ELECTROMAGNETIC DESIGN OF A RADIOFREQUENCY CAVITY

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Abstract. Electromagnetic and mechanical studies have been performed with the aim of build a RF cavity in the S-Band (2998 MHz), the design takes into consideration the relativistic change in the electron velocity through the acceleration cavity. Four cavity cases were considered at different input energies, 50 KeV, 100 KeV, 150 KeV and 200 KeV, with output energies from 350 KeV to 5 MeV, the designs show good acceleration efficiency and beam coherence comparable to the one created in the cathode.

1. Introduction

In recent years, Mexico has promoted the development of particle accelerators and radiation detectors. The lack of accelerator facilities in the country forced the researchers to perform most of the testing outside the country, one possible solution is to promote the design and construction of accelerators within the country aiming also for several industrial applications. A normal RF Linac (RFL) is under design and considered for accelerating an electron beam at 2998 MHz using the $\pi/2$ TM010 mode to accelerate in a stable condition, this design takes in to account the particle velocity within the system to set the cavity length to improve the acceleration efficiency.

The behavior of electromagnetic waves and how they resonates inside conducting walls depends strongly of the geometry itself, we designed a RF cavity geometry that enhance the particle acceleration using resonant electric fields, the code used to find the appropriated electric field to accelerate a single electron was POISSON SUPERFISH [1], with the obtained field we performed multiparticle simulations to quantify the electric field effect in the beam particle distribution including the energy gain. The goal was to find the field that preserve the beam emittance when it passes through the cavity. Therefore, an appropriated brightness is achieved when the beam is used for research activities like spectrometry of like secondary emission microscopy.

2. Cavity Dimensions

3. The electromagnetic resonance frequency $f$ in the cylindrical cavity can be obtained from the angular frequency $\omega$: 

The electromagnetic resonance frequency $f$ in the cylindrical cavity can be obtained from the angular frequency $\omega$: 

$$ f = \frac{\omega}{2\pi} $$
\[ \omega = k_r c = \frac{2.405 c}{R} \]  

(1)

Where R is the radius of the cavity, and \( \omega = 2\pi f \), and c the speed of light by setting the frequency f as fix we can calculate R [2].

The number of resonant modes will depend of the number of coupled cavities; the coupling is done ; the coupling is made by a small hole between them called the iris with a radius r. The iris radius needs to be smaller than the cavity radius in order to induce the different resonants in the cavity.

It is intended to acquire an RF generator in the S-band, 2998 MHz, with a 2 MW output power, setting this frequency as input, we can calculate the necessary radius for the cavity using the equation (1), then

\[ R = \frac{2.405 c}{2\pi f} \]  

(2)

It is important that have the same input frequency as the resonant frequency because is the one that maximize the power transmission to the cavity. There are another aspect that can impact in the generated electric field in the cavity as length and iris aperture between the multiple cavities.

In order to set an appropriated cavity length to accelerate electrons, it is necessary to take into account the relativistic speed of the electrons at the input of each cavity and take in to account the time dependence of the generated electric field within cavity to find the maximum electron acceleration.

For example, by considering two parallel infinite planes separated by a distance L with an applied sinusoidal voltage, taking z=0 as the cavity start, also assuming the electric field E in the planes to be uniform (we ignore the iris holes effect), then the field is directed in the z direction and is given by:

\[ E_z(t) = E_0 \cos(\omega t + \phi) \]  

(3)

Where \( \phi \) is the temporal phase at which the electron enters to the acceleration cavity [2],

Then the gained energy in the acceleration gap for a charge particle \( q \) that crosses it is given by the equation

\[ \Delta E = qE_0 \int_0^L \cos [\omega t(z) + \phi] \, dz \]  

(4)

Assuming that the energy gain is small enough that there is and negligible change in the electron speed through the cavity, then the total traveling time of the particle through the cavity is given by \( t(z) = \frac{z}{v} \), and then \( \omega t(z) = \omega \frac{z}{v} = 2\pi f \frac{z}{\beta c} \), now there is a direct connection between the time and the cavity frequency phase \( \omega t(z) \) of the wave that subsists inside the cavity.

In the case of three cavities coupled together there will be three resonance frequencies the \( 0, \frac{\pi}{2}, \) and \( \frac{\pi}{4} \) modes. The selected resonant acceleration mode is the \( \frac{\pi}{2} \) mode, then synchrony of the particles with the RF will be \( \omega t(z) = \frac{\pi}{2} \). So, if we choose the length of the cavity in the direction of \( z, z = L \), we can calculate the width of our cavity, if we know the energy of the electrons coming to the cavity.
\[ z = L = \frac{\beta c}{4f} \]  

(5)

Where \( \beta \) is the relativistic factor \( v/c \), once the basic cavity length is set the next steps is to make the electromagnetic calculations using simulations codes [3].

4. Electromagnetic Design

A 3-cell RF cavity has been designed to accelerate in the resonance mode because it is the one that presents the higher gradient accelerations. The electric field map density simulations results are shown in the Figure 2 where the iris connection between the cavities is set to 10 mm.

![Electric Field Distribution in the cavity center (r=0). The phase \( \phi \) is chosen in such way that the particles are accelerated by the greater part of the RF, if \( \phi \) is not chosen correctly the particles can be accelerated towards the source.](image)

Once we found the correct input phase \( \phi \) that maximize the time dependent component of the RF electromagnetic field that will interact with an electron traveling in the middle of the cavity \( r=0 \) we can set the maximum energy gain for that cavity design, Figure 3 shows the electric field experimented by the electron traveling in the cavity, as we can see the electric field is always positive leading to an optimum acceleration or energy gain.

![Electric Field in the RF Cavity for an input energy of 100 KeV, it can be observed that there is a large concentration of field in the lower corners of the first and third cavity.](image)
5. Beam Dynamics

Once the correct set of acceleration parameters are set for a single electron traveling inside the cavity the next step consist in improve the design to accelerate particle distributions, another important feature of an RF cavity is to minimize the growth of the beam longitudinal and transversal phase space area (figure 5,6), if the phase space area increase beam losses and the minimum beam size that can be focused increase, we perform a campaign of studies to improve the design and accelerate particle distributions instead.

The input beam for simulations is a 100 keV Gaussian profile (Figure 4) of 3 mm 1 σ r.m.s allowing to have particles close to the iris edges inside the cavity. We use the code Travel [4] to perform all the beam dynamics simulations.

![Figure 3](image1.png)

**Figure 3.** Cross section of the Beam at the end of the cavity. The hole in our cavity is approximately 1 cm in radius, which means that the particles do not collide with cavity walls.

The final transverse phase space shown in figure 5 has an emittance of $2.7796 \ 1 \sigma \ r.m.s$ $\pi \ mm.mrad$ with some particles showing divergence bigger than 50 mrad, this effect is due the iris edge effect than can be seen in (Figure 2), then by limit our input beam to 2 mm 1 σ r.m.s we can keep the particles divergences within our needs to have a homogeneous beam at the end of the acceleration system.

![Figure 4](image2.png)

**Figure 4.** Phase space result of the simulation. The beam divergence within the 5 mm radius shows a divergence below the 50 mrad indicating that the beam is suitable to our desired applications [5].
In the longitudinal dynamics the cavity can accelerate the particles properly the particles ±10 degrees within the energy spread of 2% that is the energy spread at the moment on introduce the particles to the acceleration cavity.

![Phase space energy distribution](image)

**Figure 5.** Phase space energy distribution in the longitudinal plane (vertical axis centered 500 KeV). The energy spread is 2% in the center of the acceleration bucket similar to the input beam energy spread.

With the obtained results from simulations we can set the transversal and longitudinal limits to for our input beam and also it works as a feedback loop to reshape the cavity in several iterations. Now the design need to be adapted to the machining tools that are available in the university workshop.

### 6. Mechanical Design

The cavity mechanical design was performed in the code Inventor AutoCAD[6] which allows us to study the mechanical strength and thermal loads of the cavity once the final design was set see Figure 6, the code also can generate G-code for the pieces to do the machining in a CNC 3-Axis Vertical Milling.

![Design of individual pieces](image)

**Figure 6.** a) Design of individual pieces and b) Assembly of parts. The pieces will be machined in copper, it is thought to make the assembly by welding.
At the moment of this workshop, we are finishing the machining of the first RF cavity and are foreseeing to finish all the cavity machining during this year (Figure 7).

Figure 7. Progress in the RF cavity machining, the pieces were created in a CNC 3-Axis vertical milling center, the polish procedure still missing.

7. Final Remarks

By taking into consideration the relativistic change in the electron speed through the acceleration cavity, we can optimize the acceleration efficiency of the single electron beam and improve the design to enhance the beam emittance in the cavity by reshaping the cavity taking into account the effect of the cavity in the beam distribution.

We are close to build the first RF cavity in Mexico, first step in the construction of the first High Energy RF accelerator (from 300KeV up to 5 MeV), with a high-brilliance beam.

References

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