Adjusted closing prices

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21–May–2010

ABSTRACT
Historical returns depend on historical closing prices and distributions. We describe how to compute adjusted closing prices from closing price/distribution data with an emphasis on spreadsheet implementation. Then the growth of a security from one date to another (1 + total return) is just the ratio of the corresponding adjusted closing prices.
1 The Growth Ratio

If $C_-$ and $C_+$ are the closing prices of a given security on two successive business days, then the value of the security grows by $\sigma_+ - 1$ (decimal, percent) over the second business day, where $\sigma_+ = C_+/C_-$. Here we assume that the second business day is not an ex-day corresponding to a declared distribution or split.

If day two is an ex-dividend day for a distribution of $D_+$ dollars per share, then the security starts from an effective price of $C_- - D_+$ at the end of the first day and grows to the $C_+$ price over the course of day two. In this case the growth ratio $\sigma_+$ should be defined as

$$\sigma_+ = \frac{C_+}{C_- - D_+} \quad \text{(1d)}$$

with the percentage growth again being $\sigma_+ - 1$. Of course equation (1d) can apply to a non-ex-day as well. Just set $D_+$ equal to zero. After an $\alpha:1$ split the effective starting price is $C_-/\alpha$, and the growth ratio is

$$\sigma_+ = \alpha \times \frac{C_+}{C_-} \quad \text{(1s)}$$

Splits can be also thought of as distributions. An $\alpha:1$ split corresponds to a $D_+ = \frac{\alpha - 1}{\alpha} \times C_-$ distribution.

2 Adjusted Closing Prices

Start with sequences of successive closing prices $C_i \ (i = 0, 1, \ldots, n)$ and distributions $D_i \ (i = 1, \ldots, n)$ of a given security. Define the corresponding growth ratios by

$$\sigma_i = \frac{C_i}{C_{i-1} - D_i} \quad (i = 1, \ldots, n). \quad \text{(1)}$$

We will refer to any sequence of positive numbers $x_i \ (i = 0, 1, \ldots, n)$ as *adjusted closing prices* for the security if

$$\frac{x_i}{x_{i-1}} = \sigma_i \quad \text{for} \quad i = 1, \ldots, n. \quad \text{(2)}$$

Then the *growth ratio* of the security for the duration under consideration is

$$\sigma = \sigma_1 \times \cdots \times \sigma_n = \frac{x_n}{x_0}, \quad \text{(3)}$$

and the percentage growth or total return is $\sigma - 1$.

See Investopedia for a less abstract description of how to compute the adjusted closing price of a stock.
3 Dividend Reinvestment

If one buys \( s_0 \) shares of a security at closing price \( C_0 \), then the value of the investment is \( s_0 C_0 \). Suppose that dividends are reinvested automatically and that the number of shares and the closing prices over the next \( n \) business days are \( s_i \) and \( C_i \) \((i = 1, \ldots, n)\), respectively. Then the growth ratio \( \sigma_i \) of the investment over the \( i \)-th business day should clearly be the ratio of the value of the investment at the close of day \( i \) to its value at the close of day \( i - 1 \):

\[
\sigma_i = \frac{s_i C_i}{s_{i-1} C_{i-1}}.
\]

By equating the \( \sigma_i \) of (1) with the \( \sigma_i \) of (4) we see that

\[
s_i = s_{i-1} + \Delta s_i, \quad \Delta s_i = \frac{D_i}{C_{i-1} - D_i}.
\]

Thus the new shares \( \Delta s_i \) are precisely what can be purchased with the dividend \( D_i \) at the “ex-closing-price” \( C_{i-1} - D_i \).

4 Spreadsheet Considerations

Suppose successive closing prices \( C \), distributions \( D \), and growth ratios \( \sigma \) (as computed by (1)) are recorded as three columns of a spreadsheet. The formulas

\[
x_+ = x_+ \times \sigma_+ \quad \text{(FILL+)}
\]

\[
x_- = x_+ \div \sigma_+ \quad \text{(FILL−)}
\]

then allow one to fill in adjusted closing prices from an arbitrary base adjusted closing price on any given business day. If the \(+ = \) time direction is up, enter the right-hand formula of the (FILL+) equation in the cell above the base price (now \( x_- \)) and “Fill Up” from there, and enter the right-hand formula of the (FILL−) equation in the cell below the base price (now \( x_+ \)) and “Fill Down” from this cell. In this way all adjusted closing prices can be in filled in, up or down, from the base adjusted closing price. (If the \( + = \) time direction is down, one should “Fill Down” with FILL+ and “Fill Up” with FILL−.)

The following spreadsheet image illustrates the procedure. The adjusted closing price of iShares Barclays 1-3Yr Treasury Bond Fund (SHY) is arbitrarily set at 100.000 on December 14, 2007. All other adjusted closing prices are filled up or down from this base price using (FILL+) or (FILL−), respectively. (The empty distribution cells are computed as zero.)
The growth of \textit{SHY} in December 2007 is measured by the ratio of its 2007-12-31 adjusted closing price to its 2007-11-30 price,

\[
\frac{100.72}{100.42} = 1.003.
\]

Thus \textit{SHY} gained 0.3\% in December 2007. The ratio, 1.003, is also the product of the December growth ratios: 1.0023 \times \cdots \times 1.0016.

\section{Data}

Here are two comma-separated-value (spreadsheet) files, \texttt{shy\_data.csv} and \texttt{eem\_data.csv}, with closing price, distribution, and adjusted closing price data for

- \texttt{SHY: iShares Barclays 1-3Yr Treasury Bond Fund}
- \texttt{EEM: iShares MSCI Emerging Markets Index Fund}
respectively. The graphs of these adjusted closing price data are shown in Figure 2.

Figure 2: Adjusted closing prices
with base price = 100 on 2006-12-29

We have chosen the bond fund \textit{SHY} because has lots distributions to work with, generally one per month. On the other hand the equity fund \textit{EEM} illustrates how splits are handled; the shares of \textit{EEM} split 3 for 1 in 2005 and again in 2008.

One can get historical closing prices for any security at \textit{Yahoo!FINANCE} or more specifically, for our funds, at

\begin{verbatim}
http://finance.yahoo.com/q/hp?s=SHY+Historical+Prices
http://finance.yahoo.com/q/hp?s=EEM+Historical+Prices
\end{verbatim}

Dividends and splits can be found here as well, but unfortunately some dividends may be missing. For example \textit{Yahoo!FINANCE} is missing the 27-Dec-2007 \textit{SHY} dividend of \$0.2794 per share shown in Figure 1.

\textit{Yahoo!FINANCE} also gives adjusted closing prices, but, because of missing dividends, these prices may be inaccurate. Consider \textit{SHY} for example. The Yahoo’s 2007-12-31 adjusted closing price is less than its 2007-11-30 adjusted closing price. Thus \textit{Yahoo!FINANCE} would have you believe that \textit{SHY} lost money in December 2007, when, in fact, \textit{SHY} gained 0.3\% in this month—as we have just seen.

The best place to get iShares distribution data is from the horse’s mouth: go to

\begin{verbatim}
http://us.ishares.com/product_info/fund/
\end{verbatim}

click on the desired fund and then on the Distributions tab. In our case

\begin{verbatim}
http://us.ishares.com/product_info/fund/distributions/SHY.htm
http://us.ishares.com/product_info/fund/distributions/EEM.htm
\end{verbatim}
will get you all distributions of SHY and EEM, respectively.

6 Closing Remarks

All of this material probably appears elsewhere. We just don’t know where.

References

Investopedia. How do I calculate the adjusted closing price for a stock? <http://www.investopedia.com/ask/answers/06/adjustedclosingprice.asp>.