RIS-Aided D2D Communications Relying on Statistical CSI with Imperfect Hardware

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Abstract—In this letter, we investigate a reconfigurable intelligent surfaces (RIS)-aided device to device (D2D) communication system over Rician fading channels with imperfect hardware including both hardware impairment at the transceivers and phase noise at the RISs. This paper has optimized the phase shift by a genetic algorithm (GA) method to maximize the achievable rate for the continuous phase shifts (CPSs) and discrete phase shifts (DPSs). We also consider the two special cases of no RIS hardware impairments (N-RIS-HWIs) and no transceiver hardware impairments (N-T-HWIs). We present closed-form expressions for the achievable rate of different cases and study the impact of hardware impairments on the communication quality. Finally, simulation results validate the analytic work.

Index Terms—Reconfigurable intelligent surface (RIS), hardware impairment, D2D communication, intelligent reflecting surface (IRS).

I. INTRODUCTION

Reconfigurable intelligent surface (RIS) is a new transmission technology that can configure the radio channel in a desired manner by optimizing the phase shift of each reflecting element [1]. Due to their appealing properties of low power consumption and low cost, RIS-aided wireless communications have attracted much research attentions [2]–[5]. Some initial attempts to study RIS-aided communication systems include RIS-aided full-duplex systems [2], RIS-aided physical layer security [3], RIS-aided wireless power transfer [4], RIS-aided mobile edge computing [5], and RIS-aided multiuser transmission [6]. The cascaded channel estimation was studied in [7].

D2D communication has received great attention to meet the rapidly increasing demand for data traffic [3]. In D2D communication in cellular networks, cellular and D2D users transmit signals simultaneously using the same spectrum of cellular users. However, there is a paucity of contributions on RIS-aided D2D communications. In [9], the authors proposed to deploy an RIS that can assist pairs of devices in their communication when the direct links are blocked by high buildings, plants and walls.

However, most of the existing contributions on D2D communication are based on the assumption of perfect hardware in the radio frequency chains [10] and the RISs [9]. Existing studies have shown that hardware impairments adversely affect the system performance of the multiple-input multiple-output systems [11], [12]. In addition, an RIS has low hardware cost [2]–[5], [9], which is prone to hardware imperfections.

Based on above, a natural question is whether we can use non-ideal low-cost hardware to assist the D2D communications. Specifically, our contributions are summarized as follows: 1) We consider an RIS-aided D2D communication system over Rician fading channels, considering hardware impairments both at the terminals and at the RISs. We assume that the hardware impairment is coupling with the transmission signal at the transceiver. Moreover, the random phase error caused by the phase noise at the RIS follows the Von Mises distribution; 2) we present the closed-form expressions for the achievable rate in the general case of both hardware impairments both at the terminals and at the RISs, which is then maximized by a genetic algorithm (GA) method. To obtain more design insights, we also study the two special cases of no RIS hardware impairments (N-RIS-HWIs) and no transceiver hardware impairments (N-T-HWIs); 3) we provide simulation results to verify our derived expressions. In addition, the exhaustive search method is used to show that our results can achieve the globally optimal solution.

The rest of the letter is organized as follows. The model for RIS-aided D2D communication system with hardware impairments is depicted in Section II. We derive the sum achievable rate’s expression and maximize it in Section III. Numerical results are presented in Section IV. And conclusions are drawn in Section V.

\[
\gamma_i = \frac{\sum_{j=1, j \neq i}^{K} \left( p_j | g_{bi}^T \Theta \Phi g_{aj} |^2 + |\mathbf{g}_{bi}^T \Theta \Phi G_a \sqrt{\Lambda} \eta_i |^2 + T_{ri} + \sigma_i^2 \right)}{p_i \lambda_i g_{bi}^T \Theta \Phi g_{ai}} \quad (10) 
\]

\[
R_i \approx \log_2 \left( 1 + \frac{p_i \lambda_i g_{bi}^T \Theta \Phi g_{ai}}{1 + \kappa_r (1 + \kappa_i) \sum_{j=1}^{K} p_j \lambda_j g_{bi}^T \Theta \Phi g_{aj}} \right) \quad \left( \frac{\varepsilon_i \beta_i \Gamma_{ri,i} + L (\varepsilon_i + \beta_i) + L (\varepsilon_i + \beta_i + 1)}{(\varepsilon_i + 1)(\beta_i + 1)} \right) \quad (13) 
\]
We consider an RIS-aided D2D communication system, where an RIS assists \( K \) pairs of devices to exchange information, as shown in Fig. 1. It is assumed that there are a total of \( K \) D2D links in the network, where the \( i \)th single-antenna transmitter (receiver) is denoted by DT\(_i\) (DR\(_i\)) for \( i = 1, \ldots, K \). In the overlay mode, the radio resources occupied by D2D links are orthogonal to that of the cellular links, which ensures no interference between D2D links and cellular links. The RIS includes the cellular links, which ensures no interference between resources occupied by D2D links are orthogonal to that of DR\(_i\).

The channel between DT\(_i\) and the RIS (the RIS and DR\(_i\)) can be written as

\[
g_{ai} = \sqrt{\alpha_{ai}} h_{ai},
\]

where \( \alpha_{ai} \) and \( \alpha_{bi} \) denote the large-scale fading coefficients, and \( h_{ai} \in \mathbb{C}^{L \times 1} \) and \( h_{bi} \in \mathbb{C}^{L \times 1} \) denote Rician fading channels, which can be expressed as

\[
h_{ai} = \frac{\varepsilon_i}{\varepsilon_i + 1} \bar{h}_{ai} + \sqrt{\frac{1}{\varepsilon_i + 1}} h_{ai},
\]

\[
h_{bi} = \frac{\beta_i}{\beta_i + 1} \bar{h}_{bi} + \sqrt{\frac{1}{\beta_i + 1}} h_{bi},
\]

where \( \varepsilon_i \) and \( \beta_i \) denote the Rician factors, \( \bar{h}_{ai} \in \mathbb{C}^{L \times 1} \) and \( \bar{h}_{bi} \in \mathbb{C}^{L \times 1} \) are non-line-of-sight components, whose entries are standard Gaussian random variables with independent and identical distribution, i.e., \( \mathcal{CN}(0, 1) \), and \( h_{ai} \in \mathbb{C}^{L \times 1} \) and \( h_{bi} \in \mathbb{C}^{L \times 1} \) are line-of-sight (LoS) components. Particularly, \( \bar{h}_{ai} \) and \( \bar{h}_{bi} \) can be written as

\[
\bar{h}_{ai}(\varphi_i^a, \varphi_i^c) = \begin{bmatrix}
\varepsilon_i e^{j2\pi(x \sin \varphi_i^a \sin \varphi_i^c + y \cos \varphi_i^a)} \\
\varepsilon_i e^{j2\pi(x \sin \varphi_i^a \sin \varphi_i^c + y \cos \varphi_i^a)} \\
\varepsilon_i e^{j2\pi(x \sin \varphi_i^a \sin \varphi_i^c + y \cos \varphi_i^a)} \\
\varepsilon_i e^{j2\pi(x \sin \varphi_i^a \sin \varphi_i^c + y \cos \varphi_i^a)}
\end{bmatrix}^T,
\]

\[
\bar{h}_{bi}(\varphi_i^a, \varphi_i^c) = \begin{bmatrix}
\varepsilon_i e^{j2\pi(x \sin \varphi_i^a \sin \varphi_i^c + y \cos \varphi_i^a)} \\
\varepsilon_i e^{j2\pi(x \sin \varphi_i^a \sin \varphi_i^c + y \cos \varphi_i^a)} \\
\varepsilon_i e^{j2\pi(x \sin \varphi_i^a \sin \varphi_i^c + y \cos \varphi_i^a)} \\
\varepsilon_i e^{j2\pi(x \sin \varphi_i^a \sin \varphi_i^c + y \cos \varphi_i^a)}
\end{bmatrix}^T,
\]

where \( 0 \leq x, y \leq \sqrt{L} - 1 \), \( \varphi_i^a, \varphi_i^c \) respectively denote the \( i \)th pair of devices’ azimuth and elevation angles of arrival (angle of departure). We assume \( d = \frac{1}{2} \) in our letter [11].

The signal transmitted from DT\(_i\) is given by

\[
x_i = \hat{x}_i + \eta_{ai},
\]

where \( \hat{x}_i \sim \mathcal{CN}(0, 1) \) represents the data symbol, \( \eta_{ai} \sim \mathcal{CN}(0, \kappa_i \mathbb{E}\{|z_{ai}|^2\}) \) denotes the impact of impairments in the transmitter hardware, and \( \kappa_i \in (0, 1) \) is the coefficient which characterizes the impairments at the transmitter.

The RIS’s hardware impairments can take the form of phase noise because the RIS is a passive device and highly accurate reflection adjustment is impractical. The phase noise matrix is denoted by \( \Phi_i = \text{diag}(e^{j\Delta\theta_1}, \ldots, e^{j\Delta\theta_L}) \), for \( \ell = 1, 2, \ldots, L \). \( \Delta\theta_\ell \) is random phase error that follows the Von Mises distribution with zero mean and concentration parameter \( \kappa_i \Delta\theta \). Additionally, \( \Delta\theta_\ell \) has a characteristic function \( \chi = \frac{1}{\kappa_i \Delta\theta} \) with \( I_\pi(\kappa_i \Delta\theta) \) being the modified Bessel function of the first kind and order \( \pi \) [12].

We exploit statistical channel state information (CSI), which varies much slowly than the instantaneous CSI and can be readily obtained. The signal received at DR\(_i\) is given by

\[
y_i = g_{bi}^T \Phi_i \Phi g_{ai} x_j + \eta_{ri} + n_i
\]

where \( g_{ai} = [g_{a1}, \ldots, g_{aK}]^T \), \( x_j \sim \mathcal{CN}(0, \sigma_j^2) \), \( \Phi \sim \mathcal{CN}(0, \kappa_i \mathbb{E}\{|z_{ai}|^2\}) \), \( \kappa_i \in (0, 1) \) is the coefficient which characterizes the impairments at the receiver, and \( n_i \sim \mathcal{CN}(0, \sigma_i^2) \) is the additive white Gaussian noise at DR\(_i\), for \( i = 1, \ldots, K \).

To analyze the achievable rates, we consider the expansion of (8) as follows

\[
y_i = \sqrt{p_j} g_{bi}^T \Phi_i \Phi g_{ai} \hat{x}_j + \sum_{j=1}^{K} \sqrt{p_j} g_{bi}^T \Phi_i \Phi g_{ai} \hat{x}_j + \eta_{ri} + n_i,
\]

where \( \eta_i \equiv [\eta_{r1}, \ldots, \eta_{rK}]^T \).

From (9), we obtain the signal-to-interference plus noise ratio at DR\(_i\) shown at the bottom of this page.

Therefore, the achievable rate for DR\(_i\) can be expressed as

\[
R_i = \mathbb{E}\{\log_2(1 + \gamma_i)\},
\]

and the sum achievable rate can be written as

\[
C = \sum_{i=1}^{K} R_i.
\]

## III. Achievable Rate Analysis

In this section, we first derive an approximate expression for the sum achievable rate in the following theorem. Then, we consider two special cases of N-RIS-HWIs and N-T-HWIs. Finally, we maximize the achievable rate with GA (genetic algorithm) method.
Theorem 1. In a D2D communication system aided by RIS, the ergodic sum achievable rate of DR, can be approximated as \( L_2 \), where \( \Gamma_{i,i} \) and \( \Gamma_{i,j} \) can be given by

\[
\Gamma_{i,j} = E \left[ \left( \frac{1}{L} \sum_{i=1}^{L} c_i^2 \left( \theta_i + \Delta \theta_i + \pi T_{i,i}^{m,n} \right) \right)^2 \right] = L + 2 \sum_{m = 0}^{N-1} \cos \left( \theta_m - \theta_m + \pi T_{i,i}^{m,n} \right), \quad m \leq n \leq L
\]

where \( T_{i,j}^{m,n} = (x_i - x_j + \gamma_{i,j} + \gamma_{i,j} + \gamma_{i,j}) \), with \( x_i = \frac{(z_i - 1)}{\sqrt{L}}, \quad y_j = \frac{(z_j - 1)}{\sqrt{L}}, \quad z \in \{m, n\}, \quad p_i = \sin \varphi_i + \sin \psi_i, \quad q_i = \cos \varphi_i + \cos \psi_i.

Proof: See Appendix A.

Corollary 1. Assuming that the RIS hardware is ideal and thus there is no phase noise, i.e., \( \Delta \theta = 0 \), for \( i = 1, 2, \cdots, N \).

It follows that \( \Phi = I_L \). The rate of DR, on N-RIS-HWIs can be approximated as \( L_2 \), where \( \Gamma_{i,j} \) can be defined as

\[
\Gamma_{i,j} = L + 2 \sum_{m = 0}^{N-1} \cos \left( \theta_m - \theta_m + \pi T_{i,i}^{m,n} \right), \quad m \leq n \leq L
\]

Proof: When \( \Phi = I_L \), we can use \( \Phi \) Eq. (20) Eq. (21), and write \( E \left[ h_{i,j}^H \Theta_{i,j} h_{i,j}^H \right] \) and \( E \left[ h_{i,j}^H \Theta_{i,j} h_{i,j}^H \right] \) as

\[
E \left[ h_{i,j}^H \Theta_{i,j} h_{i,j}^H \right] = \frac{\varepsilon_i \beta_i \Gamma_{i,j} + L (\varepsilon_i + \beta_i) + L}{\varepsilon_i + 1} (\beta_i + 1), \quad h \in \{i,j\}, \quad (16)
\]

where \( \Gamma_{i,j} \) is defined in \( (16) \).

By substituting \( (17) \) into \( (16) \), we are able to obtain the final result in \( (13) \). This completes the proof.

We aim to solve the achievable rate maximization problem by optimizing the phase shifts matrix \( \Theta \) in the special case with only one pair of users, i.e., \( K = 1 \). Without loss of generality, the user pair is referred to as user pair \( i \). We can rewrite the achievable rate as

\[
R_i^{N-RIS-HWIs} \approx \log_2 \left( 1 + \sum_{j \neq i} \frac{p_i \alpha_i \alpha_j \varepsilon_i \beta_i \Gamma_{i,j} + L (\varepsilon_i + \beta_i) + L}{\varepsilon_i + 1} (\beta_i + 1) + \sigma_i^2 \right)
\]

The rate depends on the phase shifts matrix \( \Theta \) only through the intermediate variable \( \Gamma_{i,j} \). Since \( \Gamma_{i,j} = \sum_{l=1}^{L-1} \varepsilon_l \beta_l \Gamma_{i,j} + L (\varepsilon_l + \beta_l) + L \) and \( \Gamma_{i,j} \geq 0 \), the optimization problem can be formulated as follows

\[
\max \quad R_i^{N-RIS-HWIs} \quad (19a)
\]

\[
\text{s.t.} \quad 0 \leq \Gamma_{i,j} \leq L^2. \quad (19b)
\]

The expression for the first-order derivative of SINR, \( \Gamma_{i,i} \) with respect to \( \Gamma_{i,i} \) is

\[
\frac{\partial}{\partial \Gamma_{i,i}} \left( \Gamma_{i,i} \right) = \frac{p_i \alpha_i \alpha_j \varepsilon_i \beta_i (\varepsilon_i + 1)}{\varepsilon_i + 1} \left( \varepsilon_i \beta_i \Gamma_{i,i} + L (\varepsilon_i + \beta_i) + L + \sigma_i^2 \right)^2 \geq 0.
\]

Thus, the optimal design for \( \Theta \) corresponds to setting \( \Gamma_{i,i} = L^2 \), where the optimal phase shifts of all the RIS elements are given by \( \theta_j = -\pi T_{i,i}^{m,n} + C_0, \forall \ell \), and \( C_0 \) is an arbitrary constant.

Considering this single-user system with optimal phase shift, we assume the transmit power is scaled as \( p_i = \frac{E_i}{L^2} \) and \( p_i = \frac{E_i}{L^2} \) with \( L \rightarrow \infty \). If the transmit power is scaled as \( p_i = \frac{E_i}{L^2} \), the rate becomes

\[
R_i^{N-RIS-HWIs} \rightarrow \log_2 \left( 1 + \frac{p_i \alpha_i \alpha_j \varepsilon_i \beta_i (\varepsilon_i + 1)}{\varepsilon_i + 1} (\beta_i + 1) + \sigma_i^2 \right), \quad (21)
\]

If the transmit power is scaled as \( p_i = \frac{E_i}{L^2} \), the rate becomes

\[
R_i^{N-RIS-HWIs} \rightarrow \log_2 \left( 1 + \frac{1}{\varepsilon_i \beta_i (\varepsilon_i + 1) (\beta_i + 1)} \right), \quad (22)
\]

In this case, the achievable rate only depends on the impairment coefficients at the transceiver when \( L \) grows into infinity.

Corollary 2. Assuming that the transceiver hardware is ideal, i.e., \( \kappa_i = \kappa_r = 0 \). The rate of DR, with N-T-HWIs can be approximated as

\[
R_i^{N-T-HWIs} \approx \log_2 \left( 1 + \frac{p_i \alpha_i \alpha_j \varepsilon_i \beta_i (\varepsilon_i + 1)}{\varepsilon_i + 1} (\beta_i + 1) + \sigma_i^2 \right), \quad (23)
\]

We design the phase shift with one pair of users, the achievable rate is

\[
R_i^{N-T-HWIs} \approx \log_2 \left( 1 + \frac{\epsilon_i \beta_i \Gamma_{i,j} + L (\varepsilon_i + \beta_i) + L}{\varepsilon_i + 1} (\beta_i + 1) + \sigma_i^2 \right) \quad (24)
\]

which is a monotonically increasing function of \( \Gamma_{i,i} \). The optimal design for \( \Theta \) corresponds to setting \( \Gamma_{i,i} = L^2 \), where the optimal phase shifts of all the RIS elements are given by \( \theta_j = -\pi T_{i,i}^{m,n} + C_1, \forall \ell \), and \( C_1 \) is an arbitrary constant.

Considering this single-user system with optimal phase shift, we assume the transmit power is scaled as \( p_i = \frac{E_i}{L^2} \) with \( L \rightarrow \infty \). If the transmit power is scaled as \( p_i = \frac{E_i}{L^2} \), the rate becomes

\[
R_i^{N-T-HWIs} \approx \log_2 \left( 1 + \frac{\epsilon_i \beta_i \Gamma_{i,j} + L (\varepsilon_i + \beta_i) + L}{\varepsilon_i + 1} (\beta_i + 1) + \sigma_i^2 \right) \quad (25)
\]

Due to the first-order derivative of the sum rate with respect to the phase shift \( \Theta \) is quite hard to obtain, we maximize the rate with the GA method adopted in [9] by optimizing the phase shifts. The complexity of the proposed GA algorithm is \( N_i \cdot n_i \), where \( N_i \) is the population size, and \( n_i \) is the number of generations evaluated. We take into account both continuous phase shifts (CPSs) and discrete phase shifts (DPSs). In practice, only a limited number of phase shifts can be used. We assume that the phase shift of the reflecting elements is quantized with \( B \) bits when considering DPS, and thus \( 2^B \) phase shifts can be chosen for each reflecting element.

IV. NUMERICAL RESULTS

We evaluate the impact of various parameters on the data rate performance. We set SNR = \( p_i, \varphi_i = 10 \) dB, \( \kappa = \kappa_r = \kappa_r \), \( \kappa \Delta \theta = 4, \sigma_i^2 = 1, \) and \( \varphi_i = \varphi_i \) and \( \psi_i = \psi_i \) are respectively set as \( \varphi_i \) and \( \psi_i \) in [9], for \( i = 1, \cdots, K \). Other parameters are set the same as [9].
In Fig. 2, we depict the rate in (12) versus the SNR obtained from the approximate expression in ([11]) and Monte-Carlo simulations from ([11]), when \( L = 16, K = 6, \kappa = 0.05, B = 2 \). The Monte-Carlo simulation results match the analytical expressions well, which verifies the data rate performance. Furthermore, the rate with N-RIS-HWIs performs better than the case with RIS-HWIs. As \( \kappa \Delta \phi \) decreases, the rate decreases. Additionally, when \( \kappa \Delta \phi = 0 \), the Von Mises distribution degenerates into the uniform distribution, i.e., \( \Delta \theta_\ell \sim \mathcal{U}(-\pi, \pi) \), for \( \ell = 1, 2, \cdots, L \). In this case, the random scheme curve has the same performance as that of our derived results, which demonstrate that there is no need to optimize the phase shift. The phenomenon is consistent with the result in ([12]).

Fig. 3 shows the sum rate versus the SNR when \( L = 8, K = 2, \) and \( B = 2 \). We can reduce the harmful impact of the T-HWIs by tuning the phase shifts of the RIS. Compared with the random scheme, the optimal phase shift can achieve higher rate. It is worth noting that the proposed GA method achieves similar performance to the exhaustive search, which implies that our proposed algorithm can achieve almost the globally optimal solution. Moreover, we observe that different levels of hardware impairment obtain different rate: the higher the level of the hardware impairment, the worse the performance of the rate. The rate with the ideal transceiver hardware (N-T-HWIs, \( \kappa = 0 \)) performs the best, as expected.

Fig. 4 illustrates the rate versus the Rician factor when SNR = 20 dB and \( B = 2 \). In all cases, with different values of \( L \) and \( K \), the rates approach the fixed value as Rician factor \( \varepsilon_1 \rightarrow \infty \). This is because the channels are mainly influenced by LoS component when Rician factor is large.

Fig. 5 shows the sum rate versus \( B \). The figure shows that, under the condition of DPS, three quantization bits are sufficient, which provides useful insight into the engineering design of systems aided by RISs. In addition, to resolve the issue caused by imperfect hardware, we can increase the number of low-cost reflection elements.

V. CONCLUSION

In this letter, we investigated an RIS-aided 2D communication system over Rician fading channels, considering hardware impairments both at the terminals and the RISs. We considered two special cases of N-RIS-HWIs and N-T-HWIs. We derived closed-form expressions for the achievable rate of different cases. The impact of the hardware impairment on the system performance have been observed. To resolve the issue caused by imperfect hardware, we can increase the number of low-cost reflection elements. Additionally, three quantization bits are sufficient for the DPSs setup, which provides useful insight into the engineering design of systems aided by RISs. Moreover, the extension to jointly optimizing power allocation and the phase shifts of the RIS will be left for our future work.

APPENDIX A

PROOF OF THEOREM I

Using Lemma 1 in ([13]), \( R_i \) in (11) is approximated as (23).

Next, we derive \( \mathbb{E}\left\{ |g_{il}^T \Theta \Phi g_{i'j'}|^2 \right\}, \mathbb{E}\left\{ |h_{il}^T \Theta \Phi h_{l'j'}|^2 \right\}, \mathbb{E}\left\{ |g_{il}^T \Theta \Phi G_a \sqrt{\Lambda t_h}|^2 \right\}, \) and \( \mathbb{E}\left\{ |h_{il}^T \Theta \Phi G_a \sqrt{\Lambda t_h}|^2 \right\} \). To begin with, we have

\[
\mathbb{E}\left\{ |g_{il}^T \Theta \Phi g_{i'j'}|^2 \right\} = \alpha_{il}^2 \mathbb{E}\left\{ |h_{il}^T \Theta \Phi h_{l'j'}|^2 \right\} \quad \text{and} \quad \mathbb{E}\left\{ |h_{il}^T \Theta \Phi g_{i'j'}|^2 \right\} = \mathbb{E}\left\{ |h_{il}^T \Theta \Phi h_{l'j'}|^2 \right\}.
\]

Both \( \mathbb{E}\left\{ |g_{il}^T \Theta \Phi G_a \sqrt{\Lambda t_h}|^2 \right\} \) and \( \mathbb{E}\left\{ |h_{il}^T \Theta \Phi G_a \sqrt{\Lambda t_h}|^2 \right\} \) contain the terms of (24)
\[ R_i \approx \log_2 \left( 1 + \frac{p_i \mathbb{E} \left\{ \left| g_{bi}^T \Theta \Phi g_{ai} \right|^2 \right\}}{\sum_{j=1, j \neq i}^{K} \left( p_j \mathbb{E} \left\{ \left| g_{bj}^T \Theta \Phi g_{aj} \right|^2 \right\} + \mathbb{E} \left\{ \left| g_{bi}^T \Theta \Phi g_{a} \sqrt{\Lambda} \eta_i \right|^2 \right\} + T_{ri} + \sigma_i^2 \right) \right) \]  

(23)

\[ Y_{rj} = \kappa_r \mathbb{E} \left\{ \left| g_{bi}^T \Theta \Phi g_{a} \sqrt{\Lambda} \eta_j \right|^2 \right\} = \kappa_r \mathbb{E} \left\{ g_{bi}^T \Theta \Phi g_{a} \sqrt{\Lambda} \left( \hat{x} + \eta_i \right) \left( \hat{x}^H + \eta_i^H \right) \sqrt{\Lambda} g_{ai} \Phi H^H g_{bi} \right\} \]

(26)

\[ \begin{aligned}
R_i \approx & \log_2 \left( 1 + \frac{p_i \alpha_i \mathbb{E} \left\{ \left| h_{ni}^T \Theta \Phi h_{ai} \right|^2 \right\}}{\sum_{j=1, j \neq i}^{K} \left( p_j \alpha_j \mathbb{E} \left\{ \left| h_{nj}^T \Theta \Phi h_{aj} \right|^2 \right\} + (\kappa_i + \kappa_j + \kappa_r) \alpha_i \alpha_j \mathbb{E} \left\{ \left| h_{ni}^T \Theta \Phi h_{aj} \right|^2 \right\} + \sigma_i^2 \right) \right) \\
= & \log_2 \left( 1 + \frac{p_i \alpha_i \mathbb{E} \left\{ \left| h_{ni}^T \Theta \Phi h_{ai} \right|^2 \right\}}{(1 + \kappa_r) (1 + \kappa_i) \sum_{j=1}^{K} \left( p_j \alpha_j \mathbb{E} \left\{ \left| h_{nj}^T \Theta \Phi h_{aj} \right|^2 \right\} - p_i \alpha_i \mathbb{E} \left\{ \left| h_{ni}^T \Theta \Phi h_{aj} \right|^2 \right\} + \sigma_i^2 \right) \right) \right) \end{aligned} \]

(30)

Then, \( \mathbb{E} \left\{ \left| g_{bi}^T \Theta \Phi g_{a} \sqrt{\Lambda} \eta_i \right|^2 \right\} \) can be written as follows:

\[ \mathbb{E} \left\{ g_{bi}^T \Theta \Phi g_{a} \sqrt{\Lambda} \eta_i \right\} = \kappa_i \mathbb{E} \left\{ g_{bi}^T \Theta \Phi g_{a} \Lambda g_{a}^H \Phi H^H g_{bi} \right\} . \]

Moreover, \( \gamma_{rj} \) also contains the terms \( \mathbb{E} \left\{ \left| g_{bi}^T \Theta \Phi g_{a} \Lambda g_{a}^H \Phi H^H g_{bi} \right\} \), which we will derive later.

We can find \( \mathbb{E} \left\{ g_{bi}^T \Theta \Phi g_{a} \sqrt{\Lambda} \eta_i \right\} \) and \( \gamma_{rj} \) contain the item \( \mathbb{E} \left\{ g_{bi}^T \Theta \Phi g_{a} \Lambda g_{a}^H \Phi H^H g_{bi} \right\} \), given by:

\[ \mathbb{E} \left\{ g_{bi}^T \Theta \Phi g_{a} \Lambda g_{a}^H \Phi H^H g_{bi} \right\} = \alpha_{bi} \mathbb{E} \left\{ h_{ni}^T \Theta \Phi h_{a} \sqrt{D_{a}} \Lambda \sqrt{D_{a}} H^H \Theta \Phi H^H h_{bi} \right\} \]

(29)

We obtain equality in (c) by defining \( D_{a} \triangleq \text{diag} (\alpha_{a1}, \ldots, \alpha_{aK}) \).

Substituting (24), (25), (26) and (29) into (23), we obtain (30). Employing the results of (9) Eq. (20) Eq. (21) one obtains:

\[ \mathbb{E} \left\{ \left| h_{ni}^T \Theta \Phi h_{aj} \right|^2 \right\} = \frac{\varepsilon_i \beta_j \Gamma_{i,j} + L (\varepsilon_i + \beta_j) + L}{(\varepsilon_i + 1) (\beta_j + 1)}, \quad h \in \{i, j\} . \]

(31)

With the help of:

\[ \mathbb{E} \left\{ \cos \left( \theta_n + \Delta \theta_n - \theta_m + \Delta \theta_m \right) + \pi T_{m,i} \right\} \]

(32)

\[ \Gamma_{i,j} \text{ can be further given in (12).} \]

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