Sub-wavelength Coherent Imaging of a Pure-Phase Object with Thermal Light

Minghui Zhang, Qing Wei, Xia Shen, Yongfeng Liu, Honglin Liu, Yanfeng Bai, and Shensheng Han
Key Laboratory for Quantum Optics and the Center for Cold Atom Physics of CAS,
Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Science,
P. O. Box 800-211, Shanghai, 201800, P. R. China

(Dated: April 1, 2022)

We report, for the first time, the observation of sub-wavelength coherent image of a pure phase object with thermal light, which represents an accurate Fourier transform. We demonstrate that ghost-imaging scheme (GI) retrieves amplitude transmittance knowledge of objects rather than the transmitted intensities as the HBT-type imaging scheme does.

PACS numbers: 42.30.Va, 42.50.Ar, 61.10.Dp, 42.30.Rx
Keywords: Sub-wavelength; Coherent imaging; Thermal light.

I. INTRODUCTION

In many imaging circumstances, phase information about objects plays a role as well as or even more important than intensity does, for example, when the objects are pure-phased, that is, highly transparent and absorb little light, imaging can not be simply realized by the transmitted or reflected intensity information of thermal lights. Although phase distribution about an object can be retrieved from its Fourier-transform diffraction pattern was firstly proposed by Sayre [1] and dedicated efforts described in the works like [2] demonstrated and developed the techniques, the efforts seems to be in vain if diffraction imaging applications were in hard x-ray, γ-ray, or other wavelengths where no effective lens or/and no coherent source is available. Recent works [3, 4] reported a new version of the landmark Hanbury Brown and Twiss (HBT) experiment [5] and gave lens-less Fourier-transform pattern of a Young’s double slit with thermal sources, but the phase information is not yet mentioned because the object they used was amplitude-only. In fact, as we will discuss latterly, the classical HBT-type imaging, features that the joint detection plane in the optical path passes through the object, will be invalid for retrieving phase knowledge about an object. Since middle years of last decades, ghost imaging (GI) has been enthusiastically studied[6, 7, 8, 9, 10, 11, 12, 13, 16], here, the reason for the term “ghost” used is that the image of an object, diffractive or geometrical, would appear as a function of the position in the path that actually never pass the object, and this unique feature is regarded as a key difference from classical HBT-type imaging. Although whether the entangled beams was a prerequisite once have been hotly debated[6, 7, 8, 9, 10, 13], it is generally accepted now that classical thermally emitted light can be used for GI and quantum entangled beams is not a prerequisite. In this letter, we report, for first time, the lensless retrieval of sub-wave length coherent imaging of a pure-phase object, which represents an accurate Fourier transform, within fresnel diffraction range by using GI scheme. Also, we experimentally demonstrate that ghost-imaging scheme (GI) retrieves complex amplitude transmittance of objects rather than the transmitted intensities as the HBT-type imaging scheme does.

II. THE THEORETICAL BASIS

A. Joint detection

Fig.1 shows the setup for experiment. Intensity information is recorded at plane $X_1$ and $X_2$ in two different optical paths formed by a 5/5 none-polarized beam splitter (BS). $h_{1,2}(x, x_{1,2})$ refers to the impulse response functions from thermal light plane $X$ to intensity detecting plane $X_{1,2}$, where, $x$ and $x_{1,2}$ stand for positions on plane $X$ and $X_{1,2}$, respectively.

The physics behind the joint detection for GI scheme in plane $X_1$ and plane $X_2$, as Fig.1 shows, can be explained in simplicity as follows: The two-photon ampli-
tude described by state vector $|A\rangle$ can be decomposed into weighted sum of the following three normalized basis states: $|\alpha\rangle = |vac\rangle_1 |j\rangle_2$, two photons both reflected by BS; $|\beta\rangle = |m\rangle_1 |n\rangle_2$, one photon reflected by the BS and the other transmitted the BS; and $|\gamma\rangle = |i\rangle_1 |vac\rangle_2$, two photons both transmitted the BS, i.e. $|A\rangle = 1/ \sqrt{2} |\alpha\rangle + 1/ \sqrt{2} e^{i\theta_1} |\beta\rangle + 1/ \sqrt{2} e^{i\theta_2} |\gamma\rangle$. In the equation, $\theta_1(2)$ is the phase of complex weight for state $|\beta\rangle (|\gamma\rangle)$ relative to state $|\alpha\rangle$. The expression of $|\alpha\rangle$ and $|\gamma\rangle$ can be expressed as:

$$|\alpha\rangle = |vac\rangle_1 |j\rangle_2, \quad (1)$$

and

$$|\gamma\rangle = |i\rangle_1 |vac\rangle_2, \quad (2)$$

but as for the characters of identical bosons, the two-photon state of $|\beta\rangle$ must be expanded in this way:

$$|\beta\rangle = 1/ \sqrt{2} (|m\rangle_1 |n\rangle_2 + |n\rangle_1 |m\rangle_2). \quad (3)$$

Among the Eq.(3), the subscripts 1 and 2 of state vector $|\rangle$ refer to joint detecting points in plane $X_1$ and $X_2$ respectively, and $m$, $n$ stand for two undistinguishable photons.

If we define $E^{(\pm)}(t_1, t_2, x_1, x_2)$ as the positive-frequency and negative-frequency components of the field at time-spatial point $t_1, x_1$ and $t_2, x_2$, suppose time $t_1 < t_2$, the joint detection probability per unit (time)$^2$ that one photon is recorded at $x_1$ at time $t_1$ and another at $x_2$ at time $t_2$, say, the square module of two-photon amplitude $\psi(t_1, t_2, x_1, x_2)$ corresponding to $|A\rangle$ has been described by second-order Glauber correlation function [17]:

$$G^{2}(t_1, t_2, x_1, x_2) = Tr\{\rho E^{(-)}(t_1, x_1) E^{(-)}(t_2, x_2) E^{(+)}(t_2, x_2) E^{(+)}(t_1, x_1)\},$$

where the density operator is defined as the average outer product of state $|A\rangle$: $\rho = \langle|A\rangle (A)\rangle_{av}$, i.e.,

$$\rho = 1/4 |\alpha\rangle\langle\alpha| + 1/2 |\beta\rangle\langle\beta| + 1/4 |\gamma\rangle\langle\gamma| . \quad (4)$$

Subsisting Eq.(1)-Eq.(4) into second-order Glauber correlation function, and note that $E^{(+)}(t_1, t_2, x_1, x_2) |vac\rangle = \langle vac | E^{(-)}(t_1, t_2, x_1, x_2) | vac\rangle = 0$, we find that only state $|\beta\rangle$, contributes to the joint detection as:

$$G^{(2)}(t_1, t_2, x_1, x_2) \propto \langle \beta | E^{(-)}(t_1, x_1) E^{(-)}(t_2, x_2) \times E^{(+)}(t_2, x_2) E^{(+)}(t_1, x_1) | \beta \rangle. \quad (5)$$

On the other hand, if joint detection is only performed in the same plane $X_1$ in case of HBT-type imaging, which we’ll discuss latterly, we can consider the two-photon amplitude described by state vector $|A\rangle$ as a pure state only concerning with two photons both transmitted the beam splitter, and thus the density operator of state $|A\rangle$ comes to: $\rho = |A\rangle\langle A|$, i.e. joint detection probability reduces to:

$$G^{(2)}(t_1, t_2, x_1, x_2) = \langle A | E^{(-)}(t_1, x_1) E^{(-)}(t_2, x_2) \times E^{(+)}(t_2, x_2) E^{(+)}(t_1, x_1) | A \rangle, \quad (6)$$

where, $x_1$ and $x_2$ stand for two different points in plane $X_1$. 

B. The relation between joint detection and intensity correlations

Eq.(6) and Eq.(10) states the fact that whether the joint detection performed in both planes $X_1$ and $X_2$, or in plane $X_1$ only, the joint detection probability would be presented as average production of normally ordered positive-frequency and negative-frequency operator. Quantum detection theory shows that right side of Eq.(6) and Eq.(10) is proportional to the normally ordered intensity correlation function of the optical field[18,19], i.e.,

$$G^{(2)}(t_1, t_2, x_1, x_2) \propto \langle I_1(t_1, x_1) I_2(t_2, x_2) \rangle, \quad (7)$$

and

$$G^{(2)}(t_1, t_2, x_1, x_2) \propto \langle I_1(t_1, x_1) I_2(t_2, x_2) \rangle. \quad (8)$$

The similarity of Eq.(7)and Eq.(9) means that square module of two-photon amplitude, can be measured by correlation of intensities no matter whether joint detection is performed in plane $X_1$ and $X_2$ on two different optical path formed by beam splitter or only in plane $X_1$ on the optical path transmitted the beam splitter.

C. Measuring the modulation to two-photon amplitude

Previous works[12,13] have stated that if one assumes the time window $\Delta t = t_2 - t_1 = 0$ in the experiment, and defines $\Delta I^{(2)}(r_1, r_2) = \langle I_1(r_1) I_2(r_2) \rangle - \langle I_1(r_1) \rangle \langle I_2(r_2) \rangle$, the impulse response functions for the two optical paths in Fig.1 $h_{1,2}(x, x_1, x_2)$ would be embedded into:

$$\Delta I^{(2)}(x_1, x_2) \propto \int \frac{h_1^*(x', x_1) h_2(x, x_2)}{X} < E^*(x') E(x) > d x' d x^2, \quad (9)$$

with the assumption that the thermal state of light is characterized by a Gaussian field statistics[14], among them $x$ and $x'$ denote position(s) at thermal source plane $X$ in Fig.1. According to Collins integral formula[14], there exists:

$$h_1(x, x_1) = \int_{X_0} dx_0 e^{-i k d_1} e^{-\frac{i \lambda}{\hbar \lambda_1} (x-x_0)^2} \times t(x_0) e^{-i k d_2} e^{-\frac{i \lambda}{\hbar \lambda_2} (x_1-x_0)^2}, \quad (10)$$

and

$$h_2(x, x_2) = \frac{e^{-i k d}}{\lambda d} e^{-\frac{i \lambda}{\hbar \lambda_2} (x_2-x)^2}, \quad (11)$$
and Eq. (11) into Eq. (9), we get:

\[ \Delta I^2(x_1, x_2) \propto |F\{t\left[\frac{2\pi(x_2 - x_1)}{\lambda d_2}\right]\}|^2, \quad (12) \]

Among them \( F\{ \} \) denotes Fourier transform.

In the other case, if joint detection is only performed at plane \( X_1 \) on the optical path which passes through the object, then Eq. (10) will stand for both of two impulse response functions in Eq. (9). In this situation, the equation becomes:

\[ \Delta I^2(x_1, x'_1) \propto \left| F\{t\left[\frac{2\pi(x_1 - x'_1)}{\lambda d_2}\right]\}\right|^2, \quad (13) \]

where, \( x_1 \) and \( x'_1 \) stand for position(s) at plane \( X_1 \).

Based on the theory above, the following experiment was carried out in the regime of large number photons to illustrate the behaviors of two-photon interference. Now we can easily envision the physics behind the following experiment: The modulation to the two-photon amplitude, \( \psi(x_1, x_2) \) and \( \psi(x_1, x'_1) \), by the objects was measured by correlation function of intensity fluctuations.

### III. THE EXPERIMENTAL SETUP

The experiment is set out with the use of pure phase object, which was prepared by etching two grooves with width of 150 \( \mu m \) and separating them by a 150 \( \mu m \) un-etched area on a piece of 0.75mm \( \times \) 10mm square quartz glass (JGS1), as Fig. 2 shows. The other two un-etched areas with width of 150 \( \mu m \) are left symmetrically. The depth of two grooves is arranged to be 0.47 \( \mu m \).

where, \( t(x_0) \) is the complex amplitude transmittance of objects placed on plane \( X_0 \). As for GI schemed imaging, joint detection was performed on both optical paths. Under the condition of \( d = d_1 + d_2 \), substituting Eq. (10) and Eq. (11) into Eq. (9), we get:

\[ \Delta I^2(x_1, x_2) \propto \left| F\{t\left[\frac{2\pi(x_2 - x_1)}{\lambda d_2}\right]\}\right|^2, \quad (12) \]

Choosing symmetric positions \( x_2 \) and \( x_1 = -x_2 \) to calculate \( \Delta I^2(x_1, x_2) \) with preserved data \( I_1(x_1) \) and \( I_2(x_2) \), we find what we obtain (Fig. 3a) shares the same pattern in Fig. (3b), but have double coordinate scales just as equation:

\[ \Delta I^2(x_2, -x_2) = \left| F\{t\left[\frac{2\pi(x_2 - (-x_2))}{\lambda d_2}\right]\}\right|^2 \]

\[ = \left| F\{t\left[\frac{2\pi x_2}{\lambda d_2}\right]\}\right|^2, \quad (14) \]

which derived from in Eq. (12), anticipated. The result (Fig. 3b) shows a sub-wavelength interference pattern equals to the Fraunhoffer diffraction pattern(Fig. 3a) of the same pure-phase object accomplished by a 2f system(with \( \lambda = 0.532 \mu m \) and \( f = 75mm \), but with half of the wavelength of coherent illumination. In Fig. 3a), red curve represents profile of two dimensional experimental result, and blue curve presents the standard Fourier transform of the pure-phase object’s complex amplitude transmittance under the sub-wavelength condition according to Eq. (13). Its analytical form is given by

\[ \Delta I^2(x_2, -x_2) = \text{sinc}^2\left(\frac{a}{\lambda_{sub} d_2} x_2\right)[1 - 2 \times \cos\left(\frac{2\pi a}{\lambda_{sub} d_2} x_2\right) + 4 \cos\left(\frac{4\pi a}{\lambda_{sub} d_2} x_2\right)]^2, \quad (15) \]
in which $a = 150 \mu m$, $d_2 = 75 mm$, and $\lambda_{sub} = 0.532 \mu m$. From Fig.3, we can see the experimental result fit the theoretical Fourier transform Eq. (15) quite well.

According to Ref.[3], the optical path that passes the object in our experimental setup is compatible with the classical HBT-type imaging system if joint detection is accomplished only on the arm passes the object. So, the 10,000 frames of previously preserved intensity data recorded on plane $X_1$, i.e., ensembles of $I(x_1)$ can also be utilized for HBT type of imaging. In this situation, the two points $x_1$ and $x'_1$ for joint detecting are chosen only at plane $X_1$ symmetrically with respect to the center of it, i.e., $x_1 = -x'_1$. The function $\Delta I^2(x_1, -x_1)$ contains no phase information about the object in this time, as two-dimensional pattern and its profile (red curve) in Fig.4 shows. Eq. (13) gives a straightforward explanation to the experimental result: The diffraction pattern we obtained was the Fourier transform of the square module of the transmitted function of the object. The blue curve in Fig.4 shows the analytic form of the quantity appeals to quantum lithography which was widely discussed.

\[
\Delta I^2(x_1, -x_1) = \sin^2\left(\frac{a}{\lambda_{sub}d_2}x_1\right),
\]

which also fits the experimental result very well.

Thus we find the essential differences of our GI schemed sub-wavelength coherent imaging from recently reports about sub-wavelength interference with thermal light [2, 4] is what GI type of imaging system in our experiment setup retrieved was the complex amplitude transmittance knowledge of the object rather than the transmitted intensity as the HBT schemed imaging does.

V. CONCLUSION AND DISCUSSION

The lensless scheme proposes the potential application in hard x-ray, $\gamma$-ray, or other wavelengths where no effective lens or/and no coherent source is available, and the similar idea have also been reported brilliantly, but in spatial domain[21]. On the other hand, obtains of sub-wavelength interference pattern suggest that diffraction limit can be broken through by using two-photon absorption (TPA) media if we find ways to fold the symmetric planes of $x_1$ and $x_2$ into the same one[22]. This feature appeals to quantum lithography which was widely discussed.

Apart from the unique sub-wavelength feature of the experiment result, attention shall also be paid again to the similarity and the difference between GI schemed and the classical HBT-type of imaging: They both retrieve diffraction patterns by correlation function whereas GI recovers knowledge of complex amplitude transmittance about objects rather than the transmitted intensities as HBT-type imaging does. Similar conclusion have also been reported[24], but with use of lens in both optical paths. In fact our scheme suggests a new way to perform lensless Fourier transform within frresnel diffraction region.

Unlike the other ghost imaging and ghost diffraction experiments, a pulsed thermal-light source is used instead of a continuous one, the intensity correlation can be measured even the exposure time of CCD camera is much longer than pulse width of the source. This unique impulse feature of our thermal light source enlightens us on the issue for recording intensity fluctuation by slow detector. This way suggests achieving to record intensity fluctuating much faster than the respond speed of the detecting system. As for hard x-ray imaging, there is a potential applicability to record the femto-second fluctuations of intensity by using of detecting system with response speed in nanoseconds.
Acknowledgments

The authors would like to thank Professor Kaige Wang for helpful discussion and Professor Yang-chao Tian for preparing the objects. This research is partly supported by the National Natural Science Foundation of China, Project No. 60477007, the Shanghai Optical-Tech Special Project, Project No. 034119815, and Shanghai Dengshan Project, Project No. 60JC14069.

[1] D. Sayre, Imaging Processes and Coherence in Physics, Springer Lecture Notes in Physics Vol. 112 (Springer-Verlag, Berlin, 1980), p. 229.
[2] J. R. Fienup, Appl. Opt. 21, 2758 (1982); J. Cheng, S. Han, J. Opt. Soc. Am. A 18, 1460-1464 (2001); V. Elser, J. Opt. Soc. Am. A. 20, 40-55 (2003).
[3] J. Xiong et al., Phys. Rev. Lett. 94, 173601 (2005).
[4] G. Scarcelli, A. Valencia and Y. Shih, Europhys. Lett. 68, 618 (2004).
[5] R. Hanbury Brown and R. Q. Twiss, Nature (London) 177, 27 (1956).
[6] A. V. Belinsky and D. N. Klyshko, Sov. Phys. JETF 78, 259 (1994).
[7] A. F. Abouraddy, B. E. A. Saleh, A. V. Sergienko, and M. C. Teich, Phys. Rev. Lett. 87, 123602 (2001).
[8] R. S. Bennink, S. J. Bentley, and R. W. Boyd, Phys. Rev. Lett. 89, 113601 (2002).
[9] A. Gatti, E. Brambilla, and L. A. Lugiato, Phys. Rev. Lett. 90, 133603 (2003).
[10] R. S. Bennink, S. J. Bentley, R. W. Boyd, and J. C. Howell, Phys. Rev. Lett. 92, 033601 (2004).
[11] M. D’Angelo, Y.-H. Kim, S. P. Kulik, and Y. Shih, Phys. Rev. Lett. 92, 233601 (2004).
[12] A. Gatti, E. Brambilla, M. Bache, and L. A. Lugiato, Phys. Rev. Lett. 93, 093602 (2004).
[13] J. Cheng and S. Han, Phys. Rev. Lett. 92, 093903 (2004).
[14] J. W. Goodman, Statistical Optics (Wiley, New York, 1985).
[15] S. A. Collins, J. Opt. Soc. Am. 60, 1168 (1970).
[16] G. Scarcelli, V. Berardi, and Y. Shih, Phys. Rev. Lett. 96, 063602 (2006).
[17] R. J. Glauber, Phys. Rev. 130, 2529 (1963).
[18] L. Mandel and E. Wolf, Optical Coherence and Quantum Optics, (Cambridge University Press, New York, 1995), p. 578.
[19] G. Scarcelli, V. Berardi, and Y. Shih, Appl. Phys. Lett. 88, 061106 (2006); Phys. Rev. Lett. 96, 063602 (2006).
[20] A. Gatti, E. Brambilla, M. Bache, and L. A. Lugiato, Phys. Rev. Lett. 93, 093602 (2004).
[21] J. Cheng and S. Han, Phys. Rev. Lett. 92, 093903 (2004).
[22] J. W. Goodman, in Laser speckle and related phenomena, edited by J. C. Dainty (Springer-Verlag, New York, 1984).
[23] G. Scarcelli, V. Berardi, and Y. Shih, Appl. Phys. Lett. 88, 061106 (2006); Phys. Rev. Lett. 96, 063602 (2006).
[24] A. Gatti, E. Brambilla, M. Bache, and L. A. Lugiato, Phys. Rev. Lett. 93, 093602 (2004).
[25] J. Cheng and S. Han, Phys. Rev. Lett. 92, 093903 (2004).
[26] J. W. Goodman, Statistical Optics (Wiley, New York, 1985).
[27] S. A. Collins, J. Opt. Soc. Am. 60, 1168 (1970).
[28] G. Scarcelli, V. Berardi, and Y. Shih, Phys. Rev. Lett. 96, 063602 (2006).
[29] R. J. Glauber, Phys. Rev. 130, 2529 (1963).
[30] L. Mandel and E. Wolf, Optical Coherence and Quantum Optics, (Cambridge University Press, New York, 1995), p. 578.
[31] D. F. Walls and G. J. Milburn, Quantum Optics (Springer-Verlag, Berlin, 1994), p. 39.