What can rotational splittings of low-luminosity subgiants actually tell us about the rotation profile?

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November 23, 2021

ABSTRACT

Context. Inversions of the rotation profile using rotationally induced splittings of low-luminosity subgiant stars suggest that angular momentum transport mechanisms must be 1-2 orders of magnitude more efficient than theory predicts. The lack of precise high resolution of measurements of the rotation profile limits our understanding of the physical mechanism inducing excess angular momentum transport. Rotational inversions are limited to some positions in a star and are restricted in accuracy and resolution by the number of observed splittings, the specific modes of the splittings that are observed, and their associated uncertainties. For example, state-of-the-art observations of rotational splittings of low-luminosity subgiants are limited to low degree $\ell = 1$ and 2 modes, offering only limited precision measures of the core and surface rotation rates, not the shape of the rotation profile between these positions.

Aims. We study the feasibility of making precise constraints to the rotation profile between the core and surface and the possibility of differentiating between rotation profile shapes using the observed rotational splittings of low-luminosity subgiant KIC 12508433.

Methods. We use qualitative assumptions of extreme angular momentum transport mechanisms to prescribe the shape of the five synthetic profiles with the same core and surface rotation rates. We calculate the expected rotational splittings given these five profiles and analyse the differences between them. Markov chain Monte Carlo integration of the synthetic profiles using their associated probabilities between them. Markov chain Monte Carlo integration of the synthetic profiles using their associated probabilities between them.

Results. Despite significant changes to the shape of the rotation profile, the rotational splittings deviate on a scale much smaller than the precision of splittings in current observations. We also find degeneracy between the surface rotation rate and position of strong differential rotation gradient of the inverted profiles.

Conclusions. Constraining the physical mechanism contributing to more efficient angular momentum transport during the low-luminosity subgiant phase through the shape of the profile is impossible with current observations of $\ell = 1$ and 2 rotationally split modes. We speculate that population inference using only the core and surface rotation rates from rotational splittings of a series of stars beginning to ascend the subgiant branch provides an alternative path to understanding the excess angular momentum transport.

Key words. stars: rotation – asteroseismology

1. Introduction

All stars rotate. The rate of rotation and distribution of angular momentum throughout a star evolves with the star over time. The effects of rotation on stellar structure and evolution are substantial (Heger 1998; Maeder & Meynet 2000). Rotational-based meridional flows and instabilities induce mixing through the stellar interior. Mixing brings fresh hydrogen to the stellar core, extending the main-sequence lifetime. The interior stellar rotation profile adjusts considerably with changes to the stellar structure, and mixing transports angular momentum throughout the stellar interior in response to these changes.

Measuring differential rotation of stellar interiors places essential constraints on the angular momentum transport mechanism(s) (Beck et al. 2012). Perturbations to the frequency of acoustic oscillation modes can infer the internal rotation of a star. The perturbations are referred to as rotational splittings and are proportional to the frequency of rotation. Inferring the rotation profile relies on measuring the rotational splittings and solving an inversion problem for the rotation profile using a best-fitting stellar model.

The rotational splittings in solar-like oscillators with flat solar-like rotation profiles are typically seen as mode broadenings rather than as distinct separations of the mode peaks in the power spectrum. Line width broadenings result from the combined effect of the short mode lifetimes and the slow rotation rates of solar-like stars (Chaplin et al. 2010). The magnitude of core rotation increases after the main sequence, and mode line widths decrease, leading to rotationally split frequencies that are easier to measure. We can separate the observed modes of stars into distinct flavours: pressure ($p$) modes, gravity ($g$) modes, and mixed modes. The underlying mechanisms by which $p$ and $g$ modes arise correspond to high-frequency oscillations through...
propagating in the convective envelope and low-frequency gravity waves trapped in the radiative core. Main-sequence (MS) stars do not express g-mode oscillations as they are constrained to the core. The core rotation rate of MS stars cannot be measured as a result. Evolution off the main-sequence results in subgiant and low-luminosity red giant branch (RGB) stars exhibiting mixed modes (Metcalfe et al. 2010; Bedding et al. 2011). Mixed modes arise from coupling g modes with high oscillation frequencies to pressure waves propagating through the convective envelope. Mixed modes propagate through the convective envelope resulting in the oscillation of the stellar photosphere and possible observation. Observing mixed modes allows us to constrain the structure, dynamics, and rotational frequency of stars deep radiative interiors and cores.

As a result of space-based observations from missions like CoRoT (Baglin 2003) and Kepler (Borucki & Koch 2010; Borucki et al. 2010), it is now possible to measure mode splittings of stars other than the Sun. Rotational splittings, that vary with oscillation mode spherical order, have been observed in subgiants, red giants and white dwarfs (Huber et al. 2012; Stello et al. 2013; Silva Aguirre et al. 2017; Yu et al. 2018). The core to surface rotational gradient is significant, and we can observe the splitting of modes that are sensitive to the core conditions. The radial rotation profile of subgiant stars inferred from different inversion methods are, however, not in agreement with state-of-the-art stellar modelling (Eggenberger et al. 2019; Deheuvels et al. 2020). The scale of the incongruity between the two displays a fundamental misunderstanding of the method of angular momentum transport within subgiant and low-luminosity red giant stars.

As stars transition off the main sequence, they exhibit strong radial differential rotation compared to the relatively flat solar profile exemplified in main-sequence stars (Howe 2009). Subgiants early in their transition off the main-sequence demonstrate solid body rotation (Deheuvels et al. 2020; Noll et al. 2021). However, later into their post-main-sequence evolution, the cores of more evolved subgiants, and low-luminosity RGB stars rotate much faster than their envelope. The core to surface rotation rate ratio can grow to \(\approx 20\) for stars leaving the subgiant phase (Deheuvels et al. 2014; Gehan et al. 2018; Eggenberger et al. 2019). State-of-the-art models of stellar rotation evaluate the scale of differential rotation between the core, and the surface to be one to two orders of magnitude greater than observations suggest (Fuller et al. 2015; Spada et al. 2016; Ouazzani et al. 2018). On the other hand, rotational inversions of stars later in their post-MS evolution suggest a rotation profile shape with a discontinuity deep in the radiative core (Deheuvels et al. 2014; Di Mauro et al. 2016). A rotation profile with a discontinuity deep in the radiative core is consistent with the modelled and observed rotation rates of the core and the surface.

The cores of subgiants are rotating slower than models predict, despite core contraction (Beck et al. 2012). Angular momentum transport must therefore be much more effective throughout the first ascent of the RGB to alleviate the discrepancy (Eggenberger et al. 2012; Marques et al. 2013; Ceillier et al. 2013). The total angular momentum implied by the surface rotation of white dwarfs and core rotation of RGB stars, however, agree (Gough 2015; Hermes et al. 2017). These observations imply that the excess angular momentum transport required to explain the subgiant rotation profiles will not continue past the subgiant phase.

Several modes of excess angular momentum transport could explain the disparity between models and observations: strong core-envelope coupling of first ascent RGB stars (Tayar & Pinsonneault 2013), magnetorotational instabilities in the form of the Tayler-Spruit (TS) dynamo (Cantiello et al. 2014) and the modified TS-dynamo (Fuller et al. 2019), internal gravity waves (Fuller et al. 2014), as well as angular momentum transport through mixed modes (Belkacem et al. 2015; \textsuperscript{[7]}) of these mechanisms can currently describe the rotation rates along the post-MS evolution pathway. Spada et al. 2016 parameterised the efficiency of the angular momentum transport using core-surface ratio diffusion coefficient. Their model reproduces both subgiant and RGB measurements, but the physical mechanism resulting in excess angular momentum transport still needs to be identified.

Recently proposed models of angular momentum transport cannot accurately describe measurements of the stellar rotation along the post-MS track. Disentangling the contribution of physical mechanisms to angular momentum transport requires a model that reflects the limited observations we currently have as well as either higher resolution inversions of low-luminosity subgiant rotation profiles or a higher number of low-resolution inversions of stars along the post-MS evolutionary track. We focus here on the high-resolution inversions of low-luminosity subgiant rotation profiles and how current observations limit this.

The sensitivity of each oscillation mode to the internal rotation at different layers within the star’s interior can be quantified using a so-called rotational kernel. The rotational kernels of a star are inherent to the thermodynamic structure of a star (see Aerts et al. 2010 for a derivation of these kernels). When the rotational kernel is coupled with a rotation profile, a unique signature, its rotational splittings, are expressed. The \(n, \ell, m\) rotational splitting is given by:

\[
\delta_{n,\ell,m}(\Omega) = m \beta_{n,\ell} \int_0^R K_{n,\ell}(r) \Omega(r) \, dr
\]

where, \(\delta_{n,\ell,m}\) is the \(n, \ell, m\) rotational splitting of the star, \(K_{n,\ell}\) is the rotational kernel of the \(n, \ell\) mode (determined by the stellar model), \(m\) describes the spherical harmonic order, \(\Omega(r)\) is the 1D rotation profile along the radial axis and \(\beta_{n,\ell}\) is a normalisation constant and \(R\) is the outermost radius of the star. We use the relation between the rotational splittings and the rotation profile to measure the rotation profile through a rotational inversion of the observed spectrum of rotational splittings. A detailed description of the internal rotation profile would provide us significantly more information to constrain the physical mechanism contributing to the excess of angular momentum transport in low-luminosity post-MS stars.

An example of these constraints would come from finding evidence of a strong rotational gradient in the core of low-luminosity subgiants. A strong gradient in the core of subgiants is incompatible with angular momentum transport from deep fossil magnetic fields (Gough & Thompson 1990) as they would likely smooth out sharp features. Differential rotation is damped along poloidal field lines as a result of this (Garaud 2002; Strugarek et al. 2011). Internal gravity wave angular momentum
transport is more efficient during the advanced phases of stellar evolution (Charbonnel & Talon 2008). Internal gravity waves can give birth to localised weak gradients in the profile as a result of extraction and deposit of angular momentum (Charbonnel & Talon 2005). A steep gradient could also potentially trigger magneto-rotational instabilities that would transport angular momentum (Balbus & Hawley 1994; Arlt et al. 2003; Menou & Mer 2006). Evidence of a strong angular momentum gradient towards the core of a low-luminosity subgiant constrains the excessive angular momentum transport mechanism not to include these examples.

We use the low-luminosity subgiant that has most recently left the main-sequence from the Deheuvels et al. 2014 sample, KIC 12508433 to explore rotational inversions of low-luminosity subgiant stars. Rather than attempting to invert the rotation profile of this star from the observed rotational splittings, we use KIC 12508433 as a case study to show how little information is contained in observed splittings to constrain the rotation profile. In Section 2 we construct five synthetic profiles that correspond to extreme cases of angular momentum transport. We use these and the associated rotational kernels of KIC 12508433 to determine the expected splittings. We use these splittings to show how little the shape of the profile changes the splittings relative to the associated measurement accuracy. We use Markov chain Monte Carlo (MCMC) sampling to explore the posterior distributions of parameterised rotation profiles given the splittings of three of the chosen profiles. We show evidence of degeneracy between the posterior distributions of the parameters following the sampling. From these results, we observe that attempts to constrain the shape of the rotation profile between the core and the surface, and thus the physical mechanism through which the excessive angular momentum transport occurs, is effectively impossible with current observations of \( \ell = 1 \) and 2 modes.

2. Method

2.1. Subgiant KIC 12508433

Here we investigate how measurements of the rotational splittings make it possible to infer the rotation profile. We do this by considering the splittings of the subgiant star KIC 12508433 observed in the Kepler mission. In Table 1 we provide observation details of this star, and in Figure 1 we indicate its position on the Hertzsprung-Russel Diagram relative to other stars in the Kepler sample as well as non-rotating stellar evolution tracks of similar mass, solar metallicity stars (Mathur et al. 2017). KIC 12508433 was a part of the asteroseismic rotational inversion of the rotation profiles of six subgiants and low-luminosity red giant stars in Deheuvels et al. 2014 which exemplify the angular momentum transport deficiency between stellar modelling and observations. KIC 12508433 is early in its post-MS transition, and thus the core to surface rotation rate ratio is lower than for the other stars in the sample; however, it is the earliest post-MS subgiant to show evidence of differential rotation representing a good example of additional angular momentum transport mechanism(s).

We were provided with a model of KIC 12508433 from Ball & Gizon 2017 generated using the asteroseismic module of the Modules for Experiments in Stellar Astrophysics (MESA) evolutionary code (r7624; Paxton et al. 2010, 2013, 2015, 2019).

| KIC 12508433 | Model parameters |
|--------------|------------------|
| M (M⊙)       | 1.20 ± 0.16      |
| R (R⊙)       | 2.20 ± 0.10      |
| L (L⊙)       | 3.25 ± 0.45      |
| log (g)      | 3.83 ± 0.04      |
| \( T_{\text{eff}}(K) \) | 5248 ± 130 |
| [ Fe/H ] (dex) | 0.25 ± 0.23    |
| \( \Delta \nu \) (\( \mu \)Hz) | 43.3 ± 0.2 |
| \( v_{\text{min}} \) (\( \mu \)Hz) | 793 ± 21 |
| \( \Omega_c / 2 \pi (\text{nHz}) \) | 53.2 ± 79 |
| \( \Omega_s / 2 \pi (\text{nHz}) \) | 213 ± 26 |

Table 1. Observed asteroseismic and spectroscopic properties of KIC 12508433 from Deheuvels et al. 2014 and corresponding best-fitting stellar model properties with a combined surface correction term (Ball & Gizon 2017) used in this work. The asteroseismic quantities here \( v_{\text{min}} \), and \( \Delta \nu \) are the frequency at which observed oscillation modes of a star show the strongest amplitude and the average frequency difference between oscillation modes of consecutive overtones \( n \) of differing spherical degree \( (\ell) \) respectively. Here the observed mass, radius and thus log \( g \) are obtained using asteroseismic scaling relations of \( v_{\text{min}} \) and \( \Delta \nu \).

This was done by simultaneously matching the non-seismic properties of KIC 12508433 \( (T_{\text{eff}}, \text{log} \, g, \text{[Fe/H]} \) in Table 1) and the observed mode frequencies (see Table 3 in Deheuvels et al. 2014) to those of the stellar model. Mode frequencies have been calculated using ADIPLS (Christensen-Dalsgaard 2008), with combined surface effect corrections to the frequencies, following the method of Ball & Gizon 2014. See Ball & Gizon 2017 work for a more thorough explanation of the generation of this model. The best-fitting stellar model parameters of KIC 12508433 used to generate our rotational kernels are given in Table 1.

In this model, the tachocline, the transition region between the radiative interior and the outer convective zone, occurs at \( r/R = 0.5 \) as shown in Figure 2. Efficient mixing flattens the rotation profile between the tachocline and the surface in the convective region. We adopt this assumption for the synthetic profiles by requiring constant rotation in the convective region. With this model, we generated the rotational kernels for all observable rotational splittings of KIC 12508433 as shown in Figure 3. Positions in the star with high kernel magnitude and variance between kernels (outlined by the standard deviation between kernels in red in Figure 3) offer the strongest constraints to the rotation profile. For KIC 12508433 these regions are within the core and at \( r/R > 0.8 \).

2.2. Rotation profiles of interest

The observed rotational splittings of KIC 12508433 provide measures of the core and surface rotation rates but do not constrain the shape of the rotation profile between these positions (Deheuvels et al. 2014). Deheuvels et al. 2014 report a range of possible core and surface rotation rates in the form of the measurement accuracy on these values. The uncertainty on the core and surface values reflects the limited constraints to the rotation profile through inversion made by the small set of observed rotational splittings and the measurement accuracy of those rotational splittings. The measured core and surface rotation rates of KIC 12508433 are given in Table 1 as \( \Omega_c \) and \( \Omega_s \) respectively. For this work, we have chosen to fix the core and
Fig. 1. Hertzsprung-Russell diagram of log \( L \) versus effective temperature of Kepler stars from Mathur et al. 2017. Indicated in black are the subgiants and low-luminosity red giants studied in Deheuvels et al. 2014 with KIC 12508433 indicated by a cross. Dashed lines indicate tracks of non-rotating models of different masses and solar metallicity (Choi et al. 2016; Dotter 2016).

Let us consider the limiting cases of rotation profiles for KIC 12508433. Assuming conservation of specific angular momentum, post-MS evolution leads to a fast-spinning core and a slow-spinning envelope. Models of stellar evolution show a sharp gradient in the intermediate region between these positions, where the H-burning shell lies. The position and gradient of the transition between the core and the surface rotation rates differ with angular momentum transport mechanisms and their relative strength. On the other hand, assuming an instantaneous angular momentum transport, the entire star rotates as a solid body. Small changes to angular momentum transport within a stellar model can reflect obvious changes in the rotation profile.

We consider five extreme but physically possible methods of angular momentum transport in this work. The resulting rotation profiles are shown in blue, orange, green, red and purple in Figure 4 corresponding to the radiative step, strong gradient, weak gradient, tachocline step, and shell step rotation profiles, respectively. We consider two possible effects of varying the angular momentum transport mechanism. In the first, we consider changes that arise in rotational splittings due to varying the angular momentum gradient about the same position in the star. A stronger gradient corresponds to a lower magnitude of angular momentum diffusion. The strongest angular momentum diffusion corresponds to the weak gradient rotation profile. In the second, we consider profiles containing these points. Our work reflects this. The rotational splittings owing to the synthetic rotation profiles we consider in our work do not agree with the observed rotational splittings.
Fig. 3. Rotational kernels of observed rotational splittings of the best fitting model of KIC 12508433. The red curve shows three times the standard deviation of the rotational kernels shown. Regions with large kernel deviation place strong local constraints on the rotation rate. For KIC 12508433 these regions are within the core and at $r/R > 0.8$. Inset: Same as Figure 3 but with x-axis log scaling and two example modes shown. These two modes show how different rotational kernels can probe the rotation profile of a star. Here the $\ell = 1$, $n = 9$ mode is sensitive to the core rotation rate while the $\ell = 2$, $n = 6$ is sensitive to the surface rotation rate.

a strong rotational gradient. In these models, the position of that strong gradient reflects the angular momentum transport mechanism. We consider the limiting cases of the position of the sharp gradient of the profile and consider the core and the surface rotation to align with KIC 12508433. The first, the radiative step profile, corresponds to the first in the previous set of profiles. The angular momentum transport from the core to the surface is delocalised from the core position, and as a consequence, there is angular momentum transport in the radiative zone. The exact position chosen here is arbitrary and is used only as an example. The shell step and tachocline step profiles correspond to scenarios where the strong gradient occurs at the outer radius of the g-mode cavity and the modelled position of the tachocline in KIC 12508433 respectively. The tachocline step rotation profile assumes that angular momentum transport occurs between the radiative and convective regions facilitated by rotational instabilities. In contrast, the shell step profile follows the hydrogen abundance within the star and thus follows the subgiants g-mode cavity or the core, which corresponds to the extreme conservation of specific angular momentum.

Using the synthetic rotation profiles shown in Figure 4 we calculated the expected rotational splitting frequencies using Eqn 1. The uncertainty on these rotational splittings are taken from the measurement accuracy provided in Table 3 of Deheuvels et al. 2014. See their work and references within for an explanation of the calculation of this value. The uncertainty of the rotational splittings must reflect the measurement uncertainty that would arise according to the synthetic rotation profile. The measurement accuracy of the rotational splittings varies with the rotation profile; faster rotation results in more distinct rotational splitting peaks and thus greater accuracy rotational splittings.

The use of these noise realisations is therefore consistent with the fact that we are using the observed core and surface rotation rates as KIC 12508433.

3. Results

3.1. Can current rotational splittings be used to find evidence of a strong rotational gradient in the core?

Let us first consider changes that arise in rotational splittings due to varying the angular momentum gradient about the same position in the star. The set of rotational splittings generated by varying the strength of angular momentum transport, and therefore the gradient of the profile: the radiative step, strong and weak gradient profiles, are shown in blue, orange and green in Figure 5, reflecting their colours in Figure 4. Due to our core and surface rotational rate choices, not all synthetic rotational splittings agree with the observed rotational splittings to three standard deviations.

The differences in predicted rotational splittings between the considered rotation profiles are much smaller than the measurement uncertainty. The change in the shape of the rotational splitting spectra, the set of rotational splitting frequencies versus mode frequency given a rotation profile, from the differing rotation profiles are minimal. Combining these effects indicates that the profile shape cannot be differentiated in a rotational inversion with this set of observed splittings. Indeed, measurement noise here dominates over changes apparent from varying the rotation profile shape. The observed splittings are insensitive to the gradient of the rotation profile.
However, the \( \ell \) scale as the measurement accuracy of the rotational splittings. For most of the splittings, the shape of the rotational splitting spectra does not follow that of these profiles reflect the rotation profile colour in Figure 4. The qualitative assumptions of extreme case angular momentum transport for low-luminosity post-MS stars. Cross-hatching and diagonal-hatching reflect the convective surface region and H depleted core. Here, blue, orange, green, red, and purple correspond to the radiative step, strong gradient, weak gradient, tachocline step, and shell step rotation profiles.

3.2. Can current rotational splittings be used to find the position of a strong rotational gradient in a subgiant?

We now consider changes that arise in rotational splittings due to assuming that the rotation profile contains a strong rotational gradient and varying the position of that gradient through qualitative assumptions of extreme case angular momentum transport. The synthetic profiles considered here correspond to radiative step, shell step and core step rotation profiles in Figure 4. The rotational splittings in Figure 6 generated using these profiles reflect the rotation profile colour in Figure 4. The shape of the rotational splitting spectra does not follow that of the splittings of KIC 12508433. For most of the splittings, the difference between the three synthetic profiles is on the same scale as the measurement accuracy of the rotational splittings. However, the \( \ell = 1 \) rotational splittings at 750 and 830 nHz differ by an order of magnitude greater than the measurement accuracy of the observed rotational splittings. Can significant differences between the rotation profiles for two rotational splitting modes differentiate between the extreme case rotation profiles? Moreover, can concurrent fitting of all observed rotational splittings, an inversion, differentiate between the rotation profiles? We explore the posterior distribution of the constrained rotation profile given the observed splittings to confirm that differentiating between the synthetic profiles is not possible.

3.3. Is forward modelling of the rotation profile using rotational splittings able to differentiate between extreme case rotation profiles?

We consider whether inversions can differentiate between the three synthetic profiles. The differences between the synthetic rotational splittings given the rotation profiles in Section 3.2 are more significant than those offered in Section 3.1. The larger variance in rotational splittings offers more favourable conditions to differentiate between rotation profiles following a rotational inversion. Despite the optimal conditions to differentiate between rotation profiles, we will find that minimal or no differentiation can be made. Accordingly, we conclude that the ability to differentiate between the rotation profiles in Section 3.1 are also not possible.

Our analysis follows Beck et al. 2012 and Kallinger et al. 2017 Section 8.1. They use forward modelling to infer the best fitting parameters that describe the internal rotation profile of stars using the observed rotational splittings. Here we assume that the shape of the rotation profile is a step function. We assume constant core and surface rotation rates and allow the position of the drop between the core and surface to vary. We then infer the observed rotational splittings’ constraints by considering the parameters’ posteriors that describe the profile following sampling. A step rotation profile is standard in rotating post-MS star models. The position of the rotational gradient depends on the stellar modelling parameters, specifically the angular momentum transport (Eggenberger et al. 2019). As the shape of the rotation profile is known and fixed, this is the best-case scenario in a rotation profile forward modelling. In reality, we will not know the shape. Forward modelling of the profile will be less constrained and more dependent on model
We describe the rotation profile model using three parameters of the following form:

\[
\Omega(r) = \begin{cases} 
\Omega_c & r/R \leq p \\
\Omega_s & r/R > p \end{cases}
\]

(2)

where \( \Omega_c \) and \( \Omega_s \) describe the core and surface rotation rates respectively, \( r/R \) is the scaled radius, and \( p \) corresponds to the position of the strong rotational gradient as a fraction \( r/R \). This equation describes a step function and corresponds to the assumed, known shape of the rotation profiles we investigate.

Using PyMC3 (Salvatier et al. 2016) we used MCMC to sample through 20000 sets of parameters in rotation profile space conditioned on the rotational splittings associated with the rotation profiles in Section 3.2. The standard deviation of each rotational splitting in our likelihood function was taken as the measurement error on that rotational splitting from the observations of KIC 12508433. We used the step-function model of the rotation profile described by Eqn 2 as our input model and placed a uniform prior over the model parameters. We placed wide priors over each of the rotation profile parameters. The core and surface rotation rate priors were taken as \([400:600] \) nHz and \([200:400] \) nHz respectively (a range of 100 nHz with the true values at the centre) and the range of the prior on the position of rotational discontinuity was taken as \([0:1] \) \( r/R \). The use of a wide range uniform priors here highlights the role of the data in constraining the profile and is not designed to be reflective of what one would do for inference. The posterior distribution of the inverted model parameters relative to the injected values (coloured lines) is shown in Figures 7, 8 and 9 for mock data corresponding to the rotational step, tachocline step, and shell step profiles.

Forward modelling of the rotation profile using the available rotational splittings of KIC 12508433 are not able to differentiate between the three synthetic profiles. Consider the two extremes, where the position of the strong rotation gradient occurs at the core boundary and \( r/R = 0.5 \). The posterior distributions of the model parameters are shown in Figures 8 and 9. The 99% percentile on the credible interval for \( p \) contains both \( r/R = r_{core} \) and \( r/R = 0.5 \). The posteriors also exhibit clear degeneracy between the surface rotation rate and the position of the strong rotational gradient. Strong constraints to the surface rate and the position of the gradient cannot be made simultaneously with current observations. We note that we can recover the true values if the rotational splittings are known to high precision. However, this is not the case when the rotational splittings have realistic measurement accuracy. Current observations of rotational splittings cannot make the accurate and precise constraints to the shape of the rotation profile that are required to constrain the physical mechanism behind the efficient angular momentum transport.

4. Discussion

We have shown that it is impossible to differentiate between stellar rotation profiles given the noisy observations of \( l = 1 \) and \( l = 2 \) modes of KIC 12508433. The typical shift in the mean rotational splitting frequency and variance with rotation profile shape is small relative to the measurement precision.
biased. Despite this, rotational inversions almost always use non-rotating stellar models. More accurate asteroseismic and spectroscopic constraints on the best fitting stellar model ($T_{\text{eff}}$, $\log g$, [Fe/H] and the observed mode frequencies) would provide more accurate constraints to the rotation profile. Here we have only considered the differences that arise in splittings from different choices of synthetic rotation profile relative to their measurement accuracies.

We used the averaged core and surface rotation rates for KIC 12508433 reported by Deheuvels et al. 2014. The optimally localised averaged kernels of the core of a subgiant can be well-constrained to the inner 2% of the stellar radius (see Fig. 9, in Deheuvels et al. 2014), and the surface averaged kernel is spread throughout the other 98%. We explore the effect of the degeneracy between the shape of the rotation profile and the inferred rotation rate in Section 3.3, where we find that the position of the strongest rotational gradient must be well constrained for the rotational inversion to be accurate, or vice-versa.

State-of-the-art models of rotating post-MS stars exhibit strong rotational gradients deep in their core. The synthetic rotation profiles considered here are not limited to having strong rotational gradients deep in the core. Instead, they represent the extreme cases of angular momentum transport. Despite this, we cannot differentiate between profiles given noisy measurements of $\ell = \{1, 2\}$ rotational splitting frequencies. Our work shows that extreme and potentially unphysical models of angular momentum transport are unable to be differentiated. The more nuanced differences between rotation profiles that arise from more conservative angular momentum transport mechanisms cannot be constrained using current measurements of single star rotational splittings.

Conversely, suppose the surface rotation rate was well constrained (i.e. from photometric time series analysis of stellar spot modulation), and we assume a strong rotational gradient at some position within the star. In that case, the relation between the averaged surface rotation rate from asteroseismic inversion and the photometric analysis would help constrain the position of the strong rotational gradient, as these properties are correlated. This is apparent in the posterior distributions of surface rotation and gradient position in Figures 7, 8 and 9. Santos et al. 2021 place an upper bound on the relative uncertainty of the surface rotation of subgiants to $\approx 30\%$ with most having $< 10\%$ (see their Figure 8). This places a more informative prior over the surface rotation rate than a uniform prior, subgiants with precise surface rotation measurements and asteroseismic measurements of the rotational splittings would place stronger constraints on the position of the rotational gradient than our work reflects. Though, consider a star with a photometrically measured surface rotation rate of $200/2\pi$ nHz, with a standard deviation of 10%. Figures 7, 8, and 9 show that this would still only constrain $p$ to be $< 0.5 \, \text{r}/\text{R}$ and would not differentiate between the three rotation profiles. On the other hand, stronger constraints on the rotation profile shape of even a single star would improve our understanding of the angular momentum transport and evolution. No strong constraints to the surface rotation from photometric time series analysis have been made to low-luminosity subgiants with asteroseismic observations.

We note that stars closer to the RGB (stars D and F in Deheuvels et al. 2014 and KIC 4448777 in Di Mauro et al. 2016) will have a larger number of mixed modes that can probe the core rotation more substantially and will have core rotation rates much higher than KIC 12508433. The rotational splittings
of more evolved stars will thus have smaller uncertainties and allow us to make more accurate constraints to core rotation than KIC 12508433. This results in inversions of the profile that have more substantial constraints to the core rotation rate as well as some constraints to the shape of the profile close to the core (Di Mauro et al. 2016). Rotational inversions suggest that the observed rotational splittings of more evolved stars can correctly, with probability $>90\%$, identify a strong discontinuity in the rotation profile deep in the core (Deheuvels et al. 2014). Comparatively, given the rotational splittings of a profile with a strong rotational gradient in the core of KIC 12508433, we can only constrain this gradient to be somewhere in the inner 50% of the star.

We understand the lack of constraints to the rotation profile of low-luminosity subgiant stars as a feature of their asteroseismic observations. The slow-core rotation rates, comparative to more evolved stars, and evolutionary state results in observations of low degree and low measurement accuracy rotational splittings using state-of-the-art instruments. This is reflected in the number and relative measurement accuracy of other low-luminosity subgiants in the Deheuvels et al. 2014 sample. Owing to the similarity in the rotational splitting spectra, we believe our work is qualitatively applicable to the other low-luminosity giants in Deheuvels et al. 2014 (stars B, C, and E), where the angular momentum transport discrepancy begins to become apparent. Furthermore, we find that differentiating between shapes of rotation profile is similarly impossible when increasing the measurement accuracy of rotational splittings or introducing 1 or 2 $\ell = \{1, 2\}$ rotational splittings to reproduce the observations of other low-luminosity subgiants in the sample.

Without a dedicated asteroseismology telescope, Kepler and CoRoT data are limited in constraining the shape of rotation profiles during this evolutionary phase. Rotational inversions of Kepler data have been used to probe the core and surface rotation rates of a limited number of subgiants (Deheuvels et al. 2014, 2020; Noll et al. 2021). Rotational splittings of modes up to $\ell = 10$ (Ahlborn et al. 2020) are required to accurately measure the rotation profile of subgiant stars between the core and the surface. Without strong constraints to the shape, population inference using only the core and surface rates of a series of stars ascending the subgiant branch provides an alternative path to understanding the evolution of angular momentum transport throughout this evolutionary phase. The measured rotational splittings of 33 subgiants (Li et al. 2020b,a) are expected to provide these stars core and surface rotation rates and will greatly improve our understanding of the time evolution of the angular momentum transport mechanism during the subgiant branch. Furthermore, Schunker et al. 2016b provide evidence that ensemble fittings of multiple stars using a small number of low accuracy rotational splittings make more substantial constraints to the rotation profile of solar-like oscillators. Despite their method being applied to main-sequence stars, we speculate that applying it to a series of subgiants and low-luminosity red-giants would provide a similar result.

The number of observable oscillation modes limits asteroseismic inversions of the rotation profile. The power of oscillation modes in photometric time series from integrated surface flux drops significantly for modes with $\ell > 2$ (García & Ballot 2019). Practically, long term photometry may only resolve $\ell = 1, 2$ and some $\ell = 3$ modes. This is the result of partial mode cancellation due to the physical nature of the oscillation modes and our inability to spatially resolve the stellar surface. A dedicated photometry mission should observe modes of order $\ell = 3$, albeit through extremely sensitive detectors and with high sampling frequency uninterrupted observations over year-long time scales. Furthermore, we found that the measurement uncertainties on the rotational splitting frequencies would need to be roughly an order of magnitude smaller in order to provide accurate measurements of the core and surface rotation rates at the few percent level and to confidently infer that a step in the rotation profile occurs deep in the core. Measurements of this scale of single stars are not yet feasible (Ahlborn et al. 2020). For this reason, investigating the ability to constrain the internal rotation profile of subgiant and low-luminosity red giant stars using limited noisy measurements remains a worthwhile endeavour.

Other possible constraints to the rotation profile of post-MS stars come in the form of asymmetries in rotationally split mixed modes arising from near-degeneracy effects (Deheuvels et al. 2017). A perturbative approach to probe the internal rotation of red giants using multiplet asymmetries rather than standard inversion methods of rotational splitting frequencies offers more precise constraints to the core to surface rotation rate ratio. Their work details explicitly that near-degeneracy asymmetries are significant for $\ell = 2$ mixed modes all along the RGB and not low-luminosity subgiants. Asymmetries in mixed modes arise when the frequency separation between consecutive mixed modes and the rotational splitting frequency are comparable. This is not the case for low-luminosity subgiants. However, a similar application of their work to stars along the would provide more substantial constraints to the rotation profile following the subgiant branch, offering insight into excess angular momentum transport during this phase.

Doppler imaging provides a more straightforward method to resolve $\ell > 3$ modes due to decreased atmospheric noise and the spatial resolving power (Grundahl et al. 2017). The recently developed Stellar Observations Network Group (SONG) uses a network of 1 m telescopes with high-resolution spectrographs and performs high-resolution spectroscopic measurements of stars. Spectrographic asteroseismic measurements of subgiants using this data could improve our capacity to detect $\ell \geq 3$ modes which constrain rotation rates between the core and the surface of stars better than dipole modes. However, the relative increase in constraint from octopole modes over dipole modes is minimal in constraining the angular momentum transport problem in low-luminosity post-MS stars.

Further work is required to consider the observation conditions, the number of observed modes and associated uncertainty that would allow us to make strong constraints to the shape of the rotation profile. Furthermore, excluding the measurement uncertainty on the rotation profile, varying the relative strength of angular momentum transport mechanisms can result in degenerate rotation profiles. Given more substantial constraints to the rotation profile than we currently have, we speculate that single star rotation profile measurements will not be the smoking gun in solving the angular momentum transport problem. Work is needed in terms of both observation and theoretical insight to understand the angular momentum transport problem.
5. Conclusion

Observations of subgiant stars suggest that angular momentum transport mechanisms must be 1-2 orders of magnitude more efficient than theory predicts. Constraining the shape of the rotation profile from rotational splitting measurements would help constrain the physical mechanism of angular momentum transport. We present an analysis of the current observations of rotational splittings of subgiant and low-luminosity red giant stars. Specifically, we investigate the measurement of the rotation profile shape with observations of rotational splittings to the rotation profile of KIC 12508433. We conclude that measuring the rotation of the rotation profile of KIC 12508433 is impossible given the current number and associated error of measured rotational splittings. It is nevertheless still possible to measure the core rate and an envelope-averaged rate with these observations. We understand this as a feature of the state-of-the-art observation methods of rotational splittings and the evolutionary state of low-luminosity post-MS stars. Furthermore, we believe that constraints to the rotation profile of low-luminosity post-MS stars in single-star rotational inversions using current observations of $\ell = 1$ and 2 splittings cannot differentiate between limiting case angular momentum transport mechanisms.

We considered five extreme cases of angular momentum transport through five rotation profiles. These correspond to the radiative step, strong rotation gradient, weak rotation gradient, shell step and tachocline step rotation profiles in Figure 4. The radiative step, strong gradient and weak gradient rotation profiles, considered in Section 3.1, correspond to mechanisms that resulted in differing angular momentum gradients through the radiative region. A stronger gradient corresponds to a lower magnitude of angular momentum diffusion and thus a more step-like rotation rate drop at this position in the star. In Section 3.2 the assumption of a strong rotational gradient at some position in the star in the form of a steep drop in rotation rate was made, and the position of that step was varied. These profiles correspond to: the first profile in Section 3.1, the radiative step, and the shell step and tachocline step rotation profiles in Figure 4. These correspond to transport mechanisms where: the rotational gradient follows the hydrogen abundance, the rotational gradient is delocalised from the core to the mid-position of the radiative region, and a mechanism where the rotational gradient occurs at the tachocline.

These extreme cases predict rotational splittings that differ on scales much smaller than the measurement uncertainty of the observed splittings. We verified the accuracy of measuring the rotation profile from synthetic rotational splittings using MCMC sampling. The posterior distributions cannot constrain the parameters to the true rotation profiles as in the infinitely MCMC sampling. The posterior distributions cannot constrain the rotation profile from synthetic rotational splittings using the observed splittings. We verified the accuracy of measuring the shape of low-luminosity subgiants’ internal rotation profiles. We present the challenges of observing the rotational splittings of post-MS stars without long time series for asteroseismic measurements. We speculate that population inference using the currently observed sets of rotational splittings (and thus core and surface rotation rates) of low-luminosity post-MS stars provide more stringent constraints to the evolution of angular momentum transport than inferring a single stars rotation profile from those same rotational splittings possibly can.

Acknowledgements. The authors thank Warrick Ball (University of Birmingham) for providing us with the model of KIC 12508433 integral to this work. A. R. C. is supported in part by the Australian Research Council through a Discovery Early Career Researcher Award (DE190100656). Parts of this research were supported by the Australian Research Council Centre of Excellence for All Sky Astrophysics in 3 Dimensions (ASTRO 3D), through project number CE170100001. I. M. acknowledges support from the Australian Research Council Centre of Excellence for Gravitational Wave Discovery (OzGrav), through project number CE170100004. J. M. is a recipient of the Australian Research Council Future Fellowship FT190100574. This work has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (CartographY GA, 804752). Funding for the Stellar Astrophysics Centre is provided by The Danish National Research Foundation (Grant agreement no: DNRF106).

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