We report the first model study of the net charge fluctuations in terms of $D - \text{measured}$ within the framework of the Polyakov–Nambu–Jona-Lasinio model. Net charge fluctuation is estimated from the charge susceptibility evaluated using PNJL model. A parameterization of the freeze-out curve has been used to obtain $D$ as a function of $\sqrt{s}$. We have discussed our results vis-a-vis recent experimental findings from ALICE collaboration.

PACS numbers: 12.38.Aw, 12.38.Mh, 12.39.-x

Strongly interacting matter at very high temperatures and densities is expected to undergo a transition from confined state of colored charges, the hadronic phase with broken chiral symmetry to a partonic phase in which chiral symmetry is restored and/or quarks are deconfined \cite{1}. A good understanding of this transition is relevant for studies in the fundamental interactions in particle physics, as well as for the physics of early universe and neutron stars. Thus it has become an issue of great interest in recent years, both theoretically and experimentally \cite{2}. To this end, it is essential to identify unambiguous signals which would establish the formation of a quark-gluon plasma (QGP). One such viable signal is the fluctuations of net electric charge $Q$ \cite{2,3,4}. It has been argued that this fluctuation is proportional to the square of the electric charge which takes up distinct values for the hadronic and QGP phases. While the unit of the electric charge which takes up distinct values for the hadronic and QGP phases. While the unit of the electric charge which takes up distinct values for the hadronic and QGP phases.

So the signal in fluctuations of $F$ is amplified four times in the fluctuations of $R$. Now $R$ is related to ratio of net charge fluctuation to $\langle N_{ch} \rangle$ via a quantity $D$, defined as:

$$D = \frac{\langle N_{ch} \rangle}{\langle \delta R^2 \rangle} = 4 \frac{\langle \delta Q^2 \rangle}{\langle N_{ch} \rangle^2} \approx 4 \frac{\chi_Q}{n_{ch} / T^3}$$

where $n_{ch} = \langle N_{ch} \rangle / V$ is the total charge density. $n_{ch}$ may be obtained by adding the contribution from particle and anti-particle distributions while the dimensionless charge susceptibility $\chi_Q$ may be obtained from the pressure $P$ of the system as,

$$\chi_Q \left( T, \frac{\mu_Q}{T} \right) = \frac{\partial^2}{\partial (\mu_Q / T)^2} \left( \frac{P (T, \mu_Q)}{T^4} \right),$$

where $\mu_Q$ is the electric charge chemical potential.

This definition is useful over that of the ratio fluctuations as the general form of the latter may be quite complicated. As mentioned above that only if $\langle N_{ch} \rangle >> \langle Q \rangle$ the simplified relation exists as given above. In general this condition is expected to be satisfied only for a large $T$ and very small $\mu_B$. Once the
\( \mu_B \) becomes large the net charge \( Q \) increases and the approximation would fail. However the definition of \( D \) is quite general and holds even if not so simply related to the ratio fluctuation.

A simple estimate of \( D \) was made considering the hadronic phase to be composed of pion gas and the partonic phase as computed in the Lattice Gauge Theory [1]. This gave the value of \( D \) to be \( \sim 4 \) for the hadronic phase and \( \sim 1 \) for QGP phase.

The first measurement of \( D \) in experiments have been reported recently by the ALICE collaboration [10]. They have obtained the net charge fluctuations in a rapidity window \( 0.2 < \Delta \eta < 1.6 \) using center of mass energy \( \sqrt{s} = 2.76 \) TeV in Pb-Pb run for different centralities. The \( D \) for increasing \( \Delta \eta \) continues to fall and is just short of the saturation region as expected from UrQMD simulation results [8]. The analysis of \( D \) and its variation with center of mass energies in the range of 19.6GeV < \( \sqrt{s} < 200\)GeV has also been presented in the same report. The data used were obtained by the STAR collaboration [11].

**Results in PNJL model:** Here we report on the study of net charge fluctuations in terms of the \( D - \) measure using the Polyakov loop enhanced Nambu–Jona-Lasinio (PNJL) model. In this model the quarks and Polyakov loop fields are the basic degrees of freedom. The quarks while interacting with the Polyakov loop also has a four-fermi self-interaction. Similarly the Polyakov loop fields interact via a Landau-type potential. The details of the model used for a 2 flavor system may be found in [12, 13]. The extension to 2+1 flavor system has been done in [14]. Detailed studies of fluctuations and correlations of various conserved charges were performed with the PNJL model both for 2 flavor [12, 16] and for 2+1 flavor [17–19] systems.

To compute \( D \) we evaluate \( \chi_Q \) and \( n_{ch} / T^3 \) using the PNJL model. The method of obtaining \( \chi_Q \) is quite standard as has been discussed by us earlier [13]. On the other hand \( n_{ch} \) are to be calculated from the quark distribution functions as they appear in the PNJL model.

The behavior of \( \chi_Q \) and \( n_{ch} / T^3 \) with \( T / T_c \) are shown in Fig.1 for various values of the baryon chemical potential \( \mu_B \) and for the cases of 2 flavor and 2+1 flavor systems. Here \( T_c \) is the crossover temperature at the corresponding values of \( \mu_B \). The quantities under consideration show qualitatively similar behavior. There is a sharp rise close to \( T / T_c \sim 1 \) from a negligibly small value for low temperatures. This is followed by a saturation at high temperatures close to the corresponding values for massless free quarks. The values of \( \chi_Q \) and \( n_{ch} / T^3 \), at any \( T / T_c \) are larger for larger \( \mu_B \). However as \( T \) increases they all tend towards the limit of free gas at \( \mu_B = 0 \).

We now consider the quantity \( D / D_{free} \) and study its temperature and density variations. Here \( D_{free}(T, \mu_B) \) is the temperature and chemical potential dependent limit of \( D(T, \mu_B) \) for a free massless gas of quarks. In Fig.2 we show the variation of \( D / D_{free} \) as a function of \( T / T_c \) for both 2 and 2 + 1-flavor cases. We have chosen four representative values of \( \mu_B \). It is observed that \( D \) always remains above its free field limit and approaches this limit at high enough \( T \). The sharp transition occurs near \( T = T_c \). Now both \( \chi_Q \) and \( n_{ch} \) are smaller than their corresponding free field limit below \( T = T_c \) for any \( \mu_B \). Therefore \( D > D_{free} \) implies that \( n_{ch} \) is much more suppressed than \( \chi_Q \) in the confined phase. This effect seems to be much more prominent as \( \mu_B \) is increased. In passing, let us mention that \( \langle Q \rangle / \langle N_{ch} \rangle \) in our studies varied from a value of 0.025 for high \( T \) and low \( \mu_B \) to 0.3 at the other extreme.

**Connection with heavy-ion collision experiments:** It would thus be interesting to check what happens with further increase of \( \mu_B \). The variation of \( D / D_{free} \) with \( \mu_B \) at different temperatures are shown in Fig.3. Again we find \( D \) to remain above its free field limit for all \( T \) and \( \mu_B \). For a lower temperature \( \sim 100 \) MeV, there is an initial rise and a subsequent fall with increasing \( \mu_B \). This non-monotonic behavior at low temperatures may pose a problem in using \( D \) as a direct indicator of the phase of strongly interacting matter.

Nonetheless, with an input of temperature and chemical potential from particle multiplicities at the freeze-out surface in heavy ion collision experiments, one may study the expected nature of \( D \) for different experimental conditions. The independent thermodynamic variables in the PNJL model are \( T, \mu_B, \mu_Q, \) and \( \mu_S \), where the...
latter is the strangeness chemical potential. Parameterization of the freeze-out conditions as a function of $\sqrt{s}$ are available in the literature (see e.g. [20, 21]). For a given set of the thermodynamic variables we found the variations in $\sqrt{s}$ are within 10% for different parameterizations. Here we choose the parameterization in [21] to model the freeze-out curve as:

$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4$$  \hspace{1cm} (6)$$

$$\mu_{B,Q,S}(\sqrt{s}) = \frac{d}{1 + e\sqrt{s}}$$  \hspace{1cm} (7)$$

where, $a = 0.166 \pm 0.002$ GeV, $b = 0.139 \pm 0.016$ GeV$^{-1}$, $c = 0.053 \pm 0.021$ GeV$^{-3}$ and $d$ and $e$ are given as:

|   | $d$[GeV] | $e$[GeV$^{-1}$] |
|---|---------|-------------|
| $B$ | $1.308(28)$ | $0.273(8)$ |
| $Q$ | $0.0211$ | $0.106$ |
| $S$ | $0.214$ | $0.161$ |

FIG. 2: (color online) Variation of $D$ with $T/T_c$ for different values of $\mu_B$ around $\mu_Q = 0$.

We have thus calculated $D$ as a function of $\sqrt{s}$ along the freeze-out curve. Results are shown in Fig. 4. We have varied $\sqrt{s}$ from 5 GeV to 3 TeV. $D$ picks up a value of $\sim 4$ for 2 flavor and $\sim 3.5$ for 2+1 flavor at the low $\sqrt{s}$, drops down to a value of 2.6 for 2 flavor and 2.4 for 2+1 flavor around $\sqrt{s} \sim 200$ GeV, and saturates at these values even for increasing $\sqrt{s}$. It is highly exciting to find that the general features of $D$ vs $\sqrt{s}$ curve obtained in the PNJL model are found to be similar to those obtained directly in heavy-ion collision experiments as shown in Fig. 4. (color online) $D$ as a function of $\sqrt{s}$ computed along the freeze-out curve.

FIG. 3: (color online) Variation of $D/D_{free}$ with $\mu_B$ for three values of $T$ around $\mu_B = 0$ for 2 flavors (upper panel) and 2+1 flavors (lower panel).

FIG. 4: (color online) $D$ as a function of $\sqrt{s}$ computed along the freeze-out curve.
Fig. 4 of Ref. [10]. Furthermore the numerical range of $D$ itself is exceptionally consistent.

It should however be remembered that $D$ as given in Fig. 4 is the value obtained when the system is in complete thermal equilibrium at the given values of temperature and chemical potentials. Since here we are on the freeze-out curve, we are always inside the hadronic phase, i.e. the whole of the curve in Fig. 4 is corresponding to varying environmental conditions in the hadronic phase. Thus if our results become completely consistent with experimental results the outcome will be that there is no signature of partonic phase in $D$. At present it seems that the results for STAR data are above and those of ALICE data are below our model curve. A more concrete analysis would be possible once the complete experimental data for $D$ are published.

To summarize, we presented here model study of net charge fluctuations in terms of $D$-measure using the PNJL model. We found that $D$ does not have a clear order parameter like behavior. However, given the temperature and the chemical potentials it is possible to estimate the corresponding $D$ in PNJL model and compare with experiments. Such a preliminary comparison has been done in this work giving encouraging results and indicating the possibility of detecting signatures of exotic phases in heavy-ion collisions.

This work is funded by CSIR, UGC(DRS & UPE) and DST. We thank Tapan Nayak, Anirban Lahiri and Sarbani Majumder for useful discussions.

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