Study on Mathematical Model of Water and Sediment in front of the Dam under Multi-control Boundary Conditions

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Abstract: Aiming at mathematical model of flow and sediment under multi-control boundary conditions in front of dams of hydropower station, a step-by-step solution method is proposed in this paper. In the first step, the influence of constraints such as the water intake of the power station is not considered. The water level control condition of the exit section is given according to the scheduling mode of the hydropower station, and the discharge is given by the inlet boundary. In the second step, the water level as the upstream boundary condition of the model according to the condition of the first step is obtained, and the discharge of the water intake of the hydropower station is defined as the downstream boundary condition. The mathematical model is used to perform simulation analysis on the Dashixia Reservoir. The analysis results show that the generator set, the middle desilting hole and the front area of emptying tunnel platform form scouring funnels after 50 years of operation, the sediments in front of the reservoir spillway are mainly flat silting and do not form flushing funnels. This method can reflect the influence of multiple constraint boundaries in front of the dam.

1. Introduction
There are many hydraulic constructions in front of the dam of the hydro-junction project including inlets of water diversion for power generation structure, shore spillways, ecological outlet holes, flood discharge and flush tunnels, and emptying tunnels. The main function of water diversion and power generation structure is water supply for power generation. According to the requirements of stratified water intake, water through power generation unit enters the river channel for downstream irrigation and achieves ecological water supply. Shore spillways as main outlet structure have over-discharge capacity and are responsible for discharging check flood, design flood and frequent flood. Ecological outlet holes discharge ecological flow to downstream at the initial water storage, and the initial operation has the function of emergent reduction and emptying the reservoir. Flood discharge and flush tunnel as a secondary drainage structure helping spillway can discharge the check flood, design flood and frequent flood. Emptying tunnels lowers the water level in an emergency and venting the reservoir to repair buildings such as dams. The flow pattern in front of the dam is related to the size design of the hydraulic structure and the selection of the layout point. Therefore, it is necessary to have a clear understanding of the flow pattern in front of the dam in the design process of hydro-junction. Hydraulic physical model test is often used for research in the actual project. In the model test, the water flow tests under different conditions are carried out to obtain the flow pattern in front of the dam. This method is more accurate, but the research period is longer. With the development of computers, many scholars adopt numerical simulation methods to simulate the flow patterns in front of hydraulic structures[1-3]. In addition, many new mathematical methods are also applied to the study of reservoir
scheduling, among which methods used often are fuzzy recognition theory\textsuperscript{[4-5]}, genetic algorithm\textsuperscript{[6-7]} and artificial neural network\textsuperscript{[8-9]}. The conventional numerical model only has two control conditions. Both the upstream boundary and the downstream boundary use water level or discharge as conditions. However, besides the upstream discharge condition and downstream water level condition, the water intake of hydropower station increases the orifice outflow at the downstream boundary. Because of the increase of constraint control, it is difficult to make full use of the existing control conditions according to the conventional mathematical model to simulate the flow pattern in front of the dam. Therefore, it is necessary to study the simulation method of water and sediment movement in front of dam under multiple constraints.

2. Method

2.1. Control equation

(1) The water flow continuous equation is expressed as:
\[
\frac{\partial Z}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0
\]  

(2) The water flow movement equation is expressed as:
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g \frac{\partial Z}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{u \sqrt{u^2 + v^2}}{h^{7/5}} n^2 g
\]
\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -g \frac{\partial Z}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{v \sqrt{u^2 + v^2}}{h^{7/5}} n^2 g
\]

(3) The basic equation for non-equilibrium sediment transportation under suspended load is expressed as:
\[
\frac{\partial (hs)}{\partial t} + \frac{\partial (uhss)}{\partial x} + \frac{\partial (vhss)}{\partial y} = \varepsilon \left[ \frac{\partial^2 (hs)}{\partial x^2} + \frac{\partial^2 (hs)}{\partial y^2} \right] + \alpha \omega (s_* - s)
\]

(4) The basic equation for non-equilibrium sediment transportation under bed load is expressed as:
\[
\frac{\partial (hs)}{\partial t} + \frac{\partial (uhss)}{\partial x} + \frac{\partial (vhss)}{\partial y} = \varepsilon \left[ \frac{\partial^2 (hs)}{\partial x^2} + \frac{\partial^2 (hs)}{\partial y^2} \right] + \alpha \omega (s_* - s)
\]

(5) The river bed deformation equation is defined as:
\[
\rho_s \frac{\partial Z}{\partial t} = \alpha \cdot \omega (s - s_*) + \alpha_b \cdot \omega_b (s_b - s_{b*})
\]

Where \( Z \) is the water level, \( h \) is the water depth, \( h=Z-Z_b \) and \( Z_b \) is the river bottom elevation. \( u \) and \( v \) are the components of the average flow velocity along the \( x \) and \( y \) directions. \( n \) is the roughness coefficient. \( s \) and \( s_* \) are the average sediment concentration and grouping sedimentation force of sediment along vertical direction under the non-uniform suspended load, respectively. \( \nu \) represents the turbulent viscosity coefficient. \( \varepsilon \) is the diffusion coefficient of sediment. \( \alpha \) is the recovery saturation coefficient. \( \rho_s \) is the dry weight of sediment. \( s_b \) and \( s_{b*} \) are expressed as the sediment concentration of the full water depth which the transportation rate and the effective transportation rate of sediment under bed load are converted into, respectively. \( \omega_b \) is the sedimentation coefficient of bed load. \( \omega \) is the sinking speed of bed load. In this paper, the coordinates from equations (1) to (5) are transformed to obtain and solve the equations in the orthogonal curvilinear coordinate system.

2.2. Processing method of multi-constraint boundary condition

The processing of boundary conditions with multiple constraints included the following steps:

(1) Regardless of the influence of the water intake of the power station, the discharge boundary condition was given by the inlet section of the model, and the exit section boundary adopted open
boundary condition, that is, the gradient of flow velocity was 0, and the water level control condition of the outlet section was given according to the scheduling mode of the hydropower station.

(2) Using the established two-dimensional mathematical model, the water level at the upstream boundary was calculated according to the boundary conditions in step (1).

(3) The water level calculated in step (2) was taken as the upstream inlet boundary condition of the model, and the discharge of the water intake of the hydropower station was taken as the downstream boundary condition. The planar two-dimensional mathematical model was used to simulate and obtain the flow pattern of the study area.

3. model verification

3.1. Overview of the study area

The Dashixia hydro-junction project is located on the Kumalak River, one of the source streams of the Aksu River in the Tarim River Basin in Xinjiang. The dam site is located 11km above the Dashixia Gorge exit at the junction of Wensu and Wushi County in Aksu area. It is about 37km away from the border between China and the Republic of Kyrgyzstan and about 100km away from Aksu city.

The dam site of the Dashixia hydro-junction project has a control basin area of 12,700 km² and a multi-year average runoff of 4.87 billion m³. It is recommended that the normal water storage level of Dashixia Reservoir is 1700m, the total storage capacity of the reservoir is 1.174 billion m³, Regulating storage capacity is 711 million m³, the flood control capacity is 78 million m³, the maximum dam height of the barrage is 247m, the installed capacity of the power station is 750MW, the output is guaranteed to be 86.1MW, the average annual electrical energy is 1.893 billion kW·h and annual utilization hours are 2524h. Figure 1 shows the three-dimensional topographic map in front of the dam. There are spillways, power stations, flood discharge tunnels, and emptying tunnels in front of the dam.

![Figure 1. the three-dimensional topographic map in front of the dam](image)

3.2. The model range

Two-dimensional model calculation range was selected from 37km in front of the dam to the dam site according to the topographic data of the dam area provided by the design organization. In order to reflect the river shape and hub layout in detail, the calculation area was divided by the body-fitted grid, and the construction areas such as spillway, flood discharge tunnel, power station set and emptying tunnel were partially encrypted and arranged 84,820 grids.

3.3. model verification

Figure 2 shows the ratio of the measured water level to the calculated water level under natural conditions. The calculation conditions were as follows: the upstream discharge was 154m³/s and the corresponding downstream water level was 1487.833 m. It can be seen from the calculation results that the calculated value of the model is close to the measured value, and the maximum difference is 7cm. It is preliminarily indicated that the roughness selected by the model and the model calculation
mode is in accordance with the actual situation.

4. Comparison of water surface lines along the channel

In order to analyze the calculation results of the two-step mode under multiple control boundary conditions, the mode in this paper was compared with the traditional one-step mode. Table 1 shows the water surface line values calculated by different modes, which were compared with the experimental results of the physical model. The calculation conditions were as follows: the inlet adopted discharge of 1260\(\text{m}^3/\text{s}\) once in two years, the water level in front of the dam was controlled at 1694m, the flood discharge tunnel was partially opened and the emptying tunnel was fully opened. It can be seen from the calculation results that the calculated water level of the mathematical model under the multi-control boundary conditions is higher than that of the experimental results of the physical model. It is mainly because the physical model experiment can reflect the influence of discharge and water diversion in front of the dam, and the flow velocity in the reservoir generally increases, while the planar two-dimensional mathematical model is difficult to reflect the influence of local water flow in front of the dam. Thus the calculated water level is higher than that of the experimental results of the physical model. Relatively speaking, the two-step calculation mode in this paper can initially reflect the change of flow velocity in front of the dam. The water level value calculated in this model is close to the experimental results of the physical model.

| distance from the dam (m) | Squat line (m) | experimental results of the physical model (m) | The mode in this paper (m) | Difference (m) | the traditional mode (m) | Difference (m) |
|--------------------------|---------------|-----------------------------------------------|---------------------------|---------------|-------------------------|---------------|
| 5212                     | 1502.409      | 1694.002                                     | 1694.004                  | 0.002         | 1694.015                | 0.013         |
| 15585                    | 1577.201      | 1694.005                                     | 1694.010                  | 0.005         | 1694.023                | 0.018         |
| 20742                    | 1614.008      | 1694.006                                     | 1694.010                  | 0.004         | 1694.026                | 0.020         |
| 25887                    | 1641.367      | 1694.012                                     | 1694.017                  | 0.005         | 1694.036                | 0.024         |
| 31168                    | 1666.122      | 1694.103                                     | 1694.107                  | 0.004         | 1694.132                | 0.029         |
| 33780                    | 1682.862      | 1694.319                                     | 1694.325                  | 0.006         | 1694.334                | 0.015         |
| 34820                    | 1692.597      | 1702.577                                     | 1702.582                  | 0.005         | 1702.657                | 0.080         |
| 35412                    | 1698.239      | 1706.985                                     | 1706.994                  | 0.009         | 1707.098                | 0.113         |
| 36005                    | 1703.738      | 1710.966                                     | 1710.975                  | 0.009         | 1711.086                | 0.120         |
| 36543                    | 1708.766      | 1715.410                                     | 1715.421                  | 0.011         | 1715.533                | 0.123         |
| 37081                    | 1713.768      | 1720.258                                     | 1720.271                  | 0.013         | 1720.370                | 0.112         |
5. Calculation and analysis of scouring and silting in front of the dam

In the mathematical model of the Dashixia Reservoir, the upstream boundary used a series of measured water and sediments, and the downstream boundary was controlled by the scheduling method of the Dashixia Reservoir. Figure 3 shows the sediment thickness and shape distribution in front of the dam after 30 years of the Dashixia Reservoir operation. It can be seen from the figure that the sediment siltation in front of the dam is relatively uniform, the siltation thickness in most areas is 3.4 m and the siltation thickness of the main trough is greater than that on both sides and at the dam. The siltation in front of the hydraulic structure is smaller, and the distribution pattern is closely related to the discharge condition of the building. The details are as follows: the siltation thickness of the platform in front of the power station is small with the thickness of 1.0-2.5m, the siltation thickness of the platform in front of the flush tunnel is small with the thickness of 1.0-2.8m, the siltation thickness of the platform in front of the emptying tunnel is small with the thickness less than 2m and the siltation thickness of the spillway platform is relatively uniform with the thickness of about 0.7m.

Figure 4 shows the sediment thickness and shape distribution in front of the dam after 100 years of reservoir operation. It can be seen from the figure that the siltation thickness in most areas in front of the dam exceeds 18m, and the siltation thickness in the deep shovel can reach 19.1m. The siltation thickness at the spillway is between 3m and 4m. During the operation of the reservoir, the generator set, flush tunnel and emptying tunnel can form scouring funnels. The length of the building along the water flow “front clearing” are 20m in front of the generator set and 10m in front of the 3# unit. The reason for the difference between the two length is mainly related to the scheduling mode. There are 10m in front of the middle flush tunnel and 12m in front of the emptying tunnel. After 100 years of reservoir operation, an obvious scouring funnel is formed in front of the building.

Figure 5 shows the front section position of the hydraulic structure, and Table 2 shows the morphological characteristics of scouring funnel in front of the hydraulic structure in different operating years. It can be seen from the table that although there is siltation in front of the building in the first 30 years but the siltation thickness is small and the slope is small, which can be considered that the scouring funnel is not formed. After 50 years of operation, the scouring funnel is gradually formed in front of the dam with the increase of the maximum siltation thickness, and the slope of the funnel gradually raises with the increase of the operation year of the reservoir, but the slope is basically below 40°, which is consistent with the general characteristics of the slope of the scouring funnel of the reservoir.

![Figure 3. Local siltation distribution in front of the dam after 30 years of reservoir operation](image1)

![Figure 4. Local siltation distribution in front of the dam after 100 years of reservoir operation](image2)
Figure 5. Position of the section in front of the dam

Table 2. Scouring funnel shapes in front of the hydraulic structure in different operation years

| Position | Parameters | After 10 years | After 30 years | After 50 years | After 100 years | After 150 years |
|----------|------------|----------------|----------------|---------------|-----------------|-----------------|
| section 6 | siltation height (m) | 0.8 | 1.8 | 3.5 | 7.6 | 13.0 |
|          | length (m) | 14.8 | 14.8 | 14.8 | 17.7 | 17.7 |
|          | the average slope(°) | 3.1 | 6.9 | 13.3 | 23.2 | 36.3 |
| section 9 | siltation height (m) | 0.7 | 2.2 | 4.7 | 9.6 | 17.5 |
|          | length (m) | 29.2 | 29.2 | 29.2 | 30.9 | 35.1 |
|          | the average slope(°) | 1.4 | 4.3 | 9.1 | 17.3 | 26.5 |
| section 11 | siltation height (m) | 0.9 | 2.8 | 6.2 | 13.9 | 28.6 |
|          | length (m) | 26.8 | 29.8 | 29.8 | 29.8 | 32.9 |
|          | the average slope(°) | 1.9 | 5.4 | 11.8 | 25.0 | 41.0 |
| section 13 | siltation height (m) | 1.0 | 3.3 | 7.5 | 11.1 | 20.0 |
|          | length (m) | 17.6 | 17.6 | 22.0 | 22.0 | 22.0 |
|          | the average slope(°) | 3.3 | 10.6 | 18.8 | 26.8 | 42.3 |

6. Conclusions
Aiming at the problem of multi-constrain boundary conditions in front of hydropower station dams, a step-by-step iterative method is proposed in this paper. The method can reflect the influence of multi-constraint boundary conditions in front of the dam by verification of the mathematical model. The mathematical model is used to simulate and analyze the Dashixia Reservoir. The analysis results show that after 50 years of operation, the scouring funnels are gradually formed in front of the generator set, the middle hole flush tunnel and the emptying tunnel with the increase of siltation thickness in front of the dam. The slope of the funnel gradually raises with the increase of the operation year of the reservoir, but the slope is basically below 40°. The sediment in front of the spillway of the reservoir is mainly flat silting, and does not form a scouring funnel. Due to the influence of the reservoir scheduling method, the siltation thickness is 1-2m at the end of 50 years and 3-4m at the end of 100 years. It is recommended that increasing the number of the spillway opening properly can result in less siltation during the actual operation.

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