CONTINUITY PROPERTIES OF FINELY PLURISUBHARMONIC FUNCTIONS AND PLURIPOLARITY

SAID EL MARZGUIOUI AND JAN WIEGERINCK

Abstract. We prove that every bounded finely plurisubharmonic function can be locally (in the pluri-fine topology) written as the difference of two usual plurisubharmonic functions. As a consequence finely plurisubharmonic functions are continuous with respect to the pluri-fine topology. Moreover we show that $-\infty$ sets of finely plurisubharmonic functions are pluripolar, hence graphs of finely holomorphic functions are pluripolar.

1. Introduction

The fine topology on an open set $\Omega \subset \mathbb{R}^n$ is the coarsest topology that makes all subharmonic functions on $\Omega$ continuous. A finely subharmonic function is defined on a fine domain, it is upper semi-continuous with respect to the fine topology, and satisfies an appropriate modification of the mean value inequality. Fuglede [9] proved the following three properties that firmly connect fine potential theory to classical potential theory: finely subharmonic functions are finely continuous (so there is no super-fine topology), all finely polar sets are in fact ordinary polar sets, and finely subharmonic functions can be uniformly approximated by subharmonic functions on suitable compact fine neighborhoods of any point in their domain of definition.

Another continuity result is what Fuglede calls the Brelot Property, i.e. a finely subharmonic function is continuous on a suitable fine neighborhood of any given point in its domain, [14 page 284], see also [11, Lemma 1].

Similarly, the pluri-fine topology on $\Omega \subset \mathbb{C}^n$ is the coarsest topology that makes all plurisubharmonic (PSH) functions on $\Omega$ continuous. In [8] we introduced finely plurisubharmonic functions as plurifinely upper semicontinuous functions, of which the restriction to complex lines is finely subharmonic. We will prove the analogs of two of the results mentioned above. Bounded finely plurisubharmonic functions can locally be written as differences of ordinary PSH functions (cf. Section 3), hence finely plurisubharmonic functions are pluri-finely continuous. We also prove a weak form
of the Brelot Property. Next, finely pluripolar sets are shown to be pluripolar. This answers natural questions posed e.g. by [6]. As a corollary we obtain that zero sets of finely holomorphic functions of several complex variables are pluripolar sets. Partial results in this direction were obtained in [4, 5, 8]. A final consequence is Theorem 4.5 concerning the pluripolar hull of certain pluripolar sets.

The pluri-fine topology was introduced in [13], and studied in e.g., [3, 2, 7, 8]. In the rest of the paper we will qualify notions referring to the pluri-fine topology by the prefix “$F$”, to distinguish them from those pertaining to the Euclidean topology. Thus a compact $F$-neighborhood $U$ of $z$ will be a Euclidean compact set $U$ that is a neighborhood of $z$ in the pluri-fine topology.

2. FINELY PLURISUBHARMONIC AND HOLomorphic FUNCTIONS

There are several ways to generalize the concepts of plurisubharmonic and of holomorphic functions to the setting of the plurifine topology. See e.g., [6, 8], and in particular [15] where the different concepts are studied and compared.

**Definition 2.1.** Let $\Omega$ be an $F$-open subset of $\mathbb{C}^n$. A function $f$ on $\Omega$ is called $F$-plurisubharmonic if $f$ is $F$-upper semicontinuous on $\Omega$ and if the restriction of $f$ to any complex line $L$ is finely subharmonic or $\equiv -\infty$ on any $F$-connected component of $\Omega \cap L$.

A subset $E$ of $\mathbb{C}^n$ is called $F$-pluripolar if for every point $z \in E$ there is an $F$-open subset $U \subset \mathbb{C}^n$ and an $F$-plurisubharmonic function ($\not\equiv -\infty$) $f$ on $U$ such that $E \cap U \subset \{f = -\infty\}$.

Denote by $H(K)$ the uniform closure on $K$ of the algebra of holomorphic functions in neighborhoods of $K$.

**Definition 2.2.** Let $U \subseteq \mathbb{C}^n$ be $F$-open. A function $f : U \longrightarrow \mathbb{C}$ is said to be $F$-holomorphic if every point of $U$ has a compact $F$-neighborhood $K \subseteq U$ such that the restriction $f|_K$ belongs to $H(K)$.

**Remark 2.3.** The functions defined in Definition 2.1 are called weakly $F$-PSH functions in [15], whereas the functions in Definition 2.2 are called strongly $F$-holomorphic functions. In [15] strongly $F$-PSH functions (via approximation) and weakly $F$-holomorphic functions (via holomorphy on complex lines) are defined and it is shown that the strong properties imply the weak ones.

The original definition of finely subharmonic functions involves sweeping-out of measures. If one wants to avoid this concept, one can use the next theorem as an alternative definition.
Theorem 2.4 (Fuglede [10, 12]). A function $\varphi$ defined in an $\mathcal{F}$-open set $U \subseteq \mathbb{C}$ is finely subharmonic if and only if every point of $U$ has a compact $\mathcal{F}$-neighborhood $K \subset U$ such that $\varphi|_K$ is the uniform limit of usual subharmonic functions $\varphi_n$ defined in Euclidean neighborhoods $W_n$ of $K$.

Recall also the following property, cf. [2], which will be used in the proof of Theorem 4.1 and its corollary.

Theorem 2.5. (Quasi-Lindel"of property) An arbitrary union of $\mathcal{F}$-open subsets of $\mathbb{C}^n$ differs from a suitable countable subunion by at most a pluripolar set.

3. Continuity of Finely PSH Functions

Theorem 3.1. Let $f$ be a bounded $\mathcal{F}$-plurisubharmonic function in a bounded $\mathcal{F}$-open subset $U$ of $\mathbb{C}^n$. Every point $z \in U$ then has an $\mathcal{F}$-neighborhood $O \subset U$ such that $f$ is representable in $O$ as the difference between two locally bounded plurisubharmonic functions defined on some usual neighborhood of $z$. In particular $f$ is $\mathcal{F}$-continuous.

Proof. We may assume that $-1 < f < 0$ and that $U$ is relatively compact in the unit ball $B(0,1)$. Let $V \subset U$ be a compact $\mathcal{F}$-neighborhood of $z_0$. Since the complement $\overline{C}V$ of $V$ is pluri-thin at $z_0$, there exist $0 < r < 1$ and a plurisubharmonic function $\varphi$ on $B(z_0,r)$ such that

$$\limsup_{z \to z_0, z \in \overline{C}V} \varphi(z) < \varphi(z_0).$$

Without loss of generality we may suppose that $\varphi$ is negative in $B(z_0,r)$ and

$$\varphi(z) = -1 \text{ if } z \in B(z_0,r) \setminus V \text{ and } \varphi(z_0) = -\frac{1}{2}.$$ 

Hence

$$f(z) + \lambda \varphi(z) \leq - \lambda \text{ for any } z \in U \cap B(z_0,r) \setminus V \text{ and } \lambda > 0.$$ 

Now define a function $u_\lambda$ on $B(z_0,r)$ as follows

$$u_\lambda(z) = \begin{cases} \max\{-\lambda, f(z) + \lambda \varphi(z)\} & \text{if } z \in U \cap B(z_0,r), \\ -\lambda & \text{if } z \in B(z_0,r) \setminus V. \end{cases}$$

This definition makes sense because $[U \cap B(z_0,r)] \cup [B(z_0,r) \setminus V] = B(z_0,r)$, and the two definitions of $u_\lambda$ agree on $U \cap B(z_0,r) \setminus V$ in view of (3.3).

Clearly, $u_\lambda$ is $\mathcal{F}$-plurisubharmonic in $U \cap B(z_0,r)$ and in $B(z_0,r) \setminus V$, hence in all $B(z_0,r)$ in view of the sheaf property, cf. [8]. Since $u_\lambda$ is bounded in $B(z_0,r)$, it follows from [8, Theorem 9.8] that $u_\lambda$ is subharmonic on each complex line where it is defined. It is a well known result that a bounded
function which is subharmonic on each complex line where it is defined, is plurisubharmonic, cf. [17]. In other words $u_\lambda$ is plurisubharmonic in $B(z_0, r)$.

Since $\varphi(z_0) = -\frac{1}{2}$, the set $\mathcal{O} = \{\varphi > -3/4\}$ is an $\mathcal{F}$-neighborhood of $z_0$, and because $\varphi = -1$ on $B(z_0, r) \setminus V$, it is clear that $\mathcal{O} \subset V \subset U$.

Observe now that $-4 \leq f(z) + 4\varphi(z)$, for every $z \in \mathcal{O}$. Hence

\begin{equation}
(3.5) \quad f(z) = u_4(z) - 4\varphi(z), \quad \text{for every } z \in \mathcal{O}.
\end{equation}

We have shown that $f$ is $\mathcal{F}$-continuous on a neighborhood of each point in its domain, hence $f$ is $\mathcal{F}$-continuous. □

The proof is inspired by [9, page 88-90].

**Corollary 3.2.** Every $\mathcal{F}$-plurisubharmonic function is $\mathcal{F}$-continuous.

**Proof.** Let $f$ be $\mathcal{F}$-plurisubharmonic in an $\mathcal{F}$-open subset $\Omega$ of $\mathbb{C}^n$. Let $d < c \in \mathbb{R}$. The set $\Omega_c = \{f < c\}$ is $\mathcal{F}$-open. The function $\max\{f, d\}$ is bounded $\mathcal{F}$-PSH on $\Omega_c$, hence $\mathcal{F}$-continuous. Therefore the set $\{d < f < c\}$ is $\mathcal{F}$-open, and we conclude that $f$ is $\mathcal{F}$-continuous. □

The following result gives a partial analog to the Brelot property. We recall the definition of the relative extremal function or pluriharmonicmeasure of a subset $E$ of an open set $\Omega$, cp. [1, 16]

\begin{equation}
(3.6) \quad U = U_{E, \Omega} = \sup\{\psi \in \text{PSH}^\infty_\Omega : \psi \leq -1 \text{ on } E\}.
\end{equation}

It is well known that the upper semi-continuous regularization of $U$, i.e. $U^*(z) = \limsup_{\Omega \ni v \to z} U(v)$ is plurisubharmonic in $\Omega$.

**Theorem 3.3. (Quasi-Brelot property)** Let $f$ be a plurisubharmonic function in the unit ball $B \subset \mathbb{C}^n$. Then there exists a pluripolar set $E \subset B$ such that for every $z \in B \setminus E$ we can find an $\mathcal{F}$-neighborhood $\mathcal{O}_z \subset B$ of $z$ such that $f$ is continuous in the usual sense in $\mathcal{O}_z$.

**Proof.** Without loss of generality we may assume that $f$ is continuous near the boundary of $B$. By the quasi-continuity theorem (cf. [16, Theorem 3.5.5] and the remark that follows it, see also [1]) we can select a sequence of relatively compact open subset $\omega_n$ of $B$ such that the Monge-Ampère capacity $C(\omega_n, B) < \frac{1}{n}$, and $f$ is continuous on $B \setminus \omega_n$. Denote by $\bar{\omega}_n$ the $\mathcal{F}$-closure of $\omega_n$.

The pluriharmonic measure $U^*_{\omega_n, B}$ is equal to the pluriharmonic measure $U^*_{\bar{\omega}_n, B}$, because for a PSH function $\varphi$ the set $\{\varphi \leq -1\}$ is $\mathcal{F}$-closed, thus $\varphi|_{\omega_n} \leq -1 \Rightarrow \varphi|_{\bar{\omega}_n} \leq -1$. Now, using [16, Proposition 4.7.2]

\begin{equation}
(3.7) \quad C(\omega_n, B) = C^*(\omega_n, B) = \int_{\Omega} (dd^c U^*_{\omega_n, B})^n = \int_{\Omega} (dd^c U^*_{\bar{\omega}_n, B})^n = C^*(\bar{\omega}_n, B).
\end{equation}
Let $E = \bigcap_n \tilde{\omega}_n$. By (3.7), $C^*(E, B) \leq C^*(\tilde{\omega}_n, B) \leq \frac{1}{n}$, for every $n$. Hence $E$ is a pluripolar subset of $B$.

Let $z \notin E$. Then there exists $N$ such that $z \notin \tilde{\omega}_N$. Clearly, the set $B \setminus \tilde{\omega}_N$ is an $\mathcal{F}$-neighborhood of $z$. Since $f$ is continuous on $B \setminus \omega_N$, it is also continuous on the smaller set $B \setminus \tilde{\omega}_N (\subset B \setminus \omega_N)$.

**Remark 3.4.** The above Quasi-Brelot property holds also for $\mathcal{F}$-plurisubharmonic functions, in view of Theorem 3.1.

4. $\mathcal{F}$-Pluripolar Sets and Pluripolar Hulls

In this section we prove that $\mathcal{F}$-pluripolar sets are pluripolar and apply this to pluripolar hulls.

**Theorem 4.1.** Let $f : U \to [-\infty, +\infty]$ be an $\mathcal{F}$-plurisubharmonic function ($\not\equiv -\infty$) on an $\mathcal{F}$-open and $\mathcal{F}$-connected subset $U$ of $\mathbb{C}^n$. Then the set $\{z \in U : f(z) = -\infty\}$ is a pluripolar subset of $\mathbb{C}^n$.

**Proof of Theorem 4.1.** We may assume that $f < 0$. Let $z_0 \in U$, which we can assume relatively compact in $B(0,1)$. We begin by showing that $z_0$ admits an $\mathcal{F}$-neighborhood $W_{z_0}$ such that $\{f = -\infty\} \cap W_{z_0}$ is pluripolar. If $z_0$ is a Euclidean interior point of $U$, then $f$ is PSH on a neighborhood of $z_0$ and there is nothing to prove.

If not we proceed as in the proof of Theorem 3.1. Thus, let $V \subset U$ be a compact $\mathcal{F}$-neighborhood of $z_0$, and $\varphi$ a negative PSH function on $B(z_0, r)$ such that

\begin{equation}
\varphi(z) = -1 \text{ if } z \in B(z_0, r) \setminus V \text{ and } \varphi(z_0) = -\frac{1}{2}.
\end{equation}

Let $\Phi = U_{B(z_0, r) \setminus V, B(z_0, r)}$ be the pluriharmonic measure defined in (3.6). By (4.1), we get $\varphi \leq \Phi \leq \Phi^*$. In particular $-\frac{1}{2} \leq \Phi^*(z_0)$.

Let $f_n = \frac{1}{n} \max(f, -n)$. Then $-1 \leq f_n < 0$. We define functions $v_n(z)$ on $B(z_0, r)$ as follows.

\begin{equation}
v_n(z) = \begin{cases} 
\max\{-1, \frac{1}{2} f_n(z) + \Phi^*(z)\} & \text{if } z \in U \cap B(z_0, r), \\
-1 & \text{if } z \in B(z_0, r) \setminus V.
\end{cases}
\end{equation}

Since $v_n$ is analogous to the function $u_\lambda$ in (3.4), the argument in the proof of Theorem 3.1 shows that $v_n \in \text{PSH}(B(z_0, r))$. Now for $z \in U$ such that $f(z) \not\equiv -\infty$ the sequence $f_n(z)$ increases to 0. Thus $\{v_n\}$ is an increasing sequence of PSH-functions. Let $\lim v_n = \psi$. The upper semi-continuous regularization $\psi^*$ of $\psi$ is plurisubharmonic in $B(z_0, r)$. It is a result of [1], see also Theorem 4.6.3 in [16], that the set $E = \{\psi \neq \psi^*\}$ is a pluripolar subset of $B(z_0, r)$.
We claim that $\psi^* = \Phi^*$ on $B(z_0, r)$. Indeed, $\psi \leq \psi^* \leq \Phi^*$ because the $v_n$ belong to the defining family (3.6) for $\Phi$. Now observe that $\psi = \Phi^*$ on $B(z_0, r) \setminus \{f = -\infty\}$, because $v_n = \Phi^* = -1$ on $B(z_0, r) \setminus V$. Hence

\[(4.3) \quad \{\psi^* \neq \Phi^*\} \subset B(z_0, r) \cap \{f = -\infty\}.
\]

Clearly, the set $\{\psi^* \neq \Phi^*\}$ is $F$-open. In view of Theorem 5.2 in [8] it must be empty because it is contained in the $-\infty$-set of a finely plurisubharmonic function.

Let $z \in \{\Phi^* > -\frac{2}{3}\} \cap \{f = -\infty\}$. Then it follows from the definition of $v_n$ and the claim that

$$\psi(z) = -\frac{1}{4} + \Phi^*(z) = -\frac{1}{4} + \psi^*(z).$$

Thus $z \in E$. Now $\{\Phi^* > -\frac{2}{3}\}$ is an $F$-neighborhood of $z_0$. The conclusion is that every point $z \in U$ has an $F$-neighborhood $W_z \subset U$ such that $W_z \cap \{f = -\infty\}$ is a pluripolar set. (If $f(z) \neq -\infty$ we could have chosen $W_z$ such that $W_z \cap \{f = -\infty\} = \emptyset$.)

By the Quasi-Lindelöf property, cf. Theorem 2.5 there is a sequence $\{z_n\}_{n \geq 1} \subset U$ and a pluripolar subset $P$ of $U$ such that

\[(4.4) \quad U = \bigcup_n O_{z_n} \cup P.
\]

Hence

\[(4.5) \quad \{f = -\infty\} \subset (\bigcup_n O_{z_n} \cap \{f = -\infty\}) \cup P.
\]

This completes the proof since a countable union of pluripolar sets is pluripolar. \qed

Remark 4.2. Corollary 3.2 and Theorem 4.1 give affirmative answers to two questions in [6].

A weaker formulation of Theorem 4.1 but perhaps more useful, is as follows.

Corollary 4.3. Let $f : U \rightarrow [-\infty, +\infty]$ be a function defined in an $F$-domain $U \subset \mathbb{C}^n$. Suppose that every point $z \in U$ has a compact $F$-neighborhood $K_z \subset U$ such that $f|_{K_z}$ is the decreasing limit of usual plurisubharmonic functions in Euclidean neighborhoods of $K_z$. Then either $f \equiv -\infty$ or the set $\{f = -\infty\}$ is pluripolar subset of $U$.

As a byproduct we get the following corollary which recovers and generalizes the main result in [4] to functions of several variables.

Corollary 4.4. Let $h : U \rightarrow \mathbb{C}$ be an $F$-holomorphic function on an $F$-open subset $U$ of $\mathbb{C}^n$. Then the zero set of $h$ is pluripolar. In particular, the graph of $h$ is also pluripolar.
Proof of Corollary 4.4. Let \( a \in U \). Definition 2.2 gives us a compact \( \mathcal{F} \)-neighborhood \( K \) of \( a \) in \( U \), and a sequences \((h_n)_{n \geq 0}\), of holomorphic functions defined in Euclidean neighborhoods of \( K \) such that
\[
h_n|_K \to h|_K, \text{ uniformly.}
\]
For \( k \in \mathbb{N} \) we define \( v_{n,k} = \max(\log |h_n|, -k) \) and \( v_k = \max(\log |h|, -k) \). Clearly, \( v_{n,k} \) converges uniformly on \( K \) to \( v_k \) as \( n \to \infty \). Accordingly, \( v_k \) is \( \mathcal{F} \)-plurisubharmonic on the \( \mathcal{F} \)-interior \( K' \) of \( K \). Since \( v_k \) is decreasing, the limit function \( \log |h| \) is \( \mathcal{F} \)-plurisubharmonic in \( K' \). Theorem 4.1 shows that the set \( K' \cap \{ h = 0 \} \) is pluripolar. The corollary follows now by application of the Quasi-Lindelöf property.

The pluripolar hull \( E^*_\Omega \) of a pluripolar set \( E \) relative to an open set \( \Omega \) is defined as follows.
\[
E^*_\Omega = \bigcap \{ z \in \Omega : u(z) = -\infty \},
\]
where the intersection is taken over all plurisubharmonic functions defined in \( \Omega \) which are equal to \(-\infty\) on \( E \).

The next theorem improves on Theorem 6.4 in [8].

**Theorem 4.5.** Let \( U \subset \mathbb{C}^n \) be an \( \mathcal{F} \)-domain, and let \( h \) be \( \mathcal{F} \)-holomorphic in \( U \). Denote by \( \Gamma_h(U) \) the graph of \( h \) over \( U \), and let \( E \) be a non-pluripolar subset of \( U \). Then \( \Gamma_h(U) \subset (\Gamma_h(E))^{*}_{\mathbb{C}^{n+1}} \).

**Proof.** By Corollary 4.4, the set \( \Gamma_h(E) \) is pluripolar subset of \( \mathbb{C}^{n+1} \). Let \( \varphi \) be a plurisubharmonic function in \( \mathbb{C}^{n+1} \) with \( \varphi \neq -\infty \) and \( \varphi(z, h(z)) = -\infty \), for every \( z \in E \). The same arguments as in the proof of Lemma 3.1 in [4] show that the function \( z \mapsto \varphi(z, h(z)) \) is \( \mathcal{F} \)-plurisubharmonic in \( U \). Since \( E \) is not pluripolar, it follows from Theorem 3.1 that \( \varphi(z, h(z)) = -\infty \) everywhere in \( U \). Hence \( \Gamma_h(U) \subset (\Gamma_h(E))^{*}_{\mathbb{C}^{n+1}} \). \(\square\)

5. Some further questions

**Question 1** Let \( f \) be an \( \mathcal{F} \)-plurisubharmonic function defined in an \( \mathcal{F} \)-open set \( U \subset \mathbb{C}^2 \). Suppose that for each point \( z \in U \) there is a compact \( \mathcal{F} \)-neighbourhood \( K_z \) such that \( f \) is continuous (in the usual sense) on \( K_z \). Is it true that \( f|_{K_z} \) is the uniform limit of usual plurisubharmonic functions \( \varphi_n \) defined in Euclidean neighborhoods \( W_n \) of \( K_z \)?

**Question 2** It is also interesting to figure out whether the assumption in the above question is automatically fulfilled. This would be the Brelot property for \( \mathcal{F} \)-plurisubharmonic function.

Many other questions remain open. For example, we do not know the answer to the following.

**Question 3** Is this concept of an \( \mathcal{F} \)-plurisubharmonic function biholomorphically invariant?
References

[1] Bedford, E. and Taylor, B. A: A new capacity for plurisubharmonic functions, *Acta Math.* 149 (1982), 1–40.

[2] Bedford, E. and Taylor, B. A.: Fine topology, Silov boundary and $(dd^c)^n$, *J. Funct. Anal.* 72 (1987), 225–251.

[3] Bedford, E.: Survey of pluripotential theory, Several complex variables: Proceedings of the Mittag-Leffler Inst. 1987-1988 (J-E. Fornæss, ed), Math. Notes 38, Princeton University Press, Princeton, NJ, 1993.

[4] Edigarian, E. El Marzguioui, S. and Wiegerinck, J.: The image of a finely holomorphic map is pluripolar, [arXiv:math/0701136](https://arxiv.org/abs/math/0701136).

[5] Edlund, T. and Jöricke, B.: The pluripolar hull of a graph and fine analytic continuation, *Ark. Mat.* 44 (2006), no. 1, 39–60.

[6] El Kadiri, M.: Fonctions finement plurisousharmoniques et topologie plurifine. *Rend. Accad. Naz. Sci. XLMem. Mat. Appl. (5)* 27, (2003) 77–88.

[7] El Marzguioui, S., Wiegerinck, J.: The plurifine topology is locally connected. *Potential Anal.* 25 (2006), no. 3, 283–288.

[8] El Marzguioui, S., Wiegerinck, J.: Connectedness in the plurifine topology, *Functional Analysis and Complex Analysis, Istanbul 2007*, 105–115, Contemp. Math., 481, Amer. Math. Soc., Providence, RI, 2009.

[9] Fuglede, B.: Finely harmonic functions, *Springer Lecture Notes in Mathematics*, 289, Berlin-Heidelberg-New York, 1972.

[10] Fuglede, B.: Fonctions harmoniques et fonctions finement harmonique, *Ann. Inst. Fourier, Grenoble* 24.4 (1974) 77–91.

[11] Fuglede, B.: Finely harmonic mappings and finely holomorphic functions, *Ann. Acad. Sci. Fennicae* 2 (1976), 113–127.

[12] Fuglede, B.: Localisation in fine potential theory and uniform approximation by subharmonic functions, *J. Functional Anal.* 49 (1982), 57–72.

[13] Fuglede, B.: Fonctions finement holomorphes de plusieurs variables - un essai. *Séminaire d’Analyse P. Lelong P. Dolbeault - H. Skoda*, Springer - Verlag, Lecture Notes in Math. 1198 (1986), 133–145.

[14] Fuglede, B.: Finely holomorphic functions- a survey, *Revue Roumaine Math. Pures Appl.* 33 (1988), 283–295.

[15] Fuglede, B.: Concepts of plurifinely plurisubharmonic and plurifinely holomorphic functions, preprint (2009)

[16] Klimek, M.: Pluripotential Theory, London Mathematical Society Monographs, 6, Clarendon Press, Oxford, 1991.

[17] Lelong, P.: Les fonctions plurisousharmoniques, *Ann. Sci. École Norm. Sup.* 62 (1945), 301–328.

KdV Institute for Mathematics, Universiteit van Amsterdam, Postbus 94248 1090 GE, Amsterdam, The Netherlands

E-mail address: s.elmarzguioui@uva.nl

KdV Institute for Mathematics, Universiteit van Amsterdam, Postbus 94248 1090 GE, Amsterdam, The Netherlands

E-mail address: j.j.o.o.wiegerinck@uva.nl