_N R_k$ is shown in Figure 2b. In such a scheme, the two setups of the CCXs and the two modules for the $M_N R_k$s are arranged alternatively. The CCX setups are applied for the first beam (control cebit) and the last beam (target cebit). The $M_N R_k$ modules are applied for all the beams except the first. The ellipsis in the figure indicates the omission of the beams operated by the $M_{N-1} R_k$s and $M_N R_k$s. Finally, an arbitrary $M_N R_k$ can be implemented by only Q-H-Qs and CCX setups, if one replaces the $M_{N-1} R_k$ module by the sequence composed of $M_{N-1} R_k$ modules and CCX setups, and then replaces $M_{N-2} R_k$, etc., till $M_2 R_k$. In fact, an $M_1 R_k$ is an analogy of the quantum multiplexed $R_k$ gate.\cite{61} The circuits of the quantum multiplexed $R_k$ gates are shown in the lower panels of Figure 2a (the 2-qubit case) and (b) (the recursive relation of the $N$-qubit case), and the matrix forms of them are given by Equations S8 and S9 in S1.2, Supporting Information.

Using $M_N R_k$s, an arbitrary $N$-cebit unitary operation can be expressed. Starting with the 2-cebit operation $U_{2E}$ whose setup is shown in Figure 1f, an arbitrary 3-cebit operation $U_{3E}$ can then
be implemented using four $U_{NE}$ setups, two $M_N R_N$ setups, and a $M_N R_I$ setup. In the same manner, a 4-cebit operation $U_{4N}$ can be implemented using four $U_{3N}$ setups, two $M_N R_N$ setups, and a $M_N R_I$ setup. Strictly, the recursive relation of an arbitrary $N$-cebit unitary operation $U_{NE}$ can be expressed as

$$U_{NE} = \left( I_2 \otimes U_{N-1,E} \right) \cdot Ro \cdot M_N R_N \cdot Ro' \cdot \left( I_2 \otimes U_{N-1,E} \right) \cdot Ro \cdot M_N R_N \cdot Ro' \cdot \left( I_2 \otimes U_{N-1,E} \right) \cdot Ro \cdot M_N R_N \cdot Ro' \cdot \left( I_2 \otimes U_{N-1,E} \right)$$  \hspace{1cm} (6)

where $Ro$ is the operation that swaps the first cebit and rests whole $N - 1$ cebits. $Ro'$ is the inverse of $Ro$. $U_{N-1,E}$ represents an arbitrary unitary operation on an $(N - 1)$-cebit. Such operations are easy to implement by directly changing the propagation path of the beams, and the optical routers are considered as the approaches for them in our scheme. The setup of Equation (6) is shown in the upper panel of Figure 2c. Given $N$ beams that encode an $N$-cebit state, the module of $U_{NE}$ can be given by arranging three $Ro-M_N R_N-Ro'$ setups and four $U_{N-1,E}$ modules alternatively. The $Ro-M_N R_N-Ro'$ setup operating on all beams is composed of an $M_N R_N$ setup together with two routers, one of which implements $Ro$ and the other implements $Ro'$. As shown by the figure, two $Ro-M_N R_N-Ro'$ setups and one $Ro-M_N R_N-Ro'$ setup are involved. The $U_{N-1,E}$ modules operate on all the beams except the first, and can be given by a sequence of $U_{N-2,E}$ modules and the setups of $M_{N-1} R_N$ according to Equation (6). The same goes for $U_{N-2,E}$, etc. In fact, the setup of Equation (6) is analogy of the key formula in quantum Shannon decomposition.\(^{[63]}\) The circuit of the formula is shown in the lower panel of Figure 2c (with the correspondence being marked by the arrows), and the matrix form of it is given by Equations S10 and S11 in S1.2, Supporting Information. Using Equation (6) again and again, the module of $U_{NE}$ can be finally implemented by the $U_{3E}$ modules and the $M_N R_N$ setups, which only involves the CCX setups and Q-H-Qs for single cebit operations. Then, a step-by-step manual for building a universal operation on an $N$-cebit state with the two basic setups can be obtained. A theoretical description of the above universal optical computing scheme is given in S2, Supporting Information. As indicated by our above discussions, the experimental demonstration of the proposal is relatively easy. We take 2-cebit case as an example, and explore first the CCX setup and then a universal processor in the following section.

3. The Experimental Demonstration of a Universal Two Cebit Processor

We first experimentally demonstrate the validity of simplified CCX setup shown in Figure 1e. The experimental scheme is shown in Figure 3a. The scheme is divided into three parts: the input, the CCX operation, and the output. In the input part, the beams of the 2-cebit state are set to contain two spatial modes $f_1$ and $f_2$ respectively. The source of the first beam is a 632.8 nm He-Ne laser (ThorlabsHNL210LB), and two BSs (ThorlabsBS016) are employed for producing spatial modes $f_1$ and $f_2$. A polarization sensitive beam displacer (BD) is employed for initially polarizing the laser beam, and the encoding of the cebit states is realized by QWPs and HWPs acting on the two spatial modes. As we mentioned, any 2-cebit state can be generated by the input part (the reason is given in “The Theoretical Background of the Setup for a CCX Operation and the Simplification”). In the
CCX part, the operation is realized by an interferometer. It is constructed by two PBSs (Thorlabs PBS201) and a piezoelectric ceramic element M3 for phase modification. This arrangement corresponds to the actions on $E_t$ shown in Figure 1e. Because we consider the spatial modes which are naturally distinguishable, it is not necessary to apply the MS and the CL.

In the output part, the measurement is performed by realizing the setups shown in Figure 1a,b. The LO beam is generated by splitting the beam from the previous part. This results in the interferometers shown in the output part of Figure 3a. The two interferometers are used for measuring the two spatial modes, constructed by two BSs and a piezoelectric ceramic element (M1 and M2) individually. The projective direction of the measurement is set by adjusting the polarization state of light in one arm with the wave plates. Although the measurements of two spatial modes are the same so that they can be measured using only one interferometer, we apply two in our experiments so that the visibilities can be improved independently. The light intensity is detected by using a charge coupled devices (CCD, Thorlabs BC106N-VIS/M), and the effective detection area is 8.77 mm × 6.6 mm. As shown in Figure 1e, the second beam of the cebit is not operated during the procedure. Also, according to the above setup, the measurements on it do not need to be performed together with the measurements on the first beam of cebit. Therefore, the measurement results of the first and the second beam are obtained sequentially. The results of the correlation are then calculated by multiplying the measurement data of the cebits (indicated by the multiplier in Figure 3a) according to the spatial orthonormal relation, which is equivalent to the integration of the product discussed in the previous section.

Next, we present the results of the CCX experiment. Like the benchmarking of quantum gates, we input a series of 2-cebit states and perform the correlated projection measurement on the states output by the CCX scheme of Figure 3a. The 2-cebit measurement basis is composed of sixteen different tensor products of the states in the set $\{ |h\rangle, |v\rangle, (|h\rangle + |v\rangle)/\sqrt{2}, (|h\rangle + i|v\rangle)/\sqrt{2} \}$. By referring to the quantum state tomography theory, the sixteen correlated measurement results can give the density matrix of the output state.\[62\] For example, by setting the two beams to be $(h + v)\gamma f_s/\sqrt{2} + (h + v)\gamma f_s/\sqrt{2}$ and $(h + v)\gamma f_s/\sqrt{2} + (h - v)\gamma f_s/\sqrt{2}$, the input 2-cebit state is $(|h\rangle + |v\rangle)|h\rangle/\sqrt{2}$ of the above basis set according to the definition of the cebit measurements. To measure the state output by the CCX part, the two spatial modes of the beam are measured by adjusting the wave plates of single arms of the interferometers according to the above sixteen basis states and multiplying the recorded intensity with the simulated data. Using the method provided by refs. \[62–64\], the density matrix of the output state can be obtained by the sixteen correlations after the normalization, and is shown in Figure 3b. The upper panel is the real part and the lower panel is the imaginary part. Such a density matrix corresponds to the entangled state $(|h\rangle|h\rangle + |v\rangle|v\rangle)/\sqrt{2}$. It indicates the property of a CCX for generating entanglements, corresponding to that of the quantum CNOT gate. Also, we measure the difference between the experimental density matrix and the theoretical one by referring to the quantum state fidelity,\[62\] and find it to be 0.956. Furthermore, we input other fifteen states in the above 2-cebit measurements basis set, and perform the sixteen correlated projection measurements for the outputs respectively, just like what we do when the input is $(|h\rangle + |v\rangle)|h\rangle/\sqrt{2}$. Then, the results of the entire sixteen inputs produce a 16-by-16 data matrix. By referring to the quantum process tomography theory,\[62,65\] one can obtain a 16-by-16 $\chi$ matrix from the data matrix. The entries of the $\chi$ matrix represent the indices of the expansion of a unitary matrix in the basis of Kronecker products of $\{ I, X, -iY, Z \}$. Because the expansion is unique in the basis, one unitary matrix can be precisely characterized by its $\chi$ matrix. The $\chi$ matrix of our CCX experiment is given in Figure S4, Supporting Information which shows an agreement with the theoretical one. The fidelity of implemented

**Figure 3.** The experiment of CCX operation illustrated in Figure 1e. a) The optical circuit. The light source is a 632.8 nm He-Ne laser. The beams for encoding the cebit states are generated by BSs and Q-H-Qs. The orange region in (a) marks the CCX operation, composed of two PBSs, a mirror, a piezoelectric ceramic element M3, and a filter for balancing the light intensity. The blue region marks the input and output setup. Piezoelectric ceramic elements M1 and M2 are applied for adjusting the interferometers. The legend is shown at the bottom. b) The density matrix of the output cebit when the input is $(|h\rangle|h\rangle + |v\rangle|v\rangle)/\sqrt{2}$. The upper (lower) panel shows the real (imaginary) part. The probability here is defined by the normalized correlation intensity.
CCX operations is found to be 0.989. The error mainly comes from the fluctuation of the interferometers, and the imperfect efficiency of the optical elements.

Based on the setup of the CCX, the experimental scheme for the universal 2-cebit processor is shown in Figure 4. The entire setup is also composed of three parts: the input, the processing, and the output. The input and the output are the same with the above CCX experiment. In the processing part, we realize one example of $U_{1E}$ operations defined by Equation (5).

The eight single cebit operations are set to be $U_{1E1} = R_2(\pi/2)$, $U_{1E2} = H$, $U_{1E3} = H$, $U_{1E4} = R_2(\pi/2)$, $U_{1E5} = R_2(\pi/2)$, and $U_{1E6} = U_{1E7} = U_{1E8} = I_2$. $H$ is the Hadamard operation for cebits. A general setup for $U_{1E}$ has been demonstrated in Figure 1f. Here, we apply the simplified CCX setup, so the whole implementation of $U_{1E}$ is slightly different from that in Figure 1f. For the operations on the first cebit, they are implemented by Q-H-Qs which is the same as discussed in the previous section. Specifically, the $R_2(\pi/2)$ and the $H$ on the first cebit are implemented by a QWP at 0 rad and an HWP at $\pi/2$ respectively acting on both spatial modes, shown by the yellow and the right dark-red region of Figure 4. The $R_2(\pi/2)$ can be implemented based on the equation $R_2(\pi/2) = H \cdot R_2(\pi/2) \cdot H$, while here we simplify it to merely changing the orientations of the fast axes of HWPs and QWPs.

For the operations on the second cebit, they are implemented differently. Due to the requirements of the simplified CCX setup, the second beam must be transformed to $(h + \alpha f_2)/\sqrt{2} + (h - \alpha f_2)/\sqrt{2}$ before being input to the setup. At the same time, the first beam must be also be modified so that the 2-cebit state encoded by them remains the same. Therefore, by keeping the second beam to be $(h + \alpha f_2)/\sqrt{2} + (h - \alpha f_2)/\sqrt{2}$, the operations on the second cebit can be effectively implemented by changing the physical state of the first beam. Specifically, the Hadamard on the second cebit is effectively performed by modifying the first beam according to $|p_{1,1}^H h + p_{1,1}^V v_{f1} + (p_{1,2}^H h + p_{1,2}^V v_{f2})/\sqrt{2} + (p_{1,2}^H h + p_{1,2}^V v_{f2})/\sqrt{2}|_1$. Such a modification can be implemented by an interferometer composed of two BSs and several phase-shift elements, shown by the setup of the left dark-red region in Figure 4. Similarly, $R_2(\pi/2)$ on the second cebit is effectively performed by modifying the first beam according to $(p_{1,1}^H h + p_{1,1}^V v_{f1} + (p_{1,2}^H h + p_{1,2}^V v_{f2})/\sqrt{2} + (p_{1,2}^H h + p_{1,2}^V v_{f2})/\sqrt{2}) - [(p_{1,1}^H h + p_{1,1}^V v_{f2})/\sqrt{2} + (p_{1,2}^H h + p_{1,2}^V v_{f2})/\sqrt{2}]$. Such a modification can also be implemented by an interferometer like the one for the Hadamard on the second cebit, shown by the setup of the gray region in Figure 4. Conclusively, the $U_{1E}$ operation is implemented by modifying the first beam with the above setups following the order illustrated in Figure 4. The second beam in such a scheme is also not operated except for the measurements. So, like the CCX experiment, we also use the sequential strategy to perform the measurements. The procedure is indicated by the dashed lines in the setup of Figure 4.

The evaluation of the 2-cebit processor is like what we do for CCX experiment. We input the sixteen 2-cebit states, the distinct single cebit state of which is chosen from the set $\{|0,0\rangle, |0,1\rangle, |1,0\rangle, |1,1\rangle\}$.

The results 16-by-16 data matrix can give the following projection measure using the same set with that for CCX experiment. The resultant 16-by-16 data matrix can give the performance of the quantum process tomography. The resultant $16 \times 16$ data matrix can be implemented operation $U_{1E}$ is found to be 0.989. The error mainly comes from the fluctuation of the interferometers, and the imperfect efficiency of the optical elements. Besides the example of $U_{1E}$, we consider here, other 2-cebit computation can also be performed by simply changing the orientations of the fast axes of HWPs and QWPs.
Figure 5. The process tomography results of the setup shown in Figure 4a,b are the real part and imaginary part of $\chi$ matrix obtained by experimental data. (c) and (d) are the corresponding theoretical results. The probability is also defined by the normalized the correlation intensity.

and the elements of interferometers for the effective operations on the second cebit. A thorough calculation of the whole experimental scheme is given in S3, Supporting Information. The experimental data are also provided in S4, Supporting Information.

4. Discussion and Conclusion

We summarize the core idea of our proposal for optical universal computing as follows. First, to encode the information by fully using the coherence of the light field, the counterpart of a qubit is defined and termed by the cebit. As discussed above, a cebit can be described by a complex vector in 2D Hilbert space, whose components are given by the horizontal and vertical projection of the classical beam defined by Equation (3). The correlation of the multiple cebits is defined by the integral of the products of the single beam projections. Based on the orthonormal relations below Equation (3), such a correlation exhibits a good correspondence with correlated measurement results of the multi-qubit states. Or, we can say that all $N$-cebit states form the Hilbert space of $N$-qubit states. A strict correspondence is shown by Equation (10) in “Theoretical Analysis of the Measurement Scheme for Cebits.” Second, to manipulate the cebit states, the operations on them implemented by present optical elements are discussed in detail. Like the quantum computing, a general operation on an $N$-cebit state is an element in the unitary group. A convenient way to generate the operations is to employ the concept of universal quantum gate sets. Therefore, schemes for single cebit operations (the analogy of single qubit gates) and the CCX operation (the analogy of the CNOT gate) as the elementary setups are proposed, and the combination of them is presented in Figures 1 and 2. In the above sense, we can say that the universal computation based on the cebits can be performed. The correspondence between the cebit operations and qubit gates also indicated that the number of the elementary setups for implementing an arbitrary cebit operation is equivalent to the number of gates for building the corresponding circuit in quantum computing. The foundation for implementing such a strategy is the correlation defined by Equation (8) using optical coherence.

In conclusion, we demonstrate a step-by-step manual for constructing a universal optical computing architecture. It can be considered as an analogy of universal quantum computing. Taking the 2-cebit processor as an example, we experimentally verify our results. We believe that our proposal is important from two aspects. From the aspect of optical computing, it indicates that the potential of applying optical degree of freedom for information processing is not fully developed. As mentioned in the introduction, the optical devices for computing have several advances over electric devices, such as low heat generation, high degree of parallelism, etc. However, the computing strategies in optical systems by far are either the mimic of electric computing or for dealing with special tasks, and the universal computation roadmap is not clearly drawn. Our proposal shows that in
such a system, an architecture analog to the universal quantum computing is implementable, indicating that the quantum algorithmic advances in certain cases could also be realized. More importantly, from the aspect of practical use, our proposal provides a convenient way for simulating quantum characters. In the solid systems, increasing the coherence time of quantum states is rather difficult. On the contrary, in classical optical systems, the optical coherence can be maintained for a rather long time by current techniques. Therefore, using optical coherence to perform the analogy of quantum computation would also be tricky. The complexity of the multi-qubit correlation measurement of the correlated function in a large-scale qubits system is usually time-consuming. In our scheme, the classical optical setups use a laser with medium intensity. Compared with those schemes working in the single-photon regime, the medium-intensity laser schemes would be affected less by photon number fluctuation, and thus lower the errors in the control and measurement. Last, the measurement of the correlated function in a large-scale qubit system would also be tricky. The complexity of the multi-qubit correlation grows exponentially with the qubit number, and sampling such a correlation is usually time-consuming. In our scheme, the sampling of a correlation in quantum computing is mapped to the obtaining a product of light intensity, which avoids the tedious statistical methods like event counting. Following the above, and considering that the major techniques involved in the scheme are available for current optical processing platforms, our proposal will be beneficial for achieving the high standard and near-term processors required by the era. Therefore, our results open a new way toward advanced information processing with high quality and efficiency.

5. Experimental Section

Theoretical Analysis of the Measurement Scheme for Cebits: The measurement of the cebit encoded by the multi-mode polarized beam was discussed. Using the setup shown in Figure 1a, it was easy to obtain the difference of the intensity recorded by D1 and D2,

\[ I_{D1} - I_{D2} = |E + E^{L0}|^2 - |E - E^{L0}|^2 = 2Re \{ E \cdot E^{L0}\} \]  

where Re means the real part, and \( e \) represents the conjugate complex to get. To measure the intensity of the light, one just needed to shift the phase of \( E^{L0} \) by \( \pi/2 \). In fact, term \( E \cdot E^{L0} \) had a well correspondence with the projection (\( e|q \)) of a qubit \( |q \) on \( |e \). For the analogy of an N-qubit state, \( N \) beams \( E_1 \cdots E_{N-1}, E_p, E_{N+1} \cdots E_N \) were introduced. The measurement of the \( N \) beams is given in Figure 1b, expressed by

\[ \int (e_1^* \cdot E_1) (e_2^* \cdot E_2) \cdots (e_N^* \cdot E_N) \ d\Omega = \]

\[ \int \left( e_1^* \sum_{k=1}^{p} f_{1,k} p_{1,k} \left( e_2^* \sum_{k=1}^{p} f_{2,k} p_{2,k} \right) \cdots \left( e_N^* \sum_{k=1}^{p} f_{N,k} p_{N,k} \right) \right) d\Omega = \]

\[ = \sum_{k=1}^{p} (e_1^* \cdot p_{1,k}) (e_2^* \cdot p_{2,k}) \cdots (e_N^* \cdot p_{N,k}) \]

\[ \cdot \sum_{k=1}^{p} (p_{1,k} p_{2,k} \cdots p_{N,k}) \]  

(8)

The last equality held due to the property of dyadic tensors \( e_1^* e_2^* \cdots e_N^* \) and \( p_{1,k} p_{2,k} \cdots p_{N,k} \). Also, Equation (8) indicates a good correspondence with the projection (\( e_1 |e_2| \cdots |e_N|N \) of an N-qubit state \( |qN \) on \( |e_1,e_2,\cdots,e_N|N \). Therefore, the measurement apparatus shown in Figure 1a,b can be used to perform the projection of cebits. It also indicated that the cebit space was a Hilbert space.

The conditions for the analogy relation can be strictly given. For the N- qubit state defined by Equation (3), the analogous qubit state can be expressed by,

\[ \sum_{j_1,j_2,\cdots,j_N} j_1 j_2 \cdots j_N \]

\[ q_{j_1 j_2 \cdots j_N} U j_2 \cdots j_N \]

(9)

and \( q_{j_1 j_2 \cdots j_N} \) was normalized, \( q_{j_1 j_2 \cdots j_N} = c_{j_1 j_2 \cdots j_N} \). According to the projection and the measurement given by Equation (8), the \( N \) beams that encoded the cebit state corresponding to Equation (9) must satisfy the equation set

\[ \sum_{k=1}^{p} \left( \prod_{j=1}^{N} P_{j,k} A_{j,k} \right) = q_{j_1 j_2 \cdots j_N} \]

where \( A_{j,k} \) was set to be \( H \) when \( j_k = 0 \), and to be \( V \) when \( j_k = 1 \). Notice that Equation (10) is composed of many nonlinear equations. Suppose that the orthogonality of \( j_k \) was not required. Similar results can also be obtained, while the expression will be more complicated. Besides, the above way of defining a cebit provided a formulism for simulating a qubit. Other physical systems that can be described by the similar math formulism would also support an equivalent of a qubit.

The Unitary Operation of a Single Cebit Based on Q-H-Qs: The polarization state of a single mode light beam can be arbitrarily modified by QWPs and HWPs. Specifically, a unitary transformation can be parameterized by Euler angles \( (\xi, \eta, \zeta) \), expressed by

\[ U (\xi, \eta, \zeta) = \exp (-i \frac{\xi}{2} \psi) \exp (-i \frac{\eta}{2} |Z \exp (-i \frac{\zeta}{2} |Y \]  

(11)

The Jones matrices of an HWP and a QWP (the horizontal and vertical polarization states as basis) were given by

\[ J_H (\psi) = \left( \begin{array}{cc} -\cos 2\psi & -\sin 2\psi \\ -\sin 2\psi & \cos 2\psi \end{array} \right) \]

\[ J_Q (\psi) = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} -\cos 2\psi & -i \sin 2\psi \\ -i \sin 2\psi & \cos 2\psi \end{array} \right) \]

(12)

where \( \psi \) is the orientation angle of the fast axis of the HWP or QWP. The unitary operation on the polarization can be realized by decomposing Equation (11) into the product of \( J_H \) and \( J_Q \). One strategy was the Q-H-Q decomposition given by

\[ U (\xi, \eta, \zeta) = J_Q (\xi) J_H (\eta) \]  

(13)

\[ J_Q (\xi - \frac{\zeta}{2}) \]  

(13)

The multi-mode beams that were considered for encoding cebit states can also be operated using a Q-H-Q. Apply Equation (13) to Equation (3), one has

\[ U (\xi, \eta, \zeta) E (r, t) = \sum_{k=1}^{p} f_{k} (r, t) U (\xi, \eta, \zeta) \left( p_{1,k} (r) h + p_{1,k} (r) v \right) \]

\[ = \sum_{k=1}^{p} f_{k} (r, t) \left( p_{1,k} (r) (u_0 h + u_0 v) + p_{1,k} (r) (u_1 h + u_1 v) \right) \]

(14)

\[ = \sum_{k=1}^{p} f_{k} (r, t) \left( \{u_0 p_{1,k} (r) + u_0 p_{2,k} (r)\} h + \{u_1 p_{1,k} (r) + u_1 p_{2,k} (r)\} v \right) \]
where \( u_{pq} \) is the entry of \( U(\xi, \eta, \zeta) \) in the \( (p + 1) \)th row and \( (q + 1) \)th column, \( p(q) \) equals to 0 or 1. Assume that the cebit state encoded by \( U(\xi, \eta, \zeta)E(r, t) \) is \( |c_1\rangle + |c_1\rangle |v\rangle \), then one has

\[
c'_0 = \sum_{k=1}^{p} f_k (r, t) (u_{00}p^H_{k} (r) + u_{10}p^V_{k} (r)) = u_{00}c_0 + u_{10}c_1,
\]
\[
c'_1 = \sum_{k=1}^{p} f_k (r, t) (u_{01}p^H_{k} (r) + u_{11}p^V_{k} (r)) = u_{01}c_0 + u_{11}c_1
\]

(15)

Therefore, an arbitrary unitary operation \( U_{1k} \) on cebit \( |c_1\rangle + |c_1\rangle |v\rangle \) encoded by \( E(r, t) \), can be realized by directly modifying the polarization of the beam with the Q-H-Qs.

The Theoretical Background of the Setup for a CCX Operation and the Simplification: The CCX operation on the beam cebit was calculated. First, the 2-cebit case was considered. According to Equation (10), the parameter equation of \( E_c \) and \( E_v \) can be given by

\[
\begin{pmatrix}
P^H_{c} \\
0 \\
0 \\
0 \\
P^V_{c}
\end{pmatrix}
\begin{pmatrix}
(p^H_{c})^T \\
(p^V_{c})^T \\
(p^H_{c})^T \\
(p^V_{c})^T \\
(p^H_{c})^T
\end{pmatrix}
= \begin{pmatrix}
c_0 \\
c_1 \\
c_{10} \\
c_{11}
\end{pmatrix}
\]

(16)

The CCX operation on the 2-cebit required that the output beam \( E_c' \) equaled to \( E_c \), and \( E_v' \) satisfied the equation

\[
\begin{pmatrix}
P^H_{c} \\
0 \\
0 \\
0 \\
P^V_{c}
\end{pmatrix}
\begin{pmatrix}
(p^H_{c})^T \\
(p^V_{c})^T \\
(p^H_{c})^T \\
(p^V_{c})^T \\
(p^H_{c})^T
\end{pmatrix}
= \begin{pmatrix}
c_0 \\
c_1 \\
c_{10} \\
c_{11}
\end{pmatrix}
\]

(17)

Using Equation (4) in the main text, one has

\[
F_1 \left( \begin{pmatrix}
(p^H_{c})^T \\
(p^V_{c})^T \\
(p^H_{c})^T \\
(p^V_{c})^T \\
(p^H_{c})^T
\end{pmatrix} \right) = F_2 \left( \begin{pmatrix}
(p^H_{c})^T \\
(p^V_{c})^T \\
(p^H_{c})^T \\
(p^V_{c})^T \\
(p^H_{c})^T
\end{pmatrix} \right)
\]

(18)

Because \( F_1 \) was not commonly a square matrix, the pseudo inverse was used.\(^{[59]}\) Then, as discussed in the main text, such a requirement on output beams can be realized by the setup shown in Figure 1d.

Next, the CCX operation on two cebits of an \( N \)-cebbit state was briefly discussed. Following the section “Theoretical Analysis of the Measurement Scheme for Cebits,” suppose that the \( n_{th} \) beam \( E_{n_{th}} \) encoded the control cebit, the \( n_{th} \) beam \( E_{n_{th}} \) encoded the target cebit. The requirements on their outputs \( E'_{n_{th}} \) and \( E'_{n_{th}} \) were the same with the above. Therefore, one can obtain the parameters \( p^H_{n_{th}} \) and \( p^V_{n_{th}} \) of \( E'_{n_{th}} \) by solving

\[
\sum_{k=1}^{p} p^H_{n_{th}, k} \cdots p^H_{n_{th}, k} \cdots p^H_{n_{th}, k} \cdots p^H_{n_{th}, k} = \sum_{k=1}^{p} p^V_{n_{th}, k} \cdots p^V_{n_{th}, k} \cdots p^V_{n_{th}, k} \cdots p^V_{n_{th}, k},
\]

(19)

For a sufficiently large \( P \), one can find a subspace of the solution under the constraints \( \prod_{k=1}^{N} p_{d_k} = \text{const} \). Then, Equation (19) can be simplified to

\[
\sum_{k=1}^{p} p^H_{n_{th}, k} \cdot p^H_{n_{th}, k} = \sum_{k=1}^{p} p^V_{n_{th}, k} \cdot p^V_{n_{th}, k}
\]

(20)

It was not hard to notice that Equation (20) was equivalent to Equation (17). Therefore, the CCX operation on two cebits of an \( N \)-cebbit state can also be implemented using the same setup for the 2-cebit case.

Finally, the scheme shown in Figure 1e was calculated. Using the two beams \( E_c \) and \( E_v \) defined in the main text, a 2-cebit state can be encoded under the condition

\[
\frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
|c_0\rangle \\
|c_{10}\rangle \\
|c_{11}\rangle \\
|c_{11}\rangle
\end{pmatrix}
= \begin{pmatrix}
|c_0\rangle \\
|c_{10}\rangle \\
|c_{11}\rangle \\
|c_{11}\rangle
\end{pmatrix}
\]

(21)

It is worth mentioning that Equation (21) can always be solved. So, \( E_c \) and \( E_v \) can encode an arbitrary 2-cebit state. As discussed above, the CCX operation on the 2-cebit state effectively changed the position of \( c_{10} \) and \( c_{11} \) in Equation (21). Then,

\[
\begin{pmatrix}
|c_0\rangle \\
|c_{10}\rangle \\
|c_{11}\rangle \\
|c_{11}\rangle
\end{pmatrix}
= \begin{pmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
|c_0\rangle \\
|c_{10}\rangle \\
|c_{11}\rangle \\
|c_{11}\rangle
\end{pmatrix}
\]

(22)
Therefore, the CCX operation, in this case, can be implemented by $\hat{\rho}_{c_2} \rightarrow -\hat{\rho}_{c_2}$. The setup is shown in Figure 1e and the experimental results are presented in Figure 3.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

Acknowledgements

The authors acknowledge the support of the National key R & D Program of China (2017YFA0303800) and the National Natural Science Foundation of China (91850205, 11904022).

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Y.S. provided the theoretical derivation of the work. The experiments were performed by Q.L. with the help of L.-J.K. J.S. provided support on the programs for data processing. X.Z. initiated and designed this research project.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords

classical optics, quantum information, universal computing

Received: July 27, 2022
Revised: September 18, 2022
Published online: October 31, 2022

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To obtain the density matrix of a qubit, one just need to measure certain correlations of the qubit state by the coincidence detection. The main idea to obtain the density matrix of a qubit state is the same. The only difference is that the correlation measure is equivalently done by the detection strategy given in Method 5.1.

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