Topology in full QCD with 2 colours at finite temperature and density

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We present preliminary results about topology and several other quantities at non-zero baryon density and finite temperature in full QCD with $N_c = 2$.

1. INTRODUCTION

The behaviour of the QCD vacuum at finite density can be tested in the heavy ion colliders. Therefore it is interesting to have a set of predictions from QCD and the lattice is a good means to obtain them in the non–perturbative regime.

We have analysed the model with two colours in order to make use of standard simulation algorithms. We expect that this simplification may leave at least some of the essential features of the problem.

We give a progress report of a simulation performed on a $14^3 \times 6$ lattice by using 8 flavours of staggered fermions at masses $a m = 0.05$ and $a m = 0.07$ at several values of $\beta \equiv 4/g_0^2$ where $g_0$ is the lattice bare coupling. The Monte Carlo algorithm is the standard hybrid molecular dynamics \[1\]. The action is \[2\]

\begin{align*}
S &= \frac{1}{2} \sum_n \eta_4(n) \left\{ e^{a \mu} \overline{\psi}(n) U_4(n) \psi(n + \hat{4}) \\
&- e^{-a \mu} \overline{\psi}(n + \hat{4}) U_4(n + \hat{4}) \psi(n) \right\} + \text{spatial terms,}
\end{align*}

where the “spatial terms” are those independent of the chemical potential $\mu$ and $\eta_4(n)$ are the staggered phases.

Our results are preliminary because \textit{i)} we have not yet calculated all the renormalization constants necessary for the evaluation of the physical topological susceptibility $\chi$ \[3\] and \textit{ii)} we have not calculated the physical units by determining the lattice spacing $a$.

2. PARTICLE DENSITY

In Fig. 1 we show the particle density $\rho$ as a function of the chemical potential $\mu$ for masses $a m = 0.05$ and $a m = 0.07$. In both cases we see a cubic dependence for large values of $\mu$ that is in agreement with the analytical expectation \[4\],

\[ \rho = \frac{8}{3} \left[ \mu T^2 + \frac{\mu^3}{\pi^2} \right]. \]

for $\mu \gg T$ where $T$ is the temperature.

3. TOPOLOGICAL SUSCEPTIBILITY

The topological susceptibility $\chi$ is an important parameter of the QCD vacuum. In the continuum and for the $SU(2)$ gauge group it is defined as

\[ \chi \equiv \int d^4 x \partial_\mu \langle 0 | T \{ K_\mu(x) Q(0) \} | 0 \rangle, \]

where $K_\mu(x)$ is the Chern current

\[ K_\mu(x) = \frac{g^2}{16 \pi^2} \epsilon_{\mu\nu\rho\sigma} A_\nu A_\rho A_\sigma^\alpha, \]

$g$ is the QCD coupling constant and

\[ Q(x) = g^2 \frac{g^2}{64 \pi^2} \epsilon^{\mu\nu\rho\sigma} F_\mu^\alpha(x) F_\rho^\alpha(x). \]
Eq. (3) uniquely defines the prescription for the singularity of the time ordered product when \( x \to 0 \).

On the lattice, the evaluation of \( \chi \) does not follow the above prescription. As a consequence we have to subtract the singularity which shows up as an additive renormalization. This subtraction is performed following the method of Ref. [3].

It is well-known that \( \chi \) undergoes a sharp drop at the deconfinement temperature, both for three colours [3] and for two colours [6]. We have measured this quantity as a function of the chemical potential in order to study its behaviour at finite density as well as finite temperature. In Fig. 2 we show that the behaviour is qualitatively analogous to the above-described one at finite temperature and zero density. The quantity \( Z \) in the vertical axis is the finite renormalization which relates the topological charge in the continuum and on the lattice [7]. We are presently running more statistics to sweep the sector beyond the critical density and to improve the error bars.

4. THE POLYAKOV LOOP

To investigate the deconfinement properties of the theory across the phase transition at finite chemical potential, we have measured the Polyakov loop \( \Pi \). Our results, shown in Fig. 3, indicate that the theory becomes deconfining beyond a critical density and suggest that the transition occurs at \( a\mu \sim 0.4 \) for fixed temperature. A similar trend can be observed by varying the temperature at zero chemical potential, Fig. 4, where we have traded the temperature \( T \) for the lattice coupling \( \beta \).

5. CHIRAL TRANSITION

The chiral symmetry is known to be restored at the critical temperature. This result calls for a study of the behaviour of this symmetry across the critical density. In Fig. 5 we show the chiral condensate \( \langle \bar{\psi} \psi \rangle \) as a function of the chemical potential. We see that there is a transition approximately at the same value of \( \mu \) at which the deconfinement took place. An analogous figure for \( \langle \bar{\psi} \psi \rangle \) is obtained at zero chemical potential as a function of \( T \).
6. PARTICLE SUSCEPTIBILITIES

The particle susceptibility $\chi_{\text{particle}}$ is defined as

$$\chi_{\text{particle}} \equiv \sum_{t} G(t),$$

where $G(t)$ is the propagator of such a particle at time $t$. If for example we consider a pion and a single pole dominates the propagator, then $\chi_\pi \sim Z_\pi/m_\pi^2$ where $Z_\pi$ is the pion field renormalization constant. Moreover, if $Z_{\text{particle}}$ is insensitive to the quantum properties of the particle, then $\chi_{\text{particle}}$ can provide useful information about spectroscopic level ordering [8]. Actually we shall make use of this result to compare the masses before and after the transition and we shall not need to be careful about the definition of $G(t=0)$.

In Fig. 6 the susceptibilities for pions and deltas are shown. After the transition the chirality does not matter and the masses of parity opposite particles coincide. This is another way to see that the chiral symmetry is restored. We see again that this restoration occurs at $a\mu \sim 0.4$ for $\beta = 1.5$. In Fig. 7 we see the analogous plot displaying the same phenomenon for zero density.

7. CONCLUDING REMARKS

We have presented several plots obtained from a numerical simulation of QCD with gauge group $SU(2)$ at finite temperature and density. Although our results are still preliminary, we can draw a qualitative picture of the phase diagram of the theory in the temperature–chemical potential plane (we actually substitute the temperature for the $\beta$ value, having fixed the temporal size of the lattice). There is a region where confinement, chiral symmetry breaking and a non–zero value for the topological susceptibility occurs. This region is bounded at $\mu = 0$ by $\beta \sim 1.6$ and at $T = 0$ by $a\mu \sim 0.4$ [8]. We have checked this picture from several measurements, all of them giving figures in agreement: for instance the position at $\beta = 1.5$ where the Polyakov loop raises from zero, the chiral condensate and the topological susceptibility vanish and the particle susceptibilities coalesce is always $a\mu \sim 0.4$.

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Figure 5. Chiral condensate as a function of the chemical potential at $\beta = 1.5$.

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Figure 6. Particle susceptibility as a function of the chemical potential at $\beta = 1.5$.

Figure 7. Particle susceptibility as a function of $\beta$ for $\mu = 0$. 