Properties of O(6)-U(5) transition symmetry for $^{122,124}$Cd isotopes in IBM

Huda H. Kassim$^1$, Alyaa A. Abd-Aljbar$^2$, Mushtaq Abed Al-Jubbori$^3$, Hewa Y. Abdullah$^4$, I. Hossain$^5$ and Fadhil I. Sharrad$^{1,6}$

$^1$Department of Physics, College of Science, University of Kerbala, Karbala, Iraq.
$^2$Materials Research, Ministry of Science and Technology, Baghdad, Iraq
$^3$Department of Physics, College of Education for Pure Science, University of Mosul, 41001 Mosul, Iraq
$^4$Physics Education Department, Faculty of Education, Tishk International University, 44001, Erbil, Iraq
$^5$Department of Physics, Rabigh College of Science & Arts, King Abdulaziz University, 21911 Rabigh, Saudi Arabia
$^6$College of Health and Medical Technology, Al-Ayen University, Al Nasiriya, Thi Qar 64001, Iraq

fadhil.altaie@gmail.com

Abstract.

The some properties, Energy levels, B(E2) values and potential energy surface, for even-even $^{122,124}$Cd isotopes have been studied using the interacting boson model. The predicted levels (energies, spins and parities) and B(E2) values results were reasonably consistent with the experimental data. The contour plot of the potential energy surfaces shows all interest nuclei were deformed and have O(6)-U(5) transition symmetry.

Keyword: IBM-1; neutron-rich; transition; energy levels

1. Introduction

The quadrupole collectivity in nuclear core displays particular regularities, where the atomic shape can be circular, disfigured and the circumstance in the middle. Like different models and speculations [1, 2], the Interacting Boson Model [3] has been fruitful in recreating the atomic aggregate levels regarding s and d bosons, which are basically the aggregate S and D sets of valence nucleons [4], individually. The IBM Hamiltonian has the supposed dynamical balance, and the state of quadrupole distortion can be named a circular vibrator (U(5)), pivotally symmetric disfigurement (SU(3)), and $\gamma$–unstable twisting (O(6)), if the connection qualities of the IBM Hamiltonian taken explicit
qualities. The medium-to overwhelming mass Ytterbium (Yb) isotopes are situated in the back earth mass district, a large portion of these cores were all around disfigured and it tends to be populated to high turn. Much trial informations on even-odd-mass Yb isotopes have become more plentiful [5-10]. For the heavier A=174 to 178 cores [11], past work utilizing profound inelastic responses and Gammasphere have started to uncover a lot of informations about the high-turn conduct of these neutron-rich Yb isotopes. The yrast states in the very much distorted uncommon earth area have been depicted utilizing the anticipated shell model [12-18]. In this investigation, the figurings of energy levels of even-even $^{122,124}$Cd isotopes have been finished utilizing the connecting boson model. Positive equality state energies, decrease probabilities of E2 advances (B(E2)values) and potential vitality surface were determined and contrasted and the test information.

2. Theoretical

The IBM has gotten one of the most seriously utilized atomic models, because of its capacity to portray the evolving low-lying aggregate properties of cores over a whole significant shell with a basic Hamiltonian. In the IBM the spectroscopies of low-lying aggregate properties of even-even cores were depicted regarding an arrangement of cooperating s bosons (L=0) and d bosons (L=2) [19]. Besides, the structure of low-lying levels is overwhelmed by excitations among the valence partials outside the major shut shells in this model. The quantity of proton bosons $N_p$ and neutron bosons $N_n$ were checked from the closest shut shell, and the absolute boson number $N= N_p + N_n$. The hidden structure of the six-dimensional unitary gathering SU(6) of the model prompts a basic Hamiltonian, fit for depicting the three explicit sorts of aggregate structure with traditional mathematical analogs (vibrational [20], rotational [21], and $\gamma$-unstable[22]) and furthermore the temporary cores [23] whose structure are middle.

$$H = \varepsilon_s s^+ s + \varepsilon_d (d^+ d) + \sum_{L=0,2} \sum_{s,d} C_L \left[ (d^+ d) (dd) (s^+ s) (dd) \right] + \frac{1}{\sqrt{2}} \Omega \left[ (d^+ d) (0) s^2 + (s^+ s) d^2 (dd) (0) \right]$$

where it can be written in general form as [24]

$$H = a_0 P^\dagger P + a_1 L L + a_2 Q Q + a_3 T_3 T_3 + a_4 T_4 T_4$$

where $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy. The parameters $a_0, a_1, a_2, a_3$ and $a_4$ designated the strength of the pairing, angular momentum, quadrupole, octupole and hexdecupole interaction between the bosons.

2. Results and Discussion

3.1. States

The O(6)- U(5) breaking point of the IBM-1 has been applied for the even-even $^{122,124}$Cd isotopes because of the estimations of E(41+)/E(21+) proportion as introduced in the table 1, in this manner, these isotopes have a dynamical evenness O(6)- U(5) regarding to IBM-1. The figurings have been performed with no qualification made among neutron and proton bosons. For the examination of excitation energies in Cd isotopes it was attempted to keep to least the quantity of free boundaries in Hamiltonian. The unequivocal articulation of Hamiltonian received in figurings is [24].

3.1. States

The O(6)- U(5) breaking point of the IBM-1 has been applied for the even-even $^{122,124}$Cd isotopes because of the estimations of E(41+)/E(21+) proportion as introduced in the table 1, in this manner, these isotopes have a dynamical evenness O(6)- U(5) regarding to IBM-1. The figurings have been performed with no qualification made among neutron and proton bosons. For the examination of excitation energies in Cd isotopes it was attempted to keep to least the quantity of free boundaries in Hamiltonian. The unequivocal articulation of Hamiltonian received in figurings is [24].
\[ H = \alpha n_d + a_0 P^P + a_1 L^L + a_2 Q^Q \]  

(3)

In the system of the IBM-1, the isotopic chains of Cd with Z=48 cores, having various proton bosons gaps 1, various neutron bosons particles differs from (4 to 3) for \(^{122,124}\text{Cd}\). The coefficient estimates which have a decent concurrence with the exploratory outcomes are appeared in Table 2.

**Table 1.** The values of \(E(4_1^+)/E(2_1^+)\) ratio

| A   | \(^{122}\text{Cd}\) | \(^{124}\text{Cd}\) |
|-----|--------------------|--------------------|
| R\(_{4/2}\) | 2.33              | 2.26               |

**Table 2.** IBM-1 parameters in MeV, except N

| Isotope | N | IBM | EPS | PAIR | ELL | OCT |
|---------|---|-----|-----|------|-----|-----|
| \(^{122}\text{Cd}\) | 5 | IBM | 0.530 | 0.142 | 0.013 | 0.058 |
| \(^{124}\text{Cd}\) | 4 | IBM | 0.580 | 0.153 | 0.014 | 0.063 |

The determined ground band and exploratory information of energy levels were drawing in Figure 1 for the even-even \(^{122,124}\text{Cd}\) isotope. Great understandings to the examination of the IBM-1 results (energies, spin and parity) were the trial information. In any case, it is digressed in the high turn (energies) of the exploratory information. The IBM-1 was effective to anticipate the other bands for all cores enthusiasm as appeared in Tables 3 and 4, and figure 1 individually. Levels with ‘*’ relate to cases for which the turn or potentially equality of the comparing states are not settled tentatively. The IBM-1 counts were in acceptable concurrences with the trial brings about these tables.

**Table 3.** Ground states of \(^{122,124}\text{Cd}\), The experimental data have been taken from [25]

| \(J^z\) | EXP. | IBM \(^{122}\text{Cd}\) | EXP. | IBM \(^{124}\text{Cd}\) |
|--------|------|----------------|------|----------------|
| 0\(^+\)  | 0    | 0              | 0    | 0              |
| 2\(^+\)  | 0.559| 0.564          | 0.612*| 0.632          |
| 4\(^+\)  | 1.329*| 1.300         | 1.385*| 1.453          |
| 6\(^+\)  | 2.178*| 2.198         | 2.139*| 2.449          |
| 8\(^+\)  | 2.823*| 3.252         | 2.674*| 3.099          |
Figure 1. The calculated and experimental data [25] for $^{122,124}$Cd isotopes.

Table 4. The Other states of $^{122,124}$Cd, The experimental data have been taken from [25]

| $J^\pi$ | EXP. | IBM $^{122}$Cd | EXP. $^{124}$Cd | IBM $^{124}$Cd |
|-------|-----|----------------|----------------|----------------|
| $0_2^+$ | 1.704* | 1.793 | -- | 2.100 |
| $2_2^+$ | 1.367* | 1.557 | 1.427* | 1.714 |
| $3_1^+$ | -- | 2.602 | 1.915* | 2.009 |
| $4_2^+$ | -- | 2.749 | 1.915* | 2.564 |
| $6_2^+$ | -- | 3.802 | -- | 3.008 |
| $2_3^+$ | -- | 2.556 | -- | -- |
| $4_3^+$ | -- | 3.492 | -- | -- |

3.2. The B9E2) Values

Another properties of the IBM is that the network components of the electric quadrupole administrator. The decreased framework components of the E2 administrator TE2 has the structure [26-28]
\[ T^{E2} = \alpha_2 \{ d^\dagger s + s^\dagger d \}^{(2)} + \beta_2 \{ d^\dagger d \}^{(2)} \]  

(4)

Where: \((s^\dagger, d^\dagger)\) and \((s, d)\) denoted as creation and annihilation operators corresponding to the r
s and d bosons. The parameters \(\alpha_2\) and \(\beta_2\) related to the eB parameter. Then the \(B(E2)\) values are given by

\[ B(E2) = \frac{1}{2j+1} |\langle \psi_f | \hat{\Pi} | \psi_i \rangle|^2 \]  

(5)

For the estimations of the outright \(B(E2)\) values the boundary eB were balanced by the trial \(B(E2; 2^+_1 \rightarrow 0^+_1)\), the boundaries eB are introduced in the table 5, which acquired in the current computations. The determined consequences of the diminished likelihood changes, \(B(E2)\) values, and the test information \([25]\) are introduced in Table 6 for all cores intrigue.

**Table 5.** The B9E2) parameter (in eb unit) for \(^{122-124}\)Cd isotopes.

| Isotope  | N | eB  |
|----------|---|-----|
| \(^{122}\)Cd | 5 | 0.093 |
| \(^{124}\)Cd | 4 | 0.098 Fitting= 0.098 |

**Table 6.** Comparison between theoretical and experimental \(B(E2)\) values (in \(e^2b^2\) unit) for \(^{122-124}\)Cd nuclei.

| \(J_i \rightarrow J_f \) | \(^{122}\)Cd | \(^{124}\)Cd |
|-----------------|-----------|-----------|
|                   | EXP. | IBM-1 | EXP. | IBM-1 |
| \(2^+_1 \rightarrow 0^+_1 \) | 0.093 | 0.089 | -- | 0.059 |
| \(2^+_2 \rightarrow 2^+_1 \) | -- | 0.115 | -- | 0.088 |
| \(4^+_1 \rightarrow 2^+_1 \) | -- | 0.115 | -- | 0.093 |
| \(4^+_2 \rightarrow 2^+_1 \) | -- | 0.058 | -- | 0.078 |
| \(4^+_2 \rightarrow 4^+_1 \) | -- | 0.058 | -- | 0.064 |
| \(6^+_1 \rightarrow 4^+_1 \) | -- | 0.112 | -- | 0.063 |
| \(8^+_1 \rightarrow 6^+_1 \) | -- | 0.089 | -- | 0.038 |

### 3.2. Contour Shape

Lately, the potential vitality surface by Skyrme mean field technique was planned onto the PES of the IBM Hamiltonian\([29-32]\). The desire estimation of the IBM-1 Hamiltonian with the sound state \(|\psi, \beta, \gamma\rangle\) is utilized to make the IBM vitality surface\([24]\). The state is a result of boson creation administrators \((b^\dagger)\), with

\[ |\psi, \beta, \gamma\rangle = \frac{1}{\sqrt{N}} (b^\dagger)^N |0\rangle \]  

(6)

\[ b^\dagger_c = (1 + \beta^2)^{\gamma_i} \{ s^\dagger + \beta [\cos \gamma (d^\dagger_o) + \sqrt{\bar{\gamma}} \sin \gamma (d^\dagger_2+d^\dagger_3)] \} \]  

(7)
The PES, represented by $\beta$ and $\gamma$, and can be written as: \[24\]

$$E(N, \beta, \gamma) = N\epsilon_d \beta^3/\alpha_{\beta} + \gamma(N-1)/(1+\beta)\left(\alpha_1\beta^4 + \alpha_2\beta^3 \cos 3\gamma + \alpha_3\beta^2 + \alpha_4\right)$$ \quad (8)$$

Where the $\alpha_i$'s are related to the parameters $C_L$, $\nu_2$, $v_0$, $u_2$ and $u_0$ of equation (1).

The contour shapes for Cd isotopes have been shown in Figure 2. From this figure, it was noticed that the nuclei under study have O(6)-U(5) limit.

![Figure 2](image)

**Figure 2.** (Color online) the contour shape for even-even $^{122-124}$Cd nuclei.

4. Conclusion

The IBM was utilized to figure the vitality levels (positive equality), the diminished likelihood of E2 changes and potential vitality surface for $^{122-124}$Cd isotopes. The anticipated low-lying levels (energies, twists and equalities) and the diminished likelihood of E2 changes results were sensibly reliable with the trial results. The potential vitality surfaces for Cd nuclei shows that all cores are disfigured and have O(6)-U(5) limit.

Acknowledgments

We thank the Department of Physics and University of Kerbala - College of Science - Department of Physics for supporting this work.

Reference

[1] F. I. Sharrad, I. Hossain, I. M. Ahmed, H. Y. Abdullah, S. T. Ahmad and A. S. Ahmed Braz J Phys 45, 340 (2015).
[2] F. I. Sharrad, H. Y. Abdullah, N. AL-Dahan, N. M. Umran, A. A. Okhunov and H.
Abu-Kassim, Chinese Physics C 37, 034101 (2013).
[3] I. Hossain, I. M. Ahmed, F. I. Sharrad, H. Y. Abdullah, A. D. Salman and N. Al-Dahan, Chiang Mai J. Sci. 42 996(2015).
[4] A Shelley, I Hossain, Fadhil I Sharrad, Hewa Y Abdullah and M A Saeed Prob. Atom. Sci.& Tech. 64 38 (2015).
[5] I. Hossain, H. H. Kassim, F. I. Sharrad and A. S. Ahmed, ScienceAsia 42, 22(2016).
[6] M. A. Al-Jubbori, H. H. Kassim, F. I. Sharrad and I. Hossain, Nucl. Phys. A 955, 101(2016).
[7] H. H. Khudher, A. K. Hasan and F. I. Sharrad, Ukrainian Journal of Physics 62, 152 (2017).
[8] M. O. Waheed and F. I. Sharrad, Ukrainian Journal of Physics 62, 757(2017).
[9] R. F. Casten and D. D. Warner, Rev. Mod. Phys. 60, 389 (1988).
[10] I. M. Ahmed, G. N. Flaiyh, H. H. Kassim, H. Y. Abdullah, I. Hossain and F. I. Sharrad, Eur. Phys. J. Plus 132, 84 (2017).
[11] H. H. Kassim and F. I. Sharrad, Nucl. Phys. A 933, 1(2015).
[12] M. A. Al-Jubbori, F. Sh. Radhi, A. A. Ibrahim, S. A. Abdullah Albakri, H. H. Kassim and F. I. Sharrad, Nuclear Physics A 971, 35 (2018).
[13] O. Scholten, Computer code PHINT, KVI; Groningen, Holland, (1980).
[14] M. A. Al-Jubbori, H. H. Kassim, F. I. Sharrad, A. Attarzadeh and I. Hossain, Nuclear Physics A 970, 438 (2018).
[15] A. Okhunov, F. I. Sharrad, A. A. Al-Sammarea and M. U. Khandaker, Chinese Physics C 39, 084101 (2015).
[16] H. H. Kassim, A. A. Mohammed-Ali, M. Abed Al-Jubbori, F. I. Sharrad, A.S. Ahmed and I. Hossain, J. Natn. Sci. Foundation Sri Lanka 46, 3 (2018).
[17] M. Abed Al-Jubbori, K. A. Al-Mtiuty, K. I. Saeed and F. I. Sharrad, Chinese Physics C 41, 084103 (2017).
[18] I. Hossain, F. I. Sharrad, M. A. Saeed, H. Y. Abdullah and S. A. Mansour, Maejo Int. J. Sci. Technol. 10, 95 (2016).
[19] H. H. Kassim and F. I. Sharrad, International Journal of Modern Physics E 23, (2014) 1450070.
[20] Kahtan A. Hussein, Musa K. Mohsin and Fadhil I. Sharrad, , Iranian Journal of Science and Technology, Transactions A: Science,43(2017) 1273–1285
[21] M A Al-Jubbori, H H Kassim, A A Abd-Aljbbar, H Y Abdullah, I Hossain, I M Ahmed and Fadhil I Sharrad, Indian Journal of Physics, 94(2020) 379-390
[22] Wisam N. Hussein, I. Hossain and Fadhil I Sharrad, Journal of Physics: Conf. Series, 1279 (2019) 012027
[23] Mariam O. Waheed and Fadhil I. Sharrad, Journal of Physics: Conf. Series, 1279 (2019) 012077
[24] Mahdi A. Mahdi, Fahmi Sh. Radhi, Huda H. Kassim, Mushtaq AbedAl- Jubbori and Fadhil I. Sharrad, Journal of Physics: Conf. Series, 1279 (2019) 012021
[25] http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp
[26] Mushtaq Abed Al-Jubbori, Huda H. Kassim, Fahmi Sh. Radhi, Amin Attarzadeh, I. Hossain, Imad M. Ahmed, and Fadhil I. Sharrad, Physics of Atomic Nuclei, 82 (2019) 201–211
[27] Imad Mamdouh Ahmed, Hewa Y. Abdullah, Mudhaffer Mustafa Ameen, Huda H. Kassim and Fadhil I. Sharrad, Physics of Atomic Nuclei, 81 (2018) 695–702
[28] Wisam N. Hussain and Fadhil I. Sharrad, Journal of Physics: Conference Series, 1032 (2018) 012046
[29] Imad Mamdouh Ahmed, Mushtaq Abed Al-Jubbori, Huda H. Kassim, Hewa Y. Abdullah and Fadhil I. Sharrad, Nuclear Physics A, 977 (2018)34-48
[30] Mushtaq Abed Al-Jubbori, Huda H. Kassim, Fadhil I. Sharrad and I. Hossain, International Journal of Modern Physics E, 27 (2018) 1850035.
[31] Huda H. Kassim, Amir A. Mohammed-Ali, Fadhil I. Sharrad, I. Hossain and Khalid S. Jassim, Iranian Journal of Science and Technology, Transactions A: Science, 42 (2018) 993-999.
[32] Fadhil I. Sharrad, H.Y. Abdullah, N. Al-Dahan, A. A. Mohammed-Ali, A. A. Okhunov and H. Abu Kassim, Romanian Journal of Physics, 57, (2012) 1346-1355.