Does the fear gauge predict downside risk more accurately than econometric models? Evidence from the US stock market

Chikashi Tsuji

Abstract: This paper empirically compares the usefulness of information included in the volatility index (VIX) against several generalized autoregressive conditional heteroskedasticity (GARCH) models for predicting downside risk in the US stock market. Our main findings are as follows. First, using the univariate logit and quantile regression models, we reveal that the previous day’s VIX and the forecast S&P 500 volatilities from GARCH, exponential GARCH (EGARCH), power GARCH (PGARCH), and threshold GARCH (TGARCH) models have statistically significant predictive power for large declines in the S&P 500. Second, direct comparisons with the multiple logit and quantile regression models demonstrate that the volatility forecasts from the EGARCH, PGARCH, and TGARCH models dominate the predictive power of the previous day’s VIX; and we also clarify that the predictive power of volatility forecasts from the EGARCH and TGARCH models is much stronger. Third, our additional tests further suggest that the forecast VIX, the forecast volatility of VIX, and the forecast volatility of the first log differences of VIX cannot outperform the S&P 500 volatility forecasts from econometric models in predicting US stock market downside risk. Fourth, our vector-half (VECH), Baba–Engle–Kraft–Kroner (BEKK), dynamic conditional correlation (DCC), and asymmetric DCC (ADCC) multivariate GARCH (MGARCH)
analyses demonstrate that the time-varying correlations between the previous day's VIX and the volatility forecasts from the EGARCH or TGARCH models are weaker than the correlations of volatility forecasts from the EGARCH and TGARCH models. Finally, our VECCH-, BEKK-, DCC-, and ADCC-MGARCH analyses further clarify almost perfect correlations around the US Lehman Brothers bankruptcy across all three volatility series. The key contribution of this paper is that it clarifies the superiority of volatility forecasts using econometric models compared with VIX in predicting the US stock market downside risk. The primary implications of our results are the importance of developing effective technical models and the need to use econometric model volatility forecasts in practice.

Subjects: Econometrics; Economic Forecasting; Finance

Keywords: ADCC-MGARCH model; BEKK-MGARCH model; DCC-MGARCH model; downside risk; EGARCH model; PGARCH model; S&P 500; TGARCH model; VECH-MGARCH model; VIX

1. Introduction

When and why do stock markets suddenly and sharply fall, and does this phenomenon relate to the psychology of market participants? We well know that rapid and drastic declines in stock prices strongly affect the economy. Hence, providing robust evidence as to the nature of downside risk in stock markets is a crucial concern in business, economics, and finance. The US volatility index (VIX), which is a measure of implied volatility obtained from options markets, is an important indicator of stock market risk, and is often referred to as the “fear gauge” because it is thought to reflect negative stock market psychology. The question is whether the fear gauge accurately predicts downside risk in stock markets. Alternatively, would the volatility forecasts from econometric models, especially from several generalized autoregressive conditional heteroskedasticity (GARCH) models, predict downside risk in stock markets more effectively?

As discussed later, many studies have analyzed the downside risk in stock markets by focusing on the information contained in data from options, futures, and other derivative markets (e.g. Bekaert & Hoerova, 2014; Chung, Tsai, Wang, & Weng, 2011; Fung, 2007; Li, Chen, & French, 2015). Several other studies have investigated the downside risk of stock markets by focusing on analytical techniques such as GARCH, extreme value theory (EVT), the Lévy process, and other techniques (e.g. Bates, 2012; Charles & Darné, 2014; Chesney, Reshetar, & Karaman, 2011; Wong, 2010). We note that although the effectiveness of various GARCH models for capturing asset price volatility has been suggested in previous studies (e.g. Bollerslev, 1986; Charles & Darné, 2014; Schwert, 1990; Zhang & King, 2005), there are currently very few comprehensive studies that have performed rigorous and careful empirical comparisons of different kinds of GARCH models' predictive power. Hence we consider that careful, comprehensive, and objective empirical evaluations of these models' predictive power will contribute not only to the existing body of literature but also to our future-related research. The purpose of this study is, therefore, to compare empirically the effectiveness of market data information, in the form of the VIX, and forecasts provided by modern econometric tools, by way of several GARCH models, in predicting US stock market downside risk. To the best of our knowledge, this is the first such study to compare the predictive power of the VIX and the volatility forecasts from several GARCH models thoroughly and comprehensively.

Our main findings are as follows. First, the results of both univariate logit and quantile regression models suggest that the previous day’s VIX and the volatility forecasts from the GARCH, exponential GARCH (EGARCH), power GARCH (PGARCH), and threshold GARCH (TGARCH) models have statistically significant predictive power for large declines in the US stock market. Second, direct empirical comparison demonstrates that for both multiple logit and quantile regression models, the volatility forecasts from the EGARCH, PGARCH, and TGARCH models dominate the predictive power of the previous day’s VIX; and we also reveal that the predictive power of the volatility forecasts from EGARCH and
TGARCH models is much stronger. Third, our additional tests clarify that forecast VIX, the forecast volatility of VIX, and the forecast volatility of the first log differences of VIX do not outperform the forecast volatilities from econometric models in predicting large declines in the US stock market. Fourth, our vector-half (VECH), Baba–Engle–Kraft–Kroner (BEKK), dynamic conditional correlation (DCC), and asymmetric DCC (ADCC) multivariate GARCH (MGARCH) analyses demonstrate that the time-varying correlations between the volatility forecasts from the EGARCH and TGARCH models are much higher than the correlations between the previous day’s VIX and the volatility forecasts from the EGARCH or TGARCH models. Finally, our VEC-, BEKK-, DCC-, and ADCC-MGARCH analyses also reveal almost perfect correlations around the US Lehman Brothers bankruptcy across all three volatility series. The key contribution of our work is the empirical clarification that econometric model volatility forecasts are superior to VIX in predicting the US stock market downside risk. In addition, the primary implications of our work are the importance of developing further sophisticated and effective technical models for academics and the significance of employing and applying econometric model volatility forecasts for practitioners.

The remainder of the paper is organized as follows. Section 2 reviews the existing literature. Sections 3 and 4 describe our data and research design, respectively. Sections 5–8 explain our respective models and empirical results. Section 9 discusses the interpretation and implications from our results and Section 10 summarizes our findings.

2. Literature review
We consider that we can divide the extant studies relating to downside risk in stock markets into two streams. The first stream of the literature analyzes downside risk by focusing on the information contained in market data. The second stream of the literature investigates downside risk by focusing on the application of various econometric techniques.

Starting with the first stream of the literature, Giot (2005) found a positive relation between the VIX and future S&P 100 index returns. Fung (2007) also concluded that, corresponding to a 1997 stock market crash in Hong Kong, future realized volatility was well forecast by implied volatility. Later, Coudert and Gex (2008) suggested that risk-aversion indicators, including the VIX, are superior leading indicators of stock market crises, while Chung et al. (2011) demonstrated that the information contained not in the VIX but in VIX options improved the predictive power of the returns, volatility, and density of the S&P 500.

In other work, Berger and Pukthuanthong (2012) extended the Pukthuanthong and Roll (2009) measure of market integration to provide a systemic risk estimate within international stock markets. They also suggested that an increase in their risk measure indicated a greater likelihood of global market crashes. Li et al. (2015) suggested that the information embedded in S&P 500 options and futures was useful for predicting financial crises. However, in all of these studies, comparison of the downside risk predictive power of implied volatility and that of the volatility forecasts from several GARCH models was not a focus.

In terms of the second stream of the literature, while there has been no rigorous empirical comparison of downside risk predictive power, many studies have suggested the effectiveness of the various GARCH models for modeling asset price volatility (e.g. Bollerslev, 1986; Charles & Darné, 2005; Franses & Ghysels, 1999; Sakata & White, 1998; Zhang & King, 2005). In a recent study, Wong (2010) suggested the saddle point technique to accurately back test the downside tail risk of S&P 500 returns. This study clarified that during the 1987 US stock market crash, risk models incorporating skewed and fat-tailed distributions and jumps failed to capture tail risk adequately.

Further, Chesney et al. (2011) analyzed the impact on international financial markets of terrorist events using different methods. Specifically, they used an event study, a nonparametric methodology, and a filtered GARCH–EVT approach, and suggested that, among these three alternatives, the best was the nonparametric approach. Later, Bates (2012) explored the ability of time-changed
Lévy processes to capture stochastic volatility and the fat tails observed in US stock market returns from 1926 to 2010. Charles and Darné (2014) investigated the events that strongly affected the volatility of the Dow Jones Industrial Average (DJIA) index over the period from 1928 to 2013 by applying a semiparametric test based on the Glosten–Jagannathan–Runkle (GJR) model, which is a variant of the GARCH model. Their objective was the identification of those episodes that shocked the DJIA index. Thus, they did not compare the predictive power of implied volatility and the forecast volatility from the GJR model. Other studies that have employed GARCH-type models include Engle (1982), Engle, Lilien, and Robins (1987), Nelson (1991), Ding, Granger, and Engle (1993), Engle and Ng (1993), Glosten, Jagannathan, and Runkle (1993), Bekker and Wu (2000), Alexander and Lazar (2006), Lundblad (2007), Alexander, Lazar, and Stanescu (2013), and Bekaert, Engstrom, and Ermolov (2015), among many others.

As discussed, many studies have employed various GARCH models to yield interesting evidence based on different focuses and objectives. However, as the above literature review suggests, although there are suggestions of the effectiveness of various GARCH models for capturing the dynamics of volatility in stock markets, no existing study compares the downside risk predictive power of implied volatility, VIX, with that of the volatility forecasts from various GARCH models. Accordingly, this study focuses in particular on the comparison of (1) the usability of the information embedded in the VIX for S&P 500 with (2) the ability of modern econometric tools in the form of several GARCH models, to forecast downside risk in the US stock market. By conducting a rigorous empirical examination, for what we believe is the first time, we aim to derive highly robust evidence on the relative superiority of the two alternatives in the US stock market. We thus emphasize that deriving clear evidence from comprehensive empirical examinations by rigorously applying modern econometric models as our present study will contribute not only to deepen our knowledge but also to encourage our future-related research in many fields.

3. Data and variable constructions

To analyze the predictive power for downside risk in the US, we first use the daily values of VIX close, which is a popular measure of the implied volatility from S&P 500 index options, and denote this variable as $iv$. All VIX close data are from the Chicago Board Options Exchange (CBOE). Next, to derive the other predictors of downside risk, using the daily time-series of the S&P 500 close price $p_{S&P500}$, we construct the daily percentage log return of the S&P 500, which is calculated and denoted as $dlogsp = \ln(p_{S&P500})/p_{S&P500-1} \times 100$. The data source for the daily time-series of S&P 500 close prices is the Thomson Reuters.

Applying four kinds of GARCH models, we next construct four kinds of forecast volatility series of $dlogsp$. First is the forecast volatility from a GARCH(1,1) model $\sigma^GF_{dlogsp}$; second is the forecast volatility from an EGARCH(1,1) model $\sigma^EGF_{dlogsp}$; third is the forecast volatility from a PGARCH(1,1) model $\sigma^PGF_{dlogsp}$; fourth is the forecast volatility from a TGARCH(1,1) model $\sigma^TGF_{dlogsp}$; and the above four GARCH models have generalized error distribution (GED) errors. We also include the following four control variables. First is the default spread $def$, being Moody’s Baa corporate bond yield minus Moody’s Aaa corporate bond yield. Second is $dex$, which is the first-difference series of the trade-weighted US dollar index. Third is the term premium $term$, being the difference between the 10-year US government bond yield and the 3-month Treasury bill rate. Finally, we have $dff$, which is the first-difference series of the effective federal funds rate. All data for constructing the four control variables are from the Federal Reserve Economic Data (FRED) database.

To scrutinize the various possibilities for the information contained in the VIX, we construct six forecasts as follows. First is $iv_f^{ARMA(2,1)}$, which is the forecast VIX from an autoregressive moving average ARMA(2,1) model. Second is $iv_f^{ARMA(4,4)}$, which is the forecast VIX from an ARMA(4,4) model. Third is $iv_f^{TGF}$, which is the forecast volatility of the VIX from a TGARCH(1,1) model. Fourth is $iv_f^{egf}$, which is the forecast volatility of the VIX from an EGARCH(1,1) model. Fifth is $iv_f^{pgf}$, which is the forecast volatility of the first log differences of VIX from a TGARCH(1,1) model. The final variable, $iv_{dfg}$, is the forecast volatility of the first log differences of VIX from an EGARCH(1,1) model.
### Table 1. Descriptive statistics of stock return, volatilities, and control variables for the out-of-sample period from 3 January 2006 to 28 February 2014

|                | dlogsp | iv     | $\sigma_{dlogsp}^\text{iv}$ | $\sigma_{dlogsp}^\text{egf}$ |
|----------------|--------|--------|-----------------------------|-----------------------------|
| **Mean**       | 0.0196 | 21.5143| 18.4034                     | 17.2417                     |
| **Median**     | 0.0823 | 18.5450| 14.6747                     | 14.4128                     |
| **Maximum**    | 10.9572| 80.8600| 72.4703                     | 68.4571                     |
| **Minimum**    | −9.3537| 9.8900 | 7.4181                      | 6.3010                      |
| **Standard deviation** | 1.3722 | 10.3913| 11.1986                     | 9.7996                      |
| **Skewness**   | −0.2517| 2.1062 | 2.3684                      | 2.2419                      |
| **Kurtosis**   | 12.0416| 8.7195 | 9.4006                      | 9.2608                      |

|                | $\sigma_{dlogsp}^{\text{tgf}}$ | $\sigma_{dlogsp}^{\text{egf}}$ | $\sigma_{dlogsp}^{\text{tgf}}$ | $\sigma_{dlogsp}^{\text{egf}}$ |
|----------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| **Mean**       | 17.7252                     | 18.3918                     | 1.2219                      | −0.0043                     |
| **Median**     | 14.3023                     | 14.6651                     | 1.0300                      | −0.0026                     |
| **Maximum**    | 77.1682                     | 78.6558                     | 3.5000                      | 1.6957                      |
| **Minimum**    | 6.5858                      | 7.0698                      | 0.6200                      | −3.3854                     |
| **Standard deviation** | 11.0921 | 11.6382| 0.5669                      | 0.3813                      |
| **Skewness**   | 2.4604                      | 2.4066                      | 2.3484                      | −0.5417                     |
| **Kurtosis**   | 10.2981                     | 9.7124                      | 8.1271                      | 9.3019                      |

|                | term | diff | $\sigma_{iv}^{\text{ARMA}(2,1)}$ | $\sigma_{iv}^{\text{ARMA}(4,4)}$ |
|----------------|------|------|---------------------------------|---------------------------------|
| **Mean**       | 1.9605 | −0.0020 | 21.4841                         | 21.4842                         |
| **Median**     | 2.1500 | 0.0000  | 18.5561                         | 18.5510                         |
| **Maximum**    | 3.8300 | 1.0500  | 78.8185                         | 79.0233                         |
| **Minimum**    | −0.4900 | −0.9500 | 10.0573                         | 10.0556                         |
| **Standard deviation** | 1.1951 | 0.0898  | 10.2161                         | 10.2217                         |
| **Skewness**   | −0.5126 | −0.3736 | 2.0836                          | 2.0886                          |
| **Kurtosis**   | 2.1299 | 54.8551 | 8.5284                          | 8.5680                          |

|                | $\sigma_{iv}^{\text{egf}}$ | $\sigma_{iv}^{\text{tgf}}$ | $\sigma_{dlogiv}^{\text{tgf}}$ | $\sigma_{dlogiv}^{\text{egf}}$ |
|----------------|-----------------------------|-----------------------------|---------------------------------|-----------------------------|
| **Mean**       | 6.2343                      | 5.4589                      | 6.1464                          | 5.9255                      |
| **Median**     | 3.7707                      | 3.9702                      | 5.5621                          | 5.5795                      |
| **Maximum**    | 53.6336                     | 36.4244                     | 19.2418                         | 13.8233                     |
| **Minimum**    | 1.2219                      | 0.6919                      | 4.2068                          | 3.6799                      |
| **Standard deviation** | 7.4033 | 5.0387  | 1.8559                          | 1.3839                      |
| **Skewness**   | 2.9542                      | 2.4951                      | 2.3569                          | 1.6382                      |
| **Kurtosis**   | 13.1024                     | 10.4010                     | 10.7386                         | 6.7829                      |

**Notes:** This table presents the descriptive statistics for 16 variables used in this study. More specifically, $dlogsp$ denotes the daily percentage log return of S&P 500; and $iv$ denotes the volatility index (VIX) close as to S&P 500. In addition, $\sigma_{dlogsp}^\text{iv}$ is the forecast volatility of $dlogsp$, which is derived from the GARCH(1,1) model; $\sigma_{dlogsp}^\text{egf}$ represents the forecast volatility of $dlogsp$, which is derived from the EGARCH(1,1) model; $\sigma_{dlogsp}^\text{tgf}$ denotes the forecast volatility of $dlogsp$, which is derived from the PGARCH(1,1) model; and $\sigma_{dlogsp}^\text{tgf}$ is the forecast volatility of $dlogsp$, which is derived from the TGARCH(1,1) model. Further, $\text{def}$ denotes the default spread; $\text{dex}$ is the first-difference series of the trade-weighted US dollar index; term represents the term premium; and $\text{df}$ is the first-difference series of the effective federal funds rate. Moreover, $\sigma_{iv}^{\text{ARMA}(2,1)}$ denotes the forecast VIX, which is derived from the ARMA(2,1) model; $\sigma_{iv}^{\text{ARMA}(4,4)}$ represents the forecast VIX, which is derived from the ARMA(4,4) model; $\sigma_{dlogiv}^\text{iv}$ denotes the forecast volatility of the VIX, which is derived from the TGARCH(1,1) model; $\sigma_{dlogiv}^\text{tgf}$ is the forecast volatility of the VIX, which is derived from the EGARCH(1,1) model; $\sigma_{dlogiv}^\text{egf}$ represents the forecast volatility of the first log differences of the VIX, which is derived from the TGARCH(1,1) model; and $\sigma_{dlogiv}^\text{tgf}$ denotes the forecast volatility of the first log differences of the VIX, which is derived from the EGARCH(1,1) model. All statistics in this table are those for our out-of-sample period that is from 3 January 2006 to 28 February 2014, and the number of daily observations for this period is 2,038. In this study, we first build and specify forecast models by using the samples of our in-sample period, which is from 2 January 1990 to 30 December 2005, and then empirically compare the predictive power of various variables for downside risk in the US stock market in our out-of-sample period.
Our full sample period spans 2 January 1990 to 28 February 2014, for which we use the in-sample period from 2 January 1990 to 30 December 2005 to determine the parameters for our forecast models. We then test the predictive power of the model forecasts for the out-of-sample period, 3 January 2006 to 28 February 2014. Interestingly, the time of the US Lehman Brothers collapse is included in our out-of-sample period.

Table 1 displays the descriptive statistics for the 16 variables described above. All statistics in Table 1 are for the out-of-sample period because our comparison of the predictability of the various variables is for this period. Table 1 indicates that relative to the five volatility measures of the S&P 500, \( \text{iv}, \sigma_{\text{logsp}}^{\text{iv}}, \sigma_{\text{logsp}}^{\text{egf}}, \sigma_{\text{logsp}}^{\text{pgf}}, \sigma_{\text{logsp}}^{\text{tgf}} \), the mean value of VIX is slightly higher. The values of skewness and kurtosis are almost the same in the five volatility variables. Next, the forecast values of VIX from the two ARMA models appear to be estimated accurately because the mean, standard deviation, skewness, and kurtosis values of \( \text{ivf}_{\text{ARMA}(2,1)} \) and \( \text{ivf}_{\text{ARMA}(4,4)} \) are almost the same as those for \( \text{iv} \), respectively. Further, comparing \( \sigma_{\text{iv}}, \sigma_{\text{iv}}^{\text{egf}}, \sigma_{\text{logiv}}, \sigma_{\text{logiv}}^{\text{egf}} \), their mean values are almost the same; their skewness and kurtosis values are also almost the same except for the slightly smaller values for \( \sigma_{\text{logiv}}^{\text{egf}} \). Figure 1 plots the daily time-series of the S&P 500 and VIX values. As shown, the S&P 500 quickly declines while the VIX drastically increases around the US Lehman shock on 15 September 2008.

4. Research design

We carefully design this study to include several steps in our investigation. First, to analyze the predictability of downside risk in the US stock market, we use logit models. Specifically, we compare the predictability of downside risk for the previous day’s VIX with the volatility forecasts from four GARCH models using the following models: (1) univariate logit models including either the one-day lagged VIX or the forecast volatility from one of the four GARCH models; (2) multiple logit models including both the one-day lagged VIX and the forecast volatility from one of the four GARCH models; (3) multiple logit models including the one-day lagged VIX, the forecast volatility from one of the four GARCH models, and the four control variables already discussed.

Second, our next procedure involves robustness checks using quantile regressions. More concretely, we again check the predictability of downside risk for the one-day lagged VIX and the forecast...
volatilities from the four GARCH models using: (1) univariate quantile regression models including either the one-day lagged VIX or the forecast volatility from one of the four GARCH models; (2) multiple quantile regression models including both the one-day lagged VIX and the forecast volatility from one of the four GARCH models; (3) multiple quantile regression models including the one-day lagged VIX, the forecast volatility from one of the four GARCH models, and the four control variables.

Third, our next set of analyses further investigates the usability of the information contained in the VIX. We first test the downside risk predictability of (1) the forecast VIX from the two ARMA models using univariate logit and univariate quantile regression models; and then analyze the downside risk predictability of (2) the forecast volatilities of VIX from the TGARCH and EGARCH models using univariate logit and quantile regression models. Next, we investigate the downside risk predictability of (3) the forecast volatilities of the first log differences of VIX, as derived from the TGARCH and EGARCH models, also using univariate logit and quantile regression models.

Finally, we explore the dynamic relations and their differences between the previous day’s VIX and the forecast volatilities from the more effective forecasting models, namely the TGARCH and EGARCH models. Specifically, we analyze the time-varying correlation coefficients among the three measures of volatility using the VECH-, BEKK-, DCC-, and ADCC-MGARCH models.

5. Testing downside risk predictability with logit models

5.1. Volatility forecasting models

For testing the downside risk predictability of VIX and the forecast volatilities from the econometric models, we first collect the S&P volatility forecasts from four types of univariate GARCH models. More specifically, we first employ the following GARCH(1,1) model (Bollerslev, 1986):

\[ d\log_{sp,t} = \tau_g + \kappa_{g,t}, \]

\[ \sigma_{g,t}^2 = \kappa_g \sigma_{g,t-1}^2 + \kappa_g \kappa_{g,t-1}, \]

and using this model (1), obtain the forecast volatility of S&P 500, \( \hat{\sigma}_{d\log_{sp}} \).

In addition, we also apply the following EGARCH(1,1) model (Nelson, 1991):

\[ d\log_{sp,t} = \tau_g + \kappa_{eg,t}, \]

\[ \ln(\sigma_{eg,t}^2) = \kappa_{eg} \ln(\sigma_{eg,t-1}^2) + \kappa_{eg} \left( \kappa_{eg,t-1} - 1 \right) + \kappa_{eg}\sigma_{eg,t-1}, \]

and using this model (2), obtain the forecast volatility of S&P 500, \( \hat{\sigma}_{d\log_{sp}} \).

Moreover, we further use the following PGARCH(1,1) model (Ding et al., 1993):

\[ d\log_{sp,t} = \tau_g + \kappa_{pg,t}, \]

\[ \sigma_{pg,t}^h = \kappa_{pg} \sigma_{pg,t-1}^h + \kappa_{pg} \left( \kappa_{pg,t-1} - 1 \right)^h, \]

where the parameters \( h > 0 \) and \( |\kappa_{pg}| \leq 1 \), and using this model (3), obtain the forecast S&P volatility, \( \hat{\sigma}_{d\log_{sp}} \). Furthermore, the following TGARCH(1,1) model (Glosten et al., 1993; Zakoian, 1994) (4) is also employed to obtain the S&P volatility forecasts, \( \hat{\sigma}_{d\log_{sp}} \):

\[ d\log_{sp,t} = \kappa_{tg,t}, \]

\[ \sigma_{tg,t}^2 = \sigma_{tg,t-1}^2 + \kappa_{tg} \sigma_{tg,t-1}^2 + \kappa_{tg} \kappa_{tg,t-1}^2 d_{tg,t-1}. \]
where $d_{tg}^{spt-1} = 1$ if $\kappa_{tg} < 0$ and zero otherwise. We again note that all of the parameters for these GARCH models for deriving the volatility forecasts are determined using our in-sample period from 2 January 1990 to 30 December 2005. We then use the derived forecast volatilities in various empirical tests in our out-of-sample period, which is from 3 January 2006 to 28 February 2014.

5.2. Simple tests
In order to examine the stock market downside risk predictive power of the VIX and forecast volatilities from the four GARCH models, we begin our empirical comparison by applying the following univariate logit model:

$$\Delta sp_t = m_{0,t}^{lg-k}\Delta sp_{t-j} + m_{1,t}^{lg-k}\Delta sp_{t-j} + \hat{\sigma}_{dlogsp,t}$$

$$y_t = \begin{cases} 1 & \text{if } \Delta sp_t \leq k\% \text{ VaR} \\ 0 & \text{otherwise} \end{cases}$$

where $\Delta sp_t = p_t^{S&P500} - p_{t-1}^{S&P500}$, and $x_{tg}$ takes $iv_t$, $\sigma_{dlogsp,t}^p$, $\sigma_{diaggp,t}^p$, $\sigma_{dloggp,t}^p$, or $\sigma_{diaggp,t}^p$. Further, $j$ is zero or 1 and $k$ takes a value of 99.0, 98.5, 98.0, 97.5, 97.0, or 95.0. The expression of $\Delta sp_t \leq k\% \text{ VaR}$ means a bad phenomenon exists whereby the loss amount for the S&P 500 negatively exceeds $k\% \text{ Value at Risk (VaR)}$; thus, this model (5) enables us to test the predictive power of the variable $x_{tg}$ for downside risk in the US stock market.\(^{8}\) In this model, the positive coefficient for $x_{tg}$ is the prediction power of the variable.

Table 2 provides the empirical results derived from the above model (5). The values in the table indicate the following. First, all coefficients of the one-day lagged VIX and all the forecast volatilities from the four GARCH models are statistically significantly positive, inferring that all five measures of volatility predict the downside risk of the US stock market. Second, judging by the McFadden’s $R^2$-squared values (MF-$R^2$), the predictive power of the one-day lagged VIX is less than that of the forecast volatilities from the four econometric models.

More specifically, for downside risk that exceeds 99% Value at Risk (VaR), the forecast volatility from the TGARCH model is the best predictor (MF-$R^2 = 0.250857$), and similarly for the TGARCH for 98.5% VaR (MF-$R^2 = 0.190221$), the TGARCH for 98% VaR (MF-$R^2 = 0.127788$), the TGARCH for 97.5% VaR (MF-$R^2 = 0.106875$), the EGARCH for 97% VaR (MF-$R^2 = 0.092101$), and the EGARCH for 95% VaR (MF-$R^2 = 0.082080$). Therefore, our initial simple tests show that in the context of the US stock market, the forecast volatilities derived from the TGARCH and EGARCH models have stronger downside risk predictive power than the other predictors.

5.3. Direct comparison: VIX versus forecast volatilities
We next conduct a more direct comparison. That is, we compare the downside risk predictive power of the VIX with that of the predicted volatilities from the four GARCH models by applying the following multiple logit model:

$$\Delta sp_t = s_{0,agf}^{lg-k} + s_{1,agf}^{lg-k}\Delta sp_{t-j} + s_{2,agf}^{lg-k}x_{1,t-j} + \epsilon_{agf,t}^{lg-k}\Delta sp_{t-j} + \hat{\sigma}_{agf,t}$$

$$y_t = \begin{cases} 1 & \text{if } \Delta sp_t \leq k\% \text{ VaR} \\ 0 & \text{otherwise} \end{cases}$$

where $\Delta sp_t = p_t^{S&P500} - p_{t-1}^{S&P500}$. In addition, $\Delta sp_{t-j}$ means the variable, $\sigma_{diaggp,t}^p$, $\sigma_{dloggp,t}^p$, $\sigma_{diaggp,t}^p$, or $\sigma_{dloggp,t}^p$, and $k$ takes a value of 99.0, 98.5, 98.0, 97.5, 97.0, or 95.0. Thus, in this model (6), the
Table 2. Testing the predictive power for downside risk in the US stock market by univariate logit models: results for the VIX and the forecast volatilities from several GARCH models

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| **Panel A: Downside risk predictive power of the previous day’s VIX** | | | | | | | | |
| $k = 99.0$ | $k = 98.5$ | $k = 98.0$ | | | | | | |
| $m^{p-99\%}_{tg(t)}$ | $-7.043^{***}$ | 0.000 | $m^{p-98.5\%}_{tg(t)}$ | $-6.309^{***}$ | 0.000 | $m^{p-98\%}_{tg(t)}$ | $-5.577^{***}$ | 0.000 |
| $m^{p-99\%}_{lg(t)}$ | 0.083*** | 0.000 | $m^{p-98.5\%}_{lg(t)}$ | 0.076*** | 0.000 | $m^{p-98\%}_{lg(t)}$ | 0.064*** | 0.000 |
| MF-R$^2$ | 0.188166 | MF-R$^2$ | 0.156169 | MF-R$^2$ | 0.101679 | | | |
| $k = 97.5$ | $k = 97.0$ | $k = 95.0$ | | | | | | |
| $m^{p-99\%}_{tg(t)}$ | $-5.219^{***}$ | 0.000 | $m^{p-97\%}_{tg(t)}$ | $-4.937^{***}$ | 0.000 | $m^{p-97\%}_{tg(t)}$ | $-4.372^{***}$ | 0.000 |
| $m^{p-99\%}_{lg(t)}$ | 0.060*** | 0.000 | $m^{p-97\%}_{lg(t)}$ | 0.057*** | 0.000 | $m^{p-97\%}_{lg(t)}$ | 0.057*** | 0.000 |
| MF-R$^2$ | 0.087843 | MF-R$^2$ | 0.078253 | MF-R$^2$ | 0.078673 | | | |
| **Panel B: Downside risk predictive power of the forecast volatility form the GARCH model** | | | | | | | | |
| $k = 99.0$ | $k = 98.5$ | $k = 98.0$ | | | | | | |
| $m^{p-99\%}_{lg(t)}$ | $-6.908^{***}$ | 0.000 | $m^{p-98.5\%}_{lg(t)}$ | $-6.041^{***}$ | 0.000 | $m^{p-98\%}_{lg(t)}$ | $-5.336^{***}$ | 0.000 |
| $m^{p-99\%}_{lag(t)}$ | 0.081*** | 0.000 | $m^{p-98.5\%}_{lag(t)}$ | 0.072*** | 0.000 | $m^{p-98\%}_{lag(t)}$ | 0.060*** | 0.000 |
| MF-R$^2$ | 0.223304 | MF-R$^2$ | 0.170584 | MF-R$^2$ | 0.111372 | | | |
| $k = 97.5$ | $k = 97.0$ | $k = 95.0$ | | | | | | |
| $m^{p-97\%}_{lg(t)}$ | $-4.966^{***}$ | 0.000 | $m^{p-97\%}_{lg(t)}$ | $-4.661^{***}$ | 0.000 | $m^{p-97\%}_{lg(t)}$ | $-4.054^{***}$ | 0.000 |
| $m^{p-99\%}_{lag(t)}$ | 0.056*** | 0.000 | $m^{p-97\%}_{lag(t)}$ | 0.052*** | 0.000 | $m^{p-97\%}_{lag(t)}$ | 0.050*** | 0.000 |
| MF-R$^2$ | 0.093162 | MF-R$^2$ | 0.078511 | MF-R$^2$ | 0.073811 | | | |
| **Panel C: Downside risk predictive power of the forecast volatility form the EGARCH model** | | | | | | | | |
| $k = 99.0$ | $k = 98.5$ | $k = 98.0$ | | | | | | |
| $m^{p-99\%}_{lg(t)}$ | $-7.095^{***}$ | 0.000 | $m^{p-98.5\%}_{lg(t)}$ | $-6.208^{***}$ | 0.000 | $m^{p-98\%}_{lg(t)}$ | $-5.518^{***}$ | 0.000 |
| $m^{p-99\%}_{lag(t)}$ | 0.096*** | 0.000 | $m^{p-98.5\%}_{lag(t)}$ | 0.084*** | 0.000 | $m^{p-98\%}_{lag(t)}$ | 0.072*** | 0.000 |
| MF-R$^2$ | 0.238500 | MF-R$^2$ | 0.182498 | MF-R$^2$ | 0.125834 | | | |
| $k = 97.5$ | $k = 97.0$ | $k = 95.0$ | | | | | | |
| $m^{p-97\%}_{lg(t)}$ | $-5.128^{***}$ | 0.000 | $m^{p-97\%}_{lg(t)}$ | $-4.838^{***}$ | 0.000 | $m^{p-97\%}_{lg(t)}$ | $-4.194^{***}$ | 0.000 |
| $m^{p-99\%}_{lag(t)}$ | 0.067*** | 0.000 | $m^{p-97\%}_{lag(t)}$ | 0.064*** | 0.000 | $m^{p-97\%}_{lag(t)}$ | 0.060*** | 0.000 |
| MF-R$^2$ | 0.104742 | MF-R$^2$ | 0.092101 | MF-R$^2$ | 0.082080 | | | |
| **Panel D: Downside risk predictive power of the forecast volatility form the PGARCH model** | | | | | | | | |
| $k = 99.0$ | $k = 98.5$ | $k = 98.0$ | | | | | | |
| $m^{p-99\%}_{lg(t)}$ | $-6.816^{***}$ | 0.000 | $m^{p-98.5\%}_{lg(t)}$ | $-5.978^{***}$ | 0.000 | $m^{p-98\%}_{lg(t)}$ | $-5.325^{***}$ | 0.000 |
| $m^{p-99\%}_{lag(t)}$ | 0.080*** | 0.000 | $m^{p-98.5\%}_{lag(t)}$ | 0.071*** | 0.000 | $m^{p-98\%}_{lag(t)}$ | 0.061*** | 0.000 |
| MF-R$^2$ | 0.227879 | MF-R$^2$ | 0.174846 | MF-R$^2$ | 0.119585 | | | |
| $k = 97.5$ | $k = 97.0$ | $k = 95.0$ | | | | | | |
| $m^{p-97\%}_{lg(t)}$ | $-4.955^{***}$ | 0.000 | $m^{p-97\%}_{lg(t)}$ | $-4.671^{***}$ | 0.000 | $m^{p-97\%}_{lg(t)}$ | $-4.045^{***}$ | 0.000 |
| $m^{p-99\%}_{lag(t)}$ | 0.057*** | 0.000 | $m^{p-97\%}_{lag(t)}$ | 0.053*** | 0.000 | $m^{p-97\%}_{lag(t)}$ | 0.051*** | 0.000 |
| MF-R$^2$ | 0.099698 | MF-R$^2$ | 0.086768 | MF-R$^2$ | 0.078291 | | | |
| **Panel E: Downside risk predictive power of the forecast volatility form the TGARCH model** | | | | | | | | |
| $k = 99.0$ | $k = 98.5$ | $k = 98.0$ | | | | | | |
| $m^{p-99\%}_{lg(t)}$ | $-7.008^{***}$ | 0.000 | $m^{p-98.5\%}_{lg(t)}$ | $-6.095^{***}$ | 0.000 | $m^{p-98\%}_{lg(t)}$ | $-5.389^{***}$ | 0.000 |

(Continued)
Table 2. (Continued)

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|---------------|-----------|---------|---------------|-----------|---------|---------------|-----------|---------|
| $m_{(p=99\%)}^{1-lg(t)}$ | 0.082*** | 0.000 | $m_{(p=98\%)}^{1-lg(t)}$ | 0.072*** | 0.000 | $m_{(p=96\%)}^{1-lg(t)}$ | 0.061*** | 0.000 |
| MF-$R^2$ | 0.250857 | MF-$R^2$ | 0.190221 | MF-$R^2$ | 0.127788 |
| $k = 97.5$ | $m_{(p=97.5\%)}^{1-lg(t)}$ | -5.009*** | 0.000 | $m_{(p=97\%)}^{1-lg(t)}$ | -4.702*** | 0.000 | $m_{(p=96\%)}^{1-lg(t)}$ | -4.069*** | 0.000 |
| $k = 97.0$ | $m_{(p=97.5\%)}^{1-lg(t)}$ | 0.056*** | 0.000 | $m_{(p=97\%)}^{1-lg(t)}$ | 0.052*** | 0.000 | $m_{(p=96\%)}^{1-lg(t)}$ | 0.050*** | 0.000 |
| $k = 95.0$ | MF-$R^2$ | 0.106875 | MF-$R^2$ | 0.090743 | MF-$R^2$ | 0.081729 |

Notes: This table exhibits the results of empirical comparisons of the predictive power of the one-day lagged VIX of S&P 500 and the forecast S&P 500 volatilities from various GARCH models. These tests are conducted in our out-of-sample period that is from 3 January 2006 to 28 February 2014, and all investigations are conducted by using univariate logit models. More specifically, we use the one-day lagged VIX close values and the out-of-sample forecast volatilities from various GARCH models. These tests are conducted in our out-of-sample period that is from 3 January 1990 to 30 December 2005. Further, MF-$R^2$ in this table denotes the McFadden’s $R$-squared value.

*Statistical significance of coefficients at the 10% level.
**Statistical significance of coefficients at the 5% level.
***Statistical significance of coefficients at the 1% level.

variables that have weaker predictive power should be dominated by stronger downside risk predictors. In this model, a positive coefficient again reflects the predictive power of the variable.

Table 3 details the empirical results derived from model (6). As shown, almost all the volatility forecasts from the EGARCH, PGARCH, and TGARCH models statistically significantly predict downside risk for the S&P 500. In contrast, the estimated coefficients for the one-day lagged VIX are mostly negative or insignificant. Hence, the results again suggest that the previous day’s VIX has less predictive power than the forecast volatilities from the econometric models.

More concretely, as shown in the above tests, for downside risk exceeding 99% VaR, the forecast volatility from the TGARCH model is the best predictor (MF-$R^2$ = 0.197413), the EGARCH for 98% VaR (MF-$R^2$ = 0.137424), the TGARCH for 97.5% VaR (MF-$R^2$ = 0.111233), the EGARCH for 97% VaR (MF-$R^2$ = 0.095827), and the EGARCH for 95% VaR (MF-$R^2$ = 0.082204). Based on these results, we can see that once again the forecast volatilities from the EGARCH and TGARCH models are the strongest predictors of downside risk in the US stock market.

5.4. Further tests with control variables

We further examine the downside risk predictive power of VIX and the volatility forecasts from the four GARCH models after including control variables. Namely, we next apply the following multiple logit model, which includes four control variables:

$$
\Delta s_{p_t} = \alpha_{0,xgf(t)}^{l-g-k} + \alpha_{1,xgf(t)}^{l-g-k} s_{xgf(t-1)} + \alpha_{2,xgf(t-1)}^{l-g-k} v_{t-1} + \gamma_{1,def(t-1)} + \gamma_{2,def(t-1)} \Delta s_{def(t-1)} + \gamma_{3,term(t-1)} d_{term(t-1)} + \gamma_{4,term(t-1)} d_{diff(t-1)} + \mu_{xgf(t)}^{l-g-k}
$$

(7)

$$
\gamma_{1} = \begin{cases} 
1 & \text{if } \Delta s_{p_t} \leq k\% \text{VaR} \\
0 & \text{otherwise}
\end{cases}
$$

where, as before, $\Delta s_{p_t} = p_{S&P^{500}_t} - p_{S&P^{500}_{t-1}}$ and $s_{xgf}$ denotes the variable, $\delta_{dlogsp,t}^{xgf}$, $\delta_{dlogsp,t}^{xgf}$, $\delta_{dlogsp,t}^{xgf}$ or $\delta_{dlogsp,t}^{xgf}$. Further, this model includes $def_{t-1}$, $s_{xgf}$, $term_{t-1}$, and $diff_{t-1}$ as control variables, and $k$ takes a value of 99.0, 98.5, 98.0, 97.5, 97.0, or 95.0.
### Table 3. Testing the predictive power for downside risk in the US stock market by multiple logit models: the VIX versus the forecast volatilities from several GARCH models

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|----------|---------|--------------|----------|---------|--------------|----------|---------|
| Panel A: Forecast volatility from the GARCH model versus the previous day’s VIX |
| k = 99.0 | k = 98.5 | k = 98.0 |
| $s_{y_{99}}$ | $s_{y_{98.5}}$ | $s_{y_{98.0}}$ |
| $\delta_{y_{97}}$ | $\delta_{y_{97.5}}$ | $\delta_{y_{97.0}}$ |
| MF-$R^2$ | 0.225202 | MF-$R^2$ | 0.170659 | MF-$R^2$ | 0.111381 |
| $k = 97.5$ | $k = 97.0$ | $k = 95.0$ |
| $s_{y_{97.5}}$ | $s_{y_{97.0}}$ | $s_{y_{95.0}}$ |
| $\delta_{y_{97}}$ | $\delta_{y_{97.5}}$ | $\delta_{y_{97.0}}$ |
| MF-$R^2$ | 0.093475 | MF-$R^2$ | 0.080229 | MF-$R^2$ | 0.078833 |
| Panel B: Forecast volatility from the EGARCH model versus the previous day’s VIX |
| k = 99.0 | k = 98.5 | k = 98.0 |
| $s_{y_{99}}$ | $s_{y_{98.5}}$ | $s_{y_{98.0}}$ |
| $\delta_{y_{97}}$ | $\delta_{y_{97.5}}$ | $\delta_{y_{97.0}}$ |
| MF-$R^2$ | 0.267681 | MF-$R^2$ | 0.189542 | MF-$R^2$ | 0.137424 |
| $k = 97.5$ | $k = 97.0$ | $k = 95.0$ |
| $s_{y_{97.5}}$ | $s_{y_{97.0}}$ | $s_{y_{95.0}}$ |
| $\delta_{y_{97}}$ | $\delta_{y_{97.5}}$ | $\delta_{y_{97.0}}$ |
| MF-$R^2$ | 0.110232 | MF-$R^2$ | 0.095827 | MF-$R^2$ | 0.082204 |
| Panel C: Forecast volatility from the PGARCH model versus the previous day’s VIX |
| k = 99.0 | k = 98.5 | k = 98.0 |
| $s_{y_{99}}$ | $s_{y_{98.5}}$ | $s_{y_{98.0}}$ |
| $\delta_{y_{97}}$ | $\delta_{y_{97.5}}$ | $\delta_{y_{97.0}}$ |
| MF-$R^2$ | 0.244708 | MF-$R^2$ | 0.177212 | MF-$R^2$ | 0.125031 |
| $k = 97.5$ | $k = 97.0$ | $k = 95.0$ |
| $s_{y_{97.5}}$ | $s_{y_{97.0}}$ | $s_{y_{95.0}}$ |
| $\delta_{y_{97}}$ | $\delta_{y_{97.5}}$ | $\delta_{y_{97.0}}$ |
| MF-$R^2$ | 0.101637 | MF-$R^2$ | 0.087479 | MF-$R^2$ | 0.079668 |
| Panel D: Forecast volatility from the TGARCH model versus the previous day’s VIX |
| k = 99.0 | k = 98.5 | k = 98.0 |
| $s_{y_{99}}$ | $s_{y_{98.5}}$ | $s_{y_{98.0}}$ |
| $\delta_{y_{97}}$ | $\delta_{y_{97.5}}$ | $\delta_{y_{97.0}}$ |
| MF-$R^2$ | 0.086270 | MF-$R^2$ | 0.077508 | MF-$R^2$ | 0.079668 |

(Continued)
Table 3. (Continued)

| Coefficients Estimates | p-value | Coefficients Estimates | p-value | Coefficients Estimates | p-value |
|-------------------------|---------|-------------------------|---------|-------------------------|---------|
| $\delta_{[95%]}^{\text{lag} \theta}$ | 0.172*** | 0.000 | $\delta_{[98.5\%]}^{\text{lag} \theta}$ | 0.019*** | 0.000 | $\delta_{[95%]}^{\text{lag} \theta}$ | 0.114*** | 0.000 |
| $\delta_{[5%]}^{\text{lag} \theta}$ | -0.113** | 0.022 | $\delta_{[98.5\%]}^{\text{lag} \theta}$ | -0.059 | 0.136 | $\delta_{[95%]}^{\text{lag} \theta}$ | -0.066* | 0.076 |
| MF-$R^2$ | 0.277187 | | | | | | | |
| k = 97.5 | | | | | | | | |
| $\delta_{[97.5\%]}^{\text{lag} \theta}$ | -4.697*** | 0.000 | $\delta_{[97.5\%]}^{\text{lag} \theta}$ | -4.527*** | 0.000 | $\delta_{[95%]}^{\text{lag} \theta}$ | -4.161*** | 0.000 |
| $\delta_{[5\%]}^{\text{lag} \theta}$ | 0.096*** | 0.001 | $\delta_{[97\%]}^{\text{lag} \theta}$ | 0.075*** | 0.005 | $\delta_{[95\%]}^{\text{lag} \theta}$ | 0.038* | 0.092 |
| $\delta_{[2\%]}^{\text{lag} \theta}$ | -0.049 | 0.154 | $\delta_{[97\%]}^{\text{lag} \theta}$ | -0.027 | 0.390 | $\delta_{[95\%]}^{\text{lag} \theta}$ | 0.014 | 0.581 |
| MF-$R^2$ | 0.111233 | | | | | | | |
| k = 97.0 | | | | | | | | |
| $\delta_{[97\%]}^{\text{lag} \theta}$ | 0.197413 | | | | | | | |
| k = 95.0 | | | | | | | | |
| $\delta_{[97\%]}^{\text{lag} \theta}$ | 0.135955 | | | | | | | |

Notes: This table exhibits the results of empirical comparisons of the predictive power of the one-day lagged VIX of S&P 500 and the forecast S&P 500 volatilities derived from various GARCH models. These tests are conducted in our out-of-sample period that is from 3 January 2006 to 28 February 2014, and all investigations are conducted using multiple logit models. Each multiple logit model includes a constant term, the one-day lagged VIX, and the forecast volatility derived from one of the four GARCH models. In the tests, we use the one-day lagged VIX close values and the out-of-sample forecast volatilities derived from GARCH(1,1), EGARCH(1,1), PGARCH(1,1), and TGARCH(1,1) models. All above models used to derive the forecast volatilities are specified in our in-sample period, which is from 2 January 1990 to 30 December 2005. Further, MF-$R^2$ in this table denotes the McFadden's $R$-squared value.

*Statistical significance of coefficients at the 10% level.
**Statistical significance of coefficients at the 5% level.
***Statistical significance of coefficients at the 1% level.

Table 4 provides the results using model (7). As shown, almost all the forecast volatilities from the GARCH, EGARCH, PGARCH, and TGARCH models statistically significantly predict the downside risk of the S&P 500, whereas the coefficients for the one-day lagged VIX are again mostly negative or insignificant. Therefore, once again, the previous day’s VIX is inferior to the volatility forecasts from econometric models in predicting downside risk in the US stock market.

More specifically, for downside risk exceeding 99% VaR, the forecast volatility from the TGARCH model is the best predictor (MF-$R^2$ = 0.315876), and similarly for the TGARCH for 98% VaR (MF-$R^2$ = 0.205427), the EGARCH for 97% VaR (MF-$R^2$ = 0.088127). Therefore, the above results again show that the forecast volatilities from the EGARCH and TGARCH models better predict downside risk in the US stock market than does the VIX.

6. Robustness checks using quantile regressions

This section implements robustness checks of the results presented in the previous section. As below, we conduct further analyses with different approaches using quantile regressions.

6.1. Simple tests

We begin our robustness checks by applying the following univariate quantile regression model:

$$\Delta sp_{t}^{j\%} = \beta_{0,j\%} + \beta_{1,j\%} x_{t-i} + \beta_{2,j\%} v_{t-i,-i,l} + \beta_{3,j\%} y_{t-i,l},$$

(8)

where $\Delta sp_{t}^{j\%}$ denotes the $j$-percentile point of the distribution of the S&P 500 price changes. In addition, $x_{t}$ denotes the variable, $\tilde{\alpha}_{t-i}^{\text{dgsp}}, \tilde{\alpha}_{t-i}^{\text{diagsp}}, \tilde{\alpha}_{t-i}^{\text{dgsp}}, \tilde{\alpha}_{t-i}^{\text{diasgsp}, \text{dgsp}}, \tilde{\alpha}_{t-i}^{\text{diasgsp}, \text{dgsp}}, \tilde{\alpha}_{t-i}^{\text{diasgsp}, \text{dgsp}}, \tilde{\alpha}_{t-i}^{\text{diasgsp}, \text{dgsp}}, \tilde{\alpha}_{t-i}^{\text{diasgsp}, \text{dgsp}}, \tilde{\alpha}_{t-i}^{\text{diasgsp}, \text{dgsp}}$, or $\tilde{\alpha}_{t-i}^{\text{diasgsp}, \text{dgsp}}$; $j$ takes a value of 1.0, 1.5, 2.0, 2.5, 3.0, or 5.0, and $i$ is either zero or one. Thus, model (8) enables us to test the predictive power of variable $x_{t}$ for the $j\%$ downward risk in the left tail of the price change distribution for the US stock market. In this model, a negative coefficient for a variable infers the predictive power of the tail risk.

Table 5 displays the empirical results derived from model (8). The main results are as follows. First, all of the coefficients for the one-day lagged VIX and those for the forecast volatilities from the four
Table 4. Testing the predictive power for downside risk in the US stock market by multiple logit models with control variables: the VIX versus the forecast volatilities from several GARCH models

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| Panel A: Forecast volatility from the GARCH model versus the previous day's VIX |
| $k = 99.0$ | | | $k = 98.5$ | | | $k = 98.0$ | | |
| $\beta_{99}^{\text{Log}}$ | -6.292*** | 0.000 | $\beta_{98.5}^{\text{Log}}$ | -5.438*** | 0.000 | $\beta_{98}^{\text{Log}}$ | -4.707*** | 0.000 |
| $\beta_{99}^{\text{1\,Log}}$ | 0.188*** | 0.000 | $\beta_{98.5}^{\text{1\,Log}}$ | 0.120*** | 0.002 | $\beta_{98}^{\text{1\,Log}}$ | 0.108*** | 0.002 |
| $\beta_{99}^{\text{2,Log}}$ | -0.051 | 0.283 | $\beta_{98.5}^{\text{2,Log}}$ | -0.001 | 0.979 | $\beta_{98}^{\text{2,Log}}$ | 5.66-05 | 0.999 |
| $\beta_{99}^{\text{TERM}}$ | -1.754*** | 0.003 | $\beta_{98.5}^{\text{TERM}}$ | -1.171** | 0.013 | $\beta_{98}^{\text{TERM}}$ | -1.170*** | 0.006 |
| $\beta_{99}^{\text{IV}}$ | 0.075 | 0.840 | $\beta_{98.5}^{\text{IV}}$ | 0.021 | 0.946 | $\beta_{98}^{\text{IV}}$ | -0.005 | 0.987 |
| $\beta_{99}^{\text{V}}$ | 0.208 | 0.535 | $\beta_{98.5}^{\text{V}}$ | -0.066 | 0.772 | $\beta_{98}^{\text{V}}$ | -0.084 | 0.640 |
| $\beta_{99}^{\text{EF}}$ | -2.436** | 0.028 | $\beta_{98.5}^{\text{EF}}$ | -1.986** | 0.048 | $\beta_{98}^{\text{EF}}$ | -1.747* | 0.065 |
| **MF$R^2$** | 0.299330 | 0.000 | **MF$R^2$** | 0.207457 | 0.000 | **MF$R^2$** | 0.143596 | 0.000 |
| Panel B: Forecast volatility from the EGARCH model versus the previous day's VIX |
| $k = 99.0$ | | | $k = 98.5$ | | | $k = 98.0$ | | |
| $\beta_{99}^{\text{Log}}$ | -4.411*** | 0.000 | $\beta_{98.5}^{\text{Log}}$ | -4.226*** | 0.000 | $\beta_{98}^{\text{Log}}$ | -3.960*** | 0.000 |
| $\beta_{99}^{\text{1\,Log}}$ | 0.085*** | 0.008 | $\beta_{98.5}^{\text{1\,Log}}$ | 0.066** | 0.030 | $\beta_{98}^{\text{1\,Log}}$ | 0.033 | 0.186 |
| $\beta_{99}^{\text{2,Log}}$ | 0.015 | 0.654 | $\beta_{98.5}^{\text{2,Log}}$ | 0.029 | 0.342 | $\beta_{98}^{\text{2,Log}}$ | 0.050** | 0.046 |
| $\beta_{99}^{\text{TERM}}$ | -0.949** | 0.012 | $\beta_{98.5}^{\text{TERM}}$ | -0.901** | 0.011 | $\beta_{98}^{\text{TERM}}$ | -0.594** | 0.032 |
| $\beta_{99}^{\text{IV}}$ | 0.041 | 0.884 | $\beta_{98.5}^{\text{IV}}$ | 0.139 | 0.606 | $\beta_{98}^{\text{IV}}$ | 0.110 | 0.614 |
| $\beta_{99}^{\text{V}}$ | -0.156 | 0.317 | $\beta_{98.5}^{\text{V}}$ | -0.137 | 0.323 | $\beta_{98}^{\text{V}}$ | -0.082 | 0.447 |
| $\beta_{99}^{\text{EF}}$ | -1.299 | 0.157 | $\beta_{98.5}^{\text{EF}}$ | -1.319 | 0.134 | $\beta_{98}^{\text{EF}}$ | -0.476 | 0.542 |
| **MF$R^2$** | 0.115516 | 0.000 | **MF$R^2$** | 0.101040 | 0.000 | **MF$R^2$** | 0.086715 | 0.000 |
| (Continued) |
| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|-----------------|---------|--------|-----------------|---------|--------|-----------------|---------|--------|
| $\beta_{95}$ | -0.526 | 0.135 | $\beta_{97}$ | -0.585* | 0.075 | $\beta_{95}$ | -0.429* | 0.088 |
| $\delta_{95}$ | -0.059 | 0.832 | $\delta_{97}$ | 0.686 | 0.802 | $\delta_{95}$ | 0.078 | 0.720 |
| $\beta_{95}$ | -0.108 | 0.496 | $\beta_{97}$ | -0.090 | 0.525 | $\beta_{95}$ | -0.062 | 0.575 |
| $\delta_{95}$ | -1.072 | 0.211 | $\delta_{97}$ | -1.115 | 0.179 | $\delta_{95}$ | -0.445 | 0.558 |
| MF-$R^2$ | 0.205427 | | MF-$R^2$ | 0.107021 | | MF-$R^2$ | 0.087144 | |

Panel C: Forecast volatility from the PGARCH model versus the previous day’s VIX

| k | 99.0 | 98.5 | 98.0 |
|---|-----|-----|-----|
| $\beta_{95}$ | -6.039*** | 0.000 | $\beta_{95}$ | -5.397*** | 0.000 | $\beta_{95}$ | -4.566*** | 0.000 |
| $\delta_{95}$ | 0.223*** | 0.000 | $\delta_{95}$ | 0.126*** | 0.006 | $\delta_{95}$ | 0.138*** | 0.001 |
| $\beta_{95}$ | -0.121* | 0.060 | $\beta_{95}$ | -0.031 | 0.540 | $\beta_{95}$ | -0.050 | 0.295 |
| $\delta_{95}$ | -1.259** | 0.035 | $\delta_{95}$ | -0.788* | 0.086 | $\delta_{95}$ | -0.908** | 0.029 |
| $\beta_{95}$ | -0.129 | 0.728 | $\beta_{95}$ | -0.106 | 0.736 | $\beta_{95}$ | -0.110 | 0.712 |
| $\delta_{95}$ | 0.312 | 0.338 | $\delta_{95}$ | 0.009 | 0.970 | $\delta_{95}$ | -0.007 | 0.971 |
| $\beta_{95}$ | -1.940* | 0.069 | $\beta_{95}$ | -1.762* | 0.079 | $\beta_{95}$ | -1.511 | 0.101 |
| MF-$R^2$ | 0.293792 | | MF-$R^2$ | 0.200143 | | MF-$R^2$ | 0.146952 | |

| k | 97.5 | 97.0 | 95.0 |
|---|-----|-----|-----|
| $\beta_{95}$ | -4.349*** | 0.000 | $\beta_{95}$ | -4.152*** | 0.000 | $\beta_{95}$ | -3.999*** | 0.000 |
| $\delta_{95}$ | 0.100*** | 0.009 | $\delta_{95}$ | 0.084** | 0.019 | $\delta_{95}$ | 0.027 | 0.348 |
| $\beta_{95}$ | -0.017 | 0.692 | $\beta_{95}$ | -0.002 | 0.965 | $\beta_{95}$ | 0.050 | 0.121 |
| $\delta_{95}$ | -0.708* | 0.052 | $\delta_{95}$ | -0.729** | 0.031 | $\delta_{95}$ | -0.471* | 0.064 |
| $\beta_{95}$ | -0.040 | 0.885 | $\beta_{95}$ | 0.081 | 0.763 | $\beta_{95}$ | 0.080 | 0.713 |
| $\delta_{95}$ | -0.097 | 0.537 | $\delta_{95}$ | -0.087 | 0.539 | $\delta_{95}$ | -0.070 | 0.524 |
| $\beta_{95}$ | -1.154 | 0.196 | $\beta_{95}$ | -1.197 | 0.165 | $\beta_{95}$ | -0.465 | 0.551 |
| MF-$R^2$ | 0.115641 | | MF-$R^2$ | 0.102656 | | MF-$R^2$ | 0.085669 | |

Panel D: Forecast volatility from the TGARCH model versus the previous day’s VIX

| k | 99.0 | 98.5 | 98.0 |
|---|-----|-----|-----|
| $\beta_{95}$ | -6.524*** | 0.000 | $\beta_{95}$ | -5.498*** | 0.000 | $\beta_{95}$ | -4.716*** | 0.000 |
| $\delta_{95}$ | 0.183*** | 0.000 | $\delta_{95}$ | 0.125*** | 0.000 | $\delta_{95}$ | 0.118*** | 0.000 |
| $\beta_{95}$ | -0.095* | 0.075 | $\beta_{95}$ | -0.037 | 0.391 | $\beta_{95}$ | -0.038 | 0.340 |
| $\delta_{95}$ | -0.916* | 0.071 | $\delta_{95}$ | -0.703* | 0.087 | $\delta_{95}$ | -0.756** | 0.043 |
| $\beta_{95}$ | -0.155 | 0.676 | $\beta_{95}$ | -0.103 | 0.743 | $\beta_{95}$ | -0.109 | 0.713 |
| $\delta_{95}$ | 0.322 | 0.346 | $\delta_{95}$ | 0.003 | 0.989 | $\delta_{95}$ | -0.024 | 0.896 |
| $\beta_{95}$ | -1.698* | 0.093 | $\beta_{95}$ | -1.541 | 0.103 | $\beta_{95}$ | -1.388 | 0.117 |
| MF-$R^2$ | 0.315876 | | MF-$R^2$ | 0.217889 | | MF-$R^2$ | 0.154566 | |

(Continued)
GARCH models are statistically significantly negative. This means that, yet again, all five volatility measures have predictive power for downside risk in the US stock market. Second, judging by the values of adjusted-$R^2$ in Table 5, the predictive power of the previous day’s VIX is lower than that of the forecast volatilities from the EGARCH, PGARCH, and TGARCH models.

More specifically, for downside risk of 1% left-tail risk for price changes in the S&P 500, the forecast volatility from the TGARCH model is the best predictor (adjusted-$R^2 = 0.196455$), and similarly for the EGARCH for downside tail risk of 1.5% (adjusted-$R^2 = 0.186060$), the EGARCH for downside tail risk of 2% (adjusted-$R^2 = 0.175194$), the EGARCH for downside tail risk of 2.5% (adjusted-$R^2 = 0.165310$), the EGARCH for downside tail risk of 3% (adjusted-$R^2 = 0.155252$), and the EGARCH for downside tail risk of 5% (adjusted-$R^2 = 0.124073$). Hence, similar to the results from the logit models, these results indicate that the forecast volatilities from the EGARCH and TGARCH models are stronger downside risk predictors than the previous day’s VIX.

### 6.2. Direct comparison: VIX versus forecast volatilities

We conduct a second robustness check using the following multivariate quantile regression model:

$$
\Delta sp_t^{\%} = \beta_0^{\%} + \beta_{1,spf(t)}^{\%} z_{spf(t)}^j + \beta_{2,spf(t)}^{\%} \hat{\sigma}^j + \hat{\sigma}^{\%} \hat{\varepsilon}_{spf(t),1,t} + \hat{\sigma}^{\%} \hat{\varepsilon}_{spf(t),1,t},
$$

where $\Delta sp_t^{\%}$ denotes the $j$-percentile point of the distribution of S&P 500 price changes. In addition, $z_{spf(t)}^j$ is the variable, $\hat{\sigma}^j$ is the variable, $\hat{\sigma}^j$ is the variable, $\hat{\sigma}^j$ is the variable, $\hat{\sigma}^j$ is the variable, and $j$ takes a value of 1.0, 1.5, 2.0, 2.5, 3.0, or 5.0. In this model, and as before, a negative coefficient infers that a variable has predictive power for the $j$% tail risk.

Table 6 provides the empirical results for model (9), with the values suggesting that almost all the EGARCH, PGARCH, and TGARCH model predictors statistically significantly forecast the downside risk of the S&P 500, while the coefficients of the one-day lagged VIX are mostly positive or insignificant.

### Table 4. (Continued)

| Coefficients Estimates | p-value | Coefficients Estimates | p-value | Coefficients Estimates | p-value |
|------------------------|---------|------------------------|---------|------------------------|---------|
| $\alpha^{-%97\%}$ | $-4.394^{***}$ | 0.000 | $\alpha^{-%97\%}$ | $-4.206^{***}$ | 0.000 | $\alpha^{-%97\%}$ | $-3.944^{***}$ | 0.000 |
| $\alpha^{-%97\%}$ | 0.098*** | 0.001 | $\alpha^{-%97\%}$ | 0.077*** | 0.005 | $\alpha^{-%97\%}$ | 0.040* | 0.086 |
| $\alpha^{-%97\%}$ | $-0.021$ | 0.578 | $\alpha^{-%97\%}$ | $-0.001$ | 0.984 | $\alpha^{-%97\%}$ | 0.035 | 0.232 |
| $\alpha^{-%97\%}$ | $-0.637^*$ | 0.060 | $\alpha^{-%97\%}$ | $-0.665^{**}$ | 0.036 | $\alpha^{-%97\%}$ | $-0.476^*$ | 0.057 |
| $\alpha^{-%97\%}$ | $-0.033$ | 0.906 | $\alpha^{-%97\%}$ | 0.087 | 0.746 | $\alpha^{-%97\%}$ | 0.089 | 0.683 |
| $\alpha^{-%97\%}$ | $-0.102$ | 0.513 | $\alpha^{-%97\%}$ | $-0.095$ | 0.498 | $\alpha^{-%97\%}$ | $-0.057$ | 0.600 |
| $\alpha^{-%97\%}$ | $-1.058$ | 0.212 | $\alpha^{-%97\%}$ | $-1.127$ | 0.176 | $\alpha^{-%97\%}$ | $-0.447$ | 0.552 |
| $\text{MF}^R$ | 0.123971 | | $\text{MF}^R$ | 0.106240 | | $\text{MF}^R$ | 0.088127 | |

Notes: This table exhibits the results of empirical comparisons of the predictive power of the one-day lagged VIX of S&P 500 and the forecast S&P 500 volatilities from various GARCH models. These tests are conducted in our out-of-sample period that is from 3 January 2006 to 28 February 2014, and all investigations are conducted by using multiple logit models. Each multiple logit model includes a constant term, the one-day lagged VIX, the forecast volatility from one of the GARCH models, and four control variables. Our control variables are the one-day lagged default spread, the one-day lag of the interest rate series of the trade-weighted US dollar index, the one-day lagged term premium, and the one-day lag of the first-difference series of the effective federal funds rate. All GARCH models used to derive the forecast volatilities are specified in our in-sample period, which is from 2 January 1990 to 30 December 2005. Further, $\text{MF}^R$ in this table denotes the McFadden’s $R$-squared value.

*Statistical significance of coefficients at the 10% level.

**Statistical significance of coefficients at the 5% level.

***Statistical significance of coefficients at the 1% level.
### Table 5. Testing the predictive power for downside risk in the US stock market by univariate quantile regressions: results for the VIX and the forecast volatilities from several GARCH models

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|----------|---------|--------------|----------|---------|--------------|----------|---------|
| \( \alpha_{0.01} \) & & & \( \beta_{0.01} \) & & & \( \gamma_{0.01} \) & & |
| & -9.621 & 0.178 & & -7.923 & 0.033 & & -7.852 & 0.029 |
| \( \alpha_{0.1} \) & & & \( \beta_{0.1} \) & & & \( \gamma_{0.1} \) & & |
| & -1.381 & 0.000 & & -1.304 & 0.000 & & -1.231 & 0.000 |
| Adj. \( R^2 \) & 0.168087 & Adj. \( R^2 \) & 0.157998 & Adj. \( R^2 \) & 0.148048 |

### Panel B: Downside risk predictive power of the forecast volatility form the GARCH model

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|----------|---------|--------------|----------|---------|--------------|----------|---------|
| \( \alpha_{0.01} \) & & & \( \beta_{0.01} \) & & & \( \gamma_{0.01} \) & & |
| & -20.751 & 0.000 & & -17.039 & 0.000 & & -11.400 & 0.000 |
| \( \alpha_{0.1} \) & & & \( \beta_{0.1} \) & & & \( \gamma_{0.1} \) & & |
| & -1.045 & 0.000 & & -1.058 & 0.000 & & -0.985 & 0.000 |
| Adj. \( R^2 \) & 0.165250 & Adj. \( R^2 \) & 0.157120 & Adj. \( R^2 \) & 0.145393 |

### Panel C: Downside risk predictive power of the forecast volatility form the E-GARCH model

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|----------|---------|--------------|----------|---------|--------------|----------|---------|
| \( \alpha_{0.01} \) & & & \( \beta_{0.01} \) & & & \( \gamma_{0.01} \) & & |
| & -16.446 & 0.000 & & -13.566 & 0.000 & & -11.902 & 0.000 |
| \( \alpha_{0.1} \) & & & \( \beta_{0.1} \) & & & \( \gamma_{0.1} \) & & |
| & -1.223 & 0.000 & & -1.254 & 0.000 & & -1.244 & 0.000 |
| Adj. \( R^2 \) & 0.194512 & Adj. \( R^2 \) & 0.186060 & Adj. \( R^2 \) & 0.175194 |

### Panel D: Downside risk predictive power of the forecast volatility form the P-GARCH model

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|----------|---------|--------------|----------|---------|--------------|----------|---------|
| \( \alpha_{0.01} \) & & & \( \beta_{0.01} \) & & & \( \gamma_{0.01} \) & & |
| & -12.063 & 0.000 & & -10.509 & 0.000 & & -7.745 & 0.004 |
| \( \alpha_{0.1} \) & & & \( \beta_{0.1} \) & & & \( \gamma_{0.1} \) & & |
| & -1.128 & 0.000 & & -1.128 & 0.000 & & -1.038 & 0.000 |
| Adj. \( R^2 \) & 0.165310 & Adj. \( R^2 \) & 0.155252 & Adj. \( R^2 \) & 0.124073 |

### Panel E: Downside risk predictive power of the forecast volatility form the T-GARCH model

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|----------|---------|--------------|----------|---------|--------------|----------|---------|
| \( \alpha_{0.01} \) & & & \( \beta_{0.01} \) & & & \( \gamma_{0.01} \) & & |
| & -17.997 & 0.022 & & -16.334 & 0.000 & & -14.557 & 0.000 |
| \( \alpha_{0.1} \) & & & \( \beta_{0.1} \) & & & \( \gamma_{0.1} \) & & |
| & -1.128 & 0.019 & & -1.082 & 0.000 & & -1.054 & 0.000 |
| Adj. \( R^2 \) & 0.189403 & Adj. \( R^2 \) & 0.180956 & Adj. \( R^2 \) & 0.171419 |

(Continued)
### Table 5. Testing the predictive power for downside risk in the US stock market by multiple quantile regressions: the VIX versus the forecast volatilities from several GARCH models

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| $\rho_{0.1%}$ | -16.818** | 0.012 | $\alpha_{1.5%}$ | -15.898*** | 0.022 | $\rho_{0.2%}$ | -14.515*** | 0.000 |
| $\rho_{1.5%}$ | -1.148*** | 0.005 | $\alpha_{1.5%}$ | -1.089** | 0.015 | $\rho_{2.5%}$ | -1.019*** | 0.000 |
| $\alpha_{1.5%}$ | 0.196455 | Adj. R² | $\alpha_{1.5%}$ | 0.184850 | Adj. R² | $\alpha_{2.5%}$ | 0.172468 | Adj. R² |

Notes: This table exhibits the results of empirical comparisons of the predictive power of the one-day lagged VIX of S&P 500 and the forecast S&P 500 volatilities from various GARCH models. These tests are conducted in our out-of-sample period that is from 3 January 2006 to 28 February 2014, and all examinations are conducted using univariate quantile regression models. In the tests, we use the one-day lagged VIX close values and the out-of-sample forecast volatilities from GARCH(1,1), EGARCH(1,1), PGARCH(1,1), and TARCH(1,1) models. All above models used to derive the forecast volatilities are specified in our in-sample period, which is from 2 January 1990 to 30 December 2005. Further, Adj. R² in this table denotes the adjusted R-squared value.

*Statistical significance of coefficients at the 10% level.
**Statistical significance of coefficients at the 5% level.
***Statistical significance of coefficients at the 1% level.

### Table 6. Testing the predictive power for downside risk in the US stock market by multiple quantile regressions: the VIX versus the forecast volatilities from several GARCH models

#### Panel A: Forecast volatility from the GARCH model versus the previous day’s VIX

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| $\sigma_{0.1%}$ | -16.436** | 0.023 | $\sigma_{0.1%}$ | -13.191*** | 0.001 | $\sigma_{0.2%}$ | -10.820*** | 0.003 |
| $\sigma_{0.1%}$ | -0.263 | 0.652 | $\sigma_{0.1%}$ | -0.616** | 0.034 | $\sigma_{0.2%}$ | -0.478 | 0.116 |
| $\sigma_{0.2%}$ | -0.845 | 0.291 | $\sigma_{0.2%}$ | -0.519 | 0.166 | $\sigma_{0.3%}$ | -0.685* | 0.080 |
| Adj. R² | 0.168743 | Adj. R² | 0.163054 | Adj. R² | 0.152365 |

#### Panel B: Forecast volatility from the EGARCH model versus the previous day’s VIX

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| $\sigma_{0.1%}$ | -7.769** | 0.018 | $\sigma_{0.1%}$ | -8.465** | 0.027 | $\sigma_{0.5%}$ | -8.116** | 0.017 |
| $\sigma_{0.2%}$ | -0.485 | 0.118 | $\sigma_{0.2%}$ | -0.111 | 0.762 | $\sigma_{0.5%}$ | -0.413* | 0.055 |
| $\sigma_{0.2%}$ | -0.731* | 0.057 | $\sigma_{0.2%}$ | -0.899** | 0.028 | $\sigma_{0.5%}$ | -0.441 | 0.156 |
| Adj. R² | 0.142256 | Adj. R² | 0.133822 | Adj. R² | 0.110916 |

(Continued)
Panel C: Forecast volatility from the PGARCH model versus the previous day’s VIX

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|----------|---------|--------------|----------|---------|--------------|----------|---------|
| $\phi^2_{t-1}$ | 0.386    | 0.192   | $\phi^2_{t-1}$ | 0.420  | 0.158   | $\phi^2_{t-1}$ | 0.052  | 0.892   |
| Adj. $R^2$   | 0.167572 | Adj. $R^2$ | 0.156119 | Adj. $R^2$ | 0.123669 |

Panel D: Forecast volatility from the TGARCH model versus the previous day’s VIX

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|----------|---------|--------------|----------|---------|--------------|----------|---------|
| $\phi^2_{t-1}$ | -21.119  | 0.054   | $\phi^2_{t-1}$ | -20.075 | 0.104   | $\phi^2_{t-1}$ | -15.780 | 0.010   |
| Adj. $R^2$   | 0.200419 | Adj. $R^2$ | 0.186502 | Adj. $R^2$ | 0.172670 |

Notes: This table exhibits the results of empirical comparisons of the predictive power of the one-day lagged VIX of S&P 500 and the forecast S&P 500 volatilities from various GARCH models. These tests are conducted in our out-of-sample period that is from 3 January 2006 to 28 February 2014. All examinations are conducted by using the multiple quantile regression models. Each multiple quantile regression model includes a constant term, the one-day lagged VIX, and the forecast volatility from one of the four GARCH models. In the tests, we use the one-day lagged VIX close values and the out-of-sample forecast volatilities derived from GARCH(1,1), EGARCH(1,1), PGARCH(1,1), and TGARCH(1,1) models. All above models used to derive the forecast volatilities are specified in our in-sample period, which is from 2 January 1990 to 30 December 2005. In this table, Adj. $R^2$ denotes the adjusted $R$-squared value.

*Statistical significance of coefficients at the 10% level.
**Statistical significance of coefficients at the 5% level.
***Statistical significance of coefficients at the 1% level.

Once again, the predictive power of the previous day’s VIX is weaker than that of the predictors from the econometric models.

More concretely, for left-tail downside risk of 1%, the forecast volatility from the EGARCH model predicts best (adjusted-$R^2$ = 0.200778), and similarly for the EGARCH for 1.5% left-tail risk (adjusted-$R^2$ = 0.189994), the EGARCH for 2% left-tail risk (adjusted-$R^2$ = 0.176695), the EGARCH for 2.5% left-tail risk (adjusted-$R^2$ = 0.167572), the EGARCH for 3% left-tail risk (adjusted-$R^2$ = 0.156119), and the EGARCH for 5% left-tail risk (adjusted-$R^2$ = 0.123669). Hence, our multiple quantile regression analyses using model (9) suggest that the EGARCH model most strongly predicts US stock market downside risk.
6.3. Further tests with control variables

We implement further robustness checks by applying the following multiple quantile regression model including four control variables:

$$
\Delta s_{p,t}^{q} = \chi_{0,q,ef(t)}^{q} + \chi_{1,q,ef(t)}^{q} \Delta s_{t-1}^{q} + \chi_{2,q,net(t-1)}^{q} \Delta s_{t-1}^{q} + \chi_{3,q,term(t-1)}^{q} + \chi_{d,q,def(t-1)}^{q} + \gamma_{j,q,def(t-1)}^{q} \Delta s_{t-1}^{q} + \gamma_{j,q,def(t-1)}^{q} \Delta s_{t-1}^{q} + \mu_{q,ef(t)}^{q},
$$

(10)

where, once again, $\Delta s_{p,t}^{q}$ denotes the $q$-percentile point of the distribution of S&P 500 price changes, and $\Delta s_{t-1}^{q}$ is the variable, $\Delta s_{t-1}^{q}$, or $\Delta s_{t-1}^{q}$. Further, $j$ takes a value of 1.0, 1.5, 2.0, 2.5, 3.0, or 5.0, and $\Delta s_{t-1}^{q}$, $\Delta s_{t-1}^{q}$, $\Delta s_{t-1}^{q}$, and $\Delta s_{t-1}^{q}$, are the control variables.

Table 7 details the results derived from model (10), such that the forecast volatilities from the EGARCH, PGARCH, and TGARCH models statistically significantly predict US stock market downside

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**Table 7. Testing the predictive power for downside risk in the US stock market by multiple quantile regression models with control variables: the VIX versus the forecast volatilities from several GARCH models**

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| $\chi_{0,q,ef(t)}^{q}$ | -21.689* | 0.055 | $\chi_{0,q,ef(t)}^{q}$ | -17.317* | 0.090 | $\chi_{0,q,ef(t)}^{q}$ | -19.288*** | 0.000 |
| $\chi_{1,q,ef(t)}^{q}$ | -1.060 | 0.207 | $\chi_{1,q,ef(t)}^{q}$ | -0.815 | 0.296 | $\chi_{1,q,ef(t)}^{q}$ | -0.577 | 0.303 |
| $\chi_{2,q,net(t-1)}^{q}$ | -0.620 | 0.530 | $\chi_{2,q,net(t-1)}^{q}$ | -0.953 | 0.290 | $\chi_{2,q,net(t-1)}^{q}$ | -1.144 | 0.116 |
| $\chi_{3,q,term(t-1)}^{q}$ | 11.809* | 0.059 | $\chi_{3,q,term(t-1)}^{q}$ | 12.262* | 0.062 | $\chi_{3,q,term(t-1)}^{q}$ | 16.514** | 0.016 |
| $\chi_{d,q,def(t-1)}^{q}$ | 3.941 | 0.659 | $\chi_{d,q,def(t-1)}^{q}$ | 4.282 | 0.625 | $\chi_{d,q,def(t-1)}^{q}$ | -0.347 | 0.904 |
| $\chi_{3,q,term(t-1)}^{q}$ | 1.346 | 0.243 | $\chi_{3,q,term(t-1)}^{q}$ | 1.095 | 0.330 | $\chi_{3,q,term(t-1)}^{q}$ | 1.596 | 0.104 |
| $\chi_{d,q,def(t-1)}^{q}$ | 15.486*** | 0.002 | $\chi_{d,q,def(t-1)}^{q}$ | 17.720*** | 0.001 | $\chi_{d,q,def(t-1)}^{q}$ | 16.845* | 0.053 |
| Adj. $R^2$ | 0.199998 | 0.190151 | Adj. $R^2$ | 0.190151 | 0.175999 |

**Panel A: Forecast volatility from the GARCH model versus the previous day's VIX**

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| $\chi_{0,q,ef(t)}^{q}$ | -17.518*** | 0.000 | $\chi_{0,q,ef(t)}^{q}$ | -15.791*** | 0.000 | $\chi_{0,q,ef(t)}^{q}$ | -10.663*** | 0.001 |
| $\chi_{1,q,ef(t)}^{q}$ | -0.588 | 0.218 | $\chi_{1,q,ef(t)}^{q}$ | -0.357* | 0.076 | $\chi_{1,q,ef(t)}^{q}$ | -0.475*** | 0.026 |
| $\chi_{2,q,term(t-1)}^{q}$ | -0.992* | 0.093 | $\chi_{2,q,term(t-1)}^{q}$ | -1.255*** | 0.001 | $\chi_{2,q,term(t-1)}^{q}$ | -1.043*** | 0.002 |
| $\chi_{d,q,def(t-1)}^{q}$ | 12.165** | 0.039 | $\chi_{d,q,def(t-1)}^{q}$ | 13.042*** | 0.003 | $\chi_{d,q,def(t-1)}^{q}$ | 12.272*** | 0.000 |
| $\chi_{3,q,term(t-1)}^{q}$ | -0.934 | 0.836 | $\chi_{3,q,term(t-1)}^{q}$ | 0.200 | 0.953 | $\chi_{3,q,term(t-1)}^{q}$ | 1.664 | 0.464 |
| $\chi_{d,q,def(t-1)}^{q}$ | 1.941* | 0.067 | $\chi_{d,q,def(t-1)}^{q}$ | 1.895* | 0.084 | $\chi_{d,q,def(t-1)}^{q}$ | 0.662 | 0.534 |
| $\chi_{d,q,def(t-1)}^{q}$ | 9.812 | 0.372 | $\chi_{d,q,def(t-1)}^{q}$ | 7.279* | 0.065 | $\chi_{d,q,def(t-1)}^{q}$ | 10.787 | 0.135 |
| Adj. $R^2$ | 0.165755 | 0.156493 | Adj. $R^2$ | 0.165755 | 0.125050 |

**Panel B: Forecast volatility from the EGARCH model versus the previous day's VIX**

(Continued)
Table 7. (Continued)

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| \(\beta_1\)  | 5.593     | 0.210   | \(\beta_1\)  | 8.315*    | 0.059   | \(\beta_1\)  | 8.434     | 0.245   |
| \(\beta_2\)  | 6.965     | 0.325   | \(\beta_2\)  | 2.005     | 0.607   | \(\beta_2\)  | 2.516     | 0.617   |
| \(\beta_3\)  | -0.728    | 0.639   | \(\beta_3\)  | 0.505     | 0.687   | \(\beta_3\)  | 0.803     | 0.511   |
| \(\beta_4\)  | 24.679*   | 0.064   | \(\beta_4\)  | 8.585     | 0.383   | \(\beta_4\)  | 16.638*** | 0.001   |
| Adj. R²  | 0.225475  |         | Adj. R²  | 0.203841  |         | Adj. R²  | 0.188400  |         |

Panel C: Forecast volatility from the PGARCH model versus the previous day’s VIX

| j = 2.5  | j = 3.0  | j = 5.0  |
|----------|----------|----------|
| \(\alpha_1\)  | -14.212*** | 0.000   | \(\alpha_1\)  | -15.569*** | 0.000   | \(\alpha_1\)  | -9.955***  | 0.000   |
| \(\alpha_2\)  | -1.170**  | 0.019   | \(\alpha_2\)  | -0.978***  | 0.006   | \(\alpha_2\)  | -1.041***  | 0.002   |
| \(\alpha_3\)  | -0.372    | 0.545   | \(\alpha_3\)  | -0.484     | 0.345   | \(\alpha_3\)  | -0.374     | 0.432   |
| \(\alpha_4\)  | 8.163     | 0.242   | \(\alpha_4\)  | 10.639***  | 0.001   | \(\alpha_4\)  | 9.447***   | 0.000   |
| \(\alpha_5\)  | 1.180     | 0.806   | \(\alpha_5\)  | 0.964      | 0.762   | \(\alpha_5\)  | 2.349     | 0.343   |
| \(\alpha_6\)  | 0.398     | 0.746   | \(\alpha_6\)  | 0.320      | 0.801   | \(\alpha_6\)  | -0.464     | 0.681   |
| \(\alpha_7\)  | 0.400***   | 0.002   | \(\alpha_7\)  | 7.997      | 0.380   | \(\alpha_7\)  | 5.993     | 0.144   |
| Adj. R²  | 0.176804  |         | Adj. R²  | 0.164761  |         | Adj. R²  | 0.133495  |         |

Panel D: Forecast volatility from the TGARCH model versus the previous day’s VIX

| j = 1.0  | j = 1.5  | j = 2.0  |
|----------|----------|----------|
| \(\beta_1\)  | -28.532*** | 0.000   | \(\beta_1\)  | -23.275*** | 0.000   | \(\beta_1\)  | -19.839*** | 0.000   |
| \(\beta_2\)  | -1.992*** | 0.000   | \(\beta_2\)  | -1.687***  | 0.000   | \(\beta_2\)  | -1.304***  | 0.008   |
| \(\beta_3\)  | 0.572     | 0.229   | \(\beta_3\)  | 0.296      | 0.535   | \(\beta_3\)  | -0.208     | 0.724   |
| \(\beta_4\)  | 11.675**  | 0.030   | \(\beta_4\)  | 8.986**    | 0.035   | \(\beta_4\)  | 10.736*    | 0.099   |
| \(\beta_5\)  | 5.089     | 0.239   | \(\beta_5\)  | 1.903      | 0.646   | \(\beta_5\)  | 2.209      | 0.641   |
| \(\beta_6\)  | -0.272    | 0.848   | \(\beta_6\)  | 0.157      | 0.900   | \(\beta_6\)  | 0.661      | 0.585   |
| \(\beta_7\)  | 21.705*   | 0.061   | \(\beta_7\)  | 16.511***  | 0.008   | \(\beta_7\)  | 17.502***  | 0.001   |
| Adj. R²  | 0.222179  |         | Adj. R²  | 0.203816  |         | Adj. R²  | 0.188705  |         |

| j = 2.5  | j = 3.0  | j = 5.0  |
|----------|----------|----------|
| \(\beta_1\)  | -17.228*** | 0.000   | \(\beta_1\)  | -16.114*** | 0.000   | \(\beta_1\)  | -11.659*** | 0.000   |
| \(\beta_2\)  | -1.281**  | 0.012   | \(\beta_2\)  | -1.017*    | 0.053   | \(\beta_2\)  | -0.963**   | 0.012   |
| \(\beta_3\)  | -0.166    | 0.771   | \(\beta_3\)  | -0.447     | 0.389   | \(\beta_3\)  | -0.362     | 0.463   |
| \(\beta_4\)  | 9.465     | 0.143   | \(\beta_4\)  | 11.780***  | 0.002   | \(\beta_4\)  | 10.035***  | 0.000   |
| \(\beta_5\)  | 1.353     | 0.760   | \(\beta_5\)  | 1.564      | 0.684   | \(\beta_5\)  | 2.698      | 0.413   |
| \(\beta_6\)  | 0.163     | 0.896   | \(\beta_6\)  | -0.024     | 0.986   | \(\beta_6\)  | -0.565     | 0.635   |
| \(\beta_7\)  | 16.065*** | 0.005   | \(\beta_7\)  | 10.109     | 0.530   | \(\beta_7\)  | 6.983      | 0.480   |
| Adj. R²  | 0.176892  |         | Adj. R²  | 0.164559  |         | Adj. R²  | 0.132807  |         |

(Continued)
Table 7. (Continued)

| Coefficients        | Estimates | p-value | Coefficients        | Estimates | p-value | Coefficients        | Estimates | p-value |
|---------------------|-----------|---------|---------------------|-----------|---------|---------------------|-----------|---------|
| $\gamma_{-1.5}^{t-1}$ | $-2.118^{**}$ | 0.024   | $\gamma_{-1.5}^{t-1}$ | $-1.635^{***}$ | 0.000   | $\gamma_{-2}^{t-1}$ | $-1.252^*$ | 0.090   |
| $\gamma_{2.5}^{t-1}$ | 0.680     | 0.310   | $\gamma_{-1.5}^{t-1}$ | 0.253     | 0.623   | $\gamma_{-2}^{t-1}$ | $-0.266$   | 0.665   |
| $\gamma_{1.2}^{t-1}$ | 10.459**  | 0.030   | $\gamma_{1.2}^{t-1}$ | 9.882**   | 0.033   | $\gamma_{1.2}^{t-1}$ | 12.322**  | 0.025   |
| $\gamma_{2.5}^{t-1}$ | 2.484     | 0.563   | $\gamma_{1.2}^{t-1}$ | 0.224     | 0.953   | $\gamma_{1.2}^{t-1}$ | 1.186     | 0.774   |
| $\gamma_{-1.5}^{t-1}$ | $-0.436$   | 0.803   | $\gamma_{2.5}^{t-1}$ | 0.143     | 0.908   | $\gamma_{-2}^{t-1}$ | 0.881     | 0.502   |
| $\gamma_{1.2}^{t-1}$ | 19.590    | 0.549   | $\gamma_{2.5}^{t-1}$ | 17.669    | 0.468   | $\gamma_{1.2}^{t-1}$ | 13.781    | 0.290   |
| Adj. $R^2$          | 0.224285  |         | Adj. $R^2$          | 0.206828  |         | Adj. $R^2$          | 0.190458  |         |
|                     | j = 2.5   |         |                     | j = 3.0   |         |                     | j = 5.0   |         |
| $\gamma_{-1.5}^{t-1}$ | $-19.456^{***}$ | 0.000  | $\gamma_{-1.5}^{t-1}$ | $-15.004^{***}$ | 0.000  | $\gamma_{-5}^{t-1}$ | $-11.161^{***}$ | 0.000  |
| $\gamma_{2.5}^{t-1}$ | $-1.033^{**}$ | 0.012  | $\gamma_{2.5}^{t-1}$ | $-1.023^{**}$ | 0.018  | $\gamma_{-5}^{t-1}$ | $-0.775^{***}$ | 0.002  |
| $\gamma_{2.5}^{t-1}$ | $-0.418$ | 0.410   | $\gamma_{2.5}^{t-1}$ | $-0.447$ | 0.408   | $\gamma_{2.5}^{t-1}$ | $-0.564^{**}$ | 0.195  |
| $\gamma_{1.2}^{t-1}$ | 11.382**  | 0.031   | $\gamma_{2.5}^{t-1}$ | 10.702*   | 0.053  | $\gamma_{1.2}^{t-1}$ | 11.245*** | 0.000  |
| $\gamma_{2.5}^{t-1}$ | 1.728     | 0.668   | $\gamma_{2.5}^{t-1}$ | 1.190     | 0.765   | $\gamma_{2.5}^{t-1}$ | 1.819     | 0.426  |
| $\gamma_{1.2}^{t-1}$ | 0.932     | 0.434   | $\gamma_{2.5}^{t-1}$ | 0.124     | 0.928   | $\gamma_{2.5}^{t-1}$ | $-0.727$ | 0.523  |
| $\gamma_{-1.5}^{t-1}$ | 17.043*** | 0.001   | $\gamma_{1.2}^{t-1}$ | 17.865*** | 0.002  | $\gamma_{-1.5}^{t-1}$ | 5.363*    | 0.076  |
| Adj. $R^2$          | 0.177116  |         | Adj. $R^2$          | 0.165545  |         | Adj. $R^2$          | 0.134190  |         |

Notes: This table exhibits the results of empirical comparisons of the predictive power of the one-day lagged VIX of S&P 500 and the forecast S&P 500 volatilities from various GARCH models. These tests are conducted in our out-of-sample period that is from 3 January 2006 to 28 February 2014. All examinations are conducted using multiple quantile regression models. Each multiple quantile regression model includes a constant term, the one-day lagged VIX, the forecast volatility from one of the four GARCH models, and four control variables. Our control variables are the one-day lagged default spread, the one-day lag of the first-difference series of the trade-weighted US dollar index, the one-day lagged term premium, and the one-day lag of the first-difference series of the effective federal funds rate. All GARCH models used to derive the forecast volatilities are specified in our in-sample period, which is from 2 January 1990 to 30 December 2005. Further, Adj. $R^2$ in this table denotes the adjusted $R$-squared value.

*Statistical significance of coefficients at the 10% level.
**Statistical significance of coefficients at the 5% level.
***Statistical significance of coefficients at the 1% level.

Risk, whereas the coefficients for the previous day’s VIX are mostly positive or insignificant. Hence, once again, the previous day’s VIX exhibits lower predictive power than the predictors from the econometric models.

Specifically, for 1% left-tail risk, the strongest predictor is the EGARCH model (adjusted-$R^2 = 0.225475$), and similarly for the TGARCH for 1.5% left-tail risk (adjusted-$R^2 = 0.206828$), the TGARCH for 2% left-tail risk (adjusted-$R^2 = 0.190458$), the TGARCH for 2.5% left-tail risk (adjusted-$R^2 = 0.177116$), the TGARCH for 3% left-tail risk (adjusted-$R^2 = 0.165545$), and the TGARCH again for 5% left-tail risk (adjusted-$R^2 = 0.134190$). Similar to the logit test results, our robustness checks using model (10) again demonstrate that for the downside risk in the US stock market, the forecast volatilities from EGARCH and TGARCH models are better predictors than the VIX.

Based on these results, we display the daily time-series evolution of the VIX close and the forecast volatilities of the percentage log return of the S&P 500 from the EGARCH model (Panel A) and the TGARCH model (Panel B) in Figure 2. We emphasize again that these two series from the two GARCH models display the strongest predictive power in our out-of-sample period from 3 January 2006 to 28 February 2014, a period including the Lehman shock.
7. Further scrutinies of VIX information

All results presented above robustly show that the downside risk predictive power of the VIX is weaker than that of the volatility forecasts from EGARCH and TGARCH models. However, could we use the information included in the VIX in a different way? Put differently, is there any way to derive more effectively the predictive power of the VIX? In order to examine this possibility, we further scrutinize the information included in the VIX below. We consider that these additional tests also function as robustness checks for our previous results.

7.1. Testing the predictability of the forecast VIX

Our first additional examination uses the forecast VIX derived from ARMA models. In fact, as explained earlier, ARMA(p, q) models supply good forecast values of VIX. Thus, using these forecast values, we first test the downside risk predictive power of the forecast VIX by applying the following univariate logit model:

\[
\Delta s_p_t = u_{0, p, q, i(t)} + u_{1, p, q, i(t)} \hat{V}_{t}^{ARMA(p, q)} + k_i^{ARMA(p, q)} + \epsilon_t
\]

\[
y_t = \begin{cases} 
1 & \text{if } \Delta s_p_t \leq k\% \text{VaR} \\
0 & \text{otherwise}
\end{cases}
\]
where \( \hat{V}_{t}^{ARMA(p,q)} \) denotes the forecast VIX value at time \( t \) from an ARMA\((p,q)\) model, and in accordance with the smallest AIC and SC values, we employ ARMA\((2,1)\) and ARMA\((4,4)\) models in our VIX prediction. Both ARMA models used to derive the forecast values of VIX are determined in our in-sample period from 2 January 1990 to 30 December 2005, and the tests are for our out-of-sample period from 3 January 2006 to 28 February 2014. As before, \( \Delta S_{p_t} = p_{30}S_{\text{P500}_t} - p_{30}S_{\text{P500}_{t-1}} \), and \( k \) takes a value of 99.0, 98.5, 98.0, 97.5, 97.0, or 95.0. Further, Figure 3 presents the daily time-series of the closing prices of the S&P 500 and the forecast VIX close values from the ARMA\((2,1)\) model (Panel A) and ARMA\((4,4)\) model (Panel B). We can see that these two series from the two ARMA models, shown for our out-of-sample period, display a very similar dynamic evolution.

The results for model (11) are shown in Panels A and B of Table 8, and these are to be compared with those in Table 2. As shown in Panel A of Table 8, with regard to the forecast VIX from the ARMA\((2,1)\) model, MF-\( R^2 \) = 0.187138 for 99% VaR (the highest MF-\( R^2 \) in Table 2 was 0.250857 for the TGARCH model), 0.155051 for 98.5% (0.190221 for TGARCH), 0.100719 for 98% (0.127788 for TGARCH), 0.087363 for 97.5% (0.106875 for TGARCH), 0.077690 for 97% (0.092101 for EGARCH), and 0.078521 for 95% (0.082080 for EGARCH).

Further, as shown in Panel B of Table 8, for the forecast VIX from the ARMA\((4,4)\) model, the MF-\( R^2 \) value = 0.186474 for 99% VaR (the highest MF-\( R^2 \) in Table 2 was 0.250857 for the TGARCH model), 0.155474 for 98.5% (0.190221 for TGARCH), 0.100886 for 98% (0.127788 for TGARCH), 0.087363 for 97.5% (0.106875 for TGARCH), 0.077818 for 97% (0.092101 for EGARCH), and 0.078891 for 95% (0.082080 for EGARCH). Overall, the test results from model (11) show that in terms of US stock market

\[\text{Figure 3. Dynamic relations of the S&P 500 and the forecast VIX from the ARMA models: daily time-series evolution for the period from 3 January 2006 to 28 February 2014.}\]
Table 8. Testing the predictive power for downside risk in the US stock market in terms of the forecast VIX from ARMA models: results from logit models and quantile regressions

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| **Panel A: Downside risk predictive power of the forecast VIX from the ARMA(2,1) model: results from logit models** | | | | | | | | |
| $\theta_{j-99\%}$ | -7.086*** | 0.000 | $\theta_{j-98\%}$ | -6.342*** | 0.000 | $\theta_{j-97\%}$ | -5.600*** | 0.000 |
| $\theta_{j-99\%}$ | 0.085*** | 0.000 | $\theta_{j-98\%}$ | 0.078*** | 0.000 | $\theta_{j-97\%}$ | 0.065*** | 0.000 |
| MF-R$^2$ | 0.187138 | MF-R$^2$ | 0.155051 | MF-R$^2$ | 0.100719 |
| k | 99.0 | k | 98.5 | k | 98.0 |
| **Panel B: Downside risk predictive power of the forecast VIX from the ARMA(4,4) model: results from logit models** | | | | | | | | |
| $\theta_{j-99\%}$ | -5.241 *** | 0.000 | $\theta_{j-97\%}$ | -4.957 *** | 0.000 | $\theta_{j-95\%}$ | -4.395 *** | 0.000 |
| $\theta_{j-99\%}$ | 0.061 *** | 0.000 | $\theta_{j-97\%}$ | 0.058 *** | 0.000 | $\theta_{j-95\%}$ | 0.058 *** | 0.000 |
| MF-R$^2$ | 0.186474 | MF-R$^2$ | 0.155450 | MF-R$^2$ | 0.100886 |
| k | 97.5 | k | 97.0 | k | 95.0 |
| **Panel C: Downside risk predictive power of the forecast VIX from the ARMA(2,1) model: results from quantile regressions** | | | | | | | | |
| $\theta_{j-1\%}$ | -11.817** | 0.017 | $\theta_{j-0.5\%}$ | -8.018** | 0.014 | $\theta_{j-0.25\%}$ | -8.134** | 0.014 |
| $\theta_{j-1\%}$ | -1.264*** | 0.000 | $\theta_{j-0.5\%}$ | -1.293*** | 0.000 | $\theta_{j-0.25\%}$ | -1.214*** | 0.000 |
| Adj. R$^2$ | 0.165542 | Adj. R$^2$ | 0.156664 | Adj. R$^2$ | 0.146785 |
| j | 1.0 | j | 1.5 | j | 2.0 |
| **Panel D: Downside risk predictive power of the forecast VIX from the ARMA(4,4) model: results from quantile regressions** | | | | | | | | |
| $\theta_{j-1\%}$ | -5.239 | 0.149 | $\theta_{j-0.5\%}$ | -7.506** | 0.032 | $\theta_{j-0.25\%}$ | -3.379 | 0.269 |
| $\theta_{j-1\%}$ | -1.252*** | 0.000 | $\theta_{j-0.5\%}$ | -1.042*** | 0.000 | $\theta_{j-0.25\%}$ | -1.019*** | 0.000 |
| Adj. R$^2$ | 0.137579 | Adj. R$^2$ | 0.132064 | Adj. R$^2$ | 0.106947 |
| j | 1.0 | j | 1.5 | j | 2.0 |
| (Continued)
Next, we examine the downside risk predictive power of the information contained in VIX using the following univariate quantile regression model:

\[ \Delta sp_j^{\%} = uQ^{-2.5\%}_{0,IVF(p,q)} + uQ^{-3\%}_{1,IVF(p,q)} + \frac{hQ^{-5\%}_{0,IVF(p,q)}}{uQ^{-5\%}_{0,IVF(p,q)}} + \frac{hQ^{-5\%}_{1,IVF(p,q)}}{uQ^{-5\%}_{1,IVF(p,q)}} \]  

\[ \text{(12)} \]

Figure 4. Dynamic relations of the VIX and the forecast volatilities of the VIX from the EGARCH and TGARCH models: daily time-series evolution for the period from 3 January 2006 to 28 February 2014.

Notes: This figure presents the daily time-series evolution of the VIX and the forecast volatilities of the VIX from the EGARCH(1,1) model with GED errors (Panel A) and the TGARCH(1,1) model with GED errors (Panel B). These series are exhibited for our out-of-sample period from 3 January 2006 to 28 February 2014, and this period includes the date of the Lehman Brothers bankruptcy in the US. Two kinds of GARCH models used to derive the forecast volatilities of the VIX are specified in our in-sample period, which is from 2 January 1990 to 30 December 2005.

Table 8. (Continued)

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| \( uQ^{-2.5\%}_{0,IVF(p,q)} \) | 0.075 | 0.000 | \( uQ^{-3\%}_{0,IVF(p,q)} \) | 0.011 | 0.000 | \( uQ^{-5\%}_{0,IVF(p,q)} \) | 0.254 |
| \( uQ^{-2.5\%}_{1,IVF(p,q)} \) | 0.137631 | 0.131963 | \( uQ^{-3\%}_{1,IVF(p,q)} \) | 0.000 | 0.000 | \( uQ^{-5\%}_{1,IVF(p,q)} \) | 0.000 |

Notes: This table exhibits the predictive power of the forecast VIX from ARMA(2,1) and ARMA(4,4) models. These tests are conducted in our out-of-sample period from 3 January 2006 to 28 February 2014 using univariate logit models (Panels A and B) and univariate quantile regressions (Panels C and D). Two kinds of ARMA models for deriving the forecast values of the VIX are specified in our in-sample period, which is from 2 January 1990 to 30 December 2005. In this table, MF-\( R^2 \) denotes the McFadden's R-squared value and Adj. \( R^2 \) denotes the adjusted R-squared value.

*Statistical significance of coefficients at the 10% level.
**Statistical significance of coefficients at the 5% level.
***Statistical significance of coefficients at the 1% level.
where $\hat{\sigma}_{ivf_t}^{ARMA(p,q)}$ again denotes the forecast VIX at time $t$ from an ARMA($p,q$) model. As earlier, we specify ARMA(2,1) and ARMA(4,4) models in this analysis. $\Delta sp_{ij}^k$ denotes the $j$-percentile point of the distribution of S&P 500 price changes, and $j$ takes a value of 1.0, 1.5, 2.0, 2.5, 3.0, or 5.0.

Results from the quantile regression (12) are exhibited in Panels C and D of Table 8, and these are to be compared with the earlier results in Table 5. As shown in Panel C of Table 8, with respect to the forecast VIX from the ARMA(2,1) model, the values of Adj. $R^2$ are 0.165542 for 1% left-tail risk (the highest corresponding Adj. $R^2$ in Table 5 was 0.196455 for the TGARCH), 0.156664 for 1.5% (0.186060 for EGARCH), 0.146785 for 2% (0.175194 for EGARCH), 0.137579 for 2.5% (0.165310 for EGARCH), 0.132064 for 3% (0.155252 for EGARCH), and 0.106947 for 5% (0.124073 for EGARCH).

Further, as shown in Panel D of Table 8, for the forecast VIX from the ARMA(4,4) model, the values of Adj. $R^2$ are 0.165017 for 1% left-tail risk (the highest Adj. $R^2$ in Table 5 was 0.196455 for the TGARCH), 0.156591 for 1.5% (0.186060 for EGARCH), 0.146850 for 2% (0.175194 for EGARCH), 0.137631 for 2.5% (0.165310 for EGARCH), 0.131963 for 3% (0.155252 for EGARCH), and 0.106753 for 5% (0.124073 for EGARCH). Hence, the results from the quantile regressions (12) again suggest that when predicting downside risk in the US stock market, the predictive power of the VIX forecasts from ARMA(2,1) and ARMA(4,4) models is lower than that of the forecast S&P 500 volatilities from the econometric models. This is again because all the Adj. $R^2$ values are lower than the best Adj. $R^2$ values in Table 5.

7.2. Testing the predictability of the forecast volatilities of VIX

Our second examination uses the forecast volatility of VIX derived from EGARCH and TGARCH models. Both the EGARCH and TGARCH models used to derive the volatility forecasts of VIX are determined in our in-sample period from 2 January 1990 to 30 December 2005. Using these forecast volatilities of VIX for our out-of-sample period, we further test the predictive power for downside risk by applying the following univariate logit model:

$$
\Delta sp_t = \beta_{lg-99}^0 \Delta sp_{t-1} + \beta_{lg-98.5}^0 \Delta sp_{t-1} + \beta_{lg-98}^0 \Delta sp_{t-1} + \beta_{lg-97.5}^0 \Delta sp_{t-1} + \beta_{lg-97}^0 \Delta sp_{t-1} + \beta_{lg-95}^0 \Delta sp_{t-1} + e_{lg-99}^0 \Delta sp_{t-1} + e_{lg-98.5}^0 \Delta sp_{t-1} + e_{lg-98}^0 \Delta sp_{t-1} + e_{lg-97.5}^0 \Delta sp_{t-1} + e_{lg-97}^0 \Delta sp_{t-1} + e_{lg-95}^0 \Delta sp_{t-1},
$$

where $\Delta sp_{ij}^k$ denotes the forecast volatility of VIX at time $t$ from the EGARCH or TGARCH model. In addition, $\Delta sp_t = \hat{P}_t - P_{t-1}$, and $k$ takes a value of 99.0, 98.5, 98.0, 97.5, 97.0, or 95.0. Figure 4 plots the daily time-series of VIX and the forecast volatilities of VIX from the EGARCH model (Panel A) and TGARCH model (Panel B). Plots of these series are for our out-of-sample period from 3 January 2006 to 28 February 2014.

The results from model (13) are shown in Panels A and B of Table 9, with the results to be again compared with the best results in Table 2. Panel A of Table 9 shows that for the forecast volatility of VIX from the EGARCH model, the MF-$R^2$ values are 0.198383 (vs. 0.250857 in Table 2) for 99% VaR, 0.155018 (0.190221) for 98.5%, 0.093926 (0.127788) for 98%, 0.080073 (0.106875) for 97.5%, 0.068501 (0.092101) for 97%, and 0.066565 (0.082080) for 95%.

As shown in Panel B of Table 9, for the forecast volatility of VIX from the TGARCH model, the MF-$R^2$ values are 0.183171 (vs. 0.250857 in Table 2) for 99% VaR, 0.145232 (0.190221) for 98.5%, 0.089643 (0.127788) for 98%, 0.075311 (0.106875) for 97.5%, 0.063524 (0.092101) for 97%, and 0.063225 (0.082080) for 95%. Once again, the US stock market downside risk predictive power of the forecast volatilities of VIX from the EGARCH and TGARCH models is lower than that of the forecast S&P 500 volatilities from the econometric models, again because all of the MF-$R^2$ values are lower than the best values of MF-$R^2$ in Table 2.
We also test the downside risk predictive power of the forecast VIX volatilities using the following univariate quantile regression model:

$$
\Delta s_{t}^{\%} = \rho_{0, \text{VIV-xgf}(t)}^{\%} + \rho_{1, \text{VIV-xgf}(t)}^{\%} \sigma_{t}^{\%} + \epsilon_{t}^{\%},
$$

(14)

| Table 9. Testing the predictive power for downside risk in the US stock market in terms of the forecast volatilities of the VIX from EGARCH and TGARCH models: results from logit models and quantile regressions |

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| Panel A: Predictive power of the forecast volatility of the VIX from the EGARCH model: results from logit models |
| k = 99.0 | k = 98.5 | k = 98.0 |
| $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-6.180^{***}$ | 0.000 | $\rho_{0.985, \text{VIV-xgf}(t)}^{\%}$ | $-5.464^{***}$ | 0.000 | $\rho_{0.98, \text{VIV-xgf}(t)}^{\%}$ | $-4.838^{***}$ | 0.000 |
| $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $0.165^{***}$ | 0.000 | $\rho_{0.985, \text{VIV-xgf}(t)}^{\%}$ | $0.149^{***}$ | 0.000 | $\rho_{0.98, \text{VIV-xgf}(t)}^{\%}$ | $0.122^{***}$ | 0.000 |
| MF-R² | 0.198383 | MF-R² | 0.155018 | MF-R² | 0.093926 |
| k = 97.5 | k = 97.0 | k = 95.0 |
| $\rho_{0.975, \text{VIV-xgf}(t)}^{\%}$ | $-4.523^{***}$ | 0.000 | $\rho_{0.97, \text{VIV-xgf}(t)}^{\%}$ | $-4.261^{***}$ | 0.000 | $\rho_{0.95, \text{VIV-xgf}(t)}^{\%}$ | $-3.687^{***}$ | 0.000 |
| $\rho_{0.975, \text{VIV-xgf}(t)}^{\%}$ | $0.114^{***}$ | 0.000 | $\rho_{0.97, \text{VIV-xgf}(t)}^{\%}$ | $0.107^{***}$ | 0.000 | $\rho_{0.95, \text{VIV-xgf}(t)}^{\%}$ | $0.105^{***}$ | 0.000 |
| MF-R² | 0.080073 | MF-R² | 0.068501 | MF-R² | 0.066565 |
| Panel B: Predictive power of the forecast volatility of the VIX from the TGARCH model: results from logit models |
| k = 99.0 | k = 98.5 | k = 98.0 |
| $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-5.829^{***}$ | 0.000 | $\rho_{0.985, \text{VIV-xgf}(t)}^{\%}$ | $-5.177^{***}$ | 0.000 | $\rho_{0.98, \text{VIV-xgf}(t)}^{\%}$ | $-4.628^{***}$ | 0.000 |
| $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $0.102^{***}$ | 0.000 | $\rho_{0.985, \text{VIV-xgf}(t)}^{\%}$ | $0.093^{***}$ | 0.000 | $\rho_{0.98, \text{VIV-xgf}(t)}^{\%}$ | $0.077^{***}$ | 0.000 |
| MF-R² | 0.183171 | MF-R² | 0.145232 | MF-R² | 0.089643 |
| k = 97.5 | k = 97.0 | k = 95.0 |
| $\rho_{0.975, \text{VIV-xgf}(t)}^{\%}$ | $-4.325^{***}$ | 0.000 | $\rho_{0.97, \text{VIV-xgf}(t)}^{\%}$ | $-4.075^{***}$ | 0.000 | $\rho_{0.95, \text{VIV-xgf}(t)}^{\%}$ | $-3.516^{***}$ | 0.000 |
| $\rho_{0.975, \text{VIV-xgf}(t)}^{\%}$ | $0.072^{***}$ | 0.000 | $\rho_{0.97, \text{VIV-xgf}(t)}^{\%}$ | $0.067^{***}$ | 0.000 | $\rho_{0.95, \text{VIV-xgf}(t)}^{\%}$ | $0.067^{***}$ | 0.000 |
| MF-R² | 0.075311 | MF-R² | 0.063524 | MF-R² | 0.063225 |
| Panel C: Predictive power of the forecast volatility of the VIX from the EGARCH model: results from quantile regressions |
| j = 1.0 | j = 1.5 | j = 2.0 |
| $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-30.322^{***}$ | 0.000 | $\rho_{0.995, \text{VIV-xgf}(t)}^{\%}$ | $-26.640^{***}$ | 0.000 | $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-23.060^{***}$ | 0.000 |
| $\rho_{1.01, \text{VIV-xgf}(t)}^{\%}$ | $-2.212^{***}$ | 0.000 | $\rho_{1.01, \text{VIV-xgf}(t)}^{\%}$ | $-2.017^{***}$ | 0.000 | $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-2.099^{***}$ | 0.000 |
| Adj. R² | 0.139215 | Adj. R² | 0.124371 | Adj. R² | 0.113298 |
| j = 2.5 | j = 3.0 | j = 5.0 |
| $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-22.340^{***}$ | 0.000 | $\rho_{0.995, \text{VIV-xgf}(t)}^{\%}$ | $-20.254^{***}$ | 0.000 | $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-17.245^{***}$ | 0.000 |
| $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-1.742^{***}$ | 0.000 | $\rho_{0.995, \text{VIV-xgf}(t)}^{\%}$ | $-1.836^{***}$ | 0.000 | $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-1.535^{***}$ | 0.000 |
| Adj. R² | 0.111053 | Adj. R² | 0.107057 | Adj. R² | 0.082926 |
| Panel D: Predictive power of the forecast volatility of the VIX from the TGARCH model: results from quantile regressions |
| j = 1.0 | j = 1.5 | j = 2.0 |
| $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-33.503^{***}$ | 0.000 | $\rho_{0.995, \text{VIV-xgf}(t)}^{\%}$ | $-28.533^{***}$ | 0.000 | $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-25.115^{***}$ | 0.000 |
| $\rho_{1.01, \text{VIV-xgf}(t)}^{\%}$ | $-1.409^{***}$ | 0.000 | $\rho_{1.01, \text{VIV-xgf}(t)}^{\%}$ | $-1.495^{***}$ | 0.000 | $\rho_{0.99, \text{VIV-xgf}(t)}^{\%}$ | $-1.470^{***}$ | 0.000 |
| Adj. R² | 0.141416 | Adj. R² | 0.125359 | Adj. R² | 0.114556 |
| j = 2.5 | j = 3.0 | j = 5.0 |

(Continued)
where $\hat{\sigma}_{xgf}^{iv}(t)$ denotes the forecast volatility of VIX at time $t$ from the EGARCH or TGARCH model. Further, $\Delta sp_{j}(t)$ denotes the $j$-percentile point of the distribution of S&P 500 price changes, and $j$ takes a value of 1.0, 1.5, 2.0, 2.5, 3.0, or 5.0.

Figure 5. Dynamic relations of the first log differences of the VIX and the forecast volatilities from the EGARCH and TGARCH models: daily time-series evolution for the period from 3 January 2006 to 28 February 2014.

Notes: This figure presents the daily time-series evolution of the first log differences of the VIX and the forecast volatilities from the EGARCH(1,1) model with GED errors (Panel A) and the TGARCH(1,1) model with GED errors (Panel B). These series are exhibited for our out-of-sample period from 3 January 2006 to 28 February 2014, and this period includes the date of the US Lehman Brothers bankruptcy. Two kinds of GARCH models used to derive the forecast volatilities of the VIX are specified in our in-sample period, which is from 2 January 1990 to 30 December 2005.

Table 9. (Continued)

| Coefficients | Estimates | $p$-value | Coefficients | Estimates | $p$-value | Coefficients | Estimates | $p$-value |
|--------------|-----------|-----------|--------------|-----------|-----------|--------------|-----------|-----------|
| $\rho_{\text{out-of-sample}}^{2.5\%}$ | −23.654*** | 0.000 | $\rho_{\text{out-of-sample}}^{3\%}$ | −21.361*** | 0.000 | $\rho_{\text{out-of-sample}}^{5\%}$ | −18.787*** | 0.000 |
| $\rho_{\text{out-of-sample}}^{1\%}$ | −1.319*** | 0.000 | $\rho_{\text{out-of-sample}}^{1\%}$ | −1.365*** | 0.000 | $\rho_{\text{out-of-sample}}^{1\%}$ | −1.115*** | 0.000 |

Adj. $R^2$ 0.111407 | Adj. $R^2$ 0.106773 | Adj. $R^2$ 0.084665

Notes: This table exhibits the predictive power of the forecast volatilities of the VIX from the EGARCH(1,1) model with GED errors and the TGARCH(1,1) model with GED errors. These tests are conducted in our out-of-sample period from 3 January 2006 to 28 February 2014 using univariate logit models (Panels A and B) and univariate quantile regressions (Panels C and D). Two kinds of GARCH models used to derive the forecast volatilities of the VIX are specified in our in-sample period, which is from 2 January 1990 to 30 December 2005. In this table, MF-$R^2$ denotes the McFadden's $R$-squared value and Adj. $R^2$ is the adjusted $R$-squared value.

*Statistical significance of coefficients at the 10% level.
**Statistical significance of coefficients at the 5% level.
***Statistical significance of coefficients at the 1% level.

Panel A. First log differences of the VIX and the forecast volatility from the EGARCH model

Panel B. First log differences of the VIX and the forecast volatility from the TGARCH model
The results from model (14) are in Panels C and D of Table 9, again for comparison with the best results in Table 5. Panel C of Table 9 shows that for the forecast volatility of VIX from the EGARCH model, the Adj. $R^2$ values are 0.139215 (vs. 0.196455 in Table 5) for 1% left-tail risk, 0.124371 (0.186060) for 1.5%, 0.113298 (0.175194) for 2%, 0.111053 (0.165310) for 2.5%, 0.107057 (0.155252) for 3%, and 0.082926 (0.124073) for 5%.

Table 10. Testing the predictive power for downside risk in the US stock market in terms of the forecast volatilities of the first log differences of VIX: results from logit models and quantile regressions

| Coefficients | Estimates | p-value | Coefficients | Estimates | p-value | Coefficients | Estimates | p-value |
|--------------|-----------|---------|--------------|-----------|---------|--------------|-----------|---------|
| Panel A: Predictive power of the forecast volatility of the first log differences of VIX from the EGARCH model: results from logit models |
| k = 99.0 | k = 98.5 | k = 98.0 |
| $\beta_{q,99.5}$ | $\beta_{q,98.5}$ | $\beta_{q,98.5}$ | $\beta_{q,97.5}$ | $\beta_{q,97.5}$ | $\beta_{q,97.5}$ |
| $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ |
| -8.758*** | -7.842*** | -7.016*** | 0.607*** | 0.550*** | 0.479*** |
| 0.149994 | 0.119788 | 0.086614 |
| Panel B: Predictive power of the forecast volatility of the first log differences of VIX from the TARCH model: results from logit models |
| k = 99.0 | k = 98.5 | k = 98.0 |
| $\beta_{q,99.5}$ | $\beta_{q,98.5}$ | $\beta_{q,98.5}$ | $\beta_{q,97.5}$ | $\beta_{q,97.5}$ | $\beta_{q,97.5}$ |
| $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ |
| -7.174*** | -6.418*** | -5.781*** | 0.356*** | 0.321*** | 0.278*** |
| 0.113873 | 0.087000 | 0.059770 |
| Panel C: Predictive power of the forecast volatility of the first log differences of VIX from the EGARCH model: results from quantile regressions |
| j = 1.0 | j = 1.5 | j = 2.0 |
| $\phi_{q,1.5}$ | $\phi_{q,1.5}$ | $\phi_{q,1.5}$ | $\phi_{q,1.5}$ | $\phi_{q,1.5}$ | $\phi_{q,1.5}$ |
| $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ |
| 13.538* | 10.206 | 7.397 | 7.691 | 10.253* | 7.705 |
| 0.152211 | 0.130403 | 0.115972 |
| j = 1.5 | j = 3.0 | j = 5.0 |
| $\phi_{q,3.0}$ | $\phi_{q,3.0}$ | $\phi_{q,3.0}$ | $\phi_{q,3.0}$ | $\phi_{q,3.0}$ | $\phi_{q,3.0}$ |
| $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ |
| -6.982*** | -7.172*** | -5.771*** | -6.982*** | -7.172*** | -5.771*** |
| 0.106554 | 0.095151 | 0.064069 |
| Panel D: Predictive power of the forecast volatility of the first log differences of VIX from the TARCH model: results from quantile regressions |
| j = 1.0 | j = 1.5 | j = 2.0 |
| $\phi_{q,1.5}$ | $\phi_{q,1.5}$ | $\phi_{q,1.5}$ | $\phi_{q,1.5}$ | $\phi_{q,1.5}$ | $\phi_{q,1.5}$ |
| $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ | $\phi_{egf}^{(t)}$ |
| -3.972 | -4.090 | -2.852 | -6.326*** | -6.180*** | -5.334*** |
| 0.124626 | 0.103989 | 0.090095 |
| j = 2.5 | j = 3.0 | j = 5.0 | (Continued)
Further, as shown in Panel D of Table 9, for the forecast volatility of VIX from the TGARCH model, the Adj. $R^2$ values are 0.141416 (vs. 0.196455 in Table 5) for 1% tail risk, 0.125359 (0.186060) for 1.5%, 0.114556 (0.175194) for 2%, 0.111407 (0.165310) for 2.5%, 0.106773 (0.155252) for 3%, and 0.084665 (0.124073) for 5%. Therefore, we can again see that in terms of US stock market downside risk prediction, the predictive power of the forecast volatilities of VIX from the EGARCH and TGARCH models is inferior to that of the S&P 500 volatility predictors from the econometric models. This is because, as shown in Panels C and D of Table 9, all values of Adj. $R^2$ are smaller than the best Adj. $R^2$ values in Table 5.

### 7.3. Testing the predictive power of the forecast volatilities of the first log differences of VIX

Our next set of tests uses the forecast volatilities of the first log differences of VIX from the EGARCH and TGARCH models. Once again, using our in-sample period, we determine both the EGARCH and TGARCH models to derive the forecast volatilities of the first log differences of VIX, and test the downside risk predictive power of the forecasts in our out-of-sample period by estimating the following univariate logit model:

$$\Delta s_{pt} = \rho^{\text{lg}}_{0,vdsliv} \cdot x_{gf}(t) + \rho^{\text{lg}}_{1,vdsliv} \cdot \Delta x_{gf}(t) + \nu^{\text{lg}} \cdot \Delta y_{vliv}(t),$$

(15)

$$y_{t} = \begin{cases} 1 & \text{if } \Delta s_{pt} \leq k\% \text{VaR} \\ 0 & \text{otherwise} \end{cases}$$

where $\Delta x_{vliv}$ is the forecast volatility of the first log differences of VIX at time $t$ from the EGARCH or TGARCH model. $\Delta s_{pt} = p_{t} - p_{t-1}$, and $k$ takes a value of 99.0, 98.5, 98.0, 97.5, 97.0, or 95.0. Figure 5 plots the daily time-series evolution of the first log differences of VIX from the EGARCH model (Panel A) and the TGARCH model (Panel B). The series plots are for the out-of-sample period from 3 January 2006 to 28 February 2014.

We detail the estimation results for model (15) in Panels A and B of Table 10, with the results again compared with the best results in Table 2. Panel A of Table 10 shows that for the forecast volatility of the first log differences of VIX from the EGARCH model, the MF-$R^2$ values are 0.149994 (vs. 0.250857 in Table 2) for 99% VaR, 0.119788 (0.190221) for 98.5%, 0.086614 (0.127788) for 98%, 0.064664 (0.106875) for 97.5%, 0.059250 (0.092101) for 97%, and 0.038593 (0.082080) for 95%. As shown in Panel B of Table 10, as for the forecast volatility of the first log differences of VIX from the TGARCH model, the MF-$R^2$ values are 0.113873 (vs. 0.250857 in Table 2) for 99% VaR, 0.087000 (0.190221) for 98.5%, 0.059770 (0.127788) for 98%, 0.042306 (0.106875) for 97.5%, 0.037245 (0.092101) for 97%, and 0.024952 (0.082080) for 95%. Hence, these tests again indicate that the downside risk predictive power of the forecast volatilities of the first log differences of VIX from the
EGARCH and TGARCH models is less than that of the forecast S&P 500 volatilities from the econometric models. This is because all values of MF-$R^2$ in Panels A and B of Table 10 are smaller than the best values in Table 2.

We also test the downside risk predictive power of the forecast volatilities of the first log differences of VIX using the following univariate quantile regression model:

$$\Delta sp_j^t = \gamma_0 + \gamma_1 \sigma_{dlogiv,t}^2 + \gamma_2 \sigma_{xgf,dlogiv,t}^2 + \nu_j \sigma_{dlogiv,t}^2$$  \hspace{1cm} (16)$$

where $\sigma_{dlogiv,t}^2$ denotes the forecast volatility of the first log differences of VIX at time $t$ from the EGARCH or TGARCH model, $\Delta sp_j^t$ is the $j$-percentile point of the distribution of S&P 500 price changes, and $j$ takes a value of 1.0, 1.5, 2.0, 2.5, 3.0, or 5.0.

The results from model (16) are in Panels C and D of Table 10 for comparison with the best results in Table 5. As shown in Panel C of Table 10, for the forecast volatility of the first log differences of the VIX from the EGARCH model, the $Adj.\ R^2$ values are 0.152211 (vs. 0.196455 in Table 5) for 1% left-tail risk, 0.130403 (0.186060) for 1.5%, 0.115972 (0.175194) for 2%, 0.106554 (0.165310) for 2.5%, 0.095151 (0.155252) for 3%, and 0.064069 (0.124073) for 5%.

Lastly, as shown in Panel D of Table 10, for the volatility of the first log differences of the VIX from the TGARCH model, the $Adj.\ R^2$ values are 0.124626 (vs. 0.196455 in Table 5) for 1% left-tail risk, 0.103989 (0.186060) for 1.5%, 0.090095 (0.175194) for 2%, 0.084158 (0.165310) for 2.5%, 0.076957 (0.155252) for 3%, and 0.048767 (0.124073) for 5%. In sum, the results derived from the quantile regressions again demonstrate that the forecast volatilities of the first log differences of the VIX from the EGARCH and TGARCH models cannot outperform the forecast volatilities of the S&P 500 from econometric models in predicting downside risk. This is because all values of $Adj.\ R^2$ in Panels C and D of Table 10 are smaller than the best values in Table 5.

8. Exploring the dynamic linkages between VIX and forecast volatilities

In order to inspect the dynamic linkages between VIX and the EGARCH and TGARCH volatility forecasts, we next use four multivariate GARCH models. In MGARCH models, the mean equations are as follows:

$$y_t = \mu + u_t,$$  \hspace{1cm} (17)$$

where the dependent variables are, $y_t = [y_{1,t}, y_{2,t}]$; their means, $\mu = [\mu_1, \mu_2]$; and the error terms, $u_t = [u_{1,t}, u_{2,t}]$ given we assume bivariate GARCH models in our analyses. Further, the variance-covariance matrix of the bivariate GARCH model at time $t$ is as follows:

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}. \hspace{1cm} (18)$$

The first MGARCH model used in our analyses is the following bivariate VECH model (Bollerslev, Engle, & Wooldridge, 1988):

$$vech(H_t) = W + Avech(u_{t-1}u_{t-1}') + Bvech(H_{t-1}'), \hspace{1cm} (19)$$

where $vech(\cdot)$ denotes the vector-half operator and $W, A,$ and $B$ are coefficient matrices.

Our next MGARCH model is the following bivariate BEKK model (Engle & Kroner, 1995):

$$H_t = \Pi + \Lambda u_{t-1}'u_{t-1} + \Xi H_{t-1}\Xi, \hspace{1cm} (20)$$

where $\Pi, \Lambda,$ and $\Xi$ are coefficient matrices.
We also use the following DCC- and asymmetric DCC (ADCC)-MGARCH models (Cappiello, Engle, & Sheppard, 2006; Engle, 2002):

\[ H_t = D_t R_t D_t. \]  

(21)

In the bivariate DCC and ADCC models, the dynamic changes in conditional volatility, \( \sqrt{h_{11,t}} \) and \( \sqrt{h_{22,t}} \) are derived from univariate GARCH models and then included in the diagonal matrix \( D_t \) as follows:

\[
D_t = \begin{bmatrix}
\sqrt{h_{11,t}} & 0 \\
0 & \sqrt{h_{22,t}}
\end{bmatrix}.
\]

(22)

The components of \( u_t = [u_{1,t} \ u_{2,t}] \) are standardized as \( \epsilon_{it} = u_{it}/\sqrt{h_{ii,t}} \), and the dynamic evolution of the correlations in the bivariate DCC model is given by the following matrix, \( Q_t \):

\[
Q_t = \begin{bmatrix}
q_{11,t} & q_{12,t} \\
q_{21,t} & q_{22,t}
\end{bmatrix} = (1 - \zeta - \varphi) \tilde{Q} + \zeta \epsilon_{t-1} \epsilon_{t-1} + \varphi Q_{t-1},
\]

(23)

and that in the bivariate ADCC model is given by the following matrix, \( Q_t \):

\[
Q_t = \begin{bmatrix}
q_{11,t} & q_{12,t} \\
q_{21,t} & q_{22,t}
\end{bmatrix} = (\tilde{\Theta} - \zeta^2 \tilde{Q} - \varphi^2 \tilde{Q} - g^2 \tilde{N}) + \zeta^2 \epsilon_{t-1} \epsilon_{t-1} + g^2 n_{t-1} n_{t-1} + \varphi^2 Q_{t-1},
\]

(24)

where \( \epsilon_t = [u_{1,t}/\sqrt{h_{11,t}} \ u_{2,t}/\sqrt{h_{22,t}}] \), and \( \tilde{Q} = E[\epsilon_t \epsilon_t'] \) is the unconditional variance–covariance matrix of \( \epsilon_t \). Further, as in Cappiello et al. (2006), \( \zeta \), \( \varphi \) and \( g \) are scalars; \( n_t = I[\epsilon_t < 0] \circ \epsilon_t [\cdot] \) is an indicator function which takes one if the argument is true and zero otherwise, while \( \circ \) indicates the Hadamard product; and \( \tilde{N} = E[n_t n_t'] \).

Using \( Q_t \), we can obtain the time-varying correlation coefficient matrix \( R_t \), whose diagonal elements are equal to one and nondiagonal elements are less than one by the following transformation:

\[
R_t = \text{diag}(\sqrt{q_{11,t}} \cdots \sqrt{q_{22,t}})^{-1} Q_t \text{diag}(\sqrt{q_{11,t}} \cdots \sqrt{q_{22,t}})^{-1}.
\]

(25)

8.1. MGARCH models for analyzing the dynamic linkage between VIX and forecast EGARCH volatility

We explain our MGARCH models used in this study more specifically. Denoting that \( \hat{\sigma}^2_{vg} \) is the variance of forecast volatility from the EGARCH model, \( \hat{\sigma}^2_{vg,\delta_{vg-1}} \) is the covariance of the forecast volatility from the EGARCH model and one-day lagged VIX, and \( \hat{\sigma}^2_{vg,\delta_{vg-1}} \) is the variance of the one-day lagged VIX (hereinafter, we use tilde to signify that the variable is time-varying), our bivariate (diagonal) VECM model for analyzing the dynamic linkage between VIX at time \( t - 1 \) and forecast EGARCH volatility at time \( t \) is as follows:

\[
\begin{bmatrix}
\hat{\sigma}^2_{eg} \\
\hat{\sigma}^2_{eg,\delta_{vg-1}} \\
\hat{\sigma}^2_{vg,\delta_{vg-1}}
\end{bmatrix} = \begin{bmatrix}
\pi_{eg} \\
\pi_{eg,\delta_{vg-1}} \\
\pi_{vg,\delta_{vg-1}}
\end{bmatrix} + \begin{bmatrix}
\lambda_{eg_{t-1}} & 0 & 0 \\
0 & \lambda_{eg_{t-1}} & 0 \\
0 & 0 & \lambda_{vg_{t-1}}
\end{bmatrix} + \begin{bmatrix}
\tilde{\sigma}^2_{eg_{t-1}} \\
\tilde{\sigma}^2_{eg_{t-1},\delta_{vg-1}} \\
\tilde{\sigma}^2_{vg_{t-1},\delta_{vg-1}}
\end{bmatrix},
\]

(26)
Next, our (diagonal) BEKK model for analyzing the dynamic linkage between VIX at time $t - 1$ and forecast EGARCH volatility at time $t$ is as follows:

$$
\begin{bmatrix}
\hat{\sigma}^2_{ieg, t-1} \\
\hat{\sigma}^2_{iv, t-1, eg} \\
\hat{\sigma}^2_{iv, t-1, v}
\end{bmatrix}
= 
\begin{bmatrix}
\pi_{ieg, t} & \pi_{ieg, iv, t-1} \\
\pi_{iv, t} & \pi_{iv, iv, t-1}
\end{bmatrix}
\begin{bmatrix}
\hat{\sigma}^2_{ieg, t-1} \\
\hat{\sigma}^2_{iv, t-1, eg} \\
\hat{\sigma}^2_{iv, t-1, v}
\end{bmatrix}
+ 
\begin{bmatrix}
\lambda_{ieg, t} & 0 \\
0 & \lambda_{iv, t}
\end{bmatrix}
\begin{bmatrix}
\tilde{\sigma}^2_{ieg, t-1} \\
\tilde{\sigma}^2_{iv, t-1, eg} \\
\tilde{\sigma}^2_{iv, t-1, v}
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{ieg, t} & 0 \\
0 & \varepsilon_{iv, t}
\end{bmatrix}
\begin{bmatrix}
\tilde{\sigma}^2_{ieg, t-1} \\
\tilde{\sigma}^2_{iv, t-1, eg} \\
\tilde{\sigma}^2_{iv, t-1, v}
\end{bmatrix}
\tag{27}
$$

Further, our bivariate DCC and ADCC models for investigating the dynamic relations between VIX at time $t - 1$ and forecast EGARCH volatility at time $t$ are as follows:

$$
\begin{bmatrix}
\hat{\sigma}^2_{ieg, t-1} \\
\hat{\sigma}^2_{iv, t-1, eg} \\
\hat{\sigma}^2_{iv, t-1, v}
\end{bmatrix}
= 
D^2_{t} e^{\sigma^{iv}_{t-1}} \cdot R^2_{t} e^{\sigma^{iv}_{t-1}} \cdot D^2_{t} e^{\sigma^{iv}_{t-1}},
\tag{28}
$$

where

$$
D_{t} e^{\sigma^{iv}_{t-1}} = \begin{bmatrix}
\sqrt{\hat{\sigma}^2_{ieg, t-1}} & 0 \\
0 & \sqrt{\hat{\sigma}^2_{iv, t-1, eg}}
\end{bmatrix}.
\tag{29}
$$

The diagonal components are derived using GARCH(1,1) models:

$$
\hat{\sigma}^2_{ieg, t} = \rho^{eg, t}_{0} + \rho^{eg, t}_{1} \hat{\sigma}^2_{ieg, t-1} + \rho^{eg, t}_{2} \tilde{\sigma}^2_{ieg, t-1},
\tag{30}
$$

$$
\hat{\sigma}^2_{iv, t-1, eg} = \psi^{iv, t-1}_{0} + \psi^{iv, t-1}_{1} \hat{\sigma}^2_{iv, t-2, eg} + \psi^{iv, t-1}_{2} \tilde{\sigma}^2_{iv, t-2, eg},
\tag{31}
$$

and further, $R^2_{t} e^{\sigma^{iv}_{t-1}}$ is derived as

$$
R^2_{t} e^{\sigma^{iv}_{t-1}} = \text{diag} (\sqrt{q_{11, t}}, \ldots, \sqrt{q_{22, t}})^{-1} Q^2_{t} e^{\sigma^{iv}_{t-1}} \text{diag} (\sqrt{q_{11, t}}, \ldots, \sqrt{q_{22, t}})^{-1},
\tag{32}
$$

where $Q^2_{t} e^{\sigma^{iv}_{t-1}}$ is specified using Equation (23) for DCC model and Equation (24) for ADCC model.

### 8.2. MGARCH models for analyzing the dynamic linkage between VIX and forecast TGARCH volatility

Next, denoting that $\hat{\sigma}^2_{tg, t}$ is the variance of forecast volatility from the TGARCH model, $\hat{\sigma}^2_{tg, iv, t-1}$ is the covariance of the forecast volatility from the TGARCH model and the one-day lagged VIX, and $\hat{\sigma}^2_{iv, t-1}$ is the variance of the one-day lagged VIX, our bivariate (diagonal) VECH model for analyzing the dynamic linkage between VIX at time $t - 1$ and forecast TGARCH volatility at time $t$ is as follows:

$$
\begin{bmatrix}
\hat{\sigma}^2_{tg, t} \\
\hat{\sigma}^2_{tg, iv, t-1} \\
\hat{\sigma}^2_{iv, t-1}
\end{bmatrix}
= 
\begin{bmatrix}
\pi_{tg, t} & \pi_{tg, iv, t-1} \\
\pi_{tg, iv, t-1} & \pi_{iv, t-1}
\end{bmatrix}
\begin{bmatrix}
\hat{\sigma}^2_{tg, t} \\
\hat{\sigma}^2_{tg, iv, t-1} \\
\hat{\sigma}^2_{iv, t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
\lambda_{tg, t} & 0 \\
0 & \lambda_{iv, t}
\end{bmatrix}
\begin{bmatrix}
\tilde{\sigma}^2_{tg, t} \\
\tilde{\sigma}^2_{tg, iv, t-1} \\
\tilde{\sigma}^2_{iv, t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{tg, t} & 0 \\
0 & \varepsilon_{iv, t}
\end{bmatrix}
\begin{bmatrix}
\tilde{\sigma}^2_{tg, t} \\
\tilde{\sigma}^2_{tg, iv, t-1} \\
\tilde{\sigma}^2_{iv, t-1}
\end{bmatrix}
\tag{33}
$$
We also use the bivariate (diagonal) BEKK model to analyze the dynamic linkage between VIX at time $t-1$ and forecast TGARCH volatility at time $t$ as follows:

Figure 6. Dynamic relations of the time-varying correlation coefficients from the VECH-MGARCH models: daily time-series evolution for the period from 3 January 2006 to 28 February 2014.

Panel A. Time-varying correlations between the forecast S&P 500 volatility from the EGARCH model and the one-day lagged VIX

Panel B. Time-varying correlations between the forecast S&P 500 volatility from the TGARCH model and the one-day lagged VIX

Panel C. Time-varying correlations between the forecast S&P 500 volatilities from the EGARCH and TGARCH models
Figure 7. Dynamic relations of the time-varying correlation coefficients from the BEKK-MGARCH models: daily time-series evolution for the period from 3 January 2006 to 28 February 2014.

Notes: This figure presents the daily time-series evolution of the correlation coefficients among the one-day lagged VIX and the forecast volatilities from EGARCH(1,1) and TGARCH(1,1) models with GED errors. All correlation coefficients are obtained from the BEKK-MGARCH models, and these correlations are exhibited for our out-of-sample period from 3 January 2006 to 28 February 2014, and this period includes the date of the Lehman shock in the US. More concretely, the correlation coefficients between the one-day lagged VIX and the forecast S&P 500 volatility from the EGARCH(1,1) model are shown in Panel A; the correlation coefficients between the one-day lagged VIX and the forecast S&P 500 volatility from the TGARCH(1,1) model are presented in Panel B; and those between the forecast volatility from the EGARCH(1,1) model and that from the TGARCH(1,1) model are shown in Panel C.
Moreover, our bivariate DCC and ADCC models for inspecting the dynamic relations between VIX at time $t - 1$ and forecast TGARCH volatility at time $t$ are as follows:

$$
\begin{pmatrix}
\hat{\sigma}^2_i & \hat{\sigma}^2_{iv_i} \\
\hat{\sigma}^2_{iv_i} & \hat{\sigma}^2_{iv_i-1}
\end{pmatrix} =
D_t^{iv_i} R_t^{iv_i} D_t^{iv_i-1},
$$

(35)

Notes: This figure displays the daily time-series evolution of the correlation coefficients among the one-day lagged VIX and the forecast volatilities from EGARCH(1,1) and TGARCH(1,1) models with GED errors. All correlation coefficients are derived from the DCC-MGARCH models, and these correlations are shown for our out-of-sample period from 3 January 2006 to 28 February 2014, and this period includes the date of the Lehman shock in the US. Specifically, the correlation coefficients between the one-day lagged VIX and the forecast S&P 500 volatility from the EGARCH(1,1) model are displayed in Panel A; the correlation coefficients between the one-day lagged VIX and the forecast S&P 500 volatility from the TGARCH(1,1) model are shown in Panel B; and those between the forecast volatility from the EGARCH(1,1) model and that from the TGARCH(1,1) model are exhibited in Panel C.
The diagonal components are derived using GARCH(1,1) models:

\[
\sigma_{tg,t}^2 = \sigma_{0}^2 + \rho_1 \sigma_{tg,t-1}^2 + \rho_1 \sigma_{iv,t-1}^2 + \rho_2 \sigma_{tg,t-1}^2 + \rho_2 \sigma_{iv,t-1}^2, \tag{37}
\]

Figure 9. Dynamic relations of the time-varying correlation coefficients from the ADCC-MGARCH models: daily time-series evolution for the period from 3 January 2006 to 28 February 2014.

Notes: This figure shows the daily time-series evolution of the correlation coefficients among the one-day lagged VIX and the forecast volatilities from EGARCH(1,1) and TGARCH(1,1) models with GED errors. All correlation coefficients are provided by the ADCC-MGARCH models, and these correlations are displayed for our out-of-sample period from 3 January 2006 to 28 February 2014, and this period includes the date of the Lehman shock in the US. More concretely, the correlation coefficients between the one-day lagged VIX and the forecast S&P 500 volatility from the EGARCH(1,1) model are presented in Panel A; the correlation coefficients between the one-day lagged VIX and the forecast S&P 500 volatility from the TGARCH(1,1) model are exhibited in Panel B; and those between the forecast volatility from the EGARCH(1,1) model and that from the TGARCH(1,1) model are displayed in Panel C.
\[
\tilde{\sigma}_{\nu,t-1}^2 = \nu_0^{eg,\nu_{t,1}} + \nu_1^{eg,\nu_{t,1}} \tilde{\sigma}_{\nu,t-2}^2 + \nu_2^{eg,\nu_{t,1}} \tilde{\sigma}_{\nu,t-2}^2,
\]
and further, \( R_t^{eg,\nu_{t,1}} \) is derived as
\[
R_t^{eg,\nu_{t,1}} = \text{diag}(\sqrt{q_{1,t}}, \ldots, \sqrt{q_{2,t}})^{-1} Q_t^{eg,\nu_{t,1}} \text{diag}(\sqrt{q_{1,t}}, \ldots, \sqrt{q_{2,t}})^{-1},
\]
where \( Q_t^{eg,\nu_{t,1}} \) is specified using Equation (23) for DCC model and Equation (24) for ADCC model.

**8.3. MGARCH models for analyzing the dynamic linkage between forecast EGARCH and TGARCH volatilities**

Denoting that \( \tilde{\sigma}_{eg}^2 \) is the variance of forecast volatility from the EGARCH model, \( \tilde{\sigma}_{eg,iv} \) is the covariance of the forecast volatility from the EGARCH model and forecast volatility from the TGARCH model, and \( \tilde{\sigma}_{tg}^2 \) is the variance of the forecast volatility from the TGARCH model, our bivariate (diagonal) VARCH model for investigating the dynamic relationship between forecast EGARCH volatility at time \( t \) and forecast TGARCH volatility at time \( t \) is as follows:

\[
\begin{bmatrix}
\tilde{\sigma}_{eg,t}^2 \\
\tilde{\sigma}_{eg,iv,t} \\
\tilde{\sigma}_{tg,t}
\end{bmatrix} =
\begin{bmatrix}
\pi_{eg,t} \\
\pi_{eg,iv,t} \\
\pi_{tg,t}
\end{bmatrix} +
\begin{bmatrix}
\lambda_{eg,t} & 0 & 0 \\
0 & \lambda_{eg,iv,t} & 0 \\
0 & 0 & \lambda_{tg,t}
\end{bmatrix}
\begin{bmatrix}
\tilde{\sigma}_{eg,t}^2 \\
\tilde{\sigma}_{eg,iv,t} \\
\tilde{\sigma}_{tg,t}
\end{bmatrix}
+\begin{bmatrix}
\pi_{eg,t} \\
\pi_{eg,iv,t} \\
\pi_{tg,t}
\end{bmatrix} +
\begin{bmatrix}
\lambda_{eg,t} & 0 & 0 \\
0 & \lambda_{eg,iv,t} & 0 \\
0 & 0 & \lambda_{tg,t}
\end{bmatrix}
\begin{bmatrix}
\tilde{\sigma}_{eg,t}^2 \\
\tilde{\sigma}_{eg,iv,t} \\
\tilde{\sigma}_{tg,t}
\end{bmatrix}
\]

Further, our (diagonal) BEKK model for investigating the dynamic relationship between forecast EGARCH volatility at time \( t \) and forecast TGARCH volatility at time \( t \) is as follows:

\[
\begin{bmatrix}
\tilde{\sigma}_{eg,t}^2 \\
\tilde{\sigma}_{eg,iv,t} \\
\tilde{\sigma}_{tg,t}
\end{bmatrix} =
\begin{bmatrix}
\pi_{eg,t} \\
\pi_{eg,iv,t} \\
\pi_{tg,t}
\end{bmatrix} +
\begin{bmatrix}
\lambda_{eg,t} & 0 & 0 \\
0 & \lambda_{eg,iv,t} & 0 \\
0 & 0 & \lambda_{tg,t}
\end{bmatrix}
\begin{bmatrix}
\tilde{\sigma}_{eg,t}^2 \\
\tilde{\sigma}_{eg,iv,t} \\
\tilde{\sigma}_{tg,t}
\end{bmatrix}
+\begin{bmatrix}
\pi_{eg,t} \\
\pi_{eg,iv,t} \\
\pi_{tg,t}
\end{bmatrix} +
\begin{bmatrix}
\lambda_{eg,t} & 0 & 0 \\
0 & \lambda_{eg,iv,t} & 0 \\
0 & 0 & \lambda_{tg,t}
\end{bmatrix}
\begin{bmatrix}
\tilde{\sigma}_{eg,t}^2 \\
\tilde{\sigma}_{eg,iv,t} \\
\tilde{\sigma}_{tg,t}
\end{bmatrix}
\]

Finally, our bivariate DCC and ADCC models for analyzing the dynamic linkage between forecast EGARCH volatility at time \( t \) and forecast TGARCH volatility at time \( t \) are as follows:

\[
\begin{bmatrix}
\tilde{\sigma}_{eg,t}^2 \\
\tilde{\sigma}_{eg,iv,t} \\
\tilde{\sigma}_{tg,t}
\end{bmatrix} = D_t^{eg,-tg} R_t^{eg,-tg} D_t^{eg,-tg},
\]
where

\[
D_t^{eg,-tg} = \begin{bmatrix}
\sqrt{\tilde{\sigma}_{eg,t}^2} & 0 \\
0 & \sqrt{\tilde{\sigma}_{tg,t}^2}
\end{bmatrix}.
\]

The diagonal components are derived using GARCH(1,1) models:

\[
\tilde{\sigma}_{eg,t}^2 = \nu_0^{eg,\nu_{t,1}} + \nu_1^{eg,\nu_{t,1}} \tilde{\sigma}_{\nu,t-2}^2 + \nu_2^{eg,\nu_{t,1}} \tilde{\sigma}_{\nu,t-2}^2,
\]

\[
\tilde{\sigma}_{tg,t}^2 = \nu_0^{tg,\nu_{t,1}} + \nu_1^{tg,\nu_{t,1}} \tilde{\sigma}_{\nu,t-2}^2 + \nu_2^{tg,\nu_{t,1}} \tilde{\sigma}_{\nu,t-2}^2,
\]
and further, \( R_{tgt}^{eg} \) is derived as

\[
R_{tgt}^{eg} = \text{diag}(\sqrt{q_{11,t}}, \ldots, \sqrt{q_{22,t}})^{-1} Q_{tgt}^{eg} \text{diag}(\sqrt{q_{11,t}}, \ldots, \sqrt{q_{22,t}})^{-1},
\]

where \( Q_{tgt}^{eg} \) is specified using Equation (23) for DCC model and Equation (24) for ADCC model.

**8.4. Results of the investigations**

Figures 6 and 7 display the time-varying correlations between the previous day’s VIX and the volatility forecasts from the EGARCH and TGARCH models, which in this instance we derive from bivariate VECH and BEKK models, respectively. Panels A and B of Figure 6 and Panels A and B of Figure 7 show that the time-varying correlations between the one-day lagged VIX and the forecast volatility from the EGARCH model and those between the one-day lagged VIX and the forecast volatility from the TGARCH model are generally highly correlated, although they show negative correlations in some phases.

In contrast, as shown in Panel C of Figure 6 and Panel C of Figure 7, the time-varying correlations between the forecast volatility from the EGARCH model and that from the TGARCH model are much more strongly correlated and exhibit almost no negative correlation. Interestingly, Panels A–C of Figure 6 and Panels A–C of Figure 7 demonstrate that during the Lehman shock in the US, all the three volatility series are almost perfectly correlated.

Next, Figures 8 and 9 display the time-varying correlations between the previous day’s VIX and the forecast volatilities from the EGARCH and TGARCH models, as derived from the bivariate DCC and ADCC models, respectively. Panels A and B of Figure 8 and Panels A and B of Figure 9 show that the time-varying correlations between the one-day lagged VIX and the forecast volatility from the EGARCH model and those between the one-day lagged VIX and the forecast volatility from the TGARCH model are generally highly correlated, although they again show some negative correlations in some periods, as illustrated in Figures 6 and 7. In comparison, as shown in Panel C of Figure 8 and Panel C of Figure 9, the time-varying correlations between the forecast volatility from the EGARCH model and that from the TGARCH model are much more highly correlated. It is again very interesting to note that Panels A–C of Figure 8 and Panels A–C of Figure 9 also demonstrate that during the US Lehman shock, all the three volatility series are almost perfectly correlated.

**9. Discussion—interpretation and implications**

This section discusses the interpretation and implications from our empirical study. As our empirical results showed, the previous day’s VIX does indeed have predictive power in relation to the US stock market downside risk. However, the forecast volatilities from modern econometric models, such as the EGARCH and TGARCH models, outperform the VIX in predicting large price declines in the US stock market. We interpret this inferiority of VIX for downside risk forecasting as being the result of the unstable market psychology, reflected in the so-called fear gauge derived from options markets. Econometric models, in contrast, mechanically and relatively stably produce their forecasts without being affected by the market psychology mirrored in options markets. That is, we consider that this characteristic difference between VIX and volatility forecasts from modern econometric models leads to the different forecasting power for downside risk in the US stock market. Again, we emphasize that market psychology is generally unstable, although it can usually assist in foreseeing the direction of financial markets.

Next, we discuss the implications of our study. First, as our empirical results demonstrate, the development of effective econometric models and techniques within the academic sphere is important. Therefore, our evidence should encourage academics to develop newer sophisticated statistical and econometric models that are useful and effective for analyzing and forecasting actual data in the real world. Moreover, for practitioners, such as regulators and policy-makers, our evidence implies that to consider using volatility forecasts from econometric models is important.
example, in constructing systemic risk measures, such as the development of financial stress indices by central bankers, better results will be achieved by taking into account not only VIX but also the volatility forecasts from econometric models.

10. Summary and conclusion
This paper empirically compared the downside risk predictive power contained in the VIX of S&P 500 with that of the forecast S&P 500 volatilities from various econometric models. The careful and rigorous analyses undertaken in this study provide rich, comprehensive, and robust evidence as follows.

(1) First, our analyses using both univariate logit and quantile regression models found that the previous day’s VIX and all forecast volatilities from the GARCH, EGARCH, PGARCH, and TGARCH models have statistically significant predictive power for downside risk in the US stock market.

(2) Second, by applying univariate logit and quantile regression models, we also showed that as regards downside risk in the US stock market, the predictive power of the previous day’s VIX was clearly lower than the volatility forecasts from EGARCH and TGARCH models.

(3) Third, direct comparisons of the downside risk predictive power of the previous day’s VIX with that of the forecast volatilities from the econometric models using multiple logit and quantile regression models suggested that the predictive power of the previous day’s VIX was dominated by the volatility forecasts from the EGARCH, PGARCH, and TGARCH models. In particular, the forecast volatilities from the EGARCH and TGARCH models more strongly dominated the previous day’s VIX in predicting downside risk in the US stock market.

(4) Fourth, further direct comparisons conducted by including several control variables also demonstrated that the predictive power of the previous day’s VIX was again dominated by the forecast volatilities from the EGARCH, PGARCH, and TGARCH models. Moreover, in all testing models with control variables, the forecast volatilities from the EGARCH and TGARCH models greatly dominated the predictive power of the previous day’s VIX.

(5) Fifth, our additional tests further revealed that the downside risk predictive power of (i) the forecast VIX, (ii) the forecast volatility of VIX, and (iii) the forecast volatility of the first log differences of VIX exhibited weaker predictive power than that of the forecast S&P 500 volatilities from the econometric models. These results were robustly obtained from the tests using both logit and quantile regression models.

(6) Sixth, our VECH-, BEKK-, DCC-, and ADCC-MGARCH analyses revealed that the time-varying correlations between the previous day’s VIX and the forecast volatilities from the EGARCH or TGARCH models were generally highly correlated, although they exhibited negative correlations in some periods. On the other hand, the time-varying correlations between the forecast volatilities from the EGARCH and TGARCH models were much more strongly connected.

(7) Finally, our VECH-, BEKK-, DCC-, and ADCC-MGARCH analyses further showed that, very interestingly, around the Lehman Brothers bankruptcy in the US, all three volatility series were almost perfectly correlated with each other.

As the above evidence demonstrates, VIX is indeed effective in predicting downside risk in the US stock market. However, the information included in VIX cannot outperform the forecast volatilities from modern econometric models in predicting large stock price declines in the US. We consider that our findings are highly robust as we have carefully and thoroughly scrutinized the data from a variety of angles. Therefore, the findings in this paper will contribute not only to the existing body of literature but also to our future relevant research developments. We note that the highly robust evidence we derived for the US stock market could apply to other markets. Hence, further research using other international data based on the results of this paper will be one of our future research tasks.
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Author details

Chikashi Tsuji
E-mail: mail_sec_low@minos.ocn.ne.jp
1 Faculty of Economics, Chuo University, 742-1 Higashinakano, Hachioji-shi, Tokyo 192-0393, Japan.

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Notes

1. Huang, Liu, Ghon Rhee, and Wu (2012) proposed a measure for extreme downside risk; however, they did not analyze stock market crashes. Using GARCH models, Schwert (1990) investigated the relationship between stock volatility increases and the 1987 US stock market crash. However, comparison of the predictive power of implied volatility and the forecast volatilities from GARCH models was not the focus of this particular study.
2. The results clarified that when this information was included in their models, the models well predicted stock market crisis months between 1997 and 2009.
3. Tolikas (2014) also applied EVT methods to the extreme returns of some international stock indices. The empirical results indicated that the much-celebrated generalized extreme value (GEV) distribution was not the best for capturing the fat tails of stock returns, and that the generalized logistic distribution performed much better.
4. This study found that the time-changed Carr–Geman–Madan–Yor (CGMY) process was a slightly more parsimonious alternative to the approach in Bates (2006). Bates (2006) used finite-activity stochastic-intensity jumps drawn from a mixture of normals, and also suggested that the fit of the two approaches was very similar.
5. For fair comparison of predictability, we standardize the lag orders as $p = 1$ and $q = 1$ in all four GARCH models, and all the four models always have GED errors in this study.
6. We searched for the best-fit ARMA($p$, $q$) model within the maximum lags of $p = 5$ and $q = 5$, as one week includes five business days. The Akaike information criterion (AIC) determined the best forecast model as ARMA(4,4), whereas the Schwartz (information) criterion (SC) determined the best forecast model as ARMA(2,1). Thus, we use both models in this study for forecasting VIX.
7. The previous day’s value for our measure of implied volatility, VIX, contains information on the evolution of future stock prices because this variable is derived from options markets that reflect the future expectations of market participants.
8. In all our tests by logit models in this study, we compute and regard the k% VaR as the thresholds of the logit models using our out-of-sample period, 3 January 2006 to 28 February 2014. That is, the k% points of the left tail of the actual S&P 500 return distribution realized in our out-of-sample period are the cut-offs of all our tests by logit models.
9. This is because the investigation in this study demonstrates that the EGARCH and TGARCH models are superior models in producing volatility forecasts than the simple GARCH and PGARCH models.

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