Particle emission from a black hole on a tense codimension-2 brane

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Abstract

We calculate analytically grey-body factors of Schwarzschild black-holes localized on a 3-brane of finite tension and codimension 2. We obtain explicit expressions for various types of particles emitted in the bulk as well as on the brane in both the low and high frequency regimes. In the latter case, we obtain expressions which are valid for arbitrary number of extra dimensions if the brane tension vanishes.

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I. INTRODUCTION

The possibility of producing mini black holes in high energy collisions is one of the exciting prospects of research at the LHC. It appears as one of the major implications of brane-world models, where the presence of extra dimensions opens up the possibility for a lowered Planck scale $M_*[1, 2, 3, 4, 5]$, so that gravity is much stronger in the spacetime which includes the extra dimensions and black holes can then be produced in high energy collisions $[6, 7, 8, 9]$. 

Two key signatures of such black holes would be the observation of Hawking radiation and the measurement of quasi-normal mode frequencies $[10, 11, 12, 13, 14]$. Although at the horizon of a black hole the emitted Hawking radiation is a perfect black-body radiation, for an observer at infinity the radiation appears modified. This is because in escaping from the black hole the radiation needs to cross the non-trivial potential around the black hole which implies that part of the radiation will backscatter, resulting in a frequency-dependent factor that modifies the otherwise ideal black body radiation. This is known as a grey-body radiation and has a spectrum described by a modified black-body emission rate, which in $D = 4 + n$ dimensions is of the form

$$\frac{dE(\omega)}{dt} = \sum_{\ell,\kappa} \sigma_{\ell,\kappa}(\omega) \frac{\omega}{e^{\omega/T_H} \mp 1} \frac{d^{n+3}k}{(2\pi)^{n+3}}$$

where $\ell$ stands for the angular momentum quantum number, and $\kappa$ for any other quantum numbers of the emitted particles. $\sigma_{\ell,\kappa}(\omega)$ is the grey-body factor modifying the black-body radiation at the horizon to the spectrum observed at asymptotic infinity away from the black hole. The Hawking temperature $T_H$ for a Schwarzschild black-hole is given in terms of the horizon radius $r_h$ and the number of extra dimensions $n[15]$,

$$T_H = \frac{n + 1}{4\pi r_h}$$

For such a black hole, $r_h$ is enhanced and given by

$$r_h = \frac{1}{\sqrt{\pi M_*}} \left( \frac{M_{BH}}{M_*} \right)^{\frac{1}{n+1}} \left( \frac{8\Gamma \left( \frac{n+3}{2} \right)}{n+2} \right)^{\frac{1}{n+1}}$$

where $M_{BH}$ is the mass of the black-hole.

Clearly, an experimental observation of Hawking radiation from such black holes is a very exciting prospect as it constitutes direct observational evidence of black-holes. It should provide a rich source of information about the dimensionality and structure of spacetime as
well as constitute a first step toward establishing quantum gravity as a discipline accessible to experimental investigation. One way of exploring experimental implications of such theories that can be observed at accelerators has been through event generators. Many such programs have appeared over the years [16, 17, 18], the most recent and comprehensive of which is perhaps BlackMax [19]. Based on phenomenologically realistic models free of serious problems that plague low-scale gravity, it incorporates all grey-body factors known to date along with several effects that make predictions which are as realistic as possible.

In brane-world models, we need to localize Standard Model (SM) particles on a 3-brane yet allow gravity full access to the bulk. Such a localization is often done via a projection from higher dimensional spacetime onto a four-dimensional hypersurface identified as a 3-brane representing our world. While this method succeeds in describing the geometry, it fails to capture the defining feature of the brane, which is its tension. The latter is often neglected owing to the difficulties in obtaining exact solutions of Einstein’s equations in higher dimensions in the presence of a brane of finite tension. Recently, a solution that incorporates a 3-brane of finite tension in six-dimensional spacetime was constructed by Kaloper and Kiley for static black holes [20]. It was subsequently extended to include rotating black holes [21]. In this model, at the position of the brane a conical singularity forms that acts to off-load tension thereby keeping both bulk and brane locally flat. Some work [22, 23, 24, 25, 26] has already been done to study the implications of including tension in this way, resulting in qualitative as well as quantitative modifications which may affect the interpretation of experimental results.

The present work aims at continuing in that line by studying analytically the effects of including tension on grey-body factors. In detail, Section II is a quick review of the main effects of including tension in the Schwarzschild case and the attendant modifications of the parameters of the black-hole. In Section III we obtain analytical expressions for low-frequency grey-body factors for bulk as well as brane-localized emissions, and then compare with exact numerical results. In Section IV we calculate analytically grey-body factors in the case of large imaginary frequencies, again for emissions both in the bulk and on the brane. In the tensionless limit, our expressions are valid for arbitrary number of extra dimensions. Finally, Section V contains our conclusions.
II. SCHWARZSCHILD BLACK HOLE ON A TENSE CODIMENSION-2 3-BRANE

Black-holes formed through high energy collisions are expected to evaporate through several phases (e.g. [9]). After the initial “balding” phase where the gauge hair and other asymmetries are lost, a spin-down phase ensues, with a small portion of mass being lost by radiating away the angular momentum. However, it is expected that the black-hole will spend most of its life in the Schwarzschild phase emitting spherically symmetric Hawking radiation. In what follows we will concentrate only on this phase.

In brane-world scenarios, a small black hole with radius $r_h$ which is much smaller than the size of the extra dimensions $R$ is in fact completely immersed in the full $4 + n$ dimensional spacetime. In asymptotically flat space, the metric that describes a non-rotating black-hole in higher dimensions is a generalization of the 4-dimensional Schwarzschild metric [15],

$$ds^2 = -f_n(r)dt^2 + \frac{1}{f_n(r)} dr^2 + r^2 d\Omega_{2+n}^2, \quad f_n(r) = 1 - \left(\frac{r_h}{r}\right)^{n+1}$$

where the element $d\Omega_{2+n}$ defines a $(2 + n)$-sphere. To geometrically localize fields on the brane, what has often been done is to project onto the brane by setting all angular coordinates $\theta_i$ to $\pi/2$, except for the usual 4-dimensional angles $\theta$ and $\phi$ (see, e.g., [27, 28]). The resulting 4D metric then reads

$$ds^2 = -f_n(r)dt^2 + \frac{1}{f_n(r)} dr^2 + r^2 d\Omega_2^2$$

As mentioned earlier, such treatments neglecting the tension $\lambda$ of the brane are mainly due to the difficulty of obtaining higher-dimensional solutions of Einstein’s equations in the presence of a brane of finite tension. They can be justified on the grounds that in order for a black hole to be treated semi-classically, its mass must be much bigger than the Planck scale, so that the tension of the brane does not appreciably alter the black hole background and at a scale $O(r_h)$ the solution is flat. While this is in principle true, it turns out that for a black hole to be clear of the quantum regime in this context, given a gravity scale of the order of 1 TeV, the mass of the black hole needs to be on the order of just a few TeV [9]. However, because brane tension will be generated from contributions to vacuum fluctuations due to brane-localized fields, one expects the tension to be of the same order as the fundamental scale of the theory. Therefore, in order to be realistic, the brane tension ought to be taken into account.
By introducing tension using the Kaloper-Kiley codimension-2 setup \cite{20}, the tension of the brane does not curve either the brane or the bulk, but forms a conical singularity instead, creating a deficit angle $\Delta \psi$. By adapting to brane tension, $\Delta \psi$ provides a self-tuning mechanism which leaves both bulk and brane locally flat. It is given by

$$\Delta \psi = 2\pi (1 - b) \lambda M_*^4$$

where $0 < b \leq 1$. The resulting geometry is the same as that of Schwarzschild, except that a wedge defining the deficit angle is now cut out with its edges identified, or equivalently, the coordinate $\psi$ gets rescaled. The resulting metric reads

$$d s_6^2 = -f(r) d t^2 + f(r)^{-1} d r^2 + r^2 d \Omega_4^2, \quad f(r) = 1 - \left(\frac{r_h}{r}\right)^3$$

with

$$d \Omega_4^2 = d \theta^2 + \sin^2 \theta \left[d \varphi^2 + \sin^2 \varphi (d \chi^2 + b^2 \sin^2 \chi d \psi^2)\right]$$

One significant implication that can be readily shown is that the radius of the horizon gets enhanced,

$$r_h = \left(\frac{\mu}{b}\right)^{1/3}, \quad \mu = \frac{M_{\text{BH}}}{4 \pi^2 M_*^4}$$

hence also the geometrical cross-section and production rate, which is important from an experimental point of view.

Semi-classically, we can study the propagation of particles of spin $s$ in the vicinity of a black hole by writing the corresponding field equation in the background of the metric of the black hole. The resulting equation can then be separated into a radial equation and a set of angular equations. The radial equation is most conveniently dealt with by casting it into a Schrödinger-like form,

$$\frac{d^2 \Psi (r_*)}{d r_*^2} + \left(\omega^2 - V [r (r_*)]\right) \Psi (r_*) = 0$$

where $r_*$ is the “tortoise coordinate” defined by

$$r_* = \int \frac{d r}{f(r)}$$

and $V(r)$ is an effective potential which depends on the type of particle considered. The potential also depends on the eigenvalues of the angular equations, so that in order to find the observationally significant eigenvalue $\omega$ of the radial equation, we need to first solve the
angular eigenvalue problems. An exact solution for the angular eigenvalues for integer spin was first obtained in [29]. It was subsequently derived using Jacobi polynomials for Dirac fermions in [30].

We can sum up the effects of including tension in the codimension-2 model for emissions in the bulk by the following replacement rules

$$r_h \rightarrow \frac{r_h}{\sqrt{b}}, \quad \ell_i \rightarrow \lambda_i = \ell_i + \left(\frac{1}{b} - 1\right)m, \quad \psi \rightarrow b\psi$$ (12)

where $\ell_i$ is the orbital quantum number for the $i$th angular equation, and $m$ is the magnetic quantum number. For brane-localized emissions the same rules apply except that the orbital quantum number does not get modified. In subsequent sections these rules will be the main ingredient in obtaining explicit analytical expressions for grey-body factors including brane tension.

III. LOW-FREQUENCY GREY-BODY FACTORS

Calculations of grey-body in the case of low-frequency have been performed both for bulk [27, 31, 32] and brane-localized emissions [28, 33, 34]. In this section we shall extend these results to include brane tension.

In general, one tries to find an approximate solution of the radial equation which is valid at infinity and another one valid near the horizon. Then by matching the two solutions in the common domain of validity one can extract the coefficients of reflection and transmission and hence deduce the absorption coefficient (grey-body factor) as a function of frequency.

For bulk emissions of integer spin, we can use the Ishibashi-Kodama ‘master equation’ [32, 35], in which the potential in $D = n + 4$ dimensions is given by

$$V = f(r) \left[ \frac{\ell(\ell + D - 3)}{r^2} + \frac{(D - 2)(D - 4)}{4r^2} + \frac{(1 - p^2)(D - 2)^2}{4} \frac{r^{D-3}}{r^{D-1}} \right]$$ (13)

with $p$ determining the type of field (particle) one is interested in

$$p = \begin{cases} 
0, & \text{scalar and gravitational tensor} \\
2, & \text{gravitational vector} \\
2/(D - 2), & \text{EM (gauge) vector} \\
2(D - 3)/(D - 2), & \text{EM (scalar)} 
\end{cases}$$ (14)
and $\ell$ being the angular momentum quantum number taking on values

$$
\ell = \begin{cases} 
0, 1, \ldots, \text{scalar} \\
1, 2, \ldots, \text{EM} \\
2, 3, \ldots, \text{gravitational}
\end{cases}
$$

Gravitational scalar fields are governed by a more complicated potential, but they can be described approximately using the above potential by taking $p$ to be

$$p_{\text{grav-scalar}} \sim 2 + 0.674D^{-0.5445}$$

Specifically, for the case we are interested in, $n = 2$ we have

$$p_{\text{grav-scalar}} = 2.25$$

Writing the radial equation in terms of $R(r) = r^{-1 - \frac{n}{2}}\Psi(r_*)$ for $n = 2$, we end up with

$$
\frac{f}{r^4} \frac{d}{dr} \left[ r^4 f' R(r) \right] + \left[ \omega^2 - f \left( \frac{\lambda_3 (3 + \lambda_3)}{r^2} - 4p^2 \frac{1}{r^5 r_h^3} \right) \right] R(r) = 0
$$

where we have replaced $\ell$ by $\lambda_3$ (eq. (12)) in order to account for the brane tension. Near the horizon, it is convenient to make the change of variables $r \to f(r)$ (note that $f(r) \to 0$ as $r \to r_h$). The resulting equation is

$$
(1 - f) \left( \frac{f}{r^4} \frac{d^2}{df^2} R_{NH} + \frac{d}{df} R_{NH} \right) + \left( \frac{(\omega r_h)^2}{9f(1 - f)} - \frac{\lambda_3 (3 + \lambda_3)}{9(1 - f)} + \frac{4}{9} p^2 \right) R_{NH} = 0
$$

whose general solution can be written in terms of hypergeometric functions,

$$R_{NH}(f) = A_- f^\alpha (1 - f)^\beta F(a, b, c; f) + A_+ f^{-\alpha} (1 - f)^\beta F(a - c + 1, b - c + 1, 2 - c; f)$$

where

$$
\alpha = \frac{i\omega r_h}{3}, \quad \beta = \frac{1}{2} - \frac{1}{3} \sqrt{\left( \frac{\lambda_3 + 3}{2} \right)^2 - (\omega r_h)^2},
$$

$$
a = \alpha + \beta - \frac{2}{3} p = -\frac{i\omega r_h}{3} + \frac{1}{2} - \frac{1}{3} \sqrt{\left( \frac{\lambda_3 + 3}{2} \right)^2 - (\omega r_h)^2} - \frac{2}{3} p
$$

$$
b = \alpha + \beta + \frac{2}{3} p = -\frac{i\omega r_h}{3} + \frac{1}{2} - \frac{1}{3} \sqrt{\left( \frac{\lambda_3 + 3}{2} \right)^2 - (\omega r_h)^2} + \frac{2}{3} p,
$$

$$
c = 1 + 2\alpha = 1 + \frac{2}{3} i\omega r_h
$$
Imposing the boundary condition that at the horizon the solution be purely in-going, we obtain the constraint

$$A_+ = 0 \quad (22)$$

Far from the horizon ($f \to 1$), the asymptotic solution (20) under the constraint (22) behaves as

$$R_{NH}(f) \sim A_- \left[ \left( \frac{r_h}{r} \right)^{3\beta} \frac{\Gamma(1 + 2\alpha)\Gamma(1 - 2\beta)}{\Gamma(1 + \alpha - \beta + \frac{2}{3}p)\Gamma(1 + \alpha - \beta - \frac{2}{3}p)} + \left( \frac{r_h}{r} \right)^{3(1-\beta)} \frac{\Gamma(1 + 2\alpha)\Gamma(2\beta - 1)}{\Gamma(\alpha + \beta - \frac{2}{3}p)\Gamma(\alpha + \beta + \frac{2}{3}p)} \right] \quad (23)$$

which follows from standard identities of hypergeometric functions.

In the far field region, as $r \to \infty$, or correspondingly $f \to 1$, the radial wave equation may be approximated by

$$g''(r) + \frac{1}{r}g'(r) + \left[ \omega^2 - \left( \frac{9}{4} + \lambda_3 (\lambda_3 + 3) \right) \frac{1}{r^2} \right] g(r) = 0 \quad (24)$$

where we defined $g(r) = r^{3/2}R(r)$. The solution can be written in terms of Bessel functions,

$$R_{FF}(r) = \frac{g(r)}{r^{3/2}} = \frac{1}{r^{3/2}} \left( B_+ J_{\lambda_3 + 3/2}(\omega r) + B_- Y_{\lambda_3 + 3/2}(\omega r) \right) \quad (25)$$

In the low frequency regime,

$$\omega r_h \ll 1 \quad (26)$$

the asymptotic solution (25) may be approximated near the horizon by

$$R_{FF}(r) \sim B_+ \left( \frac{\omega}{\pi} \right)^{\lambda_3 + 3/2} r^{\lambda_3} - B_- \frac{\Gamma(\lambda_3 + 3/2)}{\pi} \left( \frac{\omega}{2} \right)^{-\lambda_3 - 3/2} r^{-\lambda_3 - 3} \quad (27)$$

Matching the two asymptotic solutions in the intermediate region ((23) and (27), respectively), we obtain

$$\frac{B_+}{B_-} = -\frac{\Gamma(1 - 2\beta)\Gamma(\alpha + \beta - \frac{2}{3}p)\Gamma(\alpha + \beta + \frac{2}{3}p)\Gamma(\lambda_3 + 5/2)\Gamma(\lambda_3 + 3/2)}{\pi\Gamma(1 + \alpha - \beta + \frac{2}{3}p)\Gamma(1 + \alpha - \beta - \frac{2}{3}p)\Gamma(2\beta - 1)} \left( \frac{2}{\omega r_h} \right)^{2\lambda_3 + 3} \quad (28)$$

where we used $\beta \approx -\lambda_3/3$ (eq. (21)), on account of (26).

The reflection coefficient is

$$R = \frac{\text{outgoing amplitude}}{\text{incoming amplitude}} = \frac{B_+ - iB_-}{B_+ + iB_-} \quad (29)$$
from which we deduce the bulk absorption probability
\[ |A_{\text{bulk}}|^2 = 1 - |R|^2 = \frac{2i(B^* - B)}{BB^* + i(B^* - B) + 1}, \quad B = \frac{B_+}{B_-} \] (30)

After some algebraic manipulations, in the low frequency regime (26) we arrive at the explicit expression for the absorption probability,
\[ |A_{p\text{ bulk}}|^2 = 4\pi \left( \frac{\omega r_h}{2} \right)^{4+2\lambda_3} \left[ \frac{\Gamma \left( 1 + \frac{\lambda_3 + 2\mu}{3} \right) \Gamma \left( 1 + \frac{\lambda_3 - 2\mu}{3} \right)}{\Gamma \left( 1 + \frac{2\lambda_3}{3} \right) \Gamma \left( \lambda_3 + \frac{5}{3} \right)} \right]^2 \] (31)
which reduces to the tensionless result in the limit \( b \rightarrow 1 \) (see, e.g., [27, 31, 36]).

Evidently, the dependence of the absorption probability on the brane tension is complicated. To study its behavior, let us first look at some graphical representations of this result. Figure 1 depicts the absorption probability as a function of \( b \) with \( \omega \) fixed. As can be seen, all types of perturbation exhibit the same qualitative behavior with the probability vanishing as \( b \rightarrow 0 \) and increasing monotonically as \( b \rightarrow 1 \). This behavior persists qualitatively at high frequency as demonstrated in figure 2 which was obtained from points generated by exactly solving for absorption probability numerically. A further demonstration of how tension suppresses the absorption probability is provided in figure 3, where probability vs frequency in the case of a scalar perturbation is plotted for various values of \( b \) in the small frequency regime. The graph also has points representing exact numerical calculations. As expected, the predictions of the analytical formula diverge from the exact values with increasing frequency. Finally, figure 4 shows the effect of the magnetic quantum number \( m \) in the case of scalar emission. The qualitative behavior of all curves is the same, however the absorption probability varies over many orders of magnitude depending on \( m \). In figs. 3 and 4, the type of perturbation chosen has a negligible effect on the outcome.

Moving on to the case of brane localized modes (the photon, massless fermion and scalar), we note that because we are projecting onto the brane, angles \( \chi \) and \( \psi \) will be fixed, so that the dependence of the metric on \( b \) will only be through \( r_h \) inside the metric function \( f \). Aside from that, the results pertaining to the grey-body factors will be identical to those in the tensionless case. To find these grey-body factors we follow the same procedure as above, but instead using an equation analogous to the Teukolsky equation on the brane. For a static background, the radial wave equation is (see, e.g., [28, 37])
\[ (fr^2)^s \frac{d}{dr} \left[ (fr^2)^{1-s} \frac{dR_s}{dr} \right] + \left\{ \frac{\omega^2 f^2}{f} + 2is\omega r - \frac{is\omega}{f} (n + 1)(1 - f) \right\} - \Lambda - (2s - 1)(s - 1)(n + 1)(1 - f) \right\} R_s = 0 \] (32)
where \( \Lambda \equiv \ell(\ell+1) - s(s-1) \), and \( s \) is the spin. The corresponding angular equation reads

\[
\frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d_{s}S_{\ell}^{m}}{d\theta} \right) + \left( -\frac{2ms \cot \theta}{\sin \theta} - \frac{m^2}{\sin^2 \theta} + s - s^2 \cot^2 \theta + \Lambda \right)_{s}S_{\ell}^{m} = 0 \quad (33)
\]

where \( e^{im\varphi} \cdot s_{s}S_{\ell}^{m}(\theta) = sY_{m}^{m}(\Omega_2) \) are known as the spin-weighted spherical harmonics. For \( s = \frac{1}{2} \), the absorption probability is [28]:

\[
|A_{s=1/2}^{\text{brane}}|^2 = 4\pi \left( \frac{\omega r_h}{2^{5/3}} \right)^{2j+1} \frac{1}{\Gamma(j+1)^2} \quad (34)
\]

and for \( s = 1 \)

\[
|A_{s=1}^{\text{brane}}|^2 = \frac{1}{9} \left( 2\omega r_h \right)^{2j+2} \left[ \frac{\Gamma \left( \frac{j}{3} \right) \Gamma \left( \frac{j+1}{3} \right) \Gamma \left( j+2 \right)}{\Gamma \left( \frac{2j+1}{3} \right) \Gamma \left( 2j+2 \right)} \right]^2 \quad (35)
\]

where \( j \) is the total angular momentum in both cases.

For brane-localized scalars the result is [27]:

\[
|A_{s=0}^{\text{brane}}|^2 = \frac{16\pi}{9} \left( \frac{\omega r_h}{2} \right)^{2j+2} \left[ \frac{\Gamma \left( \frac{j+1}{3} \right) \Gamma \left( 1 + \frac{j}{3} \right)}{\Gamma \left( \frac{1}{2} + j \right) \Gamma \left( 1 + \frac{2j+1}{3} \right)} \right]^2 \quad (36)
\]

Upon comparison of (35) and (36), we obtain

\[
\frac{|A_{s=0}^{\text{brane}}|^2}{|A_{s=1}^{\text{brane}}|^2} = \left( \frac{j}{1+j} \right)^2 \quad (37)
\]

which shows that (in this frequency regime) the gauge vector and scalar behave the same way up to a multiplicative factor, which tends to unity for large \( j \). One can see from the expressions above that only the first few values of \( j \) contribute to the emission spectrum making the emission rate for scalars higher than for gauge vectors in this frequency range. This can be seen numerically in the absence of tension in the results of [38]. At higher frequencies the situation reverses, and the rate for scalars becomes smaller. Graphs analogous to those for bulk emissions are shown in figures 5 and 6 for scalars. For low frequencies, the behavior with varying tension is now reversed and emission is enhanced with increased tension. For higher frequencies the trend is not as conclusive. The difference between the two cases can be understood in terms of two competing effects: (a) the enhancement of the geometrical cross-section with tension implies an increased rate of emission for both bulk and brane-localized modes, and (b) the fact that the angular eigenvalues increase with tension and appear only in the bulk potential (causing the potential barrier to increase) results in suppression of only bulk emissions.
From an experimental point of view, it is of great interest to know the relative emissivities between brane and bulk modes. The above discussion seems to hint that the inclusion of tension favors the EHM conjecture \[39\], namely, that black holes emit mainly on the brane. To verify this in our case, let us consider the emission spectrum as defined in eq. (1) for scalars. For that, we need to write the grey-body factor \(\sigma_{\ell,k}(\omega)\) in terms of the absorption probability \(|A|^2\) and scalar multiplicities \(N_{\ell,m}\). For either bulk or brane-localized emissions it is of the form

\[
\frac{dE(\omega)}{dt d\omega} = \frac{1}{2\pi} \sum_{\ell,m} N_{\ell,m} |A_{\ell,m}|^2 \frac{\omega}{e^{\omega/T_H} - 1}.
\]

In the bulk, the scalar multiplicities are \[22\]

\[
N_{\ell_3,m}^{scal} = \begin{cases} 
  m = 0, & \frac{1}{2} (\ell_3 + 2) (\ell_3 + 1), \\
  m \neq 0, & (\ell_3 - m + 2) (\ell_3 - m + 1)
\end{cases}
\]

whereas on the brane they are equal to the standard value \(2\ell + 1\). Starting with the bulk, we numerically solve eq. (18) for the absorption probability with \(p = 0\), and using eq. (39), we end up with the spectrum shown in fig. 8. A similar calculation for brane-localized scalars is done by projecting the bulk equation on the brane, and the result is shown in fig. 9. Notice that in both figures, the emission rates are enhanced with tension. The reason for this comes primarily not from the grey-body factor, but rather from the denominator in the black-body distribution which depends on the Hawking temperature that is common to both types of emission. We can discount this effect by looking at the relative emissivity, in which we take the ratio of the emission rate in the bulk to that on the brane. Figure 10 shows this for scalars.

Similar plots are obtained for gauge vectors. On the brane, we again have the multiplicities \(2\ell + 1\), but in the bulk we have

\[
N_{\ell_3,m}^{vec} = \begin{cases} 
  m = 0, & \frac{1}{2} \ell_3 (3\ell_3 + 5), \\
  m \neq 0, & 3 (\ell_3 - m + 2) (\ell_3 - m + 1)
\end{cases}
\]

In the bulk, we solve eq. (18) with \(p = \frac{1}{2}\), and on the brane eq. (32) is solved with \(s = 1\). The results of doing this are shown in figures 11 through 13. In this case the emissivity is only partial, since the scalar degree of freedom is not included.

We can see that in both cases, in the range shown, the emission rate is greater on the brane than in the bulk, in support of the EHM claim. It should be pointed out, however, that
even though mini black-holes at the LHC will spend most of their life in the Schwarzschild phase, they first form with non-zero angular momentum. This has been shown to modify the Hawking radiation due to super-radiance in such a way that emission in the bulk at the initial phase dominates over emission on the brane [40, 41, 42].

IV. HIGH-FREQUENCY GREY-BODY FACTORS

Next we turn our attention to grey-body factors at large imaginary frequencies. We shall discuss different values of spin and take into account the extra dimensions as well as the brane tension in the codimension-2 model [20]. The calculation will follow the method of Andersson and Howls [43] which was then used by Cho [44]. It is a combination of the monodromy argument of Motl and Neitzke [45] and the standard complex-coordinate WKB method.

The first step is to rewrite the master equation (10) in a form suitable for the application of the WKB method. To this end, we introduce the redefinition of the wavefunction,

\[ \Psi = \frac{\phi}{\sqrt{f}} \]  

which reduces (10) to

\[ \phi''(r) + Q^2(r)\phi(r) = 0 \]  

The resulting WKB solution is

\[ \phi_{1,2}^{(t)} = \frac{1}{\sqrt{Q(r)}} \exp \left[ \pm i \int_t^r Q(r') \, dr' \right] \]  

with \( t \) being a reference point, and

\[ Q^2(r) = \frac{1}{f^2(r)} \left[ \omega^2 - V(r) + \frac{1}{4} (f'(r))^2 - \frac{1}{2} f(r)f''(r) \right] \]  

Integer spin particle are described by the potential (13).

Following Motl and Neitzke [45], we go around a contour in the complex \( r \)-plane and impose preservation of monodromy. Because of the exponential nature of the WKB solutions, dominance will be exchanged between the two exponentials as we move around the complex plane. This implies that some terms that are exponentially small and can be overlooked in one region of the complex plane can grow exponentially in a different region. To take
this into account, one needs to resort to the Stokes phenomenon which keeps track of these changes.

In such an analysis, we need to pay attention to the zeroes and poles of $Q(r)$. In particular, it can be shown that from each simple zero of $Q$ there emanate 3 Stokes lines along which $Q(r)dr$ is imaginary. This means that one of the two solutions $\phi^{(t)}_{1,2}$ will grow exponentially whereas the other one will decay as we move away from the reference point $t$. On the other hand, we can also define anti-Stokes lines emanating from each zero of $Q$ along which $Q(r)dr$ is real and $\phi^{(t)}_{1,2}$ are oscillatory functions. By crossing anti-Stokes lines, the exponential behaviors of the two solutions are switched, while extending the solution across a Stokes line, the linear combination defining the solution is changed in a well-defined way: the coefficient of the dominant term stays the same, but that of the sub-dominant term picks up a contribution proportional to the coefficient of the dominant term. Thus we see that the appropriate contour that can be used will trace anti-Stokes lines and cross Stokes lines. Figure 7 shows such a contour for Schwarzschild geometry assuming (large) purely imaginary frequency.

One can see from the potential (13) that for $|\Im \omega| \to \infty$, all zeroes of $Q$ approach the origin and the potential may be approximated by its expansion around $r = 0$. In this case, the $\ell$-dependent terms are negligible. They enter in subleading contributions \[46, 47\]. Near $r = 0$, keeping only the most singular term, the potential in (13), which describes integer-spin perturbations, becomes:

$$V_{\text{bulk}} \sim \frac{1}{4} (2 + n)^2 (p^2 - 1) \frac{r_h^{2(1+n)}}{r^{2(2+n)}}$$

and the resulting WKB equation is:

$$\psi''(r) + R_0(r)\psi(r) \simeq 0$$

where

$$R_0(r) = \left(\frac{r}{r_h}\right)^{2n+2} \left[ \omega^2 - \frac{1}{4r^{2(n+2)}} \left( (p^2 - 1)(n+2)^2 + (n+1)(n+3) \right) r_h^{2(n+1)} \right]$$

However, one can show \[43\] that the solutions $\phi^{(t)}_{1,2}$ so obtained by identifying $R_0(r)$ with $Q^2(r)$, do not have the right behavior near the origin when compared with the exact solution near there. This can be rectified by using a slightly different $R(r)$, which is permissible. The
modified form turns out to be:

\[ R(r) = R_0(r) - \frac{1}{4r^2} \equiv Q^2(r) = \left( \frac{r}{r_h} \right)^{2(n+1)} \left[ \omega^2 - \frac{(n + 2)^2 p^2}{4r^{2(n+2)} r_h^{2(n+1)}} \right] \]  

(48)

For \( \omega = -i|\omega| \), the zeros of \( Q^2 \) are:

\[ r_k = \left( \frac{(n + 2)p}{2|\omega|} \right)^{1/(n+2)} r_h^{(n+2)/(n+2)} e^{\frac{\pi i}{n+2}(k+\frac{1}{2})}, \quad k = 0, 1, \cdots 2(2 + n) - 1 \]  

(49)

These zeros will serve as reference points for the phase integrals, because in doing so, the

Stokes constants are either \(+i\) for traveling anti-clockwise (or \(-i\) for clockwise), and so, we will need to switch between reference points as we go around the contour:

\[ \phi^{(t_k)}_{1,2} = e^{\pm \nu_{jk}^{(t_k)}} \phi^{(t_j)}_{1,2} \]  

(50)

with

\[ \nu_{jk} = \int_{r_j}^{r_k} Q \, dr = \frac{\omega}{r_h^{n+1}} \int_{r_j}^{r_k} r^{n+1} \left[ 1 - \frac{(n + 2)^2 p^2}{4r^{2(n+2)} r_h^{2(n+1)}} \right]^{1/2} dr \]  

(51)

where \( j \) and \( k \) label two consecutive zeros. Using the substitution \( y \equiv \frac{2u r^{n+2}}{(n+2)p r_h^{n+1}} \), followed by \( y \equiv \cosh x \), we get:

\[ \nu_{jk} = \int_{r_j}^{r_k} Q \, dr = \frac{p}{2} \int_{e^{i\pi j}}^{e^{i\pi k}} \left[ 1 - \frac{1}{y^2} \right]^{1/2} \, dy = \left( \frac{e^{i\pi j} - e^{i\pi k}}{2} \right) \frac{\pi p}{2} \]  

(52)

so that:

\[ \nu_{10} = \nu_{30} \equiv \nu = -\frac{\pi p}{2} \equiv \nu_{\text{bulk}} \]  

(53)

It is also possible to consider a massless Dirac fermion propagating in the bulk, which can be relevant in split-fermion theories where rapid proton decay can be suppressed by introducing bulk fermions \cite{48, 49} that are superpartners to localized scalars and gauge bosons. A master equation potential based on a higher-dimensional Dirac equation potential was derived in \cite{49},

\[ V_{\text{bulk-Dirac}} = f \frac{d}{dr} \left[ \sqrt{f} \left( \frac{\ell + D/2}{r} \right) \right] + f \left( \frac{\ell + D/2}{r} \right)^2 \]  

(54)

By expanding near \( r = 0 \), we find this potential to be proportional to \( r^{-(7+3n)/2} \), which, for the present purpose, can be treated as zero when compared to the other bulk perturbation given by \cite{45}. At this approximation, therefore, we may use \( p = 1 \) for a bulk Dirac fermion.
Turning our attention to the actual calculation of grey-body factors, we see that since \( \omega \) in the present case is not real, the conservation of flux will take on the following generalized form:

\[
T(\omega)T(-\omega) + R(\omega)R(-\omega) = 1
\]

(55)

and therefore, we consider two separate solutions representing the boundary condition at physical infinity; one with frequency \( \omega \) and the other with \(-\omega\). In the first case, the reflected wave at infinity is represented by \( \phi_1^{(t_1)}(r) \), while the ingoing wave by \( \phi_2^{(t_1)}(r) \). For \(-\omega\), the two solutions switch roles, but the calculation is effectively the same, and we shall combine them into one by writing the wavefunction in general as:

\[
\psi = Xe^{i\theta} \phi_1^{(t_j)} + Ye^{-i\theta} \phi_2^{(t_j)}
\]

(56)

where \( \theta \) is a phase angle. In the case of \(+\omega\), \( X = R_+ \) and \( Y = 1 \), whereas for \(-\omega\), \( X = 1 \) and \( Y = R_- \). For \( \Re(\omega M) > 0 \), the outgoing wave boundary condition at spatial infinity can be analytically continued to the anti-Stokes line labeled \( a \) in the figure. To find the the \( T \)'s and \( R \)'s we need to travel a path on anti-Stokes lines starting and ending at point \( a \). The solution \( \psi_a \) at point \( a \) for \( \pm \omega \) is described by the wavefunction given in (56) with \( j = 1 \).

Since we don’t cross any Stokes lines in extending the solution to \( t_1 \), the above expression remains valid there, but in going to point \( b \) we do cross a Stokes line, and to account for the Stokes phenomenon we need to add to \( \psi_a \) a contribution proportional to \( \phi_2 \) with the constant of proportionality equal to the Stokes constant multiplied by the coefficient of the dominant function on the Stokes line, \( \phi_1 \) in this case, so that:

\[
\psi_b = \psi_a - iXe^{i\theta} \phi_2^{(t_1)} = Xe^{i\theta} \phi_1^{(t_1)} + (Ye^{-i\theta} - iXe^{i\theta}) \phi_2^{(t_1)}
\]

(57)

To extend the solution to point \( c \), we now need to change the lower limit of integration to have \( t_0 \) as reference:

\[
\psi_b = Xe^{i\theta - iv} \phi_1^{(t_0)} + (Ye^{-i\theta + iv} - iXe^{i\theta}) \phi_2^{(t_0)}
\]

(58)

In going to point \( c \) there will be no exchange in dominance, since we cross no anti-Stokes lines:

\[
\psi_c = \psi_b - iXe^{i\theta - iv} \phi_2^{(t_0)} = Xe^{i\theta - iv} \phi_1^{(t_0)} + (Ye^{-i\theta + iv} - iXe^{i\theta - iv}) \phi_2^{(t_0)}
\]

(59)
We now go to point \( c' \), and in doing so we cross no Stokes lines, so that the combination \( 59 \) remains valid, but now the integral is evaluated around a contour that loops around the pole \( r = r_h \). Replacing that contour by the one to the left of the pole, we have:

\[
\psi_{c'} = X e^{i\theta-i\nu} e^{i\Gamma} \phi_1^{(t_0)} + (Y e^{-i\theta+i\nu} - iX e^{i\theta+i\nu} - iX e^{i\theta-i\nu}) \phi_2^{(t_0)} e^{-i\Gamma} \tag{60}
\]

where \( \Gamma \) is the integral encircling \( r = r_h \) clockwise:

\[
\Gamma_{\text{bulk}} = \oint Q \, dr = -2\pi i \text{Res} \, Q = -i\pi \sqrt{1 + \left( \frac{2\omega r_h}{n+1} \right)^2} \approx -\frac{1}{2} i\pi i\beta \tag{61}
\]

where \( \beta = 1/T_H \). To connect the solution to point \( d \), we need to change the lower integration limit to \( t_3 \):

\[
\psi_{d} = \psi_{c'} + i (Y e^{-i\theta} - iX e^{i\theta-2i\nu} - iX e^{i\theta}) \phi_1^{(t_3)} e^{-i\Gamma} \tag{62}
\]

and now to connect to point \( d \), we need to cross the anti-Stokes line to get inside the loop, and this means that \( \phi_1 \) and \( \phi_2 \) will interchange dominance, further, the Stokes constant is now \( i \), since we have reversed direction:

\[
\psi_{d} = [X e^{i\theta} e^{i\Gamma} e^{-i\nu} + (iY e^{-i\theta} e^{-i\nu} + X e^{i\theta-3i\nu} + X e^{i\theta} e^{-i\nu}) e^{-i\Gamma}] \phi_1^{(t_3)} \\
+ (Y e^{-i\theta} e^{i\nu} - iX e^{i\theta-i\nu} - iX e^{i\theta} e^{i\nu}) \phi_2^{(t_3)} e^{-i\Gamma} \tag{63}
\]

Then switching back to point \( t_0 \):

\[
\psi_{d} = [X e^{i\theta} e^{i\Gamma} e^{-i\nu} + (iY e^{-i\theta} e^{-i\nu} + X e^{i\theta-3i\nu} + X e^{i\theta} e^{-i\nu}) e^{-i\Gamma}] \phi_1^{(t_0)} \\
+ (Y e^{-i\theta} e^{i\nu} - iX e^{i\theta-i\nu} - iX e^{i\theta} e^{i\nu}) \phi_2^{(t_0)} e^{-i\Gamma} \tag{64}
\]

Going back to point \( c \) and crossing a Stokes line, we encircle the pole at the origin:

\[
\psi_{c} = [(2e^{-i\nu} + e^{i\nu} + e^{-3i\nu}) e^{i(\theta-\Gamma)} X + e^{i(-\nu+\Gamma+\theta)} X + i (e^{-i\nu} + e^{i\nu}) e^{-i(\Gamma+\theta)} Y] \phi_1^{(t_0)} \\
+ e^{-i\Gamma} [-ie^{i(-\nu+\theta)} X - i e^{i(\nu+\theta)} + e^{i(\nu-\theta)}] \phi_2^{(t_0)} \tag{65}
\]

where the bar is to indicate that we have encircled the pole at the origin. Connecting to point \( b \):

\[
\psi_{b} = [(2e^{-i\nu} + e^{i\nu} + e^{-3i\nu}) e^{-i\Gamma} X + e^{i(-\nu+\Gamma)} X + i (e^{-i\nu} + e^{i\nu}) e^{-i(\Gamma+2\theta)} Y] e^{i\theta} \phi_1^{(t_0)} \\
+ [i (e^{-i\Gamma} e^{-2i\nu-i\Gamma} + e^{i\Gamma}) e^{i(\theta-\nu)} X - e^{-i(\nu+\Gamma+\theta)} Y] \phi_2^{(t_0)} \tag{66}
\]
switching to reference point $t_1$:

$$\psi_{\bar{b}} = \left[ (2 + e^{2i\nu} + e^{-2i\nu}) e^{-i\Gamma} X + e^{i\Gamma} X + i \left( 1 + e^{2i\nu} \right) e^{-i(\Gamma + 2\theta) Y} \right] e^{i\theta} \phi_{1}^{(t_1)}$$

$$+ \left[ i \left( e^{-i\Gamma} + e^{-2i\nu - i\Gamma} + e^{i\Gamma} \right) e^{i(\theta - 2\nu)} X - e^{-i(2\nu + \Gamma + \theta) Y} \right] \phi_{2}^{(t_1)}$$

and finally returning to point $a$:

$$\psi_{\bar{a}} = e^{-i\Gamma} \left[ e^{i\theta} \left( 2(1 + \cos(2\nu)) + e^{2i\Gamma} \right) X + 2ie^{i(\nu - \theta)} \cos(\nu) Y \right] \phi_{1}^{(t_1)}$$

$$+ e^{-i\Gamma} \left[ 2ie^{i(\theta - \nu)} \cos \nu \left( 1 + e^{2i\Gamma} + 2 \cos(2\nu) \right) X - e^{-i\theta} \left( 1 + 2 \cos(2\nu) \right) Y \right] \phi_{2}^{(t_1)}$$

To find $R(\omega)$ we need to properly impose the boundary conditions on $X$ and $Y$, where $Y$ will be set to unity, then we identify the coefficient of $\phi_1$ in $\psi_{\bar{a}}$ as the same coefficient appearing in $\psi_{a}$ except that because of the trip around the contour we have gained a phase:

$$e^{-i\Gamma} \left[ e^{i\theta} \left( 2(1 + \cos(2\nu)) + e^{2i\Gamma} \right) R + 2ie^{i(\nu - \theta)} \cos(\nu) \right] = e^{i\theta} X e^{-i\Gamma}$$

Solving for $R$ gives us the desired result:

$$R(\omega) = -e^{i(\nu - 2\theta)} \frac{2i \cos \nu}{e^{i\omega} + 1 + 2 \cos(2\nu)}$$

To find the corresponding $T$, we notice that the boundary condition at the horizon, with $\phi_2$ dominating can be connected to point $d$, which was reached by moving along an anti-Stokes line along which $\phi_2$ is dominant when $\omega \to -i\infty$. Therefore, we can make the following identification:

$$\psi_{d} = \cdots + T e^{-i\theta} \phi_{2}^{(t_3)} e^{-i\Gamma}$$

comparing with the expression obtained in (64), we can solve for $T(\omega)$:

$$T(\omega) = \frac{e^{i\omega} - 1}{1 + 2 \cos(2\nu) + e^{i\omega}}$$

For the case of $-\omega$, we just flip the roles of $X$ and $Y$, so that $Y = R(-\omega)$ is now obtained as follows:

$$\psi_{\bar{a}} = \cdots + R(-\omega) e^{-i\theta} \phi_{2}^{(t_1)} e^{i\Gamma}$$

with $X \equiv 1$. By comparing with (68), we find:

$$R(-\omega) = 2ie^{i(2\theta - \nu)} \cos \nu$$

Likewise, to find $T(-\omega)$, we make the identification

$$\psi_{d} = T e^{-i\theta} \phi_{1}^{(t_3)} e^{-i\Gamma} + \cdots$$
and by comparing with the expression in (64) we find that:

\[ T(\omega) = 1 \]  

(76)

The grey-body factor that results from this reads:

\[ \gamma(\omega) = T(\omega)T(-\omega) = \frac{e^{\beta \omega} - 1}{1 + 2 \cos(2\nu) + e^{\beta \omega}} \]  

(77)

This result is consistent with the asymptotic quasinormal frequencies found by Cho [44] in 4D, and it also implies Neitzke’s results [50] for scalar perturbation around a Schwarzschild hole for dimensions \( D \geq 4 \), where the only modification due to extra dimensions appears through the Hawking temperature (cf. eq. (2)). Further, in the 6D model that includes tension, one finds that an additional modification appears (cf. eq. (12)).

Moving on to brane-localized emissions, we need to use eq. (32) as the radial master equation. This would describe scalar emissions \((s = 0)\), massless Dirac fermions \((s = \frac{1}{2})\), and gauge photons \((s = 1)\). To apply our method, we need to put that equation in the Schrödinger form, in which case the potential looks like this [38]:

\[ V_s(r) = \frac{f}{r^2} \left[ A_{js} + q_{ns}(1 - f) + \frac{s^2}{4f} \left\{ (n + 1)(1 - f) - 2f \right\}^2 + sf \right] + \frac{i\omega s}{r} \left\{ (n + 1)(1 - f) - 2f \right\} \]  

(78)

where

\[ q_{ns} = (2s + n + 1)(ns + s + 1) - (s/2)(n + 1)(n + 4) \]

and

\[ A_{js} = j(j + 1) - s(s + 1) \]

To also talk about the graviton localized to the brane, we have to use another potential (e.g [51]):

\[ V_G(r) = \frac{f}{r^2} \left[ j(j + 1) - \left\{ (n + 1)^2 + 2 \right\} (1 - f) \right] \]  

(79)

However, for our purpose, what we care about is the behavior of these potentials near \( r = 0 \). Keeping only the most singular term in both cases, we can combine the two potentials into one expression:

\[ V_{brane} \sim \sigma_n r_h^{2(1+n)} \frac{1}{r^{2(2+n)}} \]  

(80)
with
\[
\sigma_n = \begin{cases} 
\frac{1}{4}s^2(n-1)^2 - \frac{1}{2}s(n-1) - (n+1), & \text{for } s = 0, \frac{1}{2}, 1 \\
(n+1)^2 + 2, & \text{for } s = 2
\end{cases} \quad (81)
\]
To find the greybody factors in this case, we duplicate our steps for bulk emissions, and we find that the only difference occurs in the \( \nu \), which we can readily find by comparing the expressions (80) and (45):
\[
\nu_{\text{brane}} = -\pi \sqrt{\frac{\sigma_n}{(n+2)^2} + \frac{1}{4}} \quad (82)
\]
In this case, the value of the contour integral \( \Gamma_{\text{brane}} \) is found to be:
\[
\Gamma_{\text{brane}} = \oint Q \, dr = -i\pi \sqrt{\frac{s(n+1) + 2i\omega r_h}{n+1}^2} \approx -\frac{1}{2}i\pi\beta \quad (83)
\]
which is asymptotically the same as \( \Gamma_{\text{bulk}} \).

These results show that the grey-body factors in all cases are governed by (77) with differences coming through the value of \( \nu \). Tables I and II summarize this for bulk and brane-localized emissions, respectively.

**TABLE I: Expressions for \( \nu \) for different bulk perturbations**

| Perturbation               | \( \nu_{\text{bulk}} \)          |
|----------------------------|-----------------------------------|
| scalar & gravi-tensor      | 0                                 |
| gravi-vector               | \(-\pi\)                           |
| EM vector                  | \(-\frac{1}{n+2}\pi\)             |
| EM scalar                  | \(-\frac{1}{n+2}\pi\)             |
| gravi-scalar               | \(~ -\frac{\pi}{4} [2 + 0.674(n + 4)^{-0.5445}] \) |
| Dirac fermion              | \(-\frac{\pi}{2}\)                |

Since the asymptotic quasinormal frequencies are the poles of the grey-body factors, we see that the standard formula still holds in form,
\[
\frac{\omega}{T_H} = 2\pi i \left( \tilde{n} + \frac{1}{2} \right) + \ln (1 + 2 \cos 2\nu) \quad (84)
\]
where \( \tilde{n} \) is the mode number, however, \( \nu \) now depends on the number of extra dimensions (as already found in [51]).
TABLE II: Expressions for \( \nu \) for different brane-localized perturbations

| Spin \( \nu \) | \( \nu_{\text{brane}} \) |
|---------------|------------------|
| 0             | \(-\pi n \)
| 1             | \(-\pi \frac{\sqrt{1+n^2}}{n+2}\) |
| 2             | \(-\pi \frac{\sqrt{n(n-2)+2}}{2(n+2)}\) |
| 1             | \(-\pi \frac{\sqrt{(n+1)^2+2}}{(n+2)^2+1}\) |

As a special case, the addition of tension in an \( n = 2 \) spacetime, will only modify \( T_H \), where it can be seen from equations (12) and (2) that the Hawking temperature decreases with brane tension,

\[
T_H \rightarrow b^{1/3} T_H
\]  
(85)

Since this occurs in an exponent (eq. (77)), even a moderate value of \( b \) will lead to \( \gamma(\omega) \rightarrow 1 \) faster with increasing frequency.

V. CONCLUSION

With the coming online of the LHC, the possibility of experimentally testing many of the theories that go beyond the Standard Model is a realistic one. Detecting black holes in high-energy collisions, if observed, would, for the first time, offer the opportunity to probe the realm of quantum gravity helping us resolve some of the most important puzzles of 20th century physics. This means that we need to be well-prepared with realistic predictions to interpret the possible observations. In the study of the creation and evaporation of mini black-holes, a main prediction of brane-world models, brane tension has often been neglected. While predictions may not be radically modified by including it, the fact that we are trying to observe something we have never observed before makes it imperative to strive to be as realistic and accurate as possible. Building on earlier work, we made an attempt to address this issue.

We explored some of the implications of the codimension-2 model on grey-body factors for various types of bulk and brane emissions from a black-hole residing on a brane of finite tension. We saw that the results presented follow primarily from two modifications: (a) the enhancement of the horizon radius, which enhanced the emission spectra with increasing
tension, and (b) the increase of the angular eigenvalues with tension, which resulted in amplified absorption probability on the brane. We calculated analytically grey-body factors in the low frequency regime. We obtained expressions that had a non-trivial dependence on the brane tension with observable implications which we discussed. In particular, we saw that the observable emission rates got enhanced for both bulk and brane emission, yet we obtained a higher rate on the brane. It should be stressed that in the case of rotating black-holes our results are modified due to super-radiance.

We also obtained analytic results in the case of large imaginary frequency. We derived expressions for grey-body factor in the bulk and on the brane for a Schwarzschild black hole in an arbitrary number of dimensions. The special case of two extra dimensions with tension was seen to follow from a simple modification of the Hawking temperature.

We compared our analytic expressions in the two asymptotic regimes of low and high frequencies, respectively, with exact numerical solutions and discussed the range of intermediate frequencies where our analytic expressions are not accurate. The size of this intermediate range varies depending on the different parameters of the setup. Interestingly, this range is shifted toward high frequencies with increasing tension indicating a wider range of validity of our analytic low frequency expressions.

In conclusion, our results show that brane tension has a pronounced effect on predictions. Although the model we discussed is not the most general one, it offers the opportunity to explore both qualitatively and quantitatively the effects of adding tension. It would be interesting to go beyond the model considered here generalizing it to other geometries and an arbitrary number of extra dimensions. We hope to report on progress in this direction soon.

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FIG. 1: Low frequency absorption probability for bulk emissions as a function of $b$. Parameter $p \in \{0, \frac{1}{2}, \frac{3}{2}, 2, 2.25\}$ from eq. (14) increases from bottom to top. Here, $\ell_3 = 2$, $m = 1$, $\omega = 0.2$, and $\mu = 1$. 
FIG. 2: High frequency absorption probability for bulk emissions as a function of $b$. Parameter $p \in \{0, \frac{1}{2}, \frac{3}{2}, 2, 2.25\}$ from eq. (14) increases from bottom to top. Here, $\ell_3 = 2$, $m = 1$, $\omega = 2.2$, and $\mu = 1$. 
FIG. 3: Absorption probability for bulk scalars ($p = 0$) as a function of $\omega$ for various values of brane tension, where $b \in \{0.4, 0.6, 0.8, 1\}$ increasing from bottom to top, with $\ell_3 = 2, m = 1, \mu = 1$. The points on the graph come from exact numerical calculations.
FIG. 4: Bulk absorption probability for scalars for different values of $m$: for $\ell_3 = 5$, and $m \leq \ell_3$, $m$ increasing top to bottom, where $b = 0.3$, $\mu = 1$. 
FIG. 5: Absorption probability for brane-localized emissions as a function of $b$. The curves from top to bottom correspond to $s = \frac{1}{2}$, $s = 0$ and $s = 1$, respectively. Here, $\ell = 2$, $\omega = 0.02$, and $\mu = 1$. 
FIG. 6: Absorption probability for brane-localized scalars ($s = 0$) as a function of $\omega$ for various values of brane tension, where $b \in \{0.4, 0.6, 0.8, 1\}$ increasing from top to bottom, with $\ell = 2, m = 1, \mu = 1$. The points on the graph come from exact numerical calculations.
FIG. 7: Stokes lines structure for the Schwarzschild black hole in 6D in the complex $r$-plane for large imaginary frequency. Empty circles are the zeros of $Q$ and the filled circle is the horizon radius.
FIG. 8: Energy emission spectrum for scalars in the bulk for various values of brane tension, with $b \in \{0.4, 0.6, 0.8, 1\}$ increasing from top to bottom.
FIG. 9: Energy emission spectrum for scalars on the brane for various values of brane tension, with $b \in \{0.4, 0.6, 0.8, 1\}$ increasing from top to bottom.

FIG. 10: Relative bulk-to-brane emissivity for scalars for different values of brane tension, with $b \in \{0.4, 0.6, 0.8, 1\}$ increasing from top to bottom.
FIG. 11: Energy emission spectrum for gauge vectors in the bulk for various values of brane tension, with $b \in \{0.4, 0.6, 0.8, 1\}$ increasing from top to bottom.
FIG. 12: Energy emission spectrum for gauge vectors on the brane for various values of brane tension, with $b \in \{0.4, 0.6, 0.8, 1\}$ increasing from top to bottom.

FIG. 13: Relative bulk-to-brane emissivity for gauge vectors for different values of brane tension, with $b \in \{0.4, 0.6, 0.8, 1\}$ increasing from top to bottom.