A comparative study on two characteristic parametrizations for high energy pp and $\bar{p}p$ total cross sections

A. Bueno$^1$ and J. Velasco$^2$

$^1$ High Energy Physics Laboratory, Harvard University, Cambridge, 02138 MA, USA
$^2$IFIC, Centro Mixto Universitat de Valencia-CSIC 46100-Burjassot, Valencia, Spain

Abstract

Available high energy data for both pp and $\bar{p}p$ total cross sections ($5 \text{ GeV} < \sqrt{s} < 1.8 \text{ TeV}$) are described by means of two well-known distinct parametrizations, characteristic of theoretical ("Regge-like" expression) and experimental ("Froissart-Martin-like" expression) practices, respectively. Both are compared from the statistical point of view. For the whole set of present data statistical analysis ($\chi^2/d.o.f.$) seems to favour a "Froissart-like" ($\ln s \approx 2$) rise of the total cross section rather than a "Regge-like" ($s'$) one.
PACS number(s): 13.85Lg, 13.85.-t

Keywords:

Cross-section, nuclear, proton, antiproton, fit, Regge, parametrization

Submitted to Physics Letters B
It is a well-established fact that hadronic total cross sections rise with energy ([1], [3], [7]) but the actual energy dependence of this rise is still an open question. We dispose of many parametrizations (see ref. [3], [4], [5], [6], which are by no means exhaustives) to describe the available data. Although different energy behaviours are proposed, as a matter of fact, most of them describe high energy data fairly well.

In this paper the explanation of how total cross sections grow and the predictions for future colliders energies play a secondary role. Our aim is to compare from a statistical point of view two characteristic parametrizations. The first is a semi-empirical parametrization, based on Regge theory and asymptotic theorems, which assumes a \((\ln s)^\gamma\) (“Froissart-like”) behaviour at high-energy. It has successively been used by experimentalists to describe their data, from the ISR [4] to the SppS [13]. Moreover it has proved to be very successful in predicting the behaviour of the total cross section from the ISR to the SppS energies, in particular its considerable rising [7]. The other one, developed by Donnachie and Landshoff [3], establishes that total cross sections grow as a power of the energy: \(s^\epsilon\). Even if we know that \(\sigma^{tot}\) cannot ultimately grow as a power of the energy, because unitarity will break down, it is claimed that it is perfectly valid in the non-asymptotic domain which has been explored up to now. It has a sound theoretical basis on Regge theory and has successfully been applied to describe a variety of hadronic processes [11]: \(\bar{p}p\), \(p\bar{p}\), \(\pi^\pm p\), \(K^\pm p\), \(\gamma p\), \(\bar{p}n\), \(pn\). It may well be considered as one of the most popular from the theoretical point of view. Its simple and compact form make it a very useful expression for representing a great variety of data as exemplified in the last version of the Review of Particle Properties [12].

In our choice we have been helped by the recent measurement of the real part of the forward elastic scattering amplitude at the SppS Collider [8]. The measurement has proved that, for the energy scale of the present accelerators, the contribution of the odd under crossing part of the scattering amplitude [9], [10] is negligible. This fact make plausible to omit all the parametrizations which include odderon effects.

Our comparative study will be done for pp and \(\bar{p}p\) processes only as they accumulate the most precise and energy-broader set of data. As far as the other hadronic processes are concerned the comparison between the two parametrizations in the high energy regime has much less interest due to the
absence of statistics in all the cases for \( \sqrt{s} > 30 \) GeV [12].

Let us assume that total cross sections can be described by any of the two following expressions, hereafter called \( P_1 \) and \( P_2 \), respectively (a detailed explanation of their motivation can be found in ref. [4] for \( P_1 \), and ref. [8] for \( P_2 \)):

\[
\sigma_{\pm}^{\text{tot}} = A_1 E^{-N_1} \mp A_2 E^{-N_2} + C_0 + C_2 \left[ \ln \left( \frac{s}{s_0} \right) \right]^\gamma \tag{1}
\]

\[
\sigma_{\pm}^{\text{tot}} = X s^\epsilon + Y_{\pm} s^{-\eta} \tag{2}
\]

where \(+(-)\) stands for pp (\( \bar{p}p \)) diffusion. \( \sigma^{\text{tot}} \) is measured in mb and energy in GeV, being \( E \) the energy measured in the lab frame. The scale factor \( s_0 \) have been arbitrarily chosen equal to 1 GeV².

In \( P_1 \) the first two terms are Regge-type terms which describe the behaviour at low energy and the difference between \( \sigma_{\bar{p}p} \) and \( \sigma_{pp} \). The remaining ones describe the high-energy behaviour. In case \( C_2 \leq \pi/m_0^2 \) and \( \gamma \leq 2 \), this parametrization is compatible with the asymptotic Froissart-Martin bound [17].

In \( P_2 \) the first term arises from pomeron exchange and the second one from \( \rho, \omega, f, a \) exchange. The coefficient \( X \) is the same for pp and \( \bar{p}p \). Both \( \epsilon \) and \( \eta \) are effective powers, slowly varying with \( s \). Previous theoretical work indicated that \( \epsilon \) should be close to 0.08 and that \( \eta \) is about 1/2. \( \epsilon \)From their analysis of pp and \( \bar{p}p \) data, Donnachie and Landshoff conclude that \( \epsilon \) and \( \eta \) can be treated as constants with values \( \epsilon = 0.0808 \) and \( \eta = 0.4525 \).

In order to compare the approaches embodied in \( P_1 \) and \( P_2 \), three different kinds of fits, using three different data sets for each fit, have been performed. On each data set the first fit, \( F_1 \), is done using parametrization \( P_1 \) and the second fit, \( F_2 \), is done with \( P_2 \). In all the cases all the parameters are allowed to vary.

Furthermore, in order to make a faithful comparison with the results obtained by Donnachie and Landshoff [14] a third fit, \( F_3 \), is carried out. It is just an \( F_2 \) fit where the parameters \( X, Y, \epsilon \) and \( \eta \) have been fixed to the
values quoted by them.

For the first fit, the experimental data set consists in all available measurements of $\sigma^{\text{tot}}$ and $\rho$, in the energy domain which spans from $\sqrt{s} = 5$ GeV up to $\sqrt{s} = 546$ GeV. The existing discrepancy between CDF ($80.6 \pm 2.3$ mb) [14] and E710 ($72.8 \pm 3.1$ mb) [15] total cross section measurements at the Tevatron ($\sqrt{s} = 1.8$ TeV) leads us to exclude these points from the fits. This means that we are left with 103 points.

The fits have been performed using the once-subtracted dispersion relations [16]

$$\rho_{\pm}(E) \sigma_{\pm}(E) = \frac{C_s}{p} + \frac{E}{\pi p} \int_{m}^{\infty} dE' E' \left[ \frac{\sigma_{\pm}(E')}{E' (E' - E)} - \frac{\sigma_{\mp}(E')}{E' (E' + E)} \right]$$

where $C_s$ is the substraction constant. It is a simultaneous fit of $\sigma^{\text{tot}}$ and $\rho$, that is, we minimize the $\chi^2$ function

$$\chi^2 = \chi^2_{\sigma_{pp}} + \chi^2_{\rho_{pp}} + \chi^2_{\sigma_{pp}} + \chi^2_{\rho_{pp}}$$

The method has proved to be very succesful in the past [4] predicting the observed rise of $\sigma^{\text{tot}}$ from the ISR to the SppS. Recently it has been used, with the last data available, to provide predictions on the behaviour of $\sigma^{\text{tot}}$ up to $\sqrt{s} = 100$ TeV [13]. A detailed study of the technique can be found in [18].

In $F_1$ we are left with eight free parameters ($A_1, N_1, A_2, N_2, C_0, C_2, \gamma, C_s$). For the $F_2$ we have six free parameters ($X, Y_+, Y_-, \eta, \epsilon, C_s$). Finally in $F_3$ we have only one free parameter ($C_s$).

Table 1 shows the values of the parameters obtained for the best $F_1$, $F_2$ and $F_3$ fits. Table 2 quotes, for these best fits, the values of $\chi^2$ and $\chi^2/d.o.f$. We have included also the predicted numerical values for $\sigma^{\text{tot}}$ for each of the three considered cases at the energies of the SppS, Tevatron and LHC. In table 3 the experimental values of $\sigma^{\text{tot}}$ at the SppS and Tevatron are compared to the predictions of $F_1$, $F_2$, and $F_3$.

It clearly appears that $F_1$, with the dominant $(\ln s)^{\gamma \approx 2}$ term at high energies ($\chi^2/d.o.f. = 0.8$) gives by far the best result. It perfectly matches
the SppS measured value. It is interesting to observe that its prediction at the Tevatron lies right at the middle (76.5 ± 2.3 mb) of the conflicting claims for $\sigma_{\text{tot}}$ of both Tevatron experiments, CDF and E710. As for $F_2$, even if it may be considered as an acceptable fit ($\chi^2/d.o.f. = 1.6$) it has, at least, two difficulties: on the one hand, the values obtained for $\epsilon$, 0.0644 ± 0.0015, and $\eta$, 0.5433 ± 0.0075, represent a challenge for the model. They are far away from their theoretical estimations quoted previously. And, on the other hand, the numerical values obtained for $\sigma_{\text{tot}}$ (see table 2), being much lower than the measured ones, rule it out. Finally $F_3$ is clearly statistically ruled out ($\chi^2/d.o.f. = 4.5$) with respect to the others fits, although its $\sigma_{\text{tot}}$ predictions might appear acceptable. Indeed, Donnachie and Landshoff considered a definite success of their parametrization the prediction, in 1985 [19], of about 73 mb for $\sigma_{\text{tot}}$ at the Tevatron. As we have said, the accepted value of $\sigma_{\text{tot}}$ at 1.8 TeV is far from clear up to now. Waiting for the measurement of $\sigma_{\text{tot}}$ at the LHC we find this parametrization statistically unsupported by present data.

In our second step, the fits are less powerful: we restrict them to the $\sigma_{pp}$ and $\sigma_{\bar{p}p}$ experimental data sample (69 points), excluding the $\rho_{pp,pp}$ measurements, which is the usual practice. Now we have seven free parameters for $F_1$ and five for $F_2$. In this case, $F_3$ is not a real fit because we do not minimize $\chi^2$.

Table 4 shows the parameters as given by the best fits and table 5 gives in addition to $\chi^2$ and $\chi^2/d.o.f.$, the values for $\sigma_{\text{tot}}$ at the energies of the SppS, Tevatron and LHC.

In figure 1 we have depicted the results of these three fits together with the experimental data on $\sigma_{\text{tot}}$. Recent cosmic rays results [20] from the Akeno Observatory obtained from the analysis of proton-air interactions at ultrahigh energies are also plotted.

The results show no sensible changes with respect to the ones of the previous step and the same discussion applies. Again $F_1$ is strongly supported by the data, which is not the case for $F_2$. $F_3$ yields, once again, $\sigma_{\text{tot}}$ predictions consistent with current experimental evidence, but its large $\chi^2/d.o.f.$ value rules it out from the statistical point of view.

Finally we investigate how the variation of the energy domain influences the results previously obtained. We restricted the fits to a smaller energy domain, in the sense of increasing energy, $10 \leq \sqrt{s} \leq 546$ GeV, but
still keeping enough data (42 points) to make results sensible.

The trend previously observed are, in spite of the bigger statistical errors due to the smaller number of data points, not altered: the results for the different fits are practically the same as an inspection of tables 6 and 7 shows. $F_1$ always gives the lowest $\chi^2/d.o.f.$ value. $F_2$ does not reproduce the experimental points and $F_3$, which does better than $F_2$, has the worst $\chi^2$ of all three fits.

In conclusion, although it is claimed that $(\ln s)^{\gamma \approx 2}$ and $s^{0.0808}$ rises adequately reproduce high-energy pp and $\bar{p}p$ data, from a careful analysis of present experimental evidence, we have shown that statistically a $(\ln s)^{\gamma \approx 2}$ fit is strongly favoured. Better $\chi^2/d.o.f.$ may be obtained, in the $s'$ model, at the price of lower $\epsilon$ values, but for these cases the resulting $\sigma^{tot}$ values are clearly unrealistic.

Acknowledgements

This research has been supported by CICYT grant number AEN93-0792.
References

[1] U. Amaldi et al., Phys. Lett. B 44 (1973) 11.
[2] S. R. Amendola et al., Phys. Lett. B 44 (1973) 119.
[3] A. Donnachie and P. V. Landshoff, Nucl. Phys. B 267 (1986) 690.
[4] U. Amaldi et al., Phys. Lett. B 66 (1977) 390.
[5] M. M. Block et al., MAD-PH 767 preprint, June 1993.
[6] R. J. M. Covolan et al., Z. Phys. C 58 (1993) 109.
[7] UA4 Collab., M. Bozzo et al., Phys. Lett. B 147 (1984) 392.
[8] UA4/2 Collab., C. Augier et al., Phys. Lett. B 316 (1993) 448.
[9] L. Lukaszuk and B. Nicolescu, Nuovo Cim. Lett. 8 (1973) 2461.
[10] P. Gauron, B. Nicolescu and E. Leader, Nucl. Phys. B 299 (1988) 640.
[11] A. Donnachie and P. V. Landshoff, Phys. Lett. B 296 (1992) 227.
[12] Review of Particle Properties, Phys. Rev. D 50 (1994), p. 1173.
[13] UA4/2 Collab., C. Augier et al., Phys. Lett. B 315 (1993) 503.
[14] CDF Collab., F. Abe et al., FERMILAB-PUB-93-234-E, Aug. 1993.
[15] E710 Collab., N. Amos et al., Phys. Rev. Lett. 68 (1992) 2433.
[16] P. Söding, Phys. Lett. 8 (1964) 285.
[17] M. Froissart Phys. Rev. 123 (1961) 1053.
A. Martin, Nuovo Cimento 42 (1966) 930; ibid. 44 (1966) 1219
[18] A. Bueno, “A precise measurement of the real part of the pp forward
elastic amplitude at CERN SppS”, Ph. D. Thesis, University of Valencia
(1994).
[19] A. Donnachie and P.V. Lanshoff, Nuc. Phys. B267 (1986) 690.
[20] M. Honda et al., Phys. Rev. Lett. 70 (1993) 525.
Figure captions

*Fig. 1* Total cross sections data from accelerators and from cosmic rays are shown together with the best $F_1$, $F_2$ and $F_3$ fits using $\sigma_{pp}^{\text{tot}}$ and $\sigma_{\bar{p}p}^{\text{tot}}$ data.
Table 1: Values of the parameters given by the best dispersion relations fits

| Fit type | Parameters |
|----------|------------|
| $(\ln s)\gamma$ | $A_1 = 42.5^{+2.0}_{-1.6}$ | $N_1 = 0.45^{+0.08}_{-0.06}$ | $A_2 = 25.5^{+0.5}_{-0.4}$ | $N_2 = 0.565^{+0.005}_{-0.004}$ |
| | $C_0 = 30.0^{+3.0}_{-4.0}$ | $C_2 = 0.10^{+0.15}_{-0.06}$ | $\gamma = 2.25^{+0.35}_{-0.31}$ | $C_s = -57.0 \pm 4.0$ |
| Regge 1 | $Y_+ = 53.90 \pm 0.1$ | $Y_- = 121.15 \pm 2.20$ | $\eta = 0.5433 \pm 0.0075$ |
| | $X = 25.16 \pm 0.29$ | $\epsilon = 0.0644 \pm 0.0015$ | $C_s = -37.7 \pm 0.7$ |
| Regge 2 | $Y_+ = 56.08$ | $Y_- = 98.39$ | $\eta = 0.4525$ |
| | $X = 21.70$ | $\epsilon = 0.0808$ | $C_s = -4 \pm 17$ |
Table 2: $\chi^2$ and $\sigma^{tot}$ values obtained by fitting with dispersion relations the available $\sigma^{tot}$ and $\rho$ experimental data for pp and $\bar{p}p$ scattering. $\sigma^{tot}$ is measured in mb.

| Fit type | $\chi^2$ | $\chi^2$/d.o.f. | $\sigma^{tot}$ (546 GeV) | $\sigma^{tot}$ (1.8 TeV) | $\sigma^{tot}$ (14 TeV) |
|----------|----------|-----------------|--------------------------|--------------------------|--------------------------|
| $(\ln s)^\gamma$ | 78.5     | 0.8             | 61.8 ± 0.7               | 76.5 ± 2.3               | 110. ± 8                  |
| Regge 1  | 153.9    | 1.6             | 56.8 ± 0.4               | 66.1 ± 0.7               | 86.0 ± 1.4                |
| Regge 2  | 456.3    | 4.5             | 60.4                     | 73.0                     | 101.5                     |
| $\sqrt{s}$ (TeV) | Data | $\sigma^{tot}$ (mb) | $F_1$ | $F_2$ | $F_3$ |
|-----------------|------|---------------------|------|------|------|
| 0.55            | UA4  | 62.2 ± 1.5          | 61.8 ± 0.7 | 56.8 ± 0.4 | 60.4 |
|                 | CDF  | 61.5 ± 1.0          |      |      |      |
| 1.8             | E710 | 72.8 ± 3.1          | 76.5 ± 2.3 | 66.1 ± 0.7 | 73.0 |
|                 | CDF  | 80.6 ± 2.3          |      |      |      |

Table 3: Experimental $\sigma^{tot}$ values in mb at the S$\bar{p}$pS (0.55 TeV) and Tevatron (1.8 TeV) and best $F_1$, $F_2$ and $F_3$ predictions.
| Fit type   | Parameters                                                                 |
|-----------|----------------------------------------------------------------------------|
| (ln s)γ   | \( A_1 = 42.5^{+2.7}_{-2.0} \), \( N_1 = 0.49^{+0.08}_{-0.11} \), \( A_2 = 25.2^{+0.1}_{-0.2} \), \( N_2 = 0.562^{+0.003}_{-0.002} \) |
|           | \( C_0 = 32.4^{+2.7}_{-6.0} \), \( C_2 = 0.063^{+0.280}_{-0.040} \), \( \gamma = 2.42^{+0.4}_{-0.6} \) |
| Regge 1   | \( Y_+ = 51.52 \pm 0.15 \), \( Y_- = 118.71 \pm 2.2 \), \( \eta = 0.5431 \pm 0.0075 \) |
|           | \( X = 25.62 \pm 0.30 \), \( \epsilon = 0.0618 \pm 0.0015 \) |
| Regge 2   | \( Y_+ = 56.08 \), \( Y_- = 98.39 \), \( \eta = 0.4525 \) |
|           | \( X = 21.70 \), \( \epsilon = 0.0808 \) |

Table 4: Values of the parameters obtained after fitting \( \sigma^{tot}_{pp} \) and \( \sigma^{tot}_{pp} \) data. Tevatron measurements are excluded from the fit. The lowest energy limit corresponds to 5 GeV.
| Fit type  | $\chi^2$ | $\chi^2/d.o.f.$ | $\sigma^{tot}$ (546 GeV) | $\sigma^{tot}$ (1.8 TeV) | $\sigma^{tot}$ (14 TeV) |
|-----------|----------|-----------------|--------------------------|-------------------------|-------------------------|
| $(\ln s)^7$ | 43.1     | 0.7             | 61.7 ± 1.3               | 76.7 ± 4.0              | 112. ± 13               |
| Regge 1   | 87.0     | 1.4             | 55.9 ± 0.4               | 64.7 ± 0.6              | 83.4 ± 1.4              |
| Regge 2   | 395.3    | 5.7             | 60.4                     | 73.0                    | 101.5                   |

Table 5: $\chi^2$ and $\sigma^{tot}$ (in mb) values obtained by fitting available data on pp and $\bar{p}p$ total cross sections. The energy range spans from 5 GeV to 546 GeV.
| Fit type | Parameters |
|----------|------------|
| (ln s)γ  | $A_1 = 49.3^{+6.4}_{-5.8}$ | $N_1 = 0.59^{+0.16}_{-0.20}$ | $A_2 = 25.4^{+0.2}_{-0.3}$ | $N_2 = 0.562^{+0.002}_{-0.001}$ |
|          | $C_0 = 34.4^{+3.1}_{-7.3}$ | $C_2 = 0.034^{+0.220}_{-0.060}$ | $\gamma = 2.64^{+0.50}_{-0.32}$ |
| Regge 1  | $Y_+ = 60.87 \pm 1.30$ | $Y_- = 124.41 \pm 7.30$ | $\eta = 0.5362 \pm 0.0190$ |
|          | $X = 24.26 \pm 0.50$ | $\epsilon = 0.0687 \pm 0.0024$ |
| Regge 2  | $Y_+ = 56.08$ | $Y_- = 98.39$ | $\eta = 0.4525$ |
|          | $X = 21.70$ | $\epsilon = 0.0808$ |

Table 6: Same as table 4, but now the lowest limit of the energy interval corresponds to 10 GeV.
| Fit type     | $\chi^2$ | $\chi^2$/d.o.f. | $\sigma^{tot}$ (546 GeV) | $\sigma^{tot}$ (1.8 TeV) | $\sigma^{tot}$ (14 TeV) |
|--------------|----------|----------------|--------------------------|--------------------------|--------------------------|
| $(\ln s)^\gamma$ | 28.5    | 0.8           | 61.7 ± 1.3               | 77.4 ± 4.2               | 116. ± 16                |
| Regge 1      | 39.5    | 1.1           | 57.8 ± 0.6               | 68.0 ± 1.0               | 90.0 ± 2.3               |
| Regge 2      | 103.4   | 2.5           | 60.4                     | 73.0                     | 101.5                    |

Table 7: Same as table 5, but now the lowest limit of the energy interval corresponds to 10 GeV.
