Gravitation Induced Monopole Radiation

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To understand how information is carried away by hawking particles from no-hair blackholes is a fundamental question in modern gravitation and quantum physics research. We show in this work behind the blackhole information paradox is a new and universal radiation mechanism, Gravitation Induced Monopole Radiation, or GIMR hereafter. This mechanism happens to all kinds of compositional objects and requires only their microscopic structure as basis. Its happening always accompanies with such inner structures’ changing and allows for explicitly hermitian hamiltonian description. In blackhole case, whose highly degenerating microstates or inner structures are embodied in the Bekenstein-Hawking entropy, by Wigner-Wiesskopf approximation we show that such a radiation has thermal spectrum exactly the same as hawking radiation; while through numeric integration, we show that variation trends of the radiation particles’ entropy exhibit all features of Page curve as expected. We also provide exact and analytic solutions to the Einstein equation describing microscopic structures required by GIMR of blackholes and show that, after quantization the degeneracy of wave functions corresponding with those solutions are consistent with the area law formula of Bekenstein-Hawking entropy.

Among his many works, discovering of blackhole radiation may be S. Hawking’s most profound contribution to modern theoretic physics [1]. Historically, calculations of the gray-body factor of black-brane radiation provide the most hard-core evidence for Anti-de Sitter/Conformal Field Theory correspondence [2]. More recently, explorations of the information missing puzzle involved in this radiation produced new ideas such as ER=EPR [3] and Island formula et al [4]. This work will, additionally, show that underlying this radiation is a universal mechanism rarely investigated in the literature and encodes rich information on blackhole microscopic structure, singularity theorem and quantum gravity. To understand this statement better, we need go back to S. Mathur’s small correction theorem [5] for a while. This theorem says that as long as Hawking radiation happens through pair production and escaping mechanism around the no-hair horizon, then no matter how ingeniously small corrections are added to the evolution of a blackhole, the entropy of its hawking particles will increase monotonically till the blackhole evaporates away. Basing on this theorem, it’s very natural to conclude that some type of new radiation mechanism is the only way to solve the information missing puzzle [6-8]. We will call this mechanism GIMR, as dubbed in the abstract of this work, see FIG. 1, for intuitions. In principle, GIMR is a universal radiation mechanism happens to all compositional objects instead of blackholes exclusively. However, in this work blackholes will be taken as proxies for all such objects and we will talk about three aspects of this mechanism, its explicitly hermitian hamiltonian description, its resolution to the information missing puzzle and its implication for blackhole inside physics and quantum gravitation theory. More technique details can be found in our previous long paper [9].

Explicitly Hermitian Hamiltonian

Different from the usual atom radiation which happens through electro-dipole coupling between the atom and photons, GIMR happens through mass-energy monopole coupling between the blackhole and to-be radiated particles.

| \( H = H_{BH} + H_{vac} + H_{int} \) |
| \( = \left( \begin{array}{c}
  w^i \\
  w^j \\
  \vdots \\
  w^q
\end{array} \right) + \sum_q k \omega_q a_q^\dagger a_q + \sum_{u^{\ldots}=\ldots} g_{u^{\ldots}=\ldots} b_{u^{\ldots}=\ldots}^\dagger a_q \) |

The logic of analogue behind this mass-energy monopole coupling and electric dipole coupling is \( M \cdot \text{Siml}\{\cdots\sim q \cdot \ell, \frac{1}{2}(M_u+M_v)\} \) plays the role of one sign charge amount while \( \text{Siml}\{\Psi[M_u(r)], \Psi[M_v(r)]\} \) plays the role of discrepancy between negative and positive charge centers. In this explicitly hermitian hamiltonian, \( w, w_\perp=w_{-1} \) et
al denote degeneracy or eigenenergies of the blackhole regarding contexts; \( i, j \) et al distinguish microstates of equal mass blackholes, i.e., \( i = 1, 2, \ldots, w, w = \exp[\frac{4\pi G N c}{\hbar}] \); \( j = 1, 2, \ldots, w_+, w_- = \exp[\frac{4\pi G N c}{\hbar}] \); the symbol \( \phi^j \) represents quantum state corresponding with the totally evaporated blackhole; \( \omega_0^j \) & \( \omega_0 \) are operators describing the vacuum fluctuation, \( b_{u,\nu}^j \) & \( a_\nu \) take responses for the blackhole energy level’s lowering or rising and the vacuum fluctuation mode \( \omega_0^j \)'s realization or inverse.

Focusing on spherically symmetric radiations only, so hawking particles’ spatial-moment can be ignored to examine exclusively, the basis of Hilbert space for an evaporating blackhole and corresponding radiations can be written as

\[
\{ u^i \otimes \phi, w^i \otimes \omega_0^1, w^i \otimes \omega_0^2, \ldots, u^n \otimes \omega_0^{p1} \cdots \omega_0^{pq} \}
\]

\[
\omega_0^{q \cdot} \cdots \omega_0^{p1} \otimes \omega_0 \otimes \omega_0^1 \otimes \omega_0^2 \cdots, \ldots
\]

\[
(4)
\]

On this basis, quantum state of a blackhole and its radiation at arbitrary middle epoch can be written as

\[
|\psi(t)\rangle = \sum_{u=0}^{\infty} \sum_{v=0}^{\infty} e^{-i\omega t-i\nu t} \langle \nu | \phi \rangle (u^i \otimes \|z\rangle)
\]

where \( \|z\rangle \equiv \{ \omega^i \cdots \omega^q \} \) is an abbreviation of the radiation particles’ quantum state, with the index \( i, j \) et al继承ating from the parent body \( w^i \), \( w^j \) and \( w^k \) et al corresponding and the total energy given by \( \omega = \cdots + p + q = w - u \). Evolutions of this wave function are determined by the standard Schroedinger equation as

\[
\begin{align*}
\hat{L}_\mu c_\mu (t) &= \sum_{u \neq u, l=1}^{v} \sum_{v} g_{u,\nu}^c c_{\nu}^\mu (t)
\end{align*}
\]

where \( \hbar \) has been set to 1 and \( \|z\rangle \) differs from \( |z\rangle \) only by the last emitted or absorbed particles. Without loss of generality, we will set

\[
\begin{align*}
\Phi^q_\nu (0) &= 1, \quad c_{u,\nu}^1 (0) = 0, \quad c_{\nu}^\mu (0) = 0
\end{align*}
\]

That is, we let our blackhole lie on eigenstate \( w^i \) at initial time \( t = 0 \).

For the first one or few particles’ radiation, we can set all \( c_{\nu}^\mu (0) = 0 \) and focus on the evolution of \( c_{u}^\nu (t) \) and \( c_{\nu}^\mu (t) \). In this case, by the standard Wignier-Wiesskopf approximation [9][10], we can easily prove that

\[
\begin{align*}
&c_{u}^\nu (t) = e^{-\Gamma t}, \quad c_{\nu}^\mu (t) = \frac{iG u w^i}{\Gamma} (e^{-\Gamma t} - 1), \quad c_{\nu}^\mu (0) = 0
\end{align*}
\]

\[
\begin{align*}
\sum_{n} g_{n}^{w,\nu,\omega} \approx \sum_{n} g_{n}^{w,\nu,\omega} (w - u - \mu) \approx e^{-\frac{8\pi G N c}{\hbar}} (9)
\end{align*}
\]

where \( \Gamma \equiv \sum_{w} \sum_{\nu} \frac{|g_{w,\nu}^2|^2}{w} \). The first step in [9] uses the fact that \( g_{w,\nu}^2 \) is approximately equal for most radiation channels. Setting \( 8\pi G N c \approx (k_B T)^{-1} \), the energy of radiated particles reads

\[
\begin{align*}
\langle E \rangle_{\nu}^{\omega} &= \sum_{\omega, n} \omega \langle c_{\nu}^\mu |^2 = \sum_{k} \frac{k_B \omega - \frac{k_B T}{2k_B T} \cdot k - \frac{k_B T}{2k_B T}}{e^{k_B T} + \cdots - 1} = \frac{\omega}{e^{k_B T} + \cdots + 1}
\end{align*}
\]

The second step assumes that the radiated particle is quantized so that \( \omega = k_B w \) with \( k = 0, 1, 2 \cdots \) for bosons. Eq[10] implies that the power spectrum of GIMR of blackholes is completely the same as Hawking radiation. It should be emphasized that GIMR happens to all kinds of compositional objects. Black hole is just such an example and the thermal spectrum is only a short-time behavior due to the exponential degeneracy of microscopic states.

While for the long term behavior of the blackhole and its GIMR, we only need to integrate equations [3][4][7] numerically. We provide in FIG.2 results of this integration explicitly. From the figure, we easily see that variations of the radiation particles’ entropy have indeed first increasing then decreasing feature, just as Page curve exhibits for unitarily evolving blackholes. New features in GIMR are, the trends have late time non-monotonic behavior. This is because in its Hamiltonian description [2], the \( u > v \) radiation and \( u < v \) absorption terms are equally allowed. This means that earlier radiated particles have always non-zero probability to come back and cause \( c_{\nu}^\mu (t) \)’s Rabi oscillation. However, as \( w \rightarrow \infty \), this late time oscillatory behavior will become more and more unimportant.

**Information missing puzzle** In GIMR and its quantum mechanical description, causes of potential information missing effects are almost transparent. Firstly, the Wigner-Wiesskopf approximation, also called Markovian approximation includes forgetting history effects. Forgetting history implies information missing very naturally. Technically [9], this happens in obtaining [10] when shifting \( c_{\nu}^\mu \)'s history out of the integration symbol, in which \( c_{\nu}^\mu (t) \approx -i \int_{0}^{t} g_{w,\nu}^u c_{\nu}^\mu (t') dt' \) (11)
This introduces non-hermiticity to the process and is the direct cause of thermal spectrum. Secondly or more importantly, in Hawking’s arguments, a blackhole’s radiation evolution is considered a sequence of events like

\[
|\psi_0\rangle \rightarrow \left| \psi_{b}\right\rangle + c_s^1 |b_{h}\rangle + c_m^1 |b_{m}\rangle + c_s^2 |b_{r}\rangle + \cdots (13)
\]

While in the true quantum world, at any given time, the blackhole size is not specifiable and the system can only be considered superposition of blackhole/radiation(BR)-configurations of different mass-ratio,

\[
|\psi_0\rangle \rightarrow \left| \psi_{b}\right\rangle + \cdots \left| \psi_{r}\right\rangle \Rightarrow \left| \psi_{t}\right\rangle (14)
\]

where PC-diagram with different length of zigzag part denotes different size blackholes and their corresponding radiations. Considerations \[13\] ignore interferences between BR-configurations of different mass-ratio, thus introducing entropies to the system artificially.

Mathematically, the process \[14\] can be written as

\[
|\psi_0\rangle \rightarrow |\psi_t\rangle \rightarrow |\psi_{b}\rangle + c_s^1 |b_{h}\rangle + c_m^1 |b_{m}\rangle + c_s^2 |b_{r}\rangle + \cdots (15)
\]

where \(|\psi_0\rangle\) and \(|\psi_t\rangle\) are the radiation-before, evaporation-after blackhole state respectively, \(|b_{h}\rangle\), \(|b_{m}\rangle\), \(|b_{r}\rangle\) are three typical but non-exhaustive intermediate state of big, median, small blackholes with their radiation. Tracing out microstates of the blackhole, we can write

\[
|\psi_t^i\rangle \sim \sqrt{\rho^b + c_s^i \sqrt{\rho^b} + c_m^i \sqrt{\rho^m} + c_s^i \sqrt{\rho^s}} (16)
\]

So entropies of the radiation product can be calculated routinely \(s = tr_{r\ldots} \langle \psi_t^i | \psi_t^i \rangle \ln \langle \psi_t^i | \psi_t^i \rangle\), with result

\[
s(t) = |c_s^j|^2 s_{b_{h}} + |c_m^j|^2 s_{b_{m}} + |c_s^j|^2 s_{b_{r}} \Rightarrow \Rightarrow \Rightarrow (17)
\]

where \(s_{b_{h}}\), \(s_{b_{m}}\), \(s_{b_{r}}\) denote entanglement entropies of the intermediate state blackholes and their radiation. At early times, \(c_s^j \gg c_m^j \& c_s^j\), so \(s(t)\) is dominated by \(s_{b_{h}}\) and increases with time. As time passes by, \(s(t)\) will be dominated by \(s_{b_{r}}\) and reach maximum on Page epoch, then decrease due to dominations of \(s_{b_{m}}\), and then Rabbi oscillates, as FIG.2 displays.

From the viewpoint of GIMR, the so called island paradox would not be a paradox. Their resolution, answers to such questions are clear, the horizon size v.s. time relation of a blackhole is measurable and encodes all information the blackhole carries initially, see FIG.2 for concrete examples.

But viewing intermediate state as different mass semiclassic blackhole and their corresponding hawking particles, one implicitly measures state of the cat-group

\[
|\bullet \bullet \rangle \rightarrow \left| \bullet \ldots \right\rangle \Rightarrow \left| \bullet \bullet \right\rangle (19)
\]

Each of this measurement gives up knowledge on the quantum state of hawking particles, especially the time interval between two successive cats’ death, which is related with the blackhole inner structures’ changing before and after the hawking particles’ emission. This changes the beginning pure state to ending mixed one or beginning mixed state to ending more highly-mixed ones and produce entropy

\[
\Delta S = (A_i - A_{i+1}) - 0 (21)
\]

where 0 denote the entropy of initial pure state, \(A_i\) and \(A_{i+1}\) are horizon area of blackholes on \(i\) and \(i+1\) measurement, their difference quantize entropy of hawking particles emitted between this two measurement. The positivity of \(\Delta S\) will cause the entropy of hawking particles increase monotonically.

**Inside Blackholes** The idea of GIMR dates back to 1970s \[13\] during which Mukhanov and Bekenstein speculated atomic physics like interpretation for hawking radiation. However, due to their simple quantization of blackhole masses basing on adiabatic invariance of the horizon area, MB derived out discrete line shape spectrum for the radiation, which contradicts Hawking’s
thermal spectrum obviously. However, just as we show in [6][17][18], microstates of blackholes can be quantized in such a way that mass spectrum of them is continuous. So contradictions plaguing MB are avoidable. To see this directly, we note that blackhole solutions to the Einstein equation with nontrivial inside horizon structure can be worked out exactly,

\[
d s^2_\text{in} = -d\tau^2 + \frac{[1 - \frac{(2GM)}{\rho^3}]^2}{\rho^2} \frac{d\rho^2}{\rho^2} + a[\tau, \rho] \rho^2 d\Omega^2_2
\]

with \( r_h = \frac{3}{2} \rho \text{max} \) and \( 2GM \text{tot} \). This means that our inside horizon metric (22) satisfies requirements of the no-hair theorem. By shifting the white hole region to the future of blackhole region and plotting the east and west semi-sphere separately, we revise the usual Penrose Carter diagram and plot our blackhole with OCO matter cores in FIG. [5]. This revision of Penrose-Carter diagram contain ideas similar with 't Hooft's antipodal identification [19]. From the figure, we easily see that our solutions respect the singularity theorem very well, i.e. all matters consisting of or falling into the blackhole will reach the singularity in finite proper time [20][22].

Quantization of the OCO structure (22)-(25) can be done canonically. Looking the core as a direct sum of many concentric shells, each moves freely under gravitational effects due to itself and more inside partners.

\[
\{ \begin{align*}
    h_i j^2 - h_i ^{-1} j^2 &= 1, \quad h_i = 1 - \frac{2GM_i}{r} \\
    \Gamma^i_{\tau \rho} d\tau^i + 0 &= \Rightarrow h_i = \gamma_i = \text{const} \\
    \Rightarrow i^2 - \gamma_i^2 + h_i = 0, \quad \gamma_i^2 &\leq 0
\end{align*} \]

where \( h_i \) is the function appearing in the effective geometry felt by the i-th shell, \( ds^2 = -h_i dt^2 + h_i^{-1} dr^2 + r^2 d\Omega^2_2 \), \( \Gamma^i_{\tau \rho} \) is the corresponding Christoffel symbol. For each i, we quantize equation (28) by looking it as an operatorised hamiltonian constraint and introduce a wave function \( \psi_i(r) \) to denote probability amplitude the shell be found of r-size, so that

\[
\left[-\frac{\hbar^2}{2m_i} \frac{\partial^2}{\partial x^2} - \frac{GM_i}{r} \frac{\gamma_i^2 - 1}{2} m_i \right] \psi_i(r) = 0, \quad 0 \leq r < \infty
\]

where \( M_i \) is the mass of i-th shell and its inner partners, \( m_i \) that of the i-th shell only. Square integrability of \( \psi_i \) requires that

\[
\psi_i = N_i e^{-x} x L_{n_i-1}^{\gamma_i} (2x), \quad x = mr(1-\gamma_i^2)^{1/2}/h
\]

\[
n_i = \frac{GM_i m_i}{h(1-\gamma_i^2)^{1/2}} = 0, \quad 1, 2, \cdots, \gamma_i^2 \leq 0
\]

where \( L_{n_i-1}^{\gamma_i}(2x) \) is the associated Lagurre polynomial and \( N_i \) its normalization in standard mathematics. We then direct-product all \( \psi_i \)'s to get wave functionals of the matter core as follows

\[
\Psi[M(r)] = \psi_0 \otimes \psi_1 \otimes \psi_2 \cdots \sum_i m_i = M_\text{tot}
\]

We will call this OCO fuzzy ball, which is pictorially similar with but mathematically more exact fuzzy ball than string theory ones. Quantization equality (31) and sum rule in (32) do not require discreteness of the black

![Diagram](Image)

**FIG. 3:** Left, in a Schwarzschild blackhole with OCO matter cores, white hole lies on the top of blackhole. Points on the diagram map to 3+1 spacetime through \((\tau, \rho^3) \otimes S^3 \otimes S^3 \otimes S^3 \rightarrow R^{3+3} \), \( S^3 \) and \( S^3 \) meaning east and west semi-sphere respectively. The five colored curves display five instantaneous of a matter shell's motion. Volume elements on the outmost shell moves along traces like A-B-C-D-E-F-G-H-A... C is the antipodal of B, E is identified with D, and et al. Right, the number of ways matters oscillating inside the horizon a blackhole is consistent with the area law formula of Bekenstein-Hawking entropy.

Outside the matter occupation region our metric joins to the usual Schwarzschild blackhole in Lemaitre coordi-
hole total mass spectrum. However, they indeed form complete constraints on how matter core of a blackhole can be considered big number of concentric shells and from what initial radial position each shell is released to freely OCO inside the horizon, i.e. degeneracies of the wave function\([32]\). We provide in FIG.3 evidences that, the number of such degeneracies is consistent with the area law formula of Bekenstein-Hawking entropy, \(\log[\#\text{degen}] \approx \frac{M_\text{tot}^2}{2} \propto \text{Area}\), see \([6, 7, 9-11, 18]\) for more details.

Obviously to allow blackhole have inner structure thus radiate as usual atoms, their central singularity in conventional cognition has to be replaced with their matter contents’ OCO motion. Just as we pointed previously, this does not violate the singularity theorem. Instead, the classic OCO picture extends notations of the singularity theorem; while the quantum OCO yields proper area law formula for Bekenstein-Hawking entropy, see FIG.3 and notations there. In this sense, we claim it a discovered structure instead of introduced picture. We expect it to appear also in a full quantum gravitation description on blackholes. It allow us understand blackhole physics the same way we do with other objects, instead of customly making as was done in island formulas \([4]\).

**Conclusion** We show in this work GIMR is a fundamental mechanism for hawking radiation and a simple resolution to the blackhole information paradox. We provide explicitly hermitian hamiltonian description for this mechanism and microscopic structure of blackholes supporting it which we call OCO fuzzy balls. The hamiltonian description indicates that hawking particles carry information away from the blackhole through changing its inner structure hence evaporation progression. In viewing evaporating blackholes as time-dependent semiclassical objects, conventional arguments ignore quantum interferences between BR-configurations of different mass-ratio thus introduce entropies to the system artificially and cause information missing puzzle. OCO fuzzy ball has similar physic picture with string theory fuzzy balls but exactly spelling-outable wave function description and countable degeneracy consistent with the Bekenstein-Hawking entropy formula. Both GIMR and OCO fuzzy ball resort no physics beyond the standard general relativity and quantum mechanics. This means that, potentials of the canonical approach to quantum gravitation is worth digging up further while new ideas such as island rules on the basic feature of quantum gravitation need be examined more deeply.

As discussion, we first note that GIMR is a universal radiation which can happen to all kinds of compositional objects. So it is very interesting to explore feature or find evidences of such a radiation in, e.g. the direct S-wave to S-wave transition of usual atoms. Secondly, tidal effects of the OCO structure of blackholes are also observable from gravitational waves following from such bodies’ binary merging \([23, 24]\). Thirdly, the OCO fuzzy ball interpretation of BH entropy is generalizable to higher dimensions and has definite predictions \([7, 9]\) for a real-number partition question defined by constraints like \([31]\) and \([32]\). So, to accept that behind the information missing puzzle happened GIMR is a big deal of small capital. Otherwise we need to answer, how to distinguish particles found around blackholes are due to Hawking Radiation or GIMR? If we refute GIMR from the beginning, anther question happens, why gravitation couplings between blackholes and those to-be radiated particles do not cause radiation while electromagnetic couplings between atoms and photons do?

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