Hairpin vortices in turbulent boundary layers

G Eitel-Amor¹, O Flores² and P Schlatter¹

¹Linné FLOW Centre, KTH Mechanics, Stockholm, Sweden
²Universidad Carlos III de Madrid, Madrid, Spain
E-mail: pschlatt@mech.kth.se

Abstract. The present work addresses the question whether hairpin vortices are a dominant feature of near-wall turbulence and which role they play during transition. First, the parent-offspring mechanism is investigated in temporal simulations of a single hairpin vortex introduced in a mean shear flow corresponding to turbulent channels and boundary layers up to $Re_\tau = 590$. Using an eddy viscosity computed from resolved simulations, the effect of a turbulent background is also considered. Tracking the vortical structure downstream, it is found that secondary hairpins are created shortly after initialization. Thereafter, all rotational structures decay, whereas this effect is enforced in the presence of an eddy viscosity.

In a second approach, a laminar boundary layer is tripped to transition by insertion of a regular pattern of hairpins by means of defined volumetric forces representing an ejection event. The idea is to create a synthetic turbulent boundary layer dominated by hairpin-like vortices. The flow for $Re_\tau < 250$ is analysed with respect to the lifetime of individual hairpin-like vortices. Both the temporal and spatial simulations demonstrate that the regeneration process is rather short-lived and may not sustain once a turbulent background has formed. From the transitional flow simulations, it is conjectured that the forest of hairpins reported in former DNS studies is an outer layer phenomenon not being connected to the onset of near-wall turbulence.

1. Introduction

Notable progress has been made over the last decades in understanding the physical processes involved in turbulent boundary layers (TBL). We have a relatively complete understanding of the kinematics of the whole TBL, from the near-wall or buffer region to the outer region. The statistics of velocity and vorticity in turbulent flows have been extensively studied, and were reviewed recently in a number of articles, see e.g. Refs. [23, 25, 35]. However, our understanding of the dynamics of TBL is more limited [15].

The region of the TBL where we have the most complete understanding of the dynamics of the flow is the buffer region. Since their first visualisation by Kline et al. [24] the near-wall streaks with alternating low and high streamwise momentum have been studied thoroughly, confirming their spanwise spacing of about 100 viscous units $\ell_\ast$. Jeong et al. [14] applied the $\lambda_2$ criterion for vortex eduction to this near-wall region, to conclude that the dominant near-wall structures are staggered quasi-streamwise vortices, surrounding the low-speed streaks. The processes that relate vortices and streaks were studied using a number of different techniques and models [10, 16, 17, 34]. There is now general agreement in that the streaks and the vortices are involved in a self-sustaining non-linear cycle, with a period of about 400 viscous time units. The sinuous instability of the streaks results in the formation of new quasi-streamwise vortices, that interact with the mean shear to generate new streaks. This non-linear cycle is autonomous and
takes place even in the absence of an overlying turbulent flow [18], and it is weakly modulated by the intensity of the overlying turbulent structures [12, 13].

On the other hand, the dynamics of the structures populating the logarithmic and outer regions of TBL are less understood. There is some agreement in the predominant role of Townsend’s attached eddies [38], although there are several models advocating for different candidates for these eddies. Probably the most widespread model is the hairpin vortex paradigm, which advocates for attached loop-like vortices, the so-called hairpin vortices. Originally proposed by Theodorsen [36], indirect evidence of these specifically shaped vortices has been found by many in experiments [2, 11, 27] and simulations [5, 28]. There exists, however, no unique quantitative definition of a hairpin, rather they describe a category of vortices consisting of a (negatively rotating) head, oriented in the spanwise direction, and two adjacent legs in the upstream and wall-ward direction [11]. From a dynamical point of view, these hairpins grow by merging [37] and by the so-called parent-offspring regeneration mechanism [40]. It is then thought that these conglomerates of similarly-oriented eddies might be responsible for the large-scale structures living in the outer region of the boundary layer [2]. In addition to complete hairpin vortices, non-symmetric incomplete variants (hooks and canes) have been found in the analysis of direct numerical simulation (DNS) data by Robinson [28].

Besides the popularity of hairpin model of TBLs, there are some open questions regarding their relevance in fully developed turbulence. Even if hairpin vortices are very common in various types of transitional flows, for example they occur in classical K and H-type transition [1, 19, 22], their predominance in fully turbulent boundary layers is unclear. In a recent simulation by Wu & Moin [39], a simulated boundary layer was shown to be densely packed with hairpin vortices. The authors classified this as the “forest of hairpins”. However, the region investigated was very close to the point of laminar-turbulent transition — in their case via travelling patches of free-stream disturbances — so it can hardly be considered an aspect of fully turbulent flow without further investigation. From stability theory it is known that a hairpin vortex created by transitional mechanisms such as e.g. wall-normal shear-layer roll-up or varicose streak instabilities, will advect downstream until it is damped by viscous effects or other (stronger) dynamics [3, 29]. Indeed, recent DNS of TBL reaching up to $Re_\theta = 4300$ [31] show no forest of hairpins downstream of transition, as opposed to Wu & Moin [39].

Besides their predominance, the regeneration process of hairpin vortices is also questionable. Zhou et al. [40] showed using DNS that an initial hairpin vortex in a laminar environment, created from a Q2 event, quickly generates a growing hairpin packet which encloses a low-speed region. Later, Kim et al. [21] found the packet-generation mechanism to be robust in the presence of background disturbances. They performed simulations where a hairpin was released in a channel flow with a turbulent mean profile disturbed by artificial random noise. Furthermore, the authors observed regeneration of a hairpin introduced in a velocity field extracted from DNS. However, the amplitude of the random noise was only about 15% of the rms known from DNS and the disturbances were not purely dissipative as one would predominantly expect in turbulence. In the case of the DNS flow field, the observation time was very limited, such that only the early stages of regeneration could be observed. The question remains of whether a dissipative background, like the one existing in a real turbulent shear flow, prevents any regeneration processes.

The aim of the present study is to determine and quantify the dynamics of the growth and interaction of hairpin vortices in TBL and to evaluate how these structures contribute to statistical quantities like mean flow and fluctuations. These findings will help answering the question whether hairpin-like structures represent a universal flow structure in TBL, and if they could consequently be used as a valid low-order model for boundary layer turbulence.

This report is split into two main parts. First, a single, artificially generated hairpin vortex is introduced in a mean shear flow and tracked while evolving downstream. A spectral flow
solver is employed in perturbation-mode which allows to maintain the base flow for all times and to choose between nonlinear and linear computations. A channel flow as well as a boundary layer with a turbulent mean profile are considered. The hairpin vortex is created by a localized volume forcing which offers more flexibility for follow-up studies. The observed dynamics will provide new details about the lifetime of the streamwise and spanwise vortex constituents in a realistic environment. Additionally, possible regeneration and package-creation processes will be investigated in the context of TBL. To investigate the effects of background noise, an eddy-viscosity approach is followed in this work which has been proposed by Flores [8]. In that regard, the first part of the present work can be viewed as an extension of published work on the evolution of hairpins in laminar flows [1, 9, 21, 40].

In the second part, the transitional region of a boundary layer is simulated using an adapted trip forcing which generates a regular pattern of hairpins with low-amplitude random modulations. The approach is partly similar to the so-called “synthetic turbulence” experimentally investigated by Coles and Barker [7] and Savas and Coles [30]. The obtained data is then analysed to answer a number of questions, e.g. how long individual hairpin vortices exist, whether span-/streamwise connection mechanisms can be observed (see [2]), and how the so-called forest of hairpins develops. Furthermore, the proposed set-up may serve as a standard flow case providing a reproducible boundary layer flow for further studies of transitional flow.

2. Hairpin vortices in parallel shear flow

The behaviour of a hairpin vortex in a mean shear flow is studied in the following. First, the numerical set-up and the employed forcing is described. Then, results are presented for a hairpin being tracked in an undisturbed mean flow extending the former study by Zhou et al. [40] with respect to higher Reynolds numbers and longer observation times. Furthermore, a temporal boundary layer simulation was performed to examine the effect of a slightly varied mean flow. In a second step, the effect of a turbulent background is included, whereas, in contrast to Kim et al. [21], the mean dissipative effect of turbulent fluctuations is accounted for. Both linear and nonlinear simulations are considered to examine the importance of nonlinearity for the regeneration process.

2.1. Numerical setup

In order to reproduce the hairpin regeneration process, a simulation setup similar to the one used by Zhou et al. [40] was used. The streamwise, wall-normal, and spanwise directions are $x$, $y$, and $z$ respectively. All computational domains were homogeneous in the streamwise and spanwise direction, i.e. periodic boundary conditions were imposed and Gauss–Lobatto collocation points were used in the wall-normal direction. The details of the computational domains are given in Tab. 1. Tests with an increased box size clearly showed that this change had no remarkable influence on the results. Numerical integration was performed using the spectral code SIMSON [6] in perturbation formulation (see Appendix), such that a turbulent mean profile could be set as a time-invariant base flow. Mean velocity profiles were taken from DNS data [20, 26], i.e. the simulation was started as a undisturbed flow with modified velocity profile. It should be stressed that the mean flow was fixed in the present study in contrast to former simulations by Zhou et al. and Kim et al. [21, 40].

The disturbance, which is required to develop a hairpin vortex, was introduced by a transient local volume forcing, causing a deceleration and ejection of fluid near the wall. This distinguishes the present study from the one by Zhou et al. [40] who employed a conditionally averaged Q2 event extracted from DNS data. However, the velocity amplitudes produced by the present forcing are quite similar to those reported by Zhou et al. [40], and the vortex structures possesses all the features described in their work. The exact form of the forcing term was derived in two
Figure 1. Distribution of the volume forcing used to produce a single hairpin vortex as described by Eq. 1. (a) top view; (b) lateral view for the plane marked by the dashed line in (a).

Table 1. Parameters for the temporal simulations of a hairpin in a mean shear flow. The channel half-height is \( h \) and the boundary layer thickness is \( \delta_{99} \). The grid resolutions \( \Delta x \), \( \Delta y \), and \( \Delta z \) are obtained from the number of collocation points in physical space.

| \( Re_{\tau} \) | Box size | Number of spectral modes | \( \Delta x^+ \) | \( \Delta y_{max}^+ \) | \( \Delta z^+ \) |
|---|---|---|---|---|---|
| 180 | \( 4\pi h \times 2h \times \frac{4}{3} \pi h \) | \( 128 \times 129 \times 128 \) | 11.8 | 4.4 | 3.9 |
| 590 | \( 2.5\pi h \times 2h \times 2h \) | \( 256 \times 257 \times 256 \) | 12.1 | 7.2 | 3.1 |
| 501 | \( 2.5\pi \delta_{99} \times 2\delta_{99} \times 2\delta_{99} \) | \( 256 \times 301 \times 256 \) | 10.3 | 5.2 | 2.6 |

steps. First, the amplitudes were chosen in order to resemble the velocity perturbation described by Zhou et al. In a second step, the spatial and temporal dimensions and gradients were tuned until a loop-vortex developed quickly from the initial counter-rotating vortex pair. It was found that a very short and intense pulse with an increased shear on the top layer gave the best results. The final form of the forcing is given by

\[
F_i = C_i \cdot \left[ -\left( \frac{z-z_0}{L_z} \right)^2 + 1 \right] \cdot \exp \left[ -\left( \frac{x-x_0}{L_x} \right)^2 - \left( \frac{y-y_0}{L_y} \right)^4 - \left( \frac{z-z_0}{L_z} \right)^2 - \left( \frac{t-t_0}{T} \right)^2 \right], \tag{1}
\]

with \( C^+ = (-4.84, 2.42, 0)^T \), \( L_x^+ = 34, L_y^+ = 75.6, L_z^+ = 54 \), and \( T^+ = 1.4 \). The values \( x_0/h = 0.5, y_0 = 0, \) and \( z_0 = 0 \) define the location of maximum forcing, where \( h \) is the channel half-height. The factor in the first parenthesis in Eq. 1 causes a weak inverse forcing on the lateral sides of the main peak which supports the onset of a counter-rotating vortex pair (compare Fig. 1). The fourth-order decay in \( y \)-direction leads to a stronger shear enhancing the evolution of the first hairpin head. Changing the parameters within a range of \( \pm 10\% \) yields very similar structures with minor differences regarding the shape of the hairpin head and the subsequent regeneration process. However, an insufficient amplitude \( C \) prevents the initial hairpin generation, whereas the threshold value strongly depends on the time coefficient \( T \). It should be noted that the vortex evolution discussed below is not specific to the exact choice of parameters, but rather very robust.
2.2. Undisturbed background

In the following, results are presented for a single hairpin vortex in a shear flow using both nonlinear and linear solution methods. First, the nonlinear case is discussed. In Fig. 2 the vortical structures resulting from the above described forcing are shown for Reynolds numbers $Re_\tau = 180$ and 590 at $t^+ = 300$. The origin of the time axis was chosen to be $t_0$, at which the initial forcing peaks (see Eq. 1). For both Reynolds numbers, the largest loop-like vortex is the one caused by the initial perturbation and is therefore denoted as primary vortex. The primary vortex is stretched by the mean shear and grows away from the wall. At $Re_\tau = 180$, the vortex head extends above the channel’s centreline at $y^+ = 180$ and is therefore bent backward. Each of the primary vortices is followed by two smaller upstream vortices induced by the first one. Using the scaled forcing described above, the obtained vortical structures also scale perfectly in inner units as evident from the figure.

Similar simulations were performed using a mean velocity profile for a zero-pressure gradient turbulent boundary layer. Here, the mean velocity was extracted from a DNS of a spatially developing TBL [33] at $Re_\tau = 500$ and implemented as a base flow with $U(y)$ and $V = 0$. Since the boundary layer is not growing in the $y$-direction, incompressibility is fulfilled. In contrast to the former simulations, a free-stream boundary condition was imposed at the upper $y$-boundary. As shown in Fig. 3, the resulting structures look very similar to those from the
channel case. Slight deviations are due to differences in the convection velocity between channel and boundary layer, with the difference becoming more evident further downstream. To quantify the temporal change of the vortical structures, the rms of the spanwise vorticity $\omega_{z,rms}$ for the whole domain, may serve as an indicator for the strength of the vortex heads. For the channel flow at $Re_T = 590$, $\omega_{z,rms}$ is plotted over time in Fig. 4(a). Directly after the injection of wall-normal momentum, there is a short increase of $\omega_{z,rms}$, which corresponds to the connection of the initial counter-rotating vortex pair. Later on, $\omega_{z,rms}$ drops monotonically but the curve flattens $t^+ > 300$, where the hairpin regeneration has come to an end.

The ensemble averaged Reynolds stress $\langle -u'v' \rangle$ passes a minimum at about $t^+ = 30$ and then grows until it reaches a maximum at about 1000 viscous time units (see Fig. 4(b)). The transient hairpin regeneration is notable from a distortion in the growth of $\langle -u'v' \rangle$ around $t^+ = 100$. For times $t^+ > 1000$, $\langle -u'v' \rangle$ slowly decays whereas the lifetime of the vortical structures is large compared to, e.g., the period of the bursting cycle in the buffer layer which is known to be about 400 viscous time units [17]. It is likely that the lifetime of the investigated vortical structures would decrease in a turbulent flow field where interactions with surrounding vortices prohibit an undisturbed growth. It should be remarked that Zhou et al. [40] observed a monotonic growth in $\langle -u'v' \rangle$ since their simulations covered a much shorter period of $t^+ = 300$. However, the initial behaviour was very similar to the present findings.

Repeating the above described simulation in linear mode proves that the hairpin regeneration is a nonlinear mechanism (cf. Figs. 4(a) and 4(b)). That is, only one initial hairpin is formed in the linear case, which propagates downstream. The vortex is elongated in the streamwise direction and no self-induction takes place. Therefore, after an initial peak $\omega_{z,rms}$ and $\langle -u'v' \rangle$ decay exponentially.

2.3. Dissipative background
Since in a developed turbulent flow each vortical structure interacts not only with the mean shear and itself but also with all surrounding fluctuations, a more realistic investigation of the regeneration process should invoke those effects. Kim et al. [21] extended the study of Zhou et al. [40] by including random noise in their channel simulations. Additionally, the initial hairpin was released in a velocity field extracted from DNS. However, the amplitude of the random
noise was only about 15% of the rms known from DNS and the disturbances were not purely dissipative. In the case of the DNS flow field, the observation period was strongly limited, such that only the early stages of regeneration could be observed. Nevertheless, the authors concluded that hairpin regeneration is a robust and sustaining process also in fully developed turbulent flow.

In Sec. 2.2 it has been shown in nonlinear simulations that a hairpin packet is formed after a rather short period of time ($t^+ < 200$) with a slow subsequent decay. It appears most likely that a dissipative turbulent background would accelerate the decay and, furthermore, nonlinear effects giving rise to hairpin regeneration should be damped or even inhibited. To investigate which influence a turbulent environment has on the hairpin regeneration process it is feasible to take into account the average dissipative effect of turbulent stresses, namely the turbulent viscosity $\nu_t = -\langle u'v' \rangle / \partial U/\partial y$. This method is limited in the sense that backscatter of turbulent energy from small to large scales is not accounted for.

In the following, the channel flow at $Re_\tau = 590$ from Sec. 2.2 is re-simulated with a total kinematic viscosity of $\nu(y) = \nu_0 + a \nu_t(y)$, where $\nu_0$ is the molecular viscosity, $\nu_t$ is extracted from DNS [26], and $a$ is a factor between 0 and 1. The respective trend of $\langle -u'v' \rangle$ is plotted in Fig. 5 for a gradually increasing factor $a$. For comparison, a linear simulation was performed in each case as well. It is clearly evidenced that an increasing turbulent viscosity strongly damps the hairpin vortices. For $a = 0.02$ one hairpin regeneration takes place, but the structures decay rapidly. Only 10% of $\nu_t$ are sufficient to completely suppress the regeneration process. The nonlinearity of the regeneration process is underlined by the fact that for increasing values of $a$ the curves for nonlinear computations more and more approach the linear solution in which no regeneration was observed. For $a \geq 0.1$ the curves for $\langle -u'v' \rangle$ almost collapse for the nonlinear and linear case (see Fig. 5). Relating back to the results of Zhou et al. and Kim et al. [21, 40], it appears that the lifetime of hairpin packets and the importance of the parent-offspring process have been overestimated in former studies. It is rather likely that hairpin reproduction occurs only for exceptionally strong loop-vortices on a very short timescale and that these structures are not eligible to sustain turbulent motion.

3. Transitional boundary layer
To investigate how hairpin vortices contribute to transition in a boundary layer and how they evolve downstream, a regular pattern of hairpins is introduced into a laminar boundary layer.
To this end, a spatially developing boundary layer is simulated in a computational domain of size $500 \delta_0^* \times 40 \delta_0^* \times 60 \delta_0^*$, with $\delta_0^*$ being the displacement thickness at the inlet, and $1024 \times 257 \times 256$ spectral collocation points. At the inflow, a Blasius profile is prescribed and a fringe region at the end of the domain ensures streamwise periodicity. The numerical approach is identical to the one used in former DNS studies \cite{31, 33} and recently Schlatter and Örlü reviewed the effect of several tripping techniques and inflow lengths on the development of wall turbulence \cite{32}. In the present study, the trip forcing is modified to place a space-filling spanwise row of six “hairpin emitters” close to the inlet at $x = 25 \delta_0^*$, using the ejection scheme described in Sec. 2.1. To be more precise, at each spanwise position an ejection was generated repeatedly with a time period $T_{hp} = 38 \delta_0^*/U_\infty$, where $U_\infty$ represents the free stream velocity. The time interval $T_{hp}$ was chosen long enough to produce separated hairpin packets with a slight random variation of the ejection time which breaks the symmetry of the simulation. The time of ejection was allowed to vary by an uncorrelated random shift $t_s$ with $|t_s| < T_{hp}/3$ for each hairpin created. The forcing parameters according to Eq. 1 are taken from the simulation of a temporal boundary layer in Sec. 2.2 and, in units of $\delta_0^*$ and $U_\infty$, read: $C = (-0.19, 0.96, 0)^T \cdot (\rho U_\infty^2/\delta_0^*), L_x = 1.80 \delta_0^*, L_y = 3.78 \delta_0^*, L_z = 2.70 \delta_0^*$, and $T = 1.56 \delta_0^*/U_\infty$.

The developed flow field after approximately three turnover times $500 \delta_0^*/U_\infty$ is shown in Fig. 6. At the inlet, where the flow is still laminar, the six lines of initial hairpins are clearly visible. They undergo a short period of regeneration with secondary and tertiary hairpins while being elongated in the direction of the mean flow. The vortex heads immediately grow towards the edge of the boundary layer and even further. At this stage, which is located around $Re_\theta = 300$, the initial hairpin heads almost disappeared (see Figs 7a and 7b). While the initial

Figure 6. Top view of the synthetic boundary layer going through stages of transition. Flow from left to right with initial hairpin vortices in the laminar region. (a) isocontours of $\lambda_1^+ = -7 \cdot 10^{-3}$; (b) isocontours of $\lambda_2^+ = -26 \cdot 10^{-3}$ and velocity contours for $u'(x, y, z) = u(x, y, z) - U(y) = -0.18 U_\infty$; (c) velocity contours for $u'(x, y, z) = -0.18 U_\infty$. The white bars demonstrate a spanwise width of 100 local wall units $\ell_\star$. 
structures are dissipated in the outer layer, there is new dynamics emerging beneath. Namely, secondary low-speed regions, which develop between the original hairpin trails, become sinuously unstable for $Re_\theta > 300$ and burst violently around $Re_\theta = 370$ (see in Figs. 6b and 6c). These unstable streaks shed a large number of vortical structures that grow to the wake region of the boundary layer. At the same time, the near-wall streaks reorganise and establish the pattern well-known for wall turbulence with a streak spacing of about 100 viscous units. From Fig. 6(b), it becomes obvious that the vortical structures near the wall are oriented in a quasi-streamwise manner, while the outer layer structures possess an arch-like shape with a certain amount of spanwise rotation. Depending on the perspective of the observer and on the contour level used for vortex visualisation, these outer structures might give the impression of a forest of hairpins (compare Figs. 7c and 7d). A closer look reveals that the arch-like structures in the outer layer are neither connected to the wall nor to the quasi-streamwise vortices in the near-wall region. Hence, a qualitative analysis of the present data implies that the flow is not dominated by wall-attached hairpins beyond $Re_\theta = 350$, which shall be further investigated in the following.

A snapshot of the fluctuations of the spanwise vorticity $\omega'_z = \omega_z - \langle \omega \rangle$ is presented in Fig. 8. To reveal the different behaviour of the flow in the initial hairpin trails and between those, two $xy$-planes are shown at the respective positions. The plane at $z/\delta^*_0 = 5$ is centred in one of the hairpin emitters and heads of the corresponding hairpins can be traced up to $x/\delta^*_0 \approx 125$, where a connection to the wall is no longer evident (compare Fig. 7b). At $z = 0$, the secondary low-speed streak, visible as a red area close to the wall, is fully developed at $x/\delta^*_0 = 100$ ($Re_\theta = 255$) and its instability reaches a maximum amplitude at approximately $x = 175 \delta^*_0$ ($Re_\theta = 360$). New arch-like vortices are shed from this point into the outer region, which becomes much
Figure 8. Instantaneous spanwise vorticity fluctuations $\omega'_z = \omega_z - \langle \omega \rangle$ in two $xy$-planes. (top) in the centre of an initial hairpin trail at $z = 5$; (bottom) between two initial hairpin trails at $z = 0$. Clockwise rotation is defined negative, therefore regions of high shear appear coloured in blue. The black line indicates the enstrophy contour $\omega^2 = 5 \cdot 10^{-4} (U_\infty / \delta^*_0)^2$ with $\omega$ being the full vorticity vector.

Figure 9. Covariances in the $xy$-plane for the transitional boundary layer. (top) turbulence intensity $\langle u'u' \rangle$; (middle) Reynolds stress $\langle -u'v' \rangle$; (bottom) variance of the spanwise vorticity $\langle \omega'_z \omega'_z \rangle$. The white lines mark the boundary layer thickness given by $u = 0.99 U_\infty$ and $\langle \omega_z \rangle^2 = 5 \cdot 10^{-4} (U_\infty / \delta^*_0)^2$, respectively.

more intermittent. Close to the wall, the newly formed streaks produce numerous wall-attached eddies.

Averaging the flow field in time and spanwise direction yields the distribution of the turbulence intensity $\langle u'u' \rangle$, the Reynolds stress $\langle -u'v' \rangle$, and the variance of the spanwise vorticity $\langle \omega'_z \omega'_z \rangle$, which are summarized in Fig. 9. The upfloating vortex heads at the inflow are visible in all three quantities as a weak outer maximum for $50 < x/\delta^*_0 < 100$. Especially the enstrophy contour indicates that the initial hairpins get lost in the wake region. Crucial for the subsequent transition are the streaks near the wall and their growing instability which manifests itself in a global maximum of $\langle u'u' \rangle$ at $x/\delta^*_0 \approx 150$. In turn, the vortices emitted in this region produce a maximum of the Reynolds stress $\langle -u'v' \rangle$ at $x/\delta^*_0 \approx 180$ and from that point the boundary layer...
thickness increases rapidly. For \( x/\delta_0^* > 200 \), no remarkable activity is found in the wake region and \( \langle u'u' \rangle \) and \( \langle -u'v' \rangle \) approach the expected \( y \)-dependence for developed wall turbulence. It can be concluded from Fig. 9 that the near-wall cycle sets in at \( x/\delta_0^* \approx 200 \) and that the arch-like vortical structures observed on top of the boundary layer (cf. Fig. 7c) are very weak compared to the near-wall dynamics. Focusing on integral and mean quantities, it becomes apparent that the boundary layer passes through transition rather quickly. Figs. 10(a) and 10(b) show the growth of the momentum thickness and the development of the mean velocity profile. Beyond \( x/\delta_0^* = 200 \), the turbulent mean velocity is well developed and \( Re_\theta(x) \) increases linearly up to approximately \( x/\delta_0^* = 400 \) where the fringe regions starts to take effect.

The shape factor \( H_{12} = \delta^*/\theta \), shown in Fig. 11, confirms that the mean velocity profile possesses the turbulent characteristics for \( x/\delta_0^* > 200 \) whereas the friction coefficient \( c_f = 2\tau_w/(\rho U_\infty^2) \) approaches the value expected for wall turbulence asymptotically after a brief overshoot.

Interestingly, during the rise of \( c_f \) there is an intermediate plateau while the secondary streaks

![Figure 10](image1.png)  
**Figure 10.** Synthetic boundary layer going through stages of transition. (a) Streamwise development of the momentum thickness Reynolds number \( Re_\theta \). The grey area marks the fringe region; (b) Mean velocity profiles at four streamwise positions indicated by dashed lines in (a) with corresponding colours. The grey lines indicate the law of the wall with the constants \( \kappa = 0.384, B = 4.17 \) for the logarithmic region.

![Figure 11](image2.png)  
**Figure 11.** Synthetic boundary layer going through stages of transition. (a) Shape coefficient \( H_{12} = \delta^*/\theta \); (b) Friction coefficient \( c_f \) from present data (solid line) compared with the Blasius solution (lower dashed line) and the Coles-Fernholz approximation for the turbulent case [4] (upper dashed line).
are formed \((60 < x/\delta^*_0 < 110)\) and \(c_f\) grows further as the streaks become unstable.

4. Conclusions
Simulations of a hairpin vortex in a mean shear flow evidenced the regeneration of hairpin vortices and proved the nonlinear nature of the parent-offspring mechanism. In contrast to former studies, the observation time was largely extended, revealing that nonlinear self induction is very short-lived, \(t^+ = 300\), the structures decay constantly. The assumption that hairpin regeneration cannot sustain in turbulent flows is further supported by simulations including a turbulent viscosity. Even a small amount of 10\% background dissipation completely suppresses the generation of new hairpins.

In a second part, a transitional boundary layer with an adapted hairpin trip forcing was set up. Using various flow field visualisations, it was shown that the initial hairpins do regenerate briefly before they are absorbed in the wake region. Therefore, it is very unlikely that hairpin vortices persist in fully developed turbulent boundary layers. Instead, the actual transition is driven by secondary low-momentum streaks which develop a growing instability and finally initiate the near-wall dynamics that sustain turbulence. This behaviour complies with mechanisms known from stability theory, which predicts transient growth of sinuous instabilities while varicose perturbations are less stable \([3, 34]\). Arch-like vortices were observed in the wake region, which may be associated with the reported phenomenon of a forest of hairpins. However, how these vortices are formed and what causes their absence at higher Reynolds numbers remain open questions.

Since the boundary conditions and the trip forcing used are reproducible, the presented boundary layer simulation could be a valuable tool for further investigations of early stages of transition in the sense of a synthetic boundary layer flow. Variation of the streamwise position of forcing and the number and strength of initial hairpin vortices would allow for a more detailed study of the processes that lead to the onset of wall-turbulence and the formation of large-scale structures.

Acknowledgments
This work was supported in part by the Multiflow program of the European Research Council. The authors would like to thank Adrián Lozano-Durán for his insightful comments on the manuscript. Additional financial support is acknowledged from the Göran Gustafsson Foundation. Computer time was provided by SNIC (Swedish National Infrastructure for Computing).

Appendix: Perturbation mode formulation
In the following, the concept of the perturbation-mode simulations is discussed in more detail. The basic principle is to decompose the velocity field into a constant base flow on the one hand and time dependent deviations from the base flow (perturbations) on the other hand. This approach eases stability analyses since distinct perturbations and their evolution can be studied in any kind of base flow. In the case of laminar channel flow, the Poiseuille profile would serve as the physically correct base flow and at the same time it represents as a stable (attracting) state of the system. That is, any artificial perturbation will be damped until the laminar profile is restored. If the Reynolds number is high enough and the initial perturbations possess certain characteristics, another stable state of the system, the turbulent regime, can be established and maintained. It is important to note that, in the turbulent state, the base flow is no longer equivalent to the mean flow. As long as, for a given geometry, the base flow is the solution of the (steady) Navier–Stokes equations, the perturbation formulation is identical to the original non-decomposed equations of motion. However, in the present work the turbulent mean profile was used as a base flow, which is not a solution of the steady Navier–Stokes equations since the
Reynolds stress terms are missing. To justify this approach, we will explain the basic concept of the perturbation formulation and discuss the consequences of using a non-physical base flow.

In the incompressible Navier–Stokes equations,

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j}, \quad (A.1)$$

the velocity can be split into a base flow $U_i$ and a perturbation $u'_i$ component

$$u_i(x, t) = U_i(x) + u'_i(x, t). \quad (A.2)$$

Rewriting Eq. (A.1) for this decomposed velocity field yields

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} + \frac{\partial u'_i}{\partial t} + U_j \frac{\partial u'_i}{\partial x_j} + u'_j \frac{\partial U_i}{\partial x_j} =$$

$$-\frac{\partial P}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{\partial p'}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u'_i}{\partial x_j \partial x_j}. \quad (A.3)$$

Subtracting the pure base flow contributions, we end up with the equations of motion in perturbation form

$$\frac{\partial u'_i}{\partial t} + u'_j \frac{\partial u'_i}{\partial x_j} + U_j \frac{\partial u'_i}{\partial x_j} + u'_j \frac{\partial U_i}{\partial x_j} =$$

$$-\frac{\partial p'}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u'_i}{\partial x_j \partial x_j}. \quad (A.4)$$

If the time-invariant base flow $U_i(x)$ fulfills

$$U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 U_i}{\partial x_j \partial x_j}, \quad (A.5)$$

then Eq. (A.4) describes the same physics as Eq. (A.1). It should be remarked that nonlinear interactions can easily be cancelled out by neglecting the second term on the left-hand side of Eq. (A.4). If a base flow is employed, that is not a Navier–Stokes solution, one inherently introduces a residual $R_i$ fulfilling

$$U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 U_i}{\partial x_j \partial x_j} + R_i \quad (A.6)$$

and therefore one solves for

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j} + R_i \quad (A.7)$$

instead of Eq. (A.1). In other words, $R_i$ is incorporated as an implicit forcing, which is not explicitly calculated but relaxes the base flow towards the prescribed form. This is similar to the procedure of manually resetting the base flow in every timestep, or explicitly calculating Reynolds stresses and including them in the equations.
References

[1] Acarlar MS and Smith CR 1987 A study of hairpin vortices in a laminar boundary layer. Part 2. Hairpin vortices generated by fluid injection J. Fluid Mech. 175 43–83
[2] Adrian RJ 2007 Hairpin vortex organization in wall turbulence Phys. Fluids 19 041301
[3] Brandt L, Schlatter P and Henningson D 2004 Transition in boundary layers subject to free-stream turbulence J. Fluid Mech. 517 167–198
[4] Chauhan KA, Monkewitz PA and Nagib HM 2009 Criteria for assessing experiments in zero pressure gradient boundary layers Fluid Dyn. Res. 41 021404
[5] Chong MS, Soria J, Perry AE, Chacin J, Cantwell BJ and Na Y 1998 Turbulence structures of wall-bounded flows using DNS data J. Fluid Mech. 357 225–247
[6] Chevalier M, Schlatter P, Lundbladh A and Henningson DS 2007 SIMSON–A Pseudo-Spectral Solver for Incompressible Boundary Layer Flow Tech. Rep. TRITA-MEK 2007:07, Royal Institute of Technology, Stockholm, Sweden
[7] Coles D and Barker SJ 1975 Some remarks on a synthetic turbulent boundary layer In Turbulent Mixing in Nonreactive and Reactive Flows, pp. 285–292 (New York: Plenum)
[8] Flores O 2007 The Dynamics of the Outer Region of Wall-Bounded Turbulence PhD thesis, Universidad Politécnica de Madrid
[9] Haidari AH and Smith CR 1994 The generation and regeneration of single hairpin vortices J. Fluid Mech. 277 135–162
[10] Hamilton JM, Kim J and Waleffe F 1995 Regeneration mechanisms of near-wall turbulence structures J. Fluid Mech. 287 317–348
[11] Head MR and Bandyopadhyay P 1981 New aspects of turbulent boundary-layer structure J. Fluid Mech. 107 297–338
[12] Hoyas S and Jiménez J 2006 Scaling of the velocity fluctuations in turbulent channels up to Reτ = 2003 Phys. Fluids 18 011702
[13] Hutchins N and Marusic I 2007 Large-scale influences in near-wall turbulence Phil. Trans. Royal Soc. A 365 647–664
[14] Jeong J, Hussain F, Schoppa W and Kim J 1997 Coherent structures near the wall in a turbulent channel flow J. Fluid Mech. 323 185–214
[15] Jiménez J 2013 Near-wall turbulence Phys. Fluids 25 101302
[16] Jiménez J and Moin P 1991 Minimal Flow unit of near-wall turbulence J. Fluid Mech. 225 213–240
[17] Jiménez J, Kawahara G, Simens MP, Nagata M and Shiba M 2005 Characterization of near-wall turbulence in terms of equilibrium and bursting solutions Phys. Fluids 17 015105
[18] Jiménez J and Pinelli A 1999 The autonomous cycle of near-wall turbulence J. Fluid Mech. 389 335–359
[19] Kachanov YS 1994 Physical mechanisms of laminar-boundary-layer transition Annu. Rev. Fluid Mech. 26 411–482
[20] Kim J, Moin P and Moser RD 1987 Turbulence statistics in fully developed channel flow at low Reynolds number J. Fluid Mech. 177 133–166
[21] Kim K, Sung HJ, and Adrian RJ 2008 Effects of background noise on generating coherent packets of hairpin vortices Phys. Fluids 20 105107
[22] Kleiser L and Zang TA 1991 The late stages of transition to turbulence in channel flow J. Fluid Mech. 245 319–348
[23] Kline S, Reynolds WC, Schraub F and Runstadler P 1967 The structure of turbulent boundary layers J. Fluid Mech. 30 741–773
[24] Marusic I, McKeon BJ, Monkewitz PA, Nagib HM, Smits AJ and Sreenivasan KR 2010 Wall-bounded turbulent flows at high Reynolds numbers: Recent advances and key issues Phys. Fluids 22 065103
[25] Moser RD, Kim J and Mansour NN 1999 Direct numerical simulation of turbulent channel flow up to Reτ = 590 Phys. Fluids 11 943–945
[26] Perry AE and Chong MS 1982 On the mechanism of wall turbulence J. Fluid Mech. 119 173–217
[27] Robinson SK 1991 Coherent motions in the turbulent boundary layer Annu. Rev. Fluid Mech. 23 601–639
[28] Sandham ND and Kleiser L 1992 The late stages of transition to turbulence in channel flow J. Fluid Mech. 245 319–348
[29] Savas Ö and Coles D 1985 Coherence measurements in synthetic turbulent boundary layers J. Fluid Mech. 160 421–446
[30] Schlatter P and Örlü R 2010 Assessment of direct numerical simulation data of turbulent boundary layers J. Fluid Mech. 659 116–126
[31] Schlatter P and Örlü R 2012 Turbulent boundary layers at moderate Reynolds numbers. Inflow length and
tripping effects. *J. Fluid Mech.* **710** 5–34

[33] Schlatter P, Örlü R, Li Q, Brethouwer G, Fransson JHM, Johansson AV, Alfredsson PH, and Henningson DS 2009 Turbulent boundary layers up to $Re_\theta = 2500$ studied through simulation and experiment *Phys. Fluids* **21** 051702

[34] Schoppa W and Hussain F 2002 Coherent structure generation in near-wall turbulence *J. Fluid Mech.* **453** 57–108

[35] Smits AJ, McKeon BJ and Marusic I 2011. High-Reynolds-number wall turbulence *Annu. Rev. Fluid Mech.* **43** 353–375

[36] Theodorsen T 1952 Mechanism of turbulence *Proc. Second Midwestern Conference on Fluid Mechanics* Ohio State University, Columbus, OH, USA.

[37] Tomkins CD and Adrian RJ 2003 Spanwise structure and scale growth in turbulent boundary layers *J. Fluid Mech.* **490** 37–74

[38] Townsend AA 1961 Equilibrium layers and wall turbulence *J. Fluid Mech.* **11** 97–120

[39] Wu X and Moin P 2009 Forest of hairpins in a low-Reynolds-number zero-pressure-gradient flat-plate boundary layer *Phys. Fluids* **21** 091106

[40] Zhou J, Adrian RJ, Balachandar S and Kendall TM 1999 Mechanisms for generating coherent packets of hairpin vortices in channel flow *J. Fluid Mech.* **387** 353–396