Tunable Aharonov-Anandan Phase in Transport Through Mesoscopic Hole Rings

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We present a theoretical study of spin-3/2 hole transport through mesoscopic rings, based on the spherical Luttinger model. The quasi-one-dimensional ring is created in a symmetric two-dimensional quantum well by a singular-oscillator potential for the radial in-plane coordinate. The quantum-interference contribution to the two-terminal ring conductance exhibits an energy-dependent Aharonov-Anandan phase, even though Rashba and Dresselhaus spin splittings are absent. Instead, confinement-induced heavy-hole–light-hole mixing is found to be the origin of this phase, which has ramifications for magneto-transport measurements in gated hole rings.

PACS numbers: 73.21.-b, 73.23.Ad, 85.35.Ds, 03.65.Vf

Introduction. Geometric quantum phases continue to be the subject of great interest because they rather elegantly elucidate quite complex fundamental microscopic properties. The well-known Aharonov-Bohm2 and Aharonov-Casher effects are pertinent examples, as is the Berry phase acquired by a quantum system during adiabatic cyclical variation of an external parameter. The more general concept of the Aharonov-Anandan phase arising in the cyclical evolution of a quantum state was later shown to subsume the above-mentioned three phenomena as special cases. Recent progress in our understanding of geometric phases has been spurred by numerous experimental and theoretical studies.

Modern nanofabrication techniques have made it possible to study quantum interference, and thus geometric quantum phases, in mesoscopic electronic devices. The first theoretical suggestions and experimental realizations of electronic quantum interference devices were Aharonov-Bohm interferometers. Subsequent theoretical studies predicted electronic signatures of Berry phases10 and the Aharonov-Casher effect11,12,13. Inspired by possible applications in the burgeoning field of spintronics,14 recent experimental efforts have been devoted to observing tunable spin-related geometric phases in semiconductor rings subject to strong spin-orbit coupling.15,16 In several of these experiments16 charge carriers moving through the ring structure are characterized by an intrinsic (spin) angular momentum equal to 3/2. This is due to the fact that states in the uppermost valence band of typical semiconductors originate from p-like atomic orbitals of the material’s constituent elements.17 The same is true for conduction-band states in HgCdTe quantum wells because of a band inversion. Here we provide a careful study of the complexities arising from the spin-3/2 character of charge carriers confined in a mesoscopic ring. We identify a nontrivial part of the Aharonov-Anandan (AA) phase that enters the interference contribution to the two-terminal ring conductance. The energy dependence of this geometric phase will result in a continuous shift of magneto-conductance (Aharonov-Bohm) oscillations as a function of carrier density in the ring, e.g., when a gate voltage is applied. We trace the origin of the anomalous AA phase to a confinement-induced coupling between heavy-hole (HH) and light-hole (LH) states that is ever-present and unrelated to (Rashba or Dresselhaus) spin splitting due to (structural or bulk) inversion asymmetry.18 Our analysis provides a framework for interpreting numerical results19 and complements previous analytical calculations20 where HH-LH mixing was neglected.

Below we describe our theoretical model for hole rings. Readers not interested in mathematical details could skip to the end of this part where results for the lowest ring subbands are presented. We then analyse the emerging energy-dependent AA phase and, in our concluding discussion, address implications for experiments.

Theoretical model for a mesoscopic hole ring. We use the Luttinger model20 in spherical approximation21 to describe electronic states in the uppermost bulk valence band. In atomic units where \( \hbar = m_0 = 1 \), it reads

\[
H_s = \left( \frac{\gamma_1}{2} + \frac{\gamma_s}{2} \left[ j_z^2 - \frac{5}{4} \right] \right) \hat{k}_\perp^2 - \frac{\gamma_s}{2} \left( \hat{k}_+^2 \hat{j}_+^2 + \hat{k}_-^2 \hat{j}_-^2 \right) + \left( \frac{\gamma_1}{2} - \gamma_s \left[ j_z^2 - \frac{5}{4} \right] \right) \hat{k}_z^2
- \sqrt{2} \gamma_s \left\{ \hat{k}_z, \hat{k}_- \right\} \left\{ \hat{j}_z, \hat{j}_- \right\} + \left\{ \hat{k}_z, \hat{k}_+ \right\} \left\{ \hat{j}_z, \hat{j}_+ \right\} \right) .
\]

where \( m_0 \) denotes the vacuum electron mass, \( \gamma_s = (3\gamma_3 + 2\gamma_2)/5 \) in terms of Luttinger parameters, \( \hat{k} \) and \( \hat{J} \) are vector oper-
Introducing a quantum-well confinement in the growth of the 2D quantum-well width in units of the effective ring width, and simply replace operators $\hat{k}_z^2$ and $\hat{k}_z$ by their respective expectation values $\pi^2/d^2$ and $0$. Using polar coordinates for in-plane motion, we obtain $\mathcal{H}_{\perp}^{(2D)} = \mathcal{H}_{sb} + \mathcal{H}_{\perp}^{(2D)}$, where

$$\mathcal{H}_{\perp}^{(2D)} = \left(\frac{\gamma_1}{2} - \gamma_s \left[\hat{j}_z^2 - \frac{5}{4}\right]\right) \left(\frac{\pi}{d}\right)^2$$

(2a)

arises from the quantised motion in $z$ direction, and the in-plane motion of holes is governed by the part

$$\mathcal{H}_{sb} = \left(\frac{\gamma_1}{2} - \gamma_s \left[\hat{j}_z^2 - \frac{5}{4}\right]\right) \left(\frac{\pi}{d}\right)^2$$

(2b)

The quasi-one-dimensional in-plane ring confinement is modeled by the singular-oscillator potential^23

$$V_{\perp}(r) = \frac{\omega^2}{2} \left(r - \frac{R}{r}\right)^2$$

(3)

which was employed before to study mesoscopic electron rings^24. In the following, the energy scale $E_0 = \pi^2\hbar^2/(2m_0d^2)$ associated with the 2D quantum-well confinement will serve as our energy unit. We also introduce the parameter $\bar{\gamma} = \gamma_s/\gamma_1$ that measures the strength of spin-orbit coupling in the valence band, the length scale $\ell_\omega = \sqrt{\gamma_1\hbar/(m_0\omega)}$ associated with the ring confinement, $\lambda_R = (R/\ell_\omega)^2$ and $\lambda_d = (2d/|\pi\omega|)^2$ representing ring radius and quantum-well width in units of the effective ring width, and the operators $\hat{\Gamma} = 1 + \bar{\gamma} \left[\hat{j}_z^2 - (5/4)\right]$ and $\hat{\vartheta} = r/(\ell_\omega \hat{\Gamma}^{1/4})$ that quantify a HH-LH splitting. With these conventions, the Hamiltonian of a mesoscopic hole ring is $H \equiv (\mathcal{H}_{sb}^{(2D)} + V_{\perp})/E_0 = H_{qw} + H_{ig}$, where

$$H_{qw} = 1 - 2\bar{\gamma} \left[\hat{j}_z^2 - \frac{5}{4}\right]$$

(4a)

arises from the HH-LH splitting in the quantum-well bound state, and the in-plane motion is governed by

$$H_{ig} = \frac{\lambda_d}{4} \hat{\Gamma}^{1/4} \left(-\partial_\vartheta^2 - \frac{\partial_\vartheta}{\vartheta} + \frac{\hat{L}_z^2}{\vartheta^2} + \left[\hat{\vartheta} - \lambda_R \hat{\Gamma}^{-1/4}\right]^2\right) + \frac{\lambda_d}{4} \bar{\gamma} \left(\hat{j}_+ \hat{\Gamma}^{-1/4} \left[\partial_\vartheta + \frac{\hat{L}_z}{\vartheta}\right]\right)^2 + \left(\hat{j}_- \hat{\Gamma}^{-1/4} \left[\partial_\vartheta - \frac{\hat{L}_z}{\vartheta}\right]\right)^2$$

(4b)

Equation (4b) suggests the wave-function ansatz

$$\psi_{n,m}(r, \varphi) = e^{mn\varphi} \begin{pmatrix} a_{n,m} \psi_{n,m}^{(3/2)}(r) \\ b_{n,m} \psi_{n,m}^{(1/2)}(r) \\ c_{n,m} \psi_{n,m}^{(-1/2)}(r) \\ d_{n,m} \psi_{n,m}^{(-3/2)}(r) \end{pmatrix}$$

(5a)

with the four spinor amplitudes given by

$$\psi_{n,m}^{(j)}(r) = \frac{\mathcal{L}_n^{(j)}}{\ell_\omega [1 + 2(|j| - 1)\bar{\gamma}])^{1/4}} e^{-\frac{r^2}{2}} \theta_j \mathcal{L}_n^{(j)}(\theta_j)$$

(5b)

$L_n^{(j)}(x)$ is an associated Laguerre polynomial, $\mathcal{L}_n^{(j)} = \sqrt{2\Gamma(n + 1)/\Gamma(n + \alpha_m^{(j)} + 1)}$, with $\Gamma(x)$ denoting the Euler Gamma function, $\theta_j = r/(\ell_\omega [1 + 2(|j| - 1)\bar{\gamma}])^{1/4}$, and $\alpha_m^{(j)} = \sqrt{(m - j)^2 + \lambda_R/(1 + 2(|j| - 1)\bar{\gamma})}$. $m$ is the eigenvalue of $\hat{M}_z$ and $n$ the oscillator-level index. Ansatz (5) diag-
onalises the first term of the Hamiltonian \((4b)\),
\[
H_{ig}^{(1)} = \lambda_d \left( n + \frac{1}{2} \right) \hat{\Gamma}^2 + \sqrt{\Gamma (m - \hat{J}_z)^2 + \lambda_R^2 - \lambda_R}.
\]
(6)

However, the second term of \(H_{ig}\) in Eq. \((4b)\) is off-diagonal in spin-3/2 space, coupling HH and LH amplitudes within subspaces spanned by \(\hat{J}_z\) eigenstates with eigenvalues \(\{\pm 3/2, \mp 1/2\}\), respectively. In addition, this term has both diagonal and off-diagonal matrix elements in the \(\psi_{n,m}\) representation, i.e., it couples states with different \(n\). We omit the lengthy analytical expressions for associated matrix elements.

In Fig. 1 for a set of realistic parameters. For comparison, we also plot dispersions obtained when HH-LH mixing is neglected but HH-LH splitting taken into account, i.e., when only \(H_{qw} + H_{ig}^{(1)}\) is considered.

**Two-terminal transport and AA phase.** At fixed energy \(E\) between the HH-like and LH-like subband bottoms shown in Fig. 1 four propagating channels exist, having different angular momenta \(m_{s\sigma}\). Here \(s = +, -\) labels the two dispersion curves \((s = \pm\) corresponding to the subspace spanned by \(\hat{J}_z\)-projection eigenstates with eigenvalues \(\{\pm 3/2, \mp 1/2\}\), respectively), and \(\sigma = +, -\) distinguishes opposite propagation directions. The condition for finding these angular momenta is \(E_{\hat{s}}(m_{s\sigma}) = E\). We now consider the following scenario, illustrated in Fig. 2. A lead attached at \(\varphi = 0\) is assumed to inject holes in a set of orthogonal initial states \(|\nu\rangle\). Here the ket refers to a normalised spin-3/2 spinor without any dependence on spatial coordinates. Ring eigenstates taken at \(\varphi = 0\) and with spatial profile neglected are given by \(\chi_{s\sigma} = (a_{0,m_{s\sigma}}, b_{0,m_{s\sigma}}, c_{0,m_{s\sigma}}, d_{0,m_{s\sigma}})\), and the injected states can be written as a superposition \(|\nu\rangle = \sum_{s\sigma} \xi_{s\sigma}^{(\nu)} \chi_{s\sigma}\).

Each spinor amplitude of a ring eigenstate with quantum numbers \(s, \sigma\) acquires a phase during propagation around a half-ring that is determined by its associated eigenvalue of the operator \(\sigma \pi (\hat{M}_z - \hat{J}_z)\). The resulting state at \(\varphi = \pi\) will be \(|\mathrm{out}\rangle = \sum_{s\sigma} \xi_{s\sigma}^{(\nu)} \chi_{s\sigma} \exp i \sigma \pi (m_{s\sigma} - \frac{3}{2})\). Assuming another lead being attached at \(\varphi = \pi\), we find the transmission probabilities from incoming-leading channel \(\nu\) to outgoing-leading channel \(\mu\). Including the effect of a finite magnetic flux \(\phi\) threading the area bounded by the ring, it reads
\[
T^\mu_\nu = \left| \sum_{s\sigma} \xi_{s\sigma}^{(\nu)} \xi_{s\sigma}^{(\mu)} \exp i \sigma \pi (m_{s\sigma} + \frac{\phi}{\pi} - \frac{3}{2}) \right|^2.
\]
(8)

In Eq. \((8)\), \(m_{s\sigma} = -m_{-s,-\sigma}\) are the angular-momentum eigenvalues found for \(\phi = 0\). The linear two-terminal ring conductance is given by \(G^\mu_\nu = \frac{e^2}{h} \sum_\nu T^\mu_\nu\).

Inspection of Eq. \((8)\) reveals the well-known signatures of the Aharonov-Bohm effect arising from the interference of counter-propagating modes with conserved quantum number \(s\). A phase difference \(\Phi^{(s)}_{\mathrm{AA}}\) of associated quantum amplitudes will be accumulated during propagation between \(\varphi = 0\) and \(\pi\) that is essentially an Aharonov-Anandan phase for holes confined in the ring. The latter can be written as the sum of a magnetic-flux-dependent part (the Aharonov-Bohm phase

![FIG. 1: Lowest hole-ring subbands (solid curves) arising from the in-plane ring-potential bound-state level with \(n = 0\), calculated for \(\tilde{\gamma} = 0.37, \lambda_d = 0.5,\) and \(\lambda_R = 10\). The dashed curves are obtained when HH-LH splitting is included but HH-LH mixing is neglected.](image1)

![FIG. 2: (Color online) Two-terminal transport. Holes in scattering states \(|\nu\rangle\) are injected via a lead attached at \(\varphi = 0\). Eigenmodes with angular momenta \(m_{s\sigma}\) provide propagation channels in the ring. Upon reaching the point \(\varphi = \pi\), interference and coupling into outgoing scattering states \(|\mu\rangle\) occurs.](image2)
2\pi(\phi/\phi_0) and a remainder \( \Phi_G^{(s)} \) that depends on the quantum number \( s \) distinguishing dispersion branches:

\[
\Phi_{AA} = 2\pi \frac{\phi}{\phi_0} + \Phi_G^{(s)}, \tag{9a}
\]

\[
\Phi_G^{(s)} = \pi (m_{s+} + m_{s-} - 3s). \tag{9b}
\]

The symmetry \( m_{s\sigma} = -m_{-s,-\sigma} \) implies \( \Phi_G^{(+)} = -\Phi_G^{(-)} \equiv \Phi_G \). We plot \( \Phi_G \) for a realistic set of parameters in Fig. 3.

**Conclusions and discussion.** We investigated spin-3/2 hole states confined in a quasi-one-dimensional ring. A number of controlled approximations were employed that can be systematically improved upon. We find a previously neglected energy-dependent contribution \( \Phi_G \) to the AA phase that results from HH-LH mixing and may be related to anomalous spin precession of spin-3/2 particles. It is likely that this phase is the origin of numerically observed magneto-oscillations of the conductance polarization in multiply connected hole nanostructures that persist even in the presence of relatively strong disorder. In a magneto-conductance experiment, the value \( \Phi_G(E_F) \) of AA phase for states at the Fermi energy would be observed as a shift in the Aharonov-Bohm oscillations. This mimicks behaviour expected in systems with a finite zero-field (Rashba or Dresselhaus) spin splitting, which was used to interpret experimental data. We show that, in general, both HH-LH mixing and spin-splitting effects need to be considered. In addition, the coupling of the ring to external leads needs to be well-understood, because the character of injected hole states will depend sensitively on the lead confinement.

**Acknowledgment.** UZ thanks P. Brusheim and D. Csontos for useful discussions.

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