The Redshift Evolution of the Tully-Fisher Relation as a Test of Modified Gravity

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ABSTRACT

The redshift evolution of the Tully-Fisher Relation probes gravitational dynamics that must be consistent with any modified gravity theory seeking to explain the galactic rotation curves without the need for dark matter. Within the context of non-relativistic Modified Newtonian Dynamics (MOND), the characteristic acceleration scale of the theory appears to be related to the current value of either the Hubble constant, i.e., $\alpha \simeq cH_0$, or the dark energy density, i.e., $\alpha \simeq (8\pi G\rho_\Lambda/3)^{1/2}$. If these relations are the manifestation of a fundamental coupling of $a_0$ to either of the two cosmological parameters, the cosmological evolution would then dictate a particular dependence of the MOND acceleration scale with redshift that can be tested with Tully-Fisher relations of high-redshift galaxies. We compare this prediction to two sets of Tully-Fisher data with redshifts up to $z = 1.2$. We find that both couplings are excluded within the formal uncertainties. However, when we take into account the potential systematic uncertainties in the data, we find that they marginally favor the coupling of the MOND acceleration scale to the density of dark energy.

1. Introduction

The relation between luminosity and spectral line widths in spiral galaxies has been widely established as a probe of the dark matter content of these galaxies (Tully & Fisher 1977). This, so called Tully-Fisher Relation (TFR), has subsequently been constructed in various forms with the stellar mass or baryonic mass of a galaxy serving as a proxy for luminosity, and similarly, with the maximum rotational velocity or asymptotic rotation velocity as a substitute for line width. Of the many parameter combinations, the tightest TFR fits occur between baryonic mass and asymptotic rotation velocity (Verheijen 2001).

Galactic velocity profiles that are asymptotically flat appear ubiquitous, yet they were not expected from Newtonian gravitational models of the baryonic matter distribution. Additional sources of the gravitational field, such as dark matter halos (Rubin et al 1985), or
modifications to the gravitational theory (e.g., Milgrom 1984) are necessary for their explanation. A successful explanation of the galactic rotation curves must also be compatible with the Tully-Fisher Relation because of the latter’s dependence on the asymptotic rotational velocities of galaxies.

The Modified Newtonian Dynamics (MOND) paradigm was first developed as a modification to gravity at low accelerations for spiral galaxies and an alternative to dark matter (Milgrom 1984; for a thorough treatment of MOND see Milgrom 2001, 2002b, 2008 and Sanders & McGaugh 2002). One of its early successes has been the fitting of galaxy rotation curves with a parametric function that is determined only by the amount of visible matter in the galaxy and a single parameter $\alpha_0$ which represents the characteristic acceleration below which gravity becomes non-Newtonian (Bekenstein & Milgrom 1984).

Non-relativistic MOND modifies Newtonian gravity such that the net acceleration experienced by a test particle in a gravitational field is $a_{\text{MOND}} = (a_N \alpha_0)^{1/2}$, where $a_N$ is the Newtonian gravitational acceleration. This phenomenological model also leads naturally to the Tully-Fisher Relation, since the velocity $V_f$ of an asymptotically flat rotation curve at large distances is related to the baryonic mass $M$ of the galaxy by (Milgrom 1984)

$$M = \frac{V_f^4}{G\alpha_0}.$$  

(1)

The most unexpected result of fitting rotation curves with MOND is that the acceleration parameter is nearly the same for all galaxies. Moreover, its value is comparable to two combinations of cosmological parameters that have dimensions of acceleration (Milgrom 2001), i.e.,

$$\alpha_0 \simeq cH_0 \simeq c\sqrt{\frac{8\pi G \rho_\lambda}{3}},$$  

(2)

where $H_0$ is the value of the Hubble constant and $\rho_\lambda$ is the cosmic density of dark energy. In the context of the Cold Dark Matter (CDM) paradigm, this similarity is believed to arise from the cosmological evolution of dark matter halos, which naturally depends on the cosmological expansion. Analytic attempts to account for the MOND acceleration scale within the CDM framework have been indeed made (Kaplinghat & Turner 2002) but also refuted (Milgrom 2002a).

MOND predictions have been examined in several other settings, from clusters of galaxies (e.g., Clowe et al. 2006) to larger cosmological scales (e.g., Scott et al. 2001). These studies have established the difficulties of using the MOND phenomenology to account for this wide set of cosmological phenomena. Even without focusing on these issues, if the MOND phenomenology describes a deviation from Newtonian gravity at galactic scales, the
numerical similarity of the various physical quantities shown in relation (2) will need to be explained.

MOND is based on a non-relativistic phenomenological function and, therefore, it cannot lead to predictions for the cosmological evolution of the Hubble parameter or of the acceleration scale $\alpha_0$. Different attempts to devise a relativistic equivalent to MOND lead to varying predictions. For example, in TeVeS, which is a Tensor-Vector-Scalar relativistic theory that accounts for a large part of the MOND phenomenology, the similarity described by equations (2) is coincidental (Bekenstein & Sagi 2008). On the other hand, in a recently proposed theory which attempts to account simultaneously for the dark matter and the dark energy problem (Zhao & Li 2008), the acceleration scale couples to the density of dark energy.

If the connection between the acceleration $\alpha_0$ and either the Hubble parameter $cH$ or the density of dark energy $\sqrt{\rho_\Lambda}$ can be proven experimentally, then this will provide important clues towards developing a successful relativistic theory that is consistent with the MOND phenomenology. Indeed, within general relativity, the Einstein equivalence principle guarantees that the cosmological expansion will affect the motion of a test particle only to second order in $(cH_0)^2$ (Peebles 1993). Therefore, the acceleration scale $\alpha_0$ will couple linearly to the Hubble parameter only if the Einstein equivalence principle is violated at cosmological scales. In this case, MOND will be a modification of inertia rather than of the gravitational field (see Milgrom 2006). On the other hand, the acceleration scale $\alpha_0$ will couple to the dark energy only if the latter is not a manifestation of a cosmological constant but rather of a dynamical field that couples non-linearly to the metric. This is true because a cosmological constant or a minimally coupled field that describes dark energy introduces a characteristic scale in curvature and not in acceleration. As a result, the relativistic version of MOND in this case will require a modification of the Einstein field equations.

The evolution of the MOND acceleration scale with redshift provides the means with which we can test, in a phenomenological way, the connection represented in equation (2) (see Milgrom 2008 and Bekenstein & Sagi 2008). This is true because the rate of the cosmological expansion $H \equiv \dot{a}/a$ and the density of dark energy have distinct dependences on redshift. In this paper, we use observations of the so-called Tully-Fisher intercept back to a redshift of $z = 1.2$ (Weiner et al. 2006; Kassin et al. 2007) as a proxy for the MOND acceleration scale (see Eq. [1]). This allows us to infer the evolution of $\alpha_0$ using only the exact phenomena that non-relativistic MOND was designed to explain, without the need for extrapolating the phenomenology to include a treatment of photon trajectories. We then compare the observations to the expected evolution of the MOND acceleration scale when coupled to the Hubble parameter or the density of dark energy, for values of the cosmological parameters.
that are consistent with recent measurements (Spergel et al. 2007).

2. The evolution of the MOND acceleration scale

For our calculations, we postulate that equation (2) represents a physical relationship between the MOND acceleration scale and either the density of dark energy or the Hubble parameter. Writing these explicitly in terms of cosmological parameters, the independent couplings take the form

\[ \alpha_0 = cH = \frac{c}{a} \]  

(3)

or

\[ \alpha_0 = \sqrt{\frac{8\pi G}{3}} \rho_\Lambda = \sqrt{\frac{8\pi G}{3}} \rho_\Lambda a^{-(1+w)} \]  

(4)

where \( a \) is the scale factor of the universe and \( w \) is a parameter that describes the dark energy equation of state, \( P_\Lambda = w \rho_\Lambda \).

We separately model the time evolution of the two couplings with the first Friedmann equation,

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} (\rho_m + \rho_\Lambda) \]  

(5)

where we assume a flat universe (\( k = 0 \)).

As previously stated, non-relativistic MOND cannot lead to predictions of the cosmological evolution of the Hubble parameter or the density of dark energy. However, the Friedmann equations have been shown through observations to be a successful predictor of cosmic evolution beyond redshift \( z = 2 \). In this context, we take the first Friedmann Equation as an empirical fit of cosmological observations back to redshift \( z = 2 \), rather than a self-consistent physical theory. For all our calculations, we use cosmological parameters from the third year WMAP results (Spergel et al. 2007), \( \Omega_\Lambda = 0.716, \Omega_{\text{mat}} = 0.268, \Omega_{\text{rad}} = 0.016 \). We solve the Friedmann equations using the forth order Runge-Kutta algorithm in order to obtain the time evolution of \( \alpha_0 \) with the MOND couplings specified by equations (3) and (4).

We now consider the results of coupling the MOND acceleration scale to the Hubble parameter and the dark energy density in turn. Figure 1 shows the results of coupling the MOND acceleration scale to the Hubble parameter for three different values of the dark energy equation of state. The acceleration scale increases with redshift independent of the equation of state, although less so for more negative values of \( w \). Figure 2 shows the same quantities with the MOND acceleration scale coupled to the density of dark energy. For \( w \)}
greater than $-1$, the acceleration scale increases as in the case of coupling to the Hubble parameter. In contrast, with $w < -1$ the acceleration scale decreases with redshift. Only when $w = -1$ does $\alpha_0$ remain constant in time, as would be expected from inspection of equation (4).

Figure 3 shows the MOND acceleration scale inferred from high-redshift TFR data (c.f. Eq. 1) along with the values predicted using select Hubble and dark energy density couplings. The Hubble parameter and dark energy density couplings are shown with $w = -1.00$ based on the constraint $w = -1.08 \pm 0.12$ imposed by the latest WMAP data (Spergel et al. 2007). We also display the $w = -1.55$ dark energy coupling line, despite the problems inherent to such a model, for easy visual comparison to the data from Weiner et al. (2006), shown as open circles, and Kassin et al. (2007), shown as filled squares.

The measurements from Weiner et al. (2006) were taken by the Team Keck Redshift Survey in the rest-frame J infrared band. They observed line of sight dispersion with the DEIMOS spectra boxcar to a limiting resolution of 56 km s$^{-1}$ in the rest frame. As in Weiner et al. (2006), the local TFR intercept from Watanabe et al. (2001) is plotted in Figure 3 for reference. These results are not corrected for stellar formation rates. However, Weiner et al. develop a model to predict the systematics in their data using an exponentially decaying star formation rate with the characteristic formation time scaled to galactic mass. With this model they predict a $\sim 0.05$ dex evolution of the TFR intercept from $z = 0.4$ to $z = 1.2$.

Kassin et al. (2007) obtained their redshift data from the All-Wavelength Extended Groth Strip International Survey (AEGIS). Their photometric measurements were provided by the Hubble Space Telescope Advanced Camera for Surveys (HST/ACS) and spectra gathered from the Deep Extragalactic Evolutionary Probe 2 Survey (DEEP2). Kassin et al. (2007) converted their luminosity data to stellar mass using the method of Lin et al. (2006) but still predicted a systematic offset of 0.05 dex between $z = 1$ and $z = 0$. This arises from considering an exponentially decreasing star formation rate in a typical spiral galaxy. In Figure 3, as in Kassin et al. (2006), we adopt the local $M_\star$-TFR intercept as found in McGaugh (2005) with the maximum disk method.

Within formal uncertainties, the data in Figure 3 are not consistent with either coupling to the Hubble parameter or dark energy density with $w = -1.00$. We find that the current high redshift Tully-Fisher data can only be fit within formal uncertainties by adopting $w \approx -1.55$, which lies well outside the constraints imposed by the WMAP third year data.

The decreasing trend in the observed Tully-Fisher intercept with redshift may be due to three sources of systematic errors: the unknown evolution of the initial mass function, of the star formation rate, and of the stellar population. The model of Weiner et al. (2006)
predicts a $-0.05$ dex decrease in the TFR intercept from $z = 0.4$ to $z = 1.2$ when taking into account all of these factors. However, this only accounts for roughly half of the observed decrease. Kassin et al. (2007) correct their data for evolution in stellar populations only and add that a $-0.05$ dex decrease in the TFR intercept from $z = 0$ to $z = 1$ would be typical based on exponentially decaying star formation models. Taking these systematic errors into account, the data from Kassin et al. (2007) are still formally inconsistent with no evolution. But, with systematic errors accounted for, coupling of the MOND acceleration parameter to the dark energy density with a realistic value of the dark energy equation of state may be consistent with both datasets. On the other hand, unreasonable values of the dark energy equation of state will still be necessary to make coupling to the Hubble parameter consistent with the data. This would appear to favor coupling to the dark energy density, but there may still be additional systematic errors unaccounted for.

As with coupling to the dark energy density, N-body dark matter halo simulations have predicted little to no evolution in the stellar mass Tully-Fisher intercept after $z = 1$ (Diemand et al. 2007). More recently, the effects of gas physics have been incorporated into these simulations, but still no evolution in the intercept is observed after $z = 1$ (Portinari & Sommer-Larsen 2008). Simulations that include gas physics do show significant mass evolution in individual galaxies along with a corresponding change in rotation velocity. This has the effect of moving galaxies along the Tully-Fisher Relation but not away from it, thereby leaving the intercept unchanged. Therefore, the predictions of Tully-Fisher intercept evolution within the $\Lambda CDM$ paradigm and the dark energy coupled MOND paradigm with $w = -1$ are indistinguishable.

3. Conclusion

In this paper, we calculated the time-evolution of the MOND acceleration parameter $\alpha_0$ when coupled to either the Hubble parameter or to the density of dark energy. For a dark energy equation of state consistent with recent data, we expect a positive evolution of $\alpha_0$ with redshift if it couples to the Hubble parameter and little to no evolution with redshift if it couples to the dark energy density. The latest measurements of the Tully-Fisher Relation support neither coupling scenario within formal uncertainties. Observations to $z = 1.2$ indicate a decrease in both the baryonic mass and J-band luminosity Tully-Fisher intercepts, which are coupled to the MOND acceleration parameter as shown in equations (3) and (4). However, the systematic uncertainties introduced by the initial mass function, stellar formation and stellar population evolution affect the data significantly. Sources of systematic uncertainty tend to shift the Tully-Fisher intercept data upward, so that the net trend has a
slight downward evolution with redshift. The corrected data would then be consistent with a coupling of $\alpha_0$ to the dark energy density with an observationally allowed equation of state with $w \approx -1$. However, the uncertainties in such a large correction for systematics also makes no evolution in the TF intercept plausible.

Definitive conclusions can only be drawn after the systematic errors are more precisely quantified. Future observations from large telescopes both on the ground and in space of galaxies at higher redshifts will be particularly helpful, since the difference between the MOND acceleration scale evolving coupled to the dark energy density or the Hubble parameter increases with redshift.

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Fig. 1.— Redshift evolution of the MOND acceleration scale normalized to the present value $\alpha_0$ if it evolves coupled to the Hubble parameter for different values of the dark energy equation of state $w$. 

\[ \frac{1}{4} \log \left( \frac{\alpha}{\alpha_0} \right) \]
Fig. 2.— Redshift evolution of the MOND acceleration scale, normalized to the present value $\alpha_0$, if it evolves coupled to the density of dark energy for different values of the dark energy equation of state $w$. 
Fig. 3.— The redshift evolution of the MOND acceleration scale, normalized to the present value $\alpha_0$, if it evolves coupled to the Hubble parameter or the density of dark energy for different values of the equation-of-state parameter $w$. The values of the MOND acceleration scale inferred from infrared J band Tully-Fisher relations by Weiner et al. (2006) are shown as open circles, where us those inferred from stellar mass TFR data from Kassin et al. (2007) are shown as filled squares. We adopt local Tully-Fisher intercept values from Watanabe (2001) and McGaugh (2005), respectively.