Abstract This paper studies the libration-free cargo transfer control of a partial space elevator where the main satellite may change its orbital state in the transfer period. The orbital motion of the main satellite in the climber transfer period is first studied. Then, a reduced-order libration-free dynamics of the partial space elevator is derived. Accordingly, a novel libration-free switching control strategy is proposed to stabilize cargo transportation with two alternating controllers. The Controller I controls the cargo speed in the libration-free mode by a shrinking horizon model predictive control based on the reduced-order libration-free dynamic mode of the partial space elevator. It leads to high computational efficiency in control. Once the libration is induced by the cargo transfer, the control turns off the Controller I and activates the Controller II to suppress the libration to zero within one time step of the Controller I by a novel prescribed-time control law based on the fixed-time control scheme. The stability of the control is proved in the Lyapunov framework. The validity and effectiveness of the proposed control strategy are demonstrated by computation simulation. Simulation results reveal that the proposed control strategy is effective in keeping stable cargo transportation while ensuring the equilibrium state at the end of transportation.

Keywords Partial space elevator · Libration-free transfer · Shrinking horizon model predictive control · Prescribed-time control

1 Introduction

Partial space elevator (PSE) is a promising technology for future cargo transportation to space stations and extra-large space structures due to its advantage in energy efficiency compared to the current space transportation by rockets [1–3]. The PSE is formed by one main satellite connected to a secondary satellite by a long tether as shown in Fig. 1 [4]. A climber moves along the tether to transfer cargo between the main and secondary satellites. The cargo transportation will not only lead to the libration of PSE due to the Coriolis effect but also the variation in orbital states of the main satellite in terms of orbital radius and angular velocity. It is well known that these will affect the stability of PSE in cargo transportation. Therefore, the in-depth knowledge of orbital and libration dynamics...
and the control strategy for stable cargo transfer are crucial for the successful operation of PSE.

The dynamic modeling and analysis of PSE have attracted many attentions from researchers. Lorenzini et al. [5] and Misra et al. [6] developed the two-piece dumbbell (TPDB) model for the PSE in a circular orbit. This model is widely used for the modeling of multibody tethered space systems, such as the sloshing fuel tethered satellites [7] and the motorized momentum exchange tether systems [8]. Based on the TPDB model, Woo and Misra [9] and Shi et al. [10] investigated the influences of the motion and parameters of climbers on the libration dynamic characteristics of PSE. Jung et al. [11] expanded the TPDB model by considering the orbital states of the main satellite and noted the orbital radius changes obviously in the cargo transfer period, even though the climber is moving at a constant speed. Li and Zhu [12, 13] proposed a high-fidelity model to include the influence of orbital states and elastic motion of tether by a nodal position finite element method. Their analysis shows that the classical TPDB model is accurate in describing the overall motions of the PSE and suitable for control law development, provided the control law is robust. Besides the theoretical studies, Yamagiwa et al. [14] investigated the PSE from an engineering aspect for space demonstration.

In the control strategy development for stable cargo transportation of PSE, Kojima and Fujii [15] proposed a “Mission Function” control law to suppress the libration of PSE in the cargo transfer period by adjusting the climber speed only. Shi and Zhu [10] proposed a flexible tension control strategy based on sliding mode control to regulate the climber speed with feedback of libration dynamic characteristics for libration suppression. Wen et al. [16, 17] and Sun et al. [18, 19] proposed tension control laws to stabilize the deployment of two-body tethered satellites, which are applicable for the libration suppression of PSE. The above works reveal that the libration of PSE can be suppressed by adjusting the climber speed in the cargo transfer period. However, it is difficult for them to enforce multiple constraints simultaneously and ensure the full state stability of PSE at the end of the cargo transfer period because the PSE is an underactuated system and is not in an equilibrium state in the transfer period due to the energy injection. Accordingly, open-loop [20, 21] or piece-wised optimal control laws [22] are applied to regulate the climber speed due to their robust global planning capability. However, the computational load is heavy compared to other control schemes.

Although effective, it is worth pointing out that the aforementioned works assumed the center of mass of PSE is approximately at the main satellite in the cargo transfer period so that the cargo transportation does not change the orbital states of the main satellite. If the total mass of the secondary satellite and the cargo is comparable to the mass of the main satellite, the orbit of the main satellite is inevitably changed because the center of mass of the PSE is no longer at the main satellite. Furthermore, the motion of the climber will change the orbital states of the PSE and the main satellite, leading to the ascending or descending of the orbital radius of the main satellite. Therefore, the additional orbital states of the main satellite

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**Fig. 1** Scheme of system description

![Scheme of system description](image_url)
exacerbate the underactuated and non-equilibrium characteristics of the PSE. In the current paper, such a scenario is called the floating main satellite. Up to date, limited tether libration control strategies are available in the literature for the PSE with a floating main satellite by simple adjustments. For instance, Zhong and Zhu [23] proposed a piecewise optimal control method to control the orbital parameters and the attitudes of an electrodynamic tether system in orbit transfer. Rao and Zhong [24] proposed a “three-stages” model predictive control scheme to simultaneously deal with libration suppression and orbital motion of a tether-tug system in deorbit process. However, a more generic control strategy is desired to achieve stable transfer of the PSE with a floating main satellite. Such a control strategy shall keep a stable state of PSE in the entire transfer period, while being computationally efficient. Shi and Zhu [4] tried a stable transfer control scheme for the PSE with two controllers acting alternately to control the thrust and tether deployment simultaneously or the tether deployment only at the secondary satellite. Although effective, the control law cannot guarantee the libration motion converging to zero in a short and fixed period. Fixed-time control theory can be used to solve this problem [25–27], but the control parameters must be adjusted following different prescribed periods.

To overcome the problem, this work models the motion of PSE by decomposing it into the global orbital motion with zero libration and the local libration motion by the Lagrangian equation. Based on the decomposed dynamics, a novel and computationally efficient switching control strategy is proposed to achieve a libration-free transfer. The cargo transfer is controlled by the primary Controller I, which is a computationally efficient shrinking horizon model predictive control (SHMPC). The controller assumes the PSE is always in the local vertical without any libration. The control action will switch to the secondary Controller II when the libration disturbance appears. The Controller II suppresses the libration to zero within one time step of the Controller I by a novel prescribed-time control (PTC) law based on the fixed-time control scheme [28–30]. The stability of the proposed control strategy is proved under the Lyapunov framework. Simulation results show that the proposed libration-free switching control strategy is highly effective in stabilizing the libration with high computational efficiency.

2 Dynamics of a PSE with floating main satellite

2.1 Dynamics of a partial space elevator with floating main satellite

Consider a floating PSE shown in Fig. 1 [4] in an ideal central gravity field where no gravity perturbations, such as the gravity gradient and Earth oblateness, are considered. The long-term environmental perturbations, such as solar radiation pressure and atmospheric drag, are neglected in the current work, because they do not have noticeable impact on the libration of the PSE in the short term. The out-of-plane motion of the PSE is ignored too because it is mainly led by the ignorable perturbances of the space environment. Their influences on the in-plane motions, which we focus on, are ignorable in practical because of [31],

\[
\left| \frac{\dot{z} \tan(\chi) L}{L} \right| < 1
\]

where \( \chi \) is the out-of-plane angle of the climber or the secondary satellite, \( L \) is the tether length. Eq. (1) denotes the ratio of the influence of the out-of-plane motion and the effect of Coriolis force on the in-plane motion. In practice, the out-of-plane motion is assumed zero or being kept at zero by the independent thrusters, such that, \( \chi = \dot{\chi} = 0 \), and Eq. (1) is zero or a small amount that much less than 1. Thus, it is safe to ignore the out-of-plane motion to make the dynamics simple. An Earth-centered inertial coordinate frame \( OXY \) is established to describe the libration motion of the PSE, see Fig. 1, where the main satellite (\( m_0 \)), the secondary satellite (\( m_2 \)), and the climber (\( m_1 \)) are simplified as mass points, the mass and elasticity of the tether are ignored.

The spatial position of the main satellite is denoted by a vector \( r_0 \) in the inertial frame. The true anomaly is denoted by \( \nu \). The generalized coordinates \( L_1 \) and \( L_2 \), together with libration angles, denote the relative positions of the climber. The tether (\( L_1 \)) is the distance between the position of the main satellite \( m_0 \) and the current position of the climber \( m_1 \). Its libration is defined by the angle \( \theta_1 \) that is measured from the vector \( r_0 \) to \( L_1 \). Similarly, the tether (\( L_2 \)) is the distance between the current position of the secondary satellite \( m_2 \) and the current position of the climber \( m_1 \). The libration angle \( \theta_2 \) denotes the libration of the tether (\( L_2 \)) measuring from the vector \( r_0 \) to the vector normal
to \( L_2 \), see Fig. 1. The total tether length can be denoted by \( L_1 + L_2 \). In the current work, the total tether length is assumed not change, i.e., \( L_1 + L_2 = \text{constant} \).

The positions of the main satellite \( r_0 \), climber \( r_1 \), and secondary satellite \( r_2 \) are defined as

\[
\begin{align*}
  r_0 &= re, \\
  r_1 &= r_0 - L_1 \cos \theta_1 e_r - L_1 \sin \theta_1 e_\theta, \\
  r_2 &= r_1 - L_2 \cos \theta_2 e_r - L_2 \sin \theta_2 e_\theta
\end{align*}
\]  

(2)

where \( r \) is the orbital radius of the main satellite, \( e_\theta \) is in the orbital motion direction of the main satellite with the angle \( \theta \), the unit vector \( e_r \) is in the orbital radius direction of the main satellite normal to \( e_\theta \).

Differentiating Eq. (2) yields the velocities of the main satellite, the climber, and the secondary satellite,

\[
\begin{align*}
  v_0 &= \frac{d}{dt} r_0 + \omega \times r_0, \\
  v_1 &= \frac{d}{dt} r_1 + \omega \times r_1, \\
  v_2 &= \frac{d}{dt} r_2 + \omega \times r_2
\end{align*}
\]  

(3)

where \( \omega = [0 \quad 0 \quad \dot{\varphi}]^T \).

Accordingly, the potential \((U)\) and kinetic energy \((K)\) of the PSE can be written as

\[
U = -\mu \left( \frac{m_0}{|r_0|} + \frac{m_1}{|r_1|} + \frac{m_2}{|r_2|} \right)
\]  

(4)

\[
K = \frac{1}{2} m_0 v_0^T \cdot v_0 + \frac{1}{2} m_1 v_1^T \cdot v_1 + \frac{1}{2} m_2 v_2^T \cdot v_2
\]

where \( \mu \) is the gravitational constant of the Earth.

Introducing the Lagrangian function \( L = K - U \) and the generalized coordinates \((q_1, q_2, q_3, q_4, q_5, q_6) = (\theta_1, \theta_2, L_1, L_2, \varphi, \theta)\), the dynamics of PSE can be derived by the Lagrange equation,

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i (i = 1, 2, 3, 4, 5, 6)
\]  

(5)

the generalized force \( Q_i \) can be derived by

\[
Q_i = \sum_{j=0}^{2} \lambda_j \frac{\partial r_0}{\partial \dot{q}_i} (j = 0, 1, 2)
\]  

(6)

where

\[
\begin{align*}
  \lambda_0 &= - T_1 \cos \theta_1 e_r - T_1 \sin \theta_1 e_\theta, \\
  \lambda_1 &= (F_1 \sin \theta_1 + T_1 \cos \theta_1 - T_2 \cos \theta_2) e_r + (F_1 \cos \theta_1 + T_1 \sin \theta_1 - T_2 \sin \theta_2) e_\theta, \\
  \lambda_2 &= (F_2 \sin \theta_2 + T_2 \cos \theta_2) e_r + (F_2 \cos \theta_2 + T_2 \sin \theta_2) e_\theta
\end{align*}
\]  

(7)

are the non-conservative forces acting on the main satellite, the climber, and the secondary satellite, respectively.

Here, \( F_1 \) and \( F_2 \) are the thrusts acting at the climber and the secondary satellite, respectively. They can be generated by the thrusters, in practice. In this work, \( F_1 \) and \( F_2 \) are assumed perpendicular to \( L_1 \) and \( L_2 \), respectively, with their positive orientations shown in Fig. 1. \( T_1 \) and \( T_2 \) are the tensions in tethers \( L_1 \) and \( L_2 \), respectively. It should be noted that in the derivation, both the \( K_k \) and \( r_k \) should be written as the vectors in the inertial coordinate frame \( OXY \) directly.

Substituting Eq. (7) into Eq. (6) yields

\[
\begin{align*}
  Q_1 &= [F_1 + F_2 \cos(\theta_1 - \theta_2)]L_1 \\
  Q_2 &= F_2 L_2 \\
  Q_3 &= F_2 \sin(\theta_1 - \theta_2) - T_1 \\
  Q_4 &= -T_2 \\
  Q_5 &= F_1 \sin \theta_1 + F_2 \sin \theta_2 \\
  Q_6 &= F_1 L_1 + F_2 [L_1 \cos(\theta_1 - \theta_2) + L_2] - r(F_1 \cos \theta_1 + F_2 \cos \theta_2)
\end{align*}
\]

(8)

Then, the system dynamics can be derived as

\[
\begin{align*}
  \ddot{\theta}_1 &= \frac{\mu \sin \theta_1}{r^2 L_1} + \frac{G_1^2 \sin \theta_1}{m_1 L_1} - \frac{T_1 \sin \theta_1}{r m_0} - \frac{T_2 \sin(\theta_1 - \theta_2)}{L_1 m_1} - 2(\dot{\varphi} + \dot{\theta}_1)L_1 + \frac{2 \dot{\varphi} \dot{\varphi} + F_1 L_1}{L_1 m_1} \\
  \ddot{\theta}_2 &= \frac{G_2 \cos \theta_2 - G_1 \sin \theta_2}{L_2 m_1} + \frac{G_1 \sin \theta_2 - G_2 \cos \theta_2}{L_2 m_2} + T_1 \sin \theta_1 + \frac{2 \dot{\varphi} \dot{\varphi} + T_1 \cos(\theta_1 - \theta_2)}{L_2 m_1} - \frac{2(\dot{\varphi} + \dot{\theta}_2)L_2}{L_2} - \frac{F_1 \cos(\theta_1 - \theta_2)}{L_2 m_1} + \frac{F_2}{L_2 m_2}
\end{align*}
\]  

(9)

\[
\begin{align*}
  \ddot{L}_1 &= \dot{\varphi}^2 L_1 - \frac{\mu \cos \theta_1}{r} - \frac{G_1 \cos \theta_1 + G_2^2 \sin \theta_1}{m_1} - \frac{T_1}{m_0} - \frac{T_1}{m_1} + \frac{T_1 \cos(\theta_1 - \theta_2)}{m_1} + \frac{T_2}{m_2} + (2 \dot{\varphi} + \dot{\theta}_1) L_1 \dot{\theta}_1
\end{align*}
\]  

(10)

\[
\begin{align*}
  \ddot{L}_2 &= \dot{\varphi}^2 L_2 - \frac{\mu \cos \theta_2}{r} - \frac{G_1 \cos \theta_1 + G_2^2 \sin \theta_1}{m_1} - \frac{T_1}{m_0} - \frac{T_1}{m_1} + \frac{T_1 \cos(\theta_1 - \theta_2)}{m_1} + \frac{T_2}{m_2} + (2 \dot{\varphi} + \dot{\theta}_1) L_1 \dot{\theta}_1
\end{align*}
\]  

(11)
\[ \ddot{L}_2 = \dot{\theta}^2 L_2 + \frac{G_1 \cos \theta_2 + G_1^3 \sin \theta_2}{m_1} \]
\[ - \frac{G_2^3 \sin \theta_2}{m_1} + \frac{T_1 \cos(\theta_1 - \theta_2)}{m_1} - \frac{T_2}{m_2} + \frac{F_1 \sin(\theta_1 - \theta_2)}{m_1} \]
\[ \ddot{r} = r \ddot{\theta} - \frac{\mu}{r^2} - \frac{T_1 \cos \theta_1}{m_0} \]
\[ \ddot{\theta} = -\frac{2 \ddot{\theta} \dot{r}}{r} - \frac{T_1 \sin \theta_1}{m_0} \]
Equations (9)–(15) describe the dynamics of the PSE [5, 6]. Equations (9)–(15) describe the dynamics of the PSE with a floating main satellite. The given dynamics are validated by the detailed model in Ref. [13] with generalized coordinates defined as a necessary and sufficient set.

It is worth noting that there is no damping effect in the dynamics of the PSE. This is because of the lack of real data on the internal damping of tether or friction in the tethered mechanism in space. Limited literature suggests that the damping or friction of tethered systems in a zero gravity environment is much lower than that on Earth [32].

Assume the total tether length \( L_0 \) is constant yields
\[ L_2 = L_0 - L_1, \quad \dot{L}_2 = -\dot{L}_1, \quad \ddot{L}_1 = -\ddot{L}_2 = u_L \] (16)

The climber’s acceleration can be regarded as a control input \( u_L \). Such that, Eqs. (10)–(12) can be rewritten as
\[ \ddot{\theta} = \frac{G^2 \cos \theta_2 - G^2 \sin \theta_2}{(L_0 - L_1)m_1} + \frac{G^2 \sin \theta_2 - G^2 \cos \theta_2}{(L_0 - L_1)m_2} \]
\[ + \frac{T_1 \sin \theta_1}{m_0} + \frac{2 \ddot{\theta} \dot{r}}{r} + \frac{T_1 \sin(\theta_1 - \theta_2)}{(L_0 - L_1)m_1} \]
\[ + \frac{2(\ddot{\theta} + \dot{\theta} \dot{L}_1)}{(L_0 - L_1)} - \frac{F_1 \cos(\theta_1 - \theta_2)}{(L_0 - L_1)m_1} + \frac{F_2}{(L_0 - L_1)m_2} \]
\[ (17) \]
\[ T_1 = m_0 \left\{ r^2 G^2 \left[ m_2 \sin \theta_2 \sin(\theta_1 - \theta_2) - m_1 \cos \theta_1 \right] \right. \]
\[ - r^2 G^2 \left[ m_1 \sin \theta_1 + m_2 \cos \theta_2 \sin(\theta_1 - \theta_2) \right] \]
\[ + m_1 \left. \{ - \mu (m_1 + m_2) \cos \theta_1 \right\} \]
\[ + r^2 \left\{ (m_1 + m_2) \left[ L_1 (\dot{\theta} + \dot{\theta}_1)^2 - u_L \right] - \cos(\theta_1 - \theta_2) \sin \theta_2 G^2 - \cos(\theta_1 - \theta_2) \cos \theta_2 G^2 \right\} \]
\[ + m_2 \cos \theta_1 \cos \theta_2 \left[ (L_0 - L_1) (\dot{\theta} + \dot{\theta}_2)^2 + u_L \right] \}
\[ + m_2 \sin \theta_1 \sin \theta_2 \left[ (L_0 - L_1) (\dot{\theta} + \dot{\theta}_2)^2 + u_L \right] \}
\[ /A \]
\[ (18) \]
\[ T_2 = \{ -2r^2 G^2 m_1 (m_1 + m_0) \cos \theta_2 \]
\[ - 2r^2 G^2 m_1 (m_1 + m_0) \sin \theta_2 \]
\[ + m_2 \left\{ 2r^2 G^2 \left[ m_0 \sin \theta_1 \sin(\theta_1 - \theta_2) + m_1 \cos \theta_2 \right] \right. \]
\[ + \sin \theta_2 \left\{ 2r^2 G^2 \left( m_1 + m_0 \cos^2 \theta_1 \right) \right. \]
\[ + 2m_1 m_0 \left\{ r^2 \left[ L_1 (\dot{\theta} + \dot{\theta}_1)^2 - u_L \right] - \mu \cos \theta_1 \right\} \}
\[ - 2m_0 \cos \theta_1 \cos \theta_2 \left\{ r^2 G^2 \left( \sin \theta_1 + m_1 \mu \cos \theta_1 \right) \right. \]
\[ - r^2 \left. \left[ L_1 (\dot{\theta} + \dot{\theta}_1)^2 - u_L \right] \right\} \}
\[ + 2r^2 m_1 (m_1 + m_0) \left[ (L_0 - L_1) (\dot{\theta} + \dot{\theta}_2)^2 + u_L \right] / (2r^2 A) \}
\[ (19) \]
where \( A = m_1 (m_1 + m_2) + m_0 \left[ m_2 \sin^2(\theta_1 - \theta_2) + m_1 \right] \).

In the cargo transfer period, the climber can be driven along the tether by the device shown in Fig. 1, or by deploying/retrieving tethers at the main satellite and the secondary satellite using the rolling wheel simultaneously in practice. The movement of the climber in the tether direction generates the Coriolis force acting on the climber. The direction of the Coriolis force is normal to the relative velocity \( \dot{L}_1 \).

Since the tether is only a tensile member and does not have bending rigidity like a beam, the tether sections at each side of the climber must be bent at the climber from its nominal straight line so that the components of tension \( T_1 \) and \( T_2 \) normal to the unbent tether will
counterbalance the Coriolis force. Mathematically, the relative effect of Coriolis force on the climber and secondary satellite can be found in Eqs. (9) and (17) as $-2(\dot{\phi} + \dot{\theta}_1)L_1/\dot{\theta}_1L_1L_1$ and $2(\dot{\phi} + \dot{\theta}_2)L_1/\dot{\theta}_2L_1(L_0 - L_1)(L_0 - L_1)$, respectively. They are always unequal in the cargo transfer period and lead to different libration motions $\theta_1$ and $\theta_2$ of the climber and the secondary satellite. As a result, the tether bends at the climber and the PSE liberates like a double pendulum with two “arms,” where the two “arms” are the two pieces of tethers (with the general coordinates of $L_1$ and $L_2$) that are regarded as massless straight cables with variable lengths. In the cargo transfer period, the lengths of these two “arms” change correspond to the changes of general coordinates $L_1$ and $L_2$. The tensions $T_1$ and $T_2$ in the tethers $L_1$ and $L_2$ can be calculated by Eqs. (18) and (19), respectively. These two equations show that the tensions are the functions of the libration angles, climber acceleration, and orbital states. The libration angles and the climber acceleration must be kept in reasonable scope to ensure $T_1, T_2 > 0$.

2.2 System in libration-free mode

The libration-free mode of a PSE is defined as the zero libration states, such that

$$\theta_1 = \dot{\theta}_1 = \ddot{\theta}_1 = 0, \quad \theta_2 = \dot{\theta}_2 = \ddot{\theta}_2 = 0 \quad (20)$$

By substituting the libration-free mode (20) into Eqs. (9) and (10), it is found that, to keep the liberation-free mode in the cargo transfer period, the disturbing effects caused by the moving climber and the orbital motions must be canceled, ensure the following conditions

$$-2(\ddot{\phi} \dot{L}_1 + \dot{\phi} \ddot{L}_1)/\dot{L}_1m_1 = 0,$$

$$2(\ddot{\phi} \dot{L}_2 - \ddot{\phi} \dot{L}_2)/\ddot{L}_2m_1 + F_{1s}/L_2m_2 = 0$$

where $F_{1s}$ and $F_{2s}$ are the state keeping thrusts and are derived from Eq. (21) as

$$F_{1s} = 2\ddot{\phi} \dot{m}_1 - 2\dot{\phi} \dot{L}_1m_1/r,$$

$$F_{2s} = -2\ddot{\phi} \dot{L}_2m_2/r \quad (22)$$

Here, it should be noted that the forms of the state-keeping thrusts are independent of the libration motion, as shown in Eq. (22). They are always required in the cargo transfer period to eliminate the coupling effect (Coriolis effect), no matter whether the PSE is running in the libration-free mode. Thus, the thrusts acted on the climber and the secondary satellite contains two parts, such that, $F_1 = F_{1s} + u_1$ and $F_2 = F_{2s} + u_2$. The first part ($F_{1s}, F_{2s}$) is the station keeping thrusts to maintain the libration-free mode. They are discussed before Eq. (27) and can be determined by Eq. (22). It corresponds to the Controller I in Sect. 3. The second part ($u_1$ and $u_2$) is determined by the Controller II, which will eliminate possible external disturbances (Coriolis forces) by the prescribed-time control within the prescribed period. This robust control law of ($u_1$ and $u_2$) will be designed in Sect. 3.3. Noticeably, when there is no libration motion, $u_1 = u_2 = 0$ and $F_1 = F_{1s}, F_2 = F_{2s}$. Substituting Eqs. (20) and (22) into Eqs. (9)–(15) yields the dynamics of the libration-free system

$$\ddot{r} = r\dot{\phi}^2 - \frac{\mu}{r^2} - \frac{T_1^0}{m_0}$$

$$\ddot{\phi} = -\frac{2\dot{\phi} \dot{r}}{r} \quad (24)$$

$$\dot{L}_1 = u_L \quad (25)$$

where

$$T_1^0 = \frac{m_0m_1}{m_0 + m_1 + m_2}\left\{\frac{\mu}{r^2} + \dot{\phi}^2 \dot{L}_1 + \frac{m_2}{m_1}\left[\frac{\mu}{(r - L_0)^2} + \frac{\mu}{r^2} + \dot{\phi}^2L_0\right] - u_L\right\}$$

Equations (23) and (25) show that the radius of the orbit of the main satellite will be influenced by the climber acceleration $u_L$ and the tether length $L_1$ directly. However, the climber motion will not influence the orbital angular acceleration of the main satellite directly, as shown by Eq. (24).

In the orbital dynamics Eqs. (13) and (14), the nonlinear coupling effects are caused by the terms of $-T_1 \cos \theta_1/m_0$ and $-T_1 \sin \theta_1/rm_0$. When the magnitudes of libration angles are small, e.g., less than 10
degrees, these two terms show a weak nonlinearity and can be approximated by zero and first-order terms. Thus, the orbital radius acceleration of the main satellite is independent of the libration motion, while the orbital circumferential acceleration of the main satellite is linearly dependent on the libration motion. This provides a theoretical foundation for the libration-free transfer control approach, where the orbital and libration motion can be approximately decoupled to simplify the libration control if the libration angle is small. Furthermore, in the libration-free mode, the libration states ($\theta_1$, $\theta_2$, $\dot{\theta}_1$, $\dot{\theta}_2$) are eliminated in the orbital motion functions, see Eqs. (23)–(26). This indicates that the orbital motion is not influenced completely by the libration states in the libration-free mode. According to the libration dynamics of Eqs. (9) and (10), when the system is in the libration-free mode, the orbital coupling terms $2\ddot{\theta}(\dot{r}/r - \dot{L}_1/L_1)$ and $2\ddot{\theta}(\dot{r}/r - \dot{L}_2/L_2)$, which are the Coriolis effect, still exist and cannot be linearized. This indicates that the libration-free cargo transfer mode is not stable because of the disturbance from the Coriolis effect and requires active state-keeping thrusts to keep this mode, such as ($F_{1x}$, $F_{2x}$) shown in Eq. (21). Thus, under the action of the state-keeping thrusts, the libration motion and the orbital motion can be decoupled completely in the libration-free mode, and the control strategy can be proposed based on this property.

Integrating Eq. (24) yields

$$\dot{\theta} = \frac{\exp(c_0)}{r^2}$$

Here, $c_0$ is a constant and can be expressed as

$$c_0 = \ln(\dot{\theta}_0) + 2 \ln(r_0)$$

where $\dot{\theta}_0 = \dot{\theta}(t_0)$ and $r_0 = r(t_0)$ are the initial $\dot{\theta}$ and $r$, respectively.

Accordingly, the order of the libration-free dynamics is reduced from 3 to 2. It should be noted that the solution in Eq. (27) is true only in the libration-free mode. Once the libration appears, the analytical solution of the main satellite’s orbital angular velocity is no longer valid because the condition in Eq. (20) does not hold anymore. Therefore, once the libration is suppressed to zero under control and Eq. (27) becomes true again, the constant $c_0$ should be refreshed based on the new initial condition of $\dot{\theta}$ and $r$.

3 Libration-free transfer control

3.1 Control scheme

The libration-free model shows a significant simplification of system dynamics in the cargo transfer period. To take this advantage, the control system is separated into two alternating parts: the libration-free subsystem (Eqs. (23), (25)–(28)) is controlled by $u_L$ and the libration subsystem (Eqs. (9) and (10)) is controlled by $u_1$ and $u_2$, depending on the orbital and libration states of the PSE. In the libration-free subsystem, the control targets are to (i) ensure two orbital states ($r$ and $\dot{r}$) converged to an equilibrium state by the end of the cargo transfer period and (ii) deal with the disturbance caused by the possible libration with one control input $u_t$ under constraints. This can be done by regulating $u_L$ with the model predictive control (MPC). In the libration subsystem, $u_1$ and $u_2$ must be designed to suppress the possible libration caused by perturbations in a fast manner to keep the PSE operating in the libration-free mode. Thus, a closed-loop control law is needed for this subsystem. The control laws about the $u_L$ and $u_1$ & $u_2$ in the two subsystems should work together seamlessly. Accordingly, a novel switching control strategy is proposed in Fig. 2 with three control inputs. It should note that $\dot{\theta} = [\theta_1, \theta_2]^T$ in Fig. 2.

The Controller I is designed for the libration-free cargo transfer, where the climber movement is regulated through $\dot{L}_1$ using the shrinking horizon model predictive control (SHMPC). Once the libration of PSE appears as the result of cargo transfer, the Controller I is deactivated, and the Controller II is activated to enforce the zero libration ($\theta$, $\dot{\theta} = 0$) condition by eliminating possible external disturbances using the prescribed-time control (PTC) within the prescribed period

$$\Delta T = \left\{ \frac{t_{i+1} - t_i}{T_s} \geq \varepsilon \right\} \left\{ \frac{t_{i+1} - t_i}{T_s} < \varepsilon \right\}$$

(29)

where $0 < \varepsilon < 1$ is a constant and $\varepsilon T_s \leq \Delta T < (\varepsilon + 1)T_s$.

It can be seen that the prescribed period depends on the PTC’s starting time $t_i$ and the predicted step of
SHMPC $T_s$ in the Controller I, where the subscript of $t_i$ denotes the number of the time interval.
If no external disturbance acts on the climber or the secondary satellite, the PSE is kept libration-free by the state keeping thrusts in Eq. (22), while the Controller I is active to plan the optimal climber speed subject to Eqs. (23) and (25) with the angular velocity function of Eq. (27). Compared with the optimal control, the computational load is much lower than to plan the state trajectory for the full state dynamics in Eqs. (9)–(14). If the libration appears and exceeds a prescribed tolerance, the control switches to the Controller II to suppress the libration within the prescribed period determined by Eq. (29). Simultaneously, the Controller I stops working and the thrust $u_L$ follows the sequence planned in the previous time interval. By the end of the prescribed period, the libration will be eliminated ($\theta, \dot{\theta} = 0$). Then, the Controller I is activated, and the Controller II enters the standby state.

### 3.2 Shrinking horizon model predictive control for orbital transfer

The objective of Controller I is to ensure that all the orbital states converge to an equilibrium state by the end of the cargo transfer period. Therefore, it is desired to use an SHMPC that the predictive horizon covers from the current state to the end of the transposition to regulate $u_L$ robustly. Since the horizon window is fixed at each step proportionally, the horizon shrinks as the end of the mission approaches as shown in Fig. 3, where $n$ is the number of the control period. Accordingly, the general objective function can be written as

$$\min J = x^T(t_f) F x(t_f) + \int_{t_i}^{t_f} (x^T Q x + u_L^T R u_L) dt$$

(30)

subjects to Eqs. (23), (25), (27) and
where $T_0$, $Q$, and $R$ are the positive semidefinite weight matrices, $x = [r, L_1]^T$, $t_i$ is the current time, $t_f$ is the predefined final time, $T_{1\max}$ and $T_{1\max}$ are the upper bounds of tension control inputs $T_1$ and $T_2$, respectively, $\dot{L}_{1m}$ and $\ddot{L}_{1m}$ are the lower and upper bounds of the climber’s accelerations, respectively.

In the $i$-th interval, the prediction horizon is $t_f - t_i = t_f - T_t (i-1)$, and the respecting action period is $t_{i+1} - t_i = T_t$. The trajectory of the states and control input $u_L$ is obtained by solving the nonlinear programming problem of Eq. (30), and the current states are used as the initial state.

### 3.3 Predefined time control for libration suppression

To suppress the libration angles of $\theta_1$ and $\theta_2$ to the desired values in a prescribed period with limited control input, a new predefined time sliding mode control is proposed to deal with the unexpected libration with limited additional thrusts. In practice, the libration states are near equilibrium if the libration is properly suppressed. Thus, the magnitudes of the libration angles are small, and the fully libration dynamics in Eqs. (9) and (10) can be linearized under the small libration assumption [33] ($\sin x = x, \cos x = 1$) and the libration dynamics can be simplified

$$\dot{\theta} = f + gu$$

$$f = \begin{bmatrix} -3\dot{\theta}^2 & 0 \\ 0 & -3\dot{\theta}^2 \end{bmatrix} \theta,$$

$$g = \begin{bmatrix} 1 \\ \frac{1}{L_1 m_1} \\ \frac{1}{(L_0 - L_1) m_1} \end{bmatrix}$$

where $\theta = [\theta_1, \theta_2]^T$, $u = [u_1, u_2]^T$, $u_1$ and $u_2$ are the additional thrusts acting on the climber and the second satellite, respectively.

A sliding mode surface with predefined time stability is designed as [29]

$$s = \theta + \operatorname{sign} \left( \frac{\text{k} \Delta T}{\pi} \dot{\theta} + \operatorname{sign} \left( \frac{\text{k} \Delta T}{\pi} \dot{\theta} \right) \right)$$

where $\operatorname{sign}^\text{n}(x) = [x_1^\text{n} \operatorname{sign}(x_1), x_2^\text{n} \operatorname{sign}(x_2), \ldots, x_j^\text{n} \operatorname{sign}(x_j)]^T$, $\Delta T$ is the predefined period, $\text{k} = \text{const} \in (0, 1)$. The designed sliding mode surfaces will approach zero when $\theta = 0$. This indicates, implemented by the sliding mode controller based on Eq. (33), the $\theta$ can be controlled to zero, and the converging period can be guaranteed within the predefined period $\Delta T$. Accordingly, Theorem 1 can be defined as follows.

**Theorem 1.** $\theta$ will converge to zero through the sliding mode surface $s$ in the predefined period $\Delta T$.

**Proof.** When $s_i = 0$, Eq. (33) can be written as.

$$\dot{s}_i = -\frac{\pi}{\text{k} \Delta T} \left[ \operatorname{sign}^\text{n}(\omega_i) + \operatorname{sign}^\text{n}(\omega_i) \right], ~ (i = 1, 2)$$

Define $\dot{\eta} = |\theta|^k/2$, then the first-order time derivation of $\eta$ can be written as

$$\dot{\eta} = \frac{\text{k}}{2} \operatorname{sign}^\text{n}(\omega) \dot{\omega}$$

Substituting Eq. (34) into Eq. (35) gets

$$\dot{\eta} = -\frac{\pi \eta}{2\Delta T} (\eta^2 + 1)$$

Solving Eq. (36) leads to the settling time

$$T = \lim_{\eta \to \infty} \frac{2\Delta T}{\pi} \int_0^\eta \frac{1}{1 + \eta^2} d\eta = \frac{2\Delta T}{\pi} \arctan(\eta)\bigg|_0^\infty = \Delta T$$

Then, Theorem 1 is proved.

Design a predefined time sliding mode control law as

$$u = -\frac{\pi}{\text{k} \Delta T} g^{-1} \left[ \Phi + \frac{\pi}{2} \operatorname{diag} \left( \frac{\theta^2}{\theta} \right) \right] - g^{-1} f$$

where

$$\Phi = \frac{2 - \text{k}}{2} \left[ \frac{\pi}{2\Delta T} \left( 2^{\text{k}} s^{1-k} + 2^{\text{k}} s^{1+k} \right) \operatorname{sign}(s) + \dot{\theta} \right]$$

$$\operatorname{diag} \left( \left[ \frac{\text{k} \Delta T}{\pi} \dot{\theta} + \operatorname{sign}^\text{n}(\omega) \right] \right)$$

(39)

Consider a candidate Lyapunov function as

$$V = \frac{1}{2} s^T s$$

(40)
Taking the derivative of $V$ yields

\[
V = s^T \cdot s \\
= s^T \cdot \left[ \frac{\theta + k}{2} \cdot \text{diag} \left( \frac{\kappa \Delta T}{\pi} \cdot \theta + \text{diag} \left( \frac{\kappa \Delta T}{\pi} \cdot 0 \right) \right) \right] \frac{\kappa \Delta T}{\pi} \cdot \text{diag} \left( \frac{\kappa \Delta T}{\pi} \cdot 0 \right) \cdot \dot{\theta} \\
= - \frac{\pi}{k \Delta T} \cdot s^T \cdot \left[ \frac{1}{2} s^T \cdot \text{diag} \left( \frac{1}{2} s^T \cdot \text{diag} \left( \frac{1}{2} s^T \cdot 0 \right) \right) \right] \\
\leq \frac{\pi}{k \Delta T} \cdot \left( \frac{1}{2} s^T \cdot \text{diag} \left( \frac{1}{2} s^T \cdot 0 \right) \right) \\
\leq \frac{\pi}{k \Delta T} \cdot \left( s^T + \text{diag} \left( \frac{1}{2} s^T \cdot 0 \right) \right) \\
= 0
\]

(41)

Therefore, the proposed control law is stable. The libration system approaches the predefined time stability in the predefined period [34].

4 Simulation and discussion

4.1 Dynamic characteristics

The dynamic characteristics of the PSE with a floating main satellite are analyzed with the following system parameters and initial states: $L_0 = 20 \text{ km}, m_0 = 50000 \text{ kg}, \quad m_1/m_0 = 0.01, \quad m_1/m_2 = 0.5, \quad L_1 = 0, \quad \theta_1(0) = \theta_2(0) = 0, \quad \dot{\theta}_1(0) = \dot{\theta}_2(0) = 0$. Here, it should be noted that, since the space experiment for a PSE is still difficult, the current work only studies the dynamics and control issues theoretically with theoretical data. All simulations were conducted by MATLAB with the RK-4 integrator. The time step is $0.001 \text{ s}$. The simulation results are shown in Figs. 4, 5, 6, 7, 8, 9, 10.

4.1.1 Effects of orbital variation in the main satellite

Let the climber move upward at constant speeds ranging from 0 to $20 \text{ m/s}$, such that $L_1(0)/L_0 = 0.01, \quad L_1(t_f)/L_0 = 0.99$ and $r(0) = 7100 \text{ km}$. The corresponding $\dot{\theta}(0)$ can be obtained from Eq. (23). The comparisons of floating and fixed main satellites on the libration of PSE in the cargo transfer period are shown in Fig. 4a and c. The trends of $\theta_1$ and $\theta_2$ are similar to those in Ref. [10], where the main satellite is in a fixed circular Kepler orbit. Figure 4b and d shows the differences of $\theta_1$ and $\theta_2$ between the floating and circular orbit cases. It shows that the difference $\theta_1$ is no more than $6.7\%$ with the peak appearing at the highest climber moving speed by the end of the transfer period. The difference of $\theta_2$ is no more than $3\%$, which is smaller than that of $\theta_1$. The results of the climber moving downward are similar to the upward case and are not presented here to avoid redundancy.

Figure 5 shows the libration caused by the floating main satellite. When the radius of the orbit of the main satellite varies near its initial state within the magnitude of $500 \text{ m}$, see Fig. 5a, the orbiting angular velocity varies in the same phase slightly, $\pm 1.5 \times 10^{-6} \text{ rad}$. The corresponding libration angles shown in Fig. 5c and d vary by magnitudes of $10^{-3} \text{ rad}$. Although the position of the climber induces some influences on the libration of the climber and the second satellite, the overall libration angles led by the floating main satellite are no more than $0.1\%$ of that induced by the Coriolis effect which is slight. Here, it should be noted that the floating magnitude $r - r_0$ of the main satellite in the numerical simulation is less than $0.1\%$ which is also slight in the pure mathematic aspect. However, for a large space structure, such floating magnitude is a concern in the engineering aspect. The cases of floating magnitudes of $100 - 1000 \text{ m}$ are studied. The results are similar to the case of $500 \text{ m}$. Thus, they are not presented to avoid redundancy.

In summary, the floating main satellite will induce additional libration motions, which is not mentioned in the literature. Although the magnitudes of additional variation in the orbital radius of the main satellite induced by the libration are small compared with the orbital radius of the main satellite $r_0$, they may have an adverse effect on the PSE structure from the engineering aspect, which should be studied in the mission planning phase.

4.1.2 Libration vs. libration-free

The effects of libration on floating the main satellite are shown in Figs. 6, 7, 8, 9. The climber moves upward at two different speed profiles with the initial and end conditions $L_1(0) = L_1(t_f) = 0$. In the libration mode, the climber and the second satellite are free to libration. In the libration-free mode, all libration is kept to zero by applying the external forces shown in Eq. (22). The effects of the libration and libration-free mode on orbital states are considered in two cases with
**Fig. 4** Comparison of floating and fixed main satellites on libration.

(a) Libration angle of the climber.

(b) Gap of $\Theta_1$ between fixed and floating cases.

(c) Libration angle of the secondary satellite.

(d) Gap of $\Theta_2$ between fixed and floating cases.
Fig. 5 Effects of the floating main satellite on libration ($r - r_0 = 500$ m).

a Variation in the orbital radius of the main satellite.
b Orbital angular velocity of the main satellite.
c Libration angle of the climber.
d Libration angle of the secondary satellite.

Fig. 6 Climber speed—trapezoid profile

Fig. 7 Climber moves in the trapezoid speed profile.

a Libration angles.
b Change of orbital radius of the main satellite.
c Orbital angular velocity of the main satellite.
d Tether length of $L_1$. 

Fig. 8 Effects of the floating main satellite on libration ($r - r_0 = 500$ m).

a Variation in the orbital radius of the main satellite.
b Orbital angular velocity of the main satellite.
c Libration angle of the climber.
d Libration angle of the secondary satellite.
Fig. 8 Climber speed—hyperbola profile

Fig. 9 Climber moves in the hyperbola speed profile. 
(a) Libration angles. 
(b) Change of the orbital radius of the main satellite. 
(c) Orbital angular velocity of the main satellite. 
(d) Tether length of $L_1$

Fig. 10 Orbital radius of the main satellite
two classical climber speed profiles. The climber speeds follow the profiles well by the open-loop control law \( u_L = \dot{L}_1 \), which is generated by the climber’s acceleration along the tether.

Let the climber move in a trapezoid speed profile as shown in Fig. 6. The acceleration and deceleration magnitudes are assumed to be the same as 0.01 m/s\(^2\) and the accelerating and decelerating lengths are the same as 0.05\(L_0\), see Fig. 7d. In such transfer mode, the magnitude of the libration angle of the climber peaks at 0.18 rad after the acceleration if no libration control is applied. By the end of the transfer period after 0.51 orbit, the PSE continues libration with a magnitude of 0.06 rad. In practice, by the end of the transfer period \( L_1 = 0 \), the climber will reach the main satellite. Thus, \( \theta_1 \) has no physical meaning and merges into \( \theta_2 \) after 0.51 orbit. The dynamic model of the PSE switches to a two-body tether system,

\[
\ddot{\theta}_2 = -\frac{3\dot{\theta}_2^2 \sin 2\theta_2}{2} + \frac{T_2 \sin \theta_2}{r(m_0 + m_1)} + \frac{2\dot{\theta}_2}{r} + \frac{F_1}{L_2 m_2},
\]

\[
\dot{r} = r \dot{\theta}_2 - \frac{T_2 \cos \theta_2}{m_0 + m_1},
\]

\[
\ddot{r} = -\frac{2\dot{\theta}_2}{r} - \frac{T_2 \sin \theta_2}{r(m_0 + m_1)}
\]

\[
T_1 = \left[ \frac{\dot{\theta}_2 L_2 - \frac{\mu \cos \theta_1}{r} - \mu \dot{\theta}_2 \sin \theta_1 - \mu (r - L_2 \cos \theta_2) \cos \theta_2 + (2\dot{\theta}_1 + \dot{\theta}_2) L_2 \dot{\theta}_2}{r^2} \right] / \left( \frac{1}{m_0 + m_1} \right)
\]

Equations (42)–(45) can be derived through the Lagrangian function without the general coordinates of \( \theta_1 \) and \( L_1 \), and \( L_2 = L_0 = \text{const} \). The variation in the orbital radius of the main satellite is presented in Fig. 7b and c. Compared to the orbital states in libration (solid line) and libration-free (dot line) modes, the magnitude and the changing trend of the radius \( r \) are obviously different. In the libration mode, the altitude of the main satellite reduces and fluctuates with a magnitude of 500 m due to the upward transportation of the climber in the libration mode. On the contrary, the orbital radius of the main satellite increases and fluctuates with a magnitude of 1100 m in the libration-free mode. This is because, in the libration-free mode, external forces act on the climber and secondary satellite to suppress the libration, which is equivalent to an extra force acting on the center of mass of the PSE aligned with the PSE orbital motion, see Eq. (22). Thus, the radius of the orbit of the center of mass will increase and, correspondingly, the radius of the orbit of the main satellite increases. This is a characteristic of the libration-free mode. On the contrary, in the downward movement phase of the climber, the altitude of the main satellite will be reduced.

When the climber speed function is changed to hyperbola, such that

\[
u_L = -\frac{1}{2} \left[ L_1(0) - L_1(t_f) \right] \left( \frac{\pi}{t_f} \right)^2 \cos \left( \frac{\pi t}{t_f} \right)
\]

The variation in the main satellite orbit states is similar to that in the trapezoid speed profile as shown in Fig. 9. After one orbit, setting \( \theta_1 = \theta_2 \), \( \dot{\theta}_1 = \dot{\theta}_2 \) in the simulation with the same reason as that in the previous case. Compared with the previous case, the magnitudes of the libration angles and the radius \( r \) are smaller. This is because, the transfer period is doubled from 0.51 to 1 orbit in the hyperbola case, and the speed curve is smoother. The comparison shows that by adjusting the climber speed profile, the orbital states of the main satellite can be controlled obviously.

In summary, (1) the main satellite’s floating influents are slight on the libration, yet the libration of the climber and the second satellite will lead to concerned orbit changes of the main satellite. (2) The libration-free transfer leads to a greater radius change of the main satellite than the climber transportation without libration suppression, as a cost of simplifying the model. (3) Different climber speed modes lead to different main satellite trajectories. Furthermore, (4) fixed speed function is difficult to ensure the full states converge to zero by the end of the transfer period. Thus, it is critical to regulate the climber’s speed carefully.

### 4.2 Control case studies

The initial conditions of the system are given in Table 1.

The transfer period is set at 5952.2 s, which equals the initial orbital period. The control laws are implemented under the proposed libration-free switching control strategy as shown in Fig. 2. The constraint
parameters of the SHMPC are given as $T_{1\text{max}} = T_{2\text{max}} = 200 \, N$, $L_{1\text{min}} = -L_{1\text{max}} = -0.01 \, m/s^2$, $n = 20$, and $T_s = t/n = 297.61 \, s$. The magnitude of the climber acceleration is not allowed to be too large to avoid overshooting. Here, it should be noted that by the end of the simulation, the climber reaches the main satellite with $L_1 = 0$, which will lead to singularity in the calculation as shown in Eq. (32). To avoid the singularity, the final $L_1(t_f)$ is set 200 m, which is a small value near zero based on previous works [4, 35]. The control parameters of the prescribed time sliding mode control are defined as $k = 0.04$ and the adapting $\Delta T$ follows Eqs. (29) with $\varepsilon = 0.7$. Both the control parameters $k$ and $\varepsilon$ can be chosen from their ranges $(0, 1)$. Greater $k$ means a faster convergence rate and higher overshooting magnitude. In the current work, the parameter $k$ was adjusted to make the control performance acceptable from a practical perspective. The parameter $\varepsilon$ and the prescribed period $\Delta T$ are positively correlated as shown in Eq. (29). They should be chosen to allow sufficient converging time based on the control capability. All simulations were carried out by MATLAB with the RK-4 integrator.

The simulation results are shown in Figs. 10, 11, 12, 13, 14, 15, 16. The solid line presents the cargo transfer in the ideal condition without libration and disturbance in the transfer period. By the end of the transfer period, the full states of the PSE converge to an equilibrium point successfully; see the solid lines in Figs. 10 and 12, 13, 14. This demonstrates the validity of SHMPC in achieving the stable cargo transfer of a PSE. In the ideal condition, only the Controller 1 works. The system dynamics is described by Eqs. (23) and (25), where the number of states is reduced from 6 to 2, and the computational efficiency has been improved greatly.

To validate the robustness of the proposed control strategy and the prescribed time sliding mode control, a sudden disturbance is added to the climber at the time point of 2950 s. Figure 11a and d shows the long-term influence of the added disturbance. To present the overall influence of the added disturbance, its influence on the secondary satellite is assumed to be

![Table 1](image)

| Parameters                                      | Values     |
|-------------------------------------------------|------------|
| Initial orbital radius of the main satellite, $r(0)$ (m) | $7.1 \times 10^5$ |
| Initial anomaly angular velocity, $\dot{\vartheta}(0)$ (rad/s) | 0.00105561 |
| Initial climber position, $L_1(0)$ (m)           | 19,800     |
| Initial climber speed, $\dot{L}_1(0)$ (m/s)      | 0          |
| Initial libration angle and angular velocity, $(\vartheta_1(0), \dot{\vartheta}_1(0))$ | (0, 0) |
| Initial libration angle and angular velocity, $(\vartheta_2(0), \dot{\vartheta}_2(0))$ | (0, 0) |

![Fig. 11](image) Prescribed-time control results.  
(a) Unsupressed libration.  
(b) Suppressed libration with PTC.  
(c) Control input.  
(d) Orbital radius of the main satellite with/without disturbing

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eliminated by the state-keeping force of the secondary satellite. The added disturbance will increase the climber libration by the magnitude of 0.1 rad, if no additional control is added to the climber in addition to the state-keeping force at the secondary satellite, see Fig. 11c. If Controller I is applied to control the climber speed based on the libration-free mode in the presence of libration, the main satellite orbit states are
obviously different from the ideal condition. Moreover, the libration will not converge to zero by the end of the transfer period. Figure 11d shows the disturbance effects on the radius of the orbit of the main satellite. For the case in which the libration disturbance effects have been eliminated, the radius will converge to a constant value by the end of the transfer period, and after 6000 s, the radius will be a constant, see the dashed line. If the libration disturbance effects are not eliminated, the radius will fluctuate near \( r_0 \); see the dot line. Such fluctuation will continue after 6000 s if no additional thrusts are applied to stabilize the orbital states of the main satellite.

When the disturbance occurs, the Controller II is activated while the Controller I begins to standby. The libration of the climber and the control inputs of the Controller II in the suppression phase from 2950 to 3273.71 s are shown in Fig. 11b, c, and Fig. 16. In the beginning, there exist \( (t_{i+1} - t_i)/T_s \leq \varepsilon \). Thus, the prescribed period is determined as \( \Delta T = t_{i+1} - t_i + T_s = 323.71 s \) per Eq. (29). Under the action of Controller II, the libration is suppressed to zero in the given period with limited additional control input successfully. This indicates that the new prescribed time sliding mode control is effective in eliminating the sudden disturbance in the prescribed period.

The states after the suppression control are shown by the dashed lines in Figs. 10, 12, 13, 14, 15, 16. After the short suppression phase, the regulated climber speed shows obvious differences from the case without disturbance if one compares the solid and dashed lines in Figs. 12, 13, and 15. Such difference leads to different orbital state trajectories in comparison with the solid and dashed lines in Figs. 10 and 14. However, the overall trends of the orbital state curves are the same. By the end of the transfer period, the main satellite’s orbital radius \( r \) and the orbital angular velocity \( \dot{\vartheta} \) converge to 7,100,572.2 m and
0.00105435 rad/s, respectively. The difference in orbital states between the disturbed and ideal cases is negligible. This is because the libration is suppressed in a fast manner. Finally, the computational efficiencies of the SHMPC in the libration mode (with full states) and the libration-free mode are compared. When the PSE operates in the libration-free mode for cargo transfer, the computational time in the first time interval, see Fig. 3, is around 1.167 s. For the PSE not working in the libration-free transportation mode, the required computational time of the SHMPC subjects to the full state in the same time interval is 51.68 s, which is about 40 times the computational time in the libration-free transfer mode. This indicates the advantage of the libration-free transfer mode in reducing the computational burden.

5 Conclusions

The nonlinear dynamic characteristics of the PSE with a floating main satellite are studied as the foundation for the development of the libration control strategy. It is found that (1) the libration of a PSE will lead to obvious orbital variations of the main satellite, while the floating main satellite influences the libration slightly; (2) the libration-free transfer leads to a different variation in the orbital radius of the main satellite than the case without libration suppression; and (3) the orbital trajectory of the floating main satellite can be controlled by regulating the climber speed. Inspired by these discoveries, a switching control strategy for libration-free transfer is proposed. The cargo transfer is performed in the libration-free mode, which makes it possible to decouple the orbital states from the libration states. The decoupling allows solving the orbital angular velocity analytically in a certain mode. Accordingly, the shrinking horizon model predictive control is used based on the advantages of the libration-free mode. Once the libration of the PSE is detected in the cargo transfer process, the control is switched to the prescribed-time control law to suppress the libration within one time step of the Controller I quickly, then the control is switched back to the shrinking horizon model predictive control. Numerical results demonstrate that the newly proposed libration-free switching control strategy is valid.

Compared to the MPC with the full-state dynamics, the computational efficiency of the new control strategy is much higher, due to the analytical solution of the orbital angular velocity and the linearization of the libration dynamics in the libration-free mode.

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Declarations

Conflict of interest The authors declare no conflict of interest in this article.

Code availability (software application or custom code) The custom codes in the current study are available from the corresponding author on reasonable requests.

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