A Non-Geometrodynamic Quantum Yang-Mills Theory of Gravity Based on the Homogeneous Lorentz Group

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Abstract Non-geometrodynamic Yang-Mills approaches toward gauge theories of gravity are relatively new. They are appealing since they put gravity on an equal footing with the weak, strong, and electromagnetic forces. In this paper, we present a non-geometrodynamic quantum Yang-Mills theory of gravity based on the homogeneous Lorentz group within the general framework of the Poincare gauge theories. The obstacles of this treatment are that first, on the one hand, the gauge group that is available for this purpose is non-compact. On the other hand, Yang-Mills theories with non-compact groups are rarely healthy, and only a few instances exist in the literature. Second, it is not clear how the direct observations of space-time waves can be explained when space-time has no dynamics. We show that the proposal is unitary and is renormalizable to the one-loop perturbation. To emphasize the non-triviality of the results, we also present a non-healthy Yang-Mills theory with the same gauge group. Although in our proposal, gravity is not associated with any elementary particle analogous to the graviton, classical helicity-two space-time waves are explained. Five observationally essential exact solutions of the field equations of our proposal are presented as well.

1 Introduction

In general, there are three categories of proposals to describe gravity as a gauge theory. The first category started with the seminal work of Utiyama [1], who used the gauge fields of the homogeneous Lorentz group to derive the equations of general relativity (GR). Later Sciama [2] and Kibble [3] extended this approach by introducing two possible independent fields which describe the dynamics of a more general geometry than the pseudo-Riemannian of GR.

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Groups other than the Poincare are also investigated in this regard [4, 5, 6, 7, 8]. The second category of proposals started after it was shown that there is a correspondence between conformal field theories and anti-de Sitter spaces. The focus of this approach is to find a correspondence between a field theory and gravity [9, 10, 11, 12]. Recently, a third category is proposed, which unlike the first two, is a non-geometrodynamic approach where gravity is described by a square of Yang-Mills theories on space-time just like the three forces of the standard model of particle physics [13, 14, 15, 16]. The characteristic feature of this approach is to derive the space-time observable effects through dynamical fields on space-time, which are distinct from space-time itself.

In this paper, we show that the framework of the first category also allows for non-geometrodynamic Yang-Mills based approaches such that the space-time observable quantities are explained by gravitational fields that live on space-time but distinct from it. Poincare gauge theories (PGT) of gravity consist of space-time and tangent spaces. In general, when torsion is not zero, both of the spaces are dynamical. If torsion is set to zero right from the beginning, we still have two conserved currents. Nevertheless, only one can have dynamics. It is often assumed that space-time is the dynamical space in the torsion-free cases, and the tangent spaces are regulated using the tetrad postulate together with the dynamics of space-time. In this paper, we alternatively propose that in the torsion-free scenarios, the tangent space should be taken dynamical, and space-time should be determined using the tetrad-postulate together with the dynamics of the tangent-spaces. Therefore, in our scenario, the observable effects of space-time are induced by a dynamical gravitational field that lives on space-time but distinct from it. In quantum physics, an indication that an observable has a quantum nature is that it satisfies a Poisson commutation relation in the classical regime. Since in our approach, unlike the geometrodynamic treatments, the space-time metric is not driven by such Poisson bracket, we can assume that it has no quantum nature without violating the principle of quantum mechanics. As a result, space-time remains a classical background even at the smallest distances.

However, we need to address a few challenges. First, unlike the other three forces that act on a subset of particles, gravity is a universal force indicating that only a few groups can be a candidate for the corresponding Yang-Mills theory. These limited options are mostly non-compact. The internal metric corresponding with such groups is not positive-definite, and the energies of the corresponding elementary particles are not bound from below. If these gauge fields represent physical particles, the theory is unacceptable. Second, the observations are in favor of a metric theory of gravity where space-time is curved by gravity. The recent direct observations of space-time fluctuations endanger any non-metric theory of gravity. The question to be answered is how the observed space-time waves can be explained if gravity is a dynamical field on space-time but not the dynamics of space-time directly?

Although the conventional Yang-Mills theories with semi-simple compact groups are shown to be renormalizable and unitary [17], the renormalizability and unitarity of a general Yang-Mills theory is not for granted and should be
studied for its particular formulation. Examples of healthy Yang-Mills theories with non-compact groups are rare in the literature. Among others, one possibility is to build an exotic but positive-definite internal metric \[15\]. Another possibility is pointed out in the Chern-Simons gauge theory with complex gauge group, where due to the general covariance the Hamiltonian remains zero and is bound from below \[19\]. One other possibility that is the subject of this paper is to utilize a group whose gauge field is not a tensor of the Lorentz group of physical observers, and as a result, does not represent elementary particles with negative kinetic energies. If we conservatively restrict ourselves to the kinematics of GR, i.e., the torsion-free PGT framework, the group whose gauge field is not a tensor of physical observers, is the homogeneous Lorentz group of the tangent spaces in which fermionic fields are defined.

One of the predictions of our scenario is that there exists no mass-less elementary particle, that plays the role of the graviton in GR. The reason is that the dynamical gauge field is not a tensor of physical observers and does not represent a physical particle. While this prediction does not violate any of the observations and is allowed, we still need to explain the observed space-time waves \[20\] that travel with the speed of light. So far, to our knowledge, this observation is solely explained through the existence of elementary particles. For instance, in GR, the graviton is associated with the space-time waves. In this paper, we present an alternative explanation. We show that even if space-time does not have a dynamics of its own, the tetrad postulate can still explain the observed space-time waves by coupling the dynamics of the non-geometrical gauge field to the space-time through a non-holonomic constraint.

In this paper, we introduce Quantum Lorentz gauge theory of gravity (QLGT), derive its Feynman rules in the path integral formalism, and calculate the irreducible one-loop diagrams of the theory and show that all of the infinities can be absorbed into its available parameters. Therefore, QLGT is renormalizable to the first loop. We also demonstrate that QLGT is unitary. To emphasize the non-triviality of the two, we also present an unacceptable Yang-Mills theory based on the same gauge group that we used, i.e., homogeneous Lorentz group. We discuss the classical field equations of Lorentz gauge theory of gravity (LGT) and show that a contraction of the field equations is the same as the divergence of Einstein equations. Using a plane wave analysis, we demonstrate that the dynamics of LGT in the tangent spaces induce space-time waves. We show that even though in GR space-time has its own dynamics and in LGT it does not, the physical modes of the space-time waves are the same in both GR and LGT, and their helicity is two. Finally, we show that the Kerr space-time, the Schwarzschild space-time in the vacuum, the Schwarzschild space-time inside the stars, the De Sitter space-time, the early universe space-time, and the space-time of the matter-dominated universe are the solutions of LGT field equations to all orders of perturbations.

The structure of this paper is as follows. An overview of the general framework of the Poincare gauge theories is presented in section 2. In the same section, we provide an overview of the quantization of geometrodynamics approaches to gravity. We construct and quantize our non-geometrodynamic
Yang-Mills theory of gravity in section 3. In the same section, we show that the theory is unitary and is renormalizable to one-loop perturbation. We also present an example of an unhealthy Yang-Mills theory based on the homogeneous Lorentz group. The classical field equations, a plane wave analysis, and a few exact solutions of LGT are presented in section 4. A conclusion is drawn in section 5.

2 An overview of the Poincare gauge theories of gravity in pseudo Riemannian geometry

This section is a summary of the literature of PGT with an emphasis on the aspects that are utilized later in the paper. For in-depth reviews, we refer the reader to [21][22][23][24]. Since the four-dimensional general linear group has no representation that transforms like a spinor under the Lorentz group, we have to define the spinor fields in the tangent spaces on space-time [25]. A set of four orthogonal vectors $e_i$ at every point of space-time defines the tangent spaces, i.e., the Lorentz frames. Here and in the rest of the paper, the Latin indices run from 0 to 3 and refer to the Lorentz frames. The orthonormality of the tetrad indicates that the metric of the tangent spaces is always Minkowskian $\eta_{ij} \equiv e_i \cdot e_j$. The Latin indices are lowered and raised by this Minkowski metric. The components of the tetrad in space-time coordinate system $e_{i\mu} \equiv e_i \cdot e_\mu$ define the space-time metric

$$g_{\mu\nu} = \eta^{ij} e_{i\mu} e_{j\nu},$$

where the Greek indices, referring to the coordinates, run from 0 to 3 and are raised and lowered by this latter metric.

Spinor fields are the scalars of space-time but the spinors of the tangent spaces

$$\tilde{\psi}(x) = \exp \left( \frac{g}{2} S^{ij} \omega_{ij} \right) \psi(x),$$
$$\psi'(x') = \psi(x).$$

(2)

Here we present a Lorentz transformation of the tetrad by a tilde and a coordinate transformation by a prime. Moreover, $g$ is a coupling constant, $\omega_{ij}$ is an arbitrary anti-symmetric tensor, and $S^{ij}$ are the six generators of the homogeneous Lorentz group in terms of the commutator of the Dirac matrices

$$S^{ij} = \frac{1}{4} [\gamma^i, \gamma^j].$$

(3)

The tetrad are the vectors of the tangent spaces but the scalars of space-time

$$\tilde{e}_i(x) = A_i^j e_j(x),$$
$$e'_i(x') = e_i(x).$$

(4)
where \( A'_{ij} \) represent the homogeneous Lorentz transformations in the vector space. If the parameter \( \omega_{ij} \) in Eq. 2 is very smaller than unity, this reads

\[
A'_{ij} \approx \delta_{ij} + \omega_{ij}.
\]

The photon, gluon, W, and Z particles are all vectors of coordinate system, but the scalars of the tangent spaces

\[
\begin{align*}
\tilde{A}_{\mu}(x) &= A_{\mu}(x), \\
A'_{\mu}(x') &= \frac{\partial x'^\nu}{\partial x^\mu} A_{\nu}(x),
\end{align*}
\]

(5)

where the internal index of the fields is not shown.

If space-time is flat and the two types of transformations are not position-dependent, the Dirac Lagrangian reads

\[
\mathcal{L}_D = \frac{i}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{i}{2} \bar{\psi} \gamma^\nu \gamma^\mu \partial_\nu \psi - m \bar{\psi} \psi.
\]

(6)

If the transformations are position-dependent, the partial derivatives should be replaced by the covariant derivatives, such that the derivatives of the fields transform as before. For spinor fields, the covariant derivatives read

\[
\begin{align*}
D_\mu \psi &= \left( \partial_\mu - \frac{1}{2} g A_{ij\mu} S_{ij} \omega_{ij} \right) \psi, \\
\bar{\psi} D_\mu &= \bar{\psi} \left( \partial_\mu + \frac{1}{2} g A_{ij\mu} S_{ij} \omega_{ij} \right),
\end{align*}
\]

(7)

where \( A_{ij\mu} \) is the gauge field of the homogeneous Lorentz group and is antisymmetric in the two Latin indices. It transforms as

\[
\begin{align*}
\tilde{A}_{ij\mu}(x) &= A_{ijm}^n A_{m\mu\nu}(x) + \partial_\mu A_{ij\nu}, \\
A'_{ij\mu}(x') &= \frac{\partial x'^\nu}{\partial x^\mu} A_{ij\nu}(x).
\end{align*}
\]

(8)

The first equation indicates that the Lorentz gauge field is not a tensor of Lorentz group of observers and therefore posses no observer-independent property like the spin and cannot represent an elementary particle.

To find the covariant derivative of the Lorentz vectors, we use the product rule to take the derivative of \( B^i \equiv \bar{\psi} \gamma^i \psi \) and use the following identity

\[
[\gamma^i, S^{mn}] = \eta^{im} \gamma^n - \eta^{in} \gamma^m.
\]

(9)

With a straightforward calculation, one can show that the covariant derivative of a vector of the tangent spaces reads

\[
D_\mu B^i = \partial_\mu B^i - g A_{ij\mu} B^j.
\]

(10)
So far, we have discussed the covariant derivatives of space-time scalars. If a field transforms like a tensor under coordinate transformations, its covariant derivative should contain space-time connections and reads

\[ D_\mu B_{j_1j_2\cdots}^{i_1i_2\cdots} = \partial_\mu B_{j_1j_2\cdots}^{i_1i_2\cdots} - gA_{k_1\mu} B_{j_1j_2\cdots}^{i_1k_2\cdots} - gA_{k_2\mu} B_{j_1j_2\cdots}^{i_2k_1\cdots} - \cdots \]

where \( \Gamma_{\mu}^{\alpha} \) is the space-time connection. Its form can be found by assuming that

\[ D_\alpha g_{\mu\nu} = \nabla_\alpha g_{\mu\nu} = 0. \]

The covariant derivative of the metric of the tangent space reads

\[ D_\mu \eta_{ij} = \partial_\mu \eta_{ij} - gA_{k_i\mu} \eta_{kj} - gA_{k_j\mu} \eta_{ik} = 0, \]

which is zero since \( \partial_\mu \eta_{ij} = 0 \), and the Lorentz gauge field is anti-symmetric in the Latin indices.

We often impose an extra condition called the tetrad postulate that reads

\[ D_\mu e_i = \partial_\mu e_i - \Gamma_{i\mu}^{\lambda} e_\lambda - gA_{\lambda_i\mu} e_\lambda = 0. \]

Note that we could have imposed this equation first and derive Eq. \ref{12}, but not vice versa. This tetrad postulate is an assumption about the equivalence of the coordinate and Lorentz frames and is imposed solely due to our physical intuition but not for mathematical consistency. Since the Christoffel symbols can be written entirely in terms of the metric, which itself is given in terms of the tetrad, the Lorentz gauge field and the tetrad were the two independent fields before imposing this condition.

The strength tensor \( F_{\mu\nu ij} \) is defined as

\[ [D_\mu, D_\nu] \psi = \frac{g}{2} F_{\mu\nu ij} S^{ij} \psi, \]

and is given by

\[ F_{\mu\nu ij} = \partial_\nu A_i - \partial_\mu A_j - gA_{k_i\mu} A_k j + gA_{k_j\mu} A_k i. \]
When expressed entirely using the coordinate indices, this is the Riemann curvature tensor

\[ R_{\mu\nu\alpha\beta} = e^i_\alpha e^j_\beta F_{\mu\nu ij}. \]  

Unlike in Yang-Mills theories, where the symmetry is uniquely associated with a single Lagrangian, in PGT, the symmetry allows more than one term. The most general form can be found in [26]

\[ \mathcal{L}_A = -\frac{1}{4} \left( c_1 F_{\mu\nu ij} e^{i\mu} e^{j\nu} e^{\kappa\sigma} + c_2 F_{\mu\nu ij} F^{\mu\sigma i\kappa} e^{j\nu} e^{\kappa\sigma} e^{\kappa\sigma} e^{\nu} + c_4 F_{\mu\nu ij} F^{\alpha\beta mn} e^{i\mu} e^{j\beta} e^{\alpha} e^{m} e^{n} + c_5 F_{\mu\nu ij} F^{\mu\nu ij} \right). \]

If we had not imposed the torsion-free condition right from the beginning, the tetrad and the connection were still independent. Therefore, to find the field equations, we needed to vary the action with respect to both of them [26]. This is not the case anymore due to the torsion-free assumption and the tetrad postulate. Instead, the field equations should be derived by varying the action with respect to either the tetrad or the Lorentz gauge field. Since the tetrad postulate is a non-holonomic constraint, the results of the variation may depend on the variation path, which is physically unacceptable. This roots back to the fact that the non-holonomic constraint can be violated along an arbitrary displaced path (variation of the variables) [27,28,29].

The issue with the non-holonomic constraints can be resolved if the displacement paths in Hamilton’s principle are restricted to those along which the constraint is not violated. To find such paths, we note that \( D_{\mu} e_{\nu} = 0 \) as the non-holonomic constraint is preserved under both coordinate and Lorentz frame transformations. Therefore, if we restrict the displacement paths to the ones generated by such transformations, the non-holonomic constraint will not be violated.

The possible classes of displacement paths are

1. option one: generated by coordinate transformations and leads to GR,
2. option two: generated by homogeneous Lorentz transformation of the tetrad frames and leads to LGT [30,31].

A more physically intuitive reason for the two options above is that while the torsion-free condition is imposed from the beginning, the framework still has two symmetries and, therefore, two independent conserved currents. In principle, each current can be used to generate a dynamic field. Since two dependent variables cannot have two independent sets of dynamical field equations, we have to choose one of the sources to drive the dynamics of the framework.
2.1 An overview of quantization of GR

In the last part of this section, we would like to review the quantization of option 1 above. GR is uniquely the simplest theory of a massless spin two elementary particle \[32,33,34,35,36\] where only \(c_1\) is kept non-zero in Eq. \[19\]. For the quantization purposes, it is easier to decompose the Lagrangian as in \[37\].

\[
S_{GR} = \int L_{GR} dt d^3x,
\]
\[
L_{GR} = \left( R^{(3)} + K_{ab} K^{ab} - K^a_a K^b_b \right) N \sqrt{g^{(3)}},
\] (20)

where \(R^{(3)}\) refers to the three-dimensional curvature, \(K_{ab}\) is the extrinsic curvature, \(g^{(3)}\) is the determinant of \(g^{(3)}_{ab}\), and the rest of the parameters are defined in the following decomposition of the space-time metric

\[
ds^2 = -(N dt)^2 + g^{(3)}_{ab} (dx^a + N^a dt) (dx^b + N^b dt).
\] (21)

The dynamical variables are \(N, N^a,\) and \(g^{(3)}_{ab}\). The corresponding canonical momenta are

\[
\pi = \frac{\partial L_{GR}}{\partial \dot{N}}, \quad \pi^a = \frac{\partial L_{GR}}{\partial \dot{N}^a}, \quad \pi^{ab} = \frac{\partial L_{GR}}{\partial \dot{g}^{(3)}_{ab}}.
\] (22)

The first two momenta are zero and define the constraints of GR. This is similar to the case of electrodynamics, where the canonical momentum of the temporal component of the vector potential is zero. The following Poisson bracket drives the dynamics of GR \[38\].

\[
\{ g^{(3)}_{ab} (x), \pi^{ij} (y) \} = \frac{1}{2} \left( \delta^i_a \delta^j_b + \delta^i_b \delta^j_a \right) \delta^3 (x - y).
\] (23)

To quantize the theory, we replace the dynamical variables by the corresponding quantum operators and the Poisson bracket by the canonical commutation relation. In the presence of the constraints mentioned above, such canonical quantization is cumbersome but informative and can be found in \[39\]. With the developments in the standard model of particle physics, and especially after the works of Feynman \[40\], Faddeev and Popov \[41\], Mandelstam \[42\], and DeWitt \[43\], the path integral formalism, also called the manifestly covariant method, became the standard method of carrying out this quantization.

In the path integral quantization, the four dimensional space-time metric as the dynamical field, is first expanded around a background, which for simplicity we take to be flat,

\[
g_{\mu\nu} = \eta_{\mu\nu} + f_{\mu\nu},
\] (24)
where $\eta$ is the Minkowski metric. The Lagrangian of GR is rewritten in terms of $f_{\mu\nu}$ and the generating functional reads

$$Z[t^{\mu\nu}] = \int Df_{\mu\nu} \exp \left( i \int d^4x [\mathcal{L}_{GR} + f_{\mu\nu} t^{\mu\nu}] \right).$$

Due to the diffeomorphism invariance of the theory, we also need to fix the gauge. A common choice is to use the harmonic coordinates where

$$C_\nu \equiv \partial^\mu f_{\mu\nu} - \frac{1}{2} \partial_\nu f^{\mu}_{\mu} = 0.$$  \hspace{1cm} (26)

Therefore, the generating functional receives a corresponding gauge fixing and a Faddeev-Popov ghost Lagrangian

$$\mathcal{L}_{GR} \rightarrow \mathcal{L}_{\text{effective}} \equiv \mathcal{L}_{GR} + \mathcal{L}_{GF} + \mathcal{L}_{FP}.$$  \hspace{1cm} (27)

The Feynman rules are subsequently read from the effective Lagrangian. These sets of rules can then be used to calculate the Feynman diagrams that contain loops. Some of the loop diagrams are divergent, as in other field theories. However, in $^{44,45}$, the authors have shown that the infinities cannot be removed by adding counterterms that have the same form as in the effective Lagrangian. Therefore, GR is a non-renormalizable theory with no falsifiable prediction for high energies. Such ultraviolet divergencies are expected in any theory like GR, whose coupling constant has a negative dimension in the mass units $^{46}$.

Even if GR was a renormalizable field theory, we still did not have a consistent quantum theory of gravity. Because, in such quantum gravity, on the one hand, time is a classical background, and on the other hand, it is a quantum operator. The problem of time in quantum geometrodynamics is extensively studied but is still open. A review of the subject can be found in $^{38,47,48}$. Moreover, unlike particle physics' standard model, GR is driven by the absolute value of energies, instead of their differences. This has led to the cosmological constant problem $^{49}$.

In the end, we refer the reader to $^{50,51,52,53}$, and the references therein, for an overview of the quantum aspects of the broader Poincare gauge theories.

### 3 Quantum LGT as a non-geometrodynamic approach to quantum gravity

In the framework of option $^{1}$ the Poisson bracket in Eq. $^{23}$ implies that the metric and its conjugate momenta are classically non-commutative. Therefore, our understanding of quantum mechanics indicates that a quantum operator at microscopic scales should replace the metric. This means that the tetrad has quantum fluctuations and can be written as

$$e_{i\mu} = \langle e \rangle_{i\mu} + e_{i\mu}^{\text{quantum}}.$$  \hspace{1cm} (28)
In the framework of option 2, however, it is the Lorentz gauge field and not the tetrad that is classically non-commutative. Therefore, at the quantum level, it is the Lorentz gauge field and not the tetrad that should be replaced by a quantum operator. We now make two further postulates in order to build a self-consistent quantum theory of gravity. First, the tetrad has no quantum fluctuation even at the smallest length scales possible

\[ e_{i\mu} = \langle e \rangle_{i\mu}, \]  

(29)

Second, the tetrad postulate is valid only at the classical level

\[ D_\mu e_{i\nu} \neq 0, \]  

(30)

\[ \langle D_\mu e_{i\nu} \rangle = \partial_\mu e_{i\nu} - \Gamma^\lambda_{\mu\nu} e_{i\lambda} - g(A_{k \mu}) e_{k\nu} = 0. \]  

(31)

We would like to mention that an assumption similar to our first postulate is the basis of the so-called quantum field theory on curved space-time. The difference is that in geometrodynamic theories, this assumption is valid only under specific semi-classical regimes but contradictory at the fundamental level due to the Poisson bracket in Eq. 23. In our non-geomerodynamics approach, the assumption can remain valid even at the fundamental level. This postulate alleviates the so-called problem of time because the time in our quantum theory of gravity has the same background nature as in quantum physics.

The reason for the second postulate is that both the metric and the connection of space-time remain classical fields even at the quantum level due to the first postulate together with Eqs. 1 and 13. On the other hand, Lorentz gauge field of the tangent spaces is fundamentally a quantum operator. Therefore, at the quantum level, the classical Christoffel symbols \( \Gamma^\alpha_{\mu\nu} \) cannot be equivalent to the quantized Lorentz gauge field \( A_{ij\mu} \) and the tetrad postulate cannot be valid.

In the Yang-Mills gauge theories based on the unitary groups, there is a unique possibility to form an invariant Lagrangian from a pair of strength tensors. The reason is that their group index is distinct from the space-time index. Space-time indices are coupled by the space-time metric while the group indices are coupled using \( \delta \equiv \text{Tr}(t \cdot t) \) with \( t \) being the generators of the group. The covariant derivative of both the space-time metric and the internal metric \( \delta \) are zero.

In LGT, in general, the Lagrangian is given in Eq. 19 which have four independent invariant terms because, in addition to the metric of space-time and the six-dimensional internal metric \( \text{Tr} \left( S^{ij} \cdot S^{mn} \right) \), there exist the tetrad that can couple the gauge and space-time indices. However, at the quantum level, the covariant derivatives of the two metrics is zero, due to how the Christoffel symbol and the Lorentz gauge field are defined. But, the covariant derivative of the tetrad is not zero according to Eq. 30. This provides a means of distinction between the terms.

Therefore, to construct a Yang-Mills theory similar to those in the standard model of particle physics, we only allow those Lagrangian terms in which the
strength tensors are coupled by objects whose covariant derivatives are zero. LGT is formally defined by

\[ L_A = \frac{1}{4} F_{\mu \nu ij} F^{\mu \nu mn} \text{Tr} \left( S^{ij} \cdot S^{mn} \right) \]
\[ = \frac{1}{4} F_{\mu \nu ij} F^{\mu \nu ij}, \]  
(32)

where in the last line we have used the six-dimensional internal metric of the homogeneous Lorentz group

\[ \delta^{ij, mn} = \text{Tr} \left( S^{ij} \cdot S^{mn} \right) = \frac{1}{2} (\eta^{im} \eta^{jn} - \eta^{in} \eta^{jm}). \]  
(33)

In the Yang-Mills theories of the standard model, the sign of the Lagrangian is the opposite of what we introduced above and is to preserve the unitarity. Later in the paper, we discuss that since the Lorentz gauge field is not a tensor of Lorentz group of physical observers and consequently cannot represent physical particles, the unitarity is preserved regardless of the sign in Eq. 32. Also, we will discuss that the definition is such that the classical field equations are in more agreement with GR.

3.1 Path integral quantization of LGT

Due to the gauge invariance of QLGT, the path integral method is the most suitable approach to quantize it. To avoid taking multiple equivalent paths, we need to introduce the gauge fixing and subsequently, the Faddeev-Popov ghost terms. The total Lagrangian of QLGT reads

\[ L_{\text{total}} = i e_i^{\mu} \bar{\psi} \gamma^{\mu} D_{\mu} \psi - i e_i^{\mu} \bar{\psi} \gamma^{\mu} D_{\mu} \bar{\psi} - m \bar{\psi} \psi \\
+ \frac{1}{4} F_{\mu \nu ij} F^{\mu \nu ij} + \frac{1}{2} \xi (\partial^\mu A_{ij})^2 - \bar{c}_{ij} \partial^\mu (D_{\mu} c_{ij}), \]  
(34)

where \( \xi \) is an arbitrary parameter, and \( c_{ij} \) with anti-symmetric indices represent the Faddeev-Popov ghosts. The generating functional, therefore, reads

\[ Z = \int DA D\bar{\psi} D\psi D\bar{c} Dc \exp \left( i \int e d^4 x L_{\text{total}} \right), \]  
(35)

where \( e \) is the determinant of the tetrad. One crucial difference between this functional and the one in GR is that the path integration on the tetrad is absent since it is a background field at the fundamental level. In quantum field theory on curved space-time also \( D_\mu c_{ij} \) is absent. Nevertheless, that is only an approximation that is valid for particular situations and is contradictory at the fundamental level.

In this section, we only quantize in a flat space-time where \( g_{\mu \nu} = \eta_{\mu \nu} \). This means that \( c = 1 \) and the Christoffel symbols are zero. Moreover, Eq. 31 indicates that \( \langle A \rangle_{ij\mu} = 0 \) just like the gauge fields of the standard model in a flat space-time.
3.1.1 Propagators

The inverse propagator of the Lorentz gauge field in the momentum space reads

\[
(PA(k)^{-1})^{ij\mu,mn\nu} = \frac{\delta^2 \mathcal{L}_{\text{total}}}{{\delta A_{ij\mu}(k)}{\delta A_{mn\nu}(k)}} \bigg|_{A=\psi=c=0},
\]  

(36)

where \(PA\) stands for the propagator for field \(A_{ij\mu}\). We use the FeynCalc package [54,55] to take the two functional derivatives, and have made the scripts available online [56]. The propagator reads

\[
PA(k)_{ij\mu,mn\nu} = i\delta_{ij,mn} \left( \eta_{\mu\nu} k^2 - (1 - \xi) \frac{k_{\mu} k_{\nu}}{k^4} \right).
\]  

(37)

The propagator for the Faddeev-Popov ghosts can be found via the same procedure

\[
Pc(p)_{ij,mn} = \frac{i\delta_{ij,mn}}{p^2}.
\]  

(38)

The fermion propagator is also known to be

\[
PF(p) = i\frac{p \cdot \gamma + m}{p^2 - m^2}.
\]  

(39)

3.1.2 Vertices

The interactions in QLGT can be found via higher-order Functional derivatives. Due to their somewhat lengthy nature, we calculate them with the FeynCalc package again and make them available in [56].

The interaction with fermions read

\[
VAFF^{ij\mu} \equiv \frac{i\delta^3 \mathcal{L}_{\text{total}}}{{\delta \bar{\psi} \delta A_{ij\mu}\delta \psi}} \bigg|_{A=\psi=c=0} =
\]  

(40)
The naming VAFF stands for the vertex of a gauge field and two fermions. The interaction of the Faddeev-Popov ghosts with the gauge field is

$$VA\bar{c}(p)^{a_1 b_1, a_2 b_2, a_3 b_3} \equiv \frac{i\delta^2 \mathcal{L}_{\text{total}}}{\delta \bar{c}(p)_{a_1 b_1} \delta A_{\alpha_2 \beta_2 \mu_2} \delta c_{a_3 b_3}} \bigg|_{A=\psi=c=0} = \ldots$$

(41)

The following is the gauge field interaction of order $g$

$$V3A(k_1, k_2, k_3)^{i_1 j_1 \mu_1, i_2 j_2 \mu_2, i_3 j_3 \mu_3} \equiv \frac{i\delta^3 \mathcal{L}_{\text{total}}}{\delta A_{i_1 j_1 \mu_1}(k_1) \delta A_{i_2 j_2 \mu_2}(k_2) \delta A_{i_3 j_3 \mu_3}(k_3)} \bigg|_{A=\psi=c=0} = \ldots$$

(42)

Finally, gauge field interaction of order $g^2$ is equal to

$$V4A^{i_1 j_1 \mu_1, i_2 j_2 \mu_2, i_3 j_3 \mu_3, i_4 j_4 \mu_4} \equiv \frac{i\delta^4 \mathcal{L}_{\text{total}}}{\delta A_{i_1 j_1 \mu_1} \delta A_{i_2 j_2 \mu_2} \delta A_{i_3 j_3 \mu_3} \delta A_{i_4 j_4 \mu_4}} \bigg|_{A=\psi=c=0} = \ldots$$

(43)
3.1.3 External lines

The total Lagrangian in flat space-time is a function of the Faddeev-Popov ghosts, the fermions, and the Lorentz gauge field. The Faddeev-Popov ghosts have wrong statistics and cannot represent physical particles. Therefore, they do not receive an external line in the Feynman diagrams. Also, following the convention, we show incoming and outgoing fermions with $u^\sigma(p)$ and $\bar{u}^\sigma(p)$ respectively, while incoming and outgoing anti-fermions with $\bar{v}^\sigma(p)$ and $v^\sigma(p)$, where $\sigma$ refers to the two spin modes.

To discuss the external lines for the Lorentz gauge field, we note that if the arbitrary parameter of the Lorentz transformation of physical observers $\omega_{ij}$ is much smaller than unity, Eq. 8 implies that the Lorentz gauge field transforms as

$$\tilde{A}_{ij\mu}(x) = A_{ij\mu}(x) + D_\mu \omega_{ij},$$  \hspace{1cm} (44)

which has an identical form as the transformation of the gauge fields in the standard model under a special unitary gauge transformation with parameter $\alpha^a$

$$\hat{A}_\mu^a(x) = A_\mu^a(x) + D_\mu \alpha^a.$$  \hspace{1cm} (45)

However, despite the similarity, there is a crucial difference. Under a Lorentz transformation of physical observers, $A'_{ij} \simeq \delta_{ij} + \omega_{ij}$, the gauge field of the unitary group has an invariant length equal to

$$\left( A_\mu^a A^\mu_a + 2 D_\mu \alpha^a A^\mu_a + D_\mu \alpha^a D^\mu a \alpha^a \right)^{\frac{1}{2}},$$  \hspace{1cm} (46)

which is independent of $\omega_{ij}$. This means that two independent experiments observe the same length for $\hat{A}_\mu^a$. On the other hand, the length of $\tilde{A}_{ij\mu}$ is equal to

$$\left( A_{ij\mu} A^{ij\mu} + 2 D_\mu \omega_{ij} A^{ij\mu} + D_\mu \omega_{ij} D^{ij\mu} \omega_{ij} \right)^{\frac{1}{2}},$$  \hspace{1cm} (47)

which depends on the parameter of the Lorentz transformation of physical observers. Therefore, the Lorentz gauge field is observer-dependent, does not have an invariant length and, consequently, cannot represent physical particles. Therefore, it receives no external line in the Feynman diagrams of QLGT. Later in this paper, we discuss that the Lorentz gauge field induces classical helicity two space-time fluctuations, which are observable fields.

3.2 The unitarity of QLGT

Since the homogeneous Lorentz group is not compact, $\delta^{ij,mn}$ in Eq. 33 is not positive definite. Instead, three of its six independent components are negative, which implies that the corresponding gauge fields have negative kinetic energies and are ghosts. Nevertheless, in this section, we will show that, since
the Lorentz gauge fields do not represent physically observable particles, the unitarity will not be violated.

To preserve probability in a quantum field theory, the S matrix has to be unitary. This means that

\[ 2 \text{Im} (T) = TT^\dagger, \]

where \( T \equiv -i(S-1) \). If \( |\phi\rangle \) is a state of the system, the equation implies that

\[ 2 \text{Im} (\langle \phi | T | \phi \rangle) = \sum_k \langle \phi | T | k \rangle \langle k | T^\dagger | \phi \rangle, \]

where \( |k\rangle \) refers to the physically observable modes of the fields and \( \sum_k |k\rangle \langle k| = 1 \). In the gauge theories of the standard model, only fermions and the transverse component of the gauge fields contribute to the \( |k\rangle \) states while the non-physical longitudinal components of the gauge fields and Faddeev-Popov ghosts are excluded.

In QLGT, the Lorentz gauge field cannot represent a physical mode and should be excluded entirely from the states of \( |k\rangle \). Therefore, the only physical states are the fermions, and consequently, Eq. 49 is non-trivial only if \( 1 = \sum_k |k\rangle \langle k| \) is intervening internal fermionic lines of Feynman diagrams. We now prove the unitarity for a rather general Feynman diagram of the form

\[ \text{\includegraphics[width=0.2\textwidth]{feynman_diagram.png}}, \]

where the question marks can be any multi-loop diagram and in the following will be presented by \( M_{1i,j\mu} \). The amplitude for diagram above reads

\[ iM = - \int \frac{d^4q}{(2\pi)^4} M_{1i,j_1\mu_1} M_{i_2,j_2\mu_2}^* \text{Tr} \left( \frac{g + m}{q^2 - m^2 + i\epsilon} \right) VAFF^{i_1,j_1\mu_1} \frac{g - k + m}{(q - k)^2 - m^2 + i\epsilon} VAFF^{i_2,j_2\mu_2} \right), \]

We now use the Cutkosky rules to derive the imaginary part of this amplitude

\[ 2 \text{Im} (\mathcal{M}) = - \int \frac{d^4q}{(2\pi)^4} M_{1i,j_1\mu_1} M_{i_2,j_2\mu_2} \times \]

\[ (-2\pi)^2 \Theta(q^0) \Theta(q^0 - k^0) \times \]

\[ \delta(q^2 - m^2) \delta((q - k)^2 - m^2) \]

\[ \text{Tr} \left( (g + m)VAFF^{i_1,j_1\mu_1} (g - k + m)VAFF^{i_2,j_2\mu_2} \right). \]
On the other hand, the right hand side of Eq. 49 is equal to
\[
\int \frac{d^3q_1}{(2\pi)^32E_{q_1}} \int \frac{d^3q_2}{(2\pi)^32E_{q_2}} (2\pi)^4 \delta^4(k - q_1 - q_2) \sum_{\text{spin}} |M_{\text{half}}|^2,
\]
where the half amplitude is equal to
\[
M_{\text{half}} = M_{1_{i_2j_2\mu_2}} \bar{\psi}^{\sigma_2}(q_2) \text{VAFF}^{i_{i_1j_1\mu_1}}(q_1)
\]
\[
= \begin{array}{c}
\end{array}
\]
\hspace{2cm} (53)
We now use the spin method to convert the spin sum of the half amplitudes into a trace
\[
\sum_{\text{spin}} |M_{\text{half}}|^2 = M_{1_{i_1j_1\mu_1}} M_{1_{i_2j_2\mu_2}}^* \times \text{Tr} \left( (q_1 + m) \text{VAFF}^{i_{i_1j_1\mu_1}} (q_2 - m) \text{VAFF}^{i_{i_2j_2\mu_2}} \right),
\]
and use
\[
\int \frac{d^4q}{2E_q} = \int d^4q \Theta(q^0) \delta(q^2 - m^2),
\]
\hspace{2cm} (55)
to show that Eq. 53 is equal to $2\text{Im} \langle M \rangle$ and, therefore, unitarity is preserved.

3.3 One-loop renormalization of QLGT

In this section, we would like to show that all of the one-loop infinities of QLGT can be absorbed in its available parameters, and the theory is renormalizable to that order. We use the FeynCalc package to carry out the calculations within the Passarino-Veltman scheme [57] and make the scripts available in [56]. For the calculations, we follow the instructions for the one-loop calculations of QED using the same computational package in [58].

3.3.1 Fermion self-energy

The correction to the fermion propagator is through the following diagram
\[
= -i\Sigma(p) = \int \frac{d^4k}{(2\pi)^4} \frac{i\delta_{ij}m\eta_{\mu\nu}}{k^2} \text{VAFF}^{i_{i_1j_1\mu_1}} \frac{i(p + k + m)}{(p + k)^2 - m^2} \text{VAFF}^{m_{j_2\nu}}.
\]
\hspace{2cm} (57)
To reduce the computational load, we rewrite the vertex in the following form

$$\text{VAFF}^{ij\mu} = \frac{-iq}{4} \epsilon^{kij\mu} \gamma^5,$$

(58)

and use the following identity

$$\delta_{ij,mn} \eta_{\mu\nu} \epsilon^{kij\mu} \epsilon^{k2mn\nu} = -6q^k k^2,$$

(59)

where $\epsilon^{kij\mu}$ is the Levi-Civita symbol. The loop reads

$$-i \Sigma(p) = iA + iB \phi,$$

(60)

where the two infinite parameters are

$$A = \frac{3g^2m}{64\pi^2} \left( 1 - 2B_0(p^2, 0, m^2) \right),$$

$$B = \frac{3g^2}{128\pi^2} \left( 1 + B_0(0, 0, m^2) - 2B_0(m^2, 0, m^2) \right).$$

(61)

Here and in the rest of the paper, $A_0(\cdots), B_0(\cdots)$ and $C_0(\cdots)$ are the Passarino-Veltman functions.

To see the corrections to the mass and the spinor field, we note that the exact fermion propagator is equal to

$$\text{PF}^{-1}(t) = \text{PF} + \text{PF} (-i \Sigma) \text{PF} + \text{PF} (-i \Sigma) \text{PF} (-i \Sigma) \text{PF} + \cdots$$

$$= \text{PF} \left( 1 + (-i \Sigma) \text{PF}(t) \right).$$

(62)

Therefore, the inverse of the propagators satisfy the following equation

$$\text{PF}^{-1}(t) = -i \left( \frac{1}{p} - m \right) + i \Sigma.$$

(63)

To absorb the two infinities of $\Sigma$, we define the renormalized parameters as

$$\psi^r = \frac{1}{\sqrt{Z_\psi}} \psi,$$

$$m^r = \frac{1}{Z_m m},$$

(64)

where $Z_{\psi/m} \equiv 1 + \delta_{\psi/m}$. Since the fermion propagator is by definition $\langle 0 | \psi \bar{\psi} | 0 \rangle$, the renormalization is equivalent to $\text{PF} \rightarrow Z_\psi^{-1} \text{PF}$, and Eq. 63 reads

$$\text{PF}^{-1}(t) = -i \left( \frac{1}{p} + \delta_\psi - B \right) - m^r - (\delta_m + \delta_\psi) m^r - A.$$

(65)

To remove the infinities, we now define

$$\delta_\psi = \text{Div}(B),$$

$$\delta_\psi + \delta_m = -\frac{1}{m^r} \text{Div}(A),$$

(66)

where Div stands for the divergent component of the expression.
3.3.2 Vacuum polarization

The corrections to the Lorentz gauge field propagator are through four loop diagrams that are calculated below. Whenever applicable, we use the Feynman’s Hooft gauge of $\xi = 1$.

The first diagram to consider is the correction by a fermionic loop

$$\equiv i\Pi_i^{ij\mu,mn\nu} = \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left( VAFF^{ij\mu} \cdot (p+k+m) \cdot VAFF^{mn\nu} \cdot (p+m) \right) \frac{((p+k)^2 - m^2)(p^2 - m^2)}{((p+k)^2 - m^2)(p^2 - m^2)}$$

$$= \frac{ig^2}{192\pi^2} B_0 \left( 0, m^2, m^2 \right) \left( 6m^2\eta_{k_1k_2} + k_1.k_2 \right) \epsilon^{k_1ij\mu} \epsilon^{k_2mn\nu},$$

where we have given a fictitious mass $\lambda$ to the gauge field and used

$$\lim_{\lambda^2 \to 0} \frac{B_0 \left( \lambda^2, m^2, m^2 \right) - B_0 \left( 0, m^2, m^2 \right)}{\lambda^2} = \frac{1}{6m^2}. \quad (68)$$

The second diagram is the correction by two first order gauge field vertices

$$\equiv i\Pi_2^{ij\mu,mn\nu} = \int \frac{d^4p}{(2\pi)^4}$$

$$V3A^{ij\mu,i_1j_1\mu_1,i_2j_2\mu_2}(k,p,-p-k)PA_{i_1j_1\mu_1,m_1n_1\nu_1}(p)$$

$$V3A^{m_1n_1\nu_1,m_2n_2\nu_2,mn\nu}(-p,p+k,-k)PA_{m_2n_2\nu_2,i_2j_2\mu_2}(p+k)$$

$$= \frac{4ig^2}{9\pi^2} k^\mu k^\nu \delta^{ij,mn}, \quad (69)$$

which is finite and needs no counter term. Note that this is a modification to the longitudinal component of the Lorentz gauge field. In the gauge theories based on the unitary groups, the correction is always to the transverse component, and the longitudinal ghost component remains suppressed. In QLGT, however, the gauge field is not a tensor, as was discussed above. Hence, neither the longitudinal nor the transverse components represent an observable particle. Therefore, the correction to the longitudinal component of the Lorentz gauge field does not have adversarial effects.
The third correction is from the second order vertex of the Lorentz gauge field
\[ i\Pi_{ij}^{3ij\mu,mn\nu} \propto \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \]
\[ = i\pi^2 A_0(0) = 0. \]
(70)
The loop is zero because the V4A vertex is momentum independent and can be taken out of the integral.

Finally, the last correction is from the ghost vertices
\[ i\Pi_{ij}^{4ij\mu,mn\nu} = \int \frac{d^4p}{(2\pi)^4} \]
\[ = -\frac{ig^2}{72\pi^2} k^\mu k^\nu \delta^{ij,mn}, \]
(71)
which is finite, and as expected, has the same form as in \( i\Pi_{ij}^{2ij\mu,mn\nu} \).

Out of the four corrections to the propagator of the Lorentz gauge field, only the fermionic loop contains infinities. To find the counterterms, we note that the only external lines in QLGT are fermions and the only possible Feynman diagrams have the following form
\[ \chi e^1 \chi e^2 \epsilon_{e_1 a_1 b_1} \epsilon_{e_2 a_2 b_2} \left( PA_{a_1 b_1} \epsilon_{a_2 b_2} + \right. \]
\[ \left. PA_{a_1 b_1} \epsilon_{i} \epsilon_{ij\mu} \cdot i\Pi_{ij\mu,mn\nu} \cdot PA_{mn\nu} \epsilon_{a_2 b_2} + O(g^4) \right), \]
(72)
where \( \chi^e \equiv -\frac{ig}{4} \bar{\varphi} \gamma^1 \gamma^5 \varphi \), and \( \varphi \) stands for any of the spinors. If we use \( \epsilon_{e_1}^{ab\sigma} \epsilon_{e_2 a} = -6\eta e_1 e_2 \), and \( \epsilon_{ij} \epsilon_{\epsilon_{mnab}} = -4\delta_{i,j \mu} \), expression above reads
\[ -6i\chi^e \chi^e \left( \frac{\eta e_1 e_2}{k^2} + \frac{6g^2}{192\pi^2 k^4} B_0(0, m^2, m^2) \right) \cdot \]
\[ (6m^2 \eta e_1 e_2 + k e_1 k e_2) + g^2 \cdot \text{finite} + O(g^4) \].
(73)
We define the renormalized parameter as

$$A_{ij\mu}^r \equiv \frac{1}{\sqrt{Z_A}} A_{ij\mu},$$

$$\xi^r \equiv \frac{1}{Z_\xi} \xi,$$

(74)

where $Z_{A/\xi} \equiv 1 + \delta_{A/\xi}$. Subsequently, the propagator of the gauge field takes the following form

$$PA(k)_{ij\mu,mn\nu} \equiv \frac{i\delta_{ij,mn}}{k^2} \left( (1 + \delta_A) \left( \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + (1 + \delta_\xi + \delta_A) \xi \frac{k_\mu k_\nu}{k^2} \right).$$

(75)

Since $\delta_A$ and $\delta_\xi$ are of order $g^2$, only the first term in the parentheses in Eq. (72) receives a correction from them while the correction to the rest of the terms are of the order of $g^4$ or higher and can be neglected. A straightforward calculation shows that the infinities can be removed if we choose

$$\delta_A + \frac{1}{3} \delta_\xi = - \frac{3g^2m^2}{16\pi^2 k^2} \text{Div} \left( B_0(0,m^2,m^2) \right),$$

$$\delta_\xi = \frac{3g^2}{32\pi^2} \text{Div} \left( B_0(0,m^2,m^2) \right).$$

(76)

We would like to emphasize that in Eq. (74) we showed that the corrections to the vacuum polarization has a form different than the one in the tree level. This is unlike any of the Yang-Mills theories of the standard model. The reason we could absorb this infinite correction by the available parameters of the theory was that the gauge field did not have an external line and the only possible Feynman diagram of the theory was given in Eq. (72) and the fortunate fact that the contraction of two Levi-Civita symbols is proportional to the six-dimensional internal metric of the homogeneous Lorentz group.

### 3.3.3 Ghost self-energy

The ghost propagator receives a corrections from the following loop

$$\ldots \Rightarrow \text{ } -i\Sigma_{ij,mn} = \int \frac{d^4k}{(2\pi)^4}$$

$$PA_{k_1 l_1 \sigma_1, k_2 l_2 \sigma_2} (k) V A c_{a_1 b_1, k_1 l_1 \sigma_1} (p + k)$$

$$P c (p) a_2 b_2 (p + k) V A c^{m n, k_2 l_2 \sigma_2} a_2 b_2 (p)$$

$$= ig^2m^2 \frac{1}{16\pi^2} \delta_{ij,mn} \left( 1 - B_0(m^2,0,0) \right).$$

(77)
The correction to the inverse of the ghost propagator can be found in the same way as for the fermion self-energy above and reads

\[ P_c(p)^{-1}(t)_{ij,mn} = -ip^2 \delta_{ij,mn} + i \Sigma_{ij,mn} \]  \hspace{1cm} (78)

Since the ghost field has no mass, we only define one renormalized parameter as

\[ \epsilon^{+ij} \equiv \frac{1}{Z_c} \epsilon^{ij} \]  \hspace{1cm} (79)

where \( Z_c \equiv 1 + \delta_c \). To remove the infinity we further assume that

\[ \delta_c \equiv - \frac{g^2}{16\pi^2} \text{Div} \left( B_0(m^2,0,0) \right). \]  \hspace{1cm} (80)

3.3.4 Fermion vertex renormalization

The correction to the fermion vertex is from the following two diagrams

\[ \Gamma^{ab\sigma} = \int \frac{d^4k}{(2\pi)^4} \text{VAFF}^{ij\mu} \cdot \text{PF}(p_2 - k) \cdot \text{VAFF}^{ab\sigma} \]

\[ \cdot \text{PF}(p_1 - k) \cdot \text{VAFF}^{mn\nu} \text{PA}(k)_{ij\mu,mn\nu} \]

\[ = A \epsilon^{ab\sigma} q \gamma^5 + B \epsilon^{ab\sigma} \gamma^5, \]

\hspace{1cm} (81)

where \( q \equiv p_1 - p_2 \), and

\[ A = - \frac{3g^3m}{128\pi^2} C_0 \left( m^2, m^2, q^2, m^2, 0, m^2 \right), \]

\[ B = \frac{3g^5}{512\pi^2} \left( -3B_0 \left( q^2, m^2, m^2 \right) + 4B_0 \left( m^2, 0, m^2 \right) \right. \]

\[ + \left( 6m^2 - 2q^2 \right) C_0 \left( m^2, m^2, q^2, m^2, 0, m^2 \right) - 1 \]. \hspace{1cm} (82)
\[ \Gamma_{ab\sigma} = \int \frac{d^4 k}{(2\pi)^4} \text{VAFF}^{ij\mu} \cdot \text{PF}(k) \cdot \text{VAFF}^{mn\nu} \]
\[ \text{PA}_{ij,ii,ij,\mu_1}(p_2 - k) \text{PA}_{mn,mm,11,\nu_1}(p_1 - k) \]
\[ \text{V3A}^{mn,1,1,ab\sigma,ij,1,\mu_1}(p_1 - k, p_2 - p_1, k - p_2) \]

\[ = \text{finite}. \] (83)

The expression for the second diagram is finite but rather lengthy and can be found in the online repository in [56]. Also, to reduce the computation load, we have used the simplifications that were described in Sec. 3.3.1 as well as \( \gamma^5 \cdot \gamma^5 = 1 \), and \( \gamma^5 \cdot \gamma^k = -\gamma^k \cdot \gamma^5 \). Since \( C_0 \) is finite, only \( B \) in the first diagram contains infinities. Therefore, the divergent correction to the fermion vertex reads

\[ \text{Div} \left( \Gamma_{ab\sigma} \right) = \text{Div} \left( B \right) \epsilon_{ab\sigma} \gamma_i \gamma^5 = 4ig^{-1} \text{Div} \left( B \right) \text{VAFF}^{ab\sigma}, \] (84)

and is proportional to the bare vertex. Hence, we can remove it by renormalizing the coupling constant

\[ g' = \frac{1}{Z_g} g, \] (85)

with \( Z_g \equiv 1 + \delta_g \), and choosing \( \delta_g \) such that

\[ \left( 4ig^{-1} \text{Div} \left( B \right) + \delta_g \right) \text{VAFF}^{ab\sigma} = 0. \] (86)

### 3.3.5 The rest of infinities

By now, we have used all of the possible parameters of QLGT to remove the infinities. On the other hand, the other three vertices in Sec. 3.1.2 also receive infinite corrections from the relevant loops. In this section, we would like to show that the gauge symmetry of QLGT implies three restrictions on the coefficients of the terms in the Lagrangian that removes the rest of infinities by the choices that we have made so far for the renormalized parameters.
Inserting all of the renormalized parameters, the total Lagrangian reads

\[ L_{\text{total}} = i \sum \bar{\psi}_i e_\mu \gamma^i \partial_\mu \psi^i - i \sum \bar{\psi}_i e_\mu \gamma^i \partial_\mu \psi^i - Z_m \bar{\psi}_i m \psi^i - \frac{ig}{4} \sum Z_i \bar{\psi}_i A_{ij\mu} \epsilon^{ij\mu} \psi^i \gamma^5 \psi^j - Z_4 A_{A^2} + Z_9 Z^2_5 g \mathcal{L}_{A^2} + Z^2_5 Z^2_3 g^2 \mathcal{L}_{A^4} - Z_6 \bar{\psi}_i c_{ij} \gamma^5 \psi^j - Z_7 \bar{\psi}_i c_{ij} \gamma^5 \psi^j + Z_8 Z^2_5 \bar{\psi}_i c_{ij} \gamma^5 \psi^j + Z_9 Z^2_5 \bar{\psi}_i c_{ij} \gamma^5 \psi^j, \]

(87)

where \( \mathcal{L}_{A^n} \) is the part of the gauge field Lagrangian containing \( n \) fields. From this renormalized Lagrangian, we can derive the corrections to the three vertices that were not directly discussed.

To validate these corrections, we write the total renormalized Lagrangian with unknown coefficients

\[ L'_{\text{total}} = i \sum \bar{\psi}_i e_\mu \gamma^i \partial_\mu \psi^i - i \sum \bar{\psi}_i e_\mu \gamma^i \partial_\mu \psi^i - Z_m \bar{\psi}_i m \psi^i - \frac{ig}{4} \sum Z_i A_{ij\mu} \epsilon^{ij\mu} \psi^i \gamma^5 \psi^j + Z_4 \mathcal{L}_{A^2} + Z_9 Z^2_5 g \mathcal{L}_{A^2} + Z^2_5 Z^2_3 g^2 \mathcal{L}_{A^4} - Z_6 \bar{\psi}_i c_{ij} \gamma^5 \psi^j - Z_7 \bar{\psi}_i c_{ij} \gamma^5 \psi^j + Z_8 Z^2_5 \bar{\psi}_i c_{ij} \gamma^5 \psi^j + Z_9 Z^2_5 \bar{\psi}_i c_{ij} \gamma^5 \psi^j + g Z_6 \bar{\psi}_i c_{ij} \gamma^5 \psi^j + g Z_8 Z^2_5 \bar{\psi}_i c_{ij} \gamma^5 \psi^j. \]

(88)

We note that the renormalized theory has to be invariant under the homogeneous Lorentz transformations. This means that the following three equations should be satisfied

\[ \frac{Z_1}{Z_4} = \frac{Z_5}{Z_4} = \frac{Z_8}{Z_4} = \sqrt{\frac{Z_6}{Z_4}}. \]

(89)

By comparison with Eq. (87) we can see that our choices for the infinities meet the enforced conditions.

### 3.4 Example of an unhealthy Yang-Mills theory based on homogeneous Lorentz group

Before ending this section, we would like to demonstrate an unhealthy Yang-Mills theory with the same gauge group as in our scenario in order to emphasize the non-triviality of the results of this section.

The construction starts by assuming that a multiplet of four fermion fields transform as follows

\[ \Psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \rightarrow \exp \left( \frac{ig}{2} S^{AB} \Omega_{AB} \right) \Psi, \]

(90)
where $S$ and $\Omega$ refer to the the six generators of the homogeneous Lorentz group and its arbitrary parameters respectively. The capital indices are to distinguish between this group and the homogeneous Lorentz group of physical frames on the tangent space. Note that each of the fermions in $\Psi$ still transforms as in Eq. 2 as well. Therefore, the model has two homogeneous Lorentz symmetries. If we localize $\Omega$ and construct the corresponding Yang-Mills theory, the strength tensor $F_{\mu \nu A \beta}$ will be defined as in Eq. 17. If we are solely interested in non-exotic Yang-Mills models, the Lagrangian also takes the form of Eq. 32.

The sole difference between this Yang-Mills model and our Yang-Mills theory is that $\Omega$ is independent of the physical frames and $A_{\mu \nu A \beta}$ refer to six tensors of the Lorentz group of physical observers and as a result represents six elementary particles; three of which are ghosts whose energies are not bound from below. Therefore, the model is not healthy.

4 The classical LGT

In this section, we discuss that since the classical tetrad postulate in Eq. 31 is a non-holonomic constraint, its implementation is not trivial. Next, we present the field equations of LGT as well as a plane wave analysis. Some exact solutions of LGT are enumerated in the end.

Hamilton’s principle is a prevalent method to derive the field equations from the action integral. In the presence of holonomic constraints, one can use the Lagrange multiplier method to add a zero term to the action and use Hamilton’s principle again. However, in the presence of non-holonomic constraints, Hamilton’s principle is not applicable in general, because the constraint can be violated along an arbitrary displaced path (variation of the variables). Therefore, we have to choose between options 1 or 2 mentioned above.

In the latter, both the tetrad and the gauge field are varied along the displacement paths generated by the homogeneous Lorentz transformation of the tangent spaces and lead to the following classical field equation for LGT [30,31]

$$D_\nu F_{\mu \nu ij} = -4\pi G \left( D_j T_{\mu i} - D_i T_{\mu j} \right),$$

(91)

where $T_{\mu i}$ is the energy-momentum tensor, and $G$ is a constant that results from the absorption of the Lagrange multiplier of the constraint (the tetrad postulate) into the field equations. The coefficients are selected, such that $G$ is equal to Newton’s constant. Therefore, unlike in geometrodynamic approaches to quantum gravity, Newton’s constant is not the fundamental coupling constant of the theory and does not affect the renormalizability of QLGT.

It should be noted that the negative sign in this equation is absent in the two references above. The sign was originally chosen to be the same as in the gauge theories of the standard model where the unitarity condition requires
it. The discussions in section 3.2 indicate that the sign of the gauge field Lagrangian of QLGT does not affect its unitarity.

If the tetrad postulate is valid, we can multiply equation above by $e^i_\alpha e^j_\beta$ to write it in the following convenient form

$$\nabla^\nu R_{\mu\nu\alpha\beta} = -4\pi G (\nabla_\beta T_{\mu\alpha} - \nabla_\alpha T_{\mu\beta}).$$

(92)

It is crucial to note that the validity of this equation is conditional upon the validity of the tetrad postulate, which does not hold at the quantum level. Also, we would like to emphasize that this equation is not a higher derivative theory since, in LGT, the connections and not the metric are the dynamical fields.

To show the relationship between LGT and GR, we multiply both sides of the equation above by $g^{\mu\alpha}$ and use the conservation law of the energy-momentum tensor to write

$$\nabla^\nu R_{\nu\beta} = -4\pi G \nabla_\beta T^\mu_\mu,$$

(93)

where $R_{\nu\beta}$ is the Ricci tensor. This equation is equal to the divergence of the Einstein equation in general relativity and shows the reason for the sign of the Lagrangian of the Lorentz gauge field.

4.1 A plane wave analysis: helicity 2 space-time wave

In geometrodynamic approaches to gravity, the observed space-time fluctuations [20] are explained via the existence of mass-less gravitons just as the electromagnetic waves are explained via the existence of photons. In our framework, however, the Lorentz gauge field does not represent a physical particle. As a result, the framework has no candidate elementary particle for gravitation. Nevertheless, in this section, we show that the unobservable Lorentz gauge field drives helicity two waves of space-time that travel with the speed of light. We will show that LGT predicts the same propagating modes of space-time wave as in GR. The analogous plane wave analysis for GR can be found in [25].

We start with the assumption that the space-time fluctuations are small and expand the tetrad as

$$e_\mu^\nu = \eta_\mu^\nu + h_\mu^\nu,$$

(94)

where $h \ll 1$. Substituting this into Eqs. 1 and 13 the metric and the Christoffel symbols read

$$g_{\mu\nu} = \eta_{\mu\nu} + 2h_{(\mu\nu)},$$

$$\Gamma^\lambda_\mu^\nu = \eta^{\lambda\sigma} (\partial_\mu h_{\sigma\nu} + \partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu}),$$

(95)
where \( h_{(\mu \nu)} \equiv \frac{1}{2} (h_{\mu \nu} + h_{\nu \mu}) \), and \( h_{\mu \nu} \equiv \delta^\rho_\mu h_{\rho \nu} \). Substituting the tetrad, the metric, and the Christoffel symbols into the classical tetrad postulate in Eq. 31, we can rewrite it as

\[
\partial_\mu h_{i\nu} - \delta_i^\sigma (\partial_\mu h_{(\sigma \nu)} + \partial_\nu h_{(\sigma \mu)} - \partial_\sigma h_{(\mu \nu)}) - g(A)_{i\nu \mu} = 0,
\]

(96)

where \( A_{i\nu \mu} \equiv \delta_i^\nu A_{ij \mu} \). We replace the fields of this equation by the following Fourier transformations

\[
h_{i\mu} = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} \Sigma_{i\mu}(p),
\]

(97)

\[
\langle A \rangle_{ij \mu} = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} \varepsilon_{ij \mu}(p).
\]

(98)

After multiplying by \( e^{ik \cdot x} \) and integrating on the position space, the classical tetrad postulate equation reads

\[
iki_{\nu} \Sigma_{[\nu i]} + ik_{\nu} \Sigma_{(i\nu)} - ik_{i} \Sigma_{(\mu \nu)} = g\varepsilon_{i\nu \mu},
\]

(99)

where the brackets around the indices indicate anti-symmetrization, and both \( \Sigma \) and \( \varepsilon \) are functions of the same momentum \( k \).

The linear form of field equation 91 in the Lorentz gauge and in the absence of matter reads 30

\[
\partial^2 (A)_{ij \mu} = 0,
\]

(100)

\[
\partial^\mu (A)_{ij \mu} = 0.
\]

(101)

Substituting Eq. 98 into these two equations indicates that

\[
k^2 = 0,
\]

\[
k^\mu \varepsilon_{ij \mu} = 0.
\]

(102)

Without loss of generality, we assume that the spatial component of the momentum vector is in the \( z \)-direction. This choice and the two conditions above leads to the following two equations

\[
k^\mu = (k, 0, 0, k),
\]

(103)

\[
\varepsilon_{j0} = -\varepsilon_{ij3}.
\]

(104)

The first equation implies that the wave is mass-less and travels with the speed of light.

So far we have fixed the Lorentz gauge using Eq. 101, we also fix the space-time coordinate in the following. Under an infinitesimal change of coordinates, we have

\[
x'^\mu \rightarrow x^\mu + \xi^\mu,
\]

\[
h'_{i\mu} \rightarrow h_{i\mu} - \partial_\mu \xi_i,
\]

\[
\Gamma'_{\mu \nu}^{\lambda} \rightarrow \Gamma_{\mu \nu}^{\lambda} - \partial_\mu \partial_\nu \xi^\lambda,
\]

\[
\langle A \rangle'_{ij \mu} \rightarrow \langle A \rangle_{ij \mu},
\]

(105)
where \( \xi, h, \) and \( \langle A \rangle \) are all taken to be very smaller than unity, and \( \xi \) is arbitrary. We use the four arbitrary parameters of the coordinate transformation such that \( g^{\mu \nu} \Gamma^\lambda_{\mu \nu} = 0 \), or equivalently

\[
2k^\mu \Sigma_{(\mu \sigma)} = k_\sigma h^{\mu \nu} \Sigma_{\mu \nu}.
\]

Inserting the components of the momentum vector, the equation implies that

\[
\Sigma_{22}^{(\sigma)} = -\Sigma_{11}, \quad \Sigma_{11} = -\Sigma_{(13)},
\]

\[
\Sigma_{(03)} = -\frac{1}{2} (\Sigma_{00} + \Sigma_{33}),
\]

\[
\Sigma_{(02)} = -\Sigma_{(23)}.
\]

Finally, equations 99, 103, and 107 indicate that

\[
\varepsilon_{013} = -\varepsilon_{010}, \quad \varepsilon_{023} = -\varepsilon_{020}, \quad \varepsilon_{033} = -\varepsilon_{030},
\]

\[
\varepsilon_{123} = -\varepsilon_{120}, \quad \varepsilon_{133} = -\varepsilon_{130}, \quad \varepsilon_{233} = -\varepsilon_{230},
\]

\[
\varepsilon_{121} = \varepsilon_{122} = \varepsilon_{031} = \varepsilon_{032} = 0,
\]

\[
\varepsilon_{012} = \varepsilon_{021}, \quad \varepsilon_{131} = \varepsilon_{011}, \quad \varepsilon_{232} = \varepsilon_{022},
\]

\[
\varepsilon_{132} = \varepsilon_{021} = \varepsilon_{231},
\]

\[
\Sigma_{32} = \frac{i}{k} \varepsilon_{233}, \quad \Sigma_{01} = \frac{i}{k} \varepsilon_{010},
\]

\[
\Sigma_{02} = \frac{i}{k} \varepsilon_{020}, \quad \Sigma_{11} = \frac{i}{k} \varepsilon_{011},
\]

\[
\Sigma_{22} = \frac{i}{k} \varepsilon_{022}, \quad \Sigma_{12} = \frac{i}{k} (\varepsilon_{021} + \varepsilon_{120}),
\]

\[
\Sigma_{21} = \frac{i}{k} (\varepsilon_{021} - \varepsilon_{120}), \quad \Sigma_{30} + \Sigma_{33} = \frac{i}{k} \varepsilon_{033},
\]

\[
\Sigma_{03} + \Sigma_{00} = \frac{i}{k} \varepsilon_{030}, \quad \Sigma_{31} = \frac{i}{k} \varepsilon_{133}.
\]

It is interesting to note that the first two lines of equations above are in agreement with Eq. 104.

At this point, we can make a conclusion. When the expectation value of the Lorentz gauge field is different from zero, space-time fluctuations are inevitable. This means that even when space-time has no dynamics of its own, the dynamics of the tangent spaces will drive classical space-time waves.

### 4.1.1 The physical modes

The physical modes of space-time fluctuations cannot be eliminated if we wish to make a second coordinate or homogeneous Lorentz transformation, which in the momentum space are

\[
\Sigma'_{\mu \nu} \rightarrow \Sigma_{\mu \nu} - i k_\mu \xi_\nu(k),
\]

\[
\varepsilon'_{ij \mu} \rightarrow \varepsilon_{ij \mu} + i k_\mu \omega_{ij}(k),
\]

(109)
where the four components of $\xi$ and the six components of $\omega$ are arbitrary. Since $k^\mu = (k, 0, 0, k)$, any of the components of $\Sigma_{ij}$ and $\epsilon_{ij\mu}$ with $\mu$ equal to 0 or 3 can be eliminated by an appropriate choice of the free parameters. This means that in Eq. 108 the physical components of space-time fluctuations are

$$
\Sigma_{11} = \frac{i}{k} \epsilon_{011},
$$

$$
\Sigma_{22} = \frac{i}{k} \epsilon_{022},
$$

$$
\Sigma_{12} + \Sigma_{21} = \frac{2i}{k} \epsilon_{021}.
$$

(110)

Note that these are the symmetrized components of the tetrad equivalent to three of the components of the metric. It is interesting to note that these are the physical components of the gravitational wave in GR.

A final point to be mentioned is that the linearized Eq. 108 is to the first order of perturbation invariant under both of the symmetry transformations of LGT. This is very important, since, for example, $\Sigma_{31}$ is not a physical mode because it is equal to $\frac{i}{k} \epsilon_{133}$, and an appropriate transformation can remove the latter. This conclusion was not possible if equality was lost after the transformation.

4.1.2 The helicity of physical modes of space-time fluctuations

Equation 107 indicates that two of the physical modes of the space-time fluctuations are dependent. Therefore, only $g_{11}$ and $g_{12}$ are independent physical modes of the metric. We define the following two fields in terms of the two physical components of the metric

$$
\Sigma_{\pm} \equiv \Sigma_{11} \mp i \Sigma_{12}. \quad (111)
$$

By performing a rotation around the direction of the motion of the wave by an angle $\theta$, we can show that

$$
\Sigma'_{\pm} \to e^{\pm 2i\theta} \Sigma_{\pm}, \quad (112)
$$

which means that the physical modes of the metric have helicity two.

4.2 A few exact solutions of LGT

In the following, we mention five space-time metrics and discuss the necessary conditions for them to be exact solutions of LGT. Calculations are all carried out using computer packages, and the corresponding scripts are available in [56].
The de Sitter space-time: The metric of the de Sitter space is

\[
ds^2 = -dt^2 + e^{Ht}|dx|^2,
\]

where \(H\) is constant. After substituting this metric into Eq. 92 we observe that the source term needs to be zero for the metric to be a solution. On the other hand, the source term is zero, even in the presence of the vacuum energy \([59]\). Therefore, unlike in GR, we not only need no dark energy to explain an accelerating expansion of the universe but also the prediction of the standard model of particle physics for the magnitude of the vacuum energy is not contradictory anymore.

The early universe solution: The so called radiation dominated universe in GR – \(\Lambda\) – CDM model has the following form

\[
ds^2 = -dt^2 + b \cdot t |dx|^2,
\]

where \(b\) is constant. We have substituted this metric into Eq. 92 and have shown that this is an exact solution with zero source term. Moreover, the source term is zero, even in the presence of radiation \([59]\). This means that, unlike in GR, the early universe solution in LGT stays stable even if other light particles exist or if the neutrinos are not hot.

The matter dominated universe: The metric in the matter dominated universe reads

\[
ds^2 = -dt^2 + b \cdot t^{\frac{5}{3}} |dx|^2.
\]

The needed energy-momentum tensor for this solution is the same in both LGT and GR. This metric would not be an acceptable solution of LGT if the sign of the Lagrangian of the Lorentz gauge field were as in \([59]\) and opposite of the choice in this paper.

The Schwarzschild solution in vacuum: The metric for this space-time is

\[
ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1}dr^2 + r^2d\Omega^2,
\]

which is an exact solution in both GR and LGT with zero energy-momentum tensor. This solution is previously discussed in \([30]\).
The Schwarzschild solution inside a star: The metric for a spherically symmetric solution inside an incompressible star reads

$$ds^2 = -\left(\frac{3}{2}\sqrt{1 - \frac{2GM}{R}} - \frac{1}{2}\sqrt{1 - \frac{2GM}{R^3}r^2}\right)^2 dt^2 + \frac{1}{1 - \frac{2GM}{R^3}r^2} dr^2 + r^2 d\Omega^2. \quad (117)$$

In both GR and LGT, this is an exact solution with the same energy-momentum tensor that reads $T^\mu_\nu = (-\rho, p, p, p)$, where $\rho$ is constant and

$$p = \rho \left(\sqrt{1 - \frac{2GM}{R}} - \sqrt{1 - \frac{2GM}{R^3}r^2}\right). \quad (118)$$

This solution is only valid for this paper’s choice of the sign of the gauge field Lagrangian. This is discussed in [31].

The Kerr metric: The Kerr space-time is crucial for describing some astrophysical observations [60], and its line element reads

$$ds^2 = -\left(1 - \frac{2GMr}{r^2 + a^2 \cos^2 \theta} \right) dt^2 - \frac{4GMar \sin^2 \theta}{r^2 + a^2 \cos^2 \theta} dtd\phi + \frac{r^2 + a^2 \cos^2 \theta}{r^2 - 2GMr + a^2} dr^2 + (r^2 + a^2 \cos^2 \theta) d\theta^2 + \left((a^4 + a^2 r^2) \cos^2 \theta + 2GMra^2 \sin^2 \theta + a^2 r^2 + r^4\right) \sin^2 \theta \frac{d\phi^2}{r^2 + a^2 \cos^2 \theta}. \quad (119)$$

This also is an exact solution of both LGT [61] and GR with zero energy-momentum tensor.

At the end of this section, we would like to mention that the classical solutions of the field equations of PGT have been studied in several places. See [62,63,64] and the references therein for an incomplete list. However, LGT field equations are different from PGT field equations in a few aspects. First, the Lagrangian of LGT is only one of the terms in PGT Lagrangian. Second, due to the complexities of the non-holonomic constraint, the source terms in the field equations are quite different.

5 Conclusion

We have presented QLGT, a quantum Yang-Mills theory of gravity based on the homogeneous Lorentz group of the physical observers in the tangent spaces of a torsion free pseudo-Riemannian manifold. Therefore, the dynamics of the
theory is defined on space-time but distinct from it. In the macroscopic world, the dynamics of the tangent spaces is coupled to space-time through the tetrad postulate as a non-holonomic constraint. Therefore, unlike the conventional Poincare gauge theory approaches, the space-time metric does not participate in any Poisson bracket in the classical regimes. This allows us to postulate, without contradicting the quantum principles, that space-time has no quantum nature at all, even in the highest possible energies. Therefore, the nature of time in QLGT is the same as in quantum physics. This alleviates the long-stood problem of time.

We have derived the Feynman rules of QLGT and calculated one-loop diagrams. We have shown that QLGT is renormalizable to the first loop approximation. The unitarity of the theory has been studied. We have shown that the probabilities are conserved in QLGT. These are non-trivial results since the homogeneous Lorentz group is non-compact, and its internal six-dimensional metric is not positive-definite. The difference between our Yang-Mills theory and a typical Yang-Mills theory of the homogeneous Lorentz group is that we are utilizing the group of physical observers. Consequently, our Lorentz gauge fields do not represent elementary particles. Therefore, no particle with negative kinetic energy is predicted in our scenario.

The classical field equations of LGT have been presented as well. Using a plane wave analysis, we have shown that the dynamics of non-physical Lorentz gauge fields in the tangent spaces are transferred to space-time using the equation of tetrad postulate. As a result, while gravity is associated with no elementary particle, space-time waves are generated by the fluctuations of an unobservable gravitational field that lives on space-time. We have shown that the physical modes of space-time wave in LGT are the same as the physical modes of space-time wave in GR, and their helicities are equal to two. Also, we have shown that the classical field equations of LGT possess the Schwarzschild solutions in the vacuum and inside the stars, the Kerr solution, the De Sitter solution, and the matter-dominated and the early universe space-time solutions in their exact forms.

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